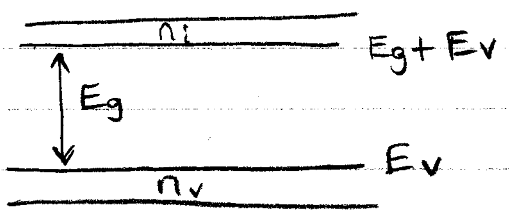
Worked Solutions May 2011 EEE 207

Q1(a)



Assume that width of levels in CB and VB is small c.f. E_g i.e. all levels are identical

$$n_v = \text{no. of } e^- \text{ in VB} = P(E_v).N_{tot} = \frac{N_{tot}}{1+\exp[(E_v - E_F)/KT]}$$

$$n_i = \text{no. of } e^- \text{ in CB} = P(E_v+E_g).N_{tot} = \frac{N_{tot}}{1+\exp[(E_v+E_g-E_F)/KT]}$$

Now, $N_{tot} = n_v + n_i$, so

$$N_{tot} = \frac{N_{tot}}{1+\exp[(E_v-E_F)/KT]} + \frac{N_{tot}}{1+\exp[(E_v+E_g-E_F)/KT]}$$

rearranging this gives:

$$\exp[(2E_v - 2E_F + E_g)/KT] = 1$$

$$\therefore \quad 2E_v - 2E_F + E_g = 0$$

$$E_F = E_v + E_g/2$$

$$\therefore \quad n_i = \frac{N_{tot}}{1+\exp[(E_v+E_g-E_v-E_g/2)/KT} = \frac{N_{tot}}{1+\exp\left(\frac{E_g}{2KT}\right)}$$

$$\approx N_{tot} \exp\left(-E_g/2KT\right) \quad \text{as } E_g \gg KT \text{ usually}$$

$$\therefore \quad n_i \propto \exp\left(A/T\right) \quad \text{where } A = -\frac{E_g}{2K}$$

6

Q1(b) $n = 10^{12}\,m^{-3}$, $p = 4\times10^{20}\,m^{-3}$, $Nd = 6\times10^{20}\,m^{-3}$

i) What is n_i? $n_i^2 = np = 10^{12}\times 4\times10^{20} = 4\times10^{32}$

$\therefore n_i = 2\times10^{16}\,m^{-3}$

ii) What is Na? $n \ll n_i$ so semiconductor is p-type with $Na - Nd \gg n_i$

$\therefore p = Na - Nd \Longrightarrow Na = p + Nd = 4\times10^{20} + 6\times10^{20}$

$Na = 10^{21}\,m^{-3}$

iii) $\sigma = 3.2\,Sm^{-1} = e(p\mu_h + \cancel{n\mu_e}) = ep\mu_h$ as $p \gg n$

$\therefore \mu_h = \dfrac{3.2}{1.6\times10^{-19}\times4\times10^{20}} = 0.05\,m^2V^{-1}s^{-1}$

6

(c) Semiconductor is p-type with $4\times10^{20}\,m^{-3}$ holes and $n_i = 2\times10^{16}\,m^{-3}$

When $n_i \approx Na - Nd$, semiconductor will lose its extrinsic p-type characteristic, $n_i \propto \exp(-Eg/2kT)$

At RT:

$$n_i = C\exp\left[\frac{-1.1\times1.6\times10^{-19}}{2\times1.38\times10^{-23}\times293}\right] = 2\times10^{16}\,m^{-3}$$

$C.\exp(-21.76) = C.\,3.53\times10^{-10} = 2\times10^{16}$

$C = 5.66\times10^{25}\,m^{-3}$

At Tcrit: $n_i = 4\times10^{20} = 5.66\times10^{25}\exp\left[\dfrac{-1.1\times1.6\times10^{-19}}{2\times1.38\times10^{-23}\times T_c}\right]$

