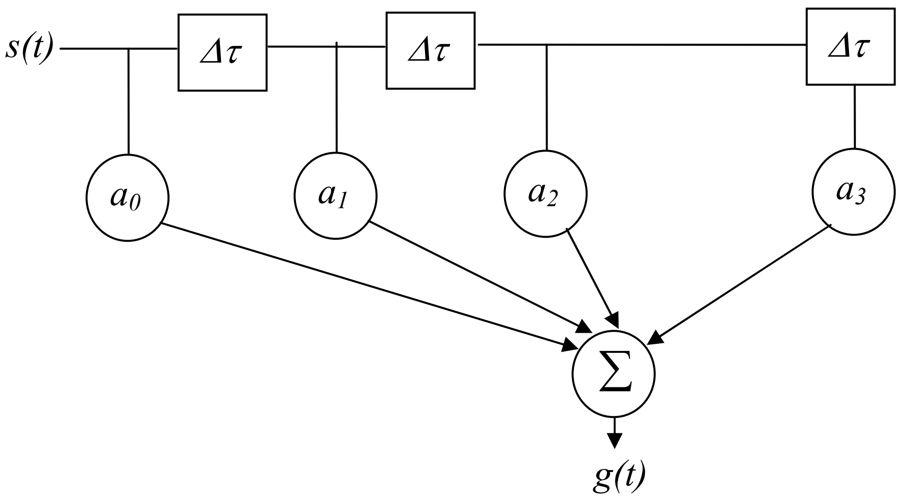
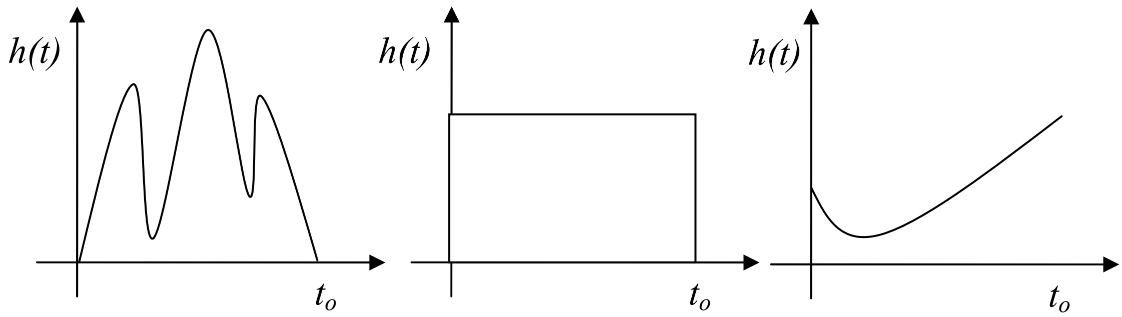
EEE 317 Tutorial answers – Matched filters

1) $h(t) = kx(t_0 - t)$

2)



3)



4) Convolution: $g(t) = \int_{-\infty}^{\infty} h(\tau) i(t-\tau) d\tau$.

Correlation: $g(t) = \int_{0}^{T} h(t) i(t) dt$

5) The time output of a matched filter is defined in terms of the convolution of the filters impulse response, *h(t)* and the incoming signal, *i(t)*

$$g(t) = \int_{-\infty}^{\infty} h(\tau) i(t-\tau) d\tau .$$

The limits of the above integral, while mathematically precise, can be simplified since we know that this is a real system. I.e. we know that nothing has happened at the filter output before we started (t = 0), and also that the filter output cannot be affected by events occurring after the time we are interested in, T, hence we can write,

$$g(T)=\int_0^T h(\tau)i(T-\tau)d\tau\ .$$

Remember that the impulse response function of the filter is $s(t-\tau)$, where $s(t)$ is the waveform which the filter is matched to, so we can write,

