Solutions

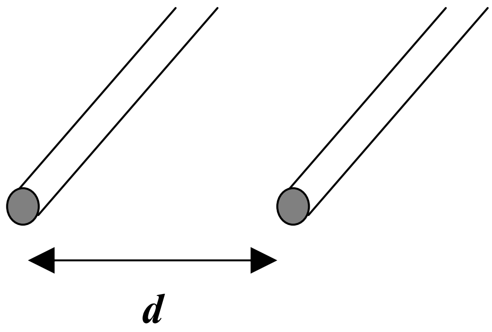
Q1 (a)

(i) **Two Wire Line**

Applications

(1) Power lines

(2) Antenna feeders (*d* small to prevent radiation losses).



(1 mark)

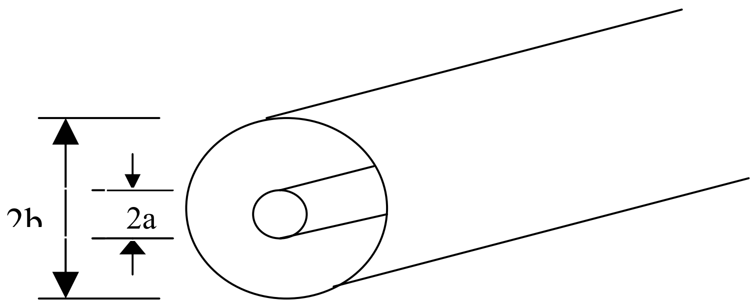
(ii) **Coaxial Line**

Applications

(1) In communications from audio up to microwave frequencies.

(2) To supply high frequency signals to test equipments where radiation of signals may cause problems.

(3) Used as the down lead from a TV antenna to the television's receiver.



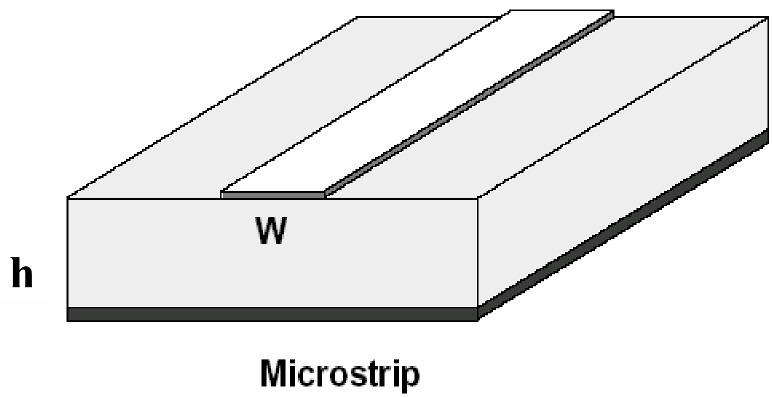
(2 marks)

(iii) **Microstrip Line**

Applications

(1) A convenient transmission line structure for probe measurements of voltage, current and waves.

(2) Communications equipment at F>1GHz, e.g. receivers.



(1 mark)

Q1(b)

A combination of lumped-constant elements consisting of resistors, capacitors, or inductors can often be used to model a circuit element. A resistor is used to characterize the power or energy lost in a circuit, while a capacitor and an inductor are used to characterize the electric and magnetic energy stored in a circuit respectively. In cases where the circuit dimensions are large with respect to a wavelength, a transmission line model consisting of resistors, capacitors, and inductors will be used to characterize the distributed parameters effect for the circuit elements. Division between lumped and distributed circuit considerations occurs when the dimension of the electronic component is greater than one-tenth of the signal wavelength (λ).

(4 marks)

Q1 (c)

Since the load needs to be match to 100 Ω, then $Z_{in} = 100\Omega$

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)}$$ **(1 mark)**

$$100(Z_o + j(80 + j20)\tan(\beta\ell)) = Z_o(80 + j20 + jZ_o \tan(\beta\ell))$$

$$100Z_o + j8000\tan(\beta\ell) - 2000\tan(\beta\ell)) = 80Z_o + j20Z_o + jZ_o^2 \tan(\beta\ell)$$

Equating real and imaginary parts to each other

$$100Z_o - 2000\tan(\beta\ell)) = 80Z_o \qquad (1)$$ **(1 mark)**

$$8000\tan(\beta\ell) = 20Z_o + Z_o^2 \tan(\beta\ell) \qquad (2)$$ **(1 mark)**

From equation (1)

$$\tan(\beta\ell) = \frac{Z_o}{100} \qquad (3)$$ **(1 mark)**

Substitue (3) in (2)

$$80Z_o = 20Z_o + \frac{Z_o^3}{100}$$

$$Z_o^2 = 6000$$

i.e.

