HEC MONTRÉAL

MATH60638A- Forecasting Methods

Report Part II:

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In this report, we present the revision made from last one and continue with the evaluation of the smoothing methods and regression methods.

1. <u>Revision of part 1</u>

Cooling Degree Day (CDD): We have rectified an issue with the Rochester station, there was a mistake while extracting and then combining the dataset. The behavior of all considered explanatory variables against demand has not changed except CDD in Fig 1.2**.** Based on the daily linear relationship between the maximum temperature (T_{max}) and peak demand starting from 72 °F (22.2 °C) upwards, the reference temperature (T_{ref}) for CDD is chosen. We now observe a linear trend between the peak demand CDD in Fig. 1.2. For CDD lag 1 in Fig. 1.3, we also observe a linear relationship with daily hourly peak demand. Windchill also shows a linear relationship with demand in figure 1.4, with more windchill coinciding with higher peak load.

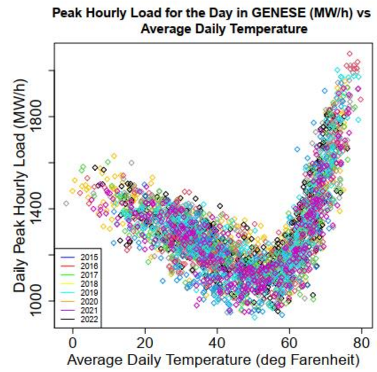

Figure 1.1: Average Temperature vs Demand

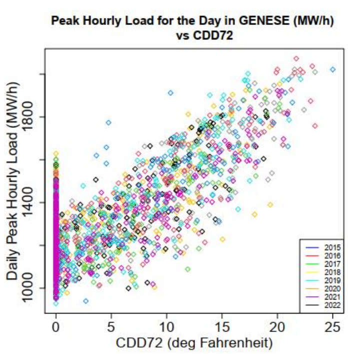

Figure 1.2: CDD vs Demand

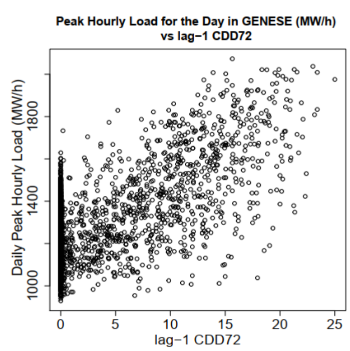

Figure 1.3: lag-1 CDD vs Demand

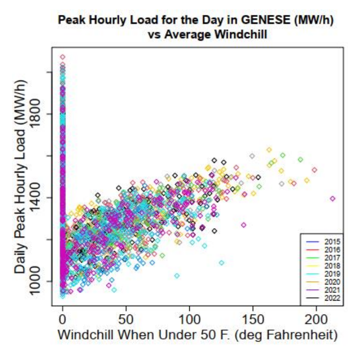

Figure 1.4: Windchill vs Demand

Dummy variables: It is important to account for the impact of holidays, days of the week, and months on electricity demand. we created dummy variables for these and included them in our regression model as additional independent variables to account for any changes in demand that may occur over these time periods. Omitting these dummy variables from the model may result in biased estimates of other variables, such as temperature, HDD, CDD.

Exploration of Hurricanes: The following plots below were adjusted to display two days before and after the disaster event as well as the week prior and after to see any distortion in demand. For hurricane Dorian event in Figure 1.5, the difference is minimal considering the week of the event represented by the black line is at the same level as any other weeks. For the tropical storm Isaias in Figure 1.6, we observed a change in level and decided to impute the day before till four days after. This is done by an average over the same day of the week from the week before and the week after. The result is presented in Figure 1.7 before the change and in Figure 1.8 after the change. The change adjusted the impact of the storm to demands under reasonable days.

