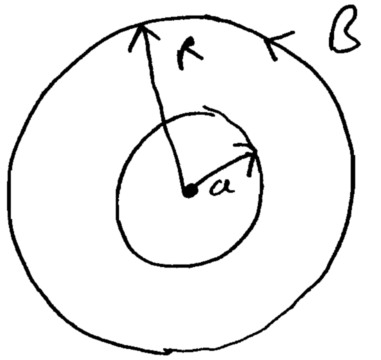
EEE 220 . 2009/10
Solutions

1 a



Due to symmetry B field can only vary with radial distance from wire

Using Ampere's law

$$\oint \underline{B} \cdot \underline{dl} = \mu_0 I, \quad I = \text{enclosed current}$$

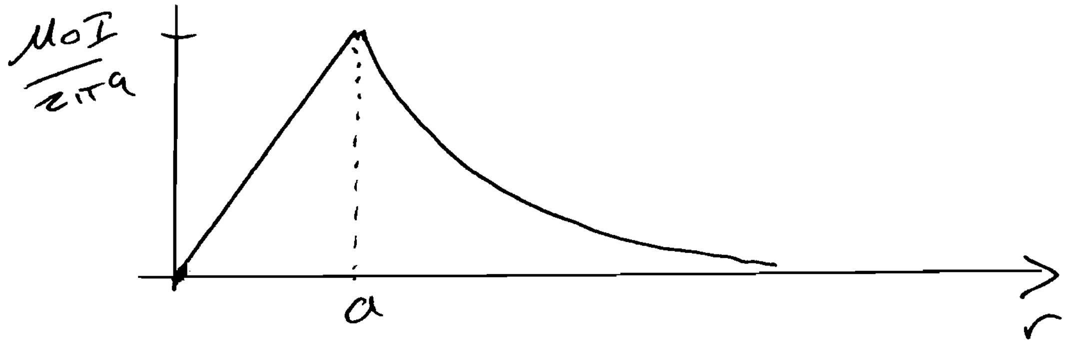
$\therefore$ outside wire $B . 2\pi r = \mu_0 I$

$$\rightarrow B = \frac{\mu_0 I}{2\pi r}$$

inside wire $I(r) = I \dfrac{\pi r^2}{\pi a^2} = I \dfrac{r^2}{a^2}$

$$\rightarrow 2\pi r B = \mu_0 I \frac{r^2}{a^2}$$

$$\rightarrow B = \frac{\mu_0 I r}{2\pi a^2}$$



(6)

b for solenoid $B = \mu_0 I N_L$

i) $B = 4\pi \times 10^{-7} \times 1 \times \frac{1000}{0.1} = \underline{0.0126T}$

ii) By definition $\psi = LI$

and $\psi = N\phi$ where $\phi = AB$ = flux

area field

hence $\psi = NAB$ and $B = \mu_0 \frac{N}{d} I$

$\therefore \psi = N^2 A \mu_0 I / d$

$\rightarrow L = \psi/_I = \mu_0 N^2 A/d$

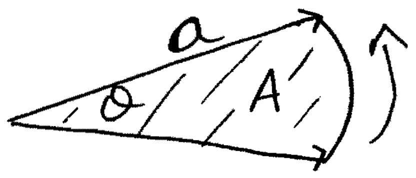
$= 4\pi \times 10^{-7} \times (1000)^2 \times \frac{\pi \times (0.01)^2}{0.1}$

$= \underline{3.9 mH}$

6

c $V = -\frac{d\phi}{dt} = -\frac{d(BA)}{dt}$ where A = area

In this problem B is constant but A changes



$dA = \frac{1}{2} a \times a d\theta = \frac{a^2 d\theta}{2}$

$\therefore \frac{dA}{dt} = \frac{1}{2} a^2 \frac{d\theta}{dt}$

we know that rod rotates at 180 rmp

$180 \text{ rpm} = 3 \text{ rps}$
$= 2\pi \times 3 \text{ rads/s} = \frac{d\theta}{dt}$

Hence $V = B . \frac{dA}{dt} = B \frac{a^2}{2} \frac{d\theta}{dt}$
$= 3\pi B a^2$

