Standard Model Lagrangian

Some Couplings and Mass terms

(A) Higgs (Scalar) Sector (ϕ):

$$V(\phi) = \mu^2 (\phi^\dagger \phi) + \lambda (\phi^\dagger \phi)^2$$

$$\mathcal{L}_{scalar} = (\mathcal{D}^\nu \phi)^\dagger (\mathcal{D}_\nu \phi) - V(\phi)$$

(B) Gauge Bosons: $SU(2)_L \times U(1)_Y$ [Electro-Weak]

We write the Higgs as a complex scalar

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\Rightarrow \mathcal{D}_\mu = \partial_\mu + i\underbrace{g \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu}_{SU(2)} + \underbrace{\frac{ig'}{2} Y B_\mu}_{U(1)}$$

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- Symmetry Breaking : $\phi_1 = \phi_2 = \phi_4 = 0$ $\Big\{$ choosing Vacuum.

$\phi_3 = v$

s.t., $|\langle\phi\rangle|^2 = \frac{2\mu^2}{\lambda}$, $\mu^2 > 0$, $\lambda > 0$

$\Rightarrow$ $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ $\longleftrightarrow$ Vacuum

★★ Now, For masses :

$$(D_\mu \phi) = \frac{1}{\sqrt{2}}\left[i g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu + \frac{i g'}{2} Y B_\mu \right] \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{i}{\sqrt{8}} \left[g(\tau_1 W_1 + \tau_2 W_2 + \tau_3 W_3) + g' Y B_\mu \right] \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{i}{\sqrt{8}} \left[g\left[\begin{pmatrix} 0 & W_1 \\ W_1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -iW_2 \\ -iW_2 & 0 \end{pmatrix} + \begin{pmatrix} W_3 & 0 \\ 0 & -W_3 \end{pmatrix} \right] + g' \begin{pmatrix} Y_{\phi_0} B_\mu & 0 \\ 0 & Y_{\phi_0} B_\mu \end{pmatrix} \right] \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{i}{\sqrt{8}} \begin{pmatrix} gW_3 + g' Y_{\phi_0} B_\mu & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g' Y_{\phi_0} B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{iv}{\sqrt{8}} \begin{pmatrix} g(W_1 - iW_2) \\ -gW_3 + g' Y_{\phi_0} B_\mu \end{pmatrix}$$

$$\Rightarrow (D^\nu \phi)^\dagger = \frac{-iv}{\sqrt{8}} \begin{pmatrix} g(W_1 + iW_2) \\ -gW_3 + g' Y_{\phi_0} B_\mu \end{pmatrix}^T$$

$$\star \Rightarrow (D^\mu \phi)^\dagger (D_\mu \phi) = \frac{v^2}{8} \left[g^2 (W_1^2 + W_2^2) + (-gW_3 + g' Y_{\phi_0} B_\mu)^2 \right]$$

Let's re-write $W^{1,2}$ as

$$W^\pm = \frac{1}{\sqrt{2}} (W_1 \pm W_2) < \begin{matrix} W^+ \\ W^- \end{matrix} \Big\} \text{ bosons}$$

by $\leftrightarrow$ $T^\pm = \frac{1}{2}(T^1 \pm iT^2)$

$$\Rightarrow \quad g^2(W_1^2 + W_2^2) = g^2(W^{+2} + W^{-2})$$
$$= 2g^2 W^+ W^-$$

* For the Y_{ϕ_0} term,

- $Y_{\phi_0} \neq 0 \Leftrightarrow$ for W_3, B_μ mixing

- mixing matrix:

$$(-g W_3 + g' Y_{\phi_0} B_\mu)^2 = (W_3 \; B_\mu) \begin{pmatrix} g^2 & -gg' Y_{\phi_0} \\ -gg' Y_\phi & g'^2 \end{pmatrix} \begin{pmatrix} W_3 \\ B_\mu \end{pmatrix}$$

- For $Y_{\phi_0} = \pm 1 \Leftrightarrow$ determinant vanishes.

