Exercise 1.1

Q.1 Find the order of the following matrices.

$$A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$$

It has 2 rows & 2 columns that's why its order is 2 - by -2

$$B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

It has 2 rows & 2 columns. So, its order is 2- by -2

$$C = \begin{bmatrix} 2 & 4 \end{bmatrix}$$

It has 1 row and 2 columns. So, its order is 1 – by -2

$$D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$$

It has 3 rows and 1 column. So, its order is 3 – by -1

$$E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

It has 3 rows and 2 columns. So, its order is 3 – by –2

$$F = \begin{bmatrix} 2 \end{bmatrix}$$

It has 1 row & 1 column. So, its order is 1- by -1

$$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

It has 3 rows and 3 columns. So, its order is 3 –by -3

$$H=\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$$

It has 2 rows & 3 columns. So, its order is 2- by -3

Q.2 Which one of the following matrices are equal?

1) $A=[3]$, **2)** $B=[3 \quad 5]$,

3) $C=[5-2]$ **4)** $D=[5 \quad 3]$

5) $E=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ **6)** $F=\begin{bmatrix} 2 \\ 6 \end{bmatrix}$

7) $G=\begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$ **8)** $H=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$

9) $I=[3 \quad 3+2]$ **10)** $J=\begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Solution:

Order of $A=[3]$ is equal to Order of $C=[5-2]$

Order of $B=[3 \quad 5]$ is equal to Order of $I=[3 \quad 3+2]$

Order of $C=[5-2]$ is equal to Order of $A=[3]$

$D=[5 \quad 3]$ has no equal matrix.

$E=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ has equal matrices.

Order of $\Rightarrow H=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ is equal to Order of $J=\begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Order of $F=\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ is equal to Order of $G=\begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$

Q.3 Find the values of a, b, c & d.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & +2d \end{bmatrix}$$

Solution:

As Matrices are equal so their corresponding entries are same.

$a+c=0 \rightarrow (1)$

$a+2b = -7 \rightarrow (2)$

$c-1=3 \rightarrow (3)$

$4d-6=+2d \rightarrow (4)$

Solving 3rd equation

c – 1 = 3

c = 3 +1

c = 4

Solving 2nd equation

a + 2b = -7

-4 + 2b = -7

2b = -7 +4

2b = -3

b = $\frac{-3}{2}$

Solving 1st equation

a + c = 0

a + 4 = 0

a = -4

Solving 4th equation

4d – 6 =2d

-6 = 2d – 4d

-6 = -2d

d = $\frac{+\cancel{6}_3}{+\cancel{2}_1}$

d = 3

Exercise 1.2

Q.1 Identify the following matrices.

$$A=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

It's all members are 0. So, it's a null matrix.

$$B = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

It has only 1 row. So, it's a row matrix.

$$C = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$$

It has only 1 column. So, it's a column matrix.

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

It is an identity matrix because its diagonal entries are 1 and non-diagonal entries are zero.

$$E = \begin{bmatrix} 0 \end{bmatrix}$$

It has only 0. So, it's a null matrix.

$$F = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

It has only 1 column. So, it's a column matrix.

Q.2 Identify the following matrices.

(1) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$

Its number of rows & columns are not equal. So, it's a rectangular matrix.

(2) $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

It has only one column. So, it's a column matrix.

(3) $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$

The number of rows & columns are equal. So, it's a square matrix.

(4) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Identity matrix – Because Diagonal entries are 1 and non-diagonal entries are 0.

(5) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

Number of rows & columns are not equal. So, it's a rectangular matrix.

(6) $\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$

It's a row matrix because it has only 1 row.

(7) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Column matrix because it has only one column.

(8) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Square matrix because number of rows & columns are equal.

(9) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Null matrix because all elements are 0.

Q.3 Identify the matrices.

(1) $A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

Scalar- matrix because it non-diagonal entries are 0 & diagonal entries are same.

(2) $B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

Diagonal matrix because its non-diagonal entries are 0.

(3) $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Unit matrix because diagonal-entries are 1.

(4) $D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$

Diagonal matrix because non-diagonal are 0.

(5) $E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$

Scalar- because diagonal are same.

Q.4 Find the negative of matrices.

(1) $A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$-A = -\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(2) $B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

$$-B = -\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

$$-B = \begin{bmatrix} -3 & +1 \\ -2 & -1 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$

$$-C = -\begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$$

(4) $D = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$

$$-D = -\begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} +3 & -2 \\ +4 & -5 \end{bmatrix}$$

(5) $E = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$

$$-E = -\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & +5 \\ -2 & -3 \end{bmatrix}$$

Q.5 Find the transpose.

(1) $A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

$$A^t = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}^t$$
$$A^t = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$$

(2) $B = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}$
$$B^t = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}^t$$
$$B^t = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$
$$C^t = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}^t$$
$$C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$$

(4) $D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$
$$D^t = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}^t$$
$$D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

(5) $E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$
$$E^t = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}^t$$
$$E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$$

(6) $F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
$$F^t = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^t$$
$$F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Q.6 Verify if $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

(i) $(A^t)^t = A$

Solution: $(A^t)^t = A$
$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$
$$A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$
$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$
$$\left(A^t\right)^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^t$$
$$\left(A^t\right)^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$
$$\left(A^t\right)^t = A$$
Hence Proved.

(ii) $\left(B^t\right)^t = B$

Solution: $\left(B^t\right)^t = B$
$$B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$
$$B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$
$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$
$$\left(B^t\right)^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}^t$$
$$\left(B^t\right)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$\left(B^t\right)^t = B$$

Hence proved

Exercise 1.3

Q.1 Which of the following are conformable for addition?

$$A=\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, \quad B=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}, \quad D=\begin{bmatrix} 2+1 \\ 3 \end{bmatrix}$$

$$E=\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}, \quad F=\begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$$

Solution:
In the above matrices following matrices are suitable for addition.

(i) A and E are conformable for addition because their order is same and both are square matrix.

(ii) B and D are conformable for addition because the order is same i.e. they have two rows and 1 Columns and both are rectangular matrices.

(iii) C and F are conformable for addition because their order is same i.e. they have three 3 rows and 2 columns and they are a rectangular matrix.

Q.2 Find the additive inverse of the following matrices:

(1) $A=\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$

Solution:
Additive inverse of a matrix is negative matrix.

$A=\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$ is

$$-A=-\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}=\begin{bmatrix} (-1)\times 2 & (-1)4 \\ (-1)(-2) & (-1)1 \end{bmatrix}$$

$$-A=\begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$$

(2) $B=\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Solution: $B=\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Its additive inverse is

$$-B=-\begin{bmatrix} +1 & 0 & -1 \\ +2 & -1 & 3 \\ +3 & -2 & 1 \end{bmatrix}$$

$$-B=\begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

Solution: $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$$-C = -\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -1\times 4 \\ -1\times -2 \end{bmatrix}$$

The additive inverse is

$$-C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

(4) $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

Solution: $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

The additive inverse is

$$-D = -\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times -3 & -1\times -2 \\ -1\times 2 & -1\times 1 \end{bmatrix}$$

$$-D = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$$

(5) $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution: $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The additive inverse of the given matrix is:

$$-E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times 0 & -1\times 1 \end{bmatrix}$$

$$-E = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

(6) $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Solution: $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Its additive inverse is

$$-F = -\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1\times\sqrt{3} & -1\times 1 \\ -1\times -1 & -1\times\sqrt{2} \end{bmatrix}$$

$$-F = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$$

Q.3 **If** $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$

$C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix},\ D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix},$

then find.

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Solution: $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The order of matrix A and the given matrix order is same. So, they can be added easily.

$$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$$

(ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Solution: $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

The order of both above matrices are same, so, they can be easily added.

$$= \begin{bmatrix} 1+(-2) \\ -1+3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & +3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

(iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

Solution: $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

As $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$$

Their orders are same so they can added

$$= \begin{bmatrix} 1+(-2) & -1+(1) & 2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1+1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Solution: $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Their orders are same. So, they can be added.

$$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$$

(v) $2A$

Solution: $2A$

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $2A$

$$= (2) \times \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(-1) & 2\times 2 \\ 2\times 2 & 2\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$$

(vi) $(-1)B$

Solution: $(-1)B$

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $(-1)B$

$$= (-1) \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)\times 1 \\ (-1)\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

(vii) $(-2)C$

Solution: $(-2)C$

As C = $\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, (-2) C

$$= (-2) \times \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (-2)(1) & (-2)(-1) & (-2)(2) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 & -4 \end{bmatrix}$$

(viii) $3D$

Solution: $3D$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $3D$

$$= (3) \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times 1 & 3\times 2 & 3\times 3 \\ 3\times -1 & 3\times 0 & 3\times 2 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$$

(ix) **3*C***

Solution: $3C$

As $C=\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, 3C

$$=(3)\times\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$$
$$=\begin{bmatrix} 3\times 1 & 3\times -1 & 3\times 2 \end{bmatrix}$$
$$=\begin{bmatrix} 3 & -3 & 6 \end{bmatrix}$$

Q.4 Perform the indicated operations and simplify the following:

(i) $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right)+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Solution: $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right)+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$$=\begin{bmatrix} 1+0 & 0+2 \\ 0+3 & 1+0 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 1+1 & 2+1 \\ 3+1 & 1+0 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$

Solution:

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$$
$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0-1 & 2-1 \\ 3-1 & 0-0 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}+\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 1-1 & 0+1 \\ 0+2 & 1+0 \end{bmatrix}$$
$$=\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

