Applied Physics For Engineers

Contents:

Kirchhoff's Voltage Law

Figure 4.6 shows a series circuit containing a voltage source and three resistors. Following the method demonstrated in Example 4.3, we can find the voltage across each resistor in the circuit:

$$I_T = \frac{E}{R_T} = \frac{20\ \text{V}}{(3 + 5 + 2)\ \Omega} = 2\ \text{A}$$

$$V_1 = (3\ \Omega)(2\ \text{A}) = 6\ \text{V} \qquad V_2 = (5\ \Omega)(2\ \text{A}) = 10\ \text{V} \qquad V_3 = (2\ \Omega)(2\ \text{A}) = 4\ \text{V}$$

These voltages are shown on the figure. Also shown are the voltages measured between each resistor terminal and the minus side of the voltage source. Notice that each of these voltages equals the voltage of the source *minus* the sum of all the voltages across resistors lying to the left of the point where it is measured. For example, the voltage between the first and second resistors is 20 V − 6 V = 14 V, and the voltage between the second and third resistors is 20 V − (6 + 10) V = 4 V. As we progress through the circuit from left to right, measuring voltages between each terminal and the minus side of the source, we notice that these voltages become smaller and smaller and finally reach zero

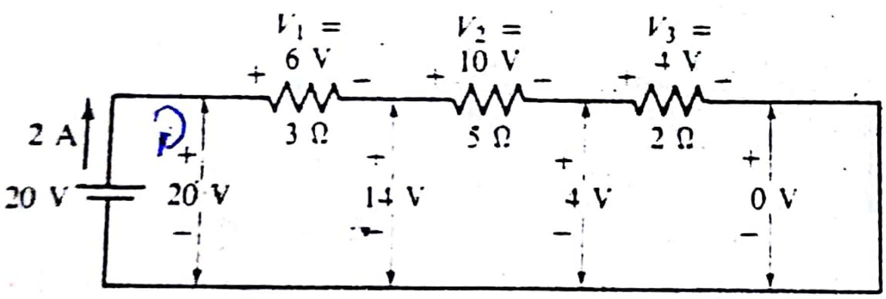


FIGURE 4.6

at the very end. We see that the voltage across each resistor causes a *voltage drop* (or potential drop) as we progress through the circuit.

In a sense, each voltage drop across a resistor in Figure 4.6 represents the amount of voltage "used up" in progressing from one resistor to the next. When we reach the end (rightmost side), we have used up the entire 20 V that was originally available. This concept is, again, very much like a water distribution system: At the pumping station, the maximum pressure in the system is available, but the pressure continually drops as we progress through the system, the lowest pressure being that at the most remote location.

If we imagine ourselves traveling around the circuit in Figure 4.6, beginning at the + terminal of the 20-V source, and measuring voltages with respect to the minus side of that source as we progress, we observe that the voltage drops *accumulate*, that is, the total drop at any point is the sum of all the drops up to that point. When we reach the end, the sum of all the drops equals the source voltage, 20 V. We can regard the 20-V source as a voltage *rise*, because if we continue around the circuit and pass through the source, we once again have 20 V to "work with."

The path we follow in traveling completely around the circuit of Figure 4.6, beginning at the + terminal of the 20-V source and ending up at the same point, is called a *closed loop*. A closed loop is any path that begins and ends at the same point. The example we have just discussed illustrates a very important fact about the voltage drops around any closed loop in an electric circuit, called *Kirchhoff's voltage law*:

The sum of the voltage drops around any closed loop equals the sum of the voltage rises around that loop.

In our example, which contains only one voltage rise, we see that the sum of the drops across the resistors, $V_1 + V_2 + V_3 = 6\text{ V} + 10\text{ V} + 4\text{ V}$, equals the 20-V rise in the source.

In our example, it is clear that the voltage drops are the voltages across the resistors and that the voltage rise occurs in the source. However, in more complex circuits, it is not always obvious which voltages represent drops and which represent rises. It is therefore convenient to establish a systematic procedure that allows us to apply Kirchhoff's voltage law in a way that does not depend on a physical interpretation of drops and rises. Figure 4.7 shows the circuit we have been discussing and shows a closed loop represented

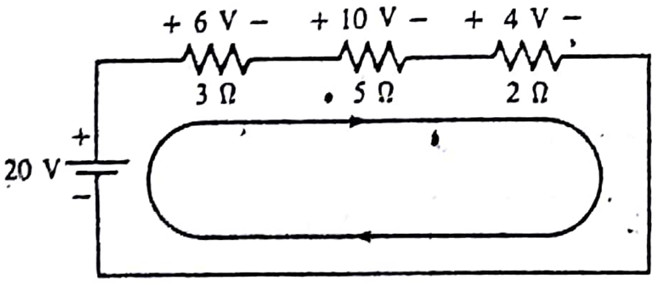
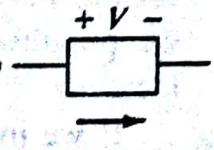
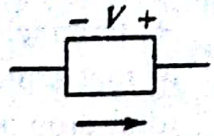


