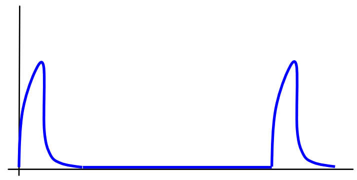
Question 1

a. *Disadvantages of Inductor-coupled Switch mode regulators*

Output is not electrically isolated from the input – under fault conditions the output could be directly coupled to the input supply.

Gain ratio of V_o/E can give a very unbalanced mark / space ratio for the switch, therefore the device is only just turned fully on before it is turned off again.

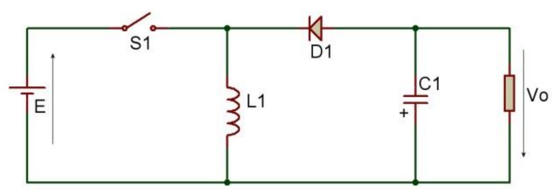


Disadvantages of Transformer-coupled Switch mode regulators

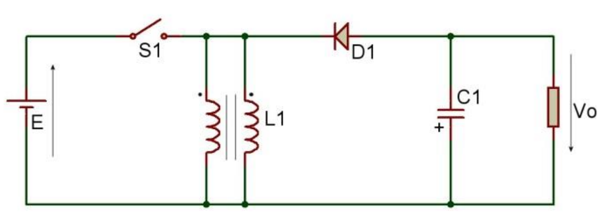
The transformer never has perfect coupling between primary and secondary, there is always some leakage inductance present. The stored energy in the leakage inductance can manifest itself as voltage spikes across the switch at turn off as the stored energy dissipates into the semiconductor switch

The transformer magnetizing inductance leads to extra current flow in the switch, over that required for the load.

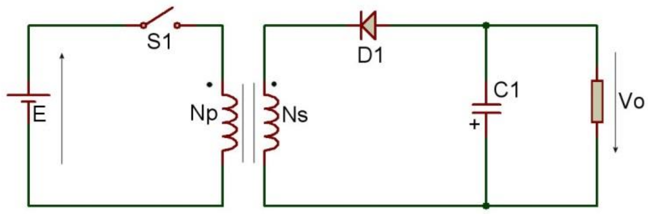
b. i) Initially starting with a Buck-Boost Converter:



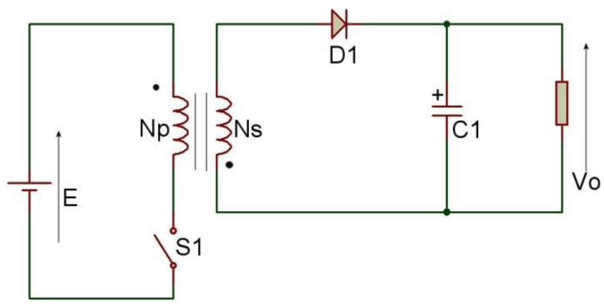
If the inductor L is produced by 2 identical windings in parallel on the same core we have:



Now, if the electrical link between the two windings is removed, electrical isolation of the input from the output is achieved, given that both of the windings are closely coupled together. Also, it is no longer necessary to have the same number of turns on each winding.



This may be re-drawn more conventionally:



ii) When switch T is on, current in the primary winding increases at a rate:

$$\frac{di_{\Pr i}}{dt} = \frac{E}{L_{\Pr i}}$$

When switch T is off the current in the secondary winding decreases at a rate:

$$\frac{di_{Sec}}{dt} = \frac{V_o}{L_{Sec}}$$

Due to the transformer coupling, the turns ratio and current transformation can be determined from:

$$N^2 = \frac{L_{\Pr i}}{L_{Sec}} \qquad V_{Sec} = \frac{V_{\Pr i}}{N} \qquad \text{and} \qquad I_{Sec} = N \cdot I_{\Pr i}$$

Equating the rate of change of winding currents:

$$\frac{N \cdot E}{L_{\Pr i}} \cdot t_{on} = \frac{N^2 \cdot V_o}{L_{\Pr i}} \cdot \left(T - t_{on}\right)$$

Cancellation and rearranging:

$$\frac{E}{V_o} = N \cdot \frac{(T - t_{on})}{t_{on}}$$

Subbing for $t_{on} = \delta T$ and inverting to get the converter gain:

$$\frac{V_o}{E} = \frac{1}{N} \cdot \frac{\delta}{1-\delta}$$

c. With a duty cycle of ~0.5 unity gain is achievable, and so N is approximately:

$(\sqrt{2} \times 230)/5$ = or 65:1.

