Department of Electronic and Electrical Engineering

First Year Laboratory:

SPECTRUM ANALYSIS

2012

INTRODUCTION

Spectrum analysis is the mathematical process of decomposing a complex periodic signal or waveform into simpler (usually sinusoidal) waveforms or harmonics. The most basic type of frequency analysis is a Fast Fourier Transform or FFT, which converts a signal from the time domain (eg. voltage amplitude vs time) into the frequency domain (magnitude of the harmonics vs frequency).

Frequency analysis is a very common technique used in many branches of engineering. For example an engine may be fitted with a vibration sensor which outputs a complex voltage signal. By performing an FFT on this signal, and monitoring any change in the magnitude of the harmonics over a period of time, it is possible to detect the early signs of bearing wear before catastrophic failure occurs.

This experiment is intended to analyse the spectrum of a number of periodic waveforms using computer aided instrumentation (National Instruments NI ELVIS).

Before attending the lab class you are required to undertake 4 sets of calculations. You should read and complete the sections marked **BEFORE THE LAB**. You should write these up in your lab book for the demonstrator to check at the start of the lab session. Failure to complete these sections will result in loss of marks.

BACKGROUND

Any periodic waveform having a frequency f can be expressed as the sum of a series of sinusoidal waveforms called harmonics having frequencies that are integer multiples of the fundamental frequency f, i.e. $2f$, $3f$, $4f$ etc.

For example, a periodic voltage waveform $v(t)$ can be written as:

$$v(t) = v_0 + \sum_{n=1}^{\infty}\left(v_{an}\cos(n\,\omega t) + v_{bn}\sin(n\,\omega t)\right) \quad (1)$$

where $\omega = 2\pi f$, and v_0, $v_{a1}, v_{b1}, \ldots\ldots, v_{an}, v_{bn}$ are called Fourier coefficients, and are calculated as follows:

$$v_o = \frac{1}{T}\int_0^T v(t)\,dt \quad (2a)$$

$$v_{an} = \frac{2}{T}\int_0^T v(t)\cos(n\omega t)\,dt \quad (2b)$$

$$v_{bn} = \frac{2}{T}\int_0^T v(t)\sin(n\omega t)\,dt \quad (2c)$$

where $T = \dfrac{1}{f}$ is the period of the waveform. Equation (1) can also be re-written as:

$$v(t) = v_0 + \sum_{n=1}^{\infty} v_n \cos(n\,\omega t + \phi_n) \tag{3}$$

Where, $v_1 = \sqrt{v_{a1}^2 + v_{b1}^2}$, …., $v_n = \sqrt{v_{an}^2 + v_{bn}^2}$, and $\phi_n = \tan^{-1}(v_{bn}/v_{an})$.

Example 1

Figure 1 shows a squarewave, which can be expressed as:

$$v(t) = \sum_{n=1,3,5,\ldots}^{\infty} \overbrace{v_{p-p}\,\frac{\sin\left(n\,\frac{\pi}{2}\right)}{\left(n\,\frac{\pi}{2}\right)}}^{v_n} \cos(n\,\omega t) \tag{4}$$

where v_{p-p} is the peak-to-peak voltage. Furthermore, for a squarewave $v_0 = 0$ and $v_n = 0$ when n is even. From figure 1 it can clearly be seen that the approximation of the squarewave as a sum of harmonics becomes more accurate as the number of considered harmonics increases. From equation (4):

The approximation using the 1st harmonic is given by:

$$v(t) = \frac{2\,v_{p-p}}{\pi}\cos(n\,\omega t) \tag{4a}$$

The approximation using the sum of the first 3 harmonics is given by:

$$v(t) = \frac{2\,v_{p-p}}{\pi}\cos(\,\omega t) - \frac{2\,v_{p-p}}{3\,\pi}\cos(3\,\omega t) + \frac{2\,v_{p-p}}{5\,\pi}\cos(5\,\omega t) \tag{4b}$$

The approximation using the sum of the first 8 harmonics is given by:

$$\begin{aligned} v(t) = &\frac{2\,v_{p-p}}{\pi}\cos(\,\omega t) - \frac{2\,v_{p-p}}{3\,\pi}\cos(3\,\omega t) + \frac{2\,v_{p-p}}{5\,\pi}\cos(5\,\omega t) \\ &- \frac{2\,v_{p-p}}{7\,\pi}\cos(7\,\omega t) + \frac{2\,v_{p-p}}{9\,\pi}\cos(9\,\omega t) - \frac{2\,v_{p-p}}{11\,\pi}\cos(11\,\omega t) \\ &+ \frac{2\,v_{p-p}}{13\,\pi}\cos(13\,\omega t) - \frac{2\,v_{p-p}}{15\,\pi}\cos(15\,\omega t) \end{aligned} \tag{4c}$$

