electromagnetic waves with and without parallel plates as support

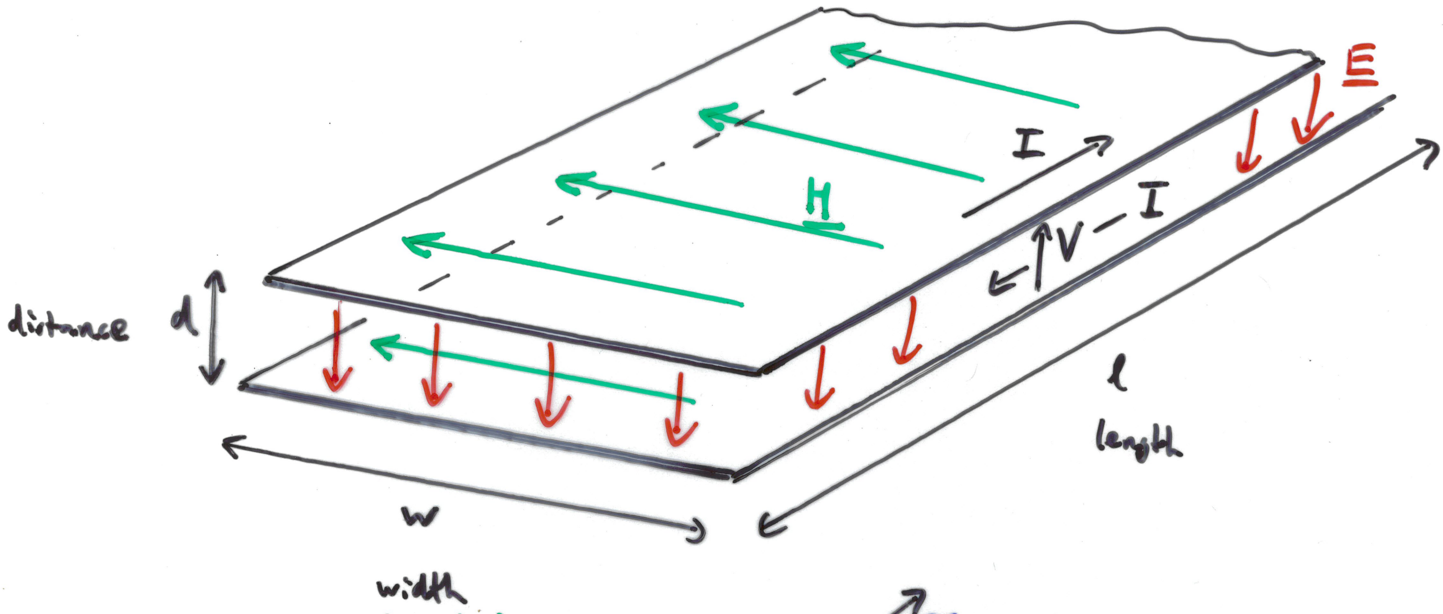
Maxwell:

$$\underline{E} \Rightarrow \operatorname{rot}\underline{E} = -\frac{\partial \underline{B}}{\partial t} \Rightarrow \underline{H}, \quad \operatorname{rot}\underline{H} = \underline{j} + \frac{\partial \underline{D}}{\partial t} \Rightarrow \underline{D}$$

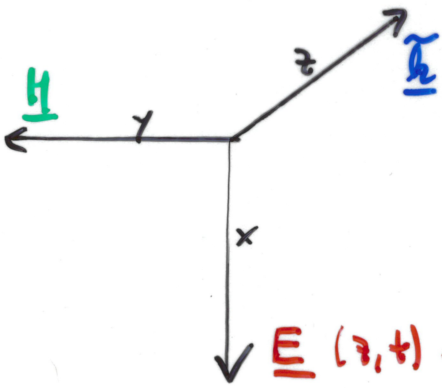
feedback: time-varying changes in $\underline{E}$ (or $\underline{H}$) generate new $\underline{H}$ (or $\underline{E}$)

fields can propagate on transmission lines for support, but can also propagate into free space without any support!

consider special case of two parallel plates:



$\underline{H}(z,t) = H_0 \underline{e_y} e^{j(\omega t - kz)}$



$\underline{E}(z,t) = E_0 \underline{e_x} e^{j(\omega t - kz)}$

$\underline{E}$, $\underline{H}$ and $\underline{k}$ form a right-handed orthogonal system (also $\underline{k}$, $\underline{E}$, $\underline{H}$)

specific capacitance and inductance of parallel plate transmission lines

calculation of C^*:

$$Q \underset{\uparrow \text{def.}}{=} \iiint \varrho \, dV \underset{\uparrow \text{Coulomb's Law}}{=} \varepsilon_0 \varepsilon_r \iiint \operatorname{div} \underline{E} \, dV \underset{\uparrow \text{Gauß's Law}}{=} \varepsilon_0 \varepsilon_r \oint_S \underline{E} \, d\underline{S} \overset{!}{=} \varepsilon_0 \varepsilon_r E \oint_S dS$$

need to find surface S of constant E:

- spheres for points
- cylinders for lines / cables
- planes for parallel plates

Then E can be taken before the integral sign.

parallel plates: $\oint_S dS = w \cdot l$

$$\Rightarrow \quad E = \frac{Q}{\varepsilon_0 \varepsilon_r w l}$$

$$\Rightarrow \quad V = -\int_0^d E \, dx = -\frac{Q d}{\varepsilon_0 \varepsilon_r w l}$$

$$\Rightarrow \quad C^* = C/l = \frac{Q}{|V| l} = \frac{\varepsilon_0 \varepsilon_r w}{d}$$

(is identical to formula $C = \varepsilon_0 \varepsilon_r \frac{w l}{d}$ for plate capacitor!)

