3.5 Axis transformations

3.5.1 ABC to αβ Transformation

As has been discussed, the torque is produced from the interaction between the combined effect of winding currents (phasor) and the rotor magnetic field. This combined effect indeed represents a magnetic motive force (mmf) in airgap, which is the vector sum of mmfs of individual phase windings.

Fig. 3.10 shows the three phase windings with equal number of turns N. The mmf produced by each winding is in the same direction of the winding axis with the magnitude being $i_x N$ (x = a,b, and c). To calculate the vector sum of the air gap mmf, we can first resolve the individual mmf vector alone two orthogonal axes α and β, α being coincident with phase a axis, and β 90^0 leading α, as shown in Fig. 3.10

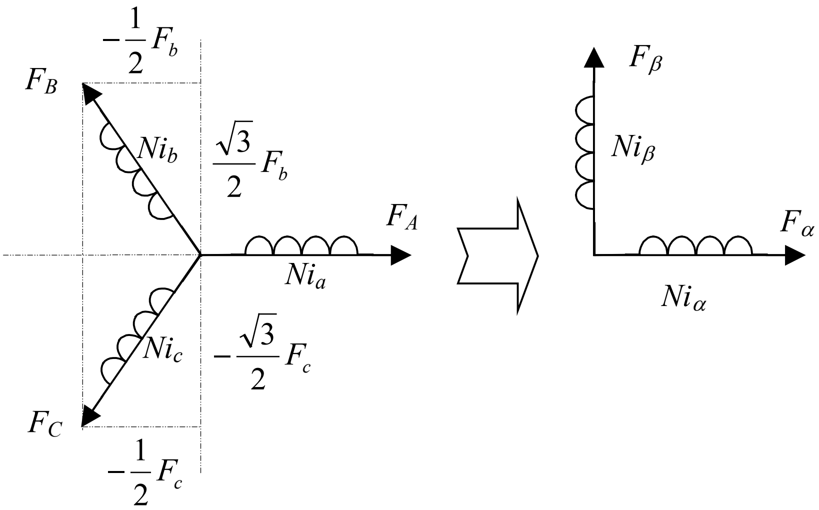

Fig. 3.10 Two axis representation of a three phase windings

	F_A	F_B	F_C
$F_\alpha = Ni_\alpha$	$F_A = Ni_a$	$-\frac{1}{2}F_B = -\frac{1}{2}Ni_b$	$-\frac{1}{2}F_C = -\frac{1}{2}Ni_c$
$F_\beta = Ni_\beta$	0	$\frac{\sqrt{3}}{2}F_B = \frac{\sqrt{3}}{2}Ni_b$	$-\frac{\sqrt{3}}{2}F_C = -\frac{\sqrt{3}}{2}Ni_c$

This implies that physically, the three-phase winding is equivalent to an orthogonal two-phase winding, as shown in Fig. 3.10. The above relationship can be generalised to a transformation matrix with a scaling factor of 2/3:

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}; \text{ define } C_{\alpha\beta\leftarrow abc} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \tag{3.23}$$

The variable x can be current, voltage, mmf, or flux, etc. It can be shown (see tutorial question) that the torque in the αβ reference frame is given by:

$$T = \frac{3}{2}p(\Psi_\alpha i_\beta - \Psi_\beta i_\alpha) \tag{3.24}$$

The interpretation of this torque production mechanism is shown in Fig. 3.11.

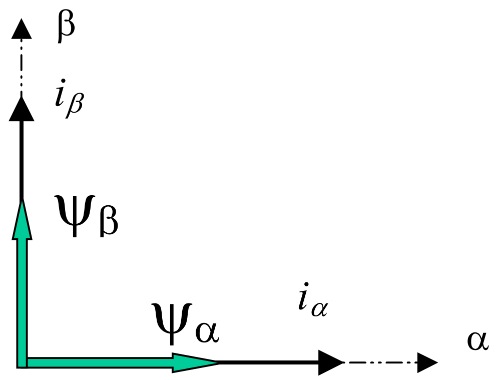

Fig. 3.11 torque production in αβ reference frame

The inverse transformation (αβ to ABC) is:

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}; \text{ define } C_{abc\leftarrow\alpha\beta} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \tag{3.25}$$

