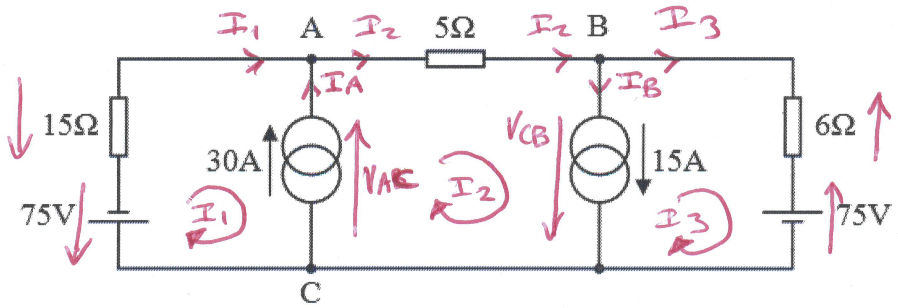
KEEPING CURRENT SOURCE POTENTIALS IN PHASE WITH CURRENT DIRECTION:

Q1(a) i



KIRCHOFF CURRENT: $I_2 = I_1 + I_A$ (1), $I_3 = I_2 - I_B$ (2)

KIRCHOFF VOLTAGE:

I_1: $15I_1 + 75 + V_{AC} = 0 \Rightarrow V_{AC} = -75 - 15I_1$ (3)

I_2: $V_{AC} + V_{CB} - 5I_2 = 0$ (4)

I_3: $V_{CB} + 75 + 6I_3 = 0 \Rightarrow V_{CB} = -75 - 6I_3$ (5)

SUB (3) IN (4): $-75 - 15I_1 + V_{CB} - 5I_2 = 0$ (6)

SUB (5) IN (6): $-75 - 15I_1 - 75 - 6I_3 - 5I_2 = 0$

$\Rightarrow 15I_1 + 6I_3 + 5I_2 = -150$ (7)

SUB (1)&(2) IN (7): $15I_1 + 6I_2 - 6I_B + 5I_1 + 5I_A = -150$

OR $15I_1 + 6I_1 + 6I_A - 6I_B + 5I_1 + 5I_A = -150$

$\Rightarrow 26I_1 = -150 - 180 + 90 - 150 = -390$

$\therefore I_1 = -15\,A$ So $I_2 = 15\,A$ & $I_3 = 0\,A$

FROM (3): $V_{AC} = -75 - 15 \times -15 = 150V$

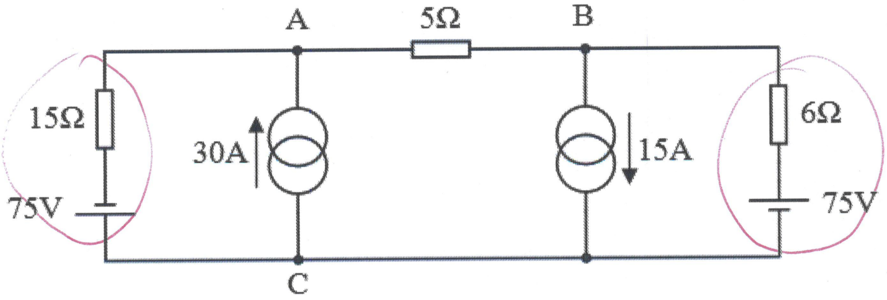
$\therefore$ POWER DELIVERED BY 30A SOURCE $= 150 \times 30$

$= 4.5\,kW$

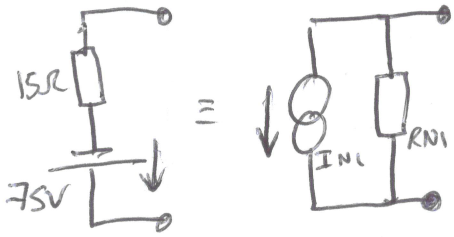
ii

SINCE $I_3 = 0A$, Batt 2 ONLY PROVIDES A VOLTAGE AND NEITHER SOURCES OR SINKS CURRENT

Q1 (b) i



TRANSFORM TO NORTON EQUIVALENT:



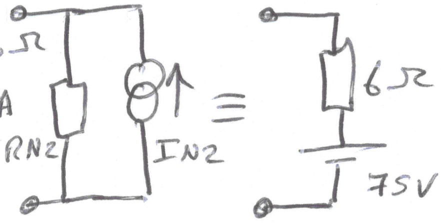
$R_{N1} = R_{th} = 15\Omega$

$I_{N1} = \frac{V_{th}}{R_{th}} = 5A$

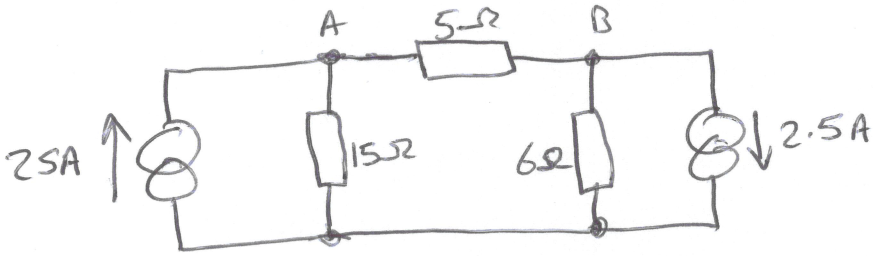
NOTE DIRECTION!

