Lecture

Boolean Algebra and Logic Simplification

Boolean Algebra

- Variable: A variable is a symbol usually an uppercase letter used to represent a logical quantity.
  - A variable can have a 0 or 1 value.

- Complement: A complement is the inverse of a variable and is indicated by a bar over the variable.
  - Complement of variable X is X*
  - If X = 0 then X*= 1 and if X = 1 then X*= 0

- Literal: A Literal is a variable or the complement of a variable. X and X* are literals

Boolean Addition & Multiplication

- Boolean Addition performed by OR gate
- Sum Term describes Boolean Addition

- Boolean Multiplication performed by AND gate
- Product Term describes Boolean Multiplication

Boolean Addition

- Sum of literals

$$A + B \quad A + \overline{B} \quad \overline{A} + \overline{B} + C$$

- Sum term = 1 if any literal = 1
- Sum term = 0 if all literals = 0

Boolean Multiplication

- Product of literals

$$A.B \qquad A.\overline{B} \qquad \overline{A}.\overline{B}.C$$

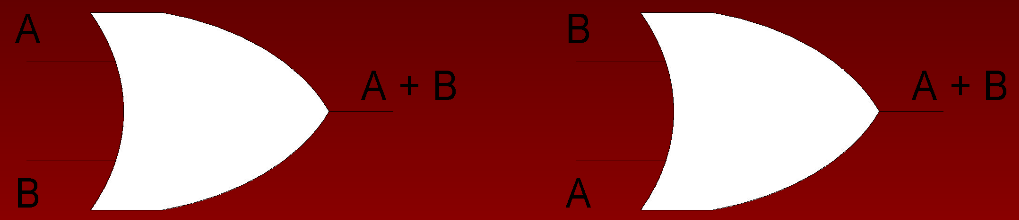
- Product term = 1 if all literals = 1
- Product term = 0 if any one literal = 0

Laws, Rules & Theorems of Boolean Algebra

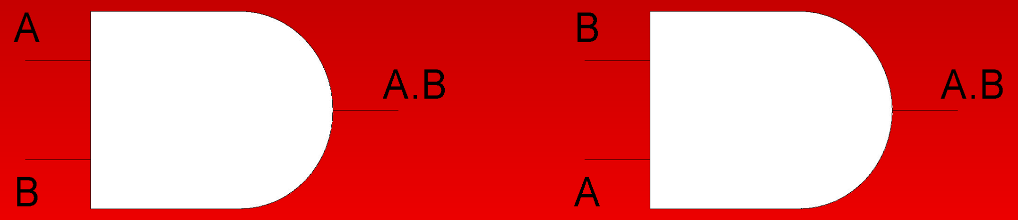
- Commutative Law
  for addition and multiplication
- Associative Law
  for addition and multiplication
- Distributive Law
- Rules of Boolean Algebra
- Demorgan's Theorems

