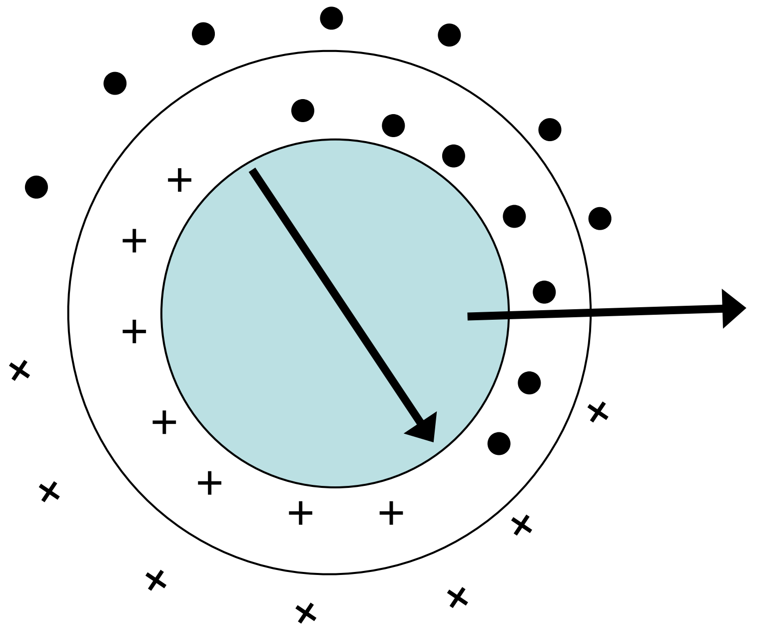
Modelling of Machines

Section 6

Rotational and transformer voltages

Consider the simplified representation of a machine



1 – Stator
2 - Rotor

L_1 and L_2 are constant and the mutual inductance between coils varies as:

$$M = M_d \cos\theta$$

The terminal voltages when the rotor are rotating is given by:

$$V_1 = i_1 R_1 + \frac{d\psi_1}{dt} = i_1 R_1 + L_1 \frac{di_1}{dt} + M_d \cos\theta \quad \frac{di_2}{dt} - M_d i_2 \sin\theta \quad \frac{d\theta}{dt}$$

and

$$V_2 = i_2 R_2 + \frac{d\psi_2}{dt} = i_2 R_2 + L_2 \frac{di_2}{dt} + M_d \cos\theta \quad \frac{di_1}{dt} - M_d i_1 \sin\theta \quad \frac{d\theta}{dt}$$

This can be expressed in matrix form as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left| \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} + \begin{bmatrix} L_1 & M_d \cos\theta \\ M_d \cos\theta & L_2 \end{bmatrix} \frac{d}{dt} + \begin{bmatrix} 0 & -M_d \sin\theta \\ -M_d \sin\theta & 0 \end{bmatrix} \frac{d\theta}{dt} \right| \quad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

