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37.

Statistics in AIDS Assignment 2

9/10

Q1. mean = 4300 acres
s.d = 750 acres
probability of between 2500 & 4200 acres will be burnt.

$P(2500 < x < 4200)$ is to be found

$\therefore$ $z = \frac{2500-\mu}{\sigma} = \frac{2500-4300}{750} = -2.40$

$z = \frac{4200-\mu}{\sigma} = \frac{4200-4300}{750} = -0.133$

$P(2500 < x < 4200) = P(-2.4 < z < -0.133)$

$= P(z < -0.133) - P(z < -2.4)$

$= 0.4483 - 0.0082$

$= 0.4401$

$\therefore$ The probability is 0.4401.

2. 38^{th} percentile.

$P(X < ?) = 0.38 \Rightarrow P(z < ?) = 0.38$

$z = -0.31$

$X = 4300 + (-0.31)(750)$

$= 4300 - 232.5$

$= 4067.5$

$\therefore$ 4067.5 is the 38^{th} percentile of the burnt acre.

Q 3. $x = 290$

$\mu = 300$

$s.d = 50$

$n = 15$

$$t = \frac{290-300}{\frac{50}{\sqrt{15}}} \qquad \frac{x-\mu}{\frac{sd}{\sqrt{n}}}$$

$= -0.774$

$\therefore$ The t-score is 0.226. (1 − t)

$\therefore$ Cummulative frequency is 0.226, for that the bulbs wont last more than 290 days.

Q 4. $x = 290$

$\mu = 300$

$S.d = 50$

$n = 15$

$$t = \frac{290-300}{\frac{50}{\sqrt{15}}} \qquad \frac{x-\mu}{\frac{sd}{\sqrt{n}}}$$

$= -0.774$

$\therefore$ t-score is −0.774.

Q 5.

FOR EDUCATIONAL USE

Q 6]

$\mu = 20$

$\sigma = 4$

Sample size $n = 64$

$P(\bar{X} < 19) = ?$

$$\text{Standard error} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{64}} = 0.5$$

$$\therefore \quad P(X < 19) = \frac{19-20}{0.5} = -2$$

$$\therefore \quad P(z < -2) = 0.0228.$$

Q7.

$n = 50$

$\mu = 112$

$\sigma = 40$

$P(110 < x < 114)$

$P(\bar{X} > 113)$

$\mu_{\bar{X}} = \mu = 112$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 5.65$$

$$\therefore P\left(\frac{110-112}{5.65} < z < \frac{110-114}{5.65}\right)$$

$\therefore P(-0.35 < z < 0.35)$
$= 0.6368 - 0.3632$
$= 0.2736$

$\therefore P(X > 113) = P\left(z > \frac{113 - \mu}{\sigma}\right)$
$= P(z > 0.18)$
$= 1 - P(z < 0.18)$
$= 1 - 0.5715$
$= 0.4286.$

Q 8.

$\mu = 30$
$\sigma = 1.5$ kg
Sample $n = 45$
Using z-score,

$\bar{\sigma} = \frac{\sigma}{\sqrt{n}}$
$= \frac{1.5}{\sqrt{45}}$
$= 6.7082$

$\therefore \frac{28 - 30}{6.7082} = P(p < 28)$
$\therefore -0.2981$

$\therefore P(z < -0.2981) = 0.3828.$

Q9. $\mu = 300$ / hour

$\therefore \mu = \frac{300}{60}$

$= 5$ / minute.

$\therefore$ a] the probability that none passes in a minute is

$\therefore P(X=0) = \frac{e^{-\lambda} (\lambda)^x}{x!}$

$\frac{e^{-5} \times 5^0}{0!}$

$\therefore e^{-5} = 0.0067$

b] Expected no. of vehichles in 2 mins is

1 min $= 5$

$\therefore$ 2 min 10 vehicles.

c]

$\frac{e^{-10} \times (10)^{10}}{10!}$ (probability that 10 vehicles pass in 2 mins)

$= 0.125$

Q10.

