Data Provided: None

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2013-14 (2.0 hours)

EEE349 Power Engineering Electromagnetics

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

Physical constants:

$\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

$\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}

1. **a.** The electric field strength at a distance r from a point charge Q in a region of permittivity ε_0 is given in spherical coordinates by:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{r}$$

where $\vec{r}$ is the unit vector in a radial direction.

Starting from this expression for electrical field strength, derive Gauss's Law. **(4)**

b. The variation with position of the electric scalar potential in a region of space of permittivity ε_0 expressed in a Cartesian coordinate system is given by:

$$V = 3x^2+xy^3+5yz \text{ (V)}$$

Calculate the following at the point (x, y, z) = (1.5, 0.5, 0.2) m:

i) The electric field strength.

ii) The charge density. **(6)**

c. Figure 1 shows a section of a high voltage cable in which the core conductor has a radius R_c, the insulation an outer radius of R_i and an overall cable length L_c. The core conductor of the cable carries a total charge Q and the insulation has a relative permittivity of ε_r. Starting from Gauss's Law derive expressions for the following:

i) The variation of electrical field in the insulation.

ii) The total voltage across the insulation layer.

iii) The capacitance of the cable

(6)

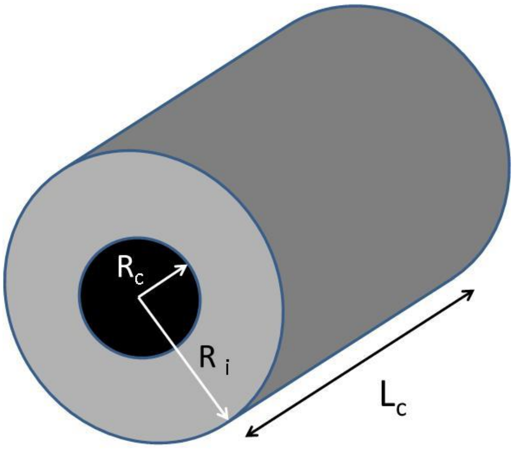

Figure 1 High voltage cable

d. The cable in part (c) has a core conductor of radius R_c=10mm and insulation thickness (i.e. R_i-R_c) of 40mm. A 10m long length of the cable carries a uniform charge density in the core conductor of 0.1275 C/m^3.

Calculate the minimum relative permittivity of the insulation to provide a factor of five safety margin over breakdown of the surrounding air. (You may assume that the breakdown field strength of the air at the particular operating conditions and altitude is 3×10^6 V/m). **(4)**

2. **a.** Figure 2 shows a simplified cross-section through a single isolated transmission line located a distance *h* above the ground. The ground can reasonably be regarded as a zero voltage equipotential. The radius of the transmission line is R_c and its length is L_c. The charge density of the transmission line is q.

Stating any assumptions that you make, and starting from Gauss's Law, show that the capacitance to ground of the transmission line is given by:

$$C = \frac{2\pi\varepsilon_0 L_c}{log_e\left(\frac{2h}{R_c}\right)}$$

(8)

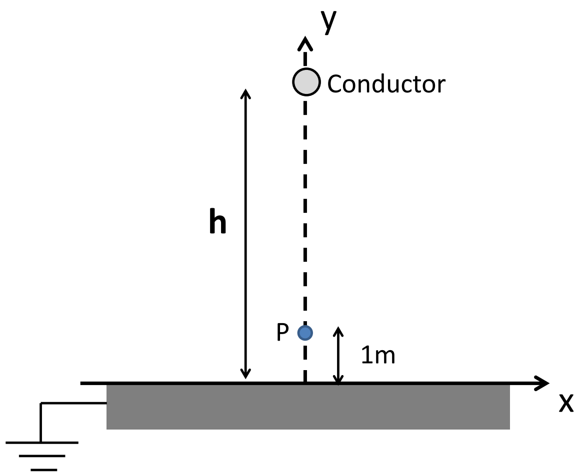

Figure 2

b. If the distance *h* is 9m, and the peak voltage on the conductor is 560 kV. Calculate the following given that the transmission line cross-section has a radius of 25mm:

i) The static charge on 1km of the transmission line.

ii) The magnitude of the electric field strength at the point **P** in Figure 2 which is directly below the transmission line and 1m above the ground. **(8)**

c. Explain briefly how you would tackle the problem of modelling the electric fields around a set of conductors which form a dual-circuit, 3-phase transmission system using an analytical approach if the pylon electrical characteristics were specified in terms of voltage and not charge density.

What other method might be more practical with such a large number of conductors specified in terms of voltage? **(4)**

3. **a.** The magnetic vector potential throughout a region of permeability μ_0 varies as a function of (x,y,z) in a Cartesian coordinate system according to the following:

$$\vec{A} = \overrightarrow{u_x}3xyz + \overrightarrow{u_y}3x^2z + \overrightarrow{u_z}z^2$$

where the vectors $\vec{u}$ are the unit vectors in x,y and z directions.

