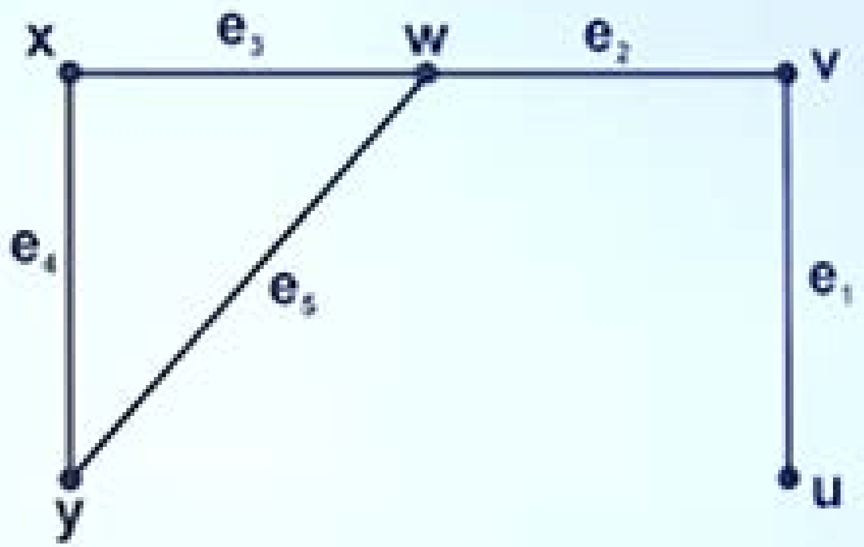
Walks Trail Circuit Paths Cycle

Euler Graph

A path in a graph is said to be an Eulerian path if it traverses each edge in the graph once and only once.

Example :

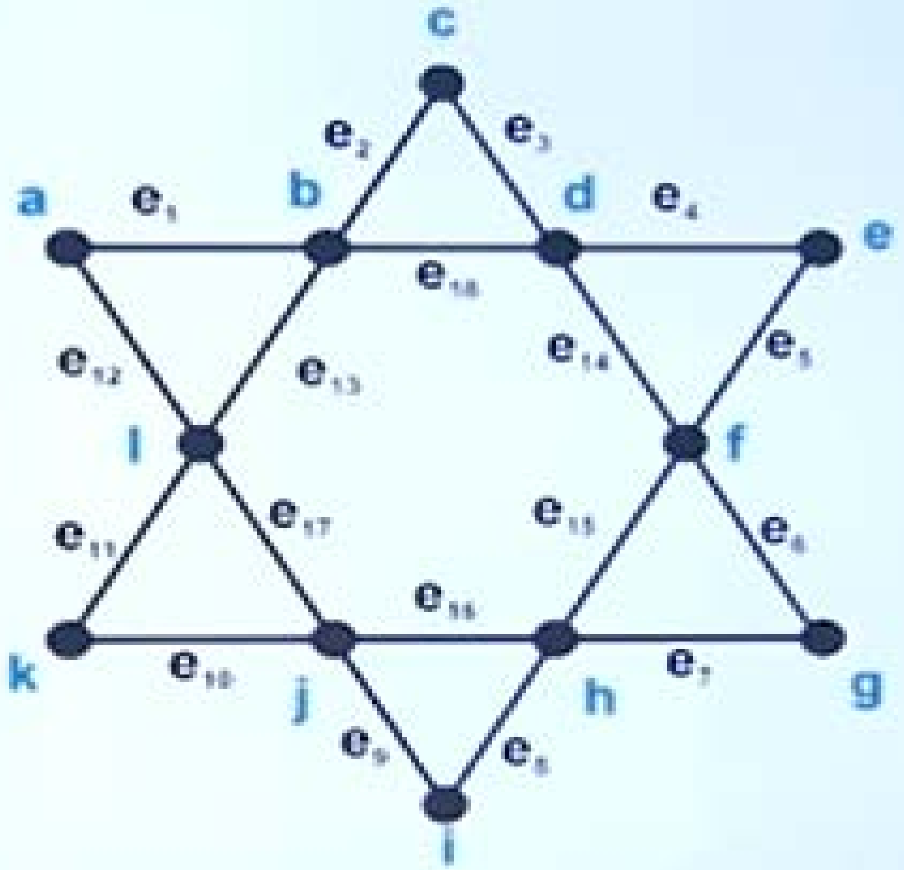


then $ue_1ve_2we_3xe_4ye_5w$

EULERIAN CIRCUIT

A circuit in a graph is said to be an Eulerian circuit if it traverses each edge in the graph once & only once.

