Topic 2

2. Optical fibre- Waveguide

2.1 Introduction

2.2 Waveguide

2.3 Optical Fibre Modes

2.4 NA & Acceptance angle

2.5 Single mode fibres

Introduction

- The direction of light can be changed when light **propagates from one medium to another** assuming the media have different refractive indices

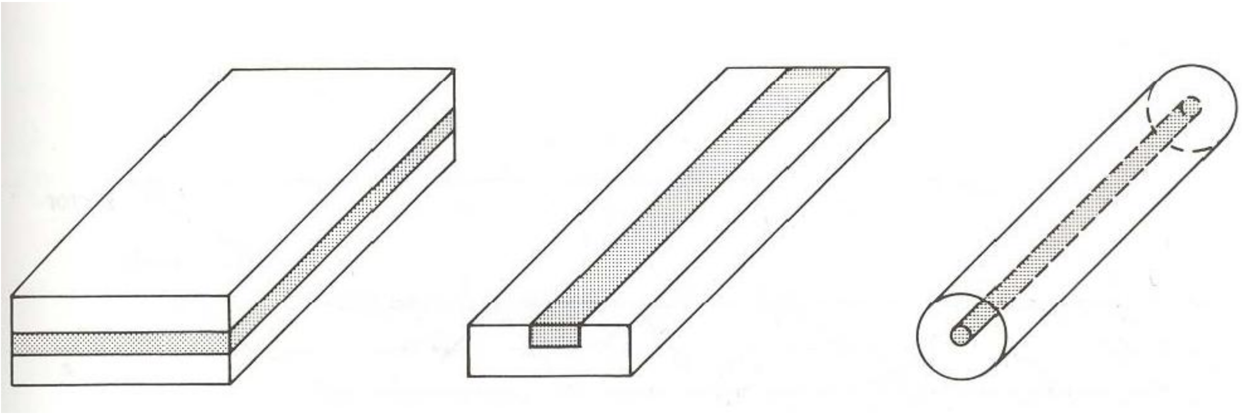
Therefore, it is possible to confine light by a system consisting of different media. This is called **optical waveguide.**

- What is a typical structure of waveguide?

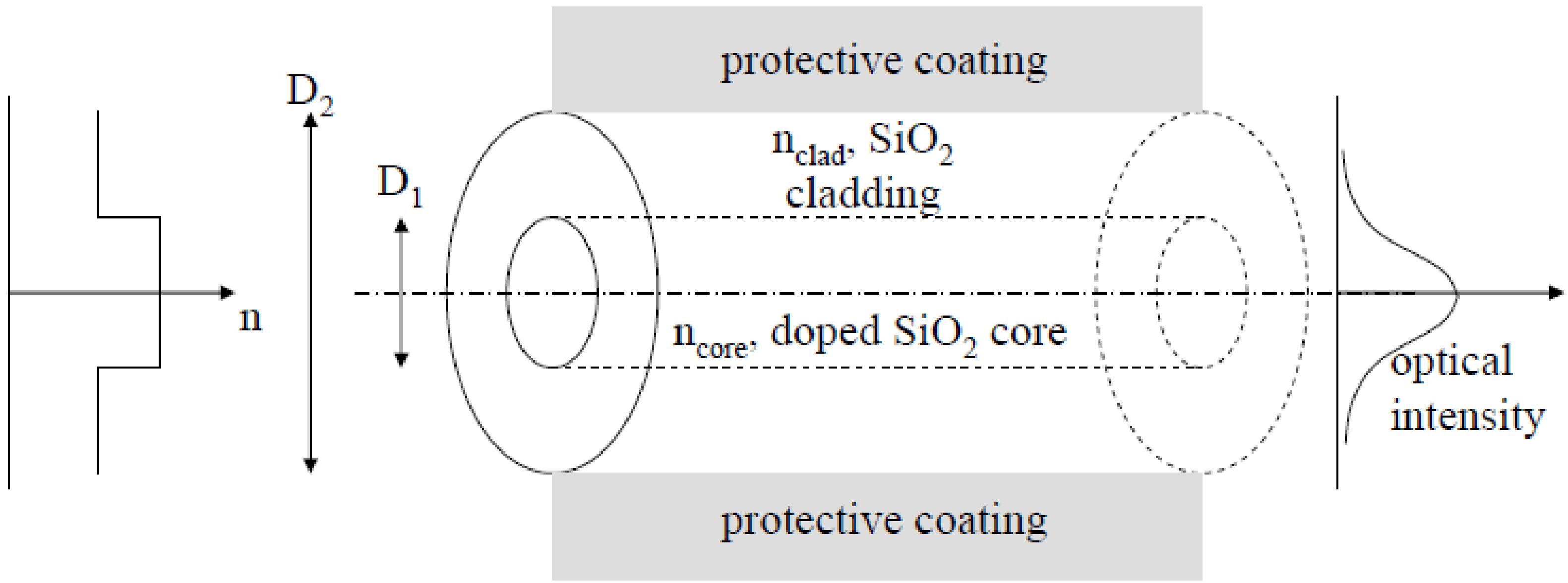
Formed by a medium which is surrounded by another medium with a lower refractive index.

