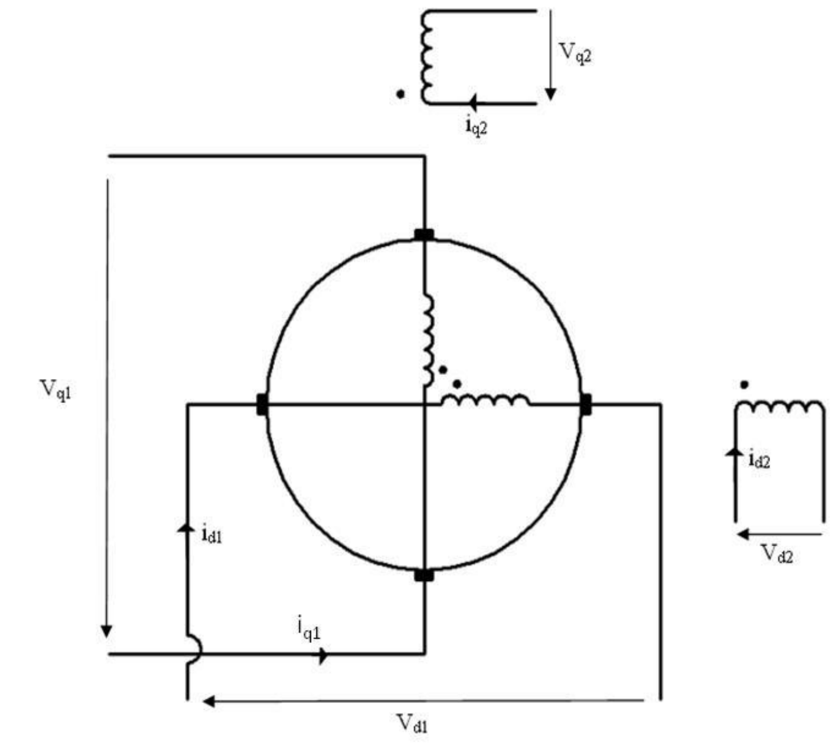
EEE409 / 6120 Modelling of Machines

Examination worked solutions 2012

1.

a)



The general form of the voltage matrix equations is given by:

$$\begin{vmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{vmatrix} = \begin{vmatrix} R_{d1}+L_{d1}p & G_{d1q1}\omega_r & M_{d1d2}p & G_{d1q2}\omega_r \\ G_{q1d1}\omega_r & R_{q1}+L_{q1}p & G_{q1d2}\omega_r & M_{q1q2}p \\ M_{d2d1}p & 0 & R_{d2}+L_{d2}p & 0 \\ 0 & M_{q2q1}p & 0 & R_{q2}+L_{q2}p \end{vmatrix} \begin{vmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{vmatrix}$$

b)

[This question has some similarities with previous questions, e.g. the universal machines question in 2006, in that it presents candidates with a mixture of machine parameters and performance data. The route to the solution is rather different, but as in many previous cases hinges around the need to link DC and AC operation via a specified torque operating point (at which the DC current and rms AC current can be equated). The first few stages do not follow a solution route which has been encountered before.]

Input power on DC = V I = 230 × 3.5 = 805W

For an efficiency of 84%

Losses (which are solely down to copper loss) = (1-0.84) × 805 = 129W

Output power = 0.84× 805 = 676W

i) $R = \frac{Losses}{(Input\ current)^2} = \frac{129}{3.5^2} = 10.5\Omega$

ii) $T = \frac{Output\ \ power}{Mechanical\, speed} = \frac{676}{\frac{18000}{60} \times 2\pi} = 0.359Nm$

Hence,

$$M = \frac{T}{(Input\ current)^2} = \frac{0.359}{3.5^2} = 0.0293H$$

At the same load torque condition, the same rms current is drawn as the DC case. Hence, the magnitude of the input current is 3.5A rms

The voltage equation for AC operation is:

$$V = (R + NX_m + j\omega_s L)\ I$$

where N is the ratio of actual speed to synchronous speed

$$\therefore \left|\frac{V}{I}\right| = |Z| = \sqrt{\left((R + NX_m)^2 + (\omega_s L)^2\right)}$$

