Data Provided: List of Useful Formulae

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2008-2009 (2 hours)

Signals and Systems

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. **a.** Consider an amplitude modulation system with a modulating signal $m(t) = A_m \cos(\omega_m t)$ and a carrier signal $c(t) = A_c \cos(\omega_c t)$, where $\omega_c >> \omega_m$.

i) Sketch and label the modulated signal $s(t) = A_c A_m \cos(\omega_c t)\cos(\omega_m t)$.

ii) Write down the corresponding expression for $s(t)$ in the frequency domain, $S(\omega)$.

iii) Sketch and label the corresponding magnitude spectrum, $|S(\omega)|$.

iv) The modulation scheme above is called double sideband-suppressed carrier. State a major advantage and a drawback of this scheme. **(8)**

b. In a different modulation scheme called double sideband, the modulated signal is given by $x(t) = \left[A_o + m(t)\right] c(t)$, where the condition, $A_o + m(t) > 0$, is used.

i) Obtain the corresponding expression for the $x(t)$ in the frequency domain, $X(\omega)$, assuming that $A_c = 1$.

ii) Sketch and label $|X(\omega)|$.

iii) State a major drawback of using this modulation scheme. **(5)**

1. **c.** An envelope detector depicted in Figure 1.1, can be used to demodulate the signal $x(t)$ in part (b). The capacitance voltages during charging and discharging are described by $e_c(t) = A_c[1 - \exp(-t/R_sC)]$ and $e_d(t) = A_d\exp(-t/R_lC)$, respectively. Assuming R_s = 75Ω, R_l = 10kΩ, $\omega_c = 2\pi\times10^5$ rad/s and ω_o = $0.01\omega_c$, work out a suitable value for C.

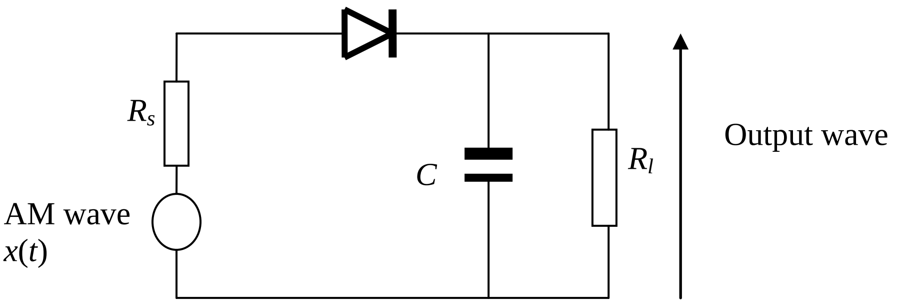

Figure 1.1

(7)

2. **a.** Consider the RC circuit depicted in Figure 2.1. $x(t)$ is the input signal while $y_0(t)$ and $y_1(t)$ are the voltages across the capacitor C and the resistor R, respectively.

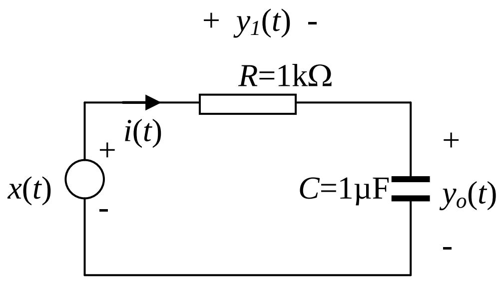

Figure 2.1

i) Obtain the transfer function and sketch the pole-zero plot if $y_o(t)$ is taken as the output signal.

ii) Obtain the transfer function and sketch the pole-zero plot if $y_1(t)$ is taken as the output signal.

iii) State whether the systems in parts (i) and (ii) exhibit low pass, band pass or high pass characteristics? **(8)**

b. Obtain the time-domain impulse responses for the systems in parts (i) and (ii). **(5)**

c. The input $x(t)$ and output $y(t)$ of a system are related by $\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 8y(t) = x(t)$. Use Laplace transforms to determine the s domain transfer function and time domain impulse response of the system assuming that $x(0) = y(0) = \left.\frac{dy(t)}{dt}\right|_{t=0} = 0$.

(7)

3. a.

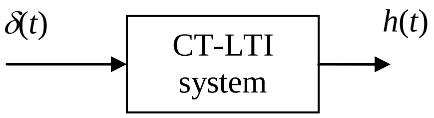

Figure 3.1

Consider a continuous time (CT) Linear Time-invariant (LTI) system with an impulse response $h(t)$ as shown in Figure 3.1. Show that the response of the LTI system to an input signal $x(t)$ is given by $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$. **(4)**

b.

