Lecture content

- Examples of Continuous Time (CT) signals
  - The exponential
  - The sinusoidal
- Manipulations of CT signals

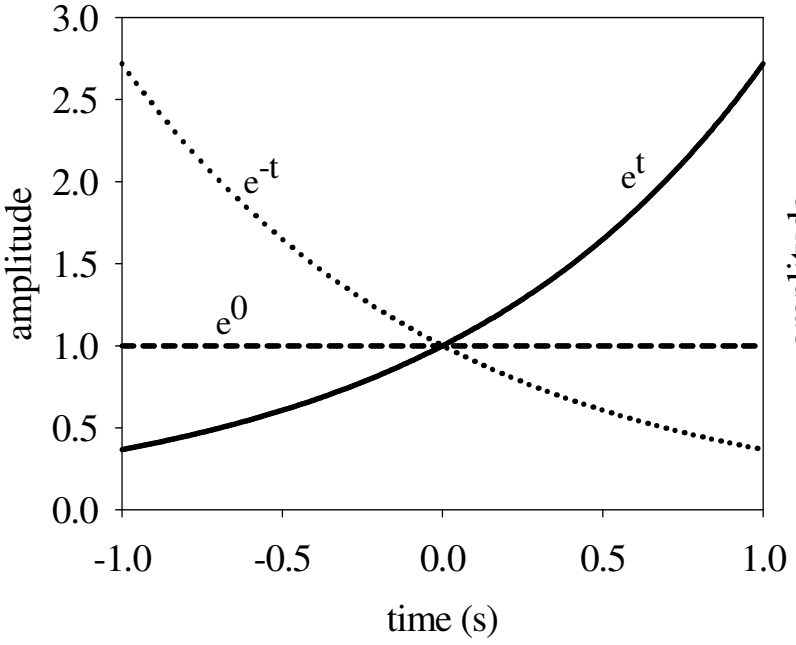
The exponential

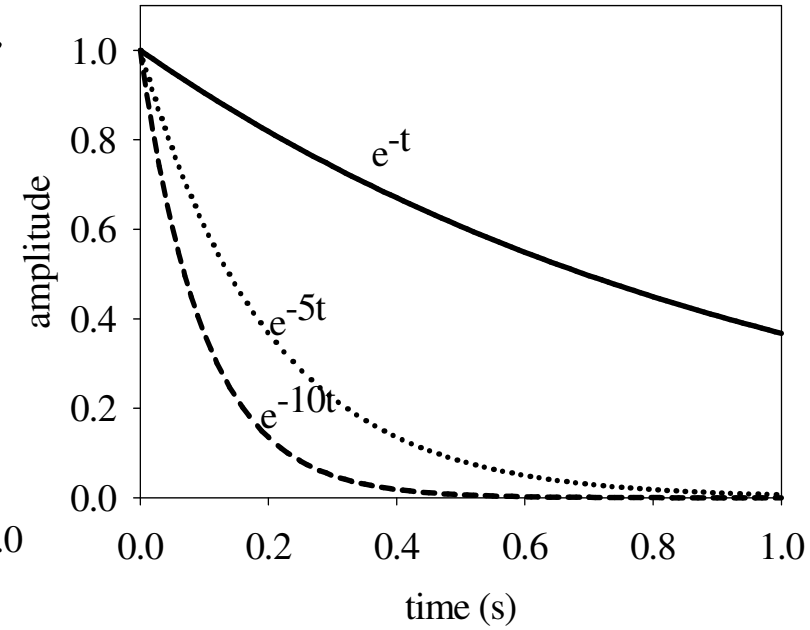
$x(t) = e^{-at}$, $t \geq 0$.

If a is positive $x(t)$ decays exponentially.

If a is negative $x(t)$ grows exponentially.

If $a = 0$, $x(t) = u(t)$.





The Exponential

At what time will $x(t) = 0$? Mathematically this happens when $t = \infty$. In practice we often consider $x(t) = 0$ if its magnitude is less than 1% of its peak magnitude.

