QUESTION 1

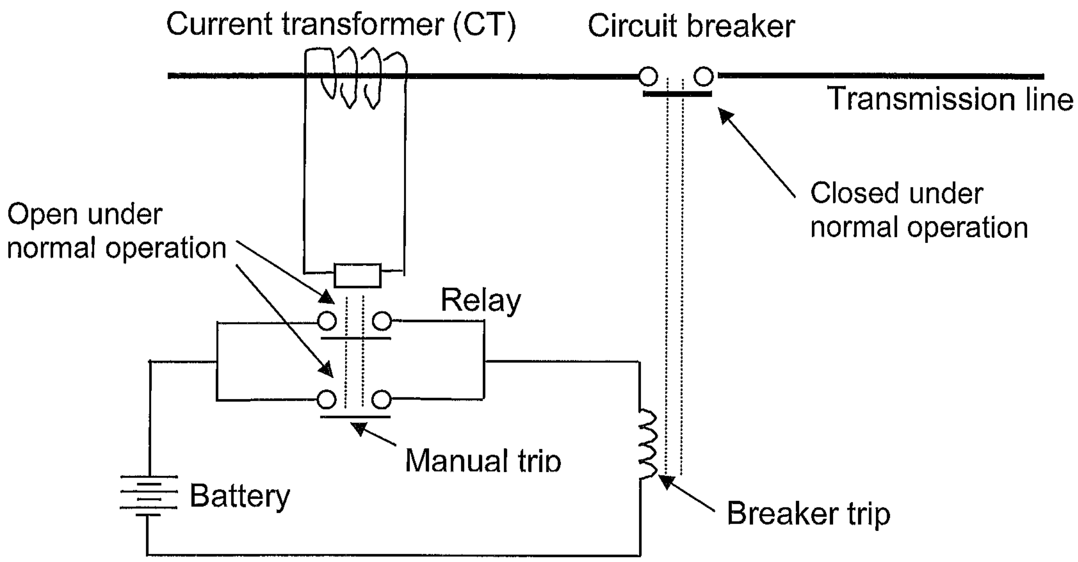
Taken from notes. Students would not necessarily be expected to go into quite as much detail, but they need to mention the key points in each section for full marks.

(a)

(i) Protection is needed to save personnel from risk of electrocution and to help prevent risk of fire or explosion and damage to plant.

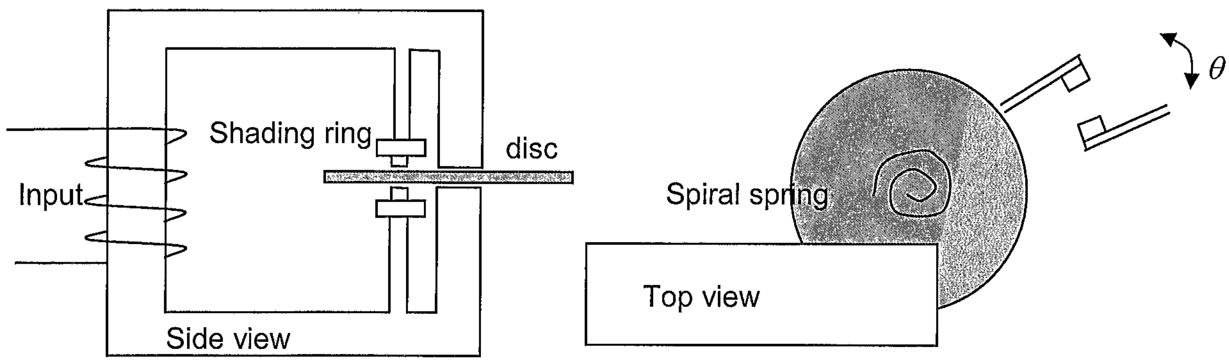
(ii) Protection systems are made up of three key components:

- Instrument transformer
- Relay
- Circuit breaker



(b) Induction relays

In an induction relay eddy currents are induced in a conducting rotor (disc), which in turn produced a flux which interacts with the stator flux to develop a torque (Similar to induction motor operation). If the input current exceeds the pickup current, the disc rotates through an angle θ to close the relay contacts. The larger the input current the faster the contact closes. After the current is removed or reduced below the pickup the spring provides resets of the contacts.



A permanent magnet may be used to produce a breaking torque. This configuration results in the operating time being dependent on operating current and distance that the disc is required to travel before closing the relay contacts. Consequently the relay exhibits an inverse definite minimum time (IDMT) characteristic – the higher the current above pickup the faster the relay will operate.

(c) Tap changing transformers are used to compensate for varying voltage drops in the system caused by changing loads and also to control reactive power flow.

