Data Provided: None

Autumn Semester 2012-13 (2.0 hours)

EEE6440 Advanced Signal Processing

Solutions:

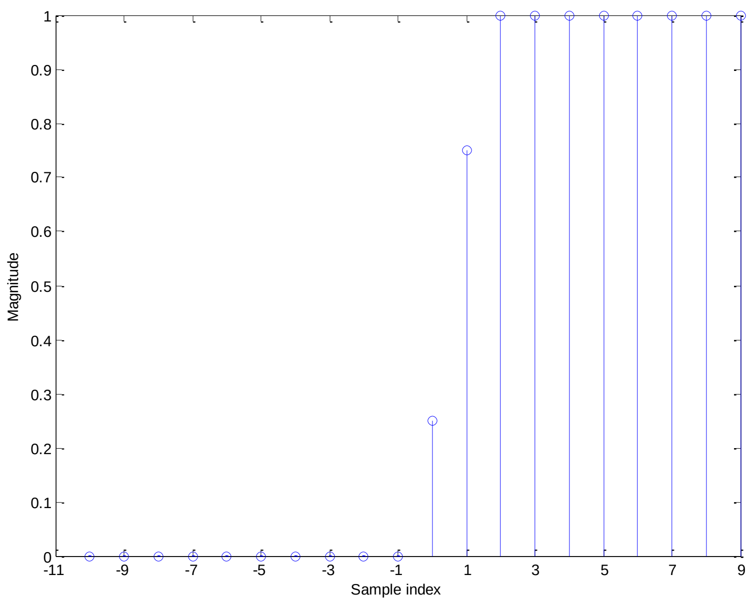
1.

a. Impulse response:

$h(n) = \{ \frac{1}{4}\ \frac{1}{2}\ \frac{1}{4} \}$ the second element is at n=0.

Step response:

Convolve the h(n) with step function u(n). In other words, taking the discrete integral of h(n). Results in { ... 0, 1/4, 3/4, 1, ...}



(2)

b.

Frequency response:

$h(-1) = \frac{1}{4}$, $h(0)= \frac{1}{2}$, $h(1)=\frac{1}{4}$

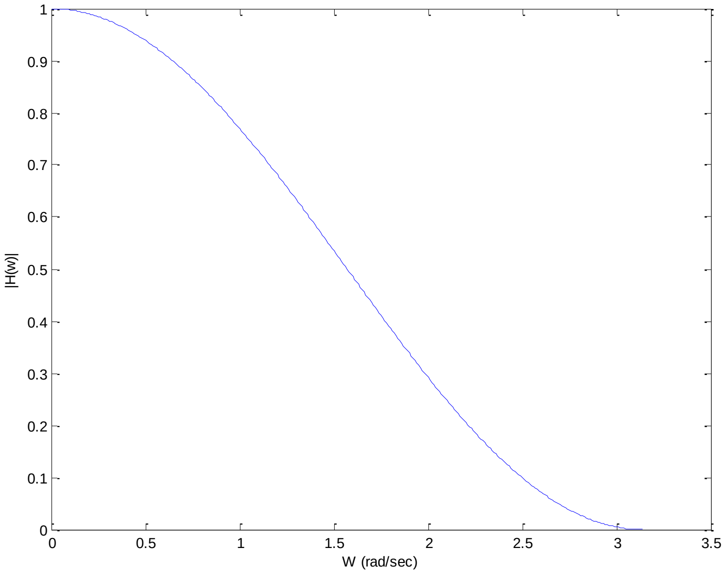
taking the z-transform

$h(z) = \frac{1}{4} (z^{-1} + 2 + z^{1})$

$z= e^{-j\omega}$,

$H(j\omega) = \frac{1}{4} (e^{-j\omega} + 2 + e^{j\omega}) = \frac{1}{4} (2+2 \cos \omega)$ **(3)**

