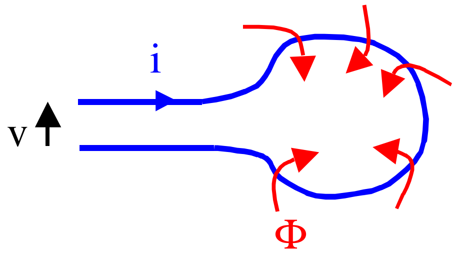
Coupled Circuits and Transformers.

Self inductance (revision of earlier notes)

The current, i, produces flux ϕ which links the circuit of n turns (N=1 turn in many circuits). If the current and hence the flux is time varying, then the induced e.m.f.:



$$e = \frac{d\varphi}{dt} = \frac{d(N\Phi)}{dt}$$

where ψ = flux linkage = $N\phi$.

If the time variation of the flux linkage, ψ, is due to a current variation with time, we can also write this as

$$e = \frac{d}{dt} N \left(\frac{Ni}{s} \right) = \frac{N^2}{s} \frac{di}{dt} = L \frac{di}{dt}$$

where L is called the self inductance:

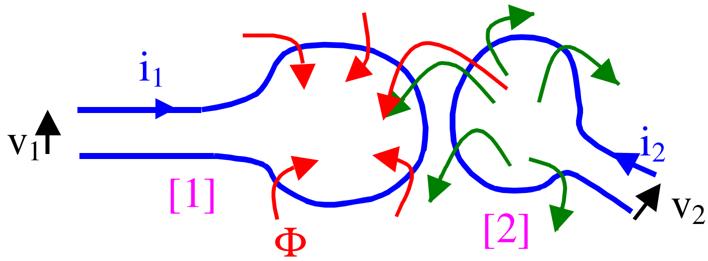
$$L = \frac{N^2}{s} \quad or \quad \frac{N\Phi}{i}$$

And neglecting circuit resistance:

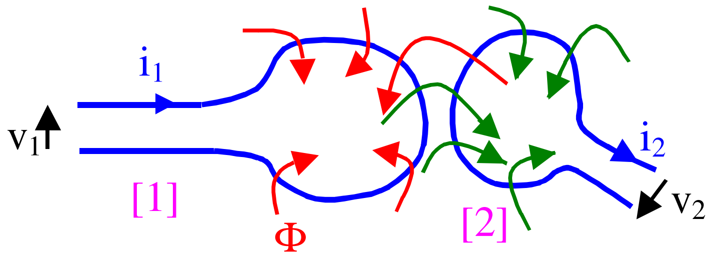
$$v = e = L di/dt$$

Mutual inductance

Mutual Flux from circuit 1 aiding the flux from circuit 2, giving a net increase in the flux passing through both circuits.



Mutual flux produced by one circuit is opposing the flux produced by the other giving a net decrease in flux through the circuits.



If part of the flux produced by a circuit links with a second circuit then they are said to have **Inductive coupling**, or have a **Mutual magnetic field**. (Note: Capacitive coupling is via a mutual electric field).

Suppose a proportion k_1 of ϕ_1 links circuit [2], and a proportion k_2 of ϕ_2 links circuit [1], then the flux linkage for each circuit is:

- for cct [1]:

$$\varphi_1 = N_1(\phi_1 \pm k_2\phi_2) \qquad (1)$$

where +ve is aiding, -ve is opposing

- for cct [2]:

$$\varphi_2 = N_2(\phi_2 \pm k_1\phi_1) \qquad (2)$$

From our previous definitions of self inductance:

$$\phi_1 = \frac{L_1 i_1}{N_1}, \quad \phi_2 = \frac{L_2 i_2}{N_2}$$

therefore:

$$\varphi_1 = L_1 i_1 \pm \left(k_2 \frac{N_1 L_2}{N_2} \right) i_2 \qquad (3)$$

and:

$$\varphi_2 = L_2 i_2 \pm \left(k_1 \frac{N_2 L_1}{N_1} \right) i_1 \qquad (4)$$

Clearly the bracketed terms must have the same dimensions (units) as L_1 and L_2 (Henry). i.e. must be inductance.
It can be shown that (by conservation of energy):

$$k_2 \frac{N_1 L_2}{N_2} = k_1 \frac{N_2 L_1}{N_1} = M \qquad (5)$$

where M is called the mutual inductance. Hence (3) and (4) become:

$$\varphi_1 = L_1 i_1 \pm M i_2 \text{ and } \varphi_2 = L_2 i_2 \pm M i_1$$

and:

$$v_1 = e_1 = \frac{d\varphi_1}{dt} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt} \qquad (6)$$

$$v_2 = e_2 = \frac{d\varphi_2}{dt} = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt} \qquad (7)$$

where the first part of the equation, $L_1 \frac{di_1}{dt}$ is called the self induced voltage, and the second part of the equation, $M \frac{di_2}{dt}$ is called the mutually induced voltage (i.e. due to the mutual flux linkage).

Coefficient of Coupling

From (5) it may be seen that:

$$M^2 = k_1 k_2 L_1 L_2 = k L_1 L_2$$

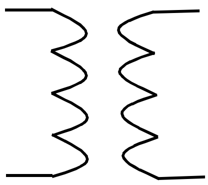
or:

$$M = k\sqrt{L_1 L_2} \quad \text{where} \quad k = \sqrt{k_1 k_2}$$

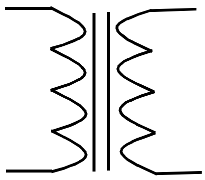
and **k** is called the coefficient of coupling, and $\mathbf{k \leq 1}$. For magnetically coupled circuits, k>0, for maximum coupling, k=1, and if you do not want 'crosstalk' between circuits, k=0. For an ideal transformer, k⇒1.

Circuit Conventions

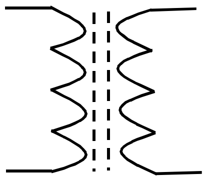
In general circuit work it is not possible to show the actual coils and core material used in coupling circuits and therefore conventions have been used to signify key information when it is important.
e.g.



