EEE337/348: Tutorial 1

1) In the absence of phonon scattering, we can estimate the energy gained by an electron when it is subjected to an external force. Consider a GaAs sample with a band structure described by

$$E(k) = \frac{\hbar^2 k^2}{2m^*} .$$

An electron at the bottom of the conduction band is subjected to an electric field pulse of magnitude 5 kV/cm for a duration of 1 ps with the GaAs sample temperature of 300 K. Calculate the energy gained by this electron. [Hint: you can start by estimating the momentum gained from the applied electric field pulse]

In the absence of scattering the electron obeys

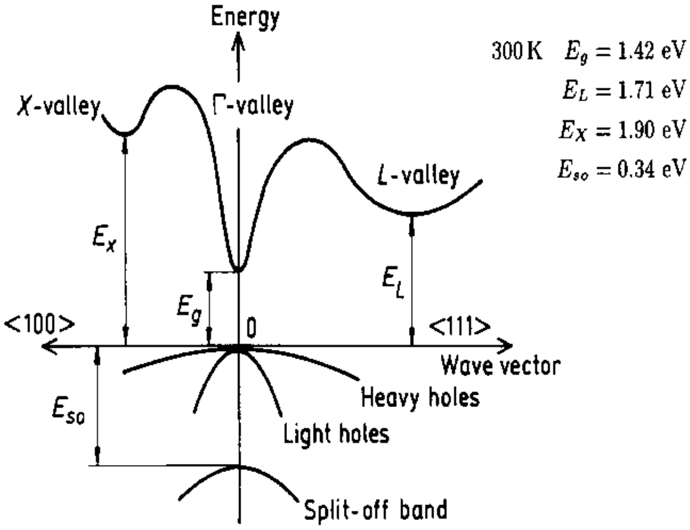
$qF = \frac{\hbar dk}{dt}$ so that after a time *t*, the change in momentum is $\hbar k = qFt$. The corresponding change in energy is

$$\Delta E = \frac{\hbar^2 k^2}{2m^*} = \frac{(qFt)^2}{2m^*} = \frac{\left[1.6\times10^{-19}\times5\times10^{5}\times10^{-12}\right]^2}{2\times0.067\times0.91\times10^{-30}} = 5.25\times10^{-20}\,J = 0.33eV .$$

2) Repeat the calculation in (1) with an electric field pulse magnitude of 50 kV/cm. Is the energy gained realistic? Explain your answer.

$$\Delta E = \frac{\hbar^2 k^2}{2m^*} = \frac{(qFt)^2}{2m^*} = \frac{\left[1.6\times10^{-19}\times50\times10^{5}\times10^{-12}\right]^2}{2\times0.067\times0.91\times10^{-30}} = 5.25\times10^{-18}\,J = 32.8eV$$

Clearly the energy gained is too large and not realistic. The scattering-free approximation is clearly not valid at such high energy. Referring to the band structure of GaAs, the L and X valleys are 0.31 and 0.48 eV above the minima of Gamma valley. Therefore electrons will transfer to these valleys as well as higher conduction bands when subjected to such as a large electrical pulse. Calculation at such a high energy is complicated and require accurate knowledge of scattering mechanisms and band structures at high energy. Note that the band structure is only well known at low energies.



3) Define i) the density of states, $N(E)$ and ii) the Fermi distribution function, f(E). In bulk semiconductors these are described by

$$N(E) = 4\pi\left(\frac{2m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} \quad \text{and} \quad f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}.$$

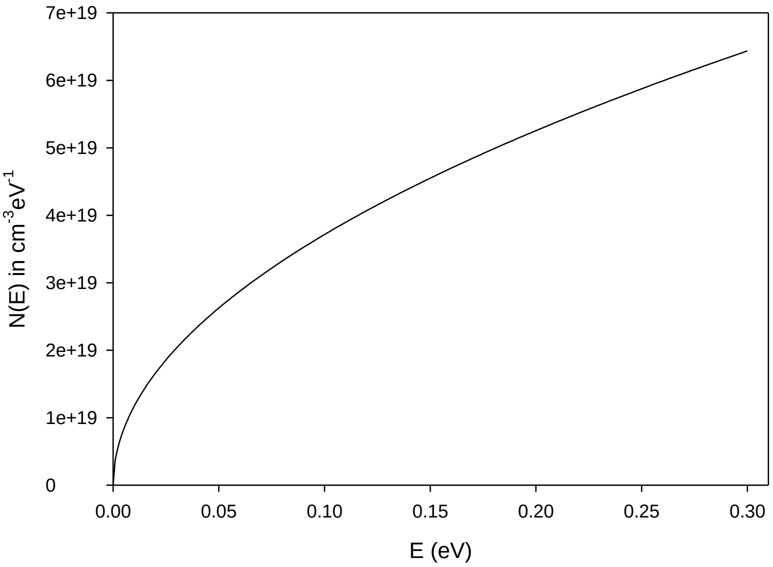
$N(E)$ is the density of allowed energy states per energy range per unit volume while f(E) is the probability of an electron occupying that energy range.

