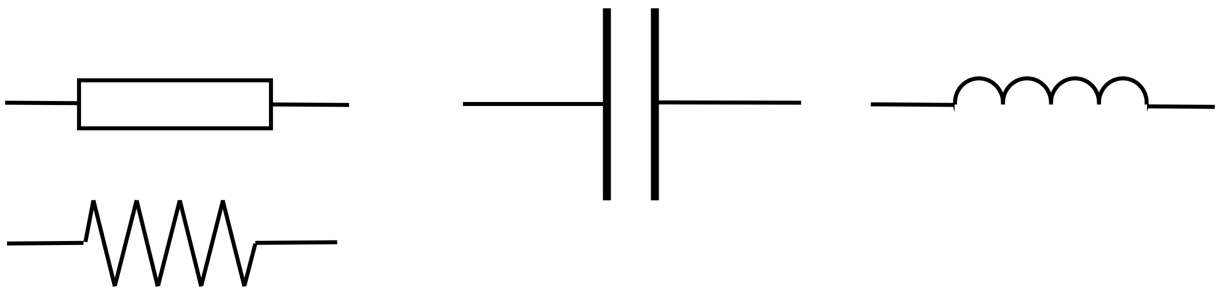
Circuit Components

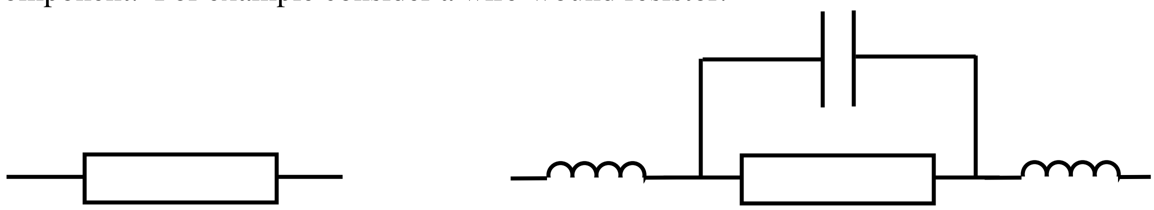
The three basic building blocks of electrical circuits are **Resistance** (***R***), **Capacitance** (***C***), and **Inductance** (***L***). Their circuit symbols are as follows:

Resistance, *R* **Capacitance, *C*** **Inductance, *L***



(Old symbol)

Normally on this course we will only consider components that are ideal, e.g. purely resistive, purely capacitive, or purely inductive. However, in practice there are elements of all three in any one component. For example consider a wire-wound resistor:



Ideal Resistor Practical Resistor

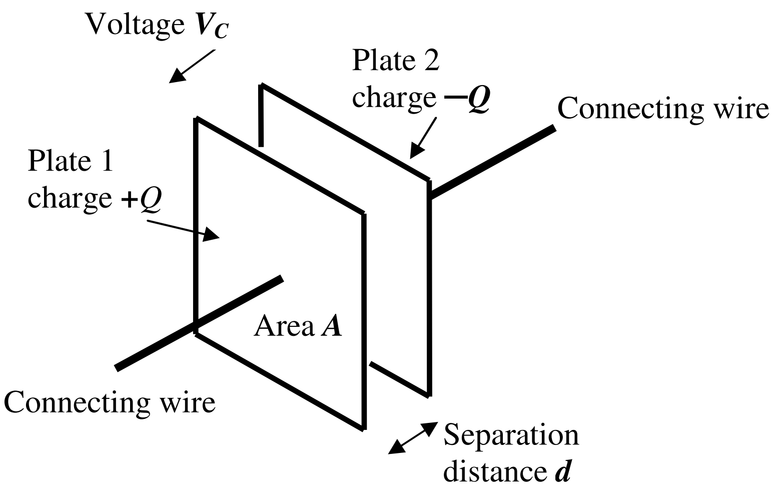
Since a wire-wound resistor is essentially a coil of high resistance wire then there will also be an element of inductance associated with it. There may also be capacitive effects between neighbouring turns. Usually care is taken to minimise these effects, so in the case of a resistor it is the resistance which is dominant.

For this course, unless specifically mentioned, it will be assumed that components purely resistive, capacitive or inductive.

