Applied Physics For Engineers

Contents:

Series Circuits

Two components are said to have a *common terminal* when there is a path of zero resistance joining a terminal of one to a terminal of the other. From a practical standpoint, components have a common terminal when there is a very low-resistance wire joining one terminal to another, or if they are soldered together, or if they are connected through some other path having negligibly small resistance. In these cases it is common practice to say that the terminals are *electrically the same*, or *electrically common*, because they have the same electrical properties, even though they may differ physically. On a schematic diagram, the zero-resistance path joining two terminals is shown by a solid line.

Two components are connected in series if they have exactly one common terminal and if no other component has a terminal that shares that common connection. Figure 4.1 shows two examples of series connections. It is easy to form a mental image of two series-connected components, since the word "series" conveys the idea of sequential.

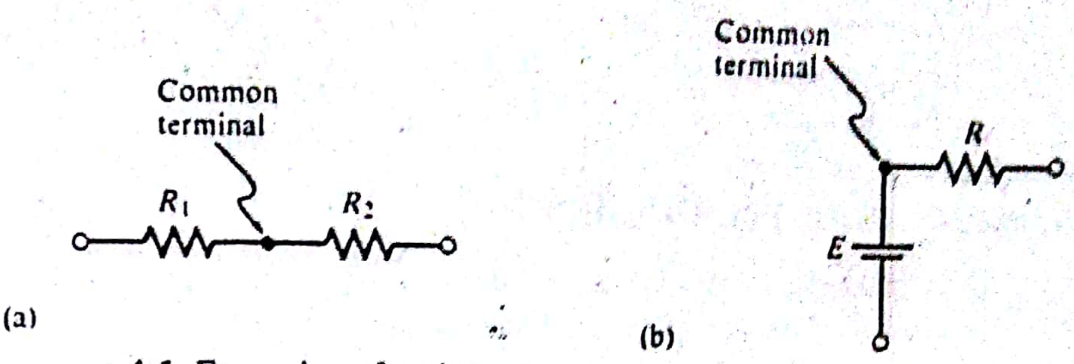


FIGURE 4.1 Examples of series-connected components. (The common terminal is darkened for emphasis, although it is not usually shown in a schematic diagram unless a third component is joined to the terminal.) (a) Resistors R_1 and R_2 are connected in series. (b) Voltage source E and resistor R are connected in series.

or one-after-the-other, and that is certainly the case here. However, beware of the simplicity of this notion: one of the most common errors in circuit analysis is to believe that two components are in series when they are not. When in doubt, make the tests implied by the formal definition: Do the components have one and only one terminal in common? Is any other component joined to that common point?

A *series path* is one in which every component in the path is in series with another component. It is conventional to say, simply, that components are "in series" if they are in the same series path, even though they may not have a common terminal. A *series circuit* is a (complete) circuit that consists *exclusively* of one series path. Lest the reader feel that we are belaboring this basically simple concept of a series connection, we emphasize again that failure to recognize it is the most frequent source of error in elementary circuit analysis.

Figure 4.2 shows some examples of series and nonseries connections, as well as some series and nonseries paths and circuits. Each example should be studied carefully and tested for compliance with the definitions we have given.

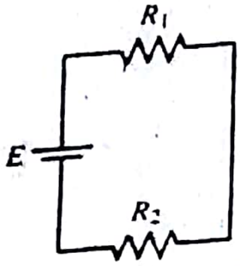
Analysis of Series Circuits

The most important property of a series path or circuit is that *the current is the same in every series-connected component*. We know from the definition that no third component can be connected to the common terminal of a series connection, so there is no way that current can be injected into or drawn away from the point where two series components are connected. Therefore, the current in one component must be exactly the same as the current in the component in series with it.