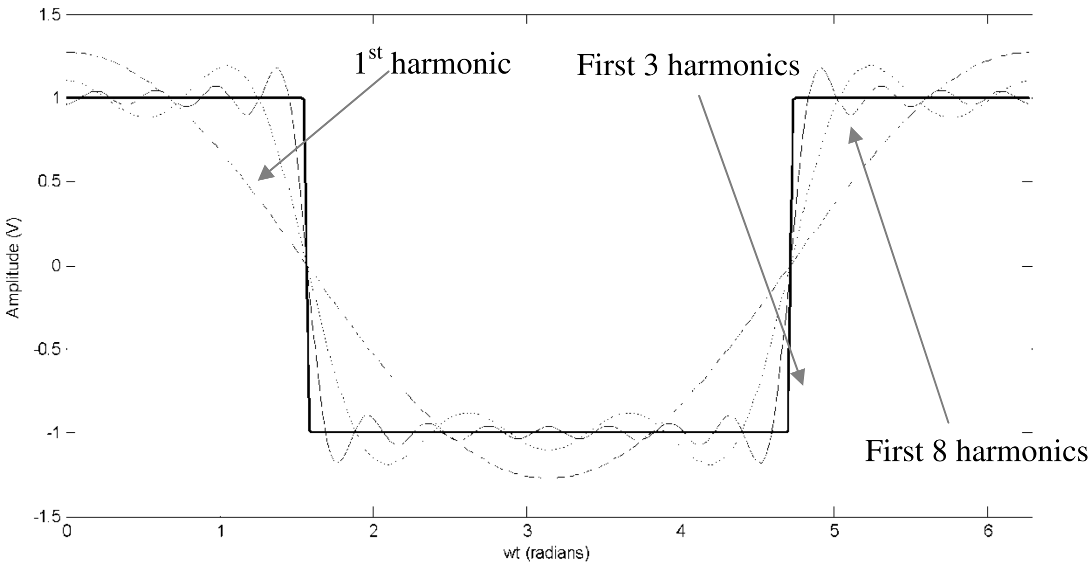


Figure 1 Squarewave for $v_{p-p} = 2$V.

Example 2

The periodic pulse train shown in figure 2, can be expressed as:

$$v(t) = \overbrace{\frac{\tau}{T} v_{p-p} + v_{\min}}^{v_0} + \sum_{n=1}^{\infty} \overbrace{\frac{2\tau}{T} v_{p-p} \left(\frac{\sin\left(\pi n \frac{\tau}{T} \right)}{\left(\pi n \frac{\tau}{T} \right)} \right)}^{v_n} \cos(n\,\omega t) \tag{5}$$

where $\frac{\tau}{T}$ is the duty cycle, and $v_{p-p} = v_{\max} - v_{\min}$ is the peak-to-peak voltage.

