Stator windings in polyphase AC machines

In order to produce optimal and smooth torque it is desirable to produce a distribution of mmf which is a close approximation to a sinusoidal current sheet. In most practical machines, the conductors are located in discrete slots in the stator core and considerable ingenuity has been applied to achieve very close approximations to the idealised sinusoidal distribution.

There are many different features of 3-phase AC stator windings:

a) **Type of coil:** concentric, lap, wave
b) **Overhang (end-winding):** diamond, multiplane, involute, mush
c) **Layers:** single, double
d) **Connection:** star, delta
e) **Phase spread:** 60^o or 120^o
f) **Slotting:** integral, fractional
g) **Coil span:** full-pitch, short-pitched
h) **Connection:** series, parallel

We will return to some of these features in detail, but will start with a simple single-layer winding

Single and double layer windings

In a single layer winding, each slot only contains one coils

- Simple coil arrangement
- End windings can be bulky
- More limited scope to tailor harmonics
- **Usually** limited to small and medium machine

In a double layer winding each slot contains two coils

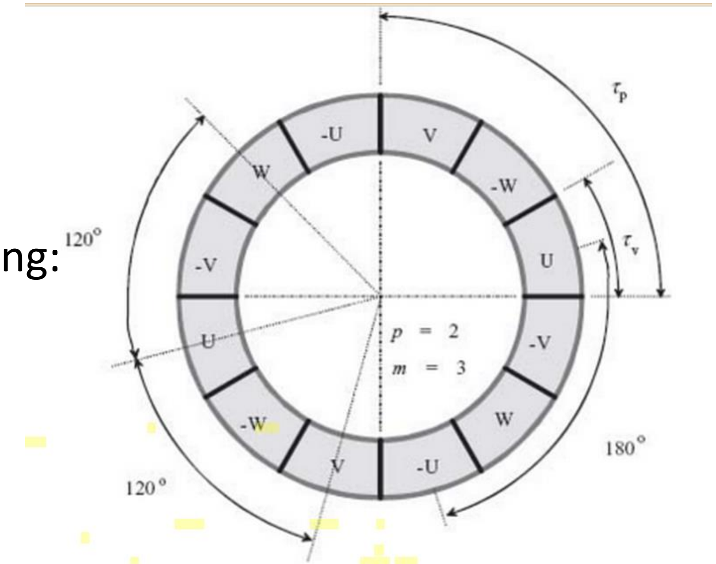
- Can give more compact end-windings
- Greater scope to tailor harmonics
- Preferred winding type for large machines

12 slot, 2-pole, double layer

12 slot, 2-pole, double layer

Integral and fractional slot windings

Consider a simple schematic of a multiphase winding:



The pole pitch of a winding is given by:

$$\tau_p = \frac{\pi D}{2p}$$

where: *D* - airgap diameter; *p* - number of pole-pairs

$$\tau_v = \frac{\tau_p}{m}$$

where: *m* – number of phases

The number of phase zones is therefore *2pm*

If the total number of slots in the stator is Q, then the number of slots in each phase zone (q) is given by:

$$q = \frac{Q}{2pm}$$

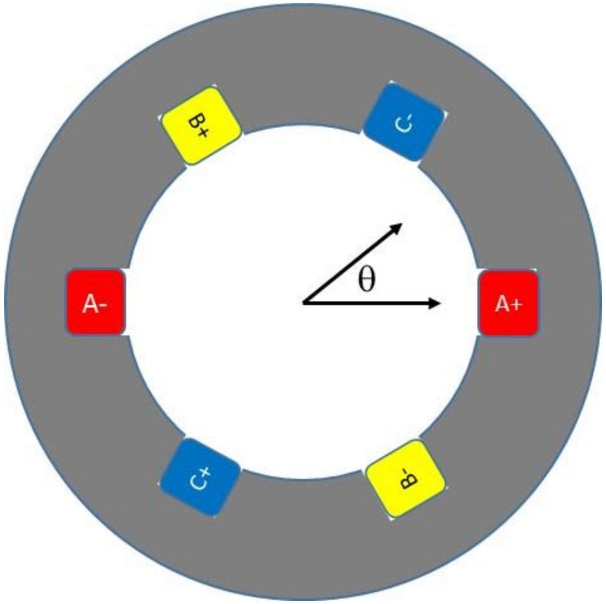
If q is an integer, then the winding is referred to as an integral slot winding, while if q is not an integer then the winding is referred as a fractional slot winding

Elementary 2 pole, 3-phase winding

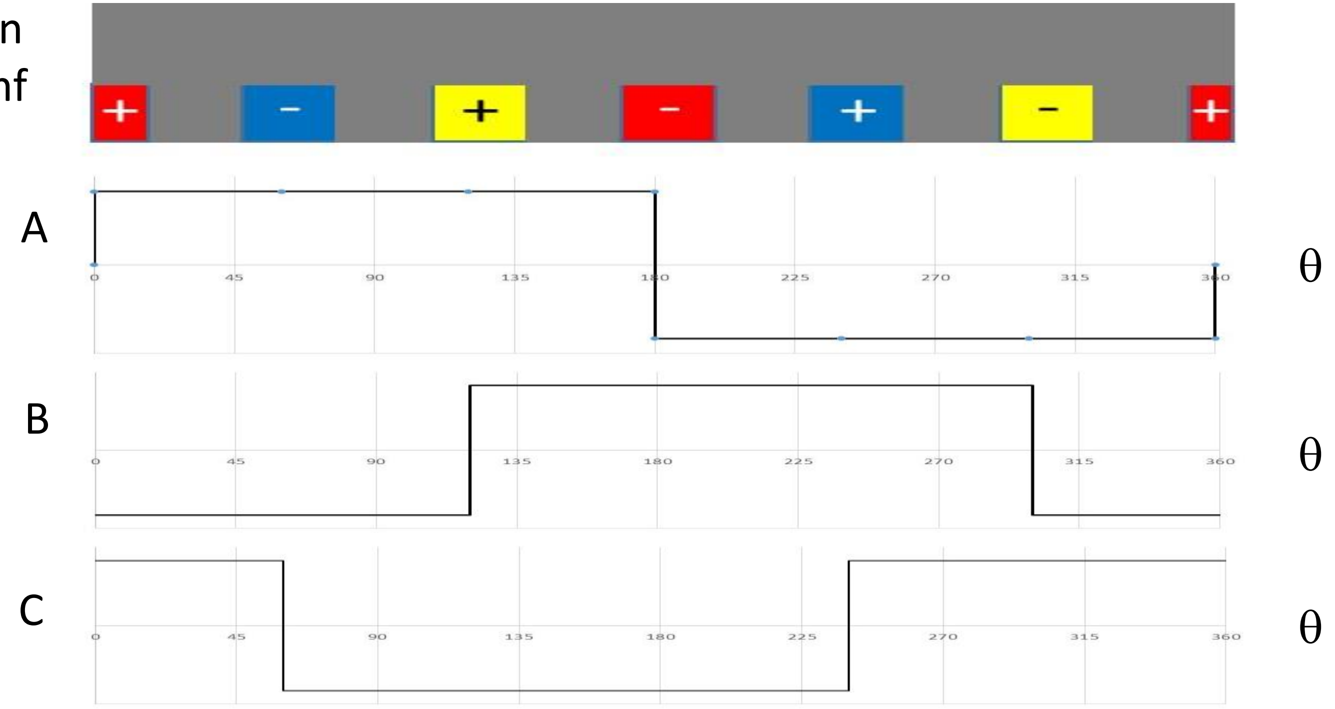
The simplest 3-phase slotted winding that produces a rotating field is:

$m = 3 \qquad p = 1 \qquad q = 1$

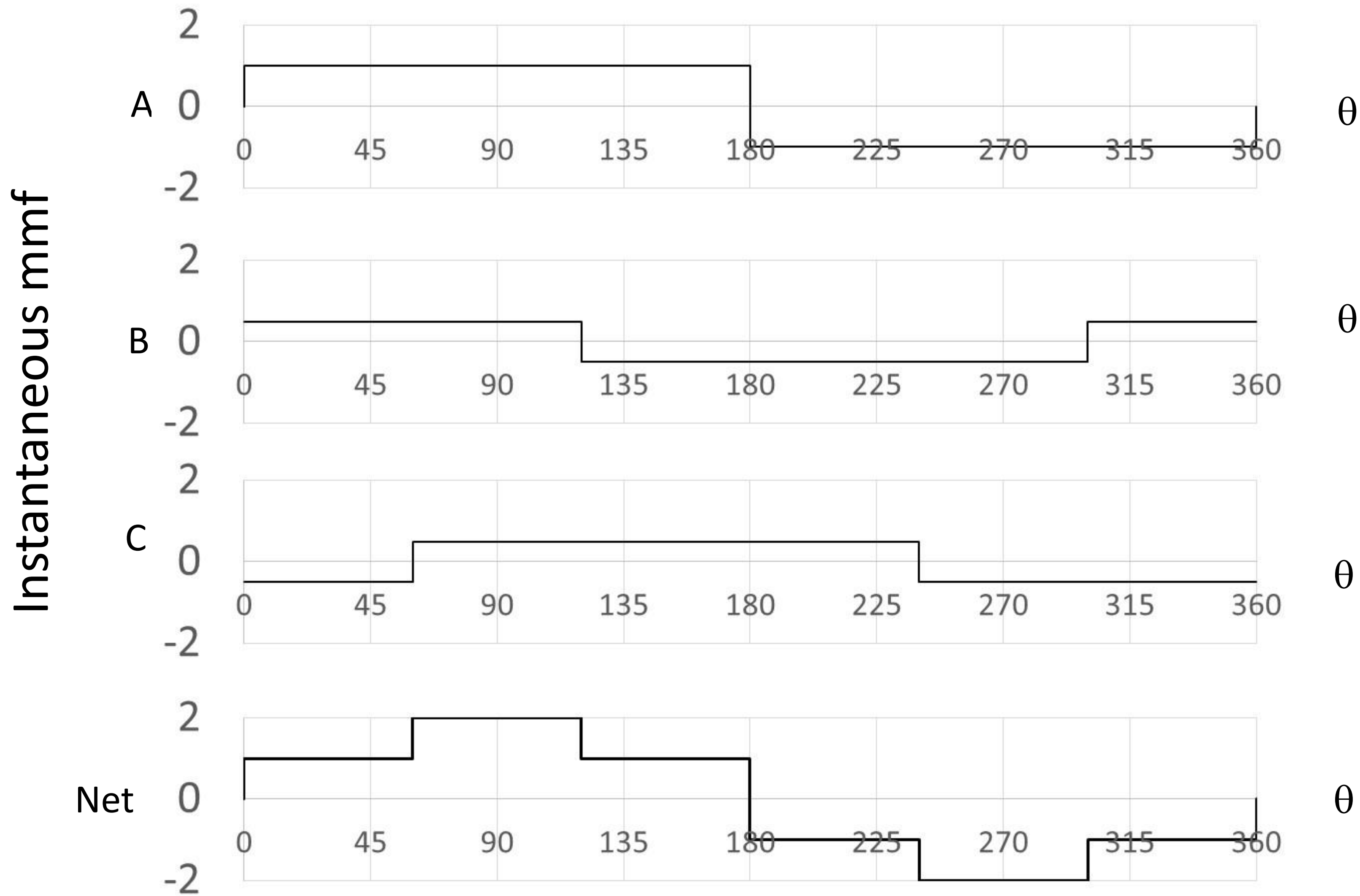
Hence: $\quad Q = 2pmq = 6$



Spatial distribution of normalised mmf



Taking an instant in time at which I_a, I_b and I_c take values of +1, -0.5 and -0.5 (equivalent to a 'time angle' of 90°) allows the instantaneous variation of mmf around the stator bore to be calculated.



The same principles can be extended to a 2 pole, 3 phase winding with more slots, noting that the phase bands span the same angle.

12 slot

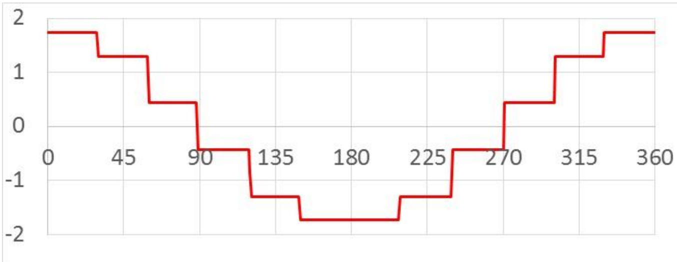
Spatial mmf distribution at instant with $I_a=1$; $I_b=I_c=-0.5$

24 slot

Spatial mmf distribution at instant with $I_a=1$; $I_b=I_c=-0.5$

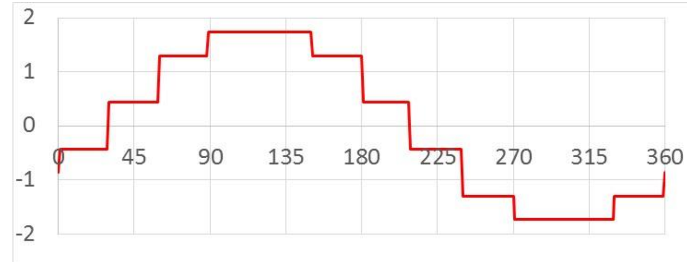
12 slot, 3-phase, 2-pole integral slot winding – variation in spatial distribution of mmf over one half cycle of excitation

