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**Forecasting Bidding Strategies for Competing Electricity Companies
in Alberta using Time Series Models**

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1 Introduction

The pricing of power in today's deregulated electricity markets has become more intricate and dynamic due to various factors, such as daily and seasonal changes in demand and supply, temperature fluctuations, and energy source availability. For instance, in the day-ahead market, prices can vary widely, ranging from 6% to 28%, compared to the relatively stable 2% to 3% for crude oil and 3% to 5% for natural gas (Simonsen, 2005; Zareipour et al., 2007). The increasing presence of intermittent renewable energy sources in recent years has added to concerns about price volatility in electricity markets (Baldick, 2012; Brancucci Martínez-Anido et al., 2016).

The complexity of price dynamics has significant implications for market participants and system operators. Developing more accurate forecasting methods has become a crucial task for investors, enabling them to plan bidding strategies to maximize utility over short, medium, and long-term perspectives. Additionally, improved forecasting can assist consumers in minimizing costs across various applications in dynamic pricing environments and responding to changes in demand (Cabral et al., 2020). Lastly, accurate forecasting supports regulatory authorities in ensuring the long-term adequacy and security of the power supply, contributing to the stability of power markets. Due to these reasons, there is a growing interest in the literature towards developing enhanced price modeling and forecasting techniques (Nowotarski & Weron, 2018; Taylor et al., 2006; Weron, 2007).

In Alberta's fluctuating electricity market, precise bidding strategy forecasting (BSF) is crucial for informed decision-making by electricity companies. This research aims to improve BSF accuracy using historical bid data, assessing specific exogenous variables' impact, and considering structural changes over time. The objective of this research is driven by regulatory considerations. The regulator wants to study the behavior of firms and improve the understanding of firms' bidding strategies in response to the fluctuating market conditions. By employing diverse time series forecasting models, our goal is to provide comprehensive insights into the dynamics of electricity bid data, offering effective modeling approaches for the bid market in Alberta, Canada."

Time series models, such as AR, ARIMA, and SARIMA, offer a practical approach to enhance accuracy by accounting for the fluctuating nature of the electricity market. Through these models, the aim is to provide valuable insights that empower decision-makers to navigate the complexities of the electricity market with greater confidence.

2 Research objectives

This research is organized around three main objectives:

Objective 1: Enhancing Bidding Strategy Forecasting Accuracy using time series models; The main objective of this project is to improve the precision of bidding strategy forecasting (BSF) in Alberta's electricity market. To attain this goal, we can consider two principal approaches. The

first involves a thorough consideration of diverse influential factors like supply, demand, weather conditions, and geopolitical events that impact electricity prices. The second centers on utilizing historical bid data, which inherently encompasses all relevant explanatory factors when accurately modeled. This research adopts the latter approach, aiming to improve forecasting precision by examining intricate patterns within the bid data.

Objective 2: Evaluating the Impact of Specific Exogenous Variables; The second objective focuses on assessing the impact of specific exogenous variables on forecasting models, with a particular emphasis on the capacity of electricity production in the next 24 hours. While acknowledging the presence of various exogenous factors influencing electricity prices, this research concentrates on the unique insight provided by the knowledge of production capacity. By exploring how this specific information contributes to forecasting accuracy, we aim to quantify the extent to which the inclusion of such targeted exogenous variables enhances the predictive capabilities of our models. This analysis will provide valuable insights into the nuanced effects of production capacity on bidding strategy forecasting within the electricity market.

Objective 3: Accounting for Structural Changes in the Electricity Market; The third objective is to assess the impact of structural changes over time in the electricity market, including regulatory policy shifts, changes in market participants, and ownership structure transformations. Analyzing electricity data across different years allows us to identify patterns influenced by changes in market dynamics, contributing to a nuanced understanding of electricity bidding dynamics in Alberta, Canada.

Through the application of diverse forecasting models and a comprehensive evaluation of their performance, this research intends to provide valuable insights into the complexities of electricity bidding functions dynamics. The subsequent sections will focus on a detailed analysis of each model's results and their implications for electricity bid market forecasting in Alberta, Canada.

3 Alberta electricity market

Alberta's electricity market transitioned to a market-based system in 2001, introducing competition in retail and wholesale segments while maintaining regulated monopolies in transmission and distribution (Olmstead & Ayres, 2014, Brown & Olmstead, 2017). This shift aimed to foster innovation, attract investment in new generation capacity, improve electricity supply reliability, and provide consumers with diversified options and competitive pricing. The distinctive feature of Alberta's electricity market is the adoption of an economic merit order dispatch system, resulting in a singular equilibrium price. This approach prioritizes economic efficiency by dispatching energy generation based on the most cost-effective sources.

Alberta's electricity generators encompass a diverse mix of coal, natural gas, hydroelectric, wind, and biomass, ensuring a reliable and flexible electricity supply. The Alberta Electric System

Operator (AESO) is tasked with market design and operation. Wholesale suppliers primarily derive revenue from payments in the short-run electricity market.

The electricity market operates as a uniform-price multi-unit procurement auction for each hour, with suppliers submitting offer bids a day ahead of physical production. Generators must offer available capacity, selecting prices ranging from \$0 to \$999.99 per MWh. In 2011, thermal plants burning coal and natural gas dominated power production, with wind, hydro, and other fossil fuels contributing to the mix. The market-clearing price is determined at every minute, reflecting the highest accepted bid price to supply realized electricity demand. Participants receive payment based on the pool price, the time-weighted average price for each hour (Benatia & Billette de Villemeur, 2023).

Table 1 details Alberta's market structure and firm characteristics. In 2010-2011, the five largest firms controlled approximately 70% of market offers, while a fringe of over 20 firms managed the rest. It is noteworthy that wind farms, excluded from market shares, receive fixed-price payments. Offer control differs from capacity ownership due to long-term bilateral contracts between suppliers.

Table 1: Alberta market and firm characteristics (Benatia & de Villemeur, 2023)

Company	Market shares (%)	Capacity (%)
TransCanada (TC)	20.9	4.2
ENMAX (EN)	18.3	6.5
Capital Power (CP)	11.8	11.8
TransAlta (TA)	10.4	36.7
ATCO (AT)	8.2	16.2
Fringe	30.4	24.5

The market's progression unfolds across three discernible periods marked by shifts in participants and control dynamics. The initial phase, spanning from 2013 to 2017, witnessed the dominance of significant strategic buyers, influencing market dynamics significantly. Subsequently, from 2017 to 2020, the Balancing Pool emerged as a prominent player, primarily engaged in providing Power Purchase Arrangements (PPAs), thereby reshaping the market landscape. In 2021, the original owners regained control over the PPAs, ushering in a new phase in the market's trajectory (Brown, Eckert, et al., 2023). This transition saw a diminishing influence of the Balancing Pool, with the owners reclaiming dominance, coinciding with a consistent upward trend in electricity prices.

This study aims to explore diverse forecasting approaches to predict electricity bid functions spanning from 2008 to 2022. Building upon the foundations laid by Benatia and de Villemeur (2023), our objective and methods are inspired by their work. Through these forecasting approaches, our intention is to uncover valuable insights into the predictability of electricity bid market and how actual changes over time impact the accuracy of forecasting models.

4 Literature review

Various methodologies are employed to predict power prices, encompassing multi-agent models, fundamental techniques, reduced-form models, statistical models, and machine learning (ML) and deep learning (DL) techniques.

In the domain of time series analysis, ARIMA models serve as valuable tools for forecasting both stationary time series and non-stationary time series that can be rendered "stationary" through differencing and other transformation techniques. The Integrated (I) component within an ARIMA model is specifically employed to address non-stationary elements inherent in a time series.

Prevalent models in univariate analysis within statistical time-series modeling include AR (Auto-Regressive), ARMA (Auto-Regressive Moving Average), and ARIMA (Auto-Regressive Integrated Moving Average) models (Zareipour, 2008). It is noteworthy that the ARMA model essentially mirrors an ARIMA model minus the integrated component, concentrating on modeling actual values rather than their differences. Widely utilized as baseline models for assessment in electricity price forecasting studies (Xie, et al., 2013), these time-series models have proven to be indispensable.

Weron (2014) emphasizes that a substantial portion of research papers in the domain of electricity price forecasting (EPF) leans towards either time series models or neural network models. This predilection is attributed to the demonstrated efficacy of statistical techniques and machine learning approaches in yielding favorable outcomes concerning forecasting accuracy.

It should be noted that almost all classes of statistical models have been used in forecasting the price of electricity: AR-models (Weron & Misiorek, 2008; (Karakatsani & Bunn, 2008), ARMA-models (Garcia, et al., 2005; Knittel & Roberts, 2005; Liu & Shi, 2013), ARIMA-models (Nogales, et al., 2002; Conejo, et al., 2005; Contreras, et al., 2003; Cuaresma, et al., 2004; Diongue, et al., 2009), ARFIMA-models (Gianfreda & Grossi, 2012; Koopman, et al., 2007), as well as partial derivatives of the listed models such as ARCH, GARCH, EGARCH, GIGARCH, etc. (Nogales, et al., 2002; Diongue, et al., 2009; Garcia, et al., 2005; García-Martos, et al., 2013; Liu & Shi, 2013). The autoregressive moving average (ARMA) model is widely used among statistical models (Shikhin, et al., 2018). This model has been chosen as it has several important advantages. ARMA models exhibit a robust mathematical and statistical foundation, rendering them among the most methodologically rigorous models within the comprehensive set of forecasting models. Another advantage is the formalized and most detailed methodology, following which you can choose the model that is most suitable for each specific TS. The procedure for checking the adequacy of the model is quite well formalized.

