Problem Sheet 6

1. (i) Given average thermal energy $= \frac{3kT}{2}$

$= \text{kinetic energy}$

$$\therefore \frac{3kT}{\cancel{2}} = \frac{1}{\cancel{2}} m v^2$$

$$v = \sqrt{\frac{3kT}{m}}$$

From $p = mv = \sqrt{\frac{3kTm^{\cancel{2}}}{\cancel{m}}} = \sqrt{3kTm}$

From de Broglie's wave equation,

$$p = mv = \frac{h}{\lambda}$$

$$\therefore \lambda = \frac{h}{\sqrt{3kTm}}$$

$$= 6.3 \times 10^{-9}\ m$$

(ii) Assuming mass of snail $\approx 10g = 0.01 kg$

" velocity " " $\approx 1 m/hr = \frac{1}{3600}\ m s^{-1}$

From ①, $p = \frac{h}{\lambda}$

$$\therefore \lambda = 2.4 \times 10^{-28}\ m$$

(Note: In an exam, if you had to guess numerical information like this, you would not be expected to get particularly close to the examiner's answer, provided your guess and answers are reasonable.)

(iii) Mass of concorde $= 185070$ kg
velocity " " $= 2179$ km/hr

source: "Civil Aircraft of the World" Swanborough (Allan, 1980)

From ①, $p = \frac{h}{\lambda}$

$$\therefore \lambda = 5.9 \times 10^{-42}\ m$$

(2)

for x-rays (photons) $E = h\nu$

$$= \frac{hc}{\lambda} \qquad \text{—— (2)}$$

$$= \frac{6.626\times10^{-34} \times 3\times10^{8}}{0.2\times10^{-9}}$$

$$= 9.94\times10^{-16}\ \mathrm{J} = eV$$

$$\therefore V = \underline{\underline{6.21\ kV}}$$

From $p = \frac{h}{\lambda}$, $E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m}$

$$= \frac{h^2}{2m\lambda^2} \qquad \text{—— (3)}$$

$$\therefore \lambda = \sqrt{\frac{h^2}{2mE}} = \left[\frac{(6.626\times10^{-34})^2}{2\times9.1\times10^{-31}\times9.94\times10^{-16}}\right]^{\frac{1}{2}}$$

$$= \underline{\underline{0.0156\ \mathrm{nm}}}$$

(3) Photon energy $= \frac{hc}{\lambda_p}$ (from (2))

Electron energy $= \frac{h^2}{2m\lambda_e^2}$ (from (3))

Equating both equations, where $\lambda_p = \lambda_e$

$$\frac{hc}{\lambda} = \frac{h^2}{2m\lambda^2}$$

$$\lambda = \frac{h}{2mc} = \frac{6.626\times10^{-34}}{2\times9.1\times10^{-31}\times3\times10^{8}}$$

$$= 1.21\times10^{-12}\ \mathrm{m}$$

$$= \underline{\underline{0.00121\ \mathrm{nm}}}$$

(4) Energy of photons needed to cause photoelectric emission = Work function of metal

$\therefore 2.4\,eV = \frac{hc}{\lambda}$ (since $E \propto \frac{1}{\lambda}$, min E $\to$ max^{m} λ)

$\lambda = \frac{6.626\times10^{-16} \times 3\times10^{8}}{2.4 \times 1.6\times10^{-19}}$

$= 5.18\times10^{-7}$ m

$= \underline{\underline{518 \text{ nm}}}$

If wavelength $= \frac{518}{2}$ nm, energy supplied by the photon is therefore twice that of the workfunction.

$\therefore$ Excess energy $= 2.4eV$.

$\therefore$ Retarding potential required to stop electron flow (current) $= \underline{\underline{2.4eV}}$

(From energy equation: $\underbrace{\hbar\omega}_{\text{energy from each photon}} = \underbrace{\phi_w}_{\text{work function}} + \underbrace{eV}_{\text{potential energy from voltage V needed to stop current flow}}$

(5) Given that $E_n = -\frac{13.6}{n^2}$ eV

$\therefore E_1 - E_2 = -13.6\left(\frac{1}{1} - \frac{1}{2^2}\right)$

$= -10.2eV$

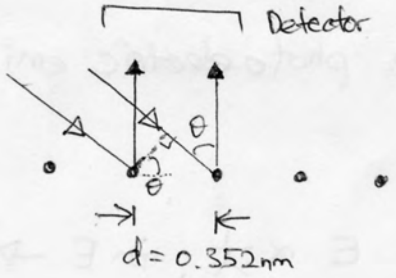
Since this is equivalent to an emitted photon with energy $= \frac{hc}{\lambda}$

$\therefore \lambda = \frac{hc}{10.2\times1.6\times10^{-19}}$

$= 1.22\times10^{-7}$ m

$= \underline{\underline{122 \text{nm}}}$

(6)



$d \sin\theta$ = path difference

$= n\lambda$

$\therefore$ peaks at detector at constructive interference

when $d \sin\theta = n\lambda$ — ①

Energy of the electron, $E = \frac{1}{2}mv^2$

$= \frac{(mv)^2}{2m} = \frac{p^2}{2m}$

$= \frac{h^2}{2m\lambda^2}$ where $p = \frac{h}{\lambda}$

$\therefore \lambda = \sqrt{\frac{h^2}{2mE}}$

$= \sqrt{\frac{(6.626 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 70 \times 1.6 \times 10^{-19}}}$

$= 14.6\text{nm}$

From ①.

$\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right)$

$= 0, \pm 24.6°, \pm 56.4°$ etc.