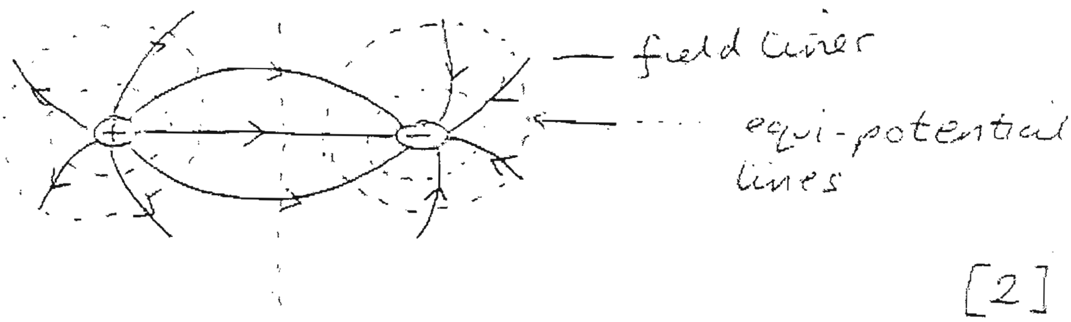
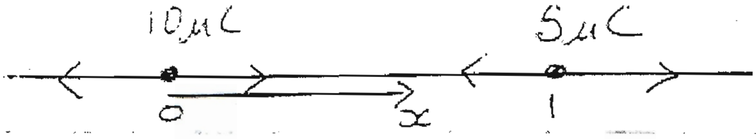
EEE 220

Question 1



[2]



As both charges +ve field cancelation point must lie between them

Total field at x is

$$E = \frac{kq_1}{x^2} - \frac{kq_2}{(1-x)^2} = 0$$

$$\rightarrow \frac{10}{x^2} - \frac{5}{(1-x)^2} = 0$$

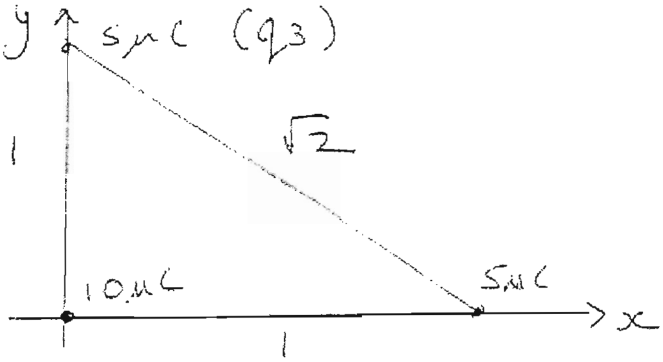
$$\rightarrow x^2 - 4x + 2 = 0$$

$$\rightarrow x = 2 \pm \sqrt{2}$$

$$\underline{x = 2 - \sqrt{2} = 0.586\,m}$$

is only valid solution [5]

Q1

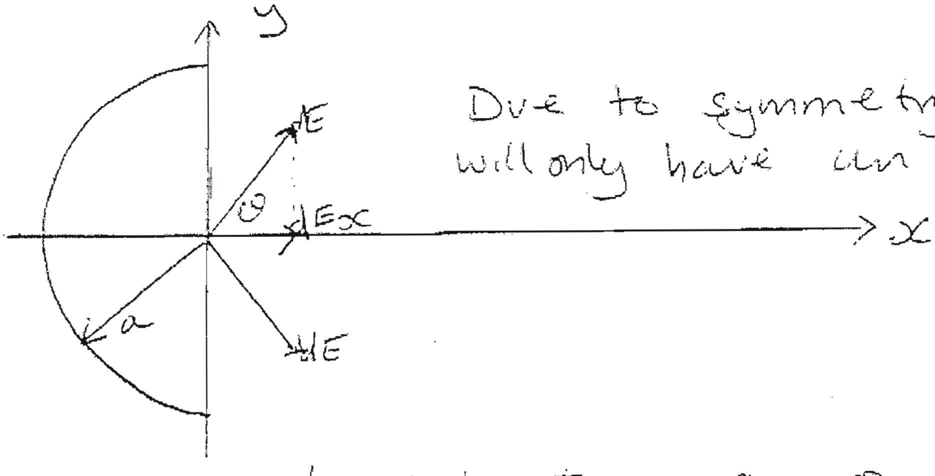


Field at q_3 is $$E = \frac{-\hat{x}}{4\pi\epsilon_0}\left[\frac{5\times10^{-6}}{(\sqrt{2})^2}\times\frac{1}{\sqrt{2}}\right]$$ $= \cos 45°$

