EEE118: Background Knowledge Problem Sheet Solutions

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Question 1

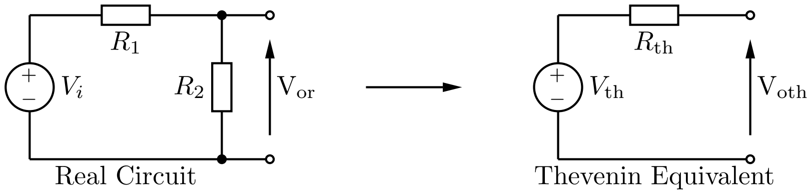

Figure 1: Left: A 'real' circuit (potential divider). Right: Thévenin equivalent.

It is necessary to find the thévenin source voltage and thévenin resistance which will make the real circuit indistinguishable from the thévenin equivalent circuit. i.e.

$$\text{Open circuit voltage of real} = \text{Open circuit voltage of thévenin}$$

$$\text{Short circuit current of real} = \text{Short circuit current of thévenin}$$

$$V_{or} = V_i \frac{R_2}{R_1 + R_2} = V_{\text{oth}}, \text{ for equivalence} = V_{\text{th}}$$

$$I_{sc} = \frac{V_i}{R_1} = \frac{V_{\text{th}}}{R_{\text{th}}} \text{ for equvalence}$$

So

$$R_{\text{th}} = \frac{V_{\text{th}} R_1}{V_i} = \frac{R_1}{V_i} \cdot V_i \frac{R_2}{R_1 + R_2} = R_1 \,//\, R_2$$

i.e.

$$V_{\text{th}} = V_i \frac{R_2}{R_1 + R_2} \text{ and } R_{\text{th}} = \frac{R_1 R_2}{R_1 + R_2}$$

Question 2

The loop analysis is shown in Fig. 2.

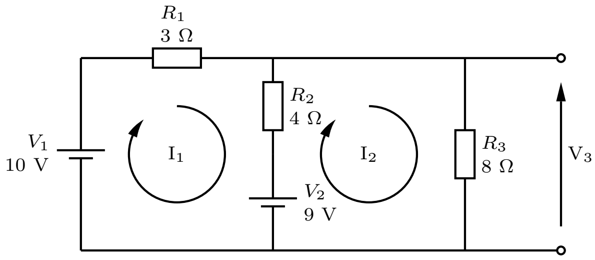

Figure 2: Question 2 with loops shown.

$$10 = I_1\,R_1 + (I_1 - I_2)\;R_2 + 9 \tag{1}$$

$$9 = R_2\;(I_2 - I_1) + R_3\,I_2 \tag{2}$$

$$V_3 = R_3\,I_2 \tag{3}$$

expanding (1)

$$10 = 3\,I_1 + 4\,I_1 - 4\,I_2 + 9 \tag{4}$$

or

$$1 = 7\,I_1 - 4\,I_2 \tag{5}$$

expanding (2)

$$9 = 4\,I_2 + 4\,I_1 + 8\,I_2 \tag{6}$$

or

$$9 = 12\,I_2 - 4\,I_1 \tag{7}$$

eliminating I_2 from (5) and (7) gives

$$9 = 12\left[\frac{7\,I_1 - 1}{4}\right] - 4\,I_1 \tag{8}$$

$$9 = 21\,I_1 - 3 - 4\,I_1 \tag{9}$$

or

$$I_1 = \frac{12}{17} = 0.706\ \text{A} \tag{10}$$

using (5),

$$1 = 7 \times \frac{12}{17} - 4\,I_2 \tag{11}$$

$$I_2 = 0.985\ \text{A} \tag{12}$$

using (3)

$$V_3 = 8\, I_2 = 7.88 \text{ V} \tag{13}$$

Superposition to find V_3

$$V_{3\ (10\ \text{V})} = V_1 \frac{R_2 \,//\, R_3}{R_1 + R_2 \,//\, R_3} = 10 \frac{{}^{32}/_{12}}{3 + {}^{32}/_{12}} = \frac{10\, {}^{8}/_{3}}{{}^{17}/_{3}} \tag{14}$$

$$= \frac{80}{17} \text{ V} \tag{15}$$

$$V_{3\ (9\ \text{V})} = V_2 \frac{R_1 \,//\, R_3}{R_2 + R_1 \,//\, R_3} = 9 \frac{{}^{24}/_{11}}{4 + {}^{24}/_{11}} = \frac{9\, {}^{24}/_{11}}{{}^{68}/_{11}} \tag{16}$$

$$= \frac{54}{17} \text{ V} \tag{17}$$

$$V_{3\,\text{Tot}} = V_{3\ (10\ \text{V})} + V_{3\ (9\ \text{V})} = \frac{134}{17} \text{ V} = 7.88 \text{ V} \tag{18}$$

