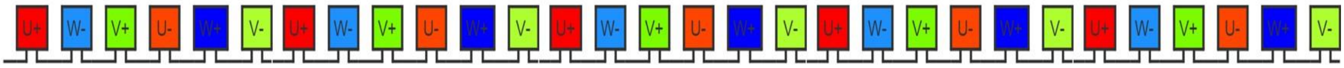
EEE6200 AC Machines

15-16 June Examination Solutions

[Commentary on the solutions is provided in italics in parentheses]

1.

a)



(4)

b) In order to calculate the Carter coefficients, it is necessary to extract several dimensional parameter from the design data provided in the question.

The stator slot pitch is given by:

$$\tau_{us} = \frac{\pi \times 0.452}{30} = 0.0473m$$

(or 0.450m – makes no meaningful difference)

The rotor slot pitch is given by:

$$\tau_{ur} = \frac{\pi \times 0.450}{10} = 0.1414m$$

The stator slot opening is given by:

$$w_{ss} = \frac{2 \times \pi \times 0.452}{360} = 0.00789m$$

And similarly for the rotor slot

$$w_{rs} = \frac{\pi \times 450 \times (1 - 0.9)}{10} = 0.0141m$$

The same procedure can be used to calculate the stator and rotor tooth widths:

The stator tooth width is 10° (from 12-2) and hence:

$$w_{st} = \frac{10 \times \pi \times 0.452}{360} = 0.0394m$$

$$w_{rt} = \frac{\pi \times 0.450 \times 0.9}{10} = 0.1272m$$

The Carter coefficient which accounts for stator slotting can now be calculated:

For the stator:

Slot width = b_1 = 7.89mm and unmodified airgap = δ = 1mm:

$$\kappa = \frac{\frac{b_1}{\delta}}{5 + \frac{b_1}{\delta}} = \frac{\frac{0.00789}{0.001}}{5 + \frac{0.00789}{0.001}} = 0.612$$

$$k_{cs} = \frac{\tau_{us}}{\tau_{us} - \kappa b_1} = \frac{0.0473}{0.0473 - 0.612 \times 0.00789} = 1.114$$

And similarly for the rotor:

Slot width = b_1 = 14.1mm and unmodified airgap = δ = 1mm:

$$\kappa = \frac{\frac{b_1}{\delta}}{5 + \frac{b_1}{\delta}} = \frac{\frac{0.0141}{0.001}}{5 + \frac{0.0141}{0.001}} = 0.738$$

$$k_{cr} = \frac{\tau_{ur}}{\tau_{ur} - \kappa b_1} = \frac{0.1414}{0.1414 - 0.738 \times 0.0141} = 1.079$$

Hence, the effective magnetic airgap is given by:

$$l_g' = l_g k_{cs} k_{cr} = 1 \times 1.114 \times 1.079 = 1.202mm$$

(6)

c) Applying Ampere's Law around each rotor slot, gives:

$$B_g = \frac{\mu_0 N I_g}{2 l_g'} = \frac{4\pi \times 10^{-7} \times 200 \times 8.5}{2 \times 0.001202} = 0.889T$$

(3)

d) The peak flux linkage can be established by integrating the airgap flux density over the full stator bore (no need to account for slot openings because of Carter coefficient)

Flux over one pole:

$$\emptyset_{pole} = \frac{B_g \pi d_{si} L_{ax}}{10} = \frac{0.889 \times \pi \times 0.452 \times 0.5}{10} = 0.0631Wb$$

But there are 5 coils per phase with 10 turns in each coil with each coil spanning one pole

(An alternative way of winding the coils would be to have 10 coils per phase, each with 5 turns and collect the flux from 10 poles – the end result is the same in terms of the flux linkage and the number of conductors in each slot – the latter is the key rather than the arrangement of the end-windings)

$$\Psi_{\max\_coil} = 10 \times 5 \times \emptyset_{pole} = 10 \times 5 \times 0.0631 = 3.16\ Wb.turns$$

[Note: Need to include N to get full marks as the question asks for flux-linkage]

(3)

e)) Flux-linkage variation between the positive and negative maximums is sinusoidal, hence flux linkage per phase:

$$\Psi = 3.16 \sin(wt)$$

$$\frac{d\Psi}{dt} = 3.16\omega\cos(\omega t)$$

$$\frac{d\Psi}{dt}\ max = 3.16\omega$$

$$\frac{d\Psi}{dt}\max = 3.16 \times 1000 \times \frac{2\pi}{60} \times 5 = \ V$$

(Factor of 5 to convert mech to elec)

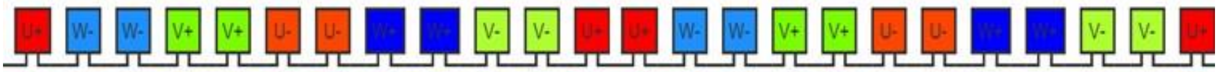
But this is the peak phase emf, hence the rms line to line voltage is given by:

$$E_{line-line} = 1652 \times \frac{\sqrt{3}}{\sqrt{2}} = 2024\,Vrms$$

(4)

2.

