Noise

1 Introduction

In general, the term "noise" is used to describe any unwanted signal but sources of noise can be divided into three main categories:

> **Man - made** - This is noise that originates in some form of human activity. Some examples are; unwanted intrusion of radio communication into audio systems, poorly suppressed commutation in brushed ac motors and the reception of terrestrially generated signals in astronomical radio telescopes.
>
> **Natural external** - This is noise that is caused by natural electromagnetic disturbances in our environment. The lightning associated with thunderstorms and high energy cosmic rays are two examples of the most energetic events.
>
> **Natural internal** - This is the hiss that you hear if you listen to your stereo with a high volume setting but no programme material. It is caused by the individual behaviour of electrons as they pass through electronic components.

Man - made noise can usually be reduced at source by the application of appropriate suppression or screening strategies. There exists a comprehensive set of legislation which defines how much noise a piece of modern equipment is allowed to emit either as radiation or back into the main power supply; considerations such as these fall under the term "electromagnetic compatability". The effects of natural external noise can be reduced by careful screening of sensitive circuitry and appropriate filtering on all input and output connections to the system. Natural internal noise is generated within the resistors, diodes and transistors that make up a system, so in that sense it is the enemy within. It is natural internal noise which is of interest in what follows. After discussing how a random noise signal is quantified and characterised, the effects of noise in circuits and systems are examined and, where possible, design approaches that minimise the effects of internal noise are mentioned.

2 Quantifying and Characterising Random Noise

(i) Amplitude

If looked at on an oscilloscope, random noise looks like a thick woolly horizontal trace with no detectable coherent shape, as shown in figure 1. The noise voltage waveform, $v_n(t)$, is a completely random variable - in other words, knowledge of its value at one instant of time offers no information about its value at the next instant and the usual concept of amplitude is meaningless. Thus changing the timebase setting of the oscilloscope will leave the trace unchanged. (This is true providing the time-

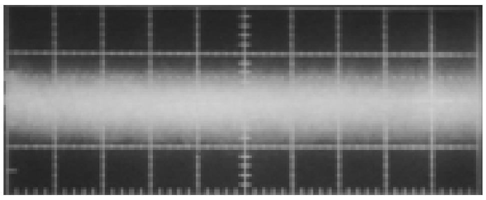

Figure 1

An analogue oscilloscope trace showing the appearance of random noise. The lack of coherence in the noise signal causes succesive traces to follow different paths across the screen. A crude idea of the distribution of noise amplitudes can be inferred from the way trace intensity changes from its centre to its extremes.

base setting is not such that the frequencies being viewed are close to the oscilloscope's bandwidth limit. The finite bandwidth of the oscilloscope gives a visible degree of coherence to the noise waveform in the immediate vicinity of the trigger point at faster timebase settings.) Digital ocilloscopes will display noise in a slightly different way.

The time average value of the sort of random waveform of interest here is zero, ie

$$\frac{1}{t_1}\int_0^{t_1} v_n(t)dt \;\to\; 0 \;\text{ as }\; t_1 \;\to\; \infty\,.$$

Although the time average value of $v_n(t)$ is zero, its instantaneous value is non - zero for most of the time and consequently it is capable of dissipating power in a resistor. This power dissipation capability is used to obtain some useful measure of noise amplitude. Power dissipation is proportional to the "mean square" value of a voltage (or current) waveform so the amplitude of $v_n(t)$ is usually expressed as a mean square value, $\overline{v_n^2}$ (with units of V^2) or as a root mean square (rms) value , $\sqrt{\overline{v_n^2}}$ (with units of V). Some writers say that the peak to peak value of random noise is six times its rms value (or peak is three times rms). They mean that from a *statistical* point of view the peak to peak value of the noise is less than six times the rms value for 99.7% of the time, but it should always be remembered that, since the instantaneous behaviour of a noise voltage cannot be predicted, this figure is only an approximation.

(ii) Frequency Distribution

Noise tends to have its most serious effects in circuits dealing with small signals, such as audio amplifier front ends for tape heads or microphone, satellite TV receivers and radar/radio systems. Since all these systems have particular bandwidths, it is useful to know how much noise energy exists within a particular bandwidth.

Being a random function of time, the noise voltage $v_n(t)$ contains energy over a wide range of frequencies. The frequency distribution of the noise is described in terms of mean square voltage per unit bandwidth. Imagine that $v_n(t)$ is put through a tuneable filter with a bandwidth δf centred on f_o, where δf is small enough to be singling out only the f_o frequency component of $v_n(t)$. Imagine also that the filter output is fed to a mean square voltmeter whose output, $\overline{v_n^2}(f_o)$, represents the noise power within the bandwidth δf. Such a hypothetical system is shown in figure 2.

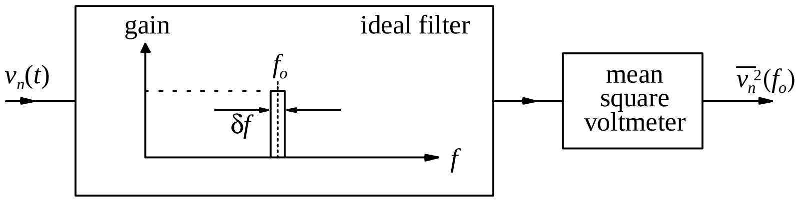

Figure 2

A notional system to measure the power spectrum of a noise signal.

If f_o is tuned over all frequency and $\dfrac{\overline{v_n^2}(f_o)}{\delta f}$, = $\overline{v_n^2}(f)$, the noise spectral density with units of $V^2\,Hz^{-1}$, is plotted as a function of frequency, the resulting curve, known as the "power spectral density" of the noise, describes how the noise power delivering capability of $v_n(t)$ varies with frequency.

In most cases of interest for natural internal noise, the power spectral density is independent of frequency. This means that the noise has the same energy for a unit bandwidth placed anywhere on the frequency scale. Such noise is called "white" by analogy with white light. There are some noise sources that have a power spectral density curve that gets smaller as frequency increases; such noise is known as "pink", again by analogy with light. Figure 3 shows the power spectral density curves for white and pink noise.

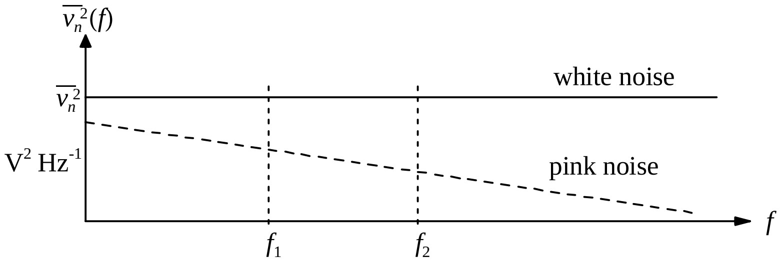

Figure 3
The power spectra of white noise and pink noise.

To find the total mean square noise voltage within a given frequency range, the power spectral density curve must be integrated over the frequency range of interest. This is straightforward for the case of white noise since its power spectral density is constant, so over a bandwidth of $(f_2 - f_1)$, the total mean square voltage is:

$$\overline{v_{nT}^2} = \int_{f_1}^{f_2} \overline{v_n^2}(f)df = \overline{v_n^2}(f_2 - f_1) = \overline{v_n^2}B = \overline{v_n^2}\Delta f \quad \mathrm{V}^2$$

where $(f_2 - f_1)$, B and Δf are three commonly used symbols for bandwidth. For pink noise the integral is the same but the result is not as straightforward because $\overline{v_n^2}(f)$ is a function of frequency. It follows that $\overline{v_n^2}(f)$ must be known in order to evaluate the integral.