$V_o' = (\sqrt{2} \times 230) / 65 = 5.004\text{V}$ so $\delta = 0.4998$. (0.5 allowable)

f = 125kHz, P_{out} = 80W @ 5V, so $\bar{I}_0$ = 16A.

The limit of continuous conduction is when the bottom of the ripple current waveform touches zero at the end of each cycle, therefore:

$$\bar{I}_{Sec} - \frac{\Delta i_{Sec}}{2} \geq 0$$

Now given that $\bar{I}_o$ is average output current, we have:

$$\bar{I}_o = \bar{I}_{Sec} \frac{(T - t_{on})}{T} \qquad \therefore \qquad \bar{I}_o = \bar{I}_{Sec}(1-\delta)$$

therefore:

$$\bar{I}_{Sec} = \frac{\bar{I}_o}{(1-\delta)}$$

so:

$$\frac{\bar{I}_o}{(1-\delta)} \geq \frac{V_o.(T - t_{on})}{2L_{Sec}}$$

Giving:

$$\bar{I}_o \geq \frac{E \cdot N \cdot \delta \cdot T \cdot (1-\delta)}{2L_{\mathrm{Pr}i}}$$

Considering 16A load at 125kHz with 50% duty cycle and rectified mains input via a 65:1 transformer, ***L* > 1.32mH**

Considering charge transferred into and out of the capacitor being equal:

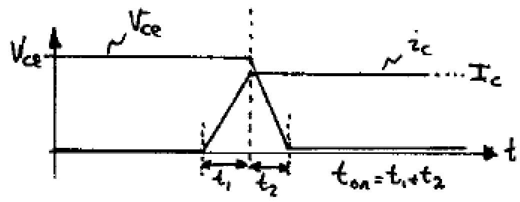
$$C = \frac{\Delta Q_c}{\Delta v_c} = \frac{\bar{I}_o \delta T}{\Delta v_c}$$

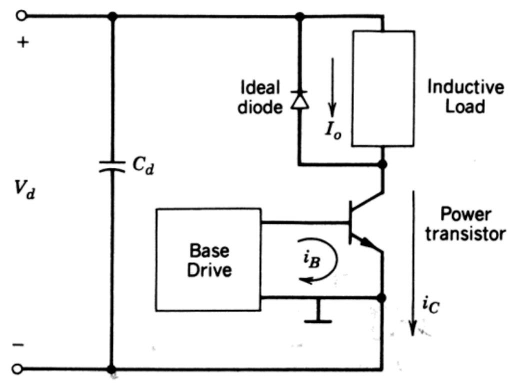
For 5% voltage ripple = 0.25V, $C = \frac{\delta T . \bar{I}_o}{\Delta v_c}$ therefore **C > 256μF**

d. If the converter was re-designed to operate at 18kHz, one of the main concerns would be audible noise generated from the windings of wound components. The switching frequency, and hence ripple current and therefore forces on windings, would be within the audio range (< 20kHz). The inductance would also need to be ~10mH so would be a very large transformer for 80W.

Question 2

a. Here, the inductive load keeps the current flowing, via the ideal freewheel diode.





The diode cannot start conducting when the BJT switches off until the voltage across the BJT has risen above V_d and forward biased the diode. Until the diode conducts, it cannot provide an alternate path for the load current, so the BJT must carry the full load current whilst the voltage across the BJT rises to forward bias the diode at switch off of the BJT. Similarly, at turn-on of the BJT, the diode cannot switch off until the current flowing through it falls to zero, therefore the voltage across the BJT is clamped at the power supply rail until the current through the BJT rises to the load current level.

