MSc(Eng) Wireless Communication Systems

Module EEE-6431: Broadband Wireless Techniques

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Syllabus Highlights

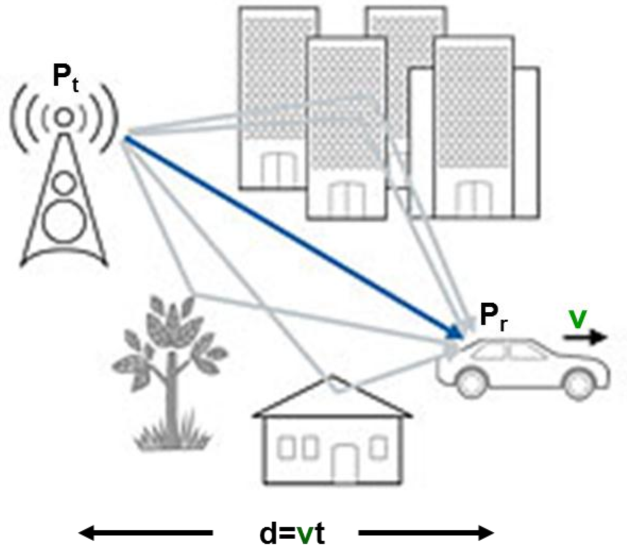
1. **Introduction - Overview of Broadband Wireless Systems**
2. **Signal Propagation, Pathloss Models and Shadowing**
3. **Statistical Fading Models: Narrowband & Wideband Fading**
4. **Capacity of Wireless Channels**
5. **Multicarrier Modulation**
6. **Spread Spectrum and CDMA**

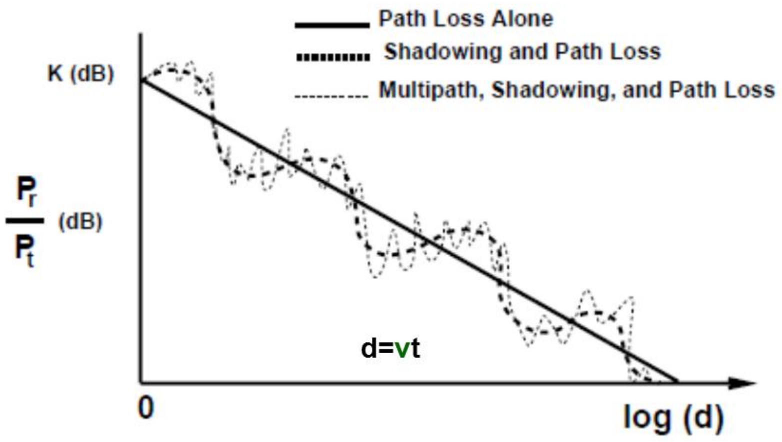
Section 2 Review

1. **Path loss models**
2. **Power fall-off with distance is proportional to d^2 in free space, d^4 in 2-ray model**
3. **Main characteristics of path loss captured in simple model $P_r = P_t K(d_0/d)^\gamma$**
4. **Random attenuation due to shadowing modelled as log-normal**
5. **Combined path loss and shadowing leads to outage**
6. **Cellular coverage area - %age of locations within a cell that are not in outage**

Section 3 Outline

1. Time-Varying Channel Impulse Response
2. Narrowband Fading Models
3. Auto/Cross Correlation & Power Spectral density
4. Envelope & Power Distributions
5. Level Crossing Rate & Average Fade Duration
6. Wideband Fading Models





3. Statistical Fading Models: Narrowband Fading

Introduction: We look at *statistical* fading models for the constructive and destructive addition of different multipath components introduced by the channel. The multipath channel is modelled by a random time-varying impulse response.

- **If a single pulse is transmitted over a multipath channel the Rx signal will appear as a *pulse train*, with each pulse in the train corresponding to the LOS component or a distinct multipath component from a distinct scatterer or cluster of scatterers.**
- **An important characteristic of a multipath channel is the *time delay spread* caused to the Rx signal. This delay spread equals the time delay between the arrival of the first received signal component (*LOS or multipath*) and the last received signal component associated with a single transmitted pulse.**
- **If the delay spread is small compared to the *inverse of the signal bandwidth,* then there is little time spreading in the Rx signal. When the delay spread is large, there is significant time spreading of the Rx signal which leads to substantial signal distortion.**
- **A multipath channel may have a time-varying nature. This time variation arises because either the transmitter or the receiver is moving, and therefore the location of reflectors in the transmission path, which give rise to multipath, will change over time.**
- **If pulses are repeatedly transmitted from a moving transmitter, changes in the amplitudes, delays, and the number of multipath components corresponding to each pulse will be observed.**

3. Statistical Fading Models: Narrowband Fading

Time varying Channel Impulse Response: From Section 2 the Tx signal was defined as

- **Transmitted Signal of bandwidth B_u & carrier frequency f_c is:** $s(t) = Re\{u(t)e^{j2\pi f_c t}\}$

$$\therefore s(t) = Re\{u(t)e^{j2\pi f_c t}\} = Re\{u(t)\}cos(2\pi f_c t) - Im\{u(t)\}sin(2\pi f_c t)$$

- **Received signal is the sum of the LoS path and all resolvable multipath components:**

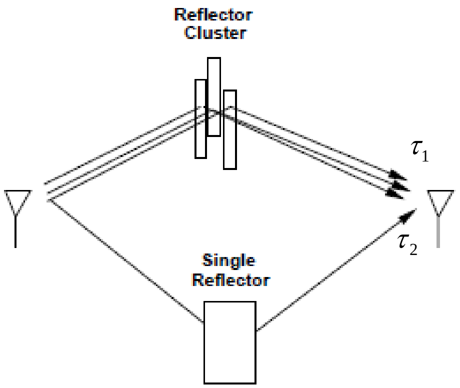
$$r(t) = \mathrm{Re}\left\{\sum_{l=0}^{L(t)-1} \alpha_l(t)u(t-\tau_l(t))e^{j\left(2\pi f_c(t-\tau_l(t))+\phi_{D_l}(t)\right)}\right\}$$

Where $l = 0$ denotes the LoS path, $L(t)$ = number of resolvable multipaths, $\tau_l(t) = \frac{r_l(t)}{c}$ denotes the *l-th* path delay for path length $r_l(t)$, $\phi_{D_l}(t)$ denotes the *l-th* path Doppler phase shift, and $\alpha_l(t)$ denotes the *l-th* path amplitude

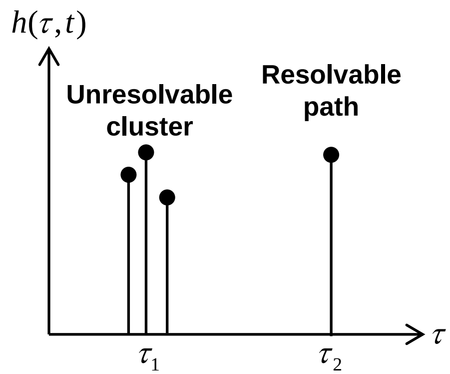
- **Resolvable Multipath Components: Two multipath components with delays τ_1 and τ_2 are resolvable if : $|\tau_1 - \tau_2| \gg \frac{1}{B_u}$ where B_u is the signal bandwidth (i.e. Rx frontend B/W)**
- **Unresolvable Multipath Components: If $|\tau_1 - \tau_2| \ll \frac{1}{B_u}$ then the two multipath components will not be resolvable. Such paths cannot be separated at the receiver making $u(t-\tau_1) \cong u(t-\tau_2)$. Such unresolvable components are combined into a single multipath component at delay $\tau \cong \tau_1 \cong \tau_2$.**
- **A narrowband channel is one with *unresolvable* multipaths while a wideband channel is one with *resolvable* multipaths.**

3. Statistical Fading Models: Narrowband Fading

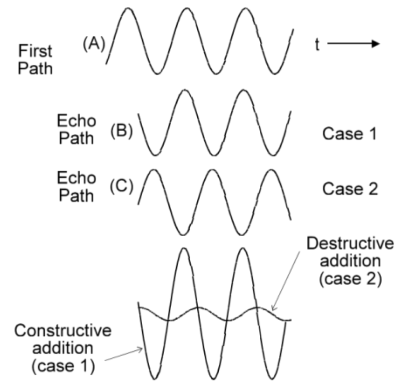
Time varying Channel Impulse Response:



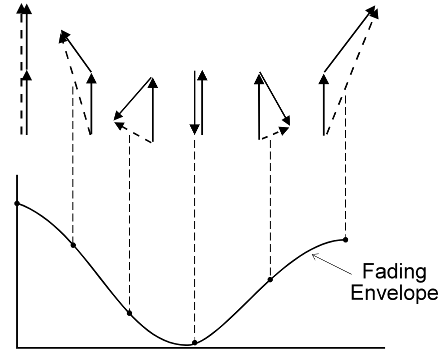
A Single Reflector & a Reflector Cluster



Discrete Channel Impulse Response



Constructive and Destructive Addition of Two Transmission Paths



Illustrating How the Envelope Fades as Two Incoming Signals Combine with Different Phases

Let $\phi_l(t) = 2\pi f_c \tau_l(t) - \phi_{D_l}(t)$ for simplification then

$$r(t) = \mathrm{Re}\left\{ \sum_{l=0}^{L(t)-1} \alpha_l(t) e^{-j\phi_l(t)} u(t - \tau_l(t)) e^{j2\pi f_c t} \right\}$$

In this expression the path amplitude $\alpha_l(t)$ is a function of path loss and shadowing while $\phi_l(t)$ depends on the delay and Doppler spread – both variables are governed by stationary, ergodic and independent random processes.

The received signal *r*(*t*) is obtained by convolving the baseband input signal *u*(*t*) with the equivalent lowpass time-varying channel impulse response $h(\tau, t)$ and then upconverting to the carrier frequency.

