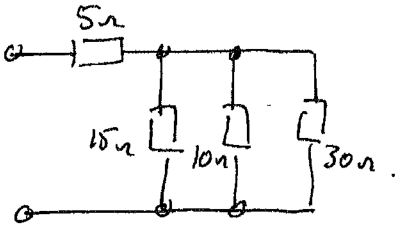
Q1 a. $R_{eff} = 5 + 15 \| 10 \| 30$

$$= 5 + \frac{1}{\frac{1}{15} + \frac{1}{10} + \frac{1}{30}}$$

$$= 5 + \frac{30}{2 + 3 + 1}$$

$$= 5 + 5 = \underline{\underline{10\,\Omega}}$$

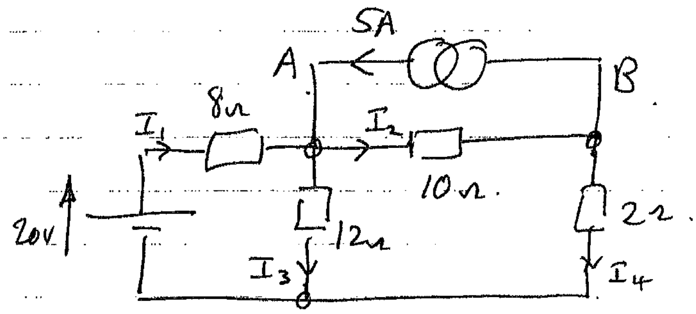


(i) current through $15\,\Omega$ $= \frac{\text{Voltage across } 15\Omega}{15} = \frac{10}{15}$

$$= \underline{\underline{0.67\,A}}$$

(ii) $P_{5\Omega} = I_{5\Omega}^2 . 5\Omega = \left(\frac{20}{10}\right)^2 . 5\Omega = \underline{\underline{20\,W}}$

b



sum currents at node A

$$I_1 + 5 = I_2 + I_3 \Rightarrow \frac{20 - V_A}{8} + 5 = \frac{V_A - V_B}{10} + \frac{V_A}{12} \quad (1)$$

sum currents at node B

$$I_2 = I_4 + 5 \Rightarrow \frac{V_A - V_B}{10} = 5 + \frac{V_B}{2} \quad (2)$$

from (1): $\frac{20}{8} + 5 = V_A \left[\frac{1}{8} + \frac{1}{10} + \frac{1}{12}\right] - \frac{V_B}{10} = 7.5$

or $V_A \frac{38}{120} - \frac{V_B}{10} = 7.5$

or $V_A . \frac{38}{12} - V_B = 75 \quad (1a)$

from (2) $5 = \frac{V_A}{10} - V_B\left[\frac{1}{10} + \frac{1}{2}\right] = \frac{V_A}{10} - \frac{6V_B}{10}.$

or $50 = V_A - 6V_B$ or $-V_B = \frac{50 - V_A}{6}.$ — (2a)

