

Section 1b: Carrier transport in semiconductors

Carrier Transport

- Various transport phenomena occur in Semiconductor devices
 - **Drift**
 - **Diffusion**
 - **Generation**
 - **Recombination**
 - Thermionic emission
 - Tunnelling
 - **Impact ionisation**
- We consider the motion of charge carriers in semiconductors under the influence of an electric field and a carrier concentration gradient

Carrier Drift

- In the absence of electric field the motion of the free charge carriers (electrons and holes) is chaotic
 - For each carrier with a velocity (v) there exists a carrier with a velocity $-v$
 - Vector sums of all the carriers is zero and
 - No net transport of charge through the crystal.
 - The average kinetic energy is given by:

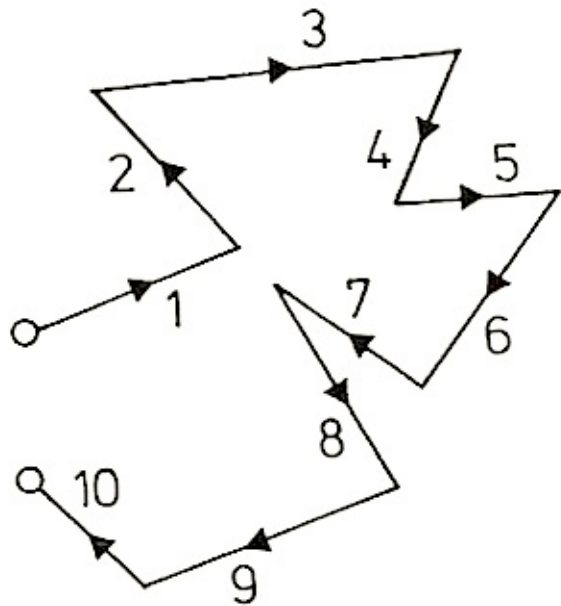
$$W_{kin} = \frac{3}{2} kT = \frac{1}{2} m^* v_{th}^2$$

Where m^* is the effective electron mass and v_{th} is the average thermal velocity, k =Boltzmann's constant T =Temperature in K

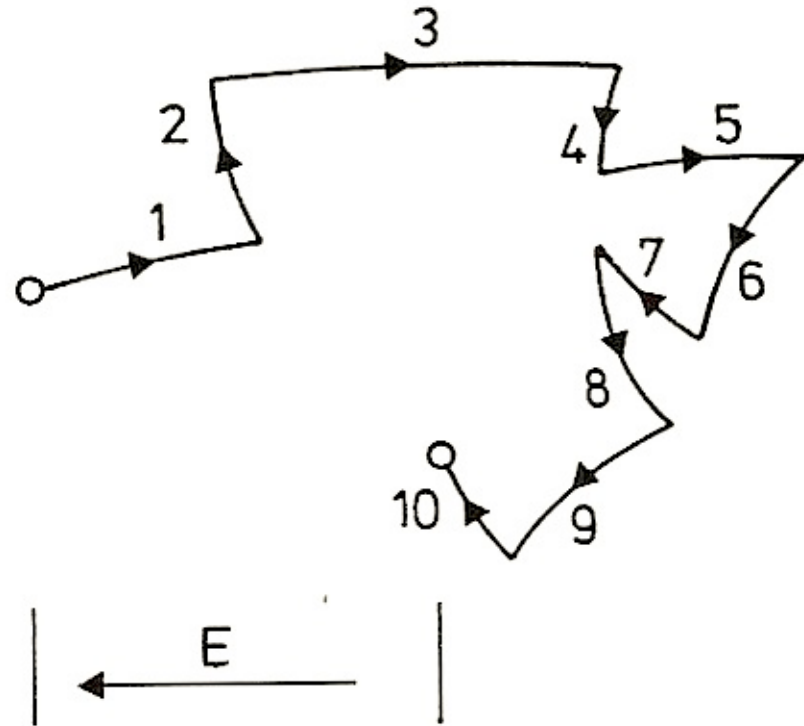
Carrier Drift

- In the presence of an electric field E , a force $F = \pm qE$ is applied on the electrons ($-q$) and holes ($+q$)
 - Carriers are initially accelerated by this force but are scattered by interactions with the crystal lattice vibrations and with imperfections in the crystal
 - As a consequence of the electric field a small average drift velocity (v_{dn} or v_{dp}) are superimposed onto the random motion of electrons and holes

Schematic path of an electron in a semiconductor



Random thermal motion



Combined thermal motion and
an applied electric field

Drift velocity and mobility

- Drift velocity can be obtained by equating the momentum (force x time) applied to an electron during free flight between collisions to the momentum gained by the electron in the same period
 - Assumption valid as in a steady state condition all momentum gained between collisions is lost to the lattice in the collision

momentum applied = momentum gained

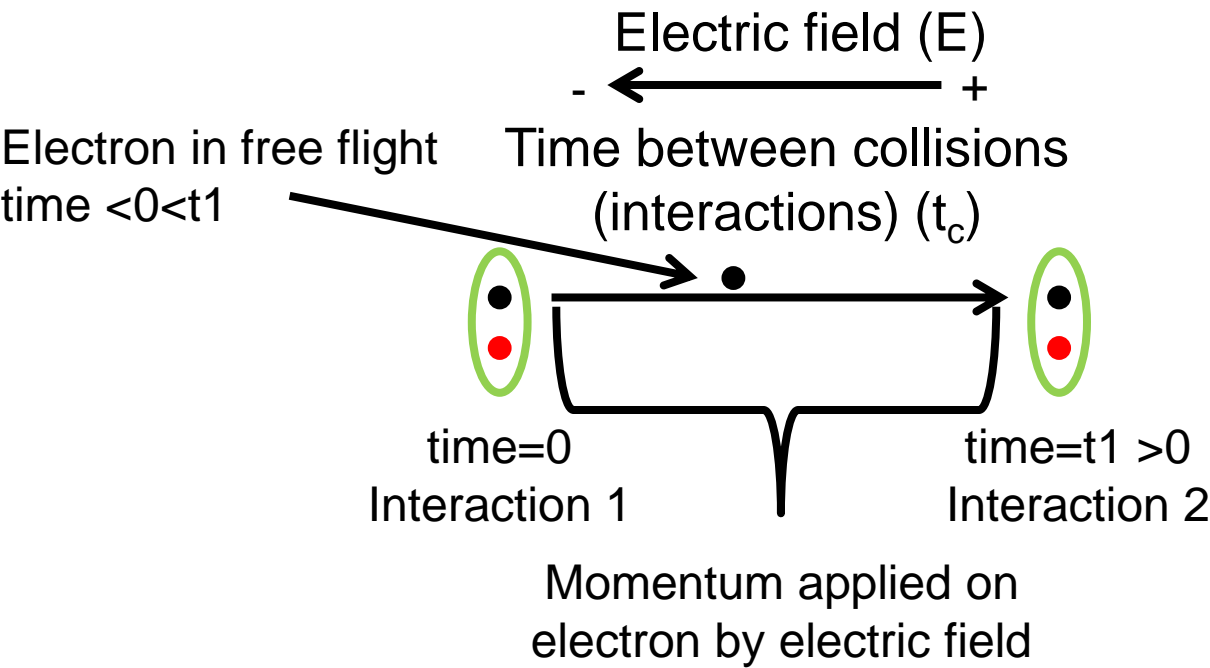
$$-qEt_c = m_n v_n \text{ or } v_n = -\left(\frac{qt_c}{m_n}\right) E$$

- Drift velocity is proportional to the applied electric field (E)
 - The proportionality factor depends upon the mean free path (t_n) and the effective mass (m_a)
 - This proportionality factor is termed electron (μ_n) or hole mobility (μ_p)

$$\mu_n \equiv \frac{qt_c}{m_n} \quad \text{thus} \quad v_n = -\mu_n E \quad v_p = \mu_p E$$

**negative sign removed for hole velocity as carrier follow the electric field

Drift velocity: detail



momentum applied = $-qEt_c$ = *momentum gained* = $m_n v_n$

Arrows point from the terms in the equation to their definitions:

- From $-q$ to 'Electron charge'
- From E to 'Electric field'
- From t_c to 'Time between Collisions or interactions'
- From m_n to 'Electron mass'
- From v_n to 'Electron velocity'

re - arranging: $v_n = -\left(\frac{qt_c}{m_n}\right)E$ *and define* $\mu_n \equiv \frac{qt_c}{m_n}$ $v_n = -\mu_n E$ $v_p = \mu_p E$

Mean free time/path calculation

- Calculate the mean free time and path of an electron having an mobility of $1450 \text{ cm}^2/\text{V-s}$ at 300K. Assume $m_a = 0.26m_o$ and $m^*=1.08m_o$ respectively

$$\tau_c = \frac{m_a \mu_n}{q} = \frac{(0.26 \times 0.91 \times 10^{-30}) \text{ kg} \times 1450 \times 10^{-4} \text{ m}^2/\text{V-s}}{1.6 \times 10^{-19} \text{ C}}$$
$$= 0.214 \text{ ps}$$

$$V_{th} = \sqrt{\frac{3kT}{m^*}} = \sqrt{\frac{(3 \times 1.38 \times 10^{-23} \times 300) \text{ J}}{(1.08 \times 0.91 \times 10^{-30}) \text{ kg}}} = 112 \text{ k m/s}$$

$$l = v_{th} \tau_c = (0.214 \times 10^{-12}) \text{ s} \times (112 \times 10^3) \text{ m/s} = 24 \text{ nm}$$

