MATH 310 - Introduction to Probability and Statistics

Practice Problems on Counting

Questions

Question 1

The IDs of the students in Habib University contains 2 letters followed by 5 digits. What are the total number of IDs possible if:

i) Repetition is allowed for both letters and digits.

ii) Repetition of letters is prohibited

iii) Repetition of digits is prohibited

iv) Repetition of both the letters and digits are prohibited.

Question 2

A tetrahedral (four-sided) die is rolled 4 times. What is the probability that all the rolls give an even number?

Question 3

A certain final exam is marked out of 50 in such a way that the obtained marks for every student is in integers. Find the probability that every student got different marks as compared to the others if:

i) There are 25 students in the class

ii) There are n students in the class (where $1 \leq n \leq 51$)

Question 4

A Book Shelf contains 2 books on Mathematics, 4 on Computer Science, 3 on Physics and 5 on Chemistry. Three Books are drawn randomly from the shelf. Find the probability that the books drawn is neither on Chemistry, nor on Physics.

Question 5

Five cards are drawn randomly from a standard 52-card deck. What is the probability that all the 5 cards are from the same suit?
(**Recall**: There are 4 suits each consisting of 13 cards)

Question 6

A coin is flipped 16 times. What is the probability that we get at least 5 tails?

Question 7

In many programming languages (such as python, C++, etc.), the name of a variable can contain numbers, letters and underscores and the name cannot start with a number. In how many ways a variable of 5 characters can be named?

Question 8

Eight cards are drawn from a standard 52-card deck. What is the probability that it contains at most two kings?

Question 9

Five people choose a number from 0 to 9. Find the probability that the number chosen by a person is one more than the number that was chosen by the person before him.

Question 10

A 52-card deck is dealt fairly to 4 players. Find the probability that each player gets an ace, king and queen.

Answer Key

Question No.	Answer
1(i)	6.76×10^7
1(ii)	6.5×10^7
1(iii)	2.0442240×10^7
1(iv)	1.9656×10^7
2	$\frac{1}{16}$
3(i)	$\frac{51!}{26! \times 51^{25}}$
3(ii)	$\frac{51!}{(51-n)!} \frac{1}{51^n}$
4	$\frac{5}{91}$
5	$\frac{33}{16660}$
6	0.9616
7	5.0602347×10^7
8	0.99064
9	$\frac{3}{50000}$
10	0.001213

Solutions

Solution for Question 1

Part (i)

Both the letters can be chosen in 26 ways. Similarly, all the 5 digits can be chosen in 10 ways. Thus the total number of ids possible are:

$$\boxed{Total\ no.\ of\ IDs = 26 \times 26 \times 10 \times 10 \times 10 \times 10 \times 10 = 6.76 \times 10^7}$$

Part (ii)

First letter can be selected in 26 ways and second in 25 ways. All the 5 digits can be chosen in 10 ways. Thus the total number of ids possible are:

$$\boxed{Total\ no.\ of\ IDs = 26 \times 25 \times 10 \times 10 \times 10 \times 10 \times 10 = 6.5 \times 10^7}$$

Part (iii)

Both the letters can be chosen in 26 ways. The First digit can be selected in 10 ways. Similarly the second, third, fourth and fifth digits can be selected in 9, 8, 7 and 6 ways. Thus the total number of ids possible are:

$$\boxed{Total\ no.\ of\ IDs = 26 \times 25 \times 10 \times 9 \times 8 \times 7 \times 6 = 2.0442240 \times 10^7}$$

Part (iv)

First letter can be selected in 26 ways and second in 25 ways. The First digit can be selected in 10 ways. Similarly the second, third, fourth and fifth digits can be selected in 9, 8, 7 and 6 ways. Thus the total number of ids possible are:

$$\boxed{Total\ no.\ of\ IDs = 26 \times 25 \times 10 \times 9 \times 8 \times 7 \times 6 = 1.9656 \times 10^7}$$

Solution for Question 2

The total number of outcomes possible are $4 \times 4 \times 4 \times 4 = 256$ ways.
Two even numbers $2, 4$ can be obtained by rolling a tetrahedral die once. Thus, by rolling the die four times, we can get an even number in $2 \times 2 \times 2 \times 2 = 16$ ways. So the Probability is:

$$P = \tfrac{16}{256}$$

$$\boxed{P = \frac{1}{16}}$$

Solution for Question 3

Part (i)

A student can obtain marks in 51 ways (from 0 to 51). Since there are 25 students, therefore the total number of ways in which marks can be obtained is,

$$51 \times 51 \times 51 \times \cdots \times 51 \;\; (25 \;\; times) = 51^{25}$$

Now for marks of the students to be different from each other, first student can have marks in 51 ways (from 0 to 50). Second student can have marks in 50 ways (since his/her marks cannot match the marks of first student). Similarly, third student can obtain marks in 49 ways and so on. For 25, students, the total number of ways in which marks can be obtained (such that each student get a different score) are:

$$51 \times 50 \times 49 \times 48 \times \cdots \times 27 = \tfrac{51!}{26!}$$

Thus, the probability is,

$$P = \tfrac{\frac{51!}{26!}}{51^{25}}$$

$$\boxed{P = \frac{51!}{26! \times 51^{25}}}$$

Part (ii)

A student can obtain marks in 51 ways (from 0 to 51). Since there are n students, therefore the total number of ways in which marks can be obtained is,

$$51 \times 51 \times 51 \times \cdots \times 51 \;\; (n \;\; times) = 51^{n}$$

Now for marks of the students to be different from each other, first student can have marks in 51 ways (from 0 to 50). Second student can have marks in 50 ways (since his/her marks cannot match the marks of first student). Similarly, third student can obtain marks in 49 ways and so on. For n, students, the total number of ways in which marks can be obtained (such that each student get a different score) are:

$$51 \times 50 \times 49 \times 48 \times \cdots \times (51 - n + 1) = \tfrac{51!}{(51-n)!}$$

