

Question 1

a.

- 1) PSR. Detects targets with a given RCS gives range, angle and speed information via processing
- 2) SSR. Cooperative radar system. Transducer on target Tx's info on location etc
- 3) Co-located Tx/Rx antennas
- 4) Tx and RX antennas at different locations

4 Marks

First need to calculate PRF which is limited by max range

$$PRF = \frac{c}{2R_{\max}} = \frac{3 \times 10^8}{2 \times 490} = 306 \text{ Hz}$$

$$\text{Duty cycle is } \frac{\tau}{T} \text{ or (pulse duration) / (pulse repetition interval)} = 140 \times 10^{-6} \times 306 = 4.28 \%$$

$$\text{Mean power} = (\text{peak power}) \cdot (\text{duty cycle}) = 56 \text{ kW} \times 4.8 = 2.4 \text{ kW}$$

$$\text{Power gain, } G = \frac{4\pi}{\Delta\theta\Delta\phi}$$

$$\Delta\theta = \frac{\lambda}{h \times \eta} = \frac{3 \times 10^8 / 1.3 \times 10^9}{6.9 \times 0.707} = 0.047 \text{ rad or } 2.7 \text{ degrees}$$

$$\Delta\phi = \frac{\lambda}{w \times \eta} = \frac{3 \times 10^8 / 1.3 \times 10^9}{11.9 \times 0.707} = 0.027 \text{ rad or } 1.57 \text{ degrees}$$

$$\text{So gain } G = \frac{4\pi}{0.027 \times 0.047} = 9820 \text{ or } 40 \text{ dB}$$

$$\text{Rotation rate} = 5 \text{ rpm} = \frac{360^\circ}{(60/5) \text{ sec}} = 30^\circ / \text{s}$$

$$\text{A point target is therefore illuminated for } \frac{\Delta\phi^\circ}{30^\circ / \text{s}} = \frac{1.57}{30} = 52.3 \text{ ms}$$

$$\text{Number of hits} = PRF \cdot (\text{illumination time}) = 0.0523 \times 306 = 16$$

$$\text{Doppler resolution} = 1 / \text{illumination time} = 1 / 0.0523 = 19 \text{ Hz}$$

10 Marks

Doppler frequency given by

$$f = f_0 \cdot 2 \cdot \frac{V_r}{c}$$

f_0 = Tx frequency
 V_r = relative velocity

CASE B: $V_r = -130 - (-80) \quad V_r = -50 \quad \text{km/h or } 13.9 \text{ m/s}$

$$f_b = \frac{2 \cdot 13.9 \cdot 10^{10}}{3 \cdot 10^8} = 926.667 \text{ Hz}$$

CASE C: $V_r = 60 - (-80) \quad V_r = 140 \quad \text{km/h or } 38.9 \text{ m/s}$

$$f_c = \frac{2 \cdot 38.9 \cdot 10^{10}}{3 \cdot 10^8} = 2.593 \times 10^3 \text{ Hz}$$

CASE D: $V_r = -100 \cdot \cos\left(\frac{\pi}{4}\right) - (-80) \quad V_r = 9.289 \quad \text{km/h or } 2.58 \text{ m/s}$

$$f_d = \frac{2 \cdot 2.58 \cdot 10^{10}}{3 \cdot 10^8} = 172 \text{ Hz}$$

CASE E: $V_r = 100 \cdot \cos\left(\frac{\pi}{4}\right) - (-80) \quad V_r = 150.711 \quad \text{km/h or } 41.8 \text{ m/s}$

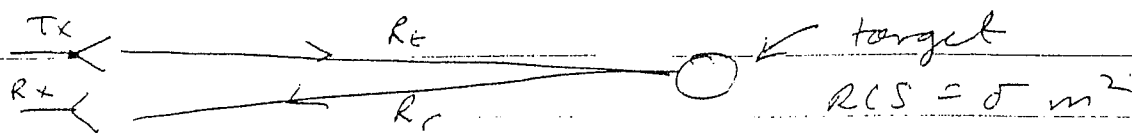
$$f_e = \frac{2 \cdot 41.8 \cdot 10^{10}}{3 \cdot 10^8} = 2.787 \times 10^3 \text{ Hz}$$

CASE F: $V_r = 0 - (-80) \quad V_r = 80$

$$f_f = \frac{2 \cdot 22.2 \cdot 10^{10}}{3 \cdot 10^8} = 1.48 \times 10^3 \text{ Hz}$$

6Marks

Q2 (A) radar range equation.