The medium with the higher refractive index acts as a "light trap". Light is confined in the waveguide due to **total internal reflection**.

- Most widely applied waveguide structure: **optical fibre**, consisting of two concentric cylinders of low-loss glass with slightly different refractive index.



Optical Fibre

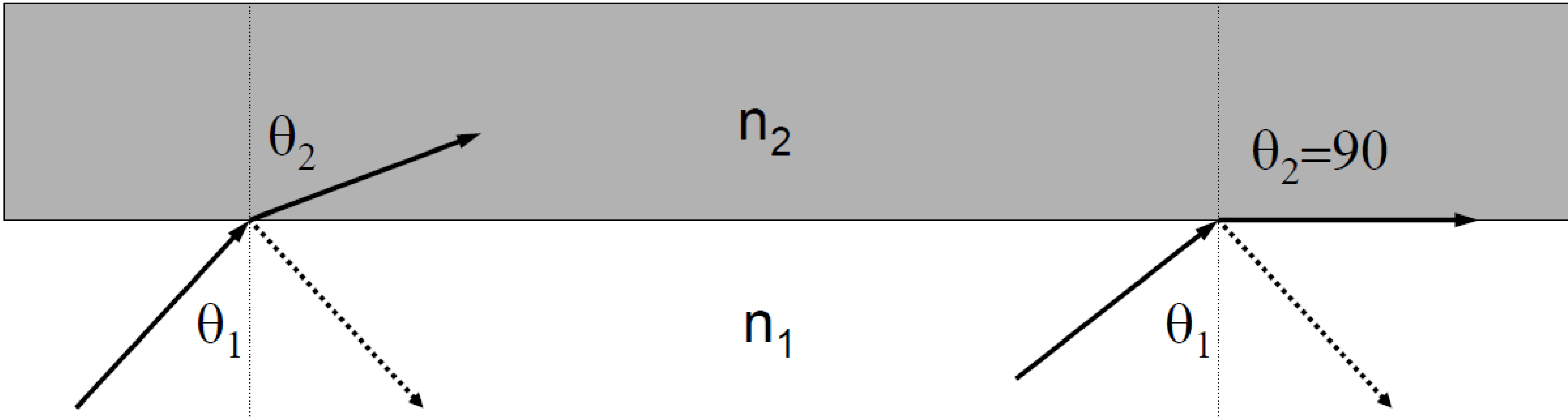


SiO_2- high purity: low loss, n_{ref} ~ 1.45

Dopants: B, F: decrease n_{ref} ,

P, Ge, Ti: increase n_{ref}

Snell's Law and guided waves



Snell's Law
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 > n_2$

Fermat principle:

Light takes a path with the shortest traversed time between two points

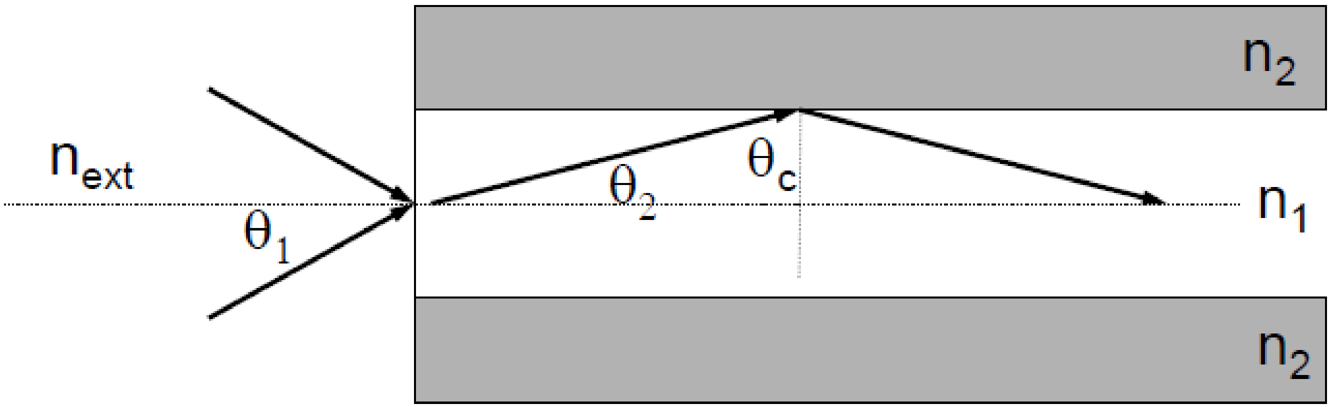
Conditions for total internal reflection (TIR):

