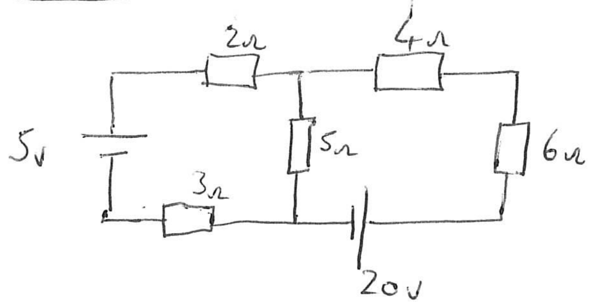
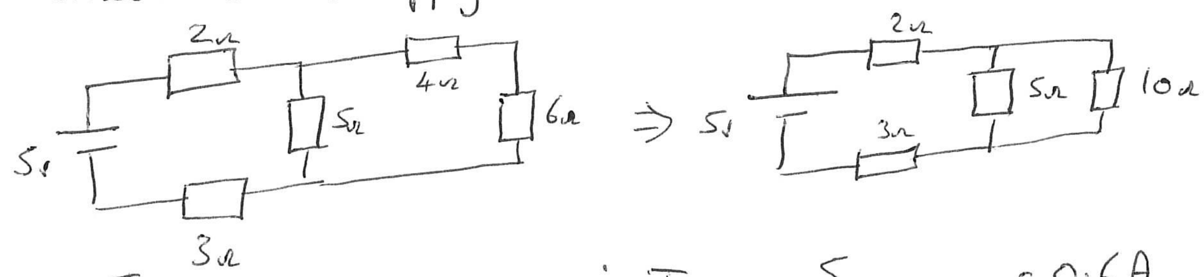
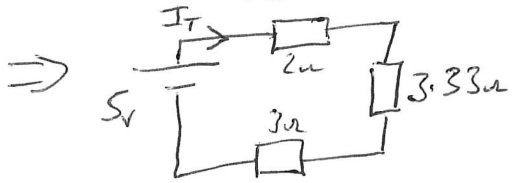
QUESTION 1

(a)



First consider the 5V supply - short out the 20v battery



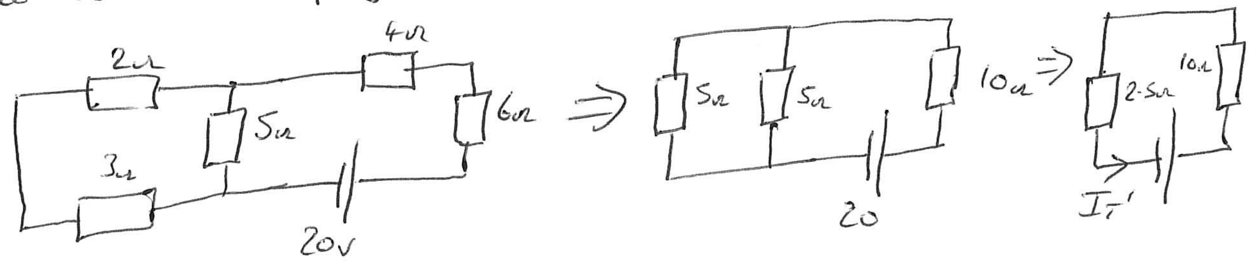


$$\therefore I_T = \frac{5}{2+3+3.33} = 0.6A$$

and hence the current through the 5Ω resistor is

$$I_{5\Omega} = \frac{10}{5+10} \cdot I_T = \frac{10}{15} \times 0.6 = 0.4A \downarrow$$

Now consider the 20v supply and short out the 5v battery:



$$I_T' = \frac{20}{12.5} = 1.6A$$

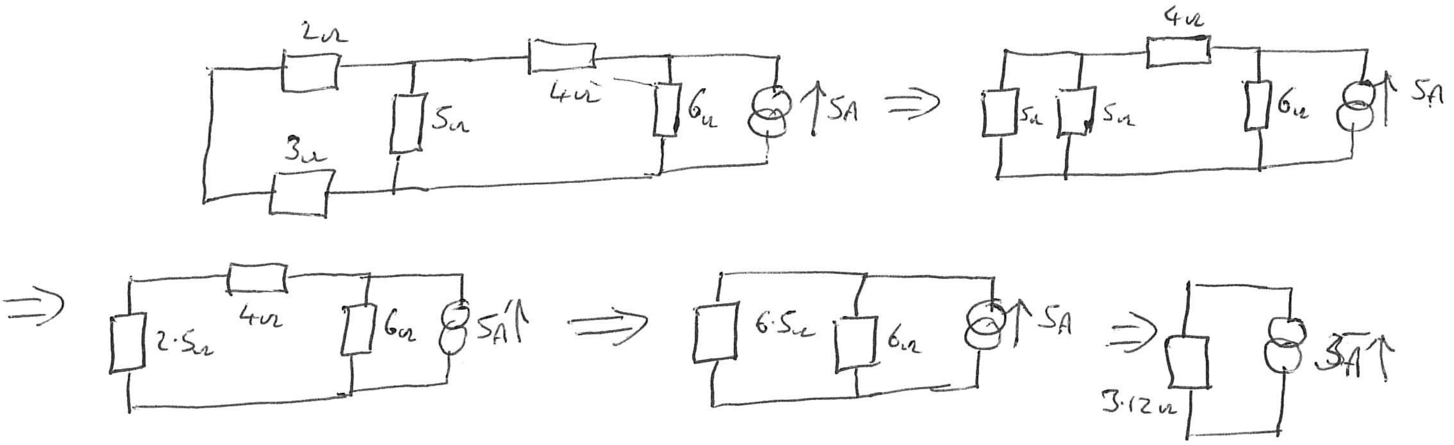
and the current through the 5Ω resistor is

$$I_{5\Omega}' = \frac{5}{5+5} \cdot 1.6 = 0.8A \downarrow$$

By superposition the total current through the 5Ω resistor is:

$$I_{5\Omega\,TOTAL} = 0.4\downarrow + 0.8\downarrow = \underline{\underline{1.2\downarrow}}$$

(b) We can use the analysis from part (a), but now need to consider the effect of the 5 A current source alone:



∴ Voltage across 3·12Ω resistor $= I.R. = 5 \times 3.12 = 15.6\,V$

Hence the current through the 6·5Ω resistor is $15.6/6.5 = 2.4\,A$.

This is the same current flowing through the 2·5Ω resistor, Hence the current flowing through the 5Ω resistor is $1.2\,A \downarrow$.

By superposition the total current flowing through the 5Ω resistor is now:

$$I'_{5\Omega\,TOTAL} = \underset{(\text{from (a)})}{1.2\downarrow} + 1.2\downarrow = \underline{\underline{2.4\,A\downarrow}}$$

c) For Thevenin we need the open circuit voltage which is equal to the voltage across the 6Ω resistor. Using the working from part (a) - for the 5V source current through 6Ω resistor (10Ω branch in network) is

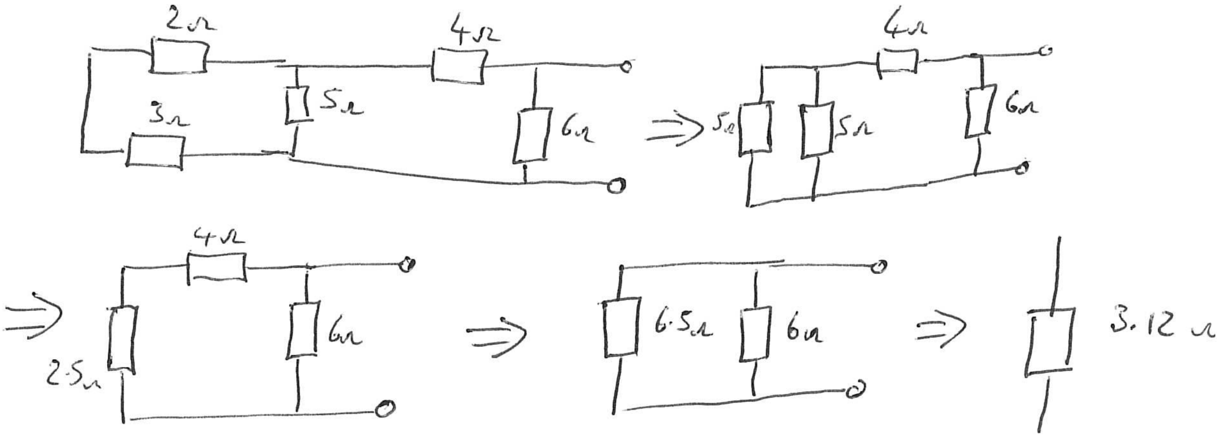
$$I_{6\Omega} = I_T \times \frac{5}{5+10} = 0.6 \times \frac{5}{15} = 0.2\,A\downarrow$$