Coils with air as the coupling mechanism

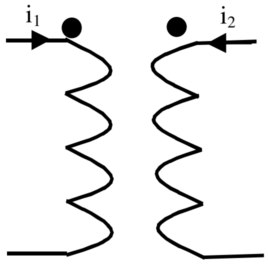


Coils with iron (laminated) as the coupling mechanism



Coils with ferrite 'pot' core

And the ‘dot’ convention to show the sign of the mutual coupling



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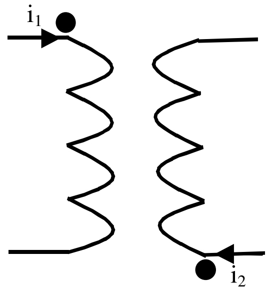
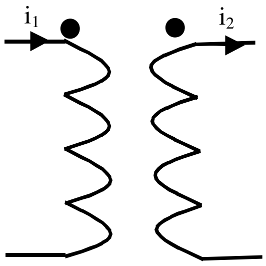
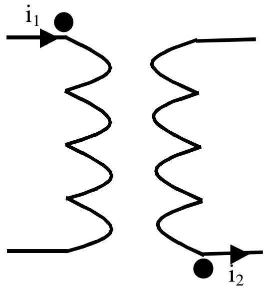


M = +ve, if currents flow into ‘dot’ on each circuit



or



M = -ve if currents flow not as above.

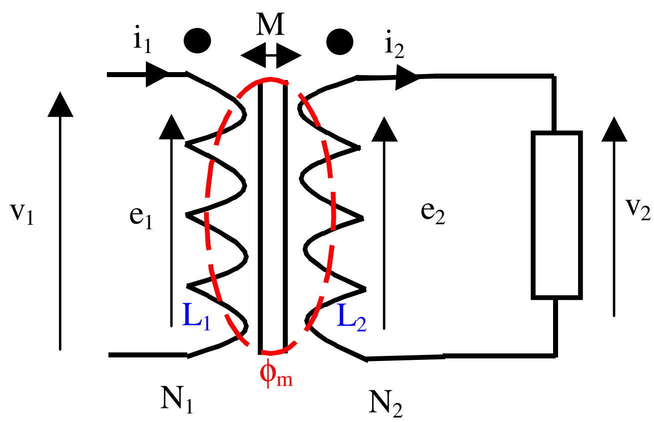
The Ideal transformer

Despite the variety of transformers in use and their varied design objectives, we still retain the concept of an ‘ideal’ transformer, which has well defined properties (c.f. R, L and C in circuits).

Properties of the ideal transformer:

1. it is a lossless device (i.e. no heat losses due to copper (I^2R) and iron loss mechanisms)
2. It has perfect coupling (k=1) between the two windings (i.e. all flux produced is **mutual** to both windings).
3. It has a magnetic core of zero reluctance (S=0)
4. There are no electric field effects (i.e. no capacitive effects)

As a consequence of the above properties, the device would have the following performance characteristics:



i) <u>On no-load</u> – ($i_2 = 0$)
Because the core has zero reluctance, then both windings have infinite self inductance $= N_1^2/S = N_2^2/S$
Therefore when the primary is connected to a supply, $\mathbf{i_1 = 0}$ **if** $\mathbf{i_2 = 0}$

[note in practice, very low reluctance in the magnetic circuit by design, minimal airgaps, but L < **∞.** Therefore a small current flows in the primary, called the magnetising current (i_m).]

If the coefficient of coupling k=1, then all the flux is mutual:

$$v_1 = e_1 = \frac{dN_1\phi_m}{dt} \quad \text{and} \quad v_2 = e_2 = \frac{dN_2\phi_m}{dt}$$

Therefore:

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

ii) <u>On load</u> – ($i_2 \neq 0$)
When both coils carry current then the resultant mmf around the core:

$$F = N_1 i_1 - N_2 i_2$$

and the resultant flux:

$$\Phi_m = \frac{N_1 i_1 - N_2 i_2}{S}$$

in the limit that $S \rightarrow 0$, then $N_1 i_1 - N_2 i_2 \rightarrow 0$ i.e. $N_1 i_1 = N_2 i_2$ or:

$$\boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1}}$$

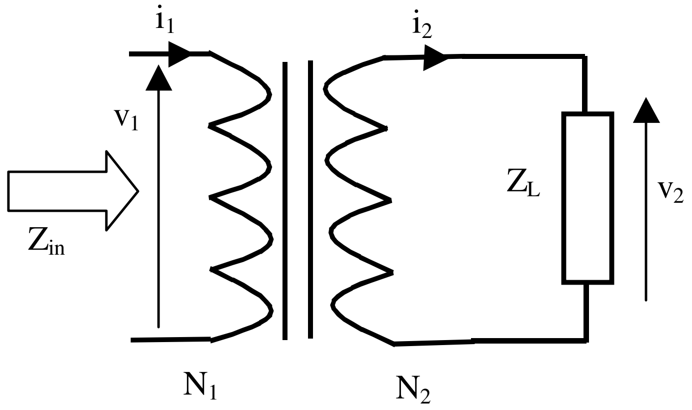
this is the mmf balance, and since the flux is unchanged:

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

as before.

Note: if $S \neq 0$ then a small current, noted in (i), will still flow (magnetising current)

iii) <u>Impedance</u>
As a result of the voltage and current relationships above, there is an impedance change through the transformer:



Clearly for the load impedance, $Z_L = v_2/i_2$, but as $v_2 = v_1 \times (N_2/N_1)$ and $i_2 = i_1 \times (N_1/N_2)$ then we have:

$$\frac{v_1}{i_1} = \frac{v_2}{i_2} \times \left(\frac{N_1}{N_2}\right)^2 = Z_L \times \left(\frac{N_1}{N_2}\right)^2$$

i.e. the load 'appears' at the input of the transformer as an impedance:

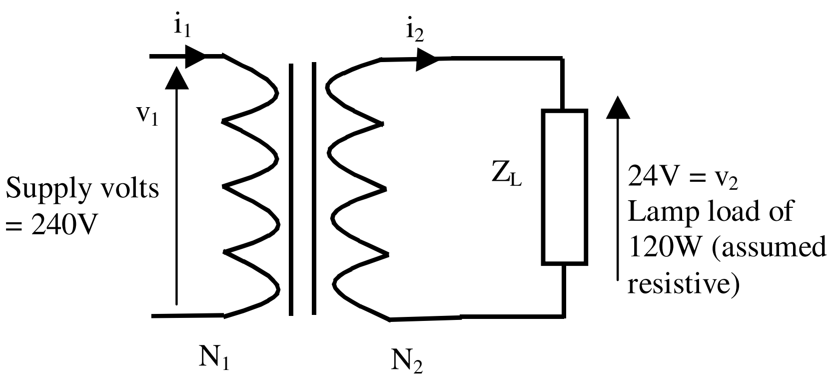
$$Z_L' = Z_L \times \left(\frac{N_1}{N_2}\right)^2$$

or we say it has a 'referred' value as seen from the primary side.

