a)

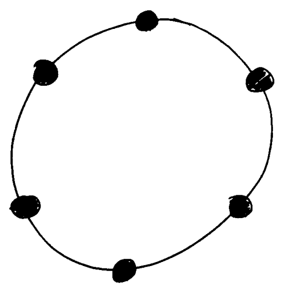
1) beads repel $\rightarrow$ beads at each end-stop

2) distribution symmetrical

3) bead 2 has the force from one bead pushing it to the right, but the forces from four beads pushing it to the left.

As $F \propto \dfrac{1}{d^2}$, bead 2 moves closer to bead 1 to balance the forces.

[In the diagram above, the effect has been exaggerated for clarity - in reality the effect is not as pronounced.]



- beads move as far away from each other as possible $\rightarrow$ beads are equally spaced around circle.

- force on each bead is purely radial - no component in the azimuthal direction

[6]

b)

i) At $(0.25, 0, 0)$ - this is inside the conducting sphere $\rightarrow$ electric field is zero.

At $(2.5, 0, 0)$ - $|\underline{E}| = \left| \dfrac{Q}{4\pi\varepsilon_0 r^2} \right|$

$$= \frac{|-4 \times 10^{-8}|}{4 \times \pi \times 8.854 \times 10^{-2} \times (2.5)^2}$$

$$= 57.5 \ \mathrm{Vm^{-1}}$$

[4]

ii) zero force $\rightarrow$ zero field

$\rightarrow$ the field from the second sphere must cancel out the field from the first at $(2.5, 0, 0)$

$$\frac{Q}{4\pi\varepsilon_0 (2.5 - y)^2} = 57.5$$

$$(2.5 - y)^2 = \frac{16 \times 10^{-8}}{4\pi \times 8.854 \times 10^{-12} \times 57.5}$$

$$y = \pm \sqrt{\frac{16 \times 10^{-8}}{4\pi \times 8.854 \times 10^{-12} \times 57.5}} - 2.5$$

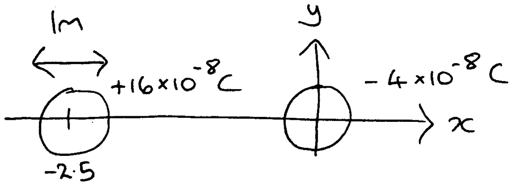
A positively charged sphere at $(7.5, 0, 0)$ m would create an electric field to the left at $(2.5, 0, 0)$ m, so would not cancel out the field caused by the sphere at the origin

$\Rightarrow$ disregard the $y = 7.5$ solution, and choose:-

$$y = -2.5 \ \mathrm{m}$$

[4]

b) iii)



To find the p.d. between the two spheres, we must integrate $\underline{E}$ along the x-axis between $x = -2$ and $x = -0.5$ m

$$E_x = \frac{Q_1}{4\pi\varepsilon_0 (x+2.5)^2} - \frac{Q_2}{4\pi\varepsilon_0 (x)^2}$$

$$V = -\int_{-2}^{-0.5} E_x \, dx$$

$$= \frac{-Q}{4\pi\varepsilon_0}\int_{-2}^{-0.5}\frac{1}{(x+2.5)^2}dx + \frac{Q}{4\pi\varepsilon_0}\int_{-2}^{-0.5}\frac{1}{x^2}dx$$

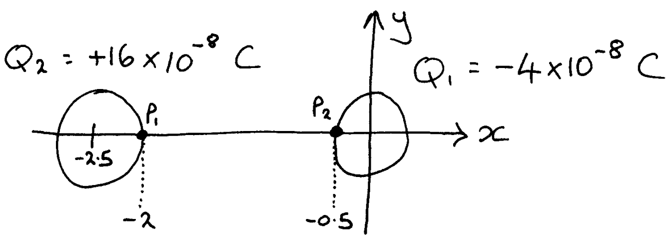
$$= \frac{-16\times10^{-8}}{4\pi\times8.854\times10^{-12}}\left[\frac{-1}{(x+2.5)}\right]_{-2}^{-0.5} - \frac{4\times10^{-8}}{4\pi\times8.854\times10^{-12}}\left[\frac{-1}{x}\right]_{-2}^{-0.5}$$

$$= \frac{-16\times10^{-8}}{4\pi\times8.854\times10^{-12}}\left[\frac{-1}{2}+\frac{1}{0.5}\right] - \frac{4\times10^{-8}}{4\pi\times8.854\times10^{-12}}\left[\frac{1}{0.5}-\frac{1}{2}\right]$$

