EEE105 2007-8 Solutions

Question 1

(a) *[Easy Bookwork]*

i) The Generation rate is the rate of so-called bond-breaking in the material, which is thermal generation of electrons from the bonding state (the valance band) to the excited state of the bond (the conduction band), where they can conduct. The electron leaves behind a hole in the valance band which can also conduct. So an electron-hole pair is said to have been generated

The Recombination rate is the rate at which free electrons in the conduction band recombine with holes causing them to annihilate each other.

[4]

ii) In any material there will be carriers being generated and recombining all the time. In equilibrium the rate of generation and recombination will be equal and hence there will be a constant number of free carriers in the material at any given time. The generation rate is a constant, independent of doping which only depends on the temperature of the material.

Considering a free electron the probability of it recombining with a hole will be dependent on the density of holes in the material. Thus the rate of recombination will be proportional to the density of holes in the material, p. By similar analogy the rate of recombination will also be proportional to the density of electrons in a material, n. Thus we can say in thermal equilibrium that:

$$G = R = Bnp = Bn_i p_i = Bn_i^2$$

in an intrinsic material, where B is a constant of proportionality.

The doped material is n-type, therefore the majority carriers are electrons and their density is given by the density of dopants (assuming $n >> n_i$). Knowing that the generation rate remains the same we can write:

$$G = Bn_i^2 = Bnp_n$$

and hence

$$p_n = {n_i^2}/{n}$$

[5]

(b) *[Fairly Simple Problem and based on a tutorial question]*

The easiest method to solve this equation is to calculate the constant C at 20°C (293 K) and then use that value to calculate n_i for the temperature of -40 °C (233 K).

$$C = n_i T^{-3/2} \exp\left(\frac{W_g}{2kT}\right)$$

Now W_g = 1.12 eV = 1.79×10^{-19} J

C=1.19×10^{22} m^{-3}K$^{-3/2}$

Substituting in the equation given for T=233 K gives

n_i =3.45×10^{13} m^{-3}

[5]

(c) *[Hidden]*

For a diode we know that the leakage current density is given theoretically by the J_0 term in the diode equation (given in table of equations) as in reverse bias the exponential part will become very small. Now from the table of equations we also have:

$$J_0 = \frac{qL_e n_p}{\tau_e} + \frac{qL_h p_n}{\tau_h}$$

As we are dealing with a p$^+$-n device we know that the hole current will dominate and hence $J_0 = \frac{qL_h p_n}{\tau_h}$

Now we know that n_i has dropped from 1.45×10^{16} to 3.45×10^{13} m^{-3} due to the fall in temperature, which is a factor of 2.38×10^{-3}. From the equation $p_n = {n_i^2}/{n}$ we can see that if n is constant then p_n must drop by a factor of $(2.38 \times 10^{-3})^2 = 5.66 \times 10^{-6}$. Assuming that the other parameters in the equation remain constant then we can say that J_0 must therefore fall by the same amount and hence we may expect a leakage current:

$$J_0(233K) = J_0(293K) \cdot 5.66 \times 10^{-6} = 5.66 \times 10^{-15} \text{ A (or 5.66 fA)}$$

[Note it is still possible to make the same deduction without simplifying the equation for the hole current, but it is a bit more messy]

[6]

Question 2

(a) *[Bookwork]*

If light shines on a semiconductor then electron-hole pairs will be generated if the light has a sufficiently short wavelength so that a photon's energy is greater than the energy required to release an electron from a bond, W_g.

Let us consider the situation where we have a p-n junction under zero bias, or reverse bias. There will be a built-in potential around the junction due to the depletion of the free electrons and holes on either side. As a result there is an electric field present. If an electron-hole pair is created in this depletion region by the absorption of the energy from a photon then the electron will be swept into the n-type material and the hole into the p-type material, respectively. These carriers can therefore appear at the device terminals and hence a current can flow.

