4. Voltage Source Inverter (VSI) and PWM techniques for AC drives

4.1 Inverter topology

The most frequently used three-phase inverter consists of three legs, one for each phase, as shown in Fig. 4.1. Each inverter leg is similar to the one used in H-bridge and the output of the each leg, for example v_{AN} (with respect to the neutral point of the DC bus) depends only on V_d, and the switching status. If the switches are assumed to be ideal, and the blanking time required in practical circuits is ignored the output voltage is independent of the output load current since one of the two switches in a leg is always on at any instant. Therefore, the inverter output voltage is independent of the direction of the load current.

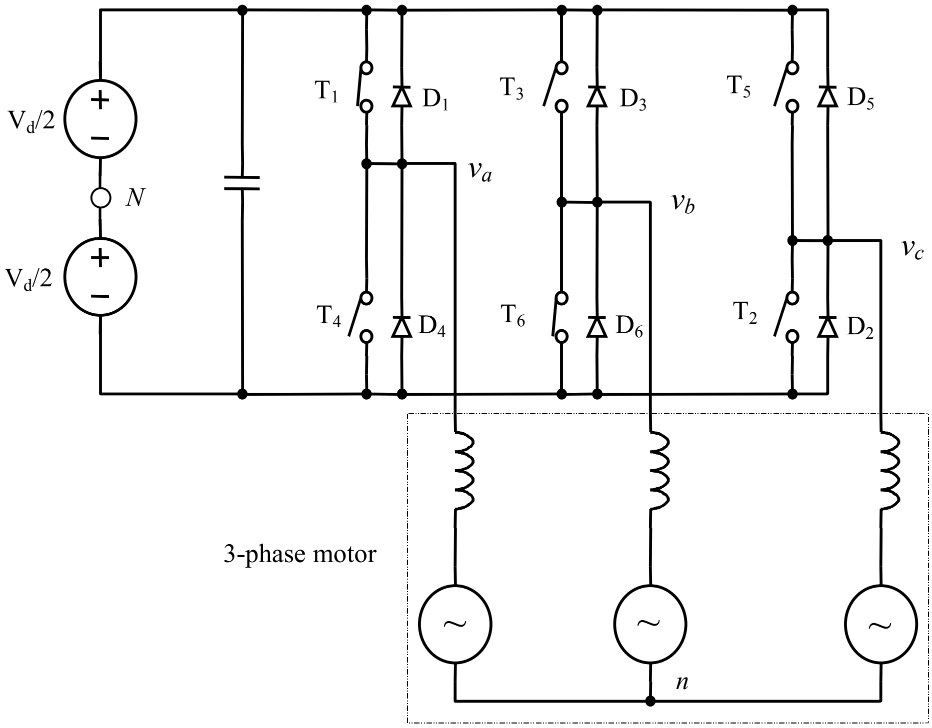


Fig. 4.1 Three phase voltage source inverter

4.2 Sinusoidal PWM method

There are different techniques to control the switches of the inverter in Fig. 4.1 in order to provide three-phase output with variable frequency and amplitude. In the sinusoidal PWM (SPWM) method, as shown in Fig. 4.2, the reference modulation waves with desired frequency and amplitude, usually from open or closed-loop controller, are compared with a triangular carrier wave, and the intersections define the switching instants. The harmonics in the output voltage appears as sidebands of switching frequency and its multiples. Therefore, a high switching frequency results in an essentially sinusoidal current (plus a superimposed small ripple at high frequency) in the motor.

Fig. 4.3(a) shows the signal flow diagram of this PWM method. With reference to Fig. 3.18 for the field oriented control of brushless PM AC motors, the command voltage vector u_s^* is transformed to v_a^*, v_b^*, v_c^*. Three comparators and a triangle carrier signal v_{tr}, which is common to all three-phase signals, generate the logic signals u_a', u_b', u_c' that control the inverter.

