JOINT PROBABILITY DISTRIBUTIONS

For both discrete and continuous random variables we will discuss the following:

- Joint Distributions (for two or more r:v:'s)
- Marginal Distributions (computed from a joint distribution)
- If X and Y are two random variables, the probability distribution that denes their simultaneous behavior is called a joint probability distribution.

- Shown here as a table for two discrete random variables, which gives

$$\mathrm{P}(X = x, Y = y)$$

X / Y	1	2	3
1	0	1/6	1/6
2	1/6	0	1/6
3	1/6	1/6	0

- If X and Y are discrete, this distribution can be described with a joint probability mass function.
- If X and Y are continuous, this distribution can be described with a joint probability density function.

Example:

- Measurements for the length and width of a rectangular plastic covers for CDs are rounded to the nearest mm (so they are discrete).
- Let X denote the length and
- Y denote the width.

The possible values of X are 129, 130, and 131 mm. The possible values of Y are 15 and 16 mm (Thus, both X and Y are discrete).

- There are 6 possible pairs (X; Y).

- We show the probability for each pair in the following table:

X / Y	139	130	131
15	0.12	0.42	0.06
16	0.08	0.28	0.04

The sum of all the probabilities is 1.

The joint probability mass function is the function $f_{(X,Y)} = P(X = x, Y = Y)$. For example, we have $f_{(X,Y)}(129,15) = 0.12$. If we are given a joint probability distribution for X and Y , we can obtain the individual probability (and these are called the Marginal Probability distribution.

Example: Continuing plastic covers for CDs

- Find the probability that a CD cover has length of 129mm (i.e. $X = 129$).
- $P(X = 129) =$
  $P(X = 129, Y = 15) + P(X = 129, Y = 16) = 0.08 + 0.12 = 0.20$

What is the probability distribution of X?

X / Y	139	130	131
15	0.12	0.42	0.06
16	0.08	0.28	0.04
	0.20	0.70	0.10

- The probability distribution for X appears in the column totals

x	139	130	131
P(x)	0.20	0.70	0.10

We can do the same for the Y random variable:

y	15	16
P(y)	0.6	0.4

Because the probability mass functions for X and Y appear in the margins of the table (i.e. column and row totals), they are often referred to as the Marginal Distributions for

X and Y .