Lecture content

- Introduction of signal
- Examples of Continuous Time (CT) signals
  - Step functions
  - Ramp function
  - Unit Impulse

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Introduction

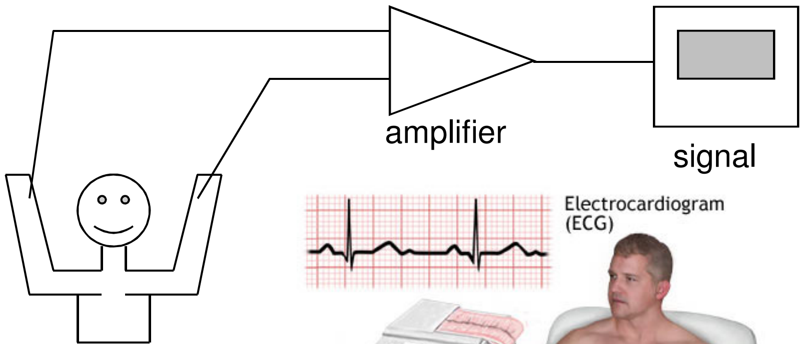
A signal $x(t)$ is formed from any physical quantity made to vary with time or other variables such as space and altitude.

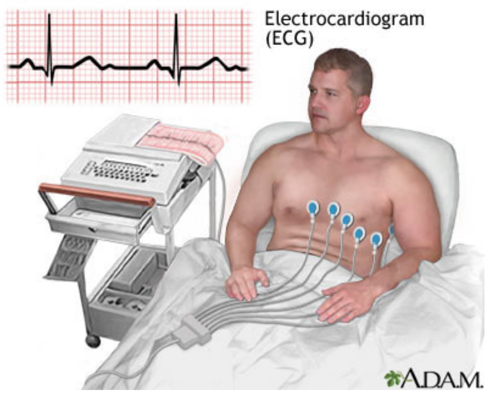
The signal incorporates and relays information.

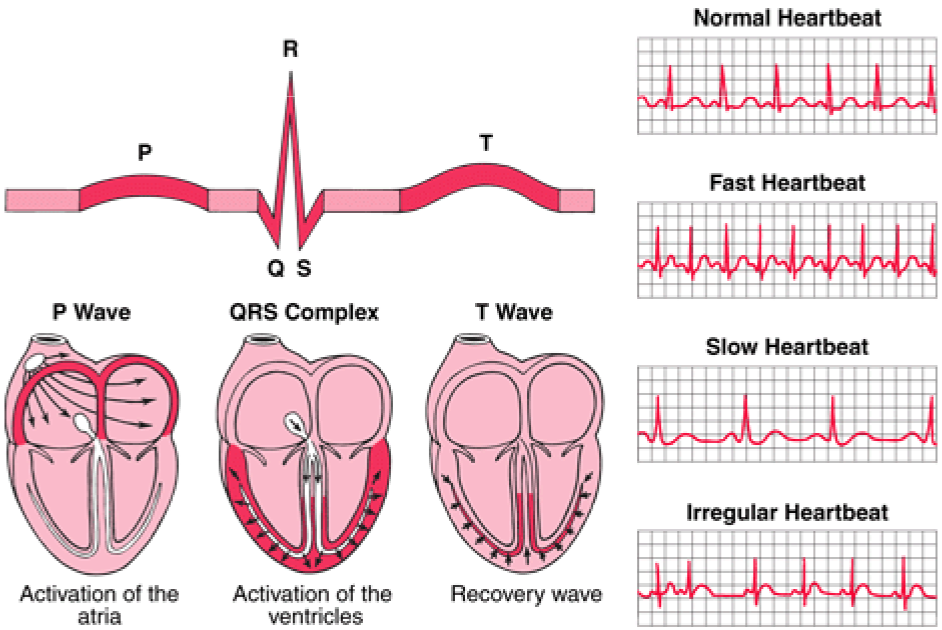
Therefore signal processing is a very important component of technology as it allows the extraction of information in a signal.

Signals: Electrocardiogram

1) Electrocardiogram (ECG) signals are used to monitor heart activity. A number of electrodes monitor different parts of the heart activity. An experienced doctor can determine whether a patient’s heart is normal by analysing the electrical signals from these electrodes





