EEE334 Solutions 2014

1 a

In surveillance radar we want an antenna with a narrow beam to give good angular resolution. However, a narrow beam means we have to scan more positions in the sky which results in a longer scan time. Hence there is a trade-off. This trade-off is significant for mechanically scanned antennas but is less critical for modern electronically scanned phased arrays.

(2)

b

Hemisphere of sky contains 2π steradians

Antenna beamwidth $\Delta\Omega = \Delta\theta\Delta\phi$

Number of beam positions $N_B = \dfrac{2\pi}{\Delta\Omega} = \dfrac{2\pi}{\Delta\theta\Delta\varphi}$

Using gain $G = \dfrac{4\pi}{\Delta\theta\Delta\phi}$ gives $N_B = \dfrac{G}{2}$

$G = 40dB = 10{,}000$ which gives $N_B = 5{,}000$

G = 30dB = 1000 which gives $N_B = 500$

Total scan time = 6s, therefore dwell time $= \frac{6s}{500} = 12\text{ms}$

(4)

c

wavelength $\lambda = \frac{c}{f} = \frac{3*10^8}{560*10^8} = 53.6cm$

PRT = pulse-width/duty-cycle $= \frac{1.3\mu S}{8.3*10^{-4}} = 1.57mS$

PRF=1/PRT = 638.5Hz

Average Power = Peak Power * Duty cycle $= 279\text{kW} \times 8.3 \times 10^{-4} = 231.6\text{W}$

Number of hits $n = \dfrac{\Delta\theta \times PRF}{6 \times RPM}$ so $\Delta\theta = \dfrac{6 \times RPM \times n}{PRF} = \dfrac{6 \times 16 \times 9.9}{638.5} = 1.49^0$

Gain $G = \dfrac{4\pi}{\Delta\theta\Delta\phi}$ with beamwidths in radians

So for beamwidths in degrees, $G = \dfrac{4\pi}{\Delta\theta\Delta\phi} \times \left(\dfrac{180}{\pi}\right)^2 = \dfrac{4 \times (180)^2}{1.49 \times 4 \times \pi} = 6920$ or 38dB

Max range $= \dfrac{c}{2 \times PRF} = \dfrac{3 \times 10^8}{2 \times 638.5} = 3.35km$

Range resolution $= \dfrac{c \times \tau}{2} = \dfrac{3 \times 10^8 \times 1.3 \times 10^{-6}}{2} = 195m$

(8)

d

Performance of radar system is proportional to the transmit power multiplied by the transmit and receive antenna gains, So we can write

$$PG^2 = K$$

Where P is transmit power, G is the antenna gain (Tx = Rx) and K is a constant.

Now we are told that

$$C = C_P + C_A = PC_k + AC_{sm}$$

Substituting for $P = \frac{K}{A^2}$ gives

$$C = \frac{KC_{kW}}{A^2} + AC_{sm}$$

Differentiating wrt A and setting to zero gives

$$C_{sm} = \frac{2KC_{kW}}{A^3}$$

But $P = \frac{K}{A^2}$ so

$$AC_{sm} = 2PC_{kW}$$

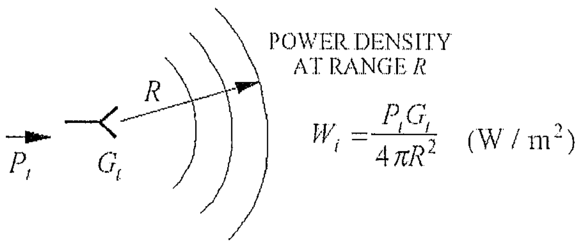
or

$$C_A = 2C_P$$

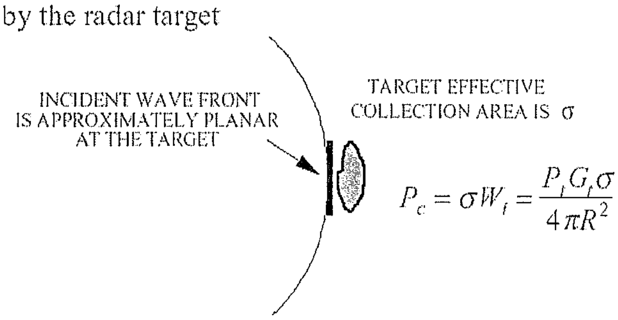
(6)