or

$$[V] = \left| [R] + [L] \frac{d}{dt} + [G]\omega \right| \quad [i]$$

Resistive terms

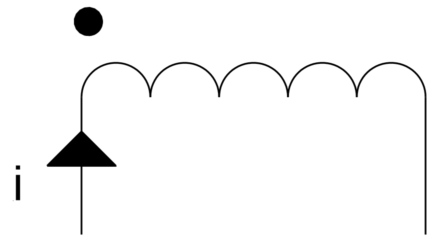
Transformer terms

Rotational terms

Analysis of Kron primitive machine

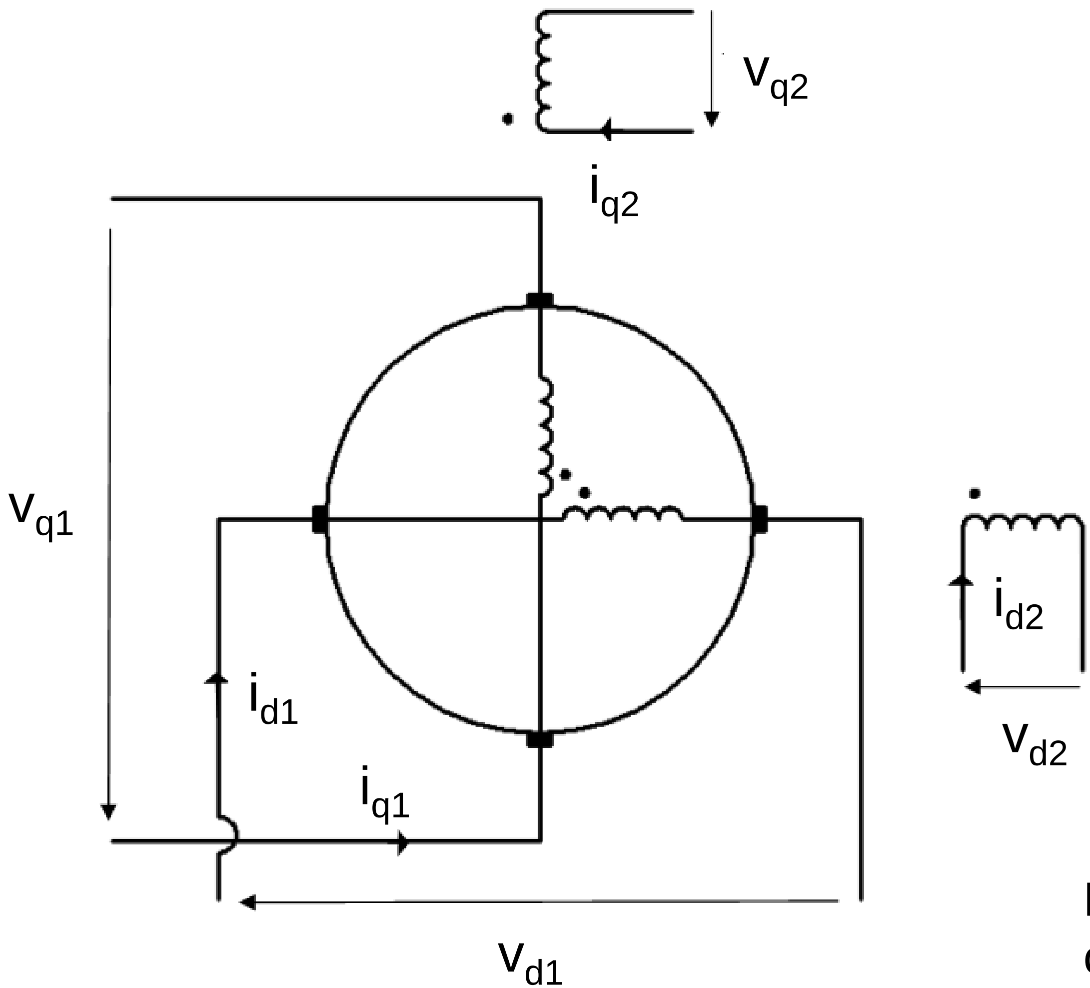
Assumptions and conventions

1) Flux is directly proportional to current (i.e. magnetic saturation)
2) Commutation is ideal (no voltage drop)
3) All types of winding can be represented by concentrated coils (i.e. coils along one axis)
4) Two axes represent axes of symmetry
5) Dot-blank notation relates current polarity to direction of flux



Primitive coil current always flows into dot end
Current in at dot end gives flux in same direction
Mutual inductance between coils on same axis is positive
Positive voltage at dot end
Power into coil from source given by positive current
Coils are arranged with dot end nearest centre

Applying the conventions and assumptions yields the generalised unconnected form of the Kron primitive