5 Methodology

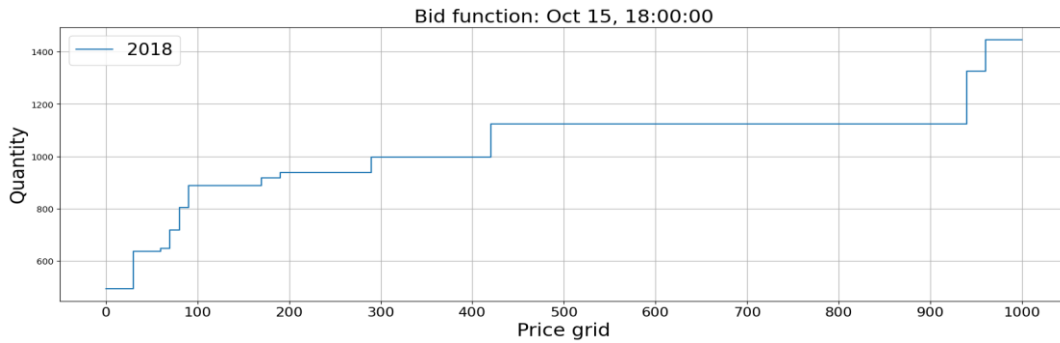
In this study, first we represent the dataset and its descriptive statistics, followed by an explanation of the preprocessing steps. Next, the proposed forecasting model is explained and visualized. Next, the optimal parameters for the model are determined, and the evaluation metrics used in the study are outlined. Lastly, the results are presented and discussed.

6 Dataset

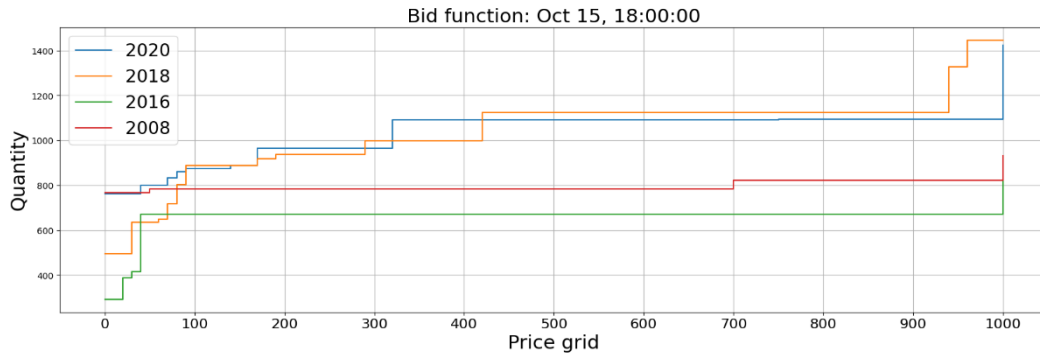
Leveraging data shared by the AESO, our dataset encompasses generator-level information on hourly bids and available capacity. Here, we focus on the hourly bid data for one firm (TransAlta) from 2008 to 2022.

In this data, we have the timestamp and the bids. There are 100 equi-spaced price values, (0, 10, 20, ..., 1000), fixed for the full sample. The quantity bids contain 100 values corresponding to each hour, and each price value on the grid. In other words, at each hour, we have 100 pair of price-quantity values (P_i, Q_i) constituting the bid function at each time step.

By plotting the price grid (P) as a function of the quantity bids (Q_i), we can visualize the bid function for each hour in the sample. Each Q_i is a series of the quantities offered at the price given by the first element of the grid. Figure 1.a shows bid function vs the price grid at a specified date, Oct 15th 2018, 18:00:00. To see the changes of function over time, we plotted bid function at the same date in different four years (Figure 1.b).



a) 2018



b) 2008, 2016, 2018, 2020

Figure1. Bid function at a same date in different years a) 2018 b) 2008, 2016, 2018, 2020

Figure 2 displays one of the series (Q1), covering hourly and daily bid values over all years of dataset. The graph depicts the significant fluctuations and variability observed in electricity bid quantities. values are subject to rapid and frequent changes, indicating a high degree of volatility in the market. The data suggests that the bid quantities can be unpredictable and subject to sudden fluctuations, making it crucial to analyze and model the bid series carefully.

The dataset spans 14 years, and to address the market's evolving behavior, we've chosen specific representative years for focused modeling: 2008, 2016, 2018, and 2020. Each selected year undergoes a tailored modeling process, concentrating on ten months of observations. This approach ensures a cost-effective and nuanced understanding of the market's dynamics, aligning our forecasting models with the distinctive characteristics of each representative period. Table 2 outlines the diverse datasets and their statistical analyses for the target variable.

Also, we decided to focus our analysis on some specific price-quantity series rather than whole series from Q1 to Q100. To do so, we selected 5 series of electricity quantities including Q1, Q20, Q50, Q70, Q90, Q100 as representative of whole 100-dimensional vector. This approach enables a reduction in cost calculations, allowing a more concentrated effort on identifying suitable forecasting models.

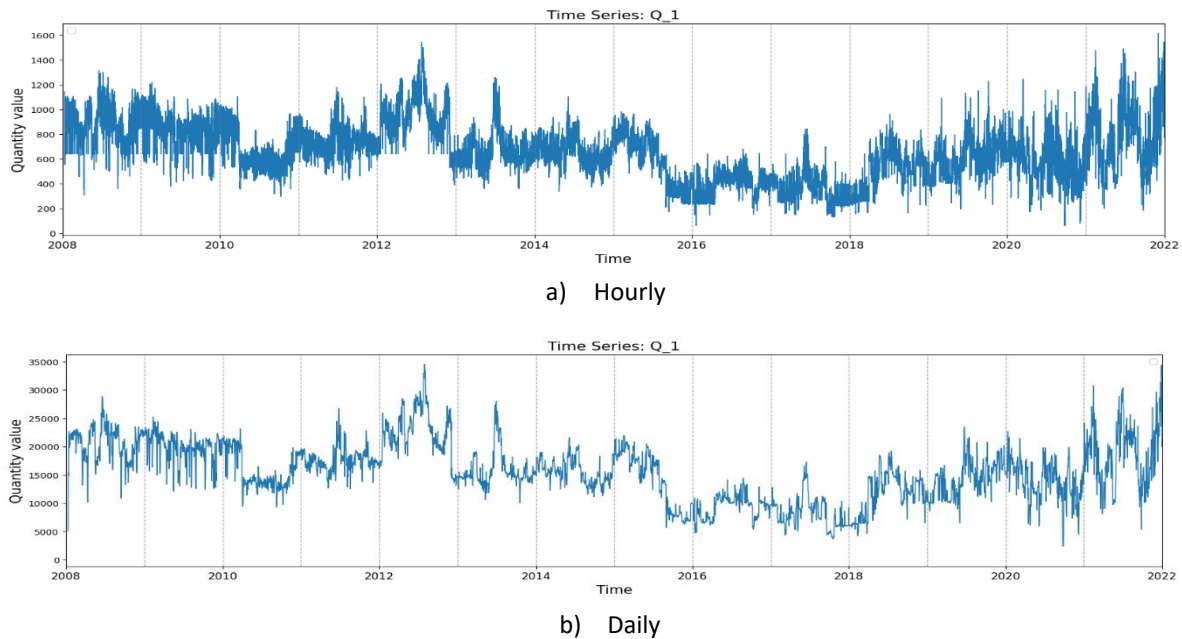


Figure 2: History of bid data over time for TransAlta company over 2008 to 2022: a) Hourly b) Daily

6.1 Train/valid/ test split

The training set is used to estimate the models whereas the valid set is used to evaluate its predictive power.

Observations of each year are split into 9 months for training and 7 days for validation. By this way, the models are compared over the validation set and the Winning model will be determined. In this project, we suffice to use valid set in order to compare our models and select the best one.

Table2: Data split of electricity bid quantities in the datasets

Year	Dataset	Time Period
2008	Train set (9 months)	Jan 1 st 2008 - Sep th 2008
	Validation set (1 week)	Oct 1 st 2008 - Oct 7 th 2008
2016	Train set (9 months)	Jan 1 st 2016 - Sep th 2016
	Validation set (1 week)	Oct 1 st 2016 - Oct 7 th 2016
2018	Train set (9 months)	Jan 1 st 2018 - Sep th 2018
	Validation set (1 week)	Oct 1 st 2018 - Oct 7 th 2018
2020	Train set (9 months)	Jan 1 st 2020 - Sep th 2020
	Validation set (1 week)	Oct 1 st 2020 - Oct 7 th 2020

★ Note: The unit of quantities are MWh.

6.2 Exogenous variable

We have access to extra information which is next 24 hours capacity of electricity production. This information is stored in a series called Q100. So, if we want to forecast for next 24 steps, we can use our information of maximum capacity as an exogenous variable with the hope of improving forecasting models and providing valuable insights into price trends and fluctuations. However, it depends whether the series in question has high or correlation with the exogenous variable (Q100). If the correlation is low, there is a chance that the exogenous will not help or even can be as a disturbing factor in the model with the reverse impact, otherwise the exogenous is expected to improve the performance of the model.

6.3 Forecasting Horizon

Based on literature, for electricity market, it is common to forecast for at most three time steps ahead. However, the main purpose of this project is trying to forecast bidding strategy for next 24 hours. Considering high volatility and fluctuations of this dataset, achieving a well-performance model to forecast for 1 step ahead is not easy and for 24 steps ahead is really challenging.

To show the performance of models in different situations, we considered two forecasting horizons: 2 hours ahead and 24 hours ahead. It is important to note that, we used the rolling-window forecasting method in order to be able to proceed over time and forecast for all periods of the validation set.

6.4 Performance criteria

While we have many criteria to report the model performance, in this research the three significant ones were considered including RMSE, MAE, MAPE.

When evaluating the performance of a prediction model for electricity price values, several loss functions are utilized as criteria. These include the mean squared error (MSE), root mean squared error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). In general, a smaller value of the loss function indicates better performance of the model. Therefore, it is essential to consider these metrics when assessing the accuracy and effectiveness of the prediction model. These functions are as follows:

$$MSE = \frac{1}{T} \sum_{i=1}^T (\hat{x}_t - x_t)^2 \quad (1)$$

$$RMSE = \sqrt{\frac{1}{T} \sum_{i=1}^T (\hat{x}_t - x_t)^2} \quad (2)$$

$$MAE = \frac{1}{T} \sum_{i=1}^T |\hat{x}_t - x_t| \quad (3)$$

$$MAPE = \frac{1}{T} \sum_{i=1}^T \left| \frac{\hat{x}_t - x_t}{x_t} \right| \quad (4)$$

Where T is the total number of observations, x_t is the actual value, and \hat{x}_t is the predicted value.

7 Data Preparation

In the realm of time series analysis, meticulous data preparation is a critical precursor to meaningful insights. This section details the steps taken to enhance the dataset's quality and completeness, laying the foundation for robust time series modeling and analysis.

7.1 Handling Missing Values:

Addressing missing values in the time series play a pivotal role in ensuring the accuracy of subsequent analyses. Notably, the dataset contained an extremely small percentage, less than 0.001 percent, of missing values, totaling 174 instances. This section outlines the approach taken to handle and replace these limited missing values.

Missing value counts: 174

To handle it, the missing values were replaced by the average of their one-step-ahead and one-step-before neighbors in the time series. This method ensures a continuous and smooth representation of the data, minimizing the impact of missing values on subsequent analysis.

7.2 Normalization:

Normalization can be beneficial when dealing with multiple series of quantities for forecasting models. Normalization, or scaling, involves transforming the data so that it falls within a specific range, often between 0 and 1. This process helps ensure that all the series have a similar scale, preventing one series with larger values from disproportionately influencing the model compared to others. Normalization is particularly relevant when the series have different units, magnitudes, or measurement scales, while this is not the case here. Experimenting with both normalized and non-normalized data can help us observe the impact on our specific forecasting model and choose the approach that yields the best results.

To do so, we did Normalization of series using the average and deviation of each series. For this study, z-score normalization is employed.

$$Z_t^i = \frac{x_t^i - \mu^i}{\sigma^i} \quad (5)$$

Where Z^i represents the normalized value of endogenous or exogenous variable i at time t , x^i represents the original value of i_{th} variable at time t , μ^i and σ^i represent the average and standard deviation of the i_{th} variable respectively. This process transforms the data so that it has a mean of zero and a standard deviation of one.

The result showed us this normalization does not change the model performance and we reach the same model performance. This is because the series share the same units and measurement scales.

7.3 Outliers

Regarding our dependent variable, outliers or quantity spikes can disrupt the historical patterns of quantity series and cause confusion or mislead forecasting models. However, spikes are also considered to be an important characteristic of the electricity market price time series. Some studies in the literature on electricity market price forecasting have reported pre-processing of spikes, in which the spikes are either removed from the data, capped at a certain value, or replaced by a weighted average of previous values for that particular hour. Other works have built forecasting models based on the original price time series data without manipulating the spikes. However, there is no conclusive evidence to support either approach in terms of the out-of-sample accuracy of the forecasting models. In this study, we conducted research and analysis on two different scenarios: one using the untransformed price series of electricity as the dependent variable, and the other using the logarithm of the quantity-price values as the dependent variable. Our goal was to compare the outcomes and determine which scenario yielded more accurate and precise results in our predictions. After careful examination, we found that in our case, for some years logarithmic version of time target variable gives us better forecasting result, while for some other years it is reverse. So, to avoid complexity, we concluded to present and discuss the result based on the scenario incorporating the original values of the electricity bid data.

8 Data Exploration & Visualization

In this section, a focused visual inspection was conducted on the bid quantity series, with specific attention to Q1, deemed representative of the entire series. This selective approach aimed to avoid cluttered plots, providing a comprehensive yet uncluttered overview of potential patterns. The visual analysis serves as a foundational step, offering insights into potential patterns and guiding subsequent in-depth analyses for enhanced comprehension of bid quantity dynamics.

8.1 Box plots

Examining the boxplots of electricity bid data across days of the week indicates minimal variations, particularly between weekdays and weekends. As shown in Figure 3, the observed slight differences suggest a relatively consistent bidding pattern throughout the week in the electricity market.

Effect on Modeling: Given the subtle distinctions between weekdays and weekends in the bid data, modeling adjustments for these specific categories may not be imperative. However, acknowledging these nuanced variations is essential during the feature selection and parameter tuning phases. Ensuring the model captures these subtle patterns can enhance its ability to accurately forecast electricity bids across different days of the week in the market.

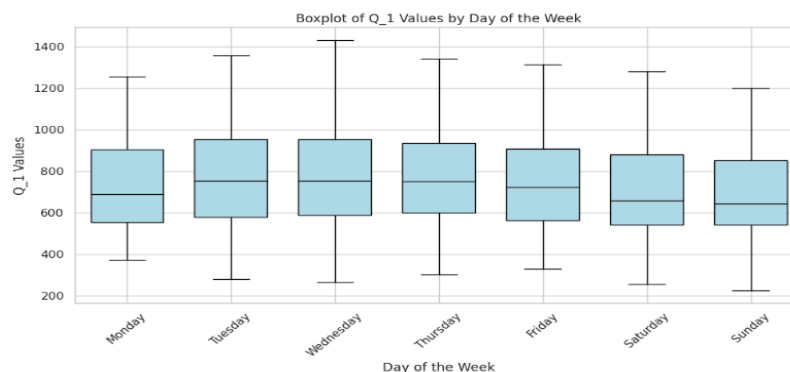


Figure3: Boxplots of Days of week for electricity quantities (Q1)

8.2 Visual Investigation on Trend & Seasonality:

In this section, we intend to have a visual inspection on the potential patterns and understand the behaviour of bid data in question. In this regard, history of bid data over time for the 2nd week and 1st month of each year were plotted in Figure 4 to 6.

Upon careful examination of the depicted electricity bid data, it was observed that daily seasonality occasionally manifests but lacks a persistent presence over time. The cyclic pattern is irregular, appearing for a few days and then fading away, with instances of more sustained daily seasonality observed in certain years, such as 2018, in contrast to shorter and less pronounced patterns, as seen in 2020.

Notably, the cyclical behavior varies throughout the years, suggesting temporal fluctuations in the series. The plots confirm the intermittent nature of the daily seasonality. Contrarily, no clear evidence of persistent weekly or yearly seasonality was identified in this electricity bid time series, highlighting the dynamic and evolving nature of the observed patterns.