So far we have been looking at circuits containing resistors. Now we will study the behaviour of capacitors and inductors in turn.

<u>Capacitance, *C*</u>

The simplest capacitor consists of two metal plates, separated by an insulating medium (called a dielectric) and has the capacity for storing electric charge. The dielectric may be air or other insulating material e.g. waxed paper, mica, plastic etc.



Assume the plates each have and area, A, (m^2) and are separated by a distance, d (m). If the charge on plate 1 is $+Q$ (C) and that on plate 2 is $-Q$ (C) then a potential difference exists between the two plates, V_C (V).

The capacitance, C, for this arrangement is then defined as:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

where ε_0 is called the permittivity of free space (8.85×10^{-12}F/m) and ε_r is called the relative permittivity and is a property of the material occupying the space between the plates. (Air has a relative permittivity of 1).

The unit of capacitance is the **Farad (F)**, but it is usual to find subdivisions, for example:

mF – millifarad ($\times 10^{-3}$ F)
μF – microfarad ($\times 10^{-6}$ F)
nF – nanofarad ($\times 10^{-9}$ F)
pF – picofarad ($\times 10^{-12}$ F)

The charge stored in the capacitor, Q, is related to the capacitance, C, and the voltage across the capacitor, V_C, by:

$$Q = C \times V_C$$

Charge = Capacitance × Voltage

And the energy stored in the capacitor is given by:

$$Energy\ stored = \frac{1}{2} C \times V_C^2$$

Energy is stored in the capacitor by virtue of the electric field.

Previously we have shown that:

$$Q = I \times t$$

i.e. the charge stored, Q, is equal to the product of the current, I, and the time, t. If the current also varies with time (i.e. $i(t)$) then we can obtain a more general expression for the charge stored:

$$Q = \int i(t)dt$$

and we also get:

$$V_C = \frac{1}{C}\int_0^t i(t)dt$$

Differentiating the equation gives an expression for the current:

$$i(t) = C\frac{dV_C}{dt}$$

Example

A 680μF capacitor is charged by a trickle current of 10mA for a period of 4 seconds. If the capacitor was initially uncharged, calculate the charge and energy stored in the capacitor and the voltage across its terminals.

The charge stored is given by:

$$Q = I \times t = 10 \times 10^{-3} \times 4 = 0.04 \text{ C}$$

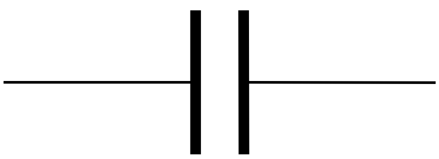
The voltage across its terminals is:

$$V_C = \frac{Q}{C} = \frac{0.04}{680 \times 10^{-6}} = 58.8 \text{ V}$$

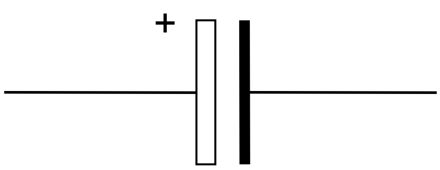
and the stored energy is:

$$E_C = \frac{1}{2}CV_C^2 = \frac{1}{2} \times 680 \times 10^{-6} \times 58.8^2 = 1.175 \text{ J}$$

The circuit symbol for a capacitor is two vertical bars (representing the plates in a capacitor):



Standard capacitor



Polarised capacitor

Types of capacitor

(a) Paper

This type consists of metal foils interleaved with paper, which may be impregnated with wax or oil and then rolled into a compact form. A variant is the metallized-paper capacitor which is very similar to the paper capacitor, but has the paper coated with a thin layer of metal on one side. Two such layers are rolled together to form a capacitor which is then placed in a sealed container. The advantage of the metallized-paper capacitor is that it is selfhealing – should a localised breakdown of the dielectric occur, the heat vaporises the metallic coating and the conductor around the problem area is thereby removed.

(b) Mica — Consists of alternate layers of mica and metal foil clamped tightly together. Sometimes the metal foils are replaced by thin layers of silver deposited on the two sides of the mica sheet. Because of the relatively high cost of this type of capacitor it is mainly used in high frequency circuits where it is necessary to minimise losses in the dielectric.