$$Z_o = 77\Omega$$ **(1 mark)**

which gives

$$\tan(\beta\ell) = 0.77$$

i.e.

$$\ell = \frac{1}{\beta}\tan^{-1}(0.77) = 0.1\lambda$$ **(1 mark)**

Q1 (d)

f=500MHz, $\lambda = \frac{c}{f} = 60\text{cm}$, $\frac{\ell}{\lambda} = 0.5$ i.e. $\ell = \frac{\lambda}{2}$

The input impedance is given by

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)}$$

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\frac{2\pi}{\lambda}\frac{\lambda}{2})}{Z_o + jZ_L \tan(\frac{2\pi}{\lambda}\frac{\lambda}{2})} = Z_L = 75 - j50\Omega$$ **(2 marks)**

at a distance of 10 cm from the load $\ell = 0.17\lambda$

i.e

$$Z_{in(\ell=10cm)} = Z_o \frac{Z_L + jZ_o \tan(0.17\lambda\beta)}{Z_o + jZ_L \tan(0.17\lambda\beta)} = 21 - j5.8\Omega$$ **(2 marks)**

$$\Gamma_{(\ell=10cm)} = \frac{Z_{in(10cm)} - Z_o}{Z_{in(10cm)} + Z_o} = -0.4 - j0.1$$ **(2 marks)**

Q2 (a)

The reflection coefficient at a distance *d* from the load is given by

$\Gamma_{(d)} = \Gamma e^{-2\gamma d}$

i.e.

$\Gamma_{(d)} = \Gamma e^{-2\alpha d} e^{-2j\beta d}$

In general then as we move from the load towards the generator $\Gamma e^{-2\alpha d}$ continuously decreases. Hence the radius of the locus in the Smith chart, Γ, no longer move around in a circle. Instead it spirals in towards the centre of the chart $z = (1 + j0)$, i.e. a long lossy line appears to be matched irrespective of its termination. On the Smith chart, move along a constant radius arc through the angle $2\beta d$ and then move radially inwards to $\Gamma e^{-2\alpha d}$.

(4 marks)

Q2 (b)

For a VSWR of 4, the VSWR circle can be plotted on the Smith chart.

(1 mark)

Since the angle of input reflection coefficient is -20°, point A represents the normalized input impedance, i.e.

$z_{in} = 2.7 - j1.75$

i.e.

$Z_{in} = 135 - j87.5$ **(2 marks)**

To find the load impedance, move a distance of 0.1λ, *towards load*, from point A to B. **(1 mark)**

The impedance at B

$z_L = 1.05 + j1.52$

i.e.

$Z_L = 52.5 + j76$ **(2 marks)**

Q2 (c)

$z_L = \frac{180 + j40}{50} = 3.6 + j0.8$ Point A **(1 mark)**

$y_L = 0.265 - j0.058$ Point B **(1 mark)**

Rotate the unit g circle *Towards Load,* by a distance $d_3 = 0.125\lambda$. **(1 mark)**

The effect of the 1st stub is to move from point B to intersect the new unit circle at point C. **(1 mark)**

The admittance point will move on the corresponding conductance circle, since the stub does not alter the real part of the admittance.

The admittance at point C is 0.26+j0.325 **(1 mark)**
compare it with that of B (j0.325+j0.058) shows that stub 1 has provided an admittance of $j0.383$. **(1 mark)**
This could be achieved by using an o.c. stub with a length of $d_1 = 0.058\lambda$, i.e. the distance from F to G on the chart. **(1 mark)**

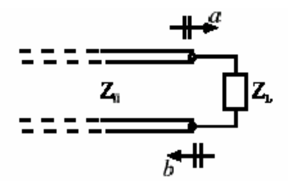
At point D, the admittance is 1+j1.58, **(1 mark)**

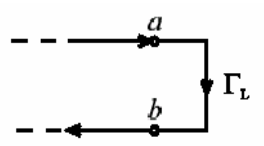
i.e. stub 2 must provide $jb_2 = -1.58$ to end up at the matched condition. **(1 mark)**
For a s.c. stub this means $d_2 = 0.09\lambda$, i.e. the distance from E to H on the chart. **(1 mark)**

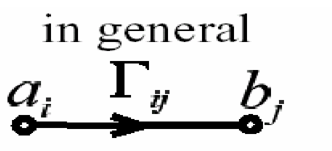
Q3 (a)

SFD can be used for a graphical representation of the interaction between the incident and scattered waves in a microwave network. **(1 mark)**
There are two basic components of any SFD:
Nodes; each port in the network has two nodes a_n and b_n. The node a_n represents a wave entering port n, while node b_n represents a wave reflected from port n. **(1 mark)**
Branches; each branch represents a direct path between a-node and b-node. For each branch there is an associated S parameter or reflection coefficient. **(1 mark)**

