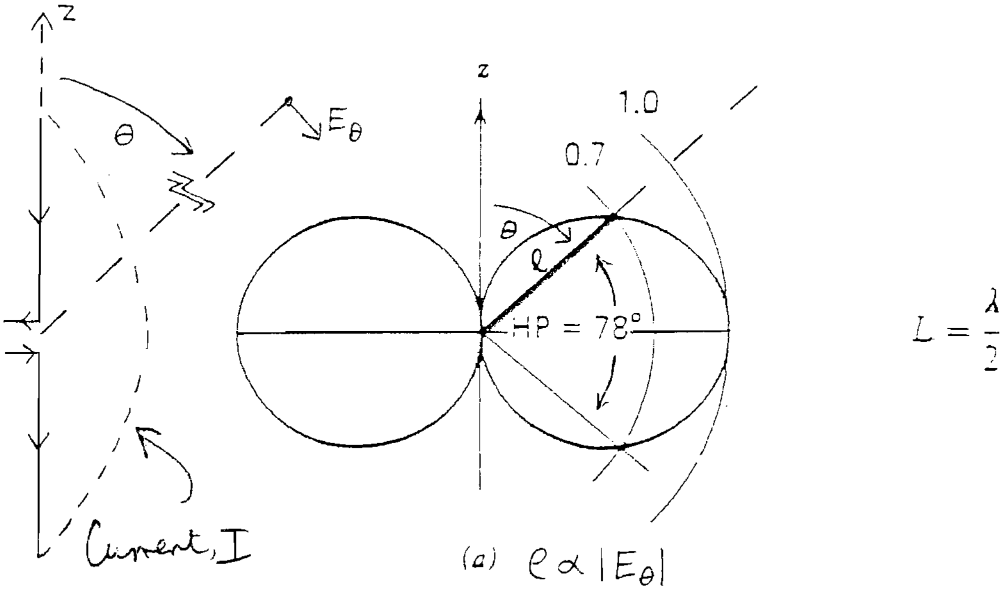
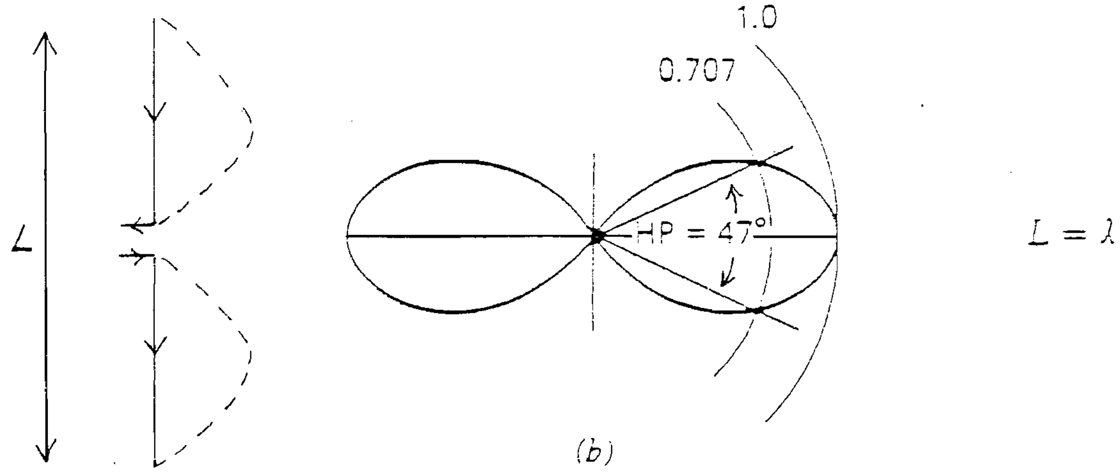
EEE406/6011 (2009) Solution to Question 1

(a)





(b)

Radiated power density of an antenna is

$$P_r = \frac{1}{2}\frac{|E_\theta|^2}{\eta} Wm^{-2} \quad (1).$$

The total radiated power over a far field sphere is then given by

$$P = \int_0^{2\pi}\int_0^{\pi} P_r r \sin(\theta) d\phi r d\theta \text{ W} \qquad (2).$$

