EEE224/227

Solutions to Tutorial Sheet 3

1. In radians

$\omega_c = 2\pi \times 25 \times 10^6 \quad = \quad 1.57 \times 10^8 \text{ rads / sec}$

$\omega_m = 2\pi \times 400 \quad = \quad 2513 \text{ rads / sec}$

The modulation index is assumed to be identical for FM and PM.

$$\beta = m_f = m_p = \frac{\text{deviation}}{f_m} = \frac{10^4}{400} = \underline{25}$$

Hence the FM and PM equations are:

(a) $v = 4 \sin\left(1.57 \times 10^8 t + 25 \sin 2513t\right)$ FM

(b) $v = 4 \sin\left(1.57 \times 10^8 t + 25 \sin 2513t\right)$ PM

Note: the difference between FM and PM is not apparent at a single modulating frequency but reveals itself in the differing behaviour of the two systems when the modulating frequency is varied. The frequency deviation is changed by the amplitude voltage of the modulating signal only.

2.

$\omega_c \quad = \quad 2\pi f_c \quad = \quad 10^8$

$\underline{\therefore\ f_c = 15.915 \text{ MHz}}$

$m_f = \beta = 3$

$\omega_m \quad = \quad 2\pi f_m \quad = \quad 10^4$

$\underline{\therefore\ f_m = 1.5915 \text{ kHz}}$

deviation $= m_f \times f_m = 4.77$ kHz

power dissipated $= ½ V^2/R = ½ \times \dfrac{100}{100} = \underline{0.5\text{W}}$

3 Using the diagram in the notes for the Armstrong modulator, it can be seen that the output carrier deviation Δf is given by the product of the starting carrier deviation Δf_1 and the multiplication ratios of the two frequency multipliers (i.e. $\Delta f = \Delta f_1 \times n_1 \times n_2$ where n_1 and n_2 are integers)

For values of Δf_1 in the range 20 - 30 Hz, there are many sets of values for n_1 and n_2 which will give Δf = 75 kHz. As an example, choose Δf_1 = 25 Hz, then $n_1 \times n_2 = 3000$ and we can choose n_1 = 50, n_2 = 60.

If we choose 500 kHz as the starting carrier frequency f_1 then at the input to the mixer, this has been increased to 0.5 x 50 = 25 MHz. The overall carrier frequency f_c = 97 MHz = $n_2[f_2 - n_1 f_1]$.
Hence $[f_2 - n_1 f_1] = f_c / n_2$ = 97/60 MHz = 1.617 MHz.
And f_2 = 1.617 + 25 = 26.167 MHz.

On a practical note, it would be better to choose a different starting carrier frequency and frequency deviation so as to allow N and M to be divisible by powers of 2 and/or 3. This would then ease the design of the frequency multiplier circuits.

4.

$\omega_c = 2\pi * 12 * 10^6$ rrrads/s
A_c=5V
df/V=25000
v_s=1.5sin(6280t)

Hence, fs=1kHz

Modulation index, m_f=df/f_s

df=25000*1.5=35700
m_f=37500/1000=37.5

Hence equation is:-

V_{FM}=5cos(24πt-37.5cos(6280t)

a). Maximum phase change is 37.5rads
b). f_m=500Hz, then

df =37.5kHz
max phase change =37500/500=75rads

c.) If v_s=3V, then df=25*3 = 75kHz

5. $$f = \frac{1}{2\pi\sqrt{LC}}$$

L and C are associated with the tuned circuit
If C increases by ΔC, f decreases by Δf

$$\therefore \ f - \Delta f = \frac{1}{2\pi\sqrt{L(C+\Delta C)}}$$

$$\therefore \ \frac{f-\Delta f}{f} = \left(1+\frac{\Delta C}{C}\right)^{-½}$$

If ΔC/C <<1, use binomial expansion

$$\frac{\Delta f}{f} \sim \frac{1}{2}\left(\frac{\Delta C}{C}\right)$$

$$\text{i.e. } \Delta C \sim 2\frac{\Delta f}{f}C$$

$\Delta f = 8 \times 10^4$ $\quad f = 100 \times 10^6$ $\quad C = 75 \times 10^{-12}$

$\therefore \ \Delta C = 0.12 \times 10^{-2} = 0.12$ pF

Since total frequency swing is ± 80 kHz

$\therefore$ Total capacitance swing = ± <u>0.12 pF</u>

6. Sidebands are spaced 10 kHz apart

$\therefore$ Number of sideband pairs in 160 kHz = 8

From Table 3 $\beta = 5$

$\therefore \ \beta f_m = \Delta f$ i.e. $\Delta f = 5 \times 10 =$ <u>50 kHz</u>.

7. 1st sidebands disappear when $J_1(\beta) = 0$

i.e. $\beta = 3.832$.

Then carrier amplitude $\propto J_o(3.832) = -{\cdot}4$

2nd sideband amplitude $\propto J_2(3.832) = {\cdot}4$

When carrier is unmodulated

$$\frac{V_c^{\ 2}}{2.50} = 100\text{ W} \quad \therefore V_c = 100\text{ V}$$

When $\beta = 3.832$

$$\text{carrier power} = \frac{V_c^2}{2R} \cdot \left[J_o(\beta) \right]^2 = 100 \times 0.16 = \underline{16\ \text{W}}$$

$\therefore$ Power in remaining sidebands = 100 - 16 = $\underline{84\ \text{W}}$

2nd sideband power

$$= \frac{2V_c^2}{2R} \left[J_2(\beta) \right]^2 = 2.100.0 \cdot 16 = \underline{32\ \text{W}}$$