$7.06\times10^{-6} = \exp\left[-\dfrac{6376.8}{T_c}\right]$

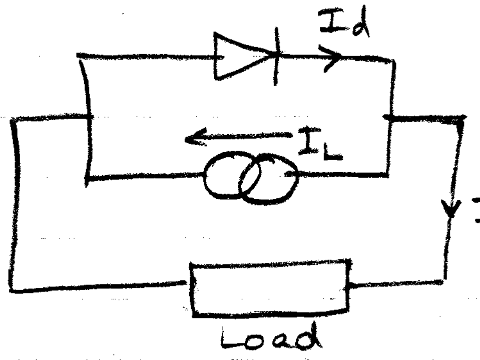
$-11.86 = -6376.8\times 1/T_c$

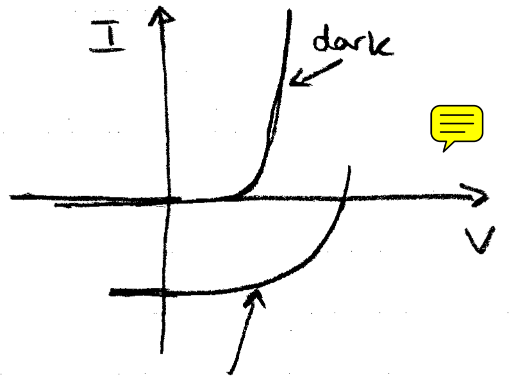
$T_c = 537\,K$ or $264^\circ C$

Q1(c) cont: Assume that any change in conductivity due to change in mobility while still p-type is small c.f. changes when it becomes intrinsic

Q 1(d) : When T decreases below RT, there will be little change in σ as p concentration will not change. At very low T, the number of free carriers would reduce due to carrier freeze out and the conductivity would reduce.

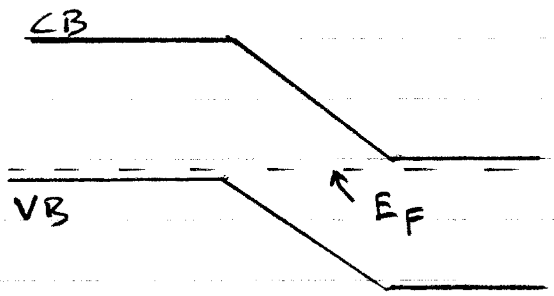
Q 2(a)



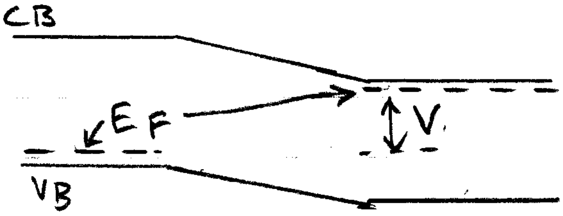


4

(b)



(i) dark



(ii) illumination

iii) Maximum voltage would be $\approx$ band-gap of semiconductor

6

(c) Longest wavelength is given by narrowest band-gap
so $\lambda_{max} = 1.24/1.42 = 0.873\,\mu m$

Detectable wavelength of 820 nm $\equiv 1.512$ eV
Total band-gap $= 1.42 + E_{1e} + E_{1h} = 1.512$ eV

$$E_{1e} = \frac{(6.63\times10^{-34})^2}{8\times0.06\times9.11\times10^{-31}\times1.6\times10^{-19}}\cdot\frac{1}{L^2} = \frac{6.28\times10^{-18}}{L^2}\,\text{eV}$$

$$E_{1h} = \frac{(6.63\times10^{-34})^2}{8\times0.45\times9.11\times10^{-31}\times1.6\times10^{-19}}\cdot\frac{1}{L^2} = 8.37\times10^{-19}\,\text{eV}$$

Q2(c) cont.

$$E_{1e} + E_{1h} = 1.512 - 1.42 = 0.092\,eV$$

$$\frac{1}{L^2}\left(6.28\times10^{-18} + 8.37\times10^{-19}\right) = 0.092\,eV$$

$$L^2 = \frac{7.12\times10^{-18}}{0.092} = 77.36\times10^{-18}$$

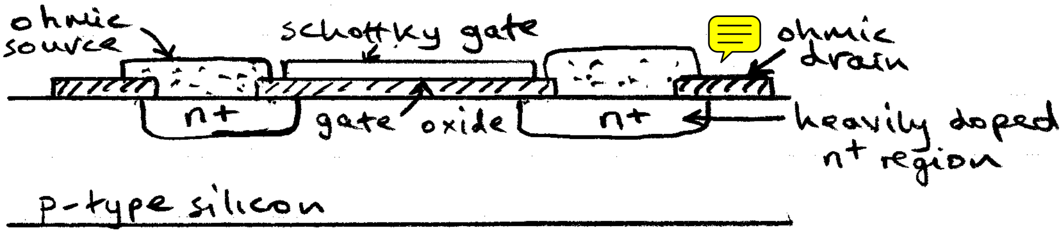
$$L = 8.8\times10^{-9}\,m = 8.8\,nm$$

8

(d) As the well width reduces, the AlGaAs barrier becomes the limit, so $\lambda_{max} = 1.24/1.8 = 0.69\,\mu m$

