Applied Physics For Engineers

Contents:

Kirchhoff's Current Law

Electrical charge is neither created nor destroyed in ordinary circuit operation. Consequently, when there is current in a component, all the charge that enters it over a period of time must equal all the charge that leaves it in the same period of time. It follows that the *rate* at which charge enters a component equals the rate at which it leaves. Recalling that electrical current is the rate of flow of charge, we conclude that the current entering a component must equal the current leaving it. This idea can be extended to include a network containing several components, where current enters and leaves via several different paths. In fact, if we draw a closed line around *any* portion of an electrical circuit, we can apply the same logic and conclude that the total current crossing that line as it enters the enclosed portion must equal the total current crossing the line in the opposite direction, regardless of the number of paths by which current can enter or leave. The portion of the circuit around which the boundary line is drawn can be as small as a

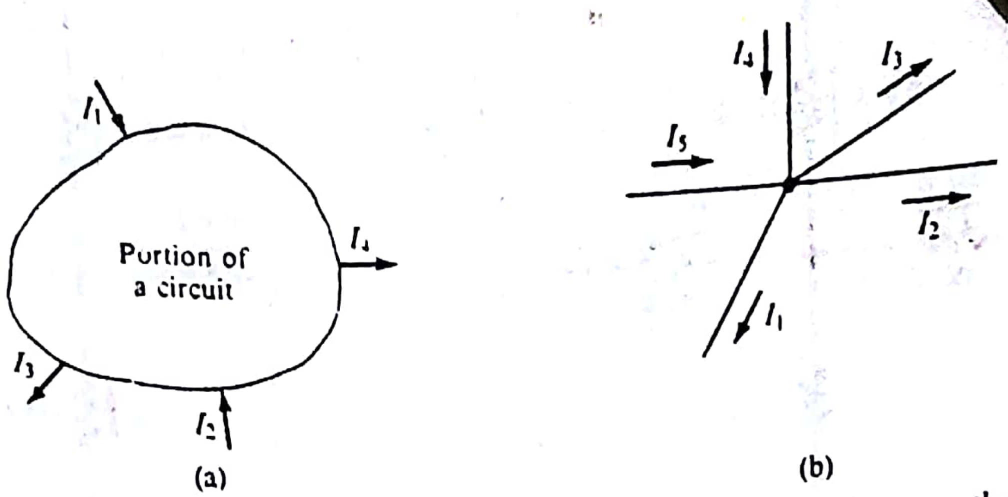

FIGURE 4.30 Examples of Kirchhoff's current law. In each case, the total current entering equals the total current leaving. (a) $I_1 + I_2 = I_3 + I_4$. (b) $I_4 + I_5 = I_1 + I_2 + I_3$.

single junction where two or more components are joined. Figure 4.30 illustrates these ideas, which are summarized by *Kirchhoff's current law*:

> **The sum of all currents entering a junction, or any portion of a circuit, equals the sum of all currents leaving the same.**

Kirchhoff's current law is often used to find an unknown current, given the magnitudes and directions of several other currents in a circuit. It is important to realize that current, like voltage, can have a negative value. The implication of a negative current is simply that its "actual" (positive) direction is the opposite of that which was assumed in a computation. For example, to say that 1 ampere of current is flowing from left to right through a resistor is *entirely equivalent* to saying that *minus* 1 ampere is flowing from right to left. If we assume left-to-right as the *reference* direction for positive current in a particular circuit, then right-to-left current is negative, and we cannot say that a positive value is "correct" while a negative value is "incorrect." The next example illustrates this point.

Example 4.15 (Analysis)

Find the current in the 150-Ω resistor in Figure 4.31(a).