$|H(j\omega)| = |(1+\cos\omega)/2|$

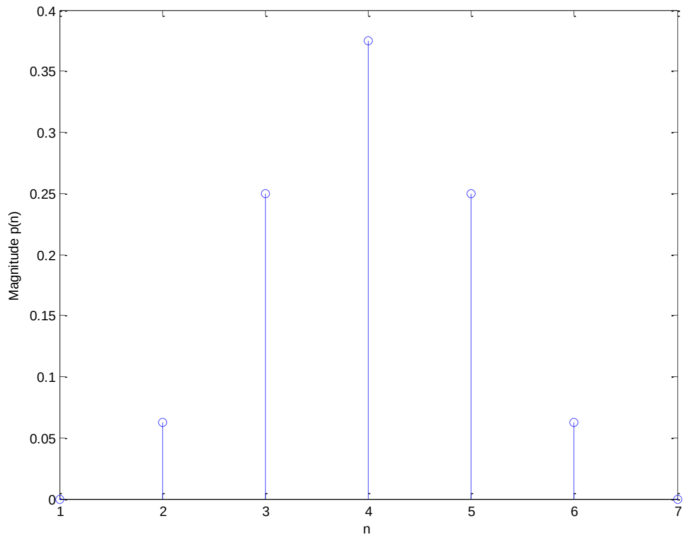


c. Convolve h(n)={ ¼ , ½ , ¼ } with itself.

p(n)= h(n) * h(n)

= { ¼ , ½ , ¼ } * { ¼ , ½ , ¼ }

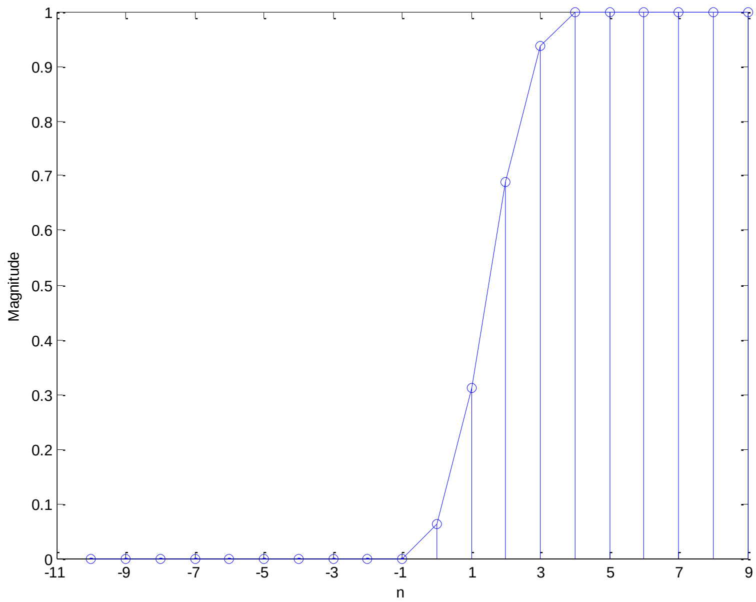
= { 1/16 , ¼ , 3/8, ¼ , 1/16}



(2)

d Time domain properties: **(3)**

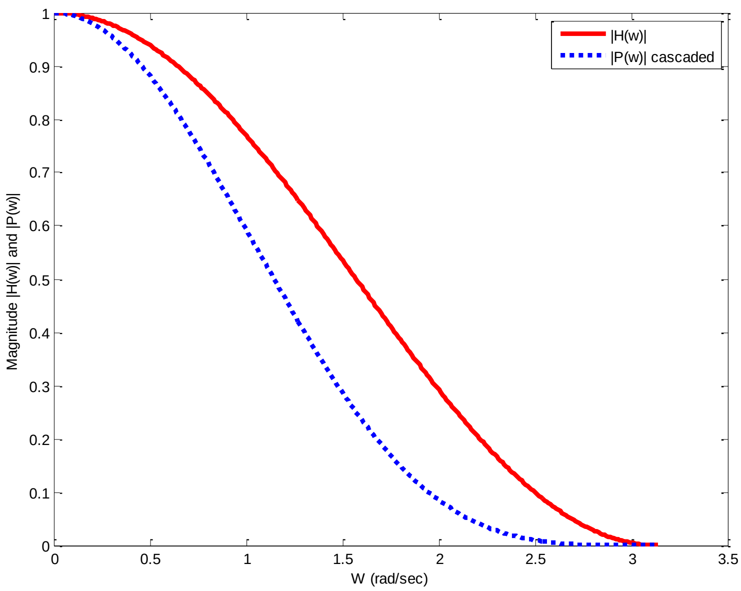
The step response is as follows:



Smooth rise. Since the kernel is larger, more emphasis is on centre data points in the filter kernel. Therefore sharp changes are preserved, while smoothing out noise.

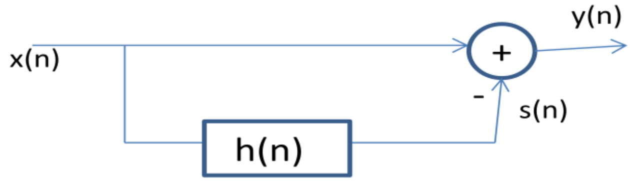
Frequency-domain performance

P = H x H in Frequency domain



Faster transition and better high frequency attenuation

e.

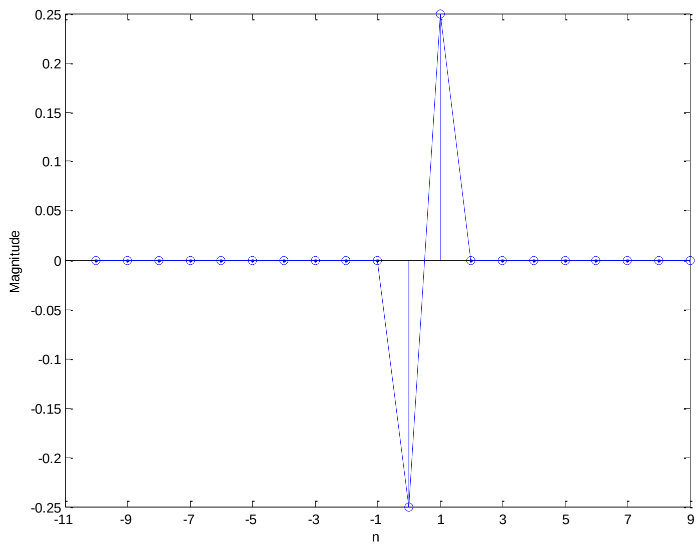


$y(n) = x(n) - h(n) * x(n)$

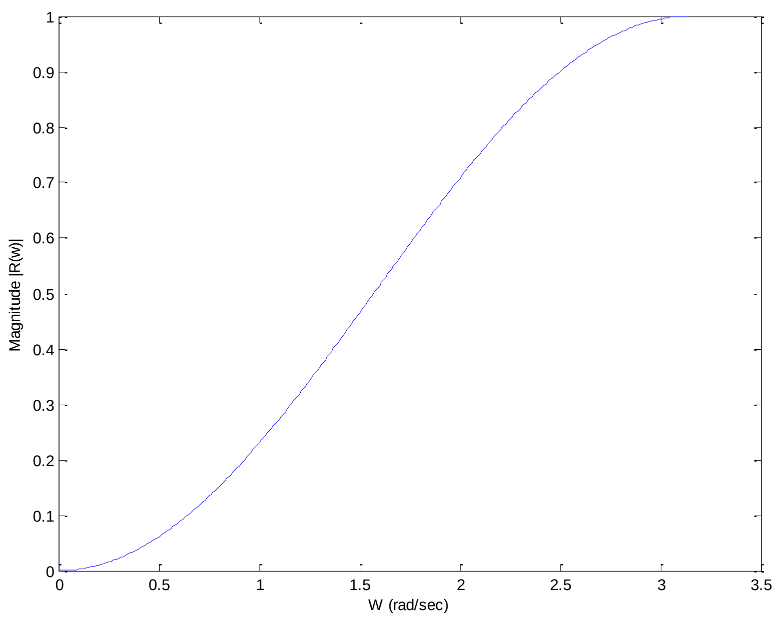
$\quad = (\delta(n) - h(n)) * x(n)$

Therefore $r(n) = (\delta(n) - h(n)) = \{ -\frac{1}{4} \;\; \frac{1}{2} \;\; -\frac{1}{4} \}$ **(2)**

f. Time domain – Step response



Frequency domain $|1 - H(w)|$



(2)

g A High pass filter – Captures high frequency components **(1)**

2.

a. The basis functions are

F1= (64 64 64 64)/128

F2=(84 35 -35 -84)/128

F3=(64 -64 -64 64)/128

F4=(35 -84 84 -35))/128

(2)

b. Show that

F1. F2 =0

F1.F3=0

F1.F4=0

F2.F3=0

F2.F4=0

F3.F4=0

F1.F1=1

F2.F2=1

F3.F3=1

F4.F4=1

(2 marks)

