

Evolutionary Game Theory

Daniel Hammer, Nicholas O'Kelley, Andrew Shelton, Faculty Advisor: Andrew Penland

Western Carolina University 

Background

- Began with simple games to learn how to develop strategies from previous data and then automate this process for larger data sets.
- Initial objective: utilize evolutionary game theory to find optimal strategies for players 1 and 2 from random initial population.
- This involved creating a generic code base capable of supporting large populations of players across different games.
- Returning to the Graph Coloring Game, we attempt to find optimal coloring strategies.

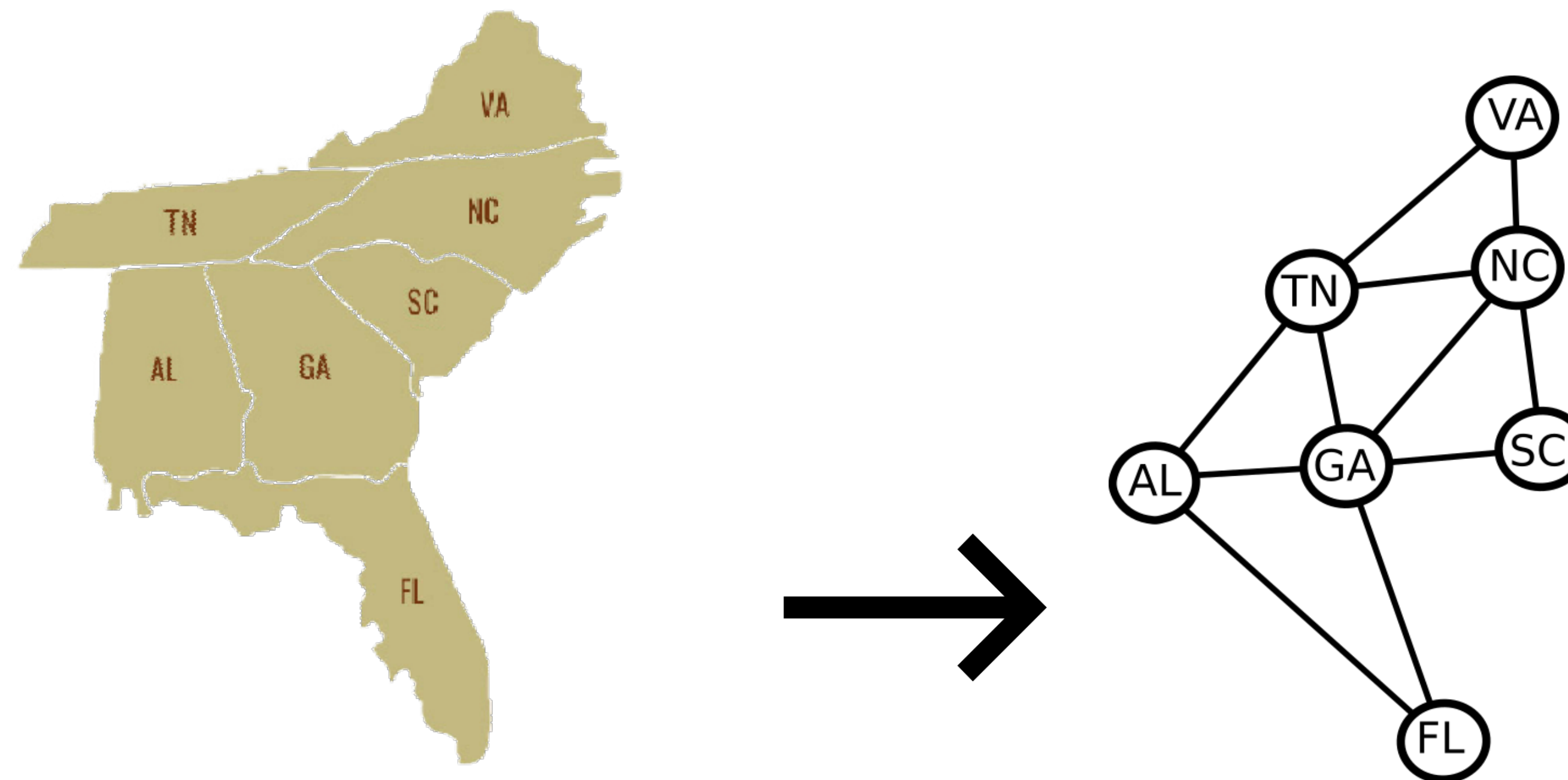
Future Work

- Find correlations between winning strategies of various games.
- Identify optimal strategies in the two-player graph coloring game.
- Streamline the process of data analysis once simulations have been run.
- Containerize the code to allow for portability for future scaling
- Scale up code base to support more game types and larger population sizes.

Acknowledgments

Built on initial research supported by the Learning Environment and Academic Research Network via the National Science Foundation under Grant No. 1524607.

