

# RATIO AND PROPORTION

## 14.1 Ratio and Proportion

We defined ratio  $a:b = \frac{a}{b}$  as the comparison of two alike quantities  $a$  and  $b$ , called the elements (terms) of a ratio. (Elements must be expressed in the same units). Equality of two ratios was defined as proportion.

That is, if  $a:b = c:d$ , then  $a, b, c$  and  $d$  are said to be in proportion.

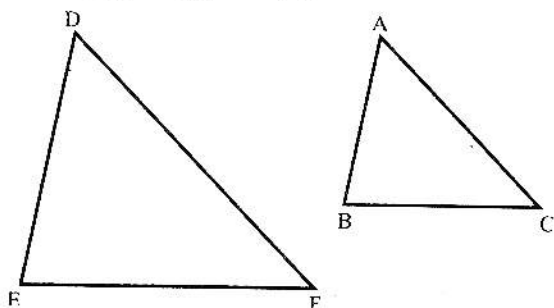
### Similar Triangles

Equally important are the similar shapes. In particular the similar triangles that have many practical applications. For example, we know that a photographer can develop prints to different sizes from the same negative. In spite of the difference in size, these pictures look like each other. One photograph is simply an enlargement of another. They are said to be similar in shape. Geometrical figures can also be similar. e.g., if

In  $\triangle ABC \longleftrightarrow \triangle DEF$

$$\angle A \cong \angle D, \quad \angle B \cong \angle E, \quad \angle C \cong \angle F,$$

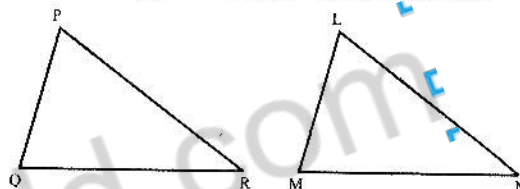
$$\text{and } \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$



then  $\triangle ABC$  and  $\triangle DEF$  are called similar triangles which is symbolically written as  $\triangle ABC \sim \triangle DEF$

It means that corresponding angles of similar triangles are equal and measures of their corresponding sides are proportional.

$\triangle PQR \cong \triangle LMN$  means that in



$$\triangle PQR \longleftrightarrow \triangle LMN$$

$$\angle P \cong \angle L,$$

$$\angle Q \cong \angle M$$

$$\angle R \cong \angle N,$$

$$\overline{PQ} \cong \overline{LM},$$

$$\overline{QR} \cong \overline{MN},$$

$$\overline{RP} \cong \overline{NL},$$

$$\text{Now as } \frac{PQ}{LM} = \frac{QR}{MN} = \frac{RP}{NL} = 1$$

$$\therefore \triangle PQR \sim \triangle LMN$$

### Note:

Two congruent triangles are similar also. But two similar triangles are not necessarily congruent, as congruence of their corresponding sides is not necessary.

**Theorem** A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.

**Given** In  $\triangle ABC$ , the line  $l$  is intersecting the sides  $\overline{AC}$  and  $\overline{AB}$  at points E and D respectively such that  $\overline{ED} \parallel \overline{CB}$ .

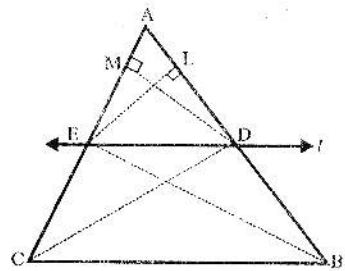
**To Prove**

$$m\overline{AD} : m\overline{BD} = m\overline{AE} : m\overline{EC}$$

**Construction**

Join B to E and C to D. From D draw  $\overline{DM} \perp \overline{AC}$  and from E draw  $\overline{EL} \perp \overline{AB}$ .

**Proof**



Statements	Reasons
In triangles BED and AED, $\overline{EL}$ is the common perpendicular.	
$\therefore \Delta BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL} \dots\dots\dots (i)$	
and $\Delta AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL} \dots\dots\dots (ii)$	Area of a $\Delta = \frac{1}{2} (\text{base}) (\text{height})$
Thus $\frac{\Delta BED}{\Delta AED} = \frac{m\overline{BD}}{m\overline{AD}} \dots\dots\dots (iii)$	Dividing (i) by (ii)
Similarly	
$\frac{\Delta CDE}{\Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}} \dots\dots\dots (iv)$	
But $\Delta BED \cong \Delta CDE$	Areas of triangles with common base and same altitudes are equal. Given that $\overline{ED} \parallel \overline{CB}$ so altitudes are equal.
$\therefore$ From (iii) and (iv), we have	
$\frac{m\overline{BD}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$ or $\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$	Taking reciprocal of both sides.
Hence $m\overline{AD} : m\overline{BD} = m\overline{AE} : m\overline{EC}$	

**Note:**

From the above theorem we also have

$$\frac{m\overline{BD}}{m\overline{AB}} = \frac{m\overline{CE}}{m\overline{AC}} \text{ and } \frac{m\overline{AD}}{m\overline{AB}} = \frac{m\overline{AE}}{m\overline{AC}}$$

### Corollaries

- a) If  $\frac{m\overline{AD}}{m\overline{AB}} = \frac{m\overline{AE}}{m\overline{AC}}$ , then  $\overline{DE} \parallel \overline{BC}$
- b) If  $\frac{m\overline{AB}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{EC}}$ , then  $\overline{DE} \parallel \overline{BC}$

### Note:

- Two points determine a line and three non-collinear points determine a plane.
- A line segment has exactly one midpoint.
- If two intersecting lines form equal adjacent angles, the lines are perpendicular.

