The University of Sheffield, Department EEE

Analogue and Digital Electronics
Problem Sheet Solutions: Operational Amplifiers

1 (i) The circuit of figure 1a is a standard non-inverting amplifier circuit. Gain is calculated by assuming $A_V >> 1$ which means $v^+ \approx v^-$. Thus

$$v^- = v_o \frac{R_1}{R_2 + R_1} \text{ and since } v_i = v^+ \text{ and } v^+ \approx v^-,$$

$$\frac{v_o}{v_i} = \frac{R_1 + R_2}{R_1} \qquad \mathbf{(1.1)}$$

the amplifier gain is thus **6 V/V**

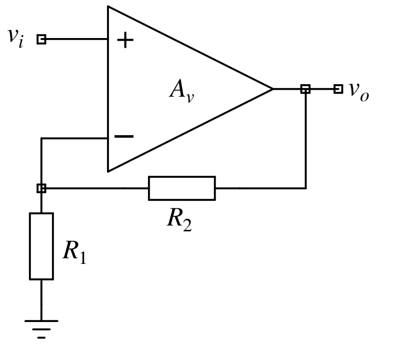


Figure 1a

(ii) The gain-bandwidth product of the amplifier is 10 MHz. The gain - bandwidth product is thus given by 6 x BW = 10 MHz. This gives a bandwidth of **1.67 MHz**. Note that this is the − 3 dB bandwidth.

(iii) To calculate the risetime of the amplifier output you should bring to mind the key relationship between the time and frequency domains for a first order system;

$$\tau = \frac{1}{\omega_0}$$, where ω_0 is a constant in the frequency domain that is 2π times f_0, the − 3 dB bandwidth expressed as a cyclic frequency. The risetime is the time taken for an exponential to rise (or fall) between 10% and 90% of its start voltage to aiming voltage travel and it is easy to show that risetime = 2.2τ. In this case the risetime is **0.21 μs**.

(iv) The network of figure 1b is the used in place of R_1; let the network be called Z_1. The amplifier circuit gain is then described by equation **(1.1)** with R_1 replaced by Z_1.

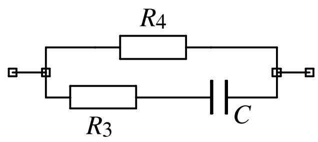


Figure 1b

Low and high frequency gains are determined by letting C approach a short circuit (0 Ω) at high frequency and approach an open circuit (∞ Ω) at low frequency. In the case of figure 1b, its low frequency impedance is R_4 (2 kΩ) and its high frequency impedance is $R_4 // R_3$ (198 Ω). So the l.f. gain is **6 V/V** and the h.f. gain is **51.5 V/V.**

(v) For this part of the question it is necessary to work out equation (1.1) with R1 replaced by figure 1b which again we can call Z1. It makes sense first to simplify Z1.

$$Z_1 = R_4 // \left(R_3 + \frac{1}{j\omega C} \right) = \frac{R_4(1 + j\omega C R_3)}{1 + j\omega C (R_3 + R_4)}$$

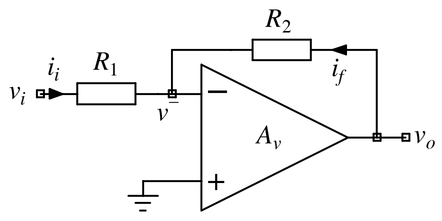
and then use this expression in place of R1 in equation (1.1) to give

$$\frac{v_o}{v_i} = \frac{R_2 + R_4}{R_4} \left(\frac{1 + j\omega C \dfrac{R_2 R_3 + R_2 R_4 + R_3 R_4}{R_2 + R_4}}{1 + j\omega C R_3} \right)$$

from which the terms requested in the question can be found by comparison with the standard form.