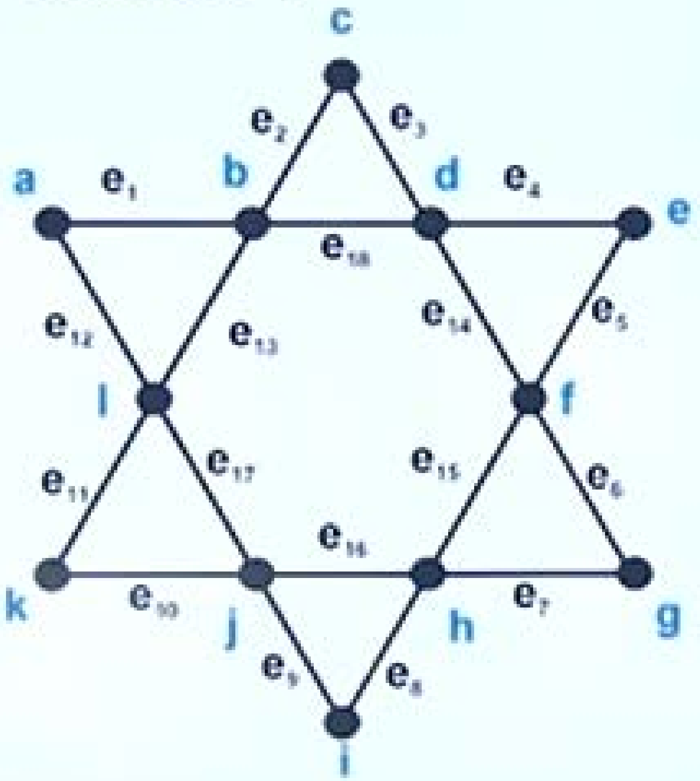
Example :

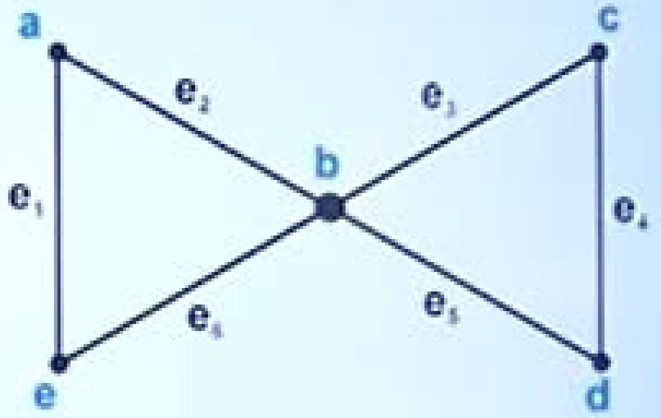


EULERIAN GRAPH

A connected graph which contain an Eulerian circuit is called Eulerian graph..

Example :

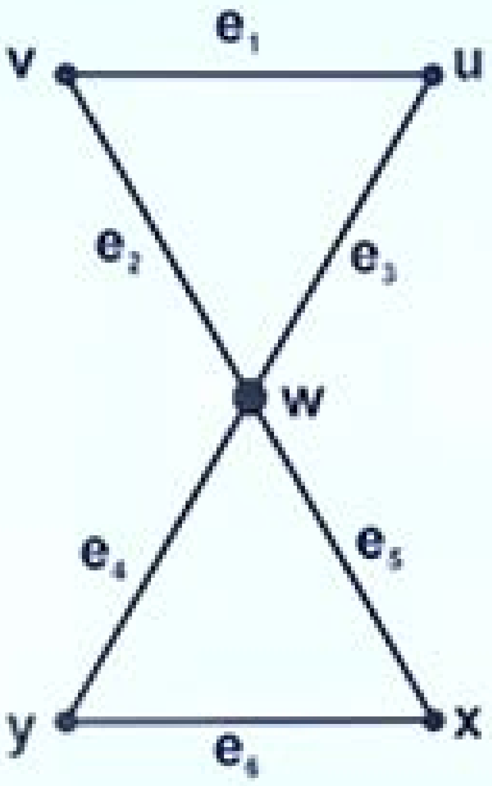




HAMILTONIAN PATH

A path which contain every vertex of a graph G exactly once is called Hamiltonian Graph.

Example :

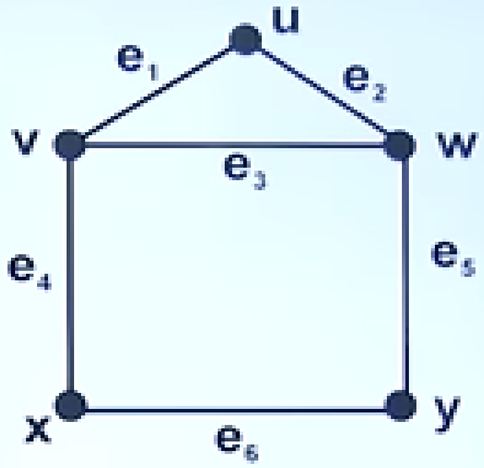


then $ue_1ve_2we_5xe_6y$

HAMILTONIAN CIRCUIT

A circuit that passes through each of the vertices in a group G exactly one except the starting vertex & end vertex is called Hamiltonian circuit.