For the 20V source the current through the 6Ω resistor is equal to $I_T' = 1.6\,A\uparrow$

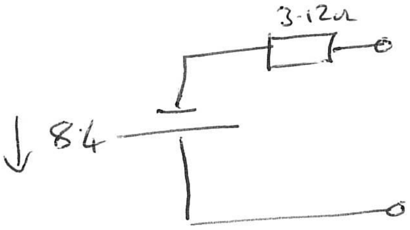
Hence total current through 6Ω resistor is $1.6\uparrow + 0.2\downarrow = 1.4\,A\uparrow$

and the Thevenin voltage is $1.4 \times 6 = 8.4\,V\downarrow$

Resistance of the network:



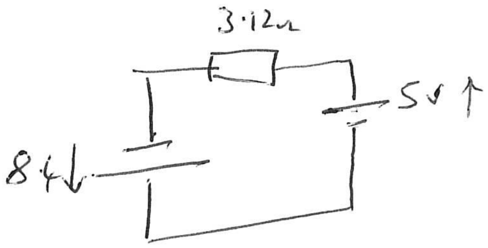
Hence the Thevenin circuit is:



When the load is connected $I = \frac{8.4}{(10+3.12)} = 0.64A \uparrow$ and the

power dissipated $= I^2R = 0.64^2 . 10 = \underline{\underline{4.096W}}$

d. When the rechargeable battery is connected:



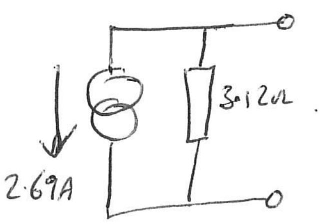
Applying K's voltage law $\quad 8.4 + 5 - 3.12I = 0$

$\therefore I = 4.29A$

The battery will act as a source.

(e) The Norton circuit can be found directly from Thevenin.

$I_N = \frac{V_{TH}}{R_{TH}} = \frac{8.4}{3.12} = 2.69A$



QUESTION 2

(a)(i) The impedance of the circuit is:

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

At resonance the imaginary terms cancel and the impedance becomes purely resistive:

Therefore

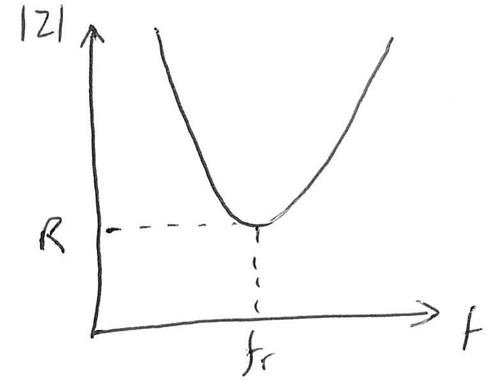
$$\omega L = \frac{1}{\omega C} \quad \text{at resonance}$$

$$\omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

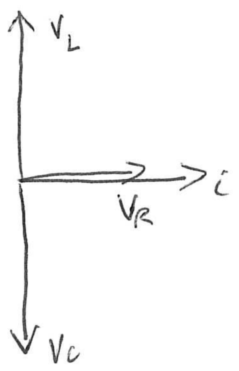
Now $\omega = 2\pi f$ hence

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

(ii)



(iii)



$|V_L| = |V_C|$ at resonance.

QUESTION 2 (CONTINUED)

(iv)

$$Q = \left| \frac{V_L}{V_R} \right| \quad \text{at resonance}$$

$$Q = \frac{I_S \omega L}{I_S R} = \frac{\omega L}{R}$$

however at resonance $\omega = \frac{1}{\sqrt{LC}}$

$$\therefore \quad Q = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \underline{\underline{\frac{1}{R}\sqrt{L/C}}}$$

(v) Using $Q = 6.86$ then since:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow 6.86 = \frac{1}{100}\sqrt{\frac{L}{C}}$$

$$\therefore \; 686^2 = \frac{L}{C} \Rightarrow 470596\,C = L \quad -①$$

Now:

$$f_r = 23200 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow LC = 4.706 \times 10^{-11} \quad ②$$

Substituting for L in ② from ①:

$$470596\,C^2 = 4.706 \times 10^{-11}$$

$$\therefore \; C = 0.01\,\mu F$$

Then back substituting:

$$L = 4.7\,mH.$$

(These are the values used in the Passive Networks lab).