(vi) The response Bode plot will be the same shape as figure 4c in the "First Order Circuits" handout with ω_0 (which corresponds to ω_1 in the figure 4c) = 53 krad s^{-1}, ω_1 (which corresponds to ω_2 in the figure 4c) = 454 krad s^{-1}, 20log$k$ = 20log6 and 20log($k\omega_2/\omega_1$) = 20log51.5.

2 Gain-bandwidth product arises as a result of the gain characteristics of the op-amp itself so you must perform an analysis which includes the frequency dependent op-amp gain. You can assume though that the amplifier is perfect in every other respect. The usual starting point is to sum currents at the inverting input node in order to find v^- in terms of v_o and v_i :



$$i_i + i_f = 0 = \frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2} \quad \text{or} \quad v^- = v_o \frac{R_1}{R_1 + R_2} + v_i \frac{R_2}{R_1 + R_2} \qquad \textbf{(2.1)}$$

Next use the op-amp equation to express v_o in terms of v^+ and v^- ....

$$v_o = A_v\,(v^+ - v^-) = A_v\, v^- \ \ (\text{since } v^+ = 0) \qquad \textbf{(2.2)}$$

Combining **(2.1)** and **(2.2)** to eliminate v^- gives,

$$\frac{v_o}{v_i} = -\frac{\dfrac{R_2}{R_1 + R_2}}{\dfrac{1}{A_v} + \dfrac{R_1}{R_1 + R_2}} \qquad \textbf{(2.3)}$$

The op-amp gain is given by

$$A_v = \frac{A_0}{1 + j\dfrac{\omega}{\omega_0}} \qquad \textbf{(2.4)}$$

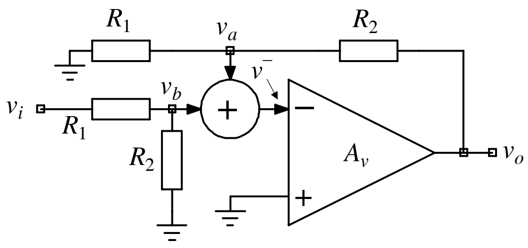
and by using **(2.4)** in **(2.3)** and forcing the result to a standard form, the circuit transfer function is

$$\frac{v_o}{v_i} = -\frac{\dfrac{R_2}{R_1 + R_2}}{\dfrac{1}{A_v} + \dfrac{R_1}{R_1 + R_2}} = -\frac{\dfrac{R_2}{R_1 + R_2}}{\left(\dfrac{1 + j\dfrac{\omega}{\omega_0}}{A_0}\right) + \dfrac{R_1}{R_1 + R_2}} = -\frac{\dfrac{A_0R_2}{R_1 + R_2}}{\left(1 + \dfrac{A_0R_1}{R_1 + R_2}\right)\left(1 + j\dfrac{\omega}{\omega_0\left(1 + \dfrac{A_0R_1}{R_1 + R_2}\right)}\right)}$$

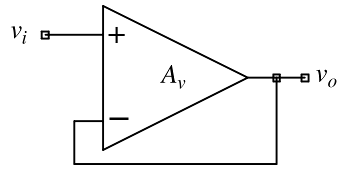
$$\equiv \frac{k}{1 + j\dfrac{\omega}{\omega_1}} \quad \text{where } k = \frac{\dfrac{A_0R_2}{R_1 + R_2}}{\left(1 + \dfrac{A_0R_1}{R_1 + R_2}\right)} \quad \text{and } \omega_1 = \omega_0\left(1 + \frac{A_0R_1}{R_1 + R_2}\right)$$

Gain-bandwidth product = $k\,\omega_1 = A_0\omega_0R_2/(R_1 + R_2)$

This result tells you that for the inverting amplifier circuit connection the gain-bandwidth product of the circuit is not independent of circuit gain. The reason for this can be seen if the circuit is drawn in a formal feedback system form. **(2.1)** suggests that v^- is the sum of two potentially divided voltages, v_a and v_b. v_a is the voltage that would appear at v^- in a non-inverting circuit; the inverting connection has an additional potential divider that converts v_i into v_b and it is this potential divider which introduces a gain dependence in the gain-bandwidth product. The effect is exactly the same as it would be if a resistive attenuator were put in front of a conventional non-inverting amplifier circuit and then evaluated the gain-bandwidth product of the combination evaluated.



