Application of Derivative.

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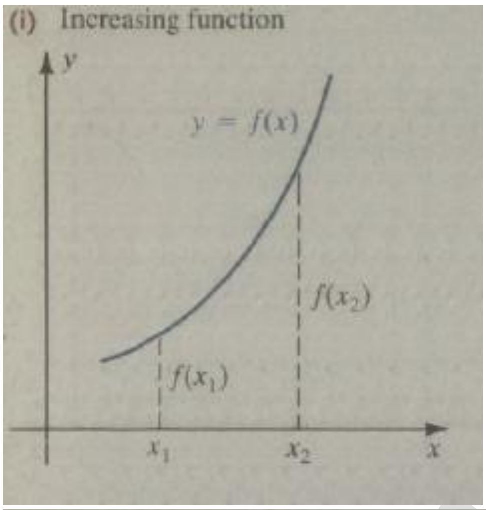
Def. 3.1

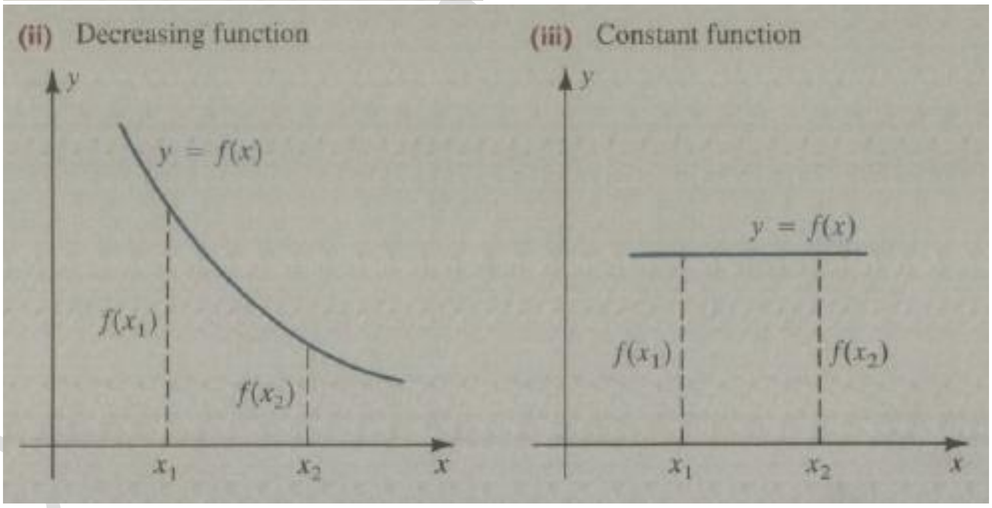
Let a function f be defined on the interval I and let x_1, x_2 denote numbers in I

I. f is increasing on I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$

II. f is decreasing on I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$

III. f is constant on I if $f(x_1) = f(x_2)$ for x_1 and x_2



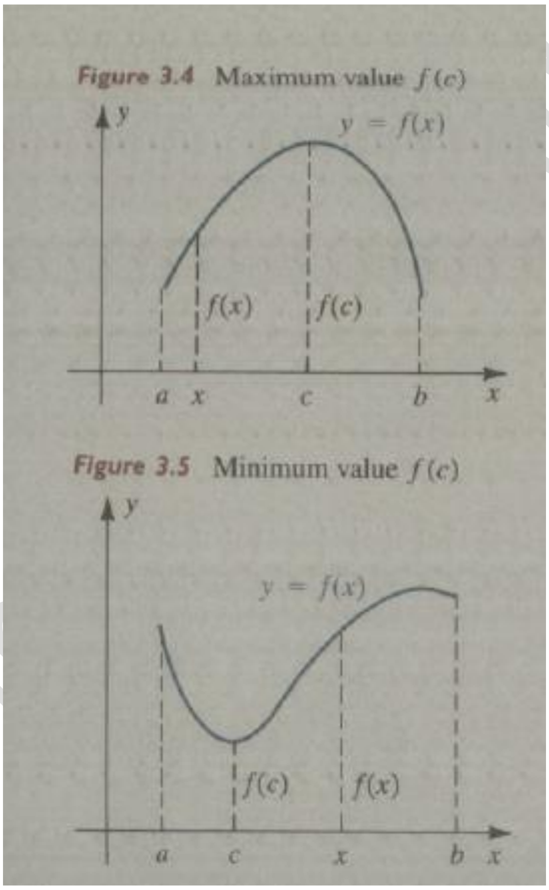


Def. 3.2

Let a function f be defined on the set of real number, and let c be a number in s

I. $f(c)$ is maximum of f on S if $f(x) \leq f(c)$ for every x in S.

II. $f(c)$ is minimum of f on S if $f(x) \geq f(c)$ for every x in S.



Def. 3.4

Let c be a number in the domain of a function f.

