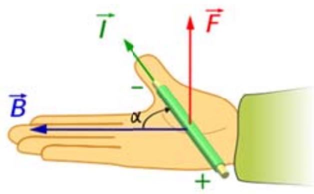
Answers to questions

Answers to question 1:

(a), If L is the rotor axial length and D is the rotor outer diameter, the Lorentz Force of:

1 conductor carrying current I (A): $F = BLI$,
2 conductors (1 turn) carrying current I (A): $F = 2BLI$,
N turns carrying current I (A): $F = 2NBLI$.



The Resultant torque for a rotor diameter D is:

$$T = F\times D/2 = 2(NI)BL\times D/2$$

If we define a new variable:

$$Q = \frac{2NI}{\pi D}$$

Then we can obtain the resultant torque:

$$\boxed{T = \frac{\pi}{2}D^2LBQ}$$

The rotor volume is $V = \frac{\pi D^2 L}{4}$, therefore the specific torque (or torque density) is

$$\frac{T}{V} = 2BQ$$

(b), In circumferentially magnetised magnet rotor, two magnets are in parallel excitation (i.e. $\Phi_g = 2\Phi_m$ and $B_gA_g = 2B_mA_m$), while in one closed-loop of magnetic circuit, the flux passes the magnet once and the airgap twice. Hence, if A_g = area of airgap surface/pole and A_m = area of magnet/pole, the actual airgap area in the magnetic circuit is only 0.5 Ag, while the actual airgap length is $2l_g$. Therefore, the expression of airgap field should be modified as:

$$B_g = \frac{B_r}{\frac{0.5A_g}{A_m} + u_r\frac{2l_g}{l_m}}$$

When the pole number is high, $A_g << A_m$ and hence the flux focusing effect is significant, it is possible $B_g > B_r$.

(c), From Gauss law and Ampere's law, without external MMF:

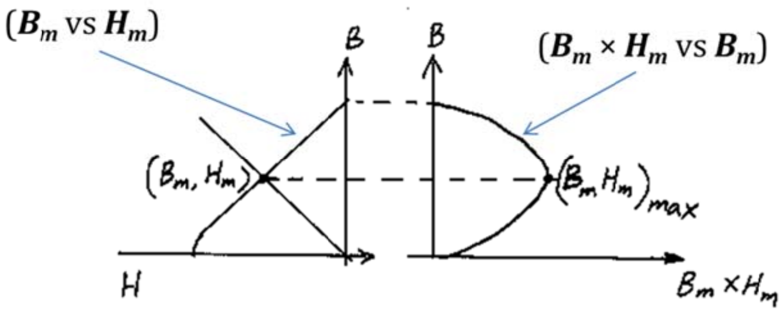
$$H_m l_m = -H_g l_g \text{ and } B_m A_m = B_g A_g$$

$$(B_m H_m)(A_m l_m) = -\frac{B_g^2}{\mu_0}(A_g l_g)$$

Or

$$(B_m H_m)V_m = -\frac{B_g^2}{\mu_0}V_g$$

where $V_m = A_m l_m$ (m^3) is the magnet volume while $V_g = A_g l_g$ (m^3) is the air-gap volume. A_m and l_m are cross section area and length of magnet, A_g and l_g are cross section area and length of airgap.



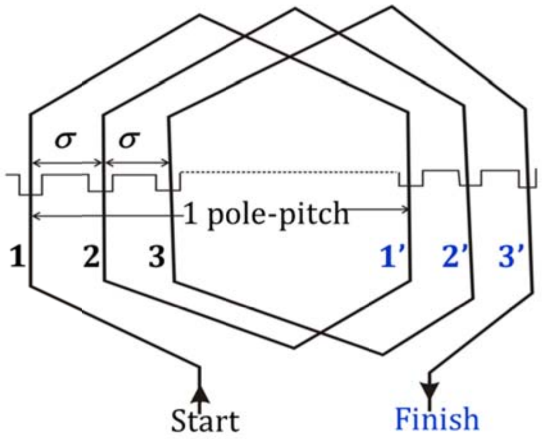
- The higher the energy product, the smaller the magnet volume,
- To produce a given flux density B_g in a gap of volume V_g, a min. volume of magnet is required when the magnet is designed to work at the point on its B-H curve when $B_m H_m$ is a max.