Commutative Law

- Commutative Law for Addition
  A + B = B + A

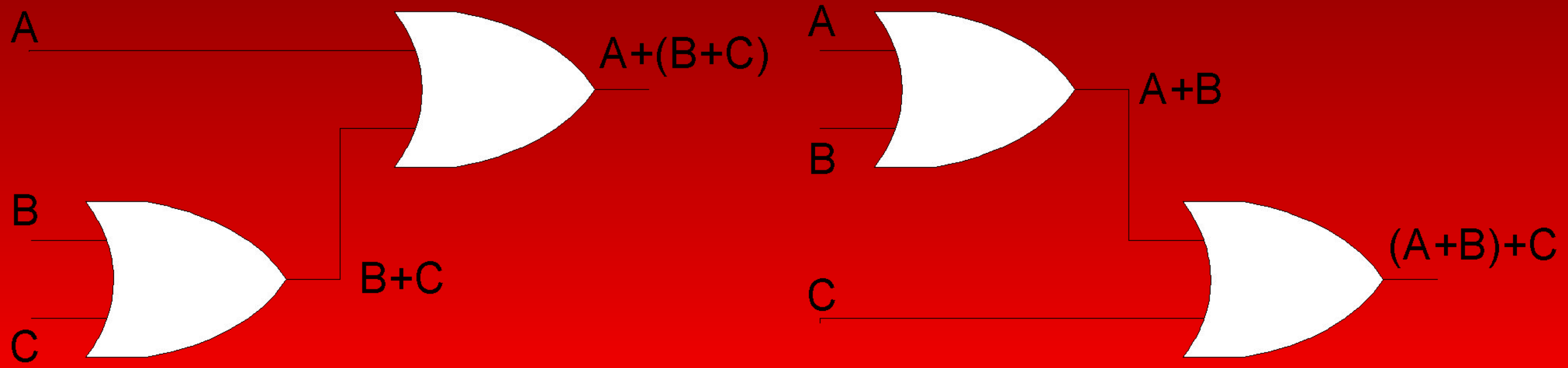


- Commutative Law for Multiplication
  A.B = B.A



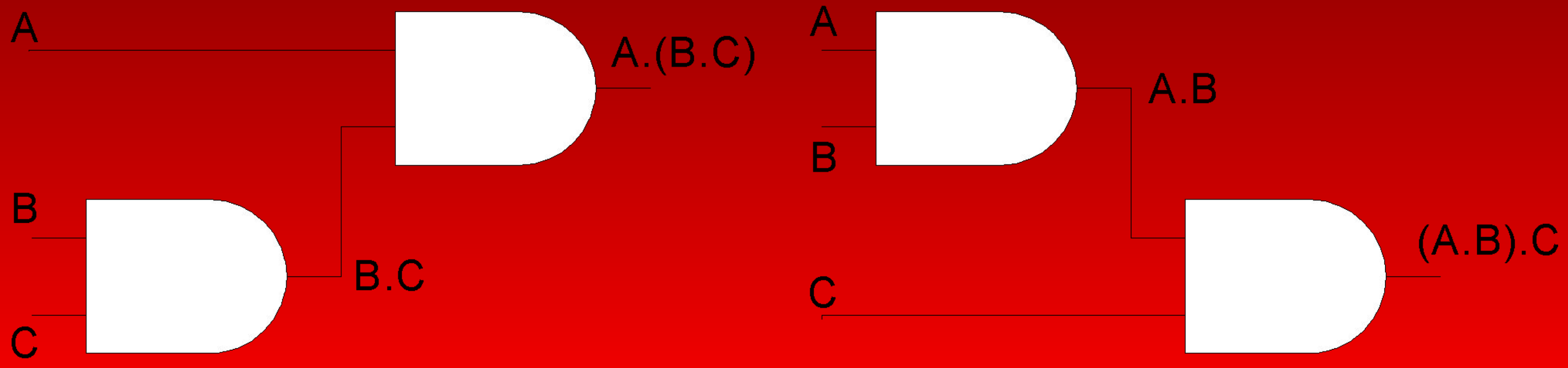
Associative Law

- Associative Law for Addition
  $A + (B + C) = (A + B) + C$



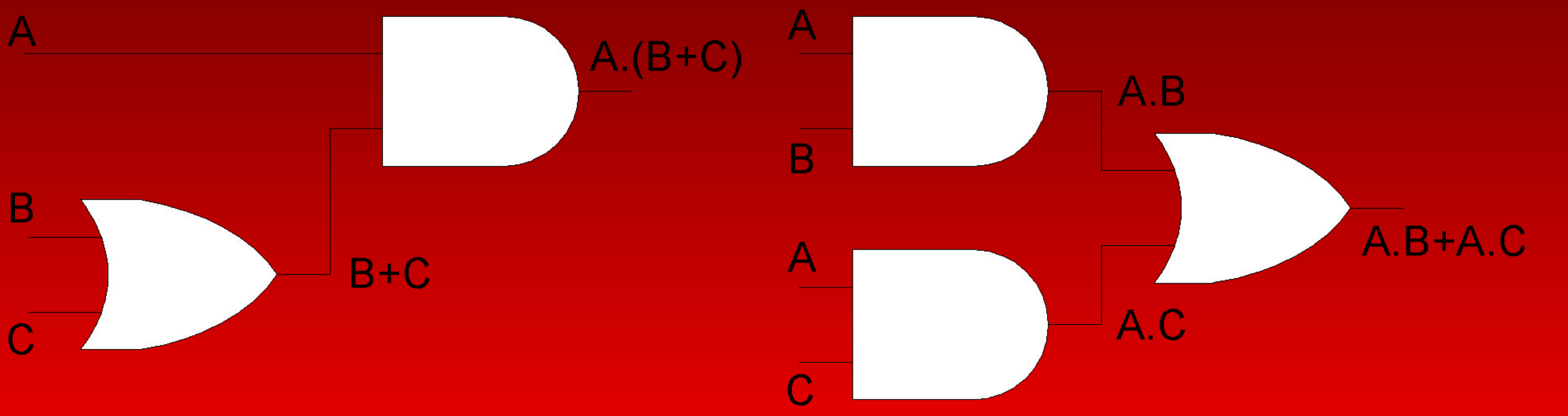
Associative Law

- Associative Law for Multiplication
  A.(B.C) = (A.B).C



Distributive Law

A.(B + C) = A.B + A.C



Rules of Boolean Algebra

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A.0 = 0$
4. $A.1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A.A = A$
8. $A.\overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + A.B = A$
11. $A + \overline{A}.B = A + B$
12. $(A+B).(A+C) = A+B.C$

Demorgan's Theorems

- First Theorem

