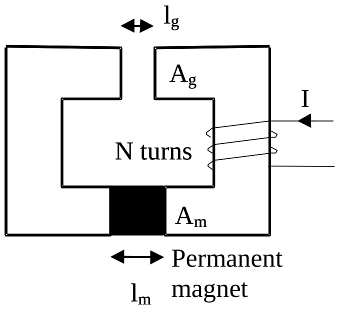
Answers to questions

Answers to question 1:

(a) As shown in the figure, from Ampere's law:

$$\oint H dl = \sum I$$

$$H_m l_m + H_g l_g = -NI \quad \textbf{(1)}$$



Gauss's law:

$$\oint B ds = 0$$

$$B_m A_m = B_g A_g \quad \textbf{(1)}$$

Demagnetisation characteristic of magnet

$B_m = \mu_0 \mu_r H_m + B_r$ (for linear part)

From (1) & (2) $B_m A_m = B_g A_g = -\mu_0 \dfrac{A_g}{l_g}(H_m l_m + NI)$

therefore

$$B_m = -\mu_0 \frac{A_g}{l_g A_m}(H_m l_m + NI) = -\mu_0 \frac{l_m A_g}{l_g A_m}\left(H_m + \frac{NI}{l_m}\right) = -\mu_0 \beta \left(H_m + \frac{NI}{l_m}\right)$$

where $\beta = \dfrac{l_m A_g}{l_g A_m}$

From (5) & (6), the magnet working point (B_m, H_m)

$$B_m = \mu_0 \mu_r H_m + B_r$$

$$H_m = -\frac{NI\beta}{l_m(\beta + \mu_r)} - \frac{B_r}{\mu_0(\beta + \mu_r)}$$

$$B_m = -\frac{\mu_0 \mu_r NI\beta}{l_m(\beta+\mu_r)} + \frac{B_r \beta}{\beta+\mu_r}$$ **(2)**

The airgap flux density on open-circuit (NI=0)

$$B_g = \frac{A_m}{A_g} B_m = \frac{B_r}{\frac{A_g}{A_m} + \mu_r \frac{l_g}{l_m}}$$ **(2)**

(b) Rotor outside diameter $D=D_i-2l_g=58$mm

Average airgap diameter, $D_g=D_i-l_g=59$mm

Average magnet diameter, $D_m=D-l_m=58-5=53$mm

$A_g/A_m=D_g/D_m=59/53=1.11$

The peak airgap flux density Bg=1.2/(1.11+1/5)=0.916T **(2)**

Magnetic loading is the average of airgap flux density B=Bg*120/180=0.61T **(1)**

Alternatively, assuming $A_g/A_m=1$, the peak airgap flux density $B_g=1.2/(1+1/5)=1$T

Magnetic loading is the average of airgap flux density $B=B_g*120/180=0.667$T

(c) Slot number $N_s=12$

Tooth pitch, $\tau_t=\pi D/N_s=3.14*58/12=15.177$mm

$$B_{tooth} = \frac{\tau_t}{w_t} B_g = \frac{15.177}{7} 0.916 = 2.168\text{T}$$ **(2)**

Which is far too high the tooth is heavily saturated (note here peak airgap flux density should be used, rather than magnetic loading). **(1)**

(d) Pole number 2p=4

Pole pitch, $\tau_p=\pi D/(2p)=3.14*58/4=45.53$mm

$$B_{core} = \frac{B_g \tau_p}{2d_c} = \frac{0.916*45.53}{2*12} = 1.74\text{T}$$ **(2)**

