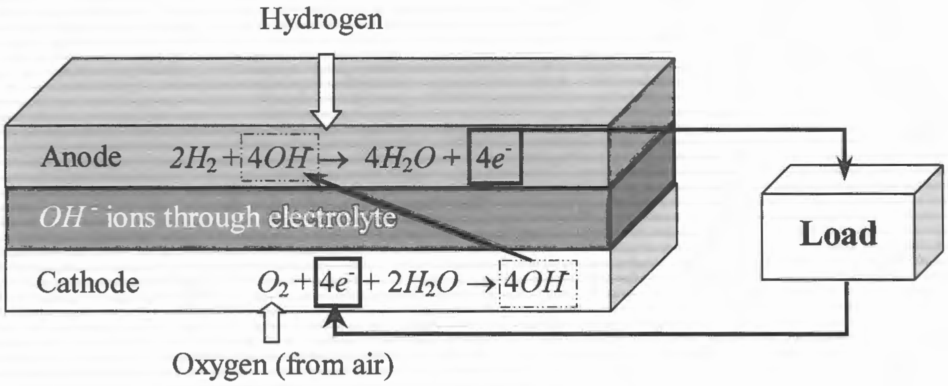
Model Solutions (EEE6021 2012-13 Exam Paper)

Question No. 1

a. The schematic of an alkaline fuel cell is shown as follows: (6)



The electrodes are usually made flat, with a thin layer of alkaline electrolyte between them as shown in the figure. The structure of the electrode is porous, so that both the electrolyte from one side and the gas from the other can penetrate it. This is to give the maximum possible contact between the electrode, the electrolyte and the gas.

In an alkaline fuel cell, alkali hydroxyl (OH^-) ions are available and mobile. At the anode these react with hydrogen, releases energy and electrons, and producing water:

$$2H_2 + 4HO^- \rightarrow 4H_2O + 4e^-$$

At the cathode oxygen reacts with electrons taken from the electrode, and water in the electrolyte, forming new OH^- ions:

$$O_2 + 4e^- + 2H_2O \rightarrow 4OH^-$$

In order for both reactions to proceed continuously, electrons produced at the anode must pass through an electrical circuit to the cathode. Also, OH ions must pass through the electrolyte. An alkaline electrolyte is a fluid with free OH^- ions, and so serves this purpose very well.

At the anode hydrogen reacts and releases energy. However, the rate at which the energy is released is limited by a number of factors since the reaction has the classical energy form. In order to release energy through the chemical reaction, the activation energy must be supplied to get over the "energy hill". If the probability of a molecule having enough energy is low, then the reaction will only proceed slowly and the current through the external load will be small. The key factors that may influence the reaction rate are:

- The use of catalysts
- Operating temperature

- The electrode area

b. If there are no losses in the fuel cell, or in chemistry term, the process is "reversible", then all Gibbs free energy of formation per mole fuel will be converted into electrical energy. (5)

Consider work to be done to move electrons through an external circuit with a voltage E. For the hydrogen fuel cell, each molecule of hydrogen releases two electrons which would pass through the external circuit. Hence, for one mole of hydrogen used, $2N$ electrons pass round the external circuit, where N is the Avagadro's number. If $-e$ is the charge on one electron, then the charge that flow is:

$-2Ne = -2F$ (Coulomb)

where F is the Faraday constant, or the charge on one mole of electrons. If E is the voltage of the fuel cell, then the electrical work done in order to move this amount of charge via the circuit is:

Electrical work done $Q\int_l \vec{E} \cdot d\vec{l}$ = charge x voltage = $-2FE$ (Joules)

For an ideal system (or has no losses), this electrical work done will be equal to the Gibbs free energy released by one mole of hydrogen. Thus:

$$\Delta \bar{g}_f = -2EF$$

or

$$E = -\Delta \bar{g}_f / 2F$$

c. The change of the Gibbs free energy of formation in the hydrogen fuel cell is the difference between the molar specific Gibbs free energy of formation of the product (water) and that of reactants (oxygen and hydrogen): (2)

$$\Delta \bar{g}_f = \left(\Delta \bar{g}_f\right)_{H_2O} - \left(\Delta \bar{g}_f\right)_{H_2} - 0.5\left(\Delta \bar{g}_f\right)_{O_2}$$

