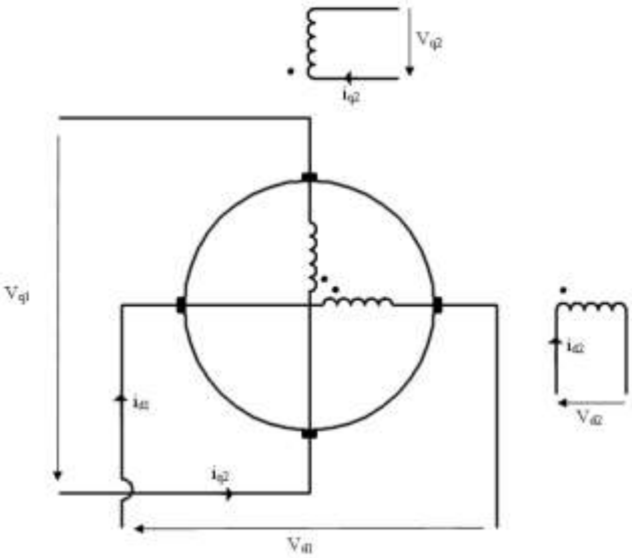
EEE 6120 Modelling of Electrical Machines
2008 Examination Solutions

1.

a)



The general form of the voltage matrix equations is given by:

$$\left|\begin{array}{c} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{array}\right| = \left|\begin{array}{cccc} R_{d1}+L_{d1}p & G_{d1q2}\omega_r & M_{d1d2}p & G_{d1q2}\omega \\ G_{q1d1}\omega_r & R_{q1}+L_{q1}p & G_{q1d2}\omega_r & M_{q1q2}p \\ M_{d2d1}p & 0 & R_{d2}+L_{d2}p & 0 \\ 0 & M_{q2q1}p & 0 & R_{q2}+L_{q2}p \end{array}\right| \left|\begin{array}{c} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{array}\right|$$

b)

On DC:

$$Input\ current = \frac{Input\ \ power}{Input\ voltage} = \frac{800}{200} = 4A$$

Losses in the machine = $I^2R = 4^2 \times 6 = 96W$

$\therefore$Output power = Input power – losses = 800-96 = 704W

$$\therefore \text{Output torque} = \frac{Output\ \ power}{rotor\ angular\ velocity} = \frac{704}{16000 \times \frac{2\pi}{60}} = 0.42Nm$$

$$But\ T = MI_{DC}^{\ 2}$$

$$\therefore M = \frac{T}{I^2} = \frac{0.42}{4^2} = 0.026H$$

On AC:

At the same load torque condition, the same rms current is drawn as the DC case. Hence, the magnitude of the input current is 4A rms

The voltage equation for AC operation is:

$$V = (R + NX_m + j\omega_s L)\ I$$

where N is the ratio of actual speed to synchronous speed

Re-arranging this equation yields:

$$N = \frac{1}{X_m}\left(\sqrt{\left(\frac{V^2}{I^2}\right) - (\omega_s L)^2}\right) - \frac{R}{X_m}$$

For the particular parameters of this motor:

$X_m = \omega M = 2\times\pi\times50\times0.026 = 8.2\Omega$

$\omega_s L = 2\times\pi\times50\times0.12 = 37.7\Omega$

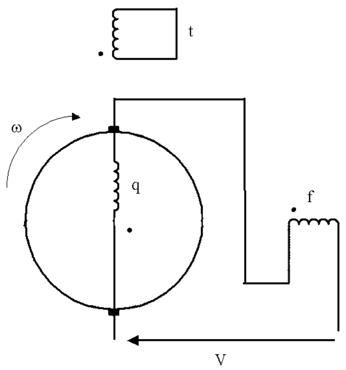
$$N = \frac{1}{8.2}\left(\sqrt{\left(\frac{230^2}{4^2}\right) - (37.7)^2}\right) - \frac{6}{8.2} = 4.56$$

