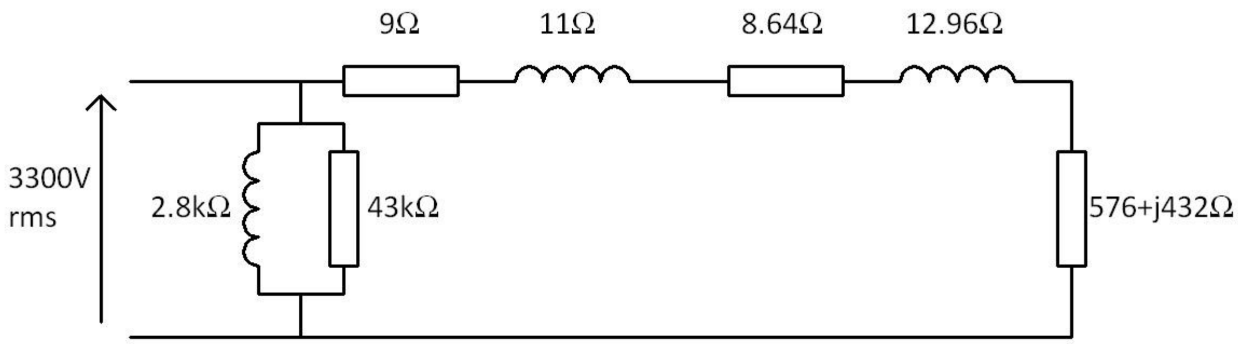
EEE102 2010/2011 – Solutions

1.

a) *[Note: It is not incorrect to draw the exact equivalent circuit – this will be awarded full marks. If the approximate equivalent circuit is used then the basis for moving the magnetising branch needs to be included].*

Since the magnetising reactance and the core loss resistance are much greater than the primary resistance and reactance, the magnetising branch can be moved to the terminals to give an approximate equivalent circuit. Referring the secondary impedances to the primary yields the following equivalent circuit:



b) On no-load the current is given by:

$I_o = I_c + I_m$

$$Ic = \frac{3300}{43000} = 0.077\ Arms$$

$$I_m = \frac{3300}{j\,2800} = -j1.18\ Arms$$

Hence, no load current is given by:

$$I_o = 0.077 + j1.18 = 1.18\angle{-86.3^\circ}\ Arms$$

c) The core loss is given by:

$$Core\ loss = \frac{3300^2}{43000} = 253W$$

d)

i) Total series impedance of the main branch is given by:

$$Z_{mb} = (9 + 8.64 + 576) + j(11+12.96+432) = 594+j456\Omega = 748\ \angle 37.5^\circ\ \Omega$$

$$I_L' = \frac{3300\angle 0^\circ}{748\angle 37.5^\circ} = 4.40\ \angle -37.5^\circ\ Arms$$

Actual load current is given by:

$$I_L = 12 \times I_L' = 52.9\angle -37.5^\circ\ Arms$$

ii) The input current is given by:

$$I_1 = I_o + I_L' = 0.077 - j1.18 + 2.66 - j3.52 = 2.74 - j4.70\ Arms$$

$$I_1 = 5.43\angle{-59.8^\circ}\ Arms$$

iii) The referred load voltage is given by:

$V_L' = I_L Z_L = 4.40\ \angle -37.5^\circ\ \times 720\angle 36.9^\circ$ =3174 $\angle$-0.66° V

Actual load voltage is given by:

$$V_L = \frac{3174\angle -37.5^\circ}{12} = 264\ \angle -0.66^\circ\ V$$

iv) Voltage regulation is given by:

$$Regulation = \frac{V_{no\ load} - V_{on\ load}}{V_{no\ load}} = \frac{275 - 264}{275} = 3.8\%$$

v) Copper losses in transformer winding

$$Copper\ loss = \ I_L^2(R_1 + R_2') = 4.40^2 \times 17.64 = 342W$$

vi) Real component of output power

$$P_{out} = 4.40^2 \times 576 = 11.19kW$$

Total losses = core loss + copper loss = 253+342 = 595W

Output power = 11195W

Efficiency = 11195 / (11195+595) = 94.9%

2. a) The total reluctance S_{TOTAL} is given by:

$$S_{TOTAL} = \frac{(\pi \times 0.12 - 1 \times 10^{-3})}{600 \times 4\pi \times 10^{-7} \times 200 \times 10^{-6}} + \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 200 \times 10^{-6}} = 6.47 \times 10^6 Hm^{-1}$$

$$Self\ inductance\ =\ L = \frac{N^2}{S_{TOTAL}} = \frac{300^2}{6.47 \times 10^6} = 13.9mH$$

b) The current is given by:

$$I = \frac{BAS_{TOTAL}}{N} = \frac{1.2 \times 200 \times 10^{-6} \times 6.47 \times 10^6}{300} = 5.18A$$

c)

i) At 380Hz:

X = 2πx380x0.0139= 33.2Ω

Z = 2 + j33.2= 33.3∠86.6°

$$I_{380} = \frac{V}{Z} = \frac{115}{33.3\angle 89.2^o} = 3.46A\angle -86.6^o\ Arms$$

At 800Hz:

X = 2πx800x0.0139= 69.9Ω

Z = 0.2 + j69.9 = 70.0∠88.4°

$$I_{800} = \frac{V}{Z} = \frac{115}{69.9\angle 89.8^o} = 1.64A\angle -88.4^o\ Arms$$

ii) At 380 Hz: Copper loss = 3.64^2x2 = 23.9W

At 800 Hz: Copper loss = 1.64^2x2 = 5.41W

iii) At 380Hz:

$$B_{peak} = \frac{NI_{peak}}{S_{TOTAL}A} = \frac{300 \times \sqrt{2} \times 3.64}{6.47 \times 10^6 \times 200 \times 10^{-6}} = 1.13T$$

And similarly at 800Hz

$$B_{peak} = \frac{NI_{peak}}{S_{TOTAL}A} = \frac{300 \times \sqrt{2} \times 1.64}{6.47 \times 10^6 \times 200 \times 10^{-6}} = 0.54T$$

d) Two ways of tacking this:

For 1.6T, the magnitude of the current is given by:

$$|I_{peak}| = \frac{B_{peak}S_{TOTAL}A}{N} = \frac{1.6 \times 6.47 \times 10^6 \times 200 \times 10^{-6}}{300} = 6.90A$$

Hence:

$$I_{rms} = \frac{6.90}{\sqrt{2}} = 4.88Arms$$

$$|Z| = \frac{115}{4.88} = 23.6\Omega$$

Hence:

$$X = \sqrt{Z^2 - R^2} = \sqrt{23.6^2 - 2^2} = 23.5\Omega$$

$$f = \frac{X}{2\pi L} = \frac{23.5}{2\pi \times 13.9 \times 10^{-3}} = 269Hz$$

Alternatively, since B is proportional to I, which is proportional to X which in turn is proportional to f, then one of the previously operating points can be scaled, e.g.

$$f_{1.6} = \frac{1.13}{1.6} \times 380 = 269Hz$$

[Full marks awarded for either solution method]

e. Doubling the inductance corresponds to halving the reluctance, i.e. to 3.24×10^{6} Hm^{-1}

The core reluctance remains the same (see hint in question) at 2.49×10^{6} Hm^{-1}

Hence, airgap reluctance = $3.24\times10^{6}-2.49\times10^{6}$ = 0.743×10^{6} Hm^{-1}

$$l_{gap} = S_{gap} \times \mu_0 \times A = 0.19mm$$

3.

a. The loads are star-connected and hence:

$I_{ph} = I_l$

$$V_{ph} = \frac{V_l}{\sqrt{3}} = \frac{1100}{\sqrt{3}} = 6350V\ rms$$

$$Z_{ph} = 16 + 6.4\Omega = 17.23\angle 21.8\,°\Omega$$

$$I_l = \frac{6350\angle 0}{17.23\angle 21.8\,°} = 368\angle - 21.8\,°\ Arms$$

b. $Real\ power = 3V_{ph}I_{ph}cos\phi = 3 \times 6350 \times 368 \times 0.928 = 6.52MW$

$$VAr = 3V_{ph}I_{ph}\sin\phi = 3 \times 6350 \times 368 \times 0.371 = 2.61MVAr$$

$$VA = 3V_{ph}I_{ph} = 3 \times 6350 \times 368 = 7.02MVA$$

[Quick check: $= \sqrt{P^2 + Q^2} = 7.02MVA$]

c. In order to improve the power factor to 0.980, it is necessary to reduce Q.