t=0



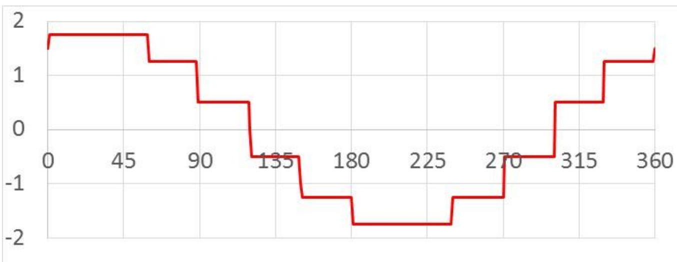
θ

t=T/3



θ

t=T/12

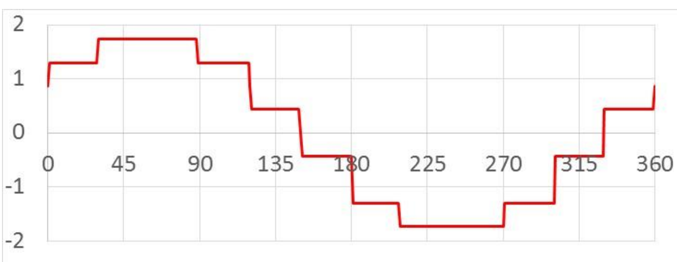


θ

t=5T/12

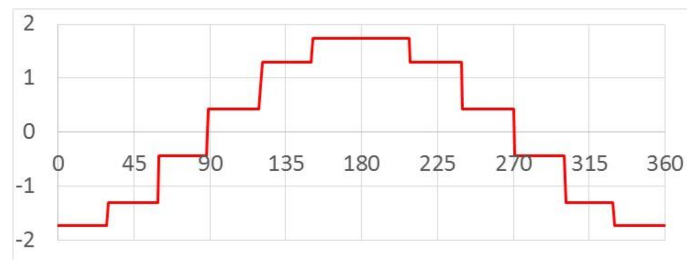
θ

t=T/6



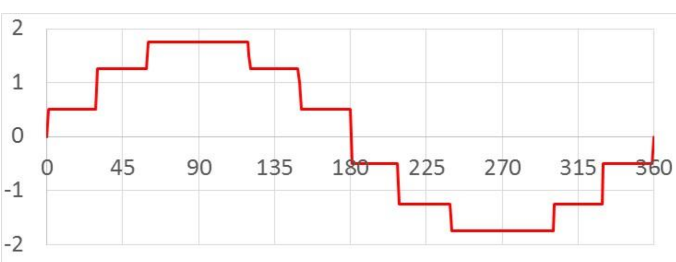
θ

t=T/2



θ

t=T/4



θ

These spatial distribution waveforms illustrate the mmf distribution rotates around the stator bore at a frequency determined by the time variation of the stator current

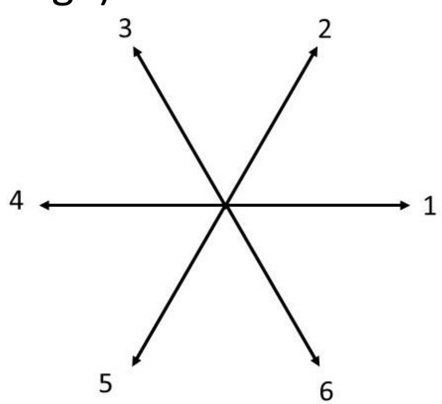
Voltage phasor analysis of coils

Although the mmf distribution provides one means of analysis, a more common approach which is used to analyse and design windings is based on voltage phasors (sometimes incorrectly referred to as voltage vectors in some notes and textbooks)

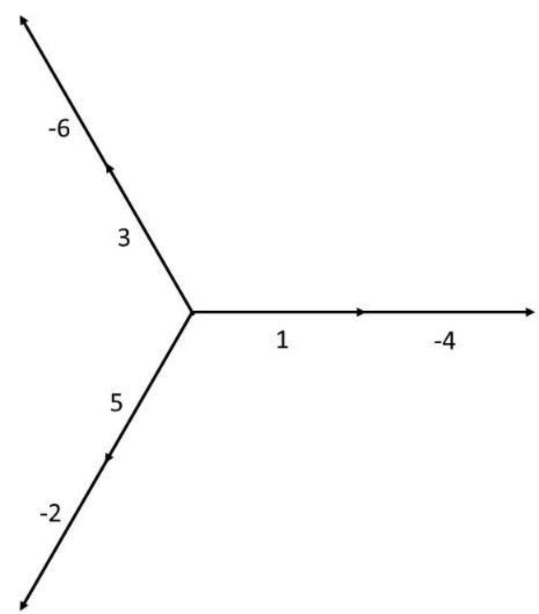
Returning to the **3-phase, 6 slot** integral pole winding studied previously:

Each successive slot is displaced in space by 60^o then it follows that the emf generated by a rotating field in each slot is phase displaced in time by the same 60^o

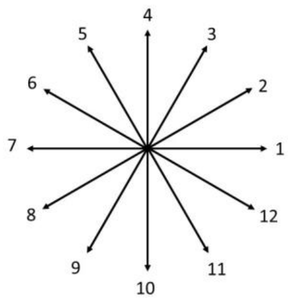
The phasor diagram for the individual slots (with no regard for polarity of connection at stage this stage) is:

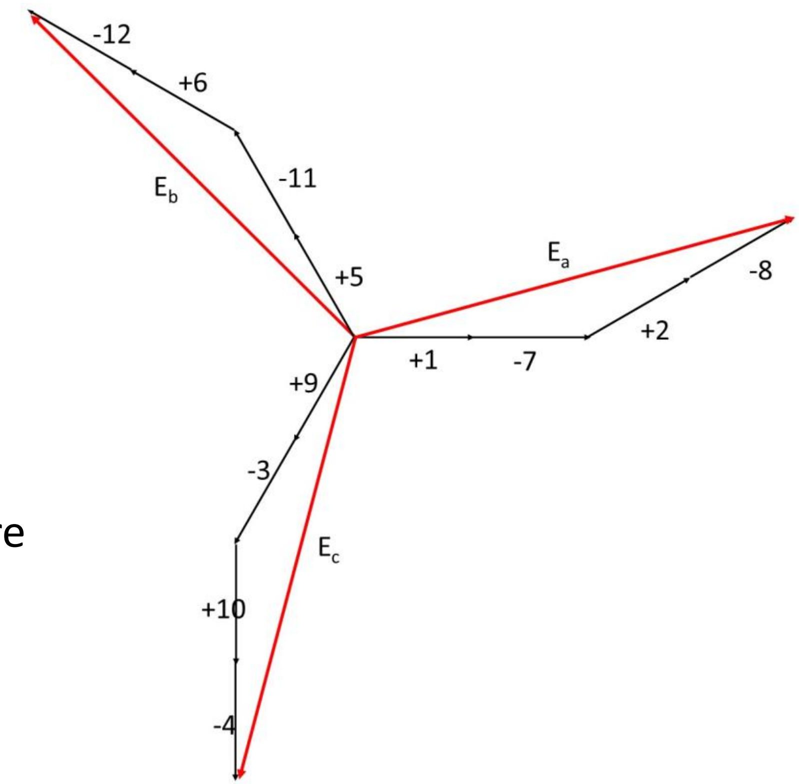


For the winding arrangement shown previously, e.g. +1 and -4 forming one phase the emf is given by:



Similarly, revisiting the **12 slot, 2 pole winding, 3-phase** integral slot winding from earlier:



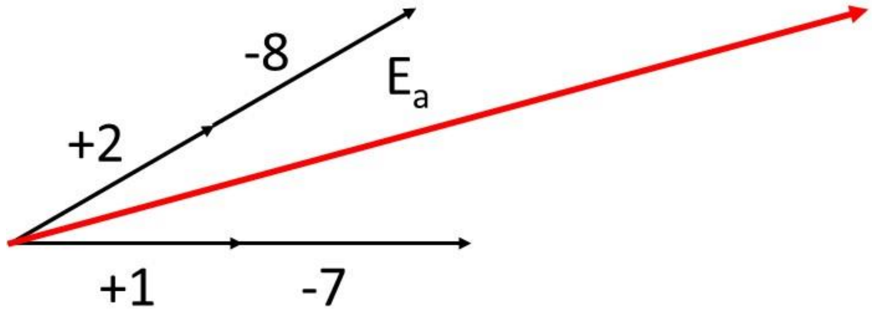


Although the individuals coils can be connected to form a 3-phase winding, there is a phase shift between the individual components within a phase – this reduces the net magnitude of the phase emf as compared to the algebraic sum of the individual slot emfs (this is the very phase shift we exploited previously to reduce the harmonic content of the spatial mmf distribution).

