EEE225 Transistor Amplifier Circuit Analysis Problem Sheet Solutions

This problem sheet builds on the analysis of the two transistor amplifier circuits EEE118. It should prepare students well to tackle general problems involving transistors in analogue circuits. The circuits used in questions 1, 2 & 3 are not directly examinable, nor are questions 8 – 10. The techniques needed to solve the first few questions are the standard techniques of circuit analysis with active devices. These techniques were first introduced in EEE118 and are further developed in EEE225. If you can solve questions 1 – 3 confidently you'll have no problem at all with questions 4 – 7 which *are* examinable. Question 7 is quite similar to the sort of questions that come up in EEE223, and some parts of it to do with crossover distortion are in EEE225 as well.

How to tackle this sheet

Do question 1 or question 2 or question 3. Do *all* of questions 4, 5 & 6. Some of question 7 is needed in EEE225 especially related to crossover distortion, the rest is needed in EEE223.

If you feel that you've not had enough practice, go back and do the other questions as well. It would certainly be a good idea to look at the past exam papers as well for practice questions. You should find the exam questions much easier than the problems in this sheet, consiquently if you can do the sheet the exam should not pose any difficulty.

Questions 8 – 10 are for students who love the topic and want to go on an adventure of their own. The solution of these questions uses many of the techniques in this course but also moves outside the scope of the course. Unless you have lots of time available having done all the other quesetions and being up to date with all your other modules I would not devote time to these questions.

If you're looking for even more analogue try Gray, Hurst, Lewis and Meyer, which is considered by many to be the standard text on the subject. Behzad Ravazi has also written some very well liked books on the topic. He also has video lectures on YouTube which covers much of EEE118 and the semiconductors and analogue aspects of EEE225 https://www.youtube.com/watch?v=yQDfVJzEymI&list=PL7qUW0KPfsIIOPOKL84wK_Qj9N7gvJX6v

Question 1: A Common Emitter Circuit

This question is about the "type 1" common emitter circuit from EEE118. Unless otherwise stated, assume that all capacitors are short circuit in the mid-band. Some solutions will be easier to reach if R_L and R_C are lumped together as R_L'. Similarly R_B may be used to represent the parallel combination of R_1 and R_2.

The objective with the small signal derivations is to show which components are in control of certain circuit parameters, therefore the final form of the answer should be manipulated to reveal this information as clearly as possible. Arranging equations in a way that reveals certain underlying relationships in the circuit parameters is something computers are not very good at, this sort of work is best done by hand.

1. Find the DC conditions of the common emitter circuit in Figure 1 assuming the base current of Q_1 can be ignored.
2. Find the DC conditions again but taking into consideration the base current. Perform your calculations for the full range of h_{FE}. Find the range of h_{FE} from the Fairchild Semiconductor BC549 datasheet.
3. Explain (briefly, using bullet points for example) the job of each component in the circuit.
4. Explain (in words) why the emitter resistor, R_{E} acts to reduce the gain of the circuit unless it is decoupled by C_{E}.
5. Draw and label the small signal equivalent circuit for Figure 1.
6. Calculate the small signal transconductance, g_{m}, and base emitter resistance, r_{be} for the range of h_{FE} given in the Fairchild Semiconductor datasheet. You may assume that the transistor stage will be operated at frequencies considerably below the transition frequency, f_T, and therefore $\beta = h_{\mathrm{FE}}$
7. Show that the mid-band voltage gain of the common emitter circuit shown in Figure 1 is given by (1).
8. Show that the mid-band output resistance of the amplifier circuit in Figure 1 is given by (2).
9. Show that the mid-band input resistance of the amplifier circuit in Figure 1 is given by (3).
10. Show that the mid-band current gain given by (4).
11. Find an expression for the transresistance v_o/i_i of the amplifier stage shown in Figure 1.
12. Draw and label the small signal equivalent circuit for Figure 1 if C_{E} is *open* circuit at all frequencies of interest, all other capacitors may be considered short circuit.

13. Assuming C_E is *open* circuit at all frequencies of interest, derive the input resistance, output resistance, voltage gain and current gain of the amplifier. The final solutions take the forms shown in (5) - (8).

14. Given your solution for the small signal properties of the stage without emitter decoupling, determine what components are in control of the voltage gain, current gain, input resistance and output resistance. Comment on the effect of emitter degeneration on the small signal parameters. For example, which components are in control of the voltage gain? Which components dominate input resistance? What are the main components which reduce current gain?

15. State the numerical values of voltage gain, current gain, power gain, input resistance and output resistance with and without emitter decoupling over the range of h_FE given in the datasheet.

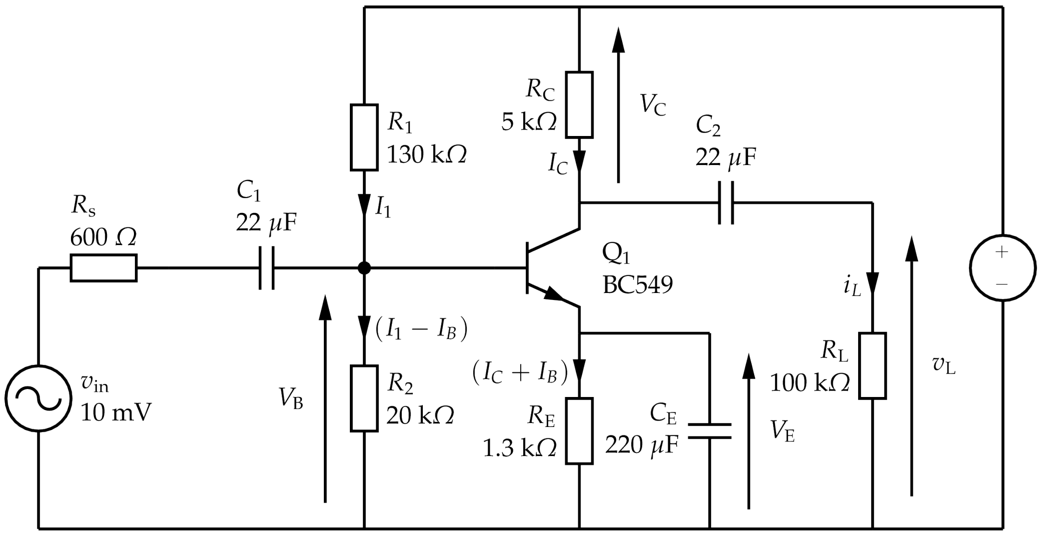

Figure 1: A common emitter amplifier circuit.

$$\frac{v_o}{v_i} = -\frac{g_m\, R_L'}{Rs\left(\frac{1}{R_B} + \frac{g_m}{\beta}\right) + 1} \tag{1}$$

$$r_o = \frac{v_o}{i_t} = R_C \tag{2}$$

$$r_i = \frac{v_i}{i_i} = \frac{1}{\frac{1}{R_B} + \frac{g_m}{\beta}} \tag{3}$$

$$\frac{i_o}{i_i} = \beta \frac{R_B}{R_B + r_{be}} \text{ or } \frac{\beta}{1 + \frac{\beta}{g_m R_B}} \tag{4}$$

$$\frac{v_o}{v_i} = -\frac{g_m R'_L}{R_S \left(\frac{1}{R_B} + \frac{g_m}{\beta} + \frac{1}{R_S} + \frac{(\beta+1)}{\beta} R_E \, g_m \left(\frac{1}{R_B} + \frac{1}{R_S} \right) \right)} \tag{5}$$

$$r_i = \frac{1 + \frac{\beta}{g_m R_E (\beta+1)}}{\frac{1}{R_B} + \frac{1}{R_E (\beta+1)} + \frac{\beta}{g_m R_E R_B (\beta+1)}} \tag{6}$$

$$r_o = R_C \tag{7}$$

$$\frac{i_o}{i_i} = -\frac{\beta}{\beta \left(\frac{1}{g_m R_B} + \frac{R_E}{R_B} \right) + \frac{R_E}{R_B} + 1} \tag{8}$$

Question 1 part 1

This part is asking for the DC conditions when I_B is ignored. I like to solve this circuit by inspection as it is reasonably easy. Technically I do (Kirchhoff's Voltage Law) around the R_E, V_{BE}, V_B loop, and KCL (Kirchhoff's Current Law) at the base node and at the collector node, but generally speaking I don't write out any algebraic loop or node equations. I just look at the circuit and draw on the voltages and currents as I go along. Any method of solution is fine as long as it works consistently. The components that set up the DC or quiescent conditions are shown in Figure 2.

$$V_B = \frac{V_S R_2}{R_1 + R_2} = \frac{15 \cdot 20}{130 + 20} = 2 \text{ V} \tag{9}$$

$$V_E = V_B - 0.7 = 1.3 \text{ V} \tag{10}$$

$$I_E = \frac{V_E}{R_E} = \frac{1.3 \text{ V}}{1.3 \text{ k}\Omega} = 1 \text{ mA} \tag{11}$$

$$I_E = I_C \because I_B = 0 \tag{12}$$

$$V_C = I_C R_C = 1 \text{ mA} \cdot 5 \text{ k}\Omega = 5 \text{ V} \tag{13}$$

$$\therefore V_{CE} + V_E = 15 - 5 = 10 \text{ V} \tag{14}$$

$$I_1 = \frac{15 \text{ V}}{130 \, \text{k}\Omega + 20 \text{ k}\Omega} = 100 \; \mu\text{A} \tag{15}$$

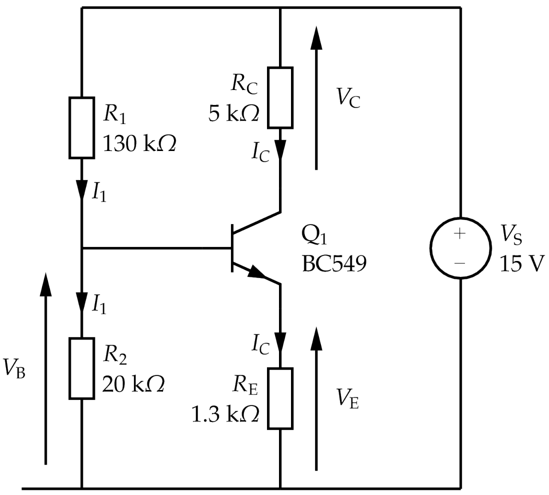

Figure 2: The common emitter amplifier circuit of question 1 showing only the components which affect the DC (quiescent) conditions. This figure assumes that the base current, I_B, can be ignored.

Question 1 part 2

Part 2 of Question 1 asks for the same analysis as part one but making no assumptions about I_B. It also requires us to do the analysis twice once for the minimum h_{FE} and once with the maximum h_{FE} that the Fairchild datasheet specifies. We should expect the minimum h_{FE} to require the largest I_B and we should design the circuit (if we were designing it as opposed to just analysing someone else's design) to use biasing resistors which will accommodate this higher value of I_B. The higher numerical value of I_B computed by using the lowest value of h_{FE} is, in fact, the *minimum* I_B we should design for. This is the value which will allow any transistor of this type to work in the circuit.

It will probably be necessary to write out some node equations for this circuit, which is shown with base current in Figure 3. From the Fairchild BC549 Datasheet $h_{\mathrm{FE}_{(\mathrm{min})}}$ is 110 and $h_{\mathrm{FE}_{(\mathrm{max})}}$ is 800. Starting with KVL around the base – emitter loop.

$$V_B = V_E + V_{BE} \tag{16}$$

KCL on the base node,

$$(I_1 - I_B)\ R_2 = (I_C + I_B)\ R_E + 0.7 \tag{17}$$

$$I_C = h_{\mathrm{FE}}\, I_B \tag{18}$$

$$(I_1 - I_B)\ R_2 = I_B\,(h_{\mathrm{FE}} + 1)\ R_E + 0.7 \tag{19}$$

KVL around the biasing loop and power supply

$$V_S = (I_1 - I_B)\ R_2 + I_1\, R_1 \tag{20}$$

$$I_E = I_C + I_B \tag{21}$$

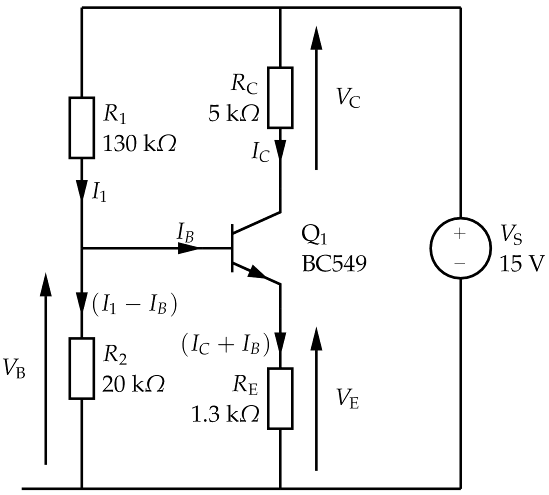

Figure 3: The common emitter amplifier circuit of question 1 showing only the components which affect the DC (quiescent) conditions without making assumptions about the base current.

Having developed (16) to (21) from the circuit using KVL and KCL we can proceed by several methods and with several end goals in mind. We may choose to solve for I_B or I_1 and then use this solution with another equation to find a consistent solution for all unknown variables. We can choose to eliminate or use substitution to solve the equations. A solution involving matrices is also possible and favoured by some. To eliminate I_1, we could multiply (19) by $(1 + R1/R2)$ then add the result of this multiplication to (20) and then solve for I_B. Another possibility is to solve (20) for I_B and then substitute the result into (19) to solve for I_1. I used the substitution method and found (22).

$$I_1 = \frac{V_S\ (1 + h_{\mathrm{FE}})\, R_E + R_2\ (V_S - V_{BE})}{(1 + h_{\mathrm{FE}})\,(R_1 + R_2)\, R_E + R_1\, R_2} \tag{22}$$

Of course the form of your equation may be different to mine, and you may choose to factorise it differently. Next we need to relate I_B and I_1 by transposing (20).

$$I_B = \frac{I_1\, R_1 - V_S}{R_2} + I_1 \tag{23}$$

We need to relate I_C to I_B. Use $I_C = h_{\mathrm{FE}}\, I_B$ in conjunction with (23) to yield,

$$I_C = \left(\frac{I_1\, R_1 - V_S}{R_2} + I_1 \right)\ h_{\mathrm{FE}} \tag{24}$$

h_{FE}	800	110
I_1 [μA]	100.164	101.072
I_B [μA]	1.228	8.043
I_C [μA]	982.399	884.718
V_C [V]	4.912	4.424
V_L [V]	10.088	10.576

Table 1: DC conditions for the common emitter amplifier circuit of question 1 for the extremes of h_{FE}. Notice how the wide variation in h_{FE} doesn't upset the DC conditions very much. This will hold as long as h_{FE} is much greater than unity and is a sign of a good circuit topology.

Use Ohm's law to find V_C in terms of I_C, $V_C = I_C\, R_C$, substituting,

$$V_C = \left(\frac{I_1\, R_1 - V_S}{R_2} + I_1 \right) h_{\mathrm{FE}}\, R_C \tag{25}$$

The voltage on the collector node with respect to ground (this is different to V_C which is the voltage across the collector resistor, R_C) is,

$$V_L = V_S - \left(\frac{I_1\, R_1 - V_S}{R_2} + I_1 \right) h_{\mathrm{FE}}\, R_C \tag{26}$$

The quiescent (DC) conditions for the common emitter amplifier circuit of question 1 are listed in Table 1.

Question 1 part 3

- V_S is the DC power supply. It supplies the current and voltage (and hence power) to run the transistor. The transistor effectively modulates the DC supply voltage with a (hopefully amplified) copy of the input waveform.

- v_{in} and R_S represent the Thévenin equivalent voltage and resistance of the circuit or system driving our transistor amplifier.

- C_1 blocks the biasing voltage on the base node from flowing into the source and also blocks any DC component that the source may have from upsetting the biasing conditions of the amplifier.

- C_2 performs a similar function to C_1 but for the output.

- R_L is the load placed on our amplifier by the circuit which it is connected to. In other words R_L represents in the input resistance of whatever is connected to the output of our amplifier.

- R_1 & R_2 set up the biasing voltage on the base of Q_1
- R_C exists to develop a voltage in response to the collector current (i.e. a resistor to develop the output voltage swing across). The size of R_C and the designers choice of I_C set the quiescent output voltage. The size of R_C (and R_L) also affects the gain of the amplifier.
- R_E allows us to make the circuit somewhat independent of the transistor h_{FE} by providing feedback from output to input.
- C_E is used to decouple the emitter node to ground such that an AC signal will choose the low impedance of the C_E over the moderate resistance of R_E in order to get to ground. The result is that from the signal's point of view, R_E does not exist and therefore can not provide feedback. The existence of C_E does not change the effect of R_E on the DC or thermal conditions however.

Question 1 part 4

R_E allows us to make the circuit operation somewhat independent of the transistor h_{FE} by providing feedback in the form of a voltage which subtracts from V_{BE} in proportion to the size of I_E (which in turn is proportional to I_C and I_B). This negative feedback will happen for DC and AC signals and so reduces the gain of the amplifier. The effects of temperature change, either due to environmental conditions or due to power dissipation in the transistor, will also be stabilised as increasing temperature increases I_C for a given V_{BE} which in turn increases V_E leaving less voltage for V_{BE}, turning the transistor off somewhat and so reducing I_C.

Question 1 part 5

The small signal model for the common emitter amplifier of Figure 1 in the mid-band (where all capacitors are short circuit) is shown in Figure 4.

Question 1 part 6

The transconductance of a BJT is given by taking the gradient of the transfer characteristic at the operating point. The transfer characteristic is given by (27). If we assume that the exponential term is much larger than unity while the diode is conducting we can approximate the expression for I_C to (28) differentiating with respect to V_{BE} yields (29). Looking at (29) it may be apparent that everything inside the square braces is actually just (28). This shouldn't be too surprising as we know that differentiating an exponential always yields an exponential multiplied by a coefficient. Substituting (28) into (29) provides the standard result (30).

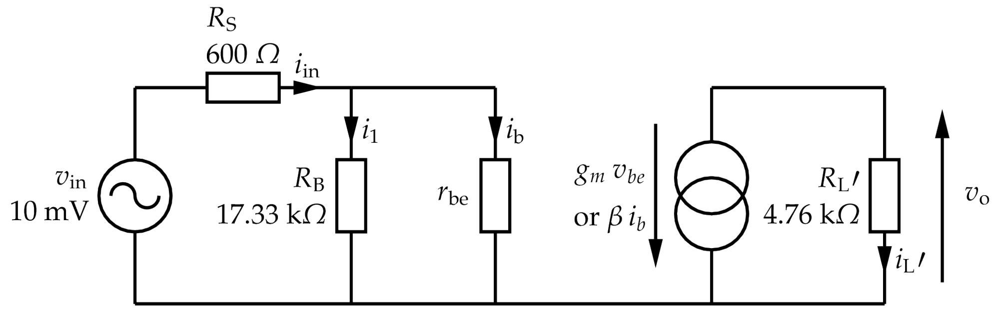

Figure 4: The small signal model of the common emitter amplifier circuit of question 1 assuming all capacitors are short circuit at signal frequencies. R_B is the parallel combination of R_1 and R_2. $R_L\prime$ is the parallel combination of R_L and R_C.

h_{FE}	g_m [mA/V]	r_{be} [kΩ]
800	37.671	21.0709
110	34.192	3.2171

Table 2: Small signal parameters of Q_1 in the common emitter amplifier of question 1.

$$I_C = I_S \left(\exp \left(\frac{q\, V_{BE}}{k\, T} \right) - 1 \right) \tag{27}$$

$$I_C = I_S \left(\exp \left(\frac{q\, V_{BE}}{k\, T} \right) \right) \tag{28}$$

$$\frac{q}{k\, T} \left[I_S \left(\exp \left(\frac{q\, V_{BE}}{k\, T} \right) \right) \right] \tag{29}$$

$$g_m = \frac{q\, I_C}{k\, T} \tag{30}$$

We can also determine the relationship between the small signal resistance looking into the base towards the emitter, r_{be} and g_m and the current gain, β. The current gain, h_{FE}, which is equal to β at low frequencies is defined as $\mathrm{d}\, I_C / \mathrm{d}\, I_B$. We can therefore write (31).

$$r_{be} = \frac{\mathrm{d}\, V_{BE}}{\mathrm{d}\, I_B} = \frac{\mathrm{d}\, I_C}{\mathrm{d}\, I_B} \cdot \frac{\mathrm{d}\, V_{BE}}{\mathrm{d}\, I_C} = \frac{\beta}{g_m} \tag{31}$$

The small signal transistor parameters are given approximately in Table 2.

Question 1 part 7

To obtain the mid-band voltage gain we can use the small signal circuit shown in Figure 4. Looking at the collector – emitter circuit we obtain (32).

$$v_o = -g_m\, v_{be}\, R_L\prime \tag{32}$$

Looking at the base – emitter circuit we can write (33)

$$v_{be} = v_i\, \frac{r_{be}||R_B}{R_S + R_B||r_{be}} \tag{33}$$

Equation 33 can be re-written in a more mathematically proper form as (34)

$$v_{be} = v_i\, \frac{1}{R_S\left(\frac{1}{r_{be}} + \frac{1}{R_B}\right) + 1} \tag{34}$$

It is sometimes instructive to replace r_{be} with β/g_m as g_m changes with I_C and temperature and β has variation with I_C and temperature and varies device to device. Leaving r_{be} in the equation can sometimes mask from the circuit designer, the effects of changing g_m and changing β on the small signal response. Making this replacement yields (35).

$$v_{be} = v_i\, \frac{1}{R_S\left(\frac{g_m}{\beta} + \frac{1}{R_B}\right) + 1} \tag{35}$$

Substituting (35) into (32) and solving for v_o/v_i yields (36)

$$\frac{v_o}{v_i} = \frac{-g_m\, R_L\prime}{R_S\left(\frac{g_m}{\beta} + \frac{1}{R_B}\right) + 1} \tag{36}$$

Question 1: part 8

The output resistance of the circuit can be found by injecting a test current into the output and measuring the voltage that the current source produces across itself to ensure the desired test current flows. Dividing this voltage by the test current yields the resistance looking into the output. A small signal model of this situation is shown in Figure 5. By inspection of Figure 5 it may be evident that the $g_m\, v_{be}$ controlled current source will pass no current as both v_{be} and i_b are zero while the input is grounded. The input is grounded when we consider the output resistance as we're interested on the effect of driving the output to see what effect a signal experiences looking inwards. Input and output resistance are examples of *driving point impedances* which are a useful tool in the analysis of many analogue and microwave circuits and systems. Another way of thinking about what should happen to the input signal source when we are determining the resistance looking into the output is to imaging we are conducting an analysis using superposition. Since the output driving current source, i_t, is the source we're interested in, all the other independent sources will be removed and replaced with their internal impedances, hence v_{in} becomes short circuit and R_S remains to represent the internal impedance of v_{in}. Bearing all this in mind, inspection of Figure 5, should show that $r_o = R_C$ without the need for algebra.

