UNIVERSITY OF SHEFFIELD
Department of Electronic and Electrical Engineering

EEE220 ELECTRIC AND MAGNETIC FIELDS
TUTORIAL QUESTIONS

ILF/AT/JLW 2006

1. What is the force between two point charges of 1C, separated by lm? What mass would have a weight of this magnitude?

   Ans. : 8.99×10^9 N, 10^6 tonnes approximately

2. (a) What is the field at (3,2,1) due to a charge $+3 \times 10^{-6}$C at (0,0,0)?
   (b) What is the field at (3,2,1) due to a charge -3×10^{-6}C at (0,0,0)?
   (c) What is the field at (1,2,3) due to a charge $+3 \times 10^{-6}$C at (0,0,0)?
   (d) What is the field at (-1,-2,-3) due to a charge $+3 \times 10^{-6}$C at (0,0,0)?
   (e) Are the fields in these four cases equal in magnitude? Explain your answer.

   Ans. : (a) : $(1.54 \times 10^3, 1.03 \times 10^3, 5.15 \times 10^2)$ Vm^{-1}
   (b) : $(-1.54 \times 10^3, -1.03 \times 10^3, -5.15 \times 10^2)$ Vm^{-1}
   (c) : $(5.15 \times 10^2, 1.03 \times 10^3, 1.54 \times 10^3)$ Vm^{-1}
   (d) : $(-5.15 \times 10^2, -1.03 \times 10^3, -1.54 \times 10^3)$ Vm^{-1}

3. (a) What is the field at (5,5,0) due to a charge $+10^{-5}$ C at (2,2,0) ?
   (b) What is the field at (5,5,0) due to a charge -2×10^{-5} C at (7,1,0) ?
   (c) What is the total field at (5,5,0) due to these two charges?
   (d) What force acts on a charge 10^{-6}C at (5,5,0) due to these two charges?

   Ans. : (a) $(3.53 \times 10^3, 3.53 \times 10^3, 0)$ Vm^{-1}
   (b) $(4.02 \times I0^3, -8.04 \times 10^3, 0)$ Vm^{-1}
   (c) $(7.55 \times 10^3, -4.51 \times 10^3, 0)$ Vm^{-1}
   (d) $(7.55 \times 10^{-3}, -4.51 \times 10^{-3}, 0)$ N

4. (a) An infinitely long wire parallel to the Z axis passes through the point (0,1,0) and carries a charge per unit length of $+3 \times 10^{-6}$ Cm^{-1}. Calculate the electric field at the point (1,0,0).
   (b) A second infinitely long wire parallel to the Z axis passes through the point (0,-1,0) and carries a charge $+10^{-6}$ Cm^{-1}. What is the total field due to the two wires at the point (1,0,0)?
   (c) What force would act on a charge of 10^{-7} C at (1,0,0) due to this total field?

   Ans. : (a) $(2.70 \times 10^4, -2.70 \times 10^4, 0)$ Vm^{-1}
   (b) $(3.60 \times 10^4, -1.80 \times 10^4, 0)$ Vm^{-1}
   (c) $(3.60 \times 10^{-3}, -1.80 \times 10^{-3}, 0)$ N

5. For a wire of length $2L$, with a charge per unit length of q_ℓ, show that the E_y component of the field would be given at (x,y) by :

$$E_y = \frac{q_\ell}{4\pi\varepsilon_0 y}\left[\frac{L+x}{\left[(L+x)^2+y^2\right]^{\frac{1}{2}}}+\frac{L-x}{\left[(L-x)^2+y^2\right]^{\frac{1}{2}}}\right]$$

The origin is at the centre of the wire, and the X-axis along the wire. [Hint : the calculation is exactly as for an infinitely long wire, except the limits of integration will be different. Therefore all that is needed is to work out the new limits to put into the result of the integration].

6. An engineer knows that a particular straight wire of length 10m carries a charge of 10^{-5} Cm^{-1}, and wants to calculate the electric field due to this at a perpendicular distance of 1m from the centre point of the wire. Unfortunately, the engineer cannot remember the formula derived in question 5 for the field due to a wire of finite length, but does remember that due to an infinitely long wire. What percentage error will result from using the latter?

