Polyphase Systems.

What is a Polyphase system?

In general a polyphase system consists of a number of ac sources of the same frequency, but at different time phases.

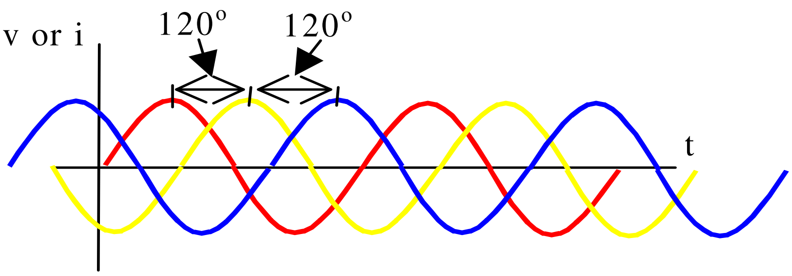
A balanced polyphase system consists of 'n' ac sources of the same amplitude and frequency, displaced in time phase by $360/n^{o}$

The most commonly used system, for reasons we shall see later is the balanced 3-phase system.

Balanced 3 Phase Systems

Representations:

(i) – Waveform of a CRO (not very useful in problem solving)



(ii) – Mathematical representations:
Instantaneous values:

$$v_r = \hat{V}\sin(\omega t)$$

$$v_y = \hat{V}\sin(\omega t - \frac{2\pi}{3})$$

$$v_b = \hat{V}\sin(\omega t - \frac{4\pi}{3})$$

(N.B subscripts 'R' or 'r', 'Y' or 'y' and 'B' or 'b' have traditionally been used to identify the phases of a 3-phase system since Red, Yellow and Blue were the historical colours employed for 3-phase wiring - Neutral was Black. However, since 1/4/06 new 'harmonised' wiring colours are to be used throughout the EU. The 3-phase wiring colours are now Brown, Black and Grey with Blue for the neutral. Take care when connecting new and old wiring!)

Phasor notoation:

$$V_r = V\angle 0 \text{ (reference)}$$
$$V_y = V\angle -120$$
$$V_b = V\angle -240$$

where:

$$V = \frac{\hat{V}}{\sqrt{2}} \quad \text{- rms value.}$$

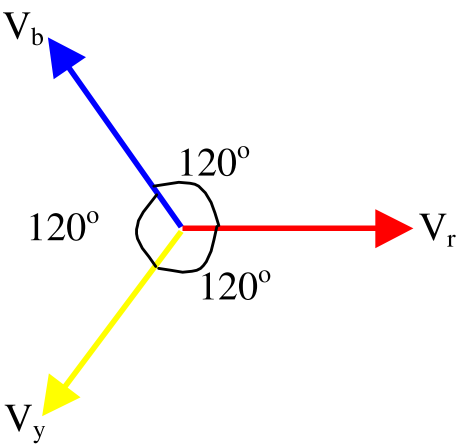
or 'j' notation for these phasors would give:

$$V_r = V$$
$$V_y = V\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$
$$V_b = V\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

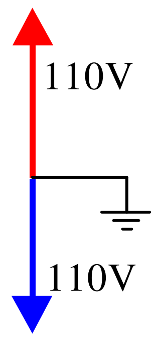
(These techniques are mainly used for numerical problem solution)

(iii) - Phasor Diagram representation:

This is a very useful method of visualising the problem prior to numerical solution. A Phasor sketch is highly recommended.



NOTE: The USA domestic supply has a 'two phase' supply where each phase provides 110V, and the two are 360/2 (180°) displaced to give either 220V or a 110V supply.



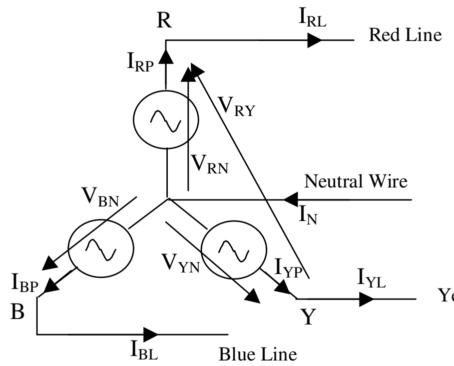
- in many systems a 'so-called' two phase supply consists of two phases (ac supplies) displaced by 90° (not 180°) and is actually an unbalanced 4 phase (360/4) system. This is historic nomenclature and should be treated with care.