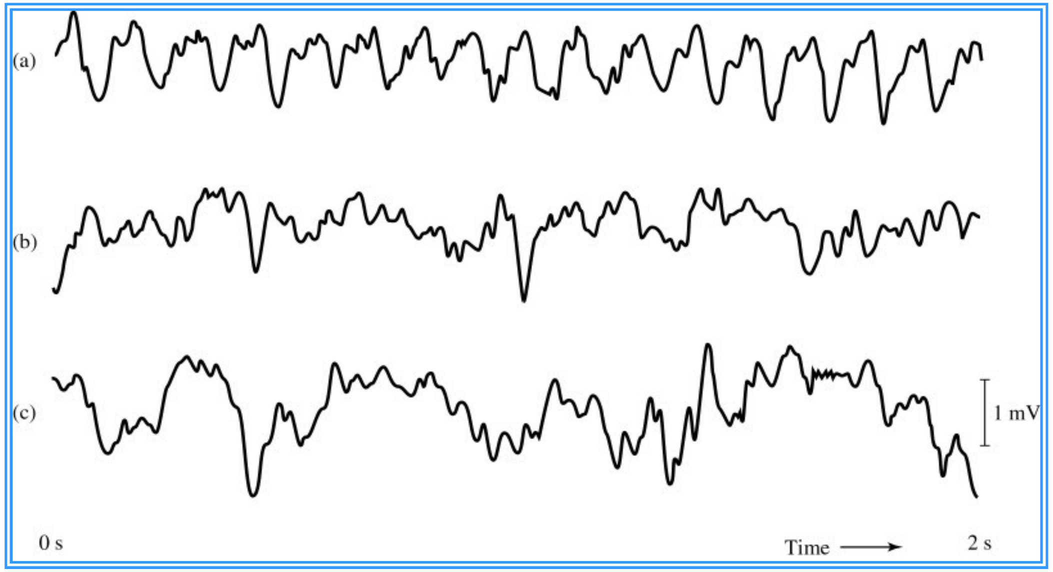


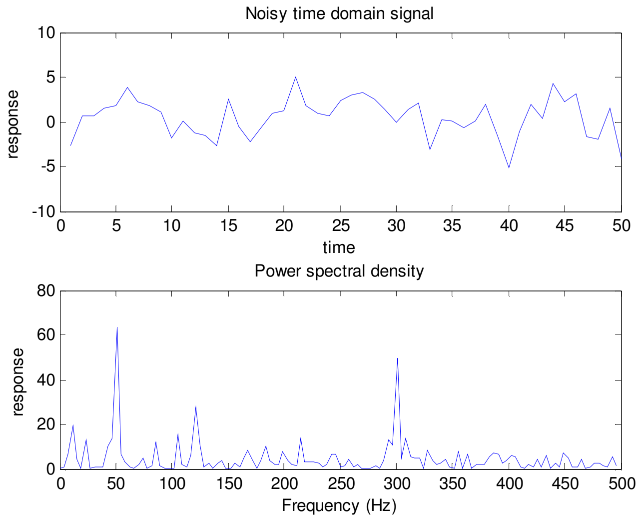
http://health.allrefer.com/health/his-bundle-electrography-ecg.html http://www.biologycorner.com/anatomy/circulatory/ecg.html

Haykin: Figure 1.9 (p. 13)

The traces shown in (a), (b), and (c) are three examples of electroencephalogram (EEG) signals recorded from the hippocampus of a rat. Neurobiological studies suggest that the hippocampus plays a key role in certain aspects of learning and memory.