FIGURE 4.7 Closed loop. When the loop passes through a component from + to −, the voltage across that component is a *drop*. When the loop passes through a component from − to +, the voltage is a *rise*.

by a path drawn with arrows superimposed on it. The arrows show that we will travel around the loop in a *clockwise* direction. As we go around this loop, notice that we pass *through* each resistor *from the positive side* of its voltage drop *to the negative side*. On the other hand, when we pass *through* the voltage source, we travel *from its negative* terminal *to its positive* terminal. These observations will be the basis for our *definitions* of voltage drops and voltage rises in *any* component:

From + to − ⟹ voltage drop 

From − to + ⟹ voltage rise 

Now that we have established definitions for drops and rises based on a purely mechanical procedure, we can apply Kirchhoff's voltage law without regard to physical interpretations. Furthermore, the law is valid *regardless of the direction in which we travel around the loop*. Figure 4.8 shows our example circuit once again, this time with a counterclockwise loop. In accordance with the definitions, notice that the voltages across the resistors are now treated as rises and the source voltage becomes a drop. It is still true that the sum of the voltage rises equals the sum of the voltage drops:

$$\underbrace{6\text{ V} + 10\text{ V} + 4\text{ V}}_{\text{rises}} = \underbrace{20\text{ V}}_{\text{drop}}$$

It should be clear from this result that *the direction of a loop does not have to be the same as the direction of the current in a circuit.*

Example 4.4 (Analysis)

Use Kirchhoff's voltage law to find the source voltage E in Figure 4.9.

SOLUTION We arbitrarily choose to draw a clockwise loop, as shown in Figure 4.9. With this choice, notice that the voltage across each resistor is a rise and the unknown voltage E is a drop. Applying Kirchhoff's voltage law, we find

$$E = 23.4 + 10.8 + 6.6 + 13.2 = 54\text{ V}$$

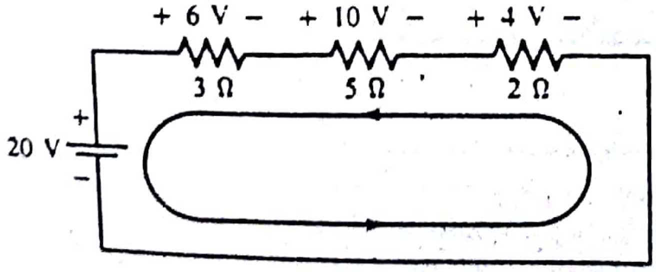


FIGURE 4.8 Same circuit as Figure 4.7, with the loop drawn in a counterclockwise direction. Notice that the drops and rises are the opposite of those in Figure 4.7.

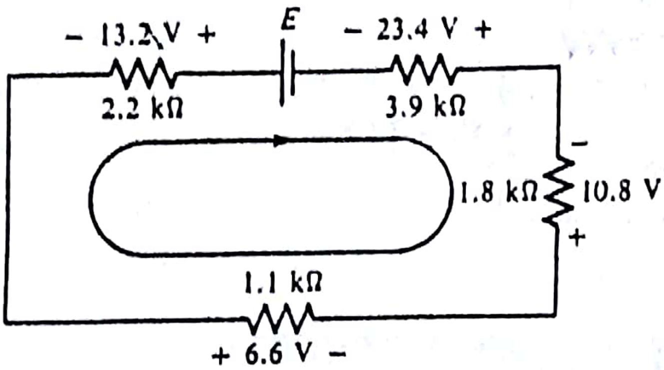


FIGURE 4.9 (Example 4.4)

Drill Exercise 4.4

Find the current in the circuit shown in Figure 4.9.

ANSWER: 6 mA. □

We can apply Kirchhoff's voltage law to determine the voltage across *any* pair of terminals in a circuit, regardless of what is or what is not connected between those terminals. Remember that voltage at any point in a circuit is always measured *with respect to* some other point in the circuit, so when we speak of the voltage across a pair of terminals, we are assuming that one of the terminals is the reference point for the voltage at the other. It is convenient to assign labels to circuit terminals as an aid in remembering which terminal is the reference for the voltage at the other. For example, we might assign the labels a and b to a pair of terminals, as shown in Figure 4.10. The notation V_{ab} then means that we regard terminal a as "positive" with respect to terminal b, but only in the sense that we measure the voltage between a and b with b as the reference terminal. In other words, we would connect the high side of a voltmeter to terminal a and the low side to terminal b. It may well be that V_{ab} is a negative voltage, in which case our voltmeter reading would be negative. In that case, reversing the voltmeter terminals (connecting the low side to terminal a) would give us the voltage V_{ba}, a positive value. Whether V_{ab} is a positive or negative value, it is *always* true that

$$V_{ab} = -V_{ba} \tag{4.3}$$

For example, if $V_{ab} = 6$ V, then $V_{ba} = -6$ V. If $V_{ab} = -20$ V, then $V_{ba} = +20$ V.