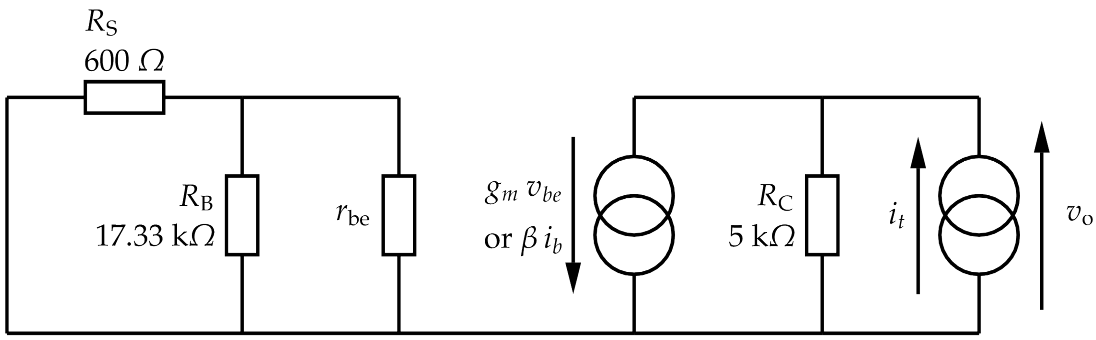

Figure 5: The small signal model for the output resistance of the common emitter amplifier in question 1. Note that R_L is the input resistance of the circuit attached to the output of our amplifier and therefore doesn't count towards the output resistance of this amplifier stage.

Question 1: part 9

The input resistance of the circuit can be found in a similar way to the output impedance except that we drive the input not the output. It is possible to drive the input with a current or voltage but since Figure 4 shows a voltage at the input we will use it and find the input current, i_{in} that flows due to the input voltage, v_{in}. We will then apply Ohm's law to v_{in} and i_{in} to yield r_i. Remember that R_S is the output resistance of the v_{in} source and therefore does not contribute to the input resistance of this amplifier. Buy inspection of the circuit we can write (37) for the input resistance.

$$r_i = r_{be}||R_B \tag{37}$$

The parallel symbol is not a real mathematical operator so we would like to remove it. It is desirable to remove r_{be} in my opinion as we can then see what effect running at higher quiescent current will have (due to its influence on g_m) we can also see what effect a change in β would have.

$$r_i = \frac{1}{\frac{1}{r_{be}} + \frac{1}{R_B}} = \frac{1}{\frac{g_m}{\beta} + \frac{1}{R_B}} \tag{38}$$

Equation (38) shows the relationship between DC conditions, β and the input resistance. Generally speaking R_B is made quite large in order to maintain as higher input resistance as possible. g_m can be made small too but g_m also appears in the numerator of the voltage gain expression so care must be taken when deciding what advantage can be obtained in one metric of performance at the expense of equal or worse disadvantage in performance of another metric. Having obtained the input resistance, careful observation of the voltage gain equation (36) will reveal that it can be re-written as (39) which may be helpful in illuminating the relationship between voltage gain and input resistance when deciding a value for g_m. Don't forget of course that changing g_m means changing the quiescent voltage on the collector which may limit the available voltage swing unless $R_L\prime$ is changed as well and this would change the gain too.

$$\frac{v_o}{v_i} = \frac{-g_m R_L\prime}{\frac{R_S}{r_i} + 1} \tag{39}$$

One way to look at (38) is to argue that the circuit cannot work without the biasing resistors. Let us assume that the transistor is perfect, in that case as $\beta \to \infty$, $r_i \to R_B$. We can then see why R_B is made large, it is essentially setting the maximum input resistance. Of course the resistors that make up R_B must pass somewhat more than ($\approx$ 10x) the DC base current in order to effectively bias the transistor. For a given collector current higher h_{FE} will yield a lower I_1 and hence permit larger R_B. In terms of small signals the g_m/β term in the denominator will decrease the input resistance as g_m (and therefore I_C) increases. r_i will increase with increasing β, up to the limit of R_B.

Question 1: part 10

The current gain is found by solving for i_o/i_i. We could change the input source to a Norton source if we wished, but I believe the solution is easier if we keep the Thévenin source. By inspection of Figure 4 we can see that the input current is divided into R_B and r_{be}. Remembering that R_S is part of the source we have (40).

$$i_b = \frac{i_i R_B}{r_{be} + R_B} \tag{40}$$

Looking at the collector side we can write (41)

$$i_o = \beta\, i_b \tag{41}$$

Substitution should be fairly straight forward. We might like to remove r_{be} to see what effect changing g_m or β will have. Substituting $r_{be} = \beta/g_m$ appropriately leads to (4).

Question 1: part 11

The transresistance can be derived directly from the small signal model of Figure 4 using the same techniques as for the other small signal parameters. Alternatively it can be produced by noting that $v_o/i_i = v_o/v_i \cdot v_i/i_i$. In other words the voltage gain multiplied by the input resistance. Assuming room temperature a numerical value between 430 kΩ and 1.68 MΩ is believable, depending on the value of β. The transresistance increases as β increases.

Question 1: part 12

Making C_E open circuit means R_E is no longer decoupled from the signal's perspective and feedback can exist between output and input. This is clearly depicted in the new small signal model of Figure 6.

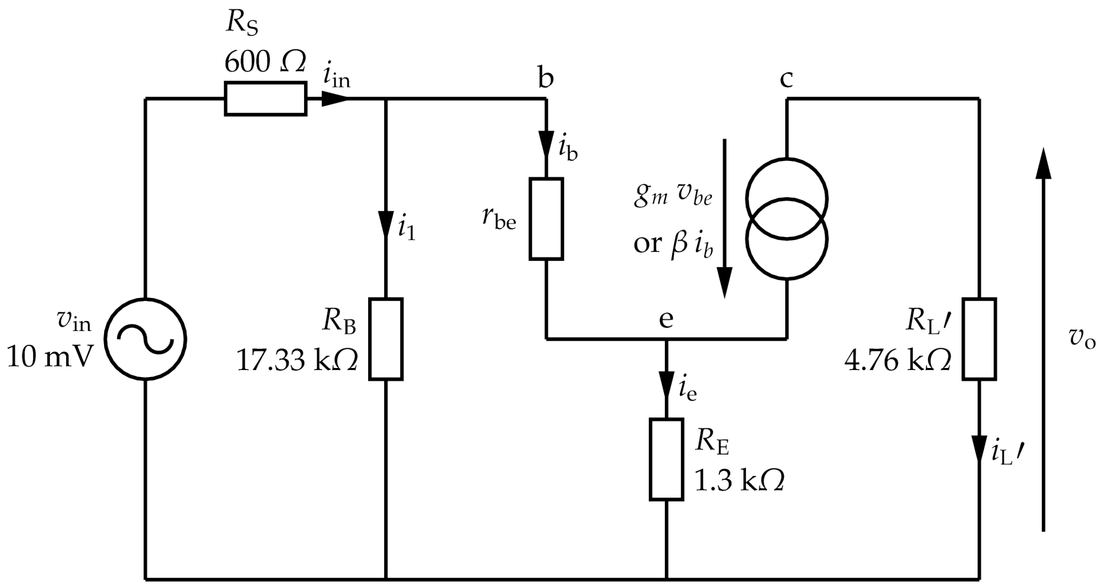

Figure 6: The small signal model for the common emitter amplifier in question 1 when assuming that R_E is not decoupled by C_E.

Question 1: part 13

Part 14 asks for a re-derivation of *all* the small signal parameters assuming R_E is not decoupled. The addition of a component which allows the input and output to depend on each other complicates the analysis somewhat. I will only show the voltage gain here. The proof of (6) – (8) are left to the reader, the methods are the same as shown for the degenerated case. To obtain the voltage gain, we can do KCL at the base and emitter nodes and KVL around the r_{be}, R_B and R_E loop.

These lead to (42) – (44).

$$\frac{v_i - v_{be} - v_e}{R_S} = \frac{v_b}{R_B} + \frac{v_{be}}{r_{be}} \tag{42}$$

$$\frac{v_e}{R_E} = \frac{v_{be}}{r_{be}} (\beta + 1) \tag{43}$$

$$v_b = v_{be} + v_e \tag{44}$$

A number of approaches are possible to solve these equations, and these are not the only equations which can be used to solve the circuit. Any valid method is acceptable. We proceed by substituting (44) into (42) to remove v_b.

$$\frac{v_i - v_{be} - v_e}{R_S} = \frac{v_{be} + v_e}{R_B} + \frac{v_{be}}{r_{be}} \tag{45}$$

We could then transpose (43) to make v_e the subject to yield (46).

$$v_e = \frac{v_{be}}{r_{be}} \, (\beta + 1) \, R_E \tag{46}$$

Substitution of (46) into (45) removes v_e to leave (47) which is an expression relating v_i to v_{be}.

$$\frac{v_i - v_{be} - \frac{v_{be}}{r_{be}} \, (\beta + 1) \, R_E}{R_S} = \frac{v_{be} + \frac{v_{be}}{r_{be}} (\beta + 1) \, R_E}{R_B} + \frac{v_{be}}{r_{be}} \tag{47}$$

Having related the output control parameter of our choice, v_{be}, (we could have chosen to use i_b instead) to the input, v_i, we seek a relationship between the output variable v_o and the control variable v_{be}. Use KCL at the collector node.

$$v_o = -g_m \, v_{be} \, R_{L'} \tag{48}$$

where $R_{L'}$ is the parallel combination of R_C and R_L. We need to transpose (47) to make v_{be} the subject. This is a little tiresome but only takes five lines of algebra, less if you can do multiple steps at once, however you get there the result is (49).

$$v_{be} = \frac{v_i}{R_S \left(\frac{1}{R_B} + \frac{1}{r_{be}} + \frac{1}{R_S} + \frac{(g_m \, r_{be}+1) \, R_E}{r_{be}} \cdot \left(\frac{1}{R_S} + \frac{1}{R_B} \right) \right)} \tag{49}$$

To obtain the final result substitute (49) into (48) to yield (50).

$$\frac{v_o}{v_i} = \frac{-g_m \, R_{L'}}{R_S \left(\frac{1}{R_B} + \frac{1}{r_{be}} + \frac{1}{R_S} + \frac{(g_m \, r_{be}+1) \, R_E}{r_{be}} \cdot \left(\frac{1}{R_S} + \frac{1}{R_B} \right) \right)} \tag{50}$$

All that remains is to use $r_{be} = \beta / g_m$ if it is appropriate to show the effects of changing g_m and β more clearly and to demonstrate which components are in control of the circuit metric. There is no ideal form for this expression but I like (51).

$$\frac{v_o}{v_i} = -\frac{g_m \, R'_L}{R_S \left(\frac{1}{R_B} + \frac{g_m}{\beta} + \frac{1}{R_S} + \frac{(\beta+1)}{\beta} \, R_E \, g_m \left(\frac{1}{R_B} + \frac{1}{R_S} \right) \right)} \tag{51}$$

Question 1: part 14

When looking at the equations that describe the metrics of the circuit's small signal performance we might ask what components would be in control if β was very large. Ideally we would like the transistor *not* to be in control of the metrics but for us to be able to set them using resistors. If this is possible it means that the particular transistor is not important for the circuit operation. This is significant because we have no control over β, but we would like our circuit to behave (more or less) the same whatever transistor we use. The method to determining what's important is to think about what's big compared to everything else in the numerator and denominator. Taking the voltage gain equation, (51) as an example, we can look at the terms in the denominator in turn. The first three terms R_S/R_B, $(R_S\, g_m)/\beta$ and 1 are not going to be large. R_S will probably not be as big as R_B because we want to make R_B big for input resistance reasons and Thévenin sources are more ideal when R_S is small. $(R_S\, g_m)/\beta$ will be less than unity with sensible numbers and R_S/R_S *is* unity. $(\beta+1)/\beta$ is more or less 1 if $\beta >> 1$ and $R_S/R_B + R_S/R_S \approx 1$ if $R_B >> R_S$ which we already said is quite likely. We can therefore reduce the last term in the denominator to $R_E\, g_m$. Looking at the numerator we have $g_m\, R_{L\prime}$. The input impedance of the next circuit should be much higher than the output impedance of this amplifier (which is R_C) so we may presume that $R_{L\prime} \approx R_C$. The g_m term in the numerator and denominator cancel leaving (52)

$$\frac{v_o}{v_i} = -\frac{g_m\, R_L'}{R_S\left(\frac{1}{R_B} + \frac{g_m}{\beta} + \frac{1}{R_S} + \frac{(\beta+1)}{\beta}\, R_E\, g_m\left(\frac{1}{R_B} + \frac{1}{R_S}\right)\right)} \approx -\frac{R_C}{R_E} \tag{52}$$

We can see this is true for the numbers in this question by comparing the exact answer with the simplification. The exact answer for $\beta = 800$ is approximately -3.629 and the simplification provides approximately -3.846. This represents about 5% error. The error with $\beta = 110$ is around 7%. Considering that all resistors have some tolerance, the level of error is quite acceptable in most cases especially considering the simplification it allows us to use.

Similar arguments can be made to show which circuit elements control the input resistance and current gain and a diligent student will attempt these explanations as they provide deep insight as to what the controlling factors are in these circuits. To provide some guidance $r_i \to R_B$ as $\beta \to \infty$ and $i_o/i_i \approx R_B/R_E$.

Question 1: part 15

The numerical parameters are given in Table 3.

	h_{FE}	v_o/v_i	i_o/i_i	r_i [kΩ]	r_o [kΩ]
non-degenerated	800	-161.243	-340.411	2.713	5.000
degenerated	800	-3.629	-12.813	17.077	5.000
non-degenerated	110	-140.002	-92.777	10.356	5.000
degenerated	110	-3.590	-11.564	15.508	5.000

Table 3: Numerical solutions for the small signal properties amplifier of question 1 with and without emitter degeneration.

Question 2: A Common Base Circuit

This question is about a capacitively coupled common base amplifier.

1. Find the DC conditions of the common base circuit in Figure 7 assuming the base current of Q_1 can be ignored.

2. Find the DC conditions again but taking into consideration the base current. Perform your calculations for the full range of h_{FE}. Find the range of h_{FE} from the On Semiconductor MJE340 datasheet.

3. Explain (briefly, using bullet points for example) the job of each component in the circuit.

4. Draw and label the small signal equivalent circuit for Figure 7.

5. Calculate the small signal transconductance, g_{m}, and base emitter resistance, r_{be} for the range of h_{FE} given in the Fairchild Semiconductor datasheet. You may assume that the transistor stage will be operated at low frequencies and therefore $\beta = h_{\mathrm{FE}}$

6. Assuming the capacitors are short circuit at all frequencies of interest, show that the input resistance of the amplifier circuit in Figure 7 is given by (53).

7. Assuming the capacitors are short circuit at all frequencies of interest, show that the output resistance of the amplifier circuit in Figure 7 is R_C.

8. Assuming the capacitors are short circuit at all frequencies of interest, show that the transresistance (output voltage / input current) gain of the common base circuit shown in Figure 7 is given by (55).

9. Derive an expression for the current gain. Solution: (56).

10. Derive an expression for the voltage gain. Solution: (57).

11. Practical transistors have a physical resistance between the active part of the base region and the transistor package leg. This is partly made from the ohmic bond-wire resistance inside the package and partly made from the ohmic resistance of the semiconductor between the position at which the bond wire is attached to the semiconductor and the position of the active part of the base material. Draw the small signal equivalent circuit assuming that this base spreading resistance, r_b, appears in series with the base leg. C_1 is still short circuit at all frequencies of interest.

12. Re-derive your small signal results so far assuming taking into account the base spreading resistance. The results are shown in (58) - (61).

13. Reflect on and then qualitatively describe (i.e. in words) the effect of the base spreading resistance on the stage's small signal parameters. Comment on the similarity of the feedback provided by lifting the base node in the common base circuit with the effects of degenerating the emitter in the common emitter circuit.

14. State the numerical values of the small signal metrics of performance with and without the base spreading resistance over the range of β. You may assume that the amplifier is operated at a low frequency and therefore $\beta = h_{FE}$

$$\frac{v_e}{i_{in}} = \frac{1}{\frac{g_m}{\beta} + g_m + \frac{1}{R_E}} \tag{53}$$

$$\frac{v_o}{i_o} = R_C \tag{54}$$

$$\frac{v_o}{i_{in}} = \frac{g_m\, R_L'}{\frac{g_m}{\beta} + g_m + \frac{1}{R_E'}} \tag{55}$$

$$\frac{\frac{\beta}{r_{be}}}{\frac{1}{r_{be}} + \frac{\beta}{r_{be}} + \frac{1}{R_E'}} \approx \alpha \tag{56}$$

$$\frac{v_o}{v_i} = \frac{g_m\, R_L'}{\left(\frac{1}{r_{be}} + g_m + \frac{1}{R_s} + \frac{1}{R_E}\right)} \tag{57}$$

$$\frac{v_e}{i_{in}} \approx \frac{r_b}{\beta} + \frac{1}{g_m} \tag{58}$$

$$\frac{v_o}{i_{in}} = \frac{R_L'}{\frac{1+\beta}{\beta} + \frac{1}{g_m\, R_E'} + \frac{r_b}{R_E'\, \beta}} \tag{59}$$

$$\frac{i_o}{i_{in}} = \frac{1}{\frac{1+\beta}{\beta} + \frac{1}{g_m\, R'_E} + \frac{r_b}{R'_E\,\beta}} \tag{60}$$

$$\frac{v_o}{v_{in}} = \frac{g_m\, R'_L}{R_s\left(\frac{g_m}{\beta} + \frac{1}{R_s} + \frac{g_m\, r_b}{\beta\, R_s} + g_m + \frac{1}{R_E} + \frac{g_m\, r_b}{\beta\, R_E}\right)} \tag{61}$$

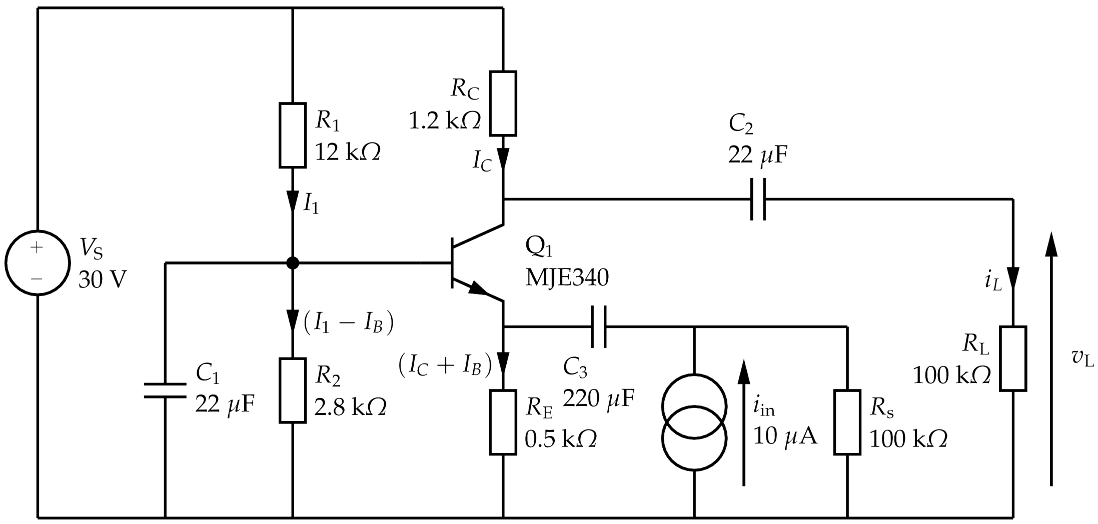

Figure 7: Common Base Amplifier Circuit

Question 2: part 1

Since the circuit topology for this common base amplifier is similar to the common emitter from question 1 we can proceed along similar lines. The base current is ignored in part 1 so we can probably solve by inspection. The voltage on the base is a potentially divided version of the supply voltage,

$$V_B = \frac{V_S\, R_2}{R_1 + R_2} = \frac{30 \cdot 2.8}{12 + 2.8} = 5.676 \text{ V} \tag{62}$$

$$V_E = V_B - 0.7 = 4.976 \text{ V} \tag{63}$$

$$I_E = \frac{V_E}{R_E} = \frac{4.976}{500} = 9.95 \text{ mA} \tag{64}$$

$$I_E = I_C \because I_B = 0 \tag{65}$$

$$V_C = I_C\, R_C = 9.95 \cdot 1.2 = 11.942 \text{ V} \tag{66}$$

$$\therefore V_{CE} + V_E = 30 - 11.942 = 18.058 \text{ V} \tag{67}$$

$$I_1 = \frac{30}{12 + 2.8} = 2.027 \text{ mA} \tag{68}$$

Question 2: part 2

Part 2 of question 2 asks for the same analysis as part one but making no assumptions about I_B. It also requires us to do the analysis twice once for the minimum h_{FE} and once with the maximum h_{FE} that the On Semiconductor datasheet specifies. We should expect the minimum h_{FE} to require the largest I_B and we should design the circuit (if we were designing it as opposed to just analysing someone else's design) to use biasing resistors which will accommodate this higher value of I_B. The higher numerical value of I_B computed by using the lowest value of h_{FE} is, in fact, the *minimum* I_B we should design for. This is the value which will allow any transistor of this type to work in the circuit.