<u>Example</u>

Show how 'first-order' idealised calculations can be done (see tutorial sheet). Such calculations are a valuable first step to check results.

A transformer is used to step down a mains supply of 240V to a safe 24V for a lighting system. If the total load is 120W, choose a suitable turns ratio for the transformer and the necessary current rating of its primary and secondary windings.



Ideally, turns ratio = voltage ratio $N_1/N_2 = 240/24 = 10:1$

(Note the actual number of turns could be 100 and 10, or 1000 and 100 etc, the ratio is the same – Actual turns determined by the magnetic circuit using $V_1=4.44fN_1\phi_m$ as in the earlier notes and tutorial sheet).

For a load of 120W @ 24V, load resistance is given by:

$$W = \frac{V^2}{R} \quad \text{or} \quad R = \frac{V^2}{W} = \frac{24^2}{120} = 4.8\Omega$$

Using the principle of 'referred' secondary quantities, then the actual load, R_L (4.8Ω) 'appears' as a load in the primary of:

$$R'_L = \left(\frac{N_1}{N_2}\right)^2 R_L \quad = \; 4.8 \times 10^2 = 480\Omega$$

$$I_1 \quad = \quad \frac{240}{480} = 0.5A$$

and:

$$\frac{I_1}{I_2} \quad = \quad \frac{N_2}{N_1} \quad \therefore I_2 = 10 I_1 = 5A$$

i.e. primary must be rated to carry 0.5A, the secondary 5A.

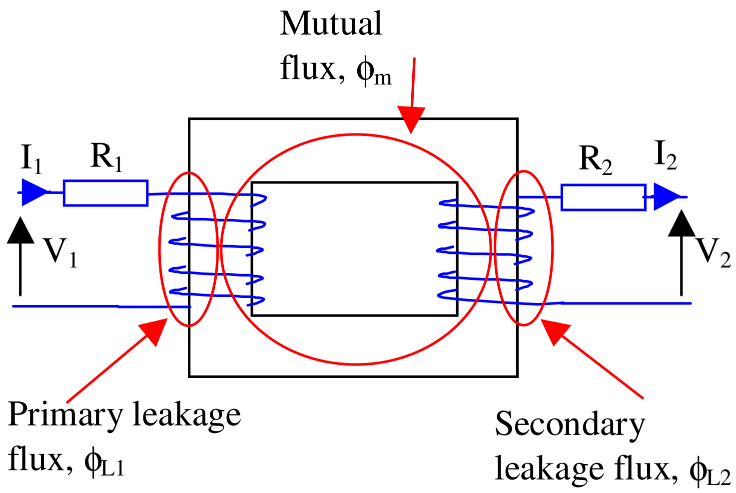
Practical transformers

Transformers or various types and sizes are used widely throughout electronic and electrical engineering systems from high frequency communications though to power systems. Basically a 'transformer' defines any device which is designed to couple two (or more) electric circuits via a mutual magnetic circuit. (Note: transformer effects, 'crosstalk' can occur as an unwanted phenomenon in many devices and circuits). Transformers can be designed for very different purposes and to meet many different specifications e.g.

1. Power transformers – designed to transmit bulk power (10's of Watts up to MW) from one voltage level to another (step up or step down) – Key specifications here are the efficiency and voltage regulation (change of voltage with load).
2. Instrumentation transformers – designed to condition ('scale') a voltage or current for safe measurements (e.g. 4kV to 5V for measuring the voltage on an overhead transmission line) – Key specifications here are the accuracy of the scaling, and the phase shift if used in conjunction with power measurements.
3. Isolating transformers – often 1:1 turns ratio used to provide a safe ('unearthed') supply (electric razor point)
4. Matching transformers – often used as part of an audio frequency system to match an amplifier to it's load (or loudspeaker) – Key specifications here are the fidelity of the signal (phase and amplitude shifts of the different frequency components)
5. Pulse transformers – used to scale a pulse rather than a sinewave – again issues such as rise times and signal droop are important.

Despite all of these variations, most will ideally behave as the 'ideal' transformer – though the designer may optimise the non-ideal features for the specific application.

In practice:-



(i) there are finite winding resistances R_1 and R_2 (and associated losses $I_1^2R_1$ and $I_2^2R_2$)

(ii) There are finite iron losses (eddy current and hysteresis)

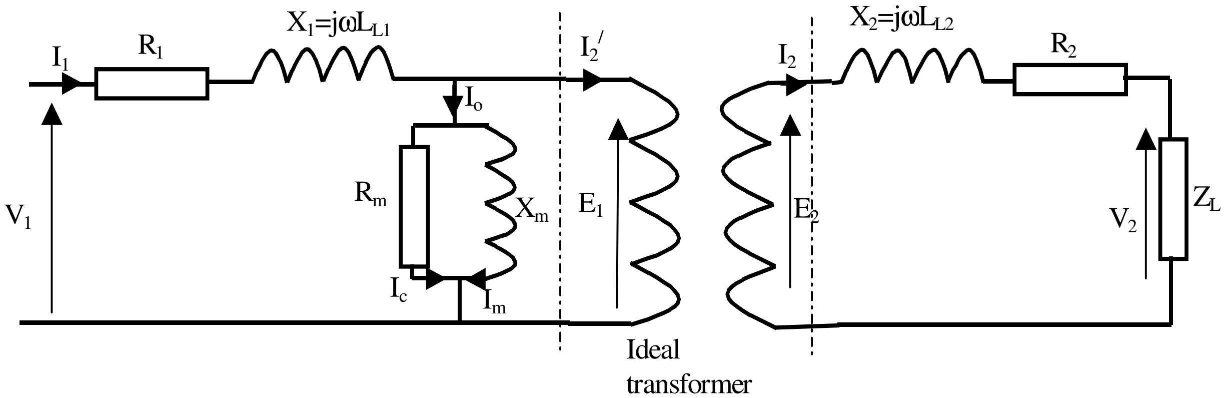
(iii) The windings are not perfectly coupled ($k \neq 1$) and hence for each winding some flux is mutual ϕ_m giving rise to the coupling and mutual inductance, and some of the flux links only one winding, ϕ_{L1}, and ϕ_{L2} for the primary and the secondary winding respectively, giving rise to the leakage inductances, L_{L1} and L_{L2} respectively.