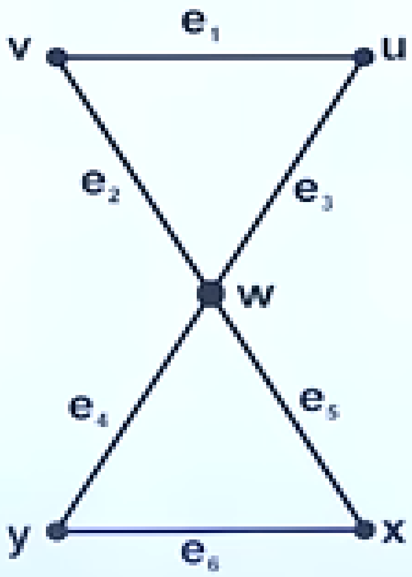
Example :



then

$ue_1ve_4xe_6ye_5we_2u$

but



then

$ue_1ve_2we_5xe_6ye_4we_3u$

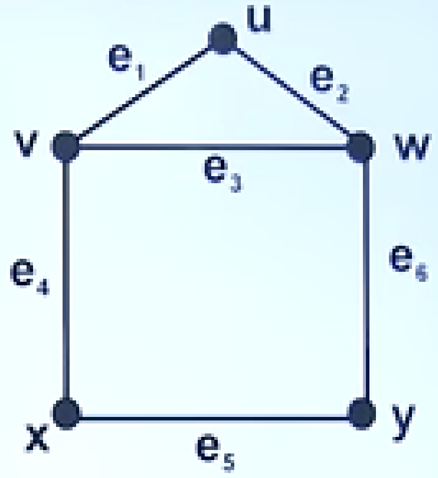
is not Hamiltonian circuit

because w vertex repeat.

HAMILTONIAN GRAPH

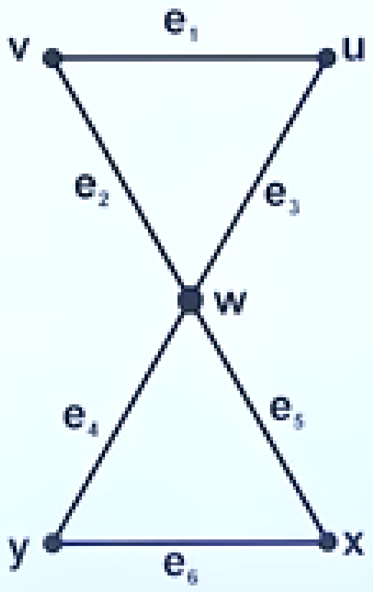
A connected graph which contain Hamiltonial circuit is called Hamiltonian Graph.

Example :



is Hamiltonian Graph

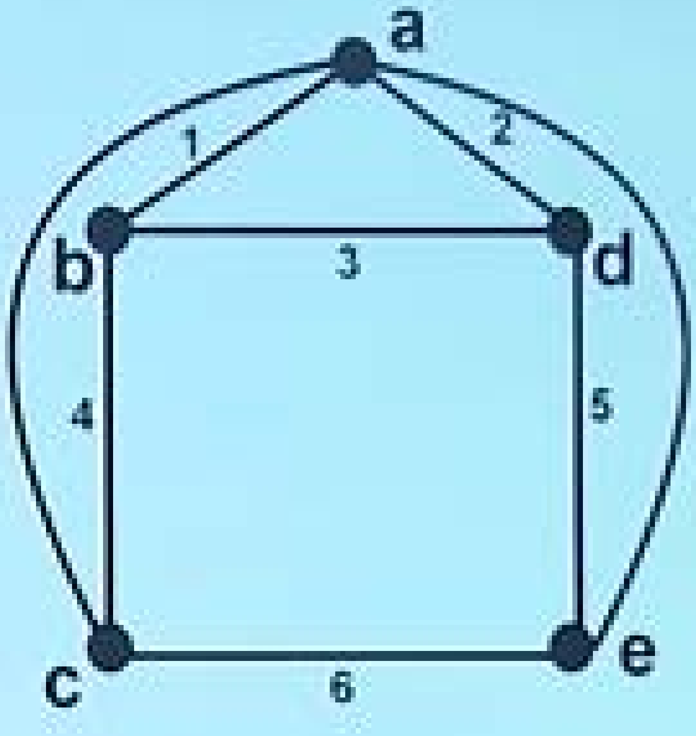
Example :



is not Hamiltonian Graph.

