LEFT HAND AND RIGHT HAND DERIVATIVE

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Slope of the tangent line

The slope m of the tangent line To the graph of a function $f(x)$ at point

$P(a, f(a)$ is defined as

$$m_a = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Derivative

Slope of the tangent line is derivative.

The derivative of a function f is the function f' whose value at x is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Provided the limit exists

Exercise No. 2.2

Exr. 1-4 (a) Use definition (2.5) to find f'. (b) Find the domain of f'. (c) Find an equation of the tangent line to the graph of f at P. (d) Find the points on the graph at which the tangent line is horizontal.

1. $f(x) = -5x^2 + 8x + 2;\quad P(-1, -11)$

(a)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \quad (1)$$

$$f(x+h) = -5(x+h)^2 + 8(x+h) + 2$$

$$= -5x^2 - 5h^2 - 10xh + 8x + 8h + 2$$

Using the values of $f(x+h) and\ f(x)$ in Eq. (1)

$$f'(x) = \lim_{h\to 0} \frac{-5x^2 - 5h^2 - 10xh + 8x + 8h + 2 + 5x^2 - 8x - 2}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{-5h^2 - 10xh + 8h}{h}$$

$$f'(x) = \lim_{h\to 0} h \times \frac{(-5h - 10x + 8)}{h}$$

$$f'(x) = \lim_{h\to 0} -5h - 10x + 8 = -10x + 8$$

b). $Domain\ of\ f' = R = (-\infty, \infty)$

c).

$$y - y_1 = m(x - x_1)$$

Here $P(x_1, y_1) = P(-1, -11), \quad and \quad m = f'(-1) = 10 + 8 = 18$

$$y + 11 = 18(x + 1)$$

$$y = 18x + 18 - 11 = 18x + 7$$

$$y = 18x + 7$$

d). The tangent line will be horizontal when $m = f'(x) = 0$

$$-10x + 8 = 0, \Rightarrow x = 4/5$$

$$y = f(x) = -5\left(\frac{4}{5}\right)^2 + 8\left(\frac{4}{5}\right) + 2 = \frac{-80 + 160 + 50}{25} = \frac{130}{25} = \frac{26}{5}$$

$$(x,y) = (\frac{4}{5}, \frac{26}{5})$$

Q. No. 4 $f(x) = x^3 - 4x,\ P(2,0)$

(a)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad (1)$$

$$f(x+h) = (x+h)^3 - 4(x+h)$$

$$= x^3 + h^3 + 3x^2h + 3xh^2$$

Using the values of $f(x+h) and\ f(x)$ in Eq. (1)

$$f'(x) = \lim_{h\to 0} \frac{x^3 + h^3 + 3x^2h + 3xh^2 - 4x - 4h - x^3 + 4x}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{h^3 + 3x^2h + 3xh^2 - 4h}{h}$$

$$f'(x) = \lim_{h\to 0} h \times \frac{(h^2 + 3x^2 + 3xh - 4)}{h}$$

$$f'(x) = \lim_{h\to 0} h^2 + 3x^2 + 3xh - 4 = 3x^2 - 4$$

b). $Domain\ of\ f' = R = (-\infty, \infty)$

c).

$$y - y_1 = m(x - x_1)$$

Here $P(x_1, y_1) =,\ P(2,0),\quad and\quad m = f'(2) = 12 - 4 = 8$

$$y - 0 = 8(x - 2)$$

$$y = 8x - 16$$

d). The tangent line will be horizontal when $m = f'(x) = 0$

$$3x^2 - 4 = 0, \Rightarrow x^2 = \frac{4}{3}, \Rightarrow x = \pm\frac{2}{\sqrt{3}}$$

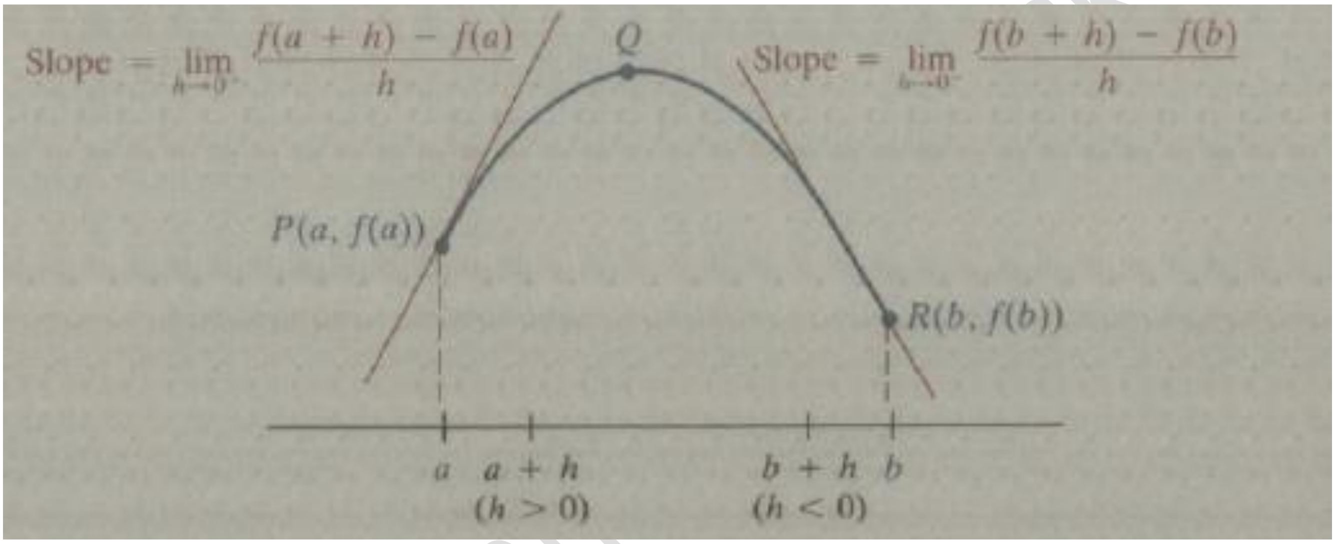
$$y = f(x) = x^3 - 4x = \frac{8}{3\sqrt{3}} - \frac{8}{\sqrt{3}} = \frac{-16}{3\sqrt{3}}$$

