The armature resistance of a permanent-magnet DC motor is 0.5 Ω. It operates from a 12V supply and runs unloaded at 1000 r/min. What will be the starting current? If the speed is 500 r/min, what will the armature current be? What will the no-load speed be if the supply voltage increases to 18 V?

On starting, the motor's back e.m.f. is zero, so $V = I_aR_a$ and the starting current is $I_a = V/R_a = 12/0.5 = 24$ A. At 1000 r/min the back e.m.f. of the unloaded motor will be equal to the supply voltage, 12 V. Thus at 500 r/min the back e.m.f. will be half this (by equation 13.16), or 6 V and the voltage drop across the armature winding is 12 - 6 = 6 V, and $I_a = 6/0.5 = 12$ A. Increasing the supply voltage to 18 V means the no-load back e.m.f. must also be 18 V, so the speed will be 1000 × 18/12 = 1500 r/min. Running at speeds much less than the no-load speed produces large power losses in the armature winding.

The power delivered by a DC motor, P_m, is EI_a, but if it rotates at ω rad/s and supplies a torque of T N/m, that power in watts must also be $T\omega$:

$$P_m = EI_a = T\omega \tag{13.17}$$

Usually speeds are given in r/min where n r/min = $n/60$ r/sec = $2\pi n/60$ rad/s = ω, then equation 13.17 becomes

$$P_m = EI_a = T\omega = 2\pi nT/60 \tag{13.18}$$

The torque developed at 500 r/min by the motor in example 13.5 must be given by

$$T = 60P_m/2\pi n = 60 \times 72/(2\pi \times 500) = 1.375 \text{ Nm} \tag{13.19}$$

We can develop an equation for the torque in terms of speed and supply voltage as follows:

$$E = Z\Phi_p n/60 = Kn \tag{13.20}$$

where $K = Z\Phi_p/60$ = constant. But the motor's mechanical power is given by

$$P_m = EI_a = E(V - E)/R_a = Kn(V - Kn)/R_a \tag{13.21}$$

And from equation 13.18,

$$T = 60P_m/2\pi n = 60K(V - Kn)/2\pi R_a \tag{13.22}$$

For the motor of example 13.5, we find $K = 12/1000$ and equation 13.22 is then

$$T = 0.229(V - 0.012n) \tag{13.23}$$

The starting torque (when $n = 0$) from equation 13.22 is

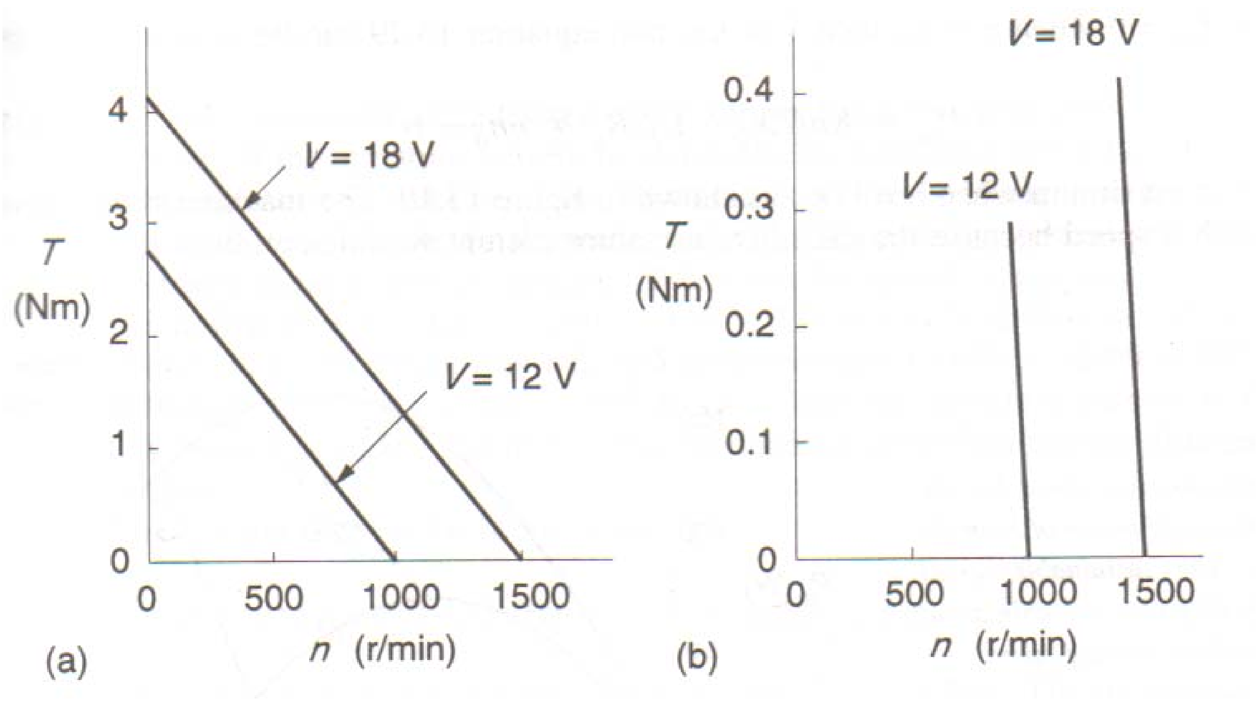
$$T_0 = 60KV/2\pi R_a \tag{13.24}$$

which is 2.75 Nm for $V = 12$ V and 4.125 Nm for $V = 18$ V.

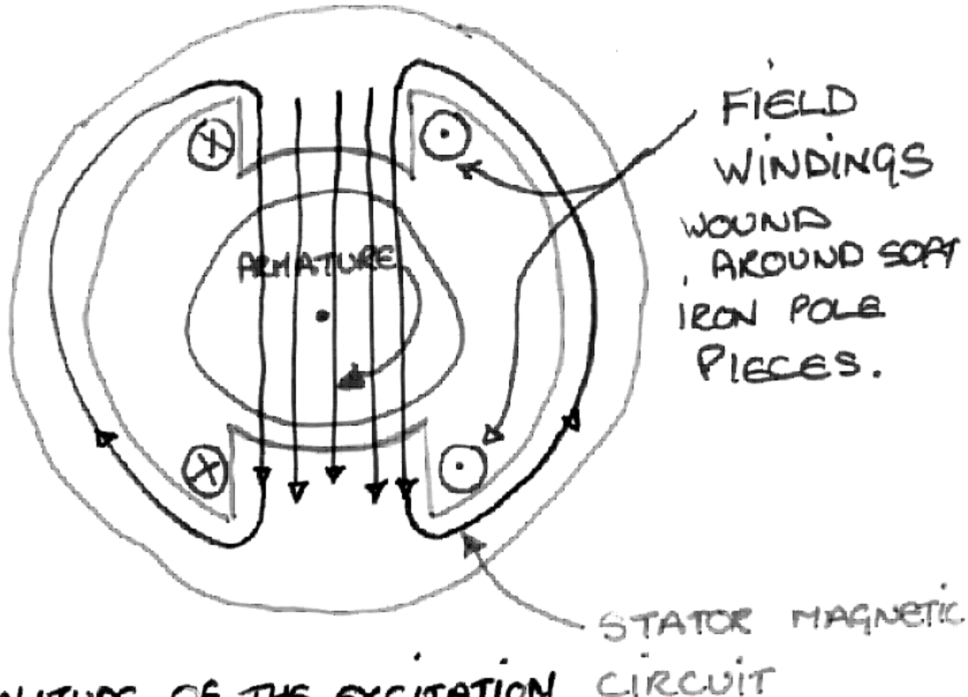
Figure 13.18a shows a graph of equation 13.23 for $V = 12$ V and $V = 18$ V. Other than for very small motors the range of sustainable speeds is small and near the zero-torque end of the graph. Since $T = P_m/\omega = EI_a/\omega$, substituting for E from equation 13.16 and ω from equation 13.18 yields the relation

