

Solutions to Tutorial Sheet 4

1. Sampling frequency = 8 kHz.
 \therefore Time between adjacent samples of same channel = 125 μ s.
Each channel requires $2 + 3 + 3 + 2 = 10 \mu$ s
 \therefore Number of 10 μ s slots in 125 μ s = 12.
i.e. 12 channels can be transmitted.

2. (a) All channels sampled equally, therefore number of samples per second = $2 \times 8 \times 10,000 = \underline{160,000}$
(b) Hence, according to Nyquist, with this number of samples we can recover a composite waveform with a maximum bandwidth of 80 kHz.
Hence LPF bandwidth = 80 kHz
(c) Total SSB bandwidth
 $= 2 \times 10 + 6 \times 3.3 = \underline{39.8 \text{ kHz.}}$
(d) New sampling scheme
Use 1 pole, 12 way rotary switch with contacts 1 and 2 connected to 10 kHz signals and contacts 3 to 8 connected to 3.3 kHz signals.
Now max frequency of input data is essentially $10/3 = 3.33 \text{ kHz.}$
 \therefore Sampling frequency for 1 channel = $2 \times 3.33 = 6.66 \text{ kHz.}$
 \therefore For 12 channels = $12 \times 6.66 = \underline{80 \text{ kHz.}}$
 \therefore Again from Nyquist, LPF bandwidth = 40 kHz.
SSB bandwidth = 40 kHz.

3. $\text{Number of quantisation levels} = 2^N$

where N = number of bits/sample.

Need know sample amplitude to 1 part in 50 ($\pm 1\%$)

\therefore Need at least 50 levels

\therefore Choose $N = 6$ ($2^6 = 64$ levels)

4. $f_c = 1.5 + (2.8 - 1.5)/2 = 2.15 \text{ kHz.}$

$$\Delta f = (2.8 - 1.5)/2 = 0.65 \text{ kHz.}$$

$$\text{Bandwidth required} = 2 \times \text{data signal bw} + 2 \times \Delta f$$

$$= 2 \times 100 + 2 \times 650 = \underline{1.5 \text{ kHz.}}$$

If data rate now 1000 bits/sec

$$\text{LF} = 2.15 - 0.65 - 1 = \underline{500 \text{ Hz.}}$$

$$\text{HF} = 2.15 + 0.65 + 1 = \underline{3.8 \text{ kHz.}}$$

5. (a) $f_{\text{IF}} = f_{\text{LO}} - f_{\text{sig}} = 2.86 - 2.8 = 0.06 \text{ GHz} = \underline{60 \text{ MHz.}}$

(b) Mutual interference between the receivers could occur if the local oscillator radiation from one receiver was picked up by the other. For example, the first receiver has an image frequency of 2.92 GHz. Hence, if the local oscillator in the second receiver was set to 2.92 GHz then this could cause interference to the first receiver. In this case the second receiver is tuned to $2.92 - 0.06 = 2.86 \text{ GHz}$.
Alternatively, if the local oscillator of the second receiver was set to 2.80 GHz, its input signal frequency would be $2.80 - 0.06 = 2.74 \text{ GHz}$.

(c) Either receiver can be tuned to receive a signal over the range 2.8 to 3.0 GHz. Hence neither receiver will interfere with the other if the IF is increased to at least 200 MHz.

6. $f_{LO} = 136 - 30 = 106 \text{ MHz.}$

$\therefore f_{\text{image}} = 106 - 30 = \underline{76 \text{ MHz.}}$ (1st IF image)

2nd IF = 10 MHz and 2nd LO > 30

i.e. $-f_{\text{sig}} + LO = 10$

\uparrow
30

$\therefore f_{2\text{nd LO}} = 40 \text{ MHz.}$

Then would have $f_{2\text{nd image}} = 30 + 20 = 50 \text{ MHz.}$

6cont $\therefore f_2 - 106 = 50 \quad \underline{f_2 = 156 \text{ MHz}}$

$106 - f_3 = 50 \quad \underline{f_3 = 56 \text{ MHz}}$

Input signals
causing 2nd IF
image problems

7. $f_{LO} > f_{\text{sig}}$

$\therefore f_{LO} - f_{\text{sig}} = f_{IF}$

$f_{IF} = 1010 - 555 = 455 \text{ kHz}$

$\therefore f_{\text{image}} = f_{\text{sig}} + 2 f_{IF}$

$= 555 + 2 \times 455 = \underline{1465 \text{ kHz.}}$

$$\rho = \frac{f_{\text{image}}}{f_{\text{signal}}} - \frac{f_{\text{signal}}}{f_{\text{image}}}$$

$$= \frac{1465}{555} - \frac{555}{1465} = 2.640 - 0.379$$

$$= 2.261$$

$$\therefore \alpha = \sqrt{1 + Q^2 \rho^2}$$

$$= \sqrt{1 + 40^2 \times 2.261^2} = 90.4$$

$$\alpha_{\text{dB}} = 20 \log_{10}(\alpha) = \underline{39.1 \text{ dB}}$$

8. $\alpha = \alpha_1 \alpha_2 \quad \text{and} \quad \alpha_1 = \alpha_2$

$\therefore \alpha = 120 = \sqrt{1 + Q^2 \rho^2}^2$

$\therefore Q^2 = \frac{120 - 1}{\rho^2} \quad \text{i.e. } Q = \frac{\sqrt{119}}{\rho}$

$$f_{\text{signal}} = 15 \text{ MHz}$$

$$IF = 450 \text{ kHz} \quad \therefore f_{LO} = 14.55 \text{ MHz.}$$

$$\begin{aligned} \therefore f_{\text{image}} &= f_{\text{sig}} - 2 \times f_{IF} \\ &= 15 - 0.9 = 14.1 \text{ MHz.} \end{aligned}$$

$$\begin{aligned} \rho &= \frac{f_{\text{image}}}{f_{\text{signal}}} - \frac{f_{\text{signal}}}{f_{\text{image}}} = \frac{14.1}{15} - \frac{15}{14.1} \\ &= 0.94 - 1.064 \end{aligned}$$

$$\therefore |\rho| = 0.124$$

$$\therefore Q = \frac{\sqrt{119}}{0.124} = \underline{88}$$

9. $S/N = 32 \text{ dB} \equiv 1585$

Then from Hartley Shannon Law

$$\begin{aligned} C &= B \log_2 (1 + S/N) \\ &= 3100 \log_2 (1 + 1585) \\ &= 3100 \times 10.63 = \underline{32953 \text{ bits/sec}} \end{aligned}$$

10. $S/N = 28 \text{ dB} \equiv 631$

$$\begin{aligned} C &= 4000 \log_2 (1 + 631) \\ &= 4000 \times 9.305 = \underline{37219 \text{ bits/sec}} \end{aligned}$$

If the S/N in a 4 kHz bandwidth is 631 this is equivalent to a noise power of 1 mW when the signal power is 631 mW. The signal power is unchanged when the bandwidth is doubled but when the bandwidth is doubled, so is the noise power.

$$\begin{aligned} \therefore C &= 8000 \log_2 (1 + 631/2) \\ &= 8000 \log_2 (316.5) = \underline{66455 \text{ bits/sec}} \end{aligned}$$