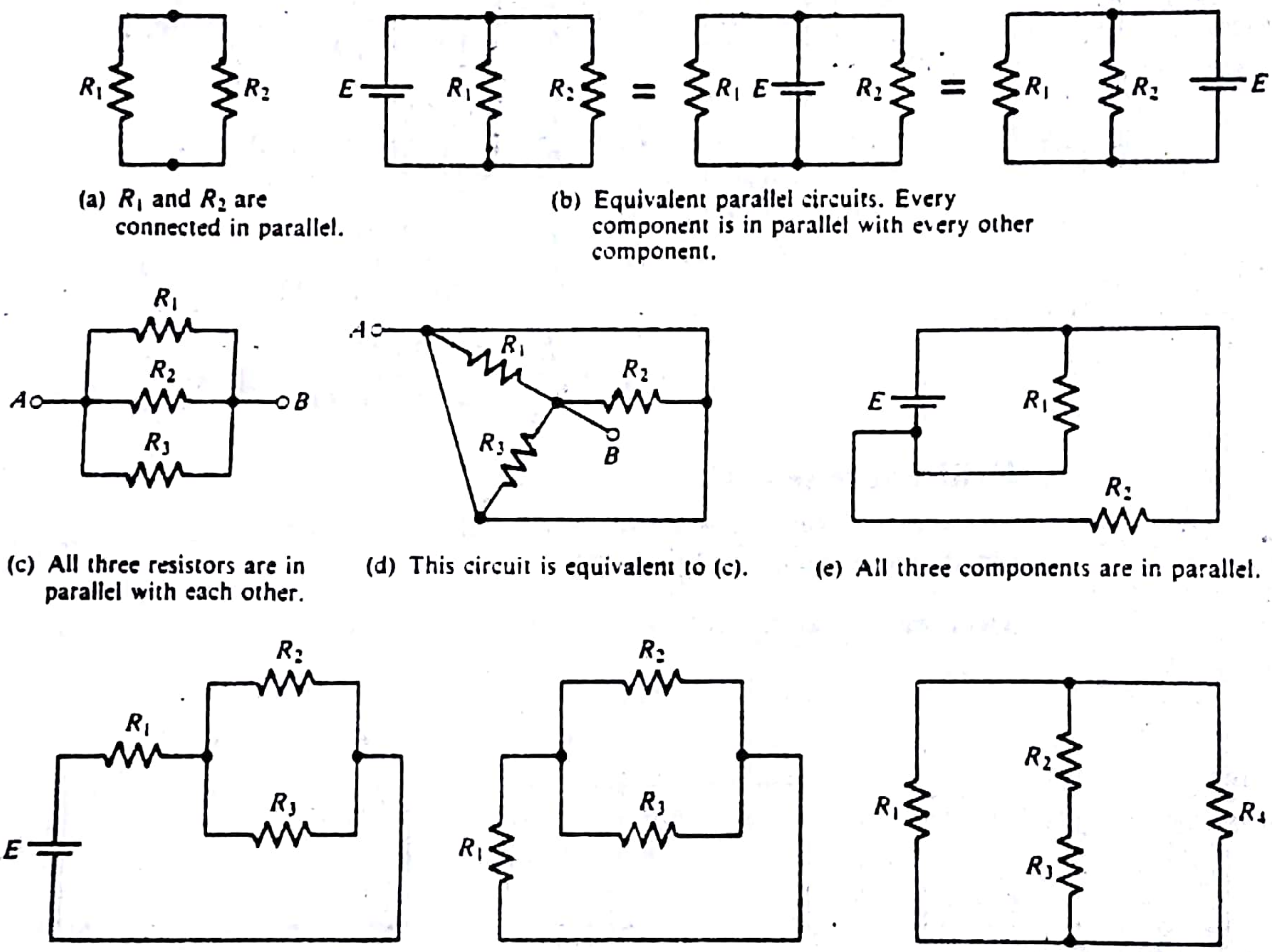
Contents:

Parallel Circuits

Two components are connected in *parallel* when they have two common terminals. A parallel circuit is one in which all components are connected in parallel. Figure 4.26 shows some examples of parallel connections and some parallel circuits. Also shown are some circuits which contain parallel circuits but are not themselves parallel circuits. Study these examples carefully. Some have been intentionally drawn in contorted ways to make it difficult (as often happens in practice) to discern the type of circuits they represent. *Always feel free to redraw a circuit in as many equivalent ways as necessary to convince yourself that it represents a circuit of a specific type* or that a series or parallel connection is truly present. Remember that solid lines represent zero-resistance paths, and they can be drawn as long, as short, or as twisted as desired, without affecting the basic nature of a circuit. It is good practice, particularly for a beginner, to redraw a circuit in a way that feels "comfortable," because only then does it become apparent



(a) R_1 and R_2 are connected in parallel.

(b) Equivalent parallel circuits. Every component is in parallel with every other component.

(c) All three resistors are in parallel with each other.

(d) This circuit is equivalent to (c).

(e) All three components are in parallel.

(f) R_2 is in parallel with R_3. R_1 is neither in series nor in parallel with R_2 or R_3. E is neither in series nor in parallel with R_2 or R_3. (E is in a series path with R_1.)

(g) All three resistors are in parallel with each other.

(h) R_1 is in parallel with R_4. R_2 is neither in series nor in parallel with either R_1 or R_4. R_3 is neither in series nor in parallel with either R_1 or R_4. (R_2 is in a series path with R_3.)

FIGURE 4.26 Examples of parallel-connected components and parallel circuits.

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which circuit analysis tools should be applied to it. Note that it is not necessary for two components to be either in series or in parallel with each other. It is often the case that they are neither in series nor in parallel. Figure 4.26(f) and (h) show examples.

Analysis of Parallel Circuits

The most important property of parallel circuits is that *every parallel-connected component has the same voltage across it*. This property is obvious when we imagine how we would measure the voltage across parallel components, and then realize that measuring across one is the same as measuring across all of them. If we know the voltage across any one parallel component, we know the voltage across each of the others and can therefore find the current in each, using Ohm's law. This computation is illustrated in the next example.

Example 4.12 (Analysis)

Find the current in each resistor in Figure 4.27.

SOLUTION It is clear that all the components in Figure 4.27 are connected in parallel, including the 32-V source. Therefore, each resistor has 32 V across it. By Ohm's law,

$$I_1 = \frac{E}{R_1} = \frac{32\text{ V}}{1.6 \times 10^3\ \Omega} = 20\text{ mA}$$

$$I_2 = \frac{E}{R_2} = \frac{32\text{ V}}{320\ \Omega} = 0.1\text{ A}$$

$$I_3 = \frac{E}{R_3} = \frac{32\text{ V}}{100 \times 10^3\ \Omega} = 320\ \overset{m}{\mu A}$$

$$I_4 = \frac{E}{R_4} = \frac{32\text{ V}}{4 \times 10^3\ \Omega} = 8\text{ mA}$$

Drill Exercise 4.12

Repeat Example 4.12 when E is changed to 48 V and each resistor has half the resistance shown in Figure 4.27.

ANSWER: $I_1 = 60$ mA, $I_2 = 0.3$ A, $I_3 = 0.96$ mA, $I_4 = 24$ mA. □

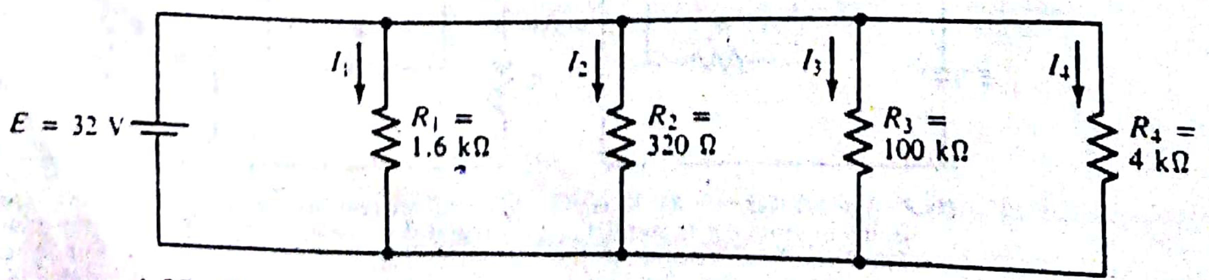

FIGURE 4.27 (Example 4.12)

