

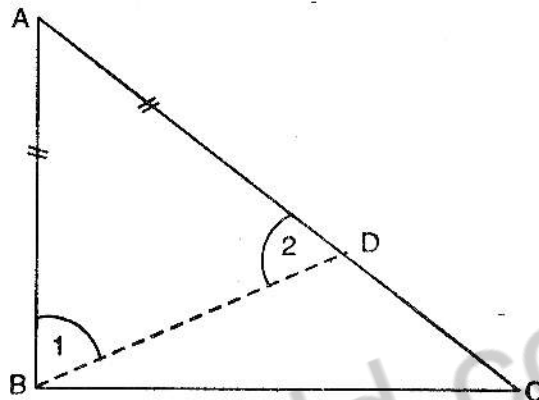
SIDES AND ANGLES OF A TRIANGLE

Theorem If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.

Given In $\triangle ABC$, $\overline{AC} > \overline{AB}$

To Prove $m\angle ABC > m\angle ACB$

Construction On \overline{AC} take a point D such that $\overline{AD} \cong \overline{AB}$. Join B to D so that $\triangle ADB$ is an isosceles triangle. Label $\angle 1$ and $\angle 2$ as shown in the given figure.

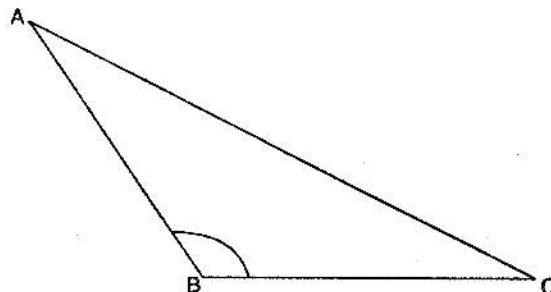


Proof

Statements	Reasons
In $\triangle ABD$	
$m\angle 1 = m\angle 2$... (i)	Angle opposite to congruent sides, (construction)
In $\triangle BCD$, $m\angle ACB < m\angle 2$	
i.e., $m\angle 2 > m\angle ACB$... (ii)	(An exterior angle of a triangle is greater than a non-adjacent interior angle).
$\therefore m\angle 1 > m\angle ACB$ (iii)	By (i) and (ii)
But	
$m\angle ABC = m\angle 1 + m\angle DBC$	Postulate of addition of angles.
$\therefore m\angle ABC > m\angle 1$ (iv)	
$\therefore m\angle ABC > m\angle 1 > m\angle ACB$	
Hence $m\angle ABC > m\angle ACB$	By (iii) and (iv) (Transitive property of inequality of real number)

Example Prove that in a scalene triangle, the angle opposite to the largest side is of measure greater than 60° . (i.e., two-third of a right-angle).

Given In $\triangle ABC$, $\overline{AC} > \overline{AB}$, $\overline{AC} > \overline{BC}$.



To Prove

$$m\angle B > 60^\circ$$

Proof

Statements	Reasons
In $\triangle ABC$	
$m\angle B > m\angle C$	$\overline{mAC} > \overline{mAB}$ (given)
$m\angle B > m\angle A$	$\overline{mAC} > \overline{mBC}$ (given)
But $m\angle A + m\angle B + m\angle C = 180^\circ$	$\angle A, \angle B, \angle C$ are the angles of $\triangle ABC$
$\therefore m\angle B + m\angle B + m\angle B > 180^\circ$	$m\angle B > m\angle C, m\angle B > m\angle A$ (proved)
Hence $m\angle B > 60^\circ$	$180^\circ/3 = 60^\circ$

Example In a quadrilateral ABCD, \overline{AB} is the longest side and \overline{CD} is the shortest side. Prove that $m\angle BCD > m\angle BAD$.

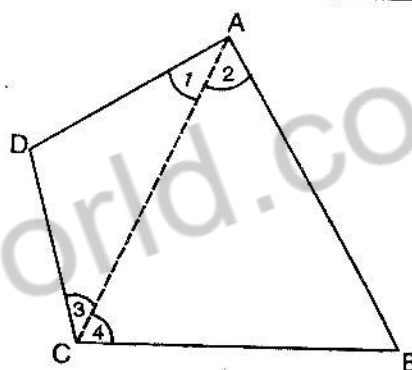
Given In quad. ABCD, \overline{AB} is the longest side and \overline{CD} is the shortest side.

To Prove $m\angle BCD > m\angle BAD$

Construction

Join A to C.