### Theorem

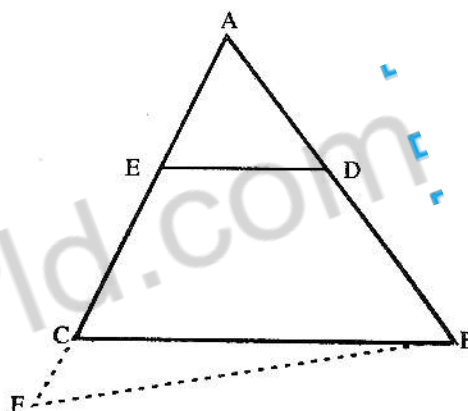
(Converse of Theorem)

If a line segment intersects the two sides of a triangle in the same ratio then it is parallel to the third side.

**Given** In  $\triangle ABC$ ,  $\overline{ED}$  intersects  $\overline{AB}$  and  $\overline{AC}$  such that  $m\overline{AD} : m\overline{BD} = m\overline{AE} : m\overline{EC}$

**To Prove**  $\overline{ED} \parallel \overline{CB}$

**Construction** If  $\overline{ED} \not\parallel \overline{CB}$ , then draw  $\overline{BF} \parallel \overline{DE}$  to meet  $\overline{AC}$  produced at F.

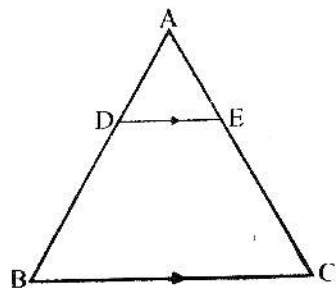


### Proof

Statements	Reasons
In $\triangle ABF$ $\overline{DE} \parallel \overline{BF}$	Construction
$\therefore \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EF}}$ .....(i)	(A line parallel to one side of a triangle divides the other two sides proportionally)
But $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$ .....(ii)	Given
$\therefore \frac{m\overline{AE}}{m\overline{EF}} = \frac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
or $m\overline{EF} = m\overline{EC}$	
which is possible only if point F is coincident with C.	(Property of real numbers)
$\therefore$ Our supposition is wrong.	
Hence $\overline{ED} \parallel \overline{CB}$	

## Exercise 14.1

1. In  $\triangle ABC$ ,  $\overline{DE} \parallel \overline{BC}$



- i)  $\overline{AD} = 1.5 \text{ cm}$ ,  $\overline{BD} = 3 \text{ cm}$ ,  
 $\overline{AE} = 1.3 \text{ cm}$  then find  $\overline{CE}$ .
- ii) If  $\overline{AD} = 2.4 \text{ cm}$ ,  $\overline{AE} = 3.2 \text{ cm}$ ,  
 $\overline{EC} = 4.8 \text{ cm}$ , find  $\overline{AB}$
- iii) If  $\frac{\overline{AD}}{\overline{DB}} = \frac{3}{5}$ ,  $\overline{AC} = 4.8 \text{ cm}$ , find  
 $\overline{AE}$
- iv) If  $\overline{AD} = 2.4 \text{ cm}$ ,  $\overline{AE} = 3.2 \text{ cm}$ ,  
 $\overline{DE} = 2 \text{ cm}$ ,  $\overline{BC} = 5 \text{ cm}$ , find  
 $\overline{AB}$ ,  $\overline{DB}$ ,  $\overline{AC}$ ,  $\overline{CE}$
- v) If  $\overline{AD} = 4x - 3$ ,  $\overline{AE} = 8x - 7$ ,  
 $\overline{BD} = 3x - 1$ , and  $\overline{CE} = 5x - 3$ , find the  
value of  $x$

In  $\triangle ABC$ ,  $\overline{DE} \parallel \overline{BC}$

$$\begin{aligned} \text{(i)} \quad \frac{\overline{mAD}}{\overline{mBD}} &= \frac{\overline{mAE}}{\overline{mEC}} \\ \frac{1.5}{3} &= \frac{1.3}{\overline{mEC}} \\ \overline{mEC} &= \frac{3 \times 1.3}{1.5} \\ &= 2.6 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \text{In } \triangle ABC, \overline{DE} \parallel \overline{BC} \\ \overline{mAB} &= \overline{mAD} + \overline{mBD} \end{aligned}$$

Let  $\overline{mDB} = x \text{ cm}$

$$\text{Now } \frac{\overline{mAD}}{\overline{mDB}} = \frac{\overline{mAE}}{\overline{mEC}}$$

$$\frac{2.4}{x} = \frac{3.2}{4.8}$$

$$x = \frac{4.8 \times 2.4}{3.2}$$

$$x = \frac{48 \times 24}{10 \times 32}$$

$$x = 3.6 \text{ cm.}$$

$$\therefore \overline{mAB} = \overline{mAD} + \overline{mBD}$$

$$\overline{mAB} = 2.4 + 3.6 = 6 \text{ cm}$$

$$\text{(iii)} \quad \frac{\overline{mAD}}{\overline{mDB}} = \frac{3}{5}, \overline{mAC} = 4.8 \text{ cm}$$

