Data Provided: Fourier Transform and Laplace Transform Pairs

The University of Sheffield

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2003-2004 (2 hours)

EEE201 Signals and Systems

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. Obtain the complex Fourier Series representation of a sampling function $p(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT)$, where T is the sampling time in seconds. Hence verify that the Fourier Transform of the sampling function is given by $P(\omega)=\frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_s)$, where ω_s is the sampling frequency in rad/s. **(6)**

b.

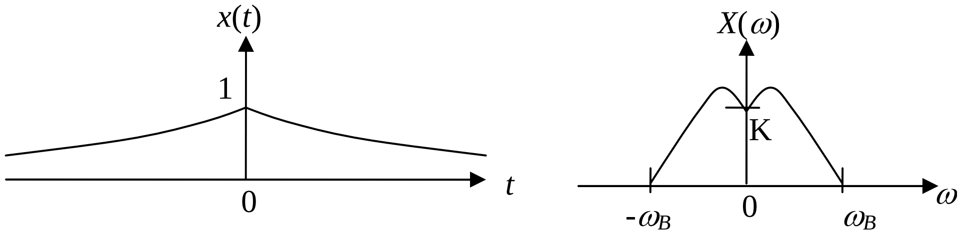

Fig. Q.1.1

The signal $x(t)$, shown in Fig.Q.1.1, is multiplied by the sampling function $p(t)$ to obtain $x_s(t)$, the sampled version of $x(t)$. Sketch and label $x_s(t)$. **(2)**

c. Show that the Fourier Transform of $x_s(t)$ is $X_s(\omega)=\frac{1}{T}\sum_{n=-\infty}^{\infty}X(\omega-n\omega_s)$, where $X(\omega)$ is the spectrum of $x(t)$ as shown in Fig.Q.1.1. **(6)**

d. Sketch and label $TX_s(\omega)$ if

i) $\omega_s < 2\omega_B$

ii) $\omega_s > 2\omega_B$.

State whether the spectrum of $x(t)$ can be recovered using a low pass filter in each case and describe the aliasing effect. **(6)**

2. **a.**

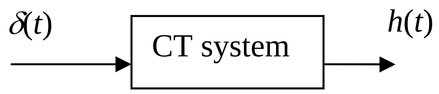

Fig. Q.2.1

Consider a continuous time Linear Time-invariant (LTI) system with an impulse response $h(t)$ as shown in Fig.Q.2.1. Prove that the response of the system LTI to an input signal $x(t)$ is given by $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$. **(4)**

b.

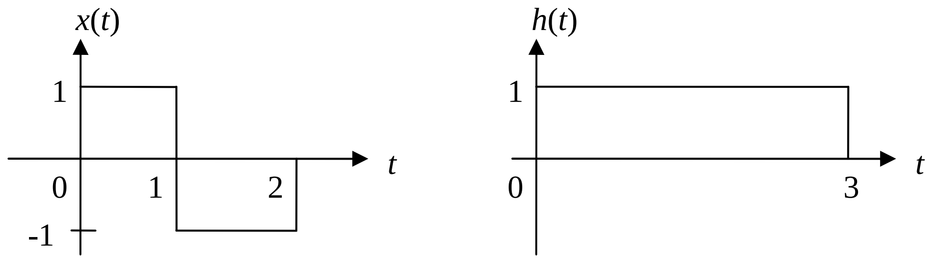

Fig.Q.2.2

Obtain the response of the LTI system using the graphical method, if the input signal $x(t)$ and the impulse response $h(t)$ are as shown in Fig.Q.2.2. Sketch and label $y(t)$. **(10)**

c. Compute the response of an LTI discrete system if the input and impulse response are described by $x[n] = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & otherwise \end{cases}$ and $h[n] = \begin{cases} e^{-n}, & 0 \le n \le 4 \\ 0, & otherwise \end{cases}$, respectively. **(6)**

3. **a.**

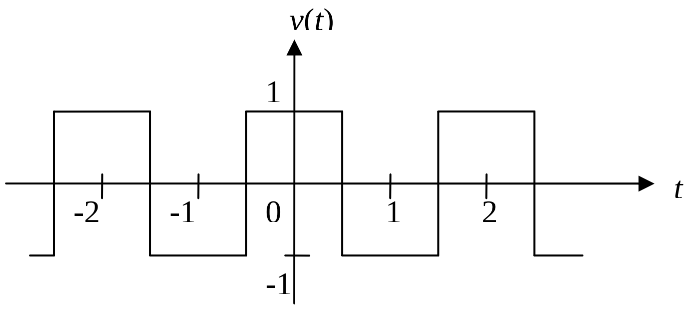

Fig.Q.3.1

Prove that the signal $v(t)$ shown in Fig.Q.3.1 can be represented by

$$x(t) = \frac{4}{\pi}\left(\cos(\pi t) - \frac{1}{3}\cos(3\pi t) + \frac{1}{5}\cos(5\pi t) - \frac{1}{7}\cos(7\pi t) + ...\right).$$

(8)

b. Calculate the average power contained in $v(t)$ within the frequency range $[-6\pi$ rad/s, 6π rad/s$]$. **(5)**

c. The signal $v(t)$ in part (a) is applied to an RC low pass filter with a transfer function $H(\omega) = \dfrac{1}{1 + j\omega/\omega_c}$, where $\omega_c = \dfrac{1}{RC}$ is the cut-off frequency. Calculate the value of the capacitance C required so that the amplitude of the fundamental component is $\dfrac{3.2}{\pi}$ after filtering. Assume R = 200kΩ. **(7)**

4. a.

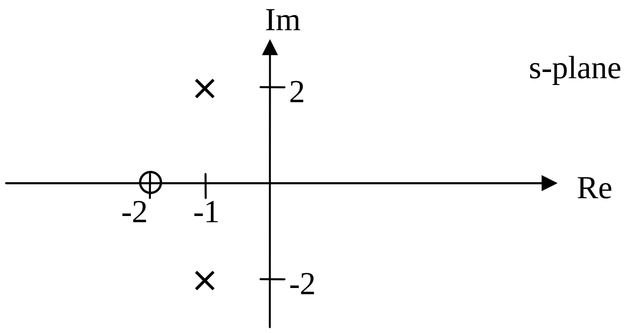

Fig.Q.4.1

Fig.Q.4.1 shows the pole-zero plot of a continuous time system. The transfer function can be expressed as $H(s) = \dfrac{N(s)}{D(s)}$. Obtain the polynomial functions $N(s)$ and $D(s)$. **(5)**

b. Find the poles, the damping factor and the natural frequency of the system shown in Fig.Q4.1. **(4)**

c. Describe and sketch the response of the system when the input is a unit step function. **(5)**

d. Find the system response $y(t)$ when the input is $x(t) = e^{-2t}u(t)$. **(6)**

CHT / SKK