Data Provided: None

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2015-16 (3.0 hours)

EEE6221 Data Coding Techniques for Communications and Storage

Answer **FOUR** questions. **No marks will be awarded for solutions to a fifth or sixth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. You are required to establish a synchronous wired connection between two digital systems. You realise that some form of line coding is necessary to avoid distortion.

i. Explain how you would go about selecting or devising a suitable line code for a particular application. **3**

ii. On installation of the cable you find that the bandwidth available is lower than expected. Characterise and contrast the two line codes depicted in the timing diagrams in Figures Q.1.a and Q.1.b below. Hence derive the bit pattern sequence that corresponds to both of the timing diagrams.
Which of the two codes would you select for your application? **8**

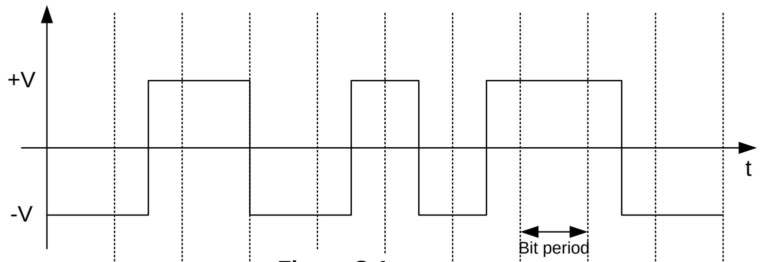

Figure Q.1.a

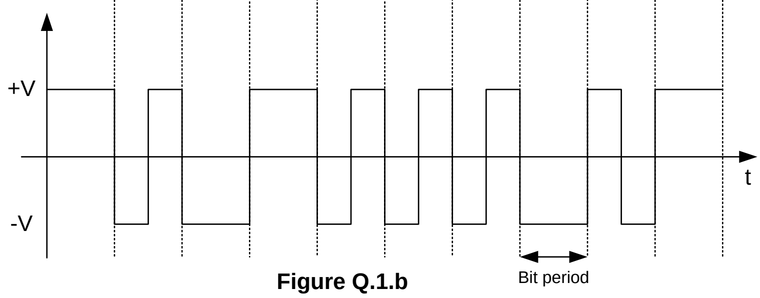

Figure Q.1.b

b. Explain briefly how pulse shaping can help improve signal quality in a digital communication system. Give an example of a popular pulse shaping filter. **3**

c. The ASCII character Z (5A Hex, 1011010 binary) is sent using an RS232 interface at a 9600 baud rate asynchronous transmission with even parity. Sketch the corresponding resulting waveform and give the resulting data rate. **4**

d. An RS232 asynchronous transmission connection is set up with odd parity and 8-bit data between a Start and a Stop bit. The master clock is set to run at a nominal rate that is a multiple ***m*** of the baud rate with a clock tolerance of **±1%**. Determine a reliable value for ***m*** for this connection, assuming the worst case conditions correspond to the transmitter's clock being fast, the receiver's clock being slow, and the start bit sampled at the receiver one cycle late. **7**

2. a. A 5-bit message ***(00110)*** is encoded using Cyclic Redundancy Check (CRC) bits generated by the generator polynomial $g(x) = x^4 + x^3 + 1$.

i) Draw a circuit diagram for this CRC encoder and explain its operation. **4**
ii) Derive the resulting codeword. **3**
iii) Give the error detection reliability figure of the CRC used. **2**

b. During transmission over a noisy channel, the CRC codeword ***(110100010)*** generated using the CRC in **a.** was corrupted by the error pattern ***(110010000).*** Perform a CRC check to check the integrity of the received message and justify your result. **5**

c. A ***(15,7)*** primitive ***BCH*** code defined over $GF(2^4)$ using the primitive polynomial $p(x) = x^4 + x + 1$, was used to encode the message:
$u(x) = x^4 + x^2 + 1$. Derive the codeword for the message $u(x)$ if the code's generator polynomial is given as $g(x) = x^8 + x^7 + x^6 + x^4 + 1$ **5**

d. After transmission over a Gaussian channel a message $m(x)$ encoded using the above **(15,7) BCH** code is received with a **single bit** error as $r(x) = x^{14} + x^{11} + x^9 + x^7 + x$. Using algebraic decoding, derive the corrected codeword. **6**

3. a. Contrast briefly the Look up table and the Meggitt decoding approaches for the non-algebraic decoding of cyclic block codes. **3**

b. A message encoded using a (15,11) single error correcting binary cyclic code generated using the generator polynomial $g(x) = x^4 + x + 1$ is received with a single error in as $r(x) = x^{13} + x^5 + x^2 + x$

i) Draw a block diagram for a decoder for this code based on the Look up table approach and explain briefly how r(x) would be corrected. **4**
ii) Draw a block diagram for a decoder for this code based on Meggitt decoding and explain briefly how r(x) would be corrected in this case. **4**

iii) Derive the correct codeword using the Meggitt decoding approach. Show all your calculations and validate your result. The syndrome corresponding to an error in the MSB is given as $S(x) = x^3 + 1$ **7**

c. A message encoded using a double error correcting binary cyclic code is received with 2- bit errors Explain how you would correct these 2 errors using Meggitt decoding. What are the implications of using Meggitt decoding for multiple error correction? **3**

d. List the steps involved in algebraic decoding of cyclic binary block codes and explain how such decoding approach overcomes the limitations of both the Look Up Table and Meggitt decoding approaches for multiple error correction. **4**

4. a. Draw a circuit to generate all non zero elements of the Galois Field $GF(2^4)$ using the primitive polynomial $p(x) = x^4 + x^3 + 1$. List all of the elements in both binary and polynomial format. **4**

b. Before transmission, a message block is encoded using an ***(n,k)RS*** (Reed-Solomon) code over the Galois Field $GF(2^4)$ in **a.** so that any 2-symbol errors from this field can be corrected.