It will probably be necessary to write out some node equations for this circuit. Eagle eyed readers will note that from a DC conditions perspective this circuit shape is identical to the common emitter circuit shape which is shown with base current in Figure 3. Another way of expressing this is to observe that the decision to call an amplifier common emitter or common base is really just an expression of where its input and output are positioned and which terminal is common to both. We can use the same equations for this problem as in part 2 of question 1. From the On Semi datasheet $h_{\mathrm{FE}_{(\mathrm{min})}}$ is 30 and $h_{\mathrm{FE}_{(\mathrm{max})}}$ is 240. Starting with KVL around the base – emitter loop.

$$V_B = V_E + V_{BE} \tag{69}$$

KCL on the base node,

$$(I_1 - I_B)\ R_2 = (I_C + I_B)\ R_E + 0.7 \tag{70}$$

$$I_C = h_{\mathrm{FE}}\, I_B \tag{71}$$

$$(I_1 - I_B)\ R_2 = I_B\,(h_{\mathrm{FE}} + 1)\ R_E + 0.7 \tag{72}$$

KVL around the biasing loop and power supply

$$V_S = (I_1 - I_B)\ R_2 + I_1\, R_1 \tag{73}$$

$$I_E = I_C + I_B \tag{74}$$

Having developed (69) to (74) from the circuit using KVL and KCL we can proceed by several methods and with several end goals in mind. We may choose to solve for I_B or I_1 and then use this solution with another equation to find a consistent solution for all unknown variables. We can choose to eliminate or use substitution to solve the equations. A solution involving matrices is also possible and favoured by some. To eliminate I_1, we could multiply (72) by $(1 + R1/R2)$ then add the result of this multiplication to (73) and then solve for I_B. Another possibility is to solve (73) for I_B and then substitute the result into (72) to solve for I_1. I used the substitution method and found (75).

$$I_1 = \frac{V_S\,(1 + h_{\mathrm{FE}})\,R_E + R_2\,(V_S - V_{BE})}{(1 + h_{\mathrm{FE}})\,(R_1 + R_2)\,R_E + R_1\,R_2} \tag{75}$$

Of course the form of your equation may be different to mine, and you may choose to factorise it differently. Next we need to relate I_B and I_1 by transposing (73).

$$I_B = \frac{I_1\,R_1 - V_S}{R_2} + I_1 \tag{76}$$

We need to relate I_C to I_B. Use $I_C = h_{\mathrm{FE}}\,I_B$ in conjunction with (76) to yield,

$$I_C = \left(\frac{I_1\,R_1 - V_S}{R_2} + I_1\right)\,h_{\mathrm{FE}} \tag{77}$$

Use Ohm's law to find V_C in terms of I_C, $V_C = I_C\,R_C$, substituting,

$$V_C = \left(\frac{I_1\,R_1 - V_S}{R_2} + I_1\right)\,h_{\mathrm{FE}}\,R_C \tag{78}$$

The voltage on the collector node with respect to ground (this is different to V_C which is the voltage across the collector resistor, R_C) is,

$$V_L = V_S - \left(\frac{I_1\,R_1 - V_S}{R_2} + I_1\right)\,h_{\mathrm{FE}}\,R_C \tag{79}$$

The quiescent (DC) conditions for the common emitter amplifier circuit of question 1 are listed in Table 4.

Question 2 part 3

Since the circuit topologies are similar for question 1 and question 2 we may presume that the purpose of many of the components are similar too. In Figure 7,

- V_S is the DC power supply. It supplies the current and voltage (and hence power) to run the transistor. The transistor effectively modulates the DC supply voltage with a (hopefully amplified) copy of the input waveform.

- i_{in} and R_S represent the Norton equivalent current source and resistance of the circuit or system driving our transistor amplifier.

h_{FE}	240	30
I_1 [mA]	2.0347	2.0800
I_B [μA]	40.5283	280.0000
I_C [mA]	9.7268	8.4000
V_C [V]	11.6722	10.0800
V_L [V]	18.3278	19.9199

Table 4: DC conditions for the common base amplifier circuit of question 2 for the extremes of h_{FE}. Notice how the wide variation in h_{FE} doesn't upset the DC conditions very much. This will hold as long as h_{FE} is much greater than unity and is a sign of a good circuit topology.

- C_1 is used to decouple the base node to ground such that an AC signal will choose the low impedance of the C_1 over the moderate resistance of R_1 and R_2 in order to get to ground. The result is that from the signal's point of view, R_1 and R_2 do not exist and therefore can not provide feedback. The existence of C_1 does not change the effect of R_1 and R_2 on the DC or thermal conditions however.
- C_2 and C_3 couple the input and output signals into the amplifier while preventing the DC component of both the source and the transistor affecting each-other in the case of C_3 and preventing the transistor stage DC conditions interfering with the operation of the next stage in the case of C_2
- R_L is the load placed on our amplifier by the circuit which it is connected to. In other words R_L represents in the input resistance of whatever is connected to the output of our amplifier.
- R_1 & R_2 set up the biasing voltage on the base of Q_1, but are also a source of negative feedback at signal frequencies if they are not decoupled by C_1.
- R_C exists to develop a voltage in response to the collector current (i.e. a resistor to develop the output voltage swing across). The size of R_C and the designers choice of I_C set the quiescent output voltage. The size of R_C (and R_L) also affects the gain of the amplifier.
- R_E allows us to make the circuit somewhat independent of the transistor h_{FE} by providing feedback from output to input.

Question 2 part 4

The small signal model for the common base amplifier of Figure 7 in the mid-band (where all capacitors are short circuit) is shown in Figure 8.

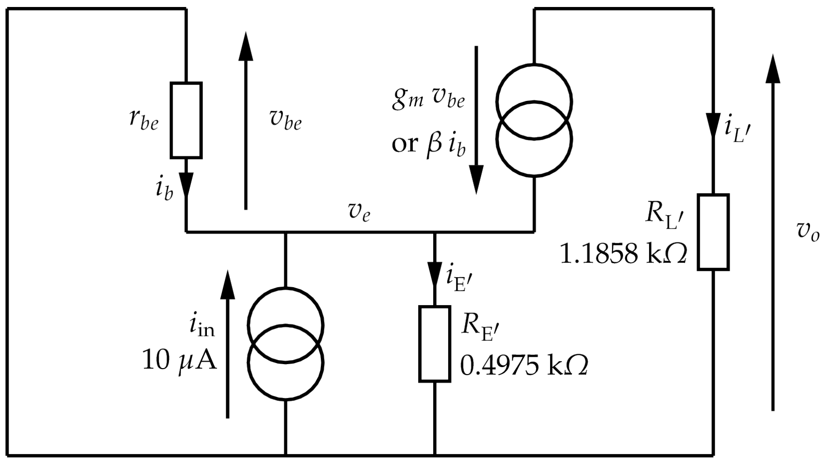

Figure 8: The small signal model of the common base amplifier circuit of question 2 assuming all capacitors are short circuit at signal frequencies. $R_{E'}$ is the parallel combination of R_E and R_S. $R_{L'}$ is the parallel combination of R_L and R_C.

h_{FE}	g_m [A/V]	r_{be} [Ω]
240	0.37591	638.442
30	0.32464	92.411

Table 5: Small signal parameters for the common base amplifier circuit of question 2 for the extremes of h_{FE}.

Question 2 part 5

The small signal parameters are computed by the same method as question 1. The results are shown in table 5.

Question 2 part 6

The analysis for the small signal metrics of performance of this amplifier, and all low frequency linear circuits are developed by the same methods - Kirchhoff's laws and Ohm's law. For the input resistance we can begin by using KCL on the emitter node,

$$i_b + g_m\, v_{be} + i_{in} = i_E \tag{80}$$

KCL on the collector node,

$$g_m\, v_{be} = i'_L \tag{81}$$

and by KVL around the base emitter loop,

$$v_e + v_{be} = 0 \tag{82}$$

we know that r_{in} is the driving point impedance looking into the emitter. If we define the voltage on the emitter with respect to ground as v_e we can write,

$$r_{in} = \frac{v_e}{i_{in}} \tag{83}$$

from (80),

$$-\frac{v_e}{r_{be}} - g_m\, v_e + i_{in} = \frac{v_e}{R_E} \tag{84}$$

collecting terms in v_e,

$$v_e \left(-\frac{1}{r_{be}} - g_m - \frac{1}{R_E} \right) = -i_{in} \tag{85}$$

dividing by i_{in},

$$\frac{v_e}{i_{in}} = -\frac{1}{-\frac{1}{r_{be}} - g_m - \frac{1}{R_E}} \tag{86}$$

tidying up the inversions,

$$\frac{v_e}{i_{in}} = \frac{1}{\frac{g_m}{\beta} + g_m + \frac{1}{R_E}} \tag{87}$$

Ideally we would like i_{in} to flow into the emitter of the transistor rather than through the emitter biasing resistor therefore R_E should be much higher than r_{in} and the $1/R_E$ term in the denominator is therefore not significant. $1/r_{be} = g_m/\beta$, assuming $\beta >> 1$ the g_m/β term in the denominator is insignificant compared to the g_m term. Therefore $r_{in} \approx 1/g_m$.

Question 2 part 7

This is the same argument as for the output resistance of Question 1. The output resistance for the common base and common emitter is only significantly affected by the transistor if r_{ce} or c_{cb} are included in the model. These are often needed to properly analyse the effect of output impedance on a current source for example.

Question 2 part 8

This analysis follows similar lines as for the input resistance. This analysis is performed using g_m and v_{be} rather than β and i_b, but the choice is yours. Start by summing currents at the emitter node,

$$i_b + g_m\, v_{be} + i_{in} - i_e = 0 \tag{88}$$

Summing currents at the collector node,

$$g_m\, v_{be} = -i_o = -\frac{v_o}{R_L'} \tag{89}$$

From (88),
$$\frac{-v_{be}}{r_{be}} + g_m\, v_{be} + i_{in} + \frac{v_{be}}{R'_E} = 0 \tag{90}$$
Make v_{be} the subject,
$$v_{be} = \frac{i_{in}}{\frac{1}{r_{be}} + g_m + \frac{1}{R'_E}} \tag{91}$$
Substituting (91) into (89),
$$v_o = \frac{-g_m - i_{in}\, R'_L}{\frac{1}{r_{be}} + g_m + \frac{1}{R'_E}} \tag{92}$$
transpose for v_o/i_{in},
$$\frac{v_o}{i_{in}} = \frac{g_m\, R'_L}{\frac{g_m}{\beta} + g_m + \frac{1}{R'_E}} \tag{93}$$
By similar arguments as for the input resistance the transresistance is approximately R_L. The g_m in the numerator and denominator cancel because all other denominator terms are insignificant.

Question 2 part 9

Part 9 asks for the current gain. This analysis is almost identical for the transimpedance except that the substitution in (89) of $i_o = -\frac{v_o}{R'_L}$ is not made such that i_o remains our output parameter and we seek i_o/i_{in}. Therefore,
$$\frac{i_o}{i_{in}} = \frac{-g_m}{\frac{1}{r_{be}} - g_m - \frac{1}{R'_E}} \tag{94}$$
If we substitute $g_m = \beta/r_{be}$ for the g_m in the numerator and denominator we have,
$$\frac{\frac{\beta}{r_{be}}}{\frac{1}{r_{be}} + \frac{\beta}{r_{be}} + \frac{1}{R'_E}} \tag{95}$$
which is,
$$\frac{\beta}{\frac{r_{be}}{r_{be}} + \frac{\beta\, r_{be}}{r_{be}} + \frac{1}{R'_E}} \tag{96}$$
If $1/R'_E$ is small compared to β (i.e. R'_E is large compared to β), (96) reduces to,
$$\frac{i_o}{i_{in}} = \frac{\beta}{1+\beta} = \alpha \tag{97}$$

Question 2 part 10

The voltage gain can be found by considering v_e/i_{in} while using a current source input or by deriving the input impedance and then using a source transformation on the current source to show that a Thévenin equivalent v_{in} is potentially divided between R_s and r_{in}. A more "brute force" approach is to derive the voltage gain using a Thévenin source directly, this is easier to understand but does involve some unnecessary work. The new small signal circuit is shown in Figure 9.

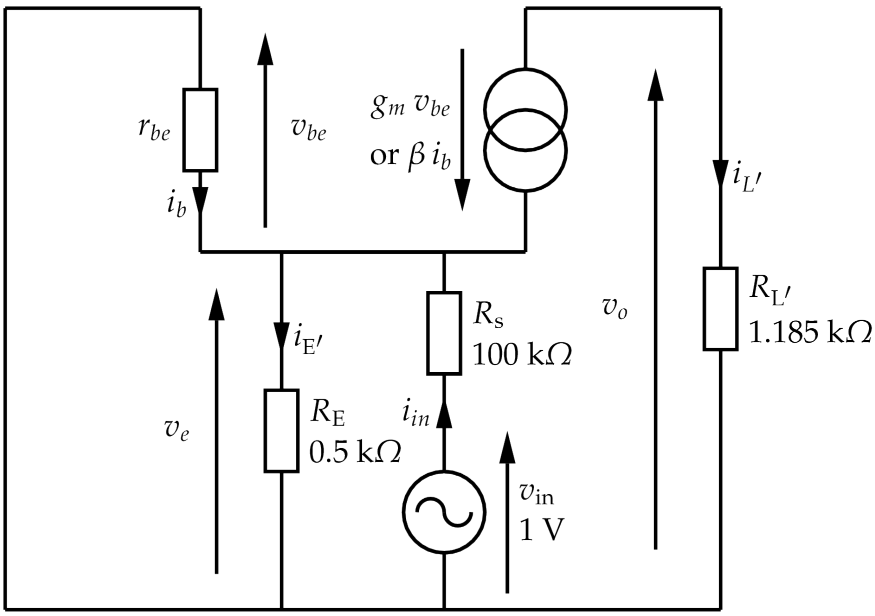

Figure 9: Common base amplifier circuit small signal model for voltage gain.

Summing currents at the emitter,

$$\frac{v_{be}}{r_{be}} + g_m\, v_{be} + \frac{v_i - v_e}{R_s} = \frac{v_e}{R'_E} \tag{98}$$

Summing voltages around the input loop,

$$v_{be} = -v_e \tag{99}$$

substituting (99) into (98),

$$\frac{v_{be}}{r_{be}} + g_m\, v_{be} + \frac{v_i + v_{be}}{r_s} = -\frac{v_{be}}{R'_E} \tag{100}$$

factorising for v_{be},

$$v_{be}\left(\frac{1}{r_{be}} + g_m + \frac{1}{R_s} + \frac{1}{R_E}\right) = \frac{-v_i}{R_s} \tag{101}$$

making v_{be} the subject,

$$v_{be} = \frac{-v_i}{R_s\left(\frac{1}{r_{be}} + g_m + \frac{1}{R_s} + \frac{1}{R_E}\right)} \tag{102}$$

Summing currents at the collector,

$$v_o = -g_m\, v_{be}\, R'_L \tag{103}$$

substituting (102) into (103) and solving for the voltage gain,

$$\frac{v_o}{v_i} = \frac{g_m\, R'_L}{\left(\frac{1}{r_{be}} + g_m + \frac{1}{R_s} + \frac{1}{R_E}\right)} \tag{104}$$

Question 2 part 11

The small signal model including the base spreading resistance is shown in Figure 10.

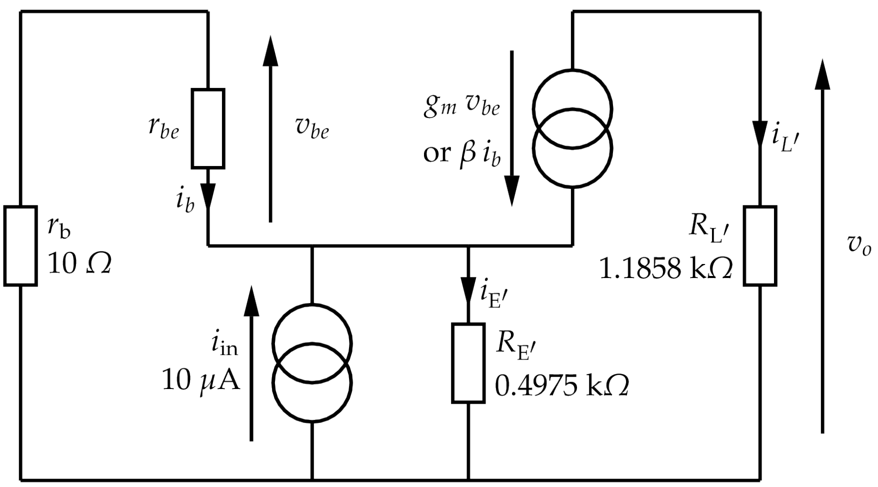

Figure 10: Common base amplifier circuit small signal model with finite base spreading resistance, r_b.

Question 2 part 12

This part of the question asks for re-derivation of all of the small signal metrics assuming r_b is non-zero. The derivations are somewhat more involved than when the base is ground, but they are instructive in showing the effect on negative feedback of the signal in the common base amplifier. Only the transimpedance and input resistance result is shown in full here. The methods to derive the other results are similar.

For the transimpedance use Figure 10. Begin by summing currents at the emitter,

$$i_{in} + i_b + \beta\, i_b = i_e \tag{105}$$

Summing voltages around the base loop,

$$v_b + v_{be} + v_e = 0 \tag{106}$$

using Ohm's law on (106)

$$i_b\, r_b + i_b\, r_{be} + i_e\, R'_E = 0 \tag{107}$$

summing currents on the collector node and using Ohm's law on R'_L,

$$v_o = -\beta\, i_b\, R'_L \tag{108}$$

solving (106) for i_e,

$$i_e = -\frac{i_b\,(r_{be} + r_b)}{R'_E} \tag{109}$$

substituting (109) into (105),

$$i_{in} + i_b + \beta\, i_b = -\frac{i_b\,(r_{be} + r_b)}{R'_E} \tag{110}$$

solve (110) for i_b,

$$i_b = \frac{-i_{in}}{1 + \beta + \frac{r_b e + r_b}{R'_E}} \tag{111}$$

substituting (111) into (107) and dividing numerator and denominator by β,

$$\frac{v_o}{i_{in}} = \frac{R'_L}{\frac{1+\beta}{\beta} + \frac{r_{be}}{\beta\, R'_E} + \frac{r_b}{R'_E\, \beta}} \tag{112}$$

then using $r_{be} = \beta / g_m$,

$$\frac{v_o}{i_{in}} = \frac{R'_L}{\frac{1+\beta}{\beta} + \frac{1}{g_m\, R'_E} + \frac{r_b}{R'_E\, \beta}} \tag{113}$$

Assuming $\beta >> 1$ then $(1 + \beta)/\beta \approx 1$. $g_m / R'_E << 1$ as we would like R'_E to be reasonably large c.f r_{in} and g_m is generally less than unity for small signal amplifiers. If β and R'_E are both reasonably large and r_b is something transistor designers like to minimise, the final term in the denominator should be much less than unity. Therefore the expression can be reduced to $v_o / i_{in} \approx R'_L$ as before but now we know that if we find ourselves in a common base application where a circuit is sensitive to r_b we can reduce its effects by choosing a transistor with a bigger β or re-designing our circuit to increase R'_E.

The current gain derivation is very similar to the transimpedance derivation but takes the form,

$$\frac{i_o}{i_{in}} = \frac{1}{\frac{1+\beta}{\beta} + \frac{1}{g_m\,R'_E} + \frac{r_b}{R'_E\,\beta}} \tag{114}$$

This simplifies to $i_o/i_{in} = \alpha$ by the same logic as used for the transimpedance above. α is sometimes called the common base current gain just as β is sometimes called the common emitter current gain.

The input impedance including r_b is derived by the same method but we seek $r_{in} = v_e/i_{in}$. Begin by summing currents at the emitter, and note that $i_e = v_e/R_E$. Also recall that the source impedance does not form part of the input impedance of an amplifier.

$$i_{in} = (\beta + 1)\; i_b = \frac{v_e}{R_E} \tag{115}$$

summing voltages around the base emitter loop

$$i_b\,r_b + i_b\,r_{be} + v_e = 0 \tag{116}$$

isolate (115) for i_b,

$$i_b = \frac{\frac{v_e}{R_E} - i_{in}}{\beta + 1} \tag{117}$$

Substituting (117) into (116),

$$\left(\frac{v_e}{R_E\,(\beta+1)} - \frac{i_{in}}{\beta+1}\right)\;(r_b + r_{be}) + v_e = 0 \tag{118}$$

after some manipulation,

$$\frac{v_e}{i_{in}} = \frac{\frac{r_b+r_{be}}{\beta+1}}{\frac{r_b+r_{be}}{R_E\,(\beta+1)} + 1} \tag{119}$$

Looking at (119) if $R_E >> 1$ and $\beta >> 1$ then the fraction in the denominator is insignificant compared to the 1. and the input resistance reduces to,

$$r_{in} = \frac{r_b + r_b e}{\beta + 1} \tag{120}$$

using $r_{be} = \beta/g_m$,

$$r_{in} = \frac{r_b + \frac{\beta}{g_m}}{\beta + 1} \tag{121}$$

and if $\beta >> 1$,

$$r_{in} = \frac{r_b}{\beta} + \frac{1}{g_m} \tag{122}$$

Therefore a non-zero r_b increases the input impedance of amplifier. This is undesirable as the common base is a current input amplifier and therefore r_{in} should be as low as possible. We can minimise the effect of r_b by ensuring we choose a transistor with a large β.

	β	v_o/v_i	i_o/i_i	r_i [Ω]	r_o [kΩ]	v_o/i_i [V/A]
non-degenerated	240	11.7460×10^{-3}	0.9906	2.635	1.200	1174.596
degenerated	240	11.7450×10^{-3}	0.9905	3.282	1.200	1174.499
non-degenerated	30	11.4068×10^{-3}	0.9620	2.963	1.200	1140.685
degenerated	30	11.3995×10^{-3}	0.9614	3.289	1.200	1139.951

Table 6: Numerical solutions for the small signal properties of the common base amplifier in question 2 with and without base degeneration.

Question 2 part 13

This question asks for a description the effect of r_b as a feedback mechanism. Essentially the effect of r_b (or a lack of decoupling of the base node) is analogous to the effect of a non-decoupled emitter resistor in the common emitter circuit. It is a form of negative feedback which acts to lower the gain of the amplifying stage. r_b drops a voltage proportional to i_b. i_b is a function of i_{in} and because the voltage across r_b will subtract from v_{be}, the larger r_b the more v_{be} is reduced and hence i_c is reduced and so the output voltage is reduced.

Question 2 part 14

The various metrics of performance are computed by inserting the resistor values and small signal parameters for the circuit of Figure 7. Numerical values are shown in Table 6. One metric that we have not considered is the transconductance (of the whole amplifier as opposed to the transconductance of the transistor). However, since we like to think about the common base amplifier as being current driven it is not very productive to ask about a metric which is i_o/v_{in}, similarly it is unlikely that one would quote the voltage gain of a common base amplifier.

Question 3: An Emitter Follower Circuit

This question is about a capacitively coupled emitter (common collector) follower amplifier, shown in Figure 11. This emitter follower stage is used to drive a 16 Ω loudspeaker represented by R_E. The DC current biasing the stage also flows through R_E. This is often not practical but for the sake of making the question easier we will assume that this is a magical speaker (from my office...) that doesn't mind having a large DC component of current flowing through it. Of course the DC current dissipates power in the speaker but this would not be useful output power (sound) it would be heat. It would also hold the voice coil away from the center position but as we have said all these problems are ignored for the sake of simplicity.

1. Find the DC conditions of the emitter follower circuit in Figure 11 assuming the base current of Q_1 can be ignored. Choose V_B such that V_L, the emitter voltage, is half way between the power supply and ground, thereby providing the largest possible output voltage swing.

2. Find the DC conditions again but taking into consideration the base current. Perform your calculations for the full range of h_{FE}. Find the range of h_{FE} from the On Semiconductor MJ15003 datasheet.

3. Explain (briefly, using bullet points for example) the purpose of each component in the circuit.

4. Sketch the output characteristic (V_{CE} *vs* I_C as a function of V_{BE} or I_B), add the operating point and the load line. On secondary axes, sketch the time dependent sinusoidal waveforms showing how the operating point moves according to the input signal, V_{in} and the output signal, V_L that results from this input.