$$T = Z\Phi_p I_a/2\pi \propto \Phi_p I_a \tag{13.25}$$

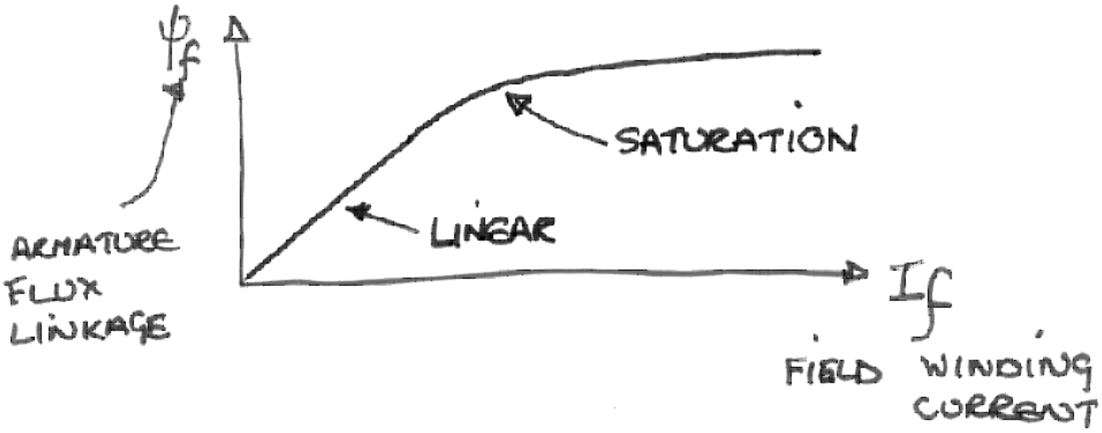
The flux/pole will be proportional to I_f in a machine with field windings, which is usually operated on the linear firt part of the magnetisation curve (unlike generators which are always operated on the saturated part of the curve to ensure a stable e.m.f.), so that $T \propto I_f I_a$.


T
(Nm)
V= 18 V
V= 12 V
n (r/min)
(a)
T
(Nm)
V= 18 V
V= 12 V
n (r/min)
(b)

WOUND FIELD MACHINES



THE MAGNITUDE OF THE EXCITATION FIELD DEPENDS UPON THE CURRENT IN THE FIELD WINDING.



CONSIDER LINEAR REGION, THEN

$$\psi_f = M I_f .$$

MUTUAL INDUCTANCE BETWEEN FIELD + ARMATURE WINDINGS.

(N.B. MUTUAL INDUCTANCE = $\frac{\text{MUTUAL FLUX LINKAGE}}{\text{AMP}}$)

$$T = M I_f I_a$$

$$E = M I_f \omega.$$

NOW 'M' IS THE MACHINE CONSTANT.

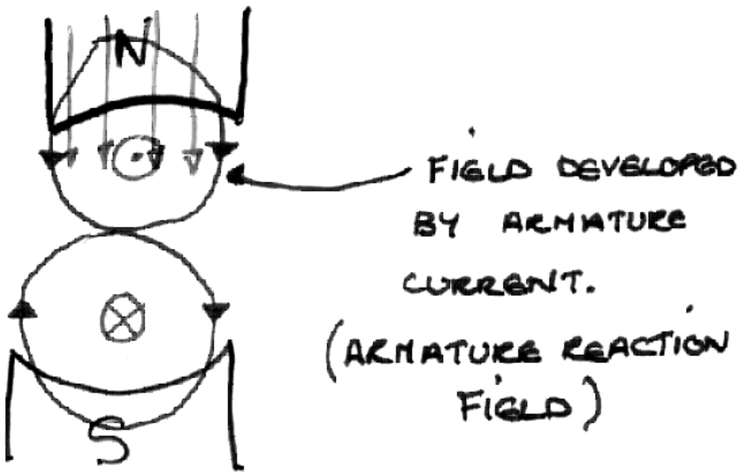
PERMANENT MAGNET SERVO MOTOR

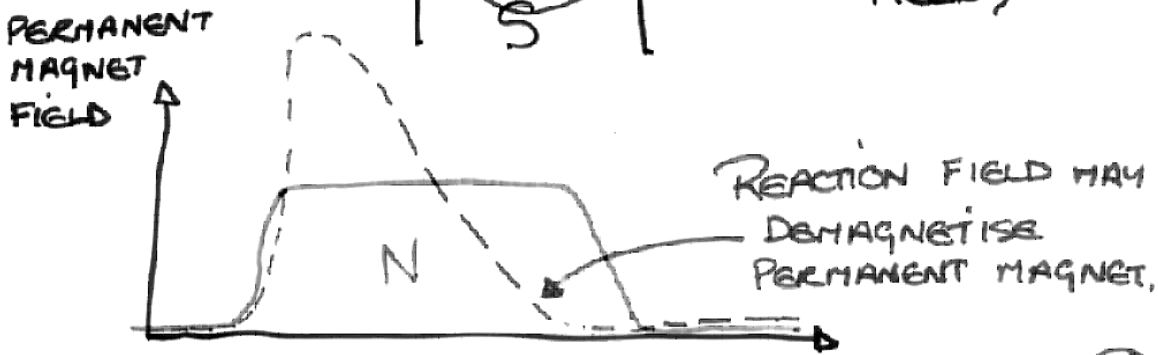
RATING PARAMETERS:

T_{STALL} – CONTINUOUS TORQUE THAT THE MOTOR CAN OUTPUT AT ZERO SPEED. THIS RATING RELATES TO THE ABILITY OF THE MOTOR TO DISSIPATE THE I^2R LOSS IN THE ARMATURE WITH THE ARMATURE STATIONARY, THIS IS THE WORST CONDITION W.R.T. COOLING ⟶ THE AIRGAP BETWEEN THE ARMATURE + STATOR ACTS AS A THERMAL BARRIER.

Ω_{max} – MAXIMUM OPERATING SPEED. LIMITED BY MECHANICAL CONSTRAINTS, ALSO THE COMMUTATOR ACTION IS SPEED LIMITED.

T_{max} – MAXIMUM PEAK TORQUE AVAILABLE FROM THE MACHINE – 5 – 10 TIMES THE CONTINUOUS RATING. LIMITED BY COMMUTATOR ACTION AT HIGH ARMATURE CURRENTS. ALSO AT HIGH ARMATURE CURRENTS THE ARMATURE REACTION FIELD MAY DE MAGNETIZE THE PERMANENT MAGNETS





N.B. A SERVO MOTOR MACHINE CONSTANT IS OFTEN EXPRESSED AS THE EMF CONSTANT IN

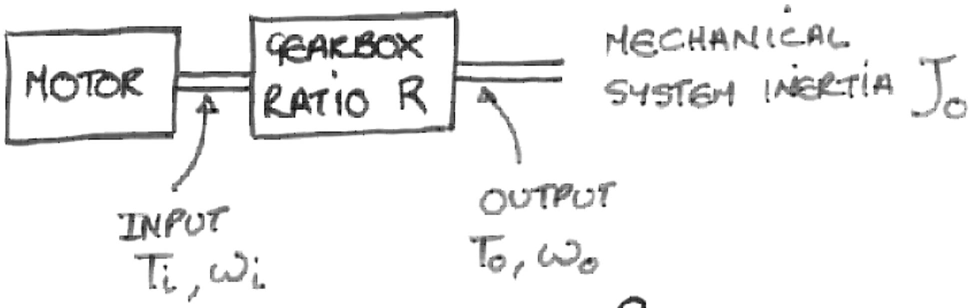
VOLTS PER 1000 rpm.

