

1)

$$a(n) = (0,2)^n u(n)$$

$$b(n) = (0,4)^n u(n)$$

$$c(n) = a(n) * b(n) = ?$$

$$a(n) * b(n) = \sum_{k=-\infty}^{\infty} a(k) \cdot b(n-k)$$

$$\Rightarrow \sum_{k=0}^n (0,2)^k \cdot (0,4)^{n-k} = \sum_{k=0}^n (0,2)^k \cdot (0,4)^n \cdot (0,4)^{-k} = \sum_{k=0}^n (0,2/0,4)^k \cdot (0,4)^n = (0,4)^n \sum_{k=0}^n (0,5)^k$$

$$\Rightarrow \sum_{k=0}^n a^k = \frac{1 - a^{n+1}}{1 - a} \Rightarrow (0,4)^n \sum_{k=0}^{\infty} (0,5)^k = (0,4)^n \frac{1 - (0,5)^{n+1}}{1 - 0,5} = (0,4)^n 2(1 - (0,5)^{n+1})$$

$$\begin{aligned} n \geq 0 &\Rightarrow (0,4)^n 2(1 - (0,5)^{n+1}) \\ \Rightarrow & \quad \quad \quad n < 0 \Rightarrow 0 \end{aligned} \quad \Rightarrow c(n) = 2(0,4)^n (1 - (0,5)^{n+1}) u(n)$$

2)

$$y(n) - 2y(n-1) + y(n-2) = x(n) + x(n-1) \quad h(n) = ?$$

$$y(n) = h(n) \Rightarrow x(n) = \delta(n) \Rightarrow$$

$$h(n) - 2h(n-1) + h(n-2) = \delta(n) + \delta(n-1)$$

$$\lambda^n - 2\lambda^{n-1} + \lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 2\lambda + 1) = 0 \quad \lambda_1 = \lambda_2 = 1$$

$$h(n) = (c_1 + nc_2)\lambda^n = c_1 + nc_2$$

$$n = 0 \Rightarrow h(0) - 2h(-1) + h(-2) = \delta(0) + \delta(-1) = 1 = c_1 + 0 \cdot c_2 = c_1 \quad (n < 0 \Rightarrow h(n) = 0 \text{ ve } n \neq 0 \Rightarrow \delta(n) = 0)$$

$$n = 1 \Rightarrow h(1) - 2h(0) + h(-1) = \delta(1) + \delta(0) \Rightarrow h(1) = 2h(0) + \delta(0) = 2 + 1 = 3 = c_1 + 1 \cdot c_2 = c_1 + c_2 \Rightarrow c_2 = 2$$

$$\begin{aligned} n \geq 0 &\Rightarrow 1 + 2n \\ n < 0 &\Rightarrow 0 \end{aligned} \Rightarrow h(n) = (1 + 2n)u(n)$$

3)

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) \quad y(-1) = y(-2) = 0 \quad x(n) = u(n)$$

a)

$$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 4\lambda + 4) = 0 \quad \lambda_1 = \lambda_2 = 2$$

$$y_d = (c_1 + nc_2)\lambda^n = (c_1 + nc_2)2^n$$

$$n = 0 \Rightarrow y(0) - 4y(-1) + 4y(-2) = 0 = (c_1 + 0 \cdot c_2)2^0 = c_1$$

$$n = 1 \Rightarrow y(1) - 4y(0) + 4y(-1) = 0 \Rightarrow y(1) = 4y(0) = 0 = (c_1 + 1 \cdot c_2)2^1 = 2(c_1 + c_2) = 2c_2$$

$$y_d = 0$$

b)

$$y_z = (c_3 + nc_4)2^n + y_0$$

$$x(n) = u(n) \Rightarrow y_0 = K \cdot u(n) \Rightarrow$$

$$Ku(n) - 4Ku(n-1) + 4Ku(n-2) = u(n) \Rightarrow K - 4K + 4K = 1 \Rightarrow K = 1 \Rightarrow y_0 = u(n)$$

$$y_z = (c_3 + nc_4)2^n + u(n)$$

$$n = 0 \Rightarrow y(0) - 4y(-1) + 4y(-2) = x(0) = 1 = (c_3 + 0 \cdot c_4)2^0 + u(0) = c_3 + 1 \Rightarrow c_3 = 0$$

$$n = 1 \Rightarrow y(1) - 4y(0) + 4y(-1) = x(1) \Rightarrow y(1) = 4y(0) + x(1) = 4 + 1 = 5 = (c_3 + 1 \cdot c_4)2^1 + u(1)$$

$$= 2(c_3 + c_4) + 1 \Rightarrow c_4 = 2$$

$$\begin{aligned} n \geq 0 &\Rightarrow n \cdot 2 \cdot 2^n + u(n) \\ n < 0 &\Rightarrow 0 \end{aligned} \Rightarrow y_z = n2^{n+1}u(n) + u(n) = (n2^{n+1} + 1)u(n)$$