3. Statistical Fading Models: Narrowband Fading

Time varying Channel Impulse Response: Contd

$$r(t) = \mathrm{Re}\left\{ \left(\int_{-\infty}^{\infty} h(\tau,t)u(t-\tau)d\tau \right) e^{j2\pi f_c t} \right\}$$

The CIR $h(\tau,t)$ represents the equivalent complex lowpass response of the channel at the *t* (when the CIR is observed at the receiver) in response to an impulse launched into the channel at time *t - τ*. Then an expression for the *discrete time varying CIR* is given by:

$$h(\tau,t) = \sum_{l=0}^{L(t)-1} \alpha_l(t) e^{-j\phi_l(t)} \delta(\tau - \tau_l(t)) = \sum_{l=0}^{L(t)-1} h_l(t)\delta(\tau - \tau_l(t))$$

The term in $h_l(t) = \alpha_l(t)e^{-j\phi_l(t)}$ is the *l-th* complex path coefficient observed at the receiver at time *t*.

For a time invariant channel $h(\tau,t) = h_l(\tau, t+T)$ which means that the channel response at time *t* to an impulse at time *t – τ* is the same as the response at time *t + T* to an impulse at time *t + T – τ*. The time invariant CIR is denoted by $h(\tau)$ and expressed as:

$$h(\tau) = \sum_{l=0}^{L-1} \alpha_l e^{-j\phi_l} \delta(\tau - \tau_l) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l)$$

3. Statistical Fading Models: Narrowband Fading

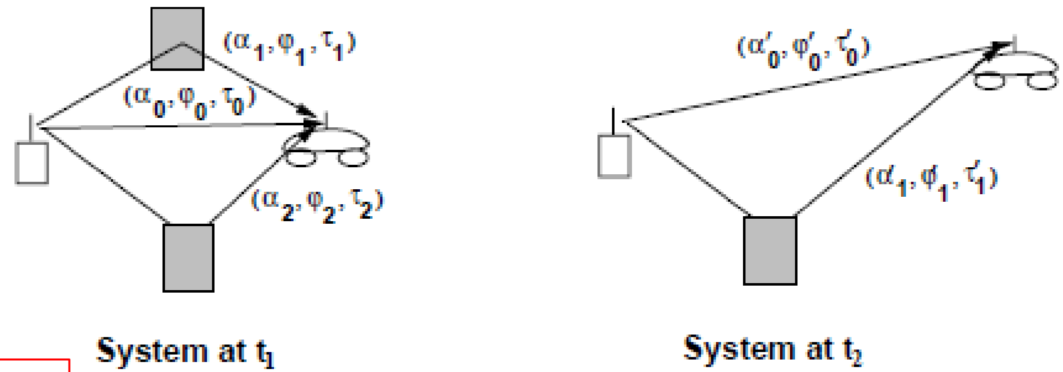
Time varying Channel Impulse Response: Example – Consider the following configurations

Time varying CIR at time t_1:

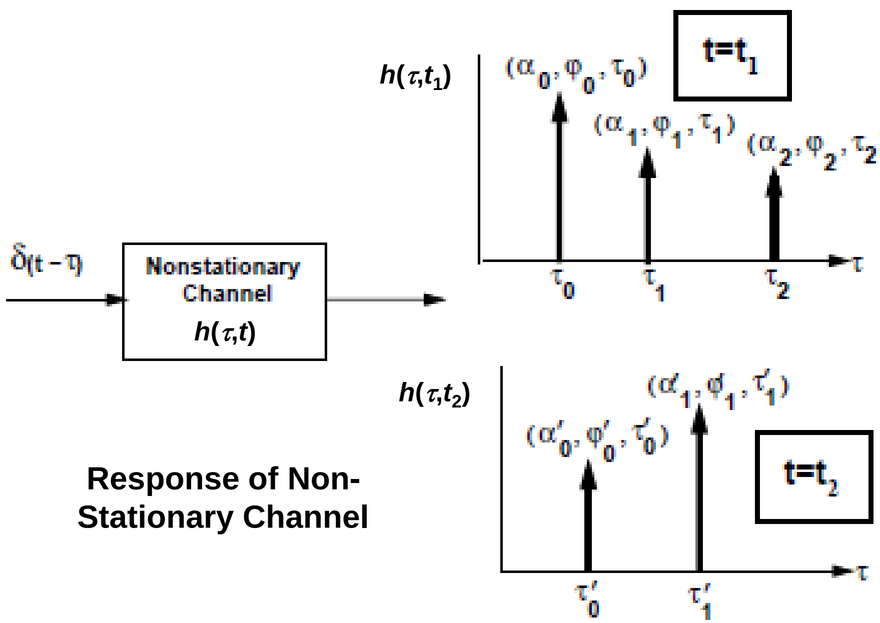
$$h(\tau,t_1)=\sum_{l=0}^{2}\alpha_l e^{-j\phi_l}\delta(\tau-\tau_l)$$

Time varying CIR at time t_2:

$$h(\tau,t_2)=\sum_{l=0}^{1}\alpha_l' e^{-j\phi_l'}\delta(\tau-\tau_l')$$



System Multipath at Two Different Measurement times



Response of Non-Stationary Channel

At typical carrier frequencies, $f_c\tau_l(t)\gg 1$ {e.g. for f_c = 1 GHz and τ_l = 50 ns then $f_c\tau_l(t)=50\gg 1$}. Then a small change in the path delay $\tau_l(t)$ can lead to a very large phase change in the *l-th* multipath component with phase:

$$\phi_l(t)=2\pi f_c\tau_l(t)-\phi_{D_l}-\phi_0$$

Rapid phase changes in each multipath component gives rise to constructive and destructive addition of the multipath components comprising the received signal, which in turn causes rapid variation in the received signal strength. *This phenomenon is called fading*

3. Statistical Fading Models: Narrowband Fading

Narrowband Fading Models: An indication of the delay spread is $T_m = max\{\tau_l - \tau_0\}, l = 0, 1 \ldots L-1$. If $T_m \ll 1/B_u$, then $u(t-\tau_l) \approx u(t)\ \forall l$ in a narrowband channel. Then -

$$r(t) = \mathrm{Re}\left\{ u(t) e^{j2\pi f_c t} \left(\sum_{l=0}^{L(t)-1} \alpha_l(t) e^{-j\phi_l(t)} \right) \right\}$$

To characterize the multipath we choose Tx signal *s*(*t*) to be an *unmodulated* carrier with random phase offset ϕ_0, making *s*(*t*) narrowband for any T_m. Then -

$$s(t) = \mathrm{Re}\left\{ e^{j(2\pi f_c t + \phi_0)} \right\} = \cos(2\pi f_c t + \phi_0)$$

And the RX signal is given by -

$$r(t) = \mathrm{Re}\left\{ \left(\sum_{l=0}^{L(t)-1} \alpha_l(t) e^{-j\phi_l(t)} \right) e^{j2\pi f_c t} \right\} = r_I(t)\cos(2\pi f_c t) - r_Q(t)\sin(2\pi f_c t)$$

Where the Inphase & Quadrature components are given by –

$$r_I(t) = \sum_{l=0}^{L(t)-1} \alpha_l(t)\cos(\phi_l(t)) \qquad r_Q(t) = \sum_{l=0}^{L(t)-1} \alpha_l(t)\sin(\phi_l(t))$$

$$\phi_l(t) = 2\pi f_c t \tau_l(t) - \phi_{D_l} - \phi_0 \quad \text{incorporates the phase off - set } \phi_0$$

For large *L*(*t*) and {$\alpha_l(t)$, $\phi_Q(t)$: independent}, central limit theory suggests that $r_I(t)$ and $r_Q(t)$ can be modelled as independent N(0, σ^2) Gaussian random processes.

3. Statistical Fading Models: Narrowband Fading

Narrowband Fading Models: Autocorrelation, cross-correlation & power spectral densities of $r_I(t)$ and $r_Q(t)$. The following key assumptions are made –

- **There is no dominant line of sight path;**
- $\alpha_l(t) = \alpha_l,\ \tau_l(t) = \tau_l$ and $f_{D_l}(t) = f_{D_l}$ **meaning the amplitudes, delays and Doppler are changing slowly over the time interval of interest. Hence** $\phi_{D_l}(t) = \int 2\pi f_{D_l}\,dt = 2\pi f_{D_l} t$ **and the phase of the *l-th* path component is** $\phi_l(t) = 2\pi f_c \tau_l - 2\pi f_{D_l} t - \phi_0$;
- **The term in** $2\pi f_c \tau_l$ **changes rapidly compared with** $2\pi f_{D_l} t$ **and** ϕ_0 **- from this assumption we can model** ϕ_0 **as a uniformly distributed r.v.** $[-\pi, \pi]$**. Then**

$$E\left[r_I(t)\right] = E[r_Q(t)] = 0 \qquad\qquad E[r_I(t) r_Q(t)] = 0$$

- **And the autocorrelation of** $r_I(t)$ **is given by**

$$\begin{aligned} A_{r_I}(t, t+\tau) &= E[r_I(t) r_I(t+\tau)] = A_{r_I}(\tau)\ (\text{i.e. WSS}) \\ &= \sum_l E[\alpha_l^2] E[\cos(\phi_l(t)) \cos(\phi_l(t+\tau))] \\ &= \frac{1}{2} \sum_l E[\alpha_l^2] E[\cos(2\pi f_{D_l} \tau)] = \frac{1}{2} \sum_l E[\alpha_l^2] E\left[\cos\left(2\pi \frac{\upsilon\tau}{\lambda} \cos(\theta_l)\right)\right] \end{aligned}$$

and $A_{r_Q}(\tau) = A_{r_I}(\tau)$ are both WSS random processes

3. Statistical Fading Models: Narrowband Fading

Narrowband Fading Models: The Cross-correlation of $r_I(t)$ and $r_Q(t)$ is -

$$A_{r_I,r_Q}(\tau) = A_{r_I,r_Q}(t,t+\tau) = E[r_I(t)r_Q(t+\tau)] = -E[r_Q(t)r_I(t+\tau)]$$

$$= -\frac{1}{2}\sum_l E[\alpha_l^2]E\left[\sin\left(2\pi\frac{\upsilon\tau}{\lambda}\cos(\theta_l)\right)\right]$$

