Tutorial Sheet 1 – Magnet and Inductance

1.1 Terms

ϕ: Magnetic Flux
B: Magnetic Flux density
S: Reluctance
μ_0: Permeability of Free Space $4\pi \times 10^{-7}$
μ_r: Relative Permeability

F: Magnetomotive Force

1.2 Equations

$$F = NI = \phi S = BAS$$

$$S = \frac{l}{\mu_0 \mu_r A}$$

$$S_{series} = S_1 + S_2 + \ldots$$

$$S_{parallel} = \frac{1}{1/S_1 + 1/S_2 + \ldots}$$

$$L = N^2/S$$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$V = IR$$

$$P = I^2 R$$

Tutorial Sheet – No 1 Answers

1 The reluctance may be calculated from:

$$S = \frac{l}{\mu_0 \mu_r A}$$

where l is the length of the flux path
A is cross-sectional area through which the flux is passing
μ_0 is the permeability of free space
μ_r is the relative permeability

Hence for the piece of steel having the rectangular cross-section:

$$S_{Rect} = \frac{l}{\mu_0 \mu_r A} = \frac{20 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 800} \times \frac{1}{\left(5 \times 10^{-2}\right)^2} = \mathbf{7.96 \times 10^4\ H^{-1}}$$

and for the steel with the circular cross-section:

$$S_{Cir} = \frac{l}{\mu_0 \mu_r A} = \frac{20 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 800} \times \frac{4}{\pi \times \left(5 \times 10^{-2}\right)^2} = \mathbf{1.01 \times 10^5\ H^{-1}}$$

The mmf is given by:

$$F = \phi \times S = B \times A \times S = B \times A \times \frac{l}{\mu_0 \mu_r A} = \frac{Bl}{\mu_0 \mu_r}$$

which is independent of the cross-sectional area. Thus the mmf across each piece of steel is the same and is equal to:

$$F = \frac{Bl}{\mu_0 \mu_r} = \frac{0.6 \times 20 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 800} = \mathbf{119.4\ AT\ (Amp\ Turns)}$$

2 For the two reluctances in series the total reluctance is given by:

$$S_{Tot} = S_{Rect} + S_{Cir} = 7.96 \times 10^4 + 1.01 \times 10^5 = \mathbf{1.81 \times 10^5\ H^{-1}}$$

The mmf is given by:

$$F = \phi \times S = 10 \times 10^{-5} \times 1.81 \times 10^5 = \mathbf{18.1\ AT}$$

3 For the two reluctances in parallel the total reluctance is given by:

$$\frac{1}{S_{Tot}} = \frac{1}{S_{Rect}} + \frac{1}{S_{Cir}}$$

or:

$$S_{Tot} = \frac{1}{\frac{1}{S_{Rect}} + \frac{1}{S_{Cir}}} = \frac{S_{Rect} \times S_{Cir}}{S_{Rect} + S_{Cir}} = \frac{7.96 \times 10^4 \times 1.01 \times 10^5}{7.96 \times 10^4 + 1.01 \times 10^5} = \mathbf{0.445 \times 10^5\ H^{-1}}$$

and the mmf is given by:

$$F = \phi \times S = 12 \times 10^{-4} \times 0.445 \times 10^5 = \mathbf{53.4\ AT}$$

4 Each quarter of the toroid is made of a different material, and since the total length of the flux path is $\pi \times D$ (where D is the diameter of the toroid) it follows that:

$$S_{Tot} = \frac{l}{\mu_0 \mu_r A} = \frac{l}{4 \mu_0 A} \times \left(\frac{1}{80} + \frac{1}{250} + \frac{1}{600} + \frac{1}{1200} \right)$$

but:

$$F = N \times I = \phi \times S_{Tot}$$

which may be rearranged to give an expression for the current:

$$I=\frac{\phi\times S_{Tot}}{N}=\frac{B\times A\times S_{Tot}}{N}=\frac{B\times A}{N}\times\frac{l}{4\mu_0 A}\times\left(\frac{1}{80}+\frac{1}{250}+\frac{1}{600}+\frac{1}{1200}\right)$$