$\therefore$ Actual speed on AC supply = 4.56 × 3000 = 13,688 rpm

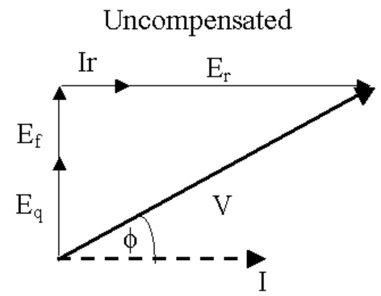
$$\text{Power factor on load} = \frac{R + NX_m}{\sqrt{\left((R + NX_m)^2 + (\omega_s L)^2\right)}} = \frac{6 + (4.56\times8.2)}{\sqrt{\left((6 + (4.56\times8.2))^2 + (37.7)^2\right)}} = 0.754\ \textit{lagging}$$

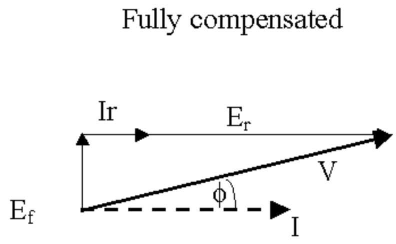
[Note: It is important to state that the power factor is **lagging]**

c) The Kron primitive equivalent of an inductively compensated series universal motor is:



d) The phasor diagrams for the uncompensated and compensated motors are:





e) The torque when operating at 16,000rpm on a 200V DC supply is 0.42Nm. For the compensated motor on a sinusoidal AC supply at the same load torque condition, the same rms current is drawn as the DC case. Hence, the magnitude of the input current is 4A rms.

With ideal compensation, the effective series inductance presented at the terminals is half that of the uncompensated machine. Hence, the speed ratio (actual : synchronous) of the compensated coil is:

$$N = \frac{1}{X_m}\left(\sqrt{\left(\frac{V^2}{I^2}\right) - (\omega_s L)^2}\right) - \frac{R}{X_m}$$

$$N = \frac{1}{8.2}\left(\sqrt{\left(\frac{230^2}{4^2}\right) - (18.85)^2}\right) - \frac{6}{8.2} = 5.89$$

$\therefore$ Actual speed on AC supply = 5.89 × 3000 = 17, 678rpm

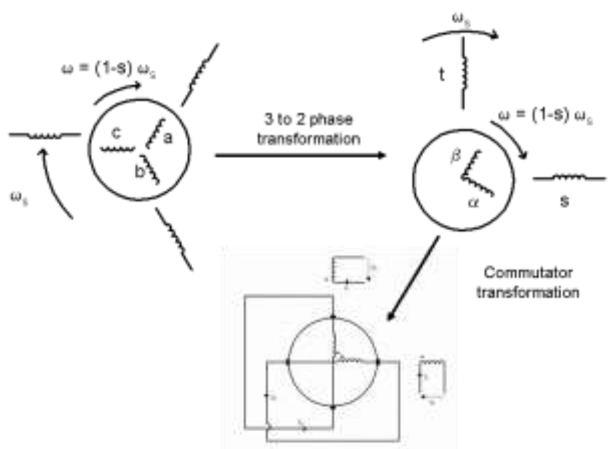
[Note: This is higher than the DC case, but it is important to note that the DC case was for 200V not 230V – for equal voltage you would expect the DC case to run faster]

$$\text{Power factor on load} = \frac{\mathrm{R} + \mathrm{NX_m}}{\sqrt{\left((\mathrm{R} + \mathrm{NX_m})^2 + (\omega_s L)^2\right)}} = \frac{6 + (4.25 \times 8.2)}{\sqrt{\left((6 + (4.25 \times 8.2))^2 + (18.85)^2\right)}} = 0.908 \ \textit{lagging}$$

(i.e power factor improves as expected)

