Signed Numbers

- Sign Bit
- 10’s & 2’s complement
- Subtraction

Signed Numbers

If the digits of the number represent the absolute value, then an extra bit is required for the sign. This is known as Signed Magnitude.

0 1100 represents the positive number +12

1 1010 represents the negative number -10

This representation leads to complex hardware for subtraction.

A format is required to represent negative numbers that is not a signed magnitude representation.

$8 - 3 =$

$7 - 5 =$

$9 - 6 =$

$8 - 3 = 5$

$7 - 5 = 2$

$9 - 6 = 3$

$8 - 3 = 5 \qquad 8 + \quad = 5$

$7 - 5 = 2 \qquad 7 + \quad = 2$

$9 - 6 = 3 \qquad 9 + \quad = 3$

$8 - 3 = 5 \qquad 8 + 7 \quad = (1)5$

$7 - 5 = 2 \qquad 7 + \qquad = \ 2$

$9 - 6 = 3 \qquad 9 + \qquad = \ 3$

$8 - 3 = 5 \qquad 8 + 7 \quad = (1)5$

$7 - 5 = 2 \qquad 7 + 5 \quad = (1)2$

$9 - 6 = 3 \qquad 9 + \quad = 3$

$8 - 3 = 5 \qquad 8 + 7 = (1)5$

$7 - 5 = 2 \qquad 7 + 5 = (1)2$

$9 - 6 = 3 \qquad 9 + 4 = (1)3$

$8 - 3 = 5 \qquad 8 + 7 = (1)5$

$7 - 5 = 2 \qquad 7 + 5 = (1)2$

$9 - 6 = 3 \qquad 9 + 4 = (1)3$

$8 - 3 = 5 \qquad 8 + 7 = (1)5$

$7 - 5 = 2 \qquad 7 + 5 = (1)2$

$9 - 6 = 3 \qquad 9 + 4 = (1)3$

10 –

Radix –

Subtraction with Complements

For two numbers X and Y, X - Y = X + (-Y), where (-Y) is the complement of Y.

The complement of a number is formed by taking it away from the next higher power of the base.

For an n-digit number, A, in base B, the complement is given by:

$$\boxed{B^n - A}$$

Example1. What is the ten's complement of 62 ?

$n = 2,\ 10^2 = 100$

$100 - 62 = 38$

Example2. Calculate 73 - 21 using ten's complement.

ten's complement of 21 is 79

73 + 79 = 152 , ignore the carry , 73 - 21 = 52

In example2, the carry indicated a positive result. In example3 there is no carry, indicating a negative result. The answer is in ten's complement.

Example3. Calculate 34 - 57 using ten's complement.

ten's complement of 57 is 43 (100 – 57 = 43)

34 + 43 = 77 no carry so this is a negative result,

still in ten's complement form

To find the magnitude of the negative result, subtract from 100

100 – 77 = 23 giving a final result of -23

Example4. Calculate 54 - 29
using ten's complement.

Example5. Calculate 42 - 93
using ten's complement.

Two's Complement

The two's complement of an n-digit binary number A is given by:

$$\boxed{2^n - A}$$

The maximum bit length of the system 'n' is chosen by the user. The left most bit is the sign bit. A sign bit of '0' represents a positive number and a sign bit of '1' represents a negative number.

What is the 2's complement of 0101_2

$2^4 = 16 = 10000_2$

$10000_2 - 0101_2 = 1011_2$

This is not ideal as we are still performing subtraction.

The 2's complement can also be formed by inverting all of the bits and adding 1.

What is the 2's complement of 0101_2

Inverting all of the bits gives 1010_2

and adding 1 gives 1011

The same process can be used to convert back from a 2's complement number to obtain the magnitude of the negative binary number.

For hand calculations, a quicker method to obtain the 2's complement is to invert all bits to the left of the first bit that is a 1 (from LSB).

What is the 2's complement of	010001101000 ?
Bits marked x are inverted	xxxxxxxxx1000
	101110011000

The 2's complement can also be formed by inverting all of the bits and adding 1.

$$2^n - A = 2^n - A - 1 + 1$$

$$\boxed{\text{So,} \quad \text{2's complement} = \underbrace{(2^n - 1)}_{\text{all 1s}} - A + 1 \quad = \; 2^n \; - \; A}$$

This works because inverting an n-bit binary number is the same as subtracting it from a number composed of all 1s, which is itself 1 less than 2^n .

The MSB (most significant bit) indicates the sign of the number.
0 indicates positive and 1 indicates negative.

A four digit number can express the following values in binary.

positive		negative	
0000	0	1000	-8
0001	+1	1001	-7
0010	+2	1010	-6
0011	+3	1011	-5
0100	+4	1100	-4
0101	+5	1101	-3
0110	+6	1110	-2
0111	+7	1111	-1

Example1: 3 + 3

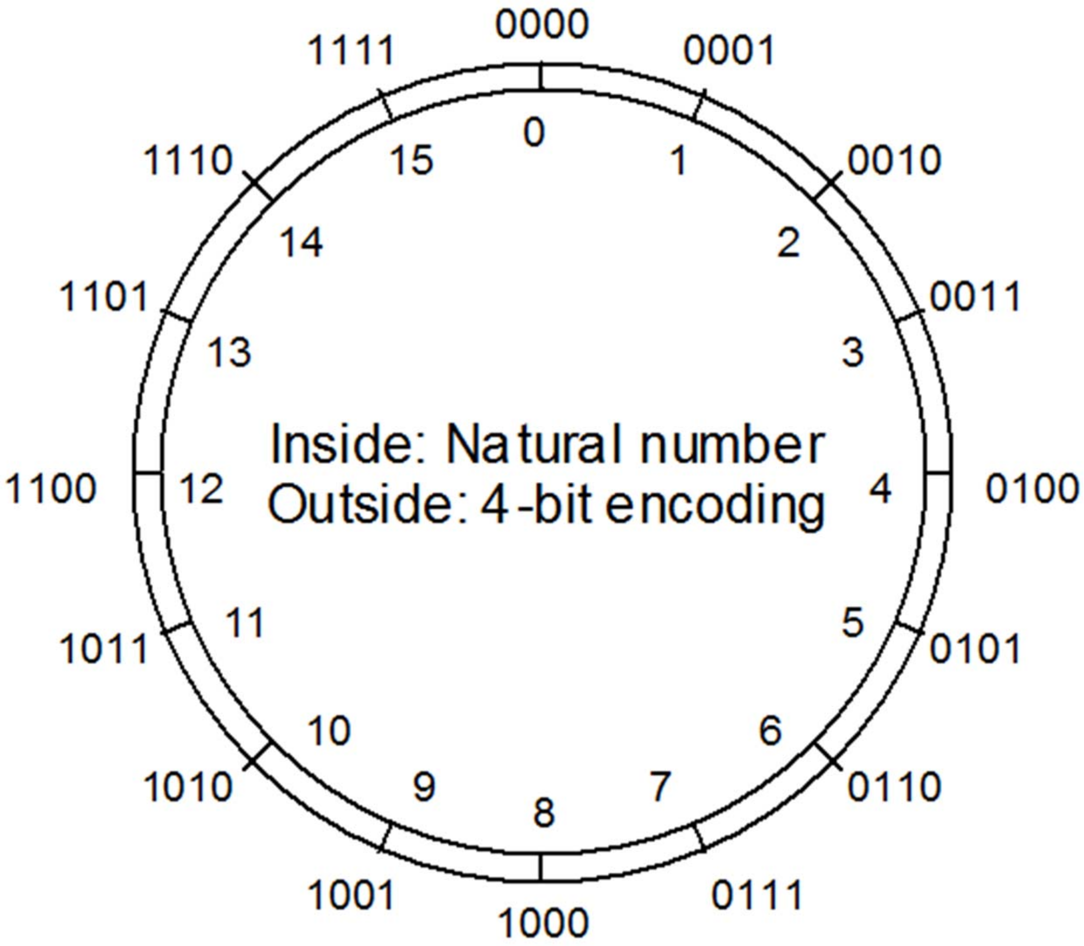
$$\begin{array}{r} 0011 \\ +\ \underline{0011} \\ 0110 \end{array}$$

Example2: 4 + 4

$$\begin{array}{r} 0100 \\ +\ \underline{0100} \\ 1000 \end{array}$$

It is important to ensure that the results of arithmetic operations do not exceed the representation.

An overflow error has occurred in Example 2.


0000
0001
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1101
1110
1111
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8
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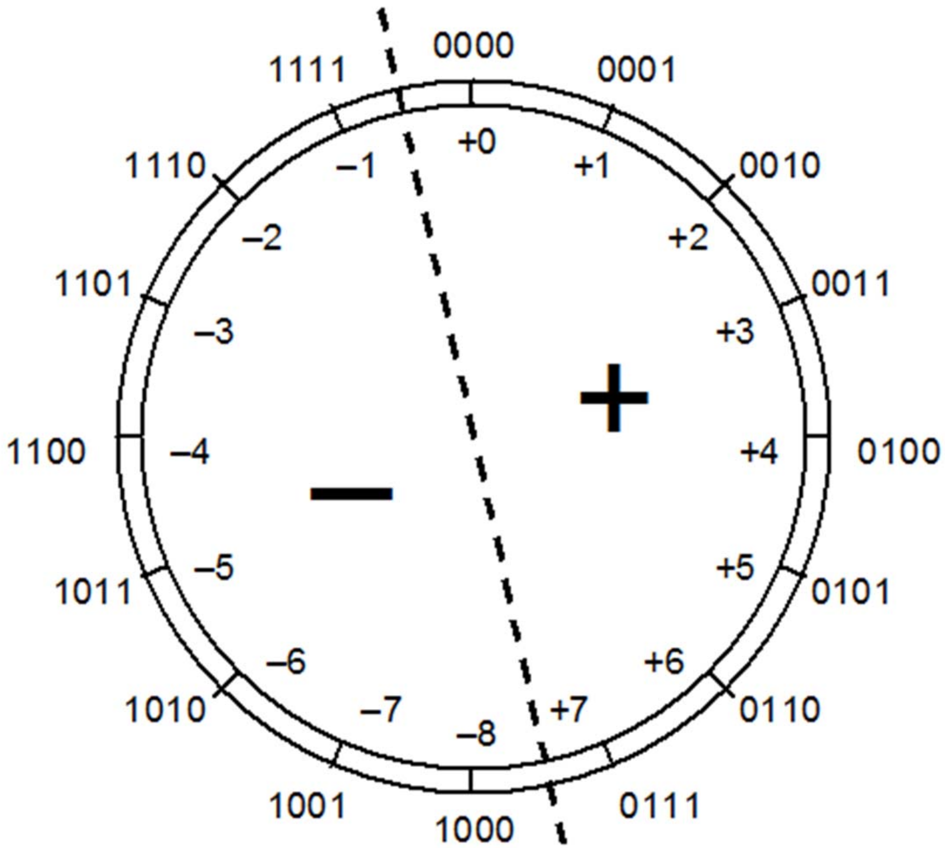
10
11
12
13
14
15
Inside: Natural number
Outside: 4-bit encoding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+1
+2
+3
+4
+5
+6
+7
–8
–7
–6
–5
–4
–3
–2
–1
+
–

For a 2's complement number, the sign bit can be taken as a negative value in the power series expansion.

$5 = 0101$

$-5 = 1011$

$-8 \quad +2 \quad +1$

This can be demonstrated by looking at some examples (using N=8):

Weight	**-128**	**64**	**32**	**16**	**8**	**4**	**2**	**1**	**Value**	
	1	0	0	0	0	0	0	0	-128 + 0 =	-128
	1	1	1	1	1	1	1	1	-128 + 127 =	-1

Examples of Binary Subtraction

Subtract 5 from 7 using binary notation then by using two's complement.

7	0111	0111
-5	-0101	+1011
+2	0010	(1)0010

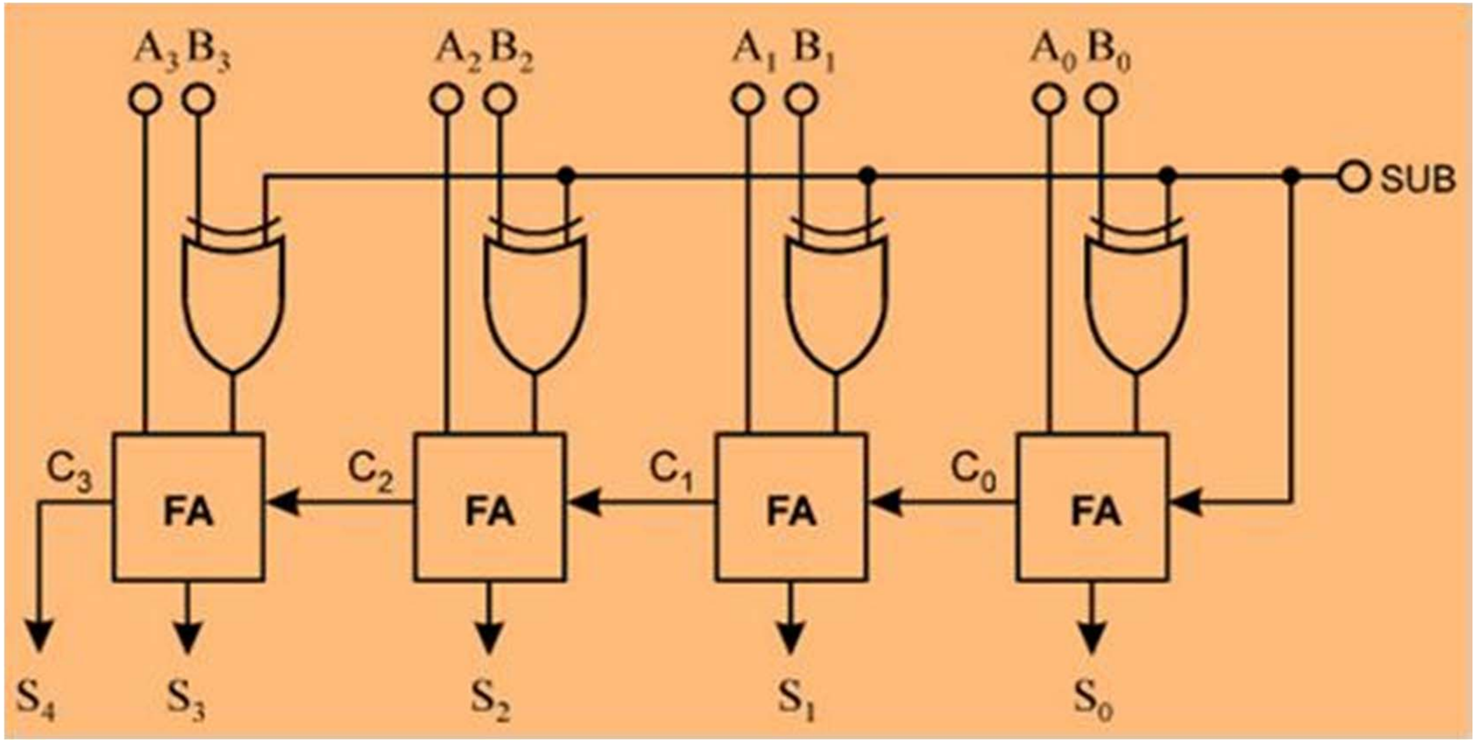
In the result, the fourth bit is the sign bit indicating positive. The overflow fifth bit is discarded.

If the chosen bit length of the arithmetic (n) is greater than the number of bits in the numbers, it is necessary to extend the sign bit to fill up the extra bits. Use 0's for positive numbers and 1's for negative numbers.

Subtract 5 from 8 using 6 bit arithmetic.

8	001000	001000
-5	-000101	+111011
+3	000011	(1)000011

Binary Adder/Subtractor Circuit



To subtract binary numbers we can use the two's complement format and a binary adder circuit. The above circuit can be used to choose between addition or subtraction using the control line 'sub'.

Summary

- Complements can be used to perform binary subtraction.
- The addition of two 1s will form a carry.
- In subtraction we borrow a 2 instead of a 10 if the subtrahend is greater than the minuend.
- A full adder can be used to add two bits plus the carry in from a previous stage. It will produce a sum and a carry out.