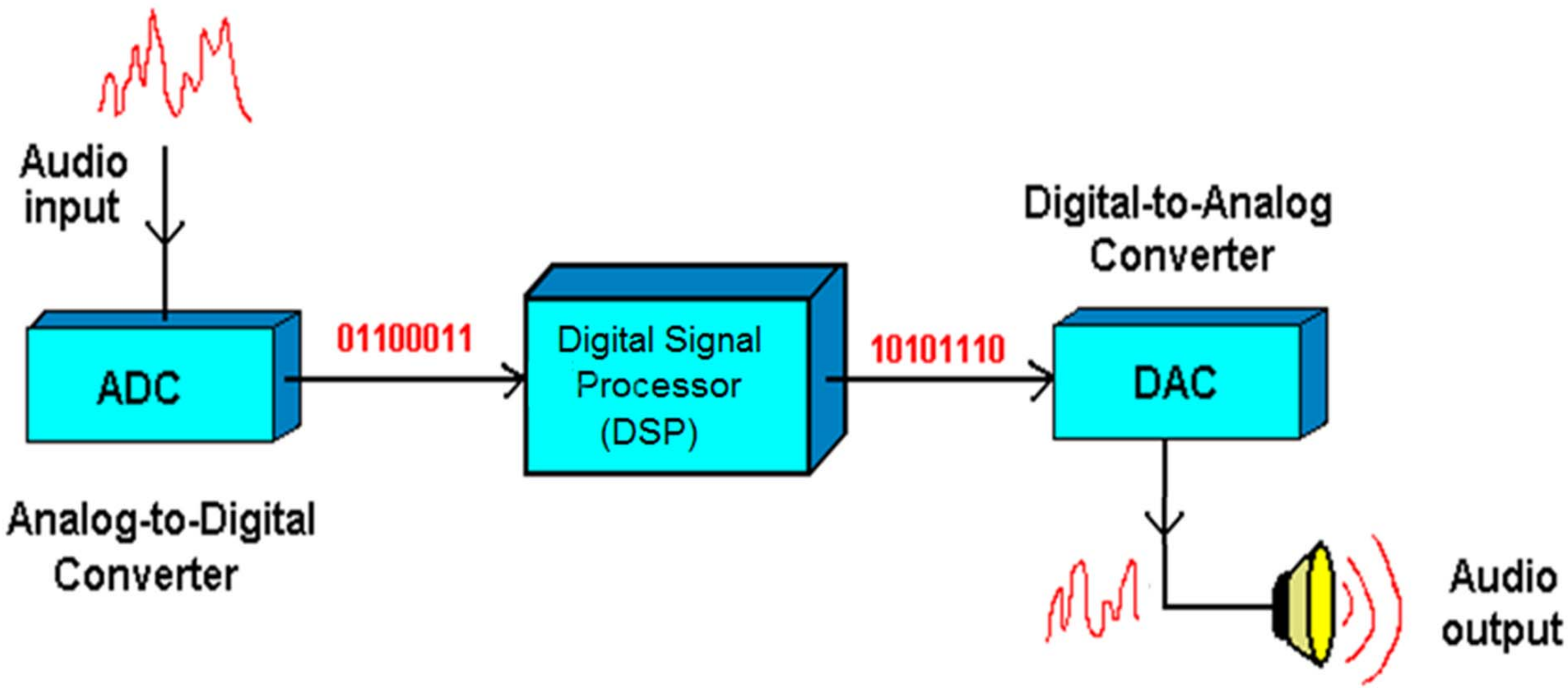
Sampled Signals

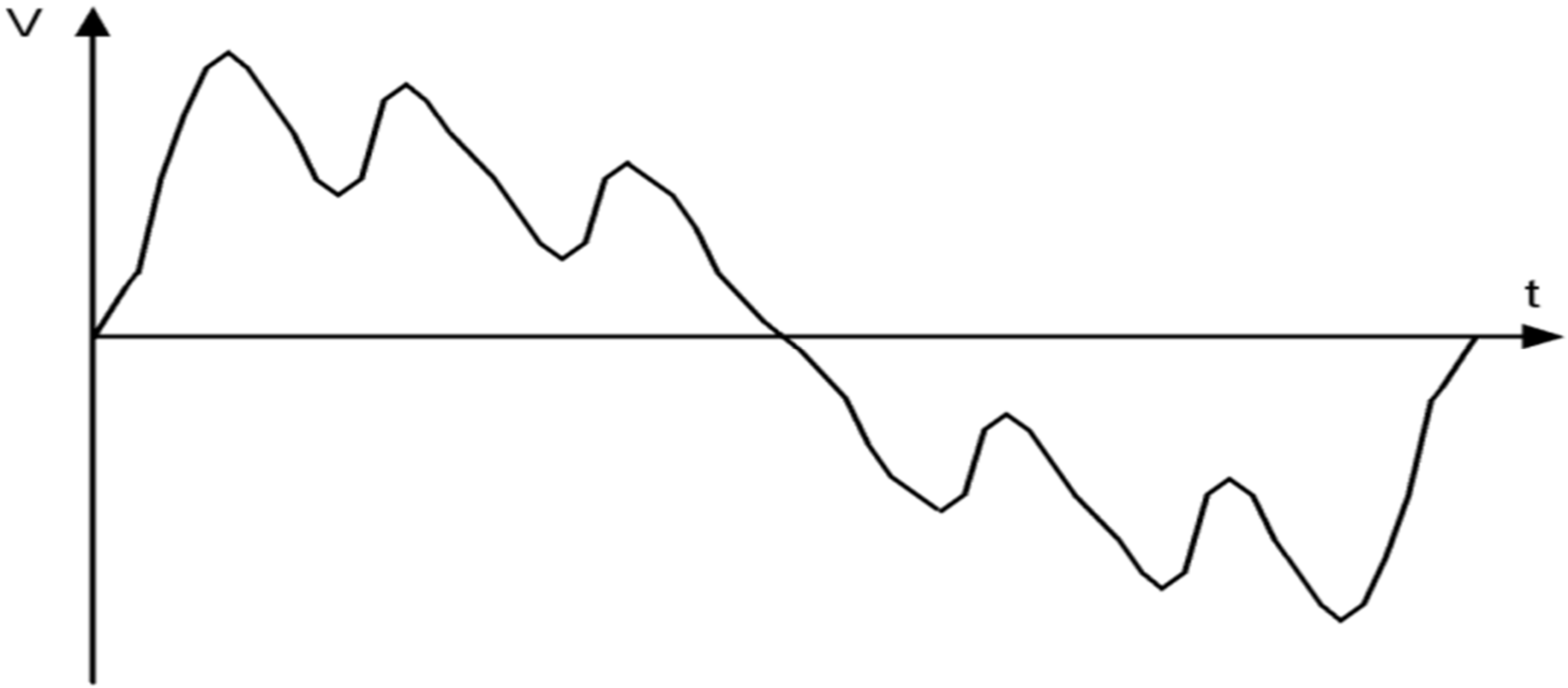
- Nyquist Frequency
- Aliasing
- Resolution