3 Phase Connections

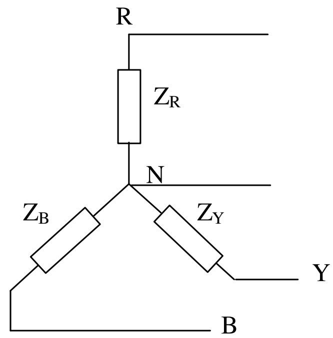
There are 2 ways of connecting a 3-phase supply or load, star (or Y) connection and delta connection.

A) STAR CONNECTION

Three phases connected to a common STAR or NEUTRAL point (N).



4-wire star connection



4-wire load connection
(Balanced if $Z_R = Z_Y = Z_B$)

- I_{RP}, I_{YP}, I_{BP} – known as PHASE CURRENT, I_P
- I_{RL}, I_{YL}, I_{BL} – known as LINE CURRENT, I_L
- $V_{RY,}$ $V_{BR,}$ V_{YB} – known as LINE VOLTAGE, V_L
- V_{RP}, V_{YP}, V_{BP} – known as PHASE VOLTAGE, V_P
- I_N – known as NEUTRAL CURRENT

NB: for a balanced system, with a balanced load, all phase quantities have the same magnitude, and all line quantities have the same magnitude.

Star Relationships:

From the diagram: Clearly it can be seen that at any node, R, Y, or B,

(i) $I_{RP} = I_{RL}$ etc. i.e. $|\text{LINE CURRENT}| = |\text{PHASE CURRENT}|$

(ii)

$$\begin{aligned}\bar{I}_N &= \bar{I}_{RP} + \bar{I}_{YP} + \bar{I}_{BP} \\ &= I_P\angle 0 + I_P\angle -120 + I_P\angle -240 \\ &= 0\end{aligned}$$

for 3 equi-spaced phasors

i.e. In a balanced system, there is no neutral current.

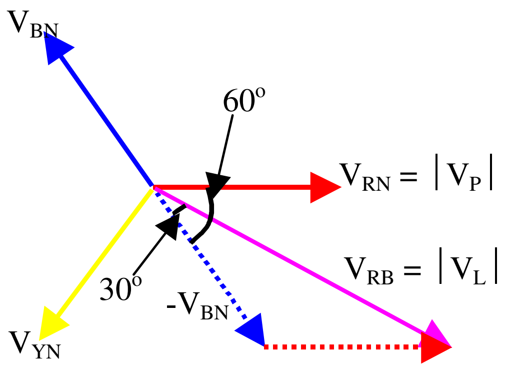
(iii) From the Phasor diagram:

Line voltage

$$\overline{V}_{RB} = \overline{V}_{RN} + \overline{V}_{NB}$$
$$= \overline{V}_{RN} - \overline{V}_{BN}$$

from Phasor Diagram

$$|V_L| = 2|V_P|\cos(30^o)$$
$$= 2|V_P| \times \frac{\sqrt{3}}{2} = \sqrt{3}V_P$$
$$\therefore |V_L| = \sqrt{3}|V_P|$$



Note that V_{RB} is 30° lagging on V_{RN} etc.

(iv) Power per phase:

$$P_p = V_p I_p \cos\phi$$

where ϕ is the phase angle of the load, which for a balanced system is identical. Therefore:

$$\text{Total power, } P = 3 \times P_p$$
$$P = 3V_P I_P \cos\phi$$

or, substituting for $|V_L| = \sqrt{3}|V_P|$ and $I_P = I_L$

$$P = \sqrt{3}V_L I_L \cos\phi$$

Similarly, kVA/Phase = $V_P I_P$ and $kVA_{total} = 3V_P I_P = \sqrt{3}\, V_L I_L$
KVAr / Phase = $V_P I_P \sin\phi$ and $kVAr_{total} = 3V_P I_P \sin\phi = \sqrt{3}\, V_L I_L \sin\phi$

Implications for Power Supplies

Typical distribution at 400V (line voltage)

Domestic (Single phase supply)

- Each customer is supplied with one line to neutral single phase supply

$$- \quad \therefore 1\phi \quad \Rightarrow \frac{400}{\sqrt{3}} = 230V \ (phase \ \ voltage)$$

- Note for single phase supplies the 4th (Neutral wire) is retained to give the required system.

