ASSIGNMENT - 5

Q.1 Sol :- we know that by the theorem" If $X \sim N(\mu, \sigma^2)$ and $Y = aX + b$, $a \neq 0$ then $Y \sim N(a\mu + b, b^2\sigma^2)$ (Linearity property of normal dist.)

$$Z = \frac{X-\mu}{\sigma} = \frac{1}{\sigma} X - \frac{\mu}{\sigma} \sim N(0,1)$$

$\boxed{X \sim N(12, 0.01)}$ $\boxed{Y = 20X + 3}$ $\boxed{\text{Here } a = 20,\ b = 3,\ \mu = 12,\ \sigma^2 = 0.01,\ \sigma = 0.1}$

by Comparison

$$Y \sim N(a\mu + b, b^2\sigma^2) = N(20 \times 12 + 3, \cancel{9 \times 0.01}\ 4)$$

$$Y \sim N(243, 4)$$

mean $= \mu$ $\quad \sigma^2 = 4$, $\sigma = 2$

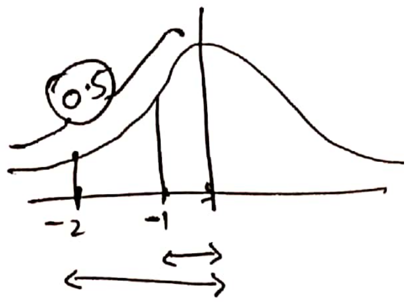
$P(239 \leq Y \leq 241)$.

put $Y = 239$, $\mu = 243$, $\sigma = 2$

$$Z = \frac{\cancel{X}-\mu}{\sigma} = \frac{239 - 243}{2} = \frac{-4}{2} = -2.$$

put $Y = 241$

$$Z = \frac{Y-\mu}{\sigma} = \frac{241 - 243}{2} = \frac{-2}{2} = -1$$



$P(239 \leq Y \leq 241) = P(-2 \leq Z \leq -1)$

Symmetrical so See Normal table In Ist Column 2.0

$= \cancel{PC}$

$= P(0 \leq Z \leq 2) - P(0 \leq Z \leq 1)$ $\to$ 1.0

$= \cancel{0.1359}$

$= 0.4772 - 0.3413$

$= 0.1359$ Ans

Q.2 $\quad X \sim B(n, p)$, Given $5\% = \frac{5}{100} = 0.05 = p$.

$B(n, p) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$n = 100$, ~~p~~

$P(\text{At least one}) = P(X \geq 1) = P(X=1) + P(X=2) \cdots P(X=100)$

or $\quad P(X \geq 1) = 1 - P(X < 1)$

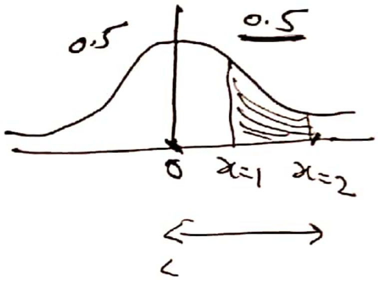
$= 1 - P(X = 0)$

$= 1 - {}^{n}C_{0}\, p^{0} q^{n-0} \rightarrow (q = 1-p)$

$= 1 - {}^{100}C_{0}\, (0.05)^{0} (1 - 0.05)^{100-0}$

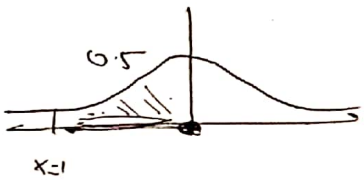
$$\boxed{P(X \geq 1) = 0.9941}$$

Q.3 $\quad [x] = \begin{cases} 1 & \text{if } 1 \leq X < 2 \\ -1 & \text{if } -1 \leq X < 0 \end{cases}$



$P(|[x]| = 1) = \phi(2) - 0.5 - ($

$P(X < 0, |[x]| = 1) = P(-1 \leq X < 0)$



$= \phi(1) - 0.5$

$(X < 0, |[x]| = 1) = \dfrac{\phi(1) - 0.5}{\phi(2) - 0.5}$ $\quad$ Ans (A)

Q.4 Poisson Distribution.

x = No. of asthma incidents in a Month.

