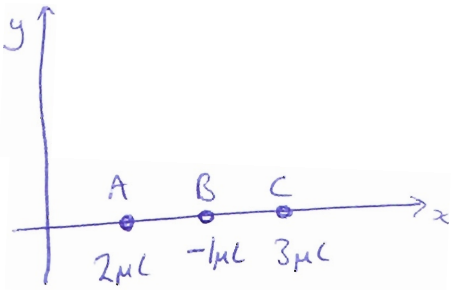
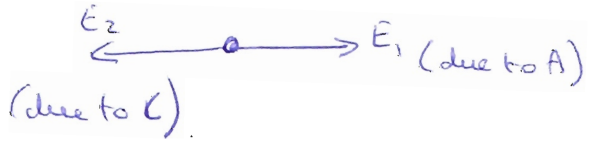
Q1.

(a)



A − (2,0,0)
B − (4,0,0)
C − (6,0,0)

At B there are two electric fields :-



$$E_1 = \frac{Q_A}{4\pi \varepsilon_0 R_{AB}^2} = \frac{2\times10^{-6}}{4\pi \times 8.854\times10^{-12} \times 2^2} \quad V/m$$

$$E_2 = \frac{Q_C}{4\pi \varepsilon_0 R_{BC}} = \frac{3\times10^{-6}}{4\pi\times 8.854\times10^{-12} \times 2^2} \quad V/m.$$

Total field (in x − direction is)

$$E = \frac{(3-2)\times10^{-6}}{4\pi \, . \, 8.854\times10^{-12} \times 4} \quad V/m$$

$$F = qE = -1\times10^{-6} \times \frac{-1\times10^{-6}}{4\pi \, . \, 8.854\times10^{-12}\times4} = 2.247\times10^{-3}\,N$$

$$\therefore \underline{\underline{F = (2.247, 0, 0) \, mN}}$$

(4)

b. (i) Treat the problem as 3 parallel plate capacitors in parallel. (2)

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{d} \qquad C_2 = \frac{\varepsilon_0 \varepsilon_{r2} A_2}{d} \qquad C_3 = \frac{\varepsilon_0 \varepsilon_{r3} A_3}{d}$$

For capacitors in parallel

$$C_T = C_1 + C_2 + C_3$$

also $A_1 = A_2 = A_3 = \frac{ab}{3}$

$$\therefore \; C_T = \frac{\varepsilon_0 ab}{3d}(\varepsilon_{r1} + \varepsilon_{r2} + \varepsilon_{r3}) \qquad (4)$$

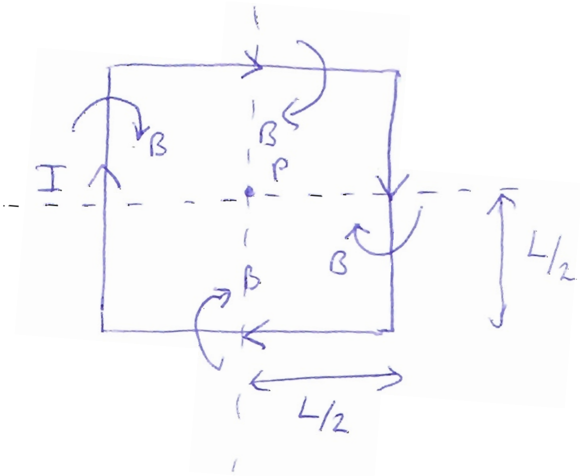
(ii) Energy stored $= \frac{1}{2}CV^2$

$$\therefore \; E = \frac{1}{2}\frac{\varepsilon_0 ab}{3d}(\varepsilon_{r1} + \varepsilon_{r2} + \varepsilon_{r3}) \times V^2$$

$$= \frac{8.854\times10^{-12} \cdot 0.015 \cdot 0.007}{6 \times 0.002}(3+6.5+4) \times 15^2 \, J$$

$$= \underline{\underline{2.353 \times 10^{-10} \, J}} \qquad (2)$$

c. (i)



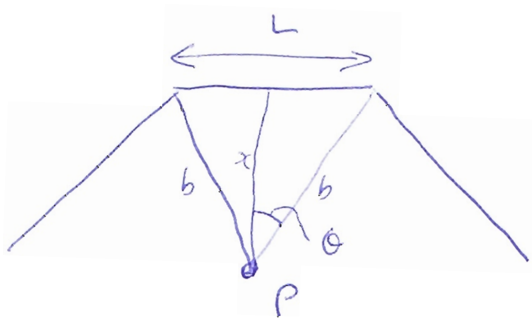
From symmetry total field at P will be 4 times field due to one wire with $x = L/2$

$$\therefore \; B_T = \frac{4\mu_0 I}{2\pi \cdot L/2}\left[\frac{1}{1+\left(\frac{L}{L}\right)^2}\right]^{1/2}$$

$$= \frac{4\mu_0 I}{\pi L}\left(\frac{1}{2}\right)^{1/2}$$

$$= \frac{4\mu_0 I}{\pi L \sqrt{2}} = \underline{\underline{\frac{2\sqrt{2}\,\mu_0 I}{\pi L}}}$$

