Data Provided: Physical constants ε_0 and μ_0 (on paper above question A1)

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2014-15 (3.0 hours)

EEE350 Electromagnetic Fields and Devices

Answer **FIVE QUESTIONS** comprising **AT LEAST TWO** each from **part A** and **part B. No marks will be awarded for solutions to a sixth question, or if you answer more than three questions from parts A or B.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

Physical constants:

$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{Fm^{-1}}$ $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$

Part A

A1. a. The electric potential in region of space with permittivity ε_0 is given by:

$V = (4z^3x^2+3y^2-6yx^2)$ V

Calculate the following at the point (x,y,z) = (4,2,3)m

i) The electric field strength **(3)**

ii) The charge density **(3)**

b. A circular conductor of radius 10mm has a static charge density of $3\times10^{-3}\mathrm{Cm^{-3}}$.

i) Starting from Gauss's Law, derive an expression for the electric field in the region outside the conductor. **(3)**

ii) Calculate the maximum value of electric field strength in the region surrounding the conductor, taking care to note its direction. **(3)**

c. Starting from Laplace's equation and listing any assumptions that you make, derive expressions for the spatial variation in the electric field strength and the electric potential between the parallel plates of a capacitor. **(8)**

A2. a. The electric field strength in a region of space of permittivity ε_0 is given by:

$$\vec{E} = 3x^2 z\,\overrightarrow{u_x} + 4xy\,\overrightarrow{u_y} + 4z^2 x\,\overrightarrow{u_z}\ V/m$$

where $\overrightarrow{u_x}, \overrightarrow{u_y}\ and\ \overrightarrow{u_z}$ are unit vectors in a Cartesian coordinate system.

Determine whether there is any charge present at the point (0,3,4) m. **(3)**

b. The electric field strength in a region of space is given by:

$$\vec{E} = 3x^2\,\overrightarrow{u_x} + 4y\,\overrightarrow{u_y}\ V/m$$

where $\overrightarrow{u_x}\ and\ \overrightarrow{u_y}$ are unit vectors in a two-dimensional cartesian coordinate system.

Calculate the electric potential difference between the points (0,0)m and (3,1)m. **(3)**

c. Figure A2 shows a cross-section through a dual circuit single-phase transmission line in which the two lines have an equal and opposite charge density q. Each conductor has a radius of R_c.

Starting from Gauss's Law, derive an expression for the voltage between the conductors in terms of the charge density and hence derive an expression for the capacitance per unit length of this conductor arrangement **(8)**

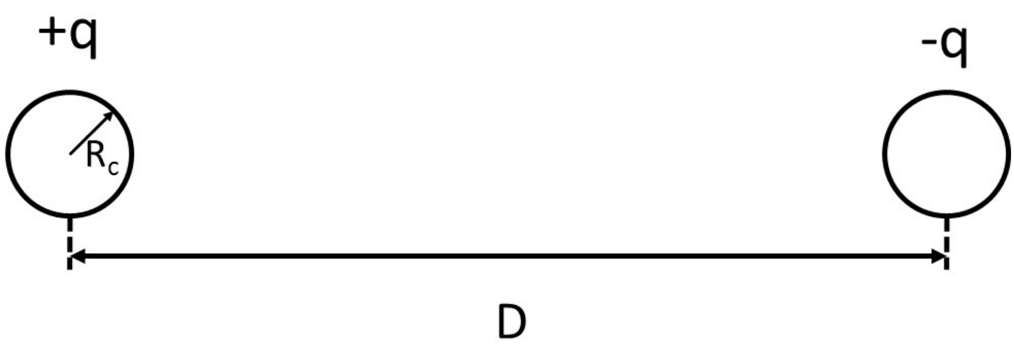

Figure A2 Dual circuit single-phase transmission line

d. A transmission system of the type shown in Figure A2 has base conductors (i.e. they have no surrounding insulation material) with a radius R_c = 20mm and are separated by a distance D of 600mm. If the break-down electric field strength of the surrounding air is 3×10^6V/m, calculate the maximum voltage difference between the conductors that can be sustained to remain below break-down conditions.