TRY THIS



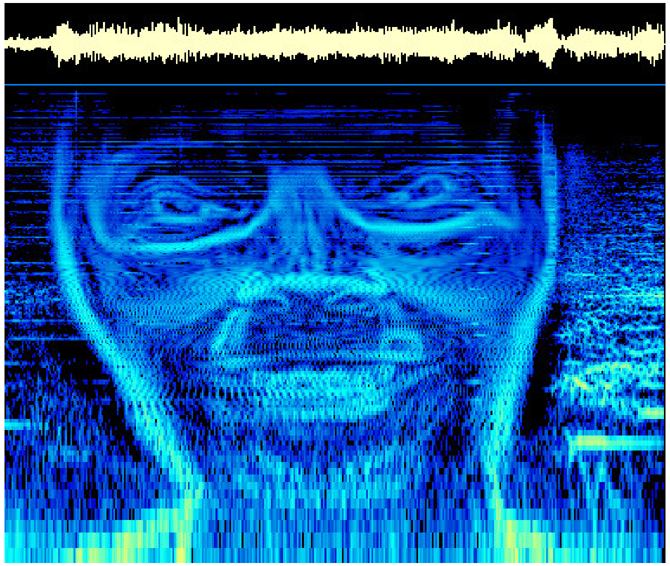


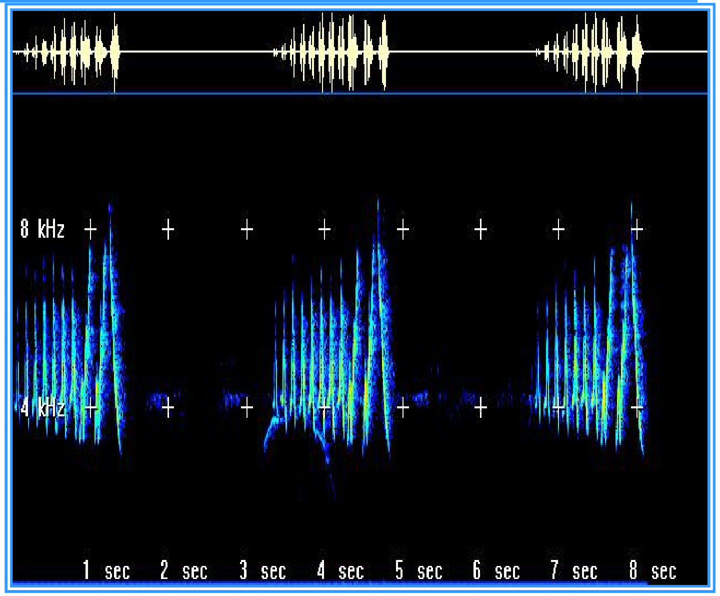

```
t = 0:.001:.25;
 x = sin(2*pi*50*t) + sin(2*pi*120*t) + sin(2*pi*300*t);
 y = x + 2*randn(size(t));
 subplot (2,1,1),plot(y(1:50))
 title('Noisy time domain signal')
xlabel('time')
ylabel ('response')

Y = fft(y,256);
 Pyy = Y.*conj(Y)/256;
 f = 1000/256*(0:127);
 subplot(2,1,2),plot(f,Pyy(1:128))
 title('Power spectral density')
 xlabel('Frequency (Hz)')
ylabel ('response‘)
```

Signals: Spectrogram

2) Audio signals such as speech waveform or music. Signal processing can be developed to characterise the speech signals in terms of their frequency spectrum. (Spectrogram)

frequency

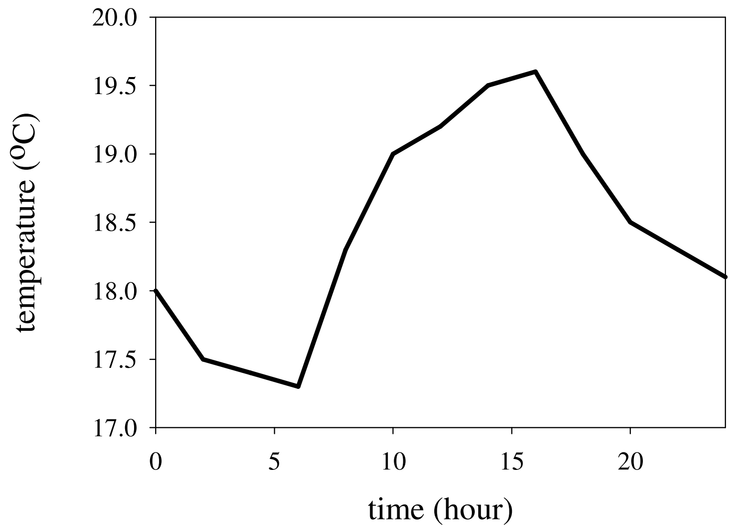




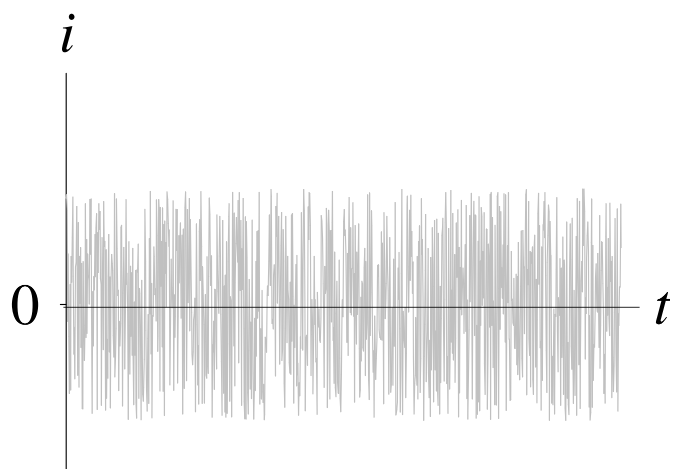
time

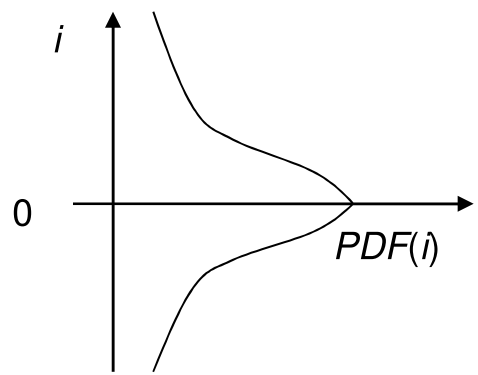
Signals: Temperature and current fluctuation

3) Temperature variation in a room during summer.



4) Current fluctuation in electronic equipments due to random motion of electrons.





Signals: Image processing

A signal $x(t)$ is real-valued or scalar valued if the value of $x(t)$ is a real number at time t.

$x(t)$ is a vector signal if it is a vector of some dimension for example the voltage at 3 points of an antenna.