a) The slot diagram should be as follows (although there is some freedom on how candidates denote the orientation of individual coil sides in individual slots)



[Variations on starting point are fine providing the sequence is correct]

(4)

b)

[Key point here is to recognise the overall winding factor is in this case is simply the distribution factor as there is no pitching factor]

The distribution factor for the fundamental:

$$k_{d1} = \frac{sin\left(\frac{n\alpha}{2}\right)}{n\,sin\left(\frac{\alpha}{2}\right)}$$

For this winding: n=2 and $\alpha = \frac{\pi}{6}$

(Note n=2 is the number of slots per pole per phase)

$$k_{d1} = \frac{sin\left(\frac{n\alpha}{2}\right)}{n\,sin\left(\frac{\alpha}{2}\right)} = \frac{\sin\left(\frac{2\pi}{12}\right)}{2\sin\left(\frac{\pi}{12}\right)} = 0.966$$

For the 5th harmonic, the distribution factor is given by:

$$k_{d5} = \frac{sin\left(\frac{n5\alpha}{2}\right)}{n\,sin\left(\frac{5\alpha}{2}\right)} = \frac{sin\left(\frac{10\pi}{12}\right)}{2\,sin\left(\frac{5\pi}{12}\right)} = 0.259$$

For the 7th harmonic, the distribution factor is given by:

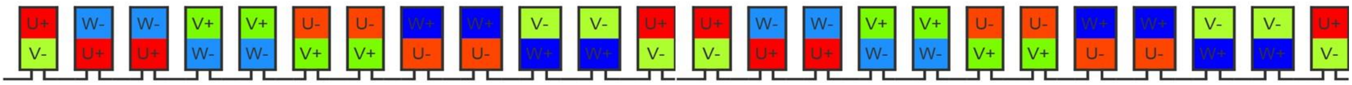
$$k_{d7} = \frac{sin\left(\frac{n7\alpha}{2}\right)}{n\ sin\left(\frac{7\alpha}{2}\right)} = \frac{sin\left(\frac{14\pi}{12}\right)}{2\ sin\left(\frac{7\pi}{12}\right)} = -0.259$$

(no concerns if minus sign missing off k_{d7})

(5)

c) A fully pitched winding has a coil span of 6 slot pitches, and hence a winding short-pitched by 2 slots has a coil span of 4 slot pitches.

The slot diagram should be as follows (although there is some freedom on how candidates denote the orientation of individual coil sides in individual slots)



[Variations on starting point are fine providing the sequence is correct]

(4)

d) [*With the revised winding design there is also a need to account for the pitch factor, but the distribution factor remains the same*]

The pitch factor for the fundamental is given by:

$$k_{p1} = sin\left(\frac{y_s}{y_f}\frac{\pi}{2}\right) = \sin\left(\frac{4}{6} \times \frac{\pi}{2}\right) = 0.866$$

The coil pitch factor for the 5th harmonic is given by:

$$k_{p5} = sin\left(\frac{5y_s}{y_f}\frac{\pi}{2}\right) = \sin\left(\frac{20}{30} \times \frac{\pi}{2}\right) = 0.866$$

The overall winding factor for the fundamental, 5th and 7th harmonic are hence given by:

$$k_{w1} = 0.966 \times 0.866 = 0.837$$

$$k_{w5} = 0.259 \times 0.866 = 0.224$$

(5)

e) A 24 slot stator can produce 2,4 and 8 poles with an integer slot winding.

(2)

3.

a) Calculating the synchronous reactance at I_f=25A

$$X_s = \frac{E_{ph}}{I_{sc}} = \frac{4700}{160} = 29.4\Omega$$

[In practice any reasonable interpolation from the graph will be accepted as will the onward propagation of these estimates through the remainder of the question]

Phase voltage is given by:

$$V_{ph} = \frac{V_{line-line}}{\sqrt{3}} = \frac{3{,}300}{\sqrt{3}} = 1{,}905V$$

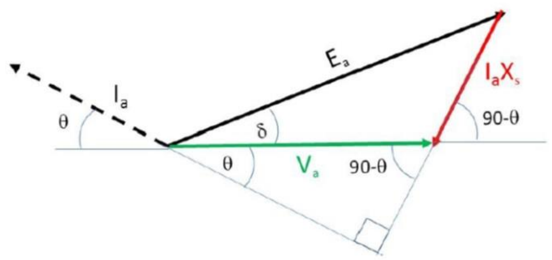
The generated power per phase is given by:

$$P_{ph} = V_{ph} I_{ph} cos\emptyset$$

$$I_{ph} = \frac{P_{ph}}{V_{ph} cos\emptyset} = \frac{-170 \times 10^3}{1905 \times 0.80} = -112A\, rms$$

$$I_{ph} X_s = -112 \times 29.4 = -\, 3277V\, rms$$

From phasor diagram:



Resolving the phasors parallel to V_a

$$E_{ph} \cos\delta + I_{ph} X_s\, sin\theta = V_{ph}$$

Rearranging gives:

$$E_{ph} \cos\delta = V_{ph} - I_{ph} X_s\, sin\theta$$

Resolving the phasors orthogonal to V_a

$$E_{ph} \sin\delta + I_{ph} X_s \cos\theta = 0$$

Rearranging gives:

$$E_{ph} \sin\delta = -\, I_{ph} X_s \cos\theta$$

$$\tan\delta = \frac{-I_{ph} X_s \cos\theta}{V_{ph} - I_{ph} X_s\, sin\theta} = \frac{+3277 \times 0.8}{1905 + 3277 \times (0.6)} = 0.677$$

Hence, $\delta = 34.1°$

$$E_{ph} = \frac{-I_{ph} X_s \cos\theta}{\sin\delta} = \frac{3851 \times 0.8}{sin(36.2^{\circ})} = \; 4675Vrms$$

From the graph, this emf is achieved at a field current of 24A.

[Again, there is some lee-way on this value given that it is estimated from the graph]

[8 marks in total, 2 for Xs, 4 for δ and 2 for I_f]

b) *[The key factor here is to recognise that X_s must be re-calculated to allow for saturation]*

At I_f = 40A:

$E_{ph\_oc} = 6080Vrms$

$I_{ph\_sc} = 260Arms$

$X_s = \frac{E_{ph}}{I_{sc}} = \frac{6080}{260} = 23.4\Omega$

[Again, there is some lee-way on this value given that it is estimated from the graph]

The maximum value of power is achieved with a load angle of 90° and is given by:

$$P_{max} = -3\frac{V_{ph}E_{ph}}{X_s}sin\delta = -3\frac{1905 \times 6080}{23.4} \times 1 = -1.48MW$$

This is not a practical operating point since it is at the limit of stability, even in steady-state. Any transient increase in load would cause the generator to lose synchronisation. Also well into saturation and hence greatly increased rotor losses.

[4 marks – 2 for new value of Xs, 1 for Pmax and 1 for discussion]

c) *[Although stability questions have been posed in past examinations, these have been based on a known step change in power. This is a challenging variant on such questions].*

From part (a)

Calculating the synchronous reactance at I_f=25A

$I_f = 25A$

$X_s = 29.4\Omega$

$E_{ph} = 4675Vrms$

$\delta = 34.1°$ (0.595 rad)

Maximum power at this field current is given by:

$$P_{max} = -3\frac{V_{ph}E_{ph}}{X_s}sin\delta = -3\frac{1905 \times 4675}{29.4} \times 1 = -908kW$$

Limit of stability can be established iteratively

Up to the critical load angle (δ_c) and critical power (Pc), the decelerating area:

$$A_{dec} = \left(P_c \times (\delta_c - 0.595)\right) - \left(908 \times 10^3 \int_{0.595}^{\delta_c} \sin\delta \, d\delta\right)$$

$$= \left(P_c \times (\delta_c - 0.595)\right) - 908 \times 10^3(\cos 0.595 - \cos\delta_c)$$

The limit of the accelerating area occurs when:

$$A_{acc} = 908 \times 10^3 \int_{\delta_c}^{\pi-\delta_c} \sin\delta \, d\delta \; - \left(P_c \times (\pi - 2\delta_c)\right)$$

$$= 908 \times 10^3(\cos\delta_c - \cos(\pi - \delta_c)) \; - \left(P_c \times (\pi - 2\delta_c)\right)$$

1st iteration:

$$\delta_c = 70° \; gives \; \frac{A_{dec} - A_{acc}}{A_{dec}} = +12.7\% \;\; hence\; unstable$$

2nd iteration:

$$\delta_c = 60° \; gives \; \frac{A_{dec} - A_{acc}}{A_{dec}} = -7.6\% \;\; hence\; stable\; but\; outside\; tolerance$$

3rd iteration:

$$\delta_c = 63° \; gives \; \frac{A_{dec} - A_{acc}}{A_{dec}} = +1.6\% \;\; hence\; unstable\; but\; inside\; tolerance$$

OK to stop here, but could iterate on until max and stableis achieved (~62°)

Hence critical power is given by:

$$P_c = 908 \times 10^3 \times sin(63) = 802kW$$

Hence, change in load is: 802-510=292 kW

[8 marks]

4.

a) Angular displacement of a normal stroke is given by:

$$\Delta\theta_s = \frac{2\pi}{Number\; of\; phases\; \times Number\; of\; rotor\; teeth} = \frac{2\pi}{3 \times 4} = \frac{\pi}{6}\; rad\; or\; 30^o$$

Hence, number of torque strokes per revolution is 12

(2)

b)

Applying the trapezium rule to integrate the area under the fully aligned curve

$$A_{0\to1} = \frac{\Psi_1}{2} = 0.30 \text{ J}$$

$$A_{1\to2} = \frac{\Psi_1 + \Psi_2}{2} = 0.81\text{J}$$

$$A_{2\to3} = \frac{\Psi_2 + \Psi_3}{2} = 1.10\text{J}$$

$$A_{3\to4} = \frac{\Psi_3 + \Psi_4}{2} = 1.21\text{J}$$

$$A_{4\to5} = \frac{\Psi_4 + \Psi_5}{2} = 1.26\text{J}$$

$$A_{0\to2} = A_{0\to1} + A_{1\to2} = 1.11J$$

$$A_{0\to5} = A_{0\to1} + A_{1\to2} + A_{2\to3} + A_{3\to4} + A_{4\to5} = 4.68J$$

