The University Of Sheffield.

Data Provided:

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ H\ m^{-1}$

Impedance of free space $Z_0 = 377\ \Omega$

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2011-12 (2.0 hours)

EEE345 Engineering Electromagnetics

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section. Where an abbreviation or symbol is not defined it can be assumed to have its usual meaning with which candidates should be familiar.**

1. **a.** Explain Maxwell's modification to Ampere's Law. **(5)**

b. Hence obtain

(i) A wave equation for a 50 *MHz* magnetic field propagating in free space,

(ii) A diffusion equation for a 50 *Hz* magnetic field propagating in a copper conductor,

explaining any assumptions made. The following relation may be of use:

$$\nabla \times \nabla \times \underline{G} = \nabla(\nabla \bullet \underline{G}) - \nabla^2 \underline{G}\ .$$

(6)

c. If a general 1D solution to the wave and diffusion equations in **b.** can be written

$$H_z = |H| exp\left(-\alpha y + j\left(\omega t - \beta y\right)\right)$$

calculate

(i) The wavelength of the 50 *MHz* field in free space,

(ii) The wavelength of the 50 *Hz* field and the skin depth in copper.

Assume that for copper $\sigma = 6 \times 10^7\ S/m$, $\mu_o = 4\pi \times 10^{-7}\ H/m$ and that $\varepsilon_o = 8.854 \times 10^{-12}\ F/m$. **(9)**

2. **a.** Write down

(i) Poisson’s equation,

(ii) Laplace’s equation. **(4)**

b. Show that the potential at any point in the *n* layer ($x > 0$) of the semiconductor *p-n* junction in Figure 1, with depletion layer width $\Delta x = 2a$ is given by:

$$V = \frac{\rho}{2\varepsilon}(2ax - x^2)$$

stating any assumptions made, and sketch this potential variation across the junction.

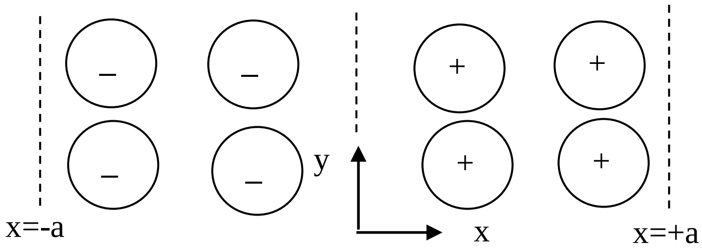

Figure 1 Semiconductor p-n junction **(10)**

c. If a *p-n* semiconductor junction has a depletion layer width of *220 nm*, a volume charge density of $4.5 \times 10^3 C/m^3$, a relative permittivity of *9* and a cross-sectional area of $10^{-9} m^2$, calculate:

(i) The voltage across the junction,

(ii) The junction capacitance.

Assume $\varepsilon_o = 8.854 \times 10^{-12} F/m$. **(6)**

3. **a.** Which of the following *h(x,t)* functions represent a travelling wave? (*x* = spatial dimension, *t* = time, *a*,*b* = constants, *f* = a function). Explain you answers.

(i) $h(x,t) = cos(2xt)$

(ii) $h(x,t) = 4x + t^2$

(iii) $h(x,t) = sin(5x + 8t)$

(iv) $h(x,t) = f(at - bx)$ **(2)**

b. What is the direction and velocity of any identified travelling waves from **3.a.**? **(2)**

c. Sketch and annotate a small elementary length *dx* of: (i) an ideal transmission line and (ii) a lossy transmission line. Indicate all the appropriate elementary resistive, capacitive and inductive components. **(2)**

3. **d.** Show that, for a transmission line with a zero resistance along the line and small but finite conductance between the lines, the propagation constant β for a fixed-frequency ω source is given by the expression below. The symbols have their usual meanings.

$$\beta = \omega(LC)^{1/2}\left\{1 - \frac{j}{2}\left(\frac{G}{\omega C}\right)\right\}$$ **(8)**

e. A 200 *MHz* voltage source delivers a sinusoidal signal of amplitude 5 *V* into a transmission line cable. The cable has the following characteristics: $C = 1\ pF\ m^{-1}$, $L = 2\ nH\ m^{-1}$, $R = 0\ \Omega\ m^{-1}$ and $G = 5\ m\Omega^{1}\ m^{-1}$. What is the maximum length of cable that can be used with a receiver that can detect pulses greater than 1 *V*? **(4)**

f. A transmission line (L = 10 $nH\ m^{-1}$, C = 1 $pF\ m^{-1}$) has effectively zero conductance and resistance. What is the impedance of the line and what is the reflection coefficient if it is connected to a load with an impedance of 200 Ω? **(2)**

4. **a.** The figure below (Fig.2) represents the cross-section of a coaxial cable, showing the direction of the current flow at time *t*.

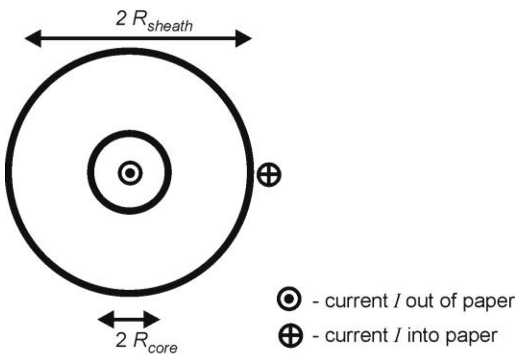

Figure 2 Sketch of coaxial cable cross-section

On two copies of this sketch, indicate the direction of a) the electric field and b) the magnetic field. **(3)**

b. The dielectric material between the core and the sheath has a relative permeability μ_r = 2. Assuming that the current flows only on the surface of the core, show that the magnetic flux density *B* at radius *r* ($R_{sheath} > r > R_{core}$) is given by:

$$B(r) = \frac{\mu_0 I}{\pi r}$$ **(3)**

c. Using this expression, show that the inductance of the cable per unit length is given by:

$$L = \frac{\mu_0}{\pi} ln\left(\frac{R_{sheath}}{R_{core}}\right)$$ **(3)**

d. For a sheath diameter of 10 *mm* and a core diameter of 1 *mm*, what capacitance per metre is required to achieve no refection losses when connecting source and load of 100 Ω? Assume a lossless transmission line. **(4)**

e. Write down equations for the sinusoidally varying electric and magnetic fields of a linearly polarized wave travelling in the positive *x*-direction in free space. **(2)**

f. Sketch the fields in **4.e.** with respect to the *xyz* axes. **(1)**

g. For an electric field ***E*** and a magnetic field ***H***, what does the vector $\mathbf{E}\times\mathbf{H}$ represent? **(1)**

h. If the RMS value of the electric field is 800 *V/m*, what is the power density on a surface perpendicular to the *x*-axis? **(3)**

GGC / GW