Applied Physics For Engineers

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Analyzing Series–Parallel Circuits

A series–parallel circuit is one that contains combinations of series- and parallel-connected components, which are in turn connected in series and/or parallel with other such combinations. Figure 5.1 is an example of a simple series–parallel circuit. Note that the voltage source E and resistor R_1 are in a series path and that R_2 and R_3 are in parallel. As shown in the figure, we may regard the parallel combination of R_2 and R_3 as being in series with E and R_1.

There are no hard and fast rules for analyzing series–parallel circuits. In general, we apply any one or more of the rules and laws developed in preceding chapters (Ohm's law, Kirchhoff's laws, and the current- and voltage-divider rules), in any sequence that leads to the solution for a particular voltage or current. There are usually many different ways to find such a solution (i.e., many different sequences in which the rules can be applied), all of which produce the same solution.

One good approach to analyzing a series–parallel circuit is to replace any combination of series- and/or parallel-connected components by a single *equivalent* resistance, thereby simplifying the circuit. Complex circuits can be progressively simplified by repeated

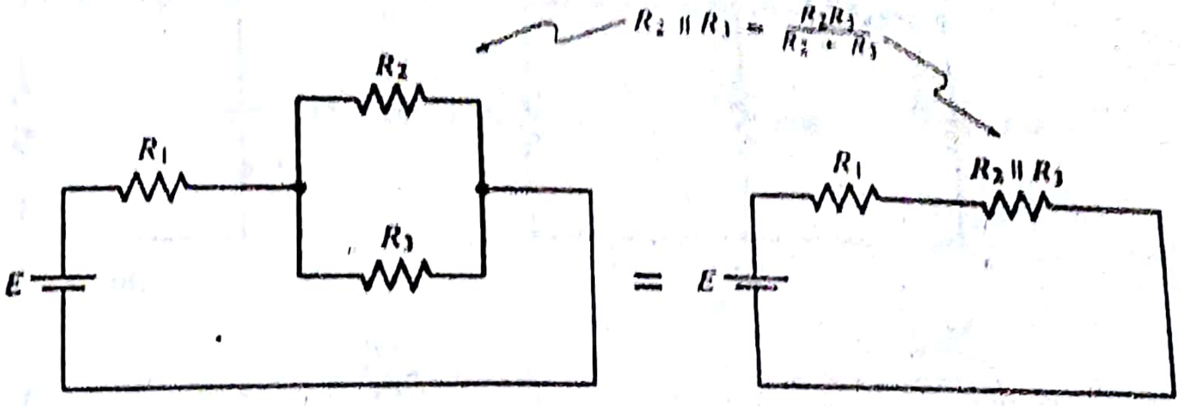

FIGURE 5.1 Example of a series–parallel circuit. The parallel combination of R_2 and R_3 is in series with E and R_1.

applications of this approach, that is, by reducing simplified equivalent circuits to even more simplified equivalent circuits, a procedure that will be demonstrated in many forthcoming examples. The next example illustrates the method and shows how two different analysis rules can be used to obtain the same solution.

Example 5.1

Find the voltage across the 60-Ω resistor in Figure 5.2 by

(a) using the current-divider rule
(b) using the voltage-divider rule

SOLUTION

(a) Figure 5.3 shows the sequence of steps that leads to a solution using the current-divider rule. We first reduce the original circuit to a simplified equivalent circuit, shown in Figure 5.3(b), where the parallel combination of the 60- and 30-Ω resistors has been replaced by a single equivalent 20-Ω resistance:

$$\frac{30 \times 60}{30 + 60} = 20\ \Omega$$

The resulting circuit is now clearly a series circuit, and can be reduced to a single equivalent resistance, as shown in Figure 5.3(c):

