MSc(Eng) Wireless Communication Systems

Module EEE-6431: Broadband Wireless Techniques

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Syllabus Highlights

1. **Introduction - Overview of Broadband Wireless Systems**
2. **Signal Propagation, Pathloss Models and Shadowing**
3. **Statistical Fading Models: Narrowband & Wideband Fading**
4. **Capacity of Wireless Channels**

5. Principles of Multicarrier Modulation

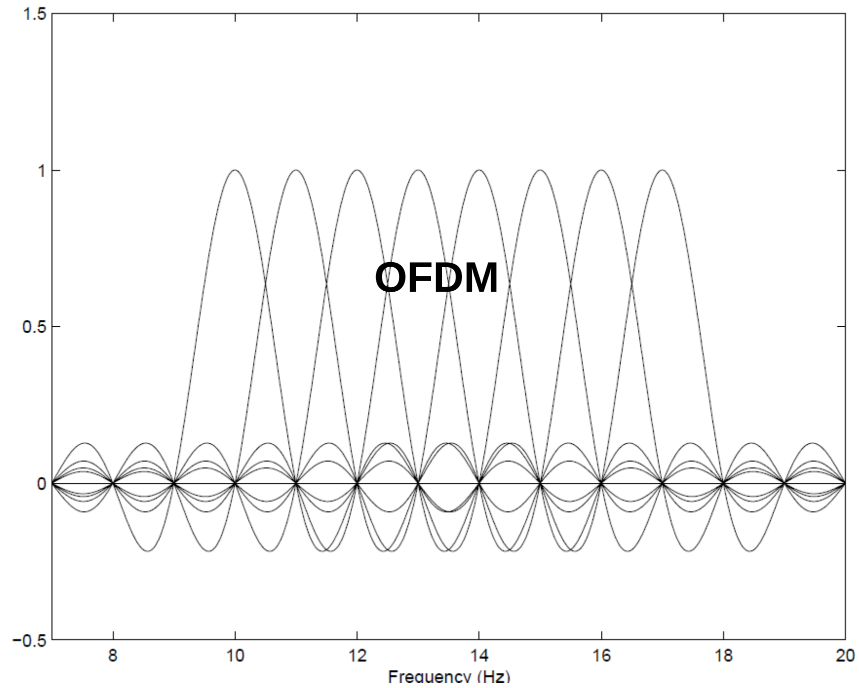
6. **Orthogonal Frequency Division Multiplexing (OFDM)**

Section 4 Review

1. Capacity of wireless channels (AWGN)
2. Capacity of flat fading wireless channels
    1. Only fading distribution known at RX
    2. Fading distribution & CSI known at RX
    3. Fading distribution & CSI known at TX as well as RX
3. The Shannon or Ergodic capacity is given by a probabilistic average
4. Optimum power allocation and method of Lagrangians
5. Optimum Shannon capacity is a “water-filling” formula in time
6. Capacity of frequency selective (FS) fading channels
7. Optimum Shannon capacity is a “water-filling” formula in frequency

Section 5 Outline

1. Data transmission using multiple carriers
2. Multicarrier modulation using overlapping subchannels
3. Mitigation of subcarrier fading



5. Multicarrier (MC) Modulation

Introduction: **The basic idea of multicarrier modulation is to divide the transmitted bit stream into many different substreams and send these over many different subchannels.**

- **Typically the subchannels are *orthogonal* under ideal propagation conditions.**
- **The data rate on each subchannel is *much less* than the total data rate.**
- **The subchannel bandwidth is *much less* than the total system bandwidth.**
- **The number of substreams is chosen to ensure that each subchannel has a bandwidth *less than the coherence bandwidth* of the channel, so the subchannels experience flat fading.**
- **Hence, the ISI on each subchannel is small.**
- **Multicarrier modulation is efficiently implemented digitally (e.g. OFDM).**

History of MC-Modulation: **First developed in 1950s for military HF radios and now commonly found in -**

