Electromagnetic Induction & Machines

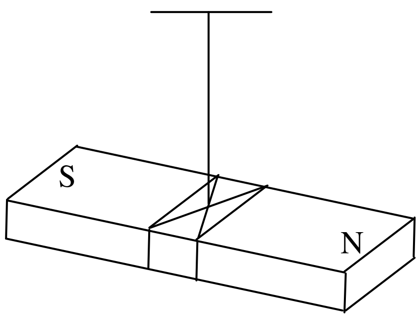
Magnetism

Magnetism and electromagnetism are very common in our everyday lives although many examples are taken for granted, for example:

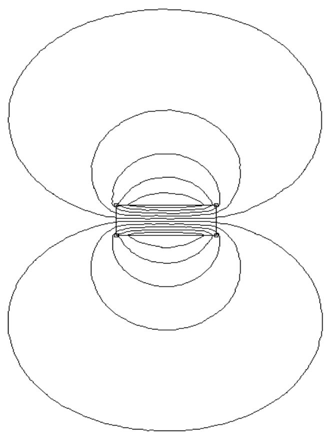
- **General magnets** - bar magnets, horseshoe magnets, compass needle
- **Household holding magnets** – noticeboard, knife rack, fridge magnets/door magnets
- **Industrial holding magnets** – switchable magnetic holding devices and chucks
- **Magnetic safety catches** – washing machine door, food mixer cover
- **Magnetic switches** – door switch for burglar alarm
- **Electromechanical devices** – (rely on interaction of an electric current with a magnetic field to produce a force of a torque) - actuators, motors, loudspeakers, relays
- **Magnetic recording and storage** – audio/video tapes, credit cards, access cards, computer disks
- **And many more!**

Magnetic Field

Take a simple bar magnet and suspend it horizontally by a piece of thread as shown in the figure (below left). The magnet will take up a position with one end pointing to the Earth's north pole (the north seeking pole or north-pole for short), the other end will point south (the south seeking pole or south-pole for short).



Simple bar magnet suspended by thread



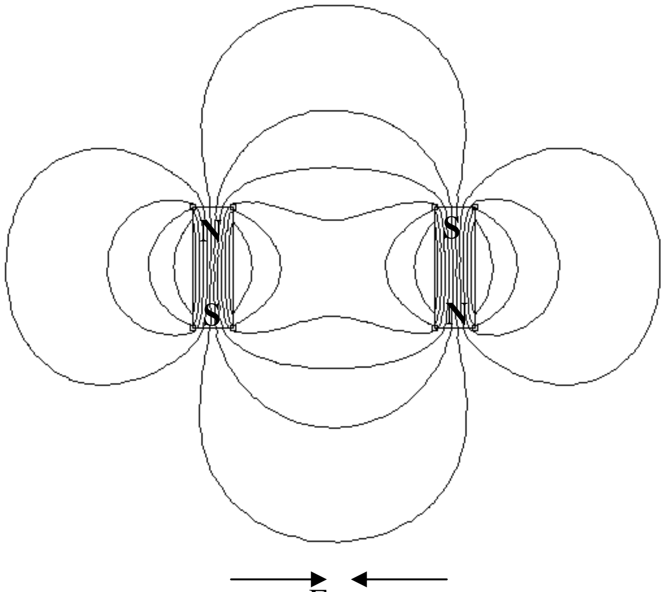
Field lines formed by iron filings

If we now place the magnet on a surface, cover it with paper, and sprinkle iron filings over the sheet, the filings will align themselves into curved chains between the 2 poles as shown in the figure (above right). This allows us to form a mental picture of the **Magnetic Field** around the bar magnet and the concept of **Lines of Magnetic Flux**.

Lines of magnetic flux have no physical existence – they are purely imaginary and introduced by Michael Faraday as a means of visualising the distribution and density of a magnetic field.

- The direction of the magnetic field is taken such that field lines emanate from a north-pole and enter a south pole.
- Each line of flux forms a closed loop.
- Lines of magnetic flux can never cross or intersect.

Lines of magnetic flux behave like stretched elastic chords – always trying to shorten themselves:

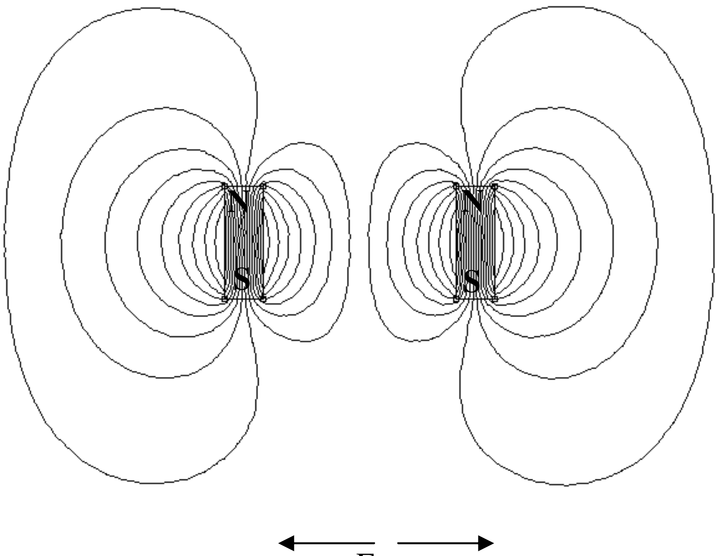


The 2 magnets will try and attract one another – <u>Opposite poles attract</u>

Force

Opposite poles facing one another

Lines of magnetic flux which are parallel and in the same direction will repel on another:



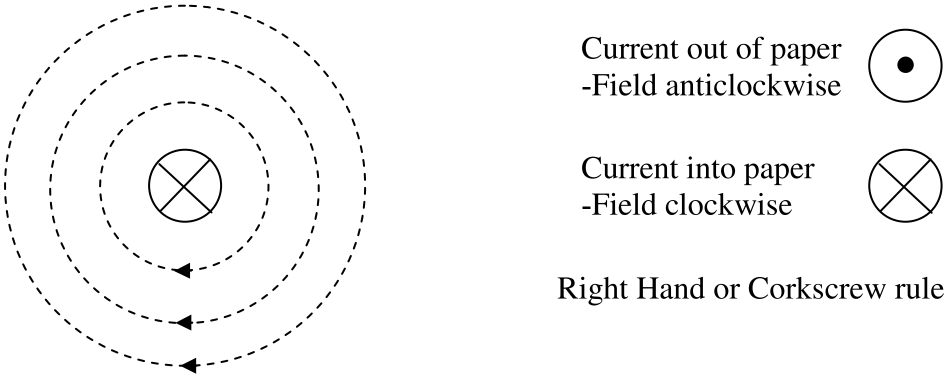
The 2 magnets will try and repel one another – <u>Like poles repel</u>

Force

Like poles facing one another

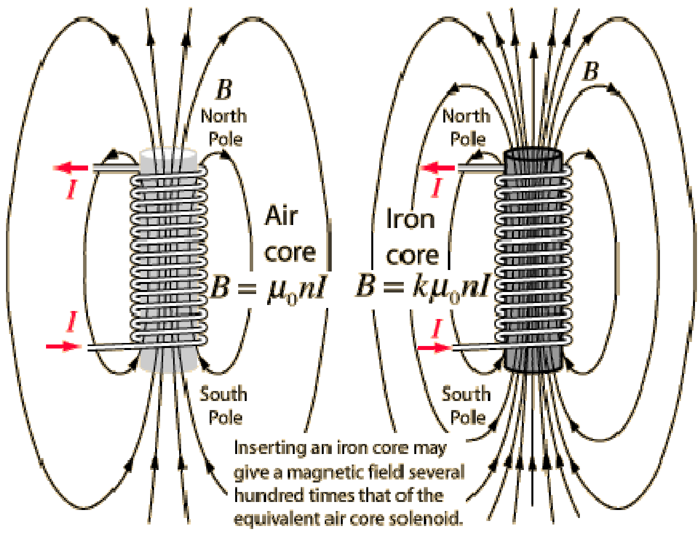
Magnetic Field due to an Electric Current

Oersted discovered that if an electric current flowed in a conductor a magnetic field is produced: around it:



Field lines around a current carrying conductor

If a current carrying wire is formed into a long straight coil then it produces a nearly uniform magnetic field similar to that of the bar magnet shown earlier. This arrangement is known as a solenoid. The **flux density**, B, within the solenoid is dependent on the number of turns, n, the current flowing in the coil, I, and the **permeability** of the 'core'. For an 'air-cored' solenoid the permeability, μ_0, is equal to $4\pi\times10^{-7}\text{Hm}^{-1}$. The magnetic field within the solenoid may be enhanced by winding the coil on a ferromagnetic core, such as iron or steel which has a relative permeability, k, of several thousand. (On this course we shall use μ_R to represent the relative permeability).



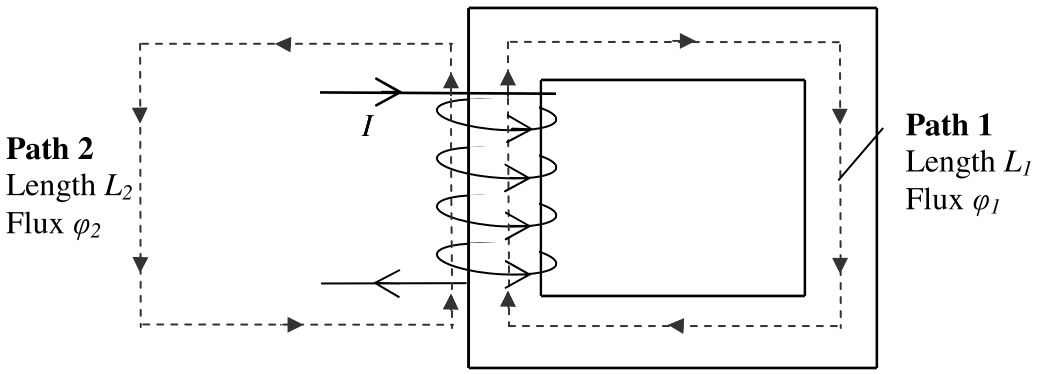
Solenoid [source: http://hyperphysics.phy-astr.gsu.edu]

Once again the corkscrew rule can be applied to find the direction of the magnetic field. Turn the corkscrew in the direction of the current and the direction it travels will be the direction of the field. Alternatively 'grip' the core with the right hand with the fingers following the direction of the current. The thumb will point in the direction of the field.