Re-arranging this equation yields:

$$N = \frac{1}{X_m}\left(\sqrt{\left(\frac{V^2}{I^2}\right) - (\omega_s L)^2}\right) - \frac{R}{X_m}$$

For the particular parameters of this motor:

N = 4.16

iii) Actual speed on AC supply = 4.16 × 3000 = 12,492rpm

iv) $\text{Power factor on load} = \frac{\mathrm{R + NX_m}}{\sqrt{\left((\mathrm{R + NX_m})^2 + (\omega_s L)^2\right)}} = 0.743\ \ lagging$

[Note: It is important to state that the power factor is **lagging]**

v) For starting torque, $\omega_r = 0$

On DC: $I = \frac{V}{R} = \frac{230}{10.5} = 21.9A$

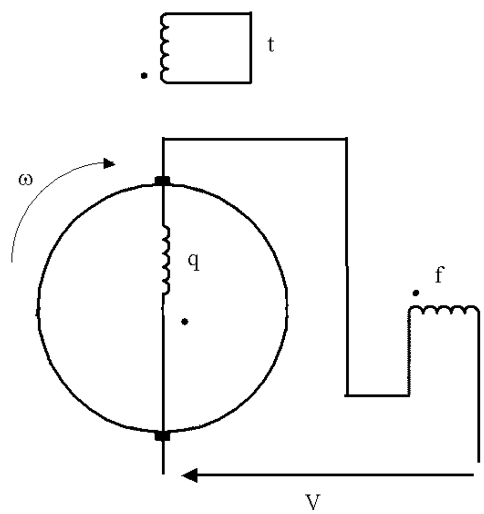
$\therefore T = MI^2 = 0.0293 \times 21.9^2 = 14.0 \;\; Nm$ on DC

On AC: $I = \frac{V}{\sqrt{R^2 + (\omega_s L)^2}} = \frac{230}{45.2} = 5.08A \; rms$

$\therefore T = MI^2 = 0.0293 \times 5.08^2 = 0.757 \;\; Nm$ on AC

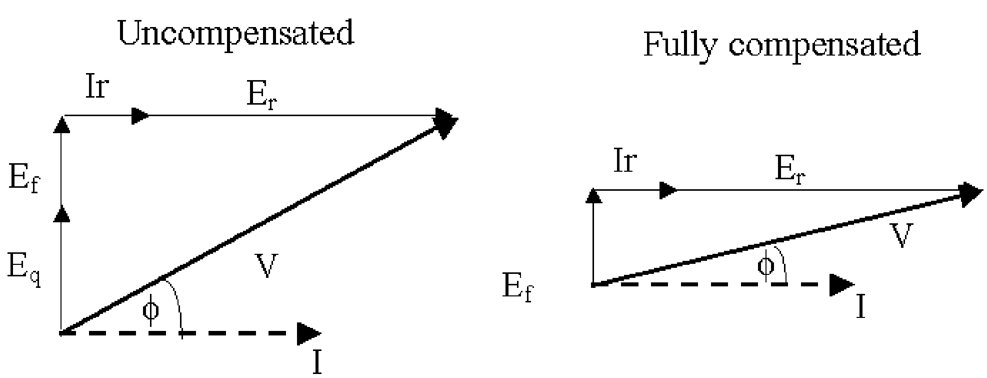
Ratio of DC starting torque to AC starting torque = 18.5

c) The Kron primitive equivalent of an inductively compensated series universal motor is:



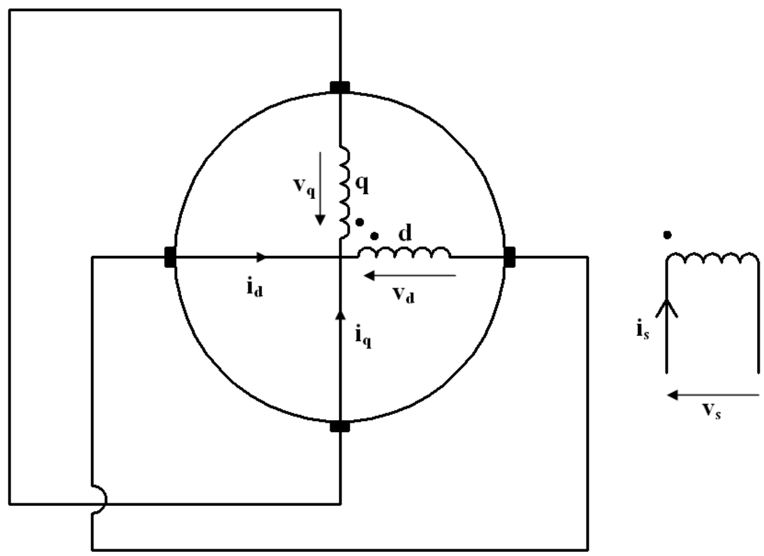
Full compensation is achieved when the q-axis coils have a coupling coefficient of 1 (this contrasts with a conductively coupled machine in which other constraints on the inductance of the compensation coil must be met).

