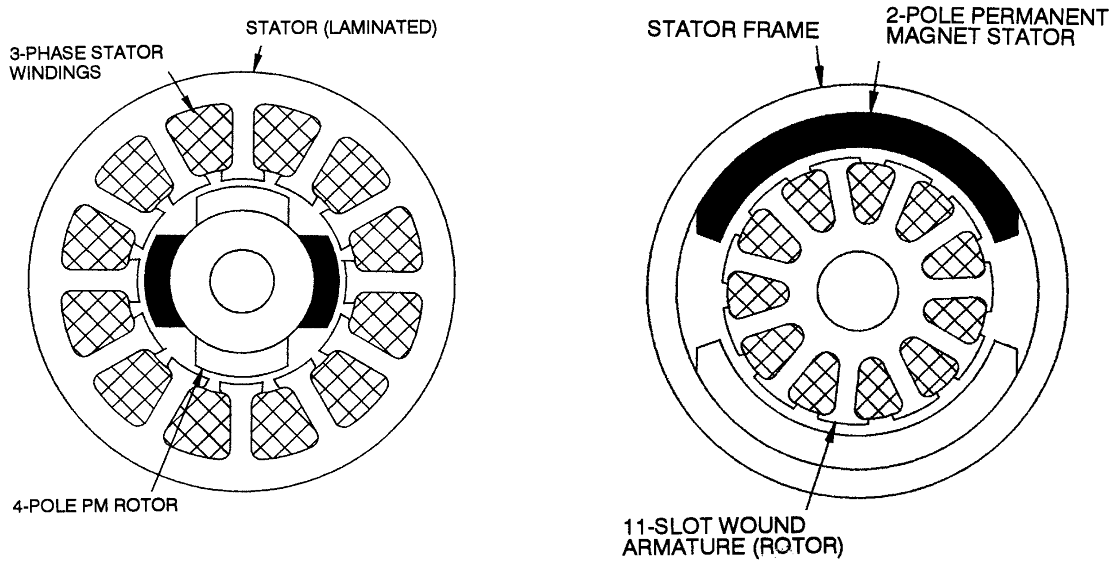
3. Synchronous Permanent Magnet Motor Drives

Synchronous permanent magnet motors provide a unique set of characteristics which make it particularly attractive to servo applications. The use of permanent magnet to generate substantial air gap magnetic flux without external excitation results in unsurpassed efficiency. Such efficiency advantages are becoming increasing valuable in many applications. This chapter discusses the basic principles of operation of synchronous permanent magnet motors and the associated control techniques for synchronous PM motor servo drives.

3.1 Synchronous Permanent Magnet Motors

Fig. 3.1(a) shows the cross-section of a typical synchronous PM motor. The magnets are on the rotating rotor. Brushes and commutator, which are the essential parts of a dc motor, is not required. For this reason synchronous PM motor is also known as brushless PM motor. Compared to a dc PM motor, which is also shown in Fig. 3.1(b), the brushless PM motor has the following advantages:



(a) brushless PM motor (b) Brushed PM dc motor

Fig. 3.1 PM motors — brushless and brushed

(a) No brushes

- Brush maintenance not required
- Brushes produce low frequency R.F. interference which would be a problem to some applications
- Brush sparking could provide a danger in flammable environments
- Brush dust can get into bearing and clog slots in commutator causing flashover.
- Brush wear dependent on environment (need different brush grades dependent on operating climates

(b) Armature winding on stator

- More area is available for windings

- Heat path to outside is easier so one can get more power out of the same frame size for a given temperature rise

(c) Absence of commutator & brush gear

- Shorter rotor --- less distance between bearings gives stiffer rotor
- Lower friction and lower rotor inertia

(d) Higher speed

- Brushed motor speed is limited by brushes.
- Brushless PM motor speed is only limited by magnet retention on the rotor. For high speed motors, a rotor is encased in a stainless steel can to keep magnets

(e) Quieter

- No audio noise from the commutator

All these advantages make brushless PM motors being increasingly used in a variety of applications. However, these advantage are not gained without a price:

(a) Need for rotor position sensing and electronic commutation
(b) Increased complexity of electronic controller

There are two basic types of brushless PM motors (i) brushless DC and (ii) brushleess AC. They differ in back-emf waveforms, and as a result, the way in which they are controlled. For most servo applications, brushless AC PM motors with sinusoidal back-emf are more commonly used as they provide smooth torque and superior performance. The following discussion therefore will focus on brushless AC PM motors.

