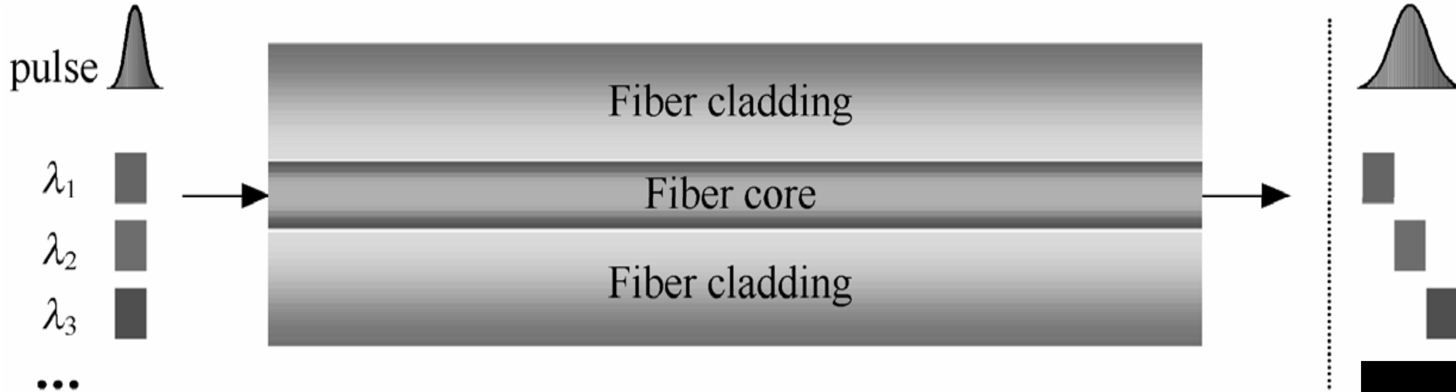
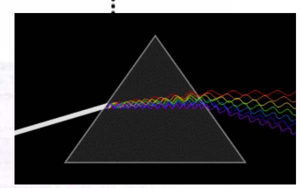
Topic 3

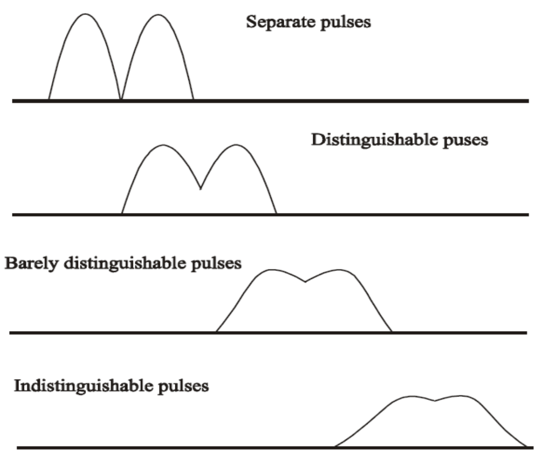
3. **Optical dispersion**

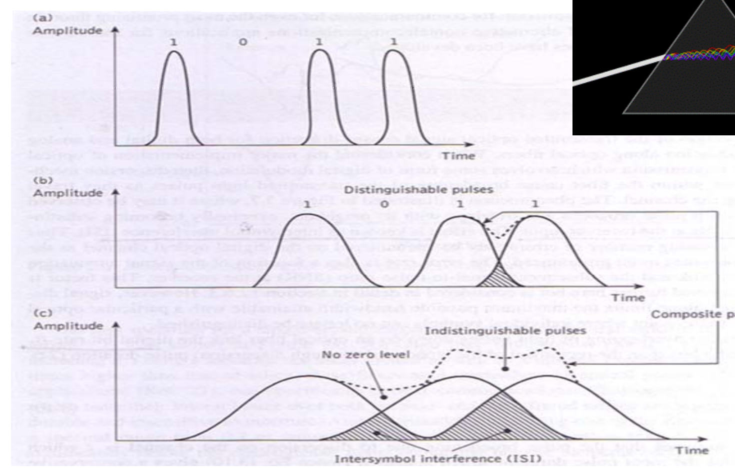
- 3.1 Introduction
- 3.2 Intermodal Dispersion
- 3.3 Intramodal Dispersion
  - Material dispersion
  - Waveguide Dispersion
  - Chromatic Dispersion in SM fibre

