
Digital Logic Design

Chapter 2

Boolean Algebra and Logic Gate

2.1 Algebras

▣ What is an algebra?

- ◆ Mathematical system consisting of
 - » Set of elements (example: $N = \{1, 2, 3, 4, \dots\}$)
 - » Set of operators (+, -, ×, ÷)
 - » Axioms or postulates (associativity, distributivity, closure, identity elements, etc.)

▣ Why is it important?

- ◆ Defines rules of “calculations”

▣ Note: operators with two inputs are called binary

- ◆ Does not mean they are restricted to binary numbers!
- ◆ Operator(s) with one input are called unary

2.2 BASIC DEFINITIONS

- A set is collection of elements having the same property.
 - ◆ S : set, x and y : element or event
 - ◆ For example: $S = \{1, 2, 3, 4\}$
 - » If $x = 2$, then $x \in S$.
 - » If $y = 5$, then $y \notin S$.
- A binary operator defines on a set S of elements is a rule that assigns, to each pair of elements from S , a unique element from S .
 - ◆ For example: given a set S , consider $\mathbf{a} * \mathbf{b} = \mathbf{c}$ and $*$ is a binary operator.
 - ◆ If (a, b) through $*$ get \mathbf{c} and $\mathbf{a}, \mathbf{b}, \mathbf{c} \in S$, then $*$ is a binary operator of S .
 - ◆ On the other hand, if $*$ is not a binary operator of S and $\mathbf{a}, \mathbf{b} \in S$, then $\mathbf{c} \notin S$.

BASIC DEFINITIONS

▣ The common postulates used to formulate algebraic structures are:

1. Closure: a set S is closed with respect to a binary operator if, for every pair of elements of S , the binary operator specifies a rule for obtaining a unique element of S .

◆ *For example, natural numbers $N=\{1,2,3,\dots\}$ is closed w.r.t. the binary operator $+$ by the rule of arithmetic addition, since, for any $a, b \in N$, there is a unique $c \in N$ such that*

» $a+b = c$

» But operator $-$ is not closed for N , because $2-3 = -1$ and $2, 3 \in N$, but $(-1) \notin N$.

2. Associative law: a binary operator $*$ on a set S is said to be associative whenever

◆ $(x * y) * z = x * (y * z)$ for all $x, y, z \in S$

» $(x+y)+z = x+(y+z)$

3. Commutative law: a binary operator $*$ on a set S is said to be commutative whenever

◆ $x * y = y * x$ for all $x, y \in S$

» $x+y = y+x$

BASIC DEFINITIONS

4. Identity element: a set S is said to have an identity element with respect to a binary operation $*$ on S if there exists an element $e \in S$ with the property that

◆ $e * x = x * e = x$ for every $x \in S$

» $0 + x = x + 0 = x$ for every $x \in I$ $I = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$.

» $1 \times x = x \times 1 = x$ for every $x \in I$ $I = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$.

5. Inverse: a set having the identity element e with respect to the binary operator to have an inverse whenever, for every $x \in S$, there exists an element $y \in S$ such that

◆ $x * y = e$

» The operator $+$ over I , with $e = 0$, the inverse of an element a is $(-a)$, since $a + (-a) = 0$.

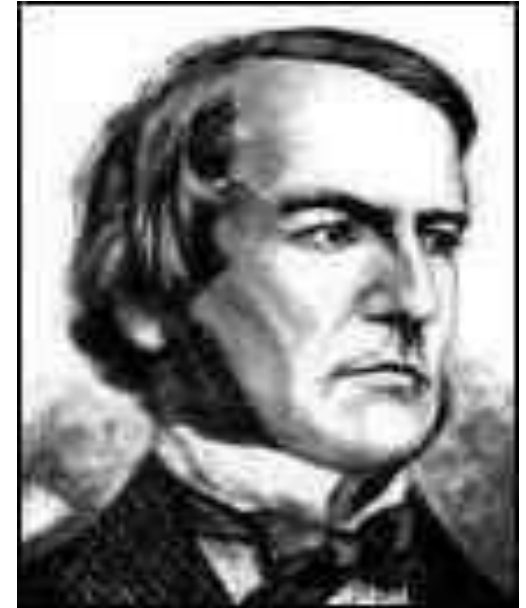
6. Distributive law: if $(*)$ and $(.)$ are two binary operators on a set S , $(*)$ is said to be distributive over $(.)$ whenever

◆ $x * (y.z) = (x * y).(x * z)$

George Boole

■ Father of Boolean algebra

- He came up with a type of linguistic algebra, the three most basic operations of which were (and still are) **AND, OR and NOT**. It was these three functions that formed the basis of his premise, and were the only operations necessary to perform comparisons or basic mathematical functions.
- Boole's system was based on a binary approach, **processing only two objects - the yes-no, true-false, on-off, zero-one approach.**
- Surprisingly, given his standing in the academic community, Boole's idea was either criticized or completely ignored by the majority of his peers.
- Eventually, one bright student, **Claude Shannon** (1916-2001), picked up the idea and ran with it



George Boole (1815 - 1864)

Boolean Algebra

□ Terminology:

- ◆ *Literal*: A variable or its complement
- ◆ *Product term*: literals connected by (\cdot)
- ◆ *Sum term*: literals connected by ($+$)

Postulates of Two-Valued Boolean Algebra

- ▣ $B = \{0, 1\}$ and two binary operations, $(+)$ and (\cdot)
- ▣ The rules of operations: AND, OR and NOT.

