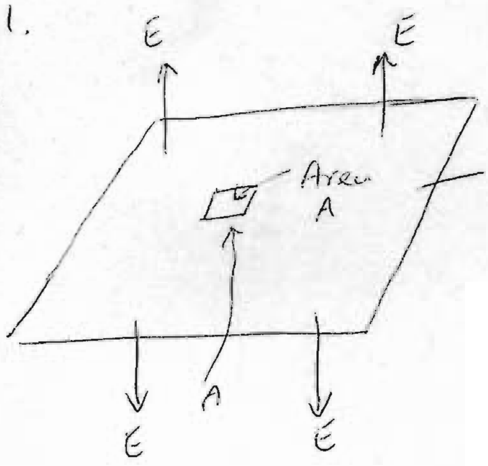
Q1.

a)



∞ sheet (Charge density / unit area $= q_s$)

By symmetry E is perpendicular to the sheet and independent of the lateral position and has the same value on either side of the sheet.

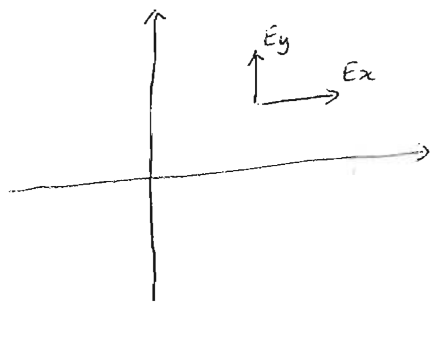
Gauss' Law $\oint_S E\,dA = \frac{q_s A}{\varepsilon_0}$ ← Enclosed charge

$$\therefore \quad \underbrace{E \times 2A}_{\text{Contribution from top and bottom surfaces}} = \frac{q_s A}{\varepsilon_0}$$

$$\therefore \quad E = \frac{q_s}{2\varepsilon_0}$$

5

b)



$E_y = E_x$ = field due to infinite sheet.

$$= \frac{q_s}{2\varepsilon_0}$$

$$\frac{q_s}{2\varepsilon_0} = \frac{5\times10^{-6}}{2\times 8.854\times10^{-12}} = 2.82\times10^{5}\ \text{V/m}$$

E field at :-

$A = (2.82,\ 2.82,\ 0) \times 10^5$ V/m

$B = (2.82,\ 2.82,\ 0) \times 10^5$ V/m

$C = (2.82,\ -2.82,\ 0) \times 10^5$ V/m

5

c) Ampères Law states that the line integral of the B-field around a closed path is equal to μ_0 times the current linking the path:

$$\oint B \cos\theta \, dl = \mu_0 I$$

Applying this to a circular path around an infinitely long wire, then at each point on the path $\theta = 0$ (i.e. $\cos\theta = 1$); and by symmetry B is constant. θ is the angle between the B-field and the direction of the path.

$$\therefore B \cdot 2\pi r = \mu_0 I$$

$$\text{or } B = \frac{\mu_0 I}{2\pi r}$$

4

d) Since the field at a distance r from the wire M is

$$B = \frac{\mu_0 I}{2\pi r}$$

The flux linking the circuit can be found by integration

$$\phi = \int B(r)\,da = \int_{z_1}^{z_2}\int_{r_1}^{r_2} B \, dr\, dz = \int_{z_1}^{z_2}\int_{r_1}^{r_2} \frac{\mu_0 I}{2\pi r} \, dr\, dz$$

$$= (z_2 - z_1) \cdot \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

The Induced emf $E = \frac{d\phi}{dt}$

$$E = (z_2 - z_1)\frac{\mu_0}{2\pi}\ln\left(\frac{r_2}{r_1}\right)\frac{dI}{dt}$$

$z_2 - z_1 = 80\times10^{-3}$ m

$\ln(r_2/r_1) = \ln 3 = 1.099$

$$\frac{dI}{dt} = \frac{d}{dt} I_0 \sin\omega t = \omega I_0 \cos\omega t$$
$$= 2\pi . 50 . 4 \text{ A s}^{-1} \text{ rms}$$

$$\therefore E = 80\times10^{-3} \times \frac{\cancel{4}^{2}\pi\times10^{-7}}{\cancel{2\pi}} \times 1.099 \times 400\pi$$

$$= \underline{\underline{22\,\mu V}} \text{ rms}$$

6

Q2

a) (i) Initially the reluctance of the magnetic circuit may be calculated from:-

$$S = \frac{l}{\mu_0 \mu_r A} = \frac{\pi D}{\mu_0 \mu_r A} = \frac{\pi \, . \, 0.2}{4\pi \times 10^{-7} \times 1000 \times 10 \times 10^{-4}} = \underline{\underline{5 \times 10^5 \, H^{-1}}}$$

(2)

(ii) The flux in the core is related to the flux density by:

$$\phi = B . A$$

But

$$NI = \phi S$$

$$\therefore I = \frac{B.A.S}{N} = \frac{1 \times 10 \times 10^{-4} \times 5 \times 10^5}{1000}$$

$$= \underline{\underline{0.5 A}}$$

(2)

(iii) Self-inductance of the coil may be obtained from:

$$L = \frac{N^2}{S} = \frac{1000^2}{5 \times 10^5} = \underline{\underline{2 H}}$$

(2)

(iv) After the slot is cut the new reluctance of the magnetic circuit is given by:-

$$S_{new} = \frac{l_{core}}{\mu_0 \mu_r A} + \frac{l_{gap}}{\mu_0 A} = \frac{1}{4\pi \times 10^{-7} \times 10 \times 10^{-4}} \times \left(\frac{\pi . 0.2 - 0.01}{1000} + 0.01 \right)$$

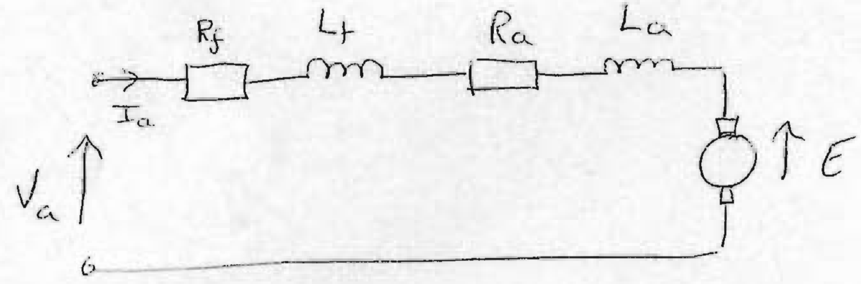
$$= 8.45 \times 10^6 \, H^{-1}$$

and the new self inductance of the coil is:

$$L_{new} = \frac{1000^2}{8.45 \times 10^6} = \underline{\underline{0.118 \, H}}$$

(3)

b)



For wound field

$I_f = I_a$

In steady-state (i.e. D.C.) L_a and L_f can be omitted.

$V = I_a(R_f + R_a) + E.$

$T = M.I_f.I_a = MI_a^2$

$E = M.I_f.\omega = MI_a\omega$

I_f = Field current

$= I_a$ = Armature current

ω = rotational speed (rad/s)

M = Mutual coupling between windings.

