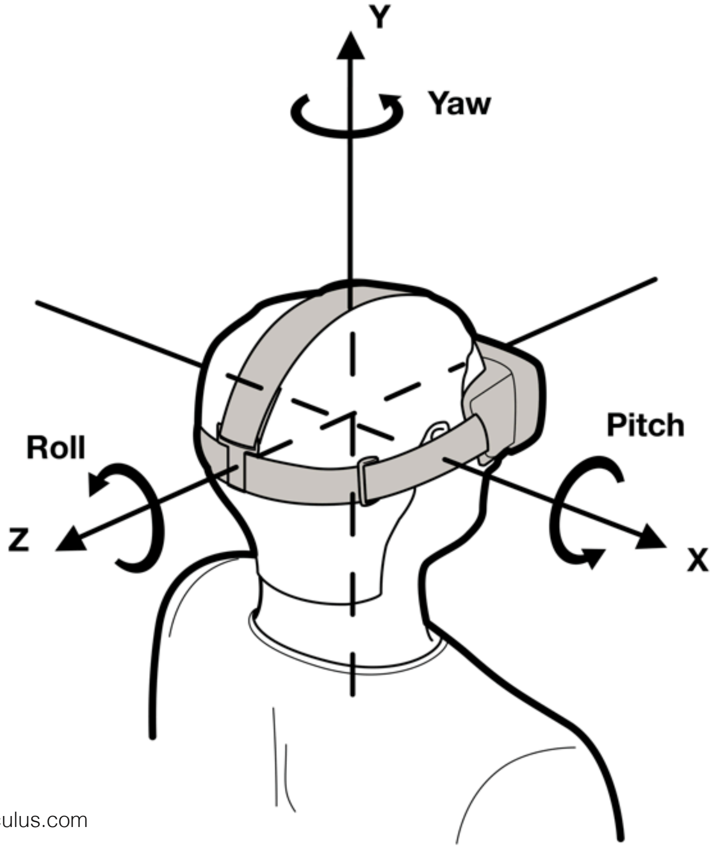
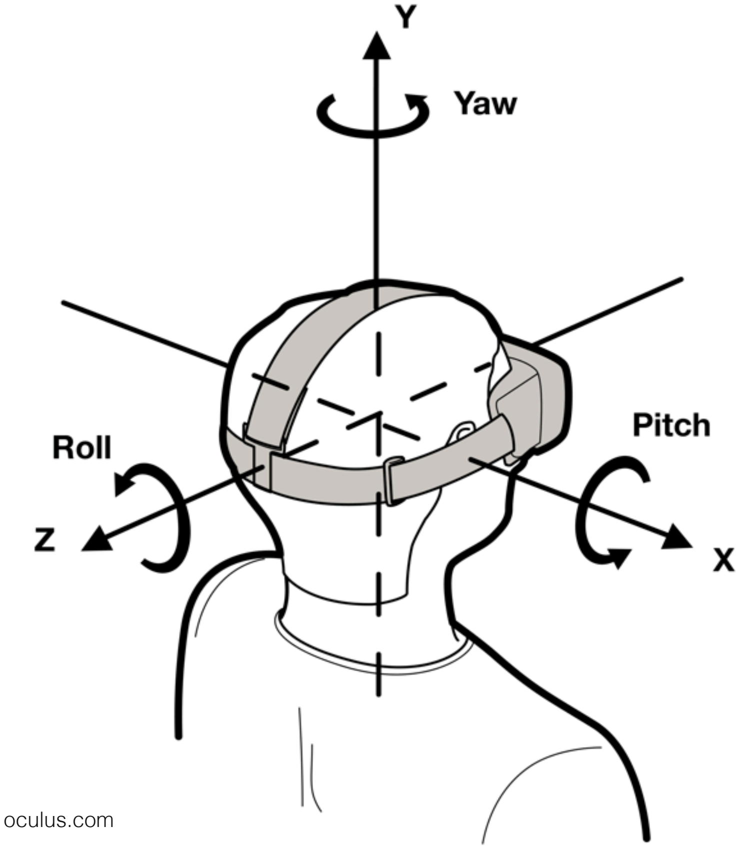


oculus.com

- primary goal: track orientation of head or other device
- orientation is the rotation of device w.r.t. world/earth or *inertial* frame
- rotations are represented by Euler angles (yaw, pitch, roll) or quaternions



- orientation tracked with IMU models relative rotation of sensor/body frame in world/ inertial coordinates
- example: person on the left looks up → pitch=90° or rotation around x-axis by 90°
- similarly, the world rotates around the sensor frame by -90° (inverse rotation)

What do Inertial Sensors Measure?

- gyroscope measures angular velocity $\widetilde{\omega}$ in degrees/sec
- accelerometer measures linear acceleration $\tilde{a}$ in m/s^2
- magnetometer measures magnetic field strength $\widetilde{m}$ in uT (micro Tesla) or Gauss → 1 Gauss = 100 uT

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ALL MEASUREMENTS TAKEN IN SENSOR/ BODY COORDINATES!