(iv) The core is not of zero reluctance and hence some mmf is needed to drive ϕ_m around the core i.e.:

$$N_1 I_m = N_1 I_1 - N_2 I_2$$

where I_m = magnetising current

As a result of (i) to (iv), a more complex model or **equivalent circuit** is require to fully characterise behaviour of the practical transformer. One such circuit is:



(i) for the ideal part, $E_1/E_2 = N_1/N_2$ and $I_2'/I_2 = N_2/N_1$ Where I_2' is the referred secondary current.

(ii) The **Primary Leakage reactance X_1** = ωL_{L1} and the **Secondary leakage reactance X_2** = ωL_{L2} are both small since the transformer is designed for low leakage.

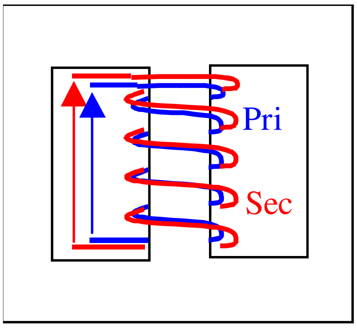
(iii) The **Primary Winding resistance, R_1**, and **the Secondary Winding resistance, R_2**, are designed to be small to reduce the winding losses.

(iv) The **Magnetising Reactance X_m** = ωL_m is designed to be high since the core is designed for low reluctance (L_m high).

(v) The **Core Loss resistor R_m** is a high value since the core is designed for low loss, and loss = E_1^2/R_m

(vi) The **no-load current I_o** is generally small when compared to I_1 and I_2' and is clearly the current which flows in the primary when there is no load on the secondary winding. Where $I_o = I_m + I_c$

(vii) On-load, total input current = $I_1 = I_2' + I_o$



Note: To minimise ϕ_{L1} and ϕ_{L2}, primary and secondary windings are usually concentric rather than on separate limbs of the magnetic circuit.

Two Simplifications (to make calculations easier!)

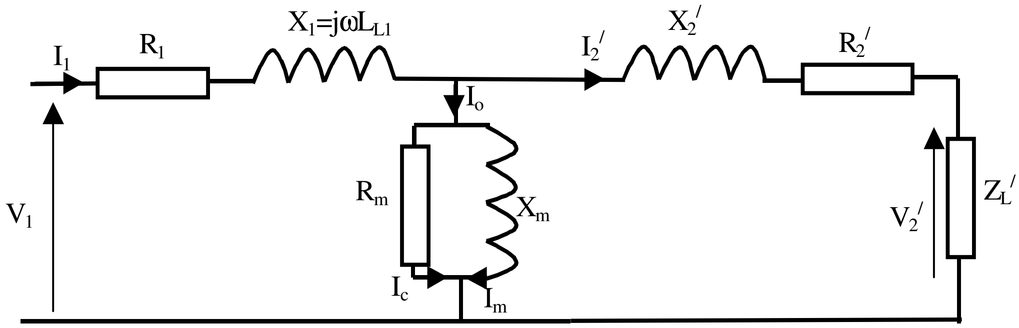
(i) Referral of impedances through the ideal transformer part

NB. This refers to the transformers own secondary resistance and leakage reactance, as well as the load.

As noted earlier, any secondary quantities (V_2, I_2, and Z_2) can be referred to the primary as:

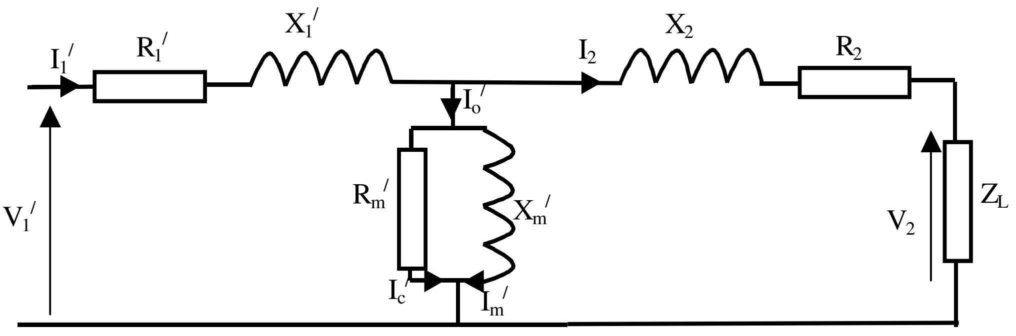
$$V_2' = V_2 \times \frac{N_1}{N_2} \qquad I_2' = I_2 \times \frac{N_2}{N_1} \qquad Z_L' = Z_L \times \left(\frac{N_1}{N_2}\right)^2$$

Hence the circuit may be re-drawn with all of the secondary quantities referred to the primary circuit:



Equivalent circuit referred to the Primary

An alternative procedure to the above may be used whereby all the primary side values are referred to the secondary.