As temperature increases, the water vapour contains more energy in the form of heat and hence the amount of energy released decreases.

d. The change in Gibbs free energy formation varies with pressure. For a hydrogen fuel, the variation is given by: (5)

$$\Delta\bar{g}_f = \Delta\bar{g}_f^0 - RT\ln\left[\frac{\left(\frac{P_{H2}}{P_0}\right)\sqrt{\frac{P_{O2}}{P_0}}}{\left(\frac{P_{H_2O}}{P_0}\right)}\right]$$

where

$\Delta\bar{g}_f^0$ — Change in molar Gibbs free energy formation at standard pressure
R — Molar gas constant, 8.314 J/(K mole)
T — Temperature (K)
P_{H2} — Partial pressure of hydrogen
P_{O2} — Partial pressure of oxygen
P_{H2O} — Partial pressure of water
P_0 — Standard pressure

If unit bar is used for pressure, $P_0 = 1.0$, the effect of pressure on the open-circuit voltage can be obtained by:

$$E = \frac{-\Delta\bar{g}_f^0}{2F} + \frac{RT}{2F}\ln\left[\frac{P_{H2}\sqrt{P_{O2}}}{P_{H_2O}}\right]$$

The maximum efficiency of the fuel cell can be evaluated by:

$$\eta_{\max} = -\frac{\Delta\bar{g}_f}{\Delta\bar{h}_f}$$

where $\Delta\bar{h}_f$ is the high heating value of the fuel.

The partial pressure of hydrogen: $P_{H2} = 2.0$

The partial pressure of oxygen: $P_{02} = 2.0\text{x}0.2095 = 0.419$

The partial pressure of water: $P_{H2O} = 1.8$

$$E = \frac{-\Delta\bar{g}_f^0}{2F} + \frac{RT}{2F}\ln\left[\frac{P_{H2}\sqrt{P_{O2}}}{P_{H_2O}}\right] = \frac{220.4\times1000}{2\times96485} + \frac{8.314\times(473)}{2\times96485}\ln\left(\frac{2.0\sqrt{0.419}}{1.8}\right)$$

$$= 1.135(V)$$

$$\eta_{\max} = -\frac{\Delta\bar{g}_f}{\Delta\bar{h}_f} = \frac{219103.91}{285840} = 0.767$$

d. When pure oxygen is supplied at 2 bars, its partial pressure becomes (2)

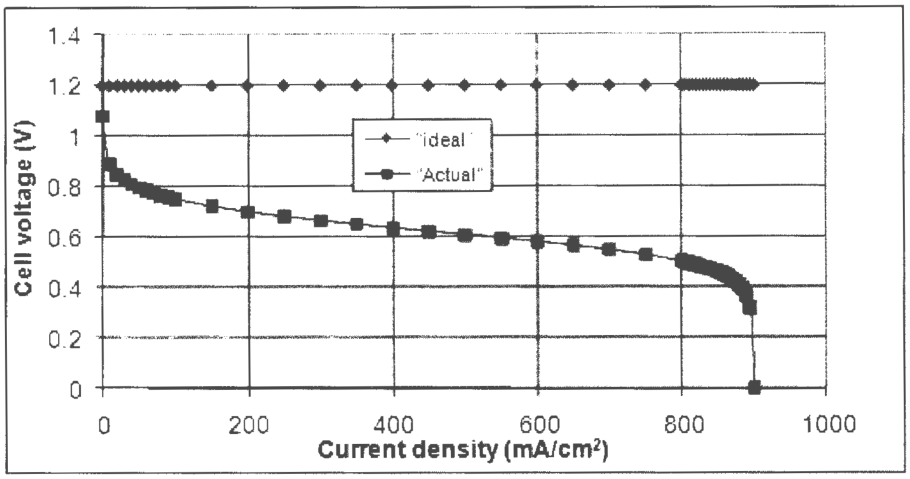
$P_{02} = 2.0\text{x}1 = 2.0$

The change in open-circuit voltage due to the variation of oxygen partial pressure from 0.419 to 2 bars at 200 ^{0}C can be calculated by:

$$\Delta E = \frac{RT}{2F}\ln\left[\frac{\sqrt{2}}{\sqrt{0.419}}\right] = \frac{8.314\times(273+200)}{4\times 96485}\ln(4.773) = 0.016(V)$$