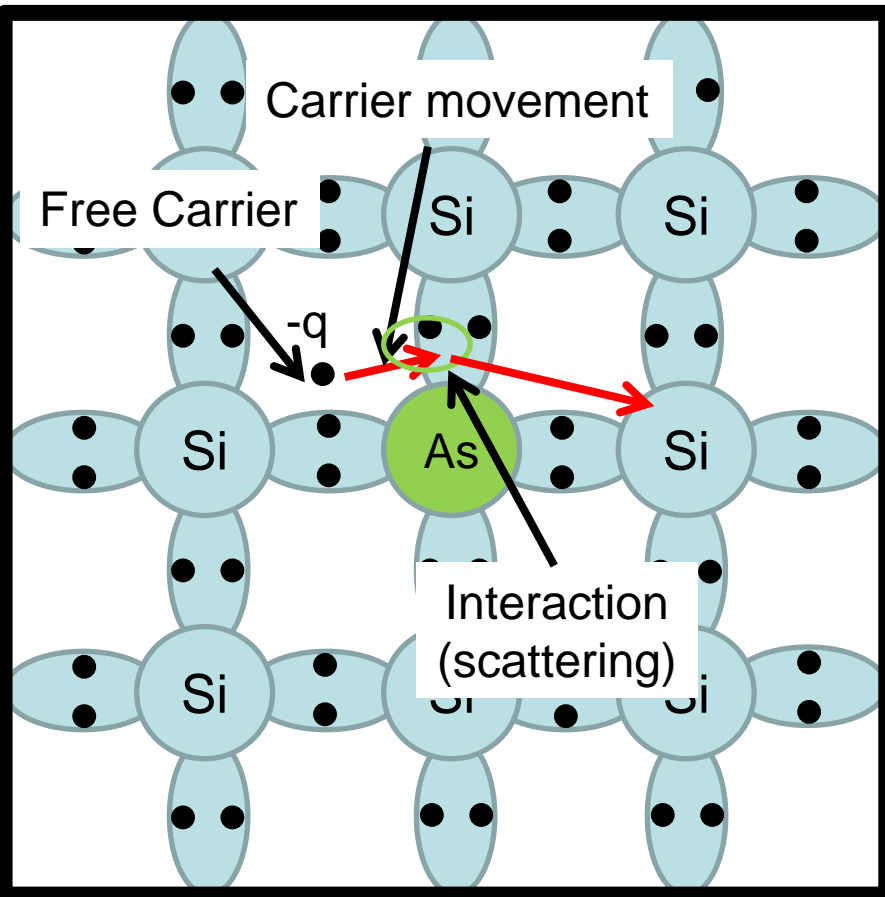
Mobility

- Mobility is directly related to the mean free time between collisions, determined by various scattering mechanisms
- Considering bulk transport, the two most important mechanisms are
 - Lattice scattering
 - Resulting in thermal vibrations of the lattice atoms at any temperature above absolute zero. These disturb the lattice periodic potential and allow energy to be transferred between the carriers and lattice
 - More significant at higher temperatures
 - Impurity scattering
 - Occurs when a charge carrier travels past an ionized dopant impurity
 - Charge carrier will be deflected owing to coulomb force interactions
 - More significant at low temperatures as faster travelling carriers have less interaction time with impurity ions

TRIVIA: What are the units of mobility?

Lattice scattering

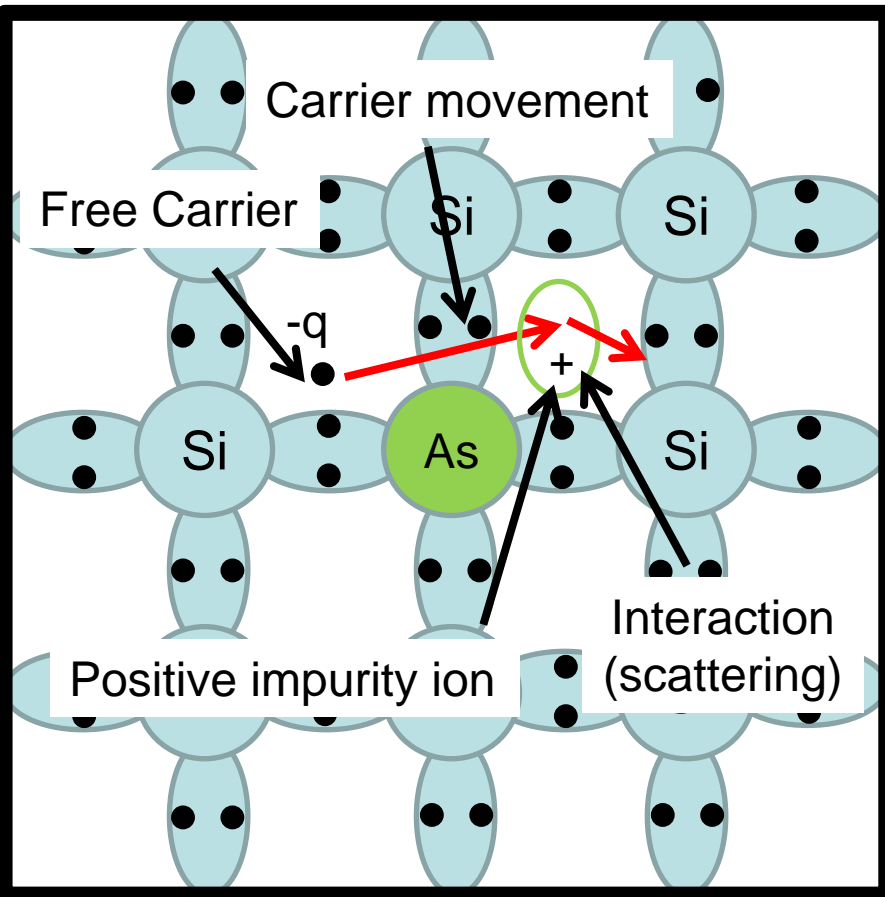
N type silicon



- Occurs when carriers interact with the crystalline lattice through thermal vibrations
- Energy transferred between the lattice and free carrier
- Since lattice vibrations increase with temperature
 - Lattice scattering shows a positive temperature coefficient
 - Scattering increases with temperature

Impurity scattering

N type silicon



- Occurs when free carriers interact with impurity ion
- Common examples are donor and acceptor impurity ions
- Scattering is due to the electrostatic forces between the carrier and ionised impurity
- Depends upon interaction time
 - Thermal velocity increases with temperature
 - Therefore interaction time decreases
 - Negative temperature coefficient i.e. as temperature increases impurity scattering reduces

Empirical mobility equations

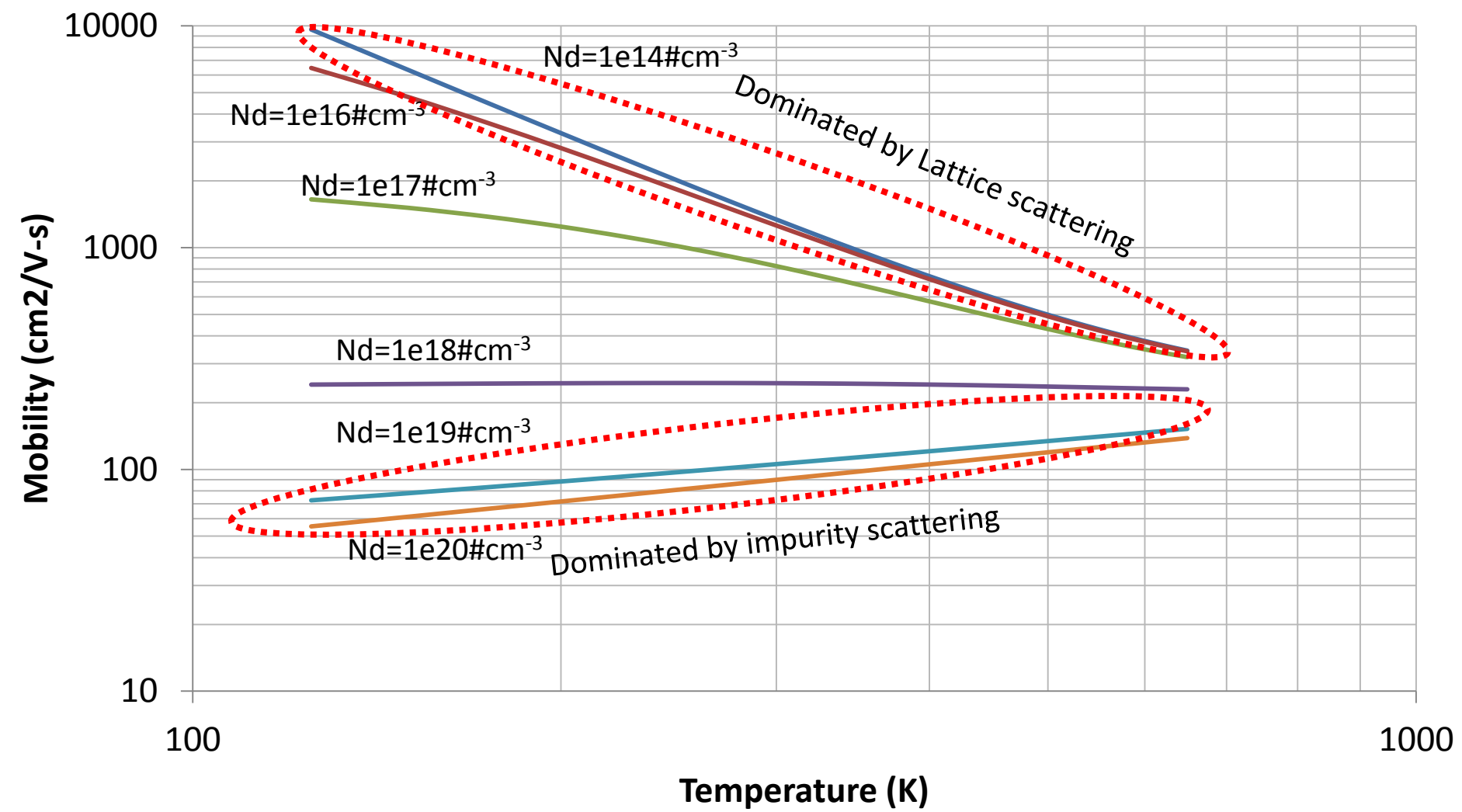
- Lattice and ionic scattering are modelled using empirical equations:
 - Lombardi model

$$\mu_n = 88 \left(\frac{T}{300} \right)^{0.57} + \frac{7.4 \times 10^8 T^{-2.33}}{1 + \frac{N}{1.26 \times 10^{17} \left(\frac{T}{300} \right)^{2.4}} \times 0.88 \left(\frac{T}{300} \right)^{0.146}}$$

