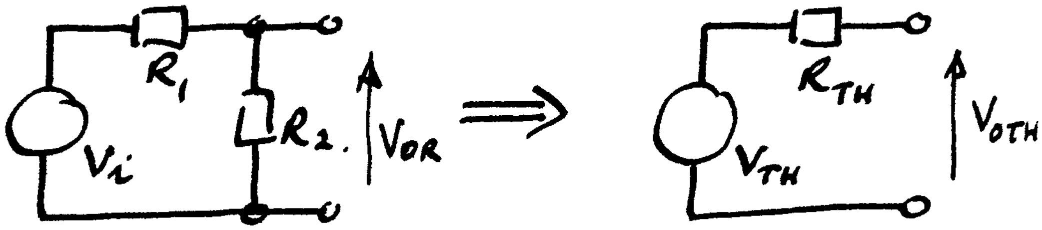
EEE103 / EEE121 / EEE141 Problem Sheet Solutions

Background Knowledge

Q1



real circuit Thevenin equivalent.

Need to find the R_{TH} + V_{TH} that will make the Thevenin equivalent indistinguishable from the real cct .... ie

output voltage of real = output voltage thevenin
short circuit output current of real = short circuit output current of thevenin

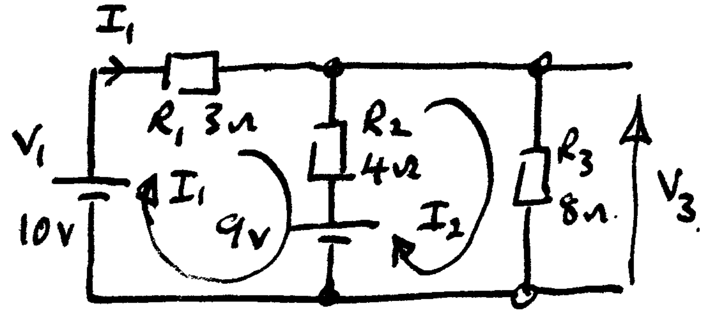
$$V_{OR} = V_i \frac{R_2}{R_1+R_2} = V_{OTH} \text{ for equivalence } = V_{TH}$$

$$I_{SCR} = V_i / R_1 = V_{TH} / R_{TH} \text{ for equivalence}$$

$$\text{so } R_{TH} = \frac{V_{TH} R_1}{V_i} = \frac{R_1}{V_i} \cdot V_i \frac{R_2}{R_1+R_2} = R_1 \| R_2.$$

$$\text{ie } V_{TH} = V_i \frac{R_2}{R_1+R_2} \text{ and } R_{TH} = \frac{R_1 R_2}{R_1+R_2}.$$

Q2. Firstly using loops:



$$10 = I_1 R_1 + (I_1 - I_2) R_2 + 9 \quad \text{———} \quad (1)$$

$$9 = R_2(I_2 - I_1) + R_3 I_2 \quad \text{——— (2)}$$

$$V_3 = R_3 I_2 \quad \text{——— (3)}$$

expanding (1)

$$10 = 3I_1 + 4I_1 - 4I_2 + 9$$

or $1 = 7I_1 - 4I_2$ ——— (4)

expanding (2)

$$9 = 4I_2 - 4I_1 + 8I_2$$

or $9 = 12I_2 - 4I_1$ ——— (5)

eliminating I_2 from (4) + (5) gives.

$$9 = 12\left[\frac{7I_1 - 1}{4}\right] - 4I_1 = 21I_1 - 3 - 4I_1$$

or $I_1 = 12/17 = \underline{\underline{0 \cdot 706 A}}$.

using (4), $1 = 7 \times \frac{12}{17} - 4I_2$ or $I_2 = 0.985A$.

and using (3), $V_3 = 8I_2 = \underline{\underline{7 \cdot 88 V}}$.