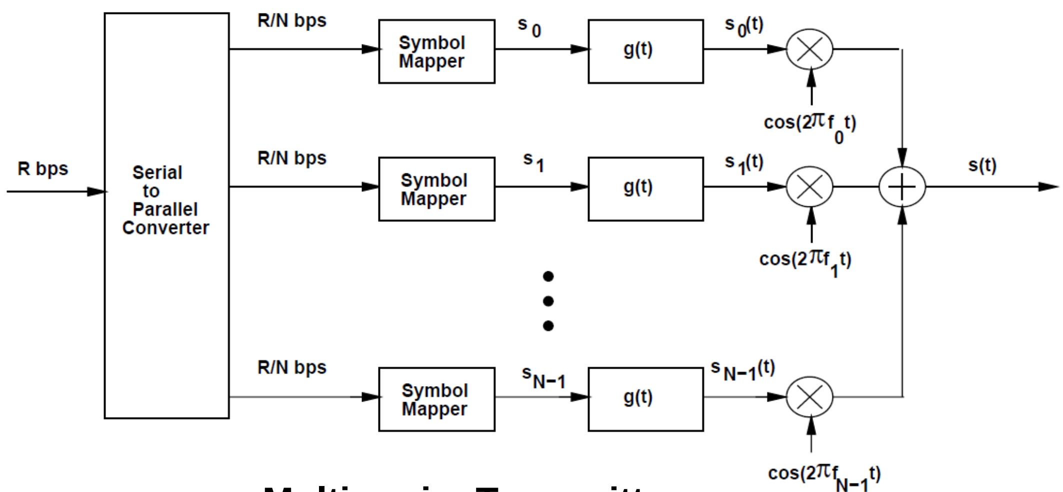
- **Digital audio and video broadcast (DAB, DVB)**
- **Digital subscriber lines (DSL) using discrete multitones**
- **Most recent generations of WLANs (IEEE802.11g/a/n)**
- **New emerging techniques include UWB (802.15), WiMax (802.16) and LTE**

5. Multicarrier Modulation

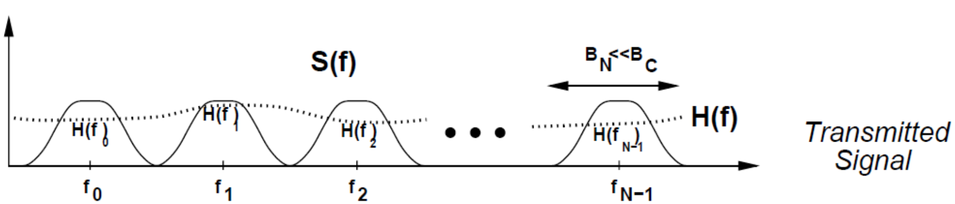
Data Transmission using Multiple Carriers: **MC modulation divides the data stream into multiple substreams that are transmitted over different orthogonal subchannels at different subcarrier frequencies.**

The number of substreams is selected to make the symbol time on each substream greater than the delay spread of the channel, i.e. the substream bandwidth is less than the channel coherence bandwidth.

This ensures that the substreams do not suffer from significant ISI



Multicarrier Transmitter



Consider a linearly modulated system of data rate *R* and bandwidth *B.*

Let channel coherence b/w $B_c < B$ meaning the wideband signal is frequency selective faded.

For *N* linearly modulated subsystems in parallel each subchannel b/w $B_N = B/N$ and rate $R_N = R/N$, then

$B_N = B/N \ll B_c$.

