EEE6081 (EEE421)
Visual Information Engineering (VIE)
Topic 5: Multi-Resolution Analysis (MRA)

- Wavelet transforms-based MRA.
  - On 1-D signals
  - On Images
  - Memory requirement
- Pyramid decompositions –based multi-resolution representations.
  - On 1-D signals
  - On Images
  - Memory requirement
- Applications
  - Image fusion
  - Image denoising
  - Information hiding

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Multi-resolution analysis (MRA)

- In the Human Visual System (HVS), the amount of details we perceive depends on the angle subtended by a scene or an object on the eye.
- Various details are visible from various distances.

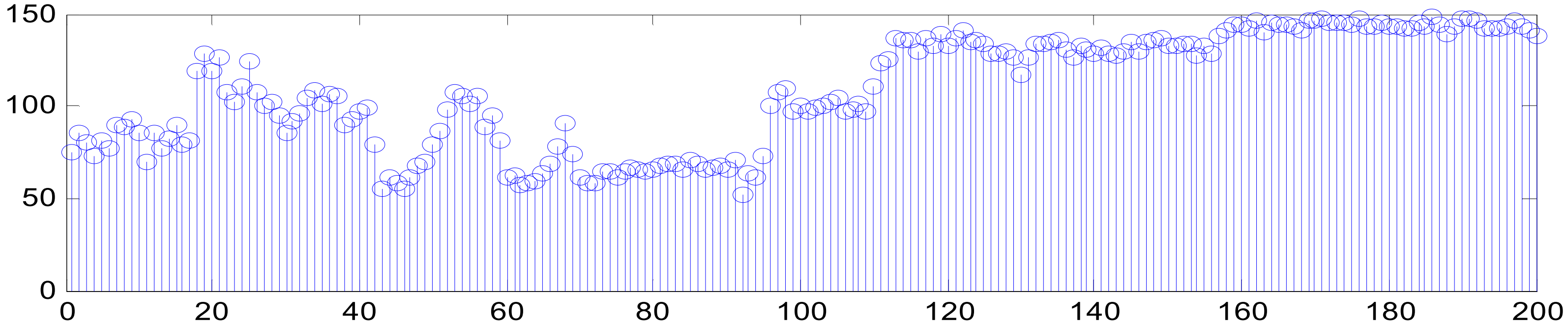
- In visual information engineering, this aspect of the HVS can be mimicked by employing a decomposition scheme that allows us to view the scene (or the image) at different space-frequency representations.
- That is at different spatial resolutions and at different frequency resolutions.

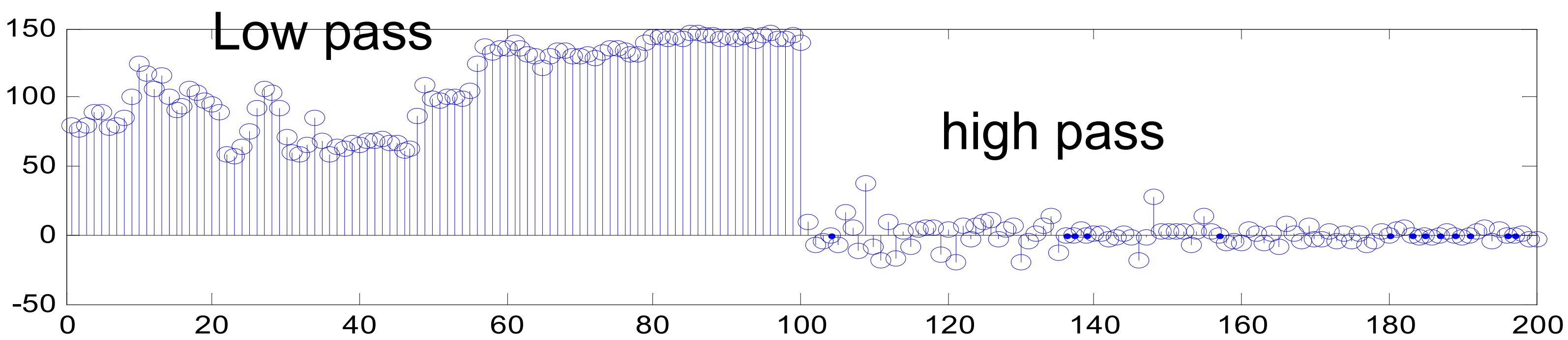
- Analysing signals (1-D or any multi-dimensional) in different space frequency resolutions for information engineering applications is called Mulri-resolution analysis (MRA)

- The Wavelet transform-based decomposition is a good example for MRA.

