Tutorial Sheet – No 3

(Sinusoidal Supplies, Power Factor and Phasors)

1 A voltage is expressed as:

$$v = 339.4\sin(100\pi t) \text{ Volts}$$

What is the peak amplitude, the rms value and the frequency of this voltage?

(339.4V; 240V_{rms}; 50Hz)

2 A sinusoidal current has an rms value of 100V_{rms} and a frequency of 200Hz. Express this in the form of an equation for the current at time *t*.

($i = 141.4\sin(400\pi t)$)

3 A sinusoidal voltage has peak-to-peak value of 198 Volts and a period of 25ms. Express this in the form of an equation for the voltage at time *t*. What is the rms value of the voltage?

($v = 99\sin(80\pi t)$; 70V_{rms})

4 A sinusoidal voltage of 240Vrms, 50Hz is applied to a series circuit consisting of two components. If the current drawn is given by:

$$i = 14.14\sin\left(100\pi t - \frac{\pi}{6}\right)$$

Find the peak current, rms current, the phase angle, and the impedance, resistance and reactance. What is the value of the component comprising the reactance.

(14.14A; 10A_{rms}; -30° or 30° lagging; 24Ω; 20.78Ω; 12Ω inductive; 38.2mH)

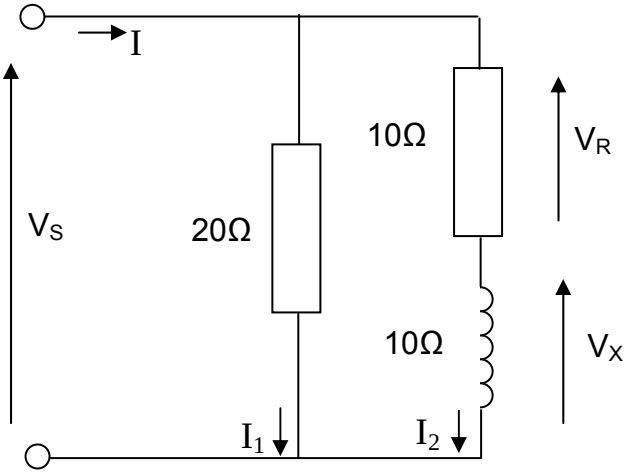
Note: It is normal convention to take the voltage as reference and describe the phase of the current with respect to the voltage i.e. -30° is 30° lagging.

5 For the circuit and supply of question 4, calculate the power factor, the average power using $P = VI\cos\phi$ and the average power using $P = I^2R$.

(0.866 lagging; 2.08kW; 2.08kW)

Note: in the above we can use the rms values of V and I and the power factor to calculate the power in a routine way, but take care, it only works if the supplies are sinusoidal.

6 A sinusoidal voltage of 100V_{rms} 50Hz is applied to the following circuit:



Calculate the rms magnitude and phase of the total input current, I, its power factor, and the total input power to the circuit.

(11.18∠ -26.6°A_{rms}; 0.89 lagging; 1kW)

7 For the circuit in question 6, calculate the rms amplitude and phase of the current and power in each of the resistors – check their sum should equal that from question 6. Calculate the two voltages V_R and V_X and explain why $|V_R| + |V_X| > |V_S|$. Sketch a phasor diagram showing the three voltages and the three currents, taking V_S as reference.

($I_1 = 5 \angle 0° A_{rms}$; $I_2 = 7.07 \angle -45° A_{rms}$; $P_1 = 500W$; $P_2 = 500W$; $V_R = 70.7 \angle -45° V_{rms}$; $V_X = 70.7 \angle 45° V_{rms}$)

8 An inductance of 0.1H and a 10Ω resistor are connected in series to a $100V_{rms}$, 50Hz source. Find the resulting current which flows in the circuit (rms magnitude and phase). What is the power factor of the circuit?

($3.03 \angle -72.3° A_{rms}$; 0.304 lagging)

9 If the components of question 8 are now connected in parallel across the same source, what is the new current and power factor?

($10.5 \angle -17.6° A_{rms}$; 0.95 lagging)

10 A coil draws a current of 4A when 24V d.c. is applied, and for the same current on a 50Hz supply the applied voltage is $40V_{rms}$. Calculate the reactance of the coil, the phase angle, the power factor and the a.c. power supplied.

(8Ω; 53.1°; 0.6 lagging; 96W)

11 A particular electric motor can be represented as a coil of reactance 30Ω and a resistance of 40Ω when connected to a supply at $200V_{rms}$, 50Hz. Calculate the current drawn from the supply. What value of capacitor must be connected in parallel to improve the overall power factor to unity, and what will be the capacitor and supply currents? What value of capacitor is required to improve the power factor to 0.9 and what will be the supply current?

($4A_{rms}$; 38.2μF; $2.4A_{rms}$; $3.2A_{rms}$; 13.5μF; $3.56A_{rms}$)

12 An electrical load consists of an inductor of 10mH in series with a resistor of 3Ω and is connected across a 50Hz alternating supply of 100V peak. Calculate the average power consumed by the load and explain how this power consumption could, in principle be increased, and calculate the maximum value.

(796W, 1.67kW)