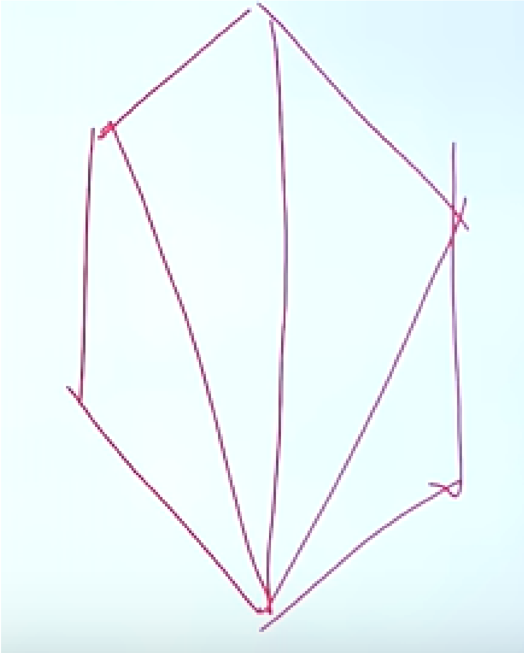
1

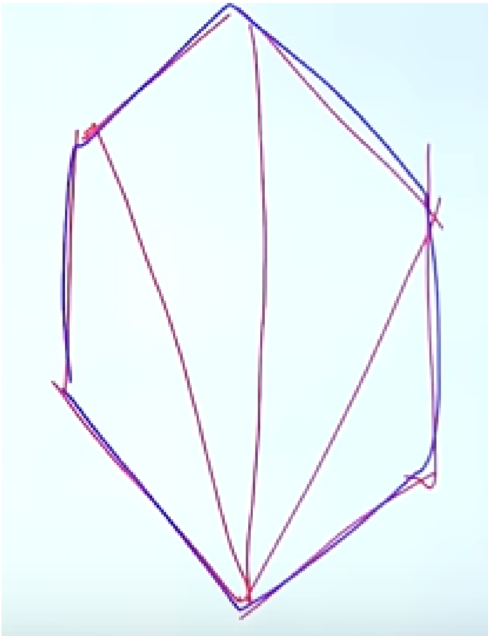
Determine a minimum Hamiltonian circuit for the graph given below.

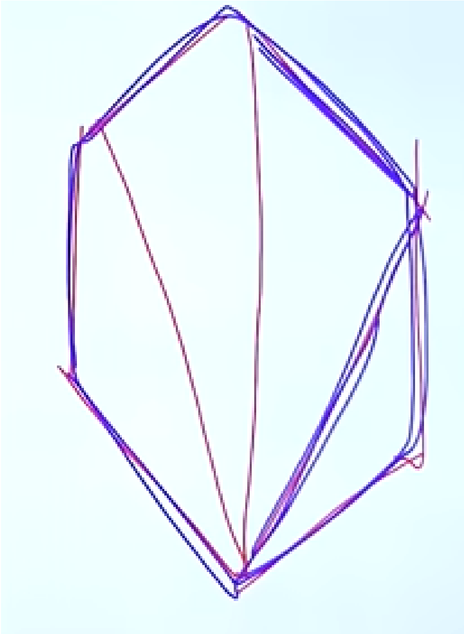


2

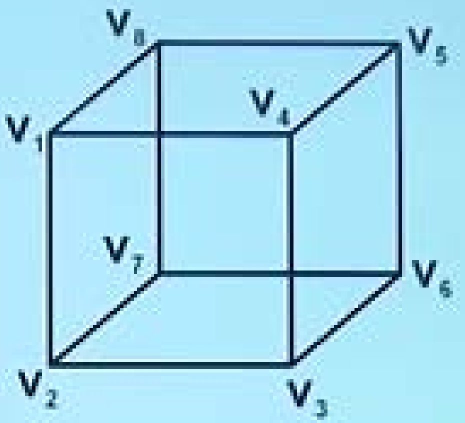
Draw a graph with six vertices containing a Hamiltonian circuit but not Eulerian Circuit.

