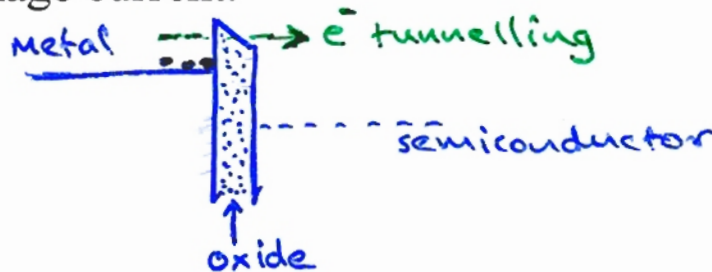Amplitude of the wave describing the electron relates to the probability of finding the electron at that point.
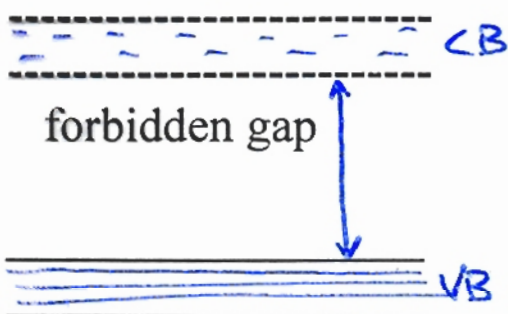
Practical examples of tunnelling:
Zener diode – see later
MOSFET – oxide between the gate and channel is very thin for modern devices as miniaturisation proceeds, resulting in tunnel leakage current.

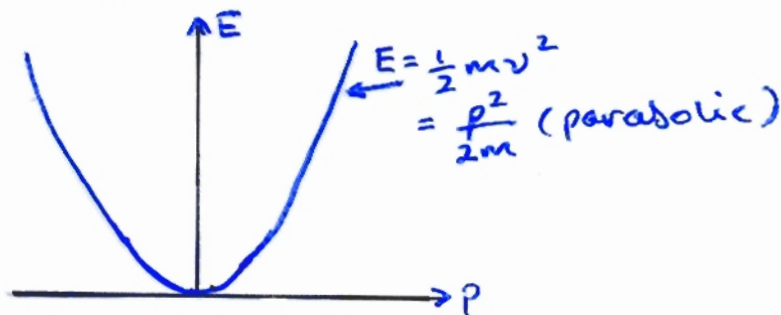## Wave Picture of Forbidden Bandgaps in Semiconductors

Before we saw that electrons were 'not allowed' in the forbidden gap between the V.B. and C.B.

How does this arise?

Consider an electron in a vacuum

$E = \frac{1}{2}mv^2 = p^2/2m$ (since momentum $p = mv$)

For an electron inside a semiconductor, the electron interaction with the lattice is taken into account by using an 'effective mass' m^* instead of m_o or m_e.
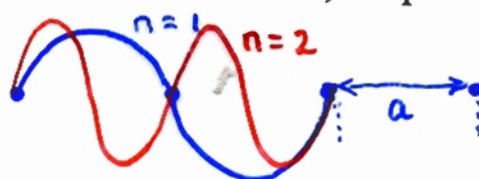Differentiating E w.r.t. p,

$$\frac{dE}{dp} = \frac{p}{m} \qquad\qquad \frac{d^2E}{dp^2} = \frac{1}{m}$$

i.e. the **mass** or effective mass is given by the **curvature** of the E-p relationship.

