Engineering Electromagnetics

Aims

The main aims of this course are to introduce a number of techniques for the calculation of electrostatic and electromagnetic field distributions and to apply them to a variety of engineering problems.

1. Mathematic Fundamentals

1.1 Coordinate systems

The choice of an appropriate coordinate system can simplify the solution of electromagnetic and electrostatic problems. The three main coordinate systems are:

a) Cartesian system
   It has three orthogonal axes, x, y, and z as shown in Fig 1.1. The basic elemental volume is a cube. The Cartesian coordinate system is often used where the field boundary of an electromagnetic problem is rectangular. e.g., Solving for the electric field between the two plates of a capacitor

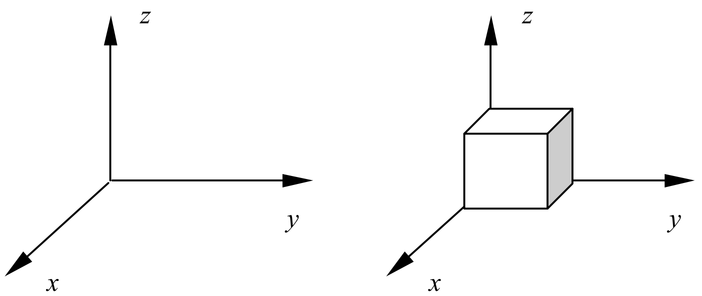


Fig. 1.1 Cartesian Coordinate System

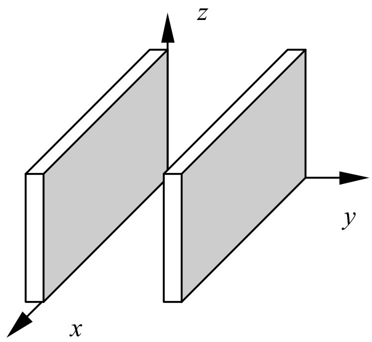


Fig. 1.2 Parallel plate capacitor

b) Cylindrical system

   It has three orthogonal axes, e_r, e_θ, and e_z as shown in Fig 1.3. Note that the coordinate of a point P is given by (r, θ, z), and the unit vectors of the three axes are in r (radial), θ (circumferential) and z directions.

   The relationship between the Cartesian and the Cylindrical coordinate systems are given by:

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ \theta &= \tan^{-1}(y/x) \\ z &= z \end{aligned} \tag{1.1}$$

or

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{aligned} \tag{1.2}$$

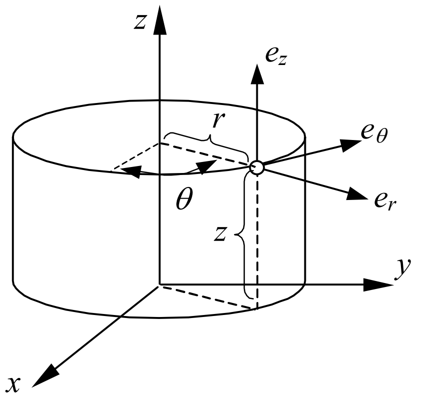


Fig. 1.3 Cylindrical coordinate system

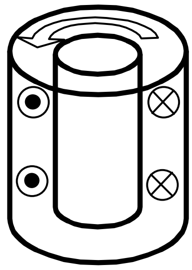


Fig. 1.4 Cylindrical coil (solenoid)

The cylindrical coordinate system is often used when the field boundary of an electromagnetic problem is cylindrical. e.g., Solving for the magnetic field produced by a cylindrical coil, as shown in Fig. 1.4

c) Spherical system

It has three orthogonal axes, e_r, e_θ, and e_ϕ as shown in Fig 1.5. Note that the coordinate of a point P is given by (r, θ, ϕ), and the unit vectors of the three axes are in r (radial), θ (latitudinal) and ϕ (longitudinal) directions.

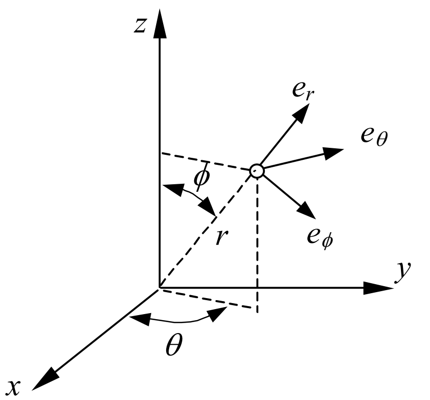


Fig. 1.5 Cylindrical coordinate system

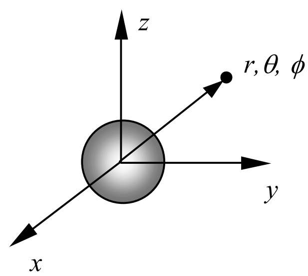


Fig. 1.6 Point charge

The relationship between the Cartesian and the Spherical coordinate systems are given by:

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ \theta &= \tan^{-1}(y/x) \\ \phi &= \tan^{-1}\left\{\sqrt{x^2 + y^2} / z\right\} \end{aligned} \tag{1.3}$$

or

$$\begin{aligned} x &= r\cos\theta\sin\phi \\ y &= r\sin\theta\cos\phi \\ z &= r\cos\phi \end{aligned} \tag{1.4}$$

The spherical coordinate system is often used when the field has a distribution of spherical symmetry. e.g., a point charge, as shown in Fig. 1.6.

d) Right and left handed coordinate systems

A rectangular or Cartesian coordinate systems has three mutually perpendicular axes, normally labelled *x, y,* and *z*. The system can be either right- or left-handed.

