Probability and Statistics
Assignment - 3 (Solution)

(1)

X	0	1	2	3	4	5	6	7	8	9
P(X)	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

$$E(X) = \sum x_i \, p(x_i)$$
$$= \frac{1}{10}\left[1+2+3+4+\cdots+9\right]$$
$$= \frac{45}{10} = 4.5$$

$$E(Y) = E\left(\frac{9X}{2}\right) = \frac{9}{2}E(X) = \frac{9}{2}\times 4.5 = 20.25$$

$$E(X^2) = \sum x_i^2 p(x_i)$$
$$= \frac{1}{10}\left[1^2+2^2+\cdots+9^2\right]$$
$$= \frac{1}{10}\cdot[285] = 28.5$$

$$E(Y^2) = E\left(\frac{81}{4}X^2\right) = \frac{81}{4}E(X^2) = 577.125$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = 577.125 - (20.25)^2$$
$$= 167.0625$$

$$S.d = \sqrt{167.0625} = 12.92526$$

(2)

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$E(Y) = \int_{-\infty}^{\infty} y f(x)\, dx$$
$$= \int_0^1 -2\log x \, dx = -2\left(x\log x - x\right)_0^1$$
$$= 2$$

Median$(X) = \frac{1}{2}$ $\left(\frac{1}{2}\text{ is median of 0 to 1}\right)$

Median$(Y) = -2\log(\text{Median}\,X) = -2\log\left(\frac{1}{2}\right) = 2\log 2$

(3) Using Binomial distribution, we get

$P(1\text{ defective}) = {}^{9}C_1 (0.95)^8 (0.05)$

(4)

x	10	11	12	13	14	15
$f(x)$	0.08	0.15	0.30	0.20	0.20	0.07

$$E(x) = \sum x_i f(x_i)$$
$$= 10\times0.08 + 11\times0.15 + 12\times0.30 + 13\times0.20 + 14\times0.20 + 15\times0.07$$
$$= 12.5$$
$$E(x^2) = \sum x_i^2 f(x_i)$$
$$= 100\times0.08 + 121\times0.15 + \ldots + 225\times0.07$$
$$= 158.1$$
$$Var(X) = 158.1 - (12.5)^2 = 158.1 - 156.25$$
$$= 1.86$$
$$S.d = \sqrt{1.86} = 1.36$$

(5) If $P(X=122)$ = 122 passengers showed up

All got seat that means 120 or less passengers showed up.

$$P(X \le 120) = 1 - P(X=121) - P(X=122)$$
$$= 1 - {}^{122}C_1 (0.9)^{121}(0.1) - {}^{122}C_0 (0.9)^{122}$$
$$= 1 - 0.00003545 - 0.0000026$$
$$1 - 0.00003805 = 0.9999$$

(6) Probability of car being defective is $p = \frac{50}{1000} = 0.05$

In binomial distribution

Mean $= np = 20 \times 0.05 = 1$

Variance $= np(1-p) = 20 \times 0.05 \times 0.95 = 0.95$

(7) $f(x) = ke^{-\frac{1}{2}|x-2|}$

As total of probability distribution should be one only

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$|x-2| = \begin{cases} 2-x & x<2 \\ x-2 & x \geq 2 \end{cases}$$

$$e^{-\infty} \to 0$$

$$\int_{-\infty}^{\infty} ke^{-\frac{1}{2}|x-2|}\,dx = 1$$

$$\int_{-\infty}^{2} ke^{-\frac{1}{2}(2-x)}\,dx + \int_{2}^{\infty} ke^{-\frac{1}{2}(x-2)}\,dx = 1$$

$$\left.\frac{k\,e^{-\frac{1}{2}(2-x)}}{\frac{1}{2}}\right|_{-\infty}^{2} + \left.\frac{k\,e^{-\frac{1}{2}(x-2)}}{-\frac{1}{2}}\right|_{2}^{\infty} = 1$$

$$2K[1-0] - 2K[0-1] = 1$$

$$4K = 1$$

$$\boxed{K = \frac{1}{4}}$$

(8) Question is wrong they have asked $P(X \geq 4)$ and finding out $P(X=4)$

For $P(X \geq 4) = 1 - P(X=2) - P(X=3)$

$= 1 - (0.15)^2 - {}^{2}C_1 (0.85)(0.15)^2$

For $P(X=4) = {}^{4}C_2 (0.85)^2 (0.15)^2 - {}^{3}C_2 (0.85)^2 (0.15)^2$

Because genes can not be detected in first three tests.

(9) Probability for 3 showing up $= P(X=3) = \frac{1}{6}$

— — — — 5, 6 showing up $: P(X \geq 5) = \frac{2}{6}$

Before 3, 5 and 6 should not show $P(X=1,2,4) = \frac{3}{6} = \frac{1}{2}$

Prob. of ~~3~~ 3 showed up at first attempt $= \frac{1}{6}$

— 3 — second attempt and 5; 6 not showing on fist attempt $= \frac{1}{2} \times \frac{1}{6}$

— 3 — third attempt $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}$

— fourth — $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}$

So Probability required $= \frac{1}{6} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{6} + \cdots$

$$= \frac{1}{6}\left[1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 - \cdots \infty\right]$$

$$= \frac{1}{6}\left[\frac{1}{1-\frac{1}{2}}\right] = \frac{1}{6} \times 2 = \frac{1}{3}$$

$\boxed{\text{Sum of GP} = \frac{a}{1-r}}$

(10) It is question of Bernoulli distribution

number of calls needed for r successes $\frac{r}{p}$

— 1 success $= \frac{1}{p}$

$= \frac{1}{0.03} = \frac{100}{3}$