Thus, the probability is,

$$P = \tfrac{\frac{51!}{(51-n)!}}{51^{n}}$$

$$\boxed{P = \frac{51!}{(51-n)!}\frac{1}{51^{n}}}$$

Solution for Question 4

Since, the books cannot be from Chemistry or Physics, therefore, we can only choose books from Computer Science and Mathematics which are 6 in total. Therefore 3 books can be chosen in $6C3 = 20$ ways.
There are 14 books in total. So, the total Number of ways in which three books can be chosen are $14C3 = 364$ ways. Therefore, the probability is,

$$P = \frac{20}{364}$$

$$\boxed{P = \frac{5}{91}}$$

Solution for Question 5

The first card can be drawn independently. So, it can be chosen in 52 ways. Now that the first card is chosen, the next cards should be from the same suit as the first card. So, the next card can be drawn in 12 ways (each suit contains 13 cards and one of the cards from the suit is already drawn). Similarly, the third, fourth and fifth cards can be drawn in 11, 10 and 9 ways respectively. Therefore, the total number of ways in which 5 cards can be drawn such that they belong to same suit is $52 \times 12 \times 11 \times 10 \times 9 = 617760$ ways.
The total number of ways in which 5 cards can be drawn is $52 \times 51 \times 50 \times 49 \times 48 = 3.118752 \times 10^8$ ways.
Hence, the probability is,

$$P = \frac{617760}{311875200}$$

$$\boxed{P = \frac{33}{16660}}$$

Solution for Question 6

There are 2 outcomes possible on each toss of a coin, either head or a tail. Thus for 16 tosses, the total number of outcomes are 2^{16}. The probability to get at least five tails can be calculated by first calculating the probability of getting at most 4 tails. This can be done as follows:

- 0 tails are possible in $16C0 = 1$ way. Intuitively, the only way we can get no tails is when we get head in all 16 flips.
- 1 tail is possible in $16C1 = 16$ ways.
- 2 tails are possible in $16C2 = 120$ ways.
- 3 tails are possible in $16C3 = 560$ ways.
- 4 tails are possible in $16C4 = 1820$ ways.

Thus, at most 4 tails are possible in $1 + 16 + 120 + 560 + 1820 = 2517$ ways. The probability to get at most 4 tails is $\frac{2517}{2^{16}}$. Lastly, by using total probability theorem, we can determine the probability to get at least 5 tails, i.e.:

$$\boxed{P = 1 - \frac{2517}{2^{16}} \approx 0.9616}$$

Solution for Question 7

First character can be taken in 27 ways (alphabets and underscore can be taken but digits cannot be taken). The remaining 4 characters can be taken in 37 ways. Thus the variable can be named in:

$$\boxed{27 \times 37 \times 37 \times 37 \times 37 = 5.0602347 \times 10^7 \ \ ways}$$

Solution for Question 8

The total number of ways in which 8 cards can be drawn from 52 cards is $52C8 = 7.5253815 \times 10^8$ ways.
There are 4 kings in 52-card deck and the total number of ways in which we can get at most two kings can be broken down into three pieces.

- For 0 kings, we have to choose 0 cards from 4 Kings and 8 from the rest 48 cards, i.e. $4C0 \times 48C8 = 3.77348944 \times 10^8$ ways.
- For 1 King, we have to choose 1 card from 4 Kings and 7 from the rest 48 cards, i.e. $4C1 \times 48C7 = 2.94516288 \times 10^8$ ways.
- For 2 Kings, we have to choose 2 cards from 4 Kings and 6 from the rest 48 cards, i.e. $4C2 \times 48C6 = 7.3629072 \times 10^7$ ways.

Summing up the numbers above, we get total number of ways such that we get at most 2 kings are 7.45494304×10^8 ways. Dividing it by total number of ways, we get:

$$\boxed{P \approx 0.99064}$$

Solution for Question 9

There are 10 numbers. So, the total number of ways in which the numbers can be chosen is $10 \times 10 \times 10 \times 10 = 10^4$ ways.
In accordance with the given condition, first player can choose a number in 6 ways, from 0 to 5. He/She cannot choose from 6 to 9 because if he does then the remaining 4 people cannot choose their numbers in such a way that their numbers are one more than the previously chosen number. The rest 4 people can only choose number in 1 way as they are bound to select the number that is one more than the previously chosen number. Therefore, the number of ways are $6 \times 1 \times 1 \times 1 \times 1 = 6$ ways.
Hence the probability is,

$$\boxed{P = \frac{6}{10^4} = \frac{3}{50000}}$$

Solution for Question 10

(Solved using Multinomial Coefficient)
The total number of ways in which 52 cards can be distributed to 4 players (each getting 13) is:

$$\frac{52!}{13!13!13!13!}$$

As there are four aces, four queens, and four kings in a 52-card deck and each player should receive one of them. Therefore, each of these can be distributed in the following number of ways:

$$\frac{4!}{1!1!1!1!}$$

The remaining 40 cards can be distributed (10 to each player) in the following number of ways:

$$\frac{40!}{10!10!10!10!}$$

Thus, the probability is,

$$P = \frac{\frac{4!}{1!1!1!1!} \times \frac{4!}{1!1!1!1!} \times \frac{4!}{1!1!1!1!} \times \frac{40!}{10!10!10!10!}}{\frac{52!}{13!13!13!13!}} \tag{1}$$

$$\boxed{P \approx 0.001213}$$