An on-load tap changing transformer can alter its voltage ratio either automatically or manually whilst carrying the load current.

An off-load tap changing transformer needs to be disconnected and isolated before the taps can be changed which can cause a disruption to the supply.

Off nominal turns ratios occur when there are numerous transformers and the rated turns ratio of the transformer is not the same as the required system base; eg:-

- A transformer with taps with at least one value of voltage changing with tap settings.
- Two different transformers are connected in parallel with different ratings.

(d) Consider the no-load voltage transformation:

Primary to secondary (supply primary, sec. open circuit)

$$V_H = I(B+C) \qquad V_L = I.B$$

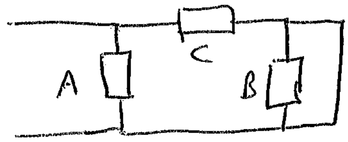
$$\text{Since } k = \frac{V_H}{V_L} = \frac{B+C}{B} \Rightarrow \frac{1}{k} = \frac{B}{B+C} \qquad (1)$$

Secondary to primary (supply secondary, primary open circuit)

$$V_L = I'(A+C) \qquad V_H = I'A$$

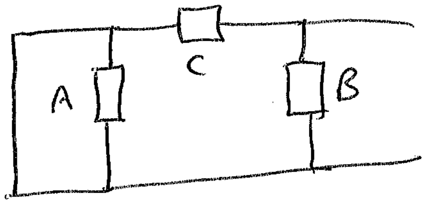
$$\therefore k = \frac{V_H}{V_L} = \frac{A}{A+C} \qquad (2)$$

Consider the short circuit impedance. Short low voltage side:



$$Z_H = \frac{AC}{A+C} \quad (3)$$

Short ~~high~~ voltage side:



$$Z_L = \frac{BC}{B+C} \quad \text{but } Z_L = \frac{Z_H}{k^2}$$

$$\therefore \frac{Z_H}{k^2} = \frac{BC}{B+C} \quad (4)$$

From (1) and (4)

$$\frac{Z_H}{k^2} = \frac{C}{k} \Rightarrow C = \frac{Z_H}{k}$$

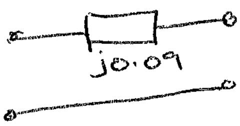
Hence from (2)

$$k = \frac{A}{A + \frac{Z_H}{k}} \Rightarrow kA = k^2 A + kZ_H \Rightarrow A = \frac{Z_H}{(1-k)}$$

Similarly from (1)

$$\frac{1}{k} = \frac{B}{B + \frac{Z_H}{k}} \Rightarrow kB + Z_H = k^2 B \Rightarrow B = \frac{Z_H}{k(k-1)}$$

(e) On the nominal tap $A = B = \infty$, $C = j0.09$ and $k = 1$



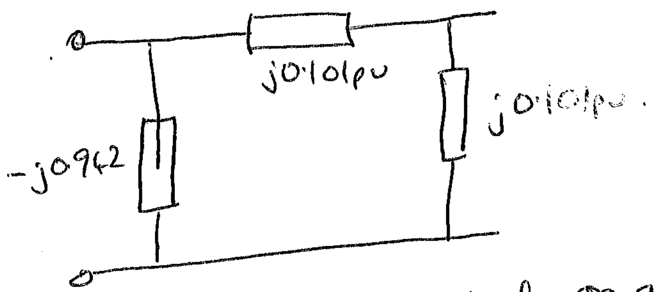
On the +12% tap: $R = 1.12$

$$Z_{HL} = 1.12^2 \times j0.09 = j0.113\,pu$$

$$A = \frac{Z_{HL}}{1-R} = \frac{j0.113}{1-1.12} = -j0.942\,pu$$

$$B = \frac{Z_{HL}}{R(R-1)} = \frac{j0.113}{1.12 \times 0.12} = j0.841\,pu$$

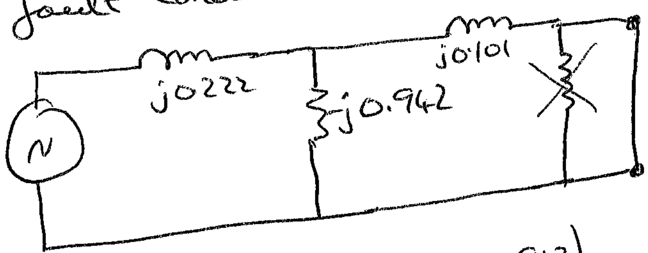
$$C = \frac{Z_{HL}}{R} = \frac{j0.113}{1.12} = j0.101\,pu.$$



