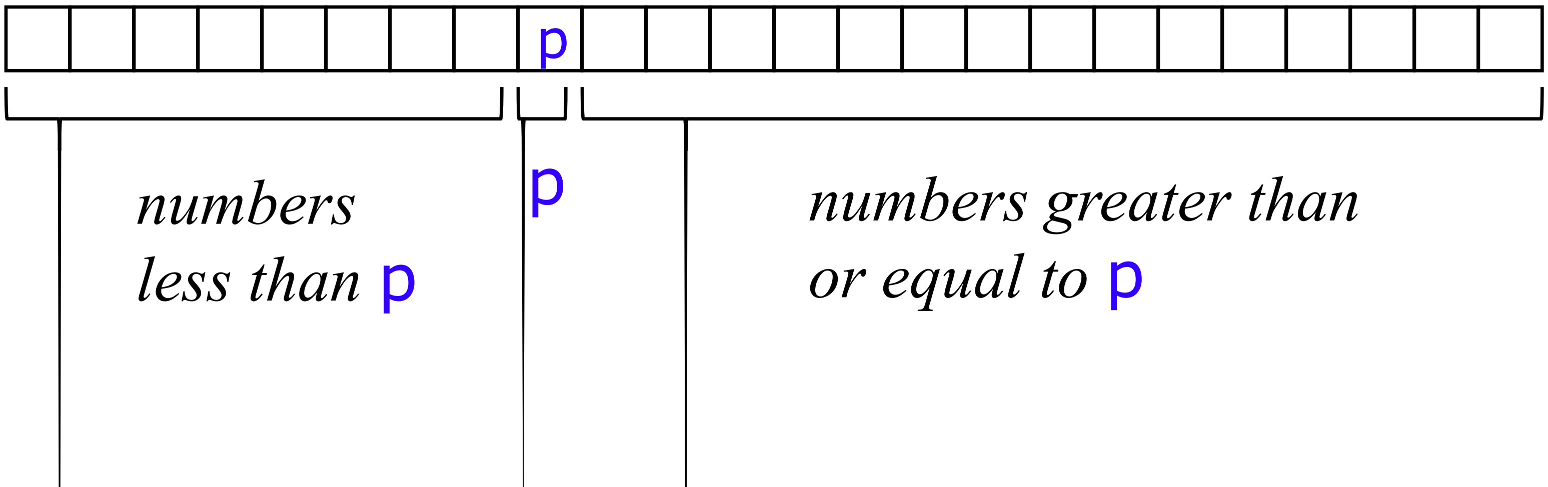
Quicksort

Quicksort I: Basic idea

- Pick some number p from the array
- Move all numbers less than p to the beginning of the array
- Move all numbers greater than (or equal to) p to the end of the array
- Quicksort the numbers less than p
- Quicksort the numbers greater than or equal to p



