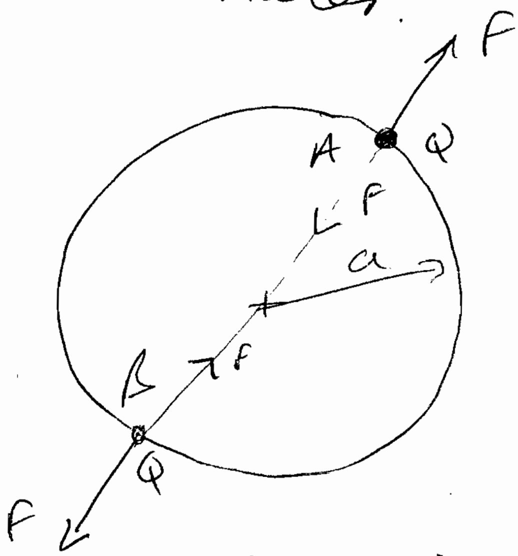
EEE220 2008/9

① a) Equilibrium position is when the two charged beads are at opposite ends of a diameter.

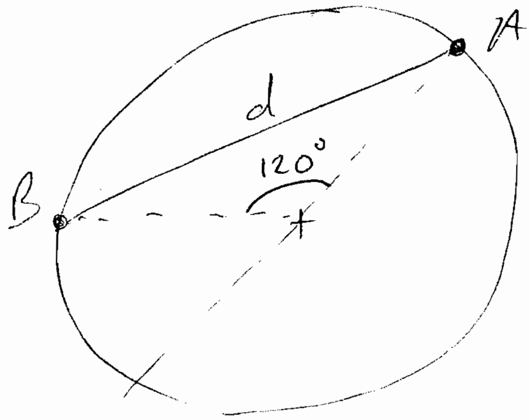


The force F on each bead is radially outwards and given by

$$F = \frac{Q^2}{4\pi\epsilon_0 (2a)^2} = \frac{Q^2}{16\pi\epsilon_0 a^2}$$ (7)

b) Potential at position B

$$\phi_1 = \frac{Q}{4\pi\epsilon_0 (2a)} = \frac{Q}{8\pi\epsilon_0 a}$$



If B moved to new position, potential given by

$$\phi_2 = \frac{Q}{4\pi\epsilon_0 d}$$

where $d = 2a \cos 30° = 2a \cdot \frac{\sqrt{3}}{2}$

$$\rightarrow \phi_2 = \frac{Q}{4\pi\epsilon_0 a\sqrt{3}}$$

(1) cont

Difference in potential

$\phi_2 - \phi_1$ and work done is

$$W_1 = Q(\phi_2 - \phi_1)$$

$$= \frac{Q^2}{\pi \epsilon_0 a}\left(\frac{1}{4\sqrt{3}} - \frac{1}{8}\right) = \frac{0.0193\,Q^2}{\pi \epsilon_0 a}$$

(7)

c) Potential at point C due to charges A and B is twice that at B due to A alone [by symmetry]

$$\therefore \phi_C = 2 \times \frac{Q}{4\sqrt{3}\,\pi \epsilon_0 a}$$

change in energy due to 3rd charge Q at C is

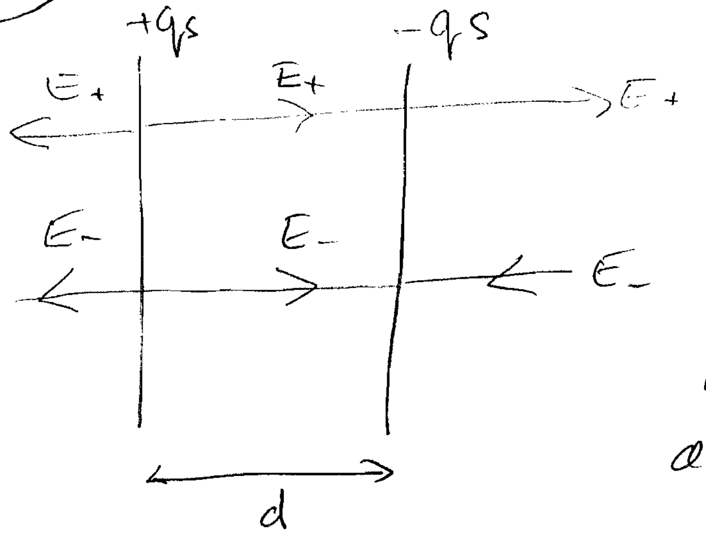
$$W_2 = Q(\phi_C - \phi_0)$$

where ϕ_0 = potential at initial position of charge 3 → infinite distance ⇒ $\phi_0 = 0$

hence $$W_2 = \frac{2Q^2}{4\sqrt{3}\,\pi \epsilon_0 a} = \frac{Q^2}{2\sqrt{3}\,\pi \epsilon_0 a}$$

(6)

(2)



outside plates fields cancel

Inside plates fields add.