This reduction in the emf as compared to the algebraic sum is quantified by means of the winding distribution factor

Winding distribution factor

- Looking more closely at the 4 **fundamental** phasors which make up phase A



- These form a bunch of 4 phasors (once appropriately reversed) with a line of symmetry. If the angle between the jth phasor is α_j, then the fundamental winding factor for a bunch of *z* phasors is given by:

$$k_{d1} = \frac{1}{z}\sum_{j=1}^{z} \cos \alpha_j$$

This can be extended to the vth harmonic to produce a more winding factor for each harmonic

$$k_{dv} = \frac{\sin \frac{v\pi}{2}}{z}\sum_{j=1}^{z} \cos \alpha_j$$

It is worth noting that the sin term only takes values of ±1 for integer values of v

Example of winding distribution factor calculations

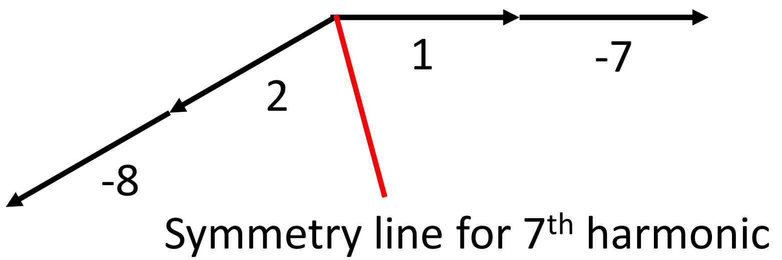
- Taking initially the fundamental of the 1 12 slot, 3-phase, integral slot winding

$$k_{d1} = \frac{sin\frac{\pi}{2}}{4}\left(cos\left(\frac{\pi}{12}\right) + cos\left(\frac{\pi}{12}\right) + cos\left(-\frac{\pi}{12}\right) + cos\left(-\frac{\pi}{12}\right)\right) = 0.966$$

- If we now consider the 7th harmonic, then determining the angles is not so straightforward as the fundamental, but taking each in turn:

Slot	Offset for fundamental	Offset for 7th harmonic	Net angle of 7th harmonic
1	0°	0°	0°
2	30°	7×30 = 210°	210°
7	180°	7×180 = 1260°	180° (1260-3×360)
8	210°	7×210 = 1470°	30° (1470-4×360)

The resulting phasor diagram (noting that the connection is set by the fundamental) is:

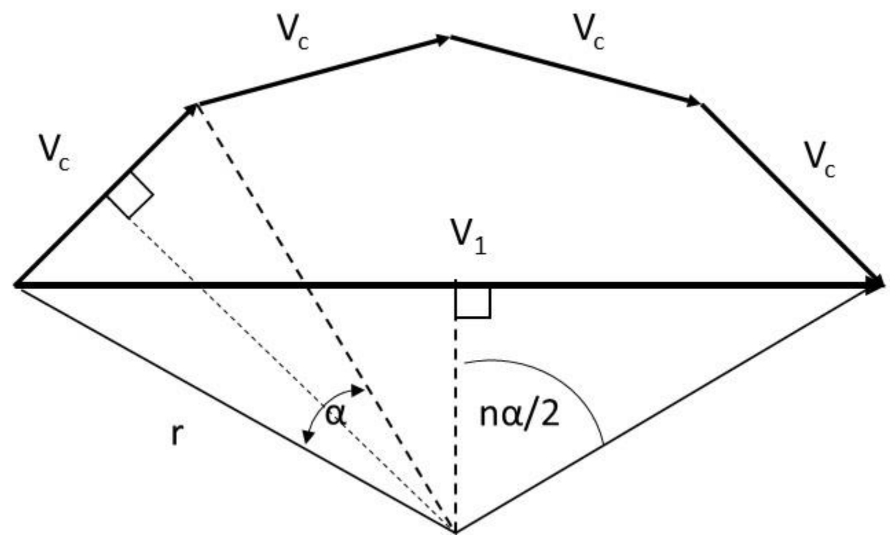


$$k_{d7} = \frac{sin\frac{7\pi}{2}}{4}\left(cos(75^o) + cos(75^o) + cos(-75^o) + cos(-75^o)\right) = -0.2558$$

The key point here is that the reduction in the 7th harmonic due to the coil distribution is much greater than the reduction in the fundamental, i.e. distributing the coil recuces the harmonic content of the emf.

Direct calculation of winding distribution factor

$$k_{d1} = \frac{phasor\ sum}{magnitude\ sum} = \frac{V_1}{nV_c}$$



For the general case of a polygon comprising n phasors each of magnitude Vc then (q=4 in the example shown), :

$$k_{d1} = \frac{2r\ sin\left(\frac{n\alpha}{2}\right)}{n2r\ sin\left(\frac{\alpha}{2}\right)} = \frac{sin\left(\frac{n\alpha}{2}\right)}{n\ sin\left(\frac{\alpha}{2}\right)}$$

The expression for k_{d1} can be re-arranged and simplified using some of the earlier relationships for a 3-phase machine:

$$k_{d1} = \frac{1}{2n\ sin\left(\frac{\pi}{6n}\right)}$$

This principle can be extended to the vth harmonic by noting that the angle is now $v\alpha$

$$k_{dv} = \frac{sin\left(\frac{nv\alpha}{2}\right)}{n\ sin\left(\frac{v\alpha}{2}\right)}$$

Direct calculation of winding distribution factor -example

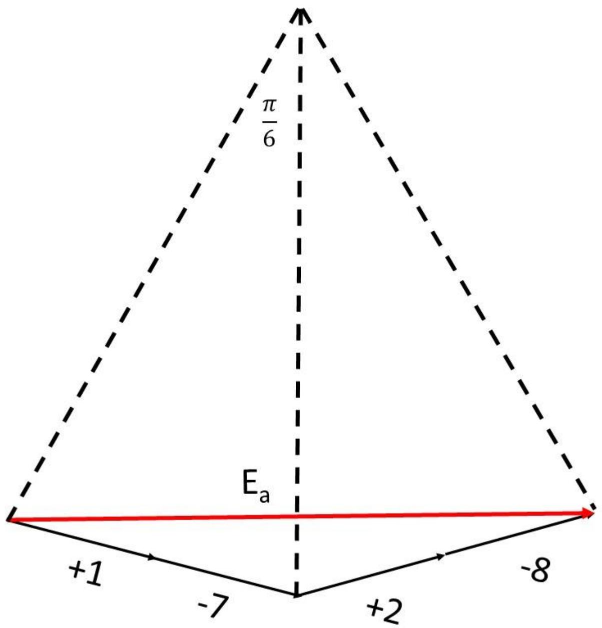
For the 12 slot, 3-phase, integral slot winding shown previously, q = 2 (there are 4 individual phasors but they form a polygon of only two sides) within a polygong of six sides and hence $\alpha = \pi/6$. Substituting for q and α yields:

$$k_{d1} = \frac{sin\left(\frac{n\alpha}{2}\right)}{n\ sin\left(\frac{\alpha}{2}\right)} = \frac{sin\left(\frac{\pi}{6}\right)}{2\ sin\left(\frac{\pi}{12}\right)} = \frac{0.5}{2 \times 0.2558} = 0.966$$

or

$$k_{d1} = \frac{1}{2n\ sin\left(\frac{\pi}{6n}\right)} = \frac{1}{4\ sin\left(\frac{\pi}{12}\right)} = 0.966$$

