8 Introduction to Fields and Electrical Physics

8.1 Coulomb's Law

In Chapter 2 we discussed the very important fact that oppositely charged particles experience a force of attraction, and that similarly charged particles experience a force of repulsion. Figure 8.1 illustrates these facts. The arrows labeled F represent the forces developed on the charged particles, and Q_1 and Q_2 represent the magnitudes of their charges. Note that each particle experiences the *same* force as its counterpart, regardless of which has the greater charge. If there are no restraints or other external forces acting on the charges, they will move in the directions indicated by the force arrows.

In Figure 8.1, r represents the distance separating the charged particles. The magnitude of the force F is directly proportional to the product of the charges, Q_1Q_2, and is inversely proportional to the *square* of the distance r separating them. These facts constitute *Coulomb's law*, which is expressed in equation form by

$$F = \frac{k\,Q_1Q_2}{r^2} \tag{8.1}$$

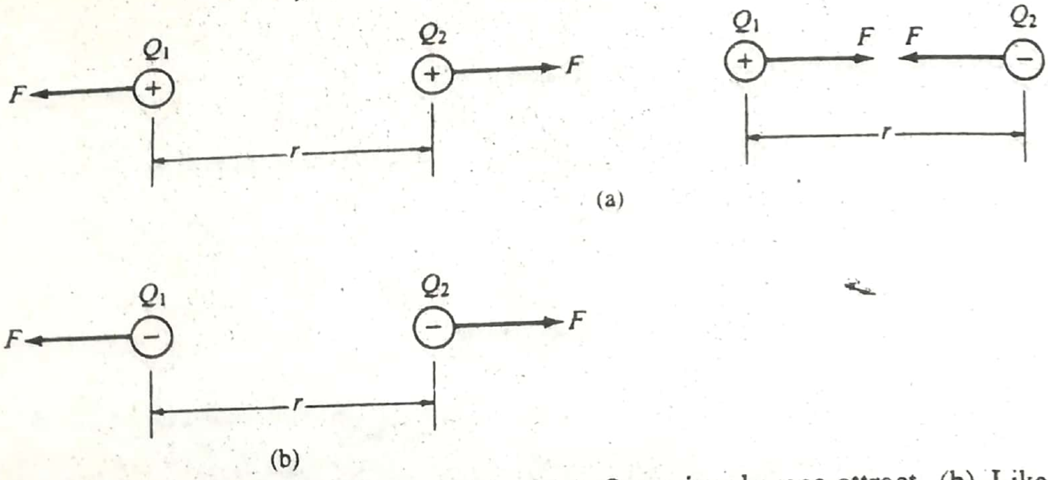


FIGURE 8.1 Forces on charged particles. (a) Opposite charges attract. (b) Like charges repel.

where F = force, newtons
k = constant, $9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$
Q_1, Q_2 = charges, coulombs
r = separating distance, meters

Example 8.1

In Figure 8.1(b), Q_1 is a 40-μC positive charge, Q_2 is a 100-μC negative charge, and r = 50 mm. Find the force of attraction between the charges.

SOLUTION From equation (8.1),

$$F = \frac{(9 \times 10^9)(40 \times 10^{-6}\ \mathrm{C})(100 \times 10^{-6}\ \mathrm{C})}{(50 \times 10^{-3}\ \mathrm{m})^2}$$
$$= 14.4 \times 10^3\ \mathrm{N}$$

Drill Exercise 8.1

What charge on each particle in Example 8.1 would develop the same force at the same distance if the two charges have equal magnitudes?

ANSWER: 63.24 μC. □

8.2 Electric Fields

Imagine a negatively charged particle fixed and isolated in space, as shown in Figure 8.2(a). If a positively charged particle is placed on the right side of the negative charge, the positive charge will be drawn to the left, as shown by the arrow, because of the force of attraction. Similarly, if the positive charge is placed above the negative charge, the positive charge will be attracted downward. The figure shows that a positive charge placed anywhere in the vicinity of the negative charge is drawn inward toward the