AND

| x | y | $x \cdot y$ |
|-----|-----|-------------|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

OR

| x | y | $x + y$ |
|-----|-----|---------|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

NOT

| x | x' |
|-----|------|
| 0 | 1 |
| 1 | 0 |

1. Closure ($+$ and \cdot)
2. The identity elements
 - (1) $x + 0 = x$
 - (2) $x \cdot 1 = x$

Postulates of Two-Valued Boolean Algebra

3. The commutative laws $x+y = y+x$, $x \cdot y = y \cdot x$

4. The distributive laws

| x | y | z | $y+z$ | $x \cdot (y+z)$ | $x \cdot y$ | $x \cdot z$ | $(x \cdot y) + (x \cdot z)$ |
|-----|-----|-----|-------|-----------------|-------------|-------------|-----------------------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

5. Complement

◆ $x+x'=1 \rightarrow 0+0'=0+1=1; 1+1'=1+0=1$

◆ $x \cdot x'=0 \rightarrow 0 \cdot 0'=0 \cdot 1=0; 1 \cdot 1'=1 \cdot 0=0$

2.4 Basic Theorems And Properties Of Boolean Algebra

Duality

- ▣ The principle of *duality* is an important concept. This says that if an expression is valid in Boolean algebra, the dual of that expression is also valid.
- ▣ **To form the dual of an expression**, replace all (+) operators with (·) operators, all (·) operators with (+) operators, all ones with zeros, and all zeros with ones.
- ▣ Form the dual of the expression **$a(b + c) = ab + ac$**
 $a(b + c) = a + (bc) = (a + b)(a + c)$
- Take care not to alter the location of the parentheses if they are present.

Basic Theorems

Table 2.1

Postulates and Theorems of Boolean Algebra

| | | |
|---------------------------|---------------------------------|-------------------------------|
| Postulate 2 | (a) $x + 0 = x$ | (b) $x \cdot 1 = x$ |
| Postulate 5 | (a) $x + x' = 1$ | (b) $x \cdot x' = 0$ |
| Theorem 1 | (a) $x + x = x$ | (b) $x \cdot x = x$ |
| Theorem 2 | (a) $x + 1 = 1$ | (b) $x \cdot 0 = 0$ |
| Theorem 3, involution | $(x')' = x$ | |
| Postulate 3, commutative | (a) $x + y = y + x$ | (b) $xy = yx$ |
| Theorem 4, associative | (a) $x + (y + z) = (x + y) + z$ | (b) $x(yz) = (xy)z$ |
| Postulate 4, distributive | (a) $x(y + z) = xy + xz$ | (b) $x + yz = (x + y)(x + z)$ |
| Theorem 5, DeMorgan | (a) $(x + y)' = x'y'$ | (b) $(xy)' = x' + y'$ |
| Theorem 6, absorption | (a) $x + xy = x$ | (b) $x(x + y) = x$ |

DeMorgan's Theorem

- ▣ Theorem 5(a): $(x + y)' = x'y'$
- ▣ Theorem 5(b): $(xy)' = x' + y'$
- ▣ By means of truth table

| x | y | x' | y' | $x+y$ | $(x+y)'$ | $x'y'$ | xy | $x'+y'$ | $(xy)'$ |
|-----|-----|------|------|-------|----------|--------|------|---------|---------|
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |

Operator Precedence

□ The operator precedence for evaluating Boolean Expression is

- ◆ Parentheses
- ◆ NOT
- ◆ AND
- ◆ OR

□ Examples

- ◆ $x y' + z$
- ◆ $(x y + z)'$

2.5 Boolean Functions

▣ A Boolean function

- ◆ Binary variables
- ◆ Binary operators OR and AND
- ◆ Unary operator NOT
- ◆ Parentheses

▣ Examples

- ◆ $F_1 = x y z'$
- ◆ $F_2 = x + y'z$
- ◆ $F_3 = x'y'z + x'y z + x y'$
- ◆ $F_4 = x y' + x' z$

Boolean Functions

- The truth table of 2^n entries (n=number of variables)

| x | y | z | F_1 | F_2 | F_3 | F_4 |
|-----|-----|-----|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 |

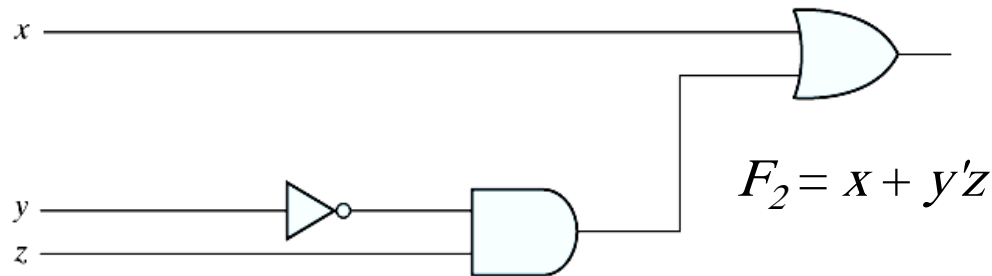
- Two Boolean expressions may specify the same function

