MSc(Eng) Wireless Communication Systems

Module EEE-6431: Broadband Wireless Techniques

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Syllabus Highlights

1. **Introduction - Overview of Broadband Wireless Systems**
2. **Signal Propagation, Pathloss Models and Shadowing**
3. **Statistical Fading Models: Narrowband & Wideband Fading**
4. **Capacity of Wireless Channels**
5. **Principles of Multicarrier Modulation**

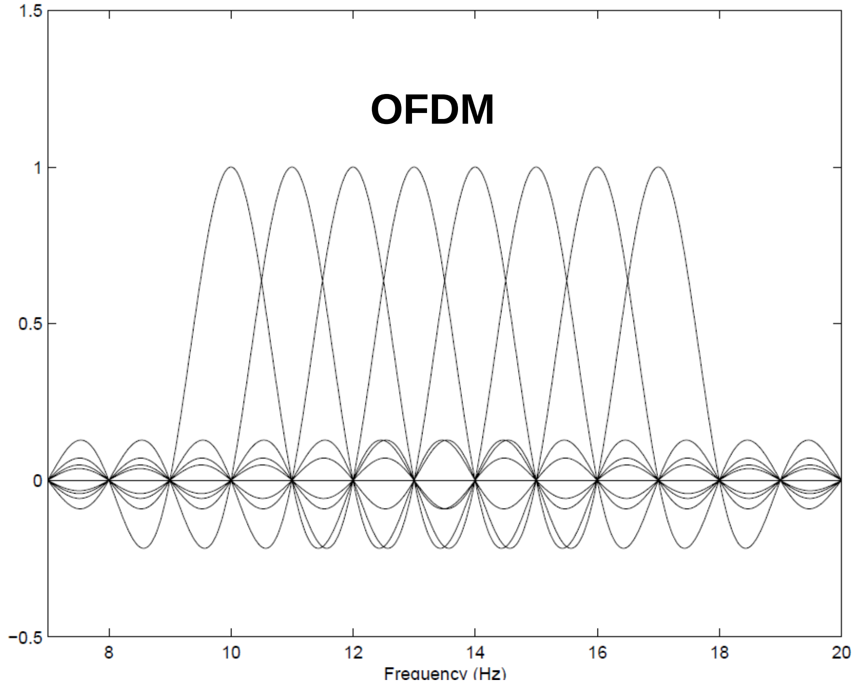
6. Orthogonal Frequency Division Multiplexing (OFDM)

Section 5 Review

1. Data transmission using multiple carriers
2. Multicarrier modulation using overlapping subchannels
3. Mitigation of subcarrier fading

Section 6 Outline

1. Discrete implementation of multicarrier modulation (DFT, cyclic prefix, OFDM)
2. Challenges in multicarrier modulation (PAPR, freq/time off-sets)
3. Case study: The IEEE 802.11a Wireless LAN standard



6. OFDM

Discrete Implementation of MC Modulation: **The development of low complexity implementations of the discrete Fourier transform (DFT) and its inverse (IDFT) has lead to the wide spread adoption of OFDM.**

Let $x[n], 0 \le n \le N-1$, denote a discrete time series. The *N*-point DFT of $x[n]$ is given by –

$$DFT\{x[n]\} = X[i] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] \exp\left(-\frac{j2\pi ni}{N}\right), \quad 0 \le i \le N-1$$

The series $x[n]$ can be recovered from its DFT using the IDFT

$$IDFT\{X[i]\} = x[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X[i] \exp\left(\frac{j2\pi ni}{N}\right), \quad 0 \le n \le N-1$$

The DFT & IDFT are implemented in hardware using the the Fast Fourier Transform (FFT) and the Inverse Fast Fourier Transform (IFFT).

Consider passing $x[n]$ through a linear time-invariant discrete-time channel $h[n]$, then the output $y[n]$ is given by the discrete-time convolutional –