In  $\triangle ABC$ ,  $\overline{DE} \parallel \overline{BC}$

$$\frac{\overline{mAD}}{\overline{mDB}} = \frac{\overline{mAE}}{\overline{mEC}}$$

$$\frac{\overline{mAD}}{\overline{mDB}} = \frac{\overline{mAC} - \overline{mCE}}{\overline{mCE}}$$

$$\frac{3}{5} = \frac{4.8 - \overline{mCE}}{\overline{mCE}}$$

$$3\overline{mCE} = 5(4.8 - \overline{mCE})$$

$$3\overline{mCE} = 24 - 5\overline{mCE}$$

$$3\overline{mCE} + 5\overline{mCE} = 24$$

$$8\overline{mCE} = 24$$

$$\overline{mCE} = \frac{24}{8} = 3 \text{ cm}$$

$$\begin{aligned} \overline{mAE} &= \overline{mAC} - \overline{mCE} \\ &= 4.8 - 3 \end{aligned}$$

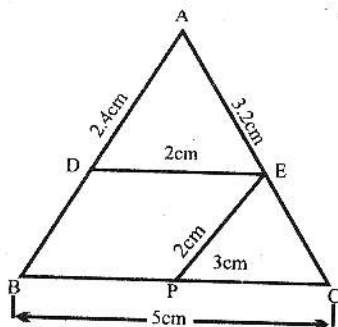
$$\overline{mAE} = 1.8 \text{ cm}$$



(iv)  $\overline{mAD} = 2.4\text{cm}$ ,

$\overline{mAE} = 3.2\text{cm}$ ,  $\overline{mDE} = 2\text{cm}$ ,  $\overline{mBC} = 5\text{cm}$ .

$\overline{mAB} = ?$   $\overline{mDB} = ?$   $\overline{mAC} = ?$   $\overline{mCE} = ?$



$\overline{EP} \parallel \overline{AB}$

DEPB is a parallelogram, then

$\overline{mPB} = \overline{mDE} = 2\text{cm}$ .

$\overline{mCP} = 5 - 2 = 3\text{cm}$

In  $\triangle ABC$ ,  $\overline{EP} \parallel \overline{AB}$

$\frac{\overline{mCE}}{\overline{mEA}} = \frac{\overline{mCP}}{\overline{mPB}}$

$\frac{\overline{mCE}}{\overline{mEA}} = \frac{3}{2}$

$\frac{\overline{mCE}}{3.2} = \frac{3}{2}$

$\overline{mCE} = \frac{3 \times 3.2}{2}$

$\overline{mCE} = 3 \times 1.6 = 4.8\text{cm}$

Now  $\overline{DE} \parallel \overline{BC}$ , in  $\triangle ABC$

$\frac{\overline{mBD}}{\overline{mAD}} = \frac{\overline{mCE}}{\overline{mAE}}$

$\frac{\overline{mBD}}{\overline{mAD}} = \frac{4.8}{3.2}$

$\frac{\overline{mBD}}{2.4} = \frac{4.8}{3.2}$

$\overline{mBD} = \frac{2.4 \times 4.8}{3.2} = 3.6\text{cm}$

$\overline{mAB} = \overline{mAD} + \overline{mDB}$

$= 2.4 + 3.6$

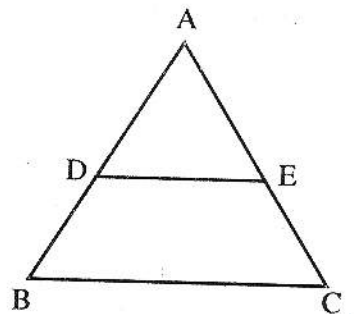
$= 6.0\text{cm}$

$\overline{mAC} = \overline{mAE} + \overline{mEC}$

$= 3.2 + 4.8$

$= 8.0\text{cm}$ .

(v) If  $\overline{AD} = 4x - 3$ ,  $\overline{AE} = 8x - 7$ ,  $\overline{BD} = 3x - 1$  and  $\overline{CE} = 5x - 3$ , Find the value of  $x$



In  $\triangle ABC$ ,  $\overline{DE} \parallel \overline{BC}$

$\frac{\overline{mAD}}{\overline{mBD}} = \frac{\overline{mAE}}{\overline{mCE}}$

$\frac{4x - 3}{3x - 1} = \frac{8x - 7}{5x - 3}$

$\frac{4x - 3}{3x - 1} = \frac{8x - 7}{5x - 3}$

$(4x - 3)(5x - 3) = (8x - 7)(3x - 1)$

$20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$20x^2 - 24x^2 - 27x + 29x + 9 - 7 = 0$

$-4x^2 + 2x + 2 = 0$

$2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x - 1) + 1(x - 1) = 0$

$(x - 1)(2x + 1) = 0$

$x - 1 = 0$  or  $2x + 1 = 0$

$x = 1$  or  $2x = -1$

$x = 1$  or  $x = \frac{-1}{2}$

But  $x = \frac{-1}{2}$  not possible

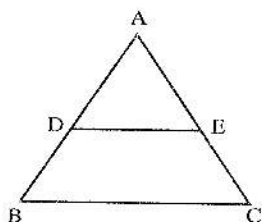
So  $x = 1$

2. If  $\triangle ABC$  is an isosceles triangle,  $\angle A$  is vertex angle and  $\overline{DE}$  intersects the

sides  $\overline{AB}$  and  $\overline{AC}$  as shown in the figure so that.

$$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$$

Prove that  $\triangle ADE$  is also an isosceles triangle.



