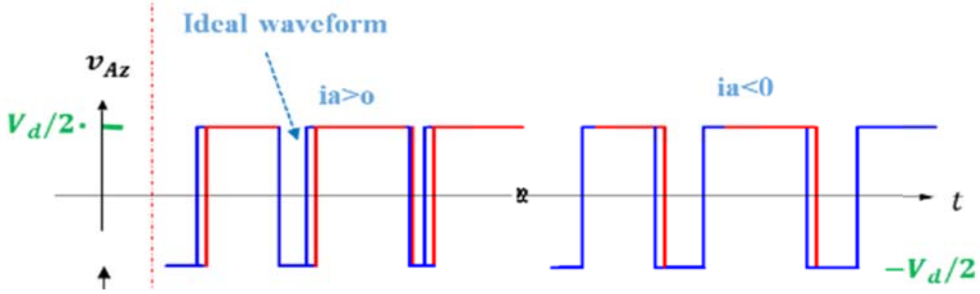
Answers EEE6205 – 2014/2015 MO

Question 1

a)

With reference to polarity of current i_a as defined in figure 1.1, dead-time effect on voltage v_{AZ} due to dead-time is as shown below.



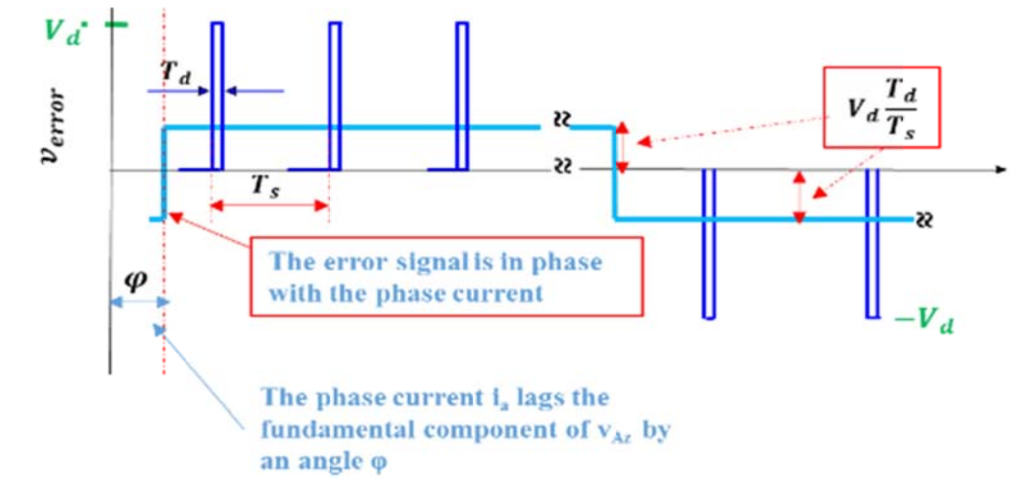
(1)

When the current is positive, as defined in figure 1.1, when switch S4 is turned off, the diode D4 will continue conducting during dead-time and the voltage at the terminal **A** will remain clamped to the negative dc bus. When the current is negative, as defined in figure 1, when S1 is switched off, during dead-time the diode D1 will continue conducting and the terminal A will remain clamped to the positive dc bus.

(2)

b)

$v_{error} = v_{Az(ideal)} - v_{Az(actual)}$



The voltage error pulses can be replaced by an equivalent rectangular waveform with the amplitude of $V_d \frac{T_d}{T_s}$ (average value over one switching period) and is in phase with the fundamental current. The amplitude of the equivalent square wave can then be obtained from: $\frac{1}{T_s}\int_0^{T_d} V_d \cdot dt = V_d \frac{T_d}{T_s}$

(4)

c)

Fourier analyses of the rectangular voltage with amplitude $V_d \frac{T_d}{T_s}$ is:

$$A_1 = \frac{2}{\pi}\int_0^{\pi} V_d \frac{T_d}{T_s} \cdot \sin(\omega t)\, d\omega t = \frac{2}{\pi} V_d \frac{T_d}{T_s}(-\cos(\omega t)) \uparrow_0^{\pi} = \frac{4}{\pi} V_d \frac{T_d}{T_s}$$

$$A_3 = \frac{2}{\pi}\int_0^{\pi} V_d \frac{T_d}{T_s} \cdot \sin(3\omega t)\, d\omega t = \frac{2}{3\pi} V_d \frac{T_d}{T_s}(-\cos(3\omega t)) \uparrow_0^{\pi} = \frac{4}{3\pi} V_d \frac{T_d}{T_s}$$

The error voltage is in phase with the current, therefore the fundamental component of the error voltage is lagging the fundamental phase voltage by φ. All even harmonics are zero for a square wave.

$$v_{err}(t) = \frac{4}{\pi} V_d \frac{T_d}{T_s}\left[\cos(\omega_o t - \varphi) + \frac{1}{3}\cos 3(\omega_o t - \varphi) + \frac{1}{5} cos\, 5(\omega_o t - \varphi) + \cdots\right]$$

ω_o – fundamental frequency

Therefore, due to dead time effect, the output voltage $v_{Az,}$ will have additional low order harmonics, such as 3rd, 5th and 7th and also will affect the fundamental component.