(b) (i) Impedance of the RL branch is:

$$Z_{RL} = R + j\omega L$$

but this is then in parallel with the capacitor:

$$\therefore \frac{1}{Z} = \frac{1}{Z_{RL}} + \frac{1}{1/j\omega C} = \frac{1}{R+j\omega L} + j\omega C = \frac{1+j\omega CR - \omega^2 LC}{R+j\omega L}$$

$$\therefore Z = \frac{R+j\omega L}{1+j\omega CR-\omega^2 LC}$$

(ii) Rationalise to the form $a+jb$ by multiplying through by the complex conjugate of the denominator $(1-\omega^2 LC - j\omega CR)$

$$\therefore Z = \frac{R+j\omega L\,(1-\omega^2 LC - j\omega CR)}{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}$$

The denominator is now real so we need to set the imaginary terms in the numerator to zero.

Multiplying out the numerator:

$$\text{Num} = R - \omega^2 RLC \underline{- j\omega CR^2} + \underline{j\omega L} \underline{- j\omega^3 L^2 C} + \omega^2 LCR$$

Setting the imaginary terms to zero:

$$-\omega CR^2 + \omega L - \omega^3 L^2 C = 0$$

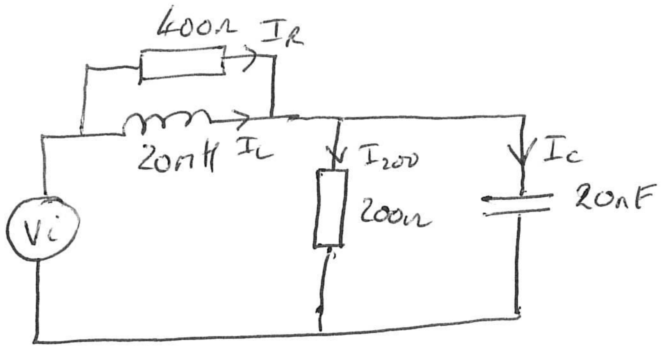
$$\therefore \omega^2 L^2 C = L - CR^2$$

$$\therefore \omega^2 = \frac{L-CR^2}{L^2 C} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

QUESTION 3

(a)



At $t=0^-$ $V_i = -10V$ Dc So:

$\underline{\underline{I_R = 0}}$ (Since there is no V across inductor)

$\underline{\underline{I_c = 0}}$ (since Vi is dc for $t<0$)

$I_L = \frac{V_i}{200} = \frac{-10}{200} = \underline{\underline{-50mA}}$

At $t=0^+$ $V_i = 20V$ so:

I_L is unchanged at $\underline{\underline{-50mA}}$

V_c will remain unchanged at $-10V$ at $t=0^+$ so

$I_R = \frac{20-(-10)}{400} = \underline{\underline{75mA}}$

$I_c = I_L + I_R - I_{200} = -50 + 75 + 50 = \underline{\underline{75mA}}$

(b) Before the switch is closed:

$Q_1 = C_1 V_1$ & $Q_2 = 0$

Then the energy in C_1 is:

$E_{C_1} = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 3\times10^{-6} \times 100^2 = \underline{\underline{15mJ}}$

(No energy stored in C_2)

After closing the switch the capacitors will share the charge equally and the voltages will be equal.

i.e. $Q_1 = Q_{1A} + Q_{2A} \Rightarrow C_1 V_1 = C_1 V_{1A} + C_2 V_{2A}$

and since $V_{1A} = V_{2A}$ then:

$$3\times10^{-6} \times 100 = 3\times10^{-6} V_{1A} + 2\times10^{-6} V_{1A}$$

$$\therefore V_{1A} = V_{2A} = 60V$$

$$\therefore E_T = E_{1A} + E_{2A} = \frac{1}{2} C_1 V_{1A}^2 + \frac{1}{2} C_2 V_{1A}^2 = \frac{1}{2} \times 3\times10^{-6} \times 60^2 + \frac{1}{2} \times 2\times10^{-6} \times 60^2$$

$$= \underline{\underline{9mJ}}$$

Energy lost as heat dissipated in the resistor.