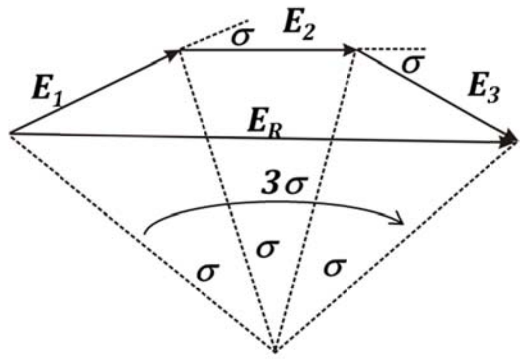
It is necessary to consider the demagnetization withstand – in general, this open-circuit magnet working point will be too low.

(d), The iron losses can be divided into hysteresis and eddy current losses. Hysteresis loss is proportional to $B^{1.5}f$, whilst eddy current loss is proportional to $\frac{B^2 d^2 f^2}{\rho}$, where B is the flux density in the iron, d and ρ are thickness and resistivity of lamination, f is the operating frequency, $f = \frac{\omega_r p}{2\pi}$ [ω_r = rad/s, p = n° of pole pair]. Both flux density and frequency affect hysteresis and eddy current losses. The material type can be another factor affecting the iron losses. For high frequency operation (high speed and/or high pole number), B may need to be reduced, and very thin laminations, alternatively powdered or ferrite cores may be employed.

<u>Answers to question 2:</u>

(a) The layout of winding and the EMF vectors of coils are shown:





$$E_R = E_1 + E_2 + E_3$$

Assuming we have m = 3 coils per phase, and $|E_1| = |E_2| = |E_3| = |E_m|$ (all the coils are identical).

Then, from the construction ($\boldsymbol{E_m} = \boldsymbol{E_1}$), we have

$$E_m = 2r\sin\frac{\sigma}{2} \quad \text{and} \quad E_R = 2r\sin\frac{m\sigma}{2}$$

The arithmetic sum of all coil EMFs: $mE_m = m2r\sin\frac{\sigma}{2}$

However, the vector sum of all coil EMFs: $E_R = 2r\sin\frac{m\sigma}{2}$

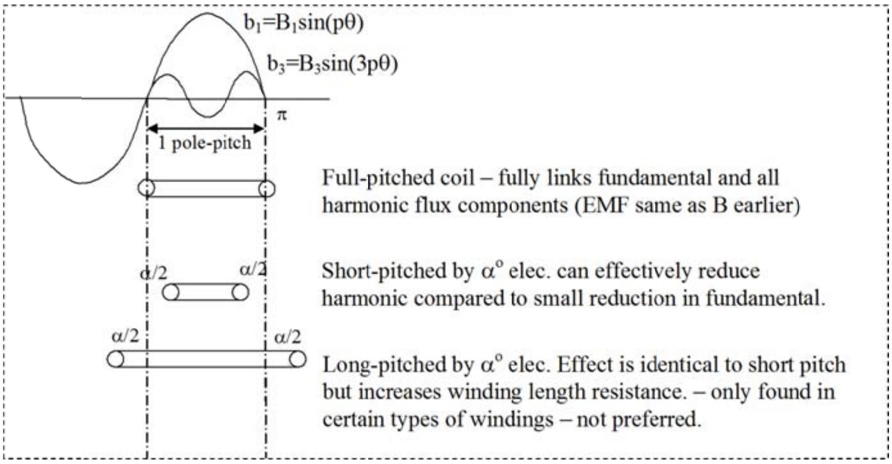
Therefore, the distribution factor for the fundamental is:

$$\boxed{k_d = \frac{effective\ induced\ emf}{arithmetic\ induced\ emf} = \frac{E_R}{mE_m} = \frac{\sin\frac{m\sigma}{2}}{m\sin\frac{\sigma}{2}}}$$

By using the similar approach, the distribution factor for the nth harmonic is:

$$\boxed{k_{dn} = \frac{\sin\frac{mn\sigma}{2}}{m\sin\frac{n\sigma}{2}}}$$

The pitch factor then can be calculated based on the following graph:



k_p is defined as: $$\frac{effective\ EMF}{EMF\ of\ \ full-pitch\ coil} \propto \frac{effective\ \ flux\ linkage}{flux\ linkage\ of\ \ full\ \ pitch\ coil} = \frac{\Psi_s}{\Psi_F}$$

