Energy and Power in Electrical Networks

Instantaneous values of time varying quantities

Note: Lower case letters are used to signify instantaneous values:

p – power (Watts, W) – (Joules per second Js^{-1})
e – Energy (Joules, J) – (Watt seconds Ws)
v – Voltage (Volts, V)
i – Current (Amps, A)

where power:

$$p = \frac{de}{dt} \quad \text{Watts (W)}$$

and energy:

$$e = \int p\,dt \quad \text{Joules (J)}$$

(note: Energy is synonymous with work, since work done equals energy expended or absorbed).

The Joule is useful unit for low power circuits, but for power systems we have larger units:

Small circuits:	= 1 Joule	= 1 Watt Second
1 'unit' in electricity tariffs	= 1 kiloWatthour	= 1kWh (1kW for 1 hour)

1 kWh = 3.6MJ (3.6×10^6J)

We also use MWh in the mains power distribution system

Aside:

Electrical energy stored in a 4700μF 25V capacitor = $0.5CV^2$ = 1.47Joules

Chemical energy stored in 1 Mars Bar! = 1.3MJ (~0.3kWh)

- Difficult to store significant amounts of energy in electrical form, therefore we use chemical storage of energy (batteries, supercapacitors etc)

NOTE: human body requires ~2kWh of energy input per day to sustain it ~ 6 Mars bars per day!)

Instantaneous Electrical Power

By definition, the potential or voltage difference between two points is the work done (energy change) per unit of charge transfer between two points, i.e.:

$$v = \frac{de}{dq} \text{ volts}$$

Also by definition, the current flow between two points is the rate of charge transfer between the two points, or:

$$i = \frac{dq}{dt} \text{ amps}$$

Hence power:

$$p = \frac{de}{dt} = \frac{de}{dq} \times \frac{dq}{dt} = vi$$

i.e. Instantaneous power is the product of the instantaneous voltage and the instantaneous current.

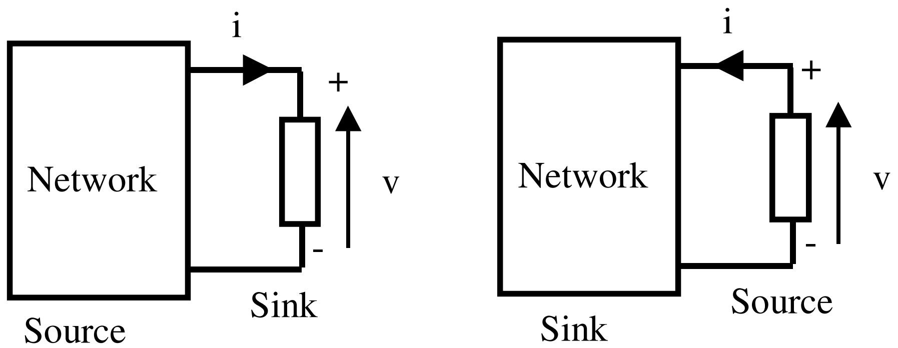
NB – THIS IS NOT, IN GENERAL, THE AVERAGE POWER – which is a more significant quantity (see later!)

And energy:

$$e = \int vidt$$

Direction of power flow

The instantaneous power absorbed or produced by a circuit is determined by the direction of the current flow through, and voltage across, the element under consideration.



Power and Energy in Network Elements

a) **Resistor, R**

- this models any device in which **energy is converted irreversibly into heat**
- energy is not stored in a resistor.

Note: v and i are lower case, instantaneous values

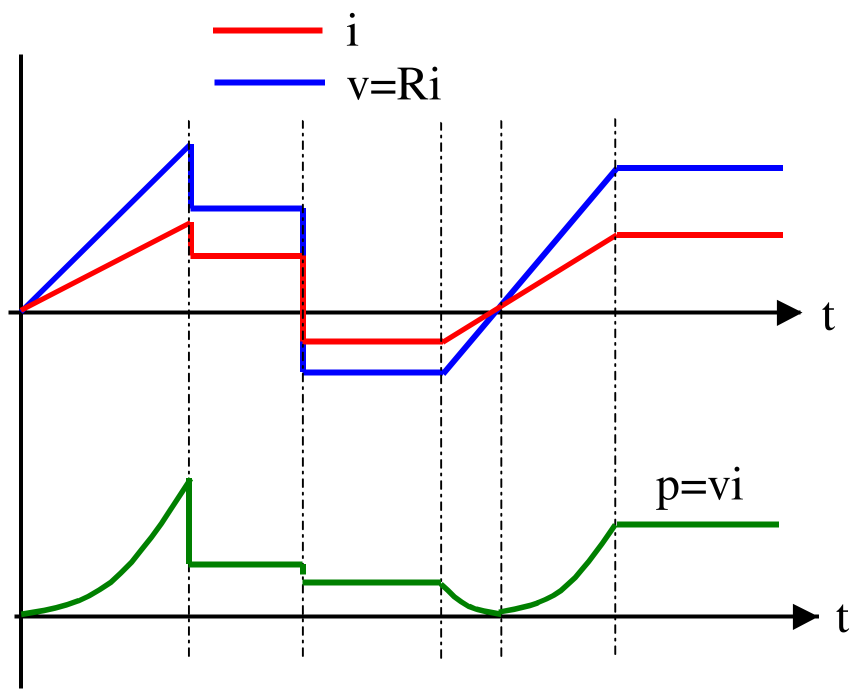
For a linear resistor:

$$v = Ri \quad \text{volts}$$

And instantaneous power:

$$p = vi = Ri^2 = \frac{v^2}{R} \quad \text{Watts}$$

Note: Even if v and i are negative, power, p, remains positive



In general, the electrical energy converted into heat in a time period $t = t_0$ to $t = t_1$ is given by:

$$e = \int_{t_0}^{t_1} vi\,dt \quad \text{Joules (Area under the p/t curve)}$$

Example

Calculate the instantaneous power drawn by a 6Ω resistor at time t = 4seconds, and the energy dissipated in the period 0 to 4 seconds, if the resistor is

(i) Connected to a 12V dc source

(ii) Connected to a $v(t) = 12t$ Volts source (t in seconds)

i) if $v = V_{dc} = 12V$ (time invariant)

then $i = I_{dc}=12/6 = 2A$ (also time invariant)

Therefore power:

$$p = vi = V_{dc}I_{dc} = 24W$$

and:

$$e = \int_0^4 24dt = 24\,t\Big|_0^4 = 96\text{ J}$$

ii) If:

$$v = 12t \text{ Volts}$$

then:

$$i = v/R = 12t/6 = 2t \text{ Amps}$$

and:

$$p = vi = 24t^2 \text{ Watts}$$

The instantaneous power then becomes:

$$@\ t = 4\text{sec},\quad p = 24\times 4^2 = 384 \text{ Watts}$$

Energy dissipated in 4 seconds

$$e = \int_0^4 24t^2 dt = \frac{24\,t^3}{3}\Bigg|_0^4 = 512\text{ J}$$

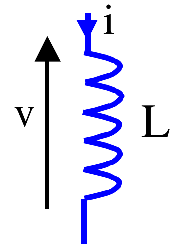
This becomes the area under the power / time curve.

