**Data Provided: None**

The University Of Sheffield.

**DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING**

**Autumn Semester 2013-14 (2.0 hours)**

**EEE6440 Advanced Signal Processing 6**

Answer **FOUR** questions (**TWO** questions from **Part A** and **TWO** questions from **Part B**). **No marks will be awarded for solutions to a third question attempted from any of the two sections.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

**PART A - Answer only TWO questions from questions 1, 2 and 3.**

**1.** Consider the filter h(n) with values { ¼ , ½ , ¼ } for n= -1, 0, 1, respectively.

**a.** Compute and draw the step response of the filter h(n). **(2)**

**b.** Compute and draw the magnitude of the frequency response of the filter h(n). **(3)**

**c.** Determine and draw the impulse response of the resulting filter kernel, p(n), if two h(n) filters are cascaded in a system. **(2)**

**d.** Sketch time-domain and frequency-domain performances of p(n) and compare them with those of h(n). **(3)**

**e.** A signal x(n) is filtered with h(n) to get the new signal s(n). Then the final output signal y(n) is computed by subtracting the signal s(n) from the original signal x(n). Draw a system block diagram to show this operation and derive the impulse response of the resulting filter r(n). **(2)**

**f.** Sketch time-domain and frequency-domain performances of r(n) **(2)**

**g.** What type of a filter is r(n)? **(1)**

**2.** An input signal $X$=($x_0$, $x_1$, $x_2$, $x_3$) is transformed into $Y$=($y_0$, $y_1$, $y_2$, $y_3$) as follows:

$$Y = HX \,,$$

where $H = \dfrac{1}{128}\begin{bmatrix} 64 & 64 & 64 & 64 \\ 84 & 35 & -35 & -84 \\ 64 & -64 & -64 & 64 \\ 35 & -84 & 84 & -35 \end{bmatrix}.$

**a.** Write down the basis functions corresponding to the above transform matrix. **(2)**

**b.** Derive the inverse transform matrix showing all steps. **(4)**

**c.** How do you compute the mean of signal $x$ using the transform domain coefficients $y$? **(2)**

**d.** How do you use this transform to remove noise from a measured signal? **(5)**

**e.** How do you use this transforms to decorrelate 2-dimensional data? **(2)**

**3. a.** Using frequency response diagrams, explain how using low pass filters in the sampling rate decimators avoids aliasing. **(4)**

**b.** A signal, sampled at 1.024 kHz, is to be decimated using a 2-stage decimator, with decimation rates $M_1$=$M_2$=4, respectively. The signal band of interest extends from 0 to 30 Hz. The overall anti-aliasing digital filtering should satisfy 0.01 dB passband deviation ($\delta_p$) and 80 dB stopband attenuation ($\delta_s$). Estimate the lengths of the low pass filters $h_1$ and $h_2$ used for the two decimations, respectively. Note that the filter length N for a low pass filter is approximated as

$N \approx \frac{-10\log(\delta_p\,\delta_s)-13}{14.6(\Delta f)} + 1$ , where $\Delta f$ is the normalised width of transition band.

**(7)**

**c.** Estimate the computational complexity of this 2-stage decimator in terms of multiplications per second.

Explain why multistage decimation is more efficient in terms of the computational complexity, compared to a single stage decimation system. **(4)**

**PART B - Answer only TWO questions from questions 4, 5 and 6.**

**4.** **a.** i) Estimate the mean, mean-square and variance of the following stationary sequence: {1.3, 1.6, 1.8, 2.7, 0.6}. (3 marks)

ii) Derive the relationship of the three averages and verify it using the above estimated results. (3 marks) **(6)**

**b.** For a 12-bit A/D converter, what is the dynamic range for a cosine wave input signal? **(3)**

**c.** Explain briefly the three modes of operation of an adaptive filter with the aid of a diagram for each mode. **(6)**

**5.** **a.** Suppose the z-transform of the cross-correlation function between the input x(n) and the output y(n) of a filter h[n] is given by $S_{xy}(z)$ and the z-transform of the autocorrelation of the input x(n) is given by $S_{xx}(z)$. Derive the relationship between these two z-transforms and show all working. **(5)**

**b.** Zero-mean white Gaussian noise with variance 2 is applied to a filter with a transfer function $H_1(z)=2-3z^{-1}$. Calculate the autocorrelation sequence of its output. **(2)**

**c.** **i)** Suppose the length of an FIR (finite impulse response) adaptive filter is N. Its input is denoted by y(n) and the training signal is denoted by x(n). Derive the LMS (least mean square) adaptive algorithm for updating the coefficients of the adaptive filter. (4 marks)

**ii)** The table below shows the input and training signal to a two-tap adaptive filter at sample numbers 10 and 11, where **h**(n) is the vector holding the two taps of the adaptive filter :

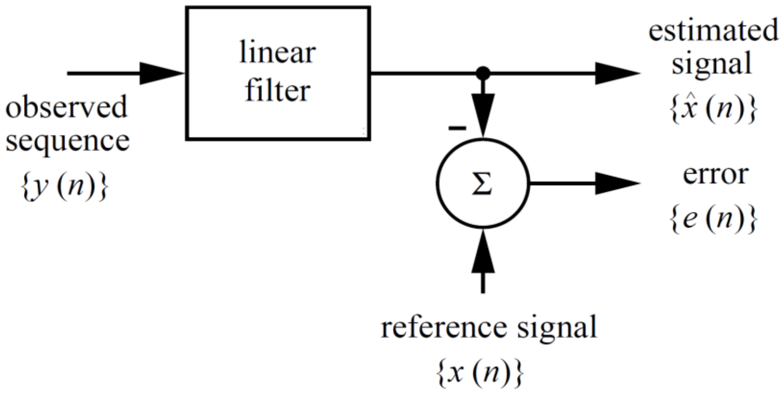
| Iteration *n* | y(*n*) | **h**(*n*) | x(*n*) |
|---|---|---|---|
| 10 | 0.25 | [1    6] | 1.2 |
| 11 | 0.3 | | -0.2 |

Using the derived LMS algorithm, evaluate **h**(11). The stepsize is fixed at 0.2. (4 marks) **(8)**

**6.** **a.** Given the correlation sequence $\phi_{xx}(m)$ of a zero-mean random sequence x(n), give the expression of its power spectral density function. Explain why this expression is given the name of "power spectral density". **(4)**

**b.** Give the cost function of the RLS algorithm with a forgetting factor, and explain why we introduce such a forgetting factor into the cost function. **(2)**

**c.** A linear estimator is shown below, where the impulse response of the linear filter is given by $h_j$, j=0, 1, …, N-1. Derive the Wiener solution for $h_j$. Show all working.




**(9)**

**GCKA / WL**