$Y_{\phi_0} = +1$ for $\mathbb{Q}$, γ coupling.

$$\Rightarrow \boxed{Y_{\phi_0} = +1}$$

$\to W_{mix}$ —/—/—

$\star$ mixing matrix: $\begin{pmatrix} g^2 & -gg' \\ gg' & g'^2 \end{pmatrix}$ | $\det(W_{mix}) = 0$

i) Eigen Value

$\lambda = 0, (g^2 + g'^2)$

For $\lambda = 0$

$$\frac{1}{\sqrt{g+g'^2}} \begin{pmatrix} g' \\ g \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}} (g' W_3 + g B_\mu) = A_\mu (\gamma)$$

For $\lambda = (g^2 + g'^2)$

$$\frac{1}{\sqrt{g+g'^2}} \begin{pmatrix} g \\ -g' \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}} (g W_3 - g' B_\mu) = Z_\mu (Z)$$

Using this to re-write Lagrangian terms,

$$(-g W_3 + g' B_\mu)^2 = (g^2 + g'^2) Z_\mu^2 + 0 \cdot A_\mu^2$$

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★ $(D^\mu \phi)^\dagger (D^\mu \phi) = \frac{1}{2} \times \frac{V^2}{4} [g^2 (W^+)^2 + g^2 (W^-)^2$
$+ (g^2 + g'^2) Z_\mu^2 + 0 \cdot A_\mu^2]$

★ Gauge Boso mass term: $\frac{1}{2} M_V^2 V_\mu^2$

$\Rightarrow \quad M_{W^+} = M_{W^-} = \frac{Vg}{2}$ ✓

$\Rightarrow \quad M_Z = \frac{V}{2} \sqrt{(g^2 + g'^2)}$ ✓

$M_\gamma = 0$ ✓

★ From scalar field definition:

$M_H = \sqrt{2\lambda V^2}$, $V \approx 246$ GeV [Muon decay]

★ <u>Mixing Angle</u>: $\theta_W = \tan^{-1}\left(\frac{g'}{g}\right)$

(c) Fermion Masses

- $-m\bar{\psi}\psi \leftrightarrow$ not allowed
- $\mathcal{L}_{\text{fermion}}(\text{mass}) = -\lambda_f[\bar{\psi}_L \phi \psi_R + \bar{\psi}_R \phi \psi_L]$

$\lambda_f \leftrightarrow$ Yukawa Coupling (y)

Lepton Masses

Not that fermions: L.H $\leftrightarrow$ SU(2) doublet
R.H $\leftrightarrow$ SU(2) singlet

:)

$$\mathcal{L}_e = -y_e \frac{1}{\sqrt{2}}\left[(\bar{\nu},\bar{e})_L \begin{pmatrix} 0 \\ v+h \end{pmatrix} e_R + \bar{e}_R (0, v+h) \begin{pmatrix} \nu \\ e \end{pmatrix}_L\right]$$

$$= \frac{-y_e(v+h)}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L)$$

$$= \frac{-y_e(v+h)}{\sqrt{2}} [\bar{e} e]$$

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$\Rightarrow \mathcal{L}_e = \underbrace{-\frac{y_e(v)}{\sqrt{2}}(\bar{e}e)}_{e^- \text{ mass term}} - \underbrace{\frac{y_e(h)(\bar{e}e)}{\sqrt{2}}}_{e^- - H \text{ coupling}}$

$$\Rightarrow \boxed{\begin{aligned} e^-(\text{mass}) &= \frac{y_e(v)}{\sqrt{2}} \\ &\text{||}^{\text{rly}} \\ \mu^-(\text{mass}) &= \frac{y_\mu(v)}{\sqrt{2}} \\ \tau(\text{mass}) &= \frac{y_\tau(v)}{\sqrt{2}} \end{aligned}}$$

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Quark Masses

- The previous mass term only gives us the mass of only one row → e, μ, τ, so we make a new doublet for the quarks.

$\Rightarrow$ $\mathcal{L}_{yukawa}$: $y_D \bar{Q}_L \phi d_R + y_u \bar{Q}_L \phi_c u_R$

$$+ \underbrace{y_e \bar{L}_L \phi e_R}_{\text{used this for lepton masses}} + h.c$$

↳ conjugate terms.