The resulting phasor diagrams are:



2.

a)



The general form of the voltage matrix equations is:

$$\begin{vmatrix} v_s \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_s + L_s p & M_{sd}p & 0 \\ M_{ds}p & R_d + L_d p & -\omega_r L_q \\ \omega_r M_{qs} & \omega_r L_d & R_q + L_q p \end{vmatrix} \begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix}$$

[Note: The use of subscripts 1 for stator and 2 for rotor would be equally correct in the above equations]

For steady-state sinusoidal AC operation, $p=j\omega_s$ and $\omega_r=(1-s)\omega_s$

The short-circuited rotor windings dictate that $V_q=V_d = 0$

Adopting subscripts 1 for stator and 2 for rotor, yields:

$$\begin{vmatrix} V_s \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m & 0 \\ jX_m & R_2+jX_2 & -(1-s)X_2 \\ (1-s)X_m & (1-s)X_2 & R_2+jX_2 \end{vmatrix} \begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix}$$

The unbalanced currents on the right hand side can be transformed to symmetrical components:

$$\begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix} = C \begin{vmatrix} Is \\ Ip \\ In \end{vmatrix}$$

Where C is the following transformation matrix:

$$C = 1 / \sqrt{2} \begin{vmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & j & -j \end{vmatrix}$$

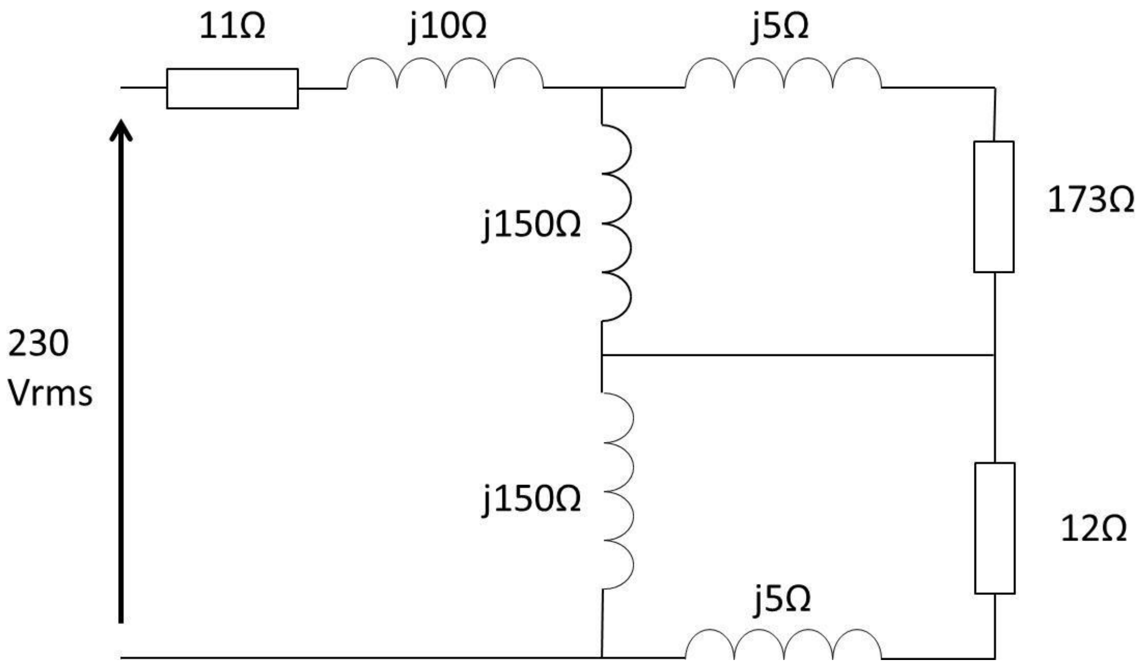
Applying the transformation matrix to the impedance matrix using:

$Z' = C_t^* \, Z \, C$

Following two stages of matrix multiplication, the final result is:

$$\begin{vmatrix} 2V_s \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 2(R_1+jX_1) & jX_m & jX_m \\ jX_m & \frac{R_2}{s} + jX_2 & 0 \\ jX_m & 0 & \frac{R_2}{(2-s)} + jX_2 \end{vmatrix} \begin{vmatrix} Is \\ \sqrt{2}\, I_p \\ \sqrt{2}\, I_n \end{vmatrix}$$

b) The equivalent circuit populated with the parameters of the machine is:



The positive sequence equivalent impedance is:

$$Z_p = \frac{\big((173 + j5) \times j150\big)}{173 + j5 + j150} = \frac{173\angle 1.65^o \times 150\angle 90^o}{232\angle 41.8^o}$$

$$= \frac{25960\angle 91.65^o}{232\angle 41.8^o} = 111.8\angle 49.8^o\Omega = 72.2 + \mathrm{j}85.4\Omega$$

Similarly, the negative sequence impedance is:

$$Z_n = \frac{\big((12 + j5) \times j150\big)}{12 + j5 + j150} = \frac{13\angle 22.6^o \times 150\angle 90^o}{155\angle 85.6^o}$$

$$= \frac{1950\angle 112.6^o}{155\angle 85.6^o} = 13\angle 27^o\Omega = 11.2 + \mathrm{j}5.7\Omega$$

Total impedance is given by:

Z_{TOTAL}= (11+72.2+11.2) + j(10+85.4+5.9) = 94.4 + j101.3Ω = 138.8 $\angle 46.7^o$ Ω

i) Input current $= \frac{V}{Z} = \frac{230\angle 0^0}{138.8\angle 46.7^0} =$1.65 $\angle$-46.7^0 Arms

ii) Power factor = cos (-46.7^0) = 0.683 **lagging** [Important to note **lagging**]

iii) Input power = VI cosϕ = 230×1.65×0.683 = 259W

iv) Voltage across positive branch:

$V_p = I_{in}Z_p = 1.65\angle{-46.7^0} \times 111.8\angle 49.8^o = 184.5\angle 3.1^o Vrms$

Current through 173Ω resistor

$$Ip = \frac{184.5\angle 3.1^o}{173\angle 1.7^o} = 1.07\ \angle 1.4^o Arms$$

And similarly across the negative branch:

$V_n = I_{in}Z_n = 1.65\angle{-46.7^0} \times 13\angle 27^o = 20.8\angle{-19.7^o} Vrms$

$$I_n = \frac{21.4\angle{-19.7^o}}{13\angle 22.6^o} = 1.60\ \angle{-42.3^o} Arms$$

$P_{mech} = \left(I_p^2 \frac{R_2'}{2s} - I_n^2 \frac{R_2'}{(2-s)2}\right)(1-s)$ =(198-30.8)×(0.87) = 145.5W

v) Mechanical torque = Mechanical power / mechanical speed

Mechanical speed = 1500×(1-0.13) = 1305 rpm

Hence, mechanical torque is given by:

$$T = \frac{144}{1305 \times \frac{2\pi}{60}} = 1.064 Nm$$

3.

a) By inspection of Figure 3, the rate of change of flux linkage has a maximum value between 17.5° and 20° with a current of 3A. The rate of change is given by:

$$\frac{\mathrm{d}\psi}{\mathrm{d}\theta} = \frac{101\times10^{-3} - 58\times10^{-3}}{2.5\times\frac{\pi}{180}} = 0.985\mathrm{Wb/radian}$$

At 4000rpm

$$\frac{\mathrm{d}\psi}{\mathrm{dt}} = \frac{\mathrm{d}\psi}{\mathrm{d}\theta}\times\frac{\mathrm{d}\theta}{\mathrm{dt}} = 0.985\times\frac{4000\times2\pi}{60} = 413\mathrm{V}$$

