

# EEE 6212

## Semiconductor Materials

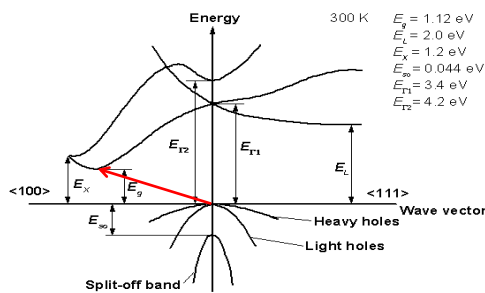
### Lecture 10: Doping

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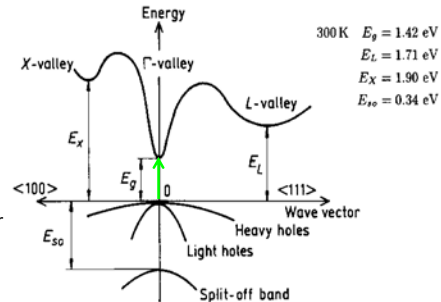
- review of band structures
- conductivity
- general formulae for charge carrier densities, DOS and Fermi distribution
- intrinsic semiconductors
- doping as electrically active, substitutional point defects in the lattice
- donators and acceptors; flat vs. deep levels
- intrinsic, saturation and extrinsic range

## typical semiconductor band structures

review of band structures at room temperature



Si



GaAs

<http://www.ioffe.ru/SVA/NSM/Semicond>

## conductivity, part 1: the basics

general equation for conductivity:

$$\sigma = e(n\mu_n + p\mu_p)$$

where  $e = 1.6022 \times 10^{-19}$  C is the elementary charge,

$n$  is the (free) electron density and

$p$  is the hole density,

$\mu$  is the mobility of charge carriers (electrons/holes)

Note that Ohm's law states  $\sigma \underline{E} = \underline{j} = -en\underline{v}_e + ep\underline{v}_p$  where  $\underline{v}_d$  denotes the drift velocity of the charge carrier type, given by  $\underline{v}_e = -\mu_e \underline{E}$  and  $\underline{v}_p = \mu_p \underline{E}$ .

## charge carrier density, DOS & Fermi distribution

charge carrier densities:

1. electron density:  $n = \int_{E_c}^{\infty} D_c(E) f(E, T) dE$

2. hole density:  $p = \int_{-\infty}^{E_v} D_v(E) [1 - f(E, T)] dE$

where  $D_{c,v}$  are the **density of states (DOS)** in the conduction and valence band and are approximately given by

$$D_c(E) \approx 4\pi/h^3 (2m_{e^*})^{3/2} \sqrt{E - E_c} \quad (\text{for } E > E_c)$$

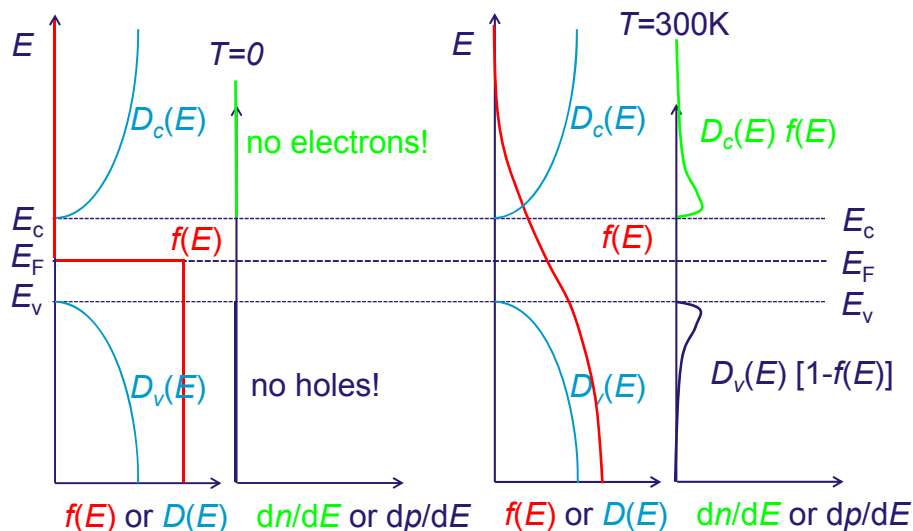
$$D_v(E) \approx 4\pi/h^3 (2m_{p^*})^{3/2} \sqrt{E_v - E} \quad (\text{for } E < E_v)$$

$$D = 0 \text{ for } E_v < E < E_c \text{ (forbidden: band-gap)}$$

and  $f(E, T) = 1 / [1 + \exp((E - E_F)/(k_B T))] \approx \exp[-(E - E_F)/(k_B T)]$

is **Fermi-Dirac distribution function** for electrons at  $E - E_F \gg 2k_B T$

## charge carrier density, DOS & Fermi distribution



## charge carrier density, DOS & Fermi distribution

insertion of Fermi-Dirac approximation yields:

$$1. \text{ electron density: } n \approx \underbrace{2 (2\pi m_{e^*} k_B T / h^2)^{3/2}}_{N_c^*} \exp [-(E_c - E_F) / (k_B T)]$$

$$2. \text{ hole density: } p \approx \underbrace{2 (2\pi m_{h^*} k_B T / h^2)^{3/2}}_{N_v^*} \exp [(E_v - E_F) / (k_B T)]$$

$$\begin{aligned} \text{note: } n \cdot p &= 4 (2\pi k_B T / h^2)^3 (m_{e^*} m_{h^*})^{3/2} \exp [-(E_c - E_v) / (k_B T)] \\ &= \underbrace{N_c^* N_v^*}_{N_c^* N_v^*} \exp [-E_g / (k_B T)] \end{aligned}$$