(both values agreeing with the value calculated previously)



Applying this to the 7th harmonic yields

$$k_{dv} = \frac{sin\left(\frac{2 \times 7 \times \frac{\pi}{6}}{2}\right)}{2\ sin\left(\frac{7 \times \frac{\pi}{6}}{2}\right)} = -0.2558\ (as\ before)$$

The preceding analysis provides a useful means of interpreting and calculating distribution factor in terms of the various individual phasors which area at play in a machine. It provides a graphical and general approach when considering various winding types including non-standard types and is applicable to harmonics.

There is an alternative expression for the ***fundamental*** distribution factor of a winding which is rather simpler and does not require consideration of phasors per se, allowing the distribution factor to be calculated simply from the number of slots per phase per pole (q) and the number of phases (m):

$$k_{d1} = \frac{sin\left(\frac{\pi}{2m}\right)}{q\ sin\left(\frac{\pi}{2mq}\right)}$$

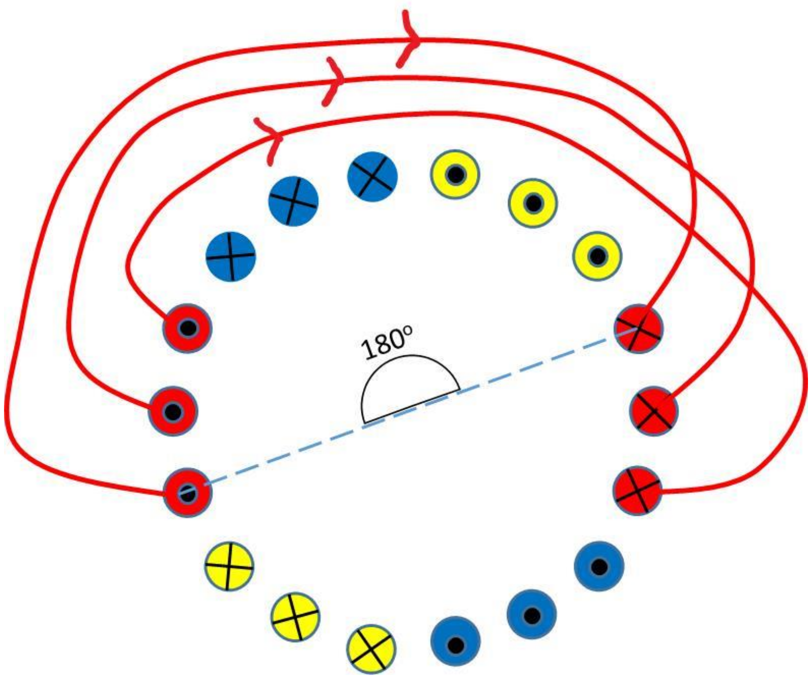
Substituting in the values from the previous example (i.e. m=3, Q=12, p=1) yields:

$$q = \frac{Q}{2pm} = \frac{12}{6} = 2 \qquad\qquad k_{d1} = \frac{sin\left(\frac{\pi}{6}\right)}{2\ sin\left(\frac{\pi}{12}\right)} = 0.966\ as\ previously$$

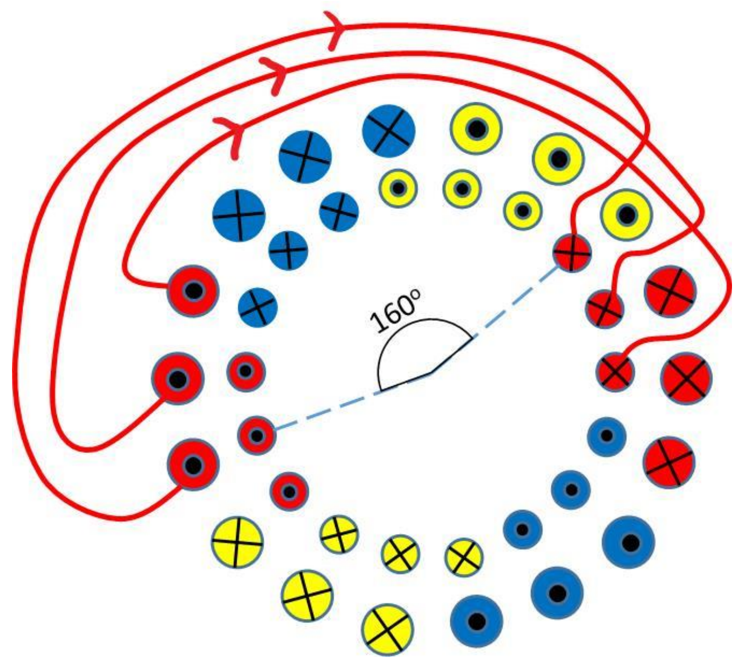
Short-pitching of coils

Consider a stator with 18 slots (Q=18) which is wound with as a 3-phase machine (m=3) producing a 2-pole field. Two possible single and double layer winding arrangements are shown below:

Single layer



Double layer



The important thing to note about the double layer winding is that the phase zones of the top and bottom layers do not have to be distributed in the same way (though of course they can be). The phase zone of the top layer can be shifted by an integer number of slots from the corresponding phase band on the bottom layer. In the double layer winding example shown, the phase zone is shifted by one slot pitch. This reduces the coil pitch from 180° to 160°, a design feature known as short-pitching

Short pitching results in a reduced flux linkage compared to a fully-pitched coil. However, by appropriate short-pitching the reduction in the harmonics can be more pronounced than in the fundamental, i.e. we are able to reduce the harmonic content of the induced emf.

If the full pitch winding spans y_f slots and the short-pitch winding spans y_s slots, then the relative shortening is y_s/y_f.

The electrical angle of the short-pitch is: $\pi\left(1-\frac{y_s}{y_f}\right)$

The pitch factor, which quantifies the reduction in emf caused by short-pitching, is given by:

$$k_p = \sin\left(\frac{y_s}{y_f}\frac{\pi}{2}\right)$$

For the v^{th} harmonic:

$$k_{pv} = \sin\left(v\frac{y_s}{y_f}\frac{\pi}{2}\right)$$

Skewing of windings

One further feature that can be incorporated into a stator to reduce the harmonic content is skewing of the stator slots. This is achieved by gradually indexing each successive lamination along the length of the stator core to produce a gradual or skew in the slot. Since the lamination thickness is much smaller than the length of most stator cores, then the skew is a good approximation to a continuous skew.

