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DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2007-2008 (2 hours)

Electric and Magnetic Fields 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. **a.** Sketch the form of the field lines and equi-potential surfaces of an electric dipole **(2)**

b. A point charge of magnitude 10μC is located at the origin and a second point charge of magnitude 5μ C is located 1 metre to the right of the origin on the horizontal x-axis. Calculate the position on the x-axis at which the magnitude of the electric field is zero. A third charge, q_3 of magnitude 5μC is positioned 1 metre above the origin on the vertical y-axis. Calculate the magnitude and direction of the force acting on q_3. **(8)**

c. A *semi*-circular ring of charge of radius ***a*** is centred at the origin in the x-y plane as shown in Figure 1. If the total charge on the ring is ***Q***, determine an expression for the electric field at the origin.

An infinite wire is now placed in the x-y plane parallel to the y-axis and a distance 2***a*** from the origin. If the wire carries a charge q_l Coulombs per unit length, calculate the total field at the origin due to the combination the infinite wire and the semi-circular ring of charge.

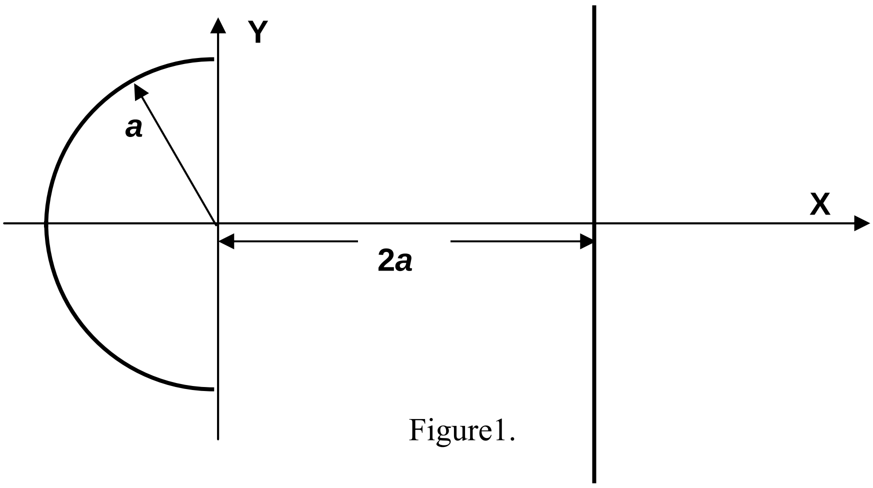


Figure1.

(10)

2. a. Use Gauss's law to derive an expression for the electric field at a distance *r* from an infinite sheet carrying a charge per unit area of q_S **(4)**

b. Two hollow spheres of radius 10cm and 20cm carry uniform charge distributions of -2μC and +3μC respectively as shown in Fig 2. Determine the magnitude of the electric field at each of the following radial distances from the origin: i) 5cm; ii) 15cm; iii) 25cm.

Sketch the form of the field lines both inside and outside the spheres

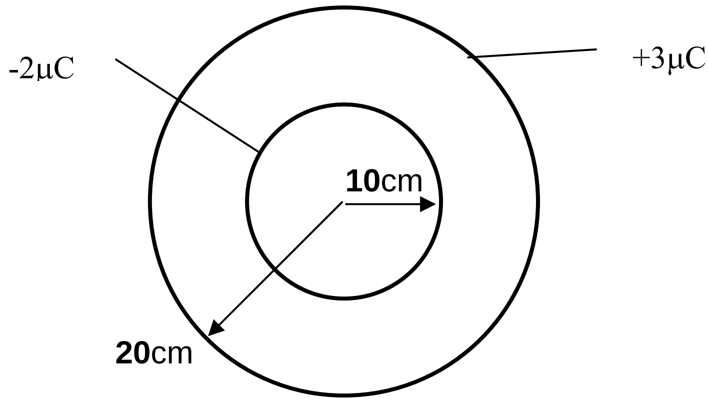


Figure 2 **(6)**

c. An air spaced parallel plate capacitor of plate area **A** and separation distance **d** is modified to include a slab of perfectly conducting material of thickness **t** and area **A,** as shown in Figure 3. The capacitor is charged to a potential **V** and the voltage source is then removed.

i) Sketch a diagram to show the charge distribution and field structure within the capapacitor

ii) Determine an expression for the capacitance of the structure, and check your answer by considering the case when **t=0**.