If $B = 6 \times 10^{-4}$ T & $a = 1.0$ m

$V = 3 \times \pi \times 6 \times 10^{-4} \times 1^2$

$= 5.65 \times 10^{-3}$ V

8

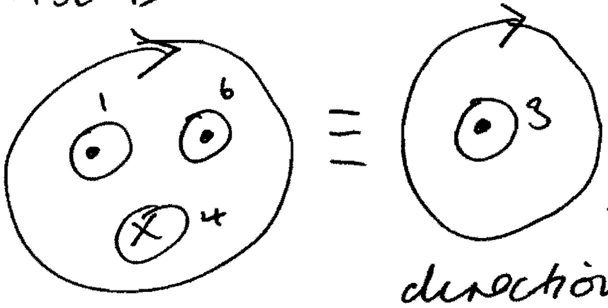
EEE220 09

Q2 a

CASE A

$I = 0 \rightarrow \oint \underline{B} \cdot d\underline{l} = 0$

CASE B

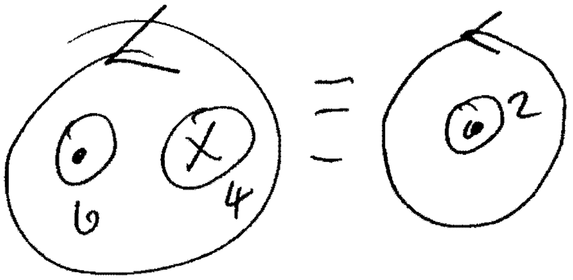


$I = 3A$ out of paper

$\rightarrow B$ is opposite to direction of contour integral shown

$\rightarrow \oint \underline{B} \cdot d\underline{l} = -3\mu_0$

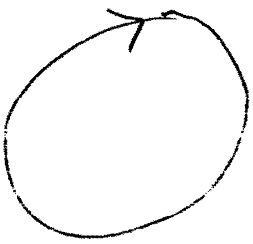
CASE C



$I = 2$

$\oint \underline{B} \cdot d\underline{l} = 2\mu_0$

CASE D



no current enclosed $\rightarrow \oint B\, d\underline{l} = 0$

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b

Biot Savart Law $\underline{H} = \frac{I}{4\pi}\int_L \frac{\underline{dl} \times \hat{r}}{r^2}$

For radial sections of loop circuit $\underline{dl}$ is ⥣ to $\hat{r}$ so $\underline{\text{no}}$ contribution.

For arc sections $\underline{dl}$ is $\perp$ to $\hat{r}$
so $\underline{dl} \times \hat{r} = dl$

Field given by $H = \frac{I}{4\pi}\int_C \frac{dl}{r^2}$ or $\frac{I}{4\pi}\int_0^\theta \frac{r\,d\theta}{r^2}$

For arc radius a, $H_a = \frac{I}{4\pi}\int_0^\theta \frac{d\theta}{a} = \frac{I\theta}{4\pi a}$

using RH rule H_a is INTO paper

For arc radius b, $H_b = \frac{I}{4\pi}\int_0^\theta \frac{d\theta}{\theta} = \frac{I\theta}{4\pi b}$

direction is OUT OF paper

Hence total field is

$H = \frac{I\theta}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right)$ OUT OF PAPER

$= \frac{I\theta}{4\pi ab}(a-b)$ "

8

c $F = \dfrac{\mu_0 I_1 I_2}{2\pi r}$

For 1A wire

$F_{2A} = \dfrac{4\pi \times 10^{-7} \times 1 \times 2}{2\pi \times 0.05} = 8 \times 10^{-6}\ Nm^{-1}$ (to right)

$F_{3A} = \dfrac{4\pi \times 10^{-7} \times 1 \times 3}{2\pi \times 0.1} = 6 \times 10^{-6}\ Nm^{-1}$ (to right)

$\rightarrow$ Total $F = 14 \times 10^{-6}\ Nm^{-1}$ (to right)

For 2A wire

$F_{1A} = \dfrac{4\pi \times 10^{-7} \times 1 \times 2}{2\pi \times 0.05} = 8 \times 10^{-6}\ Nm^{-1}$ (to left)

$F_{3A} = \dfrac{4\pi \times 10^{-7} \times 2 \times 3}{2\pi \times 0.05} = 24 \times 10^{-6}\ Nm^{-1}$ (to right)

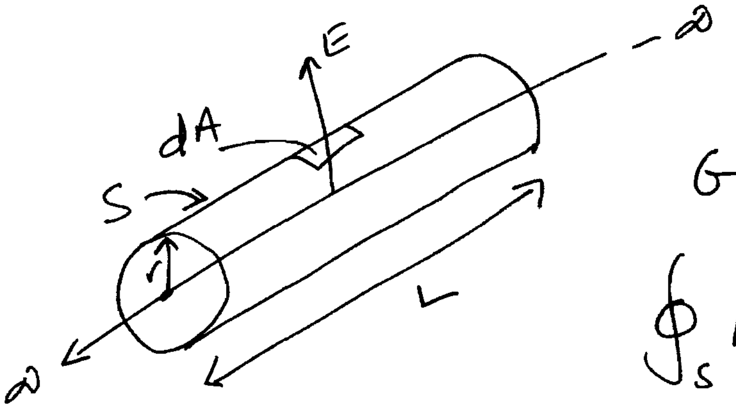
$\rightarrow$ Total $F = 16 \times 10^{-6}\ Nm^{-1}$ (to right)