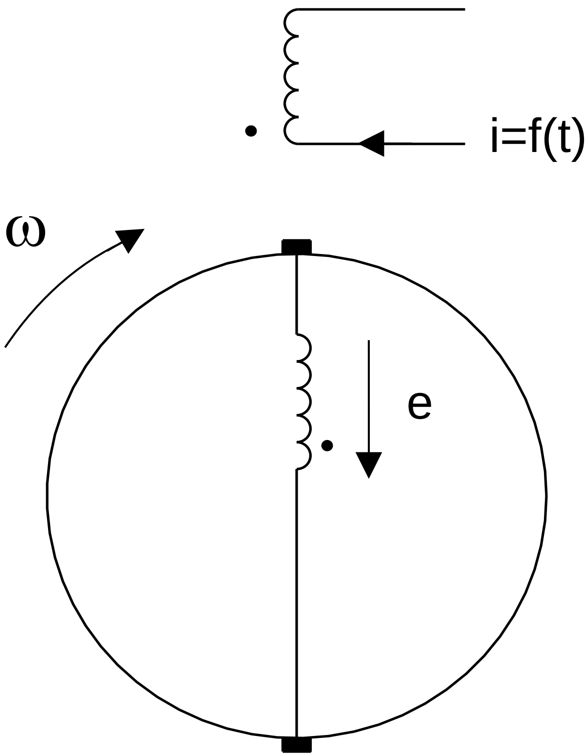
Subscripts

1 – rotor
2 - stator

N.B Note use of dot-blank convention

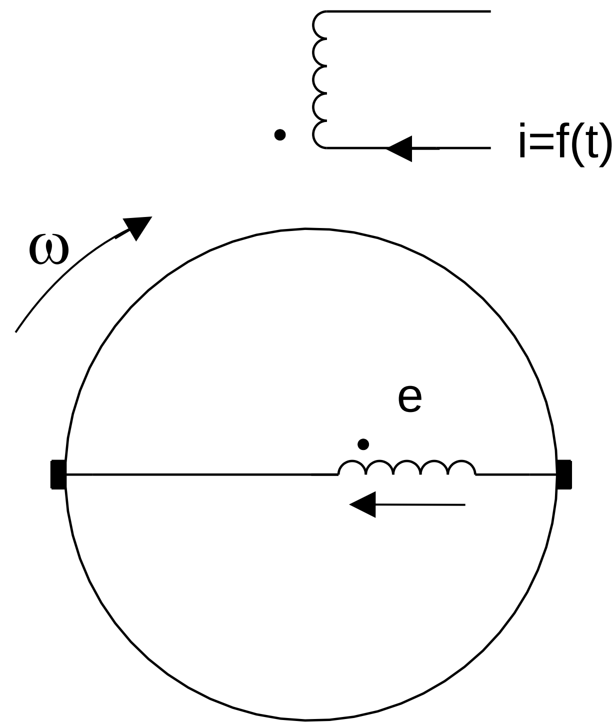
Interactions between stator and rotor coils

Coils on same axis



Transformer voltage
even when ω=0

Coils on orthogonal axes



Rotational voltage
even when $i \neq f(t)$

Consider coil d_1 in the unconnected Kron primitive

$$V_{d1} = i_{d1} R_{d1} + \frac{d\psi_{d1}}{dt}$$

$$\psi_{d1} = L_{d1} i_{d1} + M_{d1d2} i_{d2} + M_{d1q1} i_{q1} + M_{d1q2} i_{q2}$$

$$\frac{d\psi_{d1}}{dt} = L_{d1} \frac{di_{d1}}{dt} + M_{d1d2} \frac{di_{d2}}{dt} + G_{d1q1} \omega\, i_{q1} + G_{d1q2} \omega\, i_{q2}$$

Where G are rotational voltage terms

Use p to denote d/dt (note: this is simply a shorthand notation and NOT a mathematical transformation)

The voltage equations for d_1 coil can be expressed as:

$[V] = [Z]\,[I]$ where $[Z] = [R] + [L]p + [G]\omega$

Note convention on subscripts

M_{d1d2} i.e refers to voltage in d1 due to flux in d2

Seat of voltage

Seat of flux

The same voltage equations can be derived for the remaining coils – putting these into matrix for yields the **generalised form of the voltage matrix equations**:

$$
\begin{vmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{vmatrix}
=
\begin{vmatrix}
R_{d1}+L_{d1}p & G_{d1q1}\omega_r & M_{d1d2}p & G_{d1q2}\omega \\
G_{q1d1}\omega_r & R_{q1}+L_{q1}p & G_{q1d2}\omega_r & M_{q1q2}p \\
M_{d2d1}p & 0 & R_{d2}+L_{d2}p & 0 \\
0 & M_{q2q1}p & 0 & R_{q2}+L_{q2}p
\end{vmatrix}
\begin{vmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{vmatrix}
$$