3 In this question you need to recognise that a voltage follower circuit has a gain very close to unity. Under such circumstances, the gain-bandwidth product is numerically equal to the bandwidth. If you cannot see this, you must work it out ......



$$v_o = A_v\,(v^+ - v^-) = A_v\,(v_i - v_o) \quad \text{or} \quad \frac{v_o}{v_i} = \frac{1}{\dfrac{1}{A_v} + 1}$$

The op-amp gain is approximated by a first order low pass response, $A_v = A_0 /(1 + j\,\omega/\omega_0)$ so,

$$\frac{v_o}{v_i} = \frac{1}{\dfrac{1}{A_v} + 1} = \frac{A_0}{1 + A_0 + j\dfrac{\omega}{\omega_0}} = \frac{A_0}{1 + A_0}\,\frac{1}{1 + j\dfrac{\omega}{\omega_0\,(1 + A_0)}} \approx \frac{1}{1 + j\dfrac{\omega}{\omega_0\,A_0}} \quad \text{for } A_0 \gg 1$$

The -3 dB bandwidth of the voltage follower circuit, $A_0\,\omega_0$, is the gain-bandwidth product of the amplifier.

The phase associated with the voltage follower is $\varphi = -\tan^{-1}\omega/A_0\omega_0 = -\tan^{-1} f/A_0 f_0$ and the magnitude of this phase shift must not exceed 0.1° at 50 kHz. Using the phase relationship above, required gain-bandwidth product = 50 kHz / tan 0.1° = **28.6 MHz.**

4 (i) The op-amp in question has a step reponse described by a simple exponential so it is a first order system. The angular − 3 dB bandwidth of a first order system is the reciprocal of its time constant so the circuit bandwidth is $1/2.8 \times 10^{-6}$ = 357 krad s^{-1} or 56.8 kHz. We are told that the circuit gain at which the exponential response was measured was 250 V/V so the gain-bandwidth product is 56.8 kHz x 250 = **14.2 MHz.**

(ii) If the feedback resistors in the amplifier circuit were changed to give a non-inverting gain of 10 V/V the − 3 dB bandwidth would be (gain-bandwidth product) / (new gain) = **1.42MHz.**

(iii) The system time constant is related to bandwidth as explained in part **(i)**. With a gain of 10 V/V, therefore, the time constant is given by $\tau = 1 / (2\,\pi\,1.42 \times 10^6)$ = 112 ns. The circuit rise time is the time it takes the exponential step response of the circuit to travel betwee 10% and 90% of its start to aiming level range. The relationship between time constant and risetime is something you should know but if you don't, it is not difficult to work out.

t_r = risetime = 2.2τ = **246 ns**

5 (i) The op-amp will behave like a first order low pass system so if its gain-bandwidth product is 15 MHz and the non-inverting circuit gain is 10 V/V, the − 3 dB bandwidth must be 15 MHz / 10 V/V = 1.5 MHz. Its transfer function can be written as,

$$\frac{v_o}{v_i} = k\,\frac{1}{1+j\dfrac{\omega}{\omega_0}} = 10\,\frac{1}{1+j\dfrac{f}{1.5\times10^6}}$$

At a frequency of 5MHz, $|\text{gain}| = |v_o/v_i| = 10/(1+(5/1.5)^2)^{0.5} = 10/3.48 =$ **2.87 V/V**

and $\angle\,[v_o/v_i] = -\tan^{-1}\omega/\omega_0 = -\tan^{-1}5/1.5 =$ **− 73°.**

(ii) First identify the maximum rate of change of a sinusoidal waveform, $v(t) = V_P \sin \omega t$.