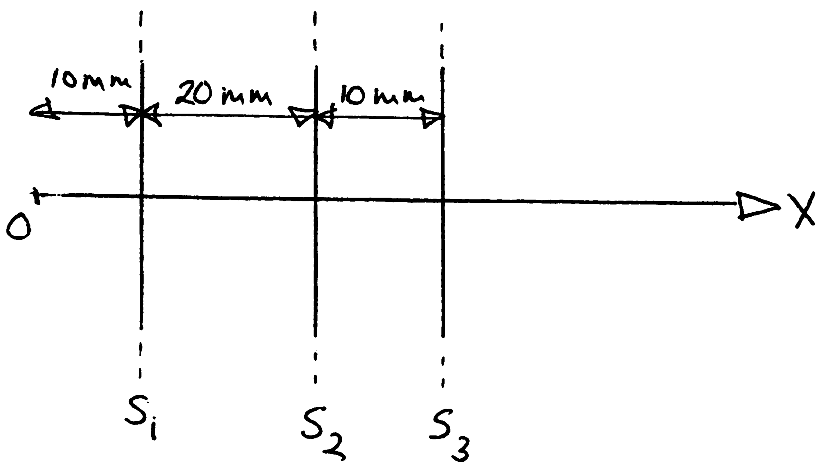
Ans. : 2%

7. Three infinite plane parallel sheets of charge S_1, S_2 and S_3 carry uniformly distributed charges of density $+10^{-7}$ Cm^{-2}, -2×10^{-7} Cm^{-2}, and $+3 \times 10^{-7}$ Cm^{-2} respectively. The sheets are perpendicular to the X-direction, with S_1 at x = 10mm, S_2 at x = 30mm, and S_3 at x = 40mm.

(a) What is the electric field due to these charge sheets?

(b) What is the potential difference between S_1 and S_2, and between S_2 and S_3?

Ans. : *(a) $E_y = 0$, $E_z = 0$. E_x depends on position relative to charge sheets; to left of S_1 $E_x = -11.3 \times 10^3$ Vm^{-1}, between S_1 and S_2 0 Vm^{-1}, between S_2 and S_3 -22.6×10^3 Vm^{-1}, and to right of S_3 11.3×10^3 Vm^{-1}*
(b) 0V; 226V



8. Two infinite plane parallel sheets separated by 10mm, carry charges of density $+10^{-6}$ Cm^{-2} and -10^{-6} Cm^{-2}. What is the force per unit area on these charge sheets?

Ans. : 5.65×10^{-2} Nm^{-2}

9. A charge $q_1 = 10^{-3}$ C is situated at *A* in the figure, and a charge $q_2 = 10^{-5}$ C at *B*.

(a) How much work has to be done to move q_2 from *B* to *C* along path 1?

(b) How much work has to be done to move q_2 from *B* to *D* along path 2?

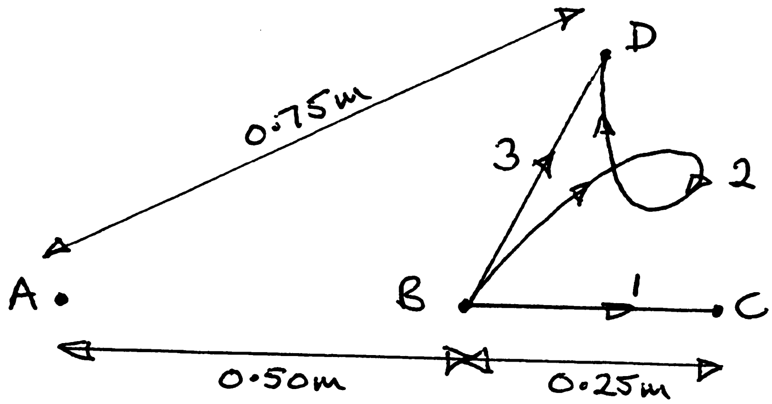
(c) How much work has to be done to move q_2 from *B* to *D* along path 3?

(d) Is the work done in moving q_2 from *B* to *C* done by the field of q_1?

(e) If $q_2 = -10^{-5}$ C how much work has to be done to move q_2 from *B* to *C*?

(f) Is work done by the field of q_1?

Ans. : (a) 60J; (b) 60J; (c) 60J; (d) yes; (e) 60J; (f) no



10. A distribution of charge produces an electric potential given by :

$$\Phi = 2x^2 + 3yz$$

What is the electric field at the points (1, 2, 3) and (-1, -1, -1) ?

