EEE407/EEE6021 2011-12 Model Solutions

1.a Combining the chemical reactions at the anode and cathode described by the following equations: **(8)**

$$Anode: 2H_2 + 2O^{2-} \rightarrow 2H_2O + 4e^-$$
$$Cathode: O_2 + 4e^- \rightarrow 2O^{2-}$$

we obtain

$$2H_2 + O_2 \rightarrow 2H_2O + 4e^-$$

If there are no losses in the fuel cell, or in chemistry term, the process is "reversible", then all Gibbs free energy of formation per mole fuel released through the reaction will be converted into electrical energy.

Consider work to be done to move electrons through an external circuit with a potential difference or voltage denoted by E. For the SO fuel cell, 2 electrons pass round the external circuit for every hydrogen molecule used. Hence, for one mole of hydrogen fuel used, $2N$ electrons pass round the external circuit, where N is the Avagadro's number. If $-e$ is the charge on one electron, then the charge that flow is:

$$-2Ne = -2F \text{ (Coulomb)}$$

where $F = eN$ is the Faraday constant, or the charge of one mole of electrons. If E is the voltage of the fuel cell, then the electrical work done in order to move this amount of charge via the circuit having electric field strength E_f is:

$$Electrical\ work\ done = Q\int_l \vec{E}_f \cdot d\vec{l} = charge \times voltage = -2FE \text{ (Joules)}$$

For an ideal system (or has no losses), this electrical work done will be equal to the Gibbs free energy released. Thus:

$$\Delta\bar{g}_f = -2EF$$

or

$$E = -\Delta\bar{g}_f / 2F$$

1.b. (i) Since the reaction uses one hydrogen molecule, and release 2 electrons. Thus for each mole of hydrogen used, 2 mole of electrons are generated, which in an ideal case would pass through an external circuit to produce the emf given by: **(2)**

$$E = -\frac{\Delta\bar{g}_f}{2F} = -\frac{-188.6\times10^3}{2\times96485} = 0.98\,(\text{V})$$

1.b. (ii) In order to compare the performance of fuel cells with heat engines, the efficiency of a fuel cell is defined as **(3)**

$$Efficieny\ \ \eta = Electrical\ \ energy\ \ produced\ \ per\ mole\ \ of\ \ fuel\ /-\Delta\bar{h}_f$$

where $\Delta\bar{h}_f$ is the change in "enthalpy of formation", or heating value of the fuel. If higher heating value of the hydrogen fuel is used, the maximum efficiency would result when all energy release in the change of Gibbs free energy of formation were converted to electrical energy. Thus:

$$Efficieny\ \ \eta = -\Delta\bar{g}_f\ /-\Delta\bar{h}_f = \frac{188.6\times10^3}{285.84\times10^3}\times100\% = 66.1\%$$

1.b. (iii) In the SO fuel cell, the equation for the chemical reaction is given by: **(5)**

$$2H_2 + O_2 \rightarrow 2H_2O + 4e^-$$

which may be rewritten as:

$$H_2 + 0.5O_2 \rightarrow 2H_2O + 2e^-$$

The Gibbs free energy of formation $\Delta\bar{g}_f$ released by one mole of fuel is dependent on the operation pressure and temperature, and the variation is given by:

$$\Delta\bar{g}_f = \Delta\bar{g}_f^0 - RT\ln\left[\frac{\left(\frac{P_{H2}}{P_0}\right)\sqrt{\frac{P_{O_2}}{P_0}}}{\left(\frac{P_{H_2O}}{P_0}\right)}\right]$$

where

$\Delta\bar{g}_f^0$ — Change in molar Gibbs free energy formation at standard pressure
R — Molar gas constant, 8.314 J/(K mole)
T — Temperature (K)

P_{H2} — Partial pressure of hydrogen
P_{O2} — Partial pressure of oxygen
P_{H_2O} — Partial pressure of water
P_0 — Standard pressure

If unit bar is used for pressure, P_0 = 1.0, the effect of pressure on the open circuit voltage can be obtained by:

$$E = \frac{-\Delta\bar{g}_f^0}{2F} + \frac{RT}{2F}\ln\left[\frac{P_{H2}\sqrt{P_{O2}}}{P_{H_2O}}\right]$$

The partial pressure of hydrogen : P_{H2} = 6.0
The partial pressure of oxygen: P_{02} = 6.0x0.2095 = 1.257
The partial pressure of water: P_{H2O} = 5