SOLUTION As shown in Figure 4.31(b), we arbitrarily assume that the current in the 150-Ω resistor is entering the junction. In other words, we take that direction as our reference. Assigning the designations I_1, I_2, I_3, and I_4 to the various currents as shown, we write Kirchhoff's current law:

$$\underbrace{I_1 + I_4}_{\text{current entering}} = \underbrace{I_2 + I_3}_{\text{current leaving}}$$

$$0.8 + I_4 = 0.2 + 0.1$$

or

$$I_4 = 0.3 - 0.8 = -0.5 \text{ A}$$

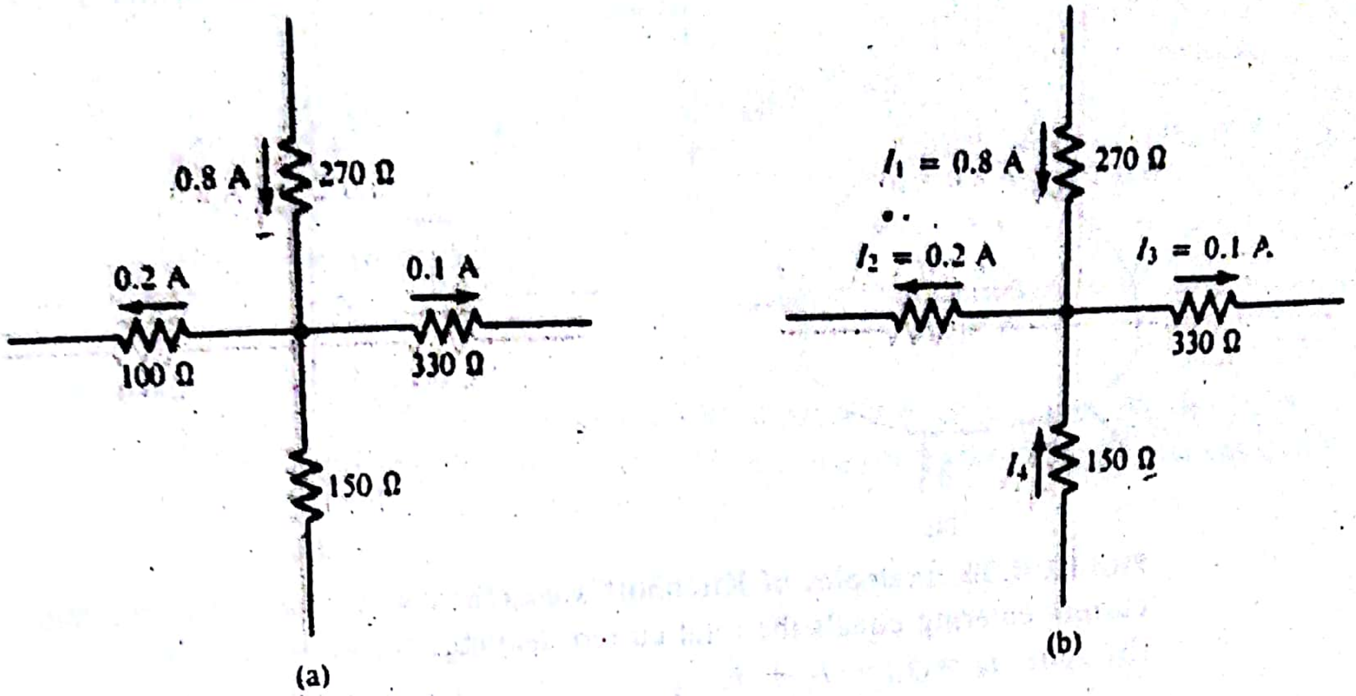

FIGURE 4.31 (Example 4.15)

Thus, the current entering the junction through the 150-Ω resistor is −0.5 A. That statement, as it stands, is perfectly correct. It is equally correct to say that the current leaving the junction through the 150-Ω resistor is +0.5 A.

Drill Exercise 4.15

Repeat Example 4.15 if the directions of the 0.8-A and 0.2-A currents in Figure 4.31(a) are reversed. □

ANSWER: 0.7 A entering the junction.