If cosϕ = 0.980 then ϕ = 11.5°
For the corrected arrangement, P is maintained at 6.52MW

$$Q = P\tan\phi = 6.52 \times 10^6 \times 0.203 = 1.32MVAr$$

Hence, Q supplied by capacitors must be:

$$Q_{cap} = 2.61 - 1.32 = 1.29MVAr$$

Hence, Q_{cap} per phase = 428 KVAr

$$X_{ph} = \frac{{V_{ph}}^2}{Q_{ph}} = \frac{6350^2}{428 \times 10^3} = 94.2\Omega$$

Hence capacitance per phase is given by:

$$C_{ph} = \frac{1}{2\pi f X_{ph}} = 33.8\mu F\ per\ phase$$

d.
VA drawn with power factor correction capacitors:

$$VA = \sqrt{P^2 + Q^2} = \sqrt{(6.52 \times 10^6)^2 + (1.32 \times 10^6)^2} = 6.65MVA$$

Hence line current is given by:

$$I_l = \frac{VA}{3V_{ph}} = \frac{6.65 \times 10^6}{3 \times 6350} = 349A$$

[As a check it is worth noting that both the above are, as expected, smaller than the original with no power factor correction capacitors].

e. During overnight operation

$$Z_{ph} = 32 + 9.4\Omega = 33.35\angle 16.4°\Omega$$

$$I_l = \frac{6350\angle 0}{33.35\angle 16.4°} = 190\angle -16.4°\ Arms$$

$Real\ power = 3V_{ph}I_{ph}cos\phi = 3 \times 6350 \times 190 \times 0.959 = 3.48MW$

$Q = P\ tan\phi = 3.48 \times 10^6 \times 0.294 = 1.02MVAr$

Hence, for unity power factor Q supplied by capacitors must be:

$Q_{cap} = 1.02MVAr$

Hence, Q_{cap} per phase = 341 KVAr

$$X_{ph} = \frac{V_{ph}{}^2}{Q_{ph}} = \frac{6350^2}{341 \times 10^3} = 118\Omega$$

Hence capacitance per phase is given by:

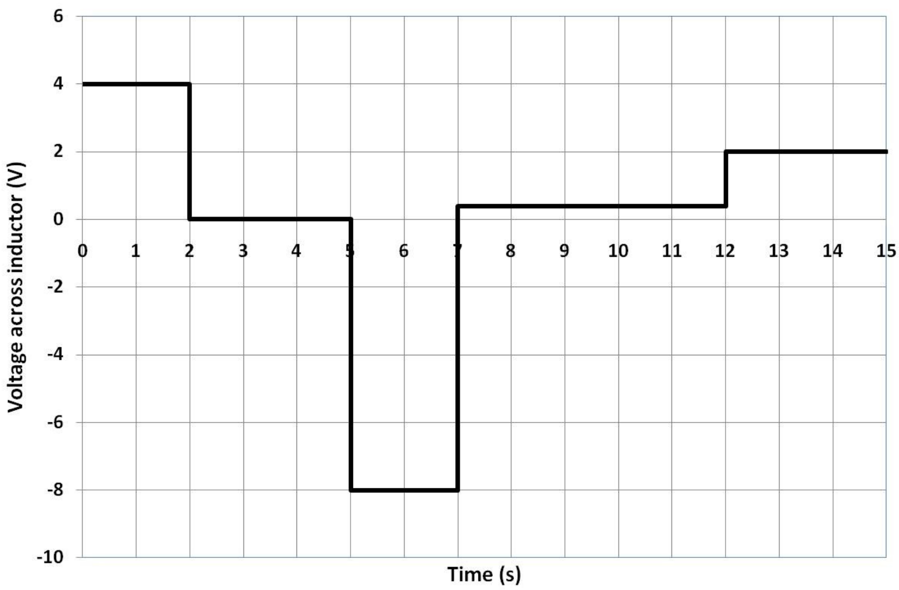
$$C_{ph} = \frac{1}{2\pi f X_{ph}} = 26.9\mu F\ per\ phase$$

