Modelling of Machines

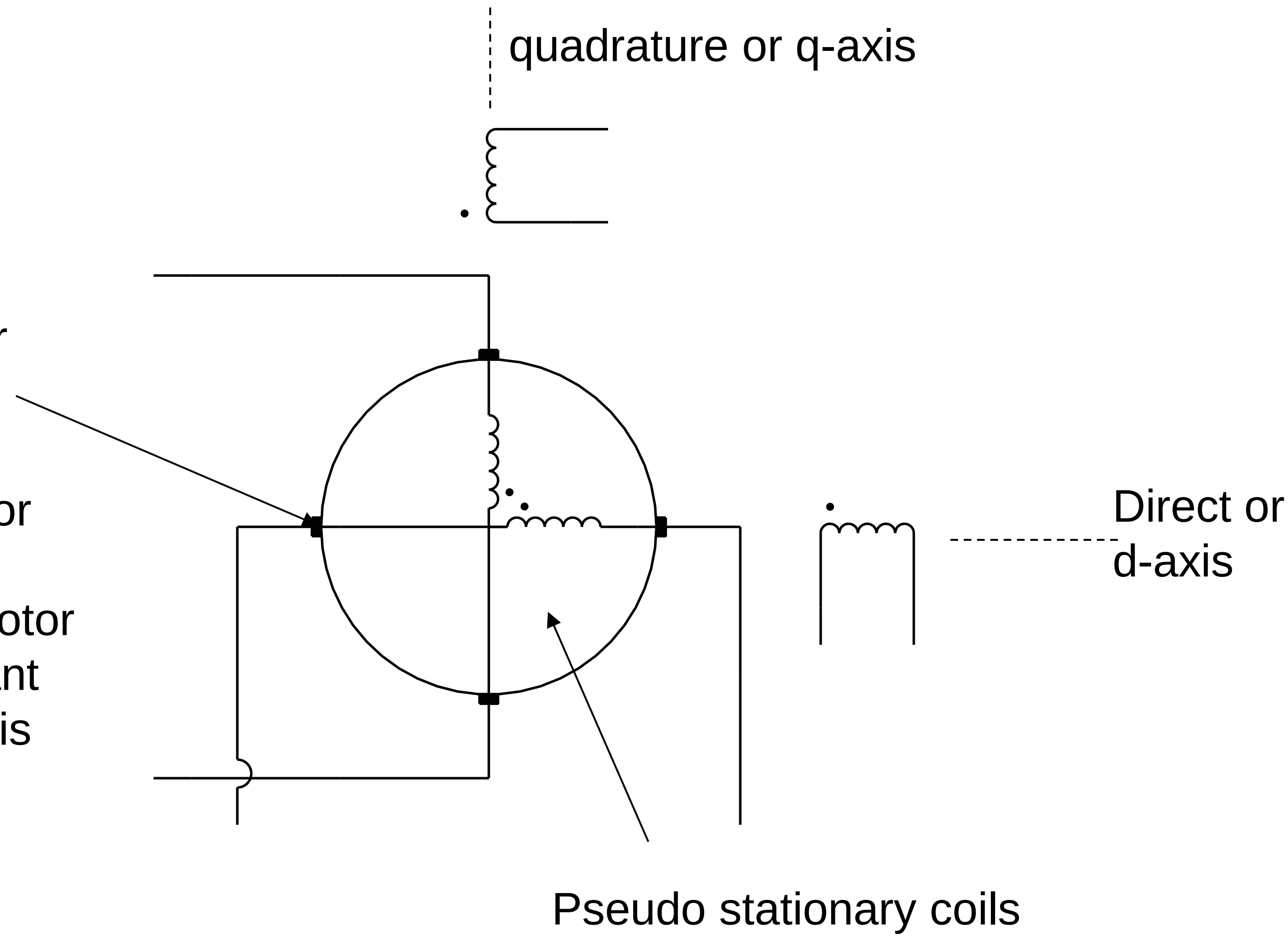
Section 5

Generalised machine theory

- Also often called Universal machine theory
- Elegant algebraic approach to modelling any kind of machine (albeit with simplifications)
- Attempts to demonstrate underlying commonality of all electrical machines
- Very popular research topic and widely used tool in 1950-1960s
- Some key features still used widely in machine control
- Nowadays, far less widely used as a machine analysis tool, but remains a very useful topic for understanding various fundamental aspects of electrical machines

