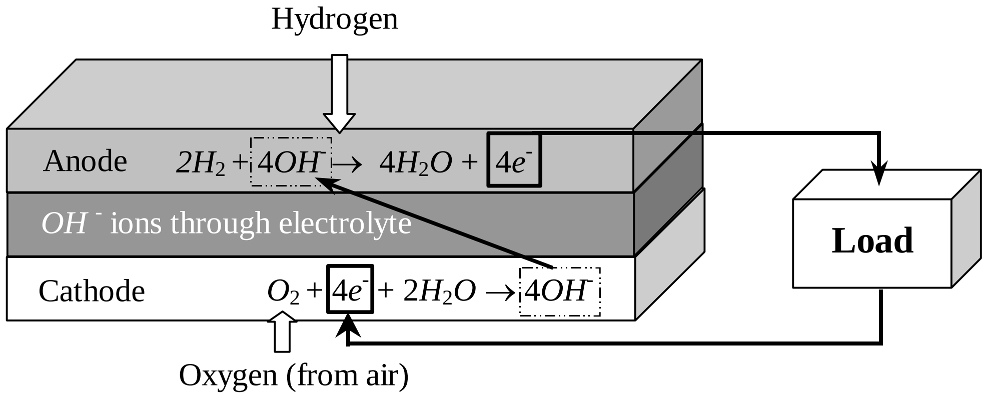
EEE6021Model Solutions 2013-14

1.

a. The schematic of an alkaline fuel cell is shown as follows: **(4)**



The electrodes are usually made flat, with a thin layer of alkaline electrolyte between them as shown in the figure. The structure of the electrode is porous, so that both the electrolyte from one side and the gas from the other can penetrate it. This is to give the maximum possible contact between the electrode, the electrolyte and the gas.

In an alkaline fuel cell, alkali hydroxyl (OH^-) ions are available and mobile. At the anode these react with hydrogen, releases energy and electrons, and producing water:

$$2H_2 + 4HO^- \rightarrow 4H_2O + 4e^-$$

With more electrons produced at the anode, the electric potential of the anode becomes the low than that of the cathode. Thus the electrons will flow from the anode to the cathode through the electrical load under the influence of the potential difference.

At the cathode oxygen reacts with electrons taken from the electrode, and water in the electrolyte, forming new OH^- ions:

$$O_2 + 4e^- + 2H_2O \rightarrow 4OH^-$$

In order for both reactions to proceed continuously, electrons produced at the anode must pass through an electrical circuit to the cathode. Also, OH ions must pass through the electrolyte. An alkaline electrolyte is a fluid with free OH^- ions, and so serves this purpose very well.

b. If there are no losses in the fuel cell, or in chemistry term, the process is "reversible", then all Gibbs free energy of formation per mole fuel will be converted into electrical energy. **(5)**

Consider work to be done to move electrons through an external circuit with a voltage *E*. For the methanol fuel cell, 12 electrons pass round the external circuit for two water molecules and two carbon dioxide molecules produced and two molecules of methanol used. Hence, for one mole of methanol used, 6*N* electrons pass round the external circuit, where *N* is the Avagadro's number. If *–e* is the charge on one electron, then the charge that flow is:

$$-6Ne = -6F \text{ (Coulomb)}$$

where *F* is the Faraday constant, or the charge of one mole of electrons. If *E* is the voltage of the fuel cell, then the electrical work done in order to move this amount of charge via the circuit is:

$$\textit{Electrical work done} = Q\int_l \vec{E}\cdot d\vec{l} = \textit{charge} \text{ x } \textit{voltage} = -6FE \text{ (Joules)}$$

