

EEE 6212

Semiconductor Materials

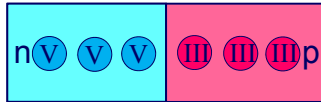
Lecture 21: pn-junctions and diodes

Lecture 21: pn-junction & diodes

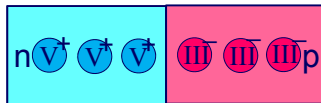
- formation of pn-junction
- solution of Poisson's equation for a pn-junction
- built-in electric field and bias
- electrical operation under forward/reverse bias
- diode characteristics
- types of diodes: rectifiers, LEDs, Zener diodes, RTDs
- principle of bipolar junction transistors (BJTs)

Formation of pn-junction

- contact two differently doped regions



- activate dopant atoms by annealing so they occupy lattice sites in tetrahedral coordination

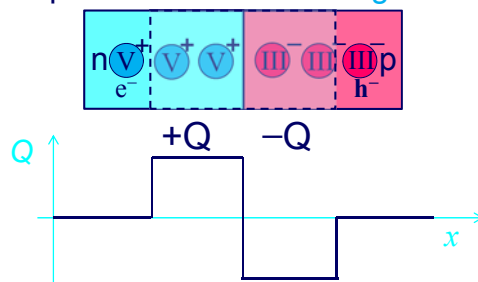


- mobile charge carriers diffuse across boundary, recombine and form a thin depletion region



Formation of pn-junction

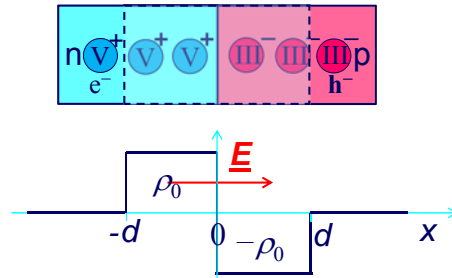
- ionised dopants create local charge density



- within the depletion, there are no free carriers any more: the depletion region is intrinsic and does not conduct



Poisson equation for pn-junction

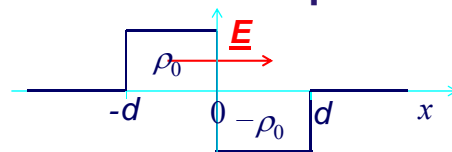


- consider step function of charge density:

$$\rho = \begin{cases} +\rho_0 & \text{for } -d < x < 0, \\ -\rho_0 & \text{for } 0 < x < d \end{cases}$$

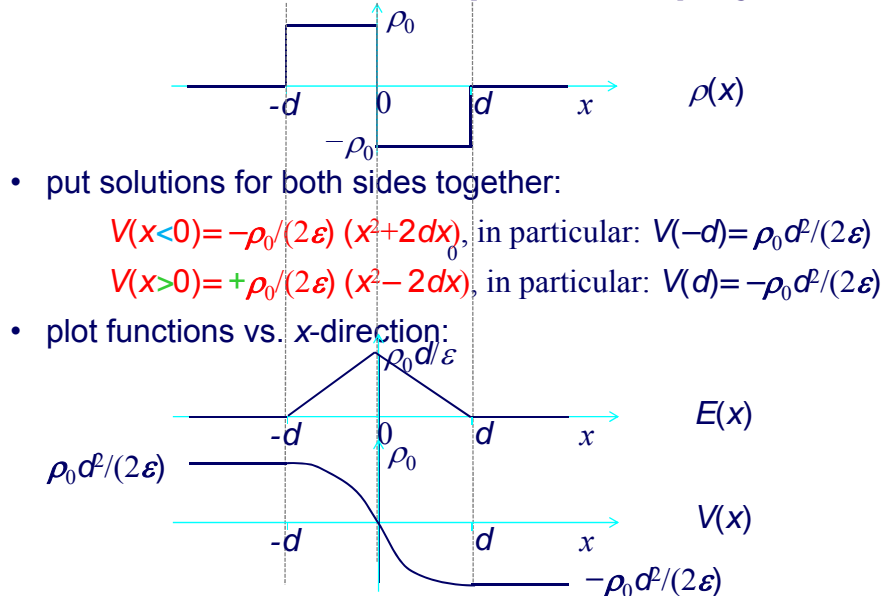
- solve Poisson equation $\nabla^2 V = \text{div grad } V = -\text{div } \underline{E} = -\rho/\epsilon$

