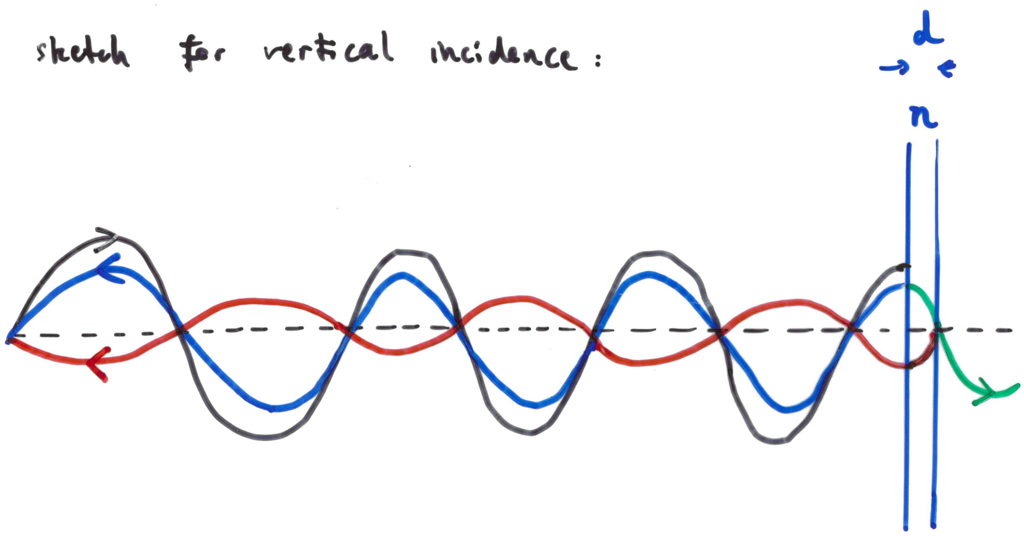
sketch for vertical incidence :



→ black = incoming
← blue = directly reflected at left interface
→ green = transmitted
← red = reflected at right interface

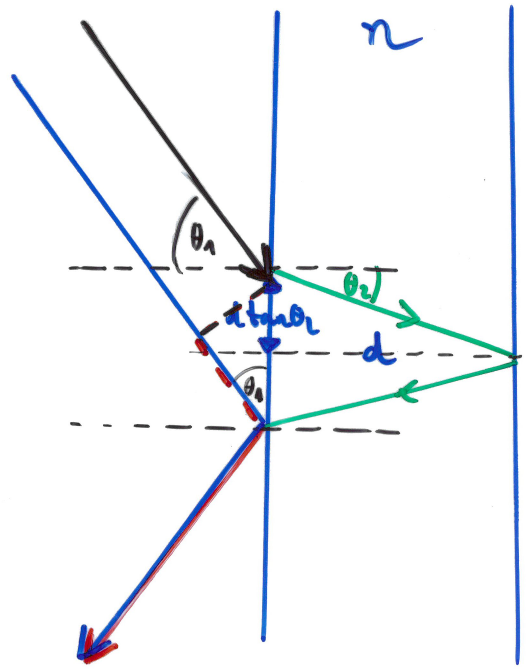
optical path difference between directly reflected and second reflected :

$$\Delta x_{\perp} = n \cdot 2d = n \cdot \frac{\lambda}{2n} = \frac{\lambda}{2}$$

$\Rightarrow$ phase difference $\varphi_{\perp} = k \cdot \Delta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi = 180°$

$\Rightarrow$ destructive interference

sketch for oblique incidence :



optical path difference between directly reflected and second reflected :

$$\Delta x_{\theta_1} = \frac{2nd}{\cos\theta_2} - \frac{2d\tan\theta_2}{\cos\theta_1}$$

can be solved with help of Snell's Law but is always $< \Delta x_{\perp}$

(3.) nano structured coatings

a) textured surfaces with pyramids / indentations / grooves act as as gradient index films if the periodicity of the surface features is <u>below</u> the wavelength: plane waves impinging on the surface see a graded transition from low to high refractive index materials

b) textured surfaces are above or at the size of the wavelength (e.g. blazed diffraction gratings) and their behaviour can be explained by geometrical optics or ray-tracing principles

(4.) combinations of graded index and interference coatings

a) single quarter-wavelength coating with refractive index smaller than that of the underlying glass

b) multiple $\lambda/4$ coatings with refractive indices increasing gradually from the surface to the glass substrate

Note: Lowest reflection is achieved if the coating has a refractive index that corresponds to the geometric mean $n_1 = \sqrt{n_0 n_2}$ between the two materials with refractive indices n_0 and n_2.

application:

crown glass: $n_2 \approx 1.52$

air: $n_0 = 1$

$\Rightarrow$ optimum anti-reflection coating would have $n_1 = \sqrt{1 \cdot 1.52} = 1.23$, however there are no natural materials that achieve this and technologists have to rely on MgF_2 ($n_1 \approx 1.38$) or synthesise complicated fluoro-polymers or mesoporous silica nano-particles, which can achieve down to $n_1 \approx 1.12$.

Note: Minimising reflections means maximising the transmittance, which for 2 films with refractive indices n_1 and n_2 between a medium with n_0 is given by

$$T_1 T_2 = (1-R_1)(1-R_2)$$

$$= \left[1-\left(\frac{n_1-n_0}{n_1+n_0}\right)^2\right]\left[1-\left(\frac{n_2-n_1}{n_1+n_2}\right)^2\right]$$

It can be shown by differentiation that this reaches a maximum for $n_{1\,opt} = \sqrt{n_0 n_2}$

The use of an anti-reflection coating with intermediate refractive index is identical to the technique of impedance matching of electrical signals on transmission lines!

example from a different frequency range:
RADAR absorbing materials

for free space propagation: $Z = \frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$

for interface to a material:

$$Z = \frac{E_0}{H_0} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} \quad \text{with} \quad \begin{array}{l} \mu_r = \mu_r' + j\mu_r'' \\ \varepsilon_r = \varepsilon_r' + j\varepsilon_r'' \end{array}$$

complex

$\rightarrow$ no reflectance if $\underline{\mu_r' + j\mu_r'' = \varepsilon_r' + j\varepsilon_r''}$,

ie. materials are to be designed with $\mu_r' = \varepsilon_r'$ and $\mu_r'' = \varepsilon_r''$,
ie. both magnetic and dielectric!

remember

$$\underline{\varepsilon_r'} + j\underline{\varepsilon_r''} = \underline{\varepsilon}_r = \underline{n}^2 = (n' + j\varkappa)^2 = \underline{(n'^2 - \varkappa^2)} + \underline{2jn'\varkappa}$$