Description:

■ This course include: Number Systems and Binary Codes; Boolean Algebra and Basic Results; Switching Functions; Minimization Techniques; Analysis and Design of Combinational and Sequential Logic circuits.

■ Course Objectives :

A student who successfully fulfils the course requirements will have demonstrated an ability to:

- Perform arithmetic operations in many number systems
- Manipulate Boolean algebraic structures
- A nalyze and design various combinational logic circuits.
- Analyze and design clocked sequential circuits

■ Textbook:

■ Digital Design - Morris Mano – 4th Edition.

References:

- ◆ Thomas L. Floyd, <u>Digital Fundamentals</u>, Merrill, imprint of Macmillan Publishing Company New York, 1994.
- ◆ M. M. Mano & C. R. Kime, <u>Logic and Computer Design Fundamentals</u>, Prentice-Hall 2001.
- B. Stephen & V. Zvonko, <u>Fundamentals of Digital Logic with VHDL</u>
 <u>Design</u>, McGraw-Hill 2000

■ Laboratory Works: :

Laboratory sessions are organized in parallel to theoretical study given in classrooms. Student required to perform at least 7 experiments and submit reports for evaluation.

■ Grading System :

Continuous A ssessment (30%)

Quizzes, Homeworks and Lab work	20
> Midterm Exam	10
Final examinations (70%)	
> Lab Exam	25
> Final Exam	45

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Chapter 1 Digital Systems and Binary Numbers

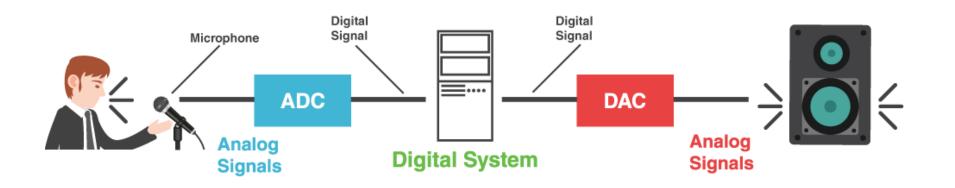
Outline of Chapter 1

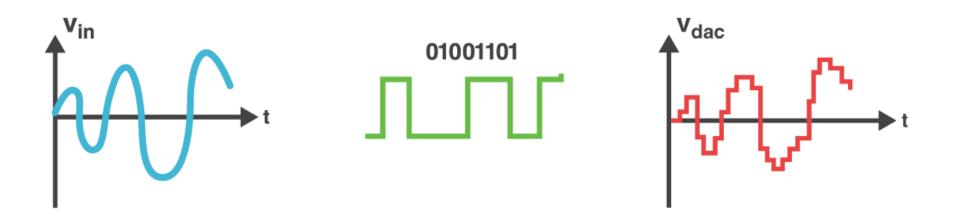
- 1.1 Digital Systems
- 1.2 Binary Numbers
- 1.3 Number-base Conversions
- 1.4 Octal and Hexadecimal Numbers
- 1.5 Complements
- 1.6 Signed Binary Numbers
- 1.7 Binary Codes
- 1.8 Binary Storage and Registers
- 1.9 Binary Logic

1.1 Digital Systems and Binary Numbers

- Digital computers
 - General purposes
 - ◆ Many scientific, industrial and commercial applications
- Digital systems (Embedded)
 - Telephone switching exchanges
 - Digital camera
 - Electronic calculators, PDA 's
 - ◆ Digital TV
- Discrete information-processing systems
 - ♦ Manipulate discrete elements of information (Factories)

Analog and Digital Signal





Analog and Digital Signal

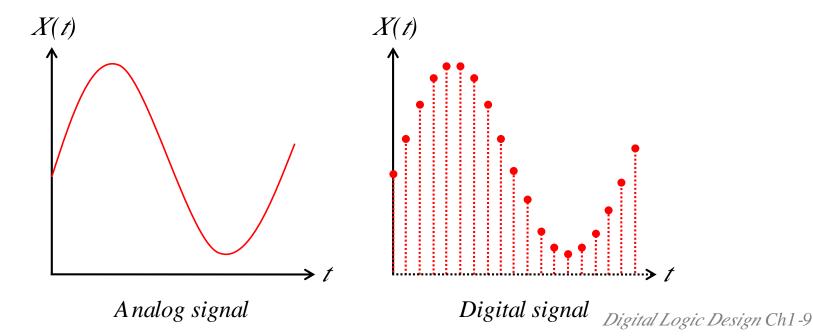
■ Analog system

 The physical quantities or signals may vary continuously over a specified range.