For an ideal system (or has no losses), this electrical work done will be equal to the Gibbs free energy released. Thus:

$$\Delta\bar{g}_f = -6EF$$

or

$$E = -\Delta\bar{g}_f / 6F$$

c. i Since the reaction uses two methanol molecules, and release 12 electrons. Thus for each mole of methanol used, 6 mole of electrons are generated, which in an ideal case would pass through an external circuit to produce the emf given by: **(2)**

$$E = -\frac{\Delta\bar{g}_f}{6F} = -\frac{-698.2\times10^3}{6\times96485} = 1.206\,(\text{V})$$

c. ii In the methanol fuel cell, the equation for the chemical reaction is given by: **(5)**

$$2CH_3OH + 3O_2 \Rightarrow 4H_2O + 2CO_2 + 12e^-$$

which may be rewritten as:

$$CH_3OH + 1.5O_2 \Rightarrow 2H_2O + CO_2 + 6e^-$$

The Gibbs free energy of formation $\Delta\bar{g}_f$ released by one mole of fuel is dependent on the operation pressure and temperature, and the variation is given by:

$$\Delta\bar{g}_f = \Delta\bar{g}_f^0 - RT\ln\left[\frac{\left(\frac{P_{CH_3OH}}{P_0}\right)\sqrt[3]{\frac{P_{O_2}}{P_0}}}{\left(\frac{P_{H_2O}}{P_0}\right)^2\left(\frac{P_{CO_2}}{P_0}\right)}\right]$$

where

$\Delta\bar{g}_f^0$ — Change in molar Gibbs free energy formation at standard pressure
R — Molar gas constant, 8.314 J/(K mole)
T — Temperature (K)
P_{CH3OH} — Partial pressure of methanol
P_{O2} — Partial pressure of oxygen
P_{H_2O} — Partial pressure of water
P_{CO2} — Partial pressure of carbon dioxide
P_0 — Standard pressure

If unit bar is used for pressure, P_0 = 1.0, the effect of pressure on $\Delta\bar{g}_f$ is simplified to:

$$\Delta\bar{g}_f = \Delta\bar{g}_f^0 - RT\ln\left[\frac{P_{CH_3OH}\sqrt[3]{P_{O_2}}}{\left(P_{H_2O}\right)^2\left(P_{CO_2}\right)}\right]$$

The reversible open-circuit voltage is given by:

$$E = -\frac{\Delta\bar{g}_f^0}{6F} + \frac{RT}{6F}\ln\left[\frac{P_{CH_3OH}\sqrt[3]{P_{O_2}}}{\left(P_{H_2O}\right)^2\left(P_{CO_2}\right)}\right]$$

The change in the open-circuit due to the change in the supply pressure of methanol is given by:

$$\Delta E = \frac{RT}{6F}\ln\left[\frac{\left(P_{CH_3OH}\right)_2}{\left(P_{CH_3OH}\right)_1}\right] = \frac{8.314\times 373}{6\times 96485}\ln 2 = 3.71\times 10^{-3}\ \text{(V)}$$

c.iii In an ideal case where all losses are neglected, the Gibbs free energy of formation release during the electro-chemical reaction in a fuel cell would be converted into electrical energy. Thus the maximum efficiency of the fuel cell can be evaluated by: **(4)**

$$\eta_{\max} = -\frac{\Delta\bar{g}_f}{\Delta\bar{h}_f}$$

where $\Delta\bar{h}_f$ is the heating value of per mole fuel. Thus the maximum theoretical efficiency of the fuel cell can be evaluated by

$$\eta_{\max} = \mu_f\left\{-\frac{\Delta\bar{g}_f^0}{\Delta h_f} + \frac{RT}{\Delta h_f}\ln\left[\frac{P_{CH_3OH}\sqrt[3]{P_{O_2}}}{\left(P_{H_2O}\right)^2\left(P_{CO_2}\right)}\right]\right\}$$

where μ_f is the fuel utilisation factor

The partial pressure of oxygen at 2 bar is P_{02} = 2.0x0.2095 = 0.419, and the maximum efficiency is:

$$\eta_{\max} = \mu_f \left\{ -\frac{\Delta \bar{g}_f^0}{\Delta h_f} + \frac{RT}{\Delta h_f} \ln \left[\frac{P_{CH_3OH} \sqrt[3]{P_{O_2}}}{(P_{H_2O})^2 (P_{CO_2})} \right] \right\}$$

$$= 0.95 \left\{ \frac{698.2}{790} + \frac{8.31 \times 373}{790 \times 10^3} \ln \frac{2\sqrt[3]{0.419}}{1.8^3} \right\} = 0.8345$$

2.

a. Key features of the output voltage characteristic: **(10)**

- The open circuit voltage is very close to the theoretical (ideal) value.
- The voltage then falls more or less linearly, but exhibiting a relative large slope which
- At a higher current density, the voltage falls rapidly away.

