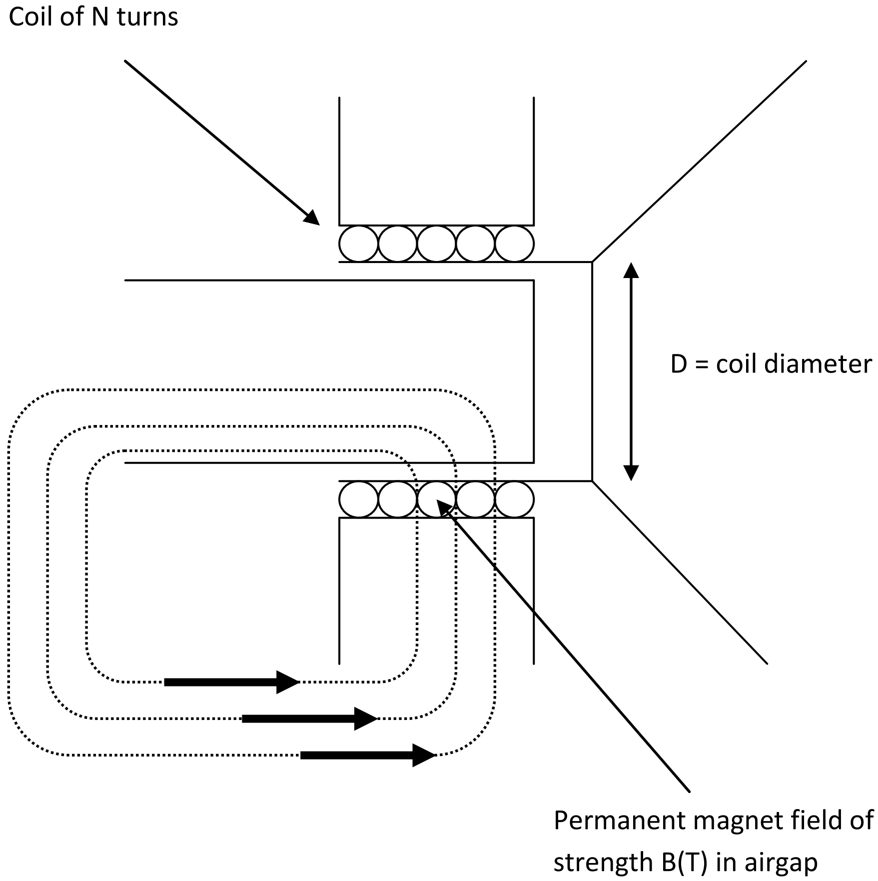
EEE202 Answers – Summer 2012-02-22

Question 1



a.
Length of interaction of coil with the magnetic field = coil circumference = πD

Thus;

$$F = BI\pi DN$$ where N is the number of series turns

$$F = (BN\pi D)I$$

or

$$F = K_e I$$ where K_e is the electromagnetic or force constant of the system

The induced voltage in the coil as a result of the motion is given as

$$E = K_e v$$ where v is the velocity of motion

And finally

$$EI = Fv$$ which gives the power balance across the electromechanical interface

b. Mass of cone and the air that it moves

$$F = Ma = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$

Substituting for F and v

$$K_eI = M\frac{d\left(E/K_e\right)}{dt}$$

$$I = \frac{M}{K_e^2}\frac{dE}{dt}$$

Thus the electrical equivalent can be given as

$$I = C\frac{dV}{dt}$$ i.e, mass is represented as a capacitance $C = \frac{M}{K_e^2}$

Spring

$$F = \sigma_s x$$

spring compliance

$$K_eI = \sigma_s\int v.dt = \sigma_s\int\left(\frac{E}{K_e}\right).dt$$

$$I = \frac{\sigma_s}{K_e^2}\int E.dt$$ or…. $$E = \frac{K_e^2}{\sigma_s}\frac{dI}{dt}$$

Giving the electrical equivalent of an inductor $L = \frac{K_e^2}{\sigma_s}$

$$V = L\frac{dI}{dt}$$

$$I = \frac{1}{L}\int V.dt$$

Mechanical loss and damping

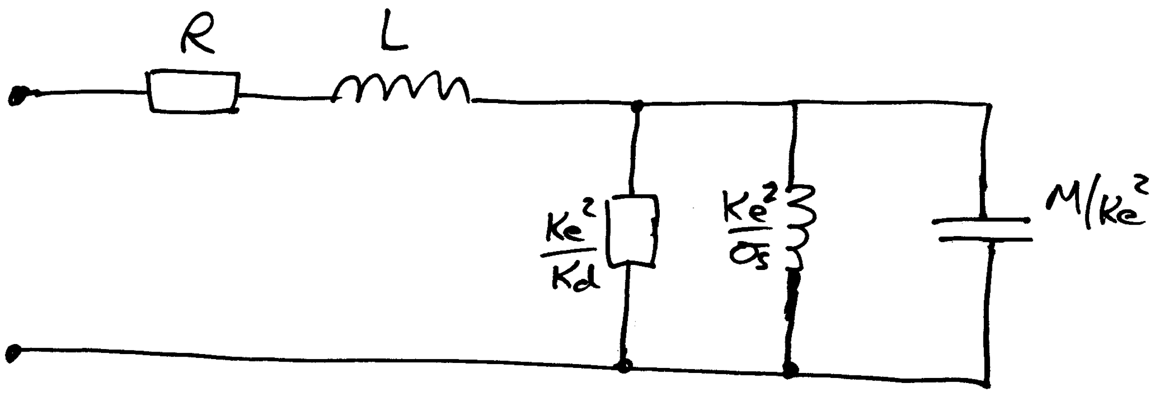
$$F = K_d v$$ where K_d is the damping coefficient

$$K_e I = K_d \left(\frac{E}{K_e} \right)$$

$$\frac{K_e^2}{K_d} I = E$$

which is analogous to a resistor, $\frac{K_e^2}{K_d} = R$

In forming the complete electrical analogue, the connection of the components needs to be considered, since force is proportional to current and the force may go one of three ways, the components are connected in parallel, with the mechanical section being in series with the electrical components



The values for the mechanical components are:

Resistor = to be ignored

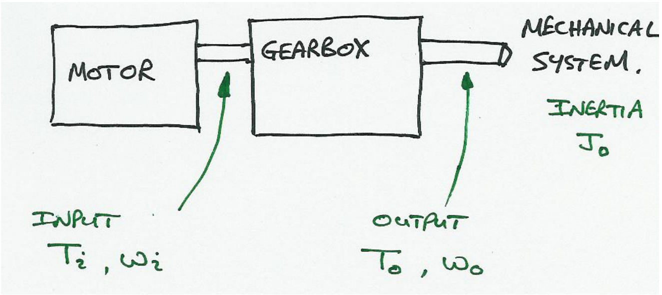
Inductor = 157.9mH

Capacitor = 158uF

c. Mechanical resonant frequency from the component values is = 31.83Hz

Question 2

a. Gearbox has a step-down ratio, R



$\omega_i = R \times \omega_o$

$T_o = R \times T_i$

Assuming a lossless system where:

$\omega_i \times T_i = \omega_o \times T_o$

The inertia the motor 'see's' through the gearbox:

Referred inertia = J_o'

From an energy balance:

$0.5\omega_i^2 J_o' = 0.5\omega_o^2 J_o$

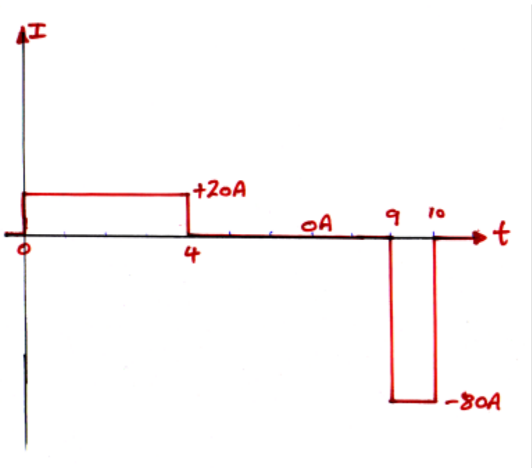
Therefore:

$$J_o' = \left(\frac{\omega_o}{\omega_i}\right)^2 .J_o$$

$$= \frac{1}{R^2}.J_o$$

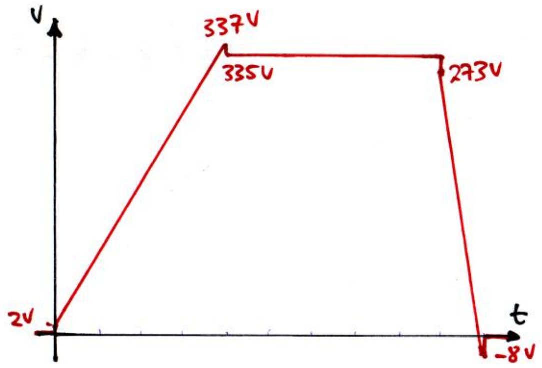
b. If the load inertia is 0.1kgm, then referring it through the gearbox gives 0.4kgm, plus the motor inertia of 0.2kgm = 0.6kgm in total. As the motor speed = load speed/2 = 2000rpm, and the motor accelerates in 4 seconds, the acceleration is 52.35rads^{-2}. From this, and the inertia, the required torque is 31.41Nm, which equates to 20A with a motor constant of 1.6Nm/A

c. If the deceleration is 4 times that of the acceleration, the required current is -80A. From this, over a 10 second cycle time, the current profile becomes:



d. The supply voltage is given from ohms law and the back emf of the motor.

20A @ 0.1Ω = 2V, 80A = 8V. The voltage requirement of the supply is 2000rpm x 2π / 60 x 1.6 = 335 + 2V = 337V dc



e. Power in the motor windings is from I^2R at 20A for 4 sec and 80A for 1 sec averaged over a 10sec cycle. = 80W total

f. Problem if motor connected straight to a 337V supply at standstill is the current will be limited by R = 0.1Ω, with zero back emf. Therefore the current aims for to 3370A. The high current could blow the fuses or demagnetise the motor

Question 3

a) As below:

R_1 = STATOR RESISTANCE PER PHASE

R_2' = REFERRED ROTOR RESISTANCE / ϕ

X_1 = STATOR LEAKAGE REACTANCE / ϕ

X_2' = REFERRED ROTOR LEAKAGE REACTANCE / ϕ

X_m = MAGNETIZING REACTANCE / ϕ

R_m = IRON LOSS RESISTANCE / ϕ (49)

V_1 = RMS SUPPLY PHASE VOLTAGE / ϕ

E_1 = INDUCED STATOR PHASE VOLTAGE

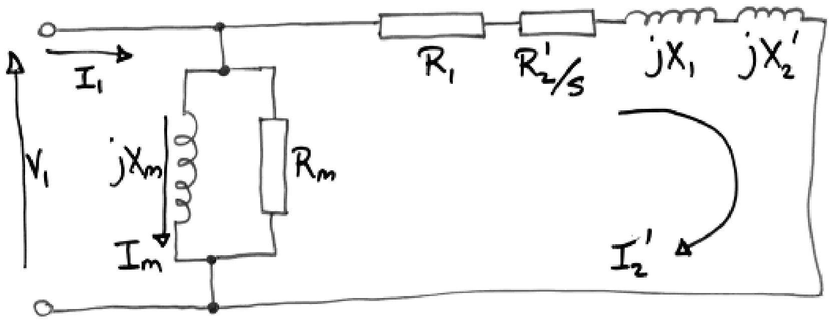
I_2' = REFERRED ROTOR CURRENT

I_m = MAGNETIZING CURRENT

I_1 = STATOR CURRENT.

APPROXIMATE EQUIVALENT CIRCUIT

ASSUME $E_1 = V_1$ ACCURACY 1-2%
FOR A TYPICAL MACHINE



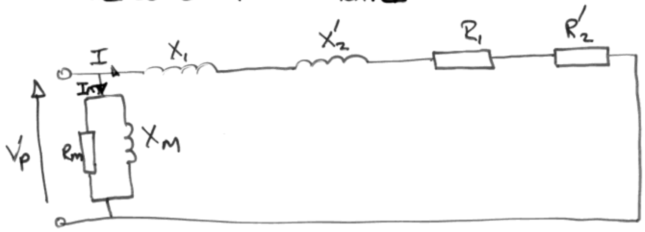
The locked rotor test is:

(12)

LOCKED ROTOR TEST

REDUCED VOLTAGE APPLIED TO INDUCTION MOTOR WITH ROTOR LOCKED TO PREVENT ROTATION.

