6.4-2) $\theta \approx \sigma^2$; $0 < \theta < \infty$; μ is known.

Details of my distribution

$$f(x;\theta) \approx \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}$$

$\theta \approx \sigma^2$ $\qquad$ $\theta\sigma^2$

Given that $\theta = \sigma^2$; $\hat{\theta}$ likehood function is

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta)$$

$$\approx \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta}} \cdot \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}$$

$$\approx \left(\frac{1}{\sqrt{2\pi\theta}}\right)^n \cdot \exp\left\{\sum_{i=1}^{n} \frac{-(x-\mu)^2}{2\sigma^2}\right\}$$

Optimize with respect to θ to find argmax

$$\frac{d}{d\sigma^2}\left[\ln(L\sigma)\right] = \cancel{-\frac{n}{2}\ln(2\pi)} - n\ln(\sigma) \quad \boxed{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2} \cdot \cancel{\ln(\exp)}}$$

$$\left(\sum_{i=1}^{n}\frac{(x-\mu)^2}{2}\right) \cdot \sigma^{-2}$$

$$\approx 0 - \frac{n}{\sigma} - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\cancel{2}} \cdot \cancel{(2)}\, \sigma^{-3}$$

$$\approx -\frac{n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\approx \cancel{(\sigma^3)}\frac{n}{\cancel{\sigma}}\cancel{\left(\frac{1}{n}\right)} = \cancel{\frac{1}{\sigma^3}} \sum_{i=1}^{n}(x_i-\mu)^2\left(\frac{1}{n}\right)\cancel{(\sigma^3)}$$

$$\theta \approx \sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n} \approx$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n}(x-\mu)^2}{n}}$$