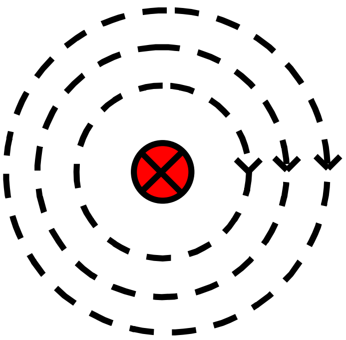
Magnetic field and Magnetic circuits: - *Background*

A 'field' describes a state of space in which there is a force on a body. i.e. the field can only be detected by it's effect.
e.g.

- Gravitational field – Force of attraction between masses
- Electric Field – Force on a charged particle
- Magnetic Field – Force on a <u>moving</u> charge. e.g. a current carrying conductor etc.

Pre 1820 – Electricity and Magnetism treated more or less independently

1820 – Oersted and Ampére discovered that the magnetic field is always associated with an electric current, and that a conductor carrying an electric current had a force acting on it when it was in a magnetic field.



Magnetic field of a straight conductor – represented by concentric circles for equipotentials

1831 – Faraday deduced and proved experimentally that when a magnetic field linking an electric circuit changed, there is a transient induced emf (and hence current flow).

Thus the electric and magnetic effects are always inter-related

Whilst all electric and magnetic phenomena of interest in electrical engineering can be explained in terms of the forces between charges (either stationary – electric field, or moving – magnetic field), we find it much easier to solve may problems by the use of <u>electric circuits</u> and <u>magnetic circuits</u> rather than the <u>field.</u>

The existence of the magnetic field associated with the electric circuit may or may not be what the electric circuit designer wants. Sometimes it is essential and useful, other times it is a pain in the …… circuit!

Examples of systems which utilise the magnetic field

- Machines – motors, generators, actuators, loudspeakers, instruments
- Transformers – power, measuring, matching
- Communication systems – H-field antenna, ferrite rod
- Inductor – Filters, tuning etc
- Electron beam devices
- μwave devices – microwave ovens etc.

Examples of systems where the effects of the magnetic field are minimised

- Screening to eliminate pickup or crosstalk
- Special windings (bifilar) to minimise inductance
- Careful PCB layout to minimise 'stray' inductance
- Careful design and 'clamping' of wires to reduce effects of unwanted forces associated with current flow – high current levels.

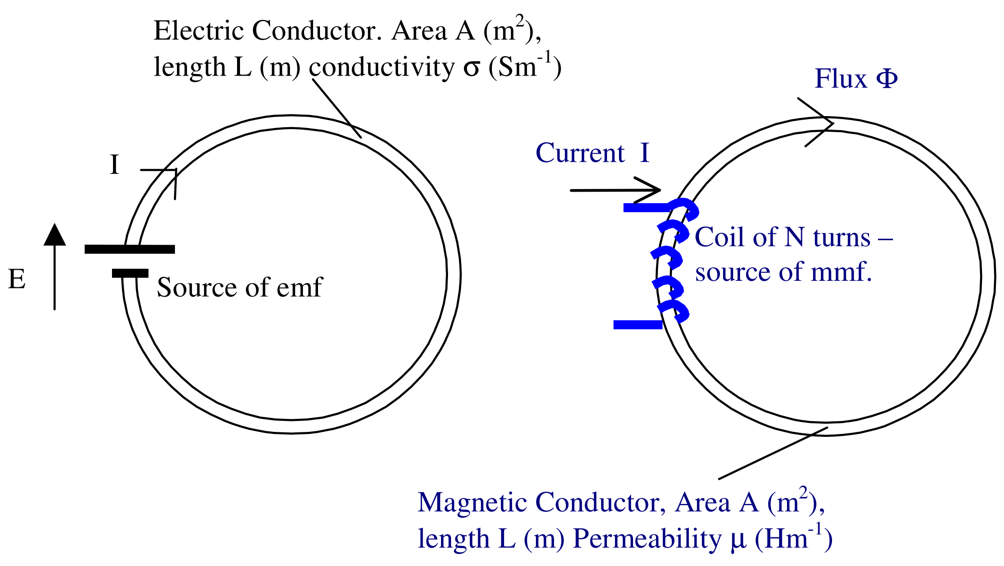
Sources of magnetic fields

All sources may be related to an equivalent current flow or charge movement

- Currents in circuits – magnetic field surrounds the conductors
- Permanent Magnets – Fields associated with electron motion in atomic structures
- High frequency 'displacement' currents in radiated fields (antennas)

Magnetic Circuits

In many practical devices the magnetic field is confined to a well-defined circuit that can be thought of as analogous to the more familiar electric circuit. **This analogy must be used with care!**



<u>Electric Circuit</u>

For a circuit made up of a number of elements we have:

$$I = \frac{E}{\Sigma R} = \frac{\text{emf}}{\text{cct resistance}} \quad \frac{(V)}{(\Omega)} \quad A$$

where for each element: $R = \dfrac{L}{\sigma A} \quad \Omega$

Magnetic Circuit

By analogy, the magnetic circuit quantities are related by:

$$\Phi = \frac{F}{\Sigma S} = \frac{\text{mmf}}{\text{cct reluctance}} \quad \text{Wb}$$

where Φ = flux in Webers (Wb)

$$S = \frac{L}{\mu A} \quad H^{-1} \quad (\text{Henry}^{-1})$$

F = magneto-motive force (mmf) = NI Amp (turns)

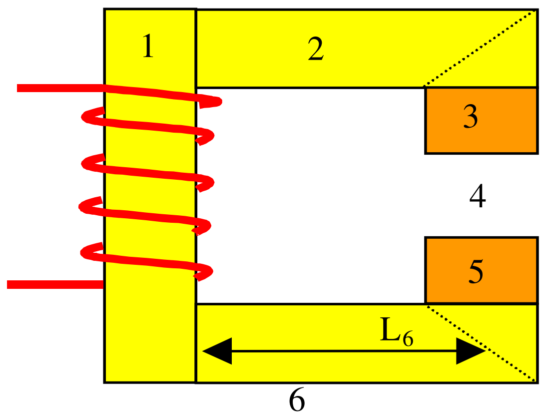
μ = permeability (Am^{-1})

Provided the magnetic field is confined to a path of known area A, length L, and permeability μ, the above method is very useful. – Note that whilst the current, I, can be associated with the 'flow of charge', the concept of flux Φ flowing around a circuit is only a useful concept. In practice, there is no physical flow involved. We can however say that the flux Φ carries the effect (the field) of the mmf. Because air is not a 'magnetic insulator', unlike electric circuits where the current can reasonably be assumed to be confined to the wires, in the magnetic circuits of some devices, significant amounts of flux 'leaks' or spreads into unwanted sections of the device.

Calculation of a 'lumped' circuit reluctance model for linear materials

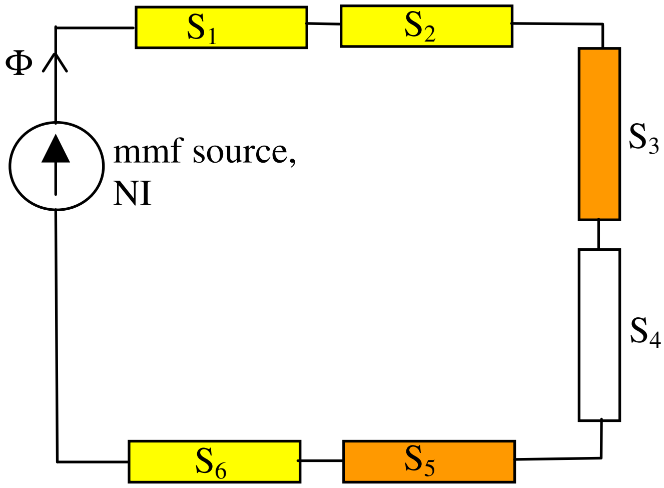
For circuits where each section can be defined in terms of sections of known length, L, area, A, and permeability, μ, then a 'lumped' reluctance model can be constructed.

Example 1 Series magnetic circuit:



Each section having a length, L_n, area, A_n, and permeability, μ_n.

For this circuit we introduce the concept of an 'effective' or 'mean' path length, L_1, L_2, etc, and areas A_1, A_2, etc. Clearly for section 1, there are longer and shorter paths through the section, and the area is not constant on the corners. However, the errors involved are very small in practice for most situations. Hence we can construct our model.



Where $S_1 = \dfrac{L_1}{\mu_1 A_1}$ etc

and flux

$$\Phi = \frac{NI}{S_1 + S_2 + S_3 + S_4 + S_5 + S_6}$$

$$= \frac{NI}{S_T}$$

or $$\Phi = \frac{NI}{\Sigma S}$$

- note that in the series circuit, the flux is continuous, and the same flux passes through all sections.

Example 2 Parallel magnetic circuit:

By the same analogy to electric circuits, we may analyse parallel magnetic circuits.

$$\frac{1}{S_T} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} \cdots$$

Magnetisation Curves and Magnetic Materials

For a material of length L, area A, and permeability μ, from the previous equations we have:

Flux = mmf / reluctance

$$\Phi = \frac{NI\,\mu\,A}{L}$$

This can be arranged into a different form to give:

$$\left(\frac{\Phi}{A}\right) = \mu\left(\frac{NI}{L}\right)$$

or

$$B = \mu H$$

There is a unique relationship for each material known as it's B/H curve, or magnetisation characteristic, independent on circuit dimensions.

where: **$B = \Phi / A$** is the **Flux Density** in **Tesla (T)** (Wbm^{-2})

and **$H = NI / L$** is the **Magnetising Force** or
Magnetic field Strength (Am^{-1})

Note: H is the cause, B is the effect

Permeability of Materials

Air (free space) – the permeability is defined as

$$\mu = \mu_0 = 4\pi \times 10^{-7} \quad Hm^{-1}$$

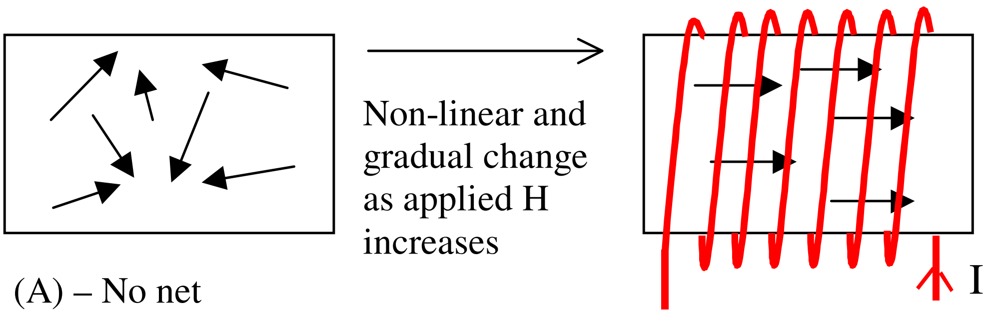
μ_o is the primary magnetic constant, and has been assigned the above value for use in the S I unit system.

'Non magnetic' Materials
On a molecular or atomic scale, materials have electron motion, which is equivalent to an atomic current. These atomic currents can be associated with both electron orbital motion about the nucleus, and the electrons spinning about their own axes. Both can give rise to magnetic fields and both can be affected by an external magnetic field. However, for most substances, these effects are either zero or so small that for most purposes such materials are classed as 'non magnetic' [Diamagnetic if $\mu < \mu_o$, Paramagnetic if $\mu > \mu_o$]

For these materials $B = \mu_o H$

Magnetic Materials - Ferromagnetism

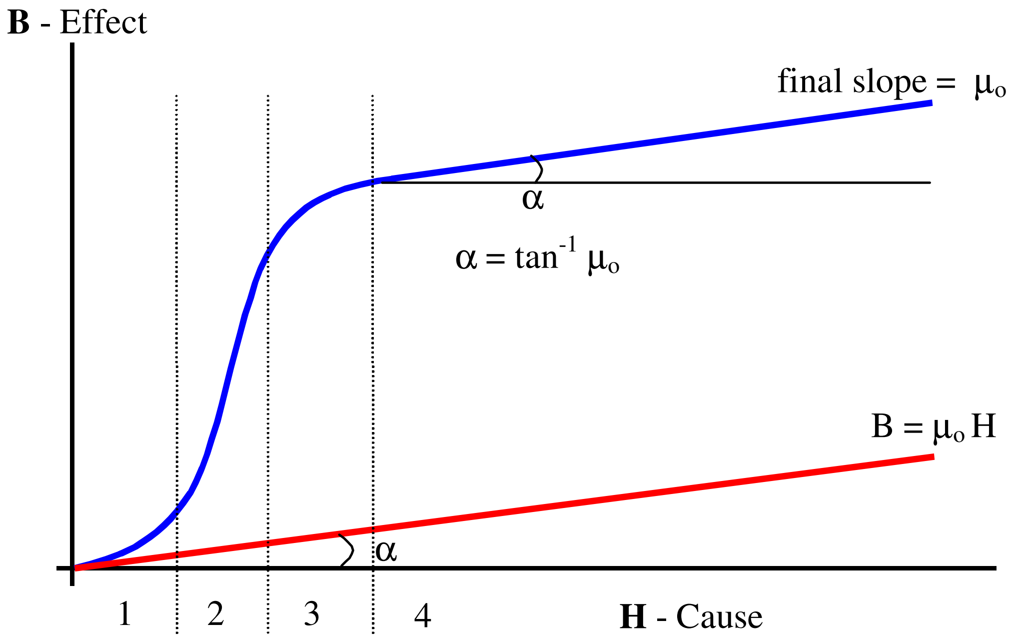
In a few elements, IRON, NICKEL, COBALT, and some of their alloys, the atoms are arranged such that their electron currents supplement each other and give a net magnetic moment. Within a specimen of such materials, the structure is usually divided up into **magnetic domains** of microscopic size within which the atomic magnets are aligned. However, the specimen as a whole may be unmagnetised due to the random or 'regular' orientation of the domains, but which can be magnetised (domains aligned) by an external applied field.



(A) – No net external effect

(B) – Domains aligned with applied field

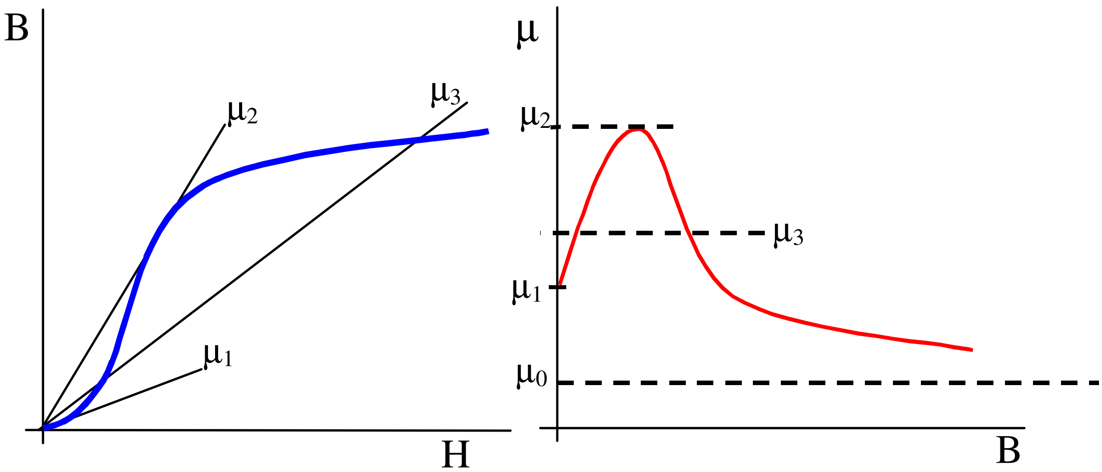
The process from A to B is non-linear and leads to a non-linear relationship known as the B/H curve. The effect of the domain alignment is that the B value obtained for a given H is much greater than that for air or a non-magnetic material.



Magnetisation curve or B/H curve

1. Initial growth of closely aligned domains
2. Rapid reversal of domains
3. final rotation of domains
4. No further contribution from domains - saturation

The alignment process is dependant on the strength of the applied magnetic field, H. Thus the B/H relationship is a curve rather than linear. <u>For these materials, μ varies with the applied field and hence the flux density in the circuit</u>.



By careful design a magnetic circuit can be operated with a high value of μ. (Other definitions of μ based on incremental slope, dB/dH, are also used for small signal models).

Saturation

A material is said to be saturated when complete alignment of the magnetic domains is achieved. This point coincides with the slope of the B/H curve having a value of μ_0.
NOTE: beyond this point B continues to increase but at a rate determined by:

$$B = \mu_0 H$$

as with non-magnetic materials.

Relative permeability

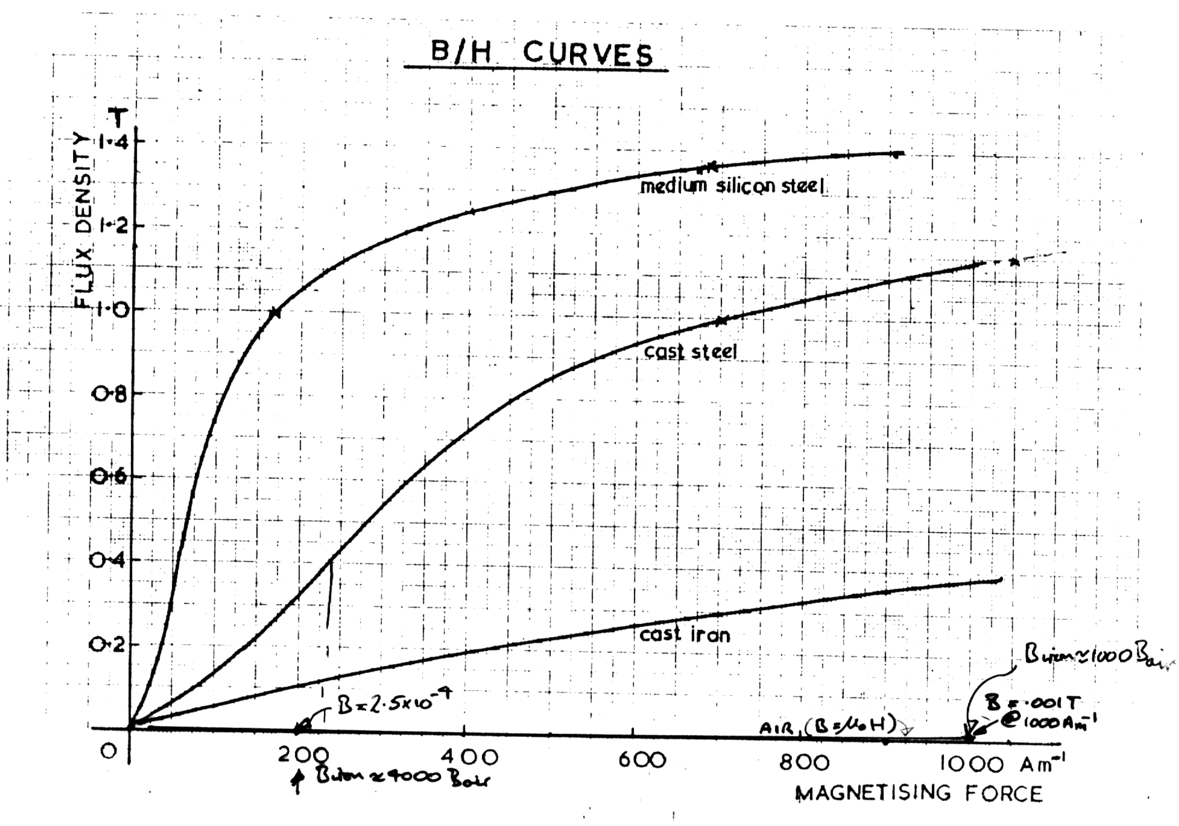
The general relationship $B = \mu.H$ is often re-written as:

$$B = \mu_r \mu_0 H$$

where:

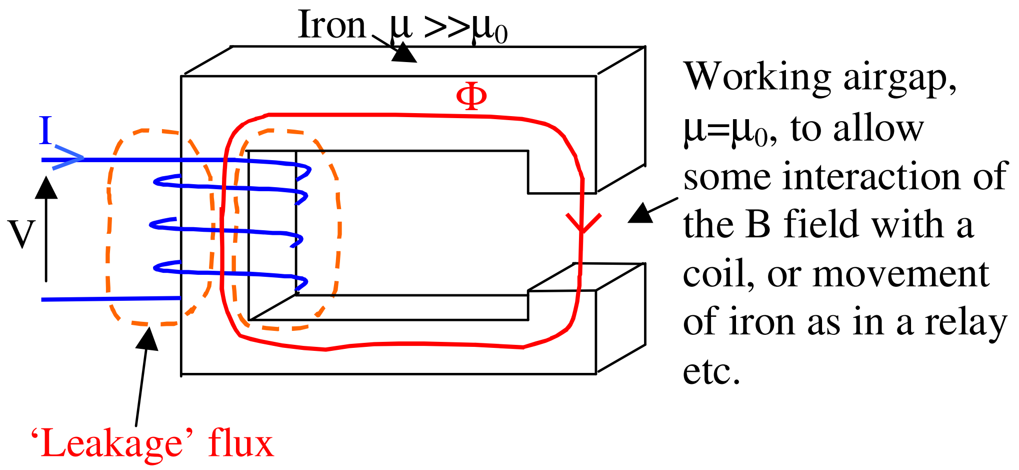
$$\mu_r = \mu / \mu_0 \text{ (dimensionless)}$$

and is effectively a measure of the 'magnification' of the B field which can be achieved if the magnetic material is used in a circuit. For typical materials, μ_r varies over the range 50 to 2000, although special materials can be produced with a μ_r of over 500,000! (suitable only for limited application).
[For curve given, at 1000A/m, for air B=0.001T !].



Circuit Analysis

This approach is particularly useful if the magnetic field is confined to paths of mainly simple geometry. The designer who wants to increase the magnetic flux for a given **mmf** is clearly going to use the multiplication available by incorporating iron or other ferromagnetic material in the circuit. In many cases the iron essentially forms the whole of the circuit, e.g. transformer and inductors, whereas in others, the iron acts to guide the flux into a 'working airgap' in the device. The latter is almost invariably the case in electromechanical devices (motors, actuators etc).



