Unit 12

LINE BISECTORS AND ANGLE BISECTORS

Right Bisector of a Line Segment:

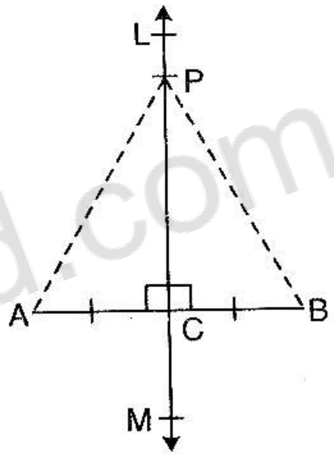
A line ℓ is called a right bisector of a line segment if ℓ is perpendicular to the line segment and passes through its mid-point.

Bisector of an Angle:

A ray BP is called the bisector of $\angle ABC$ if P is a point in the interior of the angle and $\angle ABP = \angle PBC$.

Theorem:

Any point on the right bisector of a line segment is equidistant from its end points.



Given:

A line LM intersects the line segment AB at the point C such that $\overleftrightarrow{LM} \perp \overline{AB}$ and $\overline{AC} \cong \overline{BC}$. P is a point on $\overleftrightarrow{LM}$.

To Prove: $\overline{PA} \cong \overline{PB}$

Construction:

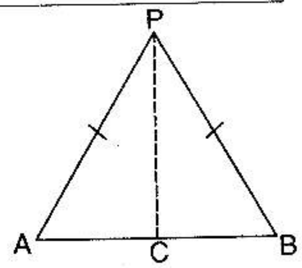
Join p to the points A and B.

Proof

Statements	Reasons
In $\Delta ACP \longleftrightarrow \Delta BCP$	
$\overline{AC} \cong \overline{BC}$	Given
$\angle ACP \cong \angle BCP$	given $\overline{PC} \perp \overline{AB}$, so that each $\angle$ at $C = 90^o$.
$\overline{PC} \cong \overline{PC}$	common
$\therefore$ $\Delta ACP \cong \Delta BCP$	S.A.S. postulate
Hence $\overline{PA} \cong \overline{PB}$	(corresponding sides of congruent triangles)

Theorem

Any point equidistant from the end points of a line segment is on the right bisector of it.



Given

$\overline{AB}$ is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$.

To Prove The Point P is on the right bisector of $\overline{AB}$.

Construction: Joint P to C, the midpoint of $\overline{AB}$.

Proof

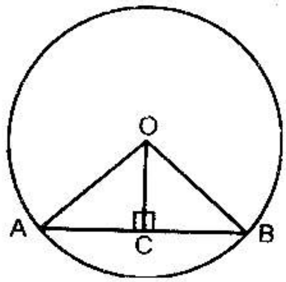
Statements		Reasons
In	$\Delta ACP \longleftrightarrow \Delta BCP$	
	$\overline{PA} \cong \overline{PB}$	Given
	$\overline{PC} \cong \overline{PC}$	Common
	$\overline{AC} \cong \overline{BC}$	Construction
	$\Delta ACP \cong \Delta BCP$	S.S.S $\cong$ S.S.S
	$\angle ACP \cong \angle BCP$ .....(i)	(corresponding angles of congruent triangles)
But	$m\angle ACP + m\angle BCP = 180^\circ$ .....(ii)	Supplementary angles
$\therefore$	$m\angle ACP = m\angle BCP = 90^\circ$	From (i) and (ii)
i.e.,	$\overline{PC} \perp \overline{AB}$ .....(iii)	$m\angle ACP = 90^\circ$ (proved)
Also	$\overline{CA} \cong \overline{CB}$ .....(iv)	
$\therefore$	$\overline{PC}$ is a right bisector of $\overline{AB}$.	construction
i.e.,	the point P is on the right bisector of $\overline{AB}$.	from (iii) and (iv)

Exercise 12.1

1. Prove that the centre of a circle is on the right bisectors of each of its chords.

Given Circle with centre O

To Prove Centre of the circle is on right bisectors of each of its chords



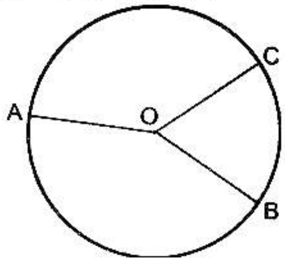
Construction

Draw any chord $\overline{AB}$. Draw $\overline{OC} \perp \overline{AB}$ join O with A and B.