(iii) $\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}+\left(\begin{bmatrix} 1 & 0 & 2 \end{bmatrix}-\begin{bmatrix} 2 & 2 & 2 \end{bmatrix}\right)$

Solution:

$$=\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}+\begin{bmatrix} 1-2 & 0-2 & 2-2 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 & 1 \end{bmatrix}+\begin{bmatrix} -1 & -2 & 0 \end{bmatrix}$$
$$=\begin{bmatrix} 2-1 & 3-2 & 1-0 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

(iv) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

Solution:

$$=\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$
$$=\begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

(v) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

Solution:

$$=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$
$$=\begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 3+0 & 1+2 & 2-1 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

(vi) $\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

Solution:

$$=\begin{bmatrix}1+2 & 2+1\\0+1 & 1+0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\1 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3+1 & 3+1\\1+1 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 4\\2 & 2\end{bmatrix}$$

Q.5 **For the matrices** $A=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$,

$B=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$ **and**

$C=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$, **verify the following rules:**

(i) $A+C=C+A$

Solutions:

L.H.S = A+C

R.H.S = C+A

LHS= A+C

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 2+0 & 3+0\\2+0 & 3-2 & 1+3\\1+1 & -1+1 & 0+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

RHS= C+A

$$=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0+2 & 0+3\\0+2 & -2+3 & 3+1\\1+1 & 1-1 & 2-0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

$A+C=C+A$

Hence proved

L.H.S = R.H.S

(ii) $A+B=B+A$

Solution: $A+B=B+A$

L.H.S = A+B

R.H.S = B+A

LHS= A+B

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-1 & +3+1\\2+2 & 3-2 & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

RHS= B+A

$$=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & -1+2 & 1+3\\2+2 & -2+3 & 2+1\\3+3 & 1-1 & 3-0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\6 & 0 & 3\end{bmatrix}$$

$A+B=B+A$

Hence proved

L.H.S = R.H.S

(iii) $B+C=C+B$

Solution: $B+C=C+B$

L.H.S = B+C

R.H.S = C+B

L.H.S = B+C

$$=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & -1+0 & 1+0\\ 2+0 & -2-2 & 2+3\\ 3+1 & 1+1 & 3+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

R.H.S = C+B

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0-1 & 0+1\\ 0+2 & -2-2 & 3+2\\ 1+3 & 1+1 & 2+3\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

L.H.S = R.H.S

B+C=C+B

Hence proved

(iv) $A+(B+A)=2A+B$

Solution: $A+(B+A)=2A+B$

L.H.S = A+ (B+A)

R.H.S = 2A+B

L.H.S= A+ (B+A)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

R.H.S = 2A+B

$$=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

L.H.S =R.H.S

A+ (B+A) =2A+B

Hence proved

(v) $(C-B)+A=C+(A+B)$

Solution: $(C-B)+A=C+(A+B)$

L.H.S = (C-B) +A

R.H.S = C+(A-B)

L.H.S = (C-B) + A

$$=\left(\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

RHS= C+ (A-B)

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & 3 & 2\\ 0 & 5 & -1\\ -2 & -2 & -3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

L.H.S = R.H.S

(C-B)+A=C+ (A-B)

Hence proved

(vi) $2A+B=A+(A+B)$

Solution: $2A+B=A+(A+B)$

L.H.S = 2A+B

R.H.S = A+ (A+B)

LHS= 2A+B

$$=2\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}2&4&6\\4&6&2\\2&-2&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}3&3&7\\6&4&4\\5&-1&3\end{bmatrix}$$

RHS= A + (A+B)

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}$$

$$=\begin{bmatrix}3&3&7\\6&4&4\\5&-1&3\end{bmatrix}$$

L.H.S = R.H.S

2A+B=A+ (A+B)

Hence proved

(vii) $(C-B)-A=(C-A)-B$

Solution: $(C-B)-A=(C-A)-B$

L.H.S = (C-B) –A

R.H.S = (C-A) –B

LHS= (C-B) – A

$$=\left(\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)-\begin{bmatrix}1&2&3\\2&3&1\\1&-&0\end{bmatrix}$$

$$=\begin{bmatrix}-2&1&-1\\-2&0&1\\-2&0&-1\end{bmatrix}-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}-3&-1&-4\\-4&-3&0\\-3&1&-1\end{bmatrix}$$

RHS= (C-A) –B

$$=\left(\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}\right)-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}-2&-2&-3\\-2&-5&2\\0&2&2\end{bmatrix}-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}-3&-1&-4\\-4&-3&0\\-3&1&-1\end{bmatrix}$$

L.H.S = R.H.S

(C-B)-A=(C-A)-B

Hence proved

(viii) $(A+B)+C=A+(B+C)$

Solution: (A+B) + C = A+ (B+C)

L.H.S = (A+B) + C

R.H.S = A + (B+C)

LHS= (A+B) +C

$$=\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$=\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$=\begin{bmatrix}1&1&4\\4&-1&6\\5&1&5\end{bmatrix}$$

R.H.S = A+ (B+C)

$$=\begin{bmatrix}1&2&3\\2&3&1\\3&-1&0\end{bmatrix}+\left(\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}\right)$$

$$=\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 1 & 4\\ 4 & -1 & 6\\ 5 & 1 & 5\end{bmatrix}$$

L.H.S = R.H.S

(A+B)+C=A+ (B+C)

Hence proved

(ix) $A+(B-C)=(A-C)+B$

Solution: $A+(B-C)=(A-C)+B$

L.H.S = A+ (B-C)

R.H.S = (A-C) + B

L.H.S = A+ (B-C)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & -1 & 1\\ 2 & 0 & -1\\ 2 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\ 4 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$$

RHS= (A-C) +B

$$=\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}\right)+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-0 & 3-1\\ 2-0 & 3+2 & 1-3\\ 1-1 & -1-1 & 0-2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 2 & 3\\ 2 & 5 & -2\\ 0 & -2 & -2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\ 4 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$$

L.H.S = R.H.S

A+ (B-C) = (A-C) +B

Hence proved

(x) $2A+2B=2(A+B)$

Solution: $2A+2B=2(A+B)$

L.H.S = 2A + 2B

R.H.S = 2(A + B)

L.H.S = 2A + 2B

$$=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+2\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}2 & -2 & 2\\ 4 & -4 & 4\\ 6 & 2 & 6\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 8\\ 8 & 2 & 6\\ 8 & 0 & 6\end{bmatrix}$$

RHS= 2 (A+B)

$$=2\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=2\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 8\\ 8 & 2 & 6\\ 8 & 0 & 6\end{bmatrix}$$

L.H.S = R.H.S

2A+2B=2(A+B)

Hence proved

Q.6 **If** $A=\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$ **and** $B=\begin{bmatrix}0 & 7\\ -3 & 8\end{bmatrix}$

find:

(i) $3A-2B$

Solution: $3A-2B$

$$3A-2B=3\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}-2\begin{bmatrix}0 & 7\\ -3 & 8\end{bmatrix}$$

$$=\begin{bmatrix}3 & -6\\ 9 & 12\end{bmatrix}-\begin{bmatrix}0 & 14\\ -6 & 16\end{bmatrix}$$

$$=\begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$$

(ii) $2A^t - 3B^t$

Solution: $2A^t - 3B^t$

When we take transpose of any matrix we change rows into columns or columns into rows.

$$A^t = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$2A^t - 3B^t = 2\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} - 3\begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$$

Q.7 If

$$2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

Solution:

$$2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 8 \\ -6 & 2a \end{bmatrix} + \begin{bmatrix} 3 & 3b \\ 24 & -12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 7 & 8+3b \\ 18 & 2a+(-12) \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$8 + 3b = 10$ __________ (i)

$2a - 12 = 1$ __________ (ii)

By solving equation (ii) we get the value of a

$2a - 12 = 1$

$2a = 1 + 12$

$2a = 13$

$$a = \frac{13}{2}$$

By solving equation (i) we get the value of b

$8 + 3b = 10$

$3b = 10 - 8$

$3b = 2$

$$b = \frac{2}{3}$$

Q.8 **If** $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

Then verify that

(i) $(A+B)^t = A^t + B^t$

Solution: $(A+B)^t = A^t + B^t$

L.H.S = $(A + B)^t$

R.H.S = $A^t + B^t$

To solve L.H.S

L.H.S = $(A + B)^t$

$$=(A+B)^t = \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\right)^t$$

$$=\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}^t$$

$$\text{R.H.H} = (A+B)^t = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

To solve R.H.S

R.H.S = $A^t + B^t$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$\text{RHS} = A^t + B^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

L.H.S = R.H.S $\Rightarrow (A+B)^t = A^t + B^t$

Hence Proved

(ii) $(A-B)^t = A^t - B^t$

Solution: $(A-B)^t = A^t - B^t$

L.H.S = $(A - B)^t$

R.H.S = $A^t - B^t$

LHS= $(A - B)^t$

$$(A - B)^t = \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\right)^t$$

$$= \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}^t$$

$$(A - B)^t = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

R.H.S $= A^t - B^t$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\text{-}1 & 0\text{-}2 \\ 2\text{-}1 & 1\text{-}0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

L.H.S = R.H.S

$(A-B)^t = A^t - B^t$

Hence proved

(iii) $A + A^+$ **is a symmetric**

Solution:

$A + A^+$ is a symmetric

To show that $A + A^t$ is symmetric, we will show that

$$\left(A + A^t\right)^t = \left(A + A^t\right)$$

$$A + A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix}$$

$$A + A^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$(A + A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$(A+A^t)^t = (A+A^t)$

Hence Proved

$A + A^t$ symmetric

(iv) $A - A^t$ **is a skew symmetric**

Solution: $A - A^t$

To show that $A\text{-}A^t$ is skew symmetric we will show that

$$(A\text{-}A^t)^t = -(A\text{-}A^t)$$

$$A - A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 1-1 \end{bmatrix}$$

$$A\text{-}A^t = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$(A\text{-}A^t)^t = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$(A\text{-}A^t)^t = -(A\text{-}A^t)$

Hence proved

$A - A^t$ is a skew symmetric

(v) $B + B^t$ **is a symmetric**

Solution: $B + B^t$

The show that $B+B^t$ is symmetric we will show that

$$(B + B^t)^t = (B + B^t)$$

$$B + B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 1+2 \\ 2+1 & 0+0 \end{bmatrix}$$

$$B + B^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$$\left(B + B^t\right)^t = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

$$\left(B + B^t\right)^t = \left(B + B^t\right)$$

Hence proved
$B + B^t$ is a symmetric

(vi) $B - B^t$ **is a skew symmetric**
Solution: $B - B^t$
To show that B – B^t is skew symmetric, we will show that
$(B\text{-}B^t)^t = -(B - B^t)$

$$B - B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix}$$

$$B - B^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(B - B^t)^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$- \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$(B - B^t)^t = -(B - B^t)$
Hence proved
$B - B^t$ is a skew symmetric.