4)

$$x(n) = (n+1)a^n u(n-1) \quad X(Z) = ?$$

$$x(n) = na^n u(n-1) + a^n u(n-1)$$

$$x_1(n) = a^n u(n) \Rightarrow X_1(Z) = \frac{1}{1 - aZ^{-1}}$$

$$x_2(n) = a^{n-1} u(n-1) \Rightarrow X_2(Z) = Z^{-1} X_1(Z) = \frac{Z^{-1}}{1 - aZ^{-1}}$$

$$x_3(n) = a^n u(n-1) \Rightarrow X_3(Z) = aX_2(Z) = \frac{aZ^{-1}}{1 - aZ^{-1}}$$

$$x_4(n) = nx_3(n) \Rightarrow -Z \frac{d}{dZ} X_3(Z) = -Z \frac{d}{dZ} \left(\frac{aZ^{-1}}{1 - aZ^{-1}} \right) = -aZ \left(\frac{-Z^{-2}(1 - aZ^{-1}) - aZ^{-2}Z^{-1}}{(1 - aZ^{-1})^2} \right)$$

$$= a \left(\frac{Z^{-1}(1 - aZ^{-1}) + aZ^{-2}}{(1 - aZ^{-1})^2} \right) = a \left(\frac{Z^{-1} - aZ^{-2} + aZ^{-2}}{(1 - aZ^{-1})^2} \right) = \left(\frac{aZ^{-1}}{(1 - aZ^{-1})^2} \right)$$

$$X(Z) = \frac{aZ^{-1}}{(1 - aZ^{-1})^2} + \frac{aZ^{-1}}{1 - aZ^{-1}}$$

$$|z| > |a|$$

5)

$$y(n) - 2y(n-1) + y(n-2) = x(n) + x(n-1) \Rightarrow H(Z) = ?$$

$$Y(Z) = X(Z)H(Z) \Rightarrow H(Z) = \frac{Y(Z)}{X(Z)} \Rightarrow$$

$$Y(Z) - 2Z^{-1}Y(Z) + Z^{-2}Y(Z) = X(Z) + Z^{-1}X(Z)$$

$$Y(Z)(1 - 2Z^{-1} + Z^{-2}) = X(Z)(1 + Z^{-1}) \Rightarrow H(Z) = \frac{1 + Z^{-1}}{1 - 2Z^{-1} + Z^{-2}} = \frac{1 + Z^{-1}}{(1 - Z^{-1})^2}$$

6)

$$h(n) = (0,5)^n u(n)$$

$$x(n) = \delta(n-3)$$

$$y(n) = ?$$

$$Y(Z) = X(Z) \cdot H(Z)$$

$$X(Z) = Z^{-3} \quad H(Z) = \frac{1}{1 - 0,5Z^{-1}}$$

$$Y(Z) = Z^{-3} \frac{1}{1 - 0,5Z^{-1}} \Rightarrow y(n) = (0,5)^{n-3} u(n-3)$$

7)

$$X(Z) = \frac{Z^{-1}}{(1 - Z^{-1})(1 + 2Z^{-1})} = \frac{A}{1 - Z^{-1}} + \frac{B}{1 + 2Z^{-1}} \Rightarrow A + B = 0 \text{ ve } 2A - B = 1$$

$$\Rightarrow A = \frac{1}{3} \text{ ve } B = -\frac{1}{3} \Rightarrow X(Z) = \frac{1}{3} \frac{1}{1 - Z^{-1}} - \frac{1}{3} \frac{1}{1 + 2Z^{-1}} = \frac{1}{3} \left(\frac{1}{1 - Z^{-1}} - \frac{1}{1 + 2Z^{-1}} \right)$$

$$a) 1 < |Z| < 2 \Rightarrow x(n) = \frac{1}{3} (u(n) + (-2)^n u(-n-1))$$

$$b) |z| > 2 \Rightarrow x(n) = \frac{1}{3} (u(n) - (-2)^n u(n))$$

8)

$$\frac{1}{2} < |Z| < 2 \quad X(Z) = \frac{\frac{3}{4}}{(1 - 0,5Z)(1 - 0,5Z^{-1})} = \frac{\frac{3}{4}}{-0,5Z(1 - 2Z^{-1})(1 - 0,5Z^{-1})}$$

$$= -\frac{\frac{3}{2}Z^{-1}}{(1 - 2Z^{-1})(1 - 0,5Z^{-1})} = \frac{A}{1 - 2Z^{-1}} + \frac{B}{1 - 0,5Z^{-1}} \Rightarrow A + B = 0 \text{ ve } 0,5A + 2B = \frac{3}{2} \Rightarrow$$

$$B = 1 \text{ ve } A = -1 \Rightarrow X(Z) = -\frac{1}{1 - 2Z^{-1}} + \frac{1}{1 - 0,5Z^{-1}} \Rightarrow$$

$$x(n) = 2^n u(-n-1) + (0,5)^n u(n)$$

9)

$$X(Z) = \frac{1 - Z^{-5}}{1 - Z^{-1}} \quad |Z| > 1$$

$$X(Z) = \frac{1}{1 - Z^{-1}} - \frac{Z^{-5}}{1 - Z^{-1}} \Rightarrow x(n) = u(n) - u(n-5)$$