2a

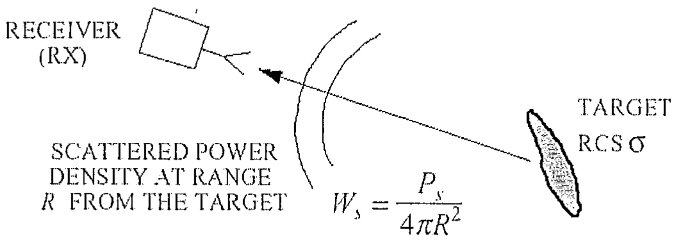
Power density incident on the target



Power collected by the radar target



The RCS gives the fraction of incident power that is scattered back toward the radar. Therefore, $P_s = P_c$ and the scattered power density at the radar is obtained by dividing by $4\pi R^2$.

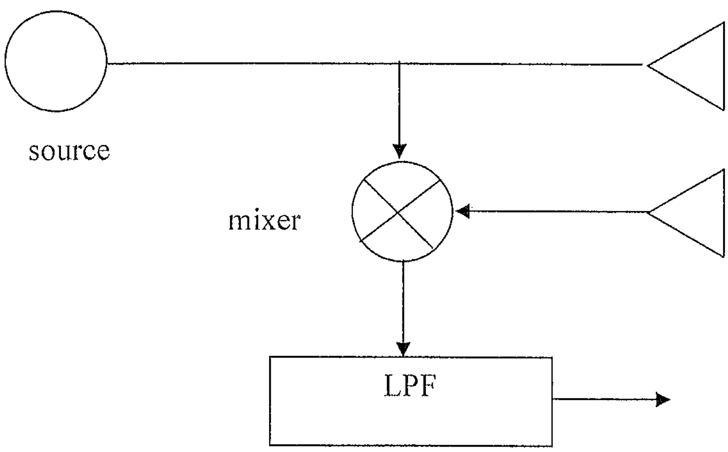


The target scattered power collected by the receiving antenna is $W_s A_{er}$. Thus the maximum target scattered power that is available to the radar is

$$P_r = \frac{P_t G_t \sigma A_{er}}{(4\pi R^2)^2} = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4}$$

(6)

2b



Tx signal = $\cos(\omega_0 t)$

Rx signal = $B\cos(\omega_d t)$

At mixer Rx is multiplied by a signal with same frequency as Tx signal

Output from mixer is $S = B\cos(\omega_d t)\cos(\omega_0 t)$

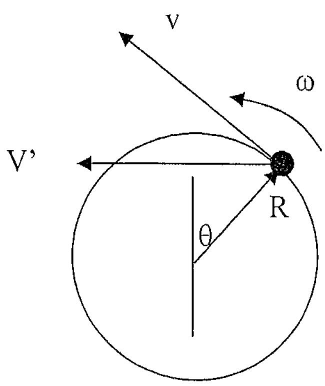
Expand to give $S = \frac{B}{2}\left[\cos\left[(\omega_d - \omega_0)t\right] + \cos\left[(\omega_d + \omega_0)t\right]\right]$

Low-pass filtering leaves only difference term i.e. $\frac{B}{2}\cos(\omega_d - \omega_0)$ where Doppler frequency $\Delta\omega = \omega_d - \omega_0$

(4)