Introduction







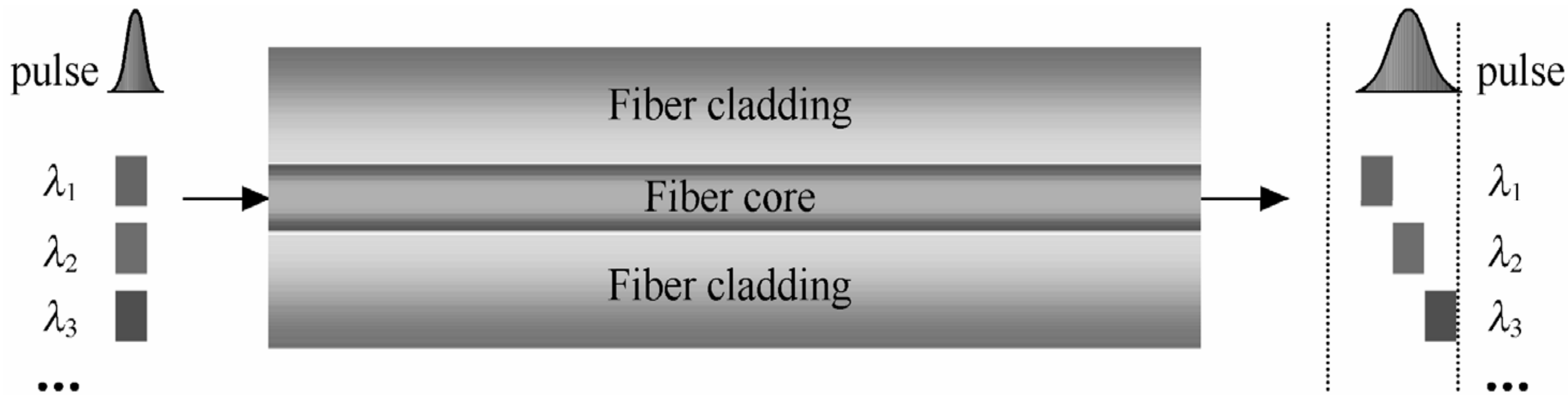


Dispersion:

(1) cause optical pulse broadening, leading to indistinguishable signal at the receiver input (i.e., so-called intersymbol interference)

(2) Cause a reduction in Bit-rate (B_T) of transmission
narrow optical pulse: more optical pulses can be carried within a given time, i.e., **high Bit-rate**

Transmission characteristics of optical fibres



Signal: **high intensity and less distortion** during transmission, which are related to **optical loss** and **dispersion**

- Origins for dispersion (causing signal distortion)

(i) Intermodal Dispersion; (ii) intramodal dispersion

- Origins for optical loss:

(i) Material absorption; (ii) Scattering loss; (iii) Fibre bending

Pulse broadening due to Dispersion

- Intermodal dispersion:
  Propagation delay differences between modes within a multiple mode optical fibre
  Depending on the transmission time between **the slowest and the fastest modes**

$\Rightarrow$ Pulse broadening
Thus it can occur in **multiple mode optical fibre**

- Chromatic Dispersion (Intramodal Dispersion):
  Due the spectral **linewidth of the optical source**, i.e., optical source does not emit just a single frequency but a band of frequencies, thus it can occur in all types of optical fibre

$\Rightarrow$ Various spectral components (wavelength): **different group velocity**
$\Rightarrow$ Pulse broadening
Thus it can occur in all types of optical fibre.

Pulse broadening due to Dispersion

- Intermodal dispersion:
  Propagation delay differences between modes within a multiple fibre
  Depending on the transmission times of the slowest and fastest modes

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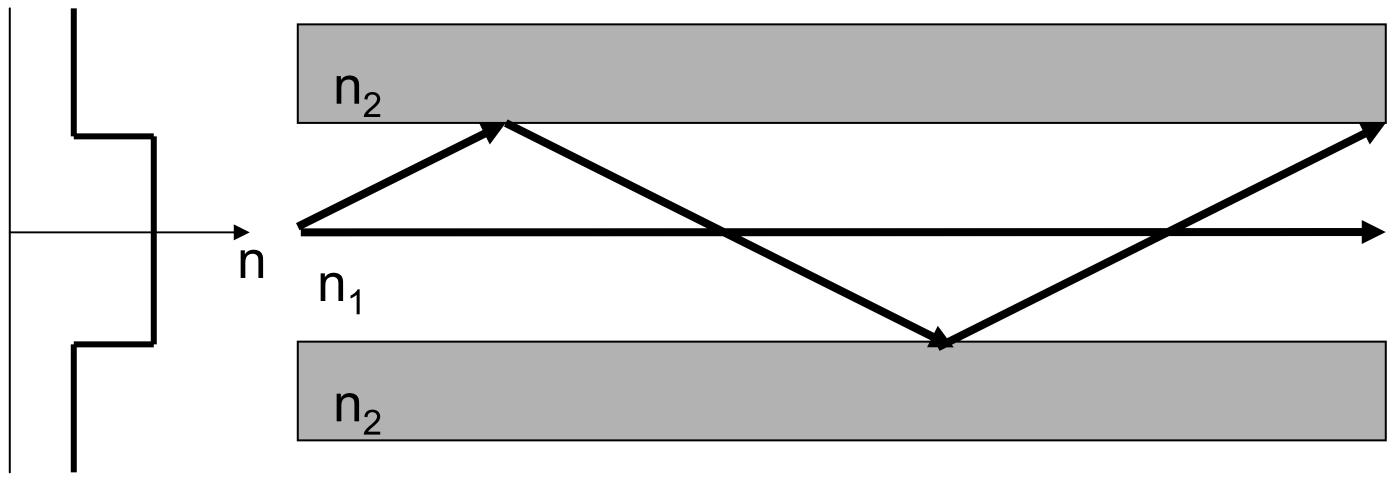
Multimode Fibre – intermodal Dispersion (1)

