[Q3a]

(i)

$$V^- = V_{out}\frac{R_2}{R_1+R_2} \qquad \& \qquad V^+ = V_{in}$$

$$V_{out} = A_V(V^+ - V^-)$$

$$V_{out} + A_V V_{out}\frac{R_2}{R_1+R_2} = A_V V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_V}{1+A_V\frac{R_2}{R_1+R_2}} = \frac{1}{\frac{1}{A_V}+\frac{R_2}{R_1+R_2}}$$

$$A_V = \frac{A_O}{1+\frac{j\omega}{\omega_o}}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\frac{1+\frac{j\omega}{\omega_o}}{A_O}+\frac{R_2}{R_1+R_2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\frac{1}{A_O}+\left(\frac{j\omega}{\omega_o A_O}\right)+\frac{R_2}{(R_1+R_2)}}$$

$$\frac{V_{out}}{V_{in}} \approx \frac{\frac{(R_1+R_2)}{R_2}}{1+\left(\frac{j\omega}{\omega_o A_O}\right)\left(\frac{(R_1+R_2)}{R_2}\right)} = \frac{k}{1+\frac{j\omega}{\omega_1}}$$

(5 marks)

(ii)

$$\text{dc gain} = k = \frac{R_1+R_2}{R_2}$$

$$\text{dc gain} = \frac{1\times10^3+100\times10^3}{1\times10^3} = 101 \text{ V/V}$$

$$\text{gain-bandwidth product} = f_0\times A_0 = f_1\times k$$

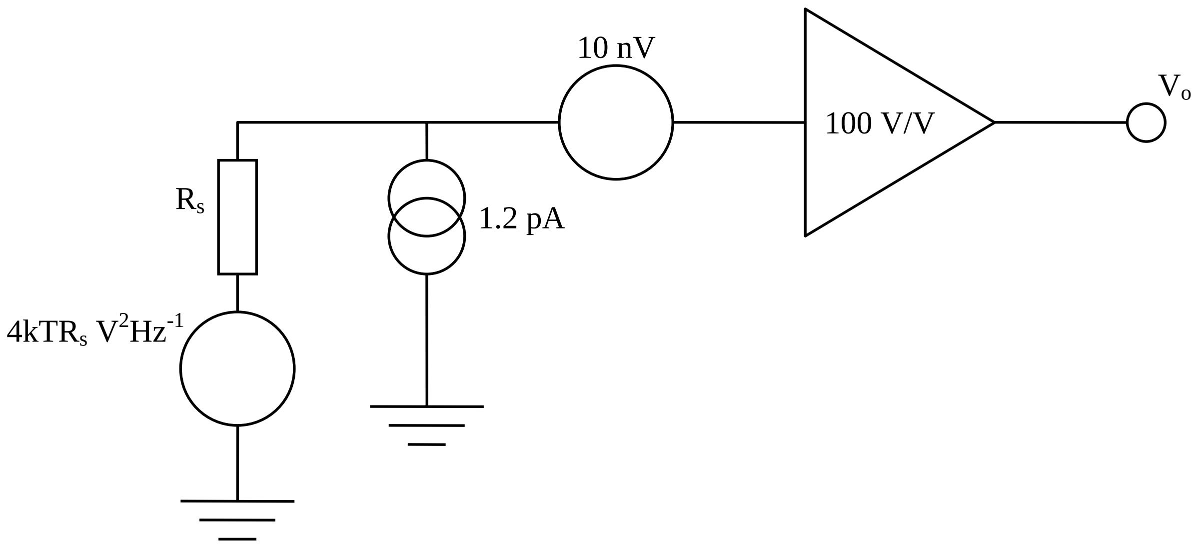
$$50\times10^6 = f_1\times101$$

$$f_1 = \frac{50\times10^6}{101} = 495 \text{ kHz}$$

(5 marks)

[Q3b]

(i)



$$\text{Noise factor} = \frac{\text{Noise power at output of real amplifier}}{\text{Noise power at output of ideal version of amplifier}}$$

$$\text{Noise factor} = \frac{\left[4kTR_s + (1.2\text{pA}\times R_s)^2 + (10\text{nV})^2\right]\times 100^2}{4kTR_s \times 100^2}$$

$$\text{Noise factor} = \frac{\left[24.8\times10^{-18} + 3.24\times10^{-18} + 100\times10^{-18}\right]\times 100^2}{24.8\times10^{-18}\times 100^2}$$

Noise factor = 5.16

(5 marks)

(ii)

$$\text{Noise at amplifier output} = \left[4kTR_s + (1.2\text{pA}\times R_s)^2 + (10\text{nV})^2\right]\times 100^2 \times 20\ \text{kHz}$$

$$\text{Noise at amplifier output} = \left[24.8 + 3.24 + 100\right]\times10^{-18}\times 100^2 \times \left(20\times10^3\right)$$