2c

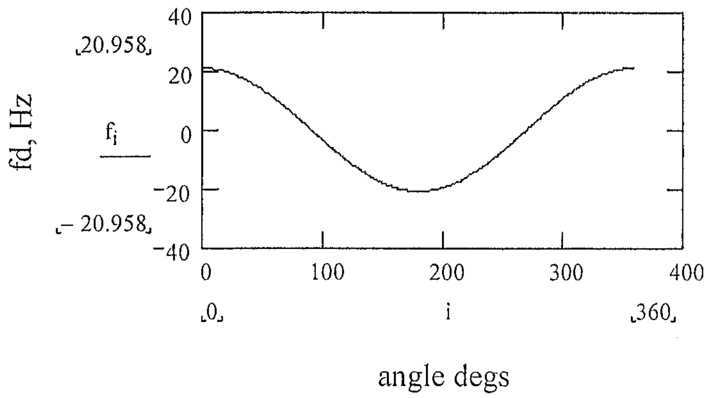
Assuming that $d \gg R$ we can assume that the target is illuminated by a plane-wave and $\alpha = 0$



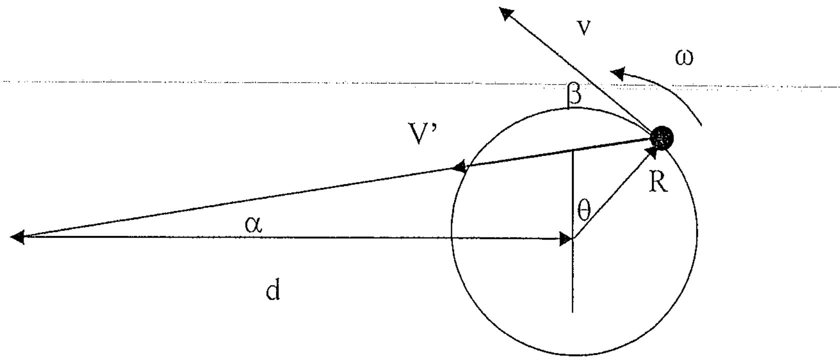
We have $v = R\omega$ and $v' = v\cos\theta$ where v' is the component of the targets velocity in the direction of the illuminating beam of the radar.

Hence Doppler shift is given by $f_d = \frac{f_0 2R\omega\cos\theta}{c}$ where f_0 is the radar frequency