$R_{N2} = R_{th} = 6\Omega$

$I_{N2} = \frac{V_{th}}{R_{th}} = 12.5A$

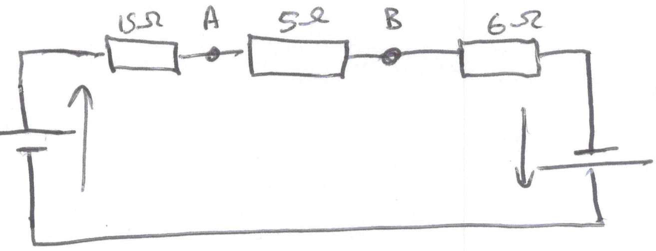


CONSIDERING CONTRIBUTIONS FROM ALL CURRENT SOURCES:



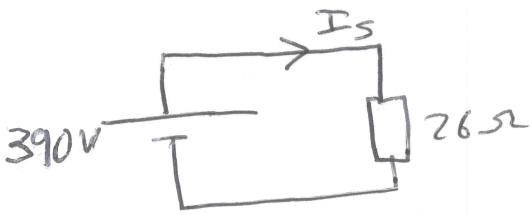
TRANSFORM TO THEVENIN EQUIVALENT:

$V_1 = 25 \cdot 15 = 375$



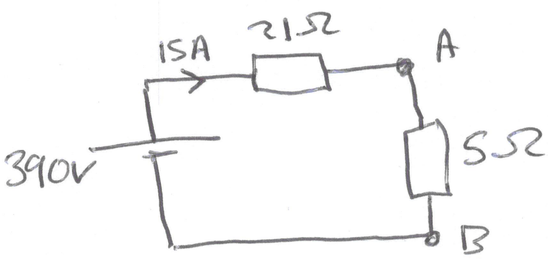
$V_2 = 2.5 \cdot 6 = 15V$

TOTAL IMPEDANCE:



$I_S = \frac{390}{26} = 15A$

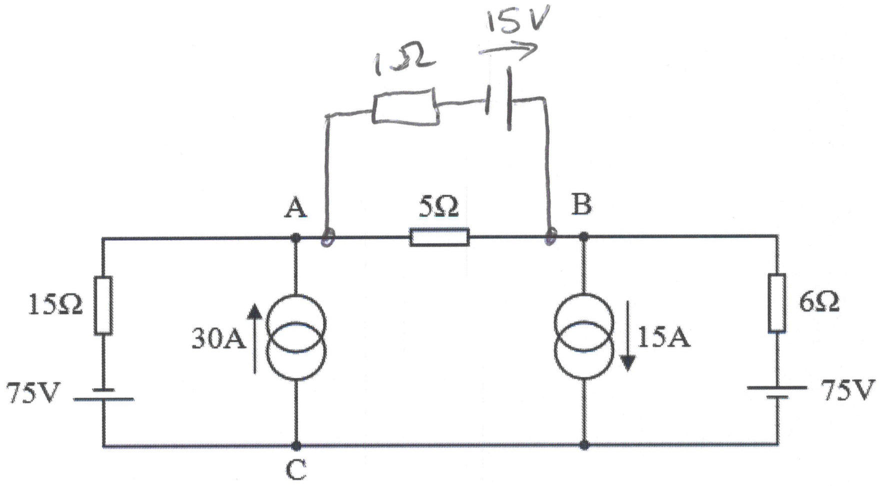
THEVENIN EQUIVALENT CIRCUIT:



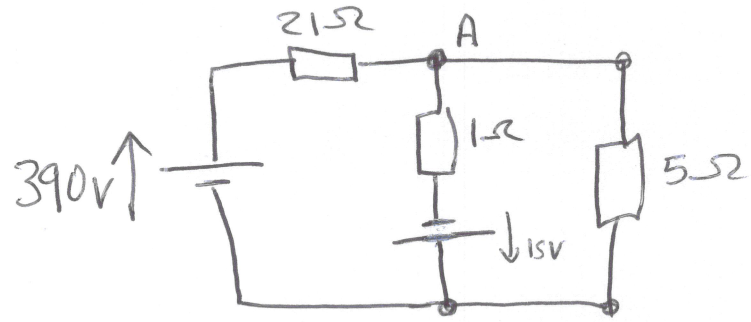
ii So POWER DISSIPATED IN 5Ω: $P = I^2 R$

$P = 15^2 \cdot 5 = 1125W$ OR 1.1kW

Q1 (c)