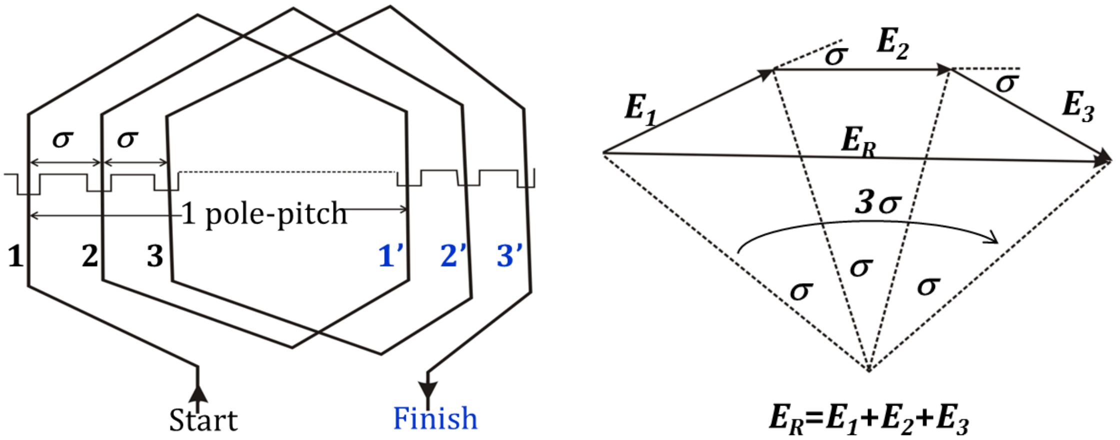
Which is reasonable **(1)**

(e) Electrical loading $Q=NI/(\pi D)=2*5000/(3.14*58)=2*27.45$ A/mm=27450A/m **(2)**

(f) Electromagnetic torque at the level of electric loading in part (e) $T = \frac{\pi}{2} D^2 LBQ$ =(3.14/2)*0.058^2*0.05*0.61*27450*2=4.42Nm **(3)**

Answers to question 2:

(a), The layout of winding and the EMF vectors of coils are shown:



(1)

Assuming we have m = 3 coils per phase, and $|E_1| = |E_2| = |E_3| = |E_m|$ (all the coils are identical).

Then, from the construction (***E$_m$*** = ***E$_1$***), we have

$$E_m = 2r\sin\frac{\sigma}{2} \quad \text{and} \quad E_R = 2r\sin\frac{m\sigma}{2}$$

The arithmetic sum of all coil EMFs: $mE_m = m2r\sin\frac{\sigma}{2}$

However, the vector sum of all coil EMFs: $E_R = 2r\sin\frac{m\sigma}{2}$ **(1)**

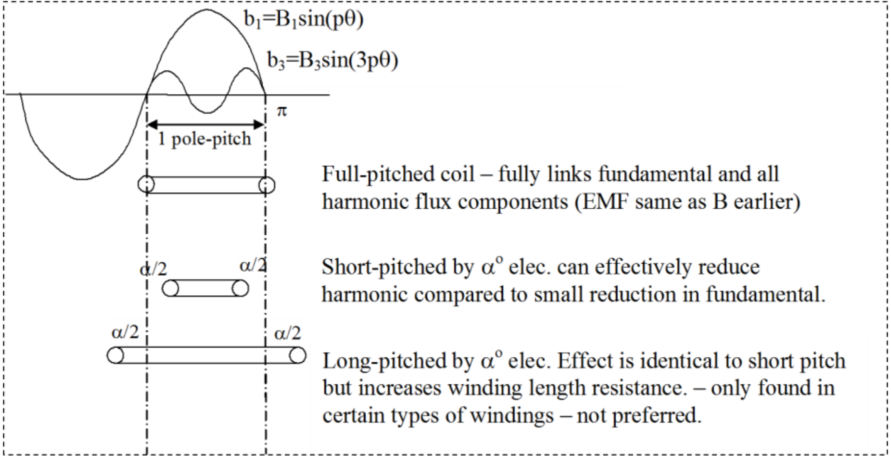
Therefore, the distribution factor for the fundamental is:

$$\boxed{k_d = \frac{\textit{effective induced emf}}{\textit{arithmetic induced emf}} = \frac{E_R}{mE_m} = \frac{\sin\frac{m\sigma}{2}}{m\sin\frac{\sigma}{2}}}$$

By using the similar approach, the distribution factor for the nth harmonic is:

$$\boxed{k_{dn} = \frac{\sin\frac{mn\sigma}{2}}{m\sin\frac{n\sigma}{2}}}$$ **(1)**

The pitch factor then can be calculated based on the following graph:



k_p is defined as: $\dfrac{effective\ \ EMF}{EMF\ of\ \ full-pitch\ coil} \propto \dfrac{effective\ \ flux\ linkage}{flux\ linkage\ of\ \ full\ \ pitch\ coil} = \dfrac{\Psi_s}{\Psi_F}$

For a short pitch coil:

$$\Psi_s = \int_{\alpha/2}^{\pi-\alpha/2} \hat{B}\sin\theta d\theta = 2\hat{B}\cos\frac{\alpha}{2} \qquad \textbf{(1)}$$