Exercise 1.4

Q.1 Which of the following product of matrices if conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ 1 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

No, these matrices cannot be multiplied because number of columns of 1st matrix is not equal to the number of rows of 2nd matrix.

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

Q.2 If $A=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ **find**

(i) AB

Solution: $AB=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$$AB=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$=\begin{bmatrix} (3\times 6)+(0\times 5) \\ (-1\times 6)+(2\times 5) \end{bmatrix}$$

$$=\begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix}=\begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii) BA **(if possible)**

Solution:

BA is not possible became number of columns of B not equal to number of rows of A.

Q.3 Find the following products

(i) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$= [(1\times4)+(2\times0)]$

$= [4+0]$

$= [4]$

(ii) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -0 \end{bmatrix}$

$= [(1\times5)+(2\times-4)]$

$= [5+(-8)]$

$= [5-8]$

$= [-3]$

(iii) $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution: $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$= [(-3\times4)+(0\times0)]$

$= [-12+0]$

$= [-12]$

(iv) $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ -0 \end{bmatrix}$

Solution: $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ -0 \end{bmatrix}$

$\begin{bmatrix} 6 & +0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$= [6\times4+(-0)(0)]$

$= [24-0]$

$= [24]$

(v) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 1\times4+2\times0 & 1\times5+2\times(-4) \\ -3\times4+0\times0 & -3(5)+0\times(-4) \\ 6(4)+(-1)(0) & 6(5)+(-1)(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 5-8 \\ -12+0 & -15-0 \\ 24-0 & 30+4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$$

Q.4 Multiply the following matrices.

(a) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

$$\begin{bmatrix} 2\times2+(3\times3) & (2\times-1)+(3\times0) \\ (1\times2)+(1\times3) & (1\times-1)+(1\times0) \\ (0\times2)+(-2\times3) & (0\times-1)+(-2\times0) \end{bmatrix}$$

$$= \begin{bmatrix} 4+9 & -2+0 \\ 2+3 & -1+0 \\ 0+-6 & 0+0 \end{bmatrix}$$

$$=\begin{bmatrix}13 & -2\\5 & -1\\0-6 & 0\end{bmatrix}$$

$$=\begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$$

(b) $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

Solution: $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

$$=\begin{bmatrix}(1\times1)+(2\times3)+(3\times-1) & (1\times2)+(2\times4)+(3\times1)\\(4\times1)+(5\times3)+(6\times-1) & (4\times2)+(5\times4)+(6\times1)\end{bmatrix}$$

$$=\begin{bmatrix}1+6+(-3) & 2+8+3\\4+15+(-6) & 8+20+6\end{bmatrix}$$

$$=\begin{bmatrix}7-3 & 13\\19-6 & 34\end{bmatrix}$$

$$=\begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$$

(c) $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

Solution: $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

$$=\begin{bmatrix}(1\times1)+(2\times4) & (1\times2)+(2\times5) & (1\times3)+(2\times6)\\(3\times1)+(4\times4) & (3\times2)+(4\times5) & (3\times3)+(4\times6)\\(-1\times1)+(1\times4) & (-1\times2)+(1\times5) & (-1\times3)+(1\times6)\end{bmatrix}$$

$$=\begin{bmatrix}1+8 & 2+10 & 3+12\\3+16 & 6+20 & 9+24\\-1+4 & -2+5 & -3+6\end{bmatrix}$$

$$=\begin{bmatrix}9 & 12 & 15\\19 & 26 & 33\\3 & 3 & 3\end{bmatrix}$$

(d) $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$

Solution: $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$

$$=\begin{bmatrix}(8\times2)+(5\times-4) & \left(8\times-\frac{5}{2}\right)+(5\times4)\\(6\times2)+(4\times-4) & \left(6\times-\frac{5}{2}\right)+(4\times4)\end{bmatrix}$$

$$=\begin{bmatrix}16+(-20) & \frac{-40}{2}+20\\12+(-16) & \frac{-30}{2}+16\end{bmatrix}$$

$$=\begin{bmatrix}16-20 & -20+20\\12-16 & -15+16\end{bmatrix}$$

$$=\begin{bmatrix}-4 & 0\\-4 & 1\end{bmatrix}$$

(e) $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Solution: $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$$=\begin{bmatrix}(-1\times0)+(2\times0) & (-1\times0)+(2\times0)\\(1\times0)+(3\times0) & (1\times0)+(3\times0)\end{bmatrix}$$

$$=\begin{bmatrix}0+0 & 0+0\\0+0 & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

Q.5 **Let** $A=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$

and $C=\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$ **verify whether**

(i) $AB=BA$

Solution: $AB=BA$

L.H.S = AB

R.H.S = BA

L.H.S =AB

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0-3) & (2\times2)+(0-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

$$\text{R.H.S}=BA=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1\times(-1)+2\times2 & 1\times3+2\times0\\ -3\times(-1)+(-5)2 & -3\times3+(-5)(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 3+0\\ 3-10 & -9-0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\ -7 & -9\end{bmatrix}$$

Since L.H.S $\neq$ R.H.S

L.H.S $\neq$ R.H.S

L.H.S $\neq$ R.H.S

(ii) $A(BC)=(AB)C$

Solution: $A(BC)=(AB)C$

L.H.S = A (BC)

R.H.S = (AB) C

L.H.S

L.H.S=A(BC)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2+2 & 1+6\\ -6+(-5) & -3+(-15)\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}4 & 7\\ -6-5 & -3-15\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}4 & 7\\ -11 & -18\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times4)+(3\times-11) & (-1\times7)+(3\times-18)\\ (2\times4)+(0\times-11) & (2\times7)+(0\times-18)\end{bmatrix}$$

$$=\begin{bmatrix}-4+(-33) & -7+(-54)\\ 8+0 & 14+0\end{bmatrix}$$

$$=\begin{bmatrix}-4-33 & -7-54\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

R.H.S = (AB)C

$$=\left(\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\right)\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(-3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & -4+0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-10\times2)+(-17\times1) & (-10\times1)+(-17\times3)\\ (2\times2)+(4\times1) & (2\times1)(4\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-20+(-17) & -10+(-51)\\ 4+4 & 2+12\end{bmatrix}$$

$$=\begin{bmatrix}-20-17 & -10-51\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

Since

L.H.S = R.H.S $\Rightarrow$ A (BC) = (AB)C

Hence proved

(iii) $A(B+C)=AB+AC$

Solution: $A(B+C)=AB+AC$

L.H.S = A (B+C)

R.H.S = AB+AC

L.H.S

LHS=A (B+C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1+2 & 2+1\\ -3+1 & -5+3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}3 & 3\\ -2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times3)+(3\times-2) & (-1\times3)+(3\times-2)\\ (2\times3)+(0\times-2) & (2\times3)+(0\times-2)\end{bmatrix}$$

$$=\begin{bmatrix}-3+(-6) & -3+(-6)\\ 6+0 & 6+0\end{bmatrix}$$

$$=\begin{bmatrix}-3-6 & -3-6\\ 6 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

R.H.S=AB+AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times+2)+(0\times-5)\end{bmatrix}$$

$$+\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-3) & -2+(-15)\\ 2+0 & +4+0\end{bmatrix}+\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & +4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & +4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10+1 & -17+8\\ 2+4 & +4+2\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

Since LHS=RHS

A (B+C) = AB+AC

Hence proved

(iv) A(B–C) = AB–AC

Solution: A (B–C) = AB–AC

L.H.S = A (B–C)

R.H.S = AB–AC

L.H.S=A(B–C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times-1)+(3\times-4) & (-1\times1)+(3\times-8)\\ (2\times-1)+(0l-4) & (2\times1)+(0\times-8)\end{bmatrix}$$

$$=\begin{bmatrix}+1+(-12) & -1+(-24)\\ -2+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}1-12 & -1-24\\ -2 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

R.H.S=AB–AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)-(3\times3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}-\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}-\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

Since L.H.S = R.H.S

A (B-C) =AB-AC, Hence **proved.**

Q.6 For the matrices $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Verify that

(i) $(AB)^t = B^t A^t$

Solution: $(AB)^t = B^t A^t$

L.H.S $= (AB)^t$

R.H.S $= B^t A^t$

$$(AB)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (+2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

LHS$= (AB)^t$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}^t$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}^t$$