And for the received signal we can write:

$$\text{For } r(t) = r_I(t)\cos(2\pi f_c t) - r_Q(t)\sin(2\pi f_c t)$$

$$\text{Then } A_r(\tau) = E[r(t)r(t+\tau)]$$

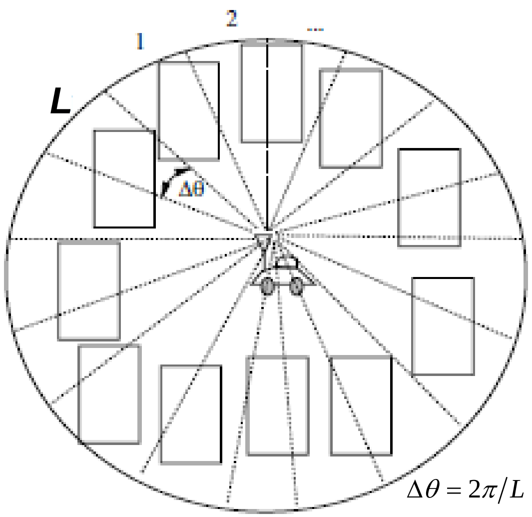
$$= A_{r_I}(\tau)\cos(2\pi f_c\tau) - A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$$

Narrowband Fading Models: The Power Spectral Densities (PSD) of $r_I(t)$ and $r_Q(t)$ -

By focusing on the *uniform scattering environment* described by Clarke where the channel consists of many scatterers densely packed as a function of angle, we assume:

- ***L* multipath components with angle of arrival $\theta_l = l\Delta\theta$ where $\Delta\theta = 2\pi/L$**
- **Each multipath component has equal power such that $E[\alpha_l^2] = 2P_r/L$**

$$\boxed{A_{r_I}(\tau) = \frac{P_r}{L}\sum_{l=1}^{L}\cos\left(2\pi\frac{\upsilon\tau}{\lambda}\cos(l\Delta\theta)\right)}$$



3. Statistical Fading Models: Narrowband Fading

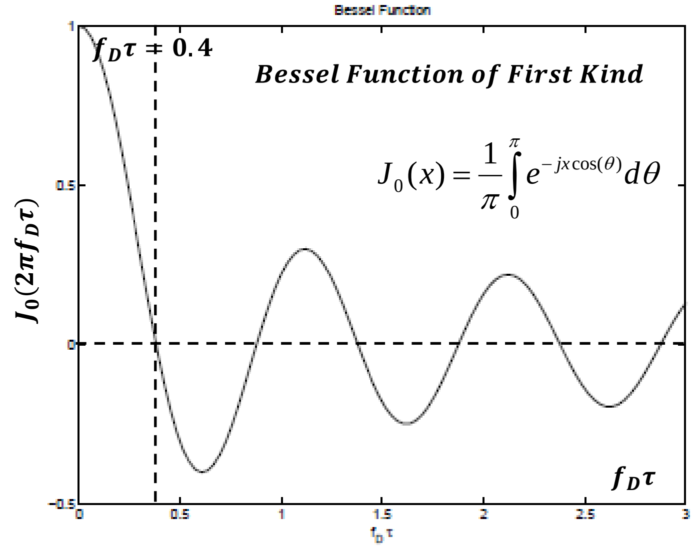
Narrowband Fading Models: The Power Spectral Densities (PSD) of $r_I(t)$ and $r_Q(t)$ - Contd

Substituting $L = 2\pi/\Delta\theta$ gives

$$A_{r_I}(\tau) = \frac{P_r}{2\pi}\sum_{l=1}^{L}\cos\left(2\pi\frac{\upsilon\tau}{\lambda}\cos(l\Delta\theta)\right)\Delta\theta$$

In the limit as $L \to \infty, \Delta\theta \to 0$

$$A_{r_I}(\tau) = \frac{P_r}{2\pi}\int\cos\left(2\pi\frac{\upsilon\tau}{\lambda}\cos(\theta)\right)d\theta = P_r J_0(2\pi f_D\tau)$$

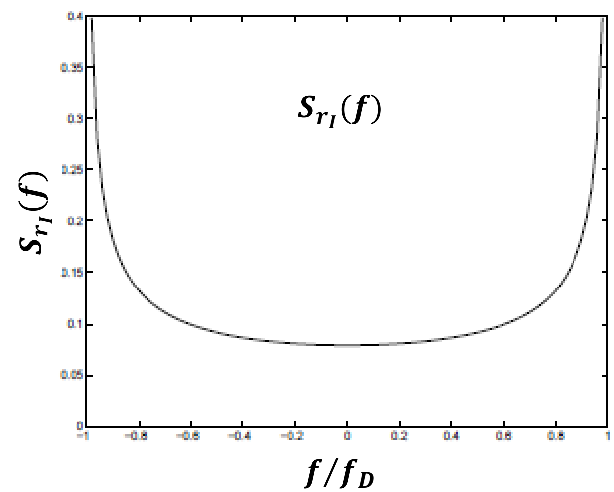


The Power Spectral Densities of $r_I(t)$ and $r_Q(t)$ are obtained by taking the Fourier transform of their respective autocorrelation functions relative to the delay parameter τ. Thus

$$\boxed{S_{r_I}(f) = S_{r_Q}(f) = \Im_T[A_{r_I}(\tau)] = \begin{cases}\frac{2P_r}{\pi f_D}\frac{1}{\sqrt{1-(f/f_D)^2}} & |f| \le f_D \\ 0 & \text{else}\end{cases}}$$

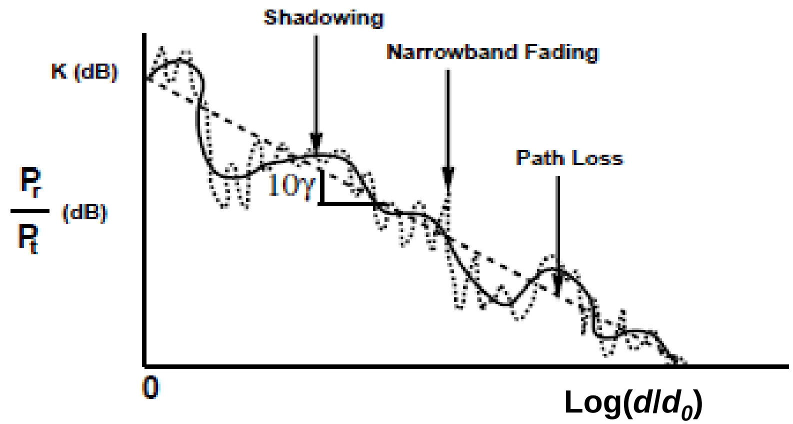
The PSD of $r(t)$ under uniform Scattering $\xrightarrow{\text{yields}}$

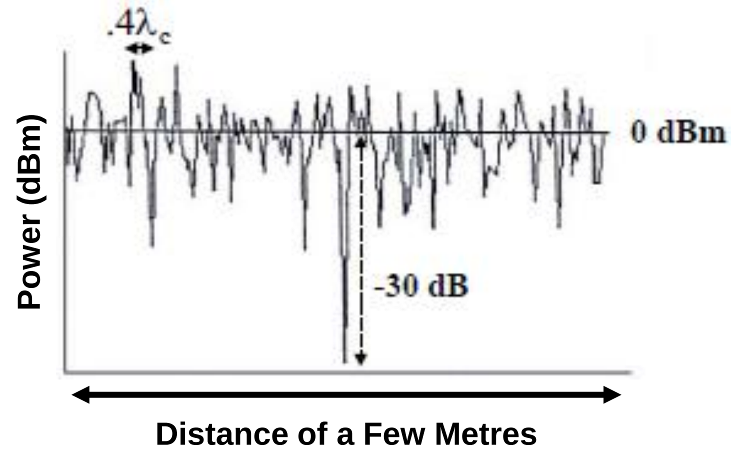
$$S_r(f) = \Im_T[A_r(\tau)] = \frac{1}{4}\left[S_{r_I}(f-f_c) + S_{r_I}(f+f_c)\right]$$
$$= \begin{cases}\frac{P_r}{2\pi f_D}\frac{1}{\sqrt{1-(|f-f_c|/f_D)^2}} & |f-f_c| \le f_D \\ 0 & \text{else}\end{cases}$$



3. Statistical Fading Models: Narrowband Fading

Narrowband Fading Models: Power Profiles -





Narrowband Fading Models: Envelope and Power Distributions -

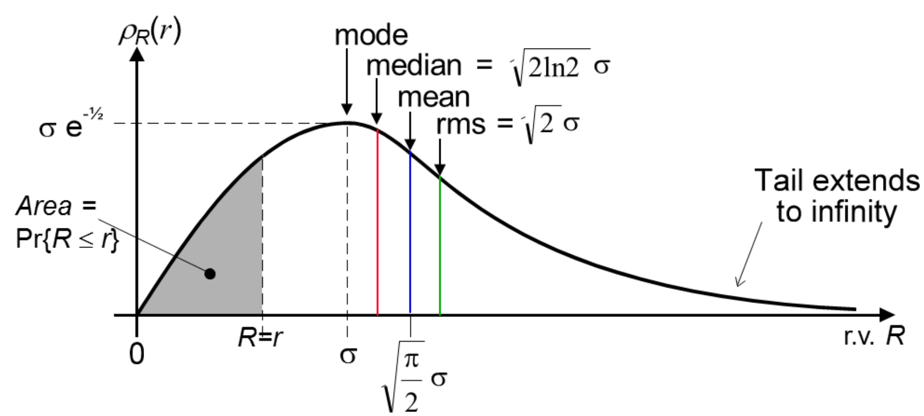
For r.v. X and Y ~ N(0,σ²), the r.v. $R = \sqrt{X^2 + Y^2}$ is *Rayleigh distributed* while the r.v. $Z = R^2$ is exponentially distributed. From earlier analysis we note that $\phi_l(t)$ is uniformly distributed $[-\pi, \pi]$ while $r_I(t)$ and $r_Q(t)$ are N(0,σ²). Then the signal envelope is given by:

$$R(t) = |r(t)| = \left|\sqrt{r_I^2(t) + r_Q^2(t)}\right| \Rightarrow \text{r.v. } R \geq 0$$

$$\text{Average Rx Power } \overline{P}_r = \sum_l E[\alpha_l^2] = \overline{r^2} = 2\sigma^2$$

$$pdf \quad \rho_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} = \frac{2r}{\overline{P}_r} e^{-r^2/\overline{P}_r}$$

$$cdf \quad F_R(r) = 1 - e^{-r^2/\overline{P}_r} = \Pr\{R \leq r\}$$



3. Statistical Fading Models: Narrowband Fading

Narrowband Fading Models: Envelope and Power Distributions

We obtain the power distribution by making The substitution $z = r^2$, thus

$$z = r^2 \Rightarrow dz = 2r dr$$

$$\text{since } \rho_Z(z)dz = \rho_R(r)dr$$

$$\rho_Z(z) = \rho_R(r)\frac{dr}{dz} = \frac{1}{2r} \times \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}$$

$$= \frac{1}{\overline{P}_r} e^{-z/\overline{P}_r}$$

The received signal power is exponentially distributed with mean $\overline{P}_r = 2\sigma^2$.