Power density at target is

$$P_D = \frac{P_t}{4\pi R_t^2} G_t$$

P_t = Tx power

G_t = Tx antenna gain

R_t = distance to target

Power reradiated by target is

$$P_{RR} = P_D \sigma \quad \sigma = RCS \text{ of target in } m^2$$

Power density back at radar is

$$P_{RD} = \frac{P_{RR}}{4\pi R_r^2} = \frac{P_D \sigma}{4\pi R_r^2} = \frac{P_t G_t \sigma}{(4\pi)^2 (R_t R_r)^2}$$

for mono-static radar $R_t = R_r = R$ hence

$$P_{RD} = \frac{P_t G_t \sigma}{(4\pi R^2)^2}$$

Power intercepted by R_x antenna with effective aperture A_e is

$$P_R = P_{RD} A_e$$

Substituting $G_R = \frac{4\pi A_e}{\lambda^2}$

where G_R = R_x antenna gain

$$P_R = \frac{P_t G_t G_R \sigma \lambda^2}{(4\pi)^3 R^4}$$

Let total systems losses be represented by L_s ($L_s < 1$) to give

$$P_R = \frac{P_t G_t G_r \sigma \lambda^2 L_s}{(4\pi)^3 R^4}$$

Let N = average system noise power so that

$$SNR = \frac{P_R}{N} = \frac{P_t G_t G_r \sigma \lambda^2 L_s}{(4\pi)^3 N R^4}$$

or

$$R = \left[\frac{P_t G_t G_r \sigma \lambda^2 L_s}{(4\pi)^3 N \cdot (SNR)} \right]^{1/4}$$

[6]

Q2 (B)

1) Approx beamwidth $\Delta = \frac{\lambda}{D}$ ← effective dimension of antenna in required plane.

At 9.4 GHz $\lambda = 0.032 \text{ m}$

$\therefore \Delta \theta \approx \frac{0.032 \times 180}{3.4 \pi} = 0.54^\circ$ — elevation

$\Delta \phi \approx \frac{0.032 \times 180}{0.75 \pi} = 2.4^\circ$ — azimuth

Antenna gain $G = \frac{4\pi A_e}{\lambda^2}$ ← effective area

$= \frac{4\pi \times (3.4 \times 0.75)}{(0.032)^2}$

$= 31293 = 45 \text{ dB}$

[note: may also use the approximation

$G \approx \frac{41253}{\Delta \theta^\circ \Delta \phi^\circ} \rightarrow 45 \text{ dB}$

[4]

11) use radar range equation (derived in part A) but should know

$P_t = 40 \text{ dBW}$

$G_{tR} = G^2 = 90 \text{ dB}$

$\sigma = 0 \text{ dBm}^2$

$\lambda^2 = -29.9 \text{ dBm}^2$

$L_s = -5 \text{ dB}$

$1/(4\pi)^3 = -33 \text{ dB}$

$1/N = +140 \text{ dBW}^{-1}$

$1/(\text{SNR}) = -13 \text{ dB}$

$R_{\text{max}}^4 = 18^{\text{th}} \text{ dBm}^4$

put all values in dB and add or convert to linear value and x and ÷

$\rightarrow \div \text{ by to give}$
 47.3 dBm

$= 53.4 \text{ km}$

[5]

24/

4 of 5

3/

Signal illuminated and detected by -20 dB
 side lobe giving total reduction in signal
 of 40 dB .
 But range reduced by $1/2 = -12\text{ dBm}^4$

→ Total increase in RCS is

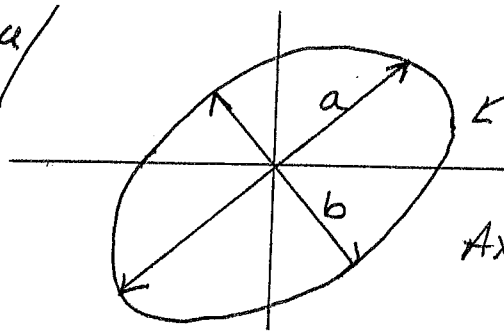
$$(40 - 12) = 28\text{ dBm}^2$$

$$\therefore \sigma' = \underset{\sigma}{\sigma} + 28\text{ dBm}^2$$

$$= \underline{28\text{ dBm}^2 \text{ or } 631\text{ m}^2}$$

[5]

3a/



Locus of E vector

Axial ratio $AR = \frac{a}{b}$

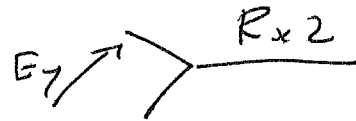
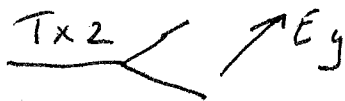
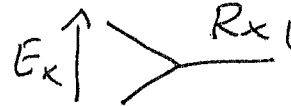
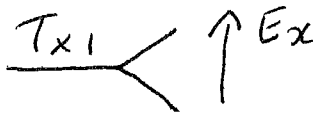
Limits $1 \leq AR \leq \infty$