- ◆ $F_3 = F_4$

Boolean Functions

Implementation with logic gates

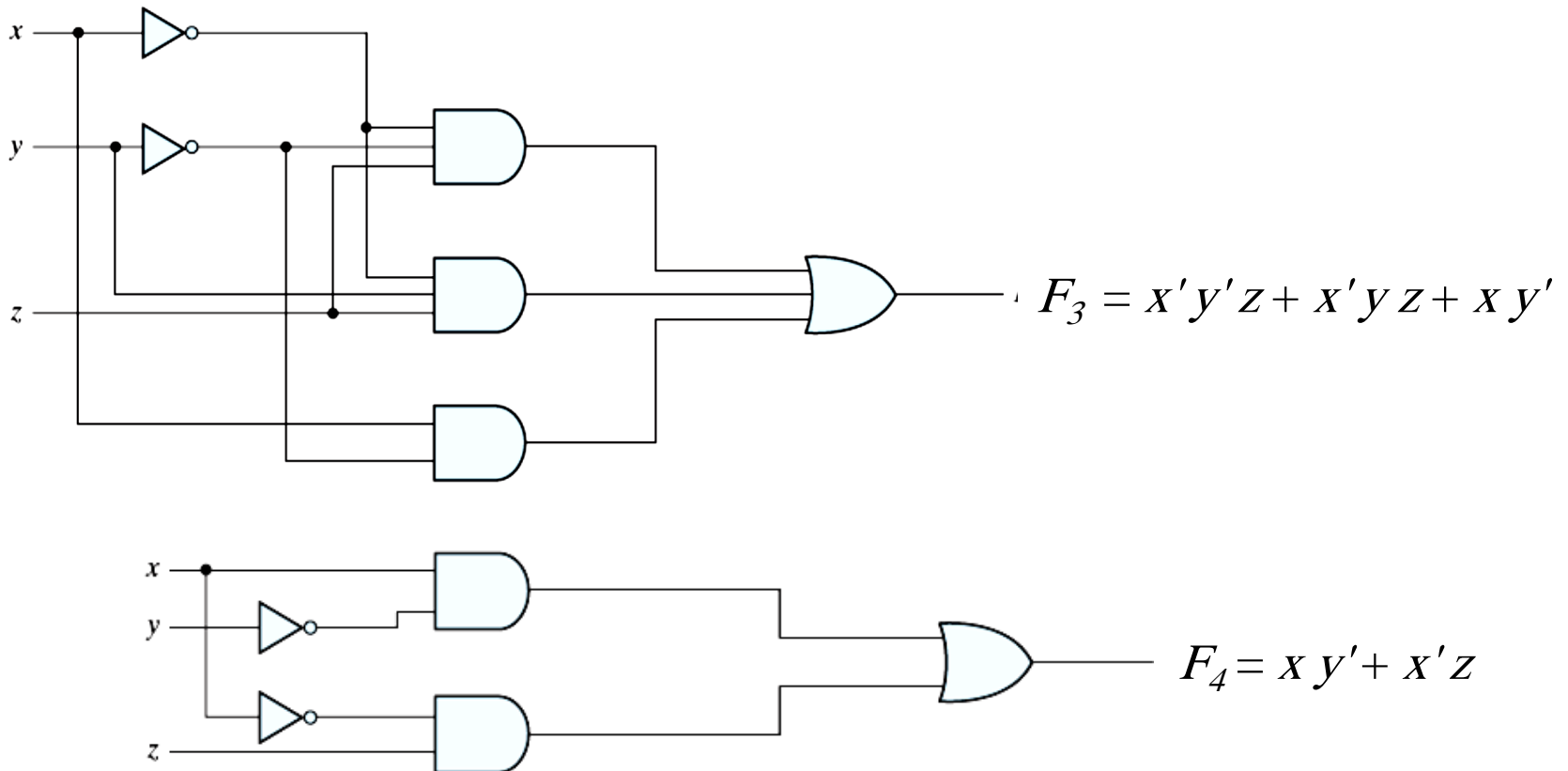
- When a Boolean expression is implemented with logic gates, each **term** requires a [gate](#) and each variable (**Literal**) within the term designates an input to the gate. (F3 has 3 terms and 8 literal)
- $F_2 = x + y'z$



Boolean Functions

Implementation with logic gates

◆ F_4 is more economical



Algebraic Manipulation

- To minimize Boolean expressions, minimize the number of literals and the number of terms → a circuit with less equipment
 - ◆ It is a hard problem (no specific rules to follow)

■ Example 2.1

$$1. x(x'+y) = xx' + xy = 0 + xy = xy$$

$$2. x+x'y = (x+x')(x+y) [\text{Distributive}] = 1(x+y) = x+y$$

$$3. (x+y)(x+y') = x+xy+xy'+yy' = x+xy+xy'+0 = x(1+y+y') = x$$

$$4. xy + x'z + yz = xy + x'z + yz(x+x') = xy + x'z + yzx + yzx' = xy(1+z) + x'z(1+y) = xy + x'z$$

