2023

DSA
PART-6

OBZTRON

C++

(SIXTH PART)

T.I.M. HAMIM

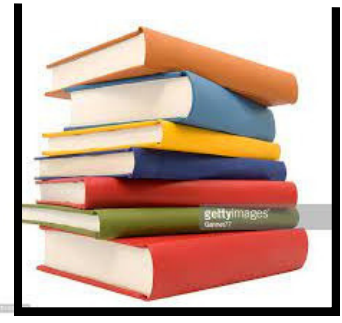
Index: DSA < Part - 2 >

1. Stacks

Definition of stack:

A stack is a linear data structure in which insertions and deletions are allowed only at the end, called the top of the stack.

As a stack is a linear data structure, we can implement it using an array or a linked list.



Stack of boocks

Stack of coins

Objects placed inside the glass jars

Both these examples have two things in common:

1. Any object (either a book or a coin) can be accessed only from the top.
2. Any object can be added only at the top.

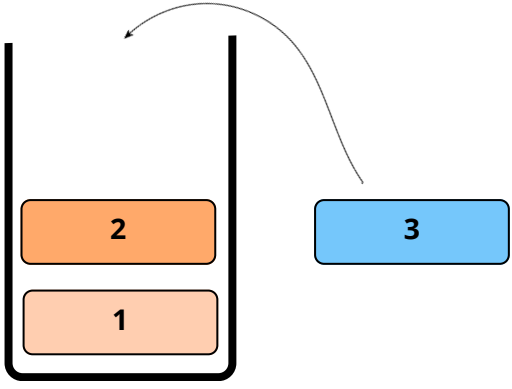
Stack as an ADT:

When we define a stack as an ADT (or Abstract Data Type), then we are only interested in knowing the stack operations from the user of view.

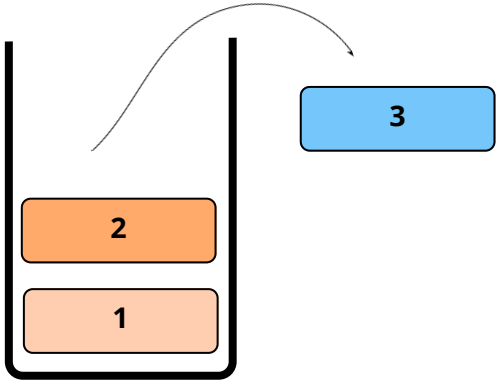
This means we are not interested in knowing the implementation details at this moment. We are only interested in knowing what types of operations we can perform on the stack.

Primary Stack Operations:

push(data): Insert data onto stack.

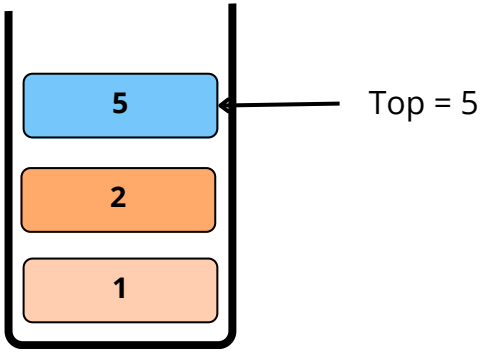


pop(): Deletes the last inserted element from the stack.

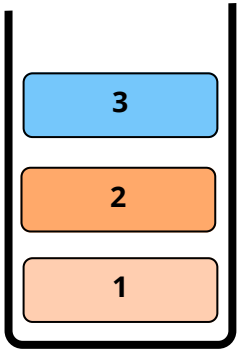


Secondary Stack Operations:

top(): returns the last inserted element without removing it..

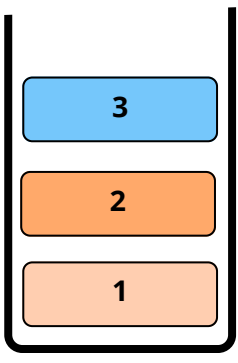


size(): returns the size or the number of elements in the stack.

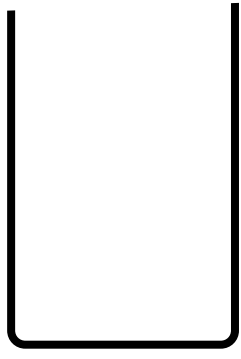


size = 3

isEmpty(): returns TRUE if the stack is empty, else returns FALSE.

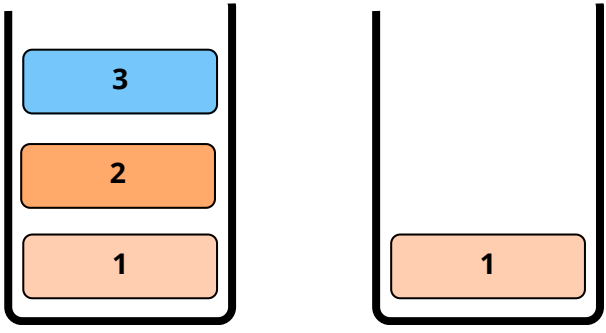


FALSE



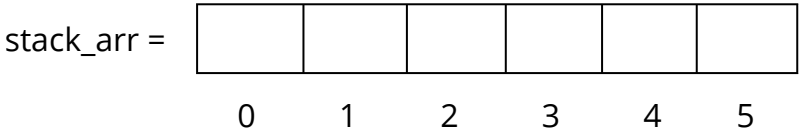
TRUE

isFull(): returns TRUE if the stack is full, else returns FALSE.



TRUE FALSE

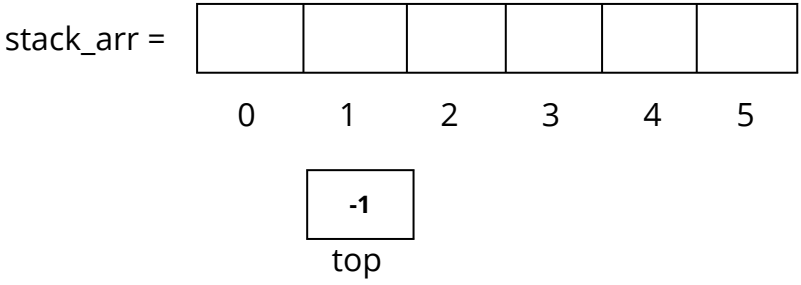
- **Array Implementation of Stacks:**



We know that stack_arr[ ] is an array and insertion and deletion operations can be performed at any place but we want the stack_arr[ ] to behave like a stack.
Hence, the insertion and deletion must be performed at the end.

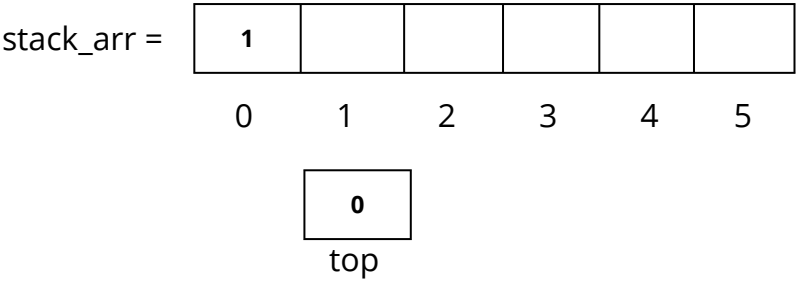
For this, we will keep a variable top which will keep track of the last inserted element or the topmost element in an array.

Empty Stack:



To indicate that the stack is empty, we will put -1 in the variable top.
Note: top = 0 indicates that the topmost element is at index 0 and this means is only one element in the stack.

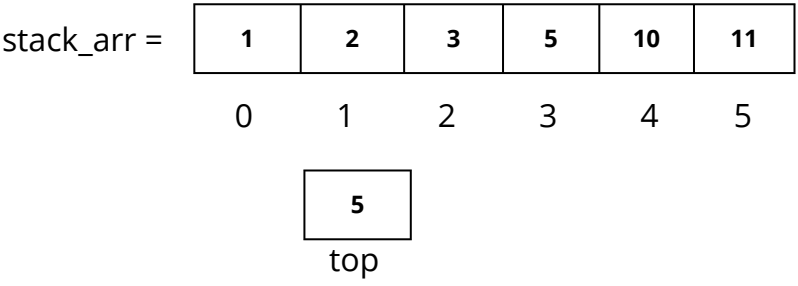
push(1):



For the push operation:

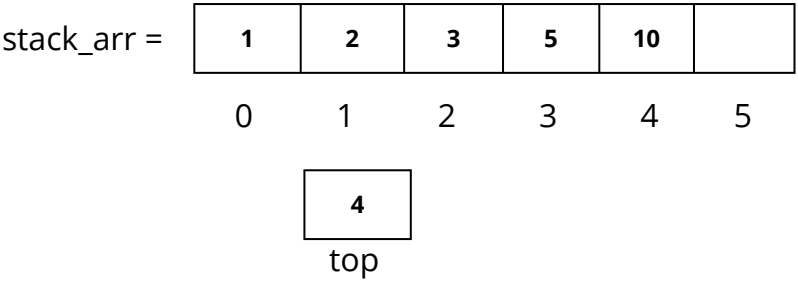
- top is incremented by 1.
- New element is pushed at the position top.

Here,



push(2)
push(3)
push(5)
push(10)
push(11)
push(12) ----------> stack overflow (New elements can't be pushed)

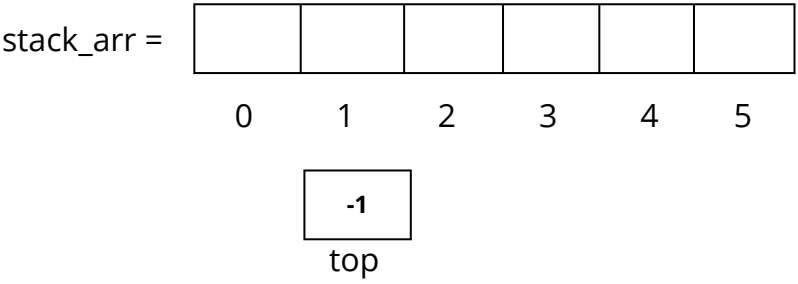
pop():



For the pop operation:

- The element at the position of top is deleted.
- top is decremented by 1.

Here,



pop()
pop()
pop()
pop()
pop()
pop() ----------> stack is empty. So, it can't be poped.

- **Array Implementation of Stacks in C :**

```
#include<stdio.h>
#include<stdlib.h>

#define MAX 4

int stack_arr[MAX];
int top = -1;

int isFull(){
   if(top == MAX - 1){
      return 1;
   }
   return 0;
}

int isEmpty(){
   if(top == -1){
      return 1;
   }
   return 0;
}

void push(int data){
   if(isFull()){
      printf("Stack overflow\n");
      return;
   }
   top = top + 1;
   stack_arr[top] = data;
}
```

```
int pop(){
    if(isEmpty()){
        printf("Stack underflow\n");
        exit(1);
    }
    int value = stack_arr[top];
    top--;
    return value;
}

int getTop() {
    if (isEmpty()) {
        printf("Stack is empty. Cannot get top.\n");
        return -1;
    }
    return stack_arr[top];
}

int size(){
    if (isEmpty()) {
        return 0;
    }
    return top+1;
}

void printStack(){
    printf("[");
    for(int i=top; i>=0; i--){
        printf("%d", stack_arr[i]);
        if(i != 0){
            printf(", ");
```

```
        }
    }
    printf("]\n");
}

int main(){
    int data;
    printStack();
    push(1);
    push(2);
    push(3);
    push(4);
    printStack();
    printf("%d is top value\n", getTop());
    printf("%d is size of the stack\n", size());
    push(5);
    data = pop();
    printf("%d get popped\n", data);
    printStack();
    printf("%d is size of the stack\n", size());
    pop();
    printStack();
    data = pop();
    printf("%d get popped\n", data);
    printStack();
    data = pop();
    printf("%d get popped\n", data);
    printStack();
    data = pop();
    printf("%d get popped\n", data);
}
```

Output:

[]
[4, 3, 2, 1]
4 is top value
4 is size of the stack
Stack overflow
4 get popped
[3, 2, 1]
3 is size of the stack
[2, 1]
2 get popped
[1]
1 get popped
[]
Stack underflow

- **Array Implementation of Stacks in C++ :**

```
#include <iostream>
using namespace std;

class Stack {
private:
  int maxSize;
  int topIndex;
  int* stackArray;

public:
  Stack(int size) {
    maxSize = size;
    stackArray = new int[maxSize];
    topIndex = -1;
  }

  ~Stack() {
    delete[] stackArray;
  }

  void push(int data) {
    if (isFull()) {
      cout << "Stack is full. Cannot push." << endl;
      return;
    }
    stackArray[++topIndex] = data;
  }
```

```
  int pop() {
    if (isEmpty()) {
      cout << "Stack is empty. Cannot pop." << endl;
      return -1;
    }
    return stackArray[topIndex--];
  }

  int top() {
    if (isEmpty()) {
      cout << "Stack is empty. Cannot get top." << endl;
      return -1;
    }
    return stackArray[topIndex];
  }

  int size() {
    return topIndex + 1;
  }

  bool isEmpty() {
    return topIndex == -1;
  }

  bool isFull() {
    return topIndex == maxSize - 1;
  }
};

int main() {
  Stack stack(5);
```

```
  stack.push(10);
  stack.push(20);
  stack.push(30);

  cout << "Top element: " << stack.top() << endl;
  cout << "Stack size: " << stack.size() << endl;

  stack.pop();

  cout << "Top element after pop: " << stack.top() <<
endl;
  cout << "Is stack empty? " << (stack.isEmpty() ?
"Yes" : "No") << endl;
  cout << "Is stack full? " << (stack.isFull() ? "Yes" :
"No") << endl;

  return 0;
}
```

Output:

```
Top element: 30
Stack size: 3
Top element after pop: 20
Is stack empty? No
Is stack full? No
```

- **Linked List Implementation of Stacks in C :**

```
//Linklist implementation with stack
#include<stdio.h>
#include<stdlib.h>
typedef struct node{
  int data;
  struct node *next;
}node;
node *top=NULL;
void push(int a){
  node *temp=(node*)malloc(sizeof(node));
  if(temp==NULL){
  printf("Stack is full\n");
  return;
  }
  temp->data=a;
  temp->next=top;
  top=temp;
}
void pop(){
  if(top==NULL){
    printf("Stack is empty\n");
    return;
  }
  node *del=NULL;
  del=top;
  top=top->next;
  free(del);

}
```

```
void display(){
 node *list=top;
 while(list!=NULL){
  printf("%d ",list->data);
  list=list->next;
 }
 printf("\n");
}

void printrev(node *p){
  if(p==NULL){
    return;
  }
  printrev(p->next);
  printf("%d ",p->data);
}

int main(){
  push(10);
  push(20);
  push(30);
  display();
  printrev(top);
  pop();
  printf("\n");
  display();
}
```

Output:

```
30 20 10
10 20 30
20 10
```

- **Linked List Implementation of Stacks in C :**

```
//Linklist implementation with stack
#include <stdio.h>
#include <stdlib.h>
#include<limits.h>

struct Node
{
  int data;
  struct Node *next;
};

typedef struct Node Node;
Node *top = NULL;

int peek()
{
  return top ? top->data : INT_MIN;
}

void push(int x)
{
  Node *temp = (Node *)malloc(sizeof(Node));
  if (temp == NULL)
  {
    printf("Error: Stack is full!!! No other element can
be inserted\n");
    return;
  }
  temp->data = x;
```

```
  temp->next = top;
  top = temp;
}

void pop()
{
  if (top == NULL)
  {
    printf("Error: Stack is Empty!! No Element to be
popped out\n");
    return;
  }
  Node *temp = top;
  top = top->next;
  free(temp);
}

void printRev(Node* p){
  if (p == NULL)return;
  printRev(p->next);
  printf(" %d", p->data);
}

void print()
{
  printf("Stack: ");
  printRev(top);
  printf(" \n");
}
```

```
int main()
{
  printf("Top element of the stack = %i\n", peek());
  print();
  push(5);
  print();
  push(7);
  print();
  push(9);
  print();
  pop();
  print();
  pop();
  print();
  printf("Top element of the stack = %i\n", peek());
  return 0;
}
```

Output:

```
Top element of the stack = -2147483648
Stack:
Stack:  5
Stack:  5 7
Stack:  5 7 9
Stack:  5 7
Stack:  5
Top element of the stack = 5
```

- **Stack Application:**

- **Application of Stacks (Nested Brackets):**

- **Expression Evaluation and Conversion:**

5 + 2
<Operand> <Operator> <Operand>

p * q
(5+2) * 6
(5+2) * (6/2)

Operator Precedence:

1. Parenthesis [ ], {}, ()
2. Exponent (right to left)
3. Division/Multiplication (left to right)
4. Addition/Subtraction (left to right)

- 5 - 2 * 3 / 6 - 10 + 2 / 7

=> 5 - { (2 * 3) / 6 } - 10 + (2 / 7)

Infix : 5*3 <operand> <op> <operand>
Prefix : *53 <op> <operand> <operand>
Postfix : 53* <operand> <operand> <op>

- **Evaluation of Postfix Expression using stack:**

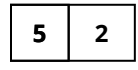
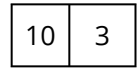
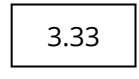
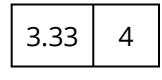
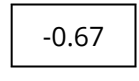
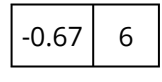
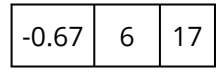
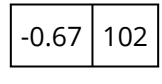
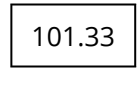
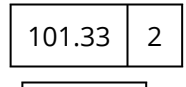
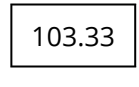
Expression:
5 * 2 / 3 - 4 + 6 * 17 + 2

- **Infix to Postfix:**

5 * 2 / 3 - 4 + 6 * 17 + 2
= { (5 * 2) / 3 } - 4 + (6 * 17) + 2
= { (52*) / 3 } - 4 + (6,17*) + 2
= { 52*3/ } - 4 + (6,17*) + 2
= (52*3/4 -) + (6,17*) + 2
= (52*3/4 - 6,17* +) + 2
= 52*3/4 - 6,17* +2+ -----> Postfix

Evaluate Postfix Expression:

52*3/4 - 6,17* +2+

Symbol	Stack
5	
52	
52*	 op1=5, op2=2 = Push res=(op1*op2)=10
52*3	
52*3/	
52*3/4	
52*3/4-	
52*3/4-6	
52*3/4-6 17	
52*3/4-6 17*	
52*3/4-6 17*+	
52*3/4-6 17*+2	
52*3/4-6 17*+2+	