- At $t = t+\tau$, $\dfrac{e^{-a(t+\tau)}}{e^{-at}} = e^{-a\tau} = e^{-1} = 0.37$ (37% of its original value).
- At $t = t+5\tau$, $\dfrac{e^{-a(t+5\tau)}}{e^{-at}} = e^{-a5\tau} = e^{-5} = 0.007$ (0.7% of its original value).

Thus, we often consider e^{-at} to reach zero after 5τ.

If $a = -j\omega_o$ (purely imaginary) $x(t) = e^{j\omega_o t}$ then is periodic.

Periodic signals with frequencies equal to integer multiples of ω_o are known as harmonics.

The sinusoidal

A sinusoidal signal is given by

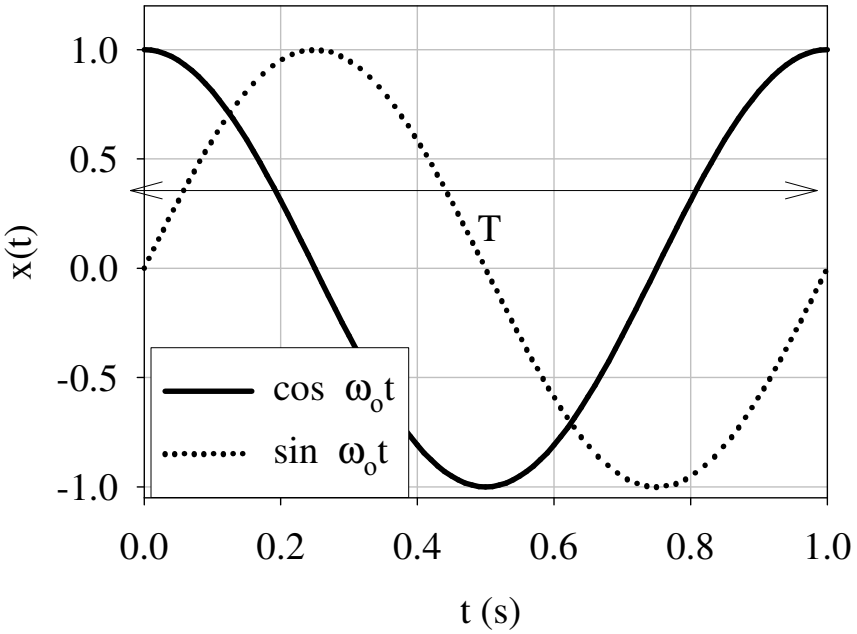
$$v(t) = V\sin(\omega_o t) = V\cos\left(\omega_o t - \pi/2\right) \qquad T = \frac{1}{f_o} = \frac{2\pi}{\omega_o}$$