Multi-resolution analysis (MRA) - Wavelets

- A 2- channel filter bank decomposes data into two sub bands: low pass and high pass.
- Non-expanding --- i.e., number of coefficients = number of data points
- The low pass signal looks the same as the original (only smoothed)
- The filter bank can be applied repeatedly on the low pass signal. This is called Dyadic decomposition





Multi-resolution analysis (MRA) - Wavelets

- At each level of decomposition, the low pass sub band represents a half resolution approximation of the low pass signal of the previous level.

- Draw the filter bank operation for the dyadic decomposition:

- We can represent the Multi-resolution analysis using wavelets as below.
  - Let the starting resolution as a_0. After one level of decomposition we have

$$\begin{tikzcd} a_0 \arrow[r] \arrow[dr] & a_{-1} \\ & d_{-1} \end{tikzcd}$$

where a_{-1} is the half resolution approximation and d_{-1} the details seen at that resolution.

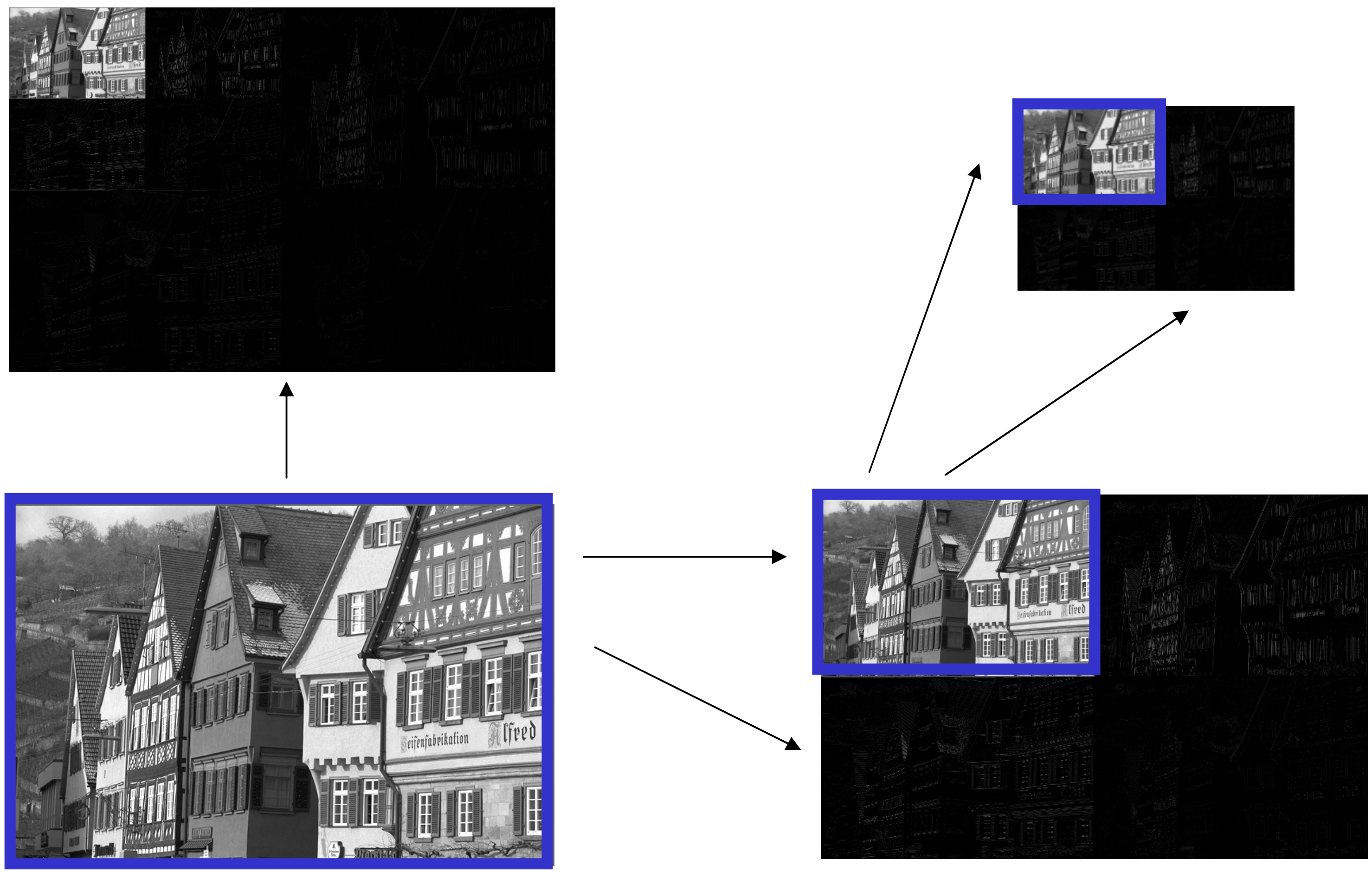
  - For n levels

$$\begin{tikzcd} a_0 \arrow[r] \arrow[dr] & a_{-1} \arrow[r] \arrow[dr] & a_{-2} \arrow[r] \arrow[dr] & a_{-3} & {} \arrow[r] \arrow[dr] & a_{-n} \\ & d_{-1} & d_{-2} & d_{-3} & & d_{-n} \end{tikzcd}$$

  - Which resolution-bands are resulted in a 2 level decomposition?
  - Specify the memory requirement (in terms of the original signal size) for a 1-D wavelet based MR representation.
  - How are the resolution bands combined to get back the original resolution data?

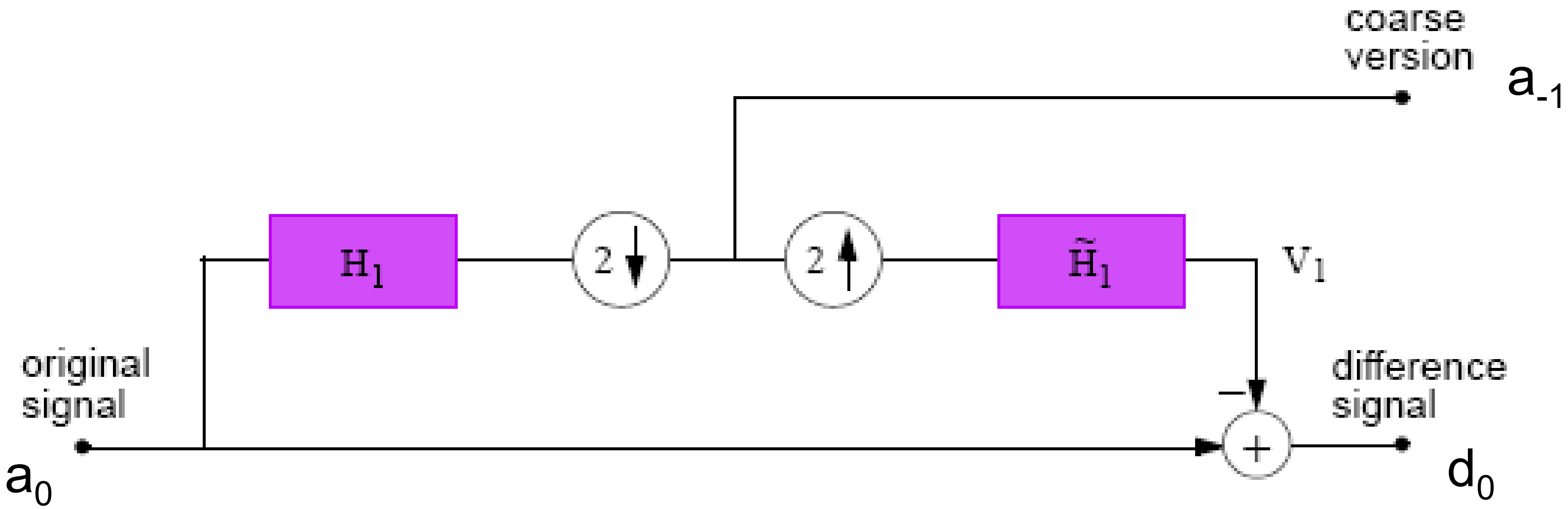
Multi-resolution analysis (MRA) - Wavelets

- What are a_{-n} and d_{-n} when a 2D wavelet transform (dyadic) is used?



