PROBLEM SHEET 2. Solutions

1) Prob. of occupancy in conduction band

$$p(E_g) = \frac{1}{1+\exp[(E_g - E_f)/kT]}$$

since E_f lies in midgap; $E_f = E_g/2$

$$\Rightarrow p(E_g) = \frac{1}{1+\exp(E_g/2kT)} \approx \exp(-E_g/2kT)$$

(i) at 0K, $p = \underline{0}$ # - no CB electrons; insulating

(ii) at 300K, $p = \exp(-0.7 \times 1.6 \times 10^{-19} / 2 \times 1.38 \times 10^{-23} \times 300)$

$= \underline{1.34 \times 10^{-6}}$ #, Small probability but enough ~~for~~ to make Ge a semiconductor.

(iii) at 500K, $p = \exp(-0.7 \times 1.6 \times 10^{-19} / 2 \times 1.38 \times 10^{-23} \times 500)$

$= \underline{3 \times 10^{-4}}$ #, greatly increased prob., increased conductivity.

NOTE: Density of states can influence electrical properties of a semiconductor.

2) Since $R \propto \sigma^{-1}$, and $\sigma = n_i e(\mu_e + \mu_h)$; $n_i = N\exp^{-E_g/2kT}$

$\therefore \sigma = N\exp^{-E_g/2kT} e(\mu_e + \mu_h)$

$\Rightarrow R \propto \exp(E_g/2kT)$; since only R, T and E_g terms are given

$\Rightarrow \ln\left(\frac{R_1}{R_2}\right) \propto \frac{E_g}{2k}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$; where $R_1 = 100\Omega$ $R_2 = 1\Omega$
$T_1 = 290K$, $T_2 = ?$

$$\Rightarrow \ln 100 = \frac{E_g}{2k}\left(\frac{1}{290} - \frac{1}{T_2}\right)$$

$$\Rightarrow 7.356 \times 10^{-4} = \left(3.45 \times 10^{-3} - \frac{1}{T_2}\right)$$

$$T_2 = 369\,K$$

$$= \underline{96°C} \; \#.$$

3) E_g, σ, and T are given.

$$\sigma = n_i e(\mu_e + \mu_n) = N \exp^{-E_g/2kT} e(\mu_e + \mu_n)$$

$$\frac{\sigma_1}{\sigma_2} = \frac{\exp - E_g/2kT_1}{\exp - E_g/2kT_2} \Rightarrow \sigma_2 = \sigma_1 \frac{\exp E_g/2kT_1}{\exp E_g/2kT_2}$$

$$= 2.13 \frac{\exp(0.72e/(2 \times 1.38 \times 10^{-23} \times 300)}{\exp(0.72e/2 \times 1.38 \times 10^{-23} \times 400)}$$

$$= \underline{69 \text{ S/m}} \ \#$$

Conductivity has increased enormously for a small temperature change.

3(i) Photon energy $E = hf = hc/\lambda$. Photon energy must be greater than E_g for electron elevation from VB into CB.

$\therefore$ for $\lambda = 1 \mu m$; $E = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^{+8}}{1 \times 10^{-6} \cdot 1.6 \times 10^{-19}}$

$\doteqdot 1.24$ eV. hence σ increases

(ii) $\lambda = 2 \mu m$ gives $E = 0.62$ eV, so no change in σ.

4) Undoped semiconductors: $n_i = N \exp^{-E_g/2kT}$

Doped semiconductors: $n = \dfrac{N}{1 + \exp[(E_g - E_f/kT)]}$

Since E_f is displaced by 10% from intrinsic position ; $E_f = 0.6 E_g$

$$n = \frac{N}{1 + \exp(E_g - 0.6E_g/kT)}$$

$$= N \exp^{-0.4 E_g/kT}$$

no. of p carriers $p = \dfrac{n_i^2}{n} = N \exp^{-0.6 E_g/kT}$ (ignore since value is small)

Hence $\sigma_n = n e \mu_e$ for doped (p is ignored since value is small)

$\sigma_i = n_i e(\mu_e + \mu_n)$ undoped

$$\frac{\sigma_n}{\sigma_i} = \frac{n e \mu_e}{n_i e(\mu_e + \mu_n)}$$

(Comment: σ is very sensitive to position of E_f)

$$= \frac{N \exp^{-0.4 E_g/kT} \ (0.13)}{N \exp^{-0.5 E_g/kT} \ (0.18)} = \underline{0.56} \ \#$$

5) $\sigma = n_i e\,(\mu_e + \mu_h)$; and $\mu_e = \mu_{eo} T^p$ and $\mu_h = \mu_{ho} T^p$

rearranging $\sigma = n_i e\, T^p\,(\mu_{eo} + \mu_{ho})$

$$\frac{1}{\rho} = N \exp^{-Eg/2kT}\, T^p e\,(\mu_{eo} + \mu_{ho})$$

$\Rightarrow$ Eg, ρ and T is given.