If the metal slab is removed

i) does the energy stored in the capacitor increase or decrease?
ii) does the voltage across the capacitor increase or decrease?

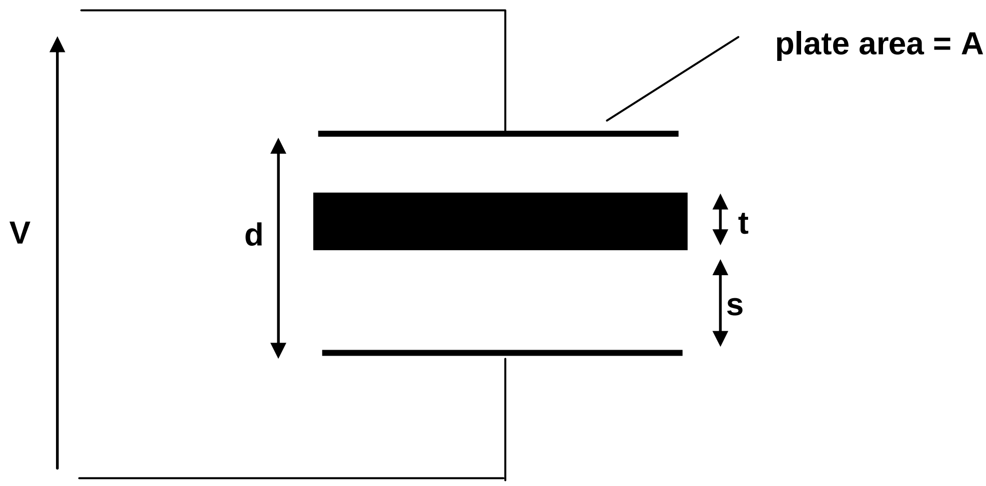


Figure 3 **(10)**

3. **a.** The magnetic flux density at a point a perpendicular distance ***x*** from the centre of a thin straight conductor of length L which is carrying a current I, is given by

$$B = \frac{\mu_0 I}{2\pi x}\left[\frac{1}{1+(2x/L)^2}\right]^{1/2}$$

Use this expression to deduce an expression for the B field at the centre of a square circuit of side L. **(6)**

b. Figure 4 shows part of a circuit in the form of a regular plane polygon of n sides carrying a current I. The distance from the centre of the polygon to the vertices is ***a***. Deduce an expression for the B field at the centre of the polygon. Use this result to find an expression for the B field at the centre of a circular loop.

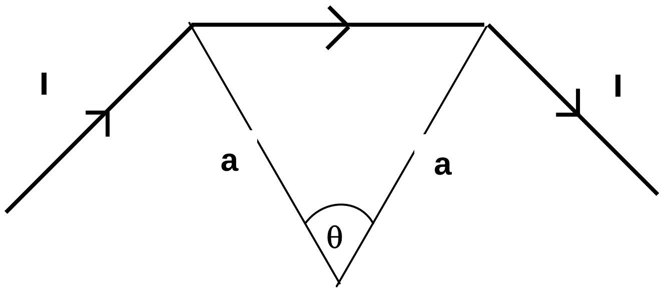


Figure 4 **(6)**

c. The circuit shown in Figure 5 consists of a square of side 2***a*** carrying a current I_s and a circular loop of radius b carrying a current I_c. The directions of the currents are defined in the diagram. Derive an expression for the magnetic field at the common centre of the two loops. If $b = a\sqrt{2}$ and $I_c = 1$, determine the value of I_s required to produce zero magnetic field at the common centre of the two loops.

