EEE6220 2015-16 exam

Q1

a

i

To solve this question we first need to know which harmonic of the source waveform will be problematic. As the victim is susceptible to frequencies in the range 130-170MHz the third harmonic of the 50MHz square wave (150MHz) will fall into this range. **2 mark**

Using Fourier the equation for a trapezoidal waveform can be simplified to

I(n)=2I/(n*pi)

If n=3 then I =0.212A **2 marks**

ii

The electric field radiated from a loop is given by:

$E = 131.6*10^{-16}\left(\frac{f^2 AI}{R}\right)\sin\theta$ **2 mark**

f=150MHz, A=0.1*0.1=0.01m^2, I=0.212A, R=3 and sin(theta) =1

Hence E=0.209V/m **2 marks**

iii

The equation needed here is:-

$$\frac{V_2}{E} = \frac{-2jZ_2 \sin\left(\frac{\beta D}{2}\right)\left[Z_0 \sin(\beta l) + jZ_1(1-\cos(\beta l))\right]}{\beta\left[(Z_0 Z_1 + Z_1 Z_2)\cos(\beta l) - j(Z_0^2 + Z_1 Z_2)\sin(\beta l)\right]}$$ **2 marks**

Hence V2/E= 9.87*10^{-3}m **3 marks**

And the voltage induced V2=9.87*10^{-3}*0.209=2.1mV **2 mark (any error calculating V2/E will be carried forward)**

B

First step is to choose the correct Reflection loss equation.

The far field can region can be approximated by $\frac{\lambda}{2\pi} = \frac{2}{2\pi} = 0.3m$, hence the shield is in the far field

So the reflection loss equation is

$$R = 168 - 10.\log_{10}\left(\left(\frac{\mu_r}{\sigma_r}\right).f\right)$$

1 mark

The reflection loss from the aluminium shield is

$$R = 168 - 10.\log_{10}\left(\left(\frac{1}{0.6}\right).150*10^6\right) = 84dB$$ **2 marks**

Hence we need at least 26dB from absorption to meet the SE requirement

$$A = 8.69\frac{d}{\delta}$$

$$\delta = \frac{1}{\sqrt{\pi\mu\sigma f}} = 7\mu m$$ **2 marks**

$$d > \frac{A*7\mu}{8.68} = 20.9\mu m$$

Q2

Assuming weak coupling system 2 does not draw significant current from system 1, hence we can consider the potential division of system 2 in isolation to system 1.

Firstly if the voltage just to the left of Cm is given as V and the impedance of system 2 is Z2 then

$$V_N = V\frac{Z_2}{Z_2 + \frac{1}{j\omega C_m}}$$ **1 marks**

$$V_N = V\frac{j\omega C_m}{\frac{1}{Z_2} + j\omega C_m}$$

The impedance of system 2 is

$$\frac{1}{Z_2} = j\omega C + \frac{1}{R_{S2}} + \frac{1}{R_{L2}}$$ **1 marks**

Hence

$$V_N = V\frac{Z_2}{Z_2 + \frac{1}{j\omega C_m}}$$

$$V_N = V\frac{j\omega C_m}{j\omega C + \frac{1}{R_{S2}} + \frac{1}{R_{L2}} + j\omega C_m} \qquad 1\,\text{mark}$$

$$V_N = V\frac{j\omega C_m}{j\omega\left[C + C_m\right] + \frac{1}{R_{S2}} + \frac{1}{R_{L2}}} \qquad 1\,\text{mark}$$

$$V_N = V\frac{j\omega C_m}{j\omega\left[C + C_m\right] + \frac{R_{S2} + R_{L2}}{R_{S2}R_{L2}}} \qquad 1\,\text{mark}$$

$$V_N = V\frac{j\omega C_m / \left[C + C_m\right]}{j\omega + \frac{R_{S2} + R_{L2}}{R_{S2}R_{L2}\left[C + C_m\right]}} \qquad 1\,\text{mark}$$