Digital Signal Processing

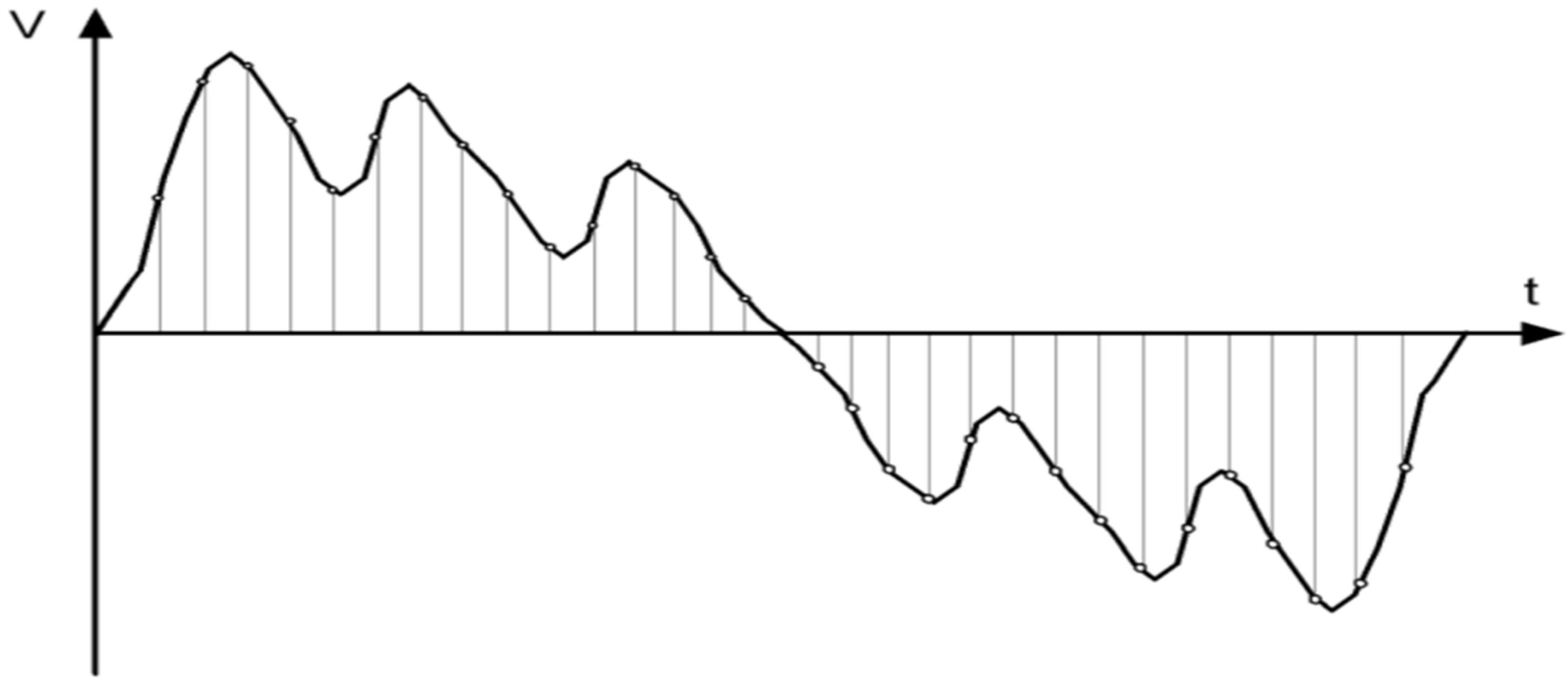
In order to digitally process naturally occurring analog signals such as sound and video, they must first be sampled and converted to a digital form. The signal can the be processed e.g. compression, noise removal.