(c) Polycarbonate — A type of plastic which can be produced in thicknesses down to about 2μm and a low dielectric loss.

(d) Ceramic — Formed by creating metallic coatings on the opposite faces of a thin slab of ceramic material. These types of capacitor tend to have a low value of capacitance, but are useful in high-frequency applications, especially in high temperature environments.

(e) Electrolytic — Very commonly used type of capacitor consisting of two aluminium foils, one with an oxide film and one without, interleaved with a material such as paper saturated with a suitable electrolyte. The finished unit is assembled in a can and hermetically sealed. The oxide film acts as a dielectric with a typical thickness of 15μm for a working voltage of 100V meaning that a very large capacitance can be obtained in a very small volume. The main disadvantage of electrolytic capacitors is that they are only suitable for use in circuits where the voltage applied to the capacitor never reverses direction (i.e. they are polarised and connecting up an electrolytic capacitor incorrectly is likely to destroy it). Their main use is where very large capacitances are required (e.g. reducing ripple voltage obtained from a rectifier in a power supply).

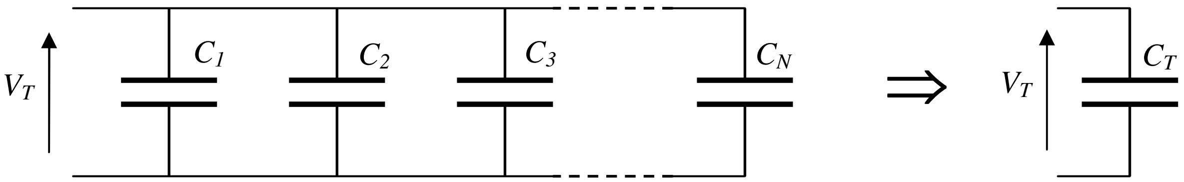
(f) Tantalum — Much smaller than the corresponding aluminium electrolytic capacitors and once again they are polarised so care must be taken to ensure they are connected the correct way round. One plate consists of pressed, sintered tantalum powder coated with an oxide layer which forms the dielectric. The case, which may be of brass, copper or silver forms the other plate. Layers of manganese dioxide and graphite form the electrolyte.

(g) Air — This type consists of one set of fixed metal plates interspaced with another set of movable plates. As the movable plates pass over the fixed plates the effective area of the capacitor changes, which results in a variable capacitance. This type of capacitor was used for tuning in older radios, but this is now done by solid-state devices.

(h) Supercapacitors — A relatively recent development which are also known as ultracapacitors or electrochemical double layer capacitors. It stores energy by polarising an electrolytic solution, although there are no chemical reactions involved with the energy storage mechanism which allows the capacitor to be charged and discharged many hundreds of thousands of times. It can be viewed as two non-reactive porous plates suspended within an electrolyte with a voltage applied across the plates. The porous carbon-base electrode material allows its effective surface area to approach 2000m^2 per gram, very much greater than can be obtained using flat or textured films and plates. The charge separation distance is determined by the size of the ions in the electrolyte and is therefore much smaller than can be achieved using a conventional electrolyte. The combination of large area and small charge separation gives the supercapacitor outstanding capacitance compared to conventional capacitors, with values of several thousand Farads being possible. However, the maximum voltage that the capacitor can withstand is fairly low, typically a few volts, however this is increasing as research into the technology is undertaken. Supercapacitors are finding uses in a variety of fields including as a replacement or supplement for batteries in hybrid-electric vehicles.

Connecting capacitors in parallel

For a group of capacitors, C_1, C_2, C_3,….. C_N connected in parallel (i.e. *with the same voltage applied across each capacitor*) find the value of a single equivalent capacitor.