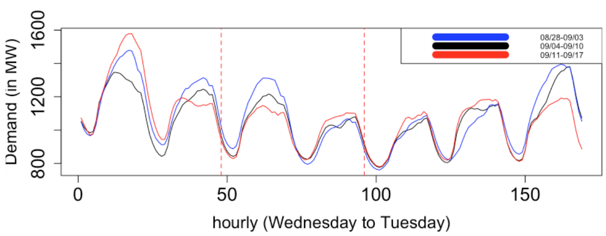

Figure 1.5: hourly demands in MW during Hurricane Dorian on Friday-Saturday September 6-7-2019

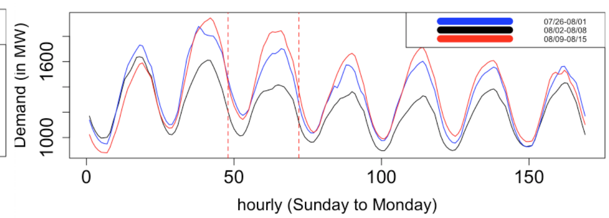

Figure 1.6: hourly demands in MW during Tropical storm Isalas on Tuesday August4, 2020

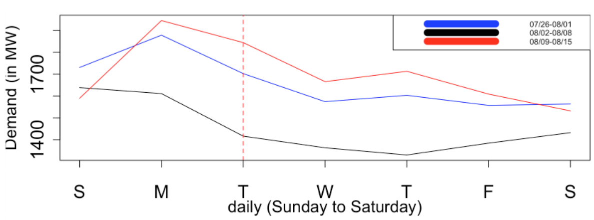

Figure 1.7: peak hourly load for the day in MW during Tropical storm Isalas on Tuesday August 4, 2020

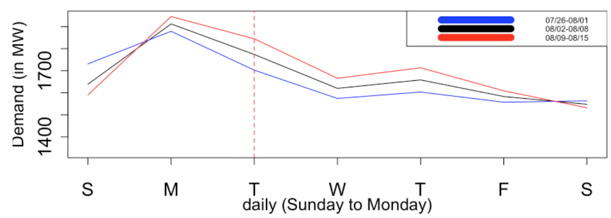

Figure 1.8: peak hourly load for the day in MW during Tropical storm Isalas on Tuesday August 4, 2020 (imputed)

Previous table on naïve forecast methods result has been updated with the corrected data. It didn't affect the statements drawn previously.

Table 1:Methods evaluation

Methods	**RMSEMAE**	**MPE**	**MAPE**
Naïve (Random Walk)	115.44 79.65	-0.35	6.03

Seasonal Naïve	184.93	125.97	-0.29	9.41
3-day moving average	140.67	99.60	-0.68	7.58
7-day moving average	146.52	101.49	-0.87	7.66

The training set encompass the years 2015 to 2019 to have a large amount of historical data. Some noticeable differences prior to 2019 are in the months of April with more variances. In the months of June to August, the span of the variances is wider compared to the years after 2019 where it spans from June and July. 2019 marks a change in economic cycle as explain in 1^{st} report.

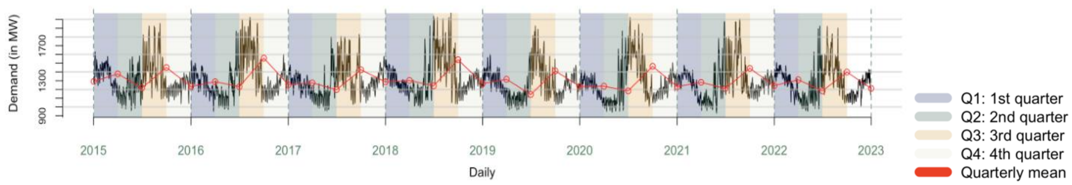

Figure 1.8: Zone B, Peak hourly load for the day in MW (2015-2022)

2. Exponential Smoothing methods

The train and validations set contain respectively 1826 and 731 observations, 2557 combine. The period covering these sets are respectively the years 2015 to 2019 and 2020 to 2021. To perform one day ahead forecast we use two types of time windows approaches. The expanding window starts with the range of 1 to 1826 days. As the day progress, the window expands by 1 until the range covers 1 to 2556 days. The rolling window start with the range of 1 to 1826 days, as the day progress it shifts by 1 until the window covers 731 to 2556 days.