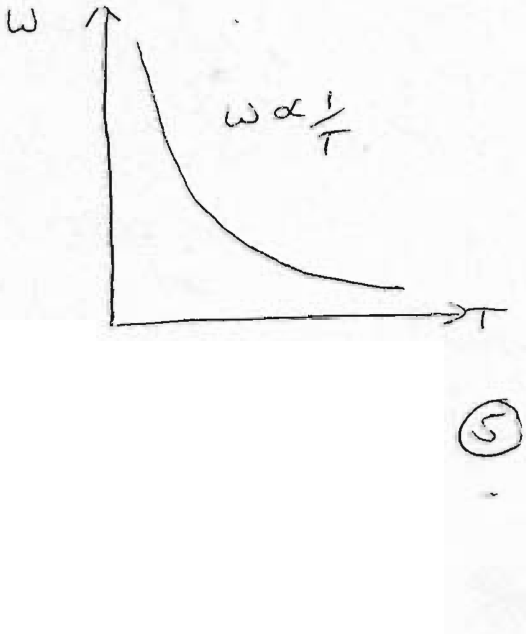
Ignoring voltage drops across R_f and R_a

$$E \simeq V_a \simeq MI_a\omega$$

$$V_a \simeq M\sqrt{\frac{T}{M}}\,\omega$$

$$\therefore \omega \simeq \frac{V_a}{\sqrt{TM}}$$

$$T \propto I_a^2$$



5

c) (i) $R_f = 1\Omega$ $R_a = 0.2\Omega$ $M = 1V/rad/s$

$T_{LOAD} = 0.01\omega^2$

$T_{MOTOR} = MI_a^2 = 1.I_a^2$

At steady-state $T_{LOAD} = T_{MOTOR}$

$0.01\omega^2 = I_a^2 \Rightarrow \omega = 10I_a$

Now $V = I_a(R_a + R_f) + M I_a \omega$

$\therefore \; 12 = I_a(R_a + R_f) + 1 . I_a . 10 I_a$

$12 = 1.2 I_a + 10 I_a^2$

$\underline{\underline{I_a = 1.037\,A}}$

$T = M I_a^2 = 1.\ 1.037^2 = 0.01\omega^2$

$\therefore \; \omega = \sqrt{\dfrac{1.037^2}{0.01}} = 10.37\,rad/s = \underline{\underline{99\,rpm}}$ 4

(ii)

For 10V Supply

$10 = 1.2 I_a + 10 I_a^2$

$\therefore \; I_a = 0.942\,A$

$\omega_{new} = \sqrt{\dfrac{0.942^2}{0.01}} = 9.42\,rad/s = 90\,rpm$

i.e. Motor slows down by 9 rpm.

3a.

p-type has resistivity $\rho = 2.5\times10^{-3}$

$$\rho = \frac{1}{ep\mu_h (+ en\mu_n)}$$

$\downarrow \rightarrow 0.$

$$\rho = \frac{1}{ep\mu}$$

$p = 5\times10^{22} = N_A \qquad n.p = n_i^2 \quad n = 2\times10^9.$

For compensated region

$n.p = n_i^2 \qquad N_A + n = p + N_d.$

$$N_A - N_d = p - n$$

$p = \frac{n_i^2}{n} \qquad N_A - N_D = \frac{n_i^2}{n} - n$

$$n^2 + (N_A - N_D)n - \frac{n_i^2}{2} = 0$$

Solution $n = \frac{N_D - N_A}{2} \pm \frac{\sqrt{(N_A - N_D)^2 - 4n_i^2}}{2}$

insignificant

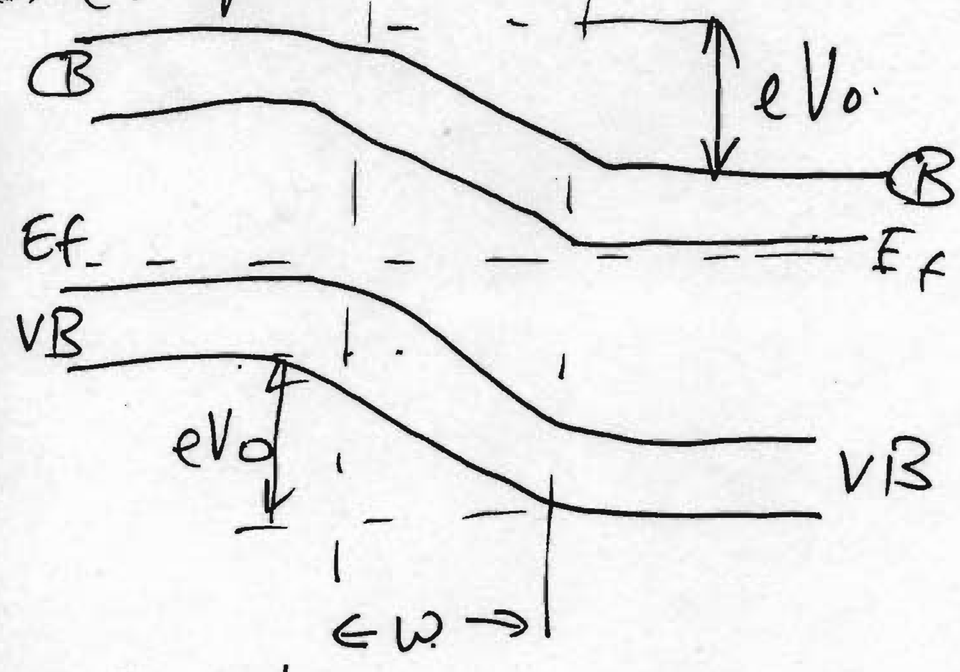
1 solution $\frac{N_D - N_A}{2} + \frac{N_A - N_D}{2} = 0$

other $n = N_D - N_A = 5\times10^{22}\ cm^{-3}$

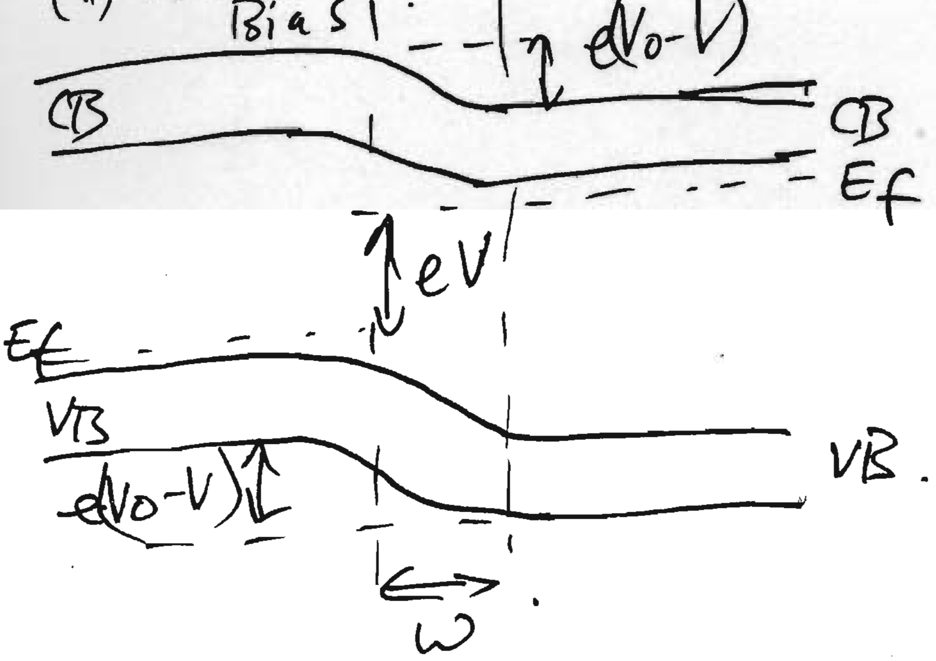
$p = 2\times10^9$

(8)

(3b) (i) Equilibrium.



