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EEE 6081 (EEE 421) Visual Information Engineering

Problem Set 1

Q1) The N-point Discrete Cosine Transform (DCT) is

$$c_n = \sqrt{\frac{e_n}{N}} \sum_{i=0}^{N-1} \left[\cos\left(\frac{(2i+1)n\pi}{2N} \right) \right] x_i$$

$$e_n = \begin{cases} 1 & \text{when } n = 0 \\ 2 & else \end{cases}$$

a) Show the basis functions of the 4 point DCT in matrix form
b) What are the DCT coefficients of the input data sequence x=[30 30 35 40]?
c) What is the reconstructed data sequence if only the first coefficient was retained and all others were set to zero? [Can you estimate this value without computing the inverse transform?]
d) What is the reconstructed data sequence if only two coefficients with the largest magnitude were retained and all others were set to zero?

Q2) A wavelet transform filter bank consists of two filters [a a] and [a –a], where a=0.7071.

e) What is the transform matrix for the first level of decomposition for the input data sequence, x=[x1 x2 x3 x4]?
f) Derive the transform matrix for the 2-level dyadic decomposition using the above filter bank?
g) Derive the transform matrix for the 2-level full tree wavelet packet decomposition using the above filter bank?
h) Verify that the transform matrix in b) forms a set of orthogonal basis functions.

Q3) The lifting factorisation of a wavelet transform is shown below:

$$\begin{bmatrix} x^f \\ y^f \end{bmatrix} = \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\beta & 1 \end{bmatrix} \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

i) What are the corresponding lifting steps of the forward transform?
j) Derive the corresponding analysis wavelet filter bank.
k) How do you obtain the integer-to-integer version of this filter bank?
l) Write down the lifting steps for the inverse transform.