Probability & Statistics: Practice Questions
Spring 2022

Conditional Probability

Question 1
In a group of kids, if one is selected at random the probability that he/she likes oranges is 0.6, the probability that he/she likes oranges and apples is 0.3. If a kid, who likes oranges, is selected at random, what is the probability that he/she also likes apples?

Question 2
A single card is drawn from a deck. A card is selected at random, find the probability of selecting a
a) King
b) red card
c) King of red card
d) King given that it is a red card
e) red card given that it is a King
f) Queen given that it is a Heart

Question 3
Two dies are thrown simultaneously and the sum of the numbers obtained is found to be 7. What is the probability that the number 3 has appeared at least once?

Total Probability Theorem

Question 4
A person has undertaken a mining job. The probabilities of completion of the job on time with and without rain are 0.42 and 0.90 respectively. If the probability that it will rain is 0.45, then determine the probability that the mining job will be completed on time.

Question 5
A problem is given to 5 students P, Q, R, S, T. If the probability of solving the problem individually is 1/2, 1/3, 2/3, 1/5, 1/6 respectively, then find the probability that the problem is solved.

Question 6
The probability that the political party A does a particular work is 30% and the political party B doing the same work is 40%. Then find the probability that the work is completed if the probability of choosing the political party A is 40% and that of B is 60%.

Multiplication Rule

Question 7
An urn contains 20 red and 10 blue balls. Two balls are drawn from a bag one after the other without replacement. What is the probability that both the balls are drawn are red?

Bayes Theorem

Question 8
Consider there are three machines. All of the machines can produce 1000 pins at a time. The rate of producing a faulty pin from Machine 1 be 10%, from Machine 2 be 20% and from Machine 3 be 5%. What is the probability that a produced pin will be faulty and it will be from the first machine?

Question 9

You have been given dice and a pack of 52 cards. You have to throw a dice and then you have to pick up a card. What is the probability that you picked up a red card and threw 6 on the dice?

Question 10
In a study, physicians were asked what the odds of breast cancer would be in a woman who was initially thought to have a 1% risk of cancer but who ended up with a positive mammogram result (a mammogram accurately classifies about 80% of cancerous tumors and 90% of benign tumors.) 95 out of a hundred physicians estimated the probability of cancer to be about 75%. Do you agree?

Solution 1
Let O: kid likes oranges, A: kid likes apples
We are given $P(O) = 0.6$, $P(A \cap O) = 0.3$

$P(A|O) = \frac{P(A \cap O)}{P(O)} = \frac{0.3}{0.6} = 0.5$

Solution 2
a) Out of the sample space of 52 cards, there are 4 Kings.
$P(King) = \frac{4}{52} = \frac{1}{13}$

b) Out of the sample space of 52 cards, there are 26 red.
$P(red) = \frac{26}{52} = \frac{1}{2}$

c) 2 cards are King of red out of the 52 cards.
$P(King \cap red) = \frac{2}{52} = \frac{1}{26}$

d) P(King given that it is a red card) $= P(King|red) = \frac{P(King \cap red)}{P(red)} = \frac{\frac{1}{26}}{\frac{1}{2}} = \frac{1}{13}$

e) P(red card given that it is a King) $= P(red|King) = \frac{P(red \cap King)}{P(King)} = \frac{\frac{1}{26}}{\frac{1}{13}} = \frac{1}{2}$

f) P(Queen given that it is Heart) $= P(Queen|heart) = \frac{P(Queen \cap Heart)}{P(Heart)} = \frac{\frac{1}{52}}{\frac{13}{52}} = \frac{1}{13}$

Solution 3
The sample space S would consist of all the numbers possible by the combination of two dies. Therefore S consists of 6 × 6 i.e. 36 events.
Event A indicates the combination in which 3 has appeared at least once.
Event B indicates the combination of the numbers which sum up to 7.