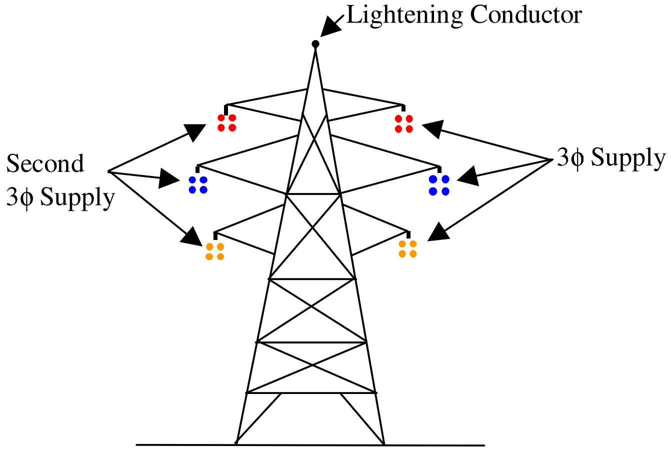
- Usually groups of houses are supplied from each phase and the net result is integrated over a large number of houses etc.

Industrial (single and 3ϕ supplies)

- Large customers usually connected directly to a 3ϕ supply (can be at a variety of voltages, 3.3kV, 11kV, 33kV or even 132kV for very large factories)
- Supply can be 3-wire or 4-wire (Neutral needed to give 1ϕ supplies within the factory.

<u>Transmission</u>

– Term used for bulk transfer of power around the country. Note the lack of a neutral wire on a typical tower configuration.

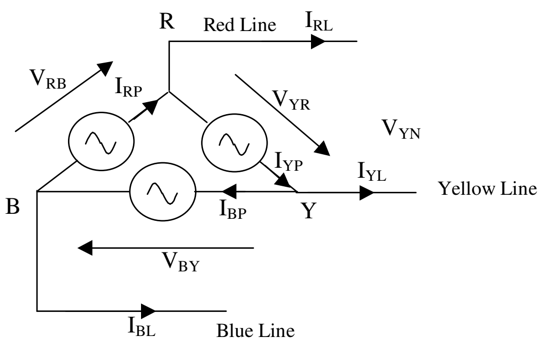


Typical Tower

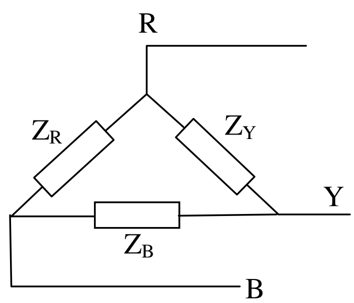
- As shown previously, if the system is balanced, no neutral wire is required. Hence compared with a 1ϕ system, where a phase wire is needed together with a neutral wire (therefore 2 wires needed), 3ϕ transmits 3× the power of the single phase connection for only 1 extra wire. – One of the reasons for use.

- Typical transmission levels 275kV, 400kV in the UK, (higher in USA where distances are longer)
- Having 2 systems in parallel improves the security of the supply

B) DELTA CONNECTION



Delta Connected Generator
(Only 3 wires possible – no Neutral)



Delta Connected Load
(Balanced if $Z_R = Z_Y = Z_B$)

Delta Relationships:

(i) From the diagram, Line voltage = Phase Voltage, i.e.:

$$|V_L| = |V_P|$$

(ii) Voltage around the delta loop

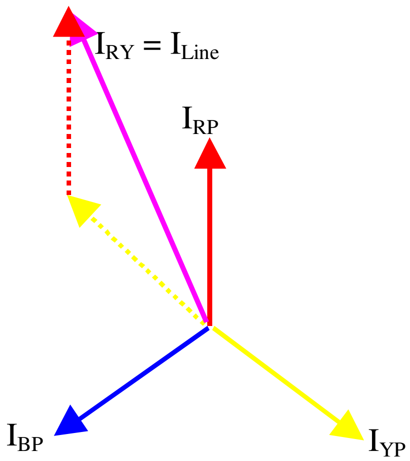
$$= V_{RB} + V_{YR} + V_{BY} = 0$$ - 3 equi-spaced Phasors

(iii) Line Current

$$\bar{I}_{RL} = \bar{I}_{RP} - \bar{I}_{YP}$$

By a similar argument to some relationships for the star voltage case:

$$|I_L| = \sqrt{3}|I_P|$$



(iii) Again power per phase:

$$P_p = V_p I_p \cos\phi$$

Therefore:

$$\text{Total power, } P = 3\times P_p$$

$$P = 3V_P I_P \cos\phi$$

or, substituting for $|V_L| = |V_P|$ and $I_L = \sqrt{3}I_P$

$$P = \sqrt{3}V_L I_L \cos\phi$$

$$\text{Total kVA} = \sqrt{3}\ V_L I_L$$

$$\text{Total kVAr} = \sqrt{3}\ V_L I_L \sin\phi \text{ etc.}$$

Examples: - Note: In year 1 **all** problems are balanced and hence all phase and line magnitudes are equal, therefore the problems may be solved on a ***per phase*** basis.

