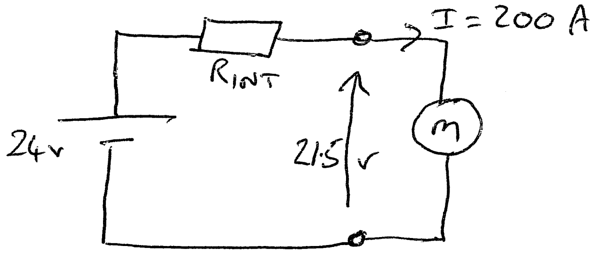
QUESTION 1

(a) Initially the battery is connected to the starter motor:



The voltage drop across the internal resistance is:

$$V_{INT} = 24 - 21.5 = 2.5V$$

Hence the internal resistance of the battery is:

$$R_{INT} = \frac{V_{INT}}{I} = \frac{2.5}{200} = \underline{\underline{0.0125\,\Omega}}$$

The power dissipated in the battery is then - $P_{INT} = I^2 R_{INT}$

$$= 200^2 \times 0.0125$$

$$= \underline{\underline{500W}}$$

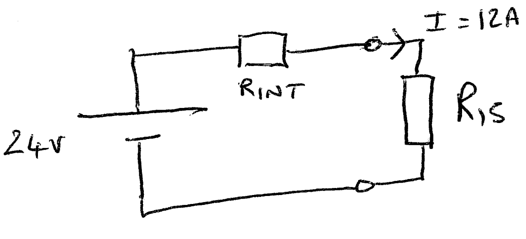
The total power supplied by the battery is:

$$P_{TOT} = 24 \times 200 = 4800\,W$$

Hence the system efficiency is:

$$\text{Efficiency} = \frac{P_{TOT} - P_{LOSS}}{P_{TOT}} \times 100 = \frac{4800 - 500}{4800} = \underline{\underline{89.6\%}}$$

(b) When the heater is first switched on it has a resistance of R_{IS} and at the final temperature it will be R_F.



$$\frac{V}{I} = R_{INT} + R_{IS} = \frac{24}{12} = 2\,\Omega$$

Hence $R_{IS} = 2 - 0.0125 = 1.988\,\Omega$

QUESTION 1 (CONTINUED)

Similarly at the final temperature:

$$R_F + R_{INT} = \frac{V}{I} = \frac{24}{9} = 2.667\Omega$$

$$\text{Hence } R_F = 2.667 - 0.0125 = 2.655\Omega$$

Now
$$\frac{R_F}{R_{20}} = \frac{\cancel{R_0}\,(1+\alpha_0 T_F)}{\cancel{R_0}\,(1+\alpha_0\ \ )}$$

$$\therefore \frac{2.655}{1.988} = \frac{(1+12.5\times10^{-3}T_F)}{(1+12.5\times10^{-3}\times15)}$$

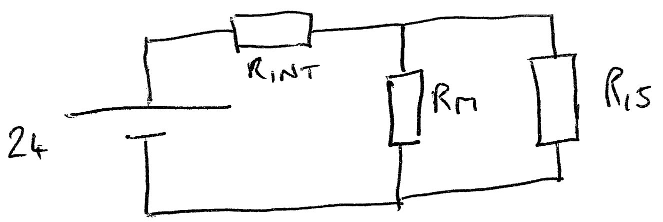
$$\therefore 1.336 = \frac{(1+12.5\times10^{-3}T_F)}{1.1875}$$

$$\therefore T_F = 46.9°C$$

(ii) At the final temperature the Power is $I^2R_F = 9^2 \times 2.655$
$= 215W$

$$\text{Hence the efficiency} = \frac{215}{24\times9}\times100\% = 99.5\%$$

(iii) When both the heater and starter are operated together:



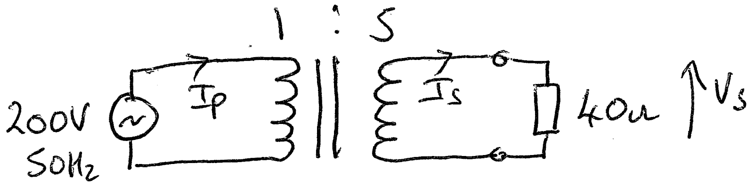
From part (a) $R_M = \frac{21.5}{200} = 0.1075\Omega$

$$R_{IS} = 1.988\Omega$$

$$\therefore R_T = R_{INT} + \frac{1}{1/R_M + 1/R_{IS}} = 0.1145\Omega$$

Hence current is $\frac{24}{0.1145} = 209.6A$

(c)(i)