Integrating out the ϕ dimension yields

$$P = 2\pi r^2 \int_0^{\pi} P_r \sin(\theta) d\theta \qquad (3)$$

and substitution of the given E field expression (1.1) then gives

$$P = 2\pi r^2 \frac{\eta I_o^2}{8\pi^2 r^2} \int_0^{\pi} \left[\frac{\cos(\pi \cos(\theta)) + 1}{\sin(\theta)} \right]^2 \sin(\theta) d\theta \quad (4).$$

The given integral (1.2) can now be identified in (4), so that

$$P = \frac{\eta I_o^2}{4\pi} \times 3.318 \qquad (5).$$

The gain of an antenna is given by

$$G = \frac{P_r|_{\theta=90^o}}{\frac{P}{4\pi r^2}} \quad (6),$$

and hence the directivity of the full-wave dipole is

$$D = \frac{\frac{\eta I_o^2}{8\pi^2 r^2}[2]^2}{\frac{3.3\eta I_o^2}{16\pi^2 r^2}} = 2.4 = 3.8dBi \quad . \quad (7).$$

(c) This is $1.7dB$ greater than the directivity of a half-wave dipole.

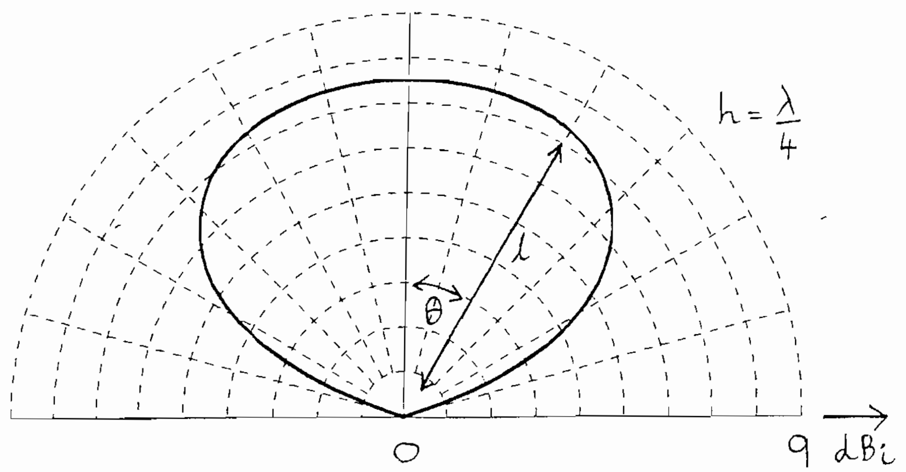
(d) The centre fed half-wave dipole will provide a good match into a feed line with $Z_o \approx 80\Omega$, however the full-wave dipole will present a very high impedance load and should therefore be fed with a line having $Z_o \approx 600\Omega$ or higher. In both cases a balanced transmission line should be used, or alternatively a balun connected between the dipole terminals and a coaxial feeder.

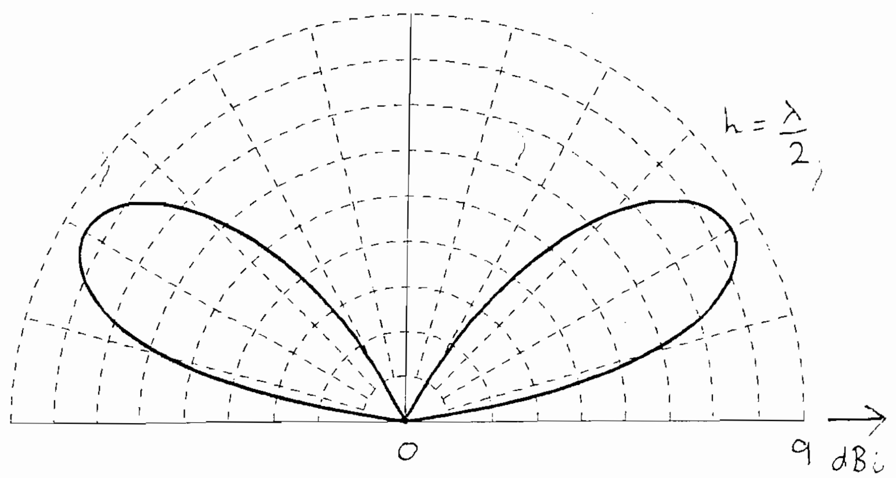
EEE406/6011 (2009) Solution to Question 2