$$R_T = 20\ \Omega + 20\ \Omega = 40\ \Omega$$

The total current drawn from the 24-V source is then

$$I_T = \frac{24\ \text{V}}{40\ \Omega} = 0.6\ \text{A}$$

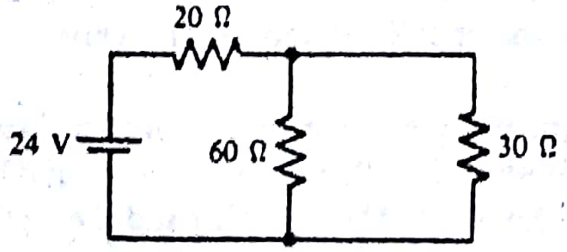

FIGURE 5.2 (Example 5.1)

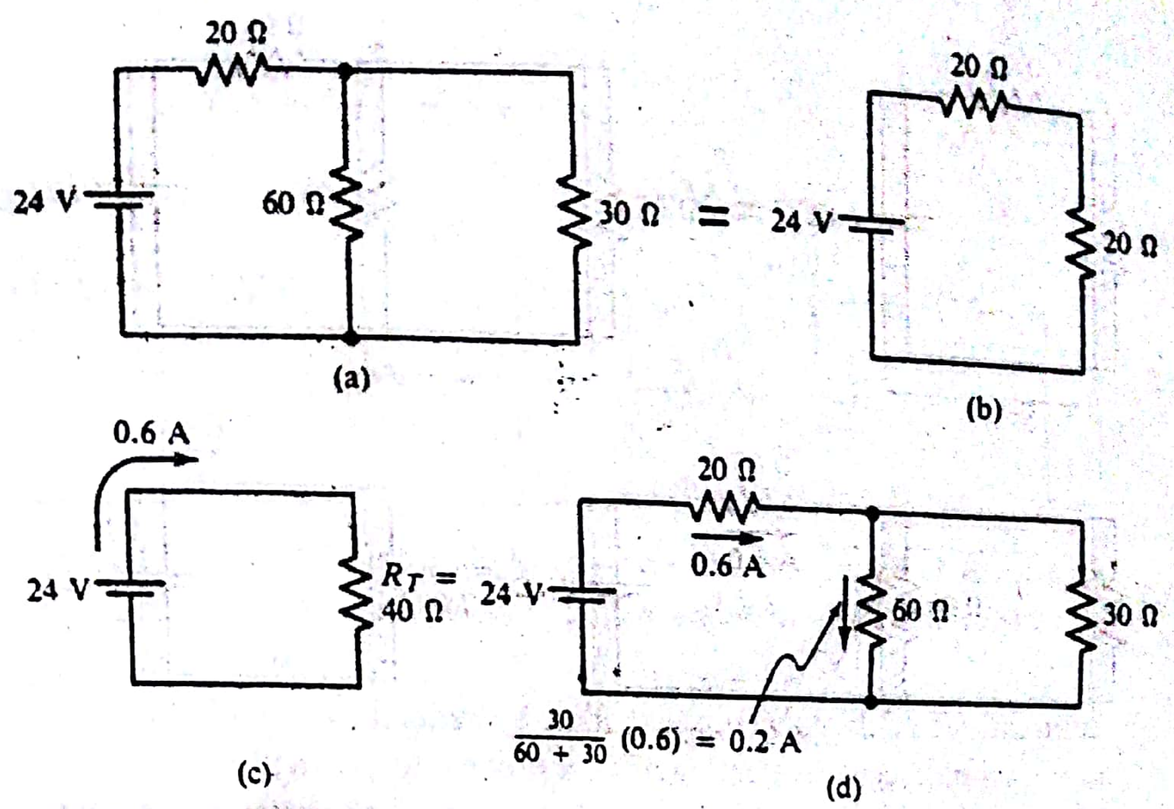

FIGURE 5.3 (Example 5.1)