$\mathbf{n_1 > n_2}$ and $\mathbf{\theta_2 \Rightarrow 90^0}$

θ_1 = critical angle θ_c when $\theta_2 = 90^0$

$\theta_c = \arcsin(n_2/n_1)$ or $\sin\theta_c = n_2/n_1$

Acceptance Angle



$n_1 > n_2$

- **TIR** is required to confine light in an optical fiber.
- Only light with **shallow angles** (< the critical angle) can propagate in a fiber.
- The question is now under which entry angle a ray can propagate along a fiber?

  Only rays that enter a fiber within an **acceptance angle** will propagate along the fiber, whereas rays outside of the cone will not be guided.

- **What is the maximal angel, i.e., Acceptance Angle θ_1?**

- **Snell's Law and TIR: $\sin\theta_c = n_2/n_1$**
- **$n_{ext}\sin\theta_1 = n_1\sin\theta_2 = n_1\cos\theta_c$**

- **Based on $\sin^2\theta + \cos^2\theta = 1$: $n_{ext}\sin\theta_1 = \sqrt{n_1^2 - n_2^2}$ = NA,**
- **NA: Numerical Aperture**

Numerical Aperture

$$\mathbf{n_{ext}Sin\theta_1 = \sqrt{n_1^2 - n_2^2} = NA}$$

- NA: a measure of the **light gathering ability** of an optical instrument

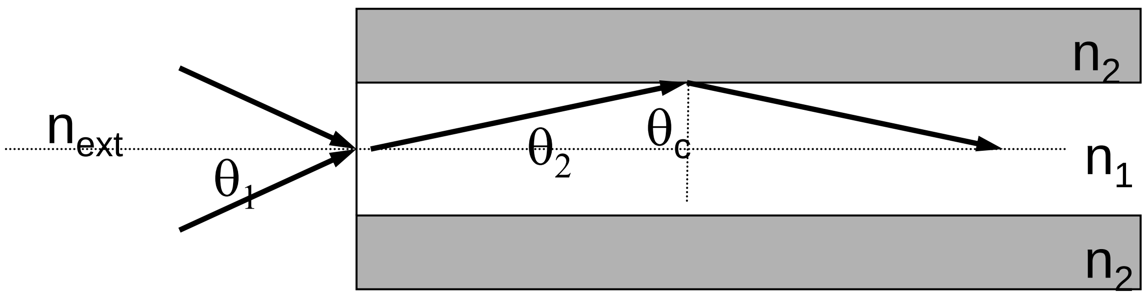
  usually used when referring to microscopes but can also be defined for optical fibres
- How much light can be accepted by a single optical fibre?

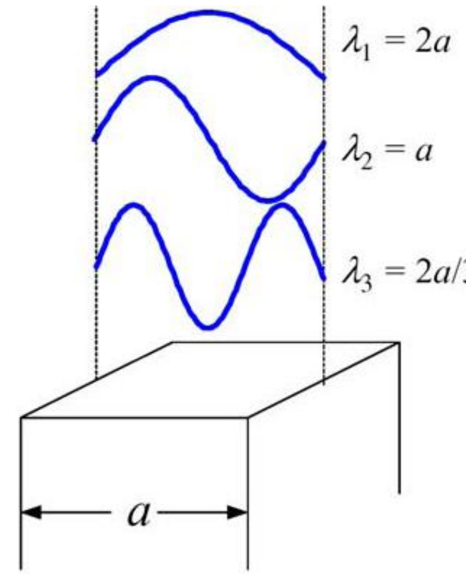
  depending upon the **contrast in refractive index** between the core and the cladding
- If fibre is immersed in air, then n_{ext} = 1, and

  NA = sin θ_1 (**N.A. is measure of acceptance angle**)
- If fibre is immersed in water or oil, then n_{ext} >1