(a)

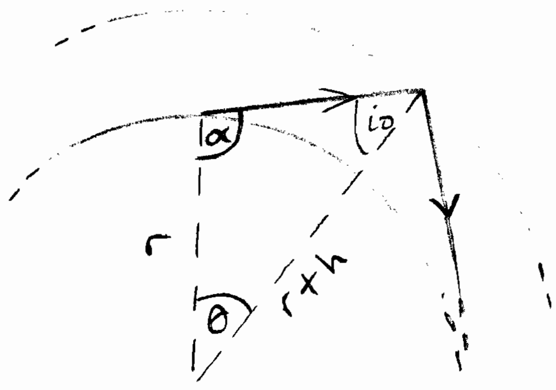
The idea behind a trapped dipole is that it is made to resonate as a half-wave at more than one frequency, by the use of traps. A trap is a parallel L-C circuit, which when resonant presents a high impedance so isolating the remaining antenna section after the trap. For the dipole in question, the traps will resonate at *~28MHz* and be positioned *~2.5m* either side of the feed. The antenna currents will see an open circuit at each trap, and thus the dipole basically behaves as a half wave at *28.4MHz*. Now, at *14.2MHz* the traps will be inductive and have significantly lower impedance, so currents will flow beyond them. The total length of the dipole is then made to be an electrical half wavelength at *14.2MHz*, taking into account the 'shortening' effect of the trap inductances.

(b)





(c)



We need to find θ. The skip distance s is then

$$s = 2r\theta \qquad (1)$$

Clearly,

$$\theta = 180 - \alpha - i_o \qquad (2)$$

and so the problem resolves into one of finding α and i_o. Now

$$\cos(i_o) = \frac{f_c}{f} \quad (3)$$

where the critical frequency is given by

$$f_c \approx 9\sqrt{N} = 9MHz \quad (4).$$

Using the sine rule,

$$\frac{\sin(180-\alpha)}{(r+h)} = \frac{\sin(i_o)}{r} = \frac{\sqrt{1-\frac{f_c^2}{f^2}}}{r} \quad (5)$$

Thus, at *14.2MHz*:

$i_o = 50.7^o$, $\alpha = 125.7^o$, $\theta = 3.6^o$, and so the skip distance is

$$s = 754km \qquad (6).$$

At *28.4MHz*:

$i_o = 71.5^o$, $\alpha = 95.2^o$, $\theta = 13.3^o$, and so the skip distance is

$$s = 2786km \quad (7)$$

(d)

At *14.2MHz* the skip distance is shorter with a launch angle of 35.7^o above the horizontal, whereas at *28.4MHz* the skip distance is significantly longer, with a launch angle of 5.2^o. Thus, for a receive destination *~2900km* away, the higher frequency only requires a single 'hop', whereas the lower requires *4* 'hops', thus there will be significantly less attenuation at *28.4MHz*. However, a very low launch angle is required at *28.4MHz* which is far removed from the dipole's main lobe.

EEE406/6011 (2009) Solution to Question 3

(a)

$$F_y = \iint_A E_y(x,y) e^{j(k_x x + k_y y)} dx dy \qquad (1)$$

where A denotes the aperture area. Since the aperture is circular it is more convenient to use polar co-ordinates, where $x = \rho\cos(\varphi)$, $y = \rho\sin(\varphi)$
and $dxdy = \rho d\rho d\varphi$. Thus

$$\begin{aligned} &k_x x + k_y y = \\ &k\sin(\theta)\cos(\phi)\rho\cos(\varphi) + k\sin(\theta)\sin(\phi)\rho\sin(\varphi) = k\rho\sin(\theta)\cos(\phi-\varphi) \end{aligned} \qquad (2)$$

and also since the aperture field is of unit amplitude (1) can be written

$$F_y = \int_0^{2\pi}\int_0^{\frac{a}{2}} e^{jk\rho\sin(\theta)\cos(\phi-\varphi)} \rho d\rho d\varphi \ (3).$$

We need to make a change of variable so that the upper limit of integration in the ρ dimension is 1, so let

$$\ell = \frac{2}{a}\rho \qquad (4)$$

then (3) becomes

$$F_y = \frac{a^2}{4}\int_0^{2\pi}\int_0^1 e^{jk\frac{a\ell}{2}\sin(\theta)\cos(\phi-\varphi)} \ell d\ell d\varphi \qquad (5).$$