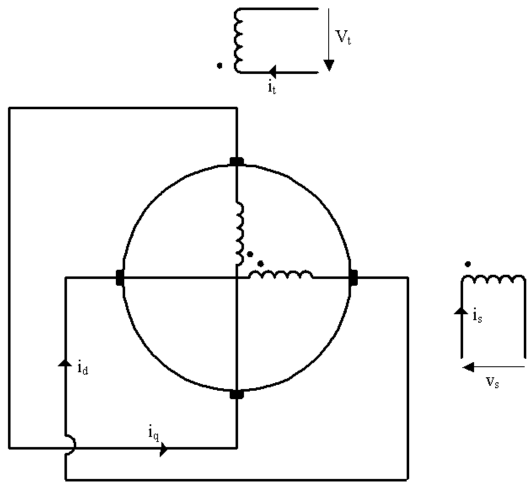
2.

(a) The diagram below shows the key steps involved in transforming a three-phase induction machine to its Kron primitive equivalent:



- Three to two phase transformation – this mathematical converts the three phase coil systems on both the rotor and the stator to two phase equivalent which produce the same rotating fields
- Commutator transformation - Converts the 2 phase rotor coil system into a pseudo-stationary commutator winding

(b) The Kron primitive equivalent of a three-phase induction motor is:



[Important to get various conventions correct – marks lost for incorrect or ambiguous labelling]

Adopting the convention of subscripts of '1' for the stator and '2' for the rotor, then the general form of the voltage matrix equations is:

$$\begin{vmatrix} v_s \\ v_t \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_1+L_1p & 0 & M_{sd}\,p & 0 \\ 0 & R_1+L_1p & 0 & M_{td}\,p \\ M_{ds}p & -M_{dt}\omega_r & R_2+L_2p & -L_2\omega_r \\ M_{qs}\omega_r & M_{qt}\,p & L_2\omega_r & R_2+L_2p \end{vmatrix} \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix}$$

For steady-state operation for a sinusoidal AC supply:

$p=j\omega_s$ and $\omega_r=(1-s)\,\omega_s$

In addition, the same magnitude of applied to the two stator coils and the two rotor coils, but with a 90^o phase difference. Hence,

$$\begin{vmatrix} V_s \\ V_t \\ V_d \\ V_q \end{vmatrix} = \begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix} = \begin{vmatrix} I_1 \\ j\,I_1 \\ I_2 \\ j\,I_2 \end{vmatrix}$$

The governing voltage equation is therefore:

$$\begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & 0 & jX_m & 0 \\ 0 & R_1+jX_1 & 0 & jX_m \\ jX_m & -(1-s)X_m & R_2+jX_2 & -(1-s)X_2 \\ (1-s)X_m & jX_m & -(1-s)X_2 & R_2+jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ jI_1 \\ I_2 \\ jI_2 \end{vmatrix}$$

But row 2 is simply row 1 ×j and row 4 is simply row 3 ×j. Hence the system can be reduced to two matrix equations:

$$\begin{array}{l} \text{Stator} \\ \text{Rotor} \end{array} \quad \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m - j(1-s)X_m & R_2+jX_2 - j(1-s)X_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix}$$

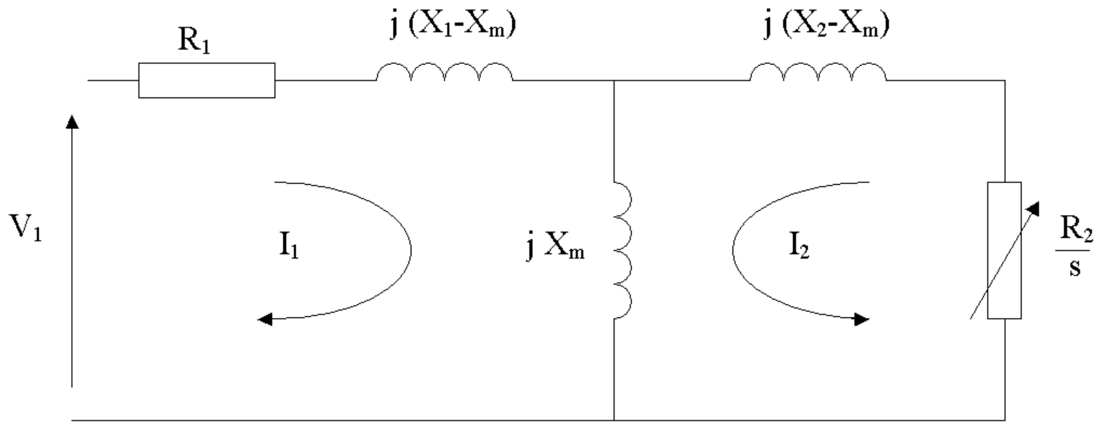
Since the rotor is short circuited, $V_2 = 0$