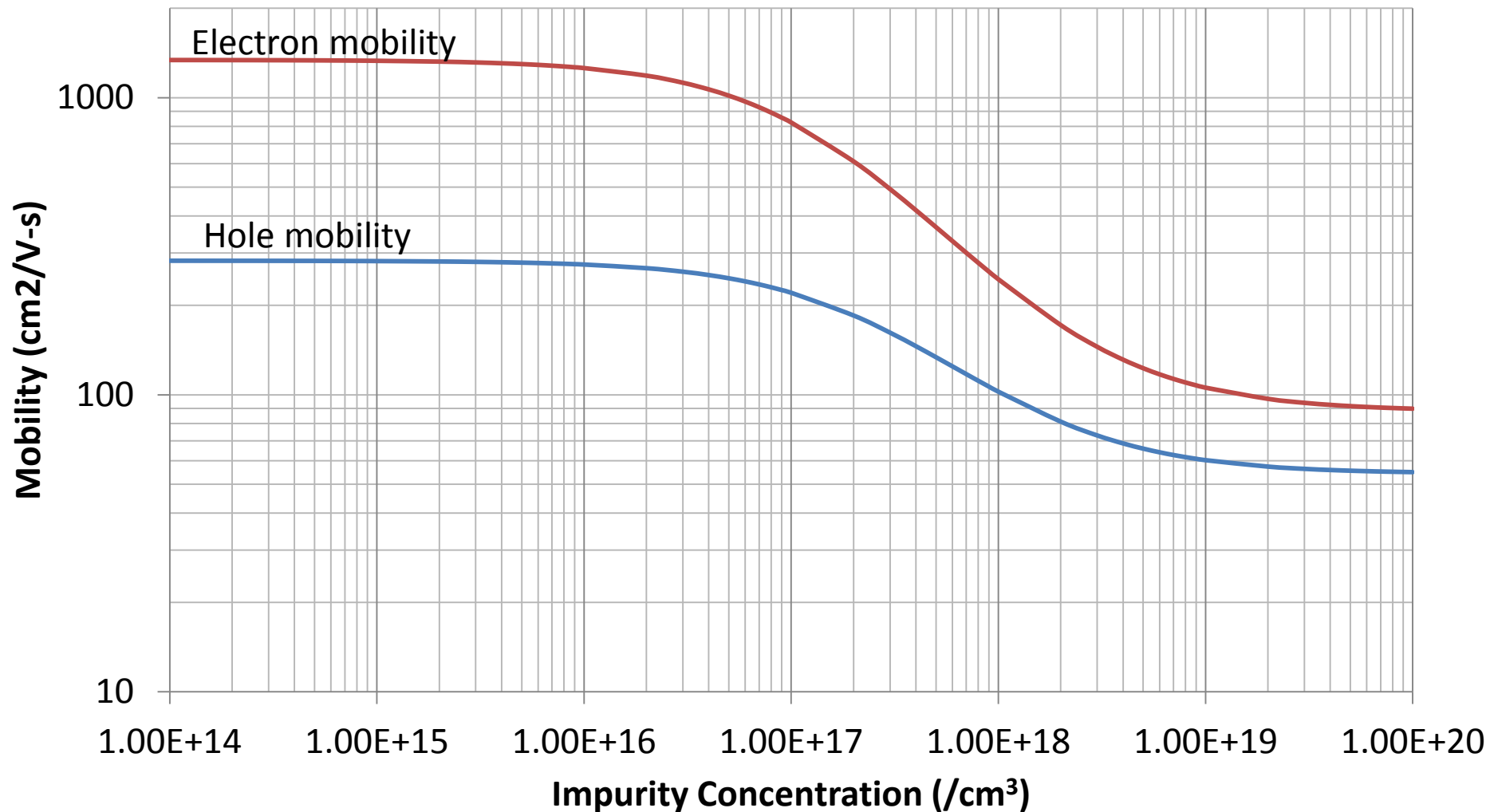
$$\mu_p = 54.3 \left(\frac{T}{300} \right)^{0.57} + \frac{1.35 T^{-2.33}}{1 + \frac{N}{2.35 \times 10^{17} \left(\frac{T}{300} \right)^{2.4}} \times 0.88 \left(\frac{T}{300} \right)^{0.146}}$$

***For reference only (not on exam)

Doping and concentration dependency of mobility: p type silicon



Room temperature electron/hole mobility



Generally there is a ration of ~3 between hole and electron mobility

Resistivity: Electrostatic potential

- When an electric field (E) is applied to the semiconductor
 - Each electron will experience a force ($-qE$) from the field
 - The force is equal to a negative gradient of potential energy:

$$-qE = -(\text{gradient of potential energy}) = -\frac{E_c}{dx}$$

- Since the bottom of the conduction band corresponds to the potential energy of an electron, the bands experience a gradient.
 - E_i is used as a reference for convenience

$$E = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_i}{dx} \quad (1)$$

- Defining related quantity electrostatic potential (Ψ) whose negative gradient equals the electric field

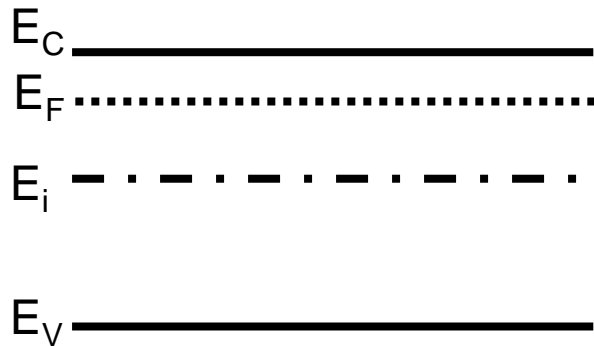
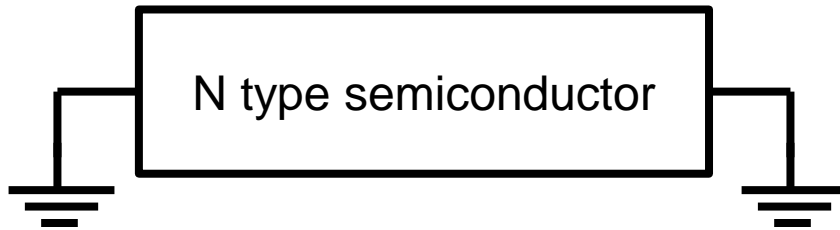
$$E \equiv -\frac{d\Psi}{dx} \quad (2) \quad \text{combining (1) and (2)} \quad \Psi = -\frac{E_i}{q}$$

Current process under electric field

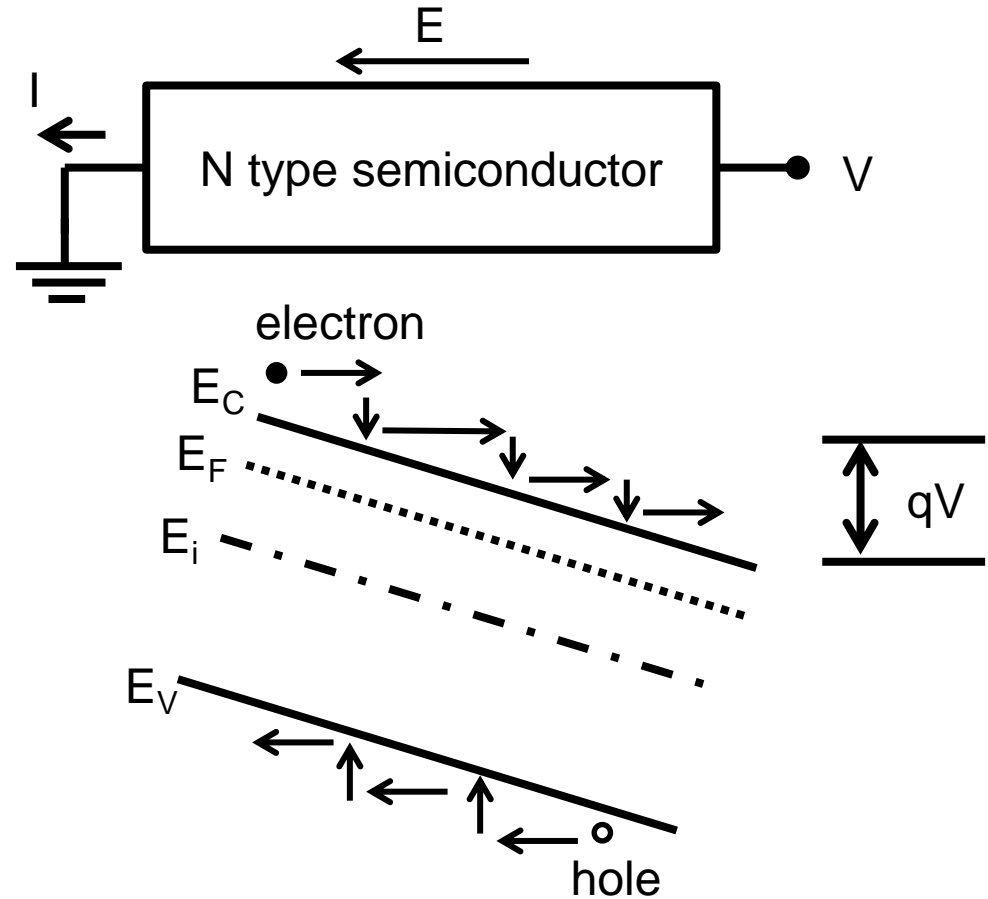
- For a homogeneous semiconductor potential energy and E_i decrease linearly with distance
 - Therefore the electric field is constant in the negative direction
 - Its magnitude is the applied voltage divided by the sample length
- The electrons in the conduction band move to the positive electrode
 - Kinetic energy - corresponds to the distance from the band edge
 - When an electron undergoes a collision it loses all or some of its Kinetic energy to the lattice and drops towards its thermal equilibrium position
 - Once it has completed this process it will continue to follow the electric field until another collision occurs
 - Holes are visualised in a similar manner but in the opposite direction
- Transport of carrier under the influence of an applied electric field is called **drift current**