The metric MAPE, mean absolute percentage error, is chosen to compare between all the applied methods for being in percentage and independent of scale. It will be used to calculate the forecast error between the true demand of the validation set and the forecasted demand. As a reminder, the baseline reference is the naïve random walk with MAPE of 6.03%. Any method with lower value than 6.03% are potential candidate. In general, methods with rolling window perform better 6 out of 9 times. When the method considers two seasonality's, alphas are near 0 apart from TBATS with one seasonality. In the table below, for each model, the smaller error of the expanding and rolling methods is in red.

Table 2:Exponential Smoothing

Method	**Expanding (MAPE)**	**Rolling (MAPE)**	**Parameters mean (expanding)**	**Parameters mean (rolling)**
Simple Exponential Smoothing	6.038855	6.038822	$\alpha = 0.9998999$	$\alpha = 0.9998909$
Double Exponential Smoothing	6.0461	6.044839	$\alpha = 0.9943176$, $\beta = 0.0003790701$	$\alpha = 0.9978903$, $\beta = 0.0004869796$
Holt Winter – additive, weekly seasonal	5.404732	5.356203	$\alpha = 0.9741401$, $\beta = 0.0006335961$, $\gamma = 0.002203585$	$\alpha = 0.9603045$, $\beta = 0.0008077293$, $\gamma = 0.002155232$
Holt Winter – multiplicative, weekly seasonal	5.290822	5.290523	$\alpha = 0.9415166$, $\beta = 0.008147679$, $\gamma = 0.01234294$	$\alpha = 0.9421597$, $\beta = 0.004188635$, $\gamma = 0.01553615$
Holt Winter – additive dampening, weekly seasonal	5.41429	5.391686	$\alpha = 0.9740042$, $\beta = 0.0001800881$, $\gamma = 0.002654317$	$\alpha = 0.9581758$, $\beta = 0.0001537726$, $\gamma = 0.002987332$
Holt Winter – multiplicative damp., weekly seasonal	5.294345	5.24684	$\alpha = 0.9767947$, $\beta = 0.000931776$, $\gamma = 0.004019256$	$\alpha = 0.9592686$, $\beta = 0.001046285$, $\gamma = 0.005204863$
Holt Winter – double seasonal (m1=7, m2=364)	5.228072	5.396669	$\alpha = 0.01956922$, $\beta = 0.02782218$ $\gamma = 0.05919245$	$\alpha = 0.06438023$, $\beta = 0.04343254$, $\gamma = 0.0822278$
TBATS (fit 7 days) (m1=7, m2=365.25)	**4.902572**	6.614688 (issue corrected)	$\alpha = 0.07965504$, $p = 2.268126$, $q = 2.076607$	$\alpha = 0.07915779$, $p = 1.915185$, $q = 1.811218$
TBATS (fit 7 days) (m1=7)	5.007801	5.910119	$\alpha = 0.07108389$, $p = 2.411765$, $q = 1.268126$	$\alpha = 0.08393824$, $p = 1.880985$, $q = 1.521204$

Simple Exponential Smoothing: This is the simplest method available involving forecasts generation for the following day by calculating a weighted average of the previous days. However, since the average alpha value is 0.9999 across 731 iterations, the forecast for the next day is nearly indistinguishable from that of the previous day, which is like the naive approach. As this method does not consider any trends or seasonal patterns, it may not perform as effectively as more advanced techniques. Its MAPE score on the validation set is 6.038%.

Exponential Smoothing (Holt): This method is based upon the simple exponential smoothing by incorporating a trend in the model. However, this is only advantageous when the data clearly exhibits a trend, which is not the case for our data. In fact, the inclusion of the trend in our model resulted in poorer performance, with a MAPE of 6.044%, which is lower than that of the single exponential smoothing. Additionally, the average alpha and beta values across all runs are 0.9978 and 0.000486, respectively, indicating that the trend is not heavily emphasized, but still enough to lead to marginally worse forecasts. Hence, the one day ahead forecasts are like the previous day.

Holt Winter: Holt-Winters' method capture the level, the trend, and the seasonal component of the timeseries. The period of seasonality chosen is weekly. This is based on the exploration analysis where we saw a demand pattern by the day of the week. Here, we present four variations of this method such as Additive, Multiplicative, Additive with dampening, and Multiplicative with dampening. The latter, being the best among with a MAPE of 5.25%. The MAPEs range from 5.25% to 5.41%.