Causes of the voltage drops

The reduction in the output voltage is due to the following losses:

- *Activation* losses. These are caused by the slowness of the reactions taking place on the surface of the electrodes. A proportion of the voltage generated is lost in driving the chemical reaction that transfers the electrons to or from the electrode. However this loss is insignificant in the SOFC due to high operating temperature which facilitates a high reaction rate.

- *Fuel crossover and internal currents***.** This energy loss results from the waste of fuel passing through the electrolyte, and, to a lesser extent, electron conduction through the electrolyte. The electrolyte should only transport ions through the cell. However, a certain amount of fuel diffusion and electron flow will always be possible. The fuel loss and current is small, and its effect is usually not very important. However, it does not have a significant effect on the open circuit voltage of the high temperature cells.

- *Ohmic losses***.** This voltage drop is the straightforward resistance to the flow of electrons through the material of the electrodes and the various interconnections, as well as the resistance to the flow of ions through the electrolyte. This voltage drop is essentially linearly proportional to current density, and so is called "Ohmic" losses. Since the resistivity of the electrodes, electrolyte and inter-connectors increases with temperature, and the Ohmic loss of this high temperature fuel cell is most significant. This is the reason why the voltage varies with current almost linearly and the $\Delta v/\Delta i$ vs is relatively large.

- *Mass transport or concentration losses***.** These result from the change in concentration of the reactants at the surface of the electrodes as the fuel is used. This is due to the fact that concentration affects voltage, and so this type of loss is sometimes called "concentration" loss. Because the reduction in concentration is the result of a failure to transport sufficient reactant to the electrode surface, this type of loss is also often called "mass transport" loss. Since there is a rate limit that the amount of fuel and oxygen can be supply to the system, as the current density increases to greater than ~800A/cm^2, the fuel cell is starved of the fuel and oxygen, and hence exhibits dramatic reduction in the output voltage.

b. The output voltage of the fuel cell under a given current density may be represented by **(4)**

$$V = E - r(i + i_n) - A\ln\left(\frac{i + i_n}{i_0}\right) + B\ln\left(1 - \frac{i + i_n}{i_l}\right) \qquad i_l - i_n > i > i_0 - i_n$$

where

E is the ideal open-circuit emf of the fuel cell

i_0 is the exchange current density representing the effect of activation energy losses

i_n is the equivalent current representing the fuel crossover, and internal leakage current.

i_l is the limiting current density i_l at which the fuel concentration will become zero

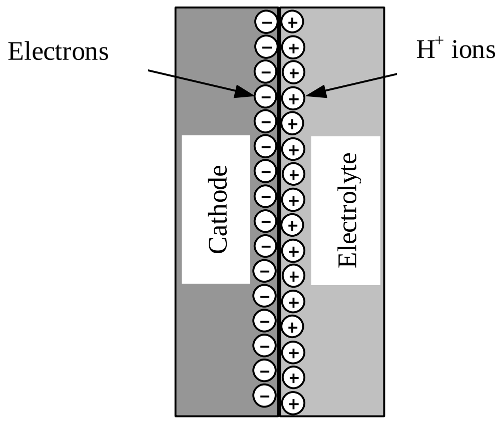
r is the ohmic resistance per unit area.

Using the constants given in Table 1, the output voltage at i = 600 (mA/cm^2) can be evaluated by:

$$V = 1.0 - 3.0\times10^{-1}(600+2)\times10^{-3} - 0.03\ln\left(\frac{600+2}{300}\right) + 0.08\ln\left(1 - \frac{600+2}{900}\right)$$
$$= 0.71(V)$$