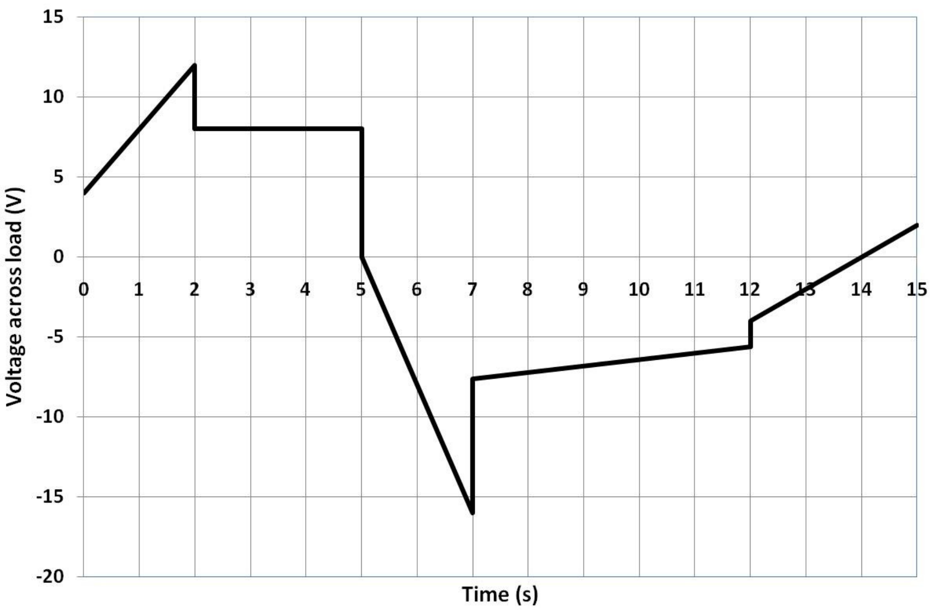
Hence capacitance that must be switched out = 33.8-26.9 = 6.88μF

4.

a) The voltage waveforms for the individual component can be derived noting V = IR and V=L di/dt

2-5s





b) Energy is only dissipated in the resistor

Taking interval in turn (re-setting time to zero at start of interval)

0-2s:

P=VI= 4t x 2t = $8t^2$

$$E = \int_0^2 8t^2 dt = \left[\frac{8}{3}t^3\right]_0^2 = 21.3J$$

2-5s

P=VI = 4 x 8 = 32

$$E = \int_0^3 32dt = [32t]_0^3 = 96J$$

5-7s

P=VI = (8-8t) x (4-4t) = 32-64t+32t^2

$$E = \int_0^2 (32 - 64\text{t} + 32\text{t}^2)dt = \left[32t - \frac{64}{2}t^2 + \frac{32}{3}t^3\right]_0^2 = 64 - 128 + 85.3 = 21.3J$$

7-12s

P=VI = (-8+0.4t) x (-4+0.2t) = 32-3.2t+0.08t^2

$$E = \int_0^5 (32 - 3.2\text{t} + 0.8\text{t}^2)dt = \left[32t - \frac{3.2}{2}t^2 + \frac{0.08}{3}t^3\right]_0^5 = 160 - 40 + 3.3 = 123.3J$$

12-15s

P=VI = (-6+2t) x (-3+t) = 18-12t+2t^2

$$E = \int_0^3 (18 - 12\text{t} + 2\text{t}^2)dt = \left[18t - \frac{12}{2}t^2 + \frac{2}{3}t^3\right]_0^3 = 54 - 54 + 18 = 18J$$

Total energy dissipated = 21.3+96+21.3+153.3+18 = 279.9 J

c) Peak energy stored in the inductor coincides with peak current, i.e. 4A

Hence, peak stored energy = 0.5x2x4^2= 16J

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11/4/11