e.g. 7.5V PER 1000 rpm.

$$E = \Psi_f \omega$$

$$7.5 = \underbrace{\Psi_f}_{\text{machine constant}} \left(1000 \times \underbrace{\frac{2\pi}{60}}_{\text{convert to radians s}^{-1}}\right)$$

GEARBOXES + REFERRAL OF INERTIA



GEARBOX HAS STEPDOWN RATIO R.

$$\omega_i = R\omega_o$$

$$T_o = RT_i$$

ASSUMES LOSSLESS SYSTEM WHERE

$$\omega_i T_i = \omega_o T_o.$$

THE INERTIA WHICH THE MOTOR SEES THROUGH THE GEARBOX

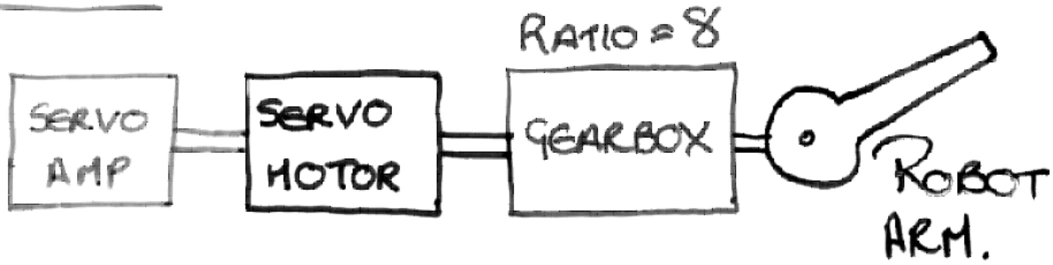
REFERRED INERTIA $= J_o'$

$$\frac{1}{2}\omega_i^2 J_o' = \frac{1}{2}\omega_o^2 J_o \qquad \text{ENERGY BALANCE}$$

$$J_o' = \left(\frac{\omega_o}{\omega_i}\right)^2 J_o$$

$$= \frac{1}{R^2} J_o$$

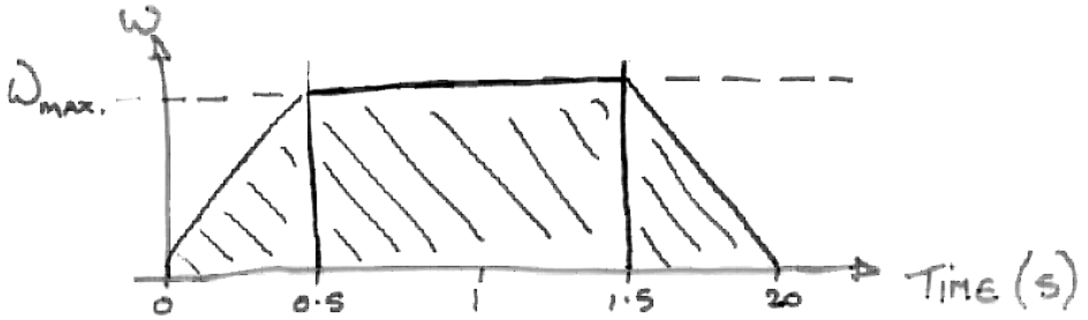
EXAMPLE



MOTOR PARAMETERS

$$\psi_f = 0.5\ \text{Nm/A.}$$
$$= 0.5\ \text{V/Rad s}^{-1}$$
$$R_A = 0.25\ \Omega$$

IT IS REQUIRED TO MOVE THE ROBOT ARM, THROUGH 135° IN 2s WITH THE FOLLOWING VELOCITY-TIME PROFILE.



7

$$\theta = \int \omega . dt$$

AREA UNDER $\omega . t = \frac{135}{360} \times 2\pi$

$$AREA = \left(\frac{1}{2}\, 0.5 .\, \omega_{MAX}\right) + \left(1 \times \omega_{MAX}\right) + \left(\frac{1}{2} . 0.5\, \omega_{MAX}\right)$$

$$= 1.5\, \omega_{MAX}.$$

$$1.5\, \omega_{MAX} = \frac{135}{360} \times 2\pi$$

$$\omega_{MAX} = \frac{2}{3} \times \frac{135}{360} \times 2\pi \quad (\text{ARM})$$

MAX SPEED OF MOTOR = 8 × ARM SPEED

ω_{MAX} FOR SERVO MOTOR $= 8 \times \frac{2}{3} \times \frac{135}{360} \times 2\pi$

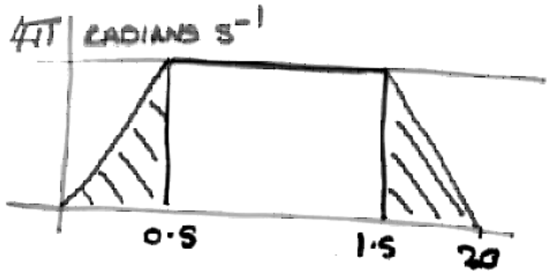
$$= \underline{4\pi \text{ RADS } s^{-1}}$$

TOTAL SYSTEM INERTIA = 0.318 $kg\,m^2$ AS SEEN BY MOTOR.

e.g. MAY BE A CONTRIBUTION OF THE ROBOT ARM INERTIA SAY 8 kg M. REFER TO MOTOR SIDE $J/R^2 = 8/8^2 = 0.125\ kg\,m^2$

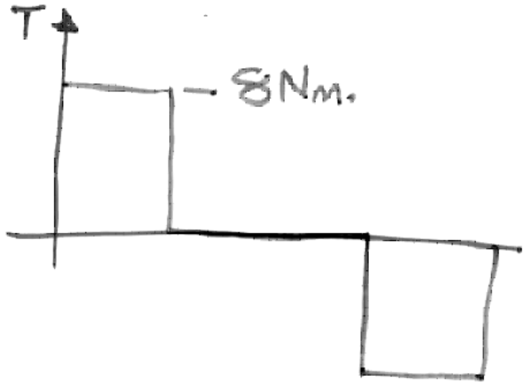
+ INERTIA OF MOTOR ITSELF + GEARBOX
$(= 0.318 - 0.125)$