- **Algorithm Evaluate Postfix Expression:**

- **Evaluation of Prefix Expression using stack:**

A*B+C*D-E
= (A*B) + (C*D) - E
= (*AB) + (*CD) - E
= { + (*AB) (*CD) } - E
= - { +(*AB) (*CD) }E
= -+*AB*CDE

[Prefix Expression]

Let,
A=2
B=3
C=4
D=5
E=7

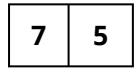
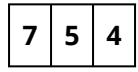
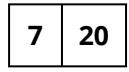
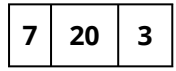
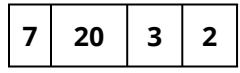
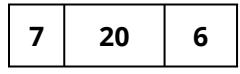
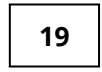
Evaluate Infix Expression:

A*B+C*D-E
= 2*3+4*5-7
= 6+20-7
= 19

Evaluate Prefix Expression:

-+*AB*CDE

-+*23*457

Symbol	Stack	
7		
5		
4		
*		op1=4, op2=5, res=20
3		
2		
*		op1=2, op2=3, res=6
+		op1=6, op2=20, res=26
-		op1=26, op2=7, res=19

- **Algorithm Evaluate Prefix Expression:**

- **Converting Expression:**

- **Infix** → Postfix, Prefix

- **Postfix** → Infix, Prefix

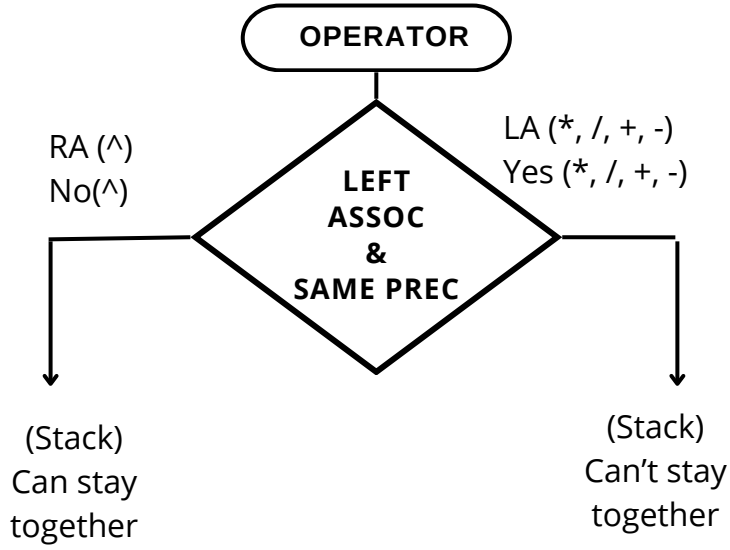
- **Prefix** → Infix, Postfix

Note:

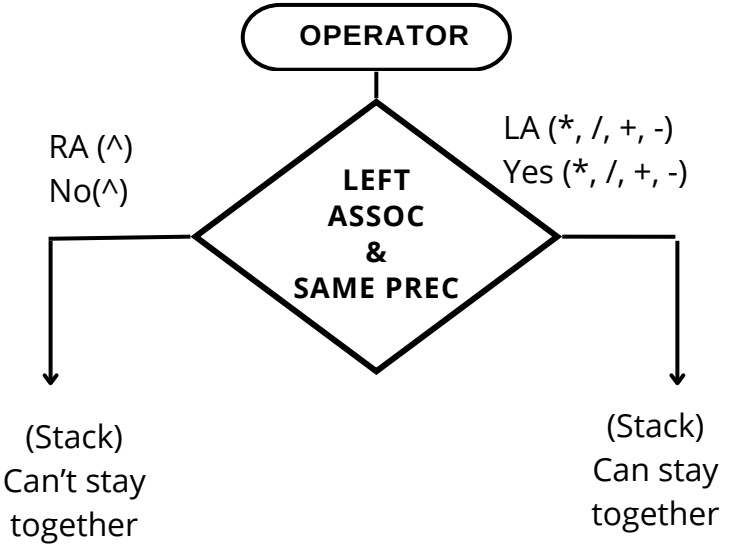
- For converting Postfix to Infix or Prefix to Infix we must need to use bracket (()) among two operands.

- For converting Postfix to Prefix or Prefix to Postfix we don't need to use bracket (()) among two operands.

- **Infix to Postfix:**



- **Infix to Prefix:**



- **Infix to Postfix:**

Scan the symbol of the expression from left to right and for each symbol, do the following:

a. If the symbol is an operand :
=> Print that symbol onto the screen.

b. If the symbol is a left parenthesis :
=> Posh it on the stack.

c. If the symbol is a right parenthesis :
=> Pop all the operators from the stack up to the first left parenthesis and store the operators in the postfix array.
=> Discard the left and right parenthesis.

d. If the symbol is an operator :
=> If the precedence of the operators in the stack are greater than or equal to the current operator, then pop the operators out of the stack and print them onto the screen, and push the current operator onto the stack.
=> else
Push the current operator onto the stack.

- **Convert Infix Expression to Postfix using Stack:**

Example:
A+B*C-D/E ---> Postfix ABC*+DE/-

Symbol	**Stack**	**Postfix**
A		A
+	+	A
B	+	AB
*	+*	AB
C	+*	ABC
-	-	ABC*+
D	-	ABC*+D
/	-/	ABC*+D
E	-/	ABC*+DE
	-	ABC*+DE/
		ABC*+DE-

- **Convert Infix Expression to Postfix using Stack:**

Example:
P*Q-D/E+(F-H)/G+(H*E)

Infix:
P*Q-D/E+(F-H)/G+(H*E)
= P*Q-D/E+(FH-)/G+(HE*)
= (P*Q)-(D/E)+{(FH-)/G}+(HE*)
= (PQ*)-(DE/)+{(FH-)G/}+(HE*)
= {(PQ*) (DE/)-} + {(FH-)G/} + (HE*)
= [{(PQ*) (DE/)-} {(FH-)G/}+] + (HE*)
= [{(PQ*) (DE/)-}{(FH-)G/}] (HE*)+
= PQ*DE/-FH-G/+HE*+
[Postfix Expression]

or,

P*Q-D/E+(F-H)/G+(H*E)
= P*Q-D/E+(FH-)/G+(HE*)
= (P*Q)-(D/E)+{(FH-)/G}+(HE*)
= (PQ*)-(DE/)+(FH-G/)+(HE*)
= (PQ*DE/-) + (FH-G/) + (HE*)
= (PQ*DE/-FH-G/+) + (HE*)
= (PQ*DE/-FH-G/+HE*+)
= PQ*DE/-FH-G/+HE*+

Conversion of Infix to Postfix Expression:

P*Q-D/E+(F-H)/G+(H*E)

Symbol	Stack	Postfix
P		P
*	*	P
Q	*	PQ
-	-	PQ*
D	-	PQ*D
/	-/	PQ*D
E	-/	PQ*DE
+	+	PQ*DE/-
(	+(	PQ*DE/-
F	+(	PQ*DE/-F
-	+(-	PQ*DE/-F
H	+(-	PQ*DE/-FH
)	+	PQ*DE/-FH-
/	+/	PQ*DE/-FH-

G	+/	PQ*DE/-FH-G
+	+	PQ*DE/-FH-G/+
(	+(	PQ*DE/-FH-G/+
H	+(	PQ*DE/-FH-G/+H
*	+(*	PQ*DE/-FH-G/+H
E	+(*	PQ*DE/-FH-G/+HE
)	+	PQ*DE/-FH-G/+HE*
		PQ*DE/-FH-G/+HE*+

- **Convert Infix Expression to Postfix using Stack:**

Example:

F*(C^A^B)

Infix:

F*(C^A^B)

= F*{C^(AB^)} [^ --> right to left Associativity]

= F*{C(AB^)^}

= FCAB^^*

Conversion of Infix to Postfix Expression:

F*(C^A^B)

Symbol	Stack	Postfix
F		F
*	*	F
(	*(	F
C	*(	FC
^	*(^	FC
A	*(^	FCA
^	*(^^	FCA
B	*(^^	FCAB
)	*	FCAB^^
		FCAB^^*

- **Convert Infix Expression to Postfix using Stack:**

Example:
P*Q-D/E+(F-H+D+Q^P^E)+(H*E/P*Q)

Infix:
P*Q-D/E+(F-H+D+Q^P^E)+(H*E/P*Q)
=
[Postfix Expression]

or,

P*Q-D/E+(F-H)/G+(H*E)
= P*Q-D/E+(FH-)/G+(HE*)
= (P*Q)-(D/E)+{(FH-)/G}+(HE*)
= (PQ*)-(DE/)+(FH-G/)+(HE*)
= (PQ*DE/-) + (FH-G/) + (HE*)
= (PQ*DE/-FH-G/+) + (HE*)
= (PQ*DE/-FH-G/+HE*+)
= PQ*DE/-FH-G/+HE*+

Conversion of Infix to Postfix Expression:

P*Q-D/E+(F-H)/G+(H*E)

Symbol	Stack	Postfix
P		P
*	*	P
Q	*	PQ
-	-	PQ*
D	-	PQ*D
/	-/	PQ*D
E	-/	PQ*DE
+	+	PQ*DE/-
(	+(	PQ*DE/-
F	+(	PQ*DE/-F
-	+(-	PQ*DE/-F
H	+(-	PQ*DE/-FH
)	+	PQ*DE/-FH-
/	+/	PQ*DE/-FH-

G	+/	PQ*DE/-FH-G
+	+	PQ*DE/-FH-G/+
(	+(	PQ*DE/-FH-G/+
H	+(	PQ*DE/-FH-G/+H
*	+(*	PQ*DE/-FH-G/+H
E	+(*	PQ*DE/-FH-G/+HE
)	+	PQ*DE/-FH-G/+HE*
		PQ*DE/-FH-G/+HE*+

- **Algorithm of Converting Infix to Postfix Expression:**

```
#include <stdio.h>
#include <string.h>
#include <stdbool.h>

#define MAX 20

int stack[MAX];
int top = -1;

bool isOperator(char ch) {
   return ch == '+' || ch == '-' || ch == '*' ||
                  ch == '/' || ch == '^';
}

int prec(char ch) {
   switch(ch) {
      case '+':
      case '-':
         return 1;
      case '*':
      case '/':
         return 2;
      case '^':
         return 3;
      default:
         return -1;
   }
}
```

```
int isEmpty() {
   return top == -1;
}

int isFull() {
   return top == MAX-1;
}

void push(int x) {
   if(isFull()) {
      printf("Error: Stack is Full!!!\n");
      return;
   }
   stack[++top] = x;
}

int pop() {
   if(isEmpty()) {
      printf("Error: Stack is Empty!!!\n");
      return -1;
   }
   return stack[top--];
}

bool isBalanced(char exp[]) {
   for(int i = 0; i < strlen(exp); i++) {
      char ch = exp[i];
      if(ch == '(') {
         push(ch);
      } else if(ch == ')') {
         if(isEmpty()) {
```

```
            return false;
         }
         pop();
      }
   }

   return isEmpty();
}

void infixToPostfix(char infix[], char postfix[]) {
   int j = 0;

   for(int i = 0; i < strlen(infix); i++) {
      char ch = infix[i];
      if(ch == '(') {
         push(ch);
      } else if(ch == ')') {
         while(!isEmpty() && stack[top] != '(') {
            postfix[j++] = pop();
         }
         pop();
      } else if(isOperator(ch)) {
         while(!isEmpty() && prec(stack[top]) >= prec(ch)) {
            postfix[j++] = pop();
         }
         push(ch);
      } else {
         postfix[j++] = ch;
      }
   }
```

```
    while(!isEmpty()) {
        postfix[j++] = pop();
    }
    postfix[j] = '\0';
}

void printStack() {
    printf("Stack:");
    for(int i = 0; i <= top; i++) {
        printf(" %i", stack[i]);
    }
    printf("\n");
}

int main()
{
    char exp[MAX];
    printf("Expression dao:\n");
    scanf("%s", exp);

    if(isBalanced(exp)) {
        printf("Sob kichu thik thak ase...");
    } else {
        printf("Bracket thik koro mia!!!...");
    }

    char infix[MAX], postfix[MAX];
    printf("Infix expression dao:\n");
    scanf("%s", infix);
```

```
    infixToPostfix(infix, postfix);
    printf("%s", postfix);

    return 0;
}
```

Output:

```
Expression dao:
(12+5)-(56/7
Bracket thik koro mia!!!...Infix expression dao:
(12+5)-(56/7)
125+567/-(
```

- **Convert Infix Expression to Prefix using Stack:**

Example:
(M+N)*D-A+B

Infix:
(M+N)*D-A+B
= (+MN)*D-A+B
= {*(+MN)}-A+B
= [-{*(+MN)D}A]+B
= +-*+MNDAB

[Prefix Expression]

Conversion of Infix to Prefix Expression:

Reverse : B+A-D*(N+M)

Symbol	Stack	Prefix
B		B
+	+	B
A	+	BA
-	+-	BA
D	+-	BAD
*	+-*	BAD

(	+-*(	BAD
N	+-*(	BADN
+	+-*(+	BADN
M	+-*(+	BADNM
)	+-*	BADNM+
	+-	BADNM+*
	+	BADNM+*-
		BADNM+*-+

After reverse, Prefix : +-*+MNDAB

- **Convert Infix Expression to Prefix using Stack:**

Example:
F*(C^A^B)

Infix:
F*(C^A^B)
= F*{C^(^AB)}
= F*{^C(^AB)}
= *F^C^AB

[Prefix Expression]

Conversion of Infix to Prefix Expression:

Reverse : (B^A^C)*F

Symbol	Stack	Prefix
(	(	
B	(	B
^	(^	B
A	(^	BA
^	(^	BA^
C	(^	BA^C

)		BA^C^
*	*	BA^C^
F		BA^C^F
		BA^C^F*

After reverse, Prefix : *F^C^AB

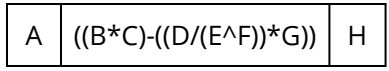
- **Algorithm of Converting Infix to Prefix Expression:**

- **Convert Postfix Expression to Infix using Stack:**

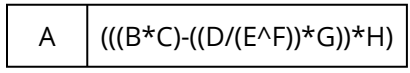
Postfix:
ABC*DEF^/G*-H*+

Symbol	Stack
A	A
B	A | B
C	A | B | **C**
*	A | (B*C)
D	A | (B*C) | D
E	A | (B*C) | D | E
F	A | (B*C) | D | E | F
^	A | (B*C) | D | (E^F)
/	A | (B*C) | (D/(E^F))
G	A | (B*C) | (D/(E^F)) | G
*	A | (B*C) | ((D/(E^F))*G)
-	A | ((B*C)-((D/(E^F))*G))

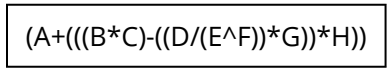
H



*



+

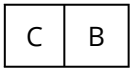
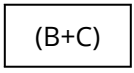
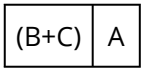
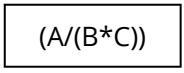


Postfix: (A+(((B*C)-((D/(E^F))*G))*H))

- **Algorithm of Converting Postfix to Infix Expression:**

- **Convert Prefix Expression to Infix using Stack:**

Prefix:
/A+BC

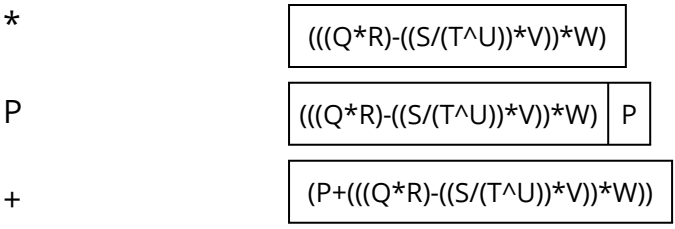
Symbol	**Stack**
C	
B	
+	
A	
/	

Prefix: (A/(B*C))

- **Convert Prefix Expression to Infix using Stack:**

Prefix:
+P*-*QR*/S^TUVW

Symbol	**Stack**
W	W
V	W | V
U	W | V | U
T	W | V | U | T
^	W | V | (T^U)
S	W | V | (T^U) | S
/	W | V | (S/(T^U))
*	W | ((S/(T^U))*V)
R	W | ((S/(T^U))*V) | R
Q	W | ((S/(T^U))*V) | R | Q
*	W | ((S/(T^U))*V) | (Q*R)
-	W | ((Q*R)-((S/(T^U))*V))

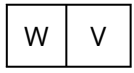
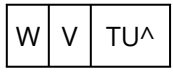
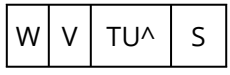
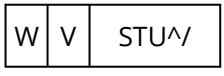
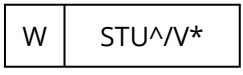
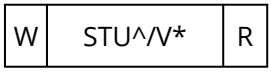
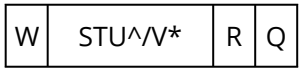
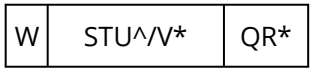


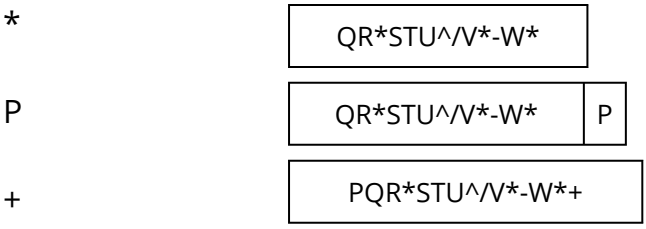
Prefix: (P+(((Q*R)-((S/(T^U))*V))*W))

- **Algorithm of Converting Prefix to Infix Expression:**

- **Convert Prefix Expression to Postfix using Stack:**

Prefix:
+P*-*QR*/S^TUVW

Symbol	**Stack**
W	
V	
U	
T	
^	
S	
/	
*	
R	
Q	
*	
-	



Postfix: PQR*STU^/V*-W*+

- **Algorithm of Converting Prefix to Postfix Expression:**

- **Convert Postfix Expression to Prefix using Stack:**

Postfix:
PQR*STU^/V*-W*+

Symbol	Stack
P	P
Q	P | Q
R	P | Q | R
*	P | *QR
S	P | *QR | S
T	P | *QR | S | T
U	P | *QR | S | T | U
^	P | *QR | S | ^TU
/	P | *QR | /S^TU
V	P | *QR | /S^TU | V
*	P | *QR | */S^TUV
-	P | -*QR*/S^TUV