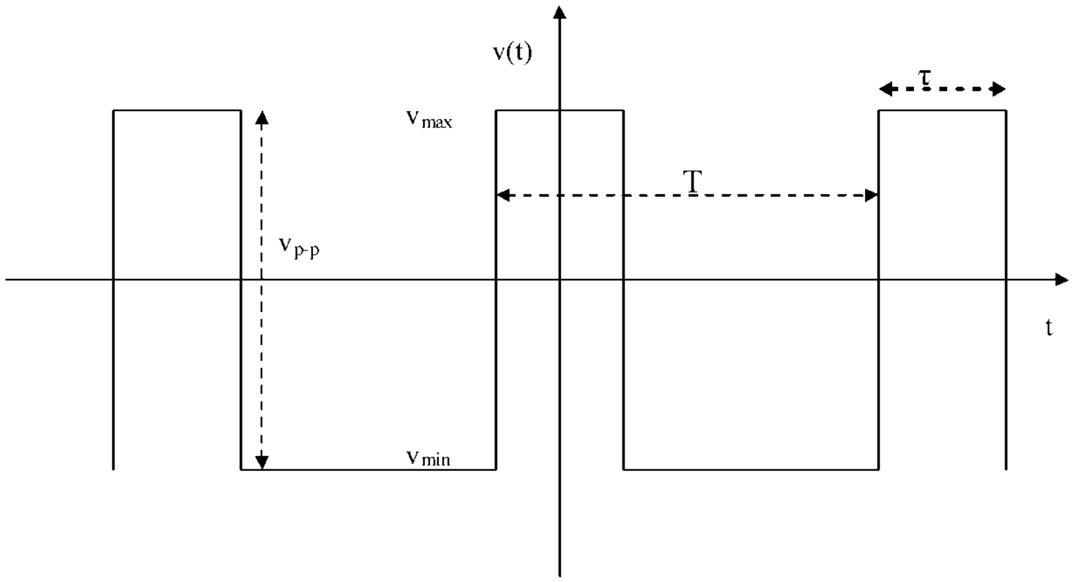


Figure 2 Periodic pulse train

The resulting harmonic spectrum is shown in figure 3.

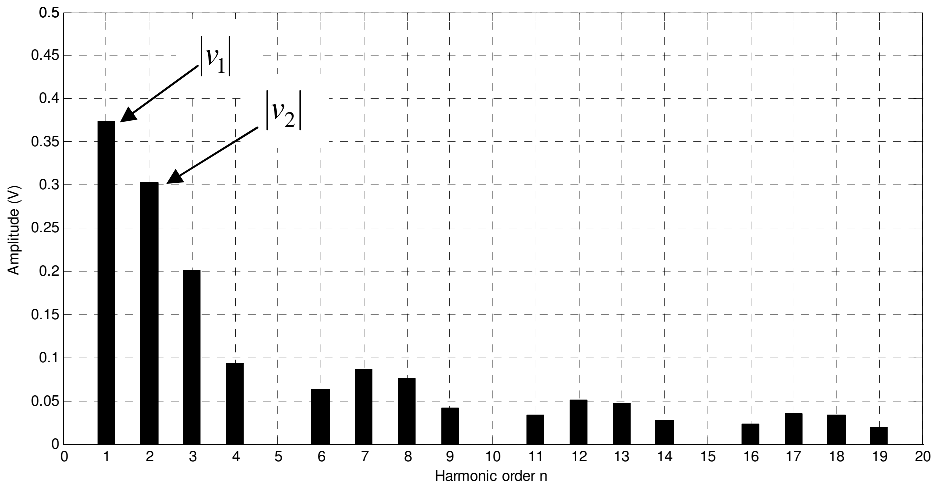


Figure 3 Harmonic spectrum of the periodic pulse train for peak-to-peak voltage v_{p-p} = 1V, and duty cycle $\frac{\tau}{T} = 0.2$.

Example 3

The periodic triangular waveform shown in figure 4, can be expressed as:

$$v(t) = \sum_{n=1,3,5,\ldots}^{\infty} \overbrace{2\, v_{p-p} \frac{(1-\cos(\pi n))}{(\pi n)^2}}^{v_n} \cos(n\, \omega t) \qquad (6)$$

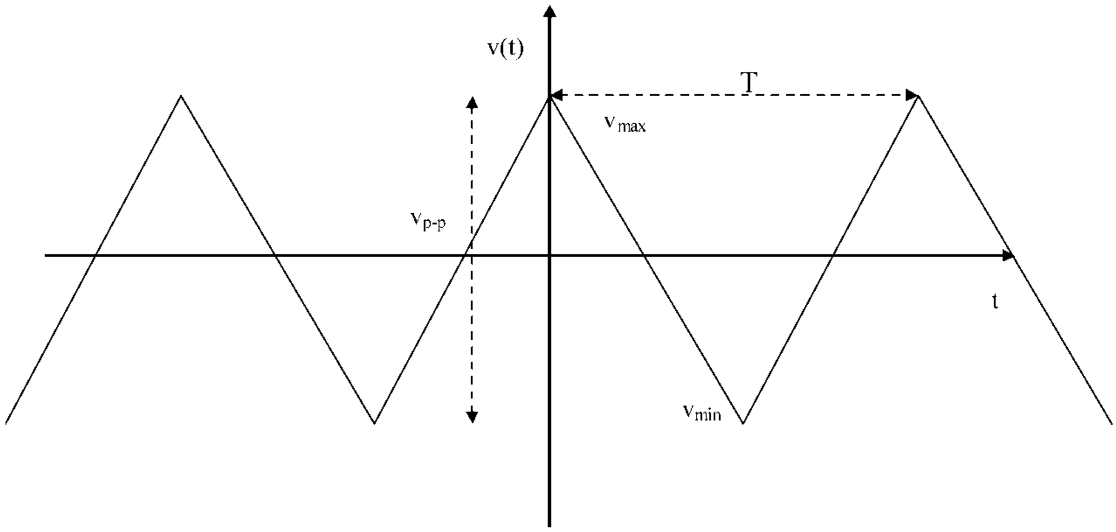


Figure 4 Periodic triangular waveform.