Euler's relation $e^{j\omega_o t} = \cos\omega_o t + j\sin\omega_o t$

$$\sin(\omega_o t) = \frac{e^{j\omega_o t} - e^{-j\omega_o t}}{2j}$$

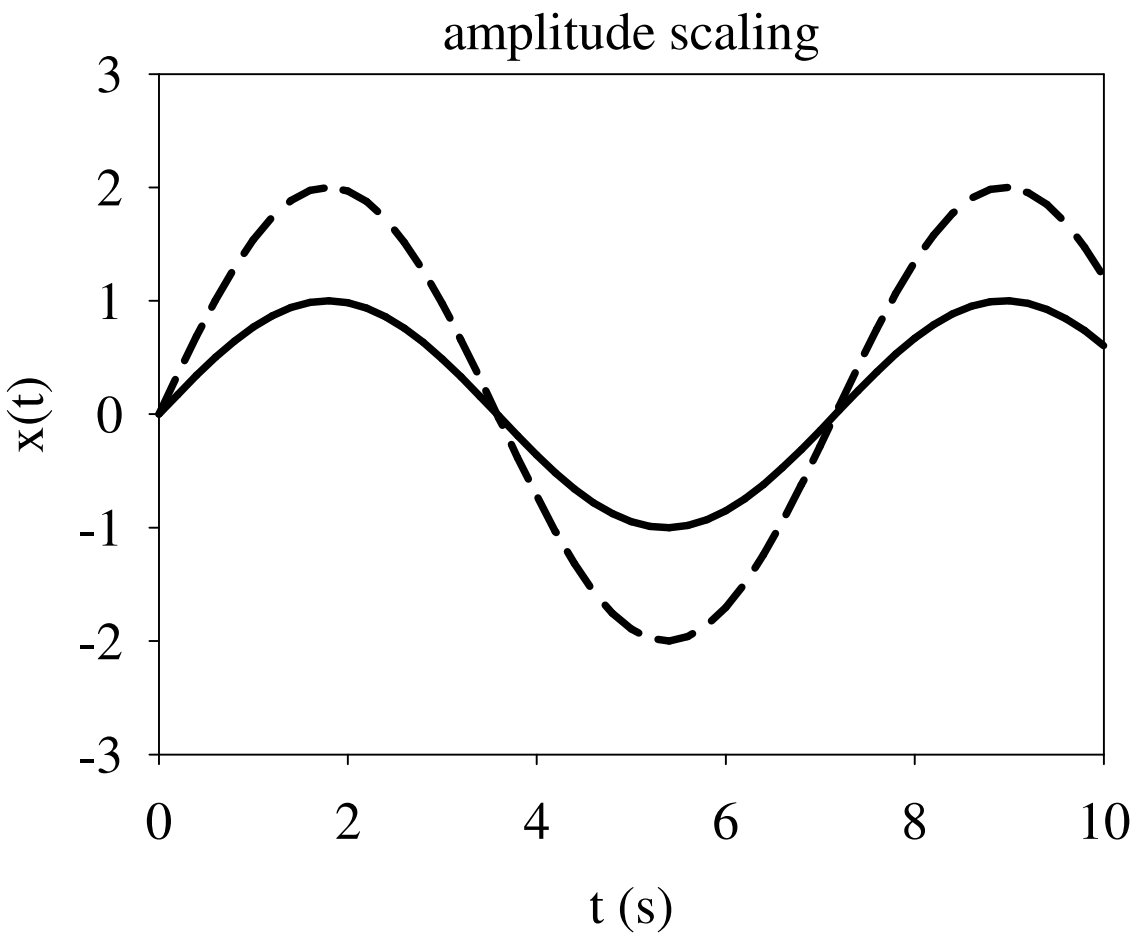
$$\cos(\omega_o t) = \frac{e^{j\omega_o t} + e^{-j\omega_o t}}{2}$$