Substituting for V_2 and dividing the rotor equations by s gives:

$$\begin{array}{l} \text{Stator} \\ \text{Rotor} \end{array} \quad \begin{vmatrix} V_1 \\ 0 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m & R_2/s + jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2' \end{vmatrix}$$

[Note I_2 is transformed to I_2']

An equivalent circuit that satisfies these voltage equations is:

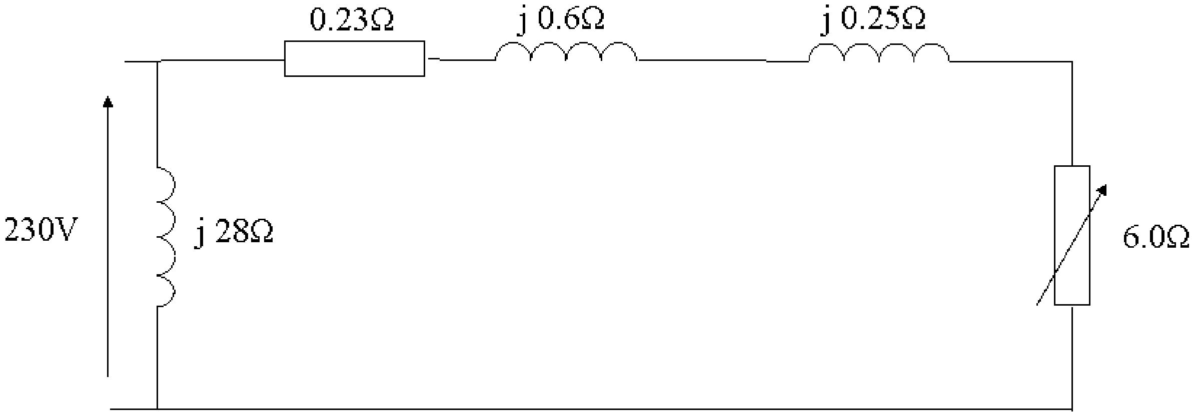


c) [This problem can be solved using either the exact or simplified equivalent circuit. The latter involves moving the magnetising branch to the terminals, but is reliant on the magnetising reactance being significantly higher than the other impedances. Providing students justify the adoption of the simplified version on the basis on the values presented in the question then the use of the simplified equivalent circuit is equally as valid in terms of the marks awarded. Simply starting with the simplified circuit without noting the basis for this will result in a deduction of marks]

The synchronous speed of a 4-pole motor on a 50Hz supply is 1500rpm. At 1470 rpm the slip is therefore (1500-1470)/1500 = 0.02

Hence, $\frac{R_2'}{2} = \frac{0.12}{0.02} = 6.0\Omega$

The approximate equivalent circuit of for one phase of the machine is therefore given by:



The impedance of the main branch of the circuit is given by:

$$Z_e = (0.23 + 6.0) + j0.85 \;\; = 6.29\angle 7.8^o \Omega$$

The current through this main path, I_1 is therefore given by:

$$I_1 = \frac{230}{6.29\angle 7.8^o} = 36.6\angle -7.8^o = 36.3 - j5.0A$$

The current in the magnetising branch is given by:

$$I_m = \frac{230}{28\angle 90^o} = 8.2\angle -90^0 = 0 - j8.2A$$

The net input current is hence given by:

$$I_{ip} = I_1 + I_m = (36.3 - j5.0) - j8.21 = 36.3 - j13.2 \;\; or \; 38.6\angle -20^o A$$