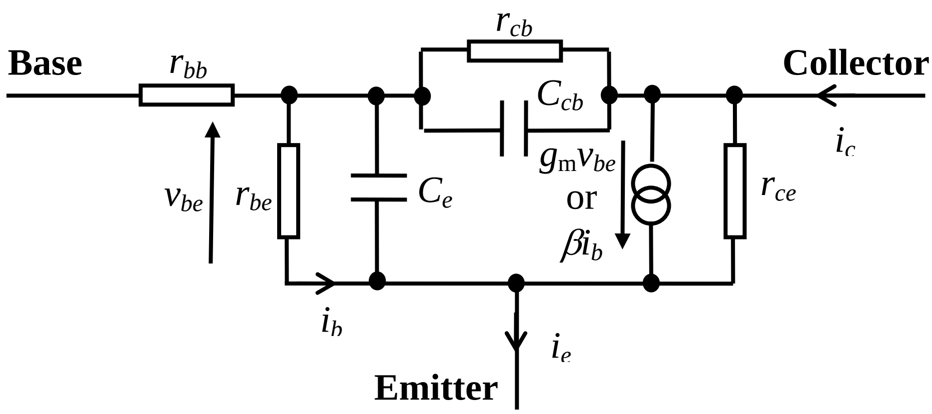
$$\text{Noise at amplifier output} = 2.56\times10^{-8}\ \text{V}^2$$

$$\text{rms reading} = \sqrt{2.56\times10^{-8}} = 160\ \mu\text{V}$$

(5 marks)

[Q4a]

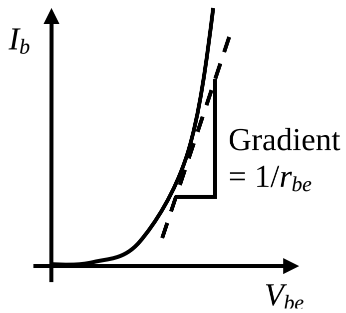
Small signal circuit for a BJT – The hybrid π model



The origins of these circuit elements are as follows:-

r_{bb} – The 'base spreading resistance', this is the series resistance between the package wire and the active part of the semiconductor material. Typically $0.5\Omega < r_{bb} < 50\Omega$, and can be ignored in most cases.

r_{be} - Base incremental resistance of the base emitter junction ($\propto 1/I_c$).



r_{bc} - Feedback resistance modelling the Early effect* ($\propto 1/I_c$). Can usually be ignored for analytical purposes, but **must** be considered if the transistor has a high impedance load.

r_{ce} - Models the small slope of the output characteristic, mostly due to the Early effect* ($\propto 1/I_c$). Again, can normally be ignored unless the transistor has a high impedance load.

C_{cb} - The base collector depletion capacitance ($\propto V_{cb}$).

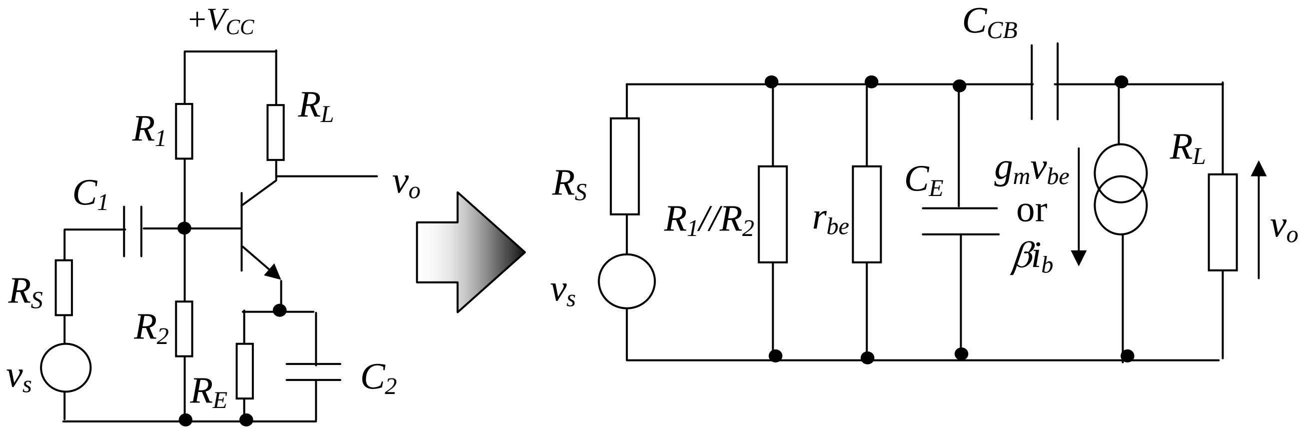
C_e - Emitter diffusion capacitance, models the transient behaviour of charge crossing a p-n junction.

