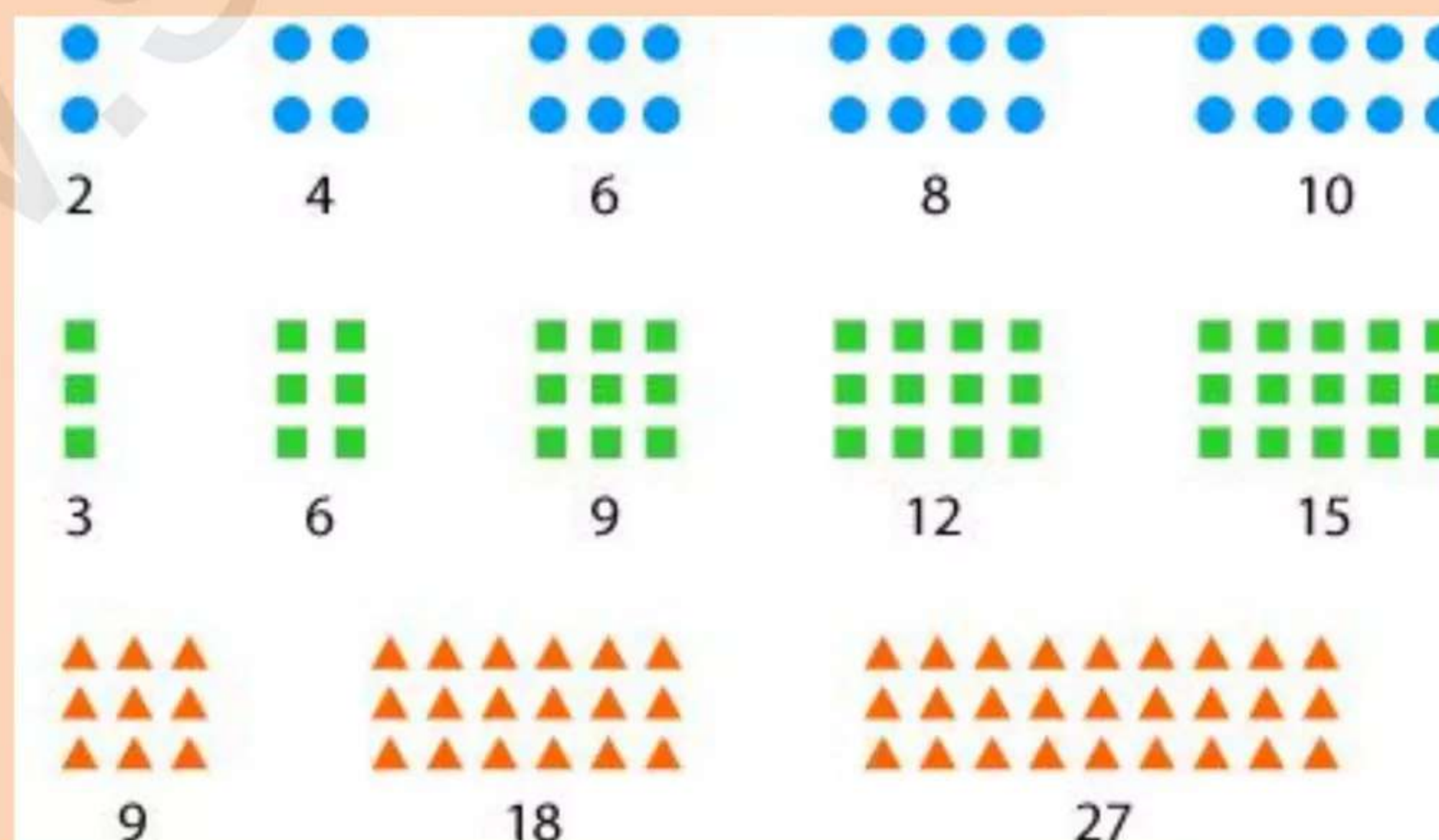