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Recall that the conductance G of a resistor is defined to be the reciprocal of its resistance: $G = 1/R$ siemens (S). *The total conductance of n parallel-connected resistors is the sum of the n individual conductances*:

$$G_T = G_1 + G_2 + \cdots + G_n \tag{4.9}$$

For example, if a 10-Ω resistor is in parallel with a 20-Ω resistor, the total conductance of the combination is $G_T = \frac{1}{10} + \frac{1}{20} = 0.1 + 0.05 = 0.15$ S. Using equation (4.9), we can derive an expression for the total equivalent resistance of n parallel-connected resistors:

$$G_T = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \tag{4.10}$$

Inverting both sides, we find

$$\frac{1}{G_T} = R_T = \frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_n} \tag{4.11}$$

Recall that Ohm's law expressed in terms of conductance is $I = EG$. Therefore, for a parallel circuit, we have

$$I_T = EG_T \tag{4.12}$$

A special case of a parallel circuit that is often encountered in circuit analysis is the combination of just two resistors, R_1 and R_2. For that case, equation (4.11) is

$$R_T = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{(R_2 + R_1)/R_1R_2}$$

or

$$R_T = \frac{R_1R_2}{R_1 + R_2} \tag{4.13}$$

Equation (4.13) shows that the total equivalent resistance of two parallel resistances is their product divided by their sum. For example, if $R_1 = 3$ kΩ and $R_2 = 6$ kΩ, then

$$R_T = \frac{(3 \times 10^3)(6 \times 10^3)}{3 \times 10^3 + 6 \times 10^3} = \frac{18 \times 10^6}{9 \times 10^3} = 2\ \text{k}\Omega$$

An important fact about parallel-connected resistances is that the total equivalent resistance is *always* less than any one, including the smallest, of the resistances in parallel. This result is evident in the example just discussed: $R_1 = 3$ kΩ, $R_2 = 6$ kΩ, and $R_T = 2$ kΩ.

Another special case of a parallel circuit is the case where all the resistors have the same value: $R_1 = R_2 = \cdots = R_n = R$. In that case, the total equivalent resistance equals the resistance of any one of them, divided by n:

$$R_T = \frac{R} {n} \qquad (n \text{ parallel resistors, each having resistance } R) \tag{4.14}$$

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For example, three 15-kΩ resistors connected in parallel have a total equivalent resistance of $R_T = 15\ \text{k}\Omega/3 = 5\ \text{k}\Omega$.

Example 4.13 (Analysis)

Find the total conductance and the total equivalent resistance of each of the networks in Figure 4.28, with respect to terminals A–B (i.e., the resistance and conductance that a source connected across A–B would see).

SOLUTION

(a) From equation (4.11),

$$R_T = \frac{1}{\frac{1}{50} + \frac{1}{100} + \frac{1}{200}} = \frac{1}{0.02 + 0.01 + 0.005}$$

$$= \frac{1}{0.035} = 28.57\ \Omega$$

$$G_T = \frac{1}{R_T} = \frac{1}{28.57\ \Omega} = 0.035\ \text{S}$$

(b) From equation (4.13),

$$R_T = \frac{(15\ \text{k}\Omega)(30\ \text{k}\Omega)}{15\ \text{k}\Omega + 30\ \text{k}\Omega} = \frac{450}{45}\ \text{k}\Omega = 10\ \text{k}\Omega$$

$$G_T = \frac{1}{R_T} = \frac{1}{10 \times 10^3\ \Omega} = 0.1\ \text{mS}$$

(c) From [illegible]

