



The
University
Of
Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2014-15 (3.0 hours)

EEE6222 Principles of Communications

Answer **FOUR** questions. **No marks will be awarded for solutions to a fifth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. For a particular communication channel (single input, single output) with a bandwidth of 4kHz and a signal to noise ratio (S/N) of 20dB:

(i) Calculate the maximum theoretical symbol rate that can be achieved without inter symbol interference (ISI) problems occurring. Give the ideal Nyquist pulse function for this case. (3 marks)

(ii) Calculate the capacity of this channel and explain how to achieve this capacity in practice. (3 marks)

(iii) Suppose an analogue signal is sampled and then quantized to 5 levels (-2, -1, 0, 1, 2). The probabilities of the samples taking the quantized values are respectively (1/2, 1/4, 1/8, 1/16, 1/16). Calculate the maximum number of samples per second we can transmit through the above channel without error.

(2 marks)

(8)

- b. What is the Shannon limit on the minimum bit energy per noise density ($\frac{E_b}{N_o}$) below which there is no error-free communication? Derive the expression for this limit and show all working.

(6)

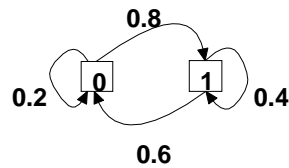
- c. x and y are two independent random variables with their probability density functions given by $f_X(x)$ and $f_Y(y)$, respectively. Derive the probability density function of their sum $z=x+y$ in detail.

(6)

2. a. In a communication system, the received signal may be processed by a matched filter. Explain, with the aid of diagrams, the operation of a matched filter.

(5)

- b. A particular information source can be represented by the following Markov model:



- (i) Determine the entropy of this source. (5 marks)

- (ii) Since this is a source with memory, extension codes can be used to develop a more efficient coding scheme. Determine such a unique, prefix-free coding scheme. (4 marks)

- (iii) Determine the coding efficiency for this coding scheme. (1 mark)

(10)

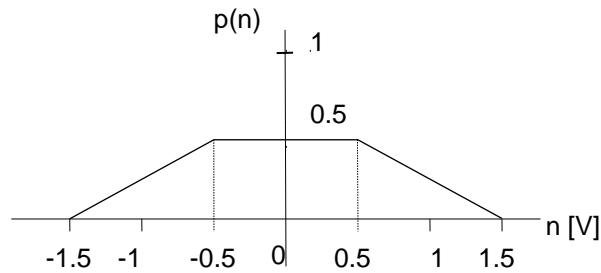
- c. (i) Configure a (4,3) even-parity error-detection code such that the parity symbol appears as the leftmost symbol of the codeword. (1 mark)

- (ii) Which error patterns can the code detect? (1 mark)

- (iii) Compute the probability of an undetected message error, assuming that all symbol errors are independent and that the probability of a channel symbol error is $p=0.002$. (3 marks)

(5)

3. a. A ternary system sends one of three different symbols, S_1 , S_2 and S_3 , with equal *a priori* probabilities, using the voltages $-2V$, $0V$ and $+2V$, respectively. The transmission channel is subject to additive noise with a distribution as shown in the following figure.



Noise probability density function.

- (i) Assuming there is no attenuation along the channel, determine the decision rule for a maximum likelihood detector. (3 marks)
- (ii) What is the overall probability of error using the above detector? (6 marks)
- (iii) If a Maximum A Posteriori (MAP) detector is used and the *a priori* probability of S_2 increases to nearly 1, i.e. $P(S_2) \rightarrow 1$, and $P(S_1), P(S_3) \rightarrow 0$, what will change in the decision rule of (i)? What is the probability of error in this case? (2 marks)

(10)

- b. The probability that a man will live 10 more years is 0.4 and the probability that his wife will live 10 more years is 0.5. Find the probability

- (i) they will both live for 10 more years; (1 mark)
- (ii) at least one will live for 10 more years; (2 marks)
- (iii) neither will live for 10 more years. (1 mark)

(4)

- c. A linear block coding system uses the following codes to send four different messages:

Message	Code word
m_1	000000
m_2	110100
m_3	011010
m_4	110011

- (i) How many bit errors per code word can be detected at the receiver? (2 marks)
- (ii) Is forward error correction possible with this code? (2 marks)
- (iii) Calculate the code rate for this code. (2 marks)

(6)

4. a. Explain the three levels of synchronisation used in a digital communication system.

(3)

- b. Explain briefly the factors that require consideration when choosing a pulse coding modulation (PCM) waveform.

(5)

- c. In a spread spectrum system the information is transmitted using a much wider bandwidth than required for the baseband data and to achieve this, pseudo-noise (PN) sequences are employed as the spread code.

(i) State the basic properties that can be applied to test the randomness of a PN sequence. (3 marks)

(ii) Give an example of a circuit that can be used to generate a PN sequence. (3 marks)

(6)

- d. Alice operates a public key encryption system that uses the RSA algorithm with $e = 3$, $d = 7$, $N = 33$. Assuming that Alice and Bob have never previously met or communicated with each other and Bob wants to send the message $M=9$ to Alice.

(i) Describe each of the steps required to send this message to Alice without anyone else being able to intercept it. (4 marks)

(ii) Explain why Alice can decipher the message correctly. (2 marks)

(6)

5. a. Let every permutation of the four symbols a_1, a_2, a_3, a_4 be equally probable. Let E_j be the event that a_j appears in the j th position, $j=1, 2, 3$, and 4 . Verify that

$$P(E_1 \cup E_3) = P(E_1) + P(E_3) - P(E_1 \cap E_3).$$

(6)

- b. Of three cards one is marked 1 on both sides, one has 0 on both sides, and the third has 1 on one side and 0 on the other. A card is selected at random and found to have 1 on one side. What is the probability that there is 1 on the other side?

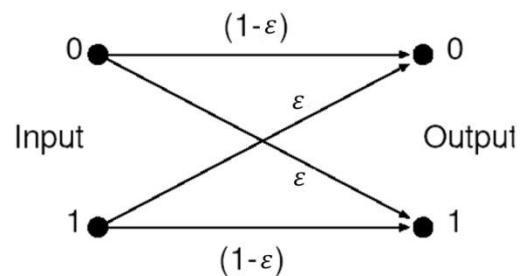
(4)

- c. Given a binary symmetric channel with crossover probability $\varepsilon=0.01$ (as shown below), with equally probable inputs, find the mutual information

(i) between a 1 transmitted and a 1 received; (3 marks)

(ii) between a 1 transmitted and a 0 received; (2 marks)

(iii) between the overall input and the overall output. (5 marks)



(10)

- 6. a.** MIMO stands for Multiple Input Multiple Output and refers to wireless systems that use more than one antenna at both the transmitter and the receiver. Give a narrowband example to show how the potential of a MIMO system can be exploited in practice.

(6)

- b.** (i) What is entropy? (2 marks)

(ii) With the aid of an example, explain on which condition a source will reach its maximum entropy. (4 marks)

(iii) Give a proof for the validity of this condition. (8 marks)

(14)**WL/XC**