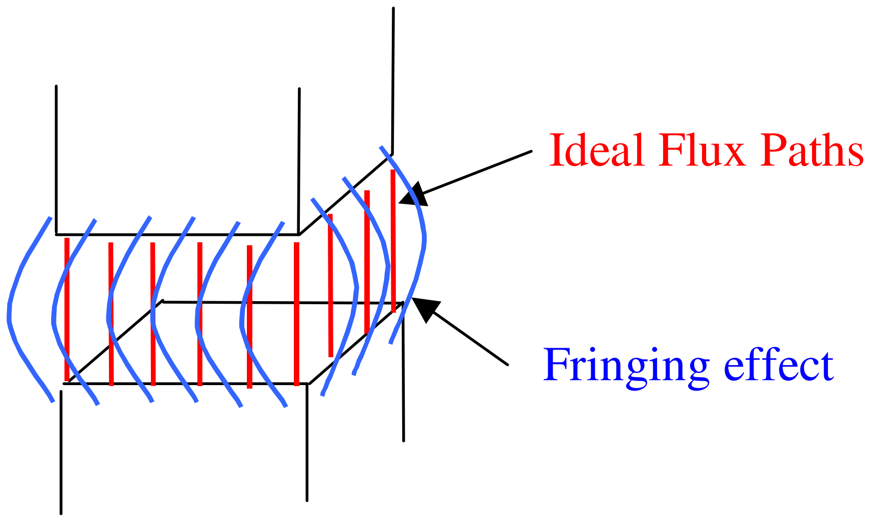
Fringing and Leakage fields

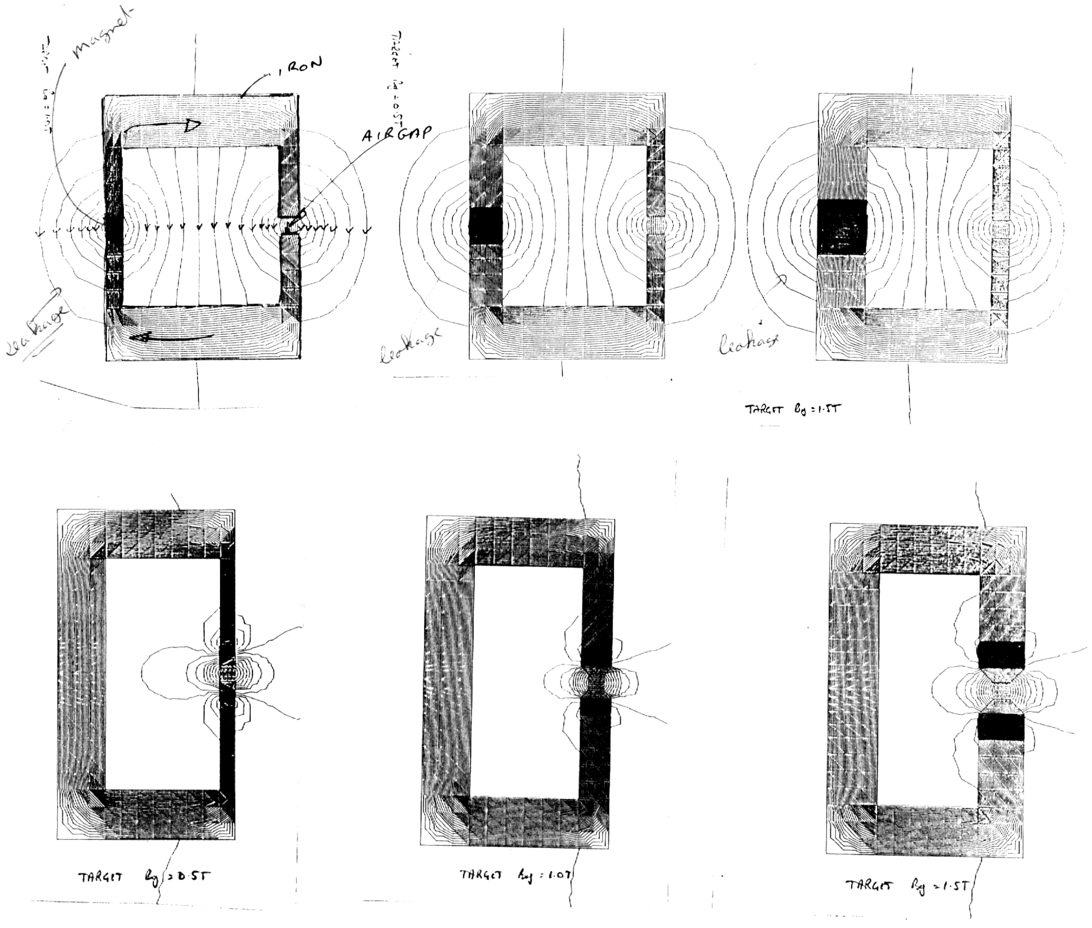
Whereas the electric current in an electric circuit can be reasonably confined to the conducting wires, the analogous effect of flux being confined to the iron is not such a reasonable assumption. This is due to the fact that:

$$\frac{\sigma_{air}}{\sigma_{copper}} << \frac{\mu_{air}}{\mu_{iron}}$$

i.e. air is not a good 'magnetic insulator'

Hence, in the circuit above, the flux would not only be produced in the working airgap, but also in the other paths shown. The flux will also tend to spread out in the working airgap – known as 'fringing effects'.




magnet
IRON
AIRGAP
leakage
leakage
leakage
TARGET Bg = 1.5T
TARGET Bg = 0.5T
TARGET Bg = 1.0T
TARGET Bg = 1.5T

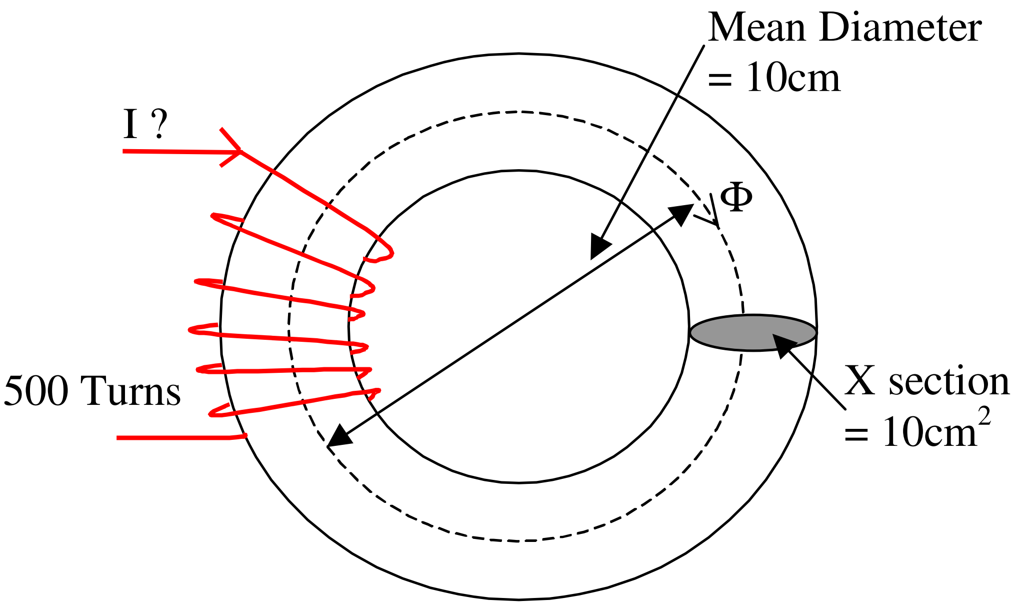
Solution of Typical Problems:

(In some cases the examples given are non-practical but demonstrate the principles)
Generally, this method of solving problems gives a useful 'first order' calculation

Example1

A mild steel ring has a cross-sectional area of $10cm^2$ and a mean diameter of 10cm. A coil of 500 turns is wound on the ring. Calculate the current required to give a total flux of 1.25mWb in the ring.

a) If the ring is of a specified relative permeability of 1560.
b) (More realistically), if a B/H curve of the material is provided.



a) μ_r specified as 1560

Φ = 1.25mWb = 1.25×10^{-3} Wb,
Area A = $10cm^2 = 10 \times 10^{-4}\ m^2$ GET UNITS INTO S.I. UNITS

Now, $B = \Phi / A = 1.25\times10^{-3} / 10^{-3} = 1.25$ T

In this problem the cross-section is uniform, and since Φ is continuous, then B is the same throughout. Also as the circuit only contains 1 material, then the relative permeability is the same throughout, therefore we can say:

$$B = \mu_r.\mu_0.H$$

where $H = 1.25 / (4\pi\times10^{-7} \text{ x } 1560) = 6.4\times10^{2}\ Am^{-1}$

and total *mmf* required:

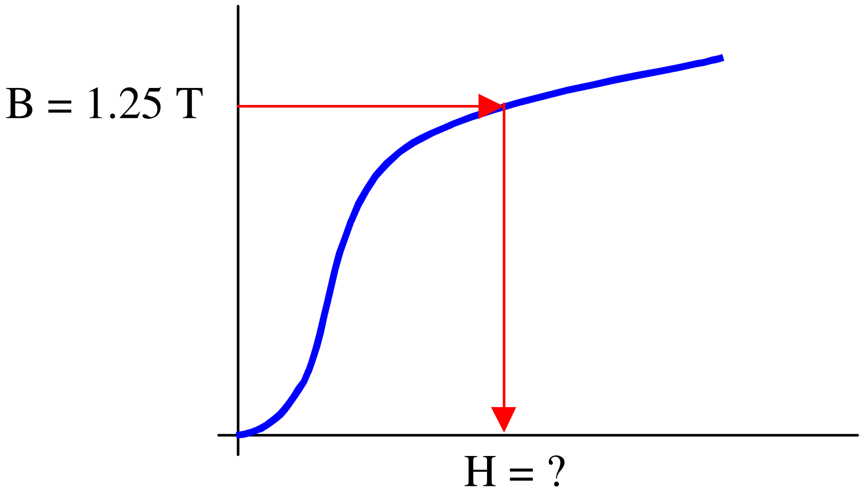
$$F = H \times L = H \times \pi D = H \times \pi \times 10\times10^{-2} = 201 \text{ A (turns)}$$

As $F = NI = 201$

$I = 201 / 500 = 0.4$ A

b) If B/H curve of material is provided

Calculate B as before, now use B/H curve to read H directly from the graph at this value of B.

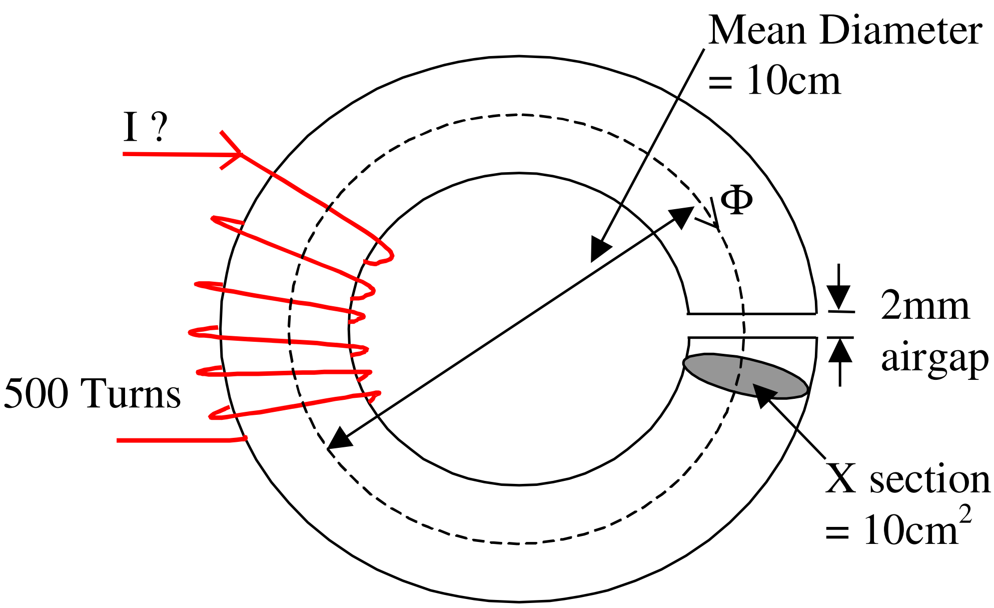


Now, given the value of H from the graph, calculate $F = H \times L$ as before.

Note: do not use the curve to give μ, as it varies over the range of B, and you don't know it accurately. Also, it takes too long!

Example 2

Same as above, but with an airgap of 2mm cut through the ring.



Now the circuit consists of 2 parts (iron and air), but the flux (Φ) is still continuous, and the cross-section (neglecting fringing) is still the same.

Can use either of two methods:

(i) $F = \Sigma H \times L$ where $H_{iron} \neq H_{air}$ and $L_{iron} \neq L_{air}$

(ii) $\Phi = NI / \Sigma S$ where $S = S_{iron} + S_{air}$

Using (i), then as before: $B_{iron} = B_{air} = 1.25$ T

$H_{air} = B_{air} / \mu_0 \quad = 9.947\times10^5$ Am^{-1}

$H_{iron} = B_{iron} / \mu_0\mu_r \quad = 6.376\times10^2$ Am^{-1} (as before)

(note $H_{air} >> H_{iron}$)

Now $F = H_{iron} L_{iron} + H_{air} L_{air}$

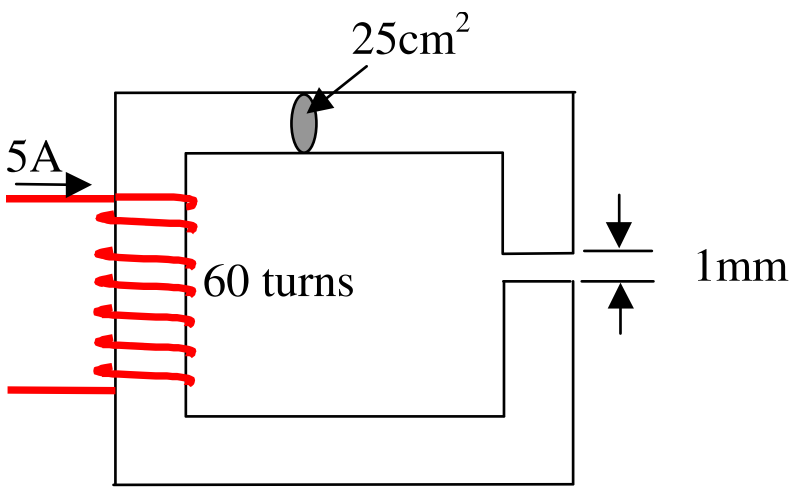
$\therefore F = 199 \quad + \quad 1989$

$\therefore F_T = 2188 = NI$

$\therefore I = 2201 / 500 = 4.38A$

Note how 2mm airgap dominates this solution and if H_{iron} had been ignored then the current, I = 4A, which would only be 10% out. i.e. In first order calculations, it is often possible to ignore the iron and assume all the *mmf* appears across the airgap.

<u>Example 3</u>



The diagram shows a magnetic circuit of constant cross-sectional area (25cm^2), with a mean path length of 15cm in the iron, and a 1mm airgap. The 60 turn coil carries 5A, calculate the flux produced:

a) if μ_r is known to be 1000

b) if a B/H curve is provided

Since B is unknown at this stage, and the subdivision of the total *mmf* between the iron and the air is also unknown, then use:

$$\Phi = \frac{F_{Total}}{S_{Total}} \quad = \frac{60 \times 5}{(S_{iron} + S_{air})}$$

As this is a series circuit, and we have the individual path lengths, L:

$$S_{Total} = \frac{L_{iron}}{\mu_r \mu_0 A_{iron}} + \frac{L_{air}}{\mu_r \mu_0 A_{air}}$$

$$\therefore S_{Total} = \frac{15 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 25 \times 10^{-4}} + \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 25 \times 10^{-4}} \quad H^{-1}$$

$$\therefore S_{Total} = 0.48 \times 10^{5} + 3.18 \times 10^{5} \quad H^{-1}$$

$$\therefore S_{Total} = 3.66 \times 10^{5} \quad H^{-1}$$

$$\Phi = \frac{60 \times 5}{3.66 \times 10^{5}} \quad = 82\text{mWb} \quad (\text{or } B = 0.33\,T)$$

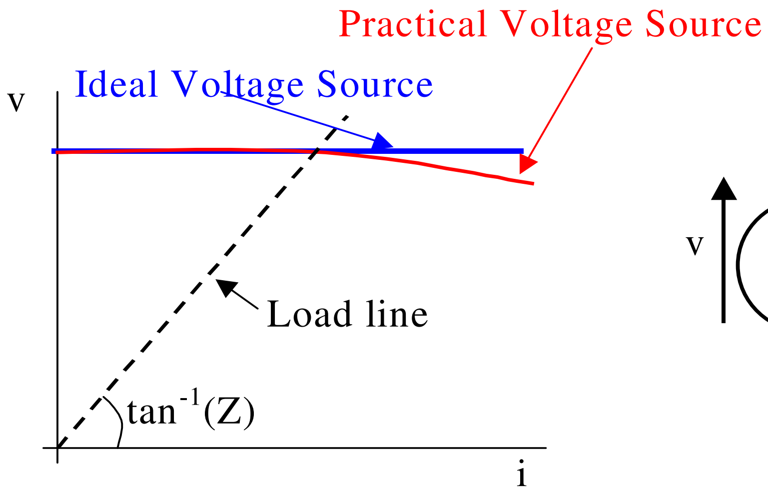
Solution if B/H curve is provided:

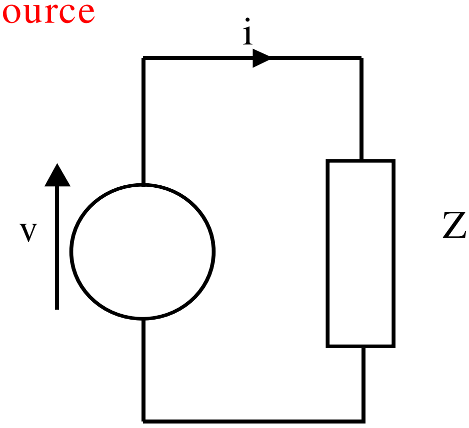
In this case there is no analytical solution and only a graphical or numerical (iterative) solution is possible. – not included in the course!

The Effects of Time Variation of Flux / Flux Density, Voltage and Current Sources

The previous analysis shows the direct links between current, I (*mmf* = NI) and Flux Φ. However he majority of electrical supplies are specified in terms of a voltage source rather than a current source. – How do we link the magnetics to the electrics in such a case ?

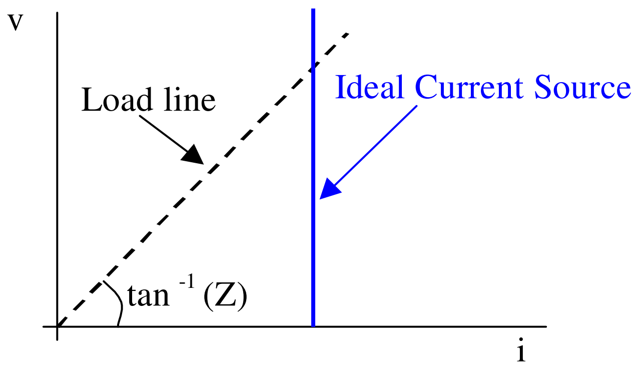
Voltage Source:

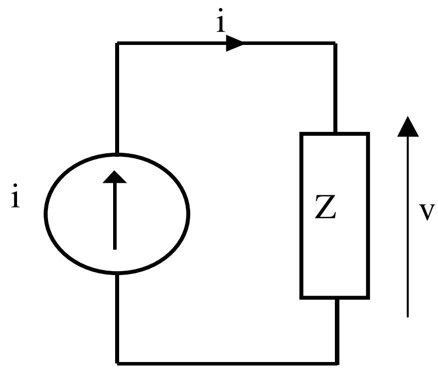




Current determined by load impedance: $i = v/Z$

Current Source:





Votage determined by load impedance: $v = i.Z$

1. Steady-State dc excitation

With no time variation the only impedance is resistive i.e.

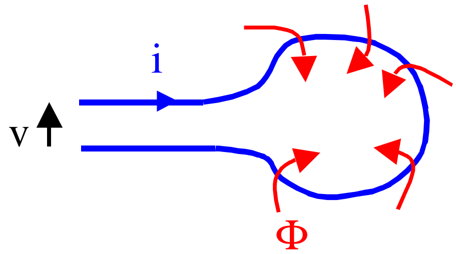
$$I = V / R$$

Hence the ink between the supply voltage, V, and the current, I, is directly via the circuit resistance (Ohm's Law) and:

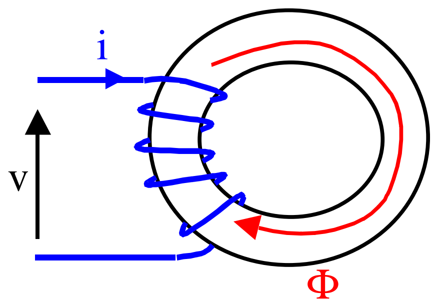
$$\Phi = N.I / S$$

as before.

2. Time Varying Excitation



Single turn circuit



Multi-turn circuit

In either case, in general we must write:

$$v = i.R + \frac{d\varphi}{dt}$$

where v, i, and φ are all instantaneous and R is the electric circuit resistance.

We can re-write this as:

$$v = i.R + e$$

where:

$$e = \frac{d\varphi}{dt}$$

is the induced *emf* due to the changes in the flux linkage with the current or coil

and φ is called the FLUX LINKAGE = $N\Phi$

i.e N is the number of turns linked by the flux Φ

Self Inductance

From previous notes we can also write:

$$\Phi = \frac{Ni}{S} \quad \text{Wb}$$

and assuming all of the N turns are linked by the flux, Φ, we say that the coil has flux linkages:

$$\varphi = N\Phi = \frac{N^2 i}{S} \quad \text{Wb (turns)}$$

Hence if φ is time varying, then by Faradays Law:

$$e = \frac{d\varphi}{dt} = \frac{d}{dt}\left(\frac{N^2 i}{S}\right) \quad \text{Volts}$$

if the current, i, is the only time-varying quantity then:

$$e = \left(\frac{N^2}{S}\right)\frac{di}{dt} \qquad \text{or} \qquad e = L\frac{di}{dt}$$

in electrical circuit models. i.e. the SELF INDUCTANCE, L, models the effect of the magnetic field surrounding an electric circuit, and in general:

$$v = i.R + L\frac{di}{dt}$$

where the inductance L can be calculated from the magnetic circuit properties, given that S is the total magnetic reluctance surrounding the N turn coil:

$$L = \frac{N^2}{S}$$

N.B. The inductance, L, is determined by the dimensions and properties of the magnetic circuit and is not a function of the current (unless the magnetic circuit saturates). i.e. circuit inductance L can be minimised by careful design if necessary.

Note that since:

$$\varphi = Li = \frac{N^2 i}{S}$$

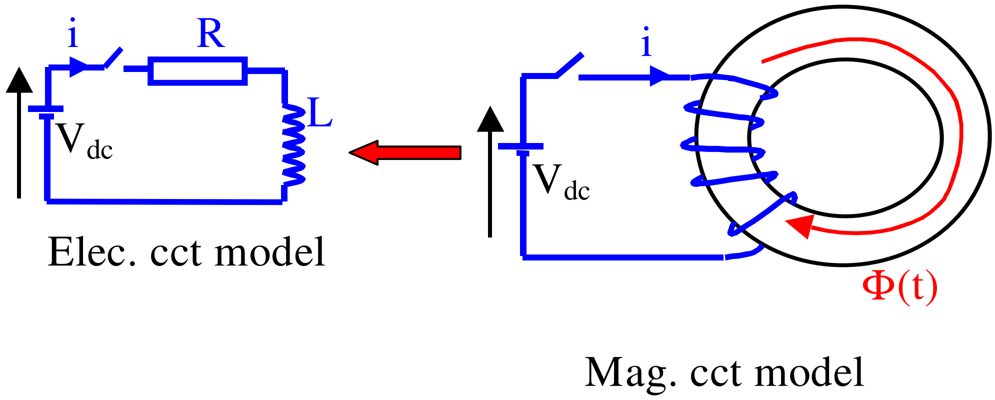
This leads to another definition:

$$L = \frac{\varphi}{i}$$

Or, the inductance may be expressed as the flux linkage per amp of current in the circuit.