$$R_T = \frac{27\ \Omega}{3} = 9\ \Omega$$

$$G_T = \tfrac{1}{9} = 0.111\ \text{S}$$

(d) From equation (4.14), the two 1.2-MΩ resistors are equivalent to $1.2\ \text{M}\Omega/2 = 0.6\ \text{M}\Omega = 600\ \text{k}\Omega$. This 600-kΩ resistance is in parallel with the remaining two 600-kΩ resistors, so

$$R_T = \frac{600\ \text{k}\Omega}{3} = 200\ \text{k}\Omega$$

$$G_T = \frac{1}{200\ \text{k}\Omega} = 5\ \mu\text{S}$$

Drill Exercise 4.13

A parallel circuit consists of four parallel-connected 480-Ω resistors in parallel with six 360-Ω resistors. What is the total resistance and total conductance of the circuit?

ANSWER: $R_T = 40\ \Omega$, $G_T = 0.025\ \text{S}$.

□

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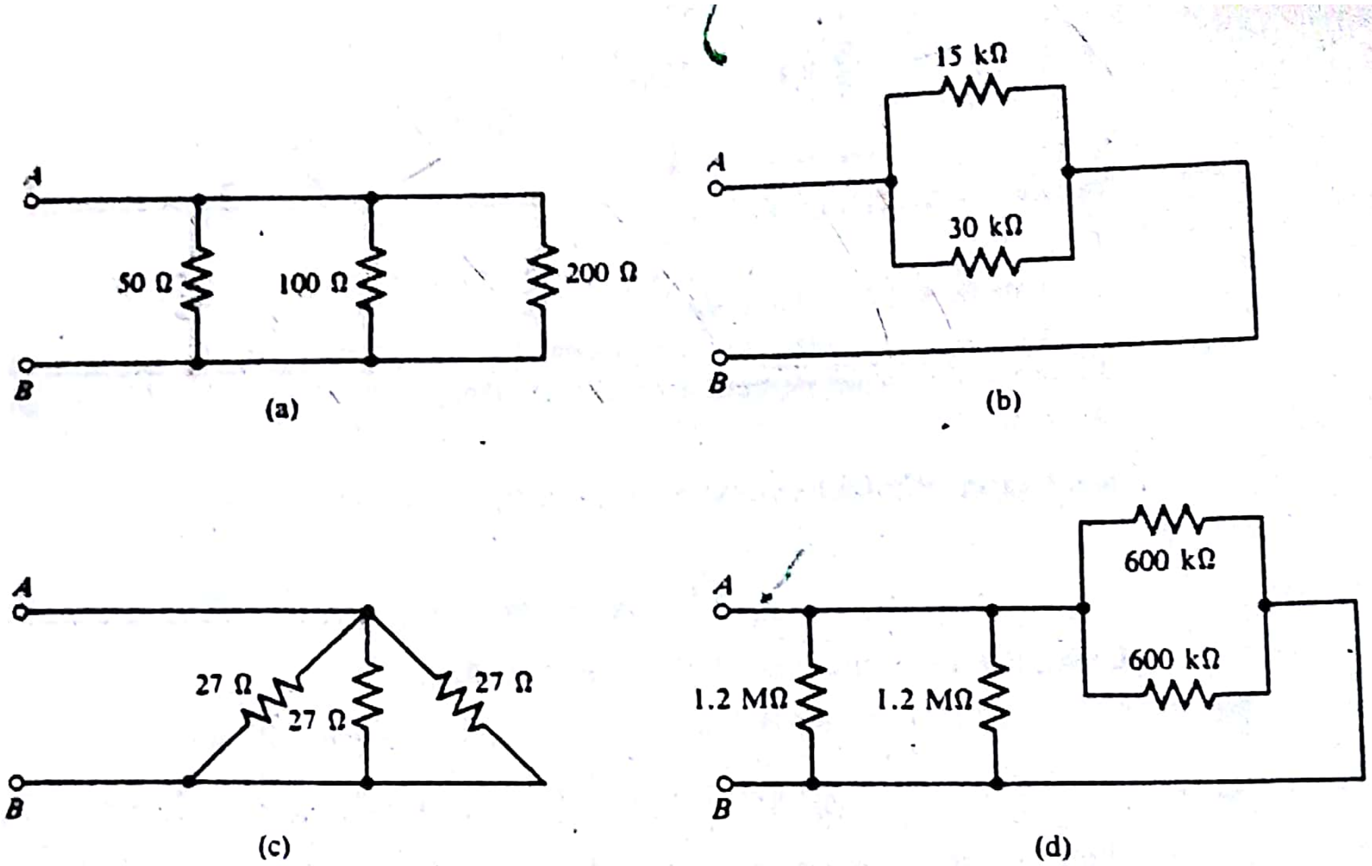