W

P	-*QR*/S^TUV	W

*

P	*-*QR*/S^TUVW

+

+P*-*QR*/S^TUVW

Prefix: +P*-*QR*/S^TUVW

- **Algorithm of Converting Postfix to Prefix Expression:**

- **Bracket problem of expression :**

```
#include <stdio.h>
#include <string.h>
#include <stdbool.h>

#define MAX 20

int stack[MAX];
int top = -1;

int isEmpty() {
   return top == -1;
}

int isFull() {
   return top == MAX-1;
}

void push(int x) {
   if(isFull()) {
      printf("Error: Stack is Full!!!\n");
      return;
   }
   stack[++top] = x;
}

int pop() {
   if(isEmpty()) {
      printf("Error: Stack is Empty!!!\n");
      return -1;
```

```
    }
    return stack[top--];
}

void printStack() {
    printf("Stack:");
    for(int i = 0; i <= top; i++) {
        printf(" %i", stack[i]);
    }
    printf("\n");
}

int isBalanced(char exp[]){
    for(int i=0; i<strlen(exp); i++)
    {
        char ch = exp[i];
        if(ch=='('){
            push(ch);
          }
        else if(ch==')'){
            if(isEmpty()){
                    printf("%d position ) beshi keno??",i);
                return 0;
            }
            pop();
        }
    }
    return isEmpty();
}
int main()
{
```

```
    char exp[MAX];
    printf("Expression dao\n");
    scanf("%s",exp);
    if(isBalanced(exp)){
        printf("All is OK!\n");
    }
    else{
        printf("braket thik kor Mia :D\n");
    }

     printf("%d",strlen(exp));



    return 0;
}
```

Output:

```
Expression dao
(1+8)-(45/
braket thik kor Mia :D
10
```

- **Bracket problem(with position) of expression :**

```
#include <stdio.h>
#include <string.h>
#include <stdbool.h>

#define MAX 20

int stack[MAX];
int top = -1;

int isEmpty() {
   return top == -1;
}

int isFull() {
   return top == MAX-1;
}

void push(int x) {
   if(isFull()) {
      printf("Error: Stack is Full!!!\n");
      return;
   }
   stack[++top] = x;
}

int pop() {
   if(isEmpty()) {
      printf("Error: Stack is Empty!!!\n");
      return -1;
   }
```

```
   return stack[top--];
}

void printStack() {
   printf("Stack:");
   for(int i = 0; i <= top; i++) {
      printf(" %i", stack[i]);
   }
   printf("\n");
}

int isBalanced(char exp[]){
   for(int i=0; i<strlen(exp); i++)
   {
      char ch = exp[i];
      if(ch=='('){
         push(ch);
        }
      else if(ch==')'){
         if(isEmpty()){
               printf("%d position ) beshi keno??",i);
            return 0;
         }
         pop();
      }
   }
   return isEmpty();
}
int main()
{
   char exp[MAX];
```

```
    printf("Expression dao\n");
    scanf("%s",exp);
    if(isBalanced(exp)){
        printf("All is OK!\n");
    }
    else{
        printf("braket thik kor Mia :D\n");
    }

    printf("%d",strlen(exp));



    return 0;
}
```

Output:

```
Expression dao
(46+7)+8-)
9 position ) beshi keno??braket thik kor Mia :D
10
```

2. Queues

Definition of queue:

A queue is a linear data structure in which elements are inserted from one end called the rear end and deleted from another end called the front end.

It follows FIFO (First In First Out) technique. Insertion in the queue is called enqueue and deletion in the queue is called dequeue.

Queues can be implemented in two ways: Array-based queues and list-based queues.

Queue Implementations:

- Queue As Array
- Queue As Circular Array
- Queue As Linked List

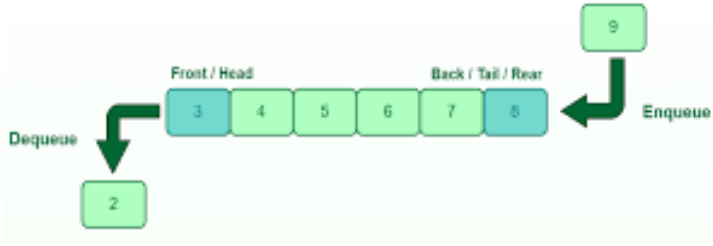
In array-based queues are implemented using the arrays. In list-based queues are implemented using a linked list.

- **Array-Based vs List-Based Queues:**

Sno.	Array-Based Queues	List-Based Queues
1.	The size of the queue should be known in advance.	It's not necessary to know the size of the queue in advance.
2.	Insertion at the end is easier but insertion at the beginning is difficult.	Insertion at both end and beginning is easy.
3.	Elements can be accessed randomly.	Elements can be accessed sequentially only.
4.	Resizing array-based queues is complex.	Resizing list-based queues is simple.
5.	Requires less memory.	Requires more memory.
6.	It is faster compared to list-based queues.	It is slower compared to array-based queues.

Property	Array-Based Queues	List-Based Queues
Underlying Data Structure	Array	Linked List
Access Time	Constant Time	O(n) Time
Memory Management	Fixed Size, May Require Resizing	Dynamic Size
Insertion/Deletion	Rear: O(1) Time, Front: O(n) Time	Rear: O(1) Time, Front: O(1) Time
Implementation	Easier to Implement	More Complex to Implement
Advantages	Constant Access Time, Efficient for Small Queues	Dynamic Size, Efficient for Large Queues
Disadvantages	Fixed Size, May Require Resizing, Inefficient for Large Queues	Slower Access Time, More Complex to Implement

Queues are linear data structures in which we add elements to one end and remove them from the other end.



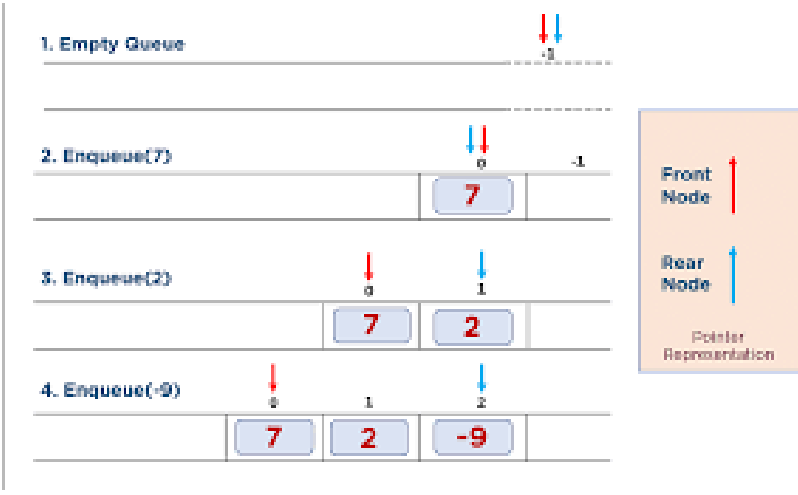
Queue Data Structure

The first item to be inserted (en-queued) is the first to be deleted (de-queued). A queue is therefore called a First In First Out (FIFO) data structure.

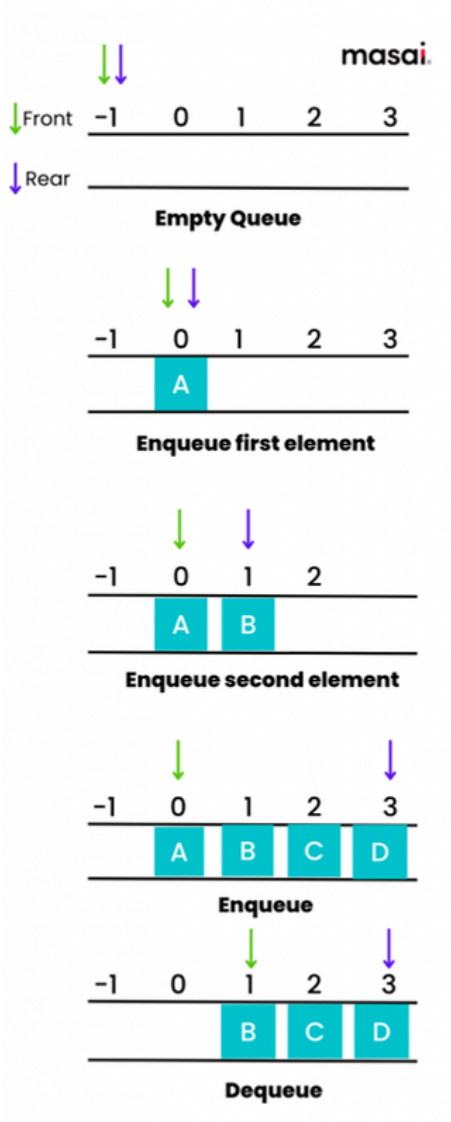
Queue operations:

- **Enqueue(value):** insert an element at the rear of the queue
- **Dequeue():** delete an element at the front of the queue
- **GetHead():** get the element at the front without deleting it

- **Enqueue(value):**



- **Dequeue():**


masai
Front
Rear
-1 0 1 2 3
Empty Queue
A
Enqueue first element
B
Enqueue second element
C D
Enqueue
Dequeue

- **Queue Implementation :**

Queue Implementations:

- Queue As Array
- Queue As Circular Array
- Queue As Linked List

- **QueueAsArray Implementation :**

```
#include <stdio.h>

#define MAX_SIZE 4

int queue[MAX_SIZE];
int front = -1;
int rear = -1;

int isFull() {
   return rear == MAX_SIZE-1;
}

int isEmpty() {
   return front == -1 && rear == -1;
}

void enqueue(int x) {
   if(isFull()) {
      printf("Error: Queue is full!!!\n");
      return;
   } else if(isEmpty()) {
      front = rear = 0;
   } else {
      rear++;
   }
   queue[rear] = x;
}
```

```
void dequeue() {
   if(isEmpty()) {
      return;
   } else if(front == rear) {
      front = rear = -1;
   } else {
      front++;
   }
}

int peek() {
   return queue[front];
}

void printQueue() {
   printf("Queue:");
   if(!isEmpty()) {
      for(int i = front; i <= rear; i++)
         printf(" %i", queue[i]);
   }
   printf("\n");
}


int main()
{
   printQueue();
   enqueue(5);
   printQueue();
   enqueue(7);
```

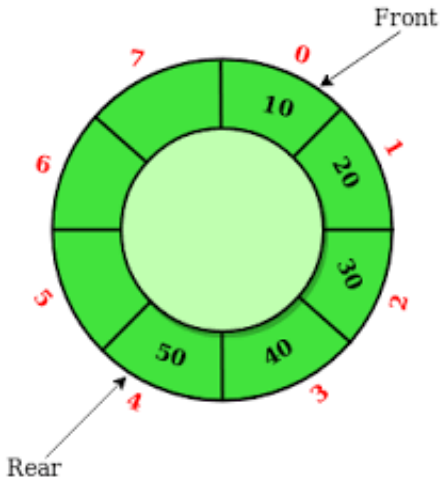
```
    printQueue();
    enqueue(4);
    printQueue();
    enqueue(5);
    printQueue();
    enqueue(17);
    printQueue();
    dequeue();
    printQueue();
    dequeue();
    printQueue();
    printf("Front Element is = %i\n", peek());

    return 0;
}
```

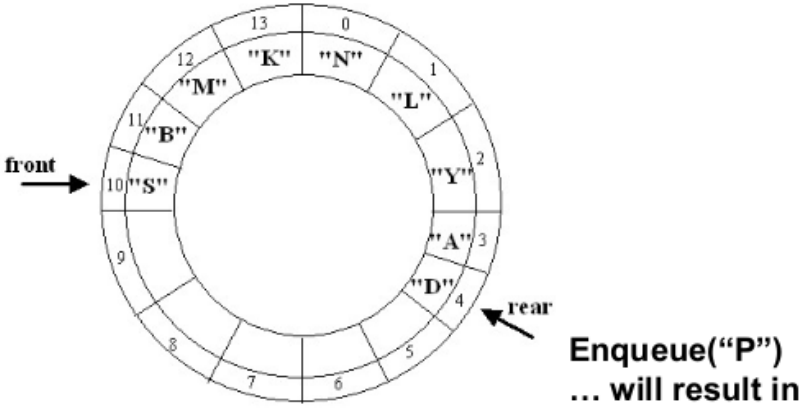
Output:

```
Queue:
Queue: 5
Queue: 5 7
Queue: 5 7 4
Queue: 5 7 4 5
Error: Queue is full!!!
Queue: 5 7 4 5
Queue: 7 4 5
Queue: 4 5
Front Element is = 4
```

- **QueueAsCircularArray Implementation :**

By using modulo arithmetic for computing array indexes, we can have a queue implementation in which each of the operations enqueue, dequeue, and getHead has complexity O(1)





DEQUEUE OPERATION

- **DynamicQueueAsArray Implementation :**

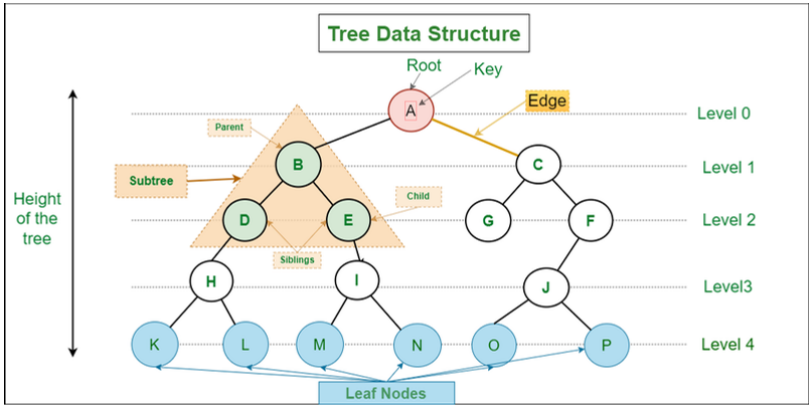
- **QueueAsLinkedList Implementation :**

3. Trees

A tree data structure is a hierarchical structure that is used to represent and organize data in a way that is easy to navigate and search. It is a collection of nodes that are connected by edges and has a hierarchical relationship between the nodes.

The topmost node of the tree is called the root, and the nodes below it are called the child nodes. Each node can have multiple child nodes, and these child nodes can also have their own child nodes, forming a recursive structure

This data structure is a specialized method to organize and store data in the computer to be used more effectively. It consists of a central node, structural nodes, and sub-nodes, which are connected via edges. We can also say that tree data structure has roots, branches, and leaves connected with one another.



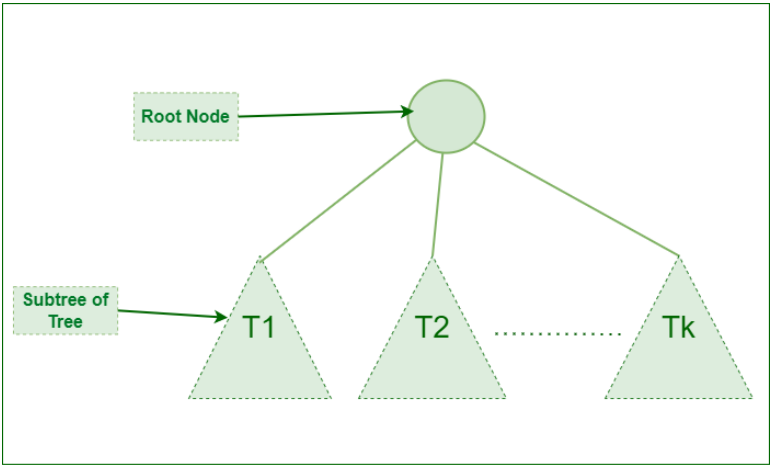
Basic Terminologies In Tree Data Structure:

- **Parent Node:** The node which is a predecessor of a node is called the parent node of that node. {B} is the parent node of {D, E}.
- **Child Node:** The node which is the immediate successor of a node is called the child node of that node. Examples: {D, E} are the child nodes of {B}.
- **Root Node:** The topmost node of a tree or the node which does not have any parent node is called the root node. {A} is the root node of the tree. A non-empty tree must contain exactly one root node and exactly one path from the root to all other nodes of the tree.
- **Leaf Node or External Node:** The nodes which do not have any child nodes are called leaf nodes. {K, L, M, N, O, P, G} are the leaf nodes of the tree.

- **Ancestor of a Node:** Any predecessor nodes on the path of the root to that node are called Ancestors of that node. {A,B} are the ancestor nodes of the node {E}
- **Descendant:** Any successor node on the path from the leaf node to that node. {E,I} are the descendants of the node {B}.
- **Sibling:** Children of the same parent node are called siblings. {D,E} are called siblings.
- **Level of a node:** The count of edges on the path from the root node to that node. Level = (height of the root - height of the node). The root node has level 0.
- **Internal node:** A node with at least one child is called Internal Node.
- **Neighbour of a Node:** Parent or child nodes of that node are called neighbors of that node.
- **Subtree:** Any node of the tree along with its descendant.
- **Height:** Maximum number of edges from the node and leaf node.

- **Representation of Tree Data Structure:**

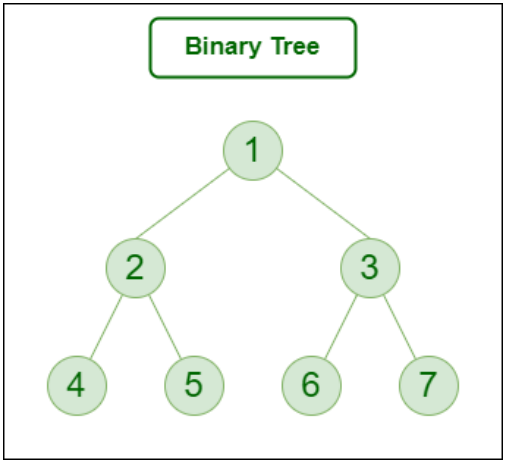
A tree consists of a root, and zero or more subtrees T1, T2, ... , Tk such that there is an edge from the root of the tree to the root of each subtree.



- **Representation of a Node in Tree Data Structure:**

```
struct Node
{
  int data;
  struct Node *first_child;
  struct Node *second_child;
  struct Node *third_child;
  .
  .
  .
  struct Node *nth_child;
};
```

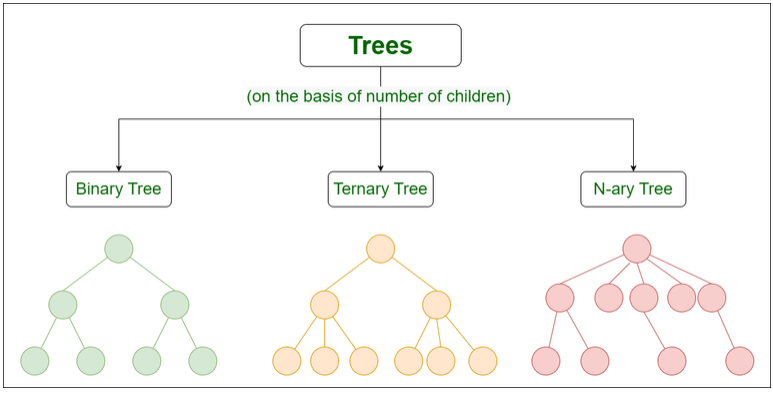
Example of Tree data structure:



Here,

- Node 1 is the root node
- 1 is the parent of 2 and 3
- 2 and 3 are the siblings
- 4, 5, 6, and 7 are the leaf nodes
- 1 and 2 are the ancestors of 5

- **Types of Tree data structures:**



1. Binary tree: In a binary tree, each node can have a maximum of two children linked to it. Some common types of binary trees include full binary trees, complete binary trees, balanced binary trees, and degenerate or pathological binary trees.