Conduction process in an n type semiconductor

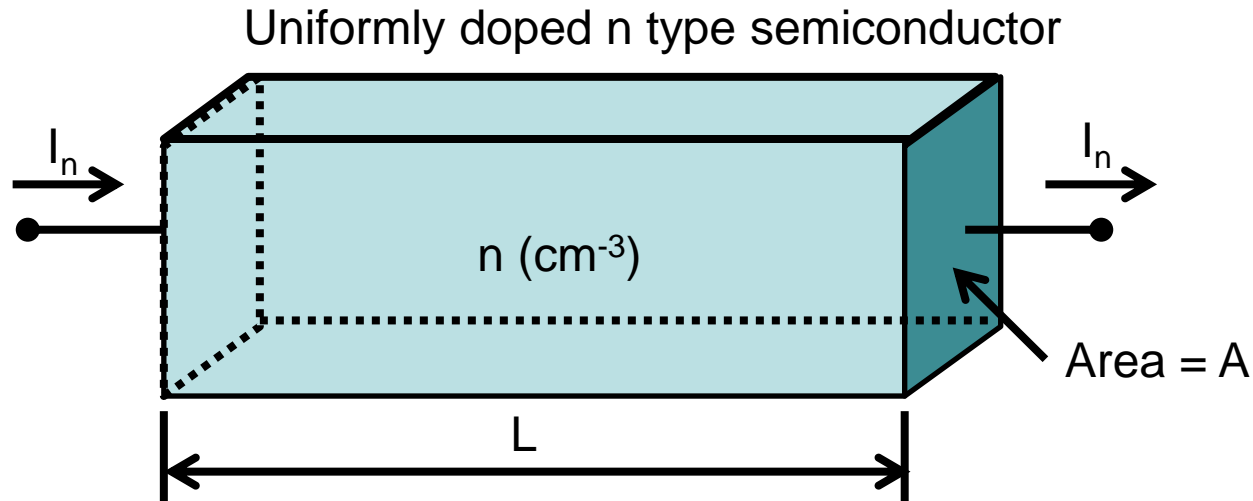
At thermal equilibrium



Under bias condition



Drift current in semiconductors



- Considering an uniformly doped n type semiconductor bar
- Supposing an electric field (E) is applied
 - Electron current density (J_n) can be found by summing the produce of charge ($-q$) on each electron times the electron velocity over all electrons per unit volume

$$J_n = \frac{I_n}{A} = \sum_{i=1}^n (-qv_i) = -qn v_n = qn\mu_n E$$

Likewise for hole current:

$$J_p = qp v_p = qp\mu_p E$$

Resistivity

- Total current density (J) is the sum of the electron and hole current densities

$$J = J_n + J_p = q(n\mu_n + p\mu_p)E$$

- ρ and σ are defined as the resistivity and conductivity of the semiconductor

$$q(n\mu_n + p\mu_p) = \sigma = \frac{1}{\rho}$$

- Generally in extrinsic material ($N_{\text{dopant}} \gg \text{intrinsic carriers}$) the minority carrier current contribution can be disregarded:

$$\text{n type: } \rho = \frac{1}{qn\mu_n} \quad \text{p type: } \rho = \frac{1}{qp\mu_p} \quad \Omega \cdot \text{cm}$$

Resistivity and resistance calculation

- Find the room temperature resistivity of an n type semiconductor doped with $1e15$ phosphorous atoms: considering an electron mobility of $1450 \text{ cm}^2/\text{V-s}$
- What is the resistance if the material had a length of $500\mu\text{m}$ and conduction area of $100\mu\text{m}^2$

Assuming complete donor ionisation: $n = N_D$

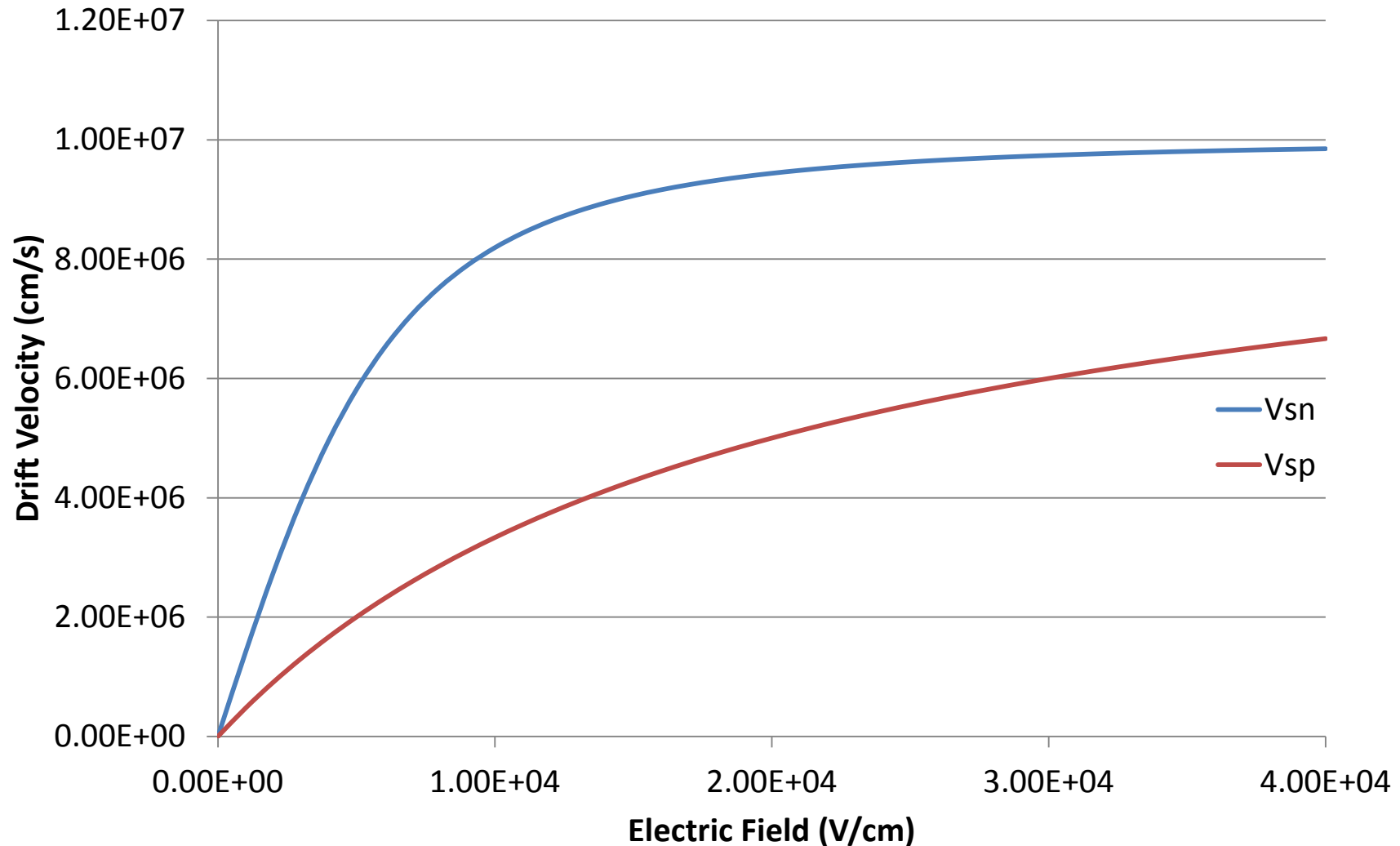
$$\rho = \frac{1}{q N_D \mu_n} = \frac{1}{1.6 \times 10^{-19} \times 10^{15} \times 1450} = 4.3 \Omega - \text{cm}$$

$$\text{resistance} = \rho \frac{L}{A} = 4.3 \times \frac{500 \times 10^{-4}}{100 \times 10^{-8}} = 215 \text{ k}\Omega$$

Velocity saturation

- At low electric-fields, the drift velocity is linearly proportional to the applied field
 - We assume that the time interval between collisions (τ_c) is independent of the applied field
 - Valid assumption as long as the drift velocity is small compared with the thermal velocity of the carriers $\sim 10^7$ cm/s for Silicon at room temperature
- As drift velocity approaches the thermal velocity its field dependency will depart from the linear relationship
 - Corresponding to a constant mobility
- Further increase in electric field the drift velocity will eventually reach its saturation velocity
- Velocity saturation is likely to occur in short channel FETs, where the channel electric fields are very high
 - Causes saturation of the current voltage characteristic
- TRIVIA: What are the benefits of high values of Velocity Saturation?

Drift velocity saturation versus electric field in Silicon

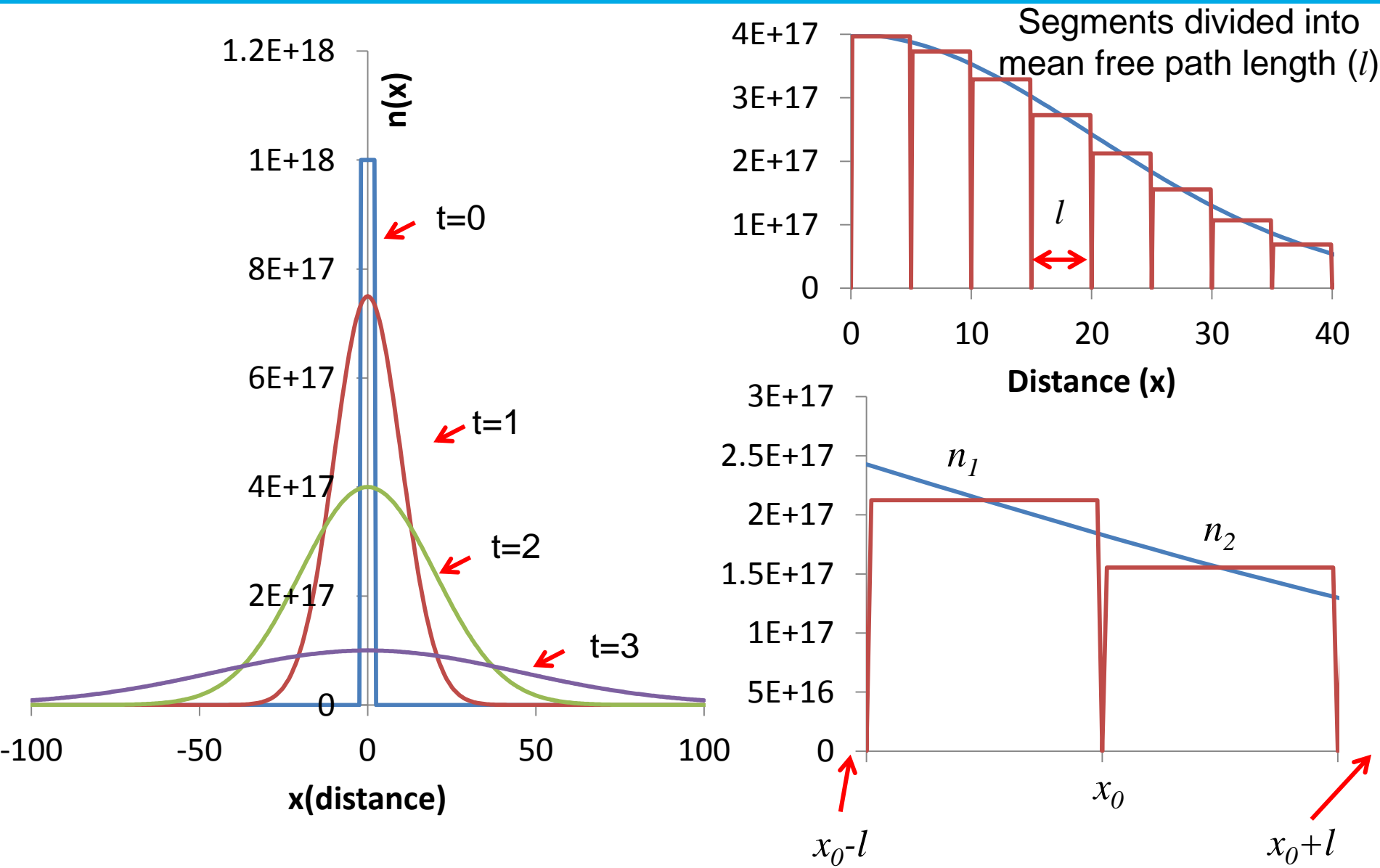


TRIVIA: Why is the velocity saturation of holes much smaller than electrons?

