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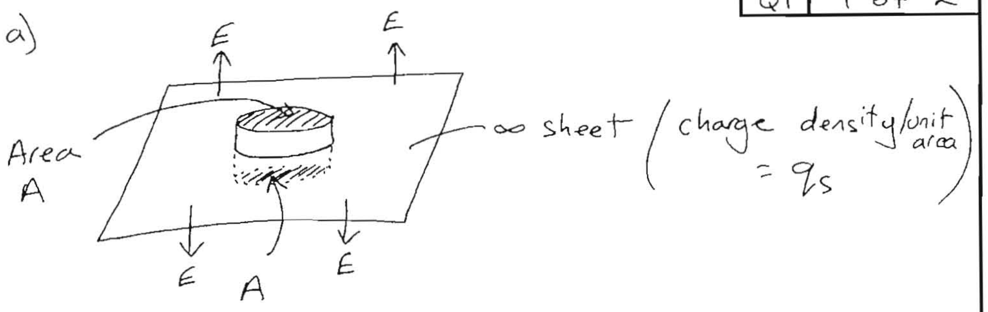
a)



By symmetry E is perpendicular to sheet and independent of lateral position and has the same value on either side.

Gauss' Law $\oint_S E\,dA = \dfrac{q_s A}{\varepsilon_0}$ ← enclosed charge

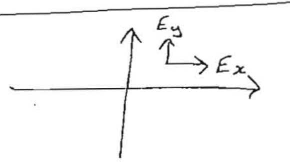
$\therefore \quad E \times 2A = \dfrac{q_s A}{\varepsilon_0}$

contribution from top and bottom surfaces.

$\rightarrow \quad E = \dfrac{q_s}{2\varepsilon_0}$

[5]

b)



$E_x = E_y$: field due to infinite sheet

$= \dfrac{q_s}{2\varepsilon_0}$

$= \dfrac{5\times10^{-6}\ C/m^2}{2\times 8.854\times10^{-12}\ F/m} = 2.82\times10^{5}\ Vm^{-1}$ (or NC^{-1})

$\Rightarrow$ E-field at:-

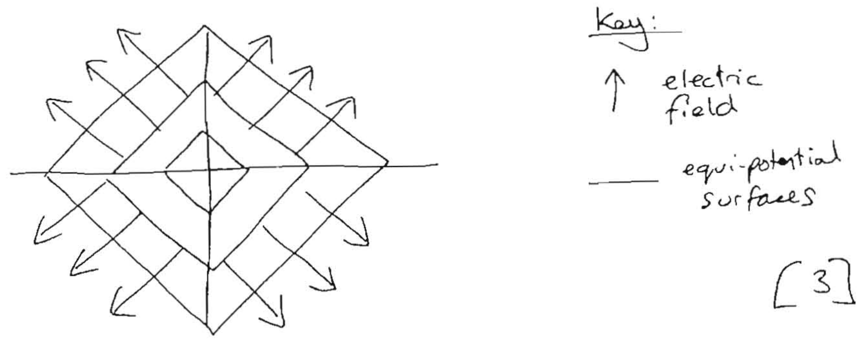
A $(2.82, 2.82, 0)\ \times10^{5}\ Vm^{-1}$

B $(2.82, 2.82, 0)\ \times10^{5}\ Vm^{-1}$

C $(2.82, -2.82, 0)\ \times10^{5}\ Vm^{-1}$

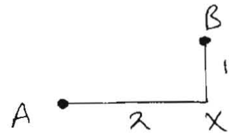
[6]

c)



[3]

d) Split problem up into 2 p.d.s



$$\text{p.d.} = \phi(B) - \phi(A) = [\phi(X) - \phi(A)] + [\phi(B) - \phi(X)]$$

$$= -\int_A^X E_x\,dl - \int_X^B E_y\,dl$$

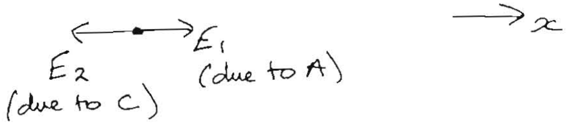
$$= -2E_x - E_y$$

$$= -8.46\times10^{5}\ V$$

[6]