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

L.H.S$=B^tA^t$

$$=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-1)+(-3\times3) & (1\times2)+(-3\times0)\\ (2\times-1)+(-5\times3) & (2\times2)+(-5\times0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & 2+0\\ -2+(-15) & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & 2\\ -2-15 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

Since L.H.S = R.H.S

$(AB)^t = B^tA^t$

Hence proved

L.H.S = R.H.S

(ii) $\mathbf{(BC)^t= C^tB^t}$

Solution: $(BC)^t= C^tB^t$

L.H.S $= (BC)^t$

R.H.S $= C^t B^t$

To solve L.H.S

$$BC=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-2)+(2\times3) & (1\times6)+(2\,\text{x}\times-9)\\ (-3\times-2)+(-5\times3) & (-3\times6)+(-5\times-9)\end{bmatrix}$$

$$=\begin{bmatrix}-2+6 & 6+(-18)\\ 6+(-15) & -18+45\end{bmatrix}$$

$$=\begin{bmatrix}4 & 6-18\\ 6-15 & 27\end{bmatrix}$$

$$=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}$$

Taking transpose of BC:-

$$(BC)^t=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}^t$$

$$\text{LHS} = (B\ C)^t \quad =\begin{bmatrix}4 & -9\\ -12 & 27\end{bmatrix}$$

To solve R.H.S =

Taking transpose of matrix C

$$C^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}$$

Taking transpose of matrix B

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

Now, multiplying matrices, $B^t C^t$

$$\text{R.H.S} = C^t B^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}\times\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-2\times1)+(3\times2) & (-2\times-3)+(3\times-5)\\ (6\times1)+(-9\times2) & (6\times-3)+(-9\times-5)\end{bmatrix}$$

$$=\begin{bmatrix} -2+6 & 6+(-15) \\ 6+(-18) & -18+45 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 6-15 \\ 6-18 & 27 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

Hence proved

L.H.S = R.H.S

Exercise 1.6

Q.1 Use of matrices, if possible to solve the following systems of linear equations.

(i) The matrices inversion method

(ii) The Cramer's rule

(i) $2x-2y=4$

$3x+2y=6$

By matrices inversion method

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Let $A=\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X=\begin{bmatrix} x \\ y \end{bmatrix} B=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$A=\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$|A|=(2)(2)-(-2)(3)$$

$$|A|=4+6$$

$$|A|=10$$

Then, solution is possible because A is non-singular matrix.

$$AdjA = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

As we know that

$$AX=B$$

$$X=A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix} 2\times 4 \quad +2\times 6 \\ -3\times 4+2\times 6 \end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix} 8+12 \\ -12+12 \end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \frac{20}{10} \\ \frac{0}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$x=2, y=0$

Solution Set = {(2, 0)}

By Cramer's rule

$$A=\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} B=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$=(2)(2)-(-2)(3)$$

$$=4-(-6)$$

$$=4+6$$

$$=10$$

$$|A_x|=\begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}$$

$$=(4)(2)-(-2)(6)$$

$$=8+12$$

$$=20$$

$$|A_y|=\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$$

$$=(2)(6)-(4)(3)$$

$$=12-12$$

$$=0$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{20}{10}$$

$$x=2$$

$$y=\frac{|A_y|}{|A|}$$

$y=\frac{0}{10}$

$y=0$

Solution Set = {(2, 0)}

(ii) $2x+y=3$

$6x+5y=1$

Matrices inversion method

$$\begin{bmatrix}2&1\\6&5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$$

Let $A=\begin{bmatrix}3&-2\\3&2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$A=\begin{bmatrix}2&1\\6&5\end{bmatrix}$$

$$|A|=\begin{vmatrix}2&1\\6&5\end{vmatrix}$$

$=(2)(5)-(1)(6)$

$=10-6$

$=4$

Solution is possible because A is non-singular matrix.

$$AdjA=\begin{bmatrix}5&-1\\-6&2\end{bmatrix}$$

$AX=B$

$X=A^{-1}B$

$$X=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}5&-1\\-6&2\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}5\times3+(-1\times1)\\-6\times3+2\times1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}15+(-1)\\-18+2\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}14\\-16\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{14}{4}\\\frac{16}{4}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{7}{2}\\-4\end{bmatrix}$$

$x=\frac{7}{2}, y=-4$

Solution Set = $\{(\frac{7}{2},-4)\}$

By Cramer's Rule

$A=\begin{bmatrix}2&1\\6&5\end{bmatrix} B=\begin{bmatrix}3\\1\end{bmatrix}$

$$|A|=\begin{vmatrix}2&1\\6&5\end{vmatrix}$$

$=(2)(5)-(1)(6)$

$=10-6$

$=4$

Solution is possible because A is non-singular matrix.

$$|A_x|=\begin{vmatrix}3&1\\1&5\end{vmatrix}$$

$=(3)(5)-(1)(1)$

$=15-1$

$=14$

$$|A_y|=\begin{vmatrix}2&3\\6&1\end{vmatrix}$$

$=(2)(1)-(3)(6)$

$=2-18$

$=-16$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{14}{4}$$

$$x=\frac{7}{2}$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{16}{4}$$

$y=-4$

Solution Set $=\left\{\left(\frac{7}{2},-4\right)\right\}$

(iii) $4x+2y=8$
$3x-y=-1$

By Matrices Inversion Method

$$\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\-1\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 2\\3 & -1\end{vmatrix}$$

$=(4)(-1)-(2)(3)$

$=-4-6$

$=-10$

Solution is possible because A is non singular matrix.

$$AdjA=\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}$$

As we know that

$AX=B$

$X=A^{-1}B$

$X=A^{-1}B$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}\begin{bmatrix}8\\-1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-8+2\\-24+(-4)\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-6\\-28\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{-6}{-10}\\\frac{-28}{-10}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{3}{5}\\\frac{14}{5}\end{bmatrix}$$

$$x=\frac{3}{5},y=\frac{14}{5}$$

Solution Set $=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$

By Cramer's rule

$A=\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix} B=\begin{bmatrix}8\\-1\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 2\\3 & -1\end{vmatrix}$$

$=(4)(-1)-(2)(3)$

$=-4-6$

$=-10$

Solution is possible because A is non singular matrix.

$$|A_x|=\begin{vmatrix}8 & 2\\-1 & -1\end{vmatrix}$$

$=(8)(-1)-(2)(-1)$

$=-8-(-2)$

$=-6$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{6}{10}$$

$$x=\frac{3}{5}$$

$$|A_y|=\begin{bmatrix}4 & 8\\3 & -1\end{bmatrix}$$

$=(4)(-1)-(8)(3)$

$=-4-24$

$=-28$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{28}{10}$$

$$y=\frac{14}{5}$$

Solution Set $=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$

(iv) $3x$-$2y$=-6
$5x$-$2y$=-10

By Matrices Inversion Method

$$\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-10\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}3 & -2\\ 5 & -2\end{vmatrix}$$
$$=(3)(-2)-(-2)(5)$$
$$=-6-(-10)$$
$$=-6+10$$
$$=4$$

Solution is possible because A is non singular matrix.

$$AdjA=\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}$$
$$X=A^{-1}B$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}\begin{bmatrix}-6\\ -10\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2\times-6+2\times-10\\ -5\times-63\times-10\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}12+(-20)\\ 30+(-30)\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}12-20\\ 30-30\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-8\\ 0\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{-8}{4}\\ \frac{0}{4}\end{bmatrix}$$
$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-2\\ 0\end{bmatrix}$$
$$x=-2,\ y=0$$

Solution Set $=\{(-2,0)\}$

By Cramer's rule

$$A=\begin{bmatrix}3 & -2\\ 5 & -2\end{bmatrix},B=\begin{bmatrix}-6\\ -10\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & -2\\ 5 & -2\end{vmatrix}$$
$$=(3)(-2)-(-2)(5)$$
$$=-6-(-10)$$
$$=-6+10$$
$$=4$$

Solution is possible because A is non singular matrix.

$$|A_x|=\begin{vmatrix}-6 & -2\\ -10 & -2\end{vmatrix}$$
$$=(-6)(-2)-(-2)(-10)$$
$$=+12-(+20)$$
$$=12-20$$
$$=-8$$
$$|A_y|=\begin{vmatrix}3 & -6\\ 5 & -10\end{vmatrix}$$
$$=(3)(-10)-(-6)(5)$$
$$=-30-(-30)$$
$$=-30+30$$
$$=0$$
$$x=\frac{|A_x|}{|A|}$$
$$x=\frac{-8}{4}$$
$$x=-2$$
$$y=\frac{|A_y|}{|A|}$$
$$=\frac{0}{4}$$
$$y=0$$

Solution Set $=\{(-2,0)\}$

(v) $3x-2y=4$
$-6x+4y=7$

By Matrices Inversion Method

$$\begin{bmatrix}3 & -2\\ -6 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ 7\end{bmatrix}$$
$$A=\begin{bmatrix}3 & -2\\ 3 & 2\end{bmatrix}X=\begin{bmatrix}x\\ y\end{bmatrix}B=\begin{bmatrix}4\\ 6\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & -2\\ -6 & 4\end{vmatrix}$$
$$=(3)(4)-(-2)(-6)$$
$$=12-(+12)$$
$$=12-12$$
$$=0$$

Solution is not possible because A is singular matrix.

