Tutorial Sheet – No 5 Answers

1 For an ideal transformer:

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

Hence:

$$I_1 = I_2 \times \frac{N_2}{N_1} = 4 \times \frac{300}{1200} = \mathbf{1A_{rms}}$$

$$V_2 = V_1 \times \frac{N_2}{N_1} = 110 \times \frac{300}{1200} = \mathbf{27.5V_{rms}}$$

The VA rating is simply the product of the voltage and current (in either winding):

$$V_1 \times I_1 (= V_2 \times I_2) = 110 \times 1 = \mathbf{110VA}$$

2 For an ideal transformer:

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

Hence:

$$V_2 = V_1 \times \frac{N_2}{N_1} = 240 \times \frac{350}{100} = \mathbf{840V_{rms}}$$

With a 1000Ω load placed across the secondary, the secondary current is given by:

$$I_2 = \frac{V_2}{R_2} = \frac{840}{1000} = \mathbf{0.84A_{rms}}$$

and:

$$I_1 = I_2 \times \frac{N_2}{N_1} = 0.84 \times \frac{350}{100} = \mathbf{2.94A_{rms}}$$

3 The transformer in question 2 is now turned round so the secondary winding now becomes the primary winding and vice-versa. The new seconday voltage is:

$$V_2 = V_1 \times \frac{N_2}{N_1} = 240 \times \frac{100}{350} = \mathbf{68.6V_{rms}}$$

and:

$$I_2 = \frac{V_2}{R_2} = \frac{68.6}{1000} = \mathbf{68.6mA_{rms}}$$

and the primary current is now:

$$I_1 = I_2 \times \frac{N_2}{N_1} = 0.0686 \times \frac{100}{350} = \mathbf{19.6mA_{rms}}$$

4 The VA rating is simply a product of the voltage and current (in either winding). We can either calculate the secondary voltage from the VA rating and refer this back to the primary, or refer the secondary current to the primary and calculate the voltage from the VA rating:

$$I_1 = I_2 \times \frac{N_2}{N_1} = 2 \times \frac{1}{6} = \mathbf{0.333A_{rms}}$$

and:

$$V_1 = \frac{VA}{I_1} = \frac{1200}{0.333} = \mathbf{3600V_{rms}}$$

For the case of a purely resistive load the power factor is unity and the current is in-phase with the supply voltage, hence:

$$I_1 = \mathbf{0.333\angle 0°A_{rms}}$$

and the power dissipated is:

$$P = V_1 \times I_1 \times \cos\phi = 3600 \times 0.333 \times 1 = \mathbf{1200W}$$

However, for the pure inductor the current lags the supply voltage by 90° and is:

$$I_1 = \mathbf{0.333\angle -90°A_{rms}}$$

For a purely inductive load the power factor is zero and no power is dissiopated:

$$P = V_1 \times I_1 \times \cos\phi = 3600 \times 0.333 \times 0 = \mathbf{0W}$$

5 Using the expression:

$$V_{1rms} = 4.44 f \times N \times \phi_{\max}$$

and the fact that:

$$\phi_{\max} = B_{\max} \times A_{core}$$

then:

$$V_{1rms} = 4.44 f \times N \times B_{\max} \times A_{core} = 4.44 \times 50 \times 1200 \times 1.2 \times 300 \times 10^{-6} = \mathbf{95.9V_{rms}}$$

6 Since:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

the output voltage is independent of frequency and hence:

$$V_2 = V_1 \times \frac{N_2}{N_1} = 240 \times \frac{1}{8} = \mathbf{30V_{rms}\ for\ all\ frequencies}$$

First obtain an expression for the core flux density in terms of frequency. Since:

$$V_{1rms} = 4.44 f \times N \times \phi_{\max}$$

and:

$$\phi_{\max} = B_{\max} \times A_{core}$$

then:

$$B_{\max} = \frac{V_{1rms}}{4.44 f \times N \times A_{core}} = \frac{240}{4.44 \times f \times 1900 \times 500 \times 10^{-6}} = \frac{56.9}{f}$$

so:

when f = 40 Hz: $B_{\max} = \frac{56.9}{40}$ = **1.42T**

when f = 50 Hz: $B_{\max} = \frac{56.9}{50}$ = **1.14T**

when f = 80 Hz: $B_{\max} = \frac{56.9}{80}$ = **0.711T**

when f = 100 Hz: $B_{\max} = \frac{56.9}{100}$ = **0.569T**

Since the core flux density is proportional to the input voltage, then the 'worst case' is when the input voltage is increased by 10% (i.e. V = 240 + 10% = 264V_{rms}) and the frequency to give the maximum flux density is:

$$f = \frac{V_{rms}}{4.44 \times B_{\max} \times A_{core} \times N} = \frac{264}{4.44 \times 1.2 \times 500 \times 10^{-6} \times 1900} = \mathbf{52.2Hz}$$

Since B_{max} is inversely proportional to frequency increasing the frequency will lower the flux density in the core so it is safe to use the transformer at any frequency above 52.2Hz.

