EEE6440
Advanced Signal Processing (ASP)

- Digital Filters I:
  - Analogue vs. Digital
  - Filter basics
  - Time/Frequency domain parameters
  - A framework for filter design

- Reference
  - Proakis: Ch8.1

- MATLAB
  - Commands: filter, freqz, pz2tf, tf2pz, fft, plot, help

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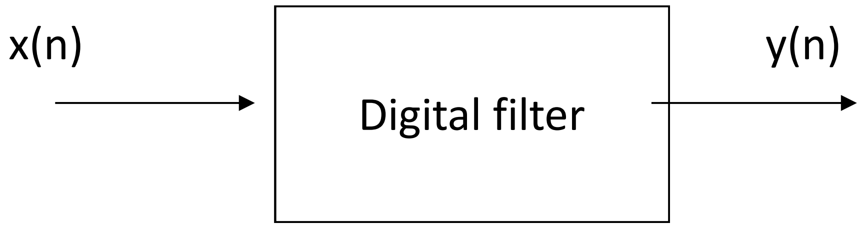
- Digital filters are used for two general purposes:
  - (1) **separation** of signals that have been combined,
    - E.g., foetus heartbeat monitoring
  - (2) **restoration** of signals that have been distorted in some way.
    - E.g., Denoising an image

- Analogue vs. Digital filters
  - Advantages of Digital filters?
  - Disadvantages of digital filters?



$$y(n) \equiv T[x(n)]$$

- Both x(n) and y(n) are digital signals.

- Usually a filter's input and output signals are in the *time domain*. (Due to A to D conversion process – sampling at equal intervals of time)

- The second most common way of sampling is at equal intervals in *space*. E.g., imaging

- Difference Equation

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \tag{1.1}$$

- Convolution

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k) \tag{1.2}$$

$$Y(z) = H(z) X(z) \tag{1.3}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}} \tag{1.4}$$

- **The impulse response**
- **The step response**
- **The frequency response.**
- Each of these responses contains complete information about the filter, but in a different form.
- If one of the three is specified, the other two are fixed and can be directly calculated.
- All three of these representations are important, because they describe how the filter will react under different circumstances.

- **The impulse response**

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$y(n) = h(n) \text{ when } x(n) = \delta(0)$$

  – The output when the input is an impulse

- **The step response**
  – The output when the input is a *step* (also called an *edge*, and an *edge response*)
  – It is the integral of the impulse response
  – Two ways to compute.

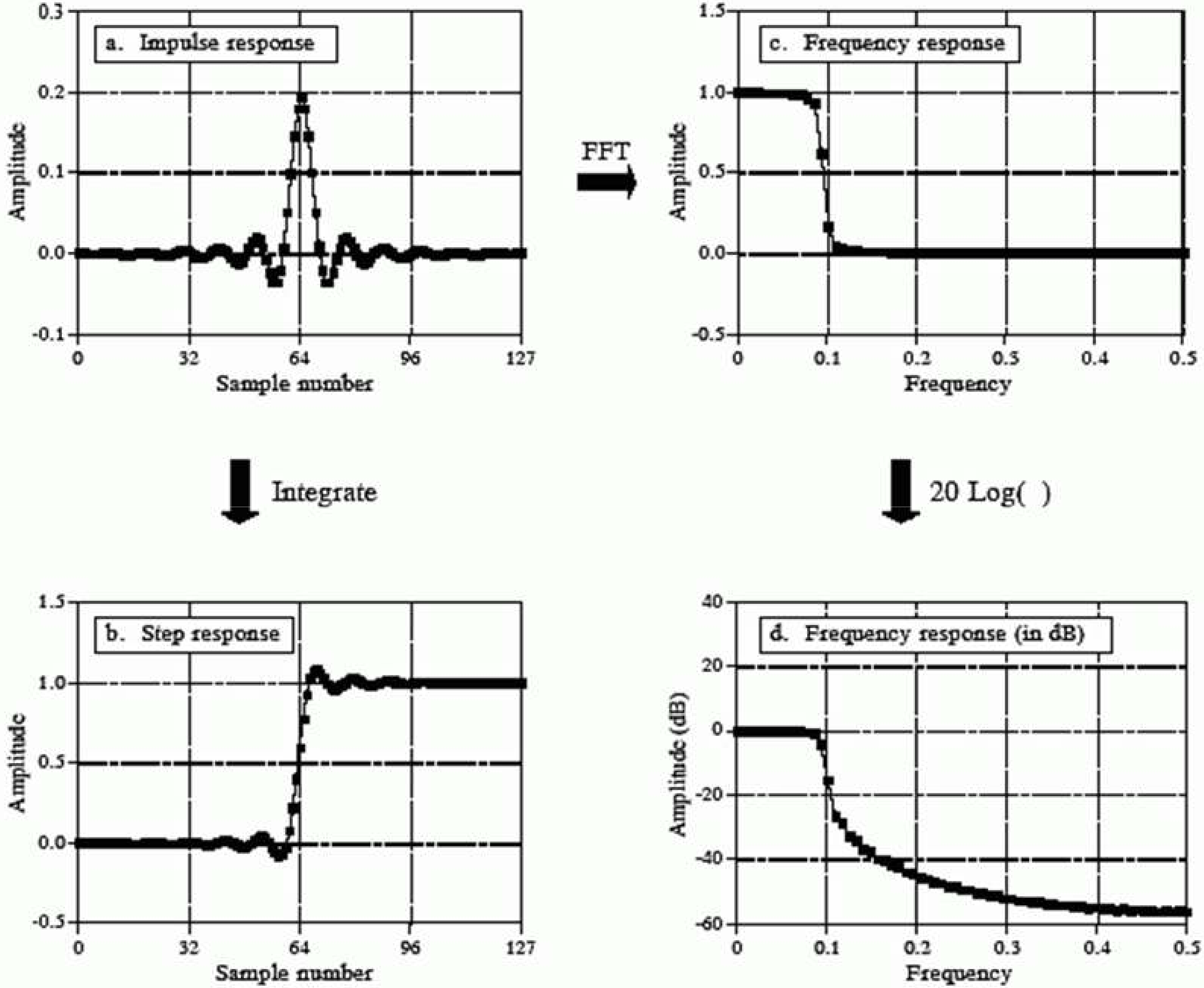
- **Frequency response**
  - Denoted by *H(jω)*

