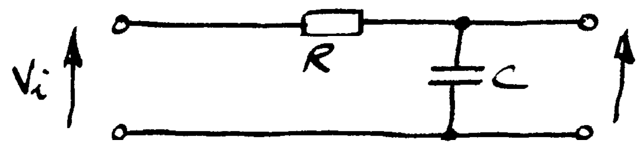
Transient behaviour of first order R-C circuits

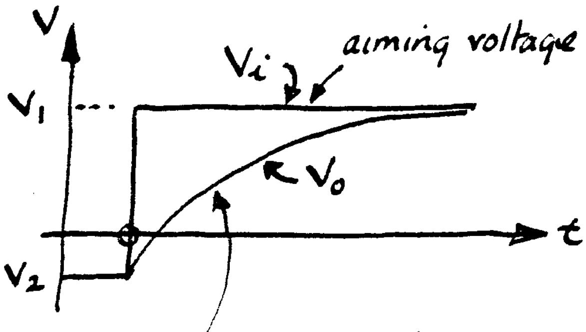
- can be derived analytically as done in the Passive Networks course.

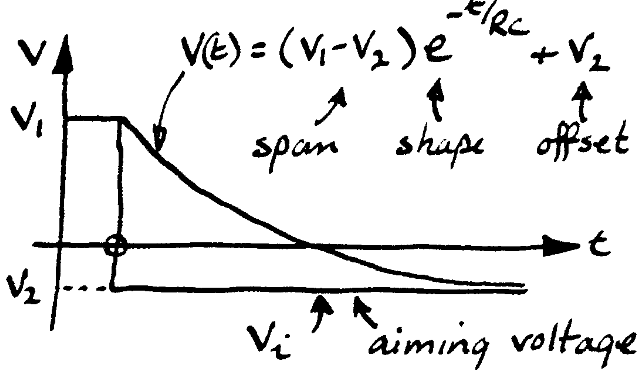
- analytical approaches are not much help when it comes to understanding how a circuit works. For this one needs to be able to visualise the behaviour of R-C circuits. You should aim to become as familiar with the behaviour of voltage and current in an R-C circuit driven by transient signals as you are with ohms law.

- <u>all</u> first order R-C circuits will have a transient response involving $e^{-t/RC}$

(1) <u>"low pass" or "simple integrator" circuit</u>







$$V(t) = (V_1 - V_2)(1 - e^{-t/RC}) + V_2$$

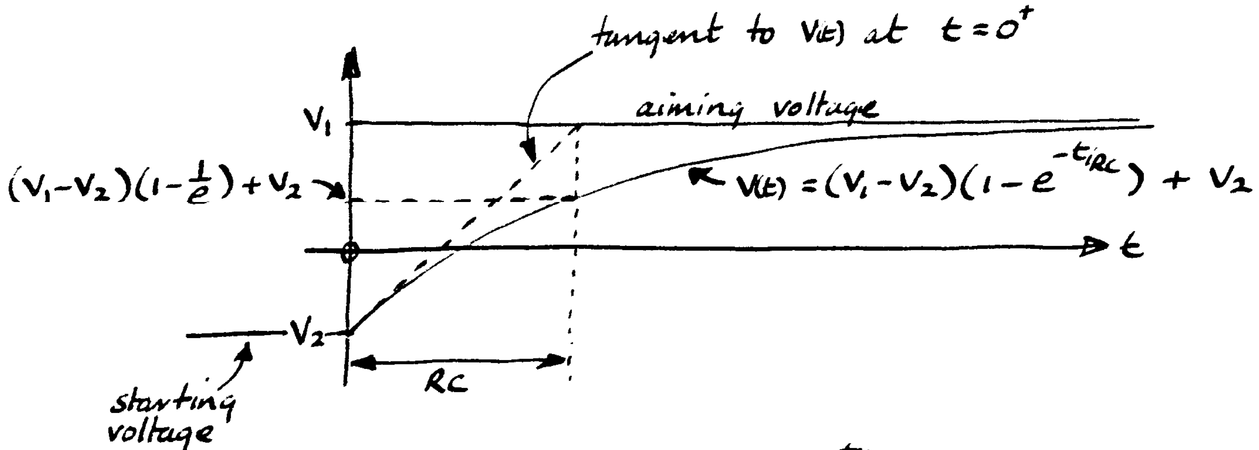
full span of the exponential — curve shape — offset.

RC is called "time constant", τ. Units = seconds

$$\left[\; RC = \text{ohms} \times \text{farads} = \frac{\text{volts}}{\text{amps}} \times \frac{\text{coulombs}}{\text{volts}} = \frac{\text{coulombs}}{\left(\frac{\text{coulombs}}{\text{seconds}}\right)} = \text{seconds} \;\right]$$

τ (= RC) has some important properties relevant to the geometry of transient responses....

response to a positive going step ....