Equivalent circuit referred to the Secondary

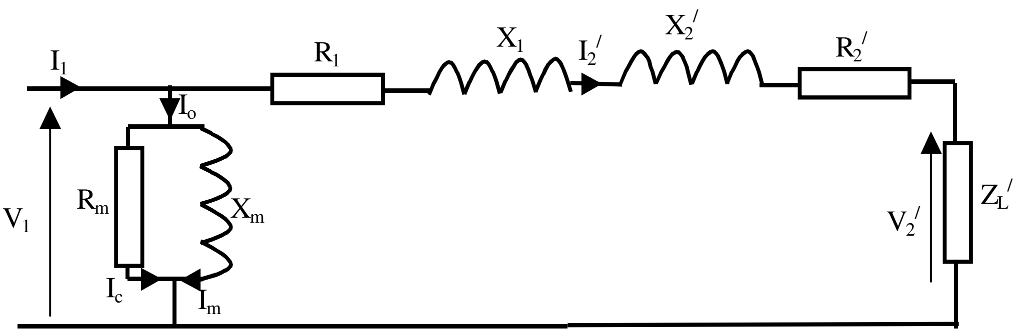
$$V_1' = V_1 \times \frac{N_2}{N_1} \qquad I_1' = I_1 \times \frac{N_1}{N_2} \qquad R_1' = R_1 \times \left(\frac{N_2}{N_1}\right)^2 \quad \text{etc}$$

Either technique reduces the problem to a single circuit without the 'ideal transformer' to complicate the situation.

(ii) An approximate equivalent circuit which is easier to solve.

This involves a small error, but significantly simplifies the solution!

Since R_m and X_m >> R_1 and X_1 for many calculations, then under the circumstances, the so-called magnetising branch can be moved to the input terminals (NB occasionally this is too inaccurate an assumption, and must not be used).



Approximate equivalent circuit referred to the primary

Operating performance of iron-cored transformers

Power transformers:
Interested particularly in efficiency and voltage ratio:

Efficiency: $$\eta = \frac{\text{output power}}{\text{input power}} \times 100\%$$

Usually more accurate to calculate:

$$\eta = \frac{\text{input power} - \text{losses}}{\text{input power}} \times 100\%$$

The losses in the transformer are:

(i) Iron losses (P_{Fe}) = $E_1^2/R_m \approx V_1^2/R_m$ $\Rightarrow$ Hysteresis and eddy current losses

(ii) Copper losses (P_{Cu}) = $I_1^2R_1 + I_2^2R_2$ $\Rightarrow$ Varies with load and constitute a heat loss – (most efficient when $P_{Fe} = P_{Cu}$)

Efficiency of very large units > 98%, e.g. 200MVA units supply 200MW with 99% efficiency and generates a loss of 2MW – cooling required!

Voltage Ratio.
Since there is a voltage drop in the transformer, the actual voltage ratio is a function of the load. [Full load or rated load defines the operating load which gives maximum permissible loss and temperature rise]. The term used to describe this is **'Regulation'**.

$$\text{regulation} = \frac{\text{'no-load' output voltage} - \text{'on-load' output voltage}}{\text{'no-load' output voltage}} \times 100\%$$

Example

- Demonstrate the method of solution.

A single-phase transformer has the following equivalent circuit parameters:

Primary winding resistance,	$R_1=2\Omega$
Primary winding leakage reactance,	$X_1=10\Omega$
Secondary winding resistance,	$R_2=0.04\Omega$
Secondary winding leakage reactance,	$X_2=0.1\Omega$
Magnetising reactance,	$X_m=2.5k\Omega$

The transformer has a no-load iron loss of 220W, and has a primary to secondary turns ratio of 10:1. the transformer is connected to a supply of 2.2kV and a load of $(3+j2)\Omega$. Calculate the following:

(i) The primary input current on no-load
(ii) The primary input current on-load
(iii) The output current and voltage on-load
(iv) The transformer output power and efficiency

Step1: Using the 'ideal' model to get order of magnitude answers

$V_1/V_2 = N_1/N_2$

Therefore:

$V_2=V_1 \times 1/10 \approx 220V$

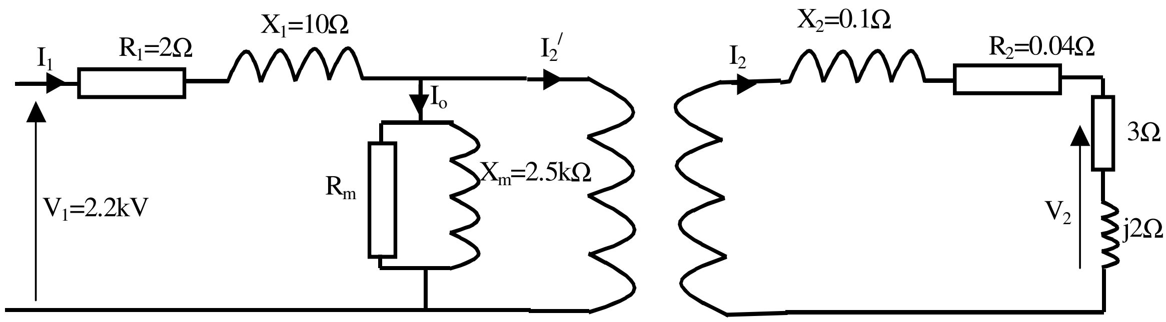
Now $I_2 \approx \dfrac{220\angle 0}{3+j2} = \dfrac{220\angle 0}{3.6\angle 33.7} = 61\angle -33.7 \quad A$

Hence $\dfrac{I_1}{I_2} = \dfrac{N_2}{N_1} \qquad \therefore I_1 = 6.1\angle -33.7 \quad A$

and output power $= I_2^2R_2 = 61^2 \times 3 =$ **11.16kW**

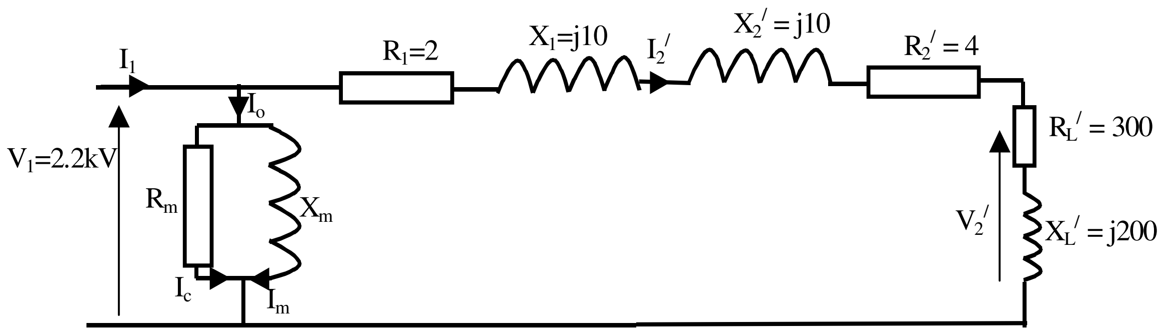
Step2: Obtain the approximate equivalent circuit (Note '**YOU**' always use this unless specifically told otherwise, or, in practice you can see that the approximations made would be inappropriate).