$$= 2.70 \text{ kV}$$

[6]

b) iii) (Alternative method)



$$\text{p.d} = \phi_{P_1} - \phi_{P_2} \qquad \phi = \frac{q}{4\pi\varepsilon_0 R}$$

$$\phi_{P_1} = \frac{Q_1}{4\pi\varepsilon_0\times2} + \frac{Q_2}{4\pi\varepsilon_0\times0.5}$$

$$= \frac{-4\times10^{-8}}{4\pi\varepsilon_0\times2} + \frac{16\times10^{-8}}{4\pi\varepsilon_0\times0.5}$$

$$= \frac{1\times10^{-8}}{4\pi\varepsilon_0}\left[-2+32\right] = 2696 \text{ V}$$

$$\phi_{P_2} = \frac{Q_2}{4\pi\varepsilon_0\times2} + \frac{Q_1}{4\pi\varepsilon_0\times0.5}$$

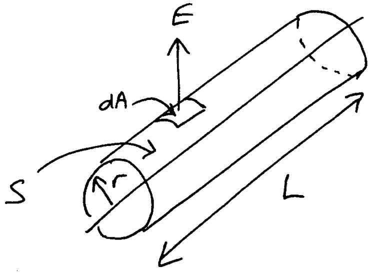
$$= \frac{16\times10^{-8}}{4\pi\varepsilon_0\times2} + \frac{-4\times10^{-8}}{4\pi\varepsilon_0\times0.5}$$

$$= \frac{1\times10^{-8}}{4\pi\varepsilon_0}\left[8-8\right] = 0$$

$$\text{p.d.} = \phi_{P_1} - \phi_{P_2} = 2.7 \text{ kV}$$

[6]

a) i)

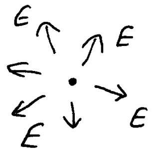


Gauss' Law

$$\oint_S E_\perp \, dA = Q/\varepsilon_0$$

Due to symmetry E cannot vary along wire (as ∞) or around wire

$\therefore$ E-field must point radially outwards



When evaluating $\oint_S E_\perp \, dA$, ends of cylinder do not contribute as dA is parallel to E

Contribution from curved part of cylinder (S) is:-

$$E_\perp \cdot \underbrace{2\pi r L}_{\text{surface area } S} = Q/\varepsilon_0$$

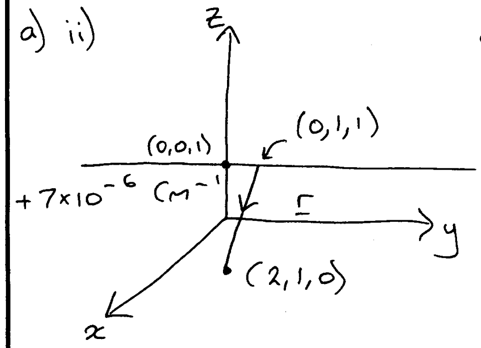
$$\therefore \quad E = \frac{Q}{L} \cdot \frac{1}{2\pi \varepsilon_0 r}$$

$$\underline{E} = \frac{q_\ell}{2\pi \varepsilon_0 r} \hat{\underline{r}}$$

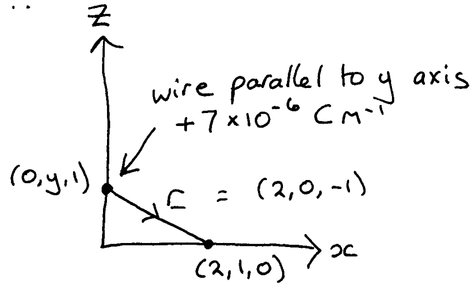
where $q_\ell = \frac{Q}{L}$ (charge per unit length)

[6]

a) ii)



or...