(f) The system can be represented on a 200 MVA base;

$$Z_{sys} = \frac{1.0 \times 200}{900} = j0.222\,pu$$

Under fault conditions on +12% tap:



$$Z_{TOT} = j0.222 + \frac{(j0.101 \times -j0.942)}{(j0.101 - j0.942)} = j0.335$$

$$I_F = \frac{1}{j0.335} = -j2.985$$

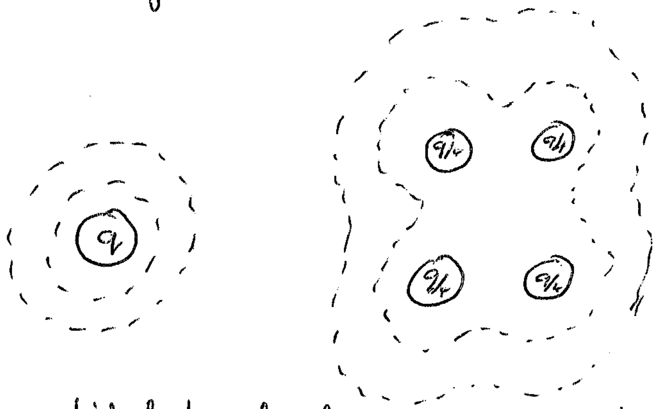
$$I_{BASE} = \frac{200 \times 10^6}{\sqrt{3} \times 275 \times 10^3} = 420A \Rightarrow I_{FAULT} = 420 \times 2.985$$

$$= \underline{\underline{1254A}}$$

(a) (i) Reduced line reactance (since the geometric mean radius < r)
Therefore reduced reactive voltage drop and increased transmission capacity

$$P_{Max} = \frac{V_S V_R}{X}$$

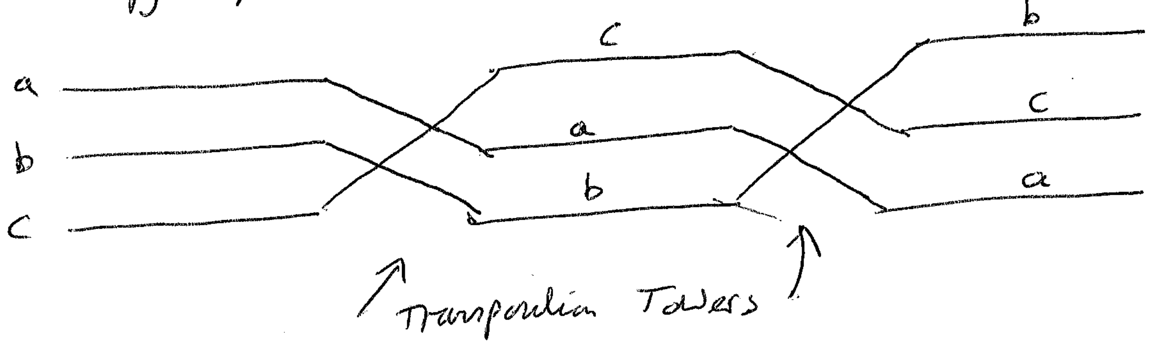
Reduced electric field (voltage gradient) compared with equivalent single conductor.



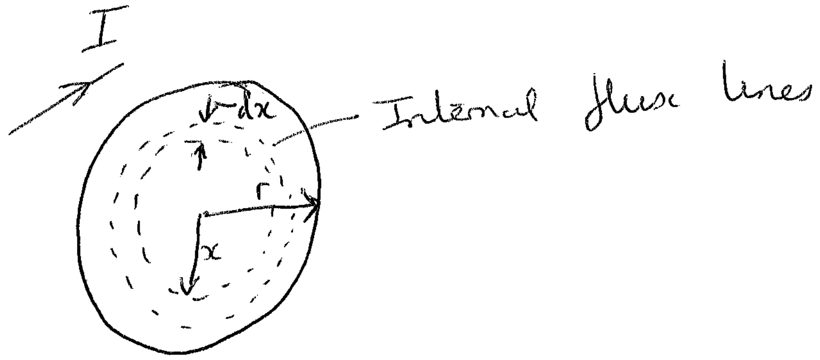
Effective radius increased therefore surface E reduced.

Reduces likelyhood of corona discharge.
Reduced skin effect compared with equivalent single conductor
Therefore reduction in line resistance and improved transmission efficiency.

(ii) Transposition is a means of making the phase inductance of an unsymmetrically disposed 3-phase line identical by interchanging the position of the conductors at regular intervals along the line. Thus all conductors occupy equal positions over equal distances.