Superposition to find I_1

$$I_{1\,(10\ \text{V})} = \frac{V_1}{(R_1 + R_2 \,//\, R_3)} = \frac{10}{3 + {}^{8}/_{3}} = \frac{30}{17} \text{ A} \tag{19}$$

$$I_{1\,(9\ \text{V})} = \frac{-V_3}{R_1} = -\frac{1}{R_1} \frac{9\, {}^{24}/_{11}}{4 + {}^{24}/_{11}} = -\frac{1}{3}\, 9\, \frac{6}{17} \tag{20}$$

$$= -\frac{18}{17} \tag{21}$$

$$\therefore I_{1\ \text{Tot}} = I_{1\,(10\ \text{V})} + I_{1\,(9\ \text{V})} = \frac{30}{17} - \frac{18}{17} = 0.706 \text{ A} \tag{22}$$

To find the Norton equivalent first short circuit the output terminals and calculate the current.

$$I_{\text{SC}\ (10\ \text{V})} = \frac{10}{3} = 3.33 \text{ A} \tag{23}$$

$$I_{\text{SC}\ (9\ \text{V})} = \frac{9}{4} = 2.25 \text{ A} \tag{24}$$

$$\therefore I_{\text{SC}\ \text{Tot}} = \frac{10}{3} + \frac{9}{4} = \frac{67}{12} = 5.58 \text{ A} \tag{25}$$

then calculate the parallel resistance which is given by ${}^{V_3}/_{I_\text{N}}$

$$= \frac{{}^{134}/_{17}}{{}^{67}/_{12}} = \frac{12 \times 134}{67 \times 17} = \frac{24}{17} = 1.41\ \Omega \tag{26}$$

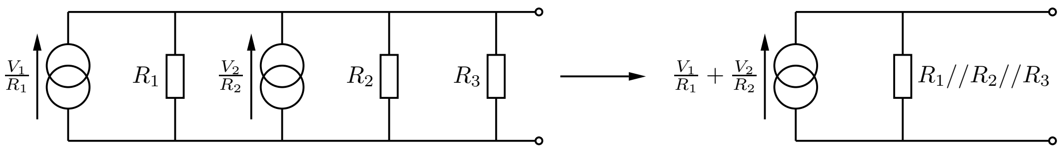

Figure 3: Alternative approach to finding the Norton equivalent.

One could also have transformed the limbs of the original circuit and then summed (see Fig. 3).

To find the value of V_2 that would make $I_1 = 0$, one can make use of the superposition process that was used to find I_1 but with 9 V replaced by V_2.

$$I_{1\ \mathrm{Tot}} = I_{1\ (10\ \mathrm{V})} + I_{1\ (\mathrm{V}_2)} = \frac{30}{17} - V_2\,\frac{2}{17} \tag{27}$$

and $I_{1\ \mathrm{Tot}} = 0$ is required, so

$$\frac{30}{17} - \frac{V_2 \times 2}{17} = 0 \text{ or } V_2 = \frac{30}{2} = 15\ \mathrm{V} \tag{28}$$

Question 3

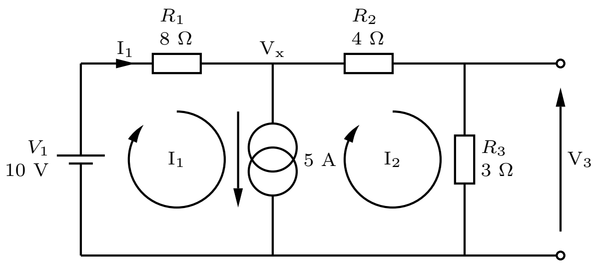

Figure 4:

The circuit under examination in question 3 is shown in Fig. 4. If loops are used to solve this question it is necessary to define a variable V_x for the unknown node voltage.