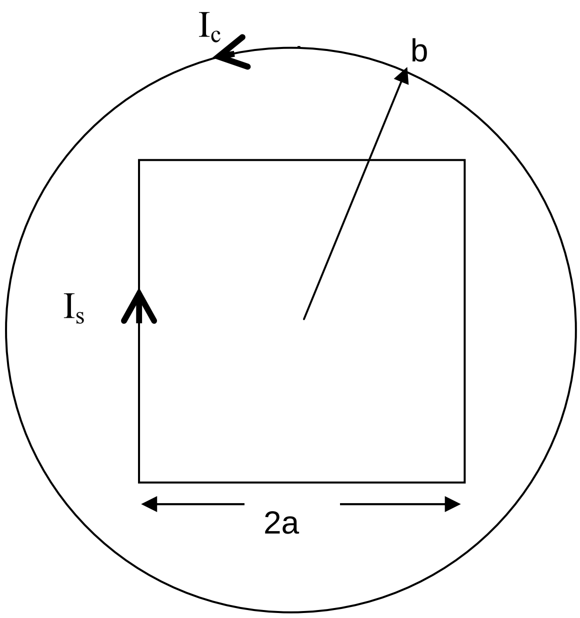


Figure 5

(8)

4. **a.** The magnetic field through the circular loop of radius ***a*** varies with time according to the equation $B = B_o(1 - e^{-\lambda t})$, as indicated in Figure 4. Derive an expression for the induced voltage in the loop. If ***a***=1cm, $\mathbf{B_o}$=500μT and $\lambda = 0.1$, calculate the induced voltage at t=20s.

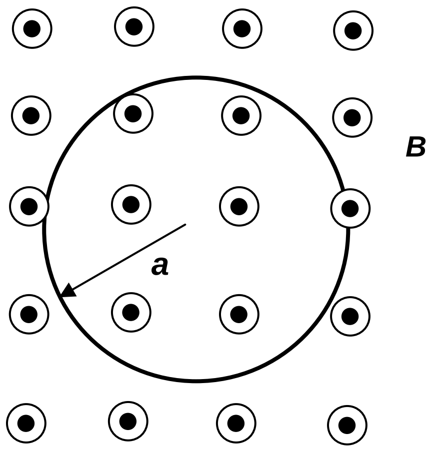


Figure 4

(8)

b. Use Ampere's law to derive expressions for the magnetic field inside *and* outside a circular wire of radius a carrying a uniformly distributed current I. Sketch the variation of the field as a function of distance from the centre of the wire.

(6)

c. A 1000 turns solenoid is 10cm long, 2cm in diameter and carries a current of 1A. Calculate

i) The magnetic field at the centre of the solenoid;
ii) The self inductance of the solenoid.

(6)

AT / JBW

UNIVERSITY OF SHEFFIELD

Department of Electronic and Electrical Engineering

EEE220 ELECTRIC AND MAGNETIC FIELDS FORMULA SHEET

ILF/AT/JLW 2007

	$\varepsilon_o = 8.854 \times 10^{-12}\ \text{Fm}^{-1}$	charge on electron = -1.6×10^{-19} C
	$\mu_o = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$	mass of electron = 9.1×10^{-31} kg

1. ELECTROSTATICS

Coulomb's Law

Force between two point charges, q_1 and q_2 has magnitude $F = \dfrac{q_1 q_2}{4\pi\varepsilon_o R^2}$ in direction along line joining charges. In vector notation $\underline{F} = \dfrac{q_1 q_2}{4\pi\varepsilon_o R^3}\,\underline{R}$ or $\underline{F} = \dfrac{q_1 q_2}{4\pi\varepsilon_o R^2}\,\hat{\underline{R}}$

