Applied Physics For Engineers

Contents:

The Voltage-Divider Rule

Consider the series circuit shown in Figure 4.18. We derive expressions for the voltages V_1 and V_2 across resistors R_1 and R_2 as follows:

$$I = \frac{E}{R_T} = \frac{E}{R_1 + R_2}$$

$$V_1 = IR_1 = \frac{E}{R_1 + R_2} R_1 \qquad (4.4)$$

$$V_2 = IR_2 = \frac{E}{R_1 + R_2} R_2 \qquad (4.5)$$

Notice that equations (4.4) and (4.5) allow us to compute the voltage drop across either resistor without first computing the current in the circuit. The equations show that the voltage across a resistance is the source voltage E times the ratio of that resistance to the total series resistance. We can generalize this result to a series circuit containing any number of resistors, as illustrated in Figure 4.19(a). The result is called the *voltage-divider rule: The voltage across any resistance in a series circuit is the source voltage times the ratio of that resistance to the total resistance of the circuit.* A series circuit can be called a *voltage divider* because the total voltage is divided among the various resistors in direct proportion to the resistance of each. In equation form, the voltage-divider rule is expressed as

$$V_x = E\frac{R_x}{R_T} \qquad (4.6)$$

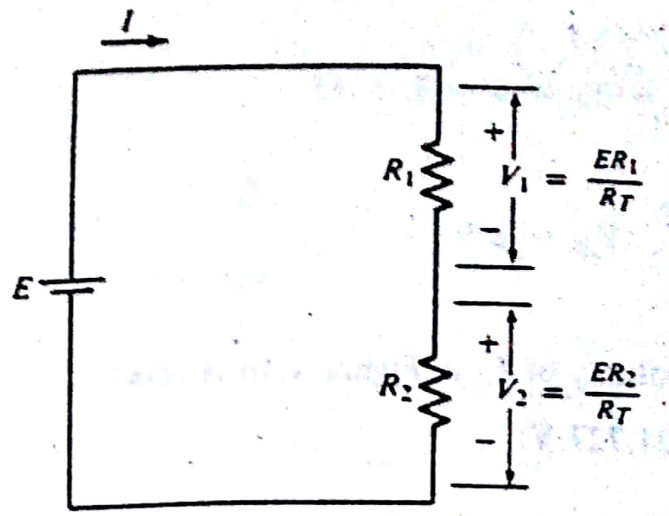

FIGURE 4.18 The voltage across each resistor in a series circuit can be found without knowing the current in the circuit. The voltage E divides between R_1 and R_2 in direct proportion to the resistance of each.

where V_x is the voltage drop across resistance R_x in the series circuit. Note that R_x may also represent the total resistance of any two or more successive resistors, in which case V_x is the voltage drop across the combination of those resistors. This interpretation is illustrated in Figure 4.19(b).