Calculate the following at the location (0.5, 2.0, 3.0) m:

i) The magnetic flux density

ii) The magnetic field strength

ii) The current density **(8)**

b. Starting from Ampere's Law derive an expression for the magnetic field strength at any point in the region surrounding an infinitely long conductor of circular cross-section carrying a steady DC current of magnitude *I*. (You do not need to derive an expression for flux density for the region within the conductor itself).

Using this expression, calculate the minimum distance that a cable in an Aluminium manufacturing plant carrying a DC current of 300,000A should be placed away from a machine operator, if the maximum flux density to which the operator can be subjected is 50mT.

Are there any additional practical measures that could be taken if the separation calculated above cannot be achieved? **(6)**

c. Figure 3 shows a region of space which has a magnetic permeability μ_0 above a thick plate of material which can be reasonably considered as having infinite magnetic permeability. There is an arrangement of coils on the surface of the plate which can be approximated by an infinitely thin current sheet at the surface of the plate. The variation in the magnitude of the surface current in the current sheet as a function of distance along the *x*-axis, *x*, is given by:

$$J(x) = J_m \sin 3x$$

where J_m is the magnitude of the peak current sheet density. You do not need to consider any time variation of the surface current. The z-component of the magnetic vector potential A_z at any point (x,y) in region 1 above the plate is given by:

$$A_z(x,y) = \frac{\mu_0 J_m}{30} sin(30x)e^{-30y}$$

If the x component of flux density at the point (x=0.15m,y=0.01m) is -0.5T, calculate the magnitude of flux density at the point (x=0.3m,y=0.05m). **(6)**

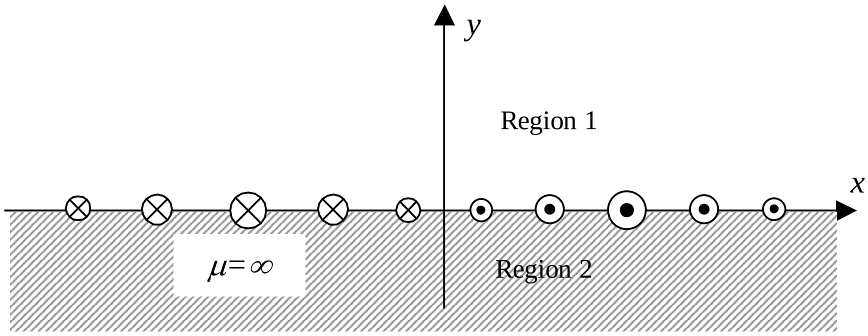

Figure 3 Two-dimensional representation of a current sheet on an infinitely permeable thick plate

4. Figure 4 shows a simplified model which represents 1-D current flow in a **thick** plate of material which has a fixed magnetic relative permeability of 1000 and an electrical conductivity of 1.0×10^{7} Sm^{-1}. The only component of the time-varying magnetic field strength is in the z-direction.

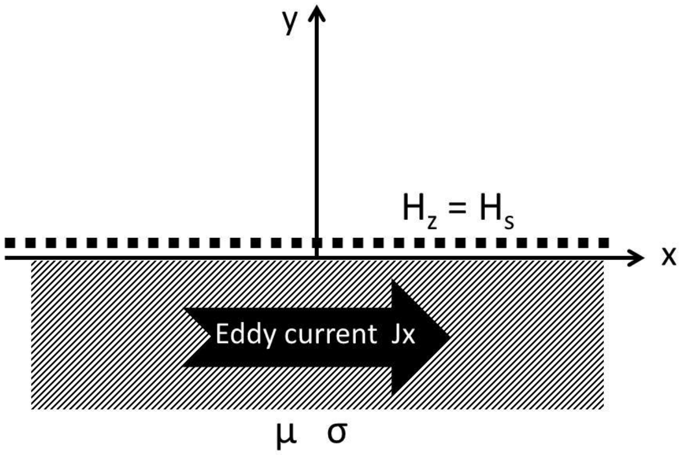

Figure 4 Simplified model which represents 1-D current flow in a thick plate

a. Explain the significance of the plate being defined as 'thick'. **(2)**

b. Calculate the frequency at which the classical skin-depth is 2mm in this material. **(3)**

c. Starting with the full three-dimensional expression for the curl of the magnetic field strength in Cartesian coordinates, demonstrate that the only component of induced current in Figure 4 is J_Z. **(3)**

d. Starting from the general diffusion equation and assuming sinusoidal excitation, show (taking care to define any symbols that you introduce) that the resulting variation with depth into the plate of the magnetic field strength is given by:

$$\frac{\partial^2 H_z}{\partial y^2} = \alpha^2 H_z$$

(4)

e. A solid 6mm diameter, 3m long circular copper conductor carries a sinusoidal current with a frequency of 15 kHz between a piece of equipment and its power supply. The copper has a relative permeability of 1.0 and an electrical resistivity of $1.78\times10^{-8}\Omega$m. Listing any assumption that you make, calculate the DC and AC resistance of this conductor. **(5)**

f. It is proposed to replace the solid conductor with a stranded conductor consisting of a number of parallel circular conductors of smaller diameter to give the same overall cross-sectional area. ***Estimate*** a suitable number of parallel conductors and their diameter if the effective AC resistance is to be reduced to a comparable level to the DC resistance. **(3)**

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