Skewing of the stator slots also results in a reduction in the emf, but again tends to be used to have a more pronounced effect on higher order harmonics, in particular the harmonics produced by the stator slots, i.e. the so-called slotting harmonics.

For a winding with a pole pitch angle of α_p and a stator skew angle *s,* the skew factor (which is ≤1) for the v^{th} harmonic is given by:

$$k_{sv} = \frac{sin\left(v\frac{s}{\alpha_p}\frac{\pi}{2}\right)}{v\frac{s}{\alpha_p}\frac{\pi}{2}}$$

Overall winding factor

- The overall winding factor for an AC winding (which is will be recalled quantifies the reduction in emf as compared to an idealised phase zone representation) for the vth harmonic is given by the product of the distribution factor, the pitch factor and the skewing factor, i.e.

  $$k_{wv} = k_{dv}\, k_{pv}\, k_{sv}$$

  The reduction in the fundamental component (which we wish to minimise) can be obtained by simply substituting v = 1

Example:

Calculate a winding factor for a double-layer winding with the following features:

Number of slots = 24
Number of stator poles = 4
Number of phases = 3
Coil pitch = 5 slots
Stator slot skew = 10° (mechanical angle)

From the above data, the number of slots per phase per pole is given by:

$$q = \frac{Q}{2pm} = \frac{24}{12} = 2 \qquad \text{i.e. an integral slot winding}$$

Consider the distribution factor for the fundamental

If we consider one electrical pole-pair (noting that there are 2 pole-pairs around the periphery) then each pole-pair spans 12 slots

$$\alpha = \frac{2\pi}{12} = \frac{\pi}{6}$$

This forms a symmetrical arrangement of slots giving rise to 6 phasors, e.g. slot 1 and slot 7 reversed. Each phase winding is therefore the combination of 2 phasors (i.e. n=2). Substituting this value of n into the expression which is specific to a 3-phase winding yields:

$$k_{d1} = \frac{1}{2n\, sin\left(\frac{\pi}{6n}\right)} = \frac{1}{4\, sin\left(\frac{\pi}{12}\right)} = 0.966$$

Quick check with the simpler formula (applicable to fundamental only) yields

$$k_{d1} = \frac{sin\left(\frac{\pi}{2m}\right)}{q\, sin\left(\frac{\pi}{2mq}\right)} = \frac{sin\left(\frac{\pi}{6}\right)}{2\, sin\left(\frac{\pi}{12}\right)} = 0.966$$

The pole pitch of the winding expressed as a number of slots = 24/4 = 6

The coil pitch of 5 is therefore a short-pitched coil and hence the coil pitch factor for the fundamental is:

$$k_p = \sin\left(\frac{y_s}{y_f}\frac{\pi}{2}\right) = sin\left(\frac{5}{6}\frac{\pi}{2}\right) = 0.966\ (coincidentally)$$

The skew factor for the fundamental (noting that s = 10^{o} and αp = 90^{o}) is given by:

$$k_{s1} = \frac{sin\left(v\frac{s}{\alpha_p}\frac{\pi}{2}\right)}{v\frac{s}{\alpha_p}\frac{\pi}{2}} = \frac{sin\left(\frac{10}{90}\frac{\pi}{2}\right)}{\frac{10}{90}\frac{\pi}{2}} = 0.994$$

The overall winding factor for the fundamental is hence given by:

$$k_{wv} = k_{dv}\,k_{pv}\,k_s = 0.966 \times 0.966 \times 0.994 = 0.927$$

i.e. distributing, short-pitching and skewing the winding results in a reduction in the fundamental of 7.3%

The benefit in terms machine performance of adopting these feature in the winding design come from the effect on the harmonics. Taking the 5th harmonic as an example:

$$k_{d5} = \frac{sin\left(\frac{n5\alpha}{2}\right)}{n\ sin\left(\frac{5\alpha}{2}\right)} \frac{sin\left(\frac{2\times 5\frac{\pi}{6}}{2}\right)}{2\ sin\left(\frac{5\frac{\pi}{6}}{2}\right)} = 0.259$$

$$k_p = \sin\left(5\frac{y_s}{y_f}\frac{\pi}{2}\right) = sin\left(5\frac{5}{6}\frac{\pi}{2}\right) = 0.259$$

For the 5th harmonic, $\alpha_p = 90/4$

$$k_{s1} = \frac{sin\left(v\frac{s}{\alpha_p}\frac{\pi}{2}\right)}{v\frac{s}{\alpha_p}\frac{\pi}{2}} = \frac{sin\left(\frac{10}{18}\frac{\pi}{2}\right)}{\frac{10}{18}\frac{\pi}{2}} = -0.216$$

Hence the winding factor for the 5th harmonic is: 0.014

Fractional slot windings

As shown earlier, the number of slots per phase zone is given by:

$$q = \frac{Q}{2pm}$$

where Q is the total number of slots, p is the number of pole-pairs and m is the number of phases

Windings with non-integer values of q are termed as fractional slot windings, e.g. a 3 phase, 6 pole winding in a stator with 27 slots gives q = 1.5

The main benefits of a fractional slot winding over an integral slot winding are:

- Greater freedom in selection of the number of slots
- Numerous alternatives for short-pitching
- For a fixed number of slots, a greater range of pole numbers can be produced
- Better suited to large machines with segmented stators
- Scope to improve voltage waveform by removing specific harmonics

Although q =1.5 in the previous example, it is useful to keep q expressed as a ratio:

$$q = \frac{z}{n}$$

where z and n are the smallest possible integers

In this specific case z = 3 and n =2

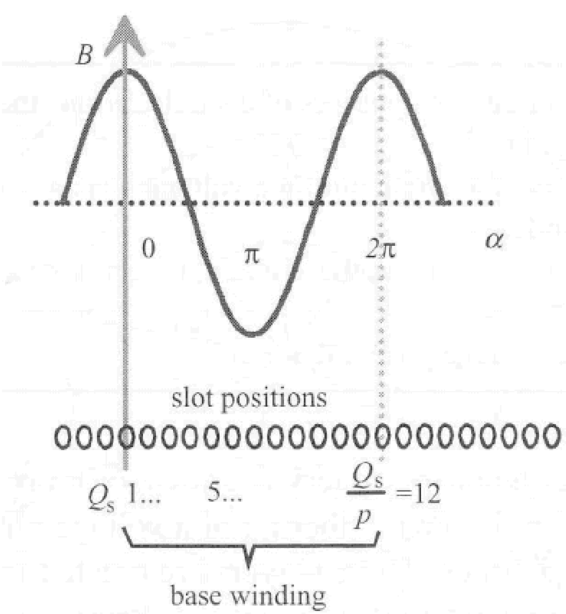
If the denominator is an odd number, then the winding is said to be a first grade winding whereas if n is even then the winding is said to be a second grade winding.

The main drawback of fractional slot windings is the presence of sub-harmonics in windings in which n≠2.
As a result, the preferred combinations for fractional slot tend to be those in which=2.