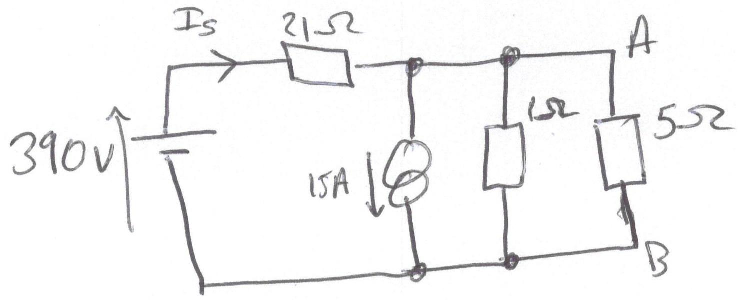


CONSIDERING ANSWER TO Q1 (b), CIRCUIT BECOMES:



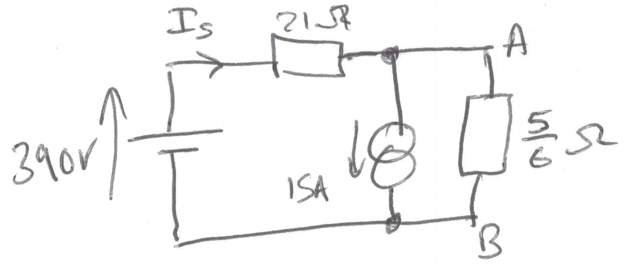
NORTON EQUIVALENT CCT OF NEW BATTERY:

$$I_N = \frac{V_{th}}{R_{th}} = \frac{15}{1} = 15\,A \quad , \quad R_N = R_{th} = 1\,\Omega$$

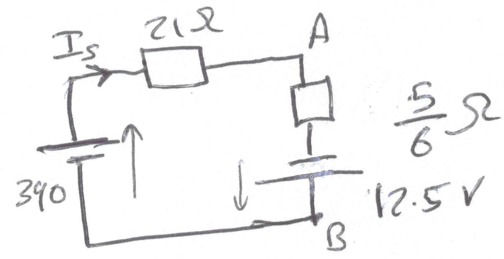


CCT 1

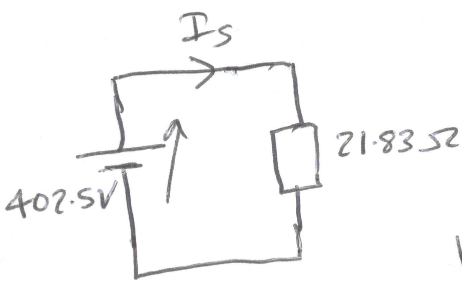
OR



$\equiv$



$\Rightarrow$



$$\therefore I_S = \frac{402.5}{21.83} = 18.44\,A$$

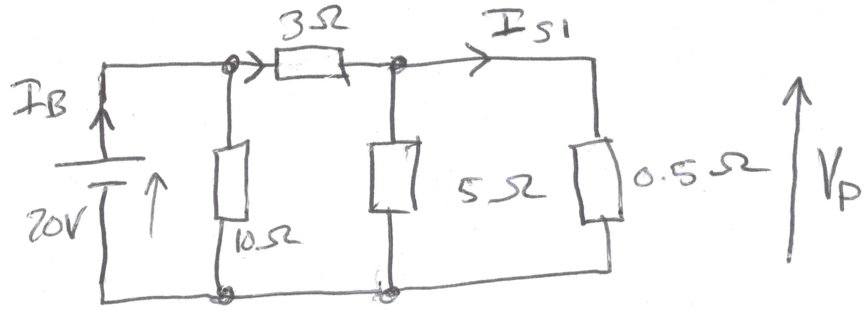
V_{AB} FROM CCT 1 $= 390 - 21 \cdot I_S$

$\therefore V_{AB} = 2.76\,V$ SO NEW POWER DISSIPATED $= 1.52\,W$

Q2 (a) i

A in STEADY-STATE :

INDUCTOR = SHORT CCT



$$R_{TOT} = 10 // (3 + 5 // 0.5) = 2.568\ \Omega$$

$$\therefore I_B = \frac{20}{2.568} = 7.788\ A$$

So CURRENT IN 3Ω RESISTOR $= I_B - \frac{20}{10\Omega} = 5.788\ A$

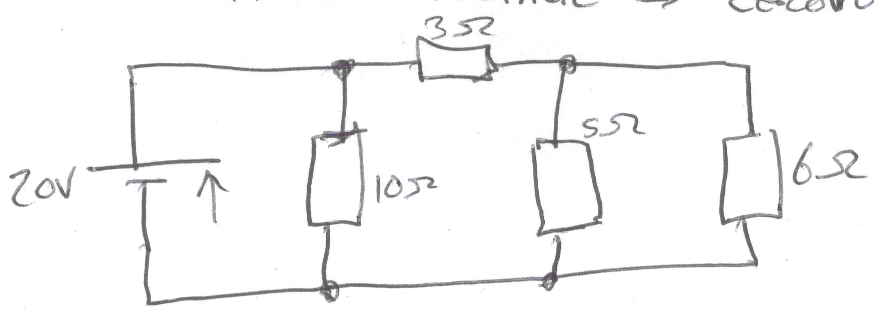
$$\therefore V_P = 20 - (3 \times 5.788) = 2.636\ V$$

$$\therefore I_{S1} = \frac{V_P}{0.5\Omega} = 5.271\ A$$

IMMEDIATELY AFTER SWITCH :