3.2 Magnetic motive force and field of 3-phase distributed windings

As shown in Fig. 3.1, in a typical distributed 3-phase winding, the conductors are laid in the slots within the stator lamination and the arrangement will have more than 1 slot per pole per phase. For simplicity, consider a basic 3-phase, 2 pole winding, as shown in Fig. 3.2

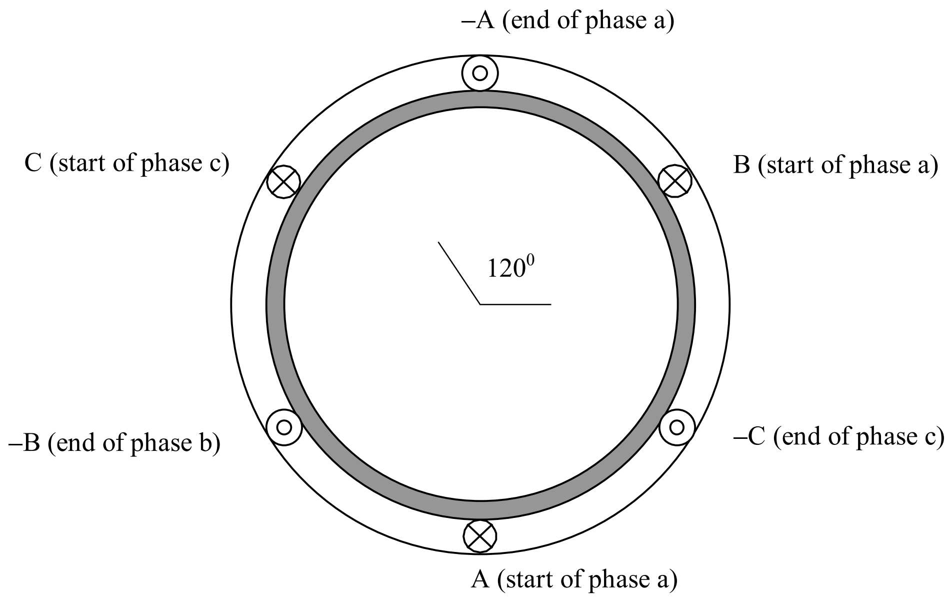


Fig. 3.2 Schematic of a basic 3-phase, 6-slot winding

Each phase has two slots spanning 180^0, and three phases are displaced symmetrically, i.e. 120^0 from each other. Now consider a field distribution created by a single phase winding, e.g., phase a, shown in Fig. 3.3