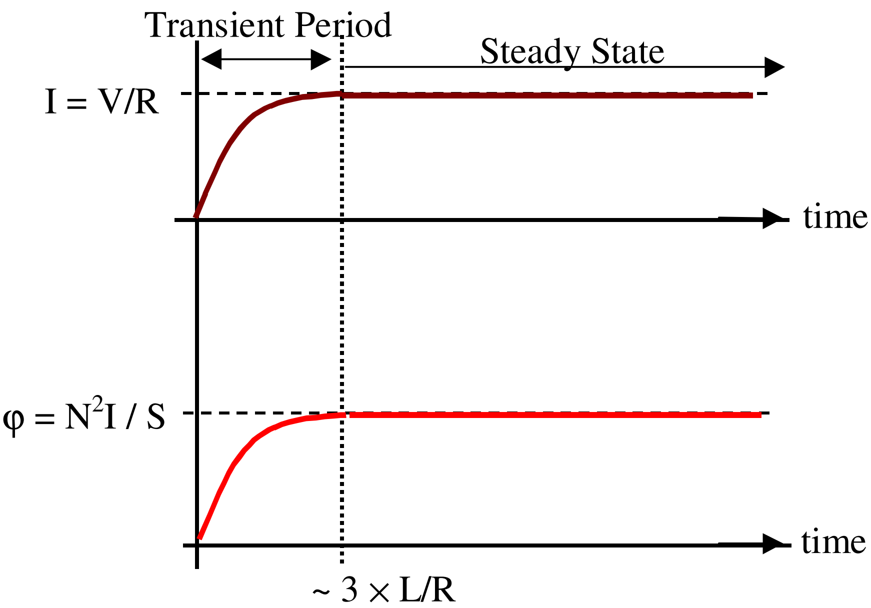
3. Inductive Electric Circuit Models with Time Variation

(i) Switched dc excitation



For this case in general we write:-

$$V_{dc} = i.R + \frac{d\varphi}{dt} \quad \textbf{OR} \quad V_{dc} = i.R + L\frac{di}{dt}$$



Stage (i) – There is a transient period during which the current rises and $di/dt \neq 0$ and:

$$\varphi = N\Phi = \frac{N^2 i}{S}$$

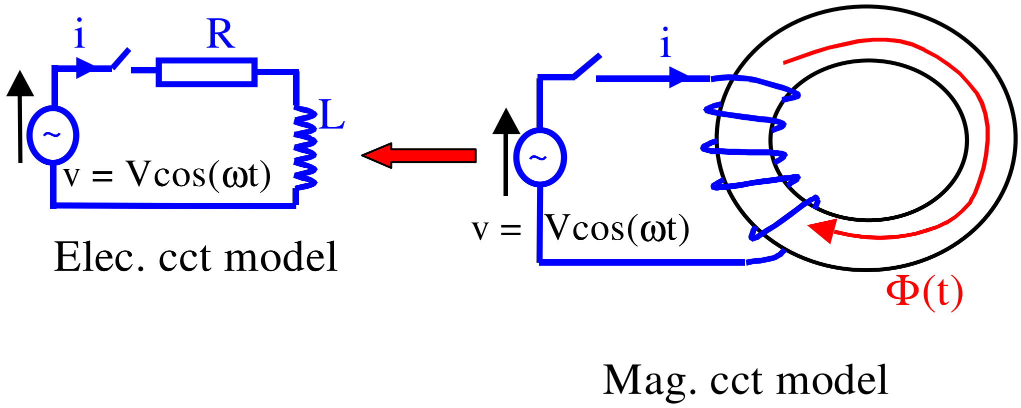
also rises.

Stage (ii) – In the steady-state, $di/dt = 0$:

$$I_{ss} = \frac{V_{dc}}{R} \text{ and } \Phi_{ss} = \frac{Ni}{S}, \quad \varphi_{ss} = \frac{N^2 i}{S} \text{ etc.}$$

Stage (i) lasts for about 3×L/R where (L/R) is called the TIME CONSTANT. After this time, the current, I, is determined entirely by the coil resistance.

(ii) Sinusoidal ac excitation



in general:

$$V_{dc} = i.R + \frac{d\varphi}{dt} \quad \textbf{OR} \quad V_{dc} = i.R + L\frac{di}{dt}$$

At initial switch on there will be a **transient** period as before, but the exact shape will depend on the point in the ac cycle at which the switch was closed.
Steady-state – Even here, v and i are not time independent. If v and i are sinusoidally time varying then we can also write the general expression in the phasor form:-

$$V = R.I + j\omega L.I \quad \textbf{OR} \quad V = (R + jX).I$$

where V and I are either peak or RMS values for the voltage and the current.
We can also write the impedance in polar form:

$$(R + jX) = Z\angle\phi$$

where we write:

$$|Z| = \sqrt{R^2 + X^2}$$

and:

$$\phi = \tan^{-1}\left(\frac{X}{R}\right)$$

In this case the link between the magnitude of the current flowing in the electric circuit and the ac voltage supply is:

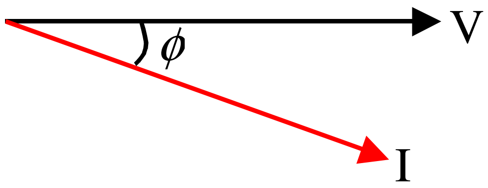
$$|I| = \frac{|V|}{|Z|}$$

where I and V are **either** peak or RMS values, where the effect of the sinusoidal time varying magnetic field surrounding the circuit is reflected in the impedance term Z (for dc, Z=R in steady-state).

If the above current of I amps flowing in a circuit of N turns, then the for a peak voltage V applied:

$$I = \frac{V}{Z}, \qquad \Phi = \frac{NI}{S}, \qquad B = \frac{\Phi}{A} \qquad \text{etc}$$

note also that the effect of the surrounding magnetic field caused the current and voltage to be out of phase by ϕ where:-



$$\phi = \tan^{-1}\left(\frac{X}{R}\right)$$

i.e. I lags V by ϕ

Special case of R<<ωL or R<<X

This is often true of devices in which the magnetic circuit is an essential part of the device (e.g. transformers, inductors, machines etc.) For many such devices we can simplify the analysis by assuming R<<ωL, giving only an induced *emf* term

$$v = \frac{d\varphi}{dt} = \frac{d(N\Phi)}{dt} \qquad \text{Faraday's Law}$$

and if v is sinusoidal, then the flux will vary sinusoidally also. i.e if Φ is of the form:

$$\Phi \sin(\omega t) \qquad \text{then} \qquad v = \frac{d(N\Phi \sin(\omega t))}{dt}$$

therefore:

$$v = \omega N \Phi \cos(\omega t)$$

i.e. π/2 out of phase with Φ which we can write as:

$$v = V\cos(\omega t)$$

where:

$$V = \omega N \Phi$$

and:

$$V_{rms} = \frac{\omega N \Phi}{\sqrt{2}} = \frac{2\pi f N \Phi}{\sqrt{2}} = 4.44f \;\; N\Phi$$

i.e. in such devices, where R is negligible (<<ωL) it is possible to estimate the peak flux, Φ, and hence the peak flux density, B=Φ/Area, directly from the applied voltage and we do not need to go through the procedure of calculating the impedance and the current (Z and I) and hence the flux Φ etc.

Example 1

A 1000 turn coil is wound on a magnetic circuit of cross-sectional area $25cm^2$. The coil has a resistance of 10Ω and the magnetic circuit has a reluctance of $5\times10^5\ H^{-1}$. Calculate the steady-state peak coil current and the corresponding peak flux density in the magnetic circuit.

(a) if the coil is supplied from a 100V dc source

(b) if the coil is supplied from a 100V(rms) sinusoidal ac supply at 50Hz.

(a) On dc:

the steady-state current is given by
$I = V/R = 100/10 = 10A$

hence the peak flux $\Phi = NI/S = 10\times1000 / 5\times10^5 = 20mWb$.

and $B_{max} = \Phi/A = 20\times10^{-3}/25\times10^{-4} = 8T$!!!

note: This is not a practical possibility since most magnetic materials saturate at B<2T. In practice, the magnetic circuit would saturate leading to a reduction in μ and an increase in the reluctance such that B would be limited to somewhere in the region of 1-2 T.

(b) On ac:

we must now include the effect of time varying flux linkage produced by the coil and the corresponding induced *emf.* i.e. now the steady-state current is obtained from:

$$V = (R + jX).I \quad \text{or} \quad |I| = \frac{|V|}{|Z|}$$

where:

$$|Z| = \sqrt{R^2 + X^2}$$

But first we need to calculate the inductance, L, from the magnetic circuit. From $L=N^2/S$, i.e.

$$L = 1000^2 / 5\times10^5 \qquad = 2H$$

And hence, $\omega L = 2\pi fL = 100\pi\times2 = 628.3\Omega$

Therefore:

$$|I| = \frac{V}{Z} = \frac{100}{\sqrt{10^2 + 628.3^2}}$$

$$= 0.16A$$ - rms since V= 100V (rms)

from this we get that the peak current: $I = \sqrt{2}.I_{rms} = 0.23A$

and hence the peak flux: $\Phi = NI/S$,

and peak flux density: $B = \Phi/\text{Area} = 0.18\text{T}$

Note that in this ac circuit model, R<<ωL. If we had approached this problem from this point of view, we could have used the relationship (see earlier)

$$V_{rms} = \frac{2\pi f N\Phi}{\sqrt{2}} \quad \text{or} \quad V_{rms} = 4.44 f N\Phi$$

which becomes:

$$V_{rms} = 4.44 \times f \times N \times B \times \text{Area}$$

this therefore gives the flux density $B = 0.18\text{T}$ directly.

The Effect of Frequency on the Size of Magnetic Devices.

From the expression:

$$V = 4.44 \times f \times N \times B \times \text{Area}$$

and recalling that for magnetic materials the flux density, B, is limited by the material properties ($B_{max} \leq 1$–2T for iron, or $B_{max} \leq 0.3$T for ferrite ‘pot’ cores), then it can be seen that to sustain a certain voltage, V, across the winding of a device we have a choice of balancing it from the product

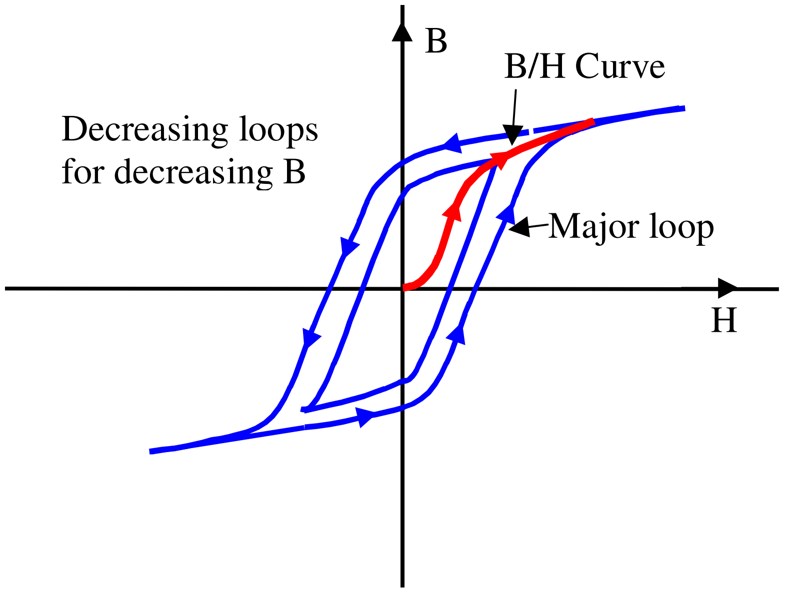
$$f \times N \times A \qquad (B_{max} \text{ limited by material})$$

Increasing N requires more turns on the coil – Increasing the overall device size
Increasing A requires a larger cross-section of the magnetic core – Increasing the overall device size
Increasing f can be used to reduce both N and A and therefore **reduce** the overall device size

This is common practice in a wide range of devices – transformers, motors, generators etc. In aircraft, a frequency of 400Hz has been used for many years in recognition of this fact. Increasing the frequency also reduces the size of smoothing inductors required in electronic circuits.

Losses in Magnetic Circuits

From a size point of view (see earlier notes about the advantages of using a higher frequency on the dimensions of magnetic devices),an increase in frequency sems very beneficial. However, there are losses in the magnetic material, which can put a limit on the frequency that can be used. These losses are usually lumped together and called 'Iron losses'. They consist of two basic loss mechanisms:



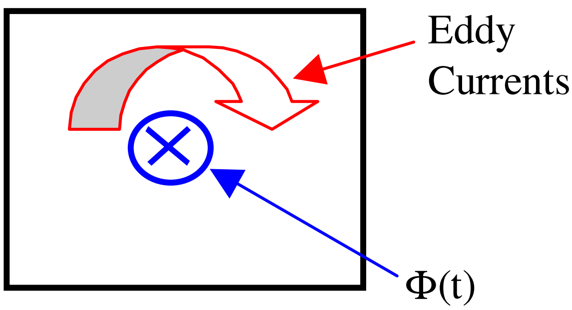
i) Hysteresis loss

All magnetic materials have a B/H loop, which characterises the relationship between the flux density in the material, B, and the *mmf*, H. The B/H curve given earlier in the notes is only relevant at steady-state (dc). On ac the material is subjected to an alternating B and H and the material cycles around the loop. Up to a limiting value of flux density, B, the loop area increases with increasing B. the loss is proportional to the loop area per cycle. Hence loss, $P_h \propto$ loop area and frequency (no of times the loop is traversed per second).

$$P_h \propto B \text{ and } f$$

ii) Eddy Current loss

If an alternating field passes through a material a voltage is induced around a given path (any material).



$$e = \frac{d\varphi}{dt} = \frac{d(NBA)}{dt}$$

i.e $e \propto B, \; A \text{ and } f$

around the loop this causes eddy currents to flow,

$i = v/R$ (R = resistance of material in loop)

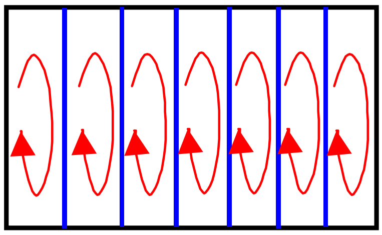
$$P_e = \oint i^2 R = \oint \frac{e^2}{R}$$

hence:

$$P_e \propto B^2, f^2, A^2, R$$

i.e. The eddy current power loss, P_e, is proportional to the square of the frequency.

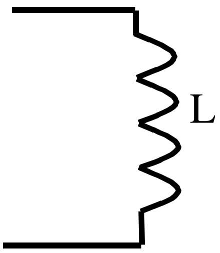
The eddy current loss my be reduced with the use of Laminations (0.5mm at 50Hz, 0.1mm at 1kHz). This reduces the area of the material per loop. Laminations of less than 0.1mm thick are difficult to produce and handle etc. Beyond 1kHz we use ferrite 'pot' cores, but now the maximum available flux density (B_{max}) is now 0.3T as compared to 1.2T for silicon steel.



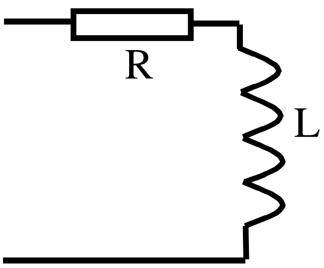
Modelling of Losses in Equivalent Circuit of Devices

To simplify the calculation of device behaviour we often use equivalent circuit models where the model application is chosen to suit the model application and accuracy required. E.g. inductor models:

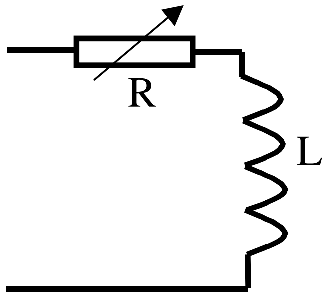
Note. R_E is a fictitious resistor, it simply gives a measure of the iron loss, = E^2/R, which gives a reasonable estimate of the watts lost due to eddy and hysteresis loss in the device.



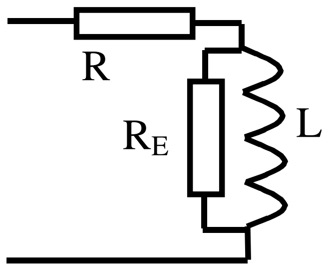
Ideal Inductor,



Model allowing for finite coil resistance (copper loss can now be calculated)



Model allowing for changes in R with frequency (Skin effect)



Model including loss in the iron core (Iron loss)

Energy and Power in Electrical Networks

Instantaneous values of time varying quantities

Note: Lower case letters are used to signify instantaneous values:

p – power (Watts, W) – (Joules per second Js^{-1})
e – Energy (Joules, J) – (Watt seconds Ws)
v – Voltage (Volts, V)
i – Current (Amps, A)

where power:

$$p = \frac{de}{dt} \quad \text{Watts (W)}$$

and energy:

$$e = \int p\,dt \quad \text{Joules (J)}$$

(note: Energy is synonymous with work, since work done equals energy expended or absorbed).

The Joule is useful unit for low power circuits, but for power systems we have larger units:

Small circuits:	= 1 Joule	= 1 Watt Second
1 'unit' in electricity tariffs	= 1 kiloWatthour	= 1kWh (1kW for 1 hour)

1 kWh = 3.6MJ (3.6×10^6J)

We also use MWh in the mains power distribution system

Aside:

Electrical energy stored in a 4700µF 25V capacitor = $0.5CV^2$ = 1.47Joules

Chemical energy stored in 1 Mars Bar! = 1.3MJ (~0.3kWh)

- Difficult to store significant amounts of energy in electrical form, therefore we use chemical storage of energy (batteries, supercapacitors etc)

NOTE: human body requires ~2kWh of energy input per day to sustain it ~ 6 Mars bars per day!)

Instantaneous Electrical Power

By definition, the potential or voltage difference between two points is the work done (energy change) per unit of charge transfer between two points, i.e.:

$$v = \frac{de}{dq} \text{ volts}$$

Also by definition, the current flow between two points is the rate of charge transfer between the two points, or:

$$i = \frac{dq}{dt} \text{ amps}$$

Hence power:

$$p = \frac{de}{dt} = \frac{de}{dq} \times \frac{dq}{dt} = vi$$

i.e. Instantaneous power is the product of the instantaneous voltage and the instantaneous current.

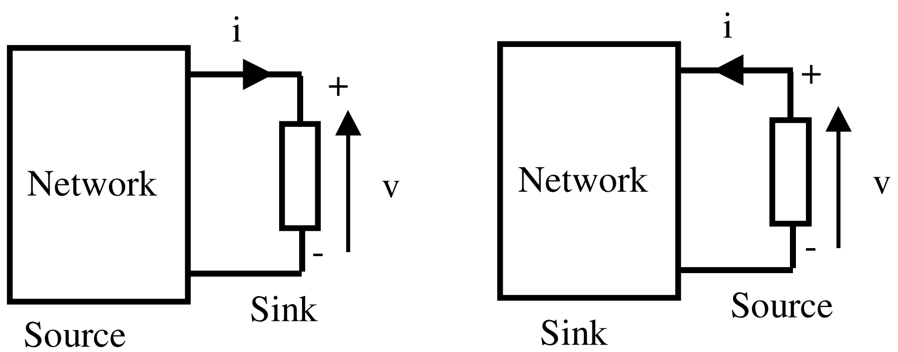
NB – THIS IS NOT, IN GENERAL, THE AVERAGE POWER – which is a more significant quantity (see later!)

And energy:

$$e = \int vi\,dt$$

Direction of power flow

The instantaneous power absorbed or produced by a circuit is determined by the direction of the current flow through, and voltage across, the element under consideration.



Power and Energy in Network Elements

a) **Resistor, R**

- this models any device in which **energy is converted irreversibly into heat**
- energy is not stored in a resistor.

Note: v and i are lower case, instantaneous values

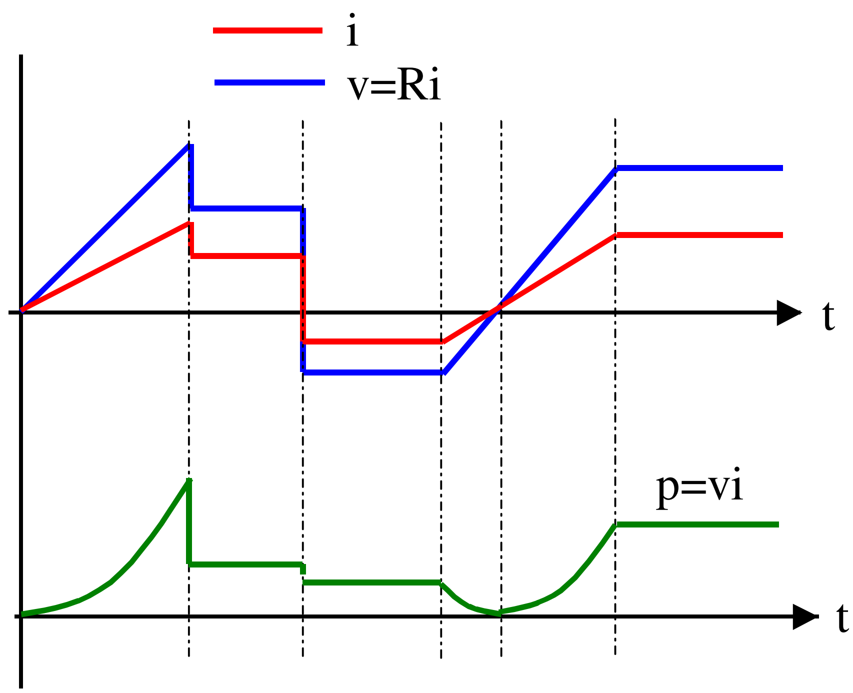
For a linear resistor:

$$v = Ri \quad \text{volts}$$

And instantaneous power:

$$p = vi = Ri^2 = \frac{v^2}{R} \quad \text{Watts}$$

Note: Even if v and i are negative, power, p, remains positive



In general, the electrical energy converted into heat in a time period $t = t_0$ to $t = t_1$ is given by:

$$e = \int_{t_0}^{t_1} vi\,dt \quad \text{Joules (Area under the p/t curve)}$$

Example

Calculate the instantaneous power drawn by a 6Ω resistor at time t = 4seconds, and the energy dissipated in the period 0 to 4 seconds, if the resistor is

(i) Connected to a 12V dc source

(ii) Connected to a $v(t) = 12t$ Volts source (t in seconds)

i) if $v = V_{dc} = 12V$ (time invariant)

then $i = I_{dc} = 12/6 = 2A$ (also time invariant)

Therefore power:

$$p = vi = V_{dc} I_{dc} = 24W$$

and:

$$e = \int_0^4 24dt = 24\,t\Big|_0^4 = 96\ J$$

ii) If:

$$v = 12t \ \text{Volts}$$

then:

$$i = v/R = 12t/6 = 2t \ \text{Amps}$$

and:

$$p = vi = 24t^2 \ \text{Watts}$$

The instantaneous power then becomes:

$$@\ t = 4sec, \quad p = 24 \times 4^2 = 384 \ \text{Watts}$$

Energy dissipated in 4 seconds

$$e = \int_0^4 24t^2 dt = \frac{24\,t^3}{3}\Big|_0^4 = 512\ J$$

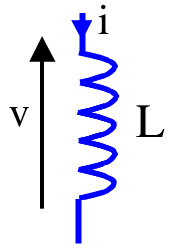
This becomes the area under the power / time curve.

