Solutions to sample exam questions for EEE349/350

1.

a.

For a conductor of length l, and applying Gauss's Law over a cylindrical surface at a radius r within the conductor cross-section itself and noting that the electric field is purely radial yields:

$$\oiint \vec{D}\,.\,\overrightarrow{ds} = \oiiint q\,dv$$

$$2\pi r l\overrightarrow{D_r} = lq\pi r^2$$

Hence,

$$\overrightarrow{D_r} = \frac{rq}{2}$$

But $\vec{D} = \varepsilon\vec{E}$ and so:

$$\overrightarrow{E_r} = \frac{rq}{2\varepsilon}$$

For the region outside the conductor

$$\oiint \vec{D}\,.\,\overrightarrow{ds} = \oiiint q\,dv$$

$$2\pi r l\overrightarrow{D_r} = \pi R_c^2 l\,q$$

Hence,

$$\overrightarrow{D_r} = \frac{R_c^2 q}{2r}$$

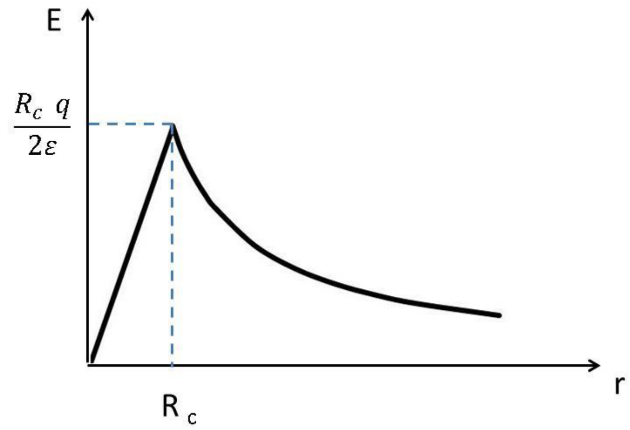
But $\vec{D} = \varepsilon\vec{E}$ and so:

$$\overrightarrow{E_r} = \frac{R_c^2 q}{2\varepsilon r}$$

In order to sketch a figure, it is necessary to establish the electric field at R_c. Application of either equation (which intuitively gives the same answer at R_c) yields:

$$\overrightarrow{E_r} = \frac{R_c\ q}{2\varepsilon}$$

Hence an appropriate sketch is:



b. The maximum electric field occurs at R_c. Setting this value to the breakdown voltage allows the maximum charge density in the conductor to be calculated.

$$\overrightarrow{E_r} = \frac{R_c q}{2\varepsilon}$$

Hence

$$q = \frac{2\overrightarrow{E_r}\varepsilon}{R_c} = \frac{2 \times 40 \times 10^6 \times 8.85 \times 10^{-12}}{10 \times 10^{-3}} = 0.0708\ Cm^{-3}$$

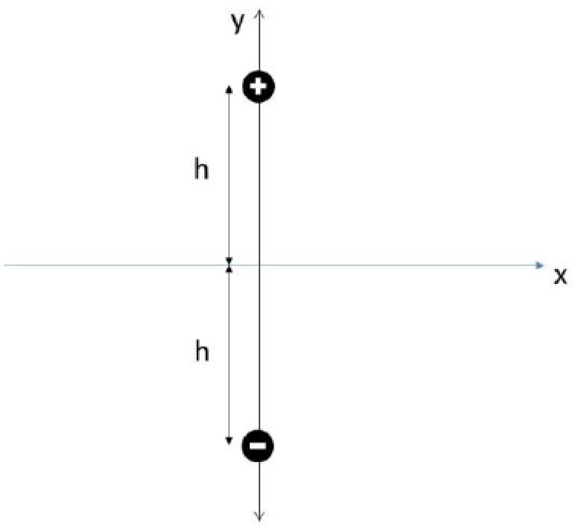
The maximum voltage across the insulation layer is given by:

$$V = \frac{R_c^2 q}{2\varepsilon} log_e\left(\frac{R_c}{R_i}\right) = \frac{(10 \times 10^{-3})^2 \times 0.0708}{2 \times 6 \times 8.85 \times 10^{-12}} log_e\left(\frac{10 \times 10^{-3}}{50 \times 10^{-3}}\right) = -107.3kV$$

c. Too difficult for an examination question.

2.

b. The method of images allows the geometry of the ground plane to be represented by an image charge as shown below.



Assume that Rc <<h.

Consider a general point outside the conductor. Applying Gauss's Law to an imaginary circular surface at a radius r yields:

$$\overrightarrow{D_r} 2\pi r L_c = \pi {R_c}^2 L_c q$$

Hence, the electric field strength is given by:

$$\overrightarrow{E_r} = \frac{\overrightarrow{D_r}}{\varepsilon_0} = \frac{{R_c}^2 q}{2\varepsilon_0 r}$$

By specifying a vertical integration path, the voltage between the conductors due to the charge on the upper transmission line (u) is given by:

$$V_{u-l,u} = \int_{R_c}^{2h} \frac{{R_c}^2 q}{2\varepsilon_0 r} dr = \left[\frac{{R_c}^2 q}{2\varepsilon_0} log_e r\right]_{R_c}^{2h} = \frac{{R_c}^2 q}{2\varepsilon_0} log_e \left(\frac{2h}{R_c}\right)$$

A similar procedure can be applied to the lower image conductor, but this will simply produce an equal and opposite voltage *[Strictly not necessary to perform the integral].*

$$V_{l-u,l} = \int_{R_c}^{2h} \frac{-{R_c}^2 q}{2\varepsilon_0 r} dr = \left[\frac{-{R_c}^2 q}{2\varepsilon_0} log_e r\right]_{R_c}^{2h} = \frac{-{R_c}^2 q}{2\varepsilon_0} log_e \left(\frac{2h}{R_c}\right)$$

Hence the voltage difference is given by:

$$V_{u-l,u} - V_{l-u,l} = \frac{{R_c}^2 q}{\varepsilon_0} log_e \left(\frac{2h}{R_c}\right)$$

However, from symmetry the potential between the upper conductor and ground is only half the voltage between the upper and lower charges.

$$V_{u-gnd} = \frac{{R_c}^2 q}{2\varepsilon_0} log_e \left(\frac{2h}{R_c}\right)$$

Hence, capacitance of transmission line to ground is:

$$C_{u-gnd} = \frac{Q}{V_{u-gnd}} = \frac{\pi {R_c}^2 L_c q}{\frac{{R_c}^2 q}{2\varepsilon_0} log_e\left(\frac{2h}{R_c}\right)} = \frac{2\pi\varepsilon_0 L_c}{log_e\left(\frac{2h}{R_c}\right)}$$