(vi) $4x+y=9$

$-3x-y=-5$

By Matrices Inversion Method

$$\begin{bmatrix}4 & 1\\ -3 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}9\\ -5\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\ 3 & 2\end{bmatrix} X=\begin{bmatrix}x\\ y\end{bmatrix} B=\begin{bmatrix}4\\ 6\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 1\\ -3 & -1\end{vmatrix}$$

$=(4)(-1)-(-1)(-3)$

$=-4+3$

$=-1$

Solution is possible because $|A|$ is non singular

$$AdjA=\begin{vmatrix}-1 & -1\\ 3 & 4\end{vmatrix}$$

As we know that

$X=A^{-1}B$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-1 & -1\\ 3 & 4\end{bmatrix}\begin{bmatrix}9\\ -5\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-9+5\\ 27+(-20)\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-4\\ 7\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{-4}{-1}\\ \frac{7}{-1}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ -7\end{bmatrix}$$

$x=4, y=-7$

Solution Set$=\{(4,-7)\}$

By Cramer's rule

$$A=\begin{bmatrix}4 & 1\\ -3 & -1\end{bmatrix}, B=\begin{bmatrix}9\\ -5\end{bmatrix}$$

$$|A|=\begin{vmatrix}4 & 1\\ -3 & -1\end{vmatrix}$$

$=(4)(-1)-(1)(-3)$

$=-4-(-3)$

$=-4+3$

$=-1$

$$|A_x|=\begin{vmatrix}9 & 1\\ -5 & -1\end{vmatrix}$$

$=(9)(-1)-(1)(-3)$

$=-9-(-5)$

$=-9+5$

$=-4$

$$x=\frac{|A_x|}{|A|}$$

$$=\frac{-4}{-1}$$

$x=4$

$$|A_y|=\begin{bmatrix}4 & 9\\ -3 & -5\end{bmatrix}$$

$=(4)(-5)-(9)(-3)$

$=-20-(-27)$

$=--20+27$

$=7$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{7}{-1}$$

$y=-7$

Solution Set$=\{(4_7,-7)\}$

(vii) $2x-2y=4$

$-5x-2y=-10$

By Matrices Inversion Method

$$\begin{bmatrix}2 & -2\\ -5 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ -10\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\ 3 & 2\end{bmatrix} X=\begin{bmatrix}x\\ y\end{bmatrix} B=\begin{bmatrix}4\\ 6\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & -2\\ -5 & -2\end{vmatrix}$$

$=(2)(-2)-(-2)(-5)$

$=-4-(+10)$

$=-4-10$

$=-14$

Solution is possible

$$AdjA = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -8+(-20) \\ 20+(-20) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -8-20 \\ 20-20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{14} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$x=2, y=0$

Solution Set $=\{(2,0)\}$

By Cramer's rule

$$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$= (2)(-2)-(-2)(-5)$

$= -4-(+10)$

$= -4-10$

$= -14$

Set is possible

$$|A_x| = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$$

$=(4)(-2)-(-2)(-10)$

$=-8-(+20)$

$=-8-20$

$=-28$

$$|A_y| = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$$

$=(2)(-10)-(4)(-5)$

$=-20-(-20)$

$=-20+20$

$=0$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{-28}{-14}$$

$x=2$

Solution Set $=\{(2,0)\}$

(viii) $3x-4y=4$

$x+2y=8$

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$=(3)(2)-(-4)(1)$

$=6-(-4)$

$|A|=6+4$

$=10$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

As we know that

$AX = B$

$X = A^{-1}B$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2\times4+4\times8 \\ -1\times+3\times8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 8+32 \\ -4+24 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$x = 4, y = 2$

Solution Set $= \{(4,2)\}$

By Cramer's rule

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$= (3)(2) - (-4)(1)$

$= 6 - (-4)$

$= 6 + 4$

$= 10$

Solution is possible

$$|A_x| = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$$

$= (4)(2) - (-4)(8)$

$= 8 - (-32)$

$= 8 + 32$

$= 40$

$$|A_y| = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$$

$= (3)(8) - (4)(1)$

$= 24 - 4$

$= 20$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{40}{10}$$

$x = 4$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{20}{10}$$

$y = 2$

Solution Set $= \{(4,2)\}$

Q.2 The length of a rectangle is 4 times it width. The perimeter of the rectangle is 150cm. Find the dimensions of the rectangle.

Solution:

Let width of rectangle $= x$

Length of rectangle $= y$

According to 1st condition

$y = 4x$

$-4x + y = 0 \qquad \rightarrow \text{...(i)}$

According to 2nd condition

2(length + Width)=Perimeter

$2(y + x) = 150$

$$y + x = \frac{\cancel{150}^{75}}{\cancel{2}}$$

$x + y = 75 \qquad \rightarrow \text{...(ii)}$

$-4x + y = 0$

$x + y = 75$

Changing into matrix form

$$\begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$X = A^{-1}B$

(By matrix inversion method)

$$\text{Let } A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$= (-4)(1) - (1)(1)$

$= -4 - 1$

$= -5$

$$AdjA = \begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix}$$

As we know that

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$= -\frac{1}{5} \begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$= -\frac{1}{5} \begin{bmatrix} 0 - 75 \\ 0 - 300 \end{bmatrix}$$

$$= \frac{1}{-5} \begin{bmatrix} -75 \\ -300 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-75}{-5} \\ \frac{-300}{-5} \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 60 \end{bmatrix}$$

$x = 15, y = 60$

Width of rectangle = x = 15cm

Length of rectangle = y = 60cm

By Cramer's rule

$$A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-4)(1) - (1)(1)$$

$$= -4 - 1$$

$$= -5$$

$$|A_x| = \begin{vmatrix} 0 & 1 \\ 75 & 1 \end{vmatrix}$$

$$= (0)(1) - (1)(75)$$

$$= 0 - 75$$

$$= -75$$

$$|A_y| = \begin{vmatrix} -4 & 0 \\ 1 & 75 \end{vmatrix}$$

$$= (-4)(75) - (0)(1)$$

$$= 0 - 300$$

$$= -300$$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-75}{-5}$$

$$x = 15$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{-300}{-5}$$

$$y = 60$$

Then

Width of rectangle = x = 15 cm

Length of rectangle = y = 60 cm

Q.3 Two sides of a rectangle differ by 3.5cm. Find the dimension of the rectangle if its perimeter is 67cm.

Solution:

Suppose Width of rectangle = x

Length of rectangle = y

According to 1st condition

$$y - x = 3.5$$

$$-x + y = 3.5 \qquad \to \ldots(i)$$

According to 2nd condition

$$2(L + B) = P$$

$$2(y + x) = 67$$

$$x + y = \frac{67}{2}$$

$$x + y = 33.5 \qquad \to (ii)$$

Changing into matrix form

$$\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

(By matrix inversion method)

Let $A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$

$$|A| = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-1)(1) - (1)(1)$$

$$= -1 - 1$$

$$= -2$$

$$AdjA = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$= \frac{1}{-2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 3.5 \\ +33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 1 \times 3.5 & 1 \times 33.5 \\ -1 \times 3.5 & 1 \times 33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 3.5(-33.5) \\ -3.5(-33.5) \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} -30 \\ -37 \end{bmatrix}$$

$$=\begin{bmatrix}\frac{30}{2}\\ \frac{37}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}15\\ \frac{37}{2}\end{bmatrix}$$

$$x=15, y=\frac{37}{2}=18.5$$

By Cramer's rule

$$A=\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}3.5\\ 33.5\end{bmatrix}$$

$$|A|=\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}$$

$$=(-1)(1)-(1)(1)$$

$$=-1-1$$

$$=-2$$

$$|A_x|=\begin{vmatrix}3.5 & 1\\ 33.5 & 1\end{vmatrix}$$

$$=(3.5)(1)-(1)(33.5)$$

$$=3.5-33.5$$

$$=-30$$

$$|A_y|=\begin{vmatrix}-1 & 3.5\\ 1 & 33.5\end{vmatrix}$$

$$=(-1)(33.5)-(3.5)(1)$$

$$=-33.5-3.5$$

$$=-37$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{-30}{-2}$$

$$x=15$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{-37}{-2}$$

$$y=\frac{37}{2}=18.5$$

Width of rectangle =x=15cm
Length of rectangle =y=18.5cm

Q.4 The third angle of an isosceles Δ is $16°$ less than the sum of two equal angles. Find three angles of the triangle.

Solution:

Let each equal angles are x and third angle is y

According to condition $y=2x-16$

$$2x-y=16 \quad \text{(i)}$$

As we know that

$$x+x+y=180$$

$$2x+y=180 \quad \text{(ii)}$$

$$2x-y=16$$

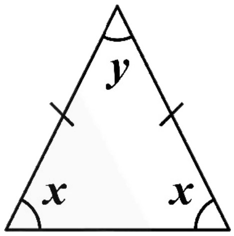
$$2x+y=180$$

Changing into matrix form

$$\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$X=A^{-1}B$$

Let $A=\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix} X=\begin{bmatrix}x\\ y\end{bmatrix} B=\begin{bmatrix}16\\ 180\end{bmatrix}$



$X=A^{-1}B$

Where

$$A=\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -1\\ 2 & 1\end{vmatrix}$$

$$=2\times1-(-1)\times2$$

$$=2+2$$

$= 4 \neq 0$ (None singular)

A^{-1} exist

$$AdjA=\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}$$

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}1\times16+1\times180\\ -2\times16+2\times180\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}16+180\\-32+360\end{bmatrix}$$
$$=\begin{bmatrix}196\\328\end{bmatrix}$$
$$=\begin{bmatrix}\frac{196}{4}\\\frac{328}{4}\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}49\\82\end{bmatrix}$$
$x = 49$

$y=82$

Cramer Rule

$$A=\begin{bmatrix}2 & -1\\2 & 1\end{bmatrix} B=\begin{bmatrix}16\\180\end{bmatrix}$$
$$|A|=\begin{vmatrix}2 & -1\\2 & 1\end{vmatrix}$$
$$=(2)(1)-(-1)(2)$$
$$=2-(-2)$$
$$=2+2$$
$$=4$$
$$|A_x|=\begin{bmatrix}16 & -1\\180 & 1\end{bmatrix}$$
$$=(16)(1)-(-1)(180)$$
$$=16+180$$
$$=196$$
$$|A_y|=\begin{bmatrix}2 & 16\\2 & 180\end{bmatrix}$$
$$=(2)(180)-(16)(2)$$
$$=360-32$$
$$=328$$
$$x=\frac{|A_x|}{|A|}$$
$$x=\frac{196}{4}$$
$$x = 49$$
$$y=\frac{|A_y|}{|A|}$$
$$y=\frac{328}{4}$$
$$y=82$$

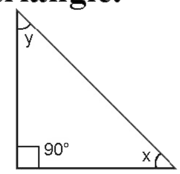
1^{st} angle = x = 49° Ans

2^{nd} angle= x = 49° Ans

3^{rd} angle =y= 82° Ans

Q.5 One acute angle of a right triangle is $12°$ more than twice the other acute angle. Find the acute angles of the right triangle.

