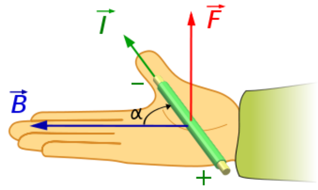
Answers to questions

Answers to question I:

(a) If L is the rotor axial length and D is the rotor outer diameter, the **Lorentz Force** of:

1 conductor carrying current I (A): $F = BLI$,
2 conductors (1 turn) carrying current I (A): $F = 2BLI$,
N turns carrying current I (A): $F = 2NBLI$.



The Resultant torque for a rotor diameter D is:

$\boldsymbol{T = F \times D/2 = 2(NI)BL \times D/2}$

If we define a new variable:

$$Q = \frac{2NI}{\pi D}$$

Then we can obtain the resultant torque:

$$\boxed{T = \frac{\pi}{2} D^2 LBQ}$$

(b) The active length of conductors now becomes:

$$L = \frac{D_o - D_i}{2}$$

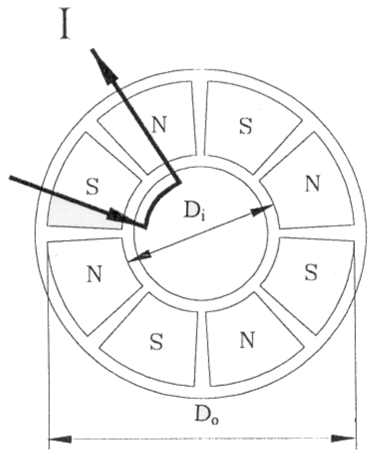
Therefore, the Lorentz force can be described by:

$$F = 2BI\frac{(D_o - D_i)}{2}$$

Hence the average torque (T in Nm)

at mean radius $(D_o+D_i)/4$:

$$\boxed{T = 2BNI\frac{(D_o - D_i)}{2}\frac{(D_o + D_i)}{4}}$$



If we define an average electrical loading (A/m):

$$Q = \frac{2NI}{\pi\left(\frac{D_o + D_i}{2}\right)}$$

Note: If Q max is used at inner diameter then different D_i/D_o can give max T,

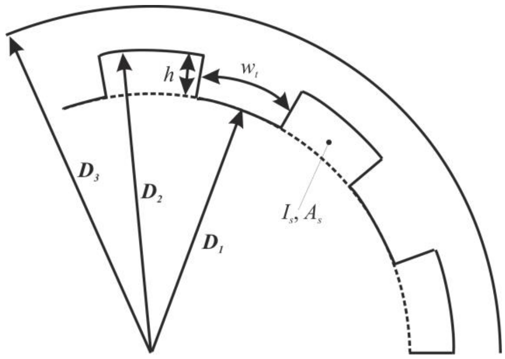
Then:

$$\boxed{T = \frac{\pi}{16}(D_o + D_i)^2(D_o - D_i)BQ}$$

And the most important differences between axial flux and radial flux machines are:

Axial-field motor	Radial-field motor
Axial air-gap flux and radial current	Radial air-gap flux and axial current
Torque is independent of axial length	Torque is proportional to axial length
Electrical loading varies with radius, max. at inner radius	Electrical loading is independent of radius

(c) If we define a total slot current *Is*, then the conductor area per slot is: $k_p A_s$



and current density (A/m^2) in slot:

$$J_s = \frac{I_s}{k_p A_s}$$

Therefore, the electrical loading can be calculated:

$$\boxed{Q = \frac{\sum I}{\pi D} = \frac{N_s A_s k_p J_s}{\pi D}}$$

If D_1, D_2, & D_3 all increased in proportion by a factor of K, then

- The slot depth : h= (D2-D1)/2 also increases by K
- Area of annulus contains slots & teeth $\frac{\pi(D_2^2 - D_1^2)}{4}$ increases by K^2
- If total tooth width ($N_s \times w_t$) increase by K for a fixed ***B*** loading, then

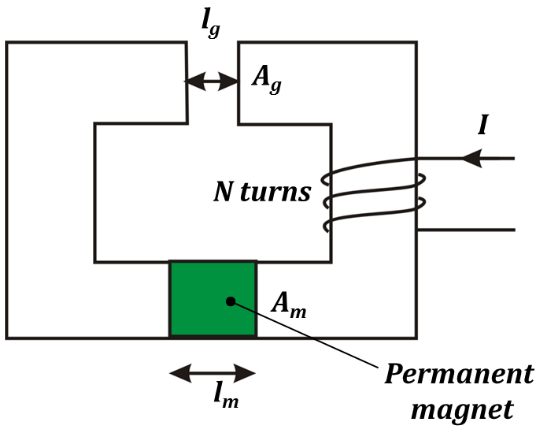
$N_s w_t h \propto K$ and areas of slots within this annulus: $N_s A_s \propto K^2$

Therefore, the electrical loading $Q = \frac{N_s A_s \bullet k_p J_s}{\pi D_1}$ increases by K

(d) The factors that influence the magnetic loading and the electrical loadings are:

(1) Magnetic loading B which is limited by saturation, magnetic remanence and iron losses,
(2) Electrical loading which is often limited by cooling and demagnetization withstand of permanent magnet.

Answers to question II:



(a) It is assumed that the magnetic saturation and the flux leakage are neglected, then based on Ampere's and Gauss's laws, we can obtain

$$H_m l_m + H_g l_g = 0 \quad (1)$$

and

$$B_m A_m = B_g A_g \quad (2)$$

the demagnetization characteristic is given by

$$B_m = \mu_0 \mu_r H_m + B_r \quad (3)$$

From (1) to (3), we can obtain the flux density in air-gap such as:

$$B_g = \frac{A_m}{A_g} B_m = \frac{B_r}{\dfrac{A_g}{A_m} + \mu_r \dfrac{l_g}{l_m}} \quad (4)$$

Based on equation (4), it can be found that the flux density in air-gap can be enhanced by:

1. Increasing Br, this means using more expensive magnet material,

2. Using thicker magnets (increase the cost) and reduce the air-gap (increase the manufacturing difficulty),

3. Reducing the ratio Ag/Am, using flux concentrating machines, however, this will increase the manufacturing difficulty and the flux leakage.

(b) From equations (1) to (2), we can obtain the energy stored in magnet such as:

$$(B_m H_m)(A_m l_m) = -\frac{B_g^2}{\mu_0}(A_g l_g)$$

And

$$(B_m H_m) V_m = -\frac{B_g^2}{\mu_0} V_g$$

Where $V_m = A_m l_m$ is the volume of magnet and $V_g = A_g l_g$ is the volume of air-gap.

Then, the volume of permanent magnet can be determined by:

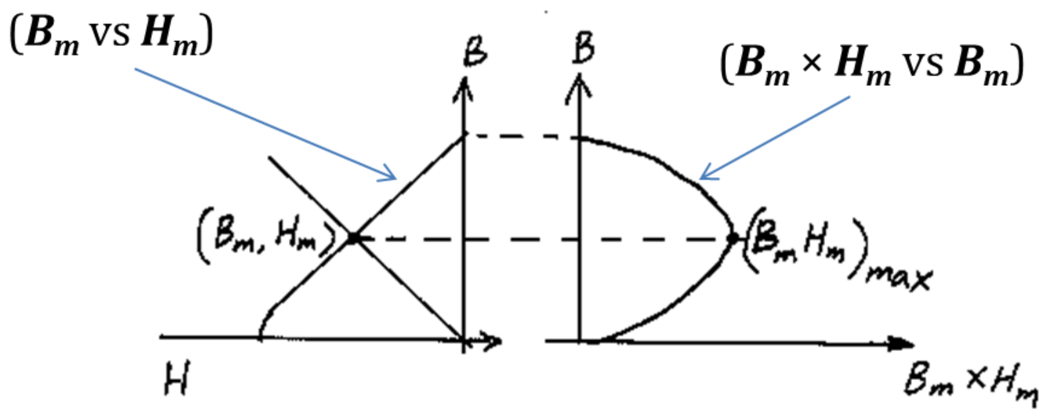
$$\boxed{V_m = -\frac{B_g^2}{\mu_0 (B_m H_m)} V_g}$$

Energy stored by permanent magnets (***B_m***×***H_m***):

