EEE204

"Electronic Devices in Circuits"

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What is the module about?

First look at R-C circuits
- transfer functions
- Bode plots
- standard forms

Operational Amplifiers
- review of ideal behaviour
- offsets
- frequency dependence
  - gain-bandwidth product
  - slew rate
- control of frequency response by using frequency dependent feedback
  - filters → an introduction to low pass filters.

- Noise
  - what is it
  - how is it modelled
  - what effect does it have on circuits
  - can its effects be minimised by design.

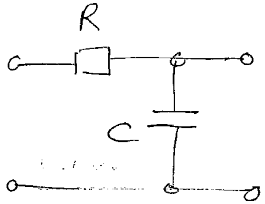
Power Amplifiers
- concentrating on output stage
- amplifier classes
- getting rid of dissipated heat.

Booklist.

Connor "Noise" (library reference)
Smith R.J. "Circuits Devices & Systems" 5th ed.
Sedra + Smith K.C. "Microelectronic circuits" 5th ed.
Horowitz + Hill "The art of electronics"
Millman "Microelectronics"
Robert Spence "Introductory Circuits"

First order circuits

(1)

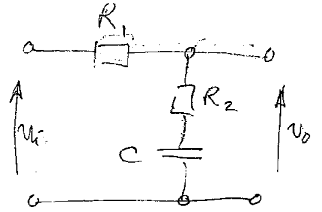


$\left(Z_C = \frac{1}{j\omega C}\right)$

lf gain → 1 as $f \to 0$

hf gain → 0 as $f \to \infty$

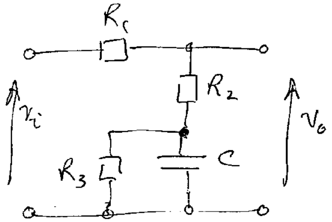
$\tau = RC$

(2) 

lf gain $\rightarrow 1$ as $f \rightarrow 0$

h.f. gain $\rightarrow \dfrac{R_2}{R_1+R_2}$ as $f \rightarrow \infty$.

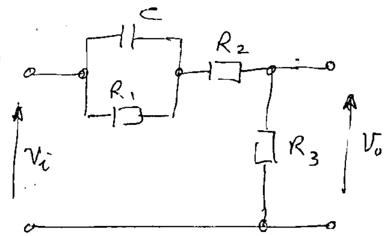
$\tau = (R_2+R_1)C$

(3) 

lf gain $\rightarrow \dfrac{R_2+R_3}{R_1+R_2+R_3}$ as $f \rightarrow 0$.

hf gain $\rightarrow \dfrac{R_2}{R_1+R_2}$ as $f \rightarrow \infty$.

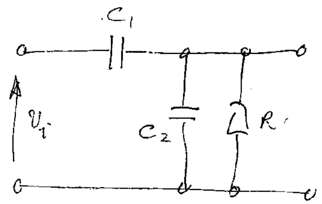
$\tau = [(R_1+R_2)/\!/R_3]C$

(4) 

lf gain $\rightarrow \dfrac{R_3}{R_1+R_2+R_3}$ as $f \rightarrow 0$

hf gain $\rightarrow \dfrac{R_3}{R_2+R_3}$ as $f \rightarrow \infty$

$\tau = [R_1 /\!/ (R_2+R_3)]C$

(5) 

lf gain $\rightarrow 0$ as $f \rightarrow 0$

hf gain $\rightarrow \dfrac{X_{C2}}{X_{C1}+X_{C2}}$

$$= \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1}+\frac{1}{j\omega C_2}} = \frac{C_1C_2}{C_1+C_2}\cdot\frac{1}{C_2}$$

$$= \frac{C_1}{C_1+C_2}$$

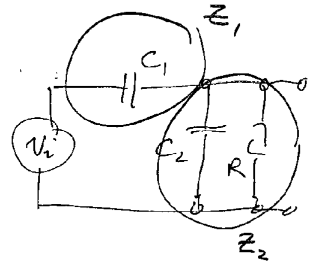
$\therefore \tau = (C_1+C_2)R.$

Working out transfer functions

Transfer function is $\frac{v_o}{v_i}$ as a function of ω

— will usually be complex.

lets look at circuit no 5.



$$\frac{v_o}{v_i} = \frac{Z_2}{Z_1+Z_2}.$$

$$Z_1 = \frac{1}{j\omega C_1}$$

$$Z_2 = \frac{1}{j\omega C_2} /\!/ R = \frac{R/j\omega C_2}{R+1/j\omega C_2} = \frac{R}{1+j\omega C_2R}$$

$$\therefore \frac{v_o}{v_i} = \frac{\frac{R}{1+j\omega C_2R}}{\frac{1}{j\omega C_1}+\frac{R}{1+j\omega C_2R}} \;\times(1+j\omega C_2R) = \frac{R}{\frac{(1+j\omega C_2R)}{j\omega C_1}+R} \;\times j\omega C_1$$

$$= \frac{j\omega C_1R}{1+j\omega C_2R+j\omega C_1R} = \frac{j\omega C_1R}{1+j\omega(C_1+C_2)R}.$$

$$= \frac{C_1}{C_1+C_2}\cdot\frac{j\omega(C_1+C_2)R}{1+j\omega(C_1+C_2)R}$$ ← this is a standard form.