2. Ternary Tree: A Ternary Tree is a tree data structure in which each node has at most three child nodes, usually distinguished as “left”, “mid” and “right”.

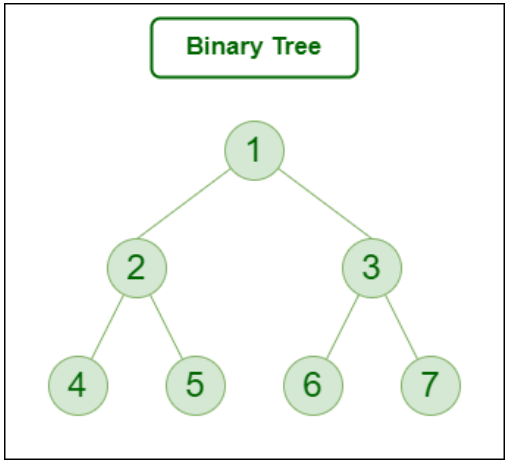
3. N-ary Tree or Generic Tree: Generic trees are a collection of nodes where each node is a data structure that consists of records and a list of references to its children(duplicate references are not allowed). Unlike the linked list, each node stores the address of multiple nodes.

1. Binary tree:

In a binary tree, each node can have a maximum of two children linked to it. Some common types of binary trees include full binary trees, complete binary trees, balanced binary trees, and degenerate or pathological binary trees.

Example:

Consider the tree below. Since each node of this tree has only 2 children, it can be said that this tree is a Binary Tree



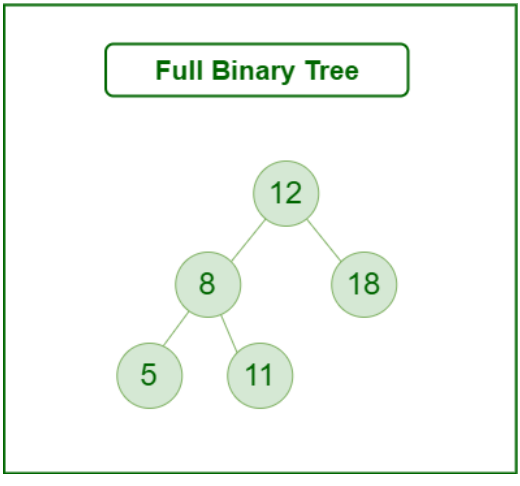
- **Types of Binary Tree:**

1. Full/ Strict Binary Tree
2. Complete Binary Tree
3. Perfect Binary Tree
4. Height Balanced Binary Tree
5. Skewed Binary Tree
6. Ordered Binary Tree

1.Full/ Strict Binary Tree:

A full binary tree is a binary tree with either zero or two child nodes for each node.

A full binary tree, on the other hand, does not have any nodes that have only one child node.



Full Binary Tree Theorem:
Let T be a nonempty, full binary tree Then:

- If T has I internal nodes, the number of leaves is L = I + 1.

This is known as the full binary tree theorem.
Facts derived from the theorem:

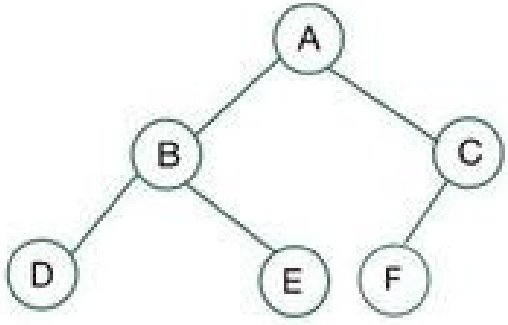
- If T has I internal nodes, the total number of nodes is N = 2I + 1.
- If T has a total of N nodes, the number of internal nodes is I = (N – 1)/2.
- If T has a total of N nodes, the number of leaves is L = (N + 1)/2.
- If T has L leaves, the total number of nodes is N = 2L – 1.
- If T has L leaves, the number of internal nodes is I = L – 1.

Some other properties:

- There are a maximum of 2k nodes in level k for every k >= 0.
- The binary tree with λ levels has maximum of (2^λ) -1 nodes.
- The binary tree with N nodes has the number of levels at least [log (N + 1)].
- The binary tree with L leaves has the number of leaves at least [log L] + 1.

2.Complete Binary Tree:

A complete binary tree is a special type of binary tree where all the levels of the tree are filled completely except the lowest level nodes which are filled from as left as possible.



Some terminology of Complete Binary Tree:

- **Root –** Node in which no edge is coming from the parent. Example -node A
- **Child –** Node having some incoming edge is called child. Example – nodes B, F are the child of A and C respectively.
- **Sibling –** Nodes having the same parent are sibling. Example- D, E are siblings as they have the same parent B.
- **Degree of a node –** Number of children of a particular parent. Example- Degree of A is 2 and Degree of C is 1. Degree of D is 0.
- **Internal/External nodes –** Leaf nodes are external nodes and non leaf nodes are internal nodes.
- **Level –** Count nodes in a path to reach a destination node. Example- Level of node D is 2 as nodes A and B form the path.
- **Height –** Number of edges to reach the destination node, Root is at height 0. Example – Height of node E is 2 as it has two edges from the root.

Properties of Complete Binary Tree:

- A complete binary tree is said to be a proper binary tree where all leaves have the same depth.
- In a complete binary tree number of nodes at depth d is 2^d.
- In a complete binary tree with n nodes height of the tree is log(n+1).
- All the levels except the last level are completely full.

3.Perfect Binary Tree:

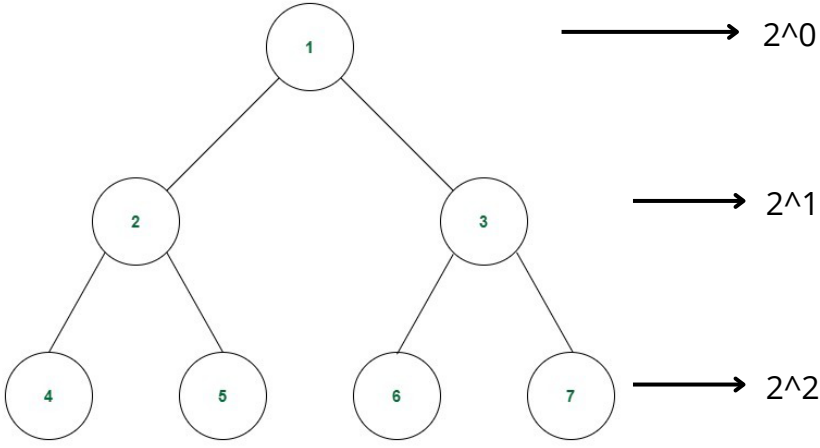
A perfect binary tree is a special type of binary tree in which all the leaf nodes are at the same depth, and all non-leaf nodes have two children. In simple terms, this means that all leaf nodes are at the maximum depth of the tree, and the tree is completely filled with no gaps.

The maximum number of nodes in a perfect binary tree is given by the formula 2^(d+1) – 1, where d is the depth/ height of the tree. This means that a perfect binary tree with a depth of n has 2^n leaf nodes and a total of 2^(n+1) – 1 nodes.

Perfect binary trees have a number of useful properties that make them useful in various applications. For example, they are often used in the implementation of heap data structures, as well as in the construction of threaded binary trees. They are also used in the implementation of algorithms such as heapsort and merge sort.

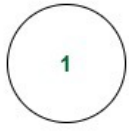
In other words, it can be said that each level of the tree is completely filled by the nodes.

Examples of Perfect Binary Tree:

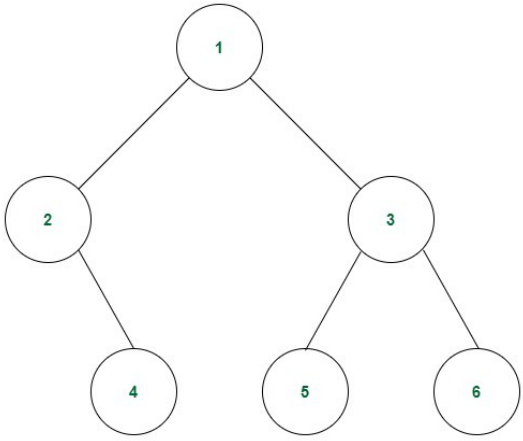


2^0 + 2^1 + 2^2 + ... + 2^h = 2^(h+1) -1
GP(Geometric Progration) series

A tree with only the root node is also a perfect binary tree.



The following tree is not a perfect binary tree because the last level of the tree is not completely filled.



Properties of a Perfect Binary Tree:

- **Degree:** The degree of a node of a tree is defined as the number of children of that node. All the internal nodes have a degree of 2. The leaf nodes of a perfect binary tree have a degree of 0.
- **Number of leaf nodes:**
  Height = h
  The number of leaf nodes = 2^h
  because the last level is completely filled.
- **Depth of a node:** Average depth of a node in a perfect binary tree is Θ(ln(n)).
- **Relation between leaf nodes and non-leaf nodes:** No. of leaf nodes = No. of non-leaf nodes +1.
- **Total number of nodes:**
  Height = h
  Total nodes = 2^(h+1) – 1.
  Each node of the tree is filled. So total number of nodes can be calculated as 2^0 + 2^1 + . . . + 2^h = 2^(h+1) – 1.
- **Height of the tree:** The height of a perfect binary tree with N number of nodes = log(N + 1) – 1 = Θ(ln(n)). This can be calculated using the relation shown while calculating the total number of nodes in a perfect binary tree.

Check whether a tree is a Perfect Binary Tree or not:

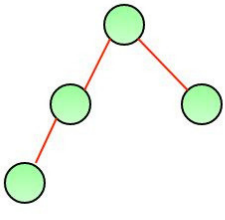
- **Check the depth of the tree:** A perfect binary tree is defined as a tree where all leaf nodes are at the same depth, and all non-leaf nodes have two children. To check whether a tree is a perfect binary tree, you can first calculate the depth of the tree.
- **Check the number of nodes at each level:** Once you have calculated the depth of the tree, you can then check the number of nodes at each level. In a perfect binary tree, the number of nodes at each level should be a power of 2 (e.g. 1, 2, 4, 8, etc.). If any level has a different number of nodes, the tree is not a perfect binary tree.

4.Height Balanced Binary Tree:

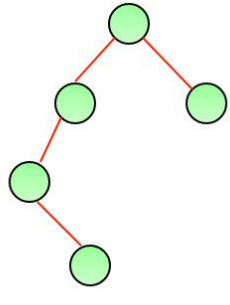
A height balanced binary tree is a binary tree in which the height of the left subtree and right subtree of any node does not differ by more than 1 and both the left and right subtree are also height balanced.

In this article, we will look into methods how to determine if given Binary trees are height-balanced

Examples:
The tree on the left is a height balanced binary tree. Whereas the tree on the right is not a height balanced tree. Because the left subtree of the root has a height which is 2 more than the height of the right subtree.



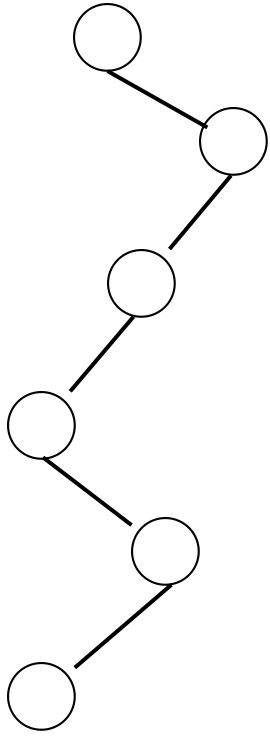
A height balanced tree



Not a height balanced tree

5.Skewed Binary Tree:

A skewed binary tree is a specific type of binary tree where each node has at most one child. In other words, it's a binary tree in which the left or right subtree of every node is empty. This results in a tree that is essentially a linked list, where each node has only one child.



The skewness of the tree depends on whether the left or right subtree is consistently chosen. If every node has a left child, it's a left-skewed binary tree; if every node has a right child, it's a right-skewed binary tree.

These skewed binary trees don't provide the advantages of a balanced binary tree in terms of search and retrieval operations, as the time complexity can degrade to O(n) in the worst case (where n is the number of nodes) instead of the more efficient O(log n) of a balanced binary tree.

6.Ordered Binary Tree:

An ordered binary tree, also known as a binary search tree (BST), is a type of binary tree with the following properties:

1. Value Ordering: Each node in the tree has a value, and for every node:

- All values in its left subtree are less than its own value.
- All values in its right subtree are greater than its own value.

2. Unique Values: No two nodes in the tree have the same value.

- These properties make searching, insertion, and deletion operations efficient, as they can be performed in O(log n) time on average, where n is the number of nodes in the tree.

- The ordered nature of the tree allows for quick searches by narrowing down the search space based on the comparison of values. It's a fundamental data structure used in many applications where efficient search is a critical requirement.

- **Uses of Binary Trees:**

1. File systems
2. Databases
3. Algorithms/ Networking
4. Maths
5. Machine Learning Algorithm
6. Future Data Structure (Graphs, Heap)

Basic Operation Of Tree Data Structure:

- **Create –** create a tree in the data structure.
- **Insert –** Inserts data in a tree.
- **Search –** Searches specific data in a tree to check whether it is present or not.
- **Traversal:**

1. **Preorder Traversal –** perform Traveling a tree in a pre-order manner in the data structure.
2. **In order Traversal –** perform Traveling a tree in an in-order manner.
3. **Post-order Traversal –** perform Traveling a tree in a post-order manner.

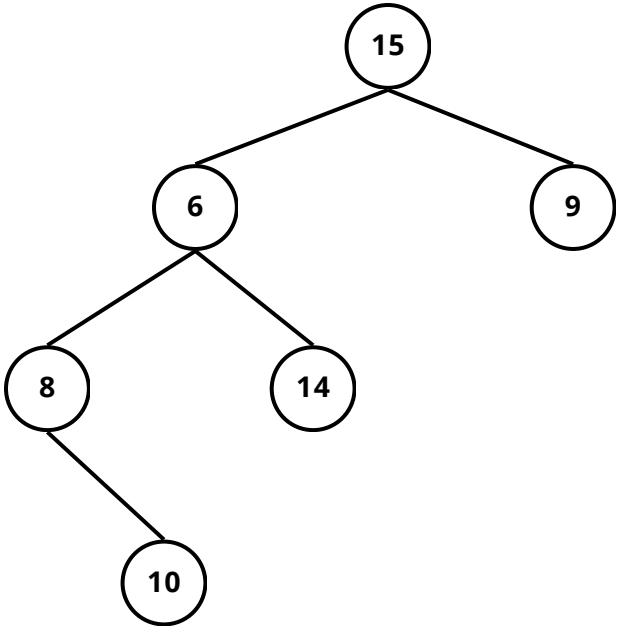
Implementation:

There are two ways to implement Trees:

1. LinkedList representation
2. Sequential representation => Using Array

1. LinkedList representation of Tree:

- **Make a program that will implement a Binary Tree and show the output of it.**



```
import java.util.Scanner;

class BinaryTree {

  public BinaryTree() {

  }

  private static class Node {
    int value;
    Node left;
    Node right;

    public Node(int value) {
      this.value = value;
    }
  }

  private Node root;

  // insert elements
  public void populate(Scanner scanner) {
    System.out.println("Enter the root Node: ");
    int value = scanner.nextInt();
    root = new Node(value);
    populate(scanner, root);
  }

  private void populate(Scanner scanner, Node
node) {
    System.out.println("Do you want to enter left of "
+ node.value);
```

```
    boolean left = scanner.nextBoolean();
    if (left) {
      System.out.println("Enter the value of the left of " +
node.value);
      int value = scanner.nextInt();
      node.left = new Node(value);
      populate(scanner, node.left);
    }

    System.out.println("Do you want to enter right of " +
node.value);
    boolean right = scanner.nextBoolean();
    if (right) {
      System.out.println("Enter the value of the right of " +
node.value);
      int value = scanner.nextInt();
      node.right = new Node(value);
      populate(scanner, node.right);
    }

  }

  // display tree's elements
  public void prettyDisplay() {
    prettyDisplay(root, 0);
  }

  private void prettyDisplay(Node node, int level) {
    if (node == null) {
      return;
    }
```

```
  prettyDisplay(node.right, level + 1);

  if (level != 0) {
    for (int i = 0; i < level - 1; i++) {
      System.out.print("|\t\t");
    }
    System.out.println("|------->" + node.value);
  } else {
    System.out.println(node.value);
  }
  prettyDisplay(node.left, level + 1);
}

public void preOrder() {
  preOrder(root);
}

private void preOrder(Node node) {
  if (node == null) {
    return;
  }
  System.out.print(node.value + " ");
  preOrder(node.left);
  preOrder(node.right);
}

public void inOrder() {
  preOrder(root);
}
```

```
  private void inOrder(Node node) {
    if (node == null) {
      return;
    }
    preOrder(node.left);
    System.out.print(node.value + " ");
    preOrder(node.right);
  }

  public void postOrder() {
    preOrder(root);
  }

  private void postOrder(Node node) {
    if (node == null) {
      return;
    }
    preOrder(node.left);
    preOrder(node.right);
    System.out.print(node.value + " ");
  }

}

public class Test {
  public static void main(String[] args) {
    Scanner scanner = new Scanner(System.in);
    BinaryTree tree = new BinaryTree();
    tree.populate(scanner);
    tree.prettyDisplay();
  }
}
```

Output:

```
Enter the root Node:
15
Do you want to enter left of 15
true
Enter the value of the left of 15
6
Do you want to enter left of 6
true
Enter the value of the left of 6
8
Do you want to enter left of 8
false
Do you want to enter right of 8
true
Enter the value of the right of 8
10
Do you want to enter left of 10
false
Do you want to enter right of 10
false
Do you want to enter right of 6
true
Enter the value of the right of 6
14
Do you want to enter left of 14
false
Do you want to enter right of 14
false
Do you want to enter right of 15
true
```

Enter the value of the right of 15
9
Do you want to enter left of 9
false
Do you want to enter right of 9
false
|------->9
15
| |------->14
|------->6
| | |------->10
| |------->8

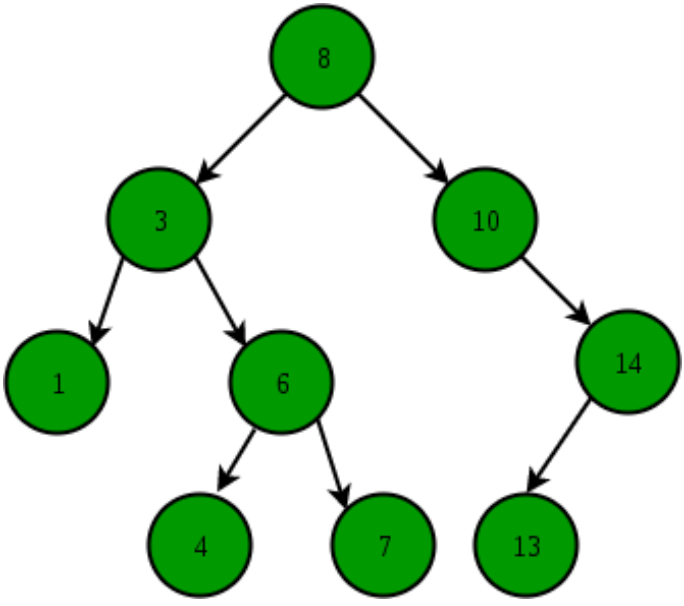
- **Binary Search Tree:**

A Binary Search Tree (BST) is a special type of binary tree in which the left child of a node has a value less than the node's value and the right child has a value greater than the node's value. This property is called the BST property and it makes it possible to efficiently search, insert, and delete elements in the tree.

