EEE606 / 6011

Question 1 Solution

(a)

The radiated power density is

$$P_r = \frac{1}{2}\frac{|E_\theta|^2}{\eta} Wm^{-2} \qquad (1).$$

The total radiated power over a far field sphere is then given by

$$P = \int_0^{2\pi}\int_0^{\pi} P_r r\, sin(\theta) d\phi r d\theta \text{ W} \qquad (2).$$

The field of a half wave dipole is given in the question and is independent of ϕ and so we may rewrite (2) thus

$$P = 2\pi r^2 \int_0^{\pi} P_r\, sin(\theta) d\theta = \frac{I_o{}^2 \eta}{4\pi}\int_0^{\pi} \frac{cos^2\left(\frac{\pi}{2}cos(\theta)\right)}{sin(\theta)} d\theta \text{ W (3).}$$

The value of the integral is given as

$$\int_0^{\pi} \frac{cos^2\left(\frac{\pi}{2}cos(\theta)\right)}{sin(\theta)} d\theta = 1.22 \qquad (4)$$

therefore the power radiated is

$$P = \frac{377 I_o{}^2}{4\pi} \times 1.22 = 36.6 I_o{}^2 \text{ W} \qquad (5).$$

We equate this power to that dissipated in a fictitious 'radiation resistance' R_r, which replaces the antenna and is connected across its terminals. Thus if I_t is the current at the antenna terminals then

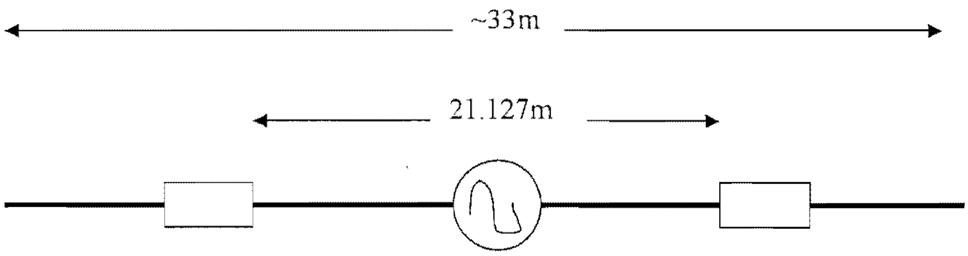
$$0.5 I_t{}^2 R_r = 36.6 I_o{}^2 \qquad (6).$$

For a half wave dipole ($L = \lambda / 2$), the current at the antenna terminals is the maximum current, so $I_t = I_o$, and hence $R_r = 73.2\Omega$. For a perfectly conducting dipole at resonance, R_r is therefore the input impedance.

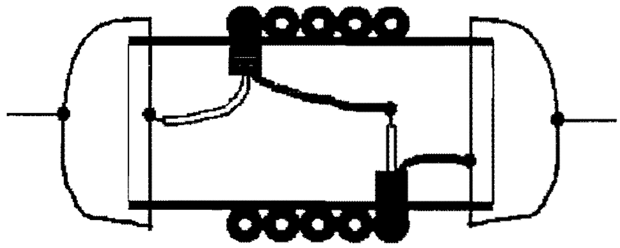
(b)

A trap is a parallel LC circuit which resonates at the higher operating frequency (7.1MHz) thus isolating the outer arms of the trapped dipole (since the impedance of a resonant parallel LC circuit is ~infinite), and making the antenna appear as a shorter dipole which is a half wavelength long at the higher frequency. At the lower frequency (3.55MHz) the trap is inductive and connects the outer arms to form a longer dipole which is *electrically* a half wavelength long at the lower frequency.

Using $\lambda = c/f$, a half wavelength at the upper and lower frequencies is 21.127m and 42.254m respectively. However, the overall wire length will be less than a half wave for resonance at the lower frequency because of the inductance of the traps.



A trap could be constructed using coaxial cable as shown below. The inner conductor and screen in series provide the inductance, and the inner conductor and screen gap the capacitance.



(c)

The trap needs to resonate at 7.1MHz, so using $7.1 \times 10^6 = \dfrac{1}{2\pi\sqrt{LC}}$,

$$C = \frac{1}{4\pi^2 \times 7.1^2 \times 10^{12} \times 17.5 \times 10^{-6}} = 28.7\,pF$$

The antenna acts as a resonant half wave dipole at both frequencies, so assuming no losses the input impedance $= R_r$. However, the 3.55MHz dipole is electrically shortened by the inductive traps, which will make R_r slightly lower than 73.2Ω at this frequency.

Question 2 Solution

(a)

(i) Typically elliptical polarization (ideally circular) would be expected from this antenna, because of the phase progression of the current along the spiral trajectory.

(ii) At resonance, the input impedance is purely real (which is the definition of resonance for any circuit) and it equals the radiation resistance assuming negligible losses.

(iii) The spiral arms are flared to increase the bandwidth. The current path has more 'choice' in movement across the conductor compared with a thin spiral, which lowers the Q.