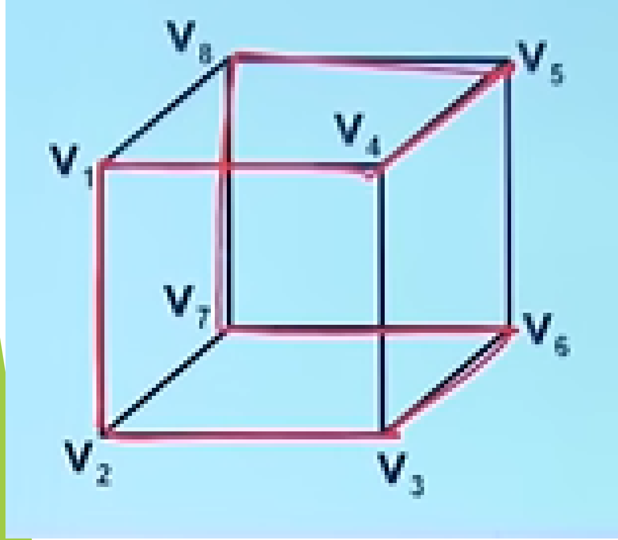


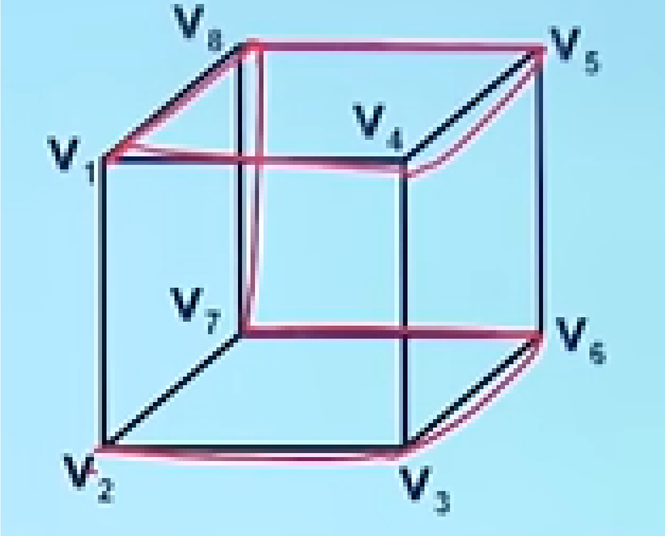




3 Check the graph is Hamiltonian or Euler.

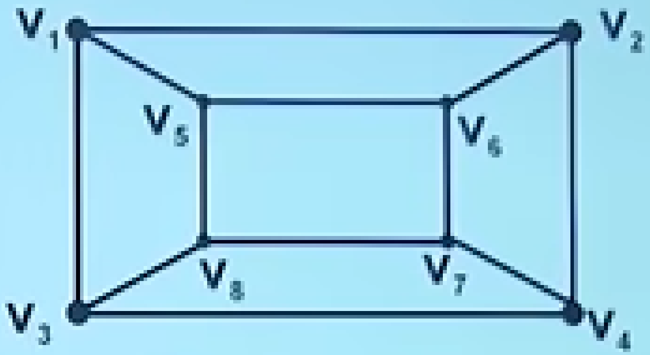






4

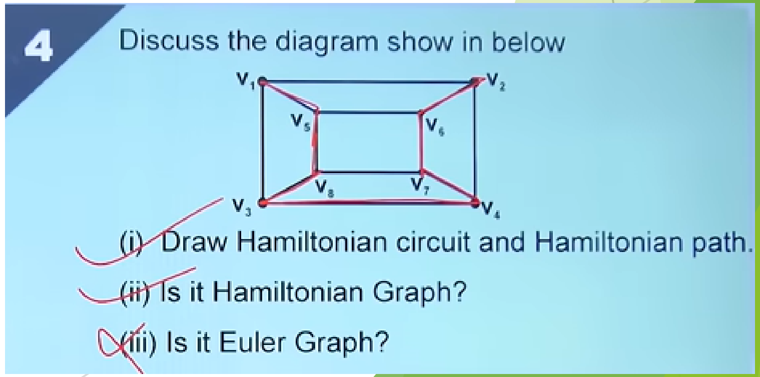
Discuss the diagram show in below



(i) Draw Hamiltonian circuit and Hamiltonian path.

(ii) Is it Hamiltonian Graph?

(iii) Is it Euler Graph?



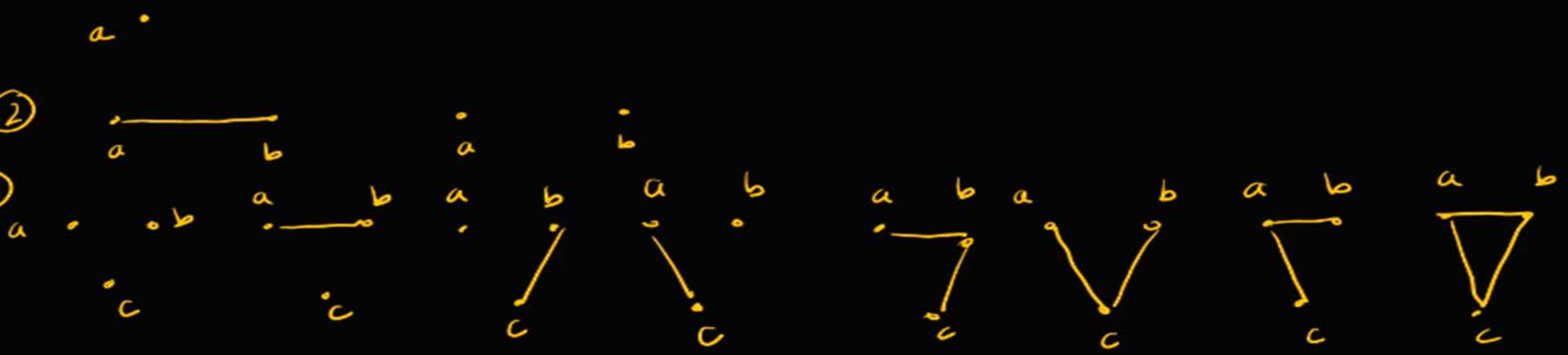
Intro To Graph Isomorphisms

How many graphs exist with n vertices where the vertices are labeled?