A Real World Example

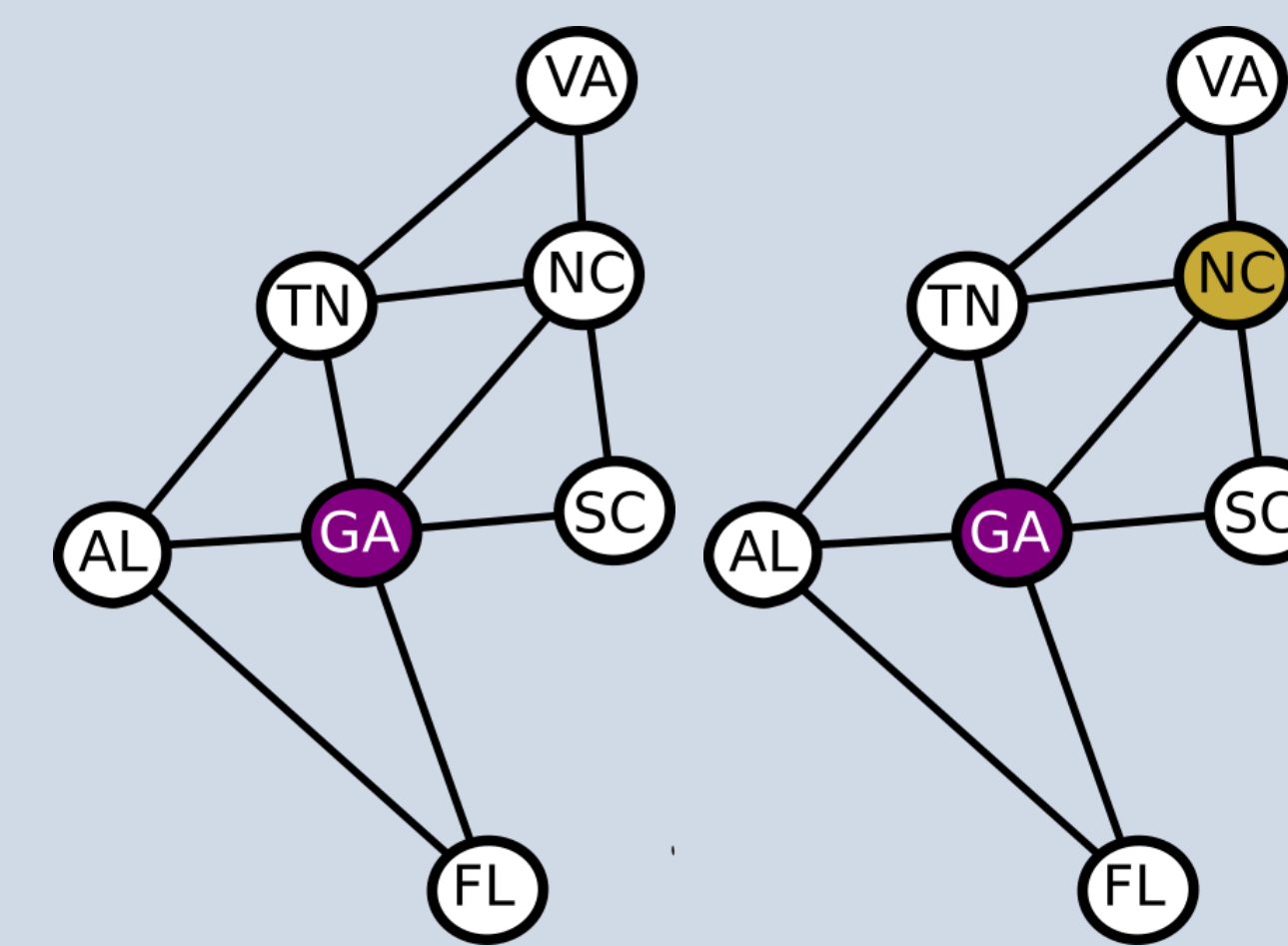


Our Work

- We began by writing the code necessary to play a generic game between two players.
- With the software designed, we run simulated graph coloring games with two competing populations.
- Winning strategies are based on fitness (win percentages) and are allowed to breed
- Strategies are represented as lists with order priority(vertex:[1,3,4,2,5], colors:[Purple,Gold])
- Breeding players is as follows:
Parent1(v[4,1,3,2], c[g,b,r,p]),
Parent2(v[1,4,2,3], c[g,r,p,b])
child1(no mutation): v[4,1,2,3], c[g,r,p,b]
child2(with mutation): v[4,2,1,3], c[b,r,p,g]
- The Child takes up to a random point from Parent1 and the rest from Parent2 for both lists
- Repeat this process and look for patterns in the data.