Hence, magnitude of phase current is 38.6A

The power factor is hence cos (-20^o) = 0.94 lagging

[Note: Important to note that power factor is ***lagging***]

The electromagnetic output power is given by:

$$P_{out} = 3\,|I_1|^2 \frac{(1-s)R_2'}{s} = 3 \times 36.6^2 \times 6.00 = 24.11kW$$

Hence the electromagnetic output torque is given by:

$$T_{out} = \frac{P_{out}}{\omega_{mech}} = \frac{23720}{1470 \times \frac{2\pi}{60}} = 156.6Nm$$

3.

a) The change in co-energy in moving from the unaligned to the aligned positions can be estimated by applying the trapezium rule to integrate the area under the aligned and unaligned curves.

Fully aligned curve:

$$A_{0\to1} = \frac{\Psi_1}{2} = \frac{0.6}{2} = 0.30 \text{ J}$$

$$A_{1\to2} = \frac{\Psi_1 + \Psi_2}{2} = \frac{0.6 + 1.02}{2} = 0.81\text{J}$$

$$A_{2\to3} = \frac{\Psi_2 + \Psi_3}{2} = \frac{1.02 + 1.18}{2} = 1.10\text{J}$$

$$A_{3\to4} = \frac{\Psi_3 + \Psi_4}{2} = \frac{1.18 + 1.24}{2} = 1.21\text{J}$$

$$A_{4\to5} = \frac{\Psi_4 + \Psi_5}{2} = \frac{1.24 + 1.28}{2} = 1.26\text{J}$$

For a current of 2A, total co-energy in aligned position:

$$A_{0\to2} = A_{0\to1} + A_{1\to2} = 1.11\text{J}$$

For a current of 5A, total co-energy in aligned position:

$$A_{0\to5} = A_{0\to1} + A_{1\to2} + A_{2\to3} + A_{3\to4} + A_{4\to5} = 4.68\text{J}$$

The same procedure could be applied to the un-aligned characteristic, but it is also reasonable to assume that the characteristic is linear, and hence the co-energies in the unaligned positions for 2A and 5A are hence given by:

$$U_{0\to2} = \frac{2\Psi_2}{2} = 0.075 \text{ J}$$

$$U_{0\to2} = \frac{5\Psi_5}{2} = \frac{5 \times 0.2}{2} = 0.5\text{J}$$

Hence the co-energy change between the unaligned and aligned positions at 2A and 5A are:

$$\Delta W_2^{'} = A_{0\to5} - U_{0\to5} = 1.03\text{J at 2A}$$

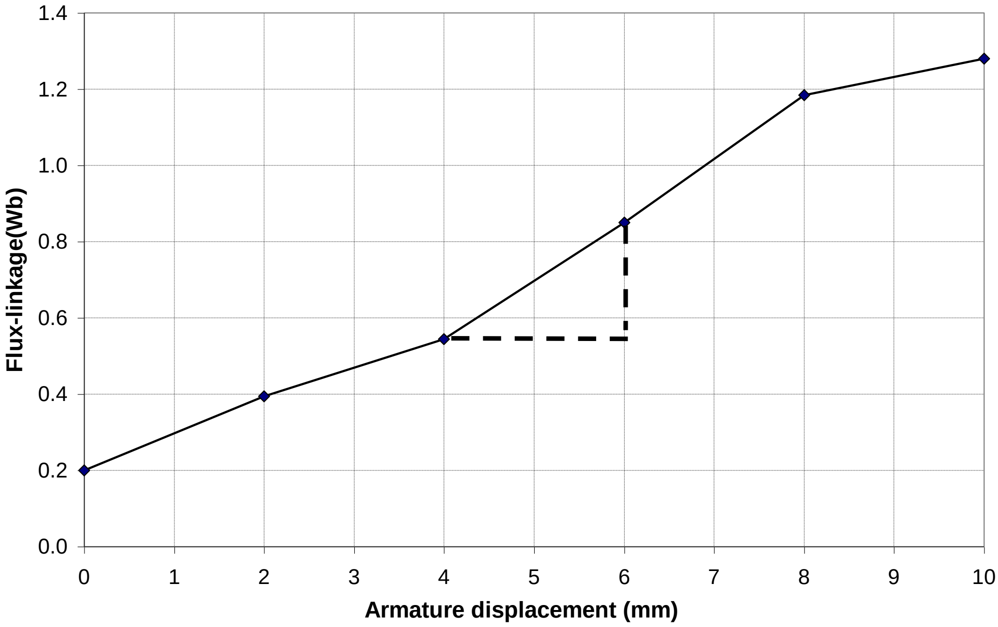
$$\Delta W^{'}{}_2 = A_{0\to5} - U_{0\to5} = 4.18\text{J at 5A}$$

Hence the forces are given by:

$$F_2 = \frac{\Delta W_2^{'}}{\Delta x} = \frac{1.03}{0.01} = 103N \text{ at 2A}$$

$$F_5 = \frac{\Delta W_2'}{\Delta x} = \frac{4.18}{0.01} = 418N \text{ at 5A}$$

b) The data for 5A can be plotted as a flux-linkage versus displacement characteristic



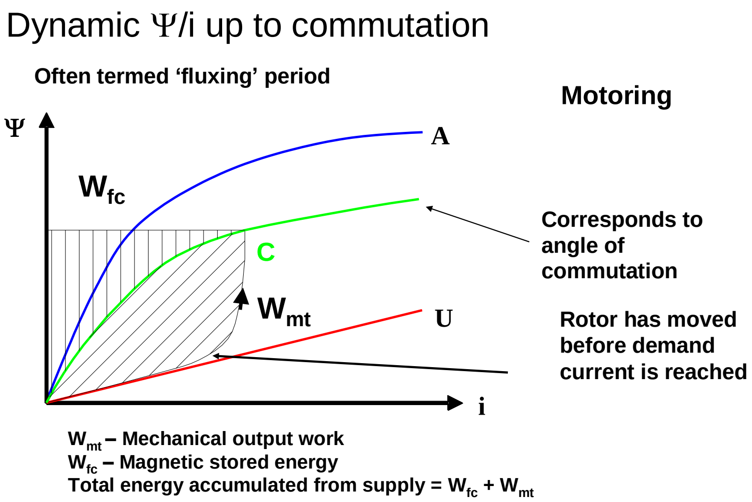
The average rate of change of flux-linkage with armature displacement can be estimated from the graph:

$$\frac{d\psi}{dx} \approx \frac{0.85 - 0.545}{0.002} = 152 \ Wb/m$$

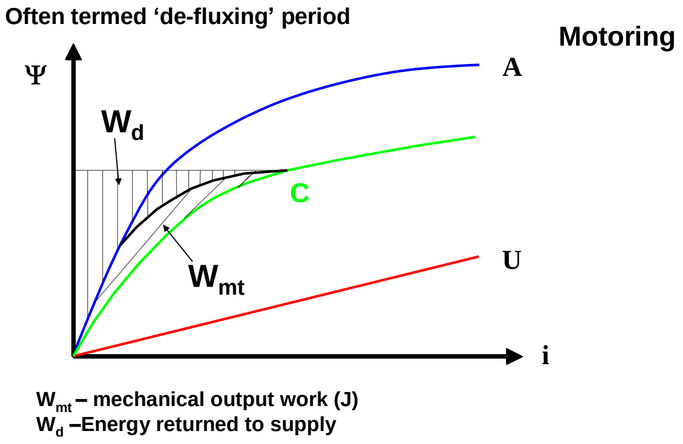
Hence, at 1.4 m/s, the induced emf is given by:

$$e = \frac{d\Psi}{dt} = \frac{d\Psi}{dx} \times \frac{dx}{dt} = 152 \times 1.4 = 213V$$

c)



Dynamic Ψ/i after commutation



d) From the aligned Ψ-I characteristic it can be seen that the onset of saturation occurs at a current of 2A (an answer based on a slightly different interpretation of saturation is equally acceptable).