Using superposition to find V_3 .....

$$V_{3(10v)} = V_1 \cdot \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} = 10 \cdot \frac{32/12}{3 + 32/12} = \frac{10 \cdot 8/3}{17/3}$$

$$= 80/17 \text{ V}.$$

$$V_{3(9v)} = V_2 \frac{R_1 \| R_3}{R_2 + R_1 \| R_3} = 9 \cdot \frac{24/11}{4 + 24/11} = 9 \cdot \frac{24/11}{68/11}$$

$$= 54/17 \text{ V}$$

$$V_{3\,TOT} = V_{3(10)} + V_{3(9)} = \frac{134}{17} \text{V} = \underline{\underline{7 \cdot 88 \text{ V}}}.$$

using superposition to find I_1 ....

$$I_{1(10)} = V_1/(R_1 + R_2 \| R_3) = \frac{10}{3 + 8/3} = \frac{30}{17} A.$$

$$I_{1(9)} = -V_3/R_1 = -\frac{1}{R_1} \cdot \frac{9 \cdot 24/11}{4 + 24/11} = -\frac{1}{3} \cdot \frac{9 \cdot 6}{17}$$

$$= -18/17.$$

$$\therefore I_{1\,TOT} = I_{1(10)} + I_{1(9)} = \frac{30}{17} - \frac{18}{17} = \underline{\underline{0.706\,A}}.$$

To find Norton equivalent....

(i) put short cct across output terminals and calculate current through it .....

$$I_{SC(10V)} = \frac{10}{3} = 3.33\,A.$$

$$I_{SC(9V)} = \frac{9}{4} = 2.25\,A.$$

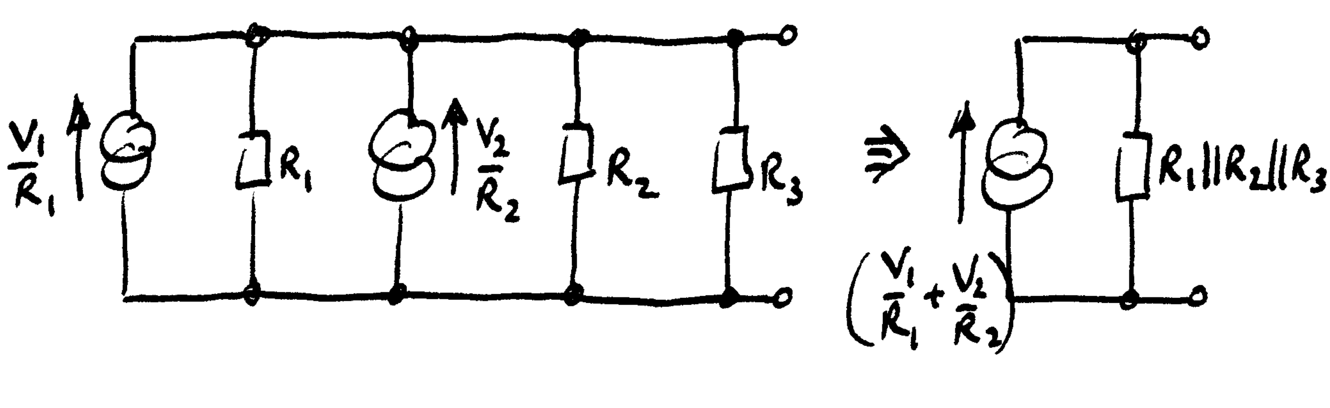
$$\therefore I_{SC\,TOT} = \frac{10}{3} + \frac{9}{4} = \frac{67}{12} = \underline{5.58\,A}.$$

$\therefore$ The Norton current source is 5.58 A.

(ii) The Norton parallel resistance is V_3/I_N.

$$= \frac{134/17}{67/12} = \frac{12 \times 134}{67 \times 17} = \frac{24}{17} = \underline{\underline{1.41\,\Omega}}.$$

One could also have transformed the limbs of the original circuit... and then summed ....



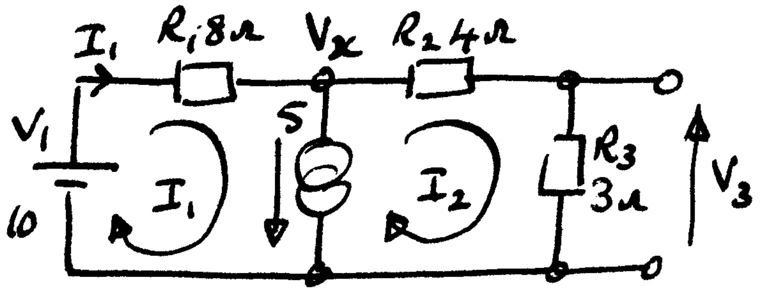
to find the value of V_2 that would make $I_1 = 0$, one can make use of the superposition process at the top of page 3 with 9V replaced by V_2 ....

