EEE105 2008-9 Examination Paper -- Solutions

1.a.[Easy Bookwork]

In a semiconductor and insulator the bonding is normally fairly similar. Essentially all the outer electrons in the material are tied up in the bonds between the atoms in the material. In order for electrons to conduct they need to escape from the bonds and they need to gain energy to do this. In a semiconductor the required energy gain is not too big and hence a small fraction of the electrons are able to escape and conduct electricity at any time. In an insulator the required energy gain is much larger and hence very few if any electrons can escape their bond at any instance in time and hence the conductivity is much lower.

1.b [Easy Bookwork]

Doping is the process where impurities are deliberately added to a semiconductor. These impurities either have one more or one less electron in their outer bonding state than for the semiconducting material. For example Si has four outer electrons, so it can be doped using an element with five or three outer electrons. If the doping element has more outer electrons then each dopant atom gives one extra electron which cannot fit in a bond and is hence free to wander around the material. Similarly a dopant with less outer electrons leaves a hole in a bond and this hole can also wander around the material. In both cases the density of charge carriers in the material is increased and hence the conductivity is also increased.

1.c.i. [Applied Bookwork]

In an extrinsic semiconductor the carrier concentration is given by the density of impurities in the material, and does not change with temperature. However the mobility will fall as the temperature rises as the density of lattice vibrations increases increasing the amount of scattering. Thus overall the conductivity will decrease.

1.c.ii. [Applied Bookwork]

In an intrinsic semiconductor the carrier concentration will rise exponentially as the temperature increases. While the mobility will fall as before, the carrier concentration rise dominates and thus overall the conductivity will increase.

1.d. [Hidden]

The resistance of the wire is 97 Ω. We first need to calculate the resistivity of the Al.

$$\rho = \frac{R_{original} A}{l} = \frac{97\Omega \cdot 1\times10^{-8} m \cdot 5\times10^{-6} m}{100\times10^{-6} m} = 4.85\times10^{-8}\Omega m$$

After ablation (removal) of the segment of Al, the resistance can be given by

$$R_{desired} = \left(4.85\times10^{-8}\Omega m\right)\cdot\left(\frac{88\times10^{-6} m}{1\times10^{-8} m \cdot 5\times10^{-6} m} + \frac{12\times10^{-6} m}{1\times10^{-8} m \cdot (5-x)\times10^{-6} m}\right)$$

Rewriting we obtain:

$$(5-x)\times10^{-6} m = \frac{12\times10^{-6} m}{1\times10^{-8} m \cdot \left[\frac{101\Omega}{\left(4.85\times10^{-8}\Omega m\right)} - \frac{88\times10^{-6} m}{1\times10^{-8} m \cdot 5\times10^{-6} m}\right]} = 3.72\times10^{-6} m$$

And hence:

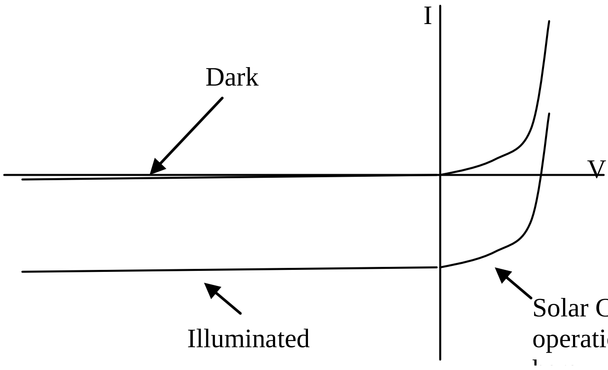
$$x = (5-3.72)\times10^{-6} m = 1.28\times10^{-6} m$$

2.a. [Bookwork]

When a piece of p-type material is brought together with a piece of n-type semiconductor then there will be a concentration gradient of electrons and holes in the junction region. As a result electrons will diffuse into the p-type material and holes into the n-type material, where they will recombine with the majority carriers. However, as the electrons and holes diffuse they will leave behind their donor and acceptor atoms, which are ionised. These atoms cannot move as they are bonded into the crystal. As the diffusion progresses so the number of exposed donors and acceptors increase. This causes an increasing electric field in the junction region which acts to oppose further electron and hole diffusion. Eventually the field is sufficiently strong to set up a drift current that exactly opposes the diffusion current of electrons and holes and the diode reaches equilibrium. The depletion region is the region where all the electrons and holes have diffused and recombined, leaving only their exposed donor and acceptors behind. The layer acts like an insulator as all the free charge carriers have recombined away.