$$\overline{A.B} = \overline{A} + \overline{B}$$

- Second Theorem

$$\overline{A + B} = \overline{A}.\overline{B}$$

Demorgan’s Theorems

- Any number of variables

$$\overline{X.Y.Z} = \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{X + Y + Z} = \overline{X}.\overline{Y}.\overline{Z}$$

- Combination of variables

$$\overline{(A + B.C).(A.C + B)} = \overline{(A + B.C)} + \overline{(A.C + B)}$$

$$= \overline{A}.\overline{(B.C)} + \overline{(A.C)}.\overline{B} = \overline{A}.(\overline{B} + \overline{C}) + (\overline{A} + \overline{C}).\overline{B}$$

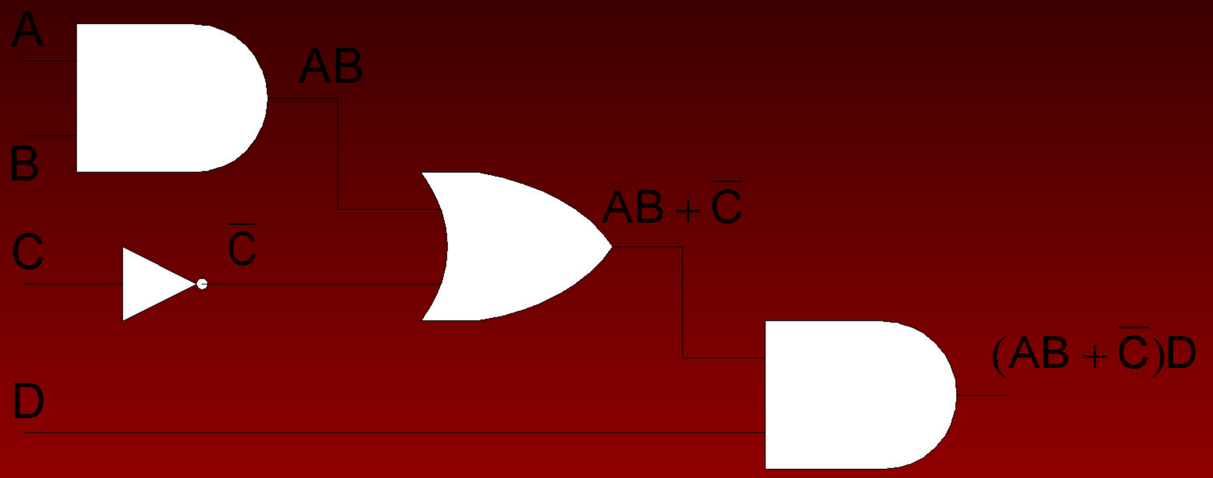
$$= \overline{A}.\overline{B} + \overline{A}.\overline{C} + \overline{A}.\overline{B} + \overline{B}.\overline{C}$$

$$= \overline{A}.\overline{B} + \overline{A}.\overline{C} + \overline{B}.\overline{C}$$