$$+\frac{\hat{y}}{4\pi\epsilon_0}\left[\frac{5\times10^{-6}}{(\sqrt{2})^2}\times\frac{1}{\sqrt{2}}+\frac{10\times10^{-6}}{1}\right]$$ $\sin 45°$

$$\rightarrow \underline{F_3 = q_3 E = -\hat{x}[0.0795] + \hat{y}[0.529] \text{ N}}$$

[3]



Due to symmetry field will only have an x component

Let charge / unit length $= q_l = \dfrac{Q}{\pi a}$

Then $$dE_x = \frac{q_l\, a\, d\theta \cos\theta}{4\pi\epsilon_0 a^2}$$

Total field $$= 2\times\int_0^{\pi/2} dE_x = 2\int_0^{\pi/2}\frac{q_l \cos\theta}{4\pi\epsilon_0 a}\,d\theta$$

Q1.

$$= \frac{q_l}{2\pi\epsilon_0 a}\Big[\sin\theta\Big]_0^{\pi/2}$$

$$= \frac{q_l}{2\pi\epsilon_0 a} = \frac{Q}{2\pi^2\epsilon_0 a^2}$$

[7]

Field at origin due to ∞ line charge is

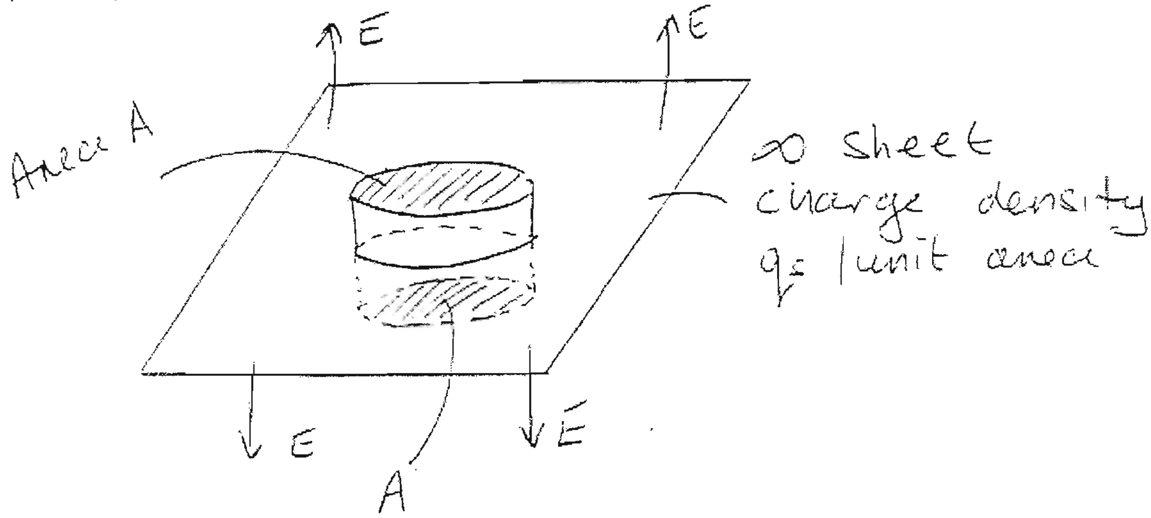
$\dfrac{q_l}{2\pi\epsilon_0(2a)}$ and is in -ve x direction.

$\Rightarrow$ Total field is

$$E = \left(\frac{Q}{2\pi^2\epsilon_0 a^2} - \frac{q_l}{4\pi\epsilon_0 a}\right)\hat{x}$$

[3]

Question 2



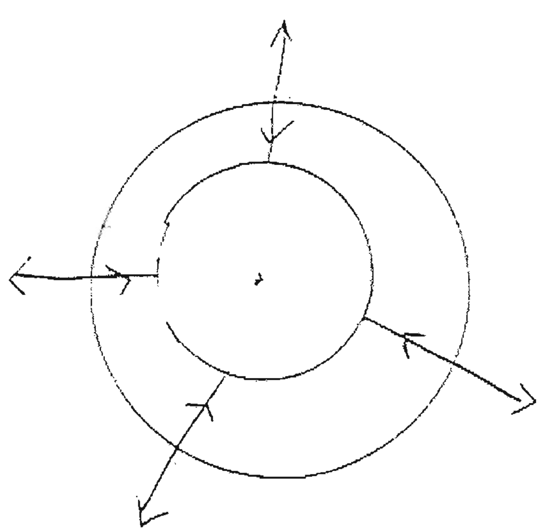
By symmetry E is $\perp$ to sheet

Gauss's Law $\oint_S E_\perp \, dA = \dfrac{q_s A}{\epsilon_0}$ ← enclosed charge

$$\therefore \quad E \times 2A = \frac{q_s A}{\epsilon_0}$$

contribution from top and bottom surfaces

$$\rightarrow \quad E = q_s / 2\epsilon_0 \qquad [4]$$



1) inside both spheres
$E = 0$

Q2

ii) 15cm is between spheres so no contribution from outer sphere but inner sphere looks like point charge with magnitude $-2\mu C$

$$\rightarrow \quad E = \frac{-2\times10^{-6}}{4\pi\epsilon_0 (0.15)^2} = 8\times10^5 \text{ V/m}$$

with direction as indicated.

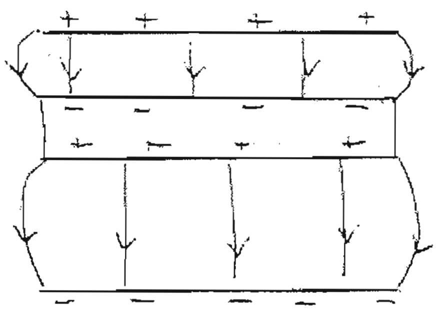
iii) at $r = 25$cm both spheres contribute to field.

$$E = \frac{1}{4\pi\epsilon_0}\left[\underbrace{\frac{-2\times10^{-6}}{(0.25)^2}}_{\text{inner}} + \underbrace{\frac{3\times10^{-6}}{(0.25)^2}}_{\text{outer}}\right]$$

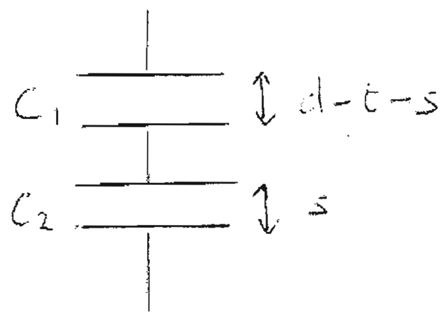
$$= 1.44\times10^5 \text{ V/m}$$

with direction as shown

[6]



$\equiv$



structure is equivalent to two capacitors in series with

$$C_1 = \frac{\epsilon A}{(d-t-s)}, \quad C_2 = \frac{\epsilon A}{s}$$

Q2

Total $C = C_1 \parallel C_2 = \frac{\epsilon A}{d-t}$

If slab removed new capacitance

$$C' = \frac{\epsilon A}{d}$$

Total charge is unchanged and is given by

$Q = CV = C'V'$ where V' is new voltage

$$\rightarrow V' = \frac{C}{C'} V$$

or $V' = \frac{\epsilon A}{d-t} \cdot \frac{d}{\epsilon A} = \frac{d}{d-t}$ → voltage increases

Energy, $\mathcal{E} = \frac{1}{2} C V^2$

New energy in system $\mathcal{E}' = \frac{1}{2} C' V'^2$

$$= \mathcal{E} \frac{C}{C'}$$

→ Energy also increases.

work is done to remove slab.

[10]

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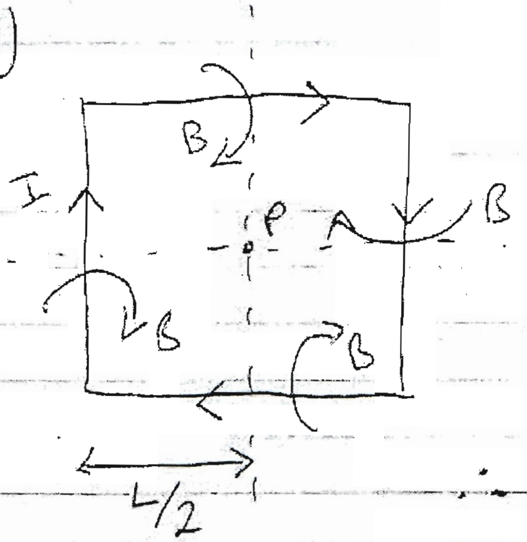
Q3 A)



From symmetry total field at P will be 4 times field due to one wire with $x = \frac{L}{2}$

$$\therefore B = \frac{4 \times \mu_0 I}{\cancel{2}\pi \times \frac{L}{\cancel{2}}} \left[\frac{1}{1 + \underbrace{\left(\cancel{2} \times \frac{L}{\cancel{2}} \times \frac{1}{L} \right)^2}_{=1}} \right]^{1/2}$$

$$\Rightarrow B = \frac{4 \mu_0 I}{\pi L} \left(\frac{1}{2} \right)^{1/2}$$

$$= \frac{4 \mu_0 I}{\pi L \sqrt{2}} \times \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = \underline{\frac{2\sqrt{2}\, \mu_0 I}{\pi L}}$$

[6]

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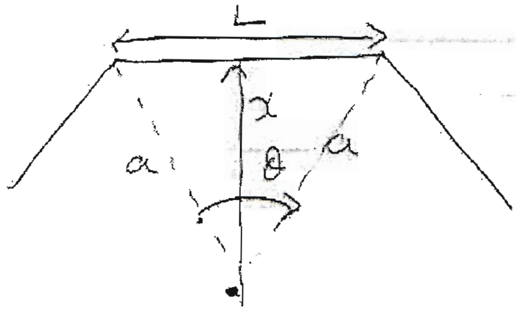
Q 3 1)



For n-sided polygon $\theta = 2\pi/n$

At centre of polygon can use same procedure as for square loop (n x field from one wire)

$$x = a\cos\frac{\theta}{2} = a\cos(\pi/n)$$

$$L = 2a\sin\frac{\theta}{2} = 2a\sin(\pi/n)$$

For n sides we have

$$B = \frac{n\mu_0 I}{2\pi a\cos(\pi/n)}\left[\frac{1}{1+\left(\frac{2a\cos(\pi/n)}{2a\sin(\pi/n)}\right)^2}\right]^{1/2}$$

$$\downarrow$$

$$\left[\frac{1}{1+\frac{1}{\tan^2(\pi/n)}}\right]^{1/2}$$

$$\downarrow$$

$$\left[\frac{\tan^2(\pi/n)}{1+\tan^2(\pi/n)}\right]^{1/2}$$

Q3B

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$$= \frac{n\mu_0 I}{2\pi a \cos(\pi/n)} \left[\tan^2(\pi/n)\cos^2(\pi/n)\right]^{1/2}$$

using $1 + \tan^2 = \sec^2$

$$= \frac{n\mu_0 I \tan(\pi/n)}{2\pi a}$$

As n becomes large, polygon tends to a circle

As $n \to \infty$, $n \underbrace{\tan(\pi/n)}$ $\to n\frac{\pi}{n} = \pi$

$\tan\theta \simeq \theta$
for θ small

$$\therefore \; B = \frac{\mu_0 I \pi}{2\pi a} = \frac{\mu_0 I}{2a}$$

[6]

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Q3 c/

From earlier we have

$$B_{cir} = \frac{\mu_0 I_c}{2a} \quad , \quad B_{sq} = \frac{2\sqrt{2}\mu_0 I_s}{\pi L}$$

for dimensions shown

$$B_{cir} = \frac{\mu_0 I_c}{2b} \quad \text{(out of page)}$$

$$B_{sq} = \frac{\sqrt{2}\mu_0 I_s}{\pi a} \quad \text{(into page)}$$

Total field $B = \dfrac{\mu_0 I_c}{2b} - \dfrac{\sqrt{2}\mu_0 I_s}{\pi a}$ (out of page)

if $b = a\sqrt{2}$ and $I_c = 1$

$$B = \frac{\mu_0}{2a\sqrt{2}} - \frac{\sqrt{2}\mu_0 I_s}{\pi a}$$

equating to zero gives

$$\frac{\mu_0}{2a\sqrt{2}} = \frac{\sqrt{2}\mu_0 I_s}{\pi a}$$

$$\rightarrow \quad I_s = \frac{\pi}{4} A \qquad [8]$$

Question 4

Flux through loop $= A B(t)$, $A =$ area of loop $= \pi a^2$

From Faraday's Law

$$V = -\frac{d\phi}{dt} = -A\frac{d B(t)}{dt} \quad , \quad B(t) = B_0(1-e^{-\lambda t})$$

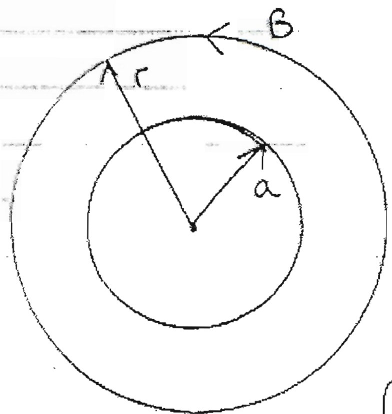
$$\rightarrow V = -AB_0\lambda e^{-\lambda t} = \underline{-\pi a^2 B_0 \lambda e^{-\lambda t}}$$

For $a = 1\,cm$, $B_0 = 500\,\mu T$ and $\lambda = 0.1$

$$V \,@\, 20s = \pi \times (0.01)^2 \times 500\times10^{-6} \times 0.1 \times e^{-0.1\times20}$$

$$= \underline{2.13\times10^{-9}\,V}$$

[8]



Due to symmetry field can only vary with radial distance from wire

Using Ampere's Law

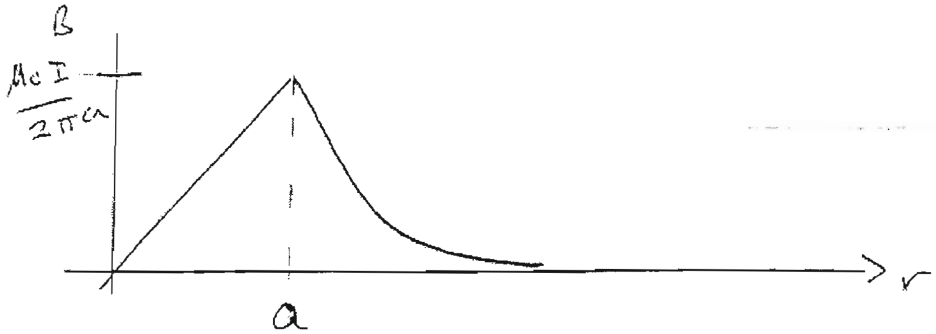
$$\oint_C \underline{B}\cdot\underline{dl} = \mu_0 I \quad , I = \text{enclosed current}$$

$\therefore$ Outside wire $\quad B.2\pi r = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi r}$

Inside wire $\quad I(r) = I\frac{\pi r^2}{\pi a^2} = I\frac{r^2}{a^2}$

$$\rightarrow 2\pi r B = \mu_0 I \frac{r^2}{a^2} \quad \rightarrow \quad B = \frac{\mu_0 I r}{2\pi a^2}$$

Q4



[6]

For solenoid $\quad B = \mu_0 I N_l$

$$\therefore B = 4\pi \times 10^{-7} \times 1 \times \frac{1000}{0.1}$$

$$= 0.158\,T$$

[3]

By definition $\quad \Psi = LI$

and $\Psi = N\phi$, where $\phi = AB$ = flux

Hence $\Psi = NAB$ but $B = \mu_0 \frac{N}{d} I$

d = length of solenoid

$$\rightarrow \Psi = N^2 A \mu_0 \frac{I}{d}$$

$$\rightarrow L = \frac{\Psi}{I} = \mu_0 N^2 \frac{A}{d}$$

$$= 4\pi \times 10^{-7} \times (1000)^2 \times \pi \times \frac{(0.01)^2}{0.1}$$

$$= 3.95\,mH$$

[3]