(3)

d.i)

$f_o = 400$ Hz, $f_{sw} = 10$ kHz, $V_d = 600$V
$\cos\varphi = 0.866 \rightarrow \varphi = 0.5236\, rad$,
Td=2μs, Ts=100μs, ma_{max}=1, $\omega_o = 2\pi \cdot 400$

$$v_{Az(actual)} = v_{Az(ideal)} - v_{error}$$

$$v_{AZ1(actual)}(t) = \frac{V_d}{2} m_a \cos(\omega_o t) - \frac{4}{\pi} V_d \frac{T_d}{T_s}\cos(\omega_o t - \varphi) \qquad \textbf{(1)}$$

$$= \frac{600}{2}\cos(2\pi \cdot 400t) - \frac{4}{\pi} \cdot \frac{2 \cdot 10^{-6}}{100 \cdot 10^{-6}} \cdot 600 \cdot \cos((2\pi \cdot 400t) - 0.5236)$$

$$= 300 \cdot \cos(2\pi \cdot 400t) - 15.3 \cdot \cos(2\pi \cdot 400t - 0.5236)$$

$$= 300 \cdot \cos(2\pi \cdot 400t) - 15.3 \cdot \{cos(2\pi \cdot 400t) \cdot \cos(0.5236) + sin(2\pi \cdot 400t) \cdot \sin(0.5236)\}$$

$$= 300 \cdot \cos(2\pi \cdot 400t) - 15.3 \cdot \{cos(2\pi \cdot 400t) \cdot 0.866 + sin(2\pi \cdot 400t) \cdot 0.5\}$$

$$= 300 \cdot \cos(2\pi \cdot 400t) - 13.25 \cdot cos(2\pi \cdot 400t) - 7.65 \cdot sin(2\pi \cdot 400t)$$

$$= 286.75 \cdot \cos(2\pi \cdot 400t) - 7.65 \cdot sin(2\pi \cdot 400t)$$

$$= \mathrm{V}' \cdot \cos(2\pi \cdot 400t - \varphi')$$

$$\mathrm{V}' = \sqrt{(286.75)^2 + (7.65)^2} = 286.75\mathrm{V}$$

$$\varphi' = atan\frac{-7.65}{286.75} = -0.027 \qquad \textbf{(3)}$$

d.ii)

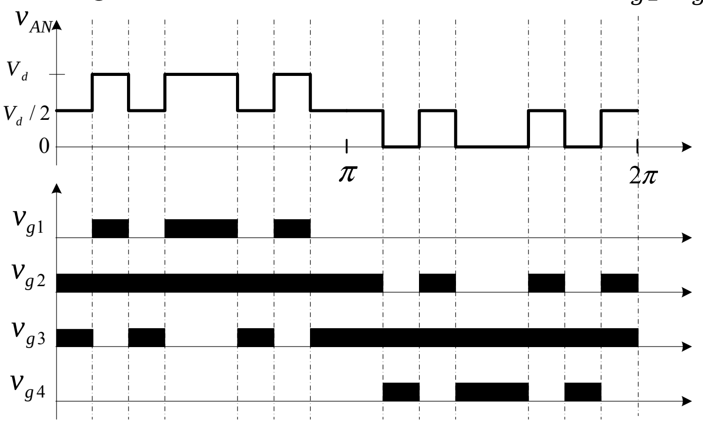
Maximum decrease in amplitude of the fundamental component of the inverter output voltage occurs at φ=0 and is equal to: $\Delta V_{Az(1)} = \frac{4}{\pi} V_d \frac{T_d}{T_s} = 15.3V$.

(4)

Question 2

a)

The output terminal voltage v_{AN} have the following three levels:V_d, $V_d/2$ and 0. Gate drive signals for switches S1, S2, S3 and S4 are v_{g1}, v_{g2}, v_{g3} and v_{g4}, respectively.



(4)

Therefore for leg A and switches S1, S2, S3 and S4, there are three possible switching combinations:

	S1	S2	S3	S4	V_{AN}
P	1	1	0	0	V_d
0	0	1	1	0	$V_d/2$
N	0	0	1	1	0

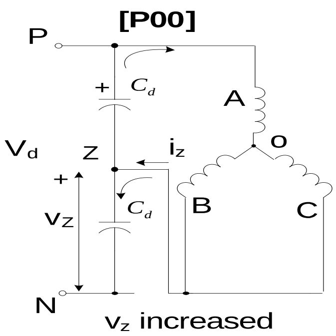
(1)

b)

Switching states [PPP], [000] and [NNN] present short circuit at the load terminals and there is no power flow between the source and load for this switching state. Any other switching state will connect the load to the dc power supply and will produce small, medium or large static output vector. The medium vectors cause deviations of voltage v_{ZN} but they are not defined. P-type small vectors make v_{ZN} rise, while N-type small vectors make v_{ZN} decline. Therefore we need to select one P-type small vector.

(3)

Switching state [P00], we can represent with the following equivalent circuit:



For the switching state [P00], the output phase voltages are: **(2)**

$$v_{Ao} = \frac{V_d}{3}$$

$$v_{Bo} = v_{Co} = -\frac{V_d}{6}$$ **(1)**

And the corresponding space-vector is:

$$\vec{V_1}(t) = \frac{2}{3}\left[v_{Ao}(t)e^{j0} + v_{Bo}(t)e^{j2\pi/3} + v_{Co}(t)e^{j4\pi/3}\right]$$

$$= \frac{2}{3}\left[\frac{V_d}{3}e^{j0} - \frac{V_d}{6}e^{j2\pi/3} - \frac{V_d}{6}e^{j4\pi/3}\right]$$

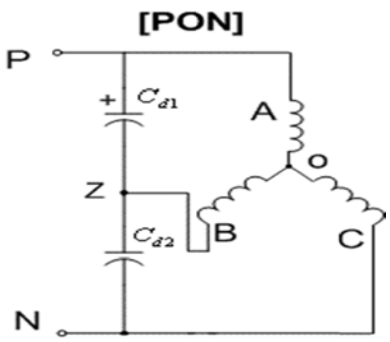
$$= \frac{2}{3}\left[\frac{V_d}{3} - \frac{V_d}{6}\cdot(-\frac{1}{2} + j\frac{\sqrt{3}}{2}) - \frac{V_d}{6}(-\frac{1}{2} + j\frac{-\sqrt{3}}{2})\right] = \frac{V_d}{3}$$

This is P-Type Small vector (amplitude Vd/3 and it is aligned with the α-axis in the α-β reference frame.