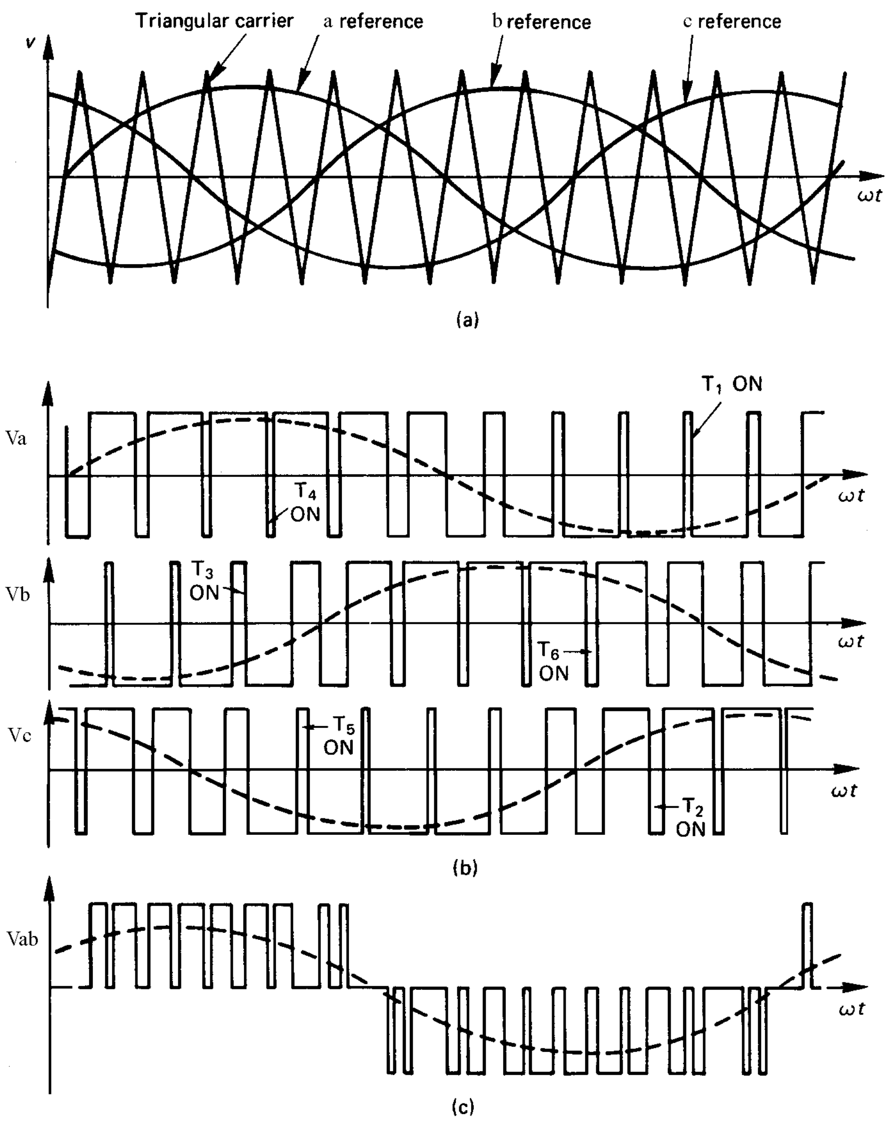
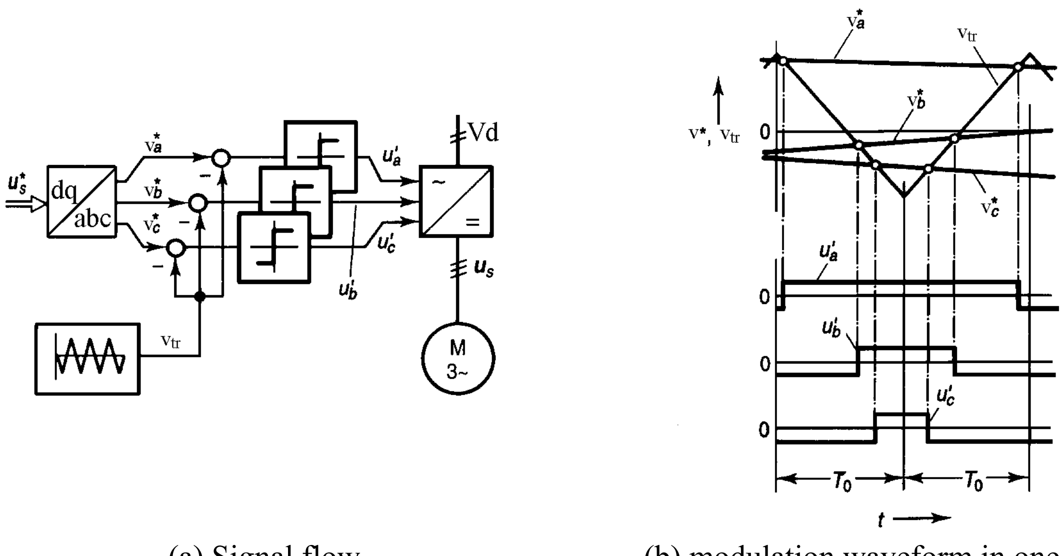


Fig. 4.2 Sinusoidal pulse width modulation



(a) Signal flow (b) modulation waveform in one cycle

Fig. 4.3 Modulation process

Fig. 4.3(b) shows the modulation process in detail, expanded over a time interval of T. Note that the three-phase switching signals u_a', u_b', u_c' are of equal sign at the beginning and at the end of each half period. The three line-to-lines voltage are then zero.

As have been discussed in Power Electronics, the amplitude modulation ratio, m_a, and the frequency modulation ratio, m_f, are defined as:

$$m_a = \frac{V_{ref}}{V_{tr}} \quad ; \quad m_f = \frac{f_s}{f_1} \tag{4.1}$$

Fig. 4.4 shows the waveforms for two different amplitude-modulation ratios. In the linear region (m_a<1.0), the fundamental frequency component in the output voltage varies linearly with the amplitude-modulation ratio m_a. From Fig 4.4, the peak value of the fundamental frequency component in one of the inverter leg is:

$$V_{aN1} = m_a \frac{V_d}{2} \tag{4.2}$$

Therefore, the line-to-line rms voltage at the fundamental frequency, due to 120^0 phase displacement between phase voltages, can be written as:

$$V_{LL1} = \frac{\sqrt{3}}{\sqrt{2}} V_{aN1} = \frac{\sqrt{3}}{2\sqrt{2}} m_a V_d \approx 0.612 m_a V_d \tag{4.3}$$

