Data Provided: List of useful formulae

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2010-2011 (2 hours)

EEE201 Signals and Systems 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. **a.** Sketch and label (i) $x(t) = u(t-1) - u(t-3)$ and (ii) $h(t) = u(t) - u(t-2)$. **(2)**

b. Consider a system with an input signal $x(t)$ and an impulse response $h(t)$ described in part (a). Sketch and label the system response. **(7)**

c. The impulse response of a simple RC low pass circuit is given by $h(t) = \frac{1}{RC} e^{-t/RC} u(t)$ where R is the resistance and C is the capacitance. Evaluate the circuit response when the input signal is a unit step function. **(3)**

d. The response of the RC circuit in part (c) is given by

$$r(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-t/RC} & 0 \le t < T \\ e^{-(t-T)/RC} - e^{-t/RC} & t \ge T \end{cases},$$

when the input signal is described by

$$m(t) = \begin{cases} 1 & 0 \le t < T \\ 0 & otherwise \end{cases}.$$

Consider a Linear Time Invariant digital communication system, in which a bit “1” is represented by $m(t)$ and a bit “0” is represented by $-m(t)$. Sketch and label the response, $y(t)$, of this system to a sequence “0 1”. Assume T = 1s and RC = 5s. Discuss whether the sequence “0 1” can be recovered from $y(t)$ in a practical system. **(8)**

2. **a.** Find the Fourier Series representation of the square-wave input signal $v(t)$ depicted in figure Q2.1.

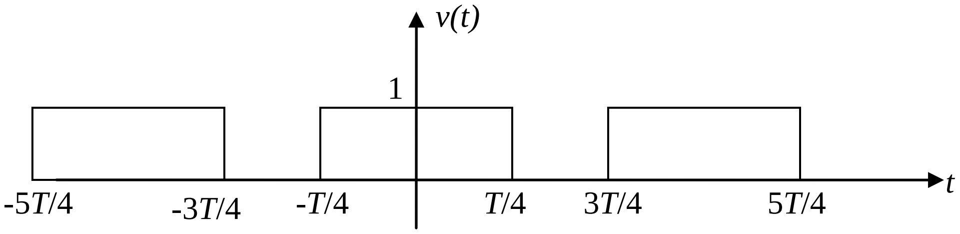

Figure Q2.1 **(8)**

b. Show that the output signal $y(t)$ of a low pass RC circuit in response to the signal $v(t)$ in part (a) is given by

$$y(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \left| \frac{10}{10 + j2n\pi} \right| \frac{1}{n\pi} \left(2\sin\left(\frac{n\pi}{2}\right) \right) \cos(2n\pi t),$$

assuming that RC = 0.1s and T=1s. **(6)**

c. Consider the output of a switching system that is represented by $v(t)$ in part (a). Compute the average power within the frequency range of -210Hz to +210Hz if

T = 0.02s. **(6)**

3. **a.** The input signal $x(t)$ and the output signal $y(t)$, of an RC circuit are related by $\frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{RC} x(t)$. Use the Laplace Transform to obtain the system transfer function $H(s)$ and the impulse response $h(t)$. [Assume zero initial conditions] **(4)**

b. Repeat part (a) if the relationship between $x(t)$ and $y(t)$ is changed to $\frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{RC} \frac{dx(t)}{dt}$. **(6)**

c. A linear feedback system consisting of two causal subsystems with transfer functions $F(s)$ and $G(s)$, is depicted in Figure Q3.1. Find the overall system transfer function $H(s)$ for this feedback system.

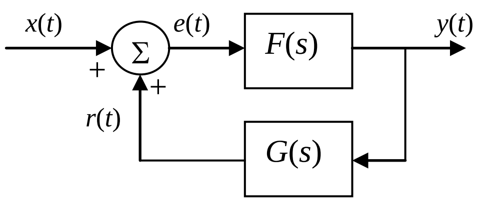

Figure Q3.1 **(4)**

d. Determine the natural oscillating frequency and damping factor of the RLC circuit in Figure Q3.2.

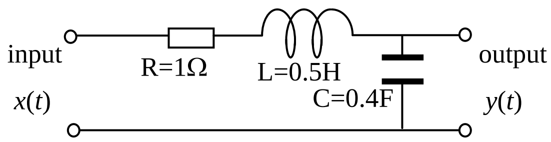

Figure Q3.2 **(6)**

4. **a.** The Fourier Transform $W(\omega)$ of the rectangular pulse $w(t)$ shown in Figure Q4.1 is given by $W(\omega) = \tau \dfrac{\sin(\omega\tau/2)}{(\omega\tau/2)}$.

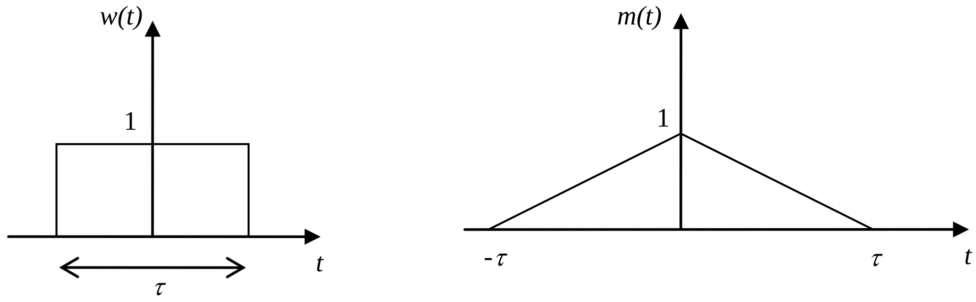

Figure Q4.1

Derive the Fourier Transform of the signal $m(t)$ using the linearity, time shift and integration properties of the Fourier Transform. **(10)**

b. **i)** Consider an amplitude modulation system with a modulating signal $m(t) = A_m \cos(\omega_m t)$ and a carrier signal $c(t) = A_c \cos(\omega_c t)$, where $\omega_c >> \omega_m$. The modulated signal is given by $s(t) = A_c[1 + \mu \cos(\omega_m t)]\cos(\omega_c t)$, where μ is the modulating factor. Sketch and label the frequency domain representation of $s(t)$. **(5)**

ii) Calculate the ratio of the average power in the side bands to the total average power assuming that the power is delivered to a 1Ω resistor. **(5)**

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