Physically based rendering

Concepts

Energy conservation :

- total amount of light reflected from surface cannot exceed total amount received .

Microfacets:

Surface composed of microfacets that reflect light independantly.

- rough : reflect light more chaotically
- smooth : more uniformity

BRDF :

Bidirectional Reflectance Distribution Function , describes how light is reflected from a surface in different directions based on material properties. typically include :

- diffuse lighting (light scattering)
- specular lighting (mirror like reflections).

Maps and textures:

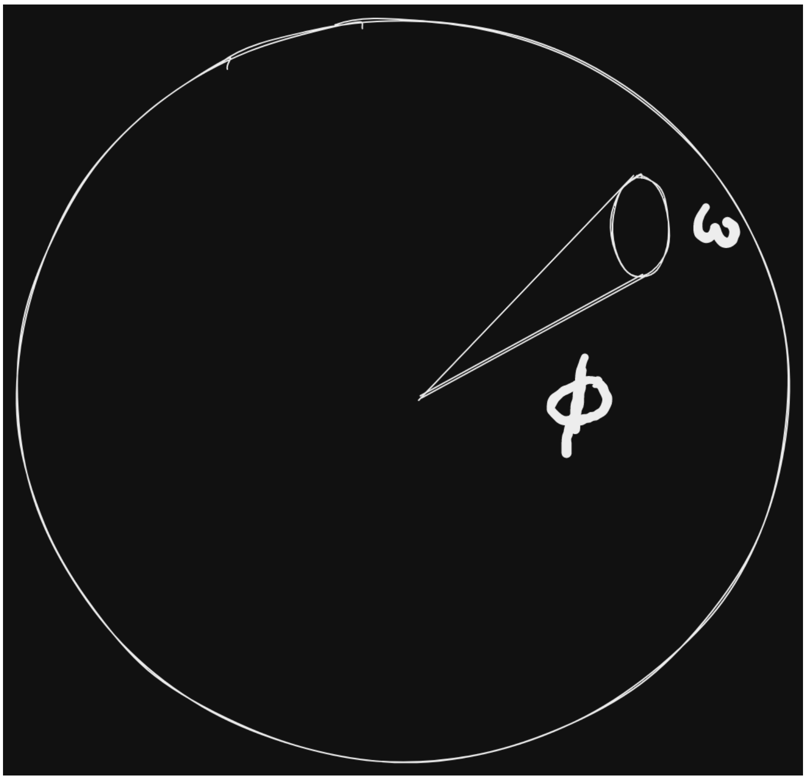
- Albedo , normal maps , metal roughness , IBL etc .

Reflectance equation :

$$L(o, ωo) = Le(o, ωo) + \int_{\Omega}BRDF(p, ωi, ωo) * L(p, ωi) * dot(n, ωi) * dω$$

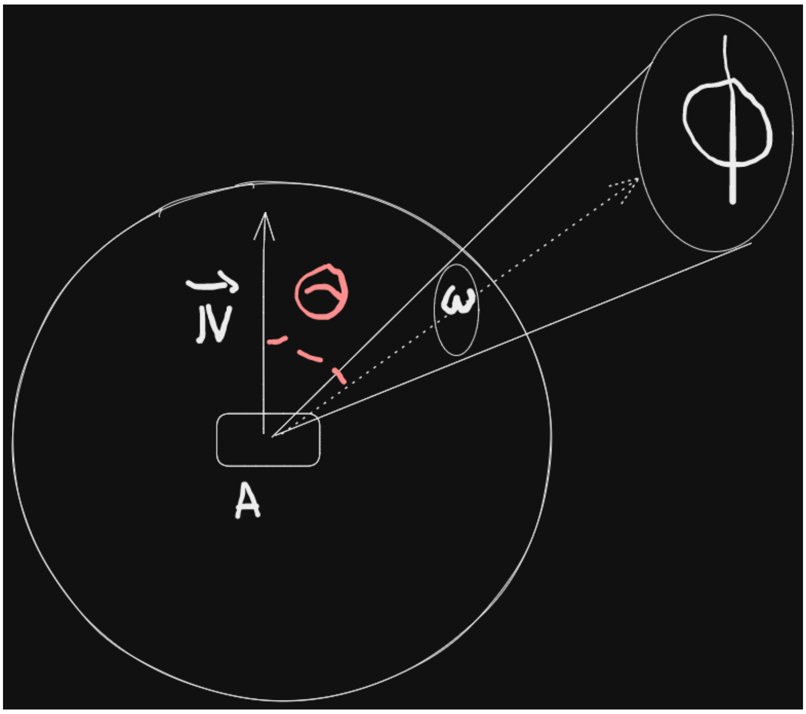
- Radiant flux ϕ: sum of wavelengths = RGB color of light .
- Solid angle ω: This simulates the incidence of multiple rays of light over a small area . It represents the projection of an area on a unit sphere.