Since $p = h/\lambda$ (de Broglie), $E = hc/\lambda$, i.e. relates energy and wavelength.
As the electron energy and hence changes, the electron wave begins to 'reflect' from the lattice, of period 'a' until:
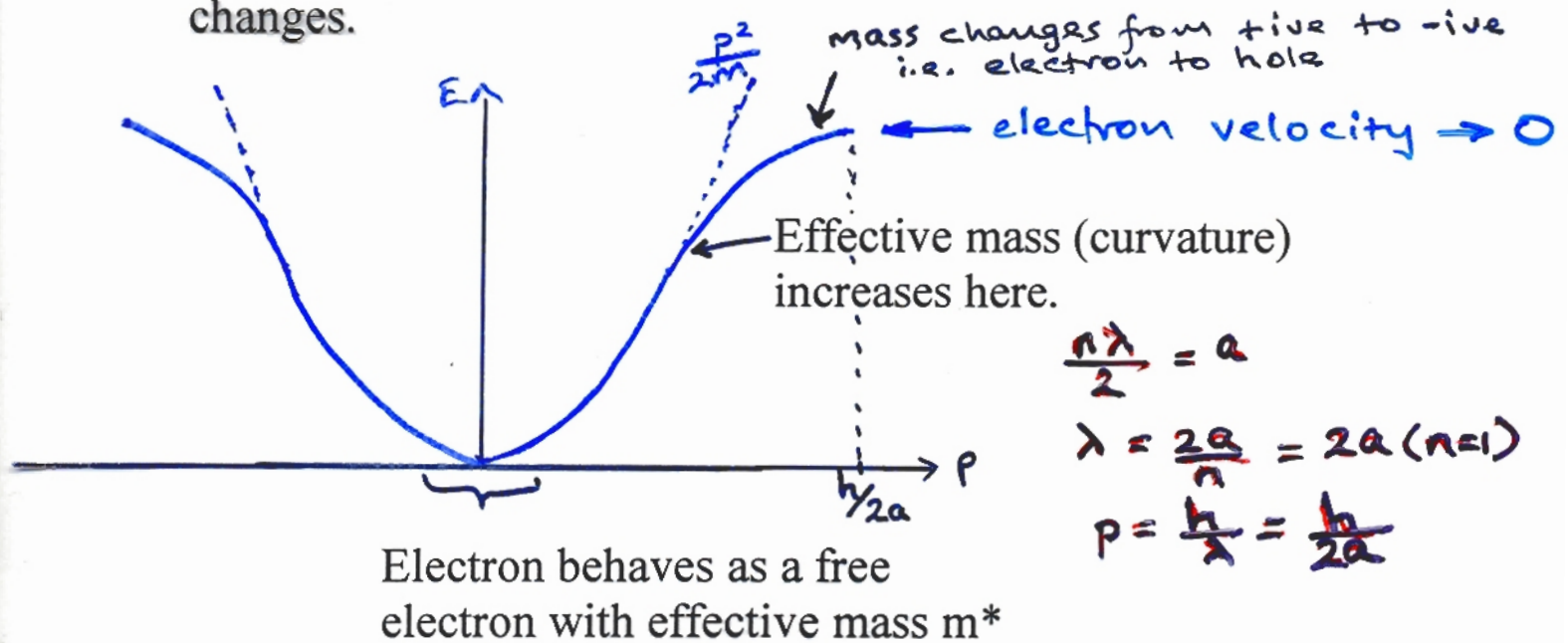
$$\frac{n\lambda}{2} = a$$

when a standing wave results and no propagation of the electron wave is possible hence we get forbidden energies. (A more thorough treatment of this requires us to understand Schrödinger's equation – *see JA, pg.85-90*). Think of an electrical analogy – if we have an air filled transmission line, with dielectric discs of permittivity ε_r spaced with a period 'a', we will find that not all wavelengths (i.e. frequencies) will propagate and we get reflections at wavelengths corresponding to $n\pi/a$.