In the time domain, we can write the symbol time T_N as

$T_N \approx 1/B_N \gg 1/B_c \approx T_{rms}$

Where T_{rms} is the channel *rms* delay spread

5. Multicarrier Modulation

Data Transmission using Multiple Carriers contd: Assume that Raise Cosine pulse shaping is used for *g*(*t*), then the symbol duration is given by $T_N = (1+\beta)/B_N$ where β is the *roll-off factor* of the pulse shape. Then the transmitted signal is given by –

$$s(t) = \sum_{i=0}^{N-1} s_i g(t) \cos(2\pi f_i t + \phi_i)$$

Where s_i is the modulation symbol associated with the *i-th* subcarrier and ϕ_i the phase off-set

For non-overlapping channels $f_i = f_0 + i \times B_N,\ \ i = 0, 1, \ldots N-1$.

The substreams occupy orthogonal subchannels of b/w B_N giving a system b/w $B = N \times B_N$ and data rate $R = N \times R_N$.

This form of MC modulation does not change the data rate or bandwidth relative to the original system, but ISI is almost completely eliminated.

This type of MC-Modulation has a number of *significant short comings:*

- **In a real implementation, the pulse shape *g*(*t*) is truncated resulting in an increased bandwidth occupancy of the subchannel. We denote this increase by ε/T_N and each subchannel is separated by $(1+\beta+\varepsilon)/T_N$ and the total bandwidth penalty due to time limiting is given by $\varepsilon N/T_N$.**
- **Then the total required system bandwidth for non-overlapping subchannels is:**

$$B = \frac{N(1+\beta+\varepsilon)}{T_N}$$

5. Multicarrier Modulation

Data Transmission using Multiple Carriers contd:

Example 1: Consider a MC system with a total passband b/w of 1 MHz. If the system operates in a city with a channel *rms* delay spread T_{rms} = 20 μs, how many non-overlapping subchannels are needed to obtain *approximately* flat fading in each subchannel?

Solution: The channel coherence b/w is given by –

$$B_c = 1/T_{rms} = \frac{1}{20\times10^{-6}} = 50 \text{ kHz}$$

For flat fading on each subchannel –

$B_N = B/N = 0.1B_c \ll B_c$. Which gives

$$N = B/0.1\, B_c = \frac{1\times10^6}{0.1\times50\times10^3} = 200 \text{ subchannels}$$

In a DFT implementation of MC modulation, we would choose *N* a power of 2 (i.e. *N* = 256)

