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# EEE331 Analogue Electronics

## 8th lecture:

- operational amplifiers (Op-Amps), part II
  - frequency behaviour of 2-stage MOS Op-Amps
  - multi-stage Op-Amps
  - feedback theory



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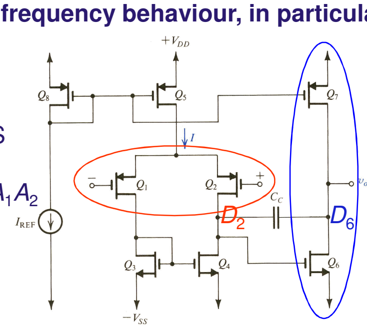
**2-stage MOS operational amplifier: frequency behaviour, I**

**Calculation of the frequency behaviour, in particular $\omega_{p1}$, $\omega_{p2}$, $\omega_t$ and $\omega_z$**

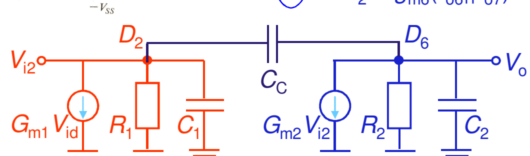
basic 2-stage CMOS op-amp with over-all open-loop gain $A_v = A_1 A_2$




1. stage: diff. amp. provides voltage gain $A_1 = -g_{m1}(r_{o2}||r_{o4})$

2. stage: class AB output stage provides voltage gain $A_2 = -g_{m6}(r_{o6}||r_{o7})$

small-signal equivalent circuit of the 2 stages



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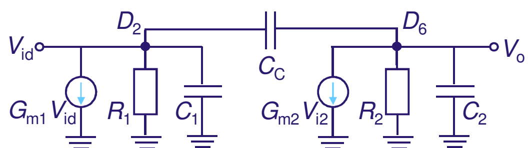
## 2-stage MOS operational amplifier: frequency behaviour, II

**Calculation of the frequency behaviour, in particular $\omega_{p1}$, $\omega_{p2}$, $\omega_t$ and $\omega_z$**

1. stage: transconductance: $G_{m1}=g_{m1}=g_{m2}$
   output resistance: $R_1=r_{o2}||r_{o4}$
   capacitance: $C_1= C_{DG2}+C_{DB2}+C_{DG4}+C_{DB4}+C_{GS6}$
   This is the total capacitance at the interface between the two stages.
2. stage: transconductance: $G_{m2}=g_{m6}$
   output resistance: $R_2=r_{o6}||r_{o7}$
   capacitance: $C_2=C_{DG6}+C_{DB6}+C_{DG7}+C_{DB7}+C_L$
   This is the total capacitance at the output node of the Op Amp and usually dominated by the load capacitance $C_L$, while $C_{DG6}<<C_C$.

small-signal equivalent circuit of the 2 stages



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## 2-stage MOS operational amplifier: frequency behaviour, III

**Calculation of the frequency behaviour, in particular $\omega_{p1}$, $\omega_{p2}$, $\omega_t$ and $\omega_z$**

equation for currents at node $D_2$: $G_{m1}V_{id}+V_{i2}/R_1+sC_1V_{i2}+sC_C(V_{i2}-V_o)=0$

equation for currents at node $D_6$: $G_{m2}V_{i2}+V_o/R_2+sC_2V_o+sC_C(V_o-V_{i2})=0$

solve 2nd equation for $V_{i2}$: $V_{i2}=V_o(1/R_2+sC_2+sC_C)/(sC_C-G_{m2})$

insert $V_{i2}$ into 1st equation to get: $V_{id}/V_o = \{sC_C/G_{m1}-[(1/R_2+sC_2+sC_C)\times(1/R_1+sC_1+sC_C)]/[(sC_C-G_{m2})G_{m1}]\}$

then re-arrange to get voltage gain and sort according to powers of $s$:

$$\boxed{V_o/V_{id}=[G_{m1}(G_{m2}-sC_C)R_1R_2] / \{1+s[C_1R_1+C_2R_2+C_C(R_1+R_2+G_{m2}R_1R_2)] +s^2 R_1R_2[C_1C_2+C_C(C_1+C_2)]\}}$$

is a transfer function with $A_{DC}=\lim_{s\to 0} V_o/V_{id}=(G_{m1}R_1)(G_{m2}R_2)$, as expected,

one transmission zero (zero of the nominator) for $\omega_Z=s_Z=G_{m2}/C_C$,

two poles $\omega_{p1}$ and $\omega_{p2}$ (roots of denominator) that we get from writing it as

