The University Of Sheffield.

Data Provided:
Laplace and z-transforms
Compensator design formulae
Performance criteria mappings

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DEPARTMENT OF AUTOMATIC CONTROL & SYSTEMS ENGINEERING
Spring Semester 2017–2018

ACS342 FEEDBACK SYSTEMS DESIGN **2 hours**

Answer ALL THREE questions.

Trial answers will be ignored if they are clearly crossed out.

All questions are marked out of 20. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

Registration number from U-Card (9 digits) — to be completed by student

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1. A feedback control system is shown in Figure 1.1.

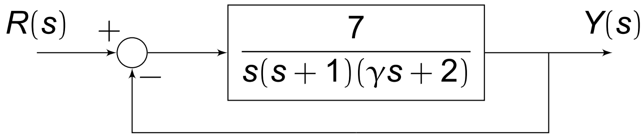

Figure 1.1

a) Write down the open-loop pole locations, and hence identify the range of γ for which the open-loop system is stable.

[2 marks]

b) Show that the closed-loop transfer function of the system is

$$\frac{Y(s)}{R(s)} = \frac{7}{\gamma s^3 + (\gamma + 2)s^2 + 2s + 7}$$

Hence, determine the range of γ for which the closed-loop system is stable.

[6 marks]

The next two parts of this question use the Bode diagram of the open-loop system (for a particular, but unknown, value of $\gamma > 0$) provided overleaf in Figure 1.2.

c) (i) Estimate the gain margin and phase margin of the system. Is the closed-loop system stable or unstable for this particular value of γ?

(ii) Estimate the rise time and overshoot of the closed-loop system.

[6 marks]

d) Design a phase-lead compensator

$$C(s) = \frac{s\alpha\tau + 1}{s\tau + 1}$$

in order to achieve a phase margin of 45° for the system. Use a safety margin of 5°, and do not attempt to use the provided transfer function of the system to perform exact calculations—your design should be done using readings from the Bode diagram in Figure 1.2.

[6 marks]

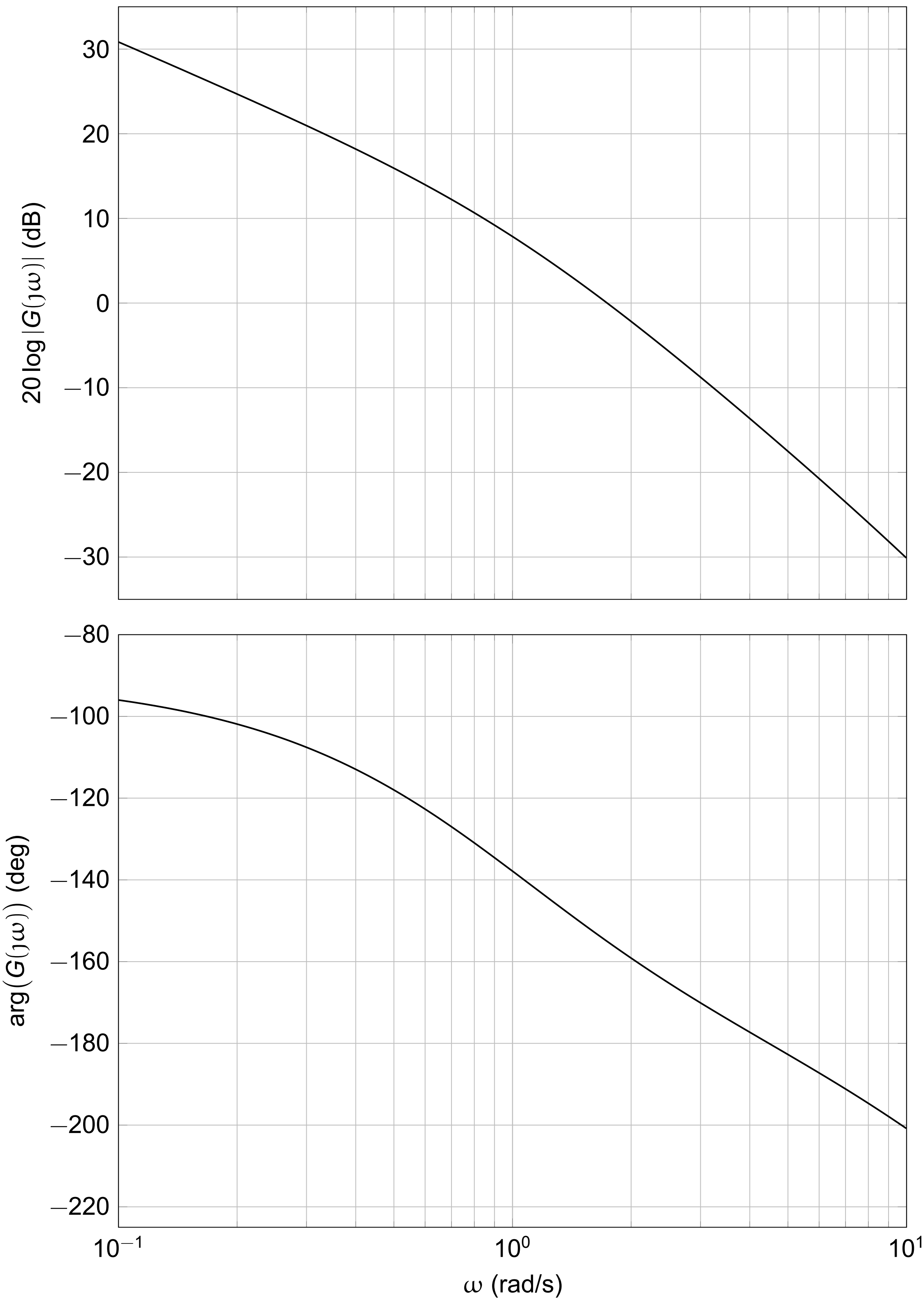

Figure 1.2: Bode diagram for Q1.