$dv(t)/dt = V_P\,\omega \cos \omega t$ is max when $\cos \omega t = 1$, so max rate of change of $v(t) = V_P\,\omega$.

Equating max rate of change to slew rate and rearranging to express frequency explicitly,

$$f_{max} = \frac{\text{slew rate}}{2\pi V_P} = \frac{15\times10^6}{2\pi\ 10} = \mathbf{398\ kHz}.$$

(iii) The rising and falling edges of a square wave of magnitude $V_{P\text{-}P}$ appearing at the output of an op-amp will be exponential in nature because of the first order system behaviour of the op-amp. There may, however, be a value of system time constant which would cause the initial rate of rise of the exponential to try and exceed the amplifier slew rate. The first thing to do therefore is identify the smallest time constant that can be tolerated by the system if slew rate limiting is to be avoided. The maximum rate of change of an exponential occurs at $t = 0$ and is easily shown by a number of means to be $V_{P\text{-}P}/\tau$. The minimum time constant is thus,

$$\tau_{min} = V_{P\text{-}P}/(\text{slew rate}) = 15 / (25\times10^6) = 600\text{ ns}$$

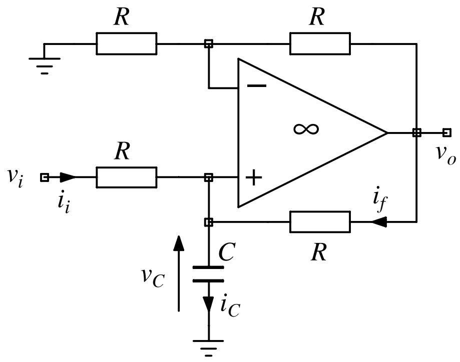
Using the known gain-bandwidth product and the relationship between τ and f_0 for a first order system the circuit gain that will give a τ of 600 ns can be found. The -3dB bandwidth that corresponds to τ = 600 ns is $f_0 = 1 / (2\,\pi\,\tau) =$ 265 kHz. The gain-bandwidth product divided by the bandwidth gives the amplifier gain that will give a first order system time constant of 600ns.

gain = (gain-bandwidth product) / (bandwidth) = 15 MHz / 265 kHz = **56 V/V**.

If the gain were halved, the bandwidth would be doubled (gain-bandwidth product is constant) and the first order system time constant would be halved. This would make the initial rate of change of the exponential waveform faster than the maximum rate of change that the amplifier could support - ie, the amplifier slew rate - and the exponential would consequently be distorted.

(iv) The question of part **(iii)** is based on the premise that the square wave is of a frequency that is sufficiently low to permit the rising and falling exponential edges of the output signal to reach their aiming level. Under such conditions the form of the exponential edges is independent of the frequency of the square wave. If the signal frequency is such that the exponential rising and falling edges are not permitted to reach their aiming levels - ie, the signal half period is of the order of, or less than, the system time constant - the problem can still be worked out but it is harder to find the functional form of the exponentials (because it is harder to identify aiming levels) and the problem becomes very dependent on the square wave frequency.

6 This question is an exercise in time domain analysis. Start as usual by identifying v^+ and v^- in terms of v_o and v_i. By inspection $v^- = v_o/2$ but to find v^+ ($= v_C$), i_C must be found by summing currents at the v^+ node:



$$i_C = i_i + i_f = \frac{v_i - v_C}{R} + \frac{v_o - v_C}{R}$$

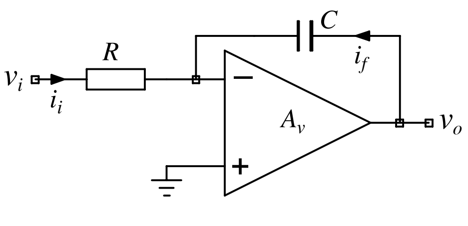
and since $A_v = \infty$, v^+ ($= v_C$) $= v^- = v_o/2$,

