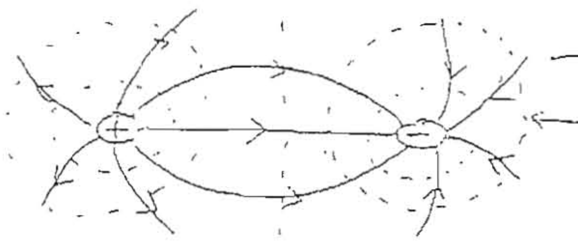
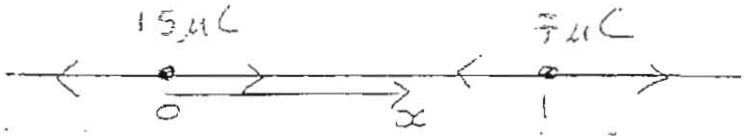
Question 1



[2]



As both charges +ve field cancelation point must lie between them

Total field at x is.

$$E = \frac{kq_1}{x^2} - \frac{kq_2}{(1-x)^2} = 0$$

$$\rightarrow \frac{15}{x^2} - \frac{7}{(1-x)^2} = 0$$

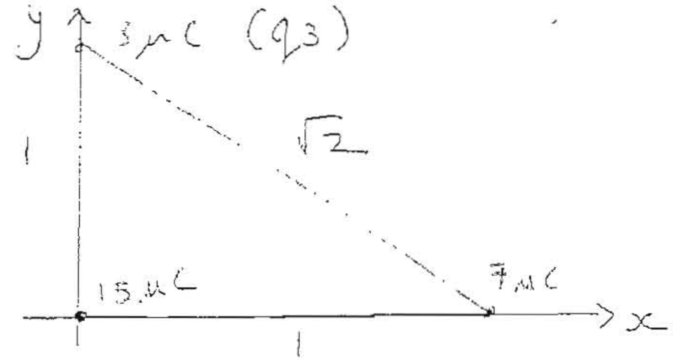
$$\rightarrow 8x^2 - 30x + 15 = 0$$

$$\rightarrow x = 1.88 \pm \sqrt{1.28}$$

$$\underline{x = 1.88 - \sqrt{1.28} = 0.743\,m}$$

is only valid selection [5]

Q1



$= \cos 45°$

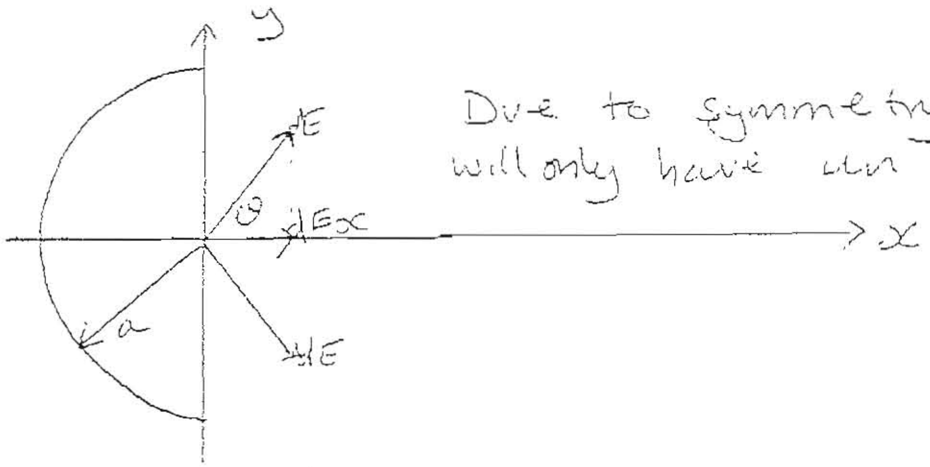
Field at q_3 is $$E = \frac{-\hat{x}}{4\pi\epsilon_0}\left[\frac{7\times10^{-6}}{(\sqrt{2})^2}\times\frac{1}{\sqrt{2}}\right]$$

$$+\frac{\hat{y}}{4\pi\epsilon_0}\left[\frac{7\times10^{-6}}{(\sqrt{2})^2}\times\frac{1}{\sqrt{2}} + \frac{15\times10^{-6}}{1}\right]$$