This equation only predicts discrete forbidden energies. In reality as the electron wave approaches this condition, more and more of the wave is reflected and the effective mass changes.
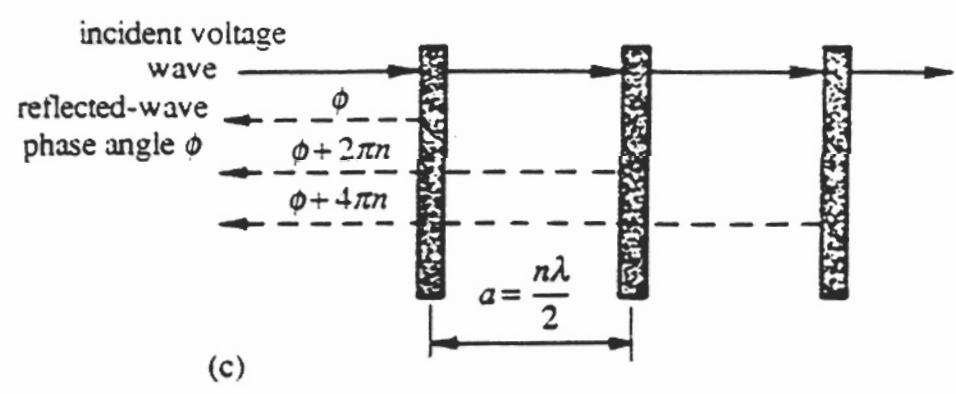

$\frac{p^2}{2m}$

mass changes from +ive to -ive
i.e. electron to hole

← electron velocity → 0

—Effective mass (curvature) increases here.

$$\frac{n\lambda}{2} = a$$

$$\lambda = \frac{2a}{n} = 2a \ (n=1)$$

$$p = \frac{h}{\lambda} = \frac{h}{2a}$$

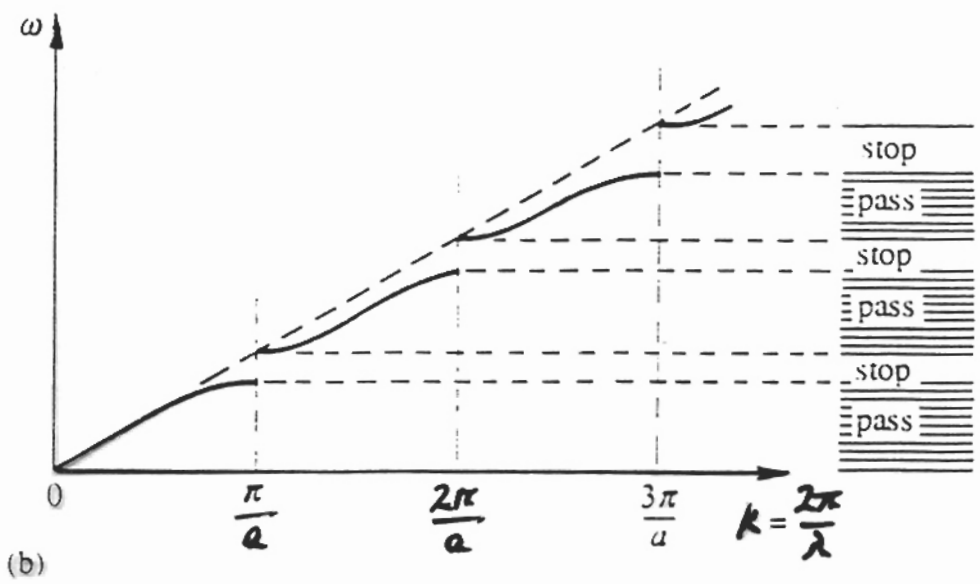Electron behaves as a free electron with effective mass m*

