

# 1. INTRODUCTION TO SERVO DRIVES

## 1.1 Introduction

Servo drives are used in a very wide power range, from a few watts to many thousands of kilowatts, in applications ranging from very precise, high-performance position controlled drives in robotics to high speed drives for processing and automated manufacturing. In all drives where the speed and position are controlled, a power electronic inverter is needed as an interface between the input power and the motor.

The following three types of motor drives will be discussed:

- ◆ DC-motor drives
- ◆ Induction-motor drives
- ◆ Synchronous permanent magnet motor drives

In servo applications of motor drives, the response time and the accuracy with which the motor follows the speed and position commands are extremely important. These servo systems require speed or position feedback for a precise control as shown in Fig. 1.1 In addition, if an ac-motor drive is used, the controller must incorporate sophistication, such as field-oriented control, to make the ac motor (through the power electronic converter) meet the servo-drive requirements.

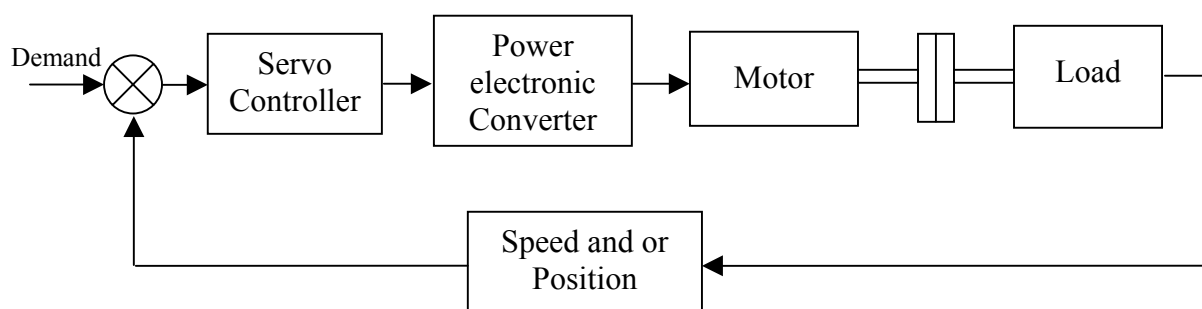


Fig. 1.1 Block diagram of servo drives

## 1.2 The mechanical system (Load)

The mechanical system is “seen” by the motor as a torque that must be applied to a shaft by the motor during a servo operation. The relation between this load torque and the motor speed must be defined. For steady state operation, this definition may be made in terms of the four-quadrant torque-speed diagram as shown in Fig. 1.2 in which  $\omega$  is the speed of the rotation of the motor, or the driven shaft, and  $T_L$  is the coupling torque developed by the motor or the load presented by the shaft of the mechanical system.

The first quadrant in Fig. 1.2 applies to normal forward driving. In the fourth quadrant the mechanical system demands a negative torque which apposes the direction of rotation to provide braking. In the third quadrant, the motor torque and direction of rotation are reversed. The operating conditions are similar to those in the first quadrant. The second quadrant may represent one of the two possible conditions. If the electrical conditions are the same as in first-quadrant driving, the mechanical system is driving the motor in a direction opposite to

that which would result from its own developed torque. This is another type of braking called “plugging”. If the electrical conditions are changed to give reverse driving in the third quadrant, any of the types of braking described for the fourth quadrant are obtained in the second quadrant.

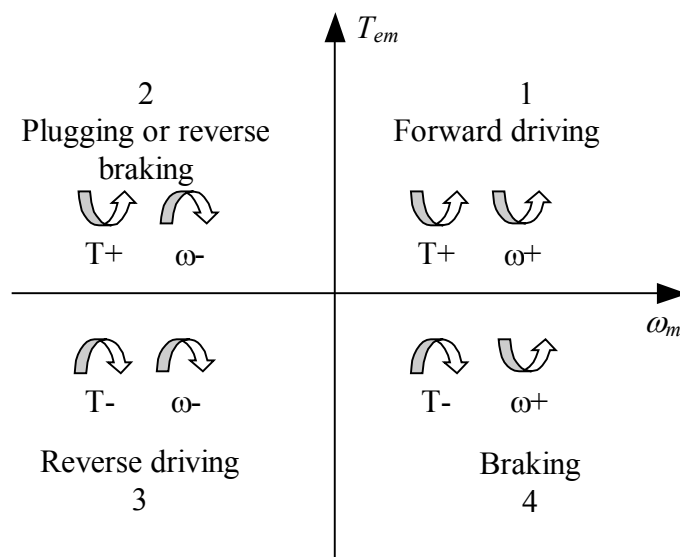


Fig. 1.2 Four quadrant, torque-speed diagram

The load torque may be consist of components due to the following:

1. Friction: Torque used to drive the mechanical system without doing additional mechanical work
2. Windage: Torque used to agitate or pump the air surrounding the moving parts of the mechanism
3. Acceleration (which may be position and negative): Torque developed under transient conditions and used to overcome the mechanical inertial of the mechnism.
4. Mechanical work