$$T = J\frac{d\omega}{dt}$$



$$T = 0{\cdot}318\left(\frac{4\pi}{0{\cdot}5}\right)$$

$$= 8\ \text{Nm.}$$

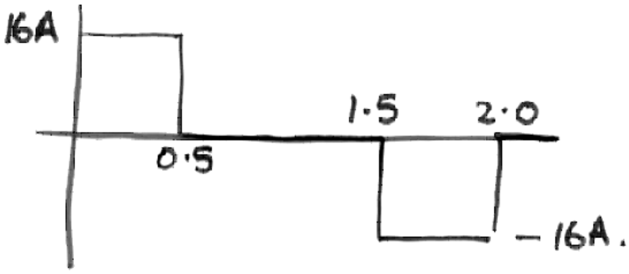


$$\psi_f = 0{\cdot}5\ \text{Nm/A.}$$

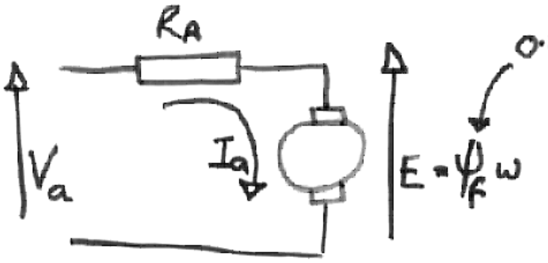
$$T = \psi_f I_a$$

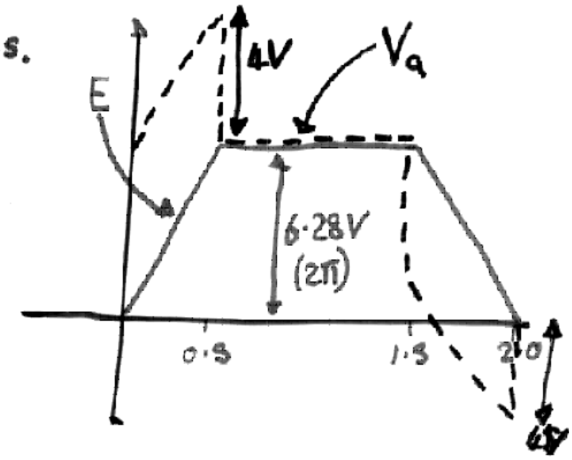
$$I_a = 8/0{\cdot}5 = 16\text{A.}$$

I.E. CURRENT TIME PROFILE REQUIRED FROM THE SERVO AMPLIFIER.



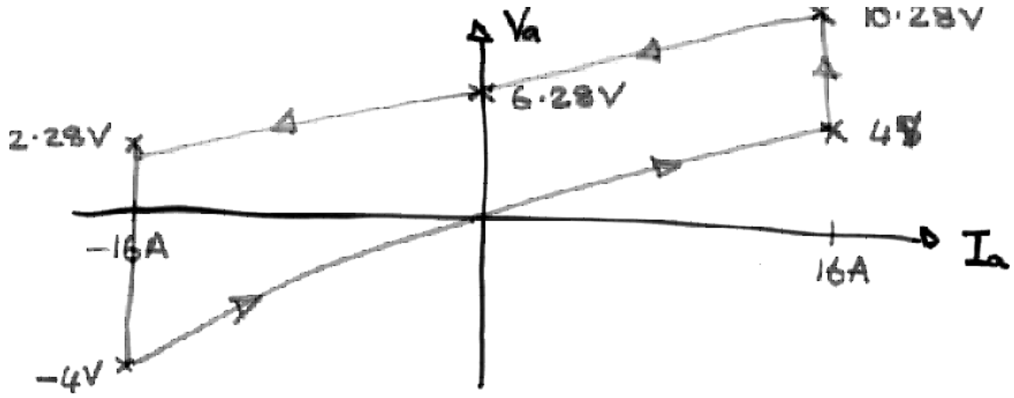
MOTOR EQUIVALENT CCT.





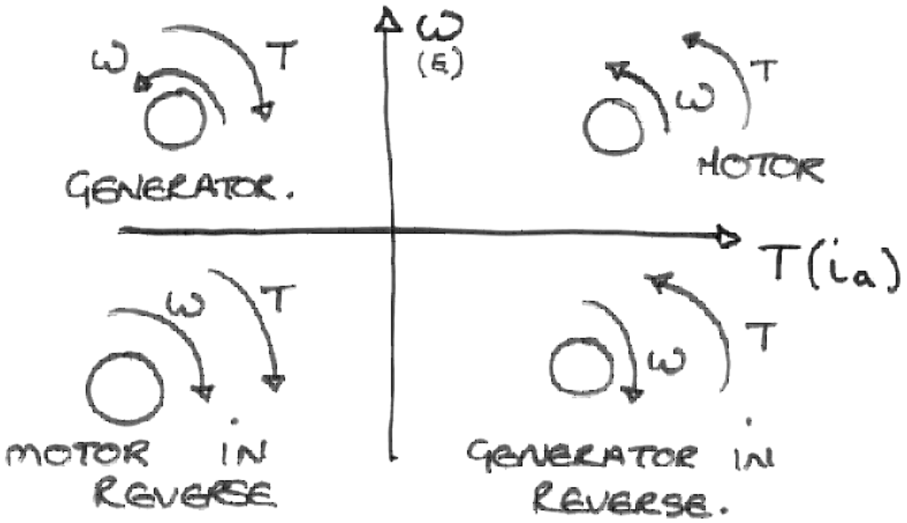
$$V_a = I_a R_a + E$$

$R_a = 0{\cdot}25\,\Omega$,
@ 16A $I_a R_a = 4V$



FOUR QUADRANT DIAGRAM

REQUIREMENTS OF A DRIVE ARE OFTEN EXPRESSED IN TERMS OF A FOUR Q DIAGRAM.

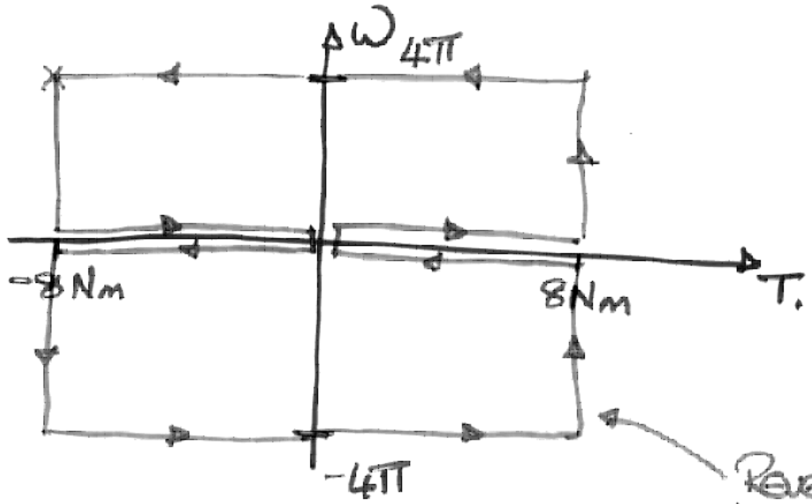


MOTOR ⟹ POWER TRANSFER FROM ELECTRICAL SUPPLY TO MECHANICAL SYSTEM.

GENERATOR ⟹ POWER TRANSFER FROM MECHANICAL SYSTEM TO ELECTRICAL.

39

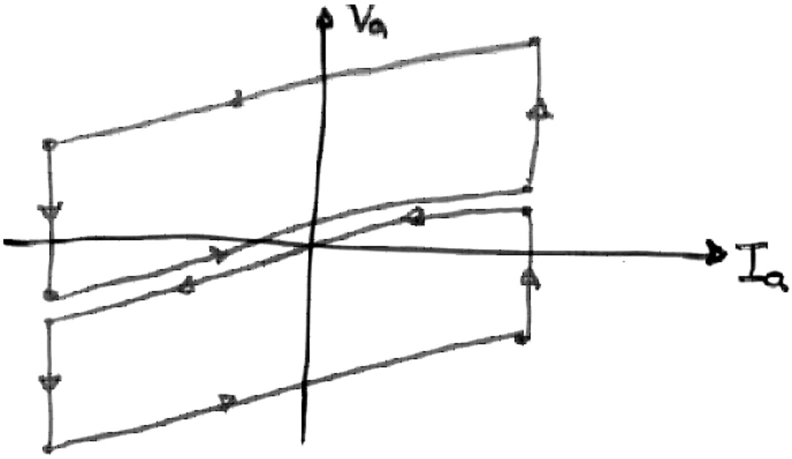
E.G. IN ROBOT APPLICATION PREVIOUSLY DESCRIBED.



