Sample solution for EEE337/348 2013-14

Q1

a)

i) Light intensity in a semiconductor is described by

$$I = I_o \exp(-\alpha x)$$

If 90% of light is absorbed within a length L, we have

$$0.1I_o = I_o \exp(-\alpha L)$$
$$\exp(-\alpha L) = 0.1$$
$$L = -\frac{1}{\alpha}\ln(0.1)$$

For GaAs, $L = -\frac{1}{5\times10^4}\ln(0.1) = 4.6\times10^{-5}cm = 0.46\mu m$

For Si, $L = -\frac{1}{2\times10^3}\ln(0.1) = 1.15\times10^{-3}cm = 11.5\mu m$

GaAs and Si layers with these thicknesses can be grown with high crystal quality and are widely available.

ii) Any two of the followings

- High purity bulk Si, with extremely low impurity concentration and long minority electron diffusion length (due to low defect density) is readily available. The former allows a large depletion region to be obtained while the latter increases the carrier collection efficiency, leading to very efficient Si solar cell.
- Large, cheap and high quality Si substrates significantly reduce the cost of Si solar cells. Si substrate size of up to 300 mm diameter (11.8 inch) is routinely used while 450 mm diameter substrate is being developed. These are much larger than the largest 150 mm (6 inch) GaAs substrate.
- The abundance of Si coupled with the state-of-the art fabrication technology and very large scale manufacturing facilities are important for manufacturing of solar cell for mass market applications. For this reason GaAs based solar cells find more application where efficiency is more important than cost.
- In Si, the dangling bonds at the surface can be successfully passivated with hydrogen. This reduces the surface recombination.
- SiO_2 which is a good electrical insulator is a native oxide that can be easily grown. When deposited with appropriate thickness it also reduces the reflection of the incident light and improves the conversion efficiency of the solar cell.

b) Using a simple case of a solar well with a depletion with of *W* and assuming that the carrier generation rate, *G*, is constant, the photocurrent in a solar cell is given by

$I_{ph} = qAG(L_e + W + L_h)$, where *L* is the minority carrier diffusion length and *A* is the area.

The depletion width is defined by the thickness of the undoped i-region. To ensure that the i-region is fully depleted, its unintentional doping needs to be very low. In the case of Si, unintentional background doping below 10^{14} cm^{-3} will ensure that a large depletion width can be achieved.

The minority carrier diffusion lengths L_e and L_h can be maximised by reducing the doping concentrations in the p and n regions. However this is not ideal since high doping concentration in the p and n-regions are required to minimise the series resistance. Therefore in practice, a relatively thin highly doped p^+ region is used to minimise the series resistance.

c) Inverted pyramid : Sunlight incident on a side slope is partially transmitted into the cell and partially reflected to the other slope, increasing the probability of light being absorbed.

Oxide layer: Designed to minimise reflection of incident sunlight. It also passivates the surface to minimise surface recombination.

p-region: Should have long minority carrier diffusion length to ensure high output current. Minority carrier lifetime in ms is routinely achieved.

P+ regions: Minority carriers are collected and is highly doped to minimise series resistance. Metal-semiconductor area (defined by the small Boron diffused p+ region) is small to minimise surface recombination loss.

N+ regions: Highly doped to minimise series resistance

Si-Oxide-Aluminium: Act as mirror to reflect light (particularly at long wavelengths) back into the cell.

d) As the name suggests, three junctions are stacked and connected in series to provide a closer match of the solar cell spectral response to that solar emission spectral. The bandgap is reduced from the top to the bottom. For example the junctions, from top to bottom, are InGaP, InGaAs and Ge with bandgaps of 1.8, 1.4 and 0.65 eV respectively. Very highly doped pn junctions are inserted between each junction. These pn junctions are designed such that carriers can tunnel through the different bandgap junctions. The triple junction can be designed to achieve high absorption efficiency from UV to 1.9 µm and achieves much higher efficiency than Si.

Q2 a) The empirical expression for avalanche multiplication factor is given by

$$M = \frac{1}{1-\left(\frac{V-IR}{V_b}\right)^{n_m}}.$$

Using the parameter values provided the multiplication factor in the dark is

$$M = \frac{1}{1-\left(\frac{29-10\times10^{-9}\times10}{30}\right)^{2}} = 15.25.$$

Note that since the dark current is low, the voltage drop across *R* is negligible compare to the bias voltage of 29 V.

