Tangent lines and rate of change

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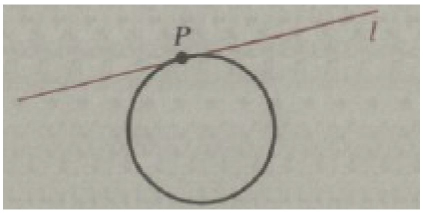
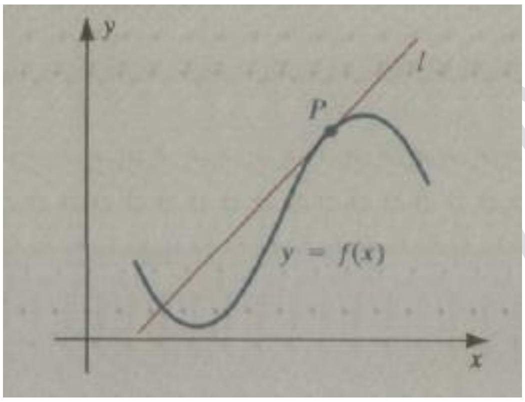
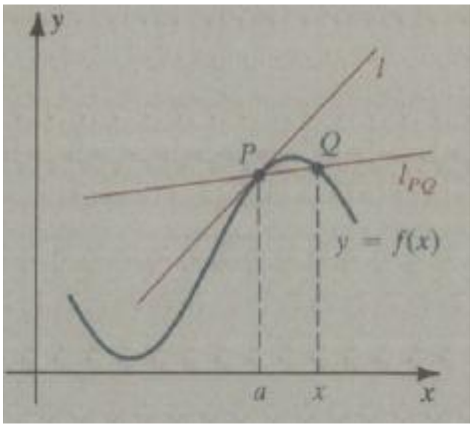
Tangent lines





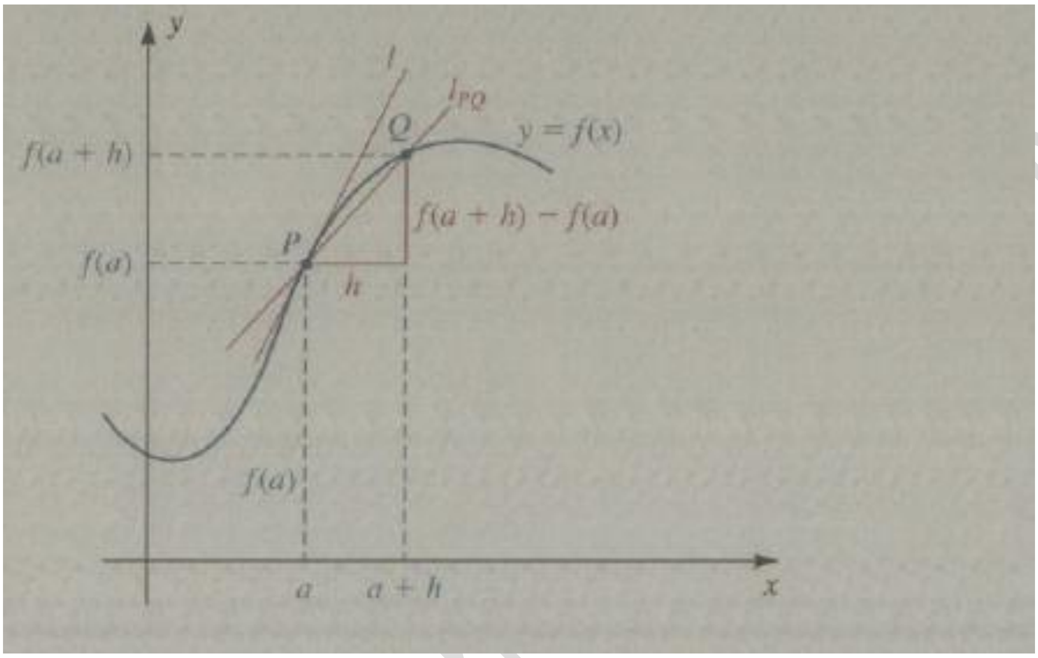
Average rate of change



$$m_{PQ} = \frac{f(x) - f(a)}{x - a}$$

Slope of the tangent line

The slope m of the tangent line To the graph of a function $f(x)$ at point $P(a, f(a)$ is defined as



$$m_a = \lim_{h \to 0} \frac{f(a+h) - f(a)}{a + h - a}$$

$$m_a = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Ex. No. 2.1

Exr. 1-6 (a) Use definition 2.1 to find the slope of tangent line to the graph of f at point $P(a, f(a))$. (b) Find an equation of the tangent line at $P(2, f(2))$.