The resulting harmonic spectrum is shown in figure 5.

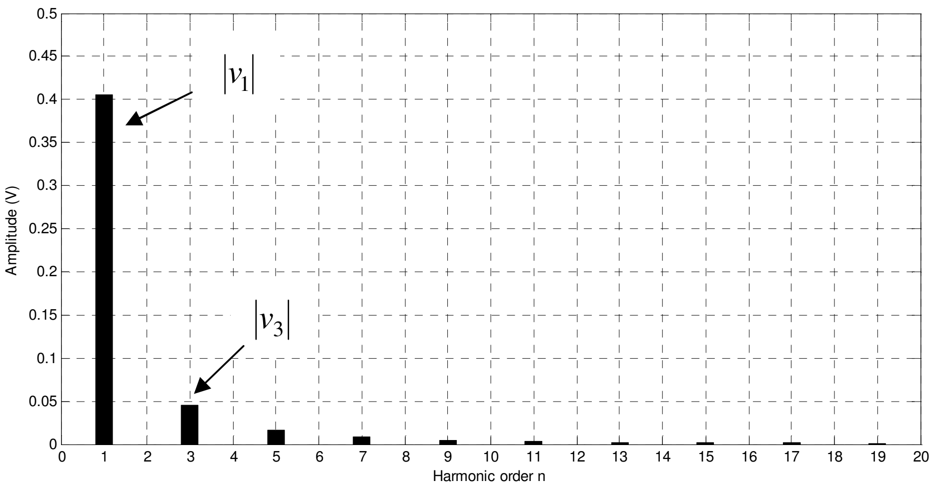
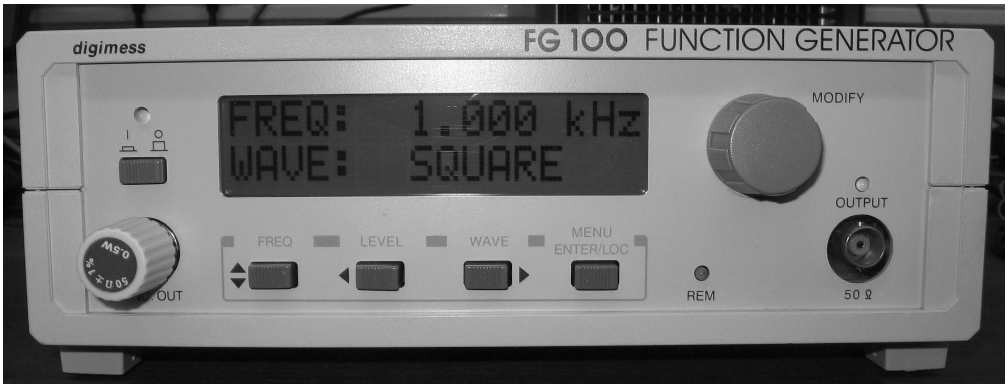


Figure 5 Harmonic spectrum of the triangular waveform for a peak-to-peak voltage $v_{p-p} = 1\text{V}$.