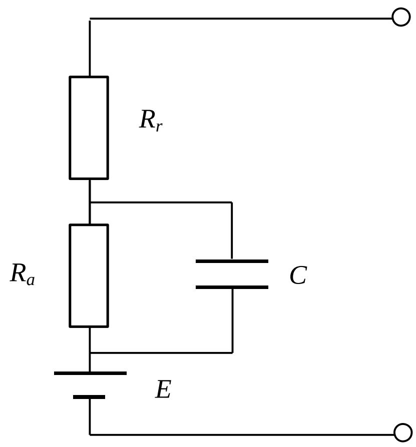
c. The electrical dynamic behaviour is mainly influence by a charge double layer which is formed at the interface between an electrode and the electrolyte. For example at the cathode of a hydrogen fuel cell, the electrons, driven through the external circuit, will collect at the surface of the electrode, and H^+ ions, which are generated at the anode, will move and be attracted to the interface surface between the cathode and the electrolyte, as shown below. These electrons and ions, together with the O_2 supplied by to the cathode, will take part in the cathode reaction: **(6)**

$$O_2 + 4e^- + 4H^+ \rightarrow 2H_2O$$



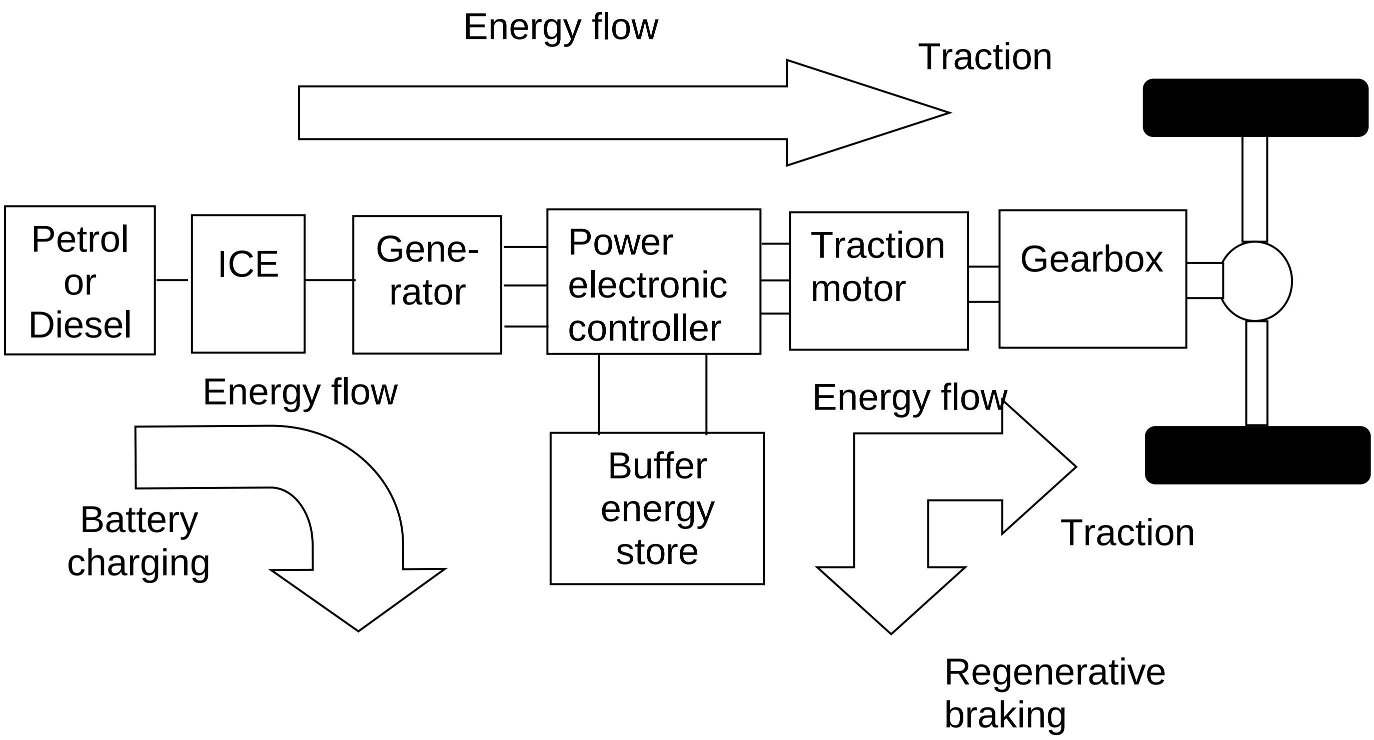
A similar charge double layer may also exist at the anode.

