Use of transformations in modelling AC machines

- The behaviour of three-phase machines is usually considered in terms of voltage and current equations
- The various coefficients in the resulting equations are time varying
- This makes the modelling challenging (but it should be stressed far from impossible) because flux linkages, voltages, and currents are all influenced by relative motion between the rotor and stator
- To simplify modelling, transformations are often used to transform the electrical quantities by referring them into to a common frame of reference, often the rotating frame
- There are many approaches to achieving this goal depending on whether the system is balanced (most straightforward and considered here) or unbalanced (more involved but useful for modelling faulted conditions, inherently unbalanced machines etc)
- For balanced conditions, two commonly used transformations are the so-called Clarke and Park transforms (which you may come across elsewhere on other modules)
- As well as being useful for modelling machines, these transformations (and variants thereof) are used to simplify the control of electrical machine

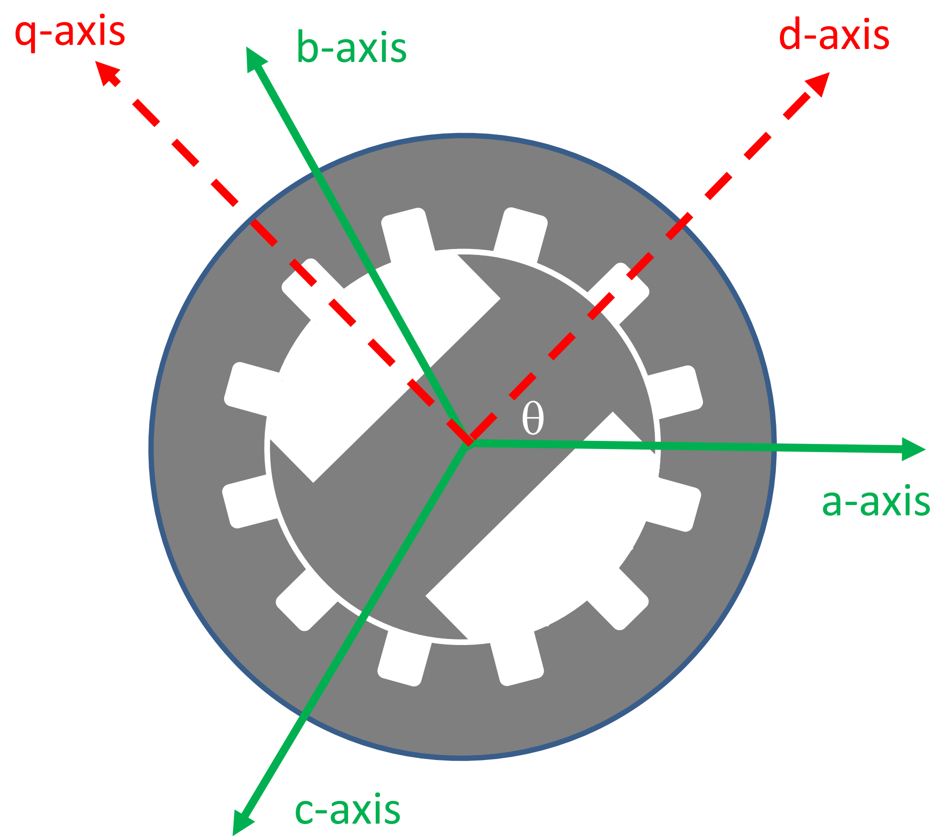
dq transformations

- The d-axis is aligned with the rotor pole
- The q-axis is orthogonal (i.e. at 90°) to the d-axis
- The a-b-c axes are aligned with the stator poles

Example shown in a 2-pole machine (i.e. 1 pole-pair)