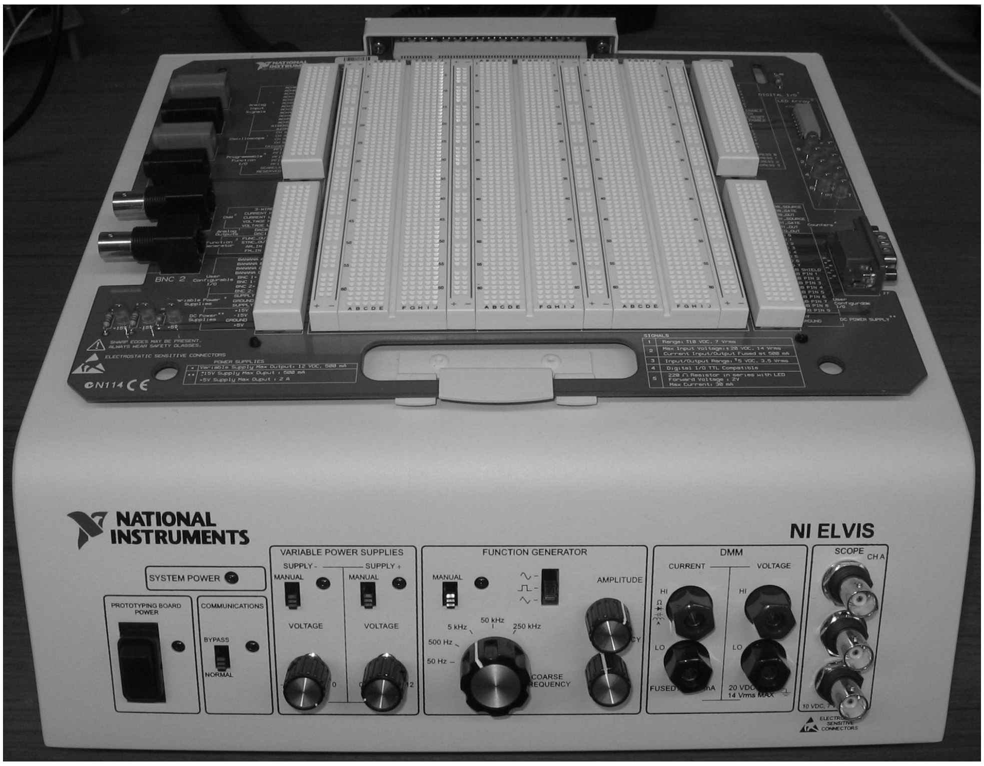
EXPERIMENTAL PROCEDURE

The experiment makes use of the National Instruments Electronic Visualisation (NI ELVIS) hardware and software platform to perform FFT's and display the results on a computer monitor. The FG100 Function Generator is used to generate the waveforms on which the FFT is performed.

The equipment to be used in this experiment is shown below.

