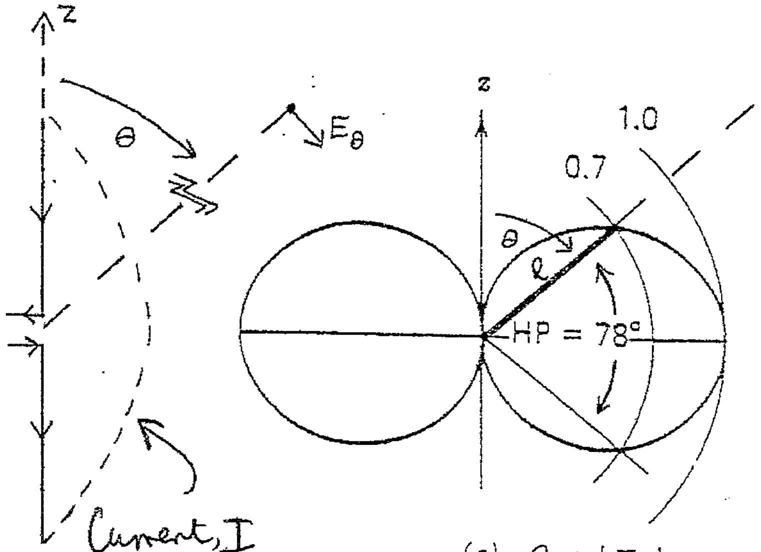
Radiation Patterns of Centre Fed Wires.

SOLUTION TO Q1

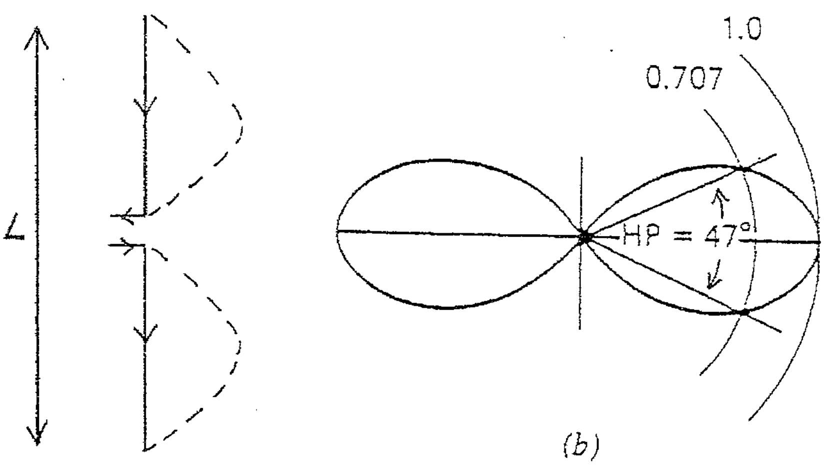
1(a)



(a) $\ell \propto |E_\theta|$

$L = \frac{\lambda}{2}$

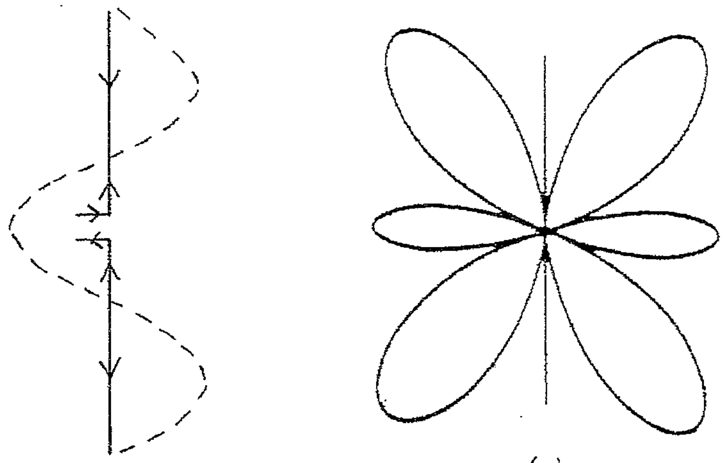
(2)



(b)