Magnetic Circuits

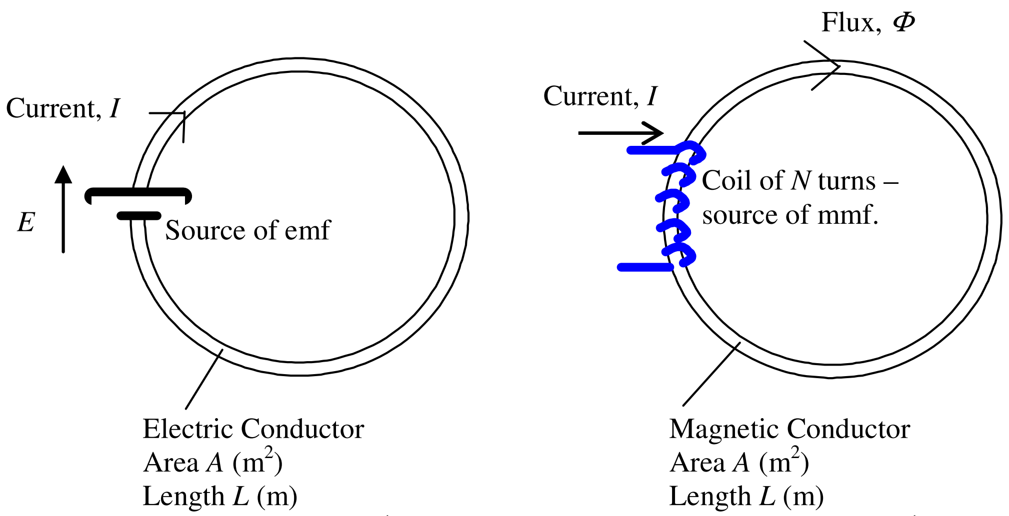
When a magnet (in the form of either a permanent magnet or current carrying coil) is present then lines of magnetic flux will flow from one pole of the magnet to the other pole through the air or other material surrounding it. The complete closed path followed by the magnetic flux lines is referred to as a **magnetic circuit**. This is similar to the concept of an electrical circuit where electric charge leaves one terminal of the supply, flows round a circuit, and returns to the other terminal of the supply. Just as the behaviour of an electrical circuit determines the amount of electric current, then the properties of a magnetic circuit determine the quantities of **magnetic flux** in its various parts.

Consider the magnetic iron core with a coil wound it which carries a current of I Amps as shown in the figure below:



The current flowing through the coil creates a magnetic flux which will flow both around the iron circuit (path 1) and also through the air (path 2). The permeability of the iron core, $\mu_R\ \mu_0$, is very much greater than that of air (which has the permeability of free space μ_0), typically the ratio will be of the order 500→1000:1 and so the majority of the flux produced by the coil will flow around path 1 (the main flux path). However air is not a perfect 'magnetic insulator' and a small amount of flux will flow round path 2 (the leakage flux path). For the purpose of this course we will ignore the leakage flux path.

Analogy with an electrical circuit



For an electrical circuit made up of a number of elements the current, *I*, is dependent on the electromotive force, *E*, and the total resistance of the circuit, *R*:

$$I = \frac{emf}{total\ circuit\ resistance} = \frac{E}{R_T}$$

where for each element: $R = \frac{L}{\sigma A} \quad \Omega$

In the magnetic circuit the current flowing through the coil of wire is known as a source of magnetomotive force or mmf for short. (A permanent magnet is also an mmf source). The mmf source will drive a flux around our magnetic circuit which has reluctance (magnetic resistance) to the flow of flux.

For the magnetic circuit we can write:

$$\phi = \frac{mmf}{total\ circuit\ reluctance} = \frac{F}{S_T}$$

where:

Φ is the flux in Webers (Wb)
$F = N\,I$ is the magnetomotive force in Amp-turns (Amps)
S_T is the reluctance of the magnetic circuit

and for each element: $S = \frac{L}{\mu A}$ Henry^{-1} (H^{-1}).

Provided the magnetic field is confined to a path of known area *A*, length *L*, and permeability μ, the above method is very useful. – Note that whilst the current, *I*, can be associated with the 'flow of charge', the concept of flux Φ flowing around a circuit is only a useful concept. In practice, there is no physical flow involved. Because air is not a 'magnetic insulator', unlike electric circuits where the current can reasonably be assumed to be confined to the wires, in the magnetic circuits of some devices, significant amounts of flux 'leaks' or spreads into unwanted sections of the device.

An electromotive force (voltage) causes a current to flow through a resistance.
A magnetomotive force causes a flux to flow through a reluctance.

emf is analogous to mmf
current is analogous to flux
resistance is analogous to reluctance

Rearranging the above equation for the magnetic circuit yields a very important expression for solving magnetic circuits:

$$Mmf = Flux \times Reluctance$$

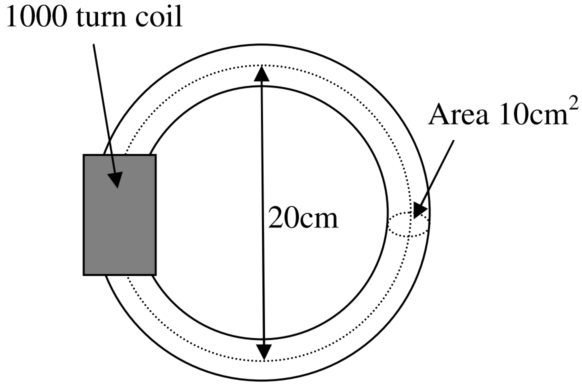
$$F = NI = \phi S$$

If the cross-sectional area through which the flux is passing is A, then we can obtain the flux density, B, as:

$$B = \frac{\phi}{A} \quad \text{Wb/m}^2 \text{ or Tesla (T)}$$

<u>Example</u>

A coil of 1000 turns carrying a current of 1A is wound on a toroidal core (circular in shape, circular in cross-section ie. a "doughnut") of mean diameter 20cm, and cross-sectional area 10cm^2 and a relative permeability, μ_r = 1000. Calculate the flux and flux density in the core.



$$\text{mmf (F)} = \text{flux } (\phi) \times \text{reluctance (S)}$$

or:

$$NI = \phi \times S$$

so:

$$\phi = \frac{NI}{S}$$

The reluctance of the magnetic circuit is:

$$S = \frac{L}{\mu_0 \mu_r A} = \frac{\pi \times D}{\mu_0 \mu_r A} = \frac{\pi \times 20 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 1000 \times 10 \times 10^{-4}} = 5 \times 10^5 \text{ H}^{-1}$$

Substituting these values in the equation for flux gives:

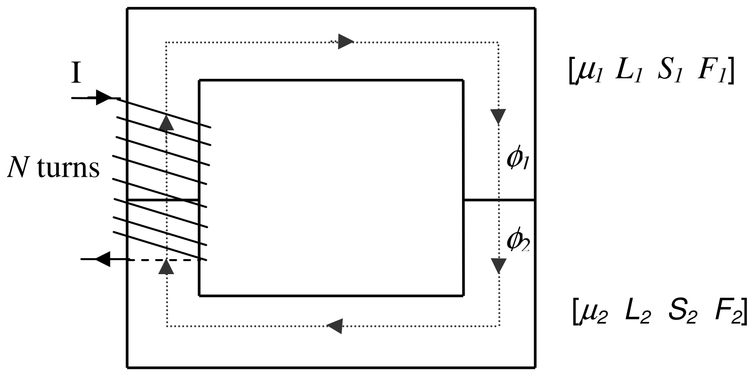
$$\phi = \frac{1000 \times 1}{5 \times 10^5} = 2 \times 10^{-3} \text{ Wb (Webers)} \quad \text{(or 2mWb)}$$

The flux density may be obtained from:

$$B = \frac{\phi}{A} = \frac{2 \times 10^{-3}}{10 \times 10^{-4}} = 2 \text{ T (Tesla)}$$

Reluctances in Series

Reluctances can be combined together in networks in a similar way to resistances in an electrical circuit. Consider a magnetic circuit made up of two parts as shown in the diagram below. A coil of N turns carrying a current of I Amps creates a flux ϕ through the two. (Ignore any leakage effects).



The two sections are in series since the flux is continuous through both parts:

$$\phi = \phi_1 = \phi_2$$

Now the total mmf is equal to the sum of the mmf in each part:

$$F_{Tot} = N \times I = F_1 + F_2$$

Now:

$$F_1 = S_1 \times \phi \quad \text{and} \quad F_2 = S_2 \times \phi$$

therefore:

$$N \times I = \phi(S_1 + S_2)$$

Now since the total mmf is equal to the product of the flux and the total reluctance, then:

$$N \times I = \phi \times S_T$$

and hence:

$$S_T = S_1 + S_2$$

If there were n sections in series then:

$$S_{To} = S_1 + S_2 + S_3 + S_4 + \cdots + S_n$$

which is analogous to an electric circuit with resistances in series:

$$R_T = R_1 + R_2 + R_3 + R_4 + \cdots + R_n$$

Reluctances in Parallel

Likewise, reluctances in parallel can be treated in a similar way to parallel resistances in an electrical circuit. For reluctances in parallel, where there are n flux paths and a common mmf:

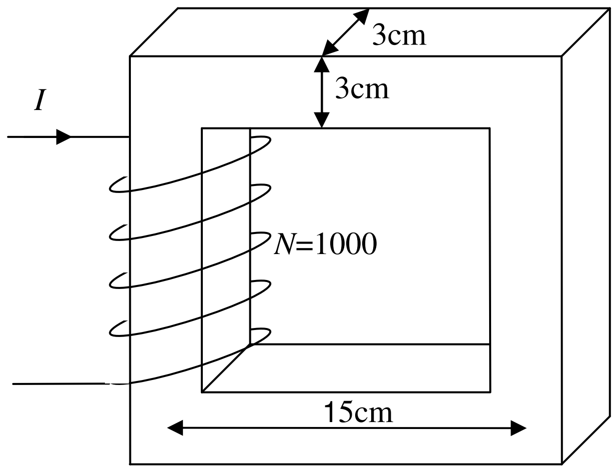
$$\frac{1}{S_T} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \frac{1}{S_4} + \cdots + \frac{1}{S_n}$$

On this course it is unlikely you will encounter devices employing flux paths in parallel.

