2. DC motor Drives

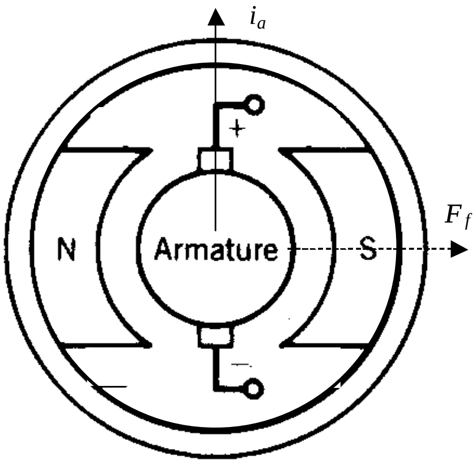
2.1 Introduction

Traditionally, dc-motor drives have been used for speed and position control applications. Recently, the use of ac-motor servo drives in these applications is increasing. In spite of that, in applications where an extremely low maintenance is not required, dc drives continue to be used because of their low initial cost and excellent drive performance.

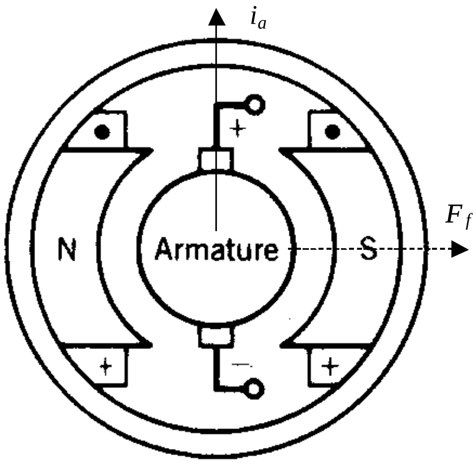
2.2. Operation and Equivalent Circuit of dc Motors

In a dc motor, the field-flux is established by the stator, either by means of permanent rnagnets as shown in Fig. 2.1(a), where F_f stays constant, or by means of a field winding as shown in Fig. 2.1(b), where the field current I_f controls F_f. If the magnetic saturation in the flux path can be neglected, then

$$F_f = k_f i_f \tag{2.1}$$



(a) permanent magnet dc-motor



(b) dc motor with field winding

Fig. 2.1 Schematic of cross section of dc motors

where k_f is a field constant of proportionality. The rotor carries in its slots the so-called armature winding, which handles the electrical power. This is in contrast to most ac motors, where the power handling winding is on the stator for ease of handling the larger amount of power. However, the armature winding in a dc machine has to be on the rotor to provide a "mechanical" rectification of voltages and currents (which alternate direction as the conductors rotate from the influence of one stator pole to the next) in the armature-winding conductors, thus producing a dc voltage and a dc current at the terminals of the armature winding. The armature winding, in fact, is a continuous winding, without any beginning or end, and it is connected to the commutator segments. These commutator segments, usually made up of copper, are insulated from each other and rotate with the shaft. At least one pair of stationary carbon brushes is used to make contact between the commutator segments (and, hence, the armature conductors), and the stationary terminals of the armature winding that supply the dc voltage and current. In a dc motor, the flux produced by the field winding **is always perpendicular** to the magnetic motive force (mmf) of the armature current. This arrangement ensures that the electromagnetic torque produced by the interaction of the field-flux F_f and the armature current i_a is at the maximum given by:

$$T_{em} = k_t F_f i_a \tag{2.2}$$

where k_t is the torque constant of the motor. In the armature circuit, a back-emf is produced by the rotation of armature conductors at a speed w_m in the presence of a field-flux F_f:

$$e_a = k_e F_f w_m \tag{2.3}$$

where k_e is the voltage constant (or back-emf constant) of the motor.

In SI units, k_t and k_e are equal (both numerically and dimensionally), which can be shown by equating the electrical power $e_a i_a$ and the mechanical power $w_m T_{em}$. The electrical power is:

$$P_e = e_a i_a = k_e F_f w_m i_a \tag{2.4}$$

And the mechanical power is

$$P_m = w_m T_{em} = k_t F_f i_a w_m \tag{2.5}$$

In steady state

$$P_e = P_m = w_m T_{em} = k_t F_f i_a w_m \tag{2.6}$$

Therefore, from the foregoing equations,

$$k_t \text{ (Nm/A·Wb)} = k_e \text{ (V/Wb·rad/s)} \tag{2.7}$$

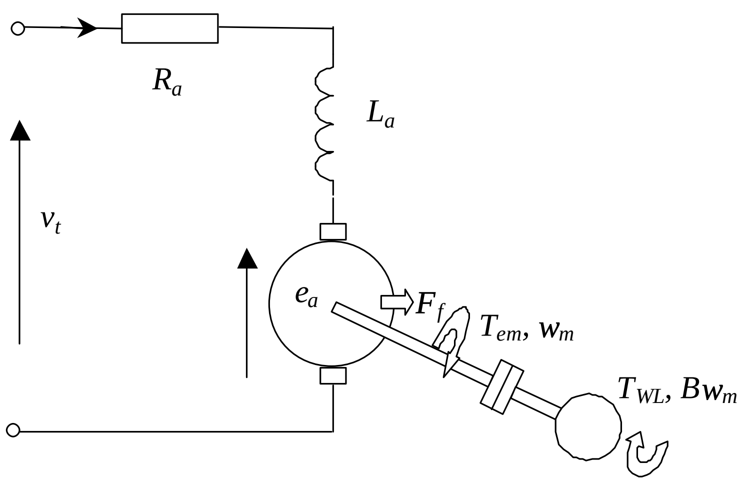


Fig. 2.2 Circuit representation of a separately excited DC machine

In practice, a controllable voltage source v_t is applied to the armature terminals to establish i_a. Therefore, the current i_a in the armature circuit is determined by v_t, the induced back-emf e_a, the armature winding resistance R_a, and the armature-winding inductance L_a:

$$v_t = e_a + R_a i_a + L_a \frac{di_a}{dt} \tag{2.8}$$

Equation (2.8) is illustrated by an equivalent circuit in Fig. 2.2. The interaction of T_{em} with the load torque, as given by Eqn. (1.14b) of Chapter 1, determines how the motor speed builds up:

$$T_{em} = J\frac{dw_m}{dt} + Bw_m + T_{WL} \tag{2.9}$$

where J and B are the total equivalent inertia and damping, respectively of the motor-load combination and T_{WL} is the equivalent working torque of the load.

DC machines are rarely used as generators. However, they act as generators while braking, where their speed is being reduced. Therefore, it is important to consider dc machines in their generator mode of operation. In order to consider braking, we will assume that the flux F_f is kept constant and the motor is initially driving a load at a speed of w_m. To reduce the motor speed, if v_t is reduced below e_a in Fig. 2.2, then the current i_a will reverse in direction. The electromagnetic torque T_{em} given by Eqn. (2.2) now reverses in direction and the kinetic energy associated with the motor-load inertia is converted into electrical energy by the dc machine, which now acts as a generator. This energy must he somehow absorbed by the source of v_t or dissipated in a resistor.

During the braking operation, the polarity of e_a does not change, since the direction of rotation has not changed. Equation (2.3) still determines the magnitude of the induced emf. As the rotor slows down, e_a decreases in magnitude (assuming that F_f is constant). Ultimately, the generation stops when the rotor comes to a standstill and all the inertial energy is extracted. If the terminal-voltage polarity is also reversed, the direction of rotation of the motor will reverse. Therefore, a dc motor can be operated in either direction and its electromagnetic torque can be reversed for braking, as shown by the four quadrants of torque speed plane in Fig. 2.3.

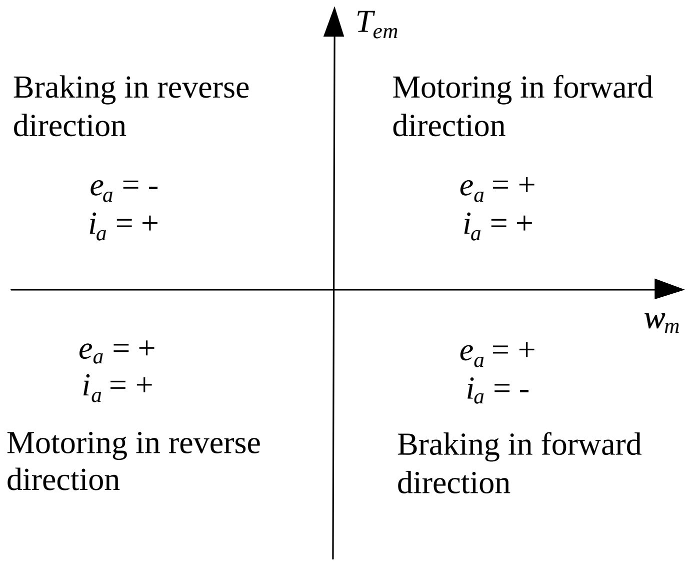


Fig. 2.3 Four quadrant operation of a dc motor

2.3 Speed Control of Permanent Magnet dc Motors

Often in small dc motors, permanent magnets on the stator as shown in Fig. 2.1(a) produce a constant field-flux F_f. In steady state, assuming a constant field-flux F_f, Eqns. (2.2), (2.3), and (1.8) result in