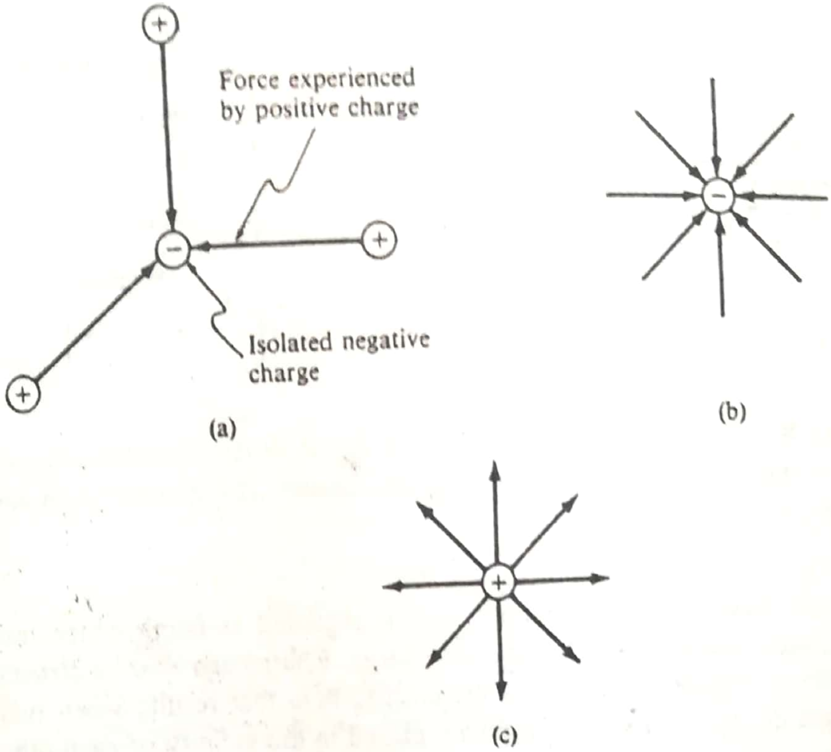


FIGURE 8.2 An electric field is represented by lines that show the direction of the force on a positive charge placed anywhere in the field. (a) Positive charges are drawn toward the isolated negative charge. (b) Electric field lines in the vicinity of an isolated negative charge. (c) Electric field in the vicinity of an isolated positive charge.

negative charge. In Figure 8.2(b), we eliminate the positive charges and simply show the lines along which positive charges would be drawn to the negative charge. The lines are a pictorial way of visualizing the reaction of any positive charge to the presence of the fixed negative charge. Of course, there are an infinite number of such lines, but we can only show several. We say that the negative charge is responsible for an *electric field*, represented by the lines in the diagram. We can use the electric field lines to predict the behavior of a *positive* charge placed anywhere in the field: the charge will experience a force in the direction shown by a line at the point where the charge is placed. Figure 8.2(c) shows the electric field established by an isolated positive charge. In this case, the lines radiate outward, because any positive charge placed in the vicinity of the isolated charge will be repelled away from it.

When two or more charged particles occupy fixed locations in space, the pattern of the electric field in the region surrounding them depends on the magnitudes of the charges and on their locations with respect to each other. For example, Figure 8.3(a) shows the electric field established by a fixed positive charge in the vicinity of a fixed negative charge. Note that lines always originate at a positive charge and terminate at a negative charge, in the sense that the direction of each line is from + to −. This direction is, again, the same as the direction that a positive charge would move if it were placed in

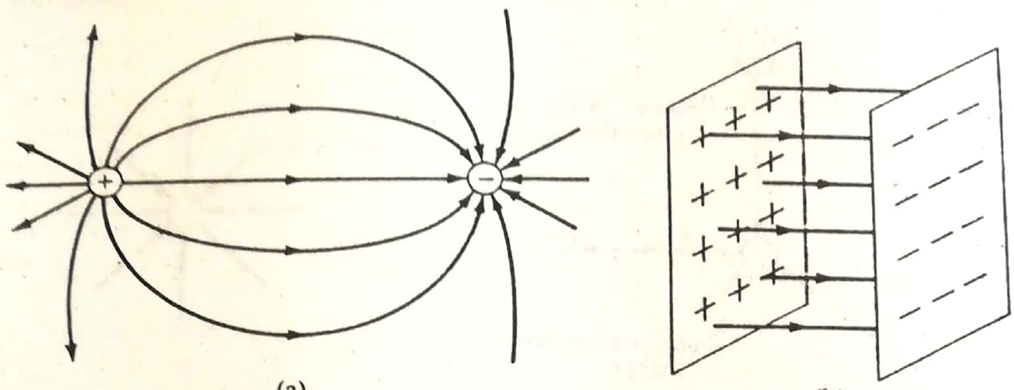


FIGURE 8.3 Electric fields established by distributions of fixed charges. (a) Electric field in the vicinity of two opposite point charges. (b) Electric field between two charged surfaces.

the field. Each charge in Figure 8.3(a) is regarded as being concentrated at a single, fixed point, and each is called a *point charge*. Charge can also be *distributed* over a line or surface. Figure 8.3(b) shows the electric field that results when two *sheet charges* (charges distributed over surfaces) are placed in the vicinity of each other.