This results in 'bands' of forbidden energy.
Note: In most semiconductors electrons exist near the bottom of the conduction band where m* is approximately constant,
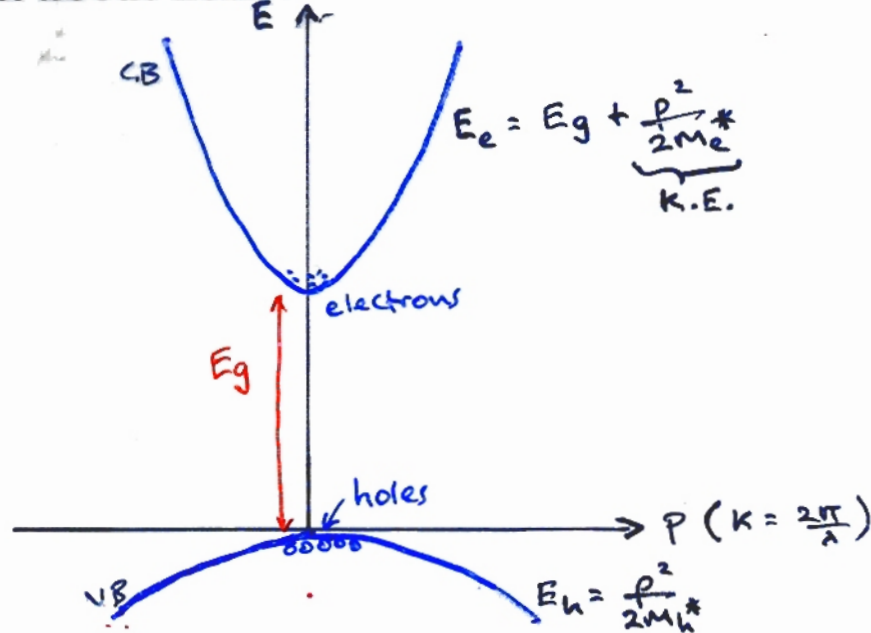i.e. we can use classical mechanics with $m_e \rightarrow m*$.

dielectric discs
(relative permittivity ε_r)
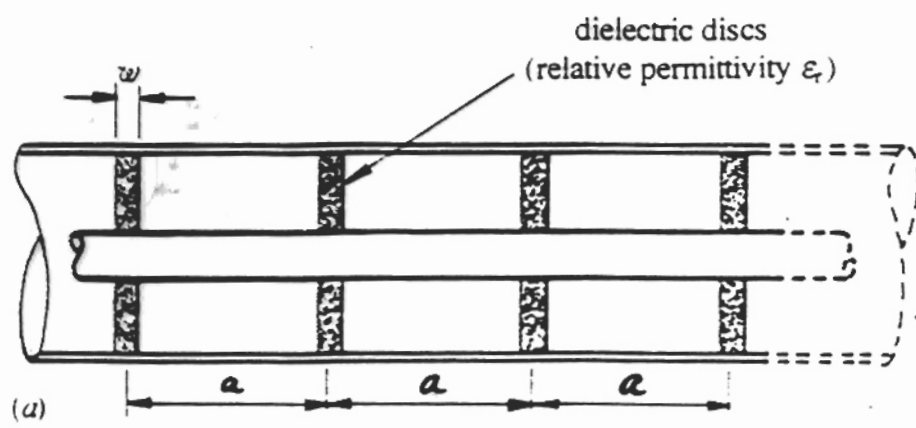
Voltage at a point x on transmission line is:

$$\frac{d^2 V}{dx^2} + \omega^2 \mu_0 \varepsilon_0 \varepsilon_r V = 0$$

(a)

(b)

stop
pass
stop
pass
stop
pass

$$k = \frac{2\pi}{\lambda}$$

incident voltage
wave

reflected-wave
phase angle ϕ

ϕ

$\phi + 2\pi n$

$\phi + 4\pi n$

$$a = \frac{n\lambda}{2}$$

(c)

5.10 (a) A periodically loaded transmission line; (b) ω–k diagram of the loaded line; (c) total
reflection occurring when $ka = n\pi$.

Stop bands arise when $ka = n\pi$ and weak reflections
from successive discs add in phase producing
total reflection - no travelling wave — only
standing wave.