And for full pitch coil:

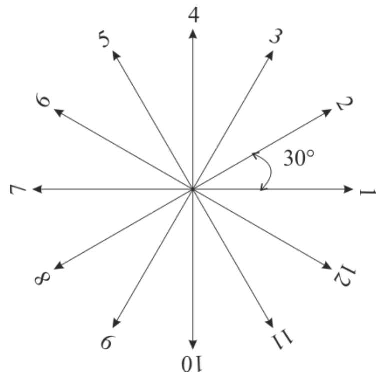
$$\Psi_F = \int_0^{\pi} \hat{B}\sin\theta d\theta = 2\hat{B} \qquad \textbf{(1)}$$

Therefore, the pitch factor is:

$$\boxed{k_p = \frac{\Psi_s}{\Psi_F} = \frac{2\hat{B}\cos\frac{\alpha}{2}}{2\hat{B}} = \cos\frac{\alpha}{2}}$$

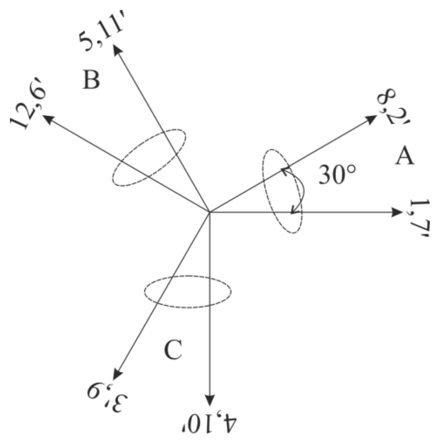
Similarly, the pitch factor for long pitch is: $k_p = \cos\dfrac{\alpha}{2}$ **(1)**

(b), For a 12-slot/14-pole double layer permanent magnet machine which has non-overlapping concentrated winding, there are 12 coils allow us to establish a 3-phase winding structure. This means each phase will only have 4 coils. The coil vectors are shown in the following graph:



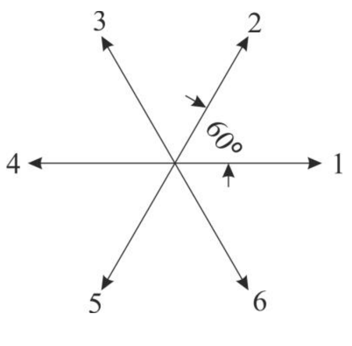
(1)

Therefore, the coil connection (some EMF vectors have been reversed to achieve the highest distribution factor) for a maximum distribution factor should be as:



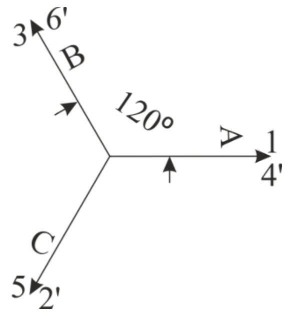
(3)

(c), For a 12-slot/14-pole alternate teeth wound permanent magnet machine which has non-overlapping concentrated winding, there are 6 coils allow us to establish a 3-phase winding structure. This means each phase will only have 2 coils. The coil vector and coil EMF vector of this machine are the same and shown in the following graph:



(1)

Therefore, the coil connection for a maximum distribution factor should be as:



(2)

(d), **Single layer windings over double layer windings:**

- Higher winding factor, **(1)**
- Higher self inductance but lower mutual inductance and hence higher fault tolerance capability, **(1)**
- Physical separation between coils
- Higher saturation level, lower torque at high phase currents **(1)**
- Potentially lower power factor **(1)**

(e), the concentrated winding often refers to the winding that is wound around only one single stator tooth, so two sides of the coil span one slot pitch, leading to short end-winding, but its MMF often contains rich harmonics. **(1.5)**

However, the distributed winding often refers to the winding that has a coil span of a few slot pitches, leading to longer end-windings. This can effectively reduce MMF harmonics and also achieve high winding factor (sometimes a winding factor of 1 can be achieved). **(1.5)**

<u>Answers to question 3:</u>

(a), From the data given, current loading $Q = \frac{N_s A_s J K_p}{\pi D} = \frac{15\times30\times8\times0.5}{\pi\times30} = 19.1 \times 10^3 A/m$