MEASURE V_1 AT I_{RATED}



USUALLY $I_m \ll I$ as $Z_m \gg Z$

$\therefore$ IGNORE THE MAGNETIZING BRANCH

MEASURE INPUT POWER

$\Rightarrow R_1 + R_2' = \dfrac{P_{LR}}{3I^2}$ $\qquad R_1$ MEASURED

$\therefore R_2'$ CALCULATED

ALSO $\dfrac{V_p}{I} = \sqrt{X_T^2 + R_T^2}$ $\qquad$ WHERE $R_T = (R_1 + R_2')$

MAY THEN FIND X_T', CAN THEN USE GIVEN EQUATIONS TO FIND MAX PULL-OUT TORQUE AT GIVEN VOLTAGE FOR MAX LOAD

b) As V_L = 80V, I = 20A, and P=2kW with R_1=0.4Ω, P=3, then:

R_2' = 1.27Ω,

Also, X_T = 1.6Ω

From these values, the pull-out torque is

$$T_{PULL-OUT} = \frac{3p V_1^2}{2\pi f_1} \frac{\sqrt{R_1^2 + (X_1 + X_2')^2}}{\left(R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2}\right)^2 + (X_1 + X_2')^2}$$

Therefore with a 20% reduction of supply voltage, the line voltage is 332V

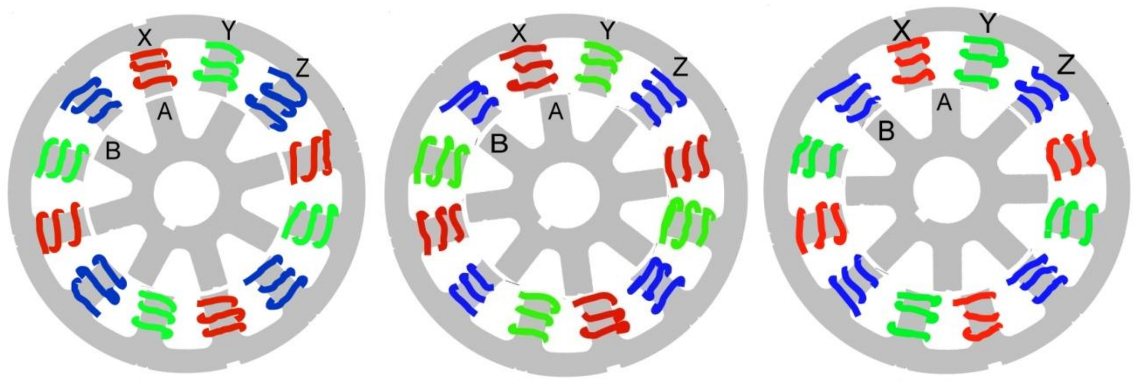
Giving a pull-out torque of $T_{Pull\ out}$ = 756.5Nm. If the load torque is lower than this, the machine will not pull out under the worst case voltage of a 20% dip in line voltage.

Qu 4.

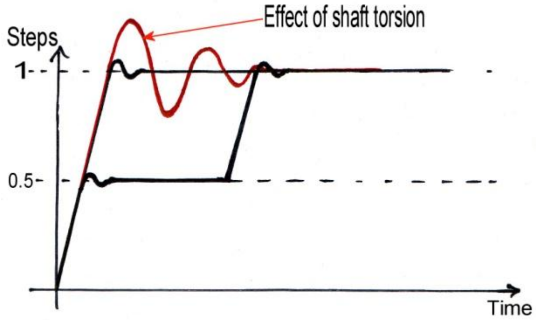
a. If an increase in the positional accuracy of the load connected to the motor is required, over that provided by full stepping, then more steps per rev are required from the motor. In this case it may be possible to operate the motor in half-step mode. The coils are now switched using the following sequence:

X	XZ	Z	ZY	Y	YX	X	XZ	Z	

Phase sequence



At the point where **Both** coils are switched on the rotor aligns to minimise the reluctance in **Both** magnetic paths. A stable position will be reaches when the rotor is aligned half way between full alignments with either phase. Operation in this mode allows twice the positional resolution to be obtained from the motor.

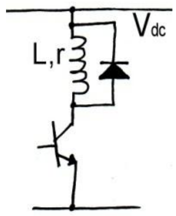


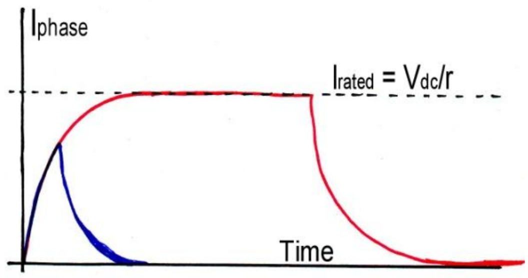
If the load is connected to the motor by a shaft which may be subject to torsion, especially if the load has a high inertia, there may be oscillations present on the load position. The use of half stepping will produce a smaller movement per step, and hence may reduce the load positional oscillations.