Using (3.4), where $\alpha = k\frac{a\ell}{2}\sin(\theta)$ and $\gamma = \varphi$ yields

$$F_y = \frac{a^2}{4}\int_0^1 2\pi J_0(k\frac{a\ell}{2}\sin(\theta))\ell d\ell \qquad (6)$$

and (3.5) where $v = \frac{ka}{2}\sin(\theta)$ and $\gamma = \ell$ then gives

$$F_y = \frac{\pi a^2}{2}\frac{J_1(\frac{ka}{2}\sin(\theta))}{\frac{ka}{2}\sin(\theta)} \qquad (7).$$

(b)

We assume here that the y co-ordinate is aligned with the Clarke belt, with $< \theta,\phi = 0^o, 0^o >$, locating the *Astra* satellite and $< \theta,\phi = 4.3^o, 0^o >$ locating *Kopernikus*. The relative strength of the interfering signal is then given by

$$20\log_{10}\left(\left|\frac{J_1(\frac{ka}{2}\sin(4.3^o))}{\frac{ka}{2}\sin(4.3^o)}\right| \times \frac{\frac{ka}{2}\sin(0^o)}{J_1(\frac{ka}{2}\sin(0^o))}\right) \qquad (8)$$

where

$$\frac{ka}{2}\sin(4.3^o) = 4.7 \qquad (9).$$

Hence, (8) can be evaluated using (3.6) and (3.7) as

$$20\log_{10}\left(\frac{0.28}{4.7} \times \frac{1}{0.5}\right)dB = -18.5dB \qquad (10)$$

Thus the interfering *Kopernikus* signal is $18.5dB$ down on the *Astra* signal.

(c)

Larger dishes are required in Scotland since the transmitted field strength decreases for wider angles off the satellite antenna main lobe direction, and the main lobe is directed towards more southerly latitudes.

From (7) the extra directivity provided by an $80cm$ reflector is given by

$$20\log_{10}\left(\frac{80^2}{60^2}\right)dB = 5dB \qquad (11).$$

EEE406/6011 (2009) Solution to Question 4

(a)

Basic Restriction	**Occupational (*public*)**	**Averaging**
SAR (whole body)	0.4 (*0.08*) W/kg	6 min
SAR in head/trunk	10 (*2*) W/kg	10g in 6 min
SAR in limbs	20 (*4*) W/kg	10g in 6 min

(b)

The body will act as a lossy monopole antenna counterpoised by the ground image, and the degree of coupling to the incident field will be a function of how close the induced current path is to a resonance. A vertical monopole will have a complementary current image in the ground, effectively doubling the electrical length and increasing coupling, since the body monopole will be electrically short at 6MHz. However a horizontal element will have an anti-phase current image which will reduce the coupling. Stronger coupling and induced SAR will be for the field polarization having the greater co-polar current path therefore.

(a)

(i) The SAR will be relatively high, with maximum levels in the lower legs. This is due to the image and body monopole being 'connected' providing a longer co-polar current path and so helping towards resonance.

(ii) The shoes will introduce capacitive reactance between the image and body monopole, thus reducing the SAR.

(iii) The 'T' monopole will increase the current path, so that the barefoot configuration will have the highest SAR of all.

(b)

The SAR will be significantly less for all configurations due to the reduced co-polar current path (2 x arm length instead of at least 2 x body height) and the anti-phase image of the horizontal current flowing between the arms. Wearing shoes won't make a great deal of difference here, but the SAR will be higher for arms outstretched due to the longer co-polar current path, with maximum levels in the arms.

As the frequency increases the wavelength shortens, so making the configurations electrically longer and increasing coupling to the incident field as resonance approaches. Thus SAR levels will generally increase up to 22MHz.

(c)

$$SAR = \frac{\sigma E^2}{D} \quad (1)$$

where E denotes the rms electric field in tissue of conductivity σ and density D. Thus

$$E^2 = E_y^{\ 2} + E_z^{\ 2} = 1000 \qquad (2)$$

and hence

$$SAR = \frac{1 \times 1000}{1000} = 1 \quad W/kg \qquad (3)$$

Assuming the field level is consistent over the whole body, then this exceeds the ICNIRP occupational safety limit.