Finding the n^{th} term of sequence

Linear Sequences

Method 1

Method 2

Quadratic Sequences

Method 1

Method 2

Cubic Sequences

Introducing sequences

In maths, we call a list of numbers in order a **sequence**.

Each number in a sequence is called a **term**.

4, 8, 12, 16, 20, 24, 28, 32, . . .

1st term **6th term**

If terms are next to each other they are referred to as **consecutive terms**.

When we write out **sequences**, **consecutive terms** are usually separated by commas.

Infinite and finite sequences

A **sequence** can be **infinite**. That means it continues forever.

For example, the sequence of multiples of 10,

10, 20 ,30, 40, 50, 60, 70, 80, 90 . . .

is infinite. We show this by adding three dots at the end.

If a sequence has a fixed number of terms it is called a **finite** sequence.

For example, the sequence of two-digit square numbers

16, 25 ,36, 49, 64, 81

is finite.

Sequences and rules

Some sequences follow a simple rule that is easy to describe.

For example, this sequence

2, 5, 8, 11, 14, 17, 20, 23, 26, 29, …

continues by adding 3 each time. Each number in this sequence is one less than a multiple of three.

Other sequences are completely random.

For example, the sequence of winning raffle tickets in a prize draw.

In maths we are mainly concerned with sequences of numbers that follow a rule.

Naming sequences

Here are the names of some sequences which you may know already:

2, 4, 6, 8, 10, . . .	**Even Numbers (or multiples of 2)**
1, 3, 5, 7, 9, . . .	**Odd numbers**
3, 6, 9, 12, 15, . . .	**Multiples of 3**
5, 10, 15, 20, 25 . . .	**Multiples of 5**
1, 4, 9, 16, 25, . . .	**Square numbers**
1, 3, 6, 10,15, . . .	**Triangular numbers**

Ascending sequences

When each term in a sequence is bigger than the one before the sequence is called an **ascending** sequence.

For example,

The terms in this ascending sequence increase in **equal steps** by adding 5 each time.

2, 7, 12, 17, 22, 27, 32, 37, . . .

+5 +5 +5 +5 +5 +5 +5

The terms in this ascending sequence increase in **unequal steps** by starting at 0.1 and doubling each time.

0.1, 0.2, 0.4, 0.8, 1.6, 3.2, 6.4, 12.8, . . .

×2 ×2 ×2 ×2 ×2 ×2 ×2

Descending sequences

When each term in a sequence is smaller than the one before the sequence is called a **descending** sequence.

For example,

The terms in this descending sequence decrease in **equal steps** by starting at 24 and subtracting 7 each time.

24, 17, 10, 3, –4, –11, –18, –25, . . .

–7 –7 –7 –7 –7 –7 –7

The terms in this descending sequence decrease in **unequal steps** by starting at 100 and subtracting 1, 2, 3, …

100, 99, 97, 94, 90, 85, 79, 72, . . .

–1 –2 –3 –4 –5 –6 –7

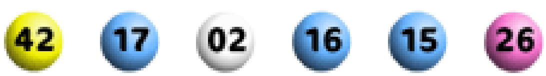
Sequences from real-life

Number sequences are all around us.

Some sequences, like the ones we have looked at today follow a simple rule.

Some sequences follow more complex rules, for example, the time the sun sets each day.

Some sequences are completely random, like the sequence of numbers drawn in the lottery.



What other number sequences can be made from real-life situations?

nth term of linear sequences

n	1	2	3	4	5	
an	4	8	12	16	20	
$an + b$	3	7	11	15	19	b
a	4	4	4	4		

$a = 4$ $b = -1$

n^{th} term is 4n - 1

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nth term of linear sequences

n	1	2	3	4	5
an	7	14	21	28	35
an + b	1	8	15	22	29
a		7	7	7	7

b

a = 7

b = -6

n^th term is 7n - 6

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nth term of linear sequences

n^{th} term : $an + b$

1	2	3	
1	9	17	...
	8	8	

1	2	3
$a + b$	$2a + b$	$3a + b$
	a	a

$a = 8$

$a + b = 1$

$8 + b = 1$

$b = -7$

n^{th} term => $8n - 7$

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Objective

Finding the nth term of a quadratic sequence

$n^2 - n + 2$

2 4 8 14 22

2 4 6 8

2 2 2

$$2n^2 - 2n - 3$$

-3 1 9 21 37

4 8 12 16

4 4 4

$an^2 + bn + c$

$a+b+c$ $4a+2b+c$ $9a+3b+c$

$3a + b$ $5a + b$

$2a$

Copy this down and learn it

$$an^2 + bn + c$$

3 4 11 24

1 7 13

6 6

$a+b+c$ $4a+2b+c$ $9a+3b+c$

$3a + b$ $5a + b$

$2a$

$$an^2 + bn + c$$

3

1

6

$a+b+c$

$3a + b$

$2a$

$$6 = 2a$$

$$1 = 3a + b$$

$$3 = a + b + c$$

$$an^2 + bn + c$$

$$3n^2 - 8n + 8$$

$a = 3$	$6 = 2a$
$b = -8$	$1 = 3a + b$
$c = 8$	$3 = a + b + c$

nd the nth terms of these sequence

3	5	9	15	23
2	10	22	38	58
-1	3	10	20	33
50	40	31	23	16

$$an^2 + bn + c$$

$a+b+c$ $4a+2b+c$ $9a+3b+c$

$3a + b$ $5a + b$

$2a$

A cubic sequence is given by

$$an^3 + bn^2 + cn + d$$

Can you find the algebraic difference table for it?

$$an^3 + bn^2 + cn + d$$

$a+b+c+d$ $8a+4b+2c+d$ $27a+9b+3c+d$ $64a+16b+4c+d$

$7a+3b+c$ $19a+5b+c$ $37a+7b+c$

$12a+2b$ $18a+2b$

$$an^3 + bn^2 + cn + d$$

$a+b+c+d$ $8a+4b+2c+d$ $27a+9b+3c+d$ $64a+16b+4c+d$

$7a+3b+c$ $19a+5b+c$ $37a+7b+c$

$12a+2b$ $18a+2b$

$6a$

Copy this down and learn it

$$an^3 + bn^2 + cn + d$$

1   -1   5   25   65

-2   6   20   40

8   14   20

6   6

$a+b+c+d$ $8a+4b+2c+d$ $27a+9b+3c+d$ $64a+16b+4c+d$ $125a+25b+5c+d$

$7a+3b+c$ $19a+5b+c$ $37a+7b+c$ $61a+9b+c$

$12a+2b$ $18a+2b$ $24a+2b$

$6a$ $6a$

$$an^3 + bn^2 + cn + d$$

1

-2

8

6

$a+b+c+d$

$7a+3b+c$

$12a+2b$

$6a$

$$6 = 6a$$

$$8 = 12a + 2b$$

$$-2 = 7a + 3b + c$$

$$1 = a + b + c + d$$

$$an^3 + bn^2 + cn + d$$

$$n^3 - 2n^2 - 3n + 5$$

$a = 1$ $\quad 6 = 6a$

$b = -2$ $\quad 8 = 12a + 2b$

$c = -3$ $\quad -2 = 7a + 3b + c$

$d = 5$ $\quad 1 = a + b + c + d$

nd the nth terms of these sequence

-11	-13	7	61	161
9	5	-19	-81	-199
-7	-3	9	35	81
17	-3	17	81	207
32	37	24	-19	-104