■ Digital system

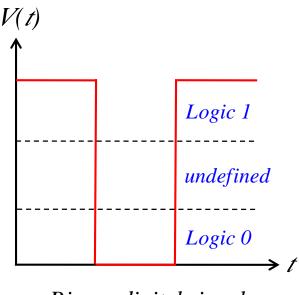
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- ◆ The physical quantities or signals can assume only discrete values.
- Greater accuracy



Binary Digital Signal

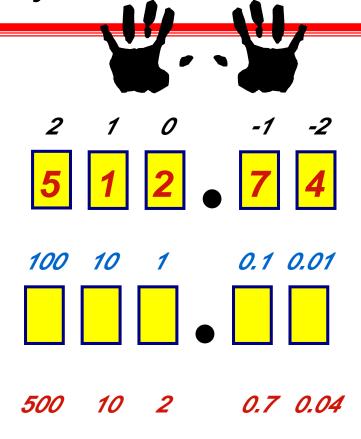
- An information variable represents a physical quantity.
- For digital systems, the variable takes on discrete values.
 - ◆ Two level, or binary values are the most prevalent values.
- Binary values are represented abstractly by:
 - Digits 0 and 1
 - ◆ Words (symbols) False (F) and True (T)
 - ◆ Words (symbols) Low (L) and High (H)
 - ◆ And words On and Off
- Binary values are represented by values or ranges of values of physical quantities.



Binary digital signal

Decimal Number System

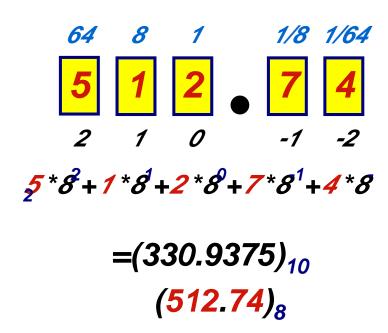
- \blacksquare Base (also called radix) = 10
 - ◆ 10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }
- Digit Position
 - ◆ Integer & fraction
- Digit Weight
 - $Weight = (Base)^{Position}$
- *Magnitude*
 - Sum of "Digit x Weight"
- Formal Notation



$$\frac{d_2^*B^2 + d_1^*B^1 + d_0^*B^0 + d_1^*B^1 + d_2^*B^2}{(512.74)_{10}}$$

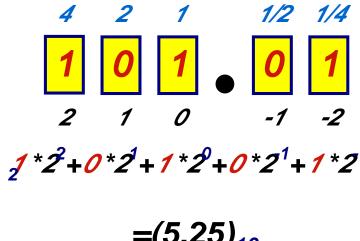
Octal Number System

- \blacksquare Base = 8
 - ♦ 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }
- Weights
 - $Weight = (Base)^{Position}$
- *Magnitude*
 - ◆ Sum of "Digitx Weight"
- Formal Notation



1.2 Binary Number System

- \blacksquare Base = 2
 - ◆ 2 digits { 0, 1 }, called binary digits or "bits"
- Weights
 - $Weight = (Base)^{Position}$
- *Magnitude*
 - Sum of "Bit x Weight"
- Formal Notation
- Groups of bits 4 bits = Nibble8 bits = Byte



 $=(5.25)_{10}$ $(101.01)_2$

1011

11000101

The Power of 2

n	2^n
0	$2^0 = 1$
1	$2^{1}=2$
2	22=4
3	23=8
4	24=16
5	$2^5 = 32$
6	$2^6 = 64$
7	2 ⁷ =128

	n	2^n
A	8	2 ⁸ =256
	9	2 ⁹ =512
	10	$2^{10} = \frac{1024}{1000}$
	11	211=2048
	12	2 ¹² =4096
	20	$2^{20} = 1M$
	30	$2^{30}=1G$
	40	2^{40} = $\overline{1T}$

Kilo

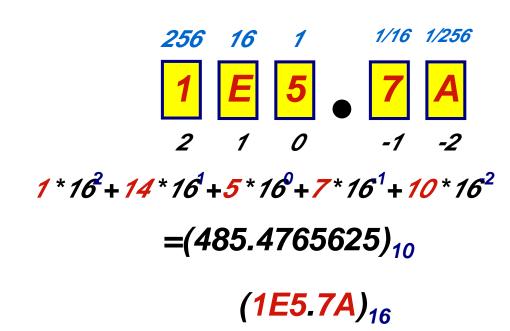
Mega

Giga

Tera

1.3 Hexadecimal Number System

- \blacksquare Base = 16
 - ◆ 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }
- Weights
 - $Weight = (Base)^{Position}$
- *Magnitude*
 - ◆ Sum of "Digitx Weight"
- Formal Notation