Use numerical values and plot. Note that 30RPM = pi rads/s



For the second part of the problem we must take into account the angle α



we now have

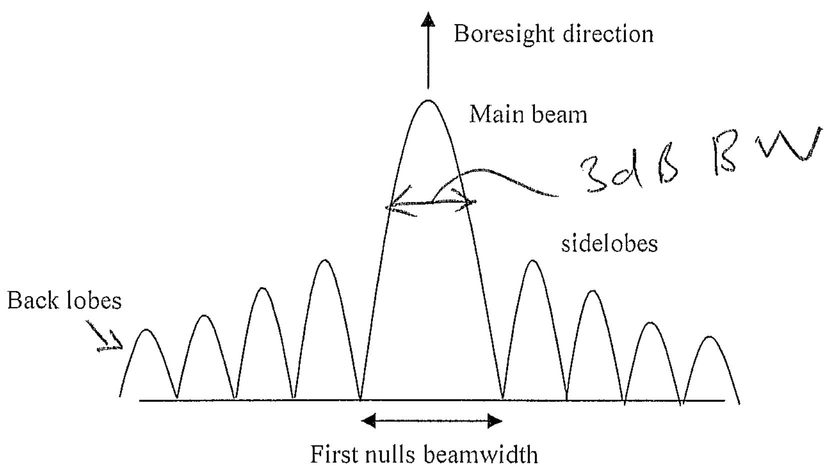
$v' = v\cos\beta$

where $\beta = \alpha + \theta$

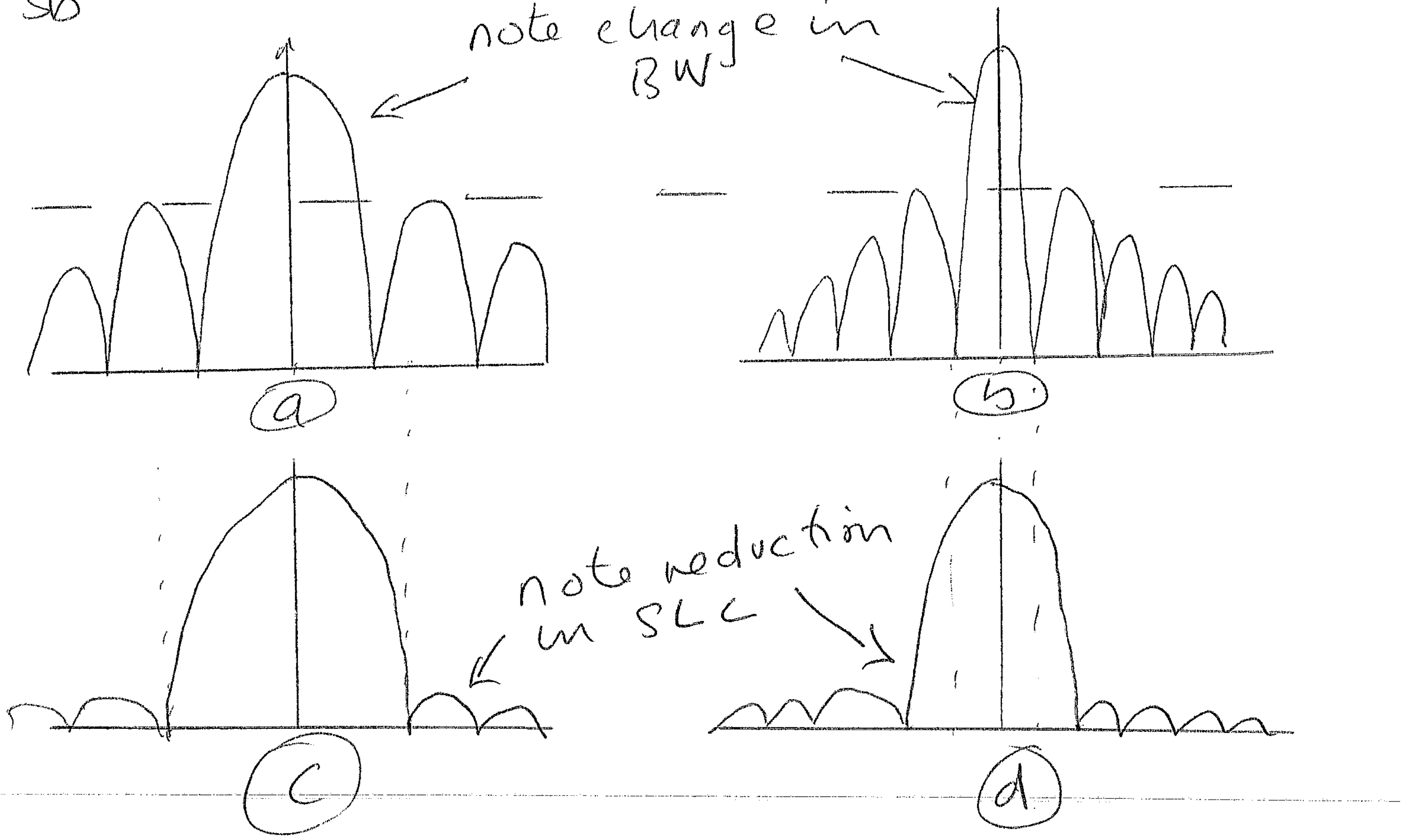
Hence

$$f_d = \frac{f_0 2R\omega\cos(\theta+\alpha)}{c}$$

3a



3b



3c

Solution is based on Friis transmission equation

$$Pr := Pt \cdot Gt \cdot Gr \left(\frac{\lambda}{4 \cdot \pi \cdot R} \right)^2$$

First calculate some addtional parameters from given information

wavelength $\lambda := \frac{3 \cdot 10^8}{10.8 \cdot 10^9}$ $\lambda = 0.028$ metres

Rx power $Pr := 500 \cdot 10^{-9}$ Watts

Range $R := 150 \cdot 10^3$ metres

Diamiter of TX dish $Dt := 2.1$ $\eta t := 0.75$ TX efficiency

Diamiter of RX dish $Dr := 1.8$ $\eta r := 0.65$ RX efficiency

Effective area of TX antenna $At := \left(\frac{Dt}{2} \right)^2 \cdot \pi \cdot \eta t$ $At = 2.598$

Gain of TX antenna $Gt := \frac{4 \cdot \pi \cdot At}{\lambda^2}$ $Gt = 4.231 \times 10^4$

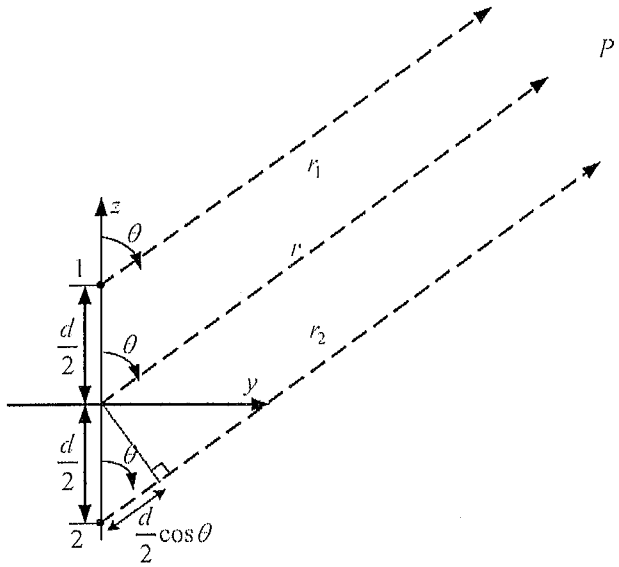
Effective area of RX antenna $Ar := \left(\frac{Dr}{2} \right)^2 \cdot \pi \cdot \eta r$ $Ar = 1.654$

Gain of RX antenna $Gr := \frac{4 \cdot \pi \cdot Ar}{\lambda^2}$ $Gr = 2.694 \times 10^4$

Therefore TX power is

$$Pt := \frac{Pr}{\left[Gt \cdot Gr \cdot \left(\frac{\lambda}{4 \cdot \pi \cdot R} \right)^2 \right]}$$ $Pt = 2.02$ Watts

3d



Field at P given by

$$E = \frac{e^{-jkr_1}}{r_1} + \frac{e^{-jkr_2}}{r_2}$$

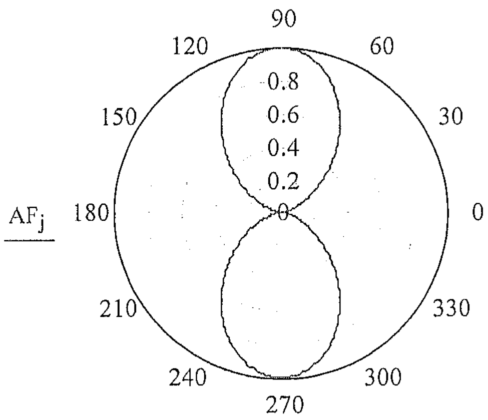
In far-field can assume that $r_1 = r_2$ for amplitude variations

For phase variations (due to difference in path length)

$$r_1 = r - \frac{d}{2}\cos\vartheta \quad \text{and} \quad r_2 = r + \frac{d}{2}\cos\vartheta$$

Field now given by $E = \frac{1}{r}\left[e^{-jk(r - d/2\cos\theta)} + e^{-jk(r + d/2\cos\theta)}\right]$

After some further manipulation this gives

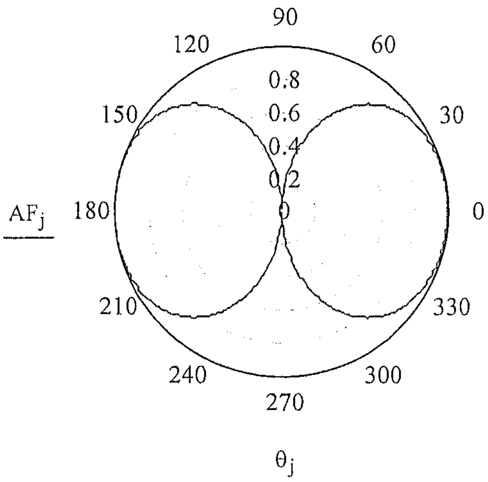


θj

$E = \frac{e^{-jkr}}{r} 2\cos\left[\frac{kd}{2}\cos\theta\right]$ and normalised

array factor given by $AF = \cos\left[\frac{kd}{2}\cos\theta\right]$

If the elements are driven in anti-phase the main beam of radiation is in the endfire direction

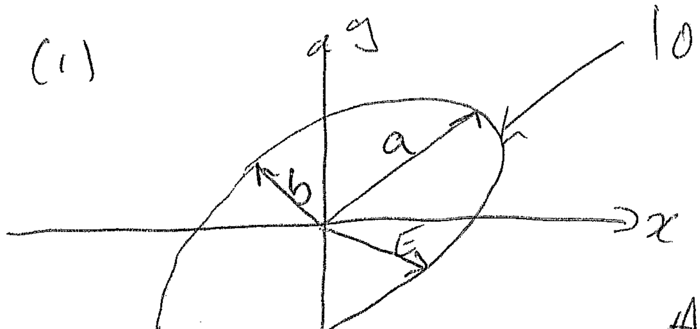


(8)

4a
Wireless links more efficient in terms of power when the communication link is over a long distance. In wireless systems the power drops off as $1/R^2$ and good cable system may have losses of 5dB per km. So, for example, if a system has a 100dB of loss at 20km doubling the distance would produce 200dB of loss in a cable system but only 106dB in a wireless system. [$1/R^2$ to $1/(2R)^2$ gives a reduction of ¼ or 6dB]

(4)

4b (i)



locus of E vector

Axial ratio

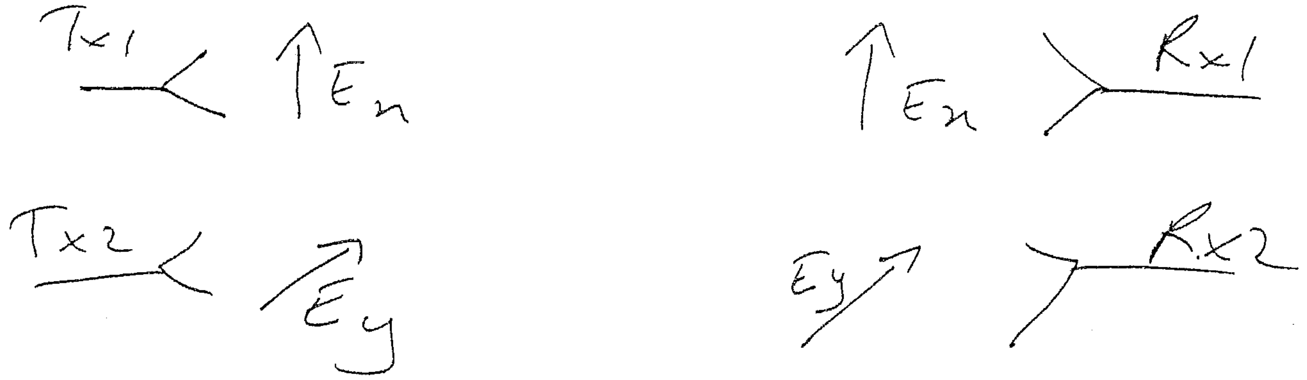
$AR = a/b$

$1 \leq AR \leq \infty$

circular pol

linear pol

9 b/(ii) Polarisation diversity.