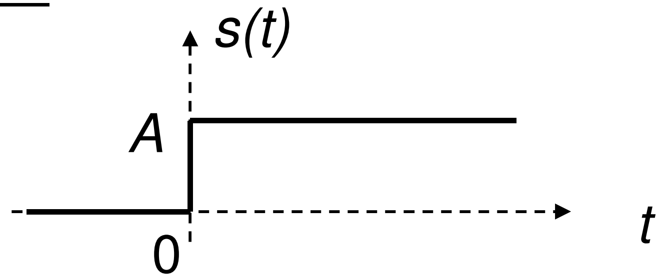
So far we have only mentioned signal that varies with time. In image processing the variation of brightness across the image is important.

Edge detection of the image

Step functions

The Step and Ramp Functions

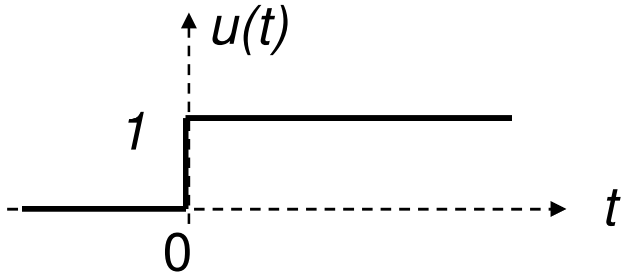
$$s(t) = \begin{cases} 0, & t < 0 \\ A, & t \geq 0 \end{cases}$$



When $A = 1$ the step function is known as a

Unit Step function

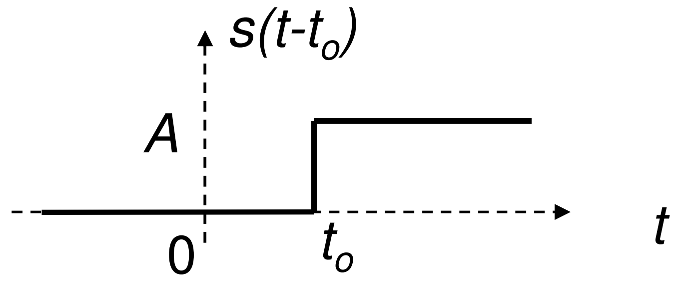
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$



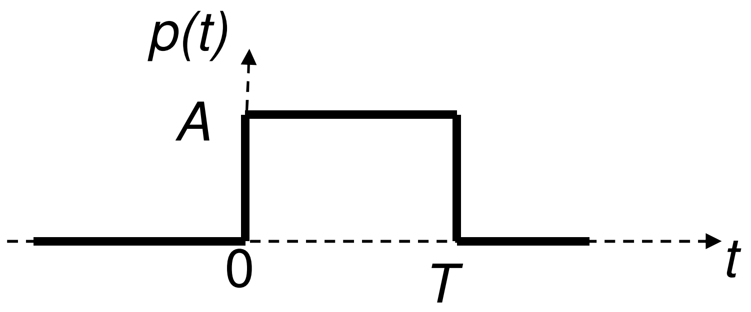
In general $s(t) = Au(t)$ where $A \neq 0$. E.g: current flow through a resistive circuit in which the switch is closed at time $t = 0$. The current is zero for $t < 0$ and has a constant value for $t \geq 0$.

Step functions

If the switch is closed at $t = t_0$ a delay step signal, $s(t- t_0)$ is obtained.