Since each of the simplified circuits is equivalent to the original, the total current supplied by the 24-V source is 0.6 A in each case. Figure 5.3(d) shows the 0.6 A in the original circuit, and we see that we can apply the current-divider rule to determine the current in the 60-Ω resistor:

$$I_{60\Omega} = \left(\frac{30}{60 + 30}\right)(0.6 \text{ A}) = 0.2 \text{ A}$$

Thus, the voltage across the 60-Ω resistor is, by Ohm's law,

$$V_{60\Omega} = (0.2 \text{ A})(60 \ \Omega) = 12 \text{ V}$$

(b) Figure 5.4 shows how the voltage-divider rule can be used to find the voltage across the 60-Ω resistor. Once again, Figure 5.4(b) is the simplified circuit that results when the parallel combination of the 30- and 60-Ω resistors are replaced by their 20-Ω equivalent. Figure 5.4(c) shows that we can use the voltage-divider rule to find the voltage across the 20-Ω equivalent resistance:

$$V_{20\Omega} = \left(\frac{20}{20 + 20}\right) 24 \text{ V} = 12 \text{ V}$$

Figure 5.4(d) shows that the 12-V drop we found across the 20-Ω resistance also appears across both the 60- and 30-Ω resistors in the original circuit, since the 20-Ω resistance is equivalent to that parallel combination. Thus, we find again that $V_{60\Omega} = 12$ V.

KVL We should note that there are still other ways for finding the voltage across the 60-Ω resistor. For example, in Figure 5.3(d), we could have calculated the drop across the 20-Ω resistor [$V = (0.6 \text{ A})(20 \ \Omega) = 12$ V] and then found the voltage

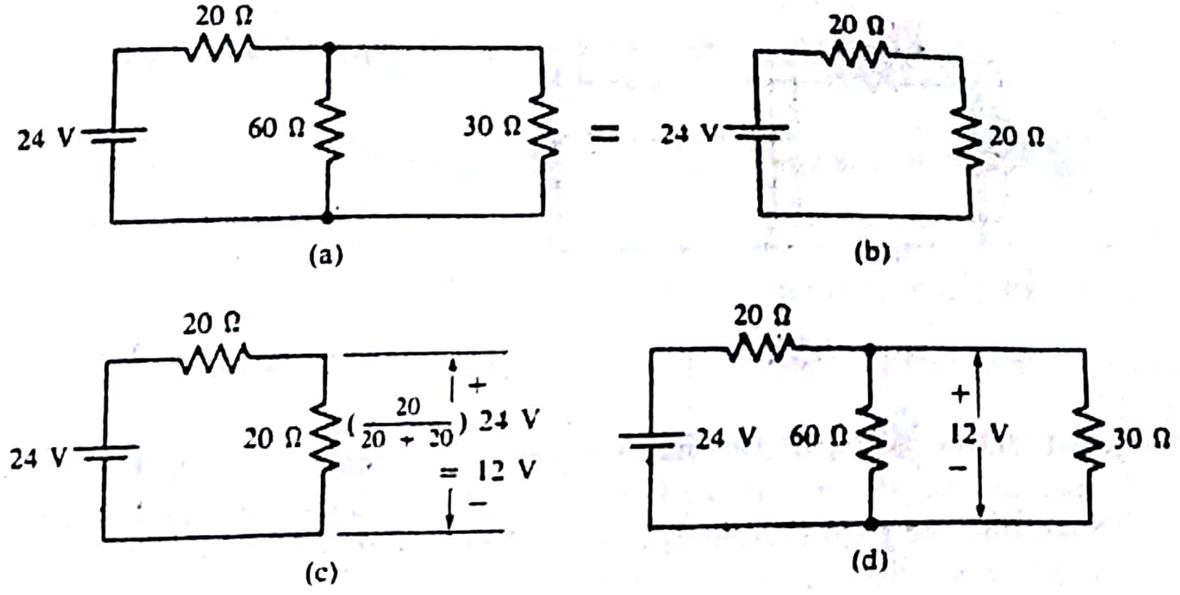

FIGURE 5.4 (Example 5.1)

across the 60-Ω resistor by applying Kirchhoff's voltage law around the loop consisting of the 24-V source, the 20-Ω resistor, and the 60-Ω resistor:

$$V_{60\,\Omega} = 24\text{ V} - 12\text{ V} = 12\text{ V}$$

Drill Exercise 5.1

If the 20-Ω resistor in Figure 5.3(a) is replaced by a 10-Ω resistor, find the voltage across that resistor.