Consider the charge on each capacitor:

On capacitor C_1 $\quad Q_1 = C_1 V_T$

On capacitor C_2 $\quad Q_2 = C_2 V_T$

On capacitor C_3 $\quad Q_3 = C_3 V_T$

$\vdots \qquad \vdots$

On capacitor C_N $\quad Q_N = C_N V_T$

Therefore the total charge in the system is:

$$Q_T = Q_1 + Q_2 + Q_3 + \cdots + Q_N$$

Now the total capacitance, C_T is:

$$C_T = \frac{Q_T}{V_T} = \frac{Q_1 + Q_2 + Q_3 + \cdots + Q_N}{V_T} = \frac{C_1 V_T + C_2 V_T + C_3 V_T + \cdots + C_N V_T}{V_T}$$

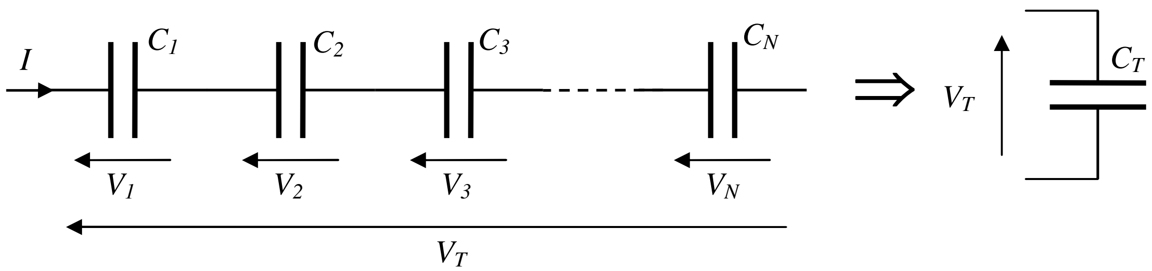
i.e:

$$C_T = C_1 + C_2 + C_3 + \cdots C_N$$

For capacitors in parallel the total equivalent capacitance is equal to the sum of the individual values of capacitance.

Connecting capacitors in series

For a group of capacitors, C_1, C_2, C_3,….. C_N connected in series (i.e. with the same current flowing through each capacitor) find the value of a single equivalent capacitor.



Since $Q = I \times t$ and the same current is flowing through each capacitor then the charge on each capacitor is the same. Hence:

$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3 = \cdots = C_N V_N$$

or rearranging the individual terms:

For capacitor C_1 $\quad V_1 = \dfrac{Q}{C_1}$

For capacitor C_2 $\quad V_2 = \dfrac{Q}{C_2}$

For capacitor C_3 $\quad V_3 = \dfrac{Q}{C_3}$

$\vdots \qquad\qquad \vdots$

On capacitor C_N $\quad V_N = \dfrac{Q}{C_N}$

Now the total voltage (which is the voltage across our equivalent capacitor, C_T) is:

$$V_T = V_1 + V_2 + V_3 + \cdots + V_N$$

and so:

$$C_T = \frac{Q}{V_T} = \frac{Q}{V_1 + V_2 + V_3 + \cdots + V_N} = \frac{Q}{\frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \cdots + \frac{Q}{C_N}}$$

which leads to the expression for calculating the total capacitance for a group of capacitors in series as:

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_N}$$

Example

Three capacitors have capacitances of 2μF, 5μF and 9μF respectively. Find the total capacitance when they are connected (i) in parallel, and (ii) in series.

(i) For a parallel connection:

$$C_T = C_1 + C_2 + C_3 = 2 + 5 + 9 = 16 \ \mu\text{F}$$

(ii) For a series connection:

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{\frac{1}{2} + \frac{1}{5} + \frac{1}{9}} = 1.23\mu\text{F}$$

Example

Two capacitors having capacitances of 6μF and 10μF respectively are connected in series across a 200V supply. Find (i) the voltage across each capacitor, (ii) the charge on each capacitor and (iii) the energy stored in each capacitor.

(i) Since the capacitors are connected in series, the charge on each is the same hence:

$$C_1V_1 = C_2V_2 \qquad \text{or} \qquad 6V_1 = 10V_2$$

and we also know that the total voltage across the series combination is 200V so:

$$V_1 + V_2 = 200$$

multiplying this equation by 10 gives:

$$10V_1 + 10V_2 = 2000$$

and substituting for V_2 gives:

$$10V_1 + 6V_1 = 2000 \qquad \text{or} \qquad V_1 = 125 \text{ V}$$

and:

$$V_2 = 200 - V_1 \qquad \text{or} \qquad V_2 = 75 \text{ V}$$

(ii) The charge on each capacitor is the same since they are connected in series:

$$Q = C_1 V_1 = 6\times10^{-6}\times125 = 750\mu\text{C} = 0.00075\text{C}$$

(iii) The energy stored in each capacitor is:

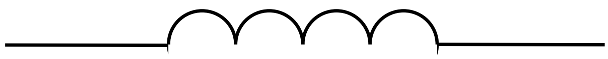
$$E_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times6\times10^{-6}\times125^2 = 46.8\text{mJ}$$

$$E_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times10\times10^{-6}\times75^2 = 28.1\text{mJ}$$

Inductance, *L*

The simplest form of inductor consists of a coil of wire wound round a former or 'core'. The core may be iron or steel, ferrite or air. An electric current always creates a magnetic field and hence energy is stored in the inductor by virtue of the magnetic field.

The circuit symbol for an inductor is a number of semi-circles which are supposed to represent a coil of wire:



The unit of capacitance is the **Henry (H)**, but it is usual to find subdivisions, for example:

mH – millihenry ($\times10^{-3}$ H)
μH – microhenry ($\times10^{-6}$ H)

And the energy stored in the inductor is given by:

$$Energy\ stored = \frac{1}{2}L\times I^2$$

Energy is stored in the inductor by virtue of the magnetic field.

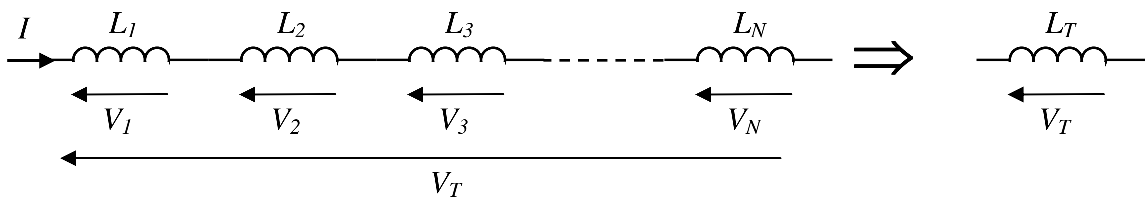
The voltage appearing across the inductor is proportional to the rate of change of current flowing through it, i.e.:

$$V_L = L\frac{dI}{dt}$$

Therefore there must be a change in the current for a voltage to be established. If the current is constant then there is no voltage across the inductor. The voltage induced is in such a direction as to oppose the change that is causing it. Thus if the current flowing through the inductor is reducing, the induced voltage will in a direction to try and increase the current (i.e. oppose the change).

Connecting inductors in series

For a group of inductors, L_1, L_2, L_3,….. L_N connected in series (i.e. with the same current flowing through each inductor) find the value of a single equivalent inductor, L_T.



For inductor L_1 $\qquad V_1 = L_1 \frac{dI}{dt}$

For inductor L_2 $\qquad V_2 = L_2 \frac{dI}{dt}$

For inductor L_3 $\qquad V_3 = L_3 \frac{dI}{dt}$

: $\qquad$:

For inductor L_N $\qquad V_N = L_N \frac{dI}{dt}$

However for our equivalent inductor:

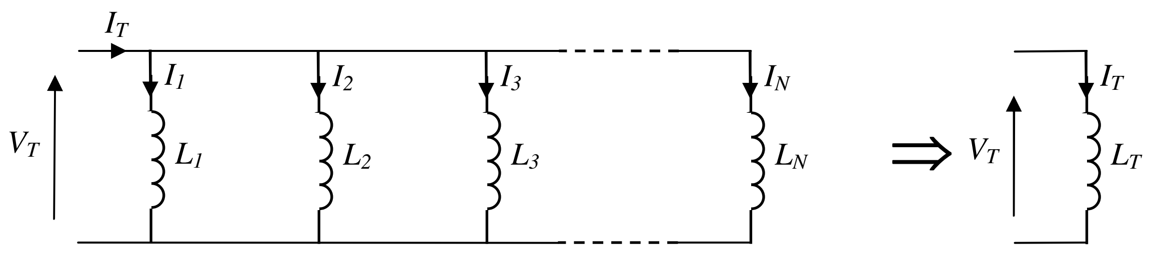
$$L_T \frac{dI}{dt} = V_T = V_1 + V_2 + V_3 + \cdots + V_N = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} + L_3 \frac{dI}{dt} + \cdots + L_N \frac{dI}{dt}$$

which leads to the expression for calculating the total inductance for a group of inductors connected in series:

$$L_T = L_1 + L_2 + L_3 + \cdots L_N$$

Connecting inductors in parallel

For a group of inductors, L_1, L_2, L_3,….. L_N connected in parallel (i.e. *with the same voltage applied across each inductor*) find the value of a single equivalent inductor.