$D(s)=(1+s/\omega_{p1})(1+s/\omega_{p2})=1+s(1/\omega_{p1}+1/\omega_{p2})+s^2/(\omega_{p1}\omega_{p2})\approx 1+ 1+s/\omega_{p1}+s^2/(\omega_{p1}\omega_{p2})$

dominant pole: $\omega_{p1}=\{R_1[C_1+C_C(1+G_{m2}R_2)]+R_2(C_2+C_C)\}^{-1} \approx 1/(R_1C_CG_{m2}R_2)$

note Miller enlargement of $C_C>>C_1$ in neg. feedback of 2nd stage whose gain is $G_{m2}R_2$

and second pole: $\omega_{p2}=(G_{m2}C_C)/[C_1C_2+C_C(C_1+C_2)] \approx G_{m2}/(C_1+C_2)$

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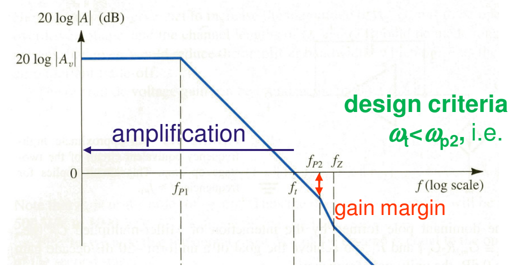
## 2-stage MOS operational amplifier: frequency behaviour, IV




**design criteria for stable Op-Amp:**
$\omega_t<\omega_{p2}$, i.e. $G_{m1}/C_C< G_{m2}/(C_1+C_2)$
**and**
$\omega_t<\omega_z$, i.e. $G_{m1}<G_{m2}$

Bode plot of loop gain

one transmission zero (zero of the nominator) for $\omega_Z=s_Z=G_{m2}/C_C$,
two poles $\omega_{p1}$ and $\omega_{p2}$ (roots of denominator) that we get from writing it as
$D(s)=(1+s/\omega_{p1})(1+s/\omega_{p2})=1+s(1/\omega_{p1}+1/\omega_{p2})+s^2/(\omega_{p1}\omega_{p2})\approx 1+s/\omega_{p1}+s^2/(\omega_{p1}\omega_{p2})$
dominant pole: $\omega_{p1}=\{R_1[C_1+C_C(1+G_{m2}R_2)]+R_2(C_2+C_C)\}^{-1}\approx 1/(R_1C_CG_{m2}R_2)$
uniform gain roll-off down to 0dB if $C_C$ is selected so that $\omega_t=\omega_{p1}\lim_{s\to 0}V_o/V_{id}\approx G_{m1}/C_C<\omega_{p2},\omega_z$
and second pole: $\omega_{p2}=(G_{m2}C_C)/[C_1C_2+C_C(C_1+C_2)]\approx G_{m2}/(C_1+C_2)$



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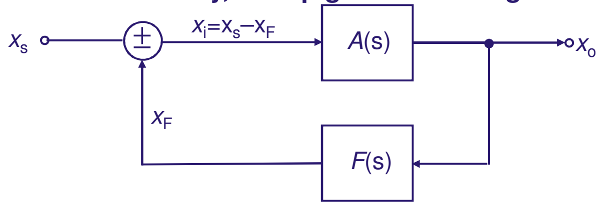
## formal feedback theory, I: loop gain and total gain




Consider a **non**-inverting amplifier where $x_s$ can be either a voltage or a current signal to be amplified. $A(s)$ denotes the -frequency dependent- amplification factor and $F(s)$ the feedback. Then we have:

$$\left.\begin{array}{l} x_i=x_s\mp x_F \\ x_o=A(s)\,x_i \\ x_F=F(s)\,x_o \end{array}\right\}\quad x_o=A(s)[x_s\mp F(s)x_o],\ \text{hence gain: } \boxed{G(s)=x_o/x_s=A(s)/[1\pm A(s)F(s)]}$$

loop gain

for large loop gain $A(s)F(s)>>1$: $G\approx A/(AF)=1/F(s)$ depends on feedback, rather than on the actual amplifier. This is a useful concept for all systems with finite input and zero output resistance (i.e. no load effects).