Example 1

A 3ϕ load is connected to a 400V, 3ϕ supply and takes a total power of 2.7kW with a line current of 60A at a lagging power factor. If the load can be considered as a series impedance per phase, calculate the load impedance components

(a) If the load is star connected

(b) If the load is delta connected.

Approach:

(i) When a 3ϕ supply is specified as 400V, this means that the magnitude of the voltage between ***any pair of lines*** is 400V – but they will be displaced by 120^o in phase.

(ii) The problem is balanced therefore all phases behave the same, but are 120^o out of phase. Hence for all the above (star or delta):

$$\text{Power per Phase} = 2.7/3 = 0.9\text{kW}$$

Also (star or delta):

$$2.7\text{Kw (Total power)} = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 60 \times \cos\phi$$

Therefore:

$$\cos\phi = (2.7\times10^3) / (\sqrt{3} \times 400 \times 60) = 0.065 \text{ lagging}$$

and the phase angle is found to be:

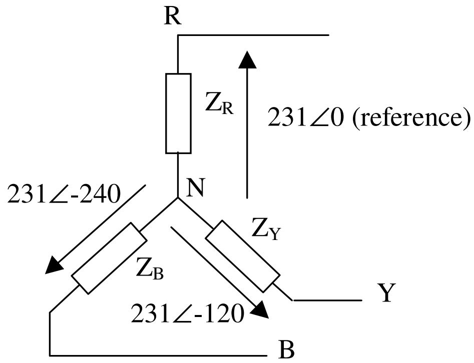
$$\phi = \cos^{-1}(0.06) = -86.3^{o}$$

Since it is lagging (current lags voltage), Impedance must be **inductive**.

(a) Star Connected Load

Load impedance connected in star across the 400V supply, therefore:

$$|V_P| = \frac{400}{\sqrt{3}} = 231\,V$$



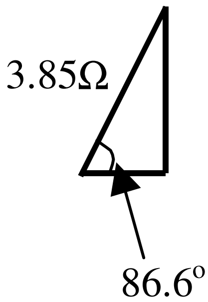
Taking the red phase as the reference and doing all the calculations on this phase (on a per-phase basis)

For Star connection:

$$|I_L| = |I_P| = 60A$$

and:

$$I_{RL} = 60\angle{-\phi} = 60\angle{-86.3}$$



therefore:

$$Z_P = V_P / I_P = (231\angle 0) / (60\angle{-86.3})$$
$$= 3.85\angle 86.3$$

then:

$$R_P = 3.85 \cos(86.3) = 0.25\Omega$$

and:

$$X_P = 3.85 \sin(86.3) = 3.84\Omega$$

(check: $P = 3 \times I_P^2 \times R_P = 3 \times 60^2 \times 0.25 = 2700$ W)

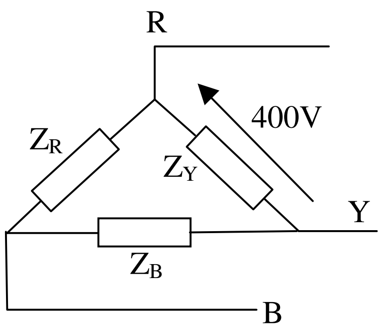
(b) Delta Connected Load

Now load impedances connected in delta connection across the 400V supply, therefore:

$$|V_{Ph}| = |V_{Line}| = 400V$$

and for Delta:

$$|I_{Ph}| = |I_{Line}|/\sqrt{3}$$
$$= 60/\sqrt{3} = 34.64A$$



Taking Red phase as the reference:

$$Z_{Ph} = (400\angle 0) / (34.64\angle{-86.6})$$
$$= 11.55\Omega \quad (3\times Z_{Ph} \text{ for the star connection})$$

and:

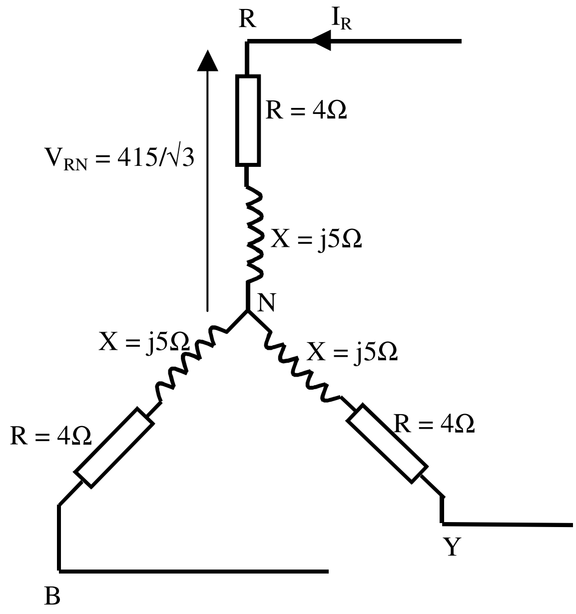
$$R_{Ph} = 3 \times (R_{Ph} \text{ for the star connection}) = 0.75\Omega$$
$$X_{ph} = 3 \times (X_{ph} \text{ for the star connection}) = 11.52\Omega$$

Example 2
More Typical (ex-exam paper question).

A 3ϕ Star-connected motor has an input impedance of (4+j5)Ω per phase and is connected to a 415V, 50Hz 3ϕ supply.

(i) Calculate the total input power, kVA and kVAr to the motor, and if it is 90% efficient, calculate the mechanical output power and power lost as heat.
If the motor is running at 1400rpm calculate the shaft torque.

(ii) A user of the motor decides to improve its power factor by inserting 700μF capacitors in each line of the supply. Calculate the new power factor. Comment on the advisability of this technique.



Solution

(i) Star Connected:

$$|V_{phase}| = |V_{Line}|/\sqrt{3}$$

$$|V_{phase}| = \frac{415}{\sqrt{3}} = 240\text{ V}$$

and Phase current = Line current:

$$|I_{Ph}| = |I_{Line}|$$

Now, Taking V_{RN} as reference = 240∠0, and solving on a per phase basis:

$$I_R = \frac{V_{RN}}{Z_{phase}} = \frac{240\angle 0}{4+j5} = \frac{240\angle 0^\circ}{6.4\angle 51.3} = 37.5\angle -51.3^\circ\ A$$

Similarly:

$$I_Y = \frac{V_{YN}}{Z_{phase}} = \frac{240\angle -120^\circ}{6.4\angle 51.3^\circ} = 37.5\angle -171.3^\circ\ A$$

etc..

Hence the power factor = cos(-51.3°) = 0.625 lagging.

Power per phase:

$$P_p = V_p I_p \cos\phi = 240\times 37.5\times 0.625$$

and total power:

$$P_T = 3P_p = \sqrt{3}\ V_L I_L \cos\phi = 415\times 37.5\times\sqrt{3}\times 0.625 = 16.85\text{kW}$$

Total kVA:

$$S_T = \sqrt{3}\times V_L\times I_L = (3\times Vp\times Ip) = 26.96\text{kVA}$$

and total kVAr:

$$Q_T = \sqrt{3}\times V_L\times I_L \sin\phi = 21\text{kVAr}$$

If motor is 90% efficient, then motor mechanical output power:

$$P_{OUT} = 0.9 \times 16.85 = 15.16\text{kW}$$

and:

$$\text{losses (heat)} = 0.1 \times 16.85 = 1.685\text{kW}$$

The shaft torque is obtained from:

$$P_{mechanical} = \text{Torque (Nm)} \times \text{Speed (rads}^{-1}\text{)} \quad \text{(Watts)}$$
$$= T \times (1400 \times 2\pi)/60$$

Hence:

$$T = P_{mechanical} \times 60 \,/\, (1400 \times 2\pi) = 15160 \times 60 \,/\, (1400 \times 2\pi)$$
$$= 103.4 \text{ Nm}$$

(ii) now consider what happens with 700µF capacitors added to each line. The new impedance per phase is given by:

$$Z'_{phase} = (R + jX_L) - jX_c$$

where:

$$X_c = 1/(2\pi fC) = 4.55\Omega$$

therefore:

$$Z'_{phase} = 4 + j5 - j4.55\ \Omega = 4 + j0.45 = 4.02\angle 6.4^{o}$$

hence new phase current:

$$I_R = \frac{240\angle 0}{4.02\angle 6.4} = 59.7\angle -6.4 \text{ A}$$

and new power factor:

$$pf = \cos(-6.4) = 0.99 \text{ lag}$$

(Note large increase in phase current through motor)