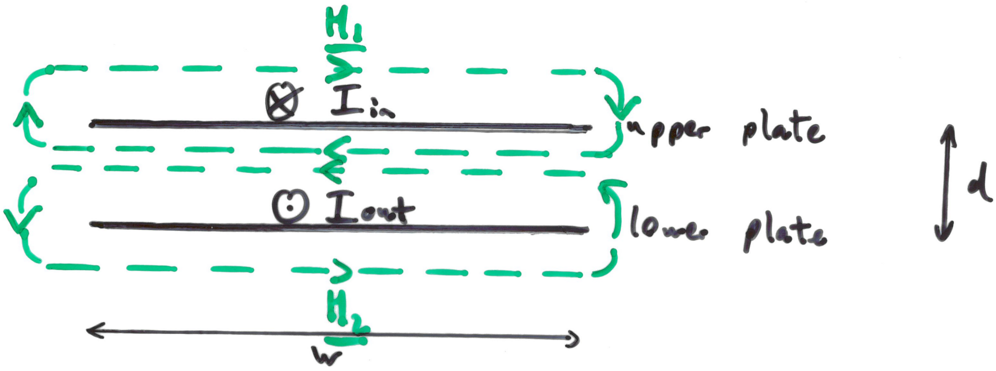
calculation of L^*:

$$I \underset{\uparrow \text{def.}}{=} \iint_S \underline{j} \, d\underline{S} \underset{\substack{\uparrow \text{Ampere's Law} \\ (\text{if } \underline{D} = \text{const.})}}{=} \iint_S \operatorname{rot} \underline{H} \, d\underline{S} \underset{\uparrow \text{Stokes's Law}}{=} \oint \underline{H} \, d\underline{r} \overset{!}{=} H \oint dr$$

need to find closed loop of constant H

- circles for coaxial cables
- rectangles for planes.

Then H can be taken before the integral sign.



$$I = I_{in} = I_{out} = H \oint dr$$

$$= 2wH_1 + 2wH_2 + \text{negligible parts of } H \text{ at the ends}$$

$$\rightarrow \Phi = \int \underline{B}\, d\underline{S} = \mu_0 \mu_r \underset{\downarrow \atop \frac{I}{2w}}{H} \underbrace{\int dS}_{2\ell d} = \mu_0 \mu_r I \frac{\ell d}{w}$$

$$\text{As } -L\frac{\partial I}{\partial t} = V_{ind} = -\frac{\partial \Phi}{\partial t}$$

$$\rightarrow L^* = \frac{L}{\ell} = \frac{\Phi}{I\ell} = \mu_0 \mu_r \frac{d}{w}$$

→ characteristic impedance of parallel plate:

$$Z_0 = \sqrt{\frac{L^*}{C^*}} = \sqrt{\frac{\mu_0 \mu_r \frac{d}{w}}{\varepsilon_0 \varepsilon_r \frac{w}{d}}} = \frac{d}{w}\sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$$

phase velocity (= wave speed):

$$v = \frac{1}{\sqrt{L^* C^*}} = \frac{1}{\sqrt{\mu_0 \mu_r \cancel{\frac{d}{w}} \cdot \varepsilon_0 \varepsilon_r \cancel{\frac{w}{d}}}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

is independent of geometry,
and for voltage / current wave travelling along $\underline{z}$:

$$\frac{E}{H} = \frac{E_0}{H_0} = \frac{V/d}{I/w} = \frac{w}{d} \underset{= Z_0}{\left(\frac{V}{I}\right)} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$$

Note: $\left.\frac{E_0}{H_0}\right|_{vac} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ \Omega$ is the characteristic impedance of free space. Both $\underline{E}$ and $\underline{H}$ satisfy wave equations of form $\ddot{\underline{f}} = v^2 \nabla^2 \underline{f}$

The wave equation

$$\ddot{\underline{f}} = \boxed{\frac{\partial^2 \underline{f}}{\partial t^2} = v^2 \operatorname{div} \operatorname{grad} \underline{f}} \equiv v^2 \nabla^2 \underline{f}$$

is general, and solutions depend on the geometry:

- plane waves ($\sin$, $\cos$, e^{j})
  in waveguides, laser cavities, optical fibres etc.
- spherical waves ($e^{j(\omega t - \underline{k}\cdot\underline{r})}/r$)
  for point emitters; e.g. atomic scatterers
- distorted wavefields that are different in near-field and far-field conditions, e.g. dipole and multi-pole antennas