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Higgs Mass

$$V(\phi) = \mu^2 (\phi^\dagger \phi) + \lambda (\phi^\dagger \phi)^2$$

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \text{ minimization: } \langle \phi \rangle = \frac{v}{\sqrt{2}}$$

$$V(\phi) = \mu^2 \left((\phi^\dagger)^2 + \phi_1^2 + \phi_2^2 \right) + \lambda \left((\phi^\dagger)^2 + \phi_1^2 + \phi_2^2 \right)^2$$

$$\phi \rightarrow \frac{v+h}{\sqrt{2}}, \quad \phi_1^\dagger, \phi_2 = 0$$

$$L_{\text{kin}}(\phi) = \frac{1}{2} (\partial_\mu (v+h)) \partial^\mu (v+h)$$

$$= \frac{1}{2} \partial_\mu (h) \partial^\mu (h)$$

$$\begin{aligned} V(\phi) &= V(\phi(v+h)) = \frac{1}{2} \mu^2 (v+h)^2 + \frac{1}{4} \lambda (v+h)^4 \\ &= \lambda v^2 h^2 + \frac{1}{2} \lambda h^4 + \lambda v h^3 \\ &\quad - \frac{1}{4} \lambda h^4 \end{aligned}$$

$$\mathcal{L}(h) = \frac{1}{2} (\partial_\mu h) (\partial^\mu h) - \lambda v^2 h^2$$

$$\Rightarrow \boxed{m_h = \sqrt{2\lambda v^2}}$$

If we can include the complex field, $\phi = i\epsilon$

$$\phi^2 = \phi^* \phi = \frac{1}{2} [(v+h)^2 + \epsilon^2], \quad \mu^2 = -\lambda v^2$$

$$\mathcal{L}_{kin} = \frac{1}{2} \partial_\mu (h + v - i\epsilon) \partial^\mu (v + h + i\epsilon)$$

$$= \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} \partial_\mu \epsilon^2, \quad \partial_\mu v = 0$$

$$V(\phi, \phi^*) = \mu^2 \phi^2 + \lambda \phi^4$$

$$= -\frac{1}{2} \lambda v^2 [(v+h)^2 + \epsilon^2] + \frac{\lambda}{4} [(v+h)^2 + \epsilon^2]^2$$

$$= -\frac{1}{4} \lambda v^4 + \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda h^4}{4} + \frac{\lambda \epsilon^4}{4}$$

$$+ \lambda v h \epsilon^2 + \frac{\lambda h^2 \epsilon^2}{2}$$

$$\mathcal{L}(h, \epsilon) \subset \underbrace{\frac{1}{2} (\partial_\mu h)^2 - (\lambda v^2) h^2}_{\text{massive scalar 'h'}}$$

$$+ \frac{1}{2} (\partial_\mu \epsilon)^2 + (c \cdot \xi^2)$$

$\phi \equiv 0$
vev.

neglecting
higher order /
constant
terms.

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