Tutorial Sheet – No 3 Answers

1 The general expression for a sinusoidal waveform is:

$$v(t) = A_{pk} \sin(2\pi ft + \phi)$$

where:

A_{pk} is the peak amplitude
f is the frequency in Hertz
φ is the phase angle

comparing

$$v = 339.4 \sin(100\pi t) \text{ Volts}$$

with the general expression gives:

A_{pk} = **339.4 V**
f = **50 Hz**

and for a sinusoidal waveform the rms value is 1/√2 times the peak value:

$$V_{rms} = \frac{A_{pk}}{\sqrt{2}} = \frac{339.4}{\sqrt{2}} = \mathbf{240V_{rms}}$$

2 Using the general expression from question 1 we can write:

$$i(t) = 100 \times \sqrt{2} \sin(2\pi \times 200 \times t + 0) = \mathbf{141.4 \sin (400\pi t)}$$

3 Remember that the peak-to-peak value is double the peak value. The frequency can be obtained from:

$$f = \frac{1}{T} = \frac{1}{25 \times 10^{-3}} = 40Hz$$

hence:

$$v(t) = \frac{198}{2} \sin(2\pi \times 40 \times t + 0) = \mathbf{99 \sin (80\pi t)}$$

and for a sinusoidal waveform the rms value is 1/√2 times the peak value:

$$V_{rms} = \frac{A_{pk}}{\sqrt{2}} = \frac{99}{\sqrt{2}} = \mathbf{70V_{rms}}$$

4 Comparing the expression for the current with the general expression shown in question 1 we can obtain the following:

I_{pk} = **14.14 V**

$I_{rms} = I_{pk}/\sqrt{2}$ = **10 V_{rms}**

f = **50 Hz**

φ = **−π/6 or −30°**

(negative sign indicates current lags behind the voltage)

Expressing the voltage and current in polar form and taking the voltage as reference gives:

$$V = 240\angle 0^\circ \quad \text{and} \quad I = 10\angle -30^\circ$$

and hence the impedance can be obtained as:

$$Z = \frac{V}{I} = \frac{240\angle 0^\circ}{10\angle -30^\circ} = 24\angle 30^\circ = 20.78 + j12 \quad \Omega$$

since the current is lagging behind the voltage this implies the circuit is inductive.

Comparing the above expression with the general expression for impedance of a circuit containing a series R L combination and looking at real and imaginary parts gives:

$$Z = R + jX_L = R + j2\pi fL$$

and:

$$R = \mathbf{20.78\ \Omega}$$

$$X_L = 12\ \Omega \quad \text{and} \quad L = \frac{X_L}{2\pi f} = \frac{12}{2\pi \times 50} = \mathbf{38.2mH}$$

5 The power factor is the cosine of the phase angle, hence:

$$pf = \cos\phi = \cos 30^\circ = \mathbf{0.866\ lagging}$$

The input power to the circuit is given by:

$$P = V_{rms} I_{rms} \cos\phi = 240 \times 10 \times \cos 30^\circ = \mathbf{2.08kW}$$

Since power can only be dissipated in a resistor then:

$$P = I_{rms}^2 R = 10^2 \times 20.78 = \mathbf{2.08kW}$$

6 The voltage of $100V_{rms}$ appears across each limb of the circuit. It is therefore easier to calculate the current in each limb and sum them to obtain the total current.
For limb 1:

$$Z_1 = 20 + j0 = 20\angle 0^\circ\ \Omega$$

$$I_1 = \frac{V}{Z_1} = \frac{100\angle 0^\circ}{20\angle 0^\circ} = 5\angle 0^\circ = 5 + j0\ \text{A}_{rms}$$

For limb 2:

$$Z_2 = 10 + j10 = 14.14\angle 45^\circ\ \Omega$$

$$I_2 = \frac{V}{Z_2} = \frac{100\angle 0^\circ}{14.14\angle 45^\circ} = 7.07\angle -45^\circ = 5 - j5\ \text{A}_{rms}$$

Summing the two currents gives:

$$I_T = I_1 + I_2 = 5 + j0 + 5 - j5 = 10 - j5 = \mathbf{11.18\angle -26.6^\circ\ A_{rms}}$$