For a short pitch coil:

$$\Psi_s = \int_{\alpha/2}^{\pi-\alpha/2} \hat{B}\sin\theta d\theta = 2\hat{B}\cos\frac{\alpha}{2}$$

And for full pitch coil:

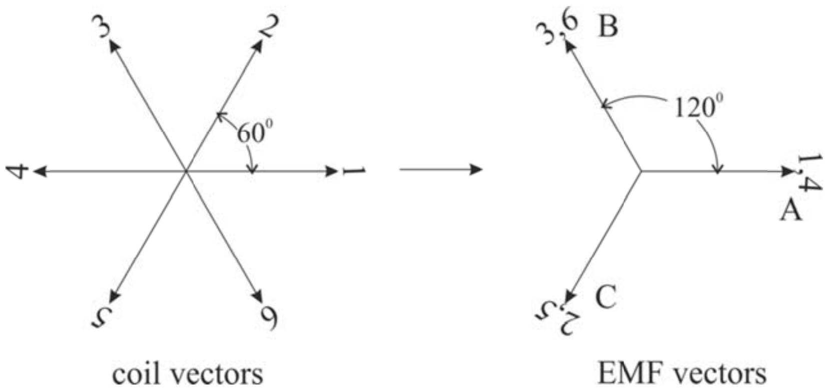
$$\Psi_F = \int_0^{\pi} \hat{B}\sin\theta d\theta = 2\hat{B}$$

Therefore, the pitch factor is:

$$\boxed{k_p = \frac{\Psi_s}{\Psi_F} = \frac{2\hat{B}\cos\frac{\alpha}{2}}{2\hat{B}} = \cos\frac{\alpha}{2}}$$

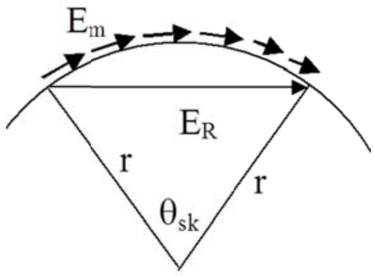
Similarly, the pitch factor for long pitch is: $k_p = \cos\frac{\alpha}{2}$

For a 12-slot/8-pole single layer surface mounted permanent magnet machine which has non-overlapping concentrated winding, there are 6 coils allow us to establish a 3-phase winding structure. This means each phase will only have 2 coils. The coil vector and coil EMF vector of this machine are the same and shown in the following graph:



(b), If the skew angle is θsk and the winding consists of m element as shown in the following graph, then we have:

$$k_{sk} = \frac{vector\ sum\ E_R}{arithmetic\ sum\ mE_m} = \frac{chord\ of\ circle}{arc\ of\ circle} = \frac{2r\sin\frac{\theta_{sk}}{2}}{r\theta_{sk}}$$



Finally, the skew factor can be calculated by:

$$\boxed{k_{sk} = \frac{\sin\frac{\theta_{sk}}{2}}{\frac{\theta_{sk}}{2}}}$$

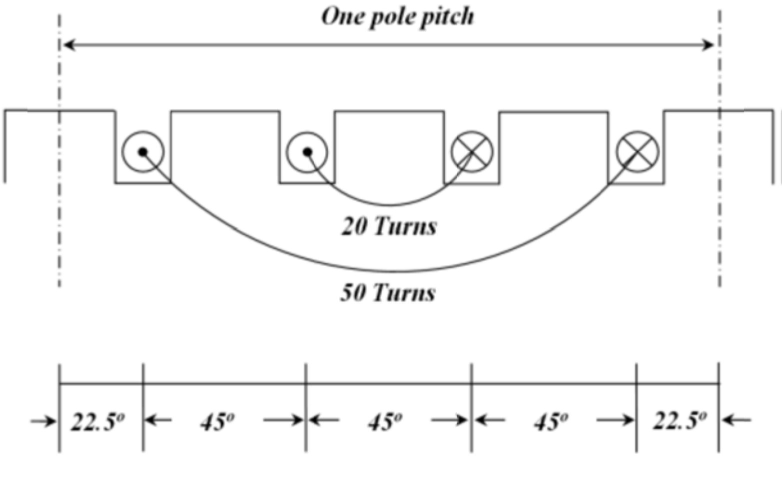
As for distribution factor, the skew factor for nth harmonic is:

$$\boxed{k_{skn} = \frac{\sin\frac{n\theta_{sk}}{2}}{\frac{n\theta_{sk}}{2}}}$$

The winding skew is a very effective approach to reduce higher harmonics. However, it reduces the fundamental as well. Moreover, due to its complex structure, it will also increase the manufacturing difficulty.