(ii) Forward Bias



(3c) $I = I_0\left(e^{\frac{eV}{KT}} - 1\right)$

Saturation current = 1nA

$T = 298K.$

$I = 1\times10^{-9}\left(e^{19.44} - 1\right)$

$= 277mA.$

3d.

$$i = \eta \frac{eP}{\hbar\omega} \quad \text{but} \quad E = \hbar\omega$$

$E = 2.2\,eV$

and $\frac{\hbar\omega}{e} =$ E in eV

$$= \eta \frac{P}{E(eV)}$$

$$= 227\,\mu A.$$

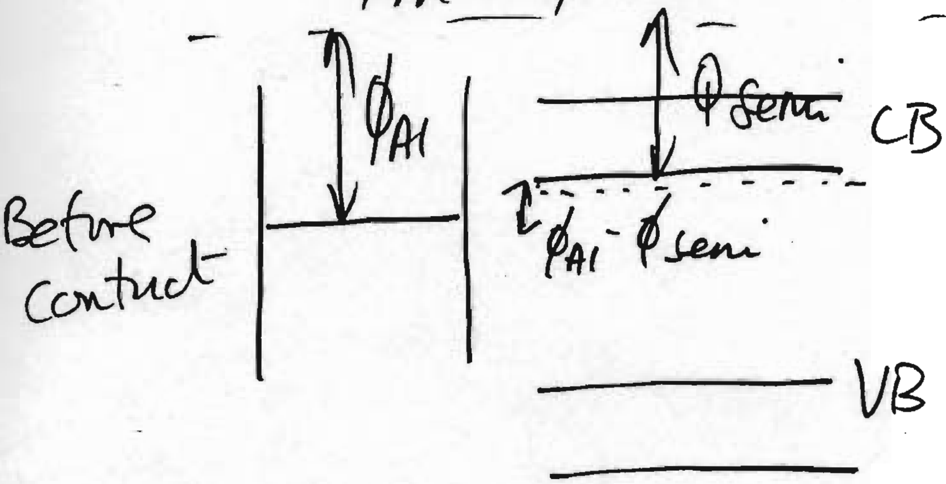
$E = 0.8\,eV \quad E < E_g \; (1.1\,eV)$

No photocurrent.

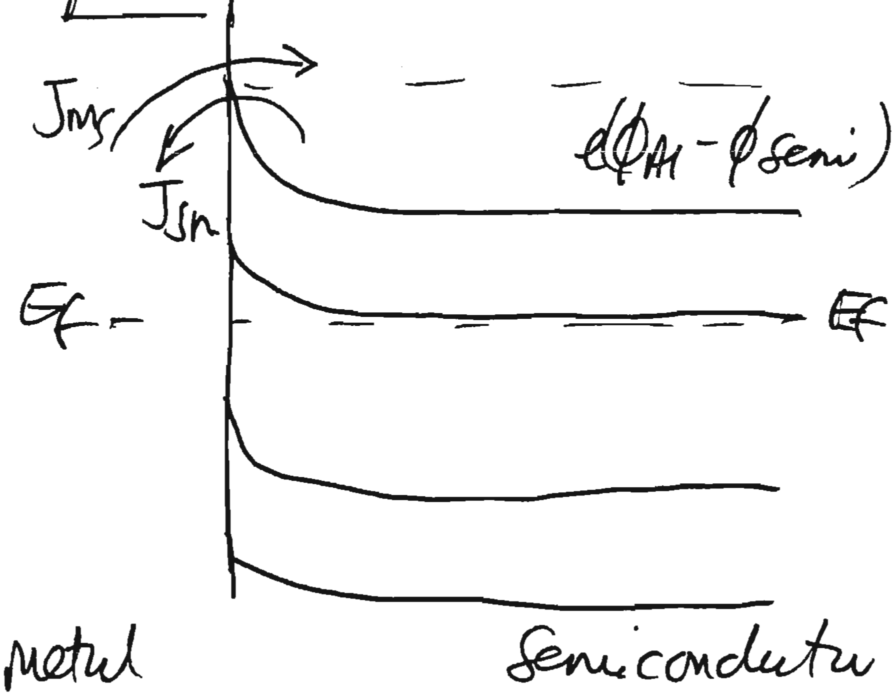
$I \rightarrow I_o \; (1 \times 10^{-9} A).$

4. $\phi_{Al} = 5.4\,eV \quad \phi_{semi} = 4.5$

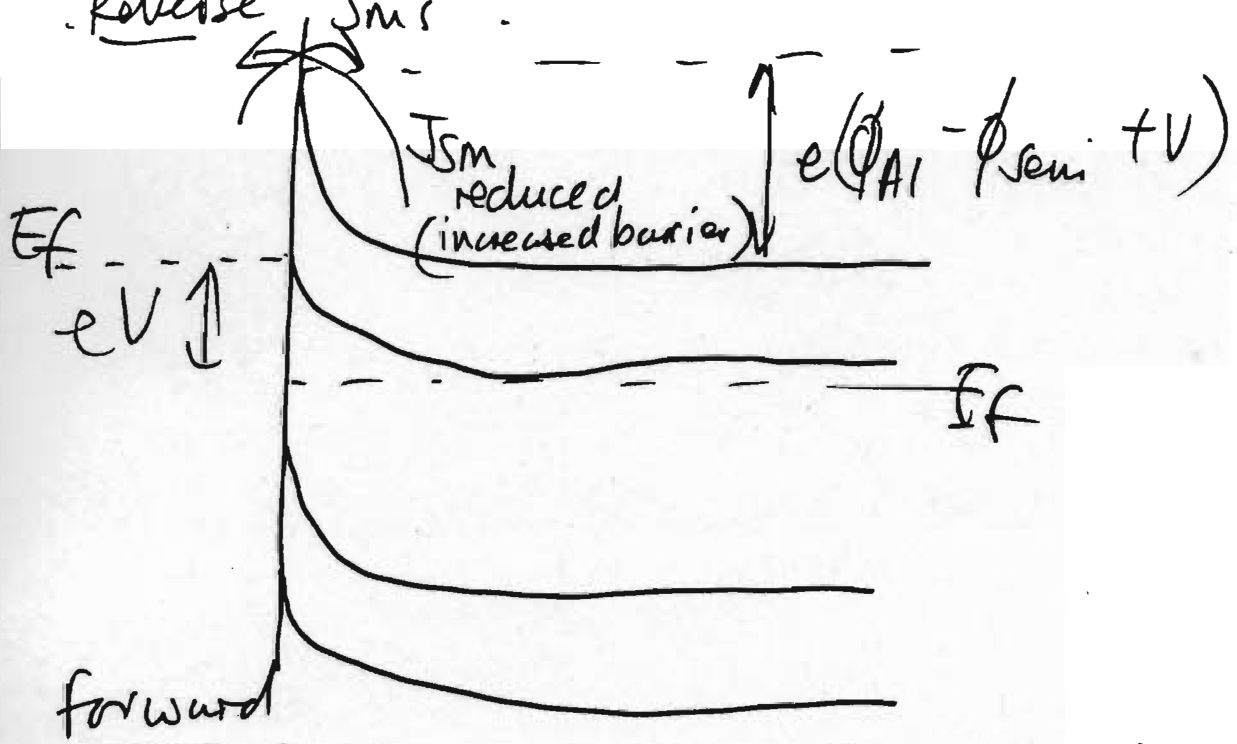
$\underline{\phi_m > \phi_{semi}} \; \therefore$ <u>schottky</u>.