Q2

a) i) At B, there are 2 E-fields



$$E_1 = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{2\times10^{-6}}{4\times\pi\times8.854\times10^{-12}\times(1)^2} \quad V/m$$

$$E_2 = \frac{3\times10^{-6}}{4\pi\times8.854\times10^{-12}\times1} \quad V/m$$

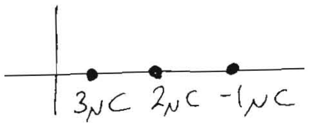
$\Rightarrow$ total field (in x-direction) is

$$\underline{E} = \left(\frac{-1\times10^{-6}}{4\pi\times8.854\times10^{-12}},\ 0,\ 0\right) \quad V/m$$

$$\underline{F} = q\underline{E}$$
$$= -1\times10^{-6}\times\underline{E}$$
$$= \left(8.99\times10^{-3},\ 0,\ 0\right) \quad N$$

[5]

a) ii) Max field at origin when...



$$-E_x = \frac{3\times10^{-6}}{4\pi\varepsilon_0\times(1)^2} + \frac{2\times10^{-6}}{4\pi\varepsilon_0\times(2)^2} - \frac{1\times10^{-6}}{4\pi\varepsilon_0\times(3)^2}$$

$$E_x = \left(-3.05\times10^4,\ 0,\ 0\right) \quad V/m$$

[5]

b) i) Treat as 3 capacitors in parallel

$$C_1 = \frac{\varepsilon_0\varepsilon_{r1}A_1}{d} \qquad C_2 = \frac{\varepsilon_0\varepsilon_{r2}A_2}{d}$$

$$C_3 = \frac{\varepsilon_0\varepsilon_{r3}A_3}{d}$$

$$C = C_1 + C_2 + C_3 \qquad \left(A_1 = A_2 = A_3 = \frac{ab}{3}\right)$$

$$C = \frac{\varepsilon_0 ab}{3d}\left(\varepsilon_{r1}+\varepsilon_{r2}+\varepsilon_{r3}\right)$$

[6]

ii) Energy stored $= \frac{1}{2}CV^2$

$$= \frac{\varepsilon_0 ab}{6d}\left(\varepsilon_{r1}+\varepsilon_{r2}+\varepsilon_{r3}\right)\times V^2$$

$$= \frac{8.854\times10^{-12}\times0.01\times0.005}{6\times0.002}\left(4+5.5+3\right)\times12^2$$

$$= 6.64\times10^{-11}\ J$$

[4]

Q4

a) Biot-Savart Law $\underline{B} = \frac{\mu_0 I}{4\pi} \int_L \frac{d\underline{l} \times \hat{\underline{r}}}{r^2}$

For radial sections of circuit $d\underline{l}$ is parallel to $\hat{\underline{r}}$ so no contribution to field.

for arc section $d\underline{l}$ is perpendicular to $\hat{\underline{r}}$ so $|d\underline{l} \times \hat{\underline{r}}| = dl$

and field magnitude is given by $B = \frac{\mu_0 I}{4\pi} \int_L \frac{dl}{r^2}$

or $\frac{\mu_0 I}{4\pi} \int_0^\theta \frac{r\,d\theta}{r^2}$

For arc radius a, $B_a = \frac{\mu_0 I}{4\pi} \int_0^\pi \frac{d\theta}{a} = \frac{\mu_0 I}{4a}$

using RHR, B_a is OUT OF the paper

for arc radius b, $B_b = \frac{\mu_0 I}{4\pi b} \int_0^{\pi/2} d\theta = \frac{\mu_0 I}{8b}$

using RHR, B_b is INTO the paper

$\Rightarrow$ Total field $= B = \frac{\mu_0 I}{4}\left[\frac{1}{a} - \frac{1}{2b}\right]$ [10]

b) The mutual inductance between the two loops is given by

$$M = \frac{\Phi}{I}$$

where Φ is the flux through the second, smaller, loop. Assuming that the B field is uniform over the area of the loop (a good approximation since $A \ll d^2$)

$$\Phi = BA$$

From formula sheet, B due to one loop is...