An example of the SFD of a one-port network is shown below







(1 mark)

Q3 (b)

To calculate S_{11}, the impedance Z_{in} is required when the network is terminated with Z_o

(i) For the series load written
$Z_{in1} = Z + Z_o$

$$S_{11} = \Gamma_{in} = \frac{Z_{in1} - Z_o}{Z_{in1} + Z_o} = \frac{Z}{Z + 2Z_o}$$ **(1 mark)**

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

For the 1st port

$$V_1 = \sqrt{Z_o}(a_1 + b_1)$$

$$V_2 = \sqrt{Z_o}\, b_2$$

Therefore

$$\frac{V_1}{V_2} = \frac{(a_1 + b_1)}{b_2} = \frac{(1 + \frac{b_1}{a_1})}{\frac{b_2}{a_1}} = \frac{1 + S_{11}}{S_{21}}$$

i.e.

$$S_{21} = \frac{V_2}{V_1}(1 + S_{11})$$ **(1 mark)**

Since

$$V_2 = V_1 \frac{Z_o}{Z + Z_o}$$

then.

$$\frac{V_2}{V_1} = \frac{Z_o}{Z + Z_o}$$

and

$$1 + S_{11} = 1 + \frac{Z}{Z + 2Z_o} = 2\frac{Z + Z_o}{Z + 2Z_o}$$

i.e.

$$S_{21} = \frac{2Z_o}{Z + 2Z_o}$$ **(1 mark)**

Therefore

$$S_{21} + S_{11} = \frac{2Z_o}{Z + 2Z_o} + \frac{Z}{Z + 2Z_o} = 1$$ **(1 mark)**

(ii) For the shunt load

$$Z_{in2} = \frac{ZZ_o}{Z + Z_o}$$

$$S_{11} = \Gamma_{in} = \frac{Z_{in2} - Z_o}{Z_{in2} + Z_o} = \frac{-Z_o}{2Z + Z_o}$$ **(1 mark)**

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

As we have proved for the series load,

$$S_{21} = \frac{V_2}{V_1}(1 + S_{11})$$

but now

$V_2 = V_1$ **(1 mark)**

i.e.

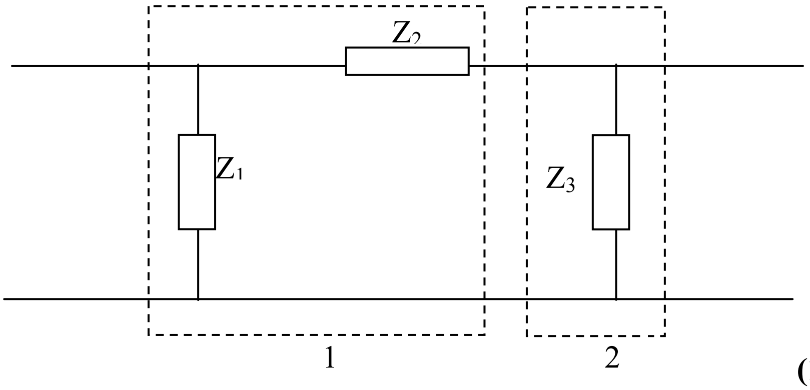
$$S_{12} = 1 + S_{11} = 1 - \frac{Z_o}{Z_o + 2Z} = \frac{2Z}{Z_o + 2Z}$$ **(1 mark)**

Therefore

$$S_{21} - S_{11} = \frac{2Z}{Z_o + 2Z} + \frac{Z_o}{Z_o + 2Z} = 1$$ **(1 mark)**

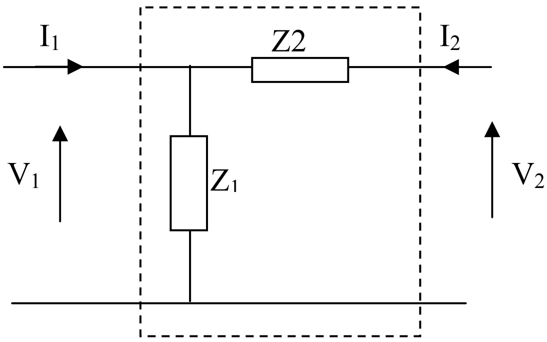
Q3 (c)

This can be done in several methods. One way to find the ABCD matrix of the given network is to divide it into two simpler networks as shown below, then finding the transmission matrix of each network. The overall ABCD matrix then can be found by cascading the two matrices



(1 mark)

For network 1



$$V_1 = I_1 Z_1 + I_2 Z_1$$

$$I_1 = \frac{V_1}{Z_1} - I_2$$

$$V_2 = I_1 Z_1 + I_2 (Z_1 + Z_2)$$

i.e.