For 3A wire

$F_{1A} = 6 \times 10^{-6}\ Nm^{-1}$ (to left)
$F_{2A} = 24 \times 10^{-6}\ Nm^{-1}$ (to left)

$\rightarrow F = 30 \times 10^{-6}\ Nm^{-1}$ (to left)

3a

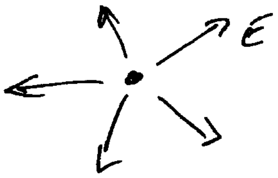


Gauss' Law

$$\oint_S E_\perp dA = Q/\epsilon_0$$

Due to symmetry E cannot vary along wire (as ∞) or around wire

∴ E-field must point radially outwards



when evaluating $\oint E_\perp dA$, ends of cylinder do not contribute as dA is parallel to E

contribution from curved part of cylinder is

$$E_\perp . \underbrace{2\pi r L}_{\text{surface area } S} = Q/\epsilon_0$$

$$\therefore \quad E = \frac{Q}{L} \cdot \frac{1}{2\pi r \epsilon_0}$$

or $E = \dfrac{q_l}{2\pi r \epsilon_0}$ where $q_l = Q/L$ charge per unit length

6

3b

i) $\underline{E} = \dfrac{q_{l_1}}{2\pi\epsilon_0 R_1^2}\underline{R_1} + \dfrac{q_{l_2}}{2\pi\epsilon_0 R_2^2}\underline{R_2}$

$= \dfrac{3\times10^{-6}}{2\pi\epsilon_0 (2)^2}(2,0,0) \quad \dfrac{-3\times10^{-6}}{2\pi\epsilon_0 (2)^2}(-2,0,0)$

$= \underline{(5.39, 0, 0)\times10^4 \ \mathrm{Vm^{-1}}}$

ii) $\underline{E} = \dfrac{3\times10^{-6}}{2\pi\epsilon_0 (\sqrt{8})^2}(-2,-2,0) - \dfrac{3\times10^{-6}}{2\pi\epsilon_0 (\sqrt{40})^2}(-6,-2,0)$

$= \underline{(-5.39, -10.8, 0)\times10^{3} \ \mathrm{Vm^{-1}}}$

iii) at (2,2,0) point is inside perfect conductor, hence

$\underline{\underline{E = 0}}$

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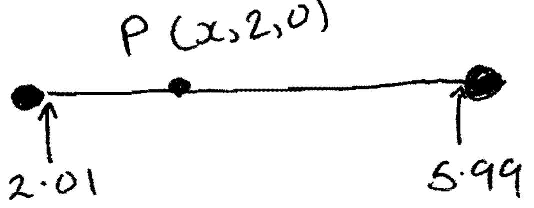

At point P $|\underline{E}|$ due to wire on LHS

$= q_l / 2\pi\epsilon_0 r = \dfrac{3\times10^{-6}}{2\pi\epsilon_0 (x-2)}$ to right

$|\underline{E}|$ due to wire on RHS

$= q_l / r / 2\pi\epsilon_0 r = \dfrac{-3\times10^{-6}}{2\pi\epsilon_0 (6-x)}$ to left

3b

$\therefore$ total $E_x = \dfrac{3\times10^{-6}}{2\pi\epsilon_0(x-2)} - \dfrac{-3\times10^{-6}}{2\pi\epsilon_0(6-x)}$

$= \dfrac{3\times10^{-6}}{2\pi\epsilon_0}\left[\dfrac{1}{x-2} + \dfrac{1}{6-x}\right]$

Potential difference $V = \displaystyle\int_{2.01}^{5.99} E_x \, dx$

$\therefore V = \dfrac{3\times10^{-6}}{2\pi\epsilon_0}\left[\displaystyle\int_{2.01}^{5.99}\dfrac{dx}{x-2} + \int_{2.01}^{5.99}\dfrac{dx}{6-x}\right]$

$= \dfrac{3\times10^{-6}}{2\pi\epsilon_0}\Big[\ln(x-2) - \ln(6-x)\Big]_{2.01}^{5.99}$

$= \dfrac{3\times10^{-6}}{2\pi\epsilon_0}\Big[\ln(3.99) - \ln(0.01) - \ln(0.01) + \ln(3.99)\Big]$