Multi-resolution analysis (MRA) - Pyramids

- Using the pyramid decomposition
  - Use the same MRA concept
  - i.e., half resolution approximation + details
  - With the wavelet transform, the details are represented at half resolution too.
  - But in pyramidal transform, the details are represented at the resolution of the previous level.
  - Takes the following form:
  - $$\begin{tikzcd} a_0 \arrow[r] \arrow[d] & a_{-1} \arrow[r] \arrow[d] & a_{-2} \arrow[r] \arrow[d] & a_{-3} \arrow[d] & \arrow[r] \arrow[d] & a_{-n} \\ d_{-1} & d_{-2} & d_{-3} & d_{-4} & d_{-n} & \end{tikzcd}$$



Multi-resolution analysis (MRA) - Pyramids

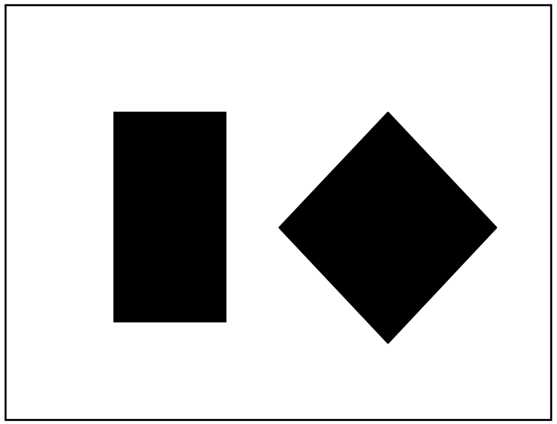
– Draw the inverse transform block diagram

– Draw a block diagram for a pyramid transform for an image.

Multi-resolution analysis (MRA) - Pyramids

– Draw the inverse transform block diagram for a pyramid transform for an image.

– For this image show the resulting sub bands for a 2 level decomposition using the pyramid transform:



Multi-resolution analysis (MRA) - Pyramids

- The memory requirement for a 2-level 2D pyramid decomposition:
  - If N is the original image size: Then the pyramidal decomposition based MRA requires $N + N/4 + N/16 = N(1 + 1/4 + 1/16)$
  - The above is greater than N. Therefore, called expansive or redundant representation.
  - Derive an expression for the required space for an n-level decomposition
  - What is the redundancy factor for a pyramidal decomposition?

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MRA Applications

- (1) Detail (high pass) sub bands contain information regarding to signal singularities and high frequency components.
  - How can we exploit this to de-noise an image?
  - How can we exploit this for edge detection?
  - How can we exploit this for image compression?
- (2) Approximation (low pass) sub bands contain a smoothed low resolution version of the original image
  - How can we exploit this to reduce complexity of some image operations?

MRA Applications: De-noising

- Original Image: a
- Distorted image: b (add noise to a)
  - For example, b=a+v*randn(size(a));
  - v is the noise variance
  - "randn" generates noise with normal distribution of zero mean and unit variance.
  - v=10 results in around 28dB noise on a gray scale image.
  - Compute the PSNR of b compared to a

- How to de-noise?
  - Do the forward wavelet transform (FWT) for image a (2 or 3 levels)
  - Do the forward wavelet transform (FWT) for image b (2 0r 3 levels)
  - Find out the difference. Which sub bands are affected due to noise?
  - Now think of a methodology to de-noise b and perform it
  - What is the PSNR of the de-noised image compared to a?