$X \sim \text{Poisson}$

$\lambda = np = 550$, $X \sim N(\mu, \sigma^2) = X \sim N(500, 500)$

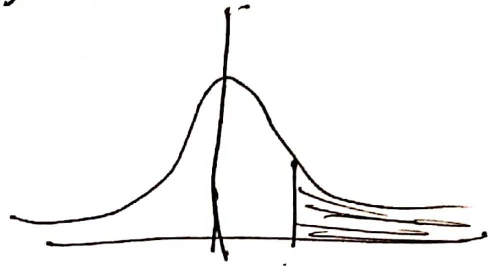
$\mu = 500, \ \sigma^2 = 500 \Rightarrow \sigma = \sqrt{500}$

$P(X \geq 550) = P(X \geq 550.5)$

$$P(X \geq 550.5) = P\left(\frac{X-\mu}{\sigma} \geq \frac{550.5-500}{\sqrt{500}}\right)$$

$$= P\left(Z \geq \frac{50.5}{\sqrt{500}}\right)$$

$$= P(Z \geq 2.2584)$$



$$= 1 - \phi(2.2584)$$

$z=2$

$$= 1 - 0.98804$$

$$= 0.01196 \text{ Ans}$$

2.25 → See Normal Table.

Column ---- 0.05

2.2 ———— 0.98804

Q.5 Mean $= \mu = 260$, $\sigma = 50$

$Q_1 \rightarrow$ Ist quartile.

$$\boxed{P(X \leq Q_1) = 0.25} = P\left(\frac{X-\mu}{\sigma}\right)$$

$$\Rightarrow P\left(\frac{X-260}{50}\right) \leq P\left(\frac{Q_1-260}{50}\right) = 0.25$$

Quartile Divides the data Four equal Parts.

I II III IV

Q_1 Q_2 Q_3

$$\Rightarrow \phi\left(\frac{Q_1-260}{50}\right) = (0.25)$$

$$\Rightarrow Q_1 = 260 + 50 \times (0.6744898)$$

$$= 226.2755 \text{ Ans}$$

(6) Let $\cancel{t}\, Y = \min\{Z, 0\}$

$$= \begin{cases} 0 & \text{if } Z > 0 \\ Z & \text{if } Z \leq 0 \end{cases}$$

so $E(Y) = E(Z)$ if $Z \leq 0$

$$= \int_{-\infty}^{0} z \cdot \frac{e^{-z^2/2}}{\sqrt{2\pi}}\, dz = \frac{-1}{\sqrt{2\pi}}$$

(7) $P(\cancel{X} \geq 110 + k) = 0.2033$

$$P\left(\frac{X-\mu}{\sigma} \geq \frac{110+k-110}{15}\right) = 0.2033$$

$$\Rightarrow P\left(Z \geq \frac{k}{15}\right) = 0.2033$$

But by Z table $P(Z > 0.83) = 0.2033$

$$\Rightarrow \frac{k}{15} = 0.83 \Rightarrow k = 12.45$$

(8) $P(0.0035 < X < 0.0065)$

$$= P\left(\frac{0.0035-0.005}{0.001} < \frac{\cancel{X}-\mu}{\sigma} < \frac{0.0015-0.005}{0.001}\right)$$

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$= P(-1.5 < Z < 1.5) = 0.86638$

(9) let $Y =$ life of semiconductor

$E(Y) =$ mean of log normal

$= e^{\mu + \sigma^2/2} = 10$

$Var(Y) = (e^{\sigma^2} - 1)\, e^{2\mu + \sigma^2} = 2.25$

$\Rightarrow \mu = 2.29146, \quad \sigma^2 = 0.02225$

$P(Y > 15) = P\left(\frac{\log Y - \mu}{\sigma} > \frac{\log 15 - 2.29146}{\sqrt{0.02225}}\right)$

$= P(Z > 2.7928) = 0.002113$

by z table

(10) $X =$ life of disc

$P(X \le 2) = 1 - e^{-2k}$ by defn of Weibull distn

$= 0.708$

$\Rightarrow e^{-2k} = 0.708 \Rightarrow k = 0.3$ (app.)