REVERSE OPERATION OF RETURNING ROBOT ARM TO ORIGINAL POSITION.

IN MOST SERVO APPLICATIONS, THERE IS A REQUIREMENT FOR 4Q OPERATION.

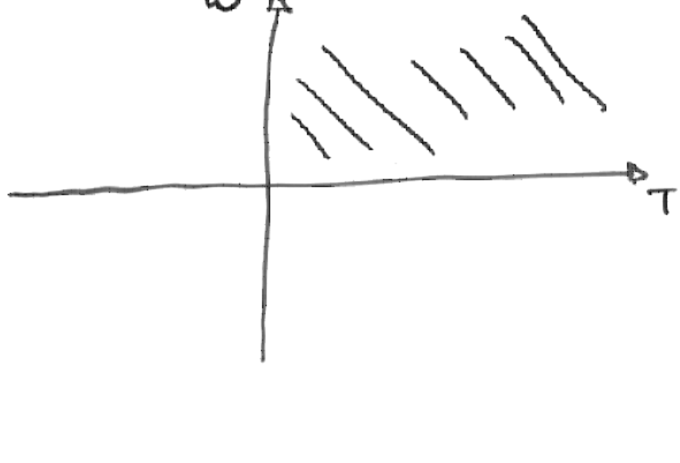
SIMILARLY A QUADRANT DIAGRAMME CAN BE DRAWN FOR THE INPUT 'TERMINAL' POWER REQUIREMENTS OF THE SERVO MOTOR, WHICH TAKES INTO ACCOUNT THE I^2R_a LOSSES IN THE ARMATURE.



THE OPERATIONAL QUADRANTS OF THE SYSTEM HAS IMPORTANT IMPLICATIONS ON THE POWER SOURCE e.g. THE SERVO AMP. WHICH DRIVES THE MACHINE.

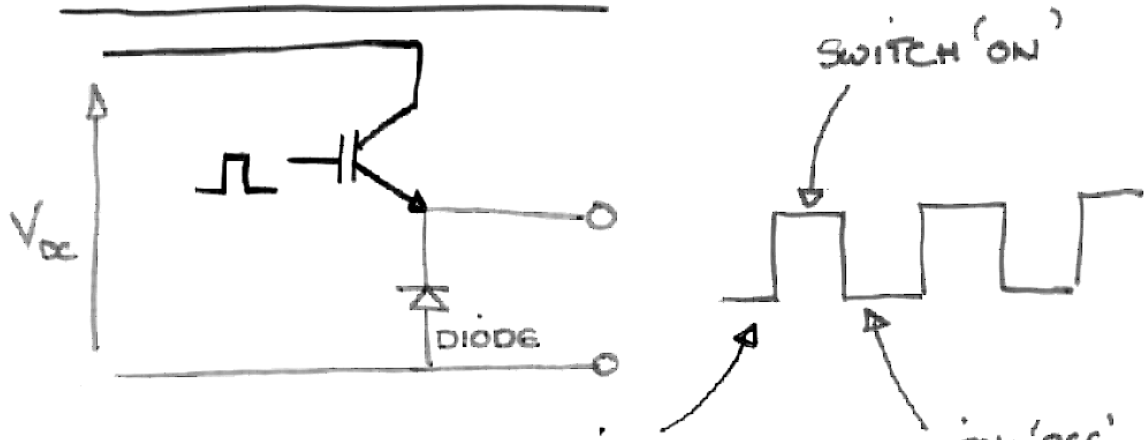
THERE ARE A NUMBER OF LESS DEMANDING APPLICATIONS WHERE A DRIVE MAY NOT NEED TO OPERATE IN ALL QUADRANTS.

e.g. A PUMP OR COMPRESSOR OR FAN DRIVE – UNIDIRECTIONAL + MOTORING OPN.



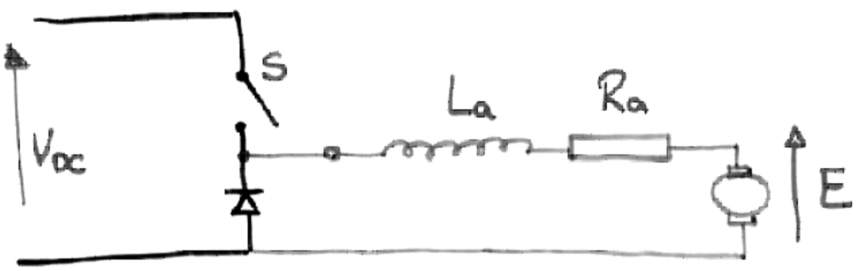
41

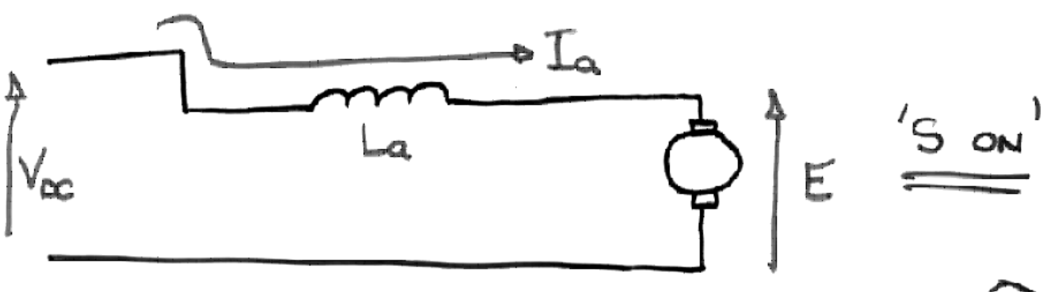
BASIC 'CHOPPER' CIRCUIT

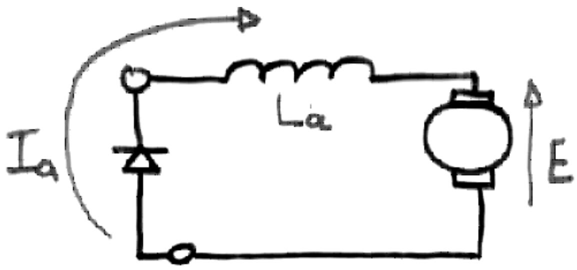


PULSE WIDTH MODULATED (PWM) OUTPUT BETWEEN V_{DC} and 0V

TYPICALLY THE OUTPUT IS SWITCHED AT MODERATE TO HIGH FREQUENCIES (2 kHz – 20 kHz). AT THESE FREQUENCIES THE INDUCTANCE OF THE MOTOR ARMATURE WINDING IS SIGNIFICANT.



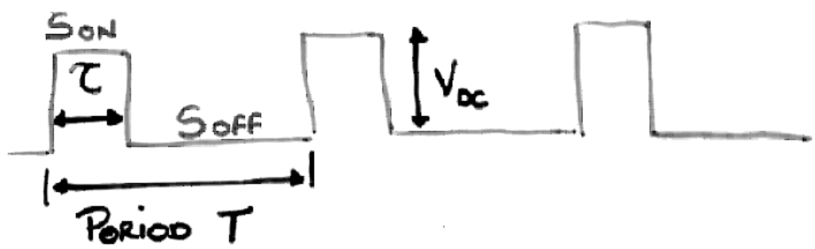




'S OFF'

ONCE A CURRENT IN THE ARMATURE INDUCTANCE HAS BEEN ESTABLISHED, THERE WILL BE A STORED ENERGY WITHIN THAT INDUCTANCE.
THE ACTION OF THE DIODE IS TO MAINTAIN A PATH FOR THIS STORED ENERGY BY ALLOWING THE ARMATURE CURRENT TO BE 'FLYWHEELED' AROUND THE CCT.