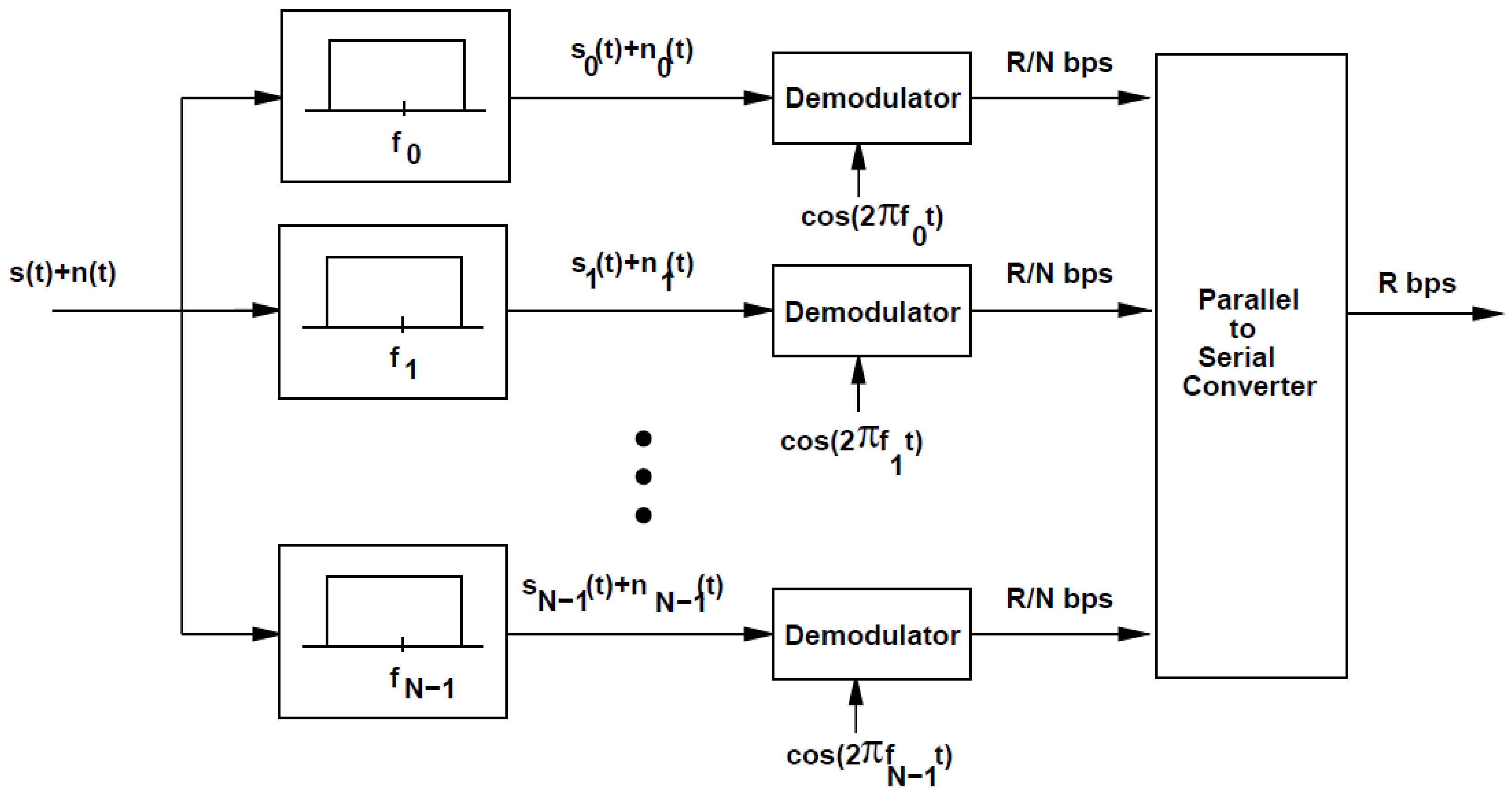
Example 2: Consider a MC system with T_N = 0.2 ms and non-overlapping subchannels: $T_N \gg T_{rms}$, where T_{rms} is the channel *rms* delay spread, ensures each subchannel experiences little ISI. If the system has *N* = 128 subchannels and uses time-limited raised cosine pulses with β = 1 and ε = 0.1, what is the total system bandwidth?

Solution: The total system b/w is given by –

$$B = \frac{N(1+\beta+\epsilon)}{T_N} = \frac{128(1+1+0.1)}{0.2\times10^{-3}} = 1.344 \text{ MHz}$$

5. Multicarrier Modulation

Data Transmission using Multiple Carriers contd: **The MC Modulation Receiver is –**



Non-overlapping MC modulation can be spectrally inefficient, requires almost ideal LPF at the receiver to maintain orthogonality of the subcarriers, and the scheme requires *N* independent modulators & demodulators which is expensive in terms of cost, power consumption & size.

5. Multicarrier Modulation

MC Modulation with Overlapping Subchannels: **Overlapping the subchannels improves the spectral efficiency of a MC system. We still require the subcarriers to be orthogonal if they are to be separated at the receiver.**

A set of approximately orthogonal subcarriers (basis functions) on the interval [0, T_N] for any {ϕ_i} is given by

$$\{cos(2\pi(f_0 + i/T_N)t + \varphi_i)\}, i = 0, 1, 2 \ldots$$

Proof:

$$\frac{1}{T_N}\int_0^{T_N} \cos\left(2\pi\left(f_0+\frac{i}{T_N}\right)t+\varphi_i\right)\cdot\cos\left(2\pi\left(f_0+\frac{j}{T_N}\right)t+\varphi_j\right)dt$$

$$=\frac{1}{2T_N}\int_0^{T_N} \cos\left(2\pi t\frac{(i-j)}{T_N}+\varphi_i-\varphi_j\right)dt+\frac{1}{2T_N}\int_0^{T_N} \cos\left(2\pi t\left(2f_0+\frac{(i+j)}{T_N}\right)+\varphi_i+\varphi_j\right)dt$$

$$\approx\frac{1}{2T_N}\int_0^{T_N} \cos\left(2\pi t\frac{(i-j)}{T_N}+\varphi_i-\varphi_j\right)dt = 0.5\partial(i-j)$$