Since $B_g \approx \dfrac{\mu_0 N_{coil} I}{2l_g}$ prior to saturation (i.e. at 1.4T) then a reasonable estimate of l_g can be obtained from this equation.

$$\therefore l_g = \frac{\mu_0 N_{coil} I}{2B_g} = \frac{4\pi \times 10^{-7} \times 200 \times 2}{2 \times 1.4} = 0.18 \ mm$$

But this is made up of two series airgaps, and hence the actual airgap between the stator and armature is 0.09mm

[a reasonable error band on this value is acceptable given the difficulty in precisely defining the onset of saturation – the method employed is the key factor in determining the marks awarded]

4.

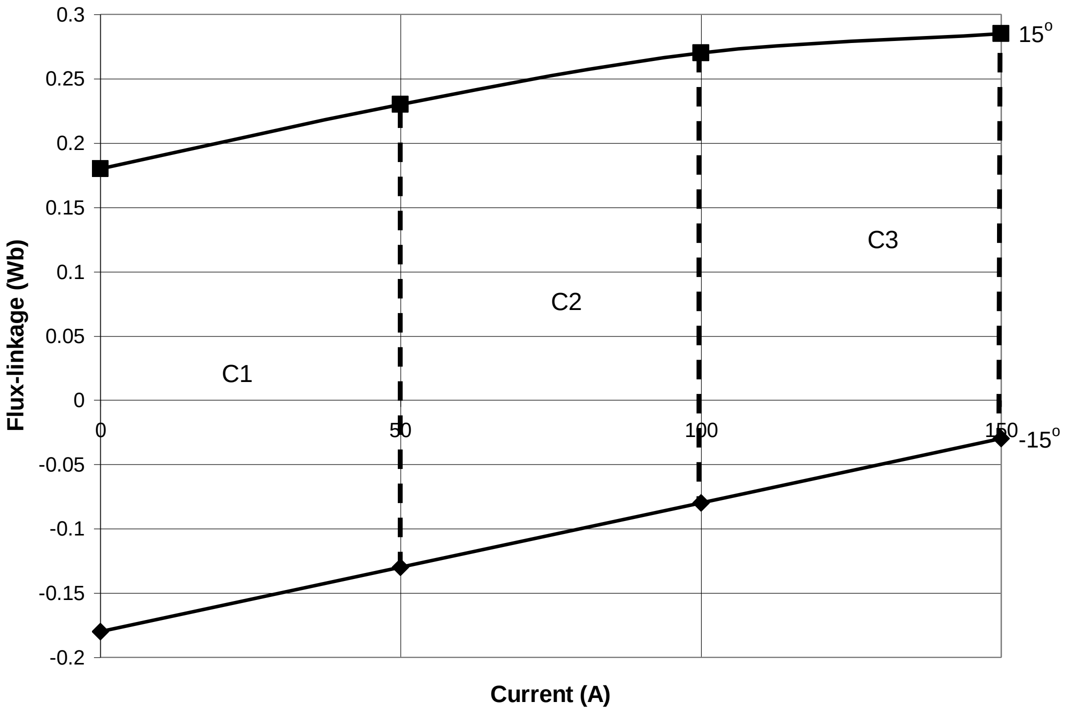
a) The flat-top of the nominally trapezoidal emf corresponds with the linear region of the flux-linkage versus rotor angular displacement characteristics. Although the slope can be taken at any point along the linear section, the region around an angular displacement of 0° mechanical is as reliable an estimate as any. The slope of the characteristic is:

$$\frac{d\psi}{d\theta} \approx \frac{0.06 + 0.06}{10 \times \dfrac{\pi}{180}} = 0.686 \ Wb/rad(mech)$$