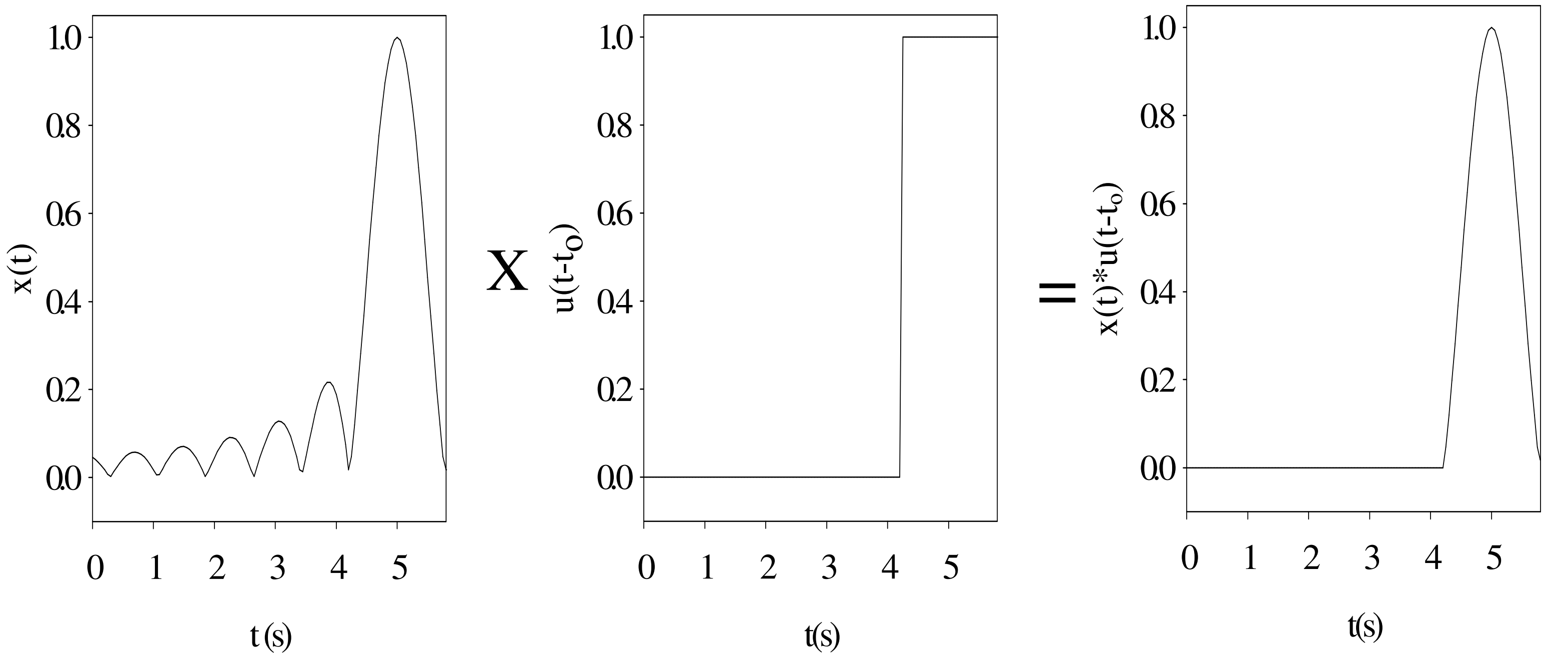
If the switch is closed at $t = 0$ and opened at time $t = T$, a pulse signal, $p(t)$ of width T is obtained.



Deleting signal using $u(t)$

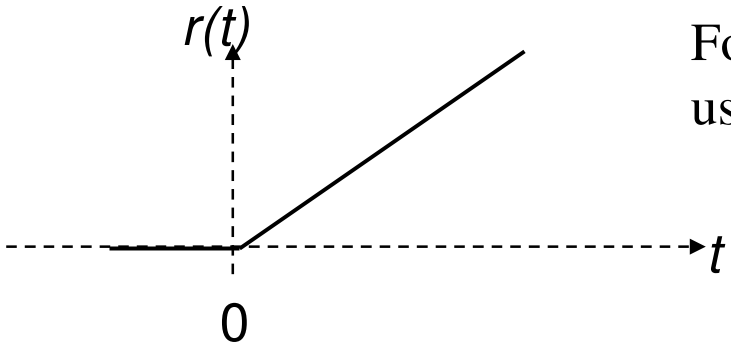
$u(t)$ is useful for deleting parts of signals in time

$$x(t)u(t-t_o) = \begin{cases} 0, & t < t_o \\ x(t), & t \geq t_o \end{cases}$$



Ramp function and periodic signal

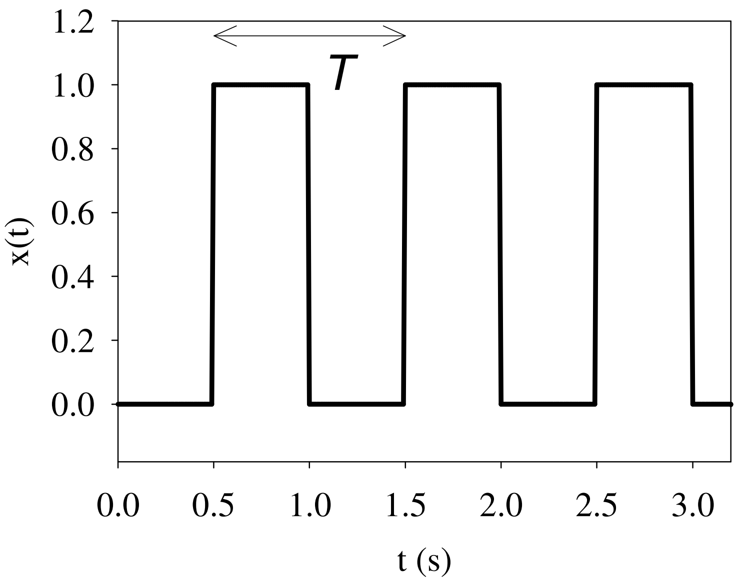
If we integrate $u(t)$, a unit ramp function, $r(t)$ is obtained.



For any slope other than unity we can use $kr(t)$ where $k \neq 0$.

$$r(t) = \begin{cases} 0, & t < 0 \\ \int_0^t 1 d\tau = t, & t \geq 0 \end{cases}$$

A CT signal $x(t)$ is periodic with period T if $x(t) = x(t+T)$ for all T.