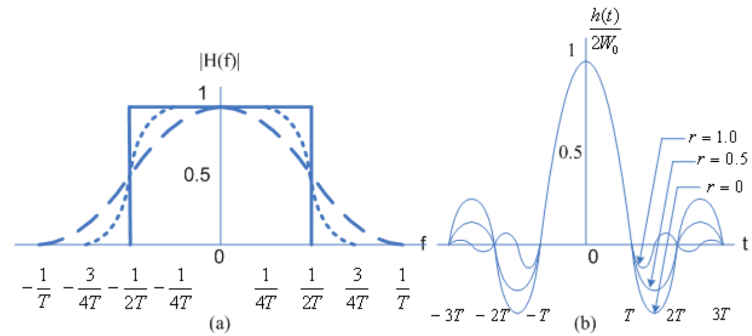
Importantly: **No set of subcarriers with a frequency separation < 1/T_N form an orthogonal basis on the interval [0, T_N] making 1/T_N the minimum subcarrier separation for orthogonality over the interval [0, T_N].**

Since the subcarriers are orthogonal then the set of functions

$$\{g(t)cos(2\pi(f_0 + i/T_N)t + \varphi_i)\}, i = 0, 1, 2 \ldots N-1$$

also form a set of orthonormal bases on the interval [0, T_N] for a suitable choice of *g(t)*.

5. Multicarrier Modulation



MC Modulation with Overlapping Subchannels:

A common choice of $g(t)$ is the family of

raised cosine pulses.

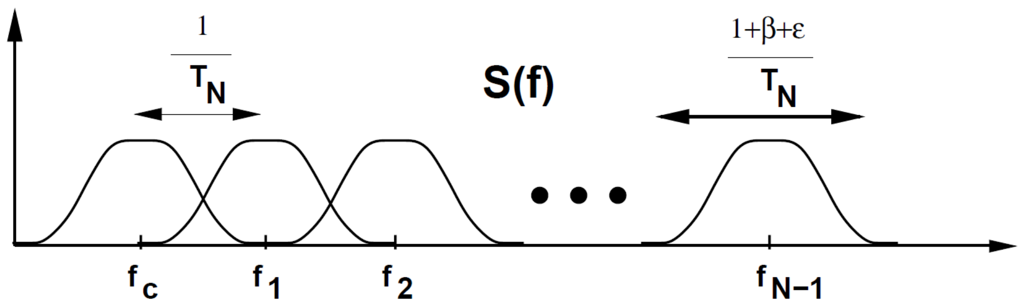
Even though the subcarriers overlap, being orthogonal allows the modulated signal in each subcarrier to be separated at the receiver.

For overlapping subchannels the total system bandwidth is given by:

$$B = \frac{N+\beta+\varepsilon}{T_N} \approx \frac{N}{T_N}$$

That is, for large N the effect of β and ε on the total system b/w is small.

Multicarrier modulation with overlapping subcarriers



Example 3: Compare the required b/w of a MC system with overlapping subchannels using the same parameters as in example 2 above.

Solution: In the previous example $T_N = 0.2$ ms, $N = 128, \beta = 1, \varepsilon = 0.1$.