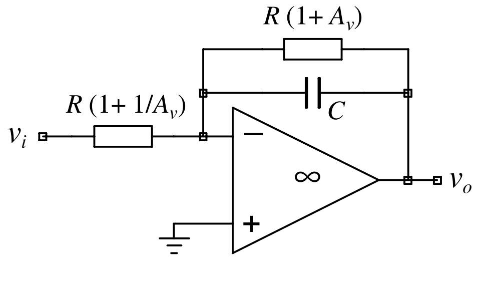
$$i_C = \frac{v_i - v_C}{R} + \frac{v_o - v_C}{R} = \frac{v_i}{R} - \frac{v_o}{2R} + \frac{v_o}{R} - \frac{v_o}{2R} = \frac{v_i}{R}$$

The voltage across C is given by the integral of the current through it:

$$v_c = \frac{v_o}{2} = \frac{1}{C}\int i_C\,\mathrm{d}t = \frac{1}{C}\int \frac{v_i}{R}\,\mathrm{d}t \text{ or } v_o = \frac{2}{CR}\int v_i\,\mathrm{d}t.$$

7 The easiest approach here is to perform a frequency domain analysis on each circuit to find their transfer functions. The equivalence of the circuits is demonstrated if they have the same transfer function. The same approach could be used with a time domain analysis but time domain analysis is often a slightly more awkward proposition than its frequency domain counterpart. Beginning with the finite gain model, summing currents at the inverting input node gives:





$$i_i + i_f = \frac{v_i - v^-}{R} + (v_o - v^-)\,sC = 0 \text{ or } v^- = \frac{v_o\,sCR + v_i}{1 + s\,CR} \qquad \textbf{(7.1)}$$

Since $v^+ = 0$, the op-amp equation, $v_o = A_v(v^+ - v^-)$ reduces to $v_o = -A_v v^-$ and this can be used to eliminate v^- from **(7.1)** to give,

$$-\frac{v_o}{A_v} = \frac{v_o\,sCR + v_i}{1 + s\,CR} \text{ or } v_o\left(\frac{1}{A_v} + \frac{sCR}{1 + s\,CR}\right) = -\frac{v_i}{1 + s\,CR} \text{ so}$$

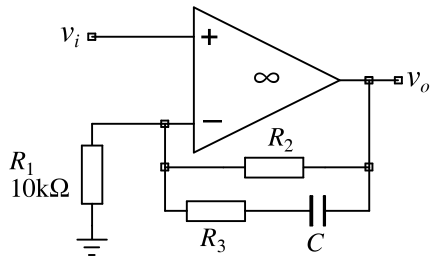
$$\frac{v_o}{v_i} = \frac{-A_v}{1 + s\,CR\,(1 + A_v)}$$

The infinite gain model can be dealt with by using the ideal expression for the gain of an inverting amplifier circuit connection:

$$\frac{v_o}{v_i} = -\frac{\dfrac{\dfrac{R\,(1+A_v)}{sC}}{R\,(1+A_v) + \dfrac{1}{sC}}}{R\left(1 + \dfrac{1}{A_v}\right)} = -\frac{\dfrac{R\,(1+A_v)}{s\,CR\,(1+A_v) + 1}}{R\left(1 + \dfrac{1}{A_v}\right)} = \frac{-R\,(1+A_v)}{(s\,CR\,(1+A_v) + 1)\,R\left(\dfrac{A_v + 1}{A_v}\right)}$$

$$= \frac{-A_v}{1 + s\,CR\,(1+A_v)} \text{ as before.}$$

8 The first thing to do in a question like this is work out the transfer function; this allows the pole and zero frequencies to be identified in terms of circuit components. Since the op-amp has infinite gain, the simple non-inverting gain expression can be used,



$$\frac{v_o}{v_i} = \frac{R_1 + \dfrac{R_2\left(R_3 + \dfrac{1}{sC}\right)}{R_2 + R_3 + \dfrac{1}{sC}}}{R_1}$$