Sketch the magnitude of $y(t) = e^{j2t} + e^{jt}$



Manipulations of CT signals

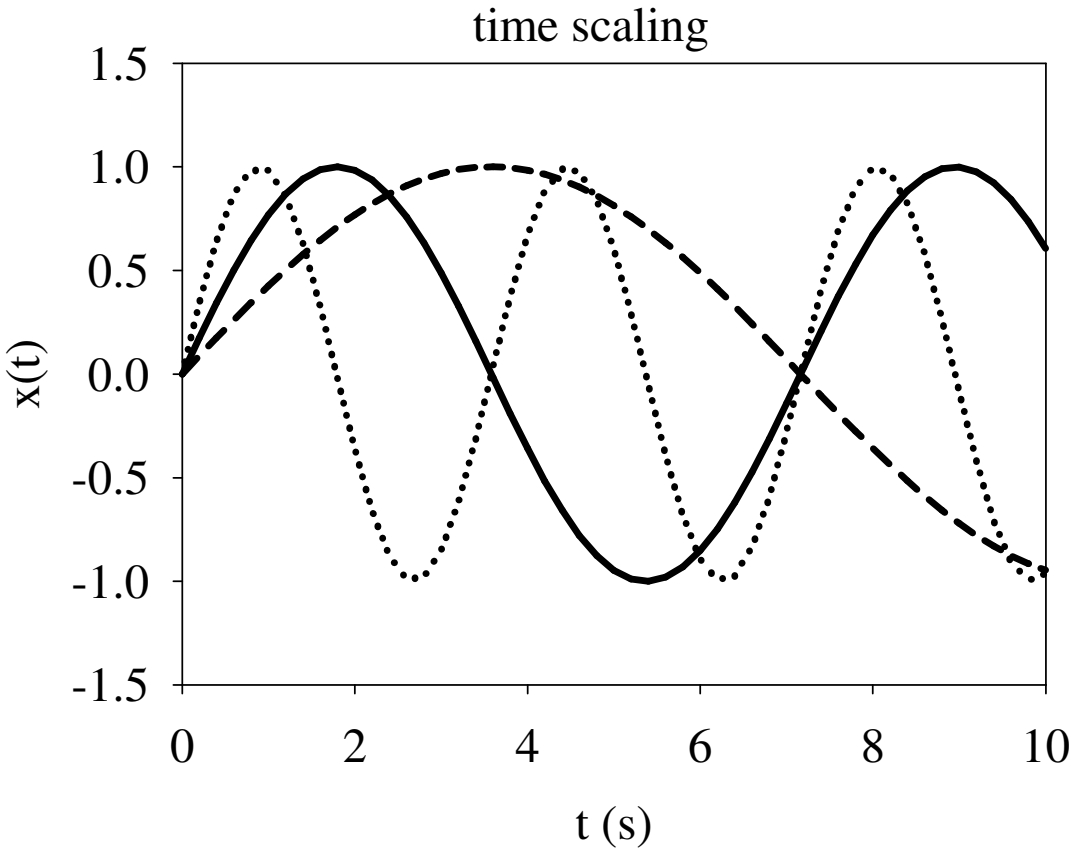
Amplitude scaling: $y(t) = Ax(t)$



Manipulations of CT signals

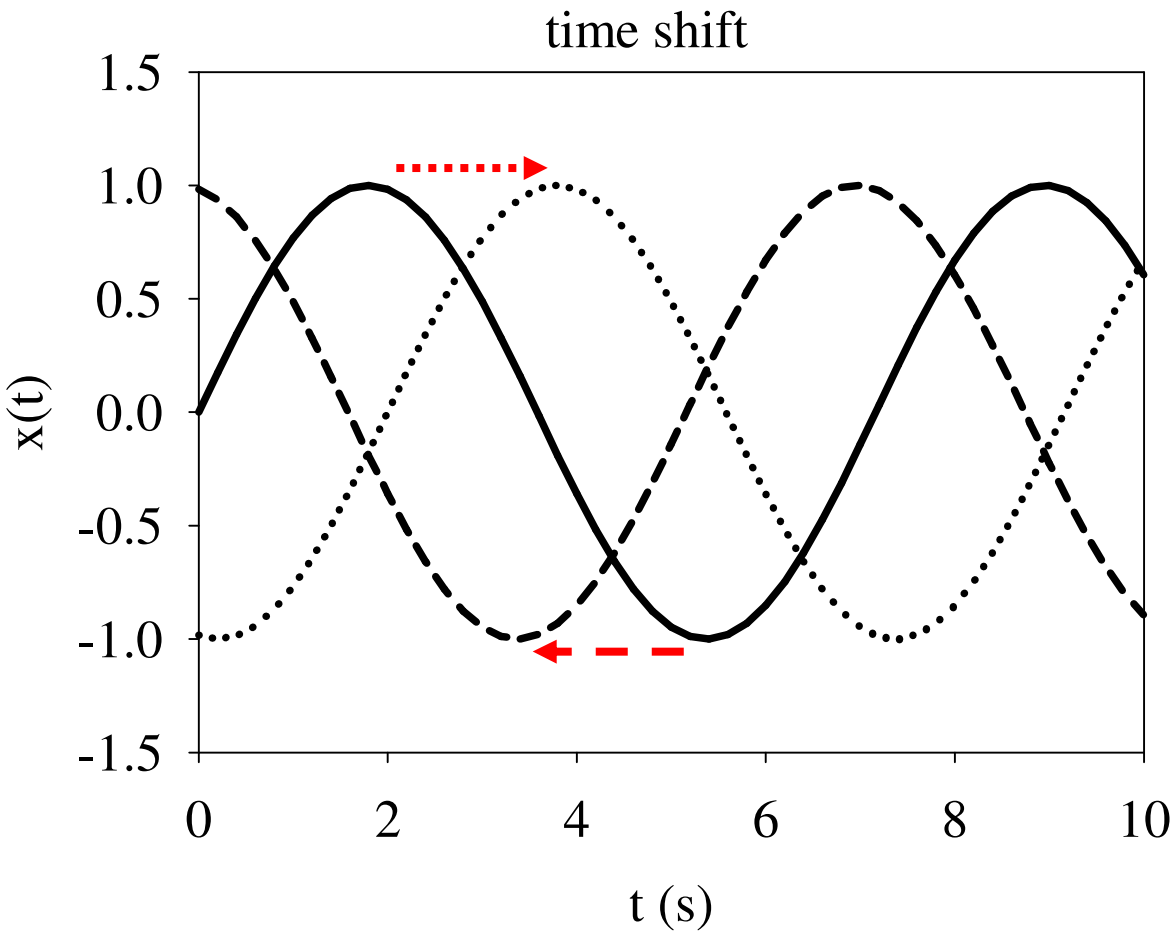
Time scaling: $y(t) = x(At)$

