The University Of Sheffield.

Department of Electronic and Electrical Engineering

First Year Laboratory:

COUPLED CIRCUITS & TRANSFORMERS

2012

AIMS

This laboratory exercise is designed to teach you some basic ideas about coupled circuits and transformers and to give you more experience in the experimental equipment and methods typically used in an electronic system environment.

OBJECTIVES

By carrying out a sequence of experiments the student is encouraged to do the following :

(i) Note the construction of the experimental coupled circuit test fixture and relate measured self and mutual inductance to the observed structure.

(ii) Compare the fixture topology with that of more conventional transformers and draw conclusions about important design criteria for power transformers.

(iii) Measure two important performance characteristics of a power transformer under different load conditions.

(iv) Observe the behaviour of a well designed transformer when subjected to over-voltage supplies and to deduce further criteria in the design process.

(v) Investigate a small signal transformer designed to operate as an impedance matching transformer for audio applications.

INTRODUCTION

Faraday's Law States that if the magnetic flux, Φ, linking any electric circuit varies with time then an emf is generated (induced) in the circuit and the magnitude of this emf is proportional to the instantaneous rate of change of the flux linkage Ψ. Flux linkage is defined as:

$$\Psi = N\Phi \qquad \textbf{(1)}$$

Depending on the application, a circuit may be designed to increase Ψ by forming it into a coil of N turns wound on a magnetic core, or alternatively it may have a single turn and be designed to minimise Ψ and any associated induced emf's. Thus in general for the circuit in figure 1 we can write:

$$v = Ri + \frac{d\Psi}{dt} \qquad \textbf{(2)}$$

where R is the circuit resistance and in this particular case $N = 1$ and $\Psi = \Phi$.

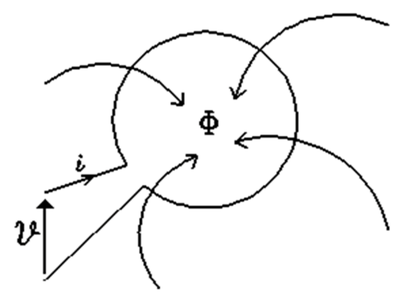


Figure 1: *Flux linkages*

It can be seen therefore that time varying flux linkage ($d\Psi/dt$) behaves like a voltage generator. If, in a particular application, the loop is short cicuited (i.e. the voltage source v in figure 1 is equal to zero) such that the $d\Psi/dt$ term becomes the only source, then the energy transferred to the circuit by the time varying flux linkage is dissipated in R by the current i.

The total flux linkages can be the sum of contributions from many flux sources. A circuit carrying current will always produce a flux which links the circuit, additional contributions may come from nearby current carrying circuits and/or permanent magnets. Time variation of these flux linkages can be due to time variation of the currents or caused by relative motion between the circuit and flux sources. Relative motion implies electromechanical energy conversion. In the coupled circuits to be tested this does not occur, and so all the induced voltages measured are due to alternating currents flowing in the circuits being tested.

Self and Mutual Inductance

Voltages induced in a circuit due to changes in the current flowing in that circuit are termed self induced. Voltages induced in a circuit due to changes in the current in other circuits are said to be mutually induced. The effects of these induced voltages can be sought deliberately i.e. by design, or they can cause unwanted effects such as 'pick up' in electronic circuits, unwanted voltage drops such as those occurring in overhead transmission lines, or damaging high voltage spikes in power electronic switches.

If a device is deliberately designed to have a large component of self induced emf it is often called an **inductor** or choke. If the circuit is designed to have a large component of mutually induced emf it is called a coupled circuit or **transformer**. Such devices frequently include multi-turn coils wound on a magnetic core of ferromagnetic or ferrite material. The cores act as magnetic 'conductors' by offering an easy path for flux and thus maximising the flux and guiding it to where it is needed. Unfortunately there is no material which will act as a magnetic insulator and hence some flux will take paths through the air surrounding the circuit. Thus all circuits will produce some unwanted flux linkages which can only be minimised by careful layout and design.