Kron primitive machine

- The basis of generalised machine theory is that any type of machine can be transformed (albeit often with very complicated mathematical transformations) to standard form known as the KRON primitive machine

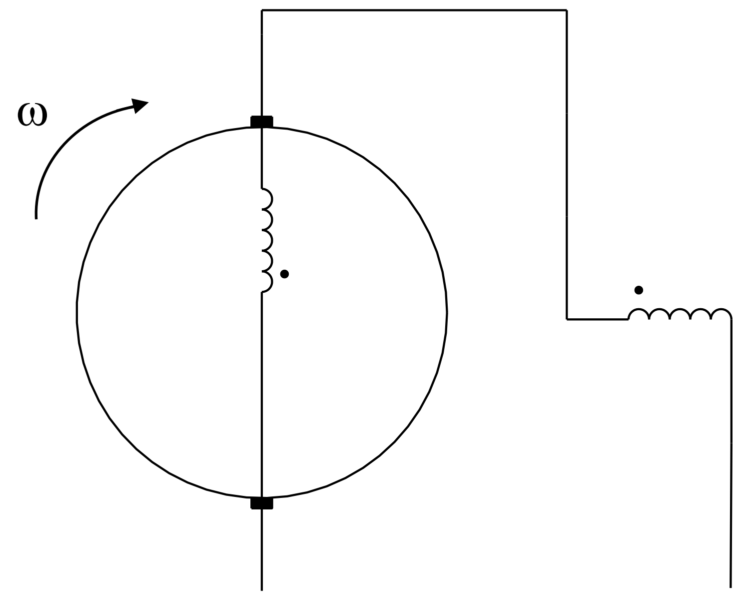


The commutator holds the flux pattern of the rotor winding constant even though it is rotating

Transformations

- Difficult though it might be to believe at this stage, any machine (e.g. a 3-phase induction machine which has no commutator) can be transformed into a Kron primitive machine
- This is performed through a series of different types of transformations (broadly divided into active and passive)

Active transformation - This allows different machines with have physical commutators (e.g. brushed DC motor) to be transformed into a Kron primitive machine by appropriate interconnection of the windings, e.g. universal motor (see later for more details on this type of machine)