The resultant open circuit voltage at 800 ^{0}C is:

$$E = -\frac{-188.6\times10^3}{2\times96485} + \frac{8.314\times(273+800)}{2\times96485}\ln\left[\frac{6\sqrt{1.257}}{5}\right] = 0.991\,(\mathrm{V})$$

1.c. For the high temperature SO fuel cell, the reaction rate is very high and therefore, the loss due to activation energy is relatively low. However, the ohmic resistance of the electrodes and electrolyte is quite high due to high temperature, and this is indeed the main cause of the losses in the fuel cell when the current density is in the middle range. **(2)**

2.a The **mass transportation loss** results from the change in concentration of the reactants at the surface of the electrodes as the fuel is used. As more oxygen and hydrogen gases are used in the fuel cells, their concentration ratio or partial pressure decreases. The decrease in partial pressure of a reactant gas, say, hydrogen, will affect the output voltage in the following manner: **(3)**

$$\Delta V = \frac{RT}{2F}\ln[P_2] - \frac{RT}{2F}\ln(P_1) = \frac{RT}{2F}\ln\left[\frac{P_2}{P_1}\right]$$

When the output current of a fuel cell is *I*, the reduction in voltage is equivalent to a loss component, *ΔVI*. Because the reduction in concentration is the result of a failure to transport sufficient reactant to the electrode surface, this type of loss is also often called "mass transportation" loss.

2.b The output voltage of a hydrogen fuel cell under a given current density may be represented by **(8)**

$$V = E - r(i + i_n) - A\ln\left(\frac{i + i_n}{i_0}\right) + B\ln\left(1 - \frac{i + i_n}{i_l}\right) \qquad i_l - i_n > i > i_0 - i_n$$

where

i_0 is the exchange current density representing the effect of activation energy losses
i_n is the equivalent current representing the fuel crossover, and internal leakage current.
i_l is the limiting current density il at which the fuel concentration will become zero
r is the ohmic resistance per unit area.

E is the ideal open-circuit emf of the fuel cell given by

$$E = -\frac{\Delta \bar{g}_f}{2F}$$

at the operating temperature of 100 0C, i.e.,

$$E = -\frac{\Delta \bar{g}_f}{2F} = \frac{225.2\times 10^3}{2\times 96485} = 1.167\ (\mathrm{V})$$

The current density of the fuel cell is

i = 250/25x25 = 0.4 A/cm^2 = 400 mA/cm^2
Using the constants given in Table 1, the output voltage at i = 400 (mA/cm^2) can be evaluated by:

$$V = 1.167 - 3.0\times 10^{-2}(400+2)\times 10^{-3} - 0.06\ln\left(\frac{400+2}{0.067}\right) + 0.05\ln\left(1 - \frac{400+2}{900}\right)$$

$$= 0.603(V)$$

The energy output of the fuel cell over one minute is

$$W_o = V\times i\times t = 0.603\times 250\times 60 = 9050.7\ \ (J)$$

The molar mass of the hydrogen is 2 amu, therefore 0.12g hydrogen fuel is equal to 0.07 mole.

The energy contained in this amount of fuel is

$$W_h = \Delta \bar{h}_f * m_h = 0.06 \times 285.84 \times 10^3 = 17150.4(J)$$

The efficiency of the fuel cell is

$$\eta = \frac{W_o}{W_h} = \frac{9050.7}{17150.4} = 0.528 = 52.8\%$$

2.c The emf E of the circuit represents the ideal open-circuit voltage of the fuel cell minus the voltage reduction due to exchange current density. **(5)**

The resistor R_r models the ohmic losses of the fuel cell, which include the resistance of the electrodes and resistance to the flow of ions in the electrolyte. A change in current gives an immediate change in the voltage drop across this resistor.

The resistor R_a models the voltage reduction due to activation loss, and is a non-linear function of the fuel cell output current. However, due to "charge double layer" effect at the surface of the electrodes, if the current changes, it will take some time for the charge stored on the surfaces (and its associated voltage) to dissipate (if the current reduces) or build-up (if there is a current increase). So, the activation voltage drop does not immediately follow the current in the way that the ohmic voltage drop does. This effect is modelled by the capacitor across this resistor. Therefore, the capacitor C represents the charge double layer effect at the electrode surfaces of the fuel cell.

