(1)

1. Let X and Y be the lifetimes of two brands of light bulbs, and suppose that X is N(2000, 40000) and Y is N(2500, 90000). Suppose that we have a 5-pack of each, and let $\bar{X}$ and $\bar{Y}$ be the sample means.
   a. ~~What are the means and variances of $\bar{X}$ and $\bar{Y}$?~~
   b. What is the distribution of $\bar{X} - \bar{Y}$?
   c. Find $P(\bar{X} > \bar{Y})$.

$N(-500, 130.000)$

5 pack of each $(\bar{x}, \bar{y})$ — Hamad Abdul

$X \approx N(2000, 40{,}000)$
$Y \approx N(2500, 90{,}000)$ * Come back

Find $\mu_{\bar{x}+\bar{y}}$; $\sigma^2_{\bar{x}+\bar{y}}$

$\mu_{\bar{x}+\bar{y}} = \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{1}{n}\sum_{i=1}^{n} y_i$

$\approx \frac{1}{5}\sum_{i=1}^{5} x_i + \frac{1}{5}\sum_{i=1}^{5} y_i$

$\mu_{\bar{x}} = \frac{1}{n}\sum_{i=1}^{n} x_i +$

$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$

$h(x) = \frac{f_i}{n(c_i - c_{i-1})}, \; c_{i-1} < x \le c_i$

2. 6.1-10

(2)

6.1-10. In 1985, Kent Hrbek of the Minnesota Twins and Dion James of the Milwaukee Brewers had the following numbers of hits (H) and official at bats (AB) on grass and artificial turf:

	Hrbek			James		
Playing Surface	AB	H	BA	AB	H	BA
Grass	204	50		329	93	
Artificial Turf	355	124		58	21	
Total	559	174		387	114	

(a) Find the batting average BA (namely, H/AB) of each player on grass.
(b) Find the BA of each player on artificial turf.
(c) Find the season batting averages for the two players.
(d) Interpret your results.

6.1 #10)
A) $BA_{H\,grass} = \frac{H}{AB} \approx 50/204 \approx .245 \approx .25$
$BA_{J\,grass} = \frac{H}{AB} \approx 93/329 \approx .2826 \approx .28$
B) $BA_{H\,art} \approx 124/355 \approx .3492 \approx .35$
$BA_{J\,art} \approx 21/58 \approx .3620 \approx .36$
C) $BA_{H\,tot} = 174/559 \approx .3112$
$BA_{J\,tot} = 114/387 \approx .2945$
D) While James had a higher batting average for just grass or artificial turf fields, he had fewer total average rates and hits than Hrbek.

(3)

6.4-2. A random sample $X_1, X_2, \ldots, X_n$ of size n is taken from $N(\mu, \sigma^2)$, where the variance $\theta = \sigma^2$ is such that $0 < \theta < \infty$ and μ is a known real number. Show that the maximum likelihood estimator for θ is $\hat{\theta} = (1/n)\sum_{i=1}^{n}(X_i - \mu)^2$ and that this estimator is an unbiased estimator of θ.

6.4-2 Show that $\theta \approx \sigma^2$ and unbiased

$\sigma^2 \approx \frac{1}{n}\sum_{n=1}^{n}(X_i - \mu)^2 \approx \frac{1}{n}\sum_{n=1}^{n} X_i^2 - \mu^2$

$\hat{\theta} \approx f(x;\theta) \approx \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$

Define Likelihood function

$L(\theta) \approx \prod_{i=1}^{n} f(x;\theta)$

$\approx \prod_{i=1}^{n} (2\pi\sigma^2)^{-1/2} \cdot \exp\left(-\frac{1}{2} \cdot \frac{(x-\mu)^2}{\sigma^2}\right)$

$\approx (2\pi\sigma^2)^{-n/2} \cdot \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right)$

$\approx \underbrace{\left(\frac{1}{\sqrt{2\pi}\cdot\sigma}\right)^n}\, \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right\}$

Essentially is a constant setting $\approx$ to α

$\approx \alpha^n \cdot \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i^2 + \mu^2 - 2\mu x_i)\right\}$

$\approx \alpha^n \cdot \exp\left\{-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n} x_i^2 + n\mu^2 - 2\mu n\bar{x}\right)\right\}$

$\approx \alpha^n \cdot \exp\left\{-\frac{n}{2\sigma^2}\left(\mu^2 - 2\mu\bar{x} + \frac{1}{n}\sum_{i=1}^{n} x_i^2\right)\right\}$

$\approx \alpha^n \cdot \exp\left\{-\frac{n}{2\sigma^2}\left(\mu^2 - 2\mu\bar{x} + \bar{x}^2\right)\right\}$

$\approx \alpha^n \cdot \exp\left\{-\frac{n}{2\sigma^2}(\mu - \mu_e)^2\right\}$

$\approx \alpha^n \cdot \exp\left\{-\frac{n}{2\sigma_e^2}(\mu^2 - 2\mu\mu_e + \mu_e^2)\right\}$

Take Ln and set equal to 0

$\ln(L\theta) \approx n\ln(\alpha) \cdot \frac{-n}{2\sigma_e}(\mu^2 - \mu_e^2) = 0$

$\approx \cancel{(n\ln\alpha)\left(\frac{-n}{2\sigma_e}\right)\sqrt{\mu^2}} = \sqrt{\mu_e^2}\,\cancel{\left(n\ln(\alpha)\left(\frac{-n}{2\sigma_e}\right)\right)}$

$\mu = \mu_e = \bar{x}$

5. The numbers below represent heights (in feet) of 3-year old elm trees, where leaves represent decimal parts of each value.

Stems	Leaves
5	1, 8
6	1, 2, 4, 7, 8, 9
7	0, 2, 3, 3, 4, 5
8	1, 1, 2, 3, 5, 6, 6, 7, 7, 9, 9
9	0, 1, 3, 4

a. Find the five number summary for the given set of data.
b. Produce a box and whisker plot for the data.

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