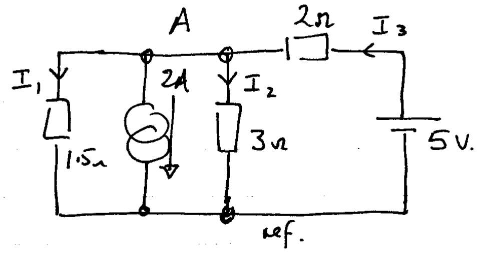
Q1 (a)



sum currents at node A

$$I_3 = I_2 + I_1 + 2A.$$

$$\frac{5 - V_A}{2\Omega} = \frac{V_A}{1.5\Omega} + \frac{V_A}{3\Omega} + 2$$

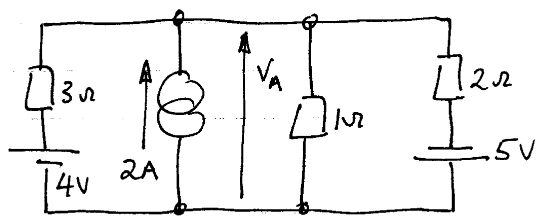
$$\frac{5}{2\Omega} - 2 = V_A\left[\frac{1}{2} + \frac{1}{1.5} + \frac{1}{3}\right] = V_A\, 1.5$$

$$2.5 - 2 = V_A . 1.5 \quad \text{or} \quad V_A = \frac{0.5}{1.5} = \underline{1/3\ V}$$

$$V(2\Omega) = 5V - 1/3\,V = \frac{14}{3}V.$$

$$P_{(2\Omega)} = V^2/R = \frac{14^2}{9} \cdot \frac{1}{2} = \frac{196}{9.2} = \frac{98}{9} = \underline{10.9W}$$

(b)



$$V_A\Big|_{4V} = \frac{4\,(1\Omega // 2\Omega)}{3 + (1\Omega // 2\Omega)} = \frac{4 \cdot 2/3}{3 + 2/3} = \frac{8/3}{11/3} = \frac{8}{11}\ V$$

$$V_A\Big|_{2A} = 2\,(3\Omega // 1\Omega // 2\Omega) = 2 \times \frac{1.2}{2.2} = \frac{1.2}{1.1} = \frac{12}{11}\ V$$

$$V\Big|_{5V} = \frac{-5\,(3//1)}{2\Omega + (3\Omega // 1\Omega)} = \frac{-5\ 3/4}{2 + 3/4} = -\frac{15/4}{11/4} = -\frac{15}{11}\ V$$

$$\therefore V_{A\,tot} = \frac{8}{11} + \frac{12}{11} - \frac{15}{11} = \underline{5/11\ V} \quad (\underline{0.455V})$$

Q1 b(i) cont....

The biggest contributing source is the $\underline{\underline{5V}}$ source

(ii) 4V source.... Since $V_A < 4V$, 4V source is driving a current into node A. $I_{4V} = \frac{(4 - 5/11)V}{3\Omega}$ $= \frac{39/11}{3\Omega} = 13/11 \text{ A} =$

$$\therefore \quad P_{(4V)} = 4V \times 13/11 \text{ A} = \underline{\underline{4.73 \text{ W}}}.$$

2A source.... Since V_A is positive, 2A driving into a positive voltage so....

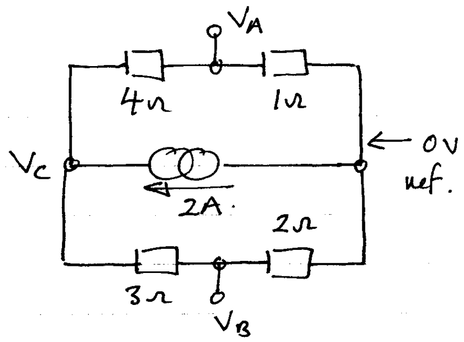
$$P_{(2A)} = 2 \times V_A = 2 \times 5/11 = \underline{\underline{0.909 \text{ W}}}$$

5V source.... Since V_A is positive, current flows from V_A into the negative side of 5v – a generating direction. $I_{5V} = \frac{V_A - (-5)}{2} = \frac{5/11 + 5}{2} = \frac{60}{22} = 2.73A.$

$$P_{(5V)} = 2.73A \times 5V = \underline{\underline{13.6 \text{ W}}}$$

The sum of these powers should equal the power dissipated in the three resistors – an extra mark to any who check!