(c), In general, for *N* coils linking the n^{th} harmonic, the winding factor for concentric winding is

$$K_{wn} = \frac{\sum_{i=1}^{N} N_i \cos\left(n\frac{\alpha_i}{2}\right)}{\sum_{i=1}^{N} N_i}$$



$N1 = 50$, $N2 = 20$, $\alpha_1 = \pi/4$, $\alpha_2 = 3\pi/4$
For the fundamental:

$$K_w = \frac{50\cos\frac{\pi}{8} + 20\cos\frac{3\pi}{8}}{50+20} = \frac{53.848}{70} = 0.77$$

(d), compared to the fully pitched distributed winding, the short-pitched concentrated winding often has shorter end-winding, therefore, smaller overall axial length and less copper losses due to lower resistance. However, the short-pitched concentrated windings could have lower pitch factor for fundamental EMF, and hence lower output torque.

Answers to question 3:

(a), The flux density in airgap can be first calculated such as

$$B_g = \frac{B_r}{1+\mu_r \frac{l_g}{l_m}}$$

Assuming $A_m = A_g$ for rectangular flux density waveform.

Then the flux per pole can be calculated using

$$\Phi = B_g \times \left(\frac{\pi DL}{2p}\right) \times \left(\frac{\alpha}{\pi}\right) = \frac{1.0}{1+1.1\frac{0.8}{5}} \times \left(\frac{\pi \times 60 \times 10^{-3} \times 40 \times 10^{-3}}{2 \times 1}\right) \times \left(\frac{110}{180}\right)$$
$$= 1.96\, mWb$$

(b), The total number of conductor is Ztotal = 1532, a = 2, Vdc = 100V, then each path has a conductor number:

Z = Ztotal/2=766

$$\boxed{K = \frac{E}{\omega_r} = \frac{Zp\Phi}{\pi} = \frac{766 \times 1 \times 1.96 \times 10^{-3}}{\pi} = 0.478 (\text{Nm/A or V/rad s-1})}$$

Under no-load condition, I = 0 and V = E, we can have

$$\omega_{NL} = \frac{V}{K} = \frac{100}{0.478} = 209\, rad/s$$

(c), R = 3Ω, Vdc = 100V, E = 0, I = (V-E)/R = V/R, the stall-torque can be calculated by

$$T_{stall} = K\frac{V}{R} = 0.478\frac{100}{3} = 15.9\, Nm$$

From Figure 1, we know that the H_{lim} is 650 kA/m, and I_{stall} = 100/3A, then

$$NI = \left(\frac{Q\pi D}{4p}\right)\frac{\alpha}{\pi} \quad and\ Q = \frac{Z_{total} I/a}{\pi D}$$

We can obtain

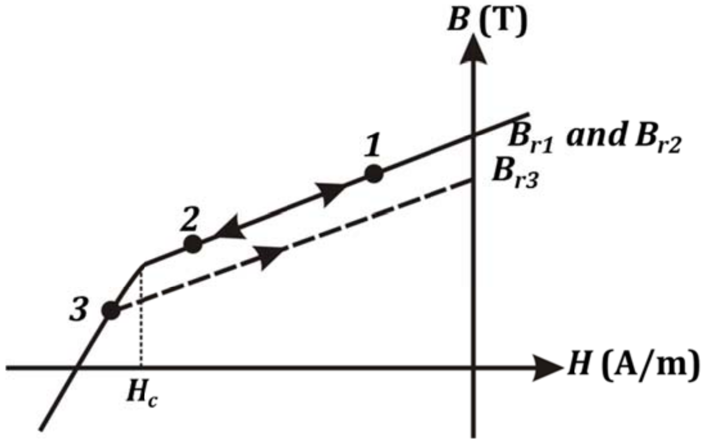
$$NI = \frac{Z_{total}}{a}\frac{I}{4p}\frac{\alpha}{\pi} = \frac{1532}{2}\frac{100/3}{4 \times 1}\frac{110}{180} = 3901.2\, ATurns$$