$$y[n] = h[n] * x[n] = x[n] * h[n] = \sum_k h[k]x[n-k]$$

And the *N*-Point *circular convolution* of $x[n]$ and $h[n]$ is defined as -

$$y[n] = h[n] \otimes x[n] = x[n] \otimes h[n] = \sum_k h[k]x[n-k]_N$$

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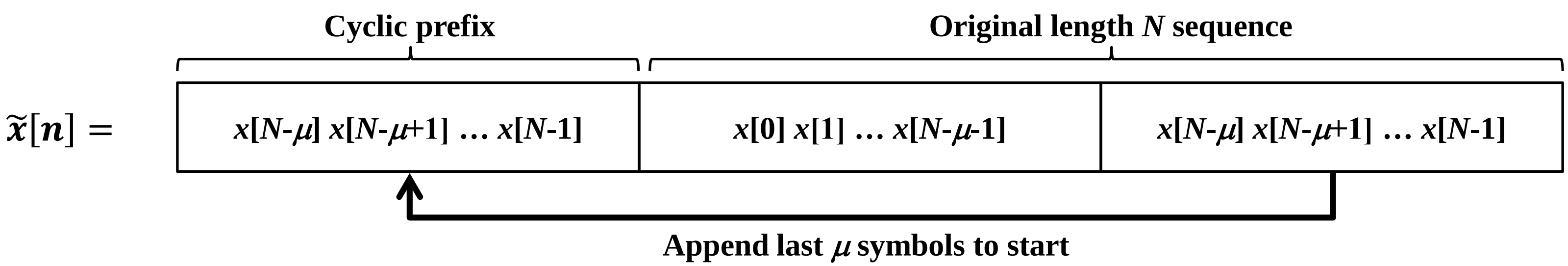
Discrete Implementation of MC Modulation Contd.: The term in $[n-k]_N$ denotes $[n-k]$ modulo N. That is, $x[n-k]_N$ is a periodic version of $x[n-k]$ with period N.

Since circular convolution in time leads to multiplication in frequency, we have

$$\mathrm{DFT}\{y[n]=h[n]\otimes x[n]\}=X[i]H[i] \quad 0\le i\le N-1$$

The linear convolution between $x[n]$ and $h[n]$ can be changed into a circular convolution by adding a cyclic prefix.

Cyclic Prefix: For a channel input sequence $x[n]=x[0],\ldots,x[N-1]$ of length N and a discrete-time channel with FIR $h[n]=h[0],\ldots,h[\mu]$ of length $\mu+1=T_m/T_s$, where T_m is the channel delay spread and T_s is the sample period, then the cyclic prefix is equal to the last μ samples of $x[n]$ -



The new sequence $\tilde{x}[n]=x[n]_N$ for $-\mu\le n\le N-1$ and we can also write $\tilde{x}[n-k]=x[n-k]_N$ for $-\mu\le n-k\le N-1$, hence

$$y[n]=\tilde{x}[n]*h[n]=\sum_{k=0}^{\mu}h[k]\tilde{x}[n-k]=\sum_{k=0}^{\mu}h[k]x[n-k]_N=x[n]\otimes h[n]$$

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Cyclic Prefix contd: By appending a cyclic prefix the linear convolution associated with $y[n]$ is transformed to a circular convolution and the DFT of the channel output is –

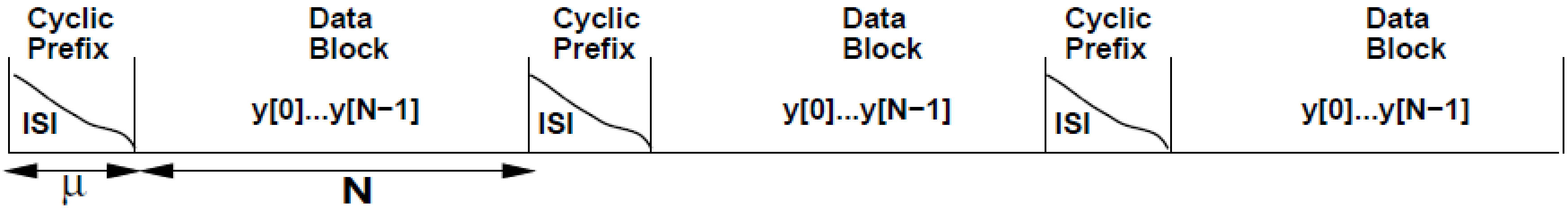
$$Y[i] = \mathrm{DFT}\{y[n] = x[n] \otimes h[n]\} = X[i]H[i] \quad 0 \le i \le N-1$$

Then the input sequence $x[n], 0 \le n \le N-1$, can be recovered from $y[n], 0 \le n \le N-1$, by

$$x[n] = \mathrm{IDFT}\left\{\frac{Y[i]}{H[i]}\right\} = \mathrm{IDFT}\left\{\frac{\mathrm{DFT}\{y[n]\}}{\mathrm{DFT}\{h[n]\}}\right\}$$

We note that $y[n], -\mu \le n \le N-1$, has length $N+\mu$ but from the above equation the first μ samples $\{y[-\mu], \dots, y[-1]\}$ are not used (i.e. they are discarded) in order to recover $x[n]$.

Specifically, if the input sequence $x[n]$ is divided into data blocks of size *N* with a cyclic prefix attached to each block to form $\tilde{x}[n]$, then the first μ samples of $y[n] = h[n] * \tilde{x}[n]$ in a block are affected by ISI from the last μ samples of the previous block.

