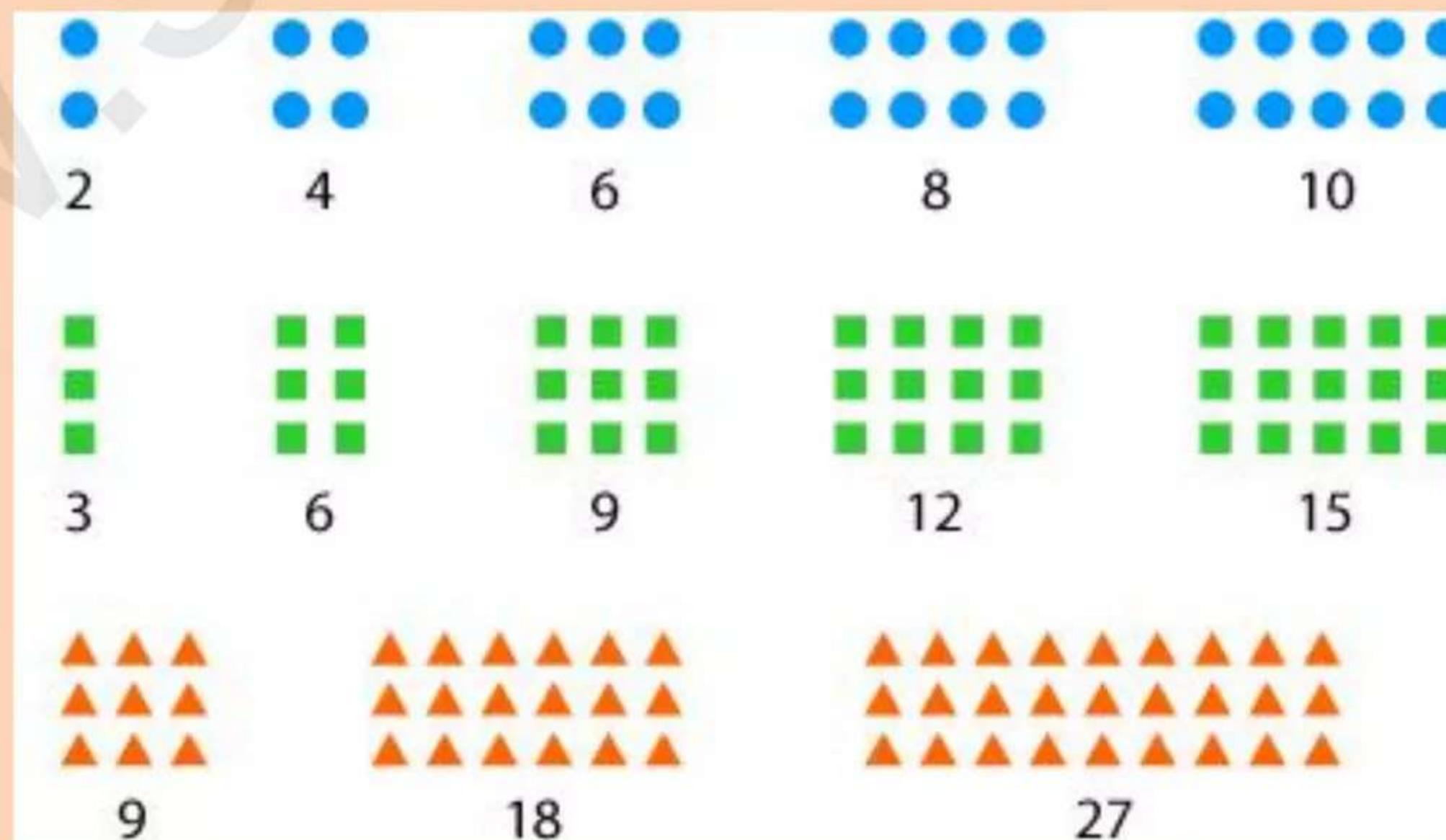
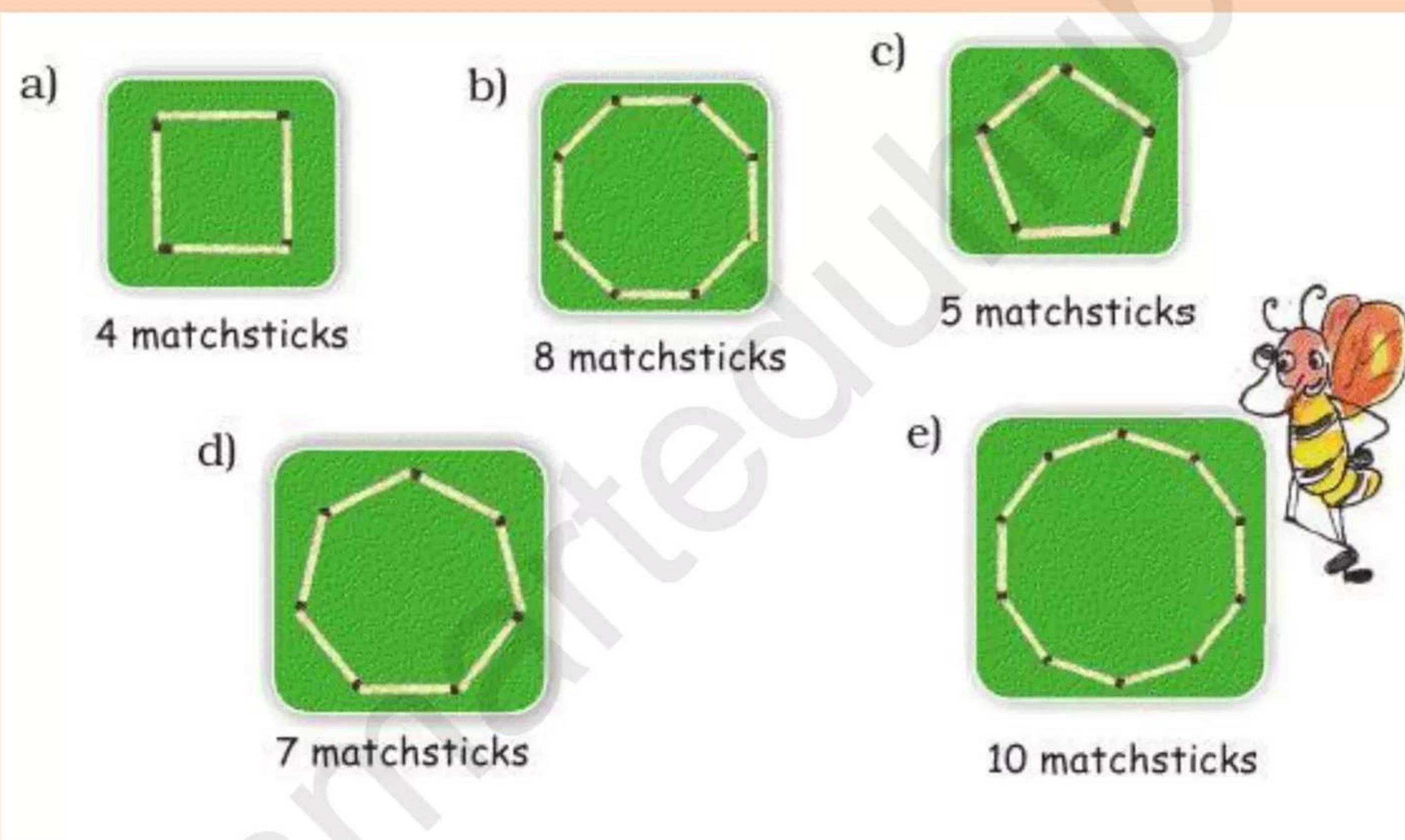