Ans. : (-4, -9, -6) Vm^{-1} , (4, 3, 3) Vm^{-1}

11. A point charge $+q_1$ is located at (0, a, 0), and a point charge $-q_1$ is located at (0, -a, 0).

(a) What is the potential due to these charges at the point (x, y, z)

(b) Simplify this expression assuming $a \quad \sqrt{x^2 + y^2 + z^2}$ and using the binomial expansion $(1 + u)^n \approx 1 + nu$

(c) Differentiate the expression for ϕ from (b) to get the field components at (x, y, z).

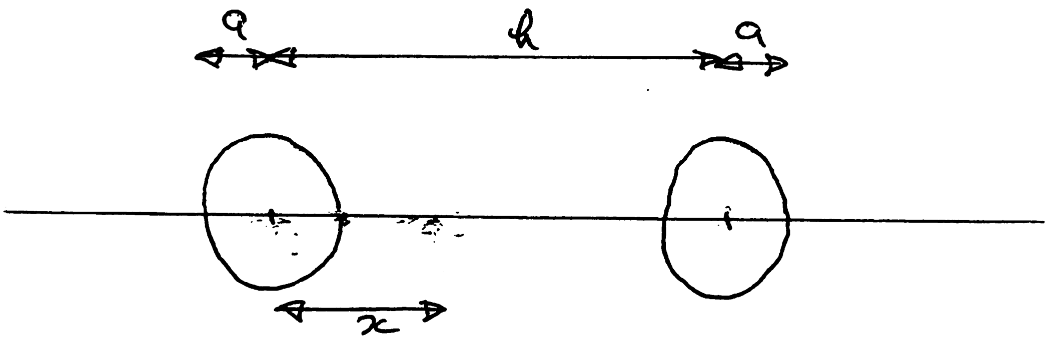
Ans. (b) : $\phi = \dfrac{q_1 a y}{2\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}}$

$$(c) \qquad E_x = \frac{q_1 a}{2\pi\varepsilon_0}\left(\frac{3xy}{\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}}\right)$$

$$E_y = \frac{q_1 a}{2\pi\varepsilon_0}\left(\frac{3y^2}{\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}} - \frac{1}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}}\right)$$

$$E_z = \frac{q_1 a}{2\pi\varepsilon_0}\left(\frac{3zy}{\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}}\right)$$

12. The figure shows a cross-section of two parallel wires each of radius a separated by a distance h in free space. One carries a charge $+q_\ell$ Cm^{-1} the other a charge $-q_\ell$ Cm^{-1}



(a) Calculate the electric field at P, a distance x from the centre of one of the conductors on a line joining their centres.

(b) Calculate the potential difference between the wires by integrating the expression for the electric field.

(c) Derive an expression for the capacitance per unit length of the wires.

$$Ans.: (a)\ \frac{q_\ell}{2\pi\varepsilon_0}\left(\frac{1}{x} + \frac{1}{h - x}\right);\ (b)\ \frac{q_\ell}{\pi\varepsilon_0}\ln\left(\frac{h-a}{a}\right);\ (c)\ \frac{\pi\varepsilon_0}{\ln\left(\frac{h-a}{a}\right)}$$

13. A plane capacitor consists of two metal foils separated by a thin dielectric layer. The relative permittivity of the dielectric is 8, and the thickness of the layer is 10^{-2} mm. If the capacitor is charged to l00V, what is the charge per unit area on the plates, and what is the electric field inside the capacitor?