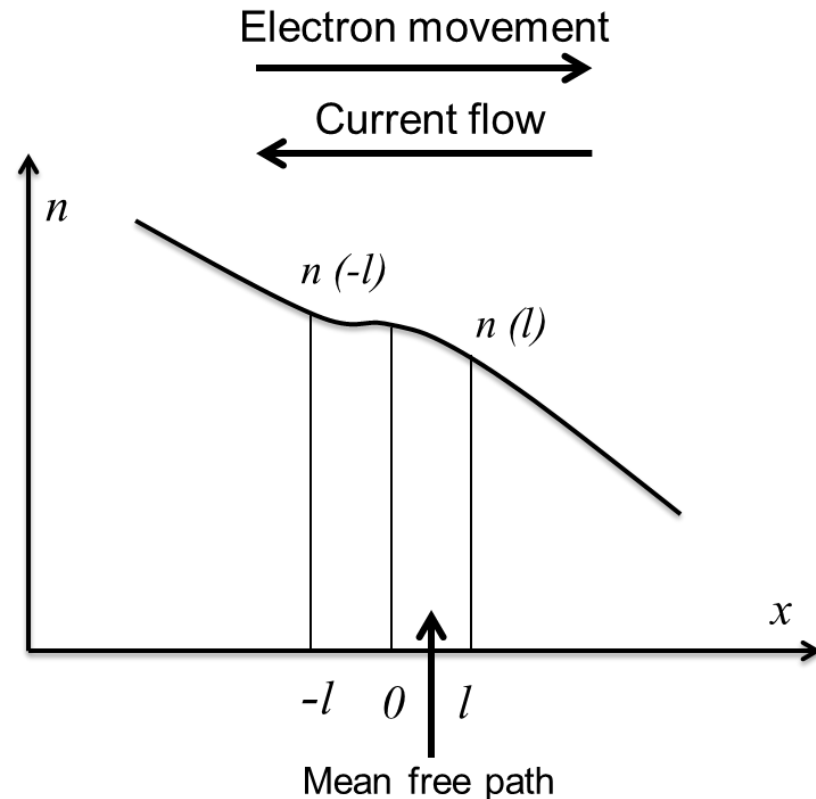
Diffusion process

- Occurs when there is a spatial variation of carrier concentration within the semiconductor
- Carriers need a non-zero temperature (Kelvin) and have an additional thermal energy that equals $kT/2$ per degree of freedom
 - Thermal energy drives the diffusion process
 - At $T=0K$: **No diffusion**
- Carrier movement via the diffusion process results in current flow
- Occurs when minority carriers are injected across junctions: Bipolar devices, or during transient conditions.
- Beware of Carrier Freeze-out at low temperatures!

Diffusion process: Spreading of excess carriers with time



Electron concentration versus distance



$$\text{mean free length } (l) = v_{th}\tau_c$$

- One-dimensional semiconductor considered where carriers can only move in one direction
 - electron density (n) only varying in the x direction
- Sample is at a uniform temperature
- Considering the thermal velocity (v_{th}), collision time (τ_c) and mean free path (l)
 - V_{th} : Average velocity of the carriers in the positive or negative directions
 - τ_c : Average time during a carrier moves with the same velocity before colliding with an atom or other carriers
 - l : Average length a carrier travels between collisions
- Electrons at one mean free path away from 0 (l) have an equal chance of moving left or right in a mean free time (τ_c) therefore half of them will move across the plane at $x = 0$

Diffusion current

- Considering one mean free path from $x=0$
 - The average carriers arriving at $x = 0$ originate either from $x=-l$ or $x=l$
- The flux at $x=0$ due to carrier originating from $x=-l$ and move from left to right:

$$\Phi_{n,left \rightarrow right} = \frac{1}{2} v_{th} n(x = -l)$$

- Factor 0.5 as only half of the carriers will move left while the other half move to the right
- The flux at $x=0$ due to carrier originating from $x=+l$ and move from right to left:

$$\Phi_{n,right \rightarrow left} = \frac{1}{2} v_{th} n(x = +l)$$

- The total flux of carriers moving from left to right at $x=0$ is therefore:

$$\Phi_n = \Phi_{n,left \rightarrow right} - \Phi_{n,right \rightarrow left} = \frac{1}{2} v_{th} [n(x = -l) - n(x = +l)]$$

Diffusion current

- Given that the mean free path is small we can write the differences in densities divided by the distance between $x=-l$ and $x=l$ as the derivative of the carrier density:

$$\Phi_n = -lv_{th} \frac{n(x=l) - n(x=-l)}{2l} = -lv_{th} \frac{dn}{dx}$$

- Electron diffusion current is equal to the flux multiplied by electron charge:

$$J_n = -q\Phi_n = qlv_{th} \frac{dn}{dx}$$

- Replacing thermal velocity and mean free path by a single parameter (diffusion coefficient D_n), so that:

$$J_n = qD_n \frac{dn}{dx}$$

- And repeating the same derivation for holes:

$$J_p = -qD_p \frac{dp}{dx}$$

Diffusion current example

- Assume that in an n type semiconductor at $T=300\text{K}$ the electron concentration varies linearly from 1×10^{18} to $7 \times 10^{17} \text{ cm}^{-3}$ over a distance of 0.1cm . Calculate the diffusion current density if the electron diffusion coefficient is $D_n=22.5 \text{ cm}^2/\text{s}$

$$J_n = qD_n \frac{dn}{dx} \approx qD_n \frac{\Delta n}{\Delta x}$$

$$1.6 \times 10^{-19} \times 22.5 \left(\frac{1 \times 10^{18} - 7 \times 10^{17}}{0.1} \right) = 10.8 \text{ A/cm}^2$$

Einstein's relation

- The diffusion equation can be written in a more useful form using the theorem for the equipartition of energy
 - i.e. molecules in thermal equilibrium have the same average energy associated with each degrees of freedom of their motion
 - In each of the three dimensions we have:

$$\text{Kinetic energy} \rightarrow \frac{1}{2} m_n v_{th}^2 = \frac{1}{2} kT \leftarrow \text{Thermal energy}$$

$$D_n = v_{th} l = v_{th} (v_{th} \tau_c) = v_{th}^2 \frac{\mu_n}{q} = \left(\frac{kT}{m_n} \right) \left(\frac{\mu_n m_n}{q} \right)$$

$$D_n = \left(\frac{kT}{q} \right) \mu_n$$

- Einstein's relation relates two important constants, **diffusion and mobility**, that characterise carrier transport by diffusion and drift in a semiconductor

Diffusivity

- Minority carriers (holes) are injected into a homogeneous n-type semiconductor at one point. An electric field of 50V/cm is applied across the sample which moves these minority carriers a distance of 1cm in 100μs. Find the drift velocity and the diffusivity of the minority carriers:

$$v_p = \frac{l}{\tau} = \frac{1cm}{100 \times 10^{-6}s} = 10^4 cm/s$$

$$\mu_p = \frac{v_p}{E} = \frac{10^4}{50} = 200 cm^2/V - s$$

$$D_p = \frac{kT}{q} \mu_p = 0.0259 \times 200 = 5.18 cm^2/s$$

Current density equations

- When an electric field is present in addition to a concentration gradient, both drift and diffusion currents will flow
- The total current density at any one point is the sum of the drift and diffusions components

$$J_n = q\mu_n nE + qD_n \frac{dn}{dx}$$

$$J_p = q\mu_p pE - qD_p \frac{dp}{dx}$$

- Total current conduction current density is given by the sum of electron and hole contributions

$$J_{cond} = J_n + J_p$$

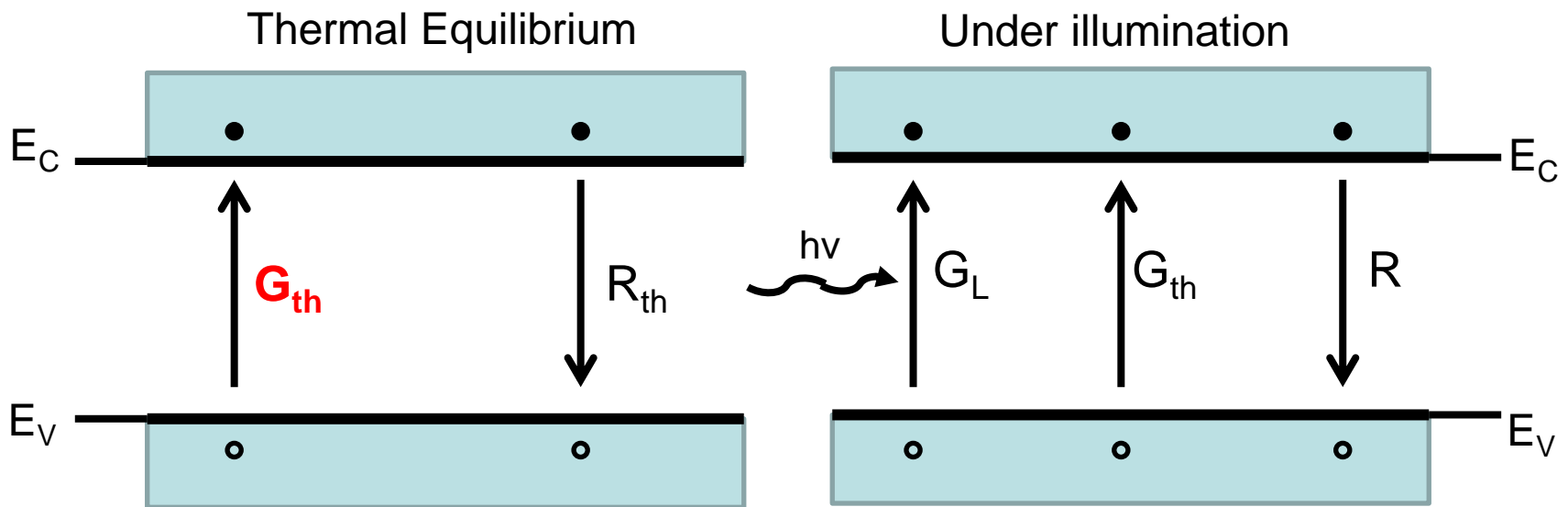
- These three equations constitute the current density equations
 - Important for analysing devices under operations for low electric fields
 - When electric fields are sufficiently high the electric field terms $\mu_n E$ and $\mu_p E$ should be replaced by the velocity saturation v_s