  N.A. unchanged, but acceptance angle (θ_1) is smaller

Waveguide Condition





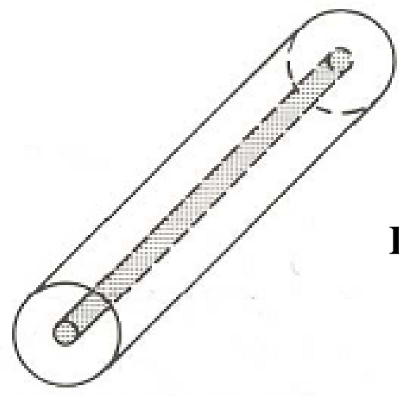
QUESTIONS:

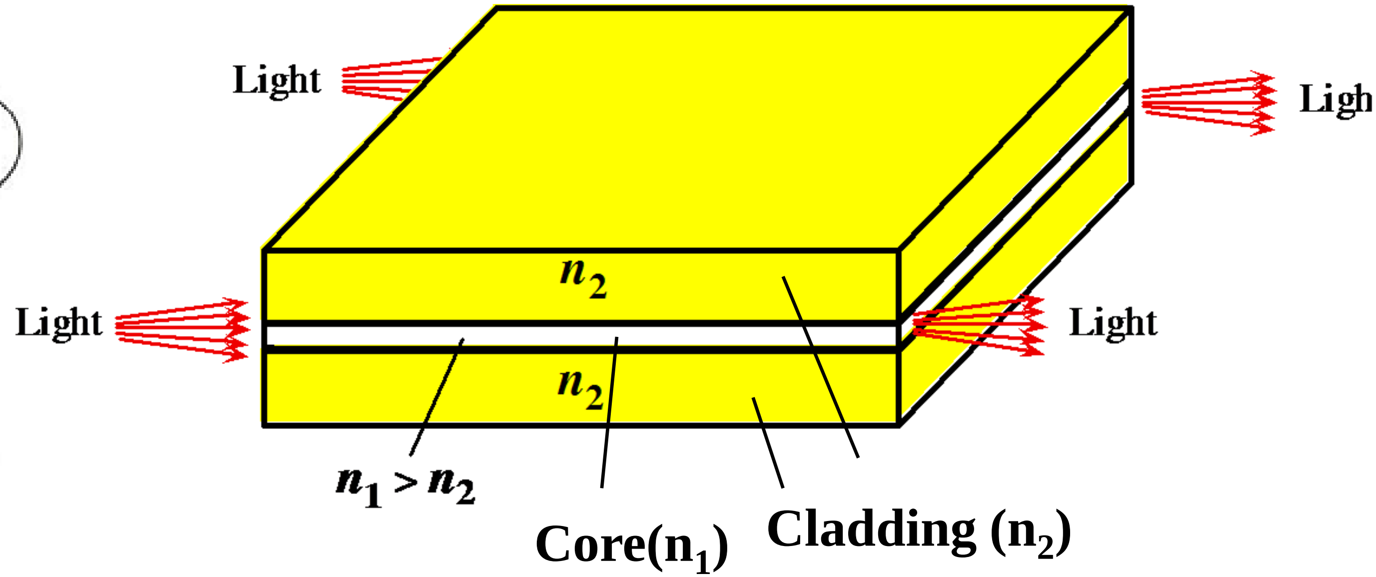
If the conditions for TIR can be met, does it mean that all such light rays can readily propagate in the guide?

$\Rightarrow$When light is confined, not all the rays with an angle < acceptance angle can propagate stably