(4)



$2\theta = \frac{2\pi}{n}$

$$L = 2b\sin\theta = 2b\sin(\pi/n)$$
$$x = b\cos\theta = b\cos(\pi/n)$$

Substituting in equation then for n sides we have:-

$$B = \frac{n\,\mu_0 I}{2\pi\, b\cos\theta}\left[\frac{1}{1+\left(\frac{2b\cos\theta}{2b\sin\theta}\right)^2}\right]^{1/2}$$

$$= \frac{n\,\mu_0 I}{2\pi b\cos\theta}\left[\frac{1}{1+\frac{1}{\tan^2\theta}}\right]^{\frac{1}{2}} = \frac{n\,\mu_0 I}{2\pi b\cos\theta}\left[\frac{\tan^2\theta}{\tan^2\theta+1}\right]^{1/2}$$

But $1+\tan^2\theta = \frac{1}{\cos^2\theta}$

$$= \frac{n\,\mu_0 I}{2\pi b\cos\theta}\left[\tan^2\theta\cos^2\theta\right]^{\frac{1}{2}} = \underline{\underline{\frac{n\,\mu_0 I}{2b\pi}\tan(\pi/n)}} \qquad (4)$$

(iii) As n becomes large polygon tends to a circle

$n \to \infty \qquad n\tan\left(\frac{\pi}{n}\right) \longrightarrow \frac{n\pi}{n} \longrightarrow \pi$

$\tan\frac{\pi}{n} \simeq \frac{\pi}{n}$
for small θ.

$$\therefore\ B = \frac{\mu_0 I\,\pi}{2b\pi} = \underline{\underline{\frac{\mu_0 I}{2b}}} \qquad (2)$$

Q2.

a. (i) The turns ratio of the transformer is $\frac{1000}{100} = 10$

$\therefore$ the referred secondary resistance is $0.02 \times 10^2 = 2\Omega$

So the total resistance referred to the primary side is $4 + 2 = \underline{\underline{6\Omega}}$ (1)

(ii) Since the transformer is rated at 20kVA and 1000Vrms the full load primary current is:

$$I_{PFL} = \frac{20000}{1000} = 20A$$ (2)

Therefore the copper losses at full load are $20^2 . 6 = \underline{\underline{2.4kW}}$

(iii) The flux in the core of the transformer may be found using:

$$V_{rms} = 4.44 f N \phi_{max}$$

$$\therefore \phi_{max} = \frac{1000}{4.44 . 50 . 2000} = 2.25 mWb.$$

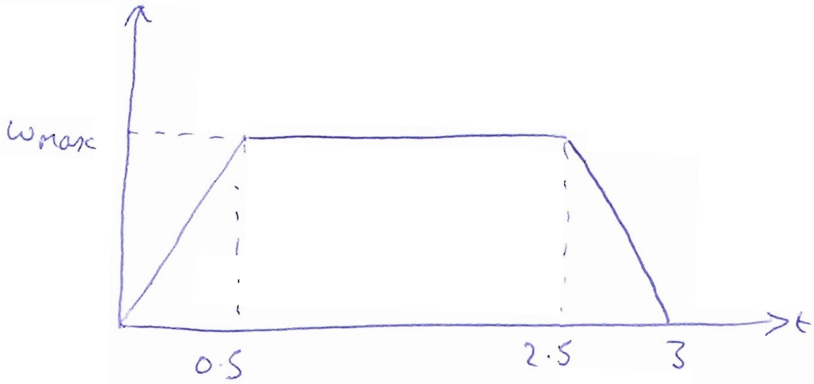
Since $B = \frac{\phi}{A}$ the Area of core $= \frac{\phi}{B} = \frac{2.25 \times 10^{-3}}{1.5}$

$= \underline{\underline{1.5 \times 10^{-3} m^2}}$ (3)

(iv) The losses in the core are obtained from the no-load current.

$$P = VI \cos\phi = 1000 . 3 . 0.3 = \underline{\underline{900W}}$$ (1)

b. (i)



The total angle traversed $\theta = \int \omega\, dt$ = area under the curve

Area under curve $= \frac{1}{2} \cdot 0.5 \cdot \omega_{max} + 2 \cdot \omega_{max} + \frac{1}{2} \cdot 0.5 \cdot \omega_{max}$

$= 2.5\,\omega_{max}$

This is equal to the angle traversed in radians $= 150 \times \frac{\pi}{180}$

$= \frac{5\pi}{6}$ rads

Equating these gives:

$$2.5\,\omega_{max} = \frac{5\pi}{6}$$

$$\therefore \omega_{max} = \frac{5\pi}{6} \cdot \frac{2}{5} = \frac{\pi}{3} \text{ rad/s}$$ (Max speed of arm)