Since the same voltage, V_T, appears across all the inductors then:

$$V_T = L_T \frac{dI_T}{dt} = L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt} = L_3 \frac{dI_3}{dt} = \cdots = L_N \frac{dI_N}{dt}$$

so for the equivalent inductor we can write:

$$\frac{V_T}{L_T} = \frac{dI_T}{dt}$$

and for similarly for the other inductors:

For inductor L_1 $\qquad \frac{V_T}{L_1} = \frac{dI_1}{dt}$

For inductor L_2 $\qquad \frac{V_T}{L_2} = \frac{dI_2}{dt}$

For inductor L_3 $\qquad \frac{V_T}{L_3} = \frac{dI_3}{dt}$

: $\qquad$:

For inductor L_N $\qquad \frac{V_T}{L_N} = \frac{dI_N}{dt}$

Applying Kirchoff's first law to the circtuit:

$$I_T = I_1 + I_2 + I_3 + \cdots + I_N$$

and hence:

$$\frac{dI_T}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt} + \cdots + \frac{dI_N}{dt}$$

Substituting values into the above equation gives:

$$\frac{V_T}{L_T} = \frac{V_T}{L1} + \frac{V_T}{L_2} + \frac{V_T}{L_3} + \cdots + \frac{V_T}{L_N}$$

which leads to the expression for calculating the total inductance for a group of inductors in parallel as:

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots \frac{1}{L_N}$$

<u>Example</u>

Three inductors have inductances of 0.015H, 3mH and 9mH respectively. Find the total inductance when they are connected (i) in series, and (ii) in parallel.

(i) For a series connection:

$$L_T = L_1 + L_2 + L_3 = 0.015 + 0.003 + 0.009 = 0.027\text{H or } 27\text{mH}$$

(ii) For a parallel connection:

$$L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}} = \frac{1}{\frac{1}{0.015} + \frac{1}{0.003} + \frac{1}{0.009}} = 0.00196\text{H or } 1.96\text{mH}$$

<u>Example</u>

A constant current of 4A flows through a series combination of two inductors having inductances of 0.5H and 2H respectively. Find (i) the total voltage across the combination and, (ii) the energy stored in each inductor.

(i) The voltage across an inductor is proportional to the rate of change of current, i.e.:

$$V_L = L\frac{dI}{dt}$$

However, in this example the current is constant (i.e. $dI/dt = 0$) and hence the voltage is zero!

(ii) The energy stored in each inductor is:

$$E_1 = \frac{1}{2}L_1 I^2 = \frac{1}{2} \times 0.5 \times 4^2 = 4\text{J}$$

$$E_2 = \frac{1}{2}L_2 I^2 = \frac{1}{2} \times 2 \times 4^2 = 16\text{J}$$

If the initial constant current of 4A is interrupted by opening a switch, and the current falls to zero in 10ms, calculate the magnitude of the voltage which appears across each inductor and the magnitude of the total voltage across the combination.
Now the voltage across an inductor depends on the rate of change of current:

$$V_L = L\frac{dI}{dt} = L \times \frac{I_{FINAL} - I_{INITIAL}}{time\,taken}$$

So:

$$V_1 = L_1\frac{dI}{dt} = 0.5 \times \frac{0-4}{0.01} = -200\text{V}$$
$$|V_1| = 200\text{V}$$

$$V_2 = L_2\frac{dI}{dt} = 2 \times \frac{0-4}{0.01} = -800\text{V}$$
$$|V_2| = 800\text{V}$$

and hence the total voltage across the two inductors is 1000V.

Check:

$$L_T = L_1 + L_2 = 0.5 + 2 = 2.5\text{H}$$

and:

$$V_T = L_T\frac{dI}{dt} = 2.5 \times \frac{0-4}{0.01} = -1000\text{V}$$
$$|V_T| = 1000\text{V}$$