# Definite Integral

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Ex. 4.4

Exr. 19-24: Given $\int_1^4 \sqrt{x}dx = \frac{14}{3}$, evaluate the integral.

19. $\int_4^1 \sqrt{x}dx$

As $\int_a^b f(x)dx = -\int_b^a f(x)dx$

Therefore $\int_4^1 \sqrt{x}dx = -\int_1^4 \sqrt{x}dx = -\frac{14}{3}$

23. $\int_4^4 \sqrt{x}dx + \int_4^1 \sqrt{x}dx$

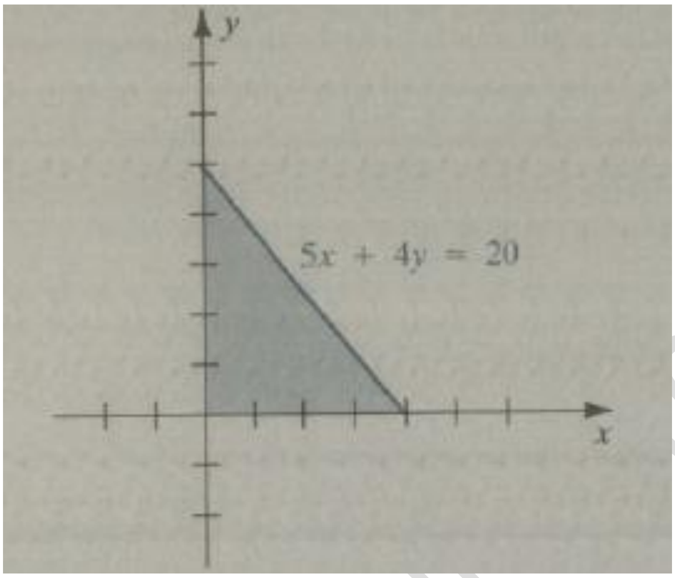
Sol: As $\int_a^a f(x)dx = 0$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_4^4 \sqrt{x}dx + \int_4^1 \sqrt{x}dx$$

$$= 0 - \int_1^4 \sqrt{x}dx = 0 - \frac{14}{3} = -\frac{14}{3}$$

Ex. 25-28: Express the region in the figure as a definite integral.




Sol:

As area of triangle= $\frac{1}{2} base \times hight = \frac{1}{2}(4) \times 5 = 2 \times 5 = 10$