4) i) Plot the density of states up to 0.3 eV in GaAs. The density of states is usually expressed in $eV^{-1}cm^{-3}$, therefore you will need to perform some unit conversion [Hint: you may use $E=mc^2$ to convert kg to $Jm^{-2}s^2$ and note that 1 J = 1/1.6×10^{-19} eV].

$$N(E) = 4\pi\left(\frac{2\times 0.067\times 9.1\times 10^{-31}}{(6.63\times 10^{-34})^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} = 4\pi\left(2.77\times 10^{35}\right)^{\frac{3}{2}} E^{\frac{1}{2}} \ \text{J}^{-3/2}\text{m}^{-3}.$$

Converting this to $eV^{-1}cm^{-3}$ we have

$$N(E) = 4\pi\left(2.77\times 10^{35}\right)^{\frac{3}{2}} E^{\frac{1}{2}} \times \left(1.6\times 10^{-19}\right)^{\frac{3}{2}} \times 10^{-6} \ \text{eV}^{-1}\ \text{cm}^{-3}$$



ii) The electron density is given by

$$n = \int_0^{E_{top}} N(E) f(E) dE$$

Assuming that the Fermi level is equal to half the bandgap of GaAs, calculate the electron density if E_{top} = 0.3 eV at 300 K.
Answer: 5.06×10^5 cm^{-3}

5) The electron density influence the current conduction in semiconductor. We know that doping of semiconductor increases the electron density. This can be shown by calculating the dependence of Fermi level on carrier concentration. Assuming that a GaAs sample is doped with an n-type dopant concentration of 10^{17} cm^{-3} and all the dopants are activated, use the Joyce-Dixon approximation given below to estimate the Fermi level at 300 K. Compare the electron density to that in 4(ii).

$$E_F = kT\left[\ln\frac{n}{N_C} + \frac{1}{\sqrt{8}}\frac{n}{N_C}\right] = 0.026\left[\ln\left(\frac{10^{17}}{4.45\times10^{17}}\right) + \frac{1}{\sqrt{8}}\left(\frac{10^{17}}{4.45\times10^{17}}\right)\right] = -0.037 \text{ eV.}$$

Clearly the Fermi level is now only 0.037 eV below the conduction band compare to 0.71 eV in the intrinsic GaAs. Repeating the calculation from 4(ii) the electron density is 9.56×10^{16} cm^{-3}. Therefore even at room temperature a large number of electrons can be found in the conduction band in this n-type GaAs.

6) The direct band gap of $Al_xGa_{1-x}As$ is given by 1.424 + 1.247x. GaAs/AlGaAs is one of the most important heterostructures used in lasers and transistors. Assuming that 60% of the band gap discontinuity is in the conduction band, calculate the conduction band and valence band barrier heights for GaAs/$Al_{0.3}Ga_{0.7}As$. Discuss whether this heterojunction is ideal for npn based heterojunction bipolar transistor (HBT).

Band gap of GaAs = 1.424 eV
Band gap of $Al_{0.3}Ga_{0.7}As$ = 1.424+1.247(0.3) = 1.80 eV.
Band gap difference = 0.376 eV.

The conduction band barrier height $=0.6\times0.376=0.226$ eV.
The valence band barrier height $= 0.4\times0.376=0.15$ eV.
For a npn HBT, ideally a large barrier height at the valence band is required to block holes from p-type base from being injected to the n-type emitter. Therefore $Al_{0.3}Ga_{0.7}As$ will work but not as an ideal wide band gap material.

Parameters for GaAs

Electron effective mass: 0.067 m_0

Temperature dependence of band gap: $1.519 - 5.405\times10^{-4}T^2/(T+204)$

At 300 K, $N_C = 4.45\times10^{17}$ cm^{-3} and $N_V = 7.0\times10^{18}$ cm^{-3}

Useful Constants

Fundamental Electronic Charge e = 1.60218 x 10^{-19} C
Electron Rest Mass m_0 = 9.1095 x 10^{-31} kg

Vacuum Permittivity Epsilon_0 = 8.85418 x 10^{-12} F.m^{-1}
Speed of Light in Vacuum c_0 = 2.99792 x 10^8 m.s^{-1}

Planck's Constant h = 6.62617 x 10^{-34} J.s
Wavelength of a 1 eV photon = 1.23977 x 10^{-6} m
1 cm^{-1} = 0.12408 meV
1 meV = 8.0593 cm^{-1}

Boltzmann's Constant k_B = 8.6174 x 10^{-5} eV.K^{-1} = 1.38066 x 10^{-23} Joules.K^{-1}
Avogadro's Constant N_A = 6.022 x 10^{26} (kgMole)$^{-1}$
Electron Volt eV = 1.60218 x 10^{-19} J