At ***<u>P = 600W @ 5000rpm</u>***, torque $T = P/\Omega$, [Ω in rad/s], then

$$T = \frac{600}{\frac{5000 \times 2\pi}{60}} = 1.15\ Nm$$

Hence, B loading required can be calculated from $T = \frac{\pi}{2} D^2 LBQ$

Giving B = 0.53 T (average) **(2)**

Since the B field is around and concentrated under the magnets, then

$B_m = B \times \frac{\pi}{\alpha} = 0.53 \times \frac{180}{140} = 0.68\ T$ **(2)**

B_m can be obtained either graphically using expression $B_m = \frac{B_r}{\left(1+\mu_r \frac{A_m l_g}{A_g l_m}\right)}$

Giving $l_m = \frac{B_m \mu_r l_g}{(B_r - B_m)} \frac{A_m}{A_g} = 1.8\,mm = 1.8 \times 10^{-3}\,m$ **(2)**

(b), the possible ways to increase air-gap flux density are as following:

- Use better magnet material that has higher remanence,
- When l_m is small, the increase in l_m leads to important increase in B_g,
- When l_m is big, B_g is almost constant,
- Increase A_m can further boost the B_g. **(3)**

(c), The flux density in airgap can be first calculated such as

$$B_g = \frac{B_r}{1+\mu_r \frac{l_g}{l_m}}$$

Assuming $A_m = A_g$ for rectangular flux density waveform. **(1)**

Then the flux per pole can be calculated using

$$\Phi = B_g \times \left(\frac{\pi DL}{2p}\right) \times \left(\frac{\alpha}{\pi}\right) = \frac{1.0}{1+1.1\frac{1}{1.8}} \times \left(\frac{\pi \times 30\times10^{-3} \times 80\times10^{-3}}{2\times2}\right) \times \left(\frac{140}{180}\right) = 0.91\,mWb$$ **(3)**

The total number of conductor is Ztotal = 1532, a = 2, Vdc = 100V, then each path has a conductor number:

Z = Ztotal/2=766

$$\boxed{K = \frac{E}{\omega_r} = \frac{Zp\Phi}{\pi} = \frac{766 \times 2 \times 0.91 \times 10^{-3}}{\pi} = 0.444\text{(Nm/A or V/rad s -1)}}$$ **(1)**

Under no-load condition, I = 0 and V = E, we can have

$$\omega_{NL} = \frac{V}{K} = \frac{100}{0.444} = 225\,rad/s$$ **(1)**

(d), R = 3Ω, Vdc = 100V, E = 0, I = (V-E)/R = V/R, the stall-torque can be calculated by

$$T_{stall} = K\frac{V}{R} = 0.444\frac{100}{3} = 14.8\,Nm$$ **(2)**

(e), the main difference between BLAC and BLDC is their waveforms of EMF and current, for BLDC, the EMF waveform is often trapezoidal while its current is often square wave. However, for BLAC, both the EMF and current are sinewave. If their EMF waveforms are not ideal, the torque and power ripples will be increased. **(3)**

<u>Answers to question 4:</u>

(a), for a single-phase winding, the resultant time and space content of the winding MMF is:

$$F(\theta,t) = \underbrace{[F_1 \sin\theta + .. + ... + F_n \sin n\theta]}_{\text{Space}} \underbrace{\sin \omega t}_{\text{Time}} \quad \textbf{(2)}$$

With $|F_n| = \frac{4H}{n\pi} k_{wn}$ and $K_{wn} = K_{dn}K_{pn}K_{sn}$

And we can rewrite the MMF by using:

$$F(\theta,t) = \frac{F_1}{2}[\cos(\theta-\omega t) - \cos(\theta+\omega t)] + ... + ... + \frac{F_n}{2}[\cos(n\theta-\omega t) - \cos(n\theta+\omega t)] \quad \textbf{(2)}$$

Consider a term such as $\cos(n\theta - \omega t)$, the peak of this terms occurs when $\cos(n\theta - \omega t) = 1$ or $(n\theta - \omega t) = 0$

Hence $\frac{d\theta}{dt} = \frac{\omega}{n}$ i.e. this term describes a MMF wave rotates in the space at a speed of $\frac{\omega}{n}$ rad/s. **(1)**

Similarly, terms such as $\cos(n\theta + \omega t)$ describes MMF wave rotates in the opposite direction at a speed of $\frac{\omega}{n}$ rad/s. **(1)**

Note: p-pole pair machine, $\theta_{elec} = \frac{\theta_{mec}}{p}$

(b), for three phase machines:

A 3-phase winding consists of 3 identical winding, each winding displaced in space by 2π/3 electrical degree and excited by 3-phase time shifted currents differing in phase by 2π/3 electrical degree.