The relation between torque due to friction in the mechanical system and speed of the driving motor is rarely simple, as shown typically in Fig. 1.3 (a). The friction torque, however, is usually small in relation to load torque and may be approximated to make analysis of the system practicable. A common approximation is shown in Fig. 1.3 (b), where

$$T_F = T_B + T_C + T_s$$

Component  $T_B$ , which is directly proportional to speed, is called *viscous friction* and is defined by:

$$T_B = B\omega = B \frac{d\theta}{dt}$$

where  $B$  = viscous damping coefficient (constant) for the system

$\theta$  = angular displacement in radians

$\omega$  = angular velocity in rad/s

The component  $T_C$ , which does not vary with speed, is called *Coulomb friction*. It apposes motion at all speeds, thus constituting a load torque for forward and reverse driving. The

small component  $T_S$  is due to static friction, or “stiction”. Its nature is that the opposing torque is large at standstill but reduces when start to move. It cannot be included in a linearised model of the system and may cause problems in terms of stability and positioning accuracy.

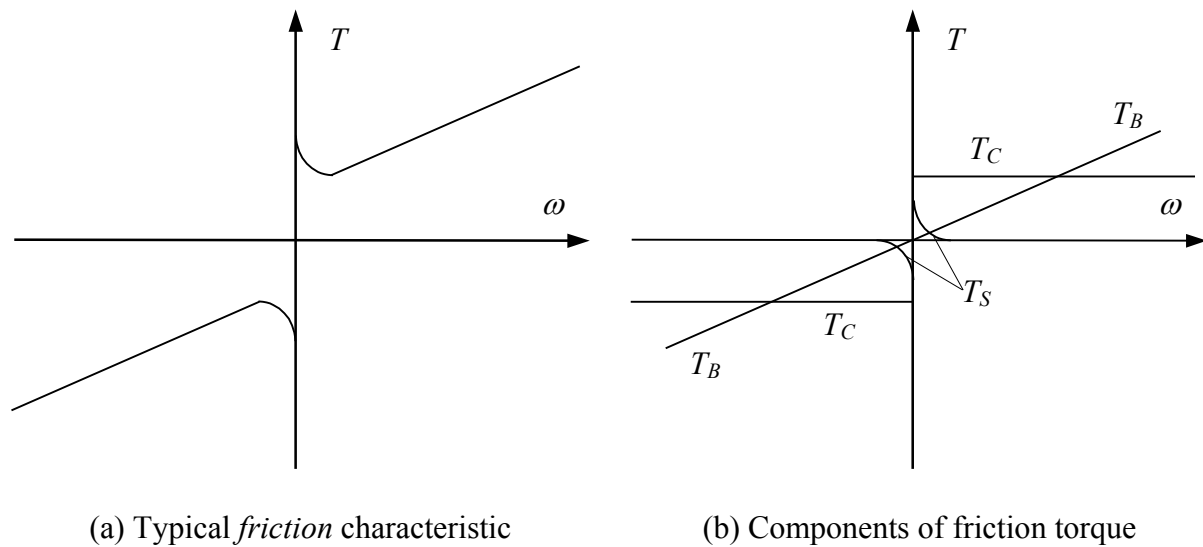


Fig 1.3 Friction model

Windage causes a component of load torque that, for rotating mechanisms, is approximately proportional to the square of the speed of rotation.

The torque required to accelerate the moving parts of the system may be expressed as

$$T_a = J \frac{d\omega}{dt} = J \frac{d^2\theta}{dt^2}$$

where  $J$  is the moment of inertia of the system.

The torque used in mechanical work will be some function of speed  $\omega$ , and may be defined by

$$T_{WL} = T(\omega)$$

Thus the output torque of the motor may be expressed as

$$T_L = J \frac{d\omega}{dt} + B\omega + T_{WL}$$

when Coulomb and static frictions are negligible.

Usually the most important component of the load torque  $T_L$  on the motor is that used in mechanical work, i.e.,  $T_{WL}$  which are dependent upon types of load.

### CONSTANT LOAD TORQUE

The torque is always in the opposite direction of rotation, but is constant, as shown in Fig. 1.4 (a). It is one of load characteristics commonly found in machine tools.

## CRANE HOIST

As shown in Fig. 1.4 (b), the torque applied to the motor shaft is unidirectional, due to the gravitational force. This scenario is also seen in a robotic manipulator lifting a heavy object. The direction of rotation needs to be bi-directional in order to lift the load up and down.

## FAN LOAD

The load torque is caused by aerodynamic drag and therefore is proportional to the square of angular velocity apposite to the direction of rotation, as shown in Fig. 1.4 (c).

## TRACTION LOAD

At low speed, the load is dominated by static and Coulomb frictions whilst at high speed the rolling resistance torque proportional to the velocity, and aerodynamic drag proportional to the square of velocity become pronounced, as shown in Fig. 1.4 (d).