Ans.: $7.08 \times 10^{-4}\ Cm^{-2}$, $10^{7}\ Vm^{-1}$

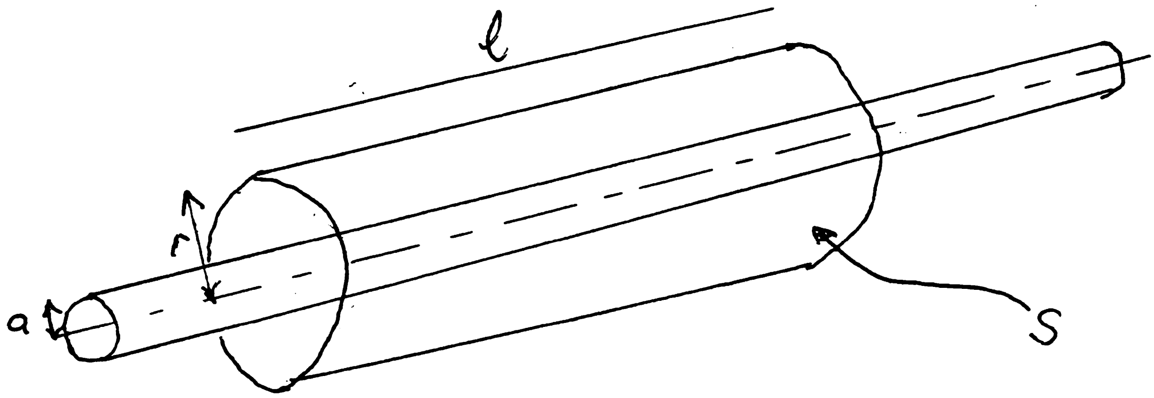
14. The figure shows a long straight rod of radius *a* and length *L*, which has a total charge *Q* uniformly distributed along its length. *L* is much greater than *a*, so end effects can be neglected. The imaginary surface *S* is cylindrical and coaxial with the rod.

(a) What is the total charge enclosed by *S*?

(b) What is the electric flux out of *S*?

(c) What is the electric field due to the rod at the surface of *S*?

Ans. : (a) $\frac{Q\ell}{L}$; (b) $\frac{Q\ell}{L\varepsilon_0}$; (c) $\frac{Q}{2\pi r L\varepsilon_0}$



15. A capacitor is made of two coaxial conducting cylinders of length *L*, separated by air. The inner cylinder is of radius *a* and carries a charge *Q*; the outer is of radius *b* and carries a charge -*Q*. Assume that *L* is much greater than *a* or *b* so that end effects can be neglected.

(a) Draw the electric field lines between the cylinders

(b) What is the electric field in the region between the two cylinders?

(c) Integrate this expression for the electric field to calculate the potential difference between the cylinders

(d) Hence deduce an expression for the capacitance between the two conductors

Ans. : (b) $\frac{Q}{2\pi r L\varepsilon_0}$; (c) $\frac{Q}{2\pi L\varepsilon_0}\ln\left(\frac{b}{a}\right)$; (d) $\frac{2\pi L\varepsilon_0}{\ln\left(\frac{b}{a}\right)}$

16. A long straight wire of diameter 1mm carries a current of 50A.

(a) Calculate the B field at the surface of the wire, and at distances of 10mm, 0.1m and 1m from the axis of the wire using the formula for an infinitely long wire.

(b) If the wire is 1m long, calculate the B field 10cm. from the axis of the wire at its mid-point. You may neglect the B field due to the current flowing in the rest of the circuit. What would be the percentage error in assuming that the formula for an infinitely long wire applied in this case?

Ans. : (a) 20mT, 1mT, 0.1mT, 10μT (b) 2%

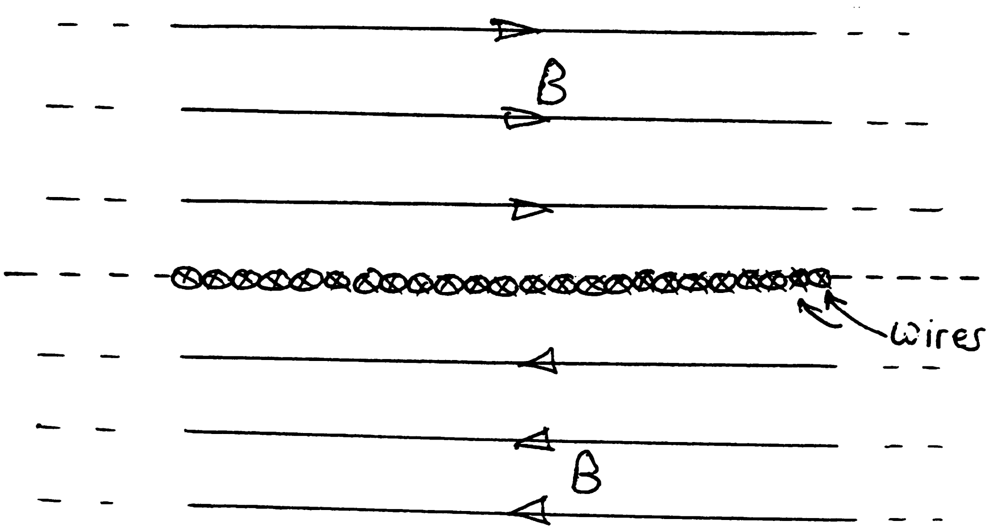
17. A long wire carrying a current of 100A is placed in a uniform field of 5mT. The wire is at right angles to field. Find where the resultant field is zero.