Gyroscopes

- gyro model: $\widetilde{\omega} = \omega + b + \eta$

Gyroscopes

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true angular velocity

additive, zero-mean Gaussian noise

bias

Gyroscopes

- gyro model: $\tilde{\omega} = \omega + b + \eta$ $\qquad \eta \sim N\left(0, \sigma_{gyro}^2\right)$

  true angular velocity — ω

  bias — b

  additive, zero-mean Gaussian noise — η

- 3 DOF = 3-axis gyros that measures 3 orthogonal axes, assume no crosstalk
- bias is temperature-dependent and may change over time; can approximate as a constant
- additive measurement noise

Gyroscopes

- from gyro measurements to orientation – use Taylor expansion

$$\theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon, \quad \varepsilon \sim O\left(\Delta t^2\right)$$

$\theta_1 = \theta_0 + (1-\alpha) * \omega$

Gyroscopes

- from gyro measurements to orientation – use Taylor expansion

$$\theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon, \quad \varepsilon \sim O(\Delta t^2)$$

$$\frac{\partial}{\partial t}\theta(t) = \omega$$

<u>have</u>: angle at last time step → $\theta(t)$

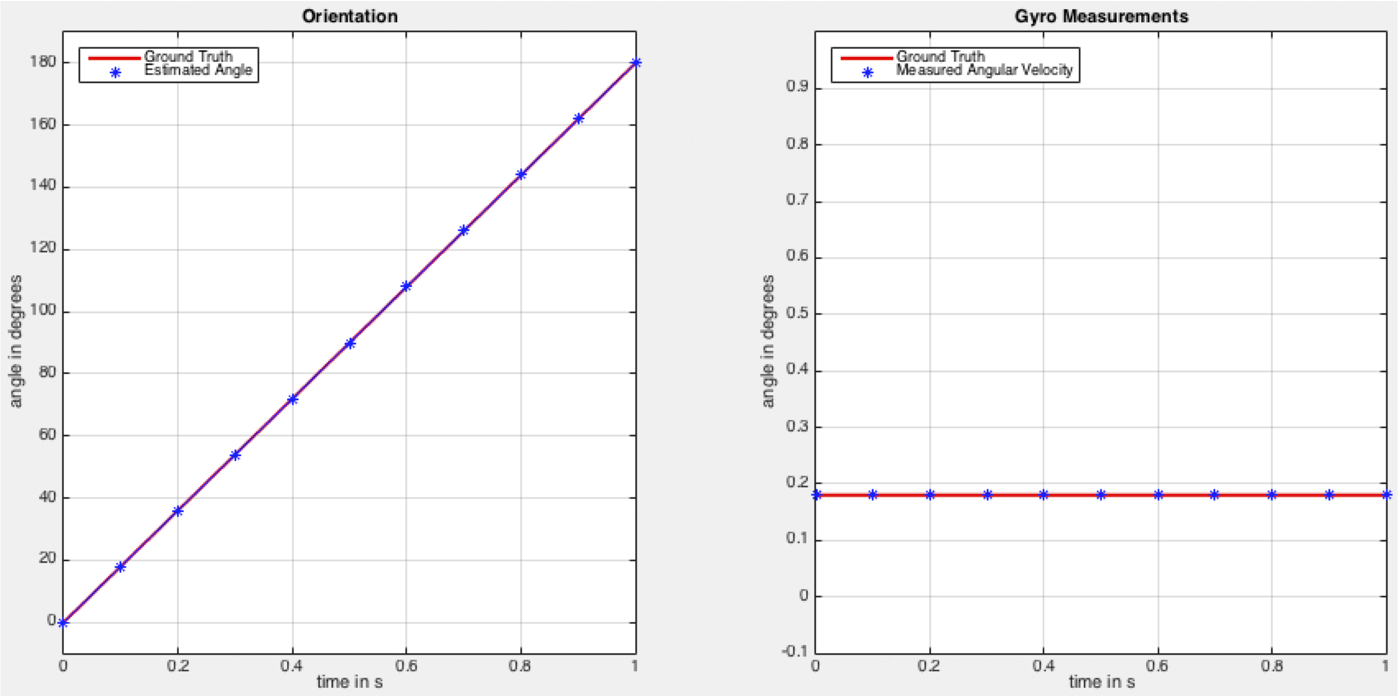
<u>have</u>: time step → Δt

<u>want</u>: angle at current time step → $\theta(t+\Delta t)$

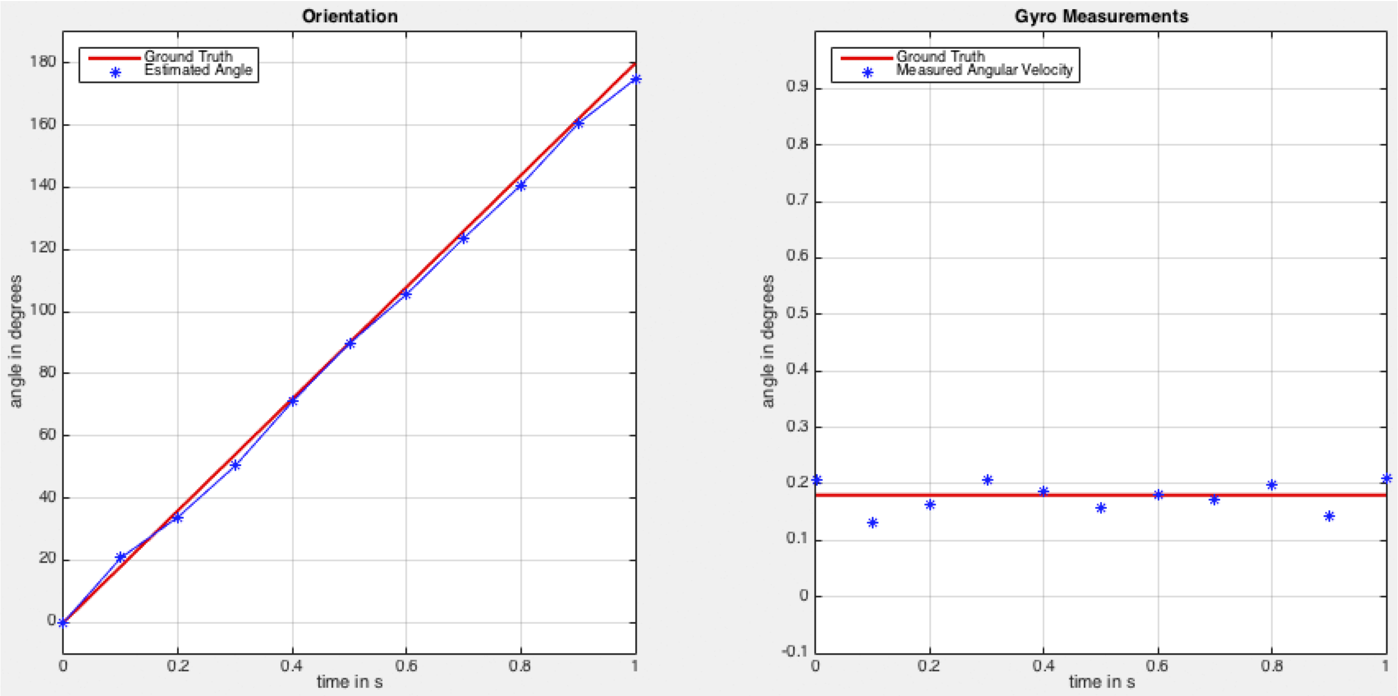
approximation error! → ε

<u>have</u>: gyro measurement (angular velocity) → ω

Gyro Integration: linear motion, no noise, no bias



Gyro Integration: linear motion, noise, no bias



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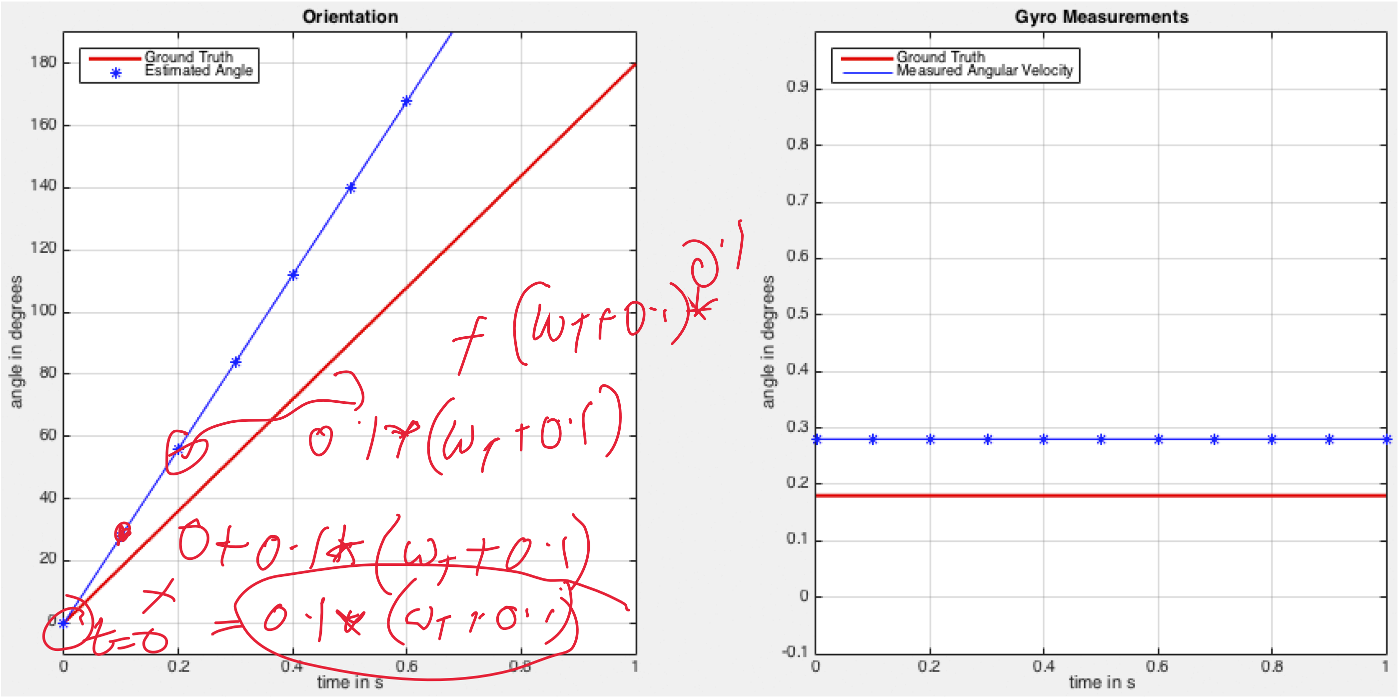