Name the angles $\angle 1, \angle 2, \angle 3$ and $\angle 4$ as shown in the figure.

**Proof**

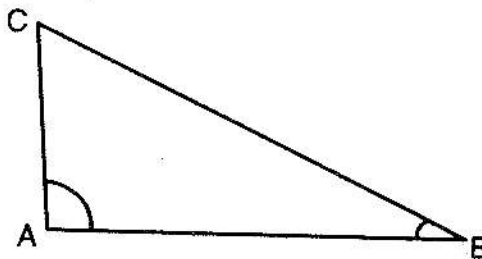
Statements	Reasons
In $\triangle ABC, m\angle 4 > \angle 2$(i)	$\overline{mAB} > \overline{mBC}$ (given)
In $\triangle ACD, m\angle 3 > m\angle 1$(ii)	$\overline{mAD} > \overline{mCD}$ (given)
$\therefore m\angle 4 + m\angle 3 > m\angle 2 + m\angle 1$	From I and II
Hence $m\angle BCD > m\angle BAD$	$\begin{cases} m\angle 4 + m\angle 3 = m\angle BCD \\ m\angle 2 + m\angle 1 = m\angle BAD \end{cases}$

Theorem:

If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.

Given In $\triangle ABC, m\angle A > m\angle B$

To Prove $\overline{mBC} > \overline{mAC}$



Proof

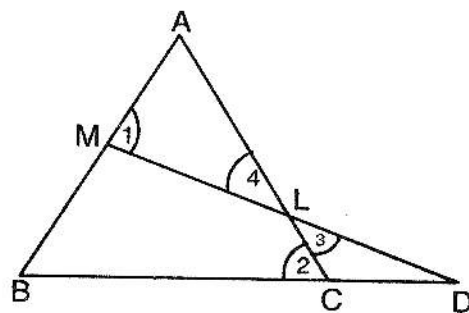
Statements	Reasons
If $m\overline{BC} > m\overline{AC}$, then either (i) $m\overline{BC} = m\overline{AC}$ or (ii) $m\overline{BC} < m\overline{AC}$	(Trichotomy property of real numbers)
From (i) if $m\overline{BC} = m\overline{AC}$, then $m\angle A = m\angle B$ which is not possible	(Angles opposite to congruent sides are congruent) Contrary to the given
From (ii) if $m\overline{BC} < m\overline{AC}$, then $m\angle A < m\angle B$ This is also not possible.	(The angle opposite to longer side is greater than angle opposite to smaller side) Contrary to the given
$\therefore m\overline{BC} \neq m\overline{AC}$ And $m\overline{BC} < m\overline{AC}$ Thus $m\overline{BC} > m\overline{AC}$	Trichotomy property of real numbers.

Note

- (i) The hypotenuse of a right angle triangle is longer than each of the other two sides.
- (ii) In an obtuse angled triangle, the side opposite to the obtuse angle is longer than each of the other two sides.

Example

ABC is an isosceles triangle with base \overline{BC} . On \overline{BC} a point D is taken away from C. A line segment through D cuts \overline{AC} at L and \overline{AB} at M. Prove that $m\overline{AL} > m\overline{AM}$.

**Given**

In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$.

D is a point on \overline{BC} away from C.

A line segment through D cuts \overline{AC} at L and \overline{AB} at M.

To Prove

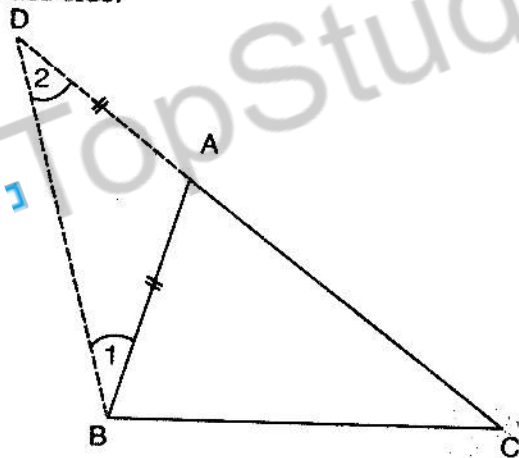
$m\overline{AL} > m\overline{AM}$

Proof

Statements			Reasons
In $\triangle ABC$			
$\angle B \cong \angle 2$...I		$\overline{AB} \cong \overline{AC}$ (given)
In $\triangle MBD$			
$m\angle 1 > m\angle B$...II		($\angle 1$ is an ext. \angle and $\angle B$ is its internal opposite \angle)
$\therefore m\angle 1 > m\angle 2$...III		From I and II
In $\triangle LCD$,			
$m\angle 2 > m\angle 3$IV		($\angle 2$ is an ext. \angle and $\angle 3$ is its internal opposite \angle)
$\therefore m\angle 1 > m\angle 3$...V		From III and IV
But $\angle 3 \cong \angle 4$...VI		Vertical angles
$\therefore m\angle 1 > m\angle 4$			From V and VI
Hence $m\overline{AL} > m\overline{AM}$			In $\triangle ALM$, $m\angle 1 > m\angle 4$ (proved)

Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

**To Prove**

- (i) $m\overline{AB} + m\overline{AC} > m\overline{BC}$
- (ii) $m\overline{AB} + m\overline{BC} > m\overline{AC}$
- (iii) $m\overline{BC} + m\overline{CA} > m\overline{AB}$

Construction

Take a point D on \overline{CA} such that $\overline{AD} \cong \overline{AB}$. Join B to D and name the angles. $\angle 1$, $\angle 2$ as shown in the given figure.

Given

$\triangle ABC$

Proof

Statements			Reasons
In $\triangle ABD$,			
$\angle 1 \cong \angle 2$...(i)		$\overline{AD} \cong \overline{AB}$ (construction)

$m\angle DBC > m\angle 1$(ii) $\therefore m\angle DBC > m\angle 2$(iii) In $\triangle DBC$, $m\overline{CD} > m\overline{BC}$ i.e., $m\overline{AD} + m\overline{AC} > m\overline{BC}$ Hence $m\overline{AB} + m\overline{AC} > m\overline{BC}$ Similarly, $m\overline{AB} + m\overline{BC} > m\overline{AC}$ And $m\overline{BC} + m\overline{CA} > m\overline{AB}$	$m\angle DBC = m\angle 1 + m\angle ABC$ From (i) and (ii) By (iii) $m\overline{CD} = m\overline{AD} + m\overline{AC}$ $m\overline{AD} = m\overline{AB}$ (construction)
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Example

Which of the following sets of lengths can be the lengths of the sides of a triangle.

- (a) 2cm, 3cm, 5cm
 (b) 3cm, 4cm, 5 cm
 (c) 2cm, 4cm, 7cm

(a) $\therefore 2+3 = 5$

\therefore This set of lengths cannot be those of the sides of a triangle.

(b) $\therefore 3+4 > 5, 3+5 > 4, 4+5 > 3$

\therefore This set can form a triangle.

(c) $\therefore 2+4 < 7$

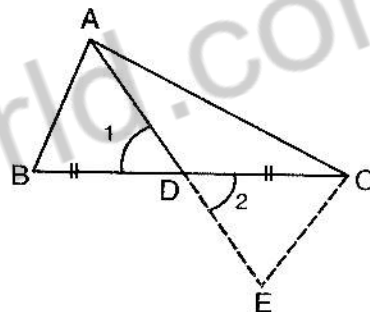
\therefore This set of lengths cannot be the sides of a triangle.

Example Prove that the sum of the measures of two sides of a triangle is

Proof

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle ECD$	
$\overline{BD} \cong \overline{CD}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{AD} \cong \overline{ED}$	Construction
$\triangle ABD \cong \triangle ECD$	

greater than twice the measure of the median which bisects the third side.



Given

In $\triangle ABC$,

median AD bisects side \overline{BC} at D.

To Prove

$m\overline{AB} + m\overline{AC} > 2m\overline{AD}$.

Construction On \overline{AD} , take a point E, such that $\overline{DE} \cong \overline{AD}$. Join C to E. Name the angles $\angle 1, \angle 2$ as shown in the figure.

$\overline{AB} \cong \overline{EC}$I	S.A.S. Postulate
$m\overline{AC} + m\overline{EC} > m\overline{AE}$II	Corresponding sides of $\cong \Delta$ s
$m\overline{AC} + m\overline{AB} > m\overline{AE}$		ACE is a triangle
Hence $m\overline{AC} + m\overline{AB} > 2m\overline{AD}$		From I and II
		$m\overline{AE} = 2m\overline{AD}$ (construction)

Example

Prove that the difference of measures of two sides of a triangle is less than the measure of the third side.