Example: Consider a channel with Rayleigh fading and average received power $\overline{P}_r$ = 20 dBm. Find the probability that the received power is below 10 dBm.

Solution: We have $\overline{P}_r$ = 20 dBm = 100mW. We want to find the probability that z < 10 dBm = 10 mW. Thus

$$\Pr\{z < 10\} = \int_0^{10} \frac{1}{\overline{P}_r} e^{-z/\overline{P}_r} dz = \int_0^{10} \frac{1}{100} e^{-z/100} dz = 0.095$$

$\Pr\{z < P_{min}\}$ defines a prob. of outage

Rician Distribution: If the channel has a fixed LOS component then $r_I(t)$ and $r_Q(t)$ are not zero-mean. The Rx signal is the superposition of a complex Gaussian component and a fixed LOS component and the signal envelope is described by a Rician prob. distribution -

$$\rho_R(r) = \frac{r}{\sigma^2} e^{-(r^2+s^2)/2\sigma^2} I_0\left(\frac{rs}{\sigma^2}\right), \quad r \geq 0$$

$$2\sigma^2 = \sum_{l, l\neq 0} E[\alpha_l^2], \text{ average power in non-Los}$$

$$s^2 = \alpha_0^2, \text{ average power in Los}$$

$$\overline{P}_r = \int_0^\infty r^2 \rho_R(r)dr = s^2 + 2\sigma^2$$

$$K = \frac{\text{Average Power in LoS}}{\text{Average Power in non-LoS}} = \frac{s^2}{2\sigma^2}$$

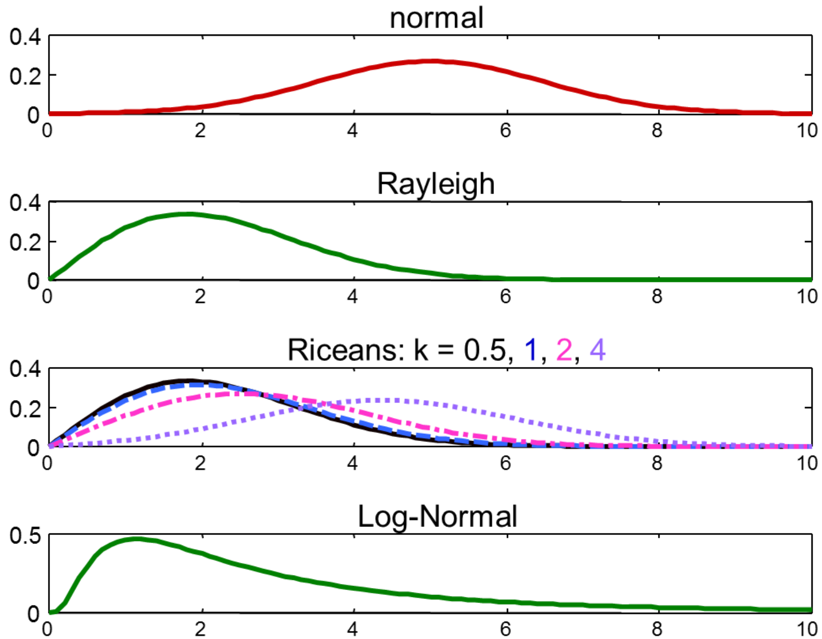
3. Statistical Fading Models: Narrowband Fading

Rician Distribution Contd:

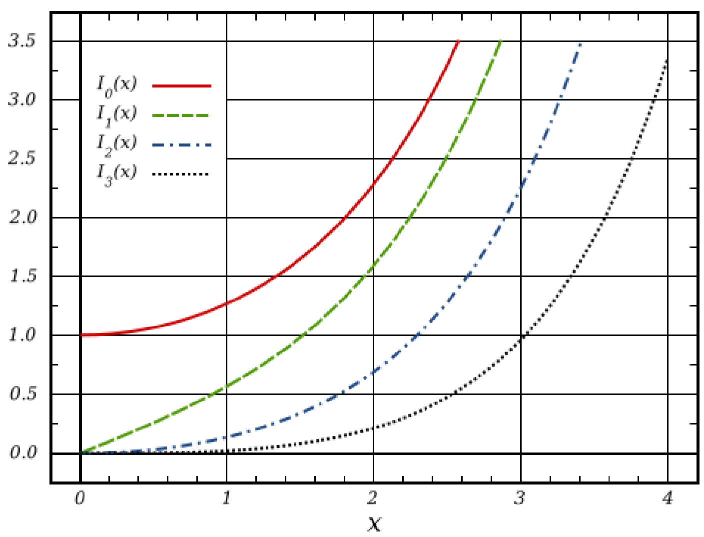
- $I_0(\cdot)$ is the modified Bessel function of zero-order
- For *K* = 0 the Rician distribution becomes a Rayleigh distribution (small *K* implies severe fading)
- For large *K* the Rician distribution tends to a Gaussian distribution (i.e. large *K* implies mild fading)!
- For *K* = ∞ there is no fading!

Substituting $s^2 = \frac{K\overline{P}_r}{(1+K)}$ and $2\sigma^2 = \frac{\overline{P}_r}{(1+K)}$

$$\rho_R(r) = \frac{2r(1+K)}{\overline{P}_r}\exp\left[-K - \frac{(1+K)r^2}{\overline{P}_r}\right] I_0\left(2r\sqrt{\frac{K(1+K)}{\overline{P}_r}}\right)$$



Modified Bessel Function of First Kind



3. Statistical Fading Models: Narrowband Fading

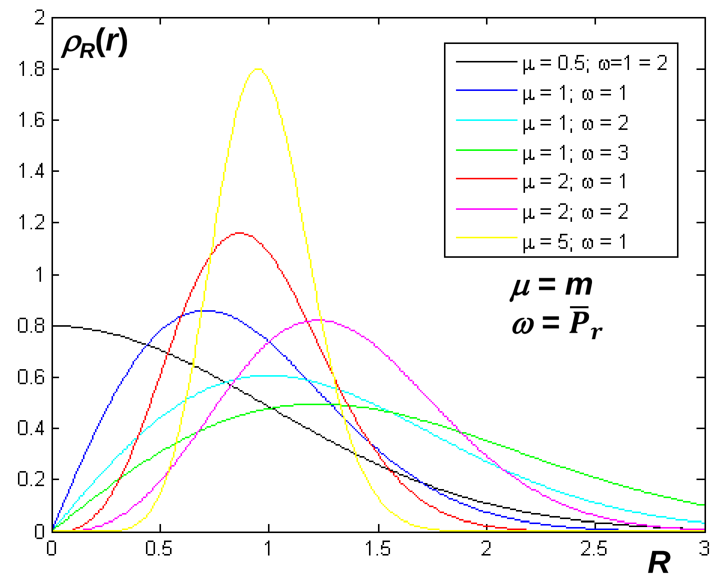
Nakagami Distribution: **The Nakagami distribution is a more general distribution whose parameters can be adjusted to fit a variety of environments.**

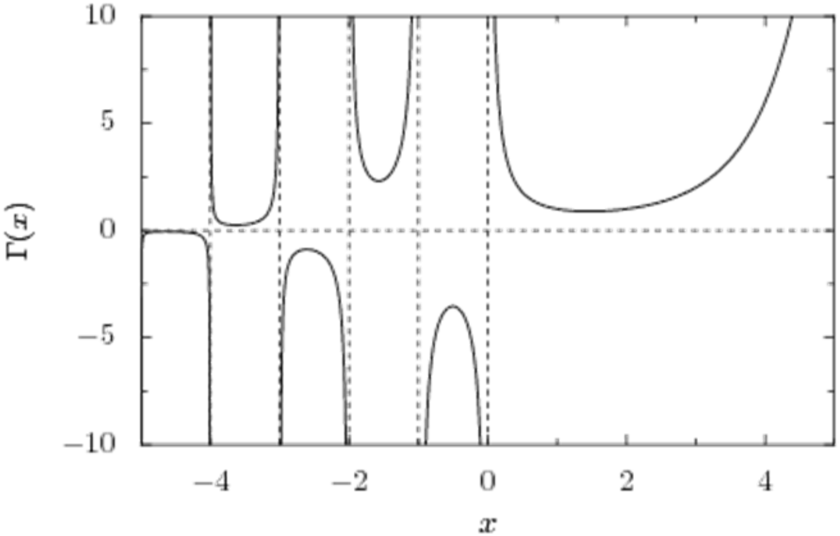
- **The distribution is parameterised by $\overline{P}_r$ and m ($m \geq 0.5$).**
- **For $m = 1$, the Nakagami distribution reduces to Rayleigh fading.**
- **For $m = (1+K)^2/(1+2K)$ the Nakagami tends to a Rician distribution.**
- **For $0.5 < m < 1$, fading is more severe than Rayleigh.**
- **For $m = \infty$ there is no fading!**

$$\rho_R(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\overline{P}_r} \exp\left[-\frac{mr^2}{\overline{P}_r}\right], \quad m \geq 0.5$$

$$\rho_Z(z) = \left(\frac{m}{\overline{P}_r}\right)\frac{z^{m-1}}{\Gamma(m)} \exp\left[-\frac{mz}{\overline{P}_r}\right], \quad z = r^2$$

$\Gamma(x) = \text{Gamma Function in } x$





3. Statistical Fading Models: Narrowband Fading

Level Crossing Rate & Average Fade Duration: The envelope level crossing rate at a specified envelope level *R*, L_R, is defined as the rate (in crossings per second) at which the envelope $r = |r(t)|$ crosses the level *R* in the positive (or negative) going direction.

