Tutorial Sheet – No 6 Answers

1 First calculate the load impedance in polar form:

$$Z_L = R + jX_L = 400 + j300 = 500\angle 36.9^\circ \ \Omega$$

Since there is no winding resistance or leakage reactance given use can be made of the equation:

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

and the output voltage is given by:

$$V_2 = V_1 \frac{N_2}{N_1} = 250 \times \frac{20}{1} = 5\text{kV}_{rms}$$

Taking the input voltage as reference:

$$V_1 = \mathbf{250 \angle 0^\circ\ V_{rms}}$$

and:

$$V_2 = \mathbf{5000 \angle 0^\circ\ V_{rms}}$$

The output current then may be calculated:

$$I_2 = \frac{V_2}{Z_L} = \frac{5000\angle 0^\circ}{500\angle 36.9^\circ} = \mathbf{10 \angle -36.9^\circ\ A_{rms}}$$

and the output power is:

$$P_{OUT} = V_2 I_2 \cos\phi = 5000 \times 10 \times \cos 36.9^\circ = \mathbf{40kW}$$

To find the total input current the load current must be referred to the primary and added to the magnetising current:

$$I_{1TOT} = I_{2REF} + I_{MAG}$$

Referring the load current:

$$I_{2REF} = I_2 \frac{N_2}{N_1} = 10\angle -36.9^\circ \times \frac{20}{1} = 200\angle -36.9^\circ = 159.94 - j120 \ \text{A}_{rms}$$

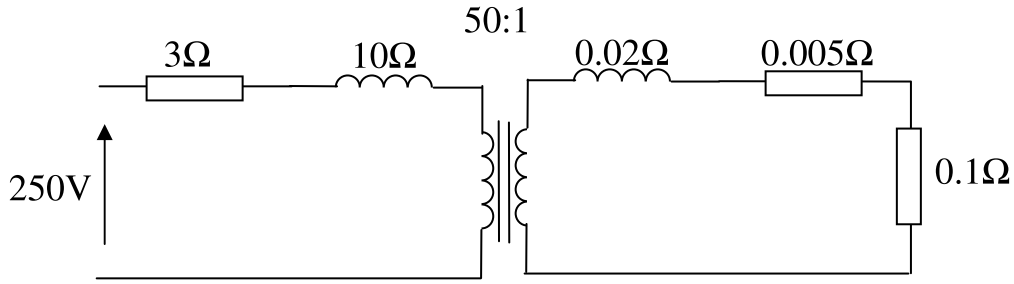
and expressing the magnetising current in complex form:

$$I_{MAG} = 30\angle -90^\circ = -j30$$

then:

$$I_{1TOT} = 159.94 - j120 - j30 = 159.94 - j150 = \mathbf{219.3 \angle -43.2^\circ\ A_{rms}}$$

2 Draw the equivalent circuit:



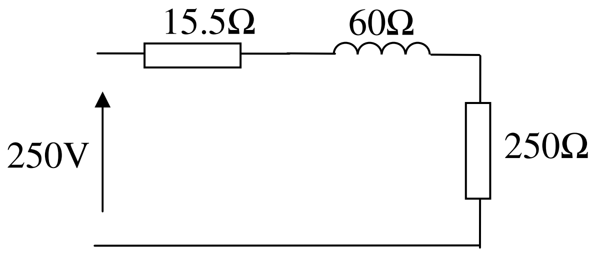
and refer all impedances to the primary side:

$$R_{2REF} = \left(\frac{N_1}{N_2}\right)^2 R_2 = \left(\frac{50}{1}\right)^2 \times 0.005 = 12.5 \ \Omega$$

$$X_{2REF} = \left(\frac{N_1}{N_2}\right)^2 X_2 = \left(\frac{50}{1}\right)^2 \times 0.02 = 50\ \Omega$$

$$R_{CABLE\_REF} = \left(\frac{N_1}{N_2}\right)^2 R_{CABLE} = \left(\frac{50}{1}\right)^2 \times 0.1 = 250\ \Omega$$

combining R_1 and R_{2REF} and X_1 and X_{2REF} gives:



The total impedance is then:

$$Z_T = 15.5 + 250 + j60 = 265.5 + j60 = 272.2\angle 12.7^\circ\ \Omega$$

and the current flowing in the primary is:

$$I_1 = \frac{V_1}{Z_T} = \frac{250\angle 0^\circ}{272.2\angle 12.7^\circ} = 0.918\angle -12.7^\circ\ \mathrm{A_{rms}}$$

The power dissipated in the cable is then:

$$P_{CABLE} = I_1^2 R_{CABLE\_REF} = 0.918^2 \times 250 = \mathbf{210.7W}$$

Note: you cannot use $P = V\,I\,\cos\varphi$ as this would include the losses in the 15.5Ω winding resistance as well as the cable.

The total losses in the transformer are the sum of the iron (core) losses and the losses in the winding resistances (copper losses):

$$P_{LOSS} = P_{Fe} + P_{Cu}$$

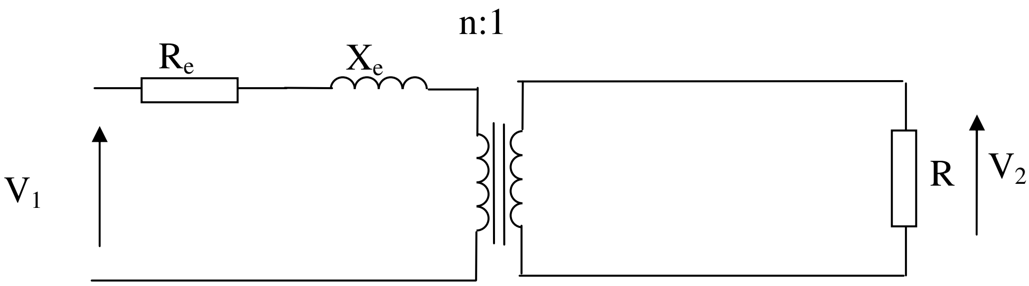
Therefore:

$$P_{LOSS} = 5 + 0.918^2 \times 15.5 = 18.06\ \mathrm{W}$$

and the efficiency is:

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{LOSS}} \times 100\% = \frac{210.7}{210.7 + 18.06} \times 100\% = \mathbf{92.1\%}$$

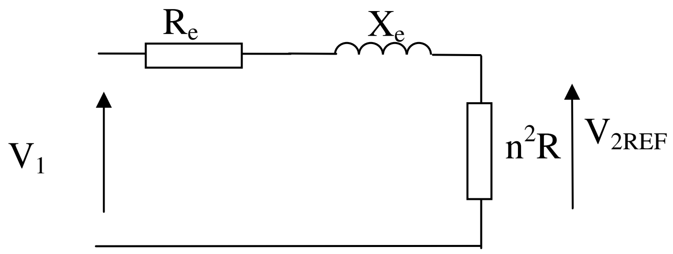
3 Draw the equivalent circuit:



Now the secondary (load) resistance can be referred to the primary side:

$$R_{REF} = \left(\frac{N_1}{N_2}\right)^2 R = n^2 R$$

So the circuit becomes:



and hence the current is:

$$I_1 = \frac{V_1}{Z_T} = \frac{V_1}{\sqrt{\left(R_e + n^2 R\right)^2 + X_e^2}}$$

and the voltage across the referred load is:

$$V_{2REF} = I_1 n^2 R = \frac{V_1 n^2 R}{\sqrt{\left(R_e + n^2 R\right)^2 + X_e^2}}$$

Now refer V_{2REF} back to the secondary side to obtain the actual load voltage:

$$V_2 = V_{2REF}\left(\frac{1}{n}\right) = \frac{V_1 n R}{\sqrt{\left(R_e + n^2 R\right)^2 + X_e^2}}$$

