Data Provided: 1 × z-transform table

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2011-12 (2.0 hours)

EEE6033 Digital Signal Processing

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

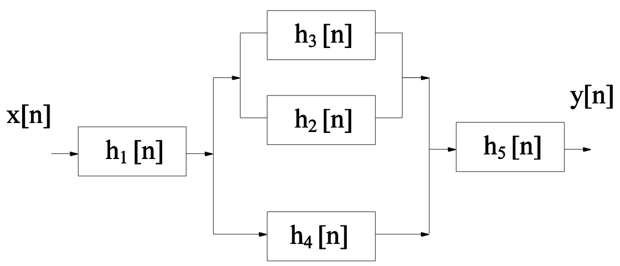
1. **a.** There are two important differences between the discrete-time and continuous-time complex exponential signals (denoted by $x[n]=e^{j\omega n}$ and $x(t)= e^{j\omega t}$, respectively). Explain in detail the two differences.

(4)

b. A sequence is said to be the eigenfunction of a linear time invariant (LTI) system, when given such a sequence at its input, its output is a simple scaled version of the same sequence. Determine whether the sequence $x[n]=\alpha^n$ (α is a nonzero constant) is the eigenfunction of an LTI system. Explain your answer.

(4)

c. Find the combined impulse response of the LTI system plotted below, which consists of five sub-systems with impulse responses $h_1[n]$, $h_2[n]$, $h_3[n]$, $h_4[n]$ and $h_5[n]$, respectively.



(4)

d. Give the transformation equations for the Fourier series (complex-valued), Fourier transform, discrete-time Fourier transform (DTFT), discrete Fourier transform (DFT). (The inverse transform equations are not required). State clearly whether it is applied to periodic or non-periodic, discrete or continuous signals, and the results after transformation are periodic or non-periodic, discrete or continuous. **(8)**

2. **a.** Consider the system function

$$H(z)=\frac{1+2z^{-1}}{1-1.5z^{-1}+0.9z^{-2}}$$

Give its direct form I and direct form II implementation structures.

(4)

b. i) Derive the z-transform of the following sequence (4 marks)

$$x[n]=(\frac{1}{2})^n u[n]+(-\frac{1}{3})^n u[n]$$

ii) Give the pole-zero plot of the z-transform, including its region of convergence (ROC) (2 marks).

(6)

c. A discrete-time system has the following transfer function

$$\frac{Y(z)}{X(z)}=\frac{2z^3-z^2+z-0.4}{z^3}.$$

Determine the output y[n] of the system for the following input x[n]

$$x[n]=\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3].$$

(5)

d. Consider a sequence $x_1[n]$ whose length is L points (nonzero for n=0, 1, …, L-1) and a sequence $x_2[n]$ whose length is P (nonzero for n=0, 1, …, P-1). A linear convolution of these two sequences will generate a third sequence $x_3[n]$. Describe the process involved in calculating this linear convolution using DFT.

(5)

3. **a.** Calculate the Discrete Fourier Transform (DFT) of the discrete series x[n]={1, 2, 2, 1}.

(4)

b. i) State the Nyquist sampling theorem and determine the minimum sampling frequency f_s required for sampling the following continuous-time signal x(t) (4 marks):

$$x(t)=\sin(10\pi t)+\cos(50\pi t)$$

ii) Suppose the discrete-time signal after sampling the above x(t) by the minimum sampling frequency is denoted by x(n). Draw the block diagram of an ideal system for recovering the original continuous-time signal and give details about the input-output relationship at each stage of the block diagram (4 marks).

(8)

c An anti-aliasing filter is to be designed for a data acquisition system and the first order lowpass filter given in the following equation is used as a prototype, where ω_b=40 rad/sec is the filter cutoff frequency.

$$H(s) = \frac{\omega_b}{s + \omega_b}$$

i. Design the digital filter using the Impulse Invariant method if the filter is implemented at a sampling frequency of 40 Hz (4 marks).
ii. Given the same sampling frequency of 40 Hz, design the digital filter using the Bilinear Transform method (4 marks)

(8)

4. **a.** Given the spectral coefficients of a filter, *H(k)*, which are symmetrical about *k*=0, the original impulse response *h*[n] can be reconstituted using the following equation, where *N* is the total number of coefficients:

$$h[n] = \frac{1}{N}\sum_{k=-(N-1)/2}^{(N-1)/2} H(k)e^{j2\pi nk/N} = \frac{1}{N}\left(H(0) + 2\sum_{k=1}^{(N-1)/2} H(k)\cos(2\pi nk/N)\right)$$

From this you are going to design a **highpass** FIR filter with *N=5* coefficients with a passband range between *0.5*kHz and 1kHz at a sampling frequency $f_s=2$kHz.

i) Use the frequency sampling method to calculate the FIR filter coefficients (6 marks).

ii) Sketch the structure of the filter using unit-delay elements (1 mark).

iii) Derive the difference equation of the filter (1 mark).

(8)

b. Consider a first-order system function of the form

$$H(z) = (1 - re^{j\theta}z^{-1}) \text{ (r<1, 0<}\theta\text{<}\pi\text{/2)}$$

i) Give its pole-zero plot and indicate the corresponding pole vector and zero vector (3 marks).

ii) Derive the magnitude response and phase response of the system function in frequency domain in terms of the pole vector and zero vector (4 marks).

(7)

c. Suppose $X_1(z)$ is the z-transform of the sequence $x_1[n]$ and $X_2(z)$ is the z-transform of the sequence $x_2[n]$. Then we have the following property:

$$x_1[n] * x_2[n] \xleftrightarrow{z-transform} X_1(z)X_2(z)$$

where * denote the convolution operation. Derive the above result.

(5)

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