

Derivation of the voltage v_{d1} for the generalised Kron primitive machine voltage matrix, followed by the full KP voltage matrix and the simplified KP voltage matrix.....

In summary:-

Transformer EMF's: $d_1 d_2, d_2 d_1, q_1 q_2, q_2 q_1$

Rotational EMF's: $d_1 q_1, d_1 q_2, q_1 d_1, q_2 d_2$

Instantaneous coil voltages may be written in matrix form: - $[v] = [z][i]$ where $[z] = [R + Lp + Gw]$

e.g. determination of v_{d1}

$$v = iR + \underbrace{L \frac{di}{dt}}_{\text{SELF INDUCED}} + \underbrace{M \frac{di}{dt}}_{\text{ONLY BETWEEN } d_1 \text{ \& \& } d_2} + \underbrace{\left[i \frac{dL}{d\theta} + i \frac{dM}{d\theta} \right] \omega}_{\substack{= 0, \\ \text{since rotor field fixed in space}}} \quad \begin{matrix} q_1 \text{ \& } q_2 \text{ both } \perp \text{ r} \\ \text{to } d_2 \end{matrix}$$

$$\Rightarrow v_{d1} = i_{d1} R_{d1} + L_{d1} \frac{di_{d1}}{dt} + M_{d1d2} \frac{di_{d2}}{dt} + \omega i_{q1} \frac{dM_{d1q1}}{d\theta} + \omega i_{q2} \frac{dM_{d1q2}}{d\theta} = 0.$$

using p to denote d/dt & $G = \frac{dM}{d\theta} + \frac{dL}{d\theta}$

$$\Rightarrow v_{d1} = (R_{d1} + L_{d1} p) i_{d1} + \underbrace{G_{d1q1} \omega i_{q1}}_{\substack{\text{Rot. Volts induced in } d_1 \\ \text{due to current in } q_1}} + M_{d1d2} p i_{d2} + \underbrace{G_{d1q2} \omega i_{q2}}_{\substack{\text{Rot. Volts induced in } d_1 \\ \text{due to current in } q_2}}$$

SUMMERISING:-

The G coefficient is POSITIVE when the FLUX axis LEADS the induced VOLTAGES in the direction of rotation.

e.g. G_{q1d1} d leads q axis \Rightarrow POSITIVE
 $\swarrow \quad \nwarrow$
 INDUCED VOLTS FLUX AXIS

\Rightarrow Matrix reduces from

$$\begin{bmatrix} V_{d1} \\ V_{q1} \\ V_{d2} \\ V_{q2} \end{bmatrix} = \begin{bmatrix} (R_{d1} + L_{d1}p) & G_{d1q1}\omega & M_{d1d2}p & G_{d1q2}\omega \\ G_{q1d1}\omega & (R_{q1} + L_{q1}p) & G_{q1d2}\omega & M_{q1q2}\omega \\ M_{d2d1}p & 0 & (R_{d2} + L_{d2}p) & 0 \\ 0 & M_{q2q1}p & 0 & (R_{q2} + L_{q2}p) \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix}$$

..... to

$$\begin{bmatrix} V_{d1} \\ V_{q1} \\ V_{d2} \\ V_{q2} \end{bmatrix} = \begin{bmatrix} (R_{d1} + L_{d1}p) & -L_{q1}\omega & M_{d1}p & -M_{q1}\omega \\ L_{d1}\omega & (R_{q1} + L_{q1}p) & M_{d1}\omega & M_{q1}p \\ M_{d1}p & 0 & (R_{d2} + L_{d2}p) & 0 \\ 0 & M_{q1}p & 0 & (R_{q2} + L_{q2}p) \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix}$$