$$10 = I_1\,R_1 + V_x \tag{29}$$

$$V_x = I_2\,R_2 + I_2\,R_3 \tag{30}$$

$$5 = I_1 - I_2 \tag{31}$$

eliminating V_x from (29) and (30)

$$10 = I_1\,R_1 + I_2\,R_2 + I_2\,R_3 \tag{32}$$

$$= 8\,I_1 + 7\,I_2 \tag{33}$$

using (31) to eliminate I_2

$$= 8\,I_1 + 7\,(I_1 - 5) = 15\,I_1 - 35 \tag{34}$$

$$I_1 = \frac{35+10}{15} = \frac{45}{15} = 3 \text{ A} \tag{35}$$

then using (31)

$$I_2 = -5 + I_1 = -2 \text{ A} \tag{36}$$

$$\therefore\ V_3 = I_2\,R_3 = -6\ V \tag{37}$$

Using superposition to find I_1

$$I_{1\,(10\text{ V})} = \frac{10}{(8+4+3)} = \frac{2}{3} \text{ A} \tag{38}$$

$$I_{1\,(5\text{ A})} = -\frac{V_x}{R_1} = -\frac{(-5\,(R_2+R_3)\,//R_1)}{R_1} \tag{39}$$

$$= \frac{5\,(R_2+R_3)}{R_1+R_2+R_3} = \frac{5\times 7}{15} = \frac{7}{3} \tag{40}$$

$$\therefore\ I_{1\ \text{Tot}} = I_{1\,(10\text{ V})} + I_{1\,(5\text{ A})} = \frac{2}{3} + \frac{7}{3} = 3 \text{ A} \tag{41}$$

To find V_3

$$V_{3\ (10\text{ V})} = 10 \cdot \frac{R_3}{R_1+R_2+R_3} = 10 \cdot \frac{3}{15} = 2 \text{ V} \tag{42}$$

$$V_{3\ (5\text{ A})} = V_x \cdot \frac{R_3}{R_2+R_3} = -5\ (R_2+R_3)\,//R_1 \cdot \frac{R_3}{R_2+R_3} \tag{43}$$

$$= -5 \cdot \frac{56}{15} \cdot \frac{3}{7} = -8 \text{ V} \tag{44}$$

$$V_{3\ \text{Tot}} = V_{3\ (10\text{ V})} + V_{3\ (5\text{ A})} = 2 - 8 = -6 \text{ V} \tag{45}$$

For the Thévenin equivalent circuit, V_{TH} is V_3 (by definition). To find R_{TH} either look into V_3's terminals with V_1 replaced by 0 Ω and I by ∞ Ω and work out the resistance or work out the current that would flow through a short circuit place across the V_3 terminals and use $R_{\text{TH}} = \frac{V_3}{I_{\text{SC}}}$.