Note the importance of expressing power, p, as a function of time in the integral (i.e. do not use e = ∫ 384dt etc – common student error!

b) **<u>Inductor, L</u>**

An inductor models the effect of the magnetic field surrounding the electric circuit. For an inductor:

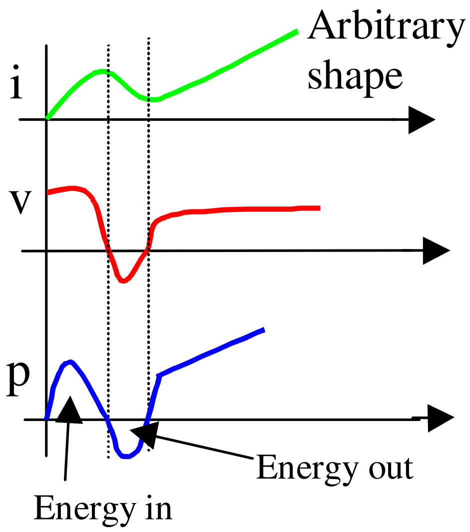
$$v = L\frac{di}{dt}$$



and instantaneous power flow:

$$p = vi = \left(L\frac{di}{dt}\right)i$$

NOTE: power flow, p, can be either positive or negative.



The energy change in the time interval, $t = t_0$ to t_1 during which the current changes from $i = i_0$ to i_1

$$\Delta e = \int_{t_0}^{t_1} Li\frac{di}{dt}dt = \int_{i_0}^{i_1} Lidi = \frac{1}{2}Li^2\Big|_{i_0}^{i_1}$$

or

$$\Delta e = \frac{1}{2}Li_1^2 - \frac{1}{2}Li_0^2$$

NOTE: if i = 0 at t = 0, then at any time, t > 0, when the current is flowing, the energy is:

$$e = \frac{1}{2}Li^2 \quad \text{Joules}$$

This is stored in the magnetic field within the inductor.

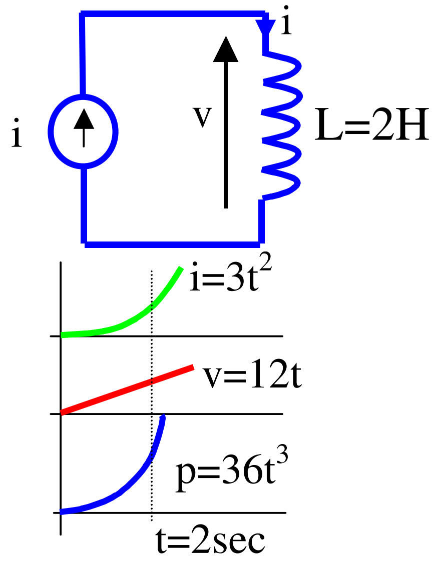
NOTES:

1) If the current is increasing then the change in energy stored is +ve, increasing. If the current is decreasing then the change in energy is –ve, and energy is recovered from the surrounding magnetic field.
2) If i=0 at t=0, then at any later time, t, the energy stored when the instantaneous current is 'i' is independent of how the current may have changed between time=0 and time = t.
3) If the current reduces to zero, then all the energy originally put into storage must come out of storage and this is independent of how fast or how slow such changes occur.
4) No energy is dissipated (converted into heat) in a **<u>pure</u>** inductor.

Example

(a) A pure inductance of 2H is connected to a current source $i=3t^2$. Calculate the instantaneous input power at t=2seconds and the total energy stored at the end of the interval t=0 to 2 seconds.

$$v = L\frac{di}{dt} = L\frac{d\left(3t^2\right)}{dt}$$
$$= 2\times 6t = 12t$$



and:

$$p = vi \quad = (12t)\times(3t^2) = 36t^3$$

at t = 2 seconds:

$$p = 36\times 2^3 = 288\ \text{W}$$
$$\text{(also } i = 12\text{A},\ v = 24\text{V)}$$

The energy input in this period from 0 to 2 seconds is given by the integral of power with respect to time:

$$e = \int_0^2 p\,dt = \int_0^2 36\,t^3 = \left.\frac{36\,t^4}{4}\right|_0^2 = 144\ \text{J}$$

(area under the power curve)

NOTE: since i = 0 at t = 0, then the energy stored at t = 2 seconds ($i = 3\times 2^2 =$ 12A) could have been obtained from:

$$\text{energy} = 0.5Li^2 = 0.5\times 2\times 12^2 = 144\ \text{J.}$$

(b) if the current is now reduced at a rate of -2As^{-1} to zero, calculate the time taken to reach zero current and the energy change over this period. For convenience, re-define t=0 as the start of this next phase, i.e. t = 0, i = 12A.

Therefore:

$$i = (12\text{-}2t)\ \text{A}$$

From this we can calculate when i = 0:

$$t = 12/2 = 6\ \text{Seconds}$$

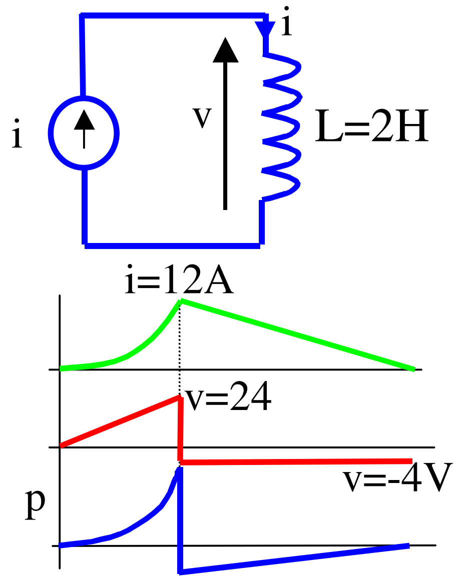
Similarly, at the new time t = 0 we have:

v =24V

and:

$$v = L\frac{di}{dt} = 2\frac{d\left(12 - 2t\right)}{dt} = -4\text{V}$$

(constant voltage)



and power:

$$p = vi \quad = -4(12-2t) = (-48 + 8t)$$

Therefore at t = 0:

$$p = -48\text{W}$$

and over the period of 6 seconds, the change in energy is given by:

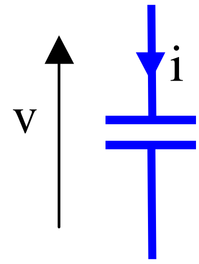
$$\Delta e = \int_0^6 (-48 + 8t)\,dt = -48t + \frac{8t^2}{2}\Bigg|_0^6 = -144\text{J}$$

(as expected! All energy has been recovered from the inductor).

c) **Capacitor, C**

A capacitor models the affect of the electric field surrounding an electric circuit. Now:

$$i = C\frac{dv}{dt} \text{ or } \quad v = \frac{1}{C}\int i\,dt$$



And a similar analysis to that used for the inductor reaches the result that energy stored:

$$e = \frac{1}{2}Cv^2$$

and this must be recovered if v = 0. There is no energy dissipated in a <u>pure</u> capacitor etc.

