Modelling of Machines

Section 8

Universal motor operation on an AC supply

The same Kron primitive and generalised form of the voltage matrix equations apply, i.e.

$$\begin{vmatrix} v_a \\ \\ v_f \end{vmatrix} \begin{matrix} = \\ \\ = \end{matrix} \begin{vmatrix} R_a + L_a p & \omega_r M \\ \\ 0 & R_f + L_f p \end{vmatrix} \begin{vmatrix} i_a \\ \\ i_f \end{vmatrix}$$

But on a sinusoidal AC supply $p = j\omega_s$

$$\begin{vmatrix} V_a \\ \\ V_f \end{vmatrix} \begin{matrix} = \\ \\ = \end{matrix} \begin{vmatrix} R_a + j\omega_s L_a & \omega_r M \\ \\ 0 & R_f + j\omega_s L_f \end{vmatrix} \begin{vmatrix} I_a \\ \\ I_f \end{vmatrix}$$

Note: V_a, V_f, I_a and I_f are now steady state AC values

Applying the same constraining equations as for DC operation (i.e. $V = V_a + V_f$ and $I = I_a = I_f$) yields:

$$V = [R_a + R_f + \omega_r M + j\omega_s (L_a + L_f)]\, I$$

Rotational angular velocity in elec rad/s

Supply angular velocity in elec rad/s

Introduce a variable N which is defined as: $N = \dfrac{Actual\ speed}{Synchronous\ speed}$

e.g a 2 pole machine rotating at 16,500rpm on a 50Hz electrical supply has a value of N = 5.5 since synchronous speed is 3000rpm (50×60)

Using this definition of N gives $\omega_r = N\omega_s$ and hence:

$$V = [r + NX_m + jX]\, I$$

Where: $r = R_a + R_f$
$X_m = \omega_s M$
$X = j\omega_s (L_a + L_f)$

Magnitude of V is given by:

$$|V| = |I| \underbrace{\sqrt{(r + NX_m)^2 + X^2}}_{|Z|}$$

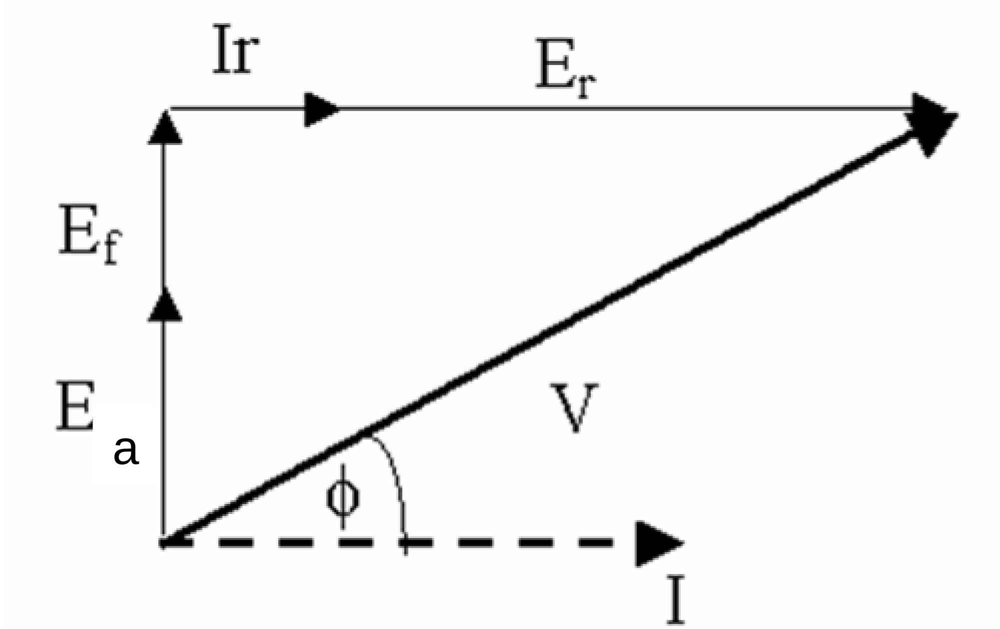
$$Power\ factor = \cos\phi = \frac{\mathrm{Re}\{Z\}}{|Z|} = \frac{r + NX_m}{\sqrt{(r + NX_m)^2 + X^2}}$$