The confined levels cannot be higher than the height of the barrier.

Q 3(a)



$V_s = 0$, V_g = +ive , V_d = +ive

At low V_g, holes repelled from under gate. At higher V_g, electrons form a conducting channel. V_{ds} causes causes current to flow between source to drain

4

(b) $$\frac{dI_d}{dV_d} = \frac{\mu_e C_g}{\ell^2}\left[V_g - V_T - V_d\right]$$

(i) to get I_d, integrate above expression:

$$I_d = \frac{\mu_e C_g}{\ell^2}\left[V_g - V_T - \frac{V_d}{2}\right] V_d$$

(ii) Saturation occurs when I_d = maximum, i.e. when $\frac{dI_d}{dV_d} = 0$ $\therefore$ $V_g - V_T - V_d = 0$

so V_d for saturation $= V_g - V_T$

To get saturation current, I_{ds}, substitute $V_d = V_g - V_T$ into expression for I_d:

$$I_{ds} = \frac{\mu_e C_g}{\ell^2} \frac{V_d^2}{2}$$

Q 3(b) (iii) transconductance, $g_m = \left.\dfrac{\partial I_d}{\partial V_g}\right|_{V_d}$, in saturation region

$$\therefore\ g_m = \frac{\mu_e C_g V_d}{l^2}$$

rearrange and substitute for I_{ds}

$$\frac{\mu_e C_g}{l^2} = \frac{g_m}{V_d} = \frac{2 I_{ds}}{V_d^2}$$

$$\therefore\ g_m = \frac{2 I_{ds}}{V_d}$$

6

(c) $R_L = 5\,K\Omega$, $I_{ds} = 50\,mA$, gain $= 25$

$$\text{gain} = g_m R_L \implies g_m = \text{gain}/R_L = 5\,mS$$

$$\frac{2 I_{ds}}{V_d} = \frac{\text{gain}}{R_L}, \quad V_d = \frac{2 I_{ds} R_L}{\text{gain}}$$

$$V_d = \frac{2 \times 50\,mA \times 5\,K\Omega}{25} = 20\,V$$

4

$\varepsilon_r = 11 \cdot 8$, $C_g = 0 \cdot 2\,pF$, $\mu_e = 0 \cdot 13\, m^2 V^{-1} s^{-1}$ (n-channel)

$$g_m = 5\,mS = \frac{\mu_e C_g V_d}{l^2}$$

$$l^2 = \frac{0 \cdot 13 \times 2 \times 10^{-13} \times 20}{5 \times 10^{-3}} \implies l = 10\,\mu m$$

For gate, $l/w = 0 \cdot 5 \implies w = l \times 2 = 20\,\mu m$, t_o = oxide thick

$$C_g = \frac{\varepsilon_o \varepsilon_r \times l \times w}{t_o} \implies t_o = (\varepsilon_o \varepsilon_r \times l \times w)/C_g$$

$$t_o = (11 \cdot 8 \times 8 \cdot 85 \times 10^{-12} \times 10 \times 10^{-6} \times 20 \times 10^{-6}) / 2 \times 10^{-13}$$

$$= 104 \cdot 4\,nm$$

6

Q4(a) Force = rate of change in momentum

$$= \frac{dp}{dt} = \hbar \frac{dk}{dt} \qquad \text{as } p = \hbar k$$

$$\text{Force} = \frac{dE}{dx} = \frac{dE}{dk} \cdot \frac{dk}{dt} \cdot \frac{dt}{dx}$$

$$\therefore \hbar = \frac{dE}{dk} \cdot \frac{dt}{dx} = \frac{dE}{dk} \cdot \frac{1}{\text{Velocity}}$$

$$\text{Acceleration} = \frac{d(\text{velocity})}{dt} = \frac{1}{\hbar} \frac{d^2E}{dk^2} \cdot \frac{dk}{dt}$$

$$= \frac{1}{\hbar^2} \cdot \frac{d^2E}{dk^2} \cdot \text{Force}$$

Force = mass × accleration, so rearranging above

$$\text{mass} = m_e^* = \hbar^2 / (d^2E/dk^2)$$

6

(b) E-K relationship is assumed parabolic, so

$E = A + Bk^2$, where A and B are constants.