IN CONTINUOUS MODE OF OPERATION THE CCT IS SWITCHED AT A CONSTANT FREQ. AND WIDTH OR PERIOD OF THE SWITCH IS VARIED.



$$\text{FREQ} = 1/T \qquad \text{AVERAGE OUTPUT VOLTAGE} = \frac{\tau}{T} V_{DC}$$

46

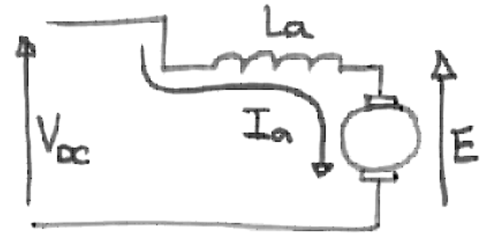
τ = S_{ON} PERIOD

T = REPETITION PERIOD.

ASSUMING MOTOR BACK EMF REMAINS CONSTANT OVER SWITCHING PERIOD (SWITCHING RATE OF ELECTRONICS >> MOTOR TIME CONSTANT) AND NEGLECTING ARMATURE RESISTANCE.

τ

ON

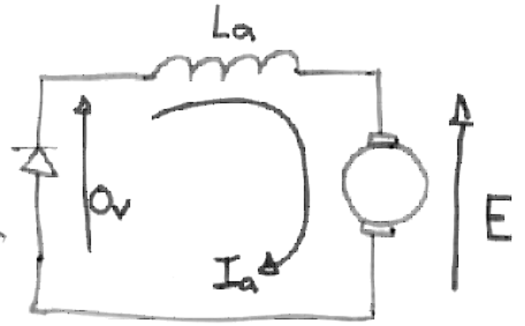


$$(V_{DC} - E) = L_a \frac{dI_a}{dt}$$

$$\frac{dI_a}{dt} = \frac{V_{dc} - E}{L_a}$$

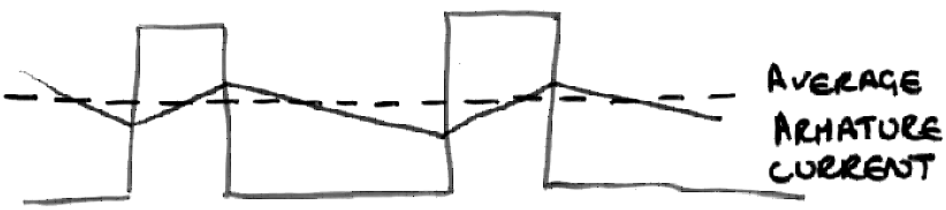
OFF

$T - \tau$



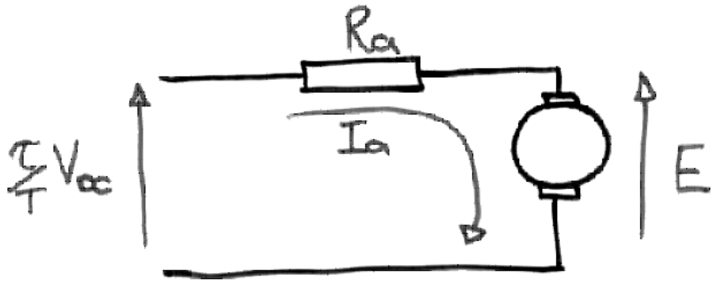
$$0 - E = L_a \frac{dI_a}{dt}$$

$$\frac{dI_a}{dt} = \frac{-E}{L_a}$$



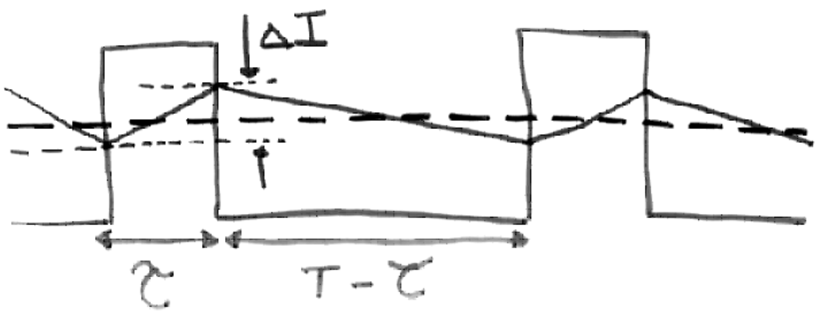
47

AVERAGE ARMATURE CURRENT CAN BE FOUND BY CONSIDERING THE AVERAGE VOLTAGE SUPPLIED TO THE MACHINE AND THE AVERAGE VOLTAGE DROP ACROSS THE ARMATURE RESISTANCE.



$$I_a \text{ average} = \frac{\frac{\tau}{T} V_{DC} - E}{R_a}$$

THE RIPPLE CURRENT ON TOP OF THE AVERAGE VALUE CAN BE FOUND FROM THE ABOVE.

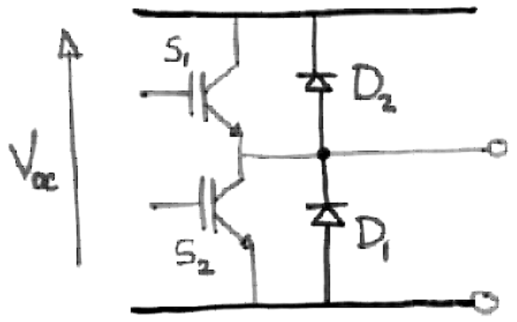


$$-E = L_a \frac{dI}{dt} \quad \text{(OFF)}$$

$$-E = L_a \frac{-\Delta I}{(T-\tau)}$$

PEAK TO PEAK RIPPLE $\Delta I = \dfrac{E(T-\tau)}{L_a}$

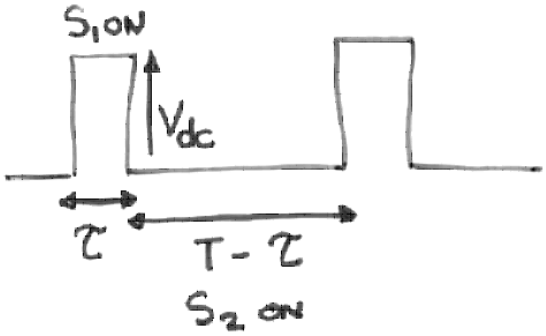
2 QUADRANT CHOPPER



S_1 AND S_2 SWITCHED IN OPPOSITION

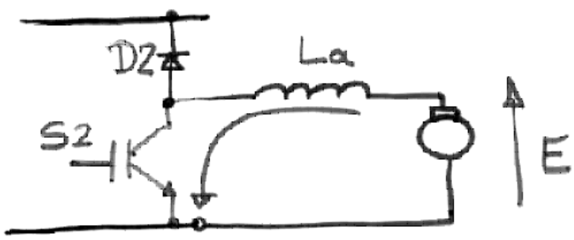
ie. S_1 ON S_2 OFF

S_1 OFF S_2 ON



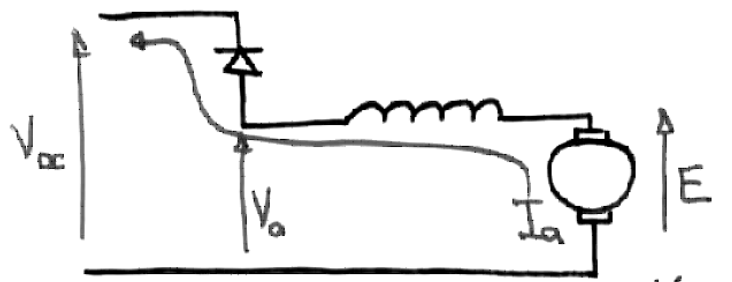
$$V_{average} = \frac{\tau}{T} V_{dc}$$

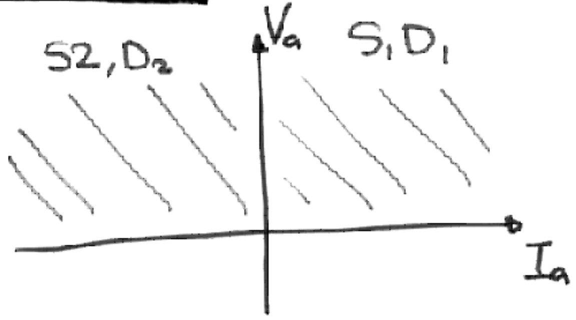
CONSIDER THE SECOND QUADRANT ACTION OF S2 AND D2.