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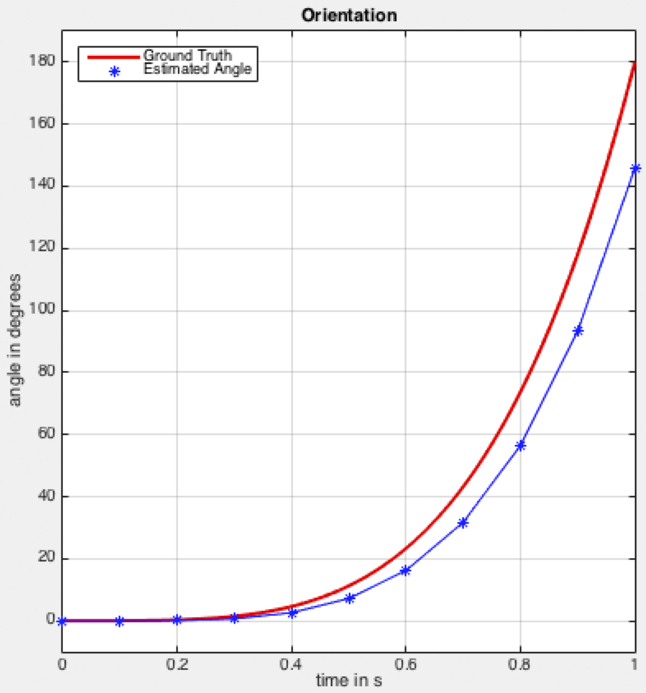
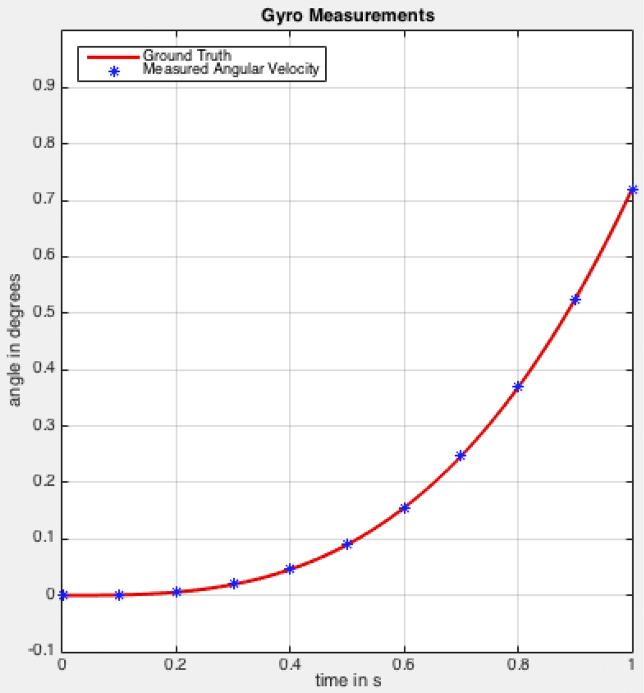
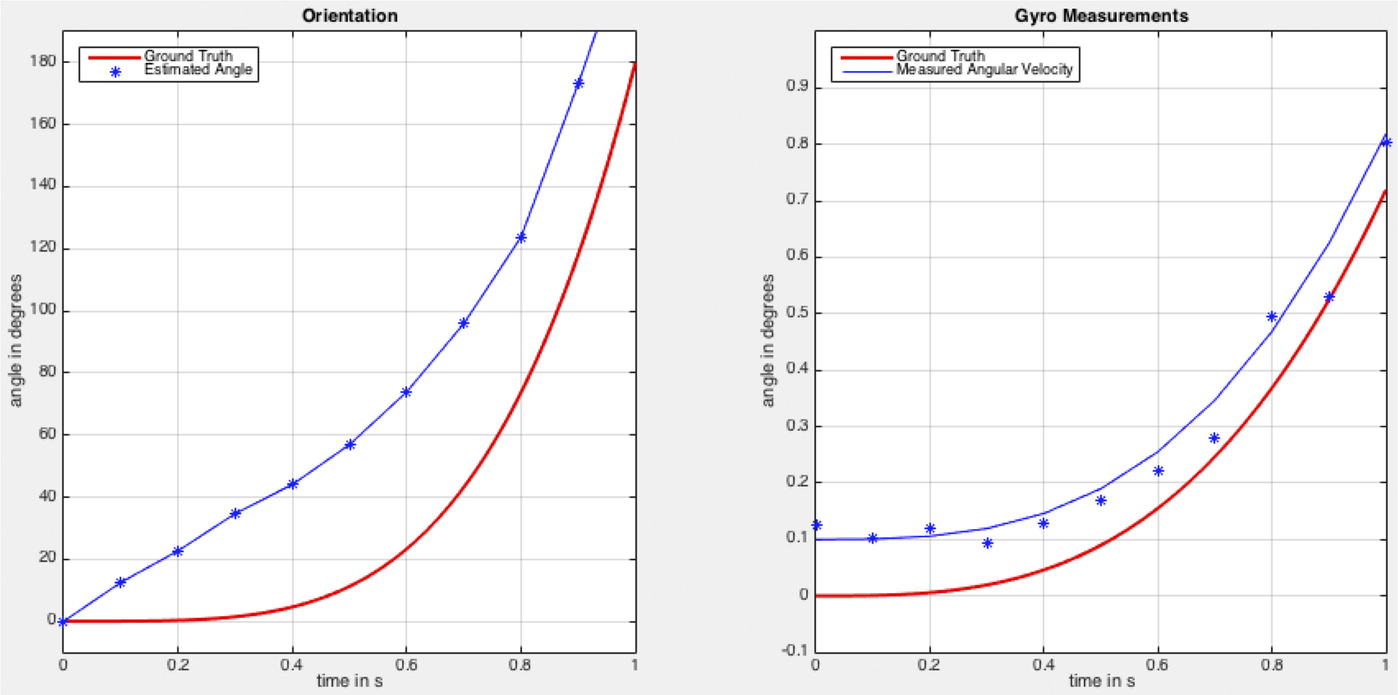