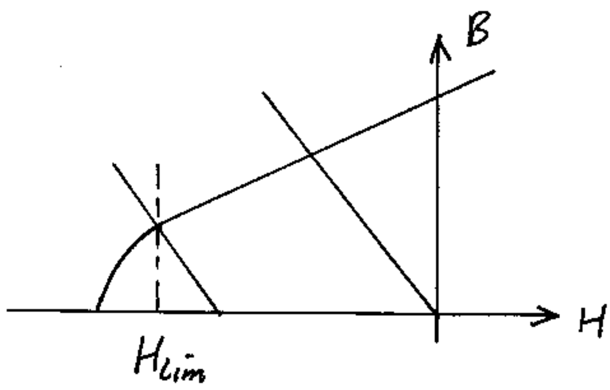
$$B_m \times H_m = \frac{B_m}{\mu_0 \mu_r}(B_m - B_r)$$

With $H_m = \frac{B_m - B_r}{\mu_0 \mu_r}$, and $\frac{d(B_m \times H_m)}{dB_m}$=0 gives the max $(B_m \times H_m)$, therefore, $\boldsymbol{B_m} = \frac{1}{2}\boldsymbol{B_r}$

Therefore, we have the graph that shows when the maximum energy storage is achieved with the variation in B_m and H_m:



(c) When current $I \neq 0$, we assume there is a demagnetizing current, therefore, the demagnetizing curves are shown (if the reversible and irreversible demagnetizing curves are given here, the marks can also be obtained):



Since the current is not equal to zero, based on Ampere's law, we can have:

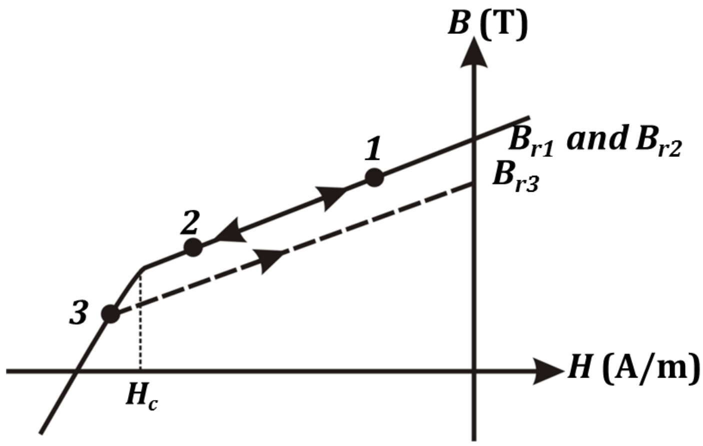
$$H_m l_m + H_g l_g = -NI \qquad (5)$$

And to avoid demagnetisation: $|H_m| < |H_{lim}|$, therefore:

$$\boxed{l_{m(\text{lim})} = -\frac{NI}{H_{\text{lim}}} - \frac{B_r}{\mu_0 H_{\text{lim}}} \times \left(l_g \frac{A_m}{A_g}\right) - \mu_r l_g \left(\frac{A_m}{A_g}\right)}$$

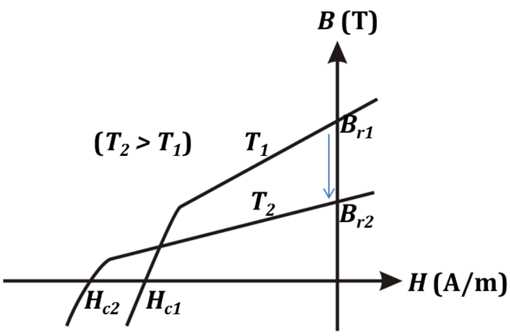
Where $\boldsymbol{H_{lim}} < 0$.

(d) The reversible and irreversible demagnetization curves are shown:

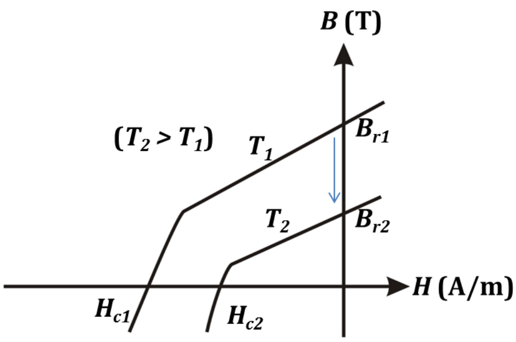


With the increase in demagnetizing field (H), the flux density in permanent magnet B decreases. However, if H does not exceed the Hc such as between points 1 and 2, the demagnetization of magnets can be recovered when –H decrease. However, if the working point of magnet is beyond the knee point (H<Hc) such as at point 3, then, when –H decrease, the demagnetization cannot be recovered and the magnet remanence reduce from Br1 = Br2 to Br3.

