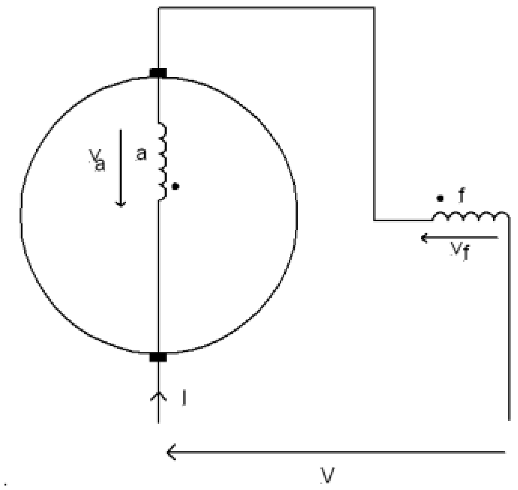
EEE6120 – Modelling of Machines

<u>2013/14 Semester 2 Examination – Worked Solutions</u>

[Notes in italics within square parenthesis are intended to provide background context to the question and/or to give further details of the methodology expected].

1.

a)



The general form of the voltage equations are:

$$\begin{vmatrix} v_a \\ v_f \end{vmatrix} = \begin{vmatrix} R_a + L_a p & \omega_r M \\ 0 & R_f + L_f p \end{vmatrix} \begin{vmatrix} i_a \\ i_f \end{vmatrix}$$

On DC: $p=0$

On AC: $p=j\omega_s$

Constraining equations:

$V = V_a + V_f$

$I = I_a = I_f$

The resulting voltage equations are:

DC operation:

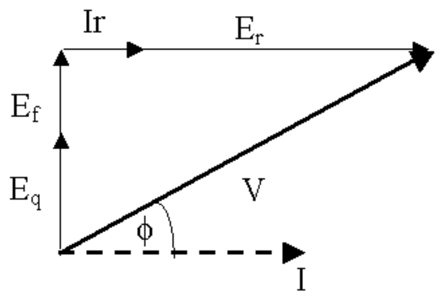
$V = I\,(R_a + R_f + \omega_r M)$

AC operation:

$V = I\,(R_a + R_f + \omega_r M + j(X_a+X_f))$

(4)

b) The phasor diagram AC operation:



Where:

$r = R_f + R_a$

$E_f = jXI_f$

$E_q = jXI_q$

$Er = I\omega_r M$

(2)

c) On an AC supply:

i)

$$T = \frac{Output\ power}{\omega_r} = \frac{840}{2094} = 0.401Nm$$

From the operating point of 0.401 Nm at 7.0Arms:

$$M = \frac{T}{I^2} = \frac{0.401}{7.0} = 8.19 \times 10^{-3}H$$

From the phasor diagram

$$Vcos\emptyset = I(R + \omega_r M)$$

Hence,

$$R = \frac{Vcos\emptyset}{I} - \omega_r M = \frac{230 \times 0.7}{7.0} - 2094 \times 8.19 \times 10^{-3} = 5.86\Omega$$

Similarly, from the phasor diagram:

$$Vsin\phi = IX$$

$$\therefore X = \frac{Vsin\phi}{I} = \frac{230 \times 0.714}{7.0} = 23.46\Omega$$

$$Z = 5.86 + j23.46\Omega$$

ii) Copper loss is given by:

$$P_{cu} = I^2R = 7.0^2 \times 5.86 = 287W$$

iii) The efficiency is given by either:

$$\eta = \frac{P_{mech}}{P_{elec}} = \frac{P_{mech}}{VIcos\phi} = \frac{840}{230 \times 7 \times 0.70} = 74.5\%$$

or

$$\eta = \frac{P_{mech}}{P_{mech} + P_{cu}} = \frac{840}{840 + 287} = 74.5\%$$

In both cases, this neglects core losses which are inherently not included in universal machine theory (unless added in post analysis)

iv) For starting torque at standstill $\omega_r = 0$

$$|I| = \frac{V}{Z} = \frac{230}{\sqrt{5.86^2 + 23.46^2}} = 9.51\ Arms$$

Hence starting torque is:

$$T = MI^2 = 8.19 \times 10^{-3} \times 9.51^2 = 0.74Nm$$

(8)

d) On DC:

i) $V = I(R + \omega_r M)$

Torque for 840Nm = 0.401Nm as before, hence $I_{DC} = 7.0A$ as before.