In  $\triangle ABC$ ,  $\angle A$  is vertical angle and  $\overline{AB} \cong \overline{AC}$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

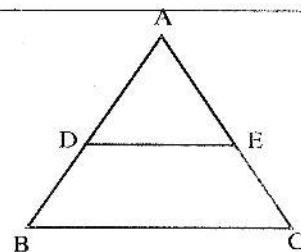
$$\frac{m\overline{DB}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$$

$$\frac{m\overline{DB} + m\overline{AD}}{m\overline{AD}} = \frac{m\overline{EC} + m\overline{AE}}{m\overline{AE}}$$

**To Prove:** Find all angles of  $\triangle ADE$

Statements	Reasons
$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$	Given
Then $\overline{DE} \parallel \overline{BC}$	Proved
$\triangle ABC$ is equilateral triangle	Corresponding angle
Then $m\angle A = m\angle B = m\angle C = 60^\circ$	
$\overline{DE} \parallel \overline{BC}$	
$m\angle 1 = m\angle B = 60^\circ$	
$m\angle 2 = m\angle C = 60^\circ$	
$m\angle A = 60^\circ$	

4. Prove that the line segment drawn through the mid-point of one side of a triangle and parallel to another side bisects the third side.



$$\frac{m\overline{AB}}{m\overline{AD}} = \frac{m\overline{AC}}{m\overline{AE}}$$

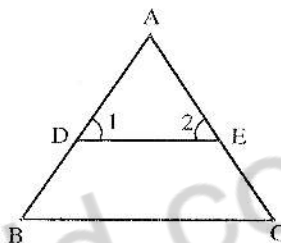
$$\text{Now } \frac{m\overline{AB}}{m\overline{AD}} = \frac{m\overline{AC}}{m\overline{AE}}$$

$\triangle ADE$  is an isosceles triangle.

3. In an equilateral triangle  $ABC$  shown in the figure.

$$m\overline{AE} : m\overline{AC} = m\overline{AD} : m\overline{AB}$$

Find all three angles of  $\triangle ADE$  and name it also.



**Given:**  $\triangle ABC$  is an equilateral triangle.

$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

Given in  $\triangle ABC$ ,  $\overline{DE}$  is such that  $\overline{mAD} = \overline{mDB}$  and  $\overline{DE} \parallel \overline{BC}$

To Prove:

$$\overline{mAE} = \overline{mEC}$$

Statements	Reasons
In $\triangle ABC$ $\overline{DE} \parallel \overline{BC}$ $\frac{\overline{mAD}}{\overline{mDB}} = \frac{\overline{mAE}}{\overline{mEC}}$ .....(i) $\overline{mAD} = \overline{mDB}$ $\frac{\overline{mDB}}{\overline{mDB}} = \frac{\overline{mAE}}{\overline{mEC}}$ $1 = \frac{\overline{mAE}}{\overline{mEC}}$ $\overline{mAE} = \overline{mEC}$	Given  Given  Put $\overline{mAD} = \overline{mDB}$ in (i)

5. Prove that the line segment joining the mid-points of any two sides of a triangle is parallel to the third side.

Given:

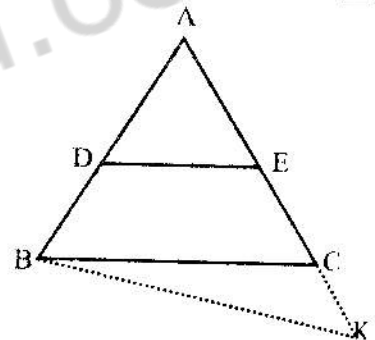
In  $\triangle ABC$ , points D, E are such that  $\overline{mAD} = \overline{mDB}$

$$\overline{mAE} = \overline{mEC}$$

$$\frac{\overline{mAD}}{\overline{mDB}} = \frac{\overline{mAE}}{\overline{mEC}}$$

To Prove:

$$\overline{DE} \parallel \overline{BC}$$



Statements	Reasons
If $\overline{DE} \parallel \overline{BC}$ Then suppose $\overline{DE} \parallel \overline{BK}$ Now $\frac{\overline{mAD}}{\overline{mDB}} = \frac{\overline{mAE}}{\overline{mEK}}$ .....(i) $\frac{\overline{mAD}}{\overline{mDB}} = \frac{\overline{mAE}}{\overline{mEC}}$ .....(ii) $\frac{\overline{mAE}}{\overline{mEK}} = \frac{\overline{mAE}}{\overline{mEC}}$ $\overline{mEK} = \overline{mEC}$	Given  From (i) and (ii)

It is possible only when point K lies on the point C.

Thus  $\overline{DE} \parallel \overline{BC}$

### Theorem

The internal bisector of an angle of a triangle divides the side opposite to it in the ratio of the lengths of the sides containing the angle.