Electric Field

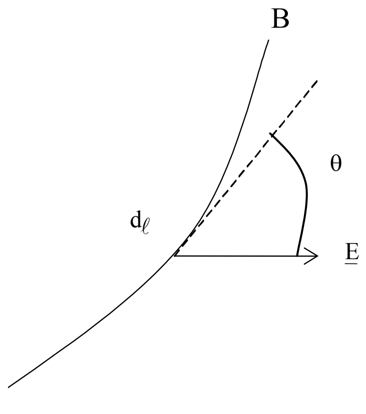
Defined by $\underline{E} = \dfrac{Q}{4\pi\varepsilon_o R^3}\,\underline{R}$, and then force is $\underline{F} = q\underline{E}$. In electrostatics we want to solve for $\underline{E}$.

Potential

Work done in moving q_1 from A to B is $W = q_1\left(\phi(A) - \phi(B)\right)$ where ϕ is potential.

Potential due to charge q is $\phi = \dfrac{q}{4\pi\varepsilon_o R}$, and ϕ and $\underline{E}$ are related by

$$\phi(B) - \phi(A) = -\int_A^B \underline{E}\cdot\underline{dl} = -\int_A^B E\cos\theta\, d\ell$$



$$\underline{E} = -\nabla\phi = \left(-\frac{d\phi}{dx},\ -\frac{d\phi}{dy},\ -\frac{d\phi}{dz}\right)$$

(a) **Gauss's Law**

Surface integral of $\underline{E}$ gives $\oint_s E\cos\theta \; da = \dfrac{Q}{\varepsilon_o}$, $Q =$ total charge enclosed by surface S.

(b) **Solving for $\underline{E}$**

Three methods possible.

(i) Use Coulomb's Law, summing all contributions with care about direction.

(ii) Calculate ϕ and then use $\underline{E} = \left(-\dfrac{d\phi}{dx}, -\dfrac{d\phi}{dy}, -\dfrac{d\phi}{dz} \right)$.

(iii) Use Gauss's Law - only works if symmetry can be employed to get $\underline{E}$ outside the integral.

(c) **Important Cases**

(i) Sheet of charge, $|\underline{E}| = \dfrac{q_s}{2\varepsilon_o}$, q_s is surface density, or charge per unit area.

(ii) Line of charge, $|\underline{E}| = \dfrac{q_\ell}{2\pi r\varepsilon_o}$, q_ℓ is charge per unit length.

(iii) Sphere of charge Q, $|\underline{E}| = \dfrac{Q}{4\pi\varepsilon_o r^2}$.

(d) **Capacitance**

Capacitance of two conductors is defined by $C = Q/V$. For parallel plate capacitor $C = \varepsilon A/d$, where $\varepsilon =$ permittivity of separating medium. Effect of dielectric medium is to increase the capacitance.

(e) **Energy**

Stored energy in capacitor is $\dfrac{1}{2}CV^2$. Energy density in electric fields is $\dfrac{1}{2}\varepsilon E^2$.

2. MAGNETIC FIELDS

(a) Force between two circuits

Force is given by Ampère's force law, but this is difficult to use. Introduce $\underline{B}$ field, and force in a circuit is $\underline{F} = \oint I \; \underline{dl} \times \underline{B}$.

(b) Biot-Savart Law

$\underline{B}$ field is given by $\underline{B} = \dfrac{\mu_o}{4\pi} \oint \dfrac{I \, \underline{dl} \times \underline{\hat{r}}}{r^2}$

Analytical results possible only for simple geometries.

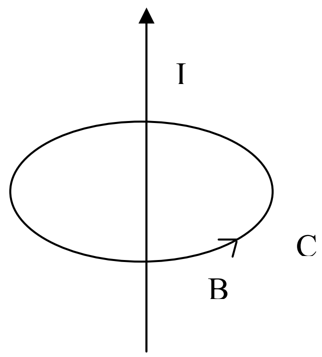
(c) Important cases of $\underline{B}$

(f) Infinitely long straight wire $B = \mu_o I/2\pi r$.