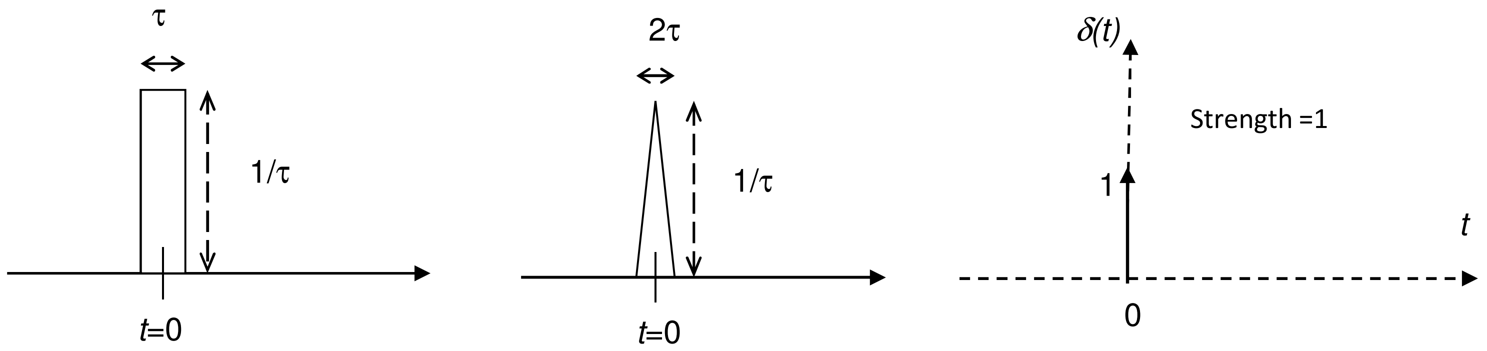
Unit Impulse

Unit impulse $\delta(t)$ is an idealisation of a signal that

- is zero for all nonzero values of t: i.e, $\delta(t) = 0$ for $t \neq 0$ and $\delta(t) = 1$ for $t = 0$.

- has an area of unity : $\int_{-a}^{a} \delta(\tau) d\tau = 1$ for any real number $a > 0$.

for example:



Unit impulse approximated by a square pulse and a triangle pulse when $\tau \to 0$. $\delta(t)$ is often represented by an arrow

For any real number K, $K\delta(t)$ is the impulse with area K.

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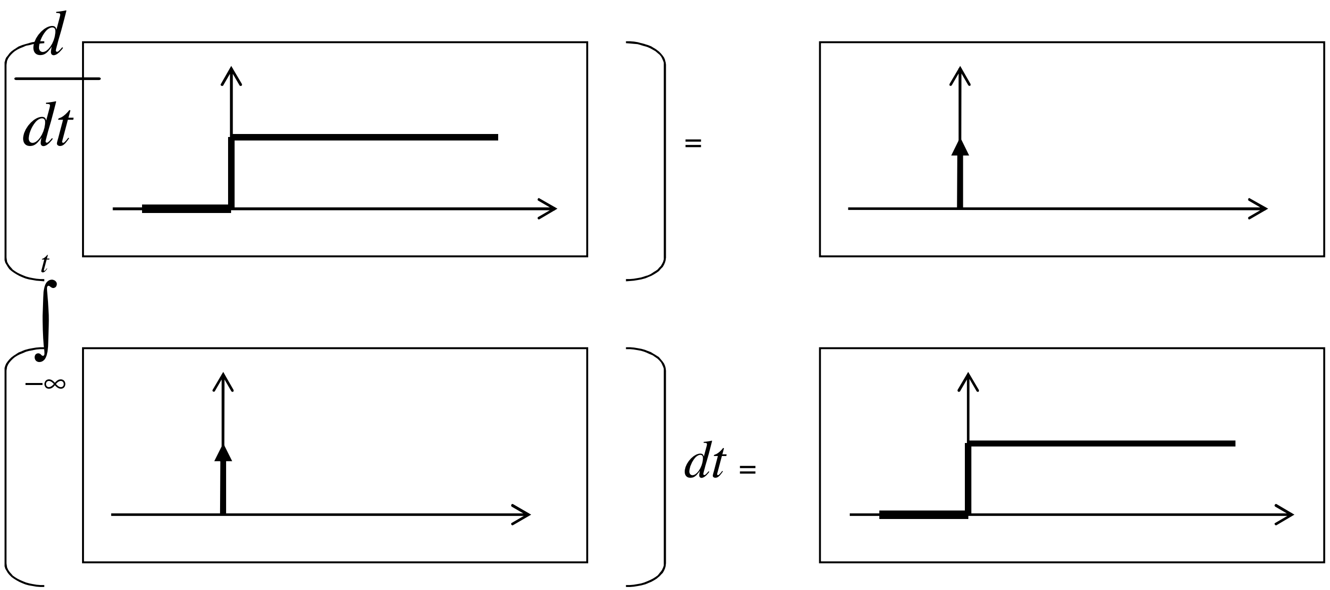
Unit Impulse