(Hint: Since D>>R_c you may assume that the localised electric field strength in the vicinity of each conductor is not influenced by the other conductor) **(6)**

A3. a. The magnetic field strength in a region of space of magnetic permeability μ_0 is given by:

$$\vec{H} = 3xyz\,\overrightarrow{u_x} + 4yz\,\overrightarrow{u_y} + 6y^2z^3\,\overrightarrow{u_z}\;\; A/m$$

where $\overrightarrow{u_x}, \overrightarrow{u_y}$ and $\overrightarrow{u_z}$ are unit vectors in a Cartesian coordinate system.

Calculate the current density at the point (3,4,2) m. **(4)**

b. The magnetic vector potential in a region of space of magnetic permeability μ_0 is given by:

$$\vec{A} = 6y^2z^2\,\overrightarrow{u_x} + 7xyz\,\overrightarrow{u_y} + xy^2z^2\,\overrightarrow{u_z}\;\; Wb/m$$

where $\overrightarrow{u_x}, \overrightarrow{u_y}$ and $\overrightarrow{u_z}$ are unit vectors in a Cartesian coordinate system.

Calculate the flux density at the point (1,2,1) m. **(4)**

c. Starting from Ampere's Law, derive an expression for the flux density in the region surrounding a circular current conductor. **(4)**

d. Figure A3 shows a region of infinite magnetic permeability, above which is a region of air with a magnetic permeability of μ_0. On the interface between the two regions, there is current sheet whose magnitude varies as a function of x as:

$$J_z = J_m sin(px)$$

The general solution for the z-component of magnetic vector potential A_z at any given combination of x and y coordinates is given by:

$$A_z(x,y) = \{C_1\sin(kx) + C_2\cos(kx)\}\{C_3e^{ky} + C_4e^{-ky}\}$$

where C_1, C_2, C_3, C_4 and k are constant

By appropriate application of boundary conditions and by noting the form of the current sheet excitation, derive expressions for the x and y component of flux density for any combination of x and y coordinates in the region above the surface. **(8)**

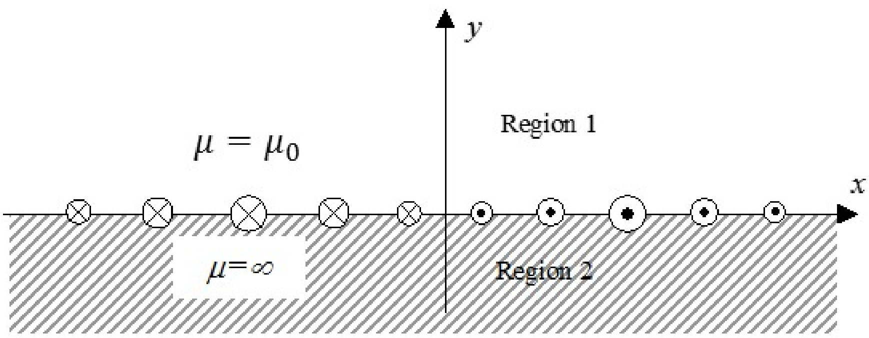

Figure A3 Region of infinite magnetic permeability with a static current sheet at the interface to region of magnetic permeability μ_0 above

A4. Figure A4 shows a simplified representation of a thick plate of electrical conductivity σ and magnetic permeability μ. The only component of magnetic field is a sinusoidal time varying magnetic field strength in the z-direction. This has a peak magnitude H_s on the surface of the plate (i.e. at y=0). The magnitude of this applied magnetic field strength does not vary as a function of x and z.