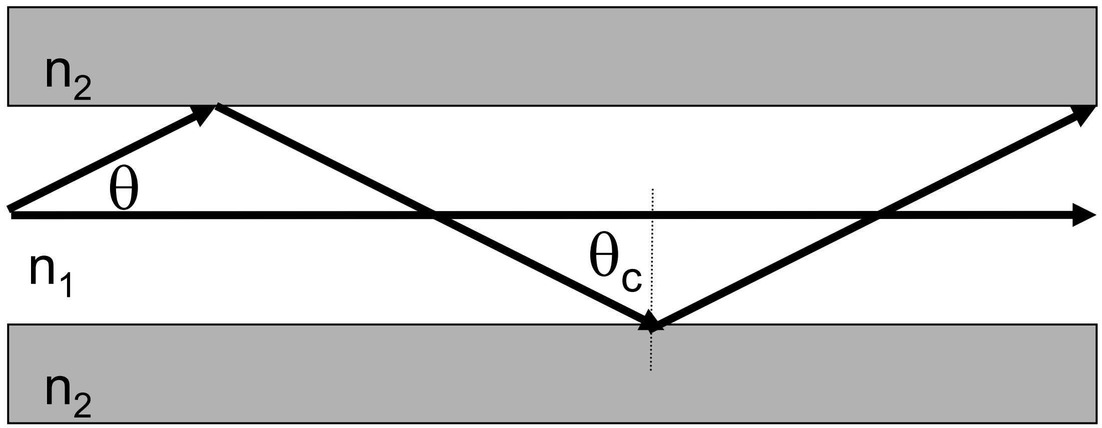
- We have known:

  Fibre with **a large diameter of Core layer**: multimode fibre (a core diameter of ~50 microns +)
- All light rays in the core travel at the same velocity (since n_{core} is constant throughout this region)



- ***Modal* dispersion due to different path lengths**

Multimode Fibre – intermodal Dispersion (2)



Time to travel distance L in a fibre for

- Meridional ray (shortest): $T_{\min} = \dfrac{L}{(c/n_1)}$
- Critical ray (longest): $T_{\max} = \dfrac{L/\cos\theta}{c/n_1}$

