1. Describe an equivalent circuit of the MOS cascode circuit in Figure 1 and find the trans-conductance, i.e. the ratio between the output current and input voltage.

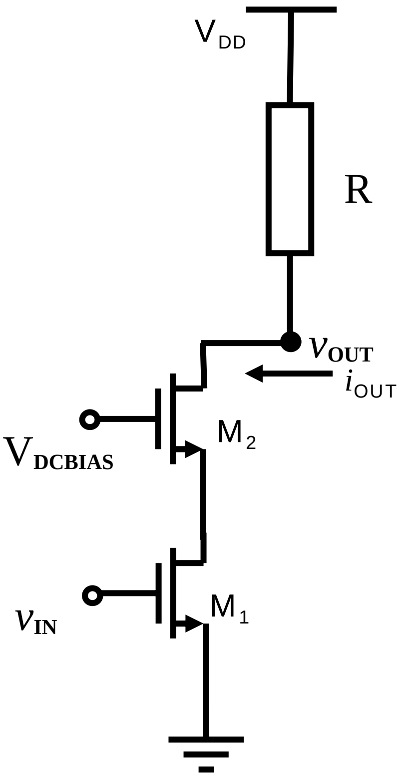


Figure 1a.

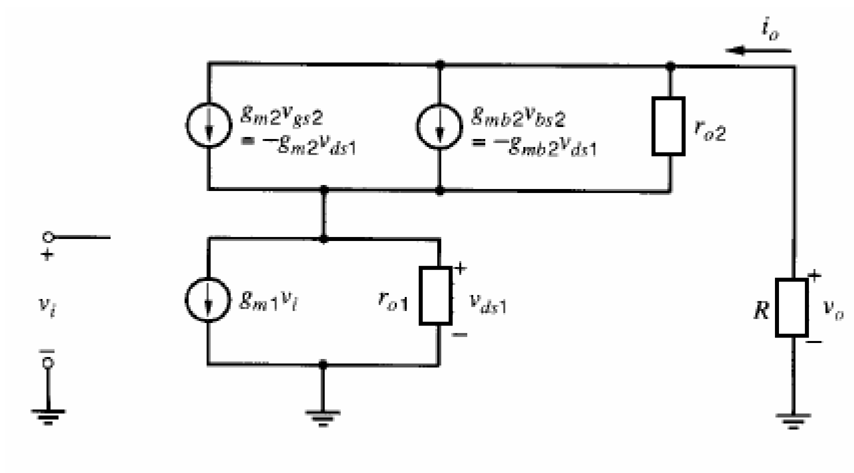


Figure 1b.

The small-small signal equivalent circuit is shown in Figure 1b. Since the input is connected to the gate of M1, the input impedance is

$$R_i \rightarrow \infty$$

To find the transconductance, set R=0 to short the output and calculate the current i_o. From KCL at the output,

$$i_o + g_{m2}v_{ds1} + g_{mb2}v_{ds1} + \frac{v_{ds1}}{r_{o2}} = 0 \qquad (1.1)$$

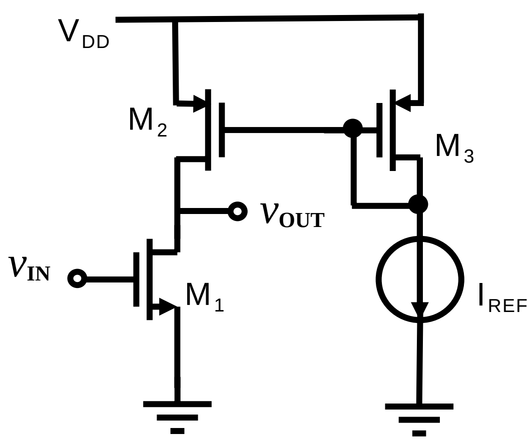
From KCL at the source of M2,

$$g_{m1}v_i + g_{m2}v_{ds1} + g_{mb2}v_{ds1} + \frac{v_{ds1}}{r_{o1}} + \frac{v_{ds1}}{r_{o2}} = 0 \qquad (1.2)$$

Solving (1.2) for vds1, substituting into (1.1), and rearranging gives

$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0} = g_{m1}\left(1 - \frac{1}{1 + (g_{m2} + g_{mb2})\,r_{o1} + \dfrac{r_{o1}}{r_{o2}}}\right) \simeq g_{m1}$$

2. State a voltage gain of common source amplifier with an active load as in Figure 2, with a representation of a small signal equivalent circuit.



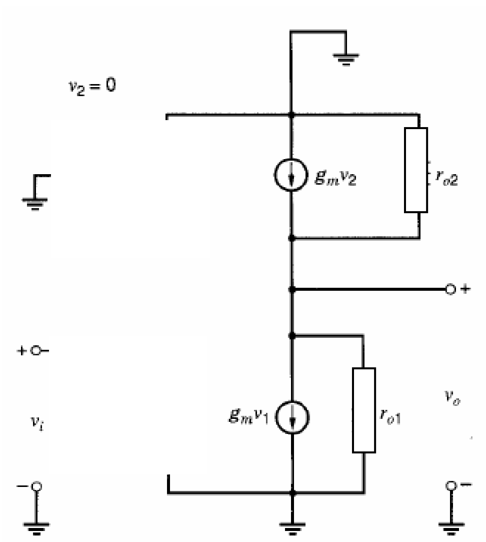


Figure 2.1 Figure 2.2

The primary characteristics of interest in the small signal analysis are the voltage-gain and output resistance when both devices operated in the active region. The small signal circuit is shown in Figure 2.2. Since I_{REF} in Figure 2.1 is assumed constant, the large-signal gate-source voltage of load transistor M2 is constant. Therefore, the small-signal voltage-controlled current $g_{m2}v_2$=0. To find the output resistance, we set the input to zero. Therefore v_1=0 and $g_{m1}v_1$=0, and the output resistance is