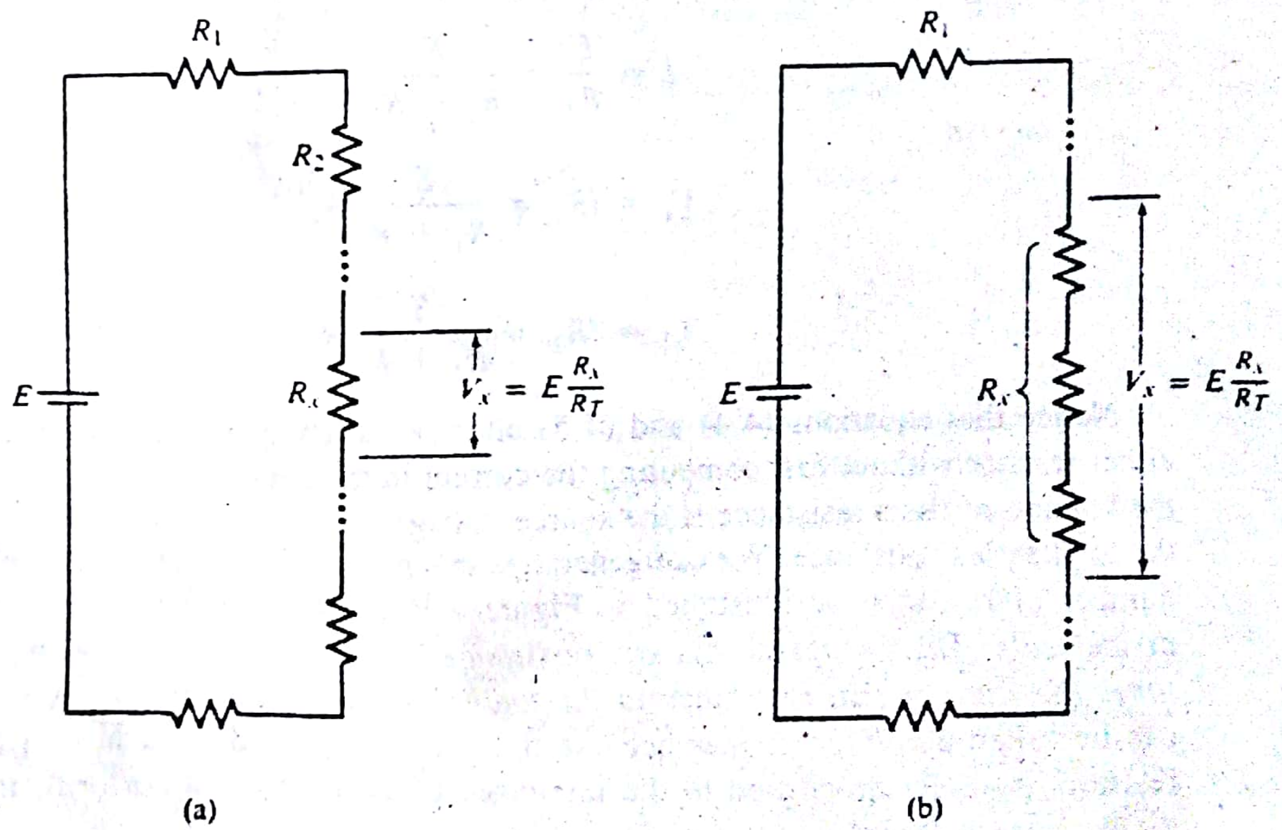

FIGURE 4.19 Voltage-divider rule. (a) V_x is the voltage across any single resistance in a series circuit. (b) V_x is the voltage across any number of successive resistances in a series circuit.

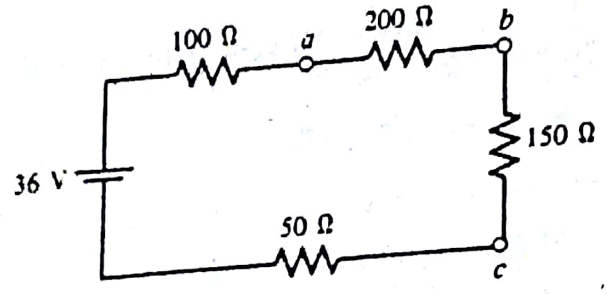

FIGURE 4.20 (Example 4.9)

Example 4.9 (Analysis)

Use the voltage-divider rule to find voltages V_{ab} and V_{ac} in Figure 4.20.

SOLUTION

$$R_T = 100\ \Omega + 200\ \Omega + 150\ \Omega + 50\ \Omega = 500\ \Omega$$

$$V_{ab} = (36\ \text{V})\left(\frac{200}{500}\right) = 14.4\ \text{V}$$

$$V_{ac} = (36\ \text{V})\left(\frac{200 + 150}{500}\right) = 25.2\ \text{V}$$

Drill Exercise 4.9

Find V_{ca} and V_{bc} in Figure 4.20.

ANSWER: $V_{ca} = -25.2\ \text{V}$, $V_{bc} = 10.8\ \text{V}$. □

Example 4.10 (Design)

A certain electronic device is activated when a voltage level of 5 V ± 10% is applied to it. In one application where it is used, the only dc power available is a 24-V source.

(a) Design a voltage divider that will provide the required activation voltage across a resistor. The voltage divider must not draw more than 10 mA from the 24-V source.

(b) Assuming that only standard-value resistors having 5% tolerance can be used in the circuit, draw the schematic diagram of the final design. Verify that it satisfies the design criteria.

SOLUTION

(a) Since the total current in the voltage divider cannot exceed 10 mA, we require

$$\frac{24\ \text{V}}{R_T} \le 10\ \text{mA}$$

or

$$R_T \ge \frac{24\ \text{V}}{10 \times 10^{-3}\ \Omega} = 2.4\ \text{k}\Omega$$

We will design a two-resistor divider so that 5 V is dropped across one of them. Let R_2 be that resistor and let R_1 be the other. Then

$$R_T = R_1 + R_2 \geq 2.4\ \text{k}\Omega$$

and, by the voltage-divider rule, we require

$$\frac{24\,R_2}{R_T} = 5\ \text{V}$$

Let us choose $R_2 = 2.4\ \text{k}\Omega$, since we know that this choice will satisfy $R_1 + R_2 \geq 2.4\ \text{k}\Omega$, whatever the value of R_1. Then

$$\frac{(24)(2.4 \times 10^3)}{R_T} = 5\ \text{V}$$

or

$$R_T = \frac{(24)(2.4 \times 10^3)}{5} = 11.52\ \text{k}\Omega$$

Finally,

$$R_1 = R_T - R_2 = 11.52\ \text{k}\Omega - 2.4\ \text{k}\Omega = 9.12\ \text{k}\Omega$$

(b) From the table of standard resistor values given in Chapter 3, we see that 2.4 kΩ is a standard 5% value and that the standard value closest to 9.12 kΩ is 9.1 kΩ. Using these values, our completed design is shown in Figure 4.21. Checking the design, we find

$$I_T = \frac{24\ \text{V}}{2.4 \times 10^3 + 9.1 \times 10^3} = 2.09\ \text{mA} \leq 10\ \text{mA}$$

$$V_1 = \frac{(2.4 \times 10^3)24\ \text{V}}{2.4 \times 10^3 + 9.1 \times 10^3} = 5.009\ \text{V} = 5\ \text{V} \pm 10\%$$

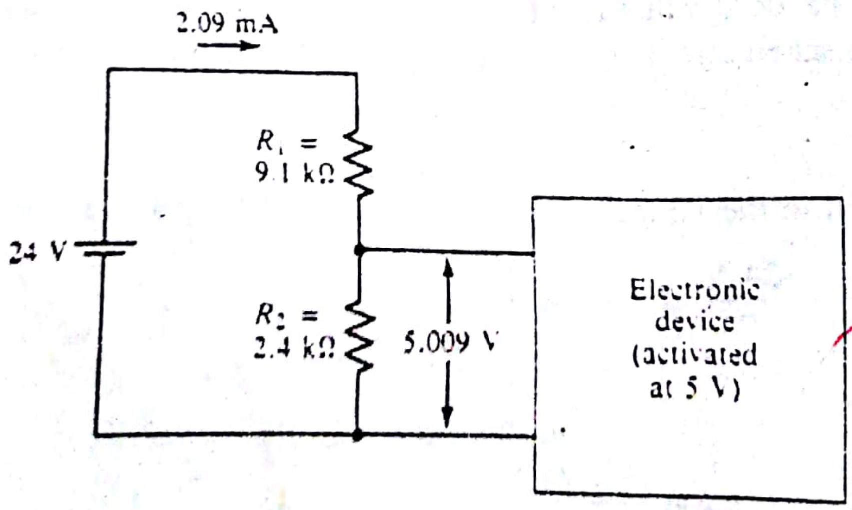

FIGURE 4.21 (Example 4.10)

Potentiometers and Rheostats

A *potentiometer* is an adjustable voltage divider that is widely used in a variety of electronic circuit applications. Figure 4.22(a) shows the schematic symbol for a potentiometer and gives some idea of its principle of operation. Notice that it is a *three-terminal* device. A voltage source E is connected across the fixed resistance R_T between terminals a and c. A terminal labeled b is attached to a movable contact (called the *wiper arm*, or *slider*) which can be adjusted so that it contacts the resistance between a and c at any point along its entire length. Visualize the contact as sliding up or down between the extreme end positions at terminals a and c. As the contact is moved down, the resistance between b and c, R_{bc}, decreases and the resistance between a and b, R_{ab}, increases. No matter where the contact is positioned,

$$R_T = R_{ab} + R_{bc} \tag{4.7}$$

As illustrated in Figure 4.22(b), we can apply the voltage-divider rule to the potentiometer to determine the voltage V_{bc} between adjustable contact b and fixed contact c:

$$V_{bc} = \frac{R_{bc}}{R_T} E \tag{4.8}$$

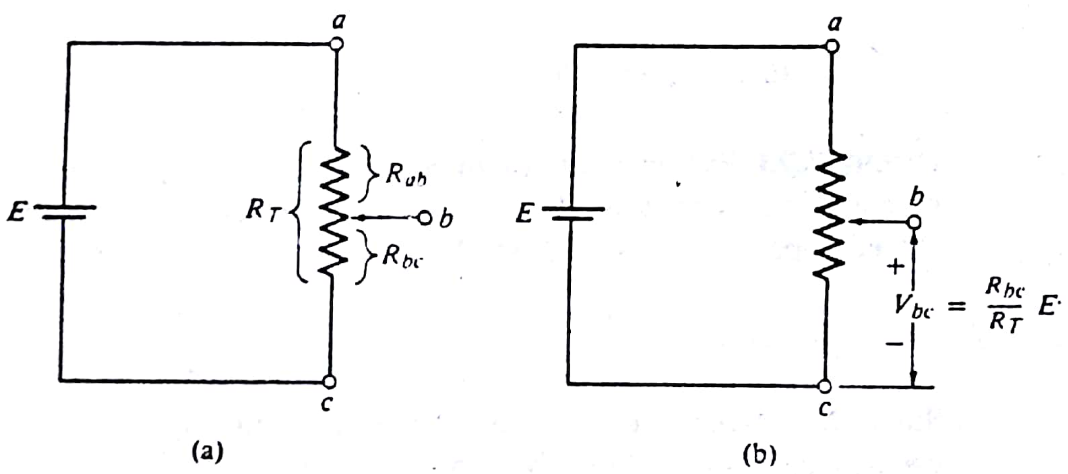

FIGURE 4.22 A potentiometer is an adjustable voltage divider. As the adjustable contact b is moved down, R_{bc} decreases and the voltage V_{bc} decreases. (a) $R_T = R_{ab} + R_{bc}$. (b) V_{bc} can be found using the voltage-divider rule.

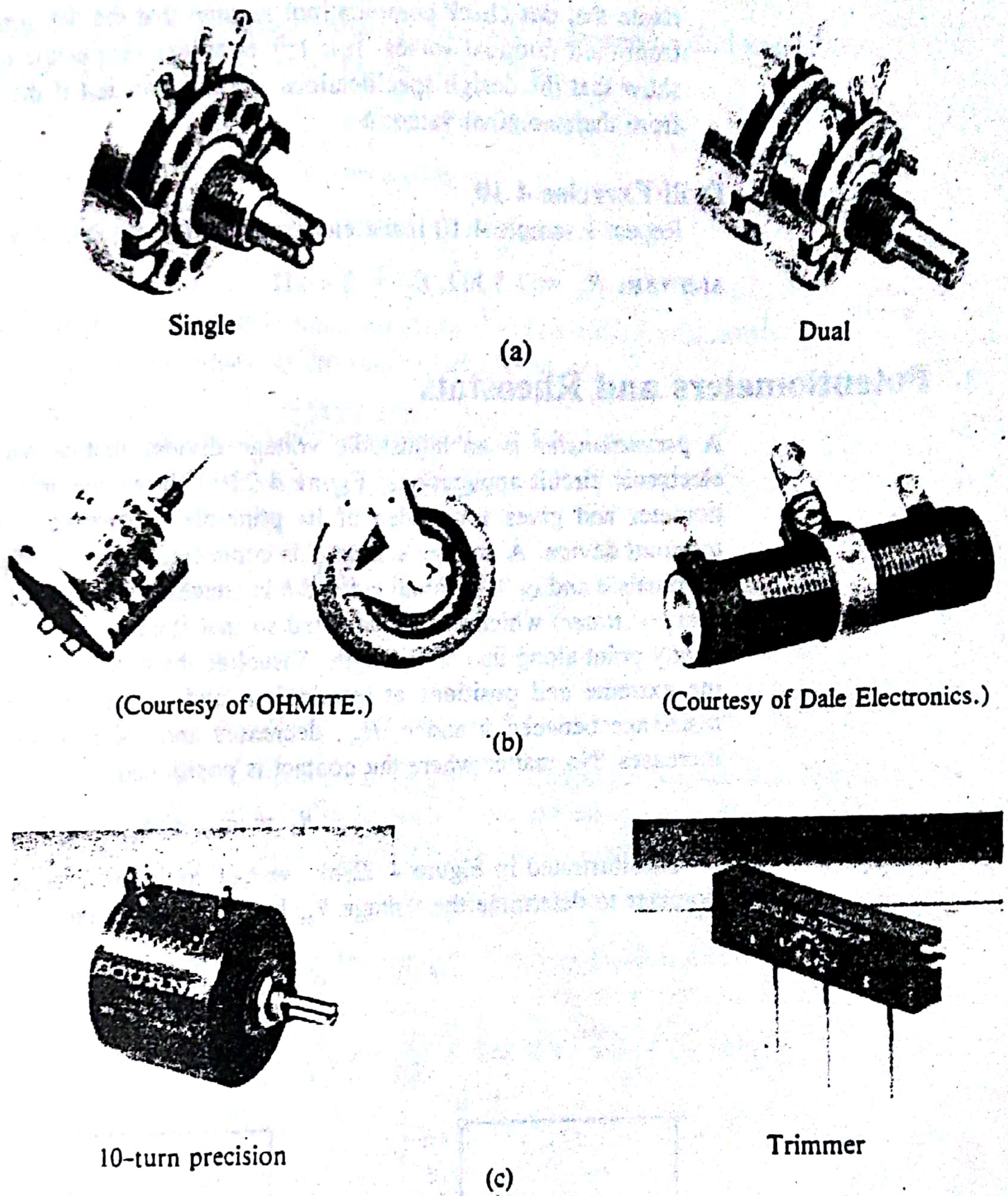

FIGURE 4.23 Examples of commercially available potentiometers. (a) Carbon composition. (Courtesy of Allen-Bradley, a Rockwell International Subsidiary.) (b) High-power, wirewound. (c) Multi-turn. (Courtesy of Bourns Precisions/Controls.)

Since R_T remains constant, the voltage V_{bc} is directly proportional to resistance R_{bc} and can be adjusted by moving the wiper arm to a desired position. When the arm is positioned at the "top," $R_{bc} = R_T$ and $V_{bc} = E$. When it is positioned at the bottom, $R_{bc} = 0$ and $V_{bc} = 0$. Thus, V_{bc} can be adjusted to any voltage between 0 and E.

Potentiometers are available in a wide variety of sizes, resistance and power ratings, and *tapers*. The latter term refers to the way the resistance between the wiper arm and

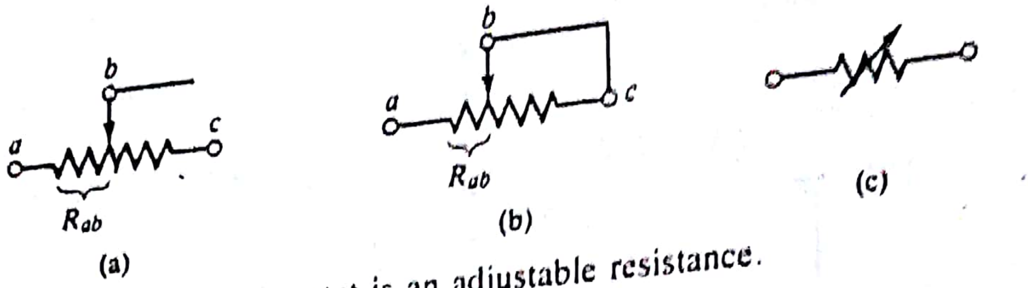

FIGURE 4.24 A rheostat is an adjustable resistance.

one end varies as the arm is moved. In some devices the variation is *linear*, meaning that R_{bc} is directly proportional to the position of the arm, while in others, the resistance may be logarithmically related to position. Figure 4.23 shows some typical, commercially available potentiometers. In practical devices, it is not possible to obtain exactly 0 V or exactly E volts at the extreme end positions, because there is always a small end resistance that the wiper arm cannot contact.

A *rheostat* is an adjustable resistance. Like a potentiometer, it has an adjustable contact that can be moved to different positions to change the resistance between a pair of terminals. However, since its sole purpose is to provide adjustable resistance, it has only two terminals: the two ends of the resistance. Rheostats are commercially available components, usually manufactured in large sizes for heavy power applications. In electronic circuits, adjustable resistance is obtained far more often by connecting a potentiometer as a rheostat. Figure 4.24 shows two ways this connection can be made. In Figure 4.24(a), terminal c is simply left open, and the adjustable resistance is R_{ab}. In Figure 4.24(b), terminal b is connected to terminal c. This connection *shorts out* R_{bc} and, as we shall see in a forthcoming discussion, the result is that the resistance between terminals b and c remains zero, regardless of wiper arm position. Thus, the adjustable resistance is again R_{ab}. Figure 4.24(c) shows the schematic symbol for a reheostat. An arrow drawn through a device symbol is the standard convention for representing adjustable components of all types.

Example 4.11 (Analysis)

Figure 4.25(a) shows a 10-kΩ potentiometer connected in a series circuit as an adjustable voltage divider, and Figure 4.25(b) shows the same device connected in a series circuit as a rheostat.

(a) What total range of voltage V_1 can be obtained in Figure 4.25(a) by adjusting the potentiometer through its entire range?