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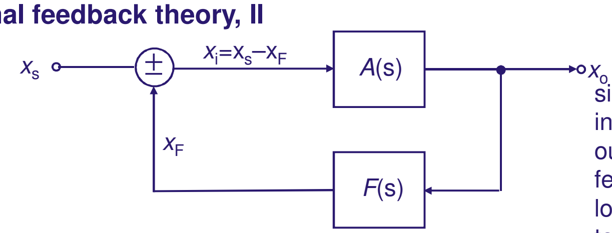
## formal feedback theory, II




signal: $x_s$
input: $x_i=x_s-x_F$
output: $x_o=A(s)\,x_i$
feedback: $x_F=F(s)x_o$
loop gain: $AF$
total gain: $A/(1+AF)$

Relationships for feedback and input signals:

$x_F=F(s)A(s)(x_s-x_F)$ -> $x_F=AF/(1+AF)\;x_s$

$x_i=x_s-F(s)A(s)\;x_i$ -> $x_i=1/(1+AF)\;x_s$

is reduced in the amplifier with negative feedback (to almost zero for large negative feedback)



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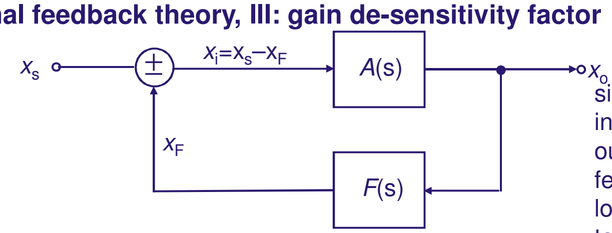
## formal feedback theory, III: gain de-sensitivity factor




signal: $x_s$
input: $x_i=x_s-x_F$
output: $x_o=A(s)\,x_i$
feedback: $x_F=F(s)x_o$
loop gain: $AF$
total gain: $A/(1+AF)$

calculation of the result of **change of gain**:

$G= A/(1+AF)$

-> $dG/dA= [1+AF-A(F+\underbrace{A\,dF/dA}_{=0 \text{ for constant } F})]\,/\,(1+AF)^2 = 1/(1+AF)^2$

-> $1/G\;dG/dA = 1/(1+AF)^2\;(1+AF)/A = 1/(1+AF)\;1/A$

-> $\boxed{dG/G = [1/(1+AF)]\;dA/A}$

The percentage change in total gain $G$ due to variations in some circuit parameter is thus smaller than the percentage change in $A$ by a factor of $(1+AF)$. This is known as the **de-sensitivity factor.**

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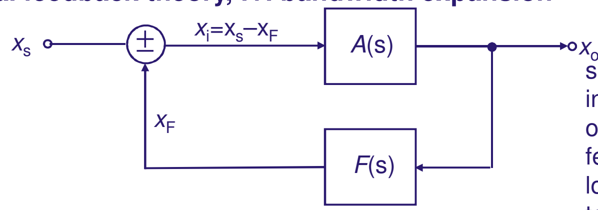
## formal feedback theory, IV: bandwidth expansion




signal: $x_s$
input: $x_i=x_s-x_F$
output: $x_o=A(s)\,x_i$
feedback: $x_F=F(s)x_o$
loop gain: $AF$
total gain: $A/(1+AF)$

calculation of the **bandwidth expansion**:

Consider an amplifier whose high-frequency response is characterised by a single pole and whose mid-band gain is $A_m$. Then the high-frequency gain is $A(s)=A_m/(1+s/\omega_H)$ where $\omega_H$ denotes the upper 3dB-frequency.