1 vertex: 1

2 vertices: 2

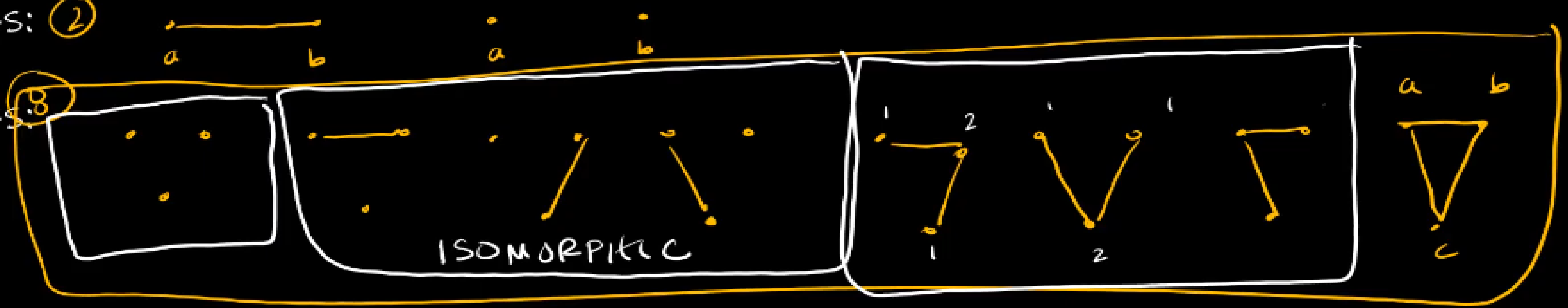
3 vertices: 8



What if we take away the labels?

2 vertices: 2

3 vertices: 8



Graph Isomorphism Definition

This brings us to the concept of isomorphism.

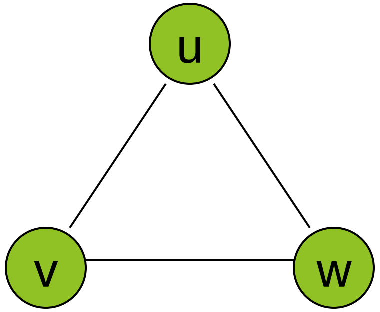
We will look at the "mathy" definition first:

Let $G_1 =< V_1, E_1 >$ and $G_2 =< V_2, E_2 >$. G_1 is isomorphic to G_2 $(G_1 \cong G_2)$ iff
$\exists\, f: V_1 \rightarrow V_2$ where:
i. f is bijective (one-to-one and onto), and
ii. $\forall\, a, b \in V_1$, $\{a, b\} \in E_1$ iff $\{f(a), f(b)\} \in E_2$

This likely doesn't make much sense, yet. Essentially, we are saying that two graphs are isomorphic if they have the exact same structural properties (number of vertices, edges, degree of vertices, etc.) Let's look at an example.

Representation- Adjacency Matrix

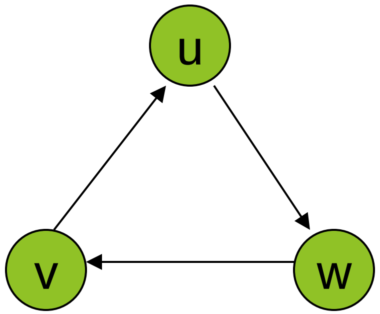
- Example: Undirected Graph G (V, E)



	v	u	w
v	0	1	1
u	1	0	1
w	1	1	0

Representation- Adjacency Matrix

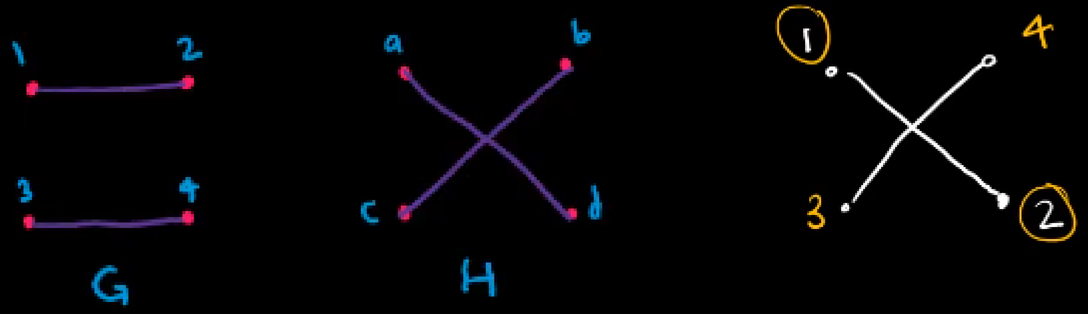
- Example: directed Graph G (V, E)



	v	u	w
v	0	1	0
u	0	0	1
w	1	0	0

Graph Isomorphism Example

Are G and H isomorphic?

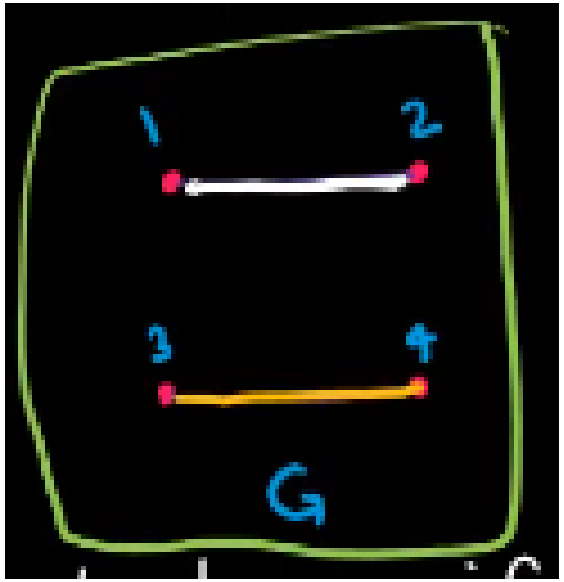


To determine this, we want to know if we can relabel H so that each graph contains the same vertices and the same edge sets. First let's look at G and H:

Graph G: $V = \{1, 2, 3, 4\},\ E = \{12, 34\}$

Graph H: $V = \{a, b, c, d\},\ E = \{ad, bc\}$

So, can I relabel graph H so that the vertex and edge sets match those for G?