$$V_{\text{TH}} = -6 \text{ V} \tag{46}$$

$$R_{\text{TH}} = R_3\,//\,(R_1+R_2) = \frac{36}{15} = 2.4\ \Omega \tag{47}$$

Yet another possibility is to perform successive transformations

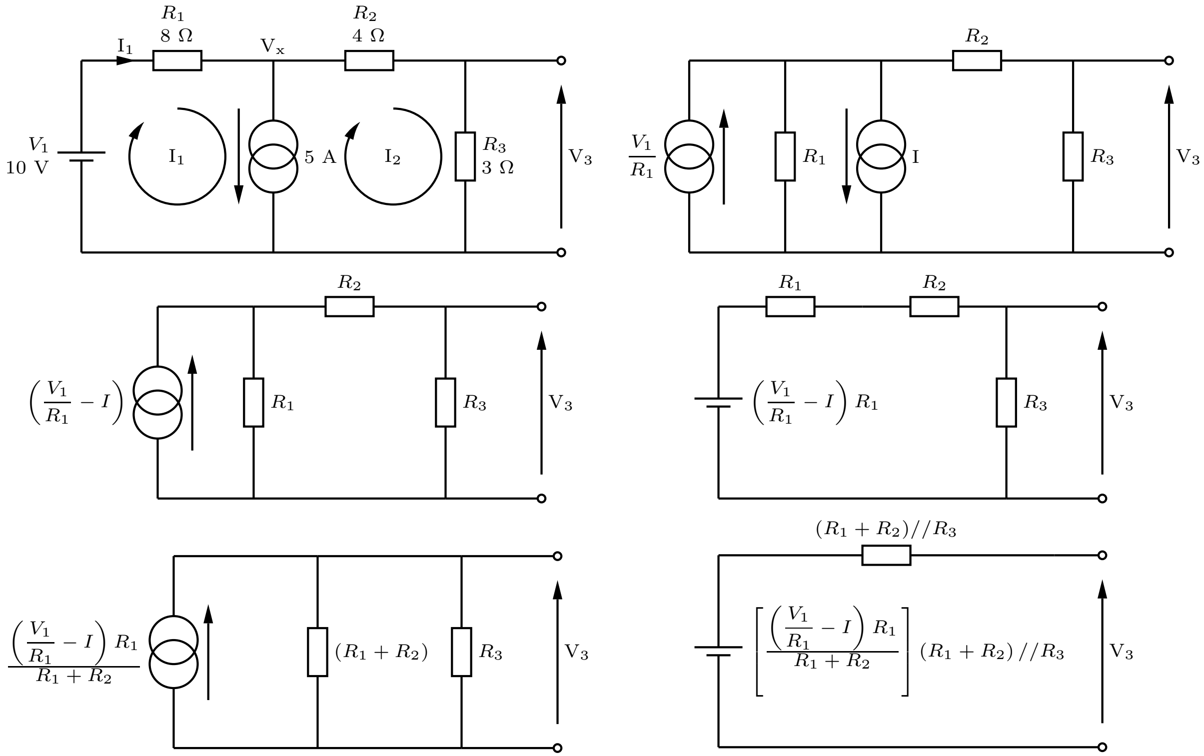

Table 1: Source transformation steps beginning at the top left and ending at the bottom right.

This method is a bit laborious but gives excellent transformation practice. To find the current that will make $V_3 = 0$, use the superposition approach ending with (45) but use I instead of 5 A.

$$V_{3\ (10\ \mathrm{V})} + V_{3\ (\mathrm{I})} = 0 = 2\ \mathrm{V} + \frac{(-I\,(R_2 + R_3)\,R_1)\,R_3}{R_2 + R_3} \tag{48}$$

$$\text{or } 2 = I\frac{R_1\,R_3}{R_1 + R_2 + R_3} = I\,\frac{24}{15}\ \therefore I = 1.25\ \mathrm{A} \tag{49}$$

Question 4

1. Everything has units of current except for the ${}^{I_5}/_{R_6}$ term.

2. The common unit is Volts. The $I_4\,(R_8 + 1)$ and R_3 terms are wrong.

3. Is correct; both sides have units of resistance.

4. The unit on both sides is Volts. All the $j\,\omega$ terms are dimensionless (and hence are correct) expect for the last one, $j\,\omega\,C_2\,R_1\,R_2$, that has units of resistance and is incorrect. Remember that ω has units of ${}^{1}/_{\mathrm{time}}$, $C\,R$ has units of time and j is dimensionless.

5. The $j\,\omega\ (C_1 + C_2)\,R^2$ term has unit of resistance and should be dimensionless.

6. The $j\,\omega\,L$ term has units of resistance and should be dimensionless.

7. Is correct; z is impedance with units of ohms, each term in the numerator of the right hand side has units of ohms, each term in the r.h.s denominator is dimensionless.

Question 5

The key relationship here is

$$V_c = \frac{1}{C}\int I\,\mathrm{d}t. \tag{50}$$

and it is quite helpful to remember that $\int I\,\mathrm{d}t. =$ charge. In the absence of impulsive currents, there are no instantaneous changes in charge and hence no sudden jumps in voltage. It is often easier in the type of problem to move the time origin to a convenient location for each piecewise linear section.