Double Seasonal Holt Winter: Holt Winter double seasonal method permits the use of an additional seasonality. It must nest with a seasonality being a multiplicative integer of the other. Here m_2 being 364 for yearly. The expanding method performed better with a MAPE of 5.23%. Within the Holt Winters category, it is the best performing method. The added cost is computing runtime. On an m1 mac, Holt Winter Multiplicative with dampening method is MAPE of 5.25% taking 2.28 minutes to run whereas for the double seasonal method, it takes around 2 hours. The difference between their MAPE is minimal at 0.02%, but the gap between the runtime is not negligible and should be a factor in the decision process.

TBATS Model: In the TBATS model, we incorporated weekly (m1=7) and yearly (m2=365.25) seasonality components to capture the complex seasonal patterns in the electricity demand data. The model parameters involved a Box-Cox transformation with an ARMA structure on the errors. It is refitted every week and updated daily with newly observed data. It is to keep the same estimated parameters to forecast the daily peak demand for the same week. In addition, it requires less time to refit the model than daily. It takes an average of 20 minutes to compute.

Based on the forecasts, compared to the double seasonal Holt Winter method, TBATS handles more effectively seasonal complexities and exhibited superior performance among other smoothing models on the validation set. The model achieved a MAPE of 4.9% on the validation set. However, the primary disadvantage of the TBATS model is the time required for training, resulting in slower forecast estimates than other smoothing alternatives.

In terms of seasonality, we tested this model in two cases. Once considering only the weekly seasonality, and once the weekly and yearly seasonality. The comparison shows that considering the annual seasonality in addition to the weekly seasonality improves the performance of the model and reduces the MAPE error from 5.007% to 4.902%.

For the TBATS with two seasonality and rolling window, there was an issue with the forecasted demands where the values were either infinite, negative, or above 10,000 MW. For the problematic forecasts, we use the model fitted from the same week and extended its horizon to reach that targeted day and replaced its value.

3. Regression methods

In this section, we will create a regression model and then analyze its performance to predict electricity demand. The explanatory variables that we used in the regression model are CDD, HDD, lag 1 of CDD and HDD, lag 2 of HDD and CDD, windchill, dummy variables for days of the week, dummy variables for holidays, dummy variables for 1 day ahead and 1 day ahead of holidays and dummy variables for months of the year.

Diagnostics for linearity

The diagnostic plots for the linear regression models, which assume normally distributed errors, revealed that the assumption of non-correlated errors was violated. Specifically, the ACF plot indicated that errors were correlated (figure 3.1). We also conducted a Durbin-Watson test on the residuals and rejected the null hypothesis of non-correlated errors (p-value $=2e^{-16}$). We concluded that the data does not meet the assumptions of a simple linear regression model, so we decided to use a linear regression model with autoregressive errors (ARMA) models.

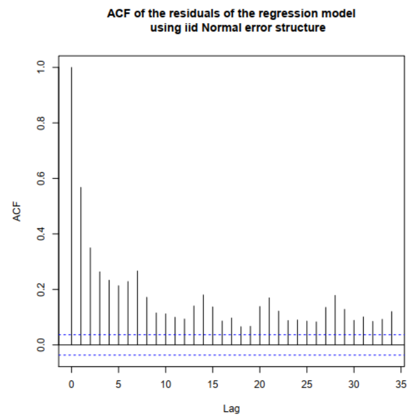

Figure 3.1: ACF of the residuals

Results for ARMA regression model

We have examined this ARMA model, which was chosen due to its valid assumptions and superior overall performance. The R code contains information on the other methods that were explored. The auto.arima function was used, and for the Arima parameters (p,d,q) we found an autoregressive structure on the errors following ARMA(5,0,0). This structure was obtained using both the training and expanding window method. Here is the equation for the fitted:

$$\hat{y}_t = 1220.11 + 0.847 * \mathrm{HDD} + 24.945 * \mathrm{CDD} + 0.6051 * \mathrm{lag1HDD} + 0.729 * \mathrm{lag2HDD} + 7.622 * \mathrm{lag1CDD} + 2.130 * \mathrm{lag2CDD} + 1.396 * \mathrm{Windchill} - 186.332 * \mathrm{Holiday} - 72.253 *$$