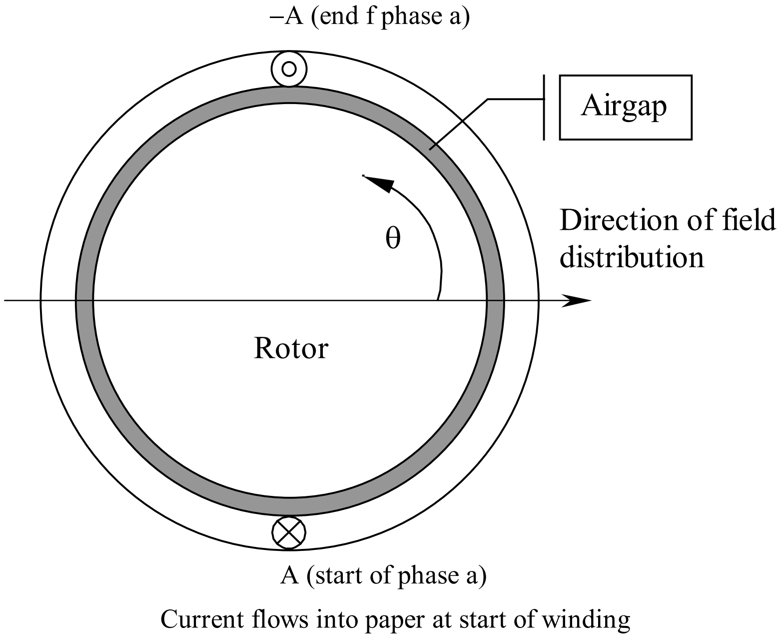


Fig. 3.3 Schematic of a basic 3-phase, 6-slot winding

We will define the current as one which flows into the start of the winding. The current in phase a creates a magnetic field like a solenoid coil whose direction coincides with the winding axis. To describe the spatial distribution of the magnetic field produced by the current, a spatial angular position is defined as a positive increasing angle from the field axis of phase a. Given a single slot winding, the magnetic motive force and field distribution as a function of the angular position θ may be shown in Fig, 3.4 for an "unrolled" motor.

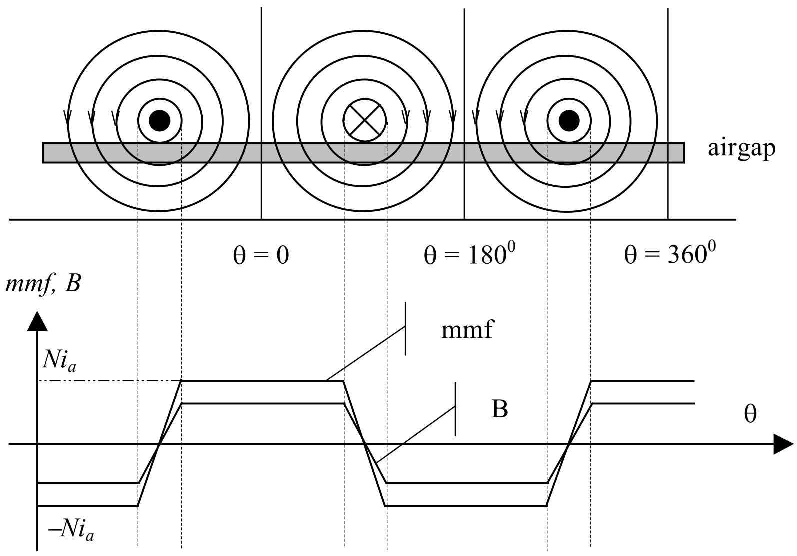
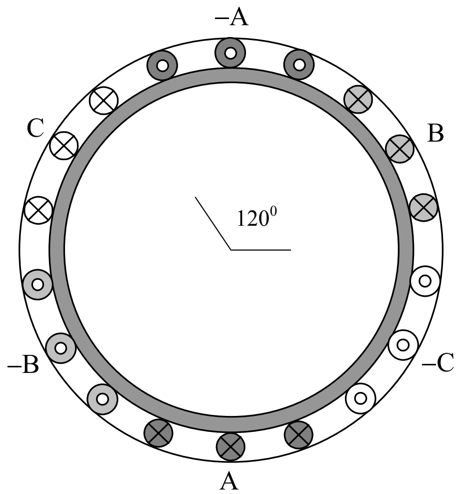


Fig. 3.4 mmf and magnetic field distribution in airgap

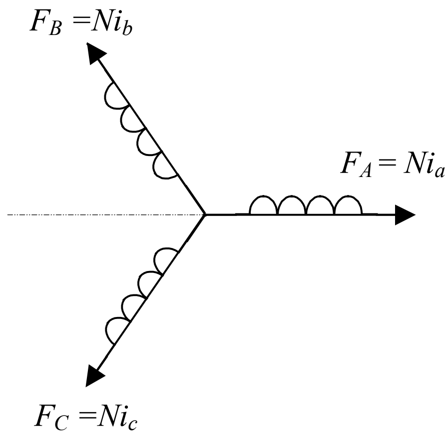
N is the number of turns of the phase windings.

