absorption in lossy materials (continued)

$$\underline{E} = E_0\, \underline{e_y}\; e^{j(\omega t - \tilde{k}x)}$$
$$\underline{H} = H_0\, \underline{e_z}\; e^{j(\omega t - \tilde{k}x)}$$

where $\tilde{k} = \omega \sqrt{L^* C^*} \sqrt{1 - j \frac{G^*}{\omega C^*}}$

with $\omega = 2\pi f$ $\quad \sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}$ $\quad G^* = \sigma \frac{w}{d}$, $C^* = \varepsilon_0 \varepsilon_r \frac{w}{d}$

1. case : weak absorption : $\sigma \ll \omega \varepsilon_0 \varepsilon_r$

Use $\sqrt{1 - jx} \approx 1 - \frac{1}{2} jx$

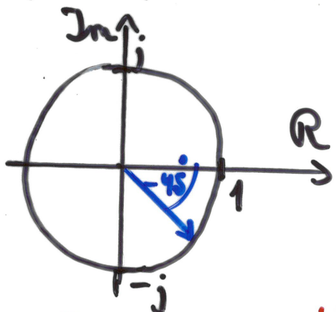
$$\Rightarrow \boxed{\tilde{k} \approx} \underbrace{\omega \sqrt{L^* C^*}}_{k_0} \left(1 - j \underbrace{\frac{\sigma}{2} \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}}_{\beta}\right) = \boxed{k_0 (1 - j\beta)}$$

β , called absorption length

$$\rightarrow \quad \underline{E} = E_0\, \underline{e_y}\; e^{j(\omega t - k_0 x)}\; e^{-\beta x}$$
$$\underline{H} = H_0\, \underline{e_z}\; e^{j(\omega t - k_0 x)}\; e^{-\beta x}$$
$$\underline{S} = \underline{E} \times \underline{H} \quad \text{with} \quad S = EH \propto e^{-2\beta x}$$

2. case : strong absorption : $\sigma \gg \omega \varepsilon_0 \varepsilon_r$

Use $\sqrt{1 - jx} \approx \sqrt{-jx} = \underbrace{\sqrt{-j}}_{\left(e^{-j\frac{\pi}{2}}\right)^{1/2} = e^{-j\frac{\pi}{4}} = \frac{1-j}{\sqrt{2}}} \cdot \sqrt{x}$



$$\Rightarrow \boxed{\tilde{k} \approx} \omega \sqrt{\cancel{\varepsilon_0 \varepsilon_r} \mu_0 \mu_r} \sqrt{\frac{\sigma}{\omega \cancel{\varepsilon_0 \varepsilon_r}}}\; e^{-j\pi/4}$$

$$= \underbrace{\sqrt{\frac{\omega \sigma \mu_0 \mu_r}{2}}}_{1/\delta} (1 - j) = \boxed{\frac{1}{\delta} (1 - j)}$$

$\Rightarrow$ $\underline{E} = E_0\, \underline{e}_y\, e^{j(\omega t - \frac{x}{\delta})}\, e^{-\frac{x}{\delta}}$

$\underline{H} = H_0\, \underline{e}_z\, e^{j(\omega t - \frac{x}{\delta})}\, e^{-\frac{x}{\delta}}$

$\underline{S} = \underline{E} \times \underline{H}$ with $S = EH \propto e^{-2\frac{1}{\delta}x}$

power decay length is $\frac{\delta}{2}$ for $1/e$ decay of S

and new wavelength is $\lambda = \frac{2\pi}{\mathrm{Im}(\tilde{k})} = 2\pi\delta$

$$\boxed{\delta = \sqrt{\frac{2}{\omega\sigma\mu_0\mu_r}}}$$

is called skin or penetration depth as the $1/e$ power decay is within $\frac{\delta}{2} = \frac{\lambda}{4\pi} \approx 0.1\,\lambda$

Note $\delta \propto \frac{1}{\sqrt{f}}$ becomes very small at high frequencies; i.e. the power (current) travels only close to the surface in a conductor (skin effect).

numerical examples:

for copper (Cu), $\sigma = 6 \cdot 10^7\,(\Omega m)^{-1}$ @ 50 Hz

$\Rightarrow$ $\delta = \sqrt{\frac{2}{2\pi f \cdot \sigma \cdot \mu_0\mu_r}} \approx 9\,\mathrm{mm}$

but @ 1 MHz: $\delta \approx 70\,\mu m$

@ 30 GHz: $\delta \approx 0.4\,\mu m$

for sea water, $\sigma = 4\,\mathrm{S/m}$, $\varepsilon_r = 80$ @ 100 MHz

i) check $\frac{\sigma}{\omega\varepsilon_0\varepsilon_r} \approx 9 \gg 1$ (just strongly absorbing!)

