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The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2015-16 (2.0 hours)

EEE349 Power Engineering Electromagnetics

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

Physical constants:

$\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$

$\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$

1. **a.** The variation in the electric flux density in a region of space of permittivity ε_0 is given by: $\vec{D} = 3x^2y\,\overrightarrow{u_x} + 4x^2y^2\overrightarrow{u_y} + 3y^2\overrightarrow{u_z}$ C/m^2

Calculate the electric field and the divergence of $\vec{D}$ at the point (1,3,4)m. **(4)**

b. A single isolated circular conductor of radius R_c carries a uniform volume charge density q. Starting from Gauss's Law, derive expressions for the variation in the electric field $\vec{E}$with distance from the centre of the conductor,r, for:
i) The region within the conductor (i.e. $r < R_c$)
ii) The region outside the conductor (i.e. $r > R_c$) **(6)**

c. Figure 1.1 shows a single-phase, two-wire power distribution line, in which the two wires have the same radius R_c and volume charge densities of $+q_1$ and $-q_2$. By application of the principle of superposition, and assuming q_1=q_2, derive an expression for the potential difference between the two wires. **(7)**

Figure 1.1 Single-phase, two wire power distribution line

d. Calculate the capacitance per unit length for the arrangement shown in Figure 1.1 if R_c=10mm and D=500mm and that the permittivity of the region outside the conductors is ε_0. **(3)**

2. **a.** The electric potential in region of space with permittivity ε_0 is given by:

$V = (3yx^3 + 5zx + 7xyz + 12)\ V$

Calculate the following at the point (x,y,z) = (3,2,1)m

i) The electric field strength **(3)**

ii) The charge density **(3)**

b. The variation in electric field strength in a region of space is given by:

$\vec{E} = 3x^2y\,\overrightarrow{u_x} + 4x^2y^2\overrightarrow{u_y}\ \ V/m$

Calculate the electric potential between the points (2,2)m and (5,5)m **(4)**

c. A sphere of R_s carries a uniform volume charge density of q and is surrounded by a region of space of permittivity ε_0.

i) Starting from Gauss's Law, derive an expression for the electric flux density at a distance r from the centre of the charged sphere

ii) If the charged sphere has a radius of 10mm and carries a uniform volume charge density of 1×10^{-3}C/m^3, calculate the distance from the centre of the sphere at which the magnitude of the electric field strength is 10,000 V/m. **(5)**

d. A 50m long high voltage cable is comprised of a central circular conductor which is surrounded by an insulating layer with a relative permittivity of 6.0 and a minimum electric field breakdown strength of $30 \times 10^6 V/m$. When this high voltage cable is installed, the 50m length has a capacitance to ground of 0.334μF. Calculate the minimum radius of the central circular conductor if the cable is to sustain a DC voltage of 30kV without breakdown of the insulation. **(5)**

3. **a.** Starting from the vector forms of Faraday's and Ampere's Laws for time varying fields shown below, and listing any assumptions that you make, derive the diffusion equation for a time varying magnetic fields at typical power system frequencies:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

You may find it useful to make use of the following identity for a general vector $\vec{F}$:

$$\nabla \times \left(\nabla \times \vec{F}\right) = \nabla\left(\nabla . \vec{F}\right) - \nabla^2 \vec{F}$$ **(7)**

b. Figure 3.1 shows a simplified cross-section through a single iron lamination which has a thickness *2b*. The lamination is exposed to a sinusoidally time varying magnetic flux which is oriented along the z-axis as shown. The maximum value of the time varying magnetic field strength at each surface is H_s. Starting from the diffusion equation, show that the x component of current density in the lamination is given by:

$$J_x = \alpha H_s \frac{sinh(\alpha y)}{\cosh(\alpha b)} e^{j\omega t}$$

where $\alpha^2 = j\omega\sigma\mu$ **(9)**

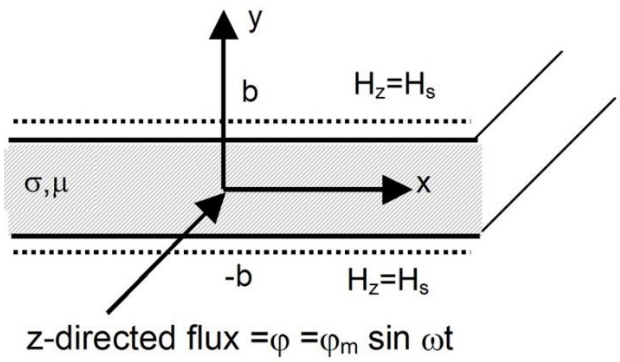

Figure 3.1 Cross-section through an iron lamination of thickness 2*b*

c. Calculate the maximum lamination thickness which it would be good design practice to use if the lamination has a *relative* permeability of 500 and an electrical conductivity of 3×10^6 Sm^{-1}, and is exposed to a applied magnetic field in the z-direction which has a frequency of 100Hz. **(4)**

4. a. The magnetic vector potential in a region of space of magnetic permeability μ_0 is given by:

$$\vec{A} = 8y^3x\,\overrightarrow{u_x} + 14xz\,\overrightarrow{u_y} + 3xy^3z^2\,\overrightarrow{u_z}\ \ Wb/m$$

where $\overrightarrow{u_x}, \overrightarrow{u_y}$ and $\overrightarrow{u_z}$ are unit vectors in a Cartesian coordinate system

Calculate the magnetic field strength, $\vec{H}$, as the point (0.1,0.7,0.15) m. **(4)**

b. The magnetic field strength in a region of space of magnetic permeability μ_0 is given by:

$$\vec{H} = 6y^2z\,\overrightarrow{u_x} + 4yx^3\,\overrightarrow{u_y} + 2x^2z^3\,\overrightarrow{u_z}\ \ A/m$$

where $\overrightarrow{u_x}, \overrightarrow{u_y}$ and $\overrightarrow{u_z}$ are unit vectors in a Cartesian coordinate system.

Calculate the current density at the point (2,5,2) m. **(4)**

c. Figure 4.1 shows the boundary between a region of air and an idealised region of iron with infinite permeability. Assuming that the problem domain is two-dimensional and there is no current present at the interface, then:
i) State two magneto-static boundary conditions that could be imposed on this boundary.
ii) State a boundary condition that can applied to the magnetic vector potential in Region 1. **(3)**

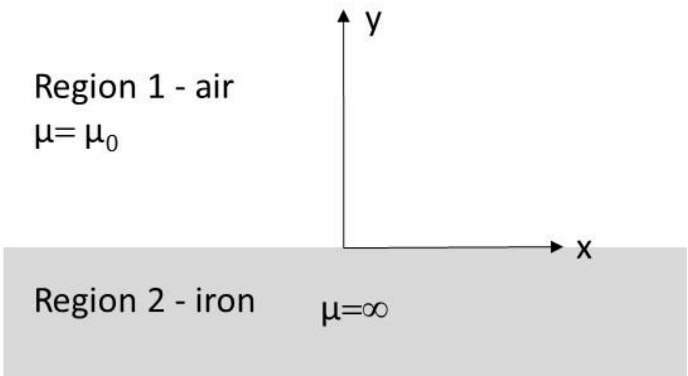

Figure 4.1. Two-dimensional interface between an air and iron region

d. A surface current density whose spatial variation with x is defined by the following expression is now introduced at the boundary shown in Figure 4.1:

$J_z = 100\ cos(x)$ A/m

Calculate the magnetic field strength component H_x in the air region at the boundary at the point x = 1.32m **(3)**

e. A large cable carrying a DC current of +5000A enters a manufacturing plant. A sensitive piece of measuring equipment in the plant must be subjected to a magnetic field of <450μT in order to function properly.

Starting from Ampere's Law derive an expression for the minimum distance that the instrument must be located away from the incoming cable in order to function properly. **(4)**

f. What two practical steps could be taken to reduce this distance? **(2)**

GWJ/ JB