Applied Physics For Engineers (PHY-121)

Lecture 02: Scientific and Engineering Notation

Scientific and Engineering Notation

Objectives

1. To show how very large or very small numbers can be expressed in scientific notation (OR in power of 10!)

Scientific Notation

- Very large or very small numbers can be expressed using scientific notation
  - The number is written as a number between 1 and 10 multiplied by 10 raised to a power.

    e.g. 7200 is 7.2×10^3
  - The power of 10 depends on:
    - The number of places the decimal point is moved.
    - The direction the decimal point is moved.

Left $\Rightarrow$ Positive exponent
Right $\Rightarrow$ Negative exponent

Scientific Notation

- Representing Large Numbers

93,000,000 miles from the Earth to the Sun
(sunlight takes 8 minutes to reach us)

93,000,000 $= 9.3 \times 10{,}000{,}000$

$= 9.3 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 9.3 \times 10^7$ (Decimal point moved 7 digits to the left)

Number between 1 and 10

Appropriate power of ten

Scientific Notation

- Representing Small Numbers

  0.000167

  To obtain a number between 1 and 10 we must move the decimal point to the right.

0 . 0 0 0 1 6 7

1 2 3 4

$$0.000167 = 1.67 \times 10^{-4}$$

10^{-4} = 1/10000 (one ten-thousandth)

Convert the following numbers between normal and scientific notation:

329

700,000

20090

0.000034

0.01023

123.4

45.607

1.7×10^{3}

2.4503×10^{5}

7.9×10^{8}

2.8×10^{-3}

7.45×10^{-1}

2.3×10^{-7}

Convert the following numbers between normal and scientific notation:

329	3.29x10^2	1.7×10^{3}	1700
700,000	7x10^5	2.4503×10^{5}	245030
20090	20.090x10^3	7.9×10^{8}	790000000
0.000034	3.4x10^-5	2.8×10^{-3}	.0028
0.01023	10.23x10^-3	7.45×10^{-1}	0.745
123.4	12.34x10^1	2.3×10^{-7}	0.0000023
45.607	4560.7x10^-2		

Scientific and Engineering Notation

Very large and very small numbers are represented with scientific and engineering notation.

Example-1 $47{,}000{,}000 = 4.7 \times 10^{7}$ (Scientific Notation)

$= 47. \times 10^{6}$ (Engineering Notation)

Prefixes used for Engineering notations

Multiplication factor (scientific notation)	Prefix	Symbol
(10^{24})	yotta	Y
(10^{21})	zetta	Z
(10^{18})	exa	E
(10^{15})	peta	P
(10^{12})	tera	T
1 000 000 000 (10^{9})	giga	G
1 000 000 (10^{6})	mega	M
1000 (10^{3})	kilo	k
100 (10^{2})	hecto	h
10 (10^{1})	deka	da
0.1 (10^{-1})	deci	d
0.01 (10^{-2})	centi	c
0.001 (10^{-3})	milli	m
0.000 001 (10^{-6})	micro	µ
0.000 000 001 (10^{-9})	nano	n
(10^{-12})	pico	p
(10^{-15})	femto	f
(10^{-18})	atto	a
(10^{-21})	zepto	z
(10^{-24})	yocto	y

Scientific and Engineering Notation

Example-2

$0.000\,027 = 2.7 \times 10^{-5}$ (Scientific Notation)

$= 27 \times 10^{-6}$ (Engineering Notation)

Example-3

$0.605 = 6.05 \times 10^{-1}$ (Scientific Notation)

$= 605 \times 10^{-3}$ (Engineering Notation)

Metric Conversions

When converting from a larger unit to a smaller unit, move the decimal point to the right. Remember, a smaller unit means the number must be larger.

Example-1

Smaller unit

$$0.47\ \text{M}\Omega = 470\ \text{k}\Omega$$

Larger number

Metric Conversions

When converting from a smaller unit to a larger unit, move the decimal point to the left. Remember, a larger unit means the number must be smaller.

Example-2

Larger unit

$$10{,}000\ \text{pF} = 0.01\ \mu\text{F}$$

Smaller number

Metric Arithmetic

When adding or subtracting numbers with a metric prefix, convert them to the same prefix first.