The root of a BST is the node that has the smallest value in the left subtree and the largest value in the right subtree. Each left subtree is a BST with nodes that have smaller values than the root and each right subtree is a BST with nodes that have larger values than the root.

Binary Search Tree is a node-based binary tree data structure that has the following properties:

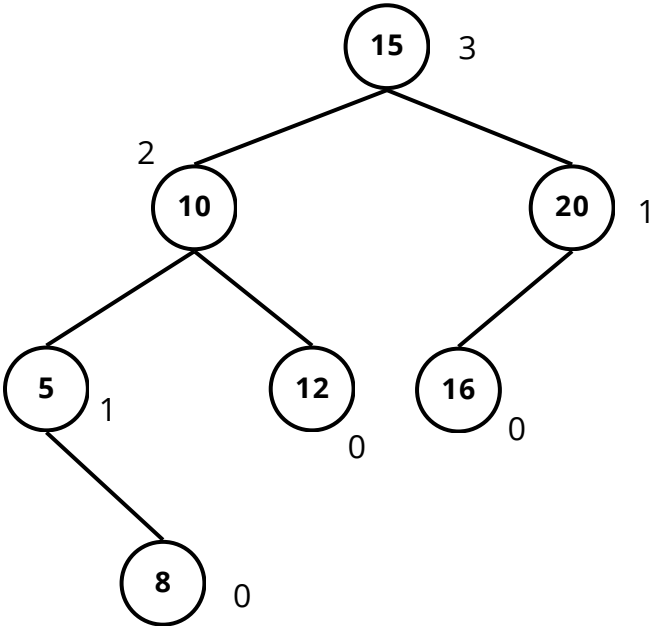
- The left subtree of a node contains only nodes with keys lesser than the node's key.
- The right subtree of a node contains only nodes with keys greater than the node's key.
- This means everything to the left of the root is less than the value of the root and everything to the right of the root is greater than the value of the root. Due to this performing, a binary search is very easy.
- The left and right subtree each must also be a binary search tree.
- There must be no duplicate nodes(BST may have duplicate values with different handling approaches)



Handling approach for Duplicate values in the Binary Search tree:

- You can not allow the duplicated values at all.
- We must follow a consistent process throughout i.e either store duplicate value at the left or store the duplicate value at the right of the root, but be consistent with your approach.
- We can keep the counter with the node and if we found the duplicate value, then we can increment the counter

- **Make a program that will implement Binary Search Tree and show the output of it.**



```
class BST {
  public class Node {
    private int value;
    private Node left;
    private Node right;
    private int height;

    public Node(int value) {
      this.value = value;
    }

    public int getValue() {
      return value;
    }
  }

  private Node root;

  public BST() {

  }

  public int height(Node node) {
    if (node == null) {
      return -1;
    }
    return node.height;
  }

  public boolean isEmpty() {
    return root == null;
  }
```

```
 public void insert(int value) {
   root = insert(value, root);
 }

 private Node insert(int value, Node node) {
  if (node == null) {
   node = new Node(value);
   return node;
  }

  if (value < node.value) {
   node.left = insert(value, node.left);
  }

  if (value > node.value) {
   node.right = insert(value, node.right);
  }

  node.height = Math.max(height(node.left),
height(node.right)) + 1;
  return node;
 }

 public void populate(int[] nums) {
  for (int i = 0; i < nums.length; i++) {
   this.insert(nums[i]);
  }
 }

 public void populatedSorted(int[] nums) {
  populatedSorted(nums, 0, nums.length);
 }
```

```
// When array is sorted
 private void populatedSorted(int[] nums, int start,
int end) {
  if (start >= end) {
   return;
  }

  int mid = (start + end) / 2;

  this.insert(nums[mid]);
  populatedSorted(nums, start, mid);
  populatedSorted(nums, mid + 1, end);
 }

 public boolean balanced() {
  return balanced(root);
 }

 private boolean balanced(Node node) {
  if (node == null) {
   return true;
  }
  return Math.abs(height(node.left) -
height(node.right)) <= 1 && balanced(node.left) &&
balanced(node.right);
 }

 public void display() {
  display(this.root, "Root Node: ");
 }
```

```
 private void display(Node node, String details) {
  if (node == null) {
   return;
  }
  System.out.println(details + node.value);
  display(node.left, "Left child of " + node.value + " : ");
  display(node.right, "Right child of " + node.value + " : ");
 }

}

public class Test {
 public static void main(String[] args) {
  BST tree = new BST();
  int[] nums = { 15, 10, 20, 12, 5, 16, 8 };
  tree.populate(nums);
  tree.display();

  BST tree1 = new BST();
  int[] nums1 = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 };
  tree1.populatedSorted(nums1);
  tree1.display();

 }
}
```

Output:

Root Node: 15
Left child of 15 : 10
Left child of 10 : 5
Right child of 5 : 8
Right child of 10 : 12
Right child of 15 : 20
Left child of 20 : 16
Root Node: 6
Left child of 6 : 3
Left child of 3 : 2
Left child of 2 : 1
Right child of 3 : 5
Left child of 5 : 4
Right child of 6 : 9
Left child of 9 : 8
Left child of 8 : 7
Right child of 9 : 10

- **Tree Traversal methods:**

1. BFS (Breath First Search) -------> Level-Order-Traversal
2. DFS (Depth First Search)

1. BFS (Breath First Search):

Breadth-First Search (BFS) is an algorithm for traversing or searching tree or graph data structures. The algorithm starts at the root node and explores the neighbor nodes at the present depth prior to moving on to nodes at the next depth level. It visits nodes level by level.

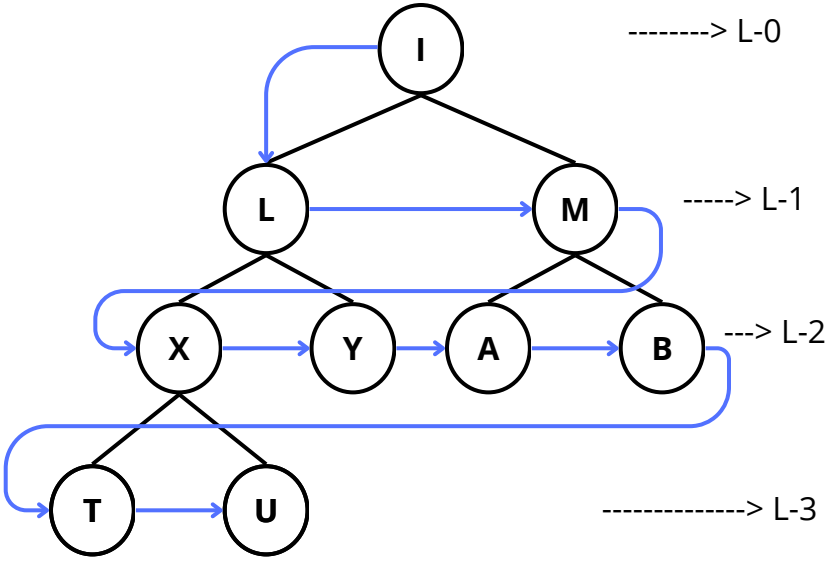
Here's a basic outline of how BFS works:

- Start at the Root: Begin at the root node of the tree or the starting node of the graph.
- Visit the Node: Process (or "visit") the current node.
- Explore Neighbors: Explore all the neighbors (adjacent nodes) of the current node. These neighbors are at the same depth level.
- Move to the Next Level: Move to the next level of the tree or graph and repeat steps 2-3 for the nodes at that level.
- Continue Until Completion: Continue this process until all nodes have been visited.

BFS uses a queue data structure to keep track of which nodes to visit next. The order in which nodes are added to and removed from the queue ensures that nodes are processed in a breadth-first manner.

BFS is often used to find the shortest path between two nodes in an unweighted graph, as it guarantees that the first time a node is encountered during the search, it is via the shortest possible path.

Here's a simple example of a tree traversal using BFS:



Output:
Level-Order:
I->L->M->X->Y->A->B->T->U

2. DFS (Depth First Search):

Depth-First Search (DFS) is another algorithm for traversing or searching tree or graph data structures. Unlike Breadth-First Search (BFS), DFS explores as far as possible along each branch before backtracking. It goes as deep as possible before exploring siblings.

Types of DFS:

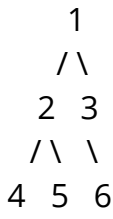
1. Pre-order Traversal
2. In-order Traversal
3. Post-order Traversal

Here's a basic outline of how DFS works:

- Start at the Root: Begin at the root node of the tree or the starting node of the graph.
- Visit the Node: Process (or "visit") the current node.
- Explore Neighbors: Recursively explore all the neighbors (adjacent nodes) of the current node. This involves going as deep as possible along each branch before backtracking.
- Backtrack: After reaching the end of a branch, backtrack to the previous node and explore its other neighbors.
- Continue Until Completion: Repeat steps 2-4 until all nodes have been visited.

DFS can be implemented using either recursion or an explicit stack data structure. When implemented with recursion, the call stack effectively takes the place of an explicit stack.

Here's a simple example of a tree traversal using DFS:



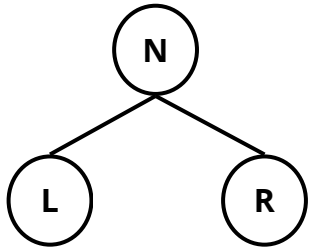
Starting from the root (1), a possible DFS traversal would be: 1, 2, 4, 5, 3, 6.

The order in which nodes are visited depends on the specific implementation and can vary. DFS has different variants, such as pre-order, in-order, and post-order traversals, which specify when the current node is visited concerning its children during the recursive exploration.

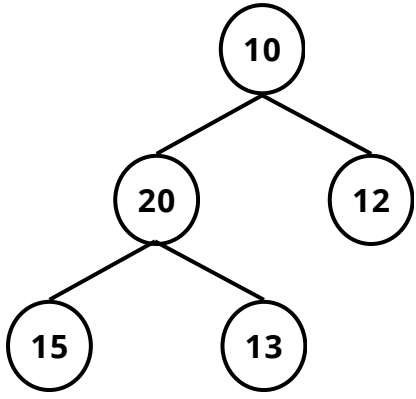
DFS is often used in problems that involve searching through possibilities and can be employed in various applications, such as maze-solving, topological sorting, and graph component analysis. However, it may not necessarily find the shortest path between two nodes in a graph, unlike BFS.

1. Pre-order Traversal:

It will display like,
Node (N) ---> Left (L) ---> Right (R)



N ---> L ---> R

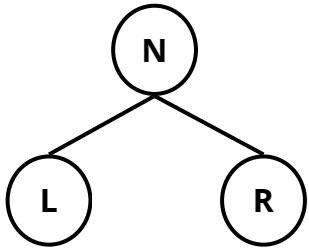


10 ---> 20 ---> 15 --> 13 --> 12

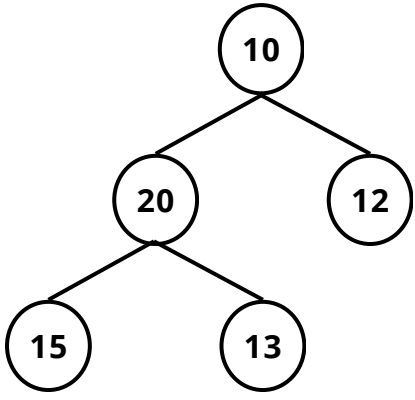
- Used for evaluating mathematical expressions or making a copy
- Serialization your array

2. In-order Traversal:

It will display like,
Left (L) ---> Node (N) ---> Right (R)



L ---> N ---> R

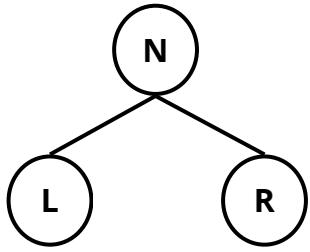


15 ---> 20 ---> 13 --> 10 --> 12

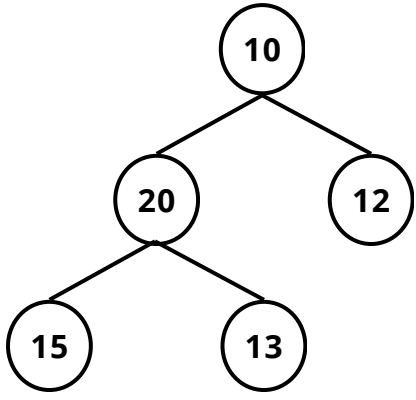
- In BST, we can visit node in sorted manner

3. Post-order Traversal:

It will display like,
Left (L) ---> Right (R) ---> Node (N)



L ---> R ---> N



15 ---> 13 ---> 20 --> 12 --> 10

- Deleting binary tree
- Bottom-up calculation

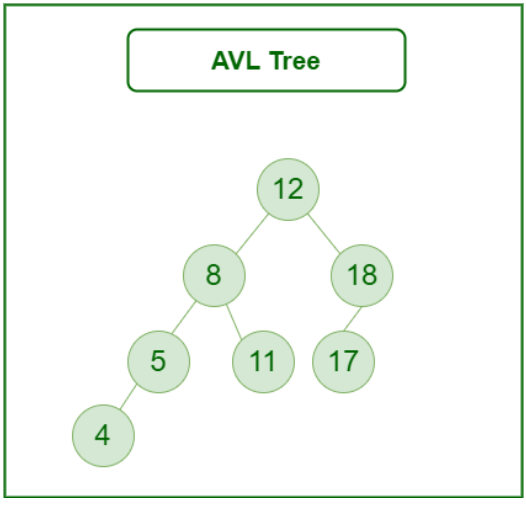
- **AVL Tree:**

An AVL tree defined as a self-balancing Binary Search Tree (BST) where the difference between heights of left and right subtrees for any node cannot be more than one.

The difference between the heights of the left subtree and the right subtree for any node is known as the balance factor of the node.

The AVL tree is named after its inventors, Georgy Adelson-Velsky and Evgenii Landis, who published it in their 1962 paper “An algorithm for the organization of information”.

Example of AVL Trees:



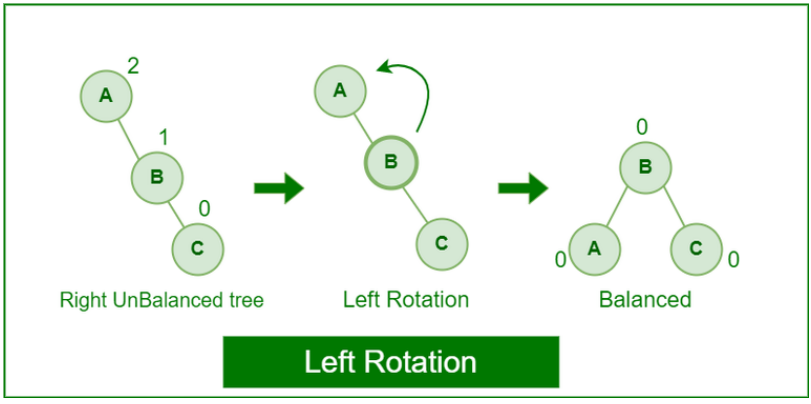
The above tree is AVL because the differences between the heights of left and right subtrees for every node are less than or equal to 1. (-1 or +1 or 0).

- **Rotating the subtrees in an AVL Tree:**

An AVL tree may rotate in one of the following four ways to keep itself balanced:

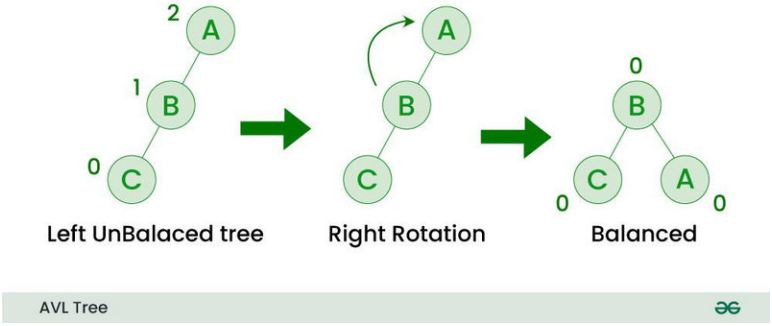
Left Rotation:

When a node is added into the right subtree of the right subtree, if the tree gets out of balance, we do a single left rotation.



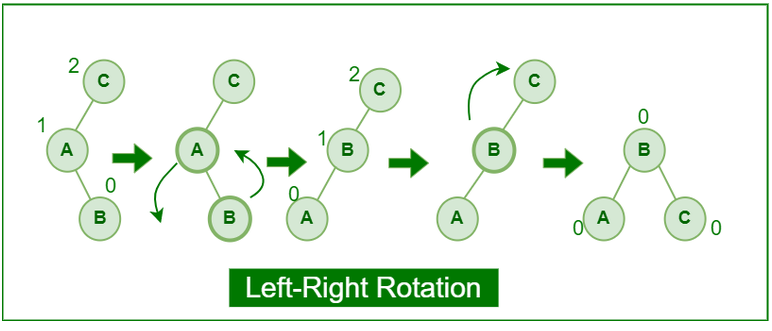
Right Rotation:

If a node is added to the left subtree of the left subtree, the AVL tree may get out of balance, we do a single right rotation.