The layer of charge on the electrode/electrolyte interface is a store of electrical charge and energy, and as such behaves much like an electrical capacitor. If the current changes it will take some time for this charge (and its associated voltage) to dissipate (if the current reduces) or build-up (if there is a current increase). So, the activation over-voltage does not immediately follow the current in the way that the ohmic voltage drop does. The result is that if the current suddenly changes the operating voltage shows an immediate change due to the internal ohmic resistance, but moves fairly slowly to its final equilibrium value. The effect of this capacitance resulting from the charge double layer may be represented in the following equivalent circuit, where E is the open-circuit voltage, R_a represents the losses due to activation, fuel cross-over and internal current, and R_r is the ohmic resistance of the fuel cell.



Solution to question No. 3

a.



- The ICE is operated at its optimum operating region.
- Improved fuel economy and reduced emissions.
- Same range as conventional vehicles.
- Requires more components then conventional vehicles, and therefore, higher price.

b.

On a flat road the forces on a vehicle of mass m are related by the following differential equation:

$$m\frac{dv}{dt}=\overbrace{F}^{\text{traction force}}-\overbrace{\lambda_f\, m\, g}^{\text{rolling resistance}}-\overbrace{\frac{1}{2}\rho_a\, C_d\, A_f\,(v+v_w)^2}^{\text{aerodynamic drag}}$$

where v is the speed of the vehicle. The above equation can be re-written as:

$$\left(\frac{2m}{C_d\, A_f\, \rho_a}\right)\frac{dv}{dt}=\left(\frac{2(F-\lambda_f\, m\, g)}{C_d\, A_f\, \rho_a}\right)-(v+v_w)^2$$

since the airspeed $v_a=v+v_w$, where v_w is the constant air speed with respect to the ground, therefore,

$\frac{dv_a}{dt}=\frac{dv}{dt}+\overbrace{\frac{dv_w}{dt}}^{=0}=\frac{dv}{dt}$, and the equation becomes:

$$\left(\frac{2m}{C_d\, A_f\, \rho_a}\right)\frac{dv_a}{dt}=\left(\frac{2(F-\lambda_f\, m\, g)}{C_d\, A_f\, \rho_a}\right)-v_a^2$$

When the traction force F is constant and $(F-\lambda_f\, m\, g)\geq 0$ the equation can be written as:

$$p\frac{dv_a}{dt}=q^2-v_a^2$$

where, $p=\frac{2m}{C_d\, A_f\, \rho_a}$ and $q^2=\frac{2(F-\lambda_f\, m\, g)}{C_d\, A_f\, \rho_a}$

However, if the traction force F is constant and $(F-\lambda_f\, m\, g)\leq 0$ the equation can then be written as:

$$p\frac{dv_a}{dt}=-q^2-v_a^2$$

$$p=\frac{2m}{C_d\, A_f\, \rho_a}\ \text{and}\ q^2=\left|\frac{2(F-\lambda_f\, m\, g)}{C_d\, A_f\, \rho_a}\right|$$

c.

The traction force delivered by the drive-train is constant and given by:

$$F = \frac{T}{R_t} \times \frac{\eta_t}{r_w} = \frac{30 \times 0.95}{0.5526 \times 0.16} = 322.34 \text{ N}$$

since the vehicle is travelling on a road with an upward inclination, the equation governing the motion of the vehicle becomes:

$$m\frac{dv_a}{dt} = \overbrace{F}^{\text{traction force}} - \overbrace{\lambda_f\, m\, g}^{\text{rolling resistance}} - \overbrace{\frac{1}{2}\rho_a\, C_d\, A_f\, v_a^2}^{\text{aerodynamic drag}}$$

$$\text{since } (F - \lambda_f\, m\, g) = (322.34 - 0.009 \times 250 \times 9.81) = 322.27 \text{ N is} > 0$$

Therefore, the equation could be written as:

$$p\frac{dv_a}{dt} = q^2 - v_a^2$$

where,

$$p = \frac{2m}{C_d\, A_f\, \rho_a} = \frac{2 \times 250}{0.6 \times 0.8 \times 1.225} = 850.34$$

$$q = \sqrt{\frac{2(F - \lambda_f\, m\, g)}{C_d\, A_f\, \rho_a}} = \sqrt{\frac{2 \times (322.34 - 0.009 \times 250 \times 9.81)}{0.6 \times 0.8 \times 1.225}} = 31.96$$