(2)

c)

The reference vector is in Sector I, region 2 and can be synthesised by two small vectors V1 and V2 and one medium vector V7. The length of the reference vector at $\theta = 30^\circ$ as shown in figure 2.2 is equal to the length of the medium vector V7 [PON] and is equal to $\frac{V_d}{\sqrt{3}}$. This can be obtained from the following analysis:



$$\vec{V_7}(t) = \frac{2}{3}\left[v_{Ao}(t)e^{j0} + v_{Bo}(t)e^{\frac{j2\pi}{3}} + v_{Co}(t)e^{\frac{j4\pi}{3}}\right] = \frac{2}{3}\left[\frac{V_d}{2}e^{j0} - 0\cdot e^{\frac{j2\pi}{3}} - \frac{V_d}{2}e^{\frac{j4\pi}{3}}\right]$$

$$= \frac{2}{3}\left[\frac{V_d}{2} - \frac{V_d}{2}(-\frac{1}{2} + j\frac{-\sqrt{3}}{2})\right] = \frac{V_d\sqrt{3}}{3}e^{j\pi/6}$$

(2)

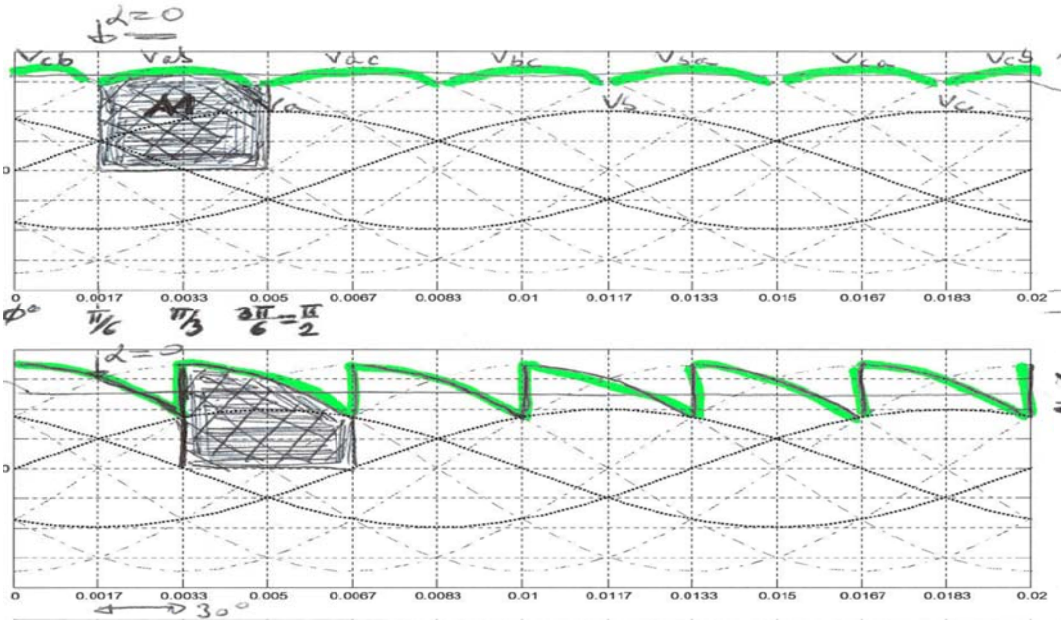
$$\Delta\theta = 2\pi f_1 \cdot T_s = 2\pi\cdot\frac{f_1}{f_s} = 2\pi\frac{50}{10000} = \frac{\pi}{100}\cdot\frac{180}{\pi} = 1.8^\circ$$

The positions of the two subsequent reference vectors are:
$\theta_1 = \theta + \Delta\theta = 30 + 1.8 = 31.8^{\circ}$
$\theta_2 = \theta_1 + \Delta\theta = 31.8 + 1.8 = 33.6^{\circ}$
The amplitude of the ref vector is constant and equal to $V_d/\sqrt{3}$.

(3)

Question 3

a)



dc link voltage for $\alpha = 0°$ and $\alpha = 30°$ is highlighted

$$V_d = \frac{area\ A1}{\pi/3} = \frac{1}{\pi/3}\int_{\pi/6+\alpha}^{\pi/2+\alpha} v_{ab} d(\omega t)$$

$$= \frac{1}{\pi/3}\int_{\pi/6+\alpha}^{\pi/2+\alpha} \sqrt{2}\cdot V_{ll_{rms}} \sin\left(\omega t + \frac{\pi}{6}\right)\cdot d(\omega t)$$

$$= \frac{3}{\pi}\sqrt{2}\cdot V_{ll_{rms}}\left\{\cos\left(\frac{\pi}{6}+\alpha+\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{2}+\alpha+\frac{\pi}{6}\right)\right\} =$$

$$\frac{3}{\pi}\sqrt{2}\cdot V_{ll_{rms}}\left\{cos\alpha\cdot cos\frac{\pi}{3} - sin\alpha\cdot sin\frac{\pi}{3} - \left(cos\frac{2\pi}{3}\cdot cos\alpha - sin\frac{2\pi}{3}\cdot sin\alpha\right)\right\}$$

$$= \frac{3}{\pi}\sqrt{2}\cdot V_{ll_{rms}}\cdot cos\alpha$$

(3)

In order to supply the required power to the load for the case when the dc link current is kept constant at 148A, the dc link voltage can be adjusted by adjusting the firing angles of the SCR devices. Losses in the circuit are neglected so:

$$P_{dc} = P_{load} = I_{dref}\cdot V_d$$

$$V_{dc} = \frac{P_{load}}{I_{dref}} = \frac{400kW}{148A} = 2702.7V$$

From the expression for the dc link voltage we have:

$$cos\alpha = \frac{V_d}{\frac{3}{\pi}\sqrt{2}\cdot V_{ll_{rms}}} = \frac{2702.7}{1.35\cdot 2300} = 0.87$$