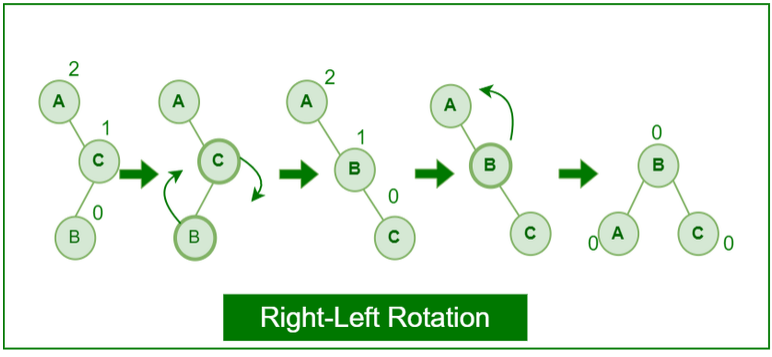
Left-Right Rotation:

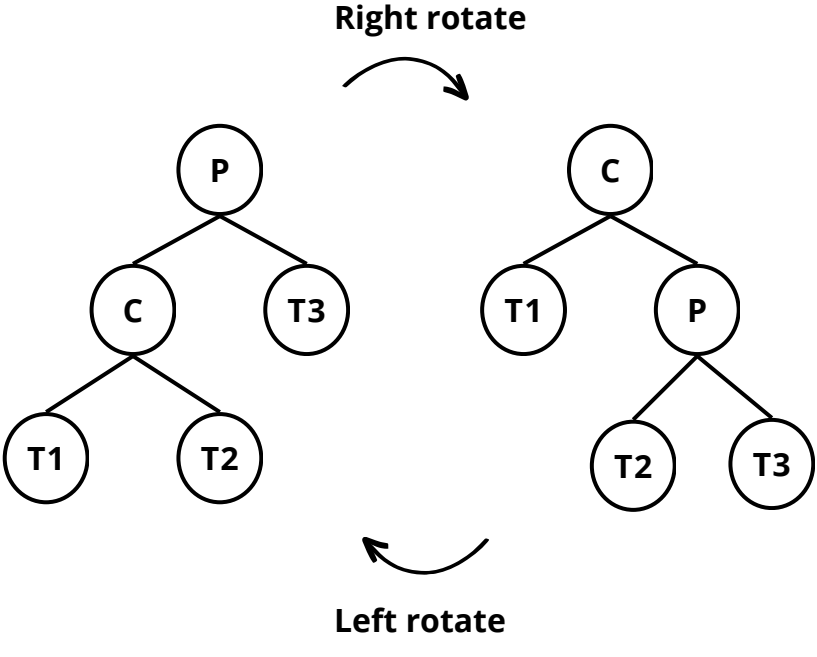
A left-right rotation is a combination in which first left rotation takes place after that right rotation executes.



Right-Left Rotation:

A right-left rotation is a combination in which first right rotation takes place after that left rotation executes.





Advantages of AVL Tree:

1. AVL trees can self-balance themselves.
2. It is surely not skewed.
3. It provides faster lookups than Red-Black Trees
4. Better searching time complexity compared to other trees like binary tree.
5. Height cannot exceed log(N), where, N is the total number of nodes in the tree.

Disadvantages of AVL Tree:

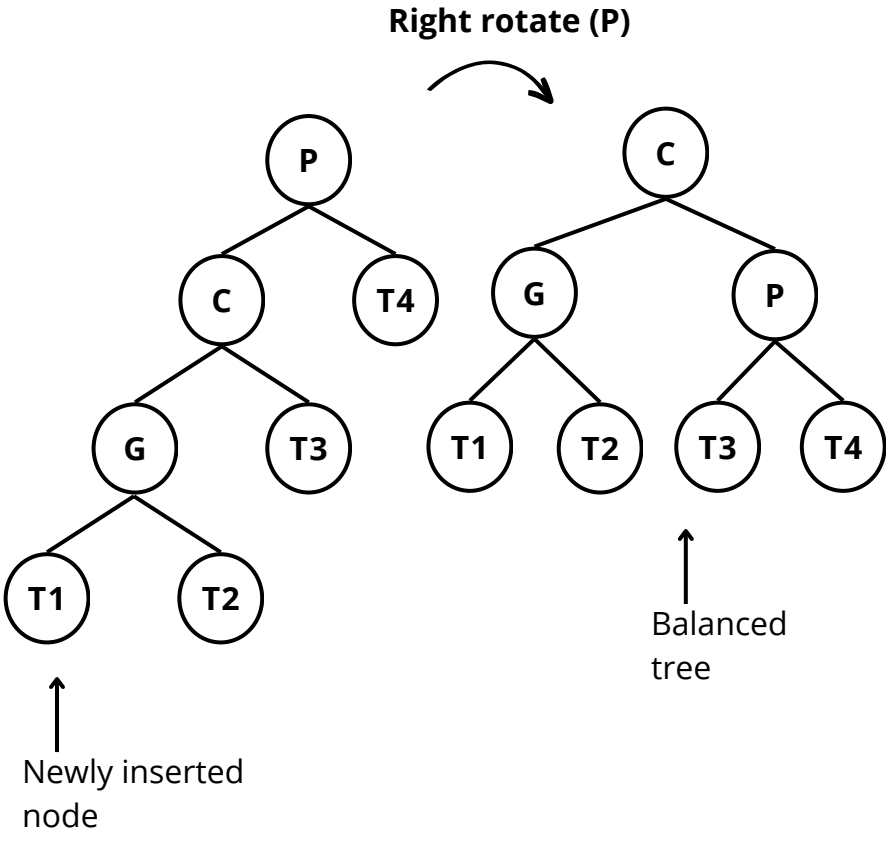
1. It is difficult to implement.
2. It has high constant factors for some of the operations.
3. Less used compared to Red-Black trees.
4. Due to its rather strict balance, AVL trees provide complicated insertion and removal operations as more rotations are performed.
5. Take more processing for balancing.

Applications of AVL Tree:

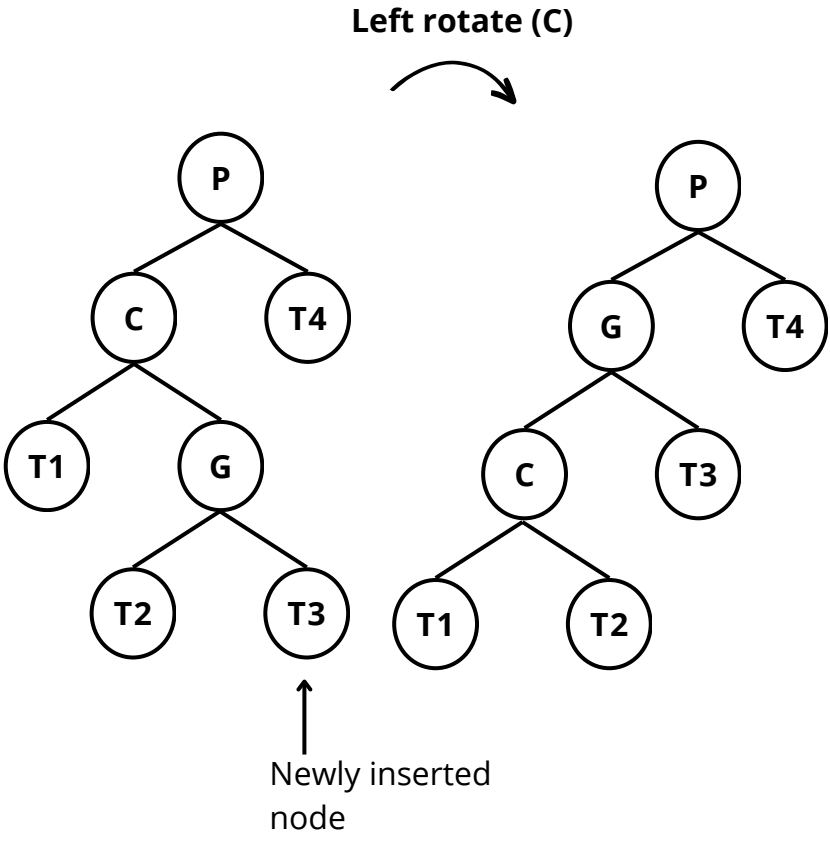
1. It is used to index huge records in a database and also to efficiently search in that.
2. For all types of in-memory collections, including sets and dictionaries, AVL Trees are used.
3. Database applications, where insertions and deletions are less common but frequent data lookups are necessary
4. Software that needs optimized search.
5. It is applied in corporate areas and storyline games.

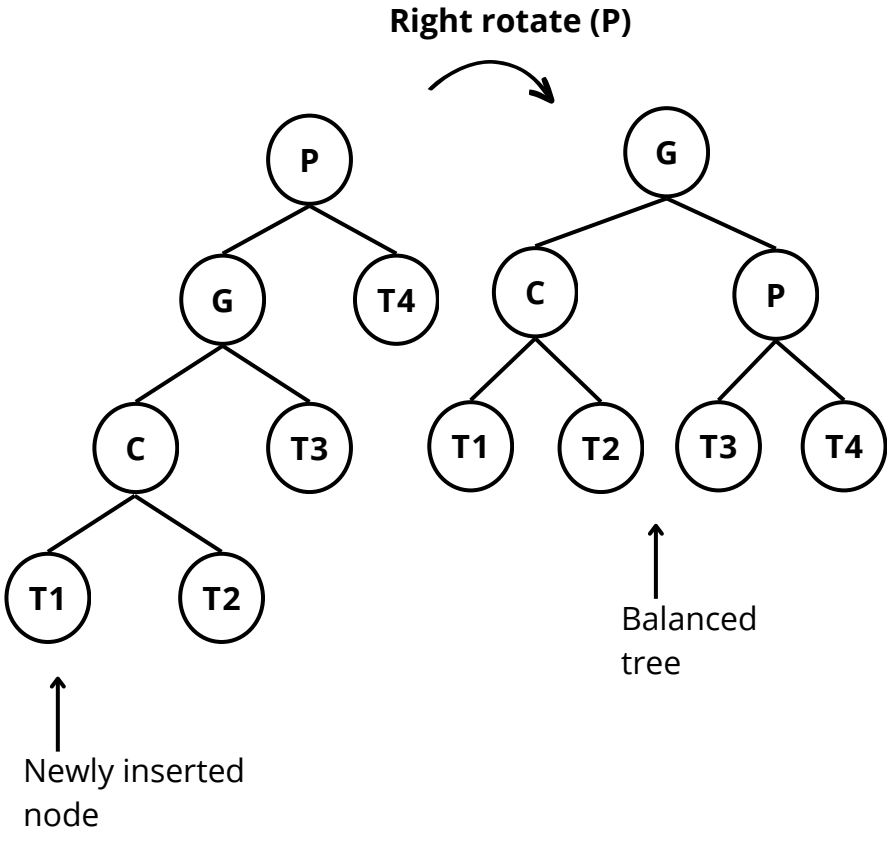
4 rules:

1. Left- Left case:

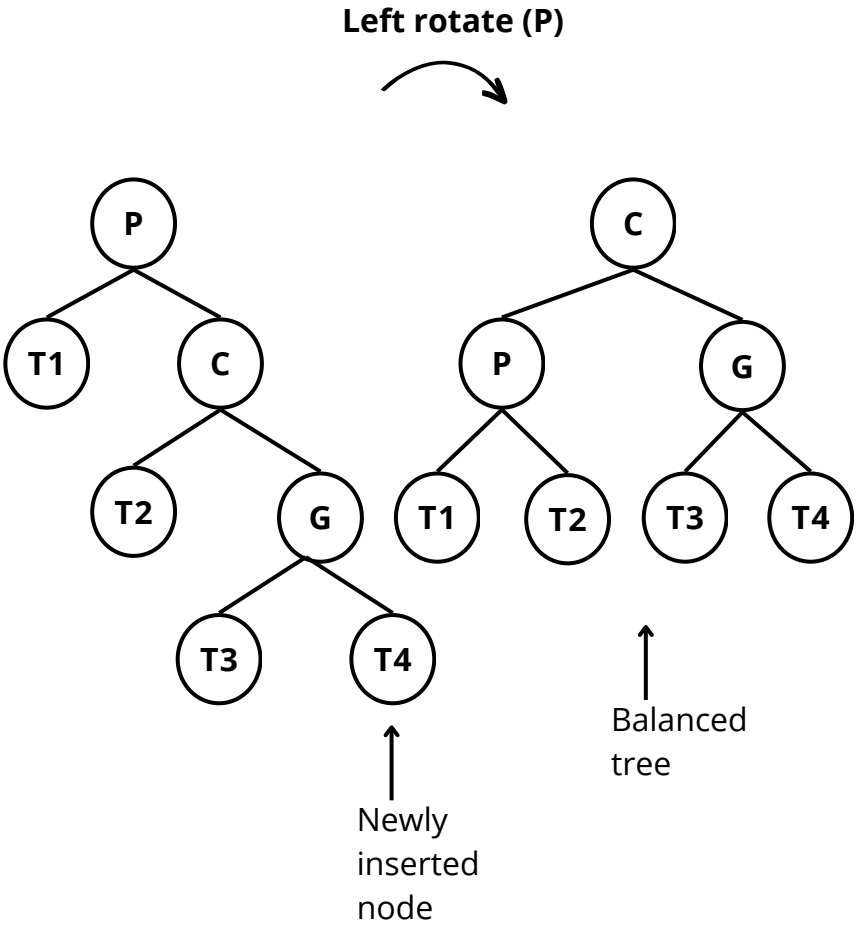


2. Left- Right case:

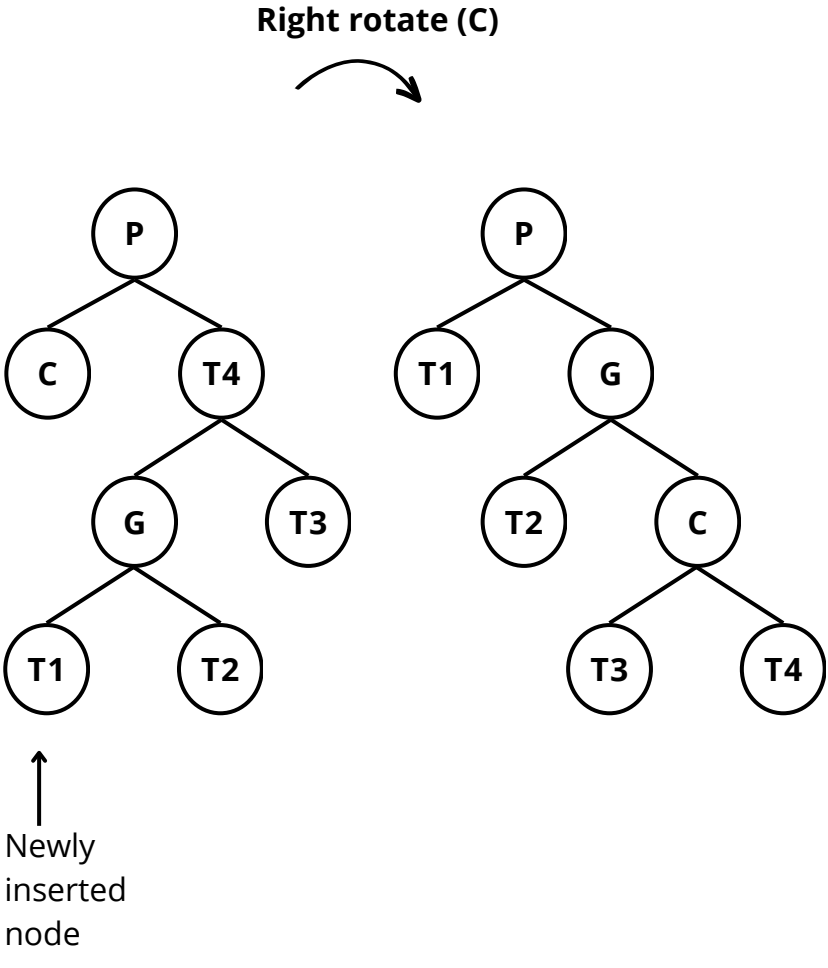


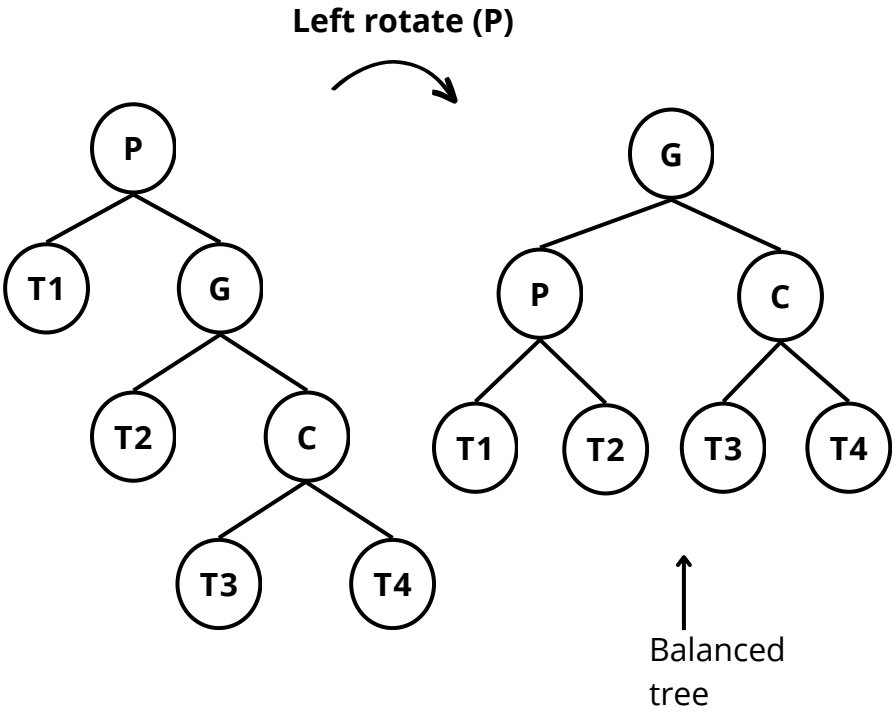

Right rotate (P)
P
G
T4
C
T3
T1
T2
G
C
P
T1
T2
T3
T4
Balanced tree
Newly inserted node