- other 2 terms are "mixed" for quark masses
- Note that $e_R \leftarrow$ singlet (always charged)

$\Rightarrow$ No neutrino's in SM ☹

- Here $\phi_c = -i\tau_2 \phi^* = \frac{-1}{\sqrt{2}} \begin{pmatrix} v+h \\ 0 \end{pmatrix}$ in vacuum

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☆ Quark Mass terms: (without h.c.)

down-type: $y_d (\bar{u}_L \ \bar{d}_L) \begin{pmatrix} 0 \\ v \end{pmatrix} d_R = y_d v \bar{d}_L d_R$

up-type: $y_u (\bar{u}_L \ \bar{d}_L) \begin{pmatrix} v \\ 0 \end{pmatrix} u_R = y_u v \bar{u}_L u_R$

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(1D) **Lagrangian Terms**

★★★ SU(2)$_L$ × U(1)$_Y$ (gauge + Higgs)

$$\mathcal{L}_{\text{Electro-Weak}} : -\frac{1}{4}\left(W^a_{\mu\nu}\right)^2 - \frac{1}{4}B^2_{\mu\nu}$$

$$+ (D_\mu \phi)^\dagger (D_\mu \phi) + m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

- $D_\mu \phi = \partial_\mu \phi - i g \tau^a . W^a_\mu \phi - \frac{1}{2} i g' B_\mu \phi$
- $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$
- $\tan \theta_W = \dfrac{g'}{g}$
- $\phi = \exp\left(2i \dfrac{\pi^a \tau^a}{v}\right) \begin{pmatrix} 0 \\ v+h \end{pmatrix} \times \dfrac{1}{\sqrt{2}}$

$\pi^a = 0 \iff$ Unitary gauge.

- $-\frac{1}{4}(W^a_{\mu\nu})^2 = \sum_{i=1}^{3} \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu}$

re-write $W^{\pm}_{\mu\nu} = \frac{1}{\sqrt{2}}(W^1_{\mu\nu} \mp i W^2_{\mu\nu})$

- $W^{1,2,3}_{\mu\nu} = \partial_\mu W^{1,2,3}_\nu - \partial_\nu W^{1,2,3}_\mu$

$\quad - g'(W^{2,3,1}_\mu W^{3,1,2}_\nu - W^{2,3,1}_\nu W^{3,1,2}_\mu)$

- $\Rightarrow W^{\pm}_{\mu\nu} = (\partial_\mu + ig' W^3_\mu) W^{\pm}_\nu$

$\quad \mp (\partial_\nu + ig' W^3_\nu) W^{\pm}_\mu$

- $W^3_{\mu\nu} = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu - ig'(W^-_\mu W^+_\nu$

$\quad - W^-_\nu W^+_\mu)$

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- i) $\sum_{i=1}^{3} \frac{1}{4} W^i_{\mu\nu} W^{i\,\mu\nu}$

$$= \frac{1}{4} W^3_{\mu\nu} W^3_{\mu\nu} - \frac{1}{2} W^-_{\mu\nu} W^{+\,\mu\nu}$$

- $$V(\phi^\dagger\phi) = m^2 h^2 + \frac{m^2 h^3}{\sqrt{2}\,v} + \frac{m^2 h^4}{8v^2} = V(h)$$

- $$D^\nu \phi = \begin{pmatrix} 0 \\ \partial^\nu h/\sqrt{2} \end{pmatrix} + \frac{ig}{2}\begin{pmatrix} 0 \\ \frac{B^\nu (v+h)}{\sqrt{2}} \end{pmatrix} + \frac{ig'}{2} \times \begin{pmatrix} \sqrt{2}\, W^+_\mu \left(\frac{v}{\sqrt{2}} + \frac{h}{\sqrt{2}}\right) \\ -W^3_\mu \left(\frac{v}{\sqrt{2}} + \frac{h}{\sqrt{2}}\right) \end{pmatrix}$$

- $$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu h\, \partial^\mu h + \frac{g'^2}{2} W^-_\mu W^{+\mu} \left(\frac{v}{\sqrt{2}} + \frac{h}{\sqrt{2}}\right)^2$$

$$+ \left[\frac{g'^2}{4} W^3_\mu W^{3\mu} + -\frac{gg'}{2} W^3_\mu B^\mu + \frac{g'}{4} B_\mu B^\mu\right] \left(\frac{v+h}{\sqrt{2}}\right)^2$$