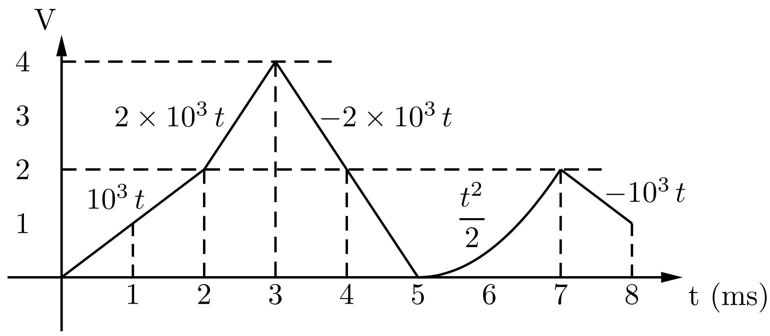

Figure 5: Capacitor voltage as a function of time.

$$\text{between 0 and 2 ms, } V = \frac{1}{C}\int 1\ \text{mA}\, \mathrm{d}t. = 10^3\, t \tag{51}$$

$$\text{between 2 and 3 ms, } V = \frac{1}{C}\int 2\ \text{mA}\, \mathrm{d}t. = 2 \times 10^3\, t \tag{52}$$

$$\text{between 3 and 5 ms, } V = \frac{1}{C}\int -2\ \text{mA}\, \mathrm{d}t. = -2 \times 10^3\, t \tag{53}$$

$$\text{between 5 and 7 ms, } V = \frac{1}{C}\int t\, \mathrm{d}t. = \frac{t^2}{2} \tag{54}$$

$$\text{between 7 and 8 ms, } V = \frac{1}{C}\int -1\ \text{mA}\, \mathrm{d}t. = -10^3\, t \tag{55}$$

Charge at the end can be found either by summing the total area under the I – t curve (integration) or by computing the charge necessary to support the 1 V final voltage which is 1 μC

Question 6

The key relationship here is

$$I = C\,\frac{\mathrm{d}V}{\mathrm{d}t} \tag{56}$$

and the only difficulties lie in the places where $\left|\frac{\mathrm{d}V}{\mathrm{d}t}\right| = \infty$. When the happens, a charge appropriate for the ΔV must enter C in zero time leading to a current pulse that is infinitely high and infinitely thin. The only thing that is defined about the pulse is its area... which, of course, is equal to the charge change caused by the voltage change.

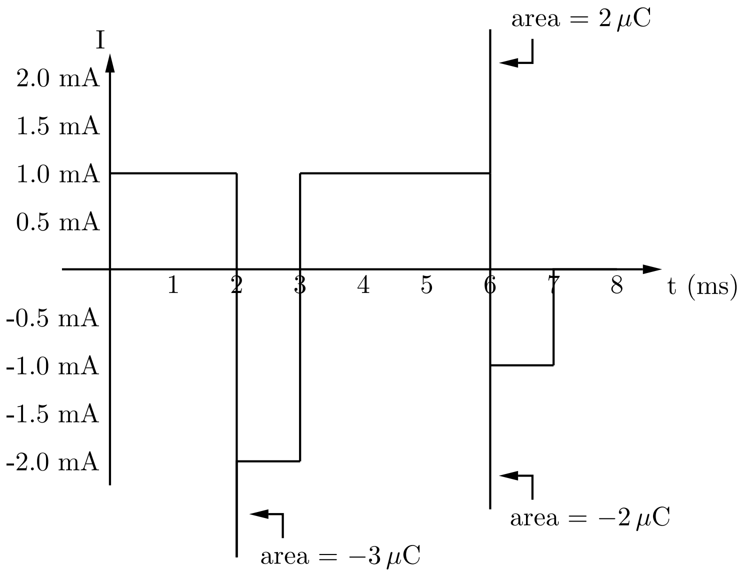

Figure 6: Capacitor current as a function of time.

$$\text{between 0 and 2 ms, } \frac{\mathrm{d}V}{\mathrm{d}t} = 10^3 \,,\, I = 1 \text{ mA} \tag{57}$$

$$\text{between 2 and 3 ms, } \frac{\mathrm{d}V}{\mathrm{d}t} = -2\times 10^3 \,,\, I = -2 \text{ mA} \tag{58}$$

$$\text{between 3 and 6 ms, } \frac{\mathrm{d}V}{\mathrm{d}t} = 10^3 \,,\, I = 1 \text{ mA} \tag{59}$$

$$\text{between 6 and 7 ms, } \frac{\mathrm{d}V}{\mathrm{d}t} = -10^3 \,,\, I = -1 \text{ mA} \tag{60}$$

$$\text{7 ms onwards, } \frac{\mathrm{d}V}{\mathrm{d}t} = 0 \,,\, I = 0 \text{ mA} \tag{61}$$