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# Linear Algebra

# Assignment 2

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## Q. What is a matrix determinant?

→ The determinant is a special number that can be calculated from a square matrix using special method.

→ It is also a function of entries of a square matrix.

There are 10 basic properties of determinants:

### 1. Reflection Property:

The determinant remains unchanged if it rows are changed into columns and vice versa.

$$\boxed{\det A^T = \det A}$$

↓↓↓

## 2. All zero property:

Incase, all elements of row or columns are zero then determinant will be zero.

**Example:** Let $A = \begin{vmatrix} 1 & 2 & 3 \\ \cancel{0} & \cancel{0} & \cancel{0} \\ 4 & 5 & 6 \end{vmatrix}$

$\det A = 1(6(0)-5(0)) - 2(6(0)-4(0)) + 3(5(0)-4(0))$

$\therefore \det.A = 0$

## 3. Repitition Property

If the elements of a row (or column) are identical to the elements of some other row (or column), then the determinant is zero.

## 4. Switching Property

The interchange of any two rows (or columns) of an determint of A will change its sign.

$$\boxed{\det B = -\det A}$$

## 5. Scalar Multiple Property

If all the elements of a row (or column) of a determinant are multiplied by a non-zero constant, then the determinant gets multiplied by the same constant.

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here first row of A will be multiplied by (2).

Example: Let $A = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$

$\det(A) = 14$, Now $\begin{vmatrix} 2(2) & 2(2) & 2(2) \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{vmatrix}$

$\det(A) = 14(2) = 28$

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## 6. Sum Property

$$\begin{vmatrix} a_1+b_1 & c_1 & d_1 \\ a_2+b_2 & c_2 & d_2 \\ a_3+b_3 & c_3 & d_3 \end{vmatrix} = \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$$

→ The sum of the product of the elements of any row (or column) with the cofactors of the corresponding elements is zero.

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## 7. Property of Invariance

The determinant remains un-changed under an operation of the form:

$$C_i \rightarrow C_i + \alpha C_j + \beta C_K, \text{ where } j, K \neq i$$

or $$R_i \rightarrow R_i + \alpha R_j + \beta R_K, \; j, K \neq i.$$

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**Example:** $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+\alpha b_1+\beta c_1 & b_1 & c_1 \\ a_2+\alpha b_2+\beta c_2 & b_2 & c_2 \\ a_3+\alpha b_3+\beta c_3 & b_3 & c_3 \end{vmatrix}$

## 8. Factor Property

If a determinant is a polynomial in $x$, then $(x-\alpha)$ is a factor of the determinant if its value is zero when we put $x=\alpha$.

## 9. Triangle Property

If all the elements of a determinant above or below the main diagonal consists of zeros, then the determinant is equal to the product of diagonal elements.

**Example:** $\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = \boxed{a_1 b_2 c_3}$

## 10. Determinant of Cofactor Matrix:

$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ then $\Delta_1 = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$

where $C_{ij}$ denotes the cofactor of the element $a_{ij}$ in $\Delta$.

## 11. Determinant of Inverse

Let A be a $n \times n$ matrix, then A is inversible iff $\det(A) \neq 0$

**Example:** let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow |A| = (4)(1) - (3)(2)$$

$|A| = -2 \neq 0$ , A is inversible

## 12. Multiplicative Property.

$$\det(AB) = \det(A) . \det(B)$$

**Example:**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} , \quad B = \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3+4 & 4+4 \\ 9+8 & 12+8 \end{bmatrix}$$

$$(AB) = \begin{bmatrix} 7 & 8 \\ 17 & 20 \end{bmatrix}$$

$$\det(AB) = (20)(7) - (8)(17) = \boxed{4}$$

$$\boxed{|AB| = 4}$$

$$|A| = (4)(1) - (3)(2) = \boxed{-2}$$

$$|B| = (3)(2) - (2)(4) = \boxed{-2}$$

$$|AB| = |A| . |B|$$ proved.

# Examples

**1. Example of Reflection Property : $(\det A^T = \det A)$**

let $A = \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}$ , $A^T = \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}$

Now $|A| = (2)(3) - (5)(4)$ , $|A^T| = (2)(3) - (5)(4)$

$|A| = 6 - 20 = \boxed{-14}$ $\quad |A^T| = \boxed{-14}$

**2. Example of Repitition property :**

let $A = \begin{bmatrix} 4 & 2 & 1 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$|A| = 4[(1)(1) - (2)(0)] - 2[(3)(1) - (4)(0)] + 1[(2)(3) - (4)(1)]$

$|A| = 4 - 6 + 2 = \boxed{0}$

**3. Example of Switching property $(\det B = -\det A)$**

let

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ , $B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

rows of B are same as of A but **switched**.

$|A| = (4)(1) - (3)(2) = \boxed{-2}$

$|B| = (3)(2) - (4)(1) = \boxed{2}$

$|B| = -|A|$

$2 = -(-2)$ proved.