Example Solutions EEE406 / 6011 (2008)

1. a.

The aperture gain is given by

$$G = \frac{P_r|_{\theta=0^\circ,\phi=0^\circ}}{\dfrac{P}{4\pi r^2}} \qquad (1.1)$$

The total power radiated by the aperture of radius a can be obtained by integrating over the aperture field, and so

$$P = \frac{1}{2} Re \int_0^{2\pi}\int_0^a (\underline{E}\times\underline{H}^*).\hat{z}\rho d\rho d\phi \qquad (1.2)$$

since we only have assumed an x directed aperture electric field here. Now for apertures of sufficient electrical size, then we can assume that the electric and magnetic fields are related by the free space wave impedance

$$\frac{E_x}{H_y} = \eta = 377\Omega \qquad (1.3)$$

and hence

$$P = \frac{1}{2\eta}\int_0^{2\pi}\int_0^a |E_x|^2 \rho d\rho d\phi \qquad (1.4).$$

From the equation given for the aperture radiation pattern we have

$$|\underline{E}(\theta=0,\phi=0)| \approx \frac{1}{\lambda r}\{\hat{\theta})|F_x|\} \quad (1.5)$$

In the far field on boresight, the same relation as (1.3) holds for the radiation fields

$$\frac{E_\theta}{H_\phi} = \eta \qquad (1.6)$$

so

$$P_r|_{\theta=0^o,\phi=0^o} = \frac{1}{2}Re(\underline{E}\times\underline{H}^*) = \frac{1}{2\eta}|E_\theta|^2 \qquad (1.7)$$

and substituting (1.5) into (1.7) gives

$$P_r|_{\theta=0^o,\phi=0^o} = \frac{1}{2\eta\lambda^2 r^2}|F_x|^2 \quad (1.8)$$

Now

$$F_x|_{\theta=0^o} = \int_0^{2\pi}\int_0^a E_x \rho d\rho d\phi \qquad (1.9)$$

Hence substituting (1.9) into (1.8) and then (1.8) and (1.4) into (1.1) gives the aperture gain as

$$G = \frac{4\pi}{\lambda^2} \frac{\left| \int_0^{2\pi} \int_0^a E_x \rho d\rho d\phi \right|^2}{\int_0^{2\pi} \int_0^a |E_x|^2 \rho d\rho d\phi} \qquad (1.10)$$

Now if the aperture is uniformly illuminated so that $E_x = C$ for instance, then (1.10) becomes

$$G = \frac{4\pi}{\lambda^2} \frac{\left(C\pi a^2\right)^2}{C^2 \pi a^2} = \frac{4\pi}{\lambda^2} A \qquad (1.11)$$

where $A = \pi a^2$.

1. b.

For a circular aperture, then

$$A = \pi \times (.45/2)^2 = 0.16m^2 \qquad (1.12)$$

Assumptions are that the aperture is uniformly illuminated by the primary feed and that the aperture is electrically large enough so that a free space aperture wave impedance can be assumed (1.3), in which case the gain is obtained from (1.11) as

$$G = \frac{4\pi}{(3 \times 10^8 / 11 \times 10^9)^2} \times 0.16 = 2703 = 34dBi \qquad (1.13)$$

1. c.

(The student needs to know the gain of a typical Yagi-Uda TV antenna. Any value from 10 to 18dBi will suffice)

Using a value of *14dBi* then $G = 25$ and re-arranging (1.11)

$$A = \frac{(3 \times 10^8 / 600 \times 10^6)^2 \times 25}{4\pi} = 0.5m^2 \qquad (1.14)$$

1. d.

If a reflector antenna was to be used for terrestrial TV reception, then from (1.14) its circular aperture would need to have a diameter of *80cm*. This would look obtrusive on a chimney compared with a Yagi, and have a significantly greater wind loading.

If a Yagi were to be used for satellite TV reception, then it would be impractical to achieve the gain in (1.13) due to the small element size at *11GHz*, the number of elements required and problems in feeding.

2. a.

The gain of an antenna G is given by

$$G = \frac{\text{Power density in direction of maximum radiation}}{\text{Power density if antenna radiates all supplied power isotropically}} \qquad (2.1)$$

Hence,

$$G = \frac{P_r\big|_{\theta=90^\circ}}{\dfrac{P}{4\pi r^2}} \qquad (2.2)$$

Now

$$P_r = \frac{1}{2}\frac{|E_\theta|^2}{\eta} \qquad (2.3)$$

where $|E_\theta|$ is given in the question, so that

$$P_r\big|_{\theta=90^\circ} = \frac{1}{2\eta}\left(\frac{2\eta I_o}{4\pi r}\right)^2 = \frac{\eta I_o^{\,2}}{8\pi^2 r^2} \qquad (2.4)$$

The power radiated into the half space above the ground plane is then

$$P = \int_0^{2\pi}\int_0^{\pi/2} P_r r\,sin(\theta)d\phi r d\theta \qquad (2.5)$$

Since the fields are invariant in ϕ, (2.5) reduces to

$$P = 2\pi\int_0^{\pi/2} P_r r^2\,sin(\theta)d\theta \qquad (2.6)$$

From (2.3)

$$P = \frac{\eta I_o^{\,2}}{4\pi r^2}\int_0^{\pi/2}\frac{cos^2\left(\dfrac{\pi}{2}cos(\theta)\right)}{sin^2(\theta)}r^2\,sin(\theta)d\theta \quad (2.7)$$

Substituting the value for the integral given in the question then yields

$$P = \frac{\eta I_o^{\,2}}{4\pi}\times 0.61 \qquad (2.8)$$

Thus from (2.4) and (2.2)

$$G = \frac{\eta I_o^{\,2}/8\pi^2 r^2}{\dfrac{0.61\eta I_o^{\,2}/4\pi}{4\pi r^2}} = 3.3 \qquad (2.9)$$

Thus the gain of the monopole in dBi is $10log_{10}(G) = 5.2dBi$

2. b.

(Here the student should recognise 37Ω *as the radiation resistance of a lossless monopole.)*

The resistor is placed at the base of the monopole, and so the terminal current I_o will flow through it. This same current flows through the radiation resistance, which also equals 37Ω, and therefore half the supplied power is dissipated in each resistance. The monopole gain therefore drops by *3dB*, to *2.2dBi*.

2. c.

If the resistor is now moved further up the monopole, less current flows through it, and therefore proportionately less power is dissipated by it. The gain will therefore be in the range $2.2dBi < G < 5.2dBi$. *(An exact value is not required)*

2. d.

If the monopole is resonant, then the reactive component of the input impedance will be zero. Hence in case (a) the input impedance equals the radiation resistance, 37Ω. In case (b) we have two 37Ω resistors in series, so the input impedance is now 74Ω.

2. e.

If the ground plane is lossy, then some of the supplied power will be dissipated by it rather than by the radiation resistance, so as in case (b) the gain will fall.

3. a.

Forces that act on an electron in an ionospheric plasma are:

(i) The force due to the electric field of a radio wave = $e\underline{E}$

(ii) The force of inertia due to movement of the electron = $-m\frac{d\underline{v}}{dt}$

(iii) The frictional or viscous force due to collisions = $-m\nu\underline{v}$

(iv) The force due to its motion in the earth's magnetic field = $e\underline{v}\times\underline{B}$

Where e is the electronic charge, m the mass of an electron, $\underline{v}$ its velocity, ν the frequency of collisions, $\underline{B}$ the earth's magnetic field and $\underline{E}$ the electric field of the radio wave.

3. b.

Since all forces must sum to zero, we have

$$e\underline{E}+e\underline{v}\times\underline{B}-m\frac{d\underline{v}}{dt}-m\nu\underline{v}=0 \qquad (3.1)$$

Neglecting the earth's magnetic field yields

$$e\underline{E}=m\frac{d\underline{v}}{dt}+m\nu\underline{v} \qquad (3.2)$$

Dropping the vector notation (assuming one Cartesian direction), and using the explicit time domain notation for electric field and velocity, namely $E=E_o e^{j\omega t}$ and $\mathrm{v}=\mathrm{v}_0 e^{j\omega t}$ where ω is the frequency of the radio wave, we can re-write (3.2) as

$$eE_o e^{j\omega t}=mj\omega v_o e^{j\omega t}+m\nu v_o e^{j\omega t} \qquad (3.3)$$

the solution to (3.3) is then

$$v=\frac{eE}{m\nu+j\omega m} \qquad (3.4)$$

Now, the current density due to the incident electric field is $J=Nev$ Am^{-2}, where N is the electron density (units m^{-3}), so

$$J=\frac{Ne^2}{m}\frac{1}{(\nu+j\omega)}E \quad (3.5)$$

Now both current density and free space displacement current give rise to a magnetic field through

$$\nabla\times\underline{H}=J+j\omega\varepsilon_o E \quad (3.6)$$

So:

$$\nabla \times \underline{H} = \frac{Ne^2}{m}\frac{1}{(\nu + j\omega)}E + j\omega\varepsilon_o E \qquad (3.7)$$

hence:

$$\nabla \times \underline{H} = \frac{Ne^2}{m}\frac{\nu}{(\nu^2+\omega^2)}E - j\omega\frac{Ne^2}{m}\frac{1}{(\nu^2+\omega^2)}E + j\omega\varepsilon_o E \quad (3.8)$$

i.e.

