Data Provided: Log 3 cycle by Log 3 cycle graph paper

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2012-13 (2.0 hours)

EEE112 Engineering Applications 1

This paper comprises **TWO** sections, **A** and **B**. You may gain up to **60 MARKS** from **SECTION A** and **40 MARKS** from **SECTION B**. Attempt **ALL** the questions in **SECTION A**. Marks will be awarded for your best **TWO** solutions in **SECTION B**. Trial answers will be ignored if they are clearly crossed out. A formula sheet is included at the end of the exam paper. **The numbers given after each section of a question indicate the relative weighting of that section.**

SECTION A

1. **a.** Simplify $\sqrt{\dfrac{x^{-5}}{x^{-2}}}$ **(1)**

b. Transpose $p = \dfrac{a^2x + a^2y}{r}$ to make ***a*** the subject. **(1)**

c. Simplify $\dfrac{2}{2v+1} - \dfrac{3}{3v+2}$ **(2)**

d. Transpose $Z = \sqrt{R^2 + \left(wL - \dfrac{1}{wC}\right)^2}$ to make ***C*** the subject. **(2)**

e. Differentiate $y = \sqrt{5x^2 - 4x - 1}$ with respect to **x**. **(4)**

2. **a.** Find the general solution of $\dfrac{dy}{dx} = \dfrac{2+y}{3+x}$ **(3)**

b. Solve the differential equation $3\dfrac{dv}{dt} = 3 + 5v$

subject to the initial conditions ***v*** =2 when ***t*** = 0, giving your answer in the form ***v*** = … **(7)**

3. **a.** A time varying current is described by the equation

$$i(t) = 6\sin\left(50pt - \frac{p}{4}\right)\text{Amps.}$$

For this equations write down:

(i) the peak-to-peak current,

(ii) the angular frequency (in radians),

(iii) the phase shift (in radians),

(iv) the period.

(4)

b. For the same time varying current as described in part **a.** sketch the waveform shape between ***t*** = -20 ms and ***t*** = +60 ms. On this sketch label clearly:

(i) all the points in time where the waveform crosses the time axis (giving the times at which this occurs) between ***t*** = -20 ms and ***t*** = +60 ms.

(ii) the amplitude of the waveform (giving it's value in Amps).

(5)

c. Re-write the same time varying current equation as described in part **a.** as a **cosine** expression rather than a **sine** expression. **(1)**

4. **a.** Express $-3.2\sin(wt) - 4.7\cos(wt)$ in the form $R\sin(wt + \alpha)$ giving $\boldsymbol{\alpha}$ in radians in the range $-\pi \le \alpha \le +\pi$.

(2)

b. An alternating current circuit has voltage $v(t) = 6\sin(wt)$ across it and current $i(t) = 4\sin(wt - \frac{p}{3})$. Given that instantaneous power can be found from the equation $p(t) = v(t).i(t)$ show, using trig. identities, that the power in this circuit can also be described as $p(t) = 6\left[1 - 2\cos(2wt - \frac{p}{3})\right]$

(8)

5. **a.** Find the value of the following determinant $A = \begin{vmatrix} 3 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{vmatrix}$

(3)

b. Solve the following simultaneous equations using the method of **<u>Gaussian elimination</u>** only, that is the method that uses an augmented matrix (NOT by Cramer's Rule and NOT by substitution) to find the values of ***x, y*** & ***z***.

$$x - 4y - 2z = 21$$
$$3x + 2y - z = -2$$
$$2x + y + 2z = 3$$

(7)

6. **a.** Exponential voltage decay in electronic circuits is sometimes described by equations of the form $v(t) = V_o e^{-t/t}$ where $\boldsymbol{V_o}$ is the starting voltage, $\boldsymbol{t}$ is the time and $\boldsymbol{\tau}$ is the time constant. How many multiples of the time constant must pass by for the voltage to have fallen to less than 1% of the starting voltage?

(1)

b. In a particular circuit voltage decays exponentially according to the equation given in part **a.** above where the starting voltage $\boldsymbol{V_o}$ = 12 V and the time constant $\boldsymbol{\tau}$ = 40ms. Find the value of time $\boldsymbol{t}$ when the voltage has fallen to 1.8 V.