And $V = V_1\frac{R_{L1}}{R_{S1} + R_{L1}}$ (assuming low resistance compared to the capacitive impedance) hence

$$V_N = V_1\frac{R_{L1}}{R_{S1} + R_{L1}}\frac{j\omega C_m / \left[C + C_m\right]}{j\omega + \frac{R_{S2} + R_{L2}}{R_{S2}R_{L2}\left[C + C_m\right]}}$$ **2 marks**

b

i

The equation of mutual capacitance is

$$C_m = \frac{0.0885\pi * L}{\cosh^{-1}(D/d)}$$ where L=10cm, D=4mm and d=1mm **0.5 mark**

$$C_m = \frac{0.0885\pi * 0.1}{\cosh^{-1}(4/1)} = 1.35pF$$

The equation of self capacitance is

$$C = \frac{2 * 0.0885\pi * L}{\cosh^{-1}(2h/d)}$$

$$C = \frac{2 * 0.0885\pi * 10}{\cosh^{-1}(6/1)} = 2.24pF$$

where h=3mm **0.5 mark**

ii

As in question 1 the Fourier series of a square wave gives

V(n)=2*V/(n*pi)

In this case if the data rate is 100Mbit/s and the probability of a 0V or 5V is equal then the overall frequency of the waveform is 50MHz hence 250MHz is the 5th harmonic

V(5)=2*5/(5*pi)=0.637V **1 mark**

iii

The equation given in part a) assumes that there is weak coupling and that the resistances of the systems are much lower than the capacitive impedance. To use this we need to check the assumptions.

$X_{cm} = -j471.6\Omega$
$X_c = -j284.2\Omega$ Both these values are comparable to the resistances in system 1 and as such the assumptions do not apply, hence, we have to solve the circuit fully. **1 mark**

The impedance of system 2 is

$$Z_2 = \left(\frac{1}{X_c} + \frac{1}{100} + \frac{1}{10M}\right) = 89 - j31.3\Omega$$ **0.5 mark**

The impedance of the parallel combination of C with RL1 is

$$Z_{C//RL1} = \left(\frac{1}{R_{L1}} + \frac{1}{X_c}\right)^{-1} = 89 - j31.3\Omega$$ **0.5 mark**

The impedance at the input to system 2 is

$$Z_{in} = \left(\frac{1}{Z_{c//RL1}} + \frac{1}{Z2 + X_{cm}}\right)^{-1} = 75.7 - j39.9\Omega$$ hence the voltage at the input to Cm is **1 mark**

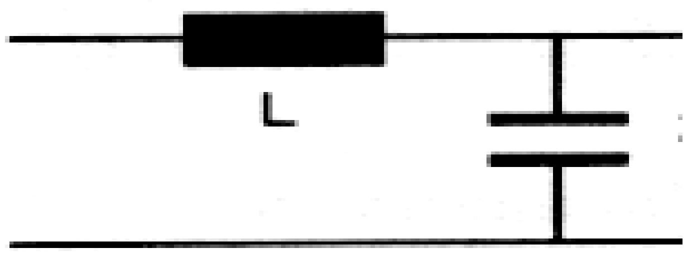
$$V = V_1\left(\frac{Z_{in}}{Z_{in} + R_{S1}}\right) = 0.637(0.46 - j0.127)$$ and the noise voltage is **1 mark**

$$V_N = V\frac{Z_2}{Z_2 + X_{cm}} = 0.637(0.46 - j0.127)(0.091 + j0.161) = 56.5mV$$ **1 mark**

As all the impedances are in parallel the voltage is dropped across all of them equally.

c

i



1 mark

The transfer function for the equation is

$$\frac{V_{out}}{V_{in}} = \frac{1}{1-\omega^2 LC}$$ **1 mark**

40 dB attenuation is 0.01 in linear, hence **1 mark**

$$0.01 = \frac{1}{1-\omega^2 LC}$$ **2 marks**

$$L = 8.2\mu H$$

Solutions

Q3a

- It is impractical to plot all values of load, or input, impedance on a rectangular plane with the co-ordinate R_L and X_L since this requires an infinite sheet of paper. **(1 mark)**
- Since the magnitude of the reflection coefficient is less than one, so it will lie within a unit circle in the reflection coefficient plane since $\Gamma = \rho\angle\theta$. Each value of reflection coefficient specifies a value of normalised impedance. **(1 mark)**
- Contours of constant normalized input resistance and input reactance can be plotted on the reflection coefficient plane (Γ, θ) **(1 mark)**
- Moving a distance *d* over the line can be expressed on the Smith chart by a rotation through an angle of $2\beta d$. Hence, Smith chart enables easy transformation of the impedance **(1 mark)**