3.5.2 αβ to dq Transformation

DEFINITION OF ROTATING dq REFERENCE FRAME

As has been discussed earlier, the rotor of a brushless PM machine produces a rotating field, which can be represented by a flux linkage vector coincident with the d axis. This axis is chosen to be the d-axis of the d-q rotating reference frame. The second axis, q, is defined to be perpendicular to the d axis according to the right hand corkscrew convention in the direction of the rotation, as shown in Fig. 3.12, where θ is the electrical angle between the α axis and the d axis. If initially the two axes coincide and the rotor rotates at a synchronous speed ω, then $\theta = \omega t$. mmf vectors in the αβ reference frame can be represented by their equivalence in the dq reference frame in the following table:

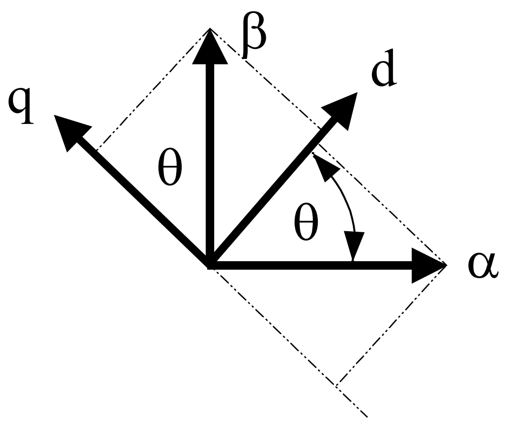

Fig. 3.12 Definition of dq rotating reference frame

	F_α	F_β
F_d	$\cos\theta$	$\sin\theta$
F_q	-$\sin\theta$	$\cos\theta$

TRANSFORMATIOMS

The relationship in the above table can be generalised to the following transformation matrix:

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} \;;\; define\;\; C_{dq\leftarrow\alpha\beta} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{3.26}$$

where the variable x can be mmf, current, voltage or flux, etc.. The inverse transformation is the transpose of $C_{dq\leftarrow\alpha\beta}$, i.e.,

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix} \;;\; define\;\; C_{\alpha\beta\leftarrow dq} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{3.27}$$

3.5.3 ABC to dq transformation and vice versa

To consider the effect of a 3-phase stator winding in the dq reference frame, we can first transform abc quantities into the αβ reference, and further to the dq frame. Thus,

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = C_{dq\leftarrow\alpha\beta} C_{\alpha\beta\leftarrow abc} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = C_{dq\leftarrow abc} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix};\; C_{dq\leftarrow abc} = C_{dq\leftarrow\alpha\beta} C_{\alpha\beta\leftarrow abc} \tag{3.28}$$

$$C_{dq\leftarrow abc} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 120^0) & \cos(\theta + 120^0) \\ -\sin\theta & -\sin(\theta - 120^0) & -\sin(\theta + 120^0) \end{bmatrix} \tag{3.29}$$

The inverse transformation is:

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \cos(\theta - 120^0) & -\sin(\theta - 120^0) \\ \cos(\theta + 120^0) & -\sin(\theta + 120^0) \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix} = C_{abc \leftarrow dq} \begin{bmatrix} x_d \\ x_q \end{bmatrix} \tag{3.30}$$

The torque equation in the dq reference frame is given by:

$$T = \frac{3}{2} p(\Psi_d i_q - \Psi_q i_d) \tag{3.31}$$

where Ψ_d and Ψ_q is the dq components of the total flux, and i_d and i_q is the dq components of the stator current. The physical interpretation of the abc to dq transformation is shown in Fig. 3.13.

