EEE6022 (2013-2014) Model solutions

1.a

When friction and the inertia of the pulley and belt are negligible, the force required to move the load of mass m is purely due to acceleration, and given by: (8)

$$F = m\frac{dv}{dt}$$

where the linear acceleration dv/dt is related to the angular acceleration, dω/dt, of the motor by:

$$dv/dt = rd\omega/dt$$

and the force reflected to the motor axis as a load torque T_L is given by:

$$T_L = F\,r$$

Thus the total motor torque required for accelerating/decelerating the drive system, expressed in terms of dv/dt is given by:

$$T_{em} = (J_m/r + r\ m)\frac{dv}{dt}$$

As can be seen, the equivalent inertia $J_{eq} = (J_m/r + rm)$ is a function of radius r, and reaches its minimum when

$$dJ_{eq}/dr = -J_m/r^2 + m = 0$$

Thus:

$$r = \sqrt{\frac{J_m}{m}}$$

1.b

The optimal pulley radius is given by: (9)

$$r = \sqrt{\frac{J_m}{m}} = \sqrt{\frac{0.002}{0.2}} = 0.1(m)$$

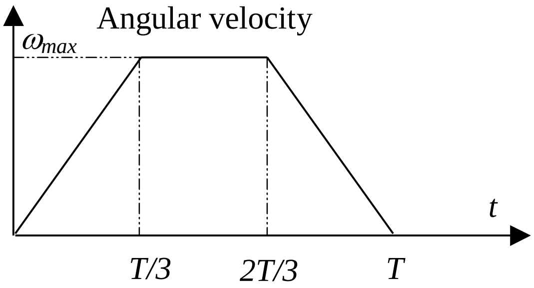
and the combined inertia on the motor axis is:

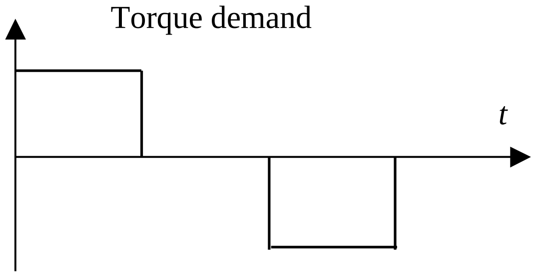
$$J_{eq} = (J_m + r^2\ m) = 2J_m = 0.004(kgm^2)$$

The total angular distance to be rotated by the motor is

$$\Theta = 0.4/0.1 = 4\ \ (rad)$$

The corresponding trapezoidal velocity profile, and torque demand is shown in the figure below:





From the velocity profile, it can be derived that the maximum speed is related to the angular distance Θ and the time period T by:

$$\omega_{\max} = 3\Theta/2T = 3*4/2/0.1 = 60\ \text{(rad/s)}$$

and the maximum angular acceleration is

$$\alpha_{\max} = \omega_{\max}/(T/3) = 3*60/0.1 = 1800\ (\text{rad/s}^2)$$

The peak torque requirement is

$$J_{eq}a_{\max} = 2.4*3 = 7.2\ \text{(Nm)}$$

and the rms torque is

$$\sqrt{2*7.2^2/3} = 5.88\ \text{(Nm)}$$

which is less than the rated rms torque of 6.0 (Nm). Thus the motor would not overheat.

1.c For the triangular velocity profile shown in below, the the maximum speed is related to the angular distance Θ and the time period T by: (3)

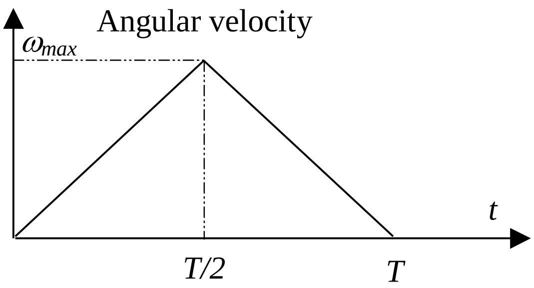
$$\omega_{\max} = 2\Theta/T = 2*4/0.1 = 80\ \text{(rad/s)}$$

