Assignment 1

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Problem 1 – Distance between two adjacent maxima If Δy is the distance between two consecutive maxima, then they are given by

$$\begin{aligned}\Delta y &= y_1 - y_2\\ &= \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d}\\ &= \frac{\lambda D}{d}\\ &= \frac{546\text{ nm} \times 55\text{ cm}}{0.12\text{ mm}}\\ &= 2.50250\text{ mm}\end{aligned}$$

Problem 2 – Angular seperation The angular seperation in double slit experiment is given by,

$$d\sin\theta = m\lambda$$

For small θ we can approximate $\sin\theta \approx \theta$, so we get the following equation,

$$d\theta = m\lambda$$

For 10% more θ we get,

$$\begin{aligned}&d\theta' = m\lambda'\\ &\implies d(1.1\theta) = m\lambda'\\ &\implies 1.1d\theta = m\lambda'\\ &\implies 1.1m\lambda = m\lambda'\\ &\implies 1.1\lambda = \lambda'\\ &\implies \lambda' = 647.9\text{ nm}\end{aligned}$$

Problem 3 – Intensity 1. The electric field produce by these electromagnetic waves would be,

$$
\begin{aligned}
\mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 \\
&= \left(E_0 \sin(\omega t) + E_0 \sin(\omega t + \phi) + E_0 \sin(\omega t + 2\phi)\right) \hat{\mathbf{n}} \\
&= E_0 \left(\sin(\omega t) + \sin(\omega t + \phi) + \sin(\omega t + 2\phi)\right) \hat{\mathbf{n}} \\
&= E_0 \left(\sin(\omega t + \phi) + \sin(\omega t) + \sin(\omega t + 2\phi)\right) \hat{\mathbf{n}} \\
&= E_0 \left(\sin(\omega t + \phi) + 2\cos\left(\frac{\omega t - \omega t - 2\phi}{2}\right) \sin\left(\frac{\omega t + \omega t + 2\phi}{2}\right)\right) \hat{\mathbf{n}} \\
&= E_0 \left(\sin(\omega t + \phi) + 2\cos(\phi) \sin(\omega t + 2\phi)\right) \hat{\mathbf{n}} \\
&= E_0 \left(1 + 2\cos(\phi)\right) \sin(\omega t + \phi) \hat{\mathbf{n}}
\end{aligned}
$$

The magnitude square of the electric field is,

$$
|\mathbf{E}|^2 = E^2 = E_0^2 \left(1 + 2\cos(\phi)\right)^2 \sin^2(\omega t + \phi)
$$

Then the intesity would be.

$$
\begin{aligned}
I &= \left\langle E^2 \right\rangle \\
&= E_0^2 \left(1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right)^2 \left\langle \sin^2(\omega t + \phi) \right\rangle \\
&= E_0^2 \left(1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right)^2
\end{aligned}
$$

I will be maximum when $\phi = 1$, so we get, $I_0 = 9E_0^2$

$$
I = \frac{I_0}{9} \left(1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right)^2
$$

2. The intensity is given by,

$$
I = \frac{I_0}{9} \left(1 + 2\cos(\phi)\right)^2
$$

Taking derivative of the intensity with respect to ϕ,

$$
\begin{aligned}
\frac{dI}{d\phi} &= \frac{d}{d\phi} \left(\frac{I_0}{9} \left(1 + 2\cos(\phi)\right)^2\right) \\
\frac{dI}{d\phi} &= -\frac{4I_0}{9} \left(1 + 2\cos(\phi)\right) \sin(\phi) \\
\implies \frac{dI}{d\phi} &= 0 \\
\implies \sin(\phi) = 0 &\implies \phi_n = n\pi \\
\cos(\phi) = -\frac{1}{2} &\implies \phi_n = \frac{2}{3}\pi + 2n\pi
\end{aligned}
$$

From second derivative test,

$$\frac{d^2I}{d\phi^2} = \frac{d}{d\phi}\left(-\frac{4I_0}{9}(1+2\cos(\phi))\sin(\phi)\right)$$
$$\frac{d^2I}{d\phi^2} = -\frac{4I_0}{9}\left(\cos(\phi)+2\cos(2\phi)\right)$$

For $\phi = \frac{2}{3}\pi + 2n\pi$, we get,

$$\frac{d^2I}{d\phi^2} = -\frac{4I_0}{9}\left(\cos\left(\frac{2}{3}\pi+2n\pi\right)+2\cos\left(2\left(\frac{2}{3}\pi+2n\pi\right)\right)\right)$$
$$\frac{d^2I}{d\phi^2} = -\frac{4I_0}{9}\left(-\frac{1}{2}+2\left(-\frac{1}{2}\right)\right)$$
$$\frac{d^2I}{d\phi^2} = \frac{2I_0}{3} > 0$$

Hence, $\phi = \frac{2}{3}\pi + 2n\pi$ is a minimum point. For $\phi = \pi n$, we get,

$$\frac{d^2I}{d\phi^2} = -\frac{4I_0}{9}\left(\cos(\pi n)+2\cos(2\pi n)\right)$$
$$\frac{d^2I}{d\phi^2} = -\frac{4I_0}{9}\left((-1)^n+2(1)\right)$$
$$\frac{d^2I}{d\phi^2} = -\frac{4I_0}{9}\left((-1)^n+2\right) < 0$$

Hence, $\phi = \pi n$ is a maximum point. Primary maxima occurs at $n = 2k$ and secondary maxima occurs at $n = 2k+1$. Therefore the ratio of the intensity of the primary maxima to the secondary maxima is,

$$\frac{I_{\text{primary}}}{I_{\text{secondary}}} = \frac{\frac{I_0}{9}\left(1+2\cos(2k\pi)\right)^2}{\frac{I_0}{9}\left(1+2\cos(\pi(2k+1))\right)^2}$$
$$\frac{I_{\text{primary}}}{I_{\text{secondary}}} = \frac{(1+2)^2}{(1-2)^2}$$
$$\frac{I_{\text{primary}}}{I_{\text{secondary}}} = 9$$