5. Draw and label the small signal equivalent circuit for Figure 11.

6. Calculate the small signal transconductance, g_{m}, and base emitter resistance, r_{be} at the operating point for the range of h_{FE} given in the On Semiconductor datasheet. You may assume that the transistor stage will be operated at low frequencies and therefore $\beta = h_{\mathrm{FE}}$. Calculate the g_m and r_{be} at the maximum and minimum collector current based on the amplitude of the input waveform. Describe the effect will the variation of g_m and r_{be} have over the course of one cycle on the shape of the voltage and current waveforms in the circuit. To simplify your discussion you may assume β has no I_C dependence and that neither β nor g_m depend on temperature (or that the transistor will not get hot - same thing).

7. Based on the size of the input signal, the DC conditions you've calculated and your knowledge of electronic circuits, how valid is the small signal assumption in this case?

8. Assuming C_1 is short circuit at all frequencies of interest, show that the input resistance of the amplifier circuit in Figure 11 is given by (123). Comment on the size of R_s compared to the input resistance, what would you expect to find when evaluating the voltage gain of this stage.

9. Assuming C_1 is short circuit at all frequencies of interest, show that the output resistance of the amplifier circuit in Figure 11 is given by (124).

10. Assuming C_1 is short circuit at all frequencies of interest, show that the voltage gain of the circuit shown in Figure 11 is approximately unity.

11. Develop an expression for the current gain, determine its maximum value and the conditions required to reach that maximum.

12. Calculate the quiescent power dissipation in Q_1 and R_E.

13. Calculate the average power dissipated in the loudspeaker, R_L in one cycle if $R_s = 0.1\ \Omega$ and if $R_s = 600\ \Omega$. Qualitatively, do these figures relate to the earlier input resistance derivation?

14. Derive an expression for the instantaneous power dissipation in the transistor, Q_1. You may assume that the power dissipated in the transistor is the product of I_C and V_{CE} which will both vary *approximately* sinusoidally given a sinusoidal input. *Hint: this involves some integration of sines and cosines.*

15. Using your derivation find the input signal amplitude which results in the highest power dissipation in the *transistor*.

16. Show that the highest possible efficiency of this circuit is 25%. You may neglect losses in R_1 and R_2.

17. What is the conduction angle of Q_1? What class of operation is this stage operating in?

$$r_{in} \approx R_B \tag{123}$$

where $R_B = R_1 || R_2$.

$$r_o \approx \frac{1}{g_m} + \frac{R_B}{\beta} \tag{124}$$

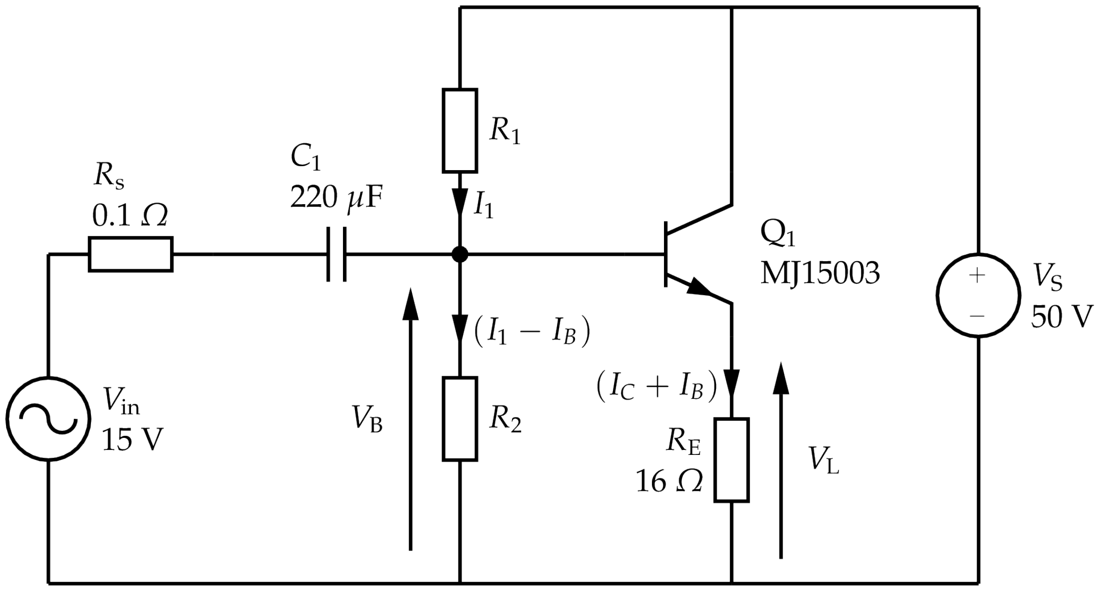

Figure 11: Emitter Follower Amplifier Circuit

Question 3 part 1

This part of the question asks for the DC conditions. This should be familliar if question 1 or question 2 has been attempted already. The question tells us that the emitter is at 25 V with respect to ground so we know $V_B = 25.7$ V. We also know that $I_E = I_C$ because the question requires us to ignore I_B.

$$I_C = \frac{V_L}{R_E} = \frac{25}{16} = 1.5625 \text{ A} \tag{125}$$

As long as we're ignoring I_B, R_1 and R_2 can be as big or small as you like but they must form a potential divider with V_S such that V_B is at 25.7 V. Let's choose $R_2 = 39\ \Omega$. The resason for the very low value will become apparent when we attack part 2 in which I_B is not neglected. The value for R_1 is given by Ohm's law,

$$R_1 = \frac{V}{I} = \frac{50 - (25 + 0.7)}{\frac{25+0.7}{39}} = 36.87\ \Omega \tag{126}$$

Question 3 part 2

Now I_B is involved we must use a relationship between I_C and I_B, the large signal common emmitter current gain h_{FE},

$$I_B = \frac{I_C}{h_{FE(\min)}} = \frac{1.5625}{25} = 62.5 \text{ mA} \tag{127}$$

The rule of thumb for base current is that it should be ten times lower than the current flowing in the biasing network. This is the reason for the very low choice of R_1 and R_2 in part 1. Keeping our choice of $R_2 = 39\ \Omega$ and summing currents at the base node.

$$I_1 - I_B = \frac{V_B}{R_2} = \frac{25 + 0.7}{39} \tag{128}$$

$$I_1 = 721.47 \text{ mA} \tag{129}$$

We can see that I_1 is slightly more than ten times I_B, so our rule of thumb is intact.

$$R_1 = \frac{50 - (25 + 0.7)}{I_1} = 33.68\ \Omega \tag{130}$$

Choosing the other extreme of h_{FE}

$$I_B = \frac{I_C}{h_{FE(\max)}} = \frac{1.5625}{150} = 10.42 \text{ mA} \tag{131}$$

$$I_1 - I_B = \frac{V_B}{R_2} = \frac{25 + 0.7}{39} \tag{132}$$

$$I_1 = 669.4 \text{ mA} \tag{133}$$

$$R_1 = \frac{50 - (25 + 0.7)}{I_1} = 36.30\ \Omega \tag{134}$$

The very low values of resistors in the biasing network pass a large DC current from the supply to ground (but this is necessary to maintain correct bias in the face of a range of h_{FE}, they disipate a good deal of power and would get hot. They also create a very low input resistance for this amplifier, which is highly undesirable. This circuit is not practical as it stands, but it is instructive to consider its faults and how we might ammeliorate them.

Question 3 part 3

This part asks for the purpose or “job” of each component in the circuit.

- There are several ways of thinking about the job of the transistor which are all compatible with eachother. It modulates the supply voltage with the input waveform to produce the correct waveshap accross R_L. Alternatively it controlls the current flowing in R_L in order to produce the correct voltage accross R_L. It provides current gain to the base current waveform which then flows in R_L thereby increasing the capacity of the input signal to deliver power to the load. It transforms the source resistance of V_S to make it possible to deliver the required power to R_L. All these describe the action of the transistor.

- The resistors R_1 and R_2 bias the transistors base (by providing a potential divider of V_S and in so doing control V_E and thefore the quiescent current in the load. They also provide the DC part of the base current required for the transistor to be in the forward active region.
- The load resistor disipates wanted, signal (AC), and quiescent, DC power. The AC component of this becomes music (programme material) the DC part is converted to heat.
- C_1 couples the input signal into the amplifier stage and prevents the DC voltage on V_B driving a DC current into the source.
- v_{in} is the signal source i.e. music or programme material of some kind, something that carries information in this case audio.
- R_S is the internal resistance of the source, v_{in}
- V_S is the power supply.

Question 3 part 4

This sketch is slightly different to the ones in the lecture notes and handouts because the question asks you to use v_{in} and V_L. This requires some thought as to how to properly represent everything because V_{CE} is on the horizontal axis of the characteristic is out of phase with the voltage across the load, V_L. This is because $V_{CE} + V_L = V_S$. If V_{CE} is rising V_L must be falling (provided V_S is constant, which it is). It is not possible to calculate the values of V_{BE} and I_B. They have been taken from a simulation (along with the output characteristic curves). But everything else can be figured out with a pad and pen and some thought. The diagram shown is linearised. This means that some liberty has been taken to ensure that the signals are sinusoidal despite the fact that real amplifiers have non-linearity. If one considers the VBE values: 659 mV - 499mV = 160 mV (negative peak to average) but 659 mV + 160 mV = 819 mV not 793 mV which is the positive peak value of V_{BE}. This discrepency is due to the exponential relationship between V_{BE} and I_C which lies at the heart of transistor opperation. The characteristic line for I_B = 8.56 mA doesn't quite meet the input and output waveforms. This is another example of how the diagram has been linearised, these points shold meet but this is only possible if the input and output waveforms have some distortion.

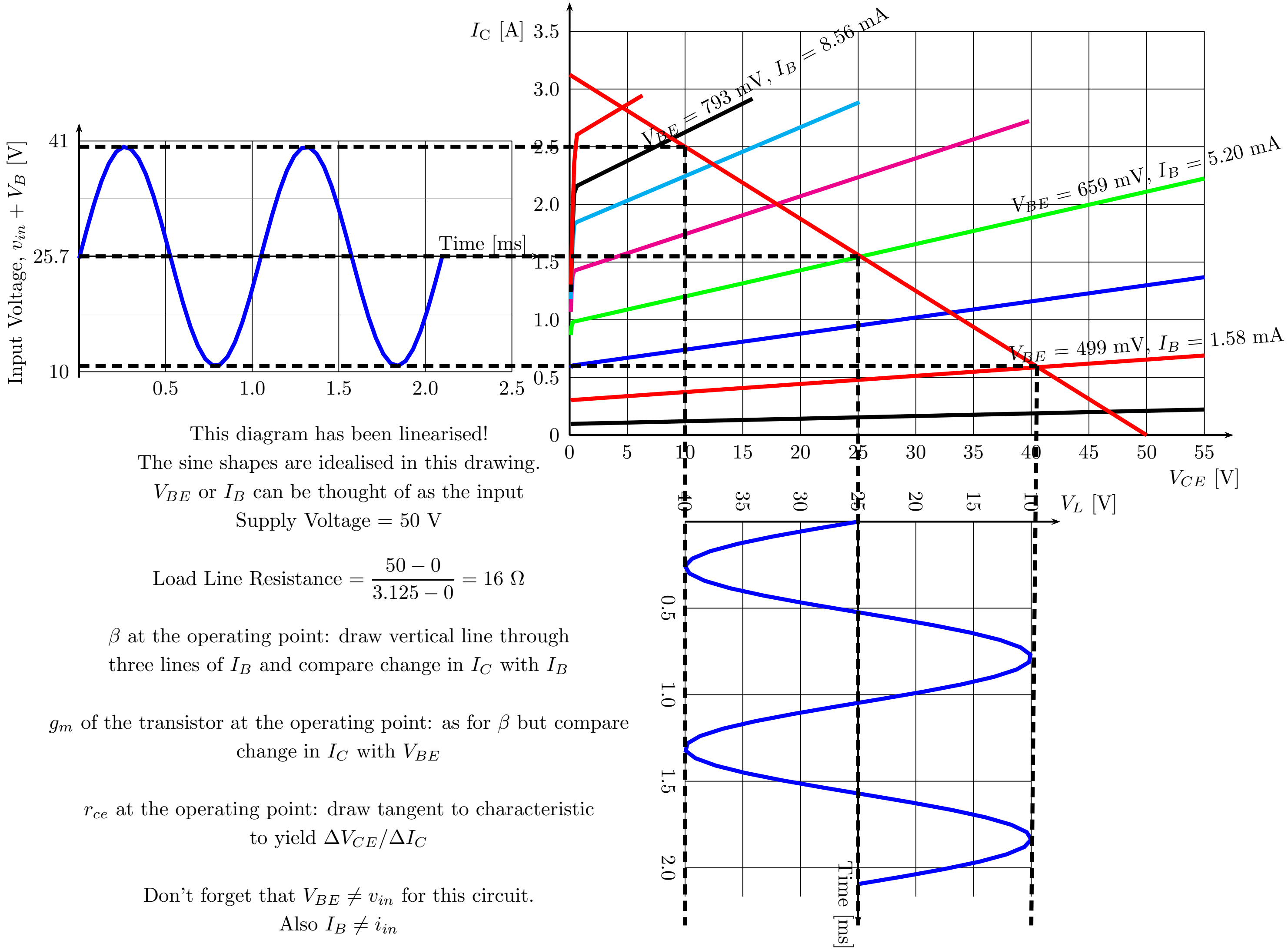

Figure 12: Emitter Follower Amplifier Circuit characteristic plot showing V_{in} and V_L, the transistor characteristics and the load line of R_E.

Question 3 part 5

The small signal model of Figure 11 is shown in Figure 13. Where the parallel combination of R_1 and R_2 is shown as $R_1||R_2$ and is called R_B for simplicity.

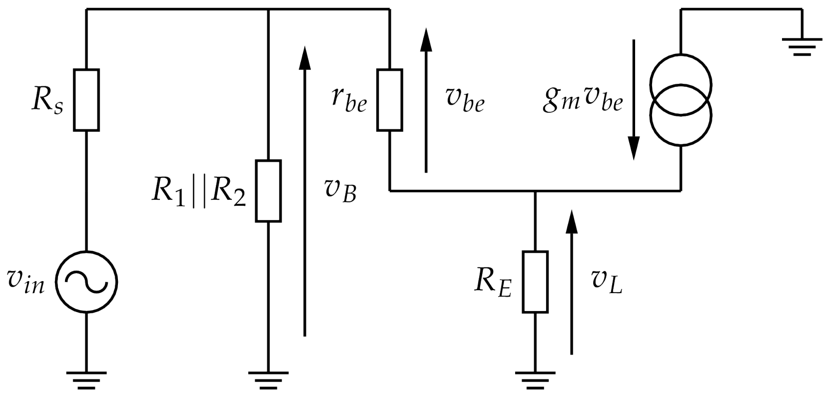

Figure 13: Small signal model of the emitter follower amplifier of Question 3.

Question 3 part 6

The transconductance is calculated by the usual method. To find it we need the maximum and minimum collector current over one full cycle of the output waveform. The transconductance at the opperating point (under quiescent conditions) is given by,

$$I_{C_Q} = \frac{V_{L_Q}}{R_L} = \frac{25}{16} = 1.5625 \text{ A} \tag{135}$$

$$g_{m_Q} = \frac{e\, I_{C_Q}}{k\, T} = \frac{1.6 \times 10^{-19} \cdot 1.5625}{1.38 \times 10^{-23} \cdot 300} = 60.386 \text{ A/V} \tag{136}$$

$$r_{be\text{min}} = \frac{\beta_{(\text{min})}}{g_m} = \frac{25}{60.386} = 0.414\ \Omega \tag{137}$$

$$r_{be\text{max}} = \frac{\beta_{(\text{max})}}{g_m} = \frac{150}{60.386} = 2.484\ \Omega \tag{138}$$

For the minimum transconductance,

$$I_{C(\text{min})} = \frac{V_{L(\text{min})}}{R_L} = \frac{25 - 15}{16} = 0.625 \text{ A} \tag{139}$$

$$g_{m(\text{min})} = \frac{e\, I_{C(\text{min})}}{k\, T} = \frac{1.6 \times 10^{-19} \cdot 0.625}{1.38 \times 10^{-23} \cdot 300} = 24.155 \text{ A/V} \tag{140}$$

$$r_{be\text{min}} = \frac{\beta_{(\text{min})}}{g_m} = \frac{25}{24.155} = 1.035\ \Omega \tag{141}$$

$$r_{be\max} = \frac{\beta_{(\max)}}{g_m} = \frac{150}{24.155} = 6.210\ \Omega \tag{142}$$

For the maximum transconductance,

$$I_{C(\max)} = \frac{V_{L(\max)}}{R_L} = \frac{25+15}{16} = 2.5\ \text{A} \tag{143}$$

$$g_{m(\min)} = \frac{e\, I_{C(\max)}}{k\, T} = \frac{1.6\times10^{-19}\cdot 2.5}{1.38\times10^{-23}\cdot 300} = 96.618\ \text{A/V} \tag{144}$$

$$r_{be\min} = \frac{\beta_{(\min)}}{g_m} = \frac{25}{96.618} = 0.259\ \Omega \tag{145}$$

$$r_{be\max} = \frac{\beta_{(\max)}}{g_m} = \frac{150}{24.155} = 1.553\ \Omega \tag{146}$$

The effect of g_m changing (and so r_{be} also changing assuming β is constant) will be a change in the input and output resistance of the amplifier over the course of one cycle of the input waveform. This will lead to some distortion of the waveform. For example as the output resistance increases some voltage will be lost accross this internal resistance and the voltage accross the load, v_L will be slightly smaller than would be expected. Similar arguements hold for the input resistance falling and loading the source somewhat (although the source impedance in this queseiton is purpously made very small to avoid the effect). The loading of the source would make the input voltage seem artificially small and hence the output voltage would be depressed as well. Since these effects depend on the part of the cycle that is presently passing into the amplifier the shape of the output waveform will be changed compared to the input. These effects can be interpreted as a change in voltage or current gain as well, but they are ultimately driven by the same underlying process which is a dependence of g_m on the signal. The effect of this change in parameters is mitigated somewhat by the large negative feedback which exists in this circuit. The circuit tries to keep the output the same as the input less the transistor V_{BE}. Even though the small signal parameters are changing significantly over one cycle the distortion introduced by the cirucit is quite small (about 0.15% for a 1 kHz fundemental when considering the first 21 harmonics i.e. the audio bandwidth, ignoring temperature effects)

Question 3 part 7

The small signal assumption is not valid. Even though the distortion is not very great g_m is changing markedly over the course of one signal cycle. This is therefore a large signal problem. This does not stop us using small signal techniques but we must remember that the answers we get from small signal analysis will vary with the signal.

Question 3 part 8

This is yet more small signal analysis, well done if you've done them all up to now, but, don't you have some other courses to work on as well? All of the analysis for this circuit is tricky because there is always feedback between the output and input. The input resistance is approximately equal to R_B in Figure 13. Using the small signal model shown in Figure 13 begin by summing voltages around the input loop,

$$v_{in} = v_e + v_{be} \tag{147}$$

Where v_e is the voltage on the emitter node ($= v_L$). We can also sume currents flowing into the base node,

$$i_{in} = i_B + i_b \tag{148}$$

where i_{in} is the current flowing from the source into the circuit, I_B is the current flowing in $R_B = R_1||R_2$ and i_b is the current flowing in r_{be}. Lastly we can sum currents flowing into the emitter node,

$$i_e = g_m\, v_{be} + \frac{v_{be}}{r_{be}} \tag{149}$$

where i_e is the signal current flowing in R_E. Use Ohm's law on (149) and transpose to obtain v_e,

$$v_e = \left(g_m\, v_{be} + \frac{v_{be}}{r_{be}} \right) R_E \tag{150}$$

and substitute into (147),

$$v_i = v_{be} \left(1 + \left(g_m + \frac{1}{r_{be}} \right) R_E \right) \tag{151}$$

Looking back at (148) we can use Ohm's law to obtain,

$$i_{in} = \frac{v_{in}}{R_B} + \frac{v_{be}}{r_{be}} \tag{152}$$

transposing for v_{be},

$$v_{be} = r_{be}\, i_{in} - \frac{v_{in}}{R_B}\, r_{be} \tag{153}$$

substituting (153) into (151) to remove v_{be},

$$v_{in} = r_{be} \left(i_{in} - \frac{v_i}{R_B} \right) \left(1 + \left(g_m + \frac{1}{r_{be}} \right) R_E \right) \tag{154}$$

Now some transposition is required where all the terms in v_{in} are arranged on one side of the equality and all the terms in i_{in} are on the other side. We will divide

v_{in} by i_{in} to obtain the input resistance r_{in}. I've skipped four lines of working and arrived at,

$$v_{in}\left(1+\frac{r_{be}}{R_B}+\frac{r_{be}\,g_m\,R_E}{R_B}+\frac{r_{be}\,R_E}{R_B\,r_{be}}\right)=i_{in}\left(r_{be}+r_{be}\,g_m\,R_E+\frac{r_{be}\,R_E}{r_{be}}\right) \tag{155}$$

dividing through,

$$r_{in}=\frac{v_{in}}{i_{in}}=\frac{r_{be}+r_{be}\,g_m\,R_E+\frac{r_{be}\,R_E}{r_{be}}}{1+\frac{r_{be}}{R_B}+\frac{r_{be}\,g_m\,R_E}{R_B}+\frac{r_{be}\,R_E}{R_B\,r_{be}}} \tag{156}$$

There are some obvious cancellations which are left in now for completeness. As usual r_{be} will be replaced with β and g_m to give,

$$r_{in}=\frac{\frac{\beta}{g_m}+R_E\,(\beta+1)}{1+\frac{\beta}{g_m\,R_E}+\frac{R_E}{R_B}\,(\beta+1)} \tag{157}$$

If we accept that the β/g_m in the numerator is not very significant comapred to $R_E\,\beta$ and that $\beta >> 1$ and that $\beta/\left(g_m\,R_E\right)$ also doesn't play a big part (put some numbers in if you don't like the idea of just getting rid of it). The expression becomes,

$$r_{in}\approx\frac{R_E\,\beta}{\frac{R_E}{R_B}\,\beta} \tag{158}$$

which simplifies to,

$$r_{in}=R_B \tag{159}$$

Taking the operating point value for g_m = 60.386 A/V, assuming β = 100 and R_B = 18.95 Ω the exact answer is r_{in} = 18.73 Ω. The approximation (that r_{in} = R_B) represents an error of about 1.18%, which is likely to be acceptable under most circumstances.