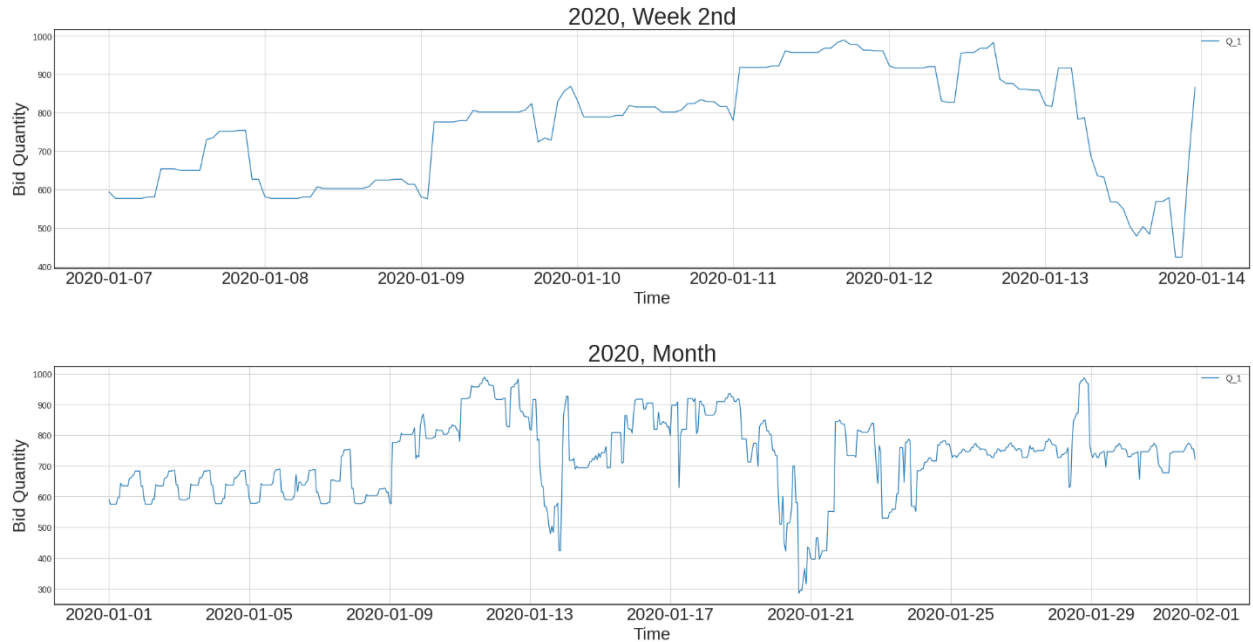


Figure 4: History of bid data; Year 2020: 2nd Week & 1st Month

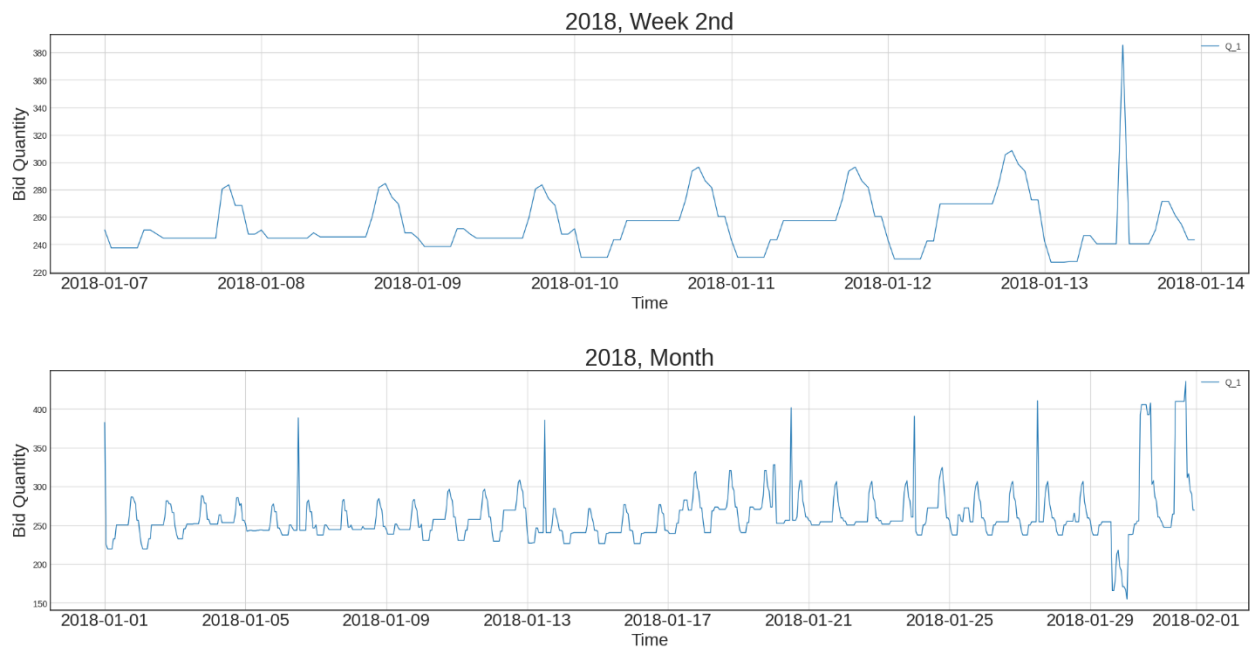


Figure 5: History of bid data; Year 2018, 2nd Week & 1st Month

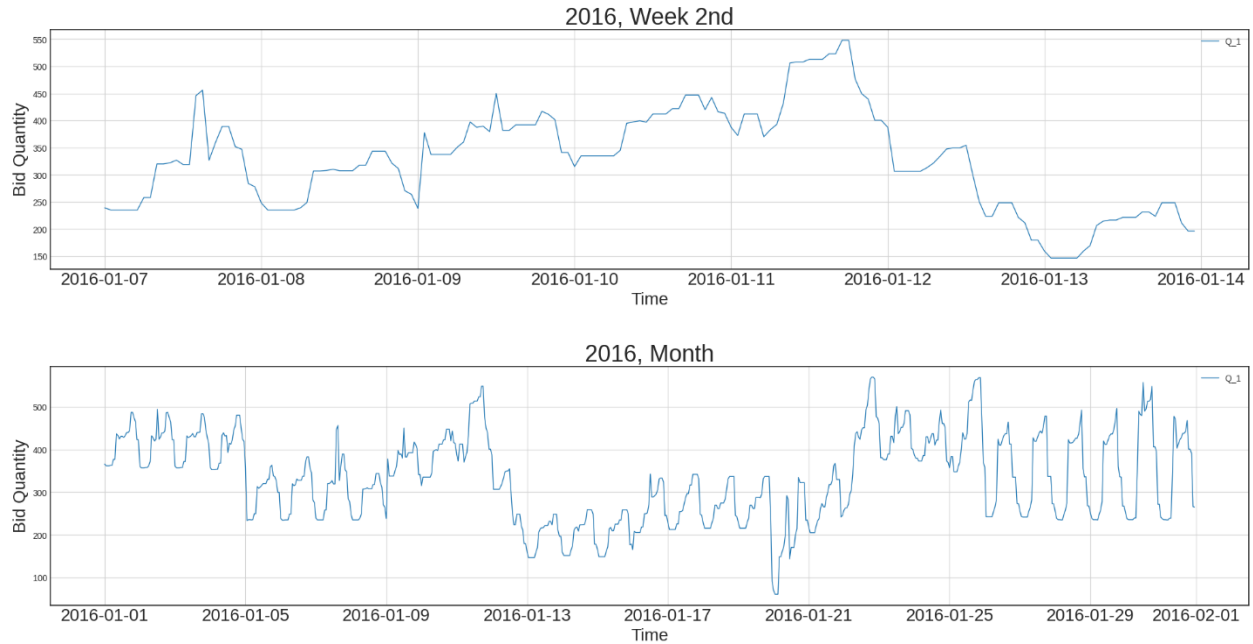


Figure 6: History of bid data; Year 2016: 2nd Week & 1st Month

8.3 Correlation matrix

We have 100 electricity bid quantity time series (Q1 to Q100) from which we have chosen some (5 of them plus one more as exogenous variable). It is important to see how much these series are correlated with each other and also how much correlation they have with exogenous variable which is the capacity electricity production.

To achieve this, we generated a correlation matrix for the bid functions(Figure 6). As bid functions exhibit distinct behaviors each year, we opted to assess the correlations on a yearly basis. The matrix ranks the series based on their correlations, reflected through a color gradient. This approach provides a clear visual representation, allowing for quick identification of Qi series with the least correlation or vice versa.

Figure 7 illustrates that in all years, the initial series (Q1) displays lower correlation with others, especially with Q100. The remaining series exhibit varying levels of correlation across different years.

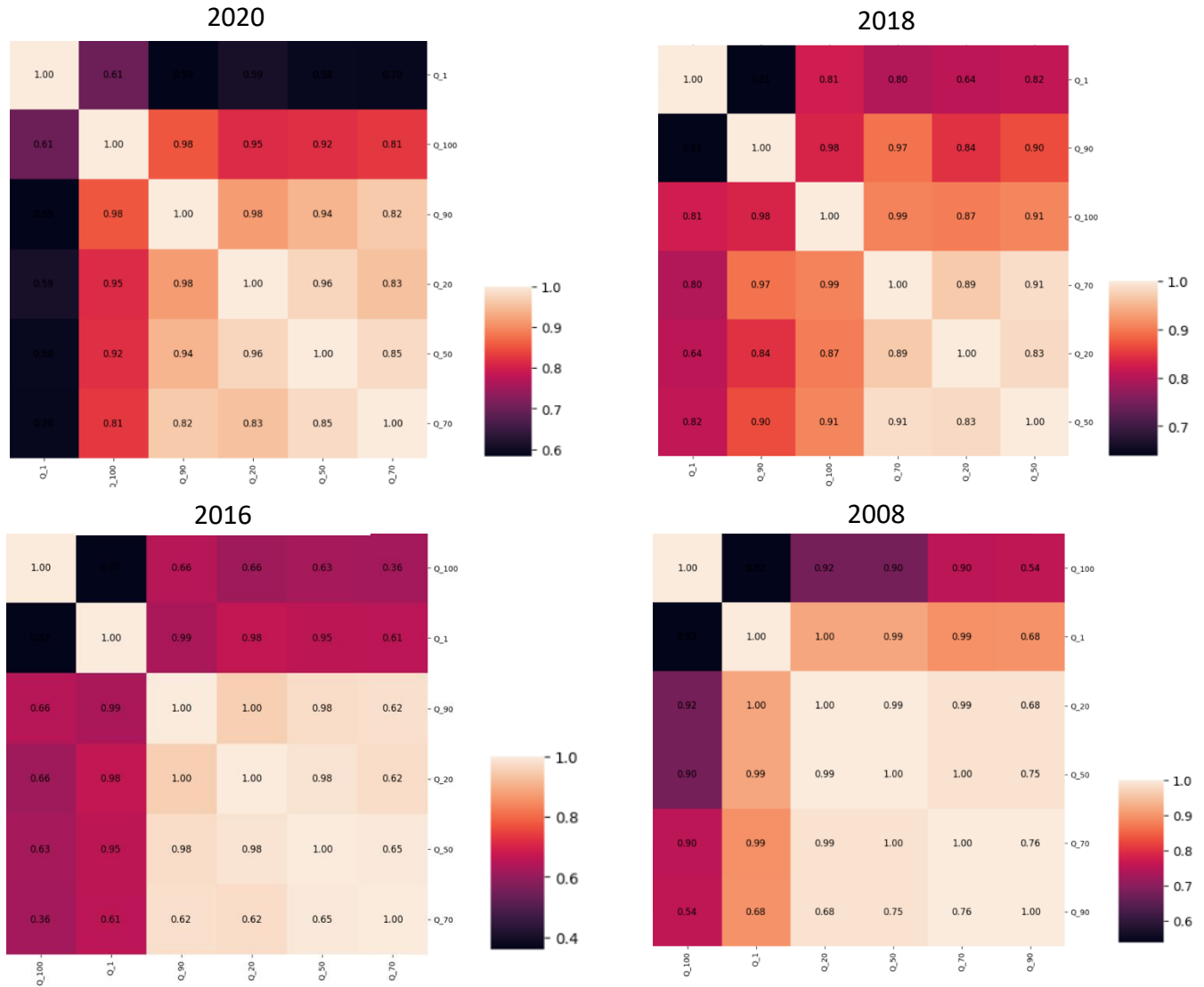


Figure 7: Correlation matrix of endogenous and exogenous series over different periods

To give a visual insight of the correlation between Q1 to Q100, we provided the plot of Q1 along with Q100 for validation week in year 2016 (Figure 8-9). As shown in Figure 1, there is a correlation which seems not be strong. In addition, the plot of some other series including Q70 and Q90 for the same period of time is presented along with Q100 in Figure 2 and shows much higher correlation between them compared to what we observed for Q1 vs Q100.

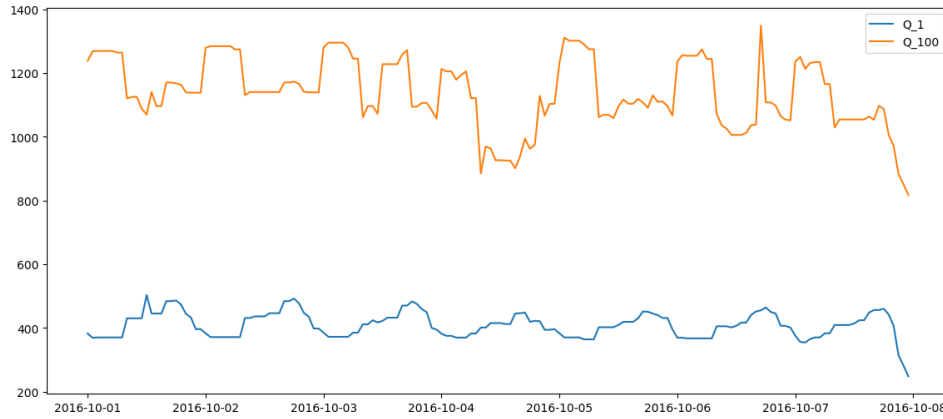


Figure 8: History of Q1 along with Q100 (exogenous variable) for validation week at year 2016

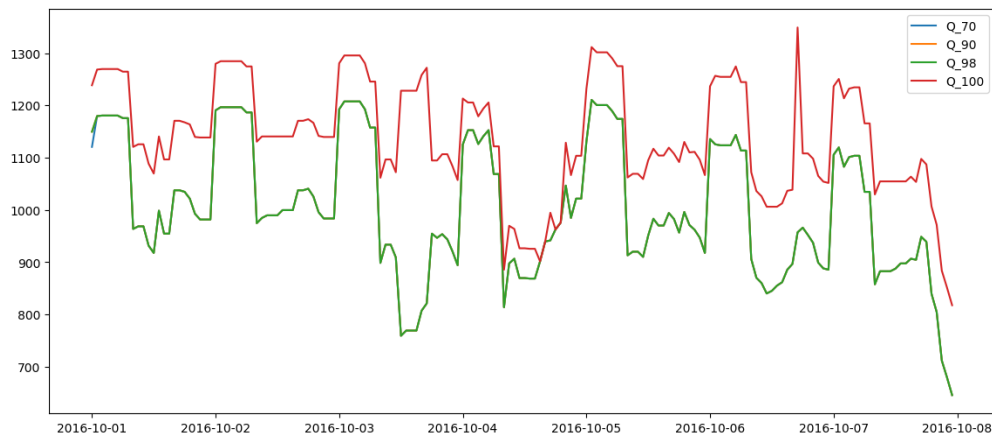


Figure 9: History of Q70 & Q90 along with Q100 (exogenous variable) for validation week at year 2016

9 Models

Since our dataset consists of multiple time series, it justifies the use of models incorporating forecasting Multiple series at the same time like VAR or VARMA. These models can also take into account the correlation between series which can add extra information to the model. However, using many correlated variables (series) results in multicollinearity which can lead to overfitting. So, we gave a try to VAR and VARMA models. In this regard, we faced two main issues: 1) multicollinearity 2) very long run time causing convergence issues specifically when adding Q100 as exogenous variable.

