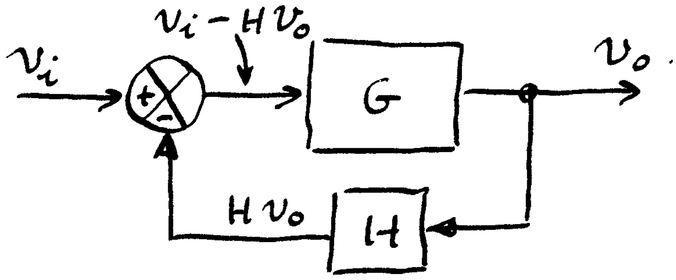
Operational Amplifiers

- most commonly used analogue building block
- been around since the 1930s (the original ones were big, heavy + power hungry)
- formed the basis of analogue computers
- designed to have
  - (i) differential input
  - (ii) very high input resistance ($> 10^9 \Omega$)
  - (iii) very low output resistance ($< 50 \Omega$)
  - (iv) very high gain. (typ 10^5)

Classical feedback system

- to understand why the op-amp is designed to have the features outlined above, consider a classical feedback system ....



If the output voltage is v_o, a fraction Hv_o is fed back to the input stage where it is subracted from v_i. This leaves $(v_i - Hv_o)$ at the input of the gain stage G, so

$$v_o = G(v_i - Hv_o).$$

or $$\frac{v_o}{v_i} = \underline{\text{system}}\ \text{gain} = \frac{G}{1+GH} \quad \text{——} (1)$$

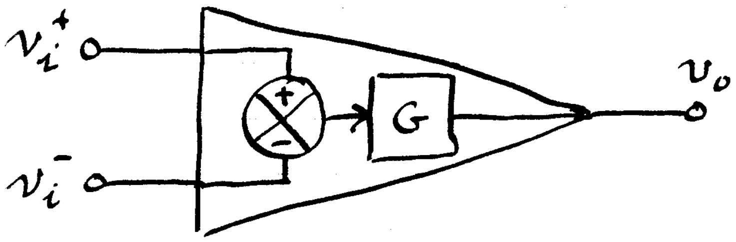
If G is very large, then $GH \gg 1$

and $\frac{v_o}{v_i} \approx \frac{G}{GH} = \frac{1}{H}$. ——— (2)

This is a very useful result because it tells the designer that if G is made large enough, system gain is dependent only on the feedback fraction H. H is usually defined by well behaved components – R and C – although in this module only R will be used.

The op-amp

– the op-amp integrates two parts of this classical feedback system ...

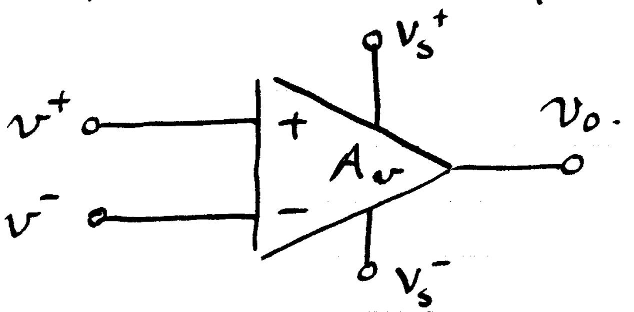


– the input resistances must be high so that the v^- input does not affect the network that defines H and so that the v^+ input does not affect the signal source.

– the output resistance must be low so that the system can drive a load without v_o being affected and so that the system can drive the network defining H without being affected.

– the reason for the differential input and the high gain are obvious from the result (2).

- the op-amp is usually drawn as



- V_s^+ + V_s^- are the power supplies. They are often not included on circuit diagrams but must be connected in the real circuit. v_o cannot go outside the range

$$V_s^+ > v_o > V_s^-$$

- v^+ is called the "non-inverting" input of the op-amp. It is identified by a "+" next to the input line, inside the op-amp triangle.
- v^- is called the "inverting input" of the op-amp and is identified by a "−".
- the output, v_o, comes from point of the amplifier symbol.
- A_v is the voltage gain (equivalent of G) which relates the output and input by the op-amp equation

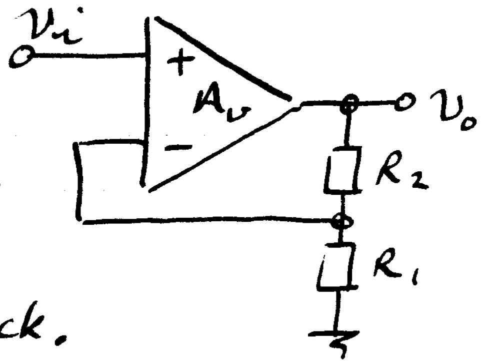
$$\boxed{v_o = A_v(v^+ - v^-)}$$

in other words, A_v operates on the difference between v^+ + v^- to produce v_o.