We have already seen that materials having a high permeability, such as iron and steel, may be used to enhance the flux density and confine the flux to a well defined circuit. However in many electromechanical devices we require movement between two parts of the magnetic circuit which means that an air-gap has to be introduced. The presence of this air-gap can have a significant effect on the magnetic circuit which is best illustrated by means of the following examples.

Example

A rectangular iron core, shown below, has a coil of 1000 turns wound on it. The mean length of each side of the core is 15cm and it has a cross-section of 3cm × 3cm. If the relative permeability of the iron is 800, calculate the coil current required to establish a flux density of 1.2T in the core.



To calculate the flux density we use the standard equation:

$$F = N \times I = \phi \times S$$

The number of turns N is specified, the current I is to be calculated and the flux ϕ and reluctance S can be obtained from the information contained in the question. Rearranging the previous equation:

$$I = \frac{\phi \times S}{N}$$

The flux circulating around the magnetic circuit for a flux density of 1.2 T in the core is obtained from:

$$\phi = B \times A = 1.2 \times 3 \times 10^{-2} \times 3 \times 10^{-2} = \mathbf{1.08 \times 10^{-3}\ Wb}$$

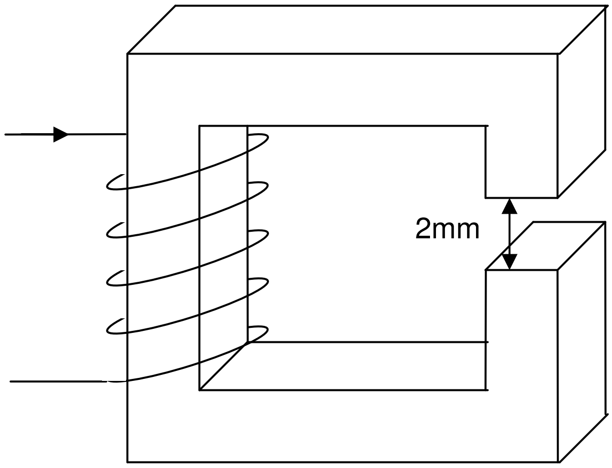
The reluctance of the magnetic circuit is obtained from:

$$S = \frac{L}{\mu_0 \mu_r A} = \frac{4 \times 15 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 800 \times 3 \times 10^{-2} \times 3 \times 10^{-2}} = \mathbf{6.63 \times 10^{5}\ H^{-1}}$$

Substituting these values in the equation for the current:

$$I = \frac{\phi \times S}{N} = \frac{1.08 \times 10^{-3} \times 6.63 \times 10^{5}}{1000} = \mathbf{0.716\ A}$$

Example
The iron core in the previous example now has a 2mm wide gap cut through one of the sides as shown in the figure below. Calculate the new value of current required to maintain a flux density of 1.2T in the air-gap.



Now the magnetic circuit is made up of two parts, namely the iron part and the air-gap. This is equivalent to two reluctances in series. The method used to find the current is the same as in the previous example except that the total reluctance is now calculated from two components.

$$S_T = S_1 + S_2$$

where:

S_1 is the reluctance of the iron part of the circuit
S_2 is the reluctance of the air-gap

The length of the iron part of the magnetic circuit has been reduced by 2mm (length of gap) and is therefore 59.8cm. Note also the relative permeability of the air-gap is unity so:

$$S_T = \frac{59.8\times10^{-2}}{4\times\pi\times10^{-7}\times800\times9\times10-4} + \frac{2\times10^{-3}}{4\times\pi\times10^{-7}\times1\times9\times10-4}$$

$$S_T = 6.61\times10^5 + 1.77\times10^6 = \mathbf{2.43\times10^6\ H^{-1}}$$

(note how the air-gap reluctance is nearly 3 times that of the iron core even though it is only 2mm wide and the length of the iron is 59.8 cm. *Thus a small air-gap dominates the magnetic circuit*). Since the cross-sectional area of the core has not changed from the previous example the flux required to produce the flux density of 1.2T remains unchanged as:

$$\phi = \mathbf{1.08\times10^{-3}\ Wb}$$

Now substituting these values to find the coil current:

$$I = \frac{\phi\times S}{N} = \frac{1.08\times10^{-3}\times2.43\times10^6}{1000} = \mathbf{2.62\ A}$$

It can therefore be seen that much more current (i.e. much more magnetomotive force) is required to drive the flux round the circuit. Maintaining the level of current from the previous example would result in a decreased flux density when an air-gap is included:

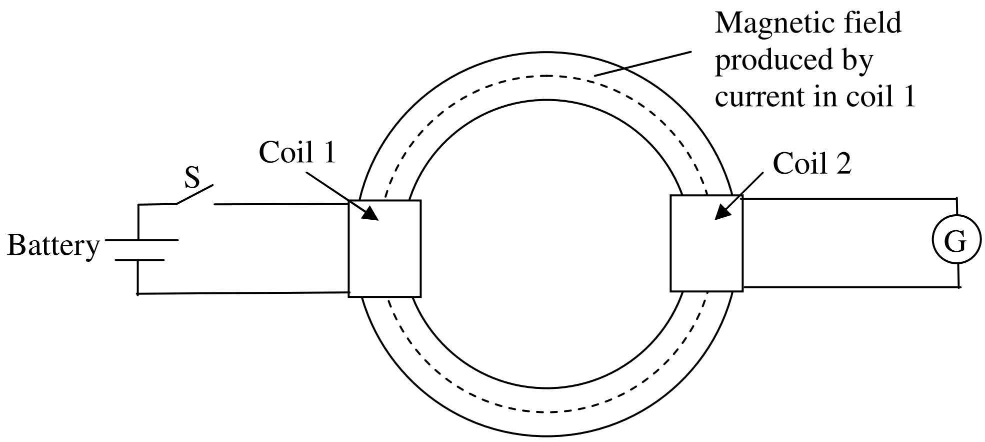
$$\phi = \frac{N\times I}{S} = \frac{1000\times0.716}{2.43\times10^6} = \mathbf{2.95\times10^{-4}\ Wb}$$

and the flux density would be:

$$B = \frac{\phi}{A} = \frac{2.95\times10^{-4}}{9\times10^{-4}} = \mathbf{0.33\ T}$$

Electromagnetic Induction

On 29th August 1831 Michael Faraday made the discovery of electromagnetic induction, i.e. a method of obtaining an electric current with the aid of a magnetic field. His experiment involved winding 2 coils on an iron ring, the coils being electrically isolated from one another.



Faraday's experiment

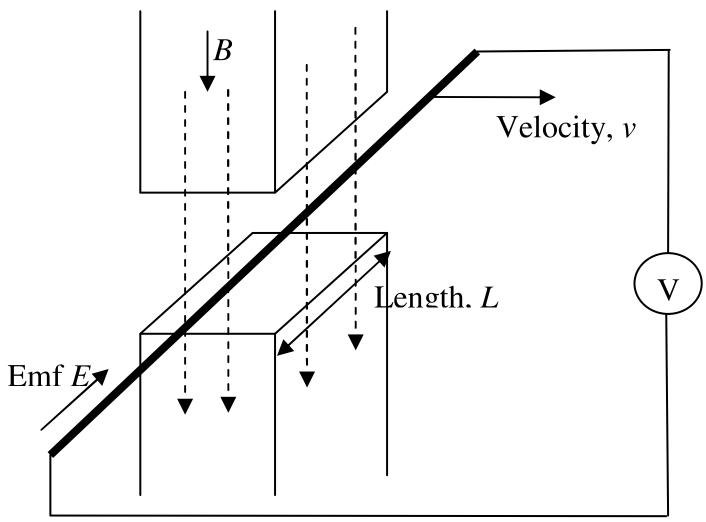
Faraday found that when switch, S, was closed the needle on the galvanometer, G, was deflected in one direction and returned to the zero point. When S was opened the needle was deflected in the opposite direction and returned to the zero point. He also noted that there was no movement of the needle for a constant current flowing in coil 1.

When the switch is closed a current builds up in coil 1 and creates a magnetic flux in the iron ring. This flux also 'links' the second coil. Faraday concluded that a voltage can be produced by a **change** in magnetic flux seen by the coil. If there is no change in the field because the current is constant then there is no voltage induced in the second coil.

Later Faraday performed another experiment where a magnet was moved relative to a coil of wire connected to a galvanometer. Once again the needle of the galvanometer moved. Either the coil or the magnet can be moved relative to one another in order to **change** the magnetic flux seen by the coil. If there is no change in the field because the flux 'linking' the coil is constant then there is no current induced in the coil.

Magnitude of the Induced emf

If a conductor moves through a magnetic field then an emf will be induced in the conductor:



The emf is induced as the conductor cuts through lines of magnetic flux (at right angles)

Emf induced in a moving conductor

If the emf E is measured in volts then:

$$E = B \times L \times v$$

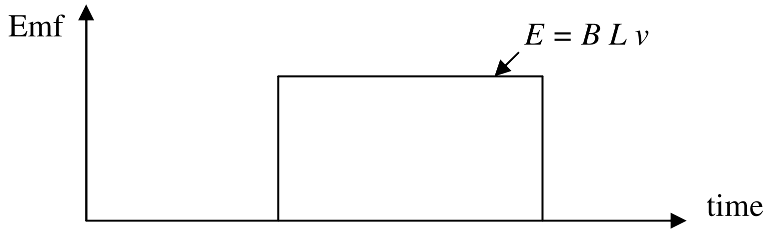
where B is in Tesla, L is in m, and v is in ms^{-1}

Assume that at velocity, v, the conductor moves a distance, d in the field, in time, t. The area swept by the moving conductor will be $A = L \times d$. But previously we have shown that $\varphi = B \times A$ hence:

$$E = BLv = BL\frac{d}{t} = \frac{BA}{t} = \frac{\phi}{t}$$

The direction of the induced emf is given by Fleming's Right Hand Rule. (Place thumb, first and second fingers of right hand at right angles to one another. If **F**irst finger points in direction of the **F**ield, the s**E**cond finger in the direction of the **E**mf, then the thu**M**b will indicate the direction of the **M**otion).