The idea behind distributing the phase winding, Fig. 3.5 (a), is to make the field distribution appear more sinusoidal, e.g. say, we were to have 3 conductors per pole per phase over 60^0, i.e. the use of 60^0 phase belt with a number of slots forming the winding gives a more sinusoidal mmf and field distribution. Thus phase a current i_a creates mmf distribution given by:

$$F_A(\theta) = Ni_a\cos\theta \tag{3.1}$$



(a) Distributed windings



(b) mmf vectors

Fig. 3.5 Distributed winding and vector representation of mmfs

This spatial distribution can be represented by a vector having the magnitude Ni_a and the direction of the winding axis, as shown in Fig. 3.5(b). The magnitude of the field (neglecting saturation) is proportional to the magnitude of mmf, and therefore to the current. Thus for phase a the airgap field is:

$$B_A(\theta) = Ki_a\cos\theta \tag{3.2}$$

where K is a constant related to the machine design

Similarly for phase b and c, which are displaced 120^0 and 240^0 relative to phase a, respectively,

$$\begin{aligned} F_B(\theta) &= Ni_b\cos(\theta - 120^0) \\ B_B(\theta) &= Ki_b\cos(\theta - 120^0) \\ F_C(\theta) &= Ni_c\cos(\theta + 120^0) \\ B_C(\theta) &= Ki_c\cos(\theta + 120^0) \end{aligned} \tag{3.3}$$

The corresponding mmf vectors are also shown in Fig. 3.5(b). Note that the above apply for a 2 pole winding. For a higher pole number we have the expressions:

$$\begin{aligned} F_A(\theta) &= Ni_a\cos(p\theta) \\ F_B(\theta) &= Ni_b\cos(p\theta - 120^0) \\ F_C(\theta) &= Ni_c\cos(p\theta + 120^0) \end{aligned} \tag{3.4}$$

where p is the number of pole pairs, and similarly for field B. For simplicity we consider here a 2-pole machine with a 3-phase balanced currents:

$$\begin{aligned} i_a(t) &= I_{pm}\sin(\omega t) \\ i_b(t) &= I_{pm}\sin(\omega t - 120^0) \\ i_c(t) &= I_{pm}\sin(\omega t + 120^0) \end{aligned} \tag{3.5}$$

The net field in the machine is the sum of contribution from each phase:

$$\begin{aligned} B_{ag} = {} & KI_{pm}\sin\omega t\cos\theta + KI_{pm}\sin(\omega t\text{-}120^0)\cos(\theta\text{-}120^0) \\ & + KI_{pm}\sin(\omega t\text{-}240^0)\cos(\theta\text{-}240^0) \end{aligned} \tag{3.6}$$

Using trigonometric identity: sinAcosB = (1/2)[sin(A+B)+sin(A-B)], we have

$$\begin{aligned} B_{ag} = 0.5KI_{pm}[& \sin(\omega t+\theta) + \sin(\omega t-\theta) + \\ & \sin(\omega t+\theta-240^0) + \sin(\omega t-\theta) + \\ & \sin(\omega t+\theta-480^0) + \sin(\omega t-\theta)] \end{aligned} \tag{3.7}$$

From the knowledge that $\sin(A) + \sin(A-120^0) + \sin(A-240^0) = 0$, Eqn. (3.7) becomes:

$$B_{ag} = 1.5KI_{pm}\sin(\omega t - \theta) \tag{3.8}$$

This expression describes a sinusoidally distributed field which rotates with the synchronous speed ω, i.e., the angular frequency of the supply. Plots of B_{ag} at different instants of t as functions of θ are shown in Fig. 3.6