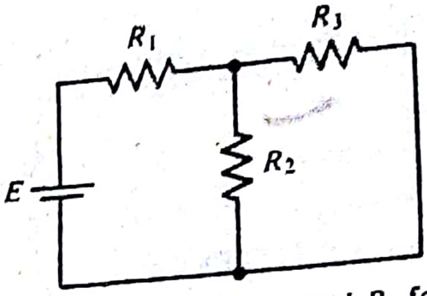
Another important fact about a series circuit is that its total resistance is the *sum* of all the series-connected resistances:

$$R_T = R_1 + R_2 + R_3 + \cdots \tag{4.1}$$

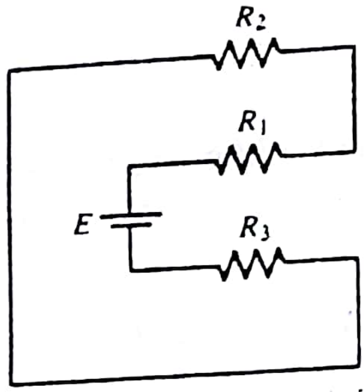
where R_T is the total resistance, and $R_1, R_2, R_3, \ldots$ are the resistances of the series-connected components. When a voltage source is connected in a series circuit, the total current produced by that source is, from Ohm's law,



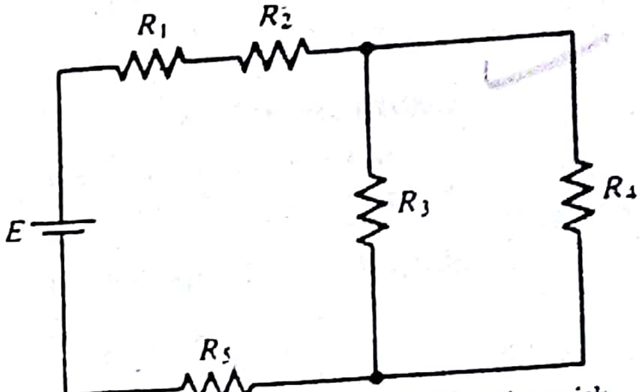
(a) E is in series with R_1. R_1 is in series with R_2. R_2 is in series with E. E, R_1, and R_2 form a series path and a series circuit. Every component is in series with every other component.



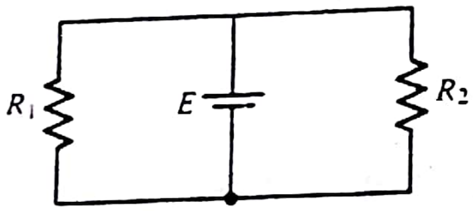
(b) E is in series with R_1. E and R_1 form a series path. R_2 and R_3 are not in series. R_1 is not in series with either R_2 or R_3. E is not in series with either R_2 or R_3. There is no series circuit.



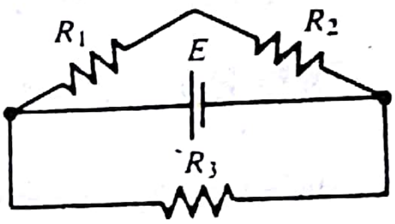
(c) E is in series with R_1. R_1 is in series with R_2. R_2 is in series with R_3. R_3 is in series with E. R_1, R_2, R_3 and E form a series path and a series circuit. Every component is in series with every other component.



(d) E is in series with R_1. R_1 is in series with R_2. R_3 and R_4 are not in series with each other or with any other component. R_5 is in series with E. R_5, E, R_1, and R_2 form a series path, so each is in series with the others. There is no series circuit.



(e) There are no series-connected components. There are no series paths or series circuits.

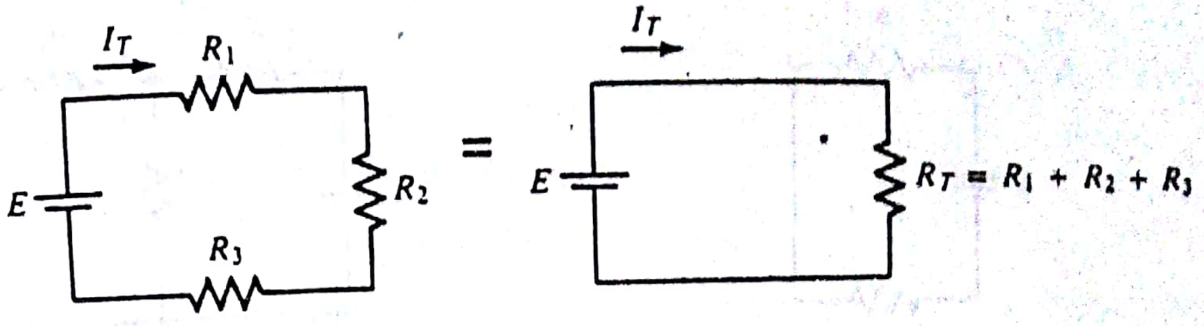


(f) R_1 is in series with R_2. R_1 and R_2 form a series path. No other components are in series. There is no series circuit.