The temperature rise will normally aggravate the irreversible demagnetization, for ferrite, it will decrease the magnet remenance but will increase the Hc. However, for rare earth such as NdFeB, the temperature reduces not only Br but also Hc, making the irreversible demagnetization much worse.



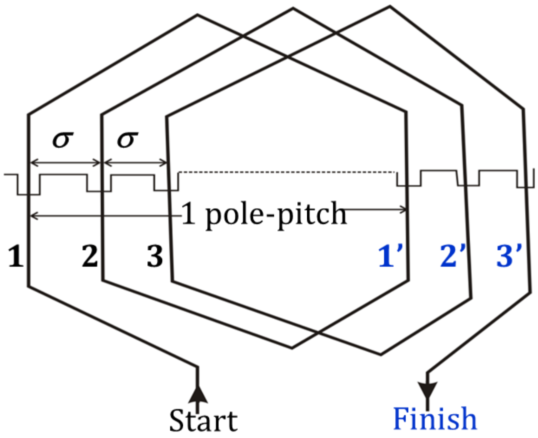
Ferrite:
$T\uparrow, B_r\downarrow, |H_c|\uparrow$

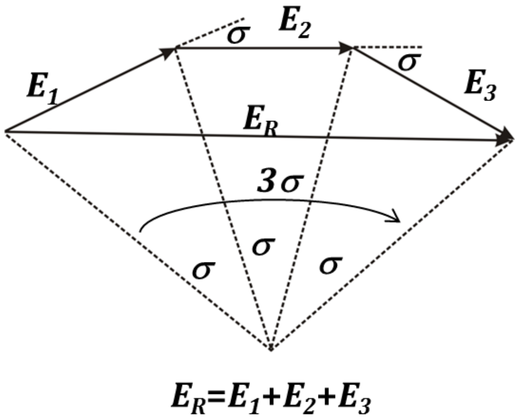


Rare earth:
$T\uparrow, B_r\downarrow, |H_c|\downarrow$

Answers to question III:

(a) The layout of winding and the EMF vectors of coils are shown:





Assuming we have m = 3 coils per phase, and $|E_1| = |E_2| = |E_3| = |E_m|$ (all the coils are identical).

Then, from the construction (***E***$\boldsymbol{E_m} = \boldsymbol{E_1}$), we have

$$E_m = 2r\sin\frac{\sigma}{2} \quad \text{and} \quad E_R = 2r\sin\frac{m\sigma}{2}$$

The arithmetic sum of all coil EMFs: $mE_m = m2r\sin\frac{\sigma}{2}$

However, the vector sum of all coil EMFs: $E_R = 2r\sin\frac{m\sigma}{2}$

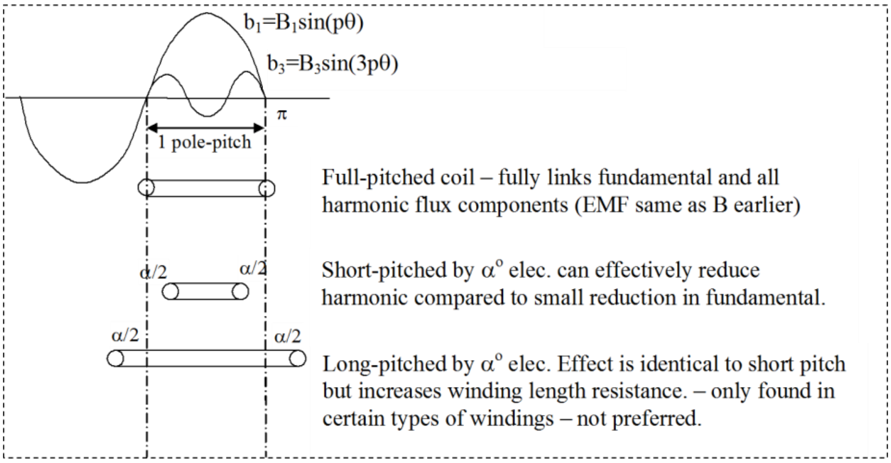
Therefore, the distribution factor for the fundamental is:

$$\boxed{k_d = \frac{effective\,induced\ emf}{arithmetic\,induced\ emf} = \frac{E_R}{mE_m} = \frac{\sin\frac{m\sigma}{2}}{m\sin\frac{\sigma}{2}}}$$

