Research Statement

Yash Tiwari
BS–MS Dual Degree Student, IISER Bhopal
yasht24@iiserb.ac.in

Research Overview

My research interests lie in nonlinear and geometric dynamical systems, with emphasis on stability theory for periodically driven and non-smooth systems. I am particularly interested in how geometric structure, perturbation theory, and averaging methods interact in systems where classical smooth frameworks fail. Broadly, I study how qualitative dynamics emerge from analytic structure and how geometric viewpoints extend stability theory beyond traditional Floquet settings.

Classical Floquet theory assumes smooth periodic coefficients and differentiable flows. However, many physically relevant systems involve impacts, frictional contacts, switching dynamics, or hybrid constraints, where the flow map fails to be C^1 and the variational equation is not globally defined. My work investigates how averaging and multi-scale perturbation methods provide generalized stability frameworks under weakened smoothness assumptions, and how these reduced systems can be interpreted geometrically as effective dynamics on slow invariant manifolds.

Current Research: Non-smooth Floquet Systems and Averaging

My current research studies periodically forced oscillators of the form

$$\ddot{x} + f(x, \dot{x}) + \varepsilon g(x, \dot{x}, t) = 0,$$

where f may contain discontinuous terms such as Coulomb friction or impact laws. In such systems, classical Floquet theory breaks down because the Jacobian of the flow contains distributional terms and the monodromy matrix is not well-defined. I analyze how averaging restores a smooth reduced dynamics that captures stability and bifurcation behavior.

Using amplitude–phase coordinates

$$x(t) = a(t)\cos(\omega t + \phi(t)),$$

and near-identity transformations, I derive averaged slow-flow systems governing (a, ϕ). Even when the original vector field is non-smooth, the averaged equations remain well-defined:

$$\dot{a} = \langle f_1(a, \phi, t)\rangle, \qquad \dot{\phi} = \langle f_2(a, \phi, t)\rangle.$$

These averaged equations define a smooth reduced dynamical system whose Jacobian determines stability in place of classical Floquet multipliers.

A key result of my work is the explicit demonstration that, in smooth limits, averaging reproduces Floquet stability to first order. For the weakly damped harmonic oscillator

$$\ddot{x} + 2\varepsilon\gamma\dot{x} + x = \varepsilon F \cos t,$$

the averaged system yields eigenvalue $-\varepsilon\gamma$, and numerical Floquet analysis gives exponents

$$\lambda_{\text{Floquet}} = -\varepsilon\gamma \pm i\omega,$$

showing exact agreement in decay rate. I extended this comparison to the weakly nonlinear Duffing oscillator, where the averaged eigenvalues match the real parts of numerically computed Floquet exponents obtained from the monodromy matrix. This establishes averaging as a quantitatively accurate stability predictor in smooth regimes.

In non-smooth systems, I show that Floquet multipliers are undefined due to non-differentiable Poincaré maps, yet the averaged slow flow remains smooth and yields meaningful stability criteria. This provides a constructive replacement for Floquet theory in hybrid systems and captures bifurcations such as border-collision transitions that lie outside classical spectral analysis.

Fourier Averaging and Multi-Harmonic Dynamics

I have also developed a Fourier-based averaging framework for multi-harmonic oscillatory systems. Writing

$$x(t) = \sum_{n=1}^{\infty} \left(a_n \cos n\Omega t + b_n \sin n\Omega t \right),$$

I derive evolution equations for harmonic coefficients using orthogonality of trigonometric modes and near-identity transformations. This produces averaged equations for each harmonic pair (a_n, b_n):

$$\begin{pmatrix} \dot{\bar{a}}_n \\ \dot{\bar{b}}_n \end{pmatrix} = \frac{1}{2n\Omega} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} R_{a_n} \\ R_{b_n} \end{pmatrix}.$$

This framework generalizes classical single-mode averaging and allows systematic stability and bifurcation analysis of multi-frequency oscillations. Geometrically, the reduced system defines a slow flow on a high-dimensional invariant torus parameterized by harmonic amplitudes.

Geometric Dynamical Systems Perspective

Alongside this analytical work, I am developing a strong foundation in geometric dynamical systems. I am particularly motivated by invariant manifold theory, metric and topological structure of flows, and Hamiltonian geometry. My guided studies in metric geometry and topological fluid dynamics have shaped my interest in viewing perturbation theory as a geometric reduction: averaging constructs effective dynamics on slow manifolds that preserve essential qualitative structure.

I am interested in how symplectic and geometric structure survive under forcing, dissipation, and hybrid constraints, and how geometric invariants constrain long-term dynamics. This perspective connects my work on averaging to broader questions in Hamiltonian dynamics, chaos, and topological aspects of flows.

Future Directions

I aim to extend these ideas to periodically driven Hamiltonian and quantum systems, including Floquet-engineered models and quantum chaos such as the kicked rotor. A central question is whether averaging-based effective dynamics can serve as geometric substitutes for classical Floquet frameworks in hybrid or non-smooth quantum-classical systems.

In the long term, I plan to pursue a PhD in nonlinear and geometric dynamical systems, working at the interface of mathematics and physics where analytic methods, geometry, and computation interact. I am particularly drawn to research environments focused on Hamiltonian systems, perturbation theory, chaos, and driven dynamics.