FIGURE 4.2 Examples of series and nonseries connections, paths, and circuits.

$$I_T = \frac{E}{R_T} \tag{4.2}$$

In other words, the current that flows in a series circuit is the same current that flows when a *single* resistor having resistance R_T is connected in series with the voltage source. We say that R_T is *equivalent* to all the series-connected resistors. As far as the voltage source is concerned, there is no difference between three series-connected 100-Ω resistors, two series-connected 150-Ω resistors, or a single 300-Ω resistor. All are equivalent to 300 Ω, and this idea is often conveyed by saying that the voltage source "sees" 300 Ω in all cases. Figure 4.3 illustrates these concepts.



$$I_T = \frac{E}{R_1 + R_2 + R_3} = \frac{E}{R_T}$$

FIGURE 4.3 The total equivalent resistance of series-connected resistors is the sum of their resistance values.

Example 4.1 (Analysis)

A 15-V source is connected in series with the following resistances: 1 kΩ, 500 Ω, 3.3 kΩ, and 2700 Ω. How much current flows in the 3.3-kΩ resistance?

SOLUTION The total resistance of the circuit is

$$R_T = 1\ \text{k}\Omega + 0.5\ \text{k}\Omega + 3.3\ \text{k}\Omega + 2.7\ \text{k}\Omega = 7.5\ \text{k}\Omega$$

Therefore, the total current is

$$I_T = \frac{E}{R_T} = \frac{15\ \text{V}}{7.5 \times 10^3\ \Omega} = 2 \times 10^{-3}\ \text{A} = 2\ \text{mA}$$

Since the circuit is a series circuit, the current is the same in every component. Thus, the current in 3.3-kΩ resistor (and every other resistor) is 2 mA.

Drill Exercise 4.1

The 2700-Ω resistor in Example 4.1 is replaced by another resistor and the total current becomes 2.5 mA. What value of resistance replaced the 2700-Ω resistor? □

ANSWER: 1.2 kΩ.

Example 4.2 (Design)

It is necessary to limit the current in a certain light-emitting diode (LED) to 5 mA. The resistance of the LED is 250 Ω and it is connected in series with a 5-V source. How much resistance should be inserted in series with the LED? (See Figure 4.4.)

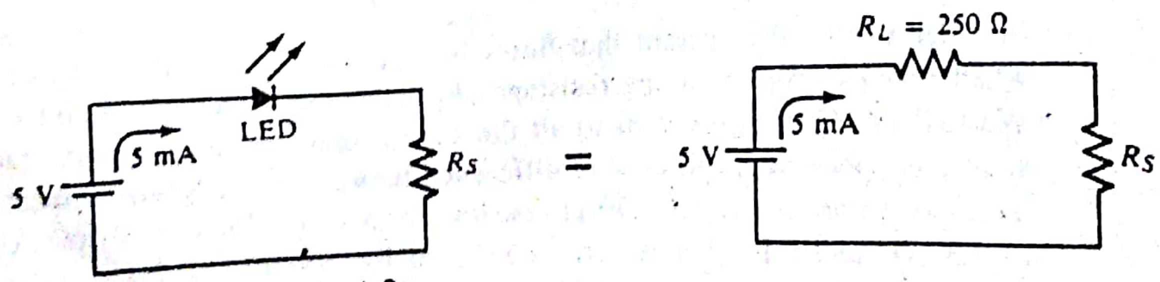


FIGURE 4.4 (Example 4.2)