With negative feedback, with a frequency-**in**dependent factor $F$, this becomes

$$G(s)=A(s)/\,[1+FA(s)]$$
$$=A_m/\{[1+s/\omega_H][1+FA_m/(1+s/\omega_H)]\}$$
$$=A_m/[1+s/\omega_H+FA_m]$$
$$=A_m/(1+FA_m)\times 1/\{1+s/[\omega_H(1+FA_m)]\}$$

gain $\times$ bandwidth =constant

reduced mid-band gain increased upper 3-dB frequency $\omega_{Hf}=\omega_H(1+FA_m)$



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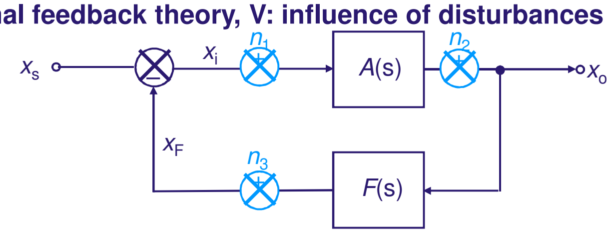
## formal feedback theory, V: influence of disturbances




consider influence of disturbances due to **additive noise**:

The **effect will depend on where in the circuit the noise is added**:

output due to $n_1$: $x_o=A(x_s-x_F+n_1)=A(x_s-Fx_o+n_1) = A/(1+AF)x_s + A/(1+AF)n_1$

output due to $n_2$: $x_o=A(x_s-x_F)+n_2=A(x_s-Fx_o)+n_2 = A/(1+AF)x_s + 1/(1+AF)n_2$

output due to $n_3$: $x_o=A(x_s-x_F)=A(x_s-Fx_o-n_3) = \underbrace{A/(1+AF)x_s}_{\text{signal}} \underbrace{-\,A/(1+AF)n_3}_{\text{noise}}$

Disturbance at the input and in the feedback loop are amplified by the same amount as the signal, disturbances at the output end are suppressed.

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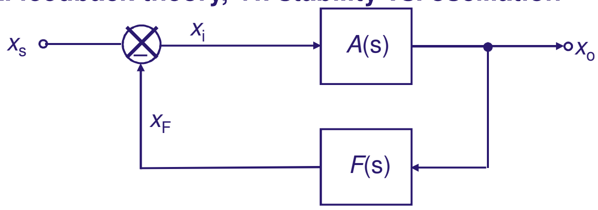
## formal feedback theory, VI: stability vs. oscillation




The closed-loop transfer function $G(s)=A(s)/[1+A(s)F(s)]$ is a complex function for physical frequencies $s=j\omega$: $G(j\omega)=A(j\omega)/[1+A(j\omega)\ F(j\omega)]$,
and so is the loop gain $A(j\omega)\ F(j\omega)=|A(j\omega)\ F(j\omega)|\ \exp\ j\phi(\omega)$
If the loop gain $|AF|\geq 1$ for any frequency where the phase shift around the loop (i.e. between $x_f$ and $x_s$) is 0º or 360º, then the system will become unstable.
The **inverting summer** which subtracts $x_F$ from $x_s$ at the input (i.e. negative feedback) **already introduces a 180º phase shift** between these signals.
Another 180º phase shift will make the feedback positive, and the circuit will start to oscillate. Hence, the **phase lag for stable amplification must be $0< |\phi(\omega)| <180º$ for all frequencies with $|A(\omega)F(\omega)|\geq 1$,** i.e. magnitude ≥ 0dB.



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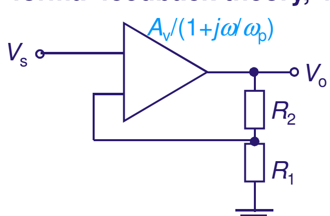
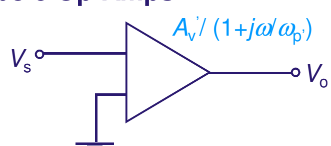
## formal feedback theory, VII: single-pole Op-Amps




voltage gain:

$$G=V_o/V_s=\frac{A_v/(1+j\omega/\omega_p)}{1+[A_v/(1+j\omega/\omega_p)]\ R_1/(R_1+R_2)} \equiv A_v'/(1+j\omega/\omega_{p'}) \quad \text{where}$$

$$= A_v/[1+j\omega/\omega_p+A_vR_1/(R_1+R_2)]$$
$$= A_v/[1+A_vR_1/(R_1+R_2)+j\omega/\omega_p]$$
$$=\frac{A_v/[1+A_vR_1/(R_1+R_2)]}{1+j\omega/\{\omega_p\ [1+A_vR_1/(R_1+R_2)]\}}$$
$$= \{A/(1+AF)\}/\{1+j\omega/[\omega_p(1+AF)]\}$$
$$= A/\ \{(1+AF)(1+j\omega/[\omega_p(1+AF)])\}$$