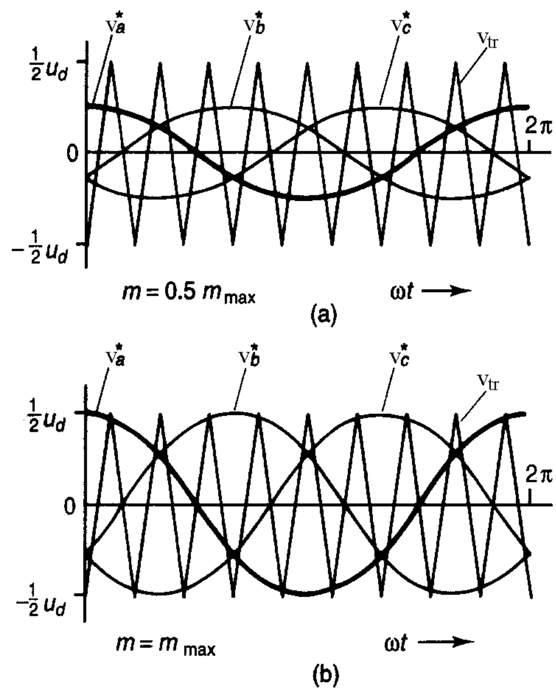


Fig. 4.4 Modulation signals and carrier signal, (a) m_a = 0.5 (b) m_a = 1.0

VOLTAGE HARMONICS

The harmonics in one of inverter legs appears as sidebands, centred around the switching frequency and its multiples, that is, around harmonics m_f, $2m_f$, $3m_f$, and so on. This general pattern holds true for all values of m_a <1.0. For a frequency modulation ratio $m_f \geq 9$ (which is always the case except for in very high power ratings), the harmonic amplitudes almost independent of m_f, though m_f defines the frequencies at which they occur. Theoretically, the frequencies at which voltage harmonics occur can be indicated as:

$$f_h = (jm_f \pm k)f_1 \tag{4.4}$$

That is, the harmonic order h corresponds to the kth sideband of the j times the frequency-modulation ratio m_f:

$$h = (jm_f \pm k) \tag{4.5}$$

The fundamental frequency corresponds to h =1. For odd values of j, the harmonics exists only for even values of k whilst for even values of j, the harmonics exist only for odd values of k.

In three-phase inverter, however, only the harmonics in the line-to-line voltages are of concern. Any triplen harmonics, i.e., h = 3, 6, 9, …, in the phase voltage will disappear in the line-to-line voltage. Table 4.1 shows the normalised line-to-line rms harmonics (V_{LLh}/V_d). as a function of the amplitude-modulation ratio m_a, assuming $m_f > 9$. Only those with significant amplitudes up to $j = 4$ are listed.

Table 4.1 Generalised Harmonics of V_{LL} for $m_f > 9$

h \ m_a	0.2	0.4	0.6	0.8	1.0
1	0.122	0.245	0.367	0.490	0.612
$m_f\pm2$ $m_f\pm4$	0.010	0.037	0.080	0.135 0.005	0.195 0.011
$2m_f\pm1$ $2m_f\pm5$	0.116	0.200	0.227	0.192 0.008	0.111 0.020
$3m_f\pm2$ $3m_f\pm4$	0.027	0.085 0.007	0.124 0.029	0.108 0.064	0.038 0.096
$4m_f\pm1$ $4m_f\pm5$ $4m_f\pm7$	0.100	0.096	0.005 0.021	0.064 0.051 0.010	0.042 0.073 0.030

OVERMODULATION

If the amplitude of modulation signals exceed the peak value of the triangular carrier, i.e., m_a >1.0, overmodulation occurs. Unlike the linear region, in this mode of operation the fundamental frequency voltage magnitude does not increase proportionally with m_a, and can be expressed by the describing function:

$$V_{LL1} = m_a \frac{\sqrt{3}V_d}{\sqrt{2}\pi}\left[\sin^{-1}\left(\frac{1}{m_a}\right)+\left(\frac{1}{m_a}\right)\sqrt{1-\left(\frac{1}{m_a}\right)^2}\right] \qquad m_a > 1.0 \qquad (4.6)$$

Fig. 4.5 shows the variation (V_{LL1}/V_d) as a function of m_a for both linear and overmodulation regions.

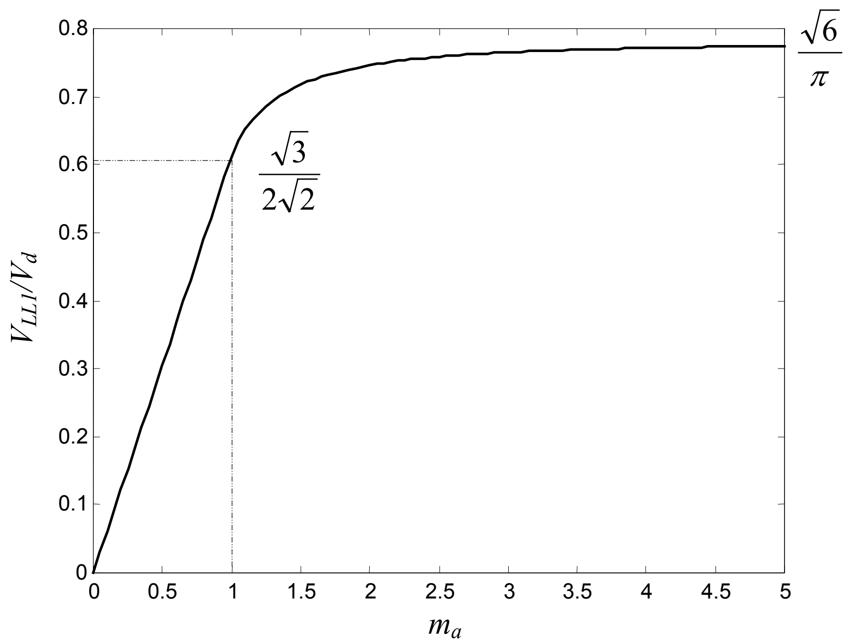
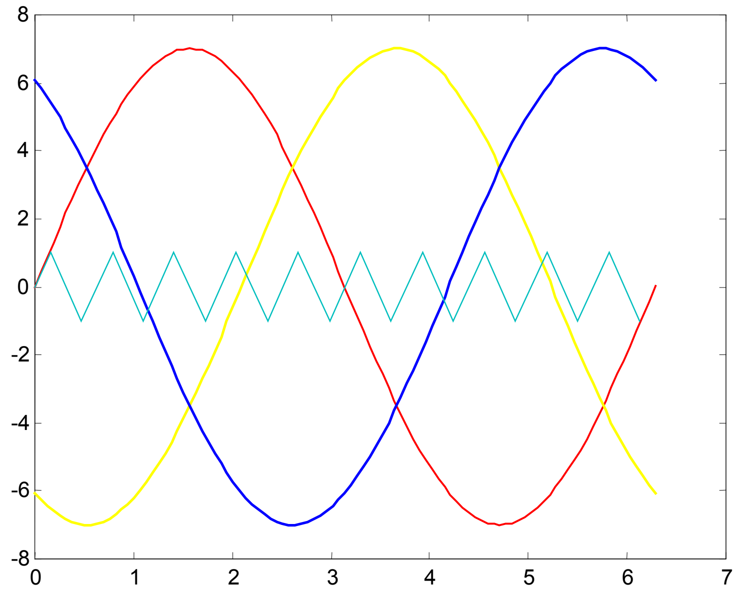


