# Basic Concepts

## Charge

**Electric charge** is the basis for describing all electrical phenomena; the separation of charge gives rise to an electric field and the movement of charge is called an electric current. Usually we use the symbol $Q$ to denote charge and the unit is the Coulomb (C).

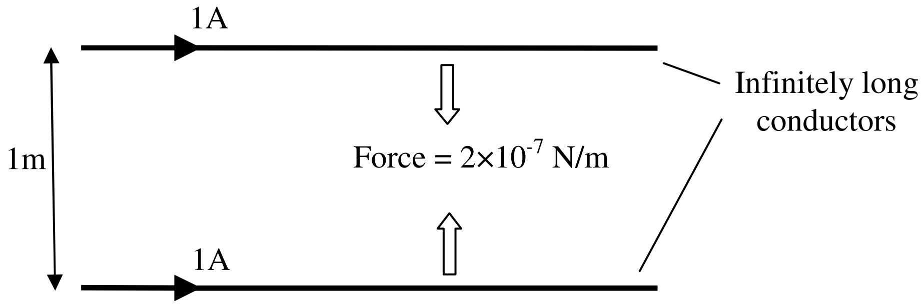
Electric charge exists in discrete quantities which are integral multiples of the charge on an electron:

- Negative charge on an electron $= -1.6 \times 10^{-19}$ C
- Positive charge on a proton $= +1.6 \times 10^{-19}$ C

Movement of charge (electrons) gives rise to an electric current for which we use the symbol, $I$, and the unit Ampère or Amp (A). Multiples of the unit exist, commonly used ones are:

1mA (milli-amp) = 0.001 A
1μA (micro-amp) = $10^{-6}$ A
1kA (kilo-amp) = 1000 A

The Ampère is formally defined as the current, which, if maintained in two infinitely long, straight, parallel conductors, of negligible cross-section, placed 1m apart in a vacuum would produce a force of $2\times10^{-7}$ N per metre length between them.




The conductors are attracted towards one another if the currents are in the same direction (as shown in the figure). If the currents are opposite in direction then they will repel one another.

The Coulomb is defined as the quantity of electricity (charge) passing a given point in a circuit when a current of 1 Ampère flows for 1 second, or:

$$Q = I \times t$$

where, $Q$ is the charge (C), $I$ is the current (A) and $t$ is the time (s). Alternatively we can say that the current is the rate of flow of charge:

$$I = \frac{Q}{t}$$

Example
A car battery is rated at 120Amp-hour (A-h). What is the total charge it can supply assuming it is initially fully charged?

120 A-h means that the battery can supply 1A for 120h, or 2A for 60h, or 30A for 4h, or 120A for 1h etc. It does not matter which one we assume in the calculation of charge as they all give the same answer.

Assume $I$ = 1A and $t$ = 120 × 60 × 60 seconds:

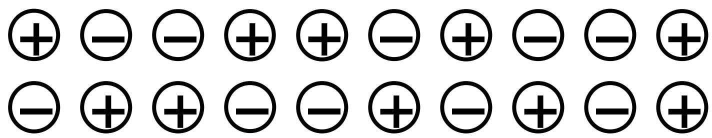
$$Q = I \times t = 1 \times 120 \times 60 \times 60 = 4.32 \times 10^5 \text{ C}$$

Check, if $I$ = 30A and $t$ = 4 × 60 × 60 seconds:

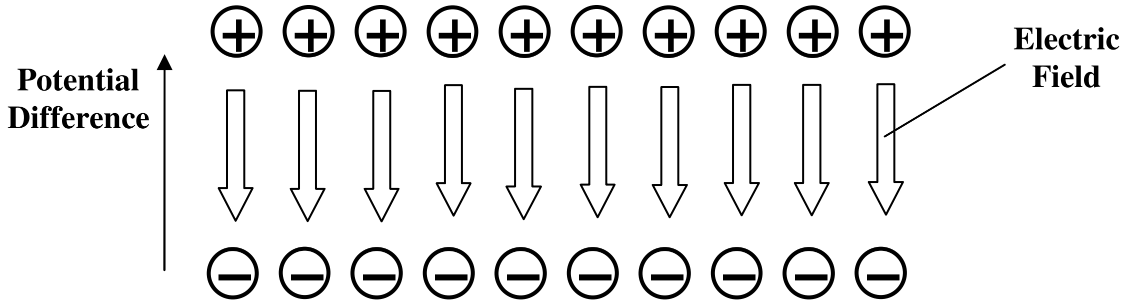
$$Q = I \times t = 30 \times 4 \times 60 \times 60 = 4.32 \times 10^5 \text{ C}$$

## Potential Difference

Normally in any material the number of positive and negative charges is equal so there is no net effect:

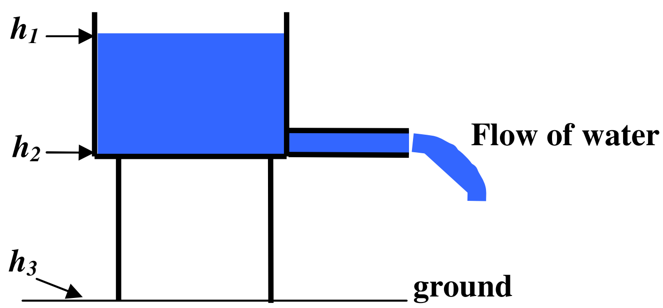


However, the charges can be separated by giving energy to the system:




There is now a **potential difference** between the positive and negative charges caused by their separation, and an **electric field** exists between the layers. I.e. work has been done in separating the charges which equals the energy given to the charges.

The potential difference is measured in Volts (V). Normally voltages or potentials are measured relative to some reference value usually 0V (known as earth or ground). This is analogous to a tank of water situated above the ground as shown in the figure below:




The potential energy due to gravity will cause the water to flow out of the pipe at a certain flow rate. The pressure, or difference in potential, that forces the water out of the pipe is directly related to the head of water ($h_1$–$h_2$) in such a way that the pressure is zero when $h_1$=$h_2$. Now if point $h_3$, corresponding to the ground level is defined as zero potential then the pressure acting on the fluid in a pipe is actually the difference in potential energy (($h_1$–$h_3$) – ($h_2$–$h_3$)) which has the same value as before. It can be seen that it is not therefore necessary to assign a precise energy value to $h_3$ and in fact it would be extremely cumbersome to do so. Clearly it is the difference in potentials that we are interested in. For an electrical system:

Voltage (potential difference) is analogous to the water pressure
Current is analogous to the water flow

So far we have seen that energy is needed to be input to our electrical system in order to separate charge and this has resulted in a potential difference. Expressing this in the form of an equation:

$$E = V \times Q$$

where $E$ is the energy input (Joules, J), $V$ is the potential difference or voltage (Volts, V) and $Q$ is the charge (Coulombs, C). Previously we have shown that:

$$Q = I \times t$$

and so:

$$E = V \times I \times t \quad \text{(Joules)}$$

or the power (rate of doing work) is:

$$P = \frac{E}{t} = V \times I \quad \text{(Watts)}$$

i.e. power is the product of the voltage and the current.

Example

A car battery rated at 120A-h has a potential difference across its terminals of 12V. What is the total energy stored when it is fully charged. For how long could the battery supply the car's radio (10W) and the headlights (200W)? What is the current drawn in each case?