- Radiant intensity : Amount of Radiant flux per solid angle.



The equation for radiant intensity is : $$I = \frac{d\phi}{d\omega}$$

- Radiance : This is the total energy on Area "A" over a solid angle ω with radiant intensity ϕ



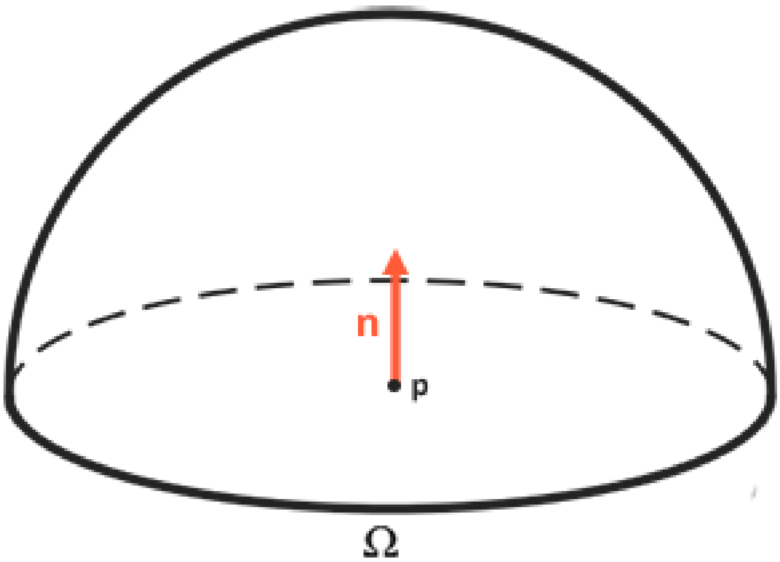
The radiance equation is :

$$L = \frac{d^2\phi}{dAd\omega cos\theta}$$

Reflectance equation for a fragment :

$$L_0(p,\vec{\omega_0}) = \int_\Omega BRDF(p , \vec{\omega_i} , \vec{\omega_0}) L_i(p , \vec{\omega_i})\vec{n}.\vec{\omega_i} d\vec{\omega_i}$$

- $\vec{\omega_0}$: Direction from the fragment to the camera.
- p : infinitesimally small point represented by area "A" , or , in this case , a fragment .
- $L_0(p,\vec{\omega_0})$: Reflected sum of all lights irradiances falling on point p , viewed by direction $\vec{\omega_0}$
- $\vec{n}.\vec{\omega_i}$: represents the $cos\theta$ between the normal $\vec{n}$ and direction $\vec{\omega_i}$
- Ω : Represents a hemisphere around the point p.



Reflectance integral resolution:

Code:

```
int steps = STEP_SIZE ;
float sum = 0.f ;
vec3 P = fragPos() ;
vec3 W0 = view_direction();
vec3 N = fragNormal();
float dW = 1.f / steps;
for(int i = 0 ; i < steps ; i++){
    vec3 Wi = -getLightDir(i);
    sum += BRDF(P , Wi , W0) * L(P , Wi) * dot(N , Wi) * dW ;
}
```

Bidirectional Reflective Distribution Function :

- Takes as input :

  1. $\vec{\omega_i}$: Light i direction.
  2. $\vec{n}$: Normal of the surface.
  3. a : Roughness surface parameter.

- Approximates how much light $\vec{\omega_i}$ contributes to the final reflected light on a surface given it's material property.

- Perfectly smooth surface returns BRDF = 0 for all $\vec{\omega_i}$, except the only ray that reflects towards $\vec{\omega_0}$ which returns BRDF = 1.

- BRDF is valid only if the reflected light is less than the sum of incoming light (energy conservation , see inequality on next equations).

Cook-Torrance model :

$$f_r=k_df_{lambert} + k_sf_{cook-torrance}$$

- f_r : Reflected light intensity at point p.
- k_d : Diffuse reflection coefficient.
- k_s : Specular reflection coefficient.
- $f_{lambert}$: Lambertian diffuse component
- $f_{cook-torrance}$: Specular reflection component.

$$\forall \vec{\omega_{i}} \in \mathbb{L} , \int_{\Omega}f_r(\vec{n} . \vec{\omega_0})d\vec{\omega_0} \leq 1 $$

The equations reads : "For all Light ω_i hitting the surface , the sum of outgoing BRDF weights in the hemisphere Ω multiplied by $cos\theta = \vec{n}.\vec{\omega_0}$ must be smaller than 1 .