Electric Flux Density

We have seen that an electric field pattern shows the *direction* of the force on a positive charge placed in the field. The pattern can also be interpreted to reveal the relative *magnitude* of the force experienced by a positive charge placed at any point in the field. In any region where the lines are close together (dense), the force on a positive charge is greater than it is in a region where the lines are less dense. Consider again the field pattern around an isolated negative charge, as shown in Figure 8.4. The closer we approach the charge, the more closely spaced are the lines. Thus, as confirmed by Coulomb's law, the closer we move a positive charge to the fixed negative charge, the greater the force of attraction on it. On the other hand, at distances well removed from the fixed negative charge, the field lines are less dense and the force is correspondingly smaller.

In one sense, the number of lines appearing on an electric field diagram is purely arbitrary: There are, after all, an infinite number of paths along which a positive charge could move. However, to develop a quantitative basis for comparing fields and for performing field computations, it is convenient to assume that the number of lines produced by a charge is the same as the charge in coulombs. Instead of "lines," the term electric *flux* is used, and is designated by the symbol ψ. Thus, ψ has the units of coulombs, and

$$\psi = Q \qquad \text{coulombs} \tag{8.2}$$

Understand that the notion of flux, and its units, serves only as a convenient basis for mathematical computations. We could not draw a field diagram showing the one one-millionth of a line corresponding to a flux of 1 μC!

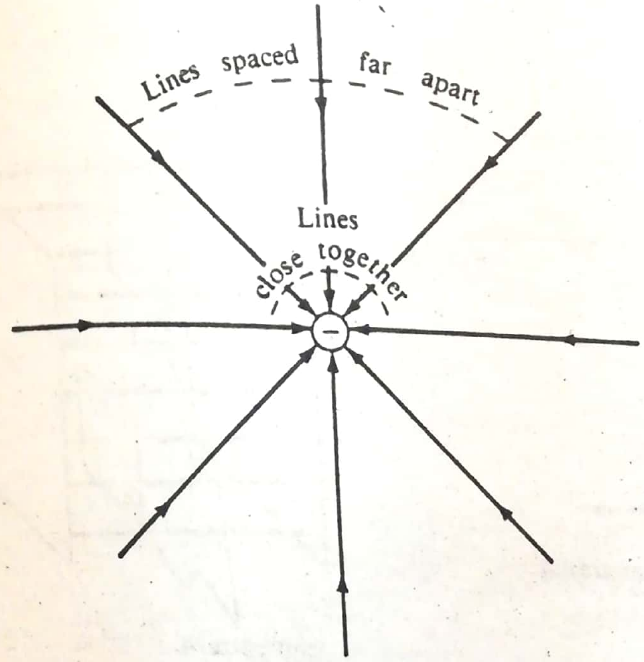


FIGURE 8.4 The more dense the field lines (the closer they are together), the greater the force on a positive charge. In the example shown, the force is greatest in the immediate vicinity of the negative charge.

We now have a basis for defining a numerical quantity that reflects how closely spaced the lines are in an electric field, that is, a measure of the *density* of the flux. We must visualize flux lines as occupying three-dimensional space around a charge, so we can picture a certain number of lines as penetrating a surface. This idea is illustrated in Figure 8.5. *Flux density D* is defined to be flux per unit area:

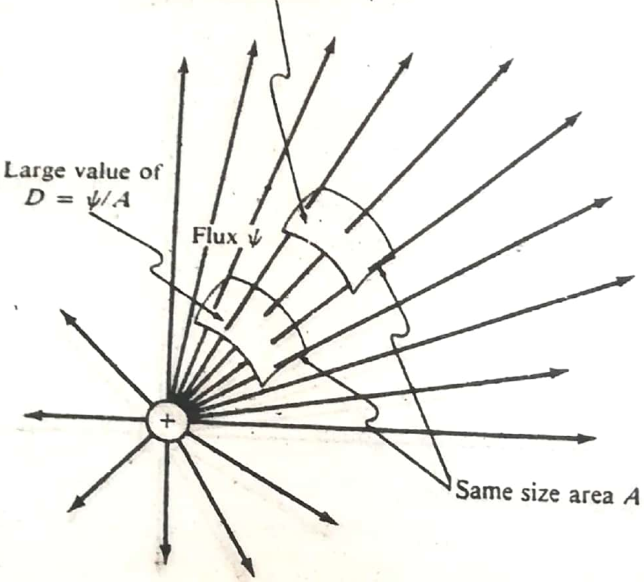
$$D = \frac{\psi}{A} \quad \text{coulombs/meters}^2 \tag{8.3}$$

