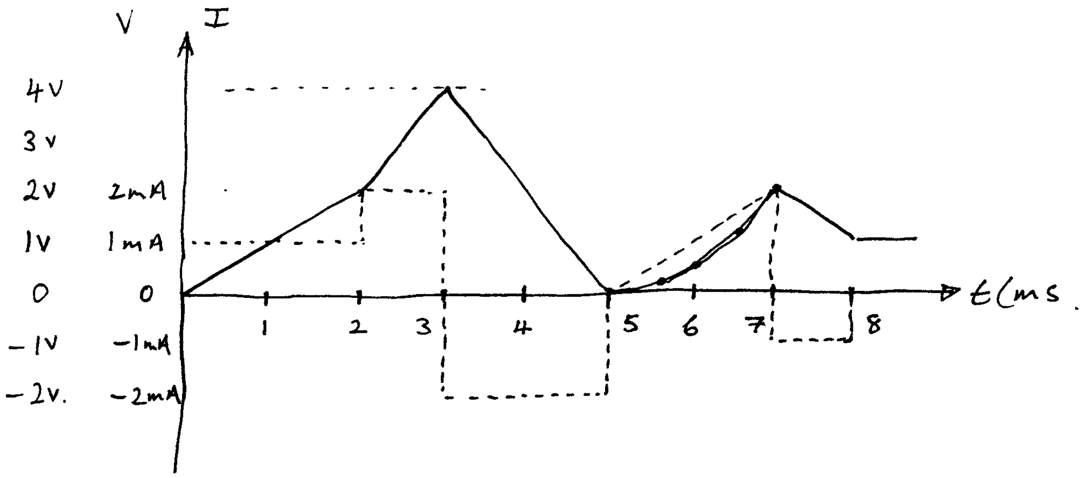
Q1 (i)



In this question, the evolution of charge in the capacitor is the area exposed under the current time graph as t increases. When I is constant, area increases at a constant rate with respect to time. ∴ Charge in and voltage across the capacitor are related by $Q = CV$

or $\Delta Q = C\Delta V$.

The only slightly tricky bit is between $t = 5ms$ + $t = 7ms$. Here the area does not change linearly with time; instead it follows a quadratic relationship. The waveform can be sketched by working out the area between 5ms + 5.5ms (= 0.125μC), between 5ms & 6ms (0.5μC), between 5ms + 6.5ms (= 1.125μC) and between 5ms and 7ms (= 2μC). Alternatively, if you are good at maths, you can say that between 5ms & 7ms $I(t) = t$ (taking 5ms as the time origin).

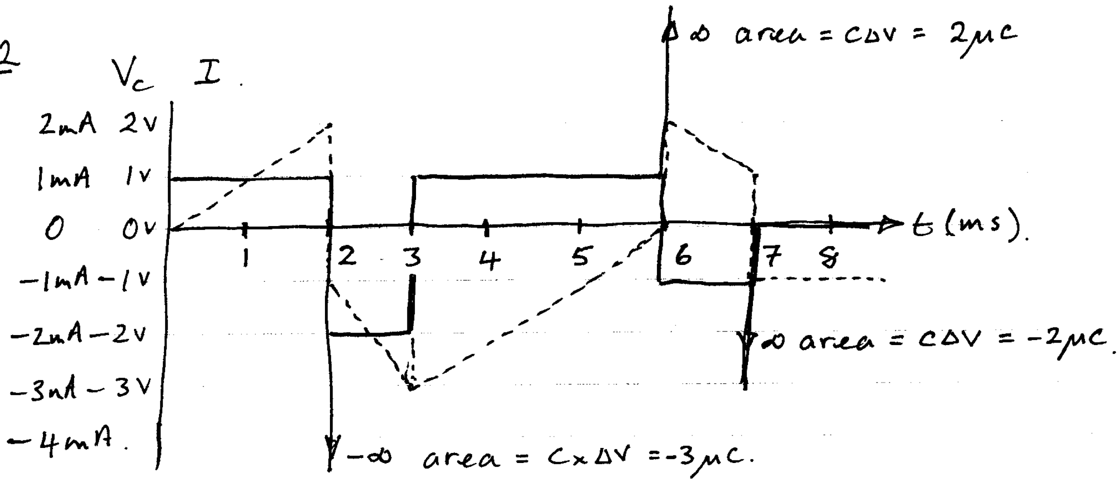
$$\text{so } V_c\Big|_{5ms-7ms} = \frac{1}{C}\int_0^{2ms} t\,dt + V_{initial}$$