The resultant MMF in air-gap will be:

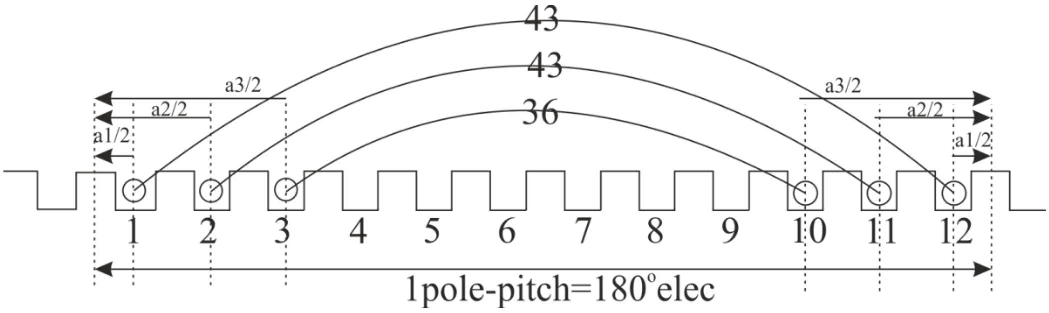
$$\boxed{F_R = \frac{3}{2}[F_1\cos(\theta-\omega t) + F_5\cos(5\theta+\omega t) + F_7\cos(7\theta-\omega t) + F_{11}\cos(11\theta+\omega t) + ...]} \quad \textbf{(2)}$$

This MMF has a forward rotating fundamental field of 3/2 amplitude of 1phase and no backward fundamental; **(1)**

Triplen harmonics have been removed; 5, 11 etc. rotating backwards; 7, 13 etc. rotating forwards. **(1)**

(c), since for each harmonic order, the single phase winding has a forward and a backward rotating MMF components, both has exactly the same amplitude and rotating at the same speed, the single phase machine only will not be able to self start. However, for three-phase system, for each harmonic order, the resultant winding harmonic has only one component, either forward rotating or backward rotating. Therefore, the three phase machines can effectively get rid of self start problems. **(3)**

(d), the machine under consideration has a pole-pair number p = 2 and a slot number N_s = 48. The winding structure of this machine is shown:



The winding factor of this winding structure can be calculated by:

$$k_{wn} = \frac{\sum_{i=1}^{N} N_i \cos \frac{n\alpha_i}{2}}{\sum_{i=1}^{N} N_i}$$

Where αn is electrical degree, by which the coil is short-pitched **(1)**

One slot-pitch angle is $\sigma = \frac{360 \times p}{N_s} = \frac{180}{12} = 15^o$, therefore, the different short-pitched angle can be calculated by:

$\alpha_1 = 15^0, \alpha_2 = 45^0, and\ \alpha_3 = 75^0,$ **(1)**

Therefore, the winding factor for the fundamental is:

$$\boxed{k_{w1} = \frac{N_1 \cos \frac{\alpha_1}{2} + N_2 \cos \frac{\alpha_2}{2} + N_3 \cos \frac{\alpha_3}{2}}{N_1 + N_2 + N_3} = 0.91} \quad \textbf{(1)}$$

And the winding factor for the third harmonic component is:

$$k_{w3} = \frac{N_1 \cos 3\frac{\alpha_1}{2} + N_2 \cos 3\frac{\alpha_2}{2} + N_3 \cos 3\frac{\alpha_3}{2}}{N_1 + N_2 + N_3} = 0.35 \quad \textbf{(1)}$$

Since $|F_n| = \frac{4H}{n\pi} k_{wn}$

Then

$\frac{F_1}{F_3} = \frac{K_{w1}}{K_{w3}/3} = \frac{3K_{w1}}{K_{w3}}$ **(1)**

And

$$F_3 = \frac{K_{w3}}{3K_{w1}} F_1 = \frac{0.35}{3 \times 0.91} F_1 = 12.8\% F_1$$