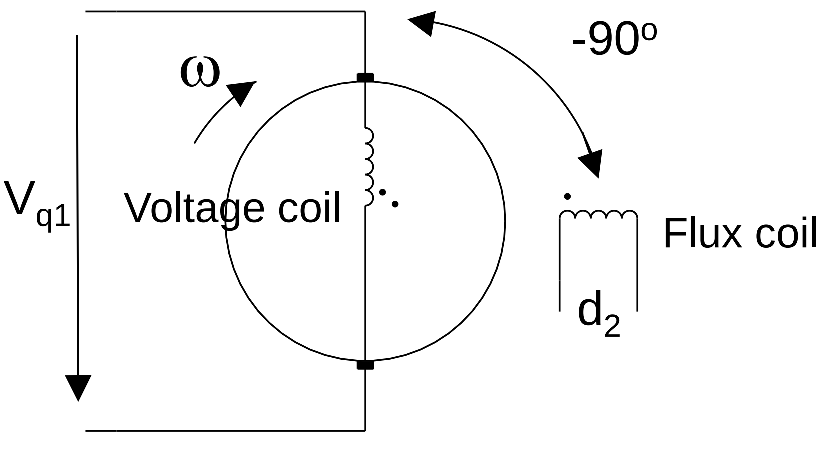
Points to note:

- Leading diagonal contains self impedance terms
- No rotational voltage induced in stator coils (d2 and q2) – hence presence of zero terms in matrix

The general form of the matrix equations can be simplified by noting the following:

$$M_{d1d2} = M_{d2d1} = M_d \quad \text{and } M_{q1q2} = M_{q2q1} = M_q$$

The G coefficients can be expressed in terms on inductances noting polarity - Consider G_{q1d2}



Sign of angle determined by angle of voltage axis relative to flux axis in direction of rotation – in this case voltage axis lags flux axis in direction of rotation – hence -90º

Consider the G term which relates the rotational voltage induced in a rotor coil by flux in a stator coil on an orthogonal axis

$$G_{q1d2} = -M_d \sin\theta$$

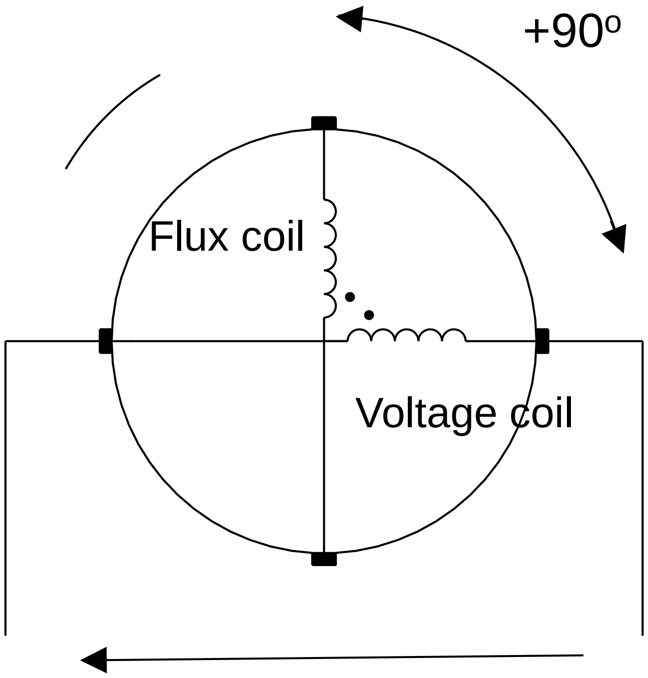
But $\theta = -90^o$ and since sin(-90) = -1 then:

$G_{q1d2} = M_d$

By a similar analysis $G_{d1q2} = - M_q$

This can consolidated into rule that the G coefficient is positive when the FLUX AXIS LEADS VOLTAGE AXIS IN DIRECTION OF ROTATION

Consider the G term which relates the rotational voltage induced in a rotor coil by flux in a **rotor** coil on an orthogonal axis



$$G_{d1q1} = -M_{q1q1} \sin(90)$$

$$= -L_{q1}$$

By a similar process:

$$G_{q1d1} = +L_{d1}$$

This leads to a simplified matrix of equations:

$$\begin{vmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{vmatrix} = \begin{vmatrix} R_{d1}+L_{d1}p & -L_{q1}\omega_r & M_d p & -M_q\omega \\ L_{d1}\omega_r & R_{q1}+L_{q1}p & M_d\omega_r & M_q p \\ M_d p & 0 & R_{d2}+L_{d2}p & 0 \\ 0 & M_q p & 0 & R_{q2}+L_{q2}p \end{vmatrix} \begin{vmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{vmatrix}$$

Notes:
The rotational terms comprise L terms when the coils are both on the rotor
The rotational terms comprise M terms when a stator coil is involed

No rotational voltages are induced in stator coils