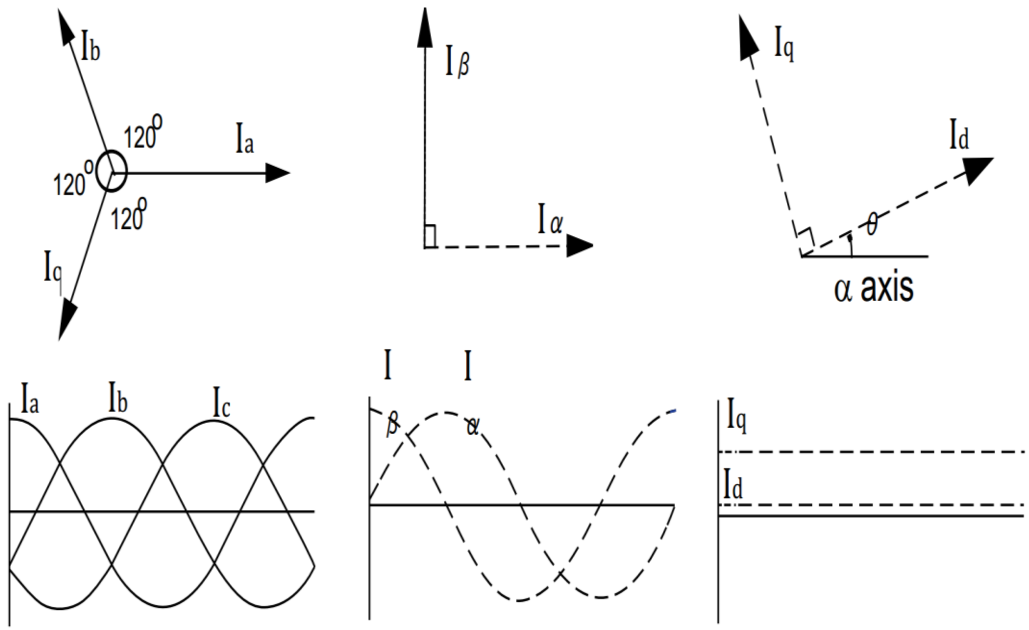
The concept can be extended to any number of pole-pairs by recognising:

$\theta_{me} = p\theta$

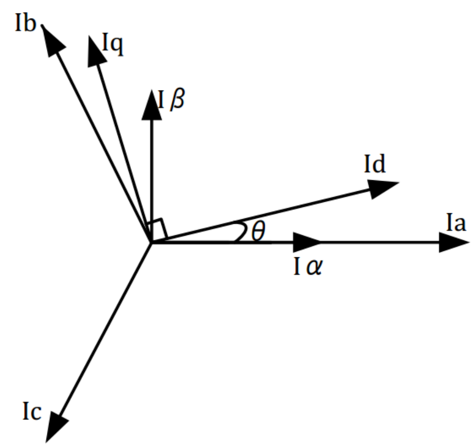


Transformation from a **balanced** three-phase system to d-q axis representation

Balanced abc representation αβ representation dq representation

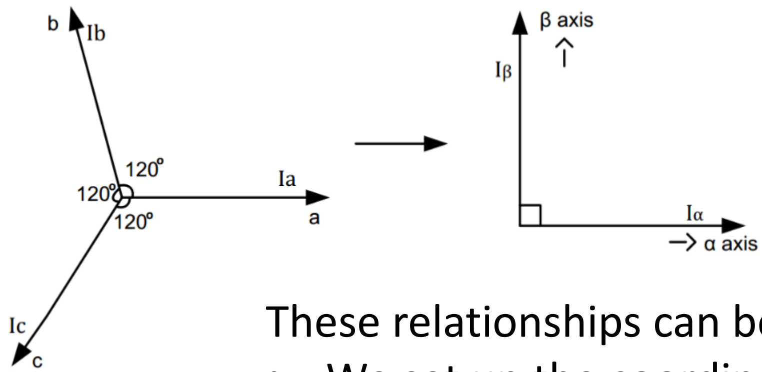


Combined phasor representation



Clarke Transformation

- Converts balanced three-phase quantities into balanced two-phase quantities quadrature quantities



$$I_\beta = \frac{2}{\sqrt{3}}(I_b - I_c)$$

$$I_\alpha = \frac{2}{3}I_a - \frac{1}{3}(I_b - I_c)$$

These relationships can be simplified if the following conditions are met:

- We set up the coordinates such that I_a and I_α are aligned
- The phase windings are star connected such that $I_a+I_b+I_c=0$

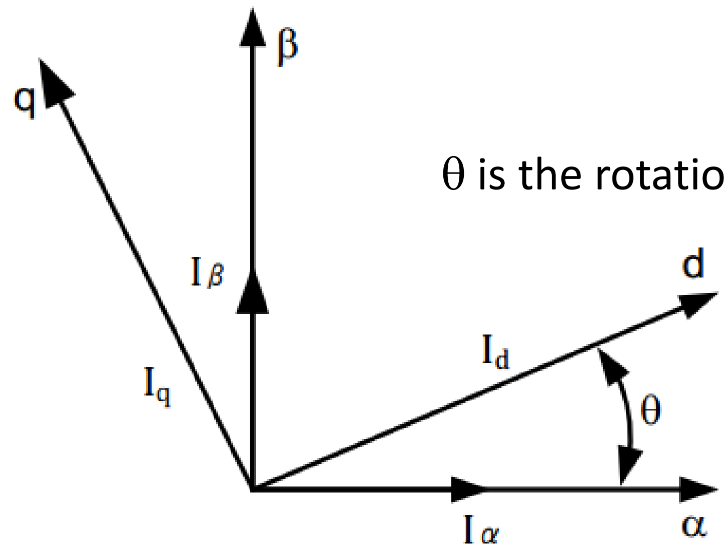
$$I_\alpha = I_a \quad \text{and} \quad I_\beta = \frac{1}{\sqrt{3}}(I_a - 2I_b)$$

Inverse Clarke Transformation

Transforms balanced quantities from a two-axis stationary reference frame to a balanced three-phase stationary reference frame

$$I_a = I_\alpha \qquad I_b = \frac{-I_\alpha + \sqrt{3}I_\beta}{2} \qquad I_c = \frac{-I_\alpha - \sqrt{3}I_\beta}{2}$$

Park and Inverse Park Transformation



θ is the rotation angle

Park Transformation

The αβ stationary reference frame quantities are transformed into the rotating dq reference frame:

$$I_d = I_\alpha cos\theta + I_\beta sin\theta$$

$$I_q = I_\beta cos\theta - I_\alpha sin\theta$$

Inverse Park Transformation

The rotating dq reference frame quantities are transformed into the stationary αβ reference frame:

$$I_\alpha = I_d cos\theta - I_q sin\theta$$

$$I_\beta = I_q cos\theta + I_d sin\theta$$

Scaling in Clarke transform

You may have noticed in the transformation that there is a degree of scaling involved in the transformations and that I_α and I_β are not simply I_a I_b and Ic resolved along the α and β axes.

In principle, there are a number of different transformations we could use (each with their own scaling) since I_α and I_β are transformed currents rather than physical currents

It is important to recognise that providing we use the appropriate formulae (with due account of scaling) for torque, power etc, then the transformation will ultimately yield the same relationship between the actual currents and torque.

The Clark transform uses a scaling of 2/3, which is more apparent if we break down the Clark transform:

$$I_\alpha = \frac{2}{3}\left(I_a - I_b cos\frac{\pi}{3} - I_c\, cos\frac{\pi}{3}\right) = \frac{2}{3}\left(I_a - \frac{1}{2}I_b - \frac{1}{2}I_c\right)$$

Scaling

Resolving phasors onto α and β

$$I_\beta = \frac{2}{3}\left(I_b cos\frac{\pi}{6} - I_c\, cos\frac{\pi}{6}\right) = \frac{2}{3}\left(\frac{\sqrt{3}}{2}I_b - \frac{\sqrt{3}}{2}I_c\right)$$

In matrix form, this can be expressed as:

$$\begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

In the Clarke transform, the 2/3 factor ensures that the magnitude of the quantities remain the same in they transformed domain.

There are several different scaling factors that you may come across in textbooks, journal papers etc, e.g. 3/2 convention, rms conventions, power invariant. All of these are correct, providing the scaling is applied consistently in the inverse transformations and in calculating output quantities.