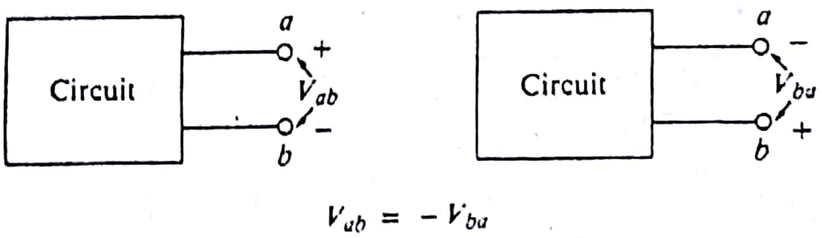
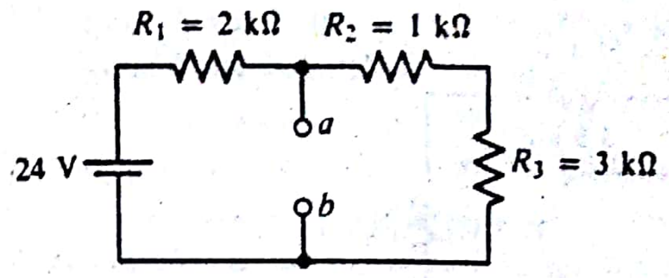


FIGURE 4.10 The convention used to show which terminal is the *reference* for a voltage across a pair of terminals. The first subscript is the "high" side, where the positive lead of a voltmeter would be connected if a voltage measurement were made. The second subscript is the "low" side, or reference terminal.



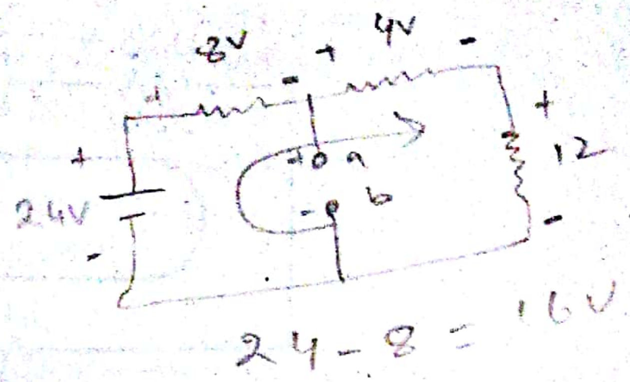


FIGURE 4.11 (Example 4.5)

Example 4.5 (Analysis)

Use Kirchhoff's voltage law to find the voltage V_{ab} in Figure 4.11.

SOLUTION We will first find the voltage drops across the resistors.

$$R_T = 2\ \text{k}\Omega + 1\ \text{k}\Omega + 3\ \text{k}\Omega = 6\ \text{k}\Omega$$

$$I_T = \frac{E}{R_T} = \frac{24\ \text{V}}{6\ \text{k}\Omega} = 4\ \text{mA}$$

$$V_1 = (4\ \text{mA})(2\ \text{k}\Omega) = 8\ \text{V} \qquad V_2 = (4\ \text{mA})(1\ \text{k}\Omega) = 4\ \text{V}$$
$$V_3 = (4\ \text{mA})(3\ \text{k}\Omega) = 12\ \text{V}$$

Figure 4.12 shows the circuit with the voltage drops across each resistor. Notice that there are *two* closed loops that pass through V_{ab}, both shown clockwise in the figure, so we can write Kirchhoff's voltage law around either one to solve for V_{ab}. Notice also that + and − polarity symbols are assigned to V_{ab} in accordance with our convention that *a* is "positive" with respect to *b* (*b* is the reference).

Writing Kirchhoff's voltage law around loop 1, we find

$$24\text{V} = 8\ \text{V} + V_{ab}$$

or

$$V_{ab} = 16\ \text{V}$$

We obtain precisely the same result in loop 2. Notice that V_{ab} is a rise in loop 2, because we pass through it from − to +:

$$V_{ab} = 4\ \text{V} + 12\ \text{V} = 16\ \text{V}$$

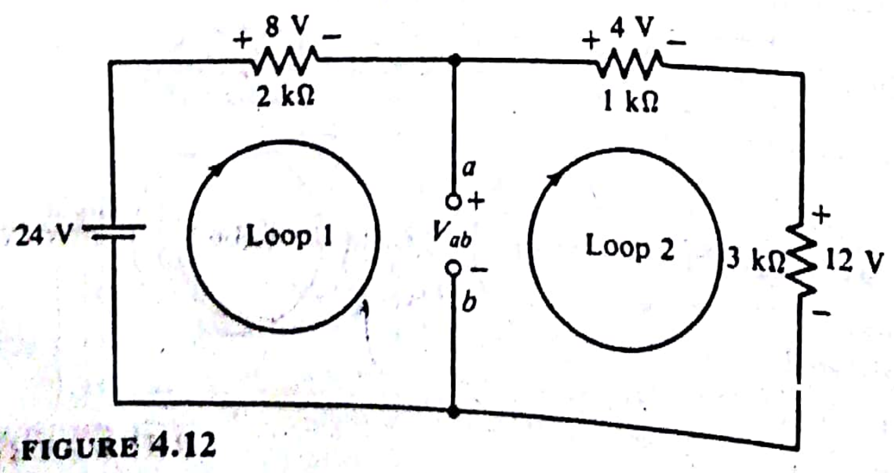


FIGURE 4.12