1. EXPERIMENTS WITH COUPLE CIRCUITS

For safety reasons, all the 50Hz experiments are supplied by a combination of variable voltage transformer (or "variac") feeding a step down isolating transformer that has an output of around 50V when its input is 230V. The variac allows you to adjust the isolating transformer output voltage between zero and about 50V. Note: In each experiment it is important to use wires of adequate current carrying capacity for each part of the circuit.

ALWAYS SWITCH POWER TO YOUR CIRCUIT WITH THE VARIAC SET INITIALLY TO 0V AND TURN IT UP SLOWLY TO GET THE VOLTAGE YOU WANT. WHEN CURRENT IS BEING MEASURED, KEEP AN EYE OF THE AMMETER READING AS YOU TURN UP THE VOLTAGE. IF CURRENT TRIES TO EXCEED 10A, TURN THE VARIAC BACK TO ZERO AND ASK FOR ADVICE FROM A DEMONSTRATOR

The equipment supplied for this experiment includes two similar coils mounted separately on a fixed, laminated, iron core and wound with blue tape.

(i) *Connect one coil to the variable 0 - 50 V supply as shown in figure 2.*

(ii) *For the full range of input voltages measure the input current (0-2A meter) and the voltage across* **both** *coils (use the Digital Multimeter (DMM) set to measure AC Volts). Take readings over the range 0.2-1.0A in steps of 0.1A and record your reading in your lab book.*

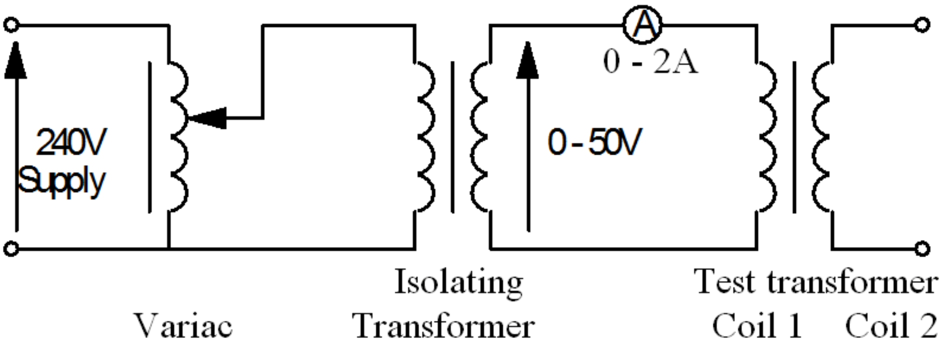
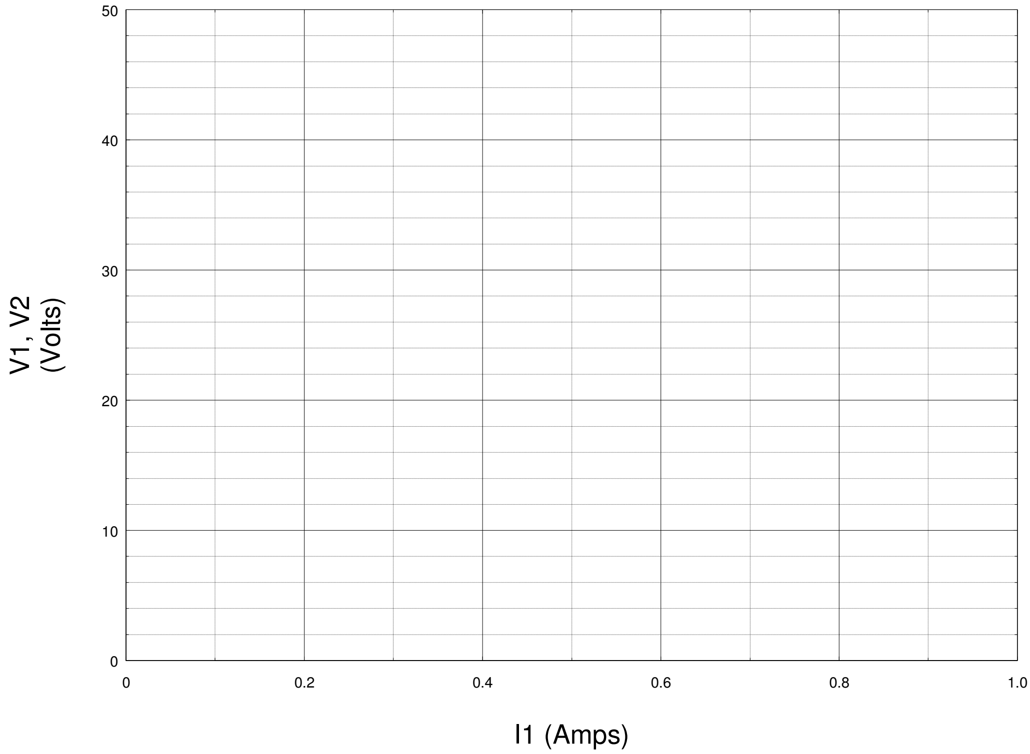