The figure below shows the ideal emf induced in a conductor moving between a pair of poles.



The level of induced emf is zero when the conductor lies outside the magnetic field and rises to a constant value whilst it is passing through a constant field at a constant velocity. In practice there will be fringing effects at the edges of the poles and the increase/decrease in the magnetic field seen by the conductor will not be so abrupt, and hence the rise and fall in E will be more gradual.

There must be relative motion between the conductor and the magnetic field for an emf to be induced; either one or the other or both must be moving relative to one another. The relative motion (or a component of it) must be at right angles to the field – i.e. the conductor must 'cut' lines of flux for an emf to be induced.

This can be summed up by Faraday's law of induction:

Whenever the flux cutting a coil is changed an emf is induced in the coil which is proportional to the rate of change of flux. The emf induced in the coil circulates a current tending to oppose the change of flux linking the coil (Lenz's law) hence the emf induced in 1 turn of the coil is:

$$\left|E_{1Turn}\right| = \frac{d\phi}{dt}$$

so if the coil has N turns then the flux will cut all these turns and:

$$\left|E_{Coil}\right| = N\frac{d\phi}{dt}$$

<u>Example</u>

A coil of 100 turns is linked by a flux of 20mWb. If the flux is reversed in 2ms calculate the average emf induced in the coil. Using:

$$\left|E\right| = N\frac{d\phi}{dt}$$

Since the flux reverses in the time period then the total change in flux will be:

$$d\phi = 20 - (-20) \ = 40\text{mWb}$$

So the emf induced in the 100 turn coil will be:

$$\left|E\right| = 100 \times \frac{40 \times 10^{-3}}{2 \times 10^{-3}} \ = 2000\text{V}$$

These laws are simple to present and understand but their consequences have had an enormous impact. The principle of induction was used to make electric generators having a small output power at first, but leading to the massive 500MW (or larger) machines in use in modern day power stations. Whether it is a small bicycle dynamo to provide a few Watts of lighting, a car alternator producing 50-300W, a portable petrol/diesel powered generator (typically producing 1kW to 50kW), or the 500/800MW steam powered turbine driven generators in a power station, the principle of operation is exactly the same. A conductor cuts magnetic flux and emfs and currents result. The size of the system and the engineering required to realise each type are the only basic differences.

Self Inductance

If a conductor, a coil of wire or a coil on a magnetic core (former), has a constant current flowing through it then the magnetic field created by the current will be constant. If the current is increased or decreased then this will mean a change in the magnetic field. A change in the magnetic field will induce an emf. Since the conductor or coil itself is affecting itself it is said to have 'self-inductance' and the induced emf is a 'self induced emf'. Inductance exists for any wire or coil which has a current flowing in it. Sometimes we wish to have a high inductance for a specific purpose (e.g. ignition coil) other times inductance is a nuisance and presents problems, e.g. the self inductance of the wiring system of a building.

The inductance of a circuit, L, is defined as:

$$L = \frac{flux\,linkages\,in\,the\,circuit}{current\,in\,the\,circuit} = flux\,linkages\ per\ amp$$

or:

$$L = N\frac{\phi}{I}$$

where N is the number of conductors or turns of wire in a coil cutting the flux ϕ. The inductance of a coil is also related to the reluctance of the magnetic circuit on which it is wound:

$$L = \frac{N^2}{S}$$

where:

$$S = \frac{l}{\mu_0 \mu_r A}$$

The self inductance of a coil has been expressed in terms of the number of turns and its physical dimensions and magnetic properties. Hence an 'air-cored' coil would be expected to have a low inductance since its reluctance is very high ($\mu_r = 1$), whilst a coil wound on an iron core has a relatively low reluctance and hence a high inductance.

Example

A coil of 200 turns is wound uniformly over a ring having a mean diameter of 200mm and a uniform cross-sectional area of 500mm^2. Determine the inductance of the coil if the ring is made of (a) plastic, and (b) iron having a permeability of 1000.

In both cases the inductance can be calculated from:

$$L = \frac{N^2}{S}$$

however, the reluctance S will be different for the plastic and iron rings.

Now since:

$$S = \frac{l}{\mu_0 \mu_r A}$$

then:

$$S_{PLASTIC} = \frac{200 \times 10^{-3} \times \pi}{4 \times \pi \times 10^{-7} \times 1 \times 500 \times 10^{-6}} = 1 \times 10^{9}\ \mathrm{H}^{-1}$$

$$S_{IRON} = \frac{200 \times 10^{-3} \times \pi}{4 \times \pi \times 10^{-7} \times 1000 \times 500 \times 10^{-6}} = 1 \times 10^{6}\ \mathrm{H}^{-1}$$

so:

$$L_{PLASTIC} = \frac{N^2}{S_{PLASTIC}} = \frac{200^2}{1 \times 10^9} = \mathbf{4 \times 10^{-5}\ H\ (=40\mu H)}$$

$$L_{IRON} = \frac{N^2}{S_{IRON}} = \frac{200^2}{1 \times 10^{-6}} = \mathbf{0.04\ H\ (=40mH)}$$

In practice coils wound on non-magnetic materials (which have the same magnetic properties as air) are termed ‘air-cored’ coils. Remember from previous lectures the electrical reactance of an inductance can be calculated from:

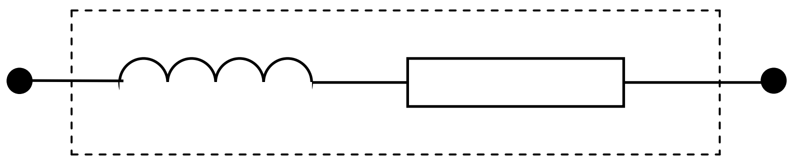
$$X_L = j\omega L$$

and so the reactance of an air-cored coil will be much less than that of an iron cored coil. The emf or voltage appearing across a coil is given by:

$$|E| = L\frac{dI}{dt}$$

This is the ‘back-emf’ of the coil. Note that there is only a voltage across the coil when there is a change of current – and hence a change in the magnetic conditions. There is no voltage across an ideal inductor with a steady (d.c.) current through it. Note that the faster the rate of change of current (i.e. the higher the value of dI / dt), then the higher the induced voltage.

In practice a coil of wire will also possess resistance, so ‘ideal’ inductors do not exist in reality. Since there is a resistance there will be an ohmic voltage drop across a real inductor when a current is flowing through it.



L = Inductance R = Resistance

Example

A coil draws a current of 2A when 12V d.c. is applied. To achieve the same value of current (i.e. $2A_{rms}$) when the coil is connected to a 50Hz supply the applied voltage is $20V_{rms}$. Calculate the inductance of the coil, the phase angle and the a.c. power dissipated.

Initially the coil is connected to a d.c. supply. Since dI / dt is zero, the voltage drop is purely across the resistance of the coil. The resistance is calculated from:

$$R = \frac{V}{I} = \frac{12}{2} = 6\Omega$$

When the coil is connected to the a.c. supply, the impedance is calculated from:

$$Z = \frac{V}{I} = \frac{20}{2} = 10\Omega$$

However, previously we have showed that for a series combination of a resistance and an inductance, the impedance is given by:

$$Z = \sqrt{R^2 + X_L^2}$$

and the reactance of the coil is obtained from:

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{10^2 - 6^2} = 8\Omega$$

Now since the reactance is:

$$|X_L| = \omega L$$

then the inductance can be calculated from:

$$L = \frac{|X_L|}{\omega} = \frac{|X_L|}{2\pi f} = \frac{8}{2\pi \times 50} = \mathbf{25.5mH}$$

The phase angle of the current relative to the supply voltage is given by:

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{8}{6} = 53.1° \text{ (lagging)}$$

Power is only dissipated in the real part of Z (ie in the resistance), so:

$$P_{AC} = I^2 R = 2^2 \times 6 = \mathbf{24W}$$

Note you can also use $P = V^2 / R$, but V here is the voltage dropped across the resistance NOT the supply voltage. This is effectively the same as the voltage in the d.c. case as for d.c. there is no voltage across L and the supply voltage is dropped across the resistance. Since the current in both cases is the same the voltage across the resistance will be the same, i.e.:

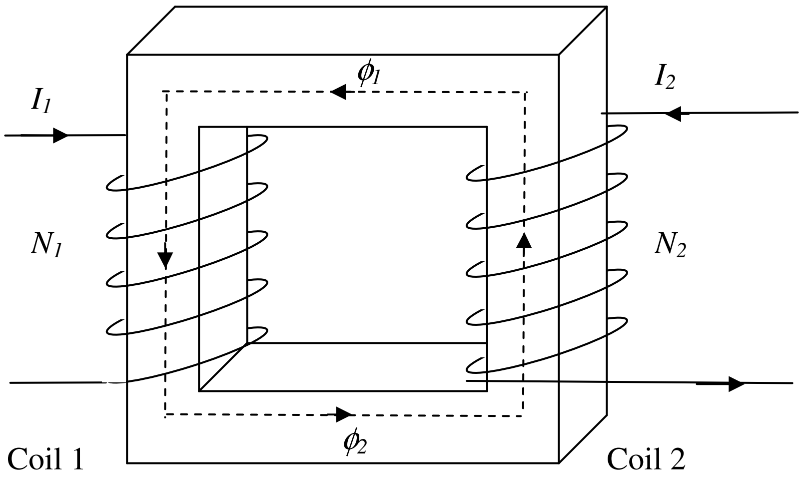
$$V_R = 12\,\text{V}$$

and:

$$P_{AC} = \frac{V^2}{R} = \frac{12^2}{6} = 24\text{W}$$

Mutual Inductance

Up to now, only one coil per magnetic circuit has been considered, but now two coils will be studied and the effects that one has on the other.