$$\text{lag1Holiday} + 44.267 \ast \text{Jan} + 42.634 \ast \text{Feb} + 41.620 \ast \text{March} + 35.632 \ast \text{April} + 0.352 \ast \text{May} + -104 \ast \text{June} + -88 \ast \text{July} - 43 \ast \text{Aug} - 96 \ast \text{Sep} - 141 \ast \text{Oct} - 177.33 \ast \text{Nov} - 121.83 \ast \text{Mond} - 53 \ast \text{Tues} - 41 \ast \text{Wed} - 65 \ast \text{Thr} + 0 \ast \text{Fri} - 79.072 \ast \text{Sat} + 0.51 e_{t-1} - 0.0311 e_{t-2} + 0.0451 e_{t-3} + 0.0436 e_{t-4} + 0.0436 e_{t-5}$$

Please note that for the sake of clarity in the presentation here, we rounded the values. By confirming that the assumptions for using this model were met, as shown by the satisfactory ACF plot (figure 3.2), we were able to proceed with using this model for forecasting. In this regard, residuals plot for HDD, CDD, Windchill are shown in figures 3.6 to 3.8 which confirms their constant deviation and zero mean. This is the case also for residual boxplots for dummy variables shown in figures 3.3 to 3.5. Our model outperformed the benchmark naïve method and all other methods discussed up to now, achieving a MAPE of 2.89%.

