EEE225: Analogue and Digital Electronics

Lecture XI

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This Lecture

Noise in Electronic Systems

- Resistors, transistors and diodes add noise to circuits. Inductors and capacitors can shape that noise, but ideal capacitors and inductors do not generate their own noise.
- To combine the contribution for every one using Thévenin and Norton sources is time-consuming and quickly becomes impractical even for small circuits.
- A method of representing the electronic system's observed noise using some simple metrics and a couple of "equivalent" noise generators is highly desirable.
- This lecture focuses on metrics of noise performance in impedance matched systems, which are usually operated above 30 MHz. Unmatched systems in Lecture 12.
- In high frequency work, amplifiers are often thought of in terms of their effect on the signal power, not voltage and current.

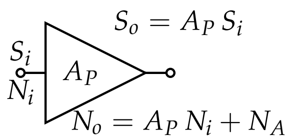
Signal to Noise Ratio: A Power Ratio

- Signal to Noise Ratio is expressed in dB $10\log_{10}\left(\frac{S}{N}\right)$
- 10 log because it is a ratio of powers.
- It is a measure of the signal quality at the point in the system where it is computed. It varies throughout a system.
- It gives no information about the actual quantity of the noise (in $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ for example). It's just a ratio.

- To assess noise performance of a system, compare the Signal to Noise Ratio at the input and output.

$$\frac{S_i}{N_i} = \frac{\text{input signal power}}{\text{input noise power}} \quad (1)$$

$$\frac{S_o}{N_o} = \frac{\text{output signal power}}{\text{output noise power}} \quad (2)$$



Noise Factor: A Power Ratio

Noise Factor is the quotient of the SNR at the input and the SNR at the output

$$F = \frac{\text{signal to noise ratio at input}}{\text{signal to noise ratio at the output}} = \frac{S_i/N_i}{S_o/N_o} \quad (3)$$

If the power gain of the system is A_p:

$$F = \frac{N_o}{A_p\,N_i} = \frac{\text{output noise power of the real amp.}}{\text{output noise power if the amp. was noiseless}} \quad (4)$$

- The idea of noise factor is useful for multi-stage impedance matched systems (satellite up/downlinks, radar, radio astronomy, cable TV).
- The noise power available at the output of an *ideal* impedance matched system is the input noise power multiplied by the system power gain.

Noise Factor in Unmatched Systems

- In unmatched systems including transistor and operational amplifier systems the idea of "power gain" is not very useful.
- Instead work out the mean square noise voltage (or current) at the nodes of interest - the ratio of mean square voltages is the same as power.
- When working with mean square voltages, the system voltage gain (should it appear) must also be squared or dimensional inconsistency will result.
- The input noise is the mean squared noise voltage (or current) at the input due to the source.
- If noise factor is used in LF systems, it's important to consider if minimising the noise factor is the most desirable outcome - it's not straight forward.

Noise Factor in Matched Systems

The output noise N_o is composed of the amplifier noise N_A and the input noise N_i multiplied by the power gain, A_P. And an ideal amplifier has no noise so its output would be the noise signal entering its input multiplied by its power gain.

$$F = \frac{A_P\,N_i + N_A}{A_P\,N_i} = 1 + \frac{N_A}{A_P\,N_i} \qquad (5)$$

If $N_A = 0$, $F = 1$. For an impedance matched system N_i is the available noise input power[1] and F can be written as

$$F = 1 + \frac{N_A}{A_P\,N_i} + 1 + \frac{N_A}{A_P\,k\,T\,\Delta f} \qquad (6)$$

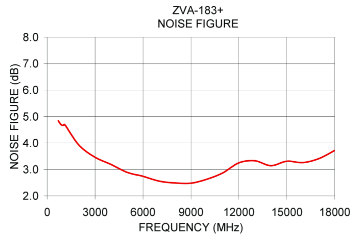
[1] Remember Lecture 10, regarding the maximum available power from one resistor into another.

Noise Figure

The noise figure is simply the noise factor, F, expressed in dB.