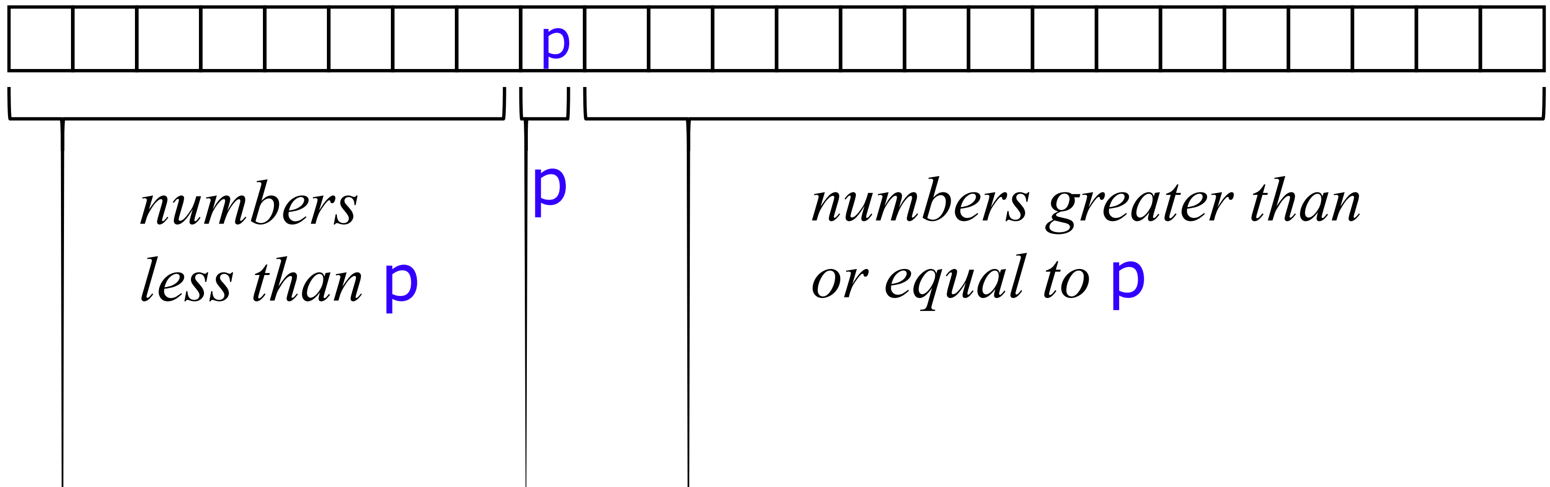
Quicksort II

- To sort a[left...right]:

1. if left < right:
    1.1. Partition a[left...right] such that:
        all a[left...p-1] are less than a[p], and
        all a[p+1...right] are >= a[p]
    1.2. Quicksort a[left...p-1]
    1.3. Quicksort a[p+1...right]
2. Terminate

Partitioning (Quicksort II)

- A key step in the Quicksort algorithm is partitioning the array
  - We choose some (any) number p in the array to use as a pivot
  - We partition the array into three parts:



Partitioning II

- Choose an array value (say, the first) to use as the pivot
- Starting from the left end, find the first element that is greater than or equal to the pivot
- Searching backward from the right end, find the first element that is less than the pivot
- Interchange (swap) these two elements
- Repeat, searching from where we left off, until done

Partitioning

- To partition a[left...right]:

1. Set pivot = a[left], l = left + 1, r = right;
2. while l < r, do
   2.1. while l < right & a[l] < pivot , set l = l + 1
   2.2. while r > left & a[r] >= pivot , set r = r - 1
   2.3. if l < r, swap a[l] and a[r]
3. Set a[left] = a[r], a[r] = pivot
4. Terminate

Example of partitioning

- choose pivot: 4 3 6 9 2 4 3 1 2 1 8 9 3 5 6
- search: 4 3 6 9 2 4 3 1 2 1 8 9 3 5 6
- swap: 4 3 3 9 2 4 3 1 2 1 8 9 6 5 6
- search: 4 3 3 9 2 4 3 1 2 1 8 9 6 5 6
- swap: 4 3 3 1 2 4 3 1 2 9 8 9 6 5 6
- search: 4 3 3 1 2 4 3 1 2 9 8 9 6 5 6
- swap: 4 3 3 1 2 2 3 1 4 9 8 9 6 5 6
- search: 4 3 3 1 2 2 3 1 4 9 8 9 6 5 6 (left > right)
- swap with pivot: 1 3 3 1 2 2 3 4 4 9 8 9 6 5 6

The partition method (Java)

```
static int partition(int[] a, int left, int right) {
    int p = a[left], l = left + 1, r = right;
    while (l < r) {
        while (l < right && a[l] < p) l++;
        while (r > left && a[r] >= p) r--;
        if (l < r) {
            int temp = a[l]; a[l] = a[r]; a[r] = temp;
        }
    }
    a[left] = a[r];
    a[r] = p;
    return r;
}
```