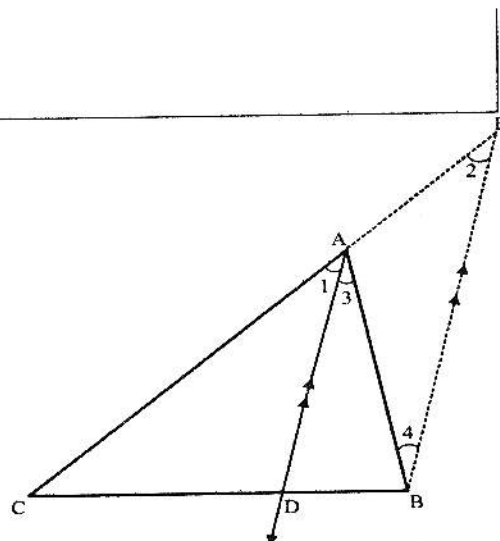
**Given:** In  $\triangle ABC$  internal angle bisector of  $\angle A$  meets  $\overline{CB}$  at the point D.

**To Prove:**  $m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$

### Construction:

Draw a line segment  $\overline{BE} \parallel \overline{DA}$  to meet  $\overline{CA}$  produced at E.

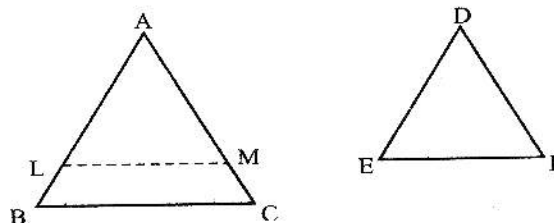
### Proof:



Statements	Reasons
$\therefore \overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersects them,	Construction
$\therefore m\angle 1 = m\angle 2$ .....(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$	
and $\overline{AB}$ intersects them,	Alternate angles
$\therefore m\angle 3 = m\angle 4$ .....(ii)	Given
But $m\angle 1 = m\angle 3$	From (i) and (ii)
$\therefore m\angle 2 = m\angle 4$	In a $\triangle$ , the sides opposite to congruent angles are also congruent.
and $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	By Theorem
or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	
Thus $m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$	$m\overline{EA} = m\overline{AB}$ (proved)

**Theorem:** If two triangles are similar, then the measures of their corresponding sides are proportional.

**Given:**  $\triangle ABC \sim \triangle DEF$





i.e.,  $\angle A \cong \angle D$ ,  $\angle B \cong \angle E$ , and  $\angle C \cong \angle F$

**To Prove:**

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

**Construction:**

i) Suppose that  $m\overline{AB} > m\overline{DE}$

ii)  $m\overline{AB} \leq m\overline{DE}$

On  $\overline{AB}$  take a point L such that  $m\overline{AL} = m\overline{DE}$

On  $\overline{AC}$  take a point M such that  $m\overline{AM} = m\overline{DF}$ . Join L and M by the line segment LM.

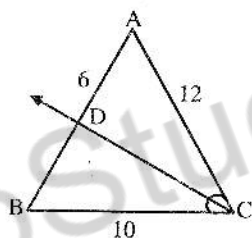
**Proof:**

Statements	Reasons
i) In $\triangle ALM \longleftrightarrow \triangle DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\triangle ALM \cong \triangle DEF$	S.A.S. Postulate
and $\angle L \cong \angle E$ , $\angle M \cong \angle F$ ,	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$ , and $\angle F \cong \angle C$	Given
$\therefore \angle L \cong \angle B$ , $\angle M \cong \angle C$ ,	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal.
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AM}}{m\overline{AC}}$	By Theorem
or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$ .....(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (construction)
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$ , we can prove that	
$\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$ .....(ii)	
Thus $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}} = \frac{m\overline{EF}}{m\overline{BC}}$	by (i) and (ii)
or $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$	by taking reciprocals
ii) If $m\overline{AB} < m\overline{DE}$ , it can similarly be	

<p>proved by taking intercepts on the sides of <math>\triangle DEF</math></p> <p>If <math>\overline{mAB} = \overline{mDE}</math>,</p> <p>then in <math>\triangle ABC \longleftrightarrow \triangle DEF</math></p> <p><math>\angle A \cong \angle D</math></p> <p><math>\angle B \cong \angle E</math></p> <p>and <math>\overline{AB} \cong \overline{DE}</math></p> <p>so <math>\triangle ABC \cong \triangle DEF</math></p> <p>Thus <math>\frac{\overline{mAB}}{\overline{mDE}} = \frac{\overline{mAC}}{\overline{mDF}} = \frac{\overline{mBC}}{\overline{mEF}} = 1</math></p> <p>Hence the result is true for all the cases.</p>	<p>Given</p> <p>Given</p> <p>A.S.A <math>\cong</math> A.S.A</p> <p><math>\overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}</math></p>
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### Exercise 14.2

1. In  $\triangle ABC$  as shown in the figure,  $\overline{CD}$  bisects  $\angle C$  and meets  $\overline{AB}$  at D,  $\overline{mBD}$  is equal to a) 5 b) 16 c) 10 d) 18