Op-Amp Circuits

There are many different circuits that are used with op-amps but there are two that are far more common than any others; the "non-inverting amplifier" and the "inverting amplifier".

Non-Inverting Amplifier



When designing an op-amp circuit it is usual to assume initially that $A_v \Rightarrow \infty$. This means that cct behaviour is completely controlled by the feedback.

If $A_v \Rightarrow \infty$, for finite v_o, $v^+ \approx v^-$ and this makes working out circuit behaviour quite straightforward....

$$v^- = v_o \frac{R_1}{R_1+R_2} \quad \text{(by potential division)}$$

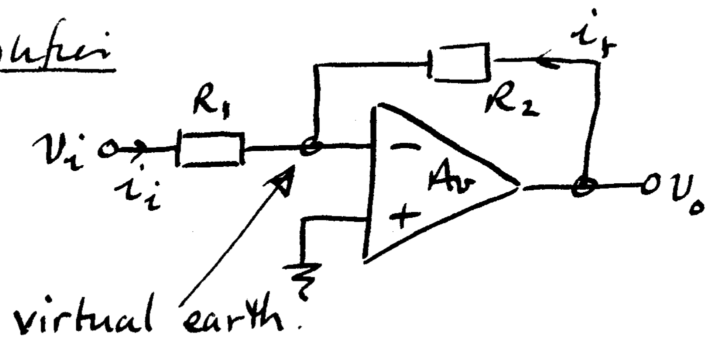
$$v^+ = v_i \quad \text{(connected by wire)}$$

$$\therefore \text{ if } A_v \Rightarrow \infty, \quad v^+ \approx v^- \text{ or } v_i \approx v_o \frac{R_1}{R_1+R_2}$$

$$\text{or } \frac{v_o}{v_i} = \frac{R_1+R_2}{R_1} \qquad (3)$$

– notice that the feedback is returned to the v^- input.

Inverting Amplifier



In the inverting amplifier connection, v^+ is grounded and v_i is applied to R_1. Again if $A_v \Rightarrow \infty$, $v^+ \approx v^-$ and since v^+ is connected to zero v^- must also be very close to zero. The v^- node in this case is sometimes called a "virtual earth" — the potential is always close to zero but the node is not actually connected to zero.

To work out gain, start by summing currents at the v^- node ....

$$i_i + i_f = 0 \quad \text{or} \quad \frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2} = 0$$

$$\text{but } v^- = 0 \quad \text{so} \quad \frac{v_i}{R_1} + \frac{v_o}{R_2} = 0$$

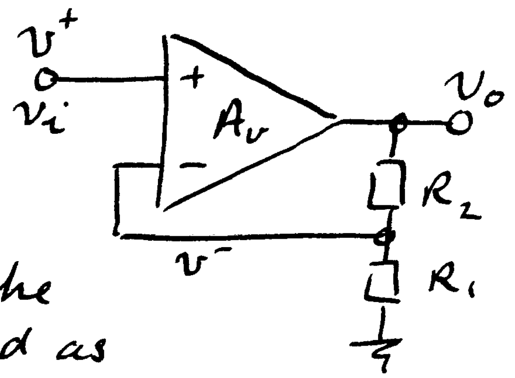
$$\text{or} \quad \frac{v_o}{v_i} = -\frac{R_2}{R_1} \qquad (4)$$

— notice the "−" sign. This means that the signal is inverted (ie phase shifted by 180°) as well as amplified. Two inverting amplifiers in series would give rise to an overall non-inverting amplifier — the first stage would invert the signal and the second would invert it back to its original phase.

Effects of finite gain:

Very occasionally it may be necessary for a designer to estimate the effect of finite op-amp gain on the overall circuit gain.

Non-inverting ...



When considering the effects of finite gain the $v^+ \approx v^-$ approximation cannot be used. Instead the analysis would proceed as follows:

$$v^- = v_o \frac{R_1}{R_1+R_2} \quad \text{(as before)}.$$

$$v^+ = v_i \quad \text{(as before)}.$$

but now the op-amp equation must be used to relate v^+, v^- + v_o ...

