The University Of Sheffield.

Data Provided:

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}

Permittivity of free space $\varepsilon_0 = 8.8 \times 10^{-12}$ F m^{-1}

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2012-13 (2.0 hours)

EEE6084 Applied Electromagnetics 6

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. The numbers given after each section of a question indicate the **relative weighting** of that section. Your answers to questions 1 and 2 should be completed in a **separate answer book** to those for questions 3 and 4. Please write the **numbers of the questions** that you have answered on the front cover of the answer book.

1. **a.** Show that the current flowing in a lossless transmission line obeys the one dimensional wave equation. **(3)**

b. i) Derive the expression for the propagation constant β of a transmission line that has small but finite resistance and conductance per unit length.

ii) A 1 GHz sine wave with peak amplitude of 5 V is launched into the lossy transmission line. Draw the time dependence of the signal at a distance of 100 m along the line. Assume the following values for the transmission line:
G = 0 S/m; R = 1 Ω/m; C = 80 pF/m; L = 20 µH/m

iii) Sketch (qualitatively) the time dependence at 100 m for a square pulse launched into the same transmission line. **(7)**

c. Figure 1 (below) represents the cross-section of a microstrip transmission line. Suggest what suitable materials would be used for A1, A2 and A3 and their approximate sizes and their purposes.

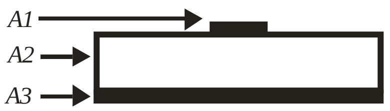

Figure 1 **(3)**

d. Assuming that the transmission line in Figure 1 is lossless and has a characteristic impedance Z_0 = 50 Ω and is connected to a load with impedance Z_L = 75 Ω. A square pulse of amplitude 5V is launched into the transmission line from a source with impedance Z_s = 100 Ω. What is the amplitude of the signal received by the load? **(4)**

e. The apparent impedance at the source end of a transmission line Z_A is given by the following equation, where the terms have their usual meanings:

$$Z_A = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{jZ_L \tan(\beta d) + Z_0}$$

i) For what length/lengths *d* of transmission line does the apparent impedance Z_A match the load impedance Z_L?

ii) If the condition in **1.e.i**. (above) is not met, what can be done in practise to match Z_A to Z_L? **(3)**

2. a. A 1 MHz TEM00-mode electromagnetic wave propagates in free space in the positive x direction.

i) Write down mathematical expressions that describe the magnetic and electric fields.

ii) Sketch the spatial relationship between the magnetic and electric fields at time t. Include an indication of the direction of propagation of the wave on your sketch. **(5)**

b. Define the Poynting vector **S**. **(2)**

c. The wave in **2.a.** has an electric field strength $E_0 = 1\times10^6$ V m^{-1} and is incident at normal incidence on a non-magnetic material with relative permittivity $\varepsilon_r = 2$. What is the electrical field strength of the wave immediately inside the interface? **(6)**

d. For the situation described in **2.c.** what is the power density of the wave within the material? **(4)**

e. The material in **2.c.** is weakly absorbing, hence the propagation constant β can be described by the following equation:

$$\beta = \omega / v - \frac{j\sigma}{2}\left(\frac{\mu\mu_0}{\varepsilon\varepsilon_0}\right)^{1/2}$$

- where the symbols have their usual meanings and the AC conductivity of the material at 1 MHz is $\sigma = 1\times10^{-5}\ \Omega$ m^{-1}. At what distance into the material does the power density fall to 50 % of its value immediately inside the interface? **(3)**

3. **a.** Using the relations

$$\nabla \bullet \overline{B} = 0,$$
$$\nabla \times \overline{H} = \overline{J}$$
$$\nabla \times (\nabla \times \overline{F}) = \nabla(\nabla \bullet \overline{F}) - \nabla^2 \overline{F}$$

where $\overline{F}$ is any vector, show that

$$\nabla^2 \overline{A} = -\mu \overline{J}$$

Define all vector quantities and give their units. **(6)**

b. A z-directed current sheet $J_z = J\cos(\beta y)$ flows on the surface (at $x = 0$) of an infinitely permeable iron conductor lying in the y-z plane as shown in Figure 2.

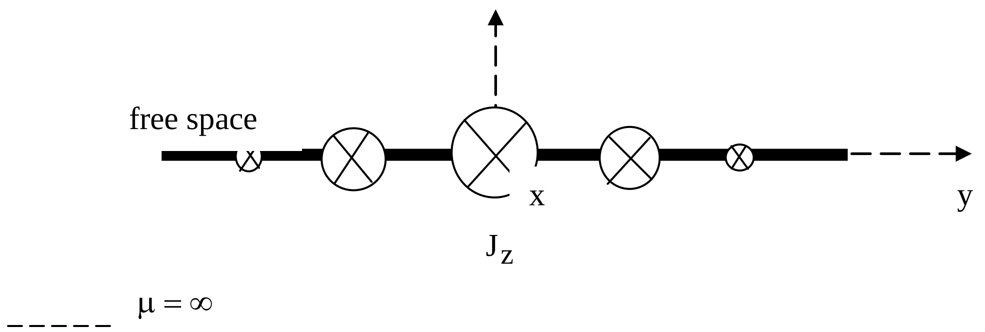

Figure 2

If

$$A_z(x, y) = \frac{\mu_o J_z}{\beta} e^{-\beta x} \quad \text{for } x \geq 0$$

(i) evaluate $\nabla^2 \overline{A}$

(ii) derive expressions for the magnetic field components

for $x \geq 0$.

The following vector operation should be of use:

$$\nabla \times \overline{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}$$

(8)

c. If $J = 1$ A/m and $\beta = 2\pi$, calculate the modulus of the magnetic field H at a distance of $x = 0$ and $x = 1$m above the current sheet and hence sketch H for:

i) $H(x, y = 0)$, $0 \leq x \leq 1$m

ii) $H(x = 0, y)$, $-1 \leq y \leq 1$m **(6)**

4. **a.** Write down Gauss's Law and briefly explain it. **(4)**

b. By considering the flux flowing through an elemental Cartesian cube containing electric charge, show that:

$$\nabla \bullet \overline{\mathrm{D}} = \rho$$

and hence write down the divergence theorem. **(10)**

c. Calculate the charge density at a point $\overline{\mathrm{P}} = \hat{\mathrm{x}} + 2\hat{\mathrm{y}} + 3\hat{\mathrm{z}}$ in space if:

(i) $\overline{\mathrm{D}} = 8\mathrm{x}^2\mathrm{yz}\,\hat{\mathrm{x}} + 4\mathrm{xy}^2\mathrm{z}\,\hat{\mathrm{y}} + 2\mathrm{xyz}^2\,\hat{\mathrm{z}}$ at $\overline{\mathrm{P}}$

(ii) $\overline{\mathrm{E}} = 8\mathrm{yz}\;\hat{x} + 4\mathrm{xz}\;\hat{y} + 2\mathrm{xy}\;\hat{z}$ at $\overline{\mathrm{P}}$ **(6)**

GLW / GGC