↑
Circular
pol

↘ Linear pol

(4)

b/



Tx_1 & Rx_1 vertically polarised

Tx_2 & Rx_2 Horizontally

Signal from Tx_1 received by Rx_1
but not by Rx_2

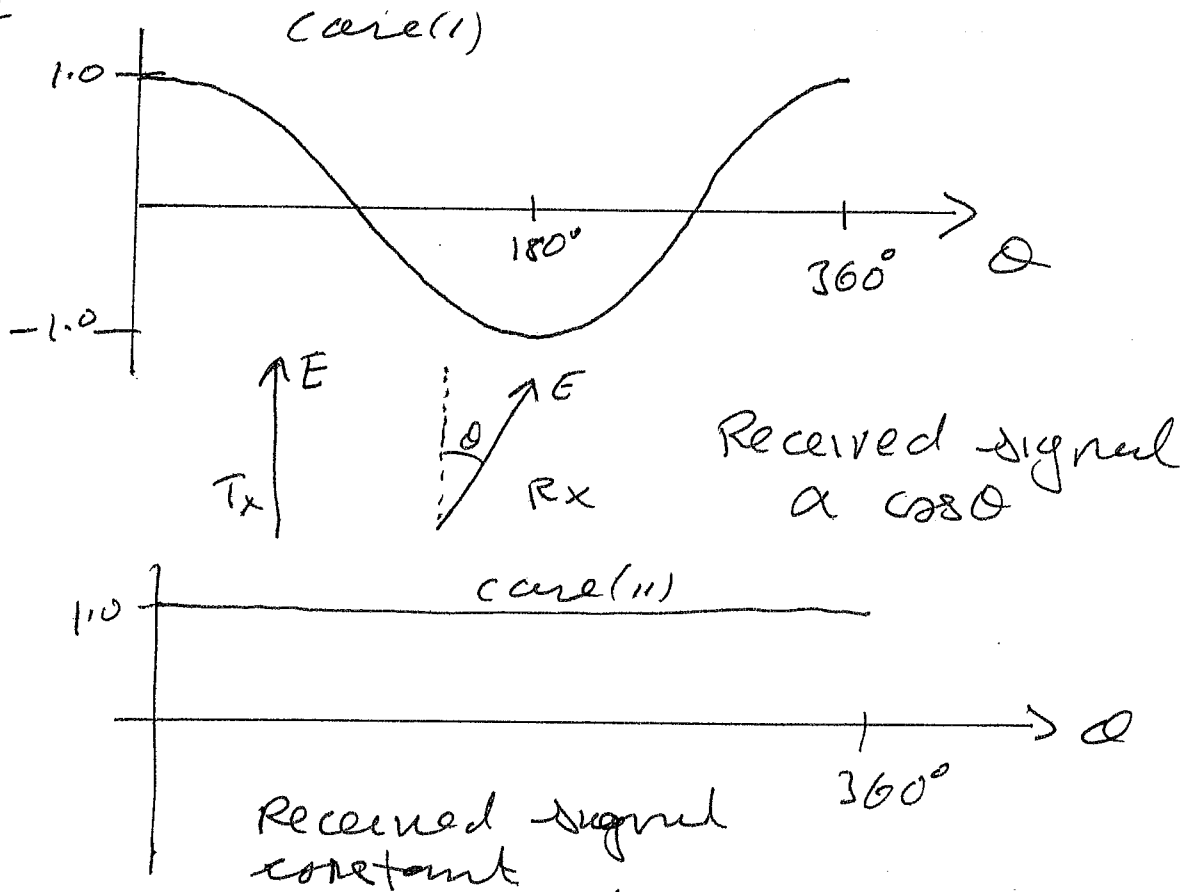
Signal from Tx_2 received by Rx_2
but not Rx_1

Hence can transmit 2 signals using
same frequency and double capacity.

can ~~also~~ also use RHC & LHC polarisation

(4)

3c



(6)

$$3D \quad P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R} \right)^2$$

$$f = 2.4 \text{ GHz} \rightarrow \lambda = 0.125 \text{ m}; \quad P_t = 5 \times 10^{-3} \text{ W}$$

$$G_t = G_r = 1; \quad R = 0.5 \text{ m}$$

$$\rightarrow P_r = 5 \times 10^{-3} \times 1 \times 1 \times \left(\frac{0.125}{4 \times \pi \times 0.5} \right)^2 = 1.98 \times 10^{-6} \text{ W}$$