[At this is based on graphical estimation of flux-linkage data from the graph some degree of tolerance in the solution is acceptable – the key is the identification of the correct data points to use]

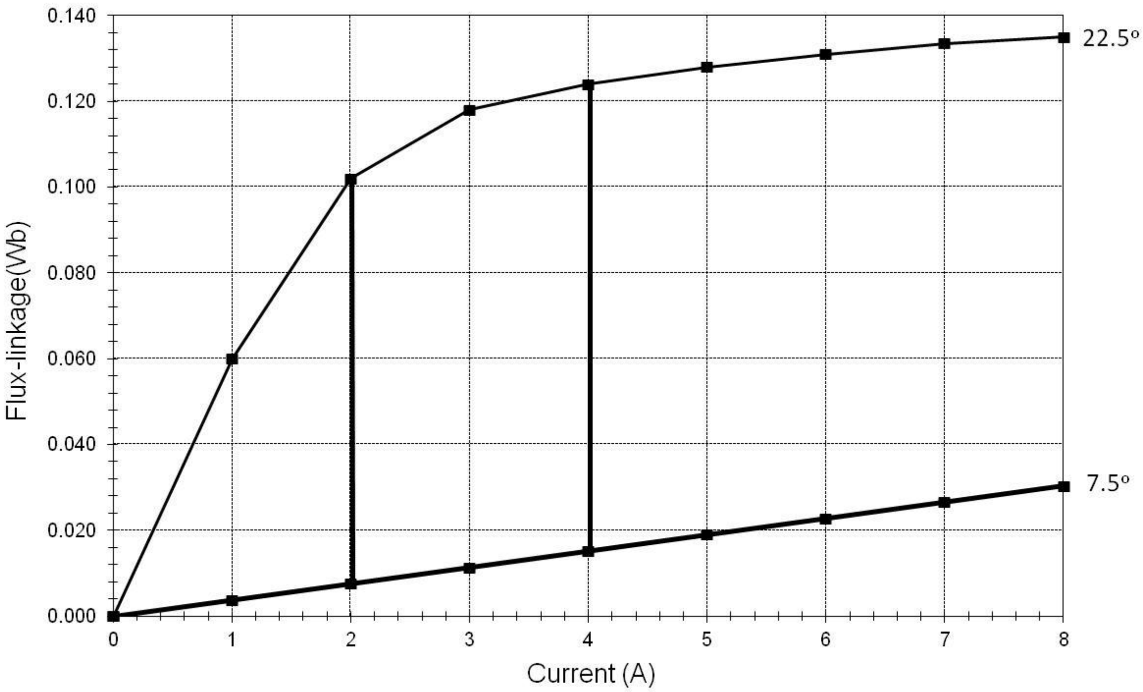
b) The normal stroke angle for a 3-phase, 12-8 SR machine is given by:

$$Angle = \frac{2\pi}{Phases\ \times Rotor\ teeth} = \frac{2\pi}{3\times 8} = \frac{\pi}{12} = 15^o mech$$

Hence, there are **24** torque strokes per mechanical revolution

c) The normal stroke therefore occurs between 7.5° and 22.5°

The resulting flux-linkage versus current characteristic is given by:



Applying the trapezium rule to integrate the area under the 22.5° curve:

$$A_1 = \frac{\Psi_1}{2} = \frac{60\times10^{-3}}{2} \qquad = 30\times10^{-3}\ \text{J}$$

$$A_2 = \frac{\Psi_1+\Psi_2}{2} = \frac{60\times10^{-3}+0.102}{2} \qquad = 81\times10^{-3}\ \text{J}$$

$$A_3 = \frac{\Psi_2+\Psi_3}{2} = \frac{0.102+0.118}{2} \qquad =110\times10^{-3}\ \text{J}$$

$$A_4 = \frac{\Psi_3+\Psi_4}{2} = \frac{0.118+0.124}{2} \qquad =121\times10^{-3}\ \text{J}$$

The area under the 7.5° curve (which can reasonably regarded as being linear) is simply given by:

$$\text{U}_{1\to2} = \frac{2\Psi_2}{2} = \frac{2\times0.008}{2} = 8\times10^{-3}\ \text{J}$$

$$\text{U}_{1\to4} = \frac{4\Psi_4}{2} = \frac{4\times16\times10^{-3}}{2} = 32\times10^{-3}\ \text{J}$$