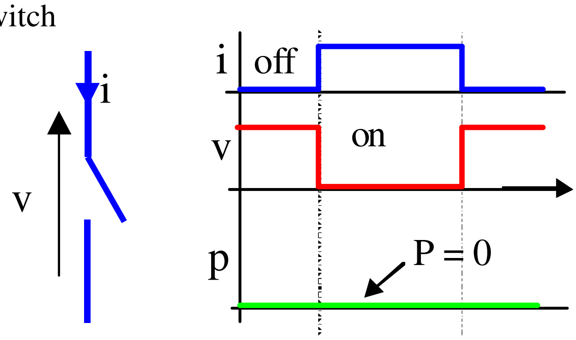
NOTE: There is a difference between a **real** and a **pure** capacitor

d) **General Circuit Device**

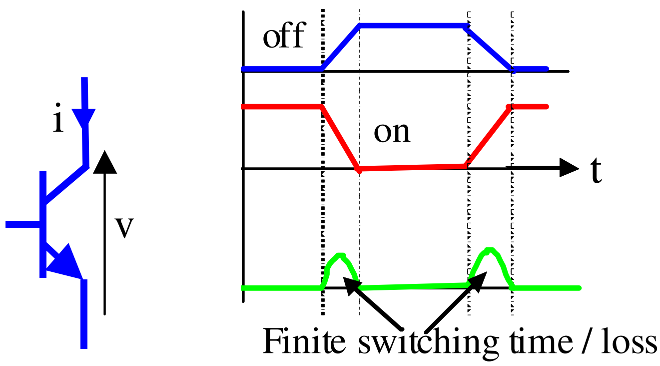
This illustrates the generality of the preceding procedures:

e.g. transistor switch / ideal switch

- Ideal switch



- Transistor Switch (see tutorial for a worked example of this. – no analytical solution will be attempted here)



In general, the instantaneous switching loss (loss during the switching time) >> on state losses or off state losses.

- Important to switch quickly, etc
- Heatsink designed to deal with the AVERAGE not peak power dissipated in device.

Comment on continuity of stored charge

Note that in capacitors and inductors the energy cannot be changed instantaneously since the power, p, required would be infinite as:

$$p = \frac{de}{dt} \rightarrow \frac{\Delta e}{\Delta t} \rightarrow \infty \quad \text{if} \;\; \Delta t \rightarrow 0$$

This idea should be familiar, w.r.t. a car. – the stored energy (Kinetic) cannot be changed instantaneously (0 to 60 mph in 0 seconds) as this would require an infinitely powerful engine etc.

i.e. For an inductor, $e = 0.5Li^2$, $di/dt \neq \infty$. Current cannot change instantaneously.

For a capacitor, $e = 0.5Cv^2$, $dv/dt \neq \infty$. Voltage cannot change instantaneously.

Average Power

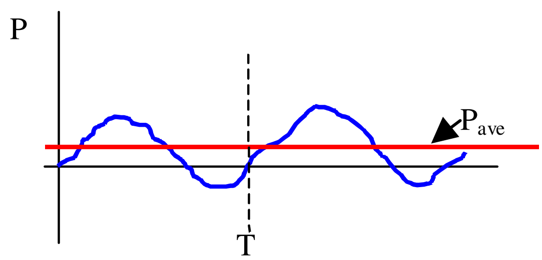
We have seen that the instantaneous power in a circuit or system is given by:

$$P = vi \qquad \text{Watts}$$

Where, in general p can be either positive or negative

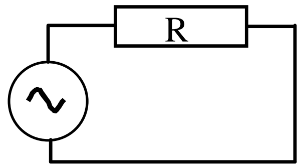
- In many systems the quantity of interest is the **average power** which is the average energy transferred per second. If the transfer is cyclic:

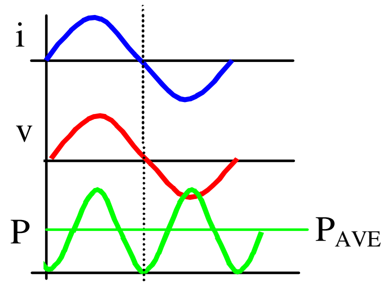
$$P_{AVE} = \frac{1}{T}\int_0^T p\,dt$$ where T is the period.



e.g. An electric lamp connected to an ac supply

Lamps form a resistive load, therefore from Ohms law, v = i.R and if i = I.sin(ωt) and v = V.sin(ωt), then, p = vi = V.I.$\sin^2(\omega t)$





$$P = \frac{VI}{2}\left(1 - \cos 2\omega t\right)$$

$$= \frac{VI}{2} - \frac{VI}{2}\cos 2\omega t$$

which comprises of a time invariant value and a pulsation at twice the supply frequency.

i.e. $P_{AVE} = VI/2$ – Time invariant.

Note:

1) Even for a resistive load, the power pulsates at twice the supply frequency, with a peak amplitude of $2 \times P_{AVE}$
2) If the thermal time constant of the lamp (or load in general) >>T, then the temperature rise or light output will only respond to the **average power**, and will not follow the instantaneous variations.
3) If T is of the same order as the thermal time constant, the temperature rise will respond to the instantaneous power input. (see tutorial sheet).

