The University Of Sheffield.

Data Provided: Laplace and z-transforms,
Performance criteria mappings

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2012-13 (2.0 hours)

ACS342 Feedback Systems Design 3

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. A system has the open-loop transfer function

$$G(s) = \frac{1 + K_M s}{s(s-1)}$$

a. Explain why it is unstable. **(2)**

b. It is placed in a feedback arrangement, as shown in **Figure 1**, with a controller $C(s)$.

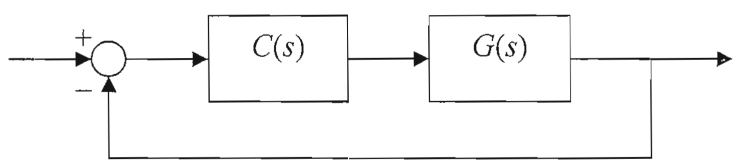

Figure 1

If the transfer function of the controller $C(s)$ is $C(s) = K$, determine the range of K and K_M required to stabilize the system. **(5)**

c. By comparing the transfer function of the closed-loop system of **Figure 1** with that of a standard second-order system, determine the natural frequency and damping ratio of the closed-loop system. **(2)**

d. For the case of $K_M = 1$, sketch the root locus diagram, indicating any important points. **(8)**

e. Owing to the zero in $G(s)$, even the critically-damped closed-loop system exhibits overshoot in response to a step input. It is proposed to modify the controller $C(s)$ to have a new transfer function

$$C(s) = \frac{K}{1 + K_M s}.$$

What effect does this controller have on the stability of the closed-loop system? **(3)**

2. A system has the open-loop transfer function

$$G(s) = \frac{K}{s(s+1)(0.1s+1)}$$

a. For the case of $K = 5$, sketch the Bode diagram. **(8)**

b. Determine the value of K required in order to achieve a gain margin of 10 dB. **(6)**

c. The system is placed in a feedback arrangement, together with a controller $C(s)$, as shown in **Figure 2**.

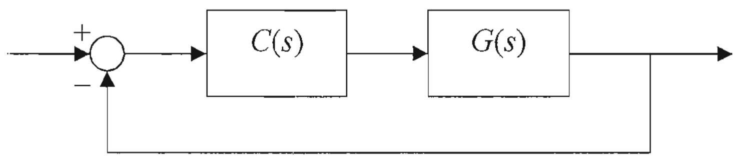

Figure 2

In order to obtain a satisfactory transient response, it is proposed that the controller $C(s)$ is a phase-lead compensator, with transfer function

$$C(s) = \frac{1}{\alpha}\frac{1+\alpha\tau s}{1+\tau s},$$

where $\alpha > 1$. The maximum phase contribution of the compensator occurs at a frequency

$$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$$

Find the values of τ and α that decrease the gain of the overall compensated system $C(s)G(s)$, with respect to that of the uncompensated system $G(s)$, by 3 dB at a frequency $\omega_m = 2$ rad/s. What is the corresponding phase contribution of the compensator? **(6)**

3. The RLC circuit shown in **Figure 3** has an input voltage $v_i(t)$. The output of interest is the voltage $v_o(t)$ across the resistor R_o.

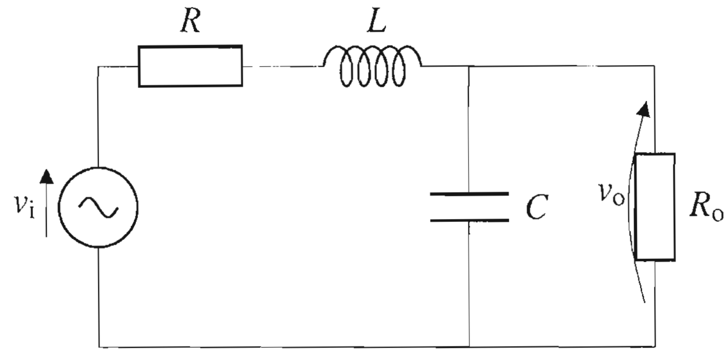

Figure 3

a. Show that the transfer function between input and output voltages is given by

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + \left(RC + \frac{L}{R_o}\right)s + \left(1 + \frac{R}{R_o}\right)}$$