$$k\cdot\frac{j\omega/\omega_o}{1+j\omega/\omega_o}$$

$$\therefore k = \frac{C_1}{C_1+C_2} \qquad \omega_o = \frac{1}{(C_1+C_2)R}$$

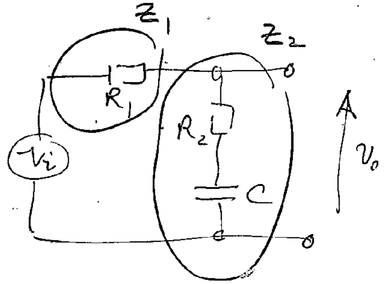
if $\omega \Rightarrow \infty$ $\quad \left|\frac{v_o}{v_i}\right| = \left|\frac{C_1}{C_1+C_2}\cdot\frac{j\omega(C_1+C_2)R}{1+j\omega(C_1+C_2)R}\right|$

$$= \frac{c_1}{c_1+c_2}\left[\frac{w^2(c_1+c_2)^2R^2}{1+w^2(c_1+c_2)^2R^2}\right]^{1/2}$$

for $w \Rightarrow \infty$ $\quad w^2(c_1+c_2)^2R^2 \gg 1$

$$\left|\frac{v_0}{v_i}\right| \Rightarrow \frac{c_1}{c_1+c_2}.$$

Consider number ②



$$\frac{v_0}{v_i} = \frac{Z_2}{Z_2+Z_1}$$

$$Z_1 = R_1$$

$$Z_2 = R_2 + \frac{1}{jwC}$$

$$\therefore \frac{v_0}{v_i} = \frac{R_2 + 1/jwC}{R_1+R_2+1/jwC}\times\frac{jwC}{jwC} = \frac{R_2 jwC + 1}{(R_1+R_2)jwC + 1}$$

$$= k.\frac{1+jw/w_1}{1+jw/w_0} \qquad w_1 = \frac{1}{CR_2} \qquad w_0 = \frac{1}{C(R_1+R_2)} \qquad k = 1$$

standard form.

called lead-lag circuit
pole-zero circuit

$$\left|\frac{1+jwR_2C}{1+jw(R_1+R_2)C}\right| = \left[\frac{1+w^2R_2^2C^2}{1+w^2(R_1+R_2)^2C^2}\right]^{1/2}$$

if $w \Rightarrow \infty$ gain $\Rightarrow \left[\frac{\cancel{w^2}R_2^2\cancel{C^2}}{\cancel{w^2}(R_1+R_2)^2\cancel{C^2}}\right]^{1/2}$ (since both w^2 terms $\gg 1$)

$$= \frac{R_2}{R_1+R_2}$$

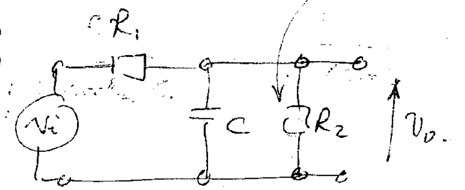
if $w \Rightarrow 0$ gain $\Rightarrow \left[\frac{1}{1}\right]^{1/2}$ since both w^2 terms $\ll 1$

$$= 1.$$

$$\left[\frac{1+jwR_2C}{1+jw(R_1+R_2)C} = \frac{1}{1+jw(R_1+R_2)C} + \frac{jwR_2C}{1+jw(R_1+R_2)C}\right]$$

modification.

modified ①



$$\frac{v_0}{v_i} = \frac{\dfrac{R_2/jwC}{R_2+1/jwC}}{R_1 + \dfrac{R_2/jwC}{R_2+1/jwC}} = \frac{\dfrac{R_2}{1+jwCR_2}}{R_1 + \dfrac{R_2}{1+jwCR_2}}$$

$$= \frac{R_2}{(1+jwCR_2)R_1 + R_2}$$

[Standard form $\quad k.\dfrac{1}{1+j\,w/w_0}$]

$$= \frac{R_2}{(R_1+R_2)+jwCR_2R_1}$$

$$= \frac{R_2}{(R_1+R_2)\left(1 + j\omega C \frac{R_2 R_1}{R_1+R_2}\right)}$$

$$\equiv k \frac{1}{1+j\omega/\omega_0}$$

notice that $j\omega$ has been kept together.

Shapes of magnitude responses

Consider $\dfrac{1}{1+j\omega/\omega_0}$

$$\left|\frac{1}{1+j\omega/\omega_0}\right| = \left[\frac{1}{1+(\omega/\omega_0)^2}\right]^{1/2}$$

what if $\omega \ll \omega_0$

$$\rightarrow \left|\frac{v_o}{v_i}\right| \Rightarrow \left[\frac{1}{1}\right]^{1/2}$$

— unity gain is an asymptote as $\omega \to 0$.

gain usually expressed in dB

$$\text{gain in dB} = 20 \log \left|\frac{v_o}{v_i}\right| = 20 \log 1$$

in this case.

$$= 0 \text{ dB}.$$

what if $\omega = \omega_0$

$$\left|\frac{v_o}{v_i}\right| = \left[\frac{1}{1+1}\right]^{1/2} = \frac{1}{\sqrt{2}}$$

in dB .... $20 \log \frac{1}{\sqrt{2}} = -3\text{dB}$.

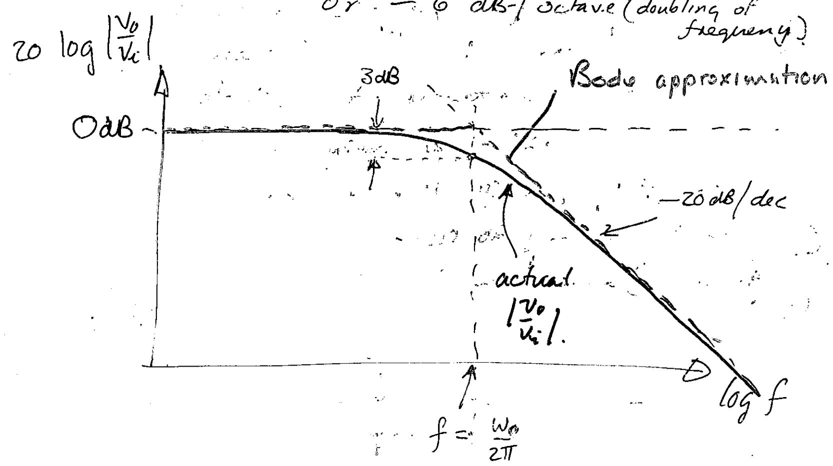
what if $\omega \gg \omega_0$

$$\left|\frac{v_o}{v_i}\right| \Rightarrow \left[\frac{1}{(\omega/\omega_0)^2}\right]^{1/2} \Rightarrow \frac{\omega_0}{\omega}.$$

— if ω increases by a factor of 10 (a decade), $\left|\frac{v_o}{v_i}\right|$ decreases by a factor of 10. A reduction of a factor 10 in gain is -20dB.