Therefore H is orthogonal.

Therefore inverse is H^t

(1 mark)

$$H^t = \frac{1}{128}\begin{bmatrix} 64 & 84 & 64 & 35 \\ 64 & 35 & -64 & -84 \\ 64 & -35 & -64 & 84 \\ 64 & -84 & 64 & -35 \end{bmatrix}$$

(1 mark) **(4)**

c. y_0=(x0+x1+x2+x3)*64/128

mean(x0+x1+x2+x3)= (x0+x1+x2+x3)/4

= (y0*128/64) / (4)

= y0 / 2

(2)

d. Divide the signal into 4 point segments **(5)**

For each segment,

{

Do the forward transform Y=TX

Keep y0

For y1, y2 and y3   keep the value only if they are greater than a threshold.

Otherwise set to 0.

Take the inverse transform of the new transform coefficients

Denoised X= KY , where K is the inverse transform matrix

}

e. Use as a separable transform

Divide data into 4 by 4 blocks

For each block

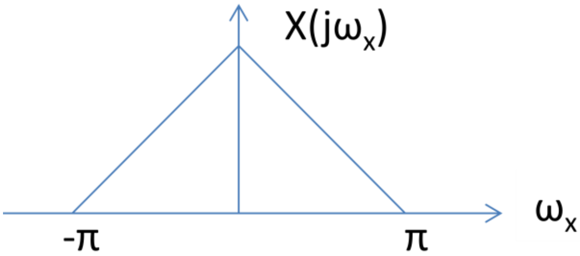
{

Apply the transform on one direction

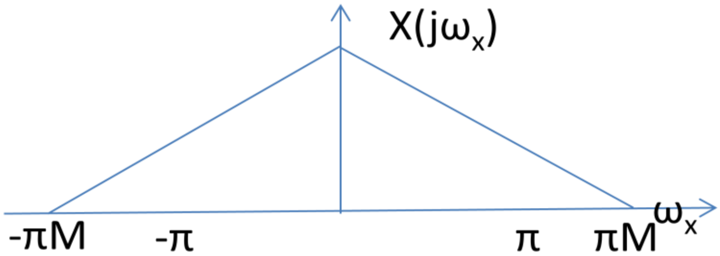
Then apply the transform in the other direction

} **(2)**

3. **a.** They are used as anti-aliasing filters.