Question 3 part 9

The output resistnace may be found by injecting a current into the emitter and observing the voltage that exists due to that current, while the input is replaced by it's internal impedance. The small signal model shown in Figure 14. Summing currents flowing into the emitter,

$$i_b\,(\beta+1)+i_e-\frac{v_e}{R_E}=0 \tag{160}$$

Note that in the diagaram R_S appears in parallel with $R_B = R_1||R_2$ therefore R_B can be re-defined as $R_B = R_1||R_2||R_s$. This makes things easier in some respects but if we wanted to know what effect an increase in R_s had on the

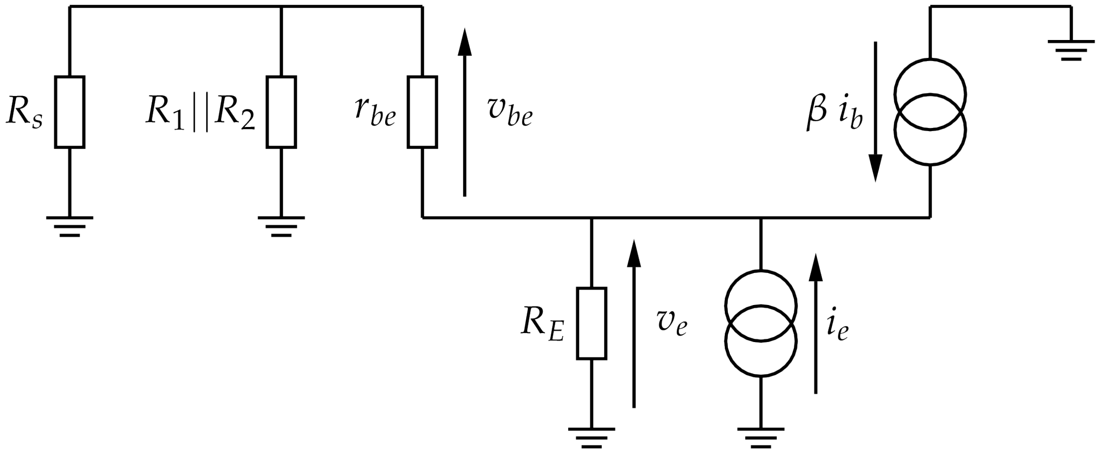

Figure 14: Small signal model of the emitter follower amplifier of Question 3 for derivation of output resistance.

output resistance we would need to avoid this simplification. Using Ohm's law, the base current is given by,

$$i_b = -\frac{v_e}{r_{be} + R_B} \tag{161}$$

substituting i_b into (161),

$$-\frac{v_e\,(\beta+1)}{r_{be} + R_B} + i_e - \frac{v_e}{R_E} = 0 \tag{162}$$

All that remains is some transposition to finr v_e/i_e,

$$-\,v_e\left(\frac{\beta+1}{r_{be} + R_B} + \frac{1}{R_E}\right) = -i_e \tag{163}$$

$$\frac{v_e}{i_e} = \frac{1}{\frac{\beta+1}{r_{be}+R_B} + \frac{1}{R_E}} \tag{164}$$

Replacing r_{be} with β and g_m

$$r_o = \frac{1}{\frac{\beta+1}{\frac{\beta}{g_m}+R_B} + \frac{1}{R_E}} \tag{165}$$

Assuming $1/R_E$ is small compared to the other term in the denominator we have,

$$r_o = \frac{\frac{\beta}{g_m} + R_B}{\beta+1} \tag{166}$$

then assuming $\beta >> 1$ this reduces to

$$r_o = \frac{\frac{\beta}{gm}}{\beta} + \frac{R_B}{\beta} \tag{167}$$

simplifying,

$$r_o = \frac{1}{g_m} + \frac{R_B}{\beta} \tag{168}$$

This is significant. Physically it means that when one looks into the emitter of a transistor one sees the source impedance, R_S (which is part of R_B in this case reduced by a factor of β. Looking at that another way it means the ability of the source to drive current into a load has increased by a factor of β. If, as in the next quesiton a darlington is involved, we can see intuetively that the βs could "stack up" somehow. In fact you are asked to show this in question 4.

Question 3 part 10

The voltage gain can be derrived using the small signal model in Figure 13. Hopefully it's evident that the voltage gain is nearly unity. Summing currents at the base node,

$$\frac{v_{in} - v_B}{R_S} = \frac{v_B - 0}{R_E} + \frac{v_{be}}{r_{be}} \tag{169}$$

summing currents at the emitter node,

$$\frac{v_{be}}{r_{be}} + g_m\, v_{be} = \frac{v_L}{R_E} \tag{170}$$

Summing voltages around the input loop,

$$v_B = v_L + v_{be} \tag{171}$$

Substituting (171) into (169),

$$\frac{v_{in} - (v_L + v_{be})}{R_S} = \frac{v_L + v_{be}}{R_B} + \frac{v_{be}}{r_{be}} \tag{172}$$

Collecting terms in v_{be}, v_L and v_{in} (I've skipped a few lines of expansion),

$$v_{be}\left(-\frac{1}{R_S} - \frac{1}{R_B} - \frac{1}{r_{be}}\right) = v_L\left(\frac{1}{R_B} + \frac{1}{R_S}\right) + v_{in}\left(-\frac{1}{R_S}\right) \tag{173}$$

An expression for v_{be} is needed to leave only v_{in}, v_L and resistors. We can transpose (170),

$$v_{be}\left(\frac{1}{r_{be}} + g_m\right) = \frac{v_L}{R_E} \tag{174}$$

substituting (174) into (173),

$$\frac{v_L\left(\frac{1}{R_B}+\frac{1}{R_S}\right)+v_{in}\left(-\frac{1}{R_S}\right)}{\left(-\frac{1}{R_S}-\frac{1}{R_B}-\frac{1}{r_{be}}\right)}\left(\frac{1}{r_{be}}+g_m\right)=\frac{v_L}{R_E} \tag{175}$$

After a further three lines of transposition,

$$\frac{v_L}{v_{in}}=\frac{\frac{1}{R_S}\left(\frac{1}{r_{be}}+g_m\right)}{\left(\frac{1}{R_B}+\frac{1}{R_S}\right)\left(\frac{1}{r_{be}}+g_m\right)+\frac{1}{R_E}\left(\frac{1}{R_S}+\frac{1}{R_B}+\frac{1}{r_{be}}\right)} \tag{176}$$

This doesn't at first glance appear to be equal to unity but working the usual magic should help. Removing r_be in favor of g_m and β,

$$\frac{v_L}{v_{in}}=\frac{\frac{1}{R_S}\left(\frac{g_m}{\beta}+g_m\right)}{\left(\frac{1}{R_B}+\frac{1}{R_S}\right)\left(\frac{g_m}{\beta}+g_m\right)+\frac{1}{R_E}\left(\frac{1}{R_S}+\frac{1}{R_B}+\frac{g_m}{\beta}\right)} \tag{177}$$

$\frac{g_m}{\beta} << g_m$ assuming $\beta >> 1$ so the bracket in the numerator and denominator $\left(\frac{g_m}{\beta}+g_m\right)$ is approximately g_m. The bracket in the denominator which is multiplied by $1/R_E$ will be small (put some numbers in if you're not convinced). This boils it down to,

$$\frac{v_L}{v_{in}}=\frac{\left(\frac{1}{R_S}\right)g_m}{\left(\frac{1}{R_B}+\frac{1}{R_S}\right)g_m} \tag{178}$$

The g_m in the numerator and denominator cancel to leave,

$$\frac{v_L}{v_{in}}=\frac{\frac{1}{R_S}}{\frac{1}{R_B}+\frac{1}{R_S}} \tag{179}$$

Ideally $R_B >> R_S$ in order that this amplifier should not unduely load the source that drives it (thereby effectivly reducing the input signal) therefore,

$$\frac{1}{R_B}+\frac{1}{R_S}\approx\frac{1}{R_S} \tag{180}$$

and (179) reduces to,

$$\frac{\frac{1}{R_S}}{\frac{1}{R_S}}=1 \tag{181}$$

The exact value for the voltage gain is approximatley 0.997 V/V so the approxiation that it is unity only has an error of 0.3% in this case, which is quite acceptable.

Question 3 part 11

The current gain of the transistor is just its β but the current gain of the circuit as a whole is defined as the current into the load divided by the current entering from the source. The small signal model of Figure 13 can be used and we begin by summing currents flowing into the base node,

$$i_{in} = \frac{V_B}{R_B} + \frac{v_{be}}{r_{be}} \tag{182}$$

Summing currents at the emitter,

$$i_L = \frac{v_{be}}{r_{be}} + g_m\, v_{be} \tag{183}$$

summing voltages around the base emitter loop,

$$v_B = v_{be} + v_L \tag{184}$$

and we also know that,

$$v_L = i_L\, R_E \tag{185}$$

Substituting (185) into (184) and the result into (182),

$$i_{in} = \frac{R_E\, i_L + v_{be}}{R_B} + \frac{v_{be}}{r_{be}} \tag{186}$$

r_{be} must be removed from (186) and i_L needs to be introduced somehow. transpose (183) for v_{be},

$$v_{be} = \frac{i_L}{\frac{1}{r_{be}} + g_m} \tag{187}$$

(187) into (186),

$$i_{in} = \frac{i_L\, R_E + \frac{i_L}{\frac{1}{r_{be}}+g_m}}{R_B} + \frac{i_L}{\left(\frac{1}{r_{be}} + g_m\right)\, r_{be}} \tag{188}$$

Collecting terms in i_L (the numerator and denominator of the first fraction on the right hand side are multiplied by $(1/r_{be} + g_m)$),

$$i_{in} = i_L \left(\frac{R_E\left(\frac{1}{r_{be}} + g_m\right) + 1}{R_B\,\left(\frac{1}{r_{be}} + g_m\right)} + \frac{1}{g_m\, r_{be} + 1} \right) \tag{189}$$

transposing for i_L/i_{in} and replacing $r_b e$ with β and g_m

$$\frac{i_L}{i_{in}} = \frac{1}{\frac{R_E\left(\frac{g_m}{\beta}+g_m\right)+1}{R_B\left(\frac{g_m}{\beta}+g_m\right)} + \frac{1}{1+\beta}} \tag{190}$$

$g_m/\beta << g_m$ if $\beta >> 1$. Similarly $1/(1+\beta) \approx \beta$ if $\beta >> 1$. If we also assume that $R_E\, g_m >> 1$ (which is probably true in most cases),

$$\frac{i_L}{i_{in}} = \frac{1}{\frac{R_E}{R_B} + \frac{1}{\beta}} \tag{191}$$

In an ideal situation R_B is much much larger than R_E in order to avoid the amplifier stage loading the source. Looking at the equation it is evident that the current gain tends to β as $R_B \to \infty$. In the case of this amplifier however the current gain is *only* 1.17, which is very bad indeed, when we consider the voltage gain is approximately unity, this transistor stage is barely increasing the source's ability to deliver power to the load, it is essentially useless. The reason is a very large current must flow in the biasing network to maintain the transistor's operating point, because the base curent is quite large. As a result the source has to drive quite a large current through the biasing network as well. In essense the R_E/R_B fraction in this case is far too close to unity (16/18.95) to allow the stage to be effective. This can also be solved with a Darlington which is the subject of question 4.

Question 3 part 12

This part asks for the quiescent power disipation of the transistor and the load. For the transistor it's the quescent V_{CE} multiplied by the quiescent I_C which is $25 \cdot 1.5625 = 39.06$ W. In the load it's $I^2\, R_E$ or $\frac{V^2}{R_E}$. $1.5625^2 \cdot 16 = 39.06$ W.

Question 3 part 13

This part asks for the power in the load for a given value of R_S. The voltage gain expression (177) will be required. The answer will depend on your value for β. If $\beta = 100$ and $R_S = 0.1\ \Omega$ the voltage gain is 0.99367. meaning the voltage appearing accross R_E due to a 15 V peak input signal (15 V comes from Figure 14) is 14.9051 V and the power disipaged in R_E is given by,

$$P_{R_E} = \frac{25^2}{16} + \frac{\left(\frac{14.9051}{\sqrt{2}}\right)^2}{16} = 46.01 \text{ W} \tag{192}$$

where the first term is the quiescent power disipation in R_E and the sceond term is the signal power. The second term is only approximatley 7 W so most of the power disipation is not related to the signal i.e. the efficiency is low.

When the source resistance increases to 600 Ω things get considerably worse. The voltage gain becomes 0.03024! The voltage appearing across the load due to the source is then, 0.4536 V and the power in the load is,

$$P_{R_E} = \frac{25^2}{16} + \frac{\left(\frac{0.4536}{\sqrt{2}}\right)^2}{16} = 39.0625 + 6.42978 \times 10^{-3} \text{ W} \tag{193}$$

In other words, almost all of the power is quiescent and the signal provides approximately 6 mW of power. This is hopefully not too supprising. Looking back at the simplified input resistance (159) which is 18.95 Ω it is clear that an unfavorable potential divider is set up between R_S and the input resistance such that the amplifier stage heavily loads the source and power transfer from the source to the amplifier is very weak. Much of the power available from the source is lost in the source's internal resistance and little is transferred to the input of the amplifier. Hence the mizerly output from the amplifier and the apparent collapse of its voltage gain.

Question 3 part 14

This part is quite taxing. It's similar to analyses found in EEE223 and does not feature in EEE225. The question asks for the instantaneous power disipation, i.e. as a function of time in the transistor and in the load resistor. For the transistor the power disipation is approximately $V_{CE} \cdot I_C$, we can write V_{CE} as,

$$V_{CE} = V_S - V_p \sin(\omega t) - V_Q \tag{194}$$

Where V_S is the DC supply voltage, V_p is the peak amplitude of the sinusoidal voltage accross the load resistor, R_E and V_Q is the quiescent voltage across the transistor. Note that since the emitter of the transistor is half way between V_S and ground V_Q is the quiescent V_{CE} and the quiescent V_E. This situation provides the maximum possible signal swing before significant distortion. The current in the transistor is the same as the current in the load (they are in series),

$$I_C = \frac{V_p \sin(\omega t)}{R_E} + \frac{V_Q}{R_E} \tag{195}$$

In this case V_Q is the quiescent voltage across the load resistor, but as we have noted it is the same as in (194) if the emitter is biased to the midpoint of the supply. Multiplying,

$$P(t) = \left(\frac{V_p \sin(\omega t)}{R_E} + \frac{V_Q}{R_E}\right)(V_S - V_p \sin(\omega t) - V_Q) \tag{196}$$

Expanding the brackets and collecting terms in $\sin(\omega t)$

$$P(t) = -\frac{V_p^2 \sin(\omega t)^2}{R_E} + \left(-\frac{V_Q V_p}{R_E} + \frac{V_p(-V_Q + V_S)}{R_E}\right) \cdot \sin(\omega t) + \frac{V_Q(-V_Q + V_S)}{R_E} \tag{197}$$

now for the tricky part, to find the average power (197) must be integrated over one period, T,

$$P = \frac{1}{T}\int_0^T P(t)\,\mathrm{d}t. \tag{198}$$

The terms in sin and the unity terms are not too hard but the $\sin^2$ term may be frustrating. The half angle formula can be used which states,

$$\sin(\omega\, t)^2 \equiv 1 - \cos(2\,\omega\, t) \tag{199}$$

Performing the integral and after several lines of tidying up,

$$P = \frac{-2\,V_Q^2 + 2\,V_Q\,V_S - V_p^2}{2\,R_E} \\ + \frac{V_p^2\,\cos(\omega\, T)\sin(\omega\, T) + 4\,V_p\,V_Q\,\cos(\omega\, T) - 2\,V_p\,V_S\,\cos(\omega\, T) - 4\,V_Q\,V_p + 2\,V_p\,V_S}{2\,T\,R_E\,\omega} \tag{200}$$

which is a bit nasty. The relation $\omega = \frac{2\pi}{T}$ is inserted and magically all the sin and cos terms become either 1 or 0, leaving,

$$P = \frac{-2V_Q^2 + 2\,V_Q\,V_S - V_p^2}{2\,R_E} \tag{201}$$

Question 3 Part 15

Having derived the average power disipation in the transistor we are required to find the signal voltage that heats the transistor the most. One possible method is to plot the average power disipation as a function of V_p given some example numbers. However we can also take the derivative and set it equal to zero to find a turning point. Taking the second derivative would show that it is a maxima. Taking the derivative of (201) with respect to V_p and setting the gradient equal to zero,

$$\frac{\mathrm{d}}{\mathrm{d}\,V_p}\left(\frac{-2V_Q^2 + 2\,V_Q\,V_S - V_p^2}{2\,R_E}\right) = -\frac{V_p}{R_E} = 0 \tag{202}$$

transposing for V_p, yields,

$$V_p = 0 \tag{203}$$

Meaning that the maximum power dissipation in the *transistor* is when there is no signal at all. If you're not keen on this idea, plot P as a function of V_p for the numbers in Figure 11 in Matlab/Maple/Mathcad etc.

Question 3 part 16

The efficiency is a bit of a trick question. As usual it's useful power out over total power, however one must be careful about the definition of useful power. The power we're interested in is the signal power in the load, not the quiescent power in the load, that is "wasted" from our perspective.

$$\eta = \frac{\frac{1}{2}\cdot\frac{V_p^2}{R_E}}{\frac{-2V_Q^2+2\,V_Q\,V_S-V_p^2}{2\,R_E} + \frac{1}{2}\frac{2\,V_Q^2+V_p^2}{R_E}} \tag{204}$$

The numerator is the signal power in R_E. Don't forget that V_p is a peak value (which is how the 1/2 comes into it). The left fraction in the denominator is just (201) which is the power in the transistor and the right fraction in the denominator is an expression for the total average power in the load (quiescent and signal). It might seem like a good idea to differentiate this with respect to V_p to find a maxima, but plotting it against V_p will show that there is only a minima. With a bit of thought it should be possible to convince yourself that the best efficiency will be when V_p is as big as it can be. Since the quiescent point, V_Q is $V_S/2$ that's as big as V_p can get without either the top or bottom of the waveform being clipped off. You may also notice that R_E cancels. Simplifying (204)

$$\eta = \frac{1}{2} \frac{V_p^2}{V_Q V_S} \tag{205}$$

Substituting in our relation for V_S and V_p, $V_p = V_S/2$,

$$\eta = \frac{\left(\frac{V_S}{2}\right)^2}{V_Q V_S} \tag{206}$$

Substituting in values ($V_S = 50$ V, $V_Q = 25$ V) leads to $\eta = 25\%$. Another approach is to say that $V_p = V_Q$ which is also true under maximum signal swing condidtions. It provides the same result. Of course it is not possible to obtain this efficiency in practice only to approach it.

Question 3 part 17

This part asks for the conduction angle. Well there is only one transistor and the current in the load, given a sinusoidal input, should be sinusoidal as well. Therefore the transistor must conduct for the full cycle (360°). There is no "proof" of this, one has to think about it. Ask yourself, 'does the transistor ever switch off?'

Question 4: A Darlington Pair

One of the many problems with the circuit in question 3 is the very low input impedance. To ameliorate this a Darlington pair is often used in operational and discrete power amplifier output stages.

1. Re-draw Figure. 11 to make use of a Darlington pair. The upper transistor will be MJE340.

2. Design suitable component values to utilize the available rail voltage appropriately, include base current and the full range of h_{FE} in your calculations.

3. Explain briefly why the Darlington is an improvement.

4. Draw and label the small signal equivalent circuit for your circuit, you may assume that $R_B = R_1||R_2$ is very large compared to R_S and can be ignored.

5. Assuming C_1 is short circuit at all frequencies of interest, develop the input resistance of the Darlington emitter follower amplifier. You may assume that $R_B = R_1||R_2 >> R_S$ and therefore can be ignored. Attempt to find a form of your equation that can show the effect of N transistors cascaded. Comment on the effects of R_S on the stage voltage gain compared to the effects of R_S on the circuit in question 3.

6. Assuming C_1 are short circuit at all frequencies of interest, develop an expression for the output resistance of the amplifier. Similarly to the input resistance, try to arrive at a form of solution which shows the effect of N transistors in cascade.

7. Assuming the biasing network, ($R_B = R_1||R_2$) can be ignored, derive an expression for the current gain.

Question 4 part 1

The redrawn diagram could look something like Figure 15.

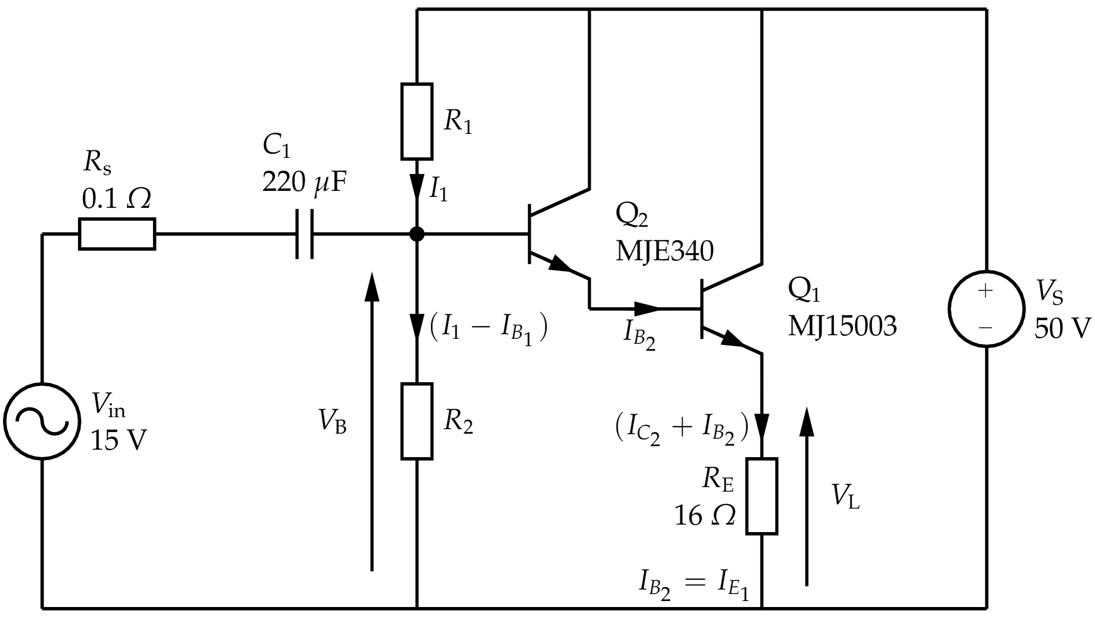

Figure 15: Emitter follower composed of a Darlington pair.