Fig. 4.5 Fundamental line-to-line rms voltage vs. m_a

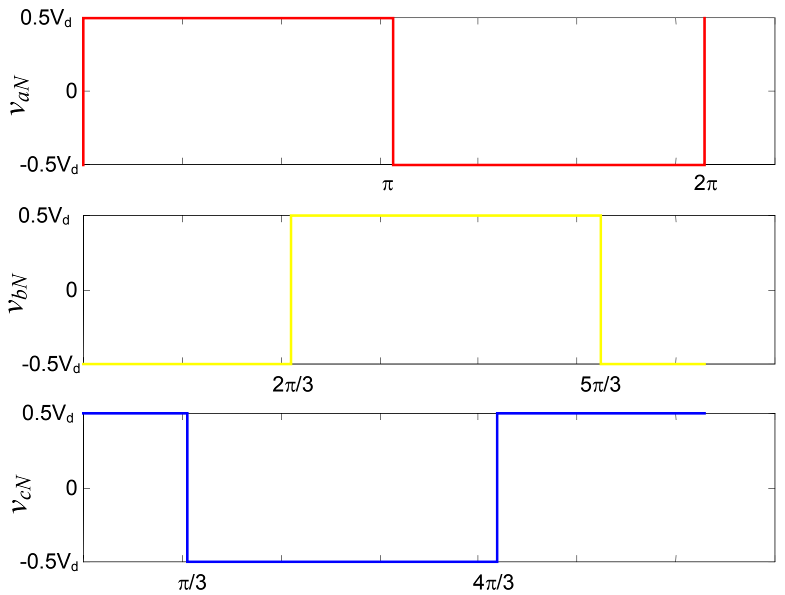
SIX-STEP OPERATION

When $m_a > 5$, PWM degenerates into square wave operation and the phase voltage waveforms with respect to the zero potential point of DC link voltage are shown in Fig. 4.6. In this mode of operation, each switch is on for 180^0(i.e., its duty ratio is 50%), and at any instant of time, three switches are on. The sequence of switching is 101, 100, 110, 010, 011,001, and back to 101, where a "1" indicates the top switch is on and the bottom off and the most significant bit denotes phase *a* inverter leg, and the least significant bit for phase *c*. This mode of operation is referred to as six-step operation, and the corresponding circuit topologies for each of the switching combination are shown in Fig. 4.7.

Although motors connected to the inverter output functions as an active component rather that a passive load, the effective impedances of each phase remain "balanced", That is, so far as voltage drops are concerned, the motor may be represented by three equivalent impedances as shown in Fig. 4.7 for the six possible connections (see tutorial example for a more rigorous proof). Note that a specific phase is alternately switched from positive pole to negative pole and that it is alternately in series with the remaining two phases connected in parallel or it is in parallel with one of the two phases and in series with the other. Hence, the voltage drop across the phase is always 1/3 or 2/3 of the DC link voltage with the polarity of the voltage drop across the phase being determined by whether it is connected to the positive or negative pole. Fig. 4.7 also shows the waveforms for the line-to-line voltages, v_{ab}, v_{bc}, v_{ca}, and line-to-neutral voltages, v_{an}, v_{bn}, v_{cn}. The presence of six "step" in the line-to-neutral voltage waveform is the reason for this type of operation being called a six-step operation. A Fourier analysis of these waveforms indicates a "square wave" type of geometric progression of the harmonics. That is, the line-to-line and line-to-neutral waveforms contain 1/5th of the 5th harmonic, 1/7th of the 7th harmonic, and 1/11th of the 11th harmonic, and so on.