sub in (1a) $V_A \frac{38}{12} + \frac{50 - V_A}{6} = 75$

$V_A 38 + 100 - 2V_A = 75 \times 12.$

$36 V_A = 800$ so $V_A = \frac{800}{36} = \frac{200}{9} = \underline{22.2\ V}.$

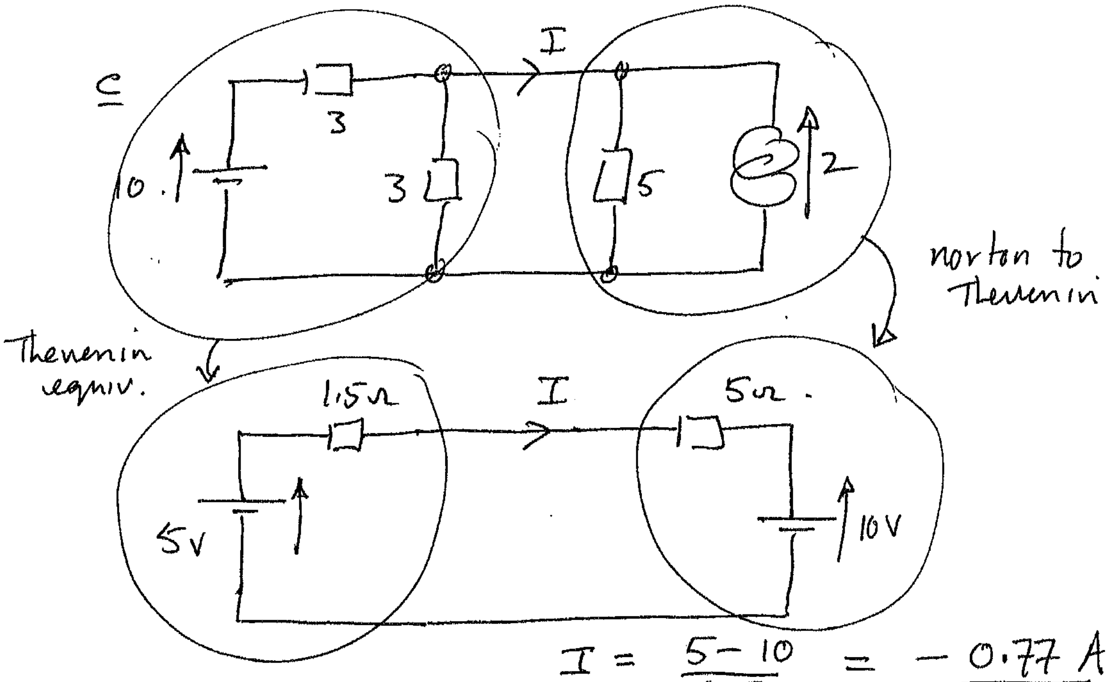
sub back in 2a. $-V_B = \frac{50 - \frac{200}{9}}{6.} = \frac{250}{54} = 4.63\ V$

or $\underline{V_B = -4.63\ V}.$

Since $V_A > 20V$, current flows into the 20V source so it is absorbing energy.

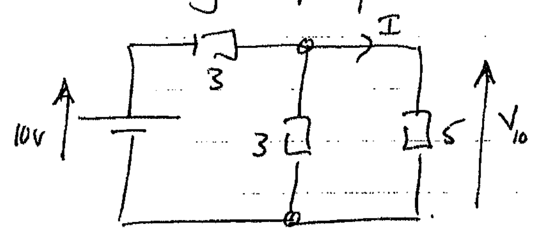
Since the 5A source is taking current from a lower potential and forcing it into a higher one, the 5A source is sourcing.

c



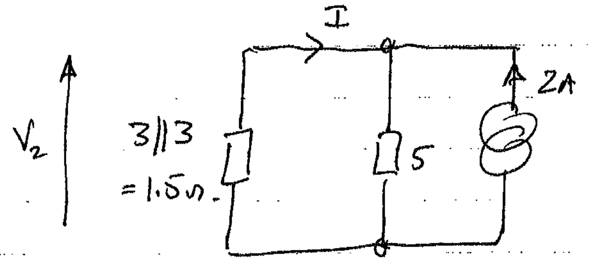
$I = \frac{5 - 10}{6.5\Omega} = \underline{-0.77\ A}$

check by superposition



$$V_{10} = 10 \times \frac{3\|5}{3+3\|5} = 10 \times \frac{15/8}{39/8} = \frac{150}{39}$$

$$I_{10} = \frac{V_{10}}{5} = \frac{150}{5 \times 39} = \frac{30}{39} = \frac{10}{13}\,A.$$



$$V_2 = 2 \times \frac{5 \times 1.5}{5+1.5} = 2 \times \frac{7.5}{6.5}.$$

$$I_2 = -\frac{V_2}{1.5\Omega} = -\frac{2 \times \cancel{7.}5}{\cancel{1.5} \times 6.5}$$

$$= -\frac{10}{6.5} = -\frac{20}{13}A.$$

$$\therefore\; I = I_{10} + I_2 = -\frac{10}{13}A = \underline{\underline{-0.77\,A}}.$$

<u>Q2 a</u> resonant frequency gives L ...

$$10\,kHz = \frac{1}{2\pi\sqrt{L \times 10^{-8}}}$$

$$\text{or } L = \frac{1}{10^{-8}(2.\pi.10kHz)^2} = \underline{\underline{25.3\,mH}}.$$