To resolve these two issues, we decided to use univariate time series models for each Q_i series (like AR, ARMA) plus using Q100 as the exogenous variable. We had two main supportive reasons for this switch:

1. The correlation among series mainly stems from their correlation with the capacity (Q100). So, having Q100 in the model represents the correlation between series in a condensed way (we verified this claim by running some models and obtained the same result compared to using multivariate models like VAR).
2. Long runtime leading to high calculation cost reduced significantly and convergence problem resolved.

So, we mainly worked on time series models including AR, ARIMA and SARIMA accompanied by an exogenous variable resulting in ARX, ARIMAX and SARIMAX. To investigate the effect of our exogenous variable, we will compare the result of models with and without the exogenous variable.

Additionally, it is essential to include a fundamental model as a baseline, such as the Naïve or Seasonal Naïve model. Therefore, a total of seven models were considered, comprising six time series models and one baseline. The selected models are detailed in Table 2, and their theoretical foundations and concepts are explained in the subsequent section.

Table2: Models

Type of model	Model name
No Exogenous	AR
	ARIMA
	ARIMA
With Exogenous	ARX
	ARIMAX
	SARIMAX
Base model	Seasonal Naïve

9.1 Theory

In this section, a brief description on the theory of forecasting models we utilized is presented.

Seasonal naïve method: The seasonal naïve method is a simple time series forecasting technique that involves using the observation from the same season in the previous year as the forecast for the current season. It is particularly useful when there is a strong seasonal pattern in the data. The fundamental equation for the seasonal naïve model can be expressed as follows:

$$y_t = y_{t-m} \quad (6)$$

where y_t is the forecasted value at time. y_{t-m} is the observed value from the same season in the previous year (where m is the seasonal period).

AutoRegressive (AR): In time-series forecasting, the Auto-Regressive (AR) method is a commonly employed technique that represents a variable through a linear formulation of its

historical values. The underlying concept is that inherent patterns in the data can be encapsulated by a set of parameters, enabling the prediction of future values (Canova, 1999). The fundamental equation for an AR model is expressed as:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \epsilon_t \quad (7)$$

where y_t represents the price at time t , c represents a constant, $\phi_1, \phi_2, \dots, \phi_p$ are coefficients, and ϵ_t represents white noise term. The parameter "p" signifies the order of the Auto-Regressive (AR) model. It's important to note that the computation of AR coefficients typically employs the least squares technique, aiming to minimize the sum of squared differences between actual and predicted values.

AutoRegressive Integrated Moving Average (ARIMA): The ARIMA method is a widely-used statistical model for time series forecasting, combining principles from AR (Auto-Regressive) and moving average (MA) methods for improved effectiveness. In ARIMA, the focus is on modeling the differences between consecutive values rather than the actual values, known as the integrated part, aiming to achieve stationarity in the time series (Zhang, 2003). The AR and MA components work collaboratively to discern patterns in these differences. The fundamental equation for an ARIMA model is expressed as:

$$y'_t = c + \phi_1 y'_{t-1} + \phi_2 y'_{t-2} + \dots + \phi_p y'_{t-p} + \vartheta_1 \epsilon_{t-1} + \vartheta_2 \epsilon_{t-2} + \dots + \vartheta_q \epsilon_{t-q} + \epsilon_t \quad (8)$$

where y'_t represents the differenced value at time t , c represents a constant, $\phi_1, \phi_2, \dots, \phi_p$ are coefficients of the AR part, $\vartheta_1, \vartheta_2, \dots, \vartheta_q$ are the moving average coefficients, and ϵ_t represents a white noise term. The parameters p , d , and q denote the order of the ARIMA model, with p representing the AR part order, d representing the integrated component order, and q representing the MA part order. In detail, p signifies the count of past time steps utilized as predictors, d indicates the number of differencing operations applied to achieve stationarity, and q denotes the quantity of lagged forecast errors employed as predictors in the model.

Vector Auto Regressive (VAR): An enhanced statistical model, known as a VAR (Vector AutoRegressive) model, builds upon the univariate AR by extending its applicability to model multiple interconnected time series. The core idea behind a VAR model is to represent each time series as a linear combination of its historical values along with those of all other time series within the model. This allows the model to learn the interdependencies between several time series (Smola & Schoellkopf, 2004). The basic equation for an AR model is represented as:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + e_t \quad (9)$$

where y_t is a vector of current values for all time series, A_1, A_2, \dots, A_p represent the VAR coefficients and e_t is a vector of error terms. Similar to the AR model, the number of lags (p), which defines how far back in past values the model will consider, needs to be determined.

9.2 Model parameters

To tune model hyperparameters, we need to go through a process using ACF/PACF of residuals, Ljung-box test etc. We applied a comparable process across various years and arrived at nearly identical sets of parameters for the models. Figure 10 presents the flowchart of process we implemented to find adequate models. In addition, the whole process for year 2021 is provided in some detailed steps:

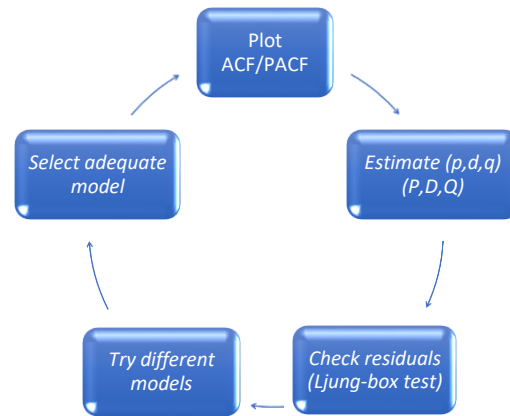
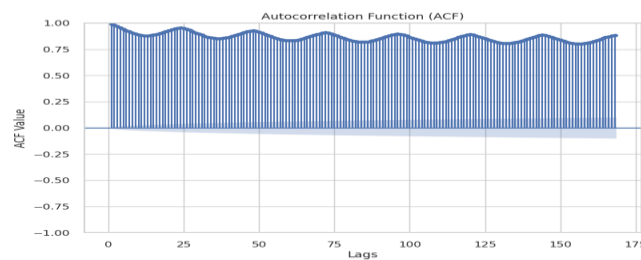


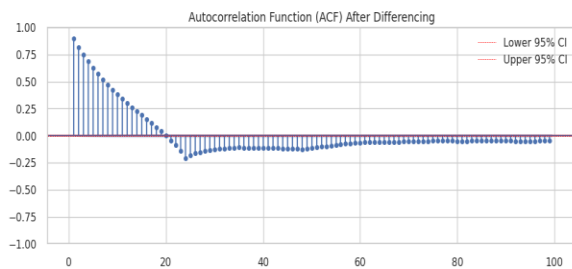
Figure. 10: Flowchart of process implemented to find adequate models

9.2.1 ACF/PACF & Stationarity

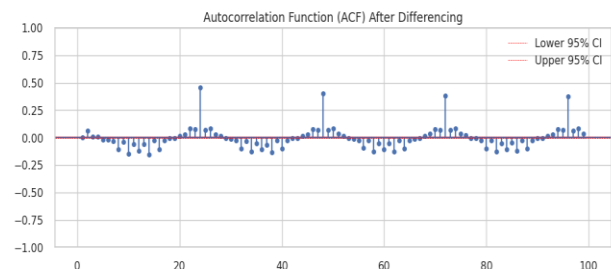
Based on ACF plot, we observe non-stationarity which could be resolved by differencing at specific lags. We see 24 hours seasonality leading us to try differencing at lag 24 (Figure 11). We also tried differencing at lag 1 which is helpful in most cases. The results are shown in fig 2 and fig 3. In both cases a big portion of nonstationary has been removed. However, we decided to go with differencing at lag 1 ($d=1$) and then handle seasonality by Seasonal ARMA terms.



(a) ACF of original series (year 2021)



(b) ACF after Differencing at lag=24



(c) ACF after Differencing at lag=1

Figure 11. ACF plots

In differenced values, spikes at lag= 24 hours represents seasonality at 24 hours.

In hourly data, we see spikes up to lag 24 for PACF and nothing after that is equivalent to term $p=24$. There is a spike at lag 1 for ACF and nothing after equivalent to term $q=1$. $q=2$ can also be checked.

To investigate in more detail, we converted series from hourly to daily series and plotted ACF/PACF for that; we observed a spike at lag 1 for PACF and nothing after which is equal to $P=1$ ($S=24$) or lag 24 for Hourly data.

9.2.2 Residual assessment

As we explored to find the right statistical model for our prediction, we carefully examined two key aspects. First, we ensured that there were no unusual patterns or dependencies in the residuals by examining any significant increases in their autocorrelation and partial correlation. This helps us avoid any hidden patterns that might affect our predictions.