(a) Modulating and carrier signals



(b) Output waveforms

Fig. 4.6 Overmodulation when m_a>5

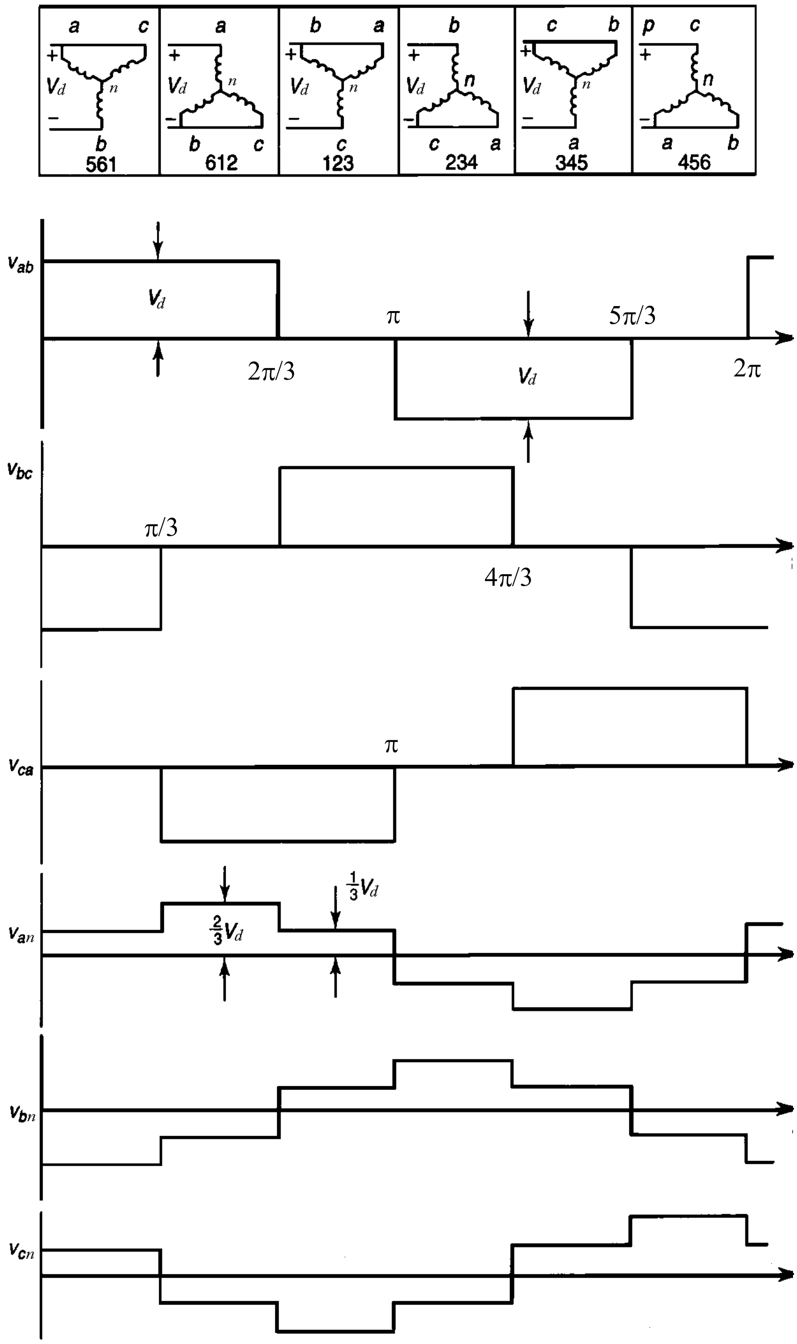


Fig. 4.7 Line-to-line and line-to-neutral voltages across the motor phases when operating in six-step mode.

4.3 Space Vector PWM Method (SVPWM)

Although the SPWM method is simple and easy to implement, it does not fully utilise the available DC link voltage as the maximum line-to-line rms voltage is $0.612V_d$ when m_a =1.0. In the following section, we will discuss a more advanced technique, Space Vector PWM.

4.3.1 Switching States and phase voltages

As has been discussed, there are six active switching states for which the corresponding phase voltages are listed in Table 4.2. In addition, there are two zero states (000) and (111) which correspond to that all three phases are connected to either negative DC or positive DC, respectively. In both cases, the net phase voltages are zero. These to zero states together with the six active states forms all possible combinations of switching states of the three-phase inverter.

Table 4.2 Switching states of three phase inverter

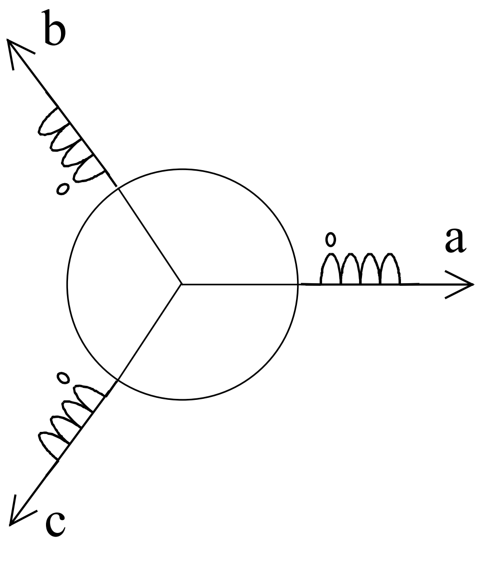
a	b	c	v_{an} (v_d)	v_{bn} (v_d)	v_{cn} (v_d)
0	0	0	0	0	0
1	0	0	2/3	-1/3	-1/3
1	1	0	1/3	1/3	-2/3
0	1	0	-1/3	2/3	-1/3
0	1	1	-2/3	1/3	1/3
0	0	1	-1/3	-1/3	2/3
1	0	1	1/3	-2/3	1/3
1	1	1	0	0	0

4.3.2 Voltage Space Vectors

Consider a symmetrical three-phase winding as shown in Fig. 4.8. The three phase axes are defined by the respective unity vectors **1**, $\boldsymbol{\alpha}$, and α^2, where $\alpha = e^{j\frac{2}{3}\pi}$, $j = \sqrt{-1}$, and the effect of three phase voltages v_{an}, v_{bn}, and v_{cn} on the airgap flux linkage of the motor can be represented by a voltage space vector defined as:

$$V_s = (2/3)\,(v_{an} + \alpha\, v_{bn} + \alpha^2\, v_{cn}) \tag{4.7}$$

It should be noted that the coefficient 2/3 is used to adjust the magnitude of the space vector so that it equals to the peak value of the phase voltage.