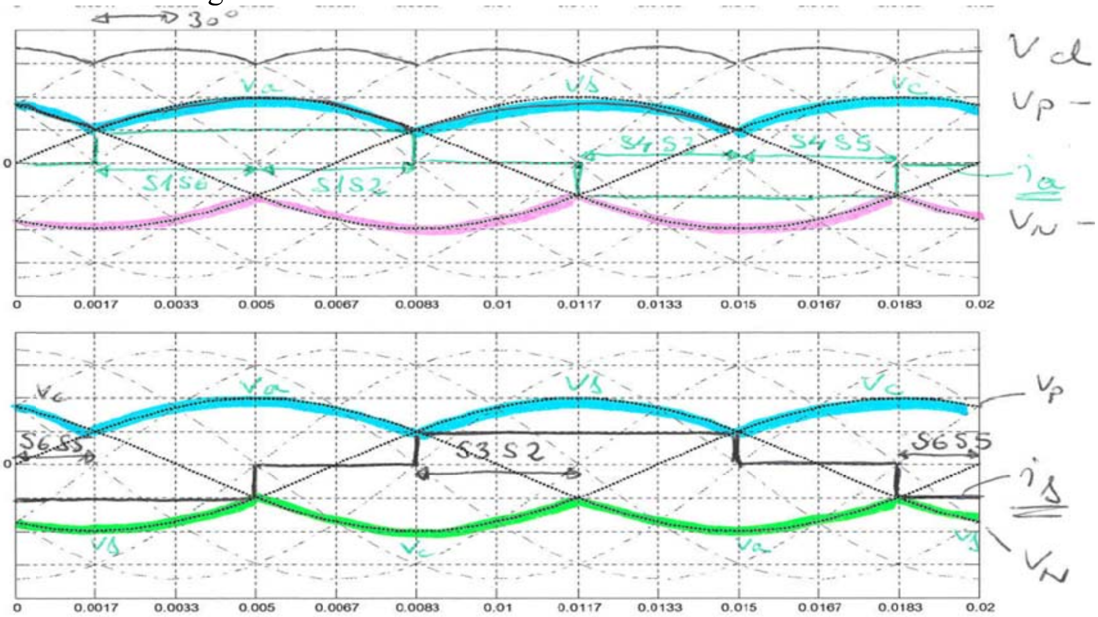
$$\alpha = 29.53^{\circ}$$

So in order to provide 400kW with the dc link current regulated at 148A, the firing angle should be approximately 30°.

(3)

b)

The dc link voltage has maximum value when $\alpha = 0°$.

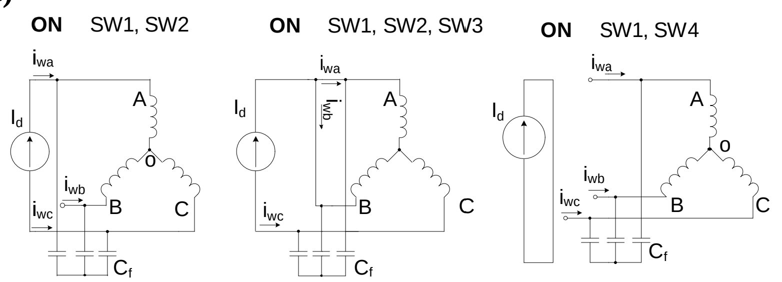


(3)

With reference to the above waveforms, the switching sequence is then: S5S6; S6S1; S1S2; S2S3; S3S4; S4S5; S5S6. Therefore each switch conducts for a period of 120° and device switching frequency is equal to the grid frequency of 50Hz.

(3)

c)



i_{wa}, i_{wb}, i_{wc} = f(I_d, switching)

i_{wa}, i_{wb}, i_{wc} $\neq$ f(LOAD)

$i_{wa}=I_d$

$I_{wb}=0$

$I_{wc}=-I_d$

i_{wa}, i_{wb} , i_{wc}= f(LOAD)

dc current source Id is bypassed

$i_{wa} = 0$

$i_{wb} = 0$

$i_{wc} = 0$

(2)

Switching Constraint: In the CSI only two switches conduct at any time instant, one in the top half of the converter and the other in the bottom half. With only one switch turned on, the continuity of the dc current is lost and a very big voltage will be induced by the dc choke Ld. This will cause damage of the switching devices. If more than two devices are on simultaneously, the PWM output current is not defined by the switching pattern. If two switches in the same leg are on at the same time, the source is bypassed. When two devices in different arms conduct, the source current, I_d, flows through the load.

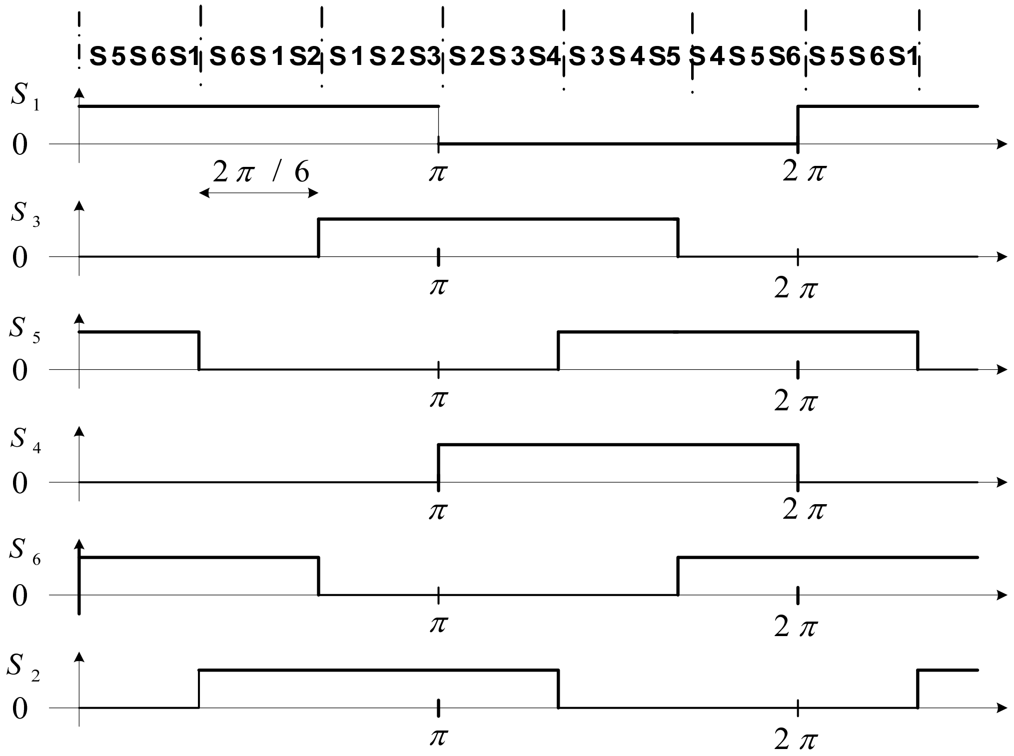
(3)