The area under the un-aligned curve (which can reasonably regarded as being linear) is simply given by:

$$U_{0\to2} = \frac{2\Psi_2}{2} = 0.03J$$

$$U_{0\to5} = \frac{5\Psi_5}{2} = 0.50J$$

Hence the change in co-energy is given by:

$$\Delta W'_{0\to2} = A_{0\to2} - U_{0\to2} = 1.08J$$

$$\Delta W'_{0\to5} = A_{0\to5} - U_{0\to5} = 4.18J$$

The average torque is therefore given by:

At 2A:

$$T_{ave} = \frac{\Delta W'_{0\to2}}{\Delta\theta} = \frac{1.08}{\pi/6} = 2.06Nm$$

At 5A:

$$T_{ave} = \frac{\Delta W'_{0\to2}}{\Delta\theta} = \frac{4.18}{\pi/6} = 7.98Nm$$

[Given that this solution involves reading a number of values from the Ψ-I characterisitc then some tolerance on the calculated values is acceptable]

(5)

c) From the aligned Ψ-I characteristic it can be seen that the onset of saturation occurs at a flux-linkage of ≈ 1Wb and a current of 2A (an answer based on a slightly different interpretation of saturation is equally acceptable). It is important to note that the flux produced by the pair of coils that constitute a phase (which have a total of N_{ph} turns) crosses 2 diametrically opposite airgaps, each of length l_g.

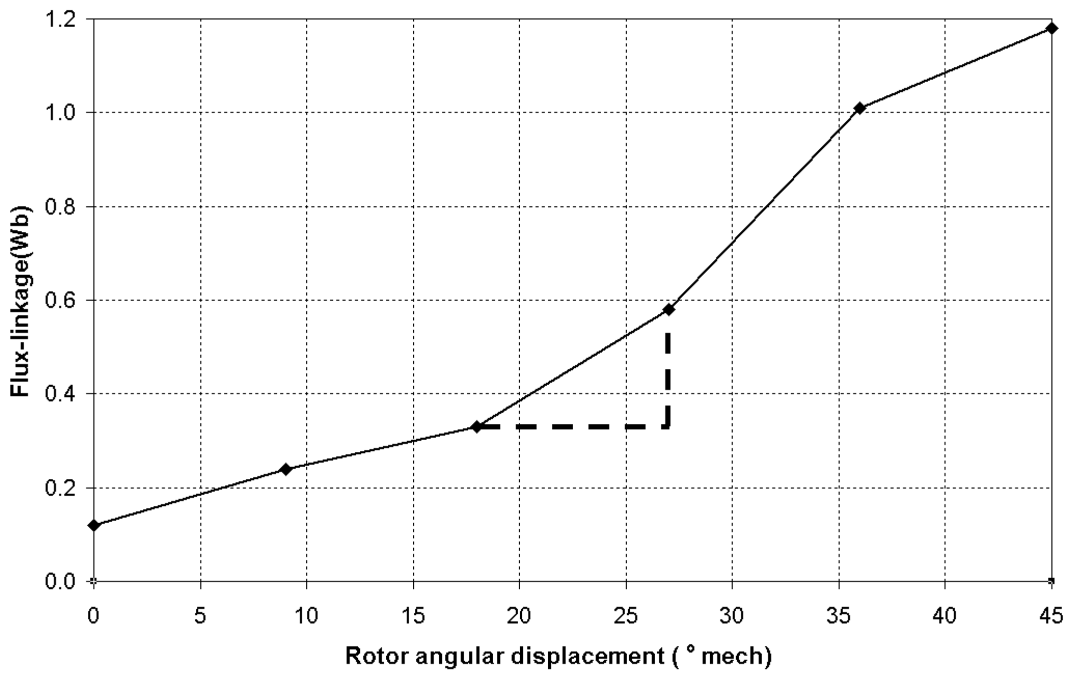
Since $B_g \approx \frac{\mu_0 N_{ph} I}{2l_g}$ prior to saturation then a reasonable estimate of l_g can be obtained from this equation.

$$\therefore l_g = \frac{\mu_0 N_{ph} I}{2B_g} = \frac{4\pi \times 10^{-7} \times 254 \times 2}{2 \times 1.6} = 0.2 \ mm$$

[a reasonable error band on this value is acceptable given the difficulty in precisely defining the onset of saturation – the method employed is the key factor in determining the marks awarded]

(4)

d) Taking the values of flux-linkage at 3A for the various and re-plotting a graph of flux-linkage versus position yields:



From the graph, the rate of change of flux-linkage with respect to rotor angular displacement around 15^o is given to a reasonable approximation by:

$$\left.\frac{d\Psi}{d\theta}\right|_{22.5} \approx \frac{\Psi_{27} - \Psi_{18}}{9\times\dfrac{\pi}{180}} \approx \frac{0.58-0.33}{9\times\dfrac{\pi}{180}} \approx 1.59\text{Wb / rad}$$