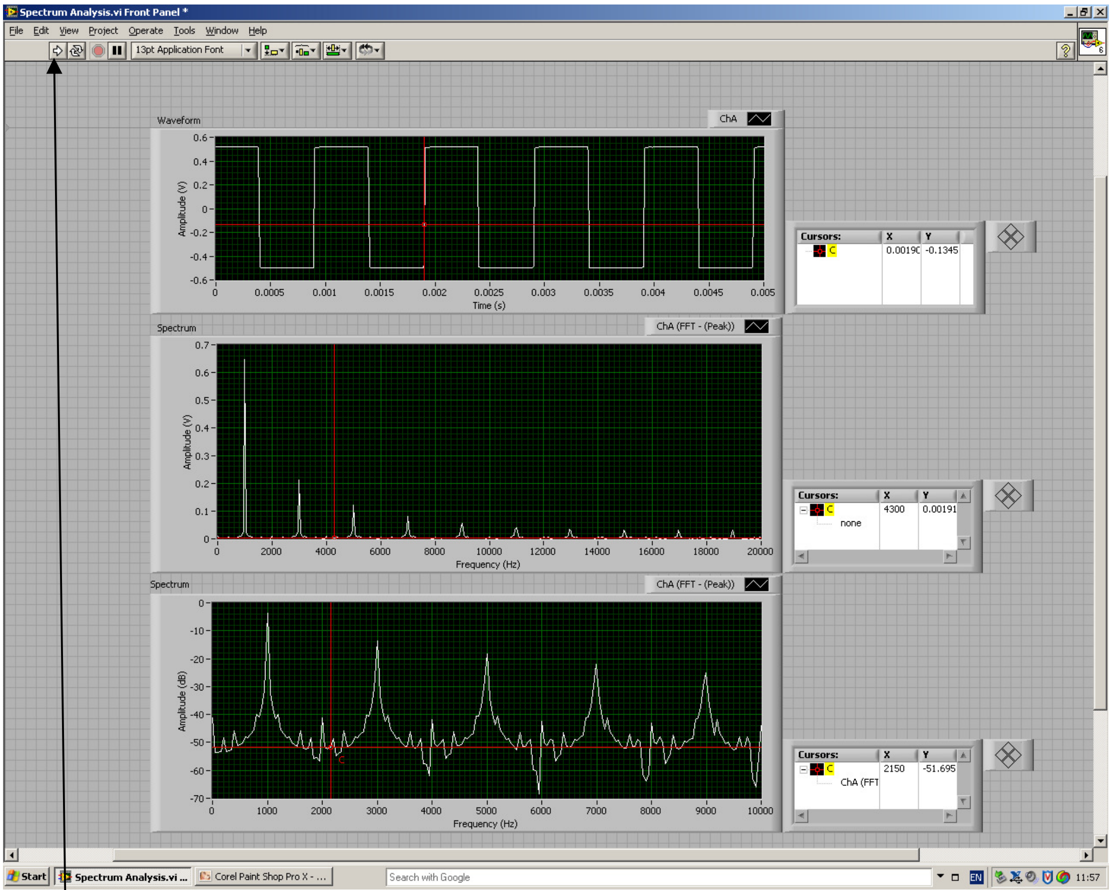


Signal/function generator



National Instruments NI ELVIS

Test circuit board (Used in experiment 3)



Run

Virtual instrument displayed
on the computer monitor

EXPERIMENT 1: SQUARE WAVE MEASUREMENTS

BEFORE THE LAB

In this section an FFT will be performed on a 1kHz, 2V peak-to-peak (v_{p-p}) square wave with a 50% duty cycle (i.e. $\frac{\tau}{T}$ = 0.5). Copy the table below into your lab book and calculate the value of $|v_n|$ for the first 5 voltage harmonics using the equation:

$$v_n = \frac{2\tau}{T} v_{p-p} \left(\frac{\sin\left(\pi n \frac{\tau}{T}\right)}{\pi n \frac{\tau}{T}} \right)$$

One value has been completed already to allow you to check your calculations.

	n=1	n=3	n=5	n=7	n=9
Measured $|v_n|$ (V)					
Calculated $|v_n|$ (V)				0.18	

DURING THE LAB

Start the computer and double click on the desktop icon Spectrum Analysis.

Set the FG100 function generator to give a 1 kHz and 2V peak-to-peak square wave. (Details of how to operate the function generator are given in Appendix 1). Connect the output of the generator to SCOPE CH A Input of NI ELVIS.

Run the vi (virtual instrument), record the amplitudes of the first 5 voltage harmonics in the table in your lab book and compare them with the calculated amplitudes. *Are they as you expect?*

EXPERIMENT 2: PULSE TRAIN MEASUREMENTS

20% DUTY CYCLE

BEFORE THE LAB

In this section an FFT will be performed on a 1kHz, 2V peak-to-peak (v_{p-p}) square wave with a 20% duty cycle (i.e. $\frac{\tau}{T}$ = 0.2). Copy the table below into your lab book and calculate the value of $|v_n|$ for the first 5 voltage harmonics using the above equation.

	n=1	n=2	n=3	n=4	n=5
Measured $|v_n|$ (V)					
Calculated $|v_n|$ (V)					

DURING THE LAB

Set the FG100 function generator to give a 1 kHz and 2V peak-to-peak square wave and set the duty cycle to 20% (1/5) (by selecting DCY from the menu of the function generator). Connect the output of the generator to SCOPE CH A Input of NI ELVIS.

Run the vi (virtual instrument), record the amplitudes of the first 5 voltage harmonics in the table in your lab book and compare them with the calculated amplitudes. *Are they as you expect?*

25% DUTY CYCLE

BEFORE THE LAB

Copy the table below into your lab book and calculate the value of $|v_n|$ for the first 5 voltage harmonics for a duty cycle of 25% (i.e. $\frac{\tau}{T}$ = 0.25) using the same equation as in the previous sections.

	n=1	n=2	n=3	n=4	n=5
Measured $|v_n|$ (V)					
Calculated $|v_n|$ (V)					

DURING THE LAB

Now set the duty cycle to 25% (1/4), run the vi, record the amplitudes of the first 5 voltage harmonics in the table in your lab book and compare them with the calculated amplitudes. *Are they as you expect?*

33% DUTY CYCLE

BEFORE THE LAB

Copy the table below into your lab book and calculate the value of $|v_n|$ for the first 5 voltage harmonics for a duty cycle of 33% (i.e. $\frac{\tau}{T}$ = 0.33) using the same equation as in the previous sections.

	n=1	n=2	n=3	n=4	n=5
Measured $|v_n|$ (V)					
Calculated $|v_n|$ (V)					

DURING THE LAB

Now set the duty cycle to 33% (~1/3), run the vi, get the magnitudes of the first 5 voltage harmonics, and compare them with the calculated amplitudes. record the amplitudes of the first 5 voltage harmonics in the table in your lab book and compare them with the calculated amplitudes. *Are they as you expect?*