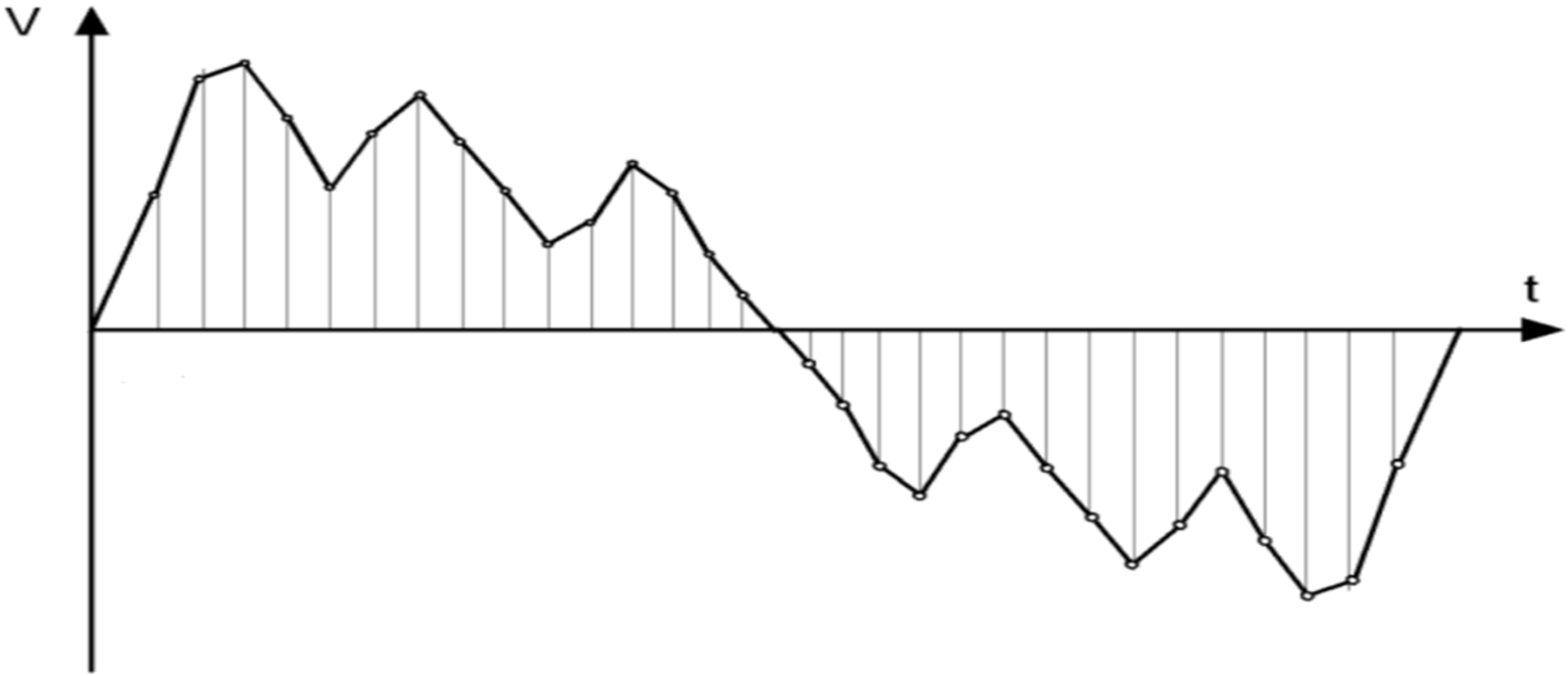
To convert the audio signal shown to a digital form, it must be sampled at fixed time intervals. Each sample is then converted to a number based on the voltage level at the sample point.