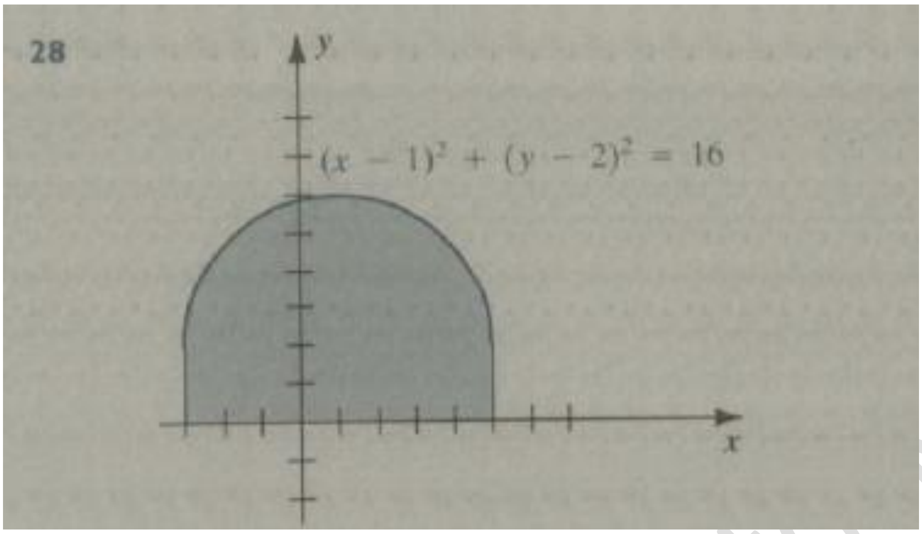
$$5x + 4y = 20,$$

$$y = \frac{20 - 5x}{4}$$

$$\int_0^4 \frac{20 - 5x}{4} dx = \int_0^4 \left(5 - \frac{5x}{4}\right) dx$$

$$= \left[5x - \frac{5x^2}{8}\right]_0^4 = 20 - 10 - 0 + 0$$

$$= 10$$




Sol: $(x-1)^2+(y-2)^2=16$

$$(y-2)^2=16-(x-1)^2$$

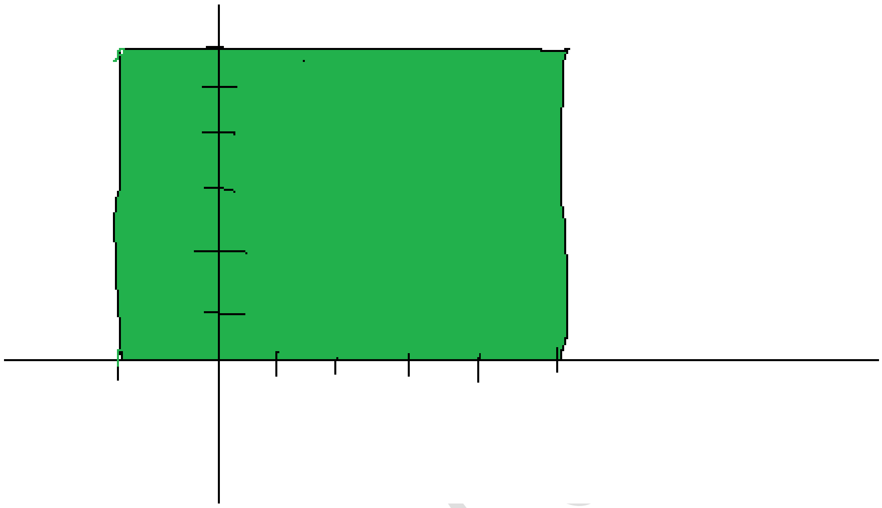
$$y-2=\pm\sqrt{16-(x-1)^2}$$

$$y=2+\sqrt{16-(x-1)^2}$$

$$\int_{-3}^{5}\left(2+\sqrt{16-(x-1)^2}\right)dx$$

Exr. 29-38: Evaluate the definite integral by regrading it as area under the graph of a function.

29. $\int_{-1}^{5} 6dx$



$$\int_{-1}^{5} 6dx = hight \times lenght = 6 \times 6 = 36$$

38: $\int_{-2}^{2}(3-\sqrt{4-x^2})dx$

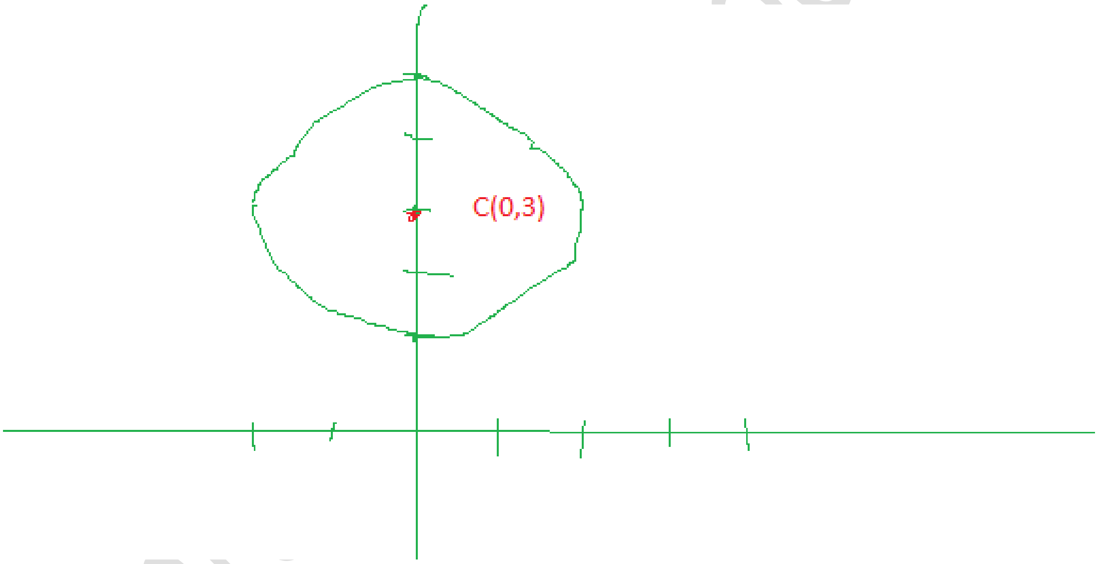
$$y = 3 - \sqrt{4-x^2}$$

$$y - 3 = -\sqrt{4-x^2}$$

$$(y-3)^2 = 4 - x^2$$

$$x^2 + (y-3)^2 = 4$$

$$C(0,3), \qquad r = 2$$




$$A = \pi \times r^2 = \pi \times (2)^2 = 4\pi = 4 \times \frac{22}{7}$$

$$= \frac{88}{7}$$