In a right-handed system, rotation from the *x* axis towards the *y* axis in the direction of fingers of right hand has the thumb pointing in the direction of the *z* axis.

It is customary to always use right-handed systems.

1.2 Vector quantities in electromagnetics

Vectors are quantities which have both magnitude and direction. There are numerous vector quantities in electromagnetics:

$\vec{E}$	— Electric field strength (sometimes referred to as simply electric field)	V/m
$\vec{D}$	— Electric flux density	C/m^2
$\vec{H}$	— Magnetic field strength	A/m
$\vec{B}$	— Magnetic flux density	T (Wb/m^2)
$\vec{J}$	— Current density	A/m^2

There are two fundamental constants in electromagnetics:

ε_o — permittivity of free space 8.85×10^{-12} (F/m)
μ_o — permeability of free space $4\pi\times10^{-7}$ (H/m)

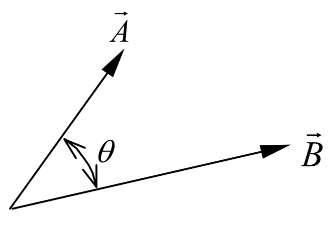
$c^2 = 1/(\varepsilon_o\mu_o)$

c — speed of light in free space (m/s)

1.3 Multiplying Vectors

a) Scalar product (or dot product)

The scalar product of two vectors is the product of the magnitudes multiplied by the cosine of the angle between the two vectors:



e. g. the scalar product of vectors $\vec{A}$ and $\vec{B}$ is:

$$\vec{A} \bullet \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos\theta$$

The scalar product of two vectors results in a scalar which is a quantity with magnitude and sign but no directions.

If the unit vectors in the Cartesian coordinate system are e_x, e_y, and e_z, respectively, then the two vectors $\vec{A}$ and $\vec{B}$ can be written as:

$$\vec{A} = A_x e_x + A_y e_y + A_z e_z$$
$$\vec{B} = B_x e_x + B_y e_y + B_z e_z$$

The scalar or dot product of these two vectors is:

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

which is a scalar.

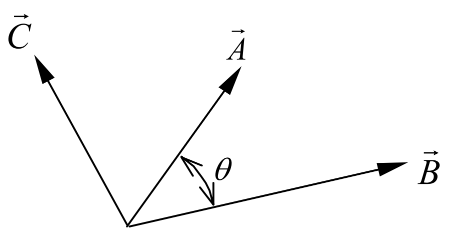
b) Vector product (or cross product)

The vector product of two vectors is a vector with a magnitude equal to the product of the two vector magnitudes multiplied by the sine of the angle between them. The direction of this third product vector is perpendicular to the plane containing the first two vectors. The two original vectors and their vector product form a right handed set. e.g.

The vector $\vec{C}$ is the vector product of $\vec{A}$ and $\vec{B}$, its magnitude $|\vec{C}|$ is given by:

$$|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \sin\theta$$

and its direction as shown so that it form a right handed set.



In vector notation

$$\vec{A} \times \vec{B} = \vec{C} = |\vec{A}| \cdot |\vec{B}| \sin\theta \; \vec{n}$$

where $\vec{n}$ is the unit vector normal to the plane which contains $\vec{A}$ and $\vec{B}$.

If the two vectors $\vec{A}$ and $\vec{B}$ are given by:

$$\vec{A} = A_x e_x + A_y e_y + A_z e_z$$
$$\vec{B} = B_x e_x + B_y e_y + B_z e_z$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) e_x + (A_z B_x - A_x B_z) e_y + (A_x B_y - A_y B_x) e_z$$

which is often expressed in determinant from as:

$$\vec{A} \times \vec{B} = \begin{vmatrix} e_x & e_y & e_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} e_x - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} e_y + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} e_x$$

Summary:

- ♦ The scalar product of two vectors yields a scalar;
- ♦ The vector product of two vectors yields another vector

1.4 Integrals of vectors

a) Line integral

Supposing we have a radial force field $\vec{F}$, i.e., it acts with a force of magnitude F on an object in the r direction as shown in Fig. 1.7:

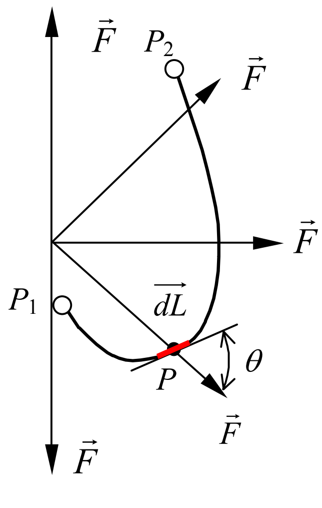


Fig. 1.7 Line integration of a vector quantity

If we moves along an arbitrary curved path between points P_1 and P_2 , the force acting at point P is $F\cos\theta$. Over a small increment of the path $\overrightarrow{dL}$, the product of the force x distance is:

$$F \cos\theta \; dL$$

which is the work done in moving through $\overrightarrow{dL}$. Note both $\vec{F}$ and $\overrightarrow{dL}$ have direction associated with them, thus,

$$dW = F\cos\theta \; dL = \vec{F} \bullet d\vec{L}$$

which is a scalar, but the right hand side is the scalar or dot product of $\vec{F}$ and $\overrightarrow{dL}$. The total work done in moving the object from P_1 to P_2 is :

$$W = \int_{P1}^{P2} dW = \int_{P1}^{P2} F\cos\theta \; dL = \int_{p1}^{P2} \vec{F} \bullet d\vec{L}$$

Line integral is useful in calculating work done. As you are probably aware from the first year course, the work done in moving from P_1 to P_2 does not depend on the path taken.

Sometimes, it is necessary to integrate around a closed loop path, e.g., calculating the potential from the electric field, so called closed loop line integrals are given in symbol:

$$V = \oint \vec{E} \bullet d\vec{l}$$

The circle indicates closed loop.

b) Surface integral

Assume that we have a uniform magnetic field with a flux density $\vec{B}$ which passes through a surface with a cross-sectional area S, as shown in Fig. 1.8.

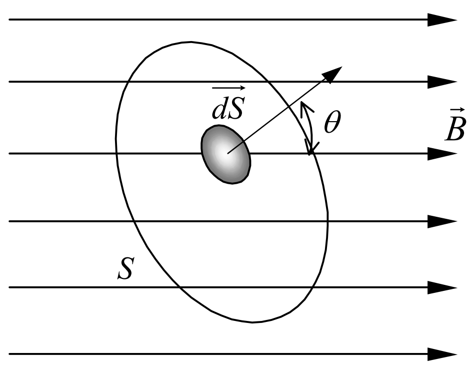


Fig. 1.8 Surface integration of a vector quantity

The amount flux, ψ, which passes through the area depends on three factors:

- Magnitude of $\vec{B}$
- Area S
- Angle between the $\vec{B}$ field and the plane of S

Thus:

$$\psi = S \cdot B\cos\theta = \vec{B} \bullet \vec{S}$$

The result is a scalar. Note the area S has a direction associated with it, i.e., $\vec{S} = S\,\vec{n}$, where $\vec{n}$ is the unit vector perpendicular to the surface *S*, commonly referred to as the normal of *S*. If the $\vec{B}$ field is not uniform, i.e., it is a function of position (*x, y, z*), we need to calculate the incremental flux dψ through an infinitesimal area *dS* at a point:

$$d\psi == \vec{B} \bullet \vec{n}dS = \vec{B} \bullet d\vec{S}$$

We calculate the total flux by integrating over the area, thus:

$$\psi = \iint d\psi == \iint_S \vec{B} \bullet d\vec{S}$$

which is a scalar. Surface integrals are often used to calculate the flux which links an area. In the above case the area was a simple plane. It is possible however to integrate any spatial varying vector over any irregular surface.

1.5 Maxwell equations

Modern electromagnetism is based on a set of four fundamental relations known as Maxwell equations:

$$\nabla \bullet \vec{D} = \rho_V \qquad (1.5a)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (1.5b)$$

$$\nabla \bullet \vec{B} = 0 \qquad (1.5c)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (1.5d)$$

where ***E*** and ***D*** are electric field quantities interrelated by ***D*** = ε***E***, with ε being the electric permittivity of the material; ***B*** and ***H*** are magnetic field quantities interrelated by ***B*** = μ***H***, with μ denoting the magnetic permeability of the material; ρ_V is the electric charge density per volume; and ***J*** is the current density per area. The equations hold in any material, including free space (vacuum), and at any spatial location (*x, y, z*). In general, all the quantities in Maxwell's equations may be a function of time t. By his formulation of these equations, published in a classic treatise in 1873, James Clerk Maxwell established the first unified theory of electricity and magnetism. His equations, which he deduced from experimental observations reported by Gauss, Ampere, Faraday, and others, not only encapsulate the connection between the electric field and electric charge and between the magnetic field and electric current, but they also define the bilateral coupling between the electric and magnetic field quantities. Together with some auxiliary relations, Maxwell's equations form the fundamental tenets of electromagnetic theory, which has profoundly influenced our society today.

You may not be familiar with some of the mathematic operators, and we shall introduce them one by one as the relevant theory is introduced and developed.