$L = \lambda$

(2)



(c)

$L = \frac{3}{2}\lambda$

(2)

1(b)

Radiated power density of an antenna is

$$\underline{P_d} = \frac{1}{2}(\underline{E} \times \underline{H}^*)\, Wm^{-2} \quad (1.1).$$

Note that in the far field (2.1) reduces to

$$P_r = \frac{1}{2} E_\theta H_\phi^* \qquad (1.2).$$

Now, since

$$\frac{E_\theta}{H_\phi} = \eta \qquad (1.3)$$

relates the only field components present, then (1.2) reduces to

$$P_r = \frac{1}{2}\frac{|E_\theta|^2}{\eta}\, Wm^{-2} \qquad (1.4).$$

1

Note (1.4) is a *power density* in Watts per Square Metre, and so to evaluate the total power radiated by the dipole we must multiply P_r by the area it flows through. The total radiated power over a far field sphere is then given by

$$P = \int_0^{2\pi}\int_0^{\pi} P_r r \sin(\theta) d\phi r d\theta \text{ W} \quad (1.5).$$

1 1

The field of a $3\lambda/2$ dipole is given in question (1.1) and is independent of ϕ and so we may rewrite (1.5) thus

$$P = 2\pi r^2 \int_0^{\pi} P_r \sin(\theta) d\theta = \frac{I_0^{\,2}\eta}{4\pi}\int_0^{\pi}\frac{\cos^2\left(\frac{3\pi}{2}\cos(\theta)\right)}{\sin(\theta)} d\theta \text{ W} \quad (1.6).$$

1 1 1

The value of the integral is given as 1.76 and therefore the power radiated is

$$P = \frac{377 I_0^{\,2}}{4\pi} \times 1.76 = 52.8 I_0^{\,2} \text{ W} \qquad (1.7)$$

1

Equating this to power flowing through a fictitious radiation resistance gives,

$$\frac{1}{2} I_0^{\,2} R_r = 52.8 I_0^{\,2} \quad (1.8)$$

1

giving a radiation resistance of

$$R_r = 105.6\,\Omega \qquad (1.9)$$

2

1(c)