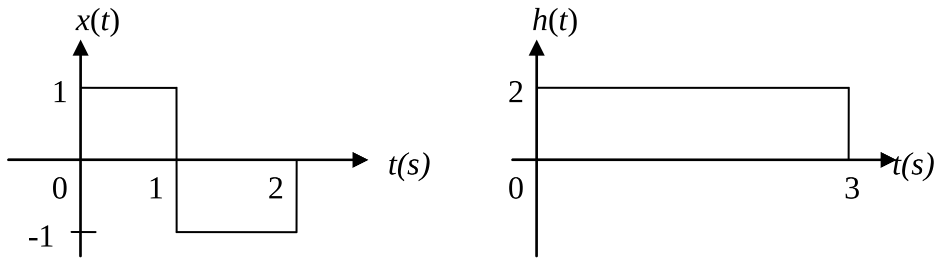

Figure 3.2

Using the graphical method, obtain the response $y(t)$ of an LTI system, if the input signal $x(t)$ and the impulse response $h(t)$ are as shown in Figure 3.2. Sketch and label $y(t)$. **(10)**

c. Compute the response of an LTI discrete system if the input and impulse response are described by $x[n] = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & otherwise \end{cases}$ and $h[n] = \begin{cases} 1/n, & 1 \le n \le 4 \\ 0, & otherwise \end{cases}$, respectively. **(6)**

4. **a.**

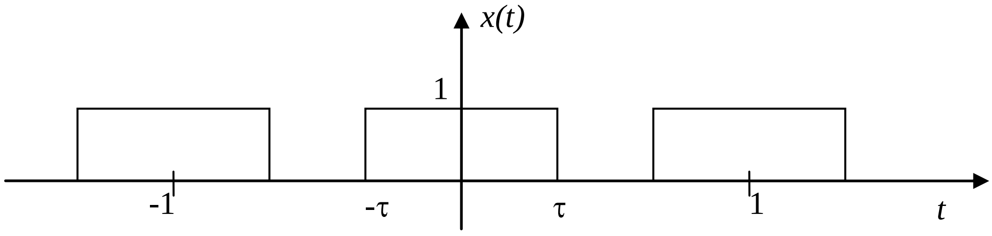

Figure 4.1

Show that the trigonometric Fourier series representation of the signal *x(t)* shown in Figure 4.1 is given by $x(t) = 2\tau + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin(2n\pi\tau)\cos(2n\pi t)$.

Assuming $\tau = 1/4$ and using up to the 3rd harmonic, write down an expression for the approximated periodic squarewave. **(8)**

b.

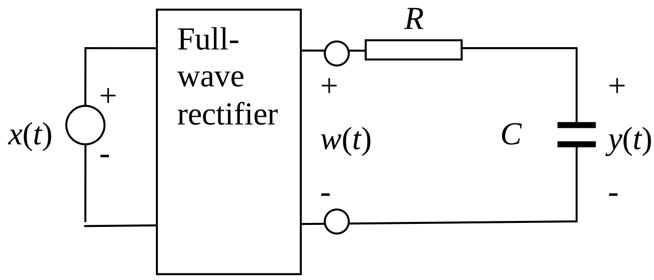

Figure 4.2

A simple dc power supply is illustrated in Figure 4.2. The output of the fullwave rectifier $w(t) = |x(t)|$ is cascaded to a RC circuit to produce an output voltage, $y(t)$. The frequency domain signal is given by $W(\omega) = \frac{4}{\pi} \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{(1-4k^2)} \delta(\omega - 100\pi k)$ where k is the harmonic number and $\delta(\omega)$ is a unit impulse function.

i) Derive an expression for the transfer function of the system, $Y(\omega)$. **(3)**

ii) Assuming that the 2nd and higher harmonics are negligible, show that the output is given by $y(t) = \frac{2}{\pi^2} + \frac{2}{3\pi^2}\left[\frac{e^{j100\pi t}}{1 + j100\pi RC} + \frac{e^{-j100\pi t}}{1 - j100\pi RC}\right]$. **(6)**

iii) The ripple in the output voltage of the circuit in Figure 4.2 can be kept small by using appropriate RC time constant. Suggest a range of values for RC that will ensure the ripple in $y(t)$ is < 2mV. **(3)**

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