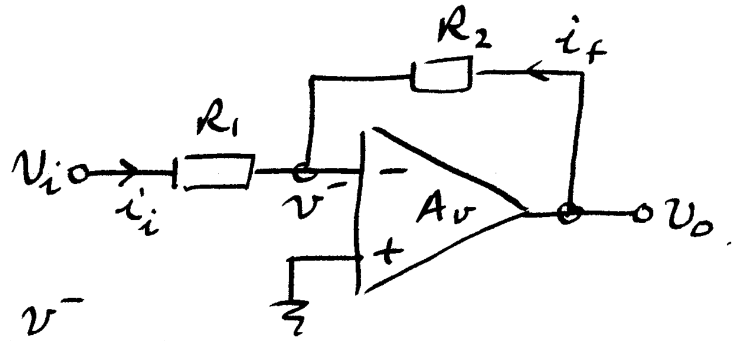
$$v_o = A_v(v^+ - v^-) = A_v\left(v_i - v_o\frac{R_1}{R_1+R_2}\right)$$

$$\text{or} \quad v_o\left[\frac{1}{A_v} + \frac{R_1}{R_1+R_2}\right] = v_i$$

$$\text{or} \quad \frac{v_o}{v_i} = \frac{1}{\frac{1}{A_v} + \frac{R_1}{R_1+R_2}} \quad \text{———} \quad (5)$$

note that if $A_v \Rightarrow \infty$, $\frac{1}{A_v}$ becomes negligible and (5) becomes the same as (3).

Inverting ...



Start as before by summing currents at the v^- node ...

$$i_i + i_f = 0 \quad \text{or} \quad \frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2} = 0.$$

which can be rearranged to give

$$v^- = v_i \frac{R_2}{R_1+R_2} + v_o \frac{R_1}{R_1+R_2}.$$

and $v^+ = 0$.

Using the op-amp equation ....

$$v_o = A_v\left(0 - \left[v_i \frac{R_2}{R_1+R_2} + v_o \frac{R_1}{R_1+R_2}\right]\right)$$

or $$v_o\left[\frac{1}{A_v} + \frac{R_1}{R_1+R_2}\right] = -v_i \frac{R_2}{R_1+R_2}$$

or $$\frac{v_o}{v_i} = -\frac{\dfrac{R_2}{R_1+R_2}}{\dfrac{1}{A_v} + \dfrac{R_1}{R_1+R_2}} \qquad \text{———} \ (6).$$

Again, if $A_v \Rightarrow \infty$, $\frac{v_o}{v_i}$ reduces to (4).

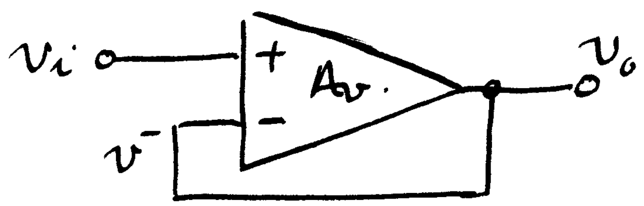
Circuit input resistance

The input to the non-inverting circuit goes straight into the op-amp so the circuit input resistance is the same as that of the op-amp — ie very high.

The inverting circuit is slightly different. Taking the $A_v \Rightarrow \infty$ case, an i_i of v_i/R_1 flows from the source. Input resistance is the ratio of applied signal to current drawn – ie, $v_i/i_i = R_1$. This is typically likely to be a few kΩ which makes inverting amplifiers unsuitable as amplifiers of signals derived from sources with a large Thevenin resistance.

Unity Gain Buffer.

– This is really a member of the non-inverting amplifier family...



Here $v^- = v_o$ so the op-amp eqn. becomes

$$v_o = A_v(v^+ - v^-) = A_v(v_i - v_o)$$

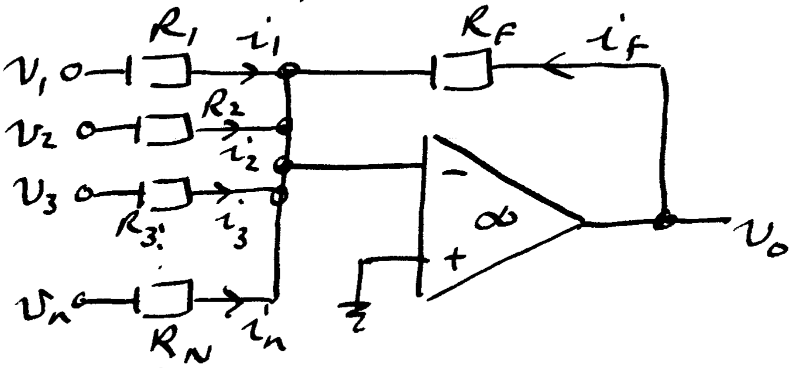
$$\text{or} \quad \frac{v_o}{v_i} = \frac{1}{\frac{1}{A_v} + 1} = \frac{A_v}{1 + A_v}$$

If A_v is large, v_o/v_i is very close to unity.