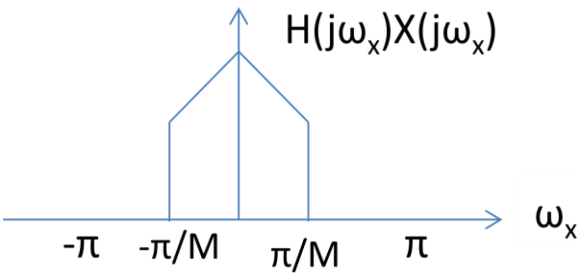


When downsampled by M , the spectrum will be spread to cause aliasing.



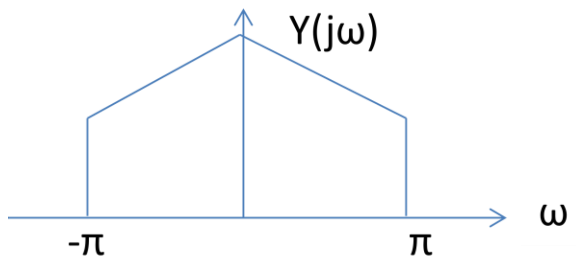
By choosing a low pass filter to restrict the signal frequency content to less than pi/M bandwidth, can avoid aliasing when the sample rate is decimated by a factor of M.

The bandwidth is limited by the low pass filtering in anti aliasing

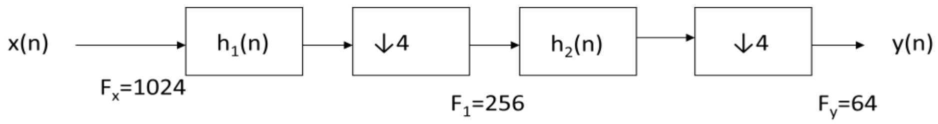


When downsampled aliasing is avoided as no frequencies higher than pi/M are presented.

(4)



b.



Passband deviation: 0.01dB → 0.00115

Stopband attenuation: 80dB → 0.0001

For both filters we choose

δ_p=0.00115/2 =0.00058

δ_s=0.0001

Filter length given by $N \approx \frac{-10\log(\delta_p\,\delta_s)-13}{14.6(\Delta f)} + 1$

$$N \approx \frac{-10\log(0.0005 \text{ x } 0.0001) - 13}{14.6(\Delta f)} + 1$$

$$N \approx \frac{4.066}{(\Delta f)} + 1$$

For h_2:

Passband 0 - 30 Hz

Stopband 32 - 64 Hz

Transition band 30Hz – 32Hz

Normalised transition bandwidth (32-30)/64 = 2/64

Therefore $N_2 \approx \frac{4.066}{\left(\frac{2}{64}\right)} + 1 = 131$

For h_1:

Passband 0 - 30 Hz

Stopband (256-32) -256 Hz = 224-256

Transition band 30Hz – 224Hz

Normalised transition bandwidth (224-30)/1024 = 194/1024 **(7)**

Therefore $N_1 \approx \frac{4.066}{\left(\frac{194}{1024}\right)} + 1 = 23$

c MPS = $\sum_{i=1}^{2} F_i N_i$ = 64x131 + 256x23 = 14 272

N is inversely proportion to Δf. If a single-stage was used Δf would have been (32-30)/1024. To make this value larger, we need to make the numerator bigger and the denominator smaller. This can be achieved by factoring F into a product of several smaller sampling rates. For each of the early stage filters, the transition bandwidth is large. This results in smaller N, hence fewer multiplications and low complexity. **(4)**

2013-2014 EEE6440 Advanced Signal Processing Solutions for Part B

Part B

Q4 a.