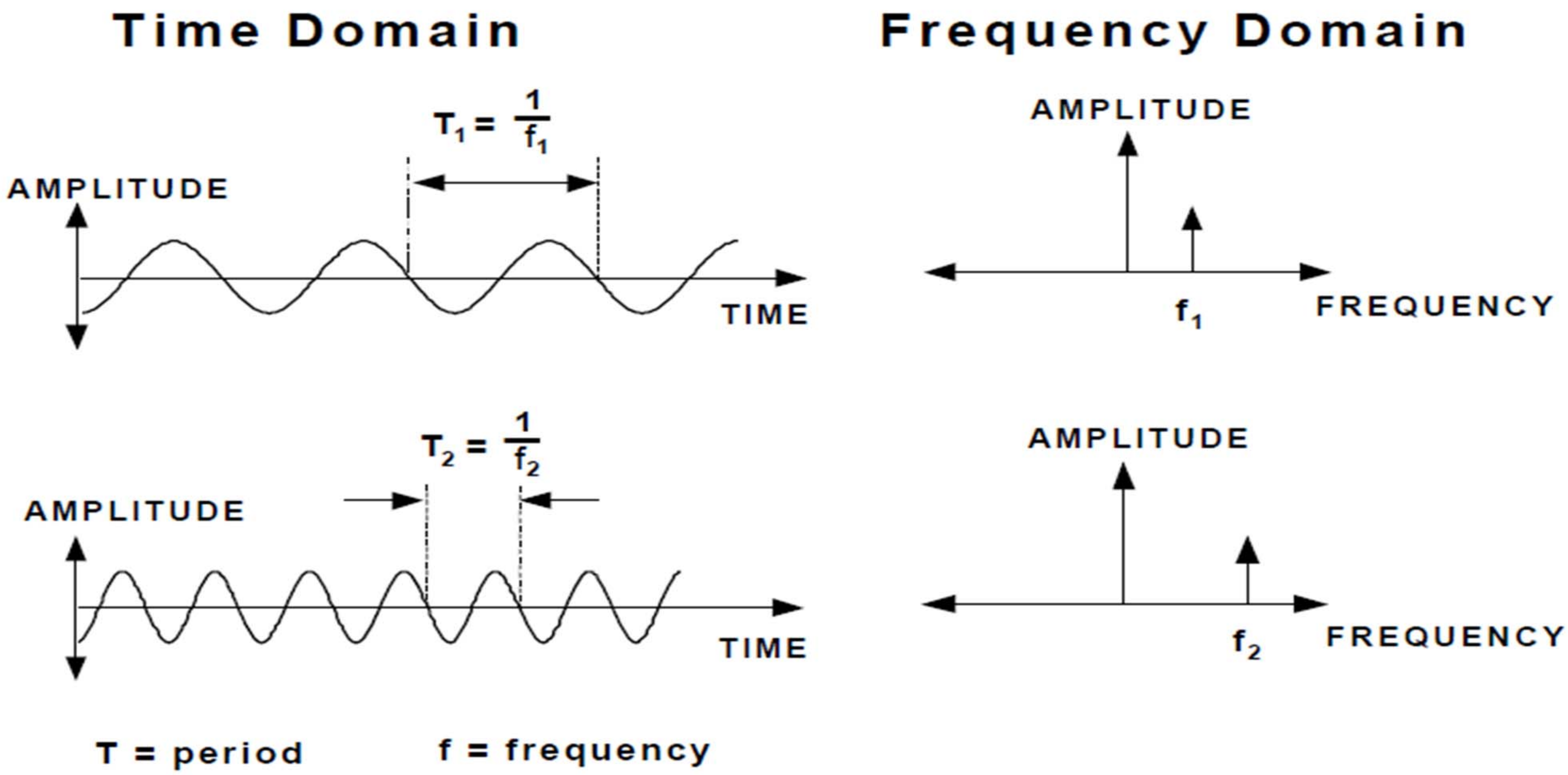


The frequency at which the signal is sampled is called the sampling rate. In an audio CD, the sampling rate is 44.1 KHz. This means that 44,100 points (voltage levels) will be captured every second. The distance between the sample points will be 1/44100 = 22.675 μs.