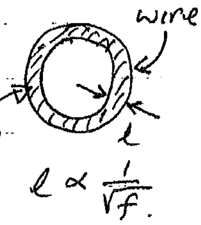
at resonance L + C cancel out so cct looks like a resistive potential divider

$$0.8 = 1 \times \frac{R}{R+R_L}. \quad \underline{\text{or}} \quad 0.8R + 0.8R_L = R.$$

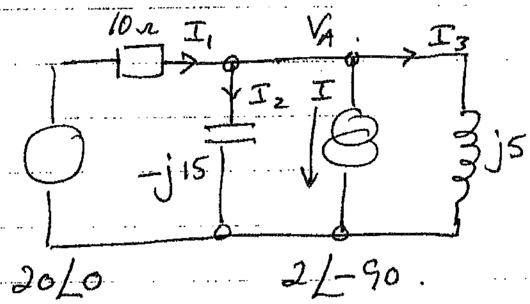
$$\underline{\text{or}} \quad R_L = \frac{0.2R}{0.8} = R/4 = \underline{\underline{25\,\Omega.}}$$

$$Q = \frac{1}{R_{TOT}}\sqrt{\frac{L}{C}} = \frac{1}{125}\sqrt{\frac{25.3 \times 10^{-3}}{1 \times 10^{-8}}} = \underline{\underline{12.7}}.$$

The value of Q varies with frequency for several reasons but the most prominent of these is usually the skin effect. As frequency increases current is carried in an annular "skin" on the outside of the wire



2b Sum currents at V_A



$$I_1 = I_2 + I + I_3$$

$$\frac{20 - V_A}{10} = \frac{V_A}{-j15} + \frac{V_A}{j5} + (-j2)$$

$$2 + j2 = V_A\left[\frac{j}{15} - \frac{j}{5} - 2j\right]$$

$$30(2+j2) = V_A(3 - j4) \quad \text{or} \quad V_A = \frac{60(1+j)}{3-4j}$$

$$\text{or } V_A = \frac{60\sqrt{2}\angle 45}{5\angle -53} = \underline{\underline{15\sqrt{2}\angle 98}} = \underline{\underline{-2.95 + j21}}.$$

$$V_{10\Omega} = 20 - V_A = 20 - (-2.95 + j21)$$

$$= 22.95 - j21$$

$$P_{10\Omega} = \frac{|V_{10}|^2}{10} = \frac{[22.95^2 + 21^2]}{10} = \frac{968}{10} = \underline{\underline{96.8\,W}}.$$

2c $\quad 10\angle 120 \equiv \underline{\underline{-5 + j8.67}}$

$\quad 20\angle 45 \equiv \underline{\underline{14.1 + j14.1}}$.

$$I = \frac{10\angle 120 - 20\angle 45}{Z} = \frac{-5 + j8.67 - 14.1 - j14.1}{Z}$$

and $Z = 2 + j\omega L = 2 + j3.14$.

$$\therefore \quad I = \frac{-19.1 - 5.43j}{2 + j3.14} = \frac{19.86\angle{-164}}{3.72\angle{57.5}}$$

$$= \underline{\underline{5.34\angle{-221.5}}} = \underline{\underline{5.34\angle{138.5}}}.$$

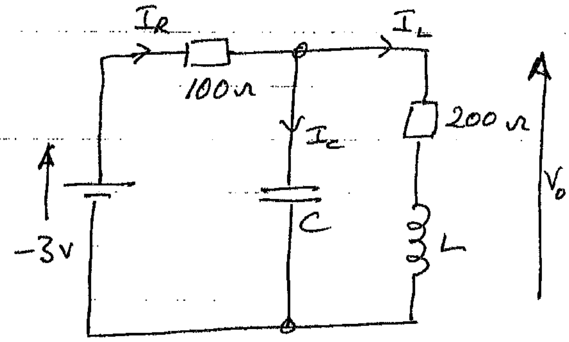
either of these will do.

$$|I_{rms}| = 5.34$$

$$\therefore \quad I_p = 5.34\sqrt{2} = 7.55A.$$

$$\therefore \quad E_p = \frac{1}{2}LI^2 = \frac{0.01 \times 7.55^2}{2} = \underline{\underline{285\ mJ}}.$$

Q3 a at $t = 0^-$ ....