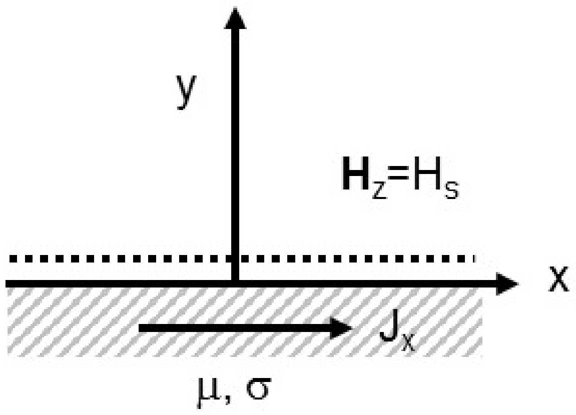

Figure A4. Simplified representation of a thick plate of electrical conductivity σ and magnetic permeability μ

a. Demonstrate mathematically that the only component of induced current in the plate is in the x-direction (J_x). **(4)**

b. Starting from the diffusion equation, derive an expression for the variation of the induced eddy current density J_x in the plate of Figure 4.1 as a function of y. Take care to explain any assumptions that you make and define any variables that you introduce. **(10)**

c. A manufacturer of steel workbenches wishes to harden the surface of the steel plate which forms the top of the bench by heating the plate to a depth of approximately 1mm. The steel has a relative magnetic permeability of 2000 and an electrical conductivity of 1.6×10^6 S/m. Assuming that the plate can be approximated as a thick plate and listing any assumptions that you make, estimate the frequency of the sinusoidally time varying magnetic field that needs to be applied to achieve this controlled heating. **(4)**

d. If the peak magnitude of the magnetic field strength applied at the surface of the bench of part (c) is 2000 A/m, calculate the loss per unit surface area of the plate. **(2)**

Part B

B1. a. From the expression of Lorentz force, show that the torque density of a cylindrical electrical machine can be expressed in terms of the magnetic and electrical loadings, $\boldsymbol{B}$ and $\boldsymbol{Q}$, respectively. List the major assumptions which you make in the derivation. **(5)**

b. Derive an expression for estimating the airgap flux density in an interrior permanent magnet machine with circumferentially magnetized magnets, such as shown in Figure B1. Explain why the air-gap flux density may be higher than the magnet remanence.

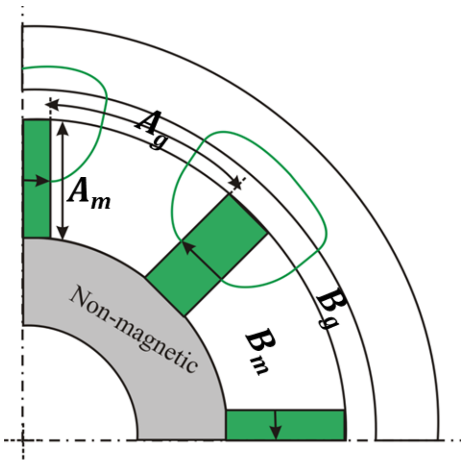

Figure B1 **(6)**

c. Neglecting all the leakage flux, show that when a magnet is required to produce a specific flux density B_g in an air-gap of length L_g and an area A_g, it is possible to minimize the magnet volume required by operating it at its maximum energy product working point $(BH)_{max}$. Explain why this design approach for the magnet is rarely adopted in practical electrical machines. **(5)**

d. List the main factors that affect the iron losses of electrical machines, and explain how the influence of iron losses on high speed machines can be minimised. **(4)**

B2. **a.** Derive general expressions for the winding pitch factor, K_p, and the distribution factor, K_d, for both the fundamental and the nth harmonic EMF components. Based on the coil EMF vectors, in order to obtain a maximum winding factor, determine the coil connections for a 3-phase, 12-slot, 8-pole surface mounted permanent magnet machine with single layer windings. **(8)**

b. Show that the winding skew factor can be derived using similar method as for the winding distribution factor. Explain the main advantages and disadvantages of using winding skew. **(4)**

c. A concentric winding which is distributed in the slots over one pole pitch in a machine having evenly spaced slots, as shown in Figure B2. Calculate the winding factor for the fundamental EMF.