→ Total field inside

$$E = E_+ + E_- = \frac{q_s}{2\epsilon_0} + \frac{q_s}{2\epsilon_0} = q_s/\epsilon_0$$

Potential difference between plates

$$V = \int_{x=0}^{d} E\,dx = \frac{q_s d}{\epsilon_0}$$

By definition $C = Q/V$

where $Q = q_s A$ — A = Area of plates

$$\rightarrow \quad C = \frac{q_s A}{q_s d/\epsilon_0} = \frac{\epsilon_0 A}{d}$$

(7)

2)

b) Potential difference between plates is determined by battery voltage V which does not change when separation of plates is changed.

we have

$$Q = CV = \frac{\epsilon_0 A V}{d}$$

$$\therefore Q_1 = \frac{\epsilon_0 A V}{d_1} \text{ and } Q_2 = \frac{\epsilon_0 A V}{d_2}$$

Change in charge is (7)

$$Q_2 - Q_1 = \epsilon_0 A V \left(\frac{1}{d_2} - \frac{1}{d_1} \right)$$ (8)

For values given

$$Q_2 - Q_1 = 8.854 \times 10^{-12} \times 20 \times 10^{-4} \times 10 \times \left(\frac{10^3}{1.1} - \frac{10^3}{1.0} \right)$$

$$= -1.61 \times 10^{-11}\ C$$

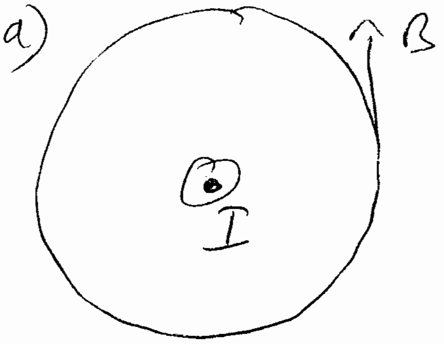
i.e. charge decreases. This takes place in 0.1s, so current is of the order

$$\left| \frac{Q_2 - Q_1}{0.1} \right| = 1.61 \times 10^{-10}\ A$$

(6)

3

a)



Ampere's law

$$\oint B\cos\theta\, dl = \mu_0 I$$

Applying this to a circular path around a long wire

$\theta = 0$ and B is constant

$$\therefore B\,2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

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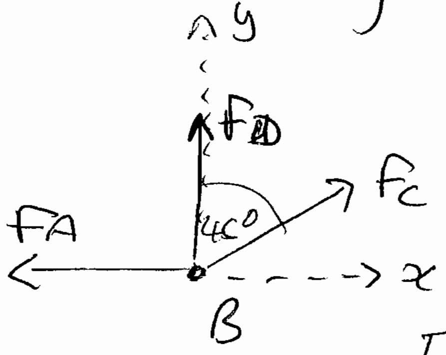
b) Force per unit length on current carrying conductors

$$f = \frac{\mu_0 I_a I_b}{2\pi d}$$



← Forces on wire B

$$F_A = \frac{\mu_0 I_1 I_2}{2\pi \epsilon}, \quad F_C = \frac{\mu_0 I_1^2}{2\pi \epsilon\sqrt{2}}$$

$$F_D = \frac{\mu_0 I_1 I_2}{2\pi \epsilon}$$

3 con't

Resolving these components we have

$$F_x = F_C \cos 45 - F_A$$

$$= \frac{\mu_o I_1^2}{2\pi t\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \frac{\mu_o I_1 I_2}{2\pi t}$$

$$= \frac{\mu_o I_1}{2\pi t}\left(\frac{I_1}{2} - I_2\right)$$

$$F_y = F_D + F_C \cos 45$$

$$= \frac{\mu_o I_1 I_2}{2\pi t} + \frac{\mu_o I_1^2}{2\pi t\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$$

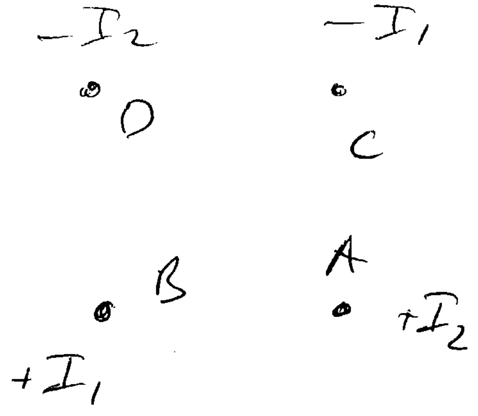
$$= \frac{\mu_o I_1}{2\pi t}\left(I_2 + \frac{I_1}{2}\right)$$

for $I_1 = 3A$, $I_2 = 1A$ and $t = 20mm$

$$F_x = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times 20 \times 10^{-3}}(1.5 - 1.0) = 1.5 \times 10^{-5}\ Nm^{-1}$$

$$F_y = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times 20 \times 10^{-3}}(1.5 + 1.0) = 7.5 \times 10^{-5}\ Nm^{-1}$$