Next, we subjected the residuals to a Ljung-Box test to assess their resemblance to white noise. In simpler terms, we checked whether they are random and unpredictable, which is essential for a good forecasting model. So, after performing these evaluations for all the statistical models we considered, Figure 12 shows sample results of Residual assessment we made:

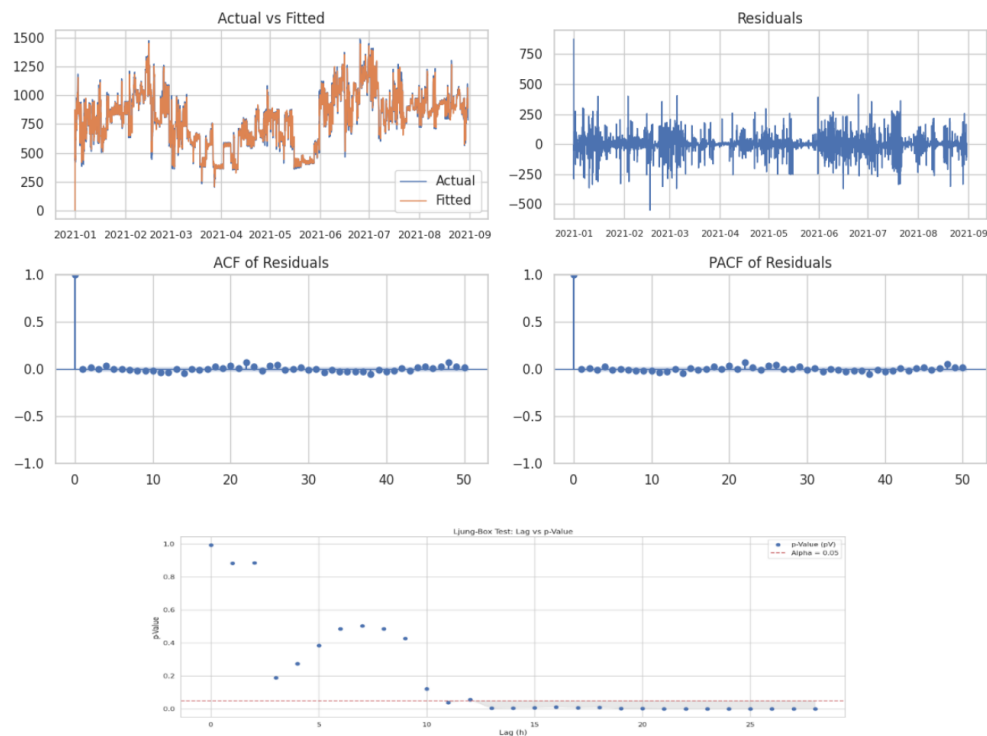


Figure 12. Residuals assessment: Residuals, ACF/PACF of Residuals, Ljung-box test

Given the initial estimations of parameters and after many try and error of various combinations for (p,d,q) & (P,D,Q) terms, we reached some combinations giving us adequate models as shown in table 3:

Table3: Model parameters

Model Parameters
AR(24)
ARIMA (24,1,1)
SARIMA (2,1,1)(1,0,0,24)
ARX(24)
ARIMAX (24,1,1)
SARIMAX (2,1,1)(1,0,0,24)
Seasonal Naïve (S=24)

These models were chosen as our main forecasting models in order to make a bigger hybrid model.

9.3 Samples (cases)

So far, we have developed our forecasting models and next step is applying those models on bid functions which consist of 5 time series of electricity bid quantities (Q1, Q20, Q50, Q70, Q90) along with Q100 as the exogenous variable. On the other hand, we have 4 separate periods of time (2008, 2016, 2018, 2020) for each of which we want to forecast the given series. By doing so, all together, we have 20 cases of forecasting modeling.

10 Result

This section presents key findings, examining the impact of incorporating exogenous variables on forecasting models and also comparing performance of all applied models against base model. The results are presented for short-term (2 steps ahead) and medium-term (24 steps ahead) forecasting horizons. To equip us for a thorough analysis, we reported our results by using some helpful factors as follows:

- **Performance criteria** including RMSE, MAE, MAPE with the focus on MAPE.
- **Winning model** representing the model with best performance.
- **Improvement rate** which is associated with the winning model and represents how much improvement we had by using Q100 as exogenous variable compared to the same model without exogenous variable.

Table 4 & 5 present results for short-term forecasting horizon and Table 6 & 7 are for medium-term forecasting horizon.

10.1 Impact of adding exogenous variable

- In most cases, models with exogenous variable (X) outperformed models without the independent variable. The addition of capacity information significantly enhanced forecasting models, evident across both short-term and medium-term horizon scenarios.
- **For medium-term horizon:** for all 4 years we see that the model with exogenous variable works better for all series except for Q1 series in year 2016 and 2020. This is justified with the fact that Q1 had low correlation with the exogenous Q100 in these two years.
- **For short-term horizon:** the output is pretty the same. For all 4 years we see that the model with exogenous variable works better for all series Q10, Q50, Q70 and Q90 except for Q1 series in two years.
- So, it shows that it is possible the low correlated exogenous variable impact the model even in a negative way. This is justified with the fact that Q1 had low correlation with the exogenous Q100 in most years. So, to avoid impact model negatively, we can conclude whenever the series has low correlation with the exogenous, we should choose among models without exogenous var and vice versa.
- **Improvement rate:** In all years, we have tangible performance improvement by using exogenous Q100. By trading the improvement rate, we observe in most years, the more correlated series with Q100 enjoyed higher improvement by using exogenous var. This improvement contains values from 2% to 68% depending on the extension of correlation. However, we have 2 negative improvement rates which are -14% and -22% among all 20 proposed cases for medium-term horizon forecasting. The same goes with short-term scenario.

10.2 Comparison with base model

- **For medium-term horizon:** except for one case (Q1 at year 2016), in all other 19 cases the basic model (seasonal Naïve) is beaten. It shows the provided forecasting models could add value and work better than simple models.
- **For short-term horizon:** in all cases the basic model (seasonal Naïve) is beaten.
- The results show that the provided forecasting models could add significant value and work better than simple basic models. In the tables, although we didn't provide the percentage of improvement we had by winning models compared to the basic model, we calculated them and they vary from 10% to more than 100% in some cases. It shows how well the implemented time series models perform over the basic model.

10.3 Comparing short-term and medium-term forecasting

- By comparing the performance metrics, we observe that the errors obtained by 2-step forecasting are much fewer than those for 24-step forecasting. As expected, the more the number of forecasting horizons, the more forecasting errors increase.

Table 4. Experimental results for next 24 hours forecasting, Year 2008 & 2016

Series	Type	Model	1 Oct 2008 - 8 Oct 2008					1 Oct 2016 - 8 Oct 2016				
			RMSE	MAE	MAPE	Winning model	Improve rate	RMSE	MAE	MAPE	Winning model	Improve rate
Q1	No Exogenous	AR	107.0	78.2	11.21%			26.2	16.6	4.24%		
		ARIMA	93.8	62.2	8.76%			23.4	13.5	3.46%		
		SARIMA	103.8	71.9	10.08%			27.5	18.4	4.59%		
	With Exogenous	ARX	101.4	73.8	10.61%			26.8	13.6	4.19%		
		ARIMAX	61.6	41.2	5.78%	ARIMAX	34%	22.6	13.6	3.50%		
		SARIMAX	62.8	42.4	5.97%			27.1	18.6	4.66%		
	Base model	Seasonal Naïve	113.5	86.5	12.45%			23.8	12.6	3.27%	Seasonal Naïve	22%
Q20	No Exogenous	AR	109.9	80.5	9.70%			94.2	74.2	7.85%		
		ARIMA	98.3	64.1	7.66%			80.1	57.2	6.15%		
		SARIMA	106.6	74.8	8.88%			80.2	58.0	6.20%		
	With Exogenous	ARX	60.6	43.8	5.34%			89.6	55.8	6.03%		
		ARIMAX	59.3	40.2	4.78%			88.8	53.9	5.84%	ARIMAX	5%
		SARIMAX	57.5	38.6	4.61%	SARIMAX	48%	94.5	64.2	6.80%		
	Base model	Seasonal Naïve	118.2	90.6	11.01%			81.7	62.2	6.68%		
Q50	No Exogenous	AR	109.9	80.5	9.70%			91.1	70.4	7.49%		
		ARIMA	98.3	64.1	7.67%			75.2	54.3	5.89%		
		SARIMA	106.7	74.9	8.89%			77.5	56.6	6.11%		
	With Exogenous	ARX	60.2	43.5	5.30%			85.5	52.7	5.78%		
		ARIMAX	59.9	39.8	4.74%			84.3	49.5	5.46%		
		SARIMAX	57.4	38.7	4.62%	SARIMAX	48%	87.6	48.8	5.42%	SARIMAX	11%
	Base model	Seasonal Naïve	118.2	90.6	11.01%			78.5	58.2	6.33%		
Q70	No Exogenous	AR	108.3	79.1	9.49%			91.8	70.8	7.53%		
		ARIMA	98.0	63.8	7.64%			76.0	55.4	5.99%		
		SARIMA	103.3	72.8	8.53%			78.6	57.7	6.21%		
	With Exogenous	ARX	72.8	52.4	6.40%			85.7	53.8	5.87%		
		ARIMAX	56.8	39.2	4.66%			84.5	49.7	5.48%	ARIMAX	8%
		SARIMAX	56.2	38.5	4.61%	SARIMAX	46%	86.5	52.2	5.62%		
	Base model	Seasonal Naïve	118.5	91.6	11.11%			86.7	59.2	6.33%		
Q90	No Exogenous	AR	105.0	76.2	9.04%			90.6	69.7	7.41%		
		ARIMA	95.5	61.4	7.28%			74.5	53.1	5.79%		
		SARIMA	102.9	72.0	8.49%			76.8	55.7	6.04%		
	With Exogenous	ARX	65.0	46.3	5.60%			85.4	52.5	5.75%		
		ARIMAX	54.0	36.4	4.31%			84.7	47.3	5.27%	ARIMAX	9%
		SARIMAX	52.8	36.4	4.30%	SARIMAX	49%	81.6	47.7	5.27%		
	Base model	Seasonal Naïve	115.5	88.7	10.65%			78.7	59.0	6.40%		