ASSUME MOTOR IS RUNNING AND GENERATING A BACK EMF. WHEN S_2 IS TURNED ON, THE ARMATURE IS SHORT - CIRCUITED AND A CURRENT WILL BUILD UP.

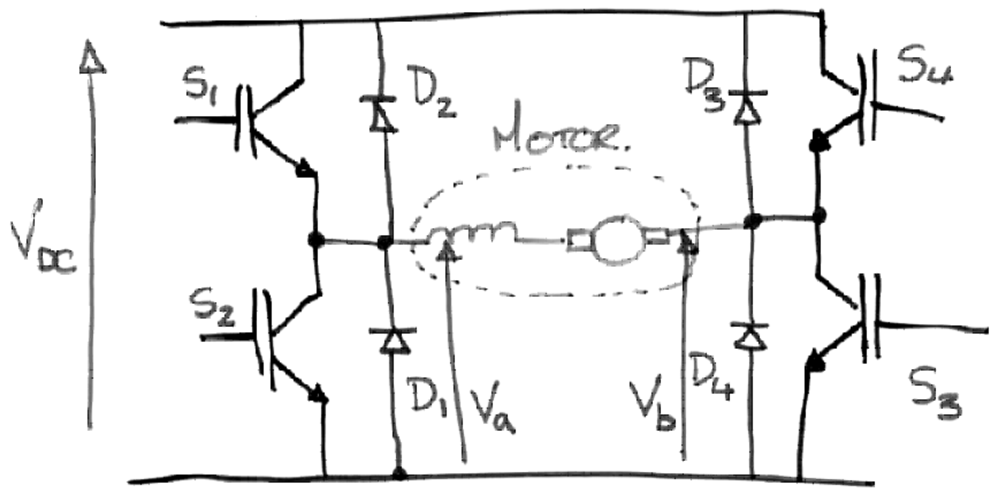
WHEN S_2 IS TURNED OFF, THIS REGENERATIVE CURRENT IS RETURNED TO THE SUPPLY VIA D_2





NOTE: THIS IS ONLY USEFUL IF THE DC SUPPLY IS CAPABLE OF ABSORBING THE REGEN. ENERGY.

FULL FOUR QUADRANT DRIVE (BRIDGE CIRCUIT)



51

S_1, S_3 ON FOR PERIOD τ
S_2, S_4 ON FOR PERIOD $(T - \tau)$

$$V_A = \frac{\tau}{T} \cdot V_{DC}$$

$$V_B = \frac{(T-\tau)}{T} \cdot V_{DC}$$

THE AVERAGE MOTOR VOLTAGE

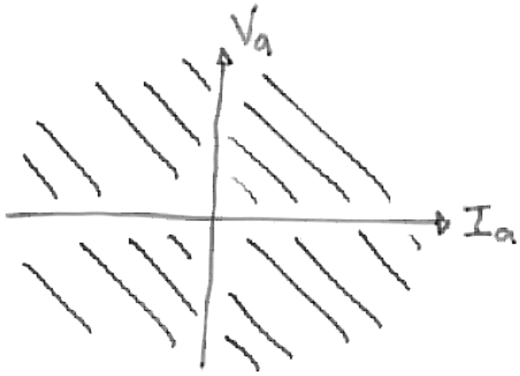
$$= V_A - V_B = \left(\frac{\tau}{T} - \frac{(T-\tau)}{T}\right) V_{DC}$$

$$= \left(\frac{2\tau}{T} - 1\right) . V_{DC}$$

$\frac{\tau}{T}$ THE DUTY VARIES FROM $0 \longrightarrow 1$

OUTPUT CHANGES FROM $-V_{DC}$ to $+V_{DC}$

$\Rightarrow$ HENCE FULL FOUR QUADRANT OPERATION.

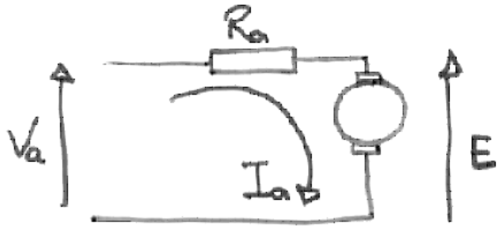


Dr Paul Stewart Dept. Electronic and Electrical Engineering

OPERATIONAL CHARACTERISTICS OF WOUND FIELD D.C. MOTORS.

MOTOR EQUATIONS

$$T = M I_a I_f.$$
$$E = M \omega I_f.$$

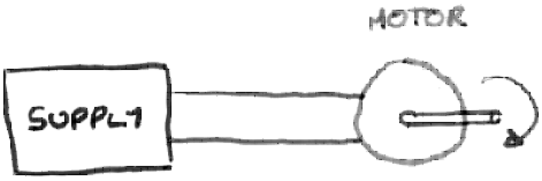


$$V_a = I_a R_a + E$$

SHUNT DC MOTOR

FIELD WINDING IS SUPPLIED SEPARATELY OR DIRECTLY FROM THE ARMATURE FIELD SUPPLY.

CONSIDER A VARIABLE SPEED APPLICATION WHERE THE DC MACHINE IS OPERATED AS A MOTOR $\longrightarrow$ TRACTION.



RATED MAX VOLTAGE
RATED MAX CURRENT

RATED MAX ARMATURE CURRENT.

53

OPERATION IS CLEARLY CONSTRAINED BY THE MAXIMUM AVAILABLE SUPPLY VOLTAGE, AND THE MAXIMUM ARMATURE CURRENT (THERMAL RATING OF ARMATURE).

MOTOR EQUATIONS

$$V_a = I_a R_a + E$$

$$T = M I_a I_f \Rightarrow I_a = T / M I_f$$

$$E = M I_f \omega$$

$$V_a = \frac{T R_a}{M I_f} + M I_f \omega$$

$$\omega = \frac{V_a}{M I_f} - \frac{R_a}{(M I_f)^2} T$$

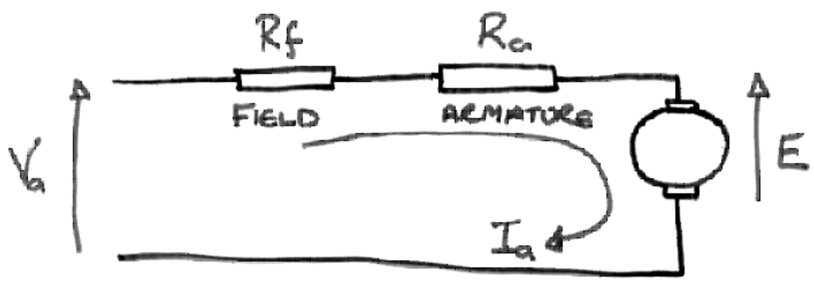
$\Rightarrow$ SPEED = NO LOAD SPEED − K TORQUE

NO LOAD SPEED AND CONSTANT K DEPEND UPON I_f.

$$T = M I_a I_f.$$

SERIES D.C. MOTOR.