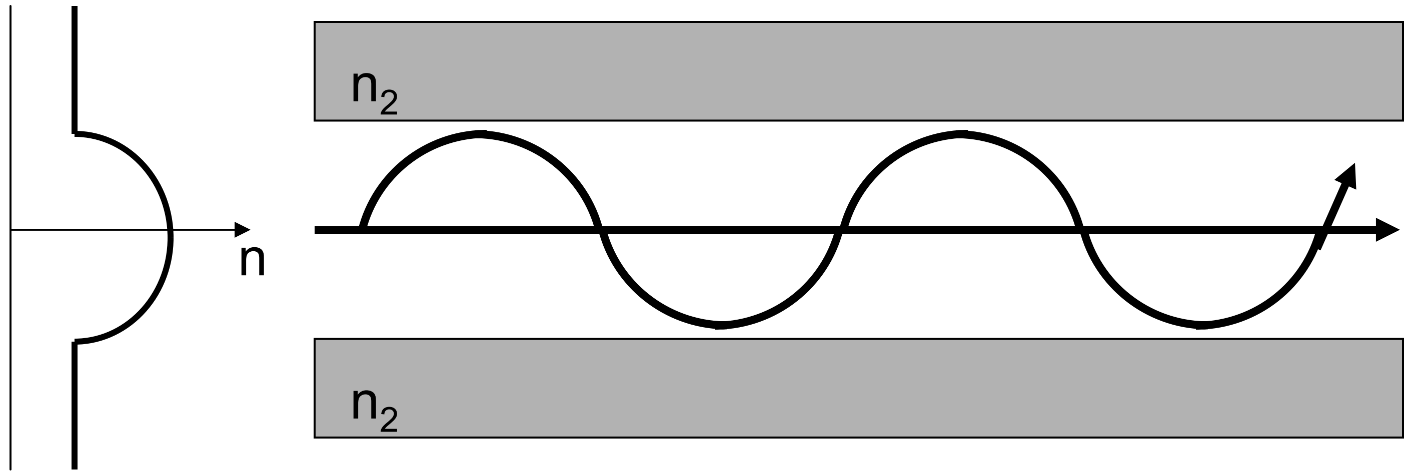
  using Snell's law $Sin\theta_c = \dfrac{n_2}{n_1} = \cos\theta$, $T_{\max} = \dfrac{L/\cos\theta}{c/n_1} = \dfrac{Ln_1^2}{cn_2}$
- Differential time delay: $\boxed{\delta T = T_{\max} - T_{\min} = \dfrac{Ln_1^2}{cn_2} - \dfrac{Ln_1}{c} = \dfrac{Ln_1^2}{cn_2}\left(\dfrac{n_1 - n_2}{n_1}\right) \approx \dfrac{Ln_1\Delta}{c}, where \quad \Delta \approx \dfrac{n_1 - n_2}{n_2}}$
- The pulse spreads in time by $\boxed{\delta t = Ln_1\Delta/c}$ in a distance L.

Graded-Index Multimode Fibre

- **Fibre has a core diameter of 50 micron**
- **Refractive Index profile:**
  - **n_{core} has a high value at the centre of the core and then gradually decreases towards the core layer & cladding layer interface.**
  - **The refractive index in the fibre core has a parabolic profile.**

$$n(r) = n_1[1 - 2\Delta(r/a)^2]^{1/2} \qquad (r < a)(core)$$

$$= n_1(1 - 2\Delta)^{1/2} \qquad (r > a)(cladding)$$

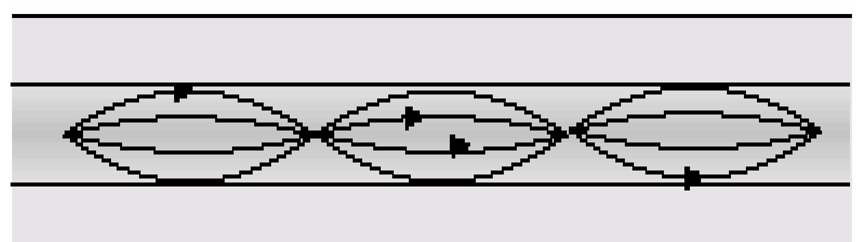


Graded-Index Multimode Fibre –Modal Dispersion

- ALL modes transit the fibre in approximately the same time
- Light travelling close to the centre of the core has the **slowest velocity** (since n is highest here, v=c/n) (Short distance and slow velocity)
- Light travelling further from the centre of the core has a higher velocity. (Longer distance but higher velocity)
- Modal dispersion only about 1 ns per km

Accurately

$$\delta T = L n_1 \Delta^2 / 8c \qquad (graded \quad index)$$
$$\delta T = L n_1 \Delta^2 / c \qquad (Step \quad index)$$



Pulse broadening due to Dispersion

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  Depending on the transmission times of the slowest and fastest modes

$\Rightarrow$ Pulse broadening
  Thus it can occur in **multiple mode optical fibre**

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  Due the finite spectral **linewidth of the optical source**, i.e., optical source do not emit just a **single frequency** but a band of frequency, thus it can occur in all types of optical fibre

$\Rightarrow$ Various spectral components: **different group velocity**

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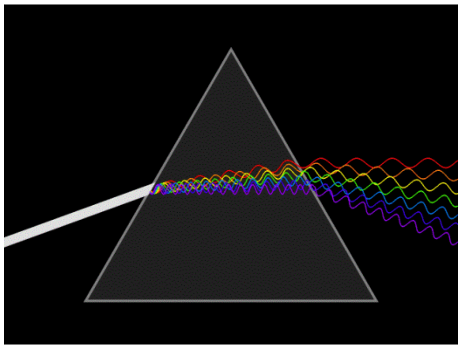
Group Index - Silica



- n=n(λ) due to the material as medium
- Introducing group index n_g

It is impossible to produce truly momochromatic light!