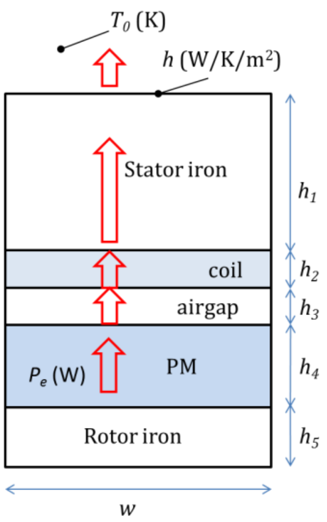
(e), the possible ways to reduce winding MMF harmonics are winding skew, winding distribution, short or long pitched winding, concentric windings. **(2 for any two of them)**

Answers to question 5:

(a), There is a requirement for smaller, cheaper and more efficient motors so an optimised design is required **(4, any four of the following)**

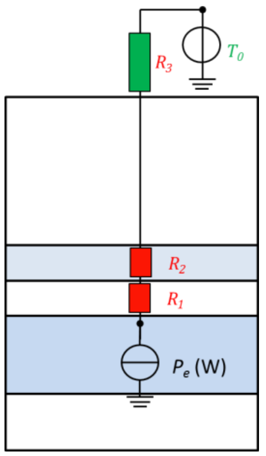
- Losses depend on temperature and temperature on losses
- Temperature influences the copper resistivity, therefore the phase resistance
- Temperature rise will significantly shorten the insulation life
- Temperature rise will demagnetise the PMs, therefore reduce the torque density
- Temperature limits the winding current density due to limited machine sizes

(b), if only PM has eddy current loss, the thermal flux path will be as following:



(3)

(c), since there is no iron loss and copper loss, and the iron and PM are very good thermal condutor, the LP model can be



(1)

Where $R_1 = \frac{h_3}{\lambda L w}$, $R_2 = \frac{h_2}{\lambda L w}$ and $R_3 = \frac{1}{h L w}$

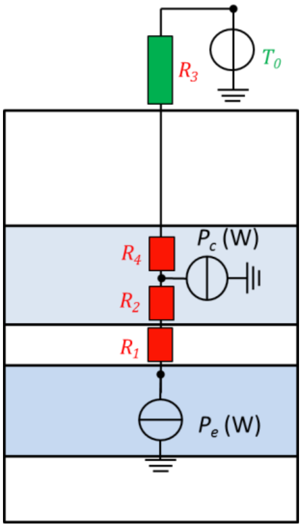
Therefore, resultant thermal resistance is

$$R = R_1 + R_2 + R_3$$ **(1)**

And the temperature within PM is

$$T = P_e \times R + T_0$$ **(1)**

(d), if there is copper losses, the LP model becomes



(2)

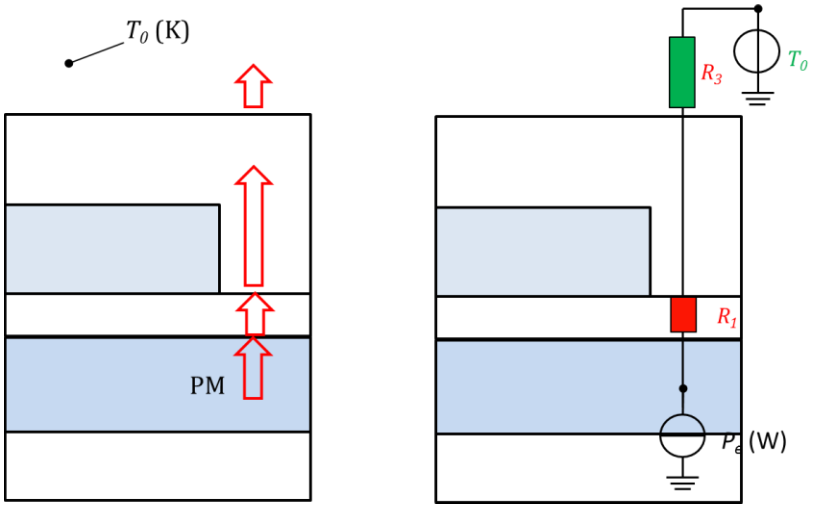
The calculation of R_1 and R_3 will be the same as in 5(c), however, since there is copper loss in windings, the thermal resistances in winding need to be calculated as:

$$R_2 = R_4 = \frac{1}{2}\frac{h_2}{\lambda L w}$$ **(1)**

And the maximum temperature within PM is:

$$T = P_e \times (R_1 + R_2) + (P_e + P_c) \times (R_4 + R_3) + T_0$$ **(1)**

(e), if the machine is slotted, since the iron has much higher thermal conductivity than equalent stator slots, the thermal flux will cross through stator iron rather than stator slots. As a result, the thermal flux path and the LP will be as following:



(2)

The calculation of R_1 and R_3 is assumed to be the same as in 5(c), therefore, the maximum temperature within the PM becomes

$$T = P_e \times (R_1 + R_3) + T_0 \qquad \textbf{(1)}$$

This means that by adopting slotted strucutre, the maximum PM temperature can be reduced. **(1)**

(f), the possible ways to reduce the PM and coil temperatures are **(2, any two of the following methods)**:

- Employing cooling fins on the outside surface of the frame to increase the exchange surface,
- Using water jactet to increase convection coefficient,
- Painting the outside surface of the frame in black to increase thermal radiation,
- Using PM segmentation to reduce the PM eddy current losses, etc.

<u>Answers to question 6:</u>

a) Assumptions:

- 1d field in the airgap, i.e. neglect fringing and leakage
- Assume permeability of the stator are armature core are infinite

The airgap flux density is given by:

$$B_g = \frac{B_r}{1 + \mu_r \frac{l_g}{l_m}} = \frac{1.23}{1 + 1.04 \times \frac{0.001}{0.008}} = 1.09T$$

(Fine if twice the magnet length and twice the gap are used)

(2)

b) If the airgap flux density if 1.09T (and neglecting the influence of armature reaction on magnetic saturation) it would be good design practice to set w_t and h_c to keep the flux density no higher than 1.8T. However, the maximum flux which passes down the central tooth is at the extremes of the ±6mm stroke (there is nominally zero next flux in the tooth body in the central position.

The maximum flux passing down the central tooth is given by:

$$\emptyset_{max} = 1.09 \times 6 \times 10^{-3} \times 40 \times 10^{-3} = 0.262\, mWb$$

To pass this flux without exceeding a flux density of 1.8T requires a tooth body width of:

$$w_t = \frac{0.262 \times 10^{-3}}{1.8 \times 40 \times 10^{-3}} = 3.63mm$$

(This could have been calculated per unit length if preferred)

(3)

c) The force can be estimated from the variation in coil flux-linkage for a displacement Δx:

$$\Delta\Psi$$

The rate of change wrt x is given by:

$$\frac{d\Psi}{dx} \approx \frac{\Delta\Psi}{\Delta x} = 2NB_gL$$

The force is given by:

$$F = I\frac{\Delta\Psi}{\Delta x} = 2NB_gLI = 2 \times 100 \times 1.09 \times 40 \times 10^{-3} \times 5 = 43.5N$$

(4)

d) The emf is given by:

$$e = 26.\frac{d\Psi}{dt} = \frac{d\Psi}{dx} \times \frac{dx}{dt} \approx \frac{\Delta\Psi}{\Delta x} \times \frac{dx}{dt} = 2NB_gL \times \frac{dx}{dt} = 2 \times 100 \times 1.09 \times 40 \times 10^{-3} \times 3$$
$$= 26.1V$$

(3)

e) For 80°C, the magnet romance reduces to:

$$B_{r80} = B_{r20}(1 - \alpha\Delta T) = 1.09(1 - 0.003 \times 60) = 1.086T$$

Force is proportional to Br and hence the current must be increased by:

$$I_{80} = 5 \times \frac{1.23}{1.086} = 5.66A$$

Copper losses increase from 2.5W to 3.2W.

(3)

f) Mass of armature given by:

$m_A = 40 \times 10^{-3} \times 60 \times 10^{-3} \times 20 \times 10^{-3} \times 8000 = 0.384kg$

Assuming no saturation, force at 25A is 5x force at 5A, i.e. 218N

Acceleration is given by:

$$a = \frac{218}{0.384} = 567\ ms^{-2}$$

(3)

g) Need to check for irreversible demagnetisation>

Take wort case temperature

Use B-H curve for that temperature

Calculate operating point at specified current

Check that operating point is not beyond the knee.

(2)