$V_{DC} = 7.0 \times (5.86 + 2094 \times 8.19 \times 10^{-3})$ =161 V

ii) Current at standstill:

$$I = \frac{V}{R} = \frac{161}{5.86} = 27.5A$$

Hence, starting torque is:

$$T = MI^2 = 8.19 \times 10^{-3} \times 27.5^2 = 6.18Nm$$

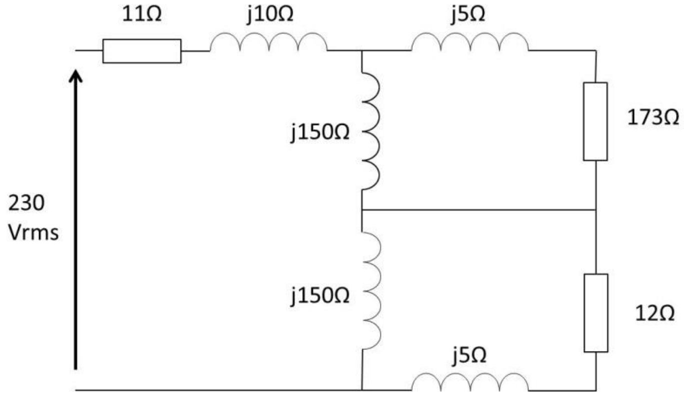
(4)

e) In practice, the starting torque obtained in part (d) would not be achieved, since the four-fold increase in current beyond the rated current of 7A is likely to result in significant magnetic saturation and hence a reduction in M.

(2)

2.

a) At 1305rpm, the slip for a 4-pole machine is 13% or 0.13. The equivalent circuit is given by:



(3)

The positive sequence equivalent impedance is:

$$Z_p = \frac{\left((173 + j5) \times j150\right)}{173 + j5 + j150} = \frac{173\angle 1.65^o \times 150\angle 90^o}{232\angle 41.8^o}$$

$$= \frac{25960\angle 91.65^o}{232\angle 41.8^o} = 111.8\angle 49.8^o\Omega = 72.2 + \mathrm{j}85.4\Omega$$

Similarly, the negative sequence impedance is:

$$Z_n = \frac{\left((12 + j5) \times j150\right)}{12 + j5 + j150} = \frac{13\angle 22.6^o \times 150\angle 90^o}{155\angle 85.6^o}$$

$$= \frac{1950\angle 112.6^o}{155\angle 85.6^o} = 13\angle 27^o\Omega = 11.2 + \mathrm{j}5.7\Omega$$

Total impedance is given by:

Z_{TOTAL}= (11+72.2+11.2) + j(10+85.4+5.9) = 94.4 + j101.3**Ω** = 138.8 ∠46.7**°** **Ω**

i) Input current $= \frac{V}{Z} = \frac{230\angle 0^0}{138.8\angle 46.7^0} =$1.65 ∠-46.7^0 Arms

ii) Voltage across positive branch:

$$V_p = I_{in}Z_p = 1.65\angle -46.7^0 \times 111.8\angle 49.8^o = 184.5\angle 3.1^o Vrms$$

Current through 173**Ω** resistor

$$Ip = \frac{184.5\angle 3.1^o}{173\angle 1.7^o} = 1.07\ \angle 1.4^o Arms$$

And similarly across the negative branch:

$$V_n = I_{in}Z_n = 1.65\angle -46.7^0 \times 13\angle 27^o = 20.8\angle -19.7^o Vrms$$

$$I_n = \frac{21.4\angle -19.7^o}{13\angle 22.6^o} = 1.60\ \angle -42.3^o Arms$$

$P_{mech} = \left(I_p^2 \frac{R_2'}{2s} - I_n^2 \frac{R_2'}{(2-s)2}\right)(1 - s)$ =(198-30.8)×(0.87) = 145.5W

Hence, the torque is given by:

$$T = \frac{P_{mech}}{\omega_r} = \frac{145.5}{\frac{1305 \times 2\pi}{60}} = 1.06Nm$$

iii) Two approaches are possible:

Electrical input power is given by:

$$P_{elec} = VIcos\emptyset = 230 \times 1.65 \times \cos(-46.7) = 259W$$

Since the model does not account for iron loss, then the difference between electrical input and mechanical is equal to the copper loss