5x



FIELD CONNECTED IN SERIES WITH THE ARMATURE.

$$\Rightarrow I_f \quad I_a$$

$$T = M I_f I_a = M I_a^2$$

$$E = M I_f \omega = M I_a \omega$$

NEGLECTING THE VOLTAGE DROP ACROSS $R_a + R_f$

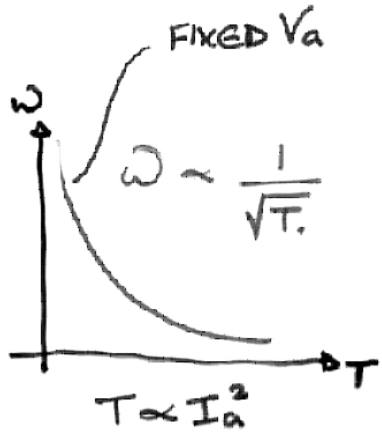
$$E \simeq \text{SUPPLY } V_a$$

$$V_a \simeq M I_a \omega$$

$$T = M I_a^2$$

$$V_a \simeq M \sqrt{T/M} \; \omega$$

$$\omega \approx \frac{1}{\sqrt{M}\sqrt{T}} V_a$$



HENCE TORQUE IS UNI DIRECTIONAL AND INDEPENDENT OF CURRENT POLARITY.
MACHINE WILL HENCE OPERATE WITH AN ALTERNATING A.C. SUPPLY.

A.C. VERSION IS CALLED THE UNIVERSAL MOTOR – COMMON IN DOMESTIC APPLIANCES e.g. WASHING MACHINES.

DC Motor Problem Sheet

1. A permanent magnet dc motor has a back emf of 50V per 1000 rpm and an armature resistance of 3Ω. If the motor is driven by an amplifier with a maximum output of 200V and 10A calculate:

(a) The maximum no-load speed of the motor
(b) The maximum torque of the motor at low speeds
(c) The maximum speed that can be achieved when providing the torque in (b)

If the amplifier current and voltage are controllable calculate the regulated volts and amps required to produce a torque of 2 Nm at 1500 rpm. Sketch the torque/speed envelope for this motor amplifier combination.

(*4000 rpm; 4.8 Nm,. 3400 rpm; 4.17 A at 87.5 V*)

2. A 500V wound field shunt dc motor has its field winding connected directly across the armature supply. At a particular load the motor runs at 750 rpm and takes an armature current of 4 A. The field current is 330 mA and the armature has a resistance of 2.5Ω. Calculate the load torque and the motor efficiency.

If the field current is reduced to 120 mA but the load torque remains the same, what is the new armature current and speed.

(*25Nm; 90.5%; 11 A; 1989 rpm*)

3. A 100V, 4-pole d.c. shunt motor runs at 750 rpm and takes an armature current of 20 A when driving a certain load. The four field windings are connected all in series across the supply and draw a field current of 5 A. If the armature resistance is equal to 0.25 Ω, find the load torque.

The four field coils are now connected all.in-parallel and the machine is run as a series motor across a 125 V supply. Calculate the new speed if the load torque remains the same.

(*24.2 Nm; 750 rpm*)

DC Motor Problem Sheet – solutions

(1a) Maximum no-load speed:

$$E = V - I_a R_a$$

However at no-load, torque and hence current are zero. Max no-load speed occurs when the applied voltage equals the back emf;

$$E = V = 200V$$

Therefore max no-load speed is

$$\omega = \frac{E}{\psi_f} = \frac{200V}{\left(\frac{50V}{1000}\right)} = 4000\,rpm$$

(1b) Maximum torque at low speeds.

Assume low speed is 500 rpm, at this speed,

$$E = \frac{50}{1000} \times 500 = 25V$$

Converting speed to radians…

$$\omega = \frac{2\pi f}{60} = 52.4\,rads^{-1}$$

Now;

$$\psi_f = \frac{E}{\omega} = \frac{25}{52.4} = 0.4775$$

With a maximum armature current of 10A, then the maximum torque is

$$T = 10 \times 0.477 = 4.8\,Nm$$

(1c) We know that *T=4.8Nm @ 10A*. Now;

$$E = V - R_a I_a = 200 - (3 \times 10) = 170V$$

And;

$$\psi_f = \frac{5}{100}$$

Therefore max speed when providing the torque in b) is:

$$\omega_{\max} = \frac{E}{\psi_f} = 170 \times \frac{100}{5} = 3400\,rpm$$

Calculate *I* and *V* to produce *2Nm @ 1500 rpm.*

$$\frac{E}{\omega} = \psi_f = \frac{T}{I_a} = 0.48$$

Therefore to calculate current

$$\frac{E}{157\,rads^{-1}} = 0.48 = \frac{2}{I_a}$$

$$E = 0.48 \times 157 = 75.4V$$

$$I_a = \frac{2\,Nm}{0.48} = 4.17\,A$$

To calculate voltage

$$V = E + I_a R_a = 75.4 + (4.17 \times 3) = 87.9V$$

(2) Load Torque:

$$E = V - I_a R_a = 500 - (2.5 \times 4) = 490V$$
$$\omega = 750 \times \frac{2\pi}{60} = 78.5 rads^{-1}$$
$$I_f M = \frac{E}{\omega} = \frac{490}{78.5} = 6.24$$

Now;

$$T = I_f \times I_a \times M = I_a \times 6.24 = 25 Nm$$

Efficiency:
Output power = speed x torque

$$P_o = 78.5 \times 25 = 1.959 kW$$

Input power = VI

$$P_i = 500 \times (4 + 0.33) = 2.165 kW$$

Efficiency = output power / input power

$$\eta = \frac{1.959}{2.165} = 90.5\%$$

New armature current
From the original parameters;

$$T = I_a I_f M$$
$$25 = 4 \times 0.33 \times M$$
$$\therefore M = \frac{25}{4 \times 0.33} = 18.94$$

For the new field current of *120 mA*

$$25 = I_a \times 0.12 \times 18.95$$
$$\therefore I_a = \frac{25}{0.12 \times 18.94} = 11A$$

New speed:

$$E = V - I_a R_a = 500 - (11 \times 2.5) = 472.5$$
$$\omega = \frac{E}{I_f} \times M = \frac{472.5}{0.12 \times 18.94} = 207.89 rads^{-1} = 1985 rpm$$

<u>(3)</u> Load torque;

$$E = V - I_a R_a = 100 - (20 \times 0.25) = 95V$$
$$I_f M = \frac{E}{\omega} = \frac{95}{78.5 rads^{-1}} = 1.21 \therefore M = \frac{1.21}{5} = 0.242$$
$$T = I_a \times I_f M = 20 \times 1.21 = 24.2 Nm$$

New speed:

$$T = \frac{I_a^2 \times M}{4 poles}$$
$$\therefore I_a \sqrt{\frac{4T}{M}} = \sqrt{400} = 20A$$
$$E = \omega \times \frac{20}{4} \times M$$
$$\therefore \omega = \frac{E \times 4}{20 \times M} = \frac{95 \times 4}{20 \times 0.242} = 78.5 rads^{-1} = 750 rpm$$