Question 4 part 2

The biasing conditions shouldn't pose too much difficulty if any of the first three quesitons have been mastered. The quiescent value of V_L should still be $V_S/2$ therefore, $V_B = 25 + 1.4$. $I_{E_2} = V_L/R_E = 25/16 = 1.5625$ A, as in question 3.

$$I_{B_2(\max)} = \frac{I_{E_2}}{\beta_{2(\min)}} = \frac{1.5625}{25} = 62.5 \text{ mA} \tag{207}$$

$$I_{B_2(\min)} = \frac{I_{E_2}}{\beta_{2(\max)}} = \frac{1.5625}{150} = 10.41 \text{ mA} \tag{208}$$

Note that $I_{B_2} = I_{E_1}$ and $I_{E_1} = I_{B_1}(1 + \beta_1)$ so,

$$I_{B_1(\max)} = \frac{I_{B_2}}{1 + \beta_{1(\min)}} = \frac{62.5 \text{ mA}}{1 + 30} = 201.6\ \mu\text{A} \tag{209}$$

$$I_{B_1(\min)} = \frac{I_{B_2}}{1 + \beta_{1(\max)}} = \frac{10.41 \text{ mA}}{1 + 240} = 43.2\ \mu\text{A} \tag{210}$$

Therefore the worst case I_{B_1} is 201.6 μA, and I_1 should be ten times greater than this value; approximately 2 mA.

$$R_2 = \frac{25 + 1.4}{2 \text{ mA}} \approx 13.2 \text{ k}\Omega \tag{211}$$

$$R_1 = \frac{50 - (25 + 1.4)}{2 \text{ mA}} \approx 11.8 \text{ k}\Omega \tag{212}$$

Question 4 part 3

The Darlington is a significant improvement because the *input resistance* and the *current gain* are both significantly increased.

Question 4 part 4

The input resistance for the darlington can be found by drawing a small signal model for both transistors and solving using the usual circuit analysis rules. However this is not a very inteligent way of attacking the problem. For example if we had a circuit with ten transistors, would we need to draw it out as a small signal model? How easy would it be to solve such a circuit? We can divide the problem into two small signal models, one for each transistor. In Figure 16 the emitter resistor r_{in_2} represents the input resistance of the lower transistor (MJ15003) while the upper transistor MJE340 is represented by the hybrid-π model. The key point about this small signal model is that the same model can be used for the lower transistor where r_{in_2} takes the place of R_S in this model. If it's not

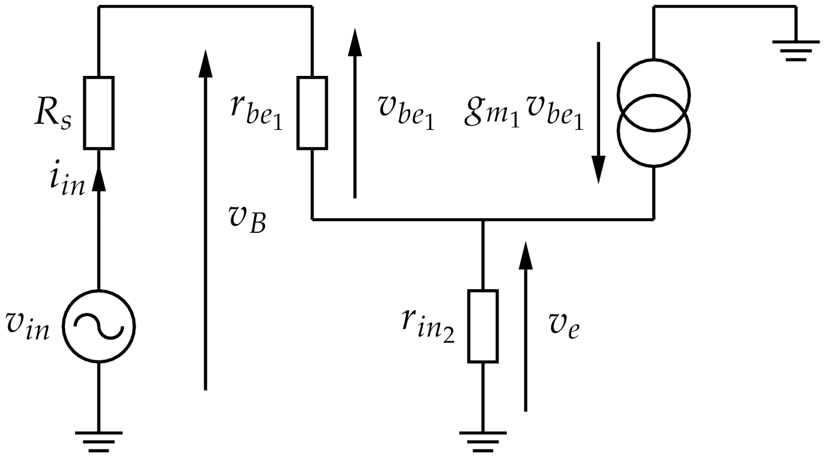

Figure 16: Partial small signal model for darlington pair input resistance

clear it may become clear as the analysis proceedes. We are asked to find the input resistance. Summing currents flowing into the base,

$$i_{in} = \frac{v_{be_1}}{r_{be_1}} \tag{213}$$

Summing voltages around the input loop,

$$V_B = v_{be_1} + v_e \tag{214}$$

Summing currents flowing into the emitter,

$$\frac{v_{be_1}}{r_{be_1}} + g_{m_1} \tag{215}$$

Transposing (214) for v_e,

$$v_e = V_B - v_{be_1} \tag{216}$$

Substituting (216) into (215) and transposing to make v_{be_1} the subject,

$$v_{be_1} = \frac{V_B}{r_{in_2}\left(\frac{1}{r_{be_1}} + g_{m_1}\right) + 1} \tag{217}$$

Substituting (217) into (213) to remove v_{be},

$$i_{in} = \frac{V_B}{r_{be_1}\left(r_{in_2}\left(\frac{1}{r_{be_1}} + g_{m_1}\right) + 1\right)} \tag{218}$$

dividing by V_B, inverting and tidying up,

$$\frac{V_B}{i_{in}} = r_{in} = r_{in_2}(1 + \beta_1) + \frac{\beta_1}{g_{m_1}} \tag{219}$$

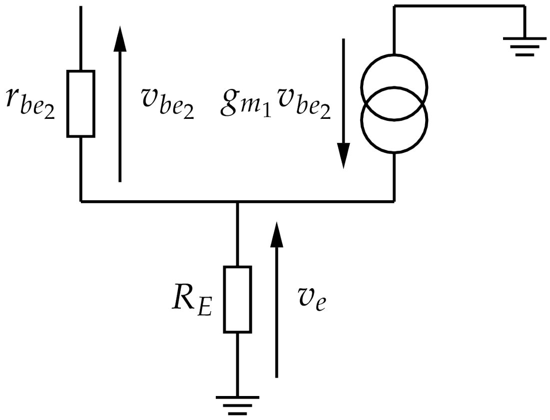

Figure 17: Partial small signal model for darlington pair input resistance, showing the lower transistor only. The content of this model is contained within r_{in_2} in Figure 17.

This is a partial result as we still have no knowledge of r_{in_2}. A small signal model may help.

This model is (ignoring the lack of a source) the same as Figure 17. It should therefore have the same input resistance,

$$r_{in_2} = R_E\left(1+\beta_2\right) + \frac{\beta_2}{g_{m_2}} \tag{220}$$

Substituting (220) into (219)

$$r_{in} = \left(R_E\ \left(1+\beta_2\right) + \frac{\beta_2}{g_{m_2}}\right)\left(1+\beta_1\right) + \frac{\beta_1}{g_{m_1}} \tag{221}$$

This can be simplified somewhat. The $\frac{\beta_x}{g_{m_x}}$ are not significant compared to the other terms so we have,

$$r_{in} = R_E\left(1+\beta_2\right)\left(1+\beta_1\right) \tag{222}$$

We can see now that the input resistance increases by a factor of approximately β (if $\beta >> 1$). If we had a Darlington of three transistors, we would not need to do any working out to know the input resisance. For the numbers in Question 3 and assuming the minimum β for both the MJ15003 and the MJE340 of 25 and 30 respectively the input resistance is approximately 12.9 kΩ. The value of R_1 and R_2 in parallel is also about 12 kΩ so we should expect an input resistance including the biasing network of about 6 kΩ, which is about ten times the maximum source resistance of 600 Ω (in question 3), which is probably acceptable in most situations.

Question 4 part 5

This question asks for the output resistance of the Darlington. If you've done question 3 this should be quite easy as it has already been derrived in Question 3 part 9 where we had,

$$r_o = \frac{1}{g_m} + \frac{R_B}{\beta} \tag{223}$$

In this situation R_B is just R_S as we're not considering the biasing network (but if we were it would just be a part of R_B). Applying the same principle as for the input resistance we can deduce r_o takes the form,

$$r_o = \frac{\frac{\beta_2}{g_{m_2}} + \frac{\frac{\beta_1}{g_{m_1}} + R_S}{\beta_1 + 1}}{\beta_2 + 1} \tag{224}$$

If you need to see the derivation it's in the solution to question 3.

Question 4 part 6

The current gain falls along very similar lines to the input resistance. The same small signal model can be used and the same partitioning approach is also possible. This time we're interested, firstly, in the current flowing in r_{in_2} due to i_{in}, which is,

$$i_{e_1} = (\beta_1 + 1)\ i_{in} \tag{225}$$

Looking at the second partial small signal model i_{e_2} is then,

$$i_{e_2} = (1 + \beta_2)\ i_{b_2} \tag{226}$$

Don't forget that $i_{e_1} = i_{b_2}$. This can be seen on the circuit diagram in Figure 15. Substituting,

$$\frac{i_{e2}}{i_{in}} = (1 + \beta_2)(\beta_1 + 1) \tag{227}$$

Question 5: Widlar Current Mirror

The circuit in Figure 18 is a Widlar current mirror. The transistors are 2N5551. You may assume that the transistors are idential.

1. Show that the current in R_L is related to the current I_S by (228).
2. If I_S is 2000 μA what is the largest value R_L that can be used without pushing Q_1 into saturation? *Hint: you will need to use the datasheet to find* $V_{\mathrm{CE(sat)}}$.
3. Draw the small signal equivalent circuit for the mirror, ensure you include r_{ce}.

4. Derive the output resistance of the mirror.
5. Derive the output resistance when emitter degeneration resistors are included.
6. By adding another transistor as in Figure. 19 a significant improvement can be made. What advantage does this circuit have over the two transistor mirror?
7. Derive the relationship between I_S and the load current in Figure 19.

$$\frac{I_S}{I_{R_L}} = \frac{h_{FE}+2}{h_{FE}} \tag{228}$$

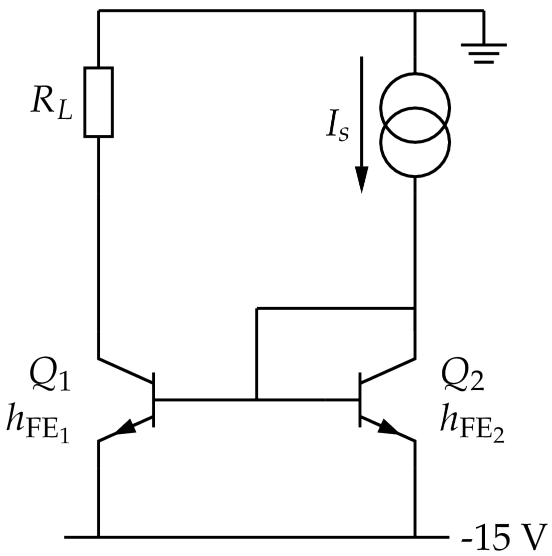

Figure 18: A Widlar current mirror circuit.

Question 5 part 1

This is the first of the current source/sink questions and is the most simple of them. It often shows up on the exam. Part one requires the derivation of the relationship between the setting current and the load current assuming both transistors are identical. This is usually a fairly good approximation if the cirucit is built in an IC process. There will also be good isothermal matching on the die as well. If it's a descrete circuit (made from individual transistors) there are some tricks we can use to help. Lots of Tectronix oscilloscopes (the "dog kennel" kind) from the early 1970's to the late 1980's (when digital scopes started to take over) had several sets of two transistors in a plastic T092 package with the flat sides

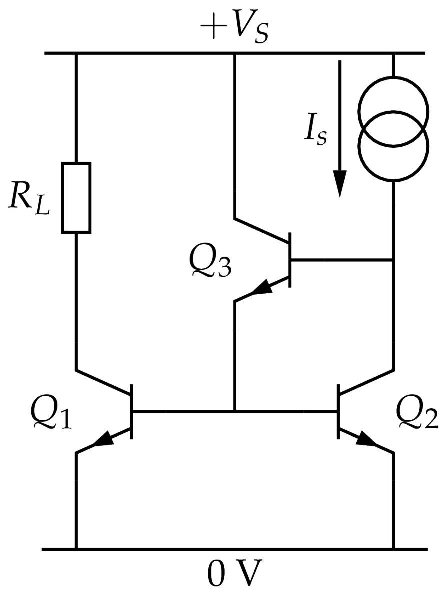

Figure 19: A current mirror circuit with helper transistor.

facing eachother and a sprung copper band wrapped round them with thermal paste between them. This was to try to keep them at the same temperature. It seems likely that the people making these oscilloscopes would have used a curve tracer to match the transistors prior to soldering them into the cirucit. In more modern times matched pairs of transistors became available, including the well known MAT-02 and LM394. However the MAT-02 is not in production and the LM394 is only made by Texas Instruments for customers buying thousands of parts. Luckily the NXP BCM846BS is a suitable matched pair. It is essentially and IC of two transistors with all the electrodes going to package legs. To perform the analysis we can sum currents flowing into the circuit nodes,

$$I_{R_L} = h_{FE}\, I_B \tag{229}$$

$$I_B = \frac{I_{R_L}}{h_{FE}} \tag{230}$$

$$I_S = I_C + 2\, I_B \tag{231}$$

$$= I_B\, (h_{FE} + 2) \tag{232}$$

$$= \frac{I_{R_L}}{h_{FE}}\, (h_{FE} + 2) \tag{233}$$

$$I_S = I_{R_L} \left(1 + \frac{2}{h_{FE}}\right) \tag{234}$$

Assuming $h_{FE} = 100$,

$$\frac{I_{R_L}}{I_S} = \frac{100}{102} \approx 0.9804 \tag{235}$$

Question 5 part 2

The datasheet for the 2N5551 doesn't give a value for $V_{CE(\text{sat})}$ for the conditions we have. The nearest is $I_C = 10$ mA, $I_B = 1$ mA and $V_{CE(\text{sat})} = 0.15$ V. Therefore apply Ohm's law,

$$R_L = \frac{V_S - V_{CE(\text{sat})}}{I_L} \tag{236}$$

The supply is 15 V below ground. The minimum h_{FE} of the 2N5551 (from the datasheet) is 80 for $I_C = 1$ mA which is nearly where we're working. The value of I_L is then,

$$I_L = I_S \frac{h_{FE}}{h_{FE} + 2} = 2 \text{ mA} \cdot \frac{80}{80 + 2} = 1.951 \text{ mA} \tag{237}$$

so,

$$R_L = \frac{V_S - V_{CE(\text{sat})}}{I_L} = \frac{15 - 0.15}{1.951} = 7.611 \text{ k}\Omega \tag{238}$$

Question 5 part 3

The small signal model is shown in Figure 20. Note that Q_2 is essentially a diode as its base and collector are connected together.

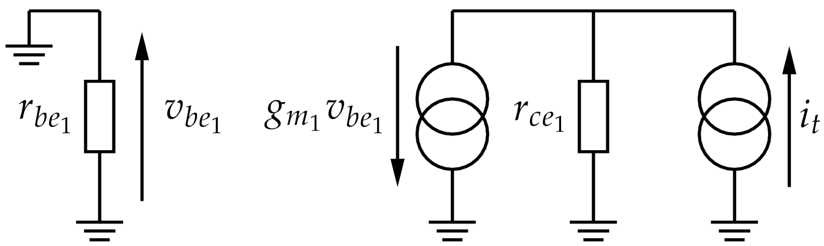

Figure 20: Small signal model of the Widlar current mirror.

Question 5 part 4

The key thing to note is that the controlled current source can only change its value if v_{be_1} changes, which it does not. The voltage on the output due to i_t is $v_t = i_t\, r_{ce_1}$ and the output resistance is simply $v_t/i_t = r_{ce_1}$.

The hybrid-π model is not the only way to look at the effects of output resistance. Several authors prefer a discussion based on DC currents to deal with the Early effect (Grey, Hurst Lewis and Meyer, "Analysis and Design of Analog Integrated Circuits", Chapter 4., 5th ed., John Wiley & Sons, 2009). Leach also prefers a DC approach http://leachlegacy.ece.gatech.edu/ece3050/notes/bjt/bjtmirr.pdf as do Analog Devices https://wiki.analog.com/university/courses/electronics/text/chapter-11. However in this course the Early effect is not studdied in sufficient detail to warrent the level of description presented in these sources. Interested readers may like to investigate on thier own.

Question 5 part 5

If emitter resistors are included the ciruict will become that of Figure 21. The derivation of the output resistance including degeneration resistors can be quite tricky but looking at the small signal model in Figure 22 it is very similar to the output resistance of the degenerated common emitter amplifier from question 1.

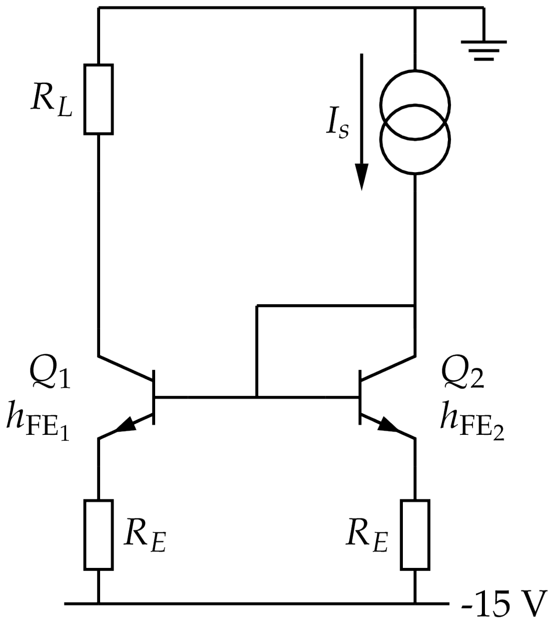

Figure 21: Current mirror with emitter degeneration resistors.

Note that in Figure 22 r_{be} is in parallel with R_E,

$$v_{be} = -i_t\ (r_{be}||R_E) \tag{239}$$

The current in r_{ce} is,

$$i_1 = i_t - g_m\, v_{be} \tag{240}$$

substituting to remove v_{be}

$$i_1 = i_t + i_t\, g_m\ (r_{be}||R_E) \tag{241}$$

The voltage on the output, v_o is,

$$v_o = -v_{be} + i_1\, r_{ce} \tag{242}$$

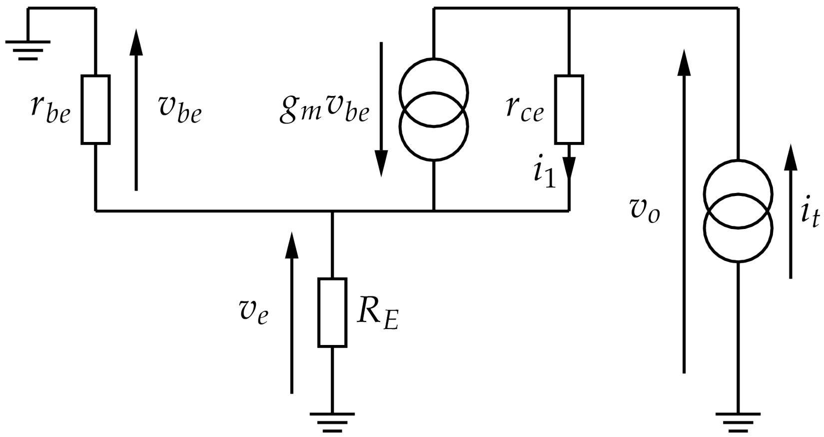

Figure 22: Small signal model of Q_1 in the current mirror with emitter degeneration.

substituting again,

$$v_o = i_t\,(r_{be}||R_E) + i_t\,r_{ce}\,(1 + g_m\;(r_{be}||R_E)) \tag{243}$$

Dividing by i_t,

$$r_o = \frac{v_t}{i_t} = r_{be}||R_E + r_{ce}\,(1 + g_m\,(r_{be}||R_E)) \tag{244}$$

The first term is small compared to the second term. Concentrating on the second term and multiplying out the parallel combination,

$$r_o \approx r_{ce}\left(\frac{g_m\,R_E\,r_{be}}{R_E + r_{be}} + 1\right) \tag{245}$$

Dividing through by r_{be},

$$r_o \approx r_{ce}\left(\frac{g_m\,R_E}{\frac{R_E}{r_{be}} + 1} + 1\right) \tag{246}$$

replacing r_{be} with g_m and β,

$$r_o \approx r_{ce}\left(\frac{g_m\,R_E}{\frac{g_m\,R_E}{\beta} +} + 1\right) \tag{247}$$

There are two interesting conditions that (247) can reduce to. If $g_m\,R_E >> \beta$, (247) becomes,

$$r_o = r_{ce}\,(\beta + 1) \tag{248}$$

If $\beta >> g_m\,R_E$ then (247) bcomes,

$$r_o = r_{ce}\,(g_m\,R_E + 1) \tag{249}$$

Question 5 part 6

The β helper is an emitter follower added to reduce the "finite gain defect" of the standard mirror – the current that is required to drive the bases making the collector current and the set current different. It does not affect the output resistance very much. It does increase the voltage required by the mirror potentially reducing headroom.

Question 5 part 7

The analysis is very similar to part 1. The result is,

$$I_S = I_{C1}\left(1 + \frac{2}{(h_{FE}+1)\ h_{FE}}\right) \tag{250}$$

Compared with (234) the defect term has been reduced by a factor of $(h_{FE}+1)$.

Question 6: Lin Style Operational Amplifier

There is a video solution to this question on the teaching resources website. The circuit of Figure 23 shows a simple form of op-amp circuit. Assuming that each transistor has a static current gain, I_C/I_B, and small signal current gain, $\Delta I_C/\Delta I_B$, of 100, that $kT/e = 0.026$ V and that each transistor has a V_{BE} of 0.7 V when conducting,

1. Estimate I_E, I_1, I_2 and I_3 assuming that $v_i = 0$ V, $v^+ = 0$ V, $v^- = 0$ V and $V_A = 0$ V.

2. Estimate the gain, v_{o1}/v_i, of the differential amplifier assuming that r_{ce} of Q_1 is very large compared to R_1. Remember to include the effects of Q_3 (ie, its input resistance) in your calculation.

3. Estimate the gain, v_a/v_{o1}, of the voltage gain stage assuming that r_{ce} of Q_3 and the input resistances of Q_4 and Q_5 are very large compared to R_{VA}.

4. Use your results from parts 2 and 3 to estimate the overall gain v_{o4}/v_i. What have you assumed in this calculation?

5. Using your powers of reasoning, identify which stage gain would be significantly improved if the small signal current gain of each transistor increased to 500.

Question 6 part 1

Each conducting base – emitter junction has 0.7 V across it.