2.d From the equivalent circuit, the frequency response of the internal impedance of a hydrogen fuel cell can be derived as: **(4)**

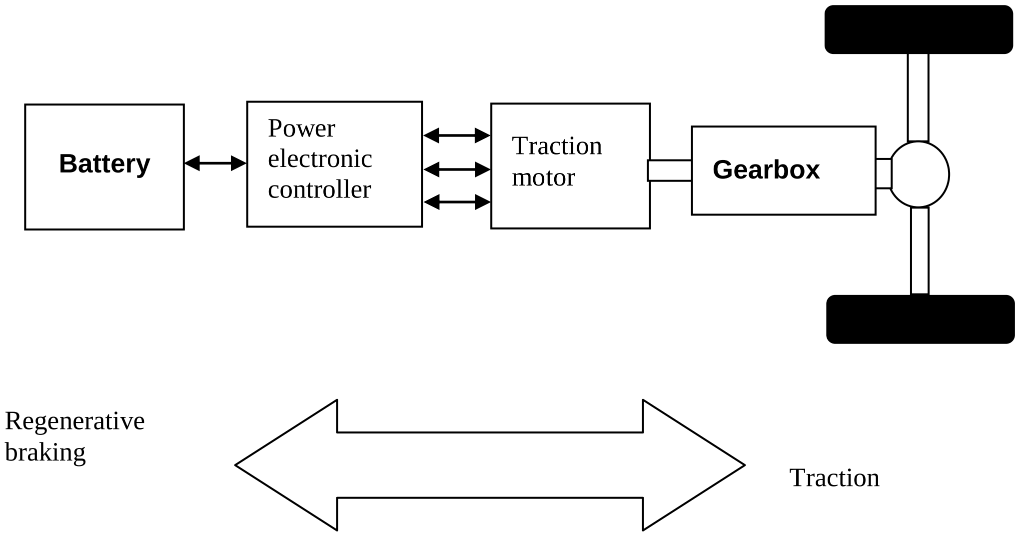
$$Z(j\omega) = R_r + \frac{R_a \frac{1}{j\omega C}}{R_a + \frac{1}{j\omega C}} = R_r + \frac{R_a}{j\omega C R_a + 1}$$

As the angular frequency approaches zero, the impedance becomes (R_r + R_a)

On the other hand, if ω goes to infinite, then $Z(j\omega)$ tends to R_r. Thus by measuring the frequency response of the fuel cell, R_r and R_a can be determined. Further the equivalent capacitance may be obtained by identifying the corner frequency of the response.

Solution to question No. 3

a.



- The battery is the only energy storage, and requires hours to be fully re-charged.
- Batteries have a low energy density, therefore, there is a trade-off between the weight of the battery and range of the vehicle.
- Energy and power requirements are coupled.
- The electric drive-train is highly efficient.
- No emissions.
- The electric motor possesses torque-speed characteristics ideal for traction.
- Only a fixed ratio transmission is required.

b.

On a road with an upward inclination α, the forces on a vehicle of mass m are related by the following differential equation:

$$m\frac{dv}{dt} = \overbrace{F}^{traction\ force} - \overbrace{\lambda_f\, m\, g \cos(\alpha)}^{rolling\ resistance} - \overbrace{\frac{1}{2} C_d\, A_f\, \rho_a\, v^2}^{aerodynamic\ drag} - m\, g \sin(\alpha)$$

which can be re-written as:

$$\left(\frac{2m}{C_d\, A_f\, \rho_a}\right)\frac{dv}{dt} = \left(\frac{2\left(F - \lambda_f\, m\, g \cos(\alpha) - m\, g \sin(\alpha)\right)}{C_d\, A_f\, \rho_a}\right) - v^2$$

therefore, when the traction force F is constant and $\geq \left(\lambda_f\, m\, g \cos(\alpha) + m\, g \sin(\alpha)\right)$, the equation can be written as:

$$p\frac{dv}{dt} = q^2 - v^2$$

where, $p = \dfrac{2m}{C_d\, A_f\, \rho_a}$ and $q^2 = \dfrac{2\left(F - \lambda_f\, m\, g\cos(\alpha) - m\, g\sin(\alpha)\right)}{C_d\, A_f\, \rho_a}$

when the vehicle is accelerating with a constant traction force $q^2 \geq v^2$, therefore,

$$p\frac{dv}{dt} = q^2 - v^2 \Rightarrow \frac{dv/p}{\left(1-(v/p)^2\right)} = \frac{p}{q}dt$$