Solution:



Let one acute angle =x

And other acute angle =y

According to 1^{st} condition

$$x=2y+12$$
$$x-2y=12 \qquad \rightarrow (i)$$

As we know

$$x+y=90 \qquad \rightarrow (ii)$$

By matrices inversion method

Changing into matrix form

$$\begin{bmatrix}1 & -2\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}12\\90\end{bmatrix}$$
$$X=A^{-1}B$$

Let $A=\begin{bmatrix}1 & -2\\1 & 1\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}12\\90\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -2\\1 & 1\end{vmatrix}$$
$$=(1)(1)-(-2)(1)$$
$$=1-(-2)$$

$=3$ (Non singular)

$\therefore A^{-1}$ exists

$$AdjA = \begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}$$

As we know that

$$X=A^{-1}B \ or$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$
$$=\frac{1}{3}\begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}12\\90\end{bmatrix}$$
$$=\frac{1}{3}\begin{bmatrix}12+180\\-12+90\end{bmatrix}$$
$$=\frac{1}{3}\begin{bmatrix}192\\78\end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{192}{3} \\ \frac{78}{3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 64 \\ 26 \end{bmatrix}$$

$x = 64, y = 26$

Then

1^{st} angle = x = 64°

2^{nd} angle = y = 26°

By Cramer's rule

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}$$

$= (1)(1) - (-2)(1)$

$= 1 - (-2)$

$= 1 + 2$

$= 3$

$$|A_x| = \begin{vmatrix} 12 & -2 \\ 90 & 1 \end{vmatrix}$$

$= (12)(1) - (-2)(90)$

$= 12 + 180$

$= 192$

$$|A_y| = \begin{vmatrix} 1 & 12 \\ 1 & 90 \end{vmatrix}$$

$= (90) - (12)$

$= 90 - 12$

$= 78$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{192}{3}$$

$x = 64$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{78}{3}$$

$y = 26$

1^{st} angle = x = 64°

2^{nd} angle = y = 26°

Q.6 Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

Suppose speed of 1^{st} car = x

Suppose speed of 2^{nd} car = y

According to 1^{st} condition

$x - y = 6 \qquad \rightarrow (i)$

According to 2^{nd} condition

Total distance = 600 km

Left distance = 123 km

Covered distance = total distance-left distance

Covered distance = 600-123

= 477 km

Total time = $4\frac{1}{2}$ hours =or $\frac{9}{2}$ hours

$$\text{Total Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$x + y = \frac{477}{\frac{9}{2}} = 477 \div \frac{9}{2} = 477 \times \frac{2}{9}$$

$$x + y = \frac{{}^{53}\cancel{477} \times 2}{\cancel{9}}$$

$x + y = 106 \qquad \rightarrow (ii)$

x - y = 6

x + y = 106

By matrices inversion method

Changing into matrix form

$$\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}6\\106\end{bmatrix}$$

$X = A^{-1}B$, where

Let $A=\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}6\\106\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -1\\1 & 1\end{vmatrix}$$

$$=(1)(1)-(-1)(1)$$

$$=1-(-1)$$

$$=1+1$$

$$=2$$

$$AdjA=\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}6\\106\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}6+106.\\-6+106\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}112\\100\end{bmatrix}$$

$$=\begin{bmatrix}\frac{112}{2}\\\frac{100}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}56\\50\end{bmatrix}$$

$x=56, y=50$

Speed of 1st car = x= 56km/h

Speed of 2nd car = y= 50km/h

By Cramer's rule

$$A=\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix} B=\begin{bmatrix}6\\106\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -1\\1 & 1\end{vmatrix}$$

$$=(1)(1)-(-1)(1)$$

$$=1-(-1)$$

$$=1+1$$

$$=2$$

$$|A_x|=\begin{vmatrix}6 & -1\\106 & 1\end{vmatrix}$$

$$=(6)(1)-(-1)(106)$$

$$=6-(-106)$$

$$=6+106$$

$$=112$$

$$|A_y|=\begin{vmatrix}1 & 6\\1 & 106\end{vmatrix}$$

$$=(106)(1)-(6)(1)$$

$$=106-6$$

$$=100$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{112}{2}$$

$$x = 56$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{100}{2}$$

$$y=50$$

Then

Speed of 1st car = x = 56km/h

Speed of 2nd car = y = 50km/h

Review Exercise 1

Q.1 Select the correct answer in each of the following.

(i) The order of matrix $\begin{bmatrix} 2 & 1 \end{bmatrix}$ is….

(a) 2-by-1 **(b)** 1-by-2
(c) 1-by-1 **(d)** 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ **is called …matrix.**

(a) Zero **(b)** Unit
(c) Scalar **(d)** Singular

(iii) Which is order of a square matrix?

(a) 2-by-2 **(b)** 1-by-2
(c) 2-by-1 **(d)** 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ **is…**

(a) 3-by-2 **(b)** 2-by-3
(c) 1-by-3 **(d)** 3-by-1

(v) Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ **is…**

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ **(d)** $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) Product of $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **is…**

(a) $[2x+y]$ **(b)** $[x-2y]$
(c) $[2x-y]$ **(d)** $[x+2y]$

(vii) If $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix}$ **=0, then x is equal to…**

(a) 9 **(b)** -6
(c) 6 **(d)** -9

(viii) If $X+\begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$**, then X is equal to…**

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
b	c	a	b	a	c	a	d

Q.2 Complete the follwoing:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called ... matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ... matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is....

(iv) In matrix multiplication, in gereral, AB ...BA.

(v) Matrix A+B may be found if order of A and B is...

(vi) A matrix is called ... matrix if number of rows and columns are equal.

ANSWER KEY

i	ii	iii	iv	v	vi
Null	Unit	$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$	$\neq$	Same	Square

Q.3 If $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$**, then find *a* and *b*.**

Solution: $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

$a+3=-3$ $\quad$ $b-1=2$

$a=-3-3$ $\quad$ $b=2+1$

$a=-6$ $\quad$ $b=3$ **Ans**

Q.4 If $A=\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, B\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$**, then find the following.**

(i) $2A+3B$

(ii) $-3A+2B$

(iii) $-3(A+2B)$

(iv) $\frac{2}{3}(2A-3B)$

Solution: (i)

$$2A+3B = 2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 3\begin{bmatrix} 5 & 4 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+15 & 6-12 \\ 2-6 & 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & -6 \\ -4 & -3 \end{bmatrix} \text{ Ans}$$

Solution: (ii)

$$-3A+2B = -3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+10 & -9-8 \\ -3-4 & 0-2 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -17 \\ -7 & -2 \end{bmatrix} \textbf{ Ans}$$

Solution: (iii)

$$-3(A+2B)=-3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}+2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$=-3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}+\begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$$

$$=-3\begin{bmatrix} 2+10 & 3-8 \\ 1-4 & 0-2 \end{bmatrix}$$

$$=\begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} -36 & 15 \\ 9 & 6 \end{bmatrix} \textbf{ Ans}$$

Solution: (iv) $\frac{2}{3}(2A-3B)$

$$=\frac{2}{3}\left(2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}-3\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$$

$$=\frac{2}{3}\left(\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}-\begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}\right)$$

$$=\frac{2}{3}\begin{bmatrix} 4-15 & 6-(-12) \\ 22(-6) & 0-(-3) \end{bmatrix}$$

$$=\frac{2}{3}\begin{bmatrix} -11 & 6+12 \\ 2+6 & 0+3 \end{bmatrix}$$

$$=\frac{2}{3}\begin{bmatrix} -11 & 18 \\ 8 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} -11\times\frac{2}{3} & 18\times\frac{2}{3} \\ 8\times\frac{2}{3} & 3\times\frac{2}{3} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{-22}{3} & 12 \\ \frac{16}{3} & 2 \end{bmatrix} \textbf{ Ans}$$

Q.5 Find the value of X, if $\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}+X=\begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$.