Since $\frac{V_p}{V_s} = \frac{N_p}{N_s} \Rightarrow V_s = V_p \cdot \frac{N_s}{N_p} = \frac{200 \times 5}{1} = \underline{\underline{1000 V_{rms}}}$

$I_s = \frac{V_s}{R_s} = \frac{1000}{40} = 25 A_{rms}$

Now $\frac{I_p}{I_s} = \frac{N_s}{N_p} \Rightarrow I_p = \frac{N_s I_s}{N_p} = \frac{5 \times 25}{1} = \underline{\underline{125 A_{rms}}}$

Power dissipated $= I_s^2 R = 25^2 \times 40 = \underline{\underline{25kW}}$

(ii) The secondary impedance is now:

$$R + j \cdot 2\pi f L = 40 + j2\pi \times 50 \times 0.15 = 40 + j47.1\Omega$$
$$= 61.8 \angle 49.7^\circ \Omega$$

$\therefore I_s = \frac{1000 \angle 0^\circ}{61.8 \angle 49.7^\circ} = 16.18 \angle -49.7^\circ A_{rms}$

$\therefore I_p = I_s \times 5 = \underline{\underline{80.9 \angle -49.7^\circ A_{rms}}}$.

Power dissipated in load $= I_s^2 \times 40 = 16.18^2 \times 40 = \underline{\underline{10.47kW}}$

(check $P = V_p I_p \cos\phi = 200 \times 80.9 \times \cos(-49.7) = 10.47kW$)

(iii) The input power factor would be $\cos(-49.7°) = \underline{\underline{0.647 \text{ lagging}}}$

The VA rating is $200 \times 80.9 = \underline{\underline{16.18\,kVA}}$

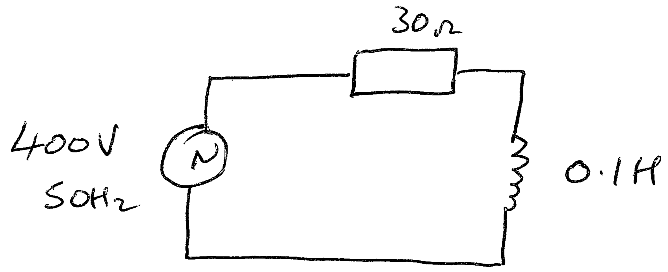
(iv) Since $V_{rms} = 4.44 f N \phi_{max}$

then $N_p = \dfrac{V_{p\,rms}}{4.44\, f.\, \phi_{max}} = \dfrac{200}{4.44 \times 50 \times 5\times10^{-3}} = \underline{\underline{180 \text{ TURNS}}}$

(v) $V_{rms} = 4.44 \times 60 \times 180 \times 5\times10^{-3} = \underline{\underline{239.8\,V_{rms}}}$

QUESTION 2

(a) (i)



The impedance of the coil is given by:

$$Z = R + j2\pi f L = 30 + j.2\pi.50.0.1$$
$$= 30 + j31.42 = \underline{\underline{43.44\angle 46.3^\circ \Omega}}$$

(ii) The current flowing in the coil is:

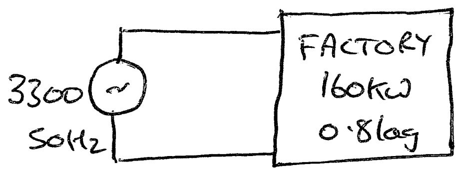
$$I = \frac{V\angle 0^\circ}{Z\angle \phi^\circ} = \frac{400\angle 0^\circ}{43.44\angle 46.3^\circ} = \underline{\underline{9.21\angle -46.3^\circ A}}$$

(iii) The real power is:

$$P = VI\cos\phi = 400 \times 9.21 \times \cos 46.3^\circ$$
$$= \underline{\underline{2545 W}}$$