Using the above equation and substituting in the values gives:

$$V_2 = \frac{250 \times 20 \times 1}{\sqrt{\left(0 + 20^2 1\right)^2 + 117^2}} = \mathbf{12V_{rms}}$$

and the current through the bulb is:

$$I_2 = \frac{V_2}{R} = \frac{12}{1} = 12\mathrm{A_{rms}}$$

and hence the power dissipated in the bulb is:

$$P_2 = I_2^2 R = 12^2 \times 1 = \mathbf{144W}$$

and the primary current is:

$$I_1 = \frac{N_2}{N_1} I_2 = \frac{1}{20} \times 12 = \mathbf{0.6A_{rms}}$$

4 (a) Use the short circuit data to find R_e and X_e:

$$P_{SC} = I_{SC}^2 R_e$$

so:

$$R_e = \frac{P_{SC}}{I_{SC}^2} = \frac{80}{25^2} = \mathbf{0.128\ \Omega}$$

and knowing the resistance the reactance can be found from the impedance:

$$Z_e = \frac{V_{SC}}{I_{SC}} = \frac{25}{25} = 1\ \Omega$$

$$X_e = \sqrt{Z_e^2 - R_e^2} = \sqrt{1^2 - 0.128^2} = \mathbf{0.99\ \Omega}$$

The open circuit (magnetising) current is the no-load current for which the magnitude is given as **1.15A$_{rms}$.** The phase angle and power factor may also be found from the no-load test data:

$$P_{OC} = V_{OC} I_{OC} \cos\phi$$

so:

$$pf = \cos\phi = \frac{P_{OC}}{V_{OC} I_{OC}} = \frac{120}{400 \times 1.15}$$ = **0.26 lagging**

and the phase angle is:

$$\phi = \cos^{-1} 0.26$$ = **74.9°**

(b) Since the transformer has a rating of 12kVA and delivers full load at unity power factor then:

$$I_1 = \frac{VA}{V_1} = \frac{12000}{400}$$ = 30 A_{rms}

and:

$$P_{OUT} = VA\cos\phi = 12000 \times 1$$ = 12000W

Now calculate the iron (core) and winding (copper) losses:

$$P_{Fe} = open\,circuit\,loss$$ = 120W

$$P_{Cu} = I_1^2 R_e = 30^2 \times 0.128$$ = 115.2W

and:

$$P_{LOSS} = P_{Fe} + P_{Cu} = 120 + 115.2$$ = **235.2W**

and the efficiency is:

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{LOSS}} \times 100\% = \frac{12000}{12000 + 235.2} \times 100\%$$ = **98.1%**

(c) Refer the secondary load to the primary by the square of the turns ratio:

$$Z_2 = 4 + j1 \ \Omega$$

so:

$$Z_{2REF} = Z_2 \left(\frac{N_1}{N_2}\right)^2 = (4 + j1) \times \left(\frac{400}{230}\right)^2 = 12.1 + j3.025 = 12.47\angle 14.04^\circ \ \Omega$$

This is then added to the winding impedance to obtain the total impedance referred to the primary side:

$$Z_T = 0.128 + j0.99 + 12.1 + j3.025 = 12.228 + j4.015 = 12.87\angle 18.18^\circ \ \Omega$$

and the primary current is:

$$I_1 = \frac{V_1}{Z_T} = \frac{400\angle 0^\circ}{12.87\angle 18.18^\circ} = 31.08\angle -18.18^\circ \ A_{rms}$$

The referred voltage across the load is then:

$$V_{2REF} = I_1 Z_{2REF} = 31.08\angle -18.18^\circ \times 12.47\angle 14.04^\circ = 387.6\angle -4.14^\circ \ V_{rms}$$

and referring this back to the secondary gives:

$$V_2 = V_{2REF}\left(\frac{N_2}{N_1}\right) = 387.6\angle -4.14^\circ \times \left(\frac{230}{400}\right)$$ = **222.9∠–4.14° V_{rms}**

The regulation is defined as:

$$Reg = \frac{no\,load\,output\,voltage - on\,load\,output\,voltage}{no\,load\,output\,voltage} \times 100\% = \frac{230 - 222.9}{230} \times 100\%$$ = **3.1%**