Solution: Given that

$$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}+X=\begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$$

$$X=\begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}-\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} 4-2 & -2-1 \\ -1-3 & -2-(-3) \end{bmatrix}$$

$$=\begin{bmatrix} 2 & -3 \\ -4 & -2+3 \end{bmatrix}$$

$$X=\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \textbf{ Ans}$$

Q.6 If $A=\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, B=\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$,

then prove that

(i) $AB \neq BA$

(ii) $A(BC)=(AB)C$

Solution: Given that

$$A=\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, B=\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

(i) $AB \neq BA$

$$\text{L.H.S}=AB=\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} 0\times(-3)+1\times5 & 0\times4+1\times(-2) \\ 2\times(-3)+(-3)\times5 & 2\times4+(-3)\times(-2) \end{bmatrix}$$

$$=\begin{bmatrix} 0+5 & 0-2 \\ -6-15 & 8+6 \end{bmatrix}$$

$$=\begin{bmatrix} 5 & -2 \\ -21 & 14 \end{bmatrix} \qquad \rightarrow(\text{i})$$

$$\text{R.H.S}=BA=\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} -3(0)+4(2) & -3(1)+4(-3) \\ 5(0)+(-2)(2) & 5(1)+(-2)(-3) \end{bmatrix}$$

$$=\begin{bmatrix} 0+8 & -3-12 \\ 0-4 & 5+6 \end{bmatrix}$$

$$=\begin{bmatrix} 8 & -15 \\ -4 & 11 \end{bmatrix} \qquad \rightarrow(\text{ii})$$

From (i) and (ii) , we get

$L.H.S \neq R.H.S$

$AB \neq BA$

Hence proved

(ii) $A(BC)=(AB)C$

Solution:

We cannot solve because matrix C is not given.

Q.7 **If** $A=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}$ **and** $B=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$,

then verify that

(i) $(AB)^t = B^t A^t$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

Solution: Given that

$A=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}$ **and** $B=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$

(i) $(AB)^t = B^t A^t$

$$AB=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}3(2)+2(-3) & 3(4)+2(-5)\\1(2)+(-1)(-3) & 1(4)+(-1)(-5)\end{bmatrix}$$

$$=\begin{bmatrix}6-6 & 12-10\\2+3 & 4+5\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2\\5 & 9\end{bmatrix}$$

$$\text{L.H.S}=(AB)^t=\begin{bmatrix}0 & 2\\5 & 9\end{bmatrix}^t$$

$$=\begin{bmatrix}0 & 5\\2 & 9\end{bmatrix} \qquad \rightarrow (\text{i})$$

$$A^t=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}^t$$

$$=\begin{bmatrix}3 & 1\\2 & -1\end{bmatrix}$$

$$B^t=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}^t=\begin{bmatrix}2 & -3\\4 & -5\end{bmatrix}$$

$$\text{R.H.S}=B^tA^t=\begin{bmatrix}2 & -3\\4 & -5\end{bmatrix}\begin{bmatrix}3 & 1\\2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}2\times3+(-3)\times2 & 2(1)+(-3)(-1)\\4\times3+(-5)\times2 & 4(1)+(-5)(-1)\end{bmatrix}$$

$$=\begin{bmatrix}6-6 & 2+3\\12-10 & 4+5\end{bmatrix}$$

$$=\begin{bmatrix}0 & 5\\2 & 9\end{bmatrix} \qquad \rightarrow (\text{ii})$$

From equal (i) and (ii) we get

L.H.S=R.H.S

$(AB)^t = B^t A^t$

Hence proved

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

$$|AB|=\begin{vmatrix}0 & 2\\5 & 9\end{vmatrix}$$

$=0\times9-2\times5$

$=0-10$

$=-10$ (Non singular)

Inverse exists

$$Adj(AB)=\begin{bmatrix}9 & -2\\-5 & 0\end{bmatrix}$$

$$\text{L.H.S}=(AB)^{-1}=\frac{1}{|AB|}Adj(AB)$$

$$=\frac{1}{-10}\begin{bmatrix}9 & -2\\-5 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\frac{9}{10} & \frac{-2}{-10}\\ \frac{-5}{-10} & \frac{0}{-10}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-9}{10} & \frac{1}{5}\\ \frac{1}{2} & 0\end{bmatrix} \qquad \rightarrow (\text{i})$$

$$B=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$$

$$|B|=\begin{vmatrix}2 & 4\\-3 & -5\end{vmatrix}$$

$=2(-5)-4\times(-3)$

$=-10+12$

$=2$ (non singular)

$\therefore B^{-1}$ exists

$$AdjB = \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} AdjB$$

$$= \frac{1}{2} \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 3(-1) - 2 \times 1$$

$$= -3 - 2$$

$= -5$ (non singular)

$\therefore A^{-1}$ exists

$$AdjA = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{-5} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

R.H.S$=B^{-1}A^{-1}$

$$= \left(\frac{1}{2} \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix} \right) \times \left(\frac{1}{-5} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix} \right)$$

$$= \frac{1}{2} \left(-\frac{1}{5} \right) \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= -\frac{1}{10} \begin{bmatrix} -5(-1)+(-4)(-1) & -5(-2)+(-4)(3) \\ 3(-1)+2(-1) & 3(-2)+2(3) \end{bmatrix}$$

$$= -\frac{1}{10} \begin{bmatrix} 5+4 & 10-12 \\ -3-2 & -6+6 \end{bmatrix}$$

$$= -\frac{1}{10} \begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{-10} & \frac{-2}{-10} \\ \frac{-5}{-10} & \frac{0}{-10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix} \qquad \rightarrow (ii)$$

From equation (i) and (ii) we get

L.H.S =R.H.S

$$(AB)^{-1} = B^{-1}A^{-1}$$

Hence proved

Exercise 1.5

Q.1 Find the determinant of following matrices.

(i) $A=\begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$

Solution:

$A=\begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$

To write the determinant form

$|A|=\begin{vmatrix}-1 & 1\\ 2 & 0\end{vmatrix}$

$=(-1)(0)-(2)(1)$
$=0-2$
$=-2$

(ii) $B=\begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$

Solution:

$B=\begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$

To write in determinant form

$|B|=\begin{vmatrix}1 & 3\\ 2 & -2\end{vmatrix}$

$=(1)(-2)-(2)(3)$
$=-2-6$
$=-8$

(iii) $C=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$

Solution:

$C=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$

To write in determinant form

$|C|=\begin{vmatrix}3 & 2\\ 3 & 2\end{vmatrix}$

$=(3)(2)-(3)(2)$
$=6-6$
$=0$

(iv) $D=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$

Solution:

$D=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$

To write in determinant form

$|D|=\begin{vmatrix}3 & 2\\ 1 & 4\end{vmatrix}$

$=(3)(4)-(2)(1)$
$=12-2$
$=10$

Q.2 Find which of the following matrices are singular or non-singular?

(i) $A=\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$

Solution:

$A=\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$

To write in determinant form

$|A|=\begin{vmatrix}3 & 6\\ 2 & 4\end{vmatrix}$

$|A|=(3)\,(4)-(2)(6)$

$|A|=12-12$

$|A|=0$

It is a singular matrix.

(ii) $B=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$

Solution:

$B=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$

To write in determinant form

$$|B|=\begin{vmatrix}4 & 1\\ 3 & 2\end{vmatrix}$$

$$|B|=(4)(2)-(3)(1)$$

$$|B|=8-3$$

$$|B|=5$$

It is non-singular matrix.

(iii) $C=\begin{bmatrix}7 & -9\\ 3 & 5\end{bmatrix}$

Solution:

$$C=\begin{bmatrix}7 & -9\\ 3 & 5\end{bmatrix}$$

To write in determinant form

$$|C|=\begin{vmatrix}7 & -9\\ 3 & 5\end{vmatrix}$$

$$|C|=(7)(5)-(3)(-9)$$

$$|C|=35+27$$

$$|C|=62$$

In not equal to zero so
It is non-singular matrix.

(iv) $D=\begin{bmatrix}5 & -10\\ -2 & 4\end{bmatrix}$

Solution:

$$D=\begin{bmatrix}5 & -10\\ -2 & 4\end{bmatrix}$$

To write in determinant form

$$|D|=\begin{vmatrix}5 & -10\\ -2 & 4\end{vmatrix}$$

$$|D|=(5)(4)-(-2)(-10)$$

$$|D|=20\text{-}20$$

$$|D|=0$$

It is singular matrix.

Q.3 Find the multiplicative inverse of each

(i) $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$

Solution:

$$A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

To write in determinant form

$$|A|=\begin{vmatrix}-1 & 3\\ 2 & 0\end{vmatrix}$$

$$|A|=(-1)(0)-(2)(3)$$

$$|A|=0-6$$

$$|A|=-6\neq 0 \text{ (Non-Singular)}$$

A^{-1}exists

To write in Adj A

$$AdjA=\begin{bmatrix}0 & -3\\ -2 & -1\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times AdjA$$

Putting the values

$$A^{-1}=\frac{1}{-6}\times\begin{bmatrix}0 & -3\\ -2 & -1\end{bmatrix}=\begin{bmatrix}0\times\frac{1}{-6} & -3\times\frac{1}{-6}\\ -2\times\frac{1}{-6} & -1\times\frac{1}{-6}\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}\frac{0}{-6} & \frac{+3}{+6}\\ \frac{+2}{+6} & \frac{+1}{+6}\end{bmatrix}$$

$$=\begin{bmatrix}0 & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{6}\end{bmatrix}$$

(ii) $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$

Solution:

$$B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

To write in determinant form

$$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$$

$$|B| = (-1)(-5) - (-3)(2)$$

$$|B| = -5 + 6$$

$|B| = 1 \neq 0$ (Non-Singular)

B^{-1} exists

$$AdjB = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times AdjB$$

Putting the values

$$B^{-1} = \frac{1}{1} \times \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \times -5 & \frac{1}{1} \times -2 \\ \frac{1}{1} \times 3 & \frac{1}{1} \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5}{1} & \frac{-2}{1} \\ \frac{3}{1} & \frac{1}{1} \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

(iii) $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

Solution:

To write in determinant form

$$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$$

$$|C| = (-2)(-9) - (3)(6)$$

$$|C| = 18 - 18$$

$|C| = 0$ Singular

C^{-1} Does not exists.