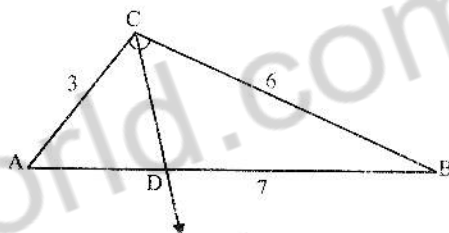
Ans.

$$\frac{\overline{mBD}}{\overline{mDA}} = \frac{\overline{mBC}}{\overline{mCA}}$$

$$\frac{\overline{mBD}}{6} = \frac{10}{12}$$

$$\overline{mBD} = \frac{10}{12} \times 6 = 5$$

2. In  $\triangle ABC$  as shown in the figure,  $\overline{CD}$  bisects  $\angle C$ . If  $\overline{mAC} = 3$ ,  $\overline{mCB} = 6$  and  $\overline{mAB} = 7$ , then find  $\overline{mAD}$  and  $\overline{mDB}$ .



Ans.

$$\overline{mAD} = x$$

$$\overline{mBD} = 7 - x$$

$$\frac{\overline{mAD}}{\overline{mDB}} = \frac{\overline{mAC}}{\overline{mCB}}$$

$$\frac{x}{7-x} = \frac{3}{6}$$

$$\frac{x}{7-x} = \frac{1}{2}$$

$$2x = 1(7-x)$$

$$2x = 7 - x$$

$$3x = 7 \Rightarrow x = \frac{7}{3}$$

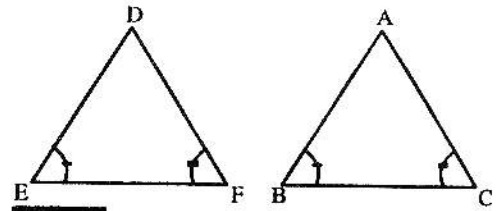
$$\overline{mAD} = \frac{7}{3}$$

$$\overline{mDB} = 7 - x$$

$$= 7 - \frac{7}{3}$$

$$= \frac{21-7}{3} = \frac{14}{3}$$

3. Show that in any correspondence of two triangles if two angles of one triangle are congruent to the corresponding angles of the other, then the triangles are similar.



**Given:** In  $\triangle ABC$  and  $\triangle DEF$

$$m\angle B = m\angle E$$

$$m\angle C = m\angle F$$

**To Prove:**  $\triangle ABC \sim \triangle DEF$

**Proof:**

Statements	Reasons
$m\angle B + m\angle C + m\angle A = 180^\circ$ ---(i)	Sum of interior angles of triangle is $180^\circ$
$m\angle E + m\angle F + m\angle D = 180^\circ$ ....(ii)	Given
$m\angle B + m\angle C + m\angle D = 180^\circ$ ...(iii)	Subtracting (i) from (ii)
$m\angle A - m\angle D = 0$	
$m\angle A = m\angle D$	
All Angles of $\triangle DEF$ and $\triangle ABC$ are congruent	
Thus $\triangle ABC \sim \triangle DEF$ .	

4. If line segments  $\overline{AB}$  and  $\overline{CD}$  intersecting at point X and  $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$  then show that  $\triangle AXC$  and  $\triangle BXD$  are similar.

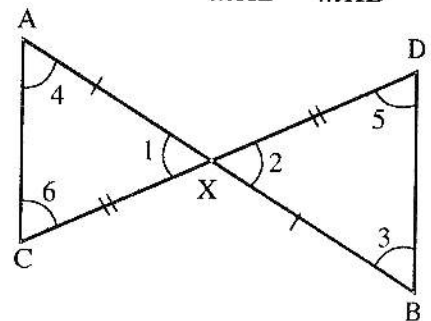
**Given:**

$\overline{AB}$  and  $\overline{CD}$  intersect each other at point x and

$$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$$

**To Prove:**

$$\triangle AXC \sim \triangle BXD$$



**Proof:**

Statements		Reasons
In	$\triangle AXC$ and $\triangle BXD$ $\angle 1 \cong \angle 2$ $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$	Vertical angles Given
Then	$\overline{AC} \parallel \overline{BD}$ $\angle 4 \cong \angle 3$ $\angle 6 \cong \angle 5$	Alternate angles
Thus	$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}} = \frac{m\overline{AC}}{m\overline{DB}}$	
Hence $\triangle AXC$ and $\triangle BXD$ are similar.		

5. Which of the following are true and which are false?

- |       |  |       |
|-------|--|-------|
| i.    | Congruent triangles are of same size and shape.          | True  |
| ii.   | Similar triangles are of same shape but different sizes. | True  |
| iii.  | Symbol used for congruent is ' $\cong$ '.                | False |
| iv.   | Symbol used for similarity is ' $\sim$ '.                | False |
| v.    | Congruent triangles are similar.                         | True  |
| vi.   | Similar triangles are congruent.                         | False |
| vii.  | A line segment has only one mid point.                   | True  |
| viii. | One and only one line can be drawn through two points.   | True  |
| ix.   | Proportion is non-equality of two ratios.                | False |
| x.    | Ratio has no unit.                                       | True  |

6. In  $\triangle LMN$  show in the figure,  $\overline{MN} \parallel \overline{PQ}$ .

i) If  $m\overline{LM} = 5\text{cm}$ ,  $m\overline{LP} = 2.5\text{cm}$ ,  $m\overline{LQ} = 2.3\text{cm}$ , then find  $m\overline{LN}$ .

ii) If  $m\overline{LM} = 6\text{cm}$ ,  $m\overline{LQ} = 2.5\text{cm}$ ,  $m\overline{QN} = 5\text{cm}$ , then find  $m\overline{LP}$ .