The total energy stored in the battery is:

$$Energy = V \times I \times t = 12 \times 120 \times 3600 = 5.18 \times 10^6 \text{ J or } 5.18 \text{ MJ}$$

The radio is rated at 10W (or 10 J/s) hence the battery could supply it for:

$$t_{RADIO} = \frac{E}{P_{RADIO}} = \frac{5.18 \times 10^6}{10} = 5.18 \times 10^5 \text{ seconds or } \frac{5.18 \times 10^5}{3600} = 144 \text{ hours}$$

and current drawn is:

$$I_{RADIO} = \frac{P}{V} = \frac{10}{12} = 0.83 \text{ A}$$

The headlights are rated at 200W (or 200 J/s) hence the battery could supply them for:

$$t_{LIGHTS} = \frac{E}{P_{LIGHTS}} = \frac{5.18 \times 10^6}{200} = 25900 \text{ seconds or } \frac{25900}{3600} = 7.2 \text{ hours}$$

and current drawn is:

$$I_{LIGHTS} = \frac{P}{V} = \frac{200}{12} = 16.67 \text{ A}$$

Check: we could calculate the total charge in the battery (see previous example) as $4.32 \times 10^5$ C. Since the current is 16.67 A or C/s the time to discharge is $4.32 \times 10^5/16.67 = 25900$ s as before.

Example

An electric heater is required to heat 15 litres of water from 12°C to 100°C. What is the electrical energy consumed (a) in megajoules (b) in kilowatt-hours (kWh). If it is required to heat the water in 10 minutes what is the power rating of the heater. Assume the specific heat capacity of water is 4200 $\text{Jkg}^{-1}\text{C}^{-1}$ and that the heater is 95% efficient.

The mass of water is 15kg and the required temperature rise, $\theta$, is 100 − 12 = 88°C. Therefore energy required is:

$$E = mass \times specific\ heat\ capacity \times \theta = 15 \times 4200 \times 88 = 5.54 \text{ MJ}$$

But the heater is only 95% efficient so only 95% of the energy we input goes into heating up the water.

(a) The amount of electrical energy required is therefore:

$$E_{ELEC} = \frac{5.54\times10^6}{0.95} = 5.83 \text{ MJ}$$

(b) The kilowatt-hour is commonly used unit for measuring the amount of electricity used. Domestic charges are based on the kWh (sometimes called the unit). This is equal to a power of 1000W being delivered for a period of 1 hour (3600s), Hence:

$$1\text{kWh} = 1000 \times 3600 = 3.6\text{MJ}$$

Hence the electrical energy required in kWh is:

$$E_{ELEC} = \frac{5.83\times10^6}{3.6\times10^6} = 1.62\text{kWh}$$

It is required to heat the water in 10 minutes or 600 s, hence:

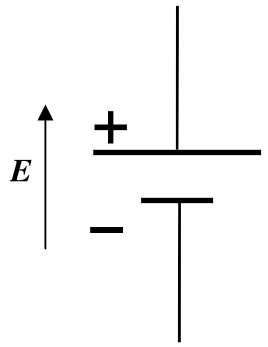
$$P = \frac{E}{t} = \frac{5.83\times10^6}{600} = 9720 \text{ W or } 9.72 \text{ kW}$$

**<u>Sources</u>**

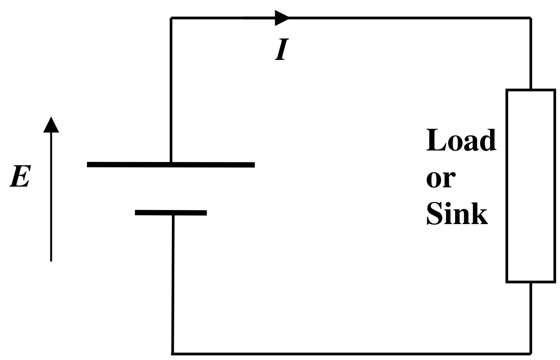
An electrical source is a device that is capable of converting non-electrical energy into electrical energy. For example:

Battery – converts chemical energy into electrical energy
Generator – converts mechanical energy into electrical energy

Normally the potential difference or voltage across the terminals of an electrical energy source is called the **Electromotive force** or emf and is measured in Volts. We sometimes use the symbol $E$ for emf but this can be confused with energy so $V_S$ is often used. The simplest form of electrical source is the cell or battery (collection of cells) which has the circuit symbol as shown below (note the + and − are usually omitted but are shown here to indicate the positive and negative terminals. Likewise $E$ and the arrow are shown to aid understanding):




Circuit symbol for a battery



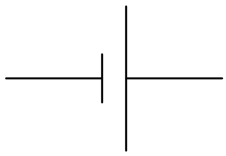

Simple circuit

When an electric circuit is formed by connecting a load (or energy sink) to the terminals of the battery an electric current will flow and energy will be transferred from the source to the sink. Convention: When energy is flowing from the source to the load positive current flows out of the positive terminal of the battery.

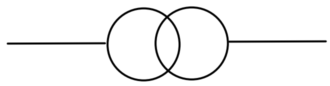
**Ideal sources**

An **ideal voltage source** delivers a prescribed voltage across its terminals independent of the current flowing through it.

An **ideal current source** delivers a prescribed current through its terminals regardless of the voltage across them.



Ideal voltage source



Ideal current source

At present we will limit our analysis of circuits to those which have a unidirectional flow of charge from a d.c. (direct current) source such as a battery or power supply. Later in the course we shall looking at circuits where the voltages and currents vary with time (transient and a.c. circuits).

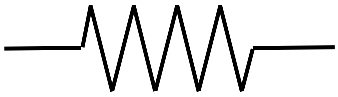
# Resistance

As electrons (charge) flow through any material they will collide with other electrons and the atoms of the material through which they are moving. This will impede their motion and some amount of electrical energy is converted to thermal energy and dissipated in the form of heat (analogous to mechanical friction). This opposition to the flow of charge is called the **Resistance** of the material.

A resistance is hence an energy sink as it converts electrical energy into thermal energy. This may be undesirable e.g. long transmission lines, supply cables etc. or may be extremely useful as in the case of heaters, cookers, light bulbs etc.

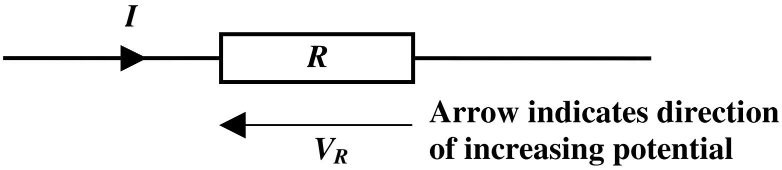
The unit of resistance is the Ohm (Ω) and the representation of a resistor (i.e. a wire or other form of material used simply because of its resistance) in a circuit diagram is:

**New symbol**

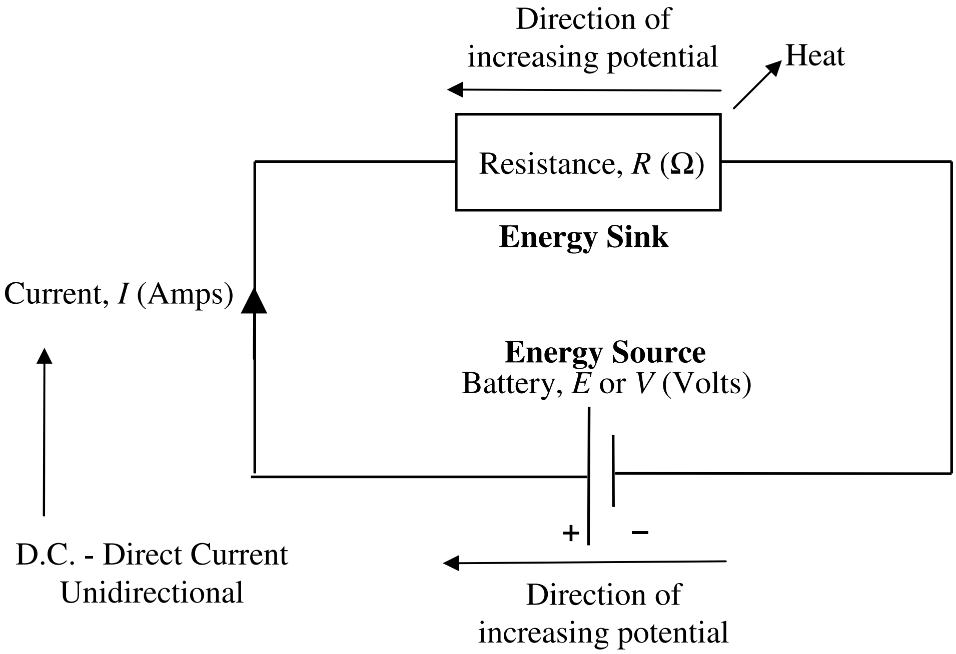


**(Old symbol)**

As an electric current (flow of charge) passes through a resistance some of the electrical energy is converted into thermal energy and therefore there is a loss in potential, hence a potential difference (voltage) appears across it:




We can now draw the simplest form of electric circuit containing a single source (battery) and a single sink (resistance).




A cell or battery is a d.c. **energy source** – it converts chemical energy to electrical energy and hence there is an increase in electrical potential as we travel from the negative (−) to positive (+) terminals. At this stage we will assume the source is d.c. having a constant potential or voltage. Later in the course we shall look at time varying voltage sources.

A resistor is an **energy sink** as it converts electrical energy into thermal energy and hence there is a drop of potential in the direction of the current flowing through it, i.e. there is an increase in potential opposing the direction of the current. Conventionally, the direction of current is such that it flows out of the positive terminal of the source. The connecting wires provide a path for the charge to flow from the source to the sink and back to the source to form a closed loop or electric circuit. In practice these wires will also have a finite resistance, but this is usually small compared with the load resistance and is ignored.

## Ohm's Law

The relationship between the electrical current, $I$, (flow of charge) passing through a particular part of a circuit and the potential difference, $V$, across that part of the circuit is defined by **Ohm's Law**:

$$\frac{V}{I} = R$$

where $R$ is the resistance measured in Ohms (Ω). This equation may be rearranged into other commonly used forms:

$$V = I \times R \qquad \text{and} \qquad I = \frac{V}{R}$$

Previously we have shown that the power, $P$, is equal to the product of the voltage, $V$, and the current, $I$:

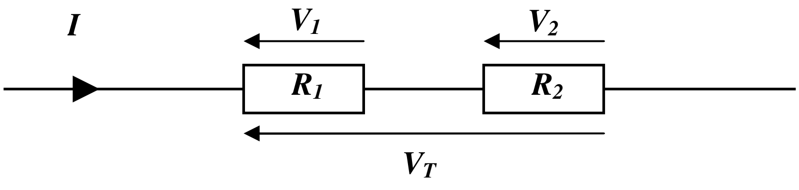
$$P = V \times I$$

hence substituting for $V$ or $I$ using Ohm's law gives:

$$P = I \times R \times I = I^2 R$$

$$P = V \times \frac{V}{R} = \frac{V^2}{R}$$

A common mistake made by students is to use the incorrect voltage in the above equations. Used correctly they all give correct answers, but the use of $P = I^2R$ is recommended!




$$P_1 = I^2 R_1 = \frac{V_1^2}{R_1} = V_1 I \qquad \text{and} \qquad P_2 = I^2 R_2 = \frac{V_2^2}{R_2} = V_2 I$$

Note the use of the voltages across individual resistances. To find the total power, $P_T$, use the total voltage or total resistance, $(R_1 + R_2)$:

$$P_T = I^2 (R_1 + R_2) = \frac{V_T^2}{(R_1 + R_2)} = V_T I$$

**Properties of materials**

For the purpose of electrical engineering, materials are classified according to their electrical resistance, i.e. their ability to allow an electric current to flow when a voltage is applied.

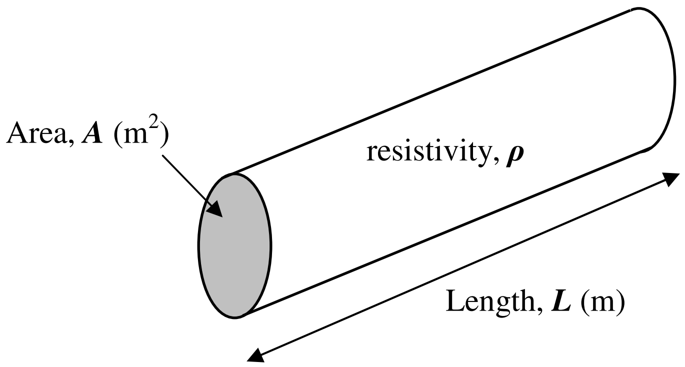
**Conductor** — a material which allows electric charge to flow freely when a voltage is applied across it. The ratio of the applied voltage to the flow of charge (or current) is the resistance of the material (Ohm's law). The majority of conductors are metallic elements with copper being the most common for electrical wiring. Gold is used for making connections within integrated circuits and connectors because it does not combine well with other materials so remains relatively pure at the surface. It also has the capability to adhere to itself which makes for very reliable connections. Where weight is a concern, such as in overhead transmission lines, aluminium is used, usually with a steel core for additional strength.

**Insulator** — a material which does not allow electric charge to flow at all. Common insulators include most plastics, rubber, glass, porcelain, wood and air.

**Semi-conductor** — a material whose electrical conductivity is between that of an insulator and a conductor and whose ability to allow electric charge to flow can be controlled. A semi-conductor's electrical properties are modified by introducing impurities by a process known as doping. Typical materials are silicon, germanium, gallium arsenide, indium phosphide etc.

Let us consider conductors……

The ability of an electrical conductor to pass an electric current is related to its dimensions and material properties (which may be temperature dependent). Consider a length, *L*, of conductor having a cross-sectional area, *A*, as shown in the figure below:




The resistance of the conductor, *R*, (Ω) is given by:

$$R = \frac{\rho L}{A}$$

where $\rho$ is the resistivity of the material and has the units Ohm-metres (Ω-m).