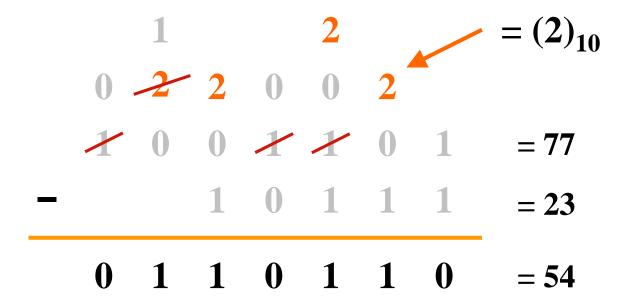


Binary Addition

■ Column Addition

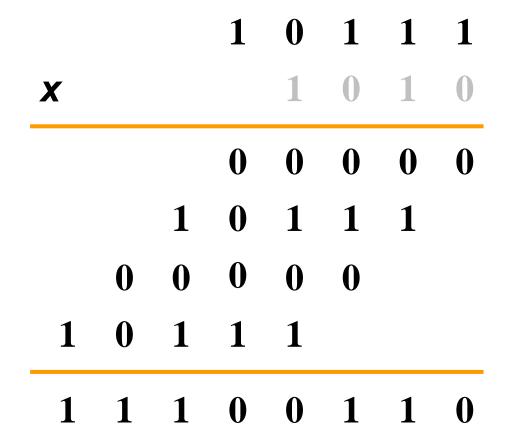
Binary Subtraction

■ Borrow a "Base" when needed

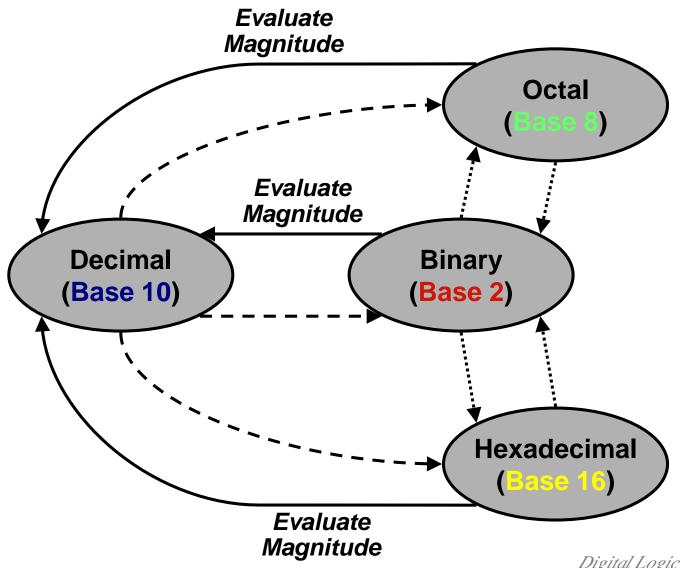


Binary Multiplication

■ Bit by bit



1.4 Number Base Conversions



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Digital Logic Design Ch1-20

Decimal (Integer) to Binary Conversion

- *Divide* the number by the 'Base' (=2)
- Take the remainder (either 0 or 1) as a coefficient
- Take the quotient and repeat the division

Example: (13)₁₀

	Quotient	Remainder	Coefficient
13 / 2 =	6	1	$a_0 = 1$
6 / 2 =	3	0	$\mathbf{a_1} = 0$
3 / 2 =	1	1	$a_2 = 1$
1 / 2 =	0	1	$a_3 = 1$
Answ	er: (1:	$(a_3 a_2)$	$a_1 a_0)_2 = (1101)_2$
		7	
		MSB	LSB

Decimal (Fraction) to Binary Conversion

- \blacksquare *Multiply* the number by the 'Base' (=2)
- Take the integer (either 0 or 1) as a coefficient
- Take the resultant fraction and repeat the division

Example: $(0.625)_{10}$

Integer Fraction Coefficient

$$0.625 * 2 = 1$$
 . 25 $a_{-1} = 1$
 $0.25 * 2 = 0$. 5 $a_{-2} = 0$
 $0.5 * 2 = 1$. 0 $a_{-3} = 1$

