EEE224/227
Solutions to Tutorial sheet 5

1. If $\omega_1 = 2\pi \times 10^5$ and $\omega_2 = 2\pi \times 5 \times 10^3$ then output from non-linear circuit is

$0.25 \sin \omega_1 t + 0.175 \sin \omega_2 t + 0.1 [0.25 \sin^2 \omega_1 t + 0.1225 \sin^2 \omega_2 t + 2 \times 0.5 \times 0.35 \sin \omega_1 t \sin \omega_2 t]$

Hence output frequencies are 5 kHz, 10 kHz, 95 kHz, 100 kHz, 105 kHz and 200 kHz

After band-pass filtering have 95, 100 and 105 kHz signals i.e. full AM where

$E_c = 0.25$ and $mE_c / 2 = 0.1 \times 0.5 \times 0.35$. Hence **m = 0.14**

If m = 0.14 when $V_{tone} = 0.7$, then **m = 1 when V_{tone} = 5 V p-p**

Maximum tone frequency is 35 kHz for AM signal to pass through the band-pass filter (in practice a bit less because of non-ideal filter characteristics).

2. For the envelope demodulator, $CR \le \dfrac{\sqrt{1-m^2}}{m\omega_m}$

(a) Hence if $\omega_m = 2\pi \times 10^3$ then for **m = 0.1, CR = 1.58 x 10^{-3}**

m = 0.3, CR = 5.06 x 10^{-4} and **m= 0.9, CR = 7.7 x 10^{-5}**

(b) From the formula, **CR = 0 when m = 1** and when ***CR = 1 / ω_m*** then **m = 0.707**

(c) Rate of discharge of C must be slow enough for voltage not to fall much between carrier peaks, yet large enough for it to follow the fastest rate of change of envelope voltage. Need f_c greater than 100 f_m.
If capacitor voltage falls too slowly, it cannot follow the envelope troughs which then become distorted. If it falls too quickly, it follows the carrier voltage, not the envelope i.e. demodulation does not occur.

(d) Rule of thumb is $\omega CR = 1$ where ω is the highest baseband frequency being detected. Here $\omega = 2\pi \times 10^4$. Hence $CR = 1.59 \times 10^{-5}$.
If m = 1, then we know CR = 0. Hence CR limits are **0 and 1.59 x 10^{-5}**

(e) Fix C first from preferred values so that R lies between 1 and 10 kΩ.
Then R' ≥ 10 R and C' ≥ 10 C

3. The modulation index β = maximum frequency deviation / modulation frequency

(i) $\beta_1 = 75 \times 10^3 / 30 =$ **<u>2500</u>**, $\beta_2 = 75 \times 10^3 / 15 \times 10^3 =$ ***<u>5</u>***

(ii) Carson bandwidth = 2 ($\Delta f + f_m$) = 2 (75 + 15) = **<u>180 kHz</u>**

(iii) LO frequency = 90 - 10.7 = **<u>79.3 MHz</u>** or 90 + 10.7 = 100.7 MHz (less likely as this falls in the FM broadcast band)

(iv) **<u>225 kHz for UK FM broadcasting</u>**

4. (i) Telephone channel bandwidth = 4 kHz and is sampled at 8 kHz

Number of samples / sec = 12 x 8 x 8000 = 768,000. Hence bandwidth = **<u>768 kHz</u>**

(ii) Bandwidth = 400 x 6 x 8000 = **<u>19.2 MHz</u>**

(iii) Each channel requires 8 x 2 x 4000 = 64,000 bits/sec

If there are N channels then the overall bit rate is N x 64,000 = 2.048×10^6

Hence the number of channels = **<u>32</u>**

In practice only 30 of these would be used for customer traffic, the other two being used for housekeeping.

The time for one frame = 32 x 8 / 2.048×10^{-6} = **<u>125 µsec</u>**

(iv) Baud rate = 1 / (time to transmit one symbol)   symbol here is a 0 or a 1

Hence time / symbol for 100, 300, 2400 and 9600 baud = 1/100 sec, 1 / 300 sec, 1 / 2400 sec and 1 / 9600 sec. Hence are assuming baud rate is equivalent to bits / sec. Then for NRZ pulses, the minimum bandwidths are **<u>100 Hz, 300 Hz, etc.</u>**

(v) Bits / sec = 1 / (bit length) e.g. 1/16 MHz, 1/125 MHz, 1/ 1024 MHz. Then for NRZ pulses, minimum bandwidth = **<u>62.5 kHz, 8 kHz, 976.6 Hz</u>**

5. (i) Quantisation intervals for 4, 6 and 8 bits / sample = **<u>(5 / 15) V, (5 / 63) V, (5 / 255) V</u>**

(ii) Largest signal = 111111, smallest signal = 000000, 2.5V ≈ 100000

(iii) 10000000 = 5 x 128 / 255 = **<u>2.5098V</u>**

01000000 = 5 x 64 / 255 = **<u>1.2549V</u>**

00011010 = 5 x 26 / 255 = **<u>0.5098V</u>**

11100101 = 5 x 229 / 255 = **<u>4.4902V</u>**

(iv) For NRZ pulses, bandwidth = 2 x 6 x 10^3 x 8 = **<u>96 kHz for 8 bit quantisation</u>**

and = $2 \times 6 \times 10^3 \times 12$ = **144 kHz for 12 bit quantisation**