Gyro Integration: nonlinear motion, noise, bias



Gyro Integration aka *Dead Reckoning*

- works well for linear motion, no noise, no bias = unrealistic

- even if bias is know and noise is zero → <u>drift</u> (from integration)

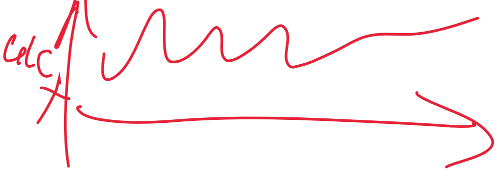
- bias & noise variance can be estimated, other sensor measurements used to correct for drift (sensor fusion)

- <u>accurate in short term, but not reliable in long term due to drift</u>

Gyro Advice

Always be aware of what units you are working with, degrees per second v radians per second!

Accelerometers



- measure linear acceleration $\tilde{a} = a^{(g)} + a^{(l)} + \eta, \quad \eta \sim N\left(0, \sigma_{acc}^2\right)$

- without motion: read noisy gravity vector $a^{(g)} + \eta$ pointing UP! with magnitude 9.81 m/s^2 = 1g

- with motion: combined gravity vector and external forces $a^{(l)}$

Accelerometers

- advantages:
    - points up on average with magnitude of 1g
    - accurate in long term because no drift and the earth’s center of gravity (usually) doesn’t move
- problem:
    - noisy measurements
    - unreliable in short run due to motion (and noise)