Q3b

$$\lambda = \frac{c}{f} = 30\text{cm}$$

Therefore

$$\beta\ell = \frac{2\pi}{\lambda} \times 5 = 1.05$$ **(1 mark)**

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)} = (49.6 - j29)\Omega$$ **(1 mark)**

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = 0.075 - j0.77$$ **(1 mark)**

$$IL = 10\log\left(1 - |\Gamma_{in}|^2\right) = -3.9\text{dB}$$ **(0.5 mark)**

$$RL = 10\log\left(|\Gamma_{in}|^2\right) = -2.2\text{dB}$$ **(0.5 mark)**

Q3c

For a quarter length transformer $Z_{in} = \frac{Z_o^2}{Z_L}$

i.e. $$Z_{in} = \frac{(75)^2}{100} = 56.25\Omega$$ **(1 mark)**

Since this is a lossless line then the power at the sending end equals the power at the receiving end, **(1 mark)**

i.e $$\frac{V_{in}^2}{Z_{in}} = \frac{V_L^2}{Z_L}$$ **(1 mark)**

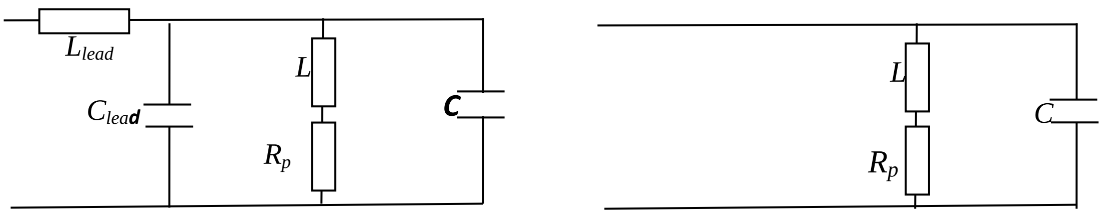
which gives

$$V_{in}^2 = \frac{Z_{in}}{Z_L} V_L^2$$ **(1 mark)**

i.e.

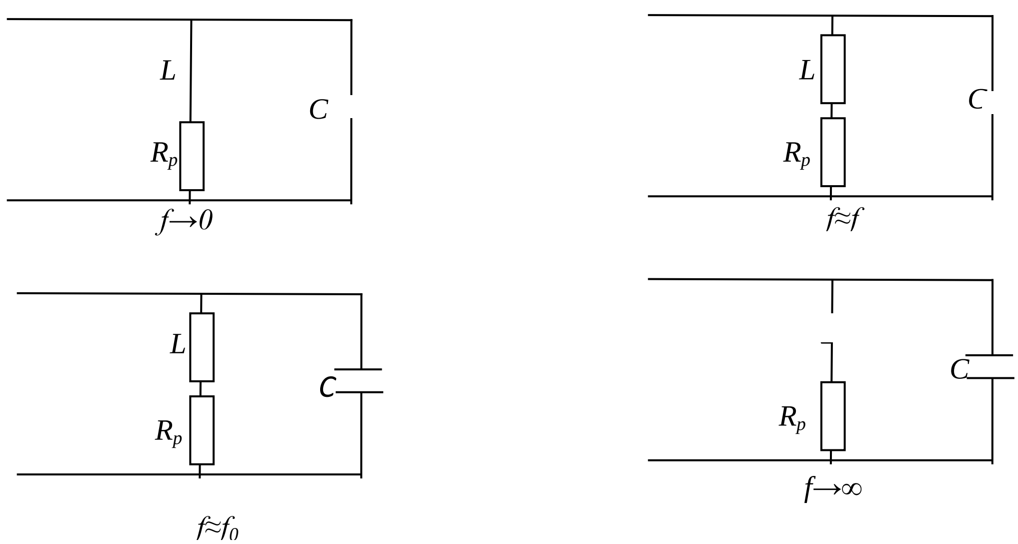
$$V_{in} = \frac{56.25}{100} \times (60)^2 = 45\text{V}$$ **(1 mark)**