Answer: $(0.625)_{10} = (0.a_{-1} a_{-2} a_{-3})_2 = (0.101)_2$

MSB LSB

Decimal to Octal Conversion

Example: $(175)_{10}$

Quotient Remainder Coefficient
$$175 \ / \ 8 = 21 \ 7 \ a_0 = 7$$
 $21 \ / \ 8 = 2 \ 5 \ a_1 = 5$ $2 \ / \ 8 = 0 \ 2 \ a_2 = 2$

Answer: $(175)_{10} = (a_2 a_1 a_0)_8 = (257)_8$

Example: $(0.3125)_{10}$

Integer Fraction Coefficient
$$0.3125*8=2.5$$
 $a_{-1}=2$ $0.5*8=4.0$ $a_{-2}=4$

Answer: $(0.3125)_{10} = (0.a_{-1} a_{-2} a_{-3})_{8} = (0.24)_{8}$

Binary – Octal Conversion

- Each group of 3 bits represents an octal digit

Example: $(10110.01)_2$

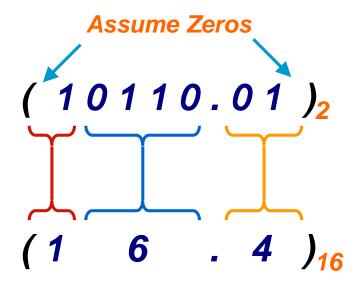
Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Works both ways (Binary to Octal & Octal to Binary)

Binary – Hexadecimal Conversion

- $\blacksquare 16 = 2^4$
- Each group of 4 bits represents a hexadecimal digit

Example:



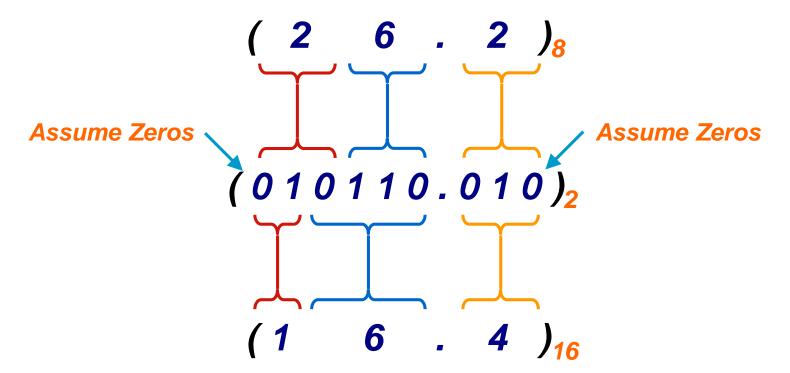
Hex	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Works both ways (Binary to Hex & Hex to Binary)

Octal – Hexadecimal Conversion

■ Convert to Binary as an intermediate step

Example:



Works both ways (Octal to Hex & Hex to Octal)

Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

1.5 Complements

- There are **two types** of complements for each base-**r** system: the radix complement and diminished radix complement.
- **Diminished Radix Complement (r-1)'s Complement**
 - ◆ Given a number N in base rhaving n digits, the (r-I)'s complement of N is defined as:

$$(r^n-1)-N$$

- **Example for 6-digit <u>decimal</u> numbers**:
 - 9's complement is $(r^n 1) N = (10^6 1) N = 9999999 N$
 - 9's complement of 546700 is 999999-546700 = 453299
- Example for 7-digit <u>binary</u> numbers:
 - 1's complement is $(r^n 1) N = (2^7 1) N = 11111111 N$
 - ◆ 1's complement of 1011000 is 1111111-1011000 = 0100111

Complements in Binary Numbers

Observations:

- Subtraction from (r^n-1) will never require a borrow
- Diminished radix complement can be computed digit-by-digit
- For binary: 1 0 = 1 and 1 1 = 0
- 1's Complement (*Diminished Radix Complement*)
 - ◆ All '0's become '1's
 - ◆ All '1's become '0's

```
Example (10110000)_2
\Rightarrow (01001111)_2
```

If you add a number and its 1's complement ...

```
\begin{array}{c} 10110000 \\ + 01001111 \\ \hline 11111111 \end{array}
```

■ Radix Complement

The r's complement of an n-digit number N in base r is defined as $r^n - N$ for $N \neq 0$ and as 0 for N = 0. Comparing with the (r-1)'s complement, we note that the r's complement is obtained by adding 1 to the (r-1)'s complement, since $r^n - N = [(r^n - 1) - N] + 1$.