[7]

(b) *[Applied Knowledge – varying degrees of difficulty – there is a similar past paper example available to students who have looked and the equation* $E_{photon} = \frac{hc}{\lambda}$ *is given in the table of equations on the examination script.]*

i) *(should be trivial).* The intensity of the light has increased 4 times. Thus the number of photons hitting the material is four times larger. Thus the photocurrent should increase by four times also to 400 μA.

ii) *(More difficult).* The intensity of the light is unchanged. However the wavelength is now shorter. This means the energy of one photon is much larger and the number of photons per mW of optical power is reduced. Thus the number of photons hitting the sample is reduced by a factor of 405/630 to 64% of the original value. The photocurrent will be 64 μA

iii) (*More difficult but a strong hint is given in part (a)).* In this case we need to show that the energy of the photon is smaller than the energy required to create an electron-hole pair in Si. As a result photons cannot be absorbed and the photocurrent should be zero.

[2+3+3=8]

(c) *[Problem - Hidden - but all the information is there]*
1 mW of light is hitting the sample. That is 1 mJ of light per second. The energy of one photon is given by $E_{photon} = \dfrac{hc}{\lambda} = 3.16 \times 10^{-19}$ J

Thus the number of photons hitting per second is: 3.17×10^{15}

Each photon should ideally contribute one electron to the current, giving a flow of electrons of 3.17×10^{15} per second. The charge on the electron is known, and our photocurrent was 100 μA. Hence the number of electrons in the photocurrent will be

$$\frac{I}{q} = \frac{1 \times 10^{-4}}{1.60 \times 10^{-19}} = 6.25 \times 10^{14}$$

Given the definition in the question the quantum efficiency is: $\dfrac{6.25 \times 10^{14}}{3.17 \times 10^{15}} = 0.197$

Thus the quantum efficiency is 0.197, or 19.7%.

[5]

Question 3

(a) *[Easy Bookwork]*

i) If a voltage is applied across two metal plates the plates will become charged creating an electric field between them. The stored charge on the plates leads to the capacitance. If an insulator is inserted between the plates and a voltage is applied, the electric field can cause the distribution of the atom cores and electrons to be modified. Thus the electron clouds around the atoms can be distorted, bonds can be modified slightly or molecules can rotate in the field. These changes or polarization causes an E-field to occur which opposes the field produced by the applied voltage on the plates. This means that a greater number of charges must be stored on the plates to bring the E-field between the plates to the same level as before, so that the voltage drop across the capacitor is the same as the applied voltage. An increase in the charge stored causes an increase in capacitance.

[5]

ii) Any two from the following: manufacturability cost of raw material, high breakdown field, small leakage current.

[2]

(b) *[Bookwork]*

This explanation is for a planar JFET, a double sided "textbook" JFET is also fine as the question does not specify. Anyone describing an n-channel device will be marked in half marks.

In a planar p-channel a layer of p-type semiconducting material is placed on a semi-insulating substrate. In a small region, where we wish the Gate we create heavily doped n-type region, giving us an n^+-p junction.

Either side of the Gate region we place a Source and Drain contact to the p-type semiconductor.

Now if we apply a small bias between Source and Drain, current will flow between the contacts through the p-type material, or p-channel.

The thickness of the channel will be given by the physical thickness of the p-type semiconductor layer, minus the depletion region thickness under the gate. As the n-type material is much more heavily doped than the p-type material the depletion region will essentially all be into the channel.

As the semiconducting material has a voltage drop across it the effect of this to mean that the reverse bias between the channel and the gate varies along the gate length. This means that the depletion region is largest (and hence the channel thinnest) in towards the drain end of the device. As magnitude of the drain voltage increases the depletion region and the drain end of the gate will get larger till it is essentially the same thickness as the channel. At this point there is only a very thin space for the carriers (holes) to flow through and we reach the pinch-off condition. Increasing the drain voltage further causes the pinched-off depletion region to extend down the gate length towards the source and it is found that the channel resistance increases in proportion to the applied voltage and hence we get a constant drain current above pinch off.

We can increase the depletion region thickness across the entire gate length by applying a reverse bias voltage to the gate. This causes the pinch-off to occur at lower magnitudes of the drain voltage and hence the value of the constant drain current decreases.

Operating the device above pinch-off we now have the situation where the drain current is only dependent on the value of the gate voltage. Thus a small input signal on the gate can control the output current flow between source and drain and we have transistor behaviour.