— slope of roll off asymptote is -20 dB / decade (of frequency)

or -6 dB / octave (doubling of frequency)



what about other forms?

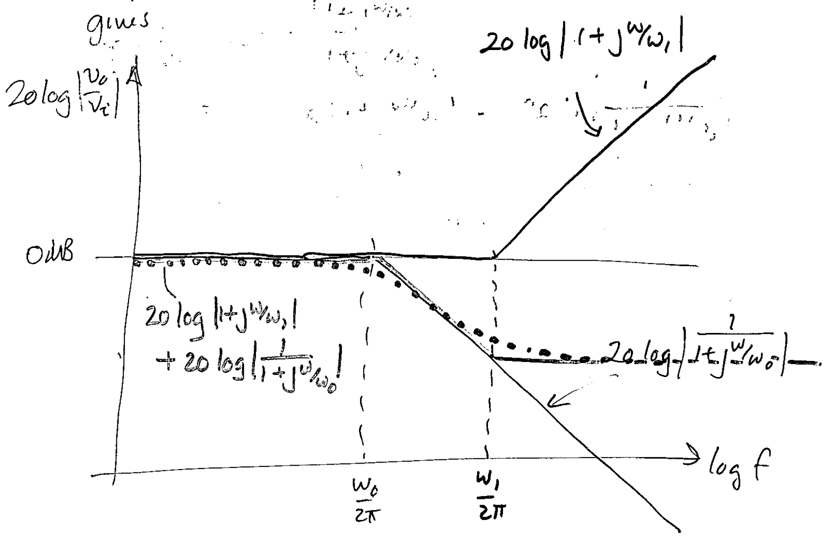
$$\frac{v_o}{v_i} = \frac{1+j\omega/\omega_1}{1+j\omega/\omega_0}$$

$$20\log\left|\frac{v_o}{v_i}\right| = 20\log\left|\frac{1+j\,\omega/\omega_1}{1+j\,\omega/\omega_o}\right|$$

$$= 20\log\left|1+j\,\omega/\omega_1\right| + 20\log\left|\frac{1}{1+j\,\omega/\omega_o}\right|$$

$$= -20\log\left|\frac{1}{1+j\,\omega/\omega_1}\right| + 20\log\left|\frac{1}{1+j\,\omega/\omega_o}\right|$$

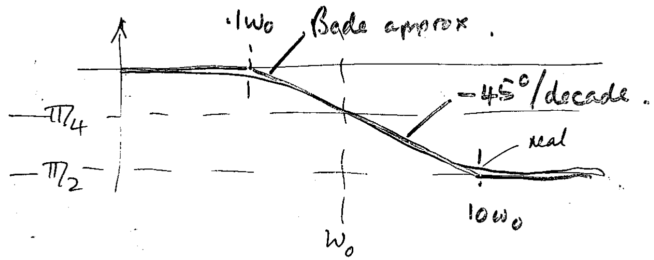
gives



Phase responses .....

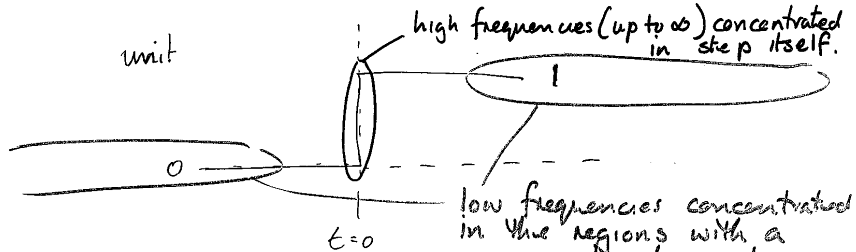
$$\frac{V_o}{V_i} = \frac{1}{1+j\,\omega/\omega_o} \qquad \phi_{\text{of } V_o \text{ w.r.t. } V_i} = -\tan^{-1}\omega/\omega_o$$

$$\frac{V_o}{V_i} = \frac{1+j\,\omega/\omega_1}{1+j\,\omega/\omega_o} \qquad \phi = \tan^{-1}\omega/\omega_1 - \tan^{-1}\omega/\omega_o.$$



Step response

- can be done by inspection for a first order system if 3 pieces of information are available
  - high frequency gain (gain as $\omega \Rightarrow \infty$)
  - low frequency gain (gain as $\omega \Rightarrow 0$)
  - time constant.



at $t = 0^-$ $V = 0$

at $t = 0^+$ $V = 1$

high frequency gain operates on the step.

low frequency gain operates on the initial voltage (ie from $t = -\infty$ to $t = 0^-$) and on final (or aiming) voltage — ie [illegible] the voltage reached at $t \Rightarrow \infty$.