Ans. : On a line parallel to the wire and 4mm from it.

18. The figure shows an infinite number of infinitely long parallel fine wires each carrying a current I, the currents all being in the same direction. Show that the B field is of the form shown in the figure and of magnitude

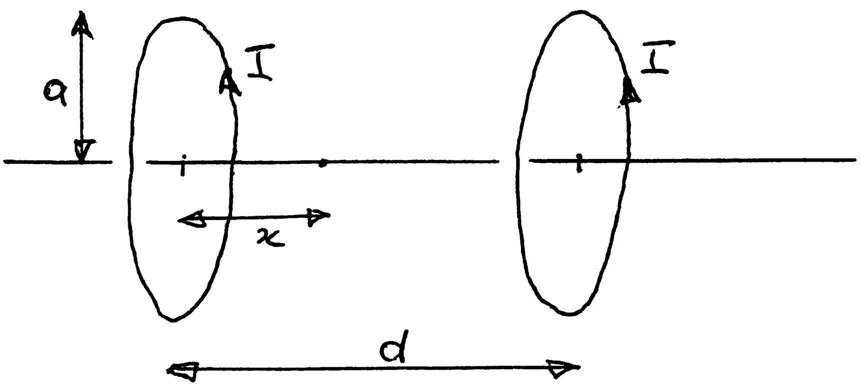
$$B = \frac{\mu_0 n I}{2}$$

where n is the number of wires per unit length.



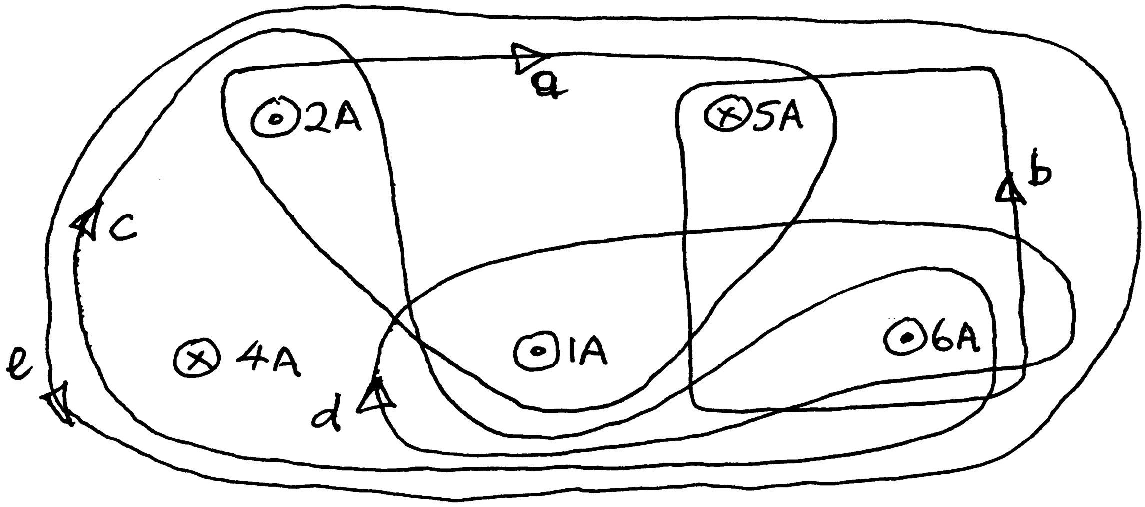
19. The figure shows two identical current loops of radius a, carrying the same current I, and separated by a distance d. What is the field at P, a distance x from the centre of one of the loops, along their common axis? If the separation of the loops is equal to their radius, i.e. $d = a$, deduce an expression for the variation of B along the axis from $x = 0$ to $x = a$. What is the value of B at the mid-point between the loops for I = 10A, and a = 0.1m? Sketch the form of the variation of B from $x = 0$ to $x = a$.

Ans. : 8.98×10^{-5} T



20. The figure shows the cross-section of several current carrying conductors; the magnitudes and directions of the currents are indicated. What is the value of the line integral of B for the paths indicated?