Generation and recombination processes

- At thermal equilibrium the relationship $n_p = n_i^2$ is valid. If excess carriers are introduced we have a non-equilibrium condition
- The process of introducing excess carriers is called carrier injection
- Whenever the thermal equilibrium condition is disturbed processes occur to restore the system to equilibrium
- The mechanism which restores equilibrium is recombination of injected carriers with the majority carriers
- Depending upon the nature of the recombination process, the released energy can be released (radiative) as either a photon or heat dissipated to the lattice (non radiative)
- Recombination process can be classified as direct and indirect processes

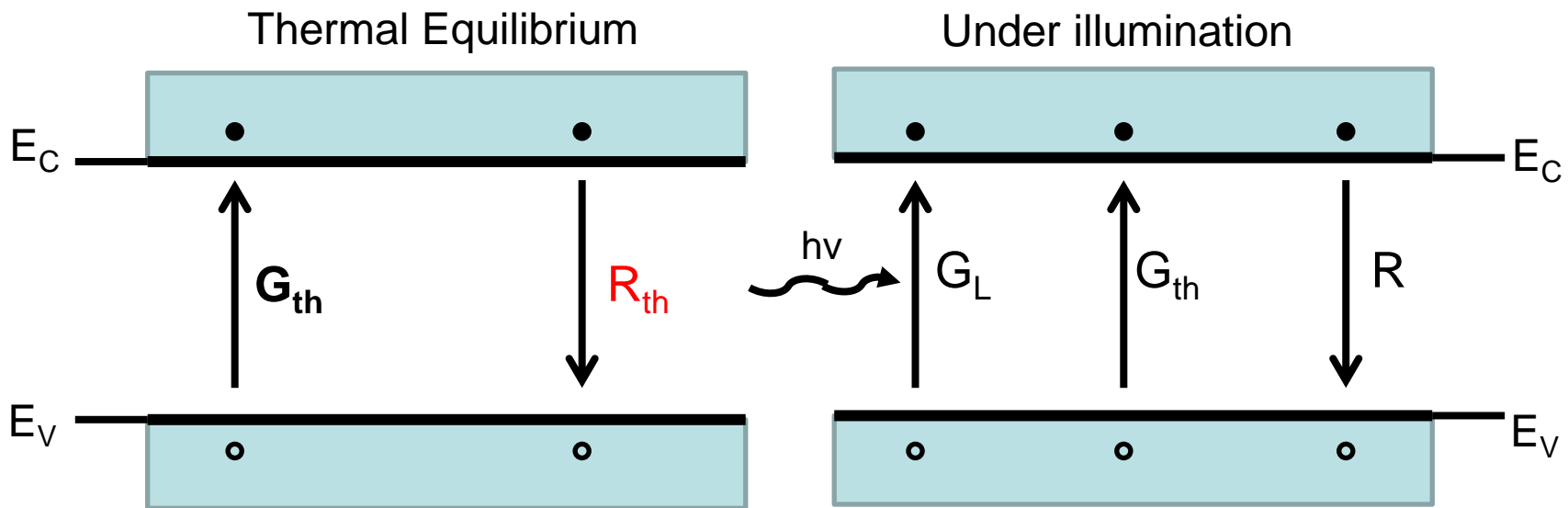
Direct recombination: Thermally generated carriers

- Dominates in direct band gap semiconductors
 - Such as Gallium Nitride, SiC, GaN
- Thermal vibration creates electron hole pairs
 - Electron promoted to conduction band hole in valence band
 - Carrier generation, represented by a generation rate G_{th}
 - Number of carriers generated per cm^3



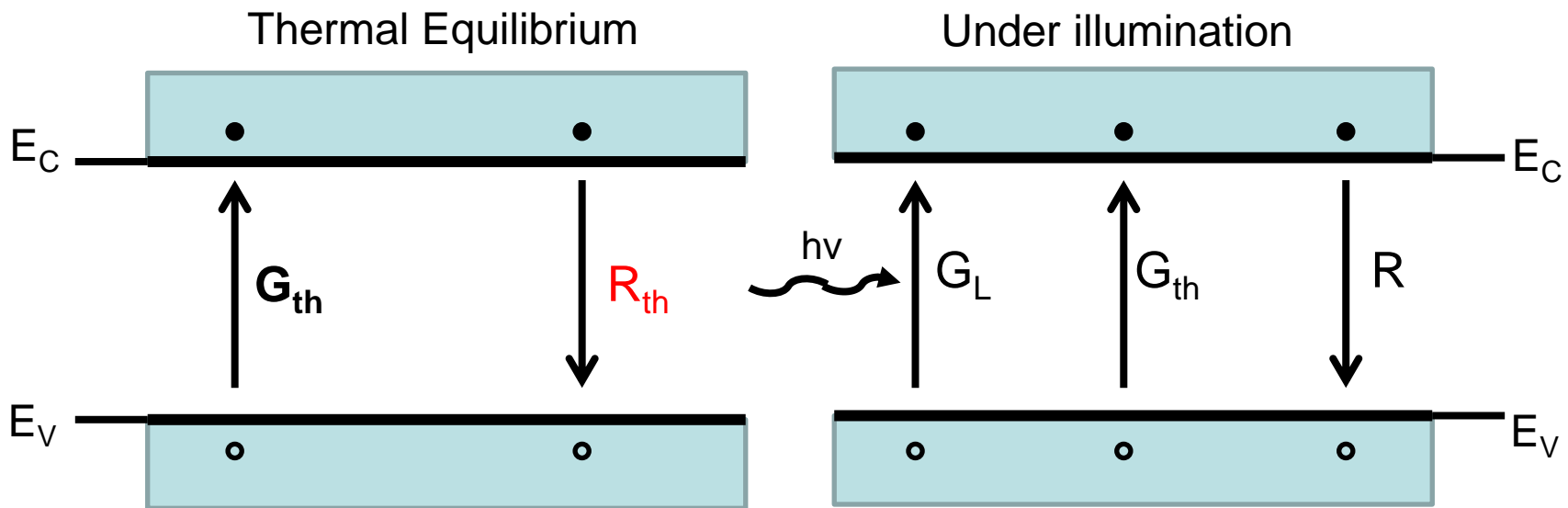
Direct recombination: Thermally generated carriers

- When an electron makes a transition downwards from the conduction band to valence band
- Electron hole pair is annihilated
 - Recombination represented by a recombination rate R_{th}
 - Under thermal equilibrium generation rate must equal recombination rate so that the carrier concentration remains constant and $np = n_i^2$ is maintained



Direct recombination: Thermally generated carriers

- When excess carriers are introduced to a direct-band gap semiconductor
- The probability is high that electrons and holes will recombine directly
 - Bottom of the conduction band and top of the valence are lined up
 - No additional momentum is required for the transition across the band gap



Generation and recombination rates

- The recombination rate (R) is proportional to the number of electrons available in the conduction band and the number of holes available in the valence band:

$$R = \beta np$$

- Where β is the recombination probability constant
- In thermal equilibrium recombination rate must be balanced by the generation rate therefore for a n type semiconductor:

$$G_{th} = R_{th} = \beta n_{no} p_{no}$$

- Note: subscript n indicates semiconductor type, subscript o indicates an equilibrium quality, therefore n_{no} and p_{no} represents equilibrium electron and hole densities in an n type semiconductor.
- Introducing excess carriers in the system: Electron hole pairs at a rate G_L are introduced resulting in a carrier concentration above their equilibrium values:

$$R = \beta n_n p_n = \beta (n_{no} + \Delta n)(p_{no} + \Delta p)$$

$$G = G_L + G_{th}$$

Net recombination

- Excess carrier concentrations, Δn and Δp , are given by:

$$\Delta n = n_n - n_{no} \quad \Delta p = p_n - p_{no}$$

- $\Delta n = \Delta p$ to maintain overall charge neutrality
- The net rate of change of hole concentration is given by:

$$\frac{dp_n}{dt} = G - R = G_L + G_{th} - R$$

- In steady state $dp_n/dt = 0$ therefore:

$$G_L = R - G_{th} \equiv U$$

- Where U is the net recombination rate
- Substituting these equations yield:

$$U = \beta(n_{no} + p_{no} + \Delta p)\Delta p$$

- For low-level injection Δp , $p_{no} \ll n_{no}$ this simplified to:

$$U \cong \beta n_{no} \Delta p = p_n - p_{no} / \frac{1}{\beta n_{no}}$$

Minority carrier lifetime

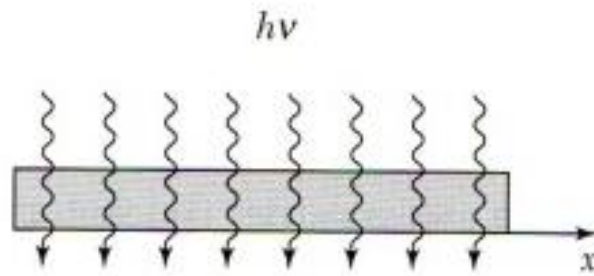
- Therefore net recombination rate is proportional to excess **minority** carrier concentration and obviously $U=0$ in thermal equilibrium.
- The probability constant $1/\beta n_{no}$ is called **lifetime** t_p of the excess or minority carriers
- Obviously $U=0$ in thermal equilibrium, or:

$$U \equiv \frac{p_n - p_{no}}{\tau_p} \quad \text{where: } t_p \equiv \frac{1}{\beta n_{no}}$$