Obtaining the level crossing rate requires the joint *pdf* $\rho(r, \dot{r})$ of the envelope level r and the envelope slope $\dot{r} = \frac{dr}{dt}$ at any time instant *t*.

In terms of the joint *pdf* $\rho(r, \dot{r})$ the expected amount of time the envelope lies in the interval $(R, R + dr)$ for a given envelope slope $\dot{r}$ and time increment *dt* is

$$\rho(R, \dot{r}) dr d\dot{r}\ dt$$

The time required for the envelope *r* to traverse the interval $(R, R + dr)$ once for a given envelope slope $\dot{r}$ is

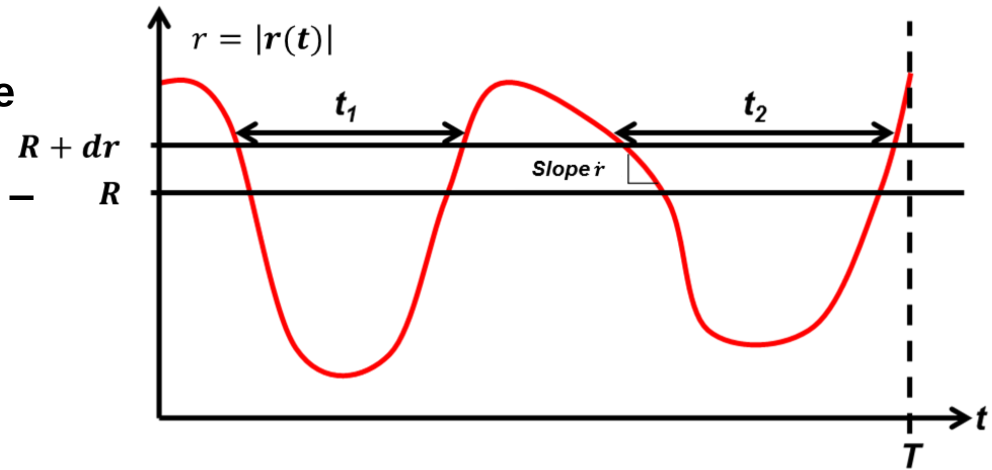
$$dr/\dot{r}$$

The ratio of these two quantities is the expected number of crossings of the envelope *r* within the interval $(R, R + dr)$ for a given envelope slope $\dot{r}$ and time increment *dt* -

$$\dot{r}\rho(R, \dot{r})\ d\dot{r}\ dt.$$

The expected number of crossings of the envelope level *R* for a given envelope slope $\dot{r}$ in a time interval of duration *T* is –

$$\int_0^T \dot{r}\rho(R, \dot{r}) d\dot{r}\ dt = \dot{r}\rho(R, \dot{r}) d\dot{r}\ T$$



3. Statistical Fading Models: Narrowband Fading

Level Crossing Rate & Average Fade Duration Contd: The expected number of crossings of the envelope level *R* with a positive slope in the time interval *T* is

$$N_R = T\int_0^\infty \dot{r}\rho(R,\dot{r})d\dot{r} = T\bar{\dot{r}}$$

Finally, the expected number of crossings per second of the envelope level *R*, or the level crossing rate applicable to any random process or signal, is

$$L_R = \frac{N_R}{T} = \int_0^\infty \dot{r}\rho(R,\dot{r})d\dot{r} = \bar{\dot{r}}$$

For Rayleigh fading, where P_R is the target power level corresponding to *r* = *R*, the level crossing rate simplifies to

$$L_R = \sqrt{2\pi} f_D \rho e^{-\rho^2}, \qquad \rho = \frac{R}{r_{rms}} = \frac{R}{\sqrt{\overline{P}_r}} = \sqrt{\frac{P_R}{\overline{P}_r}}$$

The Average Fade Duration is the average duration that the envelope remains below a specified level *R*. Although the *pdf* of the envelope fade duration is unknown, the average fade duration can be calculated. Consider a very long time interval of length *T* and let t_i be the duration of the *i-th* fade below the level *R*. The probability that the received envelope is less than *R* is

$$p(r < R) = \frac{1}{T}\sum_i t_i$$

3. Statistical Fading Models: Narrowband Fading

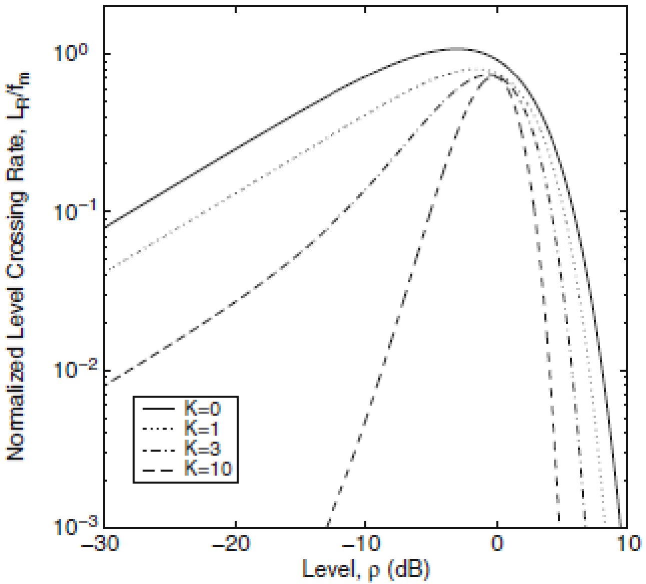
Level Crossing Rate & Average fade Duration Contd: **For T sufficiently large the average fade duration is**

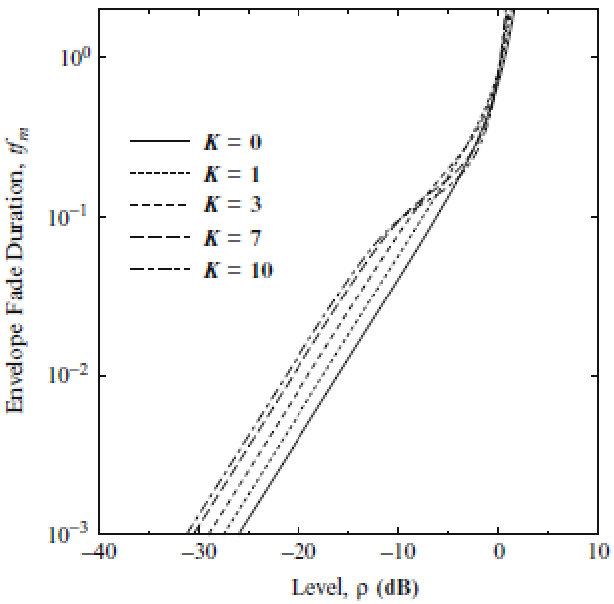
$$\bar{t}_R = \frac{1}{TL_R}\sum_{i=1}^{L_R T} t_i \approx \frac{p(r<R)}{L_R}$$

For a Rayleigh fading,

$$p(r<R) = \int_0^R \rho(r)dr = 1 - e^{-\rho^2} \Rightarrow \bar{t}_R = \frac{e^{\rho^2}-1}{\rho f_D\sqrt{2\pi}}$$

Note that since ρ represents a normalized envelope (magnitude) level, we use ρ_{dB} = $20\log_{10}(\rho)$.





The level crossing rate and average fade duration depend on the mobile speed, as $f_D = v/\lambda$.

Very deep fades occur infrequently and do not last very long. For example, at ~100 km/h and 900MHz, the f_D = 80 Hz.

In Rayleigh fading at ρ = 0 dB, there are L_R = 74 fades/s with an average fade duration of 8.5 ms.

However, at ρ = −20 dB there are only 20 fades/s with an average fade duration of 0.5 ms.

3. Statistical Fading Models: Narrowband Fading

Level Crossing Rate & Average fade Duration Contd: The average fade duration indicates the number of bits or symbols affected by a deep fade. Consider a system with bit duration T_b and suppose the probability of bit error is high when $r < R$. The following error conditions are identified –

- $T_b \approx \bar{t}_R$: the system will experience single bit error events;
- $T_b \ll \bar{t}_R$: the system will experience burst bit error events;
- $T_b \gg \bar{t}_R$: the system will experience no bit error events as fading is averaged out in Rx.

Example: A voice system has acceptable BER when the received signal power is at or above one quarter of its average value. If the BER is below this acceptable level for more than 200 ms, users will not use their mobile phone. Find the range of Doppler frequency values in a Rayleigh fading channel such that the average time duration when users have unacceptable voice quality is less than t = 100 ms.

Solution: Target received signal power is one quarter of its average value, so $P_R = \bar{P}_r/4$ giving $\rho = \sqrt{P_R/\bar{P}_r} = 1/2$. We require

$$\bar{t}_R = \frac{e^{\rho^2} - 1}{\rho f_D \sqrt{2\pi}} = \frac{e^{0.5^2} - 1}{0.5 f_D \sqrt{2\pi}} \leq t = 0.1\text{s}$$

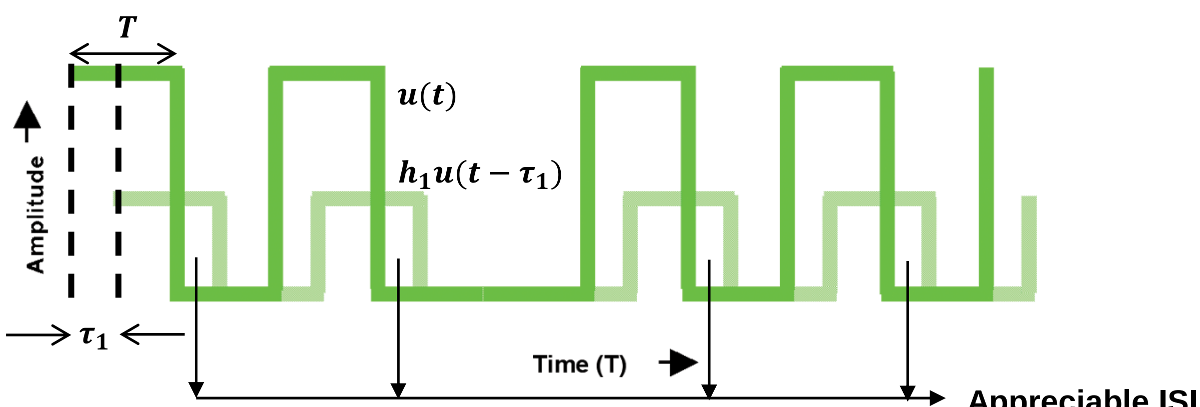
$$f_D \geq \frac{e^{0.25} - 1}{0.5 \times 0.1 \times \sqrt{2\pi}} = 2.27\text{Hz}.$$

3. Statistical Fading Models: Wideband Fading

Intersymbol Interference (ISI): If the multipath delay spread T_m is appreciable compared with the symbol period *T*, then the multipath components will interfere with subsequently transmitted pulses. This effect is known as ISI.