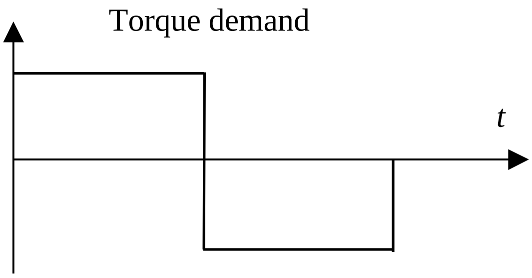
and the acceleration is

$$\alpha_{\max} = \omega_{\max}/(T/2) = 2*80/0.1 = 1600\ (\text{rad/s}^2)$$

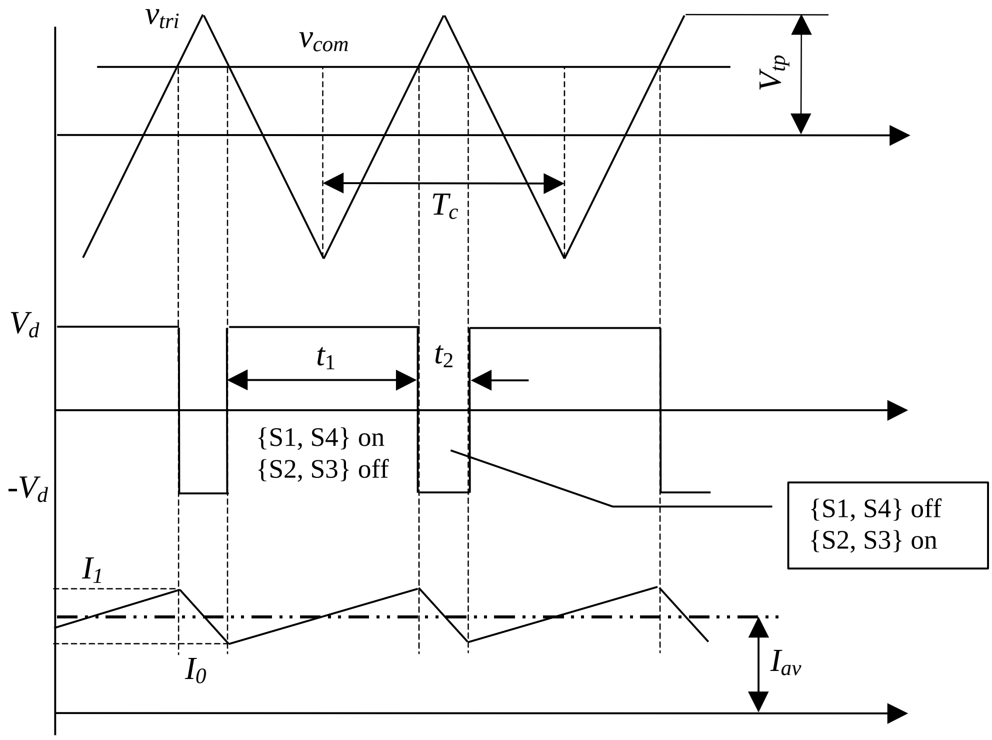
The peak torque and maximum power are given by:

$$J_{eq}a_{\max} = 1600*0.004 = 6.4\ \text{(Nm)}$$
$$P = J_{eq}a_{\max}\omega_{\max} = 1600*0.004 = 6.4*80 = 512(W)$$





2.a The waveforms for the pulse width modulation in bipolar mode are shown below. The control input v_{con} is compared with the triangle carrier, v_{tri} , if $v_{con} > v_{tri}$, S1 and S4 will be switched on while S2 and S3 are off. The output voltage is V_d, and the motor current increases under the influence of V_d. If, however, $v_{con} < v_{tri}$, S2 and S3 are on, S1 and S4 are off, and the output voltage is $-V_d$. The motor current will decreases due to the negative voltage being applied. (6)



2.b With reference to the bi-polar operation waveforms shown above, the average voltage output of the converter is given by: (3)

$$V_t = [t_1V_d - (T_c\text{-}t_1)V_d]/T_c = V_d(2D\text{-}1) \qquad (1)$$

The duty ratio D is defined as $D = t_1/T_c$

Assume that the command signal is constant within a carrier cycle, from the triangle waveform, one can obtain:

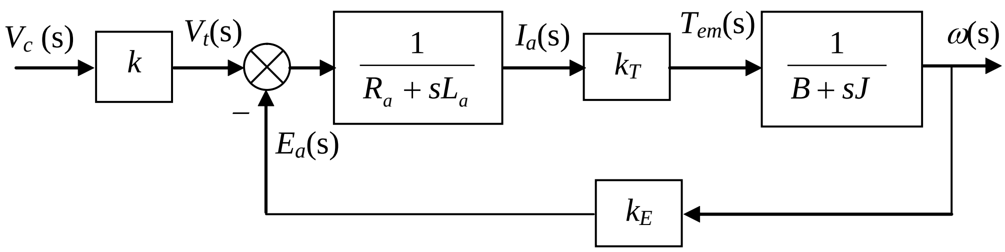
$$\frac{T_c}{2V_{tp}} = \frac{t_1}{(V_{tp} + v_{com})} \qquad \therefore \qquad 2D = 1 + \frac{v_{com}}{V_{tp}} \qquad (2)$$

Substituting (2) into (1) results in:

$$V_t = (V_d/V_{tp})v_{com} = k\, v_{com}$$

i.e., the gain of the converter is V_d/V_{tp}

2.c The transfer function block diagram between the control input, $v_c(s)$, and the motor speed, ω(s) is shown below: (5)



From the motor parameters, the parameters of various blocks are determined as follows:

k_T =10Nm/20A = 0.5= k_E; B = 0.0; armature resistance R_a = 0.37Ω

armature inductance L_a = 1.5 mH.

The moment of inertia is J = 4.0x10^{-3} (kgm^2)
H-bridge converter gain $k = V_d/V_{tp} = 20$

From the above block diagram, the transfer function between the control input and motor speed can be derived:

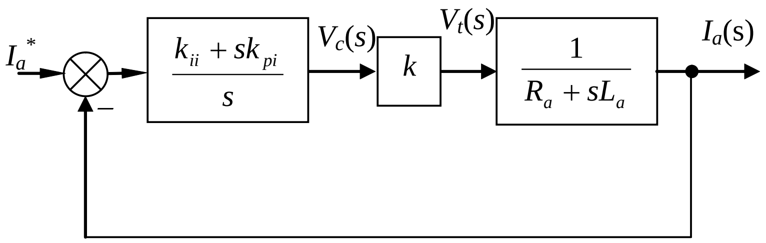
$$G(s) = \left.\frac{\omega(s)}{V_c(s)}\right| = \frac{kk_T}{(R_a + sL_a)sJ_{eq} + k_Tk_E} = \frac{k}{k_E}\frac{1}{\left(\frac{L_aJ_{eq}}{k_Tk_E}s^2 + \frac{R_aJ_{eq}}{k_Tk_E}s + 1\right)}$$

The electrical and mechanical time constants of the system are given respectively by:

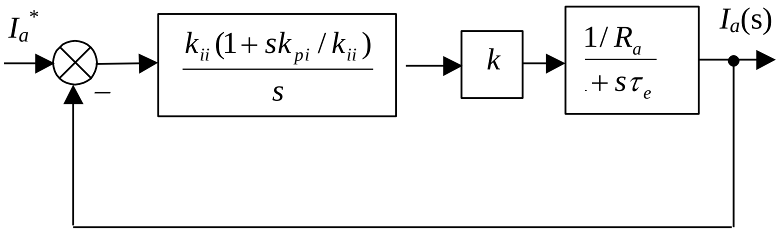
$$\text{Mechnical time constant}\ \tau_m = \frac{R_aJ_{eq}}{k_Tk_E} = \frac{0.37 \times 4.0 \times 10^{-2}}{0.5 \times 0.5} = 0.0592(s)$$

$$\text{Electrical time constant } \tau_e = \frac{L_a}{R_a} = \frac{1.5\times10^{-3}}{0.37} = 4.05\,(\text{ms})$$

2.d The block diagram of the PI current control loop, neglecting the effect of back-emf, is shown below: (6)



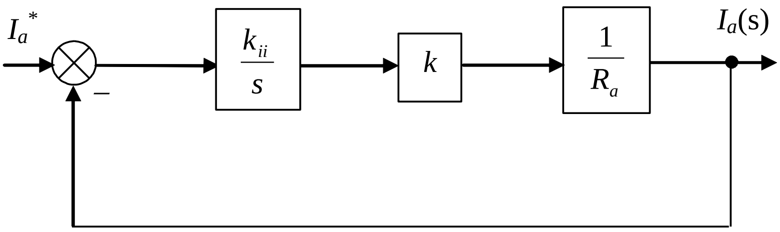
where k_{ii} and k_{pi} the integral and proportional gains of the current PI control loop , and k the gain of the H-bridge converter. The above diagram may be redrawn in the following form:



Use pole-zero cancelling,

$k_{pi}/k_{ii} = \tau_e$

the resulting transfer function block diagram is:



The closed loop current transfer function is:

$$\frac{I_a^*(s)}{I_a(s)} = \frac{\frac{k_{ii}k}{R_a s}}{1+\frac{k_{ii}k}{R_a s}} = \frac{k_{ii}k}{R_a s + k_{ii}k} = \frac{1}{\frac{R_a}{k_{ii}k}s+1}$$

To achieve the desired time constant of 1.0ms

$R_a/(k_{ii}k) = 0.001$ or $k_{ii} = R_a/(0.001*20) = 18.5$

$k_{pi} = k_{ii}\tau_e = 18.5*0.00405 = 0.075$

3.a. Let the currents and induced emfs in phases, a, b, and c of the sinusoidal waveform motor be denoted by $[i_a\ i_b\ i_c]$ and $[e_a\ e_b\ e_c]$ respectively, the electromagnetic power is given by: (6)

$$P_{em} = i_a e_a + i_b e_b + i_c e_c$$

and the electromagnetic torque is therefore given by:

$$T_{em} = P_{em}/\omega_m = \frac{1}{\omega_m}(i_a e_a + i_b e_b + i_c e_c) = \frac{1}{\omega_m}[e_a \quad e_b \quad e_c]\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

where ω_m is the angular speed of the motor.

Since

$$\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix} = \frac{d}{d\theta}\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix}\frac{d\theta}{dt} = \frac{d}{d\theta}\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix}p\omega_m$$

$$\therefore T_{em} = \frac{p\omega_m}{\omega_m}\frac{d}{d\theta}[\Psi_a \quad \Psi_b \quad \Psi_c]\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = p\frac{d}{d\theta}[\Psi_a \quad \Psi_b \quad \Psi_c]\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Represent the abc quantities of the flux-linkages and currents in their αβ counterparts

$$\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix} = C_{abc\leftarrow\alpha\beta}\begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix} \quad ; \quad \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = C_{abc\leftarrow\alpha\beta}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

$$\therefore T_{em} = p\frac{d}{d\theta}[\Psi_a \quad \Psi_b \quad \Psi_c]\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = p\frac{d}{d\theta}\left([\Psi_\alpha \quad \Psi_\beta][C_{abc\leftarrow\alpha\beta}]^T\right)C_{abc\leftarrow\alpha\beta}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

Since the transformation matrix is constant

$$\frac{d}{d\theta}\left([\Psi_\alpha \quad \Psi_\beta][C_{abc\leftarrow\alpha\beta}]^T\right) = \frac{d}{d\theta}[\Psi_\alpha \quad \Psi_\beta]\left([C_{abc\leftarrow\alpha\beta}]^T\right)$$

For a sine wave motor,

$$\begin{bmatrix}\Psi_\alpha \\ \Psi_\beta\end{bmatrix} = \Psi_m \begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix} \quad \therefore \quad \frac{d}{d\vartheta}\begin{bmatrix}\Psi_\alpha \\ \Psi_\beta\end{bmatrix} = \Psi_m \begin{bmatrix}-\sin\theta \\ \cos\theta\end{bmatrix} = \begin{bmatrix}-\Psi_\beta \\ \Psi_\alpha\end{bmatrix}$$

$$\therefore T_{em} = p\begin{bmatrix}-\Psi_\beta & \Psi_\alpha\end{bmatrix}\left(\left[C_{abc\leftarrow\alpha\beta}\right]^T\right)C_{abc\leftarrow\alpha\beta}\begin{bmatrix}i_\alpha \\ i_\beta\end{bmatrix}$$

It can be shown that

$$\left(\left[C_{abc\leftarrow\alpha\beta}\right]^T\right)C_{abc\leftarrow\alpha\beta}$$

$$= \begin{bmatrix}1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2\end{bmatrix}\begin{bmatrix}1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2\end{bmatrix} = \frac{3}{2}\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

Finally

$$\therefore T_{em} = p\begin{bmatrix}\Psi_\alpha & \Psi_\beta\end{bmatrix}\frac{3}{2}\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix}i_\alpha \\ i_\beta\end{bmatrix} = \frac{3p}{2}\begin{bmatrix}-\Psi_\beta & \Psi_\alpha\end{bmatrix}\begin{bmatrix}i_\alpha \\ i_\beta\end{bmatrix} = \frac{3p}{2}\left(\Psi_\alpha i_\beta - \Psi_\beta i_\alpha\right)$$