For a dipole made from a metal with high conductivity,

$$Z_{in} \approx R_r \qquad (1.10)$$

at resonance, so for the $3\lambda/2$ dipole

$$Z_{in} = 105.6\Omega \qquad (1.11)$$ 2

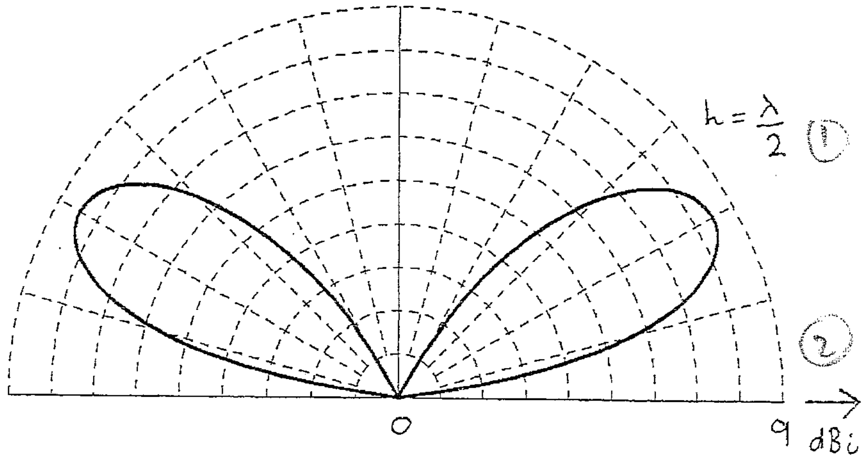
whereas a half wave dipole has

$$Z_{in} = 73.2\Omega \qquad (1.12)$$ 2

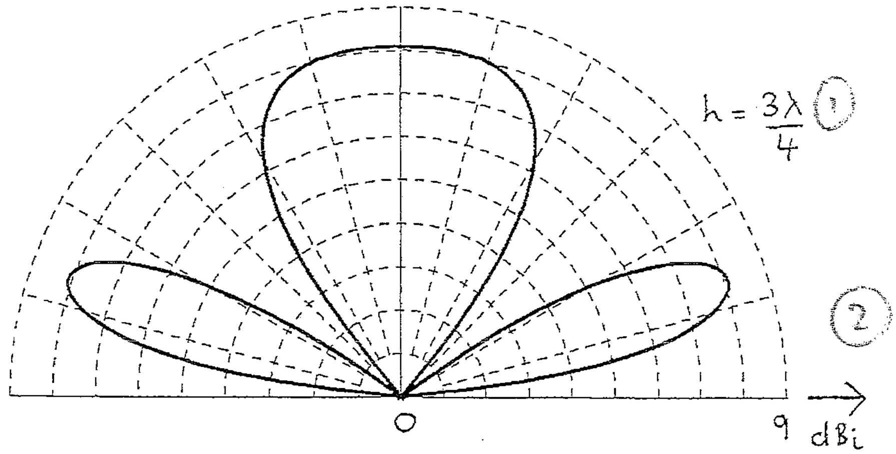
at resonance.

SOLUTION TO QUESTION 2.

2(a)

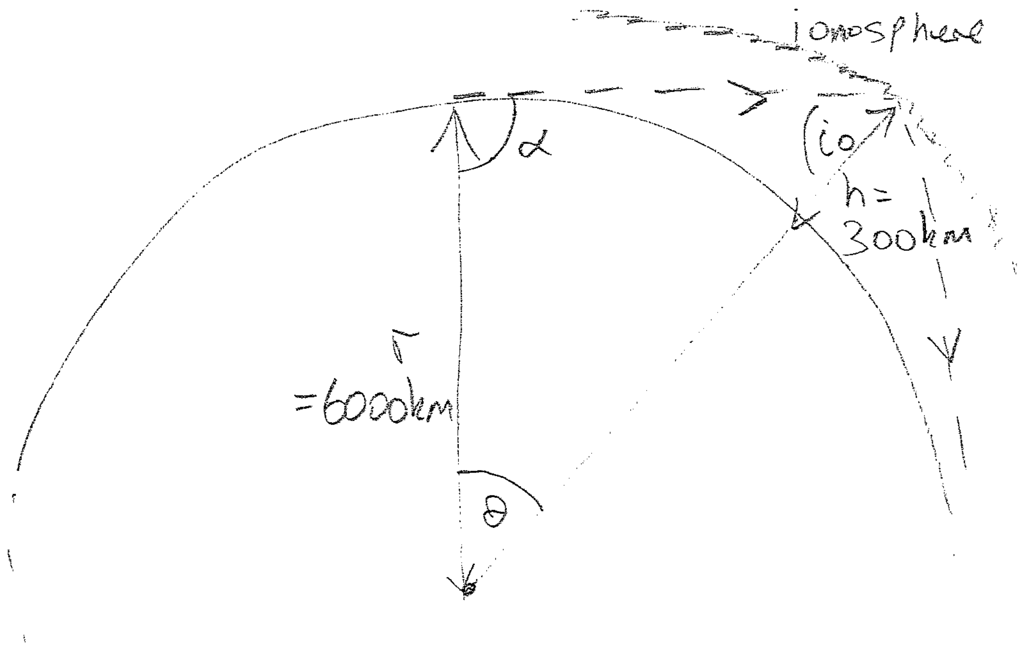


$f =$ 15MHz



$f =$ 20 MHz

2(b)



We need to find θ. The skip distance s is then

$$s = 2r\theta \qquad (2.1)$$

Clearly,

$$\theta = 180 - \alpha - i_o \qquad (2.2)$$

and so the problem resolves into one of finding α and i_o. Well, we have

$$cos(i_o) = \frac{f_c}{f} \qquad (2.3) \quad 1$$

where the critical frequency is given by

$$f_c \approx 9\sqrt{N} = 8MHz \qquad (2.4). \quad 1$$

Using the sine rule,

$$sin(180 - \alpha) = (r + h)\frac{sin(i_o)}{r} = (r + h)\frac{\sqrt{1 - \frac{f_c^2}{f^2}}}{r} \quad 1 \quad (2.5)$$