$Unity\ vector\ a\ axis : 1$

$Unity\ vetor\ b\ axis : e^{j\frac{2}{3}\pi}$

$Unity\ vetor\ c\ axis : e^{j\frac{4}{3}\pi}$

4.8. A three phase symmetrical winding

According to this definition, the trajectory of the voltage space vectors for the six active switching states forms a hexagon as shown in Fig. 4.9., where

$$\begin{aligned}\vec{V}_a &= v_{an}\mathbf{1}\\ \vec{V}_b &= v_{bn}\boldsymbol{\alpha}\\ \vec{V}c &= v_{bn}\boldsymbol{\alpha}^2\end{aligned} \tag{4.8}$$

and the two zero vectors lie at the origin of the hexagon.

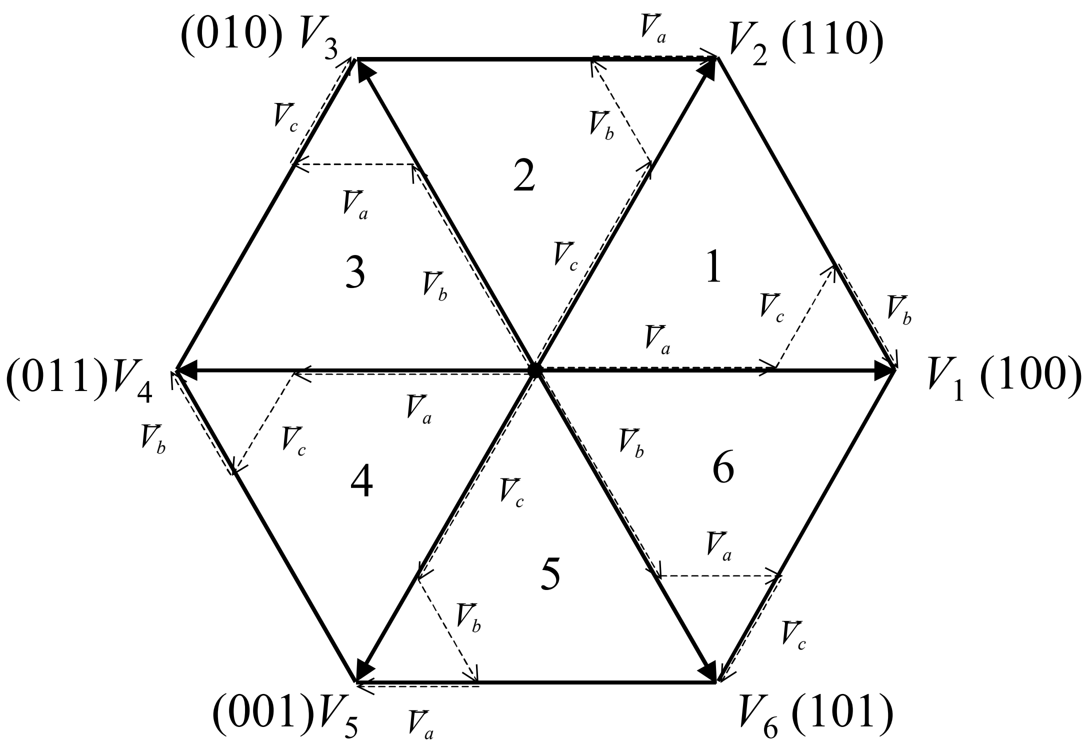


Fig. 4.9 Voltage space vectors

As can be seen, six active switching vectors form the vertices of the hexagon, and they are denoted as V_1, V_2, …, V_6 for notational convenience, each having a magnitude of $(2/3)V_d$. V_1, V_3 and V_5 are aligned with the phase a axis, phase b axis and phase c axis respectively. The six vectors also divide the hexagon into 6 regions as shown in Fig. 4.9.