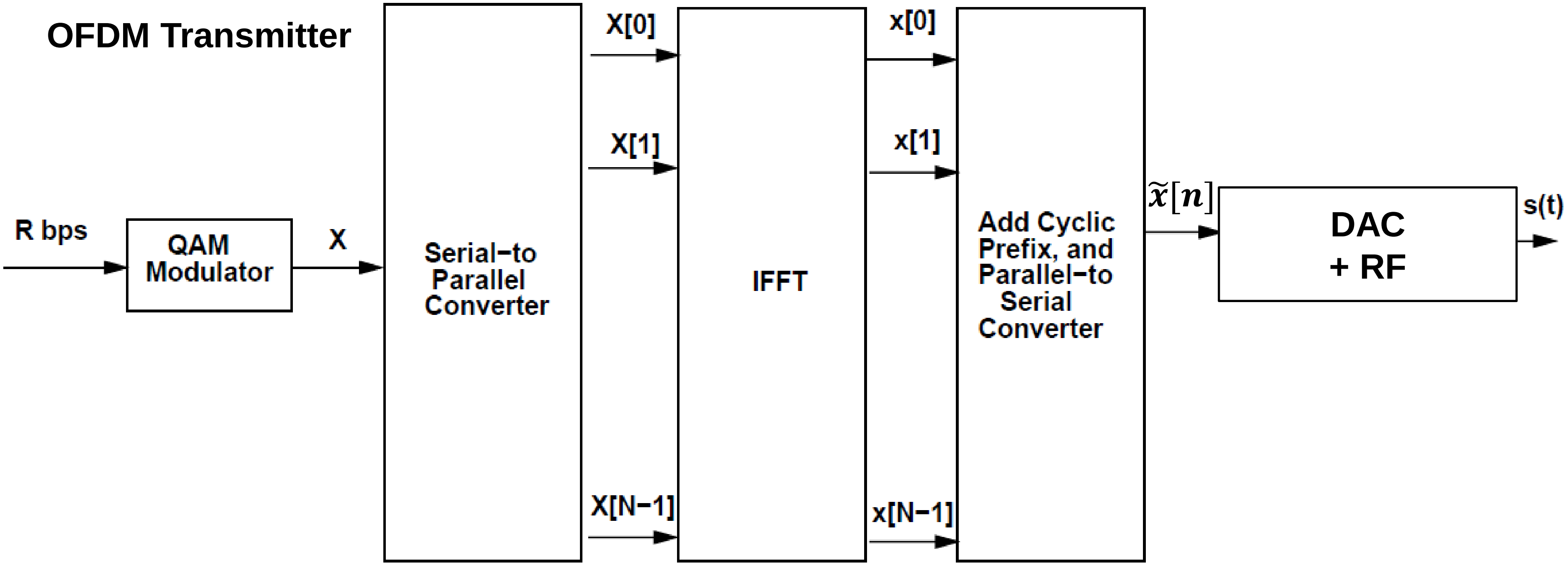


The cyclic prefix eliminates ISI *between data blocks* because the first μ samples of $y[n]$ are discarded without affecting the original information sequence.

6. OFDM

Cyclic Prefix contd: **The penalty incurred by using a cyclic prefix is an overhead of μ/N which results in a data rate reduction of $N/(\mu+N)$ and a power increase of $(\mu+N)/N$.**

Orthogonal Frequency Division Multiplexing (OFDM): **A schematic of the OFDM transmitter is shown using the IFFT implementation**