Hence, copper loss = 259-145.5 = **113.5W**

Alternatively, the copper loss in the stator is:

$$P_{cu-stator} = 1.65^2 11 = 30W$$

Positive sequence copper loss:

$$P_{Cu\_pos} = I_p^2 \frac{R_r}{2} = 1.07^2 \times 22.5 = 25.8W$$

Similarly for the negative sequence copper loss:

$$P_{Cu\_neg} = I_n^2 \frac{R_r}{2} = 1.60^2 \times 22.5 = 57.6W$$

Hence, total copper loss is:

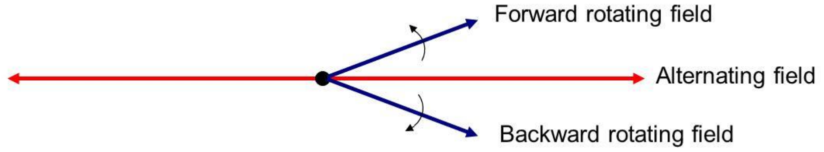
$$P_{Cu\_total} = P_{cu-stator} + P_{cu-pos} + P_{cu-neg} = 30 + 25.8 + 57.6 = 113.5\ W$$

iv) Efficiency is given by:

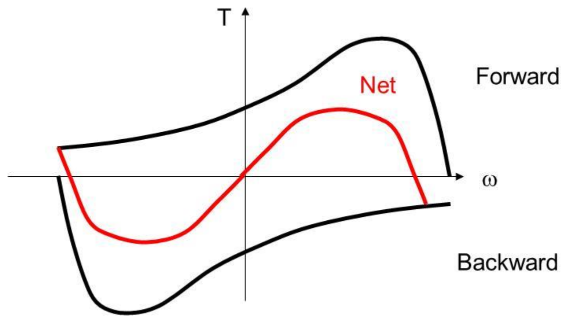
$$\eta = \frac{Output\ power}{Input\ power} \times 100\% = \frac{P_{mech}}{VIcos\emptyset} \times 100\% = \frac{145.5}{259} \times 100\% = 56.1\%$$

(10)

c) A single phase stator produces an alternating field along one axis rather than a rotating field. This alternating field can be resolved into two contra-rotating fields.



Each of the two contra-rotating fields individually produces a torque-speed characteristic which would be similar to that of a three-phase machine. Hence, net torque-speed characteristic is the sum:



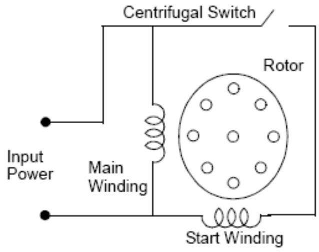
Hence, there is a net torque once the machine is rotating in one direction or the other but no starting torque.

(3)

d) <u>One</u> of the following:

<u>Split-phase motor</u>

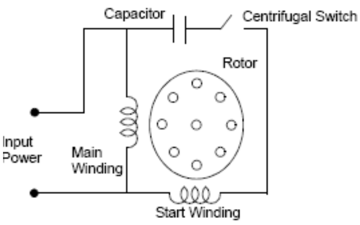
- Two stator winding (main + auxiliary starting) -displaced by 90^o in space
- Auxiliary has higher resistance to reactance ratio – hence current has different phase to main winding
- Resulting rotating field produces starting torque but inefficient – auxiliary winding is usually switched out by a centrifugal switch



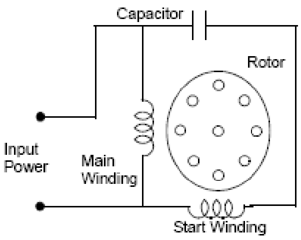
Single-phase induction motors with starting capacitors

- All variants use an auxiliary starting winding
- Use capacitor in series with auxiliary starting winding to produce a large phase shift in current and hence a phase-shifted field
- Higher efficiency and higher starting torques than split-phase machines
- Used where reasonably high starting torques required compared to rated torque
- Several different configurations used depending on applications requirements and cost / performance constraints

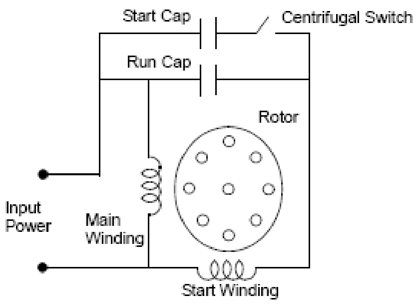
Capacitor – start motor - Capacitor only used for starting – switched out by centrifugal switch



Permanent split capacitor (PSC) -Capacitor left in circuit. Has some advantages in terms of normal running power factor and suitability for variable speed operation from power electronic inverter.