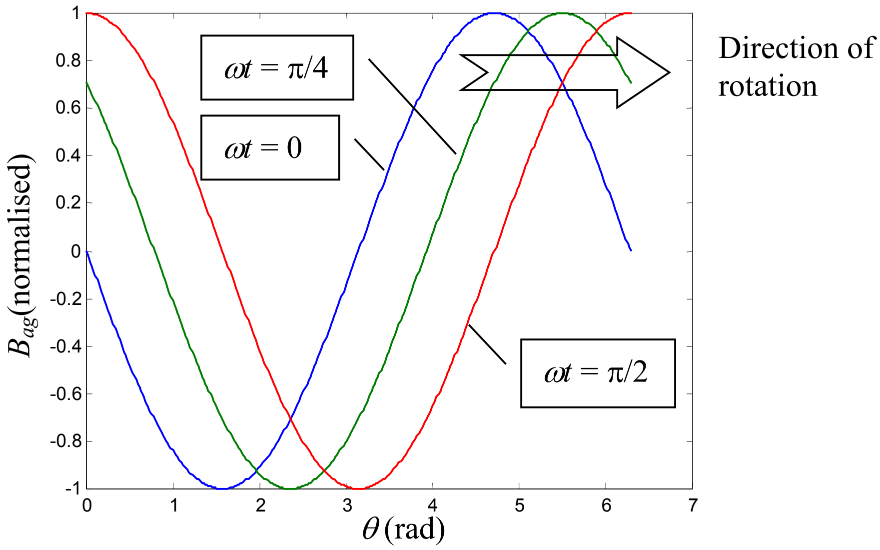


Fig. 3.6 Rotating magnetic field

3.3 Induced voltage and phasor diagram

The rotating magnetic field in the airgap will produce a flux linkage in each winding, which may be obtained by integration over the winding area. For example, for phase a, the flux linkage is given by:

$$\Psi_a = NlR_i \int_{-\pi/2}^{\pi/2} B_{ag} d\theta = NlR_i \int_{-\pi/2}^{\pi/2} 1.5KI_{pm} \sin(\omega t - \theta) d\theta \\ = L_m I_{pm} \sin \omega t \tag{3.9}$$

Where l is the axial length of the winding, R_i is the radius of the stator bore. $L_m = 3NlR_iK$ is known as the magnetising inductance of 3-phase winding.

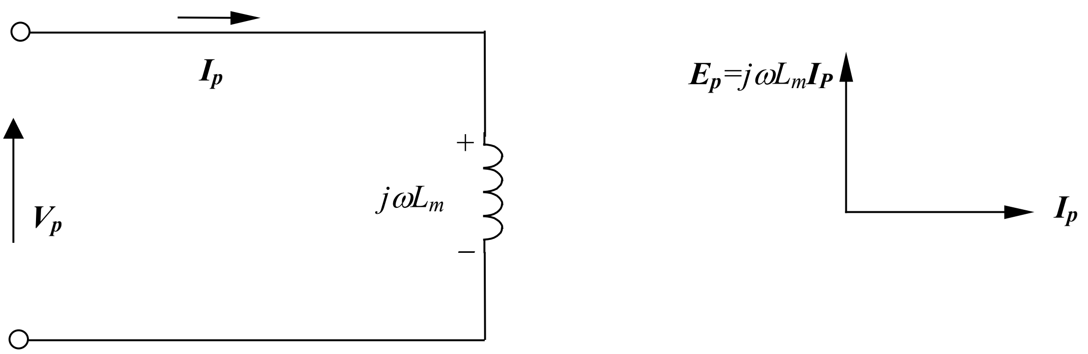
The induced voltage due to the rotating field in phase a may be found from Faraday's law:

$$e_a = \frac{d\Psi_a}{dt} = \omega L_m I_{pm} \cos \omega t \tag{3.10}$$

Similarly for phases b and c,

$$e_b = \frac{d\Psi_b}{dt} = \omega L_m I_{pm} \cos(\omega t - 120^0) \\ e_c = \frac{d\Psi_c}{dt} = \omega L_m I_{pm} \cos(\omega t - 240^0) \tag{3.11}$$

The above relationships may be represented by a per phase equivalent circuit diagram shown in Fig. 3.7(a). The phasor diagram is plotted usually at time = 0, therefore the corresponding phasor diagram is shown in Fig. 3.7 (b), note that cos(ωt) = sin(ωt+90^0). Given an idealised winding, i.e., neglecting its resistance and leakage inductance, when the 3-phase winding is connected to a sinusoidal voltage supply, a current will established in the winding. This current creates a rotating field, i.e. a winding flux linkage, which induces a voltage in the winding. If there is no other source of field, this voltage must be equal to the supply voltage V_p



(a) Equivalent circuit (b) Phasor diagram

Fig. 3.7 Equivalent circuit and phasor diagram

$V_P = \omega L_m I_{pm}$, peak flux linkage $\Psi_{pm} = L_m I_{pm} = V_p/\omega$

e.g. for a 240V rms phase winding at 50 Hz,

$\Psi_{pm} = \sqrt{2} \times 240/(2\pi 50) = 1.08 (Wb)$

3.4 Operation of brushless AC PM motors

In brushless AC PM motors, the rotor permanent magnets produce an almost sinusoidally distributed flux Φ_m in the air gap. The stator has three phases of windings, each displaced 120^0 electrical degree apart, as shown in Fig. 3.8. Assume initially the magnet (or flux) axis (referred to as d axis) coincides with phase *a* axis, the flux linkage of each phase due to permanent magnets is given by:

Fig. 3.8 Schematic of winding arrangement of a synchronous PM motor

$$\begin{aligned}\Psi_{am} &= \Psi_m \cos\theta \\ \Psi_{bm} &= \Psi_m \cos(\theta - 120^0) \\ \Psi_{cm} &= \Psi_m \cos(\theta + 120^0)\end{aligned} \tag{3.12}$$

where θ is the **electrical angle** between the d-axis and phase a winding axis and equals the number of pole pair p times the **mechanical angle** θ_m. As the rotor rotates at a synchronous speed ω_m, the flux linkages of the stator phase windings vary sinusoidally with time:

$$\begin{aligned}\Psi_{am} &= \Psi_m \cos\omega t \\ \Psi_{bm} &= \Psi_m \cos(\omega t - 120^0) \\ \Psi_{cm} &= \Psi_m \cos(\omega + 120^0)\end{aligned} \tag{3.13}$$

where $\omega = p\omega_m$ is the electrical angular speed. The induced back-emfs in the phase windings are therefore given by:

$$\begin{aligned}e_{am} &= \frac{d\Psi_{am}}{dt} = -\omega\Psi_m \sin\omega t = \omega\Psi_m \cos(\omega t + 90^0) \\ e_{bm} &= \frac{d\Psi_{bm}}{dt} = -\omega\Psi_m \sin(\omega t - 120^0) = \omega\Psi_m \cos(\omega t - 120^0 + 90^0) \\ e_{cm} &= \frac{d\Psi_{cm}}{dt} = -\omega\Psi_m \sin(\omega t + 120^0) = \omega\Psi_m \cos(\omega t + 120^0 + 90^0)\end{aligned} \tag{3.14}$$

In brushless AC PM motor drives, the stator is supplied with a set of balanced three-phase current whose frequency is controlled to be in synchronism with the rotor rotation, i.e.,

$$f = \omega/2\pi = p\omega_m/2\pi \tag{3.15}$$

The motor and supply form a balanced three-phase system which can be analysed using per phase equivalent circuit and phasor diagram. In accordance with the normal convention, the amplitudes of voltage and current phasors are represented by their rms values; the amplitudes of flux linkage phasors are represented by their peak values. The emfs and flux linkage being sinusoidal with time can be represented as phasors at ωt = zero, as shown in Fig. 3.9. Note that $\cos(\omega t) = \sin(\omega t + 90^0)$ and the flux linkage phasor is used as a reference phasor.

The magnitude of phasor $\boldsymbol{E}$ is given by

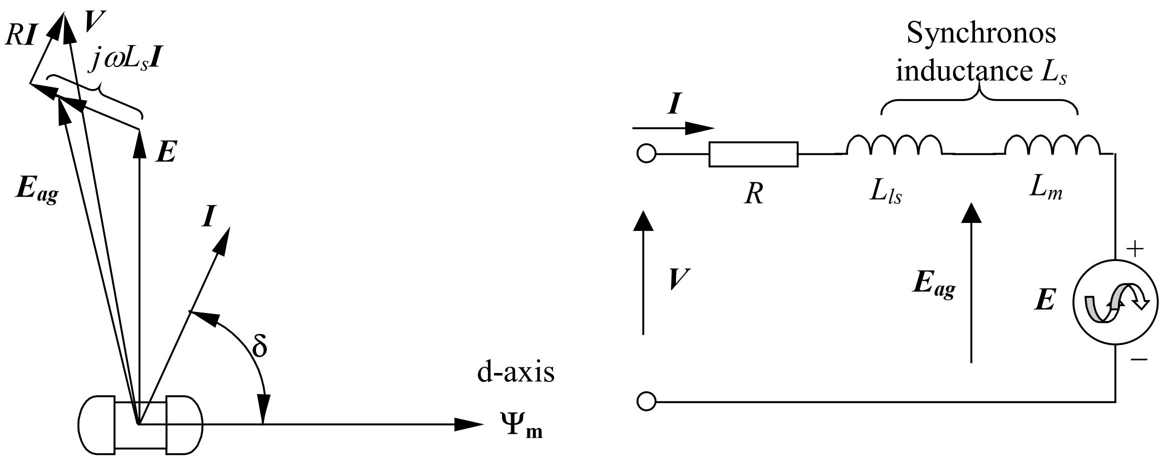
$$\boldsymbol{E} = \omega\Psi_m / \sqrt{2} \ . \tag{3.16}$$