$$= \frac{R_1R_2 + R_1R_3 + \dfrac{R_1}{sC} + R_2R_3 + \dfrac{R_2}{sC}}{R_1R_2 + R_1R_3 + \dfrac{R_1}{sC}}$$

$$= \frac{R_1 + R_2 + sC\,(R_1R_2 + R_1R_3 + R_2R_3)}{R_1(sC\,(R_2 + R_3) + 1)} = \frac{R_1 + R_2}{R_1}\,\frac{1 + sC\left(\dfrac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_2}\right)}{1 + sC\,(R_2 + R_3)}$$

$$\equiv k_L \frac{1 + j\dfrac{\omega}{\omega_1}}{1 + j\dfrac{\omega}{\omega_0}}$$ where ω_0 = the pole frequency, ω_1 = the zero frequency and k is the low frequency gain. The high frequency gain, k_H, can be worked out directly from the circuit by letting C become a short circuit or it can be deduced from the transfer function by finding the likiting value of its modulus as frequency becomes very large. Three values are defined and this leads to three equations that must be solved simultaneously:-

$$\omega_0 = \text{pole frequency} = \frac{1}{C\,(R_2 + R_3)} = 2\,\pi\,10 \qquad \textbf{(8.1)}$$

$$\omega_1 = \text{zero frequency} = \frac{R_1 + R_2}{C\,(R_1R_2 + R_1R_3 + R_2R_3)} = 2\,\pi\,500 \qquad \textbf{(8.2)}$$

$$k_H = \text{ high frequency gain } = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1\,(R_2 + R_3)} = 10 \qquad \textbf{(8.3)}$$

Combining **(8.2)** and **(8.3)** to eliminate $(R_1R_2 + R_1R_3 + R_2R_3)$ gives, $\dfrac{R_1 + R_2}{10\,C\,R_1\,(R_2 + R_3)} = 2\,\pi\,500$ and eliminating $(R_2 + R_3)$ from this using **(8.1)** leaves $\dfrac{(R_1 + R_2)\,2\,\pi\,10\,C}{10\,C\,R_1} = 2\,\pi\,500$ or $\dfrac{R_1 + R_2}{R_1} = 500$ so $\dfrac{R_2}{R_1} = 499$. R_1 is given as 10 kΩ so $\boldsymbol{R_2} = \textbf{4.99 MΩ}$.

Two of the three resistor values are now known so **(8.3)** can be used to find R_3 directly since it is the only unknown in that equation. Thus,

$$\frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1\,(R_2 + R_3)} = 10 = \frac{10^4 \times 4.99\times10^6 + 10^4 R_3 + 4.99\times10^6 R_3}{10^4\,(4.99\times10^6 + R_3)} \quad \text{or } \boldsymbol{R_3 = 91.6\ \mathrm{k\Omega}}$$

The value of C can be calculated using either **(8.1)** or **(8.2)**; **(8.1)** is numerically simpler. By either route, $\boldsymbol{C} = \textbf{3.13nF}$

The frequency response of the circuit can be drawn using the knowledge gained from problem sheet 1. Although adding a low pass circuit to the output makes the circuit second order, the additional low pass function is in series with the existing pole-zero function and thus on a log-log amplitude plot the amplitude and phase responses of the two circuits add. The overall response will be of the form:

$$\frac{v_o}{v_i} = k_L \frac{1 + j\dfrac{\omega}{\omega_1}}{\left(1 + j\dfrac{\omega}{\omega_0}\right)\left(1 + j\dfrac{\omega}{\omega_2}\right)}$$

where ω_2 is the corner frequency of the extra low-pass circuit and k_L, ω_0 and ω_1 are as before. The phase and amplitude response diagrams on the next page show the pole-zero response, the extra low-pass response and the overall response. A Bode approximation has been included on the amplitude response.