■ Example: Base-10

The 10's complement of 512398 is 487602 The 10's complement of 246700 is 753300

■ Example: Base-2

The 2's complement of 1101100 is 0010100
The 2's complement of 0110111 is 1001001

Complements Binary Numbers

- 2's Complement (*Radix Complement*)
- ◆ Take 1's complement then add 1
 - ◆ Toggle all bits to the left of the first '1' from the right

Example:

Lecture One



Subtraction with Complements

- ♦ The subtraction of two n-digit unsigned numbers M-N in base r can be done as follows:
 - 1. Add the minuend M to the r's complement of the subtrahend N. Mathematically, $M + (r^n N) = M N + r^n$.
 - 2. If $M \ge N$, the sum will produce and end carry r^n , which can be discarded; what is left is the result M N.
 - 3. If M < N, the sum does not produce an end carry and is equal to $r^n (N M)$, which is the r's complement of (N M). To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front.

■ Example 1.5

♦ Using 10's complement, subtract 72532 – 3250.

$$M = 72532$$
10's complement of $N = +96750$
Sum = 169282
Discard end carry $10^5 = -100000$
Answer = 69282

Example 1.6

◆ Using 10's complement, subtract 3250 – 72532.

$$M = 03250$$
10's complement of $N = +27468$
Sum = 30718

There is no end carry.



Therefore, the answer is -(10's complement of 30718) = -69282.

■ Example 1.7

• Given the two binary numbers X = 1010100 and Y = 1000011, perform the subtraction (a) X - Y; and (b) Y - X, by using 2's complement.

(a)
$$X = 1010100$$

 2 's complement of $Y = +0111101$
 $Sum = 10010001$
Discard end carry $2^7 = -10000000$
Answer. $X - Y = 0010001$

(b)
$$Y = 1000011$$

2's complement of $X = +0101100$
Sum = 1101111

There is no end carry.

Therefore, the answer is Y - X = -(2's complement of 1101111) = -0010001.

1.6 Signed Binary Numbers

- To represent negative integers, we need a notation for negative values.
- It is customary to represent the sign with a bit placed in the leftmost position of the number since binary digits.
- The convention is to make the sign bit 0 for positive and 1 for negative.

■ *Example*:

Signed-magnitude representation:	10001001
Signed-1's-complement representation:	11110110
Signed-2's-complement representation:	11110111

■ Table 1.3 lists all possible four-bit signed binary numbers in the three representations.

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Signed Binary Numbers

A rithmetic addition

- ◆ The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic. If the signs are the same, we add the two magnitudes and give the sum the common sign. If the signs are different, we subtract the smaller magnitude from the larger and give the difference the sign of the larger magnitude.
- ◆ The addition of two signed binary numbers with negative numbers represented in signed-2's-complement form is obtained from the addition of the two numbers, including their sign bits.
- ◆ A carry out of the sign-bit position is **discarded**.

Example:

+ 6	00000110	- 6	11111010
<u>+13</u>	00001101	<u>+13</u>	00001101
+ 19	00010011	+ 7	00000111
+ 6	00000110	-6	11111010
<u>-13</u>	<u>11110011</u>	<u>-13</u>	<u>11110011</u>
- 7	11111001	- 19	11101101

Signed Binary Numbers

- A rithmetic Subtraction
 - ♦ In 2's-complement form:
 - 1. Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including sign bit).
 - 2. A carry out of sign-bit position is discarded.

$$(\pm A) - (+B) = (\pm A) + (-B)$$
$$(\pm A) - (-B) = (\pm A) + (+B)$$

Example:

$$(-6) - (-13)$$
 (11111010 - 11110011)
 $(11111010 + 00001101)$
 $00000111 (+7)$

1.7 Binary Codes

■ BCD Code

- ◆ A number with k decimal digits will require 4k bits in BCD.
- ◆ Decimal 396 is represented in BCD with 12bits as 0011 1001 0110, with each group of 4 bits representing one decimal digit.
- ◆ A decimal number in BCD is the same as its equivalent binary number only when the number is between 0 and 9.
- ◆ The binary combinations 1010 through 1111 are not used and have no meaning in BCD.