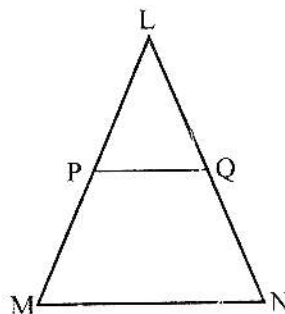
**Given:** In  $\triangle LMN$ ,  $\overline{MN} \parallel \overline{PQ}$

$m\overline{LM} = 5\text{cm}$ ,  $m\overline{LP} = 2.5\text{cm}$ ,  $m\overline{LQ} = 2.3\text{cm}$

**To Prove:**  $m\overline{LN} = ?$

**Proof:**

Statements	Reasons
$\frac{m\overline{LN}}{m\overline{LQ}} = \frac{m\overline{LM}}{m\overline{LP}}$	$\overline{PQ} \parallel \overline{MN}$ (Given)





$$\begin{aligned}\frac{m\overline{LN}}{2.3} &= \frac{5}{2.5} \\ m\overline{LN} &= \frac{5 \times 2.3}{2.5} \\ &= \frac{5 \times 23}{25} \\ &= 4.6\text{cm}\end{aligned}$$

Putting Values

(ii)

**Given:**  $\triangle LMN$ ,  $\overline{MN} \parallel \overline{PQ}$

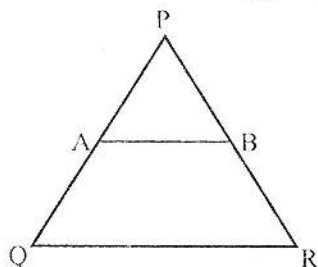
$m\overline{QN} = 5\text{cm}$ ,  $m\overline{LQ} = 2.5\text{cm}$ ,  $m\overline{LM} = 6\text{cm}$ .

**To prove:**  $m\overline{LP} = ?$

**Proof:**

$$\begin{aligned}\frac{m\overline{LP}}{m\overline{LM}} &= \frac{m\overline{LQ}}{m\overline{LN}} \\ \frac{m\overline{LP}}{6} &= \frac{m\overline{LQ}}{m\overline{LQ} + m\overline{QN}} \\ \frac{m\overline{LP}}{6} &= \frac{2.5}{2.5 + 5} \\ m\overline{LP} &= \frac{2.5}{7.5} \times 6 \\ m\overline{LP} &= \frac{1}{3} \times 6 \\ &= 2\text{cm}.\end{aligned}$$

7. In the shown figure, let  $m\overline{PA} = 8x - 7$ ,  $m\overline{PB} = 4x - 3$ ,  $m\overline{AQ} = 5x - 3$ ,  $m\overline{BR} = 3x - 1$ . Find the value of  $x$  if  $\overline{AB} \parallel \overline{QR}$ .



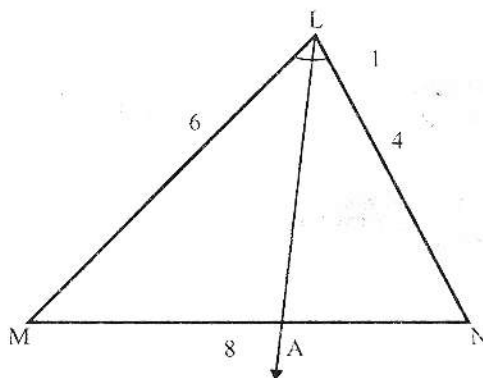
If  $\overline{AB} \parallel \overline{QR}$  then

$$\frac{m\overline{PA}}{m\overline{AQ}} = \frac{m\overline{PB}}{m\overline{BR}}$$

**Putting values**

$$\begin{aligned}\frac{8x - 7}{5x - 3} &= \frac{4x - 3}{3x - 1} \\ (8x - 7)(3x - 1) &= (5x - 3)(4x - 3) \\ 24x^2 - 8x - 21x + 7 &= 20x^2 - 15x - 12x + 9 \\ 24x^2 - 29x + 7 &= 20x^2 - 27x + 9 \\ 24x^2 - 20x^2 - 29x + 27x + 7 - 9 &= 0 \\ 4x^2 - 2x - 2 &= 0 \\ 2x^2 - x - 1 &= 0 \\ 2x^2 - 2x + x - 1 &= 0 \\ 2x(x - 1) + 1(x - 1) &= 0 \\ (2x + 1)(x - 1) &= 0 \\ 2x + 1 = 0 \text{ or } x - 1 = 0 \\ 2x = -1 \quad x = 1 \\ x &= \frac{-1}{2}\end{aligned}$$

8. In  $\triangle LMN$  shown in the figure  $\overline{LA}$  bisects  $\angle L$ . If  $m\overline{LN} = 4$ ,  $m\overline{LM} = 6$ ,  $m\overline{MN} = 8$ , then find  $m\overline{MA}$  and  $m\overline{AN}$ .