Given

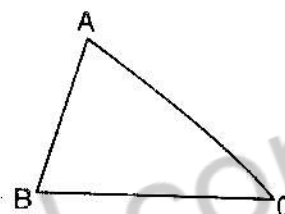
ΔABC

To Prove

$$m\overline{AC} - m\overline{AB} < m\overline{BC}$$

$$m\overline{BC} - m\overline{AB} < m\overline{AC}$$

$$m\overline{BC} - m\overline{AC} < m\overline{AB}$$



Proof

Statements	Reasons
$m\overline{AB} + m\overline{BC} > m\overline{AC}$	ABC is a triangle
$(m\overline{AB} + m\overline{BC} - m\overline{AB}) > (m\overline{AC} - m\overline{AB})$	Subtracting $m\overline{AB}$ from both sides.
$\therefore m\overline{BC} > (m\overline{AC} - m\overline{AB})$	
Or $m\overline{AC} - m\overline{AB} < m\overline{BC}$	$a > b \Rightarrow b < a$
Similarly	
$m\overline{BC} - m\overline{AB} < m\overline{AC}$	
$m\overline{BC} - m\overline{AC} < m\overline{AB}$	Reason similar to I

Exercise 13.1

1. Two sides of a triangle measure 10 cm and 15 cm. Which of the following measure is possible for the third side?

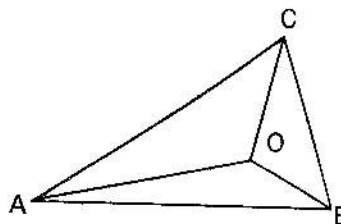
- (a) 5 cm (b) 20 cm
(c) 25 cm (d) 30 cm

Ans. 20cm.

2. O is an interior point of the ΔABC . Show that

$$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$$

Given: O is the interior point of ΔABC



To Prove:

$$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$$

Construction:

Join O with A, B and C.

Proof:

Statements	Reasons
$\triangle OAB$	
$m\overline{OA} + m\overline{OB} > m\overline{AB}$(i)	Sum of two sides > third side
Similarly	
$m\overline{OB} + m\overline{OC} > m\overline{BC}$(ii)	Sum of two sides > third side
and	
$m\overline{OC} + m\overline{OA} > m\overline{CA}$(iii)	
$2m\overline{OA} + 2m\overline{OB} + 2m\overline{OC} > m\overline{AB} + m\overline{BC} + m\overline{CA}$	Adding (i), (ii) and (iii)
$2(m\overline{OA} + m\overline{OB} + m\overline{OC}) > m\overline{AB} + m\overline{BC} + m\overline{CA}$	
$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$	

3. In the $\triangle ABC$, $m\angle B = 75^\circ$ and $m\angle C = 55^\circ$. Which of the sides of the triangle is longest and which is the shortest?

Ans: Given a $\triangle ABC$ in which

$$m\angle B = 75^\circ$$

$$m\angle C = 55^\circ$$

As $m\angle A + m\angle B + m\angle C = 180^\circ$

$$m\angle A + 75^\circ + 55^\circ = 180^\circ$$

$$m\angle A + 130^\circ = 180^\circ$$

$$m\angle A = 180^\circ - 130^\circ$$

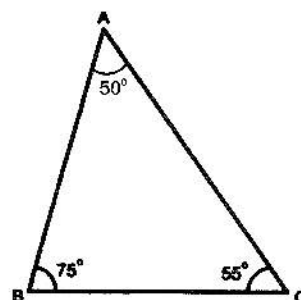
$$m\angle A = 50^\circ$$

As we know in a triangle, the side opposite to greater angle is longer than the side opposite to smaller angle

So $m\overline{AC} > m\overline{BC}$

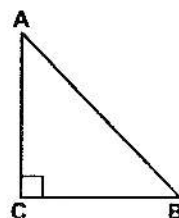
Hence longest side is \overline{AC}

and shortest side is \overline{BC}



4. Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.

Ans.



Given: $\triangle ABC$ is a right angle triangle.

Hence AB is hypotenuse of $\triangle ABC$.

To prove:

$m\angle A > m\angle C$ and $m\angle A > m\angle B$

Proof:

As $\triangle ABC$ is a right angle triangle.

So $m\angle C = 90^\circ$ is the largest angle and the remaining angles $\angle A$ and $\angle B$ are acute.

So $m\angle C > m\angle A$ and $m\angle C > m\angle B$

As the side opposite to the greater angle is longer than the side opposite to the smaller angle.