7 Using the results obtained in the previous question it can be seen that operating the transformer at higher frequencies will reduce the core flux density for a given input voltage, so:

- operating a transformer designed for 60Hz from a 50Hz supply is dangerous.
- operating a transformer designed for 50Hz from a 60Hz supply is safe.

8 Since we are told both the primary and secondary voltages the turns ratio can be found:

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{10000}{50} = \frac{200}{1} \quad \text{i.e. } \mathbf{200:1}$$

Knowing the load impedance connected across the secondary winding allows the secondary current to be found:

$$I_2 = \frac{V_2}{Z_2} = \frac{50}{5000} = 0.01\ A_{rms}$$

and the current drawn from the supply is:

$$I_1 = I_2 \times \frac{N_2}{N_1} = 0.01 \times \frac{1}{200} = \mathbf{50\mu A_{rms}}$$

9 (a) The primary (input) current is calculated from:

$$I_1 = \frac{VA}{V_1} = \frac{8000}{240} = \mathbf{33.3A_{rms}}$$

The secondary voltage and current are then easily obtained:

$$V_2 = V_1 \times \frac{N_2}{N_1} = 240 \times \frac{300}{400} = \mathbf{180V_{rms}}$$

$$I_2 = I_1 \times \frac{N_1}{N_2} = 33.3 \times \frac{400}{300} = \mathbf{44.4A_{rms}}$$

(b) Using the expressions:

$$V_{1rms} = 4.44 f \times N_1 \times \phi_{\max}$$

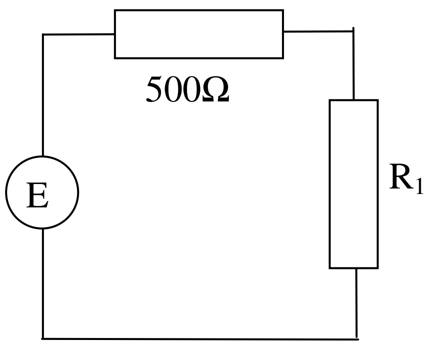
and:

$$\phi_{\max} = B_{\max} \times A_{core}$$

then:

$$A_{core} = \frac{V_{1rms}}{4.44 f \times N_1 \times B_{\max}} = \frac{240}{4.44 \times 50 \times 400 \times 1.5} = \mathbf{18cm^2}$$

10 The loudspeaker resistance may be referred to the primary circuit by the square of the turns ration. Let the referred resistance be R_1 then the circuit appears as:



and the power dissipated in the referred load is:

$$P_L = I_1^2 R_1 = \frac{E^2}{(500 + R_1)^2} R_1$$

For the maximum power transfer to the load find the condition for which:

$$\frac{dP_L}{dR_1} = 0$$

performing the differentiation:

$$\frac{dP_L}{dR_1} = \frac{E^2\left[(500+R_1)^2 - 2R_1(500+R_1)\right]}{(500+R_1)^4}$$

and setting this to zero gives:

$$500 + R_1 - 2R_1 = 0$$

or:

$$R_1 = 500\Omega$$

Maximum power transfer between source and load occurs when the internal resistance of the source is equal to the referred load resistance. Therefore the matching transformer is required to 'transform' the load resistance of 8Ω to appear as 500Ω in the primary. Now since:

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

then:

$$\left(\frac{N_1}{N_2}\right) = \sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{500}{8}} = 7.9$$

Therefore the required turns ratio is **7.9:1**.

11 The primary of the transformer is connected into the circuit carrying $50A_{rms}$ and will circulate a current of:

$$I_2 = I_1 \times \frac{N_1}{N_2} = 50 \times \frac{2}{250} = 0.4A_{rms}$$

in the secondary circuit. Since the resistor in this circuit has a value of 12.5Ω the voltage across it will be:

$$V_2 = I_2 R_2 = 0.4 \times 12.5 = \mathbf{5V_{rms}}$$

A primary current of $50A_{rms}$ will produce a voltage of $5V_{rms}$ across the load resistor, hence the sensitivity of the probe is:

$$\frac{5}{50} = \mathbf{0.1V_{rms}\ per\ A_{rms}}$$