3. Right- Right case:



4. Right- Left case:

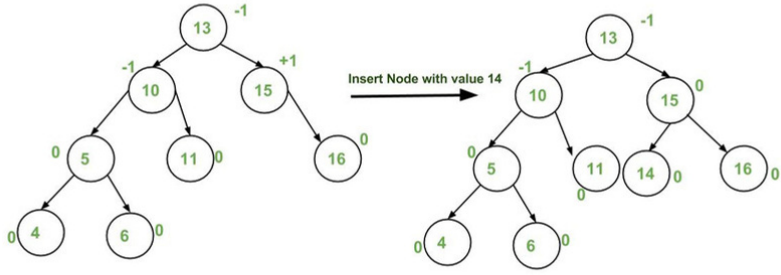



Left rotate (P)
P
T1
G
T2
C
T3
T4
G
P
C
T1
T2
T3
T4
Balanced tree

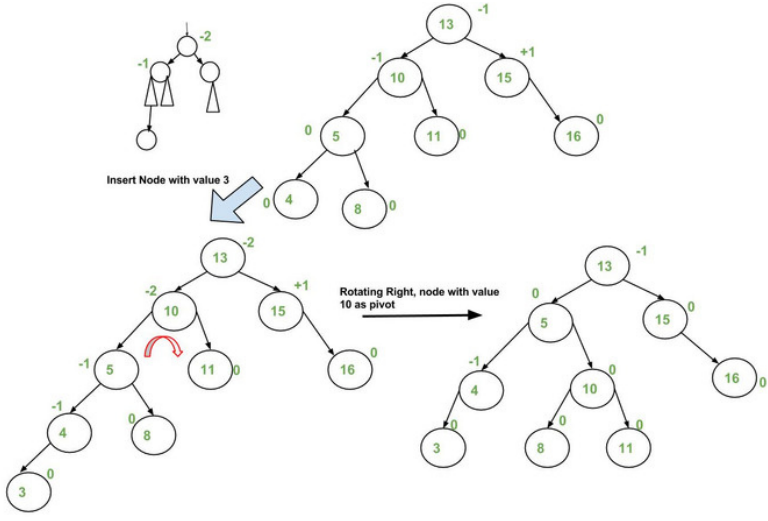
Example:

Illustration of Insertion at AVL Tree

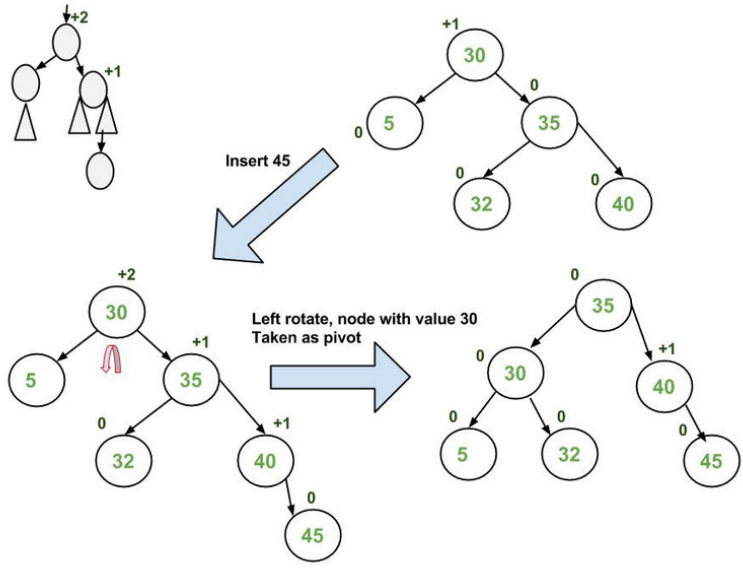
=>



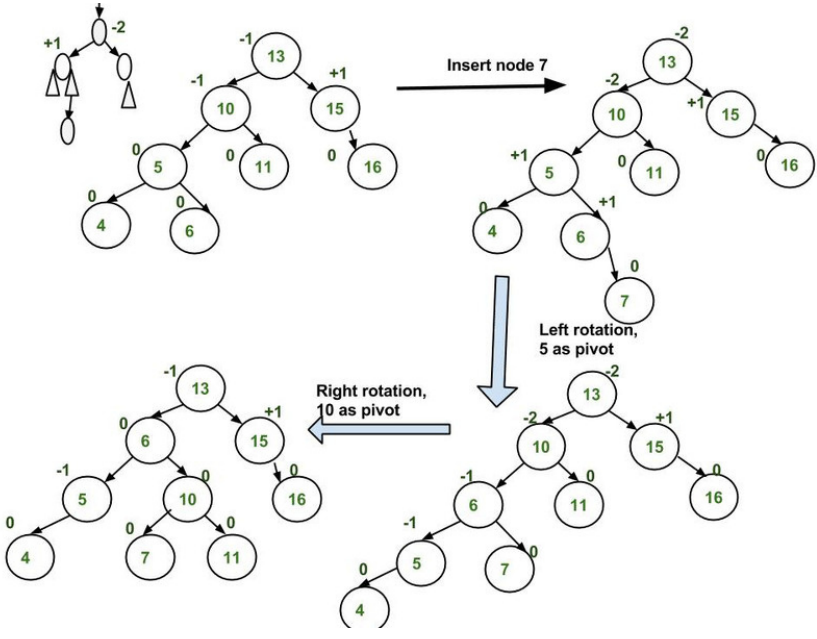
=>



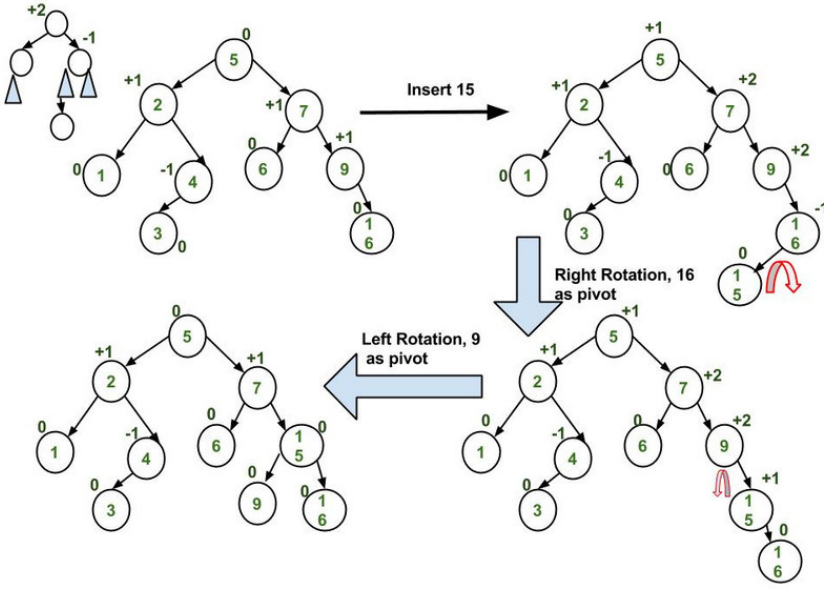
=>


+2
+1
30
5
35
32
40
Insert 45
45
Left rotate, node with value 30
Taken as pivot

=>


13
10
15
5
11
16
4
6
Insert node 7
7
Left rotation,
5 as pivot
Right rotation,
10 as pivot

=>


Insert 15
Right Rotation, 16
as pivot
Left Rotation, 9
as pivot

Operations on an AVL Tree:

- Insertion
- Deletion
- Searching [It is similar to performing a search in BST]

Algorithm of AVL Tree:

1. Insert normally node (n)
2. Start from node (n) and find the node that makes the tree unbalanced, bottom-up.
3. Using one of the 4 rules; rotate.

Time complex of AVL Tree:

O(log(n))

- **Make a AVL tree program.**

```
class AVL {

  public class Node {
    private int value;
    private Node left;
    private Node right;
    private int height;

    public Node(int value) {
      this.value = value;
    }

    public int getValue() {
      return value;
    }
  }

  private Node root;

  public AVL() {

  }

  public int height() {
    return height(root);
  }
  private int height(Node node) {
    if (node == null) {
      return -1;
```

```
    }
    return node.height;
  }

  public void insert(int value) {
    root = insert(value, root);
  }

  private Node insert(int value, Node node) {
    if (node == null) {
      node = new Node(value);
      return node;
    }

    if (value < node.value) {
      node.left = insert(value, node.left);
    }

    if (value > node.value) {
      node.right = insert(value, node.right);
    }

    node.height = Math.max(height(node.left),
height(node.right)) + 1;
    return rotate(node);
  }

 private Node rotate(Node node) {
  if (height(node.left) - height(node.right) > 1) {
   // left heavy
   if(height(node.left.left) - height(node.left.right) >
0) {
```

```
  // left left case
  return rightRotate(node);
 }
 if(height(node.left.left) - height(node.left.right) < 0) {
  // left right case
  node.left = leftRotate(node.left);
  return rightRotate(node);
 }
}

if (height(node.left) - height(node.right) < -1) {
 // right heavy
 if(height(node.right.left) - height(node.right.right) < 0) {
  // right right case
  return leftRotate(node);
 }
 if(height(node.right.left) - height(node.right.right) > 0) {
  // left right case
  node.right = rightRotate(node.right);
  return leftRotate(node);
 }
}

 return node;
}

public Node rightRotate(Node p) {
 Node c = p.left;
 Node t = c.right;

 c.right = p;
```

```
 p.left = t;

 p.height = Math.max(height(p.left), height(p.right) + 1);
 c.height = Math.max(height(c.left), height(c.right) + 1);

 return c;
}

public Node leftRotate(Node c) {
 Node p = c.right;
 Node t = p.left;

 p.left = c;
 c.right = t;

 c.height = Math.max(height(c.left), height(c.right) + 1);
 p.height = Math.max(height(p.left), height(p.right) + 1);

 return p;
}

public void populate(int[] nums) {
 for (int i = 0; i < nums.length; i++) {
  this.insert(nums[i]);
 }
}

public void populatedSorted(int[] nums) {
 populatedSorted(nums, 0, nums.length);
}
```

```
 private void populatedSorted(int[] nums, int start, int
end) {
  if (start >= end) {
   return;
  }

  int mid = (start + end) / 2;

  this.insert(nums[mid]);
  populatedSorted(nums, start, mid);
  populatedSorted(nums, mid + 1, end);
 }

 public void display() {
  display(this.root, "Root Node: ");
 }

 private void display(Node node, String details) {
  if (node == null) {
   return;
  }
  System.out.println(details + node.value);
  display(node.left, "Left child of " + node.value + " : ");
  display(node.right, "Right child of " + node.value + " : ");
 }

 public boolean isEmpty() {
  return root == null;
 }
```

```
 public boolean balanced() {
  return balanced(root);
 }

 private boolean balanced(Node node) {
  if (node == null) {
   return true;
  }
  return Math.abs(height(node.left) -
height(node.right)) <= 1 && balanced(node.left) &&
balanced(node.right);
 }

}

class Test {
 public static void main(String[] args) {
  AVL tree = new AVL();

  for(int i=0; i < 1000; i++) {
   tree.insert(i);
  }

  System.out.println(tree.height());
 }
}
```

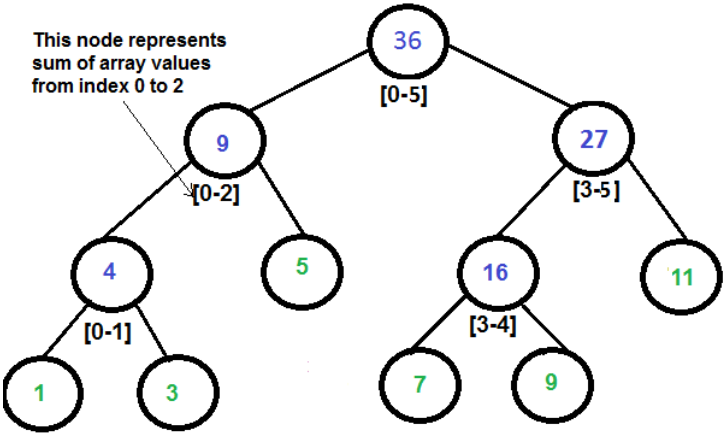
Output:

9

- **Segment Tree:**

In computer science, a Segment Tree, also known as a statistic tree, is a tree data structure used for storing information about intervals, or segments. It allows querying which of the stored segments contain a given point. It is, in principle, a static structure; that is, it's a structure that cannot be modified once it's built. A similar data structure is the interval tree.
A segment tree for a set I of n intervals uses O(n log n) storage and can be built in O(n log n) time. Segment trees support searching for all the intervals that contain a query point in time O(log n + k), k being the number of retrieved intervals or segments.

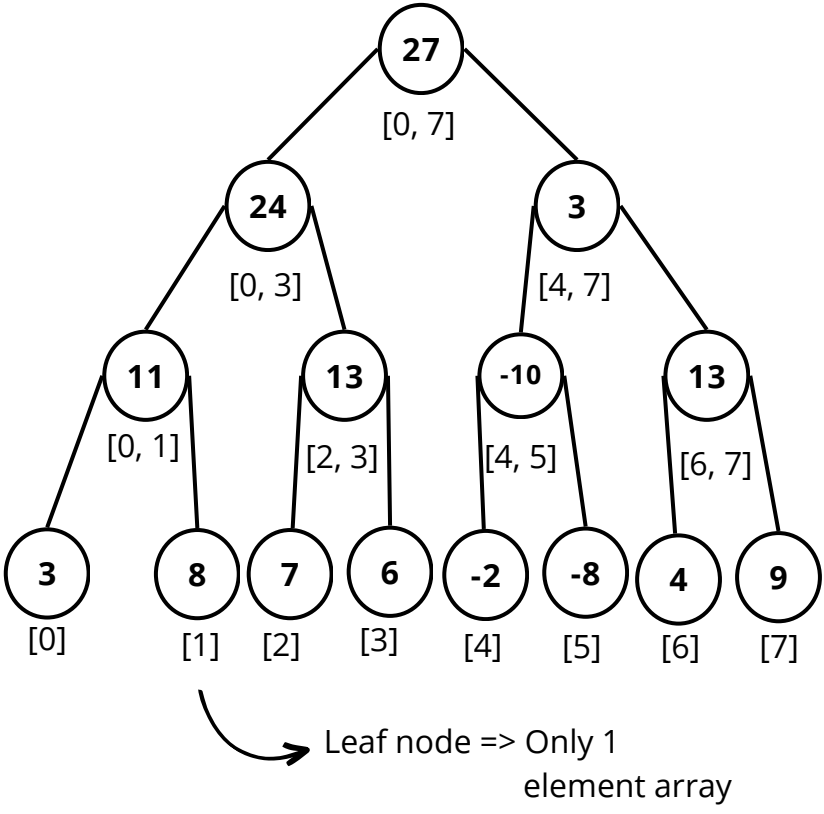
Segment => Perform query on a range (Sum, max, avg, min, product of all the number) in O(log(N))
Update => O(log(N))



Segment Tree for input array {1, 3, 5, 7, 9, 11}

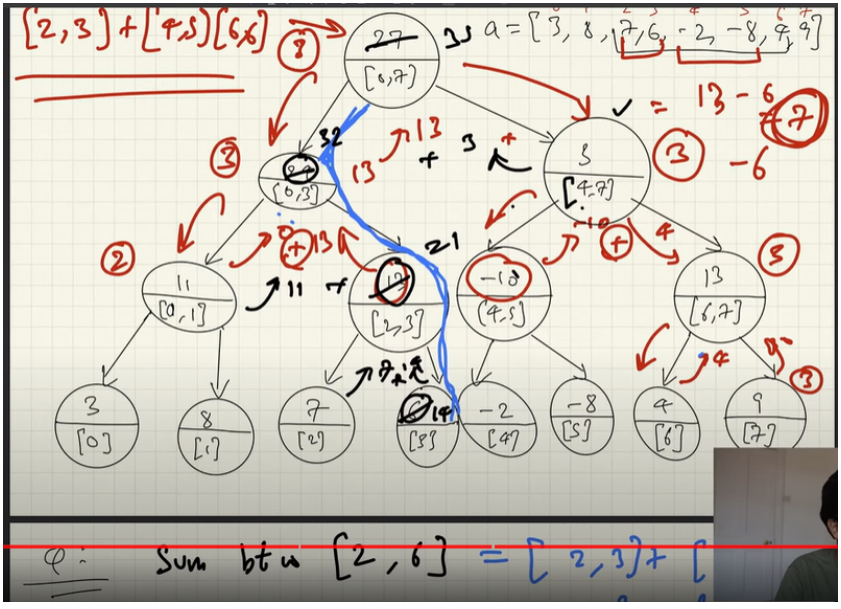
- **Some between two integer:**

arr = [3, 8, 7, 6, -2, -8, 4, 9] , N = 8



So, Segment tree is a Full Binary Tree.

Leaf nodes = N
Internal nodes = N-1
Total nodes = 2N - 1


[2,3]+[4,5][6,6]
a=[3, 8, 7, 6, -2, -8, 4, 9]
27
[0,7]
24
[0,3]
3
[4,7]
11
[0,1]
13
[2,3]
-13
[4,5]
13
[6,7]
3
[0]
8
[1]
7
[2]
[3]
-2
[4]
-8
[5]
4
[6]
9
[7]
Sum btw [2, 6] = [2, 3] + [

Cases:

1. Node interval is inside query interval
   Example: [4, 5] --> return the value

2. Noed interval is completely outside the query interval. (node start index) > (query end index) or, (node end index) > (query start index)
   [2, 6] ---> [7, 7]
   |
   |-------> return the default value of the quarry.
   In this case = 0 .

3. Overlapping:

How to update in O(log(n)) time:
Change index 3 with 14 => (3, 14)

1. Check weather index lies in interval [0, 7]
2. If yes, then check child node. If child node is out, no change in value, just return.
3. In the end you will reach leafs. Update leafs and recursion will update the tree.