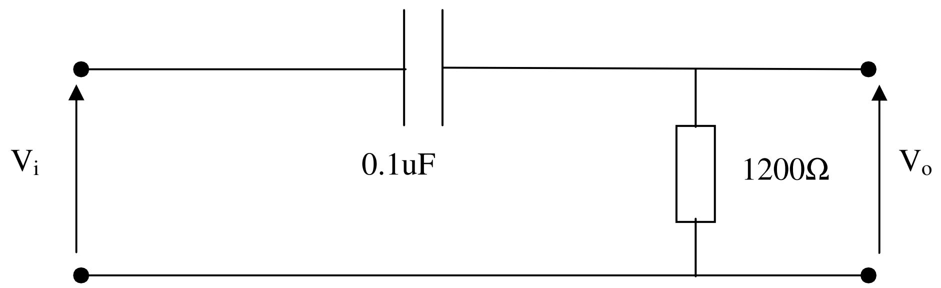
What conclusions can you draw from the above tests?

EXPERIMENT 3: RC CIRCUIT MEASUREMENTS

In this experiment we will examine the frequency content of the voltage across the resistor and capacitor of an RC circuit supplied with a square wave. This circuit is an approximation to a low-pass or a high-pass filter.

High pass filter

Connect up the RC circuit as follows:

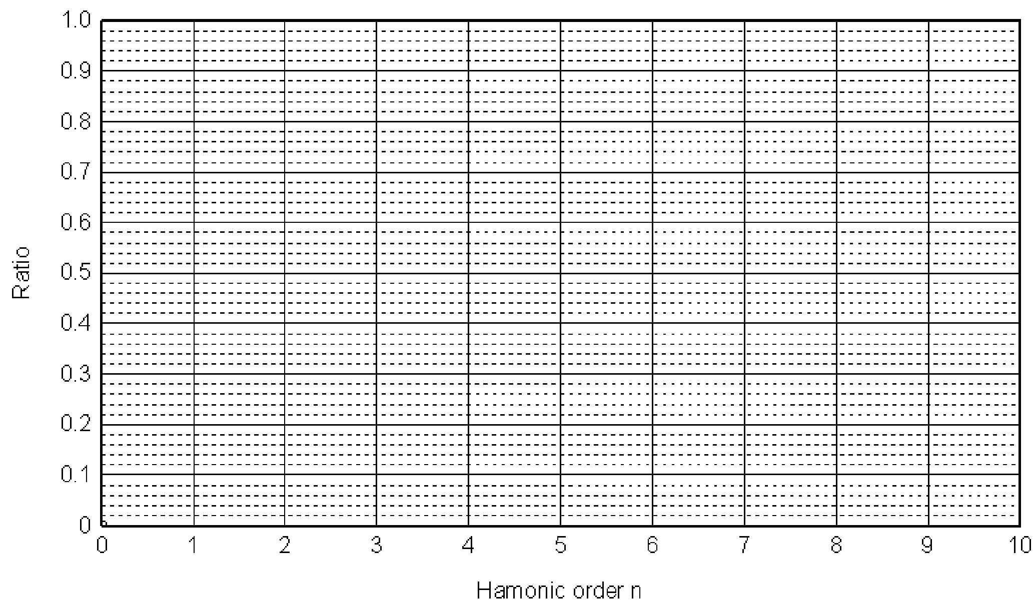


Apply a 500Hz, 4V peak-to-peak square wave to the circuit and connect its output to CHANNEL A of the NI ELVIS device.

Run the vi and record the magnitude of the first 5 input and output voltage harmonics in your lab book.

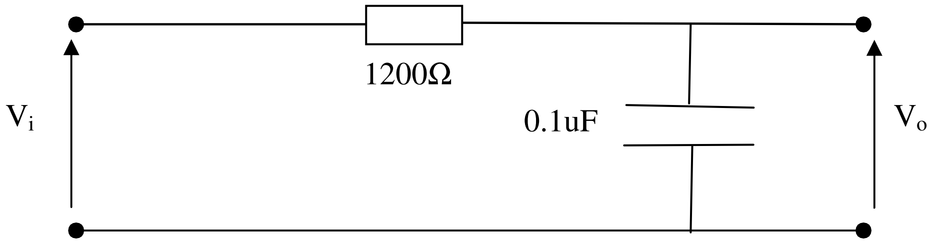
	n=1	n=3	n=5	n=7	n=9
Measured $\lvert v_{in} \rvert$ (V)					
Measured $\lvert v_{on} \rvert$ (V)					
Ratio $\frac{\lvert v_{on} \rvert}{\lvert v_{in} \rvert}$					