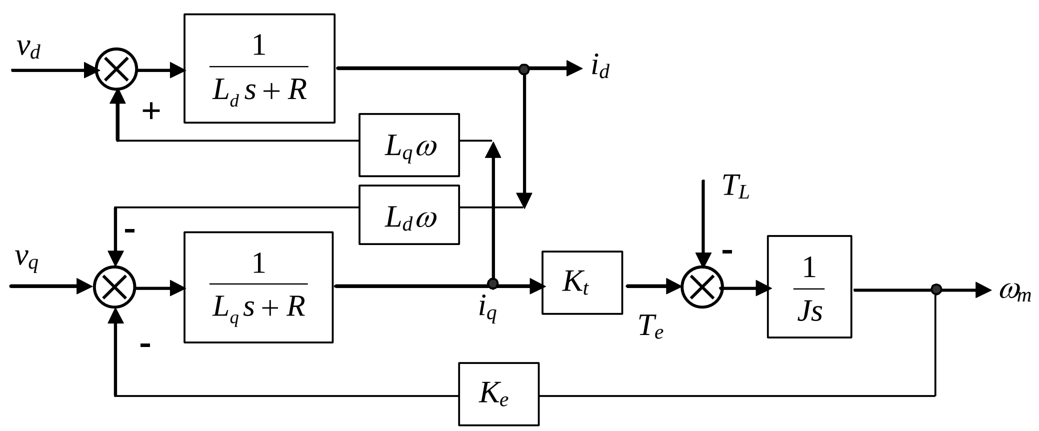
3.b From the motor data, the no-load peak flux linkage is produced by the permanent magnets and is given by: (2)

$$\Psi_m = \sqrt{2}\Psi_{rms} = \sqrt{2}E_{rms}/\omega = \sqrt{2}*25/(2\pi*2*1000/60) = 0.169 \text{ (Wb)}$$

The motor torque constant is:

$$K_T = 3p\Psi_m/\sqrt{2} = 0.716 \text{ (Nm/Arms)}$$

3.c The transfer function block diagram of the motor in dq reference frame is (4)



$L_d = L_q = L_s$ = 3/2*4.8 =7.2 (mH), $R = R_a$ = 4.2Ω,
$K_t = 3p\Psi_m/2 = 0.506$, $K_e = p\Psi_m = 0.338$, J = 0.005 kgm^2

3.d To compensate for the coupling terms in the dq axis currents, two new control inputs, $v_d^{'}$ and $v_q^{'}$ are used and given by: (8)

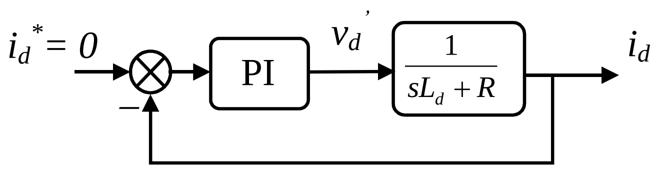
$$v'_d = v_d + \omega L_q i_q$$
$$v'_q = v_q - \omega L_d i_d$$

The resultant d, q axis current dynamics are now governed by:

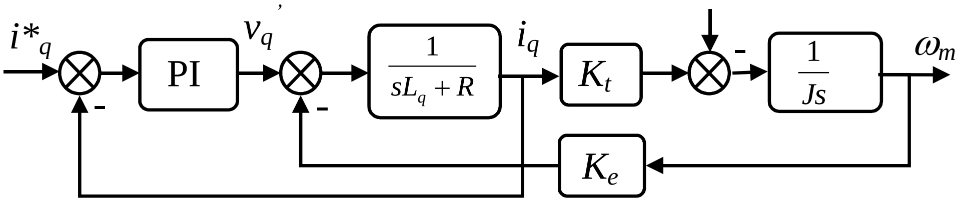
$$L_d \frac{di_d}{dt} + Ri_d = v'_d$$

$$L_q \frac{di_q}{dt} + Ri_q = v'_q - K_e \omega_m$$

Each current component may be controlled by a PI controller as shown below:



d-axis current control loop



q-axis current control loop

Since the current response is much faster than the speed response, the effect of the back emf on the q axis current may be neglected in the controller design.