3 Internal Noise Sources

There are several types of noise source in electronic devices. The two most important are thermal noise and shot noise and these are dealt with below. A third noise source, flicker noise is also mentioned below. There are sources such as partition noise which, although important in multi-electrode vacuum tubes and some solid state devices, will not be considered here.

(i) Thermal or "Johnson" noise - (white)

Thermal noise is caused by the random thermal motion imparted to electrons by collision interactions with the structure of the resistive medium through which the electrons are travelling. It is modelled as a Thevenin equivalent as shown in figure 4.

Thermal noise is given by:

$$\overline{v_n^2} = 4kTR \ \mathrm{V^2\ Hz^{-1}}$$
$$= 4kTR\,\Delta f \ \ \mathrm{V}^2$$

where:

k = Boltzmann's constant (1.38×10^{-23} J K^{-1})
T = Absolute temperature
R = Static resistance
Δf = Bandwidth

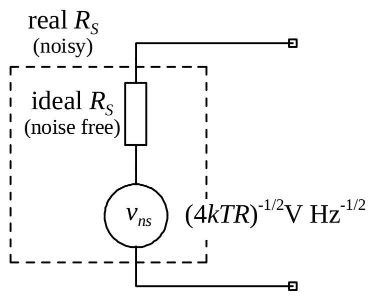

Figure 4
The noise equivalent circuit of a resistor

Note that thermal noise is **not** generated by incremental, slope, dynamic or differential resistances. Only static resistances which obey Ohm's law generate noise.

(ii) Shot noise - (white)

Shot noise is due to the fact that current is not continuous but composed of packets of charge. It is caused by the individual way in which electrons cross potential barriers in devices such as p-n junctions and thermionic diodes. The noise is modelled by a noise current generator in parallel with the noise free diode slope resistance as shown in figure 5.

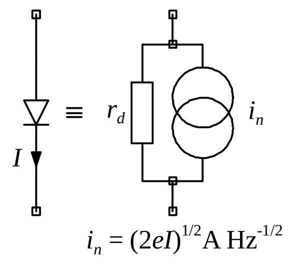

Figure 5

The noise equivalent circuit of a diode. $r_d = kT/eI$ for a p-n junction

Shot noise is given by:

$$\overline{i_n^2} = 2eI \text{ A}^2 \text{ Hz}^{-1}$$
$$= 2eI\Delta f \text{ A}^2$$

where:

e = electronic charge (1.6×10^{-19} C)
I = diode current (A)
Δf = bandwidth of interest

(iii) "1/f" or "flicker" noise - (pink)

There are many other names that have been used to describe this type of noise, partly because its origins are obscure and partly because its behaviour varies widely in different devices. The name "1/*f*" is probably the most useful of the names because it gives an approximate idea of how the noise power spectral density varies with frequency; other names tend to reflect the subjective impressions that this type of noise has created in its observers.

1/*f* noise affects all solid state devices and many other electronic components. It usually takes the form of an excess noise that increases in proportion to the current flowing through the component of interest. Evidence suggests that it is related to technological defects, since its effects have reduced as technology has improved. Manufacturers of devices such as op - amps specify 1/*f* noise by means of a

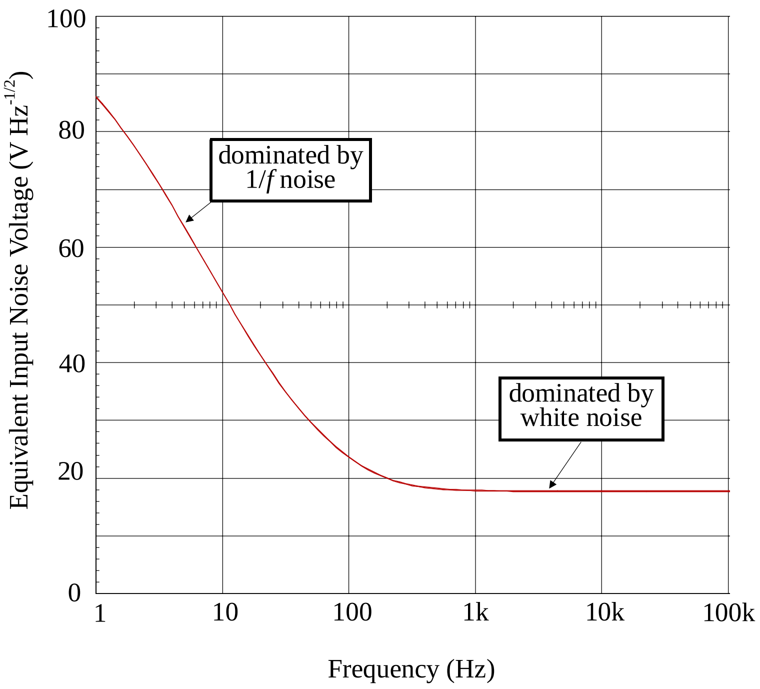

Figure 6

A typical relationship between equivalent input noise voltage and frequency as specified by op-amp manufacturers graph of equivalent input noise voltage spectral density (V $\mathrm{Hz}^{-1/2}$) against log (frequency) as shown in figure 6. Some manufacturers use a logarithmic vertical scale and others use a linear one. Figure 6 represents the sum of $1/f$ and white noise sources; at low frequencies behaviour is dominated by $1/f$ noise, while at high frequencies white noise dominates. At one particular frequency, $1/f$ and white sources contribute equally (in power terms) to the total noise and this occurs when $\sqrt{\overline{v_n^2}}$ has increased by a factor of 1.4 above its high frequency (white) value. This value of frequency is known as the "$1/f$ corner frequency" and, for modern op-amps, it lies typically between 100Hz and 1000Hz.

$1/f$ noise is described by the relationship $\overline{v_n^2}(f) = K/f^{\alpha}$ where K is a constant and α lies typically between 0.7 and 1.4, although it can occasionally be as high as 2. To work out the total noise over an interval $f_2 - f_1$ it is necessary to integrate $\overline{v_n^2}$ over that interval. Assuming $\alpha = 1$,

$$\overline{v_{nT}^2} = \int_{f_1}^{f_2} \overline{v_n^2}(f)df = \int_{f_1}^{f_2} \frac{K}{f} df = K \ln\left(\frac{f_2}{f_1}\right) \ \mathrm{V}^2$$

This result means that true $1/f$ noise has constant power per proportional bandwidth (f_2 / f_1), eg, constant power per decade or per octave, rather than the constant power per absolute bandwidth ($f_2 - f_1$) behaviour of white noise.

4 Noise Sources in Circuits

(i) Maximum Available Power

Maximum available power is the maximum noise power that can be transmitted from one resistor to another.

Consider a resistance R_S feeding a resistance R, as shown in figure 7. R_S is represented by its noise equivalent circuit consisting of a noise free R_S in series with a noise voltage generator. The questions to be answered are:

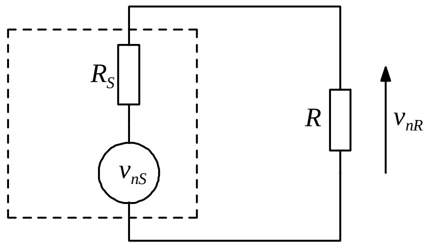

Figure 7

The circuit for the calculation of maximum available power.

a) What value of R will maximise the noise power delivered to R from R_S?
b) What is the maximum power transferred?

The answer to the first question is when $R = R_S$. The derivation of this standard result is straightforward and can be found in any textbook on circuit theory.