Example-1

$$10{,}000\ \Omega + 22\ \mathrm{k}\Omega =$$

$$10{,}000\ \Omega + 22{,}000\ \Omega = 32{,}000\ \Omega$$

Alternatively,

$$10\ \mathrm{k}\Omega + 22\ \mathrm{k}\Omega = 32\ \mathrm{k}\Omega$$

Metric Arithmetic

When adding or subtracting numbers with a metric prefix, convert them to the same prefix first.

Example-2

$$200\ \mu A + 1.0\ mA =$$

$$200\ \mu A + 1{,}000\ \mu A = 12{,}000\ \mu A$$

Alternatively,

$$0.200\ mA + 1.0\ mA = 1.2\ mA$$

Practice

- 0.0065 km -→ meters
- 1750000 micro volts ----→ kilo volts
- 5.02 x 10^3 pico seconds -> nano seconds
- 3800 cm^2 -→ m^2
- 1.84 in^3 → cm^3 → m^3 [hint: 1in=2.54cm , 1m=100cm]
- 763 mi/h → m/s { 1 mi=1.609KM }

Unit consistency and conversions

- An equation must be *dimensionally consistent*. Terms to be added or equated must *always* have the same units. (Be sure you're adding "apples to apples.")
- Always carry units through calculations.

Example 1.1 Converting speed units

The world land speed record is 763.0 mi/h, set on October 15, 1997, by Andy Green in the jet-engine car *Thrust SSC*. Express this speed in meters per second.

SOLUTION

IDENTIFY, SET UP, and EXECUTE: We need to convert the units of a speed from mi/h to m/s. We must therefore find unit multipliers that relate (i) miles to meters and (ii) hours to seconds. In Appendix E (or inside the front cover of this book) we find the equalities 1 mi = 1.609 km, 1 km = 1000 m, and 1 h = 3600 s. We set up the conversion as follows, which ensures that all the desired cancellations by division take place:

$$763.0\ \text{mi/h} = \left(763.0\ \frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(\frac{1.609\ \cancel{\text{km}}}{1\ \cancel{\text{mi}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)$$

$$= 341.0\ \text{m/s}$$

Example 1.2 Converting volume units

The world's largest cut diamond is the First Star of Africa (mounted in the British Royal Sceptre and kept in the Tower of London). Its volume is 1.84 cubic inches. What is its volume in cubic centimeters? In cubic meters?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: Here we are to convert the units of a volume from cubic inches (in.^3) to both cubic centimeters (cm^3) and cubic meters (m^3). Appendix E gives us the equality $1\ \text{in.} = 2.540\ \text{cm}$, from which we obtain $1\ \text{in.}^3 = (2.54\ \text{cm})^3$. We then have

$$1.84\ \text{in.}^3 = (1.84\ \text{in.}^3)\left(\frac{2.54\ \text{cm}}{1\ \text{in.}}\right)^3$$

$$= (1.84)(2.54)^3\ \frac{\cancel{\text{in.}^3}\ \text{cm}^3}{\cancel{\text{in.}^3}} = 30.2\ \text{cm}^3$$

Appendix F also gives us $1\ \text{m} = 100\ \text{cm}$, so

$$30.2\ \text{cm}^3 = (30.2\ \text{cm}^3)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3$$

$$= (30.2)\left(\frac{1}{100}\right)^3\ \frac{\cancel{\text{cm}^3}\ \text{m}^3}{\cancel{\text{cm}^3}} = 30.2 \times 10^{-6}\ \text{m}^3$$

$$= 3.02 \times 10^{-5}\ \text{m}^3$$

Example 1.3 Significant figures in multiplication

The rest energy E of an object with rest mass m is given by Einstein's famous equation $E = mc^2$, where c is the speed of light in vacuum. Find E for an electron for which (to three significant figures) $m = 9.11 \times 10^{-31}$ kg. The SI unit for E is the joule (J); $1\ \text{J} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2$.

SOLUTION

IDENTIFY and SET UP: Our target variable is the energy E. We are given the value of the mass m; from Section 1.3 (or Appendix F) the speed of light is $c = 2.99792458 \times 10^8$ m/s.