$$\therefore \frac{\rho_1}{\rho_2} = \frac{0.028}{0.013} = \left(\frac{T_2}{T_1}\right)^p \exp \frac{Eg}{2k}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$= \left(\frac{556}{384}\right)^p \exp \frac{0.8 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23}}\left(\frac{1}{384} - \frac{1}{556}\right)$$

$$\ln \frac{0.028}{0.013} = p \ln\left(\frac{556}{384}\right) + (\qquad)$$

$$p \ln \frac{556}{384} = \ln\left(\frac{0.028}{0.013}\right) - (\qquad)$$

$$0.37p = 3.069 - 3.736$$

$$\underline{p = -1.8}\ \#$$

6. $\sigma = (n_e \mu_e + n_h \mu_h)\, e$; since $\mu_e = \mu_h \times 10$

$N_e = 10^{19}$, $N_h = 10^{20}$

$$\therefore \sigma = \left(10^{19} \mu_e + 10^{20} \frac{\mu_e}{10}\right) e$$

$$\sigma = (10^{19} + 10^{19})\, \mu_e e$$

$$0.455 = 2 \times 10^{19} \mu_e e$$

$$\mu_e = \underline{0.142\ m^2 V^{-1} s^{-1}}\ \#$$

$$\mu_h = \underline{0.0142\ m^2 V^{-1} s^{-1}}\ \#$$

7) $\frac{1}{\rho} = n_i e(\mu_e + \mu_n)$

ρ of intrinsic Ge $= 0.6\ \Omega m$

Hence, to find n_i

$\frac{1}{0.6} = n_i e(0.38 + 0.18)$

$n_i = \underline{1.86 \times 10^{19}\ m^{-3}}$

However $N_d - N_a = 10^{20} - 7\times10^{19} = 3\times10^{19}\ m^{-3}$

$(N_d - N_a) \not\gg n_i$, both are under the same order of magnitude, hence intrinsic carriers can't be ignored.

$\therefore n = \frac{N_d - N_a}{2} + \frac{N_d - N_a}{2}\sqrt{1 + \left(\frac{2n_i}{N_d - N_a}\right)^2}$

$= 1.5\times10^{19} + 1.5\times10^{19}\sqrt{1 + \left(\frac{2\times1.86\times10^{19}}{3\times10^{19}}\right)^2}$

$n = \underline{3.89\times10^{19}\ m^{-3}}$ #. Total no. of doped n carriers + intrinsic ones

$p = \frac{n_i^2}{n} = \frac{(1.86\times10^{19})^2}{3.89\times10^{19}} = \underline{8.895\times10^{18}\ m^{-3}}$ #.

To find current density $J = \sigma E$

$= e(n\mu_e + p\mu_n)E$

$= 1.6\times10^{19}(3.89\times10^{19}\times0.38 + 8.895\times10^{18}\times0.18)(200)$

$J = \underline{524\ Am^{-2}}$ #.

8) $N_a = 4\times10^{21}$, $N_d = 3\times10^{21}$

$(N_a - N_d) = 10^{21}\ m^{-3}$

Given that $n_i^2 = 3.1\times10^{44}\ T^3 \exp\frac{-9100}{T}$

Substitute $T = 300K$; $n_i^2 = 3.1\times10^{44}(300)^3 \exp -\frac{9100}{300}$

$n_i = \underline{2.37\times10^{19}\ m^{-3}}$ #

At $T = 400k$; $n_i' = \underline{1.6\times10^{21}\ m^{-3}}$ #.

At 300k $N_a - N_d \gg n_i$, hence we can ignore the intrinsic carriers and $p = N_a - N_d = 10^{21}\ m^{-3}$

$$n = \frac{n_i^2}{p} = \frac{(2.37\times10^{19})^2}{10^{21}} = \underline{5.61\times10^{17}\ m^{-3}}\ \#$$

However at 400K, $N_a - N_d \not\gg n_i$, so intrinsic carriers must be taken into account.

Therefore; $$p = \frac{N_a - N_d}{2} + \frac{N_a - N_d}{2}\sqrt{1 + \left(\frac{2n_i}{N_a - N_d}\right)^2}$$

$$= \frac{10^{21}}{2} + \frac{10^{21}}{2}\sqrt{1 + \left(\frac{2\times1.6\times10^{21}}{10^{21}}\right)^2}$$

$$p = \frac{10^{21}}{2} + \frac{10^{21}}{2}\sqrt{11.24}$$

$$= \underline{2.19\times10^{21}\ m^{-3}}\ \#$$

$$n = \frac{n_i^2}{p} = \frac{(1.6\times10^{21})^2}{2.19\times10^{21}} = \underline{1.19\times10^{21}\ m^{-3}}\ \#.$$

Comment: At 300K, the intrinsic carriers are negligible, and the total carrier density is determined by $(N_d - N_a)$ and the sign of $(N_d - N_a)$ gives the carrier type. Intrinsic carriers -ve, so holes.
At 400K, the carriers created by the electron / hole pair process becomes comparable to the number of carriers produced by the dopant, so the total no. p, is formed by considering charge neutrality and not the general formula.

9) $N_d - N_a = 1.5\times10^{19} - 1\times10^{19} = 5\times10^{18}\ m^{-3}$.

$(N_d - N_a) \gg n_i$, so intrinsic carriers can be ignored.

$\therefore\ n = 5\times10^{18}$

$\Rightarrow\ R = \frac{\rho l}{A}$ and $\rho = \frac{1}{\sigma}$ $\therefore\ R = \frac{l}{\sigma A}$

since n-type is dominant carriers ; $\sigma = n e \mu_e$

$$\Rightarrow R = \frac{l}{n e \mu_e . A} = \frac{10^{-2}}{5\times10^{18}.\ 1.6\times10^{-19}.\ 0.12\ (10^{-2}.\ 10^{-2})} = \underline{1.04\ k\Omega}\ \#.$$