CAPACITOR OPPOSES CHANGE IN VOLTAGE → ZERO VOLTS = SHORT-CCT

So



$$R_{TOT} = 10 // (3 + 5 // 6) = 3.642\ \Omega$$

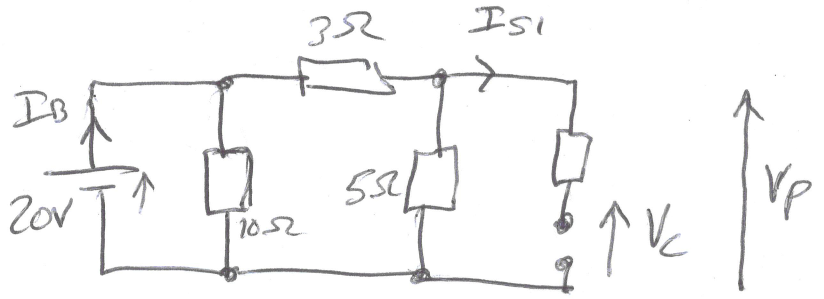
$$\therefore I_B = \frac{20}{3.642} = 5.492\ A$$

So CURRENT IN 3Ω RESISTOR $= I_B - \frac{20}{10\Omega} = 3.492\ A$

$$\therefore V_P = 20 - (3 \times 3.492) = 9.524\ V$$

$$\therefore I_{S1} = \frac{V_P}{6} = 1.587\ A$$

Q2(a)i CONT'D



B IN STEADY-STATE:

CAPACITOR = OPEN CCT

SO $I_{S1} = 0A$

Q2(a)ii

POSITION A: C DISCHARGED SO $E_C = 0J$

I IN $L = I_{S1} = 5.271A$

$\therefore E_L = \frac{1}{2}LI^2 = 0.139J$

POSITION B: L DISCHARGED IF STEADY-STATE SO $E_L = 0$ AS $I = 0$

VOLTAGE ACROSS $C = V_C = V_P$ IF OPEN-CCT

$R_{TOT} = 10 // (3+5) = 4.444\Omega$

$\therefore I_B = \frac{20}{4.444} = 4.5A$

CURRENT IN 3Ω RESISTOR $= I_B - \frac{20}{10\Omega} = 2.5A$

SO $V_P = V_C = 20 - (2.5 \times 3) = 12.5V$

$E_C = \frac{1}{2}CV^2 = 7.8mJ$

Q2(a)iii

THE INDUCTOR IS CHARGED AT COMMUTATION, HOWEVER IN POSITION B THERE IS NO DISCHARGE CIRCUIT PATH AND SO EXPERIENCES HUGE $\frac{dI}{dt}$.

THIS INDUCES HUGE VOLTAGE SPIKES INTO THE CIRCUIT AND MAY DAMAGE DEVICES / CAUSE EMI PROBLEMS

Q2(a)iv)

POSITION A: $R_{TOT} = 10 // (3 + 5 // 0.5) = 2.568\,\Omega$

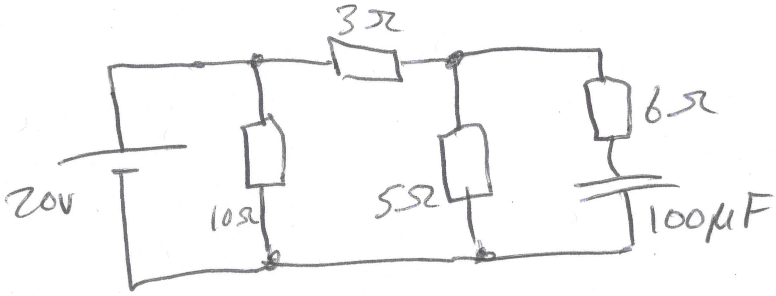
$$P = \frac{V^2}{R} = \frac{20^2}{2.568} = \underline{155.76\ W}$$

POSITION B: $R_{TOT} = 10 // (3 + 5) = 4.444\,\Omega$

$$P = \frac{V^2}{R} = \frac{20^2}{4.444} = \underline{90\ W}$$

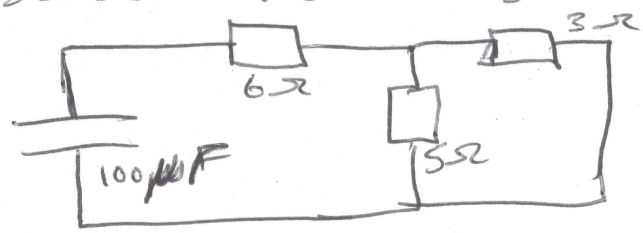
Q2(a)v

CCT:



BY INSPECTION: VOLTAGE SOURCE GOES TO SHORT-CCT

SO CAPACITOR "SEES"



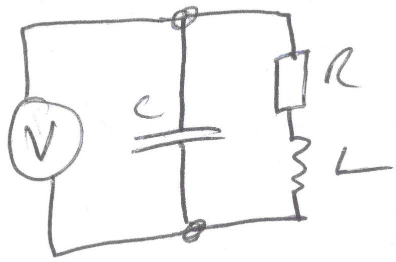
$$\text{SO } \tau = C(6 + 3//5)$$
$$= 0.788\ mS$$

REMEMBERING THAT A CAPACITOR 10% → 90% CHARGE TIME $= 2.2\tau = 1.73\ mS$

SO 2mS WOULD <u>NOT</u> BE SUFFICIENT TO ASSUME A STEADY-STATE VALUE.