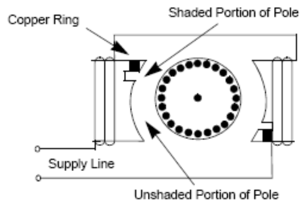


Capacitor start – capacitor run (sometimes called two value capacitor motor). Combination of capacitor start /PSC motors. Most complex but best performance as capacitor values can be optimised for start and run separately.

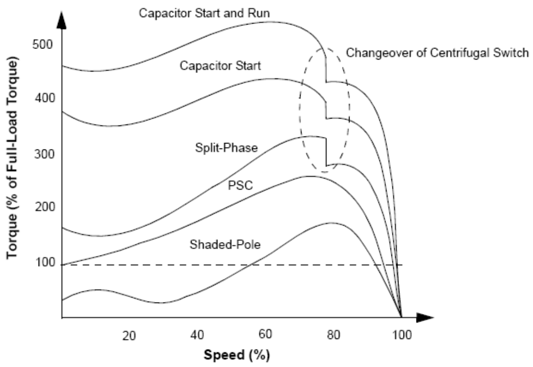


Shaded-pole motor

- Retains single stator coil
- Short-circuited copper ring placed around section of stator pole
- Induced currents flow in the ring (which oppose incident field) – results in a phase shifted flux variation under the 'shaded' region of pole
- Produces low starting torque and inefficient during running
- Only used in low cost / low power applications with predictable starting loads (e.g. cooling fans, fan heaters



+ Appropriate torque-speed curve from the following:



(4)

3.

a)

i) Number of strokes per revolution is given by:

$$Number\ of\ strokes = Number\ of\ phases\ \times Number\ of\ rotor\ teeth = 3 \times 8 = 24$$

ii) Angular displacement of one stroke is hence 360/24 = 15^o (= π/12 rads)

iii) Integrating the areas between the curves up to 5A using the trapezium rule with 1A intervals gives:

Fully aligned curve:

$$A_{0\to1} = \frac{\Psi_1}{2} = \frac{0.6}{2} = 0.30\text{ J}$$

$$A_{1\to2} = \frac{\Psi_1 + \Psi_2}{2} = \frac{0.6 + 1.02}{2} = 0.81\text{J}$$

$$A_{2\to3} = \frac{\Psi_2 + \Psi_3}{2} = \frac{1.02 + 1.18}{2} = 1.10\text{J}$$

$$A_{3\to4} = \frac{\Psi_3 + \Psi_4}{2} = \frac{1.18 + 1.24}{2} = 1.21\text{J}$$

$$A_{4\to5} = \frac{\Psi_4 + \Psi_5}{2} = \frac{1.24 + 1.28}{2} = 1.26\text{J}$$

For a current of 5A, total co-energy in aligned position:

$$A_{0\to5} = A_{0\to1} + A_{1\to2} + A_{2\to3} + A_{3\to4} + A_{4\to5} = 4.68\text{J}$$

The same procedure could be applied to the un-aligned characteristic, but it is also reasonable to assume that the characteristic is linear, and hence the co-energy in the unaligned positions for 5A is hence given by:

$$U_{0\to2} = \frac{5\Psi_5}{2} = \frac{5\times0.2}{2} = 0.5\text{J}$$

Hence the co-energy change between the unaligned and aligned positions at 5A is:

$$\Delta W' = A_{0\to5} - U_{0\to5} = 4.18\text{J}$$

Hence the average torque over the stroke for 5A is:

$$T = \frac{\Delta W'}{\theta} = \frac{4.18}{\frac{\pi}{12}} = 16Nm$$

(8)

b) In the aligned position, the cores begin to saturate appreciably at a current of ~2A. For Silicon iron this corresponds to a flux density of ~1.5T

[as with all questions concerned with an interpretation of saturation, there is a reasonable tolerance on this]