2. A unity-feedback system has the open-loop transfer function

$$KG(s) = \frac{K}{s^2 + 4s + 4}$$

a) Find the closed-loop transfer function, and hence determine the damping ratio and natural frequency of the closed-loop system as functions of K. Show that the settling time of the closed-loop step response is constant (*i.e.*, independent of K).

[5 marks]

b) Find an expression for the position error constant of $KG(s)$ in terms of K, and hence calculate the percentage steady-state tracking error (in response to a step) when K is chosen to provide an overshoot of 5%.

[5 marks]

c) Design a phase-lag compensator in order that the closed-loop system meets the following specification.

Overshoot (%)	$\leqslant 5$
Position error constant	$\geqslant 20$

You are given that the desired dominant pole location is $s^\star = -2 \pm \jmath 3.5$.

[5 marks]

d) The following continuous-time compensator is to be implemented on a digital platform.

$$C(s) = 5\frac{s+1}{s+0.1}$$

Derive a z-transform representation of the compensator's transfer function. Use a sampling time of $T = 0.1$ seconds and zero-order hold for sampling of the continuous-time input signal to the compensator.

[5 marks]

3. The attitude dynamics of a satellite are modelled by the ordinary differential equation

$$50\frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} = \tau(t)$$

where θ is the attitude (orientation) of the satellite with respect to a particular coordinate frame, and $\tau(t)$ is the torque applied to the satellite (by reaction wheels).

The aim is to design an automatic control system that can re-orient the satellite smoothly and exactly to a desired reference attitude, θ_r. In particular, the specification is as follows:

Overshoot (%)	$\leqslant 5$
Settling time (s)	$\leqslant 10$

To achieve this aim, feedback control is proposed, and a controller $C(s)$, acting on the error between θ_r and θ, is to be designed.

a) Draw a block diagram of the feedback control system.

[3 marks]

b) For the case of $C(s) = K$, determine the following for the closed-loop system (in terms of K where appropriate):

(i) the transfer function, $\Theta(s)/\Theta_r(s)$;

(ii) the pole locations;

(iii) the damping ratio and natural frequency;

(iv) the impulse response.

Hence, explain why the feedback control system is unable to meet the specification with $C(s) = K$.

[8 marks]

c) Design a PD controller

$$C(s) = K_\mathrm{P} + sK_\mathrm{D},$$

by finding suitable gains K_P, K_D in order that the closed-loop poles lead to satisfaction of the specification.

[5 marks]

d) In the real system, the following PD controller is implemented:

$$C(s) = 16 + 40s$$

Experiments with the PD-controlled closed-loop system reveal that the overshoot is significantly more (close to 20%) than that predicted from the closed-loop poles. By analysing the transfer function of the closed-loop system, identify a possible cause of this (aside from modelling errors). Explain how the excessive overshoot might be eliminated.

[4 marks]

Laplace and z-transforms

Time domain	s-domain	z-domain
$f(t)$	$F(s)$	$F(z)$
$f(t-T)$	$e^{-sT}F(s)$	$z^{-1}F(z)$
$\delta(t)$	1	–
1	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{zT}{(z-1)^2}$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{zTe^{-aT}}{(z-e^{-aT})^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\frac{z^2-z\cos(\omega T)}{z^2-2z\cos(\omega T)+1}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2-ze^{-aT}\cos(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
$f^{(n)}(t)=\frac{\mathrm{d}^n}{\mathrm{d}t^n}f(t)$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{n-1}(0)$	Various forms

Compensator design formulae

Transfer function	$\frac{s\alpha\tau+1}{s\tau+1}$ (lead) $\frac{s\tau+1}{s\alpha\tau+1}$ (lag)
Maximum phase lead/lag, ϕ_m	$\sin^{-1}\frac{\alpha-1}{\alpha+1}$
Centre frequency, ω_m	$\frac{1}{\tau\sqrt{\alpha}}$

Performance criteria mappings

2% settling time, T_s	$\dfrac{4}{\zeta\omega_n}$
10–90% rise time, T_r	$\dfrac{2.16\zeta + 0.6}{\omega_n}$ for $0.3 \leqslant \zeta \leqslant 0.8$
Percentage overshoot, O.S. (%)	$100\exp\left(\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$ for $0 < \zeta < 1$
Damping ratio from overshoot	$\zeta = \dfrac{-\ln(\text{O.S. }(\%)/100)}{\sqrt{\pi^2 + [\ln(\text{O.S. }(\%)/100)]^2}}$
Peak time, T_p	$\dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ for $0 < \zeta < 1$
Peak response, M_p	$1 + \exp\left(\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$ for $0 < \zeta < 1$
Resonant frequency, ω_r	$\omega_n\sqrt{1-2\zeta^2}$ for $0 < \zeta < \dfrac{1}{\sqrt{2}}$
Resonant peak magnitude, $M_{p\omega}$	$\dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$ for $0 < \zeta < \dfrac{1}{\sqrt{2}}$
Phase margin, ϕ_{pm}	100ζ for $0 < \zeta < \dfrac{1}{\sqrt{2}}$
Bandwidth–Rise time	$T_r = \dfrac{2.2}{\omega_B}$

END OF QUESTION PAPER