Boolean Analysis of Logic Circuits

- Boolean Algebra provides concise way to represent operation of a logic circuit
- Complete function of a logic circuit can be determined by evaluating the Boolean expression using different input combinations

Boolean Analysis of Logic Circuits



- From the expression, the output is a 1 if variable D = 1 and $(AB+\overline{C})=1$
- $(AB+\overline{C})=1$ if AB=1 or C=0

Boolean Analysis of Logic Circuits

Inputs				Output	Inputs				Output
A	B	C	D	F	A	B	C	D	F
0	0	0	0	0	1	0	0	0	0
0	0	0	1	1	1	0	0	1	1
0	0	1	0	0	1	0	1	0	0
0	0	1	1	0	1	0	1	1	0
0	1	0	0	0	1	1	0	0	0
0	1	0	1	1	1	1	0	1	1
0	1	1	0	0	1	1	1	0	0
0	1	1	1	0	1	1	1	1	1

Simplification using Boolean Algebra

Simplification using Boolean Algebra

- $AB + A(B+C) + B(B+C)$

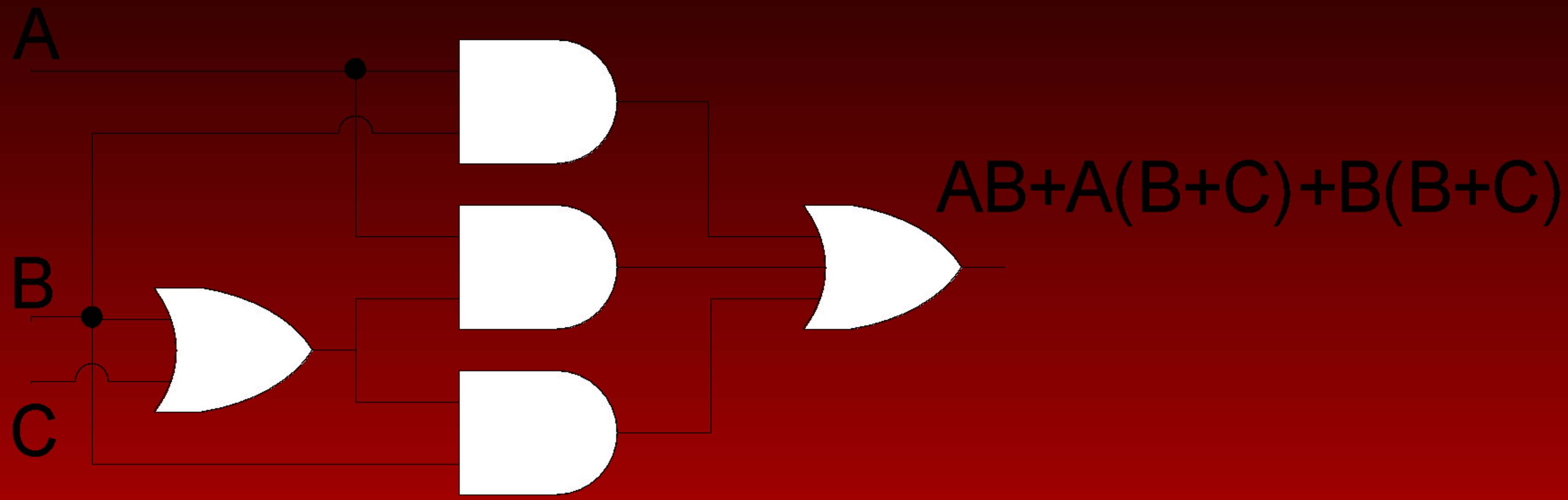
  $= AB + AB + AC + BB + BC$

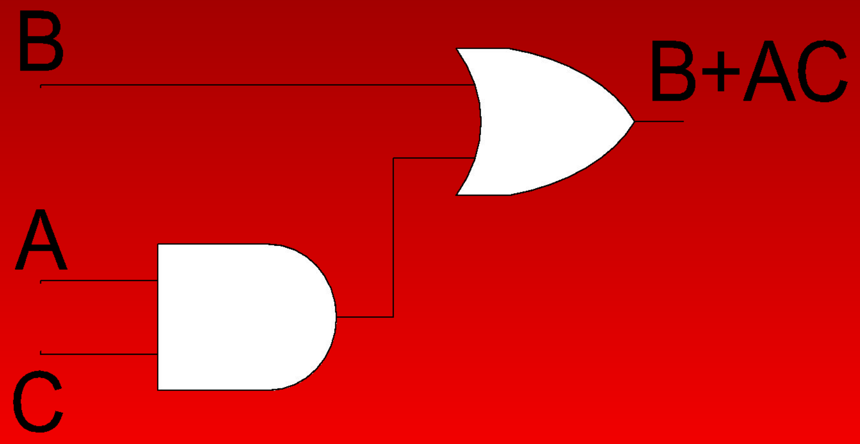
  $= AB + AC + B + BC$

  $= AB + AC + B$

  $= B + AC$

Simplified Circuit





Standard forms of Boolean Expressions

- Sum-of-Products form (Sum of Minterm)
  - When two or more product terms are summed by Boolean addition, the result is a Sum-of-Product or SOP expression
- Product-of-Sums form (Product of Maxterm)
  - When two or more sum terms are multiplied by Boolean multiplication, the result is a Product-of-Sum or POS expression