$y(t)$ is a time-compressed (speed up, if $A > 1$) or a time-expanded (slowed down, if $A < 1$) version of $x(t)$.



Manipulations of CT signals

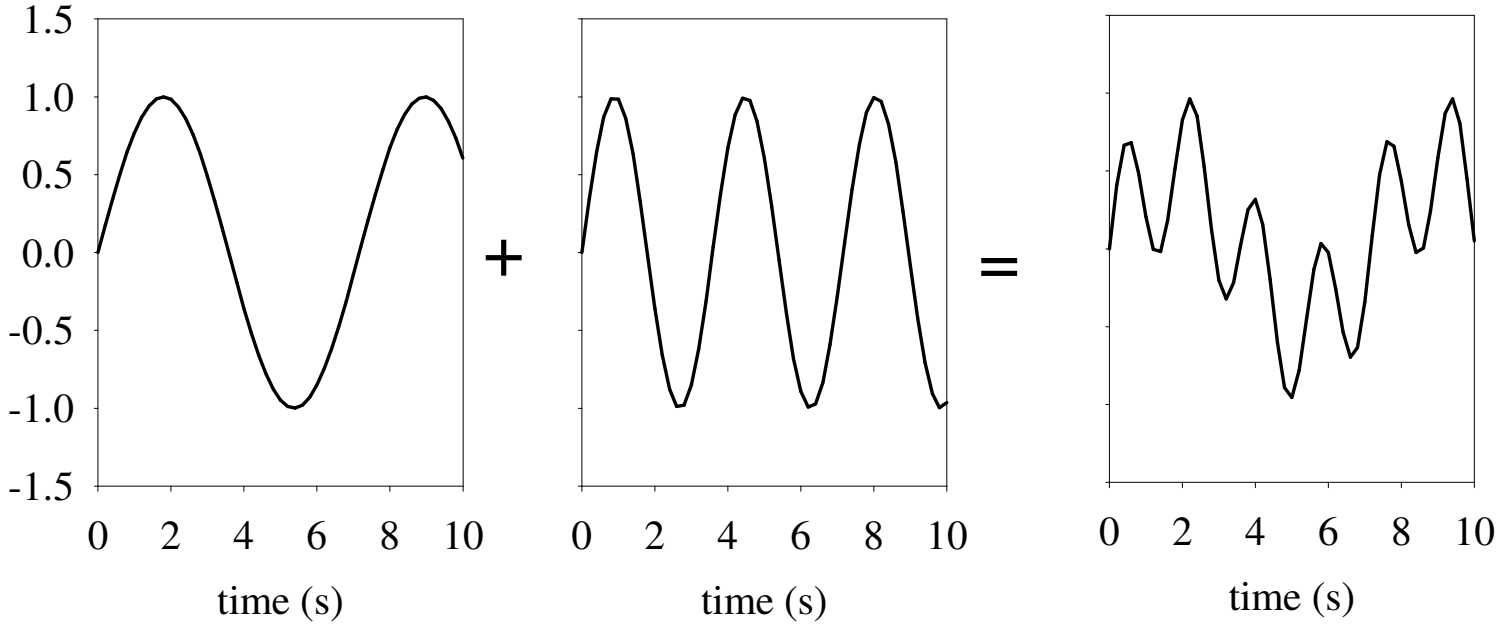
Time shifting: $y(t) = x(t\text{-}t_o)$ or $x(t+t_o)$