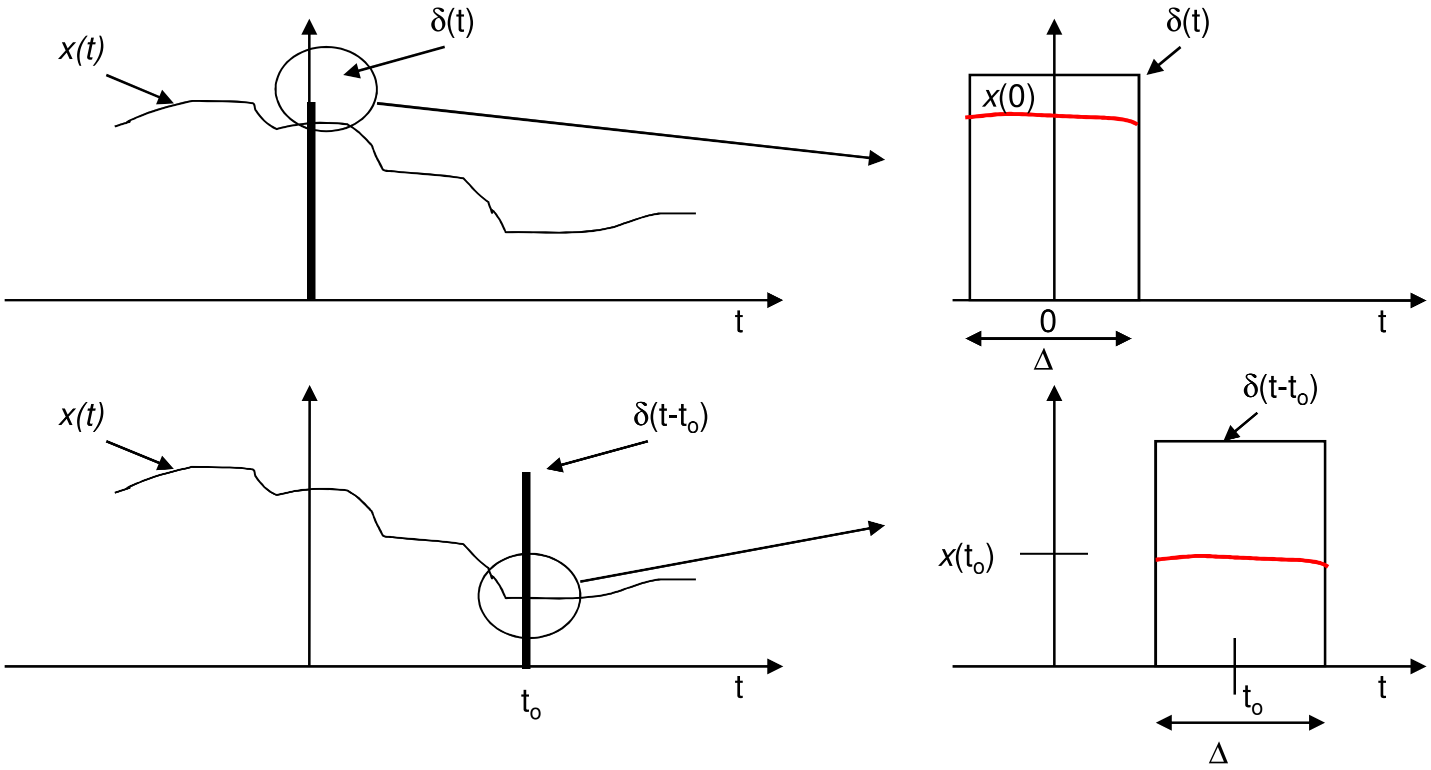
What happens if we integrate $\delta(t)$?

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t) \quad \Leftrightarrow \quad \frac{du(t)}{dt} = \delta(t)$$