Thus, at *15MHz*:

$i_o = 57.8^o$, $\alpha = 117.4^o$, $\theta = 4.8^o$, and so the skip distance is
1 1

$$s = 1005.3km \qquad (2.6). \quad 2$$

At *22.5MHz*:

$i_o = 69.2^o$, $\alpha = 101.1^o$, $\theta = 9.7^o$, and so the skip distance is
1 1

$$s = 2032km \ (2.7) \quad 1$$

2(c)

Clearly the skip distance is longer at *22.5MHz*, meaning that for a path of several thousand miles, fewer lossy ground reflections would be required to make the trip at this frequency. 1 However, it is possible that the signal could 'hop over' the remote station at either frequency if it was not approximately an integer number of hops distant. 1 The required launch elevation angles are 27.4^o and 11.1^o at *15MHz* and *22.5MHz* respectively, 1 which approximately corresponds to the position of a major lobe for the *15MHz* dipole, but not for the *22.5MHz* dipole. Thus there will be reduced signal strength into the ionosphere from the latter antenna. 1

Solution to Question 3

3 ~~4~~(a)

$$F_x = \iint_A E_x(x,y)e^{j(k_x x + k_y y)}dxdy \qquad (4.1)$$

where A denotes the aperture area. Since the aperture is circular it is more convenient to use polar co-ordinates, where $x = \rho\cos(\varphi)$, $y = \rho\sin(\varphi)$ and $dxdy = \rho d\rho d\varphi$. Thus

3

$k_x x + k_y y =$

$$k\sin(\theta)\cos(\phi)\rho\cos(\varphi) + k\sin(\theta)\sin(\phi)\rho\sin(\varphi) = k\rho\sin(\theta)\cos(\phi - \varphi) \qquad (4.2)$$

and also since the aperture field is of unit amplitude (4.1) can be written

2

$$F_x = \int_0^{2\pi}\int_0^{\frac{a}{2}} e^{jk\rho\sin(\theta)\cos(\phi-\varphi)}\rho d\rho d\varphi \quad (4.3).$$

2

We need to make a change of variable so that the upper limit of integration in the ρ dimension is 1, so let

$$\ell = \frac{2}{a}\rho \qquad (4.4)$$

1

then (4.3) becomes

$$F_x = \frac{a^2}{4}\int_0^{2\pi}\int_0^{1} e^{jk\frac{a\ell}{2}\sin(\theta)\cos(\phi-\varphi)}\ell d\ell d\varphi \qquad (4.5).$$

2

Using the given question eqns,

$$F_x = \frac{a^2}{4}\int_0^{1} 2\pi J_0\left(k\frac{a\ell}{2}\sin(\theta)\right)\ell d\ell \qquad (4.6)$$

2

$$= \frac{a^2\pi}{2}\frac{J_1\left(\frac{ka}{2}\sin(\theta)\right)}{\frac{ka}{2}\sin(\theta)} \qquad (4.7)$$

3~~4~~(b)

We assume here that the x co-ordinate is aligned with the Clarke belt, with $<\theta,\phi = 0^o, 0^o>$, locating the *Astra* satellite and $<\theta,\phi = 4.3^o, 0^o>$ locating *Kopernikus*. Assuming that $cos(4.3^o) \approx 1$, the relative strength of the interfering signal is given by 1

$$20\,log_{10}\left(\left|\frac{J_1(\frac{ka}{2}sin(4.3^o))}{\frac{ka}{2}sin(4.3^o)}\right| \times \frac{\frac{ka}{2}sin(0^o)}{J_1(\frac{ka}{2}sin(0^o))}\right) \qquad (4.8)$$ 3

where

$$\frac{ka}{2}sin(4.3^o) = 5.4 \qquad (4.9).$$ 2

Hence, (4.8) can be evaluated using question eqns as

$$20\,log_{10}\left(\frac{0.35}{5.4} \times \frac{1}{0.5}\right) dB = -17.75dB \quad (4.10)$$