By using the similar approach, the distribution factor for the nth harmonic is:

$$\boxed{k_{dn} = \frac{\sin\frac{mn\sigma}{2}}{m\sin\frac{n\sigma}{2}}}$$

The pitch factor then can be calculated based on the following graph:



k_p is defined as: $$\frac{effective\,EMF}{EMF\ of\ \ full-pitch\ coil} \propto \frac{effective\ flux\ linkage}{flux\ linkage\ of\ \ full\ \ pitch\ coil} = \frac{\Psi_s}{\Psi_F}$$

For a short pitch coil:

$$\Psi_s = \int_{\alpha/2}^{\pi-\alpha/2} \hat{B}\sin\theta d\theta = 2\hat{B}\cos\frac{\alpha}{2}$$

And for full pitch coil:

$$\Psi_F = \int_0^{\pi} \hat{B}\sin\theta d\theta = 2\hat{B}$$

Therefore, the pitch factor is:

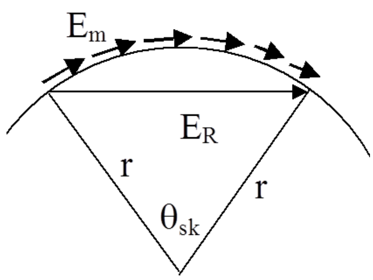
$$\boxed{k_p = \frac{\Psi_s}{\Psi_F} = \frac{2\hat{B}\cos\frac{\alpha}{2}}{2\hat{B}} = \cos\frac{\alpha}{2}}$$

Similarly, the pitch factor for long pitch is: $k_p = \cos\frac{\alpha}{2}$

Both long and short pitches are used to reduce the harmonics. However, the long pitch often leads to longer end-winding, and hence higher phase resistance. This will increase the copper losses if we assume the phase current is the same for both short-pitch and long-pitch windings. Therefore, the long pitch winding is less preferable in practical.

(b) If the skew angle is θsk and the winding consists of m element as shown in the following graph, then we have:

$$k_{sk} = \frac{vector\ sum\ E_R}{arithmetic\ sum\ mE_m} = \frac{chord\ of\ circle}{arc\ of\ circle} = \frac{2r\sin\frac{\theta_{sk}}{2}}{r\theta_{sk}}$$



Finally, the skew factor can be calculated by:

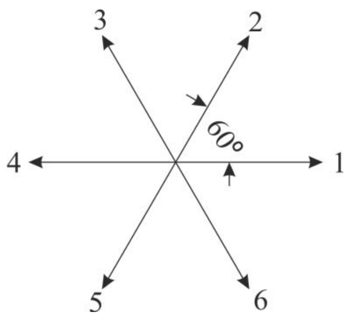
$$\boxed{k_{sk} = \frac{\sin \frac{\theta_{sk}}{2}}{\frac{\theta_{sk}}{2}}}$$

As for distribution factor, the skew factor for nth harmonic is:

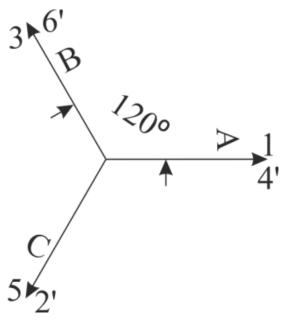
$$\boxed{k_{skn} = \frac{\sin \frac{n\theta_{sk}}{2}}{\frac{n\theta_{sk}}{2}}}$$

The winding skew is a very effective approach to reduce higher harmonics. However, it reduces the fundamental as well. Moreover, due to its complex structure, it will also increase the manufacturing difficulty.

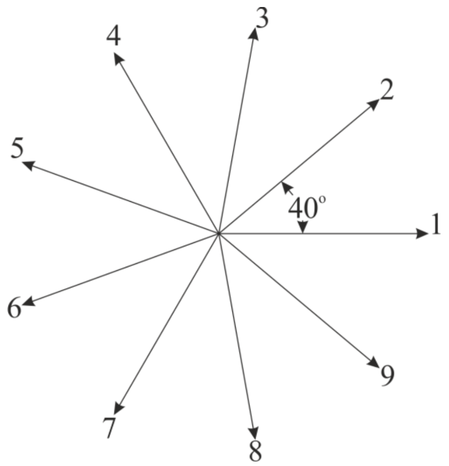
(c) For a 12-slot/14-pole alternate teeth wound permanent magnet machine which has non-overlapping concentrated winding, there are 6 coils allow us to establish a 3-phase winding structure. This means each phase will only have 2 coils. The coil vector and coil EMF vector of this machine are the same and shown in the following graph:



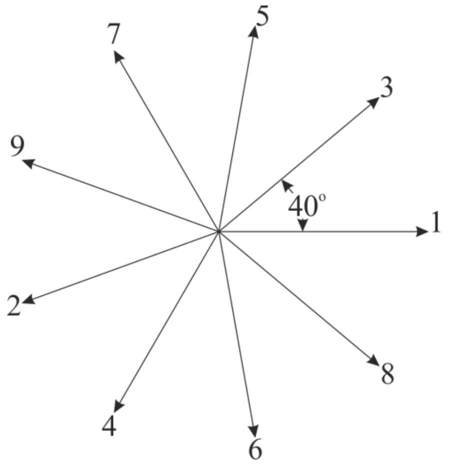
Therefore, the coil connection for a maximum distribution factor should be as:



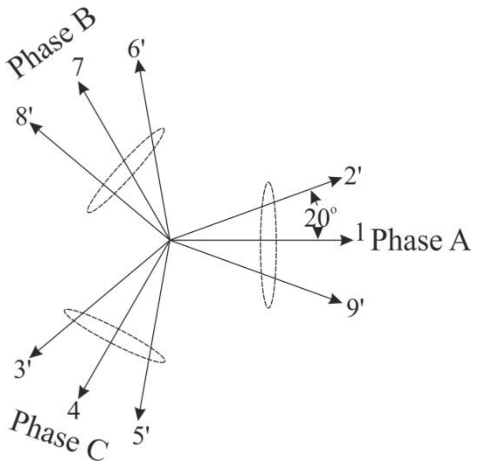
(d) For a 9-slot/10-pole all teeth wound permanent magnet machine with non-overlapping concentrated windings, there is nine coils allow us to establish a 3-phase winding structure. Therefore, each phase will have 3- coils. The coil vector is shown as:

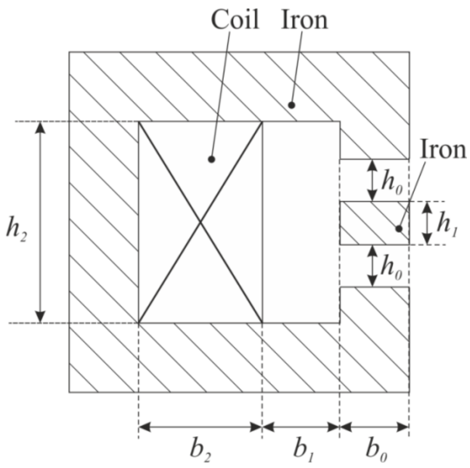


The coil back EMF vectors are then shown as



To maximize the distribution factor, the coil connection can be (don’t forget apostrophes on the figures such as 2, 9, etc., otherwise, full marks will not be given):





<u>Answers to question IV:</u>

(a) assumptions:

(1) the permeability of the iron is infinite,

(2) all flux lines are vertical
The problem can be considered as we have two air-gap here

(I) Slot opening:

mmf across opening=total slot mmf=NI,
where N=no of conductors, I=current per conductor
then, flux across opening/unit length:

$$\boxed{\Phi = \frac{mmf}{\frac{2h_0}{\mu_0 b_0}} \times (\text{unit length}) = NI\mu_0\left(\frac{b_0}{2h_0}\right)}$$

& flux linkage /unit length of machine:

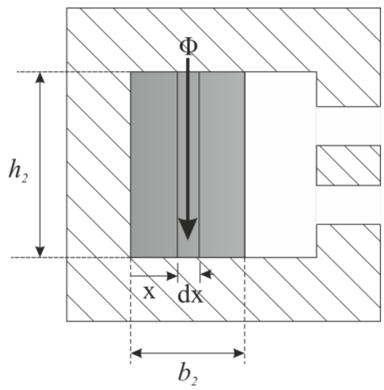
ψ = Φ × No of conductor linked (N): $= N^2 I\mu_0\left(\frac{b_0}{2h_0}\right)$

(II) Unwound section of slot (above conductors):

Again by similar analysis as slot opening

Flux linkage/unit length = $N^2 I\mu_0\left(\frac{b_1}{h_2}\right)$

(III) Wound section of slot:



In this case the MMF is distributed throughout the section

& we need to integrate across the depth $\boldsymbol{b_2}$.