(c) Before the switch is closed there is no voltage across the resistor or capacitor and the current is zero. When the switch is closed V_s appears across R and C, however the voltage across C cannot change instantaneously.

$$V_s = V_R + V_c \text{ at all times}$$

$$V_s = iR + \frac{1}{C}\int_0^t i\,dt$$

differentiating: $$\frac{dV_s}{dt} = \frac{di}{dt}.R + \frac{i}{C}$$

However V_s is constant so $\frac{dV_s}{dt} = 0$

$$\therefore \frac{di}{i} = -\frac{1}{RC}dt$$

integrating: $$\ln(i) = \frac{-1}{RC}t + A$$ (A = constant of integration)

Now when $t = 0^+$ (immediately after the switch is closed) let the current be I_o where:

$$I_o = \frac{V_s}{R}$$

Therefore
$$A = \ln I_0$$

Hence
$$\ln(i) - \ln(I_0) = -\frac{1}{RC}t$$

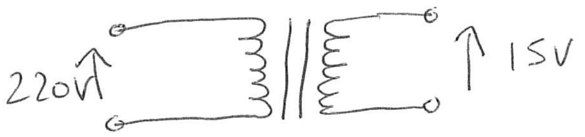
Rearranging and taking antilogs:
$$i = I_0 e^{-t/RC}$$

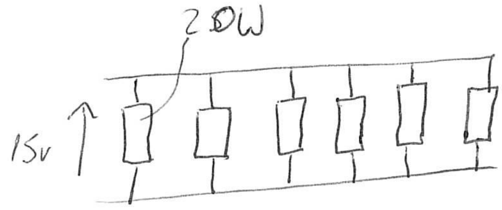
Voltage across the capacitor is:
$$V_C = \frac{1}{C}\int_0^t I_0 e^{-t/RC}\,dt = -\frac{RC}{C}I_0\left[e^{-t/RC}\right]_0^t$$
$$V_C = -I_0 R\left[e^{-t/RC} - 1\right] = I_0 R\left[1 - e^{-t/RC}\right]$$
$$\therefore \underline{\underline{V_C = V_S\left[1 - e^{-t/RC}\right]}}$$

(d) Substituting values in the above equation yields:
$$V_C = 10\left(1 - e^{-\frac{2}{10^5 \cdot 10^{-5}}}\right) = 10\left(1 - e^{-2}\right) = \underline{\underline{8.65\text{V}}}$$

QUESTION 4

(a) (i) Assuming an ideal transformer:





For six 20W bulbs the total load will be $6 \times 20 = 120W$

Since the bulbs are purely resistive then the power-factor is unity and this is also the VA rating.

Transformer rating $= \underline{\underline{120\,VA}}$

The turns ratio is given by $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{220}{15} = \underline{\underline{14 \cdot 67}}$

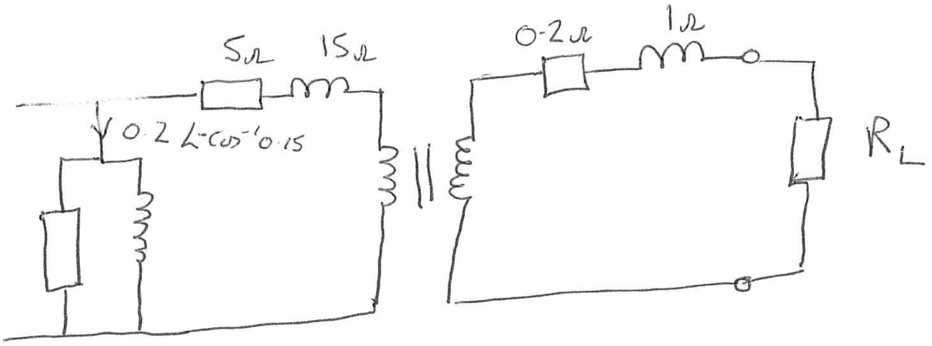
(ii) Since the power-factor = 1 and the transformer is ideal:

$$I_p . V_p = I_S V_s = VA$$

Hence $$I_p = \frac{VA}{V_p} = \frac{120}{220} = \underline{\underline{0 \cdot 545\,A}}$$

$$I_s = \frac{VA}{V_s} = \frac{120}{15} = \underline{\underline{8A}}$$

(b)(i)