2.b. [Bookwork]

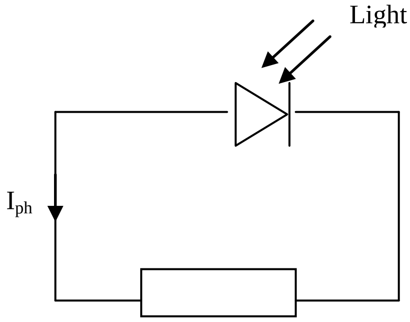
If light is shone onto the junction of a diode and absorbed, electron-hole pairs will be created. The electron-hole pairs will be spatially separated by the electric field in the p-n junction region and the holes will be driven into the p-type material and the electrons into the n-type material. This will lead to a flow of current through the diode. The current will be defined by the electron-hole pair generation rate, and should be independent of the reverse bias voltage.



For the device characteristic the photodiode can be operated under either reverse bias or zero bias. The zero bias condition is also that expected for its operation as a solar-cell.

2.c. [Bookwork-slightly applied]

The simplest circuit for using the photodiode is to connect the resistive heater in series with the photodiode. There is no need for any other power supply as the light from the sun will give the power. The current flows in the opposite direction to that for a forward biased p-n junction.



2.d. [Hidden / Difficult Applied Bookwork]

The efficiencies of the energy of a photon being converted into electrical energy is determined principally by the photon energy of the photon as compared to the Energy Gap (or Ionisation Energy), W_g, of the semiconductor material being used to make the solar cell.

If the wavelength of the light is long and the photon energy is smaller than W_g then the light will not be absorbed, no electron hole pairs are created, no current flows and the efficiency at such wavelengths is essentially zero.

If the wavelength of light is short then the light will be absorbed but each photon can only create one electron-hole pair and the potential in the circuit will be roughly related to W_g. This means the energy difference between the photon and W_g cannot be utilized. Thus the efficiency of photons with energy above W_g will be reduced as the wavelength of the light gets shorter.

Based on the above the answer to 2.d.ii will be that the highest efficiency conversion will be for photons with wavelengths that correspond to energies at or just above the Ionisation Energy, W_g.

3.a.i [Easy Bookwork]

Electron current in a p-n junction is due to electrons diffusing across the junction into the p-type material, where they will recombine with holes as they diffuse away from the junction further into the p-type material.

3.a.ii [Bookwork]

Using the current density variant, the diode equation is $J = J_0\left[\exp\left(\frac{qV}{kT}\right) - 1\right]$

where $J_0 = \frac{qL_e n_p}{\tau_e} + \frac{qL_h p_n}{\tau_h}$

Consider first the equation of the saturation current, J_0. The two terms in the equation are the electron and hole saturation currents, respectively. As the exponential terms will be the same the ratio of the electron and hole currents will be given by the ratio of the electron and hole saturation currents.

Thus we can write: $\frac{J_e}{J_h} = \frac{qL_e n_p}{\tau_e} \cdot \frac{\tau_h}{qL_h p_n}$

Now we can substitute $n_p = \frac{n_i^2}{N_a}$ and similarly $p_n = \frac{n_i^2}{N_d}$.