The **average** torque produced on an AC supply is also given by

$$T = pole\ \ pairs \times MI^2$$

- This is the same expression as the DC case, except that I is the rms value Hence a value of rms current on and AC supply produces the same average torque as the same value of DC current
- It is important to note that the torque pulsates on an AC supply

Phasor diagram for AC operation



Notes:
Power factor is lagging
Power factor ($\cos\phi$) diminishes as E_a and E_f increase

Comparison of AC and DC operation

1) Average torque on DC is the same as that on AC with the same rms current
2) DC value same as rotational emf same as rms value for AC

$$\frac{E_{R_{AC}}}{E_{R_{DC}}} = \frac{N_{AC}}{N_{DC}}$$

3) From phasor diagram

$$E_{R_{AC}} = V\cos\phi - Ir$$

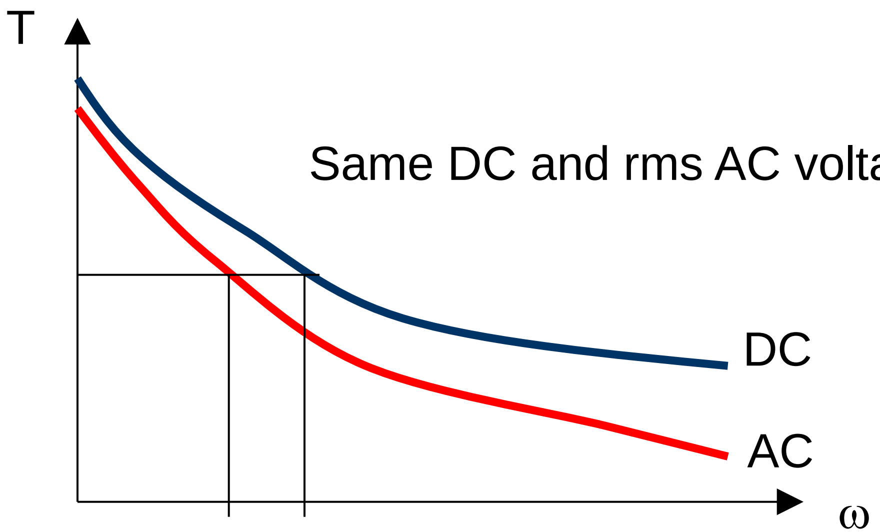
$$E_{R_{DC}} = V - Ir$$

$$\frac{N_{AC}}{N_{DC}} = \frac{\cos\phi - \frac{Ir}{V}}{1 - \frac{Ir}{V}}$$

In most design with reasonable efficiency Ir << V, hence the above expression can be simplified to:

$$\frac{N_{AC}}{N_{DC}} \approx \cos\phi$$

Machine will run slower on an AC supply for the same torque – speed difference is greater with lower power factor



Design considerations

Neglecting Ir, the angle between V and I is given by:

$$\phi = \tan^{-1}\left(\frac{E_f + E_a}{E_R}\right)$$

Hence, to minimise the angle ϕ (i.e. maximum $\cos\phi$) it is necessary to maximise E_R and minimise E_f and E_a

Smaller E_f

- lower number of field turns ($L \propto N^2$)
- lower supply frequency ($E_f \propto \omega_s$)
- low d-axis flux

Smaller E_a

- low armature coil turns
- lower supply frequency
- low q-axis flux

Smaller E_R

- high armature coil turns
- high speed
- high d-axis flux

Unfortunately, several of these are conflicting – e.g. high versus low armature turns or not possible, e.g. change 50Hz mains