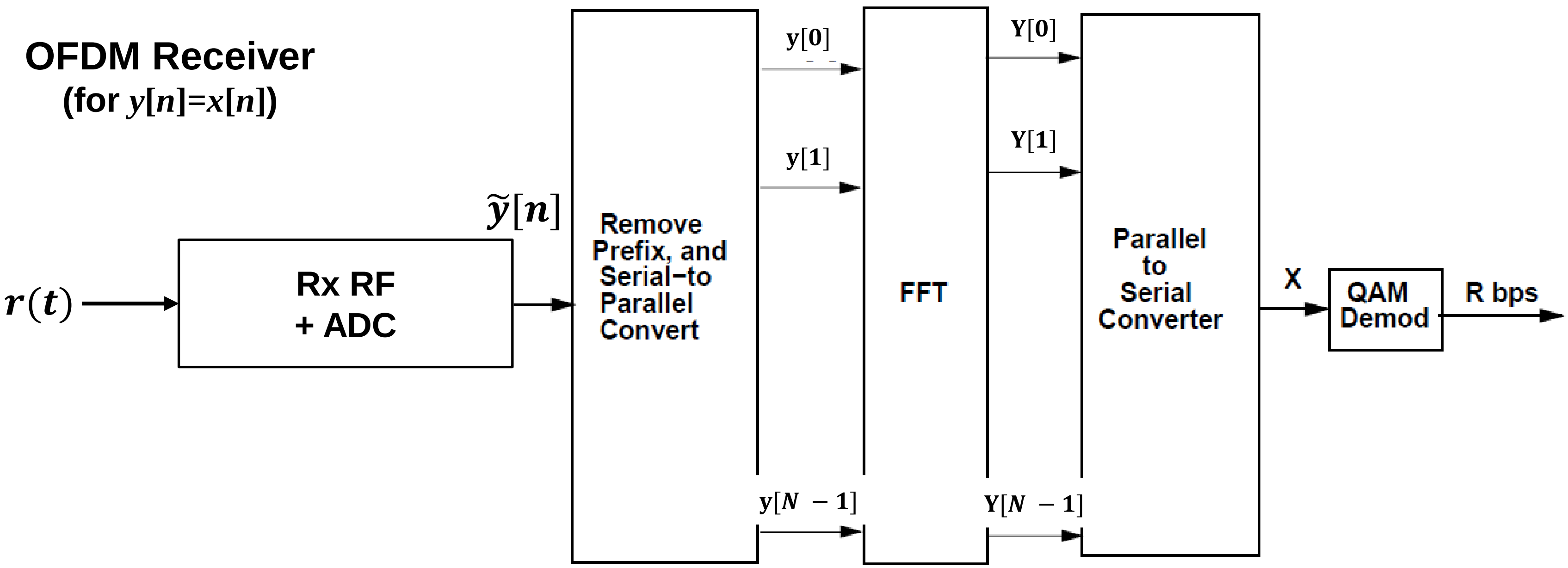


The IFFT produces the OFDM symbol consisting of the sequence $x[n] = x[0], \ldots, x[N-1]$

$$x[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X[i] \exp\left(j2\pi \frac{ni}{N} \right) \quad 0 \le n \le N-1$$

6. OFDM

Orthogonal Frequency Division Multiplexing (OFDM) contd: A schematic of the OFDM receiver is shown using the FFT implementation



The FFT reconstructs the MQAM symbols from the Rx sequence $y[n] = y[0], \ldots, y[N-1]$

$$Y[i] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y[n] \exp\left(-j2\pi \frac{ni}{N} \right) \quad 0 \le i \le N-1$$

The OFDM system effectively decomposes the wideband channel into a set of narrowband orthogonal subchannels with a different QAM symbol sent over each subchannel.

The demodulator can use the channel gains $H[i]$ to estimate the original MQAM symbols from $X[i] = Y[i]/H[i]$ known as *frequency equalisation*. This results in noise enhancement as the noise is scaled by $1/H[i]$. Pre-coding & adaptive loading are better alternatives.

6. OFDM

Orthogonal Frequency Division Multiplexing (OFDM) contd: Example: Consider an OFDM system with total bandwidth $B = 1$ MHz and $\beta = \varepsilon = 0$. A single carrier system would have symbol duration $T_s = 1/B = 1\mu s$. The channel has a maximum delay spread of $T_m = 5\mu s$, so with $T_s = 1\mu s$ and $T_m = 5\mu s$ there will be severe ISI. Assume an OFDM system using MQAM modulation on each subchannel. To keep the overhead small, the OFDM system uses $N = 128$ subcarriers to mitigate ISI. In this case $T_N = NT_s = 128\mu s$. The length of the cyclic prefix is set to $\mu = 8 > T_m/T_s$ in order to ensure no ISI between OFDM symbols.

For these parameters find: 1) the subchannel bandwidth, 2) the total transmission time of each OFDM symbol, 3) the overhead of the cyclic prefix and 4) the data rate of a 16QAM system.

Solution:

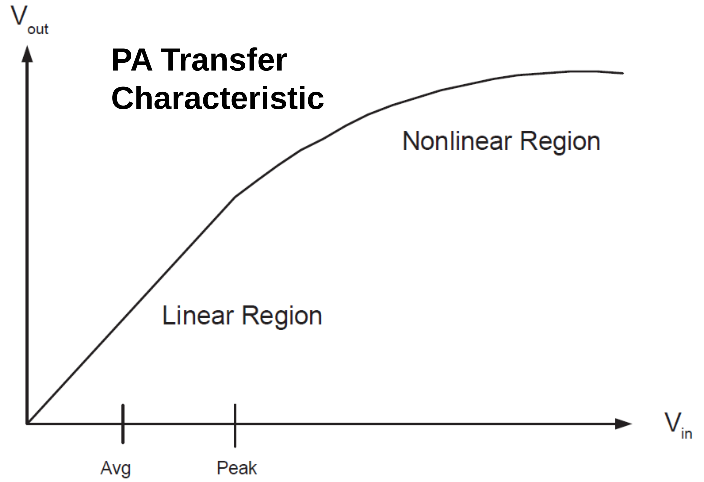
1) The subchannel bandwidth $B_N = 1/T_N = 7.812\text{kHz}$, so $B_N \ll B_c = 1/T_m = 200\text{kHz}$, thereby ensuring negligible ISI.
2) The total transmission time of each OFDM symbol is $T = T_N + \mu T_s = 128 + 8 = 136\mu s$.
3) The overhead associated with the cyclic prefix is 8/128 = 6.25%
4) The system transmits $log_2(16) = 4$ bits per subcarrier every *T* seconds, so the data rate is $128 \times 4/136 \times 10^{-6} = 3.76\text{MHz}$, which is slightly less than 4*B* = 4MHz due to the introduction of the cyclic prefix.