Hence the change in co-energy up to 2A is given by:

$$\Delta\text{W}_2^{'} = \text{A}_1+\text{A}_2-\text{U}_{1\to2} = 103\times10^{-3}\ \text{J}$$

And similarly up to 4A:

$$\Delta W_4^{'} = A_1 + A_2 + A_3 + A_4 - U_{1\rightarrow 4} = 310\times 10^{-3}\ \mathrm{J}$$

The average torque for 2A is therefore given by:

$$T_{AVE} = \frac{\Delta W^{'}}{\Delta\theta} = \frac{103\times 10^{-3}}{15\times\frac{\pi}{180}} = \underline{\mathbf{0.39Nm}}$$

And similarly for 4A:

$$T_{AVE} = \frac{\Delta W^{'}}{\Delta\theta} = \frac{310\times 10^{-3}}{15\times\frac{\pi}{180}} = \underline{\mathbf{1.18Nm}}$$

d) The maximum value of self-inductance occurs between 0 and 1A. Taking the flux-linkage at 1A as 0.06Wb, yields a phase self-inductance of:

$$L = \frac{0.06}{1.0} = 60mH$$

e) The flux-linkage versus current characteristic in the fully-aligned position shows evidence of significant magnetic saturation at ~2A *[Some spread on this value is inevitable given the rather subjective nature of interpreting saturation]*.

In the aligned position, the airgap flux density is, to a reasonable assumption, equal to the flux density in the critical parts of the core. Hence, since two coils produce the mmf that drive the flux across two-airgaps, then:

$$B_g = \frac{\mu_0 N_c I}{l_g}$$

$$N_c = \frac{B_g l_g}{\mu_0 I} = \frac{1.5\times 0.5\times 10^{-3}}{4\pi\times 10^{-7}\times 2} = 298 \text{ turns per coil}$$

4.

a) The flux-linkage characteristics for 0A is a reasonable approximation to a sine-wave [in fact the actual data is generated from a simple sin function]. It is therefore reasonable to assume that the maximum rate of change of flux-linkage will occur at angular displacements around 0^o. [*It is not necessary to identify this with a sine wave, just to recognise visually that the maximum rate of change will occur around 0^o*]. From Figure 4b, an estimate of the rate of change of flux linkage with rotor position can be made:

$$\frac{d\Psi}{d\theta} \approx \frac{\Delta\Psi}{\Delta\theta} = \frac{0.007}{20\times\frac{\pi}{180}} = 0.020 \text{ Wb/rad}$$

[These calculations have all been performed in terms of mechanical radians]

At 4500rpm

$$\frac{d\theta}{dt} = \frac{4500\times 2\times\pi}{60} = 471 \text{ rad/s} \;\; \therefore e = \frac{d\Psi}{dt} = \frac{d\Psi}{d\theta}\times\frac{d\theta}{dt} = 9.42\text{V}$$

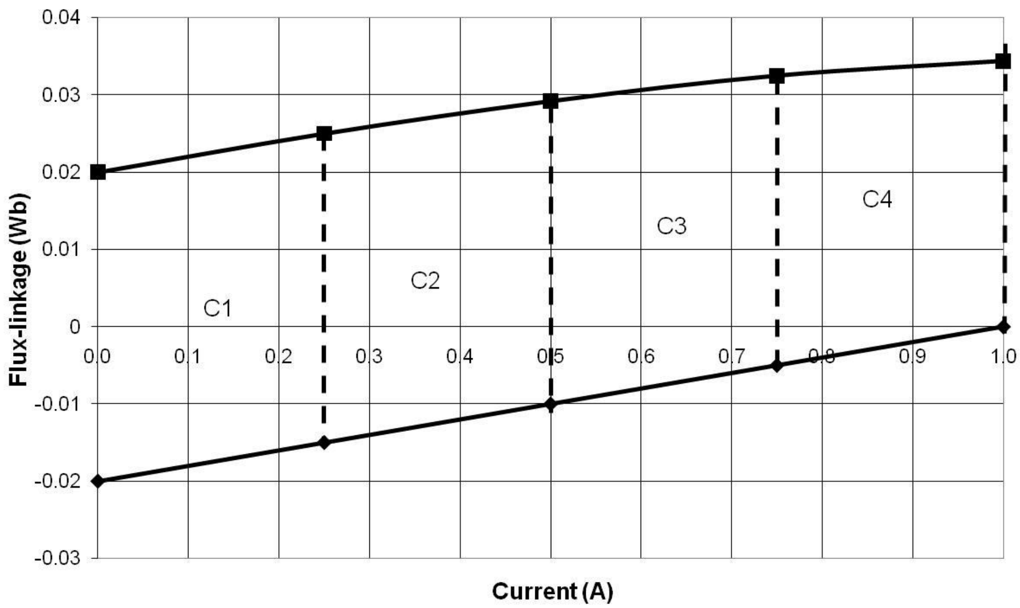
(An alternative is to note that the variation in flux-linkage can approximated as:

$\Psi = 0.02\sin(\theta)$

Hence $\frac{d\Psi}{d\theta} = 0.02\cos(\theta)$

This has a peak value of 0.02 Wb/rad - as before from graphical interpolation)

b) In order to estimate the torque for the two currents specified it is necessary to re-plot the data as a flux-linkage versus current characteristic for -90^o and $+90^o$:



The co-energy change can be estimated by trapezoidal integration of the four areas C1 to C4 shown in the graph above. Using this approach:

The change in co-energy for 0.5A is C1 + C2 = 0.010+ 0.0099= 0.0199J

The change in co-energy for 1.0A is C1+C2+C3+C4 = 0.01+ 0.0099+0.0096+0.0090=0.0385J

Change in rotor angular displacement = $180 \times \frac{\pi}{180} = \pi$ rads

The torques produced are therefore given by:

At 0.25A: $T = \frac{dW^{'}}{d\theta} \approx \frac{0.0199}{\pi} = 25.4 \ \times 10^{-3}\, Nm$

Hence, torque per amp = $25.4\times10^{-3}/0.5 = 50.8\times10^{-3}$ Nm/A

At 1.0A: $T = \frac{dW^{'}}{d\theta} \approx \frac{0.0385}{\pi} = 49.0 \times 10^{-3}\, Nm$

Hence, torque per amp = $49.0\times10^{-3}/1.0 = 49.0\times10^{-3}$ Nm/A

The reduction in torque per amp at higher current levels is due to the onset of magnetic saturation in the motor core.

c) From Figure 4b, it is apparent that saturation begins at a net coil flux of 0.03 Wb *[there is a reasonable tolerance band on this given the difficulty in unequivocally identifying the onset of saturation]*

Hence, 0.03Wb of coil flux-linkage corresponds to 1.4T in the core

At 0°, there is no net permanent magnet flux and hence the flux-linkage is entirely due to the coil. Taking the case of 1A, the flux-linkage is 0.020Wb.

Hence, with no permanent magnet flux and a 1A coil current, the core flux density is approximately given by:

$B = \frac{0.02}{0.03} \times 1.4 =$0.93T

The total effective magnetic airgap is $2l_g+2l_m$=6mm

The flux density produced by the coil at 0° is given by:

$$B_g = \frac{\mu_0 NI}{(2l_g + 2l_m)}$$

Rearranging, gives:

$$N = \frac{B_g(2l_g + 2l_m)}{\mu_0 I} = \frac{0.93 \times (6 \times 10^{-3})}{4\pi \times 10^{-7} \times 1.0} = 4440\ turns$$

d) *[The key to unlocking this problem is to recognise that the airgap flux density produced by the magnet can be obtained by scaling the flux density from the coil calculations in part (c). Once this link has been made, the calculation is itself straightforward. This process and the subsequent* *calculation of the material remanence has not featured to date in any previous examination papers, nor has it featured in any tutorial sheets]*

Maximum permanent magnet flux occurs at -90° and +90°. At these rotor angular displacements, the flux-linkage under open-circuit conditions is: 0.02Wb, which corresponds to a flux density of 0.93T.

The airgap flux is given by:

$$B_g = \frac{B_r}{1 + \mu_r \frac{l_g}{l_m}}$$

Re-arranging yields:

$$B_r = \left(1 + \mu_r \frac{l_g}{l_m}\right) B_g = 1.12\text{T}$$