Furthermore, the speed of the vehicle is given by:

$$v_a(t) = q \tanh\left(\frac{q}{p}t + C\right) \Rightarrow t = \frac{p}{q}\left(\tanh^{-1}\left(\frac{v_a}{q}\right) - C\right)$$

where, $v = 30\text{MPH} = 13.408 \text{ m/s}$ and $v_a = v + v_w = 20.113 \text{ m/s}$

$$C = \tanh^{-1}\left(\frac{v_o}{q}\right) = \tanh^{-1}\left(\frac{v_w}{q}\right) = \tanh^{-1}\left(\frac{8.939}{31.96}\right) = 0.213$$

$$t_a = \frac{850.34}{31.96} \times \left(\tanh^{-1}\left(\frac{20.113}{31.96}\right) - 0.213\right) = 14.03 \text{ s}$$

ii. The energy delivered by the drive-train is given by:

$$E=\int_0^{t_a} F\,v(t)\,dt=\int_0^{t_a} F\,(v_a(t)-v_w)\,dt=F\int_0^{t_a} v_a(t)\,dt-F\,v_w\,t_a$$
$$=F\int_0^{t_a} q\tanh\left(\frac{q}{p}t+C\right)dt-F\,v_w\,t_a$$

choosing $u=\frac{q}{p}t+C\Rightarrow dt=\frac{p}{q}du$ and, the energy would then be given by:

$$E=F\int_{u_1}^{u_2} p\tanh(u)\,du-F\,v_w\,t_a$$
$$=p\,F\,\left|\ln(\cosh(u))\right|_{u_1}^{u_2}-F\,v_w\,t_a$$

$$u_1=C=0.287;\quad u_2=\frac{q}{p}t_a+C=\frac{31.96}{850.34}\times 14.03+0.213=0.7403$$

Therefore,

$$E=F\,(p\ln(\cosh(u_2))-p\ln(\cosh(u_1))-v_w\,t_a)$$
$$=322.34\times(850.34\times\ln(\cosh(0.7403))-850.34\times\ln(\cosh(0.213))-6.704\times 14.03)=32.62\,\mathrm{kJ}$$

d.

The traction force F required for propelling the vehicle on a flat road with headwind $v_w=15\mathrm{MPH}$:

$$F=\lambda_f\,m\,g+\frac{1}{2}C_d\,A_f\,\rho_a\,(v+v_w)^2$$
$$=0.009\times 250\times 9.81+0.5\times 0.6\times 0.8\times 1.225\times(13.408+6.704)^2$$
$$F=141\,N$$

The equation governing the motion of the vehicle becomes:

$$m\frac{dv_a}{dt}=\overbrace{F}^{\text{traction force}}-\overbrace{\lambda_f\,m\,g}^{\text{rolling resistance}}-\overbrace{\frac{1}{2}\rho_a\,C_d\,A_f\,v_a^2}^{\text{aerodynamic drag}}$$

since $(F-\lambda_f\,m\,g)=(141-0.009\times 250\times 9.81)=119\ \mathrm{N}$ is >0

Therefore, the equation could be written as:

$$p\frac{dv_a}{dt}=q^2-v_a^2$$

where,

$$p = \frac{2\,m}{C_d\ A_f\ \rho_a} = \frac{2\times 250}{0.6\times 0.8\times 1.225} = 850.34$$

$$q = \sqrt{\frac{2\left(F - \lambda_f\ m\,g\right)}{C_d\ A_f\ \rho_a}} = \sqrt{\frac{2\times\left(141 - 0.009\times 250\times 9.81\right)}{0.6\times 0.8\times 1.225}} = 20.113$$

Furthermore, the speed of the vehicle is given by:

$$v_a(t) = q\tanh\left(\frac{q}{p}t + C\right) \Rightarrow t = \frac{p}{q}\left(\tanh^{-1}\left(\frac{v_a}{q}\right) - C\right)$$

where, $v = 40\text{MPH} = 17.87\ \text{m/s}$ and $v_a = v + 0 = 17.87\ \text{m/s}$

$$C = \tanh^{-1}\left(\frac{v_o}{q}\right) = \tanh^{-1}\left(\frac{13.408}{20.11}\right) = \tanh^{-1}\left(\frac{13.408}{20.11}\right) = 0.8047$$

$$t_a = \frac{850.34}{20.11}\times\left(\tanh^{-1}\left(\frac{20.11}{20.11}\right) - 0.8057\right) = 25.8\text{s}$$

Solution to question No. 4

a.