Complement of a Function

■ An interchange of 0's for 1's and 1's for 0's in the value of F

◆ By DeMorgan's theorem

$$\begin{aligned} \diamond (A+B+C)' &= (A+X)' && \text{let } B+C = X \\ &= A'X' && \text{by theorem 5(a) (DeMorgan's)} \end{aligned}$$

$$\begin{aligned} &= A'(B+C)' && \text{substitute } B+C = X \\ &= A'(B'C') && \text{by theorem 5(a)} \\ &\text{(DeMorgan's)} \end{aligned}$$

$$= A'B'C' \quad \text{by theorem 4(b) (associative)}$$

■ Generalization: a function is obtained by interchanging AND and OR operators and complementing each literal.

$$\diamond (A+B+C+D+...+F)' = A'B'C'D'...F'$$

$$\diamond (ABCD...F)' = A'+B'+C'+D'...+F'$$

Examples

▣ Example 2.2: (1) $F_1 = x'yz' + x'y'z$ (2) $F_2 = x(y'z' + yz)$

◆ $F_1' = (x'yz' + x'y'z)' = (x'yz')'(x'y'z)' = (x+y'+z)(x+y+z')$

◆ $F_2' = [x(y'z' + yz)]' = x' + (y'z' + yz)' = x' + (y'z')'(yz)'$
 $= x' + (y+z)(y'+z')$
 $= x' + yz' + y'z$

▣ Example 2.3: A simpler procedure

◆ Take the dual of the function and complement each literal

1. $F_1 = x'yz' + x'y'z$.

a) The dual of F_1 is $(x'+y+z')(x'+y'+z)$.

b) Complement each literal: $(x+y'+z)(x+y+z') = F_1'$

2. $F_2 = x(y'z' + yz)$.

a) The dual of F_2 is $x+(y'+z')(y+z)$.

b) Complement each literal: $x'+(y+z)(y'+z') = F_2'$

2.6 Canonical and Standard Forms

Minterms and Maxterms

- ▣ A minterm (standard product): an **AND term consists of all literals** in their normal form or in their complement form.
 - ◆ For example, two binary variables x and y ,
 - » $xy, xy', x'y, x'y'$
 - ◆ It is also called a standard product.
 - ◆ n variables can be combined to form 2^n minterms.
- ▣ A maxterm (standard sums): an **OR term**
 - ◆ It is also call a standard sum.
 - ◆ 2^n maxterms.

Minterms and Maxterms

- Each *maxterm* is the **complement** of its corresponding *minterm*, and vice versa.

Table 2.3
Minterms and Maxterms for Three Binary Variables

| <i>x</i> | <i>y</i> | <i>z</i> | Minterms | | Maxterms | |
|----------|----------|----------|----------|-------------|----------------|-------------|
| | | | Term | Designation | Term | Designation |
| 0 | 0 | 0 | $x'y'z'$ | m_0 | $x + y + z$ | M_0 |
| 0 | 0 | 1 | $x'y'z$ | m_1 | $x + y + z'$ | M_1 |
| 0 | 1 | 0 | $x'yz'$ | m_2 | $x + y' + z$ | M_2 |
| 0 | 1 | 1 | $x'yz$ | m_3 | $x + y' + z'$ | M_3 |
| 1 | 0 | 0 | $xy'z'$ | m_4 | $x' + y + z$ | M_4 |
| 1 | 0 | 1 | $xy'z$ | m_5 | $x' + y + z'$ | M_5 |
| 1 | 1 | 0 | xyz' | m_6 | $x' + y' + z$ | M_6 |
| 1 | 1 | 1 | xyz | m_7 | $x' + y' + z'$ | M_7 |

Minterms and Maxterms

■ An Boolean function can be expressed by

- ◆ A truth table
- ◆ Sum of **minterms** for each combination of variables that produces a (1) in the function.
- ◆ $f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$ (Minterms)
- ◆ $f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$ (Minterms)

Table 2.4

Functions of Three Variables

| x | y | z | Function f_1 | Function f_2 |
|----------|----------|----------|----------------------------------|----------------------------------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

Minterms and Maxterms

■ The complement of a Boolean function

- ◆ The **minterms** that **produce a (0)**
- ◆ $f_1' = m_0 + m_2 + m_3 + m_5 + m_6 = x'y'z' + x'yz' + x'yz + xy'z + xyz'$
- ◆ $f_2' = m_0 + m_1 + m_2 + m_4$

■ Any Boolean function can be expressed as

- ◆ A sum of minterms (“sum” meaning the ORing of terms).
- ◆ A product of maxterms (“product” meaning the ANDing of terms).
- ◆ Both boolean functions are said to be in **Canonical** form.
- ◆ $f_1 = (f_1)'$
- ◆ $= (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z)(x'+y'+z) = M_0 M_2 M_3 M_5 M_6$
- ◆ $f_2 = (x+y+z)(x+y+z')(x+y'+z)(x'+y+z) = M_0 M_1 M_2 M_4$

Sum of Minterms

- ▣ Sum of minterms: there are 2^n minterms and 2^{2n} combinations of functions with n Boolean variables.
- ▣ Example 2.4: express $F = A + B'C$ as a sum of minterms.
 - ◆ $F = A + B'C = A(B + B') + B'C = AB + AB' + B'C = AB(C + C') + AB'(C + C') + (A + A')B'C = ABC + ABC' + AB'C + AB'C' + A'B'C$
 - ◆ $F = A'B'C + AB'C' + AB'C + ABC' + ABC = m_1 + m_4 + m_5 + m_6 + m_7$
 - ◆ $F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$
 - ◆ or, built the truth table first

Table 2.5

Truth Table for $F = A + B'C$

| A | B | C | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Conversion between Canonical Forms

- ▣ The **complement** of a function expressed as the sum of minterms equals the sum of minterms **missing** from the original function.