(iii)



$$\oint H \cdot dx = I$$

$$\therefore 2\pi x H_x = \frac{I \pi x^2}{\pi r^2} \Rightarrow H_x = \frac{I x}{2\pi r^2} \text{ A/m.}$$

Since $B = \mu H$ then:

$$B_x = \frac{\mu_0 I x}{2\pi r^2}$$

Flux enclosed per metre of tubular element of flux path is:

$$d\phi = B_x dx = \frac{\mu_0 I x . 1}{2\pi r^2}$$

$\therefore$ Flux linkages caused by elemental flux (only links fraction of current)

$$d\lambda = \left(\frac{x^2}{r^2}\right) d\phi = \frac{\mu_0 I x^3 dx}{2\pi r^4}$$

$$\therefore \text{Total flux linkages (internal)} = \int_{x=0}^{r} \frac{\mu_0 I x^3 dx}{2\pi r^4} = \frac{\mu_0 I}{8\pi}$$

$$\therefore L_{INT} = \frac{\lambda}{I} = \frac{\mu_0}{8\pi}$$ ie independent of radius.

(b)(i) $gmr = \sqrt[6\times6]{(r'.d_{12}.d_{13}.d_{14}.d_{15}.d_{16})(d_{21}.r'.d_{23}.d_{24}.d_{25}.d_{26})(d_{31}.d_{32}.r'.d_{34}.d_{35}.d_{36}) \times (d_{41}.d_{42}.d_{43}.r'.d_{45}.d_{46})(d_{51}.d_{52}.d_{53}.d_{54}.r'.d_{56})(d_{61}.d_{62}.d_{63}.d_{64}.d_{65}.r')}$

Now $d_{12} = d_{23} = d_{24} = d_{25} = d_{35} = d_{36} \ldots$ etc. $= 2r$

$d_{14} = d_{16} = d_{46} = 4r$

$d_{13} = d_{26} = d_{34} = 2\sqrt{3}\, r$

$$gmr = \sqrt[36]{(0.7788r)^6(2.2.4.2\sqrt{3}.4)(2.2.2.2.2.2\sqrt{3})(2.2.2\sqrt{3}.2.2) \times (4.2.2\sqrt{3}.2.4)(2\sqrt{3}.2.2.2.2)(4.2\sqrt{3}.2.4.2)\, r^{30}}$$

$$= \sqrt[36]{(0.7788)^6\,(32\sqrt{3})^3,\,(128\sqrt{3})^3\, r^{36}} = 2.1r = \underline{\underline{8.4\,mm}}$$

b(ii) Geometric mean Distance (GMD) between phases $= \sqrt[3]{D_{12}.D_{13}.D_{23}}$

$$D_{12} = D_{13} = \sqrt{1.4^2 + 0.9^2} = 1.664m$$

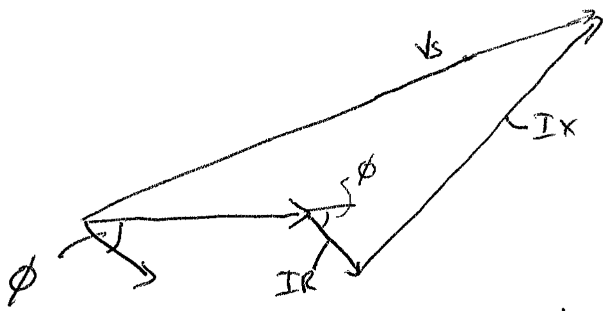
$$D_{23} = 2 \times 1.4 = 2.8m$$

$$\therefore \ GMD = \sqrt[3]{1.664 \times 1.664 \times 2.8} = 1.980m$$

$$L = 2 \times 10^{-7} \ln \frac{D_g}{R_g} = 2 \times 10^{-7} \times \ln\left(\frac{1.980}{8.4 \times 10^{-3}}\right) = 1.09\mu H/m$$

$\therefore$ Reactance of 15km line $= 2\pi . 50 . \ 1.09 \times 10^{-6} \times 15000 = \underline{\underline{5.14\,\Omega}}$