ANSWER: 8 V. □

Series–Parallel Circuit Examples

In this section we will present numerous examples of methods used to analyze series–parallel circuits, as well as some design and troubleshooting examples. For most students, a great deal of repetition is required to become proficient at solving the types of problems these examples represent. The key is practice, practice, and more practice. A good way to gain practice is first to study the solutions to the examples and then, at a later time, to solve them again without referring to the solutions provided. Remember that there is no single "right" way to analyze many of these circuits.

Hereafter, we will say that we *solve* a network when we find the voltage across and current through every component in the network. The next example illustrates that type of analysis problem.

Example 5.2 (Analysis)

Find the voltage across and current through each resistor in the circuit shown in Figure 5.5.

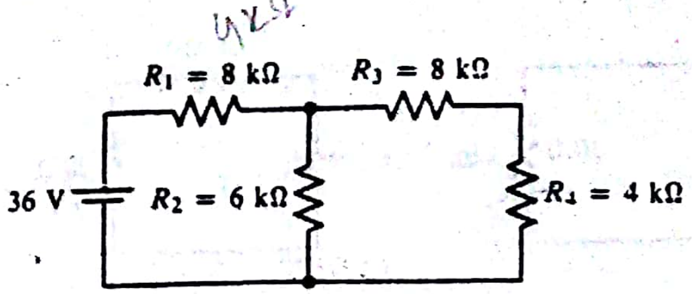

FIGURE 5.5 (Example 5.2)

SOLUTION We will construct progressively simpler equivalent circuits until we have reduced the original network of resistors to a single equivalent resistance (R_T). We can then find the total current I_T delivered by the 36-V source to the network. Finally, we will "work our way back" through the succession of equivalent circuits, computing voltages and currents along the way, until we regain the original circuit with all currents and voltages determined. The succession of equivalent circuits is shown in Figure 5.6.

In Figure 5.6(a) we see that R_3 and R_4 are in series and can therefore be combined into the single equivalent resistance $R_3 + R_4 = 8\text{ k}\Omega + 4\text{ k}\Omega = 12\text{ k}\Omega$, as shown in Figure 5.6(b). It is clear in Figure 5.6(b) that the 12-kΩ resistance is in parallel with R_2, so the combination can be replaced by

$$R_2 \| 12\text{ k}\Omega = \frac{(6\text{ k}\Omega)(12\text{ k}\Omega)}{6\text{ k}\Omega + 12\text{ k}\Omega} = 4\text{ k}\Omega$$

The new equivalent circuit that results is shown in Figure 5.6(c). It is now clear that the 4-kΩ resistance is in series with R_1, so the total resistance of the circuit is $R_T = R_1 + 4\text{ k}\Omega = 8\text{ k}\Omega + 4\text{ k}\Omega = 12\text{ k}\Omega$. Thus, the entire resistor network has been reduced to a single equivalent resistance of 12 kΩ, as shown in Figure 5.6(d).

