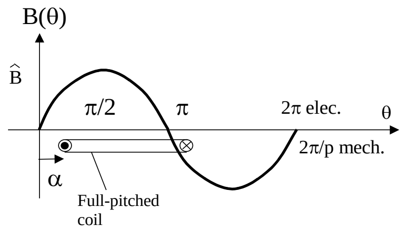
ANSWERS to EEE305 and EEE6140: 2012-2013

Question 1

1(a)

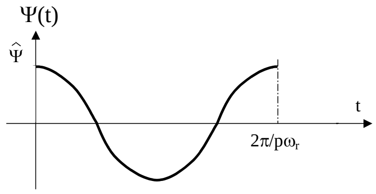
The field distributions in the brushless AC machines are assumed to be



Sinsoidal distribution

Flux per pole $\Phi_p = \dfrac{1}{\pi/p}\int_0^{\pi/p} \hat{B}\sin p\theta d\theta \times \left(\dfrac{\pi DL}{2p}\right) = B_{ave} \times \left(\dfrac{\pi DL}{2p}\right)$, where $B_{ave} = \dfrac{2}{\pi}\hat{B}$

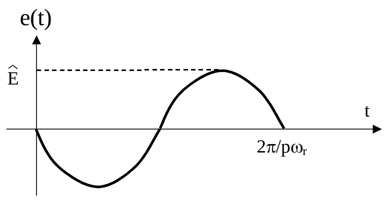
The total winding has N turns in series ($N/2p$ per pole) and the coils are full pitched (i.e. span 1 pole-pitch). If there is a relative angular velocity between coil & magnetic field of ω_r rads^{-1}. Then, flux linkage per coil will vary as shown, where $\alpha = \omega_r t$, and $\alpha = 0$ at $t = 0$



$$\Psi(t) = \hat{\Psi}\cos(p\omega_r t)$$

where peak flux linkage $\hat{\Psi} = \left(\dfrac{N}{2p}\right)\times\Phi_p$ per pole or $N\Phi_p$ for whole winding.

The induced emf for both cases is $e = \dfrac{d\Psi(t)}{dt}$



By inspection from $\Psi(t)$ diagram we can deduce the e waveform for a constant angular speed ω_r .

The output produces one cycle of emf in 1 cycle of B waveform, i.e. when $t = \dfrac{2\pi}{p\omega_r}$ sec for a p-pole-pair machine rotating at ω_r mech.rad s^{-1}

$$e = \frac{d\Psi}{dt} = (-\hat{\Psi}_p\omega_r)\sin p\omega_r t$$

i.e. $e = -\hat{E}\sin \omega t$

where $\omega = p\omega_r$

$$\hat{E} = N\Phi_p \times p\omega_r = N\frac{2}{\pi}\hat{B}\times\frac{\pi DL}{2p}\times p\omega_r$$

$$= N\hat{B}DL\omega_r$$

or $E_{rms} = \frac{2\pi f}{\sqrt{2}}N\frac{\hat{B}DL}{p} = 4.44 fN\frac{\hat{B}DL}{p}$

where f= elec. frequency= $\frac{p\omega_r}{2\pi}$

1(b)

If *N* turns (*N*=*Z*/2, *Z*=no of conductors), each carrying a current *I* then the power per phase is

assuming E & I has a phase angle ϕ between

$$P = 3E_{rms}I_{rms}\cos\phi = \frac{Z\hat{I}_{rms}}{2\sqrt{2}}\hat{B}DL\omega_r\cos\phi$$

the electrical loading $Q_{rms} = \frac{ZI_{rms}}{\pi D}$

$$P = \left(\frac{\pi D^2 L\hat{B}Q_{rms}}{2\sqrt{2}}\cos\phi\right)\omega_r$$

$$= \left(\frac{\pi D^2 L}{2}B_{av}\left(\frac{\pi}{2\sqrt{2}}\right)Q_{rms}\cos\phi\right)\omega_r = 1.11T\omega_r\cos\phi \qquad (\frac{\pi}{2\sqrt{2}} = 1.11)$$

$$T = \frac{\pi}{2}D^2 LB_{av}Q_{rms}$$

1(c) The torque density is limited by the magnetic loading B and the electrical loading Q.