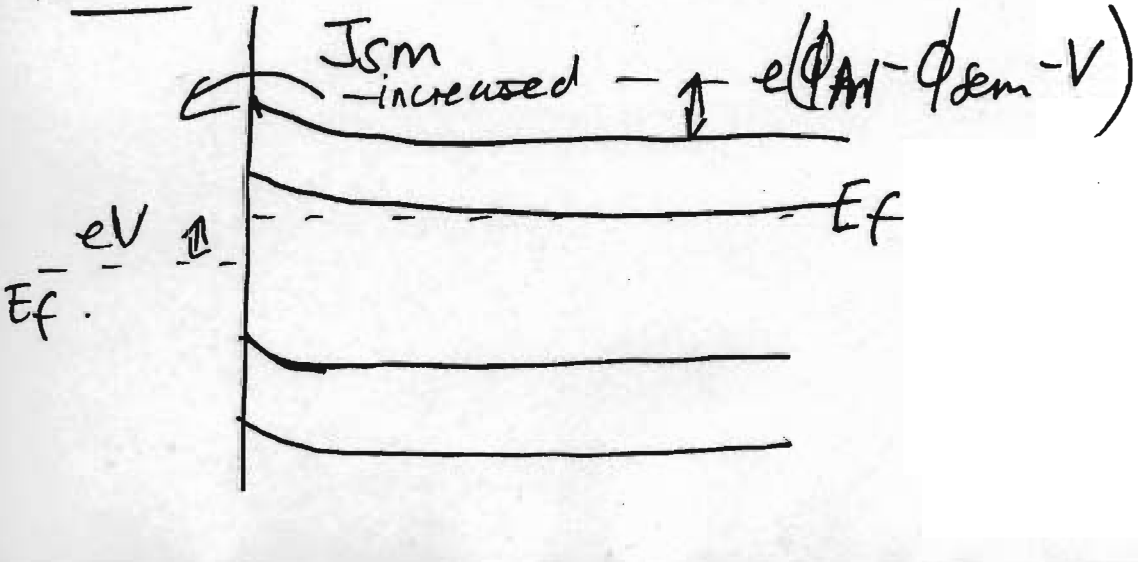
equilibrium



Reverse



forward



4b $$I_d = \frac{\mu_e C_g}{l^2} \left[V_g - V_T - \frac{V_d}{2} \right] V_d.$$

Saturation condition $V_d = V_g - V_t$.

$$[\quad] \rightarrow \left[V_g - V_T - \frac{V_g}{2} + \frac{V_t}{2} \right] (V_g - V_t)$$

$$\frac{V_g}{2} - \frac{V_t}{2} \; (V_g - V_t)$$

$$\frac{(V_g - V_t)^2}{2}.$$

$$\frac{\mu_e C_g}{l^2} = 60.$$

(i) $V_g = 1.0\,V$ $\quad V_g - V_T = 0.2V \quad \frac{(V_g - V_t)^2}{2} = 0.02$

(ii) " $2.0V$ $\quad V_g - V_T = 1.2V \quad$ " $= 0.72$

(iii) " $3.0V$ $\quad V_g - V_T = 2.2V \quad$ " $= 2.42$

(i) So $I_{ds} = 1.2$

(ii) $I_{ds} = 43.2$

(iii) $I_{ds} = 145.2$

4C

Transconductance $= \frac{dI_d}{dV_{dsat}}$. $I_{ds} = \frac{\mu_e C_g}{L^2} \frac{(V_g - V_t)^2}{2}$

Differentiate. but. $V_g - V_t = V_{dsat}$.

$$dI_{ds} = \frac{\mu_e C_g}{L^2} V_{dsat} . dV_{dsat}.$$

(ds)

$$g_m = \frac{dI_{ds}}{dV_{ds}} = \left(\frac{\mu_e C_g}{L^2}\right) V_{ds}.$$

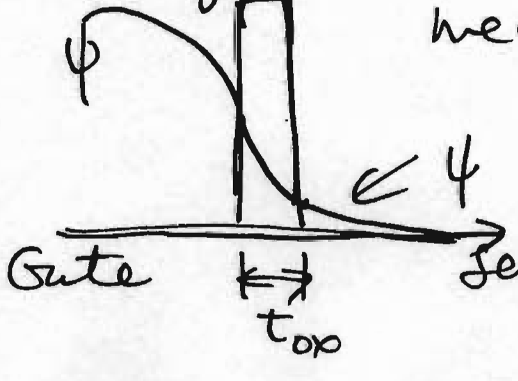
= 60 (from before)

(i) $V_g - V_T = V_{ds} = 0.2V$ $g_m = 12$

(ii) " " 1.2V $g_m = 72$

(iii) " " 2.2V $g_m = 132$

4d. Electrons will tunnel through gate oxide if thin enough. Mechanism is Quantum mechanical tunneling.

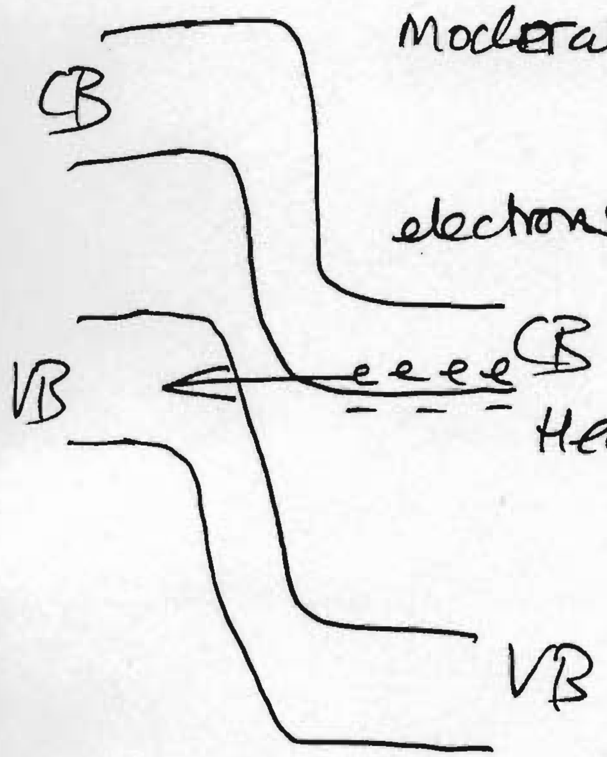


wave function within the gate oxide will take the form of an exponential decay

$$\psi(x) = Ae^{-\alpha x}.$$

constant

Other type of device is a Zener diode



Moderate reverse bias

electrons tunnelling into VB.