Advantages of Half stepping:

- Greater accuracy of load positioning
- Less likely to excite instabilities due to torque impulse exciting mechanical resonances.

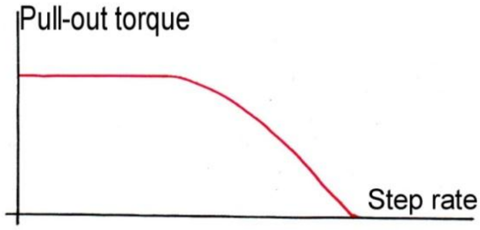
b. The student has a problem with the simple drive he is using. The simplest power drive per phase is given by a uni-polar constant voltage system:





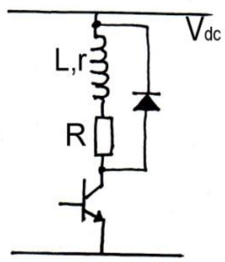
Voltage V_{dc} is chosen such that the rated current flows in the winding i.e. $V_{dc} = r \times I_{rated}$

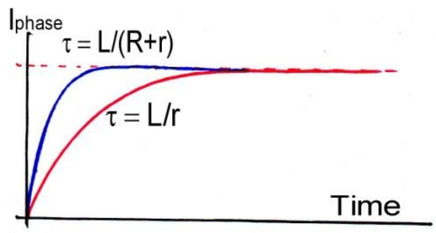
The main disadvantage with this system is the rate of rise of current in the winding. This is governed by the winding time constant ($\tau = L/r$). This is not a problem at low step rates (the winding current has plenty of time to reach its rated value). At high step rates, the current in the winding may not have time to rise to its rated value before the phase is turned off, therefore the torque will be less than the rated value.



Two main ways of overcoming the problem:

1) As the problem is related to the time constant of the winding, decreasing the time constant of the winding will increase the available torque at higher step rates. This can be achieved by changing the series resistance of the coil, and suitable changes to the applied dc voltage need to be carried out to ensure the current reaches the rated value in steady state.



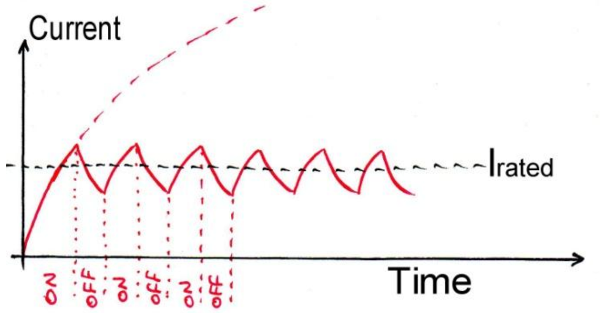


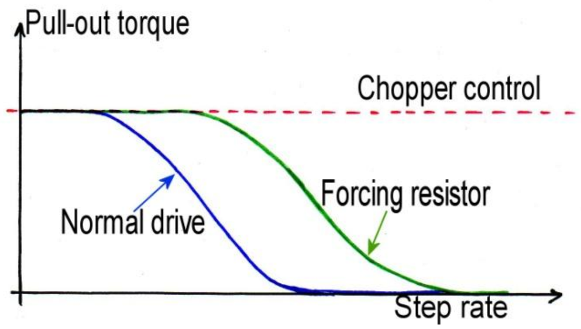
'R' may be referred to as a 'forcing' resistor. This resistor dissipates a large amount of power, however, the time constant of the circuit is reduced.

Normal we require $\tau < 1 / (10 \times \text{step frequency})$.

2) Increase V_{dc} to give a much greater steady-state current than the rated value, then monitor current and use a chopper action to limit the current to a rated value.

This allows a fast rise-time within the current range of interest, however current control is required to keep the current around the rated value.





The student should therefore consider using either a forcing resistor and a higher voltage, or a chopper drive for his robot arm.

c. The disadvantage of the variable reluctance stepper motor is the fact that there is no holding torque produced by the motor once the phases are switched off, as there is no permanent magnet flux in the system. The problem by be overcome by adding a permanent magnet along the axis of the stepper motor to give a magnetic bias to the system. This leads to a topology known as a hybrid stepper motor, which inherently exhibits a holding torque due to the permanent magnet.