Table5. Experimental results for next 24 hours forecasting, Year 2018 & 2020

Series	Type	Model	1 Oct 2018 - 8 Oct 2018					1 Oct 2020 - 8 Oct 2020				
			RMSE	MAE	MAPE	Winning model	Improve rate	RMSE	MAE	MAPE	Winning model	Improve rate
Q1	No Exogenous	AR	61.0	30.3	5.91%			118.1	80.8	14.96%		
		ARIMA	59.5	31.3	6.07%			124.8	85.2	15.88%	ARIMA	14%
		SARIMA	54.5	29.8	5.82%			133.6	89.7	16.17%		
	With Exogenous	ARX	50.9	27.1	5.27%			138.2	98.0	18.32%		
		ARIMAX	48.7	28.0	5.44%			143.8	99.5	18.54%		
		SARIMAX	45.0	25.7	5.03%	SARIMAX	14%	152.4	107.3	19.99%		
	Base model	Seasonal Naïve	72.1	33.4	6.43%			131.6	85.8	15.97%		
Q20	No Exogenous	AR	145.5	105.3	9.05%			147.0	119.0	11.82%		
		ARIMA	146.2	100.9	8.78%			143.4	115.9	11.57%		
		SARIMA	148.4	108.3	9.29%			182.8	155.2	14.92%		
	With Exogenous	ARX	117.7	81.0	7.19%			113.0	87.7	8.76%		
		ARIMAX	114.2	77.1	6.87%	ARIMAX	22%	113.9	85.5	8.41%	ARIMAX	27%
		SARIMAX	118.7	89.2	7.88%			110.5	95.0	9.31%		
	Base model	Seasonal Naïve	164.8	95.3	8.36%			191.3	138.6	13.87%		
Q50	No Exogenous	AR	155.9	112.1	9.62%			146.3	119.2	11.58%		
		ARIMA	155.8	107.2	9.30%			146.1	117.0	11.43%		
		SARIMA	155.6	112.1	9.64%			177.1	142.0	13.39%		
	With Exogenous	ARX	122.3	83.2	7.45%			89.3	59.3	5.75%		
		ARIMAX	118.8	79.3	7.15%	ARIMAX	23%	97.5	63.5	6.05%	ARIMAX	47%
		SARIMAX	118.5	88.4	7.88%			96.0	74.6	7.00%		
	Base model	Seasonal Naïve	174.3	103.8	9.19%			183.8	138.0	13.61%		
Q70	No Exogenous	AR	157.1	114.4	9.82%			123.3	100.3	9.61%		
		ARIMA	156.5	109.1	9.50%			121.0	94.6	9.18%		
		SARIMA	156.7	114.3	9.87%			170.3	139.3	12.64%		
	With Exogenous	ARX	116.9	82.3	7.29%			60.0	38.4	3.72%		
		ARIMAX	114.9	79.7	7.12%	ARIMAX	25%	59.1	37.2	3.59%	ARIMAX	61%
		SARIMAX	115.9	88.3	7.80%			65.5	48.5	4.44%		
	Base model	Seasonal Naïve	177.7	107.4	9.52%			160.0	110.8	10.81%		
Q90	No Exogenous	AR	171.9	120.8	10.30%			112.9	89.2	8.27%		
		ARIMA	176.8	119.1	10.34%			108.8	82.8	7.77%		
		SARIMA	173.0	123.7	10.56%			153.3	116.5	10.33%		
	With Exogenous	ARX	109.3	74.1	6.49%			39.8	25.2	2.30%		
		ARIMAX	110.1	72.6	6.41%	ARIMAX	38%	40.8	25.2	2.27%	ARIMAX	71%
		SARIMAX	108.5	79.5	6.98%			48.7	36.7	3.16%		
	Base model	Seasonal Naïve	187.2	115.0	10.09%			152.1	100.4	9.53%		

Table6. Experimental results for next 2 hours forecasting, Year 2008 & 2016

Series	Type	Model	1 Oct 2008 - 8 Oct 2008					1 Oct 2016 - 8 Oct 2016				
			RMSE	MAE	MAPE	Winning model	Improve rate	RMSE	MAE	MAPE	Winning model	Improve rate
Q1	No Exogenous	AR	43.2	29.9	4.20%	AR	-0.5%	19.2	12.8	3.20%		
	With Exogenous	ARX	42.8	30.0	4.22%			19.2	12.5	3.11%	ARX	2.7%
	Base model	Seasonal Naïve	48.2	32.2	4.37%			28.6	19.2	4.82%		
Q20	No Exogenous	AR	43.1	29.5	3.48%			67.7	49.7	5.23%		
	With Exogenous	ARX	37.6	26.9	3.24%	ARX	6.8%	64.2	40.7	4.40%	ARX	15.9%
	Base model	Seasonal Naïve	48.9	32.5	3.75%			97.8	62.4	6.40%		
Q50	No Exogenous	AR	43.1	29.5	3.48%			63.3	45.7	4.80%		
	With Exogenous	ARX	37.6	26.8	3.23%	ARX	7.1%	58.1	36.7	4.00%	ARX	16.8%
	Base model	Seasonal Naïve	48.9	32.5	3.75%			97.6	62.2	6.38%		
Q70	No Exogenous	AR	42.1	28.5	3.35%			62.3	46.6	4.86%		
	With Exogenous	ARX	38.2	26.9	3.22%	ARX	3.9%	56.7	36.6	3.95%	ARX	18.7%
	Base model	Seasonal Naïve	48.1	32.1	3.69%			97.6	62.2	6.38%		
Q90	No Exogenous	AR	41.1	27.5	3.19%			60.9	44.6	4.67%		
	With Exogenous	ARX	34.9	24.5	2.89%	ARX	9.5%	58.6	37.8	4.10%	ARX	12.1%
	Base model	Seasonal Naïve	46.1	31.0	3.54%			97.9	62.2	6.38%		

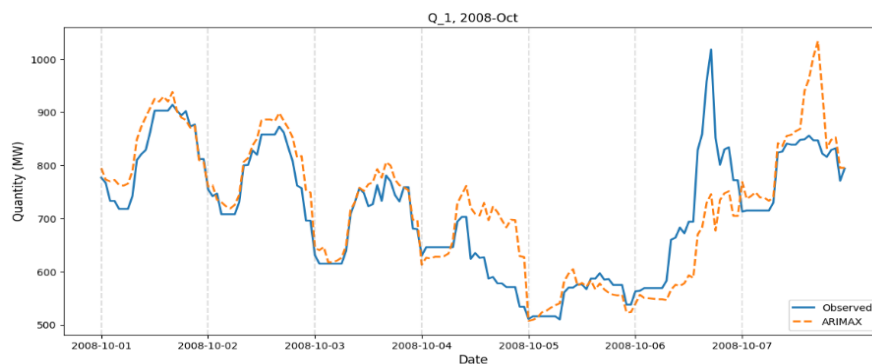
Table7. Experimental results for next 2 hours forecasting, Year 2018 & 2020

Series	Type	Model	1 Oct 2018 - 8 Oct 2018					1 Oct 2020 - 8 Oct 2020				
			RMSE	MAE	MAPE	Winning model	Improve rate	RMSE	MAE	MAPE	Winning model	Improve rate
Q1	No Exogenous	AR	24.0	12.6	2.53%			61.2	31.1	5.83%	AR	-30.7%
	With Exogenous	ARX	22.4	12.4	2.47%	ARX	2.4%	65.1	40.0	7.62%		
	Base model	Seasonal Naïve	30.4	13.9	2.81%			75.3	37.9	7.10%		
Q20	No Exogenous	AR	104.3	64.5	5.89%			90.0	58.4	5.97%		
	With Exogenous	ARX	92.2	60.1	5.48%	ARX	6.9%	67.0	39.9	4.08%	ARX	31.7%
	Base model	Seasonal Naïve	123.4	69.4	6.49%			115.3	73.5	7.61%		
Q50	No Exogenous	AR	105.3	65.3	5.94%			84.5	55.1	5.48%		
	With Exogenous	ARX	94.8	60.7	5.54%	ARX	6.8%	52.2	30.5	3.02%	ARX	44.9%
	Base model	Seasonal Naïve	121.7	67.8	6.27%			104.3	64.9	6.50%		
Q70	No Exogenous	AR	115.1	69.4	6.26%			81.5	50.3	4.83%		
	With Exogenous	ARX	92.0	58.7	4.96%	ARX	20.7%	35.9	20.3	1.95%	ARX	59.6%
	Base model	Seasonal Naïve	133.9	74.7	6.82%			105.9	65.1	6.31%		
Q90	No Exogenous	AR	106.8	63.1	5.62%			80.0	48.1	4.51%		
	With Exogenous	ARX	90.0	53.7	4.79%	ARX	14.8%	29.5	15.6	1.44%	ARX	68.0%
	Base model	Seasonal Naïve	129.0	70.7	6.37%			102.8	63.1	5.97%		

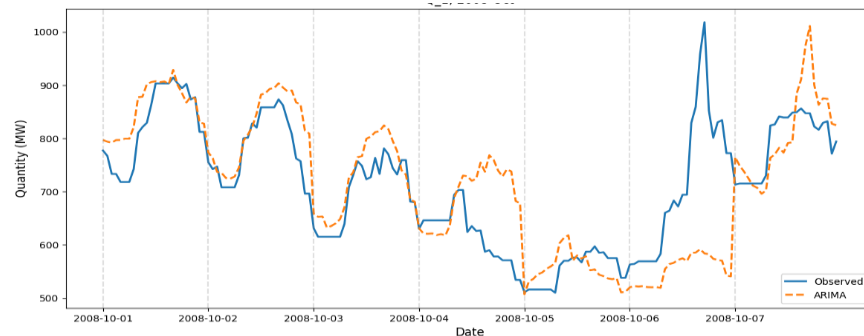
10.4 Forecasting plots

In this part, for both short-term and medium-term forecasting, plots of the forecasted values vs real bid values are depicted for the validation week in 2008. The visual forecasting results are provided for Q1 over time for three main models: Model with exogenous (ARIMAX), model without exogenous (ARIMA), baseline (Seasonal Naïve). As mentioned earlier, these plots were made using rolling-window forecasting method to be able to proceed and forecast for all period of validation set.

In 24-step ahead forecasting, better results are obtained by ARIMAX as depicted in Figure 13. Both ARIMAX and ARIMA beat Baseline. This result is compatible with the calculated errors discussed earlier provided in table 4.