Note the importance of expressing power, p, as a function of time in the integral (i.e. do not use $e = \int 384dt$ etc – common student error!

b) **<u>Inductor, L</u>**

An inductor models the effect of the magnetic field surrounding the electric circuit. For an inductor:

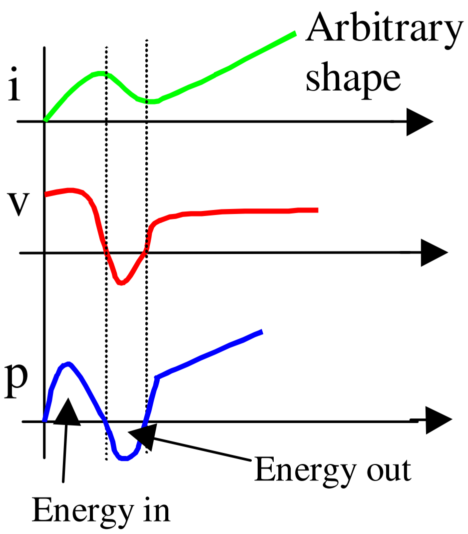
$$v = L\frac{di}{dt}$$



and instantaneous power flow:

$$p = vi = \left(L\frac{di}{dt}\right)i$$

NOTE: power flow, p, can be either positive or negative.



The energy change in the time interval, $t = t_0$ to t_1 during which the current changes from $i = i_0$ to i_1

$$\Delta e = \int_{t_0}^{t_1} Li\frac{di}{dt}dt = \int_{i_0}^{i_1} Lidi = \frac{1}{2}Li^2\Big|_{i_0}^{i_1}$$

or

$$\Delta e = \frac{1}{2}Li_1^2 - \frac{1}{2}Li_0^2$$

NOTE: if i = 0 at t = 0, then at any time, t > 0, when the current is flowing, the energy is:

$$e = \frac{1}{2}Li^2 \quad \text{Joules}$$

This is stored in the magnetic field within the inductor.

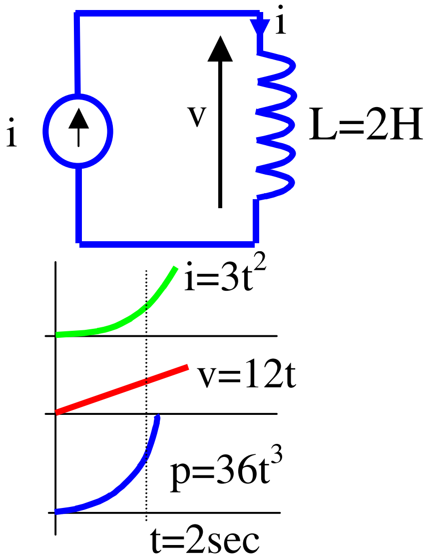
NOTES:

1) If the current is increasing then the change in energy stored is +ve, increasing. If the current is decreasing then the change in energy is –ve, and energy is recovered from the surrounding magnetic field.
2) If i=0 at t=0, then at any later time, t, the energy stored when the instantaneous current is 'i' is independent of how the current may have changed between time=0 and time = t.
3) If the current reduces to zero, then all the energy originally put into storage must come out of storage and this is independent of how fast or how slow such changes occur.
4) No energy is dissipated (converted into heat) in a **<u>pure</u>** inductor.

Example

(a) A pure inductance of 2H is connected to a current source $i=3t^2$. Calculate the instantaneous input power at t=2seconds and the total energy stored at the end of the interval t=0 to 2 seconds.

$$v = L\frac{di}{dt} = L\frac{d(3t^2)}{dt}$$
$$= 2\times 6t = 12t$$



and:

$$p = vi \quad = (12t)\times(3t^2) = 36t^3$$

at t = 2 seconds:

$$p = 36\times 2^3 = 288 \text{ W}$$
(also i = 12A, v = 24V)

The energy input in this period from 0 to 2 seconds is given by the integral of power with respect to time:

$$e = \int_0^2 pdt = \int_0^2 36t^3 = \left.\frac{36t^4}{4}\right|_0^2 = 144 \text{ J}$$

(area under the power curve)

NOTE: since i = 0 at t = 0, then the energy stored at t = 2 seconds ($i = 3\times 2^2 =$ 12A) could have been obtained from:

$$\text{energy} = 0.5Li^2 = 0.5\times 2\times 12^2 = 144 \text{ J.}$$

(b) if the current is now reduced at a rate of $-2As^{-1}$ to zero, calculate the time taken to reach zero current and the energy change over this period. For convenience, re-define t=0 as the start of this next phase, i.e. t = 0, i = 12A.

Therefore:

$$i = (12-2t)\text{ A}$$

From this we can calculate when i = 0:

$$t = 12/2 = 6 \text{ Seconds}$$

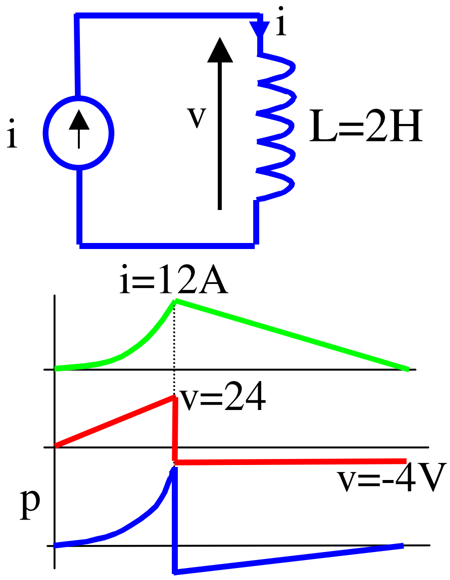
Similarly, at the new time t = 0 we have:

v =24V

and:

$$v = L\frac{di}{dt} = 2\frac{d(12-2t)}{dt} = -4V$$

(constant voltage)



and power:

$$p = vi \quad = -4(12-2t) = (-48 + 8t)$$

Therefore at t = 0:

$$p = -48W$$

and over the period of 6 seconds, the change in energy is given by:

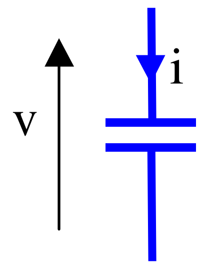
$$\Delta e = \int_0^6 (-48 + 8t)dt = -48t + \frac{8t^2}{2}\Bigg|_0^6 = -144J$$

(as expected! All energy has been recovered from the inductor).

c) **<u>Capacitor, C</u>**

A capacitor models the affect of the electric field surrounding an electric circuit. Now:

$$i = C\frac{dv}{dt} \text{ or } v = \frac{1}{C}\int idt$$



And a similar analysis to that used for the inductor reaches the result that energy stored:

$$e = \frac{1}{2}Cv^2$$

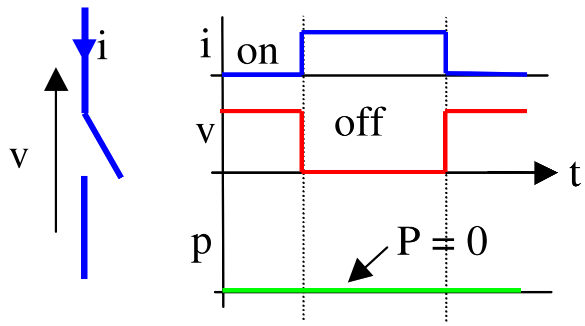
and this must be recovered if v = 0. There is no energy dissipated in a <u>pure</u> capacitor etc.

NOTE: There is a difference between a **real** and a **pure** capacitor

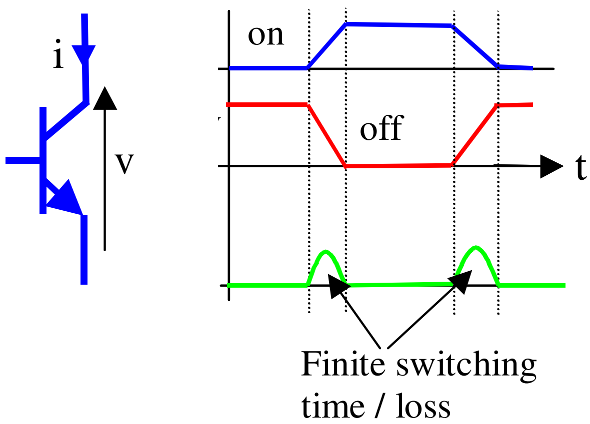
d) **<u>General Circuit Device</u>**

This illustrates the generality of the preceding procedures:
e.g. transistor switch / ideal switch

- Ideal switch



- Transistor Switch
(see tutorial for a worked example of this. – no analytical solution will be attempted here)



In general, the instantaneous switching loss (loss during the switching time) >> on state losses or off state losses.

- Important to switch quickly, etc
- Heatsink designed to deal with the AVERAGE not peak power dissipated in device.

Comment on continuity of stored charge

Note that in capacitors and inductors the energy cannot be changed instantaneously since the power, p, required would be infinite as:

$$p = \frac{de}{dt} \rightarrow \frac{\Delta e}{\Delta t} \rightarrow \infty \quad \text{if } \Delta t \rightarrow 0$$

This idea should be familiar, w.r.t. a car. – the stored energy (Kinetic) cannot be changed instantaneously (0 to 60 mph in 0 seconds) as this would require an infinitely powerful engine etc.

i.e. For an inductor, $e = 0.5Li^2$, $di/dt \neq \infty$. Current cannot change instantaneously.

For a capacitor, $e = 0.5Cv^2$, $dv/dt \neq \infty$. Voltage cannot change instantaneously.

Average Power

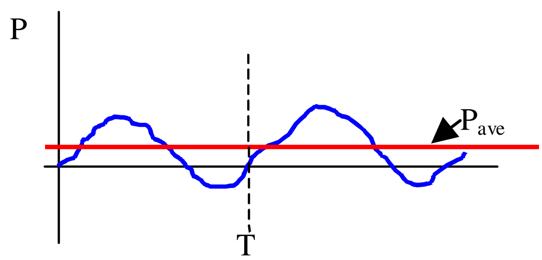
We have seen that the instantaneous power in a circuit or system is given by:

$$P = vi \quad \text{Watts}$$

Where, in general p can be either positive or negative

- In many systems the quantity of interest is the **average power** which is the average energy transferred per second. If the transfer is cyclic:

$$P_{AVE} = \frac{1}{T}\int_0^T p\,dt$$ where T is the period.



e.g. An electric lamp connected to an ac supply

Lamps form a resistive load, therefore from Ohms law, $v = i.R$ and if $i = I.\sin(\omega t)$ and $v = V.\sin(\omega t)$, then, $p = vi = V.I.\sin^2(\omega t)$

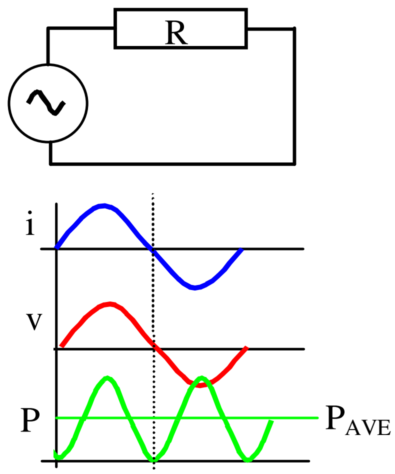
or:

$$P = \frac{VI}{2}(1 - \cos 2\omega t)$$

$$= P - P.\cos(2\omega t)$$

which comprises of a time invariant value and a pulsation at twice the supply frequency.

i.e. $P_{AVE} = VI/2$ – Time invariant.



Note:

1) Even for a resistive load, the power pulsates at twice the supply frequency, with a peak amplitude of $2 \times P_{AVE}$
2) If the thermal time constant of the lamp (or load in general) >>T, then the temperature rise or light output will only respond to the average power, and will not follow the instantaneous variations.
3) If T is of the same order as the thermal time constant, the temperature rise will respond to the instantaneous power input. (see tutorial sheet).

Note that for a resistive load:

$$P = \frac{VI}{2} = \frac{V}{\sqrt{2}} \times \frac{I}{\sqrt{2}}$$

Root Mean Square or effective value

In the above example we could also have written:

$$p = vi \equiv \frac{v^2}{R} \equiv i^2 R$$

and:

$$P_{AVE} = \frac{1}{T}\int_0^T vi\,dt \equiv \frac{1}{T}\int_0^T \frac{v^2}{R}dt \equiv \frac{1}{T}\int_0^T i^2 R\,dt$$

and if we also define the average power loss in a resistor in terms of an effective voltage and current – i.e. a voltage or current which gives the same heating (power loss) then we have:

$$P_{AVE} = \frac{V_{EFF}^2}{R} \quad \text{or} \quad I_{EFF}^2 R$$

then clearly we have:

$$V_{EFF} = \sqrt{\frac{1}{T}\int_0^T v^2 dt} \equiv V_{RMS}$$

$$I_{EFF} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \equiv I_{RMS}$$

This **Root-Mean-Square** value of v and i is a general expression which can be applied to any waveform. **However**, for a special case of a sinusoid, the above integrals give:

$$V_{RMS} = \frac{V}{\sqrt{2}} \qquad I_{RMS} = \frac{I}{\sqrt{2}}$$

Other relationships can be calculated for other waveforms – Some instruments are calibrated in RMS values, on the assumption that the waveform being measured will be a sine wave.

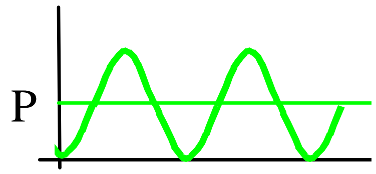
Power in Sinusoidal ac Circuits

If $i = I.\sin(\omega t)$ where $I = I_{PEAK}$ we have for various components:

(i) Resistor

We have already seen that for a resistor:

$$p = P_{Ave} - P\cos(2\omega t) \text{ Watts}$$



where:

$$P_{AVE} = \frac{VI}{2} = \frac{V}{\sqrt{2}} \times \frac{I}{\sqrt{2}} = V_{RMS} \times I_{RMS} \quad \text{etc}$$

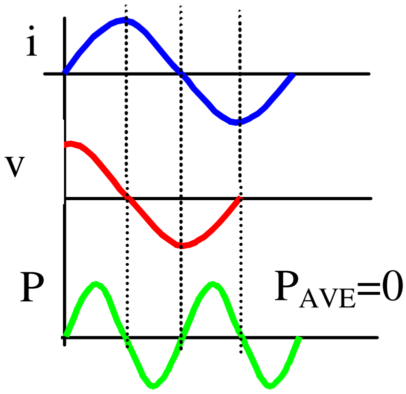
NB. Only true for pure resistance

(ii) Inductor

As:

$$i = I\sin(\omega t) \quad \text{and} \quad v = L\frac{di}{dt},$$

$$v = \omega LI\cos(\omega t)$$



which we may re-write as:

$$v = X_L I\cos(\omega t) = V\cos(\omega t) = V\sin\left(\omega t + \frac{\pi}{2}\right) \text{w}$$

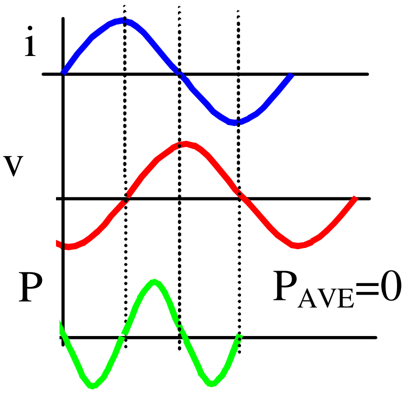
which tells us that I lags V by 90° or π/2 radians,also, as p = vi:

$$p = \frac{VI}{2}\sin(2\omega t)$$

which gives no average value, i.e.:

$$P_{AVE} = 0$$

(iii) Capacitor



Since:

$$i = I\sin(\omega t) \quad \text{and} \quad v = \frac{1}{C}\int i\,dt,$$

$$v = -\frac{1}{\omega C} I\cos(\omega t)$$

which we may re-write as:

$$v = -X_C I\cos(\omega t) = -V\cos(\omega t) = V\sin\left(\omega t - \frac{\pi}{2}\right)$$

which tells us that I leads V by 90^o or $\pi/2$ radians, also, as p = vi:

$$p = -\frac{VI}{2}\sin(2\omega t)$$

which gives no average value, i.e.:

$$P_{AVE} = 0$$

The instantaneous power in the capacitor, p, is 180^o out of phase with the instantaneous power in an inductor, for the same excitation current.

NOTE:

1) Despite the fact that both L and C have a RMS voltage, V_{rms}, and current, I_{rms}, they have no average power. I.e. $\mathbf{P_{AVE} = 0}$

Therefore, in general, P≠VI in ac circuits.

2) However, there is a continuous alternating instantaneous power flow into and out of storage. – This is known as REACTIVE POWER and is given the symbol Q to differentiate it from the REAL POWER, P.

For an Inductor:

$$|Q_L| = |I|^2 X_L$$

For a Capacitor:

$$|Q_C| = -|I|^2 X_C$$

The –ve sign indicates the 180^o phase difference.

3) Although P and Q are strictly the same units of power (Watts), to differentiate between them because they are in fact different, they are defined as:

P = REAL POWER – in Watts (W)

Q = REACTIVE POWER, in Volt Amps reactive or VAr's

Power Factor

For a circuit having a general impedance, Z = (R+jωL) or Z = ($R+X_L$), we may write the impedance as a magnitude and phase:

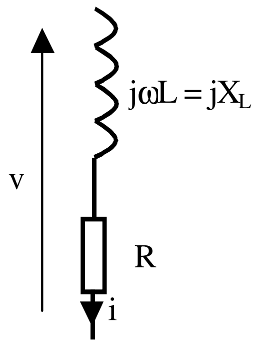
$$|Z|\angle\phi$$

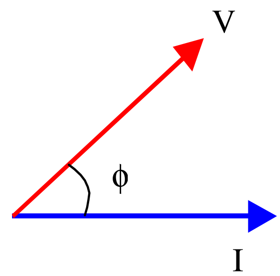
where:

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

and:

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$





As we can write i = Isin(ωt), and v = Vsin(ωt+ϕ), this gives:

$$p = v\times i = I\times V\times\sin(\omega t)\times\sin(\omega t+\phi)$$

which gives:

$$p = \frac{IV}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$$

where:

$$p = \frac{IV}{2}\cos\phi$$

is the average power, P_{AVE}, and:

$$p = \frac{IV}{2}\cos(2\omega t + \phi)$$

is a 2 × supply frequency variation. From this, it may be seen that the average power in the circuit is now given by:

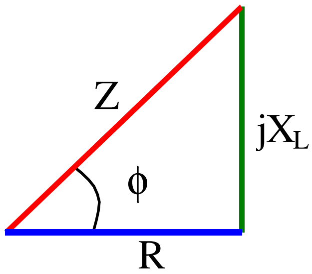
$$p = \frac{IV}{2}\cos\phi = V_{rms}I_{rms}\cos(\phi)$$

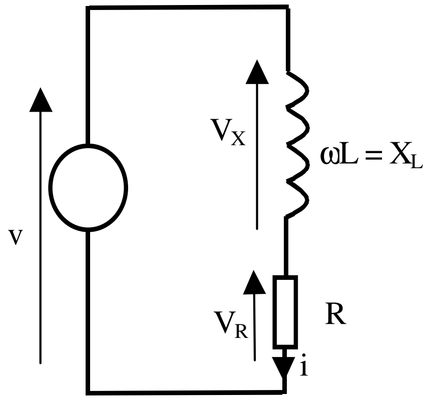
where **cos (ϕ)** is called the **POWER FACTOR**, and ϕ is the phase angle between the voltage and the current in the circuit.
Cos(ϕ) is the general factor for any ac circuit by which the supply voltage V and current I are multiplied to get the average power input.

Note: For a pure L and C, $\phi = \pm\pi/2$, cos(ϕ) = 0 and hence average power = 0 – as shown earlier.

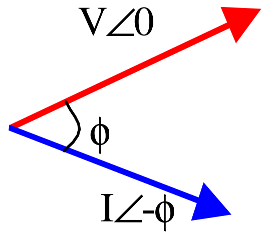
Complex power and the power triangle

As before: $Z = R + jX_L$ .We can now draw the impedance triangle as:





We can now draw a phasor diagram of the voltages and currents:



The current is then given from:

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{|V|\angle 0}{Z\angle\phi} = |I|\angle -\phi$$

Similarly, we may draw a phasor diagram for the voltages (voltage phasor diagram)
Here we see that:

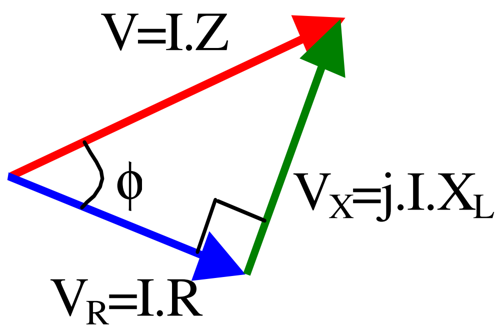
$$V_R = I\times R = V\times\cos(\phi)$$

In phase with current

and

$$V_X = j\times I\times X_L = V\times\sin(\phi)$$

90^o ahead of the current



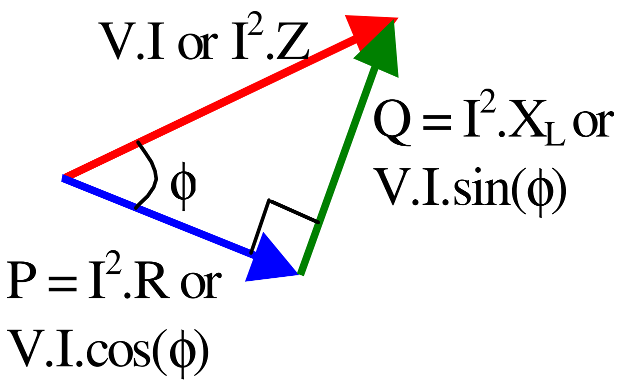
and if all the phasors are multiplied by the current, I, we produce a **Power triangle**,
Where:

$P = V.I.\cos(\phi)$ $(=I^2R)$ - Watts (W)

$Q = V.I.\sin(\phi)$ $(=I^2X_L)$ - VAr's

And the third side is called the **Volt Amps**

$S = V.I$ - VA
(called the complex power)



And we see that:

$$\mathbf{S = P + jQ}$$

- NOTE: by convention,
  Q is +ve for an inductive load,
  Q is –ve for a capacitive load

If V and I are written in phasor form, say:

$$\overline{V} = V\angle 0, \text{ and } \overline{I} = I\angle -\phi$$

then for an inductive load, (where I **lags** V), if we use complex power, S=VI, then:

$$S = \overline{V}.\overline{I} = \left|\overline{V}\right|.\left|\overline{I}\right|\angle -\phi$$

or:

$$S = V.I.\cos(\phi) - jV.I.\sin(\phi)$$

This gives the WRONG sign convention for Q.

To eliminate this difficulty, from phasor notation we always calculate:

$$S = V.I^{*} = VI\angle\phi$$

where I^{*} is called the complex conjugate of I, therefore if:

$$I = I\angle\phi \text{ then } I^{*} = I\angle -\phi$$

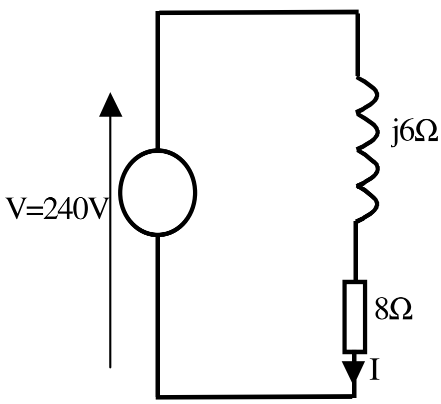
or if:

$$I = I_a + jI_b \text{ then } I^{*} = I_a - jI_b \text{ etc...}$$

Clearly, if I lags V, then Q +ve
If I leads V, then Q –ve

And they can be used to cancel each other out, or to reduce the total Q – known as **POWER FACTOR CORRECTION**.

Power Calculation Example



$Z = 8 + j6\ \Omega = 10\angle 36.87^{o}$

For this circuit calculate:

1. The circuit current
2. The VA, VAr's and Watts supplied to the load

For the circuit, taking the voltage as the reference for the phasor diagram, V=240∠0

Then:

$$\overline{I} = \frac{V}{Z} = \frac{240\angle 0}{10\angle 36.87} = 24\angle -36.87$$

i.e. I **lags** V by 36.87^o (inductive circuit)
and:

$$S = VI^* = 240\angle 0 \times 24\angle 36.87 = 5.76\angle 36.87 \text{ kVA}$$

giving:

$$|S| = |VI| = 5.76\text{kVA}$$

where:

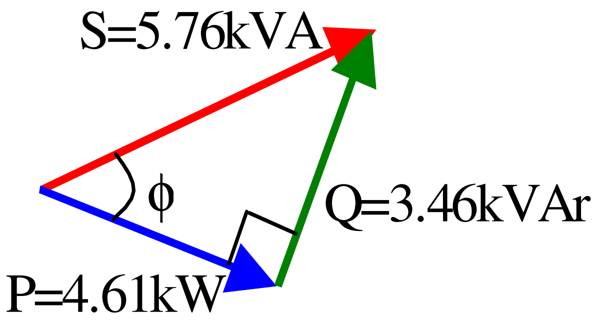
$$P = |S|\cos(\phi) = 5.76.\cos(36.87^o) = 4.61\text{kW}$$

$$[= I^2 \times R = 24^2 \times 8\ ]$$

$$Q = |S|\sin(\phi) = 5.76.\sin(36.87^o) = 3.46\text{kVAr}$$

$$[= I^2 \times X_L = 24^2 \times 6\ ]$$

This may be shown as a power triangle:



Load Specification

In the previous example, the load was specified in terms of its impedance, (R+jωL) or (8+j6Ω). In power systems the supply is usually considered to be constant (small % change can occur) at some nominal value (eg 240V, 425V, 11kV, etc.)
For these situations, it is possible to specify the load not in terms of its impedance, but in terms of the VA or Power requirements and power factor. E.g.:

(i) – Normal lightbulb is specified in terms of its wattage, 60W, 100W etc
- A kettle is specified in terms of its wattage (e.g. 2kW etc)

In these cases the load is a pure resistance (i.e. pf = 1)

(ii) - In the previous example, for a supply of 240V, the load Z_L of (8+j6)Ω could have been specified as a load of 4.76kW @0.8pf lag,
or as a load of 5.76kVA @ 0.8pf lag.

For such systems, all loads are assumed to be parallel across V.

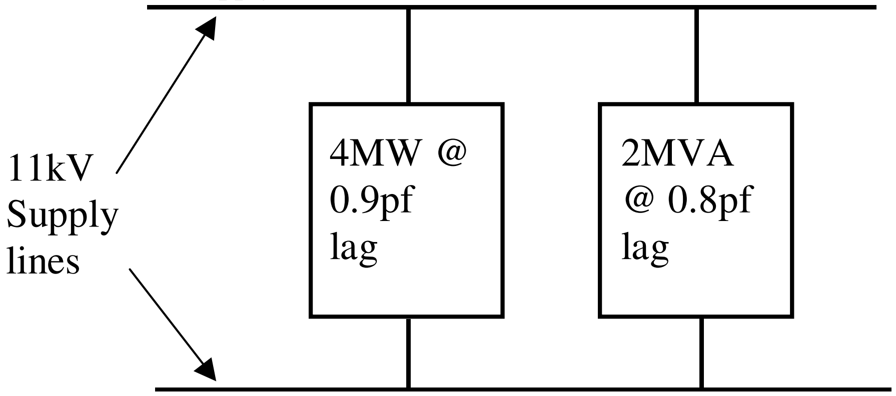
Example

The total load on an 11kV, 50Hz supply consists of the following:

i) 4MW @ 0.9pf lag

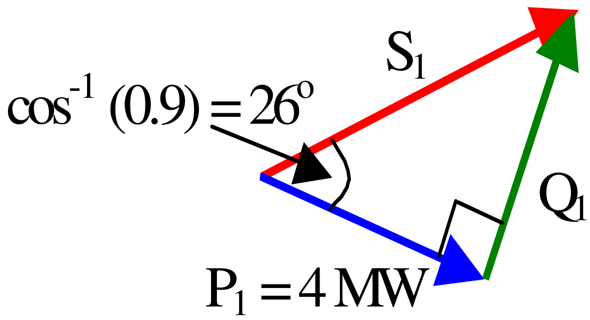
ii) 2MVA @ 0.8pf lag

Calculate the total load power, VAr and VA demands and hence the total load current drawn from the supply.



NB. All loads connected in parallel across the supply.

Load 1



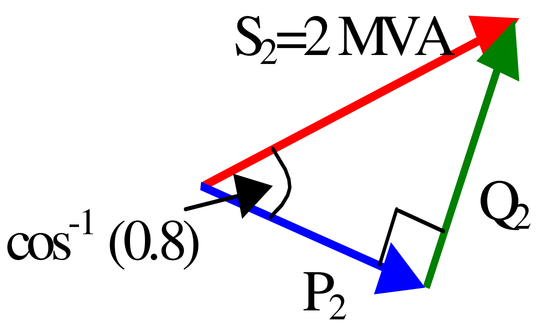
P_1 = 4MW @ 0.9pf

$\therefore$ $S_1 = P_1 / \cos(\phi) = 4.44$ MVA

and:

$Q_1 = S_1 \sin(\phi) = 1.94$ MVAr

Load 2



S_2 = 2MVA @ 0.8pf lag

$\therefore$ $P_2 = 2\cos(\phi) = 1.6$ MW

and:

$Q_2 = 2\sin(\phi) = 2 \times 0.6 = 1.2$ MVAr

Now, the total load:

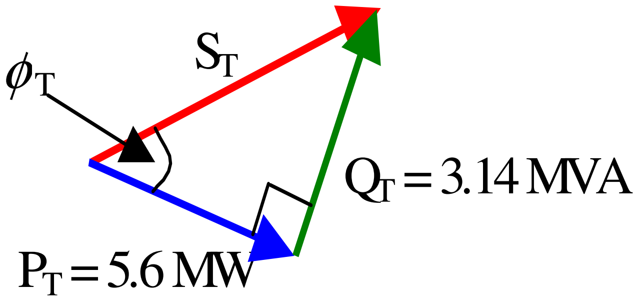
$S_T = (P_1+P_2) + j(Q_1+Q_2)$

NB. $S_T \neq S_1 + S_2$ you cannot add VA directly, only P and Q components

$\therefore$ S_T = 5.6MW + 3.14 MVAr

$$|S_T| = \sqrt{P^2 + Q^2}$$

$$= 6.42\ \text{MVA}$$



Therefore the total input current may be found from the total VA and the input voltage:

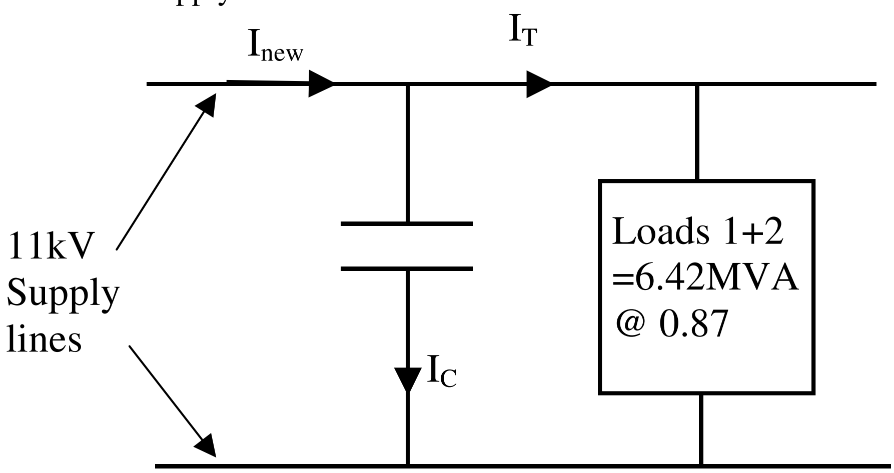
$$I_T = \frac{|S_T|}{|V|} = \frac{6.42 \times 10^6}{11 \times 10^3} = 584 \quad Amps$$

and the power factor of the total load is given by:

$$\cos(\phi_T) = \frac{5.6}{6.42} = 0.87 \quad lag$$

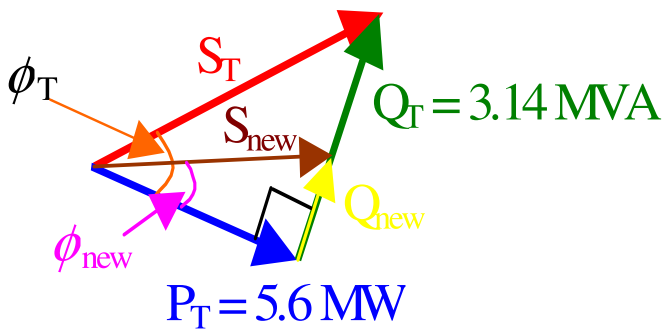
Power factor Correction

Calculate the capacitance required to improve (correct) the power factor to 0.95 lag and the new supply current.



- **Power factor correcting capacitors are always placed in parallel with the load, NOT in series**

- **Capacitors draw no power, only VAr component ∴ power remains unchanged, but the total Q reduces.**



$$\phi_T = \cos^{-1}(0.87)$$
$$\phi_{new} = \cos^{-1}(0.95) = 18.2^o$$

From the power triangle:

$$\tan(\phi_{new}) = Q_{new} / P_T = Q_{new} / 5.6$$

therefore:

$$Q_{new} = 5.6 \times \tan(18.2^o) = 1.84 \text{ MVAr}$$

So added (-ve) Q from capacitors:

$$Q_C = Q_T - Q_{new} = (3.14 - 1.84) \text{ MVAr} = 1.3 \text{ MVAr}$$

From earlier notes:

$$Q_{cap} = -I^2.X_C = 1.3 \times 10^6 = -V^2 / X_C$$

Therefore:

$$X_C = (11\times10^3) / (1.3\times10^6) = 93\Omega$$

and as:

$$X_C = 1/\omega C \quad \text{and} \quad \omega = 2\pi f = 100\pi$$

Then:

$$C = 34\mu F$$

Also, from the power diagram we have:

$$S_{new} = P / \cos(\phi_{new}) = (5.6\times10^6) / 0.95 = 5.895\text{MVA}$$

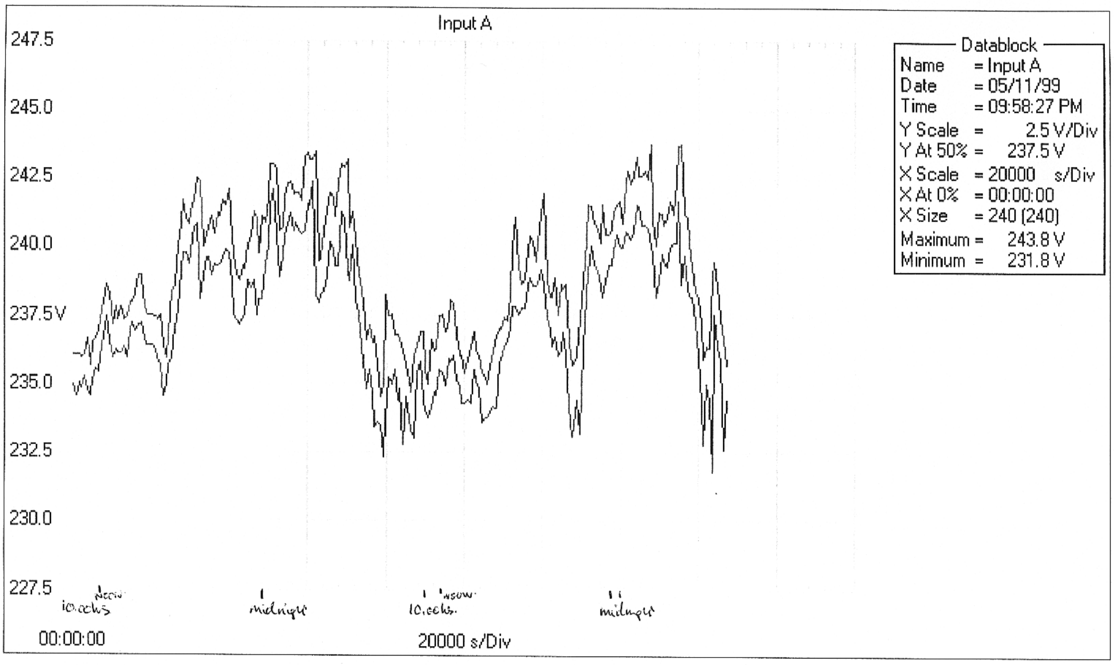
and:

$$I_{new} = \frac{|S_{new}|}{|V|} = \frac{5.895\times10^6}{11\times10^3} = 536 \quad Amps$$

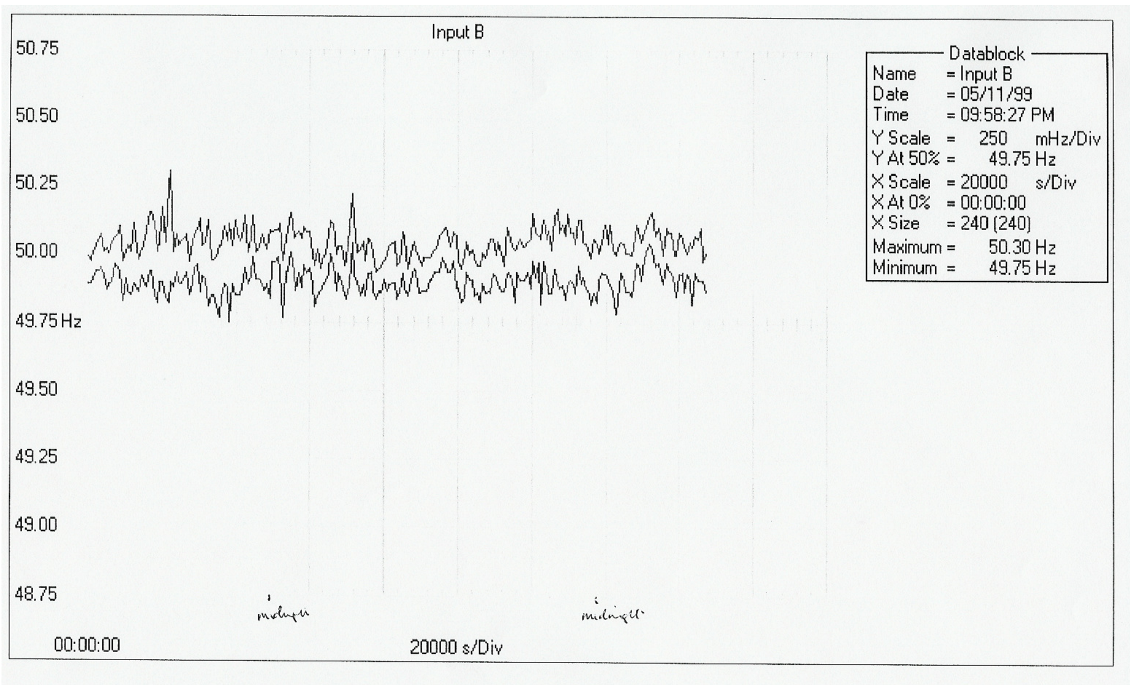
NOTE:
Supply current reduced from 584A to 536A with no reduction in power, P = 5.6MW.

Tariffs

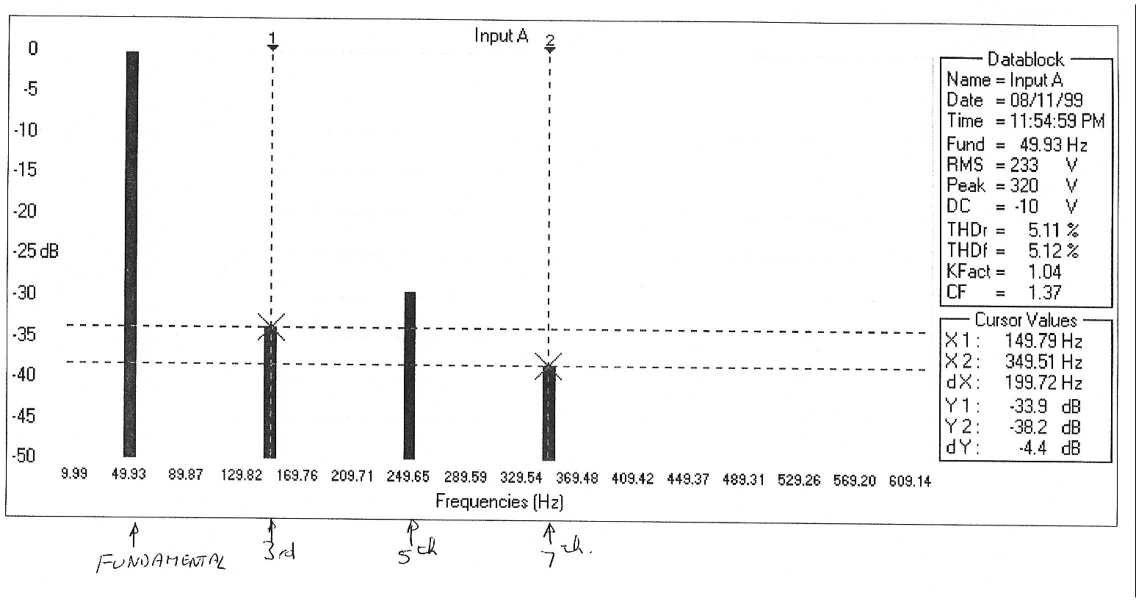
From the above it may be seen that the supply current is reduced for the same power supplied to the load. The supply company losses (in cables transmission lines, transformers etc) are reduced (= I^2R) by this technique. Hence to encourage customers to reduce the power system losses, the tariff (cost per unit of electricity supplied) charged to large power users, are based not only on the real power, P, supplied – but also on the power factor (VAr demand). By charging for Q as well as P, customers find it cost effective to install power factor correction capacitor banks at their installations.
Domestic customers are only charged for P at a fixed rate – no need to improve pf.
There are other reasons why the power companies encourage large industrial users to improve their pf. It will be seen from later in the course that the flow of Q in the network (not P) is the main cause of voltage drops through the system. Therefore by reducing Q, the voltage drops are also reduced.



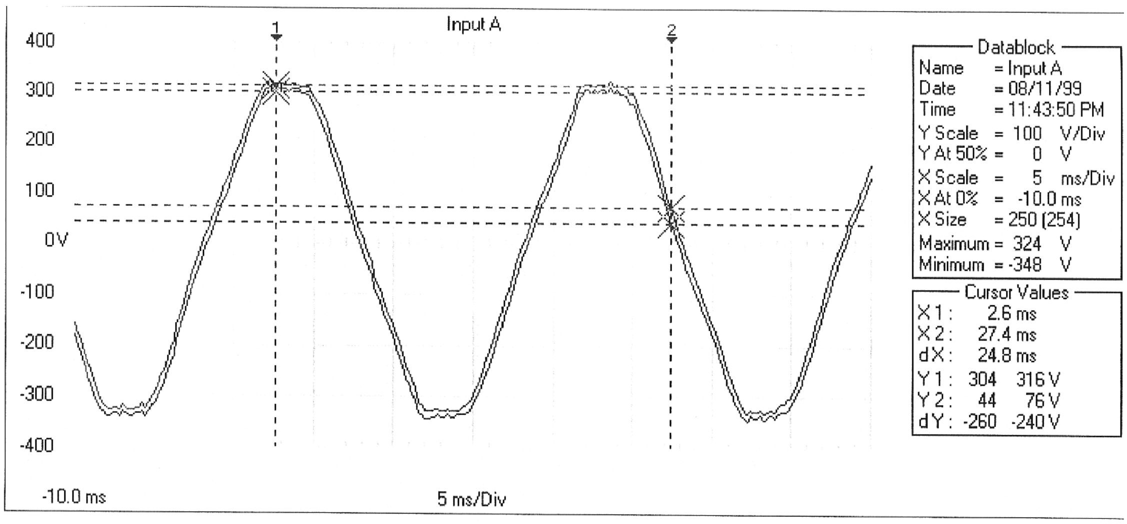
Magnitude of supply voltage over a 2 day period at St George's Complex.



Frequency of supply voltage over a 2 day period at St George's Complex.



Typical supply harmonics at St George's Complex.

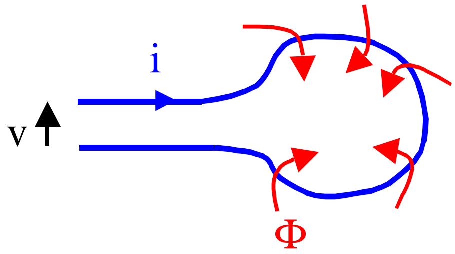


Typical supply voltage shape at St George's Complex.

Coupled Circuits and Transformers.

Self inductance (revision of earlier notes)

The current, i, produces flux ϕ which links the circuit of n turns (N=1 turn in many circuits). If the current and hence the flux is time varying, then the induced e.m.f.:



$$e = \frac{d\varphi}{dt} = \frac{d(N\Phi)}{dt}$$

where ψ = flux linkage = $N\phi$.

If the time variation of the flux linkage, ψ, is due to a current variation with time, we can also write this as

$$e = \frac{d}{dt} N\left(\frac{Ni}{s}\right) = \frac{N^2}{s}\frac{di}{dt} = L\frac{di}{dt}$$

where L is called the self inductance:

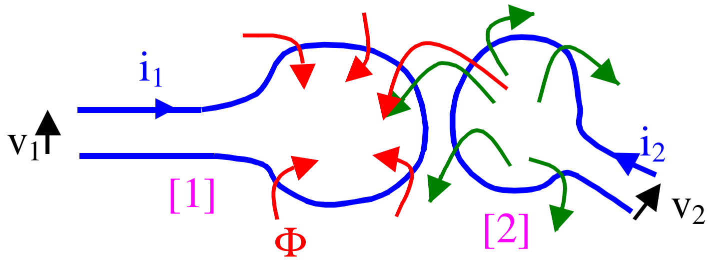
$$L = \frac{N^2}{s} \quad or \quad \frac{N\Phi}{i}$$

And neglecting circuit resistance:

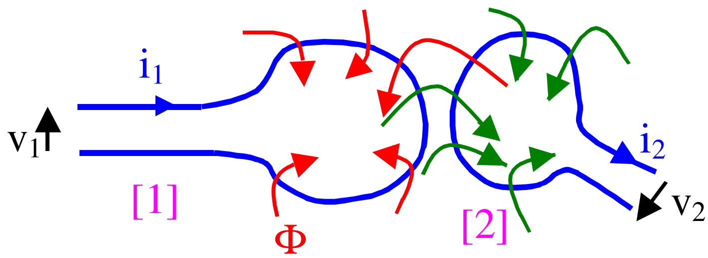
$$v = e = Ldi/dt$$

Mutual inductance

Mutual Flux from circuit 1 aiding the flux from circuit 2, giving a net increase in the flux passing through both circuits.



Mutual flux produced by one circuit is opposing the flux produced by the other giving a net decrease in flux through the circuits.



If part of the flux produced by a circuit links with a second circuit then they are said to have **Inductive coupling**, or have a **Mutual magnetic field**. (Note: Capacitive coupling is via a mutual electric field).

Suppose a proportion k_1 of ϕ_1 links circuit [2], and a proportion k_2 of ϕ_2 links circuit [1], then the flux linkage for each circuit is:

- for cct [1]:

$$\varphi_1 = N_1(\phi_1 \pm k_2\phi_2) \qquad (1)$$

where +ve is aiding, -ve is opposing

- for cct [2]:

$$\varphi_2 = N_2(\phi_2 \pm k_1\phi_1) \qquad (2)$$

From our previous definitions of self inductance:

$$\phi_1 = \frac{L_1 i_1}{N_1}, \quad \phi_2 = \frac{L_2 i_2}{N_2}$$

therefore:

$$\varphi_1 = L_1 i_1 \pm \left(k_2 \frac{N_1 L_2}{N_2} \right) i_2 \qquad (3)$$

and:

$$\varphi_2 = L_2 i_2 \pm \left(k_1 \frac{N_2 L_1}{N_1} \right) i_1 \qquad (4)$$

Clearly the bracketed terms must have the same dimensions (units) as L_1 and L_2 (Henry). i.e. must be inductance.
It can be shown that (by conservation of energy):

$$k_2 \frac{N_1 L_2}{N_2} = k_1 \frac{N_2 L_1}{N_1} = M \qquad (5)$$

where M is called the mutual inductance. Hence (3) and (4) become:

$$\varphi_1 = L_1 i_1 \pm M i_2 \text{ and } \varphi_2 = L_2 i_2 \pm M i_1$$

and:

$$v_1 = e_1 = \frac{d\varphi_1}{dt} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt} \qquad (6)$$

$$v_2 = e_2 = \frac{d\varphi_2}{dt} = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt} \qquad (7)$$

where the first part of the equation, $L_1 \frac{di_1}{dt}$ is called the self induced voltage, and the second part of the equation, $M \frac{di_2}{dt}$ is called the mutually induced voltage (i.e. due to the mutual flux linkage).

Coefficient of Coupling

From (5) it may be seen that:

$$M^2 = k_1 k_2 L_1 L_2 = k L_1 L_2$$

or:

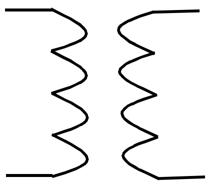
$$M = k\sqrt{L_1 L_2} \quad \text{where} \quad k = \sqrt{k_1 k_2}$$

and **k** is called the coefficient of coupling, and $\mathbf{k \le 1}$. For magnetically coupled circuits, k>0, for maximum coupling, k=1, and if you do not want ‘crosstalk’ between circuits, k=0. For an ideal transformer, k⇒1.

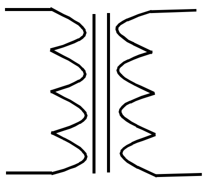
Circuit Conventions

In general circuit work it is not possible to show the actual coils and core material used in coupling circuits and therefore conventions have been used to signify key information when it is important.

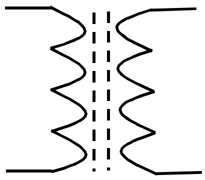
e.g.



Coils with air as the coupling mechanism

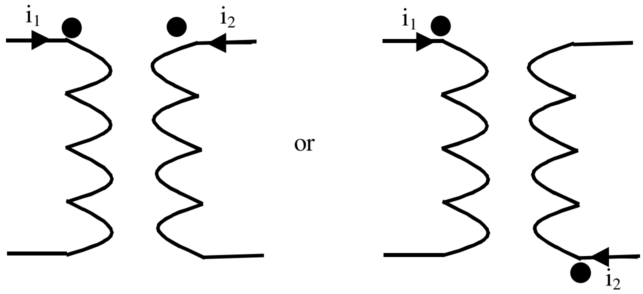


Coils with iron (laminated) as the coupling mechanism

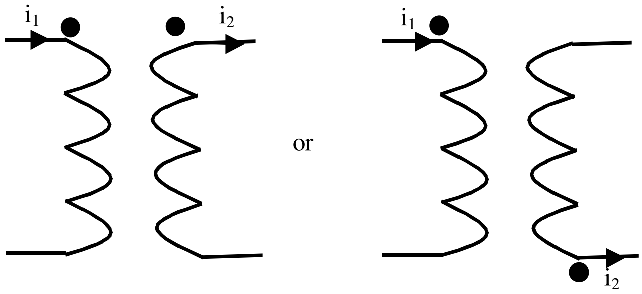


Coils with ferrite ‘pot’ core

And the ‘dot’ convention to show the sign of the mutual coupling



M = +ve, if currents flow into ‘dot’ on each circuit



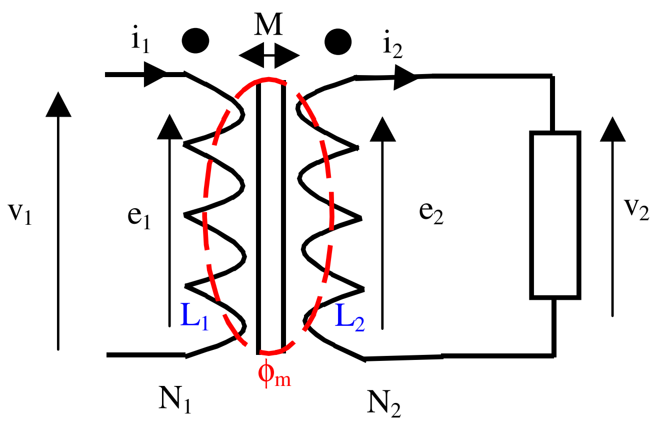
M = -ve if currents flow not as above.

The Ideal transformer

Despite the variety of transformers in use and their varied design objectives, we still retain the concept of an ‘ideal’ transformer, which has well defined properties (c.f. R, L and C in circuits).

Properties of the ideal transformer:
1. it is a lossless device (i.e. no heat losses due to copper (I^2R) and iron loss mechanisms)
2. It has perfect coupling (k=1) between the two windings (i.e. all flux produced is **mutual** to both windings).
3. It has a magnetic core of zero reluctance (S=0)
4. There are no electric field effects (i.e. no capacitive effects)

As a consequence of the above properties, the device would have the following performance characteristics:



i) On no-load – ($i_2 = 0$)
Because the core has zero reluctance, then both windings have infinite self inductance $= N_1^2/S = N_2^2/S$
Therefore when the primary is connected to a supply, $\mathbf{i_1 = 0}$ **if** $\mathbf{i_2 = 0}$

[note in practice, very low reluctance in the magnetic circuit by design, minimal airgaps, but L < **∞.** Therefore a small current flows in the primary, called the magnetising current (i_m).]

If the coefficient of coupling k=1, then all the flux is mutual:

$$v_1 = e_1 = \frac{dN_1\phi_m}{dt} \quad \text{and} \quad v_2 = e_2 = \frac{dN_2\phi_m}{dt}$$

Therefore:

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

ii) On load – ($i_2 = 0$)
When both coils carry current then the resultant mmf around the core:

$$F = N_1 i_1 - N_2 i_2$$

and the resultant flux:

$$\Phi_m = \frac{N_1 i_1 - N_2 i_2}{S}$$

in the limit that $S \to 0$, then $N_1 i_1 - N_2 i_2 \to 0$ i.e. $N_1 i_1 = N_2 i_2$ or:

$$\boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1}}$$

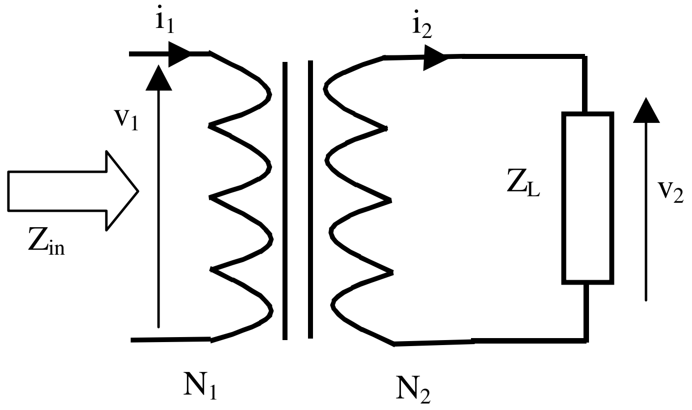
this is the mmf balance, and since the flux is unchanged:

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

as before.

Note: if $S \neq 0$ then a small current, noted in (i), will still flow (magnetising current)

iii) Impedance
As a result of the voltage and current relationships above, there is an impedance change through the transformer:



Clearly for the load impedance, $Z_L = v_2/i_2$, but as $v_2 = v_1 \times (N_2/N_1)$ and $i_2 = i_1 \times (N_1/N_2)$ then we have:

$$\frac{v_1}{i_1} = \frac{v_2}{i_2} \times \left(\frac{N_1}{N_2}\right)^2 = Z_L \times \left(\frac{N_1}{N_2}\right)^2$$

i.e. the load ‘appears’ at the input of the transformer as an impedance:

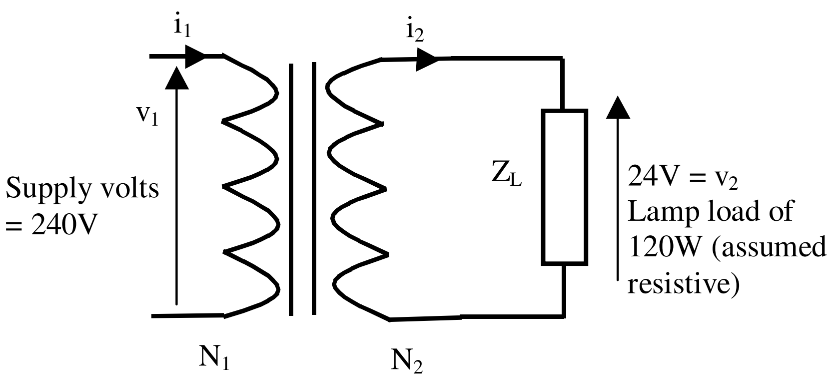
$$Z'_L = Z_L \times \left(\frac{N_1}{N_2}\right)^2$$

or we say it has a ‘referred’ value as seen from the primary side.

Example

Show how ‘first-order’ idealised calculations can be done (see tutorial sheet). Such calculations are a valuable first step to check results.

A transformer is used to step down a mains supply of 240V to a safe 24V for a lighting system. If the total load is 120W, choose a suitable turns ratio for the transformer and the necessary current rating of its primary and secondary windings.



Ideally, turns ratio = voltage ratio $N_1/N_2 = 240/24 = 10:1$

(Note the actual number of turns could be 100 and 10, or 1000 and 100 etc, the ratio is the same – Actual turns determined by the magnetic circuit using $V_1 = 4.44 f N_1 \phi_m$ as in the earlier notes and tutorial sheet).

For a load of 120W @ 24V, load resistance is given by:

$$W = \frac{V^2}{R} \quad \text{or} \quad R = \frac{V^2}{W} = \frac{24^2}{120} = 4.8\Omega$$

Using the principle of 'referred' secondary quantities, then the actual load, R_L (4.8Ω) 'appears' as a load in the primary of:

$$R'_L = \left(\frac{N_1}{N_2}\right)^2 R_L \quad = \; 4.8 \times 10^2 = 480\Omega$$

$$I_1 \quad = \; \frac{240}{480} = 0.5A$$

and:

$$\frac{I_1}{I_2} \quad = \; \frac{N_2}{N_1} \quad \therefore I_2 = 10 I_1 = 5A$$

i.e. primary must be rated to carry 0.5A, the secondary 5A.

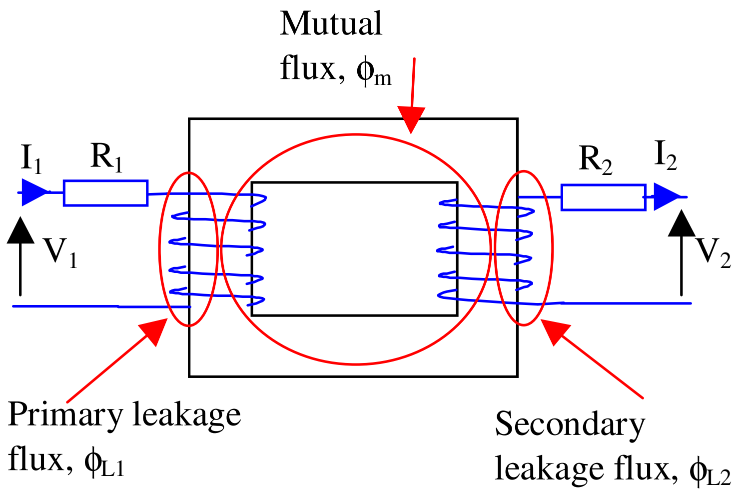
Practical transformers

Transformers or various types and sizes are used widely throughout electronic and electrical engineering systems from high frequency communications though to power systems. Basically a 'transformer' defines any device which is designed to couple two (or more) electric circuits via a mutual magnetic circuit. (Note: transformer effects, 'crosstalk' can occur as an unwanted phenomenon in many devices and circuits). Transformers can be designed for very different purposes and to meet many different specifications e.g.

1. Power transformers – designed to transmit bulk power (10's of Watts up to MW) from one voltage level to another (step up or step down) – Key specifications here are the efficiency and voltage regulation (change of voltage with load).
2. Instrumentation transformers – designed to condition ('scale') a voltage or current for safe measurements (e.g. 4kV to 5V for measuring the voltage on an overhead transmission line) – Key specifications here are the accuracy of the scaling, and the phase shift if used in conjunction with power measurements.
3. Isolating transformers – often 1:1 turns ratio used to provide a safe ('unearthed') supply (electric razor point)
4. Matching transformers – often used as part of an audio frequency system to match an amplifier to it's load (or loudspeaker) – Key specifications here are the fidelity of the signal (phase and amplitude shifts of the different frequency components)
5. Pulse transformers – used to scale a pulse rather than a sinewave – again issues such as rise times and signal droop are important.

Despite all of these variations, most will ideally behave as the 'ideal' transformer – though the designer may optimise the non-ideal features for the specific application.

In practice:-



(i) there are finite winding resistances R_1 and R_2 (and associated losses $I_1^2R_1$ and $I_2^2R_2$)

(ii) There are finite iron losses (eddy current and hysteresis)

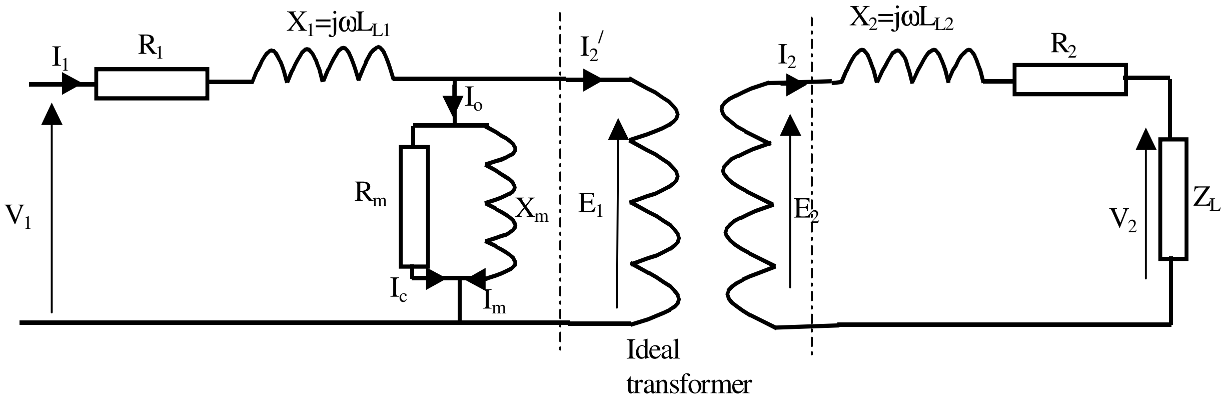
(iii) The windings are not perfectly coupled ($k \neq 1$) and hence for each winding some flux is mutual ϕ_m giving rise to the coupling and mutual inductance, and some of the flux links only one winding, ϕ_{L1}, and ϕ_{L2} for the primary and the secondary winding respectively, giving rise to the leakage inductances, L_{L1} and L_{L2} respectively.

(iv) The core is not of zero reluctance and hence some mmf is needed to drive ϕ_m around the core i.e.:

$$N_1 I_m = N_1 I_1 - N_2 I_2$$

where I_m = magnetising current

As a result of (i) to (iv), a more complex model or **equivalent circuit** is require to fully characterise behaviour of the practical transformer. One such circuit is:



(i) for the ideal part, $E_1/E_2 = N_1/N_2$ and $I_2^{/}/I_2 = N_2/N_1$ Where $I_2^{/}$ is the referred secondary current.

(ii) The **Primary Leakage reactance X_1** = ωL_{L1} and the **Secondary leakage reactance X_2** = ωL_{L2} are both small since the transformer is designed for low leakage.

(iii) The **Primary Winding resistance, R_1**, and **the Secondary Winding resistance, R_2**, are designed to be small to reduce the winding losses.

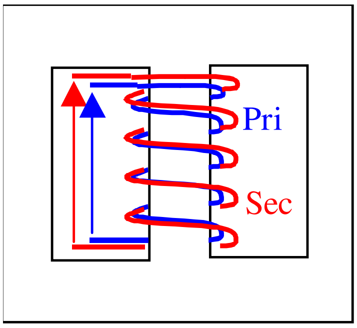
(iv) The **Magnetising Reactance X_m** = ωL_m is designed to be high since the core is designed for low reluctance (L_m high).

(v) The **Core Loss resistor R_m** is a high value since the core is designed for low loss, and loss = E_1^2/R_m

(vi) The **no-load current I_o** is generally small when compared to I_1 and $I_2^{/}$ and is clearly the current which flows in the primary when there is no load on the secondary winding. Where $I_o = I_m + I_c$

(vii) On-load, total input current = $I_1 = I_2^{/} + I_o$

Note: To minimise ϕ_{L1} and ϕ_{L2}, primary and secondary windings are usually concentric rather than on separate limbs of the magnetic circuit.



Two Simplifications (to make calculations easier!)

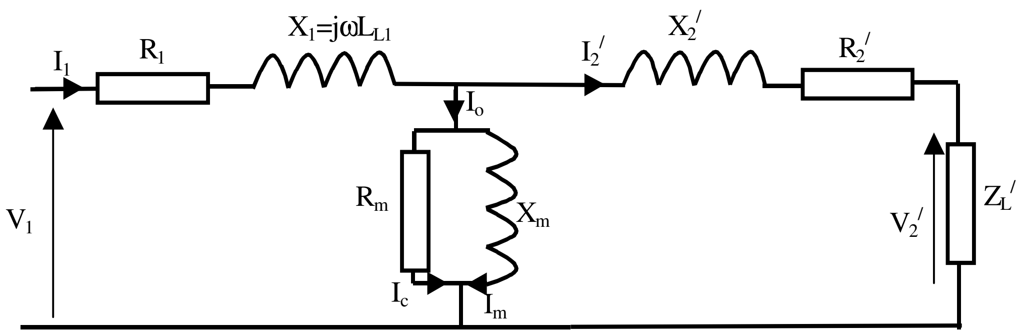
(i) Referral of impedances through the ideal transformer part

NB. This refers to the transformers own secondary resistance and leakage reactance, as well as the load.

As noted earlier, any secondary quantities (V_2, I_2, and Z_2) can be referred to the primary as:

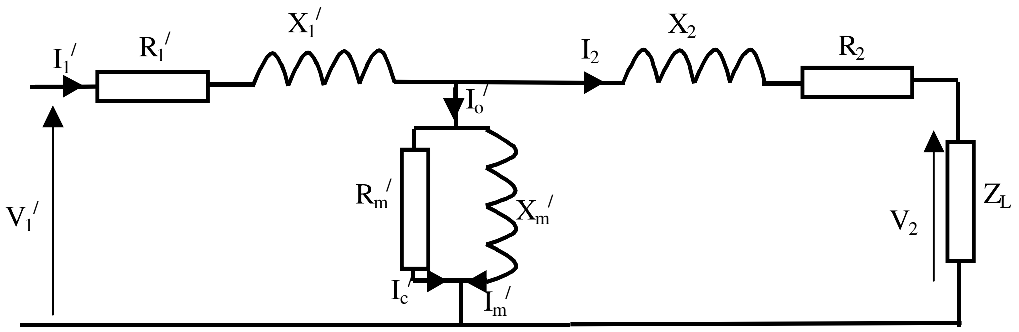
$$V_2' = V_2 \times \frac{N_1}{N_2} \qquad I_2' = I_2 \times \frac{N_2}{N_1} \qquad Z_L' = Z_L \times \left(\frac{N_1}{N_2}\right)^2$$

Hence the circuit may be re-drawn with all of the secondary quantities referred to the primary circuit:



Equivalent circuit referred to the Primary

An alternative procedure to the above may be used whereby all the primary side values are referred to the secondary.



Equivalent circuit referred to the Secondary

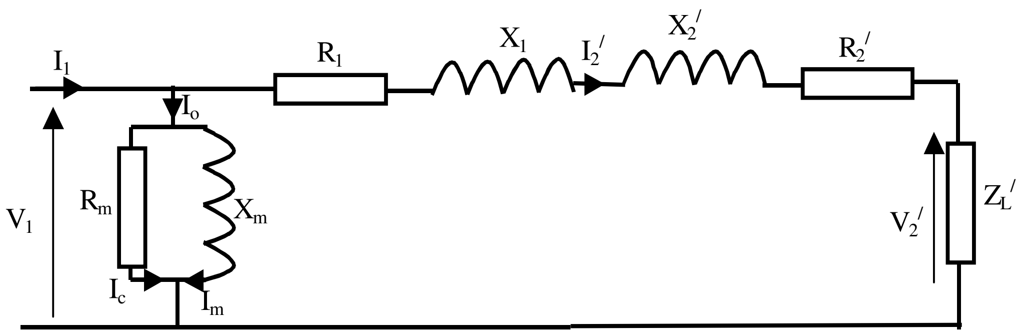
$$V_1' = V_1 \times \frac{N_2}{N_1} \qquad I_1' = I_1 \times \frac{N_1}{N_2} \qquad R_1' = R_1 \times \left(\frac{N_2}{N_1}\right)^2 \quad \text{etc}$$

Either technique reduces the problem to a single circuit without the 'ideal transformer' to complicate the situation.

(ii) An approximate equivalent circuit which is easier to solve.

This involves a small error, but significantly simplifies the solution!

Since R_m and X_m >> R_1 and X_1 for many calculations, then under the circumstances, the so-called magnetising branch can be moved to the input terminals (NB occasionally this is too inaccurate an assumption, and must not be used).



Approximate equivalent circuit referred to the primary

Operating performance of iron-cored transformers

Power transformers:
Interested particularly in efficiency and voltage ratio:

Efficiency: $$\eta = \frac{\text{output power}}{\text{input power}} \times 100\%$$

Usually more accurate to calculate:

$$\eta = \frac{\text{input power} - \text{losses}}{\text{input power}} \times 100\%$$

The losses in the transformer are:

(i) Iron losses (P_{Fe}) = $E_1^2/R_m \approx V_1^2/R_m$ $\Rightarrow$ Hysteresis and eddy current losses

(ii) Copper losses (P_{Cu}) = $I_1^2R_1 + I_2^2R_2$ $\Rightarrow$ Varies with load and constitute a heat loss – (most efficient when $P_{Fe} = P_{Cu}$)

Efficiency of very large units > 98%, e.g. 200MVA units supply 200MW with 99% efficiency and generates a loss of 2MW – cooling required!

Voltage Ratio.
Since there is a voltage drop in the transformer, the actual voltage ratio is a function of the load. [Full load or rated load defines the operating load which gives maximum permissible loss and temperature rise]. The term used to describe this is **'Regulation'**.

$$\text{regulation} = \frac{\text{'no} - \text{load' output voltage} \ - \ \text{'on} - \text{load' output voltage}}{\text{'no} - \text{load' output voltage}} \times 100\%$$

Example

- Demonstrate the method of solution.

A single-phase transformer has the following equivalent circuit parameters:

Primary winding resistance,	R_1=2Ω
Primary winding leakage reactance,	X_1=10Ω
Secondary winding resistance,	R_2=0.04Ω
Secondary winding leakage reactance,	X_2=0.1Ω
Magnetising reactance,	X_m=2.5kΩ

The transformer has a no-load iron loss of 220W, and has a primary to secondary turns ratio of 10:1. the transformer is connected to a supply of 2.2kV and a load of (3+j2)Ω. Calculate the following:

(i) The primary input current on no-load
(ii) The primary input current on-load
(iii) The output current and voltage on-load
(iv) The transformer output power and efficiency

Step1: Using the 'ideal' model to get order of magnitude answers

$$V_1/V_2 = N_1/N_2$$

Therefore:

$$V_2 = V_1 \times 1/10 \approx 220V$$

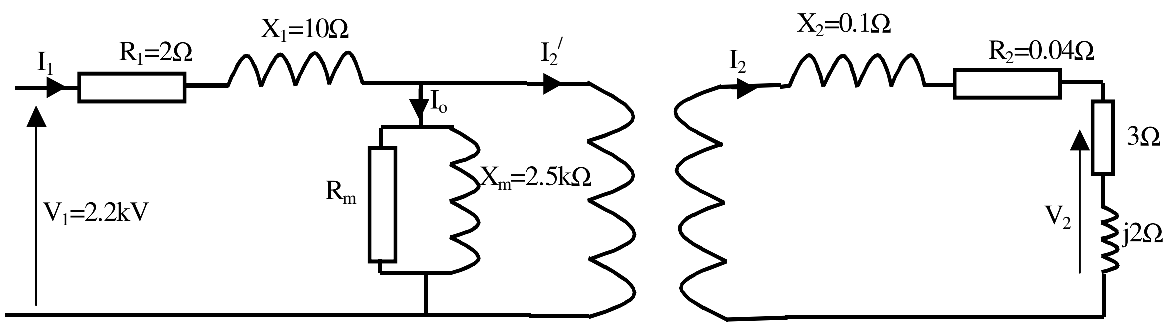
$$\text{Now } I_2 \approx \frac{220\angle 0}{3+j2} = \frac{220\angle 0}{3.6\angle 33.7} = 61\angle -33.7 \quad A$$

$$\text{Hence } \frac{I_1}{I_2} = \frac{N_2}{N_1} \qquad \therefore I_1 = 6.1\angle -33.7 \quad A$$

and output power = $I_2^2R_2 = 61^2 \times 3 =$ **11.16kW**

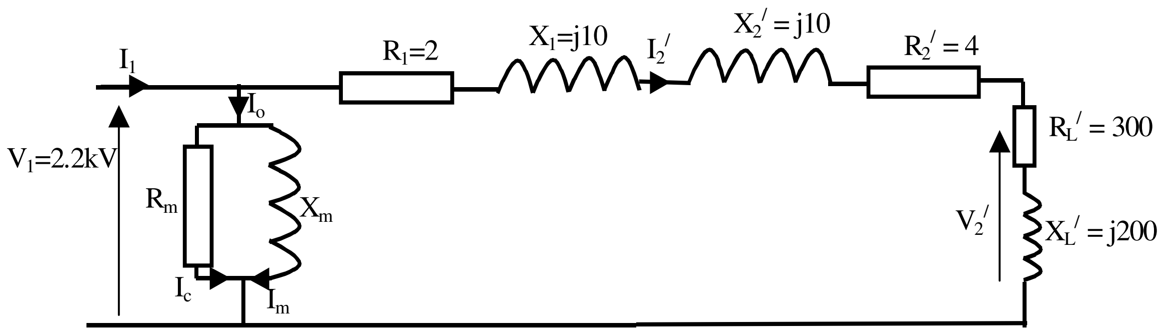
Step2: Obtain the approximate equivalent circuit (Note '**YOU**' always use this unless specifically told otherwise, or, in practice you can see that the approximations made would be inappropriate).

Starting from the un-referred, 'exact' circuit:



Referring all of the secondary quantities to the primary and at the same time moving the parallel R_m, X_m branch of the circuit to the input terminals, where:

$R_2^{/} = (N_1/N_2)^2 \times R_2 = 100R_2$, $\quad X_2^{/} = 100\, X_2$ etc.



Step3: Perform solution:

(i) Primary input current on no-load:

On no-load, $Z_L = \infty$, therefore $I_2^{/} = 0$, hence we require I_o. Since Xm = 2.5kΩ then:

$$I_m = \frac{V_1\angle 0}{jX_m} = \frac{2.2\times10^3\angle 0}{2.5\times10^3\angle 90} = 0.88\angle -90 = -j0.88 \text{ A}$$

Since the iron loss is 220W (and there is no loss in the magnetising inductance) these losses are in the resistive element, R_m. From this we can get the current, I_C:

Iron loss = $V_1 \times I_C$ (unity power factor!)

Therefore:

$$I_C = 220 / 2.2\times10^3 = 0.1\text{A}$$

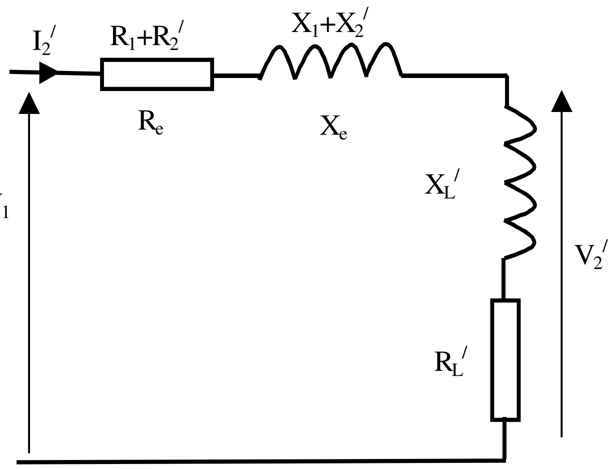
And the no-load current is:

$$\bar{I}_o = \bar{I}_C + \bar{I}_m = 0.1 - j0.88 = 0.8857\angle\text{-}83.5^o \text{ Amps}$$

(ii) The input current on-load:

NB Do not attempt to 'simplify' the circuit by combining all the parallel branches. Solve for $I_2^{/}$ and add to the value of I_o previously calculated.

We have:



Total series impedance:

$$Z_s = (R_1 + R_2^{/} + R_L^{/}) + j(X_1 + X_2^{/} + X_L^{/}) = 306 + j220 = \mathbf{376.9\angle 35.7^o}$$

Therefore:

$$I_2' = \frac{V_1\angle 0}{376.9\angle 35.7} = \frac{2200\angle 0}{376.9\angle 35.7} = 5.84\angle -35.7^o$$

or:

$$I_2^{/} = 4.74 - j3.41 \text{ A}$$

Hence the input current on-load:

$I_1 = I_o + I_2^{/} = (0.1 - j0.88) + (4.74 - j3.4) = 4.84 - j4.28 = \mathbf{6.46\angle{-41.5^o}}$

(This then compares to $I_1 = 6.1 \angle{-33.7^o}$ as calculated from the ideal model)

(iii) The output current and voltage on-load:

The output current on-load:

$$I_2 = I_2^{'} \times \frac{N_1}{N_2} = 5.84\angle -35.7 \times \frac{10}{1} = \mathbf{58.4\angle{-35.7^o}\ A}$$

The output voltage (easiest is to first calculate $V_2^{/}$):

$$V_2^{/} = I_2^{/} \times Z_L^{/} = 5.84 \angle{-35.7} \times (300 + j200)$$
$$= 5.84 \angle{-35.7} \times 360.5 \angle 33.7 = \mathbf{2105 \angle{-2.0}\ V}$$

or:

$$V_2 = V_2^{'} \times \frac{N_2}{N_1} = 2105\angle -2.0 \times \frac{10}{1} = \mathbf{210.5\angle{-2.0}\ V}$$

(which compares to $V_2 \approx 220 \angle 0$ from the ideal model).

(iv) The transformer output power and efficiency:

In this case we could use $P_o = I_2^{/} \times V_2^{/} \cos\phi$ where ϕ is the angle between $V_2^{/}$ and $I_2^{/}$ etc. or more easily, in this case is to calculate $(I_2^{/})^2 \times R_L^{/}$, therefore:

Power output $P_o = 5.84^2 \times 300 = \mathbf{10.23\ kW}$

(which compares with $P_o = 11.16$ kW from the ideal model).

Efficiency is defined for any device as:

Efficiency = (Power Output) / (Power Input) × 100%

Now:

Power input = Power output + Losses in the transformer

where:

Losses = Iron loss + Copper Loss

$$= 220 + (I_2^{/})^2 \times (R_1 + R_2^{/}) = 220 + 5.84^2 \times 6 = 220 + 204.6 = \mathbf{425\ W}$$

$$\therefore\ Efficiency = \frac{10230}{10230 + 425} \times 100\% \qquad = 96\%$$

Rating of Transformers

'Rated' means designed operating value, usually full-load. Limited by temperature (loss). Has been shown that for a.c. excited magnetic circuits, then provided coil resistance is small, the applied volts fixes the relationship between turns and flux density in the core, i.e.:

$$\mathbf{V_{rms} \approx E_{rms} = 4.44\ f\ N\ B_{max}\ A_{core}}$$

It can also be shown that the current fixes the required cross section of the copper windings, A_w, to give acceptable copper losses and heating effects. Hence the product V×I fixes the total copper and iron and hence the approximate size of the transformer for a given frequency.

V × I is called the **VOLT AMP RATING**

written as VA or kVA or MVA

FULL LOAD or RATED LOAD defines the value of the VA, voltage and current etc which gives the permissible operating temperature rise for the transformer.

NB.

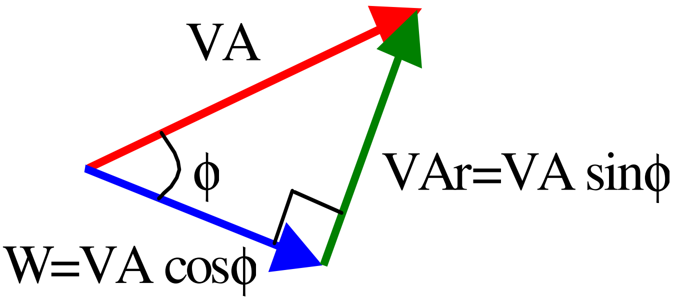
(i) Input VA ≈ Output VA (neglects magnetising current)
i.e. $V_1I_1 \approx V_2I_2$
Strictly VA usually relates to output.

(ii) The VA is not the POWER output in ac systems since V and I are not necessarily IN PHASE, depends on the load impedance .

POWER = (VA)×cosϕ

where cosϕ is the power factor.

Volt Amp Reactive, Var = (VA)×sinϕ

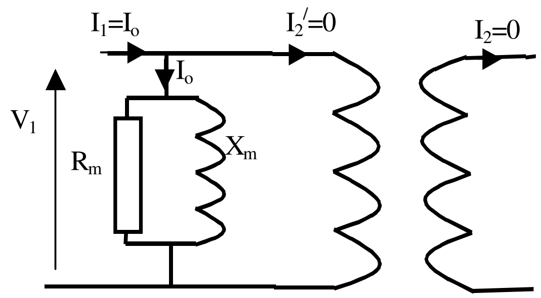


Measurement of Circuit Parameters

Transformer designers are able to calculate the equivalent circuit parameters with reasonable accuracy from the dimensions and properties of the materials used. However, all large transformers must be tested for faults and to check on their performance against the design. In many situations, testing on full-load is not possible or the performance under hypothetical load / fault conditions must be estimated. For these purposes, test values of parameters are required for accurate assessment. [There are numerous detailed tests specified in standard (NEMA) procedures]. Two relatively simple tests however, can give a reasonable level of confidence for normal sinusoidal performance of power transformers [Pulse / surge, variable frequency performance would need extra tests].

(a) Open circuit test

- Transformer tested with one winding open circuit, the other winding is connected to its **rated** [Nameplate] voltage, to give normal flux conditions in the core material. In this test, the only current flow in the excited winding is the no-load current, which is very small when compared to full-load. Therfore winding copper losses can usually be neglected. The approximate equivalent circuit then is as shown in the diagram (for a primary supplied model).



  Can also be supplied to the secondary side, where the magnetising branch has to be referred to the secondary winding via the square of the turns ratio.

<u>Test Procedure:</u>
Apply rated voltage to either winding [depends on voltage available in test lab].

<u>Measure:</u>
Input current, Input power, output voltage.

(i) Since there is no significant voltage drop in R_1, X_1 etc then
$V_1 \approx V_2^/ = N_1/N_2 \times V_2$ Hence $V_1/V_2 \approx$ Turns ratio $= N_1/N_2$

(ii) No power is absorbed in X_m only in R_m. ($R_1 << R_m$)
Hence input power = power in R_m

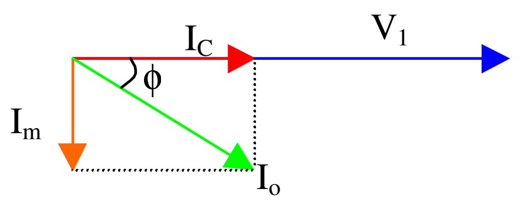
$\therefore\ W_{oc} = V_1^2/R_m, \quad R_m = V_1^2/W_{oc}$

or from a phasor diagram:

$$W_{OC} = V_1 I_o \cos\phi$$

or:

$$I_C = I_o \cos\phi = \frac{V_1}{R_m}$$



therefore:

$$R_m = \frac{V_1}{I_o \cos\phi}$$

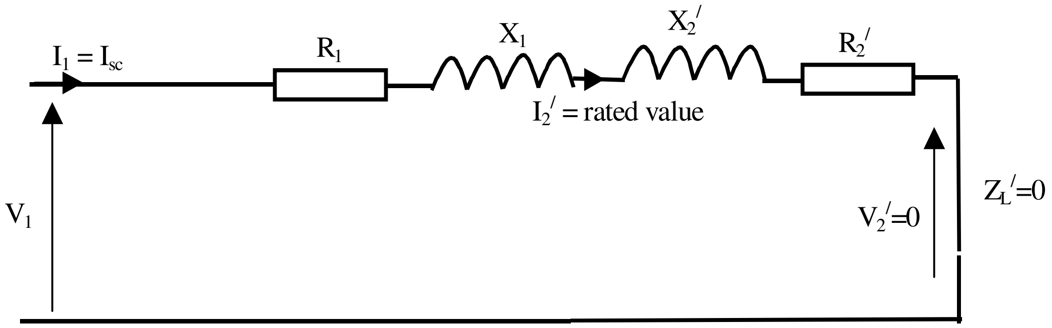
and from phasor diagram:

$$I_m = I_o \sin\phi \qquad \therefore X_m = \frac{V_1}{I_o \sin\phi}$$

Note: do not try to find the series equivalent of R_m and X_m and then sort out R_m and X_m from the test results – It takes too long!

(b) Short Circuit Test

Clearly if this was carried out at full voltage, something would give! The test is carried out at reduced voltage, but rated [full load] current. In this way, heating effects on the winding resistance and any saturation of the leakage reactances would be correctly accounted for. Again either winding can be supplied and the other shorted out. The equivalent circuit then becomes:



Primary fed, secondary short circuit.

Test Procedure:

Apply low voltage to one winding – such that the rated curent flows in the short circuit winding.

Measure:

Supply voltage, V_1, Input current, $I_1 = I_2^{/} = I_{sc}$, Input power, W_{sc}.

Since R_m and X_m are high impedances and test voltage is low, then the magnetising current is negligible. Therefore:

$$\begin{aligned}\overline{V}_1 &= \overline{I}_{sc}\left[(R_1 + R_2') + j(X_1 + X_2')\right] \\ &= \overline{I}_{sc}\left[R_e + jX_e\right]\end{aligned}$$

or:

$$\left|\overline{V}_1\right| = \left|\overline{I}_{sc}\right|\sqrt{R_e^{\,2} + X_e^{\,2}} = I_{sc} Z_e$$

Since no power consumed in X_e then input power $W_{sc} = I_{sc}^{2} \times R_e$

$$R_e = \frac{W_{sc}}{I_{sc}^{\,2}}$$

and:

$$X_e = \sqrt{Z_e^{\,2} - R_e^{\,2}}$$

Example

The following test results were obtained from a 50kVA, 3.3kV:400V transformer:

Open-circuit test: - Carried out with the primary supplied at rated volts:

Primary Volts	= 3.3kV
Secondary Volts	= 400V
Input Power	= 430W
Input Current	= 0.5A

Short-circuit test: – Again with primary winding supplied:

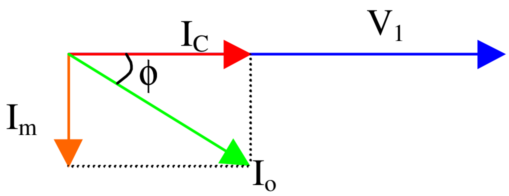
Primary Volts	= 124V
Primary Current	= 15.2A
Power Input	= 525W

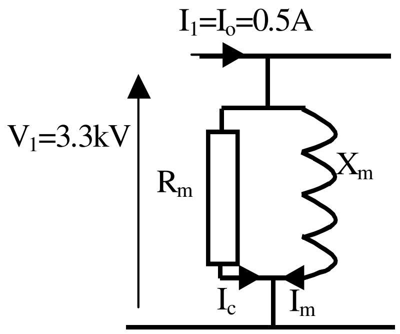
i) Calculate the equivalent circuit parameters referred to the primary

ii) Calculate the efficiency at (a) Full load 0.7pf lagging

(b) 0.5×Full load at 0.7pf lagging

i) On open-circuit the approximate equivalent circuit referred to the primary is:





$$\text{Power Input} = V_1 I_o \cos\phi = 430\text{W}$$

Therefore:

$$\cos\phi = 430 / (3.3\times10^3 \times 0.5) = 0.26$$

and:

$$\sin\phi = 0.965$$

Since:

$$I_o \cos\phi = I_c = V/R_m$$

then:

$$R_m = V / (I_o \cos\phi) = 25.4\ \text{k}\Omega$$

and:

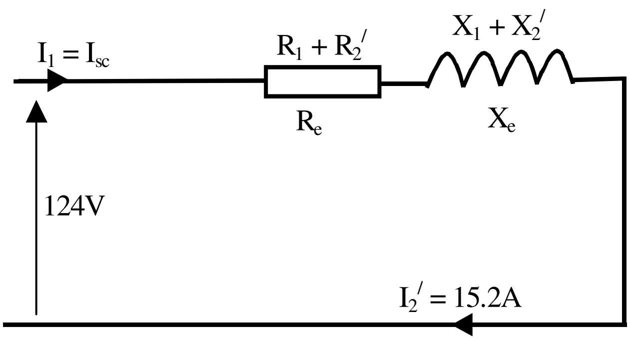
$$I_o \sin\phi = I_m = V / X_m$$

then:

$$X_m = 6.8\ \text{k}\Omega$$

On short-circuit the approximate equivalent circuit referred to the primary is:

Note: 15.2 A = 50kVA / 3.3 kV
i.e. Full load - CORRECT



Now:

$$W = (I_2^/)^2 \times R_e$$

therefore:

$$R_e = 525 / (15.2)^2 = 2.27\ \Omega$$

and:

$$|Z| = \left|\frac{V}{I}\right| = \frac{124}{15.2} = 8.16\Omega$$

and since:

$$|Z| = \sqrt{R_e^2 + X_e^2}$$

then:

$$X_e = \sqrt{8.16^2 - 2.27^2} = 7.84\,\Omega$$

ii) (a) Calculate the efficiency at full load 0.7pf lagging**:**

$$\text{Output power} = 50 \times 0.7 = 35\ \text{kW}$$

$$\begin{aligned} \text{Losses} &= \text{iron losses} + \text{copper losses} \\ &= V^2 / R_m + I_{FL}^2 \times R_e \\ &= 430\ \text{W} + 525\ \text{W} = 955\ \text{W} \end{aligned}$$

therefore:

$$\text{losses} = 0.955\ \text{kW}$$

and:

$$\text{efficiency} = 35 / (35 + 0.955) \times 100\% = 97.34\ \%$$

ii) (b) Calculate the efficiency at 0.5 × Full load at 0.7pf lagging:
On half load (17.5kW) V is constant but output current is halved therefore iron losses are as before and copper losses are reduced to 0.25 of full load value, i.e.:

$$\text{losses} = 430\ \text{W} + 525 / 4 = 561\ \text{W} = 0.561\ \text{kW}$$

and:

$$\text{output power} = 50/2 \times 0.7 = 17.5\ \text{kW}$$

therefore:

$$\text{efficiency} = 17.5 / (17.5 + 0.561) \times 100\% = 96.89\ \%$$

i.e. less efficient at reduced load.

Maximum efficiency occurs when the copper loss = iron loss. In general, a transformer that is designed for a constant load has iron loss = copper loss at full load. A transformer which is designed for a varying load has copper loss > iron loss at full load.

Polyphase Systems.

What is a Polyphase system?

In general a polyphase system consists of a number of ac sources of the same frequency, but at different time phases.

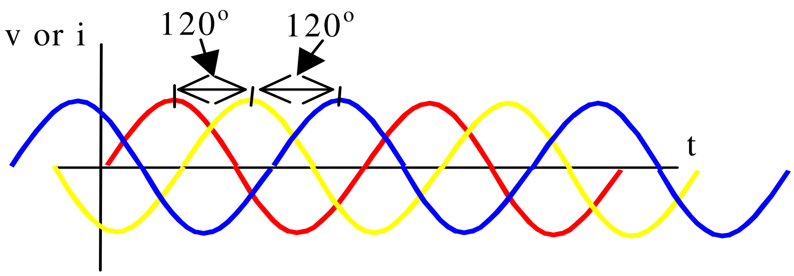
A balanced polyphase system consists of 'n' ac sources of the same amplitude and frequency, displaced in time phase by $360/n^{o}$

The most commonly used system, for reasons we shall see later is the balanced 3-phase system.

Balanced 3 Phase Systems

Representations:

(i) – Waveform of a CRO (not very useful in problem solving)



(ii) – Mathematical representations:
Instantaneous values:

$$v_r = \hat{V}\sin(\omega t)$$

$$v_y = \hat{V}\sin(\omega t - \frac{2\pi}{3})$$

$$v_b = \hat{V}\sin(\omega t - \frac{4\pi}{3})$$

(N.B subscripts 'R' or 'r', 'Y' or 'y' and 'B' or 'b' have traditionally been used to identify the phases of a 3-phase system since Red, Yellow and Blue were the historical colours employed for 3-phase wiring - Neutral was Black. However, since 1/4/06 new 'harmonised' wiring colours are to be used throughout the EU. The 3-phase wiring colours are now Brown, Black and Grey with Blue for the neutral. Take care when connecting new and old wiring!)

Phasor notoation:

$$V_r = V\angle 0 \text{ (reference)}$$
$$V_y = V\angle -120$$
$$V_b = V\angle -240$$

where:

$$V = \frac{\hat{V}}{\sqrt{2}} \quad \text{- rms value.}$$

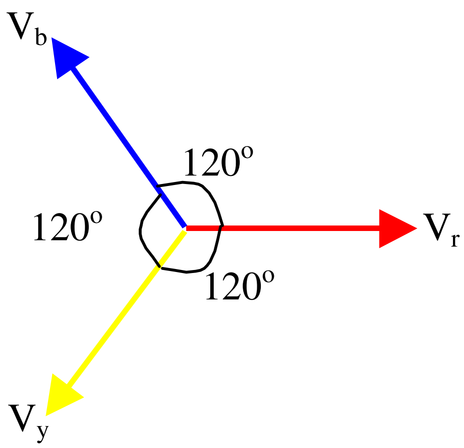
or 'j' notation for these phasors would give:

$$V_r = V$$
$$V_y = V\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$
$$V_b = V\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

(These techniques are mainly used for numerical problem solution)

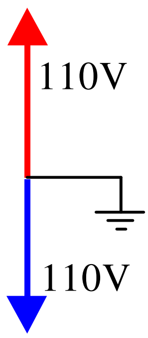
(iii) - Phasor Diagram representation:

This is a very useful method of visualising the problem prior to numerical solution. A Phasor sketch is highly recommended.



NOTE: The USA domestic supply has a 'two phase' supply where each phase provides 110V, and the two are 360/2 (180°) displaced to give either 220V or a 110V supply.

- in many systems a 'so-called' two phase supply consists of two phases (ac supplies) displaced by 90° (not 180°) and is actually an unbalanced 4 phase (360/4) system. This is historic nomenclature and should be treated with care.

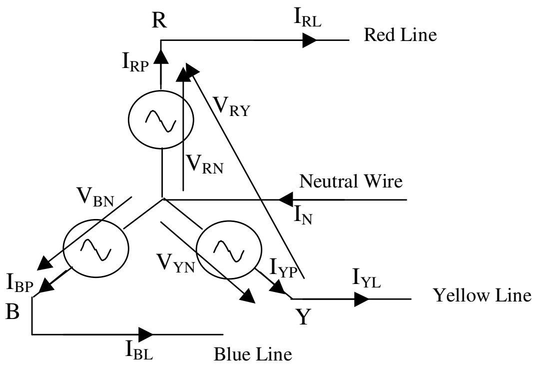


3 Phase Connections

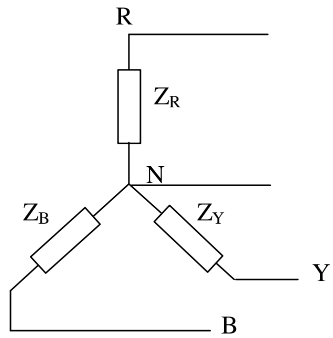
There are 2 ways of connecting a 3-phase supply or load, star (or Y) connection and delta connection.

A) STAR CONNECTION

Three phases connected to a common STAR or NEUTRAL point (N).



4-wire star connection



4-wire load connection
(Balanced if $Z_R = Z_Y = Z_B$)

- I_{RP}, I_{YP}, I_{BP} – known as PHASE CURRENT, I_P
- I_{RL}, I_{YL}, I_{BL} – known as LINE CURRENT, I_L
- V_{RY}, V_{BR}, V_{YB} – known as LINE VOLTAGE, V_L
- V_{RP}, V_{YP}, V_{BP} – known as PHASE VOLTAGE, V_P
- I_N – known as NEUTRAL CURRENT

NB: for a balanced system, with a balanced load, all phase quantities have the same magnitude, and all line quantities have the same magnitude.

Star Relationships:

From the diagram: Clearly it can be seen that at any node, R, Y, or B,

(i) $I_{RP} = I_{RL}$ etc. i.e. $|\text{LINE CURRENT}| = |\text{PHASE CURRENT}|$

(ii)

$$\begin{aligned}\bar{I}_N &= \bar{I}_{RP} + \bar{I}_{YP} + \bar{I}_{BP} \\ &= I_P\angle 0 + I_P\angle -120 + I_P\angle -240 \\ &= 0\end{aligned}$$

for 3 equi-spaced phasors

i.e. In a balanced system, there is no neutral current.

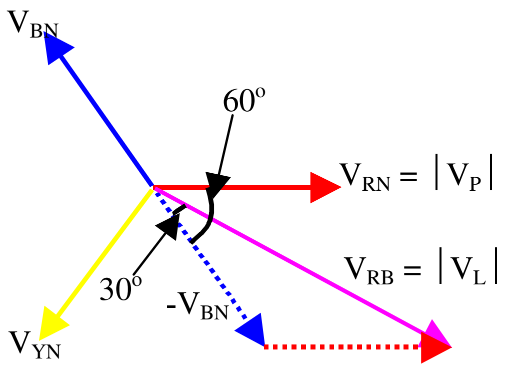
(iii) From the Phasor diagram:

Line voltage

$$\overline{V}_{RB} = \overline{V}_{RN} + \overline{V}_{NB}$$
$$= \overline{V}_{RN} - \overline{V}_{BN}$$

from Phasor Diagram

$$|V_L| = 2|V_P|\cos(30^o)$$
$$= 2|V_P| \times \frac{\sqrt{3}}{2} \quad = \quad \sqrt{3}V_P$$
$$\therefore |V_L| = \sqrt{3}|V_P|$$



Note that V_{RB} is 30^o lagging on V_{RN} etc.

(iv) Power per phase:

$$P_p = V_p I_p \cos\phi$$

where ϕ is the phase angle of the load, which for a balanced system is identical. Therefore:

$$\text{Total power, } P = 3 \times P_p$$
$$P = 3V_P I_P \cos\phi$$

or, substituting for $|V_L| = \sqrt{3}|V_P|$ and $I_P = I_L$

$$P = \sqrt{3} V_L I_L \cos\phi$$

Similarly, kVA/Phase = $V_P I_P$ and $kVA_{total} = 3V_P I_P = \sqrt{3}\, V_L I_L$
KVAr / Phase = $V_P I_P \sin\phi$ and $kVAr_{total} = 3V_P I_P \sin\phi = \sqrt{3}\, V_L I_L \sin\phi$

Implications for Power Supplies

Typical distribution at 400V (line voltage)

Domestic (Single phase supply)

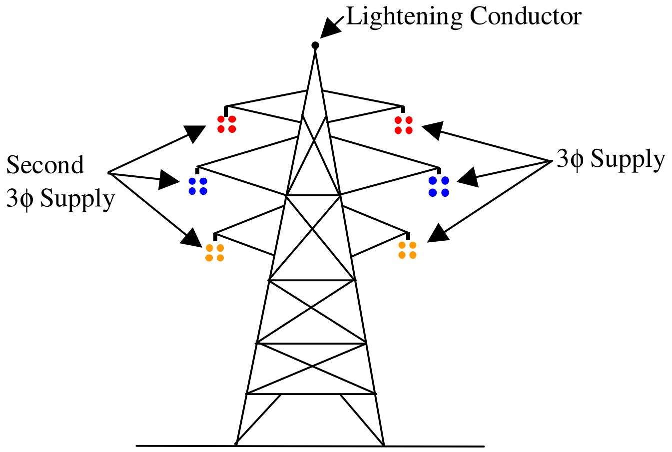
- Each customer is supplied with one line to neutral single phase supply
- $$\therefore 1\phi \quad \Rightarrow \frac{400}{\sqrt{3}} = 230\,V \;\; (\,phase \;\; voltage\;)$$
- Note for single phase supplies the 4th (Neutral wire) is retained to give the required system.
- Usually groups of houses are supplied from each phase and the net result is integrated over a large number of houses etc.

Industrial (single and 3ϕ supplies)

- Large customers usually connected directly to a 3ϕ supply (can be at a variety of voltages, 3.3kV, 11kV, 33kV or even 132kV for very large factories)
- Supply can be 3-wire or 4-wire (Neutral needed to give 1ϕ supplies within the factory.

Transmission

– Term used for bulk transfer of power around the country. Note the lack of a neutral wire on a typical tower configuration.

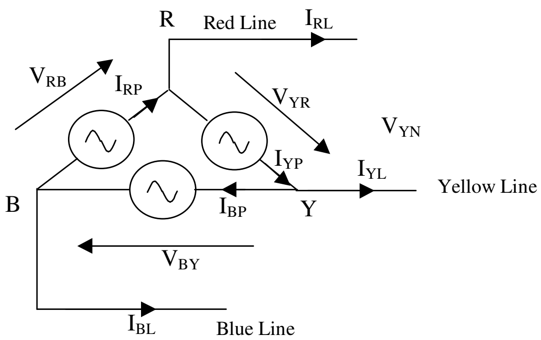


Typical Tower

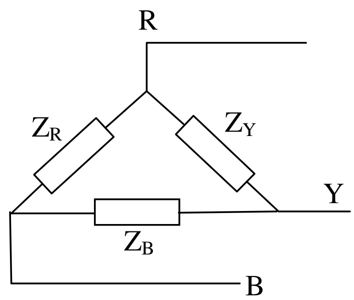
- As shown previously, if the system is balanced, no neutral wire is required. Hence compared with a 1ϕ system, where a phase wire is needed together with a neutral wire (therefore 2 wires needed), 3ϕ transmits 3× the power of the single phase connection for only 1 extra wire. – One of the reasons for use.

- Typical transmission levels 275kV, 400kV in the UK, (higher in USA where distances are longer)
- Having 2 systems in parallel improves the security of the supply

B) DELTA CONNECTION



Delta Connected Generator
(Only 3 wires possible – no Neutral)



Delta Connected Load
(Balanced if $Z_R = Z_Y = Z_B$)

Delta Relationships:

(i) From the diagram, Line voltage = Phase Voltage, i.e.:

$$|V_L| = |V_P|$$

(ii) Voltage around the delta loop

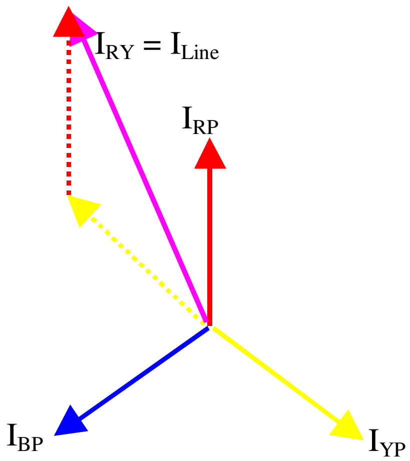
$$= V_{RB} + V_{YR} + V_{BY} = 0$$ - 3 equi-spaced Phasors

(iii) Line Current

$$\bar{I}_{RL} = \bar{I}_{RP} - \bar{I}_{YP}$$

By a similar argument to some relationships for the star voltage case:

$$|I_L| = \sqrt{3}|I_P|$$



(iii) Again power per phase:

$$P_p = V_p I_p \cos\phi$$

Therefore:

$$\text{Total power, } P = 3 \times P_p$$

$$P = 3V_P I_P \cos\phi$$

or, substituting for $|V_L| = |V_P|$ and $I_L = \sqrt{3} I_P$

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$\text{Total kVA} = \sqrt{3}\, V_L I_L$$
$$\text{Total kVAr} = \sqrt{3}\, V_L I_L \sin\phi \text{ etc.}$$

Examples: - Note: In year 1 **all** problems are balanced and hence all phase and line magnitudes are equal, therefore the problems may be solved on a ***per phase*** basis.

Example 1

A 3ϕ load is connected to a 400V, 3ϕ supply and takes a total power of 2.7kW with a line current of 60A at a lagging power factor. If the load can be considered as a series impedance per phase, calculate the load impedance components

(a) If the load is star connected

(b) If the load is delta connected.

Approach:

(i) When a 3ϕ supply is specified as 400V, this means that the magnitude of the voltage between ***any pair of lines*** is 400V – but they will be displaced by 120° in phase.

(ii) The problem is balanced therefore all phases behave the same, but are 120° out of phase. Hence for all the above (star or delta):

$$\text{Power per Phase} = 2.7/3 = 0.9\text{kW}$$

Also (star or delta):

$$2.7\text{Kw (Total power)} = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 60 \times \cos\phi$$

Therefore:

$$\cos\phi = (2.7 \times 10^3) / (\sqrt{3} \times 400 \times 60) = 0.065 \text{ lagging}$$

and the phase angle is found to be:

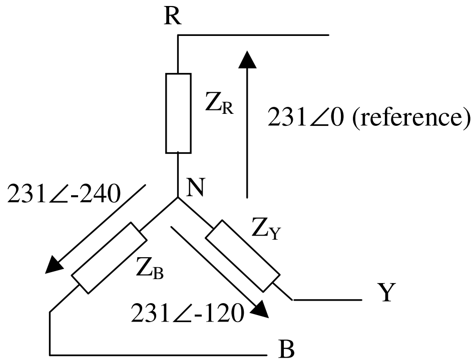
$$\phi = \cos^{-1}(0.06) = -86.3^{o}$$

Since it is lagging (current lags voltage), Impedance must be **inductive**.

(a) Star Connected Load

Load impedance connected in star across the 400V supply, therefore:

$$|V_P| = \frac{400}{\sqrt{3}} = 231V$$



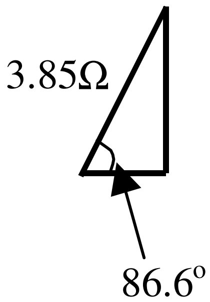
Taking the red phase as the reference and doing all the calculations on this phase (on a per-phase basis)

For Star connection:

$$|I_L| = |I_P| = 60A$$

and:

$$I_{RL} = 60\angle{-\phi} = 60\angle{-86.3}$$



therefore:

$$Z_P = V_P / I_P = (231\angle 0) / (60\angle{-86.3})$$
$$= 3.85\angle 86.3$$

then:

$$R_P = 3.85 \cos(86.3) = 0.25\Omega$$

and:

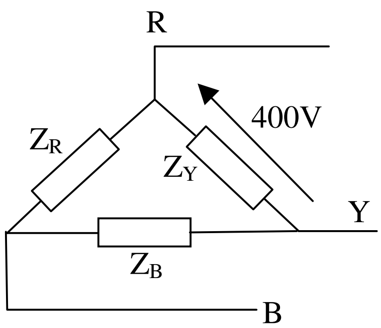
$$X_P = 3.85 \sin(86.3) = 3.84\Omega$$

(check: $P = 3 \times I_P^2 \times R_P = 3 \times 60^2 \times 0.25 = 2700$ W)

(b) Delta Connected Load

Now load impedances connected in delta connection across the 400V supply, therefore:

$$|V_{Ph}| = |V_{Line}| = 400V$$



and for Delta:

$$|I_{Ph}| = |I_{Line}|/\sqrt{3}$$
$$= 60/\sqrt{3} = 34.64A$$

Taking Red phase as the reference:

$$Z_{Ph} = (400\angle 0) / (34.64\angle{-86.6})$$
$$= 11.55\Omega \quad (3\times Z_{Ph} \text{ for the star connection})$$

and:

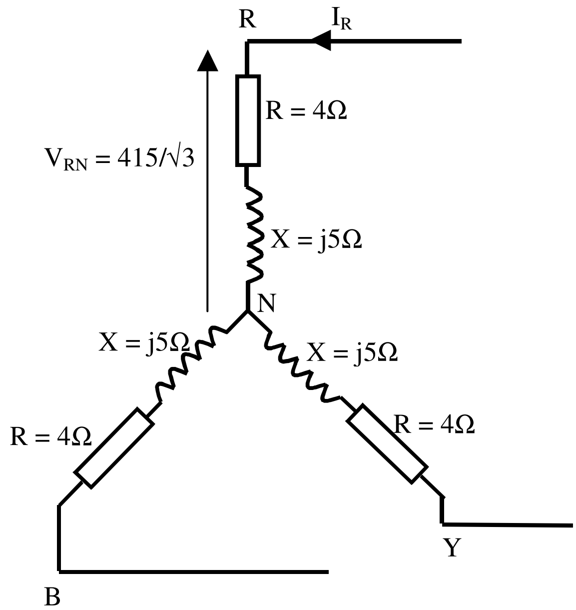
$$R_{Ph} = 3 \times (R_{Ph} \text{ for the star connection}) = 0.75\Omega$$
$$X_{ph} = 3 \times (X_{ph} \text{ for the star connection}) = 11.52\Omega$$

Example 2
More Typical (ex-exam paper question).

A 3ϕ Star-connected motor has an input impedance of (4+j5)Ω per phase and is connected to a 415V, 50Hz 3ϕ supply.

(i) Calculate the total input power, kVA and kVAr to the motor, and if it is 90% efficient, calculate the mechanical output power and power lost as heat.
If the motor is running at 1400rpm calculate the shaft torque.

(ii) A user of the motor decides to improve its power factor by inserting 700μF capacitors in each line of the supply. Calculate the new power factor. Comment on the advisability of this technique.



Solution

(i) Star Connected:

$$|V_{phase}| = |V_{Line}|/\sqrt{3}$$

$$|V_{phase}| = \frac{415}{\sqrt{3}} = 240\text{ V}$$

and Phase current = Line current:

$$|I_{Ph}| = |I_{Line}|$$

Now, Taking V_{RN} as reference = $240\angle 0$, and solving on a per phase basis:

$$I_R = \frac{V_{RN}}{Z_{phase}} = \frac{240\angle 0}{4+j5} = \frac{240\angle 0^\circ}{6.4\angle 51.3} = 37.5\angle -51.3^\circ\ A$$

Similarly:

$$I_Y = \frac{V_{YN}}{Z_{phase}} = \frac{240\angle -120^\circ}{6.4\angle 51.3^\circ} = 37.5\angle -171.3^\circ\ A$$

etc..

Hence the power factor = $\cos(-51.3^o) = 0.625$ lagging.

Power per phase:

$$P_p = V_pI_p\cos\phi = 240\times 37.5\times 0.625$$

and total power:

$$P_T = 3P_p = \sqrt{3}\ V_LI_L\cos\phi = 415\times 37.5\times\sqrt{3}\times 0.625 = 16.85\text{kW}$$

Total kVA:

$$S_T = \sqrt{3}\times V_L\times I_L = (3\times Vp\times Ip) = 26.96\text{kVA}$$

and total kVAr:

$$Q_T = \sqrt{3}\times V_L\times I_L\ \sin\phi = 21\text{kVAr}$$

If motor is 90% efficient, then motor mechanical output power:

$$P_{OUT} = 0.9\times16.85 = 15.16\text{kW}$$

and:

$$\text{losses (heat)} = 0.1\times16.85 = 1.685\text{kW}$$

The shaft torque is obtained from:

$$P_{mechanical} = \text{Torque (Nm)} \times \text{Speed (rads}^{-1}\text{)} \ \text{(Watts)}$$
$$= T \times (1400\times2\pi)/60$$

Hence:

$$T = P_{mechanical} \times 60 / (1400\times2\pi) = 15160 \times 60 / (1400\times2\pi)$$
$$= 103.4 \text{ Nm}$$

(ii) now consider what happens with 700μF capacitors added to each line. The new impedance per phase is given by:

$$Z'_{phase} = (R + jX_L) - jX_c$$

where:

$$X_c = 1/(2\pi fC) = 4.55\Omega$$

therefore:

$$Z'_{phase} = 4 + j5 - j4.55\ \Omega \ = 4 + j0.45 = 4.02\angle 6.4^o$$

hence new phase current:

$$I_R = \frac{240\angle 0}{4.02\angle 6.4} = 59.7\angle -6.4 \text{ A}$$

and new power factor:

$$pf = \cos(-6.4) = 0.99 \text{ lag}$$

(Note large increase in phase current through motor)

HOWEVER: new input power per phase:

$$P_p = V_p I_p \cos\phi = 240 \times 59.7 \times 0.99 = 14.24\text{kW}$$

Hence:

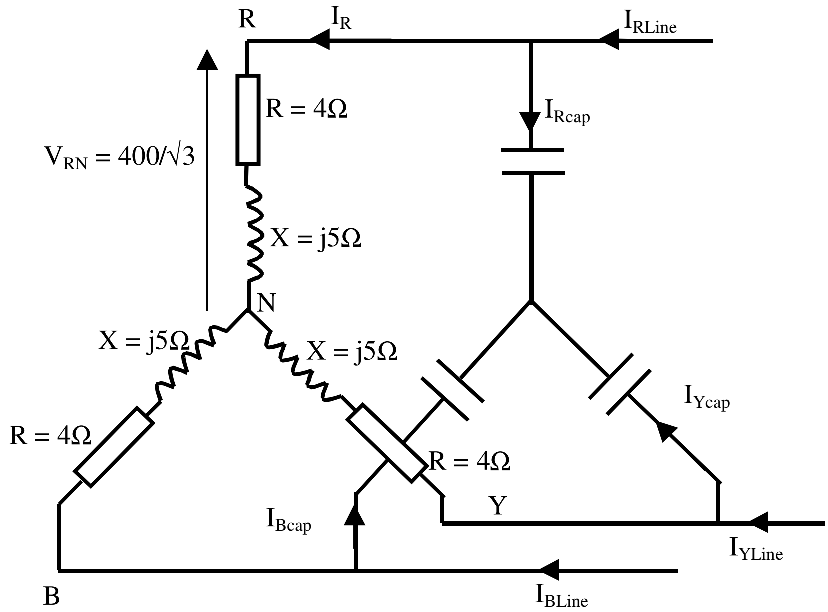
$$\text{Total input power} = 3 \times 14.24 = 42.72\text{kw (c.f. 16.85kW)}$$
$$\text{Total mechanical output power} = 0.9 \times 42.72 = 38.45\text{kW (c.f. 15.16kW)}$$
$$\text{Total losses} = 0.1 \times 42.72 = 4.27\text{kW (c.f. 1.685kW)}$$

- Hence motor would rapidly overheat!!

Correct Solution:

To obtain unity power factor operation – **Always** put capacitors in parallel with the load.

The power factor correction capacitors may either be star-connected or delta-connected. As the motor connection is now unchanged, the calculations for the currents into the motor are now the same as without the capacitors, however, the line currents into the total system is affected by the capacitors.



Again, looking at one phase because the system is balanced, from the previous calculations for the un-corrected motor above, we have power per phase:

$$P_m = V_p I_p \cos\phi = 240 \times 37.5 \times 0.625 = 5.625\text{kW}$$

and per-phase kVAr:

$$Q_m = \sqrt{3} \times V_L \times I_L \sin\phi = 21/3 = 7 \text{ kVAr}$$

This will remain un-changed after adding the pf correction capacitors in parallel. To correct the line currents to unity pf, the capacitors must provide a –ve Q equal and opposite to Q_m, i.e.:

$$Q_c = -7 \text{ kVAr}$$

and:

$$Q_c \text{ per phase} = V_{ph} I_{ph} \sin\phi \text{ } (\sin\phi = 1 \text{ for capacitors})$$

Assuming capacitors are star connected:

$$V_{ph} = V_{Line}/\sqrt{3} = 240\text{V} = V_{cap}$$

therefore required capacitor current:

$$I_{cap} = 7\times10^3 / 240 = 29.17 \text{ A}$$

giving:

$$X_c = V_{cap}/I_{cap} = 8.23\Omega = 1/2\pi fC$$

Hence the value of capacitance required is:

$$C = 387\mu\text{F / Phase}$$

- Could have used:

$$Q_c = V_c^2/X_c = 240^2/X_c \quad \text{- Same Answer}$$

If C is delta connected, then $V_c = 415$ etc. $X_c = 3\times8.2 = 24.6\Omega$ and $C = 130\mu f$