

Question 3

Part 1

$$p(w_1|x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad p(w_2|x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-4)^2}{8}}, \quad p(w_1) = p(w_2) = \frac{1}{2}$$

$$x_0: 2e^{-\frac{x^2}{2}} = e^{-\frac{(x-4)^2}{8}} \xrightarrow{\ln} 3x^2 + 8x - (16 + 8\ln 2) = 0, \quad x_0 = 1.65$$

Part 2

$$p(w_1|x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad p(w_2|x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-4)^2}{8}}, \quad p'(w_1) = \frac{1}{2}(\lambda_{12} - \lambda_{22}) = 1$$

$$p'(w_2) = \frac{1}{2}(\lambda_{21} - \lambda_{11}) = \frac{1}{2}$$

$$x'_0: 4e^{-\frac{x^2}{2}} = e^{-\frac{(x-4)^2}{8}} \rightarrow 3x^2 + 8x - (16 + 8\ln 4) = 0, \quad x'_0 = 1.95$$

کمیترت به مترات باعث کم شدن ریسک شود.

Question 2

$$cd(X, Y) = \max_{x \in X, y \in Y} d(x, y)$$

$$\text{step 1} \Rightarrow cd_{1,2} = 15, 16, \quad cd_{1,3} = 11, 22, \quad cd_{1,4} = 14, 3, \quad cd_{1,5} = 5, 5$$

$$\cancel{\text{step 2}} \quad cd_{2,3} = 17, 7, \quad cd_{2,4} = 7, 3, \quad cd_{2,5} = 13, 2$$

$$\text{step 3} \quad cd_{3,4} = 12, \quad cd_{3,5} = 9, 7, \quad cd_{4,5} = 11, 7$$

$$\min = 5, 5 \rightarrow \text{new cluster: } [1, 5]$$

$$\text{step 2} \Rightarrow cd_{2,3} = 17, 7, \quad cd_{2,4} = 7, 3, \quad cd_{3,4} = 12$$

$$cd_{2, [1, 5]} = 15, 16, \quad cd_{3, [1, 5]} = 11, 2, \quad cd_{4, [1, 5]} = 14, 3$$

$$\min = 7, 3 \rightarrow \text{combing } [1, 5] \text{ and } [2, 4] \rightarrow \text{new cluster } [1, 5], [2, 4]$$

$$\text{step 3: } cd_{3, [1, 5]} = \max(11, 2, 9, 7) = 11, 2$$

$$cd_{3, [2, 4]} = \max(17, 7, 12, 0) = 17, 7$$

$$\min = 11, 2 \rightarrow \text{new cluster} = [[1, 5], [3]], [2, 4]$$

Centroid

$$cd(X, Y) = d(\text{avg}(X), \text{avg}(Y))$$

Step 1: $cd_{1,2} = 15, 2$, $cd_{1,3} = 11, 2$, $cd_{1,4} = 14, 4$, $cd_{1,5} = 5, 5$

$$cd_{2,3} = 17, 7$$
, $cd_{2,4} = 7, 3$, $cd_{2,5} = 13, 9$

$$cd_{3,4} = 12$$
, $cd_{3,5} = 9, 8$, $cd_{4,5} = 11, 7$

Step 2: $cd_{2,3} = 17, 7$, $cd_{2,4} = 7, 3$, $cd_{3,4} = 12$

$$cd_{2, [1,5]} = cd([2, 4, 4], \text{avg}([12, 7, 7], [11, 4, 7])) = 14, 3$$

$$cd_{3, [1,5]} = cd([15, 0, 1], [11, 5, 6, 5, 8]) = 10, 2$$

$$cd_{4, [1,5]} = cd([15, 0, 1], [11, 5, 6, 5, 8]) = 12, 8$$

min is 7, 3 \rightarrow new cluster of $[1, 5], [2, 4]$

Step 3: $cd_{3, [1,5]} = cd([15, 0, 1], [11, 5, 6, 5, 8]) = 10, 17$

$$cd_{3, [2,4]} = cd([15, 0, 1],) = 14, 7$$

min is 10, 17 \rightarrow new cluster of $[[1, 5], [3]], [2, 4]$

Divisive:

$$cd(X, Y) = \min_{x \in X, y \in Y} d(x, y)$$

Step 1:

$$cd(1, \text{others}) = d_{1,5} = 5, 47$$
, $cd(2, \text{others}) = d_{2,4} = 7, 3$

$$cd(3, \text{others}) = d_{3,5} = 9, 8$$
, $cd(4, \text{others}) = d_{2,4} = 7, 3$

$$cd(5, \text{others}) = d_{1,5} = 5, 47$$
, $cd([1, 2], [3, 4, 5]) = d_{1,5} = 5, 5$

$$cd([1, 3], [2, 4, 5]) = d_{1,5} = 5, 5$$
, $cd([1, 4], [2, 3, 5]) = d_{1,5} = 5, 5$

$$cd([1, 5], [2, 3, 4]) = d_{3,5} = 9, 8$$

$$cd([2,3],[1,4,5]) = d_{2,4} = 7,3 \quad cd([2,4],[1,3,5]) = d_{4,5} = 11,7$$

$$cd([2,5],[1,3,4]) = d_{2,4} = 7,3 \quad cd([3,4],[1,2,5]) = d_{2,4} = 7,3$$

$$cd([3,5],[1,2,4]) = d_{5,1} = 5,4 \quad cd([4,5],[1,2,3]) = d_{5,1} = 5,4$$

max is 11,7 new cluster is $[2,4][1,3,5]$

Step 2:

$$cd_{2,4} = 7,3$$

$$cd(1,[3,5]) = d_{1,5} = 5,4,7$$

$$cd(3,[1,5]) = d_{3,5} = 9,7$$

$$cd(5,[1,3]) = d_{1,5} = 5,4,7$$

split $3, [1,5]$ max = 9,7

new clusters $[2,4], [1], [3,5]$

Step 3:

$$cd_{2,4} = 7,3 \quad cd_{1,5} = 5,4$$

split $2,4$

max = 7,3 \Rightarrow new cluster = $[[2],[4]], [1], [3,5]$

Step 4:

split $[3],[5] \rightarrow$ new clusters

$[[2],[4]], [1], [3],[5]]$