EEE105 - Electronic Devices
Lecture 5

Drift Velocity and Electric Field

In the last lecture we derived the general version of Ohm's Law, $J = \sigma E$, but in doing this we made the assumption that the Drift Velocity was proportional to the Electric Field. The question is: How can we show that this is reasonable?

Let us consider the processes affecting the electrons in a piece of solid material

The electrons can gain velocity by BEING ACCELERATED IN THE E-FIELD

The electrons can loose velocity when THEY GET SCATTERED OFF IMPERFECTIONS IN THE CRYSTAL.

Let us assume for the sake of argument that every time an electron gets scattered its drift velocity becomes zero. If we were to look at a single electrons drift velocity, we might see something like this:

Now we need the average velocity!

Consider a single electron again, let us consider the case where, the electron is scattered regularly, every τ seconds:

Now it is easy to see that the average velocity in the above case will be given by: $\langle v_d \rangle = {v_p}/{2}$ where v_p is the peak velocity. As the electron is being continuously accelerated by the force due to the E-field we can calculate $v_p = a\tau = \frac{F\tau}{m}$. Hence $\langle v_d \rangle = \frac{-qE\tau}{2m^*}$.

Let us consider a second example for a single electron: This time let us assume that the electron is continuously accelerated for a time 4τ seconds and them scatters four times very quickly:

Using the same analysis as before this time we get: $\langle v_d \rangle = {v_p}/{2} = \frac{-qE.4\tau}{2m^*} = \frac{-2qE\tau}{m^*}$

Now in both these arbitrary cases the AVERAGE time between scattering events is τ. However the result for the average drift velocity is very different.

The scattering of electrons off defects in a crystal lattice is a RANDOM process. It is not likely to occur regularly (as in the former example) or in any other way that can easily be described for a single electron.

To solve this problem we therefore need to consider ALL the electrons together
(or to put it in another way, we need to consider the average behaviour of the electron population)

So let us assume we have a piece of material of unit volume and with a free-electron concentration, *n* (m^{-3})

If the electon density is *n* m^{-3} and the volume of the material is 1 m^3 (unit volume) then there will be *n* electrons in our piece of material.

When our electric field, *E*, is applied the electrons will accelerate as before, where $a = \frac{-qE}{m^*}$

If the electrons are accelerating then their velcocity, and therefore their momentum, is changing. Let us consiter the rate of change of momentum with time for each electron:

$$\frac{dp_e}{dt} = \frac{d\left(m^* v_d\right)}{dt} = m^* \frac{dv_d}{dt} = m^* a = F = -qE$$

Therefore the rate of change in momentum for all the *n* electrons in our material will be: $\left(\frac{dp}{dt}\right)_{drift} = -qEn$

Where *p* is the total momentum of all the electrons.

As we know electrons accelerate until they are scattered. The chance (*or probability*) that a particular electron will be scattered in unit time is just a number (between 0 and 1). This means that, if we have a large number of electrons, some fraction of the electrons will be scattered. Thus we can write that

The number scattered per unit time $= \frac{n}{\tau}$

Where τ is a time constant (which actually turns out to be the average time between scattering events)

Aside:

The concept that the probability of a change of state occurring in a unit time being a constant value is met in many scientific and engineering situations.

It is normally expressed that if we have M (of something) present then the change in the number let unaffected is given by: $\frac{dM}{dt} = -\lambda M$, where λ is a number with dimensions in s^{-1}.

This is a so-called

which can be shown to have a solution

HOMEWORK: Prove that this solution is valid for yourself!!!

Examples where this type of analysis applies include

Back to Drift Velocity Case:

We had just said that the number scattered per unit time $= \frac{n}{\tau}$

We also we assume that each event causes the electron to loose all its momentum = $m^* v_d$ (on average)

Thus the total momentum change due to scattering events is: $\left(\frac{dp}{dt}\right)_{scatt} = -\frac{n}{\tau} m^* \langle v_d \rangle$

Where $\langle v_d \rangle$ is the average drift velocity of the population of electrons.

In equilibrium the total momentum change of the population must be

That is $\left(\frac{dp}{dt}\right)_{drift} + \left(\frac{dp}{dt}\right)_{scatt} =$

Hence

And therefore $\boxed{\langle v_d \rangle = -\frac{qE\tau}{m^*}}$ ⟵ ***drift velocity is proportional to electric field strength***

Note there is a simpler derivation in some textbooks that gives a value of ½ of this. This derivation makes the assumption that the scattering events occur REGULARLY, rather than RANDOMLY, and it is PHYSICALLY WRONG despite its simplicity, as it relies on a single particle approach.

The important parameter in the equation governing the drift velocity is τ , which is the average time between scattering events. It is affected by a number of parameters:

The equation above can also be written as: $v_d = -\frac{q\tau}{m^*}E$

From last time we can remember that we said $v_d = -\mu E$

Hence we can write that $\mu = \frac{q\tau}{m^*}$ and thus also $\sigma = \frac{nq^2\tau}{m^*}$ (using $\sigma = nq\mu$ from before)

However we rarely need these last two equations as mobility can be measured experimentally fairly easily using a technique called the Hall Effect (see EEE220 next year). Hence in most cases $\sigma = nq\mu$ is the useful equation to remember.

Key Points to Remember:

1. Electrons are scattered by RANDOM processes within a solid
    a. Scattering can occur from crystal defects including impurities, lattice vibrations and dislocations.
2. Due to the random nature of the scattering we cannot look at a single electron and extrapolate to the whole electron population.
3. In order to look at the relationship between drift velocity and E-field we must consider the electron population *as a whole*
4. We base our key argument around the fact that in a unit time a certain fraction of the electrons will be scattered.
    a. This type of analysis is very useful in many applications.
5. We can show that the drift velocity and E-field are proportional to each other
6. The drift velocity depends on the average (or mean) time between scattering events
    a. It also depends on the effective mass of the charge carrier.
7. Normally it is easier to work in terms of mobility rather than drift velocity as the mobility is a quantity that can be measured relatively easily.