$\Rightarrow$ ii) $\delta = \sqrt{\frac{2}{\omega\sigma\mu_0\mu_r}} \approx 0.025\,\mathrm{m}$

$\Rightarrow$ power penetration is half of this, i.e. $\sim 1\,\mathrm{cm}$

$\Rightarrow$ communication with sub-marines only works at lower frequencies, 10 - 200 kHz, where δ is much bigger!

for testing whether weak or strong absorption occurs,
one needs to evaluate $\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r}$ ← conductivity

↑ frequency ↑ permittivity (= „dielectric constant")

	material	ε_r	μ_r
	air	1	1
	AL	1	1
dielectrica	paper	2.3	1
	SiO_2	3.9	1
	HfO_2	25	1
	Bi_2O_3 / Nb_2O_5	> 40	1
magnetica	ferrite	1	16-600
	mu-metal	1	50,000
	99.95% pure Fe	1	200,000

examples:

glass, light (visible, e.g. red light with $\lambda = 600\,nm$):

$\rightarrow \sigma = 10^{-15}\,Sv/m$, $\varepsilon_r = 5$, $\omega = 2\pi f = 2\pi \frac{c}{\lambda} = 3 \cdot 10^{15}\,Hz$

$\Rightarrow \frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \approx 7.5 \cdot 10^{-21} \ll 1$ is <u>weakly</u> absorbing

copper, weak X-rays ($\lambda = 10\,nm$):

$\rightarrow \sigma = 6 \cdot 10^{7}\,Sv/m$, $\varepsilon_r \approx 1$, $\omega = 2\pi f = 2\pi \frac{c}{\lambda} \approx 2 \cdot 10^{17}\,Hz$

$\rightarrow \frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \approx 36 \gg 1$ is <u>strongly</u> absorbing

complex permittivity and absorption

Ampere – Maxwell – Law: $\text{rot}\, \underline{H} = \underline{j} + \frac{\partial \underline{D}}{\partial t}$

$$\underline{j} = \sigma \underline{E} \quad \text{(Ohm's Law)} \qquad \underline{D} = \varepsilon_0 \varepsilon_r \underline{E} \quad \text{(def. of flux)}$$

re-write with complex permittivity

$$\boxed{\varepsilon_r = \varepsilon_r' + j\, \varepsilon_r''}$$

$$\varepsilon_0 \varepsilon_r \frac{\partial \underline{E}}{\partial t} = \sigma \underline{E} + \varepsilon_0 \varepsilon_r \frac{\partial \underline{E}}{\partial t}$$

and use $\underline{E} = \underline{E}_0 e^{j(\omega t - \bar{k} x)}$, i.e. $\frac{\partial \underline{E}}{\partial t} = j\omega \underline{E}$

$\Rightarrow$ $\varepsilon_0 (\varepsilon_r' + j\, \varepsilon_r'')\, j\omega \cancel{\underline{E}} = (\sigma + \varepsilon_0 \varepsilon_r j\omega)\, \cancel{\underline{E}}$

compare real parts: $\boxed{\varepsilon_r' = \varepsilon_r}$ describes usual relationship between field & flux

compare imaginary parts: $\boxed{\varepsilon_r'' = -\frac{\sigma}{\omega \varepsilon_0}}$ describes absorption due to finite conductivity

If we write for the refractive index in complex notation

$$\boxed{n = n' + j\varkappa}$$

and use

$$\boxed{n^2 = \varepsilon_r}$$

$$\Rightarrow \boxed{\begin{aligned} \varepsilon_r' &= n'^2 - \varkappa^2 \\ \varepsilon_r'' &= 2n'\varkappa \end{aligned}}$$

$\Rightarrow$ Link between dielectric and optical properties.