Then

$$H_m = -\frac{NI}{l_m + \mu_r l_g} - \frac{B_r l_g}{\mu_0 (l_m + \mu_r l_g)} = -771.7\ KA/m$$

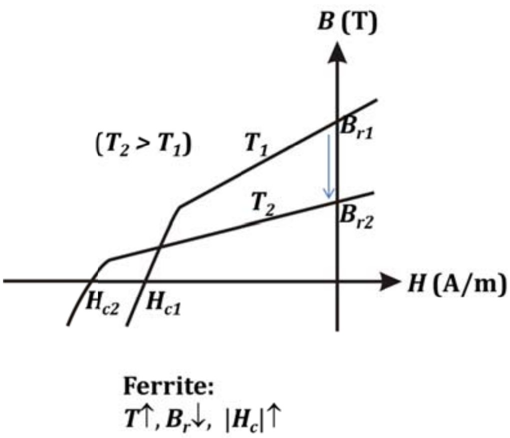
Since $|H_m| < |H_{lim}|$, the magnet will be demagnetized, especially the magnet tips.

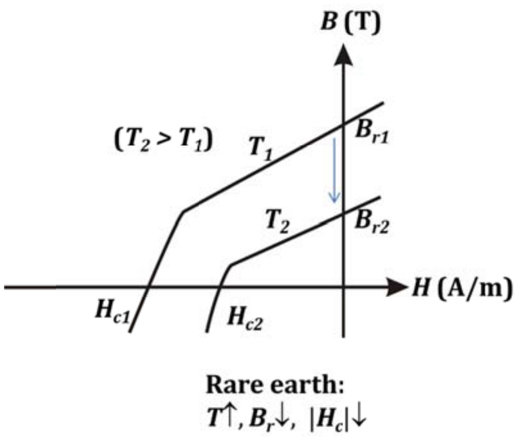
(d), The reversible and irreversible demagnetization curves are shown:



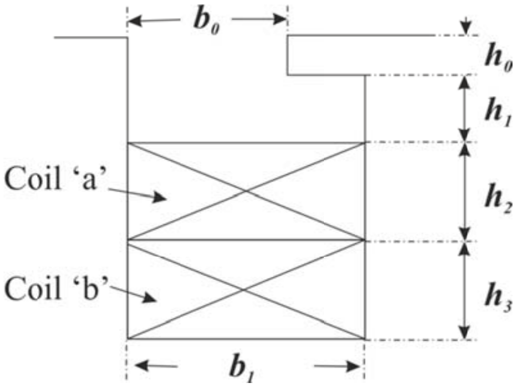
With the increase in demagnetizing field (H), the flux density in permanent magnet B decreases. However, if H does not exceed the H_c such as between points 1 and 2, the demagnetization of magnets can be recovered when $-H$ decrease. However, if the working point of magnet is beyond the knee point ($H < H_c$) such as at point 3, then, when $-H$ decrease, the demagnetization cannot be recovered and the magnet remanence reduce from $B_{r1} = B_{r2}$ to B_{r3}.

The temperature rise will normally aggravate the irreversible demagnetization, for ferrite, it will decrease the magnet remenance but will increase the H_c. However, for rare earth such as NdFeB, the temperature reduces not only Br but also H_c, making the irreversible demagnetization much worse.





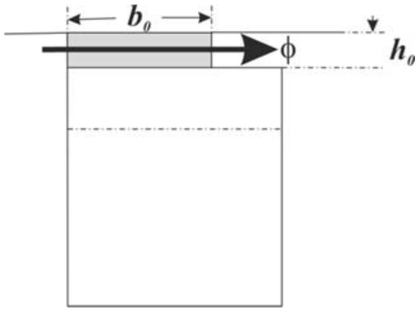
Answers to question 4:



Main assumptions:
(i) Lamination is infinitely permeable – no mmf consumed in iron
(ii) Flux paths well defined (in this case parallel to slot bottom)
(iii) Slot can be divided into easy geometric shape
(iv) Current is uniformly distributed across the wound area
(v) Winding is assumed to stack neatly each conductor occupying a 'square' as shown, in order to calculate wound depth h2 and h3
(a), coil 'a' self inductance

(1) slot opening

The MMF crossing through the slot opening is: total slot MMF = $N_a I$, where N_a is number of conductors in slot, and I is DC current per conductor (all conductors have the same current). Therefore, the flux across opening/unit length (L = 1) of machine is:



$$\Phi = \frac{MMF}{\frac{b_0}{\mu_0 h_0 L}} = N_a I \mu_0 \left(\frac{h_0}{b_0}\right)$$