Question No. 2

a. For a fuel cell operating at relatively low temperature, its output voltage characteristic is illustrated below. (10)



Even at very low current density, the output voltage is far away from the reversible open circuit voltage. The main causes of the voltage deviation are:

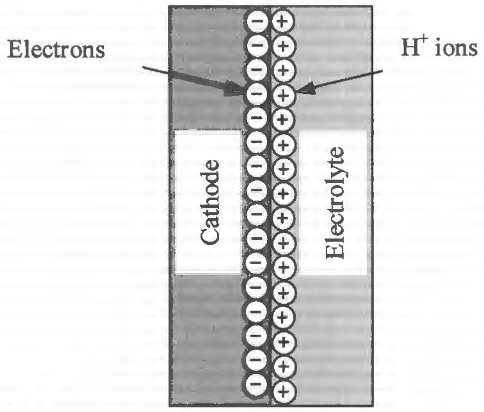
- ***Activation* losses**. These are caused by the slowness of the reactions taking place on the surface of the electrodes. A proportion of the voltage generated is lost in driving the chemical reaction that transfers the electrons to or from the electrode. This voltage drop is highly non-linear and is responsible for the initial fall of the output voltage.
- ***Fuel crossover and internal currents.*** This energy loss results from the waste of fuel passing through the electrolyte, and, to a lesser extent, electron conduction through the electrolyte. The electrolyte should only transport ions through the cell. However, a certain amount of fuel diffusion and electron flow will always be possible. The fuel loss and current is small, and its effect

is usually not very important. However, it does have a significant effect on the open circuit voltage of low temperature cells, and this is the cause of the open circuit voltage being less than the ideal value.

- ***Ohmic losses.*** This voltage drop is the straightforward resistance to the flow of electrons through the material of the electrodes and the various interconnections, as well as the resistance to the flow of ions through the electrolyte. This voltage drop is essentially proportional to current density, linear, and so is called "Ohmic" losses, or sometimes "resistive" losses. The ohmic loss manifests the linear voltage reduction over the mid current density range.
- ***Mass transport or concentration losses.*** These result from the reduction in concentration of the reactants at the surface of the electrodes as the fuel is used. Reduction in concentration leads to lower voltage, and hence this type of loss is sometimes called "concentration" loss. Because the reduction in concentration is the result of a failure to transport sufficient reactants to the electrode surfaces, this type of loss is also often called "mass transport" loss. As the current density increases to a high level, more fuel and oxygen are used, and the reactant concentration (or pressure) drops dramatically. Consequently, the voltage falls rapidly when the current density exceeds a certain level.

b. A charge double layer is formed at the interface between an electrode and the electrolyte in a fuel cell. For example at the cathode of a hydrogen fuel cell, the electrons, driven through the external circuit, will collect at the surface of the electrode, and H^+ ions, which are generated at the anode, will move and be attracted to the interface surface between the cathode and the electrolyte, as shown below. These electrons and ions, together with the O_2 supplied to the cathode, will take part in the cathode reaction: (6)

$$O_2 + 4e^- + 4H^+ \rightarrow 2H_2O$$

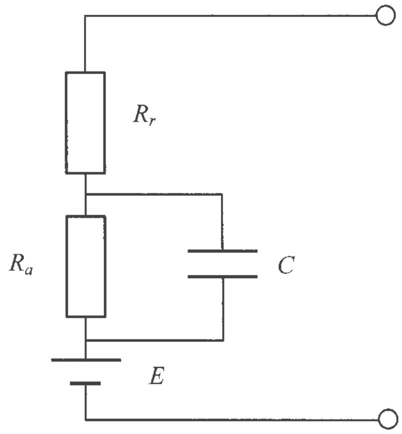


Schematic of charge double layer at Cathode

A similar charge double layer may also exist at the anode.