- <u>complementary to gyro measurements!</u>

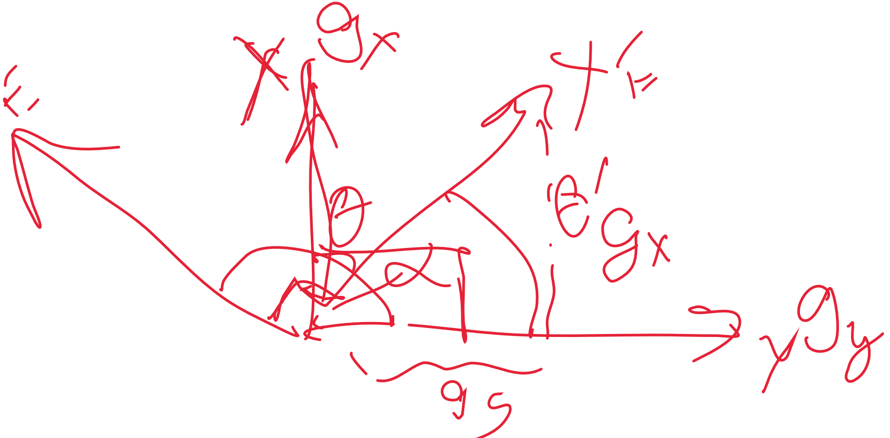
Accelerometers

- fusing gyro and accelerometer data = 6 DOF sensor fusion

- can correct tilt (i.e., pitch & roll) only – no information about yaw

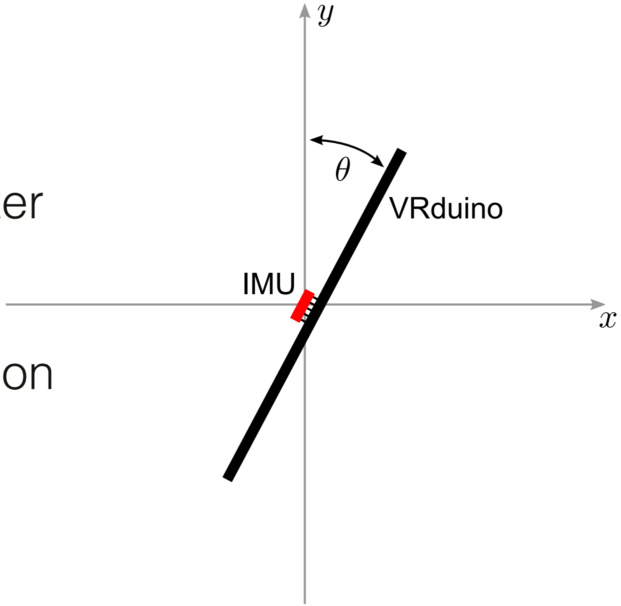
$$\theta' = \tan^{-1}(g_x/g_y)$$

$$\tan(\theta') = g_x/g_y$$



Orientation Tracking in *Flatland*

- problem: track angle $\boldsymbol{\theta}$ in 2D space
- sensors: 1 gyro, 2-axis accelerometer
- goal: understand 6-DOF sensor fusion



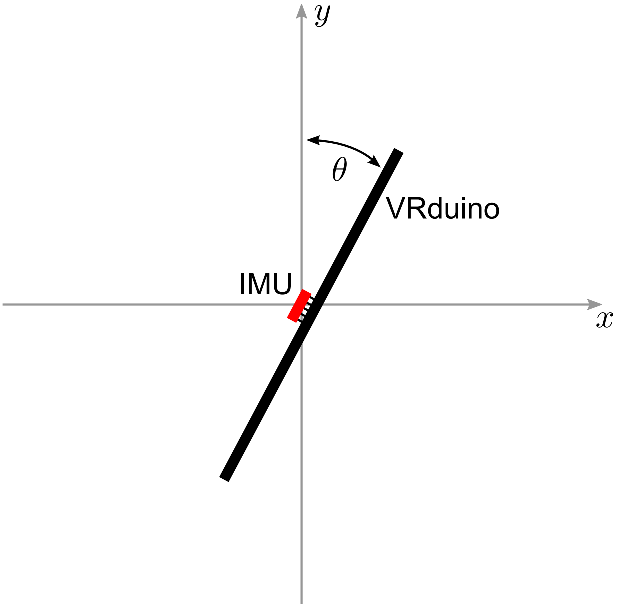
Orientation Tracking in *Flatland*

- gyro integration via Taylor series as

$$\theta_{gyro}^{(t)} = \theta_{gyro}^{(t-1)} + \tilde{\omega}\Delta t$$

- get Δt from microcontroller
- set $\theta_{gyro}^{(0)} = 0$

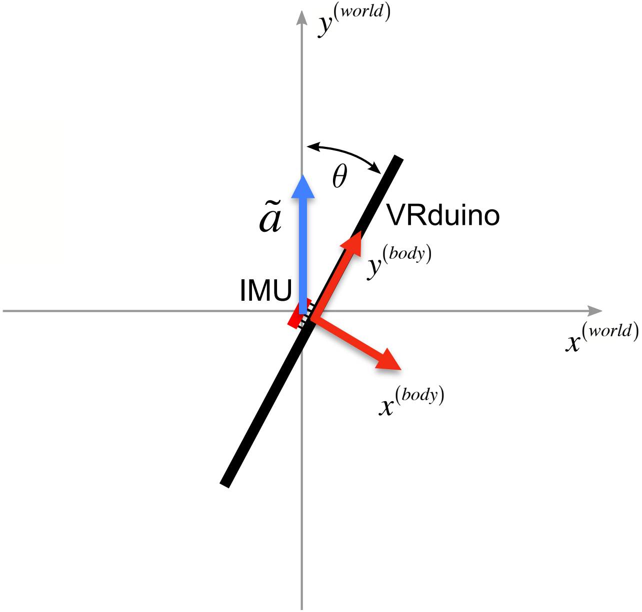
- biggest problem: drift!



