Örnek: ($n>0$ için)

1. $a = 0$
2. for $i=1$ to n
3. for $j=i$ to n
4. $a=a+1$
5. return a

algoritmasının çalışma zamanını asimtotik olarak elde ediniz.

Çözüm:

$$\sum_{i=1}^{n} \sum_{j=i}^{n} 1 = \sum_{i=1}^{n} \left(\sum_{j=1}^{n} 1 - \sum_{j=1}^{i-1} 1 \right) = \sum_{i=1}^{n} (n-i+1)$$

$$= \sum_{i=1}^{n} (n+1) - \sum_{i=1}^{n} i$$

$$= n(n+1) - \frac{n(n+1)}{2}$$

$$= n^2 + n - \frac{1}{2}n^2 - \frac{1}{2}n$$

$$= n^2 + \frac{1}{2}n = \Theta(n^2)$$

$$\sum_{i=1}^{5} i = \underbrace{1+2}_{\sum_{i=1}^{2}} + \underbrace{3+4+5}_{\sum_{i=3}^{5}} \rightarrow \sum_{i=1}^{5} - \sum$$

(Örnek 1)

Örnek: $n>0$ için

```
a=0
for i=1 to n
    for j=i+1 to n
a=a+1 ~~for k=1 to i~~
Return a
```

algoritma çalışma zamanını asimptotik olarak açıklayınız.

<u>Çözüm</u>: $$\sum_{i=1}^{n}\sum_{j=i+1}^{n}1=\sum_{i=1}^{n}\left(\sum_{j=1}^{n}1-\sum_{j=1}^{i}1\right)$$

$$=\sum_{i=1}^{n}(n-i)=\sum_{i=1}^{n}n-\sum_{i=1}^{n}i$$

$$=n^2-\frac{n(n+1)}{2}$$

$$=\frac{1}{2}n^2-\frac{1}{2}n=\Theta(n^2)$$

Örnek 2.

Örnek: $n > 0$ olmak üzere

```
a = 0
for i = 1 to n-1
for j = i+1 to n
for k = 1 to j
        a = a + 1
Return a
```

olarak verilen algoritmanın çalışma zamanını asimptotik olarak açıklayınız.

Çözüm:

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{k=1}^{j} 1 = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} j$$

$$= \sum_{i=1}^{n-1} \left(\sum_{j=1}^{n} j - \sum_{j=1}^{i} j \right) = \sum_{i=1}^{n-1} \left(\frac{n(n+1)}{2} - \frac{i(i+1)}{2} \right)$$

$$= \frac{n(n+1)(n-1)}{2} - \frac{1}{2} \sum_{i=1}^{n-1} (i^2 + i)$$

$\frac{n(n+1)(2n+1)}{6}$ idi

yerine konur

$$= \frac{n(n+1)(n-1)}{2} - \frac{1}{2} \left[\frac{n(n-1)(2n-1)}{6} \right] - \frac{1}{2} \frac{(n-1)n}{2}$$

$$= \frac{1}{2} n(n-1) \cdot \left[n+1 - \frac{(2n-1)}{6} - \frac{1}{2} \right]$$

$$= \frac{1}{12} n(n-1)(4n+4) = \frac{1}{3} n(n-1)(n+1)$$

$$= \Theta(n^3)$$

Örnek 3-

$$= (n+2)(n+1)\sum_{i=1}^{n/2} 1 - 2(n+2)\sum_{i=1}^{n/2} i - (n+1)\sum_{i=1}^{n/2} i + 2\sum_{i=1}^{n/2} i^2 -$$

$$-\frac{1}{2}\sum_{i=1}^{n/2}\left((n+1)(n+2) - i(2n+3) \cancel{+i^2} \cancel{-i^2} - i\right)$$