Poisson equation for pn-junction



- solve Poisson equation in 1D (x-direction): $\partial^2 V / \partial x^2 = -\rho_0 / \epsilon$ for left (n) side
 - double integration yields equation with 3 constants:
 $V(x) = c_1 x^2 + c_2 x + c_3$
 - then: $-E(x) = \partial V / \partial x = 2c_1 x + c_2$ and $\partial^2 V / \partial x^2 = 2c_1 \rightarrow c_1 = -\rho_0 / (2\epsilon)$
 - boundary condition: @ $x=0$: $V(0)=0 \rightarrow c_3=0$
@ $x=-d$: $E(-d)=0 \rightarrow -2c_1(-d) - c_2 = 0$
 $\rightarrow c_2 = -\rho_0 d / \epsilon$
- $\rightarrow V(x < 0) = -\rho_0 / (2\epsilon) x^2 - \rho_0 d / \epsilon x = -\rho_0 / (2\epsilon) (x^2 + 2dx)$

Poisson equation for pn-junction

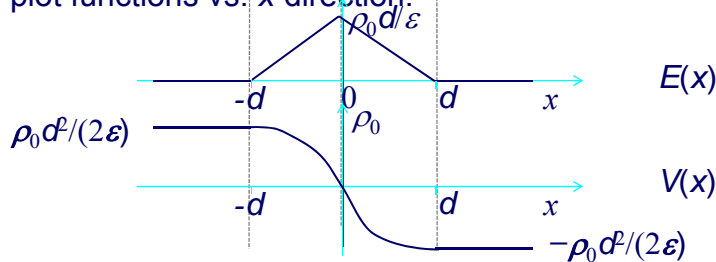


- put solutions for both sides together:

$$V(x < 0) = -\rho_0 / (2\epsilon) (x^2 + 2dx), \text{ in particular: } V(-d) = \rho_0 d^2 / (2\epsilon)$$

$$V(x > 0) = +\rho_0 / (2\epsilon) (x^2 - 2dx), \text{ in particular: } V(d) = -\rho_0 d^2 / (2\epsilon)$$

- plot functions vs. x-direction:



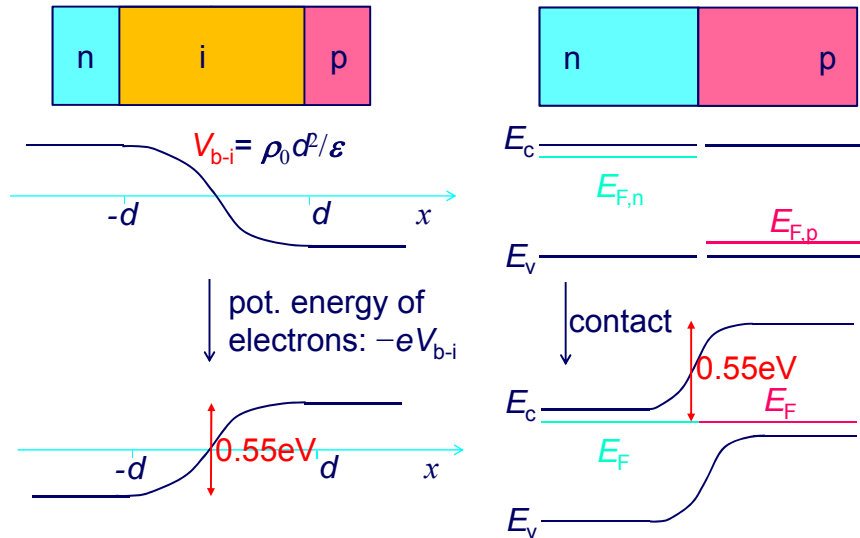
built-in potential and electric field

- some typical numbers:
- Si, $\epsilon_r = 11$, doping of 1ppm, 2/3 of dopants activated, $d = 90\text{nm}$
- $\rho_0 = 2/3 * 10^{-6} * ne = 2/3 * 10^{-6} * 8/a^3 * e = 5335 \text{ C/m}^3$
- $\epsilon = \epsilon_0 \epsilon_r = 8.8542 * 10^{-12} \text{ As/(Vm)} * 11$
- $E_{\text{max}} = \rho_0 d / \epsilon = 5.5 * 10^6 \text{ V/m}$
- $V_{\text{b-i}} = V(-d) - V(d) = \rho_0 d^2 / \epsilon = 0.55 \text{ V}$