I. $f(c)$ is local maximum of f if there exist open interval (a, b)containing c such that $f(x) \leq f(c)$ for every x in (a, b) that is the domain of f.

II. $f(c)$ is local minimum of f if there exist open interval (a, b)containing c such that $f(x) \geq f(c)$ for every x in (a, b) that is the domain of f.

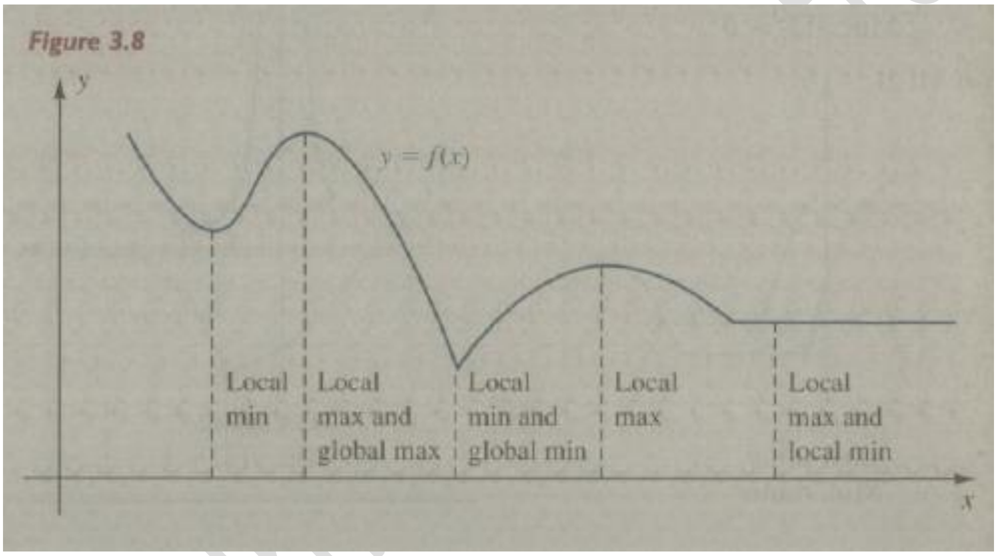
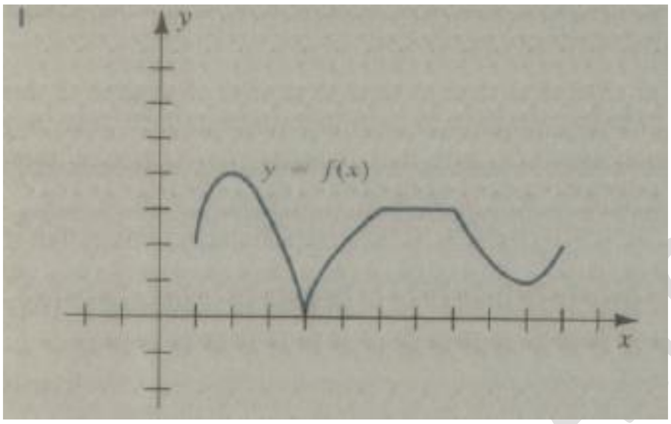


Figure 3.8

Ex. No. 3.1

Exr. 1-2: Use the graph to estimate the absolute maximum the absolute minimum and the local extrema of f.

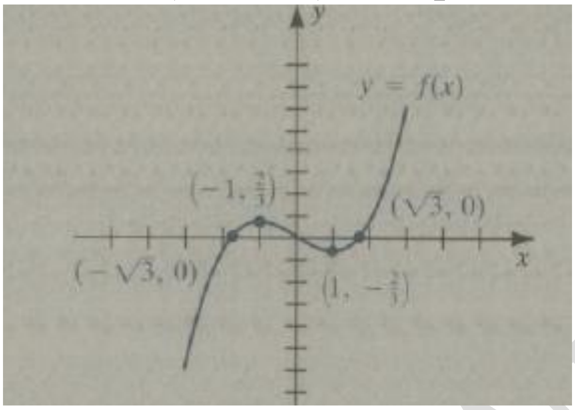


Absolute Maxima $f(2) = 4$

Absolute Minima $f(4) = 0$

Exr. 3-4: Use the graph to estimate the extreme of f on each interval

3. $(a)\ [-3,3), (b)\ \left(-3, \sqrt{3}\right), (c)\ \left[-\sqrt{3}, 1\right),\ (d)\ [0,3]$