Proof:

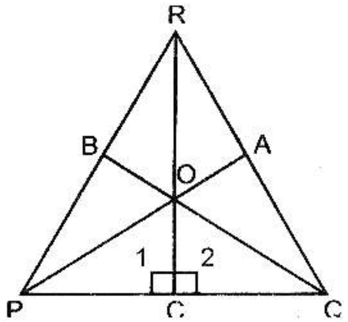
Statements	Reasons
In $\Delta OAC \leftrightarrow \Delta OBC$	
$\overline{OA} \cong \overline{OB}$	Radii of same circle
$\overline{OC} \cong \overline{OC}$	Common
$\angle ACO \cong \angle BCO$	Each of 90°
$\therefore \Delta ACO \cong \Delta BCO$	H.S $\cong$ H.S
$\therefore \overline{AC} \cong \overline{BC}$	Corresponding sides of the congruent triangles.
$\therefore \overline{OC}$ is the right bisector of $\overline{AB}$	

2. Where will be the centre of a circle passing through three non-collinear points and why?



Circle is the locus of a point which moves so that its distance from a fixed point O remains same. Otherwise no circle will be formed.

3. Three villages P, Q and R are not on the same line. The people of these villages want to make a Children Park at such a place which is equidistant from these three villages. After fixing the place, of Children park, prove that the Park is equidistant from the three villages.



Given

Three villages P, Q, R not on the same line.

To Prove

Park is equidistant from P, Q and R.

Construction

Complete the triangle PQR, draw the right bisectors of the sides $\overline{PQ}$ and $\overline{QR}$ cutting each other at O. Join O with P, Q and R. let O be the park.

Proof:

Statements	Reasons
In $\Delta OPC \leftrightarrow \Delta OQC$	
$\overline{CP} \cong \overline{CQ}$	Construction
$\overline{OC} \cong \overline{OC}$	Common
$\angle 1 \cong \angle 2$	Each of 90°
$\therefore$ $\Delta OCP \cong \Delta OCQ$	S.A.S $\cong$ S.A.S
$\therefore$ $\overline{OP} \cong \overline{OQ}$ .... (i)	Corresponding sides of congruent triangles
Similarly	
$\overline{OQ} \cong \overline{OR}$ .... (ii)	
$\therefore$ $\overline{OP} \cong \overline{OQ} \cong \overline{OR}$	

Theorem

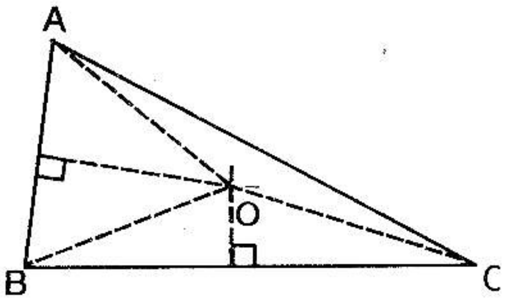
The right bisectors of the sides of a triangle are concurrent.

Given

ΔABC

To Prove

The right bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are concurrent.



Construction Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which meet each other at the point O. Join O to A, B and C.

Proof:

Statements	Reasons
$\overline{OA} \cong \overline{OB}$ ......(i)	(Each point on right bisector of a segment is equidistant from its end points)
$\overline{OB} \cong \overline{OC}$ ......(ii)	as in (i)
$\overline{OA} \cong \overline{OC}$ ......(iii)	From (i) and (ii)
$\therefore$ Point O is on the right bisector of $\overline{CA}$. ......(iv)	(O is equidistant from A and C)
But point O is on the right bisector of $\overline{AB}$ and of $\overline{BC}$. ......(v)	construction
Hence the right bisectors of the three sides of a triangle are concurrent at O.	{from (iv) and (v)}

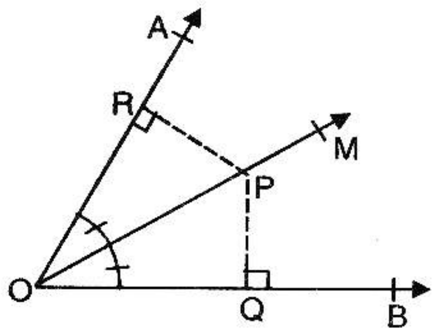
Note:

(a) The right bisectors of the sides of an acute triangle intersect each other inside the triangle.