- ◆ $F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$

- ◆ Thus, $F'(A, B, C) = \Sigma(0, 2, 3)$

- ◆ By DeMorgan's theorem

$$F(A, B, C) = \Pi(0, 2, 3)$$

$$F'(A, B, C) = \Pi(1, 4, 5, 6, 7)$$

- ◆ $m_j' = M_j$

- ▣ To convert from one canonical form to another: **interchange** the symbols Σ and Π and list those numbers **missing** from the original form

- » Σ of 1's

- » Π of 0's

□ Example

- ◆ $F = xy + x'z$
- ◆ $F(x, y, z) = \Sigma(1, 3, 6, 7)$
- ◆ $F(x, y, z) = \Pi(0, 2, 4, 5)$

Table 2.6

Truth Table for $F = xy + x'z$

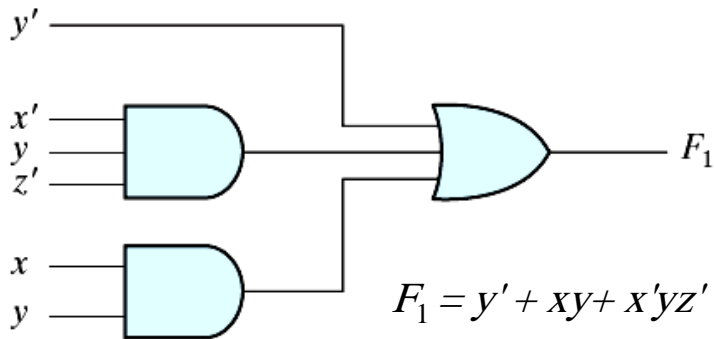
| x | y | z | F |
|----------|----------|----------|----------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Standard Forms

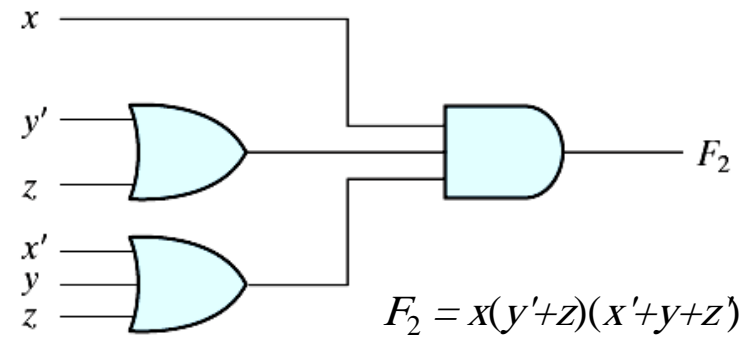
- In canonical forms each minterm or maxterm must contain **all the variables** either complemented or uncomplemented, thus these forms are very seldom the ones with the least number of literals.
- Standard forms: the terms that form the function may obtain **one, two, or any number** of literals, .There are two types of standard forms:
 - ◆ Sum of products: $F_1 = y' + xy + x'yz'$
 - ◆ Product of sums: $F_2 = x(y' + z)(x' + y + z)$
- A Boolean function may be expressed in a nonstandard form
 - ◆ $F_3 = AB + C(D + E)$
- But it can be changed to a standard form by using The distributive law
 - ◆ $F_3 = AB + C(D + E) = AB + CD + CE$

Implementation

Two-level implementation

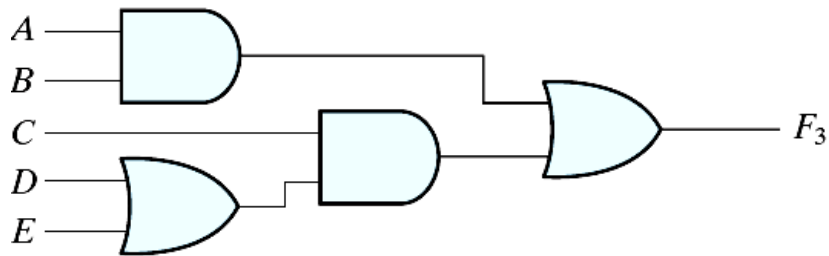


(a) Sum of Products

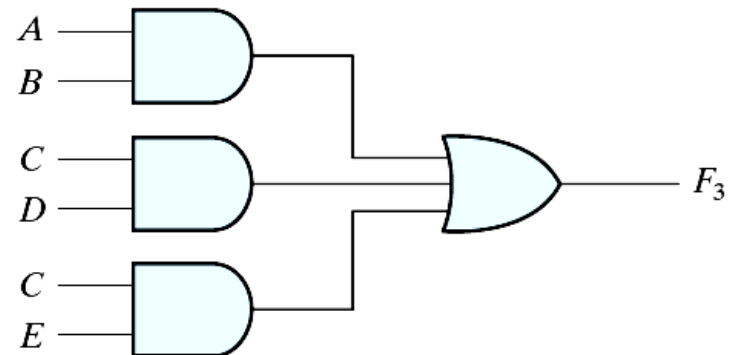


(b) Product of Sums

Multi-level implementation



(a) $AB + C(D + E)$



(b) $AB + CD + CE$

2.7 Other Logic Operations

- ▣ 2^n rows in the truth table of n binary variables.
- ▣ 2^{2^n} functions for n binary variables.
- ▣ 16 functions of two binary variables.