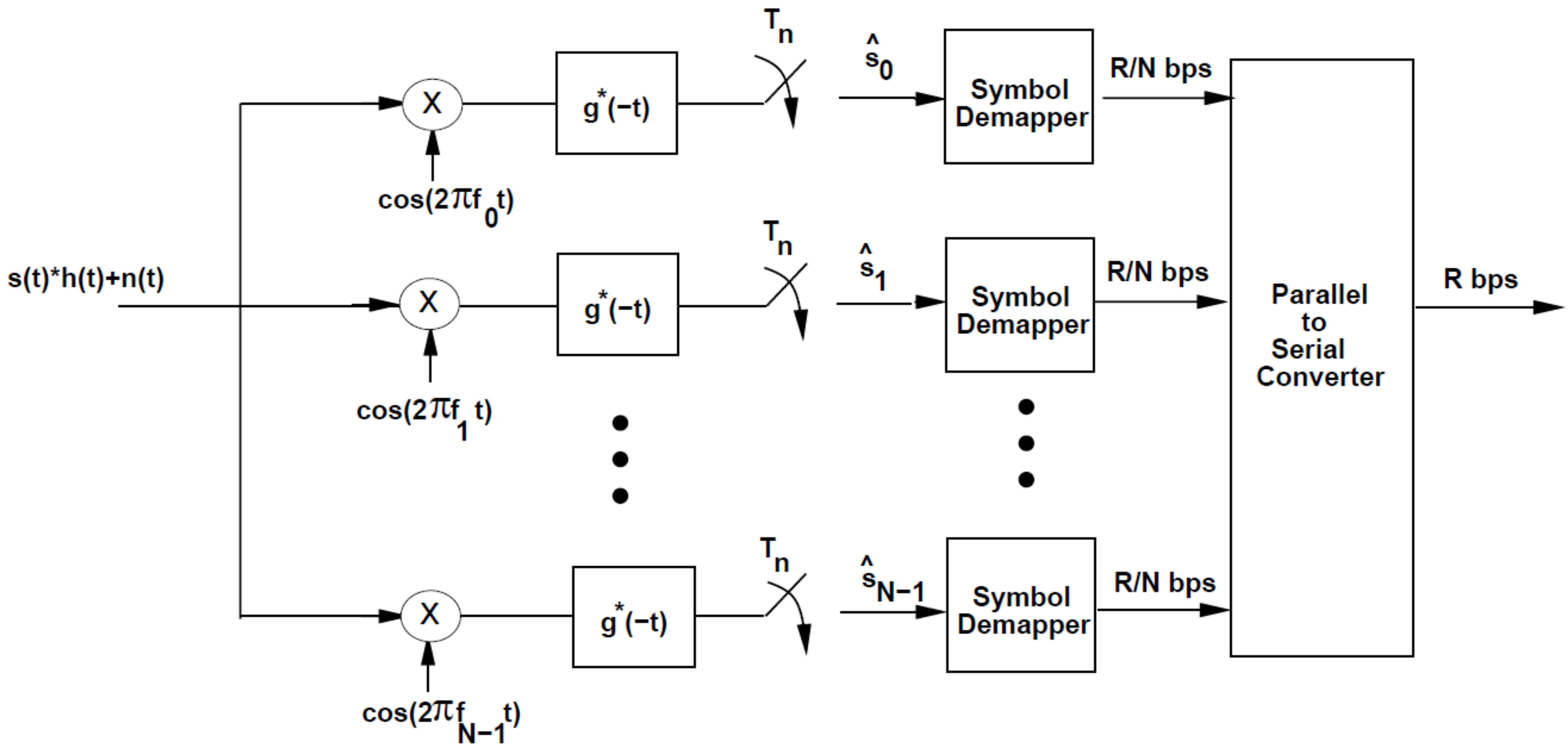
With overlapping channels $B = \frac{N+\beta+\varepsilon}{T_N} \approx \frac{128+1+0.1}{0.0002} = 645.5\text{ kHz} \approx \frac{N}{T_N} = 640\text{ kHz}$

In E.G.-2 the required b/w was 1.344 MHz, i.e. more than double the b/w cf overlapping.