First find R_L (the effective resistance of all 6 bulbs):

$P_L = 120\,W \qquad V_s = 15$

$$\therefore R_L = \frac{V_s^2}{P_L} = \frac{15^2}{120} = 1 \cdot 875\,\Omega$$

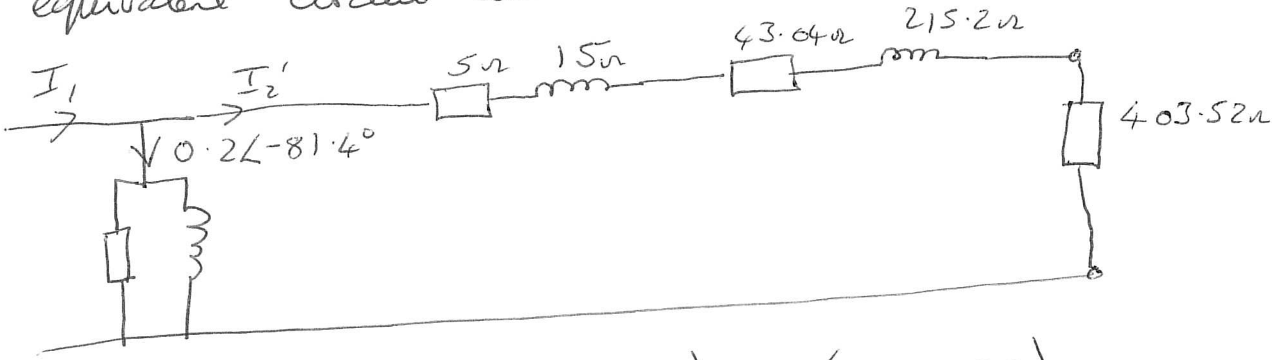
Referring all secondary quantities to the primary side:

$$R_2' = \left(\frac{N_1}{N_2}\right)^2 . R_2 = 14.67^2 \times 0.2 = 43.04\,\Omega$$

$$X_2' = \left(\frac{N_1}{N_2}\right)^2 . X_2 = 14.67^2 \times 1 = 215.2\,\Omega$$

$$R_L' = \left(\frac{N_1}{N_2}\right)^2 . R_L = 14.67^2 \times 1.875 = 403.52\,\Omega$$

The equivalent circuit can now be drawn as:



(ii) $$\therefore Z_2' = (5 + 43.04 + 403.52) + j(15 + 215.2)$$
$$= 451.56 + j\,230.2 \equiv 506.85\angle 27°\,\Omega$$

$$\therefore I_2' = \frac{220\angle 0°}{506.85\angle 27°} = 0.434\angle -27°\ A_{rms}$$

$$I_M = 0.2A \text{ @ } 0.15 \text{ pf lagging}$$
$$= 0.2\angle -81.4°\ A_{rms}$$

$$\therefore I_1 = I_M + I_2' = 0.2\angle -81.4° + 0.434\angle -27°$$
$$= 0.574\angle -43.46°\ A$$

The total input current to the transformer is:

$$\underline{\underline{0.574\angle -43.46°\ A_{rms}}}$$

(iii) The output power of the whole lighting system is:

$$P_L = I_2'^2 R_L' = 0.434^2 \times 403.52 = 76W$$

$\therefore$ Power output from each bulb is $\frac{76}{6} = \underline{\underline{12.67W}}$

Voltage across bulbs (referred to primary side) is:

$$V_B' = I_2' . R_L' = 0.434 \times 403.52 = 175.13V$$

$\therefore$ Actual voltage across each bulb is:

$$V_B = \frac{N_2}{N_1} \times V_1 = \frac{175.13}{14.67} = \underline{\underline{11.94 V_{rms}}}$$

(iv) Copper losses in transformer:

$$P_{LOSS} = I_2'^2 . (R_1 + R_2') = 0.434^2 (5 + 43.04)$$
$$= \underline{\underline{9.05W}}$$

(v) Core losses:

$$V_p . I_M \cos\phi_m = 220 \times 0.2 \times 0.15 = 6.6W$$

(Check $P_{IN} = V_p I_1 \cos\phi = 220 \times 0.574 \times \cos(-43.46)$
$= 91.66W$