Table 2.7

Truth Tables for the 16 Functions of Two Binary Variables

| x | y | F_0 | F_1 | F_2 | F_3 | F_4 | F_5 | F_6 | F_7 | F_8 | F_9 | F_{10} | F_{11} | F_{12} | F_{13} | F_{14} | F_{15} |
|-----|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|----------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |

Boolean Expressions

Table 2.8

Boolean Expressions for the 16 Functions of Two Variables

| Boolean Functions | Operator Symbol | Name | Comments |
|-------------------|------------------|--------------|---------------------------|
| $F_0 = 0$ | | Null | Binary constant 0 |
| $F_1 = xy$ | $x \cdot y$ | AND | x and y |
| $F_2 = xy'$ | x/y | Inhibition | x , but not y |
| $F_3 = x$ | | Transfer | x |
| $F_4 = x'y$ | y/x | Inhibition | y , but not x |
| $F_5 = y$ | | Transfer | y |
| $F_6 = xy' + x'y$ | $x \oplus y$ | Exclusive-OR | x or y , but not both |
| $F_7 = x + y$ | $x + y$ | OR | x or y |
| $F_8 = (x + y)'$ | $x \downarrow y$ | NOR | Not-OR |
| $F_9 = xy + x'y'$ | $(x \oplus y)'$ | Equivalence | x equals y |
| $F_{10} = y'$ | y' | Complement | Not y |
| $F_{11} = x + y'$ | $x \subset y$ | Implication | If y , then x |
| $F_{12} = x'$ | x' | Complement | Not x |
| $F_{13} = x' + y$ | $x \supset y$ | Implication | If x , then y |
| $F_{14} = (xy)'$ | $x \uparrow y$ | NAND | Not-AND |
| $F_{15} = 1$ | | Identity | Binary constant 1 |

Standard Gates

▣ Consider the 16 functions in Table 2.8

- ◆ **Two** functions produce a constant : (F_0 and F_{15}).
- ◆ **Four** functions with unary operations: complement and transfer: (F_3 , F_5 , F_{10} and F_{12}).
- ◆ The other **ten** functions with binary operators

▣ **Eight** function are used as standard gates :

complement (F_{12}), transfer (F_3), AND (F_1), OR (F_7), NAND (F_{14}), NOR (F_8), XOR (F_6), and equivalence (XNOR) (F_9).

- ◆ Complement: inverter.
- ◆ Transfer: buffer (increasing drive strength).
- ◆ Equivalence: XNOR.

Summary of Logic Gates

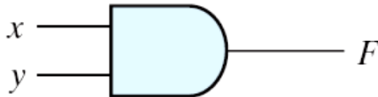
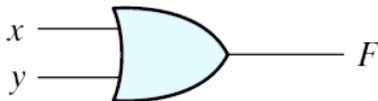
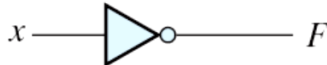
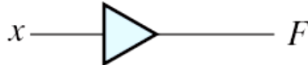
| Name | Graphic symbol | Algebraic function | Truth table | | | | | | | | | | | | | | | |
|----------|---|--------------------|---|-----|-----|-----|---|---|---|---|---|---|---|---|---|---|---|---|
| AND |  | $F = xy$ | <table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | x | y | F | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| x | y | F | | | | | | | | | | | | | | | | |
| 0 | 0 | 0 | | | | | | | | | | | | | | | | |
| 0 | 1 | 0 | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | | | | | | | | |
| OR |  | $F = x + y$ | <table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | x | y | F | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| x | y | F | | | | | | | | | | | | | | | | |
| 0 | 0 | 0 | | | | | | | | | | | | | | | | |
| 0 | 1 | 1 | | | | | | | | | | | | | | | | |
| 1 | 0 | 1 | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | | | | | | | | |
| Inverter |  | $F = x'$ | <table><tr><th>x</th><th>F</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table> | x | F | 0 | 1 | 1 | 0 | | | | | | | | | |
| x | F | | | | | | | | | | | | | | | | | |
| 0 | 1 | | | | | | | | | | | | | | | | | |
| 1 | 0 | | | | | | | | | | | | | | | | | |
| Buffer |  | $F = x$ | <table><tr><th>x</th><th>F</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr></table> | x | F | 0 | 0 | 1 | 1 | | | | | | | | | |
| x | F | | | | | | | | | | | | | | | | | |
| 0 | 0 | | | | | | | | | | | | | | | | | |
| 1 | 1 | | | | | | | | | | | | | | | | | |