Heavily doped p and n
lots of carrier.

Application is as a voltage reference
Tunnelling current holds device at a certain reverse bias.

SECTION C

5.

a. (i) The relationship between collector current, I_C, and base emitter voltage, V_{BE}, for a bipolar junction transistor is given by:

$$I_C = I_{CO}\left(\exp\left(\frac{eV_{BE}}{kT}\right)-1\right)$$

where I_{CO}, e, k and T are constants. Show that the mutual conductance, g_m, of the transistor is given by:

$$g_m = \frac{eI_C}{kT}$$ (6)

$$g_m = \frac{dI_C}{dV_{BE}} = \frac{d}{dV_{BE}}\left[I_{CO}\left(\exp\left(\frac{eV_{BE}}{kT}\right)-1\right)\right] = \frac{eI_{CO}}{kT}\exp\left(\frac{eV_{BE}}{kT}\right)$$

For forward bias, $I_C = I_{CO}\left(\exp\left(\frac{eV_{BE}}{kT}\right)-1\right) \approx I_{CO}\exp\left(\frac{eV_{BE}}{kT}\right)$

$$\therefore \frac{eI_{CO}}{kT}\exp\left(\frac{eV_{BE}}{kT}\right) \approx \frac{eI_C}{kT}$$

(ii) Show how the g_m may be expressed as $g_m = \frac{\beta}{r_{be}}$. (3)

$$r_{be} = \frac{\partial V_{BE}}{\partial I_B} \quad \text{and} \quad \beta = \frac{\partial I_C}{\partial I_B}$$

$$\therefore g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{\partial I_C}{\partial I_B} \times \frac{\partial I_B}{\partial V_{BE}} = \frac{\beta}{r_{be}}$$

b. (i) For the common emitter amplifier circuit of figure 5, assuming I_B is negligible, calculate the d.c. bias conditions, V_B, V_E, V_C and I_C, and hence the small signal parameters g_m and r_{be}.

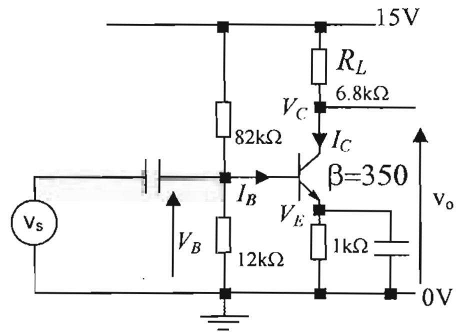


Figure 5

(6)

$$V_B = 15 \times \frac{12\text{k}\Omega}{12\text{k}\Omega + 82\text{k}\Omega} = 1.91\,\text{V}$$

$$V_E = V_B - 0.7 = 1.21\,\text{V}$$

$$I_E = \frac{V_E}{1\text{k}\Omega} = 1.21\,\text{mA}$$

If $I_C \approx I_E$ (I_C assumed negligible)$: I_C \approx 1.21\text{mA}$

$$V_C = 15 - I_C R_L = 15 - 1.62\text{mA} \times 6.8\text{k}\Omega = 6.74\,\text{V}$$

$$g_m = \frac{eI_C}{kT} = \frac{1.21\text{mA}}{0.026} = 46.5 \times 10^{-3}\ \text{A/V}$$

$$r_{be} = \frac{350}{46.5 \times 10^{-3}} = 7521 = 7.52\,\text{k}\Omega$$

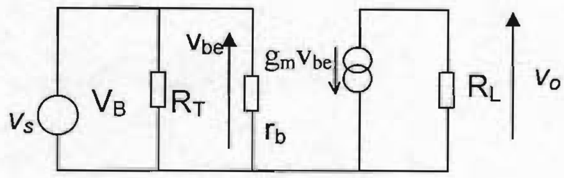
(ii) Draw a small signal equivalent circuit of the transistor and the surrounding circuitry and determine the voltage gain v_o/v_s of the amplifier. **(5)**

$$R_T = 12\text{k}\Omega // 82\text{k}\Omega = 10.47\,\text{k}\Omega$$

$$v_o = -g_m v_{be} \times R_L$$

$$\frac{v_o}{v_s} = -46.5 \times 10^{-3} \times 6.8\text{k}\Omega$$

$$\frac{v_o}{v_s} = -46.5 \times 10^{-3} \times 6.8\text{k}\Omega = 316\ \text{V/V}$$



You should assume that $V_{BE} = 0.7$V, that $kT/e = 0.026$ and that all capacitors are short circuit to a.c. signals.

6.

a. (i) Figure 6a shows a network consisting of noisy resistors, a noise voltage source and a noise current source.

Find the noise free resistance R_{Th} and the mean - square noise voltage v_{nTh}^2 (in terms of $V^2\,Hz^{-1}$) which form the Thevenin equivalent of the noisy network.

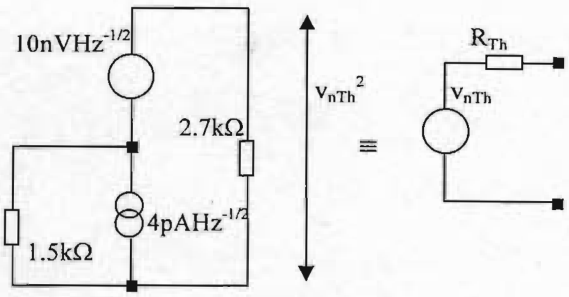


Figure 6a

(8)

R_{Th}, by inspection, is 1.5kΩ//2.7kΩ = $\frac{1.5k\Omega \times 2.7k\Omega}{1.5k\Omega + 2.7k\Omega} = 964\Omega$.