Example 4.16 (Analysis)

Find the currents I_1, I_2, I_3 and I_4 in Figure 4.32.

SOLUTION Since all components are in parallel, we know that the voltage across each resistor is 48 V. We find the current in each resistor using Ohm's law:

$$I_{12\text{ k}\Omega} = \frac{48\text{ V}}{12\text{ k}\Omega} = 4\text{ mA}$$

$$I_{8\text{ k}\Omega} = \frac{48\text{ V}}{8\text{ k}\Omega} = 6\text{ mA}$$

$$I_{9.6\text{ k}\Omega} = \frac{48\text{ V}}{9.6\text{ k}\Omega} = 5\text{ mA}$$

These currents are shown in Figure 4.33(a). The current leaving terminal *A* equals the current entering it, so

$$I_1 = 5\text{ mA}$$

The current entering junction *B* equals the sum of the currents leaving it, so

$$I_2 = I_1 + 6\text{ mA} = 5\text{ mA} + 6\text{ mA} = 11\text{ mA}$$

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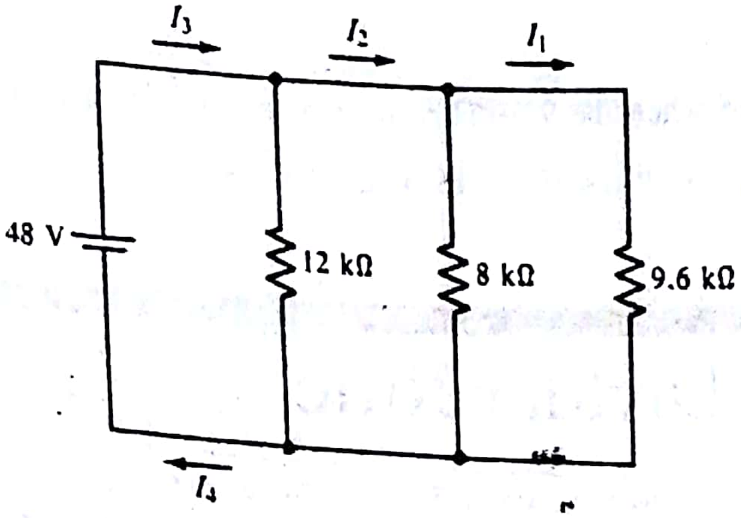

FIGURE 4.32 (Example 4.16)

Similarly, the current entering junction C is

$$I_3 = I_2 + 4\text{ mA} = 11\text{ mA} + 4\text{ mA} = 15\text{ mA}$$

Finally, the current entering the 48-V source must equal the current leaving it, so

$$I_4 = I_3 = 15\text{ mA}$$

We can verify the last result by finding the total equivalent resistance of the parallel-connected resistors and applying Ohm's law [see Figure 4.33(b)]:

$$R_T = \frac{1}{1/12\text{ k}\Omega + 1/8\text{ k}\Omega + 1/9.6\text{ k}\Omega} = 3.2\text{ k}\Omega$$

Thus,

$$I_T = \frac{E_T}{R_T} = \frac{48\text{ V}}{3.2 \times 10^3\ \Omega} = 15\text{ mA}$$

Also,

$$I_T = EG_T = (48\text{ V})\left(\frac{1}{12\text{ k}\Omega} + \frac{1}{8\text{ k}\Omega} + \frac{1}{9.6\text{ k}\Omega}\right) = 15\text{ mA}$$