$- V(h)$

$$= \frac{1}{2}\partial_\mu h \partial^\nu h + \frac{g^2}{2} W_\mu^- W^{+\mu} \left(\frac{v}{\sqrt{2}} + \frac{h}{\sqrt{2}}\right)^2$$

$$+ \frac{1}{4}(g_1^2 + g^2) Z_\mu Z^\mu \left(\frac{v}{\sqrt{2}} + \frac{h}{\sqrt{2}}\right)^2 - V(h)$$

Re-writing: $Z_\mu = W_\mu^3 \cos\theta_w - B_\mu \sin\theta_w$

$A_\mu = W_\mu^3 \sin\theta_w + B_\mu \cos\theta_w$

$\Rightarrow B_\mu = A_\mu \cos\theta_w + - Z_\mu \sin\theta_w$

$W_\mu^3 = A_\mu \sin\theta_w + Z_\mu \cos\theta_w$

$\Rightarrow B_{\mu\nu} = A_{\mu\nu} \cos\theta_w - Z_{\mu\nu} \sin\theta_w$

$W^3_{\mu\nu} = A_{\mu\nu} \sin\theta_w + Z_{\mu\nu} \cos\theta_w$

$- ig'(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+)$

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ii) $\mathcal{L}$
[Gauge + Higgs]

$$= \frac{1}{2}\partial_\mu h \partial^\mu h - m^2 h^2$$

$$- \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{4} \times \frac{v_{\nu}(g'^2 + g^2)}{2} \times Z_\mu Z^\mu$$

$$- \frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{2} [(\partial_\mu W_\nu^+)^* - (D_\nu W_\mu^+)^*]$$

$$[\partial^\mu W^{+\nu} - D^\nu W^{+\mu}]$$

$$+ \frac{1}{2} g'^2 \left(\frac{v}{v_\nu}\right)^2 W^-_\mu W^{+\mu}$$

$$+ \left(\frac{1}{4}h^2 + \frac{hv}{2}\right)\left(g'^2 W^-_\mu W^{+\mu} - \frac{1}{2}(g^2 + g'^2) Z_\mu Z^\mu\right)$$

$$- \frac{m^2 h^3}{\sqrt{2}\left(\frac{v}{v_\nu}\right)} - \frac{m^2 h^4}{8\left(\frac{v}{v_\nu}\right)^2} + g'^2 (W^-_\mu W^+_\nu - W^{-\nu} W^{+\mu})$$

$$+ \frac{ig'}{2} (A_{\mu\nu} \sin\theta_W + Z_{\mu\nu} \cos\theta_W)(W^{-\mu} W^{+\nu} - W^{-\nu} W^{+\mu})$$

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$$-g'^2\cos^2\theta_W\left(Z_\mu Z^\mu W_\nu^- W^{+\nu} - Z_\mu Z^\nu W_\nu^- W^{+\mu}\right)$$

$$+\frac{ig'^2}{2}\cos\theta_W\Big([Z_\mu W_\nu^- - Z_\nu W_\mu^-][D^\mu W^{+\nu} - D^\nu W^{+\mu}]$$

$$-(Z_\mu W_\nu^+ - Z_\nu W_\mu^+)((D^\mu W^{+\nu})^* - (D^\nu W^{+\mu})^*)\Big)$$

Substituting mass term & $e = g\sin\theta_W = g'\cos\theta_W$

$W^2 = \frac{vg}{2}$, $m_Z = \frac{v}{2}\sqrt{g^2+g'^2} = \frac{m_W}{\cos\theta_W}$,

$m_h = \sqrt{2\lambda v^2}$

$$\mathcal{L}_{gauge} = -\frac{1}{4}A_{\mu\nu}^2 - \frac{1}{4}Z_{\mu\nu}^2 + \frac{1}{2}m_Z^2 Z^\mu Z_\mu$$

$$\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-)$$

$$+ m_W^2 W_\mu^+ W_\mu^- - ie\cot\theta_W\Big[\partial_\mu Z_\nu(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-)$$

$$+ Z_\nu(-W_\mu^+\partial_\nu W_\mu^- + W_\mu^-\partial_\nu W_\mu^+ + W_\mu^+\partial_\mu W_\nu^- - W_\mu^-\partial_\mu W_\nu^+)\Big]$$

$$- ie [\partial_\mu A_\nu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)$$