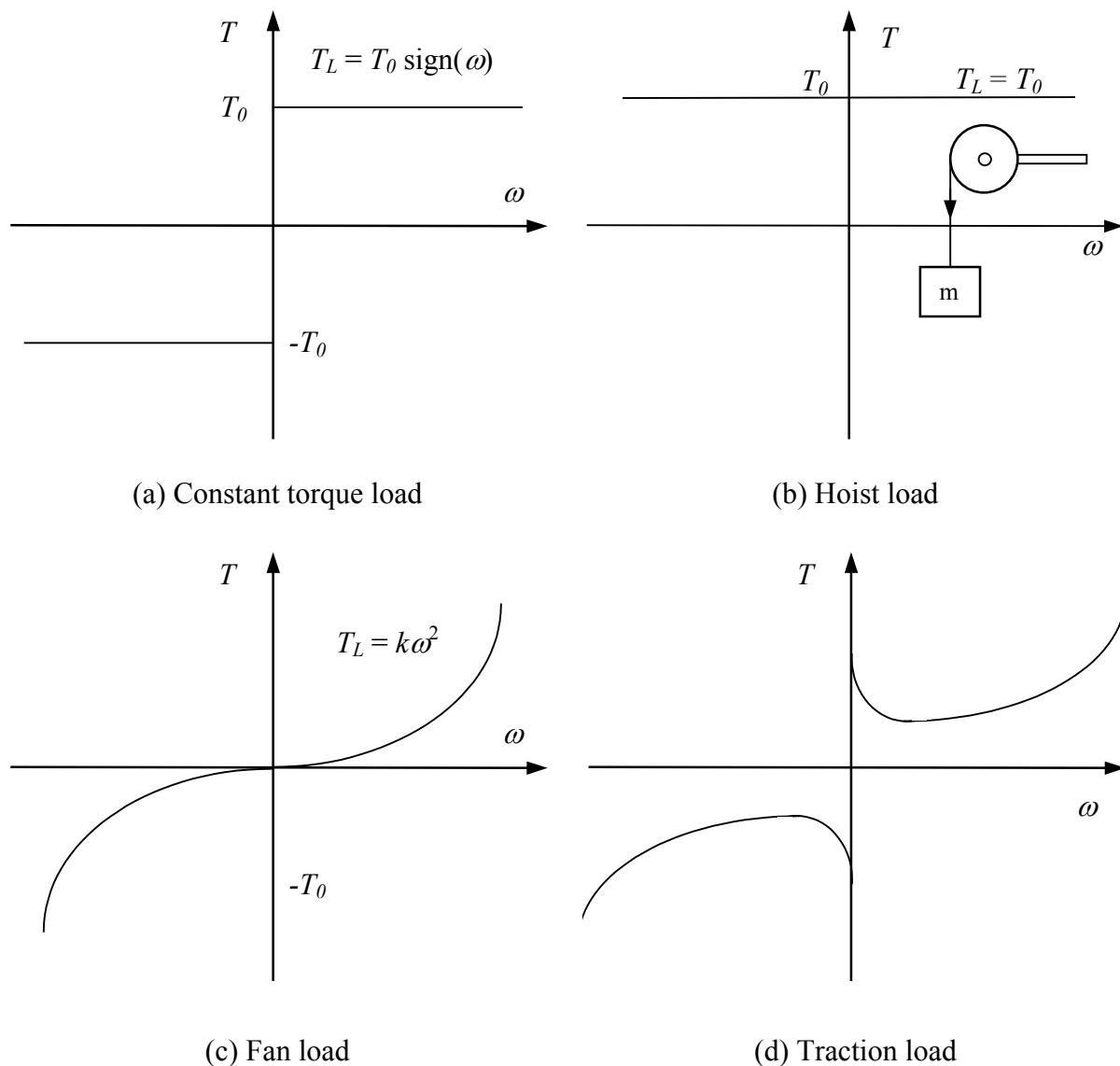


Fig 1.4 Load characteristics

### 1.3 Criteria for Selecting Drive Components

As shown in Fig. 1.1 a motor drive consists of an electric motor, a power electronic converter, and possibly a speed and/or position sensor. In this section, criteria for optimum match between the mechanical load and the drive components are discussed in general terms.

#### 1.3.1 Match Between the Motor and the Load

##### MOTION PROFILES

Many servo applications require fast and accurate point-to-point positioning of a load in a repetitive manner, a robotic arm, or an automated manufacturing assembly, for example. These applications normally involve loads which may be characterised by virtually pure inertia with negligible friction and load torque. Under these conditions, power rating of drives depends, to a large extent, on velocity profiles.

##### (a) Triangle profile

Fig. 1.5(a) shows a triangle velocity profile. Initially velocity is zero and then increases linearly to a maximum value of  $\omega_{\max}$  before decreasing linearly to zero again. The total angular distance  $\Theta$  travelled during this period  $T$  is the area of the triangle given by:

$$\Theta = \int_0^T \omega_m(t) dt = \frac{1}{2} \omega_{\max} T \quad (1.1)$$

The maximum angular velocity is therefore  $\omega_{\max} = 2\Theta/T$  and this value is proportional to the voltage requirement of drive.