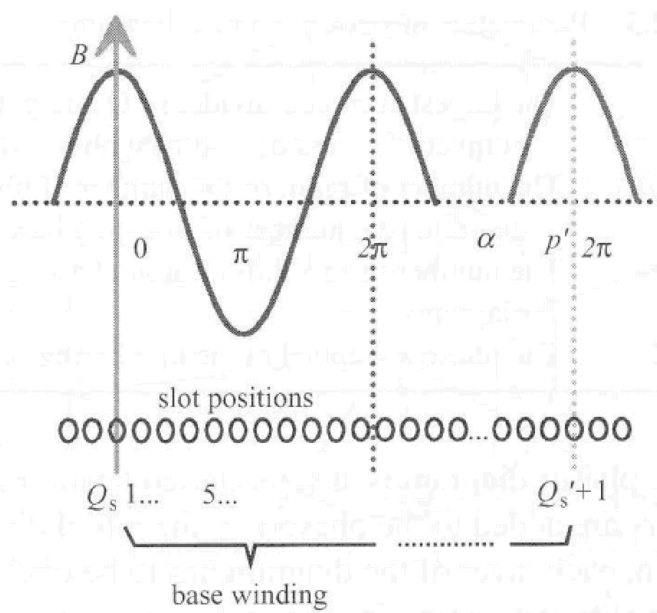
In an integral slot winding, a slot of a given phase again coincides with the peak of the flux density waveform after two pole-pitches. Hence, the so-called base winding has a length equal to two pole-pitches, i.e. one pole pair.

In a fractional slot winding, a distance greater than two pole-pitches must be traversed to arrive back at situation where the slot of a given phase coincides with the peak of the airgap flux density. Consider the example below:

Integral slot



Fractional slot



Constructing phasor diagrams for fractional slot windings

- Consider example of 27 slots, 6 pole, 3 phase winding

$$q = \frac{Q}{2pm} = \frac{27}{2 \times 3 \times 3} = \frac{27}{18} = \frac{3}{2}$$ Hence, $n = 2$

Slots per pole-pair = 27/3=9 which is an integer

$$t = 3\ (largest\ common\ divider\ of\ 27\ and\ 3)$$

This gives the number of concentric layers in the phasor diagram

$$p' = \frac{p}{t} = \frac{3}{3} = 1$$

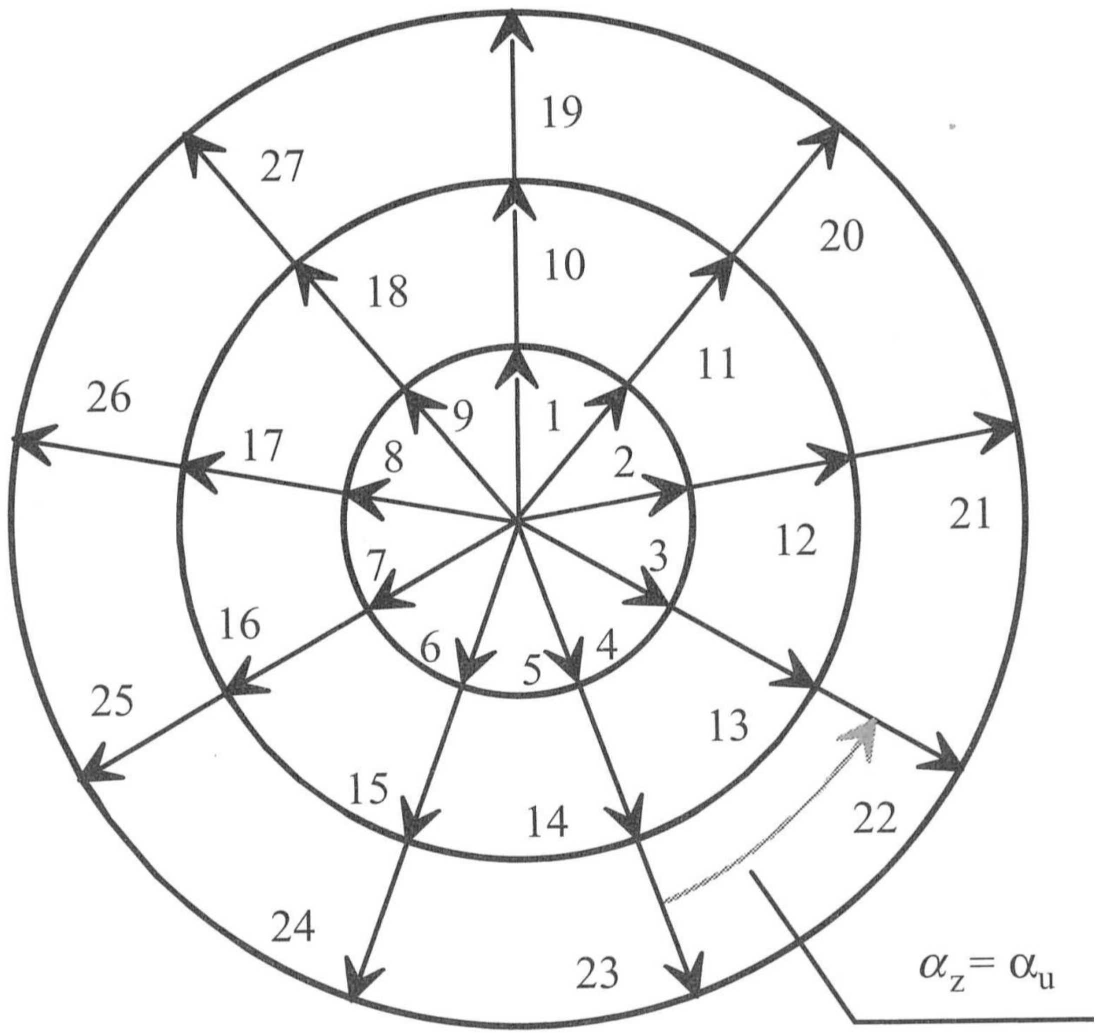
$$Q_s' = p' \frac{Q_s}{p} = \frac{27}{3} = 9$$

This gives the number of phasors in each layer

The angle between phasors is given by: $\alpha_z = \frac{2\pi}{9} = 40^o$

The slot angle is given by: $\alpha_u = \frac{p}{t}\alpha_z = \frac{3}{3} \times 40 = 40\ ^o$

The phasor diagram comprises 3 layers with 9 phasors in each layer



It is interesting to note that although this contains three-layers, each layer is in fact a repeating pattern of the innermost base layer.