It can be seen that the area cancels and the current is:

$$I=\frac{B}{N}\times\frac{l}{4\mu_0}\times\left(\frac{1}{80}+\frac{1}{250}+\frac{1}{600}+\frac{1}{1200}\right)=\frac{1}{100}\times\frac{\pi\times 0.3}{4\times 4\times\pi\times 10^{-7}}\times 0.019 \textbf{ = 35.6A}$$

The mmf across each section may be obtained from:

$$F_1=\phi\times S_1=B\times A\times\frac{l}{4\mu_0 A}\times\frac{1}{\mu_{r1}}=\frac{B\times l}{4\mu_0\mu_{r1}}$$

For the section where, $\mu_r = 80$ $F_1=\dfrac{B\times l}{4\mu_0\mu_{r1}}=\dfrac{1\times\pi\times 0.3}{4\times 4\times\pi\times 10^{-7}\times 80}$ **= 2344 AT**

For the section where, $\mu_r = 250$ $F_1=\dfrac{B\times l}{4\mu_0\mu_{r1}}=\dfrac{1\times\pi\times 0.3}{4\times 4\times\pi\times 10^{-7}\times 250}$ **= 750 AT**

For the section where, $\mu_r = 600$ $F_1=\dfrac{B\times l}{4\mu_0\mu_{r1}}=\dfrac{1\times\pi\times 0.3}{4\times 4\times\pi\times 10^{-7}\times 600}$ **= 312.5 AT**

For the section where, $\mu_r = 1200$ $F_1=\dfrac{B\times l}{4\mu_0\mu_{r1}}=\dfrac{1\times\pi\times 0.3}{4\times 4\times\pi\times 10^{-7}\times 1200}$ **= 156.3 AT**

5 Since:

$$F=N\times I=\phi\times S=B\times A\times\frac{l}{\mu_0\mu_r A}=\frac{Bl}{\mu_0\mu_r}$$

then:

$$N=\frac{Bl}{\mu_0\mu_r}\times\frac{1}{I}=\frac{0.28\times\pi\times 20\times 10^{-2}}{4\times\pi\times 10^{-7}\times 1000}\times\frac{1}{2.8} \textbf{ = 50 Turns}$$

The self-inductance is given by:

$$L=\frac{N^2}{S}=\frac{N^2\mu_0\mu_r A}{l}=\frac{50^2\times 4\times\pi\times 10^{-7}\times 1000\times 10\times 10^{-4}}{\pi\times 20\times 10^{-2}} \textbf{ = 5mH}$$

6 Using the formula derived in the previous question:

$$L=\frac{N^2}{S}=\frac{N^2\mu_0\mu_r A}{l}=\frac{250^2\times 4\times\pi\times 10^{-7}\times 1000\times 10\times 10^{-4}}{\pi\times 10\times 10^{-2}} \textbf{ = 0.25H}$$

7 Since:

$$F=N\times I=\phi\times S=B\times A\times\frac{l}{\mu_0\mu_r A}=\frac{Bl}{\mu_0\mu_r}$$

as shown previously, and the coil current is given by:

$$I=\frac{Bl}{\mu_0\mu_r}\times\frac{1}{N}=\frac{1\times 4\times 10\times 10^{-2}}{4\times\pi\times 10^{-7}\times 1000\times 1000} \textbf{ = 0.32A}$$

(note the total length of the magnetic circuit is 4 × 10cm as there are 4 sides).

8 In this example the magnetic circuit consists of two parts, the iron and the airgap created by the 1mm wide slot. The total reluctance of the magnetic circuit is the sum of the reluctance of the iron part of the circuit and the air part since they are in series:

$$S_{TOT} = S_{Iron} + S_{Air}$$

therefore, since the cross-sectional area of the airgap is assumed equal to that of the iron:

$$S_{TOT} = \frac{l_{Iron}}{\mu_0 \mu_r A} + \frac{l_{Air}}{\mu_0 A} = \frac{1}{4 \times \pi \times 10^{-7} \times A} \times \left(\frac{(0.4 - 0.001)}{1000} + \frac{0.001}{1} \right)$$