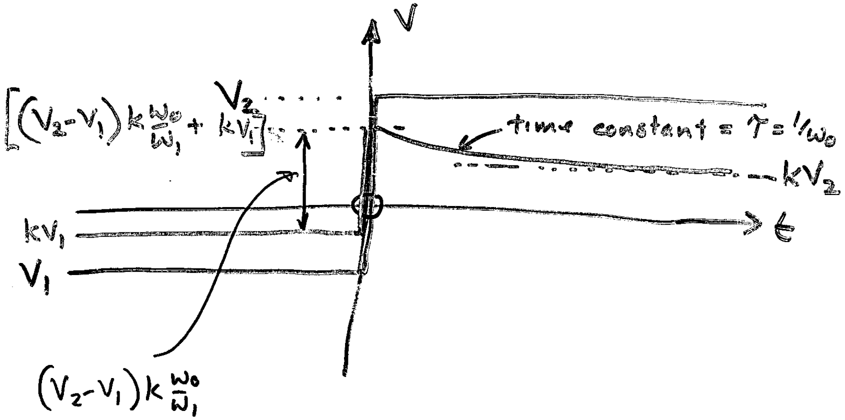
An example...

$$k \, \frac{1 + j\omega/\omega_1}{1 + j\omega/\omega_0}$$

low frequ. gain $= k$.

high frequ. gain $= k \, \dfrac{\omega_0}{\omega_1}$

$\tau = 1/\omega_0$



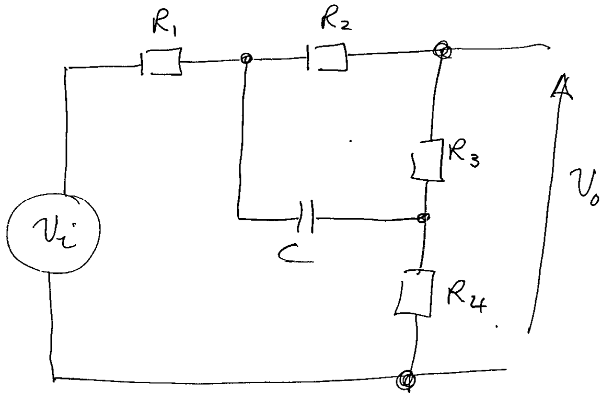
span of the exponential

$$= V_{start} - V_{finish}$$

$$= \underbrace{\left[\left[(V_2 - V_1) k \frac{\omega_0}{\omega_1} + kV_1\right] - kV_2\right]}_{V_{span}}$$

$$V(t) = \overset{V\,span}{V_{span}} \, e^{-t/\tau} + kV_2$$

TRY THIS.



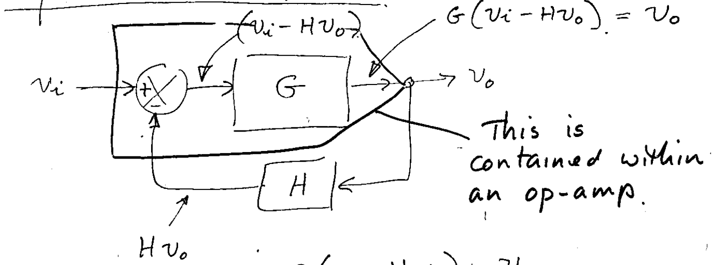
high frequency gain $\dfrac{R_4}{R_1 + R_4}$ ✓

low frequency gain $\rightarrow \dfrac{R_3 + R_4}{R_1 + R_2 + R_3 + R_4}$

time constant $\rightarrow C\,(R_1 + R_4) \,||\, (R_2 + R_3)$

$$\frac{v_o}{v_i} =$$

Operational Amplifiers



$$G(v_i - Hv_o) = v_o$$

$$Gv_i = v_o + GHv_o$$

or $$\frac{v_o}{v_i} = \frac{G}{1+GH}.$$

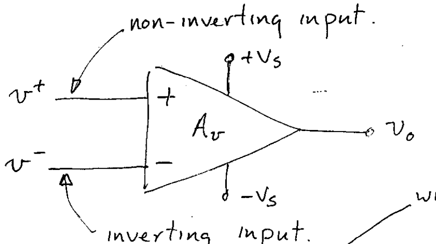
classical feedback system equation.

If G is very very large — in particular if G so big that $GH \gg 1$

$$\frac{v_o}{v_i} \approx \frac{\cancel{G}}{\cancel{G}H} \quad (\text{since } 1+GH \approx GH)$$

or $$\frac{v_o}{v_i} \approx \frac{1}{H}.$$

op-amps proper



wideband (100s MHz) op-amp.

Good quality general purpose.

A_v is typically $10^4\, V/V$ and $10^7\, V/V$

A_v operates on the difference $(v^+ - v^-)$

input impedance is designed to be very high

typically $10^5 \rightarrow 10^{15}\ \Omega$

bipolar input wideband.

CMOS input (FET input) device.

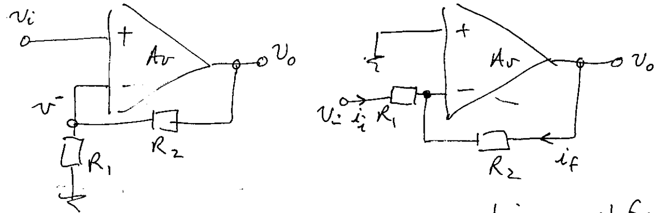
output impedance is designed to be low

typically 3Ω to $100\ \Omega$

wideband amps.

low power general purpose

Basic op-amp connections



non-inverting amplifier

inverting amplifier.

if $A_v \Rightarrow \infty$

$\rightarrow$ usual design assumption.

$$v^- = \frac{v_o R_1}{R_1 + R_2}$$

if gain is large

$v^+ - v^-$ must be ≈ 0

$$v_i \approx v^-$$

and $$v_i \approx \frac{v_o R_1}{R_1 + R_2}$$

if $A_v \Rightarrow \infty$

$$v^+ \approx v^-$$

But $v^+ = 0$

so $v^- \approx 0$

$$i_i + i_f = 0$$

(the op-amp input resistance is so high that we can ignore any current that flows into it.)