The reciprocal of the resistivity is known as the conductivity, $\sigma$, and has units of Siemens per metre (S/m), in which case:

$$R = \frac{L}{\sigma A}$$

where:

$$\sigma = \frac{1}{\rho}$$

The following table shows typical values of resistivity and conductivity for some common materials at room temperature.

| **Material** | **Resistivity (Ω-m)** | **Conductivity (S/m)** |
|---|---|---|
| Aluminium | $2.733 \times 10^{-8}$ | $36.59 \times 10^{6}$ |
| Copper | $1.725 \times 10^{-8}$ | $57.97 \times 10^{6}$ |
| Gold | $2.271 \times 10^{-8}$ | $44.03 \times 10^{6}$ |
| Iron | $9.980 \times 10^{-8}$ | $10.02 \times 10^{6}$ |
| Silver | $1.629 \times 10^{-8}$ | $61.38 \times 10^{6}$ |
| Carbon | $3.5 \times 10^{-5}$ | $0.029 \times 10^{6}$ |

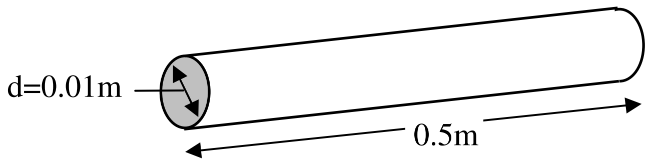
The reciprocal of resistance is called the conductance and has the symbol $G$ (units S or $\Omega^{-1}$), but this does not have widespread use:

$$G = \frac{1}{R}$$

Example

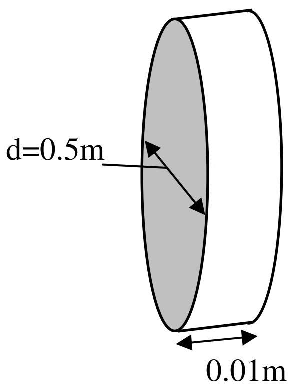
A conductor has a resistivity, $\rho = 1.725\mu\Omega$-cm.

(a) Calculate the resistance for a 0.5m length having a diameter of 1cm.
Note that the value of resistivity needs to be in SI units:
$\rho = 1.725\mu\Omega\text{-cm} = 1.725 \times 10^{-6}\Omega\text{-cm} = 1.725 \times 10^{-8}\Omega\text{-m}$




$$R_a = \frac{\rho L}{A} = \frac{\rho L}{\frac{\pi d^2}{4}} = \frac{4\rho L}{\pi d^2} = \frac{4 \times 1.725 \times 10^{-8} \times 0.5}{\pi \times (0.01)^2} = 1.098 \times 10^{-4}\ \Omega$$

(b) Calculate the resistance for a 1cm length having a diameter of 0.5m.



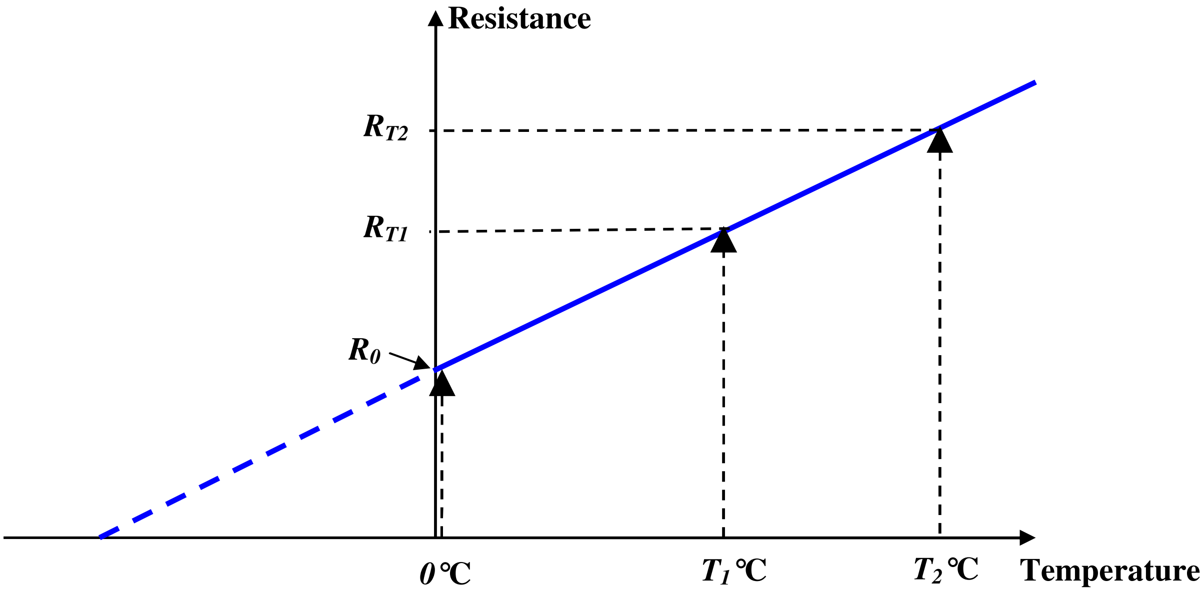

$$R_b = \frac{\rho L}{A} = \frac{\rho L}{\frac{\pi d^2}{4}} = \frac{4\rho L}{\pi d^2} = \frac{4 \times 1.725 \times 10^{-8} \times 0.01}{\pi \times (0.5)^2} = 8.78 \times 10^{-10}\ \Omega$$

Note the resistance in (a) is approximately 125000 times greater than (b).

## Variation of Resistance with temperature

The resistance of all pure metals increases with an increase in temperature, whereas the resistance of carbon, insulating materials etc. decreases with an increase in temperature. For moderate changes in temperature the change in resistance is usually linear.

The **temperature coefficient of resistance, $\alpha$,** defines the ratio of the change in resistance per degree (℃) change in temperature with reference to the resistance at a definite temperature. (For this course we will use 0°C as the reference temperature and $\alpha_0$ as the temperature coefficient of resistance). The figure below shows the variation of resistance of a typical conductor (copper) with temperature.




where:

$R_0$ is the resistance in Ohms at $0$℃
$R_{T1}$ is the resistance in Ohms at $T_1$℃
$R_{T2}$ is the resistance in Ohms at $T_2$℃

and we find that:

$$R_{T1} = R_0\left(1+\alpha_0 T_1\right)$$
$$R_{T2} = R_0\left(1+\alpha_0 T_2\right)$$

therefore:

$$\frac{R_{T1}}{R_{T2}} = \frac{R_0}{R_0}\frac{\left(1+\alpha_0 T_1\right)}{\left(1+\alpha_0 T_2\right)} = \frac{\left(1+\alpha_0 T_1\right)}{\left(1+\alpha_0 T_2\right)}$$

where $\alpha_0$ is the temperature coefficient of resistance (/℃).

A typical value for copper is 0.00426/℃.

Example

A coil of wire has a resistance of 150Ω at 20℃. It is connected to a 240V supply and after several hours the current is 1.25A. Calculate the initial current when the coil is first connected to the supply and the average temperature of the coil after several hours. Assume the temperature coefficient of resistance, $\alpha_0 = 0.00426$ /℃.