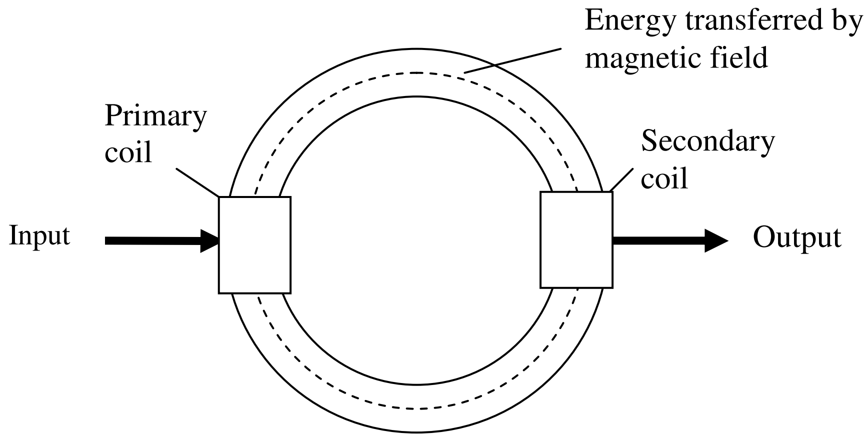


Coil 1 with current I_1 produces a flux ϕ_1
Coil 2 with current I_2 produces a flux ϕ_2

Since both coils are wound on the same magnetic circuit the flux from coil 1 will link (or couple) with coil 2 and vice-versa. If, for instance, I_1 changes then φ_1 will change which will induce a voltage in both coils. If we arrange for I_1 to be constantly changing (i.e. supplying coil 1 from an a.c. supply) then φ_1 will be alternating and a.c. voltages will be induced in both coils. (A similar argument follows for I_2). This mutual coupling of two coils, without any electrical connection between them, provides us with a very useful device known as a transformer.

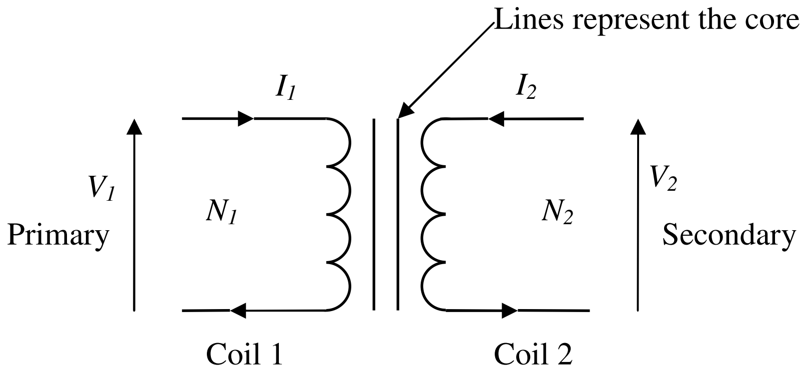
Transformers

The transformer is a device in which electrical energy is transferred from one electrical circuit to another without an electrical connection existing between the two circuits. The energy is transferred by a magnetic field which is created by current flowing through the primary coil which links with the secondary coil. Energy transfer thus occurs with electrical isolation between the two circuits.



This can be very useful, e.g. in safety critical circuits particularly in medical applications where electrical monitoring equipment connected to the patient must be isolated from the mains supply. However the main use of transformers is to alter (transform) voltage levels between circuits. Most consumer electronic products (eg. TVs, PC's, DVDs etc.) are supplied from the $230V_{rms}$ a.c. mains, but the circuits inside them operate from low voltage e.g. 12 Vd.c. supplies. A transformer can be used to lower the voltage and this is then fed into a bridge rectifier which is a device for converting a.c. voltages and currents to d.c. voltages and currents.

The equivalent circuit of a transformer may be drawn as:



Normally the winding of the transformer which is connected to the supply is called the 'primary winding' ('primary' for short) and the winding connected to the load is called the 'secondary winding' ('secondary' for short). A transformer which increases the voltage (e.g. 230V primary to 1000V secondary) is called a step-up transformer, or one which decreases the voltage (e.g. 230V primary to 10V secondary) is called a step-down transformer.

On this course we will assume an 'ideal' transformer, that is one where the magnetic coupling is perfect and there are no winding or core losses, i.e. it is 100% efficient. For such a transformer the primary and secondary voltages and currents are related by simple equations:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = Turns\ ratio$$

and:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{Turns\ ratio}$$

Combining these two equations:

$$P_{OUT} = V_2 I_2 = V_1 \frac{N_2}{N_1} \times I_1 \frac{N_1}{N_2} = V_1 I_1$$

So for an ideal transformer with no losses (ie 100% efficient) the power input to the primary winding is equal to the output power (power drawn by the load).

Example

An ideal transformer has a turns ratio of 10:1 and is connected to a 100V_{rms}, 50Hz supply. Calculate the secondary voltage, and currents in both windings when a 5Ω resistor is connected across the secondary winding.

Rearranging the voltage equation and substituting values:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad \Rightarrow \qquad V_2 = V_1 \frac{N_2}{N_1} = 100 \times \frac{1}{10} = 10\text{V}_{\text{rms}}$$

The current through the resistor is obtained using Ohm's law:

$$I_2 = \frac{V_2}{R} = \frac{10}{5} = 2\text{A}_{\text{rms}}$$

The current in the primary winding is then:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \qquad \Rightarrow \qquad I_1 = I_2 \frac{N_2}{N_1} = 2 \times \frac{1}{10} = 0.2\text{A}_{\text{rms}}$$

Power dissipated in the resistor is:

$$P_{OUT} = I_2^2 R = 2^2 \times 5 = 20\text{W}$$

or:

$$P_{OUT} = V_2 I_2 = 10 \times 2 = 20\text{W}$$

and:

$$P_{IN} = V_1 I_1 = 100 \times 0.2 = 20\text{W}$$

Example

Let us now replace the 5Ω resistor in the previous example by a load comprising a 3Ω resistor in series with a 10mH inductance.

The secondary voltage is not dependent on the load and will remain at 10V_{rms}. The total impedance is given by:

$$Z_2 = R + j\omega L = 3 + j2 \times \pi \times 50 \times 0.01 = 3 + j\,3.14 = 4.34\angle 46^\circ\ \Omega$$

Therefore the secondary current is:

$$I_2 = \frac{V_2}{Z_2} = \frac{10\angle 0^\circ}{4.34\angle 46^\circ} = 2.3\angle{-46^\circ}\ \text{A}_{\text{rms}}$$

The current in the primary winding is then:

$$I_1 = I_2 \frac{N_2}{N_1} = 2.3\angle -46^\circ \times \frac{1}{10} = 0.23\angle{-46^\circ}\ \text{A}_{\text{rms}}$$

The power dissipated in the load is (remember only the resistor dissipates power):

$$P_{OUT} = I_2^2 R = 2.3^2 \times 3 = 15.9\text{W}$$

or:

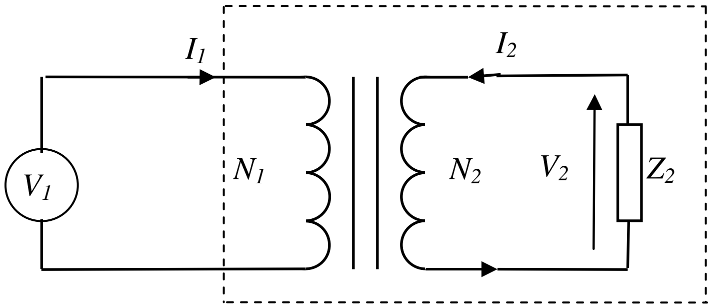
$$P_{OUT} = V_2 I_2 \cos\phi = 10 \times 2.3 \times \cos 46^\circ = 15.9\text{W}$$

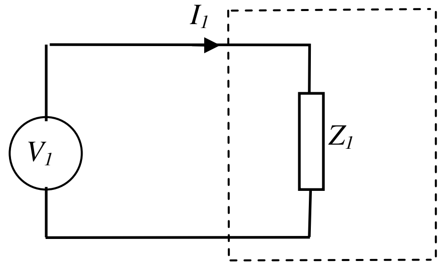
and:

$$P_{IN} = V_1 I_1 \cos\phi = 100 \times 0.23 \times \cos 46^\circ = 15.9\text{W}$$

Impedance Ratio

Consider an a.c. voltage source connected to a load. As far as the voltage source is concerned it is creating a voltage and a current flows out of it, i.e. it sees an impedance. The voltage source does not know what circuit is connected to it. Hence a transformer and its load Z_2 simply appears as a single impedance Z_1 to the source:





$$V_1 = I_1 Z_1 \quad \text{or} \quad Z_1 = \frac{V_1}{I_1}$$

Now expressing Z_1 in terms of the secondary voltage and current, since:

$$V_1 = V_2 \frac{N_1}{N_2} \quad \text{and} \quad I_1 = I_2 \frac{N_2}{N_1}$$

then:

$$Z_1 = \frac{V_1}{I_1} = V_2 \frac{N_1}{N_2} \times \frac{1}{I_2} \frac{N_1}{N_2} = \left(\frac{N_1}{N_2}\right)^2 \frac{V_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

therefore:

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

The impedance in the primary circuit equals the impedance in the secondary circuit multiplied by the square of the turns ratio. This enables impedances to be transferred (sometimes termed ‘referred’, ‘reflected’ or ‘transferred’) from the secondary to the primary and vice-versa.