$$T_{em} = k_T I_a \tag{2.10}$$

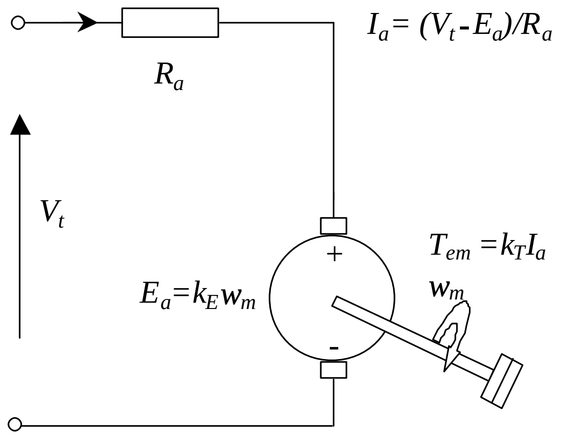
$$E_a = k_E w_m \tag{2.11}$$

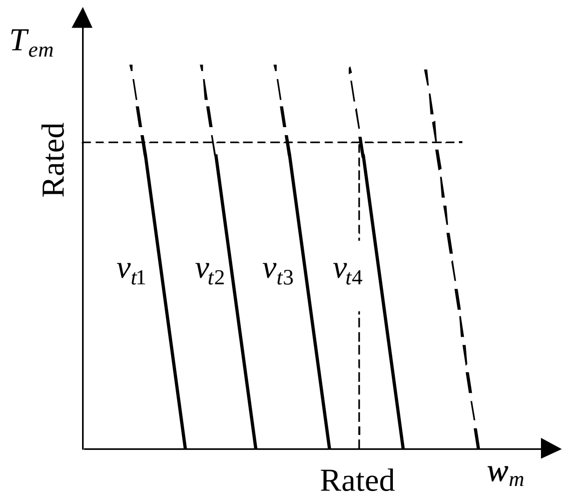
and

$$V_t = E_a + R_a I_a \tag{2.12}$$

where $k_T = k_t F_f$ and $k_E = k_e F_f$ are referred to as torque and back-emf constant of permanent magnet dc motors. Equations (2.10) through (2.12) correspond to the equivalent circuit of Fig. 2.4(a). From the above equations, it is possible to obtain the steady-state speed w_m as a function of T_{em} for a given V_t

$$w_m = \frac{1}{k_E}\left(V_t - \frac{R_a}{k_T} T_{em}\right) \tag{2.13}$$





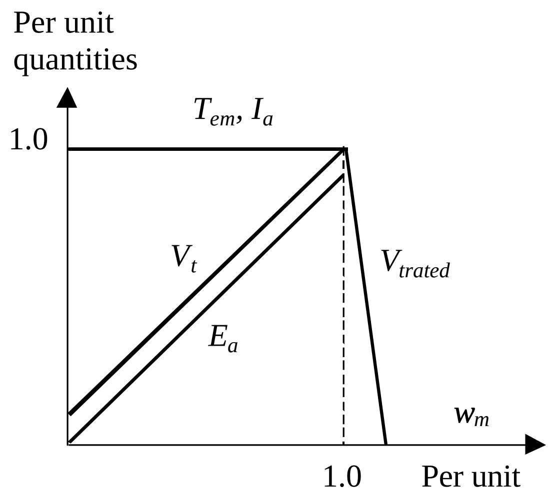


Fig. 2.4 Permanent magnet dc motor
(a) equivalent circuit
(b) torque-speed characteristics, $V_{t1} > V_{t2}, \ldots$
(c) continuous torque-speed capability

The plot of this equation in Fig. 2.4(b) shows that as the torque is increased, the torque-speed characteristic at a given V_t is essentially vertical, except for the droop due to the voltage drop I_aR_a across the armature-winding resistance. This droop in speed is quite small in integral horsepower dc motors, but may be substantial in small servo motors. More importantly, however, the torque-speed characteristics can be shifted horizontally in Fig. 2.4(b) by controlling the applied terminal voltage V_t. Therefore, ***the speed of a load with an arbitrary torque-speed characteristic can be controlled by controlling V_t in a permanent-magnet dc motor with a constant*** F_f.

In a continuous steady-state, the armature-current I_a should not exceed its rated value and, therefore, the torque should not exceed the rated torque. Therefore, the characteristics beyond the rated torque are shown as dotted in Fig. 2.4(b). Similarly, the characteristic beyond the rated speed is shown as dotted, because increasing the speed beyond the rated speed would require the terminal voltage V_t to exceed its rated value, which is not desirable. ***This is a limitation of a permanent-magnet dc motor***, ***where the maximum speed is limited to the rated speed of the motor***. The torque capability as a function of speed is plotted in Fig. 2.4(c). It shows the steady-state operating limits of the torque and current; it is possible to significantly exceed current and torque limits on a short-term basis. Figure 2.4(c) also shows the terminal voltage required as a function of speed and the corresponding E_a.

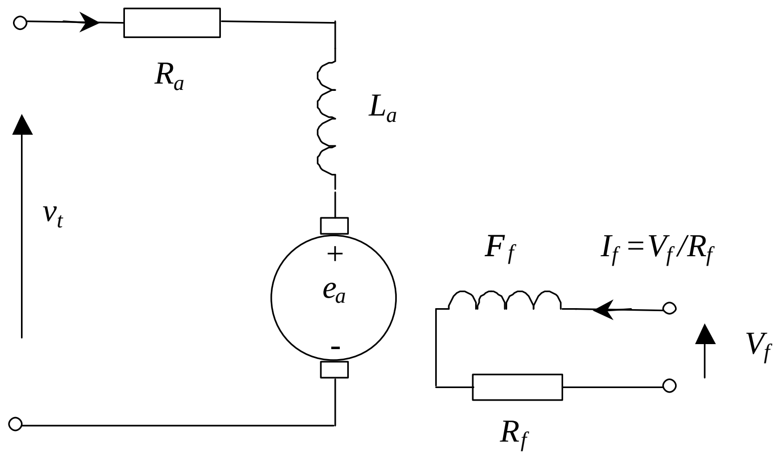
2.4 Speed Control of dc Motors with a Separately Excited Field Winding