$$I_{1TOT} = I_{1(10)} + I_{1(V_2)} = \frac{30}{17} - V_2 \cdot \frac{2}{17}$$

and $I_{1(TOT)} = 0$ is required .... so

$$\frac{30}{17} - \frac{V_2 . 2}{17} = 0 \quad \text{or} \quad V_2 = \frac{30}{2} = \underline{\underline{15V}}$$

Q3 Using loops... it is necessary to define a variable V_x for the unknown node voltage ...



$$10 = I_1 R_1 + V_x \quad \text{——— (1)}$$
$$V_x = I_2 R_2 + I_2 R_3 \quad \text{——— (2)}$$
$$I_1 - I_2 = 5 \quad \text{——— (3)}$$

eliminating V_x from (1) and (2) ...

$$10 = I_1 R_1 + I_2 R_2 + I_2 R_3$$
$$= 8 I_1 + 7 I_2$$

and using (3) to eliminate I_2 ...

$$= 8 I_1 + 7(I_1 - 5) = 15 I_1 - 35$$

$$\text{or } I_1 = \frac{35 + 10}{15} = \frac{45}{15} = \underline{\underline{3A}}$$

using (3), $I_2 = -5 + I_1 = -2A$

$$\therefore V_3 = I_2 R_3 = \underline{\underline{-6V}}$$

Using superposition to find I_1 ...

$$I_{1(10)} = {}^{10}/_{(8+4+3)} = {}^{2}/_{3}\,A.$$

$$I_{1(5A)} = -\frac{V_x}{R_1} = -\frac{(-5(R_2+R_3)\|R_1)}{R_1}$$

$$= \frac{5\,(R_2+R_3)}{R_1+R_2+R_3} = \frac{5\times 7}{15} = \frac{7}{3}.$$

$$\therefore I_{1TOT} = I_{1(10)} + I_{1(5A)} = \frac{2}{3}+\frac{7}{3} = \underline{\underline{3A}}.$$

to find V_3 ...

$$V_{3(10)} = 10 \cdot \frac{R_3}{R_1+R_2+R_3} = 10 \cdot \frac{3}{15} = 2V.$$

$$V_{3(5)} = V_x \cdot \frac{R_3}{R_2+R_3} = -5(R_2+R_3)\|R_1 \cdot \frac{R_3}{R_2+R_3}$$