$(a)\ [-3,3)$

Local Maxima $f(3) = None$
Local Minima $f(-3) = -6$

$(b)\ \left(-3, \sqrt{3}\right)$

Local Maxima $f(-1) = 2/3$
Local Minima $f(-3) = None$

$(c)\ \left[-\sqrt{3}, 1\right)$

Local Maxima $f(-1) = 2/3$
Local Minima $f(1) = None$

$(d)\ [0,3]$

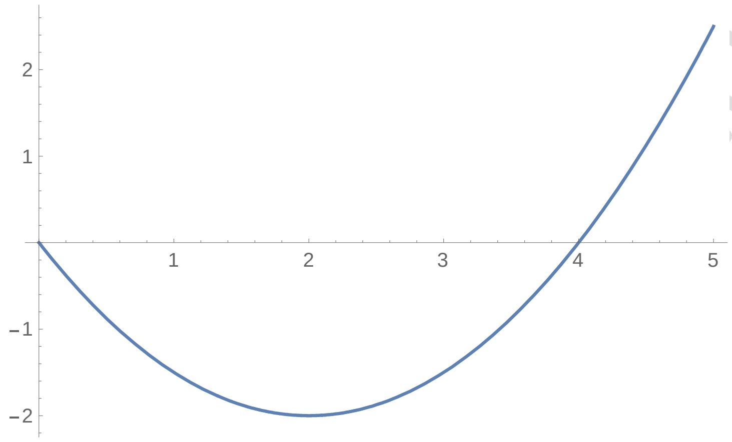
Local Maxima $f(3) = 6$
Local Minima $f(1) = -2/3$

NOTE, IF the maxima or mina lies at the point of the open, then we cant find extrema.

Exr. 5-6: Sketch graph of f and find extrema on each interval.

5. $f(x) = \frac{1}{2}x^2 - 2x$

(a) [0,5), (b) (0,2), (c) (0,4), (d) [2,5]



(a) [0,5), Local Min. $f(2) = -2$

Local Max. $f(5) = None.$

(b) (0,2) Local Min. $f(2) = None$

Local Max. $f(0) = None.$

(c) (0,4) Local Min. $f(2) = -2$

Local Max. $f(0) = f(4) = None.$

(b) [2,5] Local Min. $f(2) = -2$

Local Max. $f(5) = \frac{5}{2}$

Critical points

If a function f has a local extremum at a number c in an open interval, then either $f'(c) = 0, or\ f'(c)$ does not exist

Exr. 11-36: Find the critical numbers of the function

11. $f(x) = 4x^2 - 3x + 2$ (1)

Diff. Eq. (1) w.r.t x

$$f'(x) = 8x - 3$$

For critical points put $f'(c) = 0$

$$8c - 3 = 0$$

$$c = \frac{3}{8}$$

17. $f(z) = \sqrt{z^2 - 16}$ (1)

Diff. W.r.t z Eq. (1)

$$f'(z) = \frac{1}{2}(z^2 - 16)^{-\frac{1}{2}}(2z) = \frac{2z}{2\sqrt{z^2 - 16}} = \frac{z}{\sqrt{z^2 - 16}}$$

For critical points put $f'(c) = 0$

$$\frac{c}{\sqrt{c^2 - 16}} = 0, \Rightarrow c = 0$$

Critical points are those points where f' does not exists

$$\sqrt{c^2 - 16} = 0,$$

$$c^2 - 16 = 0, \Rightarrow c = \pm 4$$

So $c = 0, \pm 4$ are the critical points

25. $f(t) = \sin^2 t - \cos t$ (1)

Diff. Eq. (1) w.r.t t

$$f'(t) = 2\sin t \cos t + \sin t$$
$$f'(c) = 0$$
$$2\sin c \cos c + \sin c = 0$$
$$\sin c\,(2\cos c + 1) = 0$$

$$\sin c = 0, \qquad or \quad 2\cos c + 1 = 0$$
$$\sin c = 0,$$
$$\Rightarrow \; c = \sin^{-1} 0 = 0, \pi, 2\pi, 3\pi \ldots = n\pi, \quad where\ n\epsilon z$$

$$2\cos c + 1 = 0$$
$$\Rightarrow \cos c = -\frac{1}{2}$$
$$c = \cos^{-1} -\frac{1}{2} = 120,$$
$$c = \frac{2\pi}{3} + 2\pi n$$

So, all the critical points are

$c = n\pi, \frac{2\pi}{3} + 2\pi n$ where $n\epsilon z$