The answer to the second question can be found by evaluating the power delivered to R by v_{nS} when $R = R_S$. The voltage across R is given by:

$$v_{nR} = v_{nS} \ \frac{R}{R + R_S} = \frac{v_{nS}}{2} \ \text{ when } R = R_S .$$

The power dissipated in R is therefore

$$P_R = \frac{\overline{v_{nS}^2}}{4R_S} = \frac{4kTR_S}{4R_S} = kT \ \ \mathrm{W\,Hz^{-1}} \ \text{ or } \ P_R = kT\Delta f \ \ \mathrm{W}.$$

Note that the maximum available noise power is independent of the value of the source resistance, R_S, even though the noise power generated by R_S is proportional to its value. It is tempting to think that, because energy is being transferred from R_S to R , R will heat up. In fact, when the system is in thermal equilibrium, ie R and R_S at the same temperature as the environment, the same energy will flow from R to R_S as flows from R_S to R and there is no net energy transfer between the resistors. If however R_S was maintained at an elevated temperature with respect to R, there would be a net energy flow from R_S to R which would continue until the two resistors were at the same temperature. Thermal noise can be thought of as a way of moving energy around in order to achieve thermal equilibrium.

(ii) Addition of noise sources

Figure 8 shows two uncorrelated noise voltage sources. The term uncorrelated means that knowledge of the value of one source at an instant in time gives no information about the other source at any instant of time; in other words, the two sources are completely independent of one another.

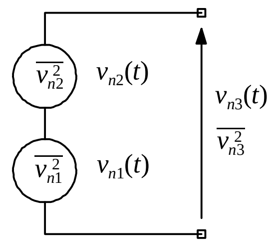

Figure 8

Noise sources in series

At any instant in time, the value of v_{n3} must be the sum of v_{n1} and v_{n2} and since that is true for all instants of time,

$$v_{n3}(t) = v_{n1}(t) + v_{n2}(t)$$

Although true, this relationship is not much help in quantifying the sum for the reasons outlined in section 2 (i). The noise is usefully quantified by its mean squared value so the question of interest is how does $\overline{v_{n3}^2}$ relate to $\overline{v_{n1}^2}$ and $\overline{v_{n2}^2}$? An answer to this question can be found as follows:

$$\overline{v_{n3}^2} = \overline{v_{n3}^2(t)} = \overline{(v_{n1}(t) + v_{n2}(t))^2} = \overline{v_{n1}^2(t)} + \overline{v_{n1}(t)v_{n2}(t)} + \overline{v_{n2}^2(t)}$$

But $\overline{v_{n1}^2(t)} = \overline{v_{n1}^2}$, $\overline{v_{n2}^2(t)} = \overline{v_{n2}^2}$ and, since $v_{n1}(t)$ and $v_{n2}(t)$ are uncorrelated, $\overline{v_{n1}(t)v_{n2}(t)} = \overline{v_{n1}(t)}\,.\,\overline{v_{n2}(t)}$. Both $\overline{v_{n1}(t)}$ and $\overline{v_{n2}(t)}$ are equal to 0, $\overline{v_{n1}(t)v_{n2}(t)} = 0$ and so

$$\overline{v_{n3}^2} = \overline{v_{n1}^2} + \overline{v_{n2}^2}$$

In other words it is the mean square values of the noise voltage sources which must be added to find the overall noise from a number of sources: noise powers rather than noise voltages add linearly. This is the same as the procedure that must be followed to work out power dissipated by the sum of two sinusoids of different frequency.

(iii) Effects of an *RC* low pass circuit on white noise

Passing white noise through a low pass filter will give the noise a pink hue because the white noise spectral density will be modified by the transmission properties of the low pass filter; high frequency components of noise will suffer some attenuation. The exact nature of the pinkness will depend upon the nature of the low pass filter's power transmission response which is the square of the modulus of the filter's amplitude reponse. The circuit of interest here is a first order *RC* low pass circuit, although the same analytical approach can be used with any frequency dependent circuit. The first order low pass *RC* circuit occurs frequently in reality and the question "What is the total output noise voltage from such a circuit?" leads to an interesting result.

Consider the circuit of figure 9. The voltage v_{onT} is the r.m.s. value of the total output noise obtained by integrating the spectral density at the output, $\overline{v_o^2}(\omega)$, over all frequency. The spectral density at the output is given by:

$$\overline{v_o^2}(\omega) = \overline{v_n^2}\left|\frac{1}{1 + j\omega C R_S}\right|^2 = \frac{\overline{v_n^2}}{1 + \omega^2 C^2 R_S^2}$$

and thus, $\overline{v_{onT}^2} = \int_0^\infty \overline{v_o^2}(\omega) = \int_0^\infty \frac{\overline{v_n^2}\,df}{1 + a^2 f^2}$

where $a = 2\pi C R_S$.

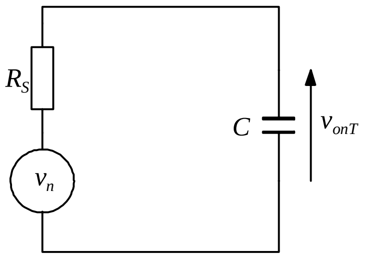

Figure 9

A low pass RC circuit fed by a white noise voltage source. The noise due to R is included in v_n

This integral is solved by using the substitution $af = \tan x$.

Thus $(1 + a^2 f^2) \to (1 + \tan^2 x) = \sec^2 x$ and $df \to \frac{\sec^2 x}{a}\,dx$, so the integral becomes:

$$\overline{v_{onT}^2} = \int_0^\infty \frac{\overline{v_n^2}\,df}{1 + a^2 f^2} = \int_0^{\frac{\pi}{2}} \frac{\overline{v_n^2}\sec^2 x\,dx}{a \sec^2 x} = \overline{v_n^2}\int_0^{\frac{\pi}{2}} \frac{dx}{a} = \frac{\overline{v_n^2}}{4C R_S}\ \mathrm{V}^2$$ which, if $\overline{v_n^2}$ = the noise $\overline{v_{nS}^2}$ due to the resistor itself, $\overline{v_{onT}^2}$ becomes:

$$\overline{v_{onT}^2} = \frac{\overline{v_{nS}^2}}{4C R_S} = \frac{4kT R_S}{4C R_S} = \frac{kT}{C}\ \mathrm{V}^2$$ where T is the temperature of the resistor.

This result is interesting because it shows that when the noise in the circuit is the thermal noise associated with the resistor, the total noise voltage across the capacitor is independent of the resistor value. All real resistors must have some capacitance associated with them - of the order of 1pF for a typical 0.25W resistor - the total mean square noise voltage across a real resistor is given by kT/C V^2. Furthermore, although in principle noise free, a capacitor on its own could be considered as being in parallel with a resistance of value approaching infinity and one would therefore expect to find a total mean square noise votage of kT/C V^2 across its terminals.

(iv) Noise temperature

If the magnitude of the noise source in figure 9 is not what would be expected from the resistor on the basis of its temperature, ie if v_n consists of the resistor noise and some other noise source, it is possible, providing the source is white, to ascribe to the resistor an effective temperature T_e such that the rms noise voltage generated by the resistor alone at T_e accounts for v_n. The temperature T_e is known as the *noise temperature* of the resistor and is simply the notional temperature to which the resistor must be raised in order to account for all the white noise sources in series with it. Thus for the circuit of figure 9 the noise temperature of R_S is found by equating the total mean square noise voltage in series with R_S, including the noise generated by R_S at its actual temperature, to the mean square noise expected from the same resistance at some elevated temperature, T_e. Remembering that v_n in figure 9 includes all the noise sources in series with R_S, T_e is given by:

$$\overline{v_n^2} = 4kT_eR_S \text{ V}^2 \quad \text{or} \quad T_e = \overline{v_n^2}/4kR_S \text{ K}$$

Thus for a circuit such as that of figure 9, the total mean square noise voltage across C can be found for any magnitude of white noise source v_n by finding the noise temperature, T_e , of R_S and using T_e as the temperature term in kT/C .