$\underline{r}$ is shortest distance from the wire to (2,1,0)

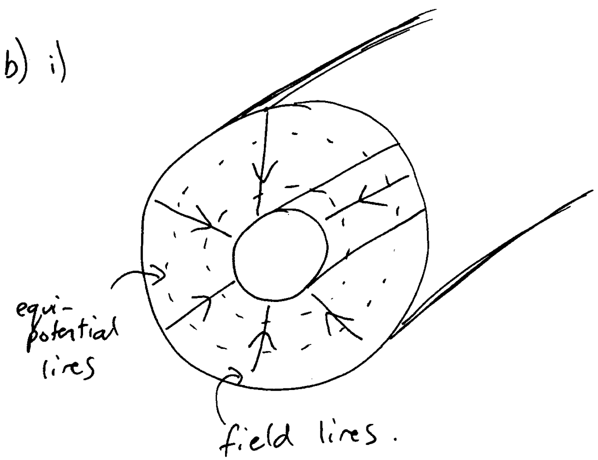
$= (2, 0, -1)$ $\qquad |\underline{r}| = \sqrt{5}$

$$\underline{E} = \frac{7\times10^{-6}}{2\times\pi\times 8.854\times10^{-12}\times\sqrt{5}} \left(\frac{2}{\sqrt{5}}, 0, \frac{-1}{\sqrt{5}}\right)$$

$$= (5.03, 0, -2.52) \times 10^4 \ \text{V/m}$$

[4]

b) i)



[2]

b) ii)

Assume that the field due to the inner conductor is same as that from an infinitely long charged wire.

$$\underline{E} = \frac{-q_\ell}{2\pi\varepsilon_0 r}\,\hat{\underline{r}}$$

$$= \frac{-Q}{2\pi\varepsilon_0 r L}\,\hat{\underline{r}}$$

Voltage between outer and inner conductors is

$$V = -\int_a^b \underline{E}\cdot dr = -\int_a^b -\hat{\underline{r}}\,\frac{Q}{2\pi\varepsilon_0 r L}\cdot\hat{\underline{r}}\,dr$$

$$= \frac{Q}{2\pi\varepsilon_0 L}\int_a^b \frac{dr}{r} = \frac{Q}{2\pi\varepsilon_0 L}\left[\ln b - \ln a\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 L}\ln\left(b/a\right)$$

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)} \quad (F)$$

$$C_\ell = \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(b/a)} \quad Fm^{-1}$$

[6]

b) iii)

$$C_\ell = \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

$$\ln(b/a) = \frac{2\pi\varepsilon_0 L}{C}$$

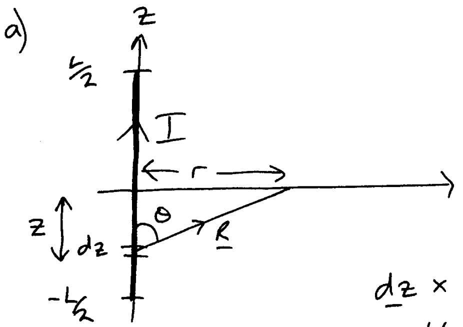
$$= \frac{2\times\pi\times 8.854\times10^{-12}\times 10}{400\times10^{-12}}$$

$$\ln(b/a) = 1.391$$

$$b = e^{1.391}\times a$$

$$= 8.04 \text{ mm}$$

a)



$$\underline{H} = \frac{I}{4\pi}\int_L \frac{d\underline{l} \times \hat{\underline{R}}}{R^2}$$

$$d\underline{z} \times \hat{\underline{R}} = dz \sin\theta \ \hat{\underline{\phi}}$$

Hence $\underline{H} = \hat{\underline{\phi}} \dfrac{I}{4\pi} \displaystyle\int_{-L/2}^{L/2} \frac{\sin\theta}{R^2} dz$

$$\sin\theta = \frac{r}{R} \quad , \quad R^2 = z^2 + r^2$$

$$\Rightarrow \underline{H} = \hat{\underline{\phi}} \frac{I}{4\pi} \int_{-L/2}^{L/2} \frac{r}{(z^2+r^2)^{3/2}} dz$$

Using the standard integral given . . .

$$\underline{H} = \hat{\underline{\phi}} \frac{IL}{2\pi r\sqrt{4r^2+L^2}}$$

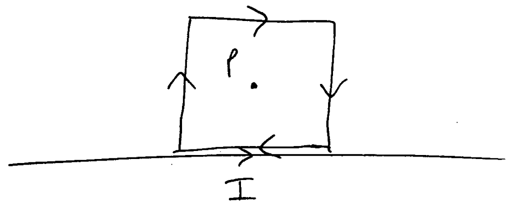
$$\underline{B} = \mu_0 \underline{H} = \hat{\underline{\phi}} \frac{\mu_0 I L}{2\pi r\sqrt{4r^2+L^2}}$$ [8]

b) $|\underline{B}| = 4 \times \dfrac{\mu_0 I L}{2\pi r\sqrt{4r^2+L^2}}$ $L = 1m$, $r = 0.5m$, $I = 5A$

$$= 5.66 \times 10^{-6} \ T$$

[4]

c) We can regard the field at P as being due to two sources, an infinitely long wire carrying a current I, and a square circuit carrying a current I.