Choosing $u = \dfrac{v}{p}$, leads to, $\dfrac{du}{\left(1-u^2\right)} = \dfrac{p}{q}dt \Rightarrow \displaystyle\int\frac{du}{\left(1-u^2\right)} = \int\frac{p}{q}dt$

since, $\displaystyle\int\frac{du}{\left(1-u^2\right)} = \tanh^{-1}(u)$, therefore,

$\displaystyle\int\frac{du}{\left(1-u^2\right)} = \int\frac{p}{q}dt \Rightarrow \tanh^{-1}(u) = \frac{p}{q}t + C \Rightarrow u = \tanh\left(\frac{a}{b}t + C\right)$; where C : integration constant.

since, $u = \dfrac{v}{p}$, therefore, $v(t) = p\tanh\left(\dfrac{p}{q}t + C\right)$

<u>Initial conditions:</u>

At $t = 0$, $v = v_o$, therefore,

$$v(t) = v_f\tanh\left(\frac{t}{\tau} + C\right) \Rightarrow \tanh(C) = \frac{v_o}{v_f} \Rightarrow C = \tanh^{-1}\left(\frac{v_o}{v_f}\right)$$

c.

i. The traction force delivered by the drive-train is constant and given by:

$$F = \frac{T}{R_t} \times \frac{\eta_t}{r_w} = \frac{72 \times 0.95}{0.17857 \times 0.2965} = 1291.9 \text{ N}$$

Since the vehicle is travelling on a road with an upward inclination, the equation governing the motion of the vehicle becomes:

$$m\frac{dv_a}{dt} = \overbrace{F}^{\text{traction force}} - \overbrace{\lambda_f\, m\, g\cos(\alpha)}^{\text{rolling resistance}} - \overbrace{\frac{1}{2}\rho_a\, C_d\, A_f\, v_a^2}^{\text{aerodynamic drag}} - m\, g\sin(\alpha)$$

since $\left(F - \lambda_f\, m\, g\cos(\alpha) - m\, g\sin(\alpha)\right)$

$= \left(1291.9 - 0.009 \times 1900 \times 9.81 \times \cos\left(2^o\right) - 1900 \times 9.81 \times \sin\left(2^o\right)\right) = 473.76$ N is > 0

therefore,

$$p = \frac{2m}{C_d\, A_f\, \rho_a} = \frac{2 \times 1200}{0.35 \times 1.6 \times 1.225} = 3498.54$$

$$q = \sqrt{\frac{2\left(F - \lambda_f \, m \, g \cos(\alpha) - m \, g \sin(\alpha)\right)}{C_d \, A_f \, \rho_a}}$$

$$= \sqrt{\frac{2 \times \left(1291.9 - 0.009 \times 1900 \times 9.81 \times \cos\left(2^o\right) - 1900 \times 9.81 \times \sin\left(2^o\right)\right)}{0.35 \times 1.6 \times 1.225}} = 37.165$$

Furthermore, the speed of the vehicle is given by: $v(t) = q \tanh\left(\frac{q}{p} t + C\right) \Rightarrow t = \frac{p}{q}\left(\tanh^{-1}\left(\frac{v}{q}\right) - C\right)$,

where C =0, since initial speed is 0.

$$t_a = \frac{5539.36}{37.165} \times \left(\tanh^{-1}\left(\frac{17.877}{37.165}\right)\right) = 78.146 \text{ s}$$

ii. The energy delivered by the drive-train is given by: $E = \int_0^{t_a} F \, v(t) dt = F \int_0^{t_a} q \tanh\left(\frac{q}{p} t\right) dt$

choosing $u = \frac{q}{p} t \Rightarrow dt = \frac{p}{q} du$ and, the energy would then be given by:

$$E = F \int_{u_1}^{u_2} p \tanh(u) du = p \, F \, \left|\ln(\cosh(u))\right|_{u_1}^{u_2}$$