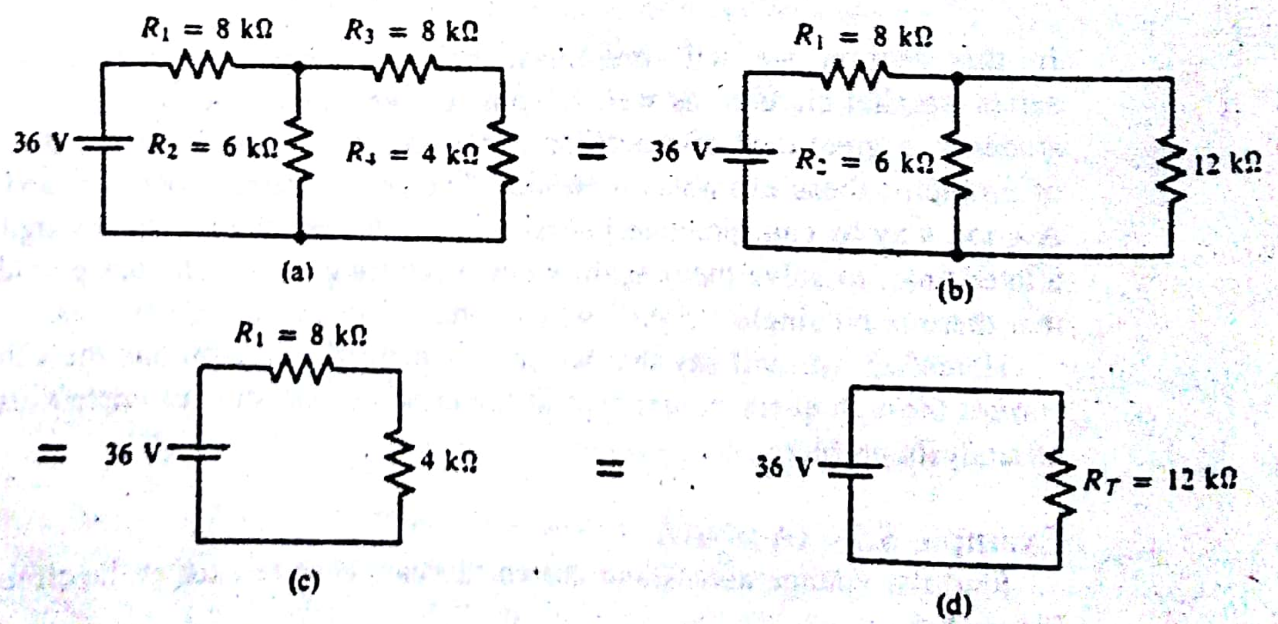

FIGURE 5.6 (Example 5.2)