The magnetic loading is limited by the magnetic saturation in the tooth and back-iron, as well as the iron losses, including hysteresis and eddy current, which will limit the magnetic loading depending on the speed. It is also limited by the permanent magnet materials in permanent magnet machines.

The electrical loading is primarily limited by the copper loss, and the demagnetisation withstand in the permanent magnet machines.

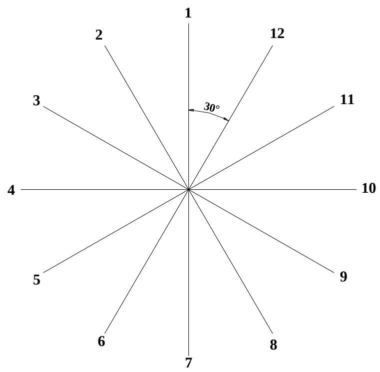
1(d)

The ideal back-emf and current waveforms for Brushless DC (BLDC) machines are trapezoidal back-emf, at least with >120 deg. elec. flat top and rectangular current waveform, while those for brushless AC (BLAC) machines are both sinusoidal. If they are non-ideal, the average torque will be reduced and the torque ripples will be increased.

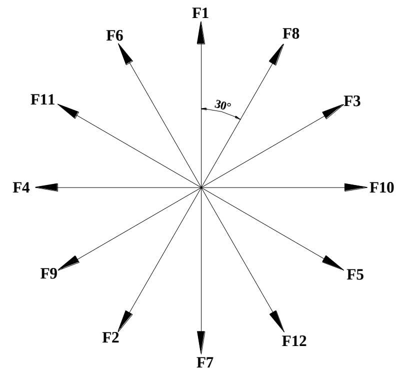
Question 2

2(a)

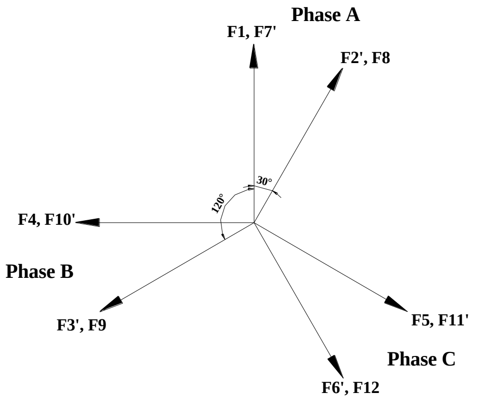
For all teeth wound motor, the coil mmf vectors and connections are shown in the figure



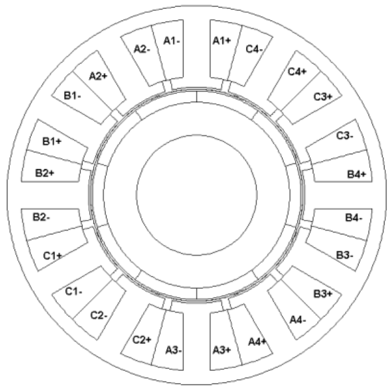
(a) coil positions (mech. deg)



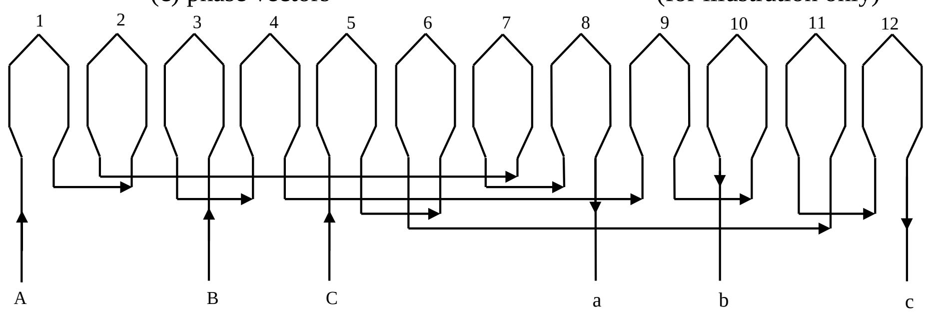
(b) coil mmf vectors (elec. deg.)