(iii) $R_{line} = 0.1 \times 15 = 1.5\,\Omega$



$$V_S^2 = (V_R + IR\cos\phi + IX\sin\phi)^2 + (IX\cos\phi - IR\sin\phi)^2$$

$$I = \frac{4 \times 10^6}{\sqrt{3} \times 11000 \times 0.8} = 262.4A$$

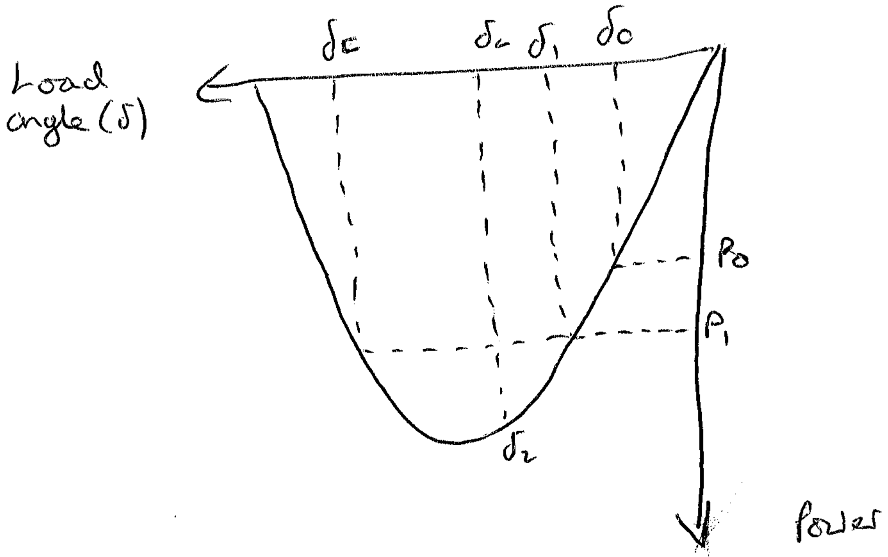
$$V_S = (6351 + 262 \times 1.5 \times 0.8 + 262 \times 5.14 \times 0.6)^2 + (262 \times 5.14 \times 0.8 - 262 \times 1.5 \times 0.6)^2$$

$$= (6351 + 314.4 + 808.0)^2 + (1077.3 - 235.8)^2$$

$\therefore \underline{\underline{V_S = 7520.6\,V}}$ (phase).

(iv) If real power remains constant but p.f. changes from 0.8 → 0.9 then I must decrease. Also ϕ decreases as p.f. increases. Hence length of IX and IR phasors in above diagram reduce and then V_S must reduce.

(a)



Assume initialy the motor is delivering mechanical power, P_0, at a load angle δ_0. The load suddenly increases to P_1. Due to the inertia of the rotor the load angle does not alter instantaneously to the value required ~~for the electrical power~~ to match the new mechanical load (i.e. δ_1)
Therefore the rotor slows down and the speed falls below Synchronous speed and δ increases towards δ_1. As δ increases towards δ_1 the decelerating power decreases. When the load angle reaches δ_1 the power difference is zero and hence the rotor is neither being decelerated or accelerated. However the rotor velocity with respect to the synchronous speed is not zero and δ will continue to increase towards δ_2. However now the electrical power input is greater than the mechanical load so the rotor will start to accelerate with a steady reduction in the rate of increase of δ. When the rotor attains synchronous speed δ stops increasing (point δ_2). Since $P_e > P_m$ the rotor speed increases above synchronous speed a δ begins to reduce. With no damping δ oscillates about δ_1. With damping the rotor will settle to δ_1, provided δ_2 has remained less than δ_C otherwise stability will have been lost.

(b) Assume the peak power is 1.0pu then

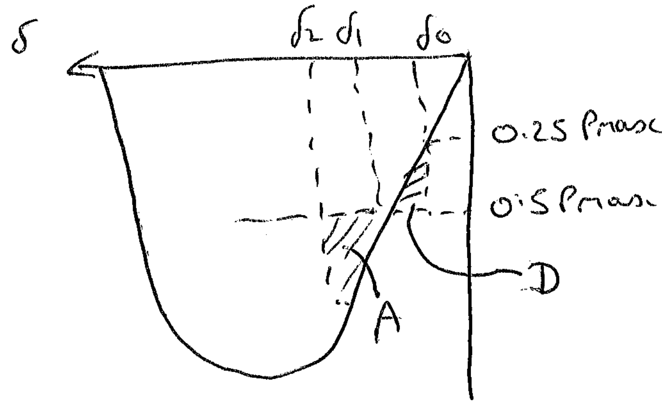
$$P_e = 1 \sin\delta$$

The initial load angle is given by:

$$0.25 = 1 \sin\delta_0 \Rightarrow \delta_0 = 14.48^\circ \equiv 0.253\text{rad}$$

The final load angle is given by:

$$0.5 = 1 \sin\delta_0 \Rightarrow \delta_0 = 30^\circ \equiv 0.524 \text{ rad.}$$