SEQUENCES



The following worksheets contain solved examples for linear, quadratic and cubic sequences.



Sequences

Sequence:

A sequence is a list of numbers (or other things) that changes according to some sort of pattern In mathematics this pattern is called a common difference or ratio.

Term:

- Each number in a sequence is called as a term.
- For example the first term is the term that occurs first in a sequence.
- The 5th term is the term that occurs in the fifth place of the sequence.
- The nth term is the term that occurs in the nth position of the sequence.

Examples of sequences:

- 1,4,9,16,25,36.....
- 0,2,4,6,8,10.....

Types of sequences:

- Arithmetic (Linear)
- Quadratic
- Cubic
- Geometric

Linear sequence:

A linear sequence is a sequence with the first difference between two consecutive terms constant.

Examples:

a. $0, 2, 4, 6, 8, 10, \dots$

$0, (0+2), (2+2), (4+2) \dots$ the common difference is 2

b. $30, 35, 40, 45, 50, 55, \dots$

$30, (30+5), (35+5), (40+5), \dots$ the common difference is 5

c. $48, 46, 44, 42, \dots$

$48, (48-2), (46-2), (44-2), \dots$ the common difference is -2

Formula for calculating the nth term of an arithmetic (linear sequence) is $a + (n-1)d$

where:

a = 1st term,

d = common difference &

n = the term you need to find out.