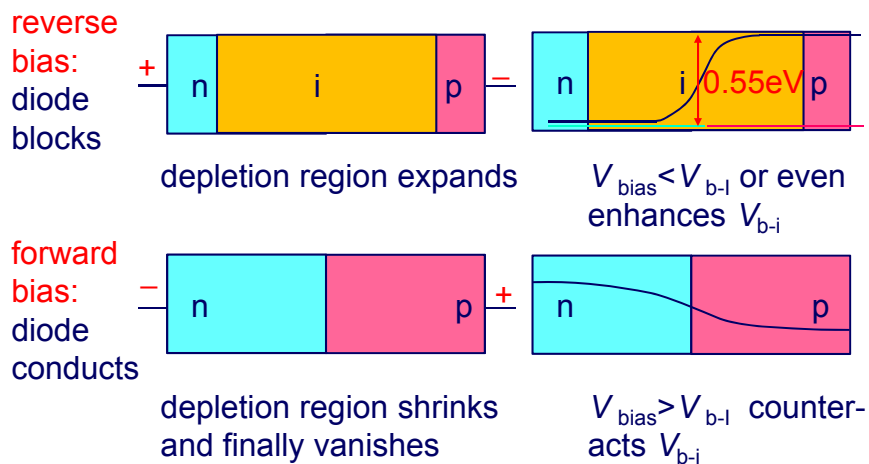
This is the **built-in potential across the pn-junction**.

For the diode to conduct, the applied voltage (**bias**) **needs to at least compensate this built-in potential**.

built-in potential and Fermi-level



qualitative diode behaviour



quantitative diode behaviour

- thermodynamic calculation of built-in potential: consider current densities due to both drift and diffusion on both sides:

$$j_n = ne\mu_e E + eD_n \frac{\partial n(x)}{\partial x}$$

$$j_p = pe\mu_p E - eD_p \frac{\partial p(x)}{\partial x}$$

- in equilibrium, there is no charge motion any more: $j_n = j_p = 0$

$$\rightarrow E = -D_n / (n\mu_e) \frac{\partial n(x)}{\partial x}$$

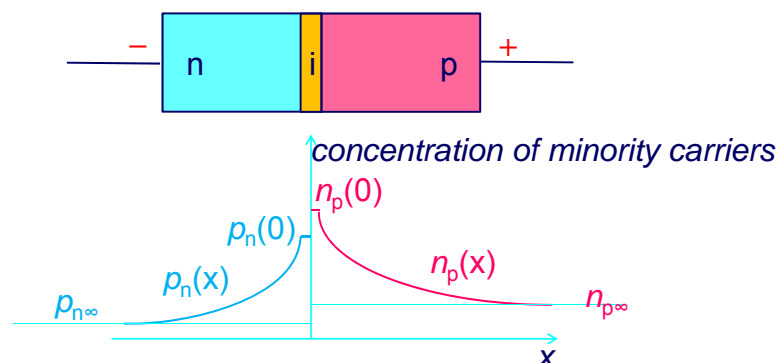
$$E = D_p / (p\mu_p) \frac{\partial p(x)}{\partial x}$$

$$\begin{aligned} \rightarrow V_{b-i} &= -\int_{-d}^d E_x dx = -\int_{-d}^d \left[\frac{D_n}{n\mu_e} \frac{\partial n(x)}{\partial x} \right] dx = \frac{D_n}{\mu_e} \int_{n_p}^{n_n} \frac{1}{n} dn(x) \\ &= \frac{D_n}{\mu_e} \ln(n_n/n_p) \\ &\approx kT/e \ln(N_D N_A / n_i^2) \end{aligned}$$

