

EEE105 Tutorial Question Set 5

Solutions

1. First we are asked to calculate the mobility. As the material is strongly n-type we can assume that the holes contribution to lowering the resistivity is minimal. [Remember that $np=n_i^2$ and that n type material $n \gg n_i$ and so $n \gg p$.] Let us use the simple equation:

$$\mu = \frac{1}{nq\rho}$$

where $\rho = 4.3 \times 10^{-3} \Omega\text{m}$ and $n = 1.2 \times 10^{22} \text{ m}^{-3}$ (given in the question) and q is the charge on the electron. Substituting gives $\mu = 0.12 \text{ m}^2\text{V}^{-1}\text{s}^{-1}$.

The second part is to calculate the mean time between collisions (or the mean scattering time). [Recall from the notes that electrons accelerate in the electric field until they hit something and scatter. τ is the average time between these scattering events.]

Now we know that this time is directly related to the mobility via

$$\mu = \frac{\tau q}{m_e^*}$$

This equation can be simply rewritten to solve for τ . The mobility was calculated above and the effective mass is given as $0.98m_e$.

Substituting in we can obtain $\tau = 6.7 \times 10^{-13} \text{ s}$ (or **0.67 ps**)

Note that the value of time between collisions is an extremely small value, as we might expect if the electrons are rattling around inside the material.

2. The point of this question is to emphasise in your mind what the Einstein relation is and what it is used for. To remind you, the Einstein relation relates diffusion coefficient to mobility by

$$D = \frac{kT}{q} \mu$$

All that is required is to plug in the values (I'll use my rule of thumb that kT at room temp is $1/40 \text{ eV}$)

For Si we can use the mobility from part 1, so $D = 0.0031 \text{ m}^2\text{s}^{-1}$

For Ge we need to refer back to Self Test 4, Question 2, where the mobility of n-type Germanium was given as $0.39 \text{ m}^2\text{V}^{-1}\text{s}^{-1}$. Substituting: $D = 0.0097 \text{ m}^2\text{s}^{-1}$

Note that you had to go searching for this last value. It is part being an engineer to be able to search out the values for things you don't know.

3. We are given the atomic weight of Ge. The mass of one Ge atom can be estimated from the atomic weight times the mass of a proton, which is listed in the table of constants on the question sheet. Thus $\text{Weight}_{\text{Ge}} = 1.215 \times 10^{-25} \text{ kg}$.

From the density we can now calculate the number of Ge atoms per metre³ as $4.45 \times 10^{28} \text{ m}^{-3}$

Hence the doping is $4.45 \times 10^{24} \text{ m}^{-3}$. The next question is to determine whether Antimony acts as a donor or an acceptor. A little research will tell you that antimony is from group V of the periodic table and therefore it's a donor. So $n = 4.45 \times 10^{24} \text{ m}^{-3}$.

To find the hole concentration we need to remember that np is a constant, i.e. that $np = n_i^2$. *The next couple of steps will be used in many questions so you really need to understand and remember them.*

Since we know n_i and have just determined n getting p is trivial:

$$p = \frac{n_i^2}{n} = \frac{(2.5 \times 10^{19})^2}{4.45 \times 10^{24}} = 1.40 \times 10^{14} \text{ m}^{-3}$$

Finally the question asks us to calculate the resistivity. Since $n \gg p$ we can ignore the contribution of holes so

Note that this is a low value of resistivity, which is perhaps not surprising since this is a high value of doping density

$$\rho = \frac{1}{nq\mu} = \frac{1}{4.45 \times 10^{24} \times 1.6 \times 10^{-19} \times 0.39} = 3.60 \times 10^{-6} \Omega m$$

4. (a) B is an acceptor in Si (you are expected to remember this!), so the Si is **p-type**

(b) This follows a similar line as part 3 above. From the atomic weight and proton mass we can get $\text{Weight}_{\text{Si}} = 4.70 \times 10^{-26} \text{ kg}$, and using the density we can now calculate the number of Si atoms per metre^3 as $4.9 \times 10^{28} \text{ m}^{-3}$

Hence $p = 4.9 \times 10^{20} \text{ m}^{-3}$.

n then follows using

$$n = \frac{n_i^2}{p} = \frac{(1.5 \times 10^{16})^2}{4.9 \times 10^{20}} = 4.6 \times 10^{11} \text{ m}^{-3}$$

$p \gg n$ and hence n can be neglected -- the answer is **yes**

$$\rho = \frac{1}{pq\mu} = \frac{1}{4.9 \times 10^{20} \times 1.6 \times 10^{-19} \times 0.046} = 0.28 \Omega m$$

$$R = \rho \frac{l}{A} = 0.28 \frac{0.1 \times 10^{-3}}{1 \times 10^{-3} \times 2 \times 10^{-3}} = 14.0 \Omega$$

$$V = IR = 2 \times 10^{-3} \times 14.0 = 0.028 \text{ V}$$

(c) To calculate the voltage we need to calculate the resistance. Since the geometry is given, then provided the resistivity can be calculated, the rest should follow:

5. This material is p-type so we know that the conductivity due to electrons is much less than that due to holes. So to start let us neglect the electrons completely. Let us then estimate the hole concentration:

$$\rho = \frac{RA}{l} = 120 \times 0.5 \times 10^{-6} / 6 \times 10^{-3} = 0.01 \Omega m$$

$$p = \frac{1}{\rho q \mu} = \frac{1}{0.01 \times 1.6 \times 10^{-19} \times 0.19} = 3.3 \times 10^{21} \text{ m}^{-3}$$

In the next part of the question we will prove the assumption that in doped material we can normally neglect the conduction due to minority carriers. To calculate the relative values, let's calculate the electron concentration

$$n = \frac{n_i^2}{p} = 1.9 \times 10^{17} \text{ m}^{-3}$$

Then to get the relative magnitudes of the conductivities

$$\frac{\sigma_p}{\sigma_n} = \frac{qp\mu_h}{qn\mu_e} = \frac{3.3 \times 10^{21} \times 0.19}{1.9 \times 10^{17} \times 0.39} = 8.4 \times 10^3$$

Thus neglecting the electron conduction gives an error of one part in 8400. In practice this is not important.

6. In order to calculate the resistivities in the doped samples we must first calculate the hole mobility from the data given about the pure material:

$$\rho = \frac{1}{n_i q (\mu_e + \mu_h)}$$

$$(\mu_e + \mu_h) = \frac{1}{n_i q \rho} = \frac{1}{1.4 \times 10^{16} \times 1.6 \times 10^{-19} \times 2 \times 10^3} = 0.22 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\therefore 1.26 \mu_e = 0.22 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\mu_e = 0.177 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\mu_h = 0.26 \times 0.177 = 0.046 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

Note here that we do have to consider electrons and holes since the electron and hole concentrations are equal (since the material is pure).

We can now apply this value of mobility to the doped material. [Note that in reality mobility can be effected by the doping level - usually it is slightly reduced due to increased scattering from the impurities introduced in doping it. That is why you are specifically told to assume that the mobility remains the same in this case]

In the doped material we can neglect the electron conductivity so if the acceptor concentration is 10^{21} m^{-3} then:

$$\rho = \frac{1}{p q \mu} = \frac{1}{10^{21} \times 1.6 \times 10^{-19} \times 0.046} = 0.136 \Omega \text{ m}$$

and if we increase the doping level by a factor of 100, then the resistivity will drop by a factor of 100.

7. First let us understand the question. Holes are flowing into some n-type material through the contact. These holes will diffuse in until they recombine. From the lecture notes we know that the hole concentration drops exponentially with distance from the contact. The excess hole concentration (over

$$\delta p = \delta p_0 \exp\left(-\frac{x}{L_h}\right)$$

the concentration that would be present naturally in the n-type material) is thus given by

where δp_0 is the excess hole concentration at the surface and L_h is the "minority carrier diffusion length". We need to find both of these to find the concentration at different distances into the material as the question requires.

$$\frac{d(\delta p)}{dx} \left(= \frac{dp}{dx} \right) = -\frac{\delta p_0}{L_h} \exp\left(-\frac{x}{L_h}\right)$$

The question gives you a hint that you should differentiate the concentration equation.

So at the surface (where $x=0$) we have:

$$\left. \frac{dp}{dx} \right|_{x=0} = -\frac{\delta p_0}{L_h}$$

Now we need to remember that the current density for diffusion current is proportional to this differential. The question tells us that the diffusion current is 509 A m^{-2} . Thus

$$J = -qD \frac{dp}{dx} = qD \frac{\delta p_0}{L_h}$$

However we also need to determine L_h . To do this remember that the average distance the carriers diffuse before they recombine is simply related to how long they diffuse for and how fast they diffuse. This relation is

Plugging this into the equation above gives

$$\delta p_0 = \frac{JL_h}{qD_h} = \frac{509 \times 1.9 \times 10^{-4}}{1.6 \times 10^{-19} \times 0.0015 \times 25 \times 10^{-6}} = 4.16 \times 10^{20} \text{ m}^{-3}$$

$$L_h = \sqrt{\frac{D_h \tau_h}{2}} = \sqrt{\frac{0.0015 \times 0.0045}{2}} = 1.9 \times 10^{-4} \text{ m}$$

This is the excess hole concentration at the surface. The question also asks what the excess concentration is at various distances below the surface. As we now have the surface concentration and L_h we can simply plug values into the exponential relation for carrier concentration to give:

$$\text{at } 0.1 \text{ mm distance the excess hole concentration} = 2.5 \times 10^{20} \text{ m}^{-3}$$

$$\text{at } 1 \text{ mm distance the excess hole concentration} = 2.5 \times 10^{18} \text{ m}^{-3}$$