Starting from the un-referred, 'exact' circuit:



Referring all of the secondary quantities to the primary and at the same time moving the parallel R_m, X_m branch of the circuit to the input terminals, where:

$R_2^{/} = (N_1/N_2)^2 \times R_2 = 100R_2, \quad X_2^{/} = 100\, X_2$ etc.



Step3: Perform solution:

(i) Primary input current on no-load:

On no-load, $Z_L = \infty$, therefore $I_2^{/} = 0$, hence we require I_o. Since Xm = 2.5kΩ then:

$$I_m = \frac{V_1\angle 0}{jX_m} = \frac{2.2\times10^3\angle 0}{2.5\times10^3\angle 90} = 0.88\angle -90 = -j0.88 \text{ A}$$

Since the iron loss is 220W (and there is no loss in the magnetising inductance) these losses are in the resistive element, R_m. From this we can get the current, I_C:

Iron loss = $V_1 \times I_C$ (unity power factor!)

Therefore:

$$I_C = 220 / 2.2\times10^3 = 0.1\text{A}$$

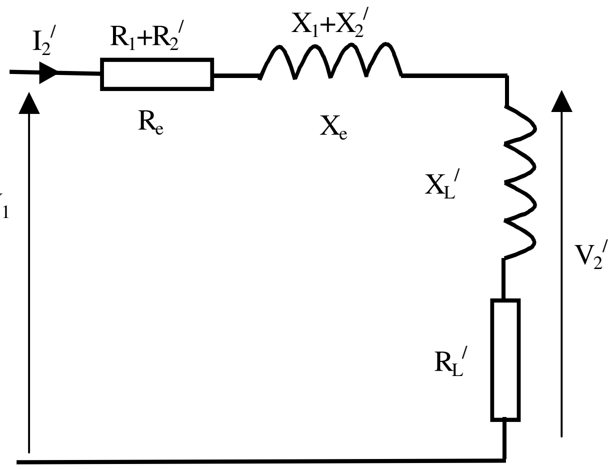
And the no-load current is:

$$\bar{I}_o = \bar{I}_C + \bar{I}_m = 0.1 - j0.88 = 0.8857\angle\text{-}83.5^o \text{ Amps}$$

(ii) The input current on-load:

NB Do not attempt to 'simplify' the circuit by combining all the parallel branches. Solve for $I_2^{/}$ and add to the value of I_o previously calculated.

We have:



Total series impedance:

$$Z_s = (R_1 + R_2^{/} + R_L^{/}) + j(X_1 + X_2^{/} + X_L^{/}) = 306 + j220 = \mathbf{376.9\angle 35.7^o}$$

Therefore:

$$I_2' = \frac{V_1\angle 0}{376.9\angle 35.7} = \frac{2200\angle 0}{376.9\angle 35.7} = 5.84\angle -35.7^o$$

or:

$$I_2^{/} = 4.74 - j3.41 \text{ A}$$

Hence the input current on-load:

$$I_1 = I_o + I_2^{/} = (0.1 - j0.88) + (4.74 - j3.4) = 4.84 - j4.28 = \mathbf{6.46\angle\text{-}41.5^o}$$

(This then compares to $I_1 = 6.1 \angle\text{-}33.7^o$ as calculated from the ideal model)

(iii) The output current and voltage on-load:
The output current on-load:

$$I_2 = I_2^{'} \times \frac{N_1}{N_2} = 5.84\angle -35.7 \times \frac{10}{1} = \mathbf{58.4\angle\text{-}35.7^o\ A}$$

The output voltage (easiest is to first calculate $V_2^{/}$):

$$V_2^{/} = I_2^{/} \times Z_L^{/} = 5.84 \angle\text{-}35.7 \times (300 + j200)$$
$$= 5.84 \angle\text{-}35.7 \times 360.5 \angle 33.7 = \mathbf{2105 \angle\text{-}2.0\ V}$$

or:

$$V_2 = V_2^{'} \times \frac{N_2}{N_1} = 2105\angle -2.0 \times \frac{10}{1} = \mathbf{210.5\angle\text{-}2.0\ V}$$

(which compares to $V_2 \approx 220 \angle 0$ from the ideal model).

(iv) The transformer output power and efficiency:
In this case we could use $P_o = I_2^{/} \times V_2^{/} \cos\phi$ where ϕ is the angle between $V_2^{/}$ and $I_2^{/}$ etc. or more easily, in this case is to calculate $(I_2^{/})^2 \times R_L^{/}$, therefore:

$$\text{Power output } P_o = 5.84^2 \times 300 = \mathbf{10.23\ kW}$$

(which compares with $P_o = 11.16$ kW from the ideal model).

Efficiency is defined for any device as:

$$\text{Efficiency} = (\text{Power Output}) / (\text{Power Input}) \times 100\%$$

Now:

$$\text{Power input} = \text{Power output} + \text{Losses in the transformer}$$

where:

$$\text{Losses} = \text{Iron loss} + \text{Copper Loss}$$

$$= 220 + (I_2^{/})^2 \times (R_1 + R_2^{/}) = 220 + 5.84^2 \times 6 = 220 + 204.6 = \mathbf{425\ W}$$

$$\therefore\ Efficiency = \frac{10230}{10230 + 425} \times 100\% \qquad = 96\%$$

Rating of Transformers

‘Rated’ means designed operating value, usually full-load. Limited by temperature (loss). Has been shown that for a.c. excited magnetic circuits, then provided coil resistance is small, the applied volts fixes the relationship between turns and flux density in the core, i.e.:

$$\mathbf{V_{rms} \approx E_{rms} = 4.44\, f\, N\, B_{max}\, A_{core}}$$

It can also be shown that the current fixes the required cross section of the copper windings, A_w, to give acceptable copper losses and heating effects. Hence the product V×I fixes the total copper and iron and hence the approximate size of the transformer for a given frequency.