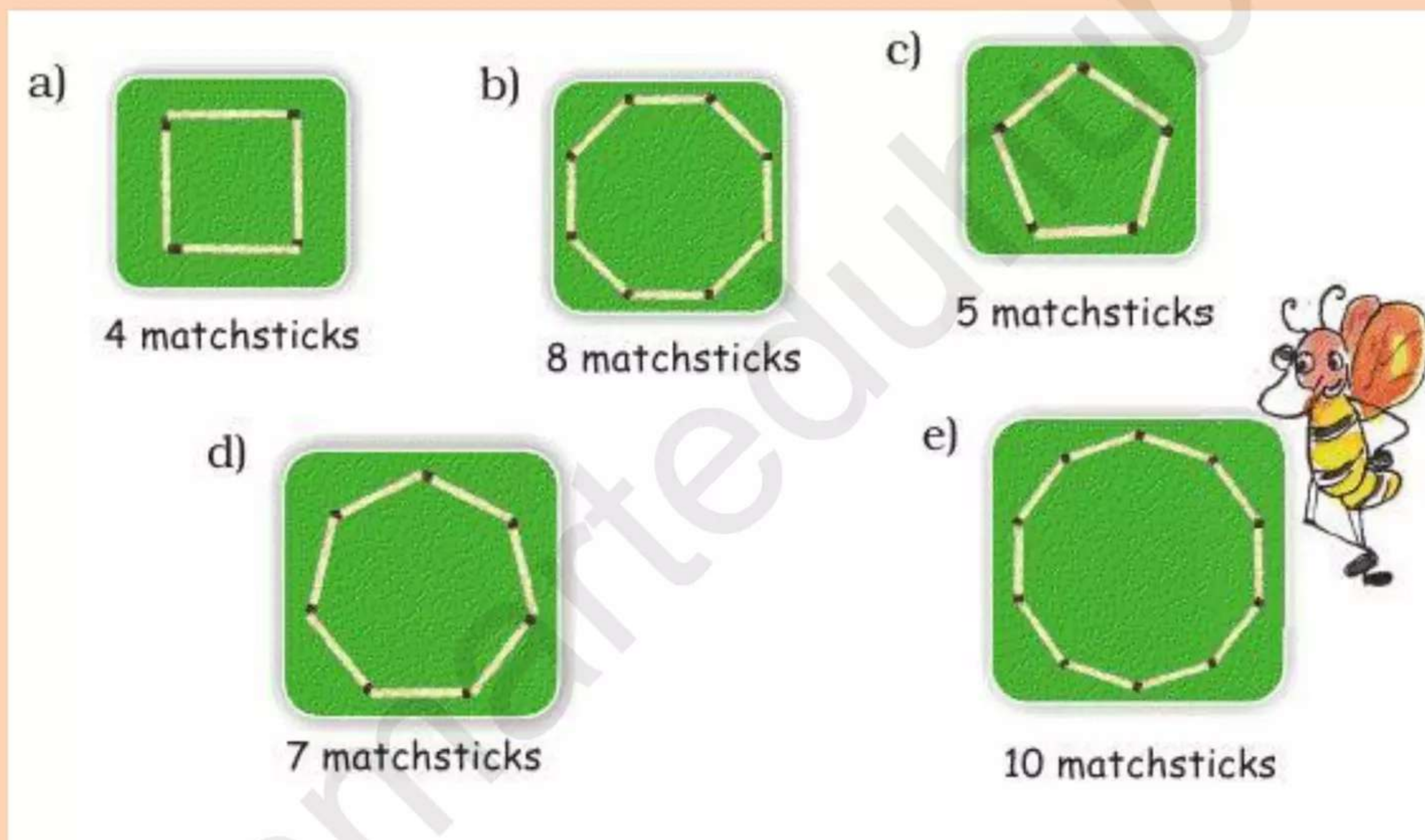