$P_{IN} = P_{OUT} + P_{LOSS} = 76 + 9.05 + 6.6 = 91.65W$)

$$\text{Efficiency} = \frac{P_{out}}{P_{IN}} \times 100\% = \frac{76}{91.66} = \underline{\underline{82.9\%}}$$

C. Using $V_{rms} = 4.44 f N \phi_{max}$

$$N_p = \frac{220}{4.44 \times 50 \times 2 \times 10^{-3}} = \underline{\underline{495 \text{ TURNS}}} \qquad N_2 = \frac{15}{4.44 \times 50 \times 2 \times 10^{-3}} = \underline{\underline{34 \text{ TURNS}}}$$

QUESTION 5

(a) Calculate the current drawn by a single induction motor:

$$I_{ph} = I_{Line} = \frac{V\angle 0^\circ}{Z\angle \phi^\circ} = \frac{400\angle 0^\circ/\sqrt{3}}{(3.6+j1.2)}$$

$$= \underline{60.86\angle -18.4^\circ\,A}$$

The power drawn by $\underline{\text{each}}$ motor is:

$$P = \sqrt{3}\,V_L I_L \cos\phi = \sqrt{3}.\,400.\,60.86\cos(-18.4^\circ)$$

$$= 40\,kW$$

Therefore total power drawn by the refrigeration system during opening hours is:

$$P_{TOT} = 2 \times 40 = \underline{\underline{80\,kW}}$$

$$P.f. = \cos -18.4^\circ = \underline{\underline{0.949\ (\text{lagging})}}$$

(b) Calculate the real and reactive power of all loads:

Lighting: $P = 30000\,kW$ $Q = 0\,kVAr$

Refrigeration $P = 80000\,kW$ (part (a))

$$Q = S\sin\phi = \frac{P}{\cos\phi}.\sin\phi = \frac{80000}{0.949}\sin 18.4^\circ$$

$$= 26.6\,kVAr$$

Miscellaneous:

$$P = S\cos\phi = 20000 \times 0.6\quad 12\,kW$$

$$Q = S\sin\phi = 20000 \times 0.8 = 16\,kVAr$$

(i) The total real power is:

$$P_{TOT} = 30000 + 80000 + 12000 = \underline{\underline{122\,kW}}$$

(ii) The total reactive power is:

$$Q_{TOT} = 0 + 26.67 + 16.0 = \underline{\underline{42.67\,kVAr}}$$

(iii) The total apparent power:

$$S_{TOT} = \sqrt{P_{TOT}^2 + Q_{TOT}^2} = \sqrt{122^2 + 42.67^2} = \underline{\underline{129.2\,kVA}}$$

(iv) The overall power factor:

$$P.f. = \frac{P_{TOT}}{S_{TOT}} = \frac{122}{129.2} = \underline{\underline{0.944}}$$

(c) During closing hours:

Lighting: $P = 4000W$ $Q = 0\,VAr$

Refrigeration $P = 40kW$ $Q = 13.3\,kVAr$

Miscellaneous $P = 8000 \times 0.6 = 4800W$

$Q = 8000 \times 0.8 = 6400\,VAr$

$$\therefore P_{TOT\_CLOSED} = 4 + 40 + 4.8 = 48.8\,kW$$

$$\therefore Q_{TOT\_CLOSED} = 0 + 13.3 + 6.4 = 19.7\,kVAr$$

Hence the total kVA when closed is:

$$S_{TOT\_CLOSED} = \sqrt{48.8^2 + 19.7^2} = \underline{\underline{52.63\,kVA}}$$

$$\text{Power factor} = \frac{P_{TOT}}{S_{TOT}} = \frac{48.8}{52.63} = \underline{\underline{0.927\ (\text{lagging})}}$$

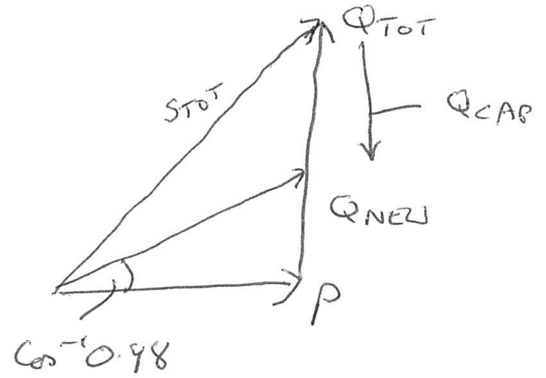
(d) During opening hours:

$P_{TOT} = 122\,kW$

$Q_{TOT} = 42.67\,kVAr$

$S_{TOT} = 129.2\,kVA$

Adding power-factor correction capacitors will not alter the real power drawn, but will effect Q and S.