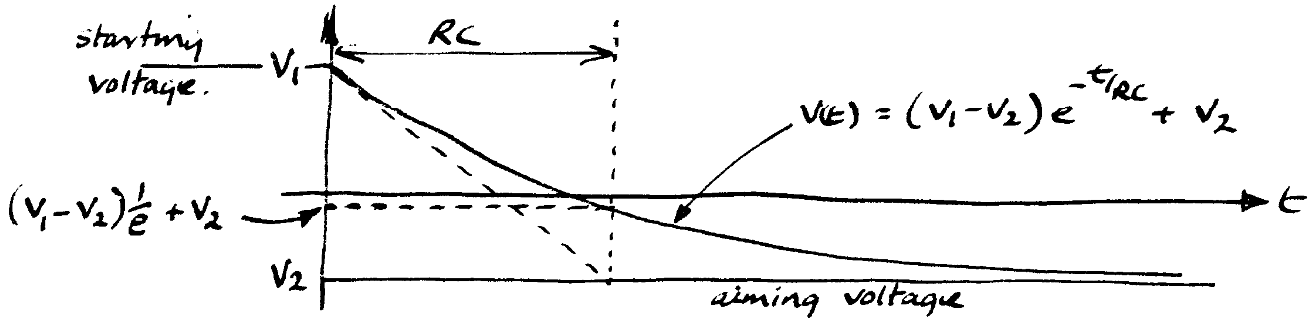


when $t = RC = \tau$, $1 - e^{-t/RC} = 1 - e^{-1} = 1 - \frac{1}{e}$

- after one time constant, the exponential has travelled $(1 - \frac{1}{e}) = 0.63$ (or 63%) of the way from its start voltage, V_2, to its finish voltage, V_1.

- a projection of the initial slope of the exponential crosses the aiming voltage at $t = RC = \tau$. (prove this for yourselves).

for a negative going step, the geometry is inverted...



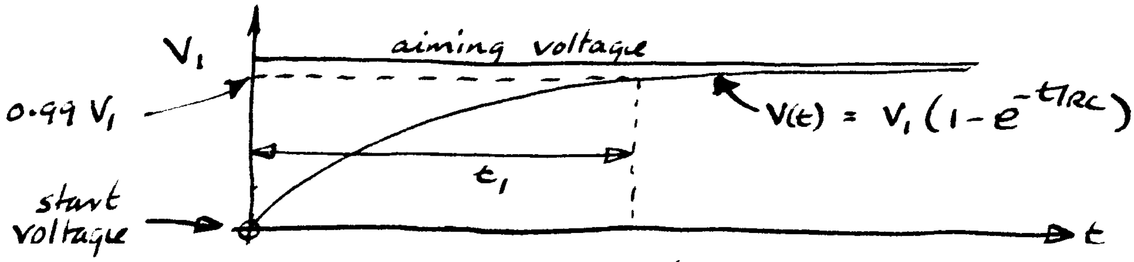
- after one time constant, the exponential has travelled 63% of the way from its start voltage to its aiming voltage (it has $1/e$ or 37% of the way still to go)

- as before, a projection of initial slope crosses the aiming voltage at $t = RC = \tau$.

- note that if exponential starts or finishes at 0V, $V_2 = 0$ and the equations are easier.

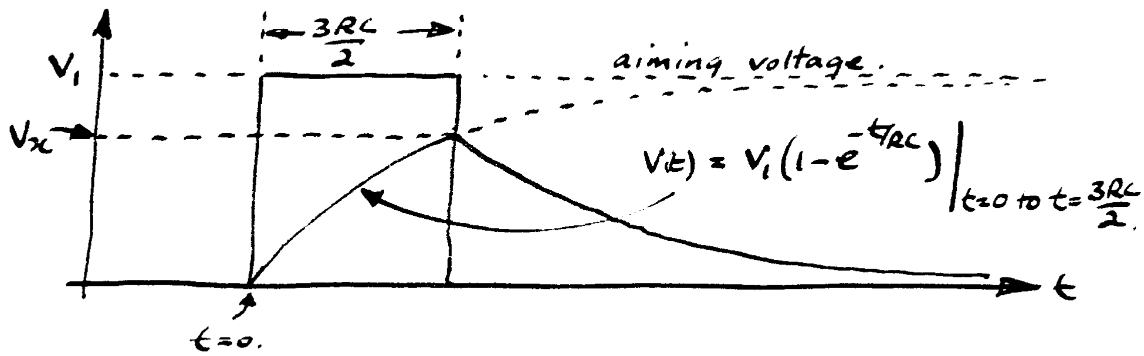
- knowledge of exponential equation, $V(t)$, describing the behaviour of V_o as function of time enables V_o at given time or time to reach a given V_o to be estimated ....

