EEE6001

Students are expected to know the following formula and information for section A of the module.

ELECTRIC FIELDS

Coulomb's Law

Force between two point charges, q_1 and q_2 has a magnitude:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_o R^2}$$

in direction of line joining charges. In vector notation:

$$\underline{F} = \frac{q_1 q_2}{4\pi\varepsilon_o R^3}\ \underline{R} \quad \text{or} \quad \underline{F} = \frac{q_1 q_2}{4\pi\varepsilon_o R^2}\ \hat{\underline{R}}$$

Electric Field

Defined by:

$$\underline{E} = \frac{Q}{4\pi\varepsilon_o R^3}\ \underline{R}$$

and then the force is:

$$\underline{F} = q\underline{E}$$

Potential

Work done in moving q_1 from A to B is:

$$q_1\ \left(\phi(A) - \phi(B)\right)$$

where φ is potential. Potential due to charge q is:

$$\phi = \frac{q}{4\pi\varepsilon_o R}$$

and φ and E are related by:

$$\phi(B) - \phi(A) = -\int_A^B \underline{E}\bullet \underline{dl} = -\int_A^B E\cos\ \theta d\ell$$

which also gives:

$$\oint E\cos\theta dl = 0$$

$$\underline{E} = -\nabla\phi = \left(-\frac{d\phi}{dx},\ -\frac{d\phi}{dy},\ -\frac{d\phi}{dz}\right)$$

Gauss's Law

Surface integral of E gives:

$$\underline{E}\text{ gives } \oint_s E\cos\theta\ da = \frac{Q}{\varepsilon_o}$$

Q is total charge enclosed by surface S.

Important Cases

Students should be able to derive these formula using expressions a variety of methods eg. Expression for the electric field, Gauss's Law.

(i) Sheet of charge:

$$\left|\underline{E}\right| = \frac{q_s}{2\varepsilon_o}$$

q_s is surface density, or charge per unit area.

(ii) Line of charge:

$$\left|\underline{E}\right| = \frac{q_\ell}{2\pi r\varepsilon_o}$$

q_l is charge per unit length.

Capacitance

Capacitance of two conductors is defined by $C = Q/V$. For parallel plate capacitor:

$$C = \varepsilon\, A/d$$

where ε = permittivity of separating medium. Effect of dielectric medium is to increase the capacitance. Stored energy in capacitor is:

$$\frac{1}{2}\,CV^2$$

MAGNETIC FIELDS

Biot-Savart Law

B field is given by:

$$\underline{B} = \frac{\mu_o}{4\pi}\oint\frac{I\underline{dl}\ \times\ \underline{\hat{r}}}{r^2}$$

Analytical results possible only for simple geometries.

Ampère's Law

$$\oint \underline{B}\bullet\underline{dl} = \oint B\cos\theta\, d\ell = \mu_o I$$

I is the total current which is enclosed by the path of integration.

Magnetic Flux

Defined by:

$$\Phi = \int B\cos\theta\, da$$

i.e. Φ is given by the integral over area of normal component of B .For uniform B,

$$\Phi = BA$$

hence B is called magnetic flux density. For a closed surface of integration:

$$\oint B\cos\theta\, da = 0$$

which implies no magnetic poles.

Important Cases

Students should be able to derive these formula using expressions a variety of methods eg. Expression for the magnetic field, Ampere's Law.

(i) Infinitely long straight wire:

$$B = \mu_o I/2\pi r$$

(ii) On axis of circular loop:

$$B = \mu_o Ia^2 / 2\left(a^2 + d^2\right)^{3/2}$$

(iii) Inside long straight solenoid:

$$B = \mu_o nI$$

MAGNETIC INDUCTION

Faraday's Law

If flux linkages through a circuit change with time, magnitude of emf induced is:

$$EMF = \frac{d\phi}{dt}$$

Polarity of *EMF* given by Lenz's Law, is such as to try to keep Φ constant.

Self-inductance

Defined by:

$$EMF = L\frac{di}{dt}$$

where L depends on geometry of circuit (and also any magnetic materials present)..
Inductance of solenoid:

$$= \frac{\mu_o N^2 A}{\ell}$$

where N is the total number A is the crosssectional area, and l is the length of the solenoid.
Energy stored in inductance is:

$$Energy = \frac{1}{2}Li^2$$

Mutual Inductance

Current change in one circuit induces emf in nearby circuit:

$$EMF = M\frac{di}{dt}$$

M is coefficient of mutual inductance, depends on geometry and materials.

EMF induced by Motion

EMF is generated by conductor moving in B field:

$$EMF = Blv\sin\theta$$

MAGNETIC CIRCUITS

Ampere's law in terms of H:

$$\oint H \cdot dl = N \times I$$

Relationship between flux density, *B* and magnetic field strength *H*:

$$B = \mu_0 \mu_r H$$

Concept of reluctance, *S*, and its calculation for simple geometries:

$$S = \frac{l}{\mu_R \mu_0 \times A}$$

where *l* is the length of the magnetic circuit and *A* is the cross-sectional area through which the flux passes. Calculations involving reluctances in series and parallel.
Relationship between MMF, *F*, flux, *φ* and reluctance, *S*:

$$F = NI = \phi \times S$$

Inductance can be expressed in terms of reluctance as:

$$L = \frac{N^2}{S}$$

TRANSFORMERS

For an ideal transformer the input power to the primary winding is equal to the ouput power drawn from the secondary winding. (N.B. Power is not the same as the VA rating).
Turns ratio and relationship between primary and secondary currents:

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

Concept of referring impedances between windings:

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

Maximum flux in the transformer core in terms of the rms supply voltage:

$$\phi_{MAX} = \frac{V_{1RMS}}{4.44 N_1 f}$$

Students should also be able to reproduce the equivalent circuit of a transformer which includes the primary and secondary winding resistances and leakage inductances and the magnetising branch.

DC MOTORS

Students should understand the basic principles of DC motors, particularly the action of the commutator and the basic equations describing their operation:

$T = MI_f I_a$ (wound field) and $T = k_t I_a$ (Permanent magnet field)

$E = MI_f \omega$ (wound field) and $T = k_e \omega$ (Permanent magnet field)

Students should be able to draw the equivalent circuit and derive the performance characteristics (torque versus speed) for both shunt and series connected wound machines.

Gearboxes and referred Inertia

For a gearbox with a step-down ratio R:

$$\omega_i = R\omega_o$$
$$T_o = RT_i$$
$$\omega_i T_i = T_o \omega_o$$
$$J_o^{"} = \frac{1}{R^2} J_o$$

For simple motion control systems employing servo motors, students should be able to draw and calculate the speed-time, torque-time, current-time, voltage-time and voltage-current diagrams and understand the concept of 4-quadrant operation.

In addition to completing the 3 tutorial sheets, students are encourage to attempt the past exam questions to be found on the EEE6001, EEE220 (Electric and Magnetic Fields) and EEE202 (Electromechanical Energy Conversion) websites. Questions relating to magnetic circuits and transformers can be found on the EEE111 website.