$$V_2 = V_1 + I_2 Z_2$$

Therefore

$V_1 = V_2 - I_2 Z_2$ **(1 mark)**

and

$$I_1 = \frac{V_2}{Z_1} - (1 + \frac{Z_2}{Z_1})I_2$$ **(1 mark)**

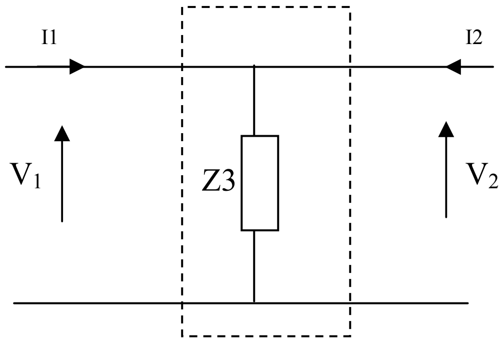
In matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & (Z_2) \\ \frac{1}{Z_1} & (1 + \frac{Z_2}{Z_1}) \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

i.e.

$$A' = 1, B' = Z_2, C' = \frac{1}{Z_1} \text{ and } D' = (1 + \frac{Z_2}{Z_1})$$ **(1 mark)**

and for network 2



$$V_1 = V_2$$

$$I_1 + I_2 = \frac{V_1}{Z_3} = \frac{V_2}{Z_3}$$

i.e. $I_1 = \frac{V_2}{Z_3} - I_2$ **(1 mark)**

In matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_3} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A'' = 1, B'' = 0, C'' = \frac{1}{Z_3} \text{ and } D'' = 1$$ **(1 mark)**

Use cascading to get the overall ABCD parameters as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$

i.e.

$$A = 1 + \frac{Z_2}{Z_3}, B = Z_2,$$

$$C = \frac{1}{Z_1} + \frac{1}{Z_3} + \frac{Z_2}{Z_1 Z_3}, D = 1 + \frac{Z_2}{Z_1}$$ **(2 marks)**

Q4(a)

Generally transistors presents a significant impedance mismatch, so matching will be achieved over a narrow frequency bandwidth. When bandwidth is an issue, then we design for a gain less than the maximum, imperfect matching, to improve bandwidth. **(2 marks)**

Sometimes it is required to design an amplifier with a specific gain, other than the maximum. Constant gain circles will be used to facilitate design for a specific gain. **(2 marks)**

Q4(b)

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = -0.18 - j0.9$$

i.e. $|\Delta| = 0.92$

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|} = 0.89$$

We have $K<1$ and $|\Delta|<1$, so the device is potentially unstable.

(1 mark)

The power gains can be calculated as follows:

First, find the source and load reflection coefficients

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.286$$ **(1 mark)**

$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} = 0.27$$ **(1 mark)**

Next the input and output reflection coefficients are computed via

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.39 - j0.64$$ **(1 mark)**

$$\Gamma_{out} = S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.828 - j0.39$$ **(1 mark)**

Which can be used to calculate

$$G_T = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)(1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2 |1 - \Gamma_s\Gamma_{in}|^2} = 8$$ **(1 mark)**

$$G_A = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)}{|1 - S_{11}\Gamma_s|^2 \left(1 - |\Gamma_{out}|^2\right)} = 32.4$$ **(1 mark)**

$$G = \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2 \left(1 - |\Gamma_{in}|^2\right)} = 16.5$$ **(1 mark)**

Q4(c)

$$N = \frac{NF - NF_{min}}{4R_N/Z_o}\left|1+\Gamma_{opt}\right|^2 = \frac{1.778\text{-}1.58}{80/50}\left|1+0.62\angle 100^\circ\right|^2 \quad = 0.145$$

(1 mark)

$$C_{NF} = \frac{\Gamma_{opt}}{(N+1)} = 0.541\angle 100^\circ$$

$$r_{NF} = \frac{\sqrt{N(N+1-\left|\Gamma_{opt}\right|^2)}}{(N+1)} = 0.29$$

(1 mark)

Next we calculate data for several input section constant gain circles

G_S	g_S	C_S	R_S
1.5dB	0.905	$0.56\angle 60^\circ$	0.204
1.7dB	0.948	$0.58\angle 60^\circ$	0.149
1.8dB	0.970	$0.59\angle 60^\circ$	0.112

The noise figure and constant gain circles are plotted on the Smith chart

(4 marks, 1 for each circle)

and the G_S =1.8dB circle just intersects the noise figure circle, and any higher gain will results in a worse noise figure.

From the Smith chart the optimum solution is then $\Gamma_S = 0.55\angle 70^\circ$ and NF =2.5dB. **(2 marks)**