Passive transformations

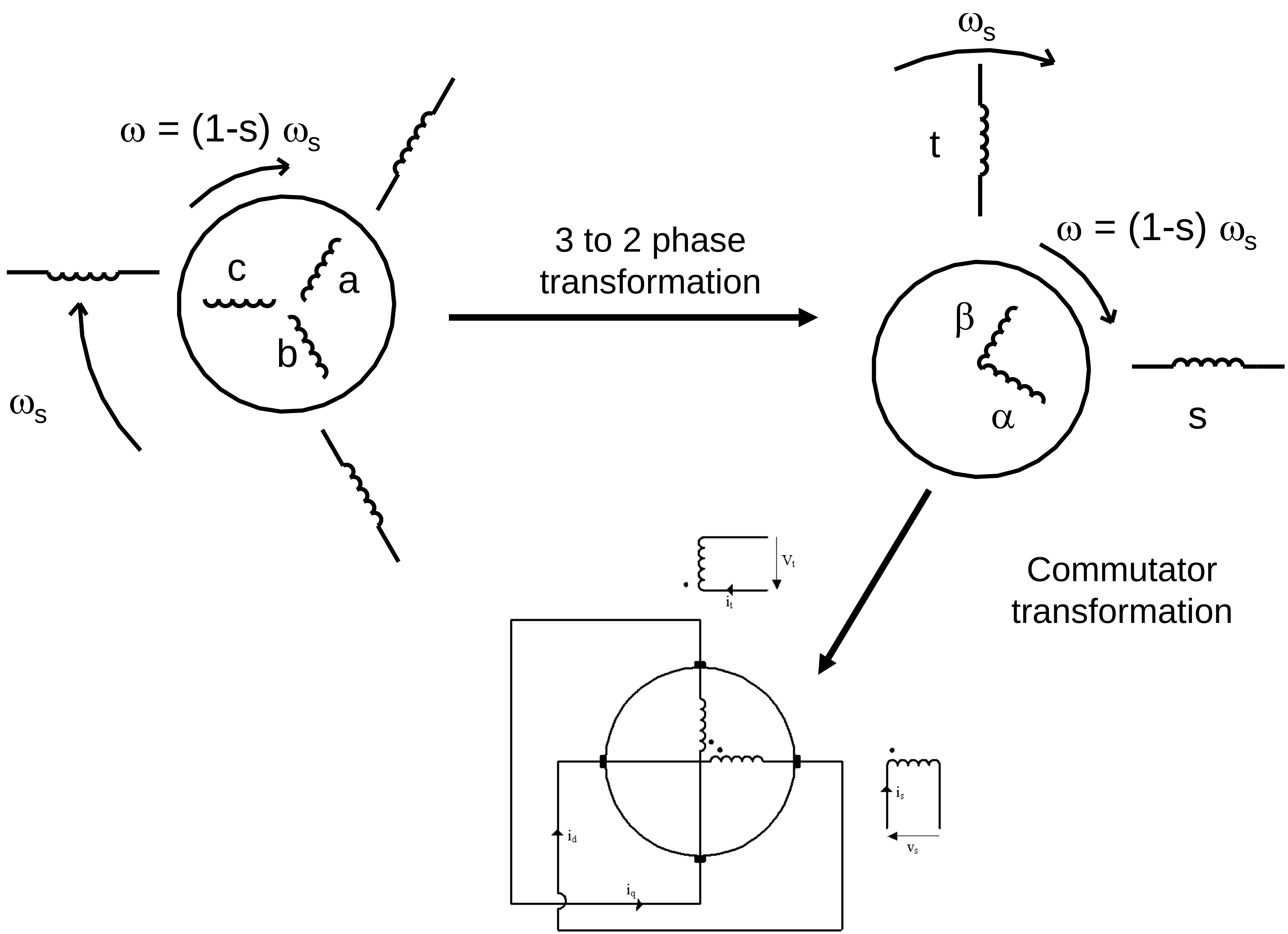
- These allow machines with no commutators, induction motors, synchronous motors etc, to be transformed to a commutator equivalent.
- This may well involve a number of steps in which one of many different transformations are applied:

a) Phase Transformation – converts a 3 phase system to a 2 phase equivalent

b) Commutator transformation – converts a 2 phase system of coils into a pseudo-stationary winding