$$+ A_\mu (- W^+_\nu \partial_\nu W^-_\mu + W^-_\nu \partial_\nu W^+_\mu$$

$$+ W^+_\nu \partial_\mu W^-_\nu - W^-_\mu \partial_\mu W^+_\nu)]$$

$$- \frac{1}{2} \frac{e^2}{\sin^2\theta_W} (W^+_\mu W^-_\mu W^+_\nu W^-_\nu + W^+_\mu W^-_\nu W^+_\mu W^-_\nu)$$

$$- e^2 \cot^2\theta_W (Z_\mu W^+_\mu Z_\mu W^-_\nu - Z_\nu Z_\mu W^+_\nu W^-_\nu)$$

$$+ e^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu)$$

$$+ e^2 \cot\theta_W [A_\mu W^+_\mu W^-_\nu Z_\nu + A_\mu W^-_\mu Z_\mu W^+_\nu$$

$$- W^+_\mu W^-_\mu A_\nu Z_\nu] \text{———} \mathrm{I}$$

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$$\mathcal{L}_{Higgs} = -\frac{1}{2} h \left(\partial^{\nu} \partial_{\nu} + m_h^2 \right) h$$

$$- \frac{g\, m_h^2}{4 m_W} h^3 - \frac{g^2}{32} \frac{m_h^2}{m_W^2} h^4$$

$$+ \frac{2h}{v} \left(m_W^2 W_{\nu}^{+} W_{\mu}^{-} + \frac{1}{2} m_Z^2 Z_{\mu}^2 \right)$$

$$+ \left(\frac{h}{v} \right)^2 \left(m_W^2 W_{\mu}^{+} W_{\mu}^{-} + \frac{1}{2} m_Z^2 Z_{\mu} Z_{\mu} \right)$$

II

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Fermion - Gauge Boson Lagrangian (SU(3) × SU(2) × U(1))

$$\mathcal{D}_\mu = \partial_\mu - ig\frac{Y}{2}B_\mu - ig'\frac{\tau^i W^i_\mu}{2}$$

$$- ig_s \frac{\lambda^a}{2} G^a_\mu$$

$$\bar{\Psi}\gamma^\mu \partial_\mu \Psi \longrightarrow \bar{\Psi}\gamma^\mu D_\mu \Psi$$

$$\mathcal{L}_{ferm} = \sum_f \bar{\Psi}\gamma^\mu D_\mu \Psi ,$$

$$\Psi \equiv f = L, e_R, Q_L, u_R, d_R$$

Leptons

U(1) terms:

$$\mathcal{L}_{ferm}(U(1))_{lepton} \subset \bar{L}\, i\gamma^\mu \left(ig\frac{Y_L}{2}B_\mu\right)L$$

$$+ \bar{e}_R\, i\gamma^\mu \left(ig\frac{Y_R}{2}B_\mu\right)e_R$$

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$$\bar{L}\gamma^{\mu}L = \bar{\nu}_L\gamma^{\mu}\nu_L + \bar{e}_L\gamma^{\mu}e_L$$

$$\Rightarrow \mathcal{L}_{ferm}(U(1), \text{lepton}) = \frac{g}{2}\Big[Y_L(\bar{\nu}_L\gamma^{\mu}\nu_L + \bar{e}_L\gamma^{\mu}e_L) + Y_R\,\bar{e}_R\gamma^{\mu}e_R\Big]B_{\mu}$$

SU(2) terms

- $\mathcal{L}_{ferm}(SU(2), \text{lepton})$

$$= \bar{L}i\gamma^{\mu}\left(ig'\frac{\tau^i}{2}W^i_{\mu}\right)L$$

$$= -\frac{g'}{2}(\bar{\nu}_L \ \ \bar{e}_L)\gamma^{\mu}\begin{pmatrix} W^3_{\mu} & W^1_{\mu} - iW^2_{\mu} \\ W^1_{\mu} + iW^2_{\mu} & -W^3_{\mu}\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$$

$$= -\frac{g'}{2}\left(\bar{\nu}_L \ \ \bar{e}_L\right)\gamma^{\mu}\begin{pmatrix} W^3_{\mu}\nu_L & -\sqrt{2}W^+_{\mu}e_L \\ -\sqrt{2}W^-_{\mu}\nu_L & -W^3_{\mu}e_L\end{pmatrix}$$

$$= -\frac{g'}{2}\Big[\bar{\nu}_L \gamma^\mu \nu_L W^3_\mu - \sqrt{2}\,\bar{\nu}_L \gamma^\mu e_L W^+_\mu$$

$$- \sqrt{2}\,\bar{e}_L \gamma^\mu \nu_L W^-_\mu - \bar{e}_L \gamma^\mu e_L W^0_\mu\Big]$$