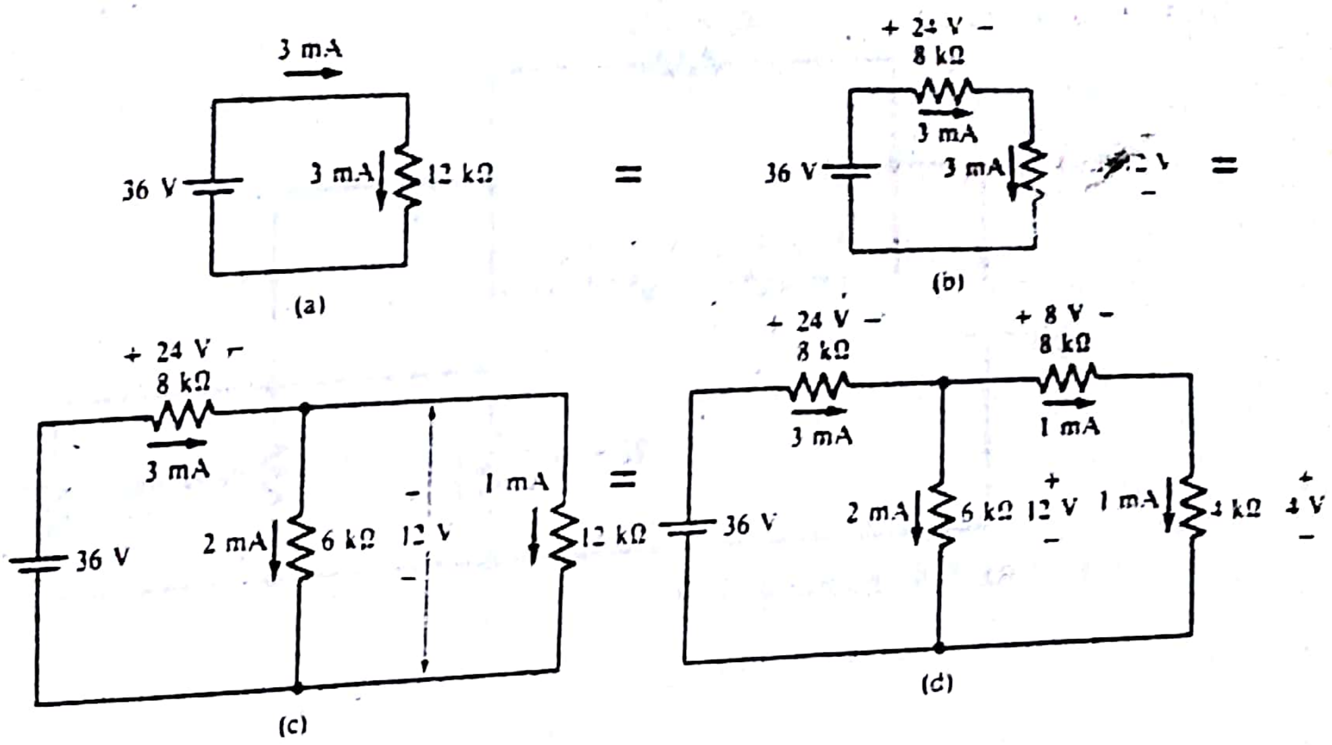

FIGURE 5.7 (Example 5.2)

Figure 5.7 shows how we work our way back through the equivalent circuits, solving for currents and voltages along the way, until the original circuit is completely solved. Note that the sequence of circuits is just the reverse of that shown in Figure 5.6. Figure 5.7(a) shows that the total current delivered by the source is

$$I_T = \frac{36 \text{ V}}{12 \text{ k}\Omega} = 3 \text{ mA}$$

As shown in Figure 5.7(b), the same 3 mA flows through the series combination of the 8-kΩ and 4-kΩ resistance, which together produced the equivalent 12 kΩ. The voltage drops across each resistance are therefore

$$V_{8\text{k}\Omega} = (3 \text{ mA})(8 \text{ k}\Omega) = 24 \text{ V}$$

$$V_{4\text{k}\Omega} = (3 \text{ mA})(4 \text{ k}\Omega) = 12 \text{ V}$$

As a check, note that Kirchhoff's voltage law is satisfied around this circuit (as it will prove to be around every loop in every equivalent circuit). Since there is 12 V across the 4-kΩ resistance, there is also 12 V across the parallel combination of the 6-kΩ and 12-kΩ resistance, because that combination produced the 4-kΩ equivalent. As shown in Figure 5.7(c), the current through each resistance can then be found:

$$I_{6\text{k}\Omega} = \frac{12 \text{ V}}{6 \text{ k}\Omega} = 2 \text{ mA}$$

$$I_{12\text{k}\Omega} = \frac{12 \text{ V}}{12 \text{ k}\Omega} = 1 \text{ mA}$$

Note that Kirchhoff's current law is satisfied in this circuit:

$$I_{8\text{k}\Omega} = 3 \text{ mA} = I_{6\text{k}\Omega} + I_{12\text{k}\Omega}$$

Finally, as shown in Figure 5.7(d), the 1-mA current in the 12-kΩ resistor of Figure 5.7(c) also flows in the 8-kΩ and 4-kΩ resistance, since those two produced the 12-kΩ equivalent. Thus,

$$V_{8k\Omega} = (1\ \text{mA})(8\ \text{k}\Omega) = 8\ \text{V}$$

$$V_{4k\Omega} = (1\ \text{mA})(4\ \text{k}\Omega) = 4\ \text{V}$$