Tx1 and Rx1 vertically polarised
Tx2 and Rx2 Horizontally "

Signal from Tx1 received by Rx1
But <u>not</u> received by Rx2

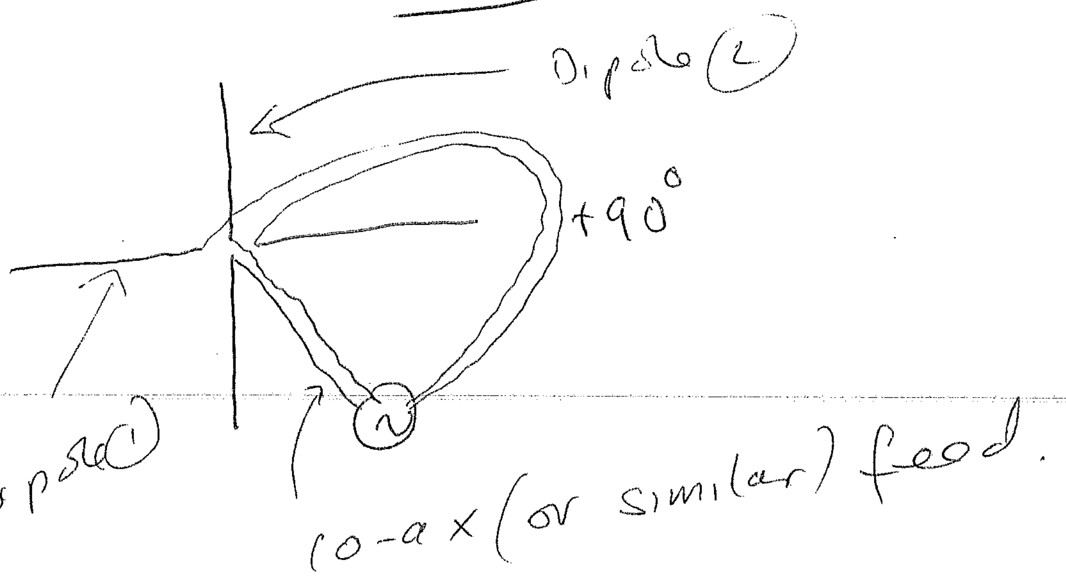
Signal from Tx2 received by Rx2
but <u>NOT</u> by Rx1

Hence can transmit 2 signals using same frequency and double capacity.

Can also use LHC + RHC pol

4b (iii)

circular pol can be generated from 2 orthogonal dipoles driven with a 90° phase difference



4c

The first step in this problem is to work out the directivity (or gain as the antenna is lossless)

Pattern maximum $U_{max} = 1$

Total radiated power $P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta d\theta d\phi = P_{rad} = 2\pi\int_0^{\pi}\sin^4\theta d\theta$

Now using the standard integral with $x = \theta$ and $a = 1$ and evaluating gives

$$P_{rad} = \frac{3\pi^2}{4}$$

Directivity given by $D = 4\pi\frac{U_{max}}{P_{ad}} = \frac{16}{3\pi} = 1.7$ or 2.3dB

Effective area of antenna given by $A_e = \frac{\lambda^2}{4\pi}D = \frac{30^2}{4\pi}\times 1.7 = 122m^2$

Power accepted by antenna $P_r = A_e W_i = 122\times 5\times 10^{-6} = 6.1\times 10^{-4} W$

The antenna is not matched to the transmission line so some power will be reflected. To calculated how much we work out the refection coefficient

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{73-50}{73+50} = 0.187$$

power transferred to coax cable $P_a = P_r\left(1-\rho^2\right) = 6.1\times 10^{-4}\times(1-0.187^2) = 5.89\times 10^{-4} W$ or 589μW

(8)