Note that for a resistive load:

$$P = \frac{VI}{2} = \frac{V}{\sqrt{2}} \times \frac{I}{\sqrt{2}}$$

Root Mean Square or effective value

In the above example we could also have written:

$$p = vi \equiv \frac{v^2}{R} \equiv i^2 R$$

and:

$$P_{AVE} = \frac{1}{T}\int_0^T vi\,dt \equiv \frac{1}{T}\int_0^T \frac{v^2}{R}dt \equiv \frac{1}{T}\int_0^T i^2 R\,dt$$

and if we also define the average power loss in a resistor in terms of an effective voltage and current – i.e. a voltage or current which gives the same heating (power loss) then we have:

$$P_{AVE} = \frac{V_{EFF}^2}{R} \quad \text{or} \quad I_{EFF}^2 R$$

then clearly we have:

$$V_{EFF} = \sqrt{\frac{1}{T}\int_0^T v^2 dt} \equiv V_{RMS}$$

$$I_{EFF} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \equiv I_{RMS}$$

This **Root-Mean-Square** value of v and i is a general expression which can be applied to any waveform. **However**, for a special case of a sinusoid, the above integrals give:

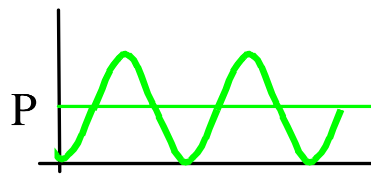
$$V_{RMS} = \frac{V}{\sqrt{2}} \qquad\qquad I_{RMS} = \frac{I}{\sqrt{2}}$$

Other relationships can be calculated for other waveforms – Some instruments are calibrated in RMS values, on the assumption that the waveform being measured will be a sine wave.

Power in Sinusoidal ac Circuits

If $i = I.\sin(\omega t)$ where $I = I_{PEAK}$ we have for various components:

(i) Resistor



We have already seen that for a resistor:

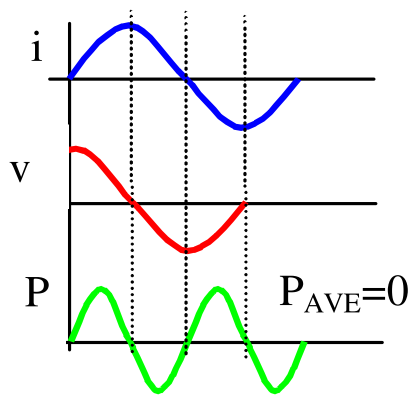
$$p = P_{Ave} - P\cos(2\omega t) \text{ Watts}$$

where:

$$P_{AVE} = \frac{VI}{2} = \frac{V}{\sqrt{2}} \times \frac{I}{\sqrt{2}} = V_{RMS} \times I_{RMS} \quad \text{etc}$$

<u>NB. Only true for pure resistance</u>

(ii) Inductor



As:

$$i = I\sin(\omega t) \quad \text{and} \quad v = L\frac{di}{dt},$$

$$v = \omega LI\cos(\omega t)$$

which we may re-write as:

$$v = X_L I\cos(\omega t) = V\cos(\omega t) = V\sin\left(\omega t + \frac{\pi}{2}\right)_w$$

which tells us that I lags V by 90^o or $\pi/2$ radians,also, as p = vi:

$$p = \frac{VI}{2}\sin(2\omega t)$$

which gives no average value, i.e.:

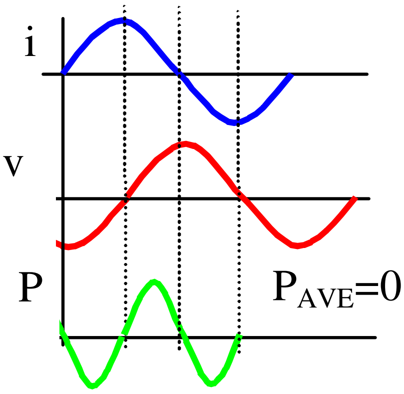
$$P_{AVE} = 0$$

(iii) Capacitor

Since:

$$i = I\sin(\omega t) \quad \text{and} \quad v = \frac{1}{C}\int i dt,$$

$$v = -\frac{1}{\omega C} I\cos(\omega t)$$



which we may re-write as:

$$v = -X_C I\cos(\omega t) = -V\cos(\omega t) = V\sin\left(\omega t - \frac{\pi}{2}\right)$$

which tells us that I leads V by 90° or π/2 radians, also, as p = vi:

$$p = -\frac{VI}{2}\sin(2\omega t)$$

which gives no average value, i.e.:

$$P_{AVE} = 0$$

The instantaneous power in the capacitor, p, is 180° out of phase with the instantaneous power in an inductor, for the same excitation current.

NOTE:

1) Despite the fact that both L and C have a RMS voltage, V_{rms}, and current, I_{rms}, they have no average power. I.e. $\mathbf{P_{AVE} = 0}$

Therefore, in general, P≠VI in ac circuits.

2) However, there is a continuous alternating instantaneous power flow into and out of storage. – This is known as REACTIVE POWER and is given the symbol Q to differentiate it from the REAL POWER, P.

For an Inductor:

$$|Q_L| = |I|^2 X_L$$

For a Capacitor:

$$|Q_C| = -|I|^2 X_C$$

The –ve sign indicates the 180° phase difference.

3) Although P and Q are strictly the same units of power (Watts), to differentiate between them because they are in fact different, they are defined as:

P = REAL POWER – in Watts (W)

Q = REACTIVE POWER, in Volt Amps reactive or VAr's

Power Factor

For a circuit having a general impedance, Z = (R+jωL) or Z = ($R+X_L$), we may write the impedance as a magnitude and phase:

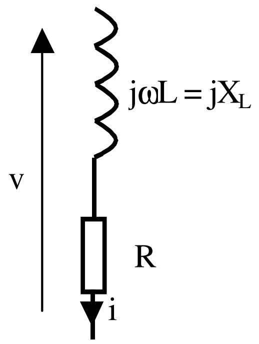
$$|Z|\angle\phi$$

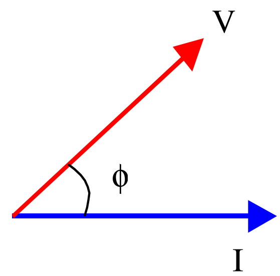
where:

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

and:

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$





As we can write i = Isin(ωt), and v = Vsin(ωt+ϕ), this gives:

$$p = v\times i = I\times V\times \sin(\omega t)\times \sin(\omega t+\phi)$$

which gives:

$$p = \frac{IV}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$$

where:

$$p = \frac{IV}{2}\cos\phi$$

is the average power, P_{AVE}, and:

$$p = \frac{IV}{2}\cos(2\omega t + \phi)$$

is a 2 × supply frequency variation. From this, it may be seen that the average power in the circuit is now given by:

$$p = \frac{IV}{2}\cos\phi = V_{rms} I_{rms}\cos(\phi)$$

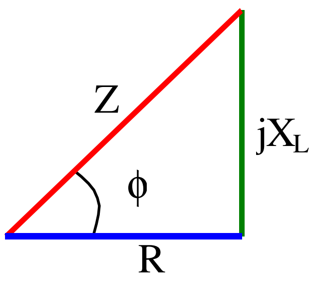
where **cos (ϕ)** is called the **POWER FACTOR**, and ϕ is the phase angle between the voltage and the current in the circuit.