Consider another example of 30 slots, 8 pole, 3 phase winding

$$q = \frac{Q}{2pm} = \frac{30}{2 \times 4 \times 3} = \frac{30}{24} = \frac{5}{4} = 1.25$$ Hence, $n = 4$

Slots per pole-pair = 30/4=7.5 which is not an integer

$$t = 2\ (largest\ common\ divider\ of\ 30\ and\ 4)$$

This gives the number of layers in the phasor diagram

$$p' = \frac{p}{t} = \frac{4}{2} = 2$$

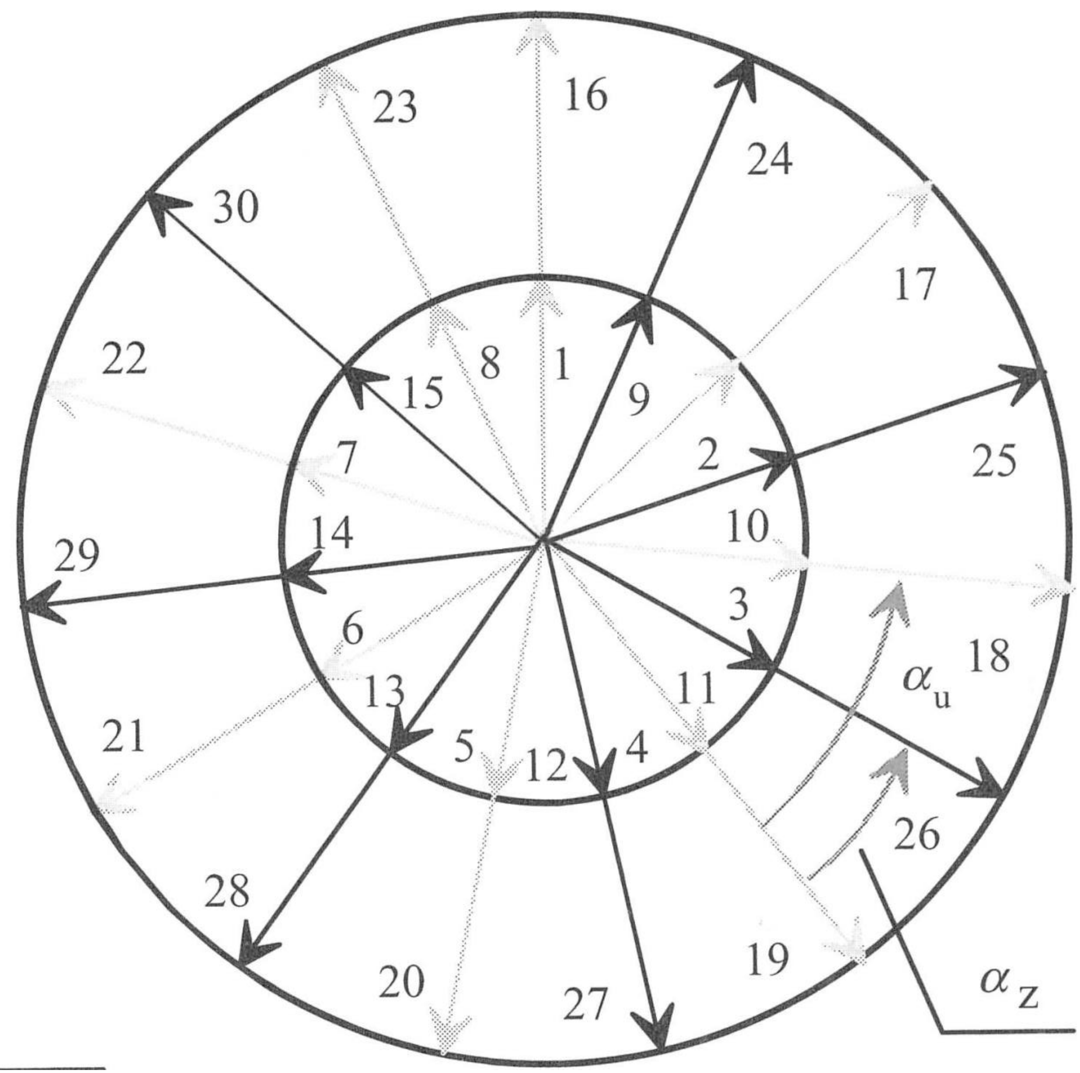
$$Q_s' = p'\frac{Q_s}{p} = \frac{2 \times 30}{4} = 15$$

This gives the number of phasors in each layer

The angle between phasors is given by: $\alpha_z = \frac{2\pi}{15} = 24^o$

The slot angle is given by: $\alpha_u = \frac{p}{t}\alpha_z = \frac{4}{2} \times 24 = 48\ ^o$

The resulting phasor diagram has 2 layers with 15 phasors in each layer



Winding connection from phasor diagrams

- In order to produce a useful fractional slot winding it is necessary to combined the appropriate phasors into 3 phase bundles
- For some cases, this procedure is relatively straightforward, e.g the first example above in which the phasor naturally group into 3 bundles.
- However in other cases, a procedure which is far from straightforward must be followed requires detailed consideration of various symmetry conditions and tables of viable connection types.
- Consider another example of 168 slots, 40 pole, 3 phase single layer winding:

$$q = \frac{Q}{2pm} = \frac{168}{2 \times 20 \times 3} = \frac{168}{120} = \frac{6}{5}$$

Slots per pole-pair = 168/20=8.4 which is not an integer

$$t = 4\ (largest\ common\ divider\ of\ 168\ and\ 20)$$

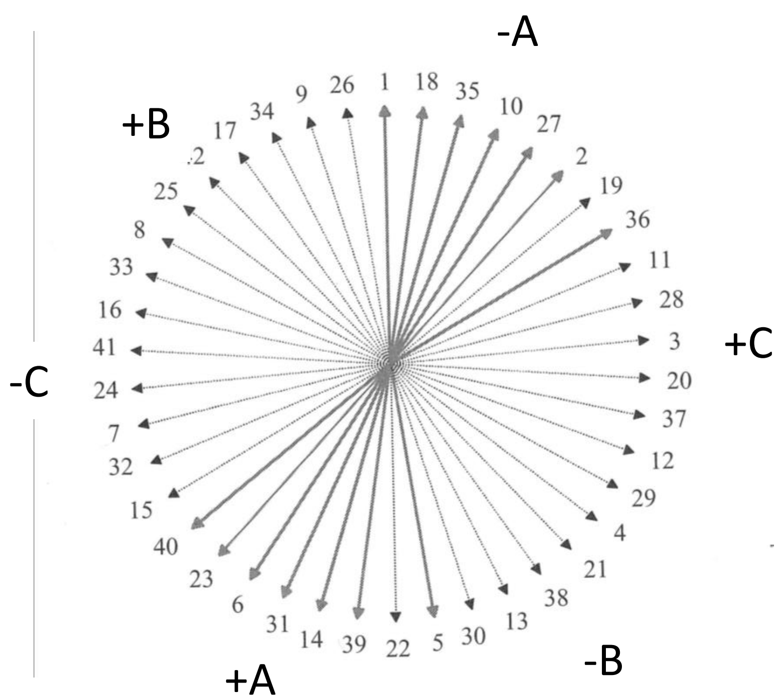
$$p' = \frac{p}{t} = \frac{20}{4} = 5$$

$$Q_s' = p'\frac{Q_s}{p} = \frac{5 \times 168}{20} = 42$$

The angle between phasors is given by: $\alpha_z = \frac{2\pi}{42} = 8.57^o$

The slot angle is given by: $\alpha_u = \frac{p}{t}\alpha_z = \frac{20}{4} \times 8.57 = 42.86\,^o$

The first layer of the phasor diagram (which in effect represents the base winding which is then repeated 4 times) has 42 phasors:



The single layer winding fore Phase A is made up from

A- 1,18,35,10,27,2,36 (not 19) A+ 40,23,6,31,14,39,5 (not 22)

The procedure can then be applied to the further 3 layers of the phasor diagram to get the full winding

The exact reasoning and process behind this is beyond the scope of this course. (There is a useful, if rather protracted description in ‘Design of Rotating Electrical Machines’ by Pyrhonen, Jokinen and Hrabovcova (Wiley) on pages 97-108. Note: this additional material is for information and interest only and is not examinable)