$\nabla \times \underline{H} = \textit{conduction current} + \textit{polarisation current} + \textit{free space displacement current}$

Now, assuming negligible collisions between electrons, $\nu \approx 0$, so the conduction current vanishes, leaving only polarisation current, thus:

$$J = -j\omega\frac{Ne^2}{m\omega^2}E \qquad (3.9)$$

3. c.

At a frequency

$$\omega^2 = \omega_c{}^2 = \frac{Ne^2}{\varepsilon_o m} \qquad (3.10)$$

then

$$\nabla \times \underline{H} = -j\omega\frac{Ne^2}{m}\frac{\varepsilon_o m}{Ne^2}E + j\omega\varepsilon_o E = 0 \qquad (3.11)$$

Thus at the critical frequency ω_c, no magnetic field is generated by the electric field, and wave propagation ceases.

3. d.

The critical frequency can be approximated by

$$f_c \approx 9\sqrt{N} = 9\sqrt{10^{12}} = 9MHz \qquad (3.12)$$

4. a.

(Only the basic start up equations requested here)

1) FDTD

Uses the differential form of Maxwell's Equations,

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t} \qquad (4.1)$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad (4.2)$$

2) FIT

Uses the integral form of Maxwell's Equations:

$$\oint \underline{H}.d\underline{l} = \iint (\frac{\partial \underline{D}}{\partial t} + \underline{J}).d\underline{S} \qquad (4.3)$$

$$\oint \underline{E}.d\underline{l} = -\iint \frac{\partial \underline{B}}{\partial t}.d\underline{S} \qquad (4.4)$$

3) MoM

Uses either the Mixed Potential Integral Equation (MPIE) for electric field, incorporating both (magnetic) vector and scalar potentials:

$$\underline{E} = -j\omega \underline{A} - \nabla V \qquad (4.5)$$

or the Electric Field Integral Equation (EFIE) which just uses the magnetic vector potential through the Lorentz condition thus:

$$\underline{E} = -\frac{j}{\omega\varepsilon\mu}(k^2 \underline{A} + \nabla(\nabla.\underline{A})) \qquad (4.6)$$

4. b.

(Marks will be earned by including the following points in the discussion, although it is not exhaustive ...)

1) The most efficient way to calculate the input impedance of a Yagi-Uda antenna is by using the EFIE in a thin wire MoM technique. This is because the environment does not need to be included in the problem, so that discretization of the problem space involves segmentation in only one dimension along the Yagi elements.

2) With the WiFi patch antenna we now have an environment around the antenna that must be included in the model. If the substrate is less than about 2 wavelengths in cross-section, then it needs to be meshed up in either an FDTD or FIT algorithm with the antenna, so that the effect of the finite sized grounded substrate on the antenna currents can be calculated. However, an electrically large substrate can be considered

infinite to a reasonable approximation, whereupon a more efficient surface patch MoM technique could be used incorporating Green's functions that include the effects of the substrate.

3) For modelling radio frequency fields inside an inhomogeneous lossy volumetric dielectric such as the head, FDTD or FIT are the only viable techniques.

4) An aircraft can be considered a perfectly conducting hollow object at radar frequencies (interior screened by skin depth, and neglecting windows and any stealthy coating etc.) and therefore any volumetric analysis such as FIT and FDTD would be inefficient. A surface patch moment method is therefore the most efficient analysis tool here.

4. c.

We require the updated electric field, and therefore use Ampere's law (4.1). Expanding the curl for the 1D case gives: *(I expect students to be able to do this – to define the curl would give too much of a hint.)*

$$-\frac{\partial H_y}{\partial z} = \varepsilon_o \frac{\partial E_x}{\partial t} \qquad (4.7)$$

giving the finite difference equation

$$E_x\big|_k^n = E_x\big|_k^{n-1} + \frac{\Delta t}{\varepsilon_o}\left(\frac{H_y\big|_{k-1/2}^{n-1/2} - H_y\big|_{k+1/2}^{n-1/2}}{\Delta z}\right) \qquad (4.8)$$

(students should realise that k is a quantized value for z)

Putting the numbers in then gives

$$E_x\big|_k^n = 0 + \frac{8.854\times10^{-10}}{8.854\times10^{-12}}(1-0.5) = 50V/m \quad (4.9)$$