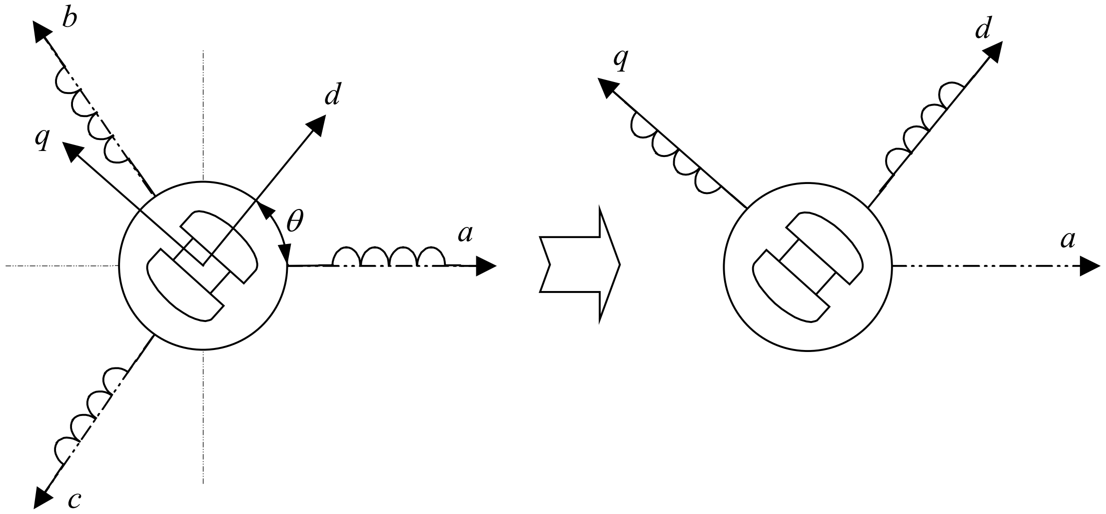

Fig. 3.13 dq equivalence of a three phase winding

3.6 Modelling of brushless permanent magnet motors in dq reference frame

3.6.1 Brushless AC motors in stator abc system

A brushlee AC motor has three stator windings and permanent magnets on the rotor. Since both the magnet and the retaining sleeves have high resistivity, rotor-induced currents can be neglected and no damper windings are modelled. Hence, the circuit equations of the three windings in phase variables are:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} L_a & M_{ab} & M_{ac} \\ M_{ab} & L_b & M_{bc} \\ M_{ac} & M_{bc} & L_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{3.32}$$

where it has been assumed that the stator resistances of all the windings are equal. The back emfs $[e_a\ e_b\ e_c]^T$ are related to the flux linkage ψ_m and electrical angular speed ω, and are given by:

$$\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \begin{bmatrix} -\omega\psi_m \sin\theta \\ -\omega\psi_m \sin(\theta - 2\pi/3) \\ -\omega\psi_m \sin(\theta + 2\pi/3 \end{bmatrix}$$

where θ is the angle between the rotor magnet axis (d-axis) and phase A winding axis. Assuming further that there is no change in the rotor reluctance with angle, then

$$L_a = L_b = L_c = L$$
$$M_{ab} = M_{bc} = M_{ac} = M$$

Hence equation (3.32) may be rewritten as:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} -\omega\psi_m \sin\theta \\ -\omega\psi_m \sin(\theta - 2\pi/3) \\ -\omega\psi_m \sin(\theta + 2\pi/3 \end{bmatrix} \tag{3.33}$$

For a three phase winding without a neutral line, the winding currents must satisfy:

$$i_a + i_b + i_c = 0$$

Therefore,

$$M\,i_b + M\,i_c = -M\,i_a; \quad M\,i_a + M\,i_c = -M\,i_b; \quad M\,i_a + M\,i_b = -M\,i_c;$$

Hence

(3.34)

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} -\omega\psi_m \sin\theta \\ -\omega\psi_m \sin(\theta - 2\pi/3) \\ -\omega\psi_m \sin(\theta + 2\pi/3 \end{bmatrix}$$

3.6.2 Brushless AC motors in dq reference frame

Multiplying the both sides of eqn. (3.34) by the abc to dq transformation matrix, $C_{dq\leftarrow abc}$, given by:

$$C_{dq\leftarrow abc} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin\theta & -\sin(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) \end{bmatrix}$$

and substituting for the phase currents with their dq axis components:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = C_{abc \leftarrow dq} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \cos(\theta - 2\pi/3) & -\sin(\theta - 2\pi/3) \\ \cos(\theta + 2\pi/3) & -\sin(\theta + 2\pi/3) \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

yields the following expression in d-q axis variables:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R & -\omega L_q \\ \omega L_d & R \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ K_e \omega_m \end{bmatrix} \tag{3.35}$$

where $L_d = L_q = L - M = 3L/2$, ω_m is the mechanical angular velocity, $K_e = p\psi_m$ is the back-emf constant, and p is the number of pole pair. The torque expressed in the d-q axis variables is given by:

$$T_e = \frac{3}{2\omega_m}(v_d i_d + v_q i_q) = \frac{3p}{2}[\psi_m i_q + (L_d - L_q) i_d i_q] = \frac{3p}{2}\psi_m i_q = K_t i_q \tag{3.36}$$

where $K_t = 3p\psi_m/2$. Note that K_t differs k_T in Eqn. (3.22) by a factor of $\sqrt{2}$. This is because i_q is corresponds to peak phase current in the abc system. As can be seen from eqn. (3.36), i_d does not contribute to the torque production, and therefore should ideally be maintained at zero in order to maximise the torque per Ampare capability. The equation of motion is:

$$J\frac{d\omega_m}{dt} = (T_e - T_L - B\omega_m) \tag{3.37}$$

where J is the moment of inertia and B is the viscous friction coefficient. The block diagram of the model representing Eqns (3.35) - (3.37) is shown in Fig. 3.14.