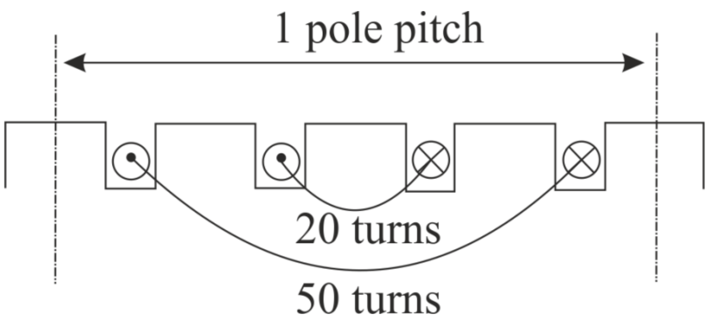

Figure B2

(5)

d. Describe the main advantages and disadvantages of the short-pitched concentrated winding, and the fully-pitched distributed winding. **(3)**

B3. A 2-pole permanent magnet DC motor, having its magnet arcs mounted adjacent to the airgap, has the following main dimensions:

Rotor active length $\boldsymbol{L}$ = 40 mm
Rotor outer diameter $\boldsymbol{D}$ = 60 mm,
Effective air-gap length $\boldsymbol{l_g}$ = 0.8 mm
Magnet radial thickness $\boldsymbol{l_m}$ = 5 mm
Magnet pole arc = 110^{o}

The magnet material has recoil permeability (μ_r) of 1.1 and its B-H characteristic is shown in Figure B3.

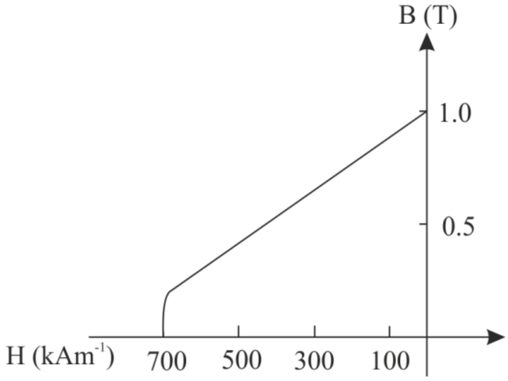

Figure B3 $\boldsymbol{H_{lim}}$ **= 650 kA/m**

a. Use analytical techniques to calculate the flux per pole for the motor. **(5)**

b. If the rotor has a total of 1532 conductors, connected in two parallel paths by the brushes, calculate the speed EMF constant for the motor and the corresponding no-load speed when operating on a 100V dc supply. **(5)**

c. The rotor winding has an effective resistance of 3 Ω, calculate the stall-torque using a dc supply of 100V supply and show whether or not the motor can tolerate the stall under these conditions without demagnetizing the magnets. **(5)**

d. Show graphically the difference between reversible and irreversible demagnetizations. Explain the influence of temperature on the irreversible demagnetization for both Ferrite and NdFeB permanent magnets. **(5)**

B4. For a slot shown in Figure B4, specify any assumptions that need to be made in order to derive analytically the expressions of:

a. The coil slot leakage inductance per-unit length for coil ‘a’ **(5)**

b. The coil slot leakage inductance per-unit length for coil ‘b’ **(3)**

c. The mutual inductance per-unit length between coils ‘a’ and ‘b’ **(7)**

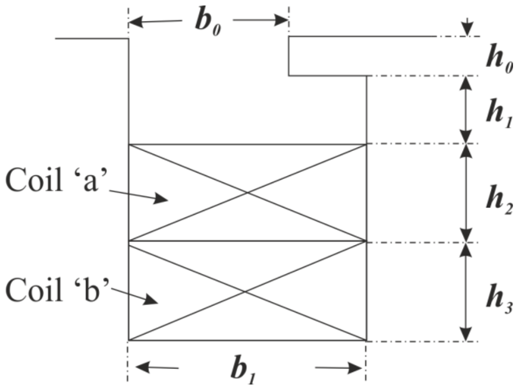

Figure B4 Numbers of turns in the slot for coil ‘a’ and ‘b’ are N_a and N_b, respectively.

d. Show graphically the difference between a single layer, concentrated winding and a double layer, concentrated winding. Describe the main advantages of the single layer, concentrated winding over the double layer, concentrated winding. **(5)**

(Full marks will not be given if the appropriate assumptions are not specified)

GJL/GWJ/JBW