Figure 2.5 Digital logic gates

Summary of Logic Gates

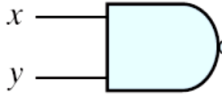
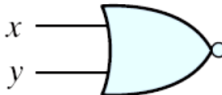
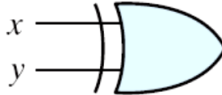
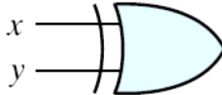
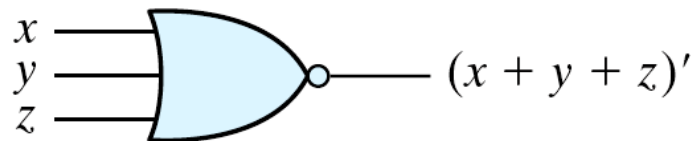
| NAND |  $F = (xy)'$ | <table> <tr> <th>x</th><th>y</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table> | x | y | F | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
|------------------------------------|--|--|-----|-----|-----|---|---|---|---|---|---|---|---|---|---|---|---|
| x | y | F | | | | | | | | | | | | | | | |
| 0 | 0 | 1 | | | | | | | | | | | | | | | |
| 0 | 1 | 1 | | | | | | | | | | | | | | | |
| 1 | 0 | 1 | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | | | | | | | | | | | | | | | |
| NOR |  $F = (x + y)'$ | <table> <tr> <th>x</th><th>y</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table> | x | y | F | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| x | y | F | | | | | | | | | | | | | | | |
| 0 | 0 | 1 | | | | | | | | | | | | | | | |
| 0 | 1 | 0 | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | | | | | | | | | | | | | | | |
| Exclusive-OR (XOR) |  $F = xy' + x'y$ $= x \oplus y$ | <table> <tr> <th>x</th><th>y</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table> | x | y | F | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| x | y | F | | | | | | | | | | | | | | | |
| 0 | 0 | 0 | | | | | | | | | | | | | | | |
| 0 | 1 | 1 | | | | | | | | | | | | | | | |
| 1 | 0 | 1 | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | | | | | | | | | | | | | | | |
| Exclusive-NOR or equivalence |  $F = xy + x'y'$ $= (x \oplus y)'$ | <table> <tr> <th>x</th><th>y</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </table> | x | y | F | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| x | y | F | | | | | | | | | | | | | | | |
| 0 | 0 | 1 | | | | | | | | | | | | | | | |
| 0 | 1 | 0 | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | | | | | | | |

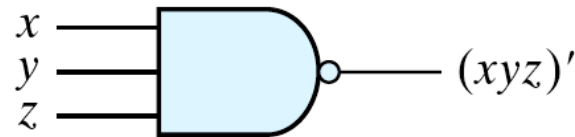
Figure 2.5 Digital logic gates

Multiple Inputs

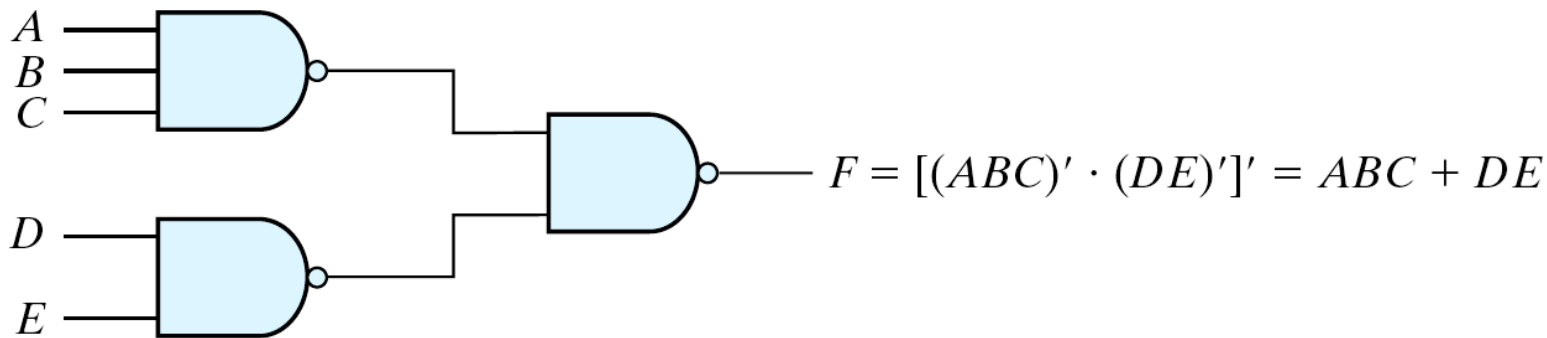
- Multiple NOR = a complement of OR gate, Multiple NAND = a complement of AND.
- The cascaded NAND operations = sum of products.
- The cascaded NOR operations = product of sums.



(a) 3-input NOR gate



(b) 3-input NAND gate

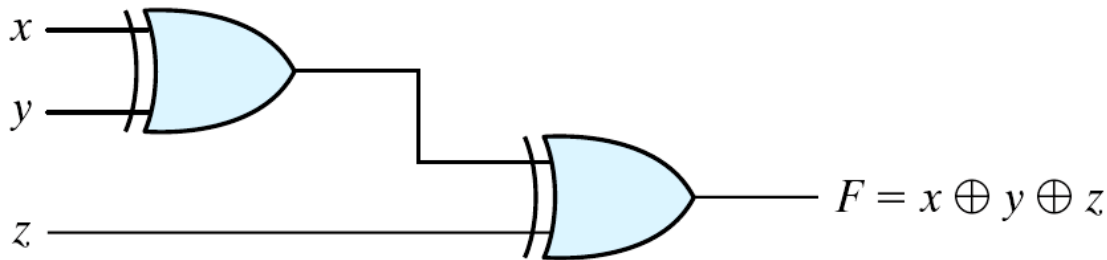


(c) Cascaded NAND gates

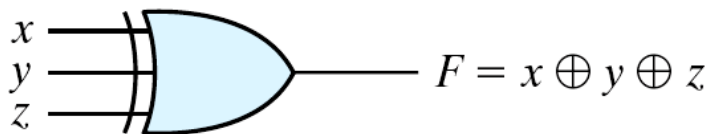
Figure 2.7 Multiple-input and cascaded NOR and NAND gates

