6.4-10

MME for geometric distribution

$$f(x;\theta) = (1-p)^{x-1} \cdot p$$

$$\mathcal{L}(p) = \prod_{i=1}^{n} p(1-p)^{x_i - 1}$$

$$\ln(\mathcal{L}(p)) = n\ln(p) \cdot \sum_{i=1}^{n}(x_i - 1)\ln(1-p)$$

Find arg max $l(p)$

$$\frac{d\,[\ln(\mathcal{L}(p))]}{dp} = 0$$

$$\approx \left\{ n\ln(p) \cdot \sum_{i=1}^{n}(x_i - 1) \cdot \ln(1-p) \right\} = 0$$

$$\approx n\left(\frac{1}{p}\right) \cdot \sum_{i=1}^{n}(x_i - 1) \cdot \left(\frac{-1}{1-p}\right) = 0$$

$$\approx \frac{n}{p} + \frac{n}{1-p} - \frac{1}{(1-p)}\sum_{i=1}^{n} x_i = 0$$

$$\approx n \frac{1}{p\cancel{(1-p)}} = \frac{-1}{\cancel{(1-p)}} \cdot \sum_{i=1}^{n} x_i$$

$$\approx n \cdot \frac{1}{p} = -\sum_{i=1}^{n} x_i$$

$$\approx \frac{1}{p} = \frac{-\sum_{i=1}^{n} x_i}{n} \approx \bar{x}$$

$$\approx p = \frac{1}{\bar{x}}$$

B) n ~~→~~ # of trails

—)