$$\frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2}$$

or $\frac{v_o}{v_i} \approx \frac{R_1+R_2}{R_1}$

but $v^- \approx 0$

$\frac{v_i}{R_1} + \frac{v_o}{R_2} = 0$ or $\frac{v_o}{v_i} = -\frac{R_2}{R_1}$

Input and output impedance

Output impedance

Output impedance of an op-amp is modified by the feedback used to control the gain

It can be shown that

$$r_{oeff} = \frac{r_o}{\left(1 + \frac{A_v R_1}{R_1+R_2}\right)} = \frac{r_o}{1+GH}$$

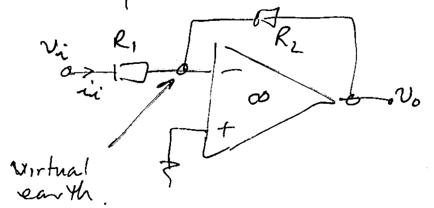
r_o — intrinsic op-amp output resistance

r_{oeff} — output resistance of feedback amplifier circuit

This is true for inverting and non-inverting amplifiers.

Input resistance

— inverting



$\frac{v_i}{i_i} = r_{ieff} = R_1$

$\frac{v_i}{i_i} = r_{ieff} = \underline{R_1}$.

— non-inverting

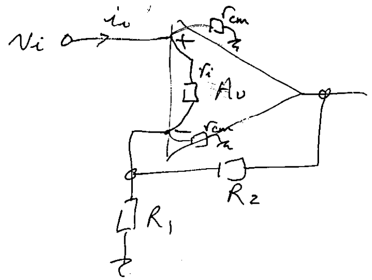
— intrinsically high input resistance because input applied directly to + input of op-amp.

But the feedback does make a difference

$$r_{ieff} = r_i\left(1 + \frac{A_v R_1}{R_1+R_2}\right)$$

r_{ieff} — effective input resistance of circuit $\frac{v_i}{i_i}$.

r_i — intrinsic op-amp input resistance



Offset effects

$$\boxed{v_o = A_v(v^+ - v^-)}$$ ← The op-amp equation.

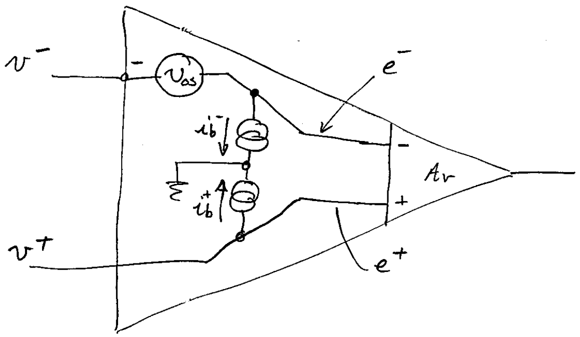
$\therefore$ if $v^+ = v^-$ $v_o = 0$ but it doesn't.

— The output that occurs when $v^+ = v^-$ is an error called an offset error.

Questions — what is the effect of offset errors on circuit performance

— what can be done - <u>by design</u> - to minimise those effects.

an offset model.



V_{os} = "equivalent input offset voltage generator"

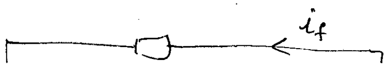
V_{os} is equal to and opposite to the voltage that must be applied between v^+ + v^- in order to get a v_o of 0V.

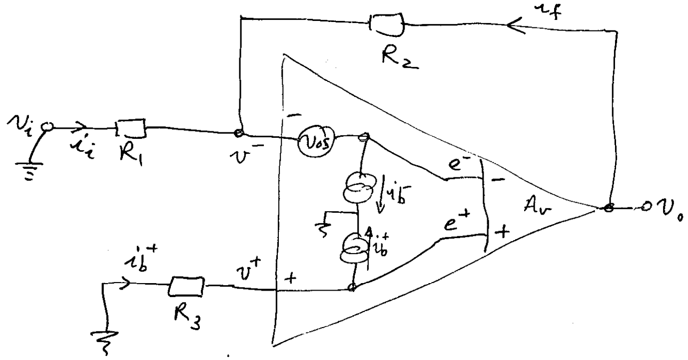
i_b^+ and i_b^- are the input bias currents required by the op-amp.

Circuits must allow i_b^+ and i_b^- to flow.

i_{os} is "equivalent input offset current generator"

$$i_{os} = |i_b^+ - i_b^-|$$





sum currents at v^- node

$$i_i + i_f = i_b^-$$

$$0 = v_i \qquad \frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2} = i_b^-$$

can be rearranged to give v^- explicitly

$$v^- = -\frac{R_1R_2}{R_1+R_2}\left[i_b^- - \frac{v_o}{R_2}\right] \quad (1)$$

$$e^- = v^- \pm v_{os} \quad (2)$$

$$e^+ = v^+ = -i_b^+R_3 \quad (3).$$

Using the op-amp equation ...

$$v_o = A_v(e^+ - e^-)$$

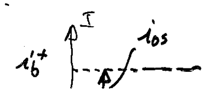
$$= A_v\Big(\underbrace{-i_b^+R_3}_{e^+} + \underbrace{i_b^-\frac{R_1R_2}{R_1+R_2} - \frac{v_oR_1}{R_1+R_2} \mp v_{os}}_{-e^-}\Big)$$

Can be neglected if $\frac{1}{A_v} \ll \frac{R_1}{R_1+R_2}$

$$\text{or} \quad v_o\left[\frac{1}{A_v} + \frac{R_1}{R_1+R_2}\right] = -i_b^+R_3 + i_b^-\frac{R_1R_2}{R_1+R_2} \mp v_{os} \approx v_o\frac{R_1}{R_1+R_2}.$$