Using superposition to find the contribution of each noise source:

Due to 10nV: $\overline{v_{n(1)}^2} = (10\text{nV})^2 \times \left(\frac{2.7\text{k}\Omega}{2.7\text{k}\Omega + 1.5\text{k}\Omega}\right)^2 = 4.133 \times 10^{-17}\,\text{V}^2\text{Hz}^{-1}$

Due to 4pA: $\overline{v_{n(2)}^2} = (4\text{pA})^2 \times \left(\frac{2.7\text{k}\Omega \times 1.5\text{k}\Omega}{2.7\text{k}\Omega + 1.5\text{k}\Omega}\right)^2 = 1.488 \times 10^{-17}\,\text{V}^2\text{Hz}^{-1}$

Due to 1.5kΩ: $\overline{v_{n(3)}^2} = 4kT(1.5\text{k}\Omega) \times \left(\frac{2.7\text{k}\Omega}{2.7\text{k}\Omega + 1.5\text{k}\Omega}\right)^2 = 1.027 \times 10^{-17}\,\text{V}^2\text{Hz}^{-1}$

Due to 2.7kΩ: $\overline{v_{n(4)}^2} = 4kT(2.7\text{k}\Omega) \times \left(\frac{1.5\text{k}\Omega}{2.7\text{k}\Omega + 1.5\text{k}\Omega}\right)^2 = 5.707 \times 10^{-18}\,\text{V}^2\text{Hz}^{-1}$

Total noise, $\overline{v_{nTh}^2} = \overline{v_{n(1)}^2} + \overline{v_{n(2)}^2} + \overline{v_{n(3)}^2} + \overline{v_{n(4)}^2} = 7.22 \times 10^{-17}\,\text{V}^2\text{Hz}^{-1}$

$\sqrt{\overline{v_{nTh}^2}} = 8.5 \times 10^{-9}\,\text{VHz}^{-1/2} = 8.5\text{nVHz}^{-1/2}$

(ii) If v_{nTh}^2 is assumed to be thermal noise generated by a noisy R_{Th}, what is the effective noise temperature of R_{Th}?

$4kR_{Th}T_E = 7.22 \times 10^{-17}\,\text{V}^2\text{Hz}^{-1}$

$\therefore T_E = \frac{7.22 \times 10^{-17}}{4 \times k \times 964} = 1356\,\text{K}$

(3)

The mean square thermal noise voltage generated by a resistor R is $4kTR$ $V^2\,Hz^{-1}$ where $k = 1.38 \times 10^{-23}$. The ambient temperature, T, is 300K throughout the question.

b. **(i)** The manufacturer's data for the single pole operational amplifier in the circuit of figure 6b specifies a gain bandwidth product of 150MHz, a slew rate of $300\text{V}\mu\text{s}^{-1}$.

What is the small signal upper -3dB frequency of the circuit of figure 6b?

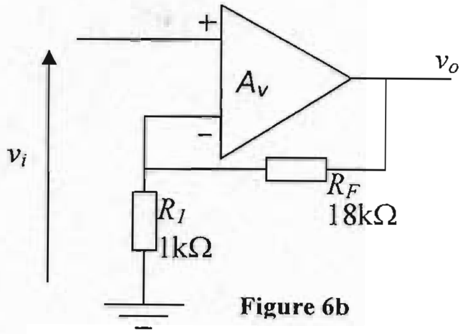


Figure 6b

(3)

Assuming A_V>>1:

Gain, $k = 1 + \frac{R_F}{R_1} = 19$

GBP = 150MHz, therefore:

-3dB bandwidth is $\frac{150\text{MHz}}{19} = 7.89\text{MHz}$

(ii) What is the maximum frequency that can be amplified by the circuit of figure 1 if a 20V peak to peak sinusoidal output signal is to remain unaffected by slew rate effects? **(4)**

Slew rate effects occur if maximum rate of change of output voltage exceeds the slew rate.

If $v(t) = \hat{V}\sin\omega t$, *then* $\frac{dv}{dt} = \hat{V}\omega\cos\omega t$

Max rate of change occurs when cos ωt = 1.

If peak to peak voltage is 20V, $\hat{V} = 10$.

$\hat{V}\omega = \text{slew rate, SR} = 300\times10^6 = 2\pi\hat{V}f$

$\therefore f = \frac{\text{SR}}{2\pi\hat{V}} = \frac{300\times10^6}{2\pi\times10} = 4.77\times10^6$

(iii) Find the gain magnitude at a frequency of 5MHz **(2)**

Circuit is first order, so transfer function can be written as:

$$\frac{v_o}{v_i} = \frac{19}{1+\frac{jf}{7.89\text{MHz}}}$$

Therefore at 5MHz, gain magnitude is $\left|\frac{v_o}{v_i}\right| = \left|\frac{19}{1+\frac{j5\text{MHz}}{7.89\text{MHz}}}\right| = \frac{19}{\sqrt{1+(0.634)^2}} = 16$

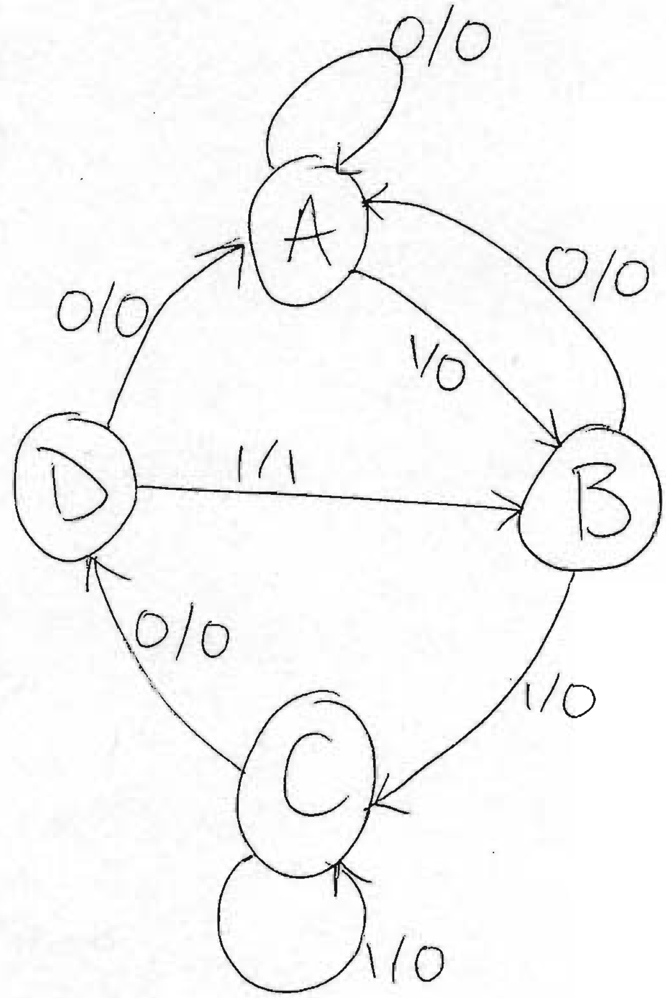
Section D

Q7

a. The outputs of a Moore machine depend solely on the current state. Therefore, more states are required, and there will be at least one cycle latency between inputs and outputs.
The outputs of a Mealey machine depend on the current state and the inputs. Therefore, fewer states are required, and there is no latency between inputs and outputs.

2

b.
(i)



3

Q7

b continued.