## intrinsic semiconductors

for intrinsic semiconductors: electron density= hole density,  
i.e.

$$n = p = n_i = 2 (2\pi k_B T / h^2)^{3/2} (m_{e^*} m_{h^*})^{3/4} \exp [-E_g / (2k_B T)]$$

insert numbers for Si at room temperature (300K):

$$E_g = 1.12 \text{ eV}, m_e \approx m_h \approx m_0 = 9.1095 \times 10^{-31} \text{ kg}$$

$$n \approx 1 \times 10^{16} \text{ m}^{-3} = 1 \times 10^{10} \text{ cm}^{-3}$$

Compare to atomic density of Si:

8 atoms per unit cell with  $a = 0.357 \text{ nm}$ :

$$n_{\text{atoms}} = 8/a^3 \approx 5 \times 10^{28} \text{ m}^{-3} = 5 \times 10^{22} \text{ cm}^{-3}$$

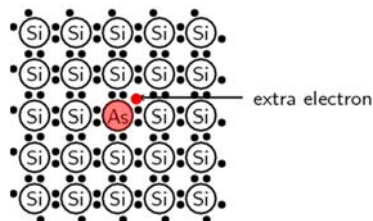
This means there is only 1 electron for  $5 \times 10^{12}$  Si atoms!

So, doping at the ppm level can change this dramatically!

## doping: electrically active point defects

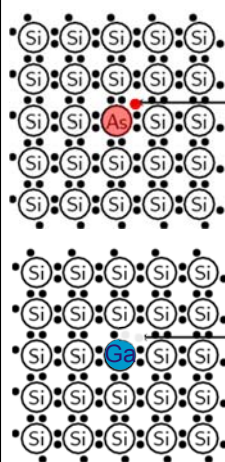
foreign atoms (impurities or intentionally implanted ions) act as dopants if they are

- integrated **substitutionally** into the host lattice on lattice sites so that they
- provide either an electron or a hole (i.e., they are **electrically active**)



an As atom has 5 valence electrons and needs only 4 for bonding in the diamond lattice, so there is **1 free electron**, which **increases  $n$  dramatically**

## donators vs. acceptors; flat vs. deep levels

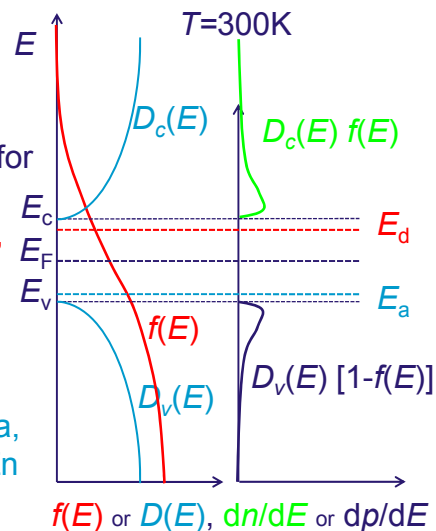


extra electron

donators for  **$n$ -doping:**  
(N), P, As,

acceptors for  **$p$ -doping:**  
B, (Al), Ga,  
Be, Mg, Zn

extra hole



**intrinsic, for high,**

**saturation and extrinsic range medium, low temperatures**

no intentional doping

all dopants have given away their charges

donor electrons ( $e^-$ ) or acceptor holes ( $p^+$ ) partially active

$$n=p=n_i$$

$$n=N_d \text{ (or } p=N_a)$$

$$n=\sqrt{\frac{1}{2}N_c^*N_d} \exp [-(E_c-E_d)/(2k_B T)] \text{ with } N_d=N_d^++N_d^0$$

$$\sigma_i=en_i(\mu_e+\mu_p)$$

$$\sigma=eN_d\mu_e \text{ (or } \sigma=eN_a\mu_p)$$

$$\sigma \propto \exp [-(E_c-E_d)/(2k_B T)]$$

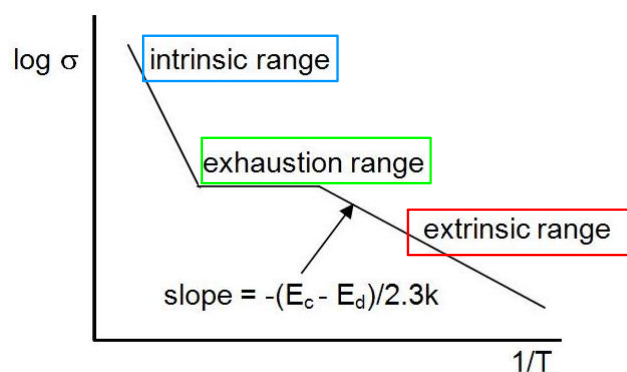
$$n_i = \sqrt{N_c^*N_v^*} \exp [-E_g/(2k_B T)]$$

$$E_{F,i} = \frac{1}{2}(E_c+E_v) + \frac{3}{4}k_B T \ln(m_h^*/m_e^*)$$

$$E_F = \frac{1}{2}(E_c+E_v) + \frac{3}{4}k_B T \ln(m_h^*/m_e^*) + k_B T \operatorname{arsinh} [N_d/(2n_i)]$$

$$E_F = \frac{1}{2}(E_c+E_d) - \frac{1}{2}k_B T \ln(2N_c/N_d)$$

## conductivity, part 2: summary for n-doping



for p-doping:  $(E_a-E_v)$  instead of  $(E_c-E_d)$