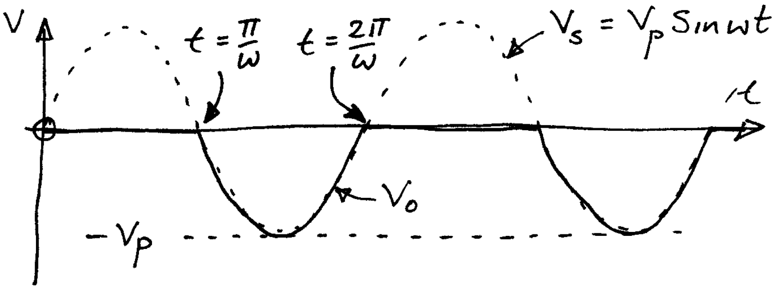
EEE123 Problem Sheet Solutions

Rectifiers and Smoothing

Q1 (i) The cct. is a half wave rectifier with a negative output.



(ii) V_s is always given as an r.m.s. quantity.

$V_{opk} = -V_{r.m.s.}\sqrt{2} = -9\sqrt{2} = \underline{\underline{-12.73\,V}}$.

[note that this assumes that the 0.7V diode drop is negligible.]

(iii) $V_{OAVE} = \frac{V_{opk}}{\pi}$ (for a half wave rectified sinusoid)

$= \underline{\underline{-4.05\,V}}$.

[If you don't know that $V_{OAVE} = V_{opk}/\pi$ for a half wave rectified sinusoid, you should know how to work it out. You need to work out the area under the waveform and divide it by the periodic time....

$$V_{AVE} = \frac{1}{T}\int_{T/2}^{T} V_p \sin\omega t\, dt = \frac{1}{(2\pi/\omega)}\int_{\pi/\omega}^{2\pi/\omega} V_p \sin\omega t\, dt$$

$$= \frac{\omega}{2\pi}\left[-\frac{V_p}{\omega}\cos\omega t\right]_{\pi/\omega}^{2\pi/\omega} = -\frac{V_p}{2\pi}\left[1-(-1)\right] = -\frac{V_p}{\pi}$$

note that the result is negative because the area under the curve for the chosen limits is negative. If the integration limit had been π/ω to 0, the answer would have been $+V_p/\pi$.]

(iv) The r.m.s (root mean square) value of a sinusoid is $V_p/\sqrt{2}$ so the mean squared value is $V_p^2/2$. A half wave signal will have half the mean squared value of a full sinusoid so the half wave mean squared (ms) value is $V_p^2/4$.

The r.m.s. value of the waveform of part (i) is therefore $V_p/2 = \underline{6.36\,V}$.

If you cannot follow the argument above, you can evaluate:

$$V_{rms} = \left[\frac{\omega}{2\pi} \int_{\pi/\omega}^{2\pi/\omega} V_p^2 \sin^2 \omega t \, dt \right]^{1/2}$$

to get the same answer.

(v) If you know the r.m.s voltage across a resistor, working out power dissipation in the resistor is straightforward....

$$P_D = \frac{V_{rms}^2}{R} = \frac{6.36^2}{100} = \underline{\underline{405\,mW}}. \ (0.405\,W)$$

(vi) If the transformer has an effective series resistance of 10Ω, the voltage across R can be found by potential division of the ideal transformer output voltage...

$$V_{pk} = V_{pk\,ideal} \times \frac{R}{10\Omega + R} = \underline{\underline{-11.75\,V}}.$$

(vii) The new power dissipated in R is

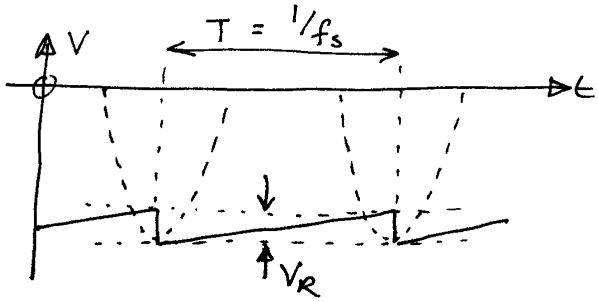
$$\frac{V_{rms\,(new)}^2}{R} = \left(\frac{11.75}{2}\right)^2 \cdot \frac{1}{100} = 335\,mW.$$