(3)

c. Exponential current growth in electronic circuits is sometimes described by equations of the form $i(t) = I_o\left(1 - e^{-t/t}\right)$ where $\boldsymbol{I_o}$ is the current after an infinite length of time, $\boldsymbol{\tau}$ is the time constant and $\boldsymbol{t}$ is the time.

For a particular circuit $\boldsymbol{I_o}$ = 40 A and $\boldsymbol{\tau}$ = 10 ms. How long will it take for the current to rise from 26 A to 38 A? **(6)**

SECTION B

7. **a.** Using loop current analysis (by defining closed current loops using Kirchoff's voltage law) derive 3 equations for loop currents $\boldsymbol{I_1}$, $\boldsymbol{I_2}$, *and* $\boldsymbol{I_3}$ for the circuit shown below in figure 1 below. **(6)**

b. Solve the 3 simultaneous equations formed in answer to part **a.** above using the method of **Cramer's Rule** only that is the method that uses determinants (NOT by Gaussian Elimination and NOT by substitution). **(11)**

c. Using the loop currents found in part **b.** above, determine the actual current flowing in the following two resistors and the direction in which it flows:

(i) the 6Ω resistor,

(ii) the 1Ω resistor. **(3)**

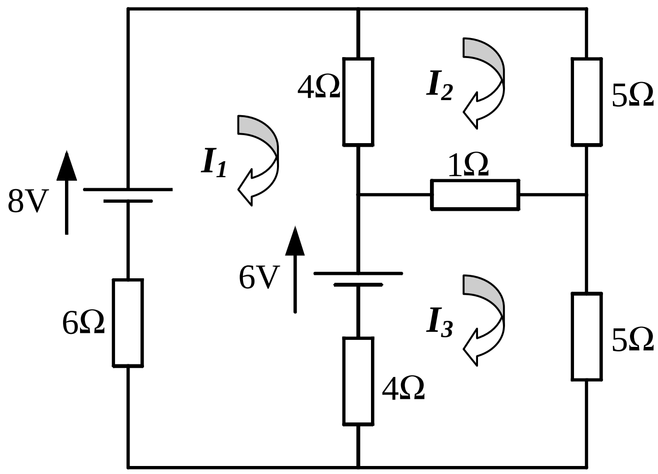

Figure 1.

8. **a.** Two complex number are given by $\boldsymbol{H} = \mathbf{1} + \boldsymbol{j}\mathbf{3}$ and $\boldsymbol{K} = \mathbf{4} - \boldsymbol{j}\mathbf{2}$. Plot both $\boldsymbol{H}$ and $\boldsymbol{K}$ on the same ARGAND diagram. **(2)**

b. Using the same $\boldsymbol{H}$ and $\boldsymbol{K}$ as in part **a.** find the results of the following two equations giving the answers in both rectangular (also called Cartesian) form (**Re+jIm**) and also in polar from ($r\angle q$):

(i) $\boldsymbol{H} - \boldsymbol{K}$
(ii) $\boldsymbol{K} / \boldsymbol{H}$ **(4)**

c. A series connected circuit consisting of two components has a total impedance of **50Ð-60°** W

(i) Determine the value of the resistance and the series connected reactance that make up this circuit giving your answers in ohms.
(ii) What sort of component will the reactance be and what will its value be if the supply frequency is 400 Hz? **(4)**

d. For the circuit shown in figure 2 below determine,

(i) the current flowing in impedance $\boldsymbol{Z_1}$, **(3)**

(ii) the value of the unknown impedance $\boldsymbol{Z_1}$ in ohms, **(5)**

(iii) the components comprising $\boldsymbol{Z_1}$ if the supply frequency is 1kHz. **(2)**