The power factor is the cosine of the phase angle, hence:

$$pf = \cos\phi = \cos 26.6^\circ = \mathbf{0.89\ lagging}$$

The input power to the circuit is given by:

$$P = V_{rms} I_{rms} \cos\phi = 100 \times 11.18 \times 0.89 = \mathbf{1kW}$$

7 Using the values for the limb current calculated in the previous question:

$$P_1 = I_{1rms}^2 R_1 = 5^2 \times 20 = \mathbf{500W}$$

$$P_2 = I_{2rms}^2 R_2 = 7.07^2 \times 10 = \mathbf{500W}$$

$$P_T = P_1 + P_2 = \mathbf{1kW}$$

Calculate the voltages across the components in limb 2:

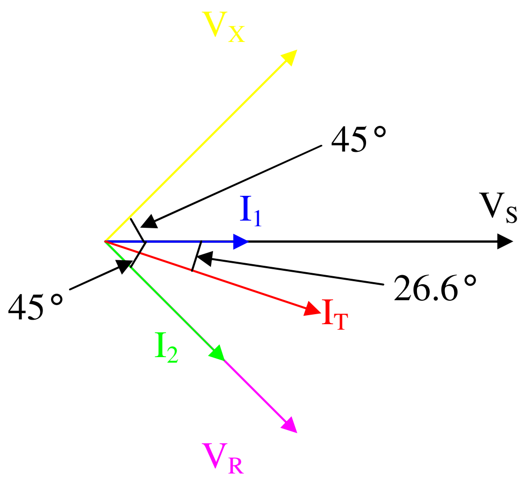
$$V_R = I_2 \times R = 7.07\angle -45^\circ \times 10 = \mathbf{70.7\angle -45^\circ\ V_{rms}}$$

$$V_X = I_2 \times X_L = 7.07\angle -45^\circ \times 10\angle 90^\circ = \mathbf{70.7\angle 45^\circ\ V_{rms}}$$

These quantities are vectors and hence their sum is:

$$V_S = \sqrt{V_R^2 + V_X^2} = \sqrt{70.7^2 + 70.7^2} = \mathbf{100\ V_{rms}}$$

The phasor diagram for the circuit is:



8 For a series combination of a resistance and an inductance:

$$Z = R + j2\pi fL = 10 + j2\pi \times 50 \times 0.1 = 10 + j31.4 = 32.96\angle 72.3°\Omega$$

Now:

$$I = \frac{V\angle 0^\circ}{Z\angle\phi} = \frac{100\angle 0^\circ}{32.95\angle 72^\circ} = \mathbf{3.03\angle -72.3° \ A_{rms}}$$

The power factor is given by:

$$pf = \cos\phi = \cos 72.3^\circ = \mathbf{0.304\ lagging}$$

9 With the two components connected in parallel the supply voltage will appear across each component, therefore it is easiest to calculate the current in R and L and then sum them.
For the resistor:

$$I_R = \frac{V}{Z_R} = \frac{100\angle 0^\circ}{10\angle 0^\circ} = 10\angle 0^\circ = 10 + j0 \ A_{rms}$$

For the inductive reactance:

$$I_X = \frac{V}{Z_X} = \frac{100\angle 0^\circ}{31.4\angle 90^\circ} = 3.18\angle -90^\circ = 0 - j3.18 \ A_{rms}$$

Summing the two currents gives:

$$I_T = I_R + I_X = 10 + j0 + 0 - j3.18 = 10 - j3.18 = \mathbf{10.5\angle -17.6° \ A_{rms}}$$

The power factor is given by:

$$pf = \cos\phi = \cos 17.6^\circ = \mathbf{0.95\ lagging}$$

10 Using Ohm's law for a dc circuit the resistance of the coil can be obtained:

$$R = \frac{V}{I} = \frac{24}{4} = 6\ \Omega$$

Now when an ac supply is used:

$$Z = \frac{V}{I} = \frac{40}{4} = 10\ \Omega$$

Let the reactance of the coil be $X_L\ \Omega = j\omega L$, then:

$$|Z| = \sqrt{R^2 + \omega^2 L^2}$$

or

$$|X_L| = \sqrt{Z^2 - R^2} = \sqrt{100-36} = \mathbf{8\ \Omega}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{8}{6}\right) = \mathbf{53.1°}$$

The power supplied is obtained from:

$$P = VI\cos\phi = 40\times4\times\cos 53.1 = \mathbf{96\ W}$$

Alternatively power dissipated in the real part of impedance (resistance):

$$P = I^2R = 4^2\times 6 = \mathbf{96\ W}$$

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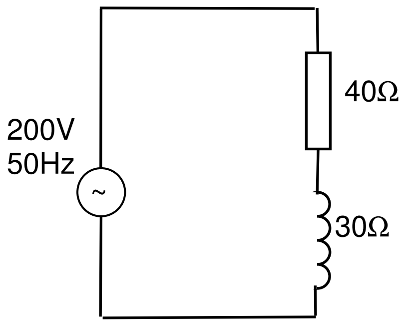