Table 1.4 *Binary-Coded Decimal (BCD)*

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Example:

◆ Consider decimal 185 and its corresponding value in BCD and binary:

$$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$$

■ *BCD* addition

■ *Example*:

• Consider the addition of 184 + 576 = 760 in BCD:

BCD	1	1		
	0001	1000	0100	184
	<u>+ 0101</u>	<u>0111</u>	<u>0110</u>	+576
Binary sum	0111	10000	1010	
Add 6		<u>0110</u>	<u>0110</u>	
BCD sum	0111	0110	0000	760

■ American Standard Code for Information Interchange (ASCII) Character Code

Table 1.7 *American Standard Code for Information Interchange (ASCII)*

				b7b6b5				
$b_4b_3b_2b_1$	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	"	2	В	R	b	r
0011	ETX	DC3	#	3	C	S	c	S
0100	EOT	DC4	\$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB		7	G	W	g	W
1000	BS	CAN	(8	Н	X	h	X
1001	HT	EM)	9	I	Y	i	У
1010	LF	SUB	201	:	J	Z	j	z
1011	VT	ESC	+	;	K	[k	{
1100	FF	FS	,	<	L	\	1	Ì
1101	CR	GS	_	=	M]	m	}
1110	SO	RS		>	N	^	n	~
1111	SI	US	/	?	O	_	O	DEL

■ ASCII Character Code

Control characters

Null	DLE	Data-link escape
Start of heading	DC1	Device control 1
Start of text	DC2	Device control 2
End of text	DC3	Device control 3
End of transmission	DC4	Device control 4
Enquiry	NAK	Negative acknowledge
Acknowledge	SYN	Synchronous idle
Bell	ETB	End-of-transmission block
Backspace	CAN	Cancel
Horizontal tab	EM	End of medium
Line feed	SUB	Substitute
Vertical tab	ESC	Escape
Form feed	FS	File separator
Carriage return	GS	Group separator
Shift out	RS	Record separator
Shift in	US	Unit separator
Space	DEL	Delete
	Start of heading Start of text End of text End of transmission Enquiry Acknowledge Bell Backspace Horizontal tab Line feed Vertical tab Form feed Carriage return Shift out Shift in	Start of heading Start of text DC2 End of text DC3 End of transmission Enquiry NAK Acknowledge SYN Bell Backspace Horizontal tab Line feed Vertical tab Form feed Carriage return Shift out Shift in SC3 DC4 DC4 ENQ3 EXAMPTE CAN EXAMPTE SYN ETB EXAMPTE EXAMPTE EXAMPTE SUB CAN EXAMPTE EXC EXC EXC EXC EXC EXC EXC EXC EXC EX

ASCII Properties

- *ASCII* has some interesting properties:
 - lacktriangle Digits 0 to 9 span Hexadecimal values 30_{16} to 39_{16}
 - \bullet Upper case A -Z span 41₁₆ to 5A₁₆
 - \bullet Lower case a-z span 61_{16} to $7A_{16}$
 - » Lower to upper case translation (and vice versa) occurs by flipping bit 6.

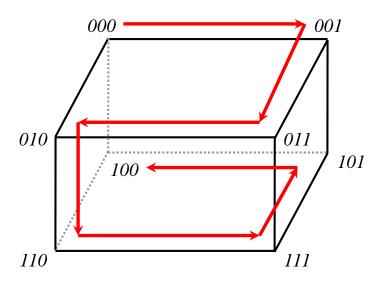
Other Decimal Codes

Table 1.5Four Different Binary Codes for the Decimal Digits

Decimal Digit	BCD 8421	2421	Excess-3	8, 4, -2, -1
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
	1010	0101	0000	0001
Unused	1011	0110	0001	0010
bit	1100	0111	0010	0011
combi-	1101	1000	1101	1100
nations	1110	1001	1110	1101
	1111	1010	1111	1110

☐ Gray Code

- ◆ The advantage is that only bit in the code group changes in going from one number to the next.
 - » Error detection.
 - » Representation of analog data.
 - » Low power design.



1-1 and onto!!

Table 1.6 *Gray Code*

Gray Code	Decimal Equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

■ Error-Detecting Code

- ◆ To detect errors in data communication and processing, an <u>eighth bit</u> is sometimes added to the ASCII character to indicate its parity.
- ◆ A parity bit is an extra bit included with a message to make the total number of 1's either even or odd.