[You could also have taken the view here that the idea Vrms of part (iv) – 6.36V – is acting on R + the 10Ω internal resistance so the power dissipated in this total resistance is

$$\frac{6.36^2}{110\Omega} = 367.7\,mW.$$

Ten elevenths of this power is being dissipated in R giving $P_D = 335\,mW$.]

Q2



assume :

(a) $I_L \approx$ constant

(b) capacitor charges instantaneously at each -ve half cycle peak.

(c) 0.7V is negligible.

Ripple behaviour governed by $I_L = C\frac{dV_o}{dt}$ and since I_L = const, $\frac{dV_o}{dt}$ is the slope of the straight line, V_R/T

$$I_L \approx \frac{V_{pk}}{R} = C\frac{V_R}{T} = C\frac{V_R}{1/f_s} = CV_Rf_s.$$

$$\text{or } C = \frac{V_{pk}}{RV_Rf_s} \approx \underline{\underline{8500\mu F}} \quad (8.5mF)$$

The power now dissipated in R is $\frac{V_{DC}^2}{R}$

$V_{DC} = V_p - V_R/2 \rightarrow$ ie, the average value of V_o.

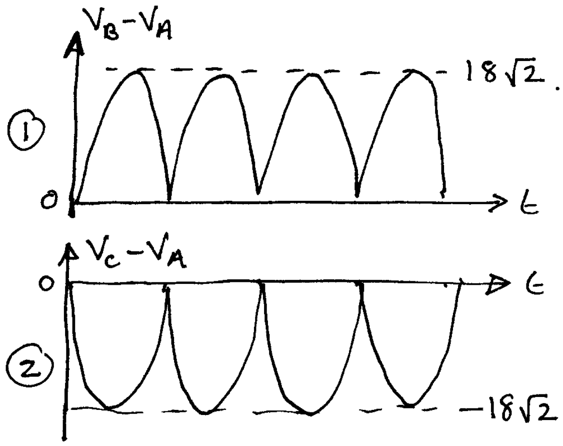
$= 12.73 - 0.15$

Thus P_D = $\underline{1.58 W}$.

[You might legitimately argue that if the 0.7V diode drop can be neglected, then so can $V_R/2$ at a mere 0.15V. Such an argument is fine (in this case) and would give a $P_D = 1.62W$. If the ripple voltage was 3V, though, ignoring it would be on the border of unreasonable.]

Q3

The outputs are as shown

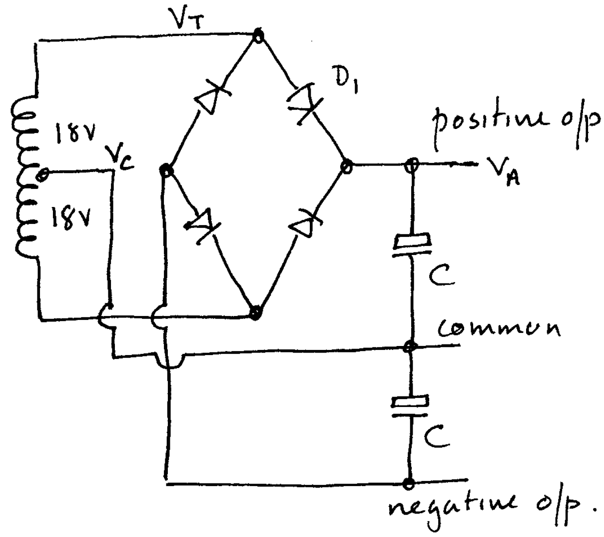


(a) $(V_B - V_A)_{pk} = \underline{\underline{18\sqrt{2}}}$

(b) $(V_A - V_C)_{pk} = -(-18\sqrt{2})$
$= \underline{18\sqrt{2}}$.