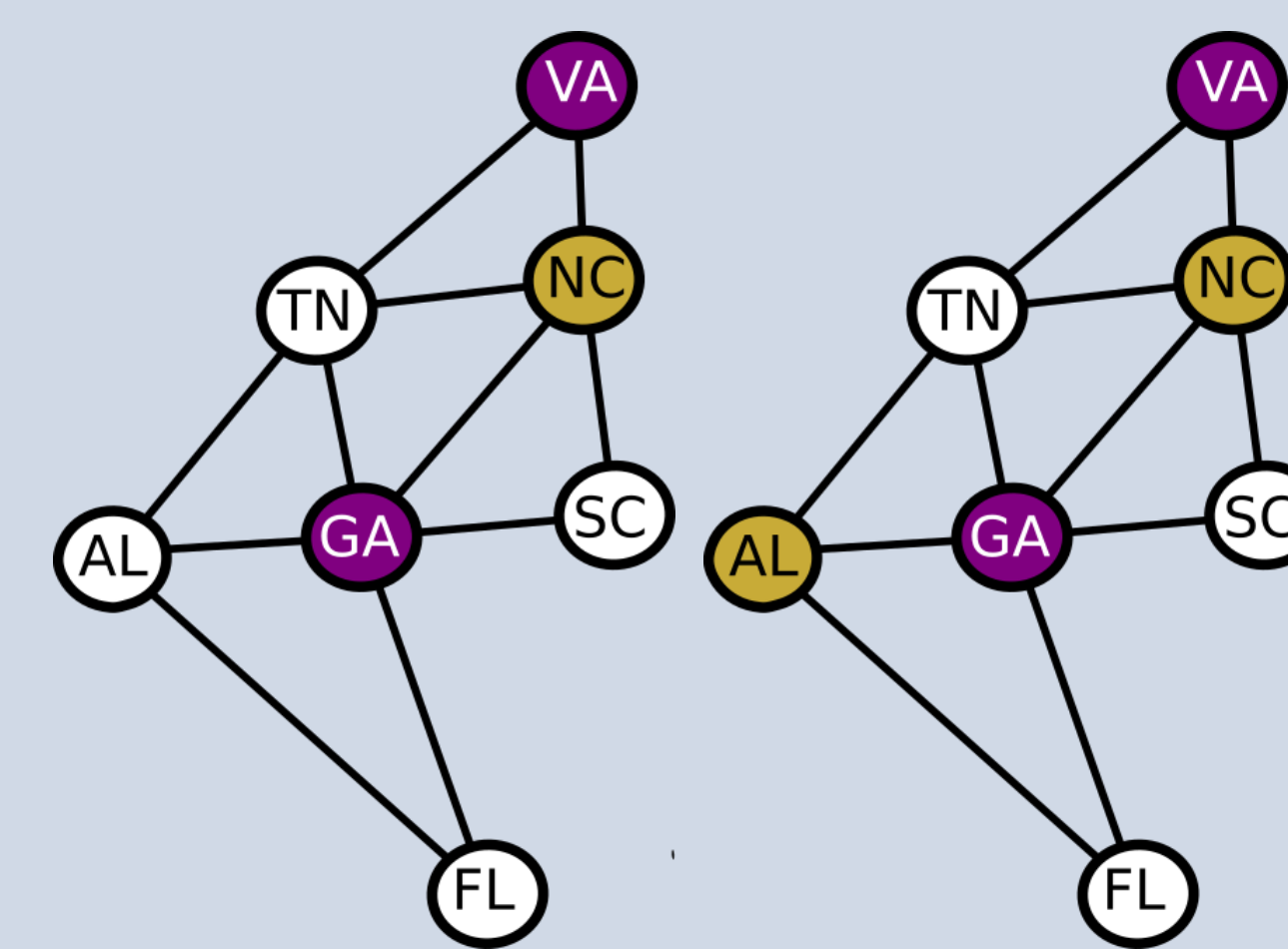
Illustrations

A sample game where k colors is 2.



Player 1

Player 2



Player 1

Player 2

Since Player 1 has no legal move, Player 2 wins!

Graph Coloring Rules

- Player 1 selects the maximum number of colors to be used in the game
- Players alternate coloring a vertex each turn
- No two adjacent vertices can be the same color
- If a proper coloring for the entire graph is achieved, Player 1 wins. If not, Player 2 wins.

Definitions

- Proper Coloring* - A coloring in which all vertices in a graph are colored and no two adjacent vertices receive the same color.
- $\chi(G)$ - Chromatic number; the smallest k such that a graph G can be properly colored with k colors.
- $\chi_g(G)$ - Game chromatic number; the minimum number of colors needed to ensure that Player 1 can win on a graph G .

Bibliography

V. Larsen, C. Dunn, and J.F. Nordstrom. Introduction to competitive graph coloring. In Valerie Peterson Anna Woodfin and Christopher Lee, editors, *A primer for undergraduate research: From groups and tiles to frames and vaccines*, pages 99–126. 2017.

H. A. Kierstead, Tomasz Bartnicki, Jarosław Grytczuk, and Xuding Zhu. The map-coloring game. *The American Mathematical Monthly*, 114(9):783–803, 2007.