Noise temperature is an important concept in communication system analysis and design and its application in that area is dealt with in the next section.

5 Noise in Systems

When dealing with the noise performance of a system, it is often impractical to analyse the system noise behaviour in terms of the individual sources of noise simply because of the number of noise sources involved. The approach usually taken when considering system noise is to represent observed noise behaviour as one or more equivalent noise generators associated with the system. This section describes the parameters used to describe noise effects in systems.

(i) Signal-to-Noise ratio

Consider the amplifier of figure 10. The input signal to noise ratio is defined as:

$$\frac{S_i}{N_i} = \frac{\text{input signal power}}{\text{input noise power}}$$

and the output signal to noise ratio as:

$$\frac{S_o}{N_o} = \frac{\text{output signal power}}{\text{output noise power}}$$

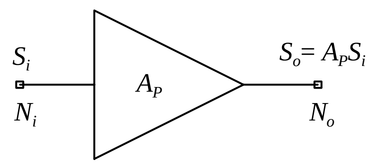

Figure 10

A signal amplifier with a power gain A_P

Signal-to-noise ratio is usually expressed in dB and since it is a power ratio, dB are found by taking 10log(S/N). It can be measured at any point in a system and is essentially a measure of signal quality at that point in the system.

Although the signal to noise ratio will vary throughout a real system, of itself it gives no information about the noise performance of the system. Indeed it gives no information about the magnitude of the noise unless the signal level at which it was measured is specified. To identify the noise performance of a system, the signal to noise ratio at input and output must be compared.

(ii) Noise factor

For a system such as that of figure 10, the noise factor, F, is defined as:

$$F = \frac{\text{signal to noise ratio at the input}}{\text{signal to noise ratio at the output}} = \frac{S_i/N_i}{S_o/N_o} \qquad \textbf{(5.1)}$$

If the power gain of the system is A_P, as in figure 10, this relationship can be written:

$$F = \frac{N_o}{A_PN_i} = \frac{\text{noise power at the output of the real amplifier}}{\text{noise power that would appear at the amplifier output if it were perfect}} \qquad \textbf{(5.2)}$$

This expression for noise factor is very useful for working out the noise factor of systems and will be used later to investigate the noise behaviour of multistage impedance matched amplifiers such as those that might be found in satellite or radar receivers. The noise power at the output of an impedance matched system is simply the available noise power at the input multiplied by the power gain.

For low frequency unmatched circuits, such as transistor and operational amplifier based amplification circuits, the concept of power gain is often not very useful. It is, however, possible to work in terms of mean-square voltages rather than powers because at a single system node the ratio of mean square voltages is the same as the ratio of powers. When working with mean square voltage quantities the system gain must be expressed in terms of the square of its voltage gain and the input noise is the mean square noise voltage appearing at the input because of the source circuit. The significance of noise factor in low frequency unmatched systems depends on the nature of the system; minimising noise factor in audio amplifiers, for example, does not necessarily lead to an amplifier with the most desirable noise properties.

The output noise, N_o, can be written as the sum of two components; the amplified input noise and the noise added by the amplifier. Thus $N_o = A_P N_i + N_A$, where N_A is called the "added noise" contributed by the amplifier. The expression describing F can thus be developed to:

$$F = \frac{N_o}{A_P N_i} = \frac{A_P N_i + N_A}{A_P N_i} = 1 + \frac{N_A}{A_P N_i} \qquad \textbf{(5.3)}$$

If $N_A = 0$ the amplifier is ideal and $F = 1$. For an impedance matched system, N_i is the available input noise power and F can be written as:

$$F = 1 + \frac{N_A}{A_P N_i} = 1 + \frac{N_A}{A_P k T \Delta f} \qquad \textbf{(5.4)}$$

(iii) Noise figure

The noise figure of a system component is simply the noise factor expressed in terms of dB, thus:

$$\text{Noise Figure, } NF = 10\log F \ \text{dB} \qquad \textbf{(5.5)}$$

It is usually the noise figure of impedance matched components, such as RF amplifier blocks, that is specified by manufacturers. In order to perform calculations using this information it is usually necessary to convert the noise figure specification into a noise factor.

(iv) Noise factor of a two stage impedance matched system.

Many analogue systems consist of a cascade of units such as a series of RF amplifier modules in a radar receiver system or a number of sequential amplification stages in a typical audio system. From a design point of view, it is important to know how each part of a system contributes to the overall system noise performance. The analysis that follows considers a combination of two amplifiers in series in an impedance matched system and works out the noise factor of the combination in terms of the noise factor of each element in the cascade. The results developed do not translate directly to the unmatched systems typically found at low frequency because in unmatched systems the input noise is not the available noise power, but the general principals revealed are true for all cascades.

Figure 11 shows two amplifiers in cascade with power gains of A_{P1} and A_{P2} respectively. The noise factors, F_1 and F_2, of the amplifiers, and the available input noise, N_I, are given by:

$$N_I = kT\Delta f$$

$$F_1 = 1 + \frac{N_{A1}}{A_{P1} kT\Delta f} \qquad \textbf{(5.6)}$$

$$F_2 = 1 + \frac{N_{A2}}{A_{P2} kT\Delta f} \qquad \textbf{(5.7)}$$

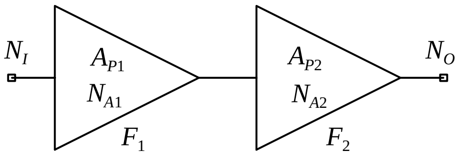

Figure 11
Two impedance matched amplifiers in cascade

To work out the noise factor of the system it is necessary to work out the noise power at the output of the real (noisy) system and divide it by the noise power that would appear at the output of an ideal system. The output noise from the real system has three components; that due to the available input noise which is amplified by both amplifiers, that due to the noise added by amplifier 1 which is amplified by amplifier 2 only and that due to the noise added by amplifier 2. The added noise powers associated with the two amplifiers can be found in terms of noise factor by rearranging equations (5.6) and (5.7). The three contributions are:

(i) output noise component due to the available input noise, N_I,

$$N_O\,(i) = A_{P1}\,A_{P2}\,N_I = A_{P1}\,A_{P2}\,kT\Delta f$$

(ii) output noise component due to the noise added by amplifier 1, N_{A1},

$$N_O(ii) = A_{P2}\,N_{A1} = A_{P2}\,(F_1 - 1)\,A_{P1}\,kT\Delta f$$

(iii) output noise component due to the noise added by amplifier 2, N_{A2},

$$N_O(iii) = N_{A2} = (F_2 - 1)\,A_{P2}\,kT\Delta f$$

The total output noise from the real amplifier is thus,

$$N_O\,(i) + N_O\,(ii) + N_O\,(iii) = A_{P1}\,A_{P2}\,kT\Delta f + A_{P1}\,A_{P2}\,(F_1 - 1)\,kT\Delta f + (F_2 - 1)\,A_{P2}\,kT\Delta f$$

$$= A_{P1}\,A_{P2}\,kT\Delta f\left(F_1 + \frac{(F_2 - 1)}{A_{P1}}\right)$$

The noise output from the ideal amplifier is simply $N_O(i)$ since in this case N_I exists but N_{A1} and N_{A2} are zero. The system noise factor is thus:

$$F = \frac{A_{P1}\,A_{P2}\,kT\Delta f\left(F_1 + \frac{(F_2 - 1)}{A_{P1}}\right)}{A_{P1}\,A_{P2}\,kT\Delta f} = F_1 + \frac{(F_2 - 1)}{A_{P1}} \qquad \textbf{(5.8)}$$

This result is extremely important for system designers wishing to minimise the effects of noise in a system. There are two major conclusions to be drawn from it:-

- The system noise factor is at least equal to the noise factor of the first stage so first stage noise performance is critical.
- The noise factor of the second stage is reduced by a factor equal to the first stage power gain before it adds to the noise figure of the first stage.