As the transmitted signal bandwidth increases so that $T_m \approx 1/B_u$, then $u(t-\tau_l) \neq u(t)$ and the received signal is the sum of all copies of the original signal where each copy is delayed by τ_l and scaled by the complex coefficient h_l.

$$r(t) = \mathrm{Re}\left\{ \sum_{l=0}^{L-1} h_l u(t-\tau_l) e^{j2\pi f_c t} \right\}$$

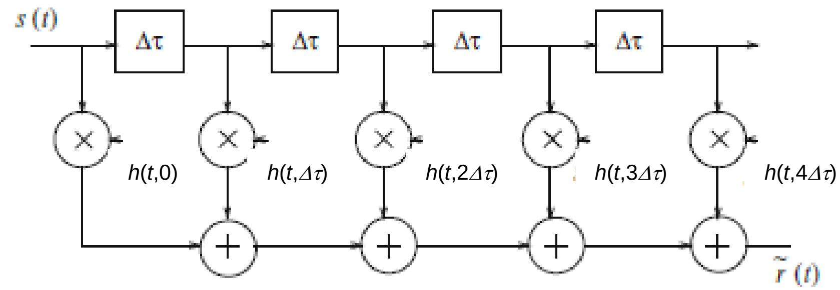


For the diagram above CIR: $h(\tau) = \delta(\tau) + h_1 \delta(\tau - \tau_1)$

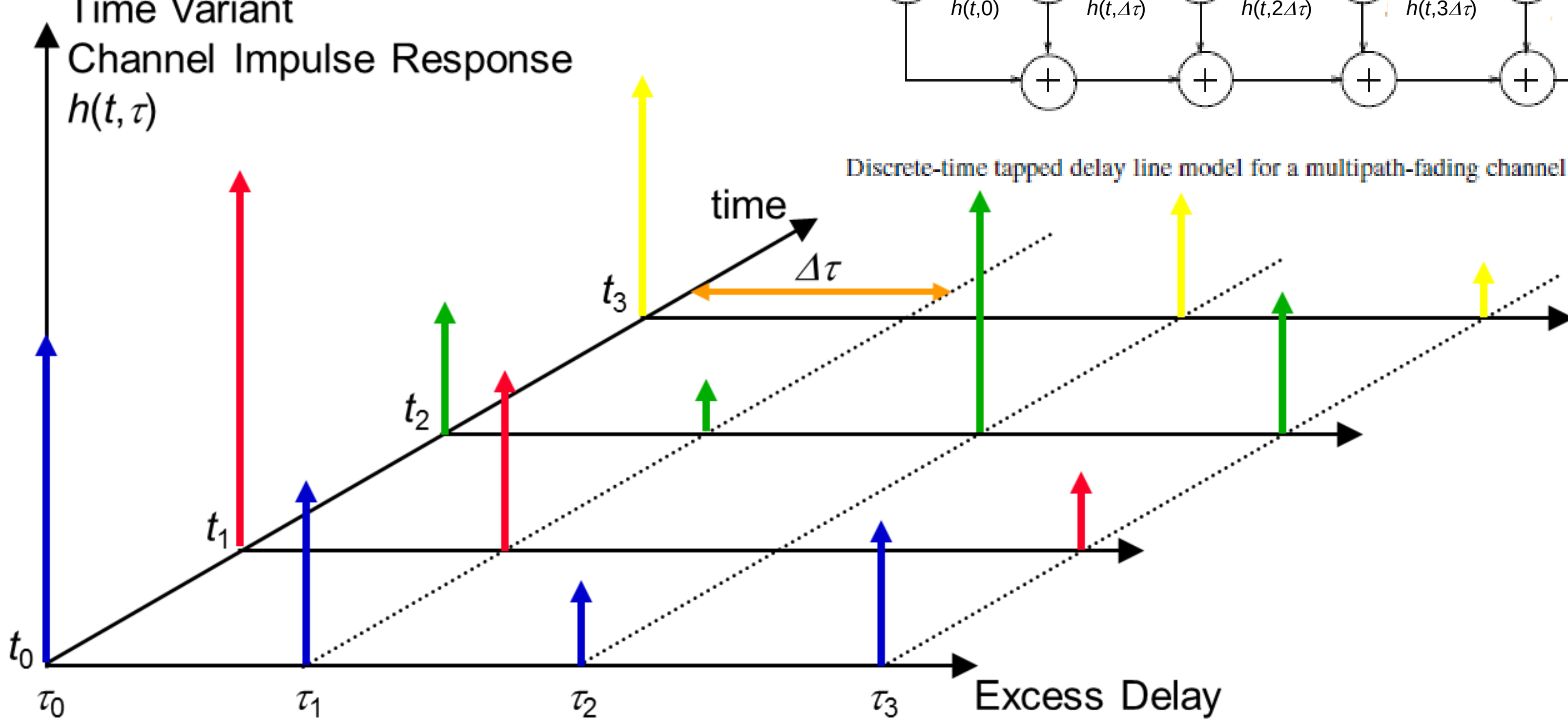
$$r(t) = Re\{u(t)e^{j2\pi f_c t} + h_1 u(t-\tau_1) e^{j2\pi f_c t}\}$$

3. Statistical Fading Models: Wideband Fading

The Time Variant Discrete Channel Impulse Response: It is useful to discretise the multipath delay axis τ into "excess delay bins" of duration $\Delta\tau$. From Nyquist, this determines the time delay resolution and specifies the useful frequency span of the model as $f_s = 2B_u = 2/\Delta\tau$.



Discrete-time tapped delay line model for a multipath-fading channel

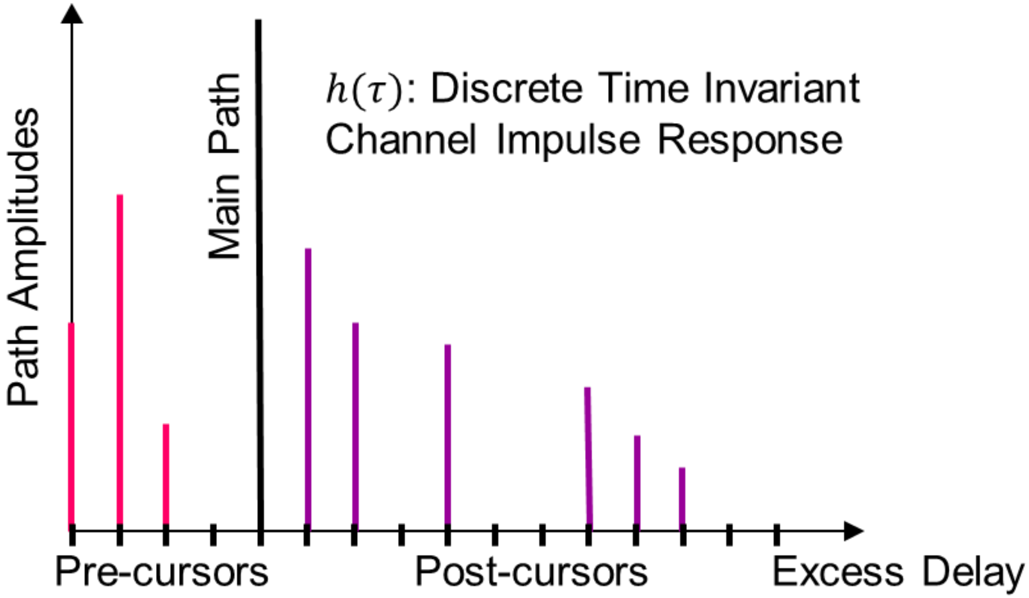


τ_0 is the excess delay of the first arriving path and is nominally set to 0 seconds

τ_i is the excess delay of the *i-th* arriving path with respect to τ_0.