$$B = \frac{\mu_0 I a^2}{2(a^2+d^2)^{3/2}}$$

thus, B due to N loops is N times this.

$$\Rightarrow \Phi = \frac{\mu_0 N I A a^2}{2(a^2+d^2)^{3/2}}$$

$$M = \frac{\mu_0 N A a^2}{2(a^2+d^2)^{3/2}}$$ [8]

For $N = 150$, $A = 10^{-4}$ m, $a = 0.2$ m, $d = 0.1$ m

$$M = 3.37 \times 10^{-7} \text{ H}$$

[2]

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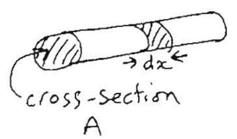
Q3.

a) – Current is due to moving charged particles



Conductor has: –

n charge carriers / unit volume

q = charge of each particle

v = velocity of charges

Amount of charge in a small volume, dx thick, : –

$$dQ = qnA\,dx$$

Force acting on this charge is: –

$$d\underline{F} = dQ\,\underline{v} \times \underline{B} = qnA\,dx\,\underline{v} \times \underline{B}$$

Writing $\underline{v}\,dx$ as $v\,d\underline{x}$ gives: –

$$d\underline{F} = qnAv\,d\underline{x} \times \underline{B}$$

Current $i = \dfrac{dQ}{dt} = qnA\dfrac{dx}{dt}$

and $\dfrac{dx}{dt} = v$

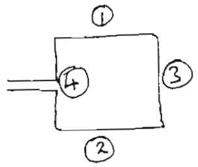
so $i = qnAv$

Substituting this into the equation above gives: –

$$d\underline{F} = i\,d\underline{x} \times \underline{B}$$

or $\underline{F} = i\int_L d\underline{l} \times \underline{B}$ [6]

b) (i)



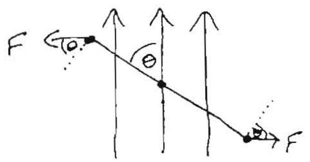
$F_{①} = ILB$
$= 0.1 \times 0.05 \times 0.5$
$= 2.5 \times 10^{-3}$ N (out of the page)

$F_{②} = ILB$
$= 2.5 \times 10^{-3}$ N (in to the page)

$F_{③} = 0$ (as wire is parallel to the field.)

[4]

(ii) Looking at the loop from the RHS...



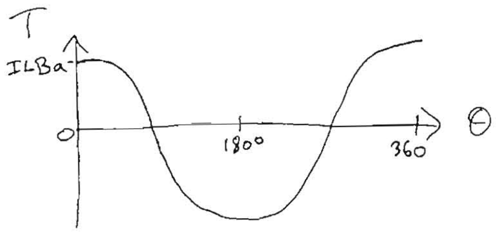
Torque = Force × distance from pivot

$\Rightarrow T = F\cos\theta \times \frac{a}{2} + F\cos\theta \times \frac{a}{2}$

$= Fa\cos\theta$

(Note that sides ③ and ④ will experience a force when $\theta > 0$, but no torque.)

$$T = ILBa\cos\theta$$



[6]

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(iii)

- for $90° < \theta < 270°$, torque is in opposite direction.
- a solution is to add a commutator to change the direction of the current every 180°
- when $\theta = 90°, 270°$, torque is zero which would prevent the motor from starting from one of these positions.
- a solution is to add a second loop at 90° to the first

[4]