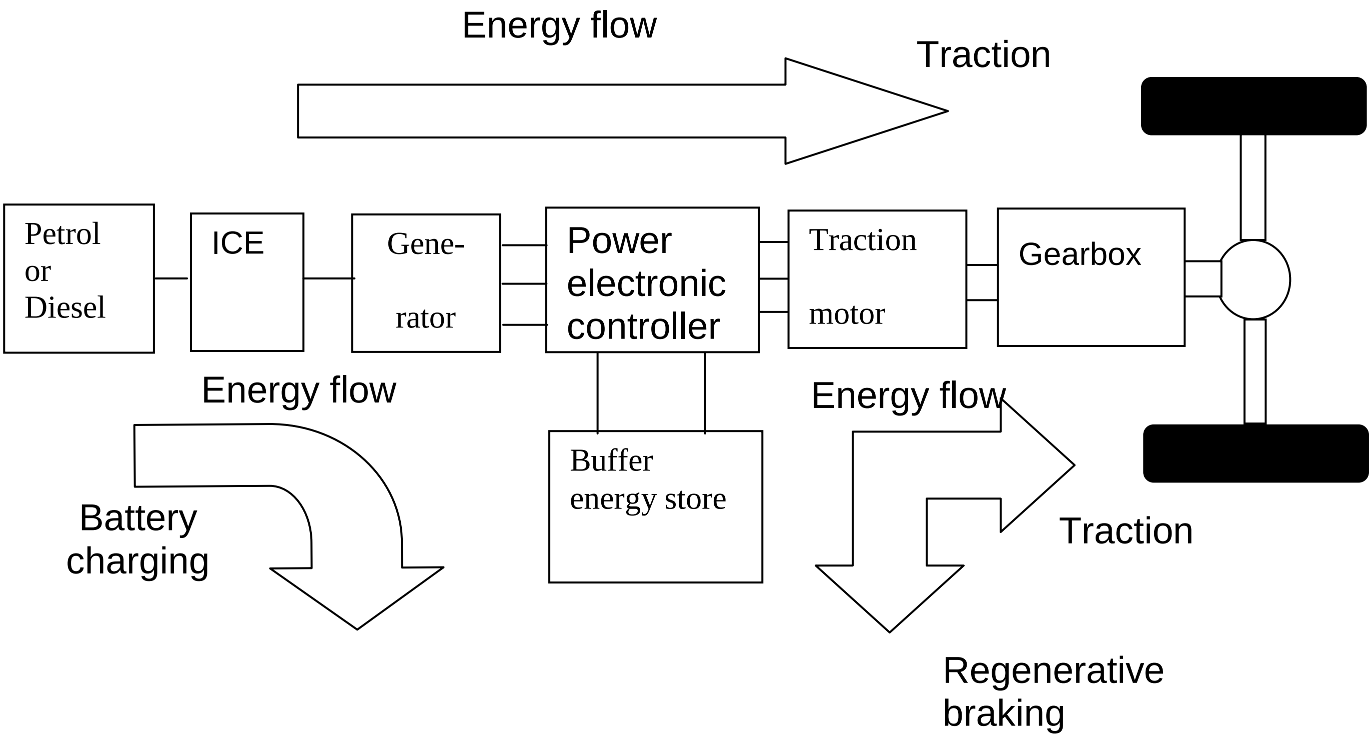
$$u_1 = 0; \quad u_2 = \frac{q}{p} t_a = \frac{37.165}{5539.36} \times 78.146 = 0.5243$$

Therefore,

$$\begin{aligned} E &= F \; p \; \left(\ln(\cosh(u_2)) - \ln(\cosh(u_1))\right) \\ &= 1291.9 \times 5539.36 \times \left(\ln(\cosh(0.5243)) - \ln(\cosh(0))\right) \\ &= 941.6\text{kJ} \end{aligned}$$

Question No. 4

a. Basic series-hybrid drive-train



- The ICE is operated at its optimum operating region.
- Improved fuel economy and reduced emissions.
- Same range as conventional vehicles.
- Requires more components then conventional vehicles, and therefore, higher price.

b. The losses of the drive-train, excluding the battery could be written as:

$$L_d = L_e + L_g$$

where,

$$L_e = 7.5T + 7.5\times10^{-2}T^2 + 3.65\times10^{-3}\,\Omega^2$$

And since the efficiency of the transmission is given by:

$$\eta_g = \frac{a + b\Omega + cT}{T} = \frac{T\,\Omega - L_e}{T\,\Omega}$$

Therefore,

$$L_e = (1-c)T\,\Omega - a\,\Omega - b\,\Omega^2$$

Therefore, the total losses of the drive-train, excluding the battery, is given by:

$$L_d = 5T + 3.75\times10^{-2}\,T^2 + (1-c)T\Omega - a\Omega + \left(3.65\times10^{-3} - b\right)\Omega^2$$

c.

