SYSTEMS

A system can be thought of as a process of transforming an input signal from one form to another as an output signal. For example hi-fi speakers form a system that transforms music in the form of electrical signal to audio signal. If the level of signal is too low or too high we can use an amplifier or an attenuator to amplify or to attenuate the signal. Another example of system is a filter that can be used to suppress noise in signal. In this course we will treat a system as a BLACK BOX with at least one input and one output.

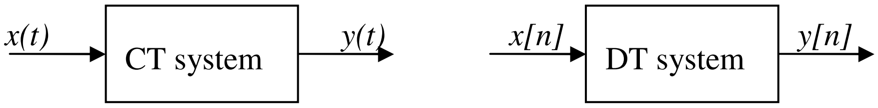

Fig 1: Continuous-time system (left) and discrete-time system (right).

A continuous-time (CT) system is a system which accepts CT signals as its input and produce CT signals as its output. Likewise, a discrete-time (DT) system is a system that transforms DT signals input to DT signals output.

Examples:

1) Consider a potential divider depicted in fig. 2(a). The output v_o is given by $v_0 = \frac{R_2 v_i}{R_1 + R_2}$. The system representation is given by fig. 2(b).

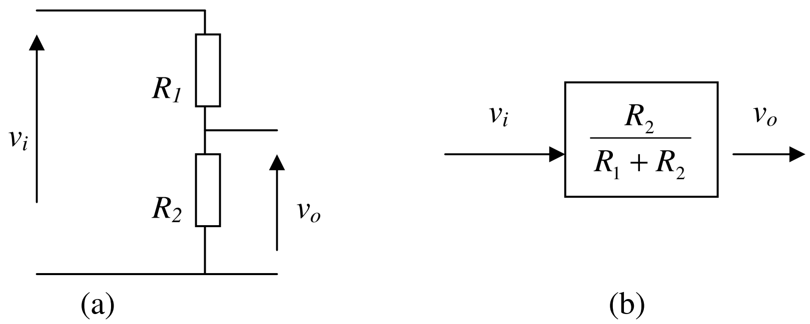

Fig.2: (a) Potential divider circuit and its system representation.

2) An op-amp can be used to form a scaling system.

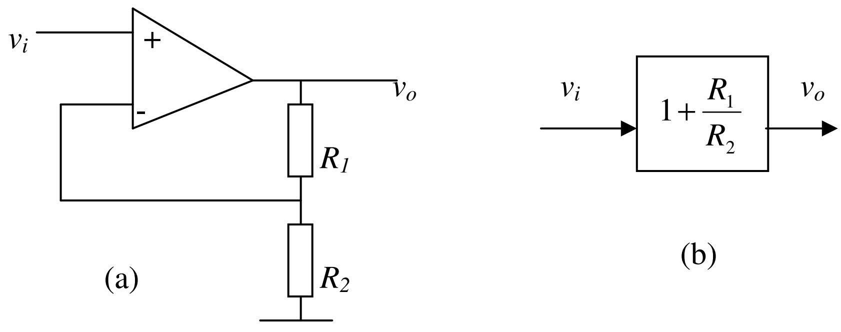

Fig.3: (a) A non-inverting op-amp circuit and (b) its system representation.

3) An op-amp can also be used to form a differentiation system.

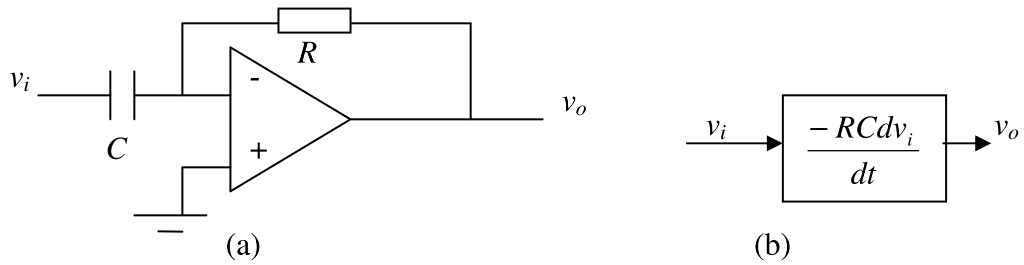

Fig.4: (a) A differentiator and (b) its system representation.