$\sin 45°$

$$\Rightarrow \underline{F_3 = q_3 E = -\hat{x}\left[0.0668\right] + \hat{y}\left[0.471\right] \text{ N}}$$

[3]



Due to symmetry field will only have an x component

Let charge / unit length $= q_l = \frac{Q}{\pi a}$

Then $$dE_x = \frac{q_l\, a\, d\theta \cos\theta}{4\pi\epsilon_0 a^2}$$

Total field $$= 2 \times \int_0^{\pi/2} dE_x = 2\int_0^{\pi/2} \frac{q_l \cos\theta}{4\pi\epsilon_0 a}\, d\theta$$

Q1

$$= \frac{q_l}{2\pi\epsilon_0 a}\Big[\sin\alpha\Big]_0^{\pi/2}$$

$$= \frac{q_l}{2\pi\epsilon_0 a} = \frac{Q}{2\pi^2\epsilon_0 a^2}$$ [7]

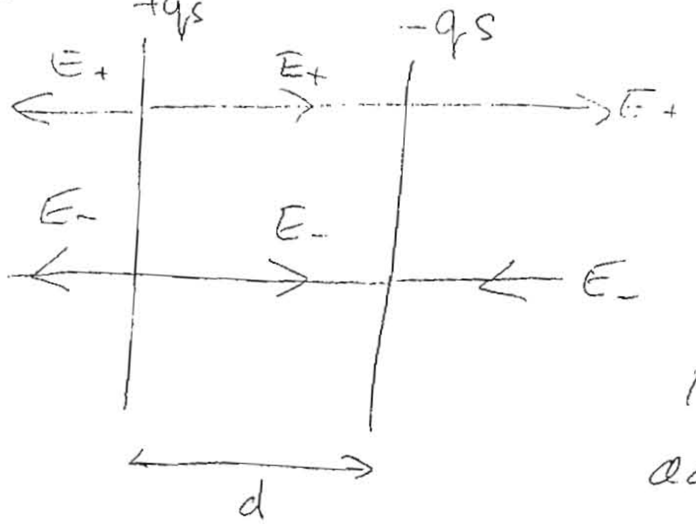
Field at origin due to ∞ line charge is

$\frac{q_l}{2\pi\epsilon_0 (2a)}$ and is in -ve x direction

$\Rightarrow$ Total field is

$$E = \left(\frac{Q}{2\pi^2\epsilon_0 a^2} - \frac{q_l}{4\pi\epsilon_0 a}\right)\hat{x}$$ [3]

(2)



outside plates fields cancel

Inside plates fields add.

$\rightarrow$ Total field inside

$$E = E_+ + E_- = \frac{q_s}{2\epsilon_0} + \frac{q_s}{2\epsilon_0} = q_s/\epsilon_0$$

Potential difference between plates

$$V = \int_{x=0}^{d} E\,dx = \frac{q_s d}{\epsilon_0}$$

By definition $C = Q/V$

where $Q = q_s A$ – A = Area of plates

$$\rightarrow \quad C = \frac{q_s A}{q_s d/\epsilon_0} = \frac{\epsilon_0 A}{d}$$

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2)

b) Potential difference between plates is determined by battery voltage V which does not change when separation of plates is changed.

we have

$$Q = CV = \frac{\epsilon_0 A V}{d}$$

$$\therefore Q_1 = \frac{\epsilon_0 AV}{d_1} \text{ and } Q_2 = \frac{\epsilon_0 AV}{d_2}$$

change in charge is

$$Q_2 - Q_1 = \epsilon_0 AV \left(\frac{1}{d_2} - \frac{1}{d_1} \right)$$

For values given

$$Q_2 - Q_1 = 8.854 \times 10^{-12} \times 8 \times 10^{-3} \times 5 \times \left(\frac{10^3}{1.2} - \frac{10^3}{1.0} \right)$$