The layer of charge on the electrode/electrolyte interface is a store of electrical charge and energy, and as such behaves much like an electrical capacitor. If the current changes it will take some time for this charge (and its associated voltage) to dissipate (if the current reduces) or build-up (if there is a current increase). So, the activation over-voltage does not immediately follow the current in the way that the ohmic voltage drop does. The result is that if the current suddenly changes the operating voltage shows an immediate change due to the internal ohmic resistance, but moves fairly slowly to its final equilibrium value.

The effect of this capacitance resulting from the charge double layer may be represented in the following equivalent circuit, where E is the open-circuit voltage, R_a represents the losses due to activation, fuel cross-over and internal current, and R_r is the ohmic resistance of the fuel cell.



c. The output voltage characteristic of fuel cells exhibits significant non-linearity. The variation of the output voltage with the output (load) current is particularly large at low or high current density. The large variation in voltage with load current is not desirable for the vehicle operation and may cause stability problem when driving at high speeds. Further the output voltage of a fuel cell stack is relatively low, which may lead to a large cable size. The use of DC/DC converter can not only increase (step-up) the output voltage, but also provide a stable voltage for the traction drive, independent of the fuel cell current. If necessary, galvanic isolation may also be incorporated into the DC/DC converter so that the fuel cell is electrically isolated from the traction drive. (4)

The power drawn by the traction drive during vehicle operations varies frequently, with the peak may times greater than the average. However, fuel cell cannot cope with fast current changes since increase or decrease in reaction rate is a slow process. In addition, fuel cells cannot absorb the energy during regenerative braking. The use of energy storage devices solve this problem, by provide peak power when required and store energy during braking.

With the energy storage devices and a DC/DC converter, the fuel cell operation can be optimally controlled and independent of vehicle drive conditions, hence, improving system efficiency.

Solution to question No. 3

a.

Advantages

- They can be charged & discharged almost indefinitely
- The recharge rate is rapid
- They can provide high discharge rate
- They require no maintenance
- They are made from non-toxic & relatively inexpensive materials

Disadvantages

- Low energy density
- Low individual cell working voltage, issue of load balance across a series string of capacitors

b.

i. on a flat road, the forces on a vehicle of mass m are related by the following differential equation:

$$m\frac{dv}{dt} = \overbrace{F}^{traction\,force} - \overbrace{\lambda_f \, m \, g}^{rolling\,resistance} - \overbrace{\frac{1}{2} C_d \, A_f \, \rho_a \, v^2}^{aerodynamic\,drag}$$

which can be re-written as:

$$\left(\frac{2m}{C_d\, A_f\, \rho_a}\right)\frac{dv}{dt}=\left(\frac{2\left(F-\lambda_f\, m g\right)}{C_d\, A_f\, \rho_a}\right)-v^2$$

therefore, when the traction force F is constant and $\geq \left(\lambda_f\, m g + m g\right)$, the equation can be written as:

$$p\frac{dv}{dt}=q^2-v^2$$

where, $p=\dfrac{2m}{C_d\, A_f\, \rho_a}$ and $q^2=\dfrac{2\left(F-\lambda_f\, m g\right)}{C_d\, A_f\, \rho_a}$

Since the gear ratio is $R_t = 10/56$, the traction force F is given by:

$$F=\eta_t\frac{T_p}{R_t\, r_w}=0.95\frac{72}{0.1786\times 0.2965}=1291.6 \text{ N}$$

ii.

$$p=\frac{2m}{C_d\, A_f\, \rho_a}=\frac{2\times 1200}{0.35\times 1.6\times 1.225}=3498.5 \text{ m}$$

And on a flat road

$$q=\sqrt{\frac{2\left(F-\lambda_f\, m g\right)}{C_d\, A_f\, \rho_a}}=\sqrt{\frac{2\times\left(1291.6-0.009\times 1200\times 9.81\right)}{0.35\times 1.6\times 1.225}}=58.79 \text{ m/s.}$$

Furthermore, $v(t)=q\tanh\left(\dfrac{q}{p}t+C\right)$, and since v_o=0, therefore, C=0, and,

$$v_c=q\tanh\left(\frac{q}{p}t\right) \Rightarrow t=\frac{p}{q}\tanh^{-1}\left(\frac{v_c}{q}\right)$$

Therefore, on a flat road,

$$t=\frac{p}{q}\tanh^{-1}\left(\frac{v_c}{q}\right)=\frac{3498.5}{58.79}\times\tanh^{-1}\left(\frac{26.82}{58.79}\right)=29.31 \text{seconds}$$

iii.