Standard forms of Boolean Expressions

- Sum-of-Products form

$$AB + ABC$$

$$ABC + CDE + \overline{B}C\overline{D}$$

$$\overline{A}B + \overline{A}B\overline{C} + AC$$

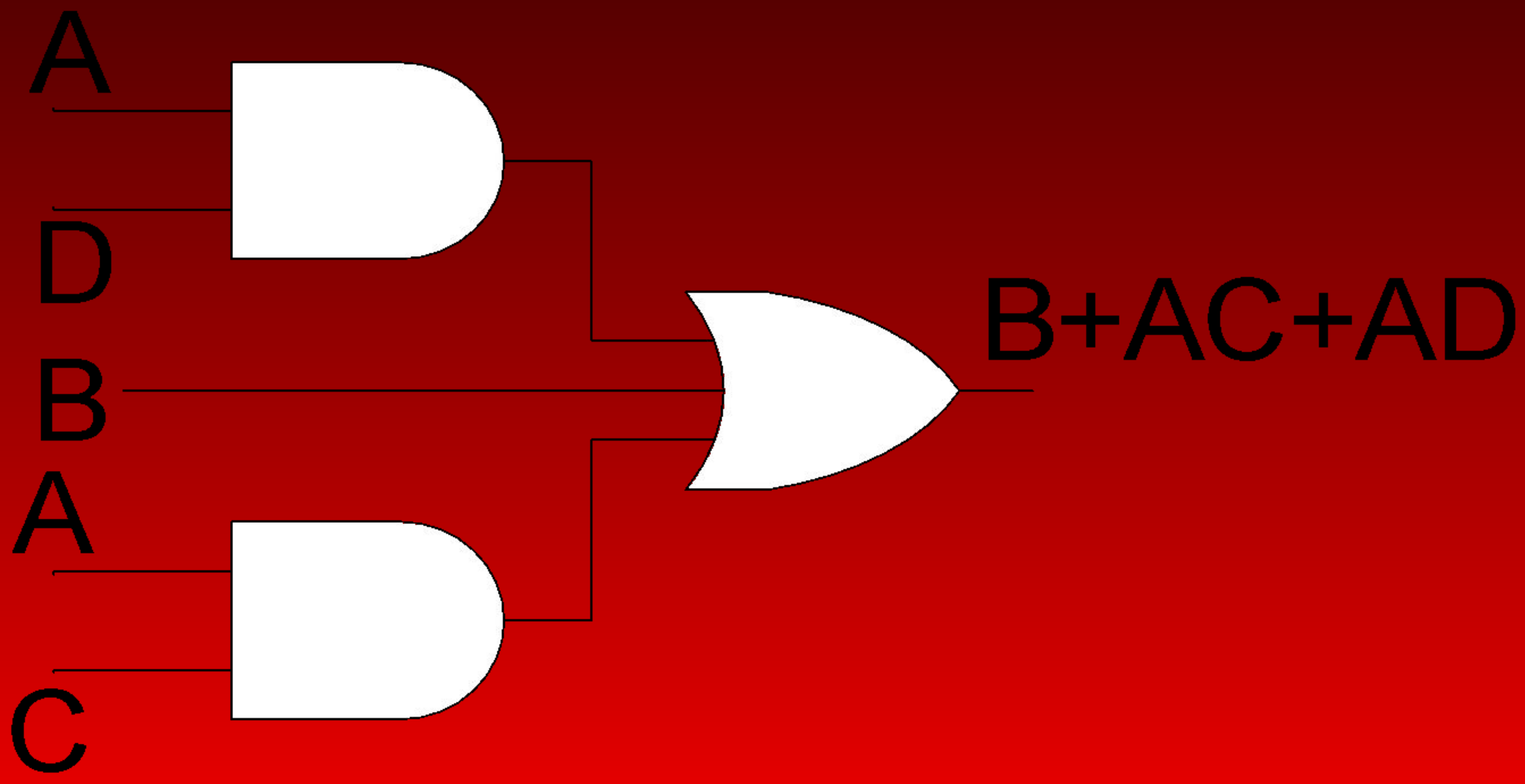
- Product-of-Sums form

$$(\overline{A} + B)(A + \overline{B} + C)$$

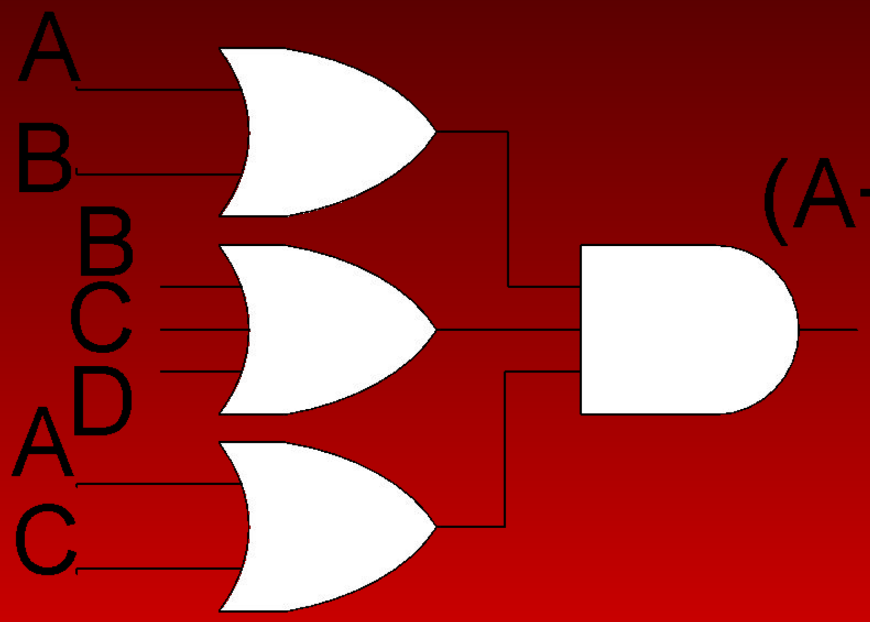
$$(\overline{A} + \overline{B} + \overline{C})(C + \overline{D} + E)(\overline{B} + C + D)$$

$$(A + B)(A + \overline{B} + C)(\overline{A} + C)$$

Implementation of SOP expression



Implementation of POS expression



$(A+B)(B+C+D)(A+C)$

Conversion of general expression to SOP form

$$AB + B(CD + EF) = AB + BCD + BEF$$

$$(A + B)(B + C + D) = AB + AC + AD + B + BC + BD$$

$$= AC + AD + B$$

$$\overline{\overline{(A + B)} + C} = \overline{\overline{(A + B)}}\,\overline{C} = (A + B)\overline{C} = A\overline{C} + B\overline{C}$$