(iv) The spiral is segmented in the model (not in practice!) to facilitate a piecewise analysis of the antenna for the purpose of calculation of current distribution using a moment method. Fields generated by current 'moments' over each triangle are evaluated at the same and every other triangle to form a set of simultaneous equations that can be solved for the current at each triangular patch.

(v) The effect of the dielectric substrate is to reduce the wavelength of the current flowing on the spiral arms, thereby increasing the electrical dimensions of the antenna, hence lowering the operating frequency compared to a free space spiral. It could also cause losses and reduce the antenna Q if it had non-zero conductivity.

(b)

(i) $K = 1$, so $\Psi = \frac{1}{0} = \infty$. This is linear polarization along the x axis.

(ii) E_x leads E_y by 90^o, so $\Delta\varphi = 90^o$, $K=0$, $\Psi = \frac{1}{1} = 1$. This is (left hand) circular polarization.

(iii) E_x leads E_y by 90^o, so $\Delta\varphi = 90^o$ as before, but they now have different amplitudes, so

$$K = \sqrt{1 + .0625 - 2 \times 1 \times 0.25} = 0.75$$

$$\Psi = \frac{\sqrt{1 + 0.25 + 0.75}}{\sqrt{1 + 0.25 - 0.75}} = \frac{1}{0.5} = 2$$

This is (left hand) elliptical polarization with the semi-major axis along x-axis.

(iv) E_x lags E_y by 45^o, so $\Delta\varphi = -45^o$,

$$K = \sqrt{0.0625 + 0.0625 + 2 \times 0.25 \times 0.25 \times 0.7071} = 0.462$$

$$\Psi = \frac{\sqrt{.25 + .25 + 0.462}}{\sqrt{.25 + .25 - 0.462}} = 5$$

This is (right hand) elliptical polarization.

(c)

When electromagnetic waves propagate through a plasma (earth's upper atmosphere) in the presence of a (earth's) magnetic field the polarization angle rotates. This is known as Faraday rotation, and is particularly relevant in satellite communications. A circularly polarized signal is less susceptible to fading than its linear counterpart over such a signal path.

Question 3 Solution
(a)

Assuming a Cartesian component of velocity and field for simplicity, let the velocity be time harmonic as

$$V = V_o \cos(\omega t) \qquad (1)$$

From equation (3.1) therefore

$$eE = -m\omega V_o \sin(\omega t) = m\omega V_o \cos(\omega t + 90^o) \qquad (2).$$

Thus

$$eE_o = m\omega V_o \qquad (3)$$

so

$$V_o = \frac{eE_o}{m\omega} \qquad (4).$$

The plasma conduction current density is given by

$$J = NeV \qquad (5)$$

so that

$$\text{(i)} \qquad J = \frac{Ne^2}{m\omega} E_o \cos(\omega t) \qquad (6).$$

From (2) the free space displacement current density is

$$\text{(ii)} \qquad \frac{\partial D}{\partial t} = -\varepsilon_o \omega E_o \cos(\omega t) \qquad (7)$$

and hence the conduction and displacement currents are 180^o out of phase. Since

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad (8)$$

the currents cancel completely and produce zero magnetic field when

$$\frac{Ne^2}{m\omega} = \varepsilon_o \omega \qquad (9)$$

and hence

$$\text{(iii)} \qquad \omega^2 = \frac{Ne^2}{m\varepsilon_o} = \omega_c^{\,2} \qquad (10)$$

is the cut off frequency.

To obtain the plasma relative permittivity, we need to express the RHS of (8) as the time differential of the D field. Substituting (6) and (7) into (8) and using (10) gives

$$\nabla \times H = \omega\varepsilon_0(\frac{\omega_c^2}{\omega^2} - 1)E_0 \cos(\omega t) \qquad (11)$$

But from (2)

$$\frac{\partial E}{\partial t} = -\omega E_0 \cos(\omega t) \qquad (12)$$

so

$$\nabla \times H = \varepsilon_0(1 - \frac{\omega_c^2}{\omega^2})\frac{\partial E}{\partial t} \qquad (13)$$

and hence the relative permittivity is

$$\text{(iv)} \quad \varepsilon_r = (1 - \frac{\omega_c^2}{\omega^2}) \qquad (14).$$

(b)