When the aerodynamic drag is considered, the energy delivered by the drive-train is given by:

$$E = \int_{t_1}^{t_2} F\, v(t)\, dt = F \int_{t_1}^{t_2} q \tanh\left(\frac{q}{p} t\right) dt$$

chosing $u = \frac{q}{p} t \Rightarrow dt = \frac{p}{q} du$ and, the energy would then be given by:

$$E = F \int_{u_1}^{u_2} p \tanh(u) du = p\, F \left|\ln(\cosh(u))\right|_{u_1}^{u_2}$$

$$u_1 = \frac{q}{p} t_1 = \frac{146.7}{9796} \times 0 = 0\,;\; u_2 = \frac{q}{p} t_c = \frac{58.79}{3498.5} \times 26.82 = 0.4507$$

Therefore,

$$\begin{aligned} E &= F\, p\, (\ln(\cosh(u_2)) - \ln(\cosh(u_1))) \\ &= 1291.6 \times 3498.5 \times (\ln(\cosh(0.4507)) - \ln(\cosh(0))) \\ &= 444.2 \text{ kJ} \end{aligned}$$

iv.

The kinetic energy of the vehicle is given by:

$$E_k = \frac{1}{2} m v_c^2 = \frac{1}{2} \times 1200 \times 26.82^2 = 431.58 \text{ kJ}$$

The kinetic energy is clearly slightly smaller than the energy delivered by the drive-train. The difference is the energy lost in the rolling resistance and aerodynamic drag.

c.

When the aerodynamic forces are considered, the traction force delivered by the drive-train is given by:

$$F = m\frac{dv}{dt} + \lambda_f\, m\, g + \frac{1}{2} C_d\, A_f\, \rho_a\, v^2$$

The traction power is then given by:

$$P = F\, v = m v \frac{dv}{dt} + \lambda_f\, m\, g\, v + \frac{1}{2} C_d\, A_f\, \rho_a\, v^3$$

Since the acceleration $\gamma = dv/dt$ is constant, therefore,

$$P = F\,v = m\,\gamma\,v + \lambda_f\,m\,g\,v + \frac{1}{2} C_d\,A_f\,\rho_a\,v^3$$

The energy delivered by the drive-train is then given by:

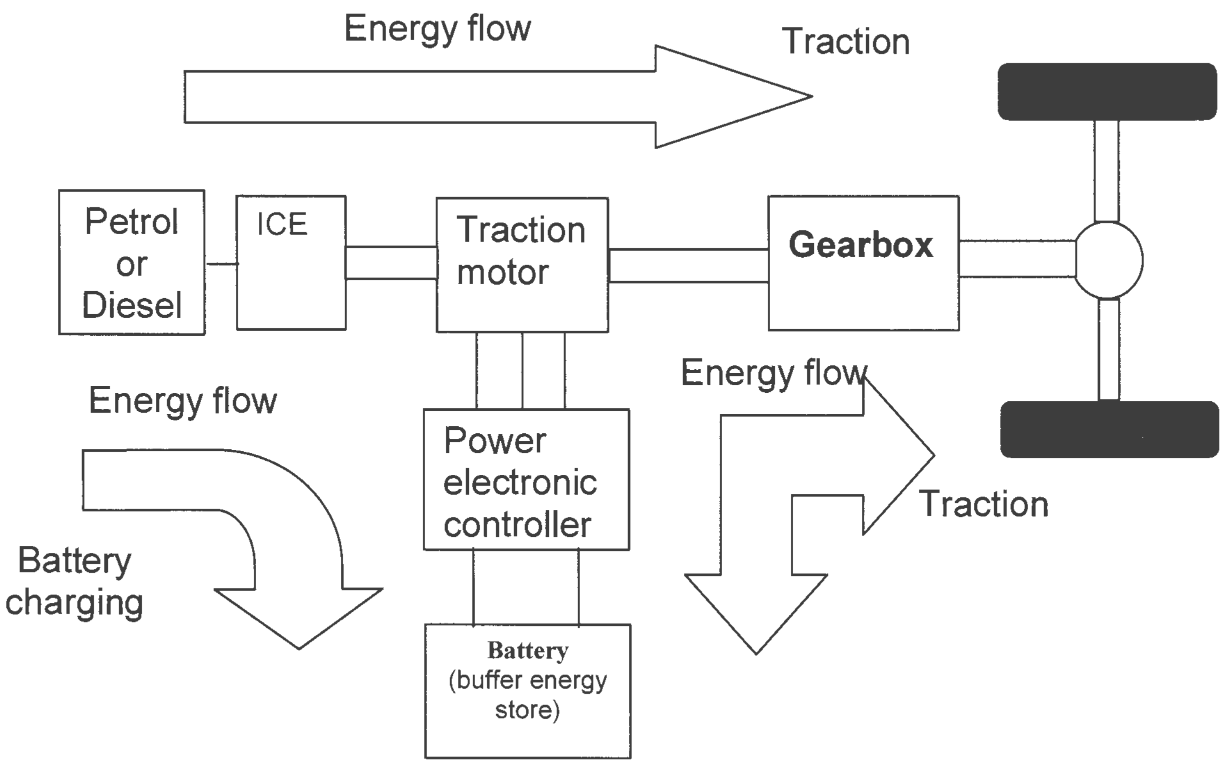
$$E = \int_0^{t_c} F\,v\,dt = \frac{1}{\gamma} \int_0^{v_c} F\,v\,dv = \frac{1}{\gamma} \int_0^{v_c} \left(m\,\gamma\,v + \lambda_f\,m\,g\,v + \frac{1}{2} C_d\,A_f\,\rho_a\,v^3 \right) dv$$

$$= \frac{1}{2} m\,v^2 + \frac{1}{2\gamma} \lambda_f\,m\,g\,v^2 + \frac{1}{8\gamma} C_d\,A_f\,\rho_a\,v^4$$

Solution to question No. 4

a.

Basic parallel-hybrid drive-train



- The power for traction can be shared between ICE and the electric motor, especially, in acceleration phases.
- Improved fuel economy and reduced emissions.
- Same range as conventional vehicles.
- Requires less components then a series hybrid.
- But still more expensive then a conventional vehicle.

b.

The traction force F required for propelling the vehicle is given by:

$$F = \lambda_f mg + \frac{1}{2} C_d A_f \rho_a (v + v_w)^2$$
$$= 0.013 \times 1900 \times 9.81 + 0.5 \times 0.37 \times 2.45 \times 1.225 \times 26.82^2$$
$$= 242.31 + 399.4$$
$$F = 641.71 N$$

The torque of the wheels, T_w, is then given by:

$$T_w = F r_w = 641.71 \times 0.2965 = 190.27 \text{ Nm}$$

and the rotational speed of the wheel, Ω_w, is given by:

$$\Omega_w = \frac{v}{r_w} = \frac{13.41}{0.2965} = 45.22 \text{ rad/s}$$

when 3[nd] gear is selected, the total gear ratio R_t, between the wheels and the traction motor is:

$$R_t = \overbrace{0.2903}^{differential} \times \overbrace{0.8857}^{gearbox} = 0.2571$$

therefore, the speed of the traction motor is given by:

$$\Omega = \frac{\Omega_w}{R_t} = \frac{45.22}{0.2571} = 175.88 \text{ rad/s}$$

If the transmission loss is considered, the torque of the traction motor is given by:

$$\eta_g = \frac{R_t T_w}{T} \Rightarrow T = \frac{R_t T_w}{\eta_g} = \frac{0.2571 \times 190.27}{0.9425} = 51.9 \text{ Nm}$$

and the loss of the power electronics and the converter are given by:

$$L_d = 5T + 3.75 \times 10^{-2} T^2 + 3.65 \times 10^{-3} \Omega^2$$
$$= 5 \times 51.9 + 3.75 \times 10^{-2} \times 51.9^2 + 3.65 \times 10^{-3} 175.88^2$$
$$= 473.42 \text{ W}$$

Therefore, the power delivered by the battery is given by:

$$P_d = T\Omega + L_d = 51.9 \times 175.88 + 473.42 = 9601.6 \text{ W}$$

c.