Orientation Tracking in *Flatland*

- angle from accelerometer

$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right)$$

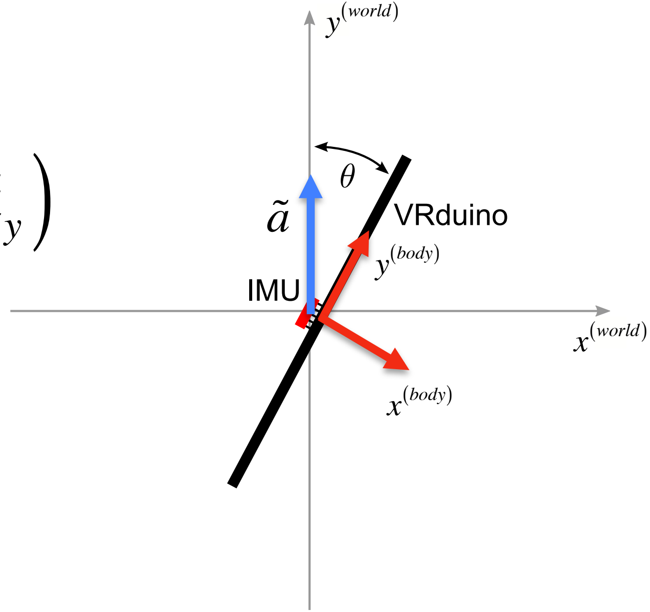


Orientation Tracking in *Flatland*

- angle from accelerometer

$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right) = \text{atan2}\left(\tilde{a}_x, \tilde{a}_y\right)$$

handles division by 0 and
proper signs, provided by most
programming languages

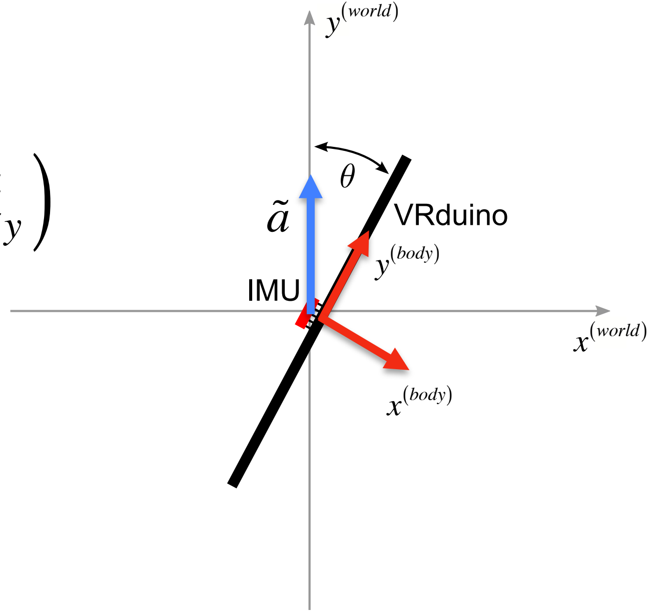


Orientation Tracking in *Flatland*

- angle from accelerometer

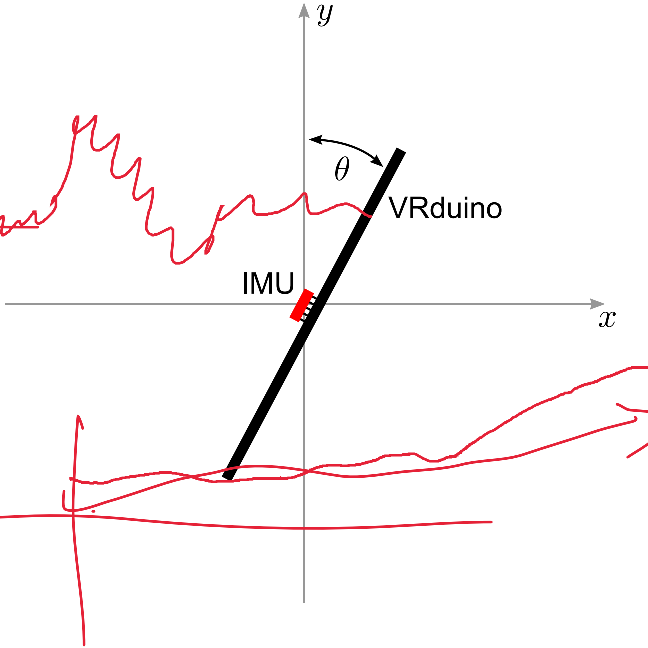
$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right) = \text{atan2}\left(\tilde{a}_x, \tilde{a}_y\right)$$

- biggest problem: noise



Orientation Tracking in *Flatland*

- sensor fusion: combine gyro and accelerometer measurements
- intuition:
  - remove drift from gyro via high-pass filter
  - remove noise from accelerometer via low-pass filter

