Data Provided: None

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2014-15 (2.0 hours)

EEE112 Engineering Applications

This paper comprises **TWO** sections, **A** and **B**. You may gain up to **60 MARKS** from **SECTION A** and **40 MARKS** from **SECTION B**. Attempt **ALL** the questions in **SECTION A**. Marks will be awarded for your best **TWO** solutions in **SECTION B**. Trial answers will be ignored if they are clearly crossed out. A formula sheet is included at the end of the exam paper. **The numbers given after each section of a question indicate the relative weighting of that section.**

SECTION A

1. **a.** Simplify $\frac{\sqrt{9x^4 4x^{-2}}}{3x^{-3}}$ **(2)**

b. Simplify $\frac{3x+6}{x^2-4}$ **(2)**

c. Rearrange $z = \frac{R.x}{\sqrt{R^2+x^2}}$ to make *x* the subject. **(2)**

d. Differentiate $y = x^3. \cos(x)$ with respect to x. **(4)**

2. **a.** Using more than one of the trig. identities available on the formula sheet (at the end of this exam paper) show how the product $sin(h)cos(g)$ can also be represented in the form $\frac{1}{2}[sin(h+g) + sin(h-g)]$ **(2)**

b. A circuit has a voltage across its terminals of $v(t) = 240\, sin(100\pi t)$ Volts and a current flowing through it of $i(t) = 5\cos(100\pi t - \frac{5\pi}{6})$ Amps. Show that the power in this circuit is given by $p = 300 + 600\sin(200\pi t + \frac{5\pi}{6})$ Watts. **(4)**

3. **a.** Given the three simultaneous equations shown below use the Gaussian Elimination method only (the method that uses an augmented matrix) to find the three unknowns *f*, *g* and *h*.

$$2f + g - h = -2$$
$$f + 2g + 3h = 7$$
$$-3f + 2g + h = 1$$

(8)

b. Find the value of the following determinant (using co-factors). Show all workings.

$$\begin{vmatrix} 4 & -3 & 2 \\ -2 & 1 & 0 \\ -1 & 0 & 3 \end{vmatrix}$$

(4)

4. **a.** Two signals (of the same frequency):

$$i_1(t) = 3\sin(\omega t) \quad \text{and} \quad i_2(t) = 2\cos(\omega t)$$

are <u>subtracted</u> from one another such that $i_3(t) = i_1(t) - i_2(t)$.

Find $i_3(t)$ expressing the result as a simple <u>cosine</u> function of time in the form

$$i_3(t) = R\cos(\omega t \pm \alpha)$$

Find *R* and α expressing α in radians. **(8)**

5. **a.** Two complex numbers are given by $c = 5 - j3$ and $d = -3 + j4$. Plot both complex numbers on the same Argand diagram and determine the modulus and argument of each one. **(4)**

b. Calculate the following quantities, expressing the answers in both rectangular (*Re*+*jIm*) and polar ($r\angle\theta$) form:

i) $c + d$

ii) cd

iii) c/d

iv) jd **(6)**

c. Three components are connected together in series. When a sinusoidal voltage of $50\angle 0^o$ Volts (at 150 Hz) is applied across this series network a current of $2.24\angle -26.57^o$ Amps flows.

(i) Calculate the combined impedance of this series network. (Give the result in both rectangular (*Re*+*jIm*) and polar ($r\angle\theta$) form).

(ii) One of the series connected components is a resistor of 20 Ω, another is a capacitor of 35.4 μF. Find the impedance of the third series connected component. **(6)**

6. **a.** Show that the equation $z = d.w^a$ (where a and d are both constants) can be put into a form equivalent to that of the general form of a straight line equation $y = mx + c$ (where m is the gradient and c is the offset) using logs. Show all your workings and label the parts of the equation representing gradient and offset clearly. **(3)**

b. Give the formula for expressing voltage gain ($\frac{V_{out}}{V_{in}}$) in terms of decibels. **(1)**

c. A system consisting of three stages of amplification (or attenuation) are connected together in cascade. The first section has a gain of +20dB, the second has a gain of +12dB and the third has a gain of -6dB. Find the overall gain of the system, express your answer in terms of both decibels and also as ($\frac{V_{out}}{V_{in}}$). **(4)**