Perrnanent-magnet dc motors are limited to ratings of a few horsepower and also have a maximum speed limitation. These limitations can he overcome if F_f is produced by means of a field winding on the stator, which is supplied by a dc current i_f 1 as shown in Fig. 2.5(*a*). To offer the most flexibility in controlling the dc motor, the field winding is excited by a separately controlled dc source v_f. As indicated by Eqn. (2.1), the steady-state value of F_f is controlled by $I_f(= V_f/R_f)$, where R_f is the resistance of the field winding. Since F_f is controllable, Eqn. (2.13) can be written as follows:

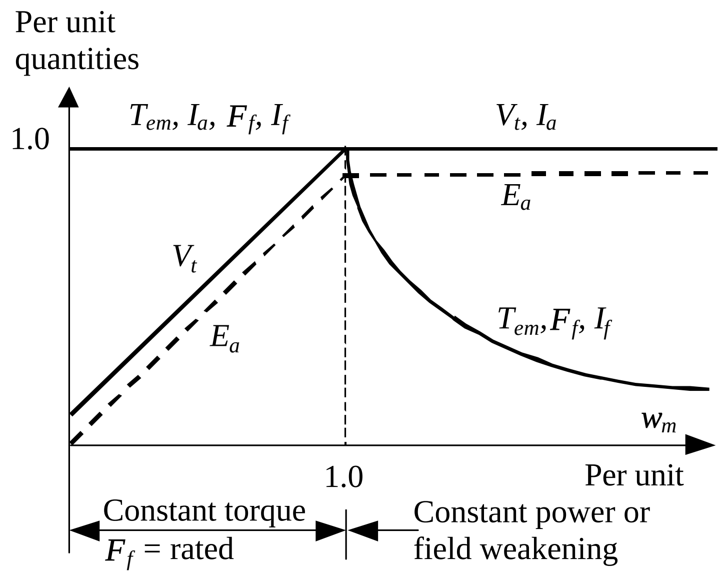
$$w_m = \frac{1}{k_e\Phi_f}\left(V_t - \frac{R_a}{k_t\Phi_f}T_{em}\right) \tag{2.14}$$

recognizing that $k_E = k_eF_f$, and $k_T = k_tF_f$. Equation (2.14) shows that in a dc motor with a separately excited field winding, both V_t and F_f can be controlled to yield the desired torque and speed. As a general practice, to maximise the motor-torque capability, F_f (hence, I_f) is kept at its rated value for speeds less than the rated speed. With F_f at its rated value, the relationships are the same as given by Eqs. (2.10) through (2.13) of a permanent-magnet dc motor. Therefore, the torque-speed characteristics are also the same as those for a permanent-magnet dc motor that are shown in Fig. 2.4(b). With F_f constant and equal to its rated value, the motor torque-speed capability is as shown in Fig. 2.5(b), where this region of constant F_f is often called the **constant torque region**. The required terminal voltage V_t in this region increases linearly from approximately 0 to its rated value, as the speed increases from 0 to its rated value. V_t and the corresponding E_a are shown in Fig. 2.5(b).

To obtain speed beyond its rated value, V_t is kept constant at its rated value and F_f is decreased by decreasing I_f. Since I_a is not allowed to exceed its rated value on a continuous basis, the torque capability declines, as F_f is reduced in Eqn (2.2). In this so called field-weakening region, the maximum power $E_a I_a$ (equal to $w_m T_{em}$) into the motor is not allowed to exceed its rated value on a continuous basis. This region, also called constant power region is shown in Fig. 2.5(b)., where T_{em} declines with ω_m, and V_t, E_a and I_a stay constant at their rated values. It should be emphasised that Fig. 2.5(b) is the plot of the maximum continuous capability of the motor in steady state. Any operating point within the region shown is, of course, permissible. In the field-weakening region, the speed may exceed by 50% to 100% of its rated value, depending on the motor characteristics.



(a) equivalent circuit



(b) Continuous torque-speed capability

Fig. 2.5 Separately excited dc motor

2.5 DC Servo Drive

Fig. 2.6 shows the block diagram of a dc servo drive system for closed-loop speed and position control. To design the proper controller that will result in high performance(i.e. quick response, low steady state error, and high degree of stability), it is important to understand the dynamics of the motor.

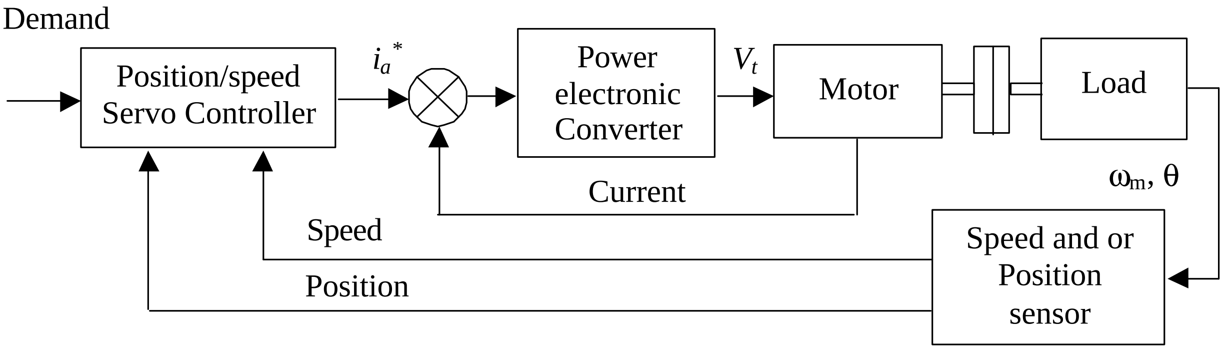


Fig. 2.6 Block diagram of a dc servo drive system

2.5.1 Transfer function model of dc motor for small signal dynamic performance

The dynamic equations which governs the dc motor operation in constant torque region are summarised as follows:

$$
\begin{aligned}
v_t &= e_a + R_a i_a + L_a \frac{di_a}{dt} \\
e_a &= k_E w_m \\
T_{em} &= k_T i_a \\
T_{em} &= J \frac{dw_m}{dt} + B w_m + T_{WL} \\
w_m &= \frac{dq_m}{dt}
\end{aligned}
\qquad (2.15)
$$

If the motor current does not exceed the value limited by the converter, then Eqn.(2.15) is a linear and may be represented by the following transfer function:

$$
\begin{aligned}
V_t(s) &= E_a(s) + (R_a + sL_a) I_a(s) \\
E_a(s) &= k_E w_m(s) \\
T_{em}(s) &= k_T I_a(s) \\
T_{em}(s) &= (Js + B) w_m(s) + T_{WL}(s) \\
w_m(s) &= s q_m(s)
\end{aligned}
\qquad (2.16)
$$

These equations for the motor-load combination can be represented by transfer-function blocks, as shown in Fig. 2.7. The input to the motor-load combination are the armature voltage V_t(s) and the load torque T_{WL}(s). Applying one input at a time by setting the other input to zero, the superposition principle yields:

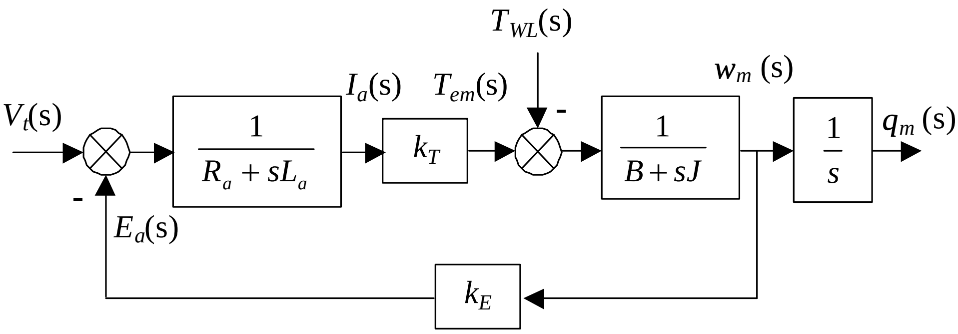


Fig. 2.7 Block diagram representation of the dc motor and load

$$w_m(s) = \frac{k_T V_t(s)}{(R_a + sL_a)(sJ + B) + k_T k_E} - \frac{(R_a + sL_a)T_{WL}(s)}{(R_a + sL_a)(sJ + B) + k_T k_E} \tag{2.17}$$

This equation results in two transfer functions:

$$G_1(s) = \left.\frac{w_m(s)}{V_t(s)}\right|_{T_{WL}(s)=0} = \frac{k_T}{(R_a + sL_a)(sJ + B) + k_T k_E} \tag{2.18}$$

and

$$G_2(s) = \left.\frac{w_m(s)}{T_{WL}(s)}\right|_{T_{WL}(s)=0} = -\frac{(R_a + sL_a)}{(R_a + sL_a)(sJ + B) + k_T k_E} \tag{2.19}$$

As a simplification to gain better insight into the dc motor behaviour, the friction term, which is usually small, will be neglected by setting $B = 0$. Moreover, considering just the motor without the load, J in Eqn. (2.18) is then the motor inertia J_m. Therefore

$$G_1(s) = \frac{k_T}{(R_a + sL_a)sJ_m + k_T k_E} = \frac{1}{k_E\left(\frac{L_a J_m}{k_T k_E}s^2 + \frac{R_a J_m}{k_T k_E}s + 1\right)} = \frac{1}{k_E(t_m t_e s^2 + t_m s + 1)} \tag{2.20}$$

where the following constants are defined:

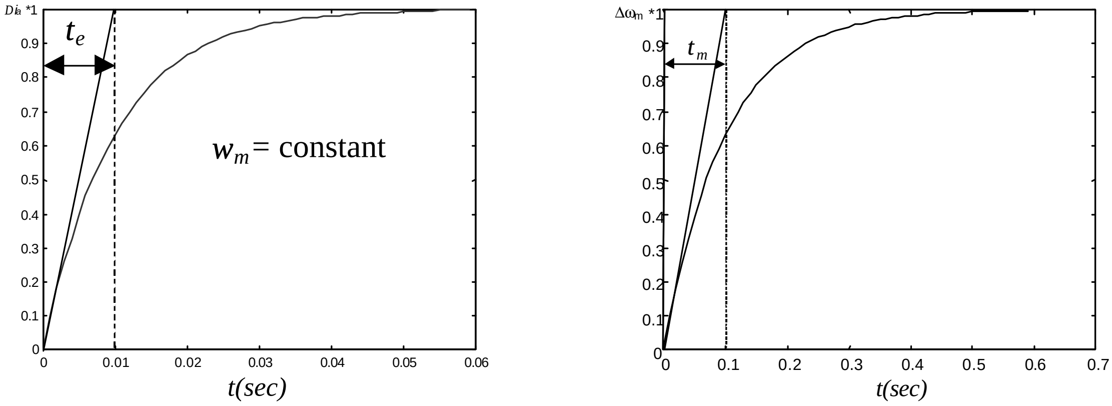
$$t_m = \frac{R_a J_m}{k_T k_E} = \text{Mechnical time constant} \tag{2.21}$$

and

$$t_e = \frac{L_a}{R_a} = \text{Electrical time constant} \tag{2.22}$$

Since in general $t_m >> t_e$, it is reasonable to approximate st_m with $s(t_m+t_e)$. Equation (2.20) becomes:

$$G_1(s) = \frac{1}{k_E(t_m s + 1)(t_e s + 1)} \tag{2.23}$$



(a) motor current response to Δv_t (b) motor speed response to Δv_t

Fig. 2.8 Current and speed response of dc motor

The physical significance of the electrical and the mechanical time constants of the motor should also be understood. The electrical time constant t_e determines how quickly the armature current builds up, as shown in Fig. 2.8(a), in response to a step change Dv_t in the terminal voltage, where the rotor speed is assumed to be constant.

The mechanical time constant t_m determines how quickly the speed builds up in response to a step change Dv_t, in the terminal voltage, provided that the electrical time constant t_e is assumed to be negligible and, hence, the armature current can change instantaneously. Neglecting t_e in Eqn. 2.23, the change in speed from the steady-state condition can be obtained as:

$$w_m(s) = \frac{V_t(s)}{k_E(t_m s + 1)} = \frac{\Delta v_t}{s} \frac{1}{(t_m s + 1)} \tag{2.24}$$

The inverse Laplace transform is:

$$w_m(t) = \frac{\Delta v_t}{k_E}\left(1 - e^{-t/t_m}\right) \tag{2.25}$$

The corresponding change in speed is plotted in Fig. 2.8(b). Note that if the motor current is limited by the converter during large transients, the torque produced by the motor is simply $k_T I_{amax}$.

2.5.2 Power Electronic Converter

Based on the previous discussion, a power electronic converter supplying a dc motor should have the following capabilities:

1. The converter should allow both its output voltage and current to reverse, in order to yield a four-quadrant operation as shown in Fig. 2.3.
2. The converter should be able to operate in a current-controlled mode by holding the current at its maximum acceptable value during fast acceleration and deceleration. The

dynamic current limit is generally several times higher than the continuous steady-state current rating of the motor.

3. For accurate control of position, the average voltage output of the converter should vary linearly with its control input, independent of the load on the motor.
4. The converter should produce an armature current with minimum current ripple so as to minimise the fluctuations in torque and speed of the motor.
5. The converter output should respond as quickly as possible to its control input, thus allowing the converter to be represented essentially by a constant gain without a dead time in the overall servo drive transfer function model.

A linear power amplifier satisfies all the requirements listed above. However, because of its low energy efficiency, this choice is limited to a very low power range. Therefore, the choice must be made between switch-mode dc-dc converters or the line-frequency controlled converters. Here, only the switch-mode dc-dc converters are described. Drives with line-frequency converters can be analysed in the same manner.

A full-bridge switch-mode dc-dc converter produces a four-quadrant controllable dc output. This full-bridge dc-dc converter is called H-bridge and the overall system is shown in Fig. 2.9, where the line-frequency ac input is rectified into dc by means of a diode rectifier and filtered by means of a filter capacitor. An energy dissipation circuit is included to prevent the filter capacitor voltage from becoming large in case of braking of the dc motor.