After the capacitors are added:

$$Q_{NEW} = P\tan\phi = P\tan\left(\cos^{-1}0.98\right)$$
$$= 122\tan\left(\cos^{-1}0.98\right) = 24.8\,kVAr$$

Hence the capacitors must provide:

$$Q_{CAP} = 42.67 - 24.8 = 17.87\,kVAr$$
$$\text{or } \frac{17.97}{3} = 5.9\cancel{8}\,kVAr/\text{phase}.$$

Since the capacitors are star connected.

$$Q_{CAP\_PH} = \frac{V_{PH}^2}{X_C} \Rightarrow X_C = \frac{400^2}{3\times 5960} = 8.95\,\Omega$$

Hence $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi . 50 . 8.95} = \underline{\underline{356\,\mu F}}$

QUESTION 6

(a) (i) Using the equation:

$$NI = S\phi = S.B.A$$

then we first need to find the reluctance, S:

$$S_T = S_{IRON} + S_{AIR}$$

$$= \frac{L_{iron}}{\mu_0 \mu_r . A} + \frac{L_{air}}{\mu_0 A} = \frac{0.478}{\mu_0 . 800 \times 900 \times 10^{-6}} + \frac{0.002}{\mu_0 . 900 \times 10^{-6}}$$

$$= 0.528 \times 10^6 + 1.768 \times 10^6 = 2.296 \times 10^6 H^{-1}$$

Now $$I = \frac{S_T \times B \times A}{N} = \frac{2.296 \times 10^6 \times 1.4 \times 900 \times 10^{-6}}{500}$$

$$= 5.785 A$$

(ii) The self-inductance is given by:

$$L = \frac{N^2}{S_T} = \frac{500^2}{2.296 \times 10^6} = 0.109 H$$

(b)(i). At a frequency of 100 Hz the reactance of the inductor is:

$$X = 2\pi f . L = 2\pi . 100 . 0.109 = 68.5\Omega$$

Hence the impedance, Z is:

$$Z = R + jX = 30 + j68.5 = 74.78 \angle 66.35° \Omega$$

and the current is 5.785 (max value)

$$\therefore V_{PK} = I.Z = 5.785 \times 74.78 = 432.6 V_{PK}$$

$$\equiv 305.9 V_{RMS}$$

(ii) The average power dissipated in the coil is

$$P = I^2_{RMS} \cdot R = \frac{I_{PK}^2}{2} \cdot R = \frac{5.785^2}{2} \times 30 = \underline{\underline{502W}}$$

(iii) Peak energy stored in the inductor is:

$$E = \frac{1}{2} L I_{PK}^2 = \frac{1}{2} \times 0.109 \times 5.785^2 = \underline{\underline{1.824J}}$$

(c)(i) If the peak current is 8A and the flux density remains at 1.4T then

$$S_{TNEW} = \frac{NI}{B.A} = \frac{500 \times 8}{1.4 \times 900 \times 10^{-6}} = 3.174 \times 10^6 H^{-1}$$

$$\therefore S_{IRONNEW} + S_{AIRNEW} = S_{TNEW}$$

Let the gap length be g:

$$\frac{1}{\mu_0 A}\left(\frac{0.48-g}{800} + g\right) = 3.174 \times 10^6$$

$$\therefore 799g = (3.174 \times 10^6 \times \mu_0 \times 900 \times 10^{-6} \times 800) - 0.48$$

$$\therefore \underline{\underline{g = 3mm}}$$

(ii) The new self inductance is:

$$L_{NEW} = \frac{N^2}{S_{TNEW}} = \frac{500^2}{3.174 \times 10^6} = \underline{\underline{78.8mH}}$$

QUESTION 6 (CONTINUED)

(iii) If the resistance of the coil is neglected then:

$$V_{RMS} = 4.44 . f . N . \phi_{max} = 4.44 f N B . A$$

$$= 4.44 \times 100 \times 500 \times 1.4 \times 900 \times 10^{-6}$$

$$= \underline{\underline{279.7 \, V_{RMS}}}$$