$$= \frac{1}{C}\left[\frac{t^2}{2}\right]_0^{2ms} + V_{initial}.$$

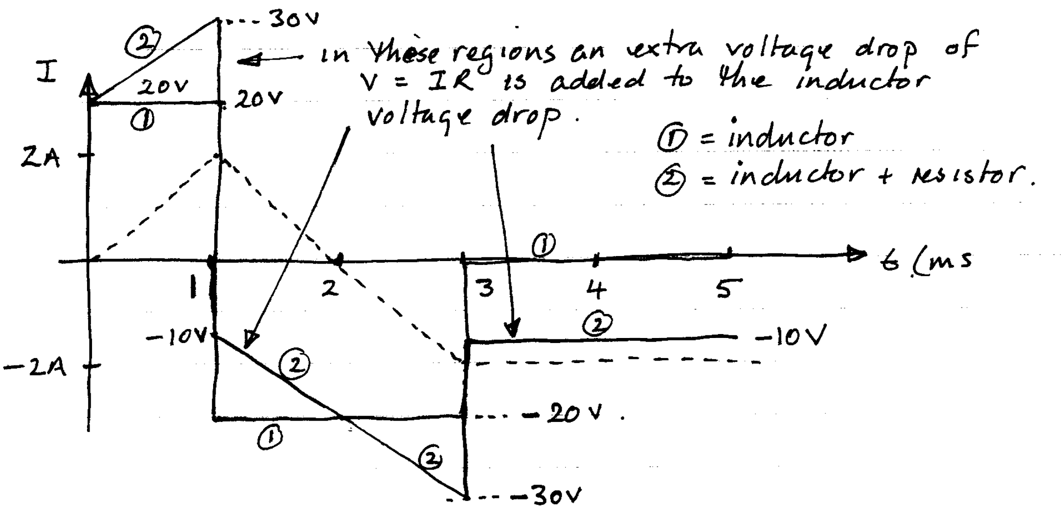
(ii) The total +ve area = 6μC ; total −ve area = 5μC.

net area @ 8ms = 6μC − 5μC = $\underline{\underline{1\mu C}}$.

Q2



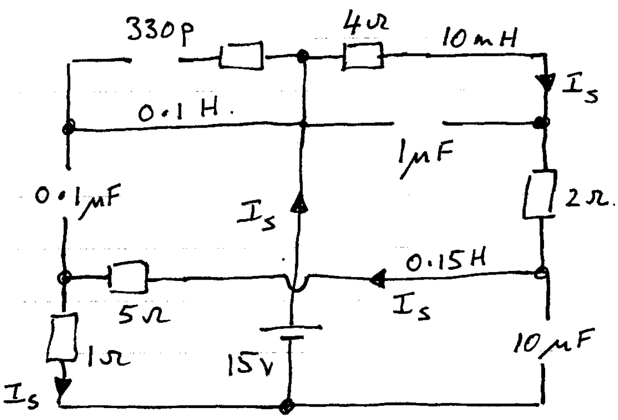
Q3



Q4 The path of I_s is indicated on the diagram. The total resistance of that path is

$4 + 2 + 5 + 1 = 12\Omega$

$\therefore I_s = \frac{15V}{12\Omega} = \underline{1.25A}$



To find the stored energies, the voltage across each C must be worked out and the current through each L must be worked out.....

Consider the inductors ....

10mH and 0.15H carry I_s so their stored energies are ....

$$E_{10mH} = \frac{1}{2} .01 \times 1.25^2 = \underline{7.8\,mJ}.$$

$$E_{0.15H} = \frac{1}{2} 0.15 \times 1.25^2 = \underline{117\,mJ}.$$

The 0.1H inductor carries a current of zero so ...

$$E_{0.10H} = \underline{0\,J}.$$

Consider the capacitors ....

The 330pF capacitor has zero volts across it

$$E_{330pF} = \underline{0\,J}$$

The 1μF capacitor has a voltage drop across it of $I_s \times 4\Omega$, ie 5V

$$E_{1\mu F} = \frac{1}{2} 10^{-6} . 5^2 = \underline{12.5\,\mu J}$$

The 0.1μF capacitor has 15V on its top terminal and $I_s \times 1\Omega$ on its bottom terminal giving a voltage difference of $15 - 1.25 = 13.75V$.

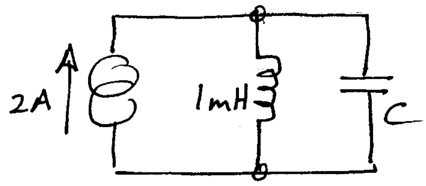
$$E_{0.1\mu F} = \frac{1}{2} 0.1\mu F \times 13.75^2 = \underline{9.45\,\mu J}.$$

The 10μF capacitor has across it the voltage generated by I_s flowing through the $5\Omega + 1\Omega$ resistors

$$E_{10\mu F} = \frac{1}{2} 10^{-5}\, 7.5^2 = \underline{281\,\mu J}.$$

Note that in this circuit the inductors are storing much more energy than the capacitors. If the resistors were all changed to kΩ instead of Ω, the capacitor energy storage would remain the same but the inductor stored energies would be a factor of 10^6 times smaller than they are here – ie nJ instead of mJ. Inductors make very effective energy stores in low voltage high(ish) current applications.

Q5



(i) Since the 2A is steady and $V_L = L\frac{dI}{dt}$, $\frac{dI}{dt} = 0$ and $\underline{V_L = 0}$

(ii) Energy stored in the inductor is $E_{1mH} = \frac{1}{2} \cdot 10^{-3} \cdot 2^2$
$\underline{= 2mJ}$

(iii) There is no voltage across the capacitor so $\underline{E_C = 0\ J}$.

(iv) $E_C = \frac{1}{2} C V^2 = 2mJ$

so for 1nF $V^2 = \frac{2 \times 2mJ}{1nF} = 4 \times 10^6$

$\therefore V_{Cmax} = \sqrt{4 \times 10^6} \underline{= 2kV}$

for 10nF $V^2 = \frac{2 \times 2mJ}{10nF} = 400 \times 10^3$

$V_{Cmax} = \sqrt{400 \times 10^3} = \underline{632\ V}$

for 100nF $V^2 = \frac{2 \times 2mJ}{100nF} = 40 \times 10^3$

$V_{Cmax} = \sqrt{40 \times 10^3} = \underline{200\ V}$