The University of Sheffield
Department of Electronic and Electrical Engineering

EEE101 Problem Sheet

Resonance and Transients

1 Measurements on the circuit of figure 1 revealed a resonant frequency of 1.59 kHz and a 3 dB bandwidth, Δf of 199 Hz.

- **(i)** What is the circuit q factor? [8]
- **(ii)** Find the value of L. [100 mH]
- **(iii)** What is the value of R_L. [25 Ω]
- **(iv)** What voltages would be measured at V_R and V_C at resonance? [$0.8V_S$, $8V_S$]
- **(v)** What is the amplitude and phase of V_L with respect to V_R? (*a positive value of phase is a phase lead*) [$8V_S$, $+88.6^{\circ}$]

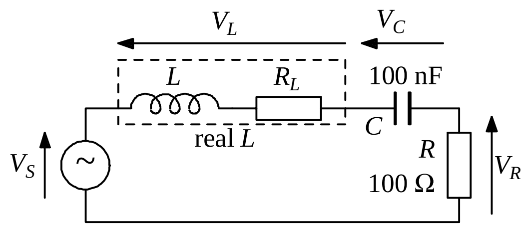

Figure 1

2 Show that the impedance of the circuit of figure 2 is

$$Z = \frac{j\omega L\,(1 + j\omega CR)}{1 - \omega^2 LC + j\omega CR}$$

and that its resonant frequency is

$$f = \frac{1}{2\pi\sqrt{LC - C^2R^2}}\,.$$

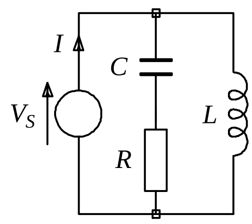

Figure 2

3 Assume initially that in the circuit of figure 3 $R_C = R_L = R$. In terms of L and C, find the value for R that will make the circuit resonant at all frequencies. (*This one is for experts*)

Show that $R_C = R_L$ is necessary for the "resonant at all frequencies" property to be achieved. (*This one is for* ***real*** *experts*)

Hint: *For both parts, get the impedance in a form of a ratio of two complex numbers of the form* $(a + jb)/(c + jd)$. *Then try to find a constant, k, such that* $k(a + jb) = (c + jd)$. *The complex numbers will then cancel out leaving you with impedance = k.*

The combination L and R_L can be used to model a wide range of electromagnetic energy converters. One example is a loudspeaker; R_L is the voice coil resistance and L is the voice coil inductance. At high audio frequencies, the impedance of the loudspeaker is primarily inductive and this can cause problems for some amplifiers. R_C and C form what is called a "Zobel" network (after its inventor) that compensates for the loudspeaker reactance and presents the power amplifier with a resistive load.

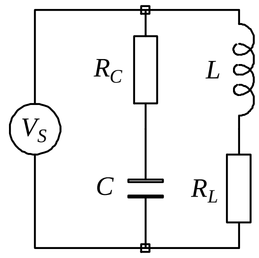

Figure 3

4 In figure 4, V_S is a step waveform changing from 0 V to 10 V at time $t = 0$. Write down

(i) I at $t = 0^-$, $t = 0^+$ and $t \Rightarrow \infty$, [0, 5 mA, 0]

(ii) V_C at $t = 0^-$, $t = 0^+$ and $t \Rightarrow \infty$. [0, 0, 10 V]

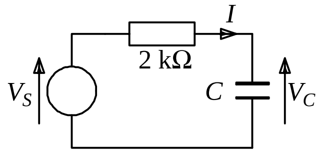

Figure 4

5 In the circuit of figure 5, V_S is a step waveform that is 3 V for all $t < 0$ and – 6 V for all $t > 0$. Write down

(i) I_L at $t = 0^-$, $t = 0^+$ and $t \Rightarrow \infty$, [1.5 mA, 1.5 mA, – 3 mA]

(ii) I at $t = 0^-$, $t = 0^+$ and $t \Rightarrow \infty$, [1.5 mA, – 7.5 mA, – 3 mA]

(iii) V_L at $t = 0^-$, $t = 0^+$ and $t \Rightarrow \infty$. [0, – 9 V, 0]

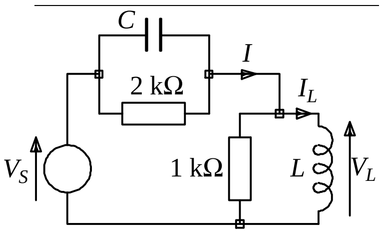

Figure 5

6 In the circuit of figure 6, V_S is a step waveform that is – 6 V for all $t < 0$ and 12 V for all $t > 0$. Write down

(i) I at $t = 0^-$, $t = 0^+$ and $t \Rightarrow \infty$, [– 1 mA, – 1 mA, 2 mA]

(ii) V_R at $t = 0^-$, $t = 0^+$ and $t \Rightarrow \infty$. [-2 V, 7 V, 4 V]

Note: You may need to do some working out for part **(ii)**.

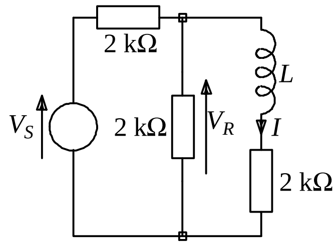

Figure 6

7 The switch in figure 7 has been in position **B** for a long time. At $t = 0$ the switch is switched to position **A**. Find I, and hence V_L, as functions of time.

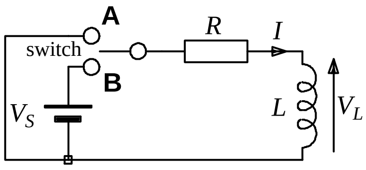

Figure 7

8 The circuit of figure 5 is fed by the step waveform shown. Derive a differential equation relating V_C to t and solve it to find V_C as a function of time.

From your expression for $V_C(t)$, derive an expression describing $I_C(t)$.

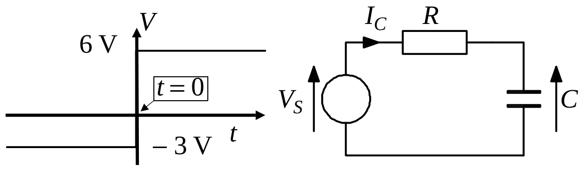

Figure 8

9 In figure 9, the switch has been in position **B** for a long time and is suddenly switched to position **A**. Find an expression that gives I_L as a function of time following the switch from **B** to **A**. Use your result to calculate the voltage across R_1 as a function of time. If $R_1 = 5$ kΩ and $R_2 = 1$ kΩ what is the peak voltage across R_1? [50 V]

After spending a long time at position **A** the switch is returned to position **B**. Find an expression that describes I_L as a function of time following the switch from **A** to **B**.

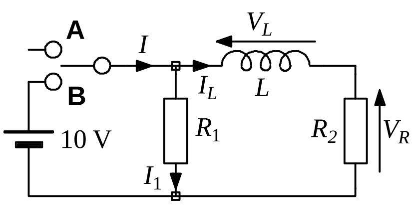

Figure 9

Treat each transient (**B** to **A** and then **A** to **B**) as occurring at $t = 0$, that is, be prepared to move your time origin to the point of interest.