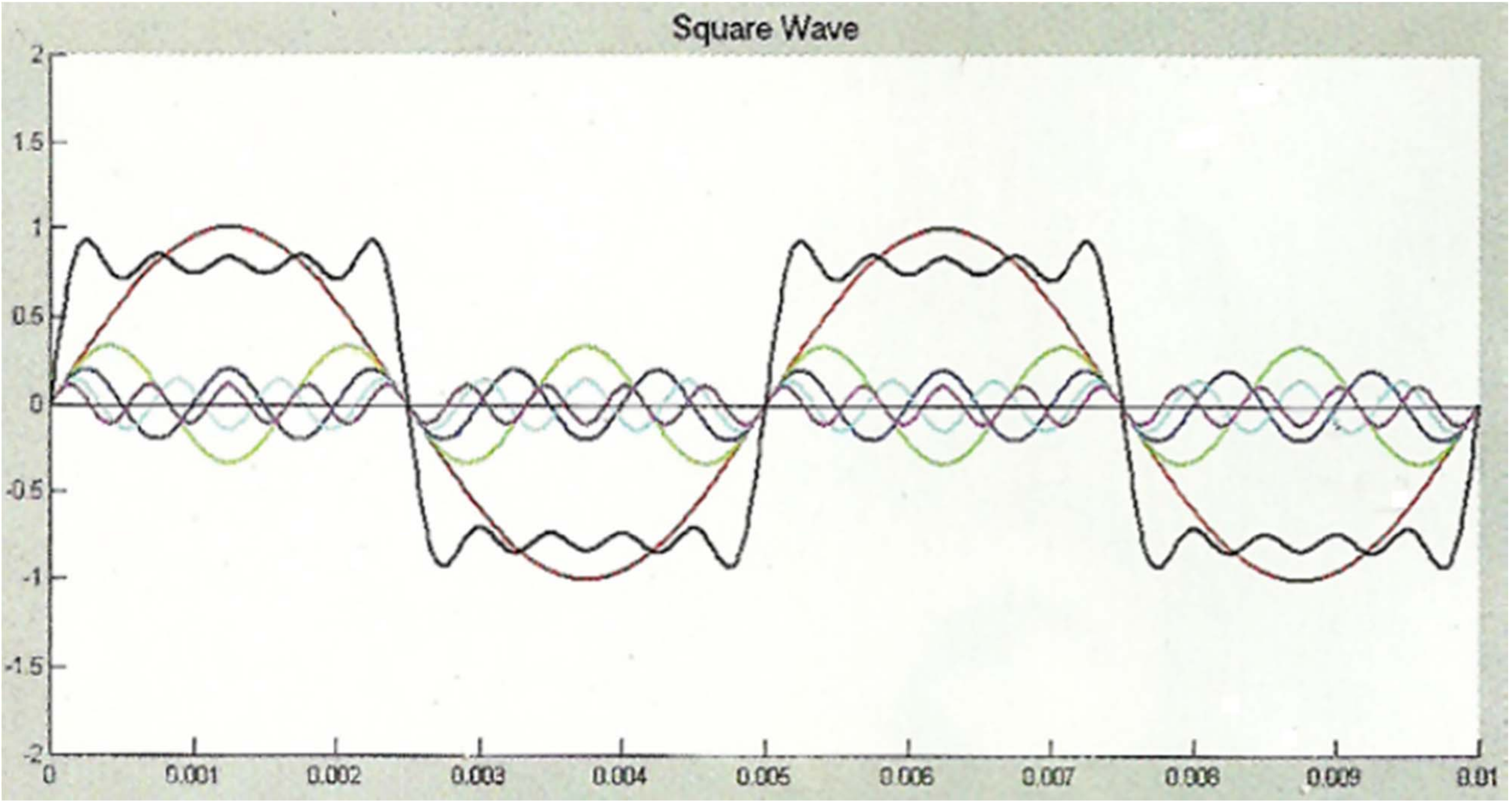


If the signal is converted back, it won't be perfect as it does not have all points from the original. The more points we have, the better will be the reconstructed signal.





Signals can be represented in both the time and frequency domain as shown. The sampling theorem tells us that the sampling frequency must be at least twice the highest frequency component of the analog signal in order to represent it. This is known as the **Nyquist frequency**.



Analog signals are composed of a spectrum of frequencies consisting of a fundamental frequency f_0 and integer multiples of f_0 known as harmonics. A square wave contains odd integer harmonics. For a complete square wave, you would have to add the odd harmonics to ∞.

- If the sampling rate is too low, the reproduction from the original signal will be poor. For an audio signal this will give poor sound quality.