Example:

In the sequence 'a' above; first term is $=a=0$, d = common difference = 2 and $n=7$ th term would be calculated in the following way:

$$t_n = a + (n-1)d$$

$$\text{So, } t_7 = 0 + (7-1)2 = 0 + 6(2) = 0 + 12 = 12$$

Do it yourself

Find the common difference and also the next two terms for the following sequences:

- $-15, -7, 1, 9, \dots$
- $-40, -240, -440, -640, \dots$
- $31, 21, 11, 1, \dots$
- $38, 138, 238, 338, \dots$

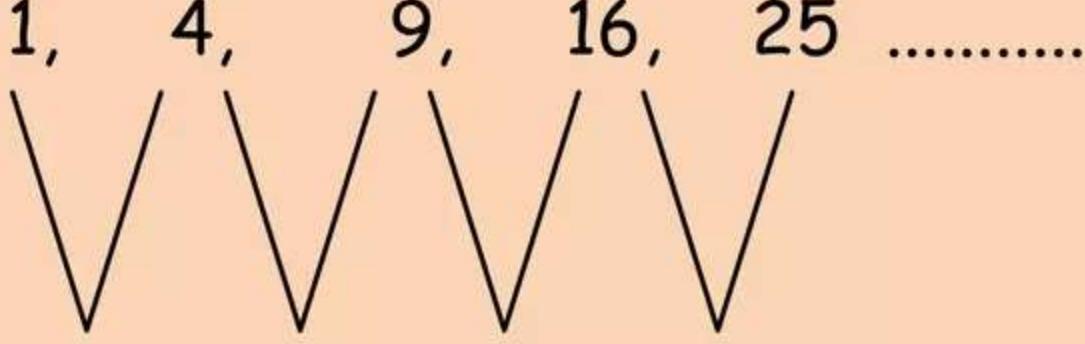
Quadratic sequence:

Whenever the second difference is constant in a sequence, the sequence is said to be a quadratic sequence.

- In such a sequence the 2nd difference is constant.
- It is of the form: $an^2 + bn + c$

Examples of quadratic sequences:

a. $1, 4, 9, 16, 25 \dots$



\rightarrow 1st difference is not constant



\rightarrow 2nd difference is constant

Method for finding the formula for the sequence:

	Sequence is of the form: $an^2 + bn + c$	$1n^2 + 1$																								
Step 1	2nd difference is 2. So to find the value of a , always halve the common difference. So value of a is $2/2=1$	$a=1$																								
Step 2	Write the original sequence and from it subtract the value of $1n^2$	<table border="1"> <tr> <td>Original seq: (S)</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td></tr> <tr> <td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr> <td>n^2</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td></tr> <tr> <td>$S - (n^2)$</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> </table>	Original seq: (S)	1	4	9	16	25	n	1	2	3	4	5	n^2	1	4	9	16	25	$S - (n^2)$	0	0	0	0	0
Original seq: (S)	1	4	9	16	25																					
n	1	2	3	4	5																					
n^2	1	4	9	16	25																					
$S - (n^2)$	0	0	0	0	0																					
Step 3	Observe the common difference (d) when you subtract $1n^2$ from the original sequence	Common difference (d) 0																								
Step 4	The remaining sequence is therefore linear and of the form $a+ (n-1)d$	Remaining sequence is : $1+ (n-1)0$ $= 1+ 0= 1$																								
Step 5	Combining steps 1 and 4 we get our required quadratic sequence	$1n^2 + 1$																								

Cubic sequences:

Sequences where the 3rd difference is constant are known as cubic sequences.
Cubic sequences are of the form:

$$an^3 + bn^2 + cn + d$$

Examples:

	0	12	10	0	-12	-20
1st difference		+12	-2	-10	-12	
2nd difference			-14	-8	-2	
3rd difference				+6		

3rd difference is constant so it is a cubic sequence .

It is of the form: $an^3 + bn^2 + cn + d = n^3 - 13n^2$

Step 1: Value of $a = 1/6 \times (3rd \text{ difference}) = 1/6 \times (6) = 1$. Hence $a=1$.

Step 2: Write original sequence and subtract an^3 i.e. n^3 from it as shown below.

S	0	12	10	0	-12	-20
$1n^3$	1	8	27	64	125	216
$s - 1n^3 = \text{our new sequence}$	-1	+4	-17	-64	-137	-236
1st Common difference		+5	-21	-47	-73	
2nd common difference			-26	-26	-26	
2nd common difference		2nd difference is constant so our new sequence is quadratic and of the form $bn^2 + cn + d$; where $b = -26/2 = -13$				
Performing this operation: $[S - (n^3 - 13n^2)]$; leaves us with a linear sequence and we get the part $cn+d$	S	0	12	10	0	-12
	n	1	2	3	4	5
	$(n^3 - 13n^2)$	$(1-13) = -12$	$8-52 = -44$	$27-117 = -90$	$64-208 = -144$	$125-325 = 200$
Finally the required sequence is	$s - (n^3 - 13n^2)$	12	56	100	144	-212
Common difference is 44. Sequence is linear		+44	+44	+44	+44	
Formula for the linear sequence is		$12 + (n-1) 44 = 12 + 44n - 44 = 44n + 32$				
Ans: Cubic sequence is		$n^3 - 13n^2 + 44n + 32$				