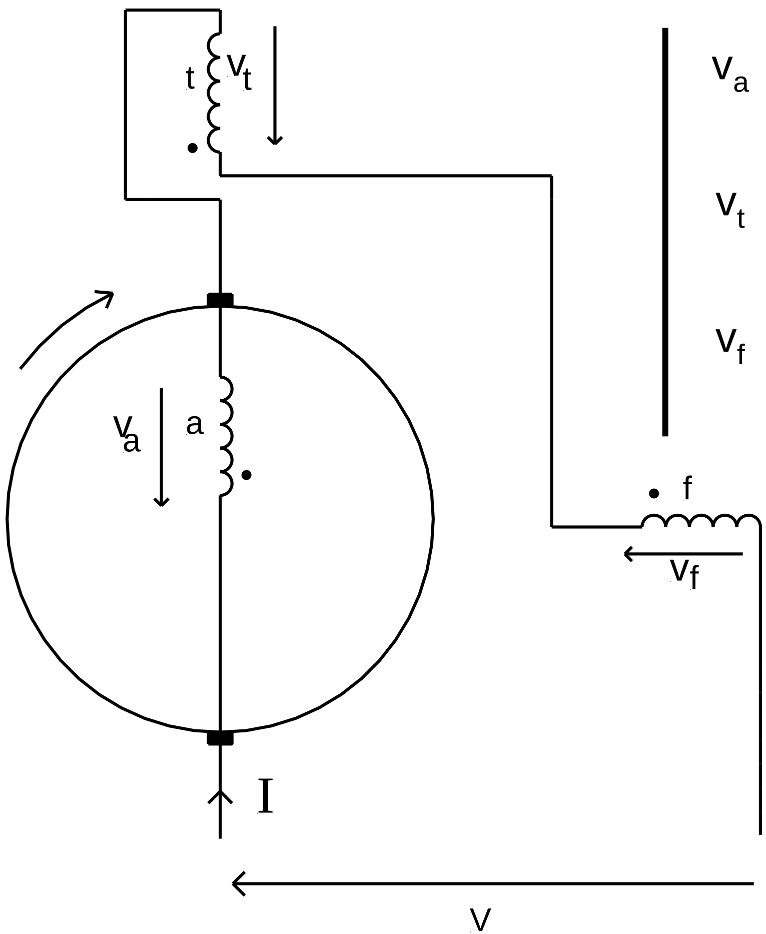
The general design guidelines (which involve compromises) are:

1) Low number of field turns – requires smallest airgap to maintain high d-axis flux
2) Low q-axis flux – use a salient structure to produce a large q-axis airgap
3) Run at high speed (15-20,000rpm common) – e.g. washing machines run with a gear ratio of 15-17 to give spin speeds of ~1200rpm

Compensation of universal motors

The performance of a universal motor can be improved by the addition of a compensating winding. to cancel E_f and E_a. Two types of compensation can be used, conductive and inductive

Conductive compensation



$$\begin{vmatrix} v_a \\ v_t \\ v_f \end{vmatrix} = \begin{vmatrix} R_a+L_a p & M_{at}\, p & M_{af}\omega_r \\ M_{ta} p & R_t+L_t p & 0 \\ 0 & 0 & R_f+L_f p \end{vmatrix} \begin{vmatrix} i_a \\ i_t \\ i_f \end{vmatrix}$$

For steady-state operation on a sinusoidal AC supply:

$p = j\omega_s$

$\omega_r = N\omega_s$

$$
\begin{vmatrix} V_a \\ V_t \\ V_f \end{vmatrix} = \begin{vmatrix} R_a+jX_a & jX_{ma} & NX_m \\ jX_{ma} & R_t+jX_t & 0 \\ 0 & 0 & R_f+jX_f \end{vmatrix} \begin{vmatrix} I_a \\ I_t \\ I_f \end{vmatrix}
$$

Constraining equations:

$$V = V_a - V_t + V_f$$
$$I = I_a = -I_t = I_f$$

$$V = I\,[R_a+R_t+R_f+NX_m+j(X_a+X_f+X_t-2X_{ma})]$$

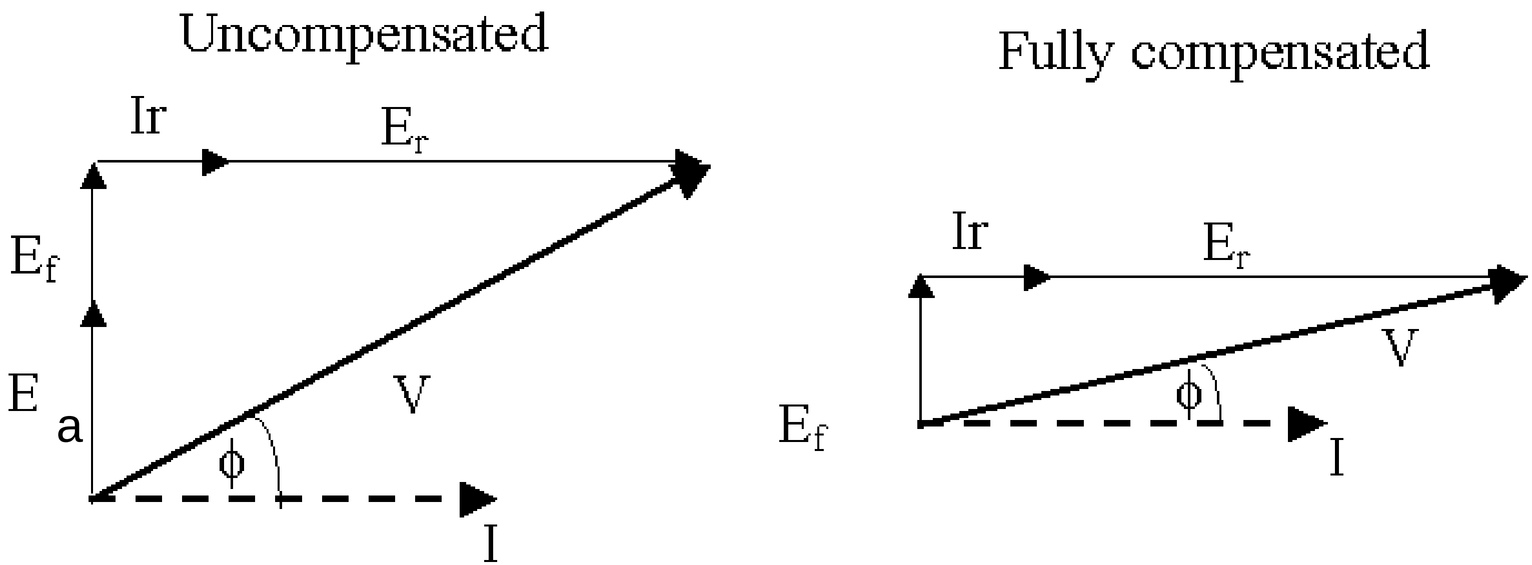
If the motor is constructed with a high coupling on the q-axis:

$$X_{ma} = K\sqrt{X_a X_t} \approx \sqrt{X_a X_t} \;\; if\;\; k \approx 1$$

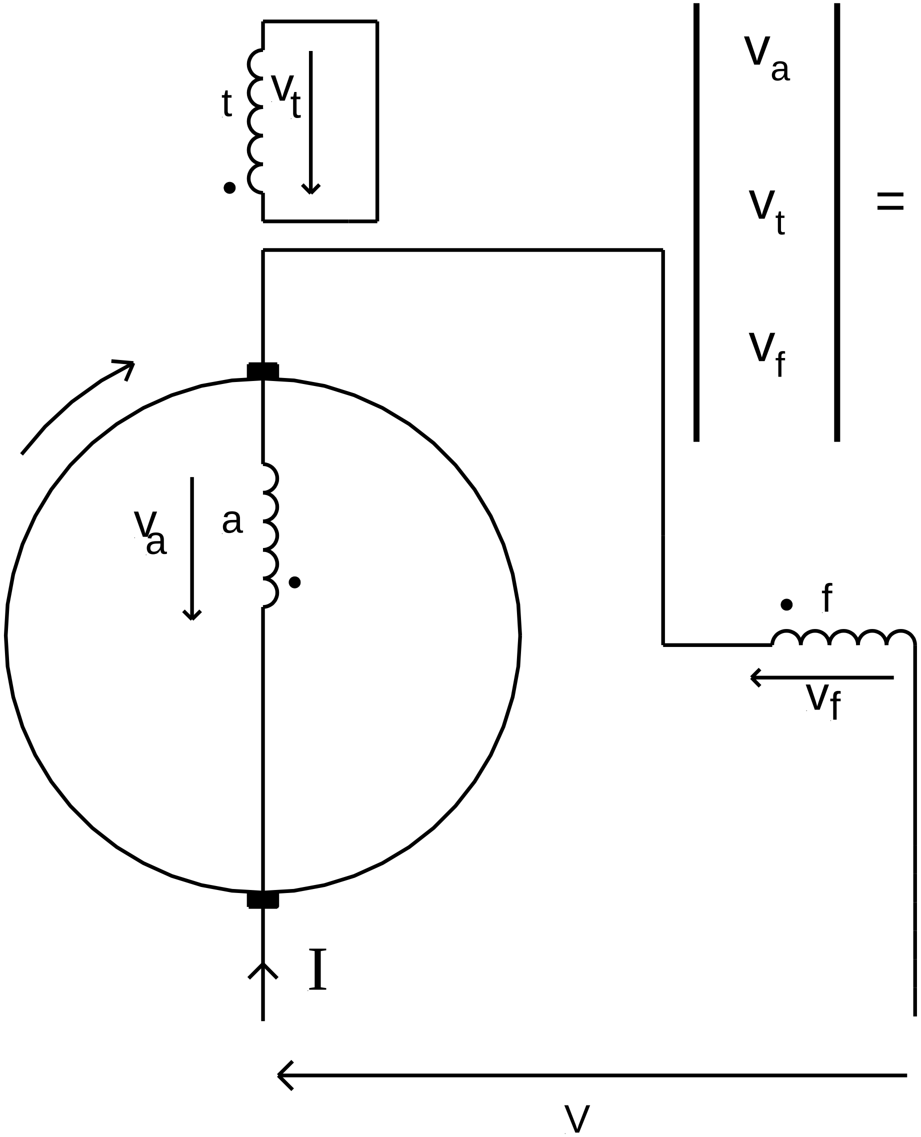
If X_t=X_a then:

$$V = I\,[r+R_t+NX_m+jX_f]$$ Where r = R_a+R_f

There is no longer a q-axis reactance term – power factor is improved



Inductive compensation



$$\begin{vmatrix} v_a \\ v_t \\ v_f \end{vmatrix} = \begin{vmatrix} R_a+L_a p & M_{at}\, p & M_{af}\omega_r \\ M_{ta} p & R_t+L_t p & 0 \\ 0 & 0 & R_f+L_f p \end{vmatrix} \begin{vmatrix} i_a \\ i_t \\ i_f \end{vmatrix}$$

Note: same equations as conductive before application of constraining equations

For steady-state operation on a sinusoidal AC supply:

$p = j\omega_s$

$\omega_r = N\omega_s$

Constraining equations:

$V = V_a + V_f$

$V_t = 0$

$I = I_a = I_f$

For t-coil

$V_t = 0 = jX_{ma}I + (R_t + jX_t)\, I_t$

Transformer coupled term

$$I_t = -\frac{jX_{ma}I}{(R_t + jX_t)}$$

Substituting for I_t in the voltage constraining equation yields:

$$V = I\left[R_f + R_a + NX_m + j\left(X_f + X_a\right) + \frac{X_{ma}^2}{R_t + jX_t}\right]$$

If the resistance of the inductive compensation coil (R_t) is negligible in comparison with its reactance (jX_t) then:

$$V = I\left[R_f + R_a + NX_m + j\left(X_f + X_a\right) - j\frac{X_{ma}^2}{X_t} \right]$$

If the coils on the q-axis are tightly coupled (i.e K≈1) then:

$$X_{ma} = K\sqrt{X_a X_t} \approx \sqrt{X_a X_t}$$

Hence, $$X_a - \frac{X_{ma}^2}{X_t} = X_a - \frac{X_a X_t}{X_t} = X_a - X_a = 0$$

Substituting for $X_a - \frac{X_{ma}^2}{X_t}$ in the voltage equation gives:

$$V = I\left[R_f + R_a + NX_m + jX_f \right]$$

Hence, complete compensation occurs without the requirement for $X_a = X_t$

Yields same phasor diagram as conductive compensation

Summary of universal machine compensation

- Both methods eliminate q-axis reactive component of voltage – hence improve power factor
- Both methods rely on close coupling of coils on q-axis (only ever approximated in practice)
- Conductive compensation requires X_t=X_a for complete compensation
- Inductive compensation – current adjusts to achieve complete compensation
- Often used in higher performance universal motors to improve performance