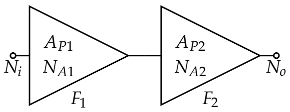
$$\text{Noise Figure, } NF = 10\,\log_{10} F \text{ dB} \qquad (7)$$

Noise Figure tends to appear on datasheets more often than noise factor for example the RF amplifier datasheet handout. To use this information it is first necessary to convert it to noise factor.



Noise Factor of a Two Stage Impedance Matched System

Many electronic systems consist of a cascade of circuits. This analysis holds for matched systems, but the underlying idea - that the noise of the first stage is the most important - holds generally.



- The available noise at the input is

$$N_i = k\,T\,\Delta f \qquad (8)$$

- The noise factor of amplifiers 1 and 2 are

$$F_1 = 1 + \frac{N_{A1}}{A_{P1}\,k\,T\Delta f}\;,\;F_2 = 1 + \frac{N_{A2}}{A_{P2}\,k\,T\Delta f} \qquad (9)$$

The output noise, N_o, has three components:

- Output noise due to the available input noise, N_i

$$N_o|_{N_i} = A_{P1}\,A_{P2}\,N_i = A_{P1}\,A_{P2}\,k\,T\,\Delta f \qquad (10)$$

- Output noise due to the noise of amplifier 1, N_{A1}

$$N_o|_{N_{A1}} = A_{P1}\,N_{A1} = A_{P2}\,(F_1 - 1)\,A_{P1}\,k\,T\,\Delta f \qquad (11)$$

- Output noise due to the noise added by amplifier 2, N_{A2}

$$N_o|_{N_{A2}} = N_{A2} = (F_2 - 1)\,A_{P2}\,k\,T\,\Delta f \qquad (12)$$

- Total *real* amplifier noise N_o (10) + (11) + (12) is,

$$A_{P1}\,A_{P2}\,k\,T\,\Delta f + A_{P1}\,A_{P2}\,(F_1 - 1)\;k\,T\,\Delta f + (F_2 - 1)\;A_{P2}\,k\,T\,\Delta f \qquad (13)$$

Which simplifies to,

$$A_{P1}\,A_{P2}\,k\,T\,\Delta f\,\left(F_1 + \frac{(F_2 - 1)}{A_{P1}}\right) \qquad (14)$$

- The noise output from the *ideal* amplifier is just (10).
- so the noise factor is

$$F = \frac{A_{P1}\,A_{P2}\,k\,T\,\Delta f\,\left(F_1 + \frac{(F_2-1)}{A_{P1}}\right)}{A_{P1}\,A_{P2}\,k\,T\,\Delta f} = F_1 + \frac{(F_2 - 1)}{A_{P1}} \qquad (15)$$

Conclusions:

- System noise factor is at least equal to the noise factor of the first stage - it is *the* most important.
- The noise factor of the second stage is reduced by a by an amount equal to the first stage power gain before it adds any noise to the system.

Always try to design the first stage to have low noise and high power gain! Much of the noise design effort of LF ICs and microwave components is focused on the first stage. It is where SNR is won and lost.

Matched System Example Question

A wideband amplifier in a matched 50 Ω system is made from two thin film amplifier modules Minicircuits ZVA-183+ followed by ZRON-8G+ with gains of 26.64 dB and 25.64 dB and noise figures of 2.56 dB and 4.29 dB respectively. The amplifier bandwidth of interest spans 1 GHz centred on 7 GHz.

- What is the gain of the series combination?
- What is the noise factor of each amplifier module?
- What is the noise figure of the combination if the ZVA-183+ module is at the input end of the amplifier?
- What is the total added noise power delivered to the load?
- What is the output signal to noise ratio if the input power is 1 pW.
- What is the effective noise temperature of the 50 Ω source resistance?

The maximum available noise power is $k\,T\,\Delta f$ W where Δf is as defined in the question.

Gain and Noise Factor

The gain of the combination is found by adding the individual gains in dB or by converting the gains in dB to linear and then multiplying.

$$A_P = 10\log_{10}\left(10^{(26.64/10)} \cdot 10^{(25.64/10)}\right) \tag{16}$$

$$= 461.32 \cdot 366.44 \tag{17}$$

$$= 1.69045 \times 10^5 \text{ W/W} = 52.28 \text{ dB} \tag{18}$$

The noise factor of the ZVA-183 @ 7 GHz is

$$F_1 = 10^{(2.56/10)} = 1.80302 \tag{19}$$

The noise factor of the ZRON-8G+ @ 7 GHz is

$$F_2 = 10^{(4.29/10)} = 2.6853 \tag{20}$$

Noise Figure of the Combination & Noise Power to the Load

To find the noise figure, first find the noise factor of the combination and then convert to dB. Use,

$$F = F_1 + \frac{F_2 - 1}{A_{P1}} = 1.80302 + \frac{2.6851 - 1}{461.32} = 1.80667 \tag{21}$$

Notice how little effect the noise factor of the second stage ZRON-8G+ has compared to the first stage. The noise figure is,

$$NF = 10^{(1.80667/10)} = 2.5688 \text{ dB} \tag{22}$$

To obtain the noise power to the load we can transpose,

$$F = 1 + \frac{N_A}{A_P\, N_i} \rightarrow N_A = A_P\, N_i\,(F - 1) \tag{23}$$

Noise Power to the Load (continued) & SNR at the Output

F = 1.80667, A_P = 169045, T = 298.15 K (25°C) &
$N_i = k\,T\,\Delta f = 1.38 \times 10^{-23} \cdot 298.15 \cdot 1 \times 10^9 = 4.11447$ pW.
Substituting into (23) yields,

$$N_A = A_P\, N_i\,(F - 1) \tag{24}$$

$$N_A = 169045 \cdot 4.11447 \times 10^{-12} \cdot (1.80667 - 1) \tag{25}$$

$$= 5.6519 \text{ nW} \tag{26}$$

To put this in perspective the signal power of the GPS system at the earth's surface is about 10 nW.
The SNR at the output can be obtained by a number of methods (as can many of these solutions), we can use,

$$F = \frac{S_i/N_i}{S_o/N_o} \tag{27}$$

SNR at the Output (continued)

We already know the noise power available at the input is $k\,T\,\Delta f = 4.11447$ pW and the question gives the input power as 1 pW. Compute the SNR at the input,

$$\frac{S_i}{N_i} = \frac{1 \times 10^{-12}}{4.11447 \times 10^{-12}} = 0.24304 \tag{28}$$

Now transpose (27) and substitute in F and (28) to yield,

$$\frac{S_o}{N_o} = \frac{S_i/N_i}{F} = \frac{0.24304}{1.80667} = 0.13452 \tag{29}$$

Comparing the input and output SNR we find the signal more corrupted when leaving the amplifier than when it entered. This is reassuring as it means the amplifier must have added some noise... and we know that it did. 0.13452 = -8.7119 dB (SNR < 1 means the noise is larger than the signal)

Noise Temperature of the Amplifier Cascade

When a resistor equal to the value of the characteristic impedance (50 Ω) is heated to a certain temperature above 0 K it will add to the signal the same noise as our amplifier cascade adds. This temperature is the effective noise temperature of the amplifier. We can take a short-cut (the proof of which I leave to you)...

$$F = 1 + \frac{T_E}{T_A} = 1 + \frac{T_E}{298.15} \tag{30}$$

where T_E is the effective noise temperature of the amplifier and T_A is the actual temperature.

$$T_E = (1.80667 - 1) \cdot 298.15 = 240.508 \text{ K} \tag{31}$$

So if we had a 50 Ω resistor at -32.642°C it would have the same noise power as our amplifier within the bandwidth under discussion.

Review

- Introduced the idea of Signal to Noise Ratio
- Discussed Noise Factor as a ratio of powers and a metric of amplifier noise performance.
- Noted some key points and restrictions of noise factor in matched and unmatched systems.
- Developed Noise Figure as noise factor expressed in dB.
- Derived the Noise factor of a two stage impedance matched electronic system.
- Drew some key conclusions 1) First stage gain should be high. First stage noise should be low. 2) Effect of second stage noise is reduced by the gain of the first stage.
- Provided a real example of the use of SNR, noise figure & Noise factor based on two MiniCircuits amplifiers.