Q3d
For a coil, the internal resistance can be represented as a parasitic element, R_p, in series with the inductor element. The proximity between the inductor turns introduces a parasitic capacitor, C_p, into the inductor equivalent circuit. The lead inductance and capacitance can be neglected since the first is much smaller than the element's inductance, and the latter is much smaller than C_p. **(1 mark)**
Therefore, the equivalent circuit can be simplified as shown in the following diagrams



$$Z_{inductor}(f) = \frac{X_C\left(X_L - jR_p\right)}{R_p + j\left(X_L - X_C\right)}$$ **(2 marks)**

At low frequencies, the parasitic resistor dominates, and hence the impedance can be expressed as $Z_{inductor}(f) \approx R_p$. When the frequency increase and exceeds $f_1 = \frac{R_p}{2\pi L}$, then the inductance element dominates. The low frequency approximation of the inductor is valid until the self resonance is reached at a frequency of $f_0 = \frac{1}{2\pi\sqrt{LC_p}}$. Above this frequency, the capacitive reactance is smaller than that of the element's inductor and parasitic resistor.
(2 marks)
Therefore, at sufficiently higher frequencies the inductor acts as a capacitor with a small reactance.



(2 marks)

Q4a
The S_{11} parameter can be determined if the output port is terminated with a matched load, i.e. $\Gamma_L=0$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$ **(1 mark)**

While the input reflection coefficient is given by

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$ **(1 mark)**

From these equations it can be seen that $S_{11} = \Gamma_{in}$ when the output port is terminated by a matched load. **(1 mark)**

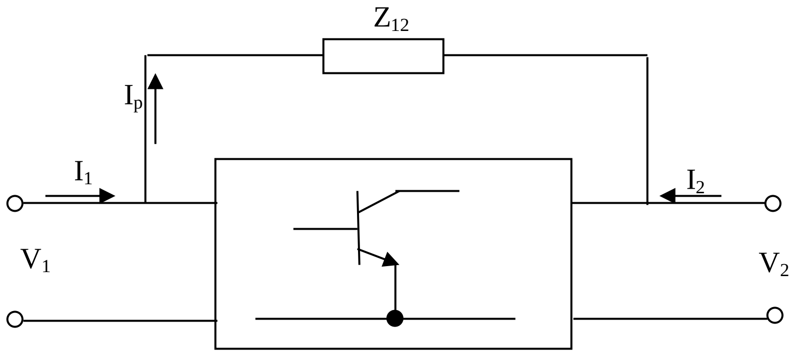
Q4b

Internal noise is generated due to the random motions of electrons as they pass through electronic components. Internal noise is generated within the resistors, diodes and transistors that make up a system, so in that sense it is the enemy within. There are three internal noise types; thermal noise, shot noise, and the flicker noise. **(2 marks)**

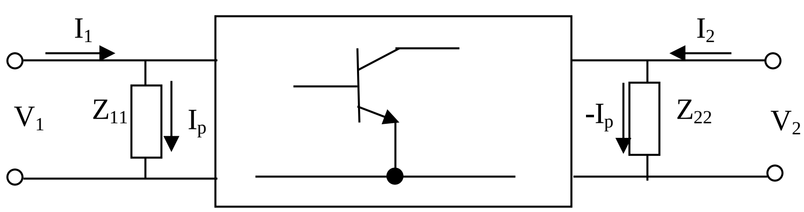
Thermal noise is also known as *Johnson,* or *Nyquist,* noise and it is caused by the thermal vibration of bound charges and it is independent of current flow. On the other hand, shot noise occurs when a current flows and causes a random flow of carriers across a *pn* junction. Finally, the flicker noise occurs in solid-state components and vacuum tubes with power that varies inversely with frequency, and so is often called as the (1/*f*) noise. **(2 marks)**