When the coil is first connected to the supply it has a resistance of 150Ω and hence the initial current may be obtained from:

$$I = \frac{240}{150} = 1.6A$$

As current flows through the coil it will heat up and its resistance will increase, hence the current will fall. This will continue until a state of equilibrium is reached – when the heat generated by the current flowing through the coil is equal to the heat dissipated to its surroundings. This is usually after a reasonable period of time – hence several hours in this question. Since we know the voltage and current we can find the final resistance of the coil at the final temperature, $\theta$℃:

$$R_\theta = \frac{240}{1.25} = 192\Omega$$

Assume the resistances at 0℃, 20℃ and $\theta$ ℃ are $R_0$, $R_{20}$ and $R_\theta$ respectively then:

$$R_{20} = R_0(1 + \alpha_0 \times 20)$$
$$R_\theta = R_0(1 + \alpha_0 \times \theta)$$

We could obtain $R_0$ from the first equation and substitute it into the second but this is not necessary:

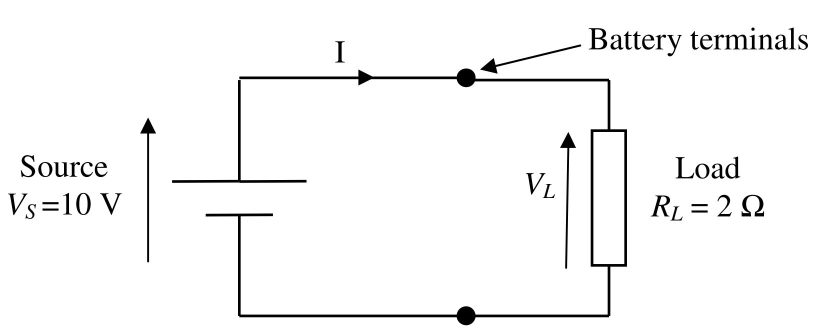
$$\frac{R_\theta}{R_{20}} = \frac{(1 + \alpha_0 \times \theta)}{(1 + \alpha_0 \times 20)}$$

Rearranging:

$$\theta = \frac{\left[\frac{R_\theta}{R_{20}}(1 + \alpha_0 \times 20) - 1\right]}{\alpha_0} = \frac{\left[\left(\frac{192}{150}(1 + 0.00426 \times 20)\right) - 1\right]}{0.00426} = \mathbf{91.3\ ℃}$$

**Connecting resistances in series**

Consider the simple circuit as shown in the figure below which consists of an ideal voltage source ($V_S$ = 10V) (e.g. battery with no internal resistance – see later) connected to a single load resistor ($R_L$ = 2Ω).




In solving electrical circuits it is always worthwhile indicating the direction of the currents and voltages using arrows as shown in the diagram. The normal convention is to have a positive current flowing out of the positive terminal of the source. The increase in potential (voltage) across a source is always from negative to positive, and the increase in potential across a sink (load) is always in the opposite direction to the current.

Since there is only one source and one sink in this circuit the source voltage is dropped across the sink, hence:

$$V_S = V_L = 10V$$

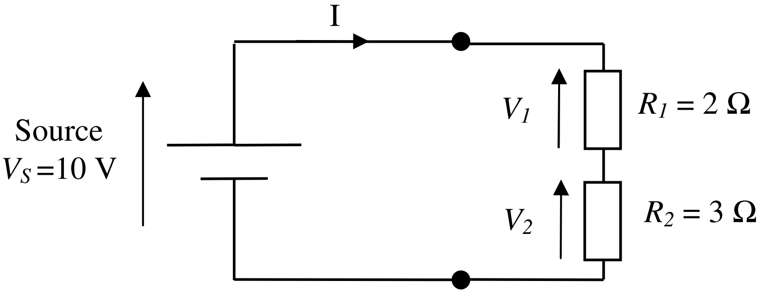
Rearranging Ohm's law the current, *I*, flowing around the circuit may be found:

$$I = \frac{V_L}{R_L} = \frac{10}{2} = 5\text{A}$$

and the power dissipated in the load resistor may be calculated:

$$P = I^2R = 5^2 \times 2 = 50\text{W} \qquad \text{or} \qquad P = VI = 10 \times 5 = 50\text{W} \qquad \text{or} \qquad P = \frac{V^2}{R} = \frac{10^2}{2} = 50\text{W}$$

Now try a circuit having two load resistors connected in series, such that the same current flows through both, as shown in the figure below:




We can apply Ohm's law to each resistor in turn:

$$V_1 = I \times R_1 \qquad \text{and} \qquad V_2 = I \times R_2$$

but now the source voltage is dropped across the pair of resistors:

$$V_S = V_1 + V_2 = I \times R_1 + I \times R_2 = I(R_1 + R_2) = I \times R_T$$

where $R_T$ is the total resistance. This means we can replace the pair of resistors with a single, equivalent resistor, $R_T = 5\Omega$ and we can find the current flowing around the circuit:

$$I = \frac{V_S}{R_T} = \frac{10}{5} = 2\text{A}$$

Knowing the current we can calculate the voltage across each resistor and the power dissipated in each.

Voltage drop across resistor $R_1$ = $V_1 = IR_1 = 2 \times 2 = 4\text{V}$

Power dissipated in resistor $R_1$ = $P_1 = I^2R_1 = 2^2 \times 2 = 8\text{W}$

Voltage drop across resistor $R_2$ = $V_2 = IR_2 = 2 \times 3 = 6\text{V}$

Power dissipated in resistor $R_2$ = $P_2 = I^2R_2 = 2^2 \times 3 = 12\text{W}$

Note: To find the power we could have used $P = VI$ or $P = V_2/R$, but we must use the voltage across the resistor of interest (***not the total voltage – this is a very common student error!***)

For resistor $R_1$:

$$P_1 = V_1I = 4 \times 2 = 8\text{W} \qquad \text{or} \qquad P_1 = \frac{V_1^2}{R_1} = \frac{4^2}{2} = 8\text{W}$$

For resistor $R_2$:

$$P_2 = V_2I = 6 \times 2 = 12\text{W} \qquad \text{or} \qquad P_2 = \frac{V_2^2}{R_2} = \frac{6^2}{3} = 12\text{W}$$

The total power dissipated in the two load resistors is 8 + 12 = 20W. This could have also been found by considering our equivalent load resistor:

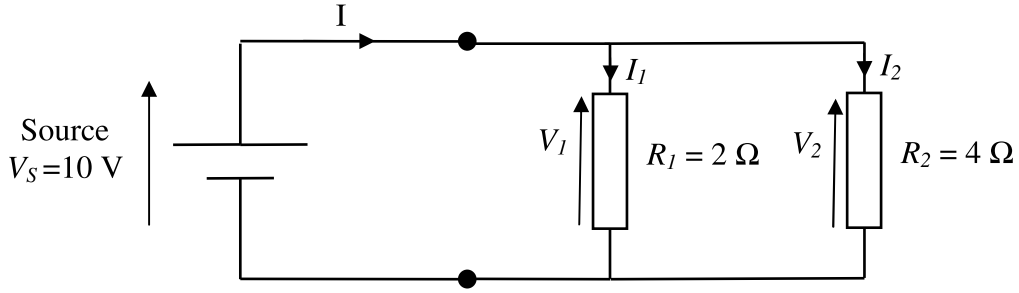
$$P = I^2 R_T = 2^2 \times 5 = 20\text{W}$$

In this example we have only considered two resistors in series, but the same method would apply to any number of resistors connected in series, i.e. with the same current flowing through each one. The equivalent resistance $R_T$ for a group of resistors in series is:

$$R_T = R_1 + R_2 + R_3 + \cdots + R_N$$

## Connecting resistances in parallel

Let us now consider a circuit in which the load consists of two resistors in parallel having the same voltage, $V_L$, across each one as shown in the figure below:




Clearly, the total current, $I$, splits into two paths with some of the current, $I_1$, flowing through resistor, $R_1$, and the remainder the current, $I_2$, flowing through resistor, $R_2$, i.e.:

$$I = I_1 + I_2$$

Clearly the voltages across the resistors are equal to the supply voltage:

$$V_S = V_1 = V_2$$

Applying Ohm's law to each resistor:

$$I_1 = \frac{V_1}{R_1} = \frac{10}{2} = 5\text{A} \quad \text{and} \quad I_2 = \frac{V_2}{R_2} = \frac{10}{4} = 2.5\text{A}$$

Hence:

$$I = I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2} = V_S\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 10\left(\frac{1}{2} + \frac{1}{4}\right) = 7.5\text{ A}$$

We could replace the pair of resistors by a single equivalent resistor, $R_T$:

$$R_T = \frac{V_S}{I} \quad \text{or} \quad \frac{1}{R_T} = \frac{I}{V_S}$$

Combining this with the previous equation gives:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \quad \text{or} \quad R_T = \frac{4}{3}\Omega$$

Check, the total current is:

$$I = \frac{V_S}{R_T} = \frac{10}{\left(\frac{4}{3}\right)} = 7.5\text{ A (as before)}$$

We can also calculate the power dissipated in each resistor:

Power dissipated in resistor $R_1$ = $P_1 = I_1^2 R_1 = 5^2 \times 2 = 50\text{W}$

Power dissipated in resistor $R_2$ = $P_2 = I_2^2 R_2 = 2.5^2 \times 4 = 25\text{W}$

Therefore the total power dissipated is 50 + 25 = 75W

Check that the equivalent resistor gives the total power:

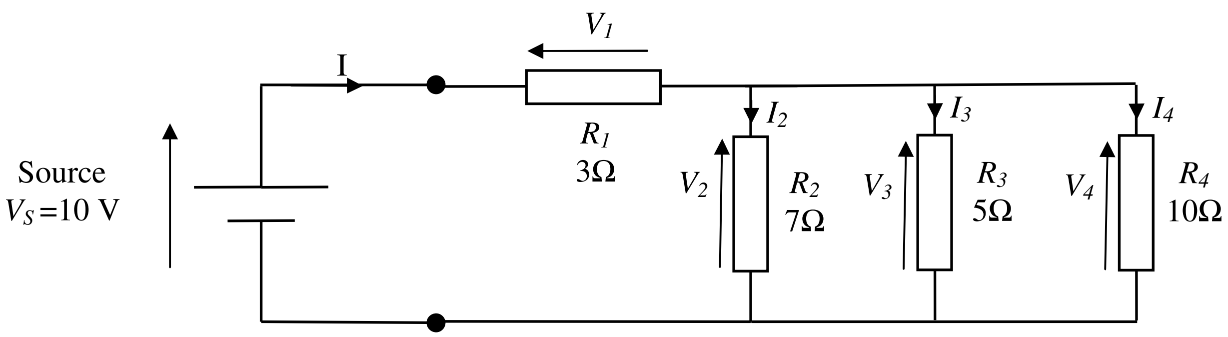
$$P_T = I^2 R_T = 7.5^2 \times \frac{4}{3} = 75\text{W}$$

We would find that if we have a number of resistors in parallel, such that the voltage across them is equal then the total resistance is obtained using:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

Example

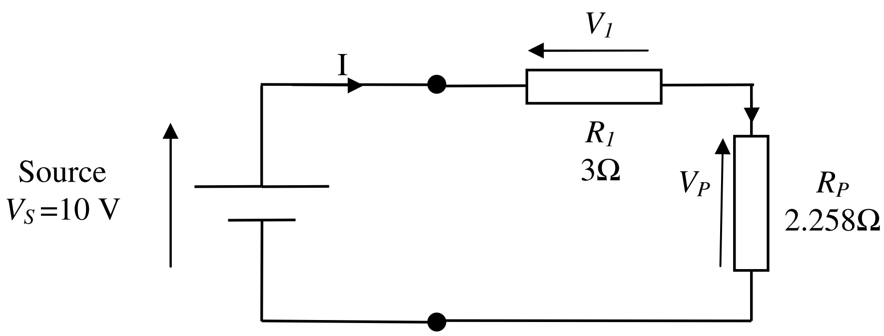
Find the total resistance of the following network of resistors, the current flowing in each resistor, the voltage across each resistor, and the power dissipated.




The key to solving this problem is to identify which resistors are in parallel and which are in series. Looking at the circuit it will be seen that the voltages across $R_2$, $R_3$ and $R_4$ are the same, therefore these resistors are in parallel and may be replaced by a single resistor $R_P$, hence:

$$R_P = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{1}{\frac{1}{7} + \frac{1}{5} + \frac{1}{10}} = 2.258\Omega$$

The circuit now reduces to:




Since the same current, $I$, flows through the two resistors these are in series and can be replaced by a single resistor:

$$R_T = R_1 + R_P = 3 + 2.258 = 5.258\Omega$$

Using this value the total current flowing out of the battery and through $R_1$ and $R_P$ can be found:

$$I = \frac{V_S}{R_T} = \frac{10}{5.258} = 1.902 \text{ A}$$

and the total power dissipated in all the load resistors is:

$$P_T = I^2 R_T = 1.902^2 \times 5.258 = 19 \text{ W}$$

The voltage across the resistors is found using Ohm's law:

$$V_1 = IR_1 = 1.902 \times 3 = 5.706 \text{ V}$$

and:

$$V_P = V_2 = V_3 = V_4 = IR_P = 1.902 \times 2.258 = 4.294 \text{ V}$$

check:

$$V_S = V_1 = V_P = 5.706 + 4.294 = 10 \text{ V}$$

Now the current in $R_2$, $R_3$ and $R_4$ and the power dissipated in each may be obtained:

Current through $R_2$ is $I_2 = \dfrac{V_P}{R_2} = \dfrac{4.294}{7} = 0.613 \text{ A}$ and $P_2 = I_2^2 R_2 = 0.613^2 \times 7 = 2.63 \text{ W}$

Current through $R_3$ is $I_3 = \dfrac{V_P}{R_3} = \dfrac{4.294}{5} = 0.859 \text{ A}$ and $P_3 = I_3^2 R_3 = 0.859^2 \times 5 = 3.69 \text{ W}$

Current through $R_4$ is $I_4 = \dfrac{V_P}{R_4} = \dfrac{4.294}{10} = 0.430 \text{ A}$ and $P_4 = I_4^2 R_4 = 0.430^2 \times 10 = 1.85 \text{ W}$

The power dissipated in $R_1$ is:

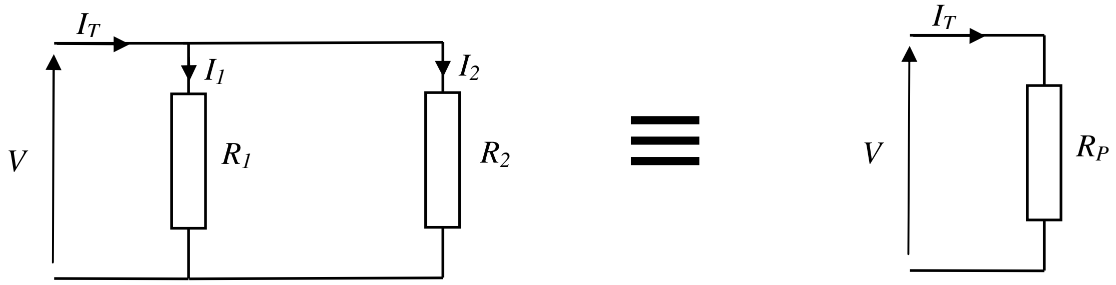
$$P_1 = I_1^2 R_1 = 1.902^2 \times 3 = 10.85 \text{ W}$$