However the arm is driven through a gearbox with a 5:1 transfer ratio

$$\therefore \omega_{motor\,max} = 5\omega_{max} = \frac{5 \cdot \pi}{3} = \underline{\underline{5.236 \text{ rad/s}}}$$ (4)

ii) The motor is required to accelerate from standstill to maximum speed in 0.5 seconds with constant acceleration.

$$\ddot{\theta} = \frac{5.236}{0.5} = 10.472 \text{ rad s}^{-2}$$

Now the total referred inertia $J = 0.5 \text{ kg m}^2$

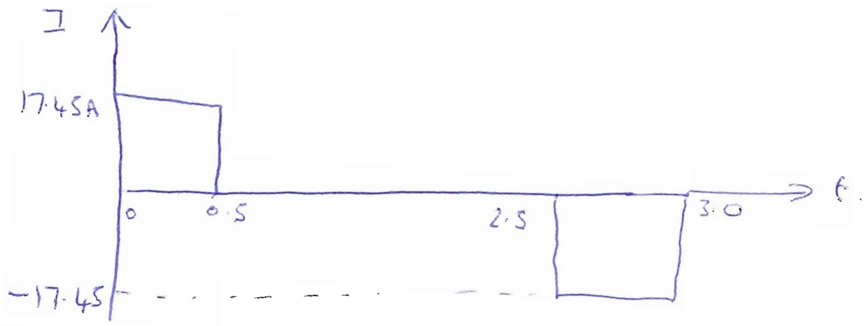
So the required torque $T = J\ddot{\theta} = 10.472 \times 0.5 = \underline{\underline{5.236 \text{ Nm}}}$

Now also since $T = k_i I$

the current I may be calculated :-

$$I = \frac{T}{k_i} = \frac{5236}{0.3} = \underline{\underline{17.45\,A}}$$

The system is assumed to be lossless, ie once the motor has accelerated to maximum speed no torque (hence current) is required to maintain it at this speed.

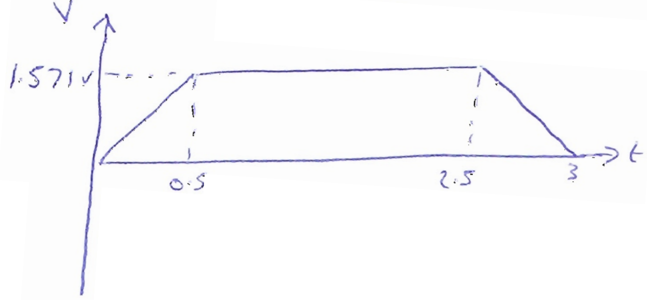


(Since acceleration and deceleration times are equal current requirements are equal)

(4)

iii) The back emf will increase linearly with motor speed from zero at standstill to E at $\omega_{motormax}$ where:

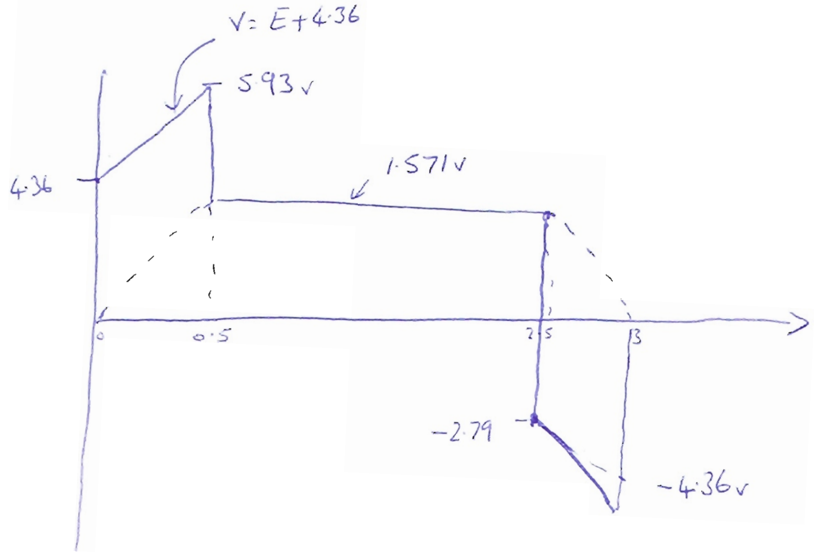
$$E = k_e\,\omega = 0.3 \times 5.236 = \underline{\underline{1.571\,V}}$$



When a current of 17.45A flows through R_A there is a voltage drop $= I.R_A = 17.45 \times 0.25 = \underline{\underline{4.36\,V}}$

Under acceleration $V = E + 4.36$

Under deceleration $V = E - 4.36V$



(5)