Sinusoidal signals: Addition

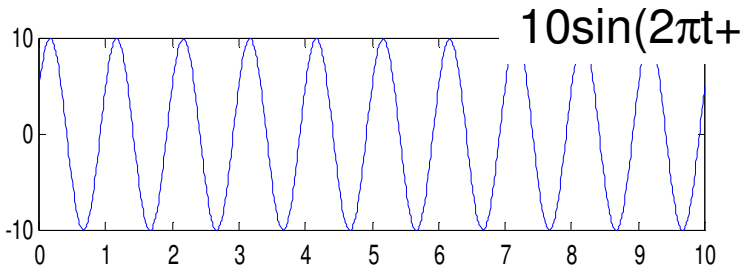
$$A \sin(\omega t + \alpha) + B \sin(\omega t + \beta) = C \sin(\omega t + \gamma)$$

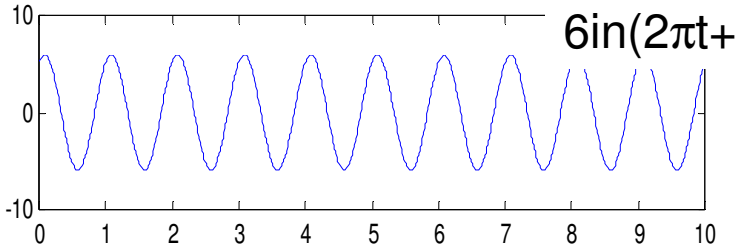
$$C = \sqrt{X^2 + Y^2}, \tan\gamma = Y/X, X = A\cos\alpha + B\cos\beta, Y = A\sin\alpha + B\sin\beta$$

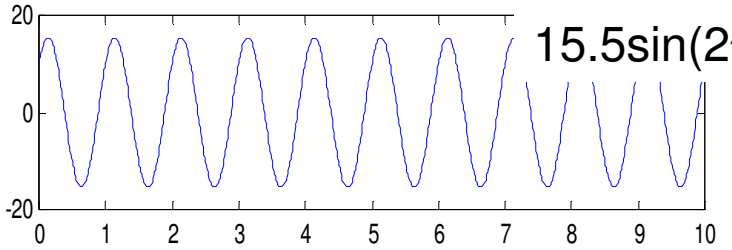


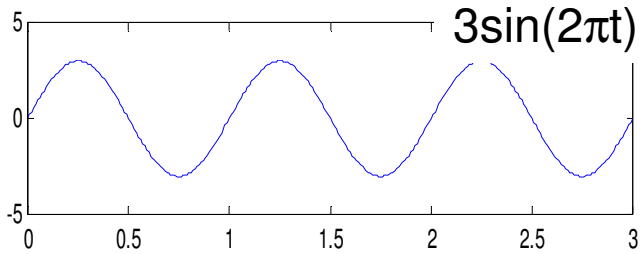
Sinusoidal signals: Addition

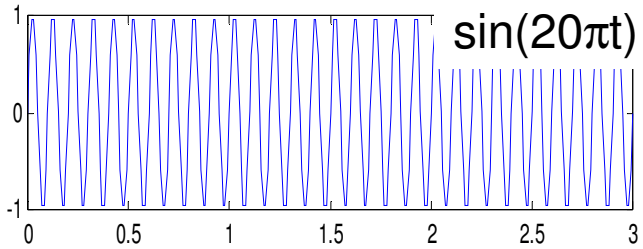
Evaluate $10\sin(2\pi t+\pi/6)+6\sin(2\pi t+\pi/3)$