The photocurrent produced is

$$I_{ph} = \frac{\eta \lambda P_{opt}}{1.24} = \frac{0.9 \times 0.85 \times 5 \times 10^{-9}}{1.24} = 3.1 \times 10^{-9} A$$

Total current flowing = 10 + 3.1 = 13.1 nA. *IR* = 0.13 µV is much smaller than 29 V. The multiplication factor is not affected. The total photocurrent is therefore 3.1 x 30 = 93 nA.

b) The noise signal at the output of the amplifier is given by

$v_{noise} = 10 \times 10^{-12} \times \sqrt{0.1 \times 10^{9}} \times G_{amp}$ where G_{amp} is the amplifier gain factor.

The photocurrent signal is $v_{photo} = 3.1 \times 10^{-9} \times M \times G_{amp}$. Therefore to overcome the amplifier noise

$$M > \frac{10 \times 10^{-12} \times \sqrt{0.1 \times 10^{9}} \times G_{amp}}{3.1 \times 10^{-9} \times G_{amp}} = 32.5 .$$

The gain of 32.5 is moderate and is below the breakdown voltage. Therefore the gain stability can easily be achieved by using a thermistor and feedback circuit to maintain the bias required to achieve the required gain.

c) Current commercial APDs for high speed optical communication employs an InGaAs absorption region and an InP multiplication region. The electric field profile is maintained by a field control layer with very small doping concentration tolerance. To achieve 100 Gb/s and above the total thickness of the APD has to be very thin. For instance to achieve a transit time limited bandwidth of 50 GHz, the thickness has to be

$$W = \frac{0.4 v_s}{f_{3dB\_tr}} \sim \frac{0.4 \times 10^{5}}{50 \times 10^{9}} = 0.8 \mu m .$$

Due to multiple transit times required to achieve the required multiplication factor, the APD has to be thinner than 0.8 µm and the field control layer and the avalanche layer will be much thinner. Growth of such APD is not mature, hence commercial APD operating at 100 Gb/s is not available.

d) To achieve 100 Gb/s operation, the diode should have a small RC time constant and a small transit time. Assuming that a minimum bandwidth of 50 GHs is required the depletion region thickness of 0.8 µm is required. Since the quantum efficiency also depends on the depletion width, a vertically illuminated photodiode is not suitable. A laterally illuminated waveguide photodiode is therefore the preferred technology. In practice InGaAs is usually used as the absorption active region. The waveguide photodiode has wide bandgap cladding layers, InP, to achieve high optical confinement. The area is typically small with width of less than 10 µm to maintain low capacitance while the contact layer is highly doped to minimise series resistance.

Q3

a)

i) In a homojunction laser the optical confinement is provided by the change of refractive index in the highly doped p and n regions. This change is relatively small, up to 1 %, and therefore light is poorly confined within the active region, leading to high opical loss. The potential barriers are also small such that carriers leak out from the active region and recombine non-radiatively in the highly doped p and n neutral regions. These factors lead to a large threshold current in the homojunction laser.

ii) In a double heterojunction laser the wide bandgap p and n regions can provide higher refractive index change of typically 5% (assuming GaAs and $Al_{0.3}Ga_{0.7}As$ combination). When combined with appropriate thickness of the active region, close to unity confinement factor can be achieved. Since light is fully confined within the active region, the photon intensity increases much more easily. The large conduction and valence band offsets also ensure that carriers are confined in the active region to increase the electrons and holes densities available for radiative recombination. Because of these the double heterojunction laser can achieve much lower threshold current than the homojunction laser.

b)

i) In a quantum well laser energy states are closely packed into sub-bands such that they can be approximated by a series of fixed energy states. In bulk, the density of states is zero at the band edge and it increases gradually with energy. On the other hand the density of states is constant at a fixed energy level. Because of the constant density of states, a group of electrons of nearly the same energy exist. There is also a corresponding group of holes in the valence band. These higher electron and hole concentration allows the population inversion to be much more easily achieved, leading to much lower threshold current. The threshold current is also much lower since the active region, d_{las}, is much thinner as described by

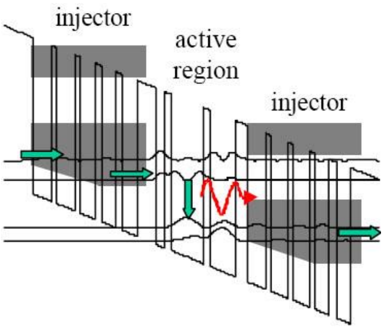
$$J_{th} = \frac{q d_{las} n_{th}}{\tau_r (J_{th})},$$

where nth is the carrier density when the cavity gain equals to the cavity loss and $\tau_r(J_{th})$ is the radiative recombination time.