$$= -5.90 \times 10^{-11} \, C$$

i.e. charge decreases. This takes pla[ce] in 0.3s, so current is of the order

$$\left| \frac{Q_2 - Q_1}{0.3} \right| = 1.97 \times 10^{-10} \, A$$

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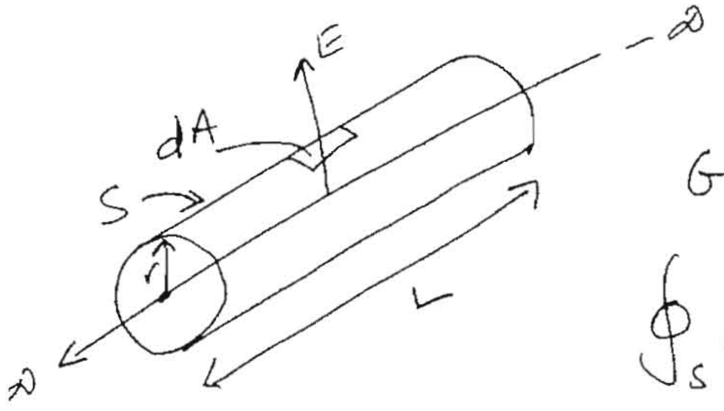
3a



Gauss' Law

$$\oint_S E_\perp \, dA = Q/\epsilon_0$$

Due to symmetry E cannot vary along wire (as ∞) or around wire

∴ E-field must point radially outwards

when evaluating $\oint E_\perp \, dA$, ends of cylinder do not contribute as dA is parallel to E

contribution from curved part of cylinder is

$$E_\perp \cdot \underbrace{2\pi r L}_{\text{surface area } S} = Q/\epsilon_0$$

$$\therefore \; E = \frac{Q}{L} \cdot \frac{1}{2\pi r \epsilon_0}$$

or $E = \dfrac{q_l}{2\pi r \epsilon_0}$ where $q_l = Q/L$ charge per unit length

6

3b

i) $$\underline{E} = \frac{q_{l1}}{2\pi\epsilon_0 R_1^2}\underline{R_1} + \frac{q_{l2}}{2\pi\epsilon_0 R_2^2}\underline{R_2}$$

$$= \frac{1.5\times10^{-6}}{2\pi\epsilon_0(\sqrt{26})^2}(5,1,0) - \frac{3\times10^{-6}}{2\pi\epsilon_0(\sqrt{2})^2}(1,1,0).$$

$$= \underline{(-21.78, -25.93, 0)\times10^{3}\ \mathrm{Vm^{-1}}}$$

ii) $$\underline{E} = \frac{1.5\times10^{-6}}{2\pi\epsilon_0(\sqrt{5})^2}(-2,1,0) - \frac{3\times10^{-6}}{2\pi\epsilon_0(\sqrt{37})^2}(-6,1,0)$$

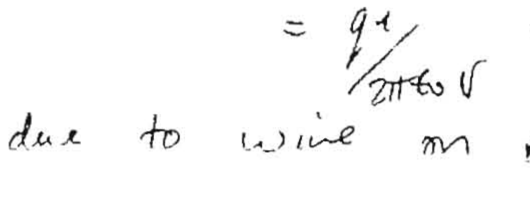
$$= \underline{(-2.04, 3.94, 0)\times10^{3}\ \mathrm{Vm^{-1}}}$$

iii) at (2,2,0) point is inside perfect conductor, hence

$$\underline{\underline{E} = 0}$$

6

3c



At point P $|\underline{E}|$ due to wire on LHS

$$= \frac{q_l}{2\pi\epsilon_0 r} = \frac{1.5\times10^{-6}}{2\pi\epsilon_0(x-2)} \quad \text{to right}$$

$|\underline{E}|$ due to wire on RHS

$$= \frac{q_l}{2\pi\epsilon_0 r} = \frac{-3\times10^{-6}}{2\pi\epsilon_0(6-x)} \quad \text{to left}$$

c)

$$\therefore \text{Total} \quad E_x = \frac{1.5 \times 10^{-6}}{2\pi \varepsilon_0 (x-2)} - \frac{3 \times 10^{-6}}{2\pi \varepsilon_0 (6-x)}$$

potential difference

$$V = \int_{2.01}^{5.99} E_x \, dx$$

$$\therefore V = \frac{1.5 \times 10^{-6}}{2\pi \varepsilon_0} \int_{2.01}^{5.99} \frac{dx}{x-2} - \frac{3 \times 10^{-6}}{2\pi \varepsilon_0} \int_{2.01}^{5.99} \frac{dx}{6-x}$$