**Given:** In  $\triangle LMN$ ,  $\overline{LA}$  is angle bisector of  $\angle L$ .

$m\overline{LM} = 6\text{cm}$ ,  $m\overline{LN} = 4\text{cm}$ ,  $m\overline{MN} = 8\text{cm}$ .

**To Prove:**  $m\overline{MA} = ?$ ,  $m\overline{AN} = ?$

**Proof:**

Let  $m\overline{AN} = x\text{cm}$

$m\overline{MA} = 8 - x\text{cm}$

$$\frac{m\overline{MA}}{m\overline{AN}} = \frac{m\overline{LM}}{m\overline{LN}}$$

Putting values

$$\frac{8-x}{x} = \frac{6}{4}$$

$$4(8-x) = 6x$$

$$32 - 4x = 6x$$

$$32 = 6x + 4x$$

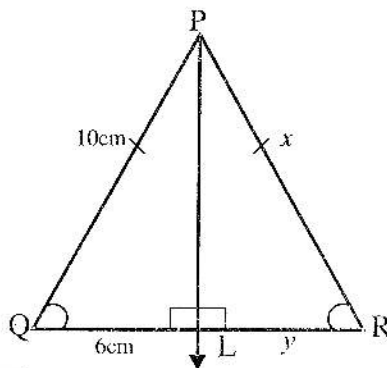
$$10x = 32$$

$$x = \frac{32}{10} = 3.2$$

$$\therefore m\overline{AN} = 3.2\text{cm}.$$

$$\begin{aligned} m\overline{MA} &= 8 - x \\ &= 8 - 3.2 \\ &= 4.8\text{cm}. \end{aligned}$$

9. In Isosceles  $\triangle PQR$  shown in the figure, find the value of  $x$  and  $y$ .



**Given:**

In  $\triangle PQR$ ,  $\overline{PQ} \cong \overline{PR}$  and  $\overline{PL} \perp \overline{QR}$ .

**To Prove:**  $x = ?$   $y = ?$

**Proof:**

In  $\triangle PRL$  and  $\triangle PQL$

$m\overline{PQ} = m\overline{PR} \dots (i)$  Isosceles triangle  
 $m\angle PLQ = m\angle PLR$  Each of right angle  
 $m\overline{PL} = m\overline{PL}$  Common  
 $\triangle PQL \cong \triangle PRL$  H.S.  $\cong$  H.S.

$$m\overline{QL} = m\overline{LR}$$

$$6 = y$$

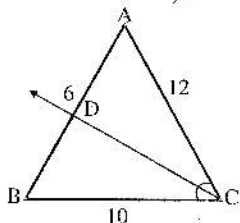
$$\Rightarrow y = 6\text{cm}.$$

$$\text{From (i) } x = 10\text{cm}.$$

## OBJECTIVE

1. In  $\triangle ABC$  as shown in figure,  $\overline{CD}$  bisects  $\angle C$  and meets  $\overline{AB}$  at D, a  $m\overline{BD}$  is equal to:

- (a) 5  
 (b) 16  
 (c) 10  
 (d) 18



2. In  $\triangle ABC$  shown in figure,  $\overline{CD}$  bisects  $\angle C$ , if  $m\overline{AC} = 3$ ,  $m\overline{CB} = 6$  and  $m\overline{AB} = 7$  then

- (i)  $\overline{AD} = \underline{\hspace{2cm}}$   
 (a)  $\frac{7}{3}$  (b)  $\frac{14}{3}$   
 (c)  $\frac{9}{2}$  (d)  $\frac{11}{2}$   
 (ii)  $m\overline{BD} = \underline{\hspace{2cm}}$   
 (a)  $\frac{7}{3}$  (b)  $\frac{14}{3}$   
 (c)  $\frac{15}{2}$  (d)  $\frac{11}{2}$

3. One and only one line can be drawn through \_\_\_\_ points:

- (a) Two (b) Three  
(c) Four (d) Five

4. The ratio between two alike quantities is defined as:

- (a)  $a : b$   
(b)  $b : a$   
(c)  $a : b = c : d$   
(d) None

5. If a line segment intersects the two sides of a triangle in the same ratio then it is parallel to the \_\_\_\_ side:

- (a) Third (b) Fourth  
(c) Second (d) None

6. Two triangles are said to be similar if these are equiangular and their corresponding sides are \_\_\_\_

- (a) Proportional  
(b) congruent

(c) concurrent

(d) None

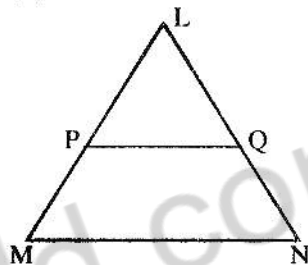
7. In  $\triangle LMN$  shown in the figure

$\overline{MN} \parallel \overline{PQ}$  if  $m\overline{LM} = 5\text{cm}$ ,

$m\overline{LP} = 2.5\text{cm}$ ,  $m\overline{LQ} = 2.3\text{cm}$  then

$m\overline{LN} = \underline{\hspace{1cm}}$ :

- (a) 4.6cm  
(b) 4.5cm  
(c) 3.5cm  
(d) 4.0



### ANSWER KEY

1.	a	2.	(i) a (ii) b	3.	a	4.	a	5.	a
6.	a	7.	a						