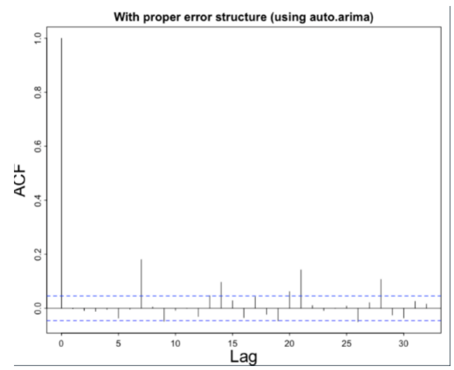

Figure 3.2: ACF of the residuals

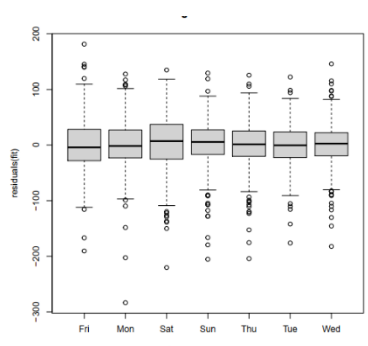

Figure 3.3: Boxplot of the residuals for Days of week

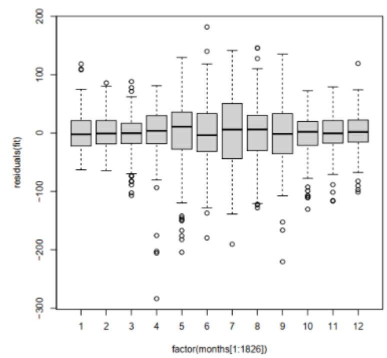

Figure 3.4: Boxplot of the residuals for months

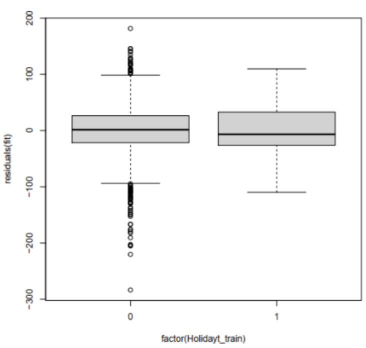

Figure 3.5: Boxplot of the residuals for Holidays

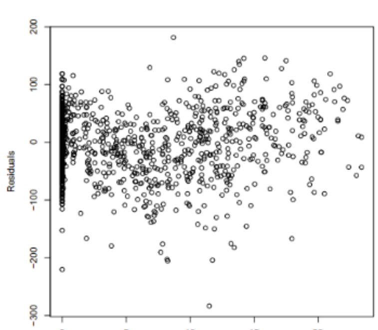

Figure 3.6: residuals for HDD

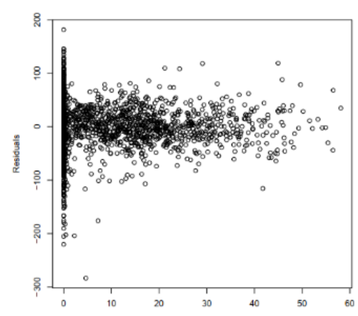

Figure 3.7: residuals for CDD

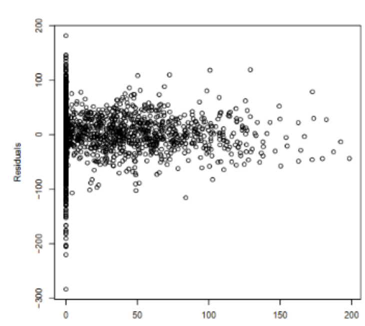

Figure 3.8: residuals for Windchill

Figure 3.9 shows the actual observed values (validation data) in black, the forecast along with a 95% confidence interval in figure3.9.a and a 80% confidence interval in figure3.9.b both shown in red colour ,respectively.

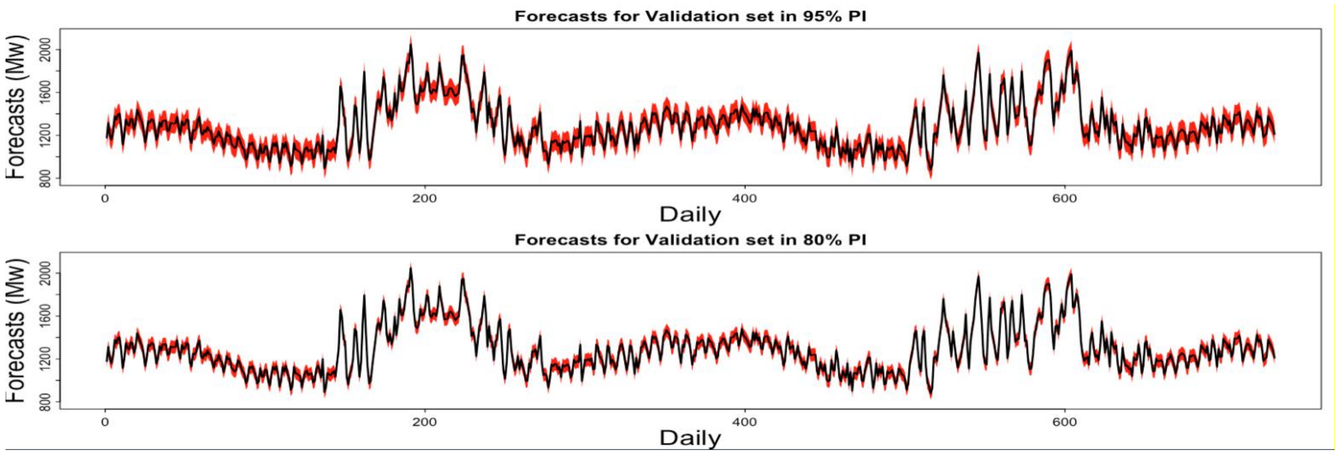

Figure 3.9: ARMA(5,0,0) Regression forecasts or validation set with a) CI=95% and b) CI=80%

We forecasted value the MAPE is justified because the forecasted value is within the prediction intervals.

The achieved MAPE 2.89% is justified because the forecasted value is within the prediction intervals. If the forecasted values are within the confidence intervals, this is a good indication that the forecast is reasonably accurate and that the prediction intervals provide a measure of uncertainty around the forecasted values. This information can be used to make informed decisions based on the forecast, and to communicate the level of uncertainty to stakeholders.

Conclusion

Table 2 summarizes the best methods for each category. ARMA performs the best with a MAPE of 2.88%, a difference of 2.02% over TBATS.

Table 2: Best methods for each category

Category	Methods	MAPE
Naïve	Random walk	6.03%
Smoothing	TBATS (m1=7, m2=365.25)	4.90%
Regression	ARMA (5, 0, 0)	2.88%

References

[1] List of New York hurricanes. (2023). In Wikipedia.

https://en.wikipedia.org/w/index.php?title=List_of_New_York_hurricanes&oldid=1141951419