Diffuse component :

$$f_{lambert} = \frac{c}{\pi}$$

- c : Albedo color

Specular component :

$$f_{cook-torrance} = \frac{DFG}{4(\vec{\omega_0}.\vec{n})(\vec{\omega_i}.\vec{n})}$$

This equation is composed of :

- $f_{cook-torrance}$: Specular component of the model.
- D : Normal Distribution Function => approximates the amount of surface microfacets aligned to the halfway vector , influenced by the roughness of the surface.
- G : Geometry Function : Describes the self shadowing properties of the microfacets .
- F : Fresnel Equations : Describes the amount of reflection at certain angles.

There are many other specular functions : here

BRDF Sub-functions :

Normal Distribution Function , "Trowbridge-Reitz GGX" :

$$ NDF(\vec{n} , \vec{h} , \alpha) = \frac{\alpha^2}{\pi ((\vec{n}.\vec{h})^2(\alpha^2 - 1) + 1)^2}$$

- $\vec{n}$: surface normal.

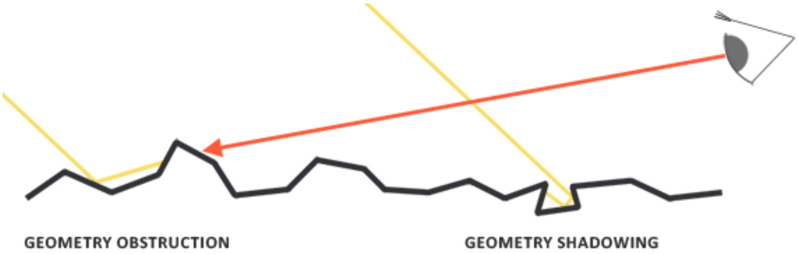
- $\vec{h}$: halfway vector.
- α : surface roughness.

Code :

```
float distributionGGX(vec3 h , vec3 n , float a){
    float aa = a*a ;
    float dot_normalized = max(dot(n , h), 0.f);
    float dot_n_h_2 = dot_normalized * dot_normalized;
    float divid = (dot_n_h_2 * (aa - 1) + 1);
    divid *= divid;
    divid *= PI ;
    return aa/divid ;
}
```

Geometry Function , Schlick-GGX:

Allows to provide "self shadowing microfacets"



$$ G_{SchlickGGX}(\vec{n} , \vec{v} , k) = \frac{\vec{n}.\vec{v}}{(\vec{n}.\vec{v})(1 - k) + k}$$

with :

- $\vec{n}$: surface normal , normlized.
- $\vec{v}$: view direction , normalized.
- k : remapping of α , defined as :

$$ k = \begin{cases} k_{direct} = \frac{(\alpha +1)^2}{8} \ k_{IBL} = \frac{\alpha^2}{2} \ \end{cases}$$

Smith Method :

Approximation of the geometry , taking a view direction + light direction vectors :

$$G(\vec{n} , \vec{v} , \vec{l} , k) = G_{sub}(\vec{n} , \vec{v} , k)G_{sub}(\vec{n} , \vec{l} , k)$$

with : $$G_{sub} = G_{SchlickGGX}$$

- The geometry function is a multiplier between [0.0, 1.0] with 1.0 (or white) measuring no microfacet shadowing, and 0.0 (or black) complete microfacet shadowing.

Code :

```
float GeometrySchlickGGX(float NdotV, float k)
{
float nom   = NdotV;
float denom = NdotV * (1.0 - k) + k;
return nom / denom;
}

float GeometrySmith(vec3 N, vec3 V, vec3 L, float k){
float NdotV = max(dot(N, V), 0.0);
float NdotL = max(dot(N, L), 0.0);
float ggx1 = GeometrySchlickGGX(NdotV, k);
float ggx2 = GeometrySchlickGGX(NdotL, k);
return ggx1 * ggx2;
}
```

Fresnel Equations :

The equations for the dielectric case : $$F_{Schlick}(\vec{h} , \vec{v} , F_0) = F_0 + (1 - F_0)(1-(\vec{h}.\vec{v}))^5 = F_0 + (1-F_0)(1-cos\theta)^5$$ $$F_0=\left(\frac{\eta_1 - \eta_2}{\eta_1+\eta2}\right)^2$$ With :

- η_1 , η_2 : Reflection and refraction coefficients .
- θ : Angle between the view direction and the half vector .

Conductor case : $$F = \frac{(\eta - 1)^2 + 4\eta(1-cos\theta)^5 + k^2}{(\eta + 1)>^2 + k^2}$$ With :

- η : Index of refraction for the conductor .
- k : Absooption coefficient for the conductor .

Code :

```
vec3 F0 = vec3(abs((1.0 - ior)/(1.0 + ior)));  //We consider the incident
ray's first medium is air.
F0 *= F0;
F0 = mix(F0 , albedo.rgb , metallic);

vec3 fresnelSchlick(float cosT , vec3 F0){
    return F0 + (1 - F0) * pow(1 - cosT , 5) ;
}
```