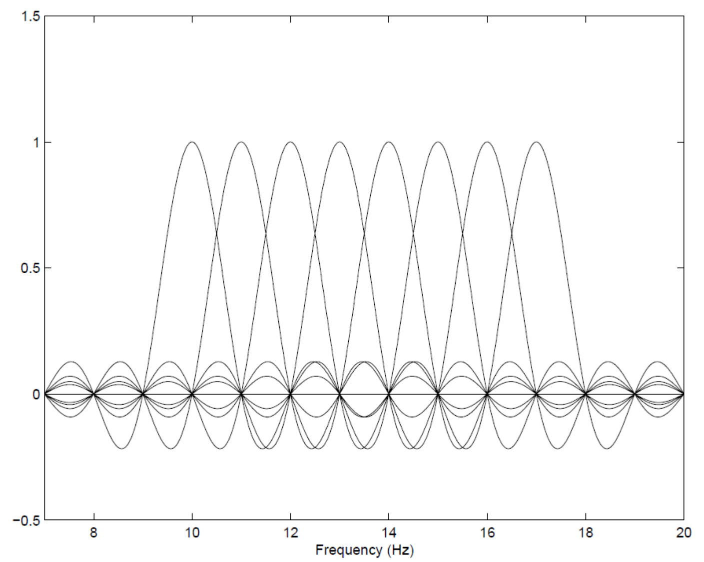
6. OFDM

Challenges in Multicarrier Systems: **Two significant challenges in MC systems such as OFDM are the "Peak-to-Average Power Ratio (PAPR)" and "Frequency & Timing Off-Set"**

Peak-to-Average Power Ratio (PAPR): **The PAPR is important in a practical communication system for a number of reasons:**

- **A high PAPR may saturate the Tx power amplifier (PA) which is a highly nonlinear device. Saturation leads to spectral regrow which causes out-of-band interference. To avoid saturation, the input signal must be significantly backed-off from the saturation point which results in low power efficiency.**
- **A high PAPR makes the Rx ADC high resolution in order to accommodate the large dynamic range which is complex, costly and power hungry.**
- **In OFDM, the PAPR increases with the number of subcarriers.**





$$PAPR_{CT} = \frac{\max_t |x(t)|^2}{E_t\left[|x(t)|^2\right]} \qquad PAPR_{DT} = \frac{\max_n |x[n]|^2}{E_n\left[|x[n]|^2\right]}$$

$$PAPR_{sqwave} = 1 \quad (0dB) \qquad PAPR_{\sin wave} = 2 \quad (3dB)$$

$$x[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X[i] \exp\left(j2\pi \frac{ni}{N} \right)$$

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Frequency & Timing Off-Set: In OFDM orthogonality between subcarriers is ensured when the frequency separation $\Delta f = 1/T_N$. For a rectangular pulse shape in time the Sinc function in frequency have nulls at multiples of $1/T_N$.

If $\Delta f \neq 1/T_N$ due to mismatched oscillators at the Tx and Rx or Doppler shift then the subcarriers will no longer be orthogonal giving rise to Intercarrier Interference (ICI).

Consider the *i-th* subcarrier expressed as: $x_i(t) = e^{j2\pi i t/T_N}$

An interfering subcarrier signal can be expressed as: $x_{i+m}(t) = e^{j2\pi(i+m)t/T_N}$

If the signal is demodulated with a frequency off-set of δ/T_N then the interference is given by: $x_{i+m}(t) = e^{j2\pi(i+m+\delta)t/T_N}$.

The ICI between the *i-th* and *m-th* subcarriers is given by:

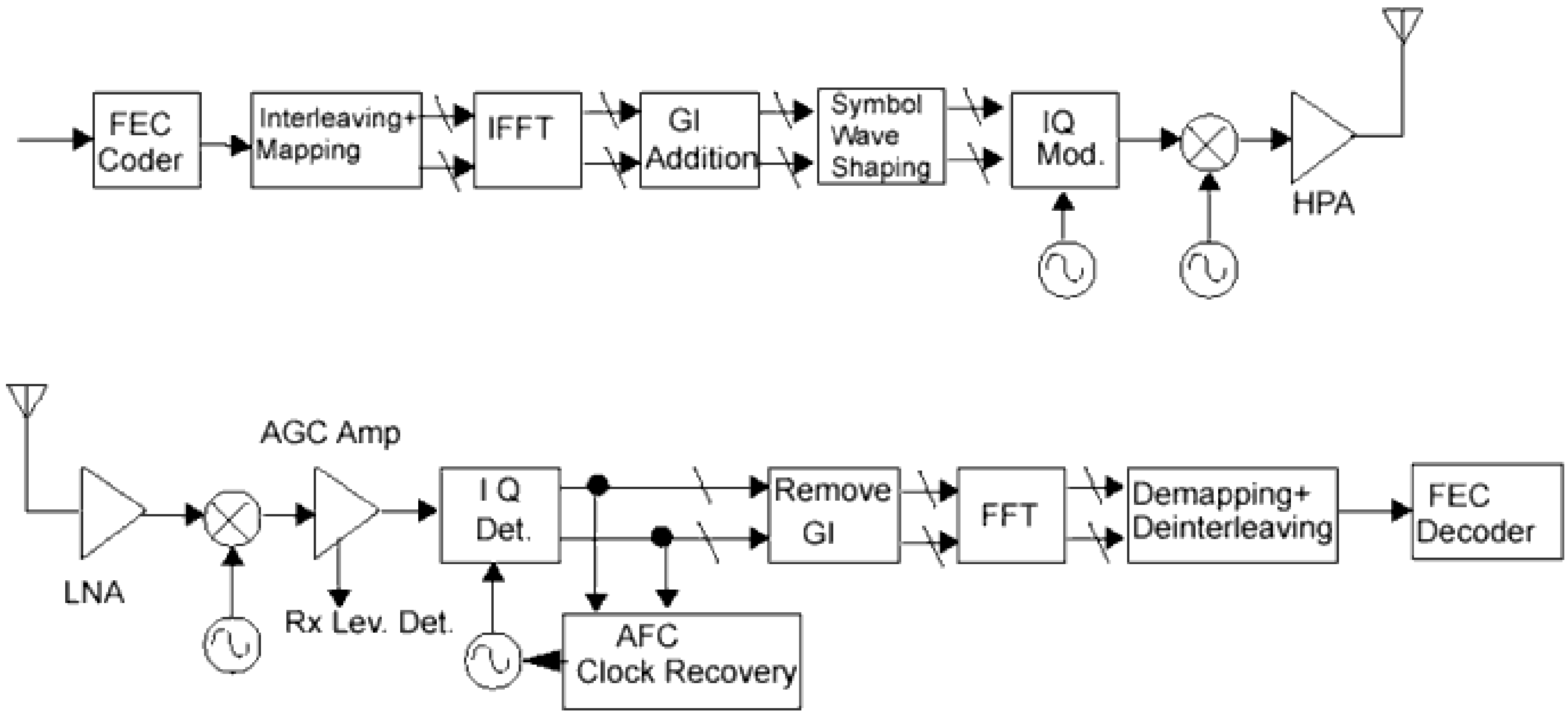
$$I_m = \int_0^{T_N} x_i(t) x_{i+m}^*(t) dt = \frac{T_N(1 - e^{-j2\pi(\delta+m)})}{j2\pi(\delta+m)}$$

And the total ICI power on the *i-th* subcarrier is given by: $ICI_i = \sum_{m \neq i} |I_m|^2 \approx const \cdot (T_N \delta)^2$

- **As T_N increases the subcarriers become narrower and more tightly packed giving more ICI. This also happens as *N* increases since $T_N = N \times T$.**
- **As δ increases so does the ICI and the growth is quadratic.**

6. OFDM

Case Study – The IEEE 802.11a Wireless LAN Standard: **The IEEE 802.11a WLAN standard is OFDM based and occupies 300 MHz of bandwidth in the 5GHz band. 802.11g is identical to .11a but operates in the 2.4GHz ISM band. The following system parameters apply to 802.11a/g:**



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Case Study – The IEEE 802.11a Wireless LAN Standard:

- *N* =64, though only 48 are used to carry data (12 are nulled and 4 are used for pilot symbols).
- The cyclic prefix uses $\mu = 16$ samples.
- Total number of samples per OFDM symbol = 64 + 16 = 80 samples
- The same channel code and modulation order is used on all subcarriers, i.e. it only changes between OFDM packets.
- Possible code rates are ½, 2/3 and ¾.
- Possible modulation orders are BPSK, QPSK, 16QAM and 64QAM.
- Each OFDM channel has a 20 MHz bandwidth (i.e. there are 300/20 = 15 non-overlapping channels. The sampling rate per channel is therefore 1/20MHz = 50ns.

Then the salient characteristics of IEEE 802.11a/g are:

1. The subcarrier bandwidth is $B_N = \frac{20\ \text{MHz}}{64} = 312.5\ \text{kHz}$
2. Since $\mu = 16$ and $1/T_s = 20\ \text{MHz}$, the max$^{\text{m}}$ delay spread for which ISI is removed is

$$T_m < \mu T_s = \frac{16}{20\ \text{MHz}} = 0.8\mu\text{s}$$

This corresponds to the delay spread of an indoor channel

6. OFDM

Case Study – The IEEE 802.11a Wireless LAN Standard Contd:

3. Including both the OFDM symbol and cyclic prefix, there are 64 + 16 = 80 samples per OFDM symbol giving the symbol time per subchannel $T = 80T_s = \frac{80}{20\text{ MHz}} = 4\mu s$.

4. The data rate per subchannel is $log_2(M)/T$. Therefore, the min^m data rate is obtained for BPSK modulation (1 bit/symbol) and ½ rate coding. Only 48 subcarriers are used –

$$R_{\min} = 48 \text{ subcarrier} \times \frac{1/2 \text{ bit}}{1 \text{ coded bit}} \times \frac{1 \text{ coded bit}}{1 \text{ subcarrier symbol}} \times \frac{1 \text{ subcarrier symbol}}{4 \times 10^{-6} \text{ seconds}} = 6 \text{ Mbit/s}$$

5. The max^m data rate is obtained for 64QAM (6 bit/symbol) and ¾ rate coding.

$$R_{\max} = 48 \text{ subcarrier} \times \frac{3/4 \text{ bit}}{1 \text{ coded bit}} \times \frac{6 \text{ coded bit}}{1 \text{ subcarrier symbol}} \times \frac{1 \text{ subcarrier symbol}}{4 \times 10^{-6} \text{ seconds}} = 54 \text{ Mbit/s}$$

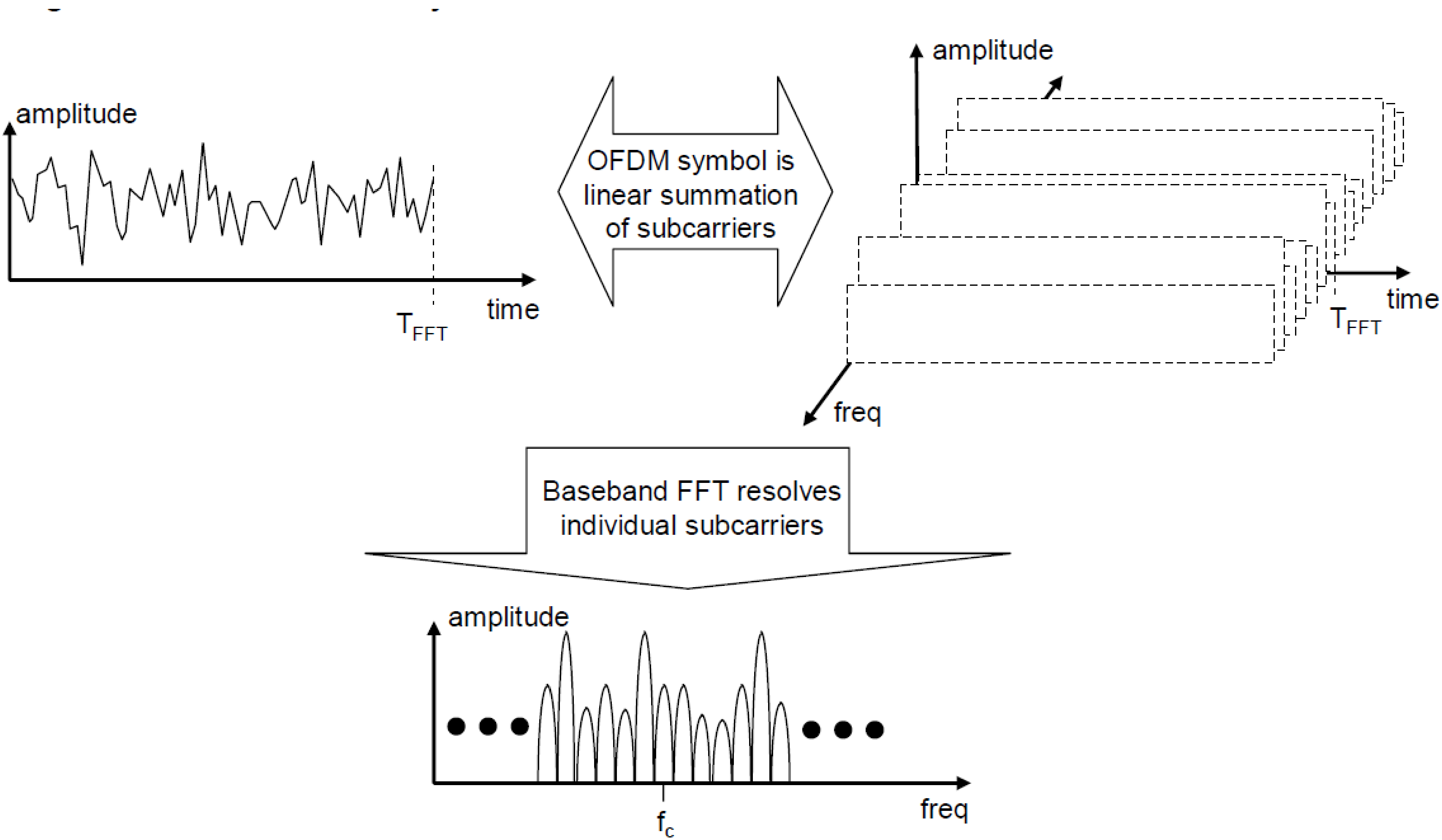
Example: Find the data rate of an 802.11a system using 16QAM and rate 2/3 coding.