Group velocity: the velocity which the envelope of the wave package travel at

Definition:

$$\boxed{\upsilon_p = \frac{c}{\left(n - \lambda \frac{dn}{d\lambda}\right)} = \frac{c}{n_g}}$$

$n_{g:}$ group index

Material Dispersion (1)

Pulse broadening: Different group velocity of the various spectral components from the optical source

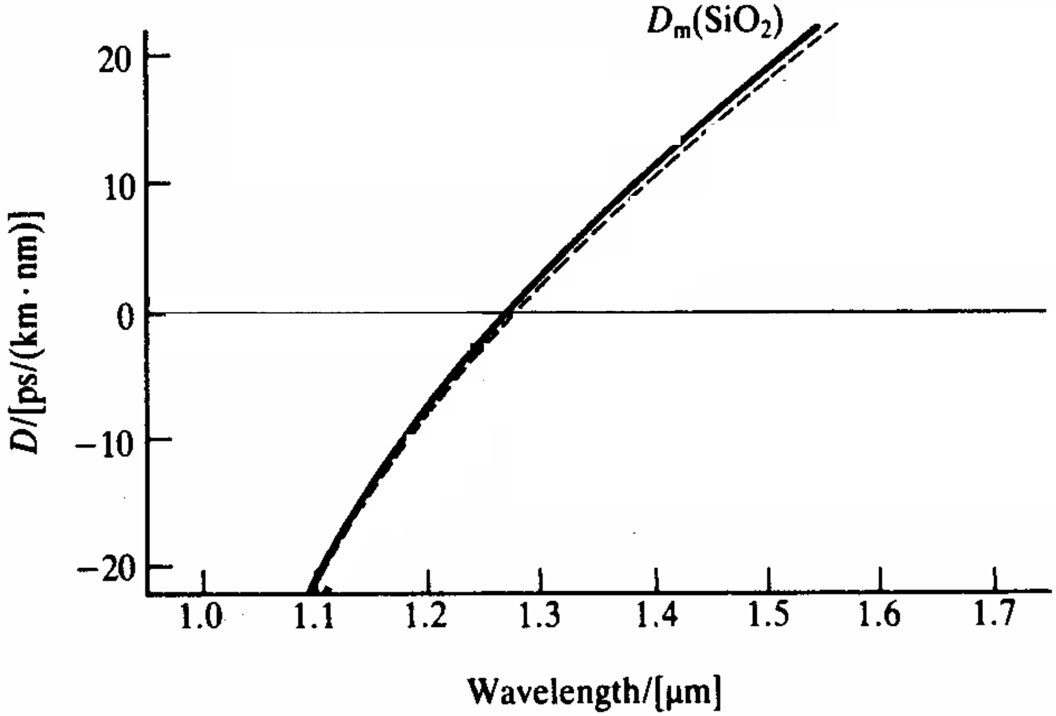
Reason: n=n(λ) due to the material as medium

Differential time delay due to material dispersion:

$$\boxed{\delta T_m = \frac{dT}{d\lambda}\Delta\lambda = \Delta\lambda L\left|\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}\right| = \Delta\lambda L\left|D_m(\lambda)\right|}$$

material dispersion coefficient: $D_m(\lambda) = \frac{\lambda}{c}\frac{d^2n}{d\lambda^2}$

Material Dispersion (2)– Silica Glass



Material Dispersion (3)- SM Fibres

•For silica – group index is a function of wavelength

Turning point at 1.25 μm

•To first order – signals with reasonable emission linewidths will propagate with no dispersion

•Away from this point - different wavelength components of emission will travel with different velocities