Unit Impulse

- It can be shown that $$x(t)\delta(t-t_o)=x(t_o)\delta(t-t_o)$$

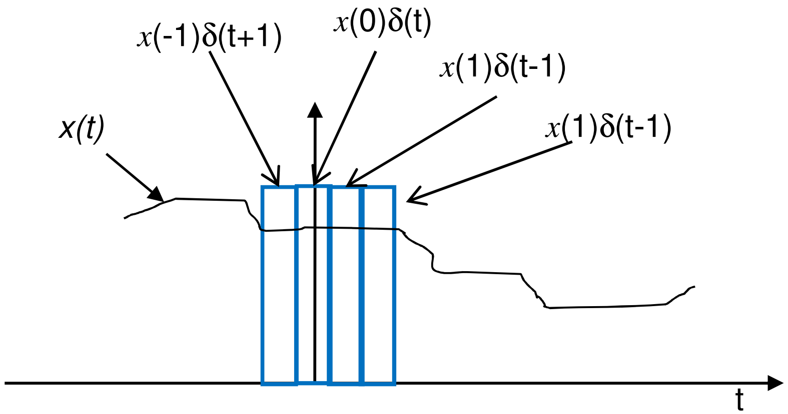


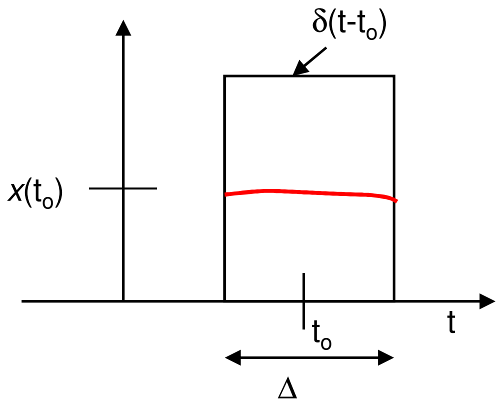
Consider the product $x(t)\delta(t)$ depicted above. If $\Delta \to 0$, $x(t)\delta(t) \approx x(0)\delta(t)$.
Using similar argument we have $x(t)\delta(t-t_o) = x(t_o)\delta(t-t_o)$.

Unit Impulse

$$x(t) = \ldots + x(-1)\delta(t+1)\,\Delta + x(0)\delta(t)\,\Delta + x(1)\delta(t-1)\,\Delta + x(2)\delta(t-2)\,\Delta + \ldots\ldots$$

$$x(t) = \sum_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\Delta$$

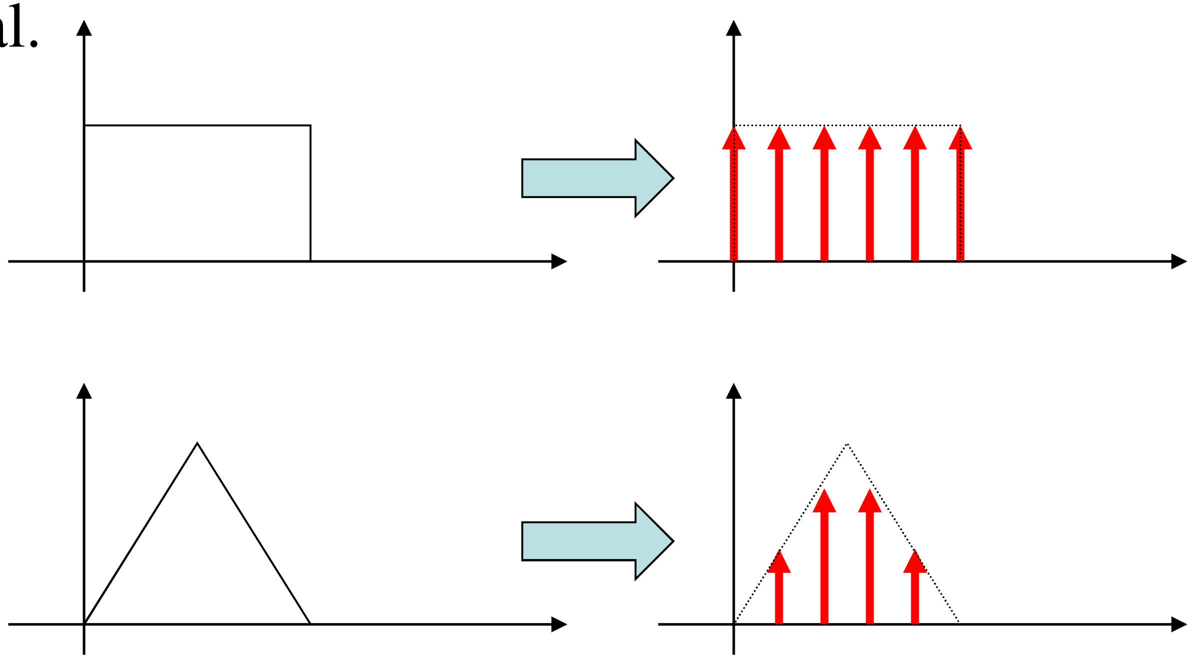




$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$$

Unit Impulse

Unit impulse can be used to construct discrete version of any CT signal.



$x(t) = \ldots x(0)\delta(t) + x(1)\delta(t-1) + x(2)\delta(t-2) \ldots\ldots$

Let τ = 0, 1, 2 …

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$$

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Unit Impulse

- The product of a signal *x(t)* with an impulse is very important in sampling of CT signals because any CT signals can be considered as a summation of an infinite number of adjacent scaled impulses, i.e

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

- Consider

$$\int_{-a}^{a} f(t)\delta(t-t_o)dt = \int_{-a}^{a} f(t_o)\delta(t-t_o)dt = f(t_o)\int_{-a}^{a} \delta(t-t_o)dt = f(t_o)$$

provided that $-a < t_o < a$.

- Therefore we can move the function *f(t)* out of the integration and substitute $t = t_o$.