If the PI current controller takes the form of

$$\frac{k_{ii} + k_{pi} s}{s}$$

and using pole-zero cancelling:

$$k_{pi}/k_{ii} = L_d/R$$

The closed loop transfer function for both d, and q axis currents is:

$$\frac{i_{d,q}(s)}{i^*_{d,q}(s)} = \frac{\frac{k_{ii}}{R\ s}}{1 + \frac{k_{ii}}{R\ s}} = \frac{k_{ii}}{R\ s + k_{ii}} = \frac{1}{\frac{R}{k_{ii}} s + 1}$$

To achieve the desired time constant of 1.0ms

$R/(k_{ii}) = 0.001$ or $k_{ii} = R/(0.001) = 4.2/0.001 = 4200$

$k_{pi} = k_{ii} \tau_e = 4200 * 0.0072/4.2 = 7.2$

Note the gains k_{pi} and k_{ii} include the dc gain of the power electronic converter. Finally, the actual control outputs for v_d and v_q are :

$$v_d = v'_d - \omega L_q i_q$$
$$v_q = v'_q + \omega L_d i_d$$

4.a From the definition for the voltage space vector, and note that (8)

$$e^{j2\pi/3} = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3} \quad ; \quad e^{j4\pi/3} = \cos\frac{4\pi}{3} + j\sin\frac{4\pi}{3}$$

$$\begin{aligned}
\vec{V}_s &= (2/3)(v_a + v_b e^{j2\pi/3} + v_c e^{j4\pi/3}) \\
&= \frac{2}{3}\left[V_m \sin\omega t + V_m \sin\left(\omega t - \frac{2\pi}{3}\right)*\left(\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}\right) + V_m \sin\left(\omega t + \frac{2\pi}{3}\right)*\left(\cos\frac{4\pi}{3} + j\sin\frac{4\pi}{3}\right)\right] \\
&= \frac{2}{3}V_m\left\{\left[\sin\omega t - \frac{1}{2}\sin\left(\omega t - \frac{2\pi}{3}\right) - \frac{1}{2}\sin\left(\omega t + \frac{2\pi}{3}\right)\right] + j\frac{\sqrt{3}}{2}\left[\sin\left(\omega t - \frac{2\pi}{3}\right) - \sin\left(\omega t + \frac{2\pi}{3}\right)\right]\right\} \\
&= \frac{2}{3}V_m\left\{\left[\sin\omega t - \sin\left(\frac{\omega t - \frac{2\pi}{3} + \omega t + \frac{2\pi}{3}}{2}\right)\cos\left(\frac{\omega t - \frac{2\pi}{3} - (\omega t + \frac{2\pi}{3})}{2}\right)\right]\right. \\
&\quad \left. + j\frac{\sqrt{3}}{2}\cos\left(\frac{\omega t - \frac{2\pi}{3} + \omega t + \frac{2\pi}{3}}{2}\right)\sin\left(\frac{\omega t - \frac{2\pi}{3} - (\omega t + \frac{2\pi}{3})}{2}\right)\right\} \\
&= \frac{2}{3}V_m\left\{\frac{3}{2}\sin\omega t - j\frac{3}{2}\cos\omega t\right\} = V_m\{\cos(\omega t - \pi/2) + j\sin(\omega t - \pi/2)\} = V_m\angle(\omega t - \pi/2)
\end{aligned}$$

The result shows that the magnitude of the voltage space vector is constant and equal to V_m, whilst its angle varies with time, i.e.,

$$\theta = \omega t - \pi/2$$

This is referred to as rotating vector, and its speed of rotation can be found by

$$\frac{d\theta}{dt} = \omega$$

i.e., equal to the frequency of the applied phase voltage.

4.b.(i) (8)

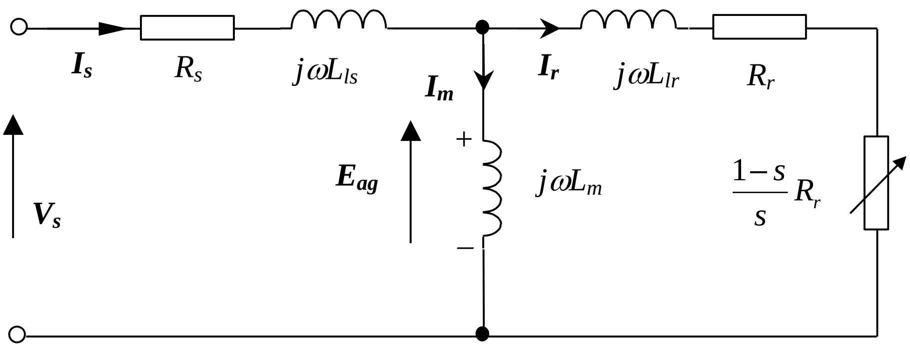
At rated speed of 1450 rpm, the slip s is given by:

s = (1500-1450)/1500 = 0.033

For small values of s, $sR_s << R_r$ and $s\omega L_l << R_r$, and the motor electromagnetic torque is proportional to slip s. Thus, at 50% load torque, the slip s is 0.0167

The rotor speed is 1500*(1-s) = 1475 (rpm)

The equivalent circuit diagram is shown below:



$R_r/s + j\omega L_{lr}$ = 0.55/0.0167+j0.95 = 32.93+ j0.95

The equivalent impedance of the parallel of the magnetising branch and the rotor branch is

$$\frac{(32.9+j0.95)j48.6}{32.9+j(0.95+48.6)} = \frac{1601.3\angle 91.65^0}{59.48\angle 56.42^0} = 26.92\angle 35.23^0 = 21.99+j15.53$$

Total impedance seen from the stator terminal:

$$0.35+j1.20+21.99+j15.53 = 22.34+j16.73 = 27.91\angle 36.82^0$$

Stator current

$$I_s = 240/27.91\angle 36.82^0 = 8.60\angle -36.82^0$$

Power factor

$$\cos\varphi = \cos 36.82^0 = 0.80$$

Induced air-gap voltage

$$E_{ag} = 240-(R_s + j\omega L_{ls})I_s = 240-(1.25\angle 73.74)8.60\angle -36.82^0 = 231.4 - j6.46$$
$$= 231.49\angle -1.6^0$$

Rotor current

$$I_r = E_{ag}/(R_r/s + j\omega L_{lr}) = 231.49\angle -1.6^0/(32.94\angle 1.65^0) = 7.03\angle -3.25^0$$

Air-gap flux linkage

$$\Psi_{ag} = E_{ag}/4.44f = 231.43/4.44/50 = 1.04(Wb)$$

The electromagnetic torque

$$T_{em} = \frac{3R_r I_r^2}{s\omega_s} = \frac{3*0.55*7.03^2}{0.0167*157.08} = 31.1(Nm)$$

Efficiency

$$\eta = P_{out} / 3I_s V_s \cos\varphi = 31.1*154.46/(3*240*8.6*0.8) = 0.97$$

Note the iron loss is not represented in the equivalent circuit, and therefore the efficiency is slightly overestimated.

4.b.(ii) (4)

At the given operation point of the rated torque at 1000 rpm, the assumption of sR_s << R_r and $s\omega L_l$ << R_r is still valid. Thus the electromagnetic torque is approximately given by:

$$T_{em} = \frac{3pV_s^2}{\omega R_r} s$$

For the same output torque, the following must be true:

$$\frac{V_s^2}{\omega} s = \text{constant} = k_1 \qquad (1)$$

On the other hand, in order to maintain approximately constant torque capability for all frequencies of motor operation, the ratio of supply voltage to angular frequency, (V_s/ω), should be kept constant, i.e.,

$$\frac{V_s}{\omega} = \text{constant} = k_2 \qquad (2)$$

Substituting $V_s = \omega k_2$ into (1) yields:

$$\omega s = k_1 / k_2^2 \quad or \quad \omega \frac{\omega - p\omega_r}{\omega} = \omega - p\omega_r = k_1 / k_2^2$$

$$\omega = k_1 / k_2^2 + p\omega_r$$

The values for k_1 and k_2 can be found at the rated operating point

$$k_1 = \frac{V_s^2}{\omega} s = \frac{240^2}{314} 0.033 = 6.05$$

$$k_2 = \frac{V_s}{\omega} = \frac{240}{314} = 0.764$$

The supply angular frequency at 1000 rpm is

$$\omega_1 = k_1 / k_2^2 + p\omega_r = 6.05 / 0.764^2 + 2*1000*2*\pi / 60 = 219.8 \text{ (rad/s)}$$

The frequency and voltage are

f_1 = 219.8/(2π) = 34.98 (Hz)
V_s = 0.764 * 219.8 = 167.93 (V)