i)
Mean: (1.3+1.6+1.8+2.7+0.6)/5=1.6
(1 mark)
Variance: ((1.3-1.6)^2+(1.6-1.6)^2+(1.8-1.6)^2+(2.7-1.6)^2+(0.6-1.6)^2)/5=0.468
(1 mark)
Mean-square: ((1.3)^2+(1.6)^2+(1.8)^2+(2.7)^2+(0.6)^2)/5=3.028
(1 mark)
ii)
The variance $\sigma_x^2(n)$, mean-square E[x^2(n)] and the mean $m_x(n)$:

$$\sigma_x^2(n) = E[(x(n) - m_x(n))^2]$$
$$= E[x^2(n) - x(n)m_x(n) - x(n)m_x(n) + m_x^2(n)]$$
$$= E[x^2(n)] - 2E[x(n)]m_x(n) + m_x^2(n)$$
$$= E[x^2(n)] - 2m_x^2(n) + m_x^2(n) = E[x^2(n)] - m_x^2(n)$$

(2 marks)
3.028-1.6^2=0.468, which verifies the above general result.
(1 mark)

Q4 b.

For cosine wave input, the dynamic range R_D of the quantiser can be calculated from the equation in Section 7.5.2 since sine wave and cosine wave have the same power given the same amplitude.
Then, for a 12-bit A/D converter (M=12):

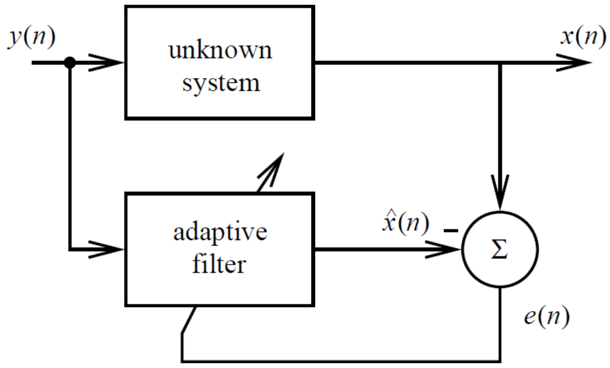
R_D=1.76+6M dB=1.76+8*12=97.76dB,

(3 marks)

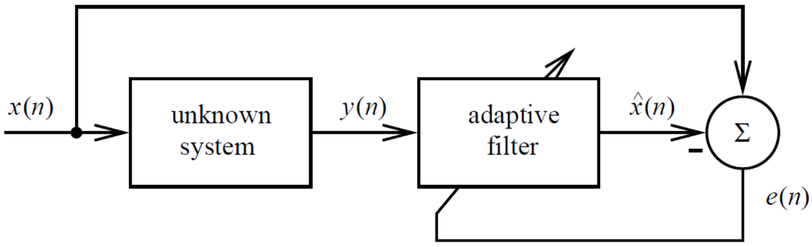
Q4 c.

The three modes of operation:

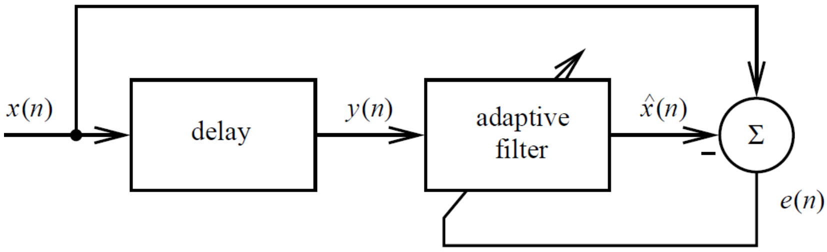
1) Direct system modelling which is typified by the application to echo cancellation where the echoes in the unknown system are duplicated in the adaptive filter and then cancelled in the summer.



2) Inverse system modelling whichis what is normally implied in the communications channel equalisation application, to overcome signal distortion and bandlimiting in the transmission channel.

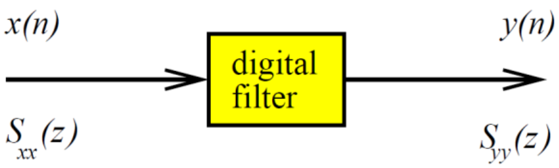


3) Linear prediction, for instance as used in autoregressive spectral analysis and in the linear predictive coding of speech.



Q5 a.

Suppose the z-transform of the filter is given by H(z).