$A_v'=A_v/[1+A_vR_1/(R_1+R_2)] \approx (R_1+R_2)/R_1$
for $A_vR_1/(R_1+R_2)>>1$
and
$\omega_p'=\omega_p[1+A_vR_1/(R_1+R_2)]$
Note again $A_v'\omega_{p'}=A_v\omega_p$=constant

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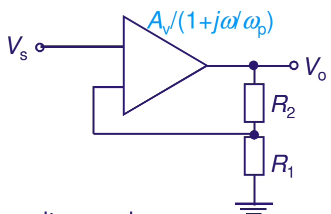
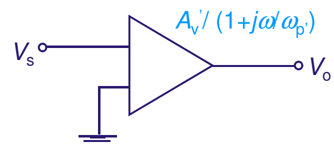
## formal feedback theory, VII: single-pole Op-Amps




voltage gain:

$$G = V_o/V_s = \frac{A_v/(1+j\omega/\omega_p)}{1+[A_v/(1+j\omega/\omega_p)]\,R_1/(R_1+R_2)}$$
$$= A_v/[1+j\omega/\omega_p + A_v R_1/(R_1+R_2)]$$
$$= A_v/[1+A_v R_1/(R_1+R_2) + j\omega/\omega_p]$$
$$= \frac{A_v/[1+A_v R_1/(R_1+R_2)]}{1+j\omega/\{\omega_p\,[1+A_v R_1/(R_1+R_2)]\}}$$
$$= \{A/(1+AF)\}/\{1+j\omega/[\omega_p(1+AF)]\}$$
$$= A/\{(1+AF)(1+j\omega/[\omega_p(1+AF)])\}$$

advantages:
- **yields max. phase shift of 90º, i.e. single-pole Op-Amps with resistive feedback cannot oscillate and are always stable!**
- hence easy to use

disadvantages:
- low unity gain bandwidth (few MHz)
- poor slewing rates



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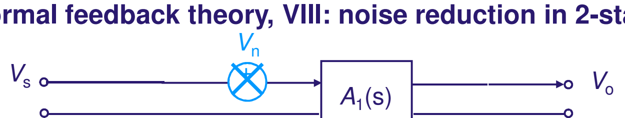
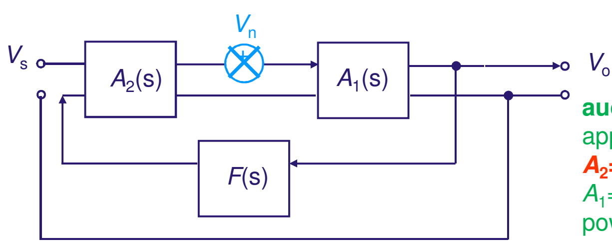
## formal feedback theory, VIII: noise reduction in 2-stage Op-Amps




signal-to-noise ratio of amplifier with input noise but without feedback: $S/N = V_s/V_n$




**audio amplifier** as an application example:
**$A_2$=noise-free pre-amp.**
$A_1$= power amp. with power-supply hum

signal-to-noise ratio of 2-stage amplifier with feedback where another noise-free stage precedes the noisy stage:
$V_o = V_s A_1 A_2/(1+A_1 A_2 F) + V_n\, A_1/(1+A_1 A_2 F)$ -> **$S/N = V_s A_2/V_n$ is $A_2$-times larger**

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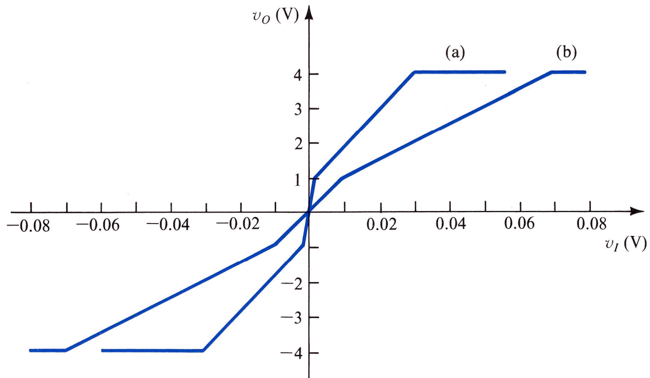
**formal feedback theory, IX: distortion reduction in 2-stage Op-Amps**