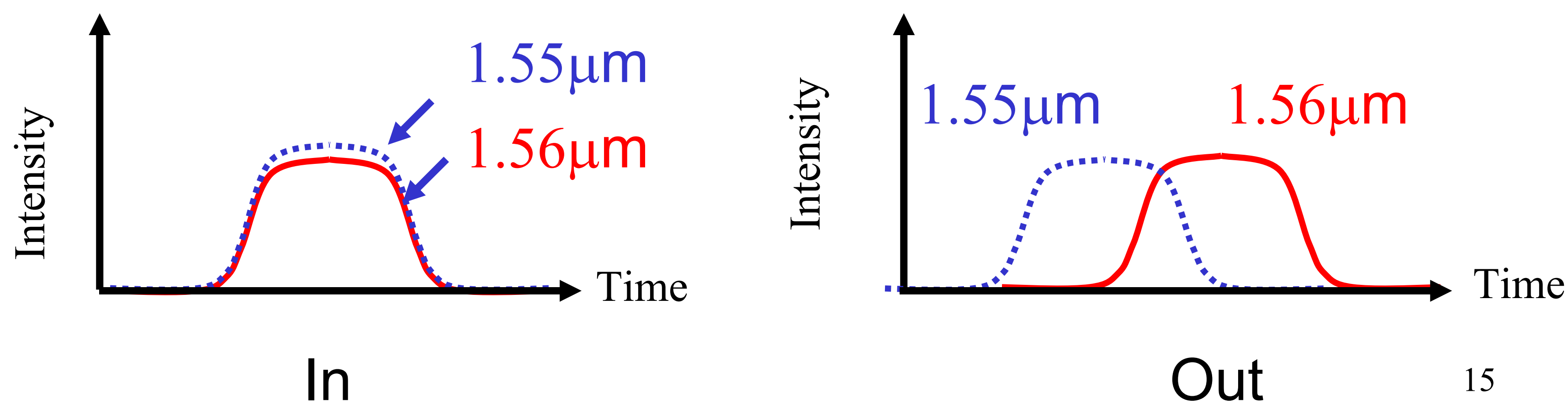
•Speed of wavepacket = c/n_g

Material Dispersion (4)

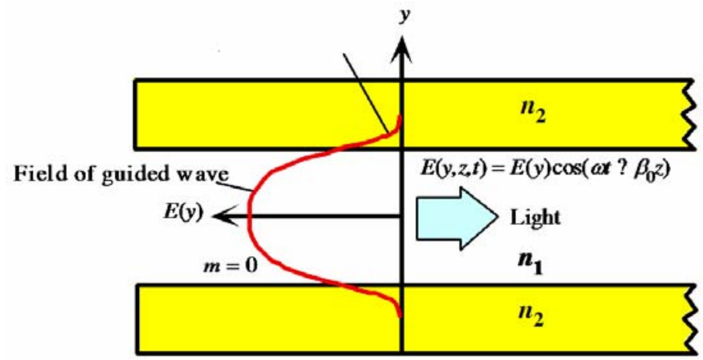
- Take case of two simultaneously injected pulses of peak wavelengths – 1.55um and 1.56um ($\Delta\lambda$ = 10nm).

- The 1.56um pulse arrives first after travelling through length L km of SiO_2 - As n_g at 1.55 um higher than at 1.56um (+ve Dispersion)

- How later does 1.55 um pulse arrive?



Waveguide Dispersion in SM Fibres



Definition: a significant fraction (~20%) of the optical power propagate in the cladding ($n_{clad} < n_{core}$), leading to the time differential delay

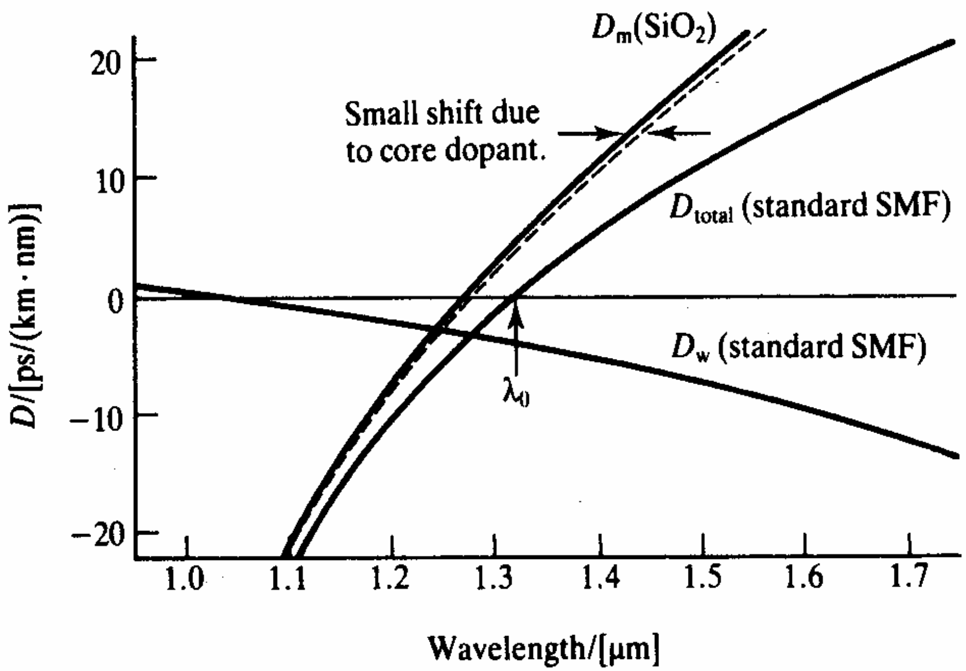
$$\boxed{\delta T_w = \left|\frac{dT_w}{d\lambda}\right|\Delta\lambda = \Delta\lambda L\left(\frac{n_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2}\right) = \Delta\lambda L\left|D_w(\lambda)\right|}$$