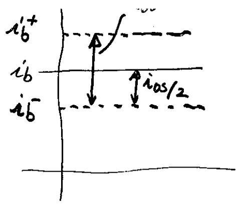
Manufacturers specify i_b = average input bias current $= \frac{i_b^+ + i_b^-}{2}$ and i_{os}.

if $i_b^+ = i_b + i_{os}/2$



if $i_b^+ = i_b + i_{os}/2$
$\quad i_b^- = i_b - i_{os}/2$

or if $i_b^+ = i_b - i_{os}/2$
$\quad i_b^- = i_b + i_{os}/2$



$$\boxed{\begin{aligned} i_b^+ &= i_b \pm i_{os}/2 \\ i_b^- &= i_b \mp i_{os}/2 \end{aligned}}$$

$$v_o \frac{R_1}{R_1+R_2} = -(i_b \mp i_{os}/2)R_3 + (i_b \pm i_{os}/2)\frac{R_1R_2}{R_1+R_2} \mp v_{os}$$

$$v_o \frac{R_1}{R_1+R_2} = i_b\left(-R_3 + \frac{R_1R_2}{R_1+R_2}\right) \pm \frac{i_{os}}{2}\left(\frac{R_1R_2}{R_1+R_2} + R_3\right) \mp v_{os}$$

$$v_o = \underbrace{i_b\left(\frac{R_1R_2}{R_1+R_2} - R_3\right)\frac{R_1+R_2}{R_1}}_{\text{can be eliminated by making } \frac{R_1R_2}{R_1+R_2} = R_3} \pm \underbrace{\frac{i_{os}}{2}\left(\frac{R_1R_2}{R_1+R_2} + R_3\right)\frac{R_1+R_2}{R_1}}_{\text{can be affected by choosing low values of resistance}} \mp \underbrace{v_{os}\frac{R_1+R_2}{R_1}}_{\text{stuck with this}}$$

putting $R_3 = \frac{R_1R_2}{R_1+R_2}$ into the result ....

$$v_o = 0 \pm \frac{i_{os}}{2}\left(\frac{R_1R_2}{R_1+R_2} + \frac{R_1R_2}{R_1+R_2}\right)\frac{R_1+R_2}{R_1} \mp v_{os}\frac{R_1+R_2}{R_1}$$

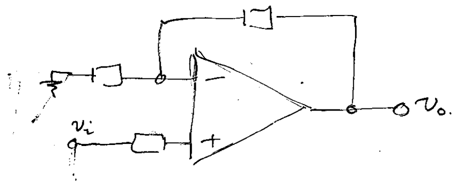
$$\pm \frac{i_{os}}{\cancel{2}}\left(\frac{\cancel{2}R_1R_2}{\cancel{R_1+R_2}}\right)\frac{\cancel{R_1+R_2}}{\cancel{R_1}}$$

$$\pm i_{os}R_2 \qquad \mp v_{os}\frac{R_1+R_2}{R_1}$$

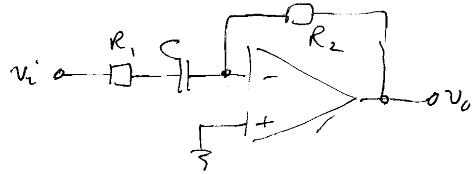
Many op-amps offer an offset null capability

Many op-amps offer an offset null capability

- offset nulls allow users to slightly alter the balance of currents through the op-amp input stages.
  - application tends to be different for different op-amps LOOK AT DATA SHEET.

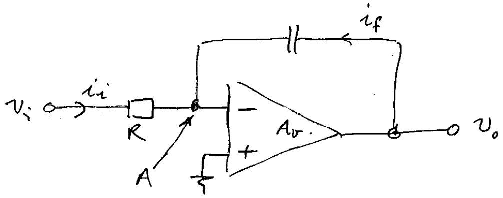


OFFSETS ARE DC EFFECTS



Frequency Dependent Feedback

The integrator



time domain

Sum currents at A

frequency domain

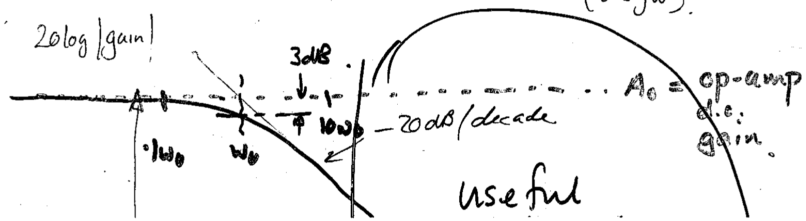
$$i_i + i_f = 0$$

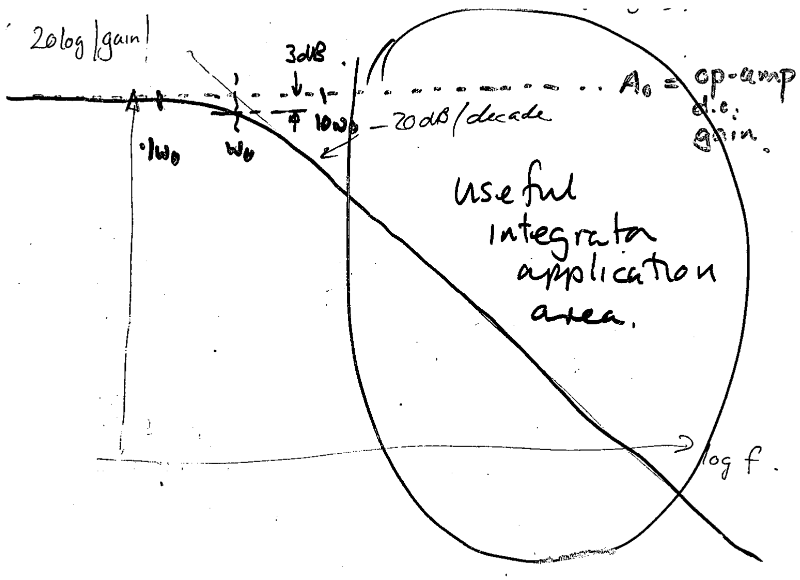
$$\frac{v_i - v^-}{R} + \frac{v_o - v^-}{1/sC} = 0 = \frac{v_i - v^-}{R} + (v_o - v^-)sC$$