eg how long does it take the output voltage of a low-pass R-C circuit to 99% of the way from its start voltage to its aiming voltage?



$$V(t) = V_1(1-e^{-t/RC})$$
$$0.99V_1 = V_1(1-e^{-t_1/RC})$$
$$e^{-t_1/RC} = 1-0.99 = 0.01$$
$$-t_1/RC = \ln(0.01)$$
$$\frac{t_1}{RC} = \ln\left(\frac{1}{0.01}\right) = \ln(100).$$
$$t_1 = RC\ln(100) = \underline{4.6RC}.$$

eg the input to a low pass RC circuit is a pulse of amplitude V_1 and duration $\frac{3RC}{2}$. What is the maximum voltage reached at the output? What is the relationship between V_o and time after the falling edge of the input pulse?



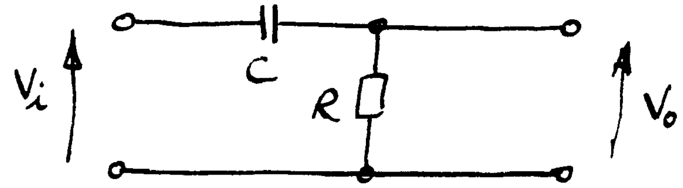
$$V_x = V_1\left(1-e^{-\frac{3RC/2}{RC}}\right) = V_1(1-e^{-3/2}) = \underline{0.777\,V_1}.$$

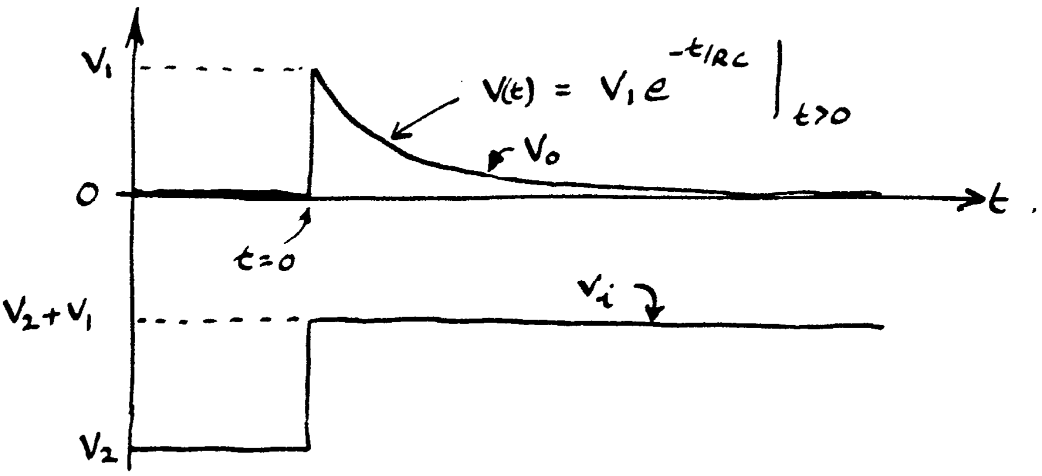
– note that expression for $V(t)$ was based on the voltage the exponential was aiming for, <u>NOT</u> the voltage at which it was truncated.

for second part, define $t=0$ at falling edge of input pulse ∴

$$V(t) = (\underbrace{0.777\,V_1}_{\text{starting voltage}} - \underbrace{0}_{\text{aiming voltage}})\,\underbrace{e^{-t/RC}}_{\text{shape}} + \underbrace{0}_{\text{offset.}}$$

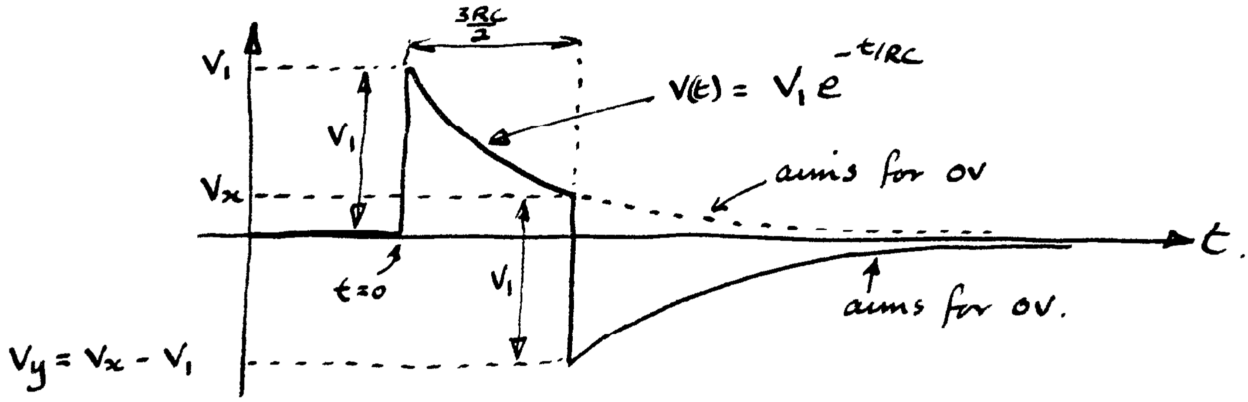
(4) <u>"High-pass" or "simple differentiator" circuit</u>