Waveguide dispersion coefficient: $D_w(\lambda) = -\frac{n_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2}$

$\Delta\lambda$: FWHM of optical source; b: normalized propagation constant; V: normalized frequency

Normally, the waveguide dispersion coefficient is negative!

Chromatic Dispersion - SM Fibres



Have **zero dispersion** at ~1.3 μm

SM fibres: total intramodal dispersion (Chromatic dispersion)

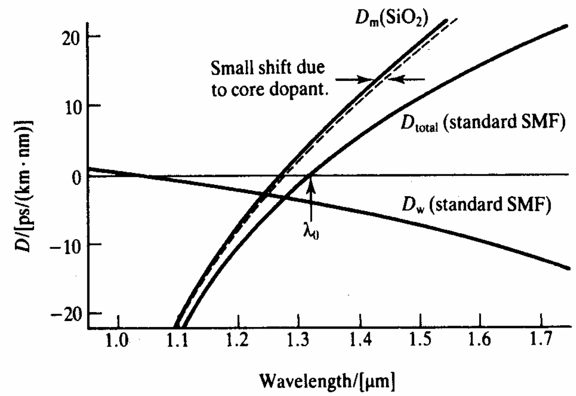
Material dispersion and **Waveguide dispersion**

Dispersion coefficient: D=D$_m$+ D$_w$= $\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}-\frac{n_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2}$

(It is possible for them to be made to cancel each other)

Dispersion Coefficient

$\Delta\delta t = L\Delta\lambda\ D(\lambda)$



Choose $\lambda = 1.31\mu m$ for $D(\lambda) = 0$.

- Note $dD/d\lambda \neq 0$, leading to second order dispersion:
- At $\lambda = 1.55\mu m$, $D(\lambda) = 15ps/(km.nm)$.
- Eg, when $\Delta\lambda = 5nm$ (Fabry-Pérot laser), $L = 10km$,

$\delta t \approx L\Delta\lambda D = 0.75ns$.

- Design single mode fibre so that '**waveguide dispersion**' (variation of speed of single mode with λ) compensates for this.

Dispersion Summary

Intermodal dispersion
(i.e., between modes)

MODAL DISPERSION

Multi-mode fibres only; minimised for graded-index fibres

Chromatic or
Intramodal dispersion
(i.e., within a single mode)