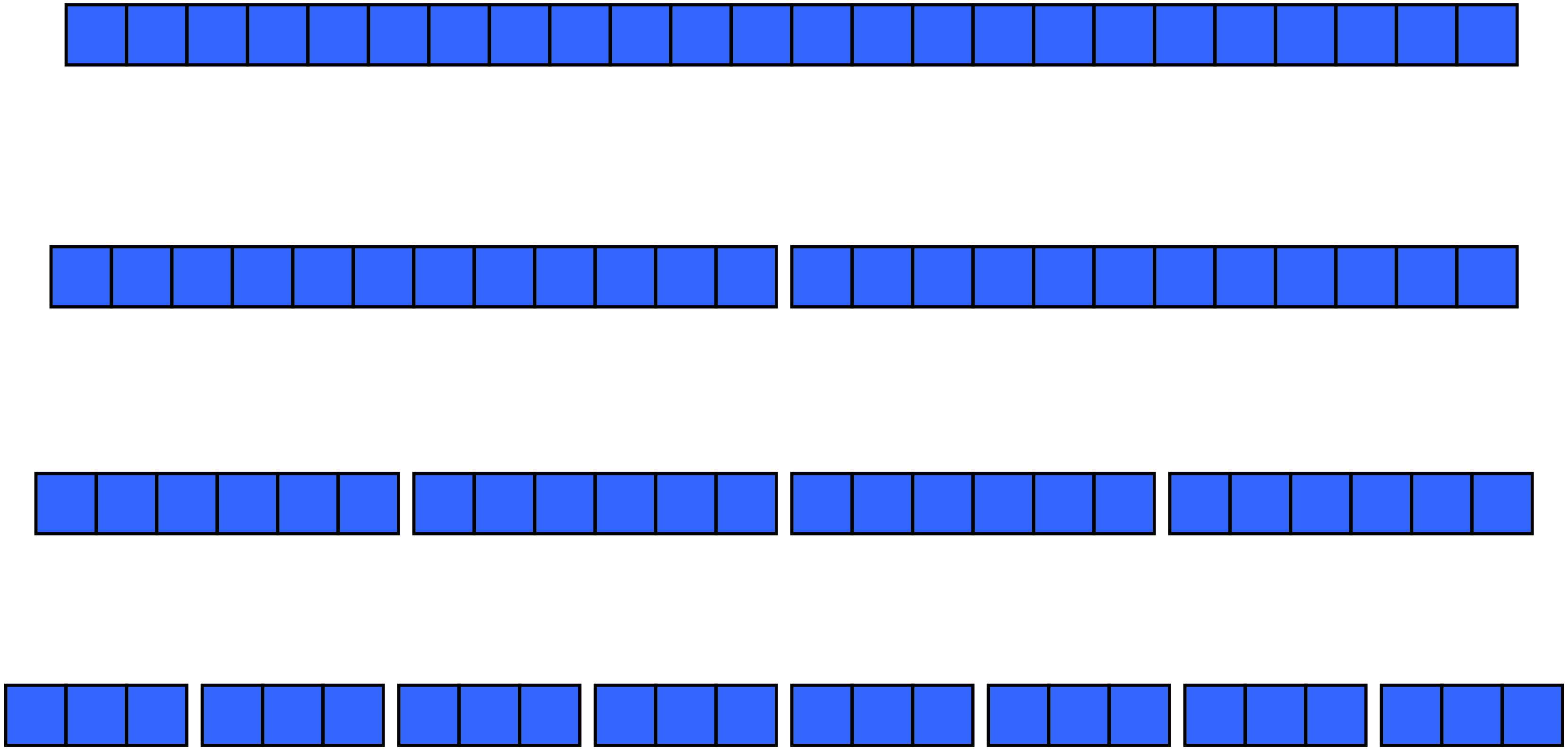
The quicksort method (in Java)

```
static void quicksort(int[] array, int left, int right) {
    if (left < right) {
        int p = partition(array, left, right);
        quicksort(array, left, p - 1);
        quicksort(array, p + 1, right);
    }
}
```