SEQUENCES



The following worksheets contain solved examples for linear , quadratic and cubic sequences.



Sequences

Sequence:

A sequence is a list of numbers (or other things) that changes according to some sort of pattern. In mathematics, this pattern is called a common difference or ratio.

Term:

- Each number in a sequence is called as a term.
- For example, the first term is the term that occurs first in a sequence.
- The 5th term is the term that occurs in the fifth place of the sequence.
- The n th term is the term that occurs in the n th position of the sequence.

Examples of sequences:

- 1, 4, 9, 16, 25, 36,
- 0, 2, 4, 6, 8, 10,

Types of sequences:

- Arithmetic (Linear)
- Quadratic
- Cubic
- Geometric

Linear sequence:

A linear sequence is a sequence with the first difference between two consecutive terms constant.

Examples:

a. 0, 2, 4, 6, 8, 10,

0, (0+2), (2+2), (4+2),the common difference is 2

b. 30, 35, 40, 45, 50, 55,

30, (30+5), (35+5), (40+5),the common difference is 5

c. 48, 46, 44, 42,

48, (48-2), (46-2), (44-2),the common difference is -2

Formula for calculating the nth term of an arithmetic (linear sequence) is $a + (n-1)d$

where:

a= 1st term,

d= common difference &

n = the term you need to find out.

Example:

In the sequence 'a' above; first term is $a=0$, d = common difference =2 and $n=7$ th term would be calculated in the following way:

$$t_n = a + (n-1)d$$

$$\text{So, } t_7 = 0 + (7-1)2 = 0 + 6(2) = 0 + 12 = 12$$

Do it yourself

Find the common difference and also the next two terms for the following sequences:

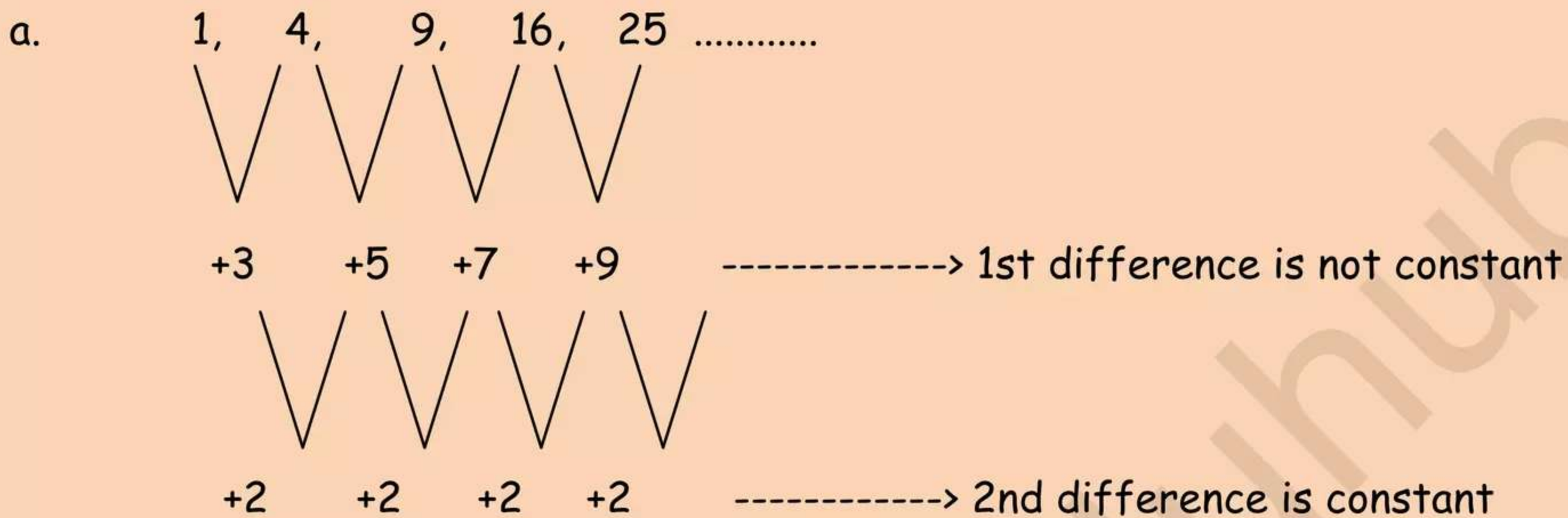
- -15, -7, 1, 9, ...
- -40, -240, -440, -640, ...
- 31, 21, 11, 1, ...
- 38, 138, 238, 338, ...