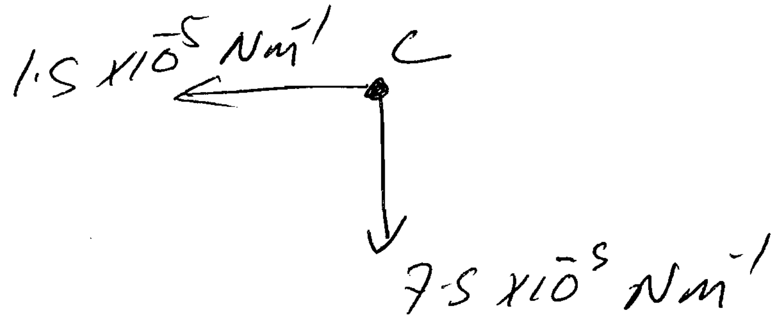
3/Cont



If rotate diagram through 180° get original problem but with every current reversed.

Hence force on C is exactly same as force on B

Hence when C is in original position we have



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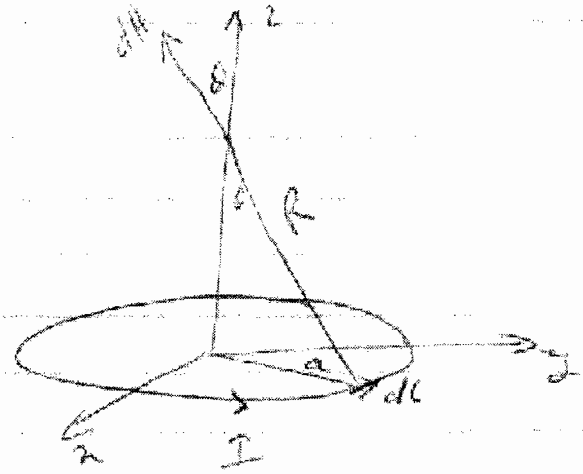
Q4 a)



Field due to small segment of wire is

$$\underline{dH} = \frac{I\,dl}{4\pi R^2}\left[\hat{z}\cos\theta - \hat{r}\sin\theta\right]$$

radial component cancelled by current segment opposite

$$\underline{H} = \int_0^{2\pi} \frac{I\,dl}{4\pi R^2}\cos\theta\;\hat{z}$$

put $R^2 = a^2 + z^2$, $\cos\theta = \frac{a}{\sqrt{a^2+z^2}}$ and $dl = a\,d\phi$

$$\rightarrow \quad \underline{H} = \hat{z}\,\frac{I a^2}{4\pi(a^2+z^2)^{3/2}}\int_0^{2\pi} d\phi$$

$$= \hat{z}\,\frac{I a^2}{2(a^2+z^2)^{3/2}}$$

For an N turn loop $\underline{H} = \frac{\hat{z}\,N I a^2}{2(a^2+z^2)^{3/2}}$

$$\underline{B} = \mu_0 \underline{H} = \frac{\hat{z}\,\mu_0 N I a^2}{2(a^2+z^2)^{3/2}}$$

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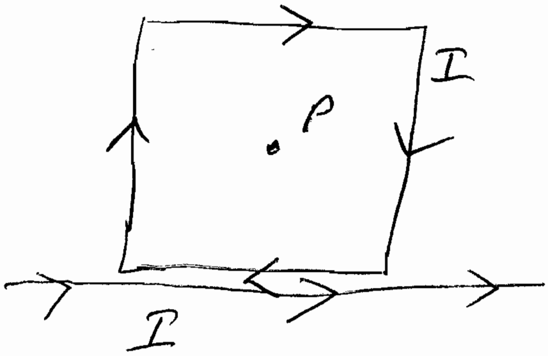
Q4

b) We can regard the field at point P as being due to an infinitely long wire carrying a current I and a square circuit carrying a current I



For the square circuit we use expression given with $y = d/2$ and $L = d$ and times by 4

i.e. $$\frac{4\mu_0 I}{\pi d}\left[\frac{1}{1+(2 \cdot d/2\,/d)^2}\right]^{1/2} = \frac{4\mu_0 I}{\sqrt{2}\,\pi d}$$

For infinite wire $y = d/2$ and $L = \infty$ to give

$$\frac{\mu_0 I}{\pi d}.$$

These two fields are in opposite direction so total field is

$$B = \frac{4\mu_0 I}{\sqrt{2}\,\pi d} - \frac{\mu_0 I}{\pi d} = \frac{\mu_0 I}{\pi d}\left(2\sqrt{2}-1\right)$$

and direction is into plane of diagram.

For $I = 10^3$ A and $d = 0.1$ m

$$B = \frac{1.828 \times 4\pi \times 10^{-7} \times 10^{3}}{\pi \times 0.1} = 7.3 \times 10^{-3}\ \text{T}$$

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