Analysis of quicksort—best case

- Suppose each partition operation divides the array almost exactly in half
- Then the depth of the recursion in $\log_2 n$
  - Because that's how many times we can halve n
- However, there are many recursions!
  - How can we figure this out?
  - We note that
    - Each partition is linear over its subarray
    - All the partitions at one level cover the array

Partitioning at various levels



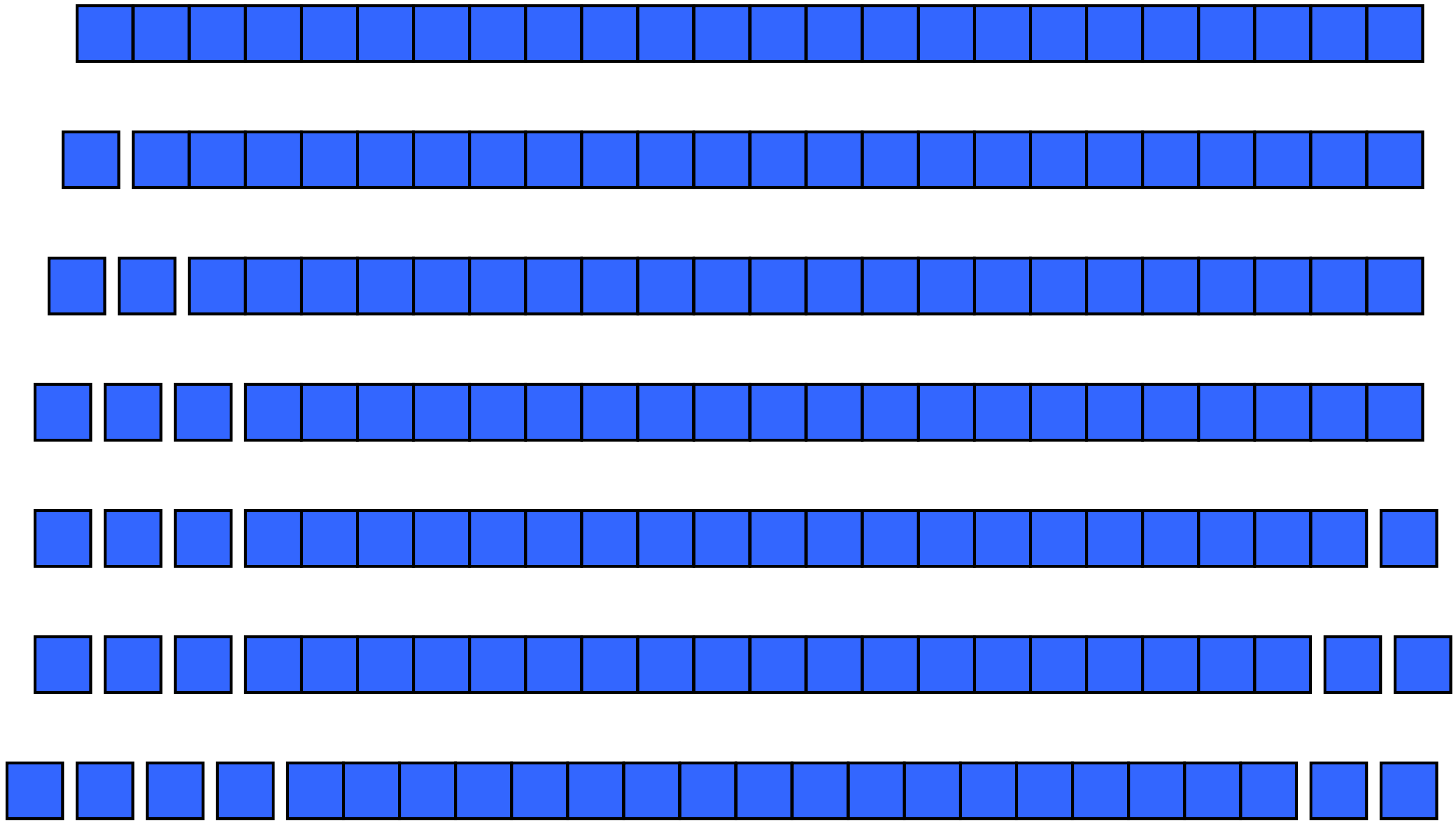
Best case II

- We cut the array size in half each time
- So the depth of the recursion in $\log_2 n$
- At each level of the recursion, all the partitions at that level do work that is linear in n
- $O(\log_2 n) * O(n) = O(n \log_2 n)$
- Hence in the average case, quicksort has time complexity $O(n \log_2 n)$
- What about the worst case?

Worst case

- In the worst case, partitioning always divides the size `n` array into these three parts:
  - A length one part, containing the pivot itself
  - A length zero part, and
  - A length `n-1` part, containing everything else
- We don't recur on the zero-length part
- Recurring on the length `n-1` part requires (in the worst case) recurring to depth `n-1`

Worst case partitioning



Worst case for quicksort

- In the worst case, recursion may be n levels deep (for an array of size n)
- But the partitioning work done at each level is still n
- $O(n) * O(n) = O(n^2)$
- So worst case for Quicksort is $O(n^2)$
- When does this happen?
  - There are many arrangements that *could* make this happen
  - Here are two common cases:
    - When the array is already sorted
    - When the array is *inversely* sorted (sorted in the opposite order)

Typical case for quicksort

- If the array is sorted to begin with, Quicksort is terrible: $O(n^2)$
- It is possible to construct other bad cases
- However, Quicksort is *usually* $O(n \log_2 n)$
- The constants are so good that Quicksort is generally the fastest algorithm known
- Most real-world sorting is done by Quicksort

Improving the interface

- We've defined the Quicksort method as
  `static void quicksort(int[] array, int left, int right) { … }`