The power-factor is $\cos\phi = \cos(-46.3) = \underline{\underline{0.69 \text{ lagging}}}$

(b)



(i) VA rating $= \frac{P}{\cos\phi} = \frac{160}{0.8} = \underline{\underline{200kVA}}$

(ii) kVAr rating $= S\sin\phi = 200.0.6 = \underline{\underline{120kVAr}}$

(iii) Magnitude of current $= \frac{S}{V} = \frac{200\times10^3}{3.3\times10^3} = 60.61A$

and the phase angle $= \cos^{-1} 0.8 = 36.87^\circ$ lagging

ie. $\underline{\underline{I = 60.61\angle -36.87^\circ}}$

(c) For the heating ovens:

$P_{L1} = 50kW \qquad Q_{L1} = 0kVAr$

For the motor load:

$$P_{L2} = S\cos\phi = 100\times10^3 \times 0.75 = 75\,\text{kW}$$

$$Q_{L2} = S\sin\phi = 100\times10^3 \times \sin(\cos^{-1}0.75) = 66.14\,\text{kVAr}$$

(i) Therefore the total factory load is:

$$P_T = P_F + P_{L1} + P_{L2} = 160 + 50 + 75 = 285\,\text{kW}$$

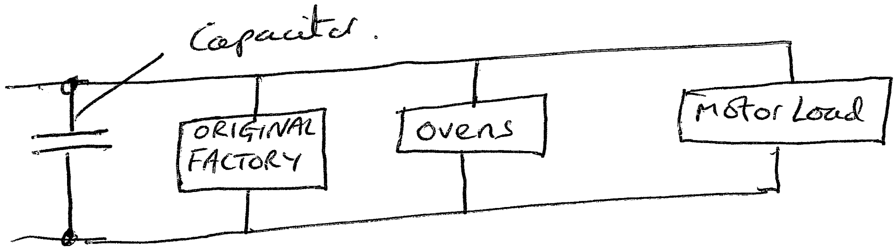
$$Q_T = Q_F + Q_{L1} + Q_{L2} = 120 + 0 + 66.14 = 186.14\,\text{kVAR}$$

$$\therefore \text{ VA rating is } \sqrt{P_T^2 + Q_T^2} = \sqrt{285^2 + 186.14^2} = \underline{\underline{340.4\,\text{kVA}}}$$

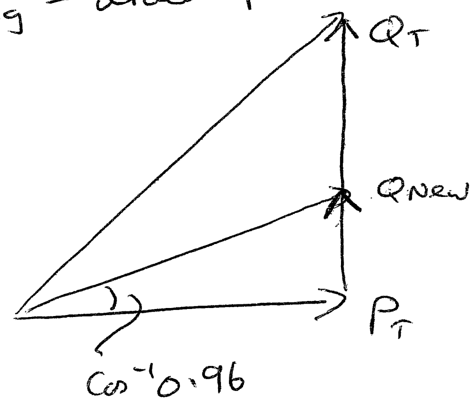
(ii) Phase angle $= \operatorname{atan}\dfrac{Q_T}{P_T} = \operatorname{atan}\dfrac{186.14}{285} = 33.15°$ lagging

$$\therefore \text{ power-factor} = \cos 33.15 = \underline{\underline{0.837 \text{ lagging}}}$$

(d) (i) Capacitor is connected across the total load in parallel:



(ii) Adding the capacitor does not change P_T. Since p.f. needs to be 0.96 lagging – draw power triangle.



$$Q_{NEW} = P_T \tan(\cos^{-1} 0.96) = 285 . \tan 16.26° = 83.12 kVAr$$

Here capacitor must provide:

$$Q_{OLD} - Q_{NEW} = 186.14 - 83.12$$
$$= \underline{\underline{103.02 kVAr}}$$

Current drawn by the capacitor:

$$I_C = \frac{103.02 \times 10^3}{3300} = \underline{\underline{31.22 A_{rms}}}$$

Since $Q_C = \dfrac{V_C^2}{X_C} \Rightarrow X_C = \dfrac{V_C^2}{Q_C} = \dfrac{3300^2}{103.02 \times 10^6}$.