$$= -5 \cdot \frac{56}{15} \cdot \frac{3}{7} = -8V.$$

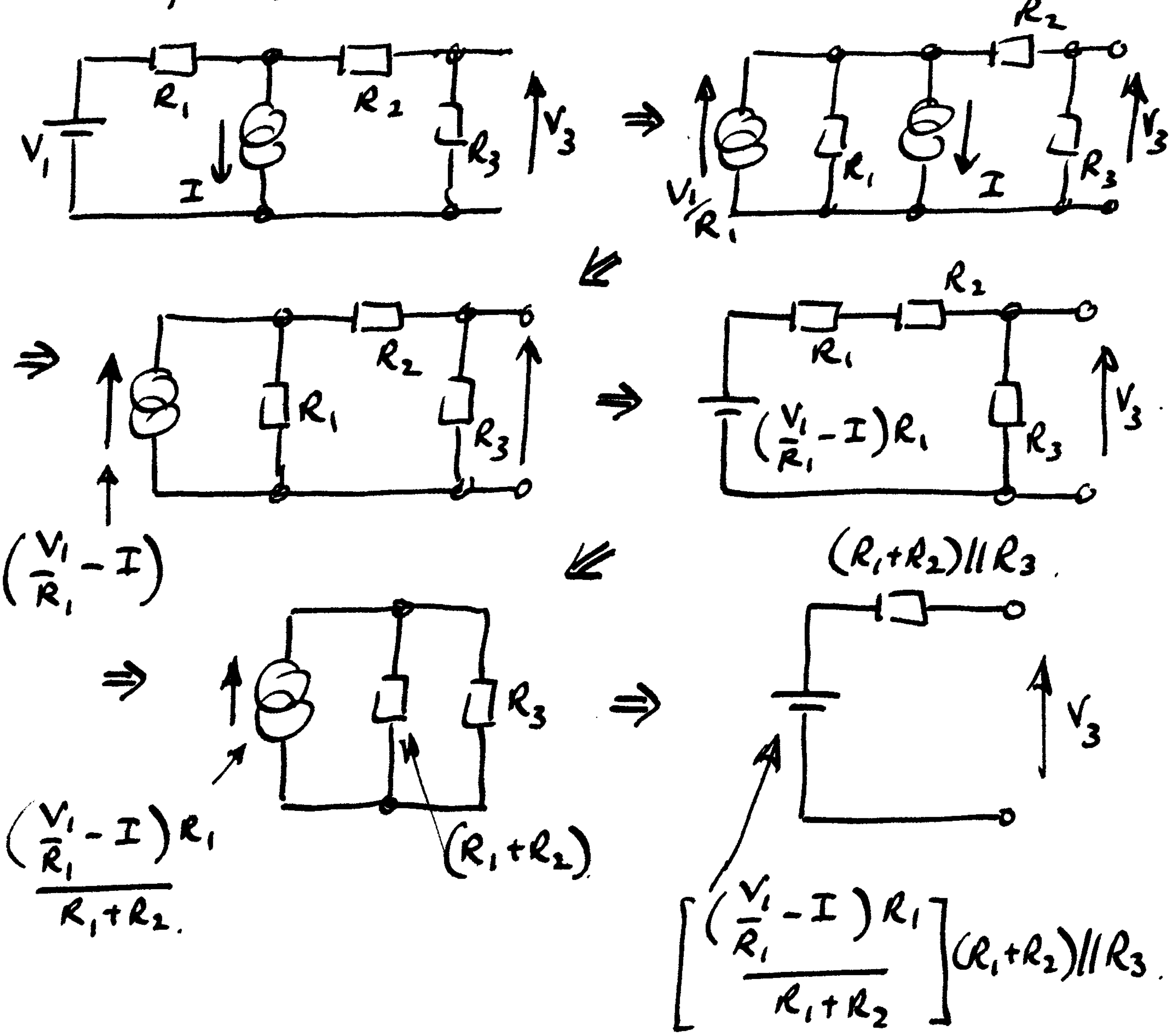
$$V_{3\,TOT} = V_{3(10)} + V_{3(5A)} = 2-8 = \underline{\underline{-6V}}.$$

For the Thevenin equivalent circuit, V_{TH} is V_3 (by definition) and to find R_{TH}, either look into V_3 terminals with V_1 replaced by 0Ω and I by $\infty\ \Omega$ and work out resistance or work out the current that would flow through a short circuit placed across the V_3 terminals and use $R_{TH} = V_3/I_{sc}$.

$$V_{TH} = \underline{\underline{-6V}}.$$

$$R_{TH} = R_3 \,\|\, (R_1+R_2) = \frac{36}{15} = \underline{\underline{2.4\,\Omega}}.$$

or yet another possibility is to do successive transformations....



This method is a bit laborious but gives excellent transformation practice.

To find I that will make $V_3 = 0$, use superposition V_3 approach on page 5 and replace 5 by I ...

$$V_{3(10)} + V_{3(I)} = 0 = 2V + \frac{\left(-I(R_2+R_3)\|R_1\right)R_3}{R_2+R_3}$$

or $$2 = I\,\frac{R_1R_3}{R_1+R_2+R_3} = I.\,\frac{24}{15}$$ $\therefore$ $\underline{\underline{I = 1.25A}}$.

Q4 (i) everything has units of current except for the I_5/R_6 term

(ii) the common unit is volts. The $I_4(R_3+1)$ and the R_3 terms are wrong.

(iii) is correct; both sides have units of R.

(iv) the unit on both sides is V. All the $j\omega$ terms are dimensionless (and hence correct) except for the last one, $j\omega C_2 R_1 R_2$, that has units of R and is incorrect. [remember ω has units of $1/\text{time}$, CR has units of time, j is dimensionless.]

(v) the $j\omega(C_1+C_2)R^2$ term has units of R and should be dimensionless.

(vi) the $j\omega L$ term has units of R and should be dimensionless.

(vii) is correct; Z is impedance with units of R, each term in numerator of right hand side has units of R, each term in r.h.s. denominator is dimensionless.