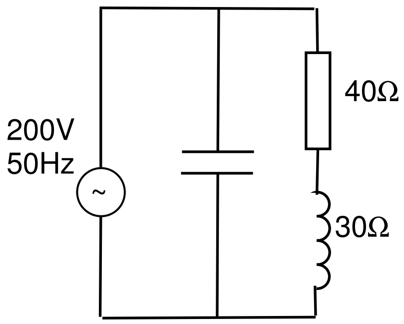
The impedance of the motor is given by:

$$Z = R + jX_L = 40 + j30 = 50\angle 36.87^\circ\ \Omega$$

The current can then be found from:

$$I = \frac{V}{Z} = \frac{200\angle 0^\circ}{50\angle 36.87^\circ} = \mathbf{4\angle{-36.87}° = 3.2 - j2.4\ A_{rms}}$$

A capacitor is now connected in parallel:



When the power factor is unity, then the phase angle is zero and the imaginary component of the supply current is zero. Therefore the capacitor current must be equal to **j2.4** or **2.4∠90° A_{rms}**. Hence:

$$|X_C| = \frac{200}{2.4} = 83.33\ \Omega$$

and:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi f X_C} = \frac{1}{2\pi\times 50\times 83.33} = \mathbf{38.2\mu F}$$

The current drawn from the supply is then:

$$I_T = I_M + I_C = 3.2 - j2.4 + 0 + j2.4 = 3.2 + j0 = \mathbf{3.2\angle 0°\ A_{rms}}$$

For an overall power factor of 0.9, the real power remains unchanged, therefore:

$$VI_{TNEW}\times 0.9 = VI_{TOLD}\times 1$$

hence:

$$I_{TNEW} = I_{TOLD} \times \frac{1}{0.9} = 3.2 \times \frac{1}{0.9} = \mathbf{3.56\ A_{rms}}$$

and since the power factor is 0.9 the current may be written as:

$$I_{TNEW} = 3.56\cos\phi - j3.56\sin\phi = 3.56 \times 0.9 - j3.56 \times \sqrt{1-0.9^2} = 3.2 - j1.55 \text{ A}_{rms}$$

The capacitor current must therefore be equal to j2.4 – j1.55 = j0.85 A_{rms}.

$$|X_C| = \frac{200}{0.85} = 253.3\ \Omega$$

and:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 253.3} = \mathbf{13.5\mu F}$$

$$P = I^2R = 4^2 \times 6 = \mathbf{96\ W}$$

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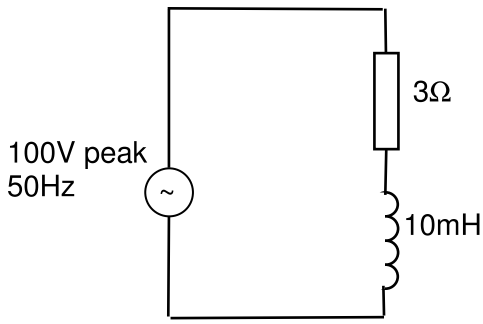

First we need to obtain the rms current:

$$Z_T = R + j\omega L = 3 + j2\pi \times 50 \times 0.01 = 3 + j3.14 = 4.34\angle 46.3°\ \Omega$$

The rms value of the voltage is related to the peak value by:

$$V_{RMS} = \frac{V_{PK}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7\ V_{rms}$$

If we take the supply voltage as reference then:

$$I_{RMS} = \frac{V_{RMS}}{Z} = \frac{70.7\angle 0°}{4.34\angle 46.3°} = 16.3\angle -46.3°\ A_{rms}$$

The input power to the circuit is given by:

$$P = V_{rms}I_{rms}\cos\phi = 70.7 \times 16.3 \times \cos 46.3° = \mathbf{796\ W}$$

The power transfer could be increased by adding a series capacitor to resonate at 50Hz with the inductor. At resonance this series RCL circuit will appear purely resistive:

$$I_{RMS} = \frac{V_{RMS}}{R} = \frac{70.7\angle 0°}{3\angle 0°} = 23.6\angle 0°\ A_{rms}$$

and the power dissipated in the 3Ω resistor would now be:

$$P = I_{RMS}^2 R = 23.6^2 \times 3 = \mathbf{1.67kW}$$