Decelerating area:

$$D = 0.5(0.524 - 0.253) - \int_{14.48}^{30} \sin\delta$$

$$= 0.1355 + \left[\cos 30^\circ - \cos 14.48^\circ\right] = 0.0333$$

Accelerating area:

$$A = \int_{30}^{\delta_2} \sin\delta - 0.5(\delta_2 - 0.524)$$

$$= -\cos\delta_2 + 0.866 - 0.5\delta_2 + 0.262$$

$$= 1.128 - \cos\delta_2 - 0.5\delta_2$$

Equal area criteria $D = A$

$$\therefore 1.128 - \cos\delta_2 - 0.5\delta_2 = 0.033$$

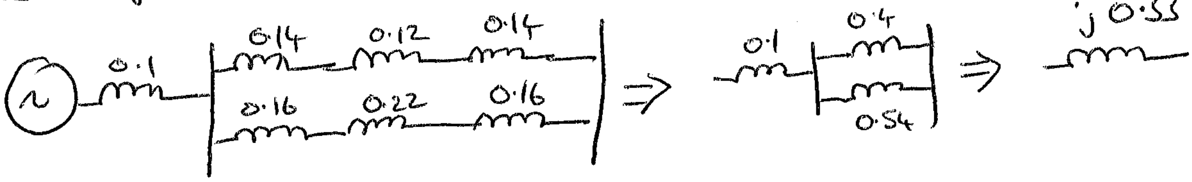
$$\therefore 0.5\delta_2 + \cos\delta_2 = 1.095$$

The solution to this equation is by 'trial & error':

δ_2	$\cos\delta_2 + 0.5\delta_2$
0.8	1.0967
0.82	1.0922
0.81	1.0945
0.809	1.0947 ←

$\therefore \delta_2 = 0.809$ rads or $\underline{\underline{46.4^\circ}}$

(c) (i) Before the fault:



$$\text{Now } P_{eb} = \frac{V_1 V_2}{X}\sin\delta = \frac{1}{0.33}\sin\delta = \underline{\underline{3\sin\delta}}$$

Expressing the load as a pu value:

$$\text{Load} = \frac{120}{150} = 0.8\,\text{pu}$$

Hence initial load angle δ_0:-

$$0.8 = 3\sin\delta \Rightarrow \delta_0 = \underline{\underline{15.5^\circ}}$$

(ii) During the fault;

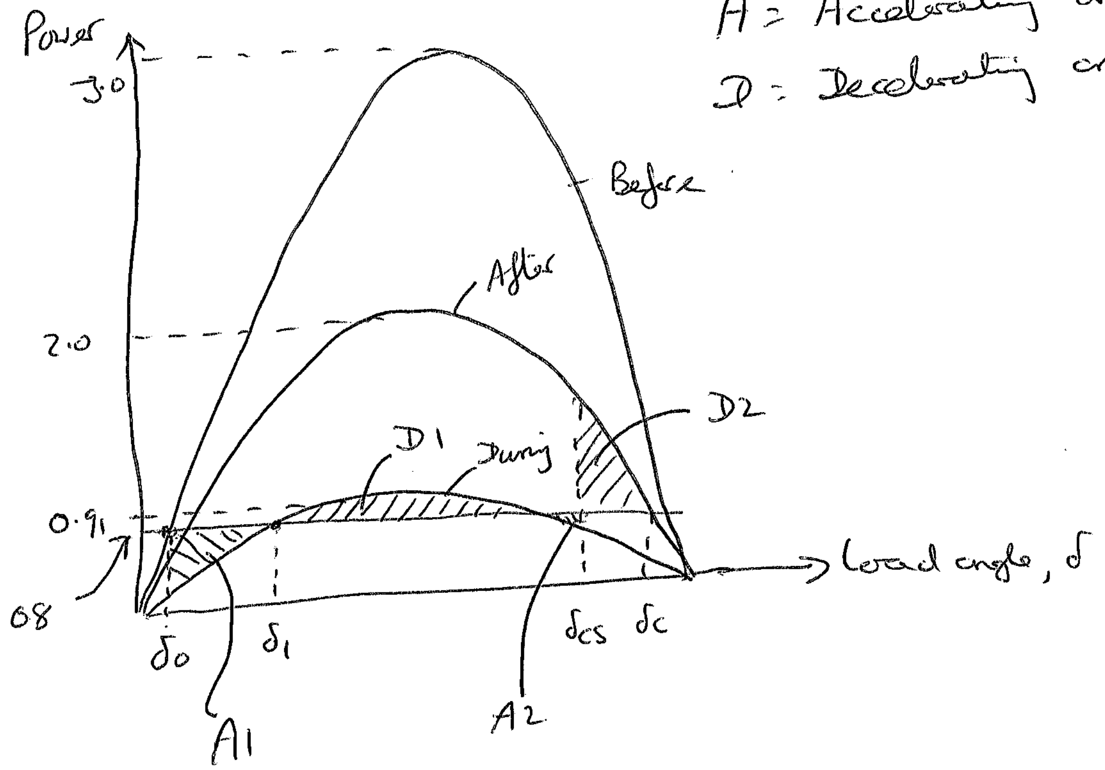
$$P_{ed} = \frac{1.0 \times 1.0}{1.1} = \underline{\underline{0.91\sin\delta}}$$

(iii) After the fault is cleared;

$$P_{ea} = \frac{1.0 \times 1.0}{0.5} = \underline{\underline{2\sin\delta}}$$

(iv)

A = Accelerating area
D = Decelerating area.