What about holes?

$$E_e = E_g + \frac{p^2}{2m_e^*} \quad \text{(K.E.)}$$

electrons

E_g

holes

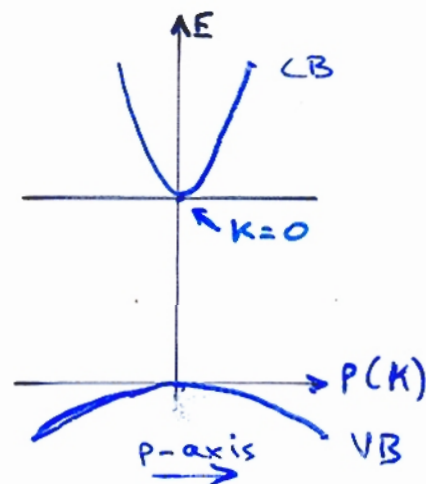$$P\left(K = \frac{2\pi}{\lambda}\right)$$

$$E_h = \frac{p^2}{2m_h^*}$$

V.B. has opposite curvature, hence m* is 'negative' – this is allowed for by assuming positively charged holes with an effective mass m_h*. As with electrons, holes in a crystal lattice can be described by classical mechanics with $m_e \rightarrow m_h^*$.
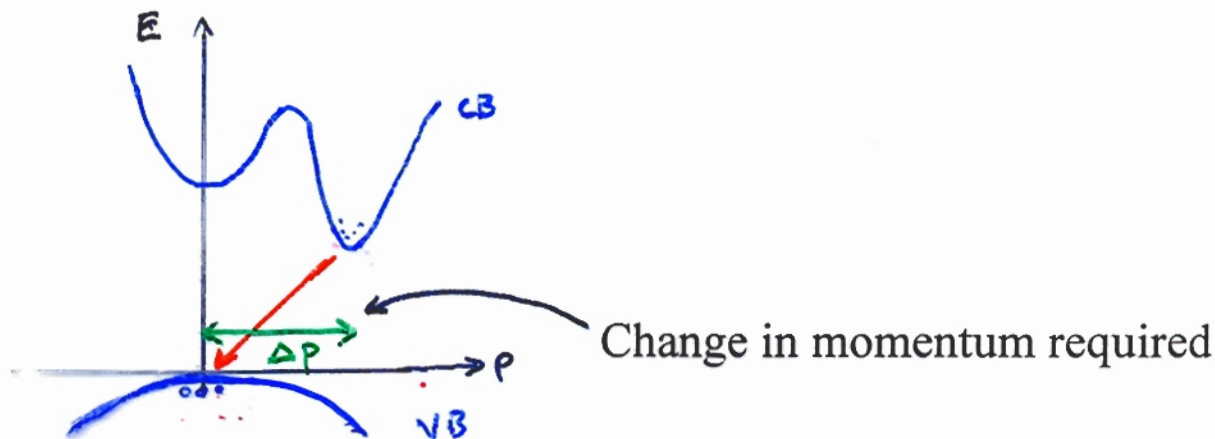
Before we drew
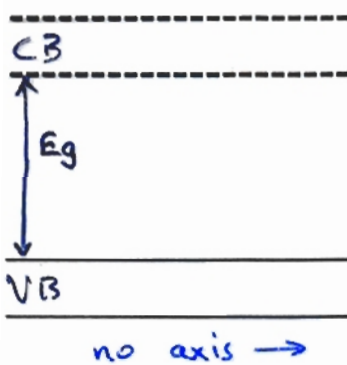
This is so called 'direct gap' semiconductor where the maximum of the valence band lies directly below the minimum of the conduction band (where most electrons are).