1. The traction force F required for propelling the vehicle is given by:

$$F = \lambda_f \, m\, g \cos(\alpha) + m\, g \sin(\alpha) + \frac{1}{2} C_d \, A_f \, \rho_a \, v^2$$
$$= 0.027 \times 1630 \times 9.81 \times \cos(0) + 1630 \times 9.81 \times \sin(0) + 0.5 \times 0.34 \times 2.1 \times 1.225 \times 13.41^2$$
$$= 431.74 + 0 + 78.64$$
$$F = 510.4\, N$$

The torque of the wheels, T_w, is then given by:

$$T_w = F\, r_w = 510.4 \times 0.3215 = 164.1\,\text{Nm}$$

and the rotational speed of the wheel, Ω_w, is given by:

$$\Omega_w = \frac{v}{r_w} = \frac{13.41}{0.3215} = 41.71\,\text{rad/s}$$

when 3nd gear is selected, the total gear ratio R_t, between the wheels and the traction motor is:

$$R_t = \overbrace{0.2703}^{differential} \times \overbrace{0.6998}^{gearbox} = 0.1891$$

therefore, the speed of the traction motor is given by:

$$\Omega = \frac{\Omega_w}{R_t} = \frac{41.71}{0.1891} = 220.6\,\text{rad/s}$$

If the transmission loss coefficients a and b are neglected, the torque of the traction motor is given by:

$$\eta_t = \frac{a + b\Omega + cT}{T} = \frac{R_t\, T_w}{T} \quad \Rightarrow \quad T = \frac{R_t\, T_w - a - b\Omega}{c} = \frac{0.1891 \times 164.1 + 0.6086 + 1.11 \times 10^{-3} \times 220.6}{0.9425}$$
$$T = 33.83\ \text{Nm}$$

and the loss of the drive-train is given by:

$$
\begin{aligned}
L_d &= 5T + 3.75\times10^{-2}T^2 + (1-c)T\Omega - a\Omega + \left(3.65\times10^{-3} - b\right)\Omega^2 \\
&= 5\times33.83 + 3.75\times10^{-2}\times33.83^2 + (1-0.9425)\times33.83\times220.6 + 0.6086\times220.6 \\
&+ \left(3.65\times10^{-3} + 1.11\times10^{-3}\right)\times220.6^2 \\
&= 1007\ \text{W}
\end{aligned}
$$

And the efficiency of the drive-train, excluding the battery, is then given by:

$$
\eta_d = \frac{T_w\,\Omega_w}{T_w\,\Omega_w + L_d} = \frac{164.1\times41.71}{164.1\times41.71+1007} = 87.2\,\%
$$

2. The traction force F required for propelling the vehicle is given by:

$$
\begin{aligned}
F &= \lambda_f\, m\, g \cos(\alpha) + m\, g \sin(\alpha) + \frac{1}{2} C_d\, A_f\, \rho_a\, (v + v_w)^2 \\
&= 0.027\times1630\times9.81\times\cos(0) + 1630\times9.81\times\sin(0) + 0.5\times0.34\times2.1\times1.225\times26.82^2 \\
F &= 746.3\,N
\end{aligned}
$$

The torque of the wheels, T_w, is then given by:

$$
T_w = F\, r_w = 510.4\times0.3215 = 240\,\text{Nm}
$$

and the rotational speed of the wheel, Ω_w, is given by:

$$
\Omega_w = \frac{v}{r_w} = \frac{13.41}{0.3215} = 41.71\,\text{rad/s}
$$

when 3[nd] gear is selected, the total gear ratio R_t, between the wheels and the traction motor is:

$$
R_t = \overbrace{0.2703}^{differential} \times \overbrace{0.6998}^{gearbox} = 0.1891
$$

therefore, the speed of the traction motor is given by:

$$
\Omega = \frac{\Omega_w}{R_t} = \frac{41.71}{0.1891} = 220.6\,\text{rad/s}
$$

If the transmission loss coefficients a and b are neglected, the torque of the traction motor is given by:

$$\eta_t = \frac{a+b\Omega+cT}{T} = \frac{R_t T_w}{T} \quad \Rightarrow \quad T = \frac{R_t T_w - a - b\Omega}{c} = \frac{0.1891\times 240 + 0.6086 + 1.11\times 10^{-3}\times 220.6}{0.9425} = 49.06\,\mathrm{N}$$