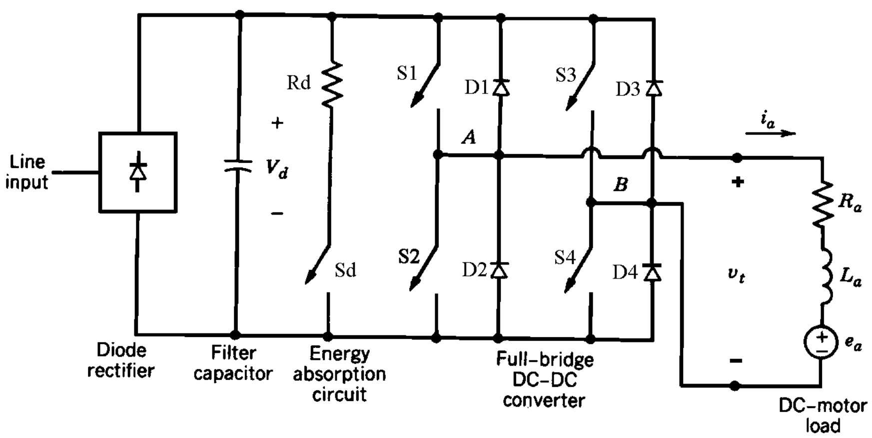


Fig. 2.9 H bridge dc-dc converter for dc motor servo drive

The H bridge dc-dc converter is capable four quadrant operation, and can be controlled in Bipolar or Unipolar PWM modes. Fig. 2.10 shows the waveform for Bipolar operation. A triangle carrier signal with period T_s is compared with a control signal $v_{control}$, which produces switching signal for all four switches. Switches (S1, S4) and (S3, S2) are controlled in pairs. During period t_1 when $v_{control} > v_{tri}$, (S1, S4) are on, and (S3, S2) off, and the motor terminal voltage is V_d, During period t_2 when $v_{control} < v_{tri}$, (S1, S4) are off, and (S3, S2) on, and the motor terminal voltage is $-V_d$. Thus the average terminal voltage is given by:

$$V_t = [t_1 V_d - (T_s - t_1)V_d]/T_s = V_d(2D-1) \qquad (2.26)$$

where $D = t_1/T_s$ is the duty ratio. From the triangle waveform, one obtains:

$$\frac{T_s}{2V_{tp}} = \frac{t_1}{(V_{tp} + v_{control})} \quad \therefore \quad 2D = 1 + \frac{v_{control}}{V_{tp}} \tag{2.27}$$

where V_{tp} is the peak value of the triangle carrier. Substitute Eqn.(2.27) into Eqn. (2.26) results in:

$$V_t = (V_d/V_{tp})v_{control} = k\, v_{control} \tag{2.28}$$

where $k = (V_d/V_{tp})$ is constant, and referred to as gain of the converter.

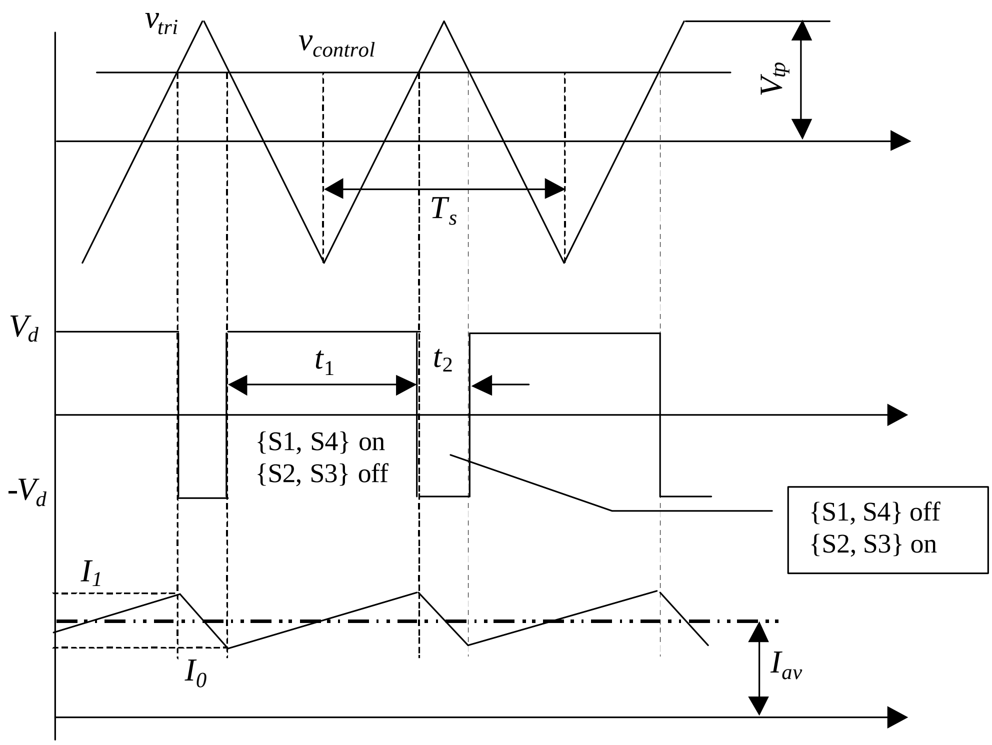


Fig. 2.10 Bipolar operation of H bridge

CURRENT RIPPLE

The switching of the terminal voltage between V_d and $-V_d$ will inevitable cause ripple in armature current, which not only incurs additional losses, but also produces pulsation torque. In steady state operation, the instantaneous speed ω_m can be assumed to be constant if there is sufficient inertia, and therefore $e_a(t) = E_a$.