** FCNC

- $\mathcal{L}_{EM} = q A_\mu \left[\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R\right]$

- Neutrino terms: $\left(-\frac{g}{2} Y_L B_\mu - \frac{g'}{2} W^3_\mu\right) \bar{\nu}_L \gamma^\mu \nu_L$

$$A_\mu = \frac{g' B_\mu - g Y_L W^3_\mu}{\sqrt{g'^2 + g^2 Y_L^2}}$$

$$Z_\mu = \frac{g Y_L B_\mu + g' W^3_\mu}{\sqrt{g'^2 + g'^2 Y_L^2}}$$

• Electron terms: $\bar{e}_L \gamma^\mu e_L \left(-\frac{g}{2} Y_L B_\mu + \frac{g'}{2} W^3_\mu \right)$

$$+ \bar{e}_R \gamma^\mu e_R \left(-\frac{g}{2} Y_R B_\mu \right)$$

$$B_\mu = \frac{g' A_\mu + g Y_L Z_\mu}{\sqrt{g'^2 + g_1^2 Y_L^2}} \; ; \; W^3_\mu = \frac{-g Y_L A_\mu + g' Z_\mu}{\sqrt{g'^2 + g^2 Y_L^2}}$$

$$\Rightarrow \frac{-A_\mu}{\sqrt{g'^2 + g^2 Y_L^2}} \left\{ \bar{e}_L \gamma^\mu e_L \left(g_1 g_2 Y_L \right) + \bar{e}_R \gamma^\mu e_R \left(g_1 g_2 Y_R \right) \right\}$$

$$- \frac{Z_\mu}{\sqrt{g'^2 + g^2 Y_L^2}} \left\{ \bar{e}_L \gamma^\mu e_L \left(g_1^2 Y_L^2 - g_2^2 \right) + \bar{e}_R \gamma^\mu e_R \left(g_1^2 Y_R Y_L \right) \right\}$$

$\because A_\mu \longleftrightarrow$ E-M current.

$$e = \frac{-g_1 g_2 Y_L}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \quad , \quad Y_R = 2 Y_L$$

For the sake of sanity
$g = g_1$ | $g_s = g_3$
$g' = g_2$

$\Rightarrow$ Choose $Y_L = -1$

$\Rightarrow$

$$e = \frac{g_1 g_2}{\sqrt{g_2^2 + g_1^2}}$$

- $\tan \theta_w = \dfrac{g_2}{g_1}$, $g_1 = e(\cos\theta_w)^{-1}$
$g_2 = e(\sin\theta_w)^{-1}$

☆☆ **ν-Z coupling**

$$-\frac{\sqrt{g_2^2 + g_1^2}}{2} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L = \frac{-g_2}{2\cos\theta_w} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L$$

$$\sqrt{g_2^2+g_1^2} = \frac{e}{\cos\theta_W \sin\theta_W}$$

e- Z coupling

$$-Z_\mu \left\{ \bar{e}_L \gamma^\nu e_L \underbrace{\left[\frac{g_1^2 - g_2^2}{2\sqrt{g_2^2+g_1^2}}\right]}_{a} + \bar{e}_R \gamma^\nu e_R \underbrace{\left[\frac{g_1^2}{\sqrt{g_2^2+g_1^2}}\right]}_{b} \right\}$$

$$a = \frac{e}{\cos\theta_W \sin\theta_W}\left(\frac{-1}{2} + \sin^2\theta_W\right)$$

$$b = \frac{e}{\cos\theta_W \sin\theta_W}(\sin^2\theta_W)$$

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☆☆ f- Z coupling

$$\frac{e}{\cos\theta_w \sin\theta_w}\left(T_3^f - Q_f \sin^2\theta_w\right)$$

$$T_3^f = \begin{cases} 0, & f = e_R, \nu_R, d_R \text{ -- (singlet)} \\ +\frac{1}{2}, & f = \nu_L, u_L \text{ -- [up]} \\ -\frac{1}{2}, & f = e_L d_L \text{ [down]} \end{cases}$$

$$Q_f = \begin{cases} -1 & , f = e \\ 0 & , f = \nu \\ \frac{2}{3} & , f = u \\ -\frac{1}{3} & , f = d \end{cases}$$

