Data Provided: 1 × z-transform table

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2012-13 (2.0 hours)

EEE6033 Digital Signal Processing 6

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. i) In the context of a discrete-time system, explain the concepts of linearity and time invariance.

(3)

ii) Explain in detail why a linear time-invariant (LTI) system can be completely characterised by its impulse response.

(6)

iii) Two LTI systems are connected in cascade and their impulse responses are denoted by $h_1[n]$ and $h_2[n]$, respectively. Explain that the overall impulse response of the cascaded system is given by the convolution of $h_1[n]$ and $h_2[n]$.

(3)

b. There are two important differences between the discrete-time and continuous-time complex exponential signals (denoted by $x[n]=e^{j\omega n}$ and $x(t)= e^{j\omega t}$, respectively). Explain in detail the two differences.

(4)

c. The impulse response h[n] of an LTI discrete-time system is given by

$$h[n]= \delta[n]+2\delta[n-1]-\delta[n-2].$$

Use z-transforms to calculate the output y[n] of the system given the input sequence

$$x[n]=\delta[n]+3\delta[n-1]-\delta[n-2].$$ **(4)**

2. **a.** i) The impulse response of an LTI system is denoted by $h_1[n]$. Give the expression for its frequency response $H_1(e^{j\Omega})$. A second LTI system has an impulse response $h_2[n]$ which is a delayed version of $h_1[n]$ by 2 samples, i.e. $h_2[n]=h_1[n-2]$. Express the frequency response of the second system in terms of $H_1(e^{j\Omega})$.

(2)

ii) If h[n] is real-valued, give a detailed proof to show that $H(e^{-j\Omega})= H^*(e^{j\Omega})$, where "*" denotes conjugation.

(3)

iii) Most signals in the real world are not in the form of complex exponentials. Explain in detail why we study the frequency response of an LTI system, i.e. the response to a complex exponential sequence?

(6)

b. i) Calculate the Discrete Fourier Transform (DFT) of the discrete series x[n]={0.5, 1, 1, 0.5}.

(4)

ii) Consider a sequence $x_1[n]$ whose length is L (nonzero for n=0, 1, …, L-1) and a sequence $x_2[n]$ whose length is P (nonzero for n=0, 1, …, P-1). A linear convolution of these two sequences will generate a third sequence $x_3[n]$. Describe the process involved in calculating this linear convolution using DFT.

(5)

3. **a.** i) State the Nyquist sampling theorem and determine the minimum sampling frequency required for sampling the following continuous-time signal x(t)

$x(t)=\cos(20\pi t)+\cos(100\pi t)$

(4)

ii) Draw the block diagram of an ideal system for recovering a general continuous-time signal x(t) from its sampled version x(n) with a sampling period T. Give details about the input-output relationship at each stage of the block diagram.

(3)

b. Consider the system function

$$H(z)=\frac{1+3z^{-1}}{1-0.5z^{-1}+0.3z^{-2}}$$

Give its direct form I and direct form II implementation structures.

(4)

c. i) Derive the z-transform of the following sequence

$$x[n]=(\frac{1}{3})^n u[n]+(-\frac{1}{4})^n u[n]$$

(4)

ii) Give the pole-zero plot of the z-transform, including its region of convergence (ROC).

(2)

d. A digital IIR lowpass filter is to be designed to remove high frequencies from a sampled input signal. The filter is to be implemented with a sampling period of *T=0.03* seconds. Currently, an analogue filter is employed with the transfer function:

$$\frac{Y(s)}{X(s)}=\frac{1}{1+s/30}$$

Design a digital filter with characteristics similar to the analogue filter, using the impulse invariant method.

(3)

4. **a.** Given the spectral coefficients of a filter, *H(k)*, which are symmetrical about *k*=0, the original impulse response *h*[n] can be reconstituted using the following equation, where *N* is the total number of coefficients:

$$h[n]=\frac{1}{N}\sum_{k=-(N-1)/2}^{(N-1)/2} H(k)e^{j2\pi nk/N}=\frac{1}{N}\left(H(0)+2\sum_{k=1}^{(N-1)/2} H(k)\cos(2\pi nk/N)\right)$$

From this you are going to design a **highpass** FIR filter with *N=5* coefficients with a passband range between 1kHz and 2kHz at a sampling frequency f_s=4kHz.

Use the frequency sampling method to calculate the FIR filter coefficients.

(6)

b. Explain the concept of circular convolution with the aid of an example. **(4)**

c. A digital FIR highpass filter is to be designed for filtering out low frequency noise. It should pass the signal with a frequency higher than 1×10^4 rad/sec (passband cutoff frequency), and attenuate any component below the frequency 1×10^2 rad/sec (stopband cutoff frequency) by more than 20dB. The sampling frequency for implementing this filter is f_s=5kHz.

i) Translate the desired highpass filter characteristic to an appropriate lowpass characteristic that is suitable for design using the 'window' method. Sketch the translated lowpass filter magnitude characteristic with the normalised stopband and passband frequencies. **(4)**

ii) Design an FIR lowpass filter to meet the lowpass filter characteristic derived in part (i), using the 'window' method with the aid of the information given below. Obtain the first 4 impulse response coefficients of the resulting lowpass filter. **(4)**

iii) From the results of part ii), derive the design result for the desired highpass FIR filter and provide the first 4 impulse response coefficients. **(2)**

Rectangular $w[n]=\begin{cases}1, & 0\le n\le N-1\\ 0, & elsewhere\end{cases}$

Bartlett $w[n]=\begin{cases}2n/(N-1), & 0\le n\le (N-1)/2\\ 2-2n/(N-1), & (N-1)/2\le n\le N-1\\ 0, & elsewhere\end{cases}$

Hanning $w[n]=\begin{cases}\{1-\cos[2\pi n/(N-1)]\}/2, & 0\le n\le N-1\\ 0, & elsewhere\end{cases}$

Hamming $w[n]=\begin{cases}0.54-0.46\cos[2\pi n/(N-1)], & 0\le n\le N-1\\ 0, & elsewhere\end{cases}$

Blackman $w[n]=\begin{cases}0.42-0.5\cos[2\pi n/(N-1)]+0.08\cos[4\pi n/(N-1)], & 0\le n\le N-1\\ 0, & elsewhere\end{cases}$

	Main-lobe width	Peak of first-lobe	Stopband attentuation factor (for lowpass filter)
Rectangular	≈4π/N	-13dB	-21dB
Bartlett	≈8π/N	-27dB	-25dB
Hanning	≈8π/N	-32dB	-44dB
Hamming	≈8π/N	-43dB	-53dB
Blackman	≈12π/N	-58dB	-74dB

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