EXECUTE: Substituting the values of m and c into Einstein's equation, we find

$$\begin{aligned}
E &= (9.11 \times 10^{-31}\ \text{kg})(2.99792458 \times 10^8\ \text{m/s})^2 \\
&= (9.11)(2.99792458)^2(10^{-31})(10^8)^2\ \text{kg} \cdot \text{m}^2/\text{s}^2 \\
&= (81.87659678)(10^{[-31+(2\times 8)]})\ \text{kg} \cdot \text{m}^2/\text{s}^2 \\
&= 8.187659678 \times 10^{-14}\ \text{kg} \cdot \text{m}^2/\text{s}^2
\end{aligned}$$

Since the value of m was given to only three significant figures, we must round this to

$$E = 8.19 \times 10^{-14}\ \text{kg} \cdot \text{m}^2/\text{s}^2 = 8.19 \times 10^{-14}\ \text{J}$$

Significant Figures

- The number of significant digits in a number is the number of digits used to express it.
- The more sig: digits used to express measured quantity the greater the Precision of that measurement.

3050. has 4 sig. figs.

Rules for Counting Significant Figures

- Nonzero integers are always significant
- Zeros
  - Leading zeros never count as significant figures
  - Captive zeros are always significant
  - Trailing zeros are significant if the number has a decimal point.

- **Nonzero integers always count as significant figures.**
- 3456 has
- 4 sig. figs.

- **Zeros**
- **Captive zeros always count as significant figures**.
- 16.07 has
- 4 sig. figs.

- **Trailing zeros are significant only if the number contains a decimal point.**
- 9.300 has 4 sig. Figs.
- 3050 has 3 sig. Figs.
- 3050. has 4 sig. figs.

Multiplication/Division with Significant Figures

- Result has the same number of significant figures as the measurement with the *smallest number* of significant figures
- Count the number of significant figures in each measurement
- Round the result so it has the same number of significant figures as the measurement with the *smallest number* of significant figures

$$\underset{\text{2 sig figs}}{4.5\ \text{cm}} \times \underset{\text{3 sig figs}}{0.200\ \text{cm}} = \underset{\text{2 sig figs}}{0.90\ \text{cm}^2}$$

Adding/Subtracting Numbers with Significant Figures

- Result is limited by the number with the smallest number of significant decimal places
- Round answer to the same decimal place

4.071 v + 2.0 v = 6.071 → 6.1 v

(round off result to make it upto 1 decimal place

Rules for Significant Figures in Mathematical Operations

- **Addition and Subtraction:** # sig. figs. in the result equals the number of decimal places in the least precise measurement. Round answer to the same decimal place

$$6.8 + 11.984 =$$

$$18.784 \rightarrow 18.8 \text{ (3 sig. figs.)}$$

Rules for Significant Figures in Mathematical Operations

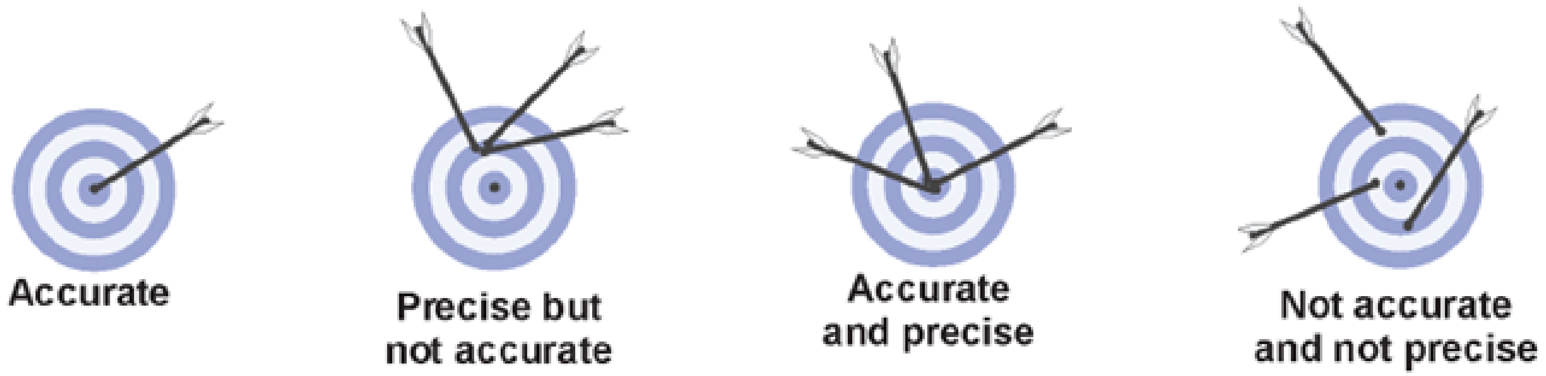
- **Multiplication and Division:** # sig. figs in the result equals the number (of significant figures) in the least precise measurement used in the calculation.