(v) $\left(\frac{\Delta t^2}{M}\right) = (0.05)^2 / 0.00025 = 10$

T	C	$C\sin\delta = P_e$	$P_a = P_m - P_e$	$10P_a$	$\Delta\delta$	δ
0 −	3	0.8	0			15.5°
0 +	0.91	0.243	0.557			15.50
0 Ave			0.278	2.78	2.78	
0.05	0.91	0.285	0.514	5.14	7.92	18.3°
0.1	0.91	0.402	0.398	3.98	11.9	26.2°
0.15	0.91	0.562	0.238	2.38	14.3	38.1°
0.2 −	0.91	0.721	0.079	0.79		52.4°
0.2 +	2.0	1.58	−0.78	−7.8		
0.2 Ave				−3.505	10.8	63.2°
0.25	2.0	1.78	−0.985	−9.85	0.95	64.1°
0.3	2.0	1.8	−1	−10	−9.05	55.05°

δ decreasing: stability maintained.

QUESTION 4

(a) Refer all pu impedances to a base of 90MVA:

Gen G1 (already on correct base) $X_+ = X_- = 0.21$ pu
$X_0 = 0.075$ pu

Trans T1 (already on correct base) $X_+ = X_- = 0.12$ pu
$X_0 = 0.075$ pu

Line L1 (132kV)

$$Z_b = \frac{(132000)^2}{90 \times 10^6} = 193.6\Omega$$

$$\therefore X_+ = X_- = \frac{8}{193.6} = 0.0413 \text{pu}$$

$$X_0 = \frac{25}{193.6} = 0.1291 \text{pu.}$$

Trans T2:

$$X_+ = X_- = 0.11 \times \frac{90}{110} = 0.09 \text{pu}$$

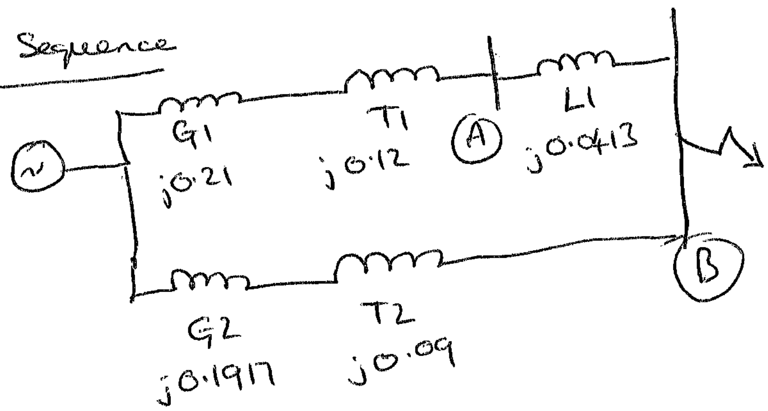
$$X_0 = 0.11 \times \frac{90}{110} = 0.09 \text{pu}$$

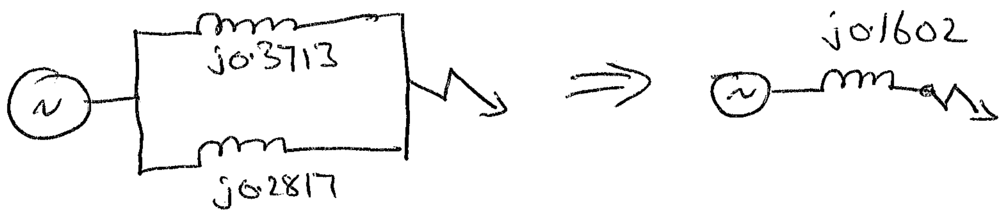
Gen G2:

$$X_+ = X_- = 0.2 \times \frac{9}{8} \times \frac{12^2}{13^2} = 0.1917 \text{pu}$$