Q4c

Miller's theorem



Actual Circuit



Equivalent Circuit

$$Ip = (V_1 - V_2)/Z_{12}$$ **(2 marks)**

For the equivalent circuit

$$Z_{11} = \frac{V_1}{Ip} = Z_{12}\left(1 - \frac{V_2}{V_1}\right)^{-1}$$

and

$$Z_{22} = \frac{V_2}{(-Ip)} = Z_{12}\left(1 - \frac{V_1}{V_2}\right)^{-1}$$ **(1 mark)**

With the aid of the following substitutions

$$Z_{12} = \frac{1}{j\omega C_\mu} \qquad Z_{11} = \frac{1}{j\omega C_{M1}} \qquad Z_{22} = \frac{1}{j\omega C_{M2}}$$

and $V_1 = v_{be}$ $\qquad V_2 = v_{ce}$ **(1 mark)**

Then it can be shown that

$$C_{Mi} = C_\mu\left(1 - (v_{ce}/v_{be})\right) \qquad \text{and} \qquad C_{M2} = C_\mu\left(1 - (v_{be}/v_{ce})\right)$$ **(1 mark)**

<u>Q4d</u>

$S_{11} = 0.61\angle -70°$, $S_{21} = 2.24\angle 32°$, $S_{12} = 0$, $S_{22} = 0.72\angle -83°$

i.e.

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = \infty$$

and

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| = 0.137$$

So the transistor is unconditionally stable. **(1 mark)**

The matching sections gain can be calculated from

$$G_{S_{max}} = \frac{1}{1 - |S_{11}|^2} = 1.59 = 2.dB$$ **(0.5 mark)**

$$G_{L_{max}} = \frac{1}{1 - |S_{22}|^2} = 2.08 = 3.18dB$$ **(0.5 mark)**

The maximum gain of the unmatched transistor

$$G_o = |S_{21}|^2 = 7.0dB$$ **(0.5 mark)**

The overall gain

$$G_{T\,max} = 3.18 + 7.0 + 2 = 12.18dB$$

This means there is a 2.18dB gain higher than the design requirements. **(0.5 mark)**

The required gain circles can be plotted using the following set of equations

$$C_S = \frac{g_S S_{11}^*}{1 - (1 - g_S)|S_{11}|^2} \qquad r_S = \frac{\sqrt{1 - g_S}\,(1 - |S_{11}|^2)}{1 - (1 - g_S)|S_{11}|^2}$$

$$C_L = \frac{g_L S_{22}^*}{1 - (1 - g_L)|S_{22}|^2} \qquad r_L = \frac{\sqrt{1 - g_L}\,(1 - |S_{22}|^2)}{1 - (1 - g_L)|S_{22}|^2}$$

$$g_S = \frac{G_S}{G_{S\,max}} \qquad g_L = \frac{G_L}{G_{L\,max}}$$

which results in

$G_S = 0.5\text{dB}$	$g_S = 0.71$	C_s=0.485∠-70°	$r_S = 0.38$
$G_S = 1.5\text{dB}$	$g_S = 0.89$	C_s=0.59∠-70°	$r_S = 0.23$
$G_L = 1.5\text{dB}$	$g_L = 0.68$	C_L=0.59∠-83°	$r_L = 0.33$
$G_L = 2.5\text{dB}$	$g_L = 0.85$	C_L=0.66∠-83°	$r_L = 0.2$

The constant gain circles are plotted on the Smith chart as shown in the figure.

(4 marks, 1 mark for each circle)

For an overall gain of 10dB, we will choose $G_S = 1.5\text{dB}$ and $G_L = 1.5\text{dB}$. We select Γ_S and Γ_L along these circles to minimise the distance from the centre of the chart. This will put Γ_S and Γ_L along the radial lines at -70° and -83° respectively. Thus $\Gamma_S = 0.33\angle -70^\circ$ and $\Gamma_L = 0.27\angle -83^\circ$. **(1 mark)**