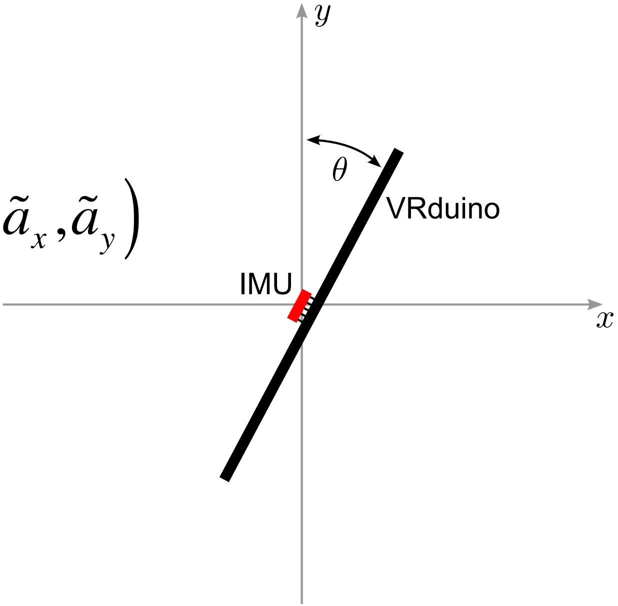


Orientation Tracking in *Flatland*

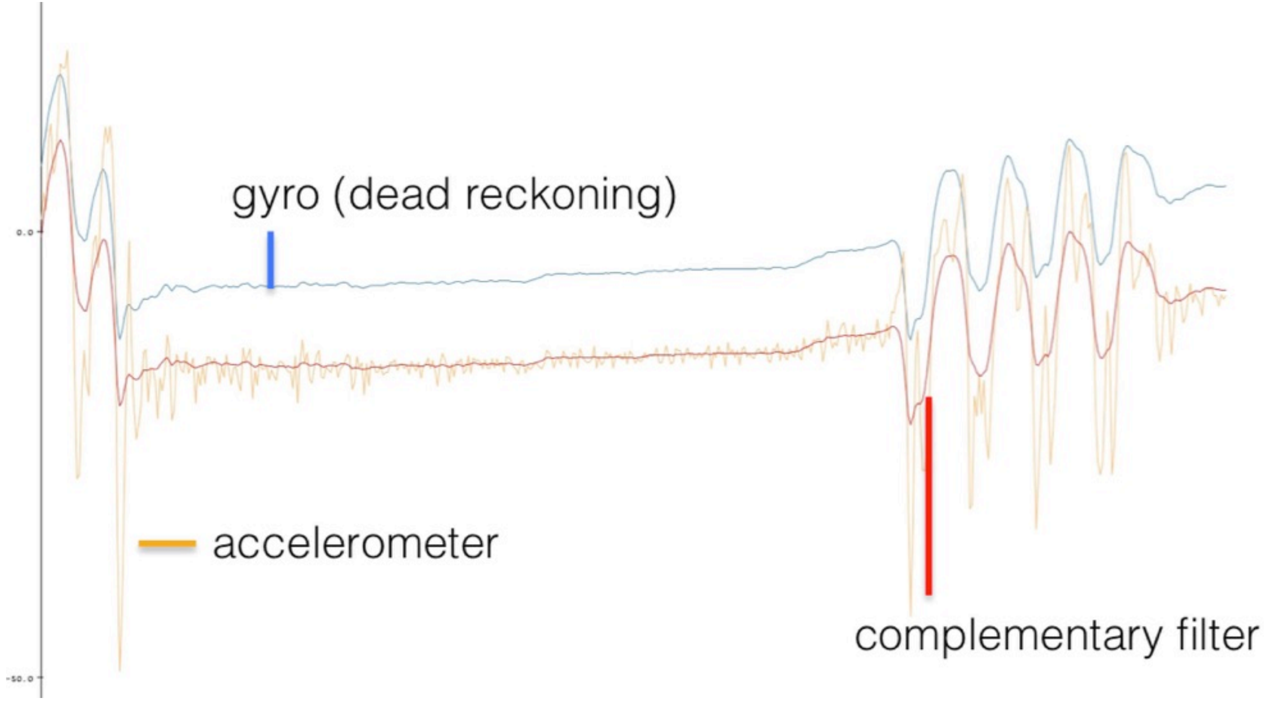
- sensor fusion with complementary filter, i.e. linear interpolation

$$\theta^{(t)} = \alpha\left(\theta^{(t-1)} + \tilde{\omega}\Delta t\right) + (1-\alpha)\,\mathrm{atan2}\left(\tilde{a}_x, \tilde{a}_y\right)$$

- no drift, no noise!



Orientation Tracking in *Flatland*



Magnetometers

- measure earth’s magnetic field in Gauss or uT
- 3 orthogonal axes = vector pointing along the magnetic field
- actual direction depends on latitude and longitude!
- distortions due to metal / electronics objects in the room or in HMD

Magnetometers

- advantages:
    - complementary to accelerometer – gives yaw (heading)
- problems:
    - affected by metal, distortions of magnetic field
    - need to know location, even when calibrated (e.g. GPS)
- together with gyro + accelerometer = 9 DOF sensor fusion