Here then, the energy per turn on switching event can be calculated from the following, assuming a linear characteristic:

$$E_{on} = V_{CE} . \int_0^{t_1} \frac{I_C . t}{t_1} dt + I_C . \int_0^{t_2} \left(V_{CE} - \frac{V_{CE} . t}{t_2} \right) dt$$

$$E_{on} = \frac{V_{CE} . I_C}{2} . (t_1 + t_2)$$

$$E_{on} = \frac{V_{CE} . I_C}{2} . t_{on}$$

Now when operating at switching frequency f:-

$$P_{AVE\,(on)} = \frac{V_{CE} . I_C . t_{on} . f}{2}$$

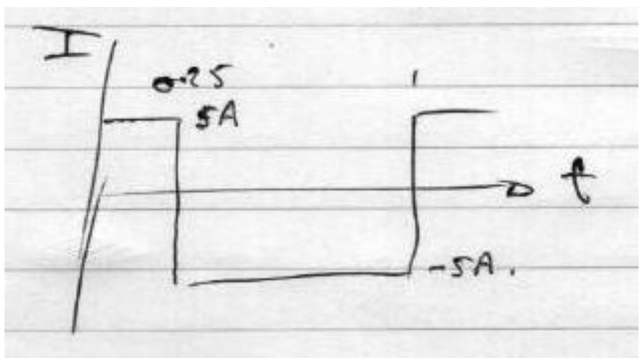
similarly, $$P_{AVE\,(off)} = \frac{V_{CE}.I_C.t_{off}.f}{2}$$

Average switching loss becomes:

$$P_{AVE\,(switching)} = \frac{V_{CE}.I_C.(t_{on}+t_{off}).f}{2}$$

b. Top left – 25%, Top right – 75%, Bottom left – 75%, Bottom right – 25%

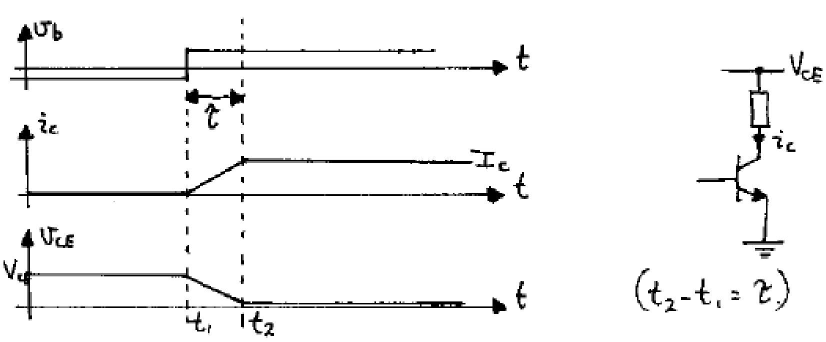
The current in the resistive load looks like:



If this is a purely resistive load, the diodes don't conduct therefore their loss can be ignored. Assuming negligible loss in the switches (V_{CEsat} << 40V), the load power can be calculated by I^2R or V^2/R to be approximately 200W since the positive and negative parts of the waveform are all positive when squared.

c. **Switching Loss:**

Purely resistive load:



Assuming a linear approximation to the actual waveforms, it is possible to calculate the switching loss for a given switching frequency:

$v_{CE} = V_{CE} - V_{CE} \times t/\tau$ and $i_C = I_C \times t/\tau$

Now power at time t,

$$P(t) = I(t).V(t) = I_C.V_{CE}.\left(\frac{t}{\tau} - \frac{t^2}{\tau^2}\right)$$

And Energy dissipated per cycle by this event is:

$$\int_0^{\tau} I_C.V_{CE}.\left(\frac{t}{\tau} - \frac{t^2}{\tau^2}\right)dt$$

Now if we are switching at a frequency, f:-

$$P_{AVE(on)} = \frac{V_{CE}.I_C.f.t_{on}}{6}$$

using similar approximations:

$$P_{AVE(off)} = \frac{V_{CE}.I_C.f.t_{off}}{6}$$

Therefore total average switching loss for a resistive load is given by:

$$P_{AVE(switching)} = \frac{V_{CE}.I_C.f.(t_{on} + t_{off})}{6}$$

and can be calculated to be 0.833W, giving a total switching loss into the heatsink of 3.33W.