A = (3, 1), (3, 2), (3, 3)(3, 4)(3, 5)(3, 6)(1, 3)(2, 3)(4, 3)(5, 3)(6, 3)

B = (1, 6)(2, 5)(3, 4)(4, 3)(5, 2)(6, 1)

$P(A) = \frac{11}{36}$

$P(B) = \frac{6}{36}$

$A \cap B = 2$
$P(A \cap B) = \frac{2}{36}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$

Solution 4
Let A be the event that the mining job will be completed on time and B be the event that it rains. We know the following:

$P(B) = 0.45$
P(no rain) $= P(B') = 1 - P(B) = 1 - 0.45 = 0.55$

By multiplication law of probability,

$P(A|B) = 0.42$, $P(A|B') = 0.90$

$P(A) = P(B)P(A|B) + P(B')P(A|B')$
$P(A) = 0.45 \times 0.42 + 0.55 \times 0.9$
$P(A) = 0.189 + 0.495 = 0.684$

Solution 5
Let A be the event that the problem is solved. Let Ei be the event that a student is chosen.

$P(A) = P(E1)P(A|E1) + P(E2)P(A|E2) + P(E3)P(A|E3) + P(E4)P(A|E4) + P(E5)P(A|E5)$

$P(A) = (\frac{1}{5})(\frac{1}{2}) + (\frac{1}{5})(\frac{1}{3}) + (\frac{1}{5})(\frac{2}{3}) + (\frac{1}{5})(\frac{1}{5}) + (\frac{1}{5})(\frac{1}{6})$

$P(A) = 0.37$

Solution 6
Let A be the event of completing the work and Ei be the event of selecting the political party.
$P(A) = P(E1)P(A|E1) + P(E2)P(A|E2) = (0.4)(0.3) + (0.6)(0.4) = 0.36$.

Solution 7
Let A and B denote the events that the first and the second balls are drawn are red balls.
$P(A) = P$(red balls in first draw) $= \frac{20}{30}$

Now, only 19 red balls and 10 blue balls are left in the bag. The probability of drawing a red ball in the second draw too is an example of conditional probability where the drawing of the second ball depends on the drawing of the first ball.
$P(B|A) = \frac{19}{29}$

By multiplication rule of probability,
$P(A \cap B) = P(A).P(B|A)$
$P(A \cap b) = (\frac{20}{30})(\frac{19}{29}) = \frac{38}{87}$

Solution 8
Let us take the probability of choosing a faulty pin randomly be represented by P(A).
Pin choose from the first machine be represented by M1.
Pin choose from the second machine be represented by M2:
Pin choose from the third machine be represented by M3:
P(choosing pin one of the three machines) $= P(M1) = P(M2) = P(M3) = \frac{1}{3}$

$P(M1|A) = \frac{P(M1)P(A|M1)}{P(M1)P(A|M1)+P(M2)P(A|M2)+P(M3)P(A|M3)}$

$P(M1|A) = \frac{\frac{1}{3}(0.1)}{\frac{1}{3}(0.1)+\frac{1}{3}(0.2)+\frac{1}{3}(0.05)}$

$P(M1|A) = \frac{2}{7}$

Solution 9
Let us denote the probability of throwing 6 on an unbiased dice be represented by P(D).
P(D) = 1/6
So, The probability of throwing another number on the dice is represented by P(E) = 5/6
The probability of picking up a red card, P(R1) = 1/2
So, The probability of not picking up a red card, P(R2) = 1 – 1/2 = 1/2
So, the probability of picking up a red card after drawing a 6 on the dice:
$P(R1|D) = \frac{P(R1)P(D|R1)}{P(R1)P(D|R1)+P(R2)P(D|R2)}$

$P(R1|D) = \frac{(\frac{1}{2})(\frac{1}{6})}{(\frac{1}{2})(\frac{1}{6})+(\frac{1}{2})(\frac{5}{6})}$

$P(R1|D) = \frac{1}{6}$

Solution 10

P = mammogram result is positive
B = tumor is benign
M = tumor is malignant
Note that $B^c = M$. We are given $P(M) = 0.01$, so $P(B) = 1 - P(M) = 0.99$.
We are also given the conditional probabilities $P(P|M) = 0.80 and P(P^c|B) = 0.90$, where the event P^c means that the result is negative, thus $P(P|B) = 0.10$. Bayes' formula in this case is
$P(M|P) = \frac{P(P|M)P(M)}{(P(P|M)P(M)+P(P|B)P(B))}$
$P(M|P) = \frac{0.80 \times 0.01}{(0.80 \times 0.01 + 0.10 \times 0.99)}$
$P(M|P) = 0.075$
So the chance would be 7.5%, which is very far from a common estimate of 75.