In case of short circuit at the inverter output terminals, the rise of the dc current will be limited by the dc choke - no additional protection is needed. **(1)**

Question 4

a)

Gate drive signals for switches S1, S3 and S5 are shifted by 120°. Switches S2, S4 and S6 work in complementary mode.

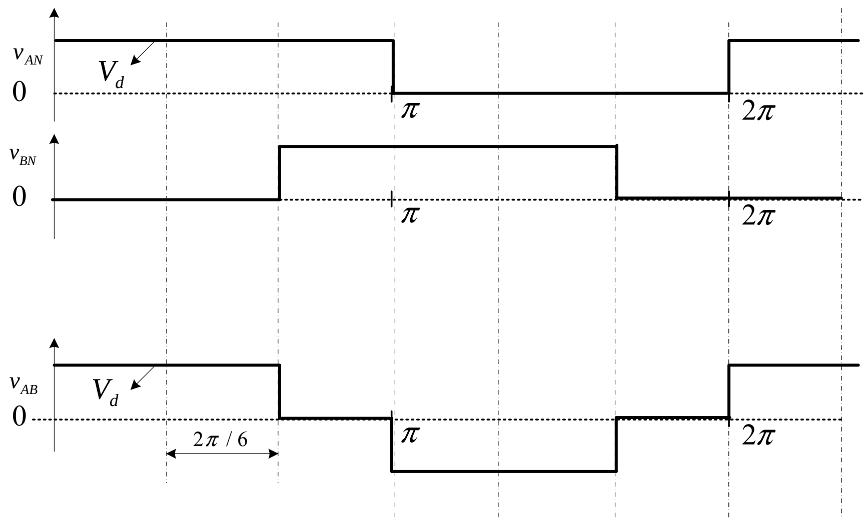


(3)

- There are six commutations in one fundamental period
- Conduction period of each switch is 120°
- There are 6 switching combinations and their sequence is:S5S6S1; S6S1S2; S1S2S3; S2S3S4; S3S4S5; S4S5S6 and the corresponding commutations are: from S5 to S2, from S6 to S3, from S1 to S4, from S2 to S5, from S3 to S6, from S4 to S1.
- Each switch turns on and off only once per fundamental period, therefore the device switching frequency is equal to the fundamental frequency.
- Conduction time of each switching combination is 60°.

(3)

b)



N-negative dc bus

From Fourier analysis, amplitude of the fundamental component of v_{AN}:

$$A_1 = \frac{1}{\pi}\int_0^{\pi} V_d \cdot \sin(\omega t)\, d\omega t = \frac{1}{\pi} V_d(-\cos(\omega t)) \uparrow_0^{\pi} = \frac{2}{\pi} V_d$$

(2)

……………………

$$v_{AN,1}(t) = V_d \frac{2}{\pi} \sin \omega_o t$$

$$v_{BN,1}(t) = V_d \frac{2}{\pi} \cdot \sin(\omega_o t - \frac{2\pi}{3})$$ **(1)**

$$v_{ab}(t) = v_{AN}(t) - v_{BN}(t)$$

$$V_{ab_{(rms)1}} = \frac{2\sqrt{3}}{\pi\sqrt{2}} V_d = 0.778 V_d$$

$$V_{ab1,rms} = 460V$$

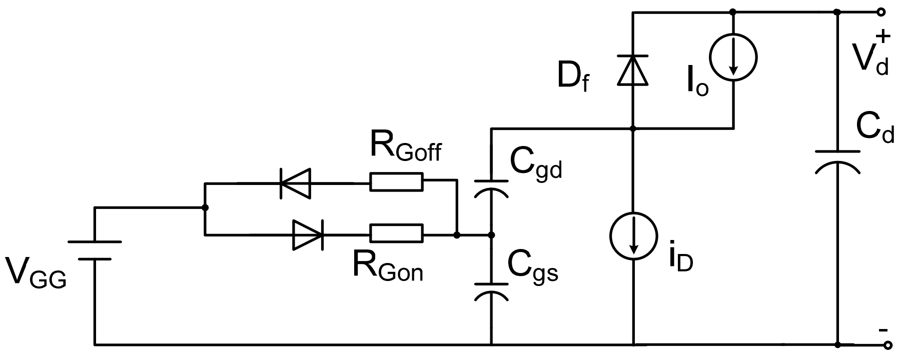
$$460 = 0.778 V_d$$

$$V_d = 591\text{V}$$

So the most appropriate value for the dc link voltage is:$V_d \approx 600\text{V}$. **(3)**