Example

Using the previous example where $Z_2 = 4.34\angle 46°\ \Omega$, refer this to the primary side:

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2 = \left(\frac{10}{1}\right)^2 \times 4.34\angle 46^\circ = 434\angle 46°\ \Omega$$

The input current will be:

$$I_1 = \frac{V_1}{Z_1} = \frac{100\angle 0^\circ}{434\angle 46^\circ} = 0.23\angle{-46°}\ \mathrm{A_{rms}} \text{ (as before)}$$

Example

Find the turns ratio to match a 3Ω loudspeaker to an amplifier having an impedance of 600Ω.

For maximum power transfer from the amplifier to the loudspeaker the ‘load’ seen by the amplifier must appear as 600Ω. (see previous notes about maximum power transfer $R_{LOAD} = R_{INT}$).

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2 \qquad \text{or} \qquad \left(\frac{N_1}{N_2}\right)^2 = \frac{Z_1}{Z_2} = \frac{600}{3} = 200$$

and the required turns ratio is given by:

$$\left(\frac{N_1}{N_2}\right) = \sqrt{200} = \mathbf{14.1}$$

VA rating

Transformers are often rated by their VA rating. This is simply the product of the rated voltage and rated full-load current of a winding, neglecting any phase difference between the voltage and current waveforms. For example a 132kVA transformer with an input voltage of 6.6kV has an input current of 132000 / 6600 = 20A. If the secondary voltage is 330V then the secondary current will be 132000 / 330 = 400A.

Input currents, voltages, and flux

Although it is a current flowing through a coil that creates the flux in the iron core, it is in fact the input voltage which determines the maximum value of the flux. It can be shown that the voltage is related to the flux in the core (for a sinusoidal supply) by:

$$V_{rms} = 4.44 N f \, \phi_{MAX} \qquad \text{or} \qquad \phi_{MAX} = \frac{V_{rms}}{4.44 N f}$$

Example

An ideal transformer is fed from 110V_{rms} 60Hz. Its maximum core flux is 3mWb. The output of the transformer needs to be 240V_{rms}. Calculate the number of turns on the primary and secondary windings. If the transformers rating is 550VA what are the primary and secondary currents?

Firstly the number of turns on the primary may be found by rearranging the equation:

$$V_{1rms} = 4.44 N_1 f \, \phi_{MAX}$$

to give:

$$N_1 = \frac{V_{1rms}}{4.44 f \, \phi_{MAX}} = \frac{110}{4.44 \times 60 \times 3 \times 10^{-3}} = 138 \text{ turns}$$

The number of turns on the secondary may be obtained from:

$$N_2 = \frac{V_2}{V_1} \times N_1 = \frac{240}{110} \times 138 = 301 \text{ turns}$$

The primary and secondary currents may be obtained from the VA rating of the transformer:

$$550VA = V_1 I_1 = V_2 I_2$$

The primary current is hence:

$$I_1 = \frac{550}{110} = 5\text{A}_{\text{rms}}$$

and the secondary current is:

$$I_2 = \frac{550}{240} = 2.3\text{A}_{\text{rms}}$$

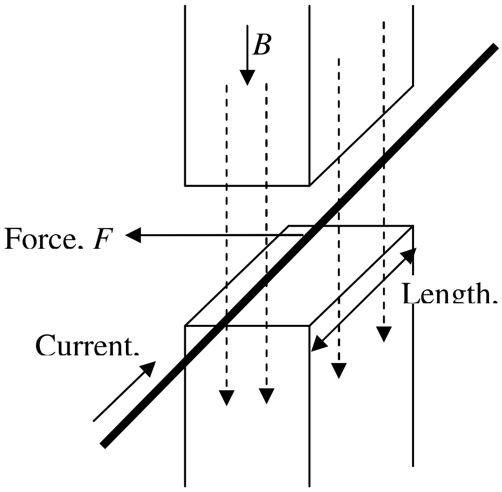
Force on a Current Carrying Conductor

When a current carrying conductor is placed in a magnetic field it will experience a force. The force will be increased by increasing:

(a) the current, *I*

(b) the strength of the magnetic field, *B*

(c) the length of the conductor at right angles to the field, *L*



Force on a current carrying conductor

Force (*F*) is proportional to the Flux Density (*B*) × Current (*I*) × Length (*L*). The direction of the force is given by Fleming's Left Hand Rule. (Place thumb, first and second fingers of left hand at right angles to one another. If **F**irst finger points in direction of the **F**ield, the se**C**ond finger in the direction of the **C**urrent, then the thu**M**b will indicate the direction of the **M**echanical force).

The unit of Flux Density is taken as the density of a magnetic field such that a conductor carrying a current of 1A at 90° to that field has a force of 1N per metre on it. The unit of Flux Density, B, is the Tesla (T) (occasionally Webers/m^2). Thus the force on the conductor is written as:

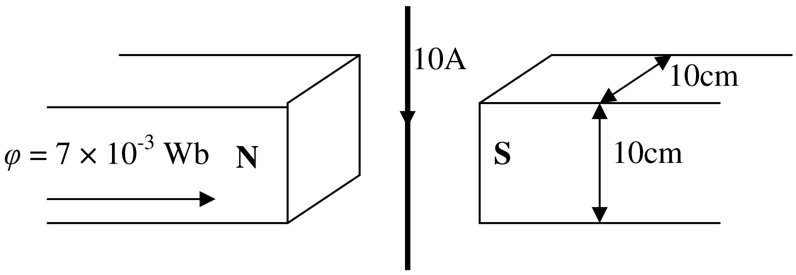
$$F = B \times I \times L$$

For a magnetic field having a cross-sectional area, *A*, and a uniform flux density, *B*, the total flux in Webers (Wb) (represented by the symbol φ) is:

$$\phi = B \times A$$

Example

For the system shown calculate the force on the conductor (neglect fringing).



Area of pole 'face', A = 10cm × 10cm = 100cm^2 = 100×10^{-4} m

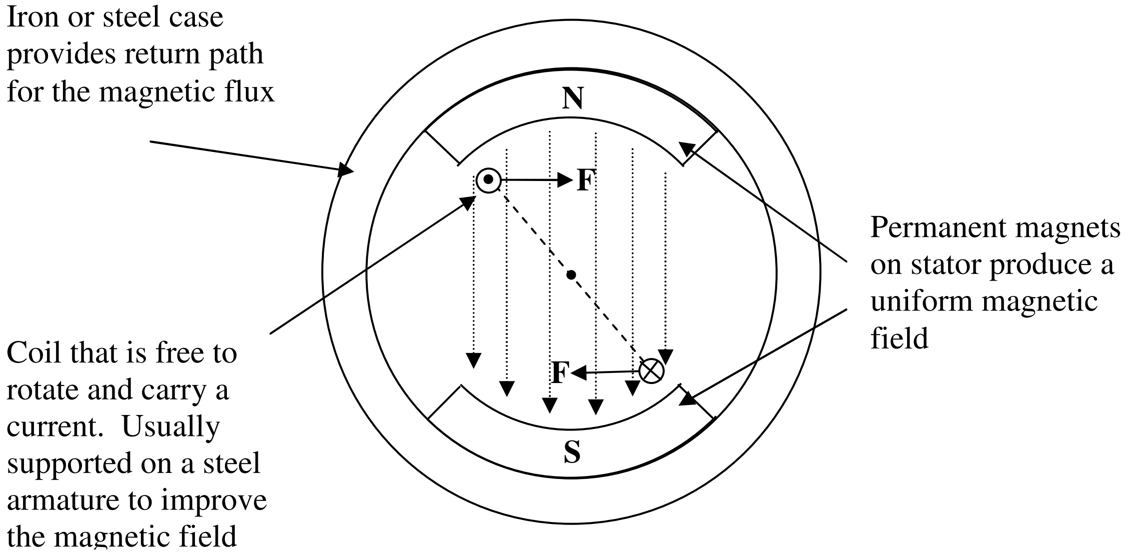
Flux density, $B = \varphi / A = 7 \times 10^{-3} / 100 \times 10^{-4} = 0.7$ T

Force = $B\,I\,L = 0.7 \times 10 \times 0.1 =$ **0.7 N**

The direction of the force is out of the paper (Fleming's left hand rule).

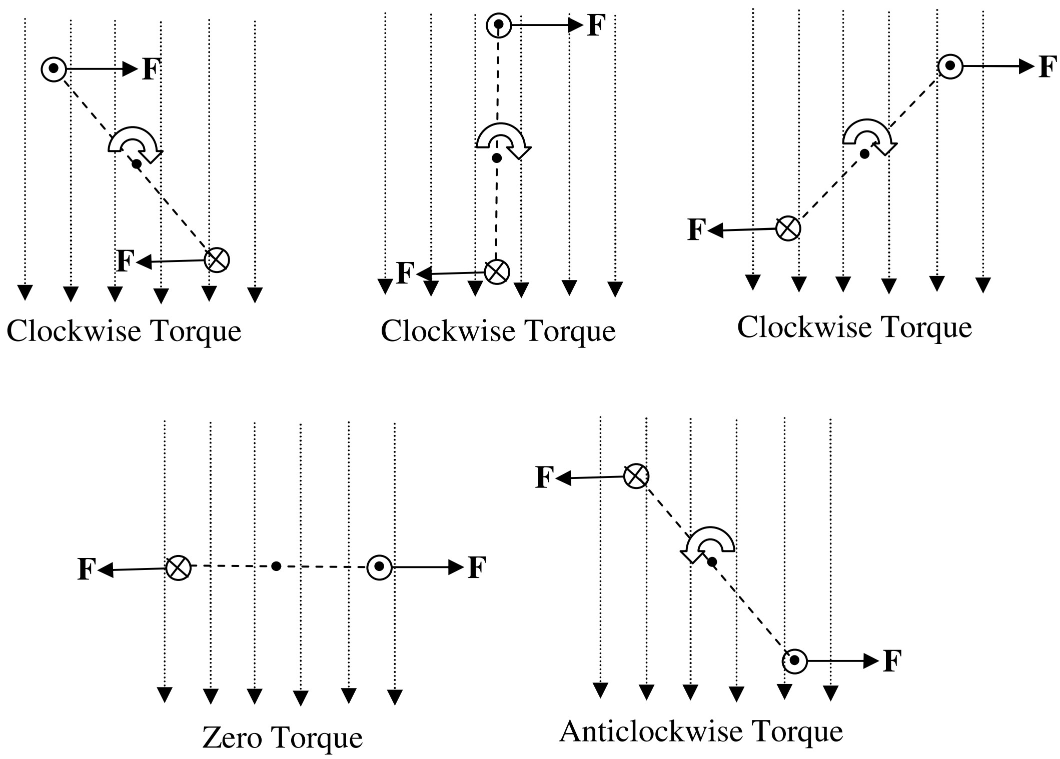
DC Motors

We have seen how a current carrying conductor placed in a magnetic field experiences a force. This concept can be extended to produce rotary motion, or a motor. Consider a coil that is free to rotate within a magnetic field produced by permanent magnets as shown in the figure below:



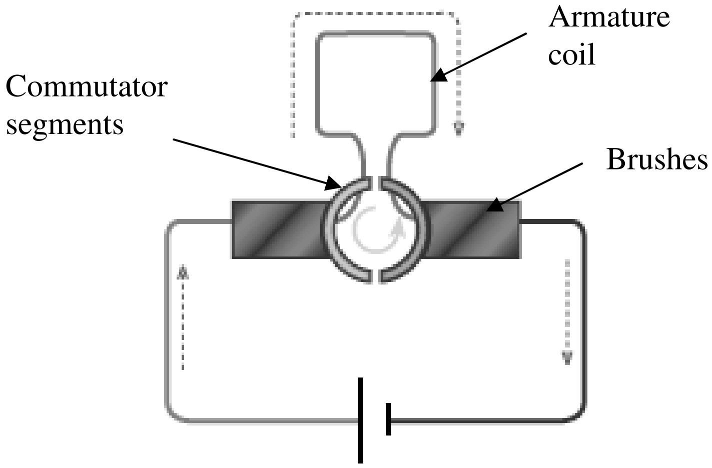
Schematic of a d.c. machine

Consider a number of positions of the coil as it rotates whilst carrying a current:



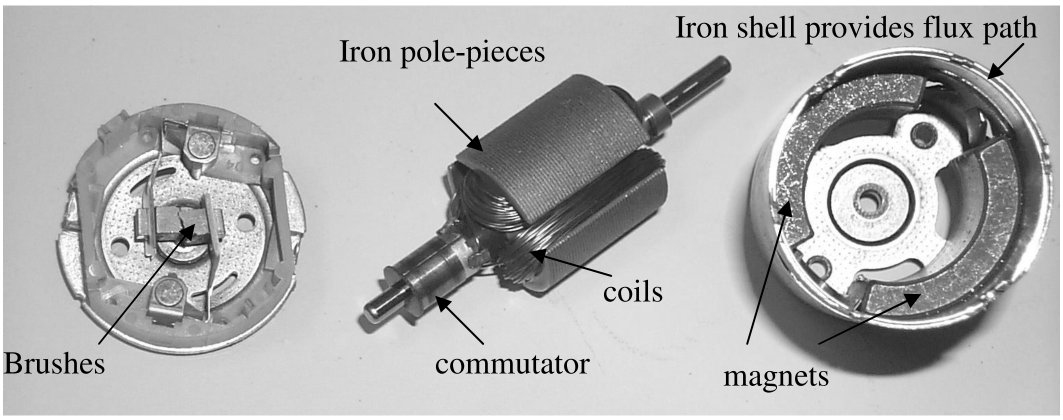
Torque on coil for various positions

Clearly, unless the direction of the current (and hence force couple) is changed, continuous rotation is not possible. This problem can be overcome by the use of a rotary switch called a commutator, which reverses the direction of the current twice every rotation. The inertia of the rotating part, which is called the rotor or armature, keeps the motor rotating during the switching periods. The commutator typically consists of a number of copper contacts fixed around the circumference of the rotating part of the machine which are connected to the coil windings, and a set of carbon brushes fixed to the stationary part of the machine which are connected to the electrical supply.



Action of the commutator

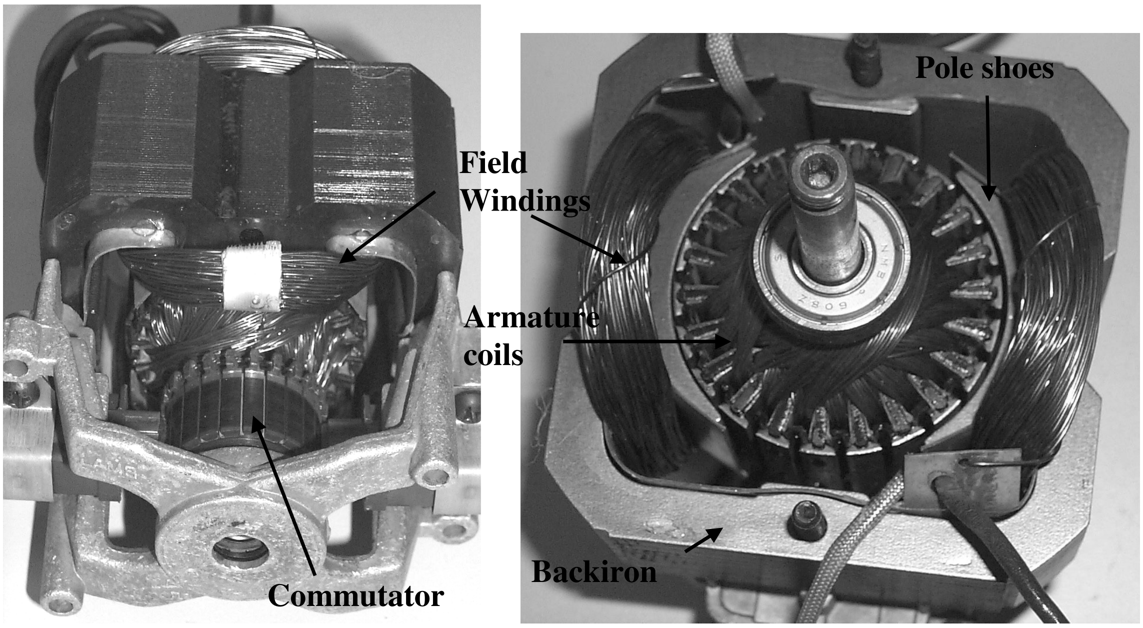
Clearly it can be seen that as the rotor rotates through 180° the commutator segments will also rotate and will connect with the brush connected to the opposite side of the supply, and hence the direction of current in the coil will reverse. By arranging the switch over point continuous motion can be achieved. In practice several coils are wound on the rotor and the commutator will thus have more segments. This is to reduce fluctuations in the torque produced, but the overall principle is as described above. The supply of current to the armature windings is from a d.c. source so this type of motor is referred to as a d.c. motor.



Small permanent magnet brushed motor

A typical example of a small brushed permanent magnet motor is shown in the figure above. The magnets are formed in arcs so as to provide a uniform magnetic field in the air-gap and the armature coils are wound on laminated iron pole-pieces to improve the flux linkage. The flux return path between the magnets is provided by the iron shell or case of the motor, which is often referred to as the 'backiron'.

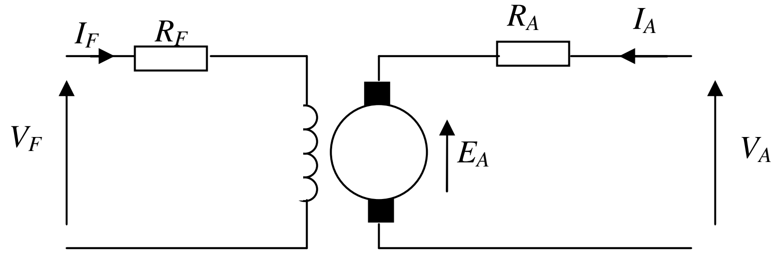
The fixed magnetic field may be either produced by a permanent magnet as shown above or may be produced electromagnetically by a current carrying coil referred to as the 'field' winding as shown below.



Wound field d.c. motor

There are several different formats of d.c. motor depending on whether the motor has permanent magnets or a wound field winding. In the case of the latter there are alternative methods of connecting this which can alter the motor's performance characteristics.

Separately excited motor



Equivalent circuit of a wound field d.c. motor

The field winding is supplied from a d.c. source, V_F, and the armature is supplied from another d.c. source, V_A, (i.e. separate supplies hence the name). The current, I_F, flowing through the field winding will produce a magnetic field which is cut by the rotating conductors on the armature and hence there will be an induced emf, E_A, (back emf) which is proportional to the magnetic field and also the speed of rotation of the armature. If the armature inductance is neglected then the armature circuit equation becomes:

$$V_A = E_A + I_A R_A$$

The induced emf is proportional to the speed of rotation (ω in rads/s) and the flux (ϕ in Wb) hence:

$$E_A = K_A \phi \omega = K I_F \omega$$

and since the voltage drop across the armature resistance is usually very small then:

$$V_A \approx K I_F \omega \quad \text{or} \quad \omega \approx \frac{V_A}{K I_F}$$

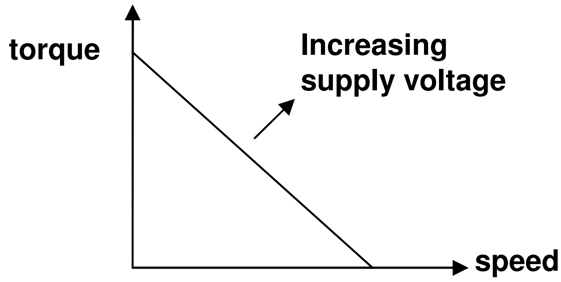
which shows that the speed of the motor varies directly with the supply voltage. The torque produced is proportional to the field and the armature currents and this can be expressed using the same constant K as above as:

$$T = K I_F I_A$$

With separate excitation both I_F and V_A can be controlled separately.