(b) The right bisectors of the sides of a right triangle intersect each other on the hypotenuse.

(c) The right bisectors of the sides of an obtuse triangle intersect each other outside the triangle.

Theorem



Any point on the bisector of an angle is equidistant from its arms.

Given

A point P is on $\overrightarrow{OM}$, the bisectors of $\angle AOB$.

To Prove

$\overline{PQ} \cong \overline{PR}$ i.e., P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Construction

Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$.

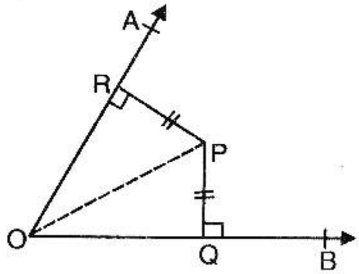
Proof:

Statements	Reasons
In $\Delta POQ \longleftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PQO \cong \angle PRO$	Construction
$\angle POQ \cong \angle POR$	Given
$\therefore$ $\Delta POQ \cong \Delta POR$	S.A.A. $\cong$ S.A.A.
Hence $\overline{PQ} \cong \overline{PR}$	(corresponding sides of congruent triangles)

Theorem

Any point inside an angle, equidistant from its arms, is on the bisector of it.

Given



Any point P lies inside $\angle AOB$ such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overrightarrow{OB}$ and $\overline{PR} \perp \overrightarrow{OA}$.

To Prove

Point P is on the bisector of $\angle AOB$.

Construction

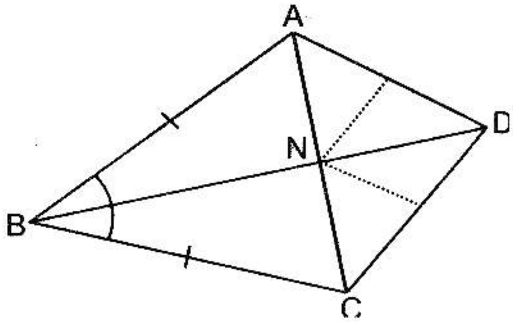
Join P to O.

Proof:

Statements	Reasons
In $\Delta POQ \longleftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$	Given (right angles)
$\overline{PO} \cong \overline{PO}$	Common
$\overline{PQ} \cong \overline{PR}$	Given
$\therefore$ $\Delta POQ \cong \Delta POR$	H.S. $\cong$ H.S.
Hence $\angle POQ \cong \angle POR$	(corresponding angles of congruent triangles)
i.e., P is on the bisector of $\angle AOB$.	

Exercise 12.2

1. In a quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}$, $\overline{CD}$ meet each other at point N. prove that $\overline{BN}$ is a bisector of $\angle ABC$.



Given Quadrilateral ABCD in which $\overline{AB} \cong \overline{BC}$. Right bisectors of $\overline{AD}$ and $\overline{CD}$ meet each other at point N.

To prove $\overline{BN}$ is a bisector of $\angle ABC$

Construction Join N with A, B, C, D

Proof:

Statements	Reasons
$\overline{NC} \cong \overline{ND}$ .... (i)	N is on the right bisector of $\overline{CD}$
$\overline{NA} \cong \overline{ND}$ .... (ii)	N is on the right bisector of $\overline{AD}$
$\overline{NA} \cong \overline{NC}$ .... (iii)	By (i) and (ii)
In $\Delta ABN \leftrightarrow \Delta CBN$	
$\overline{AB} \cong \overline{BC}$	Given
$\overline{BN} \cong \overline{BN}$	Common
$\overline{NA} \cong \overline{NC}$	Proved
$\therefore$ $\Delta ABN \cong \Delta CBN$	S.S.S $\cong$ S.S.S
$\angle ABN \cong \angle CBN$	Corresponding angles of congruent triangles.
$\therefore$ BN is a bisector of $\angle ABC$.	

2. The bisectors of $\angle A$, $\angle B$ and $\angle C$ of a quadrilateral ABCP meet each other at point O. Prove that the bisectors of $\angle P$ will also pass through the point O.

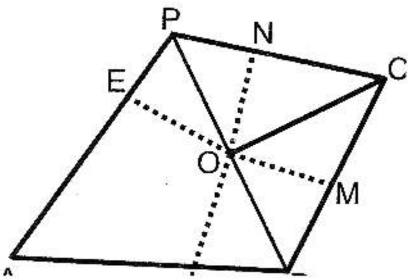
Given Bisector of the angles A, B, C meet at O.