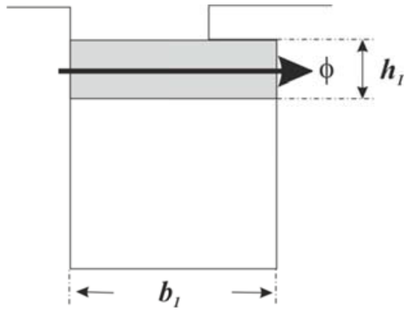
And the flux linkage/unit length of machine is:

$$\psi = \Phi \times \text{Number of conductor linked } (N_a) = N_a^2 I \mu_0 \left(\frac{h_0}{b_0}\right)$$

Inductance:

$$L = \frac{\Psi}{I} = N_a^2 \mu_0 \left(\frac{h_0}{b_0}\right)$$

(2) Unwound section of slot (above conductors)



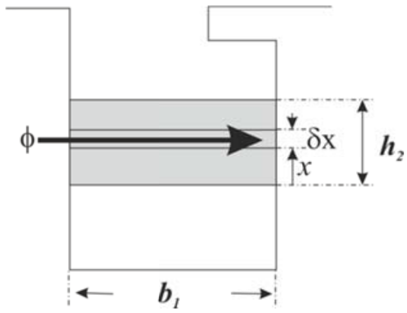
Again by same analysis as slot opening

Flux linkage/unit length = $N_a^2 I \mu_0 \left(\frac{h_1}{b_1}\right)$

Inductance:

$$L = \frac{\Psi}{I} = N_a^2 \mu_0 \left(\frac{h_1}{b_1}\right)$$

(3) Wound section of slot for coil 'a'



In this case, the MMF is distributed throughout the section, and we need to integrate across the depth $\boldsymbol{h_2}$.

Considering an elemental strip depth δ_x at x from the bottom of the winding, the MMF available below this strip is $N_a I \times \frac{x}{h_2}$, and the flux across this strip is

$$\Phi = \frac{MMF}{R} = \frac{\left(N_a I \times \frac{x}{h_2}\right)}{\left(\frac{b_1}{\mu_0 \delta_x}\right)} = N_a I \frac{x}{h_2}\left(\frac{\mu_0 \delta_x}{b_1}\right)$$

Therefore, the flux linkage of strip is

$$\Psi = \left(N_a \times \frac{x}{h_2}\right)\Phi = \left(N_a \times \frac{x}{h_2}\right) \times N_a I \frac{x}{h_2}\left(\frac{\mu_0 \delta_x}{b_1}\right) = N_a^2 I \frac{x^2}{h_2^2}\left(\frac{\mu_0 \delta_x}{b_1}\right)$$

where $\left(N_a \times \frac{x}{h_2}\right)$ is the number of conductors below the strip.

Hence, effective flux linkages for total wound section:

$$\Psi = \frac{\mu_0 N_a^2 \boldsymbol{I}}{h_2^2 b_1}\int_0^{h_2} x^2\, dx = \frac{\mu_0 N_a^2 I h_2}{3 b_1}$$

And the leakage inductance of wound area is:

$$L = \frac{\Psi}{I} = \frac{\mu_0 N_a^2 h_2}{3 b_1}$$

Hence, for a motor of unity length: Inductance per slot is

$$L_a = \frac{\Psi}{I} = N_a^2 \mu_0 \left[\frac{h_0}{b_0} + \frac{h_1}{b_1} + \frac{h_2}{3 b_1}\right]$$

(b), coil 'b' self inductance

Similar to coil 'a', it can be derived that

$$L_b = \frac{\Psi}{I} = N_b^2 \mu_0 \left[\frac{h_0}{b_0} + \frac{h_1 + h_2}{b_1} + \frac{h_3}{3 b_1}\right]$$

(c), For the flux produced by coil 'b' ($N_b I_b$), it will link the coil 'a' by flux linkage in

