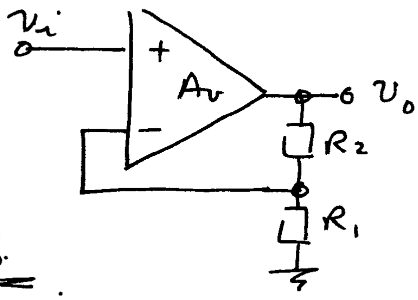
EEE123 Problem Sheet Solutions

Operational Amplifiers

Q1 (i) if $A_v \to \infty$

$v_i = v^+ = v^- = v_0 \dfrac{R_1}{R_1+R_2}$



or $\dfrac{v_0}{v_i} = \dfrac{R_1+R_2}{R_1}$

$= \dfrac{75k\Omega + 15k\Omega}{15k\Omega} = \underline{\underline{6}}$.

[You can quote $\dfrac{v_0}{v_i} = \dfrac{R_1+R_2}{R_1}$ if you can remember it]

Q2 if A_v finite ....

$v^+ = v_i$ (connected by wire)

$v^- = v_0 \dfrac{R_1}{R_1+R_2}$ (by potential division).

$v_0 = A_v(v^+ - v^-)$ (op-amp eq.)

putting v^+ + v^- into op-amp equation ...

$v_0 = A_v\left(v_i - v_0 \dfrac{R_1}{R_1+R_2}\right)$

or $v_0\left[\dfrac{1}{A_v} + \dfrac{R_1}{R_1+R_2}\right] = v_i$

or $\dfrac{v_0}{v_i} = \dfrac{1}{\dfrac{1}{A_v} + \dfrac{R_1}{R_1+R_2}}$.

a) when $R_2 = 9R_1$, ideal gain = 10.

actual gain $= \dfrac{1}{\dfrac{1}{10^5} + \dfrac{1}{10}} = 9.999$

$\therefore$ error $= \underline{\underline{-0.01\,\%}}$.

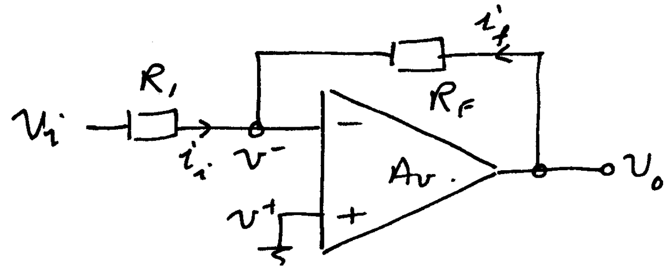
b) when $R_2 = 299 R_1$, ideal gain $= 300$

$$\text{actual gain} = \frac{1}{\frac{1}{10^5} + \frac{1}{300}} = 299.1$$

$$\text{error} = \underline{\underline{-0.3\,\%}}.$$

(both these errors are very small).

Q3



Summing currents at v^- node...

$i_i + i_f = 0$ (since op-amp input current ≈ 0).

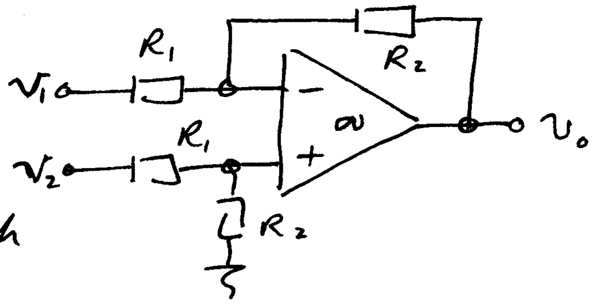
$$\frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_F} = 0.$$

but since $A_v \Rightarrow \infty$, $v^+ = 0 \approx v^-$

$$\therefore \frac{v_i}{R_1} + \frac{v_o}{R_F} = 0 \quad \text{or} \quad \underline{\underline{\frac{v_o}{v_i} = -\frac{R_F}{R_1}}}.$$

The virtual earth node is the inverting input node. It exists because $A_v \Rightarrow \infty$ so $v^+ \approx v^-$ and since v^+ is connected to 0V, v^- must always be very close to 0V.

Q4



v_o due to v_1 ... ie with v_2 grounded...