With reference to Fig. 2.10, at the beginning of period t_1, the armature current I_a is at its minimum value of I_0. Assume that the ripple current is primarily determined by the armature inductance L_a and R_a has a negligible effect, the current increase linearly to I_1 at $t = t_1$, Thus

$$I_1 = I_0 + \frac{V_d - E_a}{L_a} t_1 \tag{2.29}$$

The average current is $(I_1+I_0)/2$ and the peak-to-peak ripple is

$$\Delta I_{pp} = I_1 - I_0 = \frac{V_d - E_a}{L_a} t_1 \tag{2.30}$$

During t_2 period, the terminal voltage is negative, and the current decrease linearly, and reaches I_0 at t = t_2, Hence,

$$I_0 = I_1 - \frac{V_d + E_a}{L_a} t_2 \tag{2.31}$$

Adding Eqn. (2.29) to Eqn. (2.31) and solving for E_a yields

$$E_a = (2D - 1)V_d \tag{2.32}$$

Substitutes Eqn. (2.32) into Eqn. (2.30) and recognising $t_1 = DT_s$ one obtains

$$\Delta I_{pp} = \frac{2T_s V_d (1 - D)D}{L_a} \tag{2.33}$$

It follows that the peak-to-peak current ripple is a function of duty ratio D, and reaches its maximum at D = ½. The maximum DI_{PP} is therefore given by:

$$\Delta I_{pp} = \frac{V_d}{2L_a f_s} \tag{2.34}$$

The operation of unipolar PWM and its corresponding current ripple are illustrated in tutorial example.

REGENERATION AND BRAKING RESISTANCE

For the converter shown in Fig. 2.10, net energy flows to the motor during motoring operation. However, during deceleration, the kinetic energy in the servo system may be recovered and stored in the capacitor C. The question is how the capacitor is chosen so as to store the recovered energy without taking the DC link voltage V_d above a limited value.

Assume a nominal dc link voltage V_s, and a maximum permissible value of (V_s+V_a), V_a denoting added value due to energy recovery, the maximum recoverable energy is given by:

$$E_r = \frac{1}{2}C(V_s + V_a)^2 - \frac{1}{2}CV_s^2 = \frac{1}{2}C(2V_sV_a + V_a^2) \tag{2.35}$$

Therefore, the capacitance $C = 2E_r/(2V_sV_a+V_a^2)$. This results in an efficient system as the maximum amount of energy is recovered. However, voltage rating of the converter needs to be increased to (V_s+V_a).

In systems where the capacitor value and the voltage rating is limited, the recovered energy during braking needs to be harnessed. If the dc power supply is unable to absorb this energy, a damping resistor R_d shown in Fig. 2.10 is used. When V_d reaches the threshold value V_s, switch *Sd* is turned on discharging capacitor C, and dissipating some of energy in R_d. As a result, V_d is reduced. R_d must be chosen so that $R_dI_{max} < V_{dmax}$.

In large power systems, when dissipating energy in this way may cause various problems such as an excessively large resistor bank, overheating, poor efficiency, high running costs, etc., an active rectifier may be used to feed the recovered energy back to utility grid.

2.5.3 Transfer function block diagram of dc servo drive

The transfer function block diagram of a dc servo speed control system is shown in Fig. 2.11. It has an inner current control loop so as to improve the torque (current) response, and to reduce the influence of back-emf on the outer speed loop. This current control loop also limits the armature current to its maximum permissible value. The power electronic converter is represented by a gain of k with no delay. This representation is acceptable when the switch frequency of the converter is much higher than the current loop bandwidth. In this example, both current and speed controllers are of PI type (proportional plus integral control) although more advanced control techniques are also commonly implemented in modern servo drives.

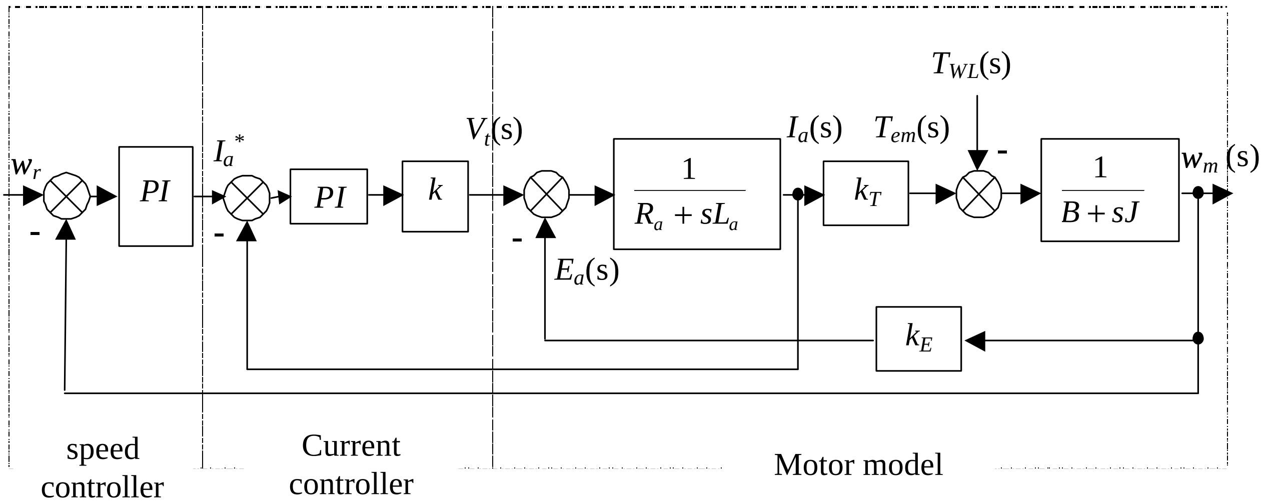


Fig. 2.7 Block diagram representation of the dc motor and load

A third position feedback loop can be wrapped around the speed loop to form a closed-loop position servo control system.