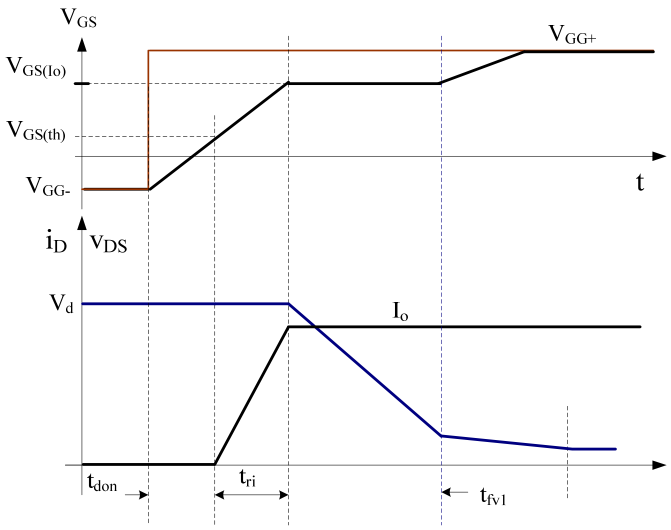
c.i)

The equivalent circuit during IGBT TURN ON is given by:



(2)

c.ii)



(1)

Once the IGBT is carrying the full load current Io but is still in the active region, the gate-source voltage becomes temporary clamped at the voltage needed to maintain load current. During the voltage fall time the entire gate current i_G flows through the equivalent gate-drain capacitance C_{gd}. This causes the drain-source voltage to drop at a rate:

$$\frac{dv_{DG}}{dt} = \frac{dv_{DS}}{dt} = \frac{V_{GG} - V_{GS(Io)}}{R_{Gon} C_{gd}}$$

$$R_{Gon} \leq R_{Gon,max} = \frac{V_{GG} - (\frac{Io}{gm} + V_{GS(th)})}{C_{gd} \cdot \left(\frac{dv_{DS}}{dt}\right)_{on}^{req}}$$ **(2)**

If we want to obtain a $\frac{dv_{DS}}{dt}$ faster than a specified minimum value, the above equation can be used to estimate gate resistance needed. Design constraints for optimisation include minimum $\frac{dv_{DS}}{dt}$ at the highest current. **(1)**

Question 5

a.i)

s_a, s_b and s_c are the switching functions for leg a, b and c, respectively.
The switching function $s_a/s_b/s_c$, is 1 when the upper switch in leg a/b/c is ON, and it is 0 when the bottom switch is ON.

(2)

For the phase voltages of the rectifier, we can write:

$$v_{ao}(t) = \frac{u_{dc}}{3} \cdot (2 \cdot s_a - s_b - s_c)$$

$$v_{bo}(t) = \frac{u_{dc}}{3} \cdot (-s_a + 2 \cdot s_b - s_c)$$

$$v_{co}(t) = \frac{u_{dc}}{3} \cdot (-s_a - s_b + 2 \cdot s_c)$$

(3)

a.ii)

For the input side of the rectifier we have:

$$e_a(t) = E_m sin(\omega t)$$

$$e_b(t) = E_m sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$e_c(t) = E_m sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$\boldsymbol{e(t)} = \begin{bmatrix} e_a(t) \\ e_b(t) \\ e_c(t) \end{bmatrix}$$

$$i_a(t) = I_m sin(\omega t - \varphi)$$

$$i_b(t) = I_m sin\left(\omega t - \frac{2\pi}{3} - \varphi\right)$$

$$i_c(t) = I_m sin\left(\omega t + \frac{2\pi}{3} - \varphi\right)$$

$$\boldsymbol{i(t)} = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$$

For the phase voltages of the rectifier, we can write:

$$\boldsymbol{v_{con}(t)} = \begin{bmatrix} v_{ao}(t) \\ v_{bo}(t) \\ v_{co}(t) \end{bmatrix}$$

<u>So the ac side can be modelled as:</u>

$$\boldsymbol{v}_{con}(t) = \boldsymbol{e}(t) - L \cdot \frac{\mathrm{d}\boldsymbol{i}(t)}{\mathrm{d}t} - R \cdot \boldsymbol{i}(t)$$ **(2)**

<u>The dc current is:</u>

$$i_{dc}(t) = s_a \cdot i_a + s_b \cdot i_b + s_c \cdot i_c$$

$$i_{dc}(t) = C \cdot \frac{\mathrm{d}u_{dc}(t)}{\mathrm{d}t} + i_L(t)$$

$$C\frac{du_{dc}}{dt} = s_a \cdot i_a + s_b \cdot i_b + s_c \cdot i_c - i_L(t)$$ **(2)**

b)

$$u_{ao}(t) = e_a(t) - L \cdot \frac{di_a(t)}{dt} - R \cdot i_a(t)$$

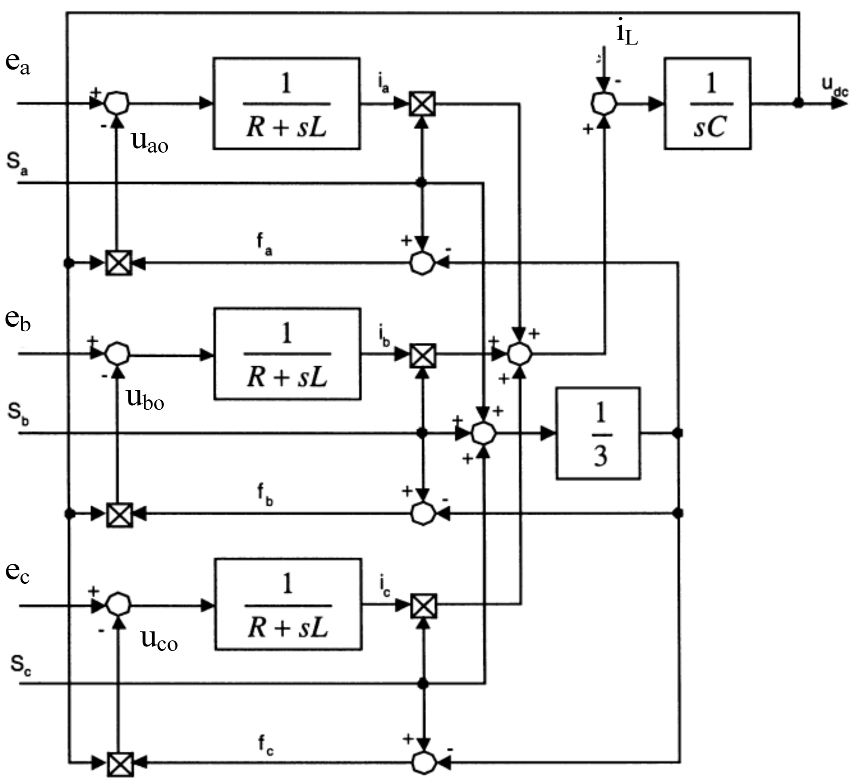
$$e_a(t) - u_{ao}(t) = L \cdot \frac{di_a(t)}{dt} + R \cdot i_a(t)$$