(1) slot opening

Inductance

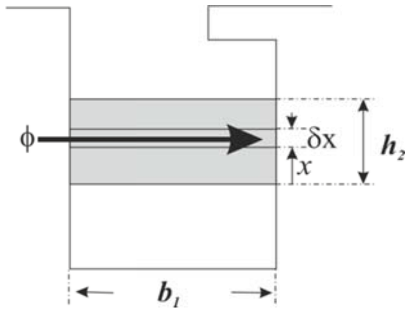
$$M = \frac{\Psi_{ab}}{I_b} = N_a N_b \mu_0 \left(\frac{h_0}{b_0}\right)$$

(2) Unwound section above the conductors

$$M = \frac{\Psi_{ab}}{I_b} = N_a N_b \mu_0 \left(\frac{h_1}{b_1}\right)$$

(3) Wound section of slot for coil 'a'

In this case, all coil 'b' MMF is linked with coil 'a', but only part of number of conductors in coil 'a' are linked throughout the section & we also need to integrate across the depth h_2. Consider elemental strip depth δ_x at x from bottom of coil 'a':



MMF available below strip (due to coil 'b' only)= $N_b I_b$

Flux Φ through strip

$$\Phi = \frac{MMF}{R} = \frac{(N_b I_b)}{\left(\frac{b_1}{\mu_0 \delta_x}\right)} = N_b I_b \left(\frac{\mu_0 \delta_x}{b_1}\right)$$

And the number of conductors linked at coil 'a' below the strip: $N_a \frac{x}{h_2}$

Therefore, the flux linkage = $\left(N_a \frac{x}{h_2}\right) N_b I_b \left(\frac{\mu_0 \delta_x}{b_1}\right)$

Hence, effective flux linkages for total section

$$\frac{N_a N_b I_b \mu_0}{h_2 b_1} \int_0^{h_2} x dx = N_a N_b I_b \mu_0 \frac{h_2}{2 b_1}$$

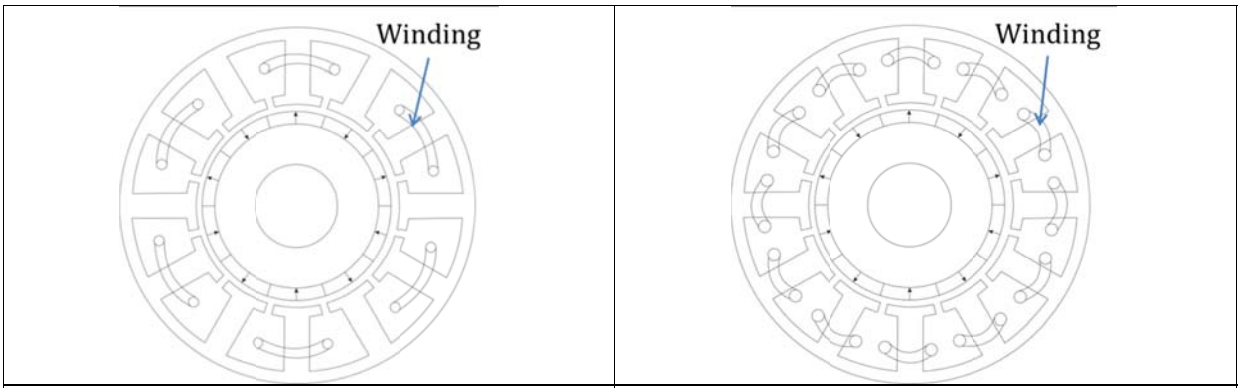
And the inductance is

$$M = \frac{\Psi_{ab}}{I_b} = N_a N_b \mu_0 \left(\frac{h_2}{2 b_1}\right)$$

Hence, for a motor of unity length:

$$M = \frac{\Psi_{ab}}{I_b} = N_a N_b \mu_0 \left[\frac{h_0}{b_0} + \frac{h_1}{b_1} + \frac{h_2}{2 b_1}\right]$$

(d), one stator slot showing single layer and double winding layer will be enough. The real machines with single layer and double layer windings can be seen:



(a) Single layer | (b) Double layer

Fig. 1 Single layer and double layer winding machines.

In general, the main advantage of single layer winding is:

- Higher winding factor (higher distribution factor and the same pitch factor),
- Higher self inductance but lower mutual inductance (less coil number but high number of turns/coil to have the same number of turns/phase and hence the same phase EMF level),
- Physical, electromagnetic and thermal separations between coils, and hence much higher fault tolerance capability, very desirable for aerospace applications,
- Higher saturation level due to more flux concentrate on each stator tooth with windings, leading to lower torque at high phase currents.