Since no base circuit information is given, base losses are neglected. Total conduction loss for 2 devices on at any one time is Ic*$V_{CE(sat)}$ = 2*5*0.85 = 8.5W total The total loss is therefore 11.83W, for a heatsink rise in temperature of 35^{o}C gives a thermal resistance of <2.96^{o}C/W.

d. i) If the load is now 5A constant current (i.e. inductive), the load sees a voltage of 40V for 25% duty and -40 for 75% duty, so the instantaneous power of the load is 200W for 25% duty and -200W for 75% duty. The average power dissipated in the load is therefore 100W, down from 200W previously.

ii) Further, 2 diodes now conduct the current during the 25% duty and 2 switches during 75%:

Diodes: 2 x 0.25 x (5 x 0.3 + 5^2 x 0.01) = 0.875W

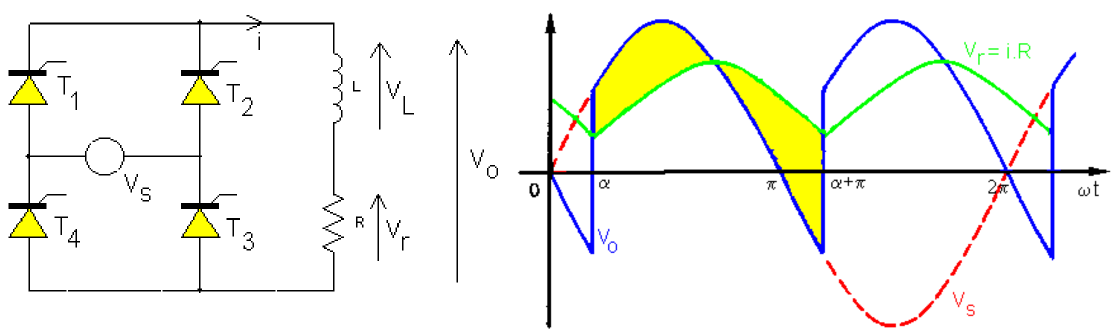
FETs: (2 x 0.75 x 5 x 0.85) + (2 x 40 x 5 x 50,000 x (200n + 300n) / 2) = 6.375 + 5

Total power loss = 12.25W

Question 3.

(a) A semi-converter utilises a mixture of diodes and thyristors in a phase controlled rectifier, whereas the full-converter uses only thyristors. The use of diodes in a semi-converter provides a free-wheel path for the output current during some of period of the input waveform, and prevents the output voltage going negative instantaneously, whereas the full converter output voltage may go negative instantaneously, as the thyristors have to conduct until the current in them falls below their holding current level (essentially zero). This implies the instantaneous output power in a full converter may be negative. The output current for a semi-converter is essentially continuous, as the diodes provide a freewheel path, whereas in a full converter, the output current is only continuous if the converter firing angle is less than the load angle, otherwise it is discontinuous.

(b) This is continuous output current operation:



Here, for example, if T1 and T3 are triggered at ωt = α, the current in the devices builds up, and the output voltage follows the input sinusoid. The voltage across the resistive part of the load is a scaled version of the current (Ohms law) and the instantaneous voltage across the inductive part of the load control the rate of change of current in the load. As the supply voltage goes negative at ωt = π, the current still flows in the devices, therefore they cannot turn off, and the instantaneous output voltage follows the supply negative. This gives an increasingly negative voltage across the inductance of the load, leading to a fall in output current. At ωt = α+π, T2 and T4 in the diagram are fired, and it may be assumed that the current flowing through T1 and T3 commutates instantaneously to T2 and T4. The output voltage thus goes positive again, and T1 and T3 turn off, as the current through them is zero and they are reverse biased.