5. Multicarrier Modulation

MC Modulation with Overlapping Subchannels:

A different *receiver structure* is needed when separating out overlapping subcarriers.



Multicarrier Receiver for Overlapping Subcarriers.

5. Multicarrier Modulation

MC Modulation with Overlapping Subchannels:

By considering the received signal only (*without noise or multipath effects*), the recovered modulation symbol is achieved by –

$$\hat{s}_i = \int_0^{T_N} \left(\sum_{j=0}^{N-1} s_j g(t) \cos\left(2\pi f_j t + \varphi_j\right) \right) \cdot g(t) \cos\left(2\pi f_i t + \varphi_i\right) dt$$

$$= \sum_{j=0}^{N-1} s_j \int_0^{T_N} g^2(t) \cos\left(2\pi \left(f_0 + \frac{j}{T_N} \right) t + \varphi_j \right) \cos\left(2\pi \left(f_0 + \frac{i}{T_N} \right) t + \varphi_i \right) dt$$

$$= \sum_{j=0}^{N-1} s_j \partial(j-i)$$

This follows because the functions $\{g(t)cos(2\pi(f_0 +$

$$= s_i$$

If the channel and noise effects are included, then the symbol in the *i-th* subchannel is scaled by the channel gain $\alpha_i = H(f_i)$ and perturbed by the noise n_i of power $N_0 B_N$.

$$\hat{s}_i = \alpha_i s_i + n_i$$

The system efficiently uses bandwidth, but orthogonality of subcarriers can be severely compromised by frequency and time off-sets.

5. Multicarrier Modulation

Mitigation of Subcarrier Fading: **In MC modulation each subchannel is rendered narrowband, however each subchannel can then experience flat fading which can cause large bit error rates.**

If the average transmit power on the *i-th* subcarrier is P_i and the subchannel fading coefficient is α_i, then the Rx SNR is given by –

$$\gamma_i = \frac{{\alpha_i}^2 P_i}{N_0 B_N}$$

Where B_N is the subchannel bandwidth.

Hence, it is important to compensate for flat-fading in the subchannels – 4 main techniques used are:

- **Forward Error Correction (FEC) coding with interleaving over time and frequency (this is the most popular);**
- **Frequency equalisation;**
- **Precoding;**
- **Adaptive loading.**

Precoding and Adaptive loading require CSI at the transmitter.

Coding with Interleaving over time & frequency: **Involves encoding data bits into codewords, interleaving the resulting coded bits over both time and frequency, and then transmitting the coded bits over different subchannels such that the coded bits within a given codeword all experience *independent fading*.**

5. Multicarrier Modulation

Coding with Interleaving over time & frequency contd: **If most of the subchannels have a** ***high SNR*****, then most coded bits will be received correctly, and the errors associated with the** ***few bad subchannels can be corrected*****.**

- **Coding across subchannels exploits** ***frequency diversity*** **inherent to a multicarrier system to correct for errors.**
- **This technique only works well if there is sufficient frequency** ***diversity across the total*** **system bandwidth.**
- **If** ***the coherence bandwidth of the channel is large*****, then the fading across subchannels will be highly correlated, which will significantly reduce the effect of coding.**

Most coding for OFDM assumes channel information in the decoder. Channel estimates are typically obtained by a two dimensional pilot symbol transmission over both time and frequency.

Note that coding with frequency/time interleaving takes advantage of the fact that the data on all the subcarriers is associated with the same user, and can be jointly processed.

Frequency equalisation: **The flat faded signal in a subchannel is equalised by dividing the received signal/symbol over the interval [0, T_N] by an estimate of the channel coefficient $\hat{\alpha}_i$.**

$$r_i = \alpha_i s_i + n_i$$

$$\hat{s}_i = \frac{r_i}{\hat{\alpha}_i} = \frac{\alpha_i s_i}{\hat{\alpha}_i} + \frac{n_i}{\hat{\alpha}_i} \approx s_i + \frac{n_i}{\hat{\alpha}_i}$$

While equalisation removes the effect of flat-fading on the signal, it enhances the noise power.