This means that the first stage dominates the noise performance of the system and should have low added noise and high power gain. A large fraction of the total design effort in noise critical systems goes into the first stage design. For systems with more than two stages a very similar analytical procedure can be followed to yield similar results. For a three stage system a similar process to that resulting in equ. (5.8) gives a system noise factor of

$F = F_1 + \dfrac{(F_2 - 1)}{A_{P1}} + \dfrac{(F_3 - 1)}{A_{P1}\,A_{P2}}$. Again, the first stage dominates and if the first stage has high gain the contributions of subsequent stages to noise factor are small for the second stage and usually negligible for the third.

In unmatched systems such as audio amplifiers the same principles apply. The first stage needs low noise because its noise performance dominates that of the whole system and it needs a high gain to ensure that the noise contributions from subsequent stages are insignificant. In all forms of electronic system dealing with very small signals the term "low noise front end" crops up and this simply reflects the importance of first stage noise. In some systems noise performance is so critical that first stages are cooled to reduce the added noise contribution of the first stage and hence improve the noise performance of the whole system.

(v) Noise temperature of a system element

Communications engineers often use the concept of noise temperature to describe the noise performance of impedance matched elements in a communication system. Figure 12a shows a real (noisy) amplifier with an added noise N_A. In figure 12b, the added noise has been taken out of the amplifier and is instead represented by an equivalent input noise source.

The input noise, N_I, is given by the available noise at the ambient temperature, T_A, because the Thevenin equivalent source resistance is equal to the input resistance of the amplifier by virtue of the system being impedance matched. Thus $N_I = kT_A\,\Delta f$. The equivalent input noise

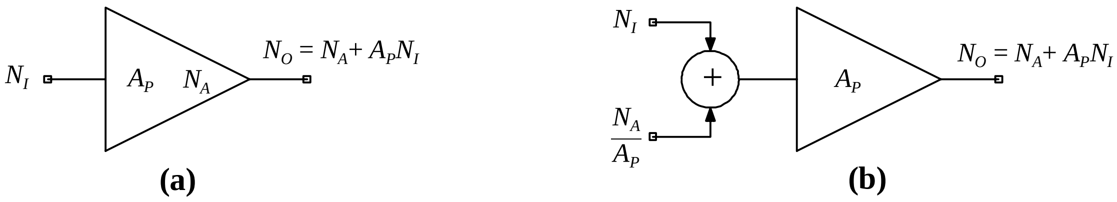

Figure 12

(a) *A noisy amplifier.* **(b)** *A noise equivalent circuit of (a) where the noise is represnted by an equivalent input generator.*

source, N_A/A_P, is also assumed to emanate from a matched resistance but its magnitude is not generally equal to the available noise at the ambient temperature. Thus an effective temperature, T_E, the amplifier noise temperature, must be ascribed to the matched resistance generating the equivalent input noise such that $\dfrac{N_A}{A_P} = kT_E\,\Delta f$. If the amplifier is perfect from a noise point of view $T_E = 0$K but if N_A is finite, $T_E > 0$. When the amplifier added noise is expressed in terms of an equivalent temperature in this way, the noise factor of the amplifier can also be expressed in terms of temperatures:

$$F = \frac{N_O\,(\text{real})}{N_O\,(\text{ideal})} = \frac{N_A + A_P\,N_I}{A_P\,N_I} = \frac{A_P\,kT_E\Delta f + A_P\,kT_A\Delta f}{A_P\,kT_A\Delta f} = 1 + \frac{T_E}{T_A} \qquad \textbf{(5.9)}$$

The noise temperature gives a direct idea of the magnitude of noise power cotributed by the system element compared to the available noise power. For example, an element with a noise temperature of 300K would contribute an added noise at the output equal to that due to the input noise. Some typical noise temperatures of elements in satellite television receivers are:

Low noise r.f. amplifier 150K

Mixer	850K
IF amplifier	400K

6 Equivalent Input Noise Generators

Representing the noise behaviour of a system by modelling the system added noise as some form of source at the input is an attractive approach for a number of reasons.

- It offers a simple representation of the noise performance of an otherwise complex systems.
- It offers a standard approach to the representation of noise sources which is convenient for comparison of noise specifications.
- It enables a standard analytical approach to system noise assessment.
- The equivalent input parameters are relatively easy to measure.

(i) The equivalent circuit

The strategy of representing system added noise by introducing at the system input an impedance matched resistive noise source at an appropriate temperature is not suitable for unmatched systems. For unmatched systems the added noise is instead represented by two generators, a series voltage generator and a shunt current generator. Two generators are necessary in order to achieve a model of system noise that is independent of the value of source impedance. In principle each system input should have equivalent generators associated with it. In systems such as operational amplifiers, the magnitude of the equivalent input generators is substantially independent of local circuit conditions but for simpler systems such as BJTs or FETs the equivalent input generator magnitudes are dependent on bias conditions.

Figure 13 shows an amplifier with a voltage gain G and added noise represented by the equivalent input noise generators, v_n and i_n. The input resistance, r_i (which can be regarded as noise free since any noise it generates is included in v_n and i_n) can usually be neglected if its value is large compared with R_S.

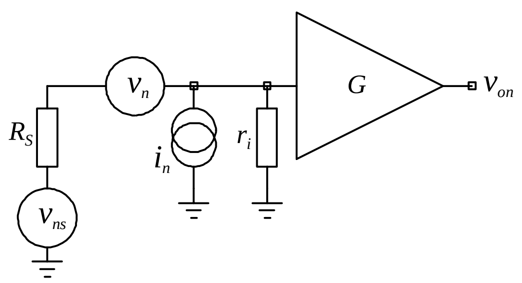

Figure 13

The equivalent input noise generators, v_n and i_n, account for all the noise added by the amplifier to any passing signal.

(ii) Quantifying the equivalent input noise generators

The equivalent input generators can be quantified by measuring the output noise voltage for different values of R_S. The measurement should be done using a true rms voltmeter with a known noise bandwidth, Δf.

If R_S is 0Ω, the voltmeter reading will be $v_{on} = \left(G^2\, \overline{v_n^2}\, \Delta f\right)^{1/2}$, ie the equivalent input current generator will have no effect and v_n is easily determined. A second measurement with a suitable finite R_S enables the equivalent input current generator to be quantified. With finite R_S,

$$\overline{v_{on}^2} = G^2\left[\overline{v_n^2}\left(\frac{r_i}{R_S+r_i}\right)^2 + \overline{v_{ns}^2}\left(\frac{r_i}{R_S+r_i}\right)^2 + \overline{i_n^2}\left(\frac{r_iR_S}{R_S+r_i}\right)^2\right]\Delta f$$
$$= G^2\left(\frac{r_i}{R_S+r_i}\right)^2\left[\overline{v_n^2} + \overline{v_{ns}^2} + \overline{i_n^2}R_S^2\right]\Delta f \qquad \textbf{(6.1)}$$

The only unknown in equation (6.1) is i_n. If r_i is very large and ill defined, as it might be in the case of a FET input op-amp, a finite R_S is necessary in order to create a well defined input voltage due to i_n. If r_i is well defined in value, R_S can be omitted and equation (6.1) simplifies to $v_{on} = \left(G^2\,\overline{i_n^2}r_i^2\,\Delta f\right)^{½}$.