Figure 2: *Coupled cicuits experiment*

(iii) Plot the input and output voltages against the input current for the coupled coils and the graph below. These graphs will be used to calculate the self inductance (section 1.2), mutual inductance and coupling coefficient (section 1.4) of the coils as described in the following sections.

A graph of input and output voltages against input current for the coupled circuits.



1.1. Calculation of Self Inductance

In the previous test only one coil carries current, hence the induced emf in that coil is self induced and the voltage induced in the open circuited coil is caused by mutual flux linkages.

For a linear magnetic circuit the flux, Φ, produced is proportional to the magnetomotive force or mmf, Ni, which depends on the number of turns on the coil, N, and the current, i:

$$\Phi = \frac{Ni}{S} \qquad \textbf{(3)}$$

Where S is defined as the circuit reluctance and is a function of the geometry and magnetic properties of the materials which make up the flux paths of the magnetic circuit surrounding the coil. For a linear system we can combine equations **(1)** and **(3)** to give:

$$\frac{d\Psi}{dt} = \frac{d}{dt}(N\Phi) = \frac{d}{dt}\left(\frac{N^2 i}{S}\right) = \frac{N^2}{S}\frac{di}{dt}$$

since the number of turns, N, and the reluctance, S, are independent of time. The **self inductance** (in Henries) is defined as:

$$L = \frac{N^2}{S}$$

Hence:

$$\frac{d\Psi}{dt} = L\frac{di}{dt} \quad \textbf{(4)}$$

where L is now an electric circuit component which models the effects of the magnetic circuit surrounding the electric circuit and the induced emf is now proportional to the rate of change of circuit current.

If the voltage and current are sinusoidal time varying then, by substituting from equation **(4)**, equation **(2)** can now be written in its phasor form as:

$$V = RI + j\omega LI = RI + j2\pi fLI$$

or:

$$\frac{V}{I} = R + j2\pi fL = Z \quad \textbf{(5)}$$

where V and I are usually referred to as rms quantities and f is the supply frequency (Hz) and:

$$|Z| = \sqrt{R^2 + (2\pi fL)^2} \quad \textbf{(6)}$$

1.2. Experimental value of Self Inductance

Use the slope of the graph 1 (V_1/I_1), in conjunction with equations **(5)** *and* **(6)***, to calculate the self inductance of coil 1:*

(a) assuming R is negligible.
(b) assuming R = 5Ω per coil.

From you results what effect does the coil resistance have on the value of the self inductance?