Q11. $\mu = 4$ mins

$\therefore \lambda = \frac{1}{\mu}$

$= \frac{1}{4}$

$= 0.25$

$\therefore$ Following exponential distibution

$P(x = 5) = \lambda e^{-(\lambda)x}$

$= 0.25 \times e^{-(0.25)(5)}$

$= 0.25 \times e^{-}$

$= 0.071$

Q12. $\mu = 8$ mins

$\lambda = \frac{1}{\mu}$

$= \frac{1}{8}$

$= 0.125$

$\therefore P(x = 5) = 0.125 \times e^{-(0.125)(5)}$

$= 0.067$

$\therefore$ Probability $= 0.067$

Q 13. $\because$ probability $= 1/4$

$n = 10$

probability that he hits exactly 3 times

$\therefore x = 3$

$p = 1/4 \quad q = 3/4.$

$\therefore P(X=3) = {}^{n}C_{x}\, p^{x}\, q^{(n-x)}$

$= {}^{10}C_{3}\, p^{3}\, q^{7}$

$= {}^{10}C_{3}\, (1/4)^{3}\, (3/4)^{7}$

$= \frac{10!}{7! \times 3!} \times \frac{3^{7}}{4^{10}}$

$= 0.2502$

$\therefore$ The probability to hit exactly 3 times is 0.2502

$\therefore$ Probability to hit atleast once

$= 1 -$ (Probability to hit not more than once)

$= 1 - [P(x=0) + P(x=1)]$

$\therefore$ Probability $= 1 - \left(\frac{3}{4}\right)^{7}$.

Q 14. ~~probabilit~~

$p = 3/100$

$= 0.03$

$n = 5$ $\therefore q = 1 - p = 0.97$

a] For $x=0$

$\therefore \; {}^5C_0 \, (0.03)^0 \, (0.97)^5$

$= \frac{5!}{5!} (0.97)^5$

$= (0.97)^5$

$= 0.858$

$\therefore P(x=0) = 0.858$

b] At least 2

$\therefore \quad P(x \geq 2) = 1 - P(x=0) - P(x=1)$

$= 1 - \left[{}^5C_0 \, (0.03)(0.97)^5 \right] - \left[{}^5C_1 \, (0.03)^1 (0.97)^4 \right]$

$= 1 - (0.97)^5 - (0.03)^1 (0.97)^4$

$= 0.0085$

c] 3.

Q 15. $n = 60$

$\mu = \$1000$

$\sigma = \$200$

Confidence level $= 95\%$

$\therefore z_\alpha = 1.96$ (Two-tail test).

$\therefore \left[\bar{x} \pm z_{\alpha} \frac{\sigma}{\sqrt{n}} \right]$

$= 1000 \pm 1.96 \times \frac{200}{\sqrt{60}}$

$= 1000 \pm 50.61$

$= 949.39, 1050.61$

$\therefore$ The 95% confident intervals are ($949.39 , $1050.61)

Q10

Household	No of children
1	2
2	3
3	1
4	0
5	5
6	2
7	1
8	4

mean $= \frac{18}{8}$

$= 2.25$

variance $= \frac{(2-2.25)^2 + (3-2.25)^2 \ldots + (4-2.25)^2}{8}$

$= 1.804$

$\therefore$ S.D $= \sqrt{1.804}$

$= 1.34$

$\therefore$ Standard ~~dev~~ error $= \dfrac{1.34}{\sqrt{n}}$

$= \dfrac{1.34}{\sqrt{8}}$

$= 0.475$

$\therefore$ C.I $= \bar{x} \pm z \dfrac{s}{\sqrt{n}}$

$= 2.25 \pm 1.96\ (0.475)$

$= [2.25 + 1.96(0.475)]\ [2.25 - 1.96(0.475)]$

$= 2.25 + 0.931\ ,\ 2.25 - 0.931$

$\therefore$ C.I $= 3.181,\ \cancel{1.8}\ 1.319$