6.38 X2.0 =

12.76 -> 13 (2 sig. figs.)

Accuracy and Precision

- accuracy
  - is the quality of being exact and free from error.
  - how close a measurement is to the true value.
- precision
  - is the degree of mutual agreement among a series of individual measurements, values, or results.



Algebra of Inequalities

An **inequality** compares two expressions using $<$, $>$, $\leq$, or $\geq$.

Symbol	Meaning	Word Phrases
$<$	is less than	Fewer than, below
$>$	is greater than	More than, above
$\leq$	is less than or equal to	At most, no more than
$\geq$	is greater than or equal to	At least, no less than

An inequality that contains a variable is an **algebraic inequality**.

Solving Inequalities

Rule #1

- ***Don't forget who the bigger number is***
  - Example:

    $$9 > x$$

  - It is okay to rewrite this statement as

    $$x < 9$$

  - If 9 is bigger than "x", that means that "x" is smaller than 9.

Rule #2

- ***When multiplying or dividing by a negative number, reverse the inequality sign.***
  - Example:

$$\frac{-5x}{-5} > \frac{15}{-5}$$

$$X < -3$$

Example 1:

$$\begin{array}{rcr} m + 14 & < & 4 \\ -14 & & -14 \\ \hline m & < & -10 \end{array}$$

Example 2:

$$\begin{array}{rcr} -7 & \geq & y - 1 \\ +1 & & +1 \\ \hline -6 & \geq & y \end{array}$$

$$y \leq -6$$

Example 3:

$$(-3)\frac{k}{-3} < 10(-3)$$

$$k > -30$$

Example 4:

$$\begin{array}{rcr} 2x + 5 & \leq & x + 1 \\ -x & & -x \\ \hline x + 5 & \leq & 1 \\ -5 & & -5 \\ \hline x & \leq & -4 \end{array}$$

Example 1.6

The speed limit on an 800-m section of a certain highway is 50 km/h. What is the minimum time required to travel that section of highway without exceeding the speed limit?

SOLUTION Letting d = distance traveled and t = time of travel, we have

$$\frac{d}{t} \le 50 \text{ km/h} = 50 \times 10^3 \text{ m/h}$$

Since t is positive, we may multiply both sides of the inequality by t to obtain

$$d \le (50 \times 10^3 \text{ m/h})t$$

Dividing both sides by 50×10^3 gives

$$\frac{d}{50 \times 10^3 \text{ m/h}} \le t \quad \text{or, equivalently,} \quad t \ge \frac{d}{50 \times 10^3 \text{ m/h}}$$

Since $d = 800$ m, we have

$$t \ge \frac{800 \text{ m}}{50 \times 10^3 \text{ m/h}} = 0.016 \text{ h} = 57.6 \text{ s}$$

Note that this inequality, stated as $t \ge 0.016$ h, is the same as stating that the *minimum* t is 0.016 h.

Drill Exercise 1.6

What is the minimum value of y that satisfies the inequality $14 \ge -6y - 1$?

ANSWER: $y = -2.5$. □

In many situations, we encounter relationships where one variable is *inversely* proportional to another variable. When the variable y is inversely proportional to the variable x, we write

$$y = \frac{K}{x}$$

where K is a constant. Notice that the value of y is maximum when the value of x is minimum, and vice versa.

Example 1.7

Given that $-R \geq -4/A$ and that A may vary from 0.5 to 10, what is the maximum value of R?

SOLUTION Multiplying both sides of the inequality by -1 gives (by rule 3)

$$R \leq \frac{4}{A}$$

Since we are seeking the maximum value of R, we consider the value of A that makes $4/A$ a maximum, namely, the minimum value of A, or $A = 0.5$:

$$R \leq \frac{4}{0.5} = 8$$

Stating that $R \leq 8$ is the same as stating that the maximum value of R is 8.