(b) What total range of voltage V_2 can be obtained in Figure 4.25(b) by adjusting the rheostat through its entire range?

SOLUTION

(a) We first find the total voltage E that appears across the end terminals of the potentiometer. By the voltage-divider rule,

$$E = \left(\frac{10\ \text{k}\Omega}{5\ \text{k}\Omega + 10\ \text{k}\Omega + 10\ \text{k}\Omega}\right) 24\ \text{V} = 9.6\ \text{V}$$

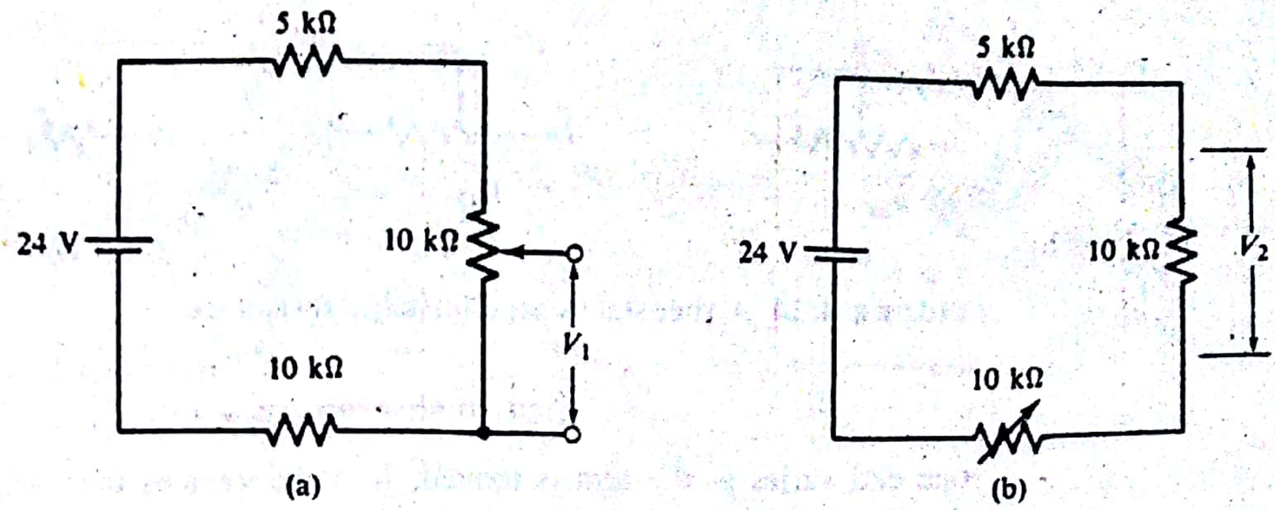

FIGURE 4.25 (Example 4.11)

When the wiper arm is at the top of the potentiometer,

$$V_1 = \left(\frac{10\text{ k}\Omega}{10\text{ k}\Omega}\right) 9.6\text{ V} = 9.6\text{ V}$$

When the wiper arm is at the bottom of the potentiometer,

$$V_1 = \left(\frac{0}{10\text{ k}\Omega}\right) 9.6\text{ V} = 0\text{ V}$$

Thus, V_1 can be adjusted between 0 and 9.6 V. We express this range by writing $0\text{ V} \le V_1 \le 9.6\text{ V}$.

(b) Let R_a be the adjustable resistance of the rheostat. By the voltage-divider rule,

$$V_2 = \left(\frac{10\text{ k}\Omega}{5\text{ k}\Omega + 10\text{ k}\Omega + R_a}\right) 24\text{ V}$$

When $R_a = 0$,

$$V_2 = \left(\frac{10\text{ k}\Omega}{15\text{ k}\Omega}\right) 24\text{ V} = 16\text{ V}$$

When $R_a = 10\text{ k}\Omega$.

$$V_2 = \left(\frac{10\text{ k}\Omega}{25\text{ k}\Omega}\right) 24\text{ V} = 9.6\text{ V}$$

Thus, V_2 can be adjusted between 9.6 and 16 V: $9.6\text{ V} \le V_2 \le 16\text{ V}$.

Drill Exercise 4.11

Repeat Example 4.11 if V_1 is measured across the 5-kΩ resistor in Figure 4.25(a) and V_2 is measured across the 5-kΩ resistor in Figure 4.25(b).

ANSWER: $V_1 = 4.8$ V at all settings of the potentiometer; (b) $4.8\text{ V} \le V_2 \le 8\text{ V}$. □