The flux density is given by:

$$B_g = \frac{\mu_0 NI}{l_g}$$

Rearranging and noting from Ampere's Law that 2 coils produce the mmf to drive the flux across two airgaps then the number of turns per coil is:

$$N = \frac{B_g l_g}{\mu_0 I} = \frac{1.5 \times 0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 2} = 298\ turns\ per\ coil$$

(3)

c) The maximum flux change at 5A occurs between 9^{o} and 12^{o}. The flux-linkage change is ~0.34 Wb. At 2000rpm, this yields an induced emf of:

$$e = \frac{\partial\psi}{\partial\theta} \times \frac{\partial\theta}{\partial t} = \frac{0.34}{3 \times \frac{\pi}{180}} \times \frac{200 \times 2\pi}{60} = 136V$$

(3)

d) *[This calculation has not been covered in notes or previous exam papers – requires some insight from candidates to put the various concepts together]*

In the un-aligned position (i.e. 0°) the inductance (which can be reasonably regarded as constant) is given by:

$$L = \frac{\psi}{I} = \frac{0.4}{10} = 0.04\ H$$

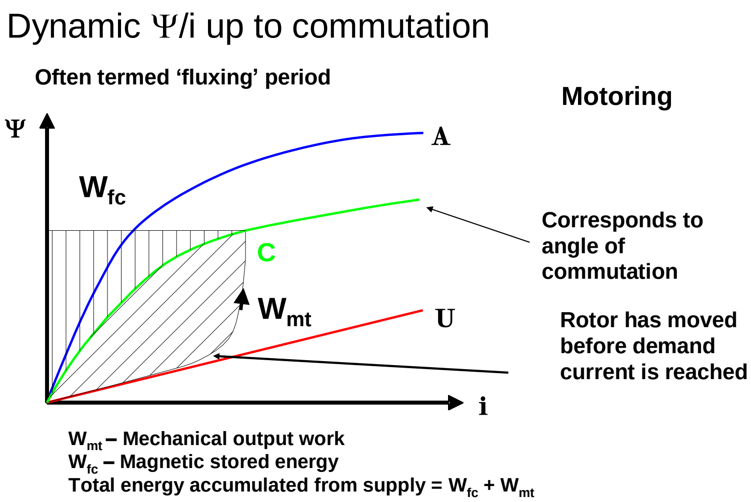
At 200V and standstill (i.e. no induced emf) then:

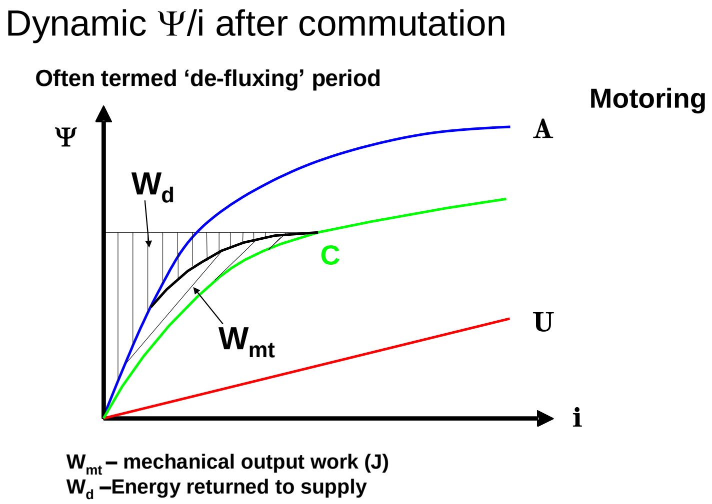
$$\frac{di}{dt} = \frac{V}{L} = \frac{200}{0.04} = 5000\ A/s$$

Time taken for current to rise to 5A is 1ms.