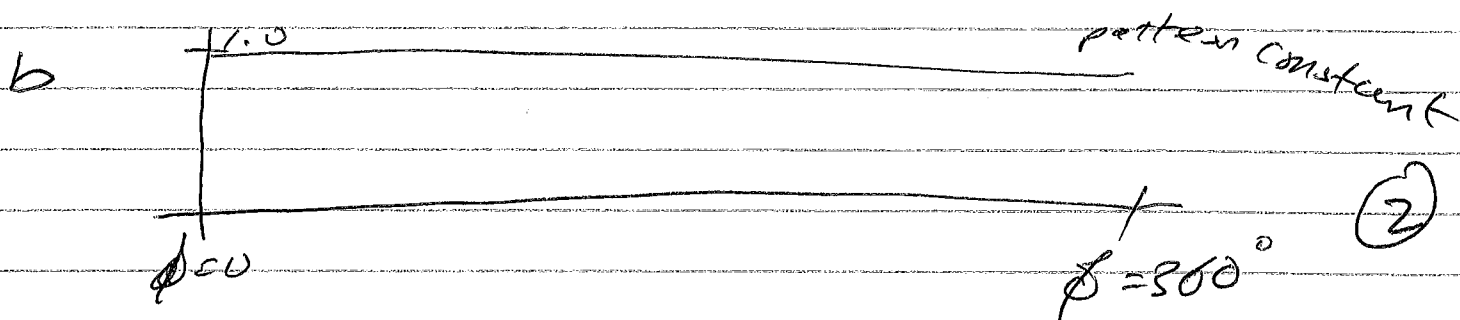
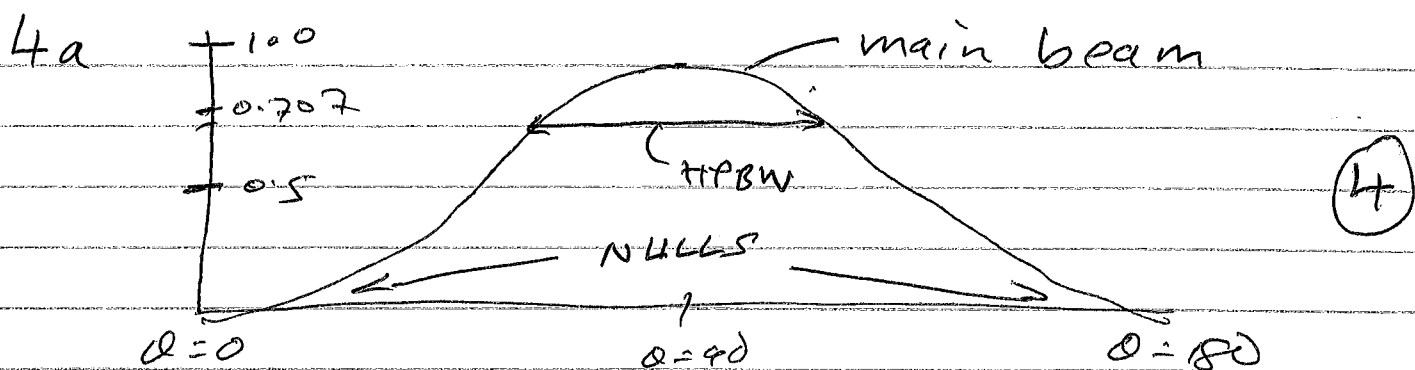
to get to dBm

$$10 \times \log_{10} \left(\frac{1.98 \times 10^{-6}}{10^{-3}} \right)$$

← mW

$$= \underline{\underline{-27 \text{ dBm}}}$$

(6)



c $\theta = 90^\circ$, $D \theta = 0^\circ + \theta = 180^\circ$ (2) + (2)

e we have $\int_0^{2\pi} \int_0^\pi D(\theta, \phi) \sin \theta d\theta d\phi = 4\pi \text{ str}$

$$\therefore \int_0^{2\pi} d\phi \int_0^\pi D_0 \sin^2 \theta \sin \theta d\theta = 4\pi$$

$$\therefore 2\pi D_0 \int_0^\pi \sin^3 \theta d\theta = 4\pi$$

$$2\pi D_0 \left[\frac{3\theta}{8} \right]_0^\pi = 4\pi \rightarrow D_0 = 16/3\pi \quad (6)$$

f) $P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R} \right)^2$, $G_t = \frac{16}{3\pi}$ as lossless

$G_r = 1.0$, $P_t = 100 \text{ W}$, $R = 10^3 \text{ m}$, $\lambda = 0.3 \text{ m}$

$$\therefore P_r = 100 \times \frac{16}{3\pi} \times 1 \times \left(\frac{0.3}{4\pi \times 10^3} \right)^2$$

$$= 9.7 \times 10^{-8} \text{ W} \text{ or } 97 \text{ nW}$$

(4)