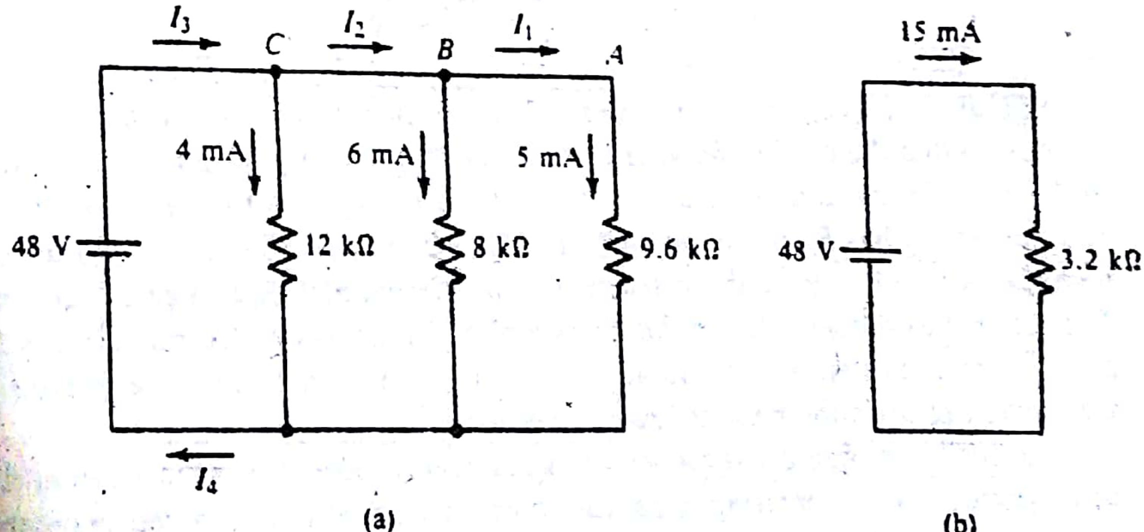

FIGURE 4.33 (Example 4.16)

The Current-Divider Rule

Consider the two parallel resistors shown in Figure 4.34, where a certain known current I enters one of the common junctions. We will derive general expressions for the currents I_1 and I_2 in each resistor. The total equivalent resistance of the parallel combination is

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Therefore, the voltage across the combination is

$$V = IR_T = \frac{IR_1R_2}{R_1 + R_2}$$

The current in R_1 is, by Ohm's law,

$$I_1 = \frac{V}{R_1} = \frac{IR_1R_2}{R_1(R_1 + R_2)}$$

or

$$I_1 = I\frac{R_2}{R_1 + R_2} \qquad (4.16)$$

Similarly,

$$I_2 = I\frac{R_1}{R_1 + R_2} \qquad (4.17)$$

Equations (4.16) and (4.17) show that the current entering a parallel combination divides so that the portion flowing in one branch is directly proportional to the resistance in the *opposite* branch. Note carefully that the expression for I_1 has R_2 in the numerator, and that for I_2 has R_1 in the numerator. Also, take special note that the denominator in both cases is the *sum* of the resistances (*not* the parallel equivalent, a common error). The reader has probably heard the oversimplified expression "current always follows the path of least resistance." It should be clear now that, in reality, the greater *portion* of the current follows the path of lesser resistance.

One easily verified consequence of the current-divider rule is that current divides into equal portions when entering a parallel combination of equal-valued resistors. For example, if 6 mA enters a junction where three 1-kΩ resistors are connected in parallel, 6 mA/3 = 2 mA will flow in each 1-kΩ resistor.

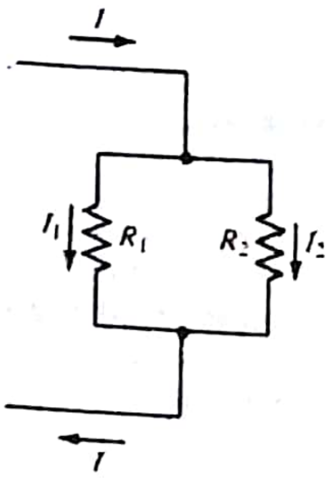

FIGURE 4.34 Current I entering a junction of two parallel resistors divides into two paths. The smaller the resistance of a path, the greater its share of the total current.

Example 4.17 (Analysis)

(a) Find the current in the 470-Ω resistor in Figure 4.35, using the current-divider rule.