For direct gap semiconductors electrons can recombine with holes easily since no change in momentum (p) is required.
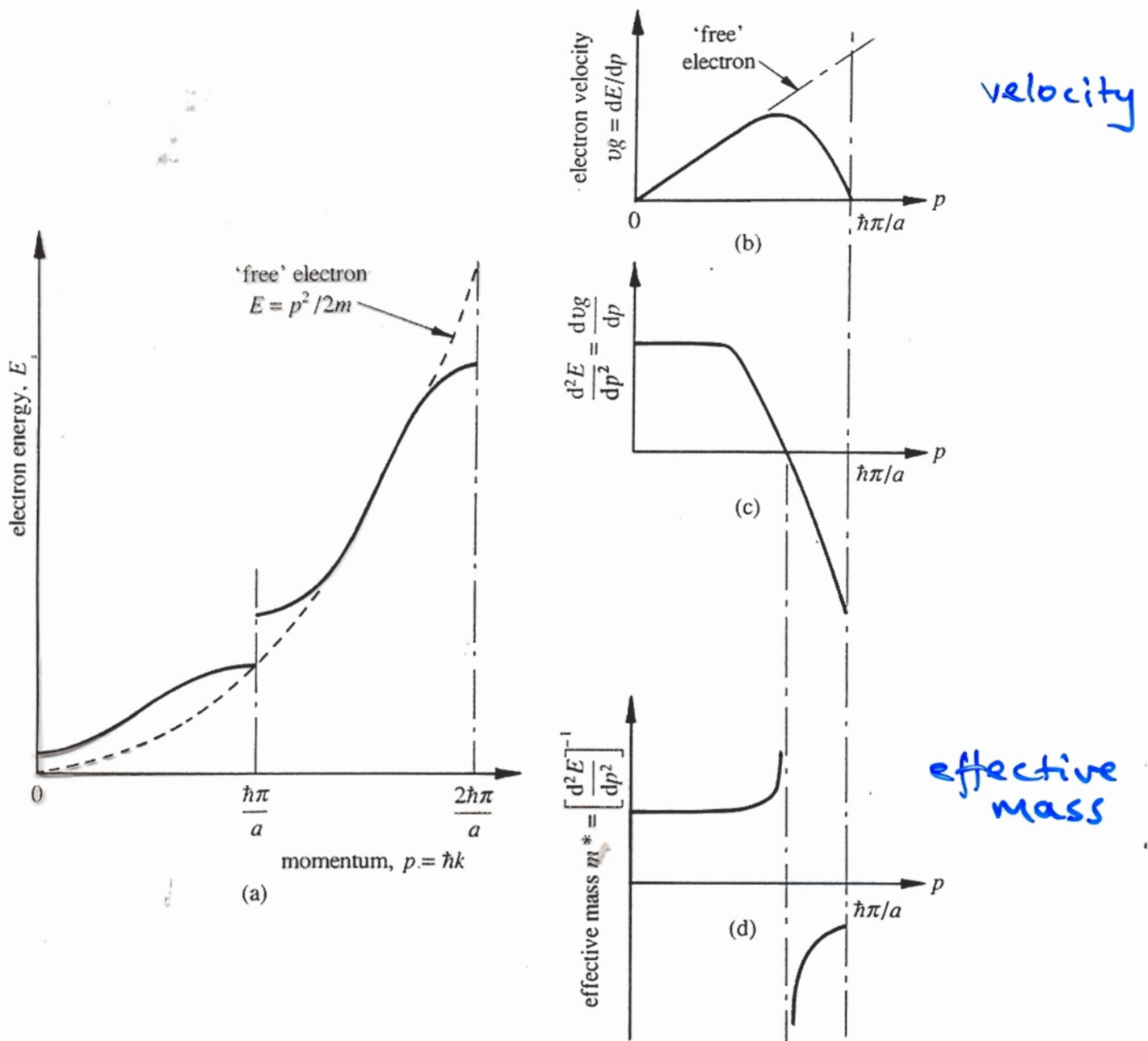
Indirect-gap semiconductors
e.g. Si

Change in momentum required

Electron-hole recombination now requires a momentum change as well. This required momentum change comes (phonons) from lattice vibrations and is less likely to happen compared to direct gap material.

Direct gap materials are good for light emission, e.g. lasers, LEDs and light detection (reverse process) – will cover later.