PREFER TO USE 5x RISE TIME OR $\approx 10 \times \tau$

Q3 (a) i



$$Z = \frac{1}{j\omega C} \Big/\Big/ (R + j\omega L)$$

$$Z = \frac{(R + j\omega L)\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$Z = \frac{R + j\omega L}{j\omega CR + (j\omega)^2 LC + 1} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega CR}$$

Q3 (a) ii

SYSTEM BECOMES RESONANT WHEN j TERMS FORCED TO ZERO:

COMPLEX CONJUGATE:

$$\frac{(R + j\omega L)(1 - \omega^2 LC - j\omega CR)}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

NUMERATOR THEREFORE IN TERMS OF J : $j\omega L(1 - \omega^2 LC) - R\, j\omega CR = 0$

$$\text{OR} \quad j\omega L = j\omega CR^2 + j\omega^3 L^2 C$$

$$L = C(R^2 + \omega^2 L^2)$$

$$\text{SO} \quad C = \frac{L}{R^2 + \omega^2 L^2}$$

Q3 (a) iii

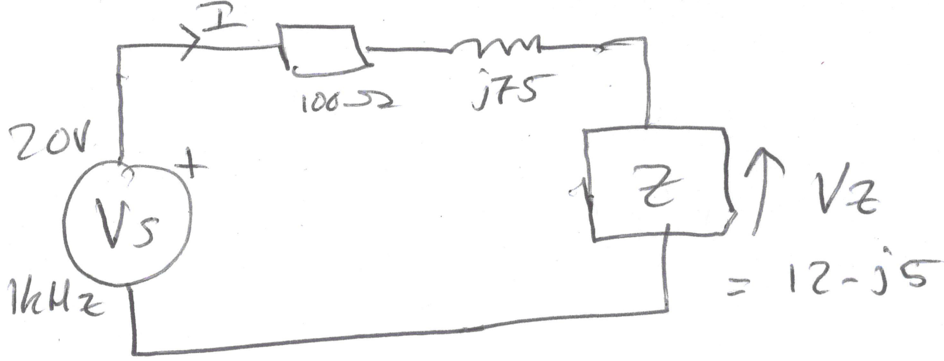
$$I_{LAMP} = \frac{V}{Z_{LAMP}} = \frac{230}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\Rightarrow |I_L| = \frac{230}{\sqrt{4 \times 10^4 + 16\pi^2 50^2}}$$

$$= \frac{230}{659.4} = 0.349\ A$$

$$\therefore P_{LAMP} = I_L^2 R = 0.349^2 \times 100 = \underline{12.2\ W}$$

Q3(b)i

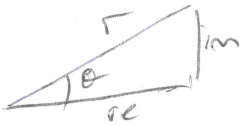


$$I = \frac{V_s - V_z}{100 + j75} = \frac{20 - 12 + j5}{100 + j75} = \frac{8 + j5}{100 + j75}$$

CONVERT TO POLAR:

$$\frac{9.43 \angle 32.0}{125 \angle 36.9} = \underline{0.0754 \angle -4.9}$$

$$= \underline{0.075 - j\,0.006}$$



$\sin\theta = \frac{opp}{hyp}$

$\tan\theta = \frac{adj}{hyp}$

$$Z = \frac{V_z}{I} = \frac{12 - j5}{0.0754 \angle -4.9} = \frac{13 \angle -22.6}{0.0754 \angle -4.9}$$

$$= \underline{172 \angle -17.7}$$

$$= \underline{164 - 52.3j}$$

Q3(b)ii

SINCE THERE IS A -VE j COMPONENT, MUST BE R & C $\rightarrow$ $\underline{R = 164\,\Omega}$

$$X_C = \frac{1}{2\pi f C} = 52.3 \Rightarrow C = \frac{1}{2\pi \cdot 1\text{kHz} \cdot 52.3} = \underline{3\mu F}$$

QUESTION 4 $\boxed{KM\ 1}$

a) [Note: It is not incorrect to draw the exact equivalent circuit - this will be awarded full marks. If the approximate equivalent circuit is used then the basis for moving the magnetising branch needs to be included.]

Since the magnetising reactance and the core loss resistance are much greater than the primary resistance and reactance, the magnetising branch can be moved to the terminals to give the approximate equivalent circuit which is easier to solve.