Quadratic sequence:

Whenever the second difference is constant in a sequence, the sequence is said to be a quadratic sequence.

- In such a sequence the 2nd difference is constant.
- It is of the form: $an^2 + bn + c$

Examples of quadratic sequences:



Method for finding the formula for the sequence:

	Sequence is of the form: $an^2 + bn + c$		$1n^2 + 1$					
Step 1	2nd difference is 2. So to find the value of a, always halve the common difference. So value of a is $2/2=1$		$a=1$					
Step 2	Write the original sequence and from it subtract the value of $1n^2$		Original seq: (S)	1	4	9	16	25
			n	1	2	3	4	5
			n^2	1	4	9	16	25
			$S-(n^2)$	0	0	0	0	0
Step 3	Observe the common difference (d) when you subtract $1n^2$ from the original sequence	Common difference (d)	0					
Step 4	The remaining sequence is therefore linear and of the form $a + (n-1)d$		Remaining sequence is : $1 + (n-1)0$ $= 1 + 0 = 1$					
Step 5	Combining steps 1 and 4 we get our required quadratic sequence		$n^2 + 1$					

Cubic sequences:

Sequences where the 3rd difference is constant are known as cubic sequences. Cubic sequences are of the form:

$$an^3 + bn^2 + cn + d$$

Examples:

	0	12	10	0	-12	-20
1st difference		+12	-2	-10	-12	
2nd difference			-14	-8	-2	
3rd difference				+6	+6	

3rd difference is constant so it is a cubic sequence .

It is of the form: $an^3 + bn^2 + cn + d = n^3 - 13n^2$

Step 1: Value of $a = \frac{1}{6} \times (\text{3rd difference}) = \frac{1}{6} \times (6) = 1$. Hence $a=1$.

Step 2: Write original sequence and subtract an^3 i.e. n^3 from it as shown below.

S	0	12	10	0	-12	-20
$1n^3$	1	8	27	64	125	216
$s-1n^3$ = our new sequence	-1	+4	-17	-64	-137	-236
1st Common difference	<div><div><div></div><div></div><div>+5</div></div><div><div></div><div></div><div>-21</div></div><div><div></div><div></div><div>-47</div></div><div><div></div><div></div><div>-73</div></div></div>					
2nd common difference	<div><div><div></div><div></div><div>-26</div></div><div><div></div><div></div><div>-26</div></div><div><div></div><div></div><div>-26</div></div><div><div></div><div></div><div></div></div></div>					
2nd common difference	2nd difference is constant so our new sequence is quadratic and of the form $bn^2 + cn + d$; where $b=-26/2 = -13$					
Performing this operation: [$S-(n^3-13n^2)$]; leaves us with a linear sequence and we get the part $cn+d$	S	0	12	10	0	-12
	n	1	2	3	4	5
	(n^3-13n^2)	$(1-13)=-12$	$8-52=-44$	$27-117=-90$	$64-208=-144$	$125-325=-200$
Finally the required sequence is	$s-(n^3-13n^2)$	12	56	100	144	-212
Common difference is 44.Sequence is linear	<div><div><div></div><div></div><div>+44</div></div><div><div></div><div></div><div>+44</div></div><div><div></div><div></div><div>+44</div></div><div><div></div><div></div><div>+44</div></div></div>					
Formula for the linear sequence is	$12 + (n-1) 44 = 12 + 44n-44 = 44n+32$					
Ans: Cubic sequence is	$n^3 - 13n^2 + 44n + 32$					