$$R_o = r_{o1} || r_{o2} \qquad (2.1)$$

Since v_2=0, $g_{m1}v_1$ flows in $r_{o1}||r_{o2}$ and

$$A_v = - g_{m1}(r_{o1} || r_{o2}) \qquad (2.2)$$

3. A CMOS differential pair with the current mirror load is shown in Figure 3.

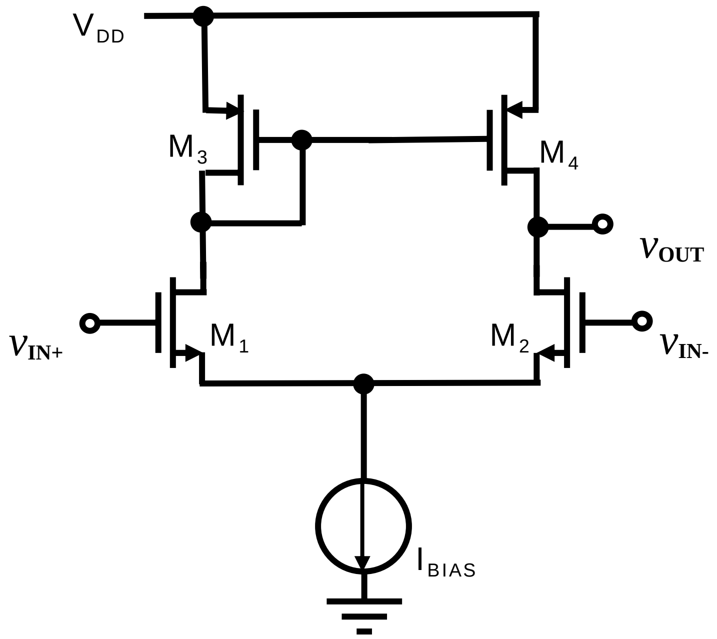


Figure 3.

3-1. Describe the circuit operation as a differential amplifier by representing an equivalent circuit.

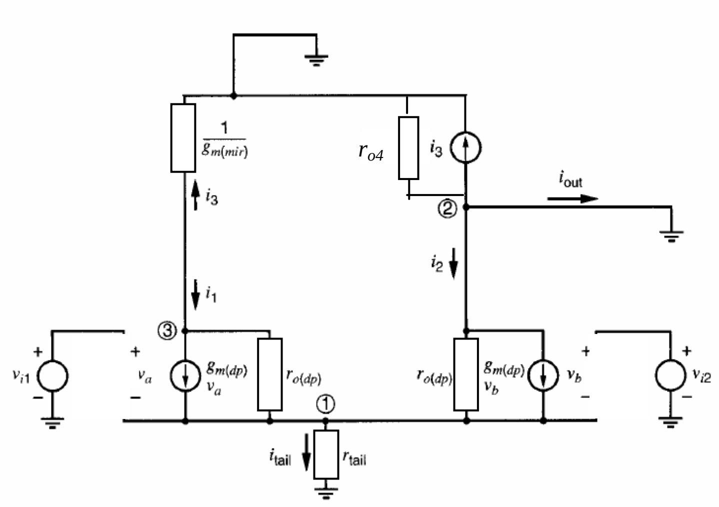


Figure 3.1

Assume $r_{tail} \rightarrow \infty$, $r_{o(\mathrm{dp})} \rightarrow \infty$, and $r_{o4} \rightarrow \infty$. The voltage v_1 at node 1 becomes the common-mode component v_{ic}, where $v_{ic}=(v_{i1}+v_{i2})/2$.

Let $v_{id}=v_{i1}-v_{i2}$ represent the differential mode of the input signal. Then $v_{i1}=v_{ic}+(v_{id}/2)$ and $v_{i2}=v_{ic}-(v_{id}/2)$. Therefore,

$$i_1 = g_{m(\mathrm{dp})}(v_{i1}-v_1)=g_{m(\mathrm{dp})}\, v_{id}/2$$

and

$$i_2 = g_{m(\mathrm{dp})}(v_{i2}-v_1)=-\, g_{m(\mathrm{dp})}\, v_{id}/2$$

(Please remember $v_1=v_{ic}$.)

If with a resistive load and a single-ended output, only i_2 will flows in the output. Due to the active loads (also acting current mirror) of M3 and M4, not only i_2 but also most of i_3 flows in the output.

Therefore transconductance G_m(dm) of a differential input transistor pair is

$$G_{m(\mathrm{dm})} = i_{out}/v_{id}$$
$$= g_{m(\mathrm{dp})}$$

where $i_{out}=-i_3-i_2$ (with active loads - current mirror as a pair of load).

3-2. State the output resistance.

The output resistance Ro is the resistance looking back into the node 2 with both inputs connected to small-signal ground:

$$R_o= r_{o4}||r_{o(\mathrm{dp})}$$

where r_{o4}, $r_{o(\mathrm{dp})}$ are output resistance of M4 and M2 (one of dp, differential pair), respectively.

4. Find the output resistance of a MOSFET. Assume the transistor operates in the active region with I_D=5μA and V_A =50V. Neglect the body effect.

The output resistance of transistor is

$$r_o = \frac{V_A}{I_D} = \frac{50\text{ V}}{5\ \mu\text{A}} = 10\text{M}\Omega$$