Q2 (a)



$$V_C = 2A \times (4+1) \| (3+2)$$
$$= 2 \times 5 \| 5$$
$$= 2 \times 2.5 = 5V.$$

$$\therefore V_A = \frac{5 \times 1}{1+4} = 1V.$$

$$V_B = \frac{5 \times 2}{3+2} = 2V.$$

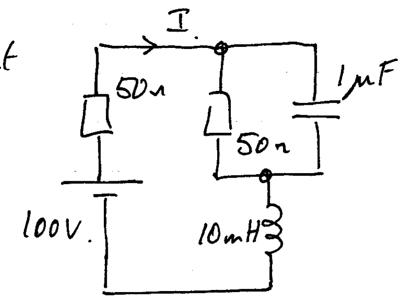
$$\therefore V_A - V_B = 1 - 2 = \underline{\underline{-1V}} = \underline{\underline{V_{Th}}}.$$

$$R_{Th} = (1\Omega + 2\Omega) \| (4\Omega + 3\Omega)$$

(replace current source by an open circuit and write down resistance between ~~A +~~ B).

$$R_{Th} = 3\Omega \| 7\Omega = \frac{21}{10} = \underline{\underline{2.1\Omega}}.$$

(b) (i) C looks like an open cct
L " " a short "



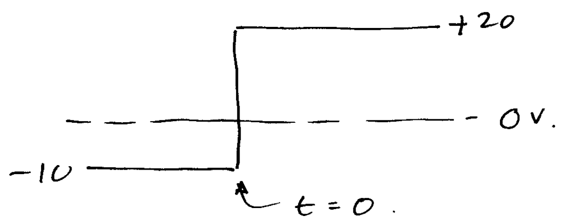
$$\therefore I = \frac{100}{50+50} = \underline{\underline{1A}}$$

(ii) $E_L = \frac{1}{2} L I^2$

$$= \frac{10^{-2} \times 1}{2} = \underline{\underline{5 mJ.}}$$

(iii) $E_C = \frac{1}{2} C V^2 = \frac{1}{2} 10^{-6} \cdot (50V)^2 = \underline{\underline{1.25 mJ.}}$

Q2 (c).



(i) at $t=0^-$ $V_1 = -10V$ so ...

$I = -10/100 = \underline{\underline{-100mA}}$.

$V_L = 0V$ (since L looks like a short cct)

$V_C = -5V$

(ii) at $t=0^+$ V_1 becomes $+20V$

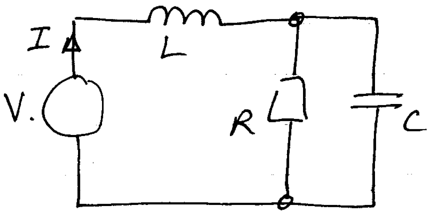
- L will not allow instantaneous change of I
- C will not allow instantaneous change of V_C

$\therefore$ $I = -10/100 = \underline{\underline{-100mA}}$. (same as $t=0^-$).

$V_C = -5V$. (same as $t=0^-$).

$V_L = 30V$ (because if I and V_C must be the same at $t=0^+$ as they were at $t=0^-$, the voltage difference between V_1 and the top of L must remain the same. Thus if V_1 rises by 30V V_L must rise by 30V also.)

Q3 (a) (i)



$$Z = V/I = j\omega L + \left(R \,\|\, \frac{1}{j\omega C}\right)$$

$$= j\omega L + \frac{R/j\omega C}{R + 1/j\omega C}$$

$$= j\omega L + \frac{R}{1+j\omega CR} = \frac{j\omega L - \omega^2 LCR + R}{1 + j\omega CR}$$

$$= R\left[\frac{1-\omega^2 LC + j\omega L/R}{1+j\omega CR}\right]$$

(ii) For resonance, j terms must disappear from Z

$$Z = \frac{R(1-\omega^2 LC + j\omega \frac{L}{R})}{1+j\omega CR} = \frac{R(1-\omega^2 LC + j\omega\frac{L}{R})(1-j\omega CR)}{1+\omega^2 C^2 R^2}$$

$$= \frac{\text{real} + j\omega\left(-(1-\omega^2 LC)CR + \frac{L}{R}\right)}{\text{real}}.$$

so to make j terms disappear ....

$$-(1-\omega^2 LC)CR + \frac{L}{R} = 0.$$

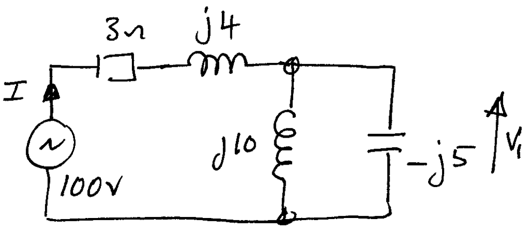
or $-CR + \omega^2 LC^2 R + L/R = 0.$

$$\omega^2 LC^2 R = CR - L/R$$

$$\omega^2 = \frac{1}{LC} - \frac{1}{C^2R^2}$$

so $\underline{\underline{\omega = \sqrt{\frac{1}{LC} - \frac{1}{C^2R^2}}}}$

Q3 (b) (i)



First find Z ...