MRA Applications: De-noising

- PSNR Computation
  - Peak Signal to Noise Ratio
  - 10 $\log_{10}$ (Peak signal power / Mean noise power)
  - Mean noise power $MSE = \frac{1}{N}\sum_i \sum_j \left(a_{i,j} - b_{i,j}\right)^2$

    N=total number of pixels

    a=original image

    b=noisy image

  - Peak signal power for a 8-bit gray scale image 255^2
  - PSNR $= 10\log_{10}\left(\frac{255^2}{MSE}\right)$

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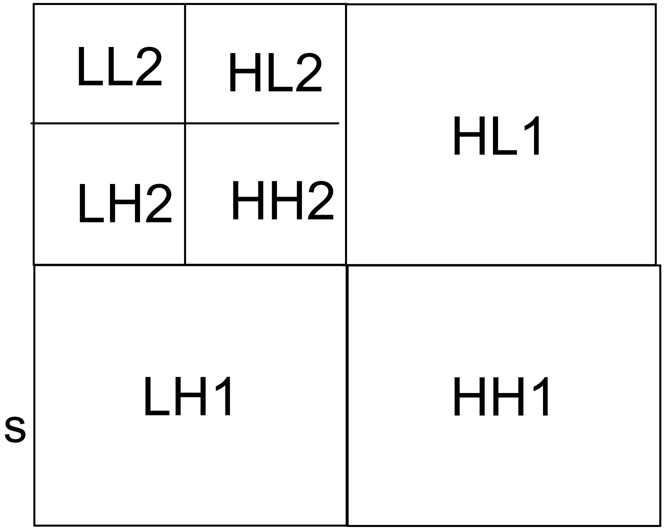
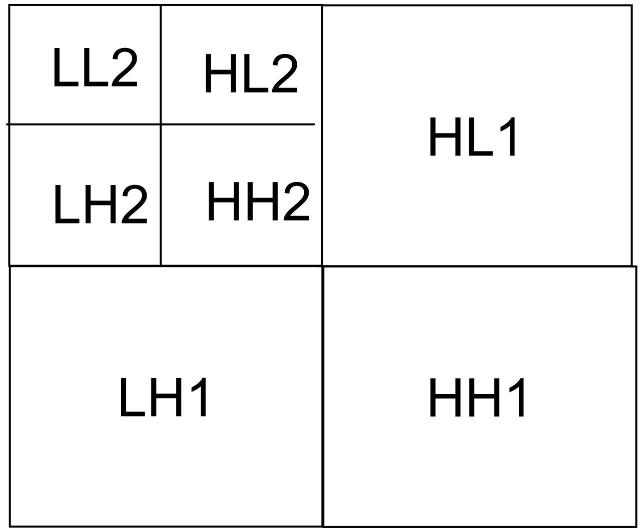
MRA Applications: De-noising



- De-noising methodology
  - Perform FWT on b
  - HL1, LH1, HH1 are the first level high frequency bands
  - HL2, LH2, HH2 are the second level high frequency bands
  - Noise is high frequency
  - and low amplitude
  - Important high frequency components (like edges) in images contain high amplitude compared to noise.
  - To de-noise set all wavelet domain coefficient magnitudes (c) less than a threshold (T) to zero.
  - $C_{i,j} = (|C_{i,j}| < T) \ ? \ 0 \ : \ C_{i,j}$
  - The threshold T can be global or can be chosen locally to each sub band
  - Now perform the inverse wavelet transform (IWT)

MRA Applications: Edge Detection

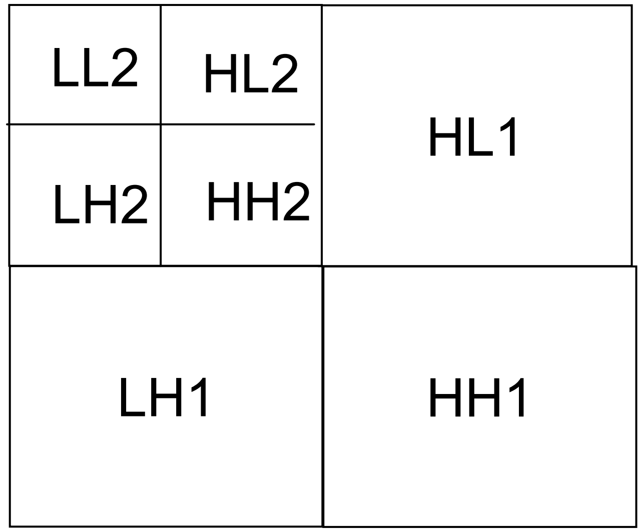


- Edge detection methodology
  - Perform FWT on b
  - HL1 and LH2 contain vertical edges
  - LH1 and LH2 contain horizontal edges
  - HH1 and HH2 contain non-horizontal and non-vertical edges