'free' electron
$E = p^2/2m$

electron velocity $vg = dE/dp$

'free' electron

$\dfrac{d^2E}{dp^2} = \dfrac{dvg}{dp}$

effective mass $m^* = \left[\dfrac{d^2E}{dp^2}\right]^{-1}$

electron energy. E

momentum, $p = \hbar k$

$\dfrac{\hbar\pi}{a}$ $\dfrac{2\hbar\pi}{a}$

$\hbar\pi/a$

velocity

effective mass

(a) (b) (c) (d)

Variation of energy, velocity and effective mass of an electron in a solid.

Accept for now that electron velocity,

$$v = \frac{dE}{dp} = \frac{dE}{\hbar dk}$$ *(actually group velocity of the wavepacket)*

At $k=0$ and at Brillouin zone edge, $v = 0$. i.e. the electronic wavefunctions become 'standing waves'.

The momentum of an electron in a solid is not simply related to its velocity or energy (like in a vacuum) due to its interactions with the lattice.

Electron velocity is a maximum at some point before the zone boundary – given by the detailed shape of the E-k (or E-p) relationship.

Effective mass (m_e^*) is initially constant (parabolic band), then increases (due to non-parabolicity), becomes infinite, then changes sign near zone boundary.
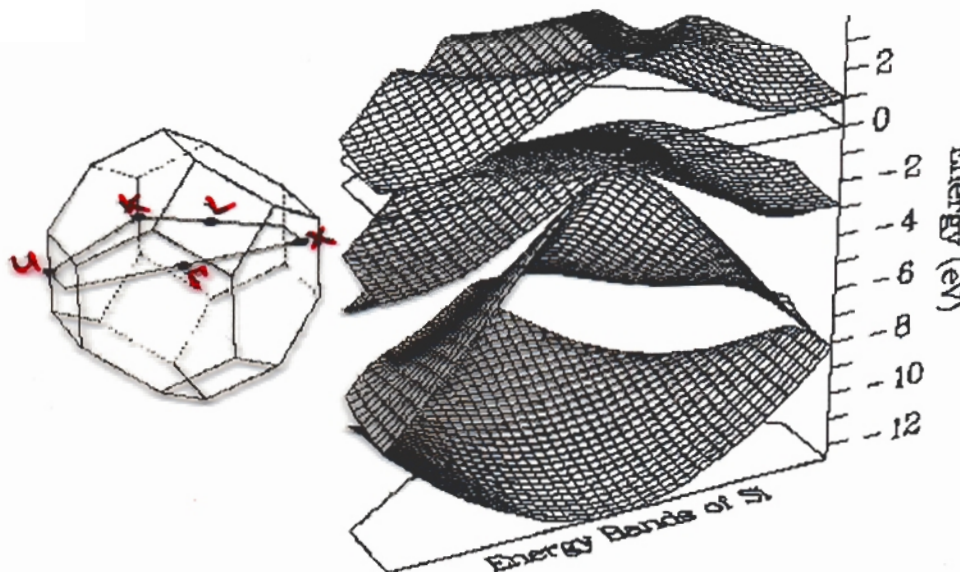Most of the time we only need to worry about electrons close to $k=0$, the zone centre.

E-p diagrams are often shown as E-k since $p=\hbar k$. The 'x' axis is sometimes shown as h/λ ($=p$) or 2π/λ ($=k$).

The Brillouin zone is

$$\frac{-\pi}{a} \geq k \geq \frac{\pi}{a}$$

where 'a' is the lattice spacing.