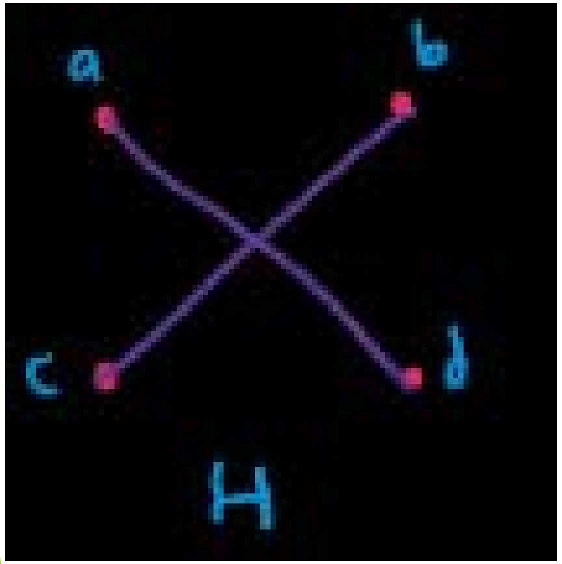
$f(1) = a$

$f(2) = d$

$f(3) = c$

$f(4) = b$

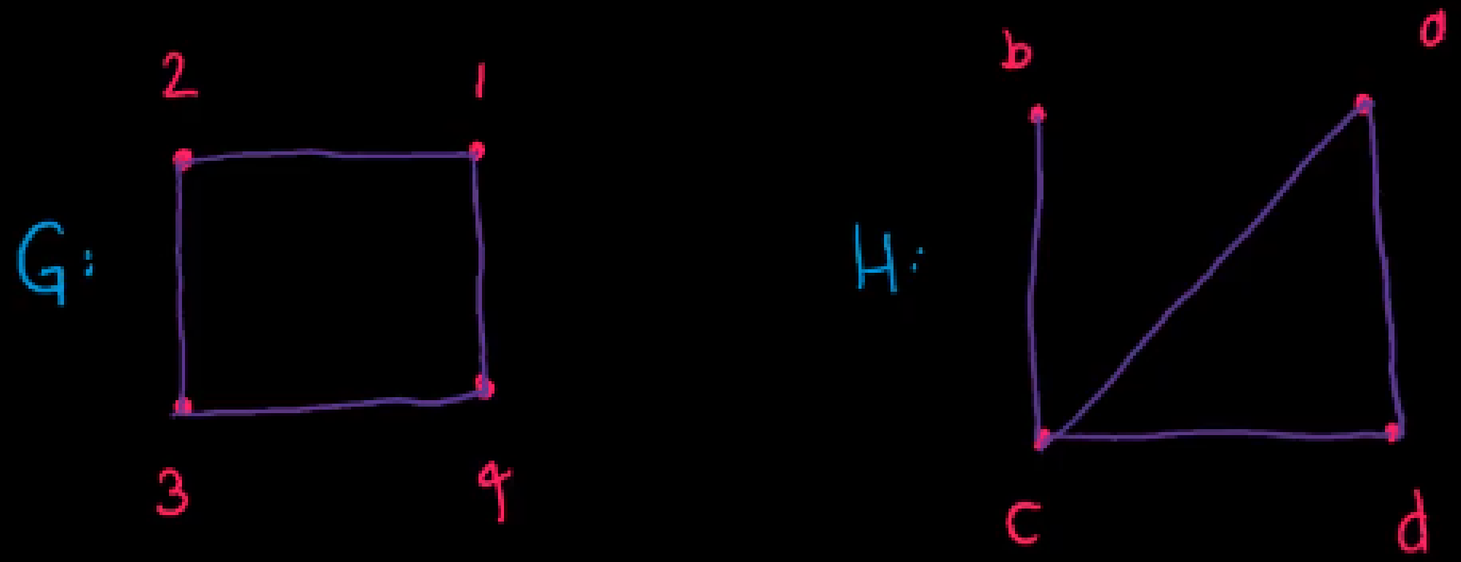
	1	2	3	4
1	0	1	0	0
2	1	0	0	0
3	0	0	0	1
4	0	0	1	0



