Readme

Dynamics-Based NOR Gate Using the Logistic Map

Abstract

We demonstrate the realization of a NOR logic gate using nonlinear discrete-time dynamics. The computation is embedded in the evolution of a chaotic logistic map combined with thresholding and excess-transfer mechanisms, illustrating how logical operations can emerge from dynamical systems without conventional Boolean circuitry.

Logistic Map Dynamics

The logistic map is a one-dimensional discrete-time dynamical system defined as

$$x_{n+1} = ax_n(1 - x_n), \tag{1}$$

where $x_n \in (0, 1)$ and a is a control parameter. In this work, the parameter is fixed at $a = 4$, corresponding to fully developed chaos. In this regime, the system exhibits strong sensitivity to initial conditions and rich symbolic dynamics, which are exploited for computation.

Encoding of Logical Inputs

Logical inputs are encoded as distinct initial conditions within the chaotic attractor. Specifically, $\text{Logical } \ 0 \to x = 0.82$,
$Logical 1 \to x = 0.86$. These values are chosen such that they lie on opposite sides of a predefined threshold while remaining within the chaotic region of the map.

Threshold Dynamics and Excess Transfer

A threshold value $x_{\text{th}} = 0.84$ is introduced. At each iteration, the state of the system is processed according to

$$excess = \{ \, 0 \ , x_n < x_{\text{th}}, x_n - x_{\text{th}}, x_n > x_{\text{th}}. \tag{2}$$

If $x_n > x_{\rm th}$, the state is clipped to $x_{\rm th}$, and the excess is transferred to the subsequent dynamical unit. This mechanism introduces nonlinear coupling between inputs and allows information to propagate through the system.

Iterative Evolution

The coupled system is iterated for a sufficiently large number of steps to eliminate transient behavior. The first unit evolves independently under the logistic map, while the second unit evolves under the combined influence of its own dynamics and the excess transferred from the first unit. This iterative process replaces conventional clocked logic with autonomous dynamical evolution.

Logical Output Detection

The logical output is determined from the excess generated by the second unit. The output is assigned according to

$$Output = \{\ 0\ , excess < \delta, 1, excess > \delta, \tag{3}$$

where $\delta = 0.14$ is a detection threshold. This procedure yields the NOR truth table, in which the output is high only when both inputs are low.

Discussion

This dynamics-based implementation of a NOR gate demonstrates that logical computation can emerge from nonlinear chaotic dynamics combined with simple thresholding rules. Such approaches highlight the potential of dynamical systems as substrates for unconventional and analog computation, with possible extensions to more complex logic circuits and hardware realizations.