1.3. Calculation of Mutual Inductance and Coupling Coefficient

When two circuits share a common or mutual part of their magnetic circuits then their induced emfs are made up of two components, a self inductance and a mutual inductance component. Thus for sinusoidal excitation the equations for the two circuits, both carrying currents, can be expressed in phasor form as:

$$V_1 = R_1I_1 + j2\pi f(L_1I_1 \pm L_{12}I_2) \quad \textbf{(7)}$$

$$V_2 = R_2I_2 + j2\pi f(L_2I_2 \pm L_{21}I_1) \quad \textbf{(8)}$$

where L_{12} is the **mutual inductance** of coil 1 to coil 2 and L_{21} is the **mutual inductance** of coil 2 to coil 1. Since, in a two circuit system, the mutual terms *must* be equal, they are usually given the symbol M. In our experiment coil 2 is open circuit ($I_2 = 0$) so equations **(7)** and **(8)** reduce to:

$$V_1 = R_1I_1 + j2\pi fL_1I_1 \text{ (as in } \textbf{(5)} \text{ above)}$$

$$V_2 = j2\pi fMI_1$$

or:

$$\left|\frac{V_2}{I_1}\right| = 2\pi fM \quad \textbf{(9)}$$

The coupling coefficient gives a measure of the fraction of the flux produced by one coil which links the second and is defined by:

$$k = \frac{M}{\sqrt{L_1L_2}} \quad \textbf{(10)}$$

k lies in the range $0 \leq k < 1$. When k = 0 there is no coupling and the circuits are independent; for most transformers one design objective is to make $k \approx 1$.

1.4. Experimental value of Mutual Inductance and Coupling Coefficient

*Use the slope of the graph 2 (V_2/I_1), in conjunction with equations **(9)** and **(10)** to calculate the mutual inductance and coupling coefficient. (You may assume that $L_1 = L_2$)*

1.5. Use of the coupled circuits as a transformer

(i) Connect up the circuit as shown in figure 3 and set the 92Ω potentiometer to half scale.
(ii) set the input voltage to 50V) and measure the input and output currents and output voltage. when the potentiometer is set at half scale.

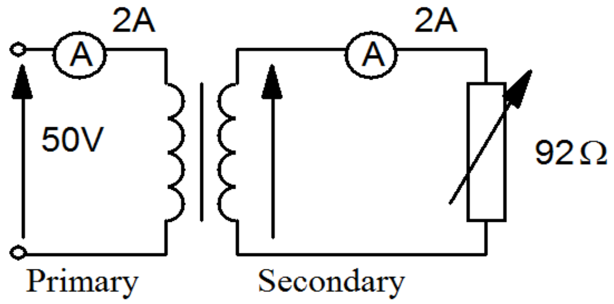


Figure 3: *Coupled circuits as a transformer*

(iii) Calculate the efficiency, η, of the coupled circuits acting as a transformer.

$$\eta = \frac{POWER_{OUT}}{POWER_{IN}} \times 100\% = \frac{V_S I_S}{V_P I_P} \times 100\%$$

TRANSFORMERS

In the previous section we saw how two coils mounted on the same magnetic core could act as a simple transformer, transferring energy from one winding without any physical electrical contact between the two windings. It will have been noted that the values of the coupling coefficient and efficiency were low. In a practical transformer we aim to maximise the magnetic coupling between the two windings and this will be demonstrated in the following sections.

Transformers are used in all areas of electrical and electronic engineering. One of their main application areas is the conversion of ac voltages from one level to another with very little loss of energy, another is the modification of the apparent impedance of a source or a load. The latter application is common in audio and communications based applications. In power management systems transformers may have to deal with electrical power flows well into the MW range at frequencies of a few tens to a few hundred Hz whilst in communication systems power levels are more likely to be in the region of μW at frequencies of MHz to low GHz. The basic behaviour of the transformer is the same in all cases although size will be different and design priorities will be different in each case.

Figure 4 shows a diagrammatic representation of a practical transformer where the iron core has been designed in such a way as to maximise coupling between the two coils.