if $A_v \gg 1$, $v^- \approx v^+ = 0$

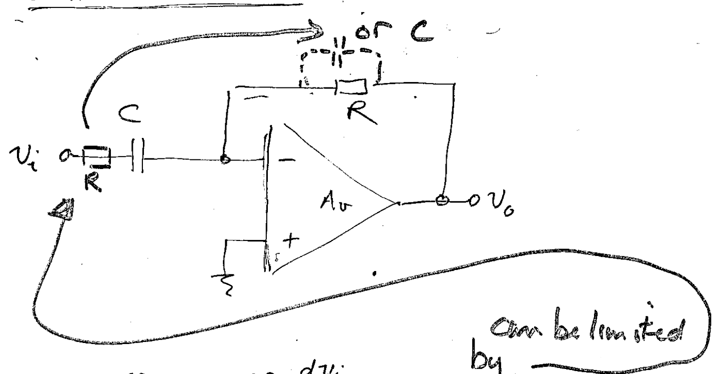
$$\frac{v_i}{R} + v_o sC = 0 \quad \text{or} \quad \frac{v_o}{v_i} = -\frac{1}{sCR}$$

$(s \equiv j\omega)$



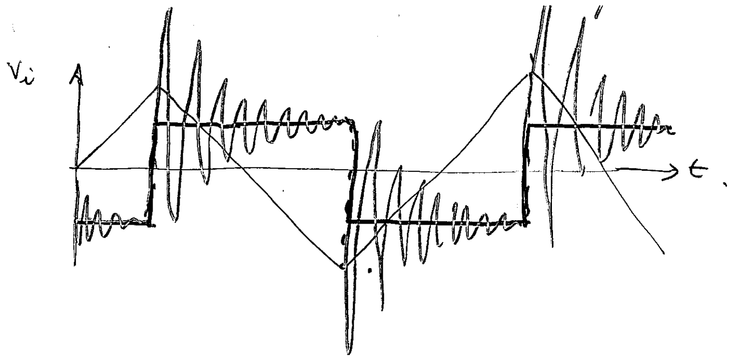


Differentiator cct

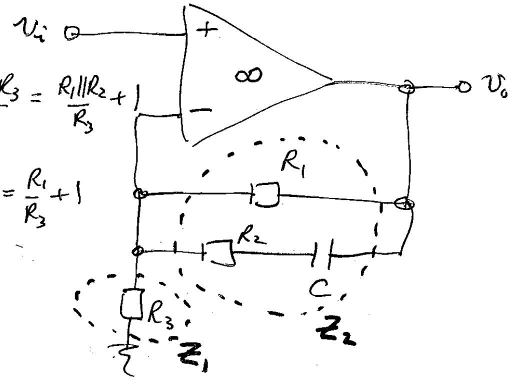


$$v_o = -CR\frac{dv_i}{dt}$$

or $v_o = -sCR$.



Pole – zero circuits



hf gain $\rightarrow$ $\dfrac{R_1 \| R_2 + R_3}{R_3} = \dfrac{R_1\|R_2}{R_3} + 1$

lf gain $\rightarrow$ $\dfrac{R_1 + R_3}{R_3} = \dfrac{R_1}{R_3} + 1$

Want to find $\dfrac{v_o}{v_i}$ as function of $s \; (\equiv j\omega)$.

$$\frac{v_o}{v_i} = \frac{Z_1 + Z_2}{Z_1}$$

$$Z_2 = R_1 \| (R_2 + 1/sC)$$

$$= \frac{R_1(R_2 + 1/sC)}{R_1 + R_2 + 1/sC} = \frac{R_1(1 + R_2 sC)}{1 + (R_1 + R_2)sC}.$$

$$Z_1 = R_3$$

$$\frac{v_o}{v_i} = \frac{R_3 + \dfrac{R_1(1+R_2 sC)}{1+(R_1+R_2)sC}}{R_3}$$

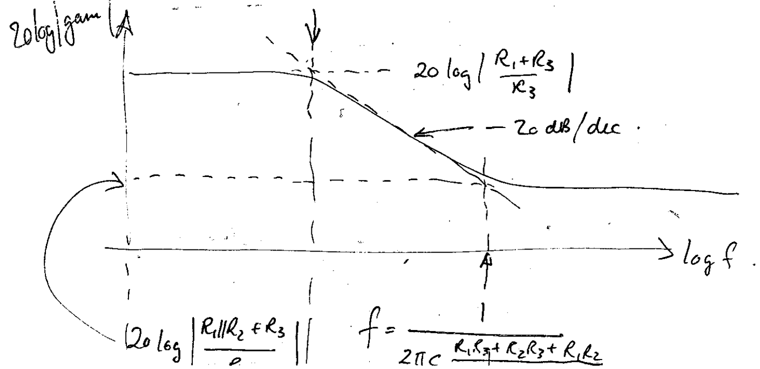
$$= \frac{R_3(1+(R_1+R_2)sC) + R_1(1+R_2 sC)}{R_3(1+(R_1+R_2)sC)}.$$