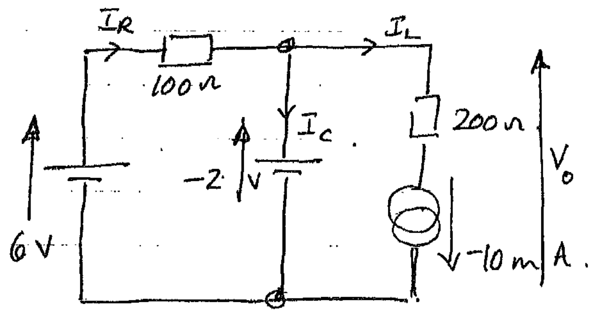
$$I_R = \frac{-3}{300} = -10mA.$$

$$I_L = I_R = -10mA$$

$$I_C = 0\ mA$$

$$V_o = I_L \times 200 = -2V$$

at $t = 0^+$ ...



$$I_R = \frac{6V - (-2V)}{100} = \underline{\underline{80mA}}.$$

$$I_L = -10mA \text{ (unchanged)}$$

$$I_C = I_R - I_L = 80mA - (-10mA)$$
$$= \underline{90mA}$$

$$V_o = V_C = -2V \text{ (unchanged)}.$$

3b Admittance, $Y = \dfrac{1}{\text{Impedance}, Z}$.

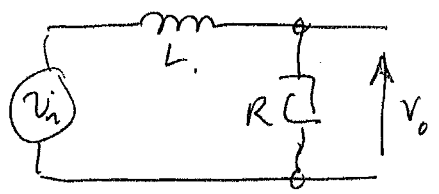
for cct (i) $Y = j\omega C + \dfrac{1}{R}$.

for cct (ii) $Y = \dfrac{1}{Z} = \dfrac{1}{R + j\omega L}$

$$= \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

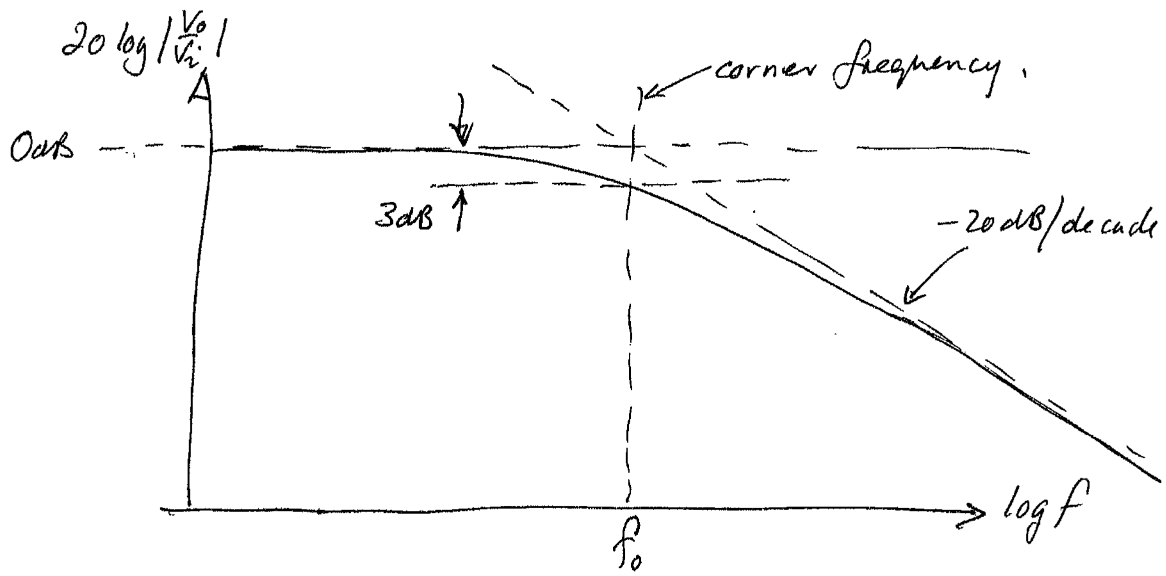
$$= \frac{R}{R^2+\omega^2 L^2} - j\frac{\omega L}{R^2+\omega^2 L^2}$$