$$e_a - u_{ao} = L \cdot s \cdot i_a + R \cdot i_a$$

$$\frac{i_a}{e_a - u_{ao}} = \frac{1}{L \cdot s + R}$$ **(1)**

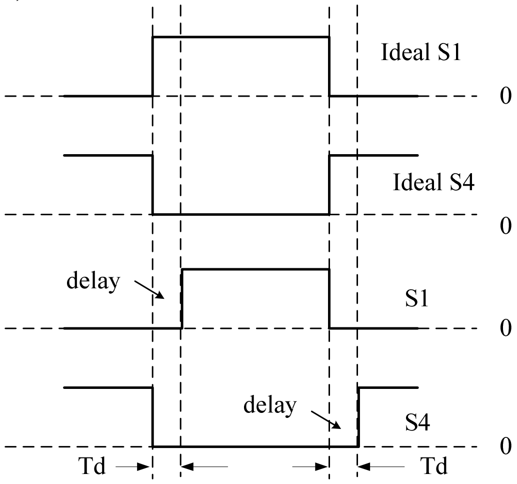
$$i_{dc}(t) = C \cdot \frac{du_{dc}(t)}{dt} + i_L(t)$$

$$\frac{u_{dc}}{i_{dc} - i_L} = \frac{1}{s \cdot C}$$ **(1)**



(3)

c)



(2)

In order to avoid shoot through effect at the dc side of the converter, it is necessary to introduce a small delay into the rising edge of the PWM signals. Considering t_{off} time of 0.65us and switching period of 100us, T_d has to be greater than t_{off}.

$1.5\mu s < T_d < 2\mu s$. **(2)**

Question 6

a)

$v_c = V_d - L \cdot \frac{di_L}{dt}$ (*)

$i_L - i_C = I_o$ (**)

$i_C = C\frac{dv_c}{dt}$ (***)

From (*) & (**):

$v_c = V_d - L \cdot \frac{di_c}{dt}$ (4*)

From (***) & (4*):

$$v_c = V_d - L \cdot C\frac{d^2v_c}{dt^2}$$

$L \cdot C\frac{d^2v_c}{dt^2} + v_c = V_d$ (!!) 2nd order linear diff. eq. **(2)**

The complete solution of (!!) can be written as:

$v_c(t) = v_{cn}(t) + v_{cf}(t) = v_{cn}(t) + V_d$

where v_{cf} is the steady state response and v_{cn} is the transient response of the system.

The homogenous equation can be obtained by setting (!!) to zero.

$$L \cdot C\frac{d^2v_c}{dt^2} + v_c = 0$$

$\frac{d^2v_c}{dt^2} + \frac{1}{L \cdot C} \cdot v_c = 0$ (++)

The characteristic equation of the above (++) is:

$\alpha^2 + \frac{1}{L \cdot C} = 0$

by defining operator $\alpha = \frac{d}{dt}$ and $v_c(t) \neq 0$.

The roots of the above eq. are then:

$\alpha_1, \alpha_2 = \pm j\sqrt{\frac{1}{C \cdot L}} = \pm j\omega_o$ - where ω_o is the resonant frequency

These two complex roots then give rise to the two linearly independent solutions:

$y_1(t) = e^{\alpha_1 t} = \cos(\omega_o \cdot t) + jsin(\omega_o \cdot t)$

$y_2(t) = e^{\alpha_2 t} = \cos(\omega_o \cdot t) - jsin(\omega_o \cdot t)$

We can obtain from them the two real-valued solutions:

$y_3(t) = \frac{1}{2}y_1(t) + \frac{1}{2}y_2(t) = \cos(\omega_o \cdot t)$

$y_4(t) = \frac{1}{2j}y_1(t) - \frac{1}{2j}y_2(t) = \sin(\omega_o \cdot t)$

The general real-valued solution of the homogenous eq. (++) is:

$v_{cn}(t) = c_1 \cdot \cos(\omega_o \cdot t) + c_2 \cdot sin(\omega_o \cdot t)$

The complete solution is then:

$v_c(t) = c_1 \cdot \cos(\omega_o \cdot t) + c_2 \cdot sin(\omega_o \cdot t) + V_d$ (!!!) **(1)**

Constants c_1 and c_2 can be found from the complete solution (!!!) and initial conditions at t = 0:

$V_{co} = c_1 + V_d \qquad \rightarrow c_1 = V_{co} - V_d$

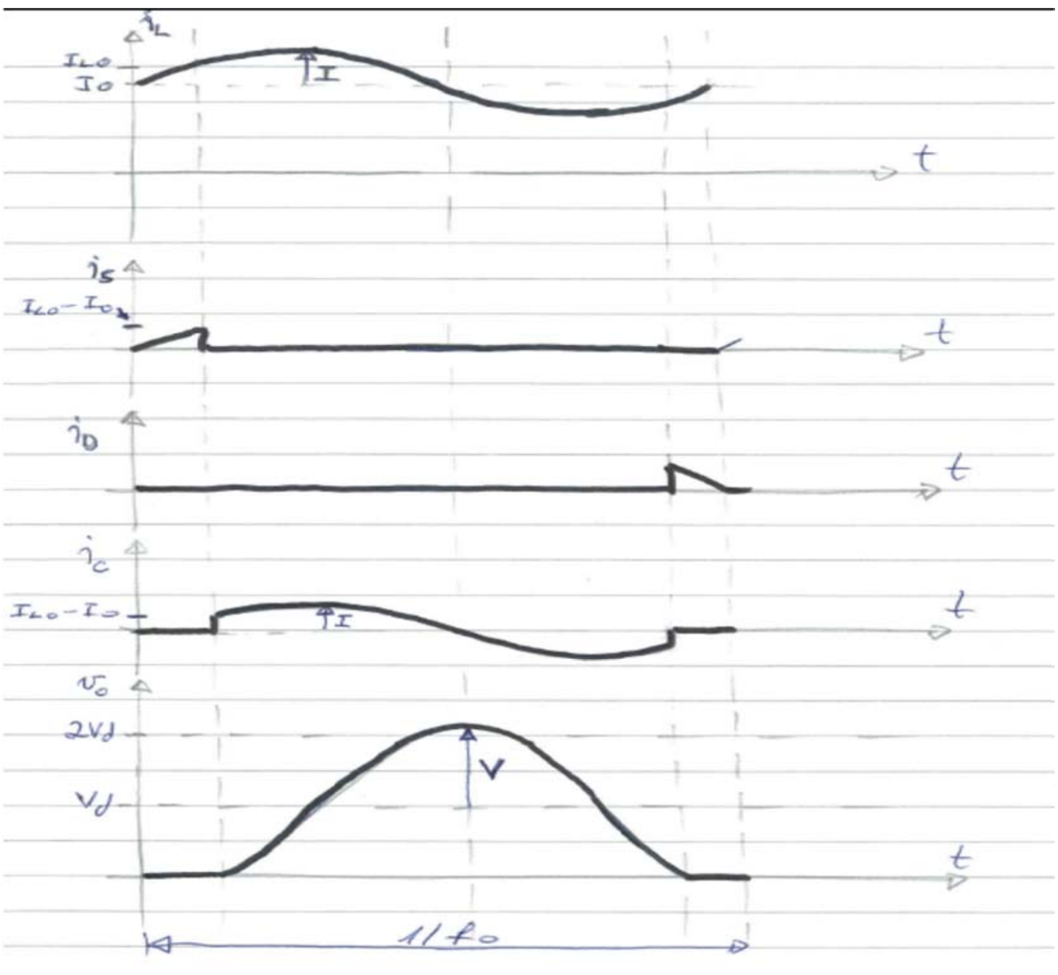
$\frac{dv_c}{dt} = c_1 \cdot \omega_o(-\sin(\omega_o \cdot t)) + c_2 \cdot \omega_o \cdot \cos(\omega_o \cdot t)$ (<>)

By substituting t = 0 in (<>): $\rightarrow \frac{i_{L0} - I_o}{C} = c_2 \cdot \omega_o$

$\boldsymbol{c_1 = V_{co} - V_d}$

$\boldsymbol{c_2 = \frac{i_{L0} - I_o}{\omega_o \cdot C} = Z_o \cdot (i_{L0} - I_o)}$ - where Z_o is characteristic impedance. **(3)**

b.i)



(2)

The current and voltage waveforms with reference to the equivalent circuit are shown above. The aim of the resonant dc link is to create an input voltage to the inverter that remains zero during a finite period of time when the inverter switches at zero voltage. The operational cycle can be divided in the following parts: 1) S is ON - charging of

L, 2) both S and D are OFF – resonance, 3) diode D conducts and clamps the inverter input voltage to ZERO. Switch S has to be turned ON before the diode ceases to conduct (this happens when its current drops to zero) to achieve zero switching.

Both the current through the inductance in dc link circuit and the load current have to be monitor, because their difference defines the peak voltage across C.

(3)

b.ii) The role of the switch and diode in the equivalent circuit is to provide controllable short circuit at the dc terminals of the inverter to allow additional charge of the inductance to compensate for the charge lost due to damping in the dc link circuit.

(1)

In the practical resonant dc link inverter this can be achieved by turning on both switches in one leg at the same time. The three legs will in turn play a role of switch S and diode D.

(2)

b.iii)

$$Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.24 \cdot 10^{-3}}{10^{-6}}} = 15.49\Omega$$

$$\omega_o = \frac{1}{\sqrt{L \cdot C}} = \frac{1}{\sqrt{240 \cdot 10^{-12}}} = 64{,}55\frac{rad}{s}$$

$$f_o = \frac{\omega_o}{2\pi} = 10.27kHz$$

$$V_{C0} = 0;\ I_{L0} = 209A;\ I_0 = 200A;\ V_d = 200V$$ **(1)**

$$i_C(t) = (I_{L0} - I_0) \cdot cos(\omega_0 t) + \frac{V_d - V_{C0}}{Z_o} \cdot sin(\omega_0 t)$$

$$i_C(t) = (209 - 200) \cdot cos(64550 \cdot t) + \frac{200}{15.49} \cdot sin(64550 \cdot t)$$ **(2)**

$$i_C(t) = I \cdot \cos(\omega_o t - \varphi_1)$$

$$\boldsymbol{I} = \sqrt{(I_{L0} - I_0)^2 + (\frac{V_d}{Z_o})^2} = \sqrt{(9)^2 + (\frac{200}{15.49})^2} = \mathbf{15.73}\boldsymbol{A}$$ **(1)**

$$\boldsymbol{\varphi_1} = \text{atan}\left(\frac{\frac{V_d}{Z_o}}{I_{L0} - I_0}\right) = \text{atan}\left(\frac{\frac{200}{15.49}}{9}\right) = \mathbf{0.962rad}$$