FIGURE 4.28 (Example 4.13)

When one resistance is much greater than another one connected in parallel with it, the equivalent resistance of the combination is very nearly equal to the *smaller* of the two:

$$\text{If} \quad R_2 >> R_1, \quad \text{then} \quad R_1 \| R_2 \approx R_1 \tag{4.15}$$

where $\|$ means "in parallel with"
$\approx$ means "approximately equal to"
$>>$ means "much greater than"

To illustrate, suppose that $R_1 = 10\ \Omega$ and $R_2 = 10\ \text{k}\Omega$. Then

$$R_1 \| R_2 = \frac{10 \times 10^4}{10 + 10^4} = \frac{10^5}{10{,}010} = 9.99\ \Omega \approx R_1$$

In this example, R_2 is 1000 times as great as R_1 and the approximation is very good. The approximation is widely used in practical design and analysis work. Even if R_2 is only 10 times as great as R_1, the assumption that $R_1 \| R_2 = R_1$ gives an error of less than 10%, which may be less than the error resulting from resistor tolerances. The standard student question is: "When can I use this approximation?" As discussed in Chapter 1, the only valid answer is that it depends on the situation. If a particular application requires calculation of a voltage or current to a high accuracy, and if all the component values are known to a high accuracy, the approximation probably should not be used. In situations such as preliminary design work, where only rough estimates are needed, the approximation can be a valuable time-saver.

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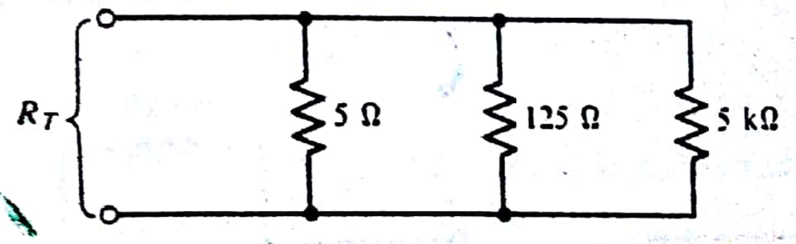

FIGURE 4.29 (Example 4.14)

Example 4.14 (Analysis)

Calculate the exact and approximate total resistance of the network shown in Figure 4.29. What percent error results from using the approximate value instead of the exact?

SOLUTION We find the exact value of R_T using equation (4.11):

$$R_T = \frac{1}{\frac{1}{5} + \frac{1}{125} + \frac{1}{5000}} = 4.803\ \Omega$$

Using the approximation $R_1 \| R_2 \simeq R_1$, we find

$$5\|125 \simeq 5\ \Omega \quad (125 \text{ is } 25 \text{ times greater than } 5)$$

$$5\|5000 \simeq 5\ \Omega \quad (5000 \text{ is } 1000 \text{ times greater than } 5)$$

Thus, the approximate value is 5 Ω. The percent error is

$$\%\ \text{error} = \frac{5 - 4.803}{4.803} \times 100\% = 4.1\%$$

Drill Exercise 4.14

Repeat Example 4.14 if the 5-Ω resistor is replaced by a 50-Ω resistor.

ANSWER: $R_T = 35.461\ \Omega$; $R_T \approx 50\ \Omega$; 41% error.

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