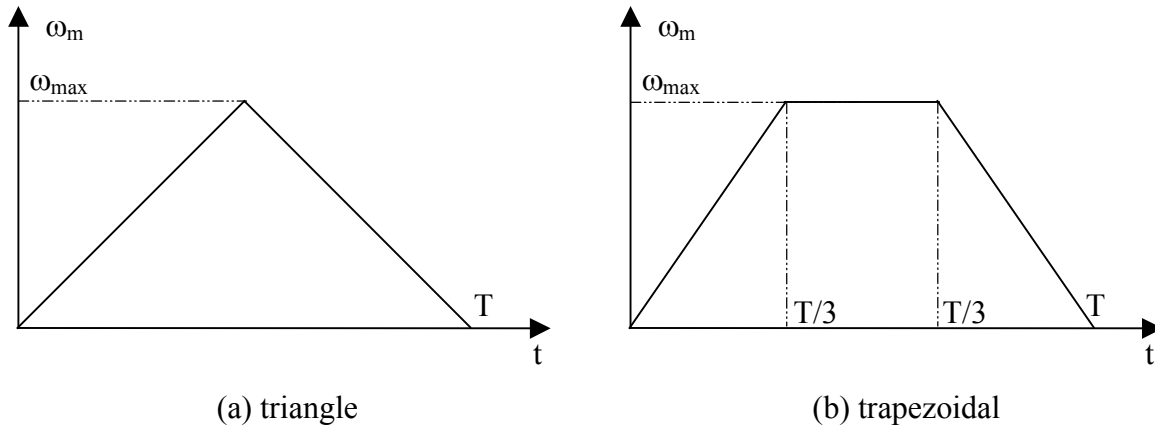


Fig. 1.5 Triangle and trapezoidal velocity profiles

The acceleration is

$$a_m = \frac{d\omega_m}{dt} = \frac{\omega_{\max}}{T/2} = \frac{4\Theta}{T^2} \quad (1.2)$$

The torque requirement of drive is proportional to the angular acceleration for pure inertia load. Thus the peak power rating of the drive is proportional to  $\omega_m a_m$  given by

$$\text{Peak power rating} \propto \omega_m a_m = 8\Theta^2/T^3 \quad (1.3)$$

and the rms torque is proportional to  $4\Theta/T^2$ .

## (2) Trapezoidal profile

As shown in Fig. 1.5 (b), the motion with a typical trapezoidal velocity profile is divided into three segments: acceleration, constant velocity and deceleration, each taking  $T/3$ . The total angular distance  $\Theta$  travelled during the period  $T$  is the area of the trapezoid given by:

$$\Theta = 2\omega_{\max}T/3 \quad (1.4)$$

Hence the maximum velocity proportional to the voltage requirement is

$$\omega_{\max} = 3\Theta/(2T) \quad (1.5)$$

and the acceleration proportional to the torque requirement is

$$a_m = \omega_{\max}/(T/3) = 9\Theta/(2T^2) \quad (1.6)$$

The peak power rating will be proportional to  $\omega_m a_m$

$$\text{Peak power rating} \propto \omega_m a_m = 27\Theta^2/4T^3 = 6.75\Theta^2/T^3 \quad (1.7)$$

During the constant velocity period, acceleration is zero so is torque. The rms torque requirement will be proportional to the rms acceleration, i.e.,

$$\text{Rms torque} \propto \text{rms } a_m = \sqrt{\frac{81\Theta^2}{4T^4} \times \frac{2T}{3}} = 3.67\Theta/T^2 \quad (1.8)$$

It follows that compared with the triangle profile for the same travel distance, the trapezoidal profile results in **(i) reduced peak power rating (ii) smaller rms torque requirement**.

## (c) Parabolic profile

It can be shown that a parabolic profile of the following form results in minimum rms torque, hence minimum losses for a purely inertial system:

$$\omega_m(t) = \frac{6\Theta}{T^2}t - \frac{6\Theta}{T^3}t^2 \quad (1.9)$$

This profile is plotted in Fig. 1.6.

The maximum velocity occurs at  $t = T/2$  and is given by  $3\Theta/(2T)$ . The acceleration is obtained as:

$$a_m = \frac{d\omega_m}{dt} = \frac{6\Theta}{T^2} - \frac{12\Theta}{T^3}t \quad (1.10)$$

The maximum acceleration occurs at  $t = 0$  and is given by:  $6\Theta/T^2$ , and the peak power is proportional to  $9\Theta^2/T^3$ . The rms acceleration can be obtained by the following integration:

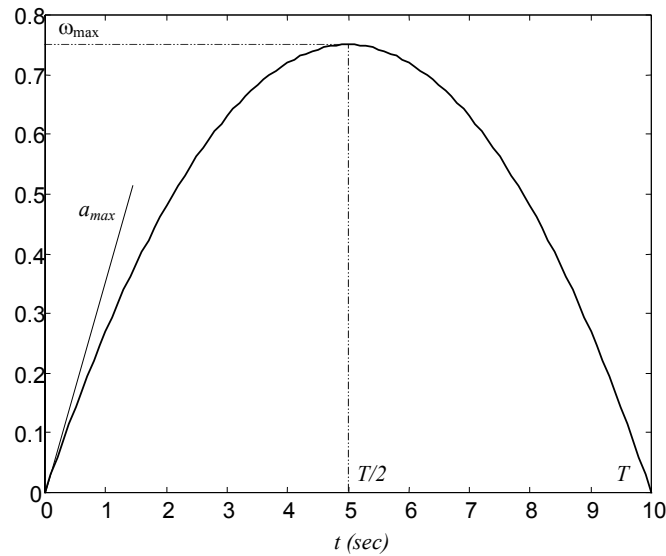


Fig. 1.6 Parabolic velocity profile

$$a_{rms} = \sqrt{\frac{1}{T} \int_0^T a_m^2(t) dt} = \sqrt{12\Theta / T^2} = 3.46\Theta / T^2 \quad (1.11)$$