SECTION B

7. **a.** A repeating voltage waveform with a period =15 seconds is shown in figure 1 below:

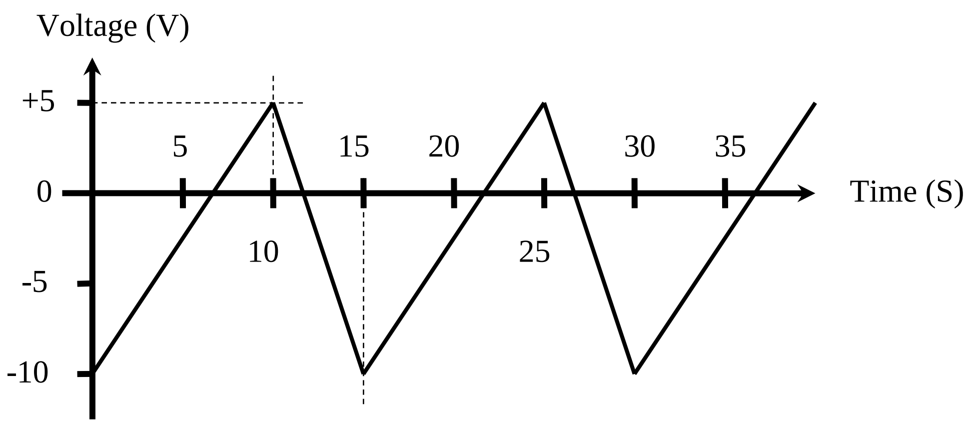

Figure 1

(i) Write down equations to describe this voltage waveform, over one period, as a function of time. (*Hint: write down one function for the period 0 to 10 seconds and another for the period 10 to 15 seconds*).

(ii) Next, with the equations derived in part (i) above and using integration, find the mean voltage of this waveform. **(10)**

b. Show that the RMS value of the current function

$$i(t) = 2\cos(\omega t) - 3$$

over the period T (where $T = \frac{2\pi}{\omega}$) is equal to $\sqrt{11}$. Show all workings. **(8)**

c. In what situations would finding the Root Mean Squared (RMS) value of a time varying waveform be more useful than finding the Mean (in other words why is RMS useful to us)? **(2)**

8. **a.** A sinusoidal voltage of $25\sin(100\pi t - 45^o)$ Volts and frequency of 50 Hz is placed across each of the following components separately. For each one calculate the current that flows, giving the result as a function of time. In addition, for each component, sketch a separate phasor diagram, showing both the voltage and current.

(i) A resistor of 5 Ω.
(ii) A capacitor of 127 μF.
(iii) An inductor of 8 mH. **(6)**

b.

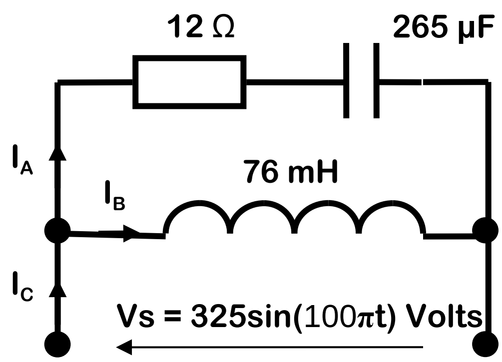

Figure 2

Consider the circuit shown as figure 2 above:

Using phasors (**<u>not</u>** complex numbers):

(i) Consider first the series connected resistor and capacitor. Find the current I_A and then the voltage across the resistor and the voltage across the capacitor. Give the magnitude and phase angle with respect to I_A in each case.
(ii) Sketch a phasor diagram showing the current and voltages found so far. Determine the phase angle between I_A and the supply voltage V_S.
(iii) Calculate I_B and add it onto the same phasor diagram created in part (i) (state the phase angle with respect to V_S).
(iv) Draw a separate phasor diagram showing both I_A and I_B. Then calculate I_C (magnitude and phase angle). Draw I_C onto the same phasor diagram. **(14)**