$$= 105.7 \Omega$$

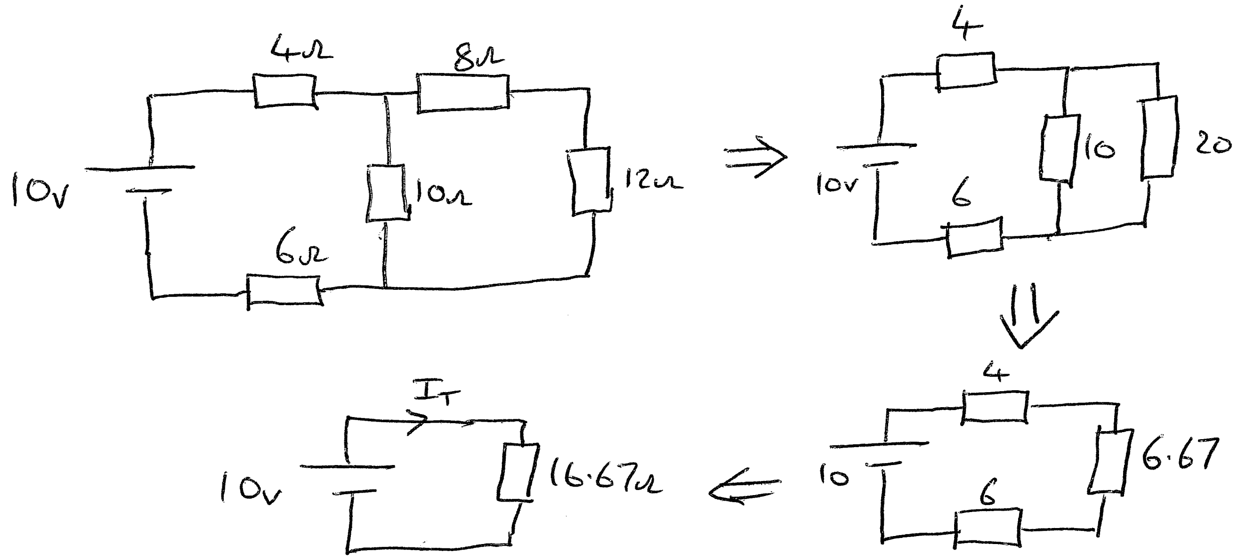
and $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi . 50 \times 105.7} = \underline{\underline{30.1 \mu F}}$

(d)(iii) The peak voltage the capacitor must withstand is

$$V_{PK} = 3300 \times \sqrt{2} = \underline{\underline{4667 V}}$$

QUESTION 3

(a) First consider the 10v source:

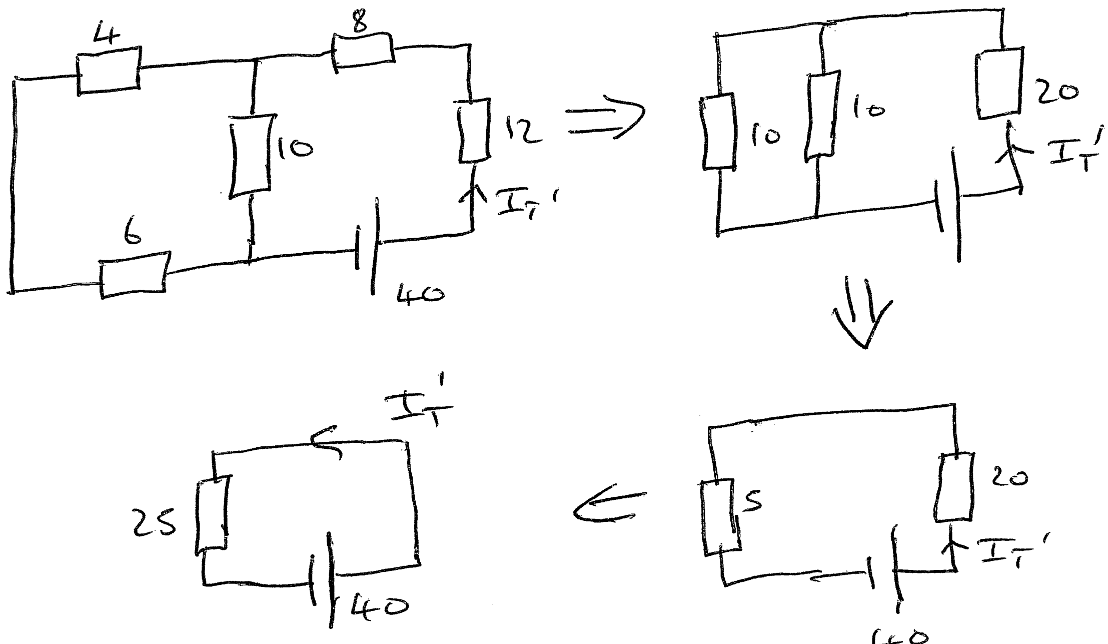


$$\therefore I_T = \frac{10}{16.67} = 0.6A$$

∴ Current through the 10Ω resistor is:

$$I_{10\Omega} = I_T \cdot \frac{20}{(10+20)} = 0.6 \times \frac{2}{3} = 0.4A \downarrow$$

Now consider the 40v supply:



$$I_T' = \frac{40}{25} = 1.6A$$

$$\therefore \text{current through the } 10\Omega \text{ resistor} = 1.6 \times \frac{10}{20} = 0.8A \downarrow$$

∴ Total current through the 10Ω resistor is:

$$I_{10\,TOTAL} = 0.4\downarrow + 0.8\downarrow = \underline{\underline{1.2A\downarrow}}$$

(b) For the Thevenin circuit we need the open circuit voltage. This is effectively the voltage across the 12Ω resistor. Using working from part (a).

For the 10V source the current through the 12Ω resistor is

$$I_{12a} = I_T \times \frac{10}{30} = 0.2A\downarrow$$

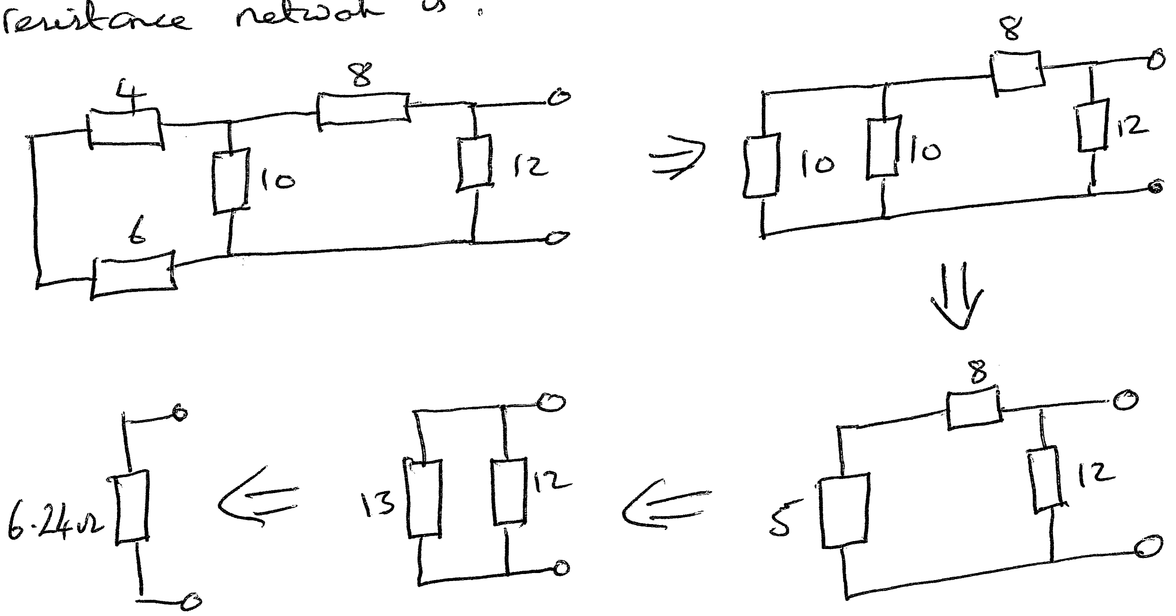
For the 40V source the current through the 12Ω resistor is equal to $I_T' = 1.6A\uparrow$

Hence the total current through the 12Ω resistor is:

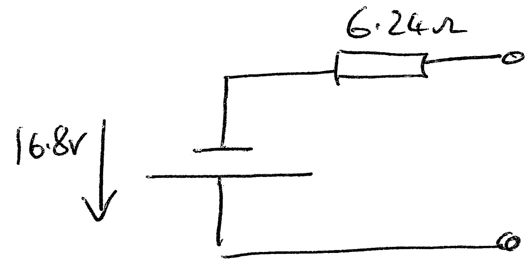
$$I_{12\,TOTAL} = 0.2\downarrow + 1.6\uparrow = 1.4\uparrow$$

Hence the Thevenin voltage is $1.4 \times 12 = 16.8V\downarrow$

The resistance network is:



Hence the Thevenin circuit is:



When the load is connected the current is:

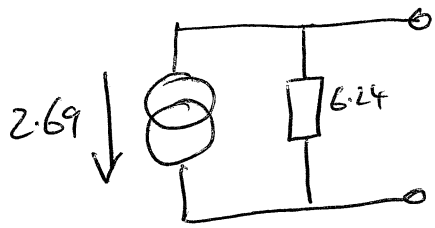
$$I_L = \frac{16.8}{(6.24+20)} = 0.64\,A$$

Hence the power dissipated in the load is:

$$P_L = 0.64^2 \times 20 = \underline{\underline{8.19W}}$$

(c) The Norton circuit can be found directly from the Thevenin circuit.

$$I_N = \frac{E_T}{R_T} = \frac{16.8}{6.24} = 2.69A$$



(d) At steady state $\frac{dI}{dt} = 0$ hence $V_{L_1} = 0$ and C_1 is fully charged. The current is therefore governed by the 2 resistors.

(i) Applying Kirchoff's law (use defined direction of Is)

$$60 + 20I_s + 10I_s = 0$$

$$\underline{\underline{I_s = -2A}}$$

QUESTION 3 (CONTINUED)

(ii) Energy stored in $L_1 = \frac{1}{2} L_1 I_S^2 = \frac{1}{2} \times 100 \times 10^{-3} \times 2^2$

$= \underline{\underline{0.2\,J}}$

The voltage across the capacitor is equal to the voltage across $R_2 = 20 \times 2 = 40\,V$

$\therefore$ Energy stored in the capacitor, C_1

$= \frac{1}{2} C V^2 = \frac{1}{2} \times 1 \times 10^{-3} \times 40^2 = 0.8\,J$

(iii) Total power dissipated in the circuit is:

$P_T = I_S^2 (R_1 + R_2) = 2^2 . 30 = \underline{\underline{120\,W}}$

EEE123 Solutions to Q4 to Q6 – 2014/15

4.

a) Load current is simply given by:

$$I_L = \frac{V_{Dc}}{R_L} = \frac{150}{60} = 2.5A$$

The load power is given by:

$$P_L = \frac{V_{DC}^2}{R_L} = \frac{150^2}{60} = 375W$$

Or alternatively,

$$P_L = I_L^2 R_L = 2.5^2 \times 60 = 375W$$

b) The minimum base current required to support this load is:

$$I_{B(min)} = \frac{I_L}{h_{FE}} = \frac{2.5}{125} = 20mA$$

This minimum value of base current is also given by:

$$I_{B(min)} = \frac{V_i - V_{BE}}{R_{B(max)}}$$

Rearranging yields:

$$R_{B(max)} = \frac{V_i - V_{BE}}{I_{B(min)}} = \frac{5 - 0.7}{0.02} = 215\Omega$$

c) In practice, it would be wise to adopt value some 2-3 time smaller to allow for device variation, different operating conditions, including for example, temperature rise of the device.

d) When the BJT transistor is fully turned on, (V_{ce}) is 1.15V. Hence the power loss in the transistor is simply:

$$P_{loss} = V_{ce} I_C = 1.15 \times 2.5 = 2.87W$$

e) For a MOSFET, the power loss is given by:

$$P_{loss} = I_L^2 R_{DS\ on}$$

Equating this to the loss for the BJT yields:

$$R_{DS\ on} < \frac{2.87}{2.5^2}$$

Hence

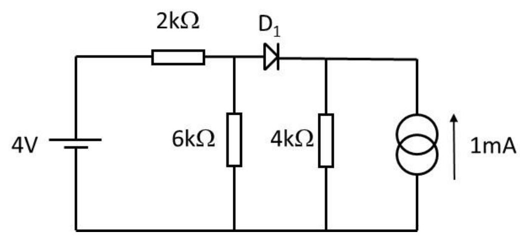
$$R_{DS\ on} < 0.46\Omega$$

f) Other advantages:

- Eliminates need for a base resistor
- Easier to drive in terms of current rating of control side electronics (notwithstanding transient)
- Lower loss on control side

5.

a)



Assume the diode is conducting and replace the diode with a +0.7V voltage supply with is positive at the anode. Replace the current source and its parallel resistance by a Thevenin equivalent and apply a Thevenin simplification to the voltage source and the 2kΩ and 6kΩ resistors.

For the voltage source and 2kΩ and 6kΩ resistors:

$$Open-circuit\ voltage\ =\left(\frac{6}{6+2}\right)\times 4=3V$$

$$Short-circuit\ current=\frac{4}{2\times 10^{3}}=2mA$$

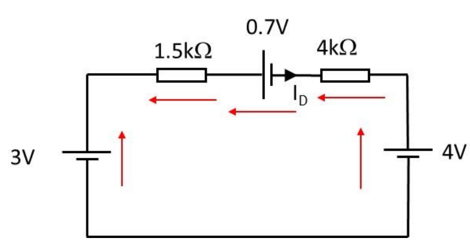
Hence $R_{TH}\ =1.5k\Omega$

For the current source and shunt resistor:

$$Open-circuit\ voltage\ =\ 1\times 10^{-3}\times 4\times 10^{3}=4V$$

$$Short-circuit\ current=\ 1mA$$

Hence $R_{TH}\ =4k\Omega$



$$3-1500I_D-0.7-4000I_D-4=0$$

Hence, I_D is given by:

$$I_D = \frac{-1.7}{5500} = -0.31mA$$

Since this current is negative, then the original assumption was incorrect, and hence the diode is non-conducting in this circuit.

This can also be solved by nodal analysis.

b) The peak AC voltage is 162.6V

Including the two on-state diode voltage drop reduces this to 161V – neglecting these diode drops does not result in any meaningful difference in this case.

$$V_{ave} = \frac{2V_p}{\pi} = \frac{2 \times 161}{\pi} = 102.5V$$

c) It is necessary to calculate Vrms applied to the load (and not use V_{ave}):

$$V_{rms} = \frac{V_p}{\sqrt{2}} = \frac{161}{\sqrt{2}} = 113.8V$$

(slightly lower than supply due to the on-state voltage drop)

Hence, average power delivered to load:

$$P_L = \frac{V_{rms}^2}{R_L} = \frac{113.8^2}{40} = 324W$$

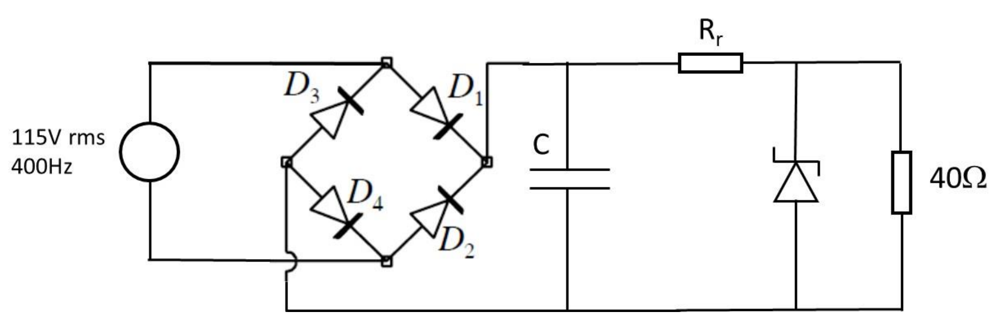
d) With a smoothed output (nominally around 161V), a slight overestimate of the load current is given by:

$$I_L = \frac{161}{40} = 4.02A$$

Assuming that the current falls linearly during the discharge period, then a slight overestimate of the capacitance is given by:

$$C_{min} = \frac{I}{dV/dt} \approx \frac{I}{\Delta V/\Delta t} \approx \frac{4.02 \times 0.00125}{6} = 0.839mF$$

