EEE225: Analogue and Digital Electronics

Lecture X

James E. Green

Department of Electronic Engineering
University of Sheffield
j.e.green@sheffield.ac.uk

This Lecture

Books/Papers about Noise

No need to buy any of these but good idea to know they exist.

1. Motchenbacher, C. D., and Connelly, J. A., Low-Noise Electronic System Design.
2. Books authored by Albert Van der Ziel (e.g. “Noise” and “Noise in Solid State Devices and Circuits” (1950’s - 1980’s try Abebooks)
3. Leach Jr., W. M., Fundamentals of Low-Noise Electronics[1]
4. W. M. Leach, Jr., Fundamentals of Low-Noise Analog Circuit Design, Proceedings of the IEEE, Vol. 82, No. 10, pp. 1515-1538, Oct. 1994. `http://dx.doi.org/10.1109/5.326411`.

[1] Quite inexpensive (as text books go) and comprehensive. Buy direct from publisher, Kendal-Hunt, 2012. Avoid the e-book! `www.kendallhunt.com`.

In this part of the course we will discuss

- Sources of noise in circuits/devices
- Methods for quantifying noise
- Limits on the available noise power
- Methods of combining noise sources in circuit analysis
- The effects of RC circuits on noise
- Noise in systems of circuits
  - Signal to noise ratio
  - Noise figure & noise factor
- Equivalent circuits for performing circuit analysis with noise sources
- Noise in Opamps

Note: Nothing will be said about BJTs or FETs and nothing about the effects of noise in oscillators. This is not an exhaustive treatment...

Noise produced by Human Activity

Noise is a term used to describe an unwanted signal regardless of origin. In general there are three sources of noise. First: Noise produced by human activity ("man-made"), this includes:

- Radio signals that are accidentally picked up.
- EMI from machines (especially brushed or AC machines)
- Poorly designed or laid out power electronics systems
- Switching Converters
- Hum Loops: poor current return path design/layout etc.

Noise produced by humans can be reduced by cunning design methods. Legislation (OFCOM, FCC etc.) exists to define how much EMI may be generated by certain devices and how much interference certain electronic systems must be able to withstand and still work properly (medical electronics especially).

Natural Sources of Electronic Noise

Natural External:

- Environmental sources such as lightning strikes (produces copious EMI)
- High energy cosmic rays.

External sources may be treated with the same methods as noise produced by human activity

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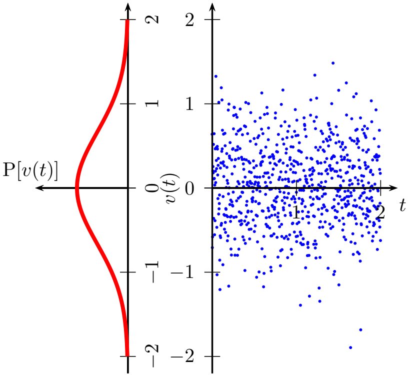
Natural Internal: This is the hiss one hears when an audio amplifier is turned up to 11 without any program material being played. It is caused by the random motion of individual electrons in components and it's what concerns us in this course. Resistors, diodes and transistors are sources of noise. *Imperfect* capacitors and inductors are also sources of noise however we do not usually treat them except in special circumstances.

Noise Amplitude Distribution

On a CRO noise is an irreducible fuzz. It doesn't have any period. Changing the timebase doesn't change its shape. As we approach the CRO bandwidth the noise appears to possess some coherence but this is a limitation of the measurement equipment and is not real. A CRO can't be meaningfully triggered from a noise signal.

$$\frac{1}{t}\int_0^t v_n\left(t\right)\,\mathrm{d}t. \rightarrow 0 \text{ as } t \rightarrow \infty$$

The value of the noise at any time provides no information about other times. The idea of "amplitude" as a metric of noise is therefore not very helpful.



- Despite having an average value of zero, the instantaneous value may not be zero so noise signals can dissipate *power*. We use this to express the quantity of noise in a meaningful way.
- Power dissipated is proportional to the "mean square" value of the noise voltage v_n or noise current i_n, $\overline{v_n}^2$ and $\overline{i_n}^2$ which have units of V^2 or A^2.
- The root mean square can also be used $\sqrt{\overline{v_n}^2}$ with units of V. (square it first, then mean, then root...)
- Occasionally you may read[2] about "crest factor". Crest factor is used as a statistical measure of the likelihood of finding a noise value above a certain threshold from the mean. For example, a Gaussian noise amplitude distribution has a peak value more than 4.9 times the RMS value only 0.0001% of the time. This is only a statistical measure, noise may take any instantaneous value.