$$X_0 = 0.06 \times \frac{9}{8} \times \frac{12^2}{13^2} = 0.0575 \text{ pu}$$

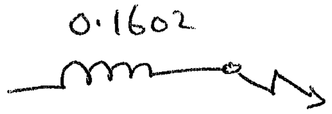
(b) Positive Sequence



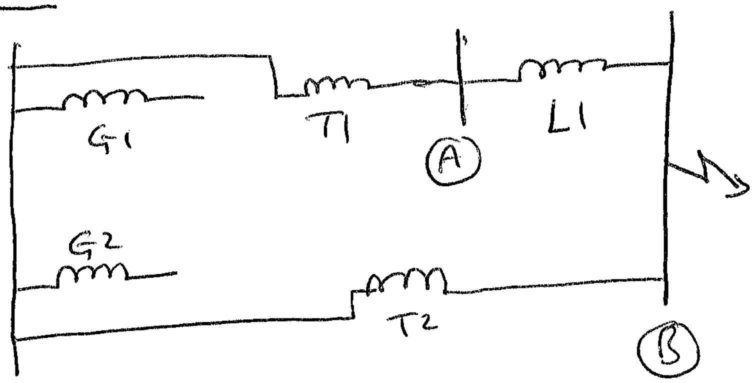


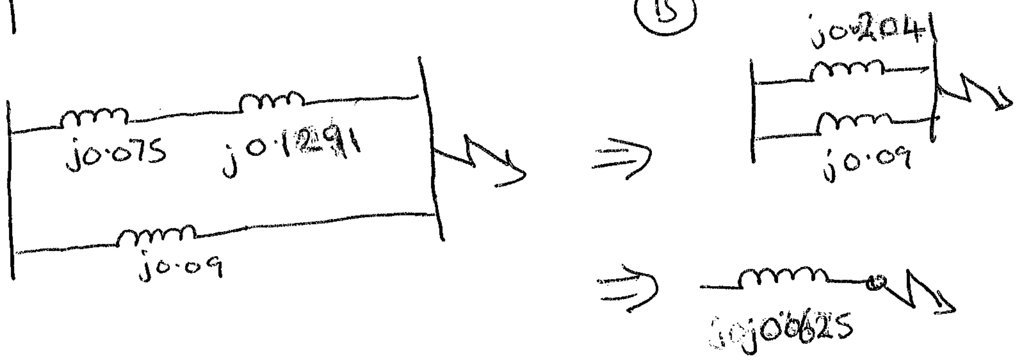
Negative Sequence

Network is identical to positive sequence both in layout and values. However there is no negative sequence voltage source.

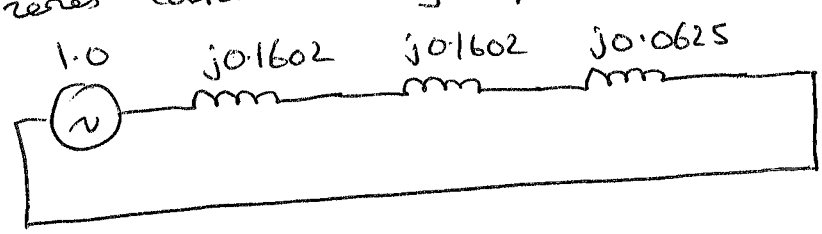


Zero sequence





(c) For a single phase to earth fault on busbar (B) assume a series connection of sequence networks:



$$I_+ = I_- = I_0 = \frac{1.0}{j(0.1602+0.1602+0.0625)} = -j2.61 \text{ pu}$$

Now fault current $= I_+ + I_- + I_0 = 3 \times (-j2.61) = -j7.83$

Now base current at busbar (B) is:

$$I_b = \frac{MVA_b}{\sqrt{3}V_b} = \frac{90\times10^6}{\sqrt{3}\times132\times10^3} = 393.6A$$

Hence actual fault current is:

$$I_{Fault} = 393.6\times7.83 = \underline{\underline{3082A}}$$

(d) Current in the overhead line is the current flowing from the left side of the diagram:

$$I_{+line} = I_+ \times \frac{j0.2817}{(j0.2817+j0.3713)} = -j2.61\times0.4314 = -j1.126$$

$I_{-line} = I_{+line}$ since the values are identical.

$$I_{o\,line} = I_o\times\frac{j0.09}{(j0.09+j0.2041)} = -j2.61\times0.306 = -j0.799$$

$$\therefore I_{A\,line} = I_{+L}+I_{-L}+I_{oL} = -j1.126-j1.126-j0.799 = -j3.051 \quad (=1201A)$$

$$I_{B\,line} = a^2I_{+L}+aI_{-L}+I_{oL} = -j1.126.1\angle240^\circ - j1.126\times1\angle120^\circ - j0.799$$
$$= j0.327pu \quad (=128.7A)$$

$$I_{C\,line} = aI_{+L}+a^2I_{-L}+I_{oL} = -j1.126\times1\angle120 - j1.126\times1\angle240^\circ - j0.799$$
$$= j0.327pu \quad (=128.7A)$$

(e) Since neither G1 nor G2 appear in the zero sequence diagram and the changes will have no effect on the positive or negative sequence diagrams, there is no change to the level of fault current.