To Prove

Bisector of $\angle P$ will also pass through O.

Construction

From O draw $\perp$ on the sides of quadrilateral BCP.

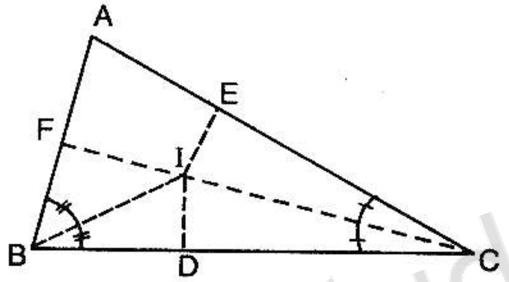


Proof:

Statements	Reasons
$\overline{OE} \cong \overline{OL}$ .... (i)	O is on the bisector of $\angle A$
$\overline{OL} \cong \overline{OM}$ .... (ii)	O is on the bisector of $\angle B$
$\overline{OM} \cong \overline{ON}$ .... (iii)	O is on the bisector of $\angle C$
$\therefore$ $\overline{OE} \cong \overline{ON}$	By (i) and (ii), (iii)
$\therefore$ O is on the bisector of $\angle P$.	$\overline{OE} \cong \overline{ON}$

Theorem

The bisectors of the angles of a triangle are concurrent.



Given

ΔABC

To Prove

The bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent.

Construction

Draw the bisectors of $\angle B$ and $\angle C$ which intersect at point I. From I, draw $\overline{IF} \perp \overline{AB}$, $\overline{ID} \perp \overline{BC}$ and $\overline{IE} \perp \overline{CA}$.

Proof:

Statements	Reasons
$\overline{ID} \cong \overline{IF}$	(Any point on bisector of an angle is equidistant from its arms)
Similarly, $\overline{ID} \cong \overline{IE}$	
$\therefore$ $\overline{IE} \cong \overline{IF}$	Each $\cong$ ID, proved.
So, the point I is on the bisector of $\angle A$ ....(i)	
Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$. ....(ii)	Construction
Thus the bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent at I.	{from (i) and (ii)}

Exercise

1. Which of the following are true and which are false?

(i) Bisection means to divide into two equal parts. **(True)**

(ii) Right bisection of line segment means to draw perpendicular which passes through the mid-point. **(True)**

(iii) Any point on the right bisector of a line segment is not equidistant from its end points. **(False)**

(iv) Any point equidistant from the end points of a line segment is on the right bisector of it. **(True)**

(v) The right bisectors of the sides of a triangle are not concurrent. **(False)**

(vi) The bisectors of the angles of a triangle are concurrent. **(True)**

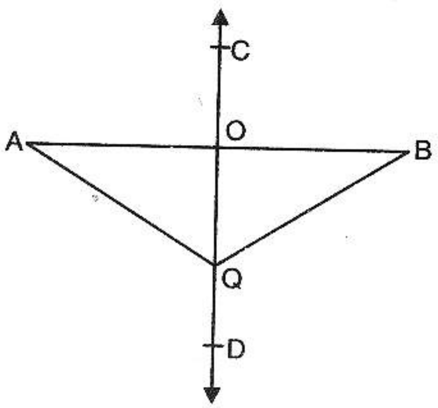
(vii) Any point on the bisector of an angle is not equidistant from its arms **(False)**

(viii) Any point inside an angle, equidistant from its arms, is on the bisector of it. **(True)**

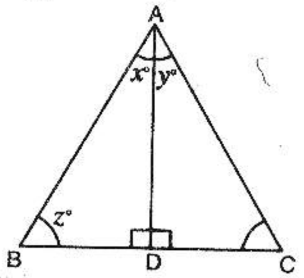
2. If $\overleftrightarrow{CD}$ is right bisector of line segment $\overline{AB}$, then:

(i) $m\overline{OA} = $ ....$m\overline{OB}$....

(ii) $m\overline{AQ} = $ ....$m\overline{BQ}$....



3. The given triangle ABC is equilateral triangle and $\overline{AD}$ is bisector of angle A, then find the values of unknowns x^o, y^o and z^o.



$\because$ ABC is an equilateral triangle.