In this definition, the surface area used in a computation must be *perpendicular* to the flux lines at every point where the flux penetrates the area. As a consequence, depending on the field pattern, the area A may in fact be that of a *curved* surface. Figure 8.5(a) shows such a case. In the figure it can be seen that the flux density is large in the region near the positive charge, because a large amount of flux penetrates the curved surface area shown. Farther away from the charge, there is less flux penetrating the same size area, so the flux density is smaller. Figure 8.5(b) shows that the flux density is the same everywhere in the region between two charged surfaces.

Example 8.2

One of the areas between the two charged surfaces in Figure 8.5(b) measures 6 mm by 8 mm, and the flux penetrating it is 96 μC. Each of the charged surfaces measures 2.5 cm by 4 cm.

(a) What is the flux density in the region between the charged surfaces?
(b) What is the total flux in the region between the charged surfaces?



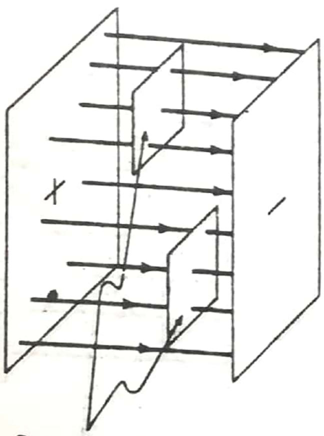


FIGURE 8.5 Flux density is flux divided by the area the flux penetrates. (a) The flux density is greatest in the region nearest the point charge. (b) The field between the charged plates is *uniform*, and the flux density is everywhere the same.

SOLUTION

(a) $A = (6\text{ mm})(8\text{ mm}) = (6 \times 10^{-3}\text{ m})(8 \times 10^{-3}\text{ m}) = 48 \times 10^{-6}\text{ m}^2$

$$D = \frac{\psi}{A} = \frac{96 \times 10^{-6}\text{ C}}{48 \times 10^{-6}\text{ m}^2} = 2\text{ C/m}^2$$

(b) $A = (2.5\text{ cm})(4\text{ cm}) = (2.5 \times 10^{-2}\text{ m})(4 \times 10^{-2}\text{ m}) = 10^{-3}\text{ m}^2$

$$\psi = DA = (2\text{ C/m}^2)(10^{-3}\text{ m}^2) = 2 \times 10^{-3}\text{ C}$$

(In the latter computation, we neglect the phenomenon called *fringing*, whereby some of the flux along the edges of the charged surfaces "escapes" from the region immediately ween the surfaces, that is, bulges outward rather than remaining parallel to the ier lines.)

Drill Exercise 8.2

Suppose that *each* dimension of the two charged surfaces in Figure 8.5(b) is doubled, but the total charge on each surface remains the same. What, then, is the flux density in the region between the surfaces?

ANSWER: 0.5 C/m².

Electric Field Intensity

Electric field intensity, also called electric field *strength*, is the ratio of the force experienced by a charge placed in the field to the magnitude of the charge itself:

$$\mathscr{E} = \frac{F}{Q} \quad \text{newtons/coulomb} \tag{8.4}$$

For example, if a 40-μC charge placed in a certain field experiences a force of 20 N, the electric field intensity at that point is $\mathscr{E} = 20\ \text{N}/40 \times 10^{-6}\ \text{C} = 5 \times 10^5\ \text{N/C}$. Notice that field intensity is a characteristic of the field itself: the "stronger" the field, the greater the force a given charge will experience when placed in the field. Furthermore, $\mathscr{E}$ can be expected, in general, to have different values at different points in the field. It is obvious that the field intensity in a region close to the positive charge in Figure 8.5(a) is greater than it is at a long distance from the charge, because a given charge will experience greater force in the region near the fixed charge, where the flux is denser.

The foregoing remarks suggest that there is a relationship between D and $\mathscr{E}$, because each is related to the magnitude of the force experienced by a charge placed in an electric field. The more dense the flux (the greater the value of D), the greater the force on such a charge. Similarly, the greater the field intensity $\mathscr{E}$, the greater the force on the charge. In fact, D and $\mathscr{E}$ are proportional to each other, as expressed by the equation

$$D = \epsilon \mathscr{E} \tag{8.5}$$

where ϵ is a constant whose value depends on the material in which the field is established (air, glass, water, etc.). ϵ is called the *permittivity* of the material and may range in value from $10^{-9}/36\pi \approx 8.84 \times 10^{-12}$ (for a vacuum) to 6.6×10^{-8} (for certain ceramics).

Example 8.3

When a 1000-μC charge is placed at a certain point in a certain electric field, it experiences a force of 28.2 N. If the field exists in a vacuum:

(a) Find the field intensity at the point where the charge is placed.
(b) Find the flux density at the point where the charge is placed.

SOLUTION

(a) From equation (8.4),

$$\mathscr{E} = \frac{F}{Q} = \frac{28.2\ \text{N}}{1000 \times 10^{-6}\ \text{C}} = 28{,}200\ \text{N/C}$$

(b) From equation (8.5),

$$D + = \epsilon \mathscr{E} = (8.84 \times 10^{-12})(28.2 \times 10^3) = 2.5 \times 10^{-7}\ \text{C/m}^2$$

Drill Exercise 8.3

What force will be experienced by a 200-μC charge placed in the field at the same point as the charge in Example 8.3 was placed?

ANSWER: 5.64 N. □

We noted that the flux density of the electric field in the region between two charged surfaces [Figure 8.3(b)] is everywhere the same. The field is said to be *uniform*. From the equation $D = \epsilon \mathscr{E}$, it follows that the electric field intensity $\mathscr{E}$ is everywhere the

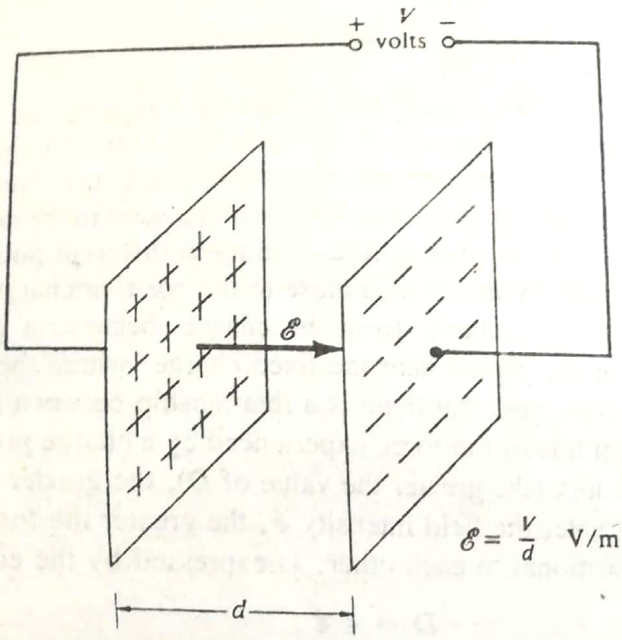


FIGURE 8.6 The electric field intensity between two charged, parallel surfaces can be computed from the voltage difference V across the surfaces.

same in the region between the charged surfaces. (These conclusions are based on ideal conditions and on two perfectly parallel surfaces, which we will assume in future discussions.) Recall from Section 2.4 that a voltage difference always exists between two oppositely charged regions. Figure 8.6 shows two charged surfaces and the voltage V that exists between them. It can be shown that the units of $\mathscr{E}$, N/C, are equivalent to volts/meter (V/m), and in the special case of Figure 8.6, the electric field intensity in the region between the two surfaces can be computed from

$$\mathscr{E} = \frac{V}{d} \quad \text{V/m} \tag{8.6}$$

where V is the voltage between the surfaces and d is their separation in meters.

Example 8.4

Two parallel surfaces are separated by 12 mm. A 600-μC charge placed between them experiences a force of 7.2 N. What is the voltage difference between the surfaces?