$= 646 \text{ kV}$

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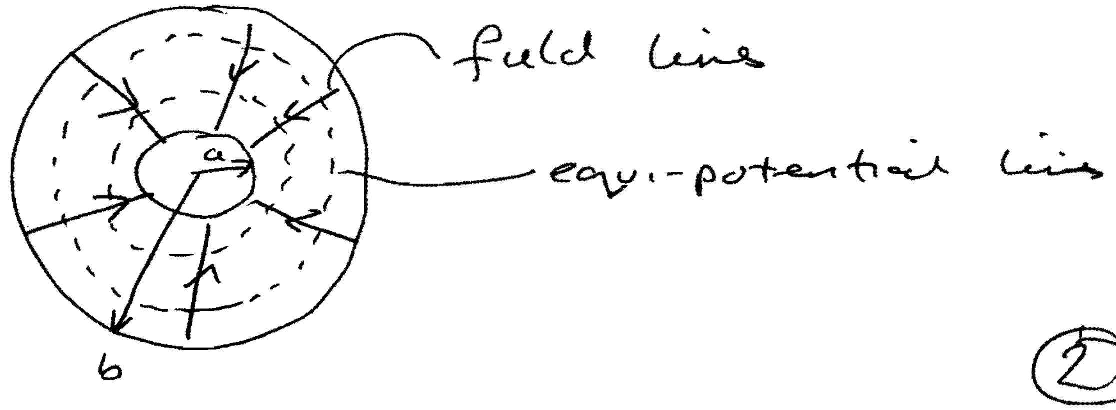
4a



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b Assuming that the field due to the inner conductor is the same as that for an infinitely long charged wire

$$\underline{E} = \frac{-q_L}{2\pi\epsilon_0\epsilon_r}\frac{\hat{r}}{r} = \frac{-Q}{2\pi\epsilon_0\epsilon_r L r}\hat{r}$$

Voltage between conductors is

$$V = -\int_a^b \underline{E}\cdot\underline{dr} = -\int_a^b -\hat{r}\,\frac{Q}{2\pi\epsilon_0\epsilon_r L r}\cdot\hat{r}\,dr$$

$$= \frac{Q}{2\pi\epsilon_0\epsilon_r L}\int_a^b \frac{dr}{r} = \frac{Q}{2\pi\epsilon_0\epsilon_r L}\left[\ln b - \ln a\right]$$

$$= \frac{Q}{2\pi\epsilon_0\epsilon_r L}\ln\left(b/a\right)$$

$$C = Q/V = \frac{2\pi\epsilon_0\epsilon_r L}{\ln(b/a)}$$

$$C_L = C/L = \frac{2\pi\epsilon_0\epsilon_r}{\ln(b/a)}$$

4c

we have

$$\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(b/a)}$$

$$\therefore \ln(b/a) = 2\pi\epsilon_0 \frac{L}{C}$$

$$= 2\pi \times 8.8 \times 10^{-12} \times \frac{10}{400 \times 10^{-12}}$$

$$\therefore \ln(b/a) = 1.391$$

$$\therefore b = e^{1.39} \times a$$

$$= e^{1.39} \times 2\,mm$$

$$= 8.04\,mm$$

2

4d $F \propto Q/x^2$

$\rightarrow F_A = \frac{k2q}{(0.2)^2} = F$ given

$F_B = \frac{k2q}{(0.1)^2}$

combining gives $F_B = F.\frac{(0.2)^2}{(0.1)^2} = 2^2F = 4F$

Hence $F_B = 4F$ direction towards B

4

EEE [illegible] 04

4e

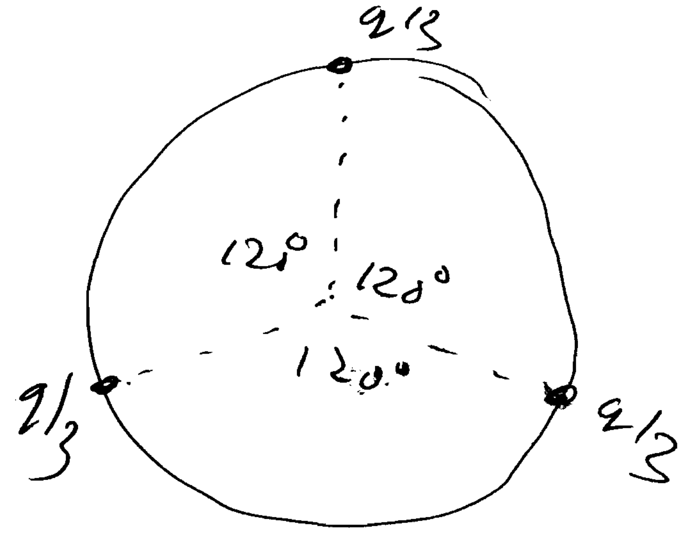
Assume starting geometry is symmetrical about charge $-q$.

Initially both +ve charges are attracted towards -ve charge.

All three charges collide
As beads are conducting, charge will redistribute evenly over all three beads.

Hence each bead will have a resultant charge of $\frac{+q-q+q}{3} = +q/3$

As all three charges now have equal +ve charge they will repel and the equilibrium state will be as shown below



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