$$(x,y) = (\frac{2}{\sqrt{3}}, \frac{-16}{3\sqrt{3}})$$

LEFT HAND AND RIGHT HAND DERIVATIVE

A function f is differentiable on a closed interval $[a, b]$ if f is differentiable on the open interval (a, b) and if the following limits exist

$$\lim_{h \to 0^+} \frac{f(a+h)-f(a)}{h} \text{ and } \lim_{h \to 0^-} \frac{f(b+h)-f(b)}{h}$$



A function f is said to be differentiable on [a,b], if right hand derivative exists at a, and left hand derivative exists at b.

Note that there is no need that these derivative limits to be equal.

Exr. 21-22. Is f differentiable on the given interval explain?

21. $f(x)=\frac{1}{x}$ (a) $[0,2]$, (b) $[1,3)$

(a) $[0,2]$

$$\lim_{h\to 0^+}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\frac{1}{h}-\infty}{h}\quad =D.N.E$$

and $\lim_{h\to 0^-}\frac{f(b+h)-f(b)}{h}=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$

$$=\lim_{h\to 0}\frac{\frac{2-2-h}{(2+h)2}}{h}$$

$$=\lim_{h\to 0}\frac{2-2-h}{h(2+h)2}$$

$$=\lim_{h\to 0}\frac{-h}{h(2+h)2}=\lim_{h\to 0}\frac{-1}{(2+h)2}=-\frac{1}{4}$$

f is not differentiable on [0,2].

(b) [1,3)

$$\lim_{h\to 0^+}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h-0}\frac{\frac{1}{1+h}-1}{h}$$

$$=\lim_{h-0}\frac{1-1-h}{h(1+h)}$$

$$\lim_{h-0}\frac{-h}{h(1+h)}=\lim_{h-0}\frac{-1}{(1+h)}=-1$$

and $\lim_{h\to 0^-}\frac{f(b+h)-f(b)}{h}=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h}$

$$=\lim_{h\to 0}\frac{3-3-h}{h(3+h)3}$$

$$=\lim_{h\to 0}\frac{-h}{h(3+h)\times 3}=\lim_{h\to 0}\frac{-1}{(3+h)\times 3}=-\frac{1}{9}$$

Hence f is differentiable on the given interval [1,3) .

Q. No. 22 $f(x) = \sqrt[3]{x}, \ \ (a)\ [-1,1], \ (b)\ [-2,-1)$

$$f(x) = x^{\frac{1}{3}}$$

$$\lim_{h\to 0^+}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0}\frac{(-1+h)^{\frac{1}{3}}+1}{h}$$

$$= \lim_{\mathrm{h}\to 0}\frac{1}{h}[-1 -$$

$$(a+b)^n = a^n + a^{n-1}b + \cdots + b^n$$

Solve in 10 minutes

Exr. 33-36: Use right hand and left hand derivatives to prove that f is not differentiable at a.

33. $f(x) = |x-5|; \quad a = 5$

$$f(x) = |x-5| = \begin{cases} (x-5), & x \geq 5 \\ -(x-5), & x < 5 \end{cases}$$

$$f(a) = f(5) = |5-5| = 0$$

$$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$\lim_{x\to a^+}\frac{f(x)-f(a)}{x-a} = \lim_{x\to 5^+}\frac{f(x)-f(a)}{x-a} = \lim_{x\to 5}\frac{x-5-0}{x-5} = \lim_{x\to 5}\frac{x-5}{x-5}$$

$$= \lim_{x\to 5} 1 = 1$$

$$\lim_{x\to a^-}\frac{f(x)-f(a)}{x-a} = \lim_{x\to 5^-}\frac{f(x)-f(a)}{x-a} = \lim_{x\to 5}\frac{-(x-5)-0}{x-5} = \lim_{x\to 5}\frac{-(x-5)}{x-5}$$

$$= \lim_{x\to 5} -1 = -1$$

As $\lim_{x\to a^-}\frac{f(x)-f(a)}{x-a} \neq \lim_{x\to a^+}\frac{f(x)-f(a)}{x-a}$

So f is not differentiable at a.