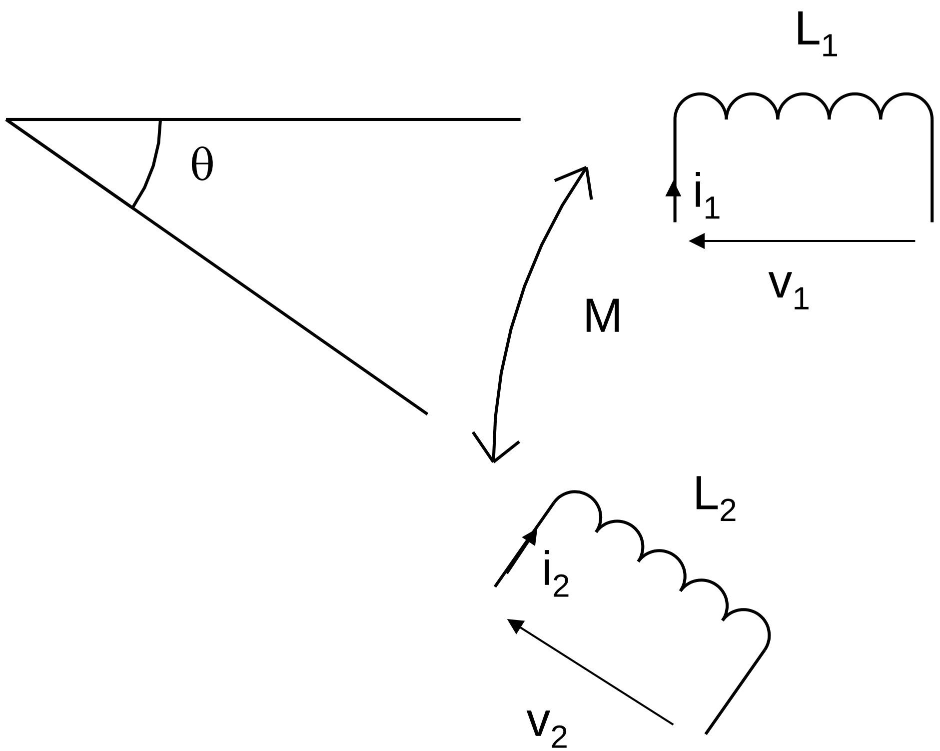
c) Symmetrical component transformation – used for modelling unbalanced systems, e.g single-phase induction motor (see later) or faulted operation

Example – transformation of a 3-phase induction motor



Fundamental processes in rotating coils

Consider the general case of two rotating coils



L and M are functions of θ

θ is a function of time

Torque producing mechanism

General case:

L_1, L_2 and M are functions of θ

From a consideration of the conservation of energy

Change in electrical energy = Change in stored magnetic energy + Change in mech output

$dW_e = dW_f + dW_m$

$$W_f = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

$$\frac{dW_f}{dt} = \frac{1}{2}\left[i_1^2\frac{dL_1}{dt} + L_1\frac{di_1^2}{dt} + i_2^2\frac{dL_2}{dt} + L_2\frac{di_2^2}{dt}\right] + Mi_1\frac{di_2}{dt} + Mi_2\frac{di_1}{dt} + i_1 i_2\frac{dM}{dt}$$

$$dW_f = \frac{1}{2}i_1^2 dL_1 + \frac{1}{2}i_2^2 dL_2 + i_1 L_1 di_1 + i_2 L_2 di_2 + Mi_1 di_2 + Mi_2 di_1 + i_1 i_2 dM$$

Torque producing mechanism (cont'd)

$$e_1 = \frac{d}{dt}(i_1 L_1) + \frac{d}{dt}(i_2 M) = \boxed{L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}} + \boxed{\left[i_1 \frac{dL_1}{d\theta} + i_2 \frac{dM}{d\theta} \right] \frac{d\theta}{dt}}$$

Transformer emf Rotational emf

Also,

$$e_2 = \frac{d}{dt}(i_2 L_2) + \frac{d}{dt}(i_1 M) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} + \left[i_2 \frac{dL_2}{d\theta} + i_1 \frac{dM}{d\theta} \right] \frac{d\theta}{dt}$$

Electrical power P_e

$$P_e = e_1 i_1 + e_2 i_2$$

$$P_e = \left(i_1^2 \frac{dL_1}{d\theta} + 2 i_1 i_2 \frac{dM}{d\theta} + i_2^2 \frac{dL_2}{d\theta}\right) \frac{d\theta}{dt} + L_1 i_1 \frac{di_1}{dt} + M i_1 \frac{di_2}{dt} + L_2 i_2 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt}$$

$$dW_e = i_1^2 dL_1 + L_1 i_1 di_1 + 2 i_1 i_2 dM + i_1 M di_2 + i_2^2 dL_2 + i_2 L_2 di_2 + i_2 M di_1$$

<u>Torque producing mechanism (cont'd)</u>

From energy balance considerations:

$$dW_m = dW_e - dW_f = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM$$

Hence, the torque is given by:

$$T = \frac{dW_m}{d\theta} = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} i_2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM}{d\theta}$$