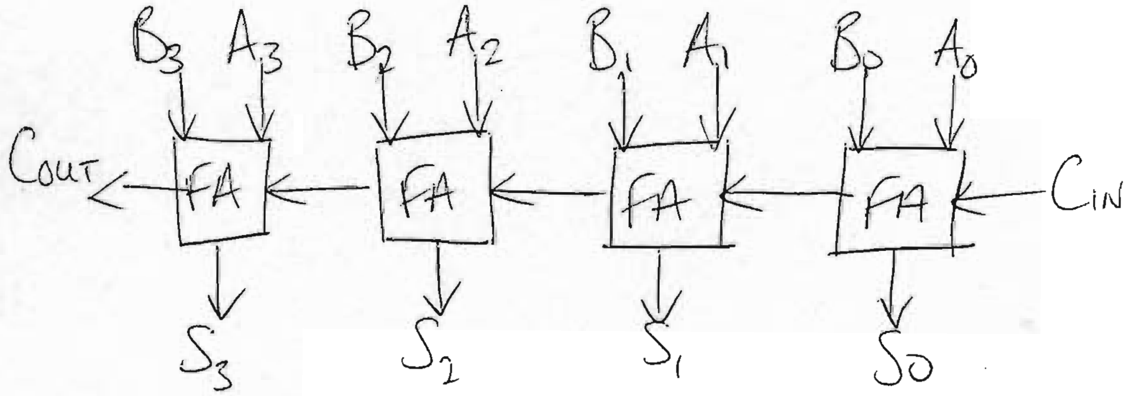
(ii)

Present State		Next State				Output	
		Input = 0		Input = 1		Input=0	Input=1
Q1	Q0	Q1	Q0	Q1	Q0	Z	Z
0	0	0	0	0	1	0	0
0	1	0	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	0	1	0	1

3

c.

(i)



2

(ii) The max. propagation delay is from B_0 or A_0 to S_3:

$$T_{XOR} + 3(T_{AND} + T_{OR}) + T_{XOR}$$

$$= 0.1 + 3(0.2) + 0.1$$

$$= 0.8 \text{ ns}$$

2

Q7

C continued.

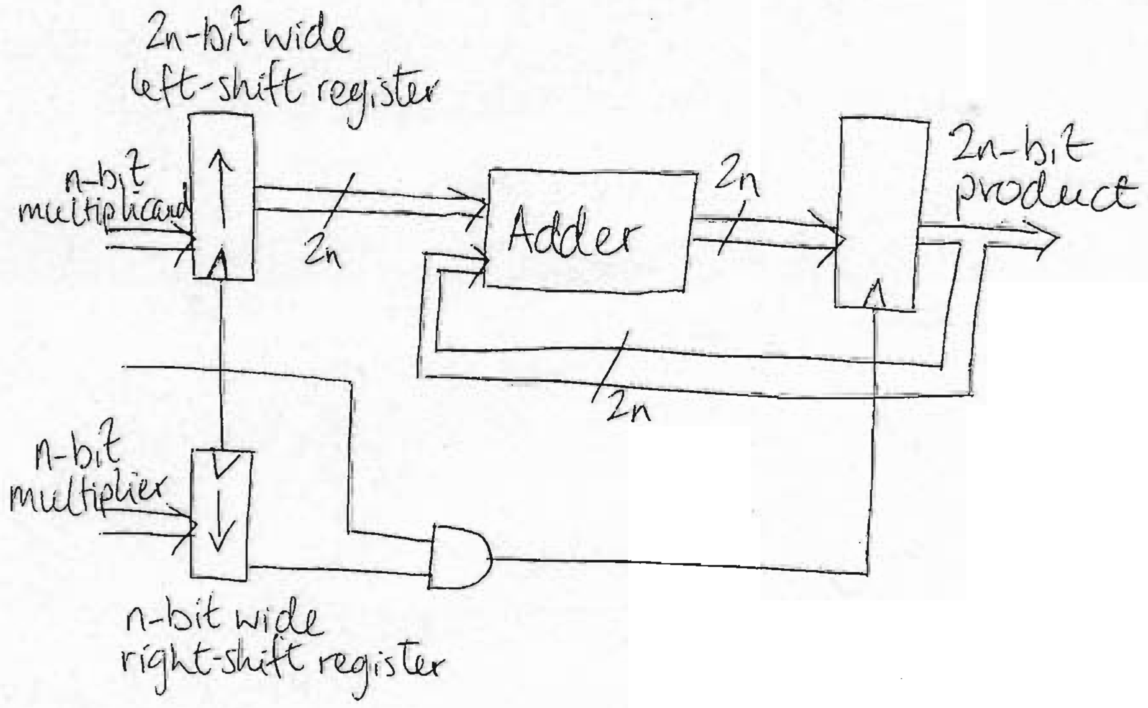
(iii) This is known as a ripple carry adder because of the way the carry signal is passed along, or ripples from one adder to the next.

The disadvantage of this is that it has a long propagation delay. This can be overcome by employing a carry lookahead adder.

The carry look ahead adder would not necessarily be of benefit in FPGA technology, as many FPGAs have dedicated support for ripple carry adders.

3

(iv)



Q7

c (iv) continued.

Initialisation:

- the n-bit multiplicand is loaded into the bottom n bits of the upper shift register, and the top n-bits are loaded with zeros.
- The n-bit multiplier is loaded into the lower shift register.
- The output register is loaded with zeros.