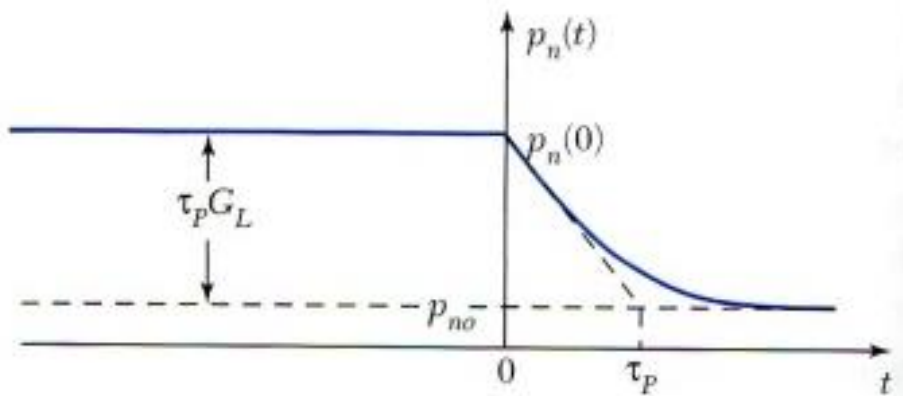
- In steady state, this gives:

$$G_L = U = \frac{p_n - p_{no}}{\tau_p}$$

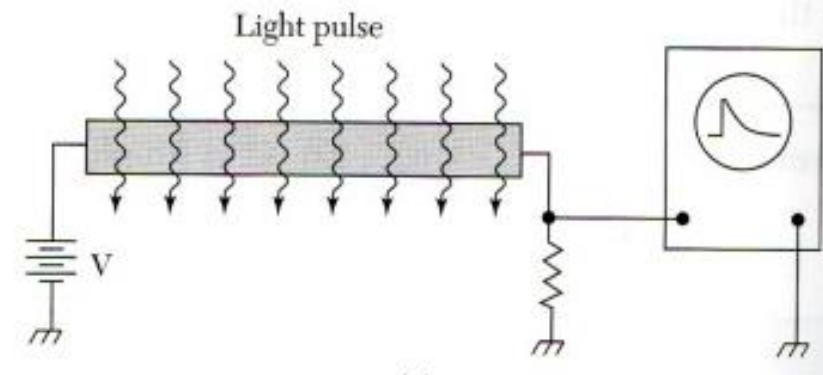
Decay of photo excited carriers



N type sample under constant illumination



Decay of minority carriers (holes) with time

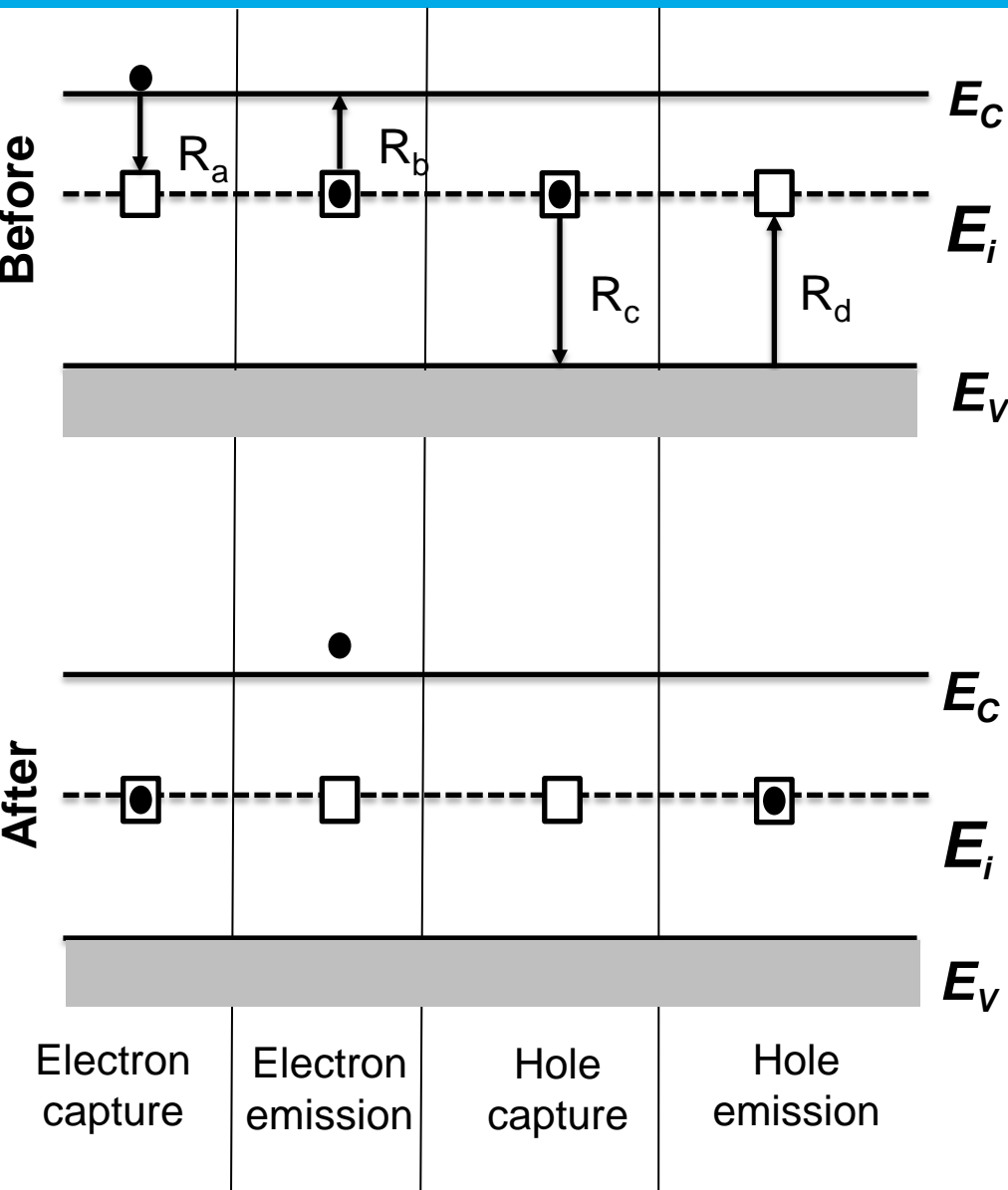


Schematic setup to measure minority carrier lifetime

Indirect recombination

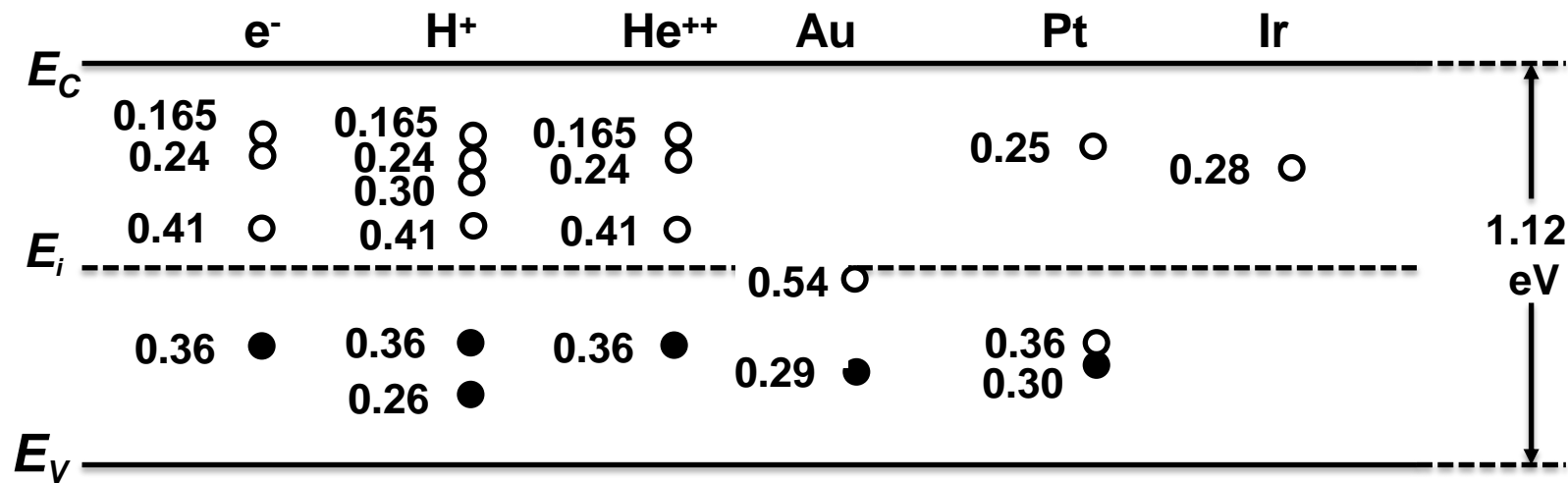
- For indirect semiconductors, such as silicon, direct recombination process is very unlikely.
 - Electrons at the bottom of the conduction band have non-zero momentum with respect to the holes at the top of the valence band
- Dominant recombination process is indirect transitions via localised energy states in the forbidden energy gap
 - These states act as a stepping stone between the conduction and valence bands

Indirect generation-recombination processes at therm. equilibrium



- Four generation – recombination process occur
 - Electron capture
 - Electron emission
 - Hole capture
 - Hole emission
- In bipolar devices recombination centres are intentionally put into the semiconductor to improve switching times
 - Irradiation
 - Ion implantation
- Termed: **Lifetime Killing**

Common lifetime killers used in silicon



- Electron irradiation (e^-) the common approach to control lifetime
 - Uniform distribution
- If profiled lifetime zones are required
 - Can be achieved by ion implantation and activation
 - Generally with helium
- Generally gold is avoided in silicon processing
 - Seen as a unwanted impurity as off state leakage currents substantially increase (mid gap recombination centre)
 - Generally used as a contact metal post high temperature fabrication

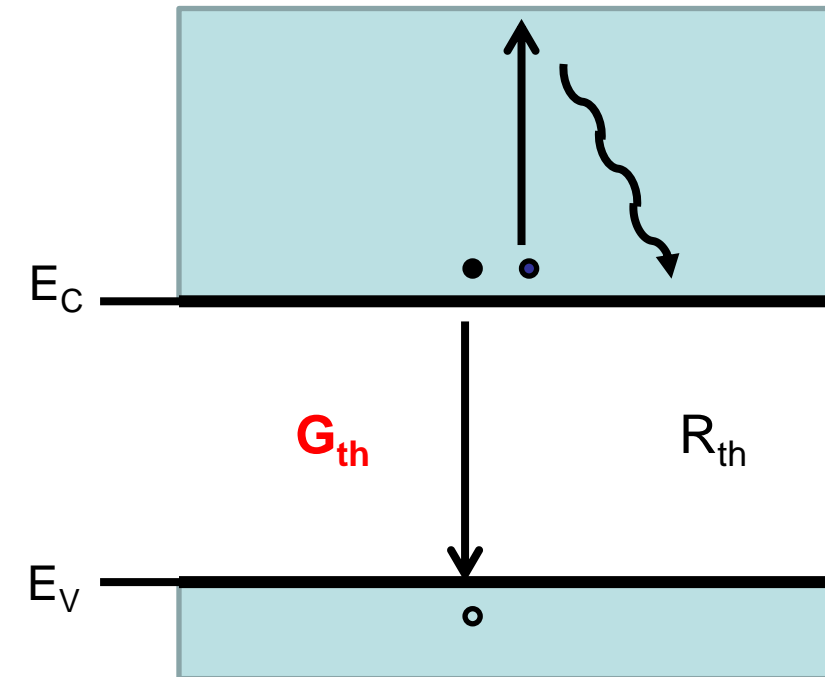
Indirect recombination model

- Modelled with the Schottky Read Hall (SRH) model:

$$U = \frac{v_{th}\sigma_n\sigma_p N_t (p_n n_n - n_i^2)}{\sigma_p \left[p_n + e^{(E_i - E_t)/kT} \right] + \sigma_n \left[n_n + e^{(E_t - E_i)/kT} \right]}$$