These examples of system show that we can use mathematical equations of systems to describe the relationships between input and output signals. Let's look at some basic system properties.

Systems with and without memory

A system is said to be ***memoryless*** if its output $y(t_o)$ depends only on the input $x(t)$, applied at $t = t_o$. $y(t_o)$ is independent of the input applied before and after $t = t_o$. For example the systems $y[n] = x[n] - 3x[n]$ and $v_o(t) = \frac{R_2}{R_1 + R_2} v_i(t)$ are memoryless. If the output value depends on past input or future input, the system is said to have ***memory***. Examples of system with memory are:

1) unit-time delay system, $y(t) = u(t-1)$.

2) voltage across a capacitor, $V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$, where C is the capacitance and $i(\tau)$ is the current.

3) an accumulator output, $y[n] = \sum_{k=-\infty}^{n} p[k]$.

We can think of the concept of memory in a system as a mechanism to "remember" or to store information about input values at times other than the present time. A unit-time advance system $y(t) = u(t+1)$ is also said to have memory because the output at time t depends on input at a future time t +1. Therefore this unit-time advance system can predict what input will be applied in the future.

Causality

A system is ***causal*** if its output at current time depends only on past and current inputs but is independent of future input. For instance the integrator system is causal or ***non-anticipatory*** because $V_c(t)$ does not depend on future input. The unit-time advance system is non-causal since its output $y(t)$ depends on future input $u(t+1)$. In practice all memoryless systems are causal.

Causality is not an important issue in applications in which time is not the variable such as in image processing. Here an algorithm is developed to make use of all the information surrounding a pixel to improve the quality of the image.

Stability

A ***stable*** system is a system in which the output does not diverge when the input to the system is bounded (i.e if its magnitude does not grow indefinitely). For example a system described by $y_1(t) = tx(t)$ is unstable. When the input $x(t) = 1$ is bounded, $y_1(t) = t$ in unbounded. A system $y_2(t) = \cos(x(t))$ is stable since the output is bounded when the input $x(t)$ is bounded.

Initial state

The output of a causal system is dependent on its past and current inputs. Consider a simple RC circuit in figure 5.

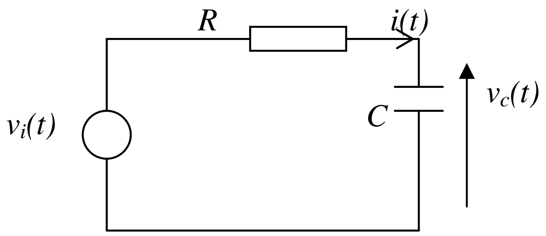

Fig. 5: RC circuit.

The current $i(t)$ and voltage across the capacitor with capacitance C are related by $i(t) = C\frac{dv_c(t)}{dt}$. Integrating the current gives $v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$. t = -∞ implies that the voltage depends on $i(t)$ from the time the circuit was first built to the time t. It is virtually impossible to find the value $i(-\infty)$. However if we know the ***initial state*** at $t = t_o$ or the value of $v_c(t_o)$ then we can evaluate the voltage as $v_c(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v_c(t_o)$.

Linearity

A system, in CT or DT, is ***linear*** if it satisfies the additivity property and the homogeneity property. Let $y_1(t)$ be the output corresponding to an input $x_1(t)$ and $y_2(t)$ be the output corresponding to an input $x_2(t)$. The system is linear if

1) The response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$ (additivity property).

2) The response to $ax_1(t)$ is $ay_1(t)$ where a is a constant (homogeneity property).

These two properties can be combined into:

CT: $ax_1(t)+ bx_2(t) \rightarrow ay_1(t)+ by_2(t)$,

DT: $ax_1[t]+ bx_2[t] \rightarrow ay_1[t]+ by_2[t]$.