3. Statistical Fading Models: Wideband Fading

Time Invariant Discrete Channel Impulse Response: **Over sufficiently short periods of time we can treat the channel impulse response as Time Invariant. Then the discrete-time channel impulse response simplifies to:**



Time Invariant CIR

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l)$$

Power Delay Profile

$$P(\tau) = \sum_{l=0}^{L-1} |h_l|^2 \delta(\tau - \tau_l)$$

Channel Output

$$r(t) = \mathrm{Re}\left\{\left(\int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau\right)e^{j2\pi f_c t}\right\}$$

$$= \mathrm{Re}\left\{\left(\sum_{l=0}^{L-1} h_l u(t-\tau_l)\right)e^{j2\pi f_c t}\right\}$$

The h_l are complex variables

$$h_l = |\alpha_l| e^{j\varphi_l}$$

$\therefore$ Average Power in $h(\tau)$ is

$$P = \sum_{l=0}^{L-1} h_l \times h_l^* = \sum_{l=0}^{L-1} |h_l|^2$$

3. Statistical Fading Models: Wideband Fading

Time Dispersion Parameters of Power Delay Profile: **The power delay profile is formed from the temporal or spatial average of consecutive channel impulse response measurements collected and averaged over a local area.**

Then the power delay profile is used to determine the *mean excess delay*, *rms delay spread* and the *maximum excess delay* (X dB). These quantities are defined as:

Mean Excess Delay $\bar{\tau}$

$$\bar{\tau} = \frac{\sum_{l=0}^{L-1} P(\tau_l)\tau_l}{\sum_{l=0}^{L-1} P(\tau_l)} = \frac{\sum_{l=0}^{L-1} |h_l|^2 \tau_l}{\sum_{l=0}^{L-1} |h_l|^2}$$

rms Delay Spread $\tau_{rms} = \sqrt{\overline{\tau^2} - \bar{\tau}^2}$

where $$\overline{\tau^2} = \frac{\sum_{l=0}^{L-1} P(\tau_l)\tau_l^{\,2}}{\sum_{l=0}^{L-1} P(\tau_l)} = \frac{\sum_{l=0}^{L-1} |h_l|^2 \tau_l^{\,2}}{\sum_{l=0}^{L-1} |h_l|^2}$$

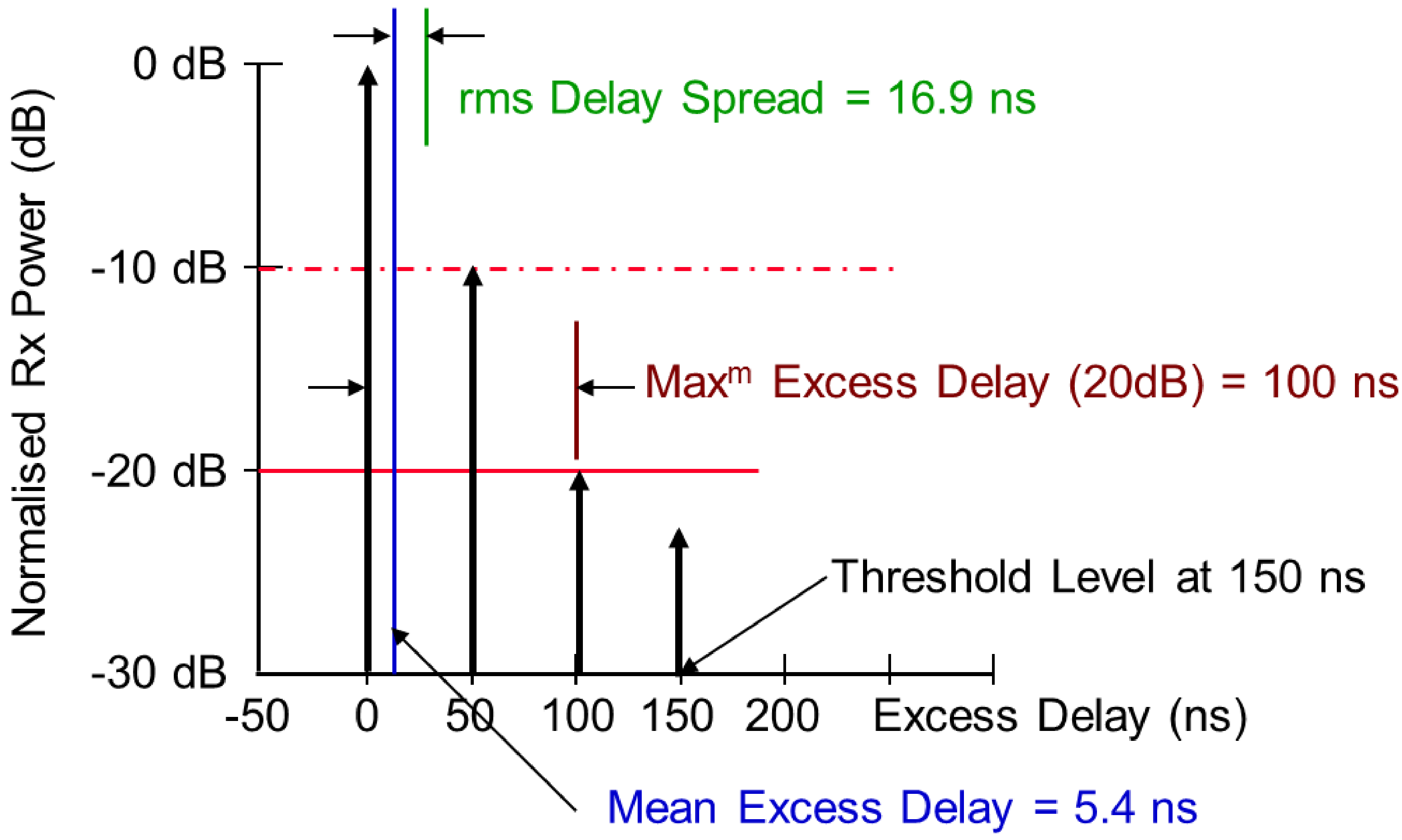
Maximum excess delay (X dB): $\boldsymbol{\tau_{max} = \tau_X - \tau_0}$

An example of an indoor power delay profile is shown below:

- **mean excess delay -** $\boldsymbol{\bar{\tau} = 5.4\ \mathrm{ns}}$
- **rms delay spread -** $\boldsymbol{\tau_{rms} = 16.9\ \mathrm{ns}}$
- **maximum excess delay (20 dB) -** $\boldsymbol{\tau_{max} = 100\ \mathrm{ns}}$
- **threshold level (effective noise floor) – 150 ns**

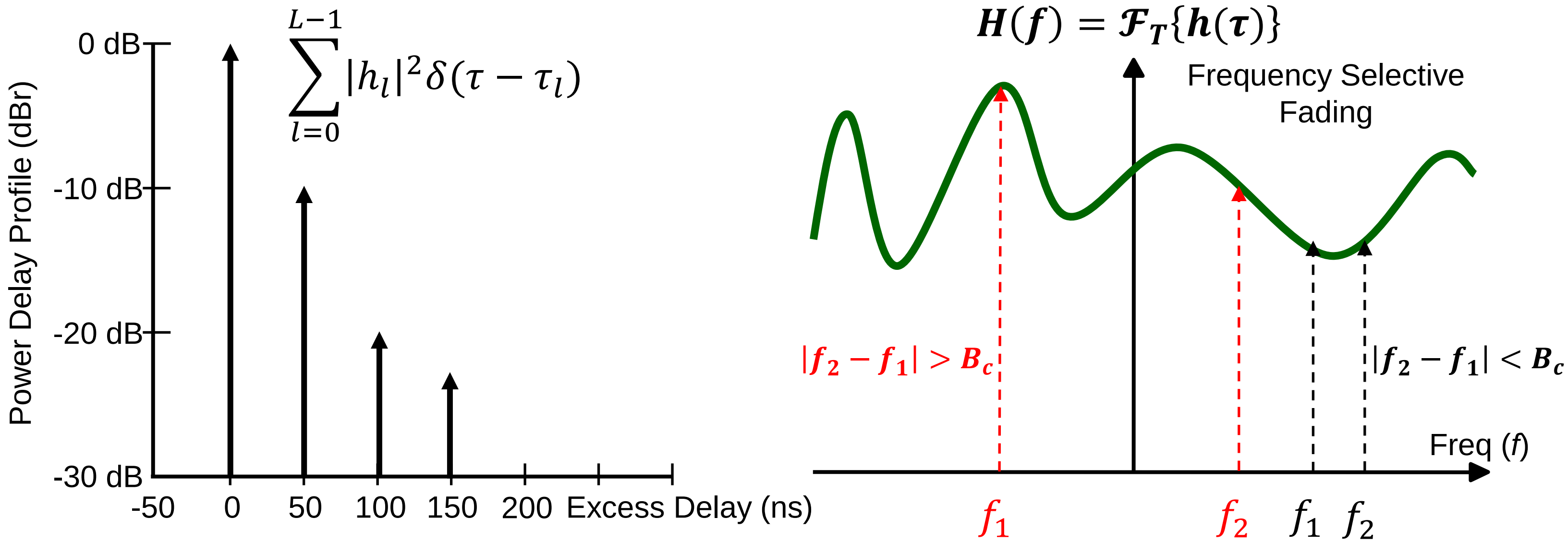
3. Statistical Fading Models: Wideband Fading

Time Dispersion Parameters of Power Delay Profile Contd:



3. Statistical Fading Models: Wideband Fading

Coherence Bandwidth: **The channel impulse response and magnitude frequency spectrum form a Fourier Transform pair - $\mathcal{F}_T\{h(\tau)\} = H(f)$.**



The bandwidth B_c where the autocorrelation *AC*(f_1, f_2) ≈ 0 for all $|f_2 - f_1| > B_c$ is called the coherence bandwidth of the channel.

The minimum frequency separation B_c for which the channel response is approximately independent is given by $B_c = 1/\tau_{rms}$. Other useful definitions of coherence bandwidth are:

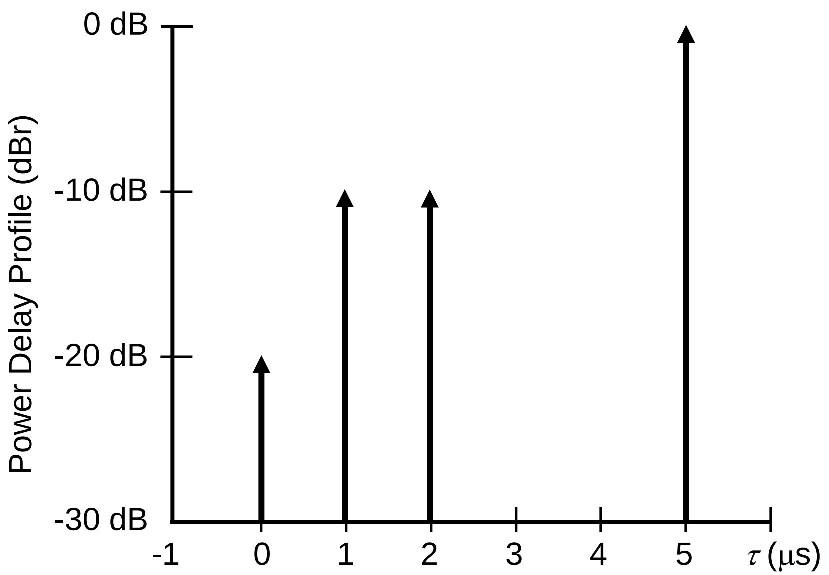
- **90% Coherence Bandwidth $B_c = 1/(50 \times \tau_{rms})$ - when *AC*(f_1, f_2) = 0.9**
- **50% Coherence Bandwidth $B_c = 1/(5 \times \tau_{rms})$ - when *AC*(f_1, f_2) = 0.5**

3. Statistical Fading Models: Wideband Fading

Coherence Bandwidth: When transmitting a narrowband signal of bandwidth $B \ll B_c$, then fading is almost equal across the entire signal bandwidth (i.e. highly correlated). This fading is referred to as *flat fading*.