Table 1.1 summarises the characteristics of three motion profiles

Table 1.1

Motion profile	$\omega_{max}$	$a_{max}$	$P_{max} \propto$	rms torque or losses $\propto$
Triangle	$2\Theta/T$	$4\Theta/T^2$	$8\Theta^2/T^3$	$4\Theta/T^2$
Trapezoid	$3\Theta/2T$	$4.5\Theta/T^2$	$6.75\Theta^2/T^3$	$3.67\Theta/T^2$
Parabolic	$3\Theta/2T$	$6\Theta/T^2$	$9\Theta^2/T^3$	$3.46\Theta/T^2$

It can be seen that the trapezoidal profile is very closed to the minimum rms losses, while having lower peak power and torque ratings than that of parabolic profile.

## MATCHING MOTOR AND LOAD

Prior to selecting the drive components, the load parameters and requirements such as the load inertia, maximum speed, speed range, and direction of motion, must be available. The motion profile as a function of time, for example as shown in Fig. 1.7(a), must also be specified. By means of modelling the mechanical system, it is possible to obtain a load--torque profile. Assuming a primarily inertial load with a negligible damping, the torque profile, corresponding to the speed profile in Fig. 1.7(a), is shown in Fig. 1.7(b). The torque required by the load peaks during the acceleration and deceleration. One way to drive a rotating load is to couple it directly to the motor. In such a direct coupling, the problems and the losses associated with a gearing mechanism are avoided. But the motor must be able to provide peak torques at specified speeds.

The other option for a rotating load is to use a gearing mechanism. A coupling mechanism such as rack-and-pinion, belt-and-pulley, or feed-screw must be used to couple a load with a linear motion to a rotating motor. A gear and a feed-screw drive are shown in Figs. 1.8(a) and 1.8(b), respectively. Assuming the energy efficiency of the gear in Fig. 1.8(a) to be 100%, the torques on the two sides of the gear are related as

$$\frac{T_m}{T_L} = \frac{\omega_L}{\omega_m} = \frac{\theta_L}{\theta_m} = \frac{n_m}{n_L} = a \quad (1.12)$$

where the angular speed  $\omega = d\theta/dt$ ,  $n_m$  and  $n_L$  are the number of teeth, and  $a$  is the coupling(gear) ratio.

In a feed-screw drive of Fig. 1.8(b), the torque and the force are related as:

$$\frac{T_m}{F_L} = \frac{v_L}{\omega_m} = \frac{x_L}{\theta_m} = \frac{s}{2\pi} = a \quad (1.13)$$

where the linear velocity  $v_L = dx_L/dt$ ,  $s$  is the pitch of the feed-screw in m/turn, and  $a$  is the coupling ratio.

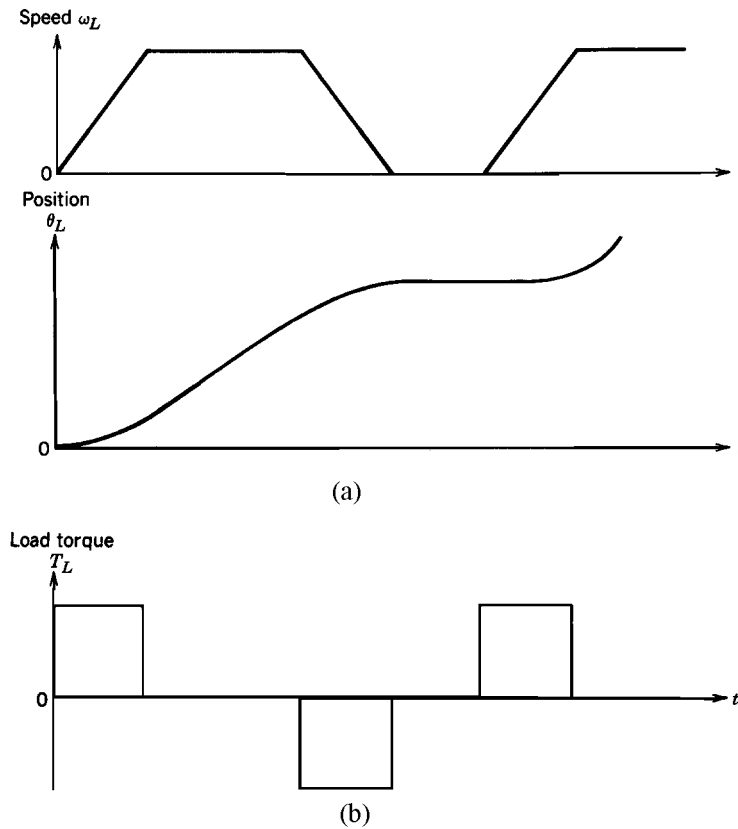


Fig. 1.7 Load profile (a) Load-motion profile, (b) Load-torque profile (assuming a purely inertia load)