$$= (n+2)(n+1)\sum_{i=1}^{n/2} 1 - (3n+5)\sum_{i=1}^{n/2} i + 2\sum_{i=1}^{n/2} i^2 - \frac{(n+1)(n+2)}{2}\sum_{i=1}^{n/2} 1 + \frac{2n+4}{2}\sum_{i=1}^{n/2} i$$

$$= \frac{n}{2}(n+2)(n+1) - (3n+5)\cdot\frac{\frac{n}{2}\left(\frac{n}{2}+1\right)}{2} + 2\cdot\frac{\left(\frac{n}{2}\right)\left(\frac{n}{2}+1\right)(n+1)}{6} - \frac{1}{2}(n+1)(n+2)\cdot\frac{n}{2} + \frac{(2n+4)}{2}\cdot\frac{\left(\frac{n}{2}\right)\left(\frac{n}{2}+1\right)}{2}$$

$$= \frac{\frac{n}{2}(n+1)(n+2)}{2} - \frac{(2n+3)\left(\frac{n}{2}\right)\left(\frac{n}{2}+1\right)}{2} + \frac{\frac{n}{2}\left(\frac{n}{2}+1\right)(n+1)}{3}$$

n çift olduğunda, $n = 2k$, ve

$$f(n) = k(k+1)\left(-\frac{1}{2} + \frac{2k+1}{3}\right) = \frac{k(k+1)(4k-1)}{6} \text{ olur}$$

n tek olursa $n = 2k+1$

$$f(n) = \frac{k(k+1)(4k+5)}{6} \text{ olur ve}$$

$$f(n) = \Theta(n^3) \text{ olur}$$

Örnek:

```
1. a = 0
2. for i = 1 to n
3.   for j = i+1 to n
4.     for k = i+j-1 to n
5.       a = a+1
6. Return a
```

a'yı hesaplayan algoritmanın çalışma zamanını f(n)'e bağlı olarak bulunuz.

Çözüm:
$$f(n) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \boxed{\sum_{k=i+j-1}^{n} 1}$$

Hatırlatma:
$$\sum_{i=t}^{n} 1 = \begin{cases} n-t+1 & \text{if } t \le n \\ 0 & \text{else} \end{cases}$$

Bu durumda
$$\sum_{k=i+j-1}^{n} 1 = \begin{cases} n-i-j+2 & \text{if } i+j-1 \le n \Leftrightarrow j \le \underline{\underline{n-i+1}} \\ 0 & \text{else} \end{cases}$$

Böylece
$$f(n) = \sum_{i=1}^{n} \sum_{j=i+1}^{n-i+1} (n+2-(i+j))$$

Aynı düşünceyle $\boxed{\begin{array}{l} i+1 > n-i+1 \\ 2i > n,\ i > \frac{n}{2} \end{array}}$ olduğunda toplam sıfır olur.

Böylece
$$f(n) = \sum_{i=1}^{n/2} \sum_{j=i+1}^{n-i+1} \left((n+2) - (i+j)\right)$$

$$= (n+2) \sum_{i=1}^{n/2} \sum_{j=i+1}^{n-i+1} 1 - \sum_{i=1}^{n/2} \left(\sum_{j=i+1}^{n-i+1} i \right) - \sum_{i=1}^{n/2} \left(\sum_{j=i+1}^{n-i+1} j \right)$$

$$= (n+2) \cdot \sum_{i=1}^{n/2} (n-i+1-(i+1)+1) - \sum_{i=1}^{n/2} i(n-i+1-(i+1)+1) - \sum_{i=1}^{n/2} \left(\sum_{j=1}^{n-i+1} j - \sum_{j=1}^{i} j \right)$$

$$= (n+2) \sum_{i=1}^{n/2} (n-2i+1) - \sum_{i=1}^{n/2} i\overset{((n+1)-(2i))}{(n-2i+1)} - \sum_{i=1}^{n/2} \left(\frac{(n-i+1)(n-i+2)}{2} - \frac{i(i+1)}{2} \right)$$