(a) ARIMAX



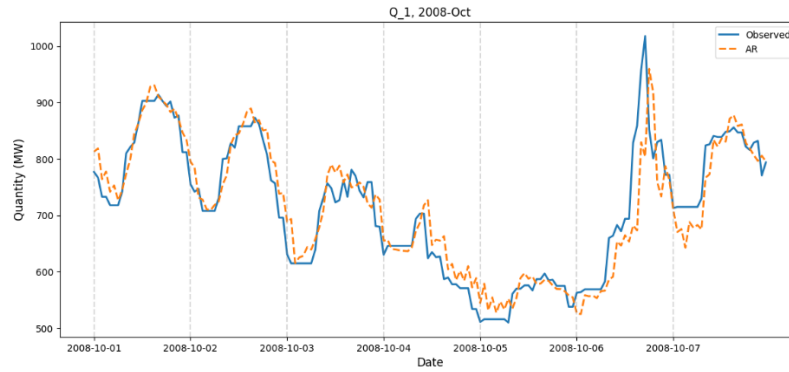
(b) ARIMA



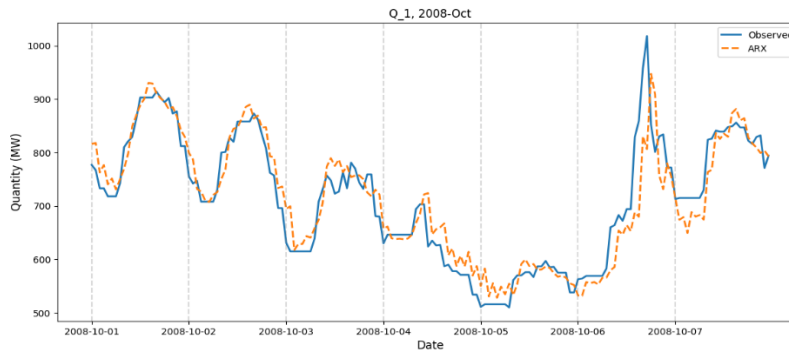
(c) Seasonal Naïve

Figure 13. 24-step ahead forecasted values vs real bid values for Q1 over validation set

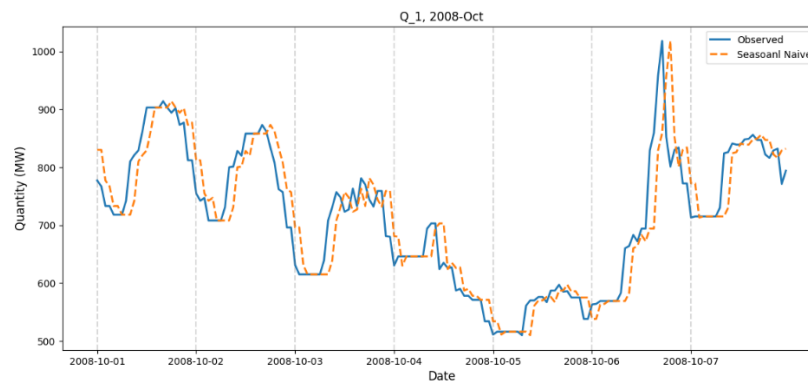
In 2-step ahead forecasting, better results are obtained by ARX as depicted in Figure 14. Both ARX and AR beat Baseline. This is compatible with the calculated errors we talked about.



(c) ARX



(d) AR



(c) Seasonal Naive

Figure 14. 2-step ahead forecasted values vs real bid values for Q1 over validation set

11 Conclusion

In conclusion of our investigation into electricity market trend prediction, we have navigated the complexities of the dataset. Through the thoughtful selection of specific years and discerning modeling, we have examined the impact of an additional factor (exogenous variable) on short and medium-term forecasting. Our findings now guide us through a straightforward comparison with a basic model, offering clear insights into the effectiveness of our forecasting approach.

- **Data Preparation and Selection:** Leveraging a comprehensive dataset spanning 14 years, we strategically focused on representative years (2008, 2016, 2018, 2020) for nuanced modeling, reducing computational costs without sacrificing insight.
- **Modeling Approach:** The complexity of multiple time series justified a shift from multivariate to univariate time series models (AR, ARIMA, SARIMA) with Q100 as an exogenous variable, addressing multicollinearity and runtime issues.
- **Exogenous Variable Impact:** Incorporating Q100 as an exogenous variable consistently improved forecasting models for both short-term and medium-term horizons, with improvement rates ranging from 2% to 68%. Notably, the level of improvement correlated with the degree of series correlation with Q100.
- **Model Comparison:** The proposed forecasting models consistently outperformed the basic model (Seasonal Naïve) in all but one case for both short-term and medium-term horizons. The improvement rates ranged from 10% to over 100%, highlighting the significant value added by the implemented time series models.
- **Forecasting Horizons:** we employed the rolling-window forecasting method to analyze two forecasting horizons. Short-term forecasting (2 steps ahead) demonstrated notably fewer errors compared to medium-term forecasting (24 steps ahead), confirming the expected increase in errors with an extended forecasting horizon.

In summary, our tailored modeling approach, leveraging univariate time series models with an exogenous variable, showcased superior forecasting capabilities and added substantial value compared to basic models. The strategic selection of representative years and careful consideration of the impact of exogenous variables contribute to the robustness and applicability of the proposed forecasting methodology.

Suggestion for future work: Recognizing the dataset's dynamic nature from 2008 to 2018, our efforts underscore the need for an adaptive model. A singular model does not suffice; instead, an adaptive approach with varying parameters across periods is essential. Future endeavors could focus on implementing an adaptive model that seamlessly accommodates the distinctive characteristics of all periods within the dataset.

It is highly possible to improve forecasting results by applying Deep learning models such as LSTM. It is recommended to consider different version of DL models and compare with the available results we achieved by time series models.

12 References

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13 Appendix

13.1 Challenges & further details of our modeling approach

To further discuss about modeling approach, we had several challenges in choosing an appropriate approach given the dataset with its specific characteristic. To do so, we required to find an answer some challenging questions:

Question1) How should we use our information of Capacity (Q100) in our model? A) Normalizing all Q_i 's by corresponding capacity Q100 at each time step t or B) Using values of Q100 as exogenous feature at time t (future).

To respond to this question, we decided to compare both scenarios. To do so, two models were considered (Normalized-ARIMA & ARIMAX) and forecasted bid quantities for Q1 over the week of validation set in year 2017 and 2021. The results are as follows:

A) Normalizing by Q100 (Normalized-ARIMA):

- *Year 2017:* IF Q1 has **low** correlation with capacity (Q100), Normalizing by capacity ($Q1/Q100$) has **high negative** impact on forecasting model (from MAPE of 7% it goes to 11.5%). This is not good and we do not want to reduce model performance.
- *Year 2021:* IF Q1 has **medium** or high correlation with capacity (Q100), Normalizing ($Q1/Q100$) Has **positive** impact on forecasting model.

B) Using Q100 as feature in future time (ARIMAX):

- *Year 2017:* If a series like Q1 has **Low** Correlation, the model performance **stays the same** or has **very limited negative** impact.

- *Year 2021:* If Q1 has **medium** or high correlation with capacity, using Q100 as feature **improves** model.

Question 2) How does using history of other series including Q2, Q3, Q10 , ... Q90, as exogenous features impact forecasting a series like Q1? (Note that here we use history (t-24) of other series as our features)

We compared two scenarios: Model with and without features (ARIMA & ARIMAX). To do so, using the models we forecasted bid quantities for Q1 over the week of validation set in year 2017. The results are as follows:

- The results showed that adding history of other Q_i 's as explanatory variable, does not add value in forecasting output. This is mainly because we are using ARIMA terms (p,q) to extract history of target series. Consequently, using history of other correlated series does not add much value in the forecasting performance.

Considering all these outputs, it seems reasonable to use only our information of Capacity (Q100) in our model as explanatory variable at future time t . By this way, this factor whether improves forecasting performance or keeps the model pretty the same. It means, when there is low correlation between the series and the feature, adding this feature does not impact the performance in a significant negative way. While, in the case of normalizing by this factor, we may impact model performance in a tangible negative way.

This is because when we first normalize by Q100, then forecast and finally undo the normalization, we are indirectly pushing the series to follow patterns existing in Q100. So, if a series like Q1 or Q20 has low correlation with Q100, applying normalization impacts their forecasting model negatively.

13.2 Visual investigation

To extend our visual investigation on our bid data, we plotted history of realizations for all selected Q_i 's in different years at a same week. As mentioned earlier, we intend to have a visual inspection on the potential patterns and understand the behaviour of the bid data in question.

Figure 1 shows history of realizations over time. for the validation week in each year. It was observed that for year 2018 and 2012, daily seasonality occasionally manifests but lacks a persistent presence over time. The cyclic pattern is irregular, appearing for a few days and then fading away. The Week at Year 2016 shows stronger daily seasonality for all Q_i 's. However, it is not a quite persistent daily pattern.

Notably, the cyclical behavior varies throughout the years, suggesting temporal fluctuations in the series . The plots confirm the intermittent nature of the daily seasonality. Contrarily, no clear evidence of persistent weekly or yearly seasonality was identified in this electricity bid time series, highlighting the dynamic and evolving nature of the observed patterns.



Figure 1: History of bid data; 2nd week of Oct; Year 2016, 2018, 2020:

History of Qi's over a month at year 2020 gives us another perspective of potential daily seasonality (Figure 2). This view of our bid data to some extent confirms the fact that there is daily seasonality but still not persistent. It gets on and off on a weekly basis. The intensity of this seasonality changes from Qi to Qi, particularly between Q1 and the rest of Qi's.

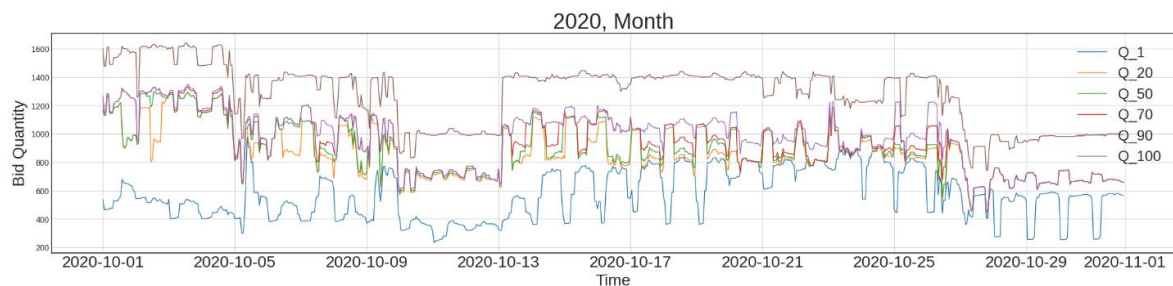


Figure 2: History of bid data; Oct 2020

Coding

Here is the file of coding for modeling and data exploration:

<https://drive.google.com/file/d/1IJChIfvFjDjMoW3xib3Y6oSAPuqb3qIDG/view?usp=sharing>