(c) phase vectors



(for illustration only)

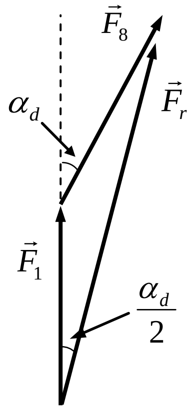


(d) coil connections to form 3-phase windings

2(b)

The stator slot pitch is $\frac{n2\pi}{N_s}$ and the pole pitch is $\frac{\pi}{p}$ where $N_s = 12$ is the slot number and p=5 is the number of pole pairs. Referring to Fig. 2(a), the coil is short-chorded by a pitch angle: $\alpha_p = \frac{\pi}{p} - \frac{n2\pi}{N_s}$. Therefore, the pitch factor is given by: $K_{pn} = \cos(p\frac{\alpha_p}{2}) = \cos\left(\frac{\pi}{2} - \frac{np\pi}{N_s}\right) = \sin\left(\frac{np\pi}{N_s}\right)$

As shown in the figure, each phase has four coil mmf vectors. Considering phase A for instance, the coil mmf vectors F_1 and F'_7 are in the same direction but displaced by an angle α_d from the mmf vectors F'_2 and F_8. All of these mmf vectors have the same magnitude because of the same number of turns wound around each tooth. The disposition angle α_d is given by: $\alpha_d = \pi - \frac{np2\pi}{N_s}$. Therefore, the distribution factor is derived as follows:



Space vectors represent the coil mmfs in phase A.

$$K_{dn} = \cos\left(\frac{\alpha_d}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{np\pi}{N_s}\right) K_{dn} = \sin\left(\frac{np\pi}{N_s}\right)$$

The winding factor is the product of the pitch factor and distribution factor:

$$K_{dpn} = K_{pn} \times K_{dn} = \sin^2\left(\frac{np\pi}{N_s}\right)$$

2(c) $\frac{|\hat{E}_n|}{|\hat{E}_1|} = \frac{\hat{B}_n K_{dpn}}{\hat{B}_1 K_{dp1}}$

$$K_{dp1} = \sin^2\left(\frac{5\pi}{12}\right) = 0.933,\ K_{dp5} = \sin^2\left(\frac{5\times 5\pi}{12}\right) = 0.067,\ K_{dp7} = \sin^2\left(\frac{7\times 5\pi}{12}\right) = 0.067$$

$$\frac{|\hat{E}_5|}{|\hat{E}_1|} = \frac{\hat{B}_5 K_{dp5}}{\hat{B}_1 K_{dp1}} = \frac{0.1\times 0.067}{0.7\times 0.933} = 0.01 = 1\%$$

$$\frac{|\hat{E}_7|}{|\hat{E}_1|} = \frac{\hat{B}_7 K_{dp7}}{\hat{B}_1 K_{dp1}} = \frac{0.08\times 0.067}{0.7\times 0.933} = 0.008 = 0.8\%$$

2(d). The most significant difference between machines having all teeth wound and alternate teeth wound is the winding distribution factor, for which $K_{dn} = 1$ for alternate teeth wound machine. This will make the back-emf waveform of the machine with all teeth wound more sinusoidal.

QUESTION 3

3(a)

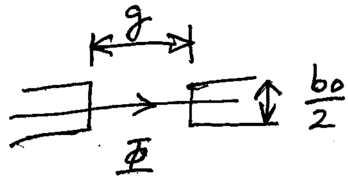
assumptions: (1) the permeability of the iron is infinite, (2) all flux lines are vertical

The problem can be regarded symmetrical about the axis and then its calculation can be equivalent to a single slot.

(1) Slot opening

mmf across opening=total slot mmf=NI,

where N=no of conductors, I=current per conductor



$\therefore$ flux across opening/unit length

$$\Phi = \frac{mmf}{\frac{2g}{\mu_0 b_0}} \text{ /unit length } = NI\mu_0\left(\frac{b_0}{2g}\right)$$

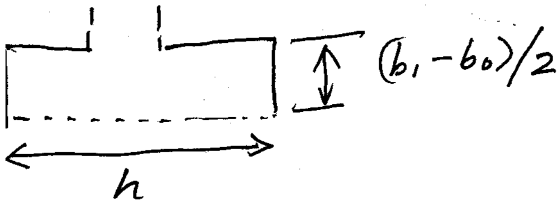
& flux linkage /unit length,

$\psi = \Phi \times$ No of conductor linked (N)= $N^2 I\mu_0\left(\frac{b_0}{2g}\right)$