Assuming that the fundamental component of the three-phase supply currents to be:

$$\begin{aligned} i_a &= \sqrt{2}I\sin(\omega t + \delta) \\ i_b &= \sqrt{2}I\sin(\omega t - 120^0 + \delta) \\ i_c &= \sqrt{2}I\sin(\omega + 120^0 + \delta) \end{aligned} \tag{3.17}$$

The stator current phasor can be represented as $\boldsymbol{I} = Ie^{j\delta}$ and the angle δ is defined later on as the torque angle of the motor. As has been discussed in section 3.2, the alternating currents will induce voltages in the stator windings. The corresponding induced voltage phasor is given by:

$$\boldsymbol{E_L} = j\omega L_m \boldsymbol{I} \tag{3.18}$$



(a) pasor diagram (b) equivalent circuit

Fig. 3.9 Per-phase representation of brushless AC PM motors

where $L_m = (3/2)L$ is the magnetising inductance of the motor, L being the self-inductance of the phase winding In addition, the stator currents will also produce voltage drops in the winding resistance R and leakage inductance L_{ls}, and the corresponding phasor is

$$\boldsymbol{V_{drop}} = (R + j\omega L_{ls})\boldsymbol{I} \tag{3.19}$$

The resulting phasor diagram and per phase equivalent circuit are shown in Fig. 3.9 (a) and (b), respectively. The combined effect of the air gap inductance and leakage inductance are lumped to together to form the synchronous inductance $L_s = L_m + L_{ls}$. $\boldsymbol{E_{ag}} = \boldsymbol{E} + \boldsymbol{E_L}$ is defined as air gap induced voltage phasor, which differs from the applied voltage phasor by $\boldsymbol{V_{drop}}$

From the per-phase equivalent circuit and the phasor diagram, the electrical power being converted into the mechanical power P_{em} is:

$$P_{em} = 3EI\cos(90^0\text{-}\delta) \tag{3.20}$$

and the electromagnetic torque is

$$T_{em} = P_{em}/\omega_m = 3EI\cos(90^0\text{-}\delta)/\omega_m \tag{3.21}$$

Using Eqn. (3.16)

$$T_{em} = (3p\Psi_m/\sqrt{2})I\sin\delta = k_T I\sin\delta \qquad (3.22$$

where $k_T = 3p\Psi_m/\sqrt{2}$ is a constant referred to as torque constant of brushless AC PM motor. Several observations can be made from Eqn. (3.22):

(a) The electromagnetic torque is a function of the angle between the current phasor and the rotor flux phasor (or d-axis), and reaches its maximum torque per Ampere value when the two phasors are orthogonal, i.e., $\delta = 90^0$.

(b) When the torque angle $\delta = 90^0$, the electromagnetic torque becomes:

$$T_{em} = k_T I$$

a relationship identical to dc motor, recall that the orthogonal relation between armature current and field is maintained by commutator and brushes in dc motors. This corresponds to motoring operation (positive torque).
when $\delta = -90^0$, $T_{em} = -k_T I$, negative torque results, corresponding to regeneration if the rotor speed stays in the same direction of rotation.

(c) To achieve maximum torque per Ampere operation, i.e., maximum efficiency, the current phasor should be control such that it is always orthogonal to the rotor flux (d axis), and by varying the magnitude and sign of the current phasor, four-quadrant torque control can be realised. The control technique, which implements this strategy, is known as **Field Oriented Control**.

In the previous analysis, the rotor saliency was ignored. This is valid assumption for surface-mounted PM motors.

The next question is how the current can be controlled to lie orthogonal to the rotor flux? The answer to this question calls for a mathematical framework, which will be discussed in the next section.