Cross-correlation

$$\phi_{xy}(m) = E[\ x(n)\ y(n+m)\ ]$$

$$S_{xy}(z) = \sum_{m=-\infty}^{\infty} \phi_{xy}(m)\ z^{-m}$$

(1 mark)

$$S_{xy}(z) = \sum_{m=-\infty}^{+\infty} E[x(n)y(n+m)]z^{-m} = \sum_{m=-\infty}^{+\infty} E[x(n)[\sum_{i=-\infty}^{+\infty} h(i)x(n+m-i)]]z^{-m}$$

$$= \sum_{m=-\infty}^{+\infty} [\sum_{i=-\infty}^{+\infty} h(i)E[x(n)x(n+m-i)]]z^{-m}$$

$$= \sum_{m=-\infty}^{+\infty} [\sum_{i=-\infty}^{+\infty} h(i)\phi_{xx}(m-i)]z^{-(m-i)}z^{-i}$$

$$= \sum_{i=-\infty}^{+\infty} h(i)z^{-i} \sum_{m=-\infty}^{+\infty} \phi_{xx}(m-i)z^{-(m-i)}$$

(2 marks)

For each fixed i, we have $\sum_{m=-\infty}^{+\infty} \phi_{xx}(m-i)z^{-(m-i)} = S_{xx}(z)$

(1 mark)

So we have

$$S_{xy}(z) = \sum_{i=-\infty}^{+\infty} h(i)z^{-i} S_{xx}(z) = H(z)S_{xx}(z)$$

(1 mark)

Q5 b.

i) $H_1(z)=2-3z^{-1}$

z-transform of the autocorrelation at the output

$S_{y_1 y_1}(z) = H_1(z)\ H_1(z^{-1})\ \sigma_x^2$

$=(2-3z^{-1})(2-3z)*2=8-12z^{-1}-12z+18=-12z+26-12z^{-1}$

(2 marks)

Inverse z-transform by inspection to give autocorrelation sequence:

$$\phi_{y_1 y_1}(m) = Z^{-1}[\ S_{y_1 y_1}(z)\ ]$$

Autocorrelation sequence: -12 for m=-1, 26 for m=0, -12 for m=1 and zero for other values of m

(1 mark)

Q5 c.

i)

A Time Recursion

$$\mathbf{h}(n) = \mathbf{h}(n-1) - \mu\ \hat{\underline{\nabla}}(n-1)\ .$$

The Exact Gradient

$$\underline{\nabla}(n) = -\ 2\ \mathrm{E}[\ \mathbf{y}(k)\ (\ x(k) - \mathbf{h}^T(n)\ \mathbf{y}(k)\ )\ ]$$

$$= -\ 2\ \mathrm{E}[\ \mathbf{y}(k)\ e(k)\ ]$$

A Simple Estimate of the Gradient

$$\hat{\underline{\nabla}}(n) = -\ 2\ \mathbf{y}(n+1)\ e(n+1)$$

The Error

$$e(n+1) = x(n+1) - \mathbf{h}^T(n)\ \mathbf{y}(n+1)$$

(3 marks)

Then the updated equation of the LMS algorithm is given by
h(n)=**h**(n-1)+2μ**y**(n)e(n)
(1 mark)

ii)

e(11)=x(11)-$\mathbf{h}^T$(10)**y**(11)=-0.2-[1 6][0.3 0.25]^T
=-2
(2 marks)
The impulse response is then updated by
h(15)=**h**(14)+2μ**y**(15)e(15)
=[1 6]^T+0.4*(-2)*[0.3 0.25]^T
=[0.76 5.8]^T
(2 marks)

Q6 a.