(iii) Optimum source resistance

Consider the amplifier of figure 13, with a voltage gain G, a noise free input resistance r_i, equivalent input noise voltage and current generators v_n and i_n and a noisy source resistance R_S. The noise factor of the system can be worked out using the relationship of equation (5.2),

$$F = \frac{N_o}{A_PN_i} = \frac{N_{o\,real}}{N_{o\,ideal}}$$

The output noise from the real amplifier is given by equation (6.1):

$$N_{o\,real} = \overline{v_{onr}^2} = G^2\left[\overline{v_n^2}\left(\frac{r_i}{R_S+r_i}\right)^2 + \overline{v_{ns}^2}\left(\frac{r_i}{R_S+r_i}\right)^2 + \overline{i_n^2}\left(\frac{r_iR_S}{R_S+r_i}\right)^2\right]$$
$$= G^2\left(\frac{r_i}{R_S+r_i}\right)^2\left[\overline{v_n^2} + \overline{v_{ns}^2} + \overline{i_n^2}R_S^2\right]$$

and since v_n and $i_n = 0$ for an ideal amplifier, that from its ideal equivalent is:

$$N_{o\,ideal} = \overline{v_{oni}^2} = G^2\,\overline{v_{ns}^2}\left(\frac{r_i}{R_S+r_i}\right)^2$$

The system noise factor is therefore:

$$F = \frac{\overline{v_n^2} + \overline{v_{ns}^2} + \overline{i_n^2}R_S^2}{\overline{v_{ns}^2}} = \frac{\overline{v_n^2} + 4kT\,R_S + \overline{i_n^2}R_S^2}{4kT\,R_S} = 1 + \frac{\overline{v_n^2}}{4kT\,R_S} + \frac{\overline{i_n^2}R_S}{4kT} \qquad \textbf{(6.2)}$$

Equation (6.2) has one term directly proportional to R_S and one inversely proportional to R_S so F will become very large for the extremes of R_S very large or R_S very small and there will be a minimum value of F for some intermediate value of R_S. To find the value of R_S that minimises F, it is necessary to differentiate equation (6.2) and equate the result to zero. F is minimised, therefore, when

$$\frac{dF}{dR_S} = -\frac{\overline{v_n^2}}{4kT\,R_S^2} + \frac{\overline{i_n^2}}{4kT} = 0 \text{ or when } R_S = \frac{v_n}{i_n}. \qquad \textbf{(6.3)}$$

It is important to realise that it is noise factor that has been minimised here, not the output noise voltage. In some applications it may not be desirable to minimise F in this way. Minimising F minimises the degradation in signal to noise ratio caused by the passage of the signal through the amplifier.

The source resistance is usually not adjustable so other steps have to be taken to achieve the minimum F condition. One method is to connect several amplifiers in parallel and another is to use impedance matching transformers.

(iv) Parallel amplifiers

Figure 14 shows N amplifiers in parallel fed by a single source with a Thevenin equivalent resistance R_S. The circuit is simplified in that r_i is assumed infinite in each amplifier; the inclusion of an r_i would lead to more terms in the intermediate parts of the analysis but the conclusions would be the same as those that follow here. Each amplifier output is potentially divided between its own R_O and the N-1 other R_Os in parallel, that is by a factor of N, before contributing to the overall output, v_{onT}. Noise at the amplifier outputs arising from a single input source adds linearly whereas noise from independent sources must add as mean squared quantities.

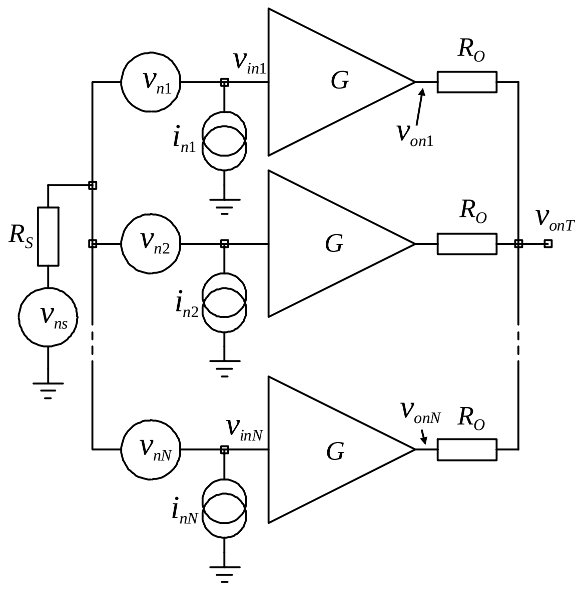

Figure 14
Identical amplifiers connected in parallel to achieve optimum source impedance matching.

The component of the output at any amplifier due to v_{ns}, the thermal noise of the source resistance, is $v_{on} = v_{ns}G$. Since vns appears at each amplifier input, these output contributions add linearly and the component of v_{onT} due to vns is $v_{onT} = Nv_{ns}G/N = v_{ns}G$.

Each of the equivalent input voltage sources affects only its own amplifier so the output at any amplifier due to its v_n is $v_{on} = v_nG$ and the component of the overall output due to each vn is $v_{onT} = v_n\, G/N$. Since each v_n is independent, their contributions must add as mean squared quantities so the v_{onT} due to all v_n is given by $\overline{v_{onT}^2} = \dfrac{\overline{v_n^2}\,G^2}{N^2}N = \dfrac{\overline{v_n^2}\,G^2}{N}$.

Each equivalent input current source affects each amplifier equally; the voltages appearing at each output due to any one current source must be combined linearly while the contributions from the different sources must be combined as mean squared additions. The overall output due to a single current source is $v_{onT} = i_n\, R_S\, G$ since i_n flowing through R_S creates a voltage which behaves in the same way as v_{ns}. The contributions from the N current sources must be added as mean squared contributions to give a total v_{onT} due to current sources of $\overline{v_{onT}^2} = N\ \overline{i_n^2}\,R_S^2\,G^2$.

The overall output noise voltage, including all contributions, is given by,

$$\overline{v_{onT}^2} = \overline{v_{ns}^2}\,G^2 + \frac{\overline{v_n^2}\,G^2}{N} + N\ \overline{i_n^2}\,R_S^2\,G^2. \tag{6.4}$$

Using the same process as that which led to equations (6.2) and (6.3) above the noise factor of the parallel system can be shown to be:

$$F = 1 + \frac{\overline{v_n^2}}{4kT\,R_S\,N} + \frac{\overline{i_n^2}\,R_S\,N}{4kT},\ \text{and this is minimum when } R_S = \frac{v_n}{N\ i_n} \tag{6.5}$$

Note that v_{ns} is in the same position in the circuit as a signal generator would be so equation (6.4) indicates that the signal gain of the parallel arrangement of figure 14 is the same as a single amplifier. Note also that equation (6.5) indicates that the parallel amplifier approach is only useful for cases where the source resistance is smaller than the optimum value required for a single amplifier. The parallel approach tends to be used with transistor amplifiers rather than operational amplifiers. The output node in the case of transistors consists of N Norton equivalent circuits connected in parallel, but Norton to Thevenin transformations on the transistor output circuits would yeild a similar circuit to figure 14, and give very similar results.