The electromagnetic torque  $T_{em}$  required from the motor can be calculated on the basis of energy considerations in terms of the inertias, required load acceleration, coupling ratio  $a$ , and the working torque or force. In Fig. 1.8(a),  $T_{WL}$  is the working torque of the load and  $\dot{\omega}_L$  is the load acceleration. Therefore



$$T_{em} = \frac{\dot{\omega}_L}{a} [J_m + a^2 J_L] + a T_{WL} + \frac{\omega_L}{a} (B_m + a^2 B_L) \quad (1.14a)$$

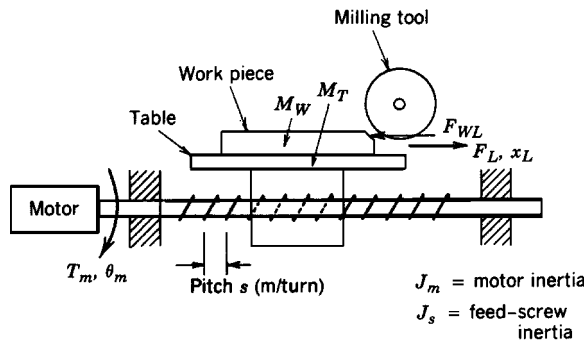
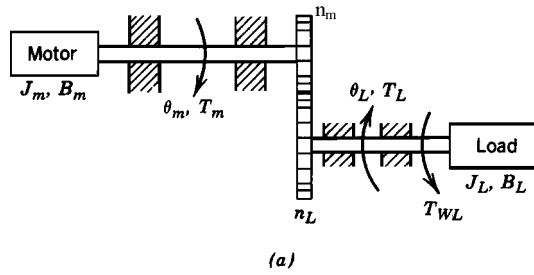


Fig. 1.8 Coupling mechanism (a) gear, (b) feed screw

This equation can be written in terms of the motor speed (recognizing that  $\omega_m = \omega_L/a$ ), the equivalent total inertia  $J_{eq} = J_m + a^2 J_L$ , the equivalent total damping  $B_{eq} = B_m + a^2 B_L$  and the equivalent working torque of the load  $T_{Weq} = a T_{WL}$ :

$$T_{em} = J_{eq} \dot{\omega}_m + B_{eq} \omega_m + T_{Weq} \quad (1.14b)$$

For a purely inertia load, Eqn. (1.14b) becomes:

$$T_{em} = \frac{\dot{\omega}_L}{a} [J_m + a^2 J_L] \quad (1.15)$$

and the minimum value of torque may be found from  $dT_{em}/da = 0$ :

$$\frac{dT_{em}}{da} = \dot{\omega}_L \left[ -\frac{J_m}{a^2} + J_L \right] = 0 \quad (1.16)$$

Thus

$$a = \sqrt{\frac{J_m}{J_L}} \quad (1.17)$$

is the optimal ratio which yields the minimum torque for a given acceleration requirement. The required motor torque at this optimal ration is:

$$T_{em} = 2\dot{\omega}_L \sqrt{J_L J_m} \quad (1.18)$$

If the peak available motor torque is given, the maximum achievable acceleration can be calculated using Eqn. (1.18) with the optimal coupling ratio.

Similarly, for the feed-screw system in Fig. 1.8(b) with  $F_{WL}$  as the working or the machining force and  $a$  as the coupling-ratio calculated in Eqn. 1.13 in terms of pitch  $s$ ,  $T_{em}$  can be calculated as

$$T_{em} = \frac{\dot{v}_L}{a} [J_m + J_s + a^2 (M_T + M_W)] + a F_{WL} \quad (1.19)$$

where  $\dot{v}_L$  is the linear acceleration of the load. As indicated by Eqns. (1.12) and (1.13), the choice of the coupling ratio  $a$  affects the motor speed. At the same time, the value of  $a$  affects the peak electromagnetic torque  $T_{em}$  required from the motor, as is indicated by Eqns. (1.14a) and (1.19). In selecting the optimum value of the coupling ratio  $a$ , the cost and losses associated with the coupling mechanism must also be included.

### 1.3.2 Thermal Considerations in Selecting the Motor

In the previous section, the match between the load and the motor is discussed that establishes the peak torque and the maximum speed required from the motor. This matching also establishes the motor-torque profile, which, for example, has the same form (but different magnitudes) as the load-torque profile of Fig. 1.7(b).

As another example, the electromagnetic torque required from the motor as a function of time is obtained as shown in Fig. 1.9(a).