Solution:

$$R = 48 \text{ subcarrier} \times \frac{2/3 \text{ bit}}{1 \text{ coded bit}} \times \frac{4 \text{ coded bit}}{1 \text{ subcarrier symbol}} \times \frac{1 \text{ subcarrier symbol}}{4 \times 10^{-6} \text{ seconds}} = 32 \text{ Mbit/s}$$

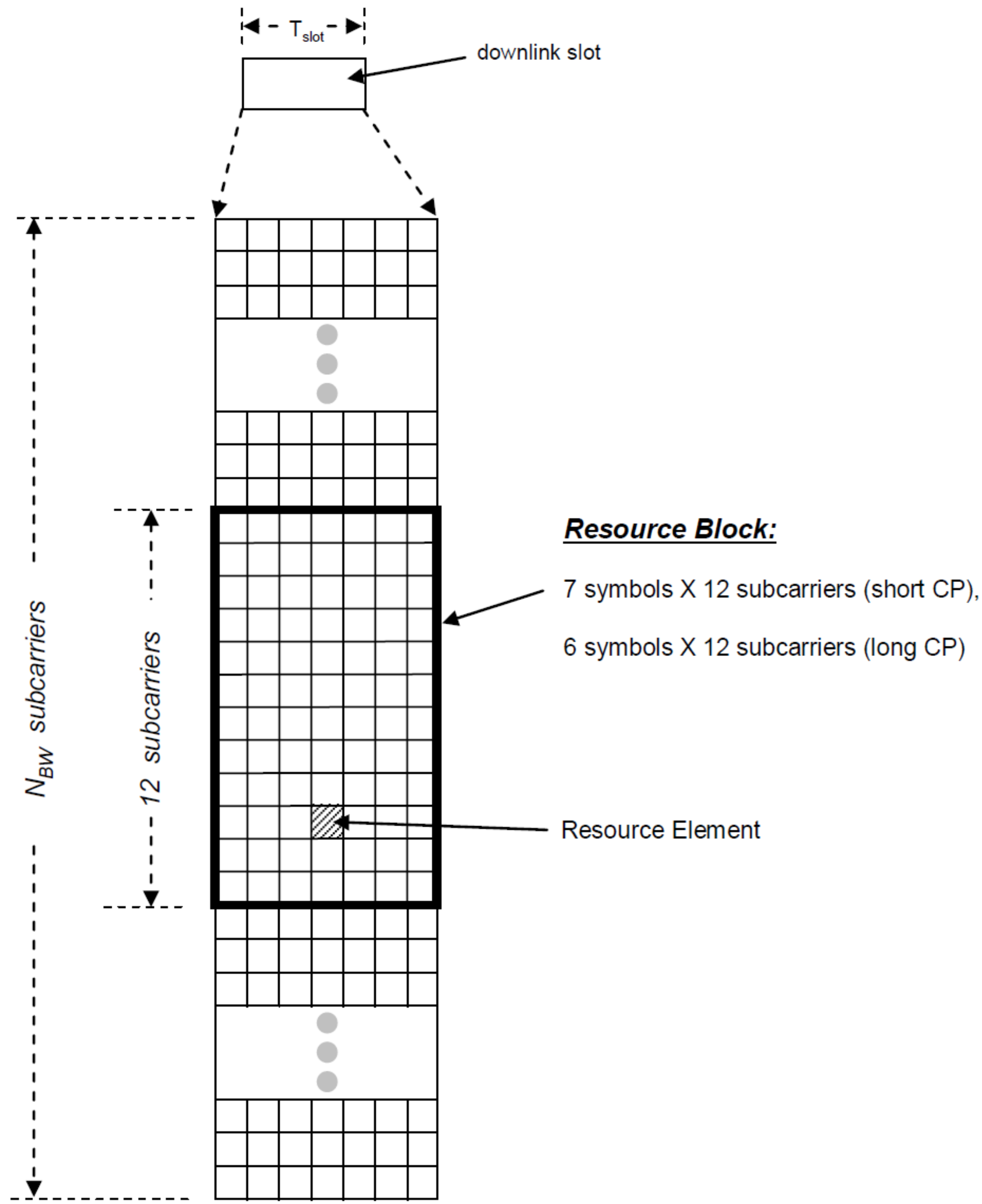
6. OFDM

LTE Framework:



6. OFDM

LTE Framework: OFDMA



6. OFDM

Summary & Main Points:

- **OFDM is readily implemented using FFT/IFFT.**
- **The cyclic prefix in OFDM maintains subcarrier orthogonality while preventing ISI between OFDM symbols.**
- **OFDM is very sensitive to PAPR effects and frequency/time off-sets.**
- **IEEE 802.11a/g uses adaptive coding & modulation to realise a high throughput WLAN for indoor use.**