(a): class B amplifier with large cross-over distortion where the voltage gain $A_v$ is piecewise linear but changes from 1000 to 100 and then 0.
(b): the gain is reduced to $G=A/(1+AF)$ by negative feedback with $F$=0.01:
$A_{f1}$=1000/(1+1000×0.01)=90.9, $A_{f2}$=100/(1+100×0.01)=50 is much better



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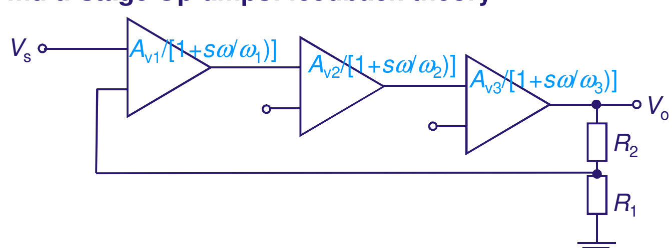
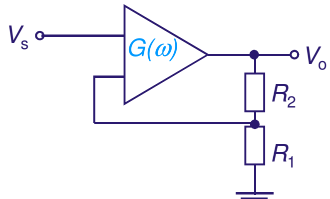
**multi-stage Op-amps: feedback theory**




gain: $G(\omega)=A_v/[(1+s/\omega_1)(1+s/\omega_2)(1+s/\omega_3)]$
where $A_v=A_{v1}\,A_{v2}\,A_{v3}$
if $\omega_1$, $\omega_2$ and $\omega_3$ are all well separated

note:
- most Op-Amps have 2 or 3 poles
- extra poles can be introduced by feedback loops

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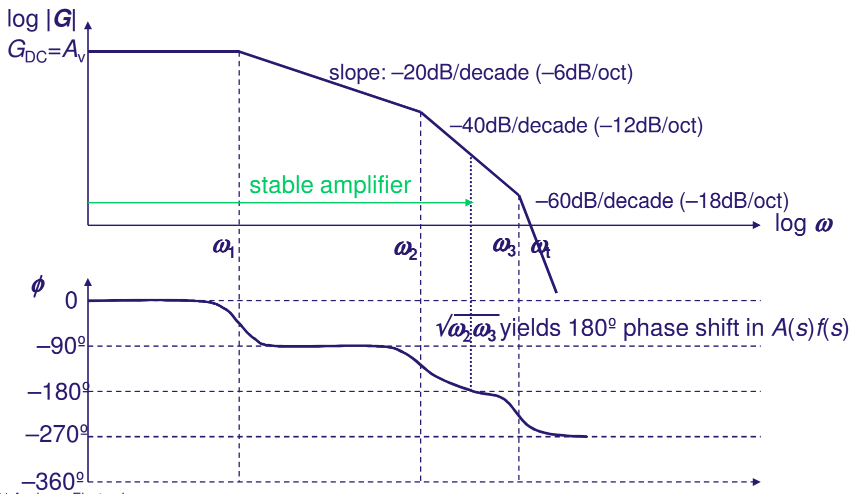
## multi-stage Op-amps: Bode plot of a bad 3-stage Op-Amp. design