- **Make a program that will implement a Segment tree of given array data and query [0, 6]**

```
class SegmentTree {

  private static class Node {
    int data;
    int startInterval;
    int endInterval;
    Node left;
    Node right;

    public Node (int startInterval, int endInterval) {
      this.startInterval = startInterval;
      this.endInterval = endInterval;
    }
  }

  Node root;

  public SegmentTree(int[] arr) {
    // create a tree using this array
    this.root = constructTree(arr, 0, arr.length - 1);
  }

  private Node constructTree(int[] arr, int start, int end) {
    if(start == end) {
      // leaf node
      Node leaf = new Node(start, end);
      leaf.data = arr[start];
      return leaf;
    }
```

```
    // create new node with index you are at
    Node node = new Node(start, end);

    int mid = (start + end) / 2;

    node.left = this.constructTree(arr, start, mid);
    node.right = this.constructTree(arr, mid + 1, end);

    node.data = node.left.data + node.right.data;
    return node;
  }

  public void display() {
    display(this.root);
  }
  private void display(Node node) {
    String str = "";

    if(node.left != null) {
      str = str + "Interval=[" + node.left.startInterval +
"-" + node.left.endInterval + "] and data: " +
node.left.data + " => ";
    } else {
      str = str + "No left child";
    }

    // for current node
    str = str + "Interval=[" + node.startInterval + "-" +
node.endInterval + "] and data: " + node.data + " <=
";
```

```
    if(node.right != null) {
      str = str + "Interval=[" + node.right.startInterval
+ "-" + node.right.endInterval + "] and data: " +
node.right.data;
    } else {
      str = str + "No right child";
    }

    System.out.println(str + '\n');

  // call recursion
  if(node.left != null) {
  display(node.left);
  }

  if(node.right != null) {
  display(node.right);
  }
  }

  // query
  public int query(int qsi, int qei) {
  return this.query(this.root, qsi, qei);
  }
  private int query(Node node, int qsi, int qei) {
  if(node.startInterval >= qsi && node.endInterval <=
qei) {
  // node is completely lying inside query
  return node.data;
  } else if (node.startInterval > qei ||
node.endInterval < qsi) {
```

```
 // completely outside
 return 0;
 } else {
 return this.query(node.left, qsi, qei) +
this.query(node.right, qsi, qei);
 }
 }

 // update
 public void update(int index, int value) {
 this.root.data = update(this.root, index, value);
 }
 private int update(Node node, int index, int value) {
 if (index >= node.startInterval&& index <=
node.endInterval){
 if(index == node.startInterval && index ==
node.endInterval) {
 node.data = value;
 return node.data;
 } else {
 int leftAns = update(node.left, index, value);
 int rightAns = update(node.right, index, value);
 node.data = leftAns + rightAns;
 return node.data;
 }
 }
 return node.data;
 }

}
```

```
public class Test {
 public static void main(String[] args) {
 int[] arr = {3, 8, 6, 7, -2, -8, 4, 9};
 SegmentTree tree = new SegmentTree(arr);
 tree.display();

 System.out.println(tree.query(1, 6));
 }
}
```

Output:

Interval=[0-3] and data: 24 => Interval=[0-7] and data: 27 <= Interval=[4-7] and data: 3

Interval=[0-1] and data: 11 => Interval=[0-3] and data: 24 <= Interval=[2-3] and data: 13

Interval=[0-0] and data: 3 => Interval=[0-1] and data: 11 <= Interval=[1-1] and data: 8

No left childInterval=[0-0] and data: 3 <= No right child

No left childInterval=[1-1] and data: 8 <= No right child

Interval=[2-2] and data: 6 => Interval=[2-3] and data: 13 <= Interval=[3-3] and data: 7

No left childInterval=[2-2] and data: 6 <= No right child

No left childInterval=[3-3] and data: 7 <= No right child

Interval=[4-5] and data: -10 => Interval=[4-7] and data: 3 <= Interval=[6-7] and data: 13
nterval=[5-5] and data: -8
nterval=[5-5] and data: -8

No left childInterval=[4-4] and data: -2 <= No right child

No left childInterval=[5-5] and data: -8 <= No right child

Interval=[6-6] and data: 4 => Interval=[6-7] and data: 13 <= Interval=[7-7] and data: 9

No left childInterval=[6-6] and data: 4 <= No right child

No left childInterval=[7-7] and data: 9 <= No right child

15

4. Heaps

Definition of Heap data structure:

A Heap is a special Tree-based Data Structure in which the tree is a complete binary tree.

Types of heaps:

Generally, heaps are of two types.

1. Max-Heap
2. Min-Heap

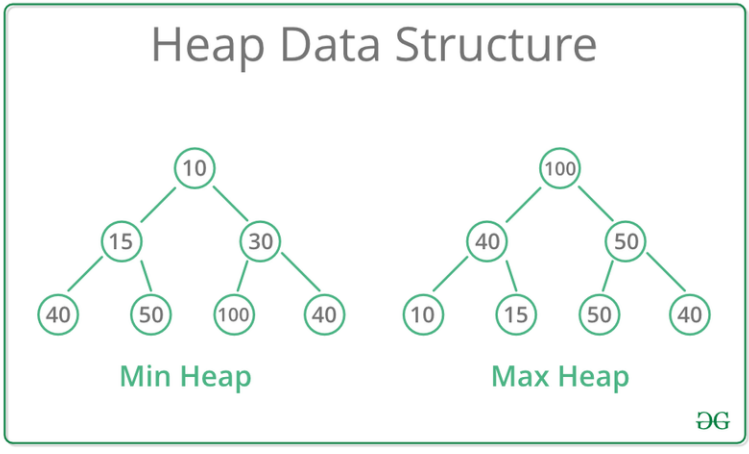
1. Max-Heap:

In this heap, the value of the root node must be the greatest among all its child nodes and the same thing must be done for its left and right sub-tree also.

The total number of comparisons required in the max heap is according to the height of the tree. The height of the complete binary tree is always logn; therefore, the time complexity would also be O(logn).

2. Min-Heap:
In this heap, the value of the root node must be the smallest among all its child nodes and the same thing must be done for its left and right sub-tree also.

The total number of comparisons required in the min heap is according to the height of the tree. The height of the complete binary tree is always logn; therefore, the time complexity would also be O(logn).



Properties of Heap:

Heap has the following Properties:

- Complete Binary Tree: A heap tree is a complete binary tree, meaning all levels of the tree are fully filled except possibly the last level, which is filled from left to right. This property ensures that the tree is efficiently represented using an array.
- Heap Property: This property ensures that the minimum (or maximum) element is always at the root of the tree according to the heap type.
- Parent-Child Relationship: The relationship between a parent node at index 'i' and its children is given by the formulas: left child at index 2i+1 and right child at index 2i+2 for 0-based indexing of node numbers.
- Efficient Insertion and Removal: Insertion and removal operations in heap trees are efficient. New elements are inserted at the next available position in the bottom-rightmost level, and the heap property is restored by comparing the element with its parent and swapping if necessary. Removal of the root element involves replacing it with the last element and heapifying down.
- Efficient Access to Extremal Elements: The minimum or maximum element is always at the root of the heap, allowing constant-time access.

Operations Supported by Heap:

Operations supported by min – heap and max – heap are same. The difference is just that min-heap contains minimum element at root of the tree and max – heap contains maximum element at the root of the tree.

Heapify:
It is the process to rearrange the elements to maintain the property of heap data structure. It is done when a certain node creates an imbalance in the heap due to some operations on that node. It takes O(log N) to balance the tree.

- For max-heap, it balances in such a way that the maximum element is the root of that binary tree and
- For min-heap, it balances in such a way that the minimum element is the root of that binary tree.

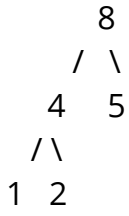
Insertion:

- If we insert a new element into the heap since we are adding a new element into the heap so it will distort the properties of the heap so we need to perform the heapify operation so that it maintains the property of the heap.

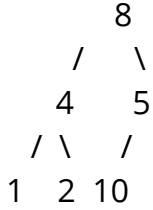
This operation also takes O(logN) time.

Examples:

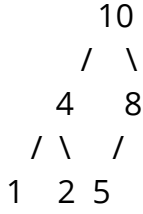
Assume initially heap(taking max-heap) is as follows



Now if we insert 10 into the heap



After heapify operation final heap will be look like this



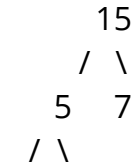
Deletion:

- If we delete the element from the heap it always deletes the root element of the tree and replaces it with the last element of the tree.
- Since we delete the root element from the heap it will distort the properties of the heap so we need to perform heapify operations so that it maintains the property of the heap.

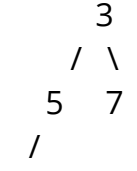
It takes O(logN) time.

Example:

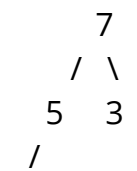
Assume initially heap(taking max-heap) is as follows



Now if we delete 15 into the heap it will be replaced by leaf node of the tree for temporary.



After heapify operation final heap will be look like this



getMax (For max-heap) or getMin (For min-heap):
It finds the maximum element or minimum element for max-heap and min-heap respectively and as we know minimum and maximum elements will always be the root node itself for min-heap and max-heap respectively. It takes O(1) time.

removeMin or removeMax:
This operation returns and deletes the maximum element and minimum element from the max-heap and min-heap respectively. In short, it deletes the root element of the heap binary tree.

Implementation of Heap Data Structure:-
The following code shows the implementation of a max-heap.

Let's understand the maxHeapify function in detail:-

maxHeapify is the function responsible for restoring the property of the Max Heap. It arranges the node i, and its subtrees accordingly so that the heap property is maintained.

- Suppose we are given an array, arr[] representing the complete binary tree. The left and the right child of ith node are in indices 2*i+1 and 2*i+2.

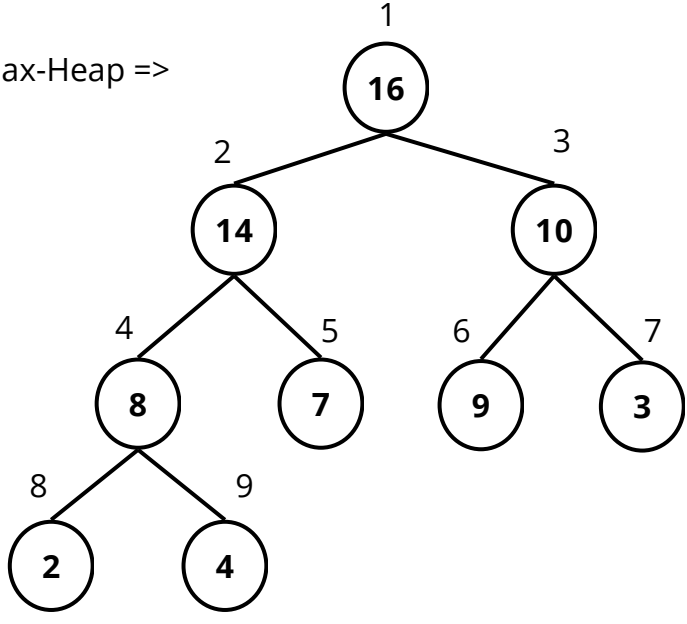
- We set the index of the current element, i, as the ‘MAXIMUM’.
- If arr[2 * i + 1] > arr[i], i.e., the left child is larger than the current value, it is set as ‘MAXIMUM’.
- Similarly if arr[2 * i + 2] > arr[i], i.e., the right child is larger than the current value, it is set as ‘MAXIMUM’.
- Swap the ‘MAXIMUM’ with the current element.
- Repeat steps 2 to 5 till the property of the heap is restored.

Heap:

1. Complete Binary Tree
2. In max-heap every node value >= All of its children node value
   or,
   In min-heap every node value <= All of its children node value
3. Root => i = 1
4. parent (i) = i/2
5. left (i) = 2 * i
6. right (i) = (2 * i) + 1
7. Height = log(N)

	1	2	3	4	5	6	7	8	9	10
arr =	**16**	**14**	**10**	**8**	**7**	**9**	**3**	**2**	**4**	

Max-Heap =>

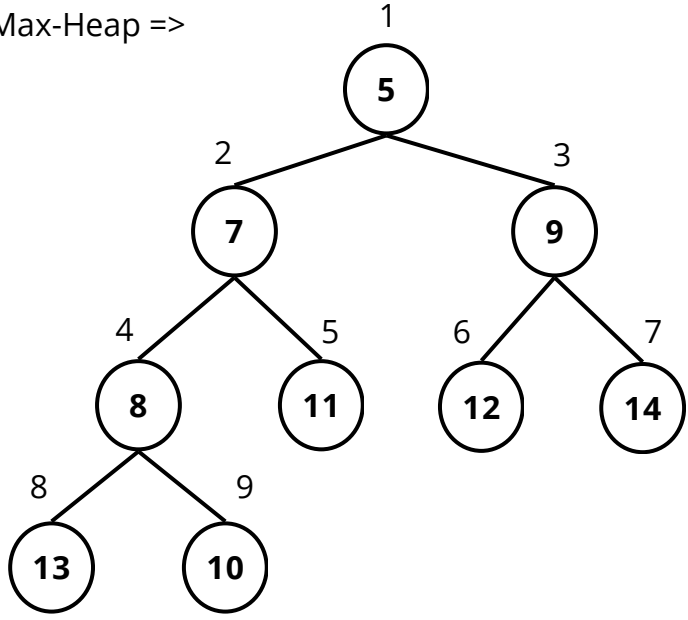


	1	2	3	4	5	6	7	8	9	10
arr =	**7**	**8**	**9**	**10**	**11**	**12**	**14**	**13**	**5**	

After insert 5:

	1	2	3	4	5	6	7	8	9	10
arr =	5	**7**	**9**	**8**	**11**	**12**	**14**	**13**	**10**	

Max-Heap =>



- **Implementation of Heap Data Structure + Heapsort + Priority Queue.**

```
import java.util.ArrayList;

class Heap<T extends Comparable<T>> {

  private ArrayList<T> list;

  public Heap() {
    list = new ArrayList<>();
  }

  private void swap(int first, int second) {
    T temp = list.get(first);
    list.set(first, list.get(second));
    list.set(second, temp);
  }

  private int parent(int index) {
    return (index - 1) / 2;
  }

  private int left(int index) {
    return index * 2 + 1;
  }

  private int right(int index) {
    return index * 2 + 2;
  }
```

```
 public void insert(T value) {
  list.add(value);
  upheap(list.size() - 1);
 }

 private void upheap(int index) {
  if(index == 0) {
   return;
  }
  int p = parent(index);
  if(list.get(index).compareTo(list.get(p)) < 0) {
   swap(index, p);
   upheap(p);
  }
 }

public T remove() throws Exception {
 if (list.isEmpty()) {
  throw new Exception("Removing from an empty
heap!");
 }

 T temp = list.get(0);

 T last = list.remove(list.size() - 1);
 if (!list.isEmpty()) {
  list.set(0, last);
  downheap(0);
 }

 return temp;
}
```

```
 private void downheap(int index) {
  int min = index;
  int left = left(index);
  int right = right(index);

  if(left < list.size() &&
list.get(min).compareTo(list.get(left)) > 0) {
   min = left;
  }

  if(right < list.size() &&
list.get(min).compareTo(list.get(right)) > 0) {
   min = right;
  }

  if(min != index) {
   swap(min, index);
   downheap(min);
  }
 }

 public ArrayList<T> heapSort() throws Exception {
  ArrayList<T> data = new ArrayList<>();
  while(!list.isEmpty()) {
   data.add(this.remove());
  }
  return data;
 }
}
```

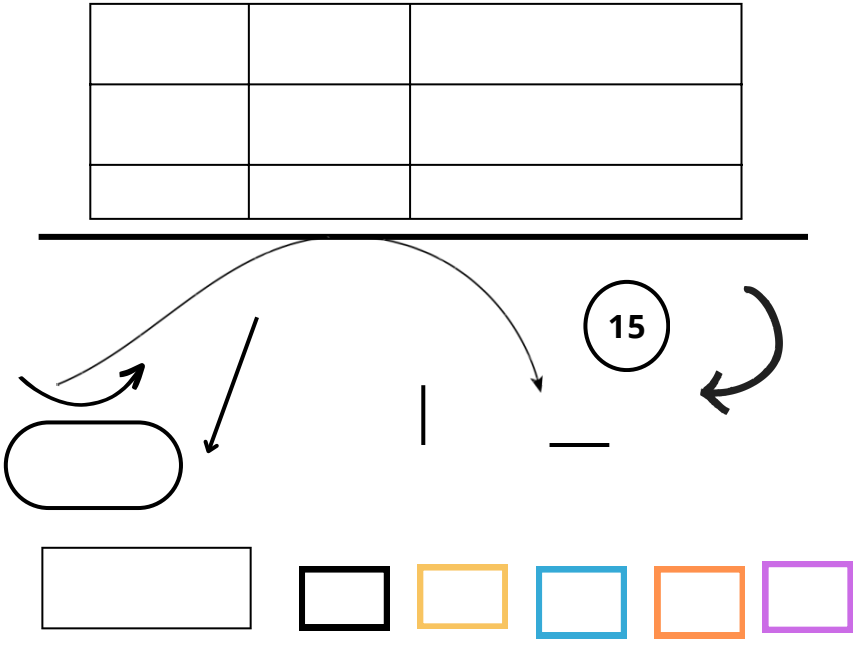
```
class Test {
 public static void main(String[] args) throws Exception{
  Heap<Integer> heap = new Heap<>();

  heap.insert(34);
  heap.insert(45);
  heap.insert(22);
  heap.insert(89);
  heap.insert(76);

  ArrayList list = heap.heapSort();
  System.out.println(list);

 }
}
```

Output:

```
[22, 34, 45, 76, 89]
```

Operator Precedence and Associativity:

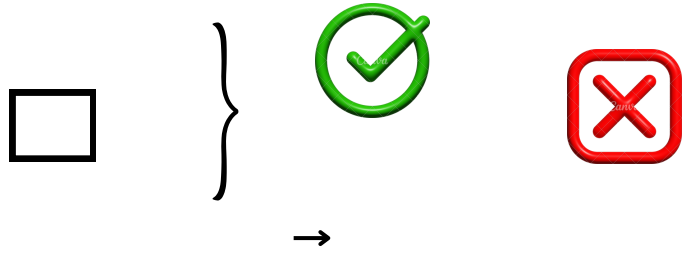
- **Make a program that will show String palindrome.**



Output:

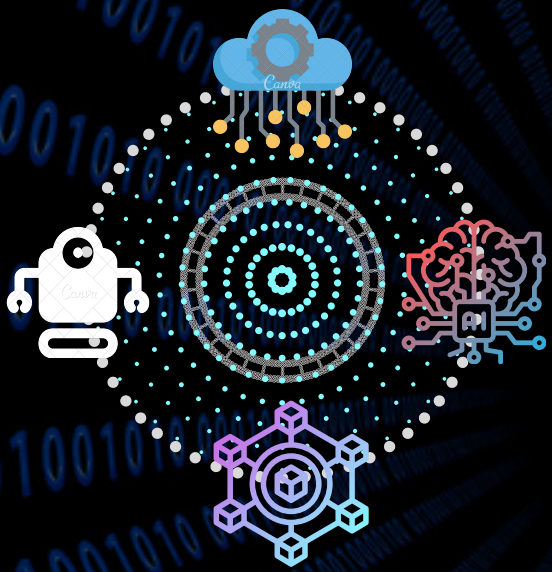
Here,

Jim



- **What is the output of the following Java program fragment:**

Operator



JAVA
(FIFTH PART)
T.I.M. HAMIM

OBZTRON
PROKASHONI