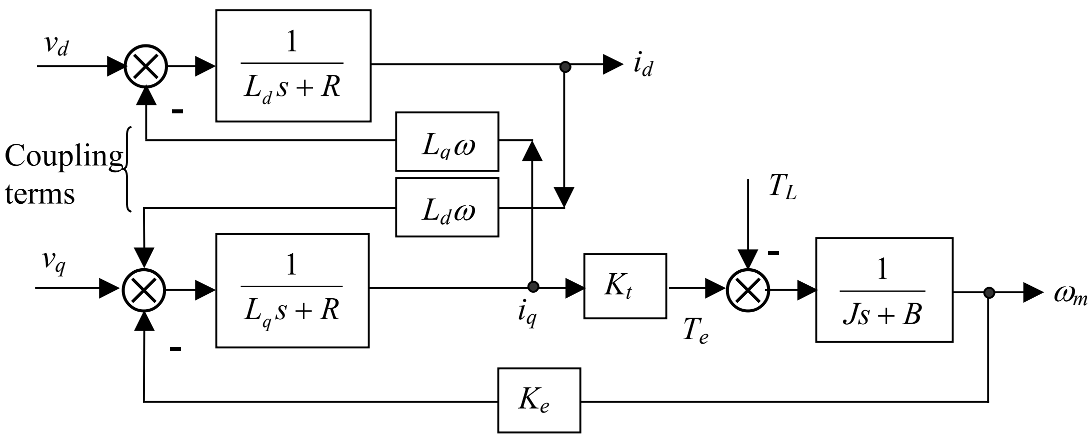

Fig. 3.14 Block diagram of d-q axis model for brushless permanent synchronous motor

It can be shown that if the rotor is salient, e.g., interior mounted, L_d and L_q are not equal, but Eqn. (3.35) is still valid.

3.7 Field Oriented Control of Brushless AC PM Motors

As has been shown, the control objective is to maintain a zero i_d and regulate i_q according to its demand. Since i_d and i_q are influenced by the v_d and v_q via the following equations:

$$\begin{aligned} L_d \frac{di_d}{dt} + Ri_d &= v_d + \omega L_q i_q \\ L_q \frac{di_q}{dt} + Ri_q &= v_q - \omega L_d i_d - K_e \omega_m \end{aligned} \tag{3.38}$$

The second terms on the right hand side of Eqn.(3.38) give rise to so-called coupling effect, i.e., change in i_d will affect i_q, and vice versa. The coupling effect is not desirable and may be removed by adding a decoupling term in v_d and v_q:

$$\begin{aligned} v_d &= v'_d - \omega L_q i_q \\ v_q &= v'_q + \omega L_d i_d \end{aligned} \tag{3.38}$$