(2)Unwound section of slot (above conductors)

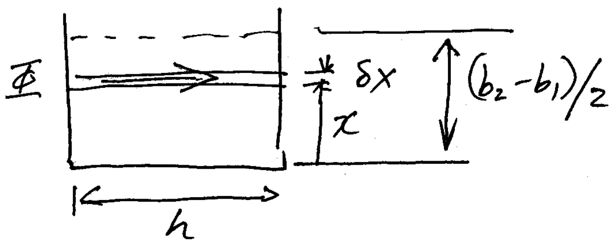
Again by same analysis as slot opening

Flux linkage/unit length= $N^2 I\mu_0\left(\frac{b_1 - b_0}{2h}\right)$



(3)For wound section of slot

In this case the mmf is distributed throughout the section & we need to integrate across the depth of $(b_2-b_1)/2$.



Consider elemental strip depth δ_x at a from bottom of winding:

Mmf available below strip= $NI \times \frac{x}{(b_2 - b_1)/2}$

Flux Φ through strip

$$= \frac{mmf}{reluc\tan ce} = NI\frac{x}{(b_2 - b_1)/2}\Big/\frac{h}{\mu_0\delta_x} \text{ /unit length} = NI\frac{x}{(b_2 - b_1)/2}\frac{\mu_0\delta_x}{h} \text{ /unit length}$$

& flux linkage $= N^2 I\left[\frac{x}{(b_2 - b_1)/2}\right]^2 \frac{\mu_0\delta_x}{h}$

(no of conductors below the strip= $N\dfrac{x}{(b_2-b_1)/2}$)

Hence effective flux linkages for total section

$$= N^2 I\frac{\mu_0}{h}\int_0^{(b_2-b_1)/2}\left[\frac{x}{(b_2-b_1)/2}\right]^2 dx = N^2 I\mu_0\left(\frac{(b_2-b_1)/2}{3h}\right)$$

So far total slot of the shape considered

Flux linkage/unit length= $N^2 I\mu_0\left[\dfrac{b_0}{2g}+\dfrac{(b_1-b_0)/2}{h}+\dfrac{1}{3}\dfrac{(b_2-b_1)/2}{h}\right]$

Hence, total inductance $\left(\dfrac{Flux\,linkage}{I}\right)$ per unit length

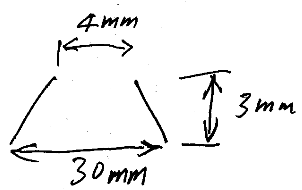
$$L = 2\times N^2\mu_0\left[\frac{b_0}{2g}+\frac{(b_1-b_0)/2}{h}+\frac{1}{3}\frac{(b_2-b_1)/2}{h}\right] \qquad [\mathrm{H}]$$

$$L = 2\times 100^2\times 4\pi\times 10^{-7}\left[\frac{6}{2\times 2}+\frac{(10-6)/2}{20}+\frac{1}{3}\frac{(20-10)/2}{20}\right] = 0.0423\,\mathrm{H}$$

3(b)

The permanent magnet can be regarded as airspace

Most of the formulae derived above can be utilized except for the tapered part.



By a similar analysis to (a), flux/unit length $\Phi = NI\mu_0\left(\dfrac{3}{(4+30)/2}\right)$

& flux linkage /unit length = $N^2 I\mu_0\left(\dfrac{3}{(4+30)/2}\right)$

Total flux linkage/unit length= $N^2 I\mu_0\left[\dfrac{5}{4}+\dfrac{3}{(4+30)/2}+\dfrac{10}{30}+\dfrac{1}{3}\dfrac{7}{30}\right]$

Hence, total inductance per unit length=

$$50^2\times 4\pi\times 10^{-7}\left[\frac{5}{4}+\frac{3}{(4+30)/2}+\frac{10}{30}+\frac{1}{3}\frac{7}{30}\right] = 0.00577\,\mathrm{H}$$

3(c)

In large electrical machines, the deep slot is often employed to accommodate high electric loading and also to increase the slot leakage in order to limit the short circuit current. The open-slot is usually employed since the conductors are large and it is easier to place the conductors in the slot.