(6)

b. Determine the steady-state output voltage $v_o(t)$ in response to a step input voltage $v_i(t) = 0.1$ V, assuming that $R_o = 100R$. **(4)**

c. The unit step response of an under-damped second-order system is given by

$$v_o(\omega_n t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} \exp(-\zeta\omega_n t) \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \arctan\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]\right)$$

where ζ is the damping ratio and ω_n is the undamped natural frequency. Show that the time t_p at which the peak magnitude occurs is given by

$$\omega_n t_p = \frac{\pi}{\sqrt{1-\zeta^2}}$$

(6)

d. Given that $R_o = 100R$, $C = 1000$ μF and $L = 2$ H, determine the value of R required so that the overshoot of the system in response to a step change in input voltage is not more than 10%. **(4)**

4. A feedback control system is to be designed to control a robot arm. The implementation of a proportional-feedback control system is shown in **Figure 4**.

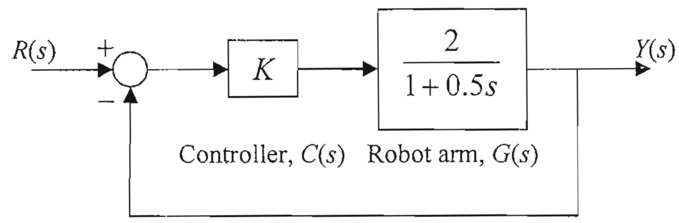

Figure 4

The input to the system is the desired rotational speed of the arm, $r(t)$, and the output is the obtained rotational speed, $y(t)$.

a. Assuming $K = 1$, determine the percentage steady-state error between desired and obtained speeds in response to a step input of 1 rad/s. What value of K is required to obtain an error within 1%? **(3)**

b. For the case of $K = 100$, obtain an expression for the output, $y(t)$, of the controlled system in response to a step input of 1 rad/s applied at time $t = 0$. Hence, determine the angular acceleration of the arm at time $t = 0_{+}$. Comment on the suitability of proportional control for this control system. **(8)**

c. It is proposed to replace the proportional controller K with a proportional plus integral (PI) controller

$$C(s) = K_{\text{p}} + \frac{K_{\text{i}}}{s}$$

Describe briefly the effect the controller has on the closed-loop step response. **(2)**

d. The control system is to be implemented in discrete time, as shown in **Figure 5**.

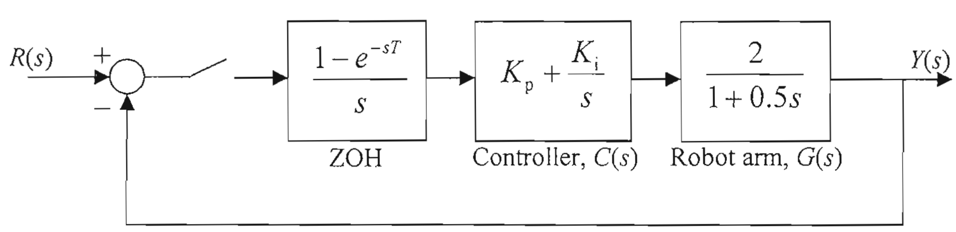

Figure 5

For the case of $K_{\text{p}} = 3$ and $K_{\text{i}} = 1$, and assuming a zero-order hold (ZOH) with a sampling period of $T = 0.5$ s, derive the discrete-time, open-loop transfer function $C(z)G(z)$. **(7)**

PT

Table of Laplace and z-transforms

Time domain	s-domain	z-domain
$f(t)$	$F(s)$	$F(z)$
$f(t-T)$	$e^{-sT}F(s)$	$z^{-1}F(z)$
1	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{zT}{(z-1)^2}$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{zTe^{-aT}}{(z-e^{-aT})^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\frac{z^2-z\cos(\omega T)}{z^2-2z\cos(\omega T)+1}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2-ze^{-aT}\cos(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$	Various forms

Performance Criteria Mappings

10—90% rise time, T_r	$\frac{2.16\zeta+0.6}{\omega_n}$ for $0.3\le\zeta\le0.8$
Percentage overshoot, O.S. (%)	$100\exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$
2% settling time, T_s	$\frac{4}{\zeta\omega_n}$
Resonant frequency, ω_r	$\omega_n\sqrt{1-2\zeta^2}$ for $\zeta<1/\sqrt{2}$
Peak resonant magnitude, M_p	$\frac{1}{2\zeta\sqrt{1-\zeta^2}}$ for $\zeta<1/\sqrt{2}$