Hence, at 1.4 m/s, the induced emf is given by:

$$e = \frac{d\Psi}{dt} = \frac{d\Psi}{d\theta} \times \frac{d\theta}{dt} = 0.686 \times \frac{8000 \times 2\pi}{60} = 576V$$

b) In brushless DC mode, currents are driven into a phase between rotor angular displacements of -60° to +60° (elec) which corresponds to -15° to +15° on the characteristics of this 8 pole machine. Re-plotting the key date points as a ψ-I cruves yields:



The co-energy changes C_1, C_2 and C_3 are given by:

$$C_1 = \frac{(0.36 + 0.36)}{2} \times 50 = 18.0J$$

$$C_2 = \frac{(0.36 + 0.35)}{2} \times 50 = 17.75J$$

$$C_3 = \frac{(0.35 + 0.315)}{2} \times 50 = 16.62J$$

Average torque during 120° commutation interval for one phase at 50A:

$$T_{ave}\Big|_{50A} = \frac{C_1}{\Delta\theta} = \frac{18}{30 \times \frac{\pi}{180}} = 34.4Nm$$

And similarly at 150A:

$$T_{ave}\Big|_{150A} = \frac{C_1 + C_2 + C_3}{\Delta\theta} = \frac{52.4}{30 \times \frac{\pi}{180}} = 100.0Nm$$

(i.e. less than 3×50A value due to saturation)

The average torque produced by the machine is 2 × values calculated above (comes from 3 × 2/3 utilisation) rather than 3 times value above (common mistake) – 2 marks deducted for this error

Hence, average machine torque at 50A is 68.8Nm and at 150A is 200Nm

c) The various inductances can be calculated from the additional flux-linkage produced by the current. For the 4 cases :

At 50A:

$$L = \frac{0.05}{50} = 1mH \quad at \ -22.5^o$$

$$L = \frac{0.05}{50} = 1mH \quad at \ +22.5^o$$

At 150A:

$$L = \frac{0.15}{150} = 1mH \quad at \ -22.5^o$$

$$L = \frac{0.088}{150} = 0.59mH \quad at \ +22.5^o$$

The difference observed at 150A and +22.5^o is a result of magnetic saturation this is the worst case angular displacement at which the magnet flux and the coil flux add to each other

d) The exact nature of the flux-paths in a multi-phase brushless DC machine is complex. However, a reasonable assumption in terms of calculating the total effective magnetic airgap (i.e. magnet length + airgap length) is to assume that each coil produces the mmf across a single combined airgap and magnet length.

Assuming that the airgap flux density provides a reasonable estimate of the flux density level in the core as a whole then:

By inspection of the flux-linkage characteristics, the core of the machine appears to be saturating at a flux-linkage of ~0.27Wb, which corresponds to a flux density of ~1.6T.

Considering the case of a rotor angular displacement of 0º (in which no net magnet flux is present). A current of 50A produces a flux-linkage of 0.05Wb, which by equivalence with 1.6T at 0.27Wb, corresponds to a flux density of 0.30 T

At this flux density level, then it is reasonable to assume that the rotor and stator cores will be infinitely permeable. The airgap flux density produced by a given coil mmf is given by:

$$B_g = \frac{\mu_0 NI}{l_{eff}}$$

Re-arranging this equation yields:

$$l_{eff} = \frac{\mu_0 NI}{B_g} = \frac{4\pi \times 10^{-7} \times 200 \times 50}{0.3} = 41.8\text{mm}$$

Therefore, length of magnet 40.8mm

[This is a very large magnet thickness – probably greater than would be used in practice but is the correct solution in this case]