The coil connected to the external driving source is called the primary. The primary coil (or winding) creates in the core the magnetic flux that induces an emf in the secondary coil (or winding). Thus electrical power can be transmitted between primary and secondary circuits even though there is no direct

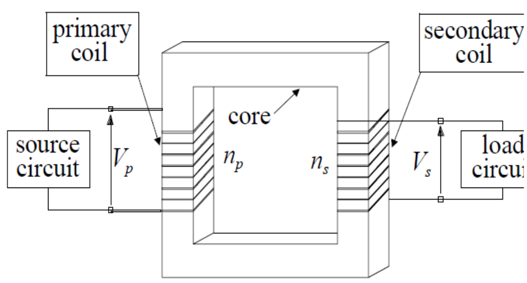


Figure 4: *Diagramatic representation of a*

electrical connection between the two windings. The capability to transmit power without direct connection, sometimes called isolation, can be exploited to reduce the risk of electric shock and hence is very important in environments such as building sites, hospitals and electrical equipment servicing facilities where the risk of injury due to electric shock is high.

Transformer action

As we have seen previously if a sinusoidal voltage is applied to a coil, a magnetic flux will be created by the current that flows. The magnetic flux will also be sinusoidal in form and will have the same frequency as the applied source voltage. Ignoring the resistance, equations **(1)** and **(2)** can be combined to give an expression for the flux set up in the core when a voltage, V_p, is applied to the primary winding having n_p turns:

$$\frac{d\Phi}{dt} = \frac{V_p}{n_p} \quad \textbf{(11)}$$

In a transformer the core is designed to make sure that as far as is possible, all the flux set up by the primary winding is routed through the secondary winding and hence the voltage across the secondary winding, V_s, is given by:

$$V_s = n_s \frac{d\Phi}{dt} \quad \textbf{(12)}$$

where n_s is the number of turns in the secondary winding. The $d\Phi/dt$ term may be eliminated from **(11)** and **(12)** to give:

$$V_s = n_s \frac{V_p}{n_p} \qquad \text{or} \qquad \frac{V_p}{V_s} = \frac{n_p}{n_s} \quad \textbf{(13)}$$

In other words, with a given input voltage a transformer designer can get whatever output voltage is required simply by defining an appropriate turns ratio, n_p/n_s . **The turns ratio is a key transformer parameter.**

The result expressed by **(13)** is based on the fundamental law **(2)** and on three assumptions

1. All the flux produced by the primary coil is guided through the secondary coils by the ferromagnetic core.
2. Setting up the flux is a lossless process
3. There are no losses in the coil wires because of currents flowing in them.

In a well designed transformer the effects of deviations from these assumptions will be small. As far as the first assumption is concerned, in a real transformer there is always a small amount of flux that manages to avoid the secondary coil and this flux is known as "leakage flux".
Leakage flux behaves like a small inductance in series with the primary coil and this is how it is modelled. The second assumption arises because the magnetic material, being imperfect, requires the primary source to expend some energy in setting up the magnetic flux. Most of this energy is stored (as it would be in an inductor and this part is modelled by an inductor) but a small fraction of it is dissipated as heat within the ferromagnetic core and this part is modelled by a resistor. The third assumption implies that the transformer coils are made of wire with zero resistance; real wire always has a finite resistance and this resistance is usually modelled by a resistance in series with the coil.

Power and current

A transformer is a passive circuit element - that is, it contains energy loss and energy storage elements only. A well designed transformer will have small power losses in comparison to the power transmitted so the approximation, ***power in = power out***, is justifiable. Thus:

$$V_p I_p = V_s I_s \qquad \text{or} \qquad \frac{V_s}{V_p} = \frac{I_p}{I_s} \qquad \textbf{(14)}$$

in other words, the current that will be drawn by the primary in response to a given secondary current can be calculated from the turns ratio. The current is important because it generates I^2R losses in the transformer windings and the maximum allowable value of this loss in a particular application determines the size of wire that must be used. Since the phase of I with respect to V is irrelevant to the calculation of I^2R losses, transformer powers are usually specified in terms of VA rather than W. VA is the product of rms voltage and rms current. V_p and V_s are inputs to the transformer design process and will be specified. Maximum allowable winding currents can be calculated from the known voltages and specified VA rating. The volume and weight, and hence the cost, of transformers is approximately proportional to their VA rating.