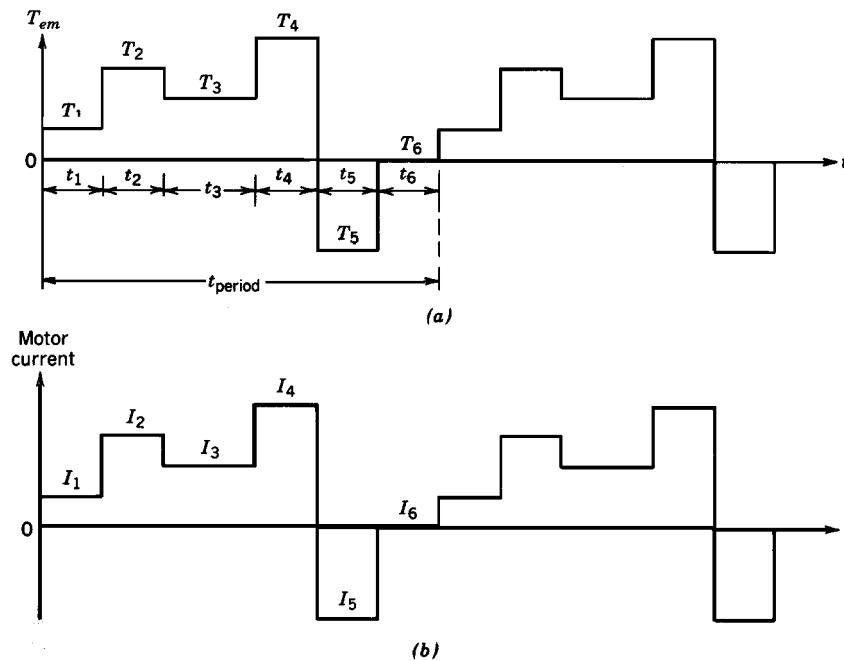


Fig. 1.9 Motor torque and current

In electric machines, the electromagnetic torque produced by the motor is proportional to the motor current  $i$ , provided the flux in the air gap of the motor is kept constant. Therefore, the motor current profile is identical to the motor-torque profile, as shown in Fig. 1.9(b). The motor current in Fig. 1.9(b) during various time intervals is a dc current for a dc motor. For an

ac motor, the motor current shown is approximately the rms value of the ac current drawn during various time intervals. The power loss  $P_R$  in the winding resistance  $R_M$  due to the motor current is a large part of the total motor losses, which get converted into heat. This resistive loss is proportional to the square of the motor current and, hence, proportional to  $T_{em}^2$  during various time intervals in Figs. 1.9(a) and (b).

If the time period  $t_{\text{period}}$  in Fig. 1.9, with which the waveforms repeat, is short compared with the motor thermal time constant, then the motor heating and the maximum temperature rise can be calculated based on the resistive power loss  $P_R$  averaged over the time period  $t_{\text{period}}$ . Therefore, in Fig. 1.9, the rms value of the current over the period of repetition can be obtained as

$$P_R = R_M I_{rms}^2 \quad (1.20)$$

where

$$I_{rms}^2 = \left( \sum_{k=1}^m I_k^2 t_k \right) / t_{\text{period}} \quad (1.21)$$

and  $m = 6$  in this example. Because of the motor current being linearly proportional to the motor torque, the rms value of the motor torque over  $t_{\text{period}}$  from Fig. 1.9 and Eqn. (1.21) is

$$T_{em,rms}^2 = k_1 \left( \sum_{k=1}^m I_k^2 t_k \right) / t_{\text{period}} \quad (1.22)$$

and therefore

$$T_{em,rms}^2 = k_1 I_{rms}^2 \quad (1.23)$$

where  $k_1$  is a constant of proportionality. From Eqns. (1.20) and (1.23), the average resistive power loss  $P_R$  is given as

$$P_R = k_2 T_{em,rms}^2 \quad (1.24)$$

where  $k_2$  is a constant of proportionality. In addition to  $P_R$ , there are other losses within the motor that contribute to its heating. These are  $P_{FW}$  due to friction and windage,  $P_{fe}$  due to iron losses within the motor laminations, and  $P_s$ , due to switching frequency ripple in the motor current, since it is supplied by a switching power electronic converter rather than an ideal source. There are always some power losses called stray power losses  $P_{\text{stray}}$  that are not included with the foregoing losses. Therefore, the total power loss within the motor is

$$P_{\text{loss}} = P_R + P_{FW} + P_{fe} + P_s + P_{\text{stray}} \quad (1.25)$$

Under a steady-state condition, the motor temperature rise  $\Delta\Omega$  in degrees centigrade is given as

$$\Delta\Omega = P_{\text{loss}} / R_{TH} \quad (1.26)$$

where  $P_{\text{loss}}$  is in watts and the thermal resistance  $R_{TH}$  of the motor is in degrees centigrade per watt. For a maximum allowable temperature rise  $\Delta\Omega_{\text{max}}$ , the maximum permissible value of  $P_{\text{loss}}$  in steady state depends on the thermal resistance  $R_{TH}$  in Eqn. (1.26). In general, the loss

components other than  $P_R$  in the right side of Eqn. (1.25) increase with the motor speed. Therefore, the maximum allowable  $P_{loss}$  and, hence, the maximum continuous motor-torque output from Eqn. (1.24) would decrease at higher speed, if  $R_{RH}$  remains constant. However, in self-cooled motors with the fan connected to the motor shaft, for example,  $R_{TH}$  decreases at higher speeds due to increased air circulation at higher motor speeds. Therefore, the maximum safe operating area in terms of the maximum rms torque available from a motor at various speeds depends on the motor design and is specified in the motor data sheets (specially in case of servo motors). For a motor-torque profile like that shown in Fig. 1.9(a), the motor should be chosen such that the rms value of the torque required from the motor remains within the motor's safe operating area in the speed range of operation.