At 200 rpm, the rate of change of angular displacement is given by:

$$\frac{d\theta}{dt} = \frac{200\times 2\pi}{60} = 20.9 \text{ rad /s}$$

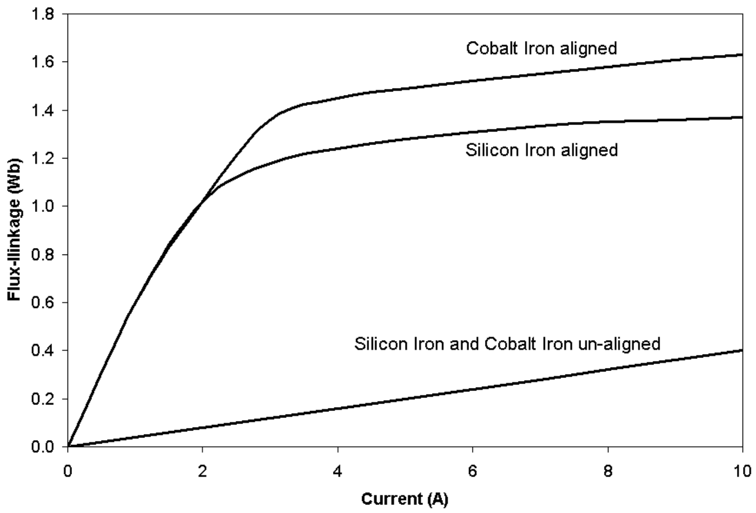
The instantaneous value of the induced emf is hence given by:

$$\frac{d\Psi}{dt} = \frac{d\Psi}{d\theta}\times\frac{d\theta}{dt} = 31.8\text{V}$$

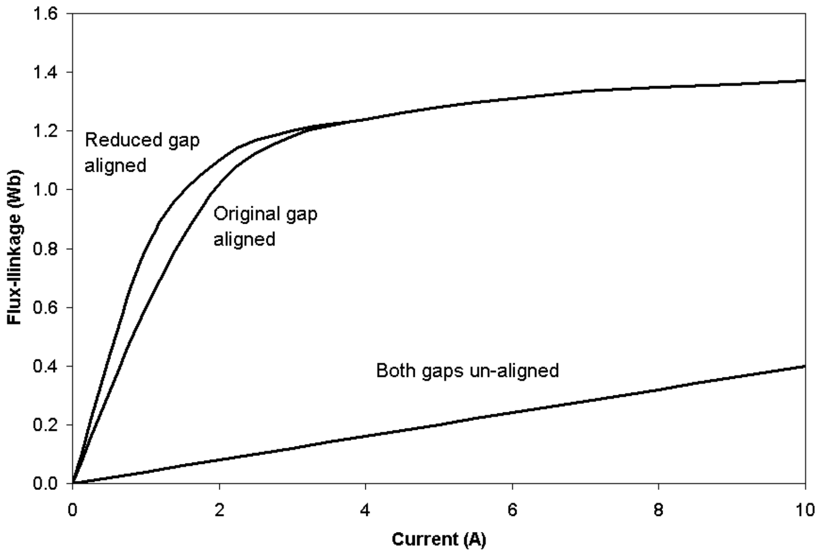
(5)

e) Two design changes that would enhance torque capability are:

i) Substitute the Silicon Iron for Cobalt Iron which has a higher saturation flux density (typically 2.35T as compared to 2.0T). Assuming that the materials have comparable permeabilities in the context of the fact that the net reluctance is dominated by the airgap even in the aligned position, then the two sets of Ψ-I characteristics will have the form:



ii) Reduce the length of the airgap. This will increase the slope of the intial linear part of the aligned characteristic, but willno discernable effect on the saturated regino of the aligned curve. Although a smaller airgap will increase the un-aligned characteristic marginally due to slightly increased leakage flux, it is reasonable to assume that the un-aligned characteristic will remain unchanged. the two sets of Ψ-I characteristics will have the form:



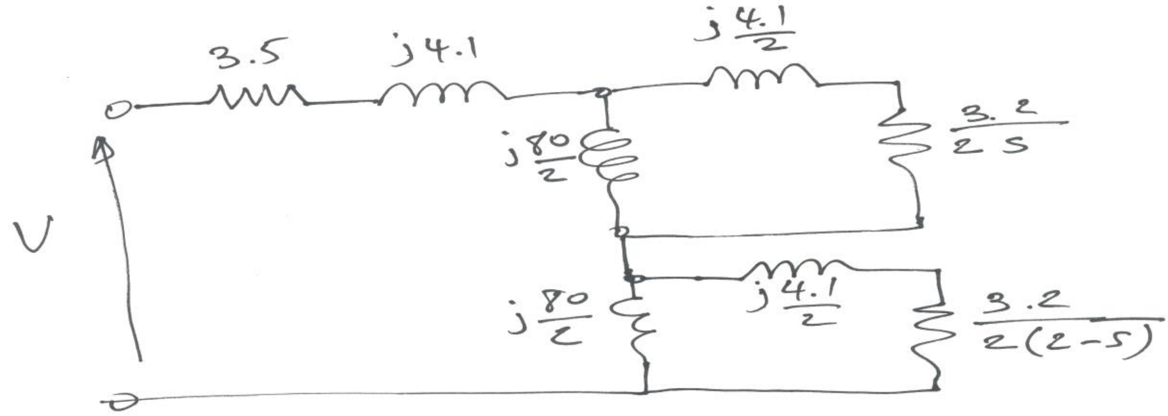
[Sketches which highlight the key features are adequate – there is no need to re-plot the graphs]