Band-gap at $k = 0$ for direct-gap semiconductor, so

$A = 0.75\,\text{eV}$

$$\frac{dE}{dk} = 2Bk \quad \text{and} \quad \frac{d^2E}{dk^2} = 2B$$

$$m_e^* = 0.04 \times 9.11 \times 10^{-31} = \hbar^2/(d^2E/dk^2) = \hbar^2/2B$$

$$B = \hbar^2/(2 \times 0.04 \times 9.11 \times 10^{-31}) = (6.626 \times 10^{-34}/2\pi)^2/(2 \times 0.04 \times 9.11 \times 10^{-31})$$

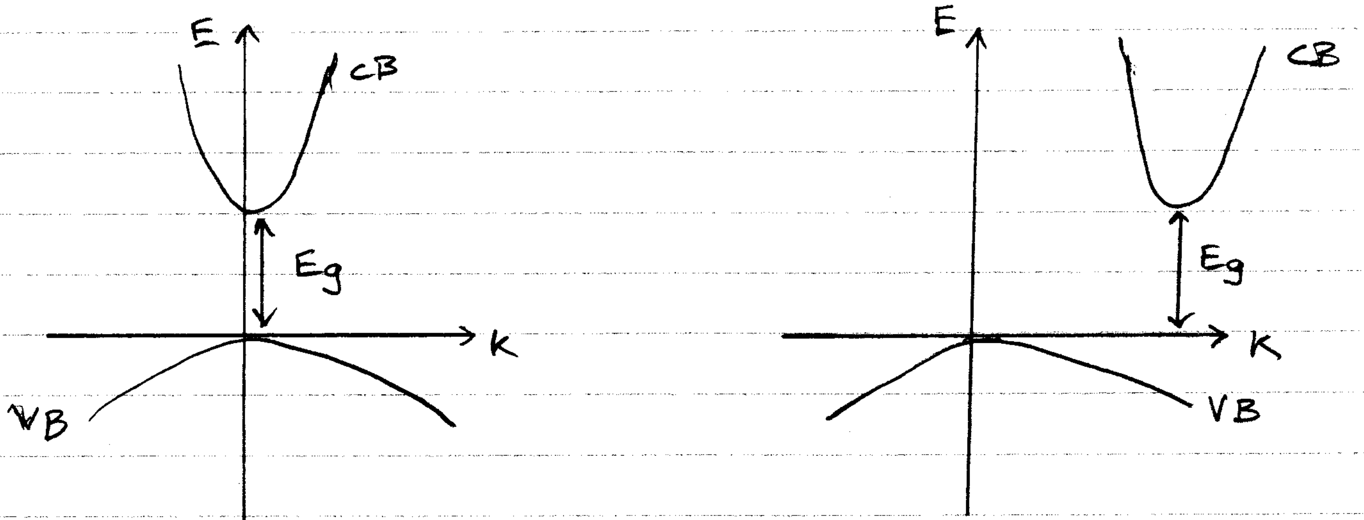
$$= 1.526 \times 10^{-37}\ \text{J m}^2 = 9.54 \times 10^{-19}\ \text{eV m}^2$$

$$\therefore E = 0.75 + 9.54 \times 10^{-19} k^2 \ \text{eV}$$

Q4(b) cont. The E-K relationship becomes non-parabolic as we move away from the zone centre and the effective mass increases.

(8)

(c)



Direct band-gap

Indirect band-gap

Recombination in direct gap semiconductor occurs at K=0 and is an efficient process. Recombination in indirect gap semiconductor involves phonons, so is inefficient. Lasers therefore are made from direct gap semiconductors

(6)