Consider elemental strip depth d_x at x from bottom of winding:

MMF available below strip = $NI \times \frac{x}{b_2}$

Flux Φ through strip:

$$\boxed{\frac{\text{MMF}}{\text{reluctance}} = NI\frac{x}{b_2}\Big/\frac{h_2}{\mu_0 d_x} \times (\text{unit length}) = NI\frac{x}{b_2}\frac{\mu_0 d_x}{h_2} \times (\text{unit length})}$$

& flux linkage = $N^2 I\frac{x^2}{b_2^2}\frac{\mu_0 d_x}{h_2}$ with number of conductors below the strip $= N\frac{x}{b_2}$)

Hence, effective flux linkages for total section:

$$\frac{N^2 I\mu_0}{b_2^2 h_2}\int_0^{b_2} x^2 dx = N^2 I\mu_0\left(\frac{b_2}{3h_2}\right)$$

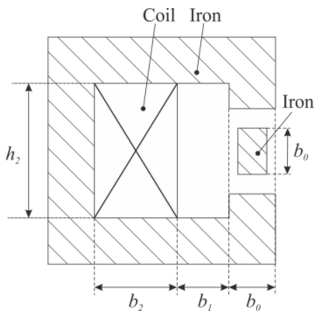
Total slot of the shape considered:

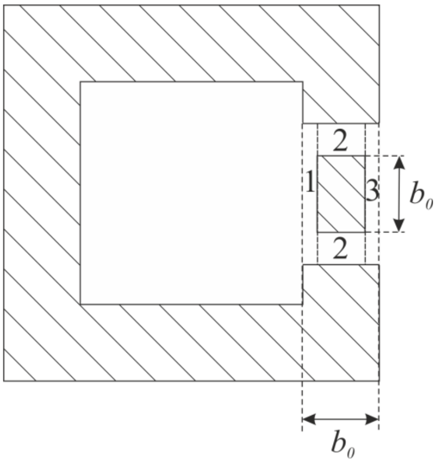
Flux linkage/unit length = $N^2 I\mu_0\left(\frac{b_0}{2h_0}\right)+N^2 I\mu_0\left(\frac{b_1}{h_2}\right)+N^2 I\mu_0\left(\frac{b_2}{3h_2}\right)$

Hence the slot inductance/unit length:

$$\left(\frac{Flux\ linkage/\ unitelength}{I}\right)=N^2\mu_0\left(\frac{b_0}{2h_0}+\frac{b_1}{h_2}+\frac{b_2}{3h_2}\right)$$

(b) When rotor rotates to another position as shown in the figure, the calculation of flux linkage in the wound as well as unwound slot sections are exactly the same as that calculated in (a). The difference is mainly in the slot opening. The slot opening now can be divided into 3 sections, which is numbered as 1, 2, 3 as shown in the following figure:





Similar to calculation for slot opening in (I),

In section 1,

the flux linkage/unit length: $=N^2 I\mu_0\left(\frac{(b_0-h_1)/2}{2h_0+h_1}\right)$

In section 3, similar results can be obtained such as

The flux linkage/unit length: $=N^2 I\mu_0\left(\frac{(b_0-h_1)/2}{2h_0+h_1}\right)$

In section 2

The flux linkage/unit length: $=N^2 I\mu_0\left(\frac{h_1}{2h_0+h_1-b_0}\right)$

The total flux linkage/unit length in slot opening:

$$N^2 I\mu_0\left(\frac{(b_0-h_1)}{2h_0+h_1}+\frac{h_1}{2h_0+h_1-b_0}\right)$$

Total slot of the shape considered:

Flux linkage/unit length = $N^2 I\mu_0\left(\frac{(b_0-h_1)}{2h_0+h_1}+\frac{h_1}{2h_0+h_1-b_0}+\frac{b_1}{h_2}+\frac{b_2}{3h_2}\right)$

Hence the slot inductance/unit length:

$$N^2\mu_0\left(\frac{(b_0-h_1)}{2h_0+h_1}+\frac{h_1}{2h_0+h_1-b_0}+\frac{b_1}{h_2}+\frac{b_2}{3h_2}\right)$$

(c) To increase the winding inductance in slot, we can increase the winding turn number this will increase the winding resistance, and hence increase copper losses. We can also decrease the air-gap length or using narrower slot or wider stator and rotor teeth widths.