We can also use the relationship that $L_e = (D_e \tau_e)^{1/2}$ and $L_h = (D_h \tau_h)^{1/2}$

Substituting gives: $\frac{J_e}{J_h} = \frac{D_e}{N_a L_e} \cdot \frac{N_d L_h}{D_h}$

From the Einstein relation we get $D \propto \mu$ and hence $\frac{J_e}{J_h} = \frac{\mu_e N_d}{\mu_h N_a}\frac{L_h}{L_e}$

From the conductivity equation we have $\sigma \propto \mu N$ (where N is the density of donors or acceptors) and hence: $\frac{J_e}{J_h} = \frac{\sigma_n}{\sigma_p}\frac{L_h}{L_e}$.

3.a iii. [Hidden – difficult – but students have access to a worked solution from a past examination paper and the target is specified in the question.]

We know from the previous part that $\frac{J_e}{J_h} = \frac{\sigma_n}{\sigma_p}\frac{L_h}{L_e}$.

We are being asked to show that $\frac{J_e}{J_h} = \left(\frac{\sigma_n}{\sigma_p}\right)^{1/2}$.

Therefore the question is really asking us to prove that $\frac{L_h}{L_e} = \left(\frac{\sigma_p}{\sigma_n}\right)^{1/2}$

Start by writing $\frac{L_h}{L_e} = \frac{(D_h \tau_h)^{1/2}}{(D_e \tau_e)^{1/2}}$

Now from before we know that $D \propto \mu$. Hence $\frac{D_h}{D_e} = \frac{\mu_h}{\mu_e}$

Furthermore the minority carrier lifetimes are given by $\tau_h = \frac{1}{Bn}$ and $\tau_e = \frac{1}{Bp}$

And hence we can get say that $\frac{\tau_h}{\tau_e} = \frac{p_{(p)}}{n_{(n)}} = \frac{N_a}{N_d}$

Substituting gives $\frac{L_h}{L_e} = \left(\frac{\mu_h N_a}{\mu_e N_d}\right)^{1/2} = \left(\frac{\sigma_p}{\sigma_n}\right)^{1/2}$

From 1.a. we had: $\frac{J_e}{J_h} = \frac{\sigma_n}{\sigma_p}\frac{L_h}{L_e} = \frac{\sigma_n}{\sigma_p} \bullet \left(\frac{\sigma_p}{\sigma_n}\right)^{1/2} = \left(\frac{\sigma_n}{\sigma_p}\right)^{1/2}$

3.b. [Bookwork from later in the course hence bit more advanced]

The Base Transport Factor is a measure of what ***fraction of the injected electron current*** manages to diffuse across the base and into the collector of the device. Some electrons will be lost due to recombination with holes in the base, which is undesirable. It can be optimized by making the base thickness as thin as is practicable.

3.c. [Problem. Note that much of the information given in the question is not relevant]

The base transport factor is a measure of the fraction of electrons that manage to cross the 0.8 μm base without recombining with a hole. Thus we can calculate it from the minority carrier diffusion length equation.

$$\frac{\delta n(x)}{\delta n_0} = \exp\left(-\frac{x}{L_e}\right)$$

Where x = the length of the base = 0.8 μm.

To solve this we need to get a value for L_e. On the front of the examination paper we are given:

$$L_e = \sqrt{D_e \tau_e}$$

$D_e = \frac{kT}{q}\mu_e$. The mobility of electrons in Si is given as $0.12\ m^2V^{-1}s^{-1}$ and we can assume thermal energy at room temperature is $0.025\ JC^{-1}$. Hence $D_e = 3\times 10^{-3}\ m^2s^{-1}$.

We are given the minority carrier lifetime for electrons in the base as 30 ns.

Hence substituting we get $L_e = \sqrt{3\times10^{-3}\cdot 3\times10^{-8}} = 9.5\times10^{-6}\ m$.