3c $V_o = V_i \dfrac{R}{R + j\omega L}$



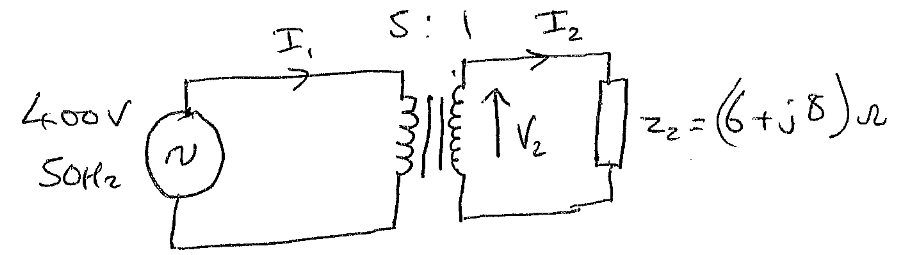
$$\frac{V_o}{V_i} = \frac{1}{1 + j\omega L/R}$$

$$\equiv \frac{1}{1 + j\,f/f_0} \quad \text{where } f_0 = \frac{1}{2\pi L/R}.$$



QUESTION 4

(a)



Assuming an ideal transformer:

(i) $\dfrac{V_1}{V_2} = \dfrac{N_1}{N_2} \Rightarrow V_2 = \dfrac{V_1 N_2}{N_1} = 400\angle 0^\circ \cdot \dfrac{1}{5}$

$= 80\angle 0^\circ \; V_{rms}$ (1)

(1)

$Z_2 = 6 + j8 = 10\angle 53.1^\circ\ \Omega$

$\therefore I_2 = \dfrac{80\angle 0^\circ}{10\angle 53.1^\circ} = 8\angle -53.1^\circ \; A_{rms}$ (1)

(ii) VA rating $= 80 \times 8 = 640\,\text{VA}$ (1)

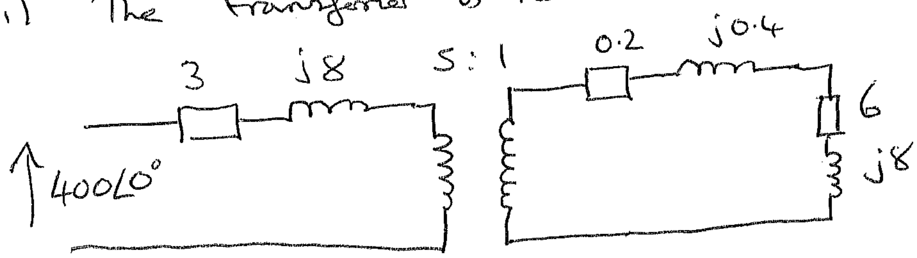
Power in load, $P_L = V_2 I_2 \cos\phi = 80 \times 8 \times \cos 53.1 = 384\,\text{W}$

(1)

OR

$P_L = I_2^2 . R_2 = 8^2 \times 6 = 384\,\text{W}$

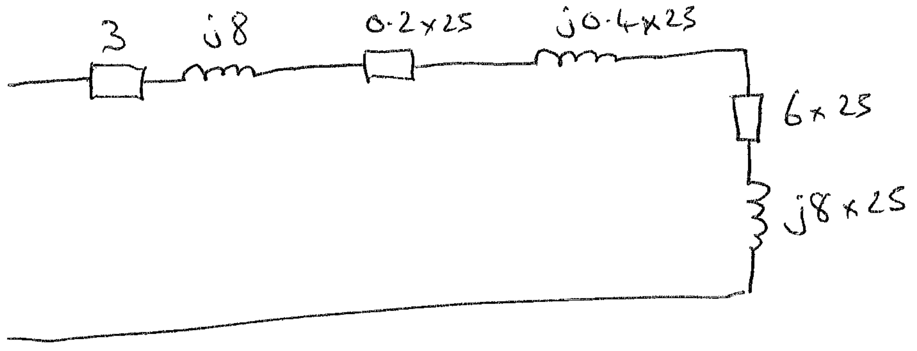
(b) (i) The transformer is not ideal:



Refer to primary side:

$Z_1 = \left(\dfrac{N_1}{N_2}\right)^2 Z_2$

$= 25 Z_2$



QUESTION 4 (CONTINUED)

$$\therefore Z_2 = R_1 + R_2' + R_L' + j(X_1 + X_2' + X_L') \ \Omega$$

$$= 3 + 5 + 150 + j(8 + 10 + 200) \ \Omega$$

$$= 158 + j218 = 269.2 \angle 54.1 \ \Omega$$ (2)

$$\therefore I_2' = \frac{400\angle 0^\circ}{269.2\angle 54.1^\circ} = 1.485\angle -54.1^\circ \text{ Arms}$$

$\therefore$ Actual load current I_2 is:

$$I_2 = \frac{N_1}{N_2} \cdot I_2' = \frac{5}{1} \times 1.485\angle -54.1^\circ = 7.43\angle -54.1^\circ \text{ Arms}$$ (2)

(1)

$$V_2 = I_2 \times Z_2 = 7.43\angle -54.1^\circ \times 10\angle 53.1^\circ = 74.3\angle -1.0^\circ \text{ Vrms}$$

(ii) To find the total input current we need to add on the magnetizing current

$$I_T = I_0 + I_2' = 0.3\angle -\cos^{-1}(0.1) + 1.485\angle -54.1^\circ \text{ Arms}$$

$$= 0.3\angle -84.3 + 1.485\angle -54.1^\circ \text{ Arms}$$

$$= 1.75\angle -59^\circ \text{ Arms}$$ (2)

(2)

Power factor $= \cos 59^\circ = 0.515$ lagging

(iii) Power supplied to load, $P_L = I_2'^2 . R_L' = 1.485^2 \times 150 = 331\text{W}$

(or $P_L = I_2^2 . R_L = 7.43^2 . 6 = 331\text{W}$) (1)

Losses (copper) $= P_{CU} = I_2'^2 (R_1 + R_2') = 1.485^2 \times 8 = 17.64\text{ W}$

Losses (iron) $= P_{FE} = V . I_0 \cos\phi = 400 \times 0.3 . 0.1 = 12\text{W}$ (3)

$$\therefore \text{Efficiency} = \frac{331}{331 + 17.64 + 12} \times 100\% = 91.8\%$$

QUESTION 4 (CONTINUED)

(c) The use of a solid iron core would significantly increase the core loss (iron loss) within the device as usually laminations are used to limit circulating eddy currents within the transformer core during operation.
These eddy currents are a result of voltages induced in the core material

Therefore whilst the device may be cheaper to produce it would be much less efficient in operation. 2

QUESTION 5

(a) (i) The impedance of the induction motor may be written as:

$$80 + j100 = 128.1 \angle 51.3^\circ \ \Omega$$

Since the motor is star connected:

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{11000}{\sqrt{3}} = 6351 \ V_{rms}$$

and $$I_{ph} = I_L = \frac{6351 \angle 0^\circ}{128.1 \angle 51.3^\circ} = \underline{49.6 \ A_{rms}} \ (-51.3^\circ)$$ (2)

The total input power to the motor is given by:

$$P_{in} = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} . 11000 . 49.6 \cos 51.3^\circ$$ (1)

$$= \underline{\underline{590.9 \ kW}}$$

and power factor $= \cos 51.3^\circ = \underline{0.625 \ lagging}$ (1)

(ii) Since the input power to the motor is 590.9kW and it is 93% efficient:

$$P_{out} = 0.93 \times 590.9 = 549.5 \ kW$$

$$\text{Speed in rpm} = 1440 = \frac{1440}{60} \times 2\pi = 150.8 \ rad/s$$

Hence $$T_{out} = \frac{P_{out}}{\omega} = \frac{549.5 \times 10^3}{150.8} = \underline{\underline{3.64 \ kN}}$$ (4)

(b) (i) For the whole factory we require the total kW and total kVAr

For the motor:

$$P_1 = 590.9 \ kW \text{ (see above)}$$

$$Q_1 = \sqrt{3} V_L I_L \sin\phi = \sqrt{3} . 11000 \times 49.6 \times \sin 51.3^\circ$$

$$= 737.5 \ kVAr.$$

QUESTION 5 (CONTINUED)