$v^+ = 0V$ because both v_2 and v_1 grounded

$$\therefore v_o = -\frac{R_2}{R_1} \cdot v_1$$

v_o due to v_2 with v_1 grounded....

v^+ is now a potentially divided version of v_2 .... $v^+ = v_2 \cdot \frac{R_2}{R_1 + R_2}$ and the amplifier operates as a non-inverting amplifier on v^+,

so $$v_o = v^+ \frac{R_1 + R_2}{R_1}$$

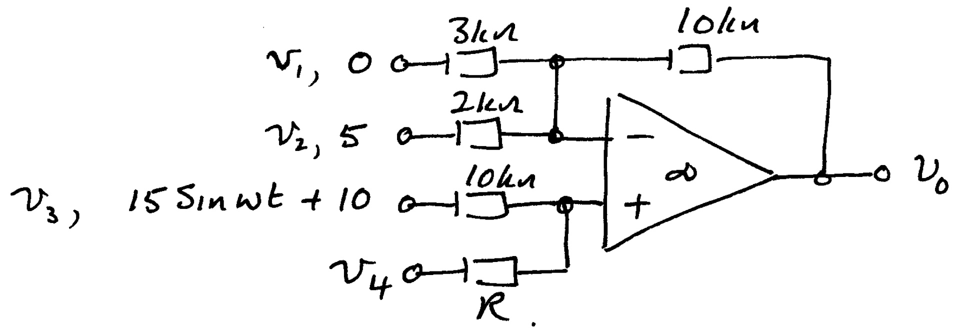
$$= v_2 \cdot \frac{R_2}{R_1 + R_2} \cdot \frac{R_1 + R_2}{R_1} = v_2\, {R_2}/{R_1}$$

$$\therefore v_{o\,TOT} = v_{o(v_1)} + v_{o(v_2)}$$

$$= -{R_2}/{R_1}\, v_1 + {R_2}/{R_1}\, v_2$$

$$= \frac{R_2}{R_1}\underline{(v_2 - v_1)}$$

Q5



Start with ac...

ground v_1, v_2, v_4 and the dc part of v_3.

v^+ is $15 \sin wt \times \frac{R}{10k + R}$.

v_o is $v^+ \cdot \frac{10k + 3k\Omega \| 2k\Omega}{3k\Omega \| 2k\Omega}$ — ie a non-inverting amp operates on v^+

So $v_{o\,ac} = \cancel{5} \cancel{\sin wt} = \overset{3}{\cancel{15}} \cancel{\sin} wt \frac{R}{10k + R} \cdot \frac{10k + 3k \| 2k}{3k \| 2k}$

$$1 = 3 \cdot \frac{R}{10k + R} \cdot \frac{11 \cdot 2k\Omega}{1 \cdot 2k\Omega}$$

or $\underline{R = 370\,\Omega}$.

considering the d.c. bit (ignore the a.c.)

$$v_{o}(v_1) = v_1 . (-) \frac{10k\Omega}{3k\Omega}$$ (since v_3 + v_4 are zero, v^+ is zero + v^- is virtual earth.)

$$v_o(v_2) = v_2 (-) \frac{10k\Omega}{2k\Omega}$$ (for same reasons).

$$v_o(v_3) = \underbrace{v_3 . \frac{370}{10k + 370}}_{v^+} \cdot \underbrace{\frac{10k + 1.2k}{1.2k}}_{\text{non-inv gain}} = 0.333\, v_3 .$$

$$v_o(v_4) = v_4 . \frac{10k}{10k + 370} . \frac{10k + 1.2k}{1.2k} = 9\, v_4$$

$$\therefore v_{oT} = -\frac{10}{3} v_1 - 5 v_2 + 0.33\, v_3 + 9\, v_4$$

$$= 0 - 25 + 3.3 + 9\, v_4 = 0$$

dc component of 0 required.

$$\therefore v_4 = \frac{21.7}{9} = \underline{2.4\,V}$$

(ii) this question is really asking what value of v_1 will put a dc offset of +5V and −5V on the output. The + v_1 will be the same as the − v_1 in magnitude terms so lets consider only the + v_1 term

We know the v_2, v_3 + v_4 contributions sum to zero so...

$$v_o = -\frac{10}{3} v_1 = 5$$

or $v_1 = -\frac{15}{10} = 1.5V$

so $v_1 = \underline{\underline{\pm 1.5V}}$ to make peak v_o reach −10 + +10 respectively.