Secondary impedances are referred to the primary winding:

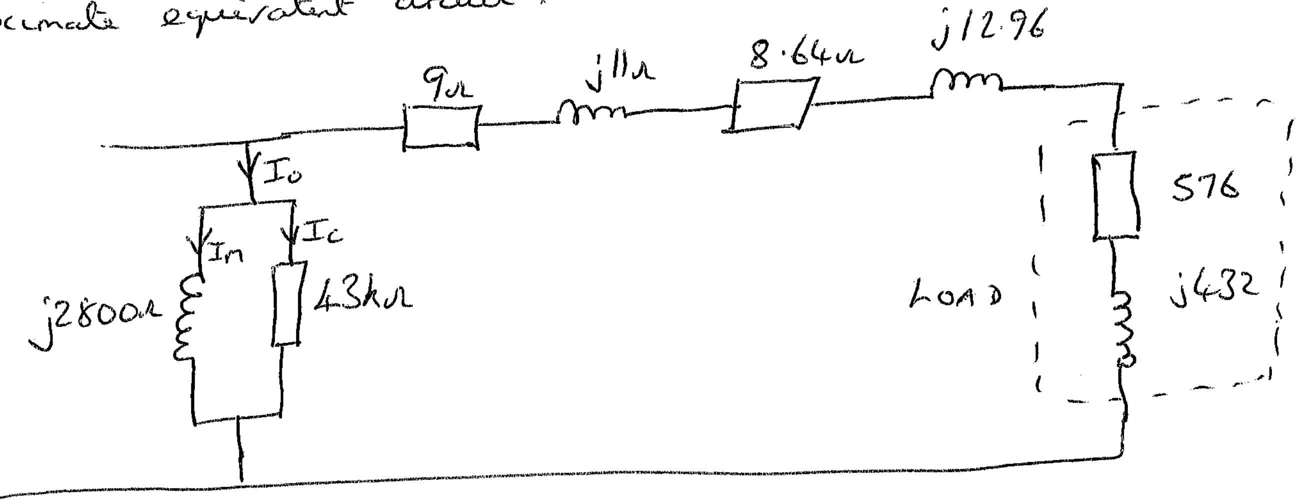
$$Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

$$\therefore \quad R_2' = (12)^2 \cdot 0.06 = 8.64\,\Omega$$

$$X_2' = (12)^2 \cdot j0.09 = j12.96\,\Omega$$

$$Z_L = (12)^2 (4 + j3) = 576 + j432\,\Omega$$

Approximate equivalent circuit:



(b) No load current $= I_0 = I_m + I_c$

$$\therefore I_0 = \frac{3300}{j2800} + \frac{3300}{43000} = 0.07674 - j1.1786$$

$$\equiv \underline{\underline{1.18\angle -86.27^\circ}}$$

QUESTION 4 (CONTINUED) KM2

(c) Core losses = Power dissipated in 43kΩ resistor:

Either $P_{FE} = I_C^2 R_C = 0.07674^2 \times 43000 = 253.2W$

or $P_{FE} = V^2/R_C = 3300^2/43000 = \underline{\underline{253.2W}}$

(d) With the load now connected:

(i) $$I_2' = \frac{V}{Z_{LTOT}} = \frac{3300}{(9+8.64+576)+j(11+12.96+432)}$$

$$= \frac{3300}{(593.64+j455.96)} = 4.41\angle{-37.52^\circ}$$

To find the actual load current this needs referring back to the secondary:

$$\frac{I_2'}{I_2} = \frac{N_2}{N_1} \Rightarrow I_2 = \frac{N_1}{N_2} \times I_2' = 12 \times 4.41\angle{-37.52^\circ}$$

$$= \underline{\underline{52.92\angle{-37.52^\circ}A}}$$

(ii) The input current to the transformer is:

$$I_1 = I_0 + I_2' = 1.18\angle{-86.27^\circ} + 4.41\angle{-37.52^\circ}$$

$$= \underline{\underline{5.263\angle{-47.22^\circ}\ A}}$$

(iii) Actual output voltage:-

$$V_2 = V_2' \cdot \frac{N_2}{N_1} = I_2' \cdot Z_2' \cdot \frac{N_2}{N_1} = 4.41\angle{-37.52} \times (576+j432) \times \frac{1}{12}$$

$$= \underline{\underline{264.5\angle{-0.65^\circ}}}\ V_{rms}$$

No load voltage: $V_2 = 3300 \times \frac{1}{12} = 275V_{rms}$

$$\therefore \text{Regulation} = \frac{\text{No-load V} - \text{On-load V}}{\text{No-load V}} = \frac{275-264.5}{275} \times 100 = \underline{\underline{3.82\%}}$$

(iv) Copper losses $= I_2'(R_1 + R_2') = 4.41^2(9 + 8.64)$

$$= \underline{\underline{343\,W}}$$

(v) Real component of output power:

$$P_{out} = 4.41^2 \times 576 = \underline{\underline{11202\,W}}$$

Total losses $= P_{Fe} + P_{Cu} = 343 + 253.2 = 596.2\,W$

Hence efficiency $= \dfrac{P_{out}}{P_{out} + P_{loss}} = \dfrac{11202}{11202 + 596.2} \times 100 = \underline{\underline{94.9\%}}$

(e) The use of a solid iron core would significantly increase the core (iron) loss within the device as usually laminations are used to limit circulating eddy currents within the core during operation. Therefore whilst the device my be cheaper to produce it would be much less efficient in operation.