Transferred impedance

Consider the circuit of figure 5 in which an ideal transformer driven by a source V_p feeds a resistive load R. The secondary current I_s is given by Ohm's law as $I_s = \frac{V_s}{R}$. By using **(14)** the primary current can be written down in terms of I_s, V_s and V_p as:

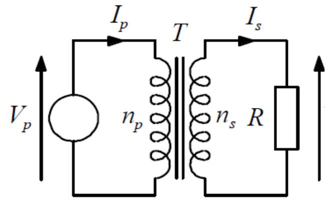


Figure 5: *An ideal transformer with resistive load*

$$I_p = I_s \frac{V_s}{V_p} = \frac{V_s}{R}\frac{V_s}{V_p} = \frac{V_s^2}{RV_p}$$

As far as the primary driving source is concerned, the effective resistance of the transformer - load combination is:

$$R' = \frac{V_p}{I_p} = V_p \frac{RV_p}{V_s^2} = R\frac{n_p^2}{n_s^2} \qquad \textbf{(15)}$$

since $V_p/V_s = n_p/n_s$. Thus the effective load impedance from the primary side is the actual impedance multiplied by the square of the turns ratio. This behaviour is commonly used in audio and communications applications to achieve virtually lossless impedance transformations.

Transformer model

In order to handle transformers analytically in a circuit environment and use normal circuit analysis methods to predict the behaviour of circuits containing transformers, it is necessary to represent all the significant effects within the transformer in terms of the elementary circuit elements of R and L. The full equivalent circuit model of a transformer is quite complicated but for most well designed transformers it is sufficient to use the model of figure 6. In figure 6 the ideal transformer is modified by the inclusion of a series impedance consisting of R_T and L_T and a shunt resistance, R_M. R_T and L_T represent the total series resistance and inductance with the secondary components transformed (by turns ratio squared) to the primary side. R_M represents the energy lost in the core as a result of setting up an alternating flux within it.

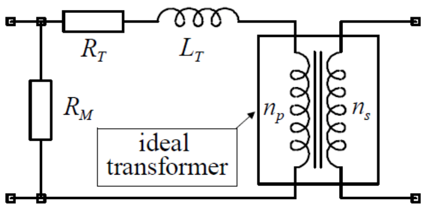


Figure 6: *An approximate transformer model*

2. EXPERIMENTS WITH AN E-SECTION POWER TRANSFORMER

The transformer to be tested is of a conventional design using 'E' and 'I' laminations. A partially built version of this and a 300 VA transformer designed using alternative 'C' cores are on display so that you can see the method of construction. Note in particular the concentric coil arrangement, to ensure a high coefficient of coupling, and the closed core of the magnetic circuit which has minimum airgaps to ensure a low reluctance and hence high winding self inductance. Note also the alternative lamination directions, both suitable for reducing unwanted eddy current losses.

2.1. Experimental evaluation of turns ratio and coupling coefficient

From equations **(7)** and **(8)** it can be seen that with one winding open circuited and the other carrying current, then, neglecting the resistance terms:

$$\text{with } I_2 = 0: \quad \frac{V_2}{V_1} = \frac{L_{21}}{L_1} \qquad \text{and with } I_1 = 0: \quad \frac{V_1}{V_2} = \frac{L_{12}}{L_2}$$

Hence from these two tests we can get an approximate value for the coefficient of coupling directly, without measuring the individual circuit inductances, since:

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} \times \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{L_{21}}{L_1} \times \frac{L_{12}}{L_2} = \frac{M}{L_1 L_{21}} = k^2 \quad \textbf{(16)}$$

(i) Locate the E-section power transformer (it can easily be identified because primary and secondary turns are given by the winding terminals). Connect the transformer primary to the isolating transformer output. Connect a voltmeter (DMM set to AC Volts) across the primary winding and adjust the variac to get a primary voltage of about 50V. Make a note of your chosen voltage and then measure the voltage appearing across the secondary terminals. Use your measured voltages and the given number of turns in each winding to check measured turns ratio with expected turns ratio. Comment on the agreement between the two values.