V × I is called the **VOLT AMP RATING**

written as VA or kVA or MVA

FULL LOAD or RATED LOAD defines the value of the VA, voltage and current etc which gives the permissible operating temperature rise for the transformer.

NB.

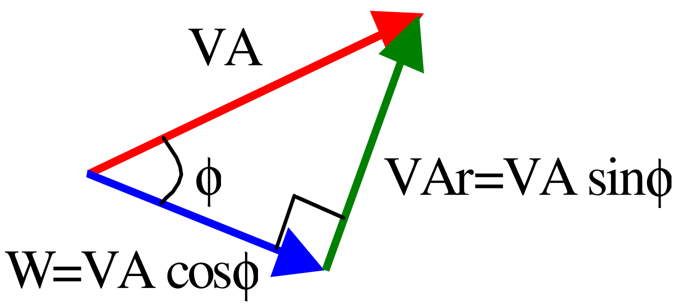
(i) Input VA ≈ Output VA (neglects magnetising current)
i.e. $V_1I_1 \approx V_2I_2$
Strictly VA usually relates to output.

(ii) The VA is not the POWER output in ac systems since V and I are not necessarily IN PHASE, depends on the load impedance .

POWER = (VA)×cosϕ

where cosϕ is the power factor.

Volt Amp Reactive, Var = (VA)×sinϕ

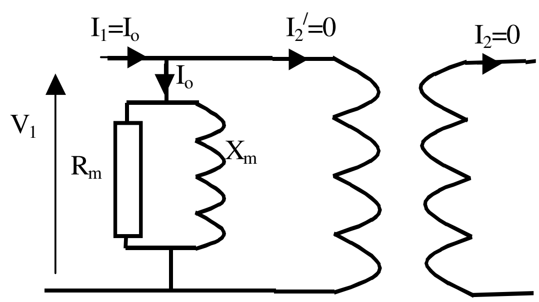


Measurement of Circuit Parameters

Transformer designers are able to calculate the equivalent circuit parameters with reasonable accuracy from the dimensions and properties of the materials used. However, all large transformers must be tested for faults and to check on their performance against the design. In many situations, testing on full-load is not possible or the performance under hypothetical load / fault conditions must be estimated. For these purposes, test values of parameters are required for accurate assessment. [There are numerous detailed tests specified in standard (NEMA) procedures]. Two relatively simple tests however, can give a reasonable level of confidence for normal sinusoidal performance of power transformers [Pulse / surge, variable frequency performance would need extra tests].

(a) Open circuit test

- Transformer tested with one winding open circuit, the other winding is connected to its **rated** [Nameplate] voltage, to give normal flux conditions in the core material. In this test, the only current flow in the excited winding is the no-load current, which is very small when compared to full-load. Therfore winding copper losses can usually be neglected. The approximate equivalent circuit then is as shown in the diagram (for a primary supplied model).



  Can also be supplied to the secondary side, where the magnetising branch has to be referred to the secondary winding via the square of the turns ratio.

Test Procedure:
Apply rated voltage to either winding [depends on voltage available in test lab].

Measure:
Input current, Input power, output voltage.

(i) Since there is no significant voltage drop in R_1, X_1 etc then
$V_1 \approx V_2^{/} = N_1/N_2 \times V_2$ Hence $V_1/V_2 \approx$ Turns ratio $= N_1/N_2$

(ii) No power is absorbed in X_m only in R_m. ($R_1 << R_m$)
Hence input power = power in R_m

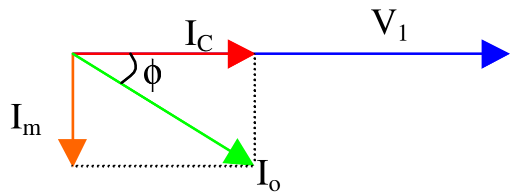
$\therefore W_{oc} = V_1^2/R_m, \quad R_m = V_1^2/W_{oc}$

or from a phasor diagram:

$$W_{OC} = V_1 I_o \cos\phi$$

or:

$$I_C = I_o \cos\phi = \frac{V_1}{R_m}$$



therefore:

$$R_m = \frac{V_1}{I_o \cos\phi}$$

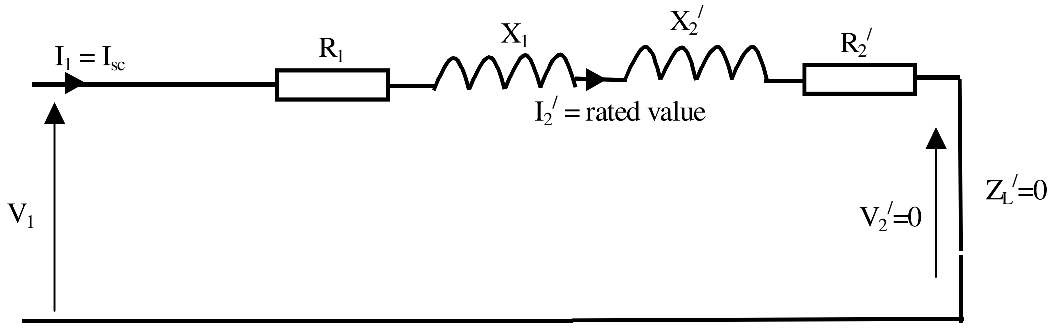
and from phasor diagram:

$$I_m = I_o \sin\phi \quad \therefore X_m = \frac{V_1}{I_o \sin\phi}$$

Note: do not try to find the series equivalent of R_m and X_m and then sort out R_m and X_m from the test results – It takes too long!