- Where v_{th} is the thermal velocity of the carriers, N_t is the trap concentration and σ is the capture cross-section ratio for holes and electrons
- σ describes the effectiveness of the trap to capture an electron or hole and is a measure of how close the electron needs to come to the centre to be captured

Auger recombination



- Auger recombination process occurs by the transfer of energy and momentum by a recombination event to a third particle (either an electron or hole)
- After the recombination process a second electron becomes an energetic electron and increases its energy level.
- It eventually loses its energy to the lattice by scattering events
- Auger recombination process occurs at very high carrier concentrations as a result of high doping or high injection levels

Lifetime example

- A silicon sample with $n_{no} = 10^{14} \text{ cm}^{-3}$ is illuminated with light and 10^{13} electron hole pairs/cm³ are created every microsecond. If $\tau_n = \tau_p = 2 \mu\text{s}$, find the change in the minority carrier concentration.

- Before illumination:

$$n_i^2 = p_{no} n_{no} \quad p_{no} = \frac{n_i^2}{n_{no}} = \frac{(9.65 \times 10^9)^2}{10^{14}} = 9.31 \times 10^5$$

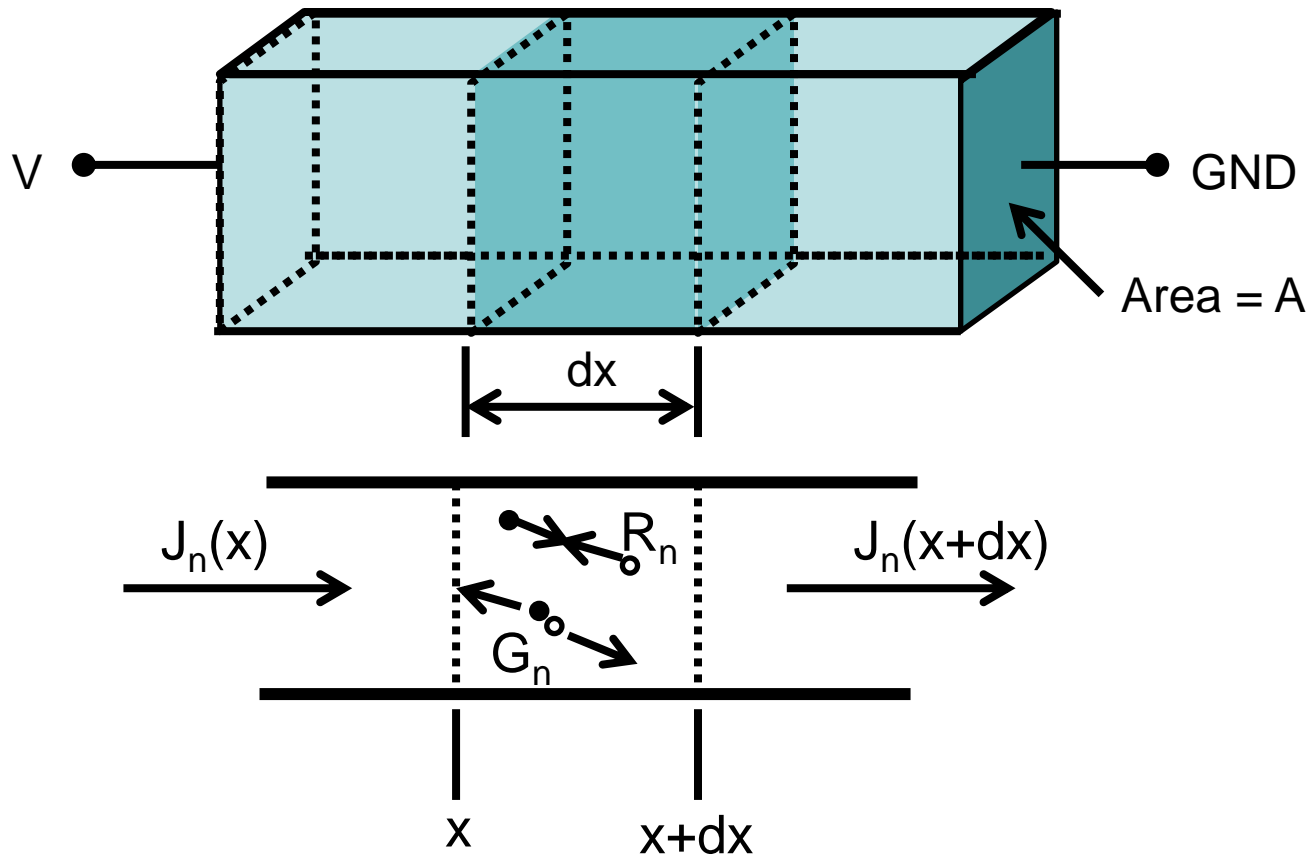
- After illumination:

$$G_L = \frac{p_n - p_{no}}{t_p} \quad p_n = p_{no} + \tau_p G_L = 9.31 \times 10^5 + 2 \times 10^{-6} \times \frac{10^{13}}{1 \times 10^{-6}} \\ = 2 \times 10^{13} \text{ cm}^{-3}$$

Continuity equation

- To this point we have only considered individual effects:
 - Carrier drift due to an electric field
 - Carrier diffusion due to concentration gradients
 - Generated carriers due to carrier injection or EHP
 - Recombination of carriers through intermediate-level recombination centres
- Considering the overall effect when these processes are occurring simultaneously, the governing equation is called the **continuity equation**

Current flow and generation-recombination processes



- Considering a slice of semiconductor of thickness dx
 - The number of electrons in the slice may:
 - increase at point dx due to the net current flow into the slice and the net generation in the slice
 - Reduce at point dx due to recombination within dx
- The overall number of electrons in the slice dx is the algebraic sum of four components:
 - The number of electrons flowing into the slice at point x ($J_n(x)$)
 - The number of electron leaving the slice at point dx ($J_n(x+dx)$)
 - Generation rate of electrons (G_n)
 - Recombination rate of electrons (R_n)

$$\frac{dn(x, t)}{dt} A dx = \left[\frac{J_n(x)A}{-q} - \frac{J_n(x + dx)A}{-q} \right] + (G_n(x, t) - R_n(x, t)) A dx$$

Where A is the cross-sectional area and $A dx$ is the volume of the slice

Hole and electron continuity equations

- Expanding the expression for current at $x+dx$ in Taylor series yields the basic continuity equation for electrons:

$$J_n(x + dx) = J_n(x) + \frac{dJ_n}{dx} dx$$

- Which can be formulated as a function of the derivation of the current:

$$\frac{dn(x, t)}{dt} = \frac{1}{q} \frac{dJ_n(x, t)}{dx} + (G_n(x, t) - R_n(x, t))$$

- And likewise for holes:

$$\frac{dp(x, t)}{dt} = -\frac{1}{q} \frac{dJ_p(x, t)}{dx} + (G_p(x, t) - R_p(x, t))$$

- Combining these with the previous definitions yields at partial differential equation for electron and hole density for a one-dimensional case:

$$\frac{dn(x,t)}{dt} = n\mu_n \frac{dE(x,t)}{dx} + \mu_n E \frac{dn(x,t)}{dx} + D_n \frac{d^2n(x,t)}{dx^2} + G_n - \frac{n(x,t) - n_o}{\tau_n}$$

$$\frac{dp(x,t)}{dt} = -p\mu_p \frac{dE(x,t)}{dx} - \mu_p E \frac{dp(x,t)}{dx} + D_p \frac{d^2p(x,t)}{dx^2} + G_p - \frac{p(x,t) - n_o}{\tau_n}$$

- A generalisation in three dimensions yields the following continuity equation for electrons and holes:

$$\frac{dn(x,y,z,t)}{dt} = \frac{1}{q} \vec{\nabla} \cdot \vec{J}_n(x,y,z,t) + G_n(x,y,z,t) - R_n(x,y,z,t)$$

$$\frac{dp(x,y,z,t)}{dt} = -\frac{1}{q} \vec{\nabla} \cdot \vec{J}_p(x,y,z,t) + G_p(x,y,z,t) - R_p(x,y,z,t)$$

Poisson's equation

- In addition to the continuity equation, Poisson's equation must be satisfied:
- Continuity equation:

$$\frac{dn(x, t)}{dt} = n\mu_n \frac{dE(x, t)}{dx} + \mu_n E \frac{dn(x, t)}{dx} + D_n \frac{d^2n(x, t)}{dx^2} + G_n - \frac{n(x, t) - n_0}{\tau_n}$$

- Poisson's equation:

$$\frac{dE(x, t)}{dx} = \frac{\rho_s}{\epsilon_s}$$

- Where ϵ_s is the semiconductor dielectric permittivity and ρ_s is the space charge density given by the algebraic sum of the charge densities and ionised impurity concentration

$$\rho_s = q(p - n + N_D^+ - N_A^-)$$

How are the equations solved?

- In principle the Continuity and Poisson's equations are solved together with appropriate boundary conditions to obtain an unique solution for any semiconductor device
- Due to their complexity these are simplified significantly to understand basic characteristic of basic device structures
- To develop “real” devices for fabrication: Technology Computer Aided Design software (TCAD) such as Silvaco Atlas or Synopsys ISE are used
 - Within numerical techniques are use to obtain convergence for a given boundary condition

Further reading

- Ben G Streetman: Solid State Electronic Devices:
ISBN: 0-13-149726-X
 - Chapters 1-3
- S.M.Sze: Semiconductor Devices: Physics and
technology: ISBN0-471-33372-7
 - Chapters 1-3
- On-line recourse
 - Colorado university
 - <http://ecee.colorado.edu/~bart/book/book/contents.htm>
 - Britney Spears guide to semiconductor physics
 - <http://britneyspears.ac/lasers.htm>