  - Important high frequency components (like edges) in images contain high amplitude compared to noise.
  - To find an edge map mark all wavelet domain coefficient magnitudes (c) higher than a threshold (T) to 1 and all the rest to 0.
  - $\text{Edge\_Map}_{i,j} = (|C_{i,j}| > T) \ ? \ 1 : 0$

  - The threshold T is usually chosen locally to each sub band or each wavelet decomposition level (e.g., T_1 for HL1, LH1, HH1 and T_2 for HL2, LH2, HH2)

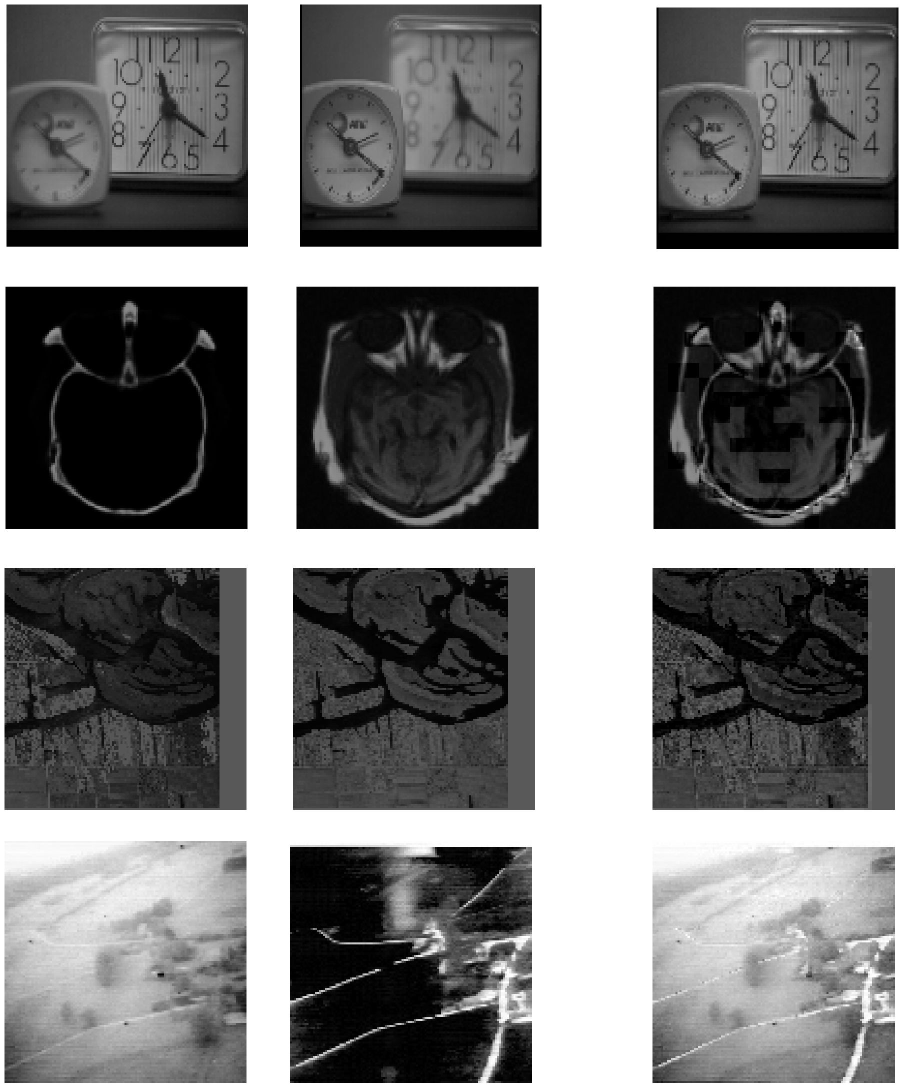
MRA Applications: Image Compression



- Image compression methodology
  - LL sub band ---- more information
    large amplitudes
  - HL, LH, HH sub bands ---- less information (mainly edges)
    low amplitudes close to zero
    large amplitudes for edges
  - Therefore quantise high frequency sub bands heavily (use fewer bits)
  - LL sub band – low quantisation (use more bits)
  - Encoder
    - Image → (FWT) → (Quantisation) → (Entropy coder) → bit stream
  - Decoder
    - bit stream →(Entropy decoder) → (De-Quantisation) → (IWT) →Image
  - More details in Topic 7

MRA Applications: Image Fusion

A B Fused

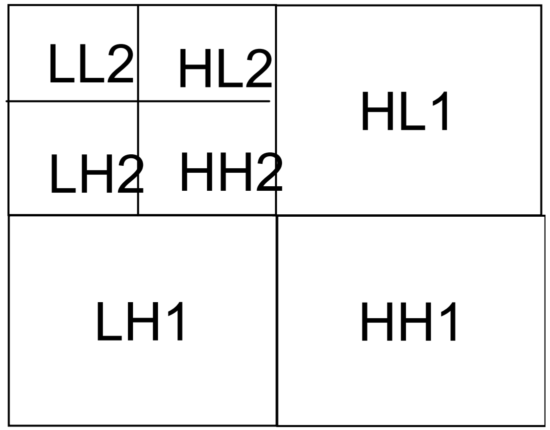


Fusion applications:

1. To get a composite image from images with different camera focus on different objects. (Multi-focus fusion)
2. To get a composite image from different imaging sources (such as MRI (image 2.A) and CT (image 2.B)) in medical imaging. (Multi-modal fusion)
3. To get a composite image from different spectral bands from hyper-spectral images in remote sensing. (multi-spectral fusion)
4. To get a composite image from different imaging sources (such as normal and IR cameras) in surveillance imaging (Multi-modal fusion)

MRA Applications: Image Fusion

- Image fusion can be