Hence $m\angle A > m\angle C$ and $m\angle A > m\angle B$

Proof

Statements	Reasons
\therefore in $\triangle ABC$ $\angle ACB > \angle ABC$ $\frac{1}{2} \angle ACB > \frac{1}{2} \angle ABC$	$\therefore \overline{AB} > \overline{AC}$
$\therefore \angle BCD > \angle DBC$ $\overline{BD} > \overline{CD}$	$\overline{CD}, \overline{BD}$ are bisectors of $\angle C, \angle B$. The bigger sides is opposite the bigger angle

Theorem From a point, outside a line, perpendicular is the shortest distance from the point to the line.

Given A line AB and a point C (not lying on \overline{AB}) and a point D on \overline{AB} such that

$\overline{CD} \perp \overline{AB}$.

To Prove

$m\angle CDB$ is the shortest distance from the point C to \overline{AB} .

Construction

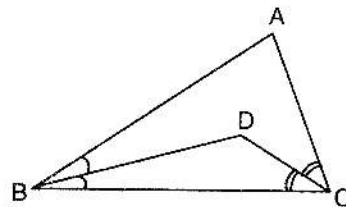
Take a point E on \overline{AB} . Join C and E to form a $\triangle CDE$

Proof:

Statements	Reasons
In $\triangle CDE$ $m\angle CDB > m\angle CED$	(An exterior angle of a triangle is greater

5. In the triangular figure, $\overline{AB} > \overline{AC}$.

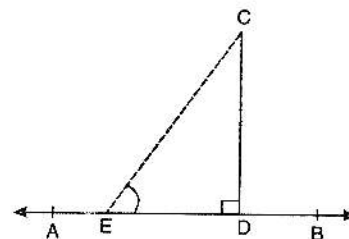
\overline{BD} and \overline{CD} are the bisectors of $\angle B$ and $\angle C$ respectively. Prove that $\overline{BC} > \overline{DC}$.



Given: $\overline{AB} > \overline{BC}$, \overline{BD} and \overline{CD} are the bisectors of the angles B and C

To Prove:

To prove = $\overline{BD} > \overline{CD}$



But $m\angle CDB = m\angle CDE$

$\therefore m\angle CDE > m\angle CED$

or $m\angle CED < m\angle CDE$

or $m\overline{CD} < m\overline{CE}$

But E is any point on AB

Hence $m\overline{CD}$ is the shortest distance from C to \overline{AB} .

than non adjacent interior angle).

Supplement of right angle.

$a > b \Rightarrow b < a$

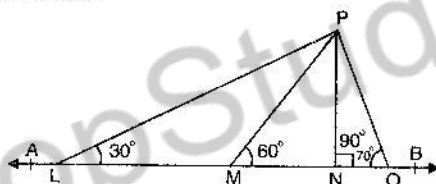
Side opposite to greater angle is greater.

Note:

- The distance between a line and a point not on it, is the length of the perpendicular line segment from the point to the line.
- The distance between a line and a point lying on it is zero

Exercise 13.2

- In the figure, P is any point and AB is a line. Which of the following is the shortest distance between the point P and the line AB.

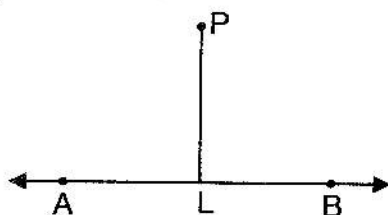


- (a) $m\overline{PL}$ (b) $m\overline{PM}$
 (c) $m\overline{PN}$ (d) $m\overline{PO}$

Ans. (c)

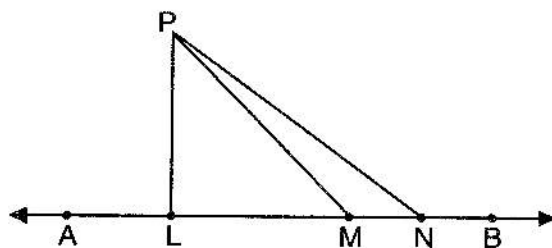
- In the figure, P is any point lying away from the line AB. Then $m\overline{PL}$ will be the shortest distance if:

- (a) $m\angle PLA = 80^\circ$
 (b) $m\angle PLB = 100^\circ$
 (c) $m\angle PLA = 90^\circ$



Ans. (c)

- In the figure, \overline{PL} is perpendicular to the line AB and $m\overline{LN} > m\overline{LM}$. Prove that $m\overline{PN} > m\overline{PM}$.