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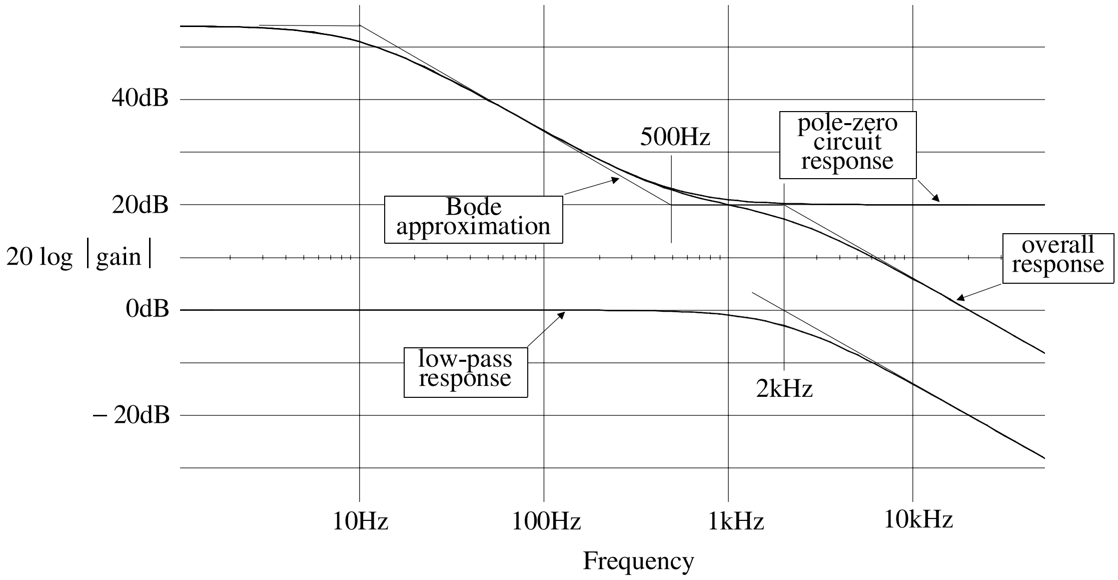
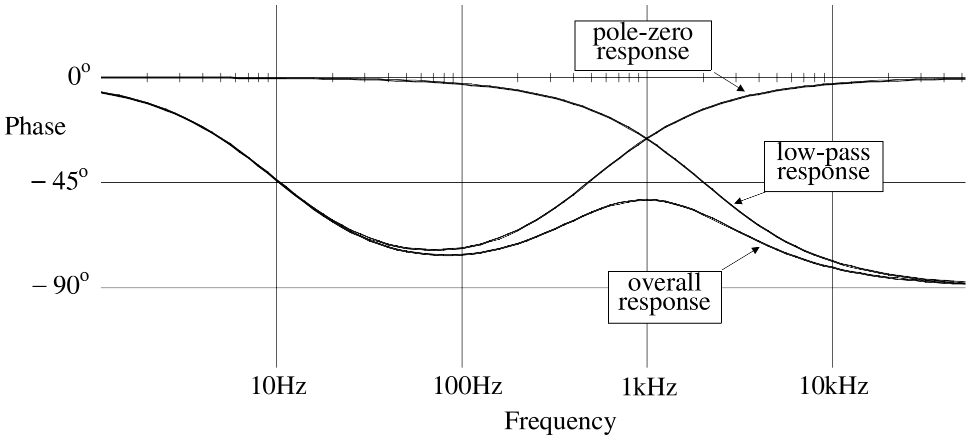

40dB
20dB
20 log |gain|
0dB
− 20dB
500Hz
pole-zero circuit response
Bode approximation
overall response
low-pass response
2kHz
10Hz
100Hz
1kHz
10kHz
Frequency


0°
Phase
− 45°
− 90°
pole-zero response
low-pass response
overall response
10Hz
100Hz
1kHz
10kHz
Frequency