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Charged Current

$$\mathcal{L}_{ferm}(\text{lept}) = \frac{g_2}{\sqrt{2}}\left[\bar{\nu}_L \gamma^\mu e_L W^+_\mu + \bar{e}_L \gamma^\mu \nu_L W^-_\mu\right]$$

$$\mathcal{L}_{ferm}(\text{quarks}) = \frac{g_2}{\sqrt{2}}\left[\bar{u}_L \gamma^\mu d_L W^+_\mu + \bar{d}_L \gamma^\mu u_L W^-_\mu\right]$$

Quark - QCD

$$\underbrace{\frac{g_3}{2}\bar{q}_\alpha \gamma^\mu \lambda^a_{\alpha\beta} G^a_\mu q_\beta \quad \begin{cases} \alpha,\beta = 1,2,3 \\ a = 1 \cdots 8 \end{cases}}_{}$$

↳ No clue!

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$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

- Lepton universality

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

- u, d universality

So all the couplings work for the other families as well.

Fermi - Gauge Lagrangian

$$\mathcal{L}_{(\text{Fermi-Gauge})} = \sum_{f=\nu,e,u,d} e\, q_f (\bar{f} \gamma^\mu f) A_\mu$$

$$+ \frac{g_2}{\cos\theta_W} \sum_{f=\nu_e,u,d} \Big[\bar{f}_L \gamma^\mu f_L (T_f^3 - q_f \sin^2\theta_W) + \bar{f}_R \gamma^\mu f_R (-q_f \sin^2\theta_W) \Big] Z_\mu$$

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$$+\frac{g_2}{\sqrt{2}}\left[(\bar{u}_L\gamma^\mu d_L + \bar{\nu}_L\gamma^\mu e_L)W_\mu^+ + (\bar{d}_L\gamma^\mu u_L + \bar{e}_L\gamma^\mu \nu_L)W_\mu^-\right]$$

$$+\frac{g_3}{2}\sum_{q=u,d}\bar{q}_\alpha\gamma^\mu\lambda^a_{\alpha\beta}q_\beta G^a_\mu$$

$\rightarrow$ III

*** <u>Fermion Mass terms</u> (Higgs-Fermion)

$$-\mathcal{L}_{yukawa} : y_d\bar{Q}_L\phi d_R + y_u\bar{Q}_L\phi_c u_R$$

$$+ y_e\bar{l}_L\phi e_R + \cdots h.c + \cdots$$

with Quark mixing;

$$-\mathcal{L}_{yukawa} = y^d_{ij}\bar{Q}^I_{Li}\phi d^I_{Rj} + y^u_{ij}\bar{Q}^I_{Li}\phi_c u^I_{Rj}$$

$$+ y^l_{ij}\bar{l}^I_{Li}\phi l^I_{Rj} , \rightarrow \text{IV}$$

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where,

Spinor field	ψ^I (Color, I_3, Y)
1) L.H quarks	Q^I_{Li} $(3, 2, +\frac{1}{3})$
2) R.H 'up' Quarks	u^I_{Ri} $(3, 1, +\frac{4}{3})$
3) R.H 'down' Quarks	d^I_{Ri} $(3, 1, +\frac{1}{3})$
4) L.H fermions	L^I_{Li} $(1, 2, -1)$
5) R.H fermions	l^I_{Ri} $(1, 1, -2)$

(E)

$$\mathcal{L}_{SM} \supset \underset{(II)}{\mathcal{L}_{(Higgs)}} + \underset{(I)}{\mathcal{L}_{(gauge)}} + \underset{(III)}{\mathcal{L}_{(fermion\text{-}gauge)}}$$

$$+ \underset{(IV)}{\mathcal{L}_{(fermion\text{-}Higgs)}}$$