■ *Example:*

Consider the following two characters and their even and odd parity:

	With even parity	With odd parity
ASCII $A = 1000001$	01000001	11000001
ASCII $T = 1010100$	11010100	01010100

■ Error-Detecting Code

- ◆ Redundancy (e.g. extra information), in the form of extra bits, can be incorporated into binary code words to detect and correct errors.
- ◆ A simple form of redundancy is parity, an extra bit appended onto the code word to make the number of 1's odd or even. Parity can detect all single-bit errors and some multiple-bit errors.
- ◆ A code word has even parity if the number of 1's in the code word is even.
- ◆ A code word has odd parity if the number of 1's in the code word is odd.
- Example:

Message A with even parity: 110001001

Message A with odd parity: 010001001

1.8 Binary Storage and Registers

Registers

- ◆ A binary cell is a <u>device</u> that possesses two stable states and is capable of storing one of the two states.
- ◆ A register is a group of binary cells. A register with n cells can store any discrete quantity of information that contains n bits.

■ A binary cell

- Two stable state
- Store one bit of information
- Examples: flip-flop circuits, capacitor

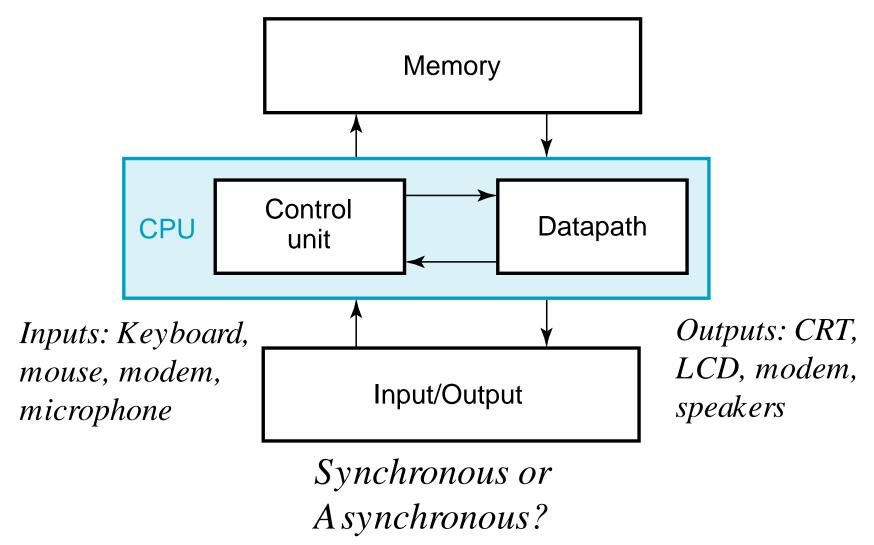
■ A register

- ♦ A group of binary cells
- ◆ *AX* in x86 *CPU*

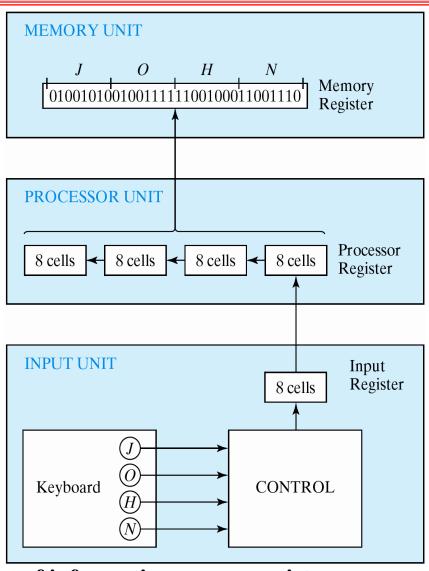
Register Transfer

- ◆ A transfer of the information stored in one register to another.
- One of the major operations in digital system.

A Digital Computer Example

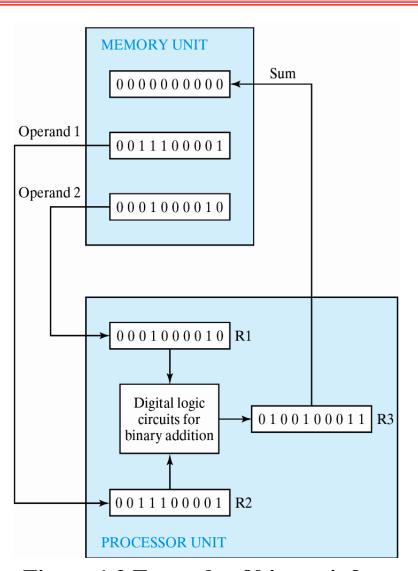


Transfer of information



8:45 Figure 1.1 Transfer of information among register

Transfer of information



- The other major component of a digital system
 - Circuit elements to manipulate individual bits of information
 - ◆ Load-store machine

```
LOAD R1;
LOAD R2;
ADD R3, R2, R1;
STORE R3;
```