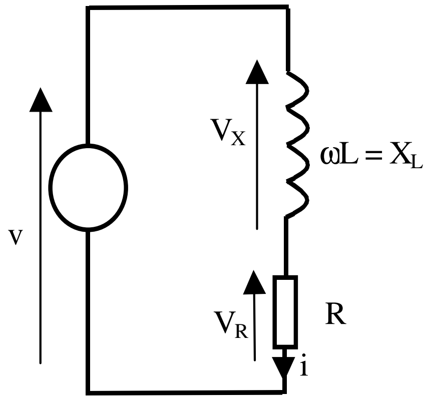
Cos(ϕ) is the general factor for any ac circuit by which the supply voltage V and current I are multiplied to get the average power input.

Note: For a pure L and C, $\phi = \pm\pi/2$, cos(ϕ) = 0 and hence average power = 0 – as shown earlier.

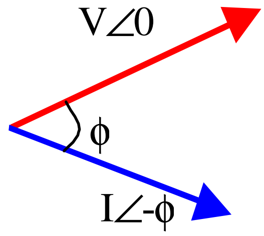
Complex power and the power triangle

As before: $Z = R + jX_L$ .We can now draw the impedance triangle as:





We can now draw a phasor diagram of the voltages and currents:



The current is then given from:

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{|V|\angle 0}{Z\angle\phi} = |I|\angle -\phi$$

Similarly, we may draw a phasor diagram for the voltages (voltage phasor diagram)

Here we see that:

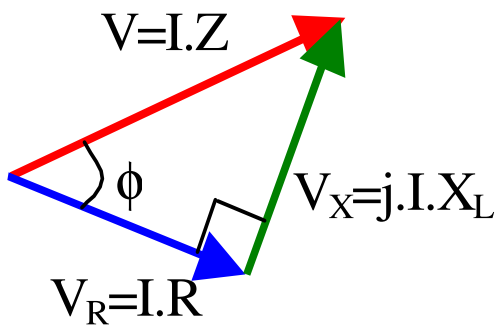
$$V_R = I\times R = V\times\cos(\phi)$$

In phase with current

and

$$V_X = j\times I\times X_L = V\times\sin(\phi)$$

90^o ahead of the current



and if all the phasors are multiplied by the current, I, we produce a **Power triangle**,

Where:

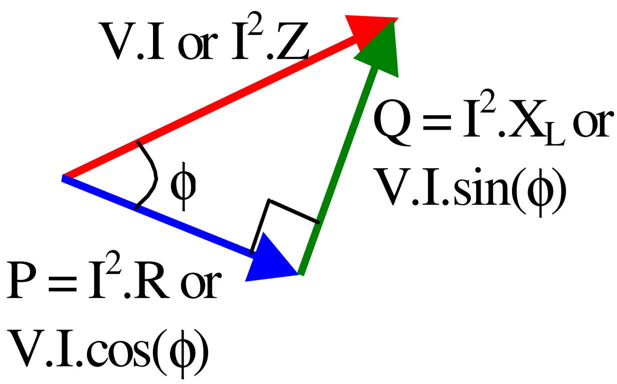
$P = V.I.\cos(\phi)$ $(=I^2R)$ - Watts (W)

$Q = V.I.\sin(\phi)$ $(=I^2X_L)$ - VAr's

And the third side is called the **Volt Amps**

$S = V.I$ - VA

(called the complex power)



And we see that:

$$\mathbf{S = P + jQ}$$

- NOTE: by convention,
  Q is +ve for an inductive load,
  Q is –ve for a capacitive load

If V and I are written in phasor form, say:

$$\overline{V} = V\angle 0, \text{ and } \overline{I} = I\angle -\phi$$

then for an inductive load, (where I **lags** V), if we use complex power, S=VI, then:

$$S = \overline{V}.\overline{I} = \left|\overline{V}\right|.\left|\overline{I}\right|\angle -\phi$$

or:

$$S = V.I.\cos(\phi) - jV.I.\sin(\phi)$$

This gives the WRONG sign convention for Q.

To eliminate this difficulty, from phasor notation we always calculate:

$$S = V.I^* = VI\angle\phi$$

where I^* is called the complex conjugate of I, therefore if:

$$I = I\angle\phi \text{ then } I^* = I\angle -\phi$$

or if:

$$I = I_a + jI_b \text{ then } I^* = I_a - jI_b \text{ etc...}$$

Clearly, if I lags V, then Q +ve
If I leads V, then Q –ve

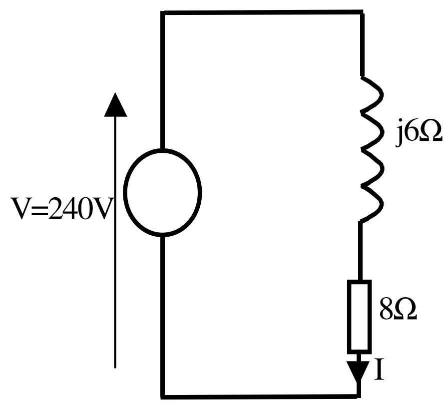
And they can be used to cancel each other out, or to reduce the total Q – known as **POWER FACTOR CORRECTION**.

<u>Power Calculation Example</u>

$Z = 8 + j6\ \Omega = 10\angle 36.87^o$

For this circuit calculate:

1. The circuit current
2. The VA, VAr's and Watts supplied to the load



For the circuit, taking the voltage as the reference for the phasor diagram, $V=240\angle 0$
Then:

$$\overline{I} = \frac{V}{Z} = \frac{240\angle 0}{10\angle 36.87} = 24\angle -36.87$$

i.e. I **lags** V by 36.87° (inductive circuit)
and:

$$S = VI^* = 240\angle 0 \times 24\angle 36.87 = 5.76\angle 36.87 \ \text{kVA}$$

giving:

$$|S| = |VI| = 5.76\text{kVA}$$

where:

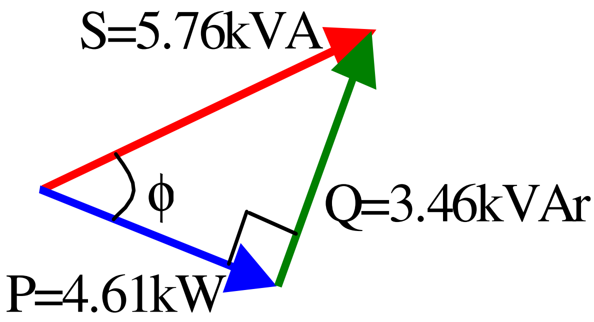
$$P = |S|\cos(\phi) = 5.76.\cos(36.87^{\circ}) = 4.61\text{kW}$$

$$[= I^2 \times R = 24^2 \times 8\ ]$$

$$Q = |S|\sin(\phi) = 5.76.\sin(36.87^{\circ}) = 3.46\text{kVAr}$$

$$[= I^2 \times X_L = 24^2 \times 6\ ]$$

This may be shown as a power triangle:



Load Specification

In the previous example, the load was specified in terms of its impedance, (R+jωL) or (8+j6Ω). In power systems the supply is usually considered to be constant (small % change can occur) at some nominal value (eg 240V, 425V, 11kV, etc.)
For these situations, it is possible to specify the load not in terms of its impedance, but in terms of the VA or Power requirements and power factor. E.g.:

(i) – Normal lightbulb is specified in terms of its wattage, 60W, 100W etc
- A kettle is specified in terms of its wattage (e.g. 2kW etc)

In these cases the load is a pure resistance (i.e. pf = 1)

(ii) - In the previous example, for a supply of 240V, the load Z_L of (8+j6)Ω could have been specified as a load of 4.76kW @0.8pf lag,
or as a load of 5.76kVA @ 0.8pf lag.

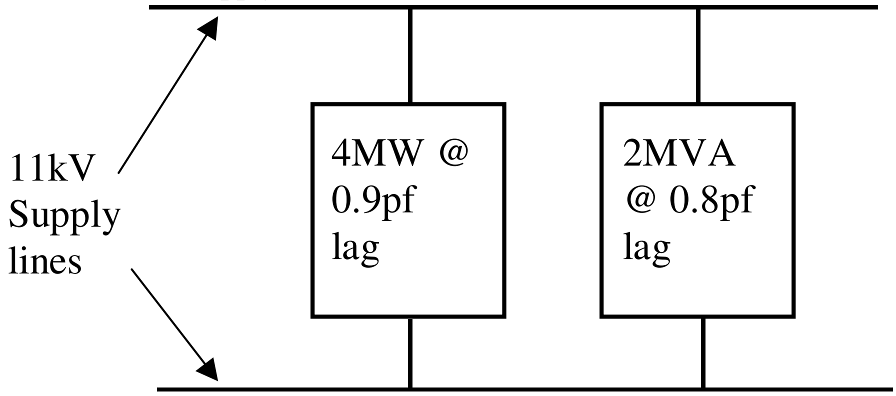
For such systems, all loads are assumed to be parallel across V.

Example

The total load on an 11kV, 50Hz supply consists of the following:

i) 4MW @ 0.9pf lag
ii) 2MVA @ 0.8pf lag

Calculate the total load power, VAr and VA demands and hence the total load current drawn from the supply.

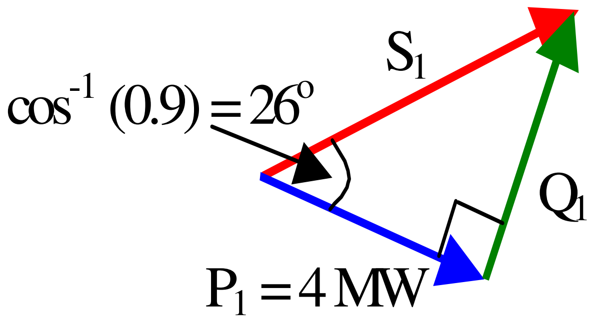


NB. All loads connected in parallel across the supply.

Load 1

$P_1 = 4\text{MW}$ @ 0.9pf

$\therefore\ S_1 = P_1 / \cos(\phi) = 4.44$ MVA



and:

$Q_1 = S_1 \sin(\phi) = 1.94$ MVAr

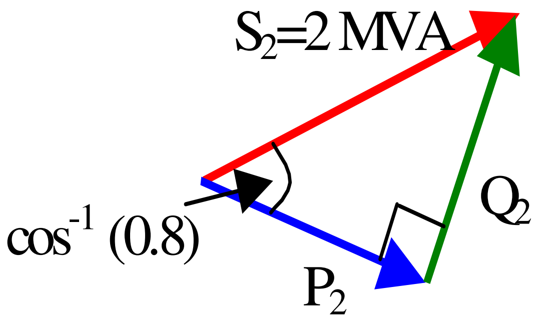
Load 2

$S_2 = 2\text{MVA}$ @ 0.8pf lag

$\therefore\ P_2 = 2\cos(\phi) = 1.6$ MW

and:

$Q_2 = 2\sin(\phi) = 2 \times 0.6 = 1.2$ MVAr



Now, the total load:

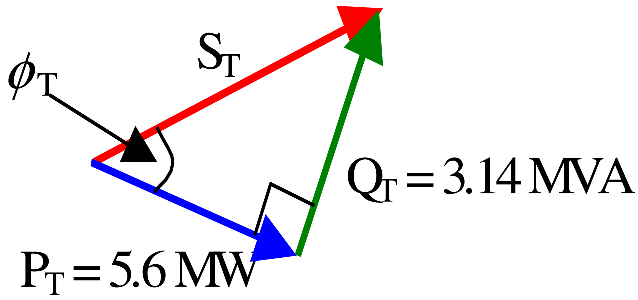
$S_T = (P_1+P_2) + j(Q_1+Q_2)$

NB. $S_T \neq S_1 + S_2$ you cannot add VA directly, only P and Q components

$\therefore\ S_T = 5.6\text{MW} + 3.14$ MVAr

$$|S_T| = \sqrt{P^2 + Q^2}$$

$$= 6.42 \text{ MVA}$$



Therefore the total input current may be found from the total VA and the input voltage:

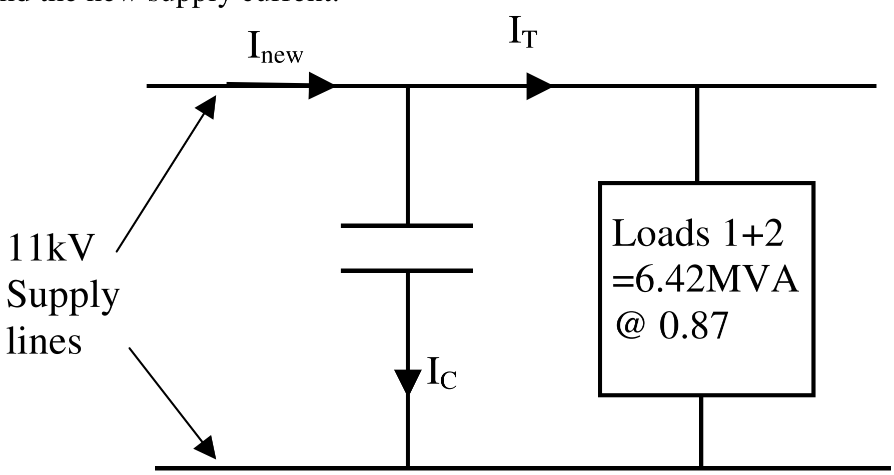
$$I_T = \frac{|S_T|}{|V|} = \frac{6.42\times10^6}{11\times10^3} = 584 \quad Amps$$

and the power factor of the total load is given by:

$$\cos(\phi_T) = \frac{5.6}{6.42} = 0.87 \quad lag$$

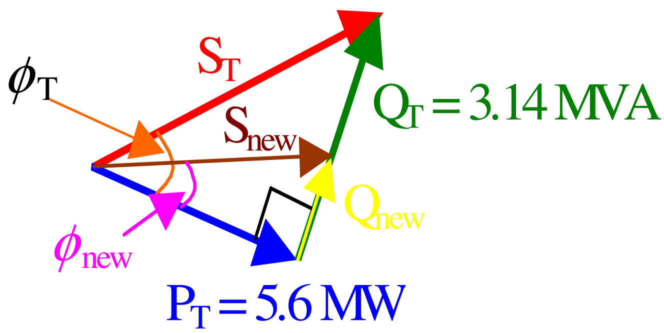
Power factor Correction

Calculate the capacitance required to improve (correct) the power factor to 0.95 lag and the new supply current.



- **Power factor correcting capacitors are always placed in parallel with the load, NOT in series**

- **Capacitors draw no power, only VAr component ∴ power remains unchanged, but the total Q reduces.**



$$\phi_T = \cos^{-1}(0.87)$$
$$\phi_{new} = \cos^{-1}(0.95) = 18.2^o$$

From the power triangle:

$$\tan(\phi_{new}) = Q_{new} / P_T = Q_{new} / 5.6$$

therefore:

$$Q_{new} = 5.6 \times \tan(18.2^o) = 1.84 \text{ MVAr}$$

So added (-ve) Q from capacitors:

$$Q_C = Q_T - Q_{new} = (3.14 - 1.84) \text{ MVAr} = 1.3 \text{ MVAr}$$

From earlier notes:

$$Q_{cap} = -I^2.X_C = 1.3 \times 10^6 = -V^2 / X_C$$

Therefore:

$$X_C = (11\times10^3) / (1.3\times10^6) = 93\Omega$$

and as:

$$X_C = 1/\omega C \qquad \text{and} \qquad \omega = 2\pi f = 100\pi$$

Then:

$$C = 34\mu F$$

Also, from the power diagram we have:

$$S_{new} = P / \cos(\phi_{new}) = (5.6\times10^6) / 0.95 = 5.895\text{MVA}$$

and:

$$I_{new} = \frac{|S_{new}|}{|V|} = \frac{5.895\times10^6}{11\times10^3} = 536 \quad Amps$$

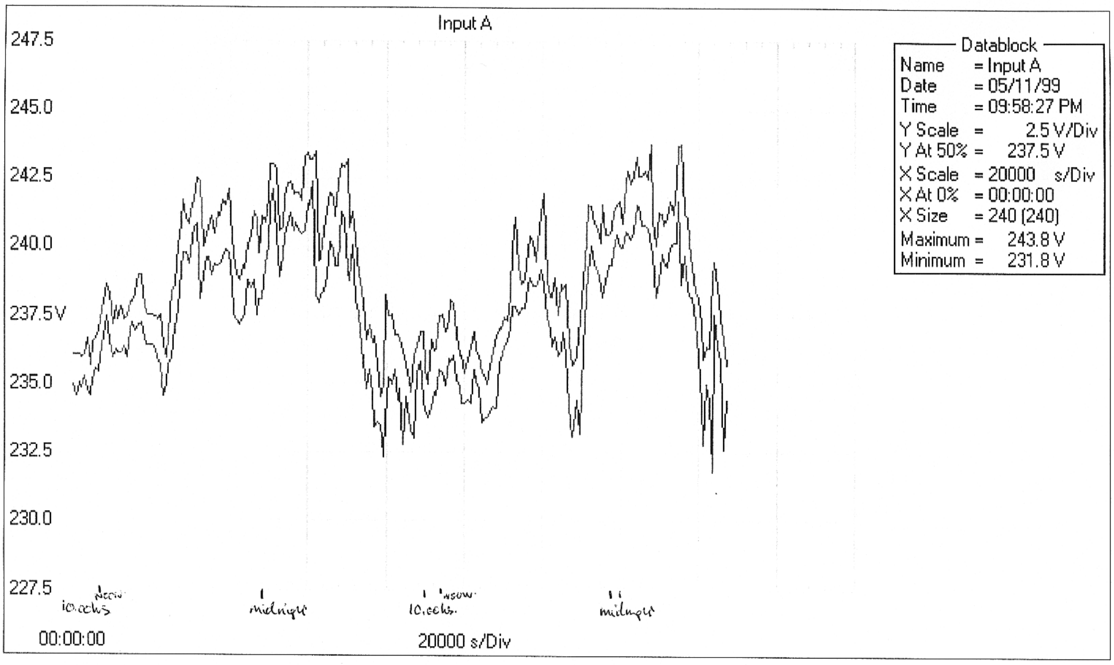
NOTE:
Supply current reduced from 584A to 536A with no reduction in power, P = 5.6MW.