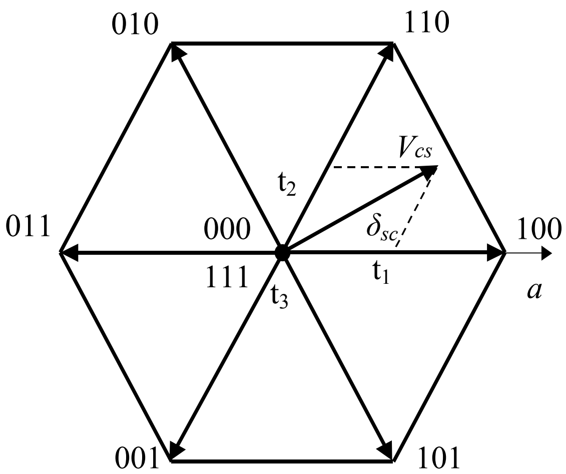
4.3.3 Space Vectors Modulation

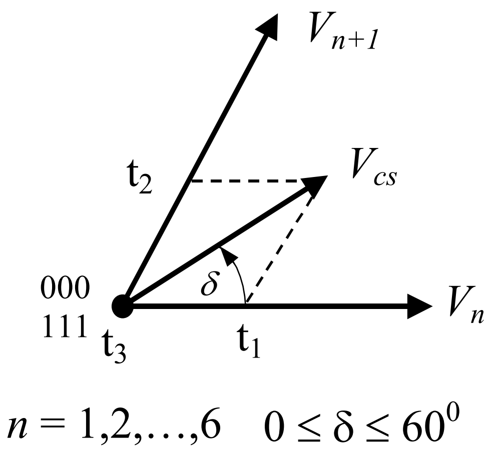
The space vector modulation technique differs from the aforementioned method in that there are not separate modulators used for each phases. Instead, the command reference voltages for three phases are expressed as a voltage vector which is processed as a whole. For example in the field oriented control of Brushless PM AC motors, the d-q command voltages from the controller output is transformed into a α-β reference frame. The resulting v_α and v_β are then expressed as a vector with amplitude V_{cs} and angle δ_{cs} using the following rectangle to polar transformation:

$$V_{cs} = \sqrt{v_\alpha^2 + v_\beta^2} = \sqrt{v_d^2 + v_q^2}$$
$$\delta_{cs} = \tan^{-1}\left(v_\beta / v_\alpha\right) = \theta + \tan^{-1}\left(v_q / v_d\right) \tag{4.9}$$

Note that Eqn. (4.9) combines the two transformations and simplifies the calculation.

A given reference command voltage vector lying within the hexagon as shown in Fig. 4.10(a) can be realised by switching between its two adjacent vectors and zero vectors (000 or 111). For example, a voltage vector in region 1 can be realised by switching between V_1, V_2 and zero. In general, if a voltage vector is located in region n (n=1,2, …, 6), the two switching vector will be V_n and V_{n+1}, as shown in Fig. 4.10(b), where $\delta = \delta_{cs} - (n-1)60^0$.





(a) Reference command within hexagon (b) Switching state vectors

Fig. 4.10 Space vector modulation

The time interval t_1, t_2 and t_3 can be determined by the following equation:

$$TV_{cs} = t_1 V_n + t_2 V_{n+1} + t_3 (0_{111 or 000}) \tag{4.10}$$

where T is the period of the PWM cycle. From Fig.4.10(b) and Eqn. (4.10), one obtains:

$$V_{cs} T \cos\delta = |V_{n1}| t_1 + 0.5 |V_{n+1}| t_2 = (2/3) V_d (t_1 + 0.5 t_2)$$
$$V_{cs} T \sin\delta = (\sqrt{3}/2) t_2 |V_{n+1}| = (V_d / \sqrt{3}) t_2 \tag{4.11}$$

Solving for t_1 and t_2 results in:

$$t_2 = mT \sin\delta$$
$$t_1 = mT \sin(\pi/3 - \delta)$$
$$m = \frac{\sqrt{3} V_{cs}}{V_d} \tag{4.12}$$

The time durations for the two zero vectors can be split into t_0 and t_7 as:

$$t_0 + t_7 = T - (t_1 + t_2) \tag{4.13}$$

In a symmetrical implementation, t_0 and t_7 are equally divided, viz., $t_0 = t_7 = 0.5[T\text{-}(t_1+t_2)]$. Having computed the on-durations of the three switching-state vectors that form one carry cycle, an adequate sequence in time of these vectors must be determined next. Associated to each switching-state vector in Fig. 4.10(a) are the switching polarities of the three inverter-legs, "1" indicating the top switch is on, the bottom one off and "0" the other way round. The zero vectors are redundant, and can be either formed as V_0 (000) or V_7 (111). In a symmetrical implementation, one carry cycle is equally divided into two sub-cycles, and the switching periods for t_0, t_1, t_2 and t_7 are also equally divided for each sub-cycle. During each carrier cycle, one can start and finish with V_0 (000) whilst V_7 (111) being inserted in the middle, or vice versa. In order to minimise the number of switching, the sequence of t_1 and t_2 depends on which two active switching-state vectors are used. For example, in region 1 V_1 and V_2 are used, and if V_0 is used at the start and finish, then the sequence $V_0(000, t_0/2) \rightarrow V_1(100, t_1/2) \rightarrow V_2(110, t_2/2) \rightarrow V_7(111, t_7) \rightarrow V_2(110, t_2/2) \rightarrow V_1(100, t_1/2) \rightarrow V_0(000, t_0/2)$ results in minimum number of switching, as shown in Fig. 4.11