For the resistive load

$$P_2 = 150\,kW \qquad Q_2 = 0$$

For the general load 300kVA at 0.75p.f. lagging:

$$P_3 = 300 \times 0.75 = 225\,kW$$

$$Q_3 = 300 \times \sin(\cos^{-1} 0.75) = 198.4\,kVAr$$

∴ The total factory load is:

$$P_T = P_1 + P_2 + P_3 = 590.9 + 150 + 225 = \underline{\underline{965.9\,kW}} \quad (2)$$

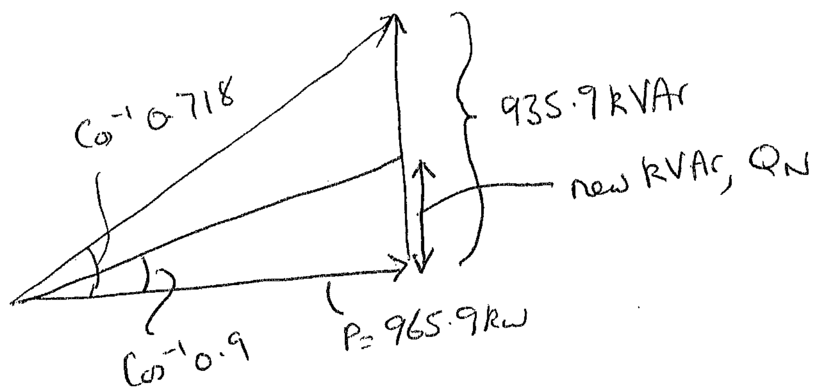
$$Q_T = Q_1 + Q_2 + Q_3 = 737.5 + 0 + 198.4 = \underline{\underline{935.9\,kVAr}} \quad (2)$$

(ii) Since $S = \sqrt{P^2 + Q^2}$

$$S = \sqrt{965.9^2 + 935.9^2} = \underline{\underline{1345\,kVA}} \quad (1)$$

$$\text{Overall power factor} = \frac{kW}{kVA} = \frac{965.9}{1345} = \underline{\underline{0.718 \text{ lagging}}} \quad (1)$$

(c) Adding the capacitor bank will not change the real power (965.9kW)



The new value of Q can be found from

$$\frac{Q_N}{P} = \tan(\cos^{-1} 0.9)$$

QUESTION 5 (CONTINUED)

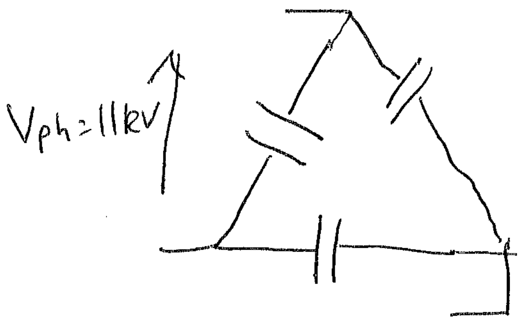
$$Q_N = 965.9 \tan(25.8^\circ) = 466.9 \text{ kVAr}$$

Therefore the capacitors have to provide $935.9 - 466.9$

$$= 469 \text{ kVAr}$$

or $\frac{469}{3} = 156.33 \text{ kVAr}$ per phase. $(= Q_{CAP})$ 3

For delta connection of capacitors:



$$V_{cap} = V_L = V_{ph}$$

$$\therefore Q_{CAP} = \frac{V_c^2}{X_c}$$

$$\therefore X_c = \frac{V_c^2}{Q_{CAP}} = \frac{11000^2}{156.33 \times 10^3} = 774\,\Omega$$

Since $X_c = \frac{1}{2\pi f C}$ $\Rightarrow$ $C = \frac{1}{2\pi f X_c} = \frac{1}{2\pi . 50 . 774}$

3

$$\therefore \underline{\underline{C = 4.11\,\mu F/\text{phase}}}$$

QUESTION 6

(a)(i) The reluctance of the magnetic circuit may be found from:

$$S = \frac{L}{\mu_0 \mu_r A}$$

$L = 4 \times 0.15 \times 10^{-2}$ m
$A = (4 \times 10^{-2})^2$ m^2
$\mu_r = 900$

$$= \frac{0.6}{\mu_0 \cdot 900 \times 16 \times 10^{-4}} = \underline{\underline{3.32 \times 10^5 \ H^{-1}}}$$ (2)

