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The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2014-15 (2.0 hours)

EEE345 Engineering Electromagnetics

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. Using Maxwell's equations for the rotation operators of the electrical and magnetic fields, the materials equations relating corresponding fluxes and fields, and the mathematical identity

$$\textbf{rot rot}\ \underline{\boldsymbol{A}} = \textbf{grad}\ \text{div}\ \underline{\boldsymbol{A}} - \nabla^2 \underline{\boldsymbol{A}} \qquad \text{(equation 1),}$$

show that in vacuum the magnetic vector potential $\underline{\boldsymbol{A}}$ obeys a wave equation. **(10)**

b. State for each of the following functions $f(x,t)$ (where x= spatial coordinate, t= time, a,b,c= constants and g= arbitrary functions) whether they represent a travelling wave, a standing wave or no wave at all. Explain your answers.

(i) $f(x,t) = \cos(3xt - a)$

(ii) $f(x,t) = \sin(2at - bx)$

(iii) $f(x,t) = 4\sin(3x)\exp(-10x)$

(iv) $f(x,t) = [g(bt - x)]^2$

(v) $f(x,t) = g(at - x^2)$ **(5)**

c. Provide a suitable sketch of the electromagnetic fields and the wave vector $\underline{\boldsymbol{k}}$ of a wave freely propagating along the z-axis. Define the Poynting vector. State its directionality and what its magnitude describes. **(5)**

2. **a.** The voltage as a function of position, x, and time, t, along a transmission line can generally be written as

$$V(x,t)=V_0^+ \exp[j(\omega t - k'x)] + V_0^- \exp[j(\omega t + k'x)] \qquad \text{(equation 2)}$$

where ω is the angular frequency and k' a propagation constant that can be written in complex notation as

$$k'=a-jb \qquad \text{(equation 3)}$$

(i) Explain what both terms on the right-hand side of equation 2 mean physically.

(ii) Assuming $V_0^-=0$ and using equation (3), show that the imaginary part of k' leads to an attenuation of the signal along the line.

(iii) For a lossy transmission line with conductance G^* per unit length, capacitance C^* per unit length, resistance R^* per unit length and inductance L^* per unit length, it can be shown that

$$k'^2 = -(G^*+j\omega C^*)(R^*+j\omega L^*) \qquad \text{(equation 4).}$$

Calculate an approximate solution for k' for the case of weak ohmic losses where $G^* \ll \omega C^*$ and $R^* \ll \omega L^*$.

(7)

b. A 50kHz signal is fed into a printed circuit board that can be described as a lossy transmission line with the characteristics of L^*=1mH/m, C^*=1nF/m, R^*=1Ω/m, G^*=0.001/(Ω m). Use your above approximate solution for k' to calculate over what length the signal can be transferred so that at least 90% of the voltage of the input signal arrives. Is this feasible for implementation? **(4)**

c. Oliver Heaviside found that if the relationship $G^*/C^*=R^*/L^*$ is obeyed, then a transmission line will show no dispersion at all.

(i) Explain what dispersion means and how it leads to signal distortion on lossy transmission lines.

(ii) Insert the above expression by Heaviside into equation 4 from Question 2**a** and calculate the signal velocity.

(iii) Compare the above result to that from equation 4 for $G^*=R^*=0$. **(9)**

3. **a.** Consider a plate capacitor of width w, length l, distance d between the plates that is filled with a dielectric of relative permittivity ε_r.

(i) Using Coulomb's Law and Gauss' Law, calculate the magnitude E of the electric field between the plates as a function of the charge Q on the plates.

(ii) From this, determine the voltage by integration.

(iii) From the above, derive the well-known relationship for the capacitance of a plate capacitor. **(10)**

b. The function

$$V(x)= (2ax-x^2)\,\rho_{\text{free}}/(2\varepsilon_0\,\varepsilon_r) \qquad \text{(equation 5)}$$

describes the potential profile across a semiconducting pn-junction of total depletion layer width $2a$ along the x-direction. Assume ε_0=8.8542×10^{-12} F/m. Choose the origin in the middle of the pn-junction to calculate for a depletion layer width of $2a$=100nm, a free charge density of ρ_{free} =8000C/m^3, a dielectric constant of ε_r=9 and a cross-sectional area of A=10^{-7} m^2

(i) the voltage drop across the whole junction and

(ii) the junction capacitance.

(iii) Compare the junction capacitance quantitatively to that of a standard plate capacitor. **(5)**

c. Consider a dielectric material with complex permittivity $\varepsilon_r = \varepsilon_r' + j\varepsilon_r''$ and complex refractive index $N=n+j\kappa$. Using the relationship $N^2=\varepsilon_r$, derive expressions for real part ε_r' and imaginary part ε_r''. What happens in the special case of $n=\kappa$ to both, and what does this mean for the relationship between the fields $\underline{\boldsymbol{E}}$ and $\underline{\boldsymbol{D}}$? **(5)**

4. **a.** Consider two parallel plates of width w and length l that are separated by a distance d. Their specific capacitance per length is given by

$$C^* = \varepsilon_0 \varepsilon_r w/d \qquad \text{(equation 6).}$$

Their specific inductance per unit length is given by

$$L^* = \mu_0 \mu_r d/w \qquad \text{(equation 7).}$$

(i) Calculate the characteristic impedance of the plates.

(ii) Show that the phase velocity of the signal wave travelling on the plates is given by $c\ (\mu_r \varepsilon_r)^{-½}$, where c is the speed of light in vacuum.

(iii) Calculate the ratio of the electric to magnetic field strength, E/H.

(iv) From this, show similarly that $E/B=c$ in vacuum. **(9)**

b. Consider light transversing from a medium with refractive index n_1 to another, denser one with refractive index n_2. Assume the sine of the angle with respect to the vertical, sin θ, is proportional to the speed of light in the corresponding medium, which is $v= c\ (\mu_r \varepsilon_r)^{-½}$. From this, derive Snell's Law of refraction, which states

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(equation 8)}$$

(3)

c. Fresnel's formula for the reflectivity of a surface for in-plane polarisation, without any absorption, is given by

$$R_\parallel = (n_2 \cos\theta_1 - n_1 \cos\theta_2)^2 / (n_2 \cos\theta_1 + n_1 \cos\theta_2)^2 \qquad \text{(equation 9)}$$

(i) Sketch what happens in the case of $\theta_1+\theta_2=90°$ to the electric field vector $\underline{\boldsymbol{E}}$ of the reflected wave.

(ii) Applying Snell's Law, derive an expression for the incidence angle θ_1 in the case of $\theta_1+\theta_2=90°$.

(iii) Describe what happens to $R_\parallel$ in the case of $\theta_1+\theta_2=90°$.

(iv) From equation 9, derive an expression for $R_\parallel$ for the case of vertical incidence. **(8)**

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