Its each angle = 60^o

$\therefore$ $z = 60^o$

$x + y = 60^o$

But $y = x$

$x + x = 60^o$

$2x = 60^o$

$x = \frac{60^o}{2}$

$x = 30^o$

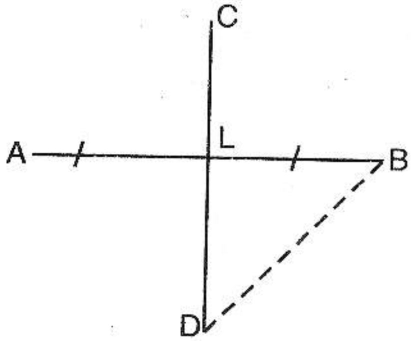
$\therefore$ $y = 30^o$

Hence $z = 60^o$

4. $\overline{CD}$ is right bisector of the line segment $\overline{AB}$.

(i) if $m\overline{AB} = 6\text{cm}$, then find the $m\overline{AL}$ and $m\overline{LB}$.

(ii) If $m\overline{BD} = 4\text{cm}$, then find $m\overline{AD}$.



Given CD is a right bisector on the line segment AB.

To find (i) $m\overline{AL}, m\overline{LB}$ when $m\overline{AB}=6cm$

(ii) $m\overline{AD}$ when $m\overline{BD}=4cm$

Construction Join B with D.

Proof:

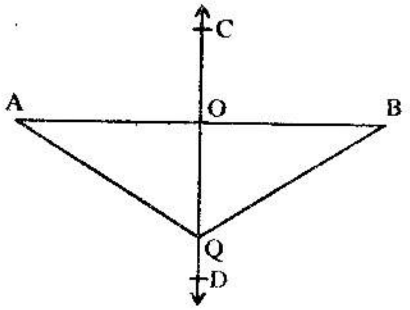
Statements	Reasons
(i) $m\overline{AL} = m\overline{LB}$	$\overline{CD}$ is a right bisector of $\overline{AB}$
$m\overline{AL} = \frac{1}{2} m\overline{AB}$	
$= \frac{1}{2}(6)$	$\because$ $m\overline{AB} = 6cm$
$= 3cm$	
$m\overline{LB} = m\overline{AL}$	
$= 3cm.$	
(ii) $m\overline{AD} = m\overline{BD}$	$\because$ $\overline{LD}$ is a right bisector of $\overline{AB}$
$\therefore$ $m\overline{AD} = 4cm$	$\because$ $m\overline{BD} = 4cm$

Objective

1. Bisection means to divide into ___ equal parts
   (a) Two (b) Three
   (c) Four (d) Five
2. ____ o f line segment means to draw perpendicular which passes through the mid-point of line segment.
   (a) Right bisection (b) Bisection
   (c) Congruent (d) mid-point
3. Any point on the _____ of a line segment is equidistant from its end points:
   (a) Right bisector (b) Angle bisector
   (c) Median (d) Altitude
4. Any point equidistant from the end points of line segment is on the ____ of it:
   (a) Right bisector (b) Angle bisector
   (c) Median (d) Altitude
5. The bisectors of the angles of a triangle are:
   (a) Concurrent (b) Congruent
   (c) Parallel (d) None
6. Bisection of an angle means to draw a ray to divide the given angle into ___ equal parts:
   (a) Four (b) Three
   (c) Two (d) Five
7. If $\overleftrightarrow{CD}$ is right bisector of line segment $\overline{AB}$ then: (i) $m\overline{OA} =$

(a) $m\overline{OQ}$ (b) $m\overline{OB}$

(c) $m\overline{AQ}$ (d) $m\overline{BQ}$



8. If $\overleftrightarrow{CD}$ is right bisector of line segment $\overline{AB}$, then $m\overline{AQ}$ =____

(a) $m\overline{OA}$ (b) $m\overline{OB}$

(c) $m\overline{BQ}$ (d) $m\overline{OD}$

9. The right bisector s of the sides of an acute triangle intersects each other ____ the triangle.

(a) Inside (b) Outside

(c) Midpoint (d) None

10. The right bisectors of the sides of a right triangle intersect each other on the ____

(a) Vertex (b) Midpoint

(c) Hypotenuse (d) None

11. The right bisectors of the sides of an obtuse triangle intersect each other ____ the triangle.

(a) Outside (b) Inside

(c) Midpoint (d) None

ANSWER KEY

1.	a	**2.**	a	**3.**	a	**4.**	a	**5.**	a
6.	c	**7.**	b	**8.**	c	**9.**	a	**10.**	c
11.	a								