$$= \frac{1.5 \times 10^{-6}}{2\pi \varepsilon_0} \Big[\ln(x-2) \Big]_{2.01}^{5.99}$$

$$- \frac{3 \times 10^{-6}}{2\pi \varepsilon_0} \Big[-\ln(6-x) \Big]_{2.01}^{5.99}$$

$$= \frac{1.5 \times 10^{-6}}{2\pi \varepsilon_0} \Big[\ln(3.99) - \ln(0.01) \Big]$$

$$- \frac{3 \times 10^{-6}}{2\pi \varepsilon_0} \Big[-\ln(0.01) + \ln(3.99) \Big]$$

$$= \frac{3 \times 10^{-6}}{2\pi \varepsilon_0} \Big[\ln(3.99)/2 - \ln(0.01)/2$$

$$+ \ln(0.01) - \ln(3.99) \Big] \underline{\underline{= -161 \text{ KV}}}$$

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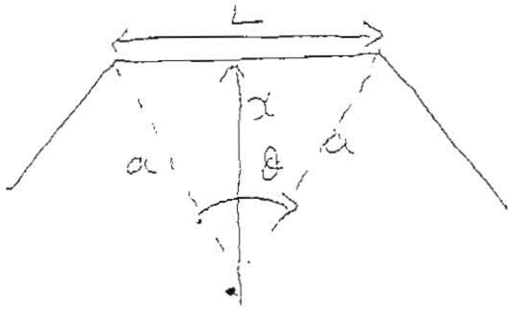
Q4 B)



For n-sided polygon $\theta = 2\pi/n$

At centre of polygon can use same procedure as for square loop (n x field from one wire)

$$x = a\cos\frac{\theta}{2} = a\cos(\pi/n)$$

$$L = 2a\sin\frac{\theta}{2} = 2a\sin(\pi/n)$$

For n sides we have

$$B = \frac{n\mu_0 I}{2\pi a\cos(\pi/n)}\left[\frac{1}{1+\left(\frac{2a\cos(\pi/n)}{2a\sin(\pi/n)}\right)^2}\right]^{1/2}$$

$$\downarrow$$

$$\left[\frac{1}{1+\frac{1}{\tan^2(\pi/n)}}\right]^{1/2}$$

$$\downarrow$$

$$\left[\frac{\tan^2(\pi/n)}{1+\tan^2(\pi/n)}\right]^{1/2}$$

Q4 B

EEE220

$$= \frac{n\mu_o I}{2\pi a \cos(\pi/n)} \left[\tan^2(\pi/n)\cos^2(\pi/n)\right]^{1/2}$$ using $1+\tan^2 = \sec^2$

$$= \frac{n\mu_o I \tan(\pi/n)}{2\pi a}$$

As n becomes large, polygon tends to a circle

As $n \to \infty$, $\underbrace{n\tan(\pi/n)}_{\tan\theta \simeq \theta \text{ for } \theta \text{ small}} \to n\frac{\pi}{n} = \pi$

$$\therefore B = \frac{\mu_o I \pi}{2\pi a} = \frac{\mu_o I}{2a}$$

$\boxed{6}$

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Q4 c/

from earlier we have

$$B_{cir} = \frac{\mu_0 I_c}{2a} \quad , \quad B_{sq} = \frac{2\sqrt{2}\mu_0 I_s}{\pi L}$$

For dimensions shown

$$B_{cir} = \frac{\mu_0 I_c}{2b} \quad \text{(out of page)}$$

$$B_{sq} = \frac{2\sqrt{2}\mu_0 I_s}{\pi 3a} \quad \text{(into page)}$$

Total field $B = \frac{\mu_0 I_c}{2b} - \frac{2\sqrt{2}\mu_0 I_s}{\pi 3a}$ (out of page)

if $b = a\sqrt{2}$ and $I_c = 4.5$

$$B = \frac{4.5\mu_0}{2a\sqrt{2}} - \frac{2\sqrt{2}\mu_0 I_s}{\pi 3a}$$

equating to zero gives

$$\frac{4.5\mu_0}{2a\sqrt{2}} = \frac{2\sqrt{2}\mu_0 I_s}{\pi 3a}$$

$$\rightarrow \quad I_s = \frac{13.5\pi}{8}$$

[8]