- If the sampling rate is too high, the reproduced signal will be very good but a large amount of storage space may be required for the samples. Time and energy will also be wasted in processing the extra data.

How is the best sampling rate chosen ?

Consider an analog waveform with a maximum frequency component $f_{a(max)}$

For a Nyquist frequency $f_{a(max)}$:

$$\boxed{f_{sample} > 2 \times f_{a(max)}}$$

For an audio CD with a sampling rate of 44.1 KHz, frequencies up to about 22 KHz can be captured. (specification for common audio equipment is 20 KHz)

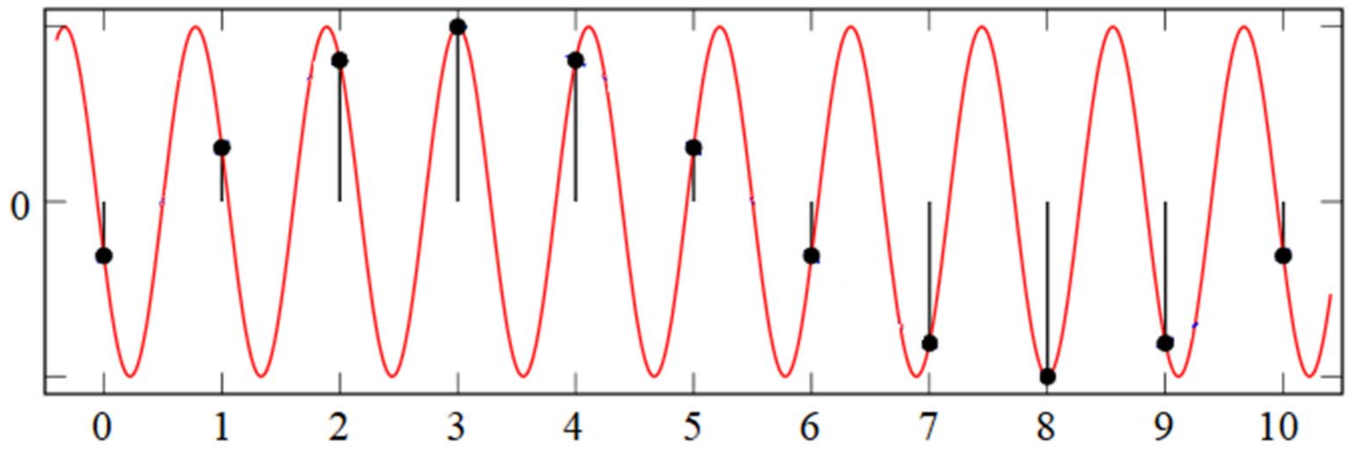
An analog signal contains harmonics of the frequencies present. These are integer multiples of the fundamental frequency.

For a fundamental frequency f , there are harmonics at 2f, 3f, 4f etc

If frequencies higher than the Nyquist frequency are present, then an unwanted condition known as aliasing will occur.