$$Z = 3 + j4 + \frac{j10 \cdot -j5}{j10 + (-j5)}$$

$$= 3 + j4 + \frac{50}{j5} = 3 + j4 - j10 = 3 - j6$$

$$= 3(1 - j2) \equiv 6.7\angle -63.4^\circ$$

$$I = \frac{V}{Z} = \frac{100}{6.7\angle -63.4} = \underline{\underline{14.9\angle +63.4}}$$

or $$I = \frac{100}{3(1-j2)} = \frac{\cancel{100}^{20}}{3}\,\frac{1+j2}{\cancel{5}} = \underline{\underline{\frac{20}{3}(1+j2)}}$$

$$V_1 = I\,\frac{j10 \cdot -j5}{j10 + (-j5)} = I \cdot -j10$$

$$= 149\angle 63.4 - 90 = \underline{\underline{149\angle -26.6}}$$

$$= \frac{20}{3}(1+j2) \cdot -j10 = \frac{200}{3}(-j + 2)$$

$$= \underline{\underline{\frac{200}{3}(2 - j)}}$$

(ii) Z from above $= \underline{\underline{6.7\angle -63.4^\circ}}$

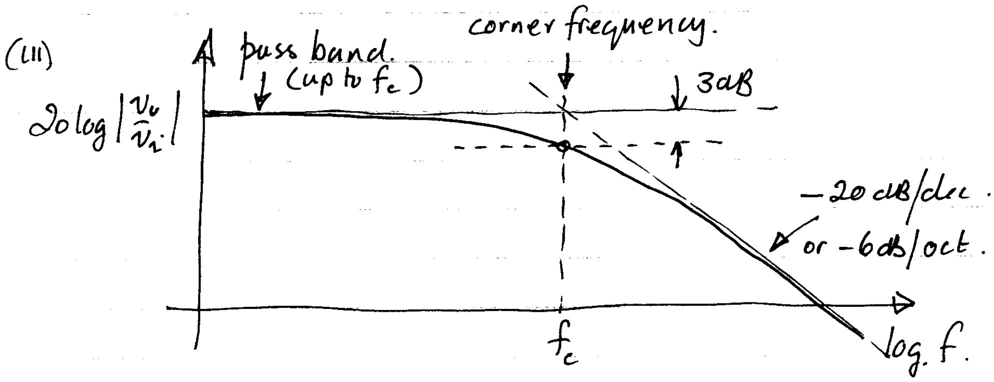
(iii) If C is $-j10\,\Omega$ then $j10 \,\|\, -j10 = \infty$ and the parallel combination is resonant.

Thus $\underline{\underline{I = 0}}$ and $V_1 = V_{source} = \underline{100\,V\angle 0}$

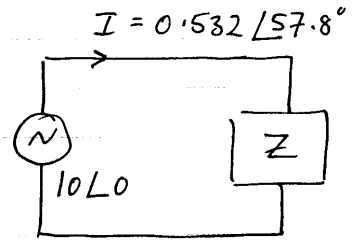
Q4 (a) (i) A low pass filter is a circuit that passes low frequencies — ie frequencies below a particular frequency and attenuates signals at frequencies above this particular frequency.

(ii) Circuits 4a (ii) and 4a (iii) are low pass filters.

(iii)



(b) If current leads voltage in a series combination of two components by an angle of other than 90°, the two components must be R + C



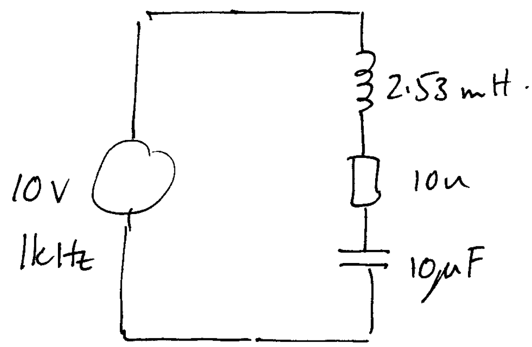
$$Z = \frac{10}{0.532\angle 57.8} = 18.8\angle -57.8 \equiv 10 - j15.9$$

$$10 - j15.9 = R + X_C = R - \frac{j}{\omega C}.$$

$$\therefore \; \underline{R = 10\,\Omega} \qquad \omega C = \frac{1}{15.9} \quad \text{or} \quad C = \frac{1}{2.\pi.10^3.15.9}$$

$$= \underline{10\mu F}.$$

Q4(b) (ii) ...



$$Z = j\,2.\pi.10^3 L + 10\Omega - \frac{j}{2.\pi.10^3.C}$$

$$= j15.9 + 10\Omega - j15.9 = 10\Omega$$

$\therefore$ cct is resonant.

$$I = \frac{10V}{10\Omega} = 1A \qquad \therefore \quad P_D = \underline{\underline{10W}}.$$

$$V_L = IX_L = 1A \times 15.9\Omega = \underline{\underline{15.9V}}.$$