### 1.3.3 Match Between the Motor and the Power Electronic Converter

A match between the load and the various characteristics of the motor, such as its inertia, and the peak and the rms torque capability, have been discussed in the previous two sections. Depending on the power rating, speed of operation, operating environment, reliability, various other performance requirements by the load, and the cost of the overall drive, one of the following four types of motor drive may be selected: dc motor drive, induction-motor drive, synchronous-motor drive, and the step-motor drive.

The power electronic converter topology and its control depend on the type of motor drive selected. In general, the power electronic converter provides a controlled voltage to the motor in order to control the motor current and, hence, the electromagnetic torque produced by the motor. Some of the considerations in matching the power electronic converter to the motor are discussed in the following subsections.

#### CURRENT RATING

As we discussed previously, the rms value of the torque that a motor can supply depends on its thermal characteristics. However, a motor can supply substantially larger peak torques (as much as four times the continuous maximum torque) provided that the duration of the peak torque is small compared with the thermal time constant of the motor. Since  $T_{em}$  is proportional to  $i$ , a peak torque requires a corresponding peak current from the power electronic converter. The current capability of the power semiconductor devices used in the converter is limited by the maximum junction temperature within the devices and other considerations. A higher current results in a higher junction temperature due to power losses within the power semiconductor device. The thermal time constants associated with the power semiconductor devices are in general much smaller than the thermal time constants of various motors. Therefore, the current rating of the power electronic converter must be selected based on both the rms and the peak values of the torque that the motor is required to supply.

#### VOLTAGE RATING

In both dc and ac motors, the motor produces a back-emf  $e$  that opposes the voltage  $v$  applied to it, as shown by a simplified generic circuit of Fig. 1.10. The rate at which the motor current and, hence, the torque can be controlled is given by:

$$di/dt = (v-e)/L \quad (1.27)$$

where  $L$  is the inductance presented by the motor to the converter. To be able to quickly control the motor current and, hence, its torque, the output voltage  $v$  of the power electronic converter must be reasonably greater than the back-emf  $e$ . The magnitude of  $e$  in a motor

increases linearly with the motor speed, with a constant flux in the air gap of the motor. Therefore, the voltage rating of the power electronic converter depends on the maximum motor speed with a constant air-gap flux.

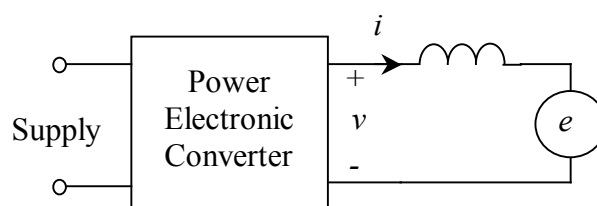


Fig. 1.10 Simplified circuit of a motor drive

### *SWITCHING FREQUENCY AND THE MOTOR INDUCTANCE*

In a servo drive, the motor current should be able to respond quickly to the load demand, thus requiring  $L$  to be small in Eqn. (1.27). Also, the steady-state ripple in the motor current should be as small as possible to minimize the motor loss  $P_s$ , in Eqn. (1.26), and the ripple in the motor torque. A small current ripple requires the motor inductance  $L$  in Eqn. (1.27) to be large. Because of the conflicting requirements on the value of  $L$ , the ripple in the motor current can be reduced by increasing the converter switching frequency. However, the switching losses in the power electronic converter increase linearly with the switching frequency. Therefore, a reasonable compromise must be made in selecting the motor inductance  $L$  and the switching frequency.

#### **1.3.4 Selection of Speed and Position Sensors**

In selecting the speed and position sensors, the following items must be considered: direct or indirect coupling, sensor inertia, possibility and avoidance of torsional resonance, and the maximum sensor speed.

To control the instantaneous speed within a specified range, the ripple in the speed sensor should be small. This can be understood in terms of incremental position encoders, which are often used for measuring speed as well as position. If such a sensor is used at very low speeds, the number of pulse outputs per revolution must be large to provide instantaneous speed measurement with sufficient accuracy. Similarly, an accurate position information will require an incremental position encoder with a large number of pulse outputs per revolution.

#### **Recommended Books**

1. N. Mohan, T. M. Undeland, and W. P. Robbins: "Power Electronics: Converters, Applications and Design", John Wiley & Sons Inc, 1989
2. S. B. Dewan, G. R. Slemon and A. Straughen, "Power Semiconductor Drives", John Wiley & Sons Inc, 1984
3. T. J. E. Miller, "Brushless Permanent-Magnet and Reluctance Motor Drives"- 2nd (Corr.)impression, Oxford University Press: Clarendon, 1993
4. P. Vas, "Vector control of AC Machines", Oxford University Press: Clarendon, 1990
5. B. K. Bose, "Power Electronics and Variable Frequency Drives --- Technology and Applications", IEEE Press, 1997
6. W. Leonhard, "Control of Electrical Drives", 3<sup>rd</sup> Edition, Springer, 2001.