Aliasing

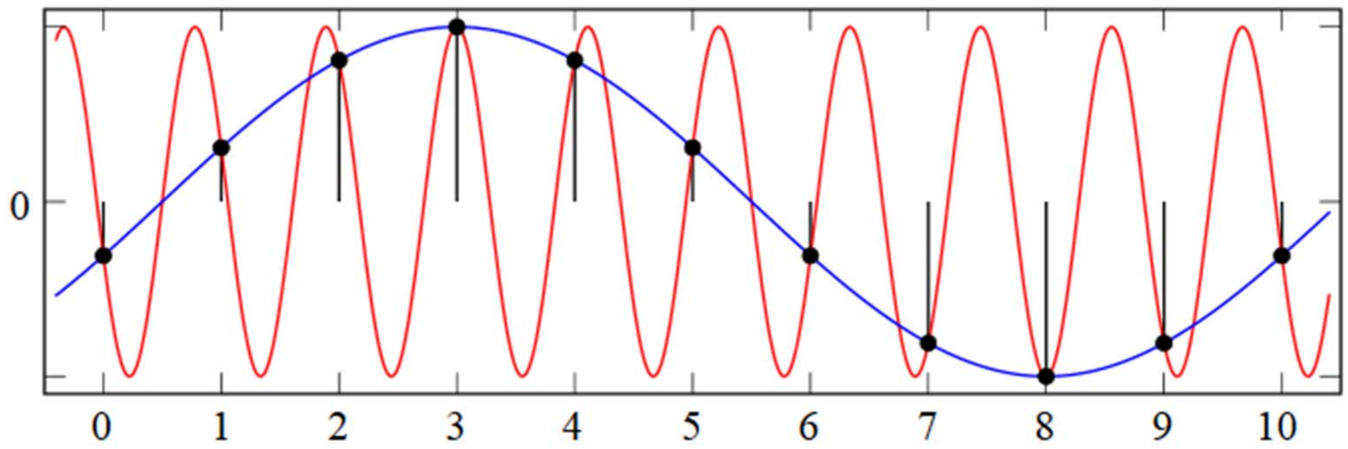
Aliasing is a limitation of discrete time sampling and occurs when higher frequencies than the Nyquist frequency are present.



The alias is an unwanted frequency that is not part of the original waveform. To eliminate aliasing, a low pass filter is used to remove frequencies above the Nyquist frequency.

Aliasing

Aliasing is a limitation of discrete time sampling and occurs when higher frequencies than the Nyquist frequency are present.



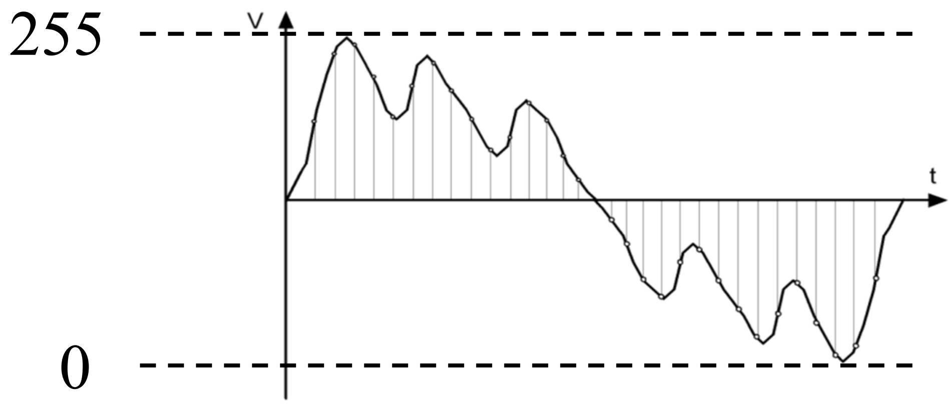
The alias is an unwanted frequency that is not part of the original waveform. To eliminate aliasing, a low pass filter is used to remove frequencies above the Nyquist frequency.