(c) i) With a full converter, the average output voltage is given by:

$$V_o = \frac{2\sqrt{2}V_s}{\pi}\cos(\alpha)$$

Therefore for a 80V average from a 230V rms input, **α=1.19rad (67.27°)**

ii) An average output voltage of 80V across a 4Ω load gives a load current of **20A**, as the average output voltage only appears across the resistive part of the load. Given the load inductance is 50mH, the load angle **$\phi = \tan^{-1}(\omega L/R)$ = 1.32rad (75.7^o)**.

Here, $\alpha < \phi$ therefore the current in the output is continuous.

Question 4.

(a) From: $\begin{array}{l} i_L^\bullet = \dfrac{dv_i}{L} - \dfrac{Ri_L}{L} + \dfrac{CRv_o^\bullet}{L} \\ v_o^\bullet = \dfrac{i_L}{C} - \dfrac{v_o}{CR} \end{array}$ if we perturb the duty cycle, the output voltage and the inductor current we get $I_L^\bullet + \tilde{i}_L = \dfrac{(d+\tilde{d}).V_i}{L} - \dfrac{R}{L}.(I_L + \tilde{i}_L) + \dfrac{CR}{L}.(V_o^\bullet + \tilde{v}_o^\bullet)$

Now, splitting into steady state and transient terms, the transient terms give:

$$\therefore \quad \tilde{i}_L^\bullet = \frac{\tilde{d}V_i}{L} - \frac{R\tilde{i}_L}{L} + \frac{CR\tilde{v}_o^\bullet}{L}$$

Also, $V_o^\bullet + \tilde{v}_o^\bullet = \dfrac{I_L}{C} + \dfrac{\tilde{i}_L}{C} - \dfrac{V_o}{CR} - \dfrac{\tilde{v}_o}{CR}$, so extracting the transient terms from this gives: $\boxed{\tilde{v}_o^\bullet = \dfrac{\tilde{i}_L}{C} - \dfrac{\tilde{v}_o}{CR}}$ which may be transformed into the Laplace domain to give: $si_L = \dfrac{dv_i}{L} - \dfrac{Ri_L}{L} + \dfrac{sCRv_o}{L}$

and: $sv_o = \dfrac{i_L}{C} - \dfrac{v_o}{CR} \quad \therefore \quad v_o\left(s + \dfrac{1}{CR}\right) = \dfrac{i_L}{C}$

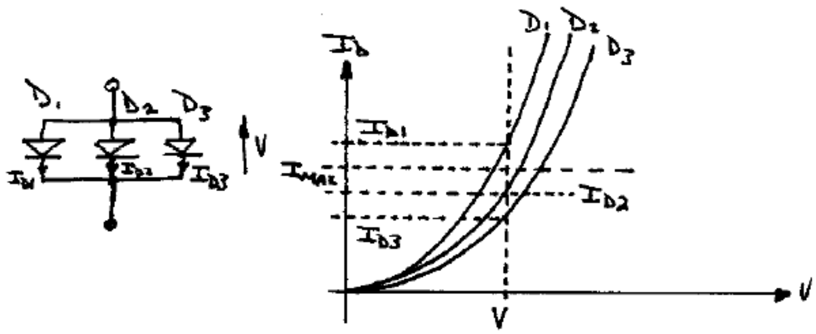
$$\left(sCv_o + \frac{v_o}{R}\right)(sL + R) = dv_i + sCRv_o$$

$$\therefore \quad v_o LC\left(s^2 + \frac{s}{CR} + \frac{1}{LC}\right) = dv_i$$

Giving what was required:

$$\boxed{\therefore \quad \frac{v_o}{d} = \frac{v_i}{LC\left(s^2 + \dfrac{s}{CR} + \dfrac{1}{LC}\right)}}$$