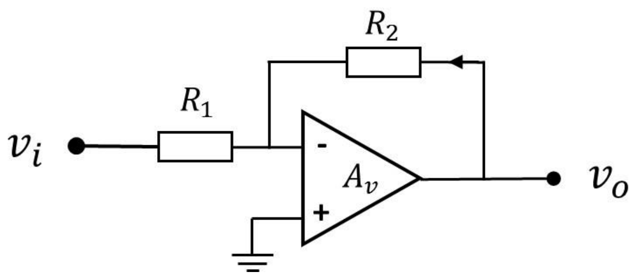
e)



The regulator resistor R_r drops the appropriate voltage between the capacitor voltage and the load which is determined by the total current flowing.

6.

a)



Assumptions:

- The op-amp gain $A_v \to \infty$
- The op-amp has infinite input resistance

In order to determine the effective gain of this circuit, we start by summing the currents into the v^- node:

$$i_i + i_f = 0$$

(since an op-amp has close to infinite input impedance, i.e. no current flows into the op-amp input)

This can be written as:

$$\frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2} = 0$$

But from the discussion above $v^- = 0$ (because it is a virtual earth) and so:

$$\frac{v_i}{R_1} + \frac{v_o}{R_2} = 0$$

Hence, the effective gain is given by:

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}$$

b) To get an input resistance of $10k\Omega$, then $R_1 = 10k\Omega$ and hence:

$$R_2 = -gain \times R_1 = 100 \times 10{,}000 = 1M\Omega$$

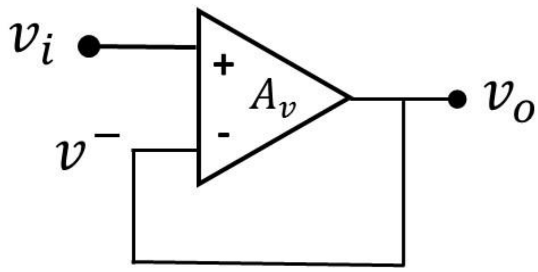
c) With a source impedance of $2k\Omega$, there is a potential divider action at play in setting v_i at the input stage. Specifically:

$$v_i = \frac{10{,}000}{10{,}000 + 2{,}000} v_s$$

Where v_s is the

Hence the actual voltage present at v_i is only 83.3% of the source voltage, i.e. a 16.7% error.

d) This can be remedied by including a unity buffer which has an input impedance close to that of the op-amp itself and a low output impedance (10s of Ω). The circuit diagram is:



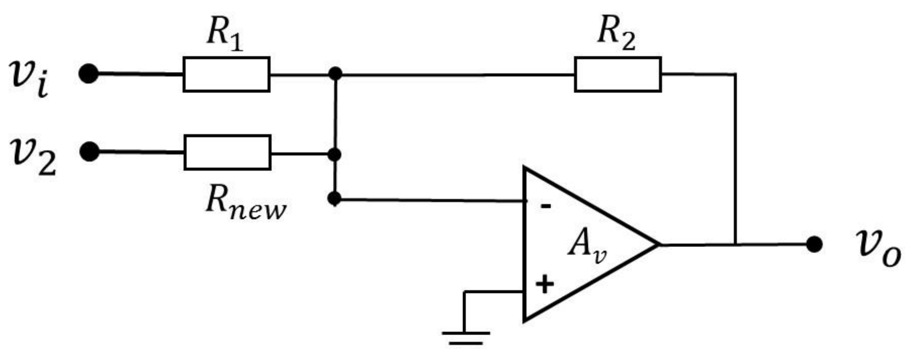
[Important to get correct polarity of input terminals]

e) With the finite op-amp gain:

$$\frac{v_o}{v_i} = -\frac{\frac{R_2}{R_1 + R_2}}{\frac{1}{A_v} + \frac{R_1}{R_1 + R_2}} = 99.8$$

i.e. 0.2% change in effective gain.

f) The modified circuit (which is in effect a summer) is:



Assuming R_2 remains fixed at 1MΩ, then the new resistor is 20kΩ to give a gain of -50.