(v) Transformer matching

The second technique used for presenting an amplifier with the source resistance that will minimise its noise factor involves the use of a transformer as an impedance changer. Figure 15 shows a transformer coupled amplifier together with its equivalent circuit. The transformer has a turns ratio of 1:n with the n being on the amplifier side. The circuit of figure 15(b) is the same

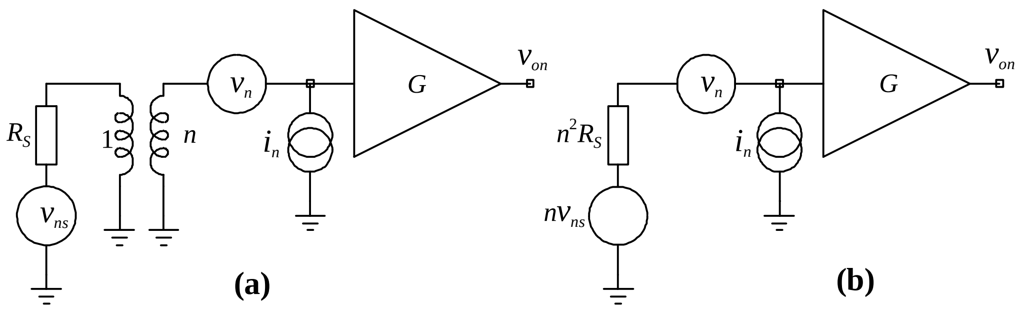

Figure 15
(a) *A transformer coupled input stage and* **(b)** *its equivalent circuit*

as figure 13 with the exception that r_i does not exist so the analysis of section 6(iii), with appropriate modification to symbols, can be used here. Thus, R_S becomes n^2R_S and v_{ns} becomes nv_{ns} to give:

$$F = \frac{\overline{v_n^2} + n^2\,\overline{v_{ns}^2} + \overline{i_n^2}\,n^4R_S^2}{n^2\,\overline{v_{ns}^2}} = \frac{\overline{v_n^2} + 4kT\,n^2R_S + \overline{i_n^2}\,n^4R_S^2}{4kT\,n^2R_S} = 1 + \frac{\overline{v_n^2}}{4kT\,n^2R_S} + \frac{\overline{i_n^2}\,n^2R_S}{4kT} \qquad \textbf{(6.6)}$$

In this case R_S is fixed but since n, the turns ratio, can be controlled by design, equation (6.6) must be differentiated with respect to n to find the value of n that minimises F. What is actually happening is that as n changes, the source resistance presented to the amplifier changes. The appropriate value of n could equally well be found by evaluating the optimum source resistance for the amplifier of interest and then working out the turns ratio needed to transform the actual source resistance into the optimum value. By either method, F is minimised when:

$$n = \left(\frac{v_n}{i_n\,R_S}\right)^{1/2}$$

This approach to optimising noise performance is commonly used in audio systems to match low impedance microphones to low noise amplifiers.

7 Noise in Operational Amplifiers

Operational amplifier ICs, op-amps, are relatively sophisticated analogue subsystems and it would be impractical to evaluate their noise performance by examining each separate noise source within the IC. Instead it is the macroscopic noise performance of the whole amplifier that is modelled by abstracting all internal noise sources and representing them as equivalent input noise generators. Much the same approach is taken to the modelling of dc offset effects.

(i) Op-amp noise equivalent circuits.

Op-amps are usually used in impedance unmatched systems so equivalent input noise generators are used to model the op-amp noise behaviour. Three equivalent circuits are commonly used and these are shown in figure 16. Figure 16(a) is the most detailed of the three circuits, using two noise generators at each input. This model must be used in situations where the op-amp input current is too high to be ignored - in other words when the input resistance is low. Providing the input resistance of the op-amp is very high - so high that currents flowing into the inputs are negligible in comparison to other currents in the circuit - only one equivalent input voltage source placed in series with either the inverting or non-inverting input of a standard op-amp is necessary and the model of figure 16(b) results. Since the assumption of very high input resistance is valid for most modern op-amps, the single voltage source model is the one which will be followed here. A third model, shown in figure 16(c), has a single current source connected between the inputs. Connecting the current noise generator between the inputs models the current noise in the two inputs as correlated and thus, at any instant, the noise current in one input is minus that in the other. Note that in the models of figures 16(b) and 16(c) the equivalent input voltage generator can be put in series with either input without affecting the model behaviour.

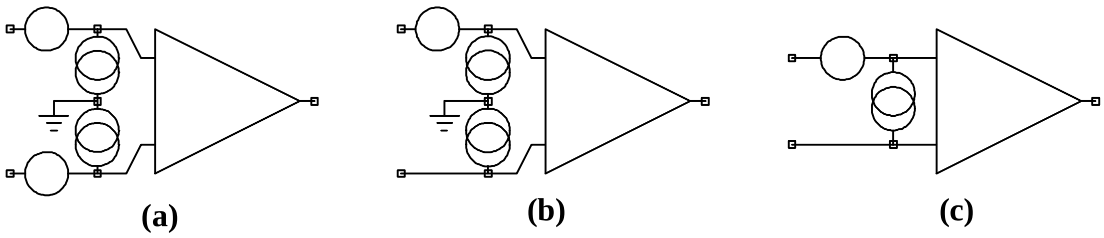

Figure 16

Three noise models for op-amps. The + and - inputs have not been marked because it makes no difference from a noise point of view which is which. **(a)** *The most accurate of the three but more accurate than necessary in most cases.* **(b)** *Suitable for amplifiers with very high input resistance - ie, most op-amps.* **(c)** *Sometimes used for op-amps but makes the two equivalent input noise currents coherent.*

(ii) Noise analysis of op-amp circuits

Figure 17a shows the op-amp noise equivalent circuit and figure 17b shows an op-amp circuit with only noise sources included - if a noise analysis is being performed, signal sources should be replaced by their Thevenin equivalent impedances. For an inverting amplifier the signal source would be inserted in series with R_1 and for a non-inverting amplifier the signal source would be connected in series with R_3; the noise equivalent circuit is the same for both cases. Since the objective here is a noise analysis all other aspects of amplifier performance are assumed to be ideal. Analysis of figure 17b gives:-

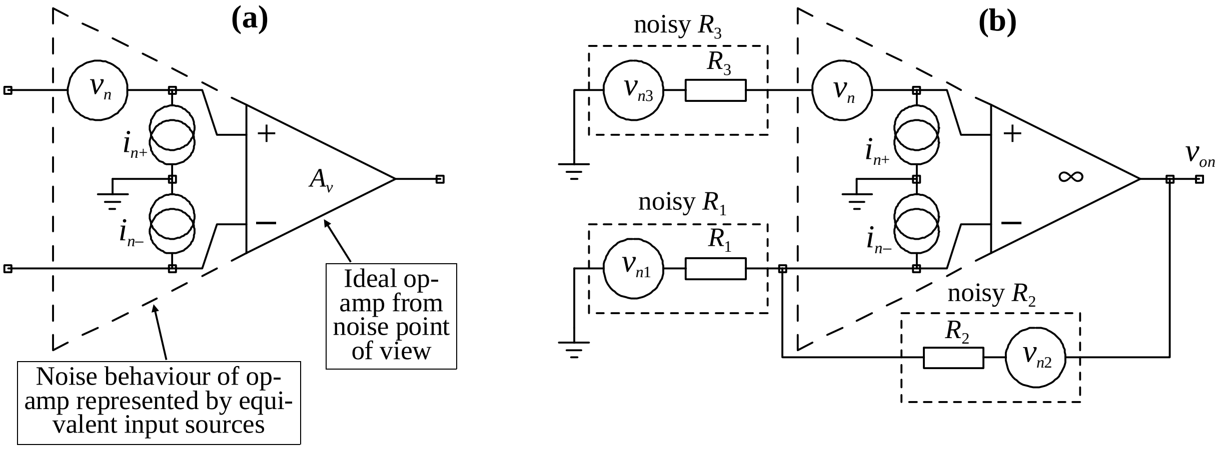

Figure 17

(a) *The noise equivalent circuit of an op-amp.* **(b)** *The noise equivalent circuit appropriate for inverting and non-inverting amplifier circuit connections.*