– notice that V_2 can have any value – it is the height of the step that is important.

– a negative going step will produce an inverted version of the V_o shown above eg A high pass RC circuit is driven by an input pulse of amplitude V_1 and width $\frac{3RC}{2}$. What is the most negative voltage reached by the output? What is the equation of the exponential following the negative edge of the input pulse?



first find V_x ....

$$V(t) = V_1 e^{-t/RC}$$

$$V(x) = V_1 e^{-\frac{3RC}{2}/RC} = V_1 e^{-3/2} = 0.223 V_1$$

then find V_y ...

$$V_y = V_x - V_1 = 0.223 V_1 - V_1$$

$$= -V_1(1 - 0.223)$$

$$= \underline{\underline{-0.777 V_1}}$$

to find equation of exponential after falling edge of input pulse, let $t=0$ at falling edge

$$V(t) = \underbrace{[0}_{\text{aiming voltage}} - \underbrace{(-0.777V_1)]}_{\text{starting voltage}}\underbrace{[1 - e^{-t/RC}]}_{\text{shape}} + \underbrace{(-0.777V_1)}_{\text{offset}}$$

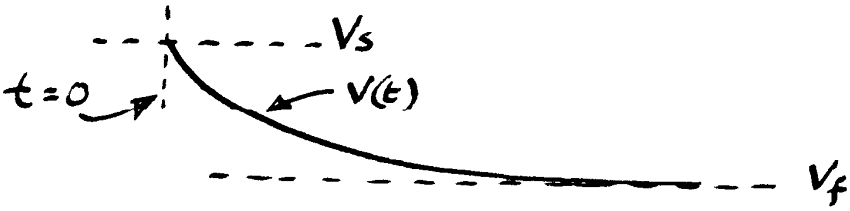
$$= 0.777 V_1 (1 - e^{-t/RC}) - 0.777 V_1$$

[$= -0.777 V_1 e^{-t/RC}$ which is to be expected since the shape could be regarded as an inverted $e^{-t/RC}$ shape]

(iii) a general approach to both high + low pass

- the main difference between the high + low pass cases is the sign of the exponential term. Rather than defining the two shapes $(1-e^{-t/\tau})$ and $e^{-t/\tau}$, it is possible to use a general formula based on the $e^{-t/\tau}$ shape which is the fundamental first order response shape.

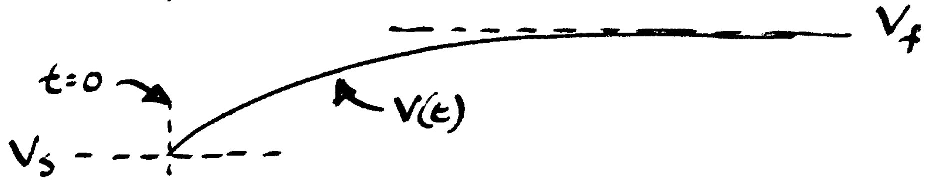
- Consider an exponential shape starting at V_s and finishing at V_f



V(t) is given by:

$$V(t) = (V_s - V_f)\,e^{-t/\tau} + V_f .$$

This is exactly the same expression as used on pages 1 and 2 for the trailing edge of the low pass response, and on page 4 for the high pass response. If the exponential is redrawn with $V_s < V_f$ .....



the same equation must hold because no condition was placed on the relationship between V_s + V_f ....

$$V(t) = (V_s - V_f)\,e^{-t/\tau} + V_f$$

this can be written as:

$$\begin{aligned} V(t) &= -(V_f - V_s)\,e^{-t/\tau} + (V_f - V_s) - (V_f - V_s) + V_f \\ &= (V_f - V_s)(1 - e^{-t/\tau}) + V_s \end{aligned}$$

– ie, the same as the equation used on page 1 + 2 to describe the rising exponential of the low pass response.