i) Find suitable n and k values and hence define the corresponding ***RS*** code to enable this error correction capability. How many bits can this code correct. **3**

ii) Derive the generator polynomial $g(x)$ for your defined ***RS*** code in i). **6**

iii) Find the **RS** codeword for the message $m(x) = x + 1$. **6**

c. A 3-symbol message is encoded using the a ***(7,3)RS*** (Reed-Solomon) code defined over $GF(2^3)$ using the primitive polynomial $p(x) = x^3 + x^2 + 1$. After transmission over a noisy channel, the corresponding 7-symbol ***RS*** codeword is received with one symbol error as $r(x) = \alpha^2 x^5 + \alpha x^4 + \alpha x^2 + \alpha^6 x + 1$. Using algebraic decoding, derive the corrected codeword given that for a single symbol error correction the error location, X_1, and the error magnitude, Y_1, are given respectively by the equations: **6**

$$X_1 = \sigma_1 = \frac{S_2}{S_1} \quad \text{and} \quad Y_1 = \frac{S_1^{\,2}}{S_2}$$

where S_1 and S_2 are the first 2 syndromes.

3

5. **a.** Contrast briefly losseless and lossy compression explaining why is it possible to compress data in each case. **3**

b. Explain briefly how Vector Quantisation is used for image data compression stating its the advantages and drawbacks. **3**

c. List and explain, briefly, the charateristics of the Discrete Cosine Transform (DCT) which are attractive for image and video compression. How is compression achieved in DCT-based systems. What are the potential limitations of DCT- based compression? **4**

d. Determine the **DCT** of the two-dimensional data given below explaining any shortcuts made. Comment on the energy distribution in the 2-D DCT obtained, and explain any possible implications in a typical DCT-based compression system. **8**

$$\begin{bmatrix} 10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 10 & 0 & 10 & 0 \end{bmatrix}$$

The ***k-th/n-th DCT/IDCT*** pair of an ***N***-sample block input is given by:

$$X_k = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \alpha_k x_n \cos\left[\frac{(2n+1)k\pi}{2N}\right]$$

$$x_n = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} \alpha_k X_k \cos\left[\frac{(2n+1)k\pi}{2N}\right] \qquad \alpha_0 = \frac{1}{\sqrt{2}} \quad \alpha_k = 1 (k \neq 0)$$

e. Consider the following ***DCT*** coefficients block obtained after the quantisation step in baseline **JPEG**:

$$\begin{bmatrix} 47 & 28 & -3 & 0 & 0 & 0 & 0 & 0 \\ -22 & -10 & -2 & 0 & 0 & 0 & 0 & 0 \\ 12 & 10 & 1 & 0 & 0 & 0 & 0 & 0 \\ 9 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The DC value is encoded separately and requires 5 bits; what is the total number of bits taken by the AC coefficients given the Huffman codewords for the Run/size combinations shown below in table Q.5. Find the average bit rate and hence the compression rate achieved in this case. **7**

Run/Size	1/2	0/5	0/4	0/2	0/1	0/0
Huffman code	11011	11010	1011	01	00	1010

Table Q.5

6. **a.** Compare block codes and convolutional codes, as applied to error correction, giving typical situations where each one might be employed. **3**

b. Explain briefly the frequency domain approach to Reed-Solomon (RS) encoding and decoding. **4**

c. A (7,3)RS (Reed-Solomon) code defined over **GF(2^3))** using the primitive polynomial $\boldsymbol{p(x) = x^3 + x + 1}$ was used to encode a 3-symbol message into a 7-symbol codeword using frequency domain encoding. The encoded codeword was received as c'= (α^3, α^2, α, α^6, 1, α^2, α^3); a frequency domain decoding is adopted for the error correction and the spectrum of c' was hence first computed and found to be: C'= (α^4, α^2, α, α, α^5, α, α^2).
The recursive circuit shown in Figure Q.6.a below was then used to determine the Error spectrum E.

i) Derive the complete error spectrum E in this case. **5**
ii) Hence, derive the time domain corrected message c. **6**

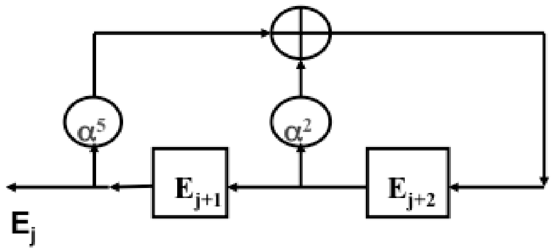

Figure Q.6.a

d. Using the (*n*,*k*,*m*) convolutional encoder circuit shown in Figure Q.6.b below, a binary data sequence was encoded and transmitted, in the order (Q_1Q_0) from left to right, through a noisy channel and received with <u>no more than 3</u> bit errors as:

11 01 11 00 01 01 10 00 01 11

Correct the received sequence <u>giving all solutions</u> and explaining how you might select the right one. Hence determine the original transmitted data **7**

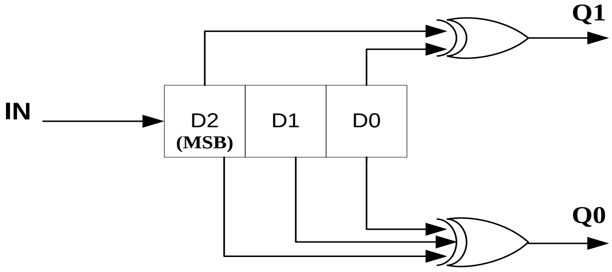

Figure Q.6.b

MB/CKA