i. The traction force F required for propelling the vehicle is given by:

$$\begin{aligned}
F &= \lambda_f\, m\, g \cos(\alpha) + m\, g \sin(\alpha) + \frac{1}{2} C_d\, A_f\, \rho_a\, v^2 \\
&= 0.027\times1630\times9.81\times\cos(0) + 1630\times9.81\times\sin(0) + 0.5\times0.34\times2.1\times1.225\times13.41^2 \\
&= 431.74 + 0 + 78.64 \\
F &= 510.4\,N
\end{aligned}$$

The torque of the wheels, T_w, is then given by:

$$T_w = F\, r_w = 510.4\times0.3215 = 164.1\,\mathrm{Nm}$$

and the rotational speed of the wheel, Ω_w, is given by:

$$\Omega_w = \frac{v}{r_w} = \frac{13.41}{0.3215} = 41.71\,\mathrm{rad/s}$$

when 3rd gear is selected, the total gear ratio R_t, between the wheels and the traction motor is:

$$R_t = \overbrace{0.2703}^{differential} \times \overbrace{0.6998}^{gearbox} = 0.1891$$

therefore, the speed of the traction motor is given by:

$$\Omega = \frac{\Omega_w}{R_t} = \frac{41.71}{0.1891} = 220.6\,\text{rad/s}$$

If the transmission loss coefficients a and b are neglected, the torque of the traction motor is given by:

$$\eta_g = \frac{a + b\Omega + cT}{T} = \frac{R_t T_w}{T} \quad \Rightarrow \quad T = \frac{R_t T_w - a - b\Omega}{c} = \frac{0.1891 \times 164.1 + 0.6086 + 1.11 \times 10^{-3} \times 220.6}{0.9425}$$
$$T = 33.83\text{ Nm}$$

and the loss of the drive-train is given by:

$$\begin{aligned} L_d &= 5T + 3.75 \times 10^{-2} T^2 + (1-c)T\Omega - a\Omega + \left(3.65 \times 10^{-3} - b\right)\Omega^2 \\ &= 5 \times 33.83 + 3.75 \times 10^{-2} \times 33.83^2 + (1 - 0.9425) \times 33.83 \times 220.6 + 0.6086 \times 220.6 \\ &+ \left(3.65 \times 10^{-3} + 1.11 \times 10^{-3}\right) \times 220.6^2 \\ &= 1007\text{ W} \end{aligned}$$

therefore, the efficiency of the drive-train is given by:

$$\eta_d = \frac{T_w \Omega_w}{T_w \Omega_w + L_d} = \frac{164.1 \times 41.71}{164.1 \times 41.71 + 1007} = 87.17\%$$

ii. The power delivered by the battery can be expressed as:

$P_d = E_o I - R_i I^2$, therefore, current is solution of the quadratic equation : $R_i I^2 - E_o I + P_d = 0$

$\Delta = E_o^2 - 4 R_i P_d$ and the solutions could be expressed as :

$$I_1 = \frac{E_o - \sqrt{E_o^2 - 4 R_i P_d}}{2 R_i} \text{ and } I_2 = \frac{E_o + \sqrt{E_o^2 - 4 R_i P_d}}{2 R_i}$$

I_2 is not a valid solution, since $I_2 \neq 0$, when $P_d = 0$, therefore, $I = I_1 = \dfrac{E_o - \sqrt{E_o^2 - 4 R_i P_d}}{2 R_i}$

When Q_d=50%,

$$E_o = 200 - 0.25 Q_d = 200 - 0.25 \times 50 = 187.5\,\text{V}$$

and,

$$R_i = 100 + 1.25 Q_d = 100 + 1.25 \times 50 = 162.5\,\text{m}\Omega$$

Therefore,

$$I = \frac{E_o - \sqrt{E_o^2 - 4 R_i P_d}}{2 R_i} = \frac{187.5 - \sqrt{187.5^2 - 4 \times 162.5 \times 10^{-3} \times 7852}}{2 \times 162.5 \times 10^{-3}} = 43.52\text{ A}$$

The terminal voltage of the battery is then given by:

$$V = E - R_i I = 180.43 \text{ V}$$

iii. The efficiency of the battery is given by:

$$\eta_b = 100 \times \frac{V}{E} = 100 \times \frac{180.43}{187.5} = 96.23\%$$

And the efficiency of the drive-train including the battery is then given by:

$$\eta_t = \eta_d \times \eta_b = 0.8717 \times 0.9623 = 83.88\ \%$$