2

Solution to Question 4

4 **~~3~~(a)**

The time dependences of x and y polarized electric fields may be written

$$E(t) = E_i \cos(\omega t + \varphi_i) \quad (3.1)$$

where $i = x, y$. The magnitude of the electric field at a given instant is then

$$|\underline{E}| = \sqrt{E_x^{\,2} \cos^2(\omega t + \varphi_x) + E_y^{\,2} \cos^2(\omega t + \varphi_y)} \qquad (3.2)$$

(i) If the phase shift $\varphi_x - \varphi_y = n\pi$ then the polarization is *linear*, and is generated by a dipole antenna for example. 2

1

(ii) If $\varphi_x - \varphi_y = (2n-1)\frac{\pi}{2}$ and $|E_x| = |E_y|$ then the polarization is *circular*, and this is generated by orthogonal 'crossed' x-y dipoles for example, each fed with the same power but with a 90^o phase shift. A spiral antenna also generates such a field. 2 1

(iii) If $\varphi_x - \varphi_y = (2n-1)\frac{\pi}{2}$ and $|E_x| \neq |E_y|$ or $\varphi_x - \varphi_y \neq (2n-1)\frac{\pi}{2}$ then the polarization is *elliptical*. Such a field can also be generated using crossed dipoles, feeding different power and phasing to each. 2

4 **~~3~~(b)**

The *axial ratio* (Ψ) is defined as

$$\Psi = \frac{L_1}{L_2} = \frac{|\underline{E}|_{max}}{|\underline{E}|_{min}} \quad (3.3)$$ 1

To determine the max and min values of $|\underline{E}|$ in Eq 3.2 we solve

$$\frac{\partial}{\partial t}|\underline{E}| = 0 \qquad (3.4)$$ 1

to find expressions for the field at the times of the max and min turning points. From Eq 3.4,

$$E_x^{\,2} \sin 2(\omega t + \varphi_x) + E_y^{\,2} \sin 2(\omega t + \varphi_y) = 0 \qquad (3.5)$$ 1

Eq 3.5 can be re-written

Eq 3.5 can be re-written

$$\tan 2\omega t = \frac{-(E_x^2 \sin 2\varphi_x + E_y^2 \sin 2\varphi_y)}{E_x^2 \cos 2\varphi_x + E_y^2 \cos 2\varphi_y} \qquad (3.6)$$ 1

from which

$$\cos 2\omega t = \frac{E_x^2 \cos 2\varphi_x + E_y^2 \cos 2\varphi_y}{H} \qquad (3.7)$$ 1

and

$$\sin 2\omega t = -\frac{E_x^2 \sin 2\varphi_x + E_y^2 \sin 2\varphi_y}{H} \qquad (3.8)$$ 1

and

$$H = \sqrt{E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos 2\Delta\varphi} \qquad (3.9)$$ 1

where

$$\Delta\varphi = \varphi_x - \varphi_y \qquad (3.10)$$

Substitution of Eqs 3.7 and 3.8 into Eq 3.2 then yields

$$|\underline{E}| = \sqrt{\frac{E_x^2}{2} + \frac{E_y^2}{2} + \frac{1}{2}\sqrt{E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos 2\Delta\varphi}} = L_1 \qquad (3.11)$$

An alternative solution to Eq 3.6 is $\tan(2\omega t + \pi)$ which yields 1

$$|\underline{E}| = \sqrt{\frac{E_x^2}{2} + \frac{E_y^2}{2} - \frac{1}{2}\sqrt{E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos 2\Delta\varphi}} = L_2 \qquad (3.12)$$

Thus, Eqs 3.11 and 3.12 represent the max and min field values respectively, whose ratio defines the axial ratio in Eq 3.3.

4 ~~3~~**(c)**

If $\Delta\varphi = 90^o$ Eqs 3.3, 3.11 and 3.12 yield 2

$$\psi = \frac{E_x}{E_y} = 2 \qquad (3.13)$$ 2