When transmitting a wideband signal of bandwidth $B \gg B_c$, the channel amplitude values at frequencies separated by more than the coherence bandwidth are independent (i.e. highly uncorrelated). Then the channel amplitude varies widely across the signal bandwidth. This fading is referred to as *frequency-selective fading*.

Example: Calculate the mean excess delay, the *rms* delay spread and the maximum excess delay (20 dB) for the MP power delay profile shown. Then estimate the 50% coherence bandwidth of the channel. Would this channel be suitable for a TACs or GSM service without the use of an equaliser?



Maximum Excess Delay (20dB): $\tau_{\max} = 5\,\mu\text{s}$

Mean Excess Delay

$$\bar{\tau} = \frac{(0.01)(0) + (0.1)(1) + (0.1)(2) + (1)(5)}{0.01 + 0.1 + 0.1 + 1} = 4.38\,\mu\text{s}$$

Mean Square Excess Delay

$$\overline{\tau^2} = \frac{(0.01)(0^2) + (0.1)(1^2) + (0.1)(2^2) + (1)(5^2)}{0.01 + 0.1 + 0.1 + 1} = 21.07\,\mu\text{s}^2$$

3. Statistical Fading Models: Wideband Fading

Example Contd:

$$rms \text{ Delay Spread } \tau_{rms} = \sqrt{\overline{\tau^2} - \bar{\tau}^2} = \sqrt{21.07 - (4.38)^2} = 1.37\ \mu\text{s}$$

$$50\% \text{ Coherence Bandwidth } B_c = 1/5\tau_{rms} = 1/(5 \times 1.37\ \mu\text{s}) = 146 \text{ kHz}$$

Since B_c > 25 kHz, TACS will work without an equaliser. However, B_c < 200 kHz, therefore GSM would need an equaliser.

Doppler Power Spectrum and Coherence Time: Coherence time and Doppler (frequency) spread describe the time varying nature of the mobile channel.

Coherence time T_c is a statistical measure of the time duration over which the channel impulse response does not change – i.e. the period of time over which the signal envelope remains constant.

If $f_D = v/\lambda$ is the maximum Doppler shift then: $T_c \approx 1/f_D$

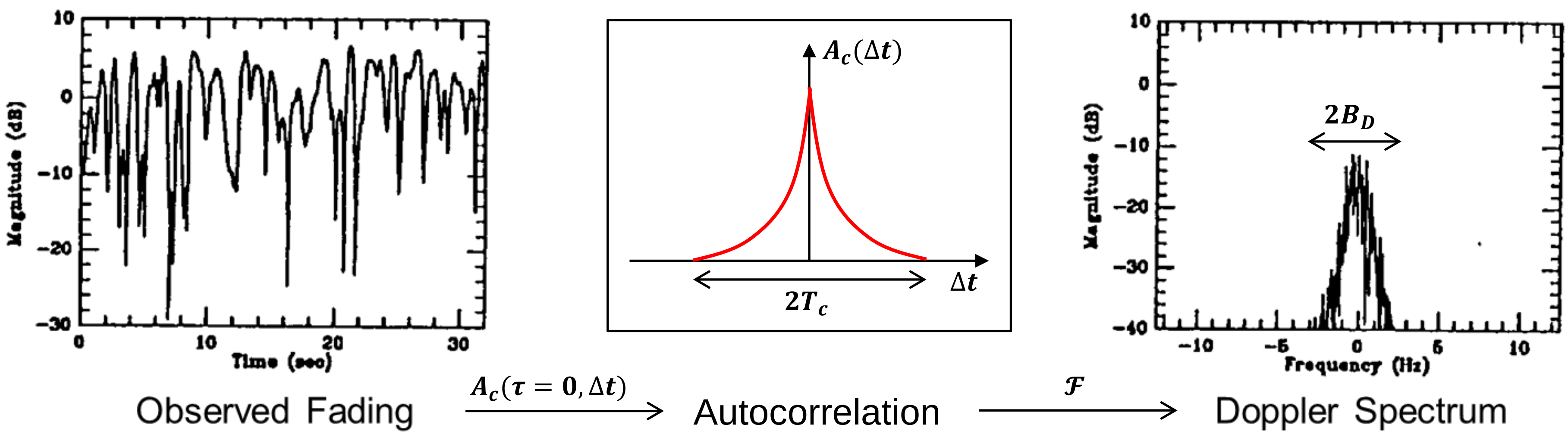
We can define a 50% autocorrelation coherence time as $T_c \approx 9/16\pi f_D$

Often for digital transmission, the coherence time is taken as the geometric mean of the two previous equations giving $T_c \approx \sqrt{9/16\pi {f_D}^2} = 0.423/f_D$

Doppler spread B_D measures the spectral broadening due to the time rate of change of the fading envelope. B_D is defined as the range of baseband frequencies over which the received Doppler spectrum is none-zero. Then $B_D \cong 1/T_c\ (\cong f_D)$.

3. Statistical Fading Models: Wideband Fading

Doppler Power Spectrum and Coherence Time Contd:



Example:

Consider a mobile travelling at 100 km/h using a 900 MHz carrier –

$$f_D = v/\lambda = 27.78/0.333 = 83.3\ \text{Hz}$$

$$T_c(50\%) = 9/16\pi f_D = 2.15\ \text{ms}$$

Then providing the symbol rate in a digital transmission link $R_s > 1/T_c = 1/2.25ms = 465$ symbols/s, the channel will not cause symbol distortion due to Doppler spread (NB this does not mean that the symbol will not undergo ISI).

3. Statistical Fading Models: Wideband Fading

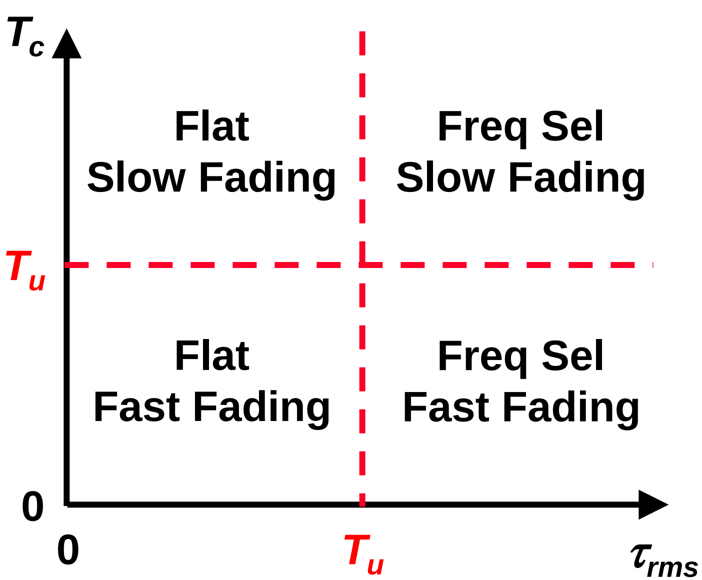
Classification of Small-Scale Fading:

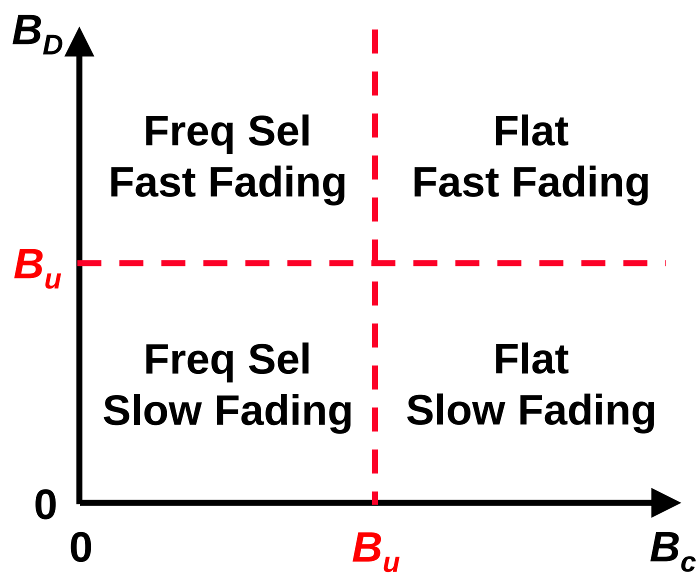
MP Time Delay Spread Small Scale fading

- **Flat Fading**: $\tau_{rms} < T_u$, $B_c > B_u$
- **Freq Selective Fading**: $\tau_{rms} > T_u$, $B_c < B_u$

Doppler Spread Small Scale fading

- **Fast Fading**: $T_c < T_u$, $B_D > B_u$
- **Slow Fading**: $T_c > T_u$, $B_D < B_u$





Summary & Main Points:

- **The multipath channel is modelled by a random time-varying impulse response (CIR)**
- **The ability of a receiver to resolve multipaths determines if the channel is narrowband or wideband**
- **For the narrowband channel we use Clarke's scattering model to demonstrate the Doppler spectrum**
- **The narrowband channel can be statistically characterised by the Rayleigh, Rician and Nakagami distributions**
- **We use these distributions to determine outage probability due to multipath fading as well as level crossing rate and average fade duration (i.e. QoS metrics)**
- **The wideband channel is characterised by intersymbol interference (ISI) and the time variant discrete channel impulse response**
- **The wideband channel is also known as a frequency selective channel parameterised by the mean excess delay and the *rms* delay spread**
- **The frequency selective channel exhibits a coherence bandwidth that depends on the *rms* delay spread and a Doppler spread that depends on the Doppler bandwidth**
- **The types of multipath fading channels can be classified depending on delay spread, coherence bandwidth, Doppler spread and Doppler bandwidth.**