Examples:

1. Determine whether the system $y(t) = K\frac{dx(t)}{dt}$ is linear.

2. Show that the system $y(t) = e^{x(t)}$ is non-linear.

3. Determine whether $y(t) = 3x(t) + 4$ is a linear system.

<u>Time Invariance</u>

If the characteristics of a system are independent of time it is said to be ***time invariant***. RC circuit in figure 5 is time invariant because the values of R and C are constant over time. A time shift in the input signal will result in an identical shift in the output signal of a time invariant system.

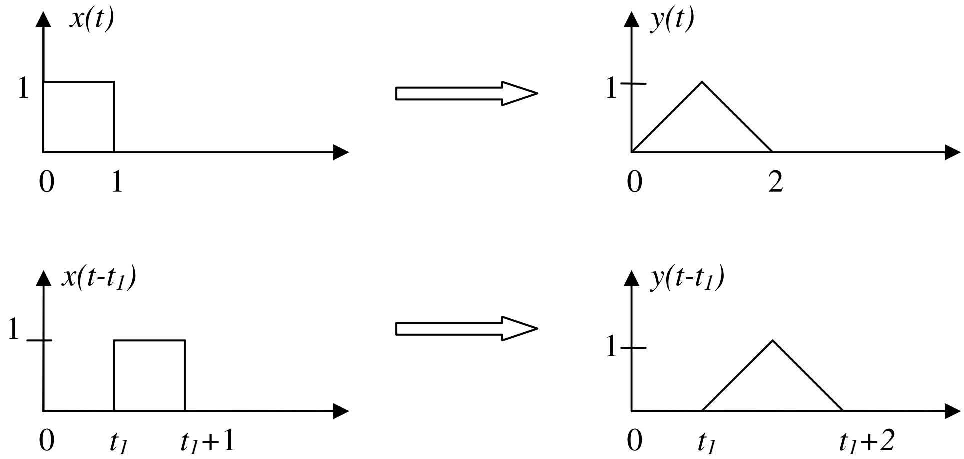

Figure.6: Time shift property.

A system that is linear and time-invariant is therefore known as a ***Linear Time Invariant*** (LTI) system. The potential divider in figure 2 is an example of an LTI system. If we know the input-output ($x(t)$-$y(t)$) pair of an LTI system, then we can compute the response of the system to any signal that can be constructed from $x(t)$.

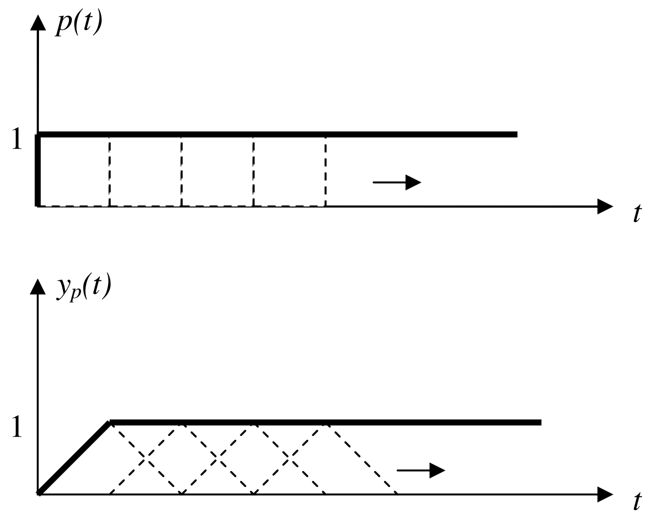

Figure 7: Response to input p(t) can be computed using input-output pair in figure 6.
We can compute the response of the LTI system in fig. 6 to an input $p(t)$ because $p(t) = x(t) + x(t-1) + x(t-2) + x(t-3) +$……… and the output is $y_p(t) = y(t) + y(t-1) + y(t-2) + y(t-3) +$……….

Notes: