

### Properties of Fourier Transform

Property	Aperiodic signal, $x(t)$	Fourier Transform, $X(\omega)$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(\omega) + bX_2(\omega)$
Time Shifting	$x(t - t_o)$	$e^{-j\omega t_o} X(\omega)$
Frequency Shifting	$e^{j\omega_o t} x(t)$	$X(\omega - \omega_o)$
Time Scaling	$x(at)$	$\frac{1}{a} X\left(\frac{\omega}{a}\right)$
Differentiation in Time	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Differentiation in Frequency	$tx(t)$	$j \frac{dX(\omega)}{d\omega}$
Integration in time	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{X(\omega)}{j\omega} + \pi X(0) \delta(\omega)$
Convolution	$x(t) * h(t)$	$X(\omega) \cdot H(\omega)$
Multiplication in time	$x(t) \cdot h(t)$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\lambda) H(\omega - \lambda) d\lambda$
Parseval's Theorem	$E = \int_{-\infty}^{\infty}  x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty}  X(\omega) ^2 d\omega$	

### **List of useful formulae**

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k]$$

$$x(t) = \int_{-\infty}^{\infty} x(\lambda) \delta(t-\lambda) d\lambda$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) d\lambda$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_0 = \frac{1}{T} \int_{\langle T \rangle} x(t) dt$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt$$

$$a_n = 2 \operatorname{Re}[c_n] = \frac{2}{T} \int_{\langle T \rangle} x(t) \cos n\omega_0 t dt$$

$$b_n = -2 \operatorname{Im}[c_n] = \frac{2}{T} \int_{\langle T \rangle} x(t) \sin n\omega_0 t dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(\omega) = 2 \int_0^{\infty} x(t) \cos \omega t dt$$

$$X(\omega) = -j2 \int_0^{\infty} x(t) \sin \omega t dt$$

$$X(s) = \int_0^{\infty} x(t) e^{-st} dt$$

$$x(t) = \frac{1}{j2\pi} \int_{c-j\infty}^{c+j\infty} X(s) e^{st} dt$$