But the current is given by (note A again cancels):

$$I = \frac{\phi \times S}{N} = \frac{B \times A}{N} \times \frac{1}{4 \times \pi \times 10^{-7} \times A} \times \left(3.99 \times 10^{-4} + 10^{-3}\right) = \mathbf{1.1A}$$

9 The first thing to note in this question is that there are 2 main flux paths, as indicated on the diagram. Certain assumptions have to be made in this problem:

- there is no leakage flux (all flux leaving the end of the plunger crosses the airgap horizontally)
- there is no airgap between the core and the plunger at the right hand side
- an approximation has to be made as to the length of the two flux paths in the iron core. Both are assumed to be of length 15+10/2 = 20cm since we are not given any further details of the dimensions of the plunger, other than its area
- The area of the airgap is assumed equal to that of the plunger

Firstly the reluctance of the circuit needs to be calculated for both cases of the plunger being fully out (as shown in the figure) and fully in (when there is no airgap beween the end of the plunger and the iron core at the left hand side).

$$S_{Out} = S_{Core} + S_{Pl\_Out} + S_{Air}$$

$$S_{In} = S_{Core} + S_{Pl\_In}$$

However the reluctance of the core is that of the upper and lower limbs in parallel. Since both limbs are identical, the total reluctance of the core will be equal to half the reluctance of a single limb:

$$S_{Core} = \frac{1}{2} \times \frac{l_{Core}}{\mu_0 \mu_r A_{core}} = \frac{1}{2} \times \frac{20 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 500 \times 10 \times 10^{-4}} = 1.59 \times 10^5 \text{ H}^{-1}$$

Now calculate the reluctance of the remaining components:

$$S_{Pl\_Out} = \frac{l_{Pl\_Out}}{\mu_0 \mu_r A_{Pl}} = \frac{11 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 500 \times 5 \times 10^{-4}} = 3.5 \times 10^5 \text{ H}^{-1}$$

$$S_{Pl\_In} = \frac{l_{Pl\_In}}{\mu_0 \mu_r A_{Pl}} = \frac{13 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 500 \times 5 \times 10^{-4}} = 4.14 \times 10^5 \text{ H}^{-1}$$

$$S_{Air} = \frac{l_{Air}}{\mu_0 A_{Pl}} = \frac{2 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 5 \times 10^{-4}} = 3.18 \times 10^7 \text{ H}^{-1}$$

Combining these elements:

$$S_{Out} = 1.59 \times 10^5 + 3.5 \times 10^5 + 3.18 \times 10^7 = 3.23 \times 10^7 \text{ H}^{-1}$$

$$S_{In} = 1.59 \times 10^5 + 4.14 \times 10^5 = 5.73 \times 10^5 \text{ H}^{-1}$$

The inductance can now be calculated for each position from:

$$L = \frac{N^2}{S}$$

so:

$$L_{Out} = \frac{N^2}{S_{Out}} = \frac{1000^2}{3.23 \times 10^7} = 30.95\text{mH}$$

$$L_{In} = \frac{N^2}{S_{In}} = \frac{1000^2}{5.73 \times 10^5} = 1.745\text{H}$$

The impedance can then be calculated for each position from:

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

so:

$$|Z_{Out}| = \sqrt{120^2 + (2\pi \times 50 \times 30.95 \times 10^{-3})^2} = 120.4\Omega$$

$$|Z_{In}| = \sqrt{120^2 + (2\pi \times 50 \times 1.745)^2} = 561.2\Omega$$

Now the current in each position may be obtained:

$$|I_{Out}| = \frac{V}{|Z_{Out}|} = \frac{240}{120.4} = 1.99\text{A}_{rms}$$

$$|I_{In}| = \frac{V}{|Z_{In}|} = \frac{240}{561.2} = 0.427\text{A}_{rms}$$

The power dissipated in the 120Ω resistance can be calculated as:

$$P_{Out} = I_{Out}^2 \times R = 1.99^2 \times 120 = 475.2\text{W}$$

$$P_{In} = I_{In}^2 \times R = 0.427^2 \times 120 = 21.88\text{W}$$

Therefore the ratio of the powers is:

$$ratio = \frac{P_{Out}}{P_{In}} = \frac{475.2}{21.88} = \mathbf{21.7}$$