The circuit is used to isolate high impedance sources from low impedance loads; it has a high power gain.

Circuits with multiple inputs

Summers



Assume $A_v \Rightarrow \infty$ so $v^- \Rightarrow$ virtual earth. $= 0V$.

(this assumption will always be valid in a practically sensible circuit).

then

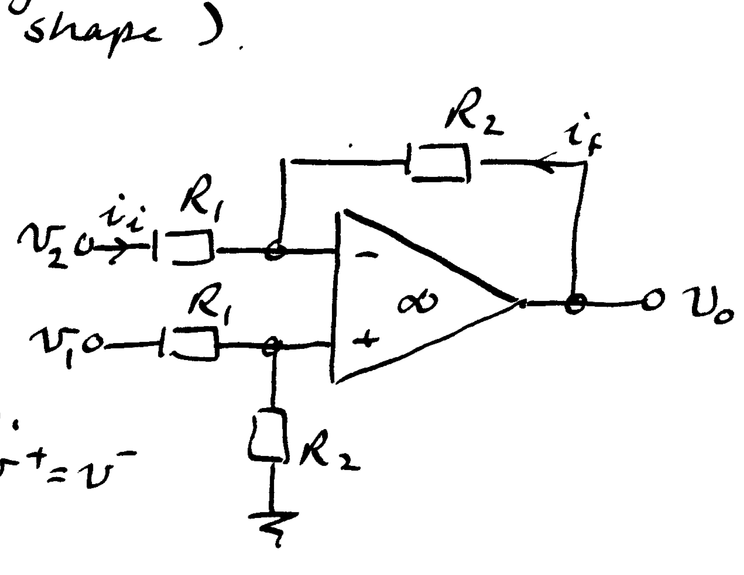
$$i_f + i_1 + i_2 + i_3 + \ldots\ldots + i_n = 0.$$

or
$$\frac{v_o}{R_F} + \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \cdots + \frac{v_n}{R_n} = 0.$$

or
$$v_o = -\left[v_1\frac{R_F}{R_1} + v_2\frac{R_F}{R_2} + \cdots + v_n\frac{R_F}{R_n}\right]$$

(many analogue audio "mixers" use this circuit shape).

Subtractors.



There are a couple of ways of solving a circuit like this.

Since $A_v \Rightarrow \infty$, $v^+ = v^-$

so here we will work out v^+ + v^- and then equate them to get v_o in terms of v_1 + v_2.

summing currents at v^- node ...

$$i_i + i_f = 0 \quad \text{or} \quad \frac{v_2 - v^-}{R_1} + \frac{v_o - v^-}{R_2} = 0.$$

and this can be rearranged to give

$$v^- = v_2 \frac{R_2}{R_1+R_2} + v_o \frac{R_1}{R_1+R_2}.$$

v^+ is a potentially divided version of v_1 ....

$$v^+ = v_1 \frac{R_2}{R_1+R_2}.$$

equating v^+ + v^-...

$$v_2 \frac{R_2}{R_1+R_2} + v_o \frac{R_1}{R_1+R_2} = v_1 \frac{R_2}{R_1+R_2}.$$

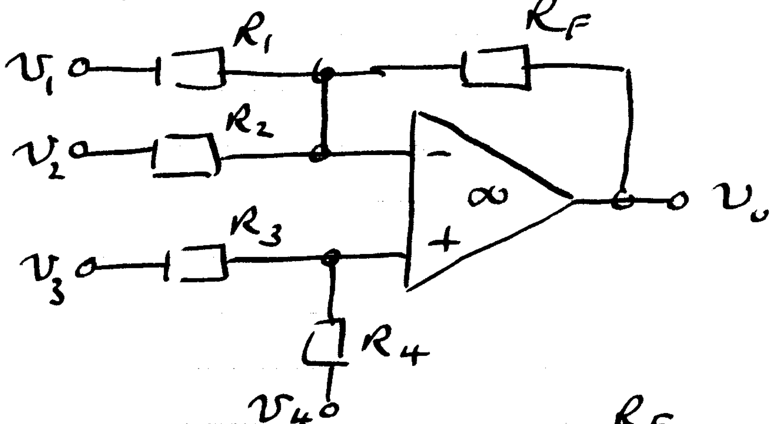
$$\text{or} \quad v_o \frac{R_1}{R_1+R_2} = v_1 \frac{R_2}{R_1+R_2} - v_2 \frac{R_2}{R_1+R_2}$$