$$= \frac{R_3 + (R_1+R_2)sCR_3 + R_1 + R_1R_2 sC}{R_3\;(1+(R_1+R_2)sC)}.$$

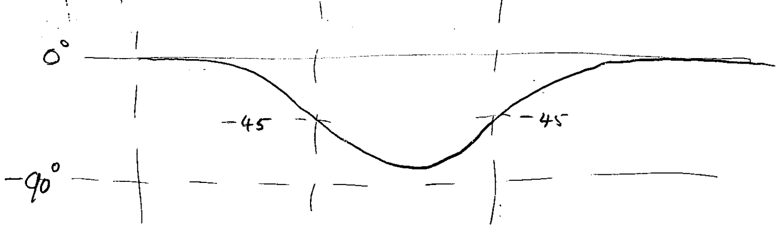
$$= \frac{(R_1+R_3) + sC(R_1R_3 + R_2R_3 + R_1R_2)}{\sim}$$

$$= \frac{R_1+R_3}{R_3} \circ \frac{1 + sC\dfrac{(R_1R_3 + R_2R_3 + R_1R_2)}{R_1+R_3}}{1 + sC(R_1+R_2)}.$$

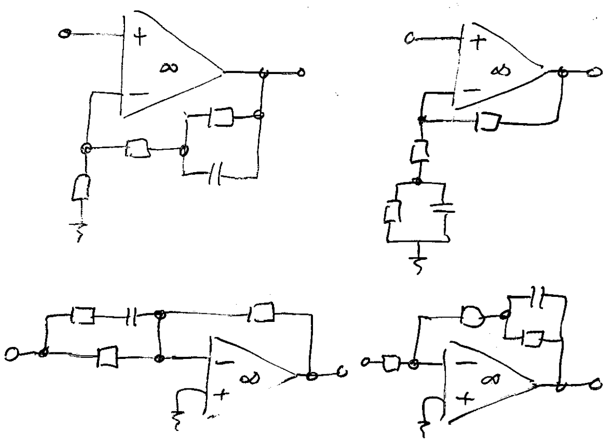
$$f = \frac{1}{2\pi C(R_1+R_2)}$$



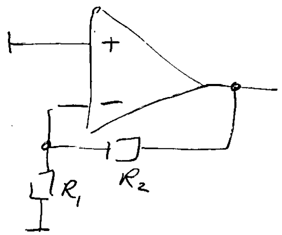
$20\log\left|\frac{R_1 \| R_2 + R_3}{R_3}\right|$ $\quad f = \frac{1}{2\pi C \frac{R_1R_3 + R_2R_3 + R_1R_2}{R_1+R_3}}$

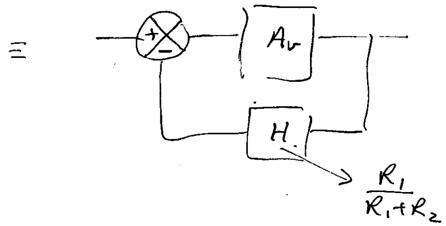


Can occur in a range of different shapes ...:



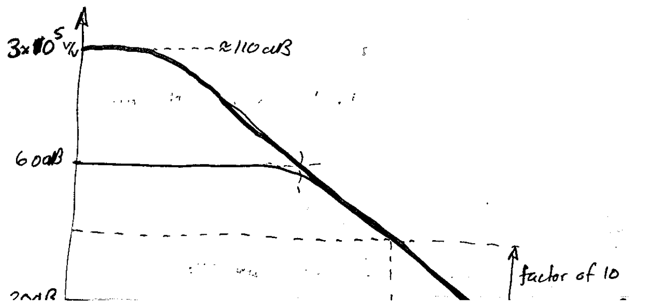
Intrinsic frequency response of op-amp.

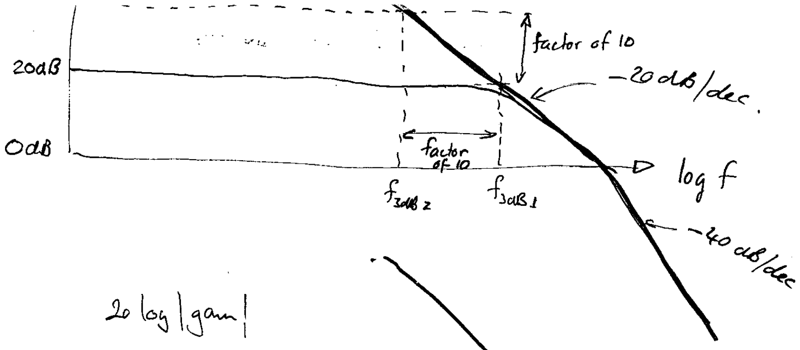


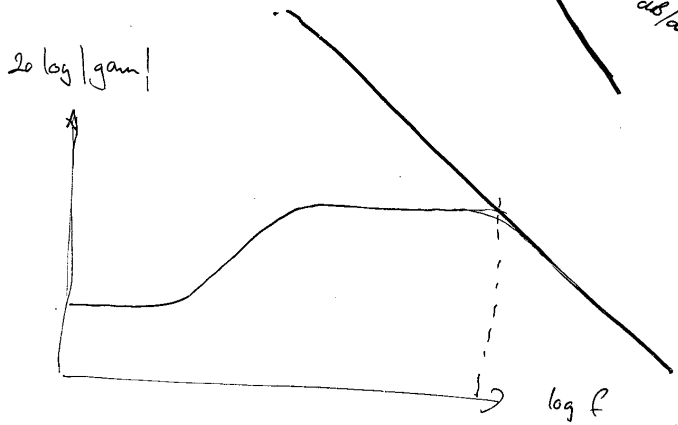


— op-amp is designed to be first order by the manufacturer so that sales will be big.

— a few uncompensated amplifiers with 2nd and occasionally 3rd order responses are available for specialist use.