  - Multiview fusion (taken from the same camera under different conditions)
  - Multimodal fusion (taken from different sensors)
  - Multi-temporal fusion (images taken at different times)
  - Multi-spectral fusion (same image bus different spectral bands)
  - Multi-focus fusion (different regions are in focus or out of focus)



- Image pixel domain technologies:
  - A simple strategy: Compare each pixel from two images A and B to get the fused image (F).

    $F_{i,j} = (A_{i,j} > B_{i,j})\ ?\ A_{i,j} : B_{i,j}$ Or $F_{i,j} = a A_{i,j} + b B_{i,j}$; with a+b=1
  - Any drawbacks?

- MRA-based approach
  - A → (FWT) → C^A
  - B → (FWT) → C^B
  - Now fusion rule $C^F_{i,j} = (|C^A_{i,j}| > |C^B_{i,j}|)\ ?\ C^A_{i,j} : C^B_{i,j}$ (For high pass bands)
  - $C^F_{i,j} = aC^A_{i,j} + bC^B_{i,j}$ with a+b=1 (For the low pass band)
  - C^F → (IWT) → F

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MRA Applications: Data Hiding (watermarking)

LL2	HL2	HL1
LH2	HH2	
LH1		HH1

- Watermarking strategy
  - Our Requirements
    - Imperceptible
    - Robust
  - Data hiding requires modification of pixel values
  - That means distortion
  - **Imperceptibility** – The human eye is more susceptible to distortion in low frequency areas than high frequency areas like edges. Therefore choose pixels in high frequency areas to hide data.
  - We can find these pixels in wavelet transform domain.
  - i.e., the high frequency sub bands.
  - **Robustness** – Should be robust to compression. In compression low amplitude high frequency data is quantized (removed). So watermarked information can be lost). Therefore the choice of pixels should consider this too.
  - Methodology
    - Image → (FWT) → (Sub band and coefficient selection) → (Coefficient modification) → (IWT) → watermarked image