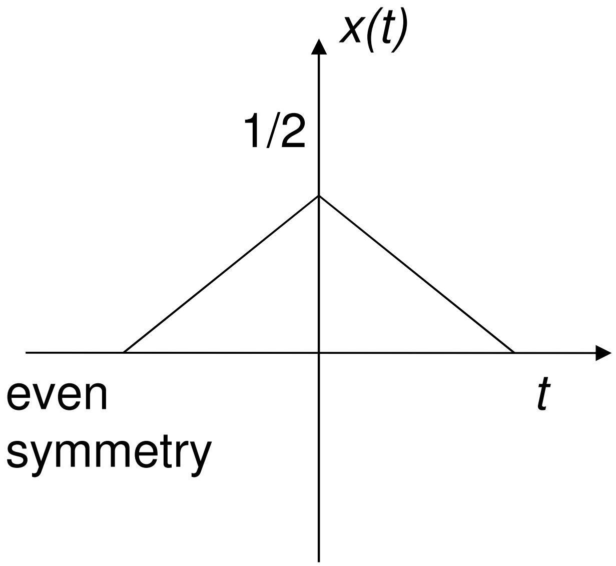
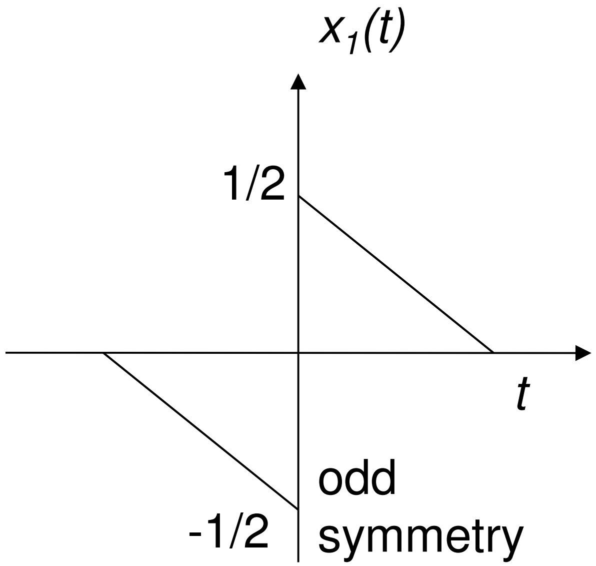
Lecture content

- Signal symmetry: Even and odd signals
- Piecewise Continuous Signals
- Discrete Time (DT) signals
  - Step function
  - Ramp function
  - Impulse

Even and odd signals

A signal $x(t)$ has an even symmetry if $x(t) = x(-t)$. On the other hand if $x(t) = -x(-t)$ the signal $x(t)$ is said to have an odd symmetry

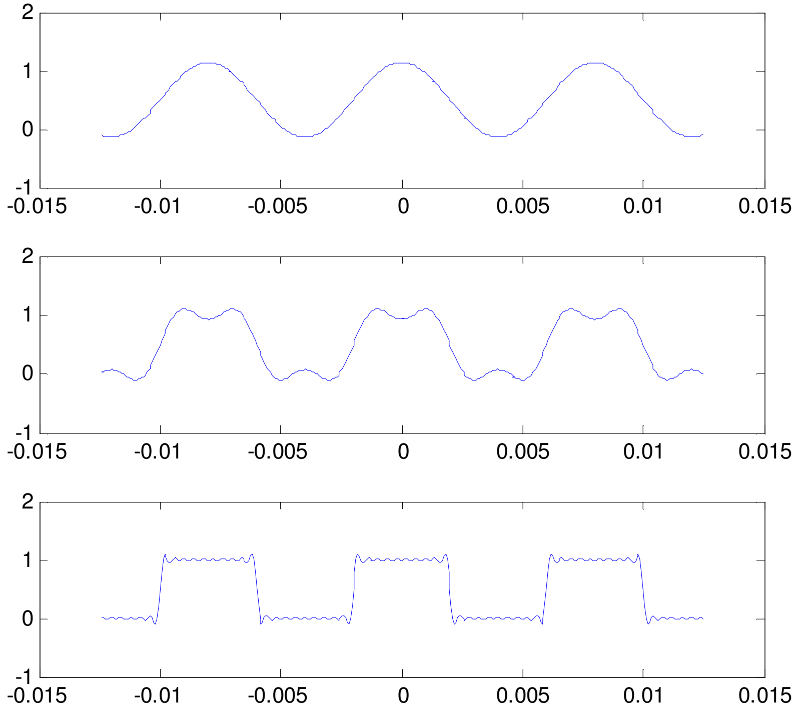


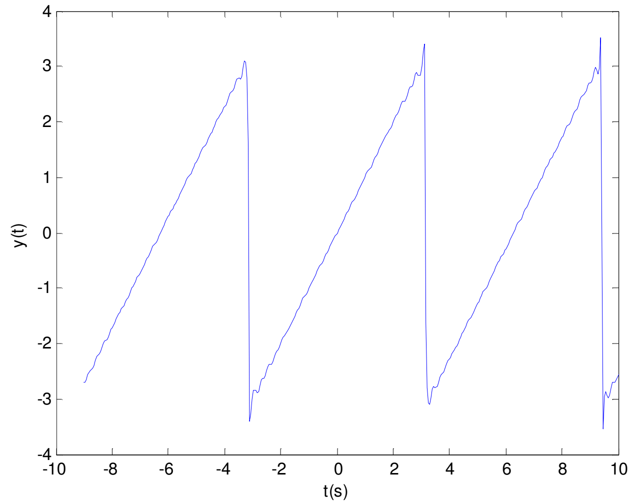


Fourier Series Representation

$$x(t) = \frac{1}{2} + \sum_{n=1}^{N} \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos(2n\pi t)$$

$$x(t) = \sum_{n=1}^{N} \frac{2}{n} (-1)^{n+1} \sin(nt)$$



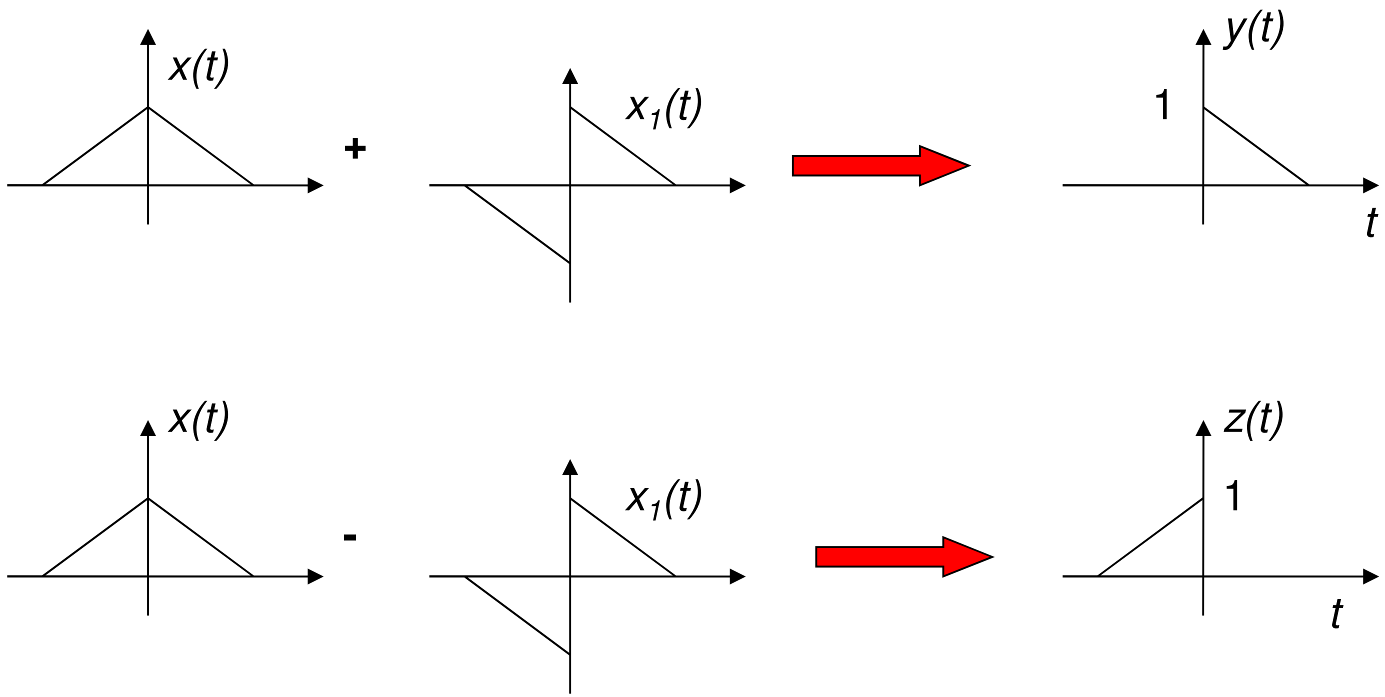


ODD

EVEN

Even and odd signals

Signals can be broken into sums of an even and an odd signal. e.g: $y(t) = x(t) + x_1(t)$ and $z(t) = y(-t) = x(t) - x_1(t)$



Even and odd signals

The even component of a signal $y(t)$ is given by

$$y_{even}(t) = \frac{1}{2}\left[y(t) + y(-t)\right]$$

The odd component of a signal $y(t)$ is given by

$$y_{odd}(t) = \frac{1}{2}\left[y(t) - y(-t)\right]$$

If $y_1(t)$ and $y_2(t)$ are even signals, $y_1(t) \pm y_2(t)$, $y_1(t) \times y_2(t)$ and $y_1(t) \div y_2(t)$ are even signals.

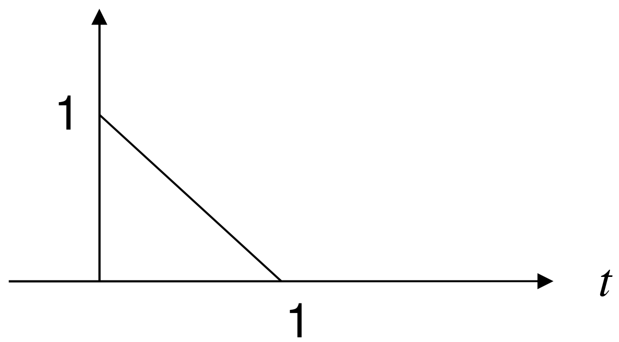
If $y_1(t)$ and $y_2(t)$ are odd signals, $y_1(t) \pm y_2(t)$ is odd but $y_1(t) \times y_2(t)$ and $y_1(t) \div y_2(t)$ are even signals.

Exercise

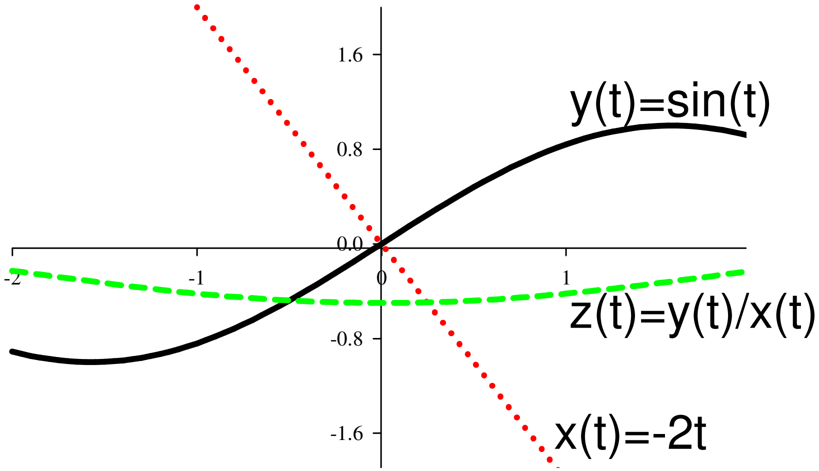
Find the even and odd parts of these functions

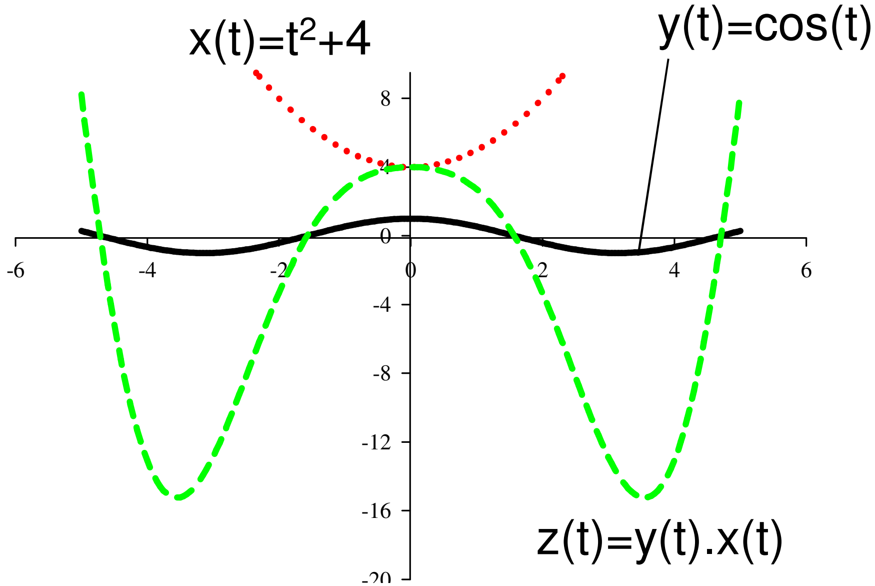
i) $g(t) = t^2 - 2t + 4$

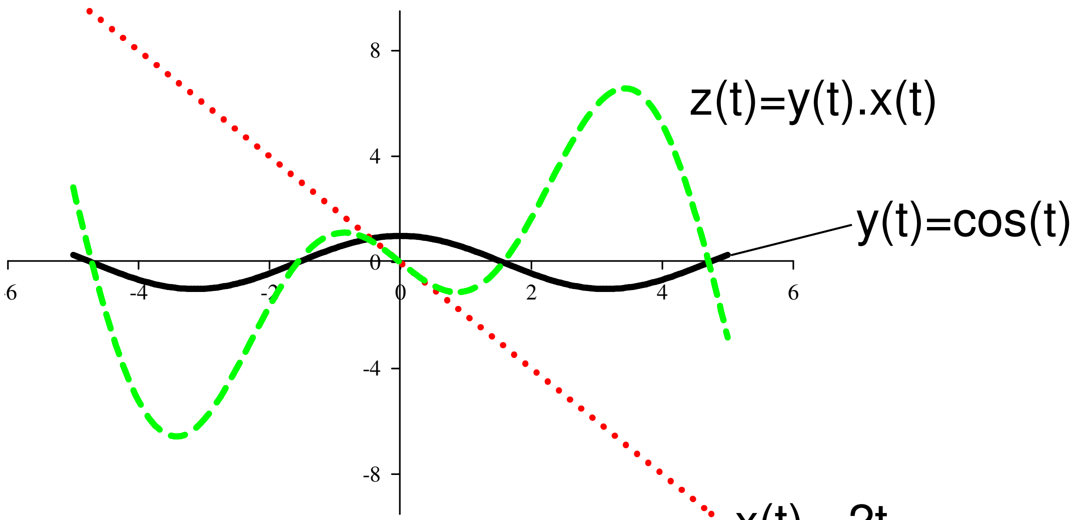
ii) $g(t)$



Even and odd signals







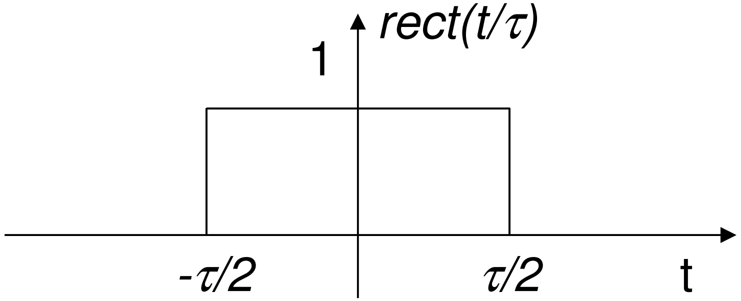
Piecewise continuous signals

Consider,

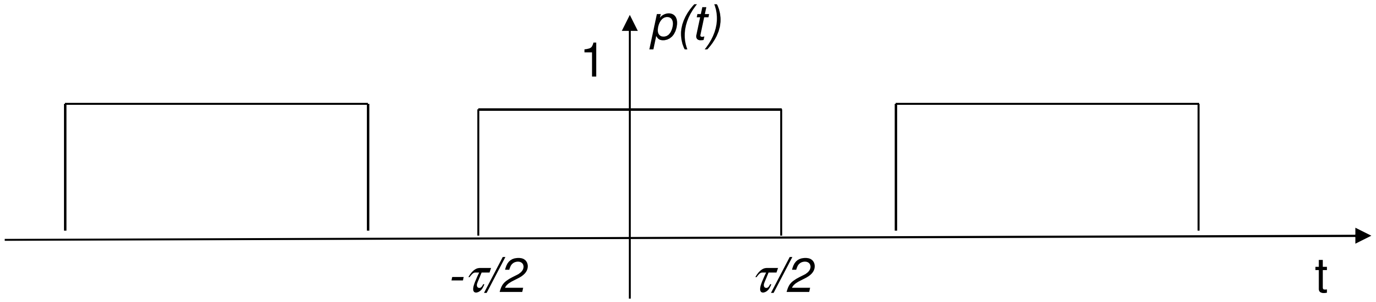
$$rect(t/\tau) = \begin{cases} 1 & \dfrac{-\tau}{2} \le t < \dfrac{\tau}{2} \\ 0 & t < \dfrac{-\tau}{2}, t \ge \dfrac{\tau}{2} \end{cases}$$

where τ is a fixed positive number – the pulse duration.

$rect(t/\tau)$ is discontinuous at $t = \pm \dfrac{\tau}{2}$.

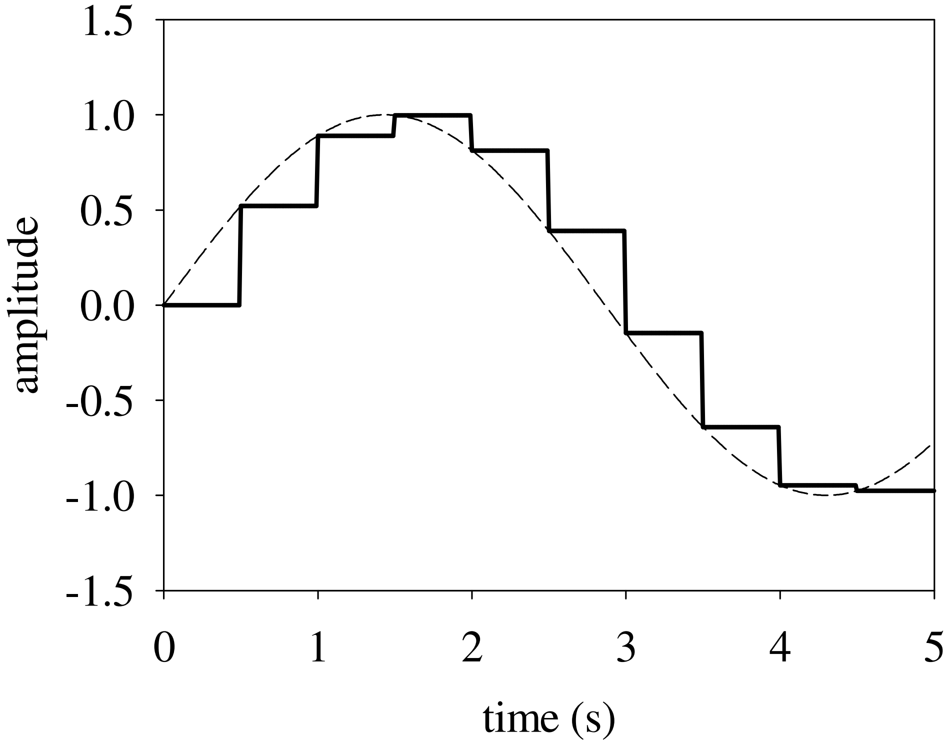


A **piecewise continuous function** is **continuous at all *t* except at a finite collection of *t_1***, e.g: train of pulses.



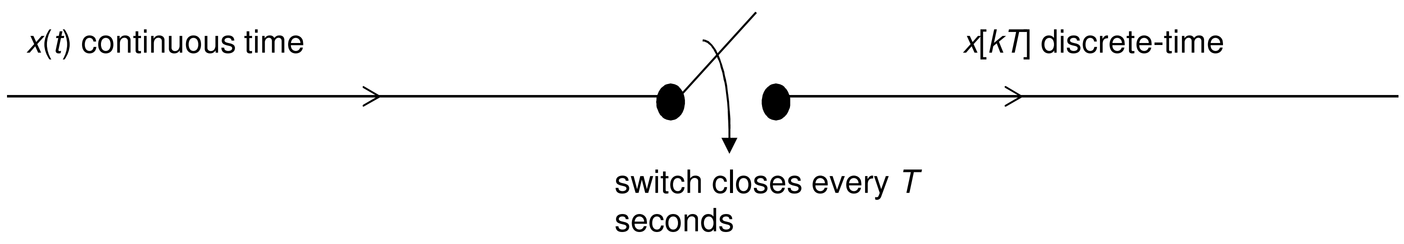
Piecewise continuous signals

A CT signal can be approximated by piecewise-constant or stair-step function.



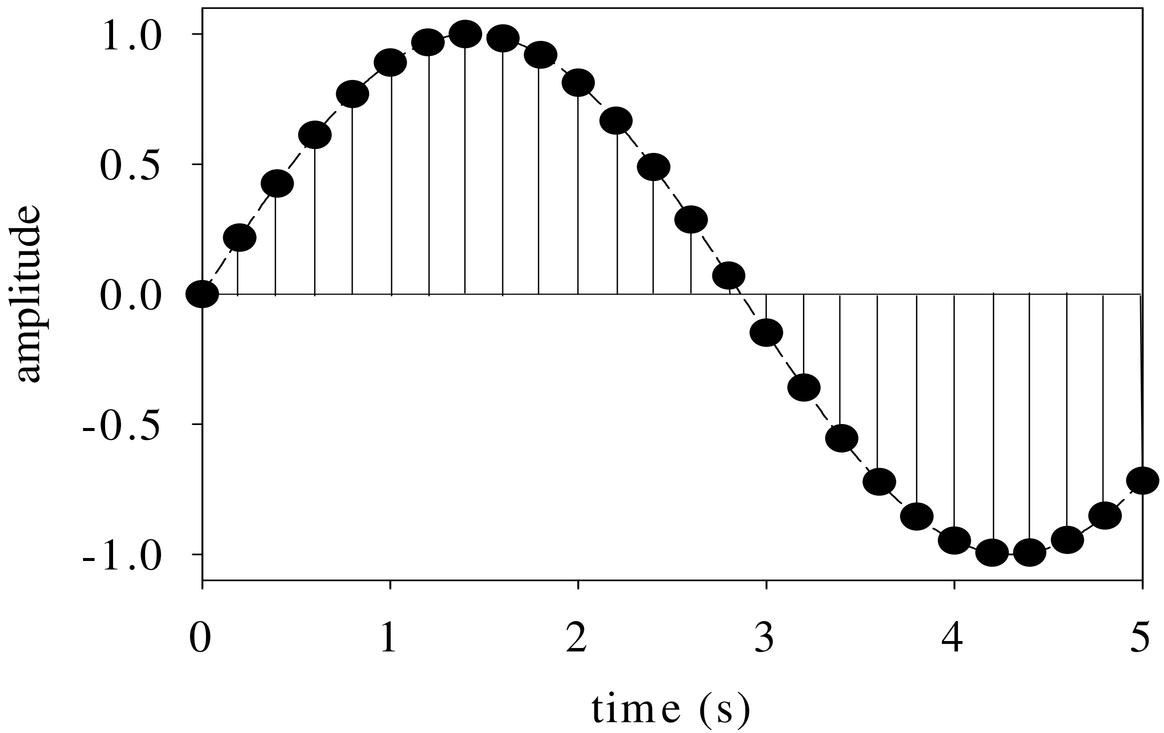
Discrete Time signals

We can make time take on discrete values by only allowing t values to be given by $t = kT$ where k is an integer, T is fixed, positive and real. A discrete signal $x[kT]$ is defined only at kT and has values only at 0, $\pm T$, $\pm 2T$ etc. (use square brackets to remind ourselves we are using discrete-time signals).



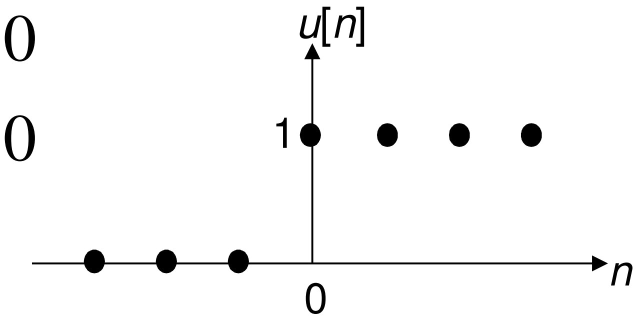
Discrete Time signals

Discrete-time signals often arise from sampling a continuous-time signal (in which case we call it a **sampled** version of the original). In the example below the sinusoidal signal is sampled every 0.2s.



Discrete Time signals

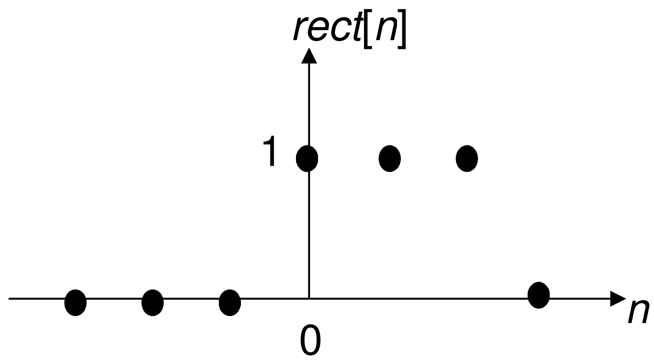
<ins>Unit step function</ins> $u[n] = \begin{matrix} 0 & n < 0 \\ 1 & n \geq 0 \end{matrix}$



$u[n\text{-}N]$ is the delayed unit step.

