University of Bahri

College of Computer Sciences and Mathematics

Second Semester - Academic Year 2021/2022

Department	General	**Porgram**	B.Sc.
Course Title	Ordinary Differential Equations	**Level**	2^{nd} Year
Course Instructor	Salma Omar A.Adam	**Lecture NO.**	1

1 Introduction

Definition 1 *An equation involving derivatives or differentials of one or more dependent variables with respect to one or more independent variables is called a differential equations.*

For examples of differential equations we list the following

$$dy = (x + \sin x)dx \tag{1}$$

$$\frac{d^4x}{dt^4} + \frac{d^2x}{dt^2} + (\frac{dx}{dt})^5 = e^t \tag{2}$$

$$y = \sqrt{x}\frac{dy}{dx} + \frac{k}{\frac{dy}{dx}} \tag{3}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{4}$$

1.1 Ordinary Differential Equation

A differential equation involving derevatives with respect to a single independent variable is called an ordinary differential equation (see equations (**??**),(**??**) and (**??**)).

1.2 Partial Differential Equation

A differential equation involving partial derevatives with respect to more than one independent variables is called a partial differential equation (see equations (**??**)).

1.3 Order of a Differential Equation

The order of the highest order derivative involved in a differential equation is alled the order of the diferential equation. e.g equation (**??**) if of the fourth order and equation (**??**) of the first order.

1.4 Degree of a Differential Equation

The degree of a differential equation is the degree of the highest derivative which occurs in it, after the differential equation has been mad free from radicals and fractions as far as the derivative are concerned.
Note that the above definition of degree does not require variables $x, t, u, \cdots$ ect to be free from radicals and fractions.

1.5 Linear and Non-linear Differential Equations

A differential equation is called linear if

i. Every dependent variable and every derivative involved occurs in the first degree only, and

ii. No products of dependent variabls and/or derivatives occur.

A differential equation which is not linear is called a non-linear differential equation. e.g equation (**??**) is linear and equations (**??**), (**??**) are non-linear.

1.6 Solution of a Differential Equation

Any relation between the dependent and independent variables , when substituted in the differential equation reduces it to an identity is called a solution or integral of the differential equation. It should be noted that a soluton of a differential equation does not involve the derivatives of the dependent variable with respect to the independent variable or variables.
For example, $y = ce^{2x}$ is a solution of $y' = 2y$ because by putting $y = ce^{2x}$ and $y' = 2y$, the given differential equation reduces to the identity $2ce^{2x} = 2ce^{2x}$.

2 Equations of First Order and First Degree

There are two standard forms of differential equations of first order and first degree, namely

(i) $\dfrac{dy}{dx} = f(x, y)$

(ii) $M(x, y)dx + N(x, y)dy = 0$

In what follows we shall see that an equation in one of these forms may readily be written in the other form. It will assumed that the necessary conditions for the existance of solutions are satisfied. We now discuss various methods to solve such equations.

2.1 Separation of Variables

If a differential equation of the first order and first degree is of the form

$$f_1(x)dx = f_2(y)dy \tag{5}$$

where $f_1(x)$ is a function of x only and $f_2(y)$ is a function of y only, then we say that the variales are seprable in the given differential equation. Such equations are solved by integrating both sides of (**??**) and adding an arbitrary constant of integration to any one of the two sides. Thus the solution of (**??**) is

$$\int f_1(x)dx = \int f_2(y)dy + C \tag{6}$$

Remark. To simplify the solution (**??**), the constant of integration C can be selected in any suitable form. For example, C can be replaced by $\frac{c}{3}, \log C, \tan C, \tan^{-1} C, e^C, \cdots etc$

2.1.1 Example

Solve

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

Solution

2.1.2 Example

Solve

$$\ln\left(\frac{dy}{dx}\right) = ax + by$$

Solution

2.1.3 Example

Solve

$$y - x\frac{dy}{dx} = a\left[y^2 + \frac{dy}{dx}\right]$$

<u>Solution</u>

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