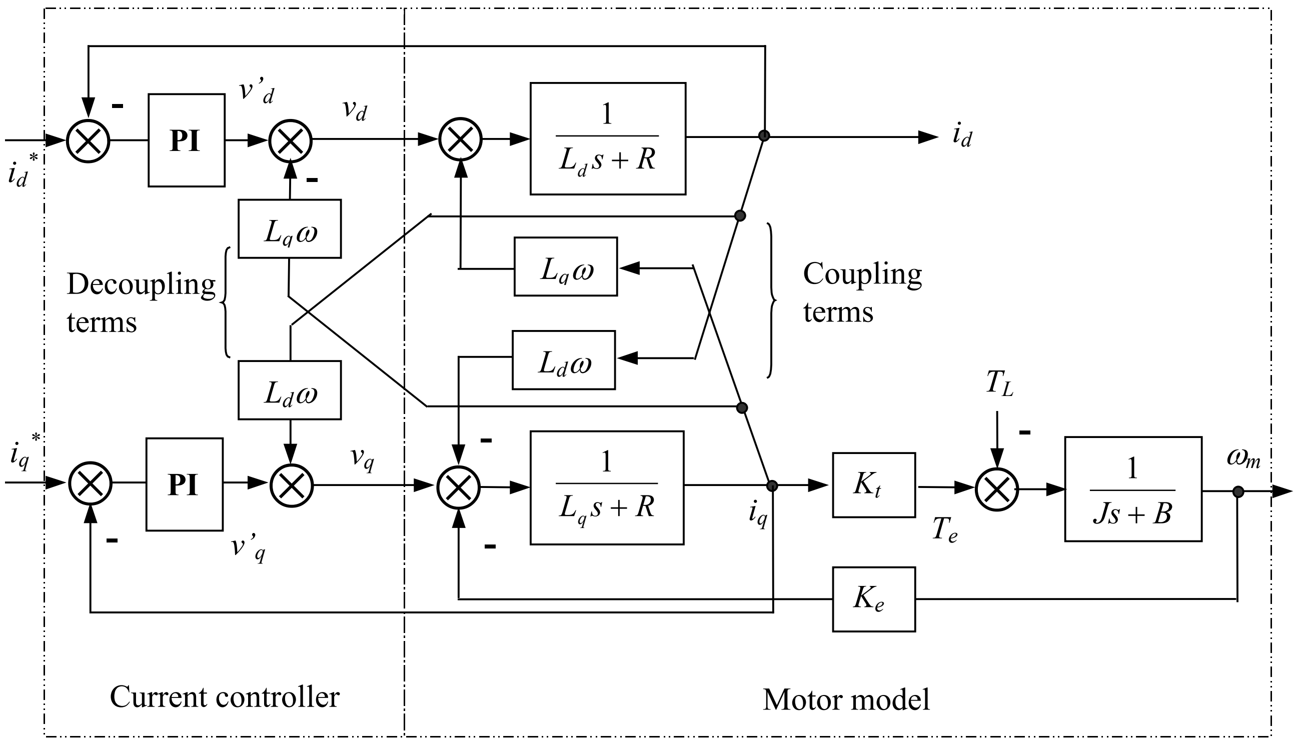

Fig. 3.15 A field oriented control scheme in d-q reference frame

Hence,

$$\begin{aligned} L_d \frac{di_d}{dt} + Ri_d &= v'_d \\ L_q \frac{di_q}{dt} + Ri_q &= v'_q - K_e \omega_m \end{aligned} \tag{3.39}$$

In Eqn. (3.39), i_d and i_q are decoupled with respect to the control input v'_d and v'_q. A typical field oriented current control scheme with PI current controllers and decoupling is shown in Fig. 3.15, which can be design separately for i_d and i_q:

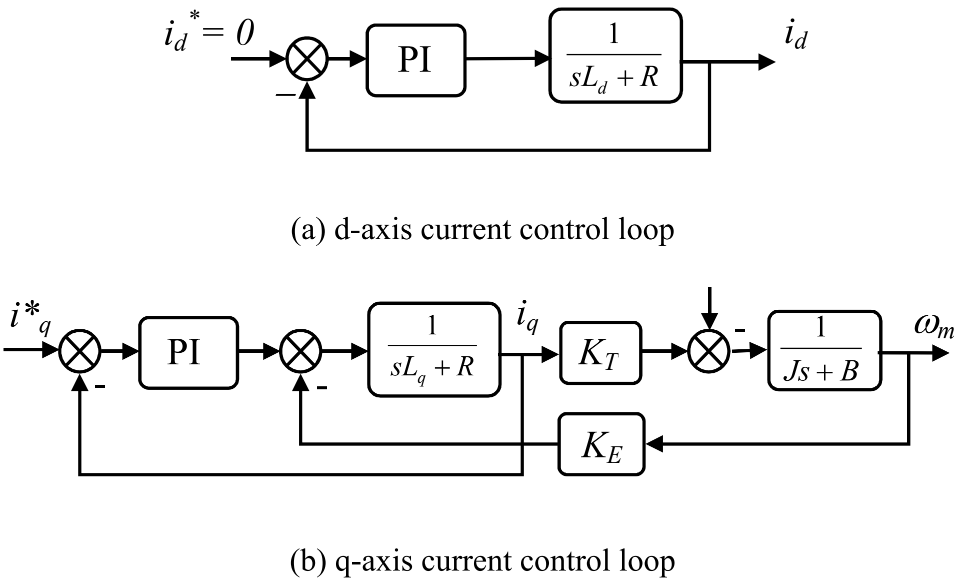

Fig. 3.16 Decoupled dq current control loop

The demand for the d axis current is usually set to zero. If the controller in Fig. 3.15 achieves ideal decoupling and current control over a wide speed range, then the motor dynamics with the closed-loop current controller may be simplified as:

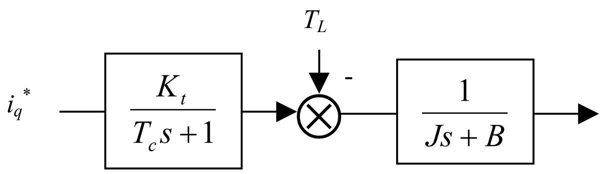

Fig. 3.17 Simplified drive model

where T_c is the time constant of the current loop. Note that q axis current control loop and the resulting current dynamics are the same as that for dc motor drives.

PHYSICAL IMPLEMENTATION

The above discussion only focuses on the control requirement. In order to implement this control scheme, it is necessary to form i_d and i_q for the current feedback loop, and the control outputs that can be realised in the stator reference. The block diagram for physical implementation is shown in Fig. 3.18.

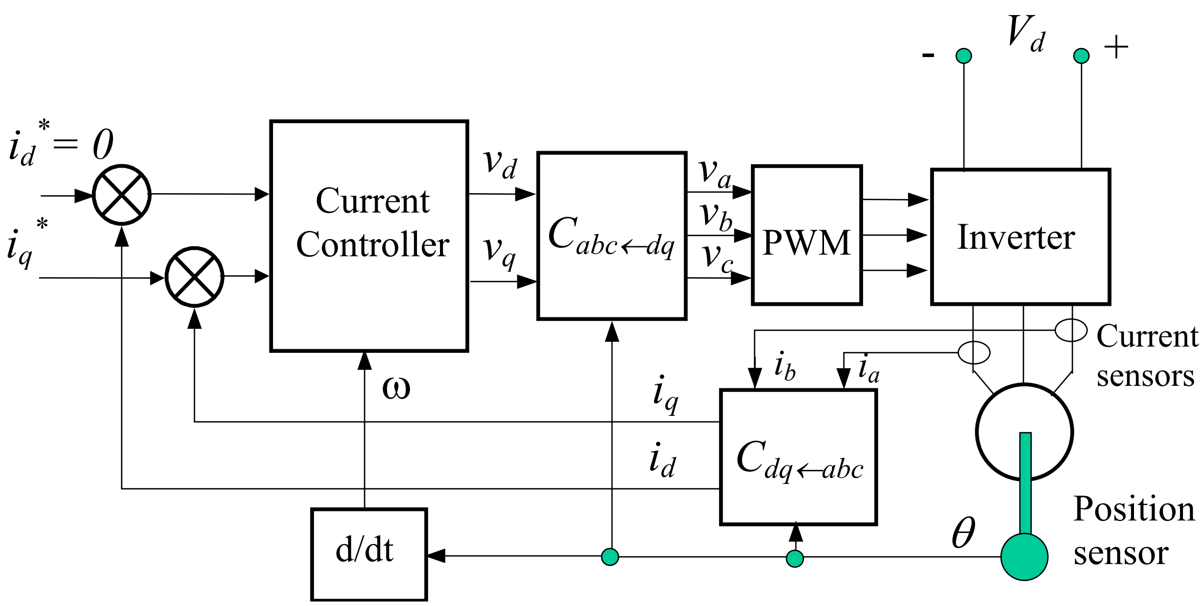

Fig. 3.18 Block diagram of Brushless PM AC drives with Field Oriented Control

As can be seen, the phase currents are measured (the third phase current is obtained from $i_c = -(i_a + i_b)$), and transformed into d, q components. The θ angle required for the transformation is obtained from the position sensor/transducer, such as an encoder or resolver. The angular velocity information is needed for the current loop decoupling, and is usually obtained by differentiating the position signal. The output of the current controller is the v_d and v_q, which are transformed back to voltage signals in the *abc* system. These signals are then used to control a power electronic inverter via a PWM modulation scheme, so that the required phase voltages are applied to the motor terminals to achieve the Field Oriented Control. The question now is how the PMW signals are generated. This issue will be discussed in the next chapter.