Silicon Band Structure

Perspective plot of the energy band structure of silicon. The figure to the left shows the Brillouin zone, and the two-dimensional section over which the energy bands are displayed. The energy bands are plotted to the right. The four surfaces lying below 0 eV are the valence bands, and the upper $k = 0$, which on this figure is the center of the front boundary of the Brillouin-zone section. The minimum conduction-band energy occurs along the front boundary of the section, near the left and right ends. Thus, Si has an indirect-gap band structure.
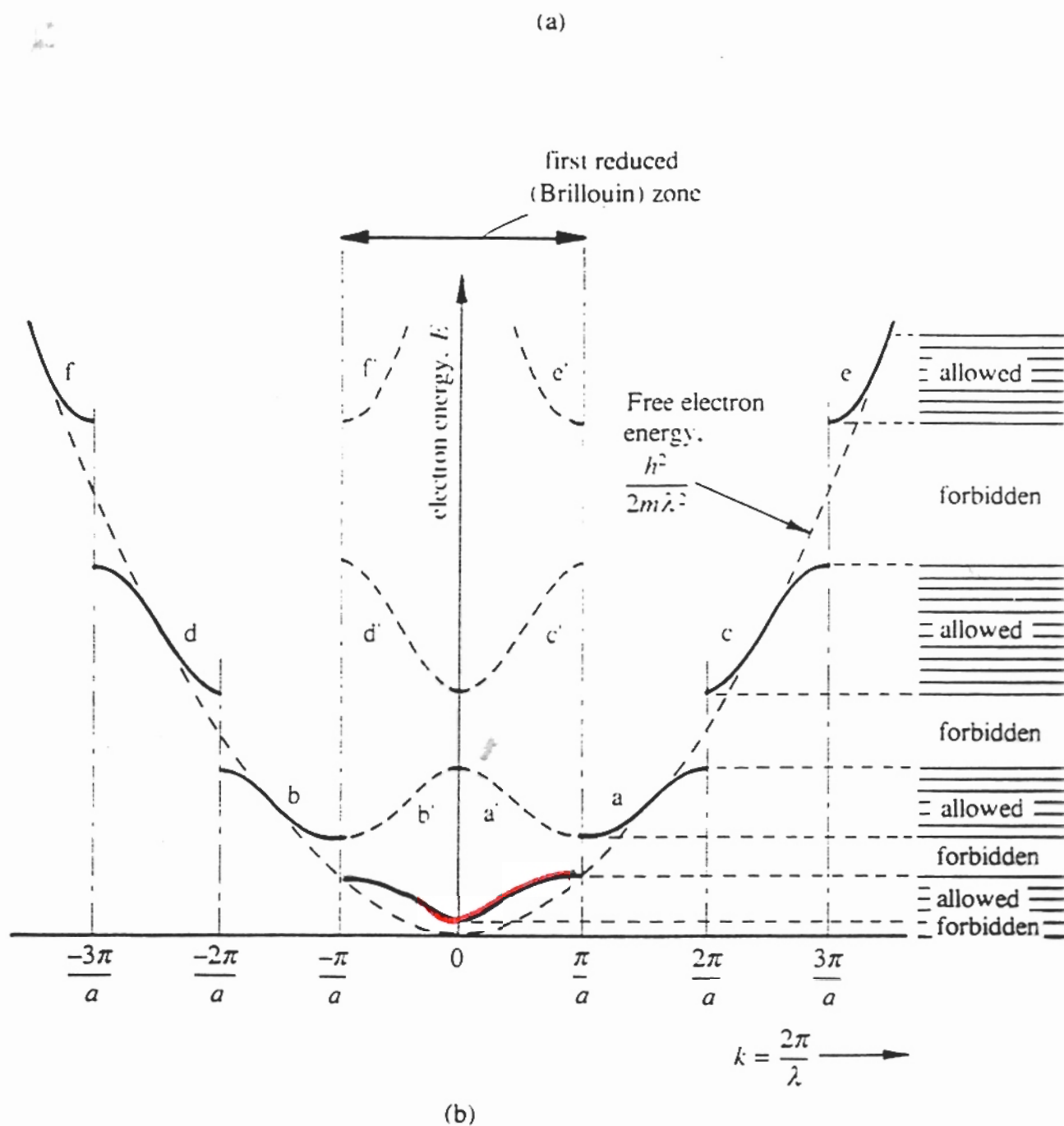
(a)

Fig. 5.9 (a) Range of possible solutions of Eq. (5.9). (b) Electronic energy as a function wavelength and the energy band structure of a hypothetical solid. ———reduced z representation.