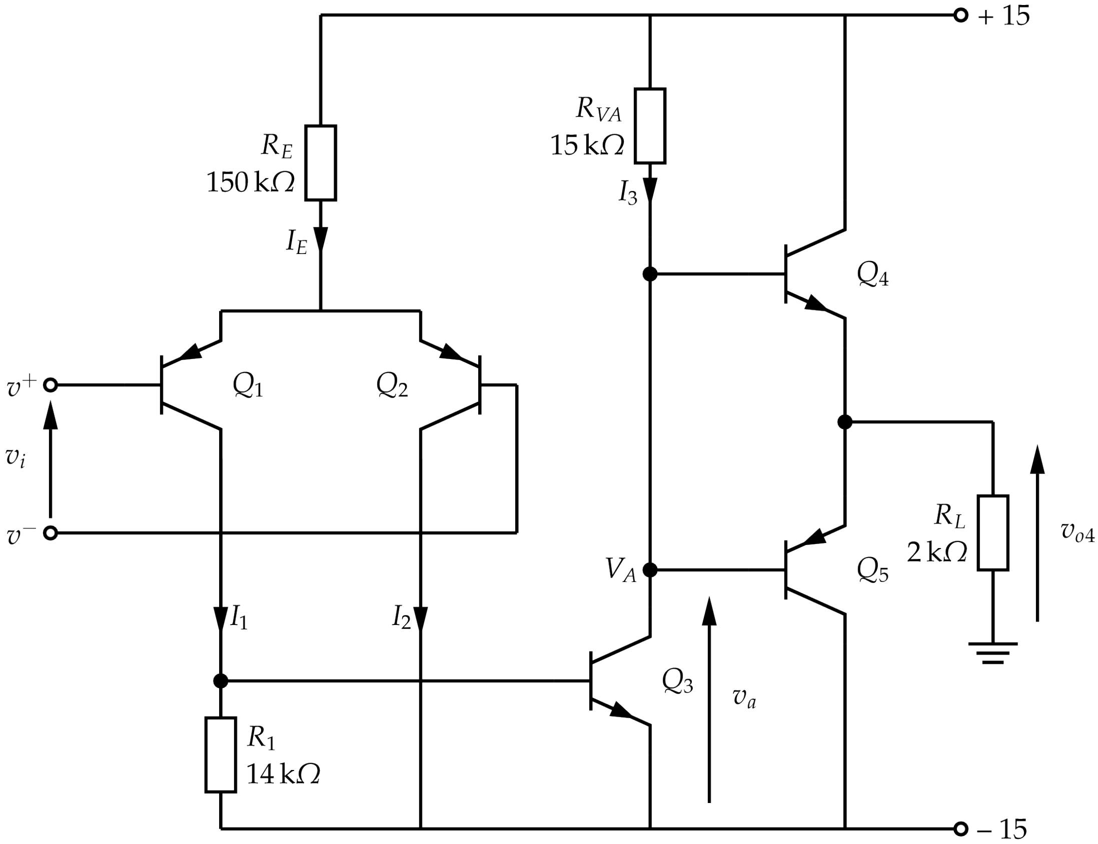

Figure 23: Simplified operational amplifier circuit.

- For I_E, $V_{E1} = V_{E2} = 0.7$ V. The voltage drop across R_E is therefore $(15 - 0.7)$ V which is 14.3 V. Ohm's law then gives $I_E = 14.3/R_E = 95.3\ \mu$A.

- For I_3, the current through R_{VA} is given by Ohm's Law as $I_3 = (15 - V_A)/R_{VA} = 1.00$ mA

- For I_1, if $V_A = 0$, neither Q_4 or Q_5 are conducting (since $V_{BE} = 0$ V for both transistors) so $I_{C3} = I_3$. $I_{B3} = I_{C3}/h_{FE_3} = 10\ \mu$A and the current through $R_1 = 0.7/R_1 = 50\ \mu$A. Using KCL, $I_1 = I_{R_1} + I_{B_3} = 60\ \mu$A.

- Leaving $I_2 = I_E - I_1 = 35.3\ \mu$A.

Question 6 part 2

For small signal gain calculations we must use the small signal equivalent circuit that describes how the transistors in the circuit behave as far as signals are concerned. The full small signal circuit is shown in Figure 24. For small signal gain calculations we must use the small signal equivalent circuit that describes how the transistors in the circuit behave as far as signals are concerned. The full small signal circuit is shown in Figure 24. In Figure 24, Q_2 is encircled by a box showing that apart from ground there is only one connection between Q_2 and the rest of the circuit. This means that Q_2 can be represented as an equivalent resistance to ground. To find out what that resistance is we. In Figure 24, Q_2 is encircled by a box showing that, apart from ground, there is only one connection between Q_2 and the rest of the circuit. This means that Q_2 can be represented as an equivalent resistance to ground. In this situation Q_2 can be thought of as a common base amplifier. To find out what that resistance is we need to consider the circuit of Figure 25. Here is is the current flowing into the emitter terminal of Q_2 and v_s is the voltage imposed on the emitter of Q_2 by the rest of the circuit. The ratio v_s/i_s is the effective resistance looking into Q_2 as far as the rest of the circuit is concerned. First sum currents at the emitter node,

$$i_{b2} + g_{m2}\, v_{be2} + i_s = 0 \tag{251}$$

or,

$$\frac{v_{be2}}{r_{be2}} + g_{m2}\, v_{be2} + i_s = 0 \tag{252}$$

Inspection of the circuit reveals that $v_s = -v_{be2}$ and using this in the equation above gives,

$$\frac{v_s}{i_s} = \frac{1}{\frac{1}{r_{be2}} + g_{m2}} \tag{253}$$

which approximates to,

$$\frac{v_s}{i_s} = \frac{1}{g_{m2}} = r_{e2} \tag{254}$$

since $1/r_{be} = g_m/\beta$ and, for a small signal amplifier transistor β is often much greater than unity.

The effect of the next (voltage amplification) stage, Q_3, is taken into account by r_{be3}, the input resistance of Q_3. Using r_{e2}, the equivalent circuit simplifies to that shown in Figure 26. In this circuit we need to find $g_{m1}\, v_{be1}$ i.e. v_{be1}, in terms of v_i because v_{o1} is proportional to $g_{m1}\, v_{be1}$. The approach is a nodal analysis at the emitter node – sum currents at the emitter node – together with a voltage sum around the input loop. The two resulting equations are,

$$i_{b1} + g_{m1}\, v_{be1} = i_e \tag{255}$$

and

$$v_i = v_e + v_{be1} \tag{256}$$

or

$$\frac{v_{be1}}{r_{be1}} + g_{m1}\, v_{be1} = \frac{v_e}{(R_E||r_{e2})} = \frac{v_i - v_{be1}}{(R_E||r_{e2})} \tag{257}$$

Transposing,

$$v_{be1} = \frac{v_i}{R_E||r_{e2}\left(\frac{1}{r_{be1}} + g_{m1} + \frac{1}{R_E||r_{e2}}\right)} \tag{258}$$

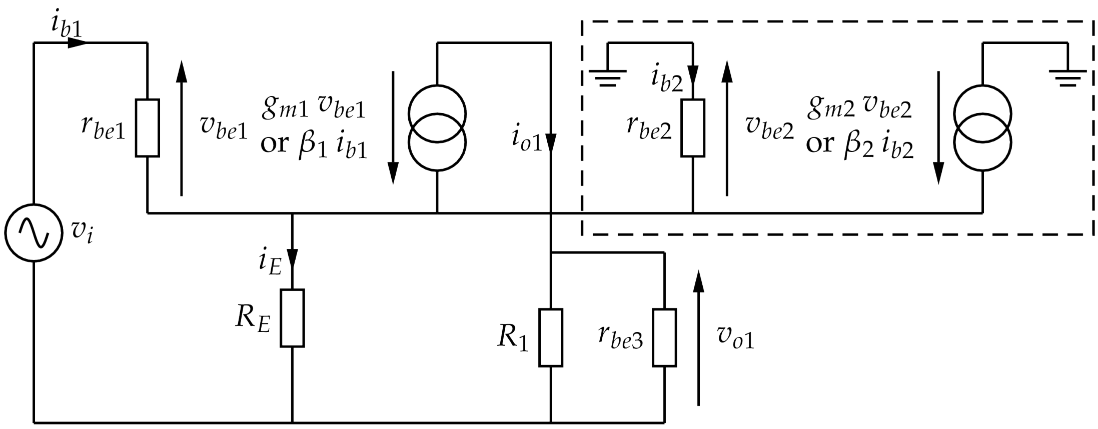

Figure 24: Small signal model of Q_1, Q_2 and Q_3 in Figure 23.

This expression can be simplified by recognising that $g_{m1} >> 1/r_{be1}$ because

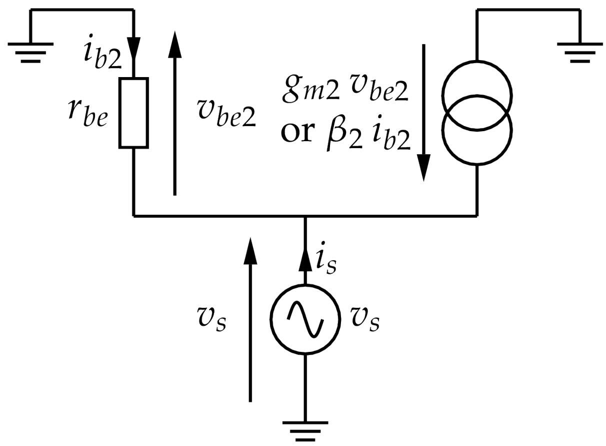

Figure 25: Small signal model of Q_2 in Figure 23 reduced from Figure 24.

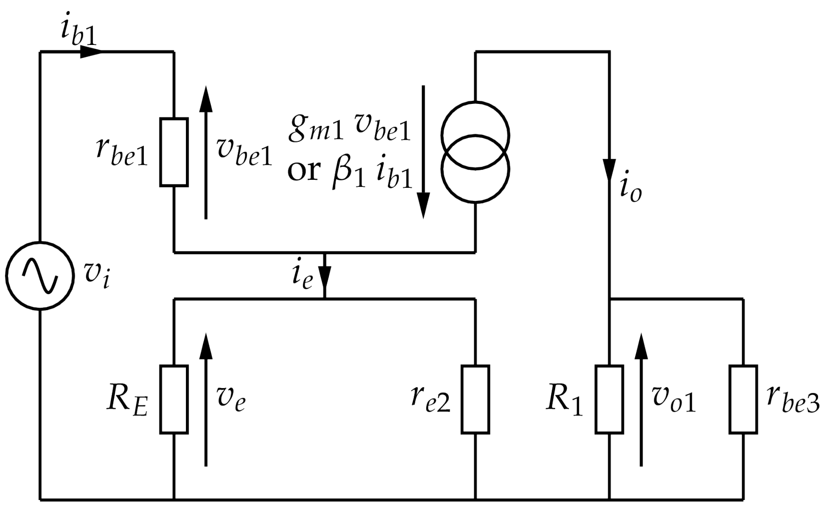

Figure 26: Simplified differential stage and voltage amplifier input small signal model.

$\beta >> 1$ and $R_E || r_{e2} \approx r_{e2}$ since $R_E >> r_{e2}$. The various parameter values are,

$$R_E = 150\ \mathrm{k\Omega} \tag{259}$$

$$R_1 = 14\ \mathrm{k\Omega} \tag{260}$$

$$g_{m1} = \frac{e\, I_{C1}}{k\, T} = (60\ \mu\mathrm{A}/0.026\ \mathrm{V}) = 2.3\ \mathrm{mA\,V^{-1}} \tag{261}$$

$$r_{be1} = \frac{\beta_1}{g_{m1}} = 100/0.0023 = 43.5\ \mathrm{k\Omega} \tag{262}$$

$$r_{e2} = \frac{k\, T}{e\, I_{C2}} = (0.026\ \mathrm{V}/35.3\ \mu\mathrm{A}) = 737\ \Omega \tag{263}$$

$$r_{be3} = \frac{\beta_3}{g_{m3}} = 2.6\ \mathrm{k\Omega} \tag{264}$$

These values give values of v_{be1}/v_i of 0.371 and 0.370 for the simplified and unsimplified forms of the equation respectively. The closeness of these results is confirmation that the simplifications are appropriate. At the output,

$$v_{o1} = i_o\ (R_1 || r_{be3}) \tag{265}$$

since

$$i_o = g_{m1}\, v_{be1} \tag{266}$$

we have

$$v_{o1}/v_{be1} = -g_{m1}\ (R_1 || r_{be3}) = -5.04 \tag{267}$$

Therefore

$$\frac{v_{o1}}{v_i} = \frac{v_{o1}}{v_{be1}} \cdot \frac{v_{be1}}{v_i} = -5.04 \cdot 0.370 = -1.87 \tag{268}$$

Question 6 part 3

To make this estimate a small signal equivalent circuit of Q_3 is required. Q_3 is connected in a common emitter configuration so its equivalent behaviour is much more straightforward than was the case for the differential amplifier stage. From Figure 27,

$$v_a = i_{o3}\, R_{VA} = -g_{m3}\, v_{be3}\, R_{VA} = -g_{m3}\, v_{o1}\, R_{VA} \tag{269}$$

or

$$\frac{v_a}{v_{o1}} = -g_{m3}\, R_{VA} = -0.0384 \cdot 15\mathrm{k\Omega} = -577 \tag{270}$$

The assumptions in the question do not make any difference to the complexity of the small signal equivalent circuit of Figure 27, they simplify the collector circuit of the transistor to one single resistor, R_{VA}. In the absence of this simplification, R_{VA} would consist of a parallel combination of resistors, one static (R_{VA}) and the rest incremental.

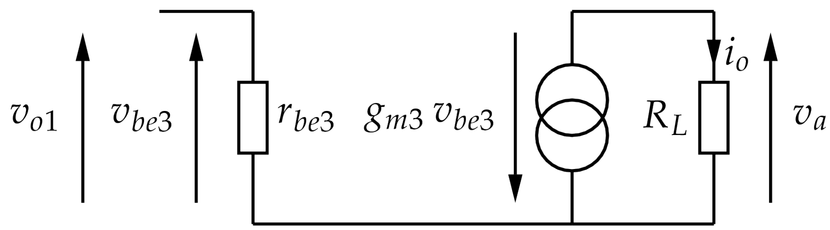

Figure 27: Voltage amplifier stage small signal model.

Question 6 part 4

The gain of stages 1 and 2 together is the product of their individual gains, i.e,

$$\frac{v_a}{v_i} = \frac{v_a}{v_{o1}} \cdot \frac{v_{o1}}{v_i} = (-577) \cdot (-1.87) = 1079 \tag{271}$$

This equals the overall gain *if the output stage gain is assumed to be very close to unity* – an assumption usually true for emitter follower circuits.

Question 6 part 5

This change of current gain would not affect the transistor g_m values very much because it would not affect the values of I_C much except for a slight improvement in the balance between Q_1 and Q_2 collector currents due to the reduced I_B requirement of Q_3. The biggest effects would be in the values of r_{be} of the transistors and this would have the most significance for gain in terms of the effect r_{be3} has on first stage gain. r_{be3} would change from 2.6 kΩ to 13 kΩ – you can look at your answer to part 2 to see what effect this would have on first stage gain.

Question 7: Push Pull Emitter Follower

1. Concisely describe the cause of crossover distortion in class B push-pull amplifiers.

2. Use a sketch to show the effects of crossover distortion on a triangle or sinusoidal waveform, taking particular care with your representation of the crossover region.

3. Sketch a circuit diagram of a voltage amplifier and push pull stage which largely overcomes the problems of crossover distortion and describe the operation of your circuit.

4. Calculate the quiescent power dissipation in one of the output transistors in your circuit.

5. Calculate the average power dissipated in the load resistor of your circuit.

6. Derive expressions for the instantaneous power dissipation in one of the output transistors. You may assume that the power dissipated in a transistor is the product of I_C and V_{CE} which will both vary *approximately* sinusoidally given a sinusoidal input. *Hint: this involves some integration of sines and cosines.*

7. Using your derivation find the signal voltage amplitude across the output which results in the highest power dissipation in the *transistor*.

8. Show that the highest possible efficiency of this circuit is approximately 70%.

9. The push-pull stage may operate in class C, B or A depending on the quiescent current flowing in the output transistors, which in turn is related to the voltage between the bases of the two output transistors. Sketch the load voltage and collector current waveforms of the two output transistors for each class, noting the salient features.

10. For each class of operation above, what angle of current conduction exists in each class and what *approximate* range of voltages must exist between the bases of the output transistors?

Question 7: part 1

Crossover distortion arises when conduction is transferred from one transistor to the other in the output stage of a push pull class B amplifier. Notwithstanding the 0.7 V across the base emitter required to enter the forward active region, the problem is caused by the dependence of the device transconductance on the collector (or drain) current which in turn leads to a change in output resistance that is a function of the collector (or drain) current and hence output voltage. This is similar to the arguements in question 3 part 6. In other words g_m is not constant but changes considerably over the course of one cycle of the waveform changing all of the small signal parameters considerably as a function of or time. In other words, analysis of this circuit requires the use of a large signal approach to the transistor operation. Even if g_m was independent of I_C (I_D) there would still be a problem because the output resistance would change during the crossover region unless the biasing was perfect (i.e. output devices never off or on together).

Question 7: part 2

Figure 28 shows the shape of a crossover distorted sinusoidal waveform as the components of the collector currents in the push-pull stage.

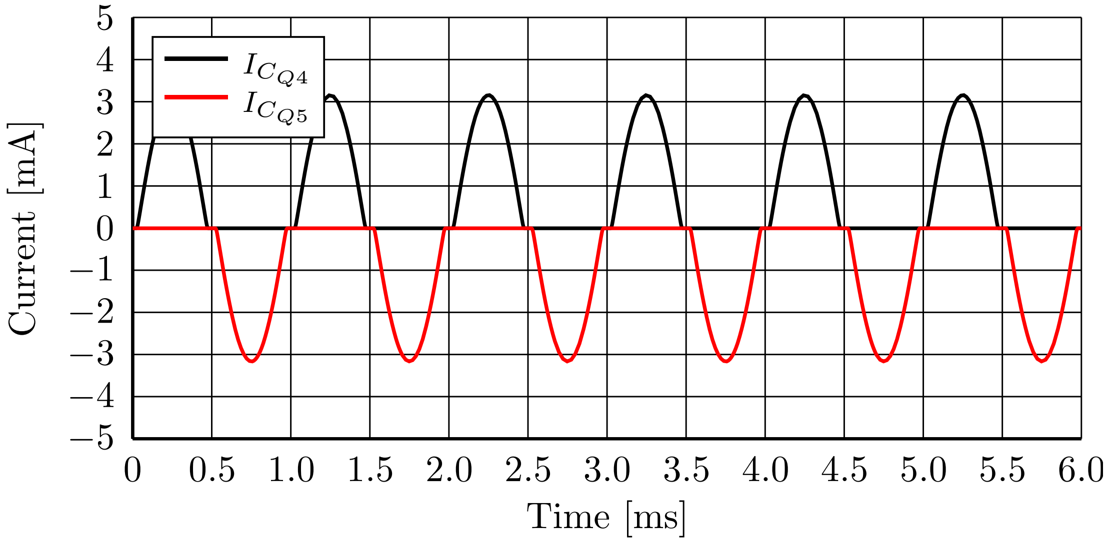

Figure 28: Crossover distortion of a sinusoidal waveform passing through an under-biased push pull stage. Collector current for the upper, NPN, transistor shown in black. Collector current for the lower, PNP, transistor shown in red.

Question 7: part 3

In a real circuit the problem can be reduced to a point where it is no longer significant by arranging for some overlap of conduction of the output devices in the crossover region. A circuit that can achieve this is shown in Figure 29. R_9, R_{10} and Q_{10} act as a floating fixed voltage source which, in conjunction with R_{E4} and R_{E5} control the quiescent bias current in Q_4 and Q_5 and so control the angle of conduction overlap.

Question 7: part 4

To calculate a numerical value for the quiescent power dissipation of an output transistor one must make some choices about the circuit, specifically the power supply voltage and the quiescent output stage current are needed however these are not given so we may presume that an algebraic answer is required. Let the power supply be V_S^+, the voltage across the external load resistor be V_L and the quiescent current in the transistor's collector be I_Q. Also assume the base current is negligible. The quiescent power dissipation in each output stage transistor is then,

$$P_Q = \left(V_S^+ - V_L\right)\, I_Q \tag{272}$$

Question 7: part 5

The calculation of average power dissipation does not feature in EEE225, but it is often important in power electronic circuits, machine drives and power systems.

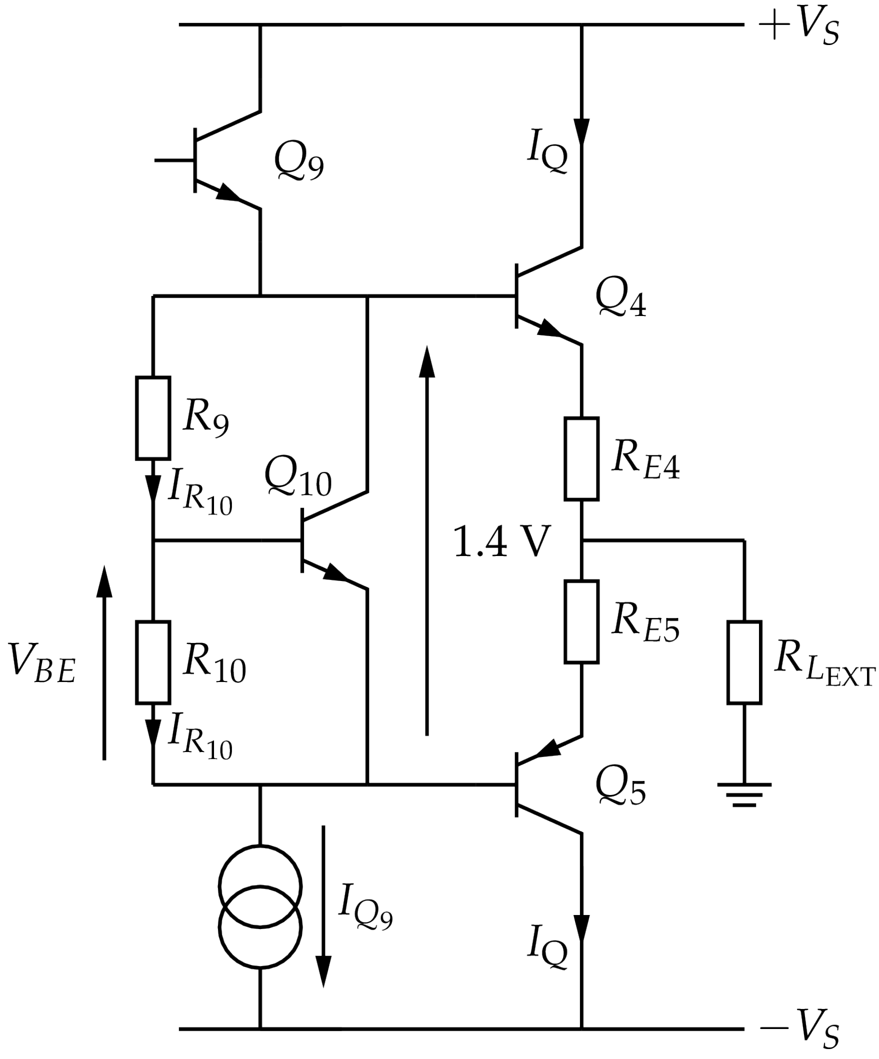

Figure 29: The circuit including the 'amplified diode' structure (Q_{10}, R_9 and R_{10}) attempts to bias the base of Q_4 and Q_5 appropriately and maintain a small quiescent current through Q_4's collector, R_{E4}, R_{E5} and Q_5's collector. The g_m of Q_4 and Q_5 still depends on their respective collector currents, so small signal parameters such as input and output impedance of the stage are still a function of these currents and change over the course of a signal cycle.