[Features such as improved cooling, higher packing factor coils etc although clearly advantageous for enhancing torque density are not the answers being sought in this case – largely since they have no direct bearing on the flux-linkage versus current characteristic but rather influence the degree to which a given level of current can be thermally sustained]

(4)

5.

(i)



Equivalent circuit of the single-phase induction machine.

(ii)

At the slip s=0.05, the impedance of the positive branch is:

$$Z_2^+ = \frac{R_2'}{2\,s} + j\,\frac{X_2'}{2} = 32 + j\;2.05$$

And the negative branch:

$$Z_2^- = \frac{R_2'}{2\,(2-s)} + j\,\frac{X_2'}{2} = 0.82 + j\;2.05$$

Therefore, the total impedance is given by:

$$\begin{aligned} Z_t &= R_1 + j\,X_1 + \left(\frac{1}{Z_2^+} + \frac{1}{Z_m}\right)^{-1} + \left(\frac{1}{Z_2^-} + \frac{1}{Z_m}\right)^{-1} \\ &= 3.5 + j\;4.1 + 35.6 + j\;25.14 + 0.772 + j2.0 = 22.57 + j21.97 \\ &= 31.5\;e^{j44.23}\;\Omega \end{aligned}$$

Therefore, the input current is given by:

$$I = \frac{V}{Z_t} = \frac{120}{31.5e^{j44.23}} = 3.81e^{-j44.23}$$

And the power factor is given by:

$$cos(44.23) = 0.716 \text{ Lagging}$$

(ii)

The currents in the forward and backward branches are given by:

$$I_2^+ = I\;\frac{\frac{Z_m}{2}}{Z_2^+ + \frac{Z_m}{2}} = (2.73 - j2.66)\;\frac{\frac{j80}{2}}{32 + j2.05 + \frac{j80}{2}} = 2.85 - j0.355$$

And,

$$I_2^- = I \frac{\frac{Z_m}{2}}{Z_2^- + \frac{Z_m}{2}} = (2.73 - j2.66) \frac{\frac{j80}{2}}{0.82 + j2.05 + \frac{j80}{2}} = 2.63 - j2.48$$

Furthermore, the speed of motor at the slip s=0.05 is

$$\omega_r = 2\,\pi\,f\,(1 - s) = 358.14 \text{ rad/s}$$

And the forward torque is given by:

$$T_f = (I_2^+)^2 \frac{R_2'}{2} \frac{(1 - s)}{s\,\omega_r} = 2.88^2 \times \frac{3.2}{2} \times \frac{(1 - 0.05)}{0.05 \times\ 358.14} = 0.704 \text{ Nm}$$

And the backward torque is given by:

$$T_b = (I_2^-)^2 \frac{R_2'}{2} \frac{(1 - s)}{(2 - s)\,\omega_r} = 3.61^2 \times \frac{3.2}{2} \times \frac{(1 - 0.05)}{(2 - 0.05) \times\ 358.14} = 0.028 \text{ Nm}$$

And the total electromagnetic torque is given by:

$$T_e = T_f - T_b = 0.767 - 0.028 = 0.676 \text{ Nm}$$

(iii)

The copper losses of the motor are given by:

$$P_c = R_1 I^2 + (I_2^+)^2 \frac{R_2'}{2} + (I_2^-)^2 \frac{R_2'}{2} = 3.5 \times 3.81^2 + 2.88^2 \times \frac{3.2}{2} + 3.61^2 \times \frac{3.2}{2} = 84.93 \text{ W}$$

The total drag torque from the bearings and the fan is given by:

$$T_d = 0.02 + \left(\frac{\omega_r}{\omega_s}\right)^2 \times 0.05 = 0.02 + 0.045 = 0.065 \text{ Nm}$$

And the mechanical losses are given by:

$$P_m = T_d \omega_r = 0.065 \times 358.14 = 23.28 \text{ W}$$

Therefore, the output power is given by:

$$P_{out} = (T_e - T_d)\,\omega_r = (0.676 - 0.065) \times\ 358.14 = 218.8 \text{ W}$$

And the efficiency is given by:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{218.8}{218.8 + 84.93 + 23.28} = 67.1\ \%$$

b.