Thus the base transport factor = $\frac{\delta n\left(8\times10^{-7}\right)}{\delta n_0} = \exp\left(-\frac{8\times10^{-7}}{9.5\times10^{-6}}\right) = 0.92$

4.a. [Bookwork]

Description of MOSFET in n-layer with p^+ regions for source and drain. Silicon Oxide between gate and semiconductor surface. The answer should include picture(s) showing this structure and describing how a negative gate bias increases the n-depletion region under the gate and then pulls carriers in from the p^+ regions to form an p-channel. If the magnitude of the (negative) drain voltage is sufficiently high then the channel will be pinched off at the drain end as the field between gate and drain is smaller than that between source and gate. In this region it is found that the channel resistance ideally increases in proportion to the drain voltage and hence the current flow between source and drain will depend only on the bias applied to the gate. The greater the negative bias on the gate the greater the current flow between source and drain.

The threshold voltage is the critical minimum voltage for the channel to form, ie where sufficient bias is applied to the gate for electrons to be pulled from the source, drain n-type regions to form the channel

4.b. [Bookwork]

The transconductance is the rate of change of drain current with gate voltage. It is usually measured at some particular value of gate voltage in a device, as it can vary a little (although in the ideal textbook device it does not).

$$g_m = \left.\frac{\partial I_d}{\partial V_g}\right|_{V_g}$$

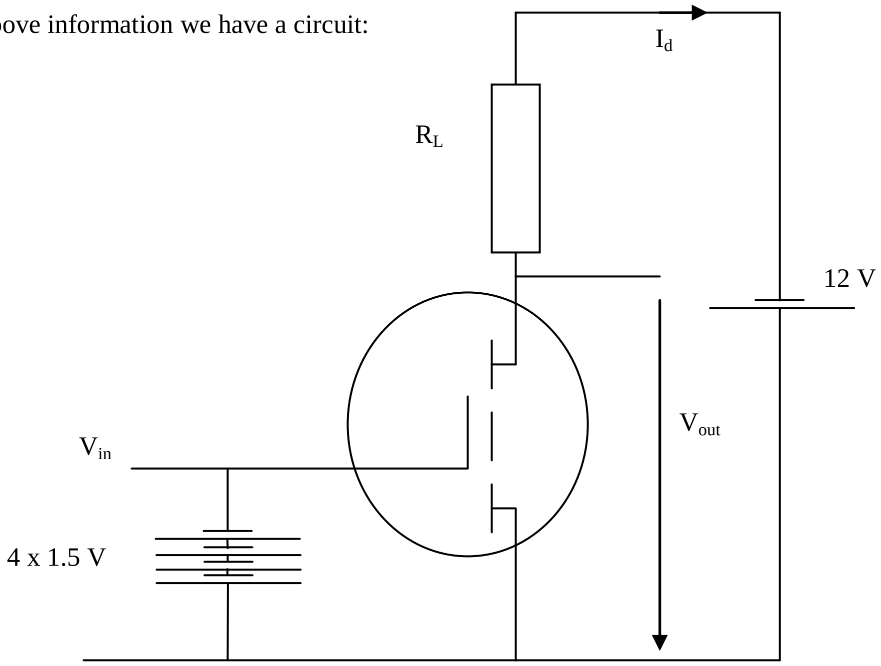
4.c. [Hidden – similar problems have been set in the past]

The signal is +/-10 mV and it needs to be converted to give a value +/-2.5 V. Thus the voltage gain of the circuit is 250.

We must decide for a simple amplifier to set the quiescent point for the gate voltage. We are told that the power supply available is 12 V. Using 1.5 V batteries we can set the quiescent point at 6 V using four 1.5V batteries (the ideal value in the middle of the range).

Note that as the voltage required to feed the D-A converter is +/-2.5V we can also choose use 2, 3, 5 or 6 batteries to set a quiescent point at 3 V, 4.5 V, 7.5 V or 9 V, respectively. All these values of quiescent point also allow the signal to be amplified without clipping distortion.

Using the above information we have a circuit:



Now we need a maximum peak to peak output of V_{gs}= 5 V for a 20 mV input

Using the equation: $i_d = g_m v_{gs}$ we have 5 mS x 20 mV = 100 μA.

Thus we need a load resistor with a voltage drop difference of 5 V for a current difference of 100 μA through it.
Therefore R_L = 50 kΩ