(c) $(V_C - V_B)_{pk} = -18\sqrt{2} - 18\sqrt{2}$
$= \underline{\underline{-36\sqrt{2}}}$

Q4 (i)



(ii) using the same approximations as in Q2 but recognising that I_L is given explicitly and the circuit is full wave (ie the charging interval is $1/2f_s$)

$$I_L = C\frac{dV_o}{dt} = \frac{CV_R}{1/2f_s} = 2f_s C V_R.$$

$$\text{Thus gives } C = \frac{I_L}{2f_s V_R} = \frac{2}{2.50.1} = \underline{\underline{20\,mF}}.$$

Since the +ve & -ve outputs have the same load conditions, this value of C applies to both.

(iii) A variety of answers are acceptable here depending on assumptions made ... these must be stated if used. The answer given is based on

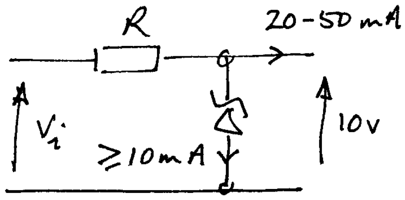
$$V_{odc} = V_{pk} - 0.7 - V_R/2 = \underline{24.3\,V}$$

If you choose to neglect the 0.7V or the $V_R/2$ or both — all reasonable approximations in this case, you would get marginally higher answers.

(iv) Consider D_1. When V_T is at its +ve peak w.r.t. V_c, D_1 conducts and C is charged to almost $18\sqrt{2}$ V. V_T then falls until at the peak of the next half cycle it $= -18\sqrt{2}$. But V_A is still $\approx +18\sqrt{2}$ so the peak reverse voltage across D_1 is $2\times18\sqrt{2} \approx \underline{51V}$.

Q5. (i) 10V (should be obvious!)

(ii) A range of answers is possible here depending on the assumptions made in determining the output voltage from Q4.



The answer given here uses the instantaneous minimum value of V_o taking diode drop and ripple into account....

$$R_{max} = \frac{V_{i\,min} - V_o}{I_{L\,max} + I_{Z\,min}} = \frac{(18\sqrt{2} - 0{\cdot}7 - 1) - 10}{50\text{mA} + 10\text{mA}}$$

$$= \underline{\underline{230\,\Omega}}.$$

[If you neglected the 0·7 and V_R, $R = 258$. Ignoring one or the other gives an answer between $230\,\Omega$ + $258\,\Omega$]

(iii) If the load is disconnected, all 60mA will flow through the zener diode

$$\therefore P_D = 10\text{V} \times 60\text{mA} = \underline{\underline{600\text{ mW}}}.$$

(iv) This answer will be slightly dependent on your assumptions about V_i and your value of R. The answer given assumes an average dc V_i of 25V (ie the 0·7v drop is ignored)

$$P_D = \frac{25 \times 25}{230} = \underline{\underline{2{\cdot}72\text{ W}}}.$$

(v) Under normal conditions

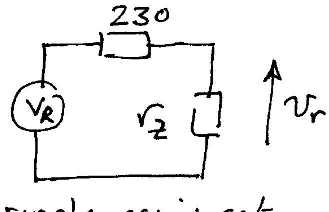
$$P_{D(zener)} = 10\text{V} \times 40\text{mA} = \underline{400\text{mW}}.$$

(note that the minimum I_L condition represents the worst case normal operation for the zener).

$$P_{D\,res.} = 15^2/230 = \underline{\underline{980\text{mW}}}.$$

(this value depends on R and Vi assumptions).

(vi) $v_r = V_R \dfrac{r_z}{230 + r_z}$



ripple equiv cct.

$v_r = 10\text{mV}$

$V_R = 1\text{V}$

$\longrightarrow$ gives $r_{z\,max} = \underline{\underline{2{\cdot}3\,\Omega}}.$