9. **a.** For the following differential equation:

$$\frac{5}{y}.\frac{dy}{dx} = 2x$$

(i) Find the general solution.
(ii) Given the initial conditions $y(0) = 4$ find the particular solution. **(6)**

b. Consider the circuit shown in figure 3 below:

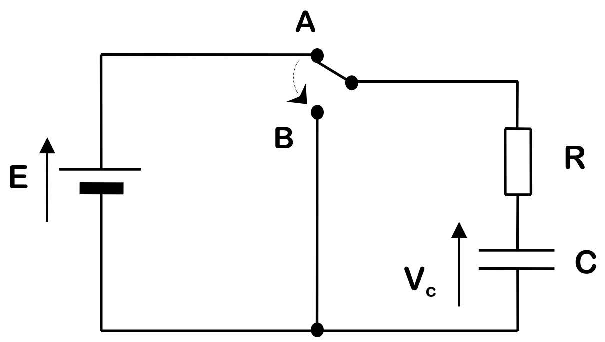

Figure 3

The switch has been in position **A** for a long time. At time $t = 0$ the switch moves to position **B**. After time $t = 0$ the voltage across the capacitor can be described by the following differential equation:

$$V_C = -CR\frac{dv_C}{dt}$$

(i) Find the general solution for the equation above.
(ii) Using the boundary conditions at $t = 0$ when $V_c = E$ find the particular solution. **(10)**

c. For the circuit shown in figure 3 the following equation describes the behaviour of the circuit when the switch has been in position B for a long time and then is returned to position **A**:

$$V_c = E\left(1 - e^{\frac{-t}{RC}}\right)$$

(i) Rearrange the equation to make *t* the subject.
(ii) If $E = 30$ V, $R = 4$ kΩ and $C = 20$ μF, find the time taken for V_c to rise from 0 to 15 V. **(4)**

10. A particular sinusoidal voltage waveform is described by the following function:

$$v(t) = 50\sin\left(360\pi t + \frac{\pi}{3}\right) Volts$$

a. For the waveform described above:

(i) what is the peak-to-peak voltage?
(ii) what is the phase shift?
(iii) what is the frequency (in Hz)?
(iv) what is the period of the waveform? **(4)**

b. Convert the voltage waveform function given at the top of this question into the equivalent cosine function. **(2)**

c. A current $i(t) = 5\sin\left(628t - \frac{3\pi}{4}\right) Amps$ flows through each of the following components. Find the voltage (as a function of time) across each of the components separately (including the phase). Give the phase in the range $\pi > 0 > -\pi$.

(i) A resistor of 20 Ω
(ii) An inductor of 16 mH
(iii) A capacitor of 53 µF **(9)**

d. Two sinusoidal waveforms, both of frequency 24 Hz, are summed (added) together. The first waveform has an amplitude of 16 V, the second waveform has an amplitude of 19 V. The first voltage has a phase angle of zero, the second has a phase angle 180 degrees behind the first. Write down the resulting waveform as a function of time. **(5)**

FORMULA SHEET

Trig. Identities

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)$$

$$\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta)$$

Logarithmic Laws

$$\log_a x^n = n\log_a x$$

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Differentiation

$$\frac{d(\sin(x))}{dx} = \cos(x)$$

$$\frac{d(\cos(x))}{dx} = -\sin(x)$$

$$\frac{d(\sin(kx))}{dx} = k\cos(x)$$

Integration for $f(x)$

$$\int \sin x = -\cos x + c$$

$$\int \sin k.x = -\frac{1}{k}\cos k.x + c$$

$$\int \cos x = \sin x + c$$

$$\int \cos k.x = \frac{1}{k}\sin k.x + c$$

$$\int \frac{1}{x} = \ln(x) + c$$

PLJ/AM

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