Linear Approx.

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Linear Approx.

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0)}{\Delta x}$$

$$f(x_0 + h) - f(x) \approx +\Delta x f'(x_0)$$

$$f(x_0 + h) \approx f(x_0) + \Delta x f'(x_0)$$

Ex. 2.8

Exer. 1–8: Use a linear approximation to estimate $f(b)$ if the independent variable changes from a to b.

1 $f(x) = 4x^5 - 6x^4 + 3x^2 - 5; \quad a = 1, \quad b = 1.03$

Q. 1. $f(x) = 4x^5 - 6x^4 + 3x^2 - 5, \quad a = 1, \ b = 1.03$

$$f(x_0 + h) \approx f(x_0) + \Delta x f'(x_0)$$

$$f(b) \approx f(a) + \Delta x f'(a) \quad (1)$$

$$f(1) = 4 - 6 + 3 - 5 = -4$$

$$\Delta x = b - a = 0.03$$

$$f(x) = 4x^5 - 6x^4 + 3x^2 - 5$$

$$f'(x) = 20x^4 - 24x^3 + 6x$$

$$f'(1) = 20 - 24 + 6 = 2$$

Using the values of $f(1), f'(1), and\ \Delta x$ in Eq. (1)

$$f(1.03) \approx -4 + (0.03)(2) = -4 + 0.06 = -3.94$$

5 $f(\theta) = 2\sin\theta + \cos\theta; \qquad a = 30°, \quad b = 27°$

$$(x_0 + h) \approx f(x_0) + \Delta x f'(x_0)$$

$$f(\theta) = 2\sin\theta + \cos\theta$$

$$x_0 + h = 27$$

Choosing $x_0 = 30$

$$h = -3°$$

$$180° = \pi$$

$$-3° = \frac{-3\pi}{180} = -\frac{\pi}{60}$$

$$f(27) \approx f(30) + \Delta x f'(30)$$

$$f'(\theta) = 2\cos\theta - \sin\theta$$

$$f'(30) = \frac{2\sqrt{3}}{2} - \frac{1}{2} = \sqrt{3} - \frac{1}{2} = 1.232$$

$$f(30) = 2\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2} = 1.866$$

$$f(27) \approx f(30) + \Delta x f'(30)$$

$$= 1.866 + \left(-\frac{\pi}{60}\right)(1.232) =$$

Q. 21. $f(x) = \sqrt[3]{65}$

$$f(x_0 + h) \approx f(x_0) + \Delta x f'(x_0)$$

$$f(x) = (x)^{\frac{1}{3}}$$

$$x_0 + h = 65$$

Choosing $x_0 = 64$

$$f(65) \approx f(64) + \Delta x f'(64)$$

$$\Delta x = h = 65 - 64 = 1$$

$$f(x) = (x)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$$

$$f'(64) = \frac{1}{3\,(64)^{\frac{2}{3}}} = \frac{1}{3 \times 16} = \frac{1}{48}$$

$$f(64) = (x)^{\frac{1}{3}} = 4$$

$$f(x_0 + h) \approx f(x_0) + \Delta x f'(x_0)$$

$$\sqrt[3]{65} \approx 4 + \left(\frac{1}{48}\right) = 4.02$$

Q. 25. $f(x) = \cos 59$

$$f(x_0 + h) \approx f(x_0) + \Delta x f'(x_0)$$

$$f(x) = \cos x$$

$$x_0 + h = 59$$

Choosing $x_0 = 60$

$$h = -1°$$

$$180° = \pi$$

$$-1° = \frac{-\pi}{180}$$

$$f(59) \approx f(60) + \Delta x f'(60)$$

$$f'(x) = -\sin x, \quad f'(60) = -0.866$$

$$f(x_0 + h) \approx f(x_0) + \Delta x f'(x_0)$$

$$\cos 50 \approx 0.5 + \left(\frac{-\pi}{180}\right) \times (-0.866) =$$

Quiz 2

From ex 2.8

1 to 8

21-26

QUIZ # 2

A girl starts at a point A and runs east at a rate of 10 ft/sec. One minute later, another girl starts at A and runs north at a rate of 8 ft/sec. At what rate is the distance between them changing 1 min after the second girl starts?

30 MINUTES

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A girl starts at a POINT A and runs East at a rate of 10FT/SEC 10 feet second .one minute later

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