Modelling of Machines

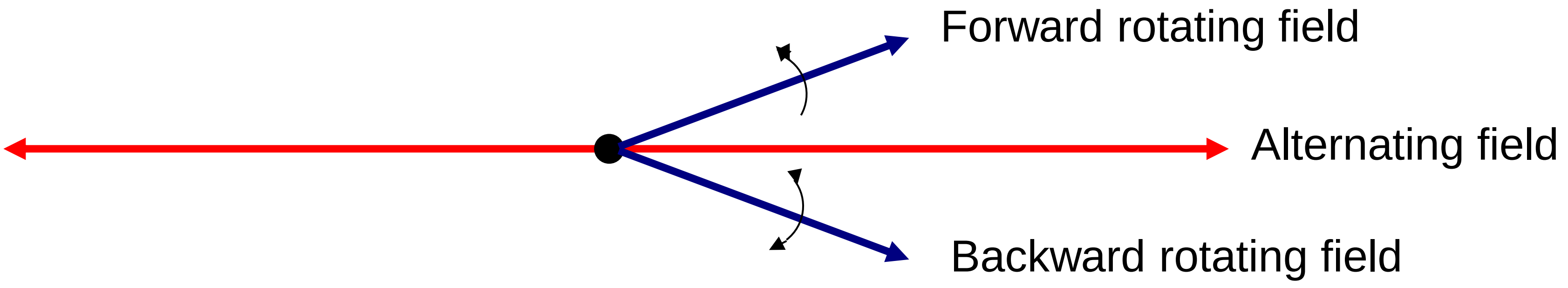
Section 10

Single-phase induction machines

- Three-phase induction machines are the dominant machine type for industrial drives from ~2kW to 20MW
- However, they require a 3-phase supply and so are not suitable for domestic applications nor necessarily cost-effective for low power, light industrial facilities
- Hence, single-phase induction machines which operate from a normal domestic AC mains socket are widely used with powers up to 3kW or so
- Often preferred to universal motors in higher power applications (>750W or so) or continuous running type applications (e.g. central heating pumps) where brush wear in universal machines would be a problem

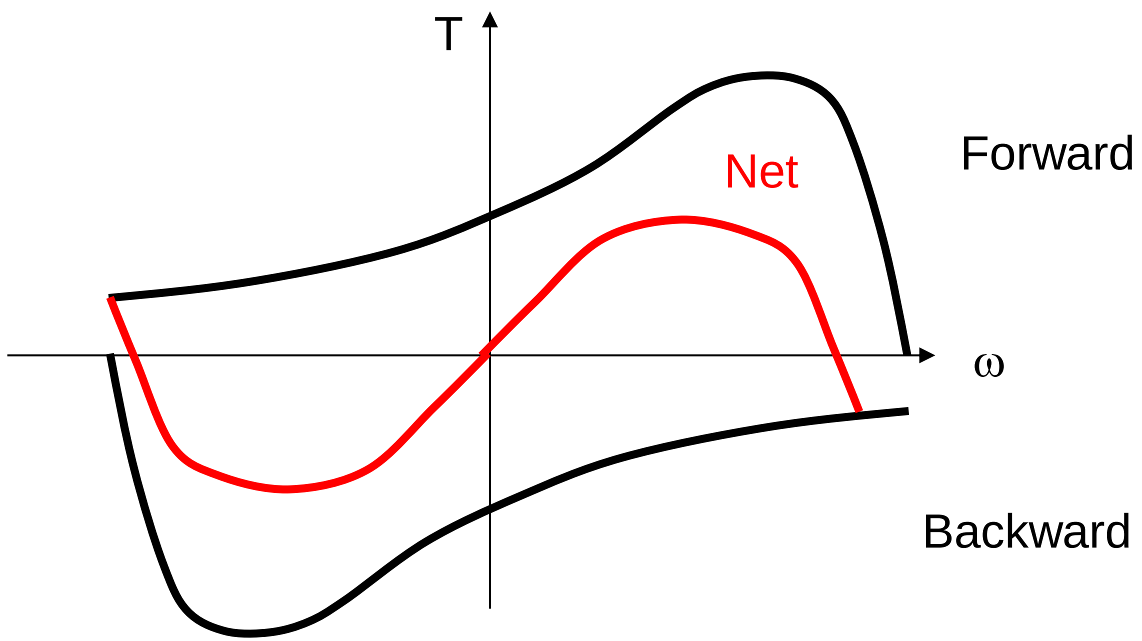
Stator field

- Very similar operating mechanism to a 3-phase machine (i.e. currents induced in short-circuited rotor) except that the single-phase stator winding does not produce a rotating field
- A single phase stator produces an alternating field along one axis rather than a rotating field
- However, this can be resolved into two contra-rotating fields



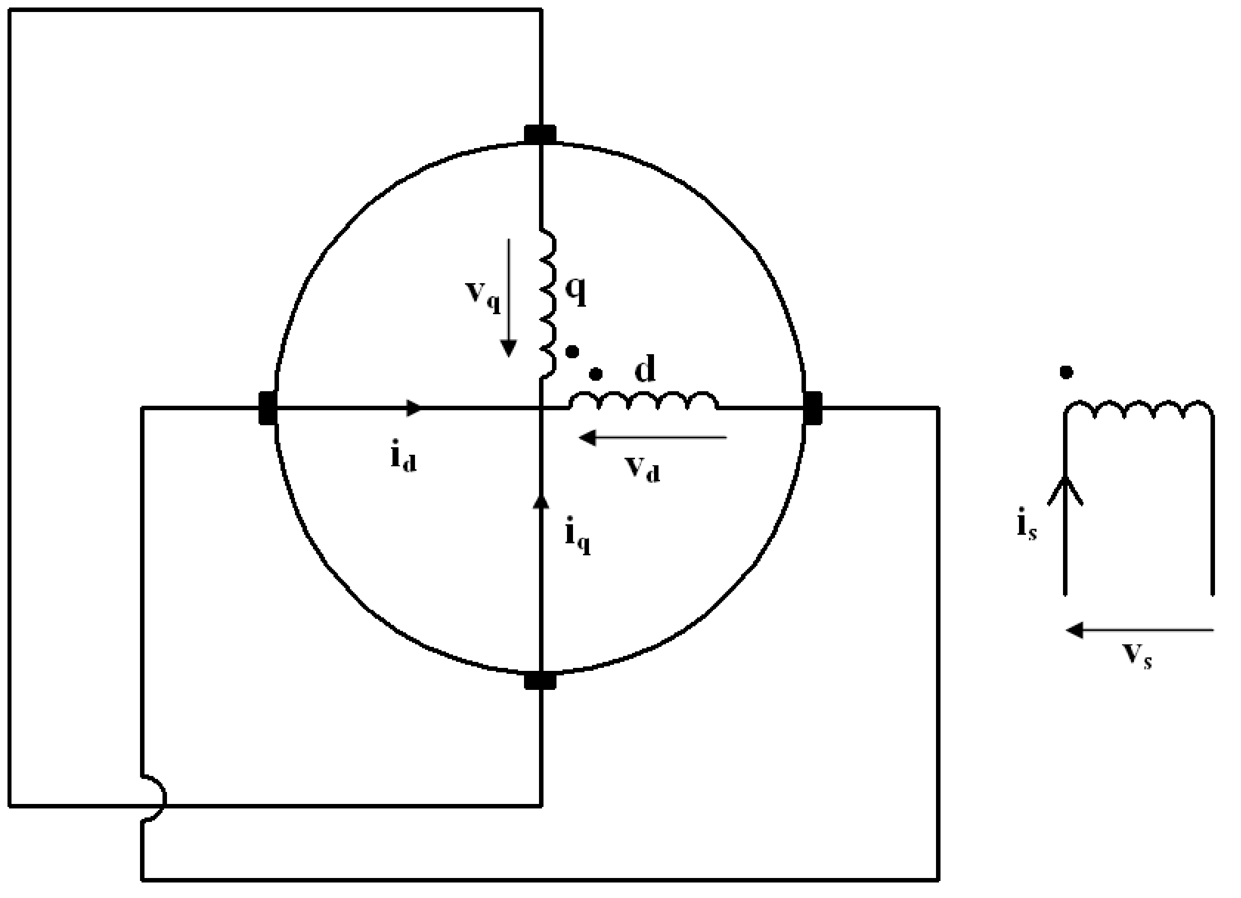
Stator field

- Each of the two contra-rotating fields individually produces a torque-speed characteristic which would be similar to that of a three-phase machine.
- Hence, net torque-speed characteristic is the sum:



•Hence, there is a net torque once the machine is rotating in one direction or the other but no starting torque

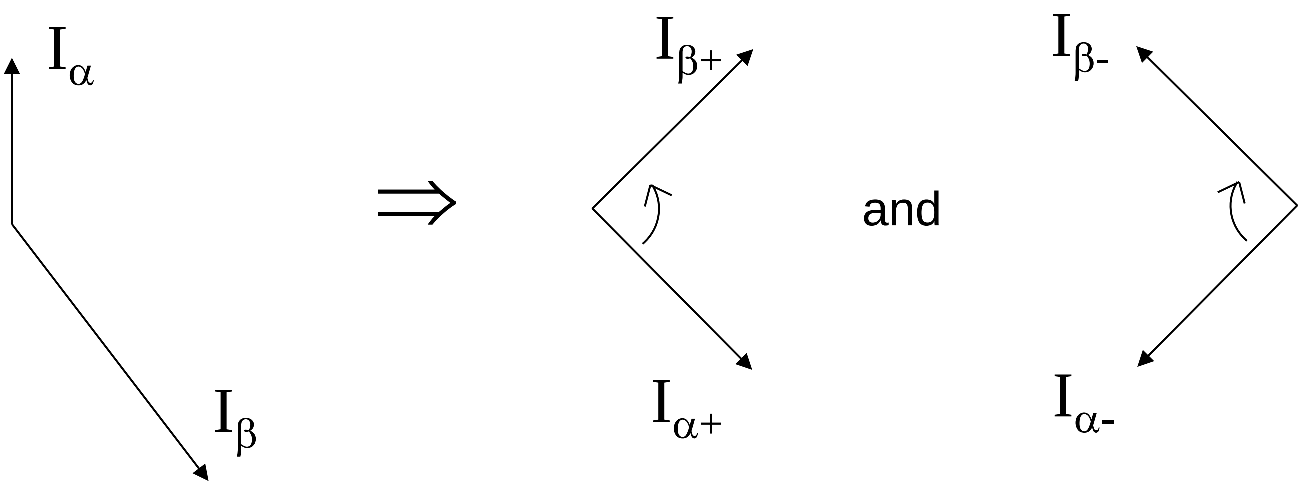
Kron primitive equivalent



(Rotor has same basic configuration as that of a 3-phase induction machine)

Symmetrical component theory

- The two-phase rotor currents in the rotor of a single phase induction machine are **<u>unbalanced</u>**
- Hence, In order to analyse single-phase induction machines it is necessary to introduce so-called ‘symmetrical component’ transformation
- This is a widely used tool for transforming unbalanced currents to a balanced system of so-called positive and negative sequence currents



Returning to the Kron primitive, the general form of the voltage matrix equations is:

$$\begin{vmatrix} v_s \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_s + L_s p & M_{sd}p & 0 \\ M_{ds}p & R_d+L_d p & -\omega_r L_q \\ \omega_r M_{qs} & \omega_r L_d & R_q+L_q p \end{vmatrix} \begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix}$$

For steady-state sinusoidal AC operation, $p=j\omega_s$ and $\omega_r=(1-s)\omega_s$

The short-circuited rotor windings dictate that $V_q=V_d = 0$

Adopting subscripts 1 for stator and 2 for rotor, yields

$$\begin{vmatrix} V_s \\ 0 \\ 0 \end{vmatrix} = \underbrace{\begin{vmatrix} R_1+jX_1 & jX_m & 0 \\ jX_m & R_2+jX_2 & -(1-s)X_2 \\ (1-s)X_m & (1-s)X_2 & R_2+jX_2 \end{vmatrix}}_{\text{Z – impedance matrix}} \begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix}$$

The unbalanced currents on the right hand side can be transformed to symmetrical components:

$$\begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix} = C \begin{vmatrix} I_s \\ I_p \\ I_n \end{vmatrix}$$

Where C is the zero sequence transformation matrix. In order to the transformation matrix on the impedance matrix it is necessary to also use Ct* which is the transpose of the complex conjugate of C (effectively swap rows and columns and change signs of all imaginary terms).

$$C = 1/\sqrt{2} \begin{vmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & j & -j \end{vmatrix} \quad \text{and} \quad C_t^* = 1/\sqrt{2} \begin{vmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & -j \\ 0 & 1 & j \end{vmatrix}$$

Note – both these transformations leave I_s unchanged (simply multiply by √2/√2)

The impedance matrix Z can be transformed to a modified impedance matrix Z’ using: Z’ = C_t* Z C

$$Z' = 1/\sqrt{2}\begin{vmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & -j \\ 0 & 1 & j \end{vmatrix} \begin{vmatrix} R_1+jX_1 & jX_m & 0 \\ jX_m & R_2+jX_2 & -(1-s)X_2 \\ (1-s)X_m & (1-s)X_2 & R_2+jX_2 \end{vmatrix} 1/\sqrt{2}\begin{vmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & j & -j \end{vmatrix}$$

Multiplying out the first pair of matrices, i.e. C_t* and Z, yields :

$$Z' = 1/2\begin{vmatrix} \sqrt{2}(R_1+jX_{1)} & \sqrt{2}jX_m & 0 \\ jX_m - j(1-s)X_m & R_2+jX_2-j(1-s)X_2 & -(1-s)X_2-j(R_2+jX_2) \\ jX_m+j(1-s)X_m & R_2+jX_2+j(1-s)X_2 & -(1-s)X_2+j(R_2+jX_2) \end{vmatrix} \begin{vmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & j & -j \end{vmatrix}$$

Multiplying out the remaining matrices yields:

$$Z' = 1/2\begin{vmatrix} 2(R_1+jX_1) & \sqrt{2}jX_m & \sqrt{2}jX_m \\ \sqrt{2}j\ s\ X_m & \begin{matrix} R_2+jX_2-j(1-s)X_2- \\ j^2(R_2+jX_2)-j(1-s)X_2 \end{matrix} & \begin{matrix} R_2+jX_2-j(1-s)X_2+j(1- \\ s)X_2+j^2(R_2+jX_2) \end{matrix} \\ \sqrt{2}(jX_m+j(1-s)X_m) & \begin{matrix} R_2+jX_2+j(1-s)X_2-j(1- \\ s)X_2+j^2(R_2+jX_2) \end{matrix} & \begin{matrix} R_2+jX_2+j(1-s)X_2+j(1- \\ s)X_2-j^2(R_2+jX_2) \end{matrix} \end{vmatrix}$$

This impedance matrix can be greatly simplified by noting the many cancellations of various terms (noting that $j^2 = -1$). This tidying up process yields:

$$Z' \quad = 1/2 \begin{vmatrix} 2(R_1+jX_1) & \sqrt{2}jX_m & \sqrt{2}jX_m \\ \sqrt{2}j\,s\,X_m & 2(R_2+jsX_2) & 0 \\ \sqrt{2}j(2-s)X_m & 0 & 2(R_2+j(2-s)X_2) \end{vmatrix}$$

Hence, substituting back from Z' into the voltage equations and multiplying both sides gives:

$$\begin{vmatrix} V_s \\ 0 \\ 0 \end{vmatrix} = 1/2 \begin{vmatrix} 2(R_1+jX_1) & \sqrt{2}jX_m & \sqrt{2}jX_m \\ \sqrt{2}j\,s\,X_m & 2(R_2+jsX_2) & 0 \\ \sqrt{2}j(2-s)X_m & 0 & 2(R_2+j(2-s)X_2) \end{vmatrix} \begin{vmatrix} I_s \\ I_p \\ I_n \end{vmatrix}$$

Dividing row 2 by √2s throughout and row 3 by √2s(2-s) yields:

$$
\begin{vmatrix} 2V_s \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 2(R_1+jX_1) & \sqrt{2}jX_m & \sqrt{2}jX_m \\ jX_m & R_2/s+jX_2 & 0 \\ jX_m & 0 & R_2/(2-s)+jX_2 \end{vmatrix} \begin{vmatrix} I_s \\ \sqrt{2}\, I_p \\ \sqrt{2}\, I_n \end{vmatrix}
$$