(ii) on axis of circular loop, $B = \mu_o Ia^2 / 2\left(a^2 + d^2\right)^{3/2}$.

(iii) Inside long straight solenoid $B = \mu_o nI$.

(d) Ampère's Law



$$\oint_C \underline{B} \cdot \underline{dl} = \oint_C B \cos\theta \, d\ell = \mu_o I$$

I is the current which threads the path of integration.

Direction given by right-hand rule

(e) Magnetic Flux

Defined by $\Phi = \int B \cos\theta \, da$, i.e. Φ is given by the integral over area of normal component of $\underline{B}$. For uniform *B*, Φ = *BA*, hence B is called magnetic flux density. For a closed surface of integration $\oint B \cos\theta \, da = 0$, which implies no magnetic poles.

3. MAGNETIC INDUCTION

(a) Faraday's Law

If flux linkages through a circuit change with time, magnitude of emf induced is $\varepsilon = \frac{d\Phi}{dt}$. Polarity of ε given by Lenz's Law, is such as to try to keep Φ constant.

(b) Self-inductance

Defined by $\varepsilon = L\frac{di}{dt}$ where L depends on geometry of circuit (and also any magnetic materials present). In a circuit L causes current to lag voltage.

Inductance of solenoid $= \frac{\mu_o N^2 A}{\ell}$, where N is the total number of turns, A is the cross-sectional area, and ℓ is the length of the solenoid.

(c) Magnetic Energy

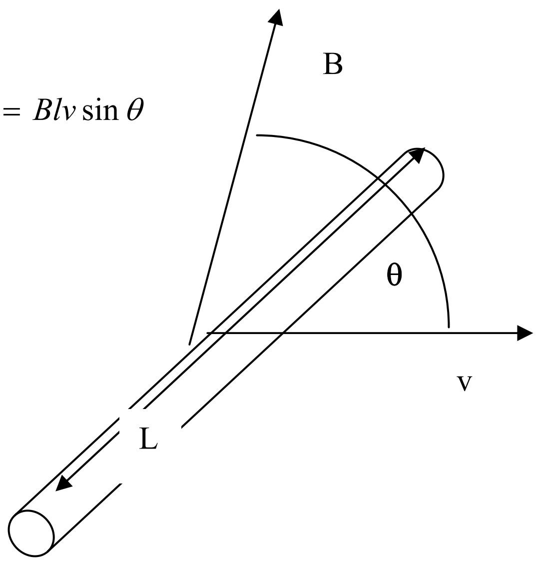
Energy stored in inductance is $\frac{1}{2}Li^2$. Energy per unit volume in magnetic fields is $\frac{B^2}{2\mu_o}$ or $\frac{B^2}{2\mu}$ if magnetic material of permeability μ is present.

(d) Mutual Inductance

Current change in one circuit induces emf in nearby circuit $\varepsilon = M\frac{di}{dt}$. M is coefficient of mutual inductance, depends on geometry and materials. M is reciprocal.

(e) EMF induced by Motion

EMF is generated by conductor moving in B field, $\varepsilon = Blv\sin\theta$



4. MAGNETIC FORCES

(a) Force between parallel wires

Force per unit length is $f = \mu_o \, I_1 I_2 / 2\pi p$, where p is distance between wires. Like currents attract, unlike repel. The unit of current (Ampere) is defined from this relation.

(b) Force on Linear Conductor

$F = BIl \sin\theta$ or in vector notation $\underline{F} = I\underline{l} \times \underline{B}$

(c) Torque on Current Loop

$T = NIBA \sin\alpha$

Applications include motor and meter.

(d) Force on Charged Particle

$\underline{F} = q\left(\underline{v} \times \underline{B}\right)$ is at right angles to both $\underline{B}$ and $\underline{v}$.

Gives Hall effect and gyration of charges about field lines.