The power spectral density function:

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} \phi_{xx}(m) \exp(-j\omega m\Delta t)$$

(1 mark)

Its inverse transform is given by:

$$\phi_{xx}(m) = \frac{\Delta t}{2\pi} \int_0^{2\pi/\Delta t} S_{xx}(\omega) \exp(\, j\omega m\Delta t)\, d\omega$$

(1 mark)

Then, for a zero mean stationary random process, its variance (the average power) is given by

$$\sigma_x^2 = \phi_{xx}(0)$$
$$= \frac{\Delta t}{2\pi} \int_0^{2\pi/\Delta t} S_{xx}(\omega)\, d\omega$$

(1 mark)
The average power is the integral of $S_{xx}(\omega)$ over the whole frequency range.

$S_{xx}(\omega)$ is the distribution of average power with respect to frequency - the POWER SPECTRAL DENSITY.
(1 mark)

Q6 b.

In many applications where the underlying processes are non-stationary, it is often more appropriate to minimise the effect of old data by progressively reducing the contribution to the squared error cost function. This is akin to assuming that the processes are stationary over short data records and can be realised by providing a `forgetting mechanism' using an exponentially weighted cost function as follows ($0<\alpha<=1$)

$$\xi(n)=\sum_{k=0}^{n}(x(k)-\hat{x}(k))^2\alpha^{n-k}$$

Q6 c.

$$e(n) = x(n) - \hat{x}(n)$$

The mean-square error (MSE) cost function

$$\xi(n) = E[\ e^2(n)\ ]$$

(1 mark)

$$\hat{x}(n) = \sum_{i=0}^{N-1} h_i\, y(n-i)$$

$$= [\ h_0\ h_1 \cdots h_{N-1}\ ]\begin{bmatrix} y(n) \\ y(n-1) \\ \cdot \\ \cdot \\ \cdot \\ y(n-N+1) \end{bmatrix}$$

$$= \mathbf{h}^T\ \mathbf{y}(n) = \mathbf{y}^T(n)\ \mathbf{h}$$

Differentiate

$$\frac{\partial \xi}{\partial h_j} = \frac{\partial}{\partial h_j} E[\ \{\ e^2(n)\ \}\ ]$$

$$= E[\ \frac{\partial}{\partial h_j}\ \{\ e^2(n)\ \}\ ]$$

$$= E[\ 2\ e(n)\ \frac{\partial e(n)}{\partial h_j}\ ]$$

$$= E[\ 2\ e(n)\ \frac{\partial}{\partial h_j}\ \{\ x(n) - \mathbf{h}^T\ \mathbf{y}(n)\ \}\ ]$$

$$= E[\ 2\ e(n)\ \frac{\partial}{\partial h_j}\ \{\ -h_j\ y(n-j)\ \}\ ]$$

$$= E[\ 2\ e(n)\ y(n-j)\ \}\ ]$$

$$= 0$$

for j=0, 1, …, N-1.

In vector form, the gradient is given by

$$\underline{\nabla} = -2\ E[\ \mathbf{y}(n)\ e(n)\ ]$$

$$= -2\ E[\ \mathbf{y}(n)\ (\ x(n) - \mathbf{y}^T(n)\ \mathbf{h}\ )\ ]$$

$$= -2\ E[\ \mathbf{y}(n)\ x(n)\ ] + 2\ E[\ \mathbf{y}(n)\mathbf{y}^T(n)\ ]\ \mathbf{h}$$

$$= -2\ \Phi_{yx} + 2\ \Phi_{yy}\ \mathbf{h}$$

$$= \underline{0}$$

where

Autocorrelation matrix

$$\Phi_{yy} = E[\ \mathbf{y}(n)\ \mathbf{y}^T(n)\ ]$$

Cross-correlation vector

$$\Phi_{yx} = E[\ \mathbf{y}(n)\ x(n)\ ]$$

Optimal Solution

$$\Phi_{yy}\ \mathbf{h}_{opt} = \Phi_{yx}$$

Alternative formulation

$$\mathbf{h}_{opt} = \Phi_{yy}^{-1}\ \Phi_{yx}$$

GCKA / WL