Ans. : (a) $2\mu_0 A$; (b) $1\mu_0 A$; (c) $-4\mu_0 A$; (d) $-7\mu_0 A$; (e) 0



21. A wire of radius 2mm carries a current of 10A. Assuming the wire is infinitely long, so that the field is symmetric about the wire, use Ampère's law to calculate the B field at distances of 50mm, 10mm, 2mm and 1mm from the axis of the wire. For the last distance assume that the current is uniformly distributed inside the wire.

Ans. : 4×10^{-5} T; 2×10^{-4} T; 10^{-3} T; 5×10^{-4} T

22. The figure shows the cross-section of a long coaxial cable, which consists of two concentric conductors, shown shaded, insulated from each other. A current *I* flows out of the figure, in the inner conductor, and into the figure in the outer conductor, the currents being assumed to be distributed uniformly within each conductor. Calculate the variation of the B field with position,

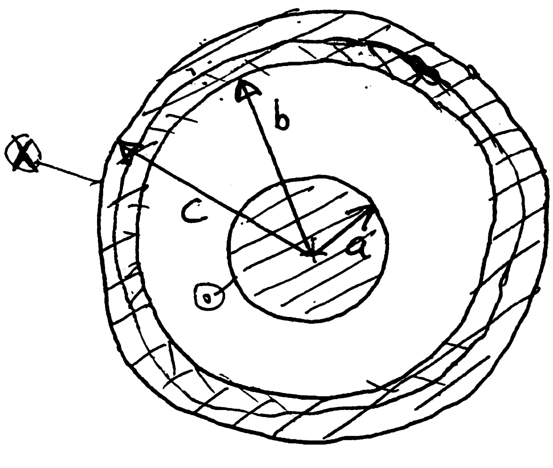
(a) inside the inner conductor, i.e. $r \le a$

(b) between the two conductors, i.e. $a \le r \le b$

(c) within the outer conductor, i.e. $b \le r \le c$

(d) outside the outer conductor, i.e. $c \le r$

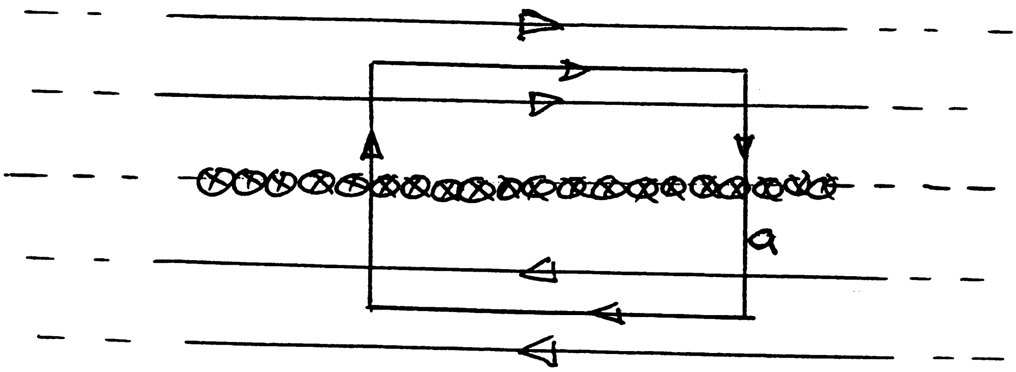
Ans. : (a) $\dfrac{\mu_0 I r}{2\pi a^2}$; (b) $\dfrac{\mu_0 I}{2\pi r}$; (c) $\dfrac{\mu_0 I\left(c^2 - r^2\right)}{2\pi r\left(c^2 - b^2\right)}$; (d) 0