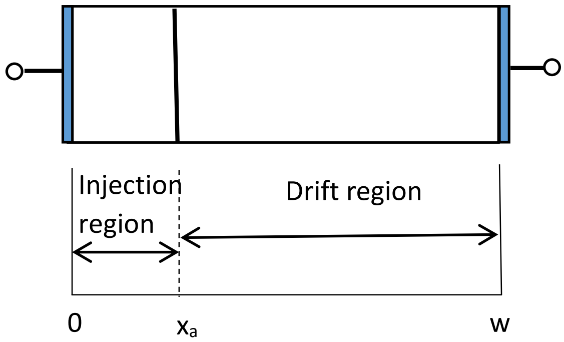
ii) The energy states can also easily be modified by changing the width of the well. Although less desirable using high current injection conditions, excited states will also be occupied and can contribute to radiative recombination. Therefore more than one wavelength can be emitted. When combined with cavity of different lengths or with Bragg reflectors, a wide lasing wavelengths can be easily obtained.

c) Auger recombination increases exponentially as the bandgap decreases. Hence it is a very dominant non-radiative recombination mechanism in narrow bandgap materials such as InSb. In addition there is no wide bandgap material that is lattice matched to InSb. Therefore it is not possible to fabricate an InSb based laser.

d) The quantum cascade laser (QCL) is an important laser for the 3-5 μm. The band diagram is shown below. In the injector region, the quantum wells are designed such that under appropriate biasing condition, the energy states are aligned to allow electron to tunnel through the barriers. This enables the electron to move from a low energy state to a higher energy state. The electron is subsequently injected into the active region containing quantum well with energy state separation that will produce the desired wavelength when the electron make the intra-band transition from the high to low energy states. After the photon emission the electron from the low energy state is transported to the injector region and the process is repeated. Typically the injector and active regions are repeated for 20-100 times to produce a QCL.

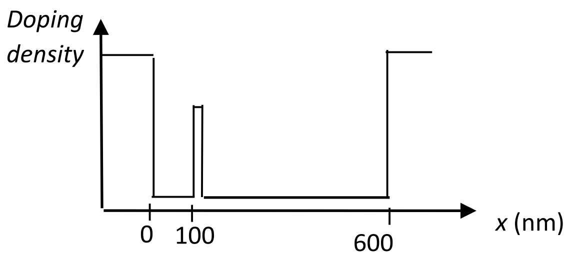


Q4.



A schematic of an idealised IMPATT diode is shown above. Charge carriers are generated by impact ionisation in the injection region. The IMPATT diode is biased such that the field in the injection region is very close to the breakdown field. These charges are then injected into the drift region which has low electric field to ensure that the charges do not experience impact ionisation while they drift. The time delay created by the impact ionisation and drift process causes a phase lag between the output current and the input voltage, leading to a negative resistance.

b)



i) The oscillation frequency is given by

$$f_{IMP} = \frac{v_{sat}}{2(w - x_a)} = \frac{1 \times 10^5}{2 \times 500 \times 10^{-9}} = 100\,GHz\,.$$

ii) The breakdown voltage is given by

$$V_B = E_m x_a + \left(E_m - \frac{qQ_c}{\varepsilon_s} \right)(w - x_a)$$

$$= (600 \times 10^3 V/cm) \times (100 \times 10^{-7} cm) + \left(600 \times 10^3 V/cm - \frac{q \times 3 \times 10^{12}}{12.4 \times 8.85 \times 10^{-14}} \right)(500 \times 10^{-7})$$

$$V_B = 6.0 + 8.13 = 14.13V$$

iii) The voltage drop across the injection region is 6V. Hence the voltage across the drift region is 8.13 V. The electric field is therefore 8.13/500x10^{-7} = 163 kV/cm.

c) To operate at 1 THz, the drift region has to be very thin.

$$w - x_a = \frac{v_{sat}}{2(1 \times 10^{12})} = \frac{1 \times 10^5}{2 \times 10^{12}} = 50nm.$$

To ensure that the impact ionisation occurs over a short duration than the drift time the injection region and the field control region will have to much shorter than 50 nm. Accurate control of doping across a few nm of field control layer is challenging. In addition if the injection region is below 50 nm, very high band to band tunnelling current will dominate making the IMPATT diode very noisy.

d) A tunnel diode consists of a p-n junction in which both p and n regions are highly doped such that the Fermi levels are in the valence band in the p region and in the conduction band in the n region. Because of the very high doping the depletion region is very thin and hence the tunnelling distance is only 5-10 nm thick.

Under forward bias there is a band of energy states that are fully occupied in the n side and a corresponding empty states in the p side. The electrons can tunnel from n to p side. A peak current is produced when the tunnelling is maximum. At higher forward bias, the occupied and empty states are not fully “aligned” and hence the tunnelling current drops. At even higher forward bias the occupied and empty states are no longer aligned. Electrons will have to overcome the potential barrier via thermionic effect and normal thermal current flows.