[2] yes, you have to *read* about electronics.

Noise Frequency Distribution

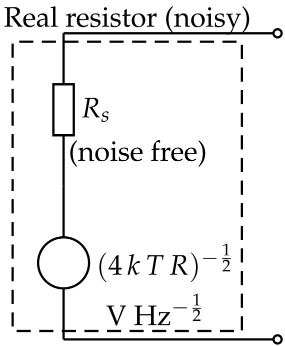
- Certain noise processes inside electronic devices are effective at different frequencies.
- That is to say, the mean square value of noise changes as a function of f.
- This is not related to the amplitude distribution (Gaussian, Poison etc.)
- To assess the effect of frequency dependent noise we use "noise power spectral density" - think spectrum as in radio spectrum and spectrum analyser.
- Power spectral density is the mean squared noise amplitude *per Hz* (i.e. in one unit of frequency or bandwidth). Measured in V^2/Hz "volts squared per Hertz" or $\mathrm{V}/\sqrt{\mathrm{Hz}}$ "volts per root Hertz". It is how we commonly describe noise sources.

Johnson noise is caused by the random motion of electrons in resistive media. It has constant power spectral density - “white” because all frequencies are represented equally, as in white light and can be modelled by a Thévenin source, in which the voltage represents the noise v_n and the series resistor is a noise free equivalent of the resistance being modelled.

$$\overline{v_n{}^2} = 4\,k\,T\,R \quad \mathrm{V}^2/\mathrm{Hz} \qquad (1)$$

$$= 4\,k\,T\,R\,\Delta f \quad \mathrm{V}^2 \qquad (2)$$

where k is Boltzmann's constant, T is the absolute temperature, R is the resistance and Δf is the bandwidth of the measurement. Thermal noise is *not* generated by incremental resistances such as r_{be}.



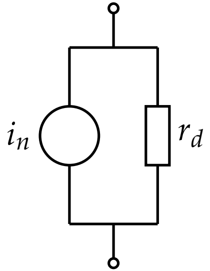
Shot Noise

Shot noise is caused by the discontinuous nature of current flowing in pn junctions and thermionic valves. It has a constant power spectral density (it is “white”). and is modelled by a Norton source in parallel with a resistance which models the dynamic resistance of the pn junction. Since it is an incremental resistance it is noise free.

$$\overline{i_n}^2 = 2\, q\, I_{DC} \quad \mathrm{A}^2/\mathrm{Hz} \tag{3}$$

$$= 2\, q\, I_{DC}\, \Delta f \quad \mathrm{A}^2 \tag{4}$$

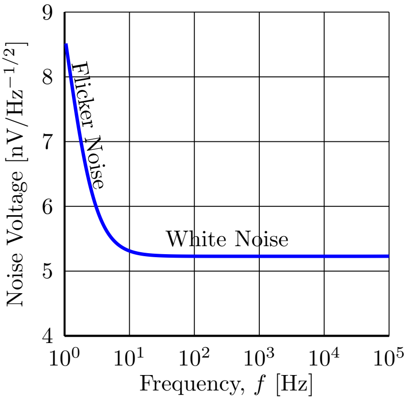
where q is the magnitude of the electron charge, I_{DC} is the quiescent or large signal current in the junction and Δf is the bandwidth of the measurement.



$$i_n = (2\, q\, I_{DC})^{-1/2}\ \mathrm{A\ Hz}^{-1/2}$$

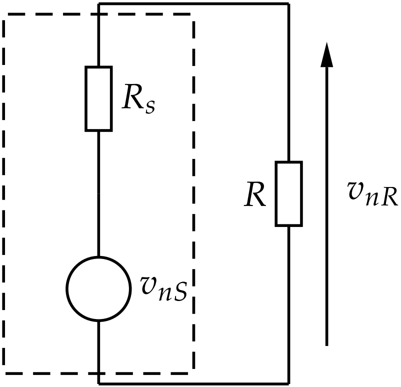
Flicker noise also called 1/f affects both resistors and semiconductor devices as well as many other physical processes. The magnitude of 1/f noise is proportional to the magnitude of the DC current flowing in a component and its power spectral density approximately obeys a 1/f relationship. Manufacturers specify 1/f by means of a graph, and often a "corner frequency" where 1/f gives way to white noise. This corner lies at the frequency where $\sqrt{\overline{v_n}^2}$ has risen to 1.4 times it's high frequency value.

It's origins are not properly understood. It appears to be related to process quality as 1/f has reduced as silicon processing has matured in the last 60 years.



Maximum Available Power

Find the maximum power that can be transferred from a noisy resistor to a noise free resistor. Optimum value of R?