HOWEVER: new input power per phase:

$$P_p = V_p I_p \cos\phi = 240 \times 59.7 \times 0.99 = 14.24\text{kW}$$

Hence:

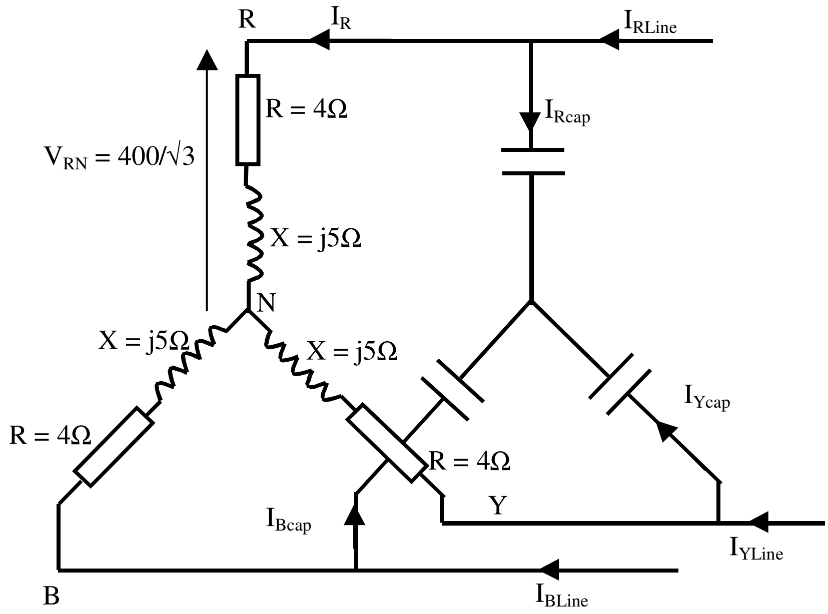
$$\text{Total input power} = 3 \times 14.24 = 42.72\text{kw} \text{ (c.f. 16.85kW)}$$
$$\text{Total mechanical output power} = 0.9 \times 42.72 = 38.45\text{kW} \text{ (c.f. 15.16kW)}$$
$$\text{Total losses} = 0.1 \times 42.72 = 4.27\text{kW} \text{ (c.f. 1.685kW)}$$

- Hence motor would rapidly overheat!!

Correct Solution:

To obtain unity power factor operation – **Always** put capacitors in parallel with the load.

The power factor correction capacitors may either be star-connected or delta-connected. As the motor connection is now unchanged, the calculations for the currents into the motor are now the same as without the capacitors, however, the line currents into the total system is affected by the capacitors.



Again, looking at one phase because the system is balanced, from the previous calculations for the un-corrected motor above, we have power per phase:

$$P_m = V_p I_p \cos\phi = 240 \times 37.5 \times 0.625 = 5.625\text{kW}$$

and per-phase kVAr:

$$Q_m = \sqrt{3} \times V_L \times I_L \sin\phi = 21/3 = 7 \text{ kVAr}$$

This will remain un-changed after adding the pf correction capacitors in parallel. To correct the line currents to unity pf, the capacitors must provide a –ve Q equal and opposite to Q_m, i.e.:

$$Q_c = -7 \text{ kVAr}$$

and:

$$Q_c \text{ per phase} = V_{ph} I_{ph} \sin\phi \text{ } (\sin\phi = 1 \text{ for capacitors})$$

Assuming capacitors are star connected:

$$V_{ph} = V_{Line}/\sqrt{3} = 240\text{V} = V_{cap}$$

therefore required capacitor current:

$$I_{cap} = 7\times10^3 / 240 = 29.17 \text{ A}$$

giving:

$$X_c = V_{cap}/I_{cap} = 8.23\Omega = 1/2\pi fC$$

Hence the value of capacitance required is:

$$C = 387\mu\text{F / Phase}$$

- Could have used:

$$Q_c = V_c^2/X_c = 240^2/X_c \quad \text{- Same Answer}$$

If C is delta connected, then $V_c = 415$ etc. $X_c = 3\times8.2 = 24.6\Omega$ and $C = 130\mu$f