Permanent magnet (PM) motor

This type of motor has permanent magnets in place of the field coils on the stator and hence it can be viewed as a separately excited machine with a fixed magnetic field. It is more efficient than a wound field machine since there are no losses in the field winding. The ideal torque speed curve is shown below:



Torque-speed characteristic for a PM brushed d.c. motor

Since the magnetic field in a permanent magnet motor cannot be altered then the equations for the torque and back emf may be simplified:

$$V_A = E_A + I_A R_A$$

The induced emf is proportional to the speed of rotation (ω) and the flux (ϕ) hence:

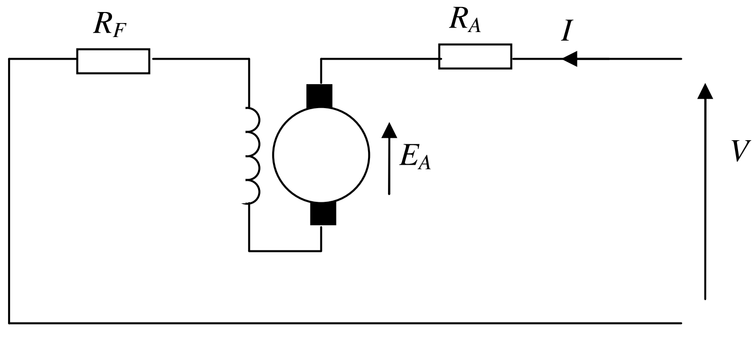
$$E_A = K_e \omega$$

$$T = K_T I_A$$

The constants K_e and K_t are equal in magnitude and have units of V/rad/s and Nm/A respectively.

Series d.c. motor

If the field winding consists of a few turns capable of carrying the full armature current then it may be connected in series with the armature winding:



Series connected wound field motor

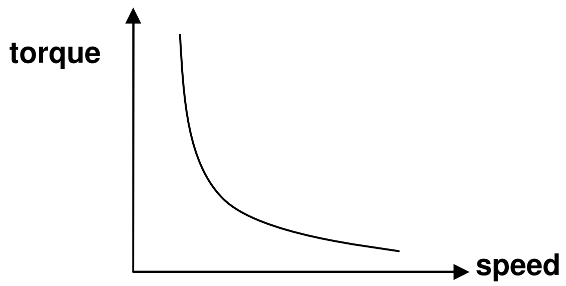
This now means that the field current and the armature current are equal and so:

$$T = KI^2 \quad \text{or} \quad I = \sqrt{\frac{T}{K}}$$

and:

$$\omega = \frac{V}{\sqrt{KT}}$$

which means that the torque and speed are no longer independent and the resultant torque speed curve is:



Torque-speed characteristic for a wound field series connected d.c. motor

This type of characteristic is very useful for a vehicle traction motor where the machine can develop high accelerating torques at low speeds and as the speed rises the torque reduces until it is just sufficient to maintain the speed of the vehicle. Speed control is achieved by means of variation of the supply voltage since for a given torque it may be seen from the equation above that the speed is directly proportional to the voltage.

Note that for a very lightly loaded series machine the current flowing is low and hence the magnetic field produced by the field winding is low and the speed will increase to account for this. In small machines usually friction (e.g. from the brushes sliding over the commutator, the bearing and windage losses especially if a cooling fan is fitted) will limit this speed to a safe value, but care may need to be taken with larger machines to ensure the rotor speed is maintained to a safe level.

Note when starting d.c. motors the back emf will be zero and very large currents will flow. In small motors the armature resistance is usually sufficient to limit this to a safe value, but in large motors precautions must be taken. This can be achieved by including an additional starting resistance which is switched out of circuit once the motor is running or in the case of modern electronic motor controllers these usually incorporate a 'soft-start' facility.

Universal motor

A variant of the wound field d.c. motor is the universal motor. The name derives from the fact that it may use an a.c. or d.c. supply, although in practice they are nearly always used with a.c. supplies. The principle is that in a wound field a.c. motor the current in both the field and the armature (and hence the resultant magnetic fields) will alternate (reverse polarity) at the same time, and hence the mechanical torque generated is always the same. In practice the magnetic circuit is laminated to reduce losses and the resultant motor is generally less efficient than an equivalent pure d.c. motor.

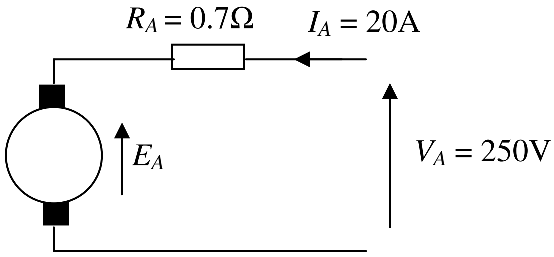
The advantage of the universal motor is that a.c. supplies may be used on motors which have the typical characteristics of d.c. motors, specifically high starting torque and very compact design if high running speeds are used. The negative aspect is the maintenance and reliability problems caused by the commutator, and as a result such motors will rarely be found in industry but are the most common type of a.c. supplied motor in domestic equipment such as electric drills, vacuum cleaners, mixers etc. which are only used intermittently.

Example

A permanent magnet d.c. motor runs at 500rpm and draws an armature current of 20A when connected to a 250V d.c. supply. The motor has an armature resistance of 0.7 Ω. Calculate the output torque, power and efficiency.

To use the derived equations we need the speed in radians/s:

$$500rpm = \frac{500}{60} rps = \frac{500}{60} \times 2\pi \ = \mathbf{52.36\ rad/s}$$



Since:

$$V = E + I_A R_A$$

then:

$$E = V - I_A R_A = 250 - 20 \times 0.7 \ = \mathbf{236V}$$

Now for a permanent magnet motor:

$$E = k_e \omega \qquad \text{and} \qquad T = k_t I \qquad \text{and} \qquad k_e = k_t$$

So:

$$k_e = \frac{E}{\omega} = \frac{236}{52.36} \ = 4.507\text{V/rad/s}$$

and:

$$k_t = 4.507\text{Nm/A}$$

So the motor torque is:

$$T = k_t I = 4.507 \times 20 \ = \mathbf{90.14Nm}$$

and the output power is:

$$P = T \times \omega = 90.14 \times 52.36 \ = \mathbf{4720\ W}$$

The efficiency is given by:

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100 = \frac{P_{OUT}}{P_{OUT} + P_{LOSSES}} \times 100$$

The losses are in the form of heat in the 0.7Ω resistance of the armature windings:

$$P_{LOSSES} = I^2 R = 20^2 \times 0.7 = 280 \text{ W}$$

Therefore the efficiency is:

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{LOSSES}} \times 100 = \frac{4720}{4720 + 280} \times 100 \ = \mathbf{94.4\%}$$

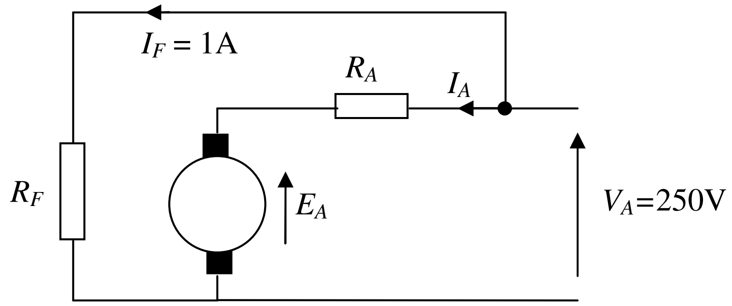
Check:

$$P_{IN} = V \times I = 250 \times 20 \ = 5000 \text{ W}$$

$$P_{IN} = P_{OUT} + P_{LOSSES} = 4720 + 280 = 5000 \text{ W}$$

The permanent magnets are replaced by a field winding connected across the supply, which draws a current of 1A to produce the same level of field as the magnets. Calculate the efficiency of this new motor.

Since the magnetic field is the same, the induced emf, current, power output and losses in the armature winding will remain unchanged, however there is now a power loss in the conductors of the field winding which will effect the efficiency.



$$P_{LOSS\_FIELD} = 250 \times 1 = 250 \text{ W}$$

Alternatively:

$$R_F = \frac{V}{I_F} = \frac{250}{1} = 250\ \Omega$$

$$P_{LOSS\_FIELD} = I_F^2 \times R_F = 1^2 \times 250 = 250 \text{ W}$$

New efficiency is:

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{LOSSES}} \times 100 = \frac{4720}{4720 + 280 + 250} \times 100 = \mathbf{89.9\%}$$

Brushless motors

One of the major problems with a brushed d.c. machine is the disadvantages associated with the commutator and brush gear. The carbon brushes wear and therefore require regular replacement, and the residue (tiny particles of carbon) can contaminate the bearings and the surrounding environment. The need for a mechanical contact between the brush face and commutator introduces a braking torque which has to be subtracted from the motor output torque. There is also sparking at the surface of the commutator as the brushes move from one segment to another, which can present problems in certain environments and produce electromagnetic interference.

With the recent advances in electronics it is now possible to replace the mechanical switching of the currents in the winding by semiconductor switches. The windings of a brushless motor are placed on the fixed stator and consist of a number of coils which are displaced at regular intervals around the stator. By switching the currents in these coils in a certain order a magnetic field which appears to rotate can be created. Permanent magnets, which are now fixed to the rotating rotor will follow this rotating field and hence produce rotary motion at the output shaft. In order for the system to function correctly the position of the rotor must be known at any instant so that the currents in the stator windings can be switched in the correct order and at the correct speed. This position sensing is usually achieved using Hall effect devices (electronic switches which react to a magnetic field) or optical sensors. The information from the sensors is passed to the electronic drive which then switches the correct windings.

The brushless d.c. motor hence has a number of advantages over its brushed counterpart. Since the commutator and brushes have been eliminated then they are virtually maintenance free, more efficient, since there is no brush drag and therefore may be operated up to much higher speeds. Since the windings are stationary it is possible to place the stator outside the rotor so that it is easier to dissipate the heat from the resistance loss (copper loss or I^2R loss). This is the preferred topology, although brushless motors where the rotor is external stator do exist for certain applications. The problem of sparking of brushes has been eliminated so this type of machine may be used in a hazardous environment and electromagnetic interference is reduced.