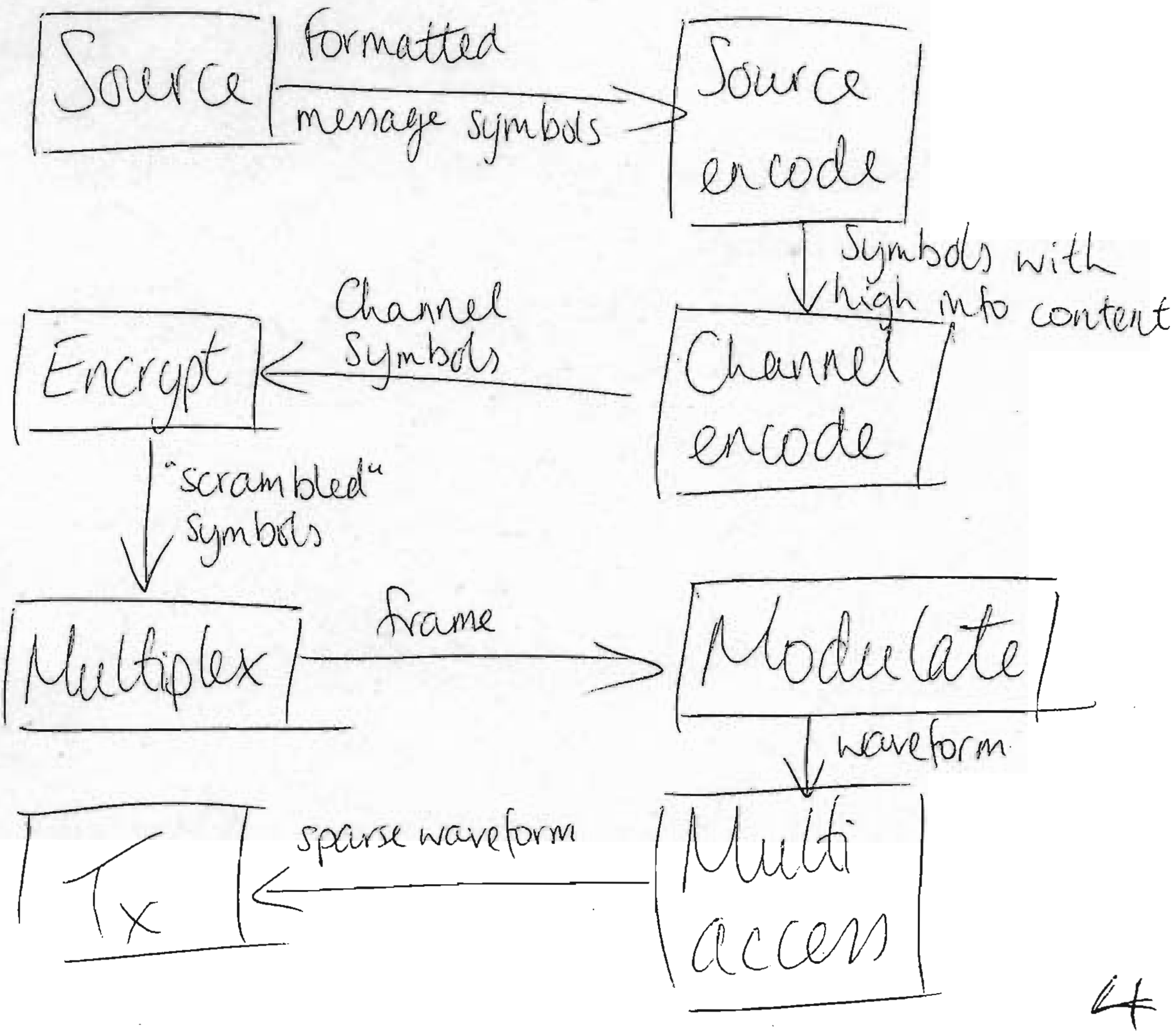
- At each stage the current LSB of the multiplier determines whether the current multiplicand value is added to the total.

- The process takes n clock cycles, excluding initialisation.

5

Q8.

a.



4

b. (i) Average Codeword Length

$\bar{L} = 0.4\times1 + 0.25\times2 + 0.2\times3 + 0.1\times4$
$+ 0.05\times4$
$= 2.1$

Compression Ratio $CR = 3/2.1 = 10/7$ 2

(ii) Entropy: $\sum_{j=1}^{5} P(m_j)\log_2 P(m_j) = 2.041$

Coding efficiency $= \dfrac{2.041}{2.1}\times100 = 97.2\%$

2

Q8.

b continued.

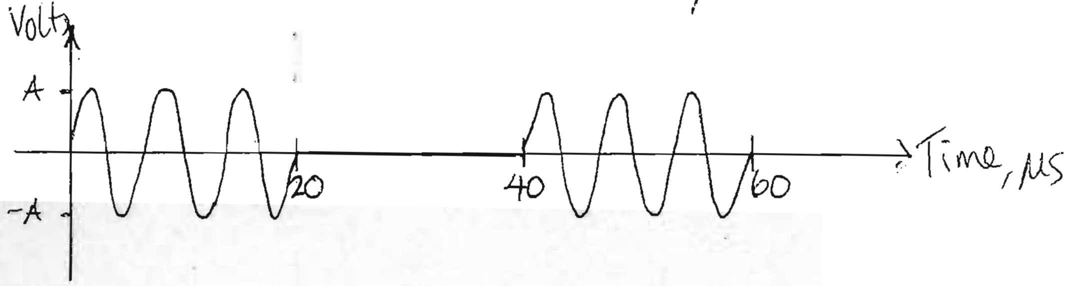
(iii)

If the average codeword length is less than the entropy, some of the information will be lost.
Such coding techniques are called "lossy" coding.

2

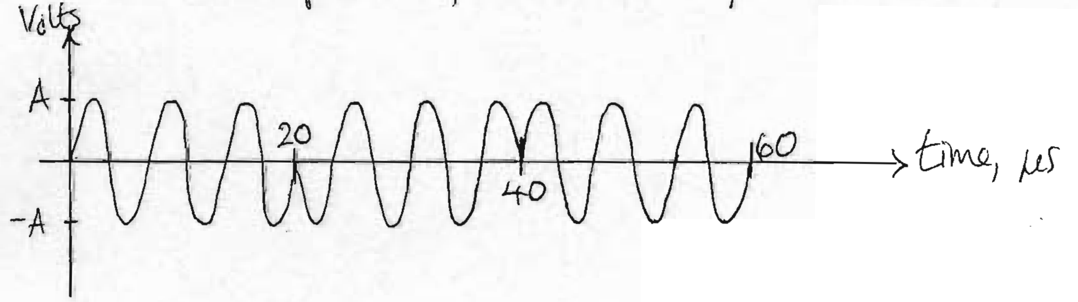
c.

(i) ASK output for the sequence "101".



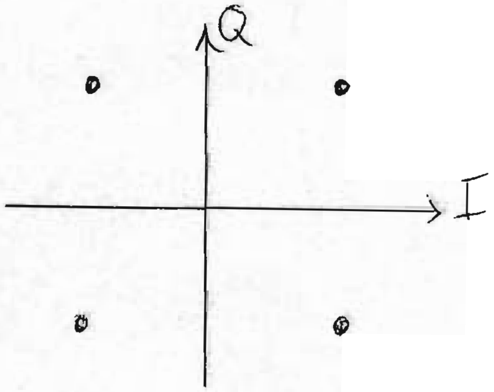
2

(ii) BPSK output for the sequence ~~"101"~~ "101".



2

(iii)

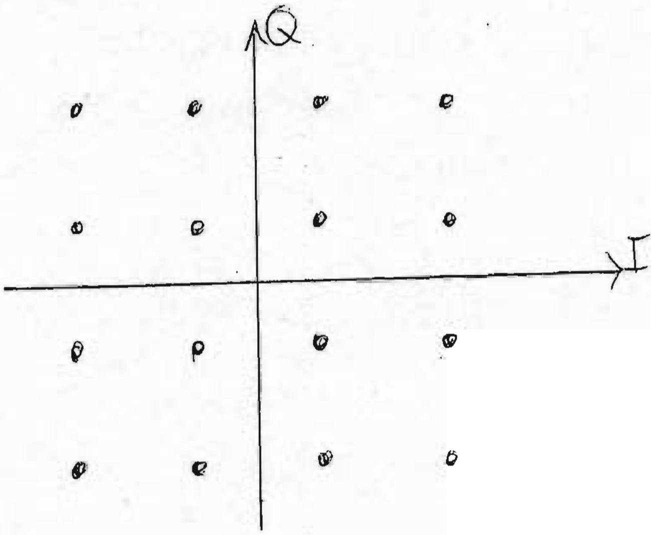


QPSK Constellation Diagram

3

Q8.
C continued.
(iv)

QAM16 will achieve higher data rates than 8-PSK.



QAM16 Constellation Diagram

QAM16 is used in modems and digital TV.

3