SOLUTION From equation (8.4),

$$\mathscr{E} = \frac{F}{Q} = \frac{7.2 \text{ N}}{600 \ \mu\text{C}} = 12 \times 10^3 \text{ N/C} = 12 \times 10^3 \text{ V/m}$$

From equation (8.6),

$$V = \mathscr{E}d = (12 \times 10^3 \text{ V/m})(12 \times 10^{-3} \text{ m}) = 144 \text{ V}$$

Example 8.5

The voltage across two oppositely charged surfaces separated by a 0.12 mm thickness of porcelain is being increased by increasing the amount of charge on them. At what voltage will the electrons on the negative surface penetrate the porcelain and neutralize the positive surface?

SOLUTION The negative surface *discharges* when the voltage is sufficient to break down the porcelain insulator. By equation (8.6),

$$\mathscr{E} = \frac{V}{d}$$

From Table 8.1, porcelain breaks down at $\mathscr{E} = 7 \times 10^6$ V/m, so

$$7 \times 10^6 \text{ V/m} = \frac{V}{0.12 \times 10^{-3} \text{ m}}$$

$$V = (7 \times 10^6 \text{ V/m})(0.12 \times 10^{-3} \text{ m}) = 840 \text{ V}$$

Drill Exercise 8.5

What thickness of porcelain would be required in Example 8.5 if breakdown were not permitted to occur for voltages up to 35 kV?

ANSWER: 5 mm. □

8.4 Voltage

In the SI system, voltage is derived from power and current. Specifically, 1 volt is defined to be the potential difference that exists between two points when the current between the points is 1 ampere and the rate of energy consumption (power) is 1 watt. Of course, this definition follows directly from the now familiar power equation, $P = VI$:

$$V = \frac{P}{I} \qquad 1 \text{ V} = \frac{1 \text{ W}}{1 \text{ A}}$$

However, a definition based on power reveals little about the true nature of voltage, or potential difference. We wish now to develop the notion of voltage in a more insightful way, using some of the concepts that have been introduced in connection with electric fields.

Recall from Chapter 2 that we described voltage as a measure of the ability of a source to produce current. In order to produce current, a two-terminal voltage source creates a force that repels electrons from its negative terminal and attracts them to its positive terminal. As we now know, there is an electric field between the two oppositely charged terminals. The greater the field intensity, the greater the force on a charge placed in the field. One way we could measure how effective that field is in forcing charge to move through it would be to determine how much work would have to be performed

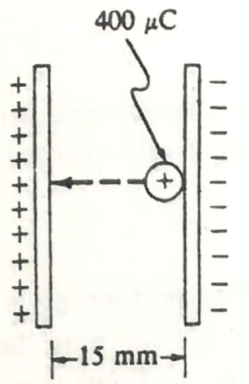


FIGURE 8.7 (Example 8.6)

(i.e., how much energy would have to be expended) to move a charge in *opposition* to the field. For example, the greater the work required to move a positive charge from the negative to the positive terminal of a voltage source, the greater the voltage of the source. We say that the potential difference between two points is 1 volt when 1 joule of work must be performed to move 1 coulomb of charge from one point to the other. In other words, voltage is the ratio of work to charge:

$$\text{(volts) } V = \frac{W}{Q} = \frac{\text{joules of work}}{\text{coulombs of charge}} \tag{8.7}$$

We see that 1 volt is the same as 1 joule per coulomb.

Regarding voltage as the ratio of work to charge helps build an intuitive understanding of the concept, because it conveys the idea of potential as an ability to accomplish something useful: Voltage can put charge in motion. Similarly, a voltage *drop* is the loss of ability, or potential, to move charge. The greater the drop, the less energy is available to move charge from one point to another. Understand, however, that this notion of voltage is *equivalent* to 1 volt = 1 watt/ampere; it is not an alternative definition. The units volts, watts/ampere, and joules/coulomb are all equivalent and can all be reduced to the same SI *base* units.

Example 8.6

24×10^{-3} J of energy is required to move the 400-μC positive charge shown in Figure 8.7 from the negatively charged surface to the positively charged surface.

(a) What is the potential difference between the two charged surfaces?
(b) What is the electric field intensity in the region between the surfaces?

SOLUTION

(a) The number of joules of energy required to move the charge equals the amount of work that must be performed on it. By equation (8.7),

$$V = \frac{W}{Q} = \frac{24 \times 10^{-3}\ \text{J}}{400 \times 10^{-6}\ \text{C}} = 60\ \text{V}$$

(b) By equation (8.6),

$$\mathscr{E} = \frac{V}{d} = \frac{60\ \text{V}}{15 \times 10^{-3}\ \text{m}} = 4000\ \text{V/m}$$