(i)

Under the locked rotor tests, when the slip s=1, the magnetising branches are very large compared to the positive and negative branches, and can be treated as open-circuit. Therefore,

$$V = I_{lr}\left(R_1 + j\,X_1 + \frac{R_2'}{2} + j\frac{X_2'}{2} + \frac{R_2'}{2} + j\frac{X_2'}{2}\right)$$

Therefore, the current is given by:

$$I_{lr} = \frac{V}{\left(R_1 + R_2' + j\,(X_1 + X_2')\right)} = \frac{42}{\left(3.5 + 3.2 + j\,(4.1 + 4.1)\right)} = 2.51 - j\,3.07$$
$$= 3.96\,e^{-j50.73}\ \text{(A)}$$

And the power factor is given by:

$$cos(50.73) = 0.633 \text{ lagging}$$

(ii)

Under the no-load test the slip is very small and close to 0. Therefore, for the positive circuit, the impedance of the positive branch is much larger than the magnetising branch, whist for the negative circuit, the impedance of the negative branch is much smaller than the corresponding magnetising branch. Therefore,

$$V = I_{oc}\left(R_1 + j\,X_1 + j\frac{X_m}{2} + j\frac{X_2'}{2} + \frac{R_2'}{4}\right)$$

Therefore,

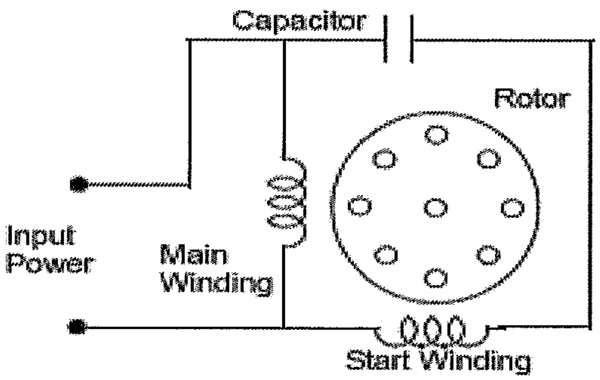
$$I_{oc} = \frac{V}{\left(R_1 + \frac{R_2'}{4} + j\left(X_1 + \frac{X_m}{2} + \frac{X_2'}{2}\right)\right)} = \frac{120}{\left(3.5 + \frac{3.2}{4} + j\left(4.1 + \frac{80}{2} + \frac{4.1}{2}\right)\right)} = 0.24 - j2.58$$
$$= 2.59\,e^{-j84.7}\ \text{(A)}$$

And the power factor is given by:

$$cos(84.7) = 0.092 \text{ lagging}$$

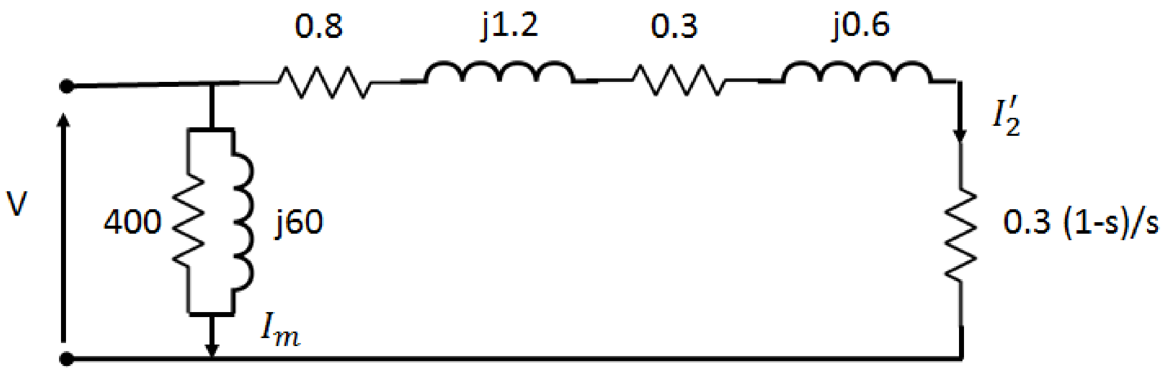
d.

Capacitor left in circuit. Has some advantages in terms of normal running power factor and suitability for variable speed operation from a power electronic inverter.



6.

a)



Per-phase equivalent circuit diagram

b.

(i)

The total output power of the 3-phase induction motor is given by:

$$T \times \omega_r = \frac{R_2' \,(1-s)}{s} \; {I_2'}^2$$

Where T is the torque and $\omega_r = (1-s)\,\omega_s$ is the speed of the rotor. Therefore,

$$T = \frac{R_2'}{s\,\omega_s} \; {I_2'}^2$$

Furthermore, from the circuit diagram in **a.**

$$I_2' = \frac{V}{\sqrt{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2}}$$

Therefore, from the above equations, the torque T is given by:

$$T = \frac{3\,V^2}{\omega_s} \times \frac{R_2'/s}{\left(R_1 + R_2'/s\right)^2 + (X_1 + X_2')^2}$$

(ii)

The maximum torque of the motor occurs when the maximum airgap power occurs. Using the circuit diagram above, the maximum airgap power occurs when:

$$\frac{R_2'}{s} = |R_1 + j\,(X_1 + X_2')|$$

Therefore, the slip at which the maximum torque occurs is given by:

$$s = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2)^2}} = \frac{0.3}{\sqrt{0.8^2 + (1.2 + 0.6)^2}} = 0.152$$