	a	d	c	b
a	0	1	0	0
d	1	0	0	0
c	0	0	0	1
b	0	0	1	0

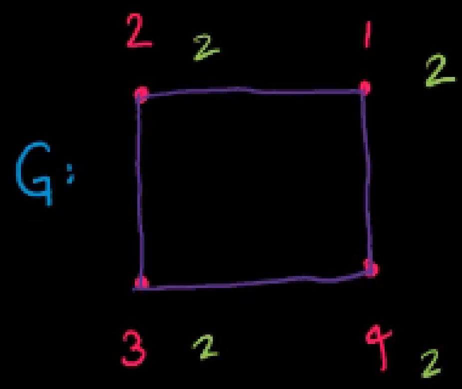
Graph Isomorphism Practice

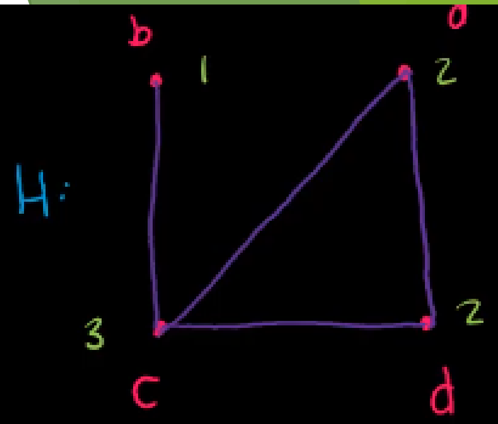
Is $G \cong H$? Tell why or why not. If so, provide the mapping.



Graph Isomorphism Theorem

$G \ncong H$





If $\boldsymbol{G}$ is isomorphic to $\boldsymbol{H}$:

- G and H have the same number of edges and vertices. ✓

- G and H have the same degree sequence structure

  $D = d_0, d_1, d_2 \ldots$ where $d_i =$# of vertices of degree $\boldsymbol{i}$

  $D_G: 0, 0, 4, 0, \ldots$ $\qquad$ $D_H: 0, 1, 2, 1, 0 \ldots$

- G and H have the same cycle structure

  G: 4-cycle

  H: 3-cycle

A graph is called *planar* if it can be drawn in the plane without any edges crossing (where a crossing of edges is the intersection of the lines or arcs representing them at a point other than their common endpoint). Such a drawing is called a *planar representation* of the graph.

A graph may be planar even if it is usually drawn with crossings, because it may be possible to draw it in a different way without crossings.

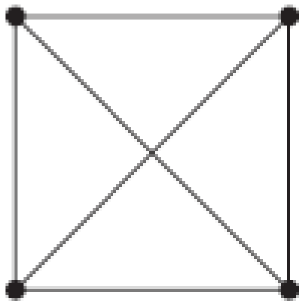


FIGURE 2 The Graph K_4.

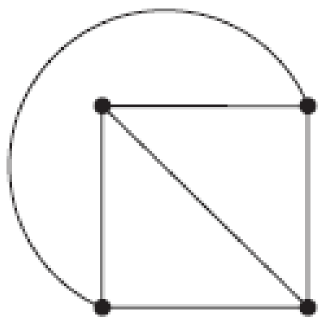


FIGURE 3 K_4 Drawn with No Crossings.

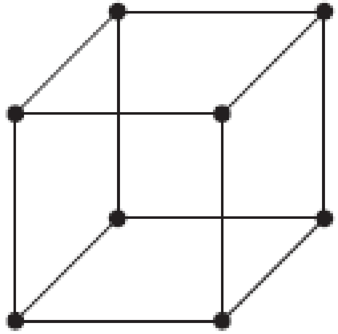


FIGURE 4 The Graph Q_3.

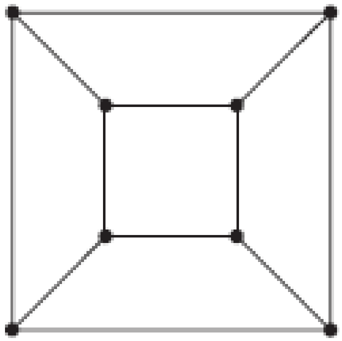


FIGURE 5 A Planar Representation of Q_3.

Corollary 10.1.2

The total degree of a graph is even.

Proposition 10.1.3

In any graph there are an even number of vertices of odd degree.

Suppose G is any graph, and suppose G has n vertices of odd degree and m vertices of even degree, where n is a positive integer and m is a nonnegative integer. *[We must show that n is even.]* Let E be the sum of the degrees of all the vertices of even degree, O the sum of the degrees of all the vertices of odd degree, and T the total degree of G. If $u_1, u_2, \ldots, u_m$ are the vertices of even degree and $v_1, v_2, \ldots, v_n$ are the vertices of odd degree, then

$$E = \deg(u_1) + \deg(u_2) + \cdots + \deg(u_m),$$
$$O = \deg(v_1) + \deg(v_2) + \cdots + \deg(v_n), \quad \text{and}$$
$$T = \deg(u_1) + \cdots + \deg(u_m) + \deg(v_1) + \cdots + \deg(v_n) = E + O.$$

Now T, the total degree of G, is an even integer by Corollary 10.1.2. Also E is even since either E is zero, which is even, or E is a sum of the numbers $\deg(u_i)$, each of which is even. But

$$T = E + O,$$