1. $f(x) = 5x^2 - 4x$

$$f(a+h) = 5(a+h)^2 - 4(a+h)$$

$$f(a) = 5a^2 - 4a$$

$$m_a = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$m_a = \lim_{h\to 0} \frac{5(a+h)^2 - 4(a+h) - (5a^2 - 4a)}{h}$$

$$= \lim_{h\to 0} \frac{5(a^2 + h^2 + 2ah) - 4(a+h) - (5a^2 - 4a)}{h}$$

$$= \lim_{h\to 0} \frac{5a^2 + 5h^2 + 10ah - 4a - 4h - 5a^2 + 4a}{h}$$

$$= \lim_{h\to 0} \frac{+5h^2 + 10ah - 4h}{h}$$

$$= \lim_{h\to 0} h \frac{(5h + 10a - 4)}{h}$$

$$= \lim_{h\to 0} 5h + 10a - 4 = 10a - 4$$

b. put $a = 2$

$$m_2 = 20 - 4 = 16$$

Eq. of the tangent line is determined as

$$y - y_1 = m_2(x - x_1)$$

Here $x_1 = 2$

$$y_1 = 5x^2 - 4x = 20 - 8 = 12$$

$$y - 12 = 16(x - 2)$$

$$y = 12 + 16x - 32$$

$$y = 16x - 20$$

3. $f(x) = x^3$

$$m_a = \lim_{h\to 0}\frac{f(a+h) - f(a)}{h}$$

$$f(a+h) = (a+h)^3, \qquad f(a) = a^3$$

$$m_a = \lim_{h\to 0}\frac{(a+h)^3 - a^3}{h}$$

$$m_a = \lim_{h\to 0}\frac{a^3 + h^3 + 3ah(a+h) - a^3}{h}$$

$$m_a = \lim_{h\to 0}\frac{a^3 + h^3 + 3a^2h + 3ah^2 - a^3}{h}$$

$$m_a = \lim_{h\to 0}\frac{h^3 + 3a^2h + 3ah^2}{h}$$

$$m_a = \lim_{h\to 0} h \times \frac{(h^2 + 3a^2 + 3ah)}{h}$$

$$m_a = \lim_{h\to 0}(h^2 + 3a^2 + 3ah) = 3a^2$$

At $P(2, f(2))$.

$$m_2 = 12$$

b. $y - y_1 = m_2(x - x_1)$

Here $x_1 = 2, y_1 = x^3 = 8$

$$y - 8 = 12(x - 12)$$

$$y = 12x - 144 + 8$$

$$y = 12x - 136$$

15. Rescue helicopter drops a crate of supply from a height of 160 feet . After t seconds ,the crate is $160 - 16t^2$ feet above the ground

a. find the velocity of the crate at $t = 1$

b. with what velocity does the crate strike the ground ?

$s(t) = 160 - 16t^2$

$$v_a = \lim_{h\to 0} \frac{s(a+h) - s(a)}{h}$$

$$s(a+h) = 160 - 16(a+h)^2$$

$$s(a) = 160 - 16a^2$$

$$v_a = \lim_{h\to 0} \frac{160 - 16(a+h)^2 - (160 - 16a^2)}{h}$$

$$= \lim_{h\to 0} \frac{160 - 16(a^2 + h^2 + 2ah) - (160 - 16a^2)}{h}$$

$$= \lim_{h\to 0} \frac{160 - 16a^2 - 16h^2 - 32ah - 160 + 16a^2}{h}$$

$$= \lim_{h\to 0} \frac{-16h^2 - 32ah}{h}$$

$$= \lim_{h\to 0} -16h - 32a =$$

$$v_a = -32a \qquad (1)$$

$$t = a = 1$$

$$v_1 = -32\ ft/sec$$

b. When the great strike the ground $s = 0$

$s(t) = 160 - 16t^2$

$$160 - 16t^2 = 0, \quad t = \sqrt{10}$$

Using $t = \sqrt{10}\ in\ (1)$

$$v = -32\sqrt{10} ft/sec$$