- So we would have to call it as
  `quicksort(myArray, 0, myArray.length)`
- That's ugly!
- Solution:

```
static void quicksort(int[] array) {
    quicksort(array, 0, array.length);
}
```

- Now we can make the original (3-argument) version private

Tweaking Quicksort

- Almost anything you can try to “improve” Quicksort will actually slow it down
- One *good* tweak is to switch to a different sorting method when the subarrays get small (say, 10 or 12)
  - Quicksort has too much overhead for small array sizes
- For large arrays, it *might* be a good idea to check beforehand if the array is already sorted
  - But there is a better tweak than this

Picking a better pivot

- Before, we picked the *first* element of the subarray to use as a pivot
  - If the array is already sorted, this results in $O(n^2)$ behavior
  - It's no better if we pick the *last* element
- We could do an *optimal* quicksort (guaranteed $O(n \log n)$) if we always picked a pivot value that exactly cuts the array in half
  - Such a value is called a median: half of the values in the array are larger, half are smaller
  - The easiest way to find the median is to *sort* the array and pick the value in the middle (!)

Median of three

- Obviously, it doesn't make sense to sort the array in order to find the median to use as a pivot
- Instead, compare just *three* elements of our (sub)array—the first, the last, and the middle
  - Take the *median* (middle value) of these three as pivot
  - It's possible (but not easy) to construct cases which will make this technique $O(n^2)$
- Suppose we rearrange (sort) these three numbers so that the smallest is in the first position, the largest in the last position, and the other in the middle
  - This lets us simplify and speed up the partition loop

Final comments

- Quicksort is the fastest known sorting algorithm
- For optimum efficiency, the pivot must be chosen carefully
- "Median of three" is a good technique for choosing the pivot
- However, no matter what you do, there will be some cases where Quicksort runs in $O(n^2)$ time

The End