The power delivered by the battery can be expressed as:

$P_d = E_o\, I - R_i\, I^2$, therefore, current is solution of the quadratic equation : $R_i\, I^2 - E_o\, I + P_d = 0$

$\Delta = E_o^2 - 4\, R_i\, P_d$ and the solutions could be expressed as :

$$I_1 = \frac{E_o - \sqrt{E_o^2 - 4\, R_i\, P_d}}{2\, R_i} \text{ and } I_2 = \frac{E_o + \sqrt{E_o^2 - 4\, R_i\, P_d}}{2\, R_i}$$

I_2 is not a valid solution, since $I_2 \neq 0$, when $P_d = 0$, therefore, $I = I_1 = \dfrac{E_o - \sqrt{E_o^2 - 4\, R_i\, P_d}}{2\, R_i}$

When Q_d=50%,

$E_o = 200 - 0.25\, Q_d = 200 - 0.25 \times 50 = 187.5\,\mathrm{V}$

and,

$R_i = 100 + 1.25\, Q_d = 100 + 1.25 \times 50 = 162.5\,\mathrm{m\Omega}$

Therefore,

$$I = \frac{E_o - \sqrt{E_o^2 - 4\, R_i\, P_d}}{2\, R_i} = \frac{187.5 - \sqrt{187.5^2 - 4 \times 162.5 \times 10^{-3} \times 1000}}{2 \times 162.5 \times 10^{-3}} = 43.5\ \mathrm{A} \ \text{ for } P_d = 7850\ \mathrm{W}$$

The efficiency of the battery is then given by:

$$\lambda_b = 100 \times \frac{E - R_i\, I}{E} = 100 \times \frac{187.5 - 162.5 \times 10^{-3} \times 43.5}{187.5} = 96.2\%$$

d.

If the vehicle is going downhill without applying mechanical or re-generative braking, the only braking force is due to the rolling resistance, the aerodynamic drag, the iron losses in the permanent magnet machine and the losses of the gearbox.

The speed of the machine at 30MPH is $\Omega = 175.88\,\mathrm{rad/s}$, therefore the iron loss of the machine is $P_{iron} = 3.65 \times 10^{-3}\, \Omega^2 = 3.65 \times 10^{-3}\, 175.88^2 = 112.91\,\mathrm{W}$ and the torque produced by the machine is given by:

$$T = \frac{P_{iron}}{\Omega} = \frac{112.91}{175.88} = 0.62\text{Nm}$$

And since the efficiency of the gearbox when 3rd gear is selected is 94.25%, the braking torque on the wheel due to losses in the machine and the transmission is given by:

$$T_w = \frac{T}{\eta_g R_t} = \frac{0.62}{0.9425 \times 0.2571} = 2.56\text{Nm}$$

And the braking force is given by:

$$F_m = \frac{T_w}{r_w} = \frac{2.56}{0.2965} = 8.63\,\text{N}$$

Furthermore, the force braking force produced by the aerodynamic drag at 30MPH with a headwind of 20MPH is given by:

$$F_d = \frac{1}{2} C_d A_f \rho_a (v + v_w)^2 = 0.5 \times 0.37 \times 2.45 \times 1.225 \times 26.82^2 = 399.4\,\text{N}$$

Therefore, the component of weight pulling the vehicle downhill should be equal to the braking forces:

$$m g \sin(\alpha) = F_m + F_d + \lambda_f m g$$

Therefore,

$$\alpha = \sin^{-1}\left(\frac{F_m + F_d + \lambda_f m g}{m g}\right) = \sin^{-1}\left(\frac{8.63 + 399.4 + 242.3}{1900 \times 9.81}\right) = 2\,\text{degrees}$$

This justifies the assumption that the effect rolling resistance of the inclination angle on the rolling resistance in negligible.