Resolution

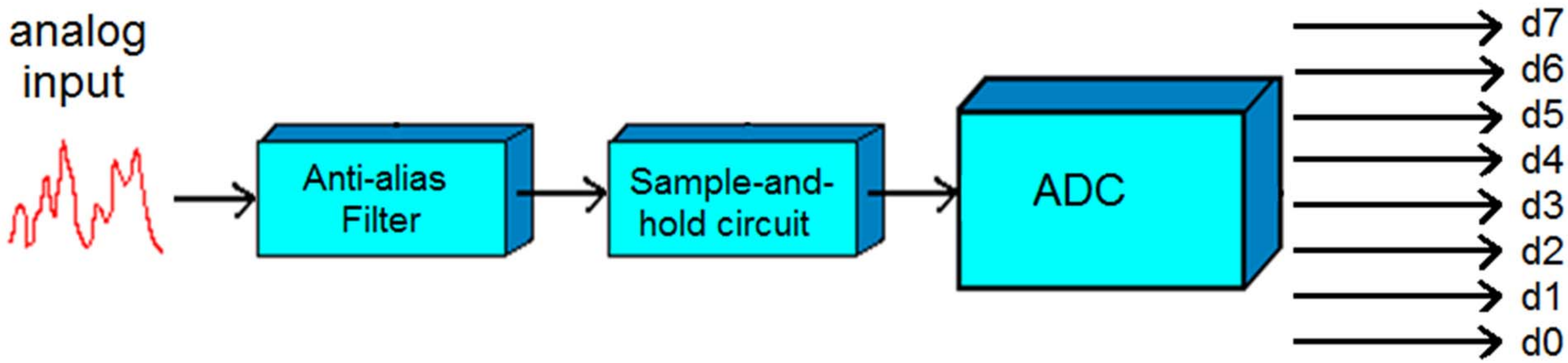
Each sample can be stored with a variable number of bits.

e.g. for 8 bits , $2^8 = 256$

each sample can take a value from 0 to 255

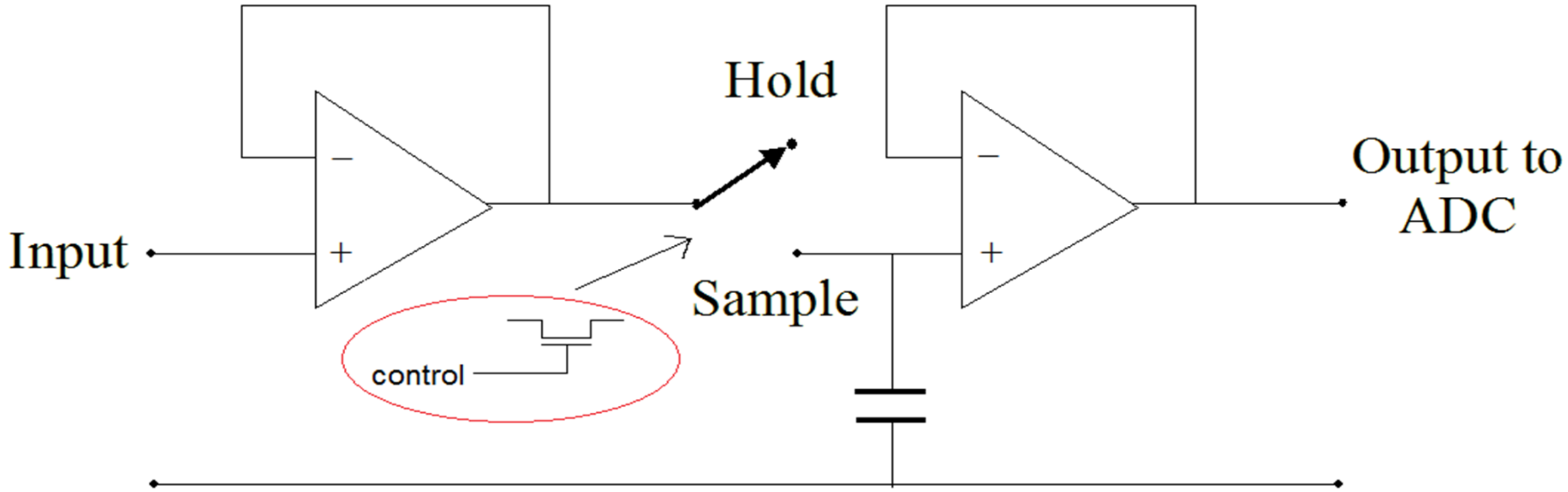


More bits will give a higher quality at the cost of more storage.



Holding the Sampled Value

The sampled value must be held constant until the next sample occurs in order for the ADC to have sufficient time to convert the sample to the digital format.



The input from the anti-alias filter is buffered by a op-amp configured as a unity-gain voltage follower. A positive pulse on the nmos switch will allow the capacitor to charge to the instantaneous value of the input voltage. When the switch turned off , the voltage will be held on the capacitor until the next pulse due to the high input impedance of the buffer before the output. This results in a 'stairstep' approximation of the original waveform.