[7]

(b) *[Hidden/Problem – gate channel capacitance is not discussed in the lectures, although depletion region thickness and junction capacitance are]*

In this case we need to first recognise that the depletion region will form the insulating region between the conducting n^+ gate and the p-channel

We can calculate the capacitance under the gate from $C = \dfrac{\varepsilon A}{d}$.

We can get the permittivity as ε_r is given as 12 for Si.

The area of the "plate" can just be given by the product of the gate length and width:

$A = w \cdot l = 2\times10^{-6} \cdot 5\times10^{-4} = 1\times10^{-9}\,\text{m}^2$

This assumes that the depletion region is much smaller than the gate length

Now we are told that V_d is small hence we will assume that the depletion region is the same thickness along the entire length of the gate.

We are given the formula $d_j = \left(2\varepsilon_0\varepsilon_r V_j / qN_a\right)^{0.5}$ adapted for a n^+-p rather than p^+-n junction.

Substituting we get $d_j = 2.82\times10^{-7}\,\text{m}$

We can now get the $C = \dfrac{8.85\times10^{-12} \cdot 12 \cdot 1\times10^{-9}}{2.82\times10^{-7}} = 3.77\times10^{-13}\,\text{F}$, or 0.38 pF

[6]

Question 4

(a) *[Bookwork]*

The transistor is p-n-p. Half marks for descriptions of n-p-n device.

The solution should show a diagram of the device, showing a thin n-type base between an n^+ emitter and n collector. The principles of operation are as follows:

In order for the device to operate we wish for holes to be emitted from the emitter and collected by the collector and for that hole current to be controlled by the the base-emitter voltage (or by the current in the base). To achieve this we first need to consider how to get holes injected from the emitter into the base. This can be obtained by doing the emitter to a much higher density than the base. This will ensure that hole current will dominate when the junction is under forward bias, as the ratio of electron to hole current can be shown to be (approximately) equal to the ratio of the conductivites of the n-type material to p-type material. Ideally we would like the electron current in this p-n junction to be as close to zero as possible. *(Answer to part ii)*

The holes injected into the n-type base will diffuse away from the base-emitter junction, and potentially will recombine with electrons as they go. We want most of the holes to diffuse right across the base and then be swept by the reverse biased base collector junction into the collector. This will give us a large collector current and small base current. To achieve this we need to ensure that the base is thin compared to the hole minority carrier diffusion length. This leads us to the definition of the base transport factor, which is the fraction of the holes emitted into the base that reach the collector, which is given by the ratio I_C/I_e. *(Answer to part iii)*

The current flowing into the collector is governed entirely by the current flowing through the base (as the base emitter voltage governs the forward bias of that junction), and if the base is thin then the base current will be small compared to the collector current. Thus a small base current is controlling the large collector current. This means we have current gain as small changes in the base current give large collector current changes. *(Answer to part i)*

A diagram for part iv) should show a single transistor with the emitter terminal on a common earthed line. The base terminal should be biased to -0.7 V. The collector terminal should be connected to a load resistor and thence to a large (eg -20 V) negative bias compared to the earth line. The input signal should be shown between the base terminal and the base biasing power supply. *(Answer to part iv)*

[11]

(b) *[Hidden Problems]*

i)

In the device we need to know what fraction of the excess holes injected into the base will travel the 1 μm across the base without recombining. We know (from table of equations that

$$\partial p = \partial p_0 \exp\left(\frac{-x}{L_h}\right)$$

Hence:

$$\frac{\partial p(x)}{\partial p_0} = \exp\left(\frac{-x}{L_h}\right)$$

Substituting we get

$$\frac{\partial p(x)}{\partial p_0} = \exp\left(\frac{-1}{15}\right) = 0.94$$

[4]

ii)

This part requires us to remember that the ratio of electron to hole current is approximately given by the ratio of conductivities in the material.

We can simply calculate the conductivity of the emitter and base material using

$\sigma_n = nq\mu_e = 1.92 \times 10^4$

$\sigma_p = pq\mu_h = 7.2 \times 10^5$

Thus the electron current will 2.7% the size of the hole current. As the hole current is much larger we can say that the electron current will be 2.7% of the total current and hence the hole current will be 97.3% of the total current

The emitter injection efficiency is thus 0.97.

[5]