$R_s = R$ to yield maximum power transfer. This is from EEE118 lecture 2.

The voltage across R is,

$$v_{nR} = v_{nS}\frac{R}{R + R_s} = \frac{v_{nS}}{2} \qquad (5)$$

Power is then,

$$P_R = \frac{\overline{v_{nS}^2}}{4\,R_s} = \frac{4\,k\,T\,R_s}{4\,R_s} = k\,T \qquad (6)$$

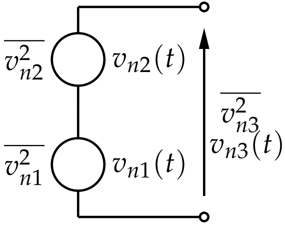
W/Hz or $P_R = k\,T\Delta f$ W.

Power available is therefore independent of R_s.

Combining Noise Sources

We often need to combine several sources of noise to find the total noise voltage between a given node and ground or the total noise current flowing through a branch.

Adding $v_{n2}(t)$ to $v_{n1}(t)$ would be correct, but wouldn't help much...



$$\begin{aligned}\overline{v_{n3}^2} = \overline{v_{n3}^2(t)} &= \overline{(v_{n1}(t) + v_{n2}(t))^2} \\ &= \overline{v_{n1}^2(t)} + 2\overline{v_{n1}(t)\, v_{n2}(t)} + \overline{v_{n2}^2(t)} \end{aligned} \quad (7)$$

$$\overline{v_{n1}^2(t)} = \overline{v_{n1}^2},\ \overline{v_{n2}^2(t)} = \overline{v_{n2}^2} \quad (8)$$

since $v_{n1}(t)$ and $v_{n2}(t)$ are *uncorrelated*,

$$\overline{v_{n1}(t) v_{n2}(t)} = \overline{v_{n1}(t)} \cdot \overline{v_{n2}(t)} \quad (9)$$

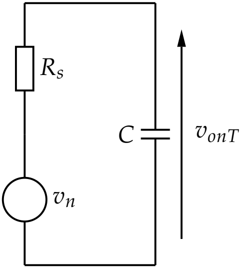
$$\overline{v_{n1}(t)}\ \&\ \overline{v_{n2}(t)} = 0,\ \overline{v_{n1}(t) v_{n2}(t)} = 0$$

$$\overline{v_{n3}^2} = \overline{v_{n1}^2} + \overline{v_{n2}^2} \quad (10)$$

Effect of a Parallel RC network on White Noise

Filtering white noise leaves it somewhat more pink than white. The *power* transmission response of the filter determines the result.

$$\overline{v_{onT}^2} = \frac{k\ T}{C} \quad (11)$$



- If v_n is the thermal noise of the resistor, the mean square output is independent of the value of resistance.
- All real resistors have some capacitance in parallel with them. Approximately 1 pF for a 0.25 W resistor.
- Considering only the capacitor, it is in parallel with a resistance of $\infty\ \Omega$.
- Expect $kT/C\ \mathrm{V}^2$ across its terminals.
- Charge transfer can be found from $\overline{q_n^2} = C^2\ \overline{v_n^2}$ because $\overline{q_n^2} = k\ T\ C\ \mathrm{C}^2$.

Noise Temperature

- Often devices display more noise than the theory predicts, due to physical manufacturing constraints.
- In resistors (and some other places) this is called "excess noise" (EEE118, Lecture 1).
- One way to account for the extra noise is to adjust the temperature we use in the calculations to a higher value to make up for the extra noise.
- This new temperature is the *effective noise temperature*, T_e.
- Noise temperature often used as a metric of quality for low noise amplifiers esp. in satellite and other microwave applications.
- To obtain the effective noise temperature equate $\overline{v_n^2} = 4\,k\,T_e\,R_s\ \mathrm{V}^2$ so $T_e = \frac{\overline{v_n^2}}{4\,k\,R_s}\ \mathrm{K}$. where $\overline{v_n^2}$ is the combination of all of the noise sources in series with R_s. If it's only R_s, and R_s is noisy but otherwise ideal, $T_e = T$.

Review

- Enumerated some sources of noise.
- Introduced the idea of electronic noise as a random disturbance due to electron motion.
- Considered the problems associated with quantifying noise.
- Introduced the amplitude distribution and the frequency distribution.
- Listed three noise sources commonly found in electronic circuits, discussed colour.
- Derived the total available noise power from a resistance in a specified measurement bandwidth.
- Derived how uncorrelated noise sources should be combined.
- Discussed the effect of low pass filtering a white noise source, and showed something interesting about the noise across a capacitor due to other sources.
- Introduced the concept of effective noise temperature.