(ii) Now reconnect the transformer so the secondary winding is connected to the supply. Adjust the supply to about 25V and record the voltages across both windings. From you measurements, in conjunction with those from (i), use equation **(16)** *to calculate a value for the coupling coefficient. How does this value compare with the value obtained for the two coils in section 1.4? Can you explain any discrepancy?*

2.2. Transformer efficiency and regulation

(i) ***<u>Remember to reconnect the transformer so that the primary winding is connected to the supply.</u>*** *Connect a 10A ammeter and 5Ω variable resistor across the secondary winding of the E-section power transformer, as shown in figure 7. Adjust the variable resistance to its maximum value and carefully adjust the variac (starting from zero) to get a V_p of 50V.* ***While doing this remember to monitor the ammeter - it should reach a current of about 5A when V_p is 50V. If the current significantly exceeds 5A at any point during variac adjustment, check that the variable resistor is set to its maximum value. IF IN DOUBT ASK A DEMONSTRATOR****.*

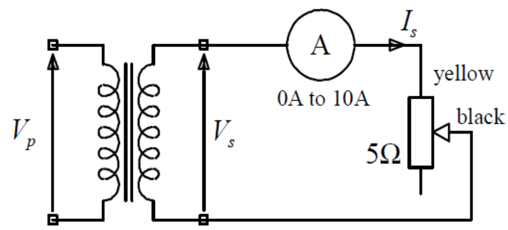


Figure 7: *"Yellow" and "black" indicate the variable resistor terminals that should be used*

*(ii) Maintaining V_P at 50V by adjusting the variac as necessary, measure V_s for the values of I_s in the range 5A to 9A in 0.5A increments. You should change I_s by altering the variable resistor (**Note this can get quite hot**). Currents in excess of 8A, are overloading the transformer and variable resistor somewhat so take these measurements as quickly as possible. Record your measurements in your labbook.*

(iii) For each set of readings calculate values for the regulation and efficiency (see below) and plot these against the load current on graphs 2 and 3 on the next page.

The output power, P_{LOAD}, and losses, P_{LOSS}, are given by:

$$P_{LOAD} = V_s I_s \text{ Watts}$$

$$P_{LOSS} = I_s^2 R_s + P_m \text{ Watts}$$

where R_s is the winding resistance referred to the secondary side of the transformer and P_m is the power lost in the iron core. For this transformer these values are R_s = 0.18Ω and P_M = 9.0W respectively. The efficiency, η, is then given by:

$$\eta = \frac{P_{OUTPUT}}{P_{INPUT}} = \frac{P_{LOAD}}{P_{SUPPLIED}} = \frac{P_{LOAD}}{P_{LOAD}+P_{LOSS}} \times 100 \ \%$$

The regulation (which is a measure of how the output voltage changes with load) is given by:

$$regulation = \frac{V_{s\,no\,load} - V_{s\ loaded}}{V_{s\,no\,load}}$$

where $V_{s\ no\ load}$ can be calculated from V_p and the known turns ratio and $V_{s\ loaded}$ is the measured V_s.

	Graph 2 (Experiment 2(iii)) **Efficiency vs Load current**
	Graph 3 (Experiment 2(iii)) **Regulation vs Load current**

3. EXPERIMENTS WITH SMALL AUDIO TRANSFORMER

3.1. Impedance transformation

(i) Using a laboratory oscillator, or the in-built signal generator of the digital oscilloscope, as a driving source, apply a 1kHz input signal of anywhere between 2V and 5V peak to the primary winding of the small signal transformer with yellow tape around its windings. Measure V_p and V_s using an oscilloscope and work out the transformer turns ratio, N.