This is related to Optical Mode

Planar dielectric slab waveguide as an example

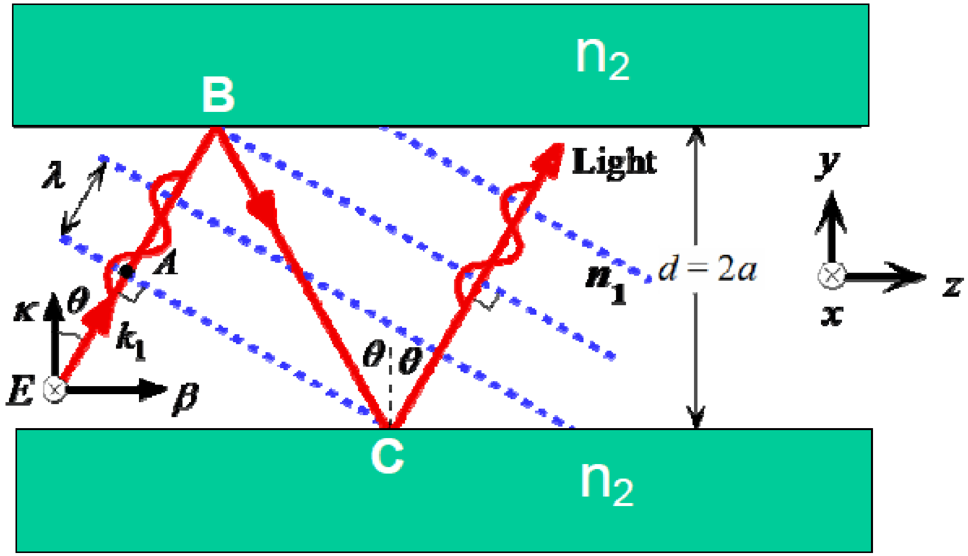




- NOTE: Actual optical fibre is a **cylindrical waveguide**, not a **slab waveguide**
- A slab waveguide: **Core layer** (refractive index n_1) sandwiched between two **cladding layers** (refractive index of n_2);
- Conditions required for TIR: $n_2 < n_1$

QUESTION: in which angle light ray can readily propagate in the guide?
Corresponding to Optical Fibre Mode

Formation of optical modes (1)

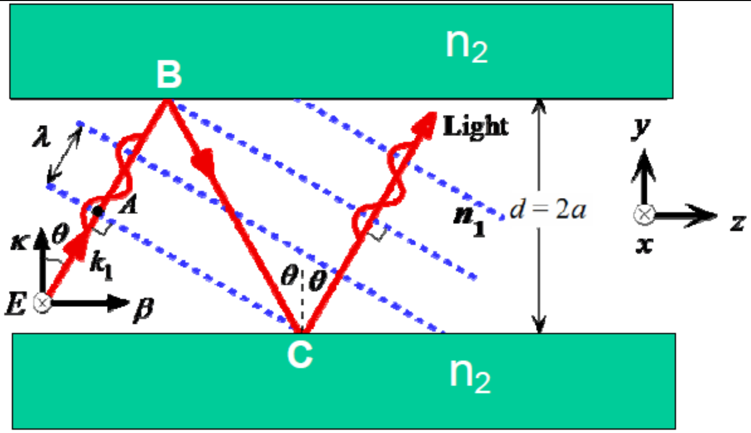


- **Calculation of Phase: wavenumber x transmission distance= K• L**
  Free space: $k = 2\pi/\lambda$; Medium (n_1): $k_1 = (2\pi/\lambda)n_1$

- **Phase change due to extra length from A to B and then C**

$$\Delta\phi(AC) = \underbrace{k_1(AB + BC)}_{\text{Optical distance difference}} - \underbrace{2\phi}_{\text{Reflection}} = m(2\pi), \quad m = 0,1,2,\cdots \quad (1)$$

Formation of optical modes (2)



(1) Path length difference

$$\boxed{\begin{aligned} &AB + BC = BC\cos(2\theta) + BC = BC[\cos(2\theta)+1] = BC[(2\cos^2\theta - 1)+1] \\ &= \frac{d}{\cos\theta}[2\cos^2\theta] = 2d\cos\theta \end{aligned}}$$

<u>Phase difference 1</u>: $k_1 2d\cos\theta$

(2) Reflection:

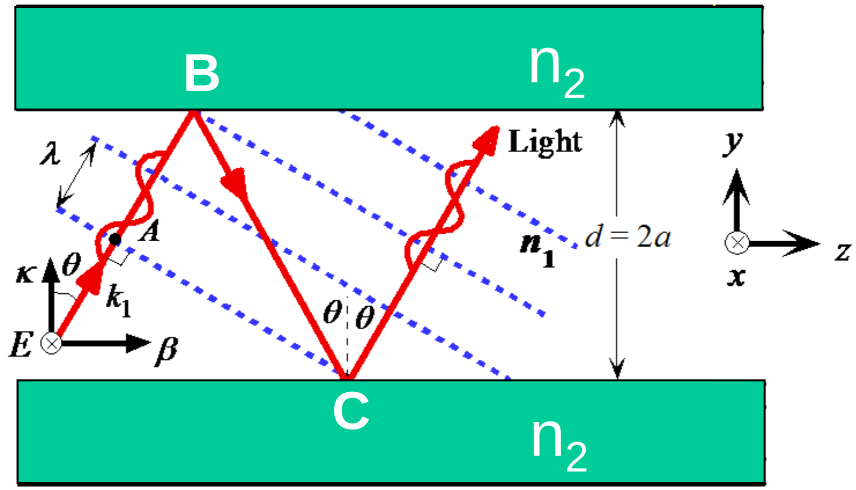
<u>Phase difference 2</u>: 2 ϕ a function of incidence angle θ_m

In total: Phase change: $\boxed{k_1(2d\cos\theta) - 2\phi}$

Is there any other simple way to calculate it?