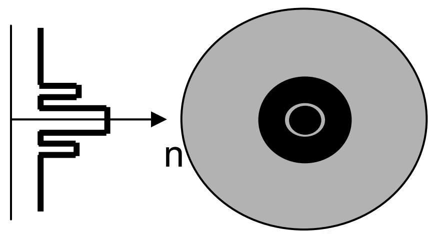
MATERIAL DISPERSION

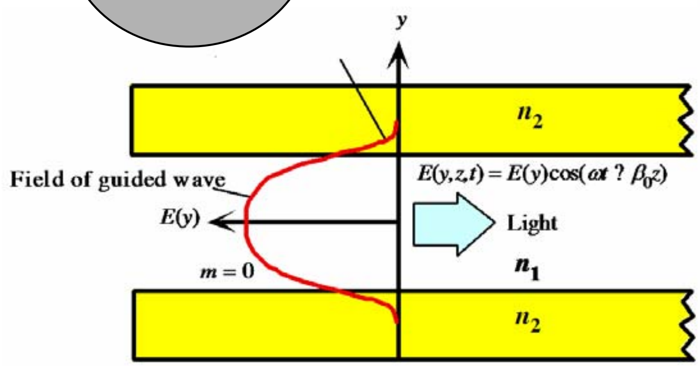
WAVEGIUDE DISPERSION

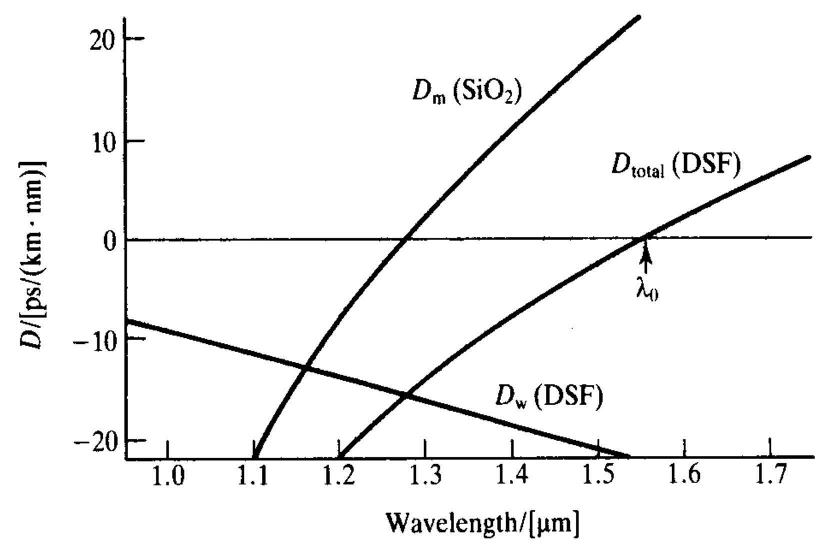
Single or multi-mode fibres but only significant for single mode fibres.

These dispersions are wavelength dependent

Dispersion Shifted SM Fibre







A change in SMF refractive index profile: tune zero-dispersion wavelength point to a specific wavelength. The changes include: reduce the core diameter and increase difference in refractive index between core layer and cladding layer.

Mechanism: change in optical confinement, leading to the change in fraction of the optical power propagating in the cladding **(Δn↑, confinement ↑, fraction of optical power in the cladding ↓)**

$\Rightarrow$ Change the waveguide dispersion (i.e., makes it more or less **negative**). For example, the material dispersion is now cancelled at longer wavelengths (i.e. zero dispersion where attenuation is also a minimum: 1550nm).

T3 Summary

- **Multimode Fibre**
- Consider ray picture for light propagation
- Snell's law – a ray of light incident at a particular angle to a step change in refractive index will be totally internally reflected
- Allows acceptance angle and numerical aperture to be determined

- **Inter-Modal Dispersion**
- Considering critical ray and meridional ray – transit time down a length of fibre is different for the two different modes
- This intermodal dispersion severely limits distance-bit rate product
- Can be partially overcome by using graded index fibre – extra path compensated by lower n, higher v_{group}

T3 Summary

- **Single Mode Fibre – Chromatic Dispersion**
- Cannot use ray picture – solving Maxwell's equations gives a single mode which sits within the core *and* the cladding region
- As only one mode intermodal dispersion is eliminated - Dispersion is now due to $n_{group}(\lambda)$
- Chromatic Dispersion of single mode fibre made up of two components – material ($n_{group}(\lambda)$) and waveguide dispersion (average n_{group} the mode sees is a function of λ as mode size is fn of λ)
- Can engineer waveguide dispersion by fibre design to give zero dispersion at a chosen wavelength

T3 Tutorial Questions

T 3.1 Draw the refractive index profile of step-index multimode fibre with core index n_1, cladding index n_2, and core diameter D. Explain the terms “meridonal ray”, “skew ray”, “axial ray”, and “critical ray”. Trace the path of a ray which crosses the fibre axis at an angle before entering the fibre to be guided by total internal reflection . Define Numerical Aperture and show that it is given by $NA= \sqrt{(n_1^2-n_2^2)}$. Explain how NA determines the ability of the fibre to collect light. What is modal dispersion? Explain this in terms of an axial and critical ray and show that a pulse spreads in time by $(Ln_1/c)(n_1-n_2)/ n_2$. Explain how a multimode GRIN fibre reduces this modal dispersion.

T3 Tutorial Questions

T3.2 A step index multi-mode fibre has a core index of refraction of 1.5 and an index step Δ of 0.02. Calculate the maximum angle which a guided ray may have relative to the axis (a) inside the fibre, (b) in air outside the fibre before launching. (c) Calculate the maximum time delay difference per kilometer between the axial ray and a guided non-axial. What is the value of the numerical aperture of the fibre?

Answers – (a) 11.5° to axis; (b) 17.4° to axis; (c) 10^{-7} s/km

(d) NA = 0.3

T3.3 What is meant by intermodal and intramodal dispersion?

T3.4 Describe, using figures if necessary the origin of chromatic dispersion in a single-mode fibre.

Exercise

Question-1: calculate the number of modes of waveguide

For an planar waveguide described as above, $d = 2a = 100\ \mu m$; $n_1 = 1.490$; $n_2 = 1.470$, if the waveguide is designed for a light with an wavelength of $\lambda = 1\ \mu m$, please estimate the **number of modes**.