Multiple Inputs

- The XOR and XNOR gates are commutative and associative.
- Multiple-input XOR gates are uncommon?
- XOR is an odd function: it is equal to 1 if the inputs variables have an odd number of 1's.



(a) Using 2-input gates



(b) 3-input gate

| x | y | z | F |
|-----|-----|-----|-----|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

(c) Truth table

Figure 2.8 3-input XOR gate

Positive and Negative Logic

Positive and Negative Logic

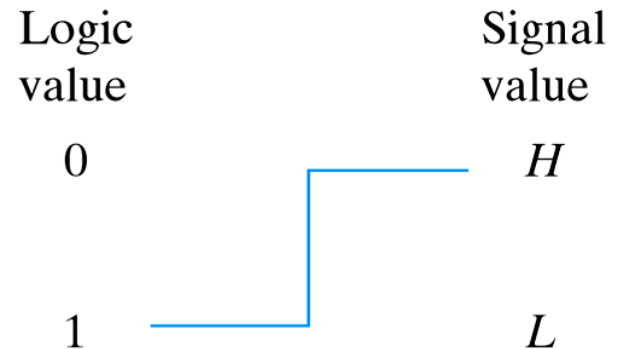
- ◆ Two signal values \Leftrightarrow two logic values
- ◆ Positive logic: $H=1$; $L=0$
- ◆ Negative logic: $H=0$; $L=1$

Consider a TTL gates

- ◆ A positive logic AND gate
- ◆ A negative logic OR gate



(a) Positive logic



(b) Negative logic

Figure 2.9 Signal assignment and logic polarity

Positive and Negative Logic

| x | y | z |
|-----|-----|-----|
| L | L | L |
| L | H | L |
| H | L | L |
| H | H | H |

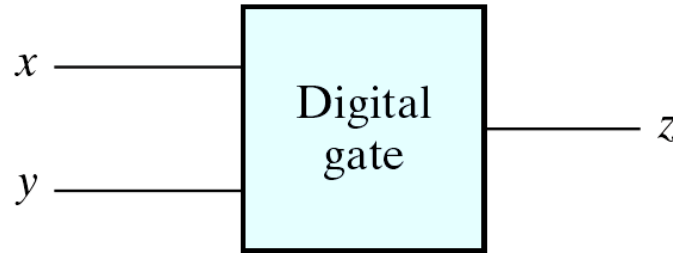
(a) Truth table with H and L

| x | y | z |
|-----|-----|-----|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

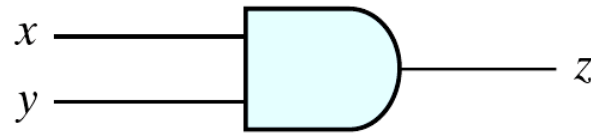
(c) Truth table for positive logic

| x | y | z |
|-----|-----|-----|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

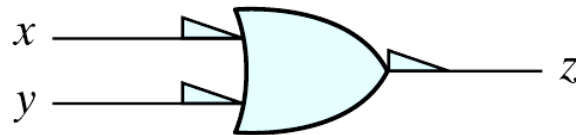
(e) Truth table for negative logic



(b) Gate block diagram



(d) Positive logic AND gate



(f) Negative logic OR gate

Figure 2.10 Demonstration of positive and negative logic

2.9 Integrated Circuits

Level of Integration

- An IC (a chip)

- Examples:

- ◆ Small-scale Integration (SSI): < 10 gates
- ◆ Medium-scale Integration (MSI): $10 \sim 100$ gates
- ◆ Large-scale Integration (LSI): $100 \sim \text{xk}$ gates
- ◆ Very Large-scale Integration (VLSI): $> \text{xk}$ gates

- VLSI

- ◆ Small size (compact size)
- ◆ Low cost
- ◆ Low power consumption
- ◆ High reliability
- ◆ High speed

Digital Logic Families

- ▣ Digital logic families: circuit technology
 - ◆ TTL: transistor-transistor logic (dying?)
 - ◆ ECL: emitter-coupled logic (high speed, high power consumption)
 - ◆ MOS: metal-oxide semiconductor (NMOS, high density)
 - ◆ CMOS: complementary MOS (low power)
 - ◆ BiCMOS: high speed, high density

Home Work (3)

Digital Design (4th)- Morris Mano-Page 66-
Problems:

2.3 d,f

2.4 d,e

2.6 Only for (2.3 d,f)

2.7 Only for (2.4 d,e)

2.9

2.20

2.22

Home Work (4)

Digital Design (4th)- Morris Mano-Page 66-
Problems:

2.13

2.14

2.15

2.27

2.28