$$|\underline{B}|_{square} = 4 \times \frac{\mu_0 I}{\pi d\sqrt{2}}$$

$$|\underline{B}|_{wire} = \frac{\mu_0 I}{\pi d}$$

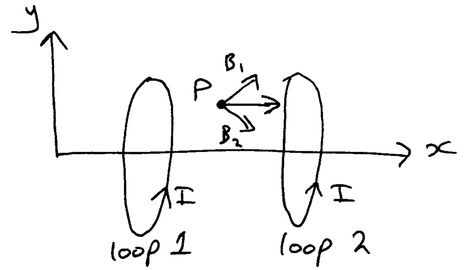
$$|\underline{B}|_P = \frac{\mu_0 I}{\pi d} - \frac{4\mu_0 I}{\sqrt{2}\,\pi d}$$

$$= \frac{\mu_0 I}{\pi}\left[\frac{1}{d} - \frac{4}{\sqrt{2}d}\right]$$

$|\underline{B}| = 2.93 \times 10^{-3}$ T (into the page)

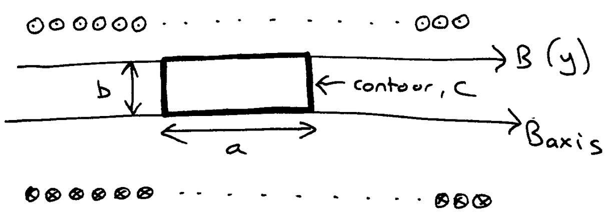
a) Along the axis $B = \mu_0 n I$

Assume solenoid consists of an infinite number of turns. For any point P, there will be as many turns to the left as to the right. Consider one such pair of turns, equidistant from P



At point P, y-components of B_1 and B_2 cancel, but x-components add.

$\Rightarrow$ B-field will always be parallel to the axis



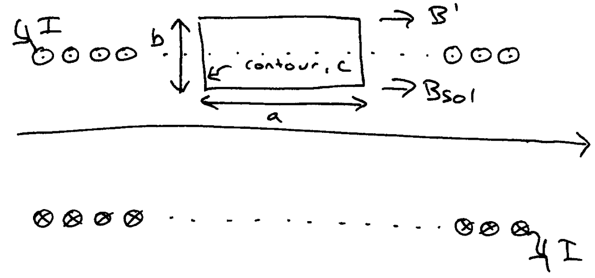
Apply Ampere's Law around contour C

$$\oint \underline{B} \cdot d\underline{l} = \mu_0 I = 0 \quad \text{(no current through contour C)}$$

$$B_{axis} \times a + 0 \times b - B(y) \times a + 0 \times b = 0$$

$$B(y) = B_{axis} = \mu_0 n I$$

$\Rightarrow$ B-field is uniform throughout solenoid



Current through contour $= I' = Ina$

$$\oint \underline{B} \cdot d\underline{l} = \mu_0 I'$$

$$a \times B_{sol} - a \times B' = \mu_0 I n a$$

but $B_{sol} = \mu_0 n I$

$$\Rightarrow B' = 0$$

$\Rightarrow$ No field outside solenoid. [10]

b) i) $B_{sol} = \mu_0 n I$

$$= 4\pi \times 10^{-7} \times \frac{2000}{0.15} \times 1$$

$$= 16.8 \text{ mT}$$ [2]

ii) $$L = \frac{\mu_0 N^2 A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 2000^2 \times \pi \times 0.015^2}{0.15}$$

$$= 23.7 \text{ mH}$$ [2]

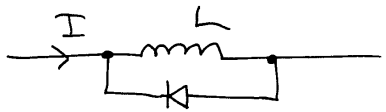
i) $W = \frac{1}{2} L I^2$

$= \frac{1}{2} \times 23.7 \times 10^{-3} \times \left(\frac{12}{10}\right)^2$

$= 1.71 \times 10^{-2}$ J [2]

ii) When the battery is disconnected, this energy causes a voltage spike known as a "back emf" to be produced. This voltage acts to oppose the change in current, and can be very large, damaging sensitive components (e.g. a transistor operating a relay.)

To protect against this, a diode can be connected in parallel with the coil to safely discharge any back emf.



[4]