$T = 49.06$ Nm

and the loss of the drive-train is given by:

$$L_d = 5T + 3.75\times 10^{-2}\,T^2 + (1-c)T\Omega - a\Omega + \left(3.65\times 10^{-3} - b\right)\Omega^2$$
$$= 5\times 49.06 + 3.75\times 10^{-2}\times 49.06^2 + (1-0.9425)\times 49.06\times 220.6 + 0.6086\times 220.6$$
$$+ \left(3.65\times 10^{-3} + 1.11\times 10^{-3}\right)\times 220.6^2$$
$$= 1324.5\,\mathrm{W}$$

And the efficiency of the drive-train, excluding the battery, is then given by:

$$\eta_d = \frac{T_w\,\Omega_w}{T_w\,\Omega_w + L_d} = \frac{164.1\times 41.71}{164.1\times 41.71 + 1324.5} = 83.8\,\%$$

b.

The power delivered by the battery can be expressed as:

$P_d = E_o\,I - R_i\,I^2$, therefore, current is solution of the quadratic equation : $R_i\,I^2 - E_o\,I + P_d = 0$

$\Delta = E_o^2 - 4\,R_i\,P_d$ and the solutions could be expressed as :

$$I_1 = \frac{E_o - \sqrt{E_o^2 - 4\,R_i\,P_d}}{2\,R_i} \text{ and } I_2 = \frac{E_o + \sqrt{E_o^2 - 4\,R_i\,P_d}}{2\,R_i}$$

I_2 is not a valid solution, since $I_2 \neq 0$, when $P_d = 0$, therefore,

$$I = I_1 = \frac{E_o - \sqrt{E_o^2 - 4\,R_i\,P_d}}{2\,R_i}$$

When Q_d=20%,

$$E_o = 200 - 0.25\,Q_d = 200 - 0.25\times 20 = 195\,\mathrm{V}$$

and,

$$R_i = 100 + 1.25\,Q_d = 100 + 1.25\times 20 = 125\,\mathrm{m\Omega}$$

Therefore,

$$I = \frac{E_o - \sqrt{E_o^2 - 4\,R_i\,P_d}}{2\,R_i} = \frac{195 - \sqrt{195^2 - 4 \times 125 \times 10^{-3} \times 10000}}{2 \times 125 \times 10^{-3}} = 53.1\,\text{A} \;\text{ for }\; P_d = 10000\;\text{ W}$$

The efficiency of the battery is then given by:

$$\lambda_b = 100 \times \frac{E - R_i\,I}{E} = 100 \times \frac{195 - 125 \times 10^{-3} \times 53.1}{195} = 96.6\,\%$$

<u>**c.**</u>

If the vehicle is going downhill, without applying mechanical, the only braking force is due to the rolling resistance, the aerodynamic drag and the power absorbed by the battery.

Furthermore, the force braking force produced by the aerodynamic drag at 30MPH is given by:

$$F_d = \frac{1}{2} C_d\,A_f\,\rho_a\,(v)^2 = 0.5 \times 0.34 \times 2.1 \times 1.225 \times 13.41^2 = 78.6\,\text{N}$$

Therefore, the power related to component of weight pulling the vehicle downhill should be equal to the braking forces:

$$m\,g\,\sin(\alpha)v = P_b + F_d\,v + \lambda_f\,m\,g\,\cos(\alpha)v$$

Therefore, the power absorbed by the battery is given by:

$$\begin{aligned} P_b &= m\,g\,\sin(\alpha)v - F_d\,v - \lambda_f\,m\,g\,\cos(\alpha)v \\ &= 1630 \times 9.81 \times \sin\left(5^o\right) \times 13.41 - 78.6 \times 13.41 - 0.027 \times 1630 \times 9.81 \times \cos(5) \times 13.41 \\ &= 11867\;\text{W} \end{aligned}$$