Tutorial 3

1. Find the Fourier Transforms of the following signals:
(i) $x(t) = 1$ (use duality property) (ii) $x(t) = e^{j\omega_o t}$ (use frequency shift property)
(iii) $x(t) = \delta(t-t_o)$ (use time shift property)

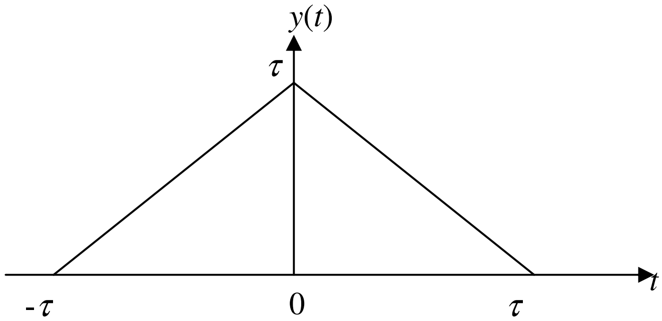
2. Verify that the Fourier Transform of a train of impulse $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$, is given by $P(\omega) = \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$, where T_s is the sampling time and $\omega_s = 2\pi/T_s$.

3. Prove the convolution property of Fourier Transform, $\mathcal{F}[x(t)*h(t)] = X(\omega).H(\omega)$.

4. Show that $\mathcal{F}[x(t).h(t)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega')H(\omega - \omega')d\omega'$.

5. The Fourier Transform of a signal $x(t) = \begin{cases} 1, & |t| < \tau \\ 0, & |t| > \tau \end{cases}$, is $X(\omega) = \frac{2\sin\omega\tau}{\omega}$. Use this Fourier Transform pair and the duality property to find the Fourier Transform of a signal described by $y(t) = \frac{\sin t}{\sqrt{\pi t}}$. Calculate the total energy contained in $y(t)$ using Parseval's theorem.

6. Using the integration property and the Fourier Transform of the rectangular pulse, derive the Fourier Transform of the triangular signal shown below.



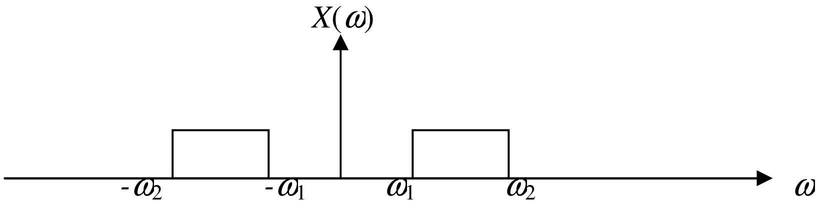
7. The carrier frequency used in an AM wave is typically in the range of 0.535-1.605 MHz. A superheterodyne receiver, consisting of a product modulator and a local oscillator followed by a bandpass filter, is usually used as the receiver. Obtain the tuning frequency range of the oscillator that is required to translate an input AM wave, with a bandwidth of 8 kHz, to a frequency band with an intermediate frequency of 0.455 MHz.

8. In a pulse amplitude modulation system, an analogue signal $x(t)$ is multiplied by a periodic train of rectangular pulses, $p(t)$. The Complex Fourier Series representation

of $p(t)$ is given by $p(t)=\sum_{n=-\infty}^{\infty}\left(\frac{\tau\sin(n\omega_s\tau/2)}{T(n\omega_s\tau/2)}\right)e^{jn\omega_s t}$, where τ is the pulse width and $\omega_s=\frac{2\pi}{T}$ is the repetition frequency of $p(t)$. Find the spectrum of the modulated signal, $m(t)$.

9. Consider a continuous time signal, $x(t)$, that lies in the frequency band $|\omega| < 10\pi$ rad/s. Due to inadequate shielding the signal is contaminated by a large sinusoid with a frequency of 38π rad/s. This contaminated signal is now sampled at a frequency of 5π rad/s.
i) At what frequencies does the interfering sinusoid appear after sampling?
ii) A low pass filter is used to reduce aliasing. A sufficient condition is to attenuate the interfering sinusoid by a factor of 100. Work out the RC time constant required to achieve this.

10. Consider a continuous time signal $x(t)$ with a magnitude spectrum shown below.



i) Based on the Nyquist Theorem, state the sampling interval, T_s, required to avoid aliasing.
ii) Assuming that $\omega_1 > \omega_2-\omega_1$. Work out the maximum sampling interval such that it is still possible to reconstruct $x(t)$ perfectly. (Note that in this case T_s can be smaller than in part (i)).