In many power electronic and machine drive applications the current and/or voltage waveforms are not sinusoidal. *Average* power will be the product of RMS (root mean square) current and RMS voltage. It is not meaningful to take the RMS of a power waveform because RMS is simply a way of relating the heating effect of a time dependent signal with an equivalent DC quantity i.e. one which gives rise to the same heating effect. For the push-pull circuit in this question, let us assume a sine wave voltage signal, $V(t) = V_P \sin(\omega\, t)$ across the external load resistor, R_L. Where V_P is the *peak* magnitude of the sinusoidal waveform across the load. The instantaneous power is given by the usual formula,

$$P_L(t) = \frac{V(t)^2}{R_L} = \frac{V_P^2 \sin(\omega\, t)^2}{R_L} \tag{273}$$

To integrate $\sin(\omega\, t)^2$ we can remember the "half-angle formula" or if we can't remember that far back, a table of integrals can be used to confirm that,

$$\sin(\omega\, t)^2 \equiv 1 - \cos(2\,\omega\, t) \tag{274}$$

one can read this as "sine squared ω t" or "sine ω t squared", they are the same. Our average power integral is then,

$$P_L(t) = \frac{1}{{}^{T}/_{2}} \cdot \frac{V_P^2}{R_L} \int_0^{{}^{T}/_{2}} 1 - \cos(2\,\omega\, t)\ \mathrm{d}t \tag{275}$$

It is only necessary to integrate over half a period because the shape of the load power waveform repeats every half period. It's not so hard to see why this might be. Consider the sine shape of the voltage across the load, now remember that we squared it, effectively taking the absolute value and scaling by a factor. Therefore there will be two regions of high peak power in every cycle of the voltage waveform and these will be identical, therefore only half a voltage cycle needs to be considered to capture a full cycle of the power waveform. Performing the integral,

$$P_L(t) = \frac{1}{{}^{T}/_{2}} \cdot \frac{V_P^2}{R_L} \left[\frac{t}{2} - \frac{2\,\omega\, \sin(2\,\omega\, t)}{4\,\omega}\right]_0^{{}^{T}/_{2}} \tag{276}$$

Inserting limits,

$$P_L = \frac{1}{{}^{T}/_{2}} \cdot \frac{V_P^2}{R_L} \left(\left[\frac{{}^{T}/_{2}}{2} - \frac{\sin\left(2\,\omega\, {}^{T}/_{2}\right)}{2}\right] - \Big[0\Big]\right) \tag{277}$$

We can substitute $\omega = {}^{2\,\pi}/_{T}$ and do some cancellations,

$$P_L = \frac{2\,V_P^2}{T\,R_L} \left(\frac{T}{4} - \frac{\sin\left(2\,\frac{\cancel{2}\pi}{\cancel{T}}\frac{\cancel{T}}{\cancel{2}}\right)}{2}\right) \tag{278}$$

this leaves,

$$P_L = \frac{2\,V_P^2}{T\,R_L}\left(\frac{T}{4} - \frac{\sin\left(2\,\pi\right)}{2}\right) = \frac{\cancel{2}\,V_P^2}{\cancel{T}\,R_L}\cdot\frac{\cancel{T}}{\cancel{4}_2} \tag{279}$$

because $\sin\left(2\,\pi\right) = 0$. Tidying up,

$$P_L = \frac{V_P^2}{2\,R_L} \tag{280}$$

If you're concerned by the 2 in the denominator, don't forget that V_P is the *peak* value of the load voltage and that V_P is squared. Many students will not need to derive this result as it is widely remembered that for a sinusoidal voltage and a purely resistive load $P = V^2/R$, where V is measured in V_{RMS}. Nevertheless if the shape of the waveform is not sinusoidal a different answer will result. For example similar analysis for a triangle wave whose rising edge is of the form,

$$V(t) = \frac{2\,V_P}{\pi}\left(t - {}^{\pi}/_{2}\right) \tag{281}$$

where V_P is the maximum peak amplitude of the triangle yields,

$$P_L = \frac{1}{T}\left(\frac{4\,{V_P}^2\,T^3}{3\,\pi^2\,R_L} - \frac{2\,{V_P}^2\,T^2}{\pi\,R_L} + \frac{{V_P}^2\,T}{R_L}\right) \tag{282}$$

which reduces to,

$$P_L = \frac{V_P^2}{3\,R_L} \tag{283}$$

This result is probably less familiar than the sine wave case. Diligent students will show this by performing the integral. It is not difficult, just tedious. A little thought will show that the period of the power waveform, $T = \pi$ in this case too if the triangle voltage waveform has period $2\,\pi$.

Question 7: part 6

The dissipation in the transistors in a push-pull output stage follows a similar principle as for the power in the load. It is also not considered in EEE225 but is a good example of the sort of analysis that one often faces in power electronics problems. One integrates the power waveform across one period in order to find the average value. Since each transistor will conduct for half of the collector current waveform period we can perform the integral across half a collector current cycle and still yield the average power from *both* transistors simply by multiplying our end result by 2.

$$P_D(t) = \frac{1}{T}\int_0^{{}^{T}/_{2}} V_D\,\sin\left(\omega\,t\right)\;I_P\,\sin\left(\omega\,t\right)\;\mathrm{d}t \tag{284}$$

where V_D is the maximum (i.e. peak) voltage amplitude across the power transistor (i.e. V_{CE}) and I_P is the maximum current amplitude in both the transistor and the external load resistor.

$$P_D(t) = \frac{1}{T} \int_0^{T/2} (V_S - V_P \sin(\omega t))\, I_P \sin(\omega t)\, dt \tag{285}$$

where V_S is the power supply voltage and V_P is the peak voltage amplitude across the external load resistor. If you don't see why this is the case look at Figure 29, we are ignoring the small voltage drop across R_{E4} and R_{E5}. The integral can be multiplied out to yield,

$$P_D(t) = \frac{1}{T} \int_0^{T/2} \frac{V_S V_P}{R_L} \sin(\omega t) - \frac{V_P^2}{R_L} \sin(\omega t)^2\, dt \tag{286}$$

Using the "half angle formula" again the second part of the integral becomes,

$$P_D(t) = \frac{1}{T} \int_0^{T/2} \frac{V_S V_P}{R_L} \sin(\omega t)\, dt - \frac{V_P^2}{T R_L} \int_0^{T/2} \frac{1 - \cos(2\omega t)}{2\omega}\, dt \tag{287}$$

The second integral becomes,

$$\frac{V_P^2}{2 T R_L} \int_0^{T/2} 1 - \cos(2\omega t)\, dt \tag{288}$$

We've already solved this in part 5,

$$\frac{V_P^2}{2 T R_L} \left(\Big[t \Big]_0^{T/2} - \left[\frac{\sin(2\omega t)}{2\omega} \right]_0^{T/2} \right) \tag{289}$$

Combining this result with the first part of the power integral,

$$P_D(t) = \frac{1}{T} \int_0^{T/2} \frac{V_S V_P}{R_L} \sin(\omega t)\, dt - \frac{V_P^2}{2 T R_L} \Big[t \Big]_0^{T/2} + \frac{V_P^2}{2 T R_L} \left[\frac{\sin(2\omega t)}{2\omega} \right]_0^{T/2} \tag{290}$$

Performing the remaining integral,

$$P_D = -\frac{V_S V_P}{T R_L} \left[\frac{\cos(\omega t)}{\omega} \right]_0^{T/2} - \frac{V_P^2}{2 T R_L} \Big[t \Big]_0^{T/2} + \frac{V_P^2}{2 T R_L} \left[\frac{\sin(2\omega t)}{2\omega} \right]_0^{T/2} \tag{291}$$

Inserting, $\omega = \frac{2\pi}{T}$,

$$P_D = -\frac{V_S V_P}{T R_L} \left[\frac{\cos\left(\frac{2\pi}{T} t\right)}{\frac{2\pi}{T}} \right]_0^{T/2} - \frac{V_P^2}{2 T R_L} \Big[t \Big]_0^{T/2} + \frac{V_P^2}{2 T R_L} \left[\frac{\sin\left(2 \frac{2\pi}{T} t\right)}{2 \frac{2\pi}{T}} \right]_0^{T/2} \tag{292}$$

Applying the limits of integration,

$$P_D = -\frac{V_S\,V_P}{\cancel{T}\,R_L}\left(\left[\frac{\cos\left(\frac{\cancel{2}\pi}{\cancel{T}}\frac{\cancel{T}}{\cancel{2}}\right)}{\frac{2\pi}{\cancel{T}}}\right]-\left[\frac{\cos\left(\frac{2\pi}{T}\,0\right)}{\frac{2\pi}{\cancel{T}}}\right]\right)-\frac{V_P^2}{2\cancel{T}\,R_L}\left(\left[\frac{\cancel{T}}{2}\right]-\left[0\right]\right)$$
$$+\frac{V_P^2}{2\cancel{T}\,R_L}\left(\left[\frac{\sin\left(\cancel{2}\,\frac{2\pi}{\cancel{T}}\frac{\cancel{T}}{\cancel{2}}\right)}{2\,\frac{2\pi}{\cancel{T}}}\right]-\left[\frac{\sin\left(2\,\frac{2\pi}{T}\,0\right)}{2\,\frac{2\pi}{\cancel{T}}}\right]\right) \quad (293)$$

This expression looks fairly horrid, but canceling appropriately it simplifies readily to,

$$P_D = \frac{V_S\,V_P}{\pi\,R_L} - \frac{V_P^2}{4\,R_L} \quad (294)$$

Don't forget that this result is for one transistor, double it for both transistors.

Question 7: part 7

The next part of the question asks for the load voltage which results in the highest power dissipation in the output transistors. This voltage is not necessarily the same as the load voltage which dissipates the highest power in the load. V_P can not exceed V_S in fact V_P can only approach V_S in theory. In practical circuits there is some *headroom* required which lowers the efficiency below the theoretical maximum. This headroom required is often related to the driver stage (Q_9) and current source (I_{Q_9}). requirements. Looking at (294), the term in V_P^2 will begin to dominate as V_P increases and the terms are nearly equal in magnitude when $V_P = V_S$. We may therefore expect a maximum value when V_P is somewhere around half of V_S. Since we seek a maximum value we also seek a turning point and therefore can differentiate to find an expression for the maximum power dissipation in terms of the load voltage V_P,

$$\frac{\mathrm{d}P_D(t)}{\mathrm{d}V_P} = \frac{V_S}{\pi\,R_L} - \frac{2\,V_P}{4\,R_L} = 0 \quad (295)$$

solving the derivative equal to zero yields,

$$V_P = \frac{2\,V_S}{\pi} \quad (296)$$

The maximum average power dissipation in the transistor is found by substitution of (296) into (294).

$$P_{\mathrm{MAX}} = \frac{V_S\frac{2\,V_S}{\pi}}{\pi\,R_L} - \frac{\left(\frac{2\,V_S}{\pi}\right)^2}{4\,R_L} \quad (297)$$

which simplifies to,

$$P_{D_{\mathrm{MAX}}} = \frac{V_S^2}{\pi^2\,R_L} \quad (298)$$

Question 7: part 8

The efficiency is simply the desired output (load) power upon the total power (load + device dissipation),

$$\eta = \frac{P_L}{P_L + P_D} = \frac{\frac{V_P^2}{2\,R_L}}{\frac{V_P^2}{2\,R_L} + 2 \cdot \left(\frac{V_S\,V_P}{\pi\,R_L} - \frac{V_P^2}{4\,R_L}\right)} \tag{299}$$

It might be useful to plot a graph of this function to see that, over the range of physically sensible parameters, there is no turning point. Instead we can deduce that the highest efficiency will be when $V_P = V_S$. If you're not happy with that, plot the curve for some representative values (e.g. $V_S = 1$, $V_P = 0$ to 1) in Matlab or similar. R_L cancels so its value is not important. Inserting $V_P = V_S$ and canceling,

$$\eta = \frac{\pi}{4} \cdot 100 = 78.54\% \tag{300}$$

In practice the highest practically realizable efficiency is about 70%, principally due to $V_{CE(\text{sat})}$ and headroom requirements of the driver stage. This result only holds with a sine wave and the maximum efficiency is only obtained when the output voltage is maximized, which is not always desirable. Other waveforms generate different results. However, the sinusoid, running at the V_L which develops the maximum power in the output transistors is the worst case condition for a given V_S.

Question 7: part 9

Current waveforms representative of the different classes of operation graphs are presented in the lecture notes but are shown in Figures 30 - 34 as well.

Question 7: part 10

The angle of conduction and the class of a stage are listed in Table 7. The biasing voltages required are somewhat open to interpretation, the feature that identified the class of an amplifier stage is the angle of current conduction. This could also be expressed very approximately as a relationship between the load current and quiescent current. However both the biasing voltages and the quiescent current to max load current ratio are rough rules of thumb, not to be relied upon.

Questions 8 – 10

If you genuinely have a solution you want to discuss, email it to me. If you're making an serious attempt at answering these last three questions, you will probably not need my help as you will be in a position to help yourself should you get stuck. Also you can call on Grey if you're stuck.

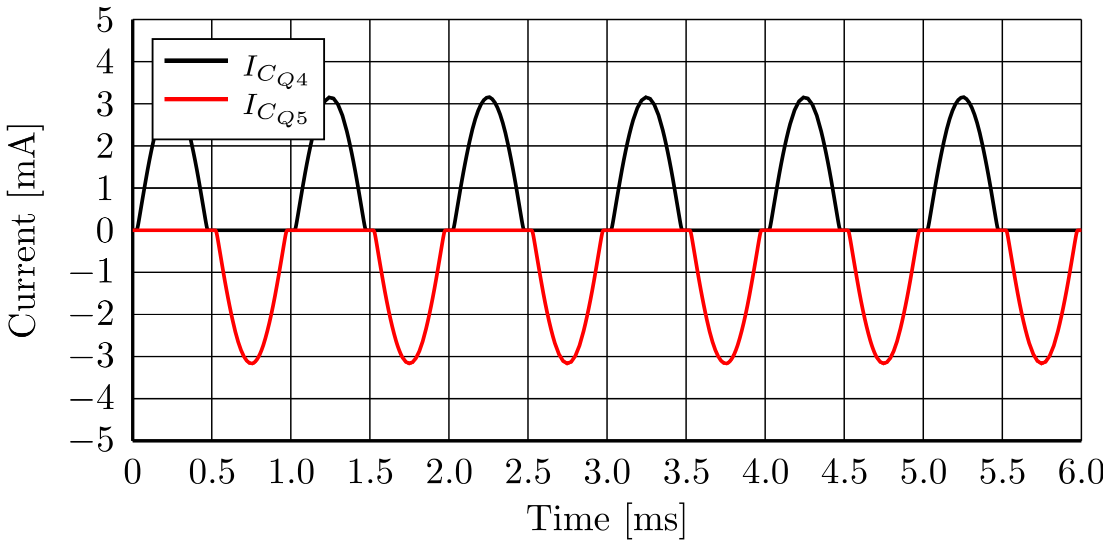

Figure 30: Conduction angle < 180°. Class C. Heavy crossover distortion.

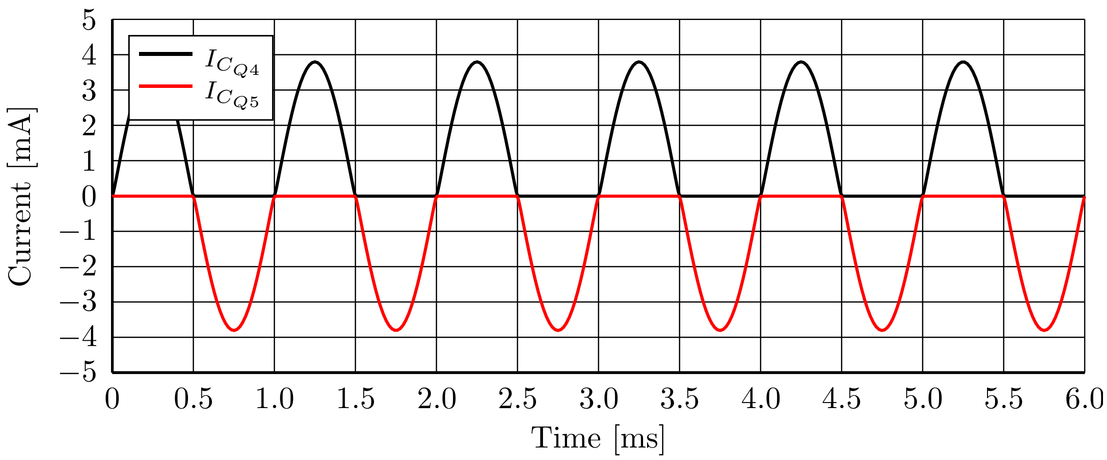

Figure 31: Conduction angle is 180°. Class B. Ideally no crossover distortion, but in practice there is still some due to biasing circuit limitations, temperature effects and differences in the output devices.

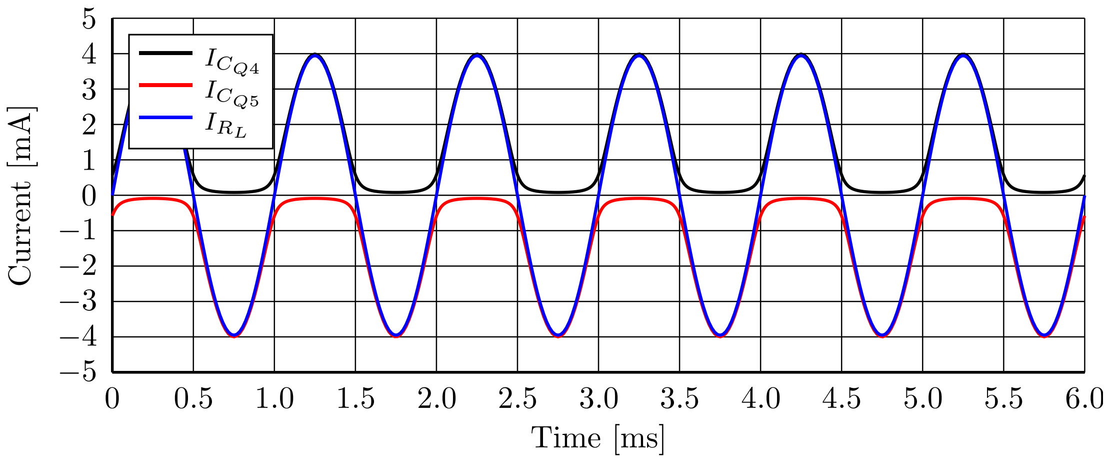

Figure 32: Conduction angle > 180° but < 360°. Class AB. Some crossover distortion, The extended crossover region changes output resistance over a wider range of the signal, increasing distortion.

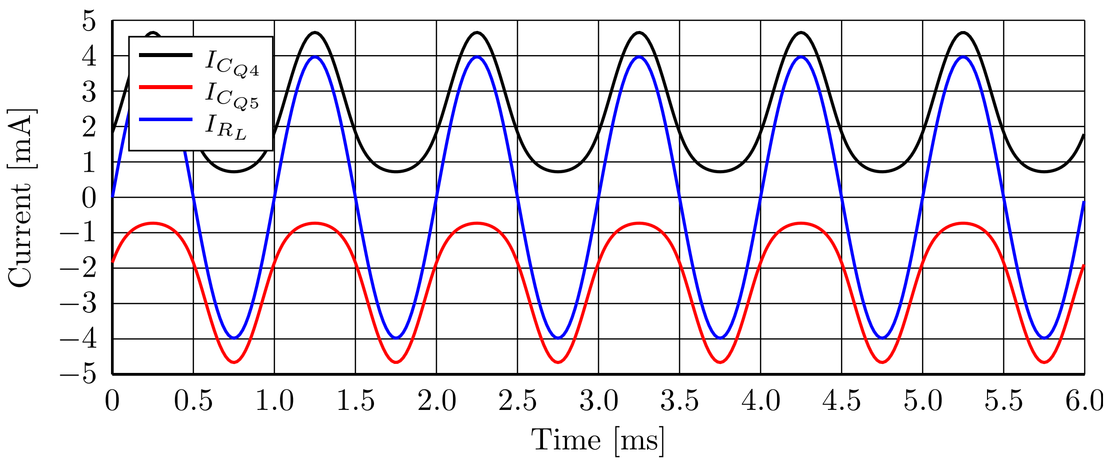

Figure 33: Conduction angle 360°. Class A. No crossover distortion, but other distortion mechanisms still exist. Output resistance still a function of collector current.

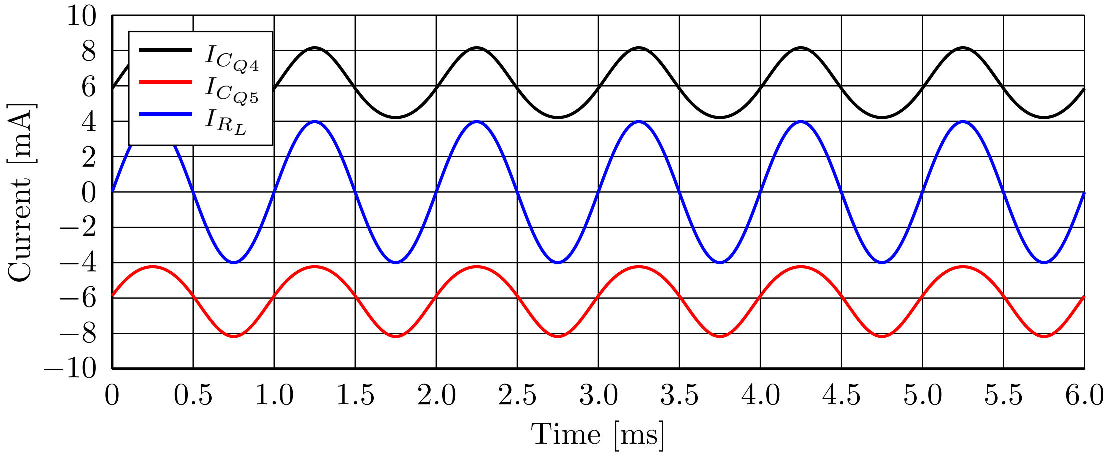

Figure 34: Conduction angle 360°. Class A. No crossover distortion, and higher bias compared to Figure 33 means the signal is a better approximation to a *small signal* in this case than in Figure 33 therefore the change in small signal parameters is less marked and distortion mechanisms are reduced further at the expense of very poor efficiency.

Class	Conduction Angle	Push Pull Bias [V]	$I_Q : I_L$
Class A	360	>> 1.4 V	$I_Q > I_{L_{\text{MAX}}}$
Class AB	< 360 and > 180	> 1.4	$I_Q \approx 0.1 I_{L_{\text{MAX}}}$
Class B	180	1.4 V	$I_Q < 0.1 I_{L_{\text{MAX}}}$
Class C	< 180	< 1.4 V	$I_Q << 0.1 I_{L_{\text{MAX}}}$

Table 7: The relationship between current conduction angle and the class of an amplifier stage. Also shown are rules of thumb for biasing of a push-pull stage and for load to quiescent current relationships.