(b) If three diodes are hard wired in parallel, Kirchoff's law says they must have the same voltage drop across each of them. However, as diode characteristics are never identical, even if they are of the same type:



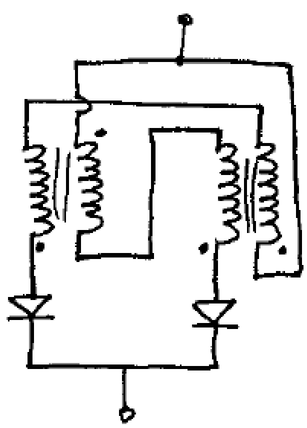
Parallel operation of devices may lead to different currents in each of the devices. This may lead to failure of diode D1 in this case. The simplest way to overcome this is to (one method required only)

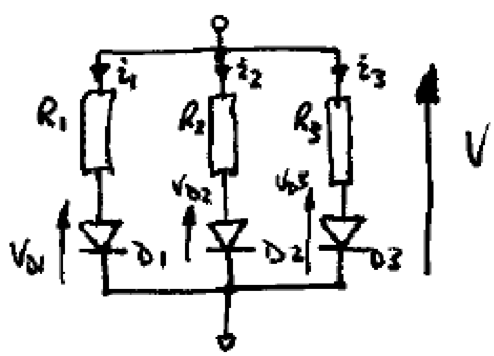
i) de-rate the diode by the use of a paralleling factor, where:

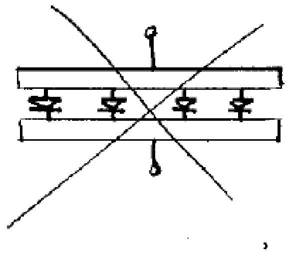
$$\frac{\textit{current in average diode}}{\textit{current in heavily loaded diode}} = \textit{paralleling factor}$$

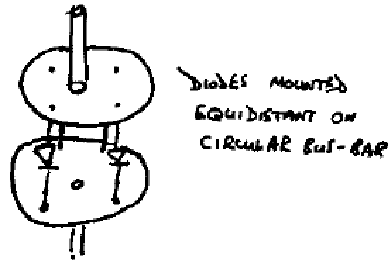
ii) All forward characteristics of a diode are dependent on temperature. Therefore all diodes should be mounted on a common heatsink to ensure a uniform temperature between the devices. Also, the path lengths in each parallel leg should be the same, to equalize bus-bar inductance. This helps prevent un-equal current sharing due to unequal inductive impedance in series with each diode.

iii) The use of coupled reactances or series resistors will ensure both dynamic and static current sharing.

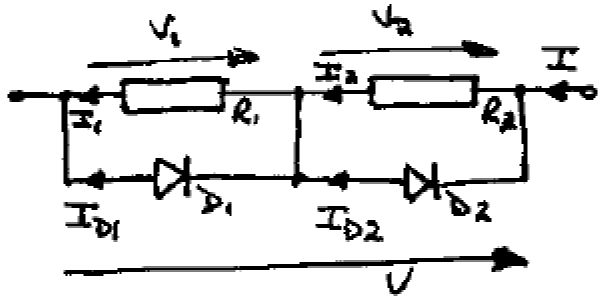








c. Diodes connected in series will carry the same reverse leakage current (Kirchoff). However, differences in device characteristics will then lead to uneven voltage sharing of the reverse voltage, to a point where one of the diodes may fail (possibly to a short circuit causing a chain failure!).



Max leakage current = 400μA, given maximum of 10% difference in shared reverse voltage (V_{max} reverse = 1000V), $\Delta V = 100V$, therefore:

$$R = \frac{\Delta V}{\Delta I_L} = \frac{100}{400 \times 10^{-6}} = 250k\Omega$$

Given a maximum voltage across one of the resistors of 550V, the power in the resistor is given by V^2/R = 1.21W, nominally 2W.