Data Provided: Laplace Transform pairs and Properties of Fourier Transform

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2006-2007 (2 hours)

Signals and Systems

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a.

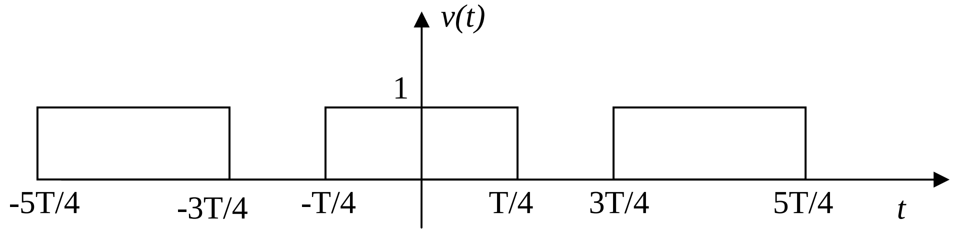

Figure Q.1.1

Show that the Trigonometric Fourier Series coefficient of the signal *v(t)* shown in Figure Q.1.1 is given by $a_n = \dfrac{2\sin\left(\dfrac{n\pi}{2}\right)}{n\pi}$, where *n* is the harmonic number. Hence write down an expression for the Fourier Series representation of *v*(*t*). **(9)**

b. Sketch and label the amplitude spectrum, up to the 9^{th} harmonic, of the signal *v(t)* in Figure Q.1.1. **(3)**

c. In a simple dc to ac converter, the output is obtained by opening and closing a switch at 50 Hz such that the amplitude changes from 0 to 1 V. What is the average power of the output signal within the frequency range of -200 Hz to 200 Hz? **(8)**

2. a. Write the equation describing the time domain convolution of two continuous time signals, *h(t)* and *x(t)*. Briefly outline the steps required to perform a graphical convolution. **(4)**

b. Consider a linear time-invariant system with an impulse response given by $h(t) = e^{-at}u(t)$. Sketch the graphical convolution process between *h*(*t*) and a unit step function *u*(*t*) and show that the **unit step response** is given **(8)**

by $y(t) = \frac{1}{a}\left(1 - e^{-at}\right)u(t)$.

c. Consider a signal $x(t)$ shown in Figure Q.2.1.

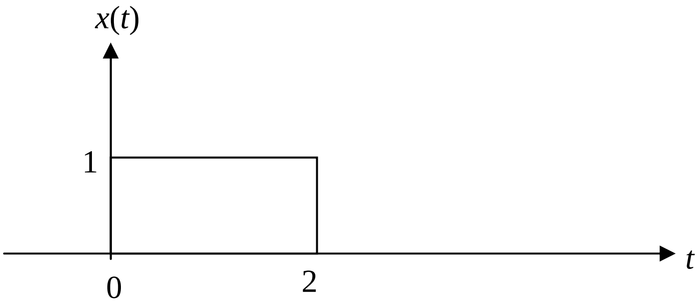

Figure Q.2.1

The signals $x(t)$ and $h(t)$, from 2(b), are sampled at every 0.5s in order to produce discrete signals $x[n]$ and $h[n]$, respectively. Write down the expressions for $x[n]$ and $h[n]$. Compute the response $y[n] = x[n]*h[n]$ assuming that $h[n] = 0$ for $n > 2$ and $a = 2$. **(8)**

3. a.

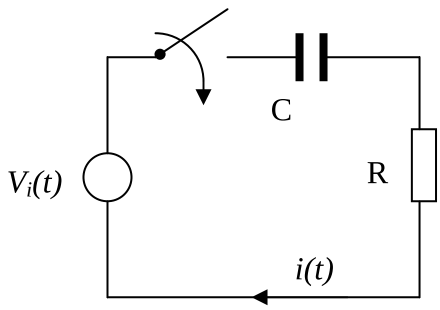

Figure Q.3.1

Use the Laplace transform to show that $i(t) = \frac{A}{R}e^{-t/RC}u(t)$ in the RC circuit shown in Figure Q.3.1 assuming that the initial voltage across the capacitor, $V_c(0)$ = 0 and $V_i(t) = Au(t)$. [Hint: $V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$]. **(8)**

b. Write down the initial value for $i(t)$ in Figure Q.3.1 and verify your answer using the initial value theorem. **(4)**

c. Find the time required for $V_c(t)$ to reach 0.5 V if A = 1 V, R = 10 Ω and C = 0.1 F. **(8)**

4. a.

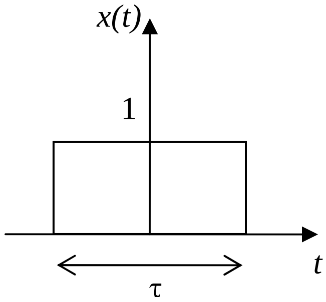

Figure Q.4.1(a)

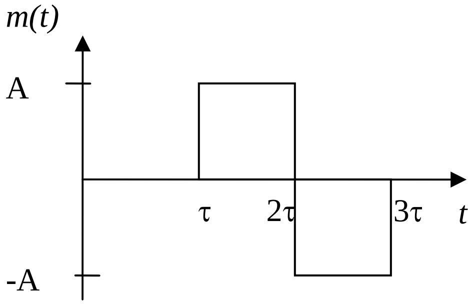

Figure Q.4.1(b)

Prove that the Fourier transform of the signal *x(t)* shown in figure Q.4.1(a) is given by $X(\omega) = \tau \dfrac{\sin\left(\dfrac{\omega\tau}{2}\right)}{\left(\dfrac{\omega\tau}{2}\right)}$. **(5)**

b. Obtain the Fourier transform of the signal *m(t)* shown in figure Q.4.1(b) by using the spectrum *X(ω)* in part (a) and the time shift property of the Fourier transform. **(5)**

c.

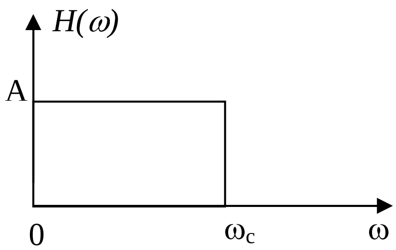

Figure Q.4.2

The magnitude spectrum of an ideal low pass filter is shown in Figure Q.4.2.

i) Obtain the time domain expression for a signal, $h(t)$, that will produce the spectrum shown in Figure Q.4.2. **(4)**

ii) Calculate the peak amplitude of $h(t)$. **(2)**

iii) Work out the value of ω_c if $h(t)$ first becomes zero at t = 1ms.. **(4)**

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