(8 marks)

[Q4b]

(i)

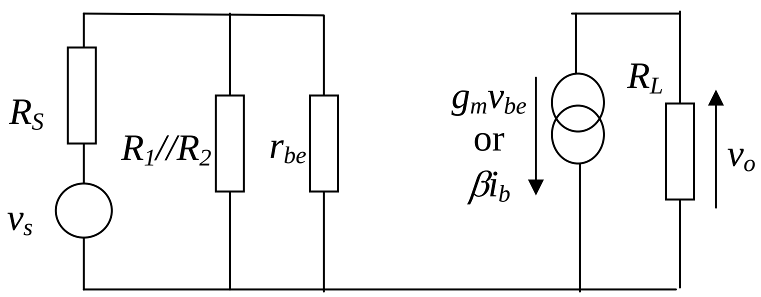
Since it is likely that $R_L >> r_{ce}$ and r_{bc} and $R_S >> r_{bb}$, we shall ignore r_{bb}, r_{bc} and r_{ce}.



(2 marks)

in the mid frequency range:-
bypass capacitors C_1 and C_2 will be short circuits **(1 mark)**
parasitic capacitances C_{cb} and C_e will be open circuits **(1 mark)**

Hence the mid-frequency ss circuit is,



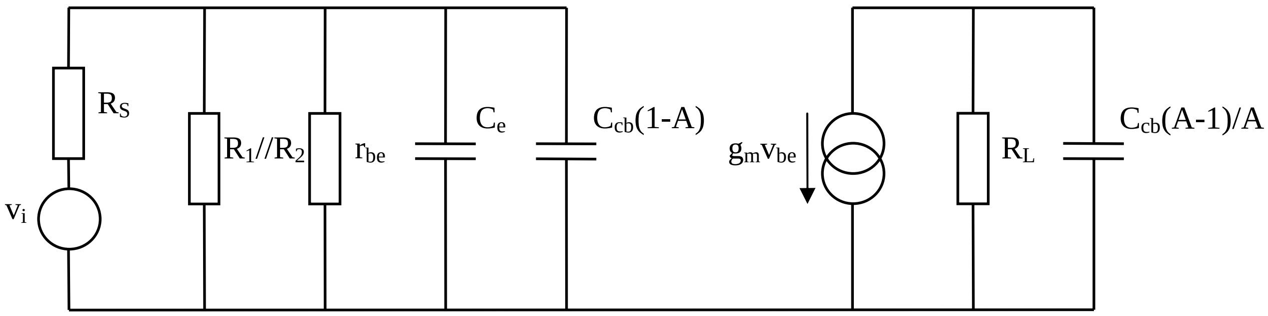
(1 mark)

The mid-frequency gain is thus,

$$A = \frac{v_0}{v_s} = \frac{v_s}{v_{be}}\frac{v_{be}}{v_0} = \frac{R_1 // R_2 // r_{be}}{R_1 // R_2 // r_{be} + R_S}\left(-g_m R_L\right)$$

(1 mark)

(ii)



(2 marks)

(iii)

$$A = -g_m R_L$$

$$A = -0.04 \times 5000 = -200$$

$$C_{cb}(1-A) = \left(5 \times 10^{-12}\right) \times 201 = 1 \text{ nF}$$

$$f_{upper,input} = \frac{1}{2\pi\left(R_S // R_1 // R_2 // r_{be}\right)\left(C_e + C_{cb}(1-A)\right)}$$

$$f_{upper,input} = \frac{1}{2\pi\left(10 \times 10^3 // 200 \times 10^3 // 100 \times 10^3 // 12 \times 10^3\right)\left(20 \times 10^{-12} + 1 \times 10^{-9}\right)}$$

$$f_{upper,input} = 30.9 \text{ kHz}$$

$$f_{upper,output} = \frac{1}{2\pi R_L C_{cb} \frac{(A-1)}{A}}$$

$$f_{upper,output} = \frac{1}{2\pi \times \left(5 \times 10^3\right) \times \left(5 \times 10^{-12} \times \frac{201}{200}\right)}$$

$$f_{upper,output} = \frac{1}{2\pi \times \left(5 \times 10^3\right) \times \left(5 \times 10^{-12} \times \frac{201}{200}\right)}$$

$$f_{upper,output} = 6.33 \text{ MHz}$$

Therefore:-

$$f_{upper} = 30.9 \text{ kHz}$$

(4 marks)