QUESTION 4:

4(a)

If D is the rotor diameter, L is the rotor axial length, and B, the magnetic loading, is the average airgap flux density.

Force of 1 conductor carrying current I, F = BLI [N]

……... .. 2 ……………………………………, *F = 2BLI* (1 turn)

Force…..N turns……………………………, *F = 2NBLI*

Resultant torque for a rotor diameter D, $T = F \times \frac{D}{2} = 2(NI)BL\frac{D}{2}$

Electric loading is defined as number of current carrying conductors/unit of periphery $Q = \frac{2NI}{\pi D}$, $[Am^{-1}]$

Then $T = \frac{\pi}{2} D^2 LBQ$

Hence the torque density is $\frac{T}{D^2 L} = \frac{\pi}{2} BQ$

4(b) In circumferentially magnetised magnet rotor, two magnets are in parallel excitation (i.e. $\Phi_g = 2\Phi_m$ and $B_g A_g = 2B_m A_m$), while in one closed-loop of magnetic circuit, the flux passes the magnet once and the airgap twice. Hence, if A_g= area of airgap surface/pole and A_m = area of magnet/pole, the actual airgap area in the magnetic circuit is only $0.5A_g$, while the actual airgap length is $2l_g$. Therefore, the expression of airgap field should be modified as:

$$B_g = \frac{B_r}{\frac{0.5A_g}{A_m} + \mu_r \frac{2l_g}{l_m}}$$

When the pole number is high, $A_g << A_m$ and hence the flux focusing effect is significant, it is possible $B_g > B_r$

4(c)

From Gauss law and Ampere's law, without external mmf

$H_m l_m = -H_g l_g$ and $B_m A_m = B_g A_g$

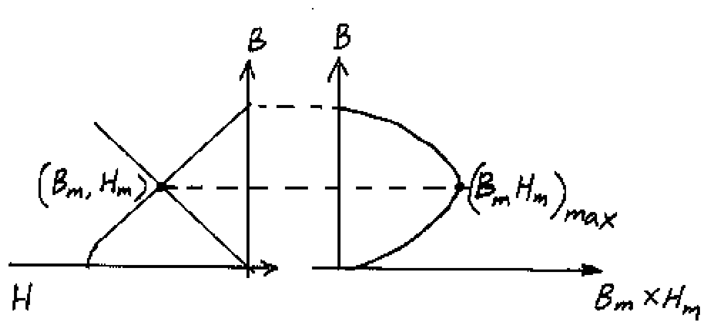
$$(B_m H_m)(A_m l_m) = -\frac{B_g^2}{\mu_0}(A_g l_g)$$

$$(B_m H_m)V_m = -\frac{B_g^2}{\mu_0}V_g \quad \text{and} \quad V_m = -\frac{B_g^2}{\mu_0 (B_m H_m)}V_g$$

$V_m = A_m l_m$ - volume of magnet (A_m and l_m are cross section area and length of magnet)

$V_g = A_g l_g$ - volume of airgap (A_g and l_g are cross section area and length of airgap)

$B_m H_m$ - magnet energy product



- The higher the energy product, the smaller the magnet volume
- To produce a given flux density B_g in a gap of volume V_g, a min. volume of magnet is required when the magnet is designed to work at the point on its B-H curve when $B_m H_m$ is a max.

It is necessary to consider the demagnetization withstand – in general, this open-circuit magnet working point will be too low.

4(d)

The iron losses can be divided into hysteresis and eddy current losses. Hysteresis loss is proportional to $B^{1.5} f$, whilst eddy current loss is proportional to $\frac{1}{\rho} d^2 B^2 f^2$, where B is the flux density in the iron, d and ρ are thickness and resistivity of lamination, f is the operating frequency, $f = \frac{\omega_r p}{2\pi}$ [ω_r = rads^{-1}, p = no of pole pair]. Both flux density and frequency affect hysteresis and eddy current losses. For high frequency operation (high speed and/or high pole number), B may need to be reduced, and very thin laminations, alternatively powdered or ferrite cores may be employed.