$$H(j\omega) = H(z)\big|_{z=e^{j\omega}} = \frac{b_0 + b_1 e^{-j\omega} + \ldots + b_M e^{-jM\omega}}{1 + a_1 e^{-j\omega} + \ldots + a_N e^{-jN\omega}}$$

  - Magnitude and Phase responses

$$H(j\omega) = |H(j\omega)| e^{-j\Phi(\omega)} = |H(j\omega)| \angle H(j\omega)$$

$$|H(j\omega)| = |H(-j\omega)|$$
$$\Phi(\omega) = -\Phi(-\omega)$$


a. Impulse response
Amplitude
Sample number
FFT
c. Frequency response
Frequency
Integrate
20 Log( )
b. Step response
d. Frequency response (in dB)
Amplitude (dB)

Exercise: 2.1

- Consider the difference equation

  $y(n) = 0.5y(n-1) + x(n) - x(n-2)$

- Find out
  - Impulse response
  - Step response
  - Frequency response

Filter Implementation

- The most straightforward way to implement a digital filter is by **convolving** the input signal with the digital filter's *impulse response*.
- All possible linear filters can be made in this manner.
- When the *impulse response* is used in this way, filter designers give it a special name: the **filter kernel**.
- There is also another way to make digital filters, called **recursion**.

- When a filter is implemented by convolution, each sample in the output is calculated by *weighting* the samples in the input, and adding them together.

- Recursive filters are an extension of this, using previously calculated values from the *output*, besides points from the *input*.

- Instead of using a filter kernel, recursive filters are defined by a set of **recursion coefficients**.

- Remember, all linear filters have an impulse response, even if you don't use it to implement the filter.

- To find the impulse response of a recursive filter, simply feed in an impulse, and see what comes out.
- The impulse responses of recursive filters are composed of sinusoids that exponentially decay in amplitude.
- In principle, this makes their impulse responses *infinitely long*. However, the amplitude eventually drops below the round-off noise of the system, and the remaining samples can be ignored.
- Because of this characteristic, recursive filters are also called **Infinite Impulse Response or IIR** filters.
- In comparison, filters carried out by convolution are called **Finite Impulse Response or FIR** filters.

IIR Filters

- Duration length of h(n) is infinite. That means h(n)≠0 when n→∞.
- IIR filters are recursive. That means at least one coefficient $a_k \neq 0$.
- Causal IIR filters can be represented as

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

- IIR filters have at least one nonzero finite pole

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}} = \frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{\left(1 - p_1 z^{-1}\right)\ldots\left(1 + p_N z^{-N}\right)}$$

FIR Filters

- Duration length of h(n) is finite. Length is M+1 < ∞. That means h(n)=0 when n→∞.
- FIR filters are non-recursive. That means all coefficients a_k=0 when k>0 and a_0=1.
- Causal FIR filters can be represented as

$$y(n)=\sum_{k=0}^{M}h(k)x(n-k)=\sum_{k=0}^{M}b_k x(n-k)$$

- FIR filters are all zero filters. That means all poles are either 0 finite pole

$$H(z)=b_0+b_1z^{-1}+\ldots+b_M z^{-M}$$

Information in signals

- There are many ways that information can be contained in a signal.
  - information represented in the time domain,
  - information represented in the frequency domain
- Information represented in **the time domain** describes when something occurs and what the amplitude of the occurrence is.
- Each sample contains information that is interpretable without reference to any other sample. Even if you have only one sample from this signal, you still know something about what you are measuring.
- This is the simplest way for information to be contained in a signal.
- The *step response* describes how information represented in the *time domain* is being modified by the system

Information in signals

- In contrast, information represented in the frequency domain is more indirect.
- Many things in our universe show periodic motion. By measuring the frequency, phase, and amplitude of this periodic motion, information can often be obtained about the system producing the motion.
- A single sample, in itself, contains no information about the periodic motion. The information is contained in the *relationship* between many points in the signal.
- The *frequency response* shows how information represented in the *frequency domain* is being changed.