Plot the ratio $\frac{\lvert v_{on} \rvert}{\lvert v_{in} \rvert}$ on the graph below:



Low pass filter

Connect up the RC circuit as follows:

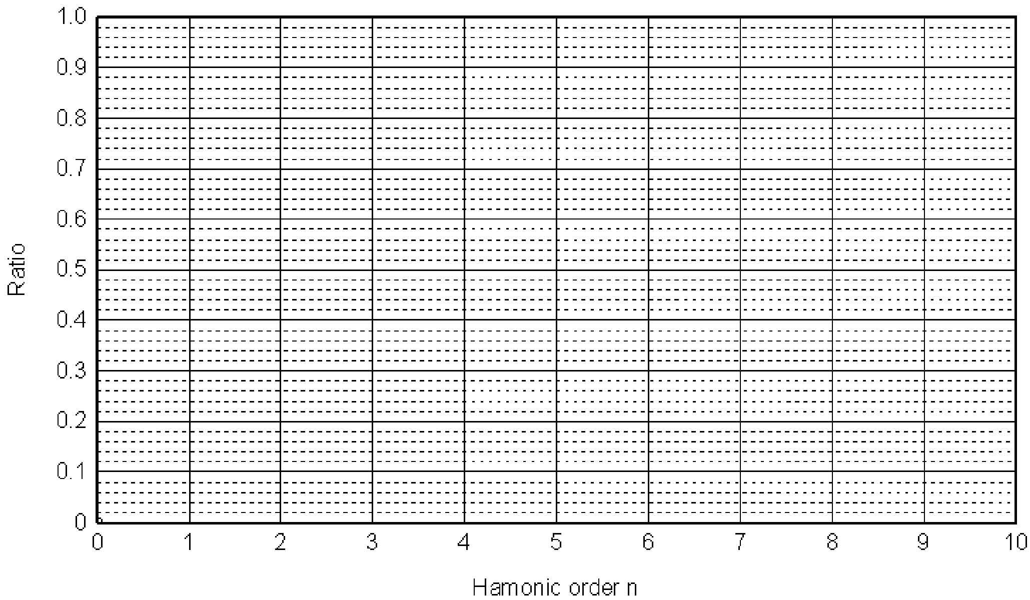


Apply a 500Hz, 4V peak-to-peak square wave to the circuit and connect its output to CH A of the NI ELVIS device.

Run the vi and record the magnitude of the first 5 input and output voltage harmonics in your lab book.

	n=1	n=3	n=5	n=7	n=9
Measured $\lvert v_{in} \rvert$ (V)					
Measured $\lvert v_{on} \rvert$ (V)					
Ratio $\frac{\lvert v_{on} \rvert}{\lvert v_{in} \rvert}$					

Plot the ratio $\frac{\lvert v_{on} \rvert}{\lvert v_{in} \rvert}$ on the graph below:



Are the graphs for the high and low pass filters as you would expect?

APPENDIX: OPERATION OF THE FG100 FUNCTION GENERATOR

The FG100 has 4 dual purpose function buttons:-

FREQ:	**1.000kHz**
WAVE:	**SQUARE**

FREQ/*cursor up and down (F1)*
LEVEL/*cursor left (F2)*
WAVE/*cursor right (F3)*
MENU/ENTER *(F4)*

The rotary switch is used to set parameters and to scroll forwards or backwards through the menu in use. When the power switch is turned on, the 'REM' red LED indicator will be illuminated while an internal initialisation test takes place. Once the indicator goes out, the FG 100 is ready for operation and the message (above right) is an example of what may appear on the display.

To set frequency, press the **FREQ** button (F1) and the cursor will appear on the display under the last position of the frequency value. With the cursor buttons (F2 and F3) select the parameter position to be changed and then use the rotary switch to obtain the required frequency value.

Note: Once a function has been selected in this way (e.g. frequency), the **MENU** button (F4) must be pressed again before selecting another function (e.g. output level), in order to re-activate the function buttons F1, F2 and F3.

To set output level, press the **LEVEL** button F2, and use the cursor buttons (F2 and F3) and the rotary switch to obtain the required output value.