$A = \pi r^2 - \pi b^2$

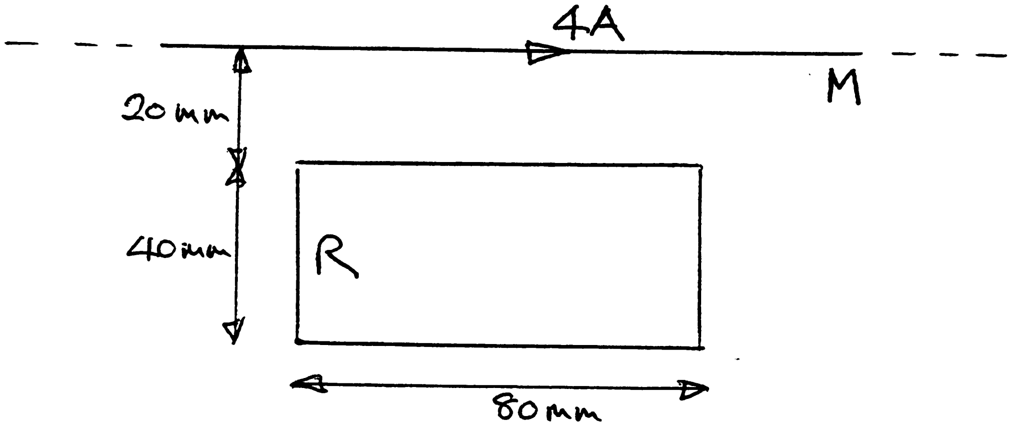
$\pi(r^2 - b^2)$

23. The figure shows an infinite number of parallel fine wires, n per unit length, each carrying a current I. The form of the B field is as shown. Using Ampere's law for the path labelled (a), show that the B field on either side of the wires is $\mu_0 nI / 2$.



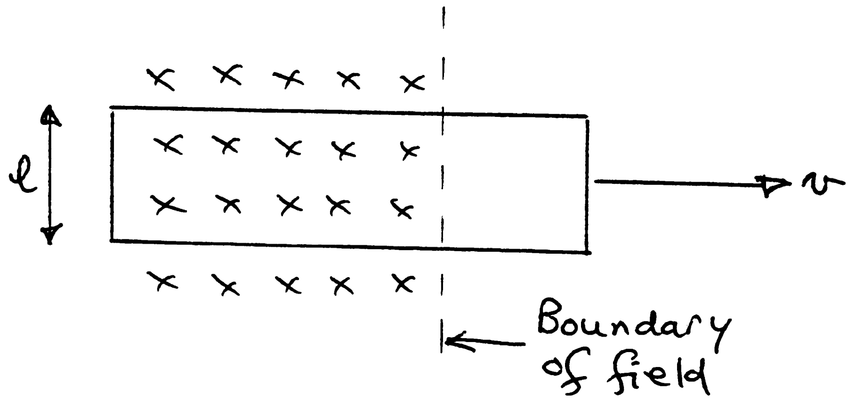
24. The figure shows a wire M carrying a 50Hz, 4A, rms current from the mains, which runs close to a circuit R which a student has constructed. This circuit is shown for simplicity as a rectangle but could well consist of resistors, transistors, etc. mounted on a printed circuit board. What emf is induced in the circuit R?

Ans. : 22μV rms



25. The figure shows a rectangular loop which is drawn out of an applied uniform magnetic field B with a constant velocity v. If $\ell = 100\text{mm}$, B = 1T, and $v = 10\text{ms}^{-1}$, what emf is induced?

Ans. : 1V



26. Estimate the magnitude of the force per unit length on a mains cable carrying a current of 1A rms, assuming that the centres of the live and neutral leads are 2mm apart. In which direction does the force act?

Ans. : 10^{-4} Nm^{-1}

27. (a) Estimate the magnitude of the force per unit length between the cables of an overhead transmission line if the cables are 1m apart. Assume that the current is 1000A.

(b) Estimate the magnitude of the force per unit length on one of these cables due to the earth's magnetic field of 10^{-4} T.

(c) Are the forces calculated in parts (a) and (b) above comparable to the gravitational forces on the cables?

Ans. : (a) 0.2 Nm^{-1}*; (b) 0.1* Nm^{-1}

28. (a) For the linear electrical generator, show that the power consumed by the resistor R is

$$\frac{B^2 \ell^2 v^2}{R}$$

(b) The work done in moving the slider A_1A_2 is (force) × (distance) where the force arises from the interaction of the current in the circuit with the B field. Show that the rate of doing work is $BI\ell v$.

(c) Show that the rate of doing mechanical work is equal to the power consumed by the resistor.

29. A copper wire of square cross-section, side 1mm, carries a current of 0.1A perpendicular to a magnetic field of 0.8T. What is the Hall voltage if the number of charge carriers in copper is 8.4×10^{28} m^{-3} ?

Ans. : 5.95 nV

30. The electrons in a cathode ray tube (CRT) are accelerated by a 5kV voltage to a velocity of about 4×10^7 ms^{-1}. Estimate the magnitude of the force on each electron due to the earth's magnetic field of 10^{-4} T. How does this force compare to the gravitational force on the electron? What electric field would produce a force of magnitude of that due to the magnetic field?

Ans. : 6.4×10^{-16} *N,* 8.9×10^{-30} *N,* 4×10^3 Vm^{-1}