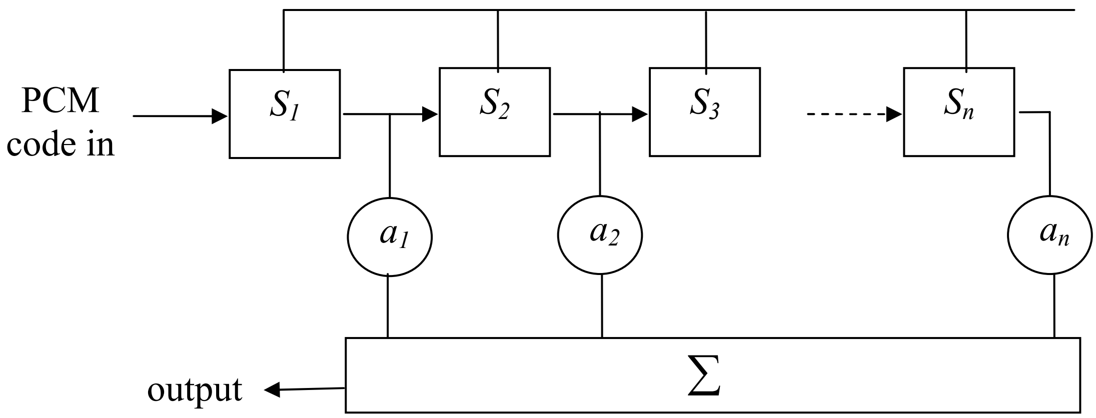
$$g(T)=\int_0^T s(T-\tau)i(T-\tau)d\tau$$

We can simplify the above by letting $\alpha = T - \tau$, hence $d\alpha = -d\tau$, $\tau = 0 \rightarrow \alpha = T$ and $\tau = T \rightarrow \alpha = 0$.

$$g(T)=-\int_T^0 s(\alpha)i(\alpha)d\alpha=\int_0^T s(\alpha)i(\alpha)d\alpha\ .$$

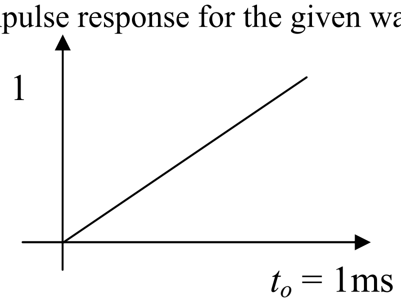
The above is equivalent to $g(T)=\int_0^T s(t)i(t)dt$, which is the correlation of the signal waveform to the received (signal + noise) waveform.

6) A typical PCM matched filter is sketched below. The code words are multiplied by the tap weights and the results added to give an output which is then fed to some control logic to decide as to whether a bit has been received or not. The last bit in (most significant bit) is multiplied by a_1 and the first bit in (least significant bit) is multiplied by a_n.



Typically there will be many of these filters, one for each of the valid codewords. Let us consider a system using four bit codewords, numbered 0 to 15 consequetively. If 4 and then 7 was transmitted the bit strem would be 0**100,0**111. If the reciever was out of sync and thus sampled the bit stream one bit too late then the message receieved would be 1000 (highlighted previously in bold) which is the message for 8. Hence it is vital that such PCM matched filters are properly synchronised.

7) The impulse response for the given waveform is…



The values of the impulse response function at the four equal delay intervals are,

t = 0ms $h(t)$ = 0;

t = 0.33ms $h(t)$ = 0.33;

t = 0.66ms $h(t)$ = 0.66;

t = 1ms $h(t)$ = 1;

From the notes we recall that the tap weights are given by $a_k = h(k\Delta\tau).\Delta\tau$

and as $\Delta\tau$ = 0.33ms we can write down the tap weights directly as,

$a_0 = 0\times0.33\times10^{-3} = 0;$

$a_1 = 0.33\times0.33\times10^{-3} = 1.1\times10^{-4};$

$a_2 = 0.66\times0.33\times10^{-3} = 2.2\times10^{-4};$

$a_3 = 1\times0.33\times10^{-3} = 3.3\times10^{-4};$

8) The integrate and dump matched filter can be used on any generic rectangular waveform. The PCM filter by contrast gives an extra level of matching between signal and filter by being specific to a particular sequence of binary levels. As a result a PCM filter is wholly inappropriate for use for any other digital waveform other than the one it was designed for.

9) To obtain the maximum amount of signal power, one would have to have a transparent pass band filter at the frequencies of interest. To obtain optimum noise performance, by contrast, one would require an opaque blocking filter at the frequencies of interest. The conflict arises because the noise and the signal share the same bandwidth, and so optimising a receiver for one of these concerns will seriously affect the other. Therefore the best any designer can hope for is a compromise. Usually we try to maximise the signal to noise ratio in order to make the signal as easy to detect as possible over and above the noise floor. A matched filter gives the maximum signal to noise ratio possible for a given signal waveform.