(b) Find the current in the 330-Ω resistor using the current-divider rule, and verify the result using Kirchhoff's current law.

SOLUTION

(a) $$I_1 = I\frac{R_2}{R_1 + R_2} = 160\text{ mA}\,\frac{330}{330 + 470} = 66\text{ mA}$$

(b) $$I_2 = I\frac{R_1}{R_1 + R_2} = 160\text{ mA}\,\frac{470}{330 + 470} = 94\text{ mA}$$

Notice that the larger current (94 mA) flows in the smaller resistor (330 Ω). To verify part (b) using Kirchhoff's current law, note that

$$I = I_1 + I_2$$

so

$$I_2 = I - I_1 = 160\text{ mA} - 66\text{ mA} = 94\text{ mA}$$

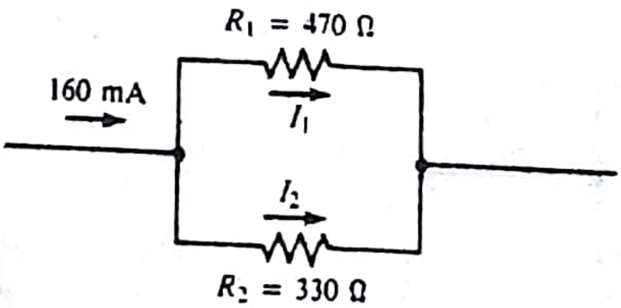

FIGURE 4.35 (Example 4.17)

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The current-divider rule can be generalized to include the case where current I enters a junction of an arbitrary number of parallel resistors:

$$I_x = \frac{IR_T}{R_x} \tag{4.18}$$

This came from?

$IR_T = V$ by ohm's law

so $I = V/R$ so $I_x = \frac{V}{R_x}$

where I_x is the current in resistor R_x and R_T is the total equivalent resistance of the parallel combination of all the resistors.

Example 4.19 (Design)

What should be the value of R in Figure 4.37 if the current in it must be 0.1 A?

SOLUTION The current from the 0.4-A current source divides among the three parallel resistors. By equation (4.18), the current in R is

$$I_x = 0.1 = \frac{0.4R_T}{R}$$

or

$$R_T = 0.25R$$

Now

$$R_T = 30\ \Omega \| 60\ \Omega \| R$$

But

$$30\ \Omega \| 60\ \Omega = \frac{(30)(60)}{30 + 60}\ \Omega = 20\ \Omega$$

so

$$R_T = 20 \| R = 0.25R$$

$\left(\because \frac{20R}{20+R}\right)$

Thus

$$\frac{20R}{20 + R} = 0.25R$$

$$20R = 5R + 0.25R^2$$

$$15R = 0.25R^2$$

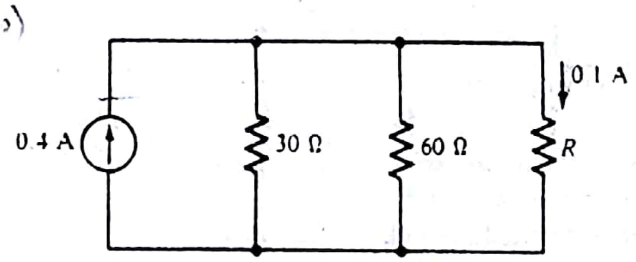
$$R = \frac{15}{0.25} = 60\ \Omega$$

Drill Exercise 4.19

What current will flow in resistor R in Figure 4.37 when R is made equal to 30 Ω?

ANSWER: 0.16 A. □

$I_{60\Omega} = \frac{0.4(30\|60)}{60 + (30\|60)} = 0.1A$



$I_{30\Omega} = \frac{0.4(60\|R)}{30 + (60\|R)} = \frac{0.4(30)}{30 + 20} = 0.2A$

FIGURE 4.37 (Example 4.19)