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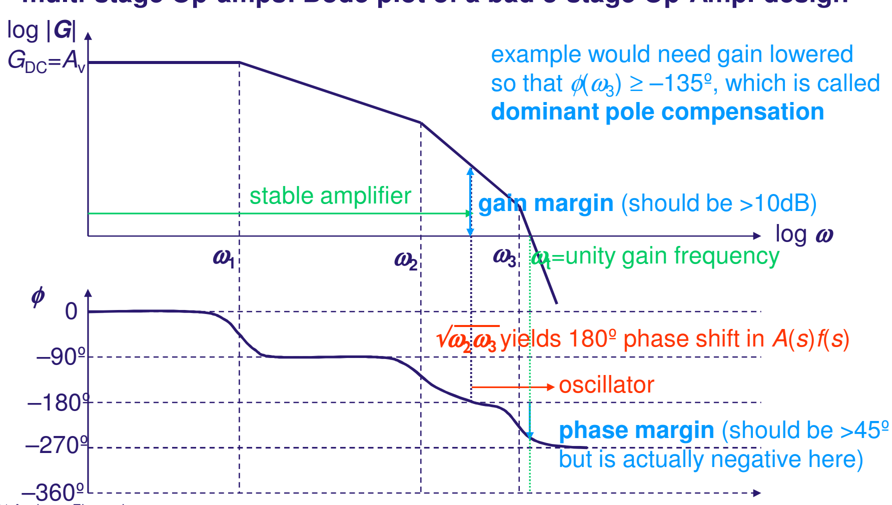

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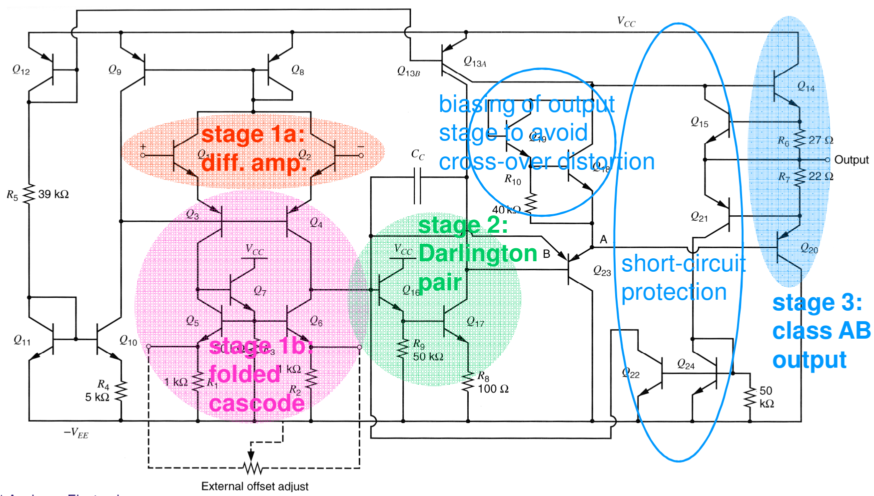
## multi-stage Op-Amps: the 741 Op-Amp



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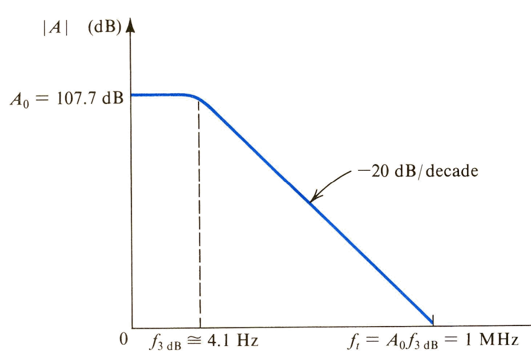
## multi-stage Op-Amps: the 741 Op-Amp




small-signal voltage gain:

$G=v_o/v_i= (v_{i2}/v_i)\,(v_{o2}/v_{i2})\,(v_o/v_{o2})$

$= -G_{m1}\,(R_{o1}||R_{i2})$

$\times(-G_{m2}R_{o2})\,G_{vo3}\,R_L/(R_L+R_{out})$

$=(-483)\times(-527)\times0.964$

$=2.44\times10^5$ V/V $=$ **107.7dB**

for $G_{m1}$=0.19 mA/V, $G_{m2}$=6.5 mA/V, $G_{vo3}\approx1$, $R_{o1}$=6.7MΩ, $R_{i2}$=4.1MΩ, $R_{o2}$=81kΩ, $R_{out}\approx75\Omega$, $R_L$=2kΩ

dominant pole frequency:

$f_p=1/(2\pi C_{in}R_t)$=**4.1Hz**

where $C_{in}=C_C(1+G_{m2})$=15.5nF

and $R_t=(R_{o1}||R_{i2})$=2.54MΩ

unit-gain bandwidth:

$f_t=Gf_p$=**1.0MHz**