QUESTION 5 $\boxed{KM\ 4}$

(a) (i) Phase impedance $Z = 8.1 + j6.7\ \Omega$

Phase voltage $= \dfrac{6600}{\sqrt{3}} = 3810.5\ V_{rms}$

$\therefore$ Phase current $= \dfrac{V\angle 0^\circ}{Z} = \dfrac{3810.5\angle 0^\circ}{(8.1 + j6.7)} = \underline{\underline{362.5\angle -39.6^\circ\ A}}$

Since the system is star-connected phase current = line current, hence

Line current $= \underline{\underline{362.5\angle -39.6^\circ\ A_{rms}}}$

(ii) Real power $= \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 6600 \times 362.5 \cos 39.6^\circ$

$= \underline{\underline{3.193\,MW}}$

Reactive power $= \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 6600 \times 362.5 \sin 39.6^\circ$

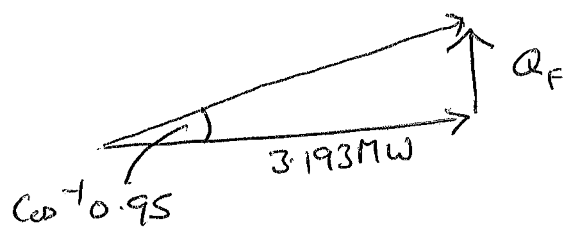
$= \underline{\underline{2.641\ MVAR}}$

Apparent power $= \sqrt{3}\ V_L I_L = \sqrt{3} \times 6600 \times 362.5$

$= \underline{\underline{4.144\,MVA}}$

(b) (i) Adding the capacitor bank will not affect the real power.

After the capacitors are added:



$\phi = \cos^{-1} 0.95 = 18.19^\circ \Rightarrow \tan\phi = 0.3287$

Hence $Q_F = 3.193 \times \tan\phi = 3.193 \times 0.3287 = 1.05\,MVAR$

QUESTION 5 (CONTINUED)

Therefore the capacitor bank must provide:

$$2.641 - 1.05 = 1.591\ \text{MVAR} \text{ or } 0.5303\ \text{MVAR/phase.}$$

Capacitor voltage = 6600 V_{rms} since they are delta connected.

Now $Q_{c\text{-phase}} = \dfrac{V_{phase\text{-}rms}^2}{X_c} \Rightarrow X_c = \dfrac{6600^2}{0.5303 \times 10^6} = 82.14\Omega$

Hence $C = \dfrac{1}{2\pi f X_c} = \dfrac{1}{2\pi \times 50 \times 82.14} = \underline{\underline{38.75\mu F\ /\text{phase}}}$.

(b)(ii) $I_{ph\text{-}cap} = \dfrac{6600}{82.14} = 80.35\ \text{Arms}$

$I_{line\text{-}cap} = \sqrt{3} \times 80.35 = 139.17\ \text{Arms.}$

Original plant current $= 362.5\angle -39.6° \equiv 279.31 - j231.1$ Arms

$\therefore$ New current with caps $= 279.31 - j231.1 + j139.17$
$= 279.31 - j91.93 \equiv \underline{\underline{294.05\angle -18.2°\ \text{Arms}}}$

(Alternatively $P = \sqrt{3} V_L I_L \cos\phi$

$\therefore I_L = I_{ph} = \dfrac{P}{\sqrt{3} V_L \cos\phi} = \dfrac{3.193 \times 10^6}{\sqrt{3} \times 6600 \times 0.95} = \underline{294\ \text{Arms}}$

and phase angle $= \cos^{-1} 0.95 = -18.2°$ (– since p.f. lagging))

(iii) Maximum voltage rating of capacitors = peak voltage
$= \sqrt{2} \times 6600 = \underline{\underline{9334\ V}}$

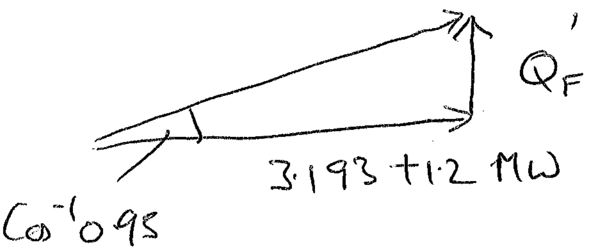
QUESTION 5 (CONTINUED)

KM6

(iv) Either: Reduce cable/line losses

Reduce voltage drops along lines.

