① Let X_1, X_2, X_3 and X_4 be independent r.v. with respective means $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. Then $Y = \min(X_1, X_2, X_3, X_4)$ has an exponential distribution with the mean equal to ?

Let $X_1, X_2, \dots X_n$ be independent r.v. with X_i having exponential(λ_i) distribution.

Then distribution of $\min(x_1, x_2 \dots x_n)$ is exponential $(\lambda_1 + \lambda_2 + \dots + \lambda_n)$.

Thus mean of $\min(x_1, x_2, x_3, x_4) = \dfrac{1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}$

$\lambda_1 = \frac{1}{E(x_1)} = \frac{1}{1} = 1$, $\lambda_2 = \frac{1}{E[x_2]} = \frac{1}{1/2} = 2$, $\lambda_3 = 3$, $\lambda_4 = 4$

$\therefore$ mean of $(x_1, x_2, x_3, x_4) = \dfrac{1}{1+2+3+4} = \dfrac{1}{10} = 0.1$

② Test results from an electronic circuit board indicates that 50% of board failures are caused by assembly defect, 40% are due to electrical component and 10% are due to mechanical defects. Suppose that 10 boards fails independently. Let the r.v. X, Y and Z denote the number of assembly, electrical and mechanical defects, among the 10 boards. Calculate $P(X=5, Y=3, Z=2)$.

<u>Ans:</u> The r.v.s X, Y and Z have multinomial distribution with $n=10$, $P_1 = 0.5$, $P_2 = 0.4$, $P_3 = 0.1$

$\therefore P(X=5, Y=3, Z=2) = \dfrac{10!}{5!\,4!\,1!} \times (0.5)^5 (0.4)^3 \cdot (0.1)^2$

$= 0.0252$

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(3) Soft drink cans are filled by an automated filling machine. Assume that the fill volumes of the cans are independent, normal random variables with the ~~mean~~ standard deviation 15 mL. Suppose the probability that the mean of a sample of 100 cans is below 350 mL is 0.005. What should be the mean fill volume be equal to?

Ans $\sigma = 15$, $n = 100$ → sample size.

Let $\bar{X}$ = The mean fill volume of sample of 100 cans,

∴ Let μ be the mean.

The standard deviation is $\frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{100}} = 1.5$ mL.

$$\boxed{z = \frac{\bar{x} - \mu}{1.5}}$$

Given $P(\bar{X} < 350) = 0.005$

$$\Rightarrow P\left(Z < \frac{350 - \mu}{1.5}\right) = 0.005$$

From Normal distribution table, we have $Z = -2.58$

$$\therefore -2.58 = \frac{350 - \mu}{1.5}$$

$$\Rightarrow \mu = 350 + 2.58\,(1.5)$$

$$= 353.87 \rightarrow \boxed{353.8637}\ \boxed{B}$$

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Que: (4) A plastic casting for a magnetic disk is composed of two halves. The thickness of each half is normally distributed with a mean of 2 millimeters and standard deviation of 0.1 millimeter and the two halves are independent. What is the probability that the total thickness exceed 4.3 millimeters?

Ans: $T = X + Y$, T: Total Thickness
X, Y are thickness of two halves.

Given $E(X) = E(Y) = 2$

and $\sigma_x = \sigma_y = 0.1$

$\therefore$ Variance of X = variance of $Y = (0.1)^2$

we need to find $P(T > 4.3)$

Now $E(T) = E(X+Y) = E(X) + E(Y) = 4$

$V(T) = V(X+Y) = V(X) + V(Y) = (0.1)^2 + (0.1)^2$
$= 0.02$

$\therefore \sigma_T = \sqrt{V(T)} = 0.141 = \sqrt{0.02}$

$\therefore P(T > 4.3) = P\left[\frac{X+Y-E(X+Y)}{\sqrt{V(X+Y)}} > \frac{4.3 - E(T)}{\sqrt{V(T)}}\right]$

$= P\left[z > \frac{4.3-4}{\sqrt{0.02}}\right]$

$= 1 - P[z < 2.12132]$

$= 1.06195$

(5) The joint density function of r.v X and Y is given by

$$f(x,y) = \begin{cases} x+y & 0<x<1, \ 0<y<1 \\ 0 & \text{otherwise.} \end{cases}$$

$\boxed{\frac{1}{6}}$.

Let $U = X - Y$, find $\text{var}(U)$.

Ans: (B) 1/6

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6. Let $X_1, X_2, \dots X_n$ be n random variables, then

$$V\left[\sum_{i=1}^{n} a_i X_i\right] = \sum_{i=1}^{n} a_i^2 V(X_i) + 2\sum_{\substack{i=1 \\ i<j}}^{n}\sum_{j=1}^{n} a_i a_j \operatorname{cov}(X_i X_j)$$

<u>Ex</u>: Let $X_1, X_2 \dots X_n$ be r.v. such that the variance of each variable as 1 and co-relation between each pair of different variable is $\frac{1}{4}$. Then $\operatorname{Var}(X_1 + X_2 + \dots + X_n) = ?$

<u>Ans</u>: $$V[X_1 + X_2 + \dots + X_n] = V(X_1) + V(X_2) + \dots + V(X_n)$$
$$+ 2\Big[\operatorname{cov}(X_1 X_2) + \operatorname{cov}(X_1 X_3) + \dots + \operatorname{cov}(X_1 X_n)$$
$$+ \operatorname{cov}(X_2 X_3) + \dots \operatorname{cov}(X_2 X_n)$$
$$+ \dots$$
$$+ \operatorname{cov}(X_{n-1} X_n)\Big]$$

$$= n + 2\left[\frac{n-1}{4} + \frac{n-2}{4} + \frac{n-3}{4} + \dots + \frac{1}{4}\right]$$

$$= n + \frac{2}{4}\left[1 + 2 + 3 + \dots + (n-1)\right]$$

$$= n + \frac{1}{2}\,\frac{(n-1)\,n}{2} = n + \frac{n(n-1)}{4} = \cancel{\frac{4n+n^2}{4}}$$

$$= n\left[1 + \frac{n-1}{4}\right] = n\left[\frac{4+n-1}{4}\right] = \frac{n(n+3)}{4} \quad \underline{(\text{Ans})}$$

Dr. Satyaban Panigrahi

⑨ Suppose $X_1, X_2 \cdots X_{10}$ are i.i.d random variables following an exponential distribution with the mean 9. Find $P(Y>7 \mid Y>4)$, where $Y = \min\{X_1, X_2 \cdots X_{10}\}$?

Ans Mean of $X_i = 9 = \frac{1}{\lambda_i}$

$$\Rightarrow \lambda_i = \frac{1}{9}$$

Hence for $Y = \min\{X_1, X_2 \cdot X_{10}\}$,

$$\lambda = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \cdots + \frac{1}{\lambda_{10}} = \frac{10}{9}$$

Now $P(Y>7 \mid Y>4) = P(Y>3)$

$$= e^{-\lambda \cdot 3} = e^{-\frac{10}{9}\times 3}$$

$$= e^{-10/3} \approx 0.035673$$

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