Tariffs

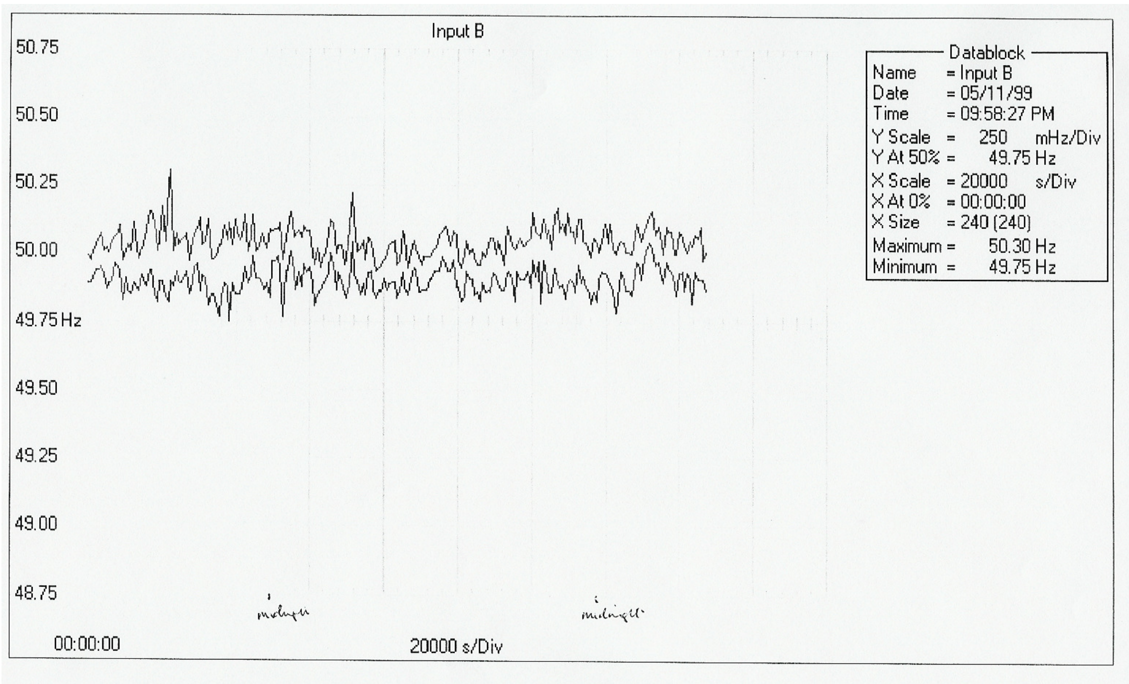
From the above it may be seen that the supply current is reduced for the same power supplied to the load. The supply company losses (in cables transmission lines, transformers etc) are reduced (= I^2R) by this technique. Hence to encourage customers to reduce the power system losses, the tariff (cost per unit of electricity supplied) charged to large power users, are based not only on the real power, P, supplied – but also on the power factor (VAr demand). By charging for Q as well as P, customers find it cost effective to install power factor correction capacitor banks at their installations.

Domestic customers are only charged for P at a fixed rate – no need to improve pf.

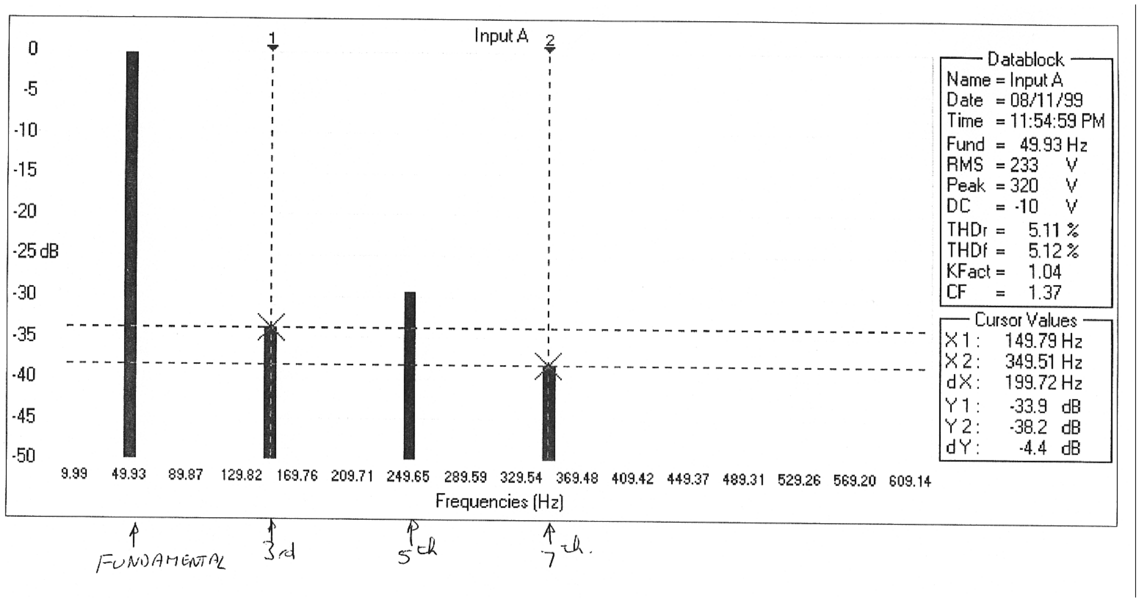
There are other reasons why the power companies encourage large industrial users to improve their pf. It will be seen from later in the course that the flow of Q in the network (not P) is the main cause of voltage drops through the system. Therefore by reducing Q, the voltage drops are also reduced.



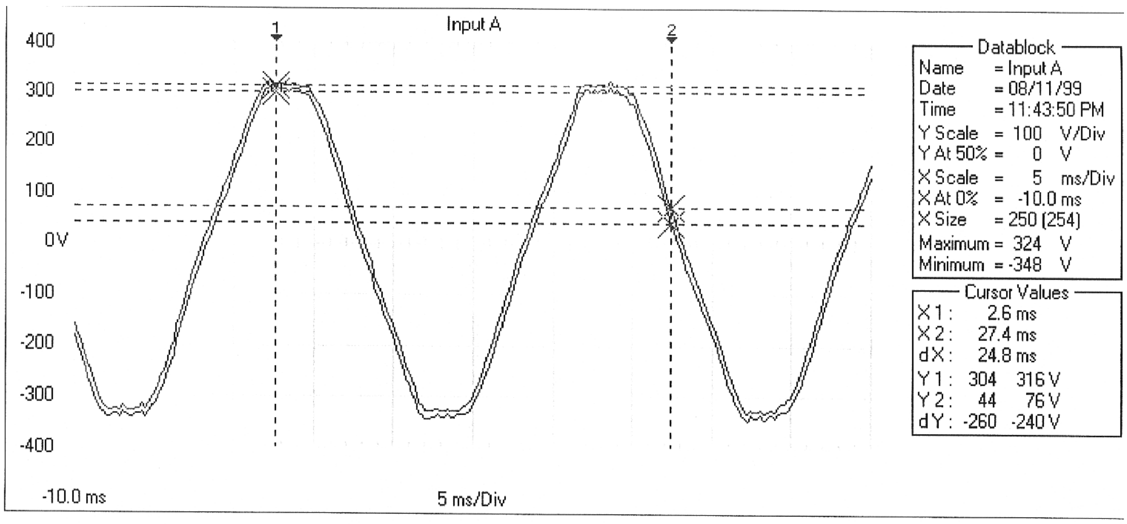
Magnitude of supply voltage over a 2 day period at St George's Complex.



Frequency of supply voltage over a 2 day period at St George's Complex.



Typical supply harmonics at St George's Complex.



Typical supply voltage shape at St George's Complex.