(3)

e)

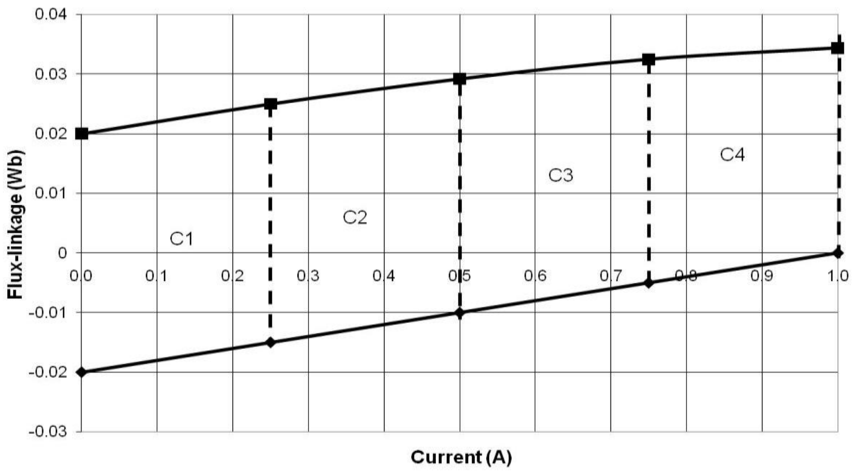




(3)

4.

a) Re-plotting the data at -90° and +90° as a flux-linkage versus current characteristic yields:



The co-energy change can be estimated by trapezoidal integration of the four areas C1 to C4 shown in the graph above. Using this approach:

The change in co-energy for 1.0A is C1+C2+C3+C4 = 0.01+0.0099+0.0096+0.0090=0.0385J

Change in rotor angular displacement = $180 \times \dfrac{\pi}{180} = \pi$ rads

The torque produced is therefore given by:

At 1.0A: $T = \dfrac{dW'}{d\theta} \approx \dfrac{0.0385}{\pi} = 49.0 \times 10^{-3}\, Nm$

(7)

b) The flux-linkage characteristics for open-circuit conditions (i.e 0A) is a reasonable approximation to a sine-wave [*in fact the actual data is generated from a simple sin function*]. It is therefore reasonable to assume that the maximum rate of change of flux-linkage will occur at angular displacements around 0^o. [*It is not necessary to identify this with a sine wave, just to recognise visually that the maximum rate of change will occur around 0^o*]. From Figure 4b, an estimate of the rate of change of flux linkage with rotor position can be made:

$$\frac{\mathrm{d}\Psi}{\mathrm{d}\theta} \approx \frac{\Delta\Psi}{\Delta\theta} = \frac{0.007}{20 \times \dfrac{\pi}{180}} = 0.020 \text{ Wb/rad}$$

[*These calculations have all been performed in terms of mechanical radians*]

At 6000rpm

$$\frac{d\theta}{dt} = \frac{6000 \times 2 \times \pi}{60} = 628 \ rad/s \quad \therefore \mathrm{e} = \frac{\mathrm{d}\Psi}{\mathrm{dt}} = \frac{\mathrm{d}\Psi}{\mathrm{d}\theta} \times \frac{\mathrm{d}\theta}{\mathrm{dt}} = 12.6\mathrm{V}$$

(An alternative is to note that the variation in flux-linkage can approximated as:

$\Psi = 0.02\sin(\theta)$

Hence $\frac{d\Psi}{d\theta} = 0.02\cos(\theta)$

This has a peak value of 0.02 Wb/rad - as before from graphical interpolation)

(3)

c) From the ψ/θ curves, the material begins to saturate at a flux-linkage of 0.034Wb which corresponds to a flux density of 1.5T.

The peak flux-linkage due to the magnet alone is 0.02Wb, which by equivalence corresponds to a flux density of:

$B_{oc\_peak} = \frac{0.02}{0.034} \times 1.5 = 0.88T$

The airgap flux density is given by:

$$B_g = \frac{B_r}{1 + \mu_r \frac{2l_g}{2l_m}}$$

Re-arranging yields:

$$l_m = \frac{\mu_r l_g}{\left(\frac{B_r}{B_g} - 1\right)} = \frac{1.05 \times 0.5 \times 10^{-3}}{\left(\frac{1.25}{0.88} - 1\right)} = 1.25mm$$

(6)

d) The total effective magnetic path length is $2\left(l_m + l_g\right) = 3.5mm$

Taking a zero magnet flux condition with no saturation, e.g. 0^{o} at 1A gives a flux-linkage of 0.02Wb, which by the equivalence established in part (c) corresponds to a flux density of 0.88T at 1A. Hence:

$$N = \frac{B_g l_g}{\mu_0 I} = \frac{0.88 \times 3.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 1} = 2450\ turns$$

(4)