Check:

Total current is $I_2 + I_3 + I_4 = 0.613 + 0.859 + 0.430 = 1.902 \text{ A}$

Total power is $P_1 + P_2 + P_3 + P_4 = 10.85 + 2.63 + 3.69 + 1.85 = 19 \text{ W}$

**<u>Quick calculation for two resistors in parallel</u>**




Two resistors in parallel may be replaced by an equivalent resistor, $R_P$, where:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

hence:

$$R_P = \frac{R_1 R_2}{R_1 + R_2}$$

Find $I_1$ and $I_2$ in terms of the total current $I_T$. Using Ohm's law:

$$V = I_1 R_1 = I_2 R_2 = I_T R_P = I_T \left( \frac{R_1 R_2}{R_1 + R_2} \right)$$

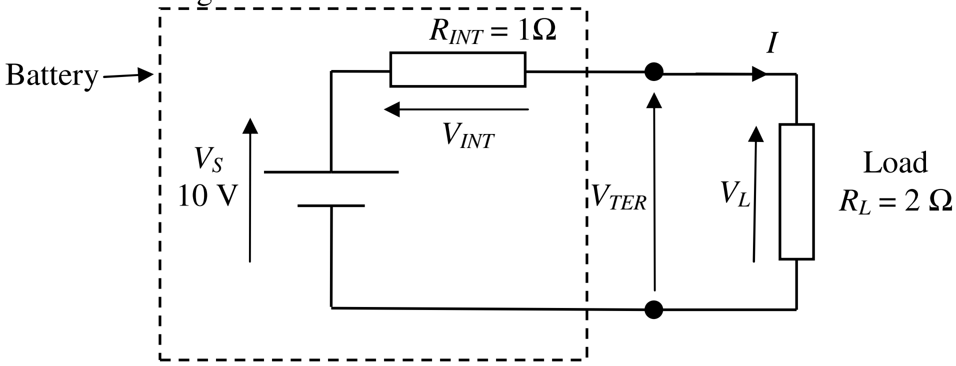
from which $I_1$ and $I_2$ may be found:

$$I_1 = \frac{I_T}{R_1} \left( \frac{R_1 R_2}{R_1 + R_2} \right) = I_T \frac{R_2}{R_1 + R_2} \qquad \text{and} \qquad I_2 = \frac{I_T}{R_2} \left( \frac{R_1 R_2}{R_1 + R_2} \right) = I_T \frac{R_1}{R_1 + R_2}$$

**Non-ideal voltage sources**

So far in our analysis of circuits we have assumed 'ideal' voltage sources, that is to say they will always produce their rated voltage no matter what current is being drawn from them. In practice cells and batteries possess **internal resistance** from the materials and chemicals they are constructed from. This means the voltage at the terminals will now be a function of the current drawn and there will also be power dissipation within the battery. A visible example of this is the dimming of a car's headlights when the engine is cranked on the starter motor.

Let us consider the example we solved earlier, but now assume the battery has a 1Ω internal resistance as shown in the figure:




The current, $I$, flows through both the internal resistance, $R_{INT}$, and the load resistor, $R_L$, and so the two resistors are in series:

$$R_T = R_{INT} + R_L = 1 + 2 = 3\Omega$$

and the current is:

$$I = \frac{V_S}{R_T} = \frac{10}{3} = 3.33 \text{ A}$$

A current flowing through a resistor (energy sink) will dissipate power as heat. The energy dissipated in the load, which maybe a light bulb, heater etc. is useful energy, whereas the energy dissipated within the battery is waste energy as it simply heats up the battery itself.

Power dissipated in load (useful energy) is $P_L = I^2 R_L = 3.33^2 \times 2 = 22.2$ W

Power dissipated in battery (waste energy) is $P_{INT} = I^2 R_{INT} = 3.33^2 \times 1 = 11.1$ W

The total power dissipated in our system is 22.2 + 11.1 = 33.3 W.

The voltage across the terminals of the battery, $V_{TER}$, is equal to the voltage across the load, $V_L$:

$$V_{TER} = V_L = IR_L = 3.33 \times 2 = 6.67 \text{ V}$$

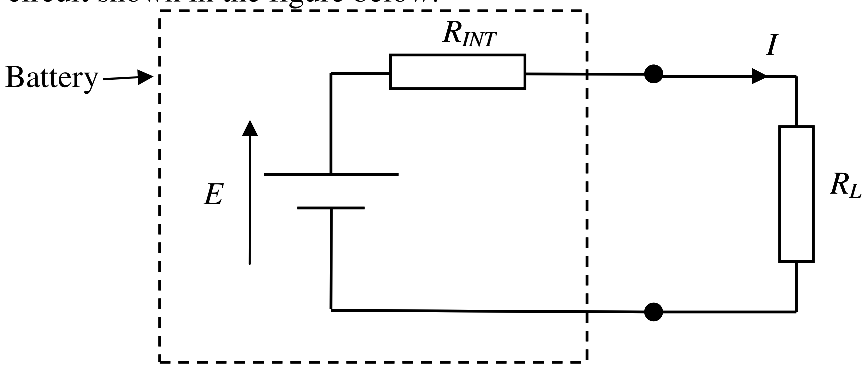
Alternatively we could have calculated the voltage drop across the internal resistance and subtracted this from $V_S$:

$$V_{TER} = V_S - IR_{INT} = 10 - 3.33 \times 1 = 6.67 \text{ V}$$

$V_S$ is called the **open-circuit**, or **no-load** voltage, of the battery since if the load was disconnected no current would flow and hence $V_{INT}$ would be zero and $V_S$ would appear across the terminals. Often we use the symbol $E$ (open-circuit emf) to depict this.

This example has shown that to transfer 22.2W of power to our load requires a total power of 33.3W with 11.1W being dissipated in the battery itself as waste energy. Let us now look at the condition for maximum power transfer between the source and the load.

Consider the circuit shown in the figure below:




The current, $I$, in the circuit is given by:

$$I = \frac{E}{R_{INT} + R_L}$$

and the power transmitted to the load is:

$$P_L = I^2 R_L = \left( \frac{E}{R_{INT} + R_L} \right)^2 R_L = \frac{E^2 R_L}{R_{INT}^2 + 2R_{INT} R_L + R_L^2} = \frac{E^2}{\frac{R_{INT}^2}{R_L} + 2R_{INT} + R_L}$$

We are looking for the value of $R_L$ which will give maximum power transfer. This will occur when the denominator in the above equation is minimum, i.e.when:

$$\frac{d}{dR_L}\left\{ \frac{R_{INT}^2}{R_L} + 2R_{INT} + R_L \right\} = 0$$

$$-\frac{R_{INT}^2}{R_L^2} + 1 = 0$$

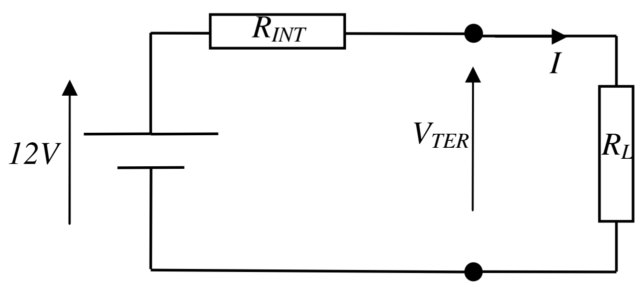
hence:

$$R_L = R_{INT}$$

This means that the condition for maximum power transfer between source and load is when the load resistance is equal to the internal resistance of the source. This is also known as resistance matching.