Ans. Here it is given $m\overline{PL}$ is perpendicular to line \overline{AB} and $m\overline{LN} > m\overline{LM}$

Proof:

Here $m\overline{PN} > m\overline{PM}$

As \overline{PL} is the shortest distance from P to line \overline{AB} . So

$\overline{PL} \perp \overline{AB}$

As we go away from point L, the distance from points to L increases Hence

$m\overline{PN} > m\overline{PM}$

4. Which of the following are true and which are false?

(i) The angle opposite to the longer side is greater. **TRUE**

(ii) In a right-angled triangle greater angle is of 60° . **FALSE**

(iii) In an isosceles right-angled triangle, angles other than right angle are each of 45° . **TRUE**

(iv) A triangle having two congruent sides is called equilateral triangle. **FALSE**

(v) A perpendicular from a point to a line is shortest distance. **TRUE**

(vi) Perpendicular to line form an angle of 90° . **TRUE**

(vii) A point out-side the line is collinear. **FALSE**

(viii) Sum of two sides of triangle is greater than the third. **TRUE**

(ix) The distance between a line and a point on it is zero. **TRUE**

(x) Triangle can be formed of lengths 2 cm, 3 cm and 5 cm. **FALSE**

5. What will be angle for shortest distance from an outside point to the line?

Ans. 90°

6. If 13 cm, 12 cm, and 5 cm are the lengths of a triangle, then verify that difference of measures of any two sides of a triangle is less than the measure of the third side.

Ans: (i) $13 - 12 = 1 < 15$

(ii) $12 - 4 = 7 < 13$

(iii) $13 - 5 = 8 < 12$

So verified

7. If 10 cm, 6 cm and 8 cm are the lengths of a triangle, then verify that sum of measures of two sides of a triangle is greater than the third side.

Ans. (i) $10 + 6 = 16 > 8$

(ii) $6 + 8 = 14 > 10$

(iii) $10 + 8 = 18 > 6$

8. 3 cm, 4 cm and 7 are not the lengths of the triangle. Give the reason.

Ans: $3 + 4 < 7$

9. If 3 cm and 4 cm are lengths of two sides of a right angle triangle then what should be the third length of the triangle.

Ans. Third length = $\sqrt{3^2 + 4^2}$
 $= \sqrt{25} = 5\text{cm}$

OBJECTIVE

1. Which of the following sets of lengths can be the lengths of the sides of a triangle:

(a) 2cm, 3cm, 5cm

(b) 3cm, 4cm, 5cm

(c) 2cm, 4cm, 7cm

(d) None

2. Two sides of a triangle measure 10cm and 15cm. Which of the following measure is possible for the third side!

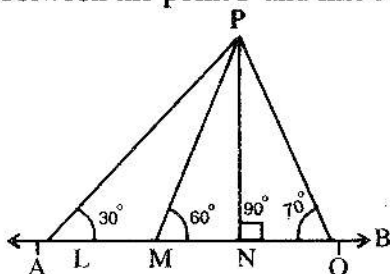
(a) 5cm

(b) 20cm

(c) 25cm

(d) 30cm

3. In the figure, P is any point and AB is a line. Which of the following is the short distance between the point P and line AB.



- (a) \overline{PL}
 (b) \overline{PM}
 (c) \overline{PN}
 (d) \overline{PO}
4. In the figure, P is any point lying away from the line AB. Then \overline{PL} will be shortest distance if:
-
- (a) $m\angle PLA = 80^\circ$
 (b) $m\angle PLB = 100^\circ$
 (c) $m\angle PLA = 90^\circ$
 (d) None
5. The angle opposite to the longer side is:
- (a) Greater
 (b) Shorter
 (c) Equal
 (d) None
6. In right angle triangle greater angle of:

- (a) 60°
 (b) 30°
 (c) 75°
 (d) 90°
7. In an isosceles right-angled triangle angles other than right angle are each of:
- (a) 40°
 (b) 45°
 (c) 50°
 (d) 55°
8. A triangle having two congruent sides is called ____ triangle.
- (a) Equilateral
 (b) Isosceles
 (c) Right
 (d) None
9. Perpendicular to line form an angle of ____
- (a) 30°
 (b) 60°
 (c) 90°
 (d) 120°
10. Sum of two sides of triangle is ____ than the third.
- (a) Greater
 (b) Smaller
 (c) Equal
 (d) None
11. The distance between a line and a point on it is ____
- (a) Zero
 (b) One
 (c) Equal
 (d) None

ANSWER KEY

1.	b	2.	b	3.	c	4.	c	5.	a
6.	d	7.	b	8.	a	9.	c	10.	a
11.	a								