(b) Short Circuit Test

Clearly if this was carried out at full voltage, something would give! The test is carried out at reduced voltage, but rated [full load] current. In this way, heating effects on the winding resistance and any saturation of the leakage reactances would be correctly accounted for. Again either winding can be supplied and the other shorted out. The equivalent circuit then becomes:



Primary fed, secondary short circuit.

Test Procedure:

Apply low voltage to one winding – such that the rated curent flows in the short circuit winding.

Measure:

Supply voltage, V_1, Input current, $I_1 = I_2^{/} = I_{sc}$, Input power, W_{sc}.

Since R_m and X_m are high impedances and test voltage is low, then the magnetising current is negligible. Therefore:

$$\overline{V}_1 = \bar{I}_{sc}\left[(R_1 + R_2') + j(X_1 + X_2')\right]$$
$$= \bar{I}_{sc}\left[R_e + jX_e\right]$$

or:

$$\left|\overline{V}_1\right| = \left|\bar{I}_{sc}\right|\sqrt{R_e^{\,2} + X_e^{\,2}} = I_{sc} Z_e$$

Since no power consumed in X_e then input power $W_{sc} = I_{sc}^{\,2} \times R_e$

$$R_e = \frac{W_{sc}}{I_{sc}^{\,2}}$$

and:

$$X_e = \sqrt{Z_e^{\,2} - R_e^{\,2}}$$

Example

The following test results were obtained from a 50kVA, 3.3kV:400V transformer:

Open-circuit test: - Carried out with the primary supplied at rated volts:

Primary Volts	= 3.3kV
Secondary Volts	= 400V
Input Power	= 430W
Input Current	= 0.5A

Short-circuit test: – Again with primary winding supplied:

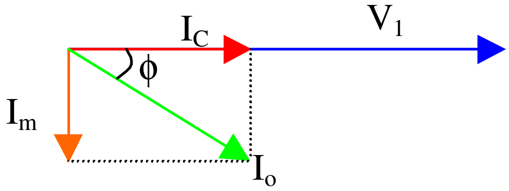
Primary Volts	= 124V
Primary Current	= 15.2A
Power Input	= 525W

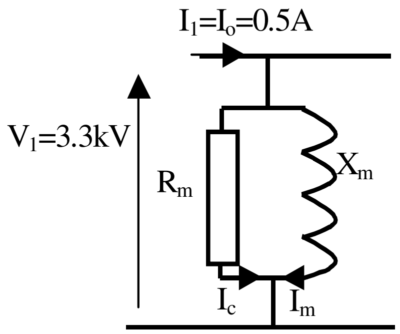
i) Calculate the equivalent circuit parameters referred to the primary

ii) Calculate the efficiency at (a) Full load 0.7pf lagging

(b) 0.5×Full load at 0.7pf lagging

i) On open-circuit the approximate equivalent circuit referred to the primary is:





$$\text{Power Input} = V_1 I_o \cos\phi = 430\text{W}$$

Therefore:

$$\cos\phi = 430 / (3.3\times10^3 \times 0.5) = 0.26$$

and:

$$\sin\phi = 0.965$$

Since:

$$I_o \cos\phi = I_c = V/R_m$$

then:

$$R_m = V / (I_o \cos\phi) = 25.4\ \text{k}\Omega$$

and:

$$I_o \sin\phi = I_m = V / X_m$$

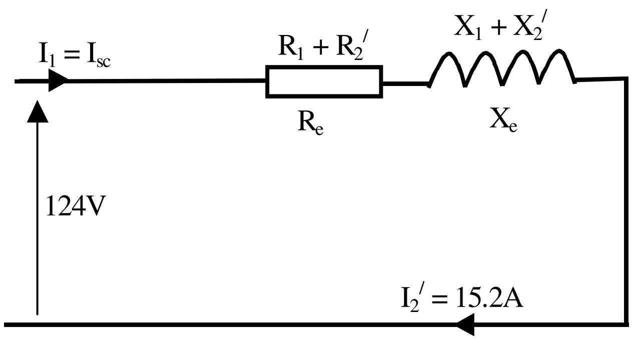
then:

$$X_m = 6.8\ \text{k}\Omega$$

On short-circuit the approximate equivalent circuit referred to the primary is:

Note: 15.2 A = 50kVA / 3.3 kV

i.e. Full load - CORRECT



Now:

$$W = (I_2^{/})^2 \times R_e$$

therefore:

$$R_e = 525 / (15.2)^2 = 2.27\ \Omega$$

and:

$$|Z| = \left|\frac{V}{I}\right| = \frac{124}{15.2} = 8.16\Omega$$

and since:

$$|Z| = \sqrt{R_e^{\ 2} + X_e^{\ 2}}$$

then:

$$X_e = \sqrt{8.16^2 - 2.27^2} = 7.84\,\Omega$$

ii) (a) Calculate the efficiency at full load 0.7pf lagging**:**

$$\text{Output power} = 50 \times 0.7 = 35\ \text{kW}$$

$$\begin{aligned}\text{Losses} &= \text{iron losses} + \text{copper losses}\\ &= V^2 / R_m + I_{FL}^{\ 2} \times R_e\\ &= 430\ \text{W} + 525\ \text{W} = 955\ \text{W}\end{aligned}$$

therefore:

$$\text{losses} = 0.955\ \text{kW}$$

and:

$$\text{efficiency} = 35 / (35 + 0.955) \times 100\% = 97.34\ \%$$

ii) (b) Calculate the efficiency at 0.5 × Full load at 0.7pf lagging:
On half load (17.5kW) V is constant but output current is halved therefore iron losses are as before and copper losses are reduced to 0.25 of full load value, i.e.:

$$\text{losses} = 430\ \text{W} + 525 / 4 = 561\ \text{W} = 0.561\ \text{kW}$$

and:

$$\text{output power} = 50/2 \times 0.7 = 17.5\ \text{kW}$$

therefore:

$$\text{efficiency} = 17.5 / (17.5 + 0.561) \times 100\% = 96.89\ \%$$

i.e. less efficient at reduced load.

Maximum efficiency occurs when the copper loss = iron loss. In general, a transformer that is designed for a constant load has iron loss = copper loss at full load. A transformer which is designed for a varying load has copper loss > iron loss at full load.