$$\overline{v_{on}^2} = G^2\left[\overline{i_{n+}^2}R_3^2 + \overline{i_{n-}^2}\left(\frac{R_1R_2}{R_1+R_2}\right)^2 + \overline{v_n^2} + \overline{v_{nf}^2} + \overline{v_{n3}^2}\right] \tag{7.1}$$

where

G = closed loop gain, $(R_1 + R_2)/R_1$
$\overline{v_{n3}^2}$ = noise due to R_3, $4kTR_3$ V^2 Hz^{-1}
$\overline{v_{nf}^2}$ = noise due to the feedback resistors R_1 and R_2, $4kTR_1R_2/(R_1+R_2)$ V^2 Hz^{-1}
$\overline{v_n^2}$ = op-amp's equivalent input noise voltage generator
$\overline{i_{n+}^2}$ = op-amp's equivalent input noise current generator at the non-inverting input
$\overline{i_{n-}^2}$ = op-amp's equivalent input noise current generator at the inverting input

The derivation of equation (7.1) provides an excellent exercise in the analysis of circuits with multiple uncorrelated noise sources. The easiest way of arriving at the solution is probably to use the superposition principle to consider the output voltage due to each individual source in turn and then sum the squares of each output component in order to arrive at the total output voltage. This approach also has the advantage that it is easy to compare the relative magnitudes of contributions from various parts of the circuit and hence identify any problem areas. Note that the non-inverting closed loop gain has been taken as a factor from each term in equation (7.1). Since it is the non-inverting closed loop gain that operates on noise sources at the input, irrespective of the type of circuit connection (ie, inverting or non-inverting), the non-inverting gain is often called the "noise gain".

Inspection of equation (7.1) will tell the op-amp circuit designer whether it is possible to affect the noise performance of the circuit by careful design. Consider the terms as they appear in equation (7.1):

(i) The first term is due to the voltage developed across R_3 by i_{n+}. If R_3 is made equal to zero, the noise contribution due to i_{n+} disappears. Remember, though, that R_3 may be there to minimise offset effects or alternatively it may be a Thevenin equivalent source resistance, so it may not be sensible or possible to change its value. Under such circumstances the magnitude of the first term can only be reduced by choosing an op-amp with a very low i_{n+}.

(ii) The second term is due to the voltage developed across the parallel combination of

R_1 and R_2 by i_{n-}. Choice of an op-amp with a low i_{n-} will reduce this term, as will a reduction in the value of the parallel combination of R_1 and R_2. R_1 and R_2 determine the circuit gain so the ratio of their values is defined by the gain requirement. A high value of gain means $R_2 \gg R_1$ and then the parallel combination approximates to R_1. The lowest values that can be used may be limited by offset considerations or by the acceptable magnitude of signal current drawn through R_1 and R_2; the output current capability of an op-amp is limited and signal current wasted down the feedback path is not available for driving a load.

(iii) The third term is due to the op-amp's equivalent input noise voltage generator. There is nothing that can be done to reduce this component except choosing an op-amp with a low value of v_n.

(iv) The fourth term is caused by the thermal noise generated in the feedback resistors, R_1 and R_2. As for term (ii), R_1 and R_2 are effectively in parallel and their combined value approximates to R_1 if the circuit gain is high. Making the parallel combination as small as possible will minimise the noise contributed by the feedback resistors but the constraints are the same as for term (ii).

(v) The fifth term is due to the thermal noise generated in R_3. It can be reduced by reducing R_3. Considerations limiting the smallness of R_3 are discussed in term (i)

A general purpose op-amp will have an equivalent input noise voltage generator of around 20nV $Hz^{-1/2}$ and at room temperature the resistance that will generate a thermal noise of the same value is around 24kΩ. In many cases it is possible to reduce R_3 and the parallel combination of R_1 and R_2 to values well below 24kΩ and thus ensure that terms (iv) and (v) make a small contribution to total noise. Op-amps with FET input devices have equivalent input current noise generators of the order of 0.01pA $Hz^{-1/2}$, as opposed to the significantly larger 0.4 pA $Hz^{-1/2}$ typical of their BJT input counterparts, and hence represent a good choice for reducing terms (i) and (ii).

Wide bandwidth op-amps tend to have lower equivalent input noise voltage generators and higher equivalent input noise current generators than general purpose op-amps and there are other devices specially designed to minimise noise. Remember though that a device designed to minimise one undesirable effect may well be worse than average in some other respects.

(iii) A simplified op-amp noise model

If a circuit such as figure 17b has been designed such that the noise associated with the feedback resistors has been made negligible, that is terms (ii) and (iv) in equation (7.1) have been made small compared to the other noise sources in the circuit, and the circuit connection is non-inverting, the simplified equivalent circuit of figure 18 can be used. The circuit of figure 18 is also appropriate for the small number of special amplifiers that have built in dc bias on the non-inverting input and built in feedback components. Such special amplifiers are often optimised for low noise performance and are typically intended for amplification in ac coupled environments such as audio systems, where designer access

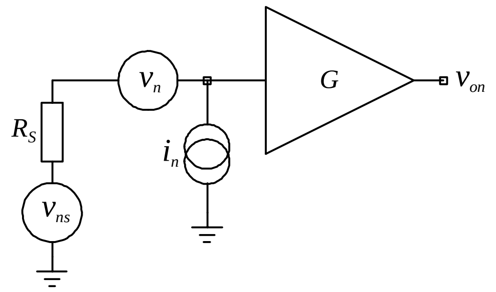

Figure 18
A simplified amplifier noise model

to differential inputs is not usually essential.

The gain of the amplifier is now that defined by the manufacturers or by the feedback. Any noise introduced by the feedback components is either negligible by design or included in the equivalent input generators, v_n and i_n. If the amplifier has an input resistance, which may or may not be the case, its effect is to potentially divide the voltage sources and provide a parallel path for current sources and these effects must be taken into account when calculating output noise. The input resistance, if it exists, will be noise free since its noise contribution will be accounted for by the equivalent input generators.

8 Concluding Comments

This handout was intended to provide you with an introductory working knowledge of noise and how to handle it. The physical processes that give rise to the noise and the derivations of the expressions describing the magnitudes of thermal noise and shot noise have not been mentioned. The sources considered have all been independent and hence uncorrelated although many situations exist where the sources are partially correlated. BJTs and FETs have been mentioned in passing but the details of their noise behaviour have not been discussed. The list below provides some sources of further information on noise:

W.M. Leach: "Fundamentals of Low-Noise Analog Circuit Design", IEEE Proc. **82**, pp 1515 - 1538, Oct 1994. (*This is a tutorial paper concentrating on device modelling and circuit design techniques for low noise*.)

C.D. Motchenbacher and J.A. Connelly: "Low noise Electronic System Design", Wiley, 1993, ISBN 0-471-57742-1. (*A general book about noise in circuits and systems.)*

A. Van Der Ziel: "Noise in Solid State Devices and Circuits", Wiley, 1986. (*Van Der Ziel has written many titles on noise; this is one of the more recent ones. The books tend to be written from a solid state rather than a system viewpoint.*)

D.A. Bell: "Electrical Noise", Van Nostrand, 1960. (*Concentrates on physical mechanisms.*)