reflection, reflectance and anti-reflection coatings

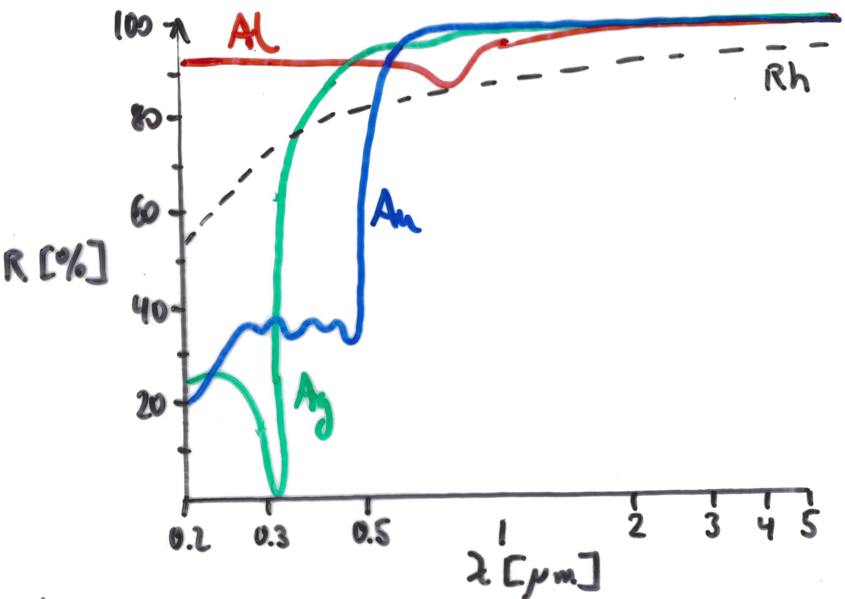
definitions:

reflection = phenomenon that occurs when electromagnetic radiation passes from medium with refractive index $\underline{n_1}$ to another medium with $\underline{n_2} \neq \underline{n_1}$

reflectivity = ratio $R = \left(\frac{|E_r|}{|E_i|}\right)^2 = r^2 \in \mathbb{R}^+$
is a materials property defined for interfaces between thick films, given by Fresnel's equations for $R_\perp$ and $R_\parallel$

reflectance is defined as above, but for multiple thin films, and thus depends on layer thicknesses, periodicity, geometry and surface roughnesses

reflection coefficient = ratio $r = \frac{E_r}{E_i} \in \mathbb{C}$
may be a complex number



for reflection of unpolarised light on a metal surface to vacuum:

$$R = \frac{(n-1)^2 + \varkappa^2}{(n+1)^2 + \varkappa^2} \quad , \quad T = \frac{4(n^2+\varkappa^2)}{(n+1)^2 + \varkappa^2}$$

$$A = 1 - R$$

note for strong absorption: $\sigma \gg \omega \varepsilon_0 \varepsilon_r$

if

$$\underline{E} = E_0 \underline{e_y} e^{j(\omega t - \frac{1}{\delta}x)} e^{-\frac{1}{\delta}x}$$

with skin depth

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu_0 \mu_r}}$$

then

$$I \propto |\underline{S}| \propto |\underline{E}|^2 = E_0^2 e^{-\frac{2}{\delta}x} = E_0^2 e^{-2\varkappa k x}$$

$\Rightarrow \frac{1}{\delta} = \varkappa k$

$$\varkappa = \frac{1}{k\delta}$$

The absorptive part, $\varkappa$, of the refractive index $\underline{n} = n + i\varkappa$ is large if the skin depth, δ, is small: the radiation decays quickly in the material so that most of the energy is reflected.

example for metals: IR: $\delta = 10-50$ nm

$\Rightarrow$ $\varkappa$ big

$\Rightarrow$ $R \to 1$

$\Rightarrow$ $A \to 0$

① multiple layer reflections and index matching

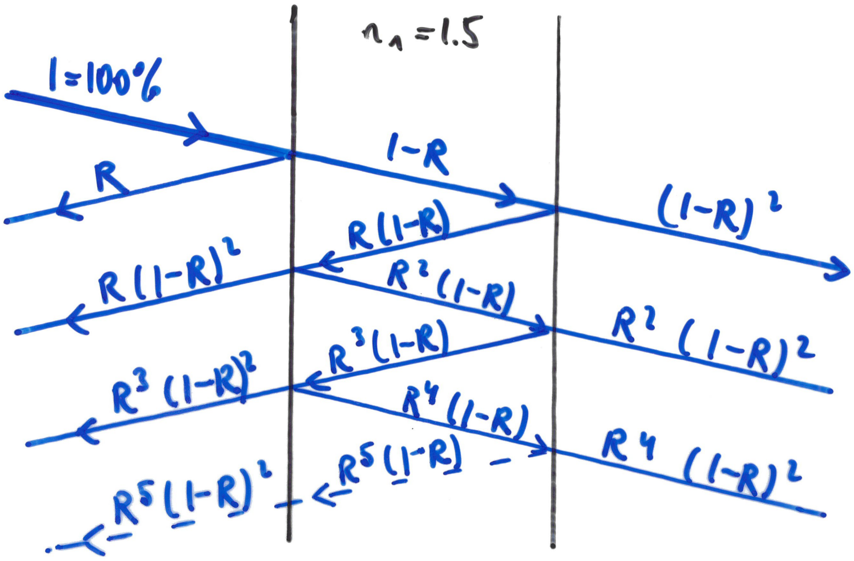
consider interfaces between materials without absorption (i.e. only n_i, all $\varkappa_i = 0$ for $i = 1,\dots,n$)

single layer: $n_1 = 1.5$ vs. vacuum

$$R \approx \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{n_1 - 1}{n_1 + 1}\right)^2 = \left(\frac{0.5}{2.5}\right)^2 = 0.04$$

$\Rightarrow$ 4% reflection, 96% transmission

if only single reflection on surface is considered if multiple reflections at glass / vacuum interface are considered



total reflectance :

$$R + R(1-R)^2 + R^3(1-R)^2 + R^5(1-R)^2 + \ldots$$

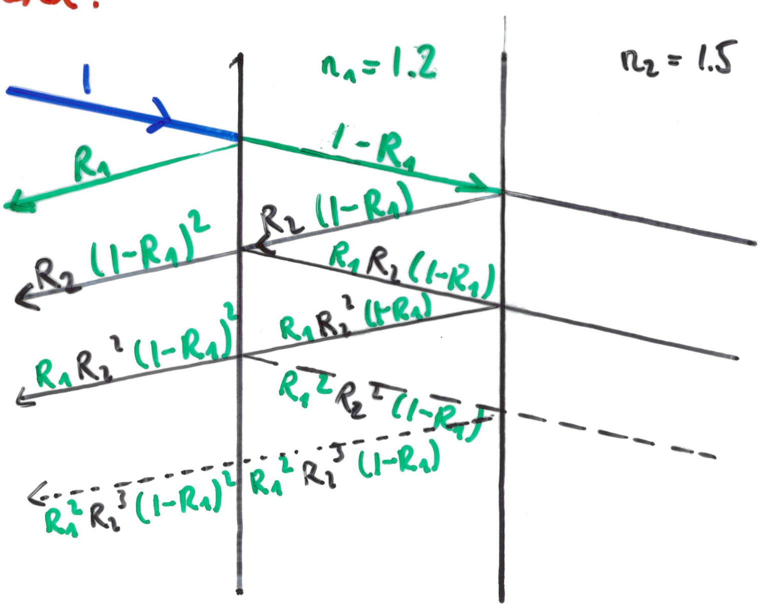
$$= R + R(1-R)^{\cancel{2}} \underbrace{\left[1 + R^2 + R^4 + \ldots\right]}_{\frac{1}{1-R^2} = \frac{1}{(1+R)\cancel{(1-R)}}} \quad \text{(geometric series)}$$

$$= R\left(1 + \frac{1-R}{1+R}\right)$$

$$= \frac{2R}{1+R}$$

$$= 0.077 = 7.7\%$$

if single reflection at double layer with $n_1 = 1.2$, $n_2 = 1.5$ is considered :



$$R_{total} \approx R_1 + R_2\,(1-R_1)^2$$

$$= \left(\frac{1.2-1}{1.2+1}\right)^2 + \left(\frac{1.5-1.2}{1.5+1.2}\right)^2 \left[1-\left(\frac{1.2-1}{1.2+1}\right)^2\right]^2$$

$$= 0.0083 + 0.0123 \cdot 0.9835$$

$$= 0.0204 \quad \text{instead of } 0.04$$

if all reflections at both interfaces are considered:

$$R_{total} = R_1 + R_2(1-R_1)^2 + R_1R_2^{\,2}(1-R_1)^2 + R_1^{\,2}R_2^{\,3}(1-R_1)^2 + \dots$$

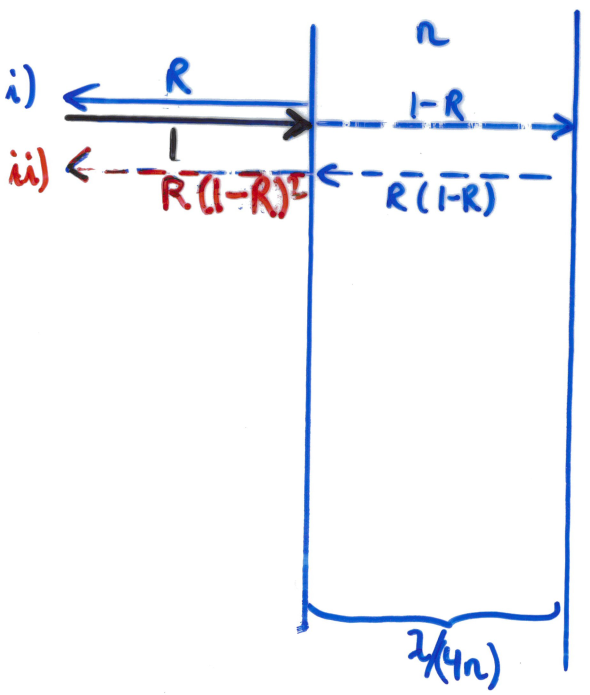
$$= R_1 + R_2\,(1-R_1)^2 \underbrace{\left[1 + R_1R_2 + R_1^{\,2}R_2^{\,2} + \dots\right]}_{\frac{1}{1-R_1R_2}}$$

$$= R_1 + \frac{(1-R_1)^2\,R_2}{1-R_1R_2}$$

$$= 0.020408 \quad \text{instead of } 0.077$$

$\Rightarrow$ in both cases, the additional layer with intermediate refractive index reduces the total reflectance because this is proportional to the square of the difference between n_1 and n_2

② interference coatings by $\lambda/4$ Layers



The directly reflected wave R and the wave reflected at the backside of the layer, $R(1-R)^2$, can interfere destructively if the layer thickness for vertical incidence is given by $\lambda/(4n)$ where λ is the wave length in the medium of refractive index n. This only works for one wavelength and one direction (here: vertical incidence).