<ins>The rectangular pulse signal</ins>

$$\begin{aligned} rect[N] &= u[n] - u[n\text{-}N] \\ &= 0,\ n < 0 \\ &= 1,\ 0 \leq n \leq N\text{-}1 \\ &= 0,\ n \geq N \end{aligned}$$

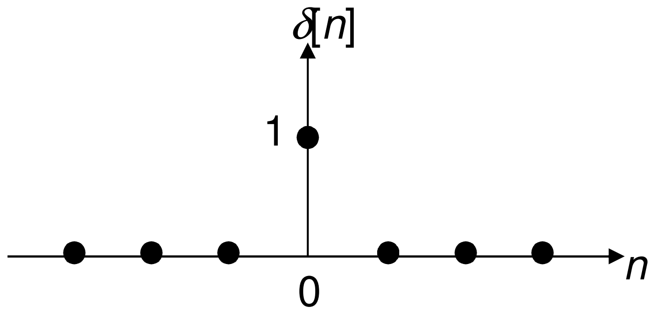


Discrete Time signals

The unit impulse function

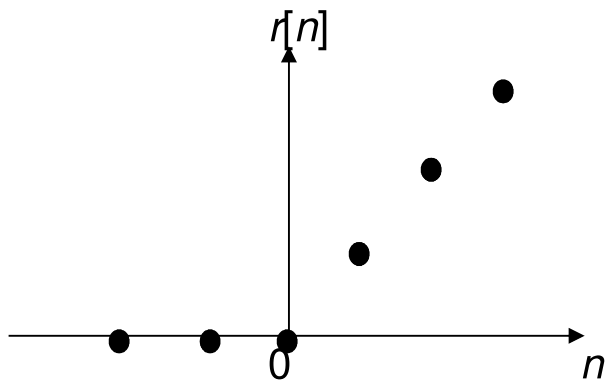
$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

$\delta[n] = u[n] - u[n-1]$



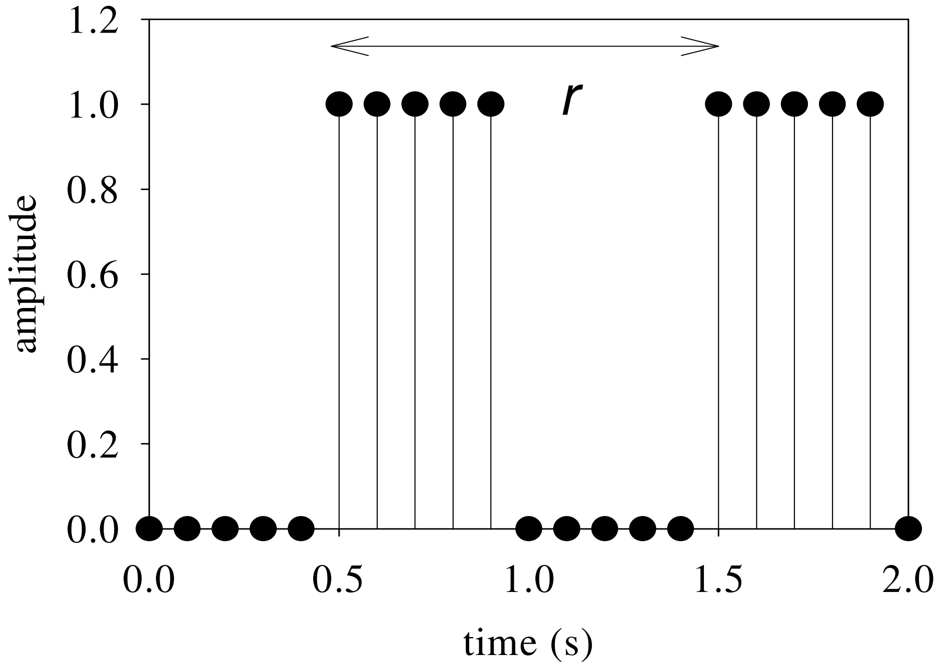
The unit ramp function

$r[n] = nu[n]$ i.e n times unit step value.



Discrete Time signals

A DT signal $x[n]$ is periodic if $x[n] = x[n+r]$ for all integers n and r is a positive integer



DT signals and CT signals have many similarities such as the amplitude and time scaling, time shift and addition and multiplication operations of signals. DT exponentials and sinusoidal signals can be obtained from their CT equivalent by sampling.