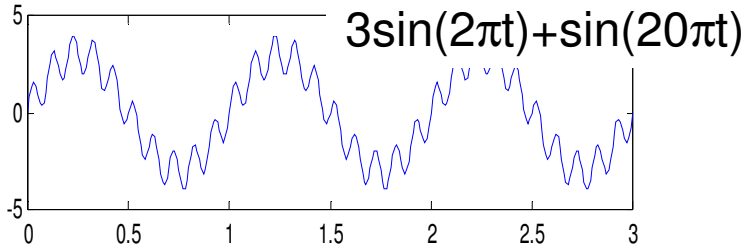




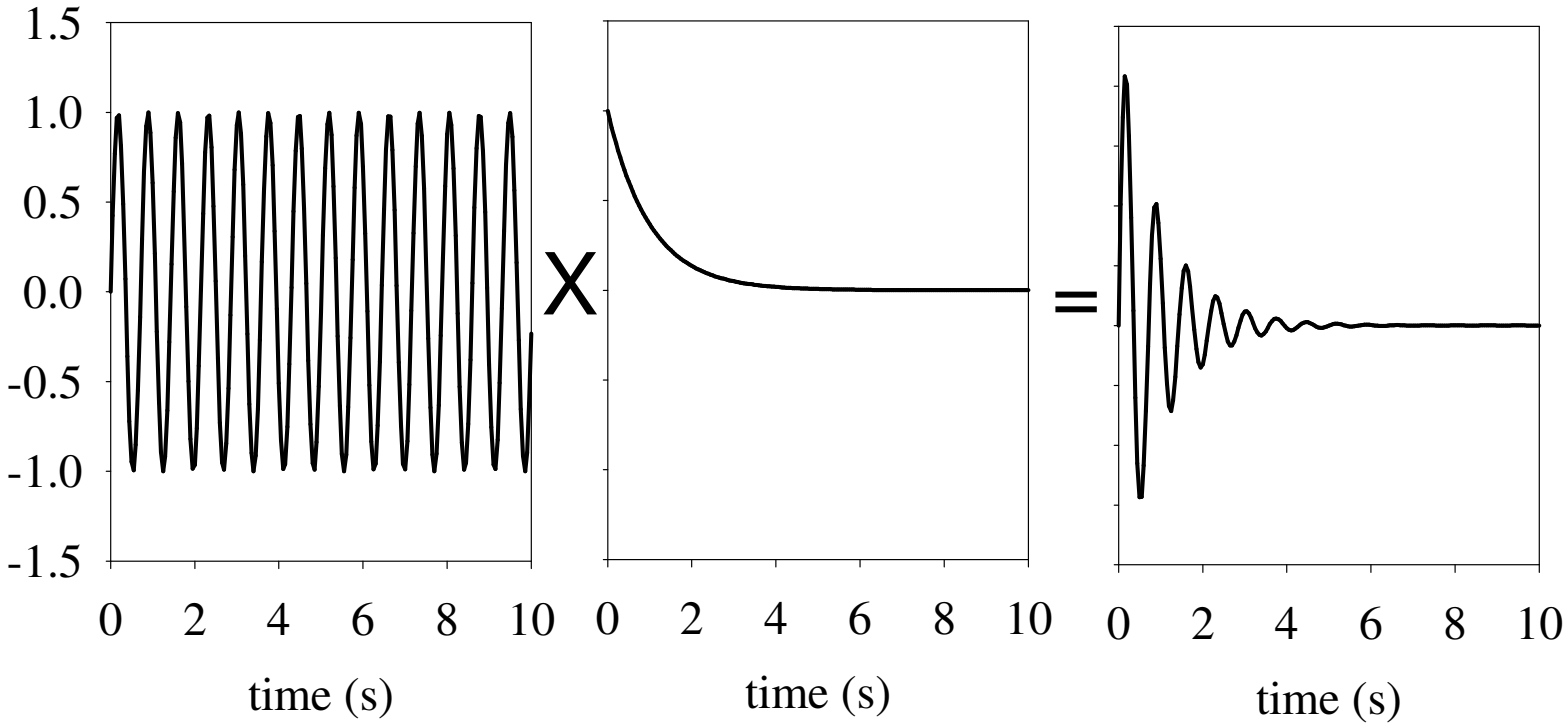








Sinusoidal signals: Multiplication



Sinusoidal signals multiplied by an exponential are usually known as damped sinusoids