$$\text{or} \quad v_o = R_2/R_1 \, (v_1 - v_2).$$

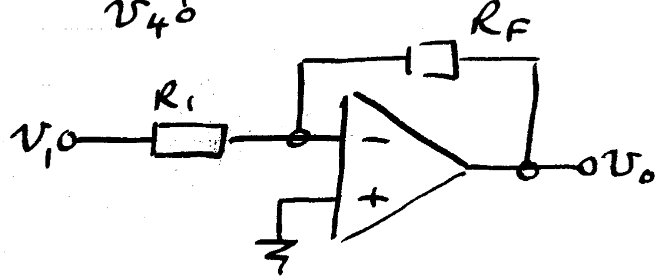
Note that the accuracy of the subtraction depends upon the matching of the two R_1s and R_2s.

General multiple input circuit

The subtractor circuit can be generalised to allow more than two inputs. Such a circuit could be analysed by finding v^+ and v^- and equating them, or by using the principle of superposition. Superposition has the advantage that at each stage the circuit is reduced to a relatively simple single input circuit.



Consider first the output due to v_1. v_2, v_3 + v_4 are grounded. The cct becomes:



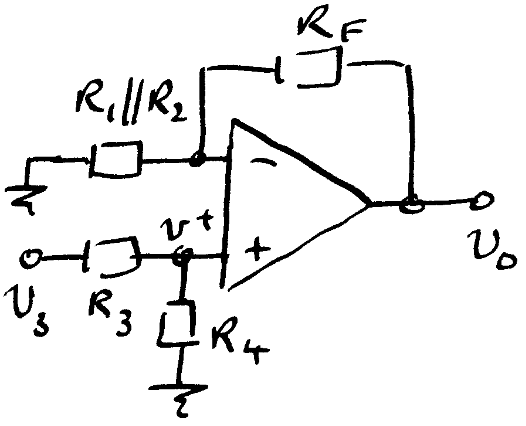
note – since both v_3 + v_4 are zero, v^+ is zero and v^- is a virtual earth. Since v^- is a virtual earth, no current flows throng R_2 so it has no effect on the circuit.

$$v_o\Big|_{\text{due to } v_1} = v_1\left(-R_F/R_1\right).$$

By a very similar argument (change of variable names only)

$$v_o\Big|_{\text{due to } v_2} = v_2\left(-R_F/R_2\right).$$

The output due to v_3 leads to a more complicated circuit ...



Here, v_1 + v_2 are grounded so R_1 is effectively in parallel with R_2. v^+, the non-inverting amplifier input is a potentially divided version of v_3 ....

so $$\frac{v_o}{v^+} = \frac{R_F + R_1 \| R_2}{R_1 \| R_2} \quad \left(\begin{array}{l}\text{ie non-inv.}\\ \text{amp. gain}\end{array}\right).$$

and $$\frac{v^+}{v_3} = \frac{R_4}{R_3 + R_4} \quad (\text{by pot. div.})$$

$$\therefore \frac{v_o}{v_3} = \frac{v_o}{v^+} \times \frac{v^+}{v_3} = \frac{R_4}{R_3 + R_4} \cdot \frac{R_F + R_1 \| R_2}{R_1 \| R_2}$$

or $$v_o \Big|_{\text{due to } v_3} = v_3 \frac{R_4}{R_3 + R_4} \cdot \frac{R_F + R_1 \| R_2}{R_1 \| R_2}.$$

By a very similar argument...

$$v_o \Big|_{\text{due to } v_4} = v_4 \frac{R_3}{R_3 + R_4} \cdot \frac{R_F + R_1 \| R_2}{R_1 \| R_2}.$$

$$v_{o\,TOT} = v_o \Big|_{v_1} + v_o \Big|_{v_2} + v_o \Big|_{v_3} + v_o \Big|_{v_4}.$$

And remember that if any of the vs consist of d.c. + sinusoid, those two parts can be treated separately.