For silicon with $N_A = N_D = 5 \times 10^{15} \text{ cm}^{-3}$, $n_i = p_i \approx 10^{10} \text{ cm}^{-3}$, room temp.:

$$V_{b-i} \approx 0.026V \ln(25 \times 10^{10}) \approx 0.68V$$

diode characteristics



exponential decay in p-region: $n_p(x) = n_{p\infty} + [n_p(0) - n_{p\infty}] \exp(-x/L)$

exponential decay in n-region: $p_n(x) = p_{n\infty} + [p_n(0) - p_{n\infty}] \exp(x/L)$

approximations: applied voltage (bias) drops almost exclusively across depletion region and Maxwell-Boltzmann distribution

$$n_n/n_p = p_p/p_n = \exp[eV_{b-i}/(kT)] \text{ is valid}$$

diode characteristics

exponential decay in p-region: $n_p(x) = n_{p\infty} + [n_p(0) - n_{p\infty}] \exp(-x/L_n)$

exponential decay in n-region: $p_n(x) = p_{n\infty} + [p_n(0) - p_{n\infty}] \exp(x/L_p)$

approximations: applied voltage (bias) drops almost exclusively across depletion region and Maxwell-Boltzmann distribution

$n_n/n_p = p_p/p_n = \exp[eV_{b-i}/(kT)]$ is valid

-> $n_n = n_p \exp[eV_{b-i}/(kT)] \approx n_p(0) \exp(-x/L) \exp[eV_{b-i}/(kT)]$

$\approx n_p(0) \exp[e(V_{b-i} - V_d)/(kT)]$ where V_d is the applied bias

-> diffusion currents in depletion region:

$$j_n|_{x=0} = eD_n \partial n_p / \partial x|_{x=0} = eD_n n_p(0)/L_n [\exp(eV_d/(kT)) - 1]$$

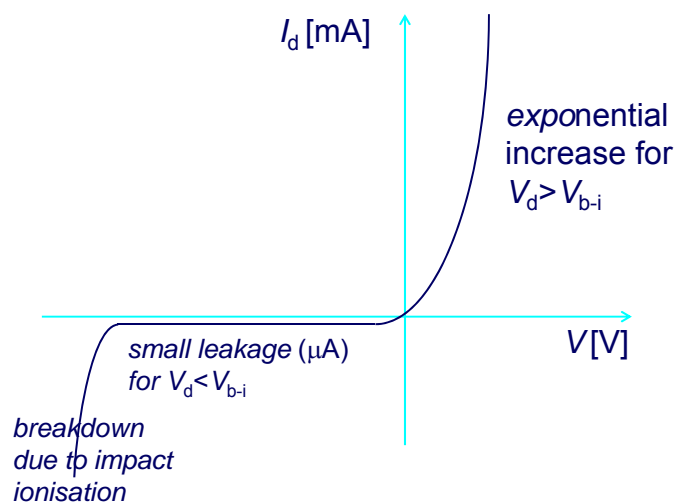
$$j_p|_{x=0} = -eD_p \partial p_n / \partial x|_{x=0} = eD_p p_n(0)/L_p [\exp(eV_d/(kT)) - 1]$$

-> total diode current through area A is $I_d = A(j_n + j_p)$, hence

$$I_d = I_S [\exp(eV_d/(kT)) - 1] \text{ where } I_S = eA[D_p p_n(0)/L_p + D_n n_p(0)/L_n]$$

diode characteristics

$I_d = I_S [\exp(eV_d/(kT)) - 1]$ is known as **Shockley's diode equation**.



types of diodes and applications

- **rectifying** diode: blocks for $V < V_{b-i}$ and conducts for $V > V_{b-i}$
- **biasing** diode: yields voltage drop of V_{b-i} ($\sim 0.6V$ for silicon)
- **LEDs**: light-emitting diodes when a direct band-gap semiconductor pn-junction is **forward biased with $eV > E_g$** so electrons and holes are injected from opposite ends and recombine near the contact area, creating photons (light)
- **Zener** diodes: **reverse biased**, yield constant voltage drop independent of current flow
- **APDs**: avalanche photodiodes are operated under **very strong reverse bias** (so normally only leakage current); if a high-energy photon or X-ray strikes, it produces e-h pairs that can be separated in the field and due to their high energy produce an avalanche of further charge carriers (\rightarrow high current pulse)

principle of bipolar junction transistor (BJT)

principle: connect 2 diodes with same doping back-to-back so you get a 3-terminal device and operate **one diode just under forward bias (base-emitter, BE)** and **the other under reverse bias (base-collector, BC)**

nnp: $V_{BE} \sim 0.7V$, $V_{CB} \gg 1V$

npn: $V_{BE} \sim -0.7V$, $V_{CB} \ll -1V$