<u>Example</u>

The voltage across the terminals of a 12V car battery drops to 9V when supplying a current of 300A to the starter motor. Find the internal resistance of the battery and the load resistance for maximum power transfer to the load. How does the system efficiency vary with $R_L$?




First we need to find the internal resistance of the battery. When the current is 300A the terminal voltage, $V_{TER}$, is 9V so 3V must be dropped across the internal resistance, hence:

$$R_{INT} = \frac{V_{INT}}{I} = \frac{3}{300} = 0.01\Omega$$

For maximum power transfer to the load:

$$R_L = R_{INT} = 0.01\Omega$$

Under these conditions the new current will be:

$$I = \frac{E}{R_{INT} + R_L} = \frac{12}{0.01 + 0.01} = 600 \text{ A}$$

and the power in the load is:

$$P_L = I^2 R_L = 600^2 \times 0.01 = 3.6 \text{ kW}$$

However, since the internal resistance carries the same current and has the same value then there is also 3.6 kW dissipated in the internal resistance. (This is equivalent to a 3 bar electric fire heating up the battery!).

Batteries are often specified in terms of Ampere-hours (A-h). If our battery is 60A-h it can deliver (fully charged) 60A for 1hour or 600A for 0.1 hour (6 minutes). (Unfortunately the electrolyte would probably boil after 30 seconds!).

The efficiency of a system is defined as:

$$\eta = \frac{P_{OUTPUT}}{P_{INPUT}} \times 100\% = \frac{P_{OUTPUT}}{P_{OUTPUT} + P_{LOSSES}} \times 100\% = \frac{P_L}{P_L + P_{INT}} \times 100\%$$

In the example we have just looked at, where $R_L$ and $R_{INT}$ are equal, then:

$$\eta = \frac{P_L}{P_L + P_{INT}} \times 100\% = \frac{3600}{3600 + 3600} \times 100\% = 50\%$$

Alternatively:

$$\eta = \frac{P_L}{P_L + P_{INT}} \times 100\% = \frac{I^2 R_L}{I^2 R_L + I^2 R_{INT}} \times 100\% = \frac{R_L}{R_L + R_{INT}} \times 100\% = \frac{0.01}{0.01 + 0.01} \times 100\% = 50\%$$

What happens to the efficiency when the load resistance is say 1Ω?

$$I = \frac{E}{R_{INT} + R_L} = \frac{12}{0.01 + 1} = 11.88 \text{ A}$$

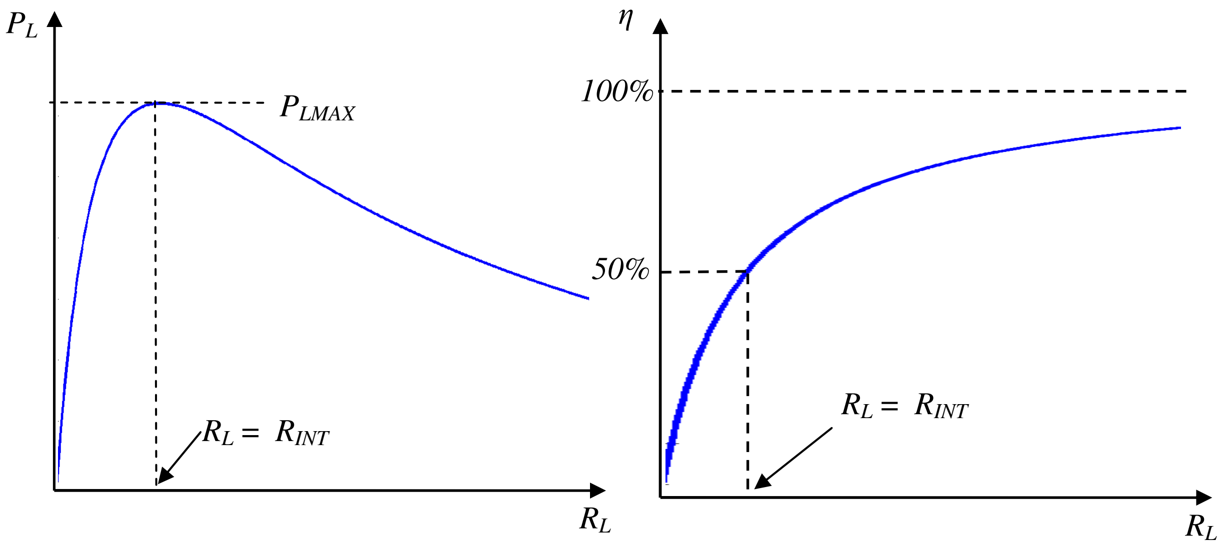
$$P_L = I^2 R_L = 11.88^2 \times 1 = 141 \text{ W}$$

$$P_{INT} = I^2 R_{INT} = 11.88^2 \times 0.01 = 1.41 \text{ W}$$

$$\eta = \frac{P_{LOAD}}{P_{LOAD} + P_{INT}} \times 100\% = \frac{141}{141 + 1.41} \times 100\% = 99\%$$

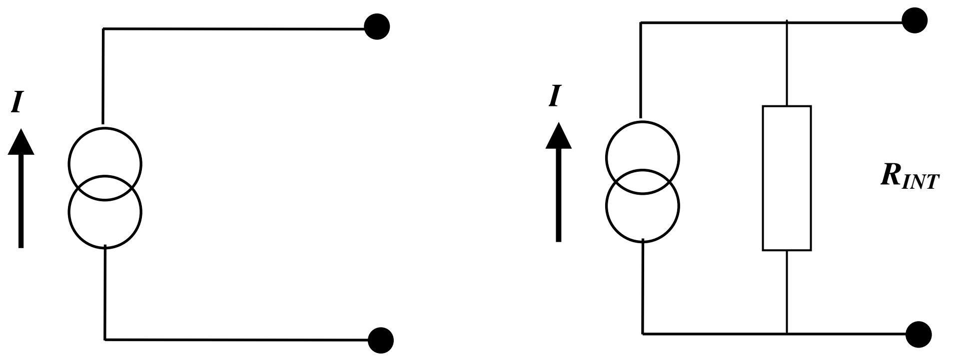
What happens if $R_L$ is zero i.e. the terminals of the battery are shorted out? The current is now only limited by the internal resistance of the battery and a current of 1200 A would flow. The power dissipated in the battery would be 14.4kW and the power in the load would be zero and hence the efficiency would be zero. This is likely to cause serious damage to the battery and should never be done.

The condition for maximum power transfer to the load is when the load resistance is equal to the internal resistance but the efficiency is only 50%. We could repeat the above calculations for a range of loads and obtain the curves shown in the figure below.




## Current sources

An ideal current source is a device which will generate a prescribed current independent of the circuit to which it is connected. To do so it must be able to generate an arbitrary voltage across is terminals. A more practical representation has an internal resistance connected in parallel with its terminals. On this course only limited use is made of circuits containing current sources.




**Ideal current source** **Practical current source**