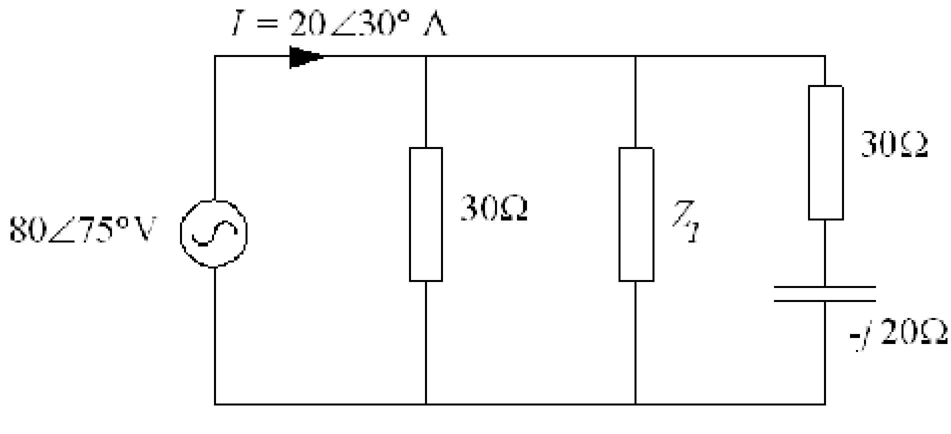

Figure 2.

9. **a.** Write down the general expression for the **mean** (sometimes called the **average**) value of a periodic current ***i(t)*** of period **T**. **(2)**

b. Several cycles of a voltage waveform are shown in Figure 3 below. Calculate both the mean value and rms value for this waveform.

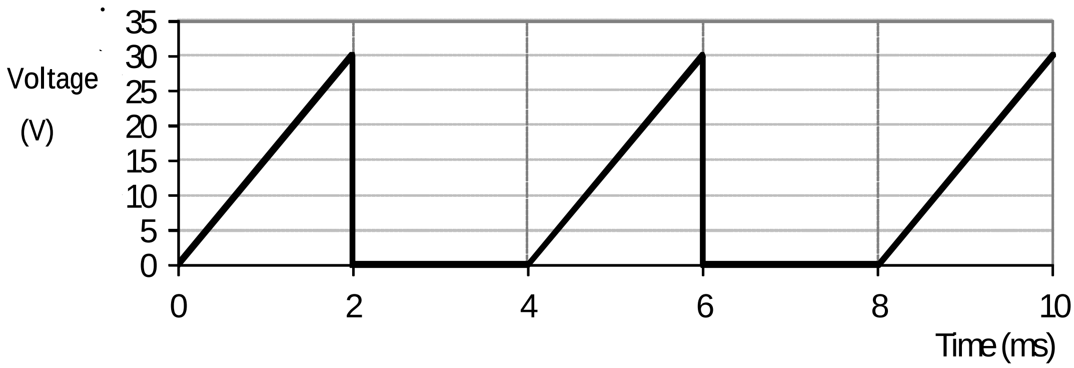

Figure 3.

(10)

c. A voltage is described by $v(t) = \cos(t) - 3$ (Volts). Calculate the r.m.s value of the voltage waveform over the period $t = 0$ to $t = 2$p. **(8)**

10. **a.** Show that when data behaves according to a relationship of the form $y = a.x^n$ (where ***a*** and ***n*** are both constants) it can be plotted on logarithmic graph paper as a straight line, by manipulating the equation. (Hint: take log.s).

(2)

b. The following data is believed to behave according to the relationship of the form $y = a.x^n$ (where ***a*** and ***n*** are both constants). Using the graph paper provided, show that the data does indeed correspond with this relationship.

x	y
1	3
3	11
8	36
20	109
50	328
120	937

c. Using the data given in part **b.** above find the values of:

(i) the constant ***n***

(ii) the constant ***a***.

FORMULA SHEET

Trig. Identities

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\sin A\cos B = \frac{1}{2}\sin(A+B) + \sin(A-B)$$

$$\cos A\cos B = \frac{1}{2}\cos(A-B) + \cos(A+B)$$

$$\sin A\sin B = \frac{1}{2}\cos(A-B) - \cos(A+B)$$

$$\sin^2 q = \tfrac{1}{2}(1-\cos 2q)$$

$$\cos^2 q = \tfrac{1}{2}(1+\cos 2q)$$

Logarithmic Laws

$$\log_a x^n = n\log_a x$$

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Integration for $f(x)$

$$\int \sin x = -\cos x + c$$

$$\int \sin k.x = -\frac{1}{k}\cos k.x + c$$

$$\int \cos x = \sin x + c$$

$$\int \cos k.x = \frac{1}{k}\sin k.x + c$$

$$\int \frac{1}{x} = \ln(x) + c$$

PLJ