Truth Tables, Boolean Expressions, and Logic Gates

AND

X	y	Z
0	0	0
0	1	0
1	0	0
1	1	1

$$z = x \cdot y = xy$$

$$y = \int_{z}^{z} z$$

OR

X	y	Z,
0	0	0
0	1	1
1	0	1
1	1	1

\mathcal{X}	<i>)</i>	Z
0	0	0
0	1	1
1	0	1
1	1	1

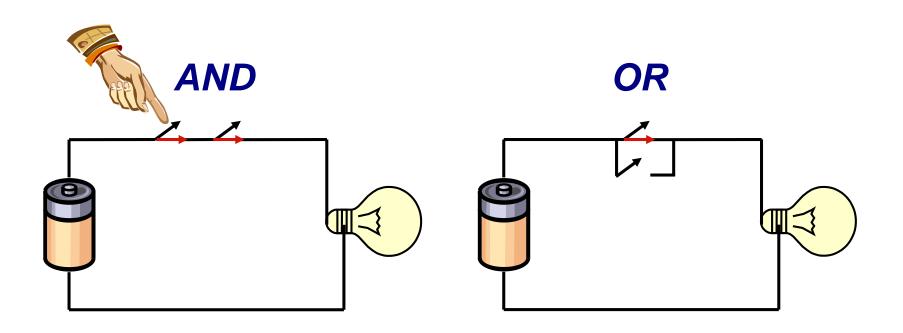
$$z = x + y$$

x	Z
0	1
1	0

$$z = \overline{X} = X'$$

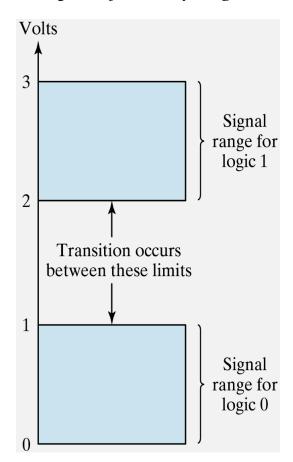
$$x \longrightarrow z$$

Switching Circuits



Logic gates

♦ Example of binary signals



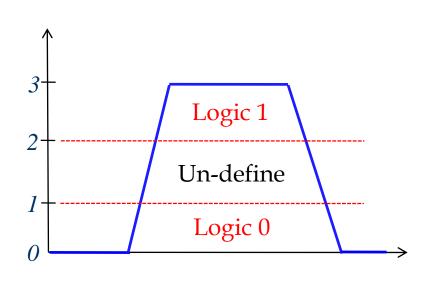


Figure 1.3 Example of binary signals

Logic gates

◆ Graphic Symbols and Input-Output Signals for Logic gates:

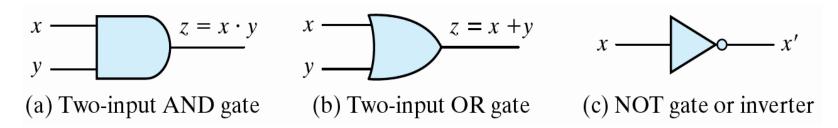


Fig. 1.4 Symbols for digital logic circuits

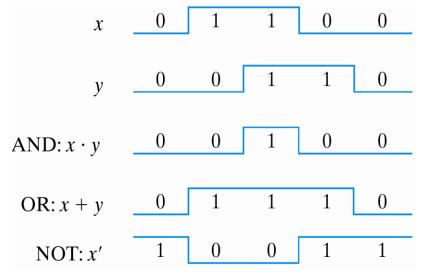
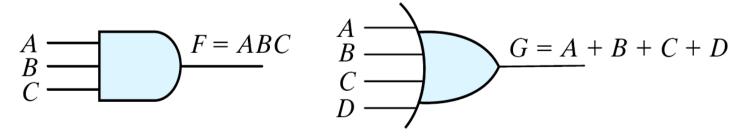


Fig. 1.5 Input-Output signals for gates

■ Logic gates

Graphic Symbols and Input-Output Signals for Logic gates:



(a) Three-input AND gate

(b) Four-input OR gate

Fig. 1.6 Gates with multiple inputs

THE END

```
HomeWork (1):
Digital Design (4th)- Morris Mano-Page 32-
Problems:
     1.8
     1.9
     1.13
     1.14
     1.18
```

THE END

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HomeWork (2):
Digital Design (4th)- Morris Mano-Page 32-
Problems:
     1.20
     1.22
     1.29
     1.30
     1.34
```