And the maximum torque is then given by:

$$T = \frac{3\,V^2}{\omega_s} \times \frac{R_2'/s}{\left(R_1 + R_2'/s\right)^2 + (X_1 + X_2')^2}$$

$$= \frac{3 \times 1500^2}{314.16} \times \frac{0.3/0.152}{\left(0.8 + 0.3/0.152\right)^2 + (1.2 + 0.6)^2} = 7.76 \text{ kNm}$$

(iii)

The slip at speed ω_r=1485rpm is s= 0.01, therefore the output power is given by:

$$T \times \omega_r = 3\,V^2 \times \frac{(1-s) \times R_2'/s}{\left(R_1 + R_2'/s\right)^2 + (X_1 + X_2')^2}$$

$$= 3 \times 1500^2 \times \frac{(1-0.01) \times 0.3/0.01}{\left(0.8 + 0.3/0.01\right)^2 + (1.2 + 0.6)^2} = 210 \text{ kW}$$

The total copper losses are the sum of the copper losses in the stator and rotor, and are given by:

$$P_{copper} = 3\,R_1\,{I_2'}^2 + 3\,R_2' {I_2'}^2$$

Where,

$$I_2' = \frac{V}{\sqrt{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2}} = \frac{1500}{\sqrt{\left(0.8 + \frac{0.3}{0.01}\right)^2 + (1.2 + 0.6)^2}} = 48.6 \text{ A}$$

Therefore, the copper losses are given by:

$$P_{copper} = 3\,R_1\,I_2^2 + 3\,R_2' I_2^2 = 3 \times 0.8 \times 48.6^2 + 3 \times 0.3 \times 48.6^2 = 7.8 \text{ kW}$$

The iron loss is given by:

$$P_{iron} = \frac{3\,V^2}{R_m} = \frac{3 \times 1500^2}{400} = 16.9\text{ kW}$$

The efficiency is then given by:

$$\eta = \frac{210}{210 + 7.8 + 16.9} = \frac{210}{210 + 7.8 + 16.9} = 89.47\ \%$$

c.

(i)

The electromagnetic torque is given by:

$$T = \frac{3\,V^2}{\omega_s} \times \frac{R_2'/s}{\left(R_1 + R_2'/s\right)^2 + (X_1 + X_2')^2}$$

However, at a frequency of 40Hz the synchronous speed and the values of the inductive components of the equivalent circuit will change. Therefore, at 40Hz:

$$\omega_s = 2 \times \pi \times \frac{40}{2} = 125.66\frac{rad}{s} = 1200\text{ rpm}$$

$$X_1 = \frac{40}{50} \times 1.2 = 0.96\ \Omega$$

$$X_2' = \frac{40}{50} \times 0.6 = 0.48\ \Omega$$

Furthermore, in order to ensure a constant flux operation, the ratio $\frac{V}{f}$ is kept constant and at 40Hz, the voltage V=1200 Vrms. The electromagnetic torque is then calculated as:

$$T = \frac{3\,V^2}{\omega_s} \times \frac{R_2'/s}{\left(R_1 + R_2'/s\right)^2 + (X_1 + X_2')^2}$$

$$= \frac{3 \times 1200^2}{125.66} \times \frac{0.3/0.03}{\left(0.8 + 0.3/0.03\right)^2 + (0.96 + 0.48)^2} = 2.89\text{ kNm}$$

(ii)

The iron loss resistance and the magnetising reactance also vary with frequency:

$$X_m = \frac{40}{50} \times 60 = 48\ \Omega$$

And since the iron loss varies linearly with frequency:

$$R_m = \frac{50}{40} \times 400 = 500\ \Omega$$

The current I_m:

$$I_m = \frac{V}{R_m} + j\frac{V}{Xm} = \frac{1200}{500} + j\frac{1200}{48} = 2.4 - j\ 25\ \text{(A)}$$

The current I_2':

$$I_2' = \frac{V}{\left(R1 + \frac{R_2'}{s}\right) + j\ (X_1 + X_2)} = \frac{V}{\left(R1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2)^2}\left(\left(R1 + \frac{R_2'}{s}\right) - j\ (X_1 + X_2)\right)$$

$$= \frac{1200}{\left(0.8 + \frac{0.3}{0.03}\right)^2 + (0.96 + 0.48)^2}\left(\left(0.8 + \frac{0.3}{0.03}\right) - j\ (0.96 + 0.48)\right)$$

$$= 109.2 - j\ 14.6\ \text{(A)}$$

The total current is given by:

$$I = I_m + I_2' = 111.6 + j\ 39.4 = 118.3\ e^{-j19.5}\quad \text{(A)}$$

Therefore, the power factor is 0.94.

(iii)

At a slip s=0.03, the fan will be producing a torque:

$$T_f = C_f\ \omega_f^2 = C_f\ (1-s)^2\omega_s^2 = 0.155\ \times (1 - 0.03)^2 \times 125.66^2 = 2.3\ \text{kNm}$$

The torque is less than the electromagnetic torque produced by the motor at s=0.03, therefore, when driving the fan, the motor will be running a slip smaller than 0.03.