From (10),

$$f_c = \frac{1}{2\pi}\sqrt{\frac{e^2}{m\varepsilon_0}}\sqrt{N} \qquad (15)$$

hence

$$f_c = \frac{1}{2\pi} \times 56 \times 10^6 = 8.9MHz \quad (16)$$

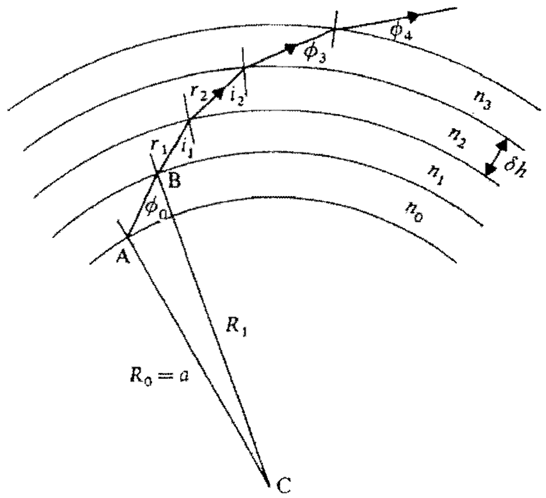
The definition of critical frequency is that no magnetic field is generated by changing electric field, and hence the relative permittivity is zero. Below f_c the permittivity is negative, meaning that no propagation occurs within the plasma, and it reflects radio waves, whereas above f_c the permittivity is positive and propagation occurs.

(c)

A plasma antenna is one where the host for the rf scattering charge carriers is not a metal, but rather a gas or semiconductor. For instance free electrons are produced in gas discharge tubes (e.g. fluorescent lights) or by biasing surface PIN diodes in a piece of silicon, and charge densities commensurate with metals are obtainable.

Question 4 Solution

(a)



$$R_O = a = 6370km \text{ and } r_j = 90 - \phi_j$$

From Snell's law:

$$n_0 \sin i_0 = n_1 \sin r_1 = n_1 \cos\phi_1,\ n_1 \sin i_1 = n_2 \sin r_2 = n_2 cos\phi_2 \quad \ldots \qquad (1)$$

and from the sine rule:

$$\frac{R_0}{\sin i_0} = \frac{R_1}{\sin(180 - r_0)} = \frac{R_1}{\cos\phi_0},\ \frac{R_1}{\sin i_1} = \frac{R_2}{\sin(180 - r_1)} = \frac{R_2}{\cos\phi_1} \quad \ldots \qquad (2)$$

so that using (2) to substitute for $sin i_o$ in (1) gives

$$n_0 R_0 \cos\phi_0 = n_1 R_1 \cos\phi_1 = n_2 R_2 \cos\phi_2 \quad etc\ldots \qquad (3)$$

Thus at any given height h where the refractive index is n

$$an_0 \cos\phi_0 = n(a+h)\cos\phi \qquad (4)$$

so (4) can then be re-written for a 'planar' earth as

$$n_0 \cos\phi_0 = n^* \cos\phi \qquad (5)$$

where the modified refractive index is

$$n^* = \frac{(a+h)}{a} n \approx n + \frac{h}{a} \qquad (6)$$

Thus n^* can be used in a 'flat earth' model and takes account of the earth's curvature. From (6)

$$\frac{\partial n^*}{\partial h} = \frac{\partial n}{\partial h} + \frac{1}{a} \qquad (7)$$

(b)

$$\partial n / \partial h = -39 \times 10^{-6}\, km^{-1} \qquad (8)$$

so that

$$\frac{\partial n^*}{\partial h} = -39 \times 10^{-6} + \frac{1}{6370} = 118 \times 10^{-6}\, km^{-1} \qquad (9)$$

The radius of curvature R of the propagation path is given by

$$\frac{1}{R} \approx -\frac{\partial n^*}{\partial h} \quad (10),$$

so

$$R = 8474576m \qquad (11)$$

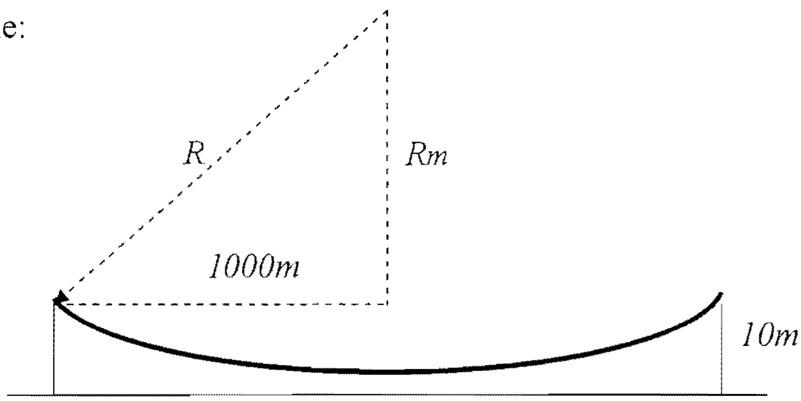
and with reference to the figure below,

$$R_m = \sqrt{R^2 - 1000^2} = 8474568\text{m} \qquad (12).$$

Hence the minimum height of the signal path is

$$10 - (R - R_m) = 2m \qquad (13).$$

Not to scale:



(c)

Typically a high pressure weather system will cause a temperature inversion, whereby the air gets warmer with height rather than cooler. This means that $\partial n^* / \partial h$ becomes negative, causing a positive radius of curvature of the signal. This has the effect of greatly increasing the propagation distance, since waves are 'bent' back down to earth in the troposphere.