(iv) $D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$

Solution:

To write in determinant form

$$D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$$

$$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix} = \frac{1}{2} \times 2 - \frac{3}{4} \times 1$$

$$= 1 - \frac{3}{4}$$

$$= \frac{4-3}{4}$$

$|D| = \frac{1}{4} \neq 0$ (Non Singular)

D^{-1} exists

$$AdjD = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$D^{-1} = \frac{1}{|\mathrm{D}|} \times AdjD$$

By putting the values

$$= \frac{1}{\frac{1}{4}} \begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \div \frac{1}{4} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \times \frac{4}{1} \begin{bmatrix} 2 & \frac{-3}{2} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 4 \begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Q.4 **If** $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ **and** $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$,

then

Then verify that

(i) A(AdjA)=(AdjA)A=(detA)I

Solution: A(AdjA)=(AdjA)A=(detA)I

$$A=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$$

$$\text{AdjA}=\begin{bmatrix}6 & -2\\ -4 & 1\end{bmatrix}$$

$$\text{detA}=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$$

$=1\times 6 \quad 2\times 4$

$=6-8$

$=-2$

$$A(AdjA)=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}\begin{bmatrix}6 & -2\\ -4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & (-2)+2\\ 24-4 & -8+6\end{bmatrix}$$

$$\text{A (Adj A)}=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_ (i)}$$

$$(AdjA)A=\begin{bmatrix}6 & -2\\ -4 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$$

$$(AdjA)A=\begin{bmatrix}(6)\times 12\times 4 & (6)\times 2+(-2)\times 6\\ (-4)\times 1+(1)(4) & (-4)(2)+(1)(6)\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & 12-12\\ -4+4 & -8+6\end{bmatrix}$$

$$\text{(AdjA)A}=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_ (ii)}$$

$$(\det A)\text{I}=-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-2\times 1 & 0\times 2\\ -2\times 0 & 1\times -2\end{bmatrix}$$

$$\text{(detA)I}=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_ (iii)}$$

Hence proved

From eq (i), (ii) and (iii)

A(AdjA)=(AdjA)A=(detA)I

(ii) $BB^{-1} = I = B^{-1}B$

Solution: $BB^{-1} = I = B^{-1}B$

To write in determinant form

$$|B|=\begin{vmatrix}3 & -1\\ 2 & -2\end{vmatrix}$$

$=-6-(-2)$

$=-6+2$

$=-4\neq 0$ (None singular)

$=B^{-1}$ exists.

To write in AdjB

$$AdjB=\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}$$

$$B^{-1}\frac{1}{|B|}Adj$$

$$B^{-1}=\frac{1}{-4}\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}$$

New

$$BB^{-1}=\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}\times\frac{1}{-4}\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}$$

$$=-\frac{1}{4}\begin{bmatrix}-6+2 & 3-3\\ -4+4 & 2-6\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-4 & 0\\ 0 & -4\end{bmatrix}$$

$$=\begin{bmatrix}\frac{4}{4} & 0\\ 0 & \frac{4}{4}\end{bmatrix}$$

$$BB^{-1}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

and

$$B^{-1}B=\frac{1}{-4}\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-6+2 & 2-2\\ -6+6 & 2-6\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-4 & 0\\ 0 & -4\end{bmatrix}$$

$$=\begin{bmatrix} \frac{-4}{-4} & 0 \\ 0 & \frac{-4}{-4} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$B^{-1}B = I$

From (i) and (ii)

$BB^{-1}=I=B^{-1}B$

Hence proved

Q.5 Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

Solution: $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 21+(-20) & -15+15 \\ 28+(-28) & -20+21 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} -3+4 & 2+(-2) \\ -6+6 & 4+(-3) \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Given matrices are multiplicative inverse of each other

Q.6

(i) $(AB)^{-1} = B^{-1}A^{-1}$

Solution: $(AB)^{-1} = B^{-1}A^{-1}$

$$A=\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}, B=\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$AB=\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 4\times(-4)+0(1) & 4\times(-2)+0(-1) \\ -1\times(-4)+2(1) & -1\times(-2)+2(-1) \end{bmatrix}$$

$$=\begin{bmatrix} -16+0 & -8+0 \\ 4+2 & 2+(-2) \end{bmatrix}$$

$$=\begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$$

To write in determinant form

$$|AB|=\begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$$

$$|AB|=0-(-48)$$

$$|AB|=48$$

To write in Adj (AB)

$$\text{Adj (AB)} = \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|}\times AdjAB$$

$$=\frac{1}{48}\times\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$=\begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

To solve R.H. S

To write in determinant form

$$|B|=\begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$$

$|B| = 4 - (-2)$

$|B| = 4 + 2$

$|B| = 6$

To write in Adj B

$$AdjB = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times AdjB$$

By putting value

$$B^{-1} = \frac{1}{6} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$$|A| = 8 - (-0)$$

$$|A| = 8$$

To write in Adj A

$$AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{8} \times \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

To solve R.H.S

$$B^{-1}A^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \times \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{6} \times \frac{1}{8}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{48}\begin{bmatrix} -2+2 & 0+8 \\ -2-4 & 0-16 \end{bmatrix}$$

$$= \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \times \frac{1}{48} & 8 \times \frac{1}{48} \\ -6 \times \frac{1}{48} & -16 \times \frac{1}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

Hence proved

L.H.S = R.H.S

Unit 1: Matrices and Determinants

Overview

Matrix:

A rectangular array of real numbers enclosed within brackets is said to form matrix.

Rows of a Matrix:

In matrix, the entries presented in horizontal way are called rows.

i.e. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 8 \\ 7 & 1 & 5 \end{bmatrix} \begin{array}{l} \to R_1 \\ \to R_2 \\ \to R_3 \end{array}$.

Columns of a Matrix:

In matrix, all the entries presented in vertical way are called columns of matrix.

$$\begin{array}{ccc} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 8 \\ 7 & 1 & 5 \end{bmatrix} \\ \downarrow \quad \downarrow \quad \downarrow \\ C_1 \quad C_2 \quad C_3 \end{array}.$$

Order of a Matrix:

The number of rows and columns in a matrix specifies its order. If a matrix M has m rows and n columns then M is said to be of order, $m-by-n$.

i.e. $\begin{bmatrix} 0 & 8 & 0 \\ 0 & 4 & 8 \\ 7 & 1 & 5 \end{bmatrix}$ the order matrix is $3-by-3$

Equal Matrix's:

Let A and B be two matrices. Then A is said to be equal to B, and denoted by $A = B$, if and only if;

(i) The order of A = the order of B

(ii) Their corresponding entries are equal.

i.e. $A = \begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2+1 \\ -4 & 4-2 \end{bmatrix}$ are equal matrices.

Rectangular Matrix:

A matrix M is called rectangular if, the number of rows of M is not equal to the number of columns of M.

e.g., $B=\begin{bmatrix} a & b & c \\ d & e & d \end{bmatrix}$.

Square Matrix:

A matrix is called a square matrix if its number of rows is equal to its number of columns.

i.e, $A=\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$

Null or Zero Matrix:

A matrix M is called a null or zero matrix if each of its entries is 0.

e.g. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Transpose of a Matrix:

A matrix obtained by interchanging the rows into columns or columns into rows of a matrix is called transpose of that matrix.

Negative of a Matrix:

Let A be matrix. Then its negative, $-A$ is obtained by changing the signs of all the entries of A,

i.e. If $A=\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, then $-A=\begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$

Symmetric Matrix:

A square matrix is symmetric if it is equal to its transpose i.e., matrix A is symmetric if $A^t = A$.

Skew-Symmetric Matrix:

A square matrix A is said to be skew-symmetric if $A^t = -A$.

Diagonal Matrix:

A square matrix A is called a diagonal matrix if at least any one of the entries of its diagonal is not zero and non-diagonal entries must all be zero.

i.e. $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Scalar Matrix:

A diagonal matrix is called a scalar matrix, if all the diagonal entries are same and non-zero. For example $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ where k is a constant $\neq 0,1$

Identity Matrix:

A diagonal matrix is called identity (unit) matrix if all diagonal entries are 1 and it is denoted by I.

e.g., $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ is a 3-by-3 identity matrix.

Addition of Matrices:

Let A and B be any two matrices with real number entries. The matrices A and B are conformable for addition, if they have the same order.

Subtraction of Matrices:

If A and B are two matrices of same order then subtraction of matrix B from matrix A is obtained by subtracting the entries of matrix B from the corresponding entries of matrix A and it is denoted by $A-B$.

Multiplication of Matrices:

Two matrices A and B conformable for multiplication, giving product AB if the number of columns of A is equal to the number of rows of B.

Determinant of a 2-by-2 Matrix:

Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ be a 2-by-2 square matrix. The determinant of A, denoted by **det A** or $|A|$ is defined as.

$$|A| = \det A = \det\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \begin{vmatrix}a & b\\ c & d\end{vmatrix} = ad-bc=\lambda \in R$$

Singular Matrix:

A square matrix A is called singular if the determinant of A is equal to zero.

For example, $A=\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix}$ is a singular matrix, since det $A=1\times 0-0\times 2=0$.

Non-Singular Matrix:

A square matrix A is called non-singular if the determinant of A is not equal to zero.

For example $A=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}$ is non-singular, since $\det A = 1\times 2-0\times 1=2\neq 0$.

Adjoint of a Matrix:

Adjoint of a square matrix $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of other entries. Adjoint of matrix A is denoted as Adj A.

i.e. Adj A $=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$