5. Multicarrier Modulation

Precoding: **This technique is similar to frequency equalisation, except that the fading is inverted at the Tx instead of the Rx. The transmitter needs to know the subchannel flat-fading gains $\{\alpha_i : i = 0, 1, 2 \ldots N-1\}$. These are obtained by estimation at the Rx and subsequent transmission to the Tx.**

$$t_i = \frac{s_i}{\hat{\alpha}_i} \quad \text{and} \quad r_i = \alpha_i t_i + n_i$$

$$\therefore \hat{s}_i = \frac{\alpha_i s_i}{\hat{\alpha}_i} + n_i \approx s_i + n_i$$

Precoding is frequently used on wireline MC systems such as ADSL

Adaptive Loading: **This technique is based on adaptive modulation techniques. It is commonly used on slowly changing channels where CSI can be passed from the receiver to the transmitter before the channel changes significantly.**

From Section 4 the capacity of a Frequency Selective (FS) Fading channels of total power constraint *P*, *N* independent subchannels and subchannel bandwidth B_N is given by -

$$C = \max_{P_i : \sum_i P_i \le P} \sum_{i=0}^{N-1} B_N \log_2\left(1 + \frac{\alpha_i^2 P_i}{N_0 B_N}\right)$$

The power allocation P_i that maximises this expression is water-filling over frequency, i.e.

$$\frac{P_i}{P} = \begin{cases} \frac{1}{\gamma_c} - \frac{1}{\gamma_i} & \gamma_i \ge \gamma_c \\ 0 & \gamma_i < \gamma_c \end{cases}$$

γ_c defines a cut-off SNR and in general $\gamma_i = \frac{\alpha_i^2 P}{N_0 B_N}$

5. Multicarrier Modulation

Adaptive Loading contd: Substituting the power adaptation formula into the power constraint gives -

$$C=\sum_{i:\gamma_i\geq\gamma_c} B_N \log_2\left(\frac{\gamma_i}{\gamma_c}\right)$$

For a Gray coded MQAM modulation scheme, we can adapt the transmit power P_i subject to the average total power constraint P and an instantaneous BER constraint $P_b(\gamma_i)=P_b$

$$P_b(\gamma_i)\leq\frac{1}{5}\exp\left[\frac{-3\gamma_i}{2\left(M(\gamma_i)-1\right)}\frac{P_i}{P}\right],\quad M(\gamma_i)=1+K\gamma_i\frac{P_i}{P},\quad K=\frac{-3}{2\ln(5P_b)}\leq 1$$

The sum data rate is given by –

$$R=\sum_{i=0}^{N-1} B_N \log_2\left(1+\frac{K\gamma_i P_i}{P}\right)$$

The optimal power constraint and corresponding data rate are given by -

$$\frac{KP_i}{P}=\begin{cases}\dfrac{1}{\gamma_K}-\dfrac{1}{\gamma_i} & \gamma_i\geq\gamma_K\\ 0 & \gamma_i<\gamma_K\end{cases}\qquad R=\sum_{i:\gamma_i\geq\gamma_K} B_N \log_2\left(\frac{\gamma_i}{\gamma_K}\right)$$

$\gamma_K=\frac{\gamma_c}{K}$ defines the cut-off fade depth for an MQAM scheme.

Summary & Main Points:

- **Non-overlapping MC modulation is conceptually straight forward but can be b/w inefficient and too complex.**
- **Overlapping MC modulation significantly improves the bandwidth efficiency but is sensitive to frequency and time off-sets.**
- **The min^{m} frequency separation for orthogonal overlapping subcarriers is $\Delta f = 1/T_N$.**
- **Each subchannel suffers flat fading but symbol ISI is avoided.**
- **Frequency selective effects over the system bandwidth are mitigated by channel coding & interleaving, frequency equalisation, precoding and adaptive loading.**