(ii) To establish a flux density of 1.2T in the core:

$$\phi = B \cdot A = 1.2 \times 16 \times 10^{-4} = 1.92 \times 10^{-3} \ Wb$$

Now since $NI = S\phi \Rightarrow I = \frac{S\phi}{N} = \frac{3.32 \times 10^5 \times 1.92 \times 10^{-3}}{600}$

$$= \underline{\underline{1.06 A}}$$ (2)

(iii) The self inductance is given by:

$$L = \frac{N^2}{S} = \frac{600^2}{3.32 \times 10^5} = \underline{\underline{1.084 H}}$$ (1)

(b) For this new case where there is an airgap in the core the reluctance consists of 2 components.

$$S_T = S_{CORE} + S_{AIR}$$

$$= \frac{0.6 - 0.004}{\mu_0 \times 900 \times 16 \times 10^{-4}} + \frac{0.004}{\mu_0 \cdot 16 \times 10^{-4}} \ H^{-1}$$

$$= 3.29 \times 10^5 + 1.99 \times 10^6 = 2.32 \times 10^6 \ H^{-1}$$

QUESTION 6 (CONTINUED)

Now since L has to equal $0.2\,H$ and $L = \dfrac{N^2}{S}$

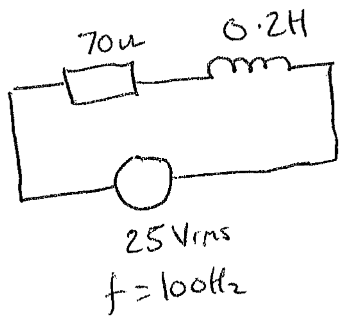
$$\therefore N = \sqrt{LS} = \sqrt{0.2 \times 2.32 \times 10^6} = \underline{\underline{681 \text{ TURNS}}}$$ 3

(ii) From $NI = \phi S$ and $B = \dfrac{\phi}{A}$ we have

$$B = \frac{NI}{SA} = \frac{681 \times 1}{2.32 \times 10^6 \times 16 \times 10^{-4}} = \underline{\underline{0.183\,T}}$$ 2

(iii) Because most core materials have a non-linear B-H characteristic then any inductor without an airgap would have an inductance which would vary with the level of current in the coil. This would be caused by the change in current leading to a change in the mmf and hence B resulting in changes in the permeability and hence reluctance of the core. 2

(c) (i)



$$Z = 70 + j2\pi.100.0.2$$
$$= 70 + j125.7 \equiv 143.9\angle 60.9^\circ\,\Omega$$

$$\therefore |I_{rms}| = \frac{25}{143.9} = 0.173\,A_{rms}$$

$$\therefore |I_{pk}| = \sqrt{2} \times 0.173 = 0.246\,A.$$

Since $\phi S = NI \Rightarrow \phi = \dfrac{NI}{S}$

$$\therefore \phi = \frac{681 \times 0.246}{2.32 \times 10^6} = 7.22 \times 10^{-5}\,Wb$$

$$\therefore B_{pk} = \frac{\phi}{A} = \frac{7.22 \times 10^{-5}}{16 \times 10^{-4}} = \underline{\underline{0.045\,T}}$$

(4)

(ii) The power dissipated in the coil

$$P = I_{rms}^2 . R = 0.173^2 . 70 = \underline{\underline{2.09\,W}}$$

(2)

The peak energy stored in the inductor

$$E = \frac{1}{2} L I_{pk}^2 = \frac{1}{2} . 0.2 \times 0.246^2 = \underline{\underline{6.05\,mJ}}$$

(d) Since the source is dc the initial current is

$$I_{INIT} = \frac{V}{R} = \frac{40}{70} = 0.571A$$

The switch is opened in 1.5 mS hence $\frac{dI}{dt} = \frac{0.571}{1.5 \times 10^{-3}} = 381.7\ As^{-1}$

$$\therefore V = L\frac{dI}{dt} = 0.2 \times 381.7 = \underline{\underline{76.1\,V}}$$

(2)