Actually there exists an optical confinement in the y direction only

Waveguide condition



$$k_1(2d\cos\theta) - 2\phi = m(2\pi)$$

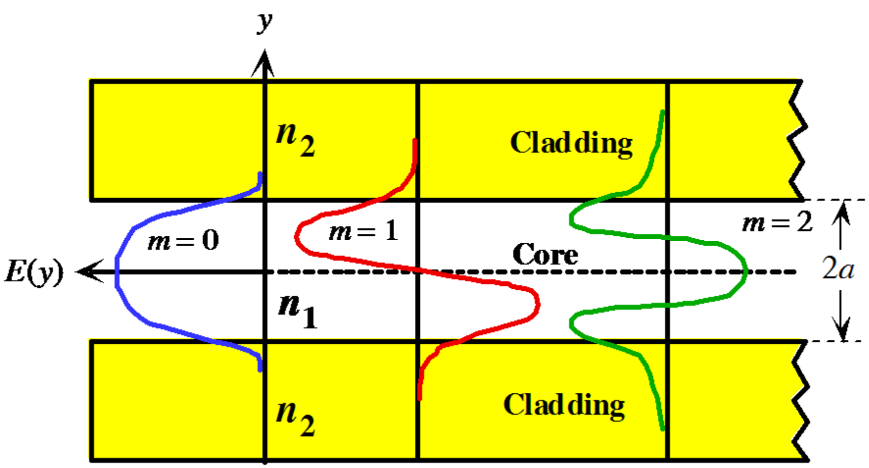
$$\Rightarrow k_1 d\cos\theta_m - \phi_m = m\pi$$

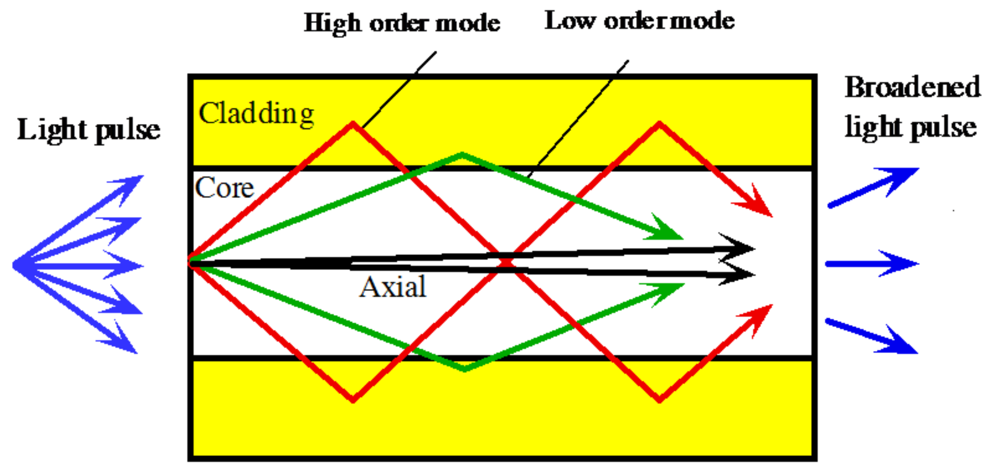
$$\boxed{\left[\frac{2\pi n_1 d}{\lambda}\right]\cos\theta_m - \phi_m = m\pi}$$

$$\boxed{\sin\theta_m \geq \sin\theta_c = \frac{n_2}{n_1}}$$

- Each *m: corresponding to each* θ_m, *allowing* a particular wave to travel stably in the *z*-direction
- *m*: **mode number**
- Small θ_m means larger *m*

Optical modes in planar waveguide





$$[\frac{2\pi n_1 d}{\lambda}]\cos\theta_m - \phi_m = m\pi$$

Therefore

$$m = \frac{1}{\pi}[2\frac{\pi d}{\lambda}n_1\sqrt{1-\sin^2\theta_m} - \phi_m]$$

$$\boxed{\sin\theta_m \geq \sin\theta_c = \frac{n_2}{n_1}}$$

$$\leq \frac{1}{\pi}[2\frac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^2} - \phi_m]$$

$$\Rightarrow m \leq \frac{1}{\pi}[2V - \phi_m].where V = \frac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^2}$$

- Higher modes: penetrate much more into the cladding

- Maximal number for modes, depending on the size of core (multiple-mode)

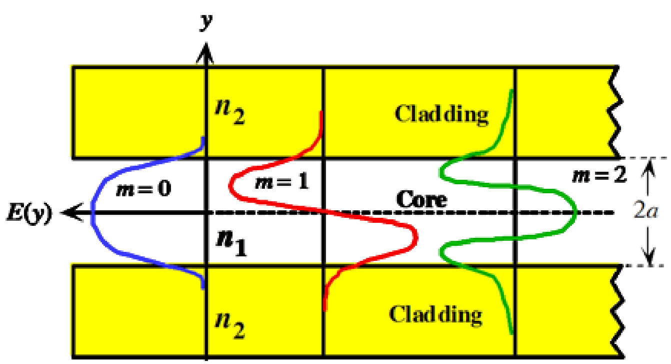
  Small size (a) meaning small number of modes

  If the size is small enough, allowing m=1, this is a single-mode

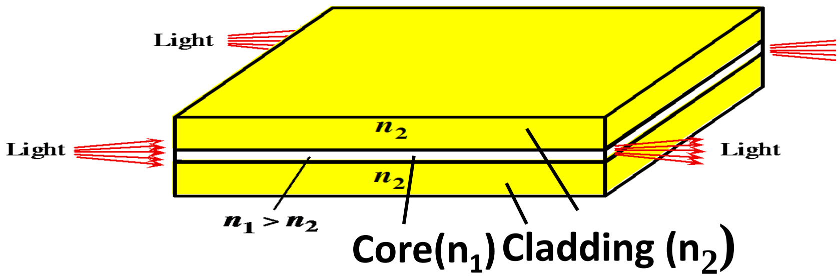
Single Mode Fibre (planar waveguide)

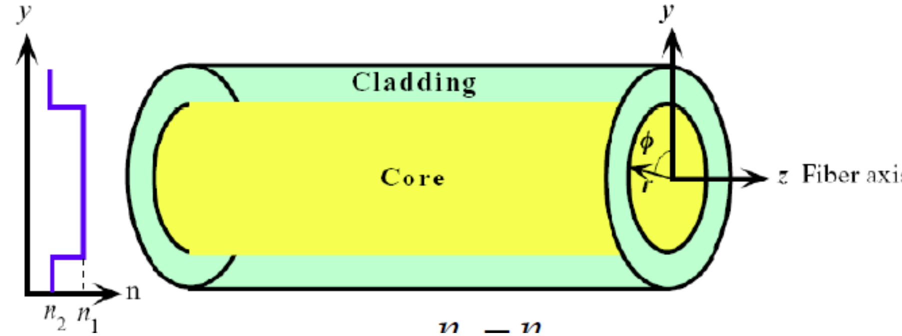
$$m \le \frac{1}{\pi}[2V - \phi_m].where V = \frac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^2}$$

- **m depends on the thickness of core layer,**
- **Small thickness means less number of optical modes**
- **When the thickness is small enough, it will allow one mode to exist. This is a single mode optical fibre, normally, Fibre has a core diameter of ~ few μm**
- **RAY model breaks down** when core ~ wavelength of light
- Need wave model to correctly explain SM fibre behaviour (Maxwell's equations):
- **Some parts of wave extend to cladding layer even under TIR, which is very important when we discuss dispersion**



Optical fibre (Step Index Fibers)





- **So far, we discussed about a planar waveguide**
- **Actual optical fibre is a cylindrical waveguide, but not a planar waveguide**
- **The general ideas for guided wave propagation in planar waveguides can be extended to step indexed optical fibers with certain modifications.**
- **Major difference:**
- **The planar waveguide is bounded only in one dimension. Distinct modes are labelled with one integer, *m.***
- **The cylindrical fiber *is bounded* in two dimensions. Two integers, *l and m, are required to label all the possible* guided modes**

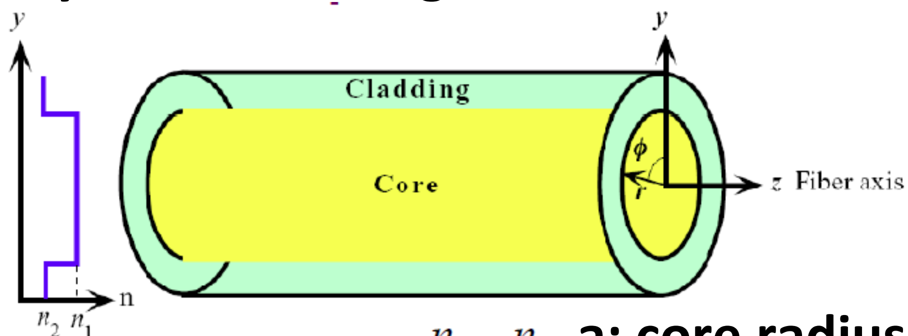
Cylindrical optical fibre (Step Index Fibre)

- **How to determine a single mode planar waveguide?**

$$m \le \frac{2V-\phi}{\pi} \approx \frac{2V}{\pi}$$

- **How to determine a single mode cylindrical waveguide?**

$$\boxed{V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = 2.405}$$



a: core radius

- Calculation indicates: for **a fixed wavelength**, when the V ≤ **2.405, only the fundamental mode (LP_{01})** can propagate through the fiber core, namely, **single mode fiber.**
- For **a fixed core radius**, **the cut-off wavelength** above which the fiber becomes single mode is given by the above equation
- **The number of modes M in a step index fiber:**
  (**compare with the equation for a planar waveguide**)

$$\boxed{m \approx \frac{V^2}{2}}$$

Cylindrical optical fibre (Step Index Fibre)

•Example: A Multimode Fiber

A step index fiber with a core diameter of 100 µm has a core of refractive index 1.468 and a cladding of refractive index of 1.447. For a transmitter of 1.55 µm, how many modes that are allowed

$$\boxed{\begin{aligned} &V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{\pi 100}{1.55}\sqrt{1.468^2 - 1.447^2} = 50.14 \\ &m \approx \frac{V^2}{2} = \frac{50.14^2}{2} = 1250 \end{aligned}}$$

•Example: A Single Mode Fiber

Determine the core radius of a single mode fiber that has a core refractive index of 1.468 and a cladding refractive index of 1.447, assuming that a transmitter of 1.3 µm is used?

$$\boxed{\begin{aligned} &V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{1.3}\sqrt{1.468^2 - 1.447^2} \le 2.405 \\ &a \le 2.01 \mu m \end{aligned}}$$

Cylindrical optical fibre (Step Index Fibre)

Example: Single Mode Cut-Off Wavelength

•Determine the cut-off wavelength for single mode operation for a fiber that has a core with a diameter of 10 μm, a refractive index of 1.458, and a cladding of refractive index of 1.452?

•Determine the *V-number* when operating at λ= 1.3 μm?

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{\pi 10}{\lambda}\sqrt{1.458^2 - 1.452^2} \leq 2.405$$

$$\lambda \geq 1.72\mu m$$

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{\pi 10}{1.3}\sqrt{1.458^2 - 1.452^2} = 3.19$$

Transmission characteristic of optical fibres

Signal: high intensity and less distortion during transmission, which are related to **optical loss** and **dispersion**

- Origins for dispersion: distortion

(i) Intermodal Dispersion; (ii) intramodal dispersion

- Origins for optical loss:

(i) material absorption; (ii) scattering loss; (iii) Fibre bending

Exercises & Homework

Question 1: calculate the number of optical modes of a waveguide

Question 2:

For an planar waveguide described as above, d = 100 μm; n_1 = 1.490; n_2 =1.470, if the waveguide is designed for a light with an wavelength of λ = 1 μm, please estimate the **number of optical modes**.

T2 Summary

- Optical fibre
- Waveguide
- Total internal reflection
- Consider ray picture for light propagation
- Snell's law – a ray of light incident at a particular angle to a step change in refractive index will be totally internally reflected
- Allows acceptance angle and numerical aperture to be determined
- Single Mode
- Multimode Fibre

T2 Tutorial Questions

T 2.1 Draw the refractive index profile of step-index multimode fibre with core index n_1, cladding index n_2, and core diameter D. Explain the terms “meridonal ray”, “skew ray”, “axial ray”, and “critical ray”. Trace the path of a ray which crosses the fibre axis at an angle before entering the fibre to be guided by total internal reflection . Define Numerical Aperture and show that it is given by $NA= \sqrt{(n_1^2-n_2^2)}$. Explain how NA determines the ability of the fibre to collect light. What is modal dispersion?

T2 Tutorial Questions

T2.2

A step index multi-mode fibre has a core index of refraction of 1.5 and an index step Δ of 0.02. Calculate the maximum angle which a guided ray may have relative to the axis (a) inside the fibre, (b) in air outside the fibre before launching. (c) What is the value of the numerical aperture of the fibre?

Answers – **(a) 11.5° to axis**

(b) 17.4° to axis

(d) NA = 0.3