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SOLUTION Let R_S be the required series resistance and R_L be the LED resistance. The total resistance of the series combination will then be

$$R_T = R_S + R_L = R_S + 250$$

Therefore, $R_S = R_T - 250$. The total resistance, R_T, must limit the current to 5 mA:

$$I_T = \frac{E}{R_T}$$

$$5 \text{ mA} = \frac{5 \text{ V}}{R_T} \Rightarrow R_T = \frac{5 \text{ V}}{5 \text{ mA}} = 1 \text{ k}\Omega$$

Therefore,

$$R_S = R_T - 250 = 1000 - 250 = 750 \ \Omega$$

Drill Exercise 4.2

What *additional* resistance should be inserted in series with the value of R_S found in Example 4.2 if it is desired to limit the current in the LED to 2.5 mA?

ANSWER: 1 kΩ. (As $I = \frac{V}{R}$ so $I \propto \frac{1}{R}$) □

Once we have found the current in a series circuit, we can find the voltage across each resistance in the circuit using Ohm's law. The next example illustrates this fact.

Example 4.3 (Analysis)

Find the current in and voltage across each resistor in the circuit of Figure 4.5.

SOLUTION

$$R_T = R_1 + R_2 + R_3 + R_4 = 12 \ \Omega + 6 \ \Omega + 10 \ \Omega + 20 \ \Omega = 48 \ \Omega$$

$$I_T = \frac{E}{R_T} = \frac{24 \text{ V}}{48 \ \Omega} = 0.5 \text{ A}$$

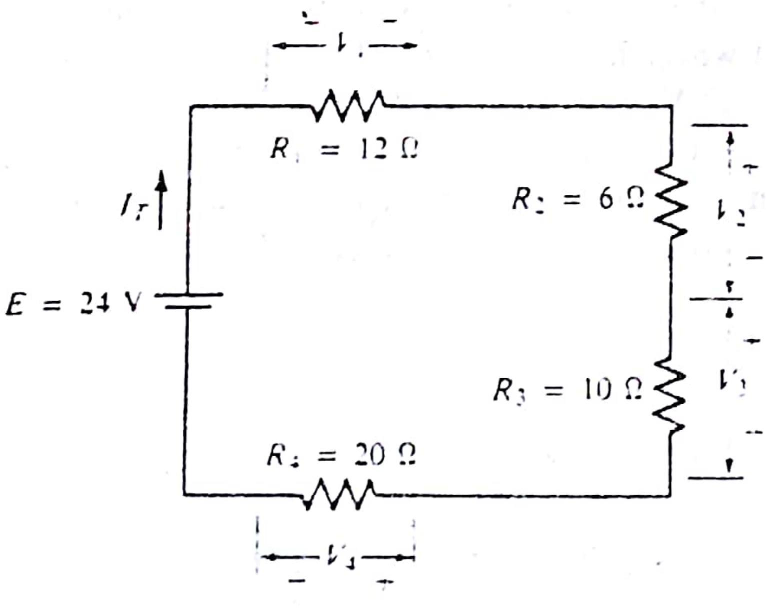

FIGURE 4.5 (Example 4.3)

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Since the circuit is a series circuit, the current in each resistor is 0.5 A. Applying Ohm's law to each, we find

$$V_1 = (I_T)(R_1) = (0.5 \text{ A})(12 \ \Omega) = 6 \text{ V}$$
$$V_2 = (I_T)(R_2) = (0.5 \text{ A})(6 \ \Omega) = 3 \text{ V}$$
$$V_3 = (I_T)(R_3) = (0.5 \text{ A})(10 \ \Omega) = 5 \text{ V}$$
$$V_4 = (I_T)(R_4) = (0.5 \text{ A})(20 \ \Omega) = 10 \text{ V}$$

Notice that Figure 4.5 shows + and − polarity symbols attached to each of the voltage designations V_1 through V_4. These are the polarities that should be observed when connecting a voltmeter to measure the voltage across each resistor. In each case we say that the voltage is *referenced* to the side where the minus sign appears.

Drill Exercise 4.3

If the resistance of R_1 in Figure 4.5 were doubled, what would be the new voltage across it?

ANSWER: 9.6 V. □

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