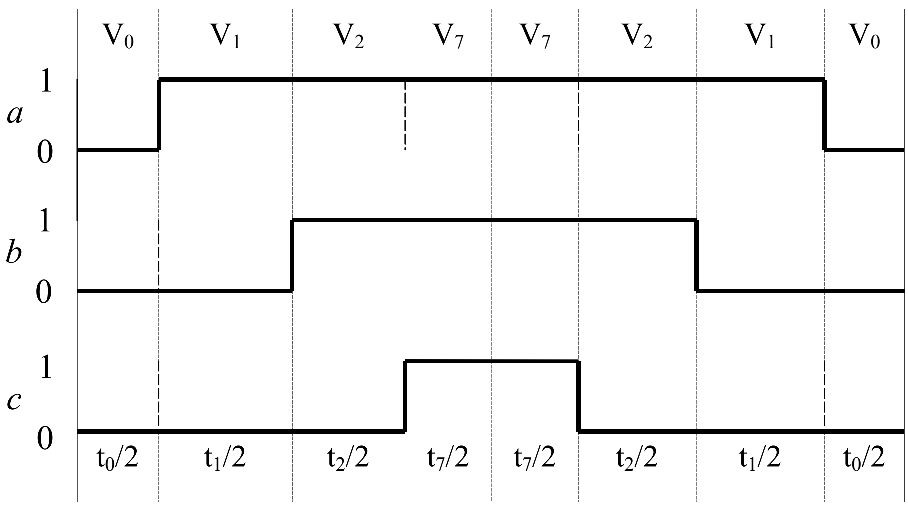


Fig. 4.11 Per carrier cycle switching sequence for region 1 $0<\delta_{cs}<60^0$

On the other hand, in region 2 V_2 and V_3 are used, and the sequence, V_0 (000, $t_7/2$) $\rightarrow$ V_3 (010, $t_2/2$) $\rightarrow$ V_2 (110, $t_1/2$) $\rightarrow$ V_7 (111, t_7) $\rightarrow$ V_2 (110, $t_1/2$) $\rightarrow$ V_3 (010, $t_2/2$) $\rightarrow$ V_0 (000, $t_0/2$), results in a minimum number of switching, as shown in Fig. 4.12. In general, for the odd regions, the time sequence should be $t_0/2 \rightarrow t_1/2 \rightarrow t_2/2 \rightarrow t_7 \rightarrow t_2/2 \rightarrow t_1/2 \rightarrow t_0/2$, whilst for the even regions, $t_0/2 \rightarrow t_2/2 \rightarrow t_1/2 \rightarrow t_7 \rightarrow t_1/2 \rightarrow t_2/2 \rightarrow t_0/2$, i.e., t_1 and t_2 are swapped.

The switching sequences for the other regions are left as a tutorial question.

When the reference command-voltage vector locates inside the inner circle of the hexagon, as shown in Fig. 4.13, the output of the SVPWM is linear with the modulation ration *m.* The maximum peak phase voltage and the rms line-to-line voltage occur at m = 1, and are given respectively by:

$$V_p = V_d / \sqrt{3}$$
$$V_{LL1} = \left(\sqrt{3} \big/ \sqrt{2} \right) V_p = V_d / \sqrt{2} = 0.707 V_d \tag{4.14}$$

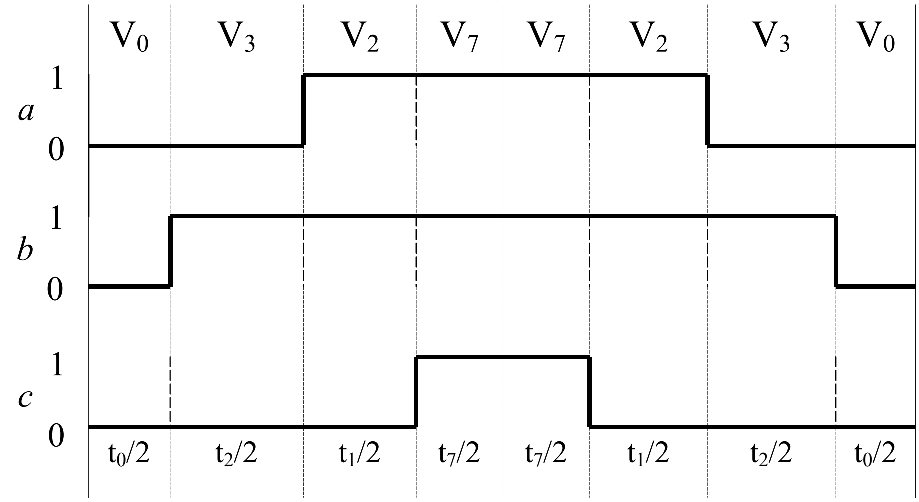


Fig. 4.12 Per carrier cycle switching sequence for region 2 $60^0 < \delta_{cs} < 120^0$

Hence compared with the SPWM technique (V_{LL1}=0.612V_d), the SVPWM method produces 15.5% higher output voltage.

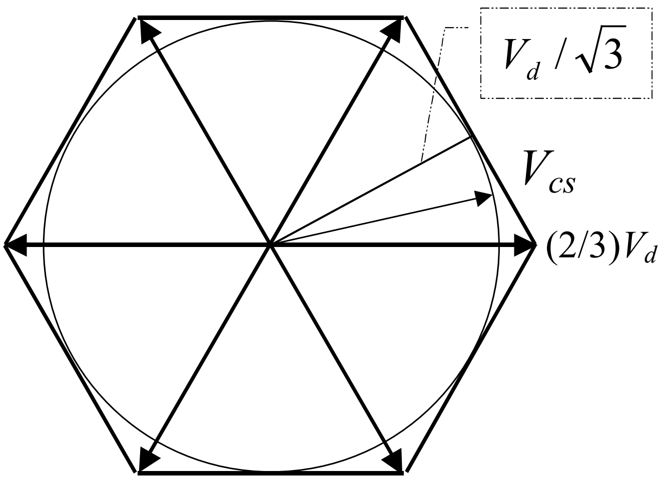


Fig. 4.13 Linear region of SVPWM

As the modulation ratio *m* increases from 0 to 1, the duration of zero vectors decreases. Over-modulation occurs when the reference voltage vector is located outside the hexagon. Some form of over-modulation strategy may be applied to increase the rms output of the line-to-line voltage output.