(c) (i) With the synchronous machine the new power demand will be:



$$Q_F' = 4.393 \tan(\cos^{-1} 0.95) = 1.444 \text{MVAR.}$$

Hence the synchronous machine provides $2.641 - 1.444 = \underline{1.197 \text{MVAR}}$

(ii) The synchronous machine MVA:

$$S = \sqrt{P^2 + Q^2} = \sqrt{1.2^2 + 1.197^2} = \underline{1.695 \text{MVA}}$$

and it operates at a power factor of:

$$\cos\phi = \text{p.f.} = \frac{1.2}{1.695} = \underline{0.708 \text{ leading}}$$

(iii) $$I_L = I_{ph} = \frac{S_{SM}}{\sqrt{3} V_L} = \frac{1.695 \times 10^6}{\sqrt{3} \times 6600} = 148.27 \angle +44.93^\circ \ A_{rms}$$

$$\text{Hence } Z_{ph} = \frac{\frac{6600}{\sqrt{3}} \angle 0^\circ}{148.27 \angle 44.93^\circ} = 25.7 \angle -44.93^\circ = \underline{18.19 - j18.15 \ \Omega}$$

QUESTION 6

KM 7

(a) (i) The total flux in the airgap is:

$$\phi = B.A = 0.8 \times 100\times10^{-6} = 8\times10^{-5}\,Wb$$

Hence the total reluctance is:

$$S_T = \frac{NI}{\phi} = \frac{200\times4}{8\times10^{-5}} = 1\times10^{7}\,H^{-1}$$

Now the reluctance can also be calculated as:

$$S_T = S_{IRON} + S_{AIR}$$

$$= \frac{\pi.0.12 - g}{2000\mu_o \times 100\times10^{-6}} + \frac{g}{\mu_o \times 100\times10^{-6}}$$

$$\therefore \frac{\pi.0.12 - g}{2000} + g = 1\times10^{7}\times\mu_o\times100\times10^{-6}\times2000$$

$$\therefore 1999g + 0.377 = 1.257\times10^{-3}\times2000$$

$$\therefore \underline{\underline{g = 1.07\,MM}}$$

(ii) The inductance is given by:

$$L = \frac{N^2}{S} = \frac{200^2}{1\times10^7} = \underline{\underline{4\,mH}}$$

(b) The peak current of the AC supply must be 4 since from part (a) 4A DC produced a flux density of 0.8T in the airgap. Hence the rms current $= 4/\sqrt{2}$ A.

The impedance of the device is:

$$Z = R + j2\pi fL = 5 + j.2\pi f.0.004$$

$$\text{or } |Z| = \sqrt{5^2 + (0.025f)^2}$$

QUESTION 4 (CONTINUED)

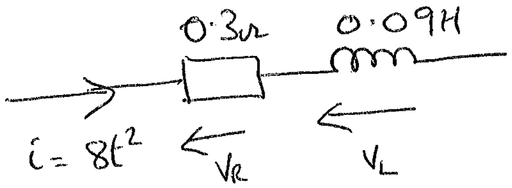
KM 8

$$\therefore \quad \frac{V_{rms}}{Z} = \frac{100}{\sqrt{5^2+(0.025f)^2}} = \frac{4}{\sqrt{2}}$$

$$\therefore \quad 25 + (0.025f)^2 = \left(\frac{100\sqrt{2}}{4}\right)^2$$

$$\therefore \quad \underline{\underline{f = 1.4\,kHz}}$$

(C)(i)



At $t = 1.5s$ $\quad i = 8t^2 = 18A$

$V_R = 0.3 \times 18 = 5.4v$

$V_L = L\frac{di}{dt} = 0.09 . 16t = 2.16v$

Hence total voltage is $5.4 + 2.16 = \underline{\underline{7.56v}}$

(ii) Since both V_R and V_L will be at their highest value at $t = 4.5s$ then:

$V_R = 0.3 . 8t^2 = 2.4t^2 = 2.4 \times 4.5^2 = 48.6v$

$V_L = 0.09 \times 16t = 0.09 \times 16 \times 4.5 = 6.48v$

$\therefore$ Maximum voltage required $= 48.6 + 6.48 = \underline{\underline{55.08V}}$

(iii) Power dissipation in the resistor $= I^2R$

$P = (8t^2)^2 \times 0.3 = 19.2t^4$

at $t = 2s$

$$\underline{\underline{P = 307W}}$$

QUESTION 4 (CONTINUED)

KM9

(iv) Since when $t=0$ there is no energy stored in the inductor, then the total energy stored in the inductor at $t=4.5s$ can be found from:

$$E = \frac{1}{2} L i^2$$

$$\therefore \quad E = \frac{1}{2} L \left(8t^2\right)^2 = 2.88 \times 4.5^4 = 1181 J$$

Energy dissipated in the resistor over the 4.5s period:

$$E_R = \int_0^{4.5} V_R \cdot i \, dt = \int_0^{4.5} i^2 R \, dt = \int_0^{4.5} 0.3 \times 64 t^4 \, dt$$

$$\therefore \quad E_R = 0.3 \left[\frac{64t^5}{5}\right]_0^{4.5} = 7086 J$$

$\therefore$ Total energy supplied from source $= 7086 + 1181$

$= \underline{\underline{8267 J}}$

$\therefore$ Average power drawn $= \dfrac{8267}{4.5} = \underline{\underline{1837 W}}$