(ii) Using a multimeter set to measure resistance, measure the winding resistance of the primary, R_p, and secondary, R_s, windings of your transformer and record this in your labbook.

(iii) Set up the circuit of figure 8 using a secondary load resistance R_L of 15Ω, measure the effective resistance of the primary of the loaded transformer as follows. Measure V_i and V_p for values of R_a of 22kΩ, 18kΩ, 13kΩ, 8.2kΩ and 4.7kΩ and record these in your labbook.

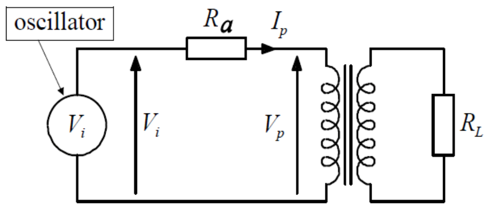


Figure 8: *Circuit for testing audio transformer*

The transformer and its load behave like a resistance R_{EFF} as far as the source circuit is concerned so, by potential division:

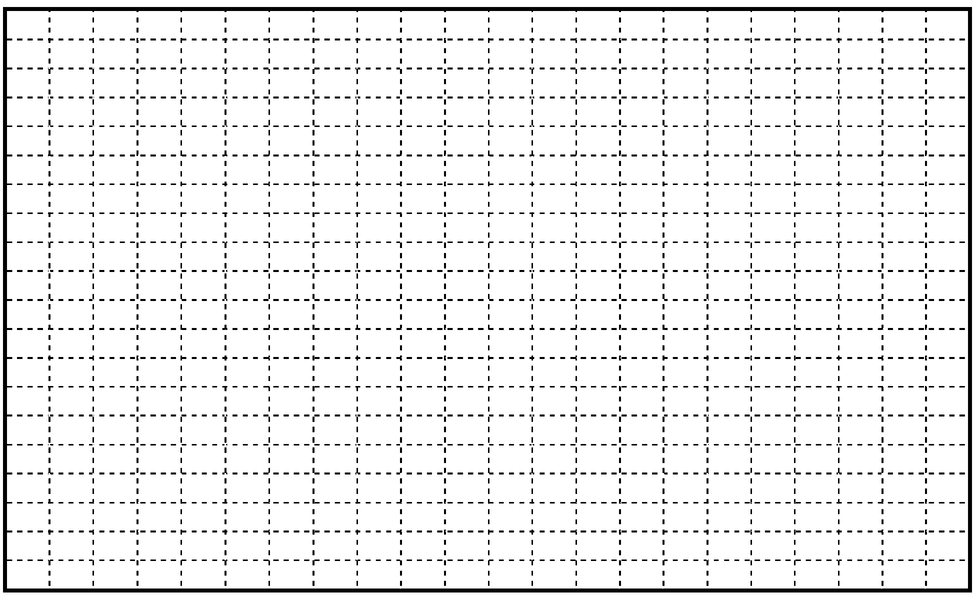
$$V_p = V_i \frac{R_{EFF}}{R_a + R_{EFF}}$$

This expression can be rearranged to give:

$$\frac{V_i}{V_p} = \frac{R_a}{R_{EFF}} + 1$$

so if V_i / V_p is plotted as a function of R_a the result should be a straight line with a slope of $1/R_{EFF}$.

(iv) Using the measured values of V_i and V_p values calculate the ratio V_i / V_p for each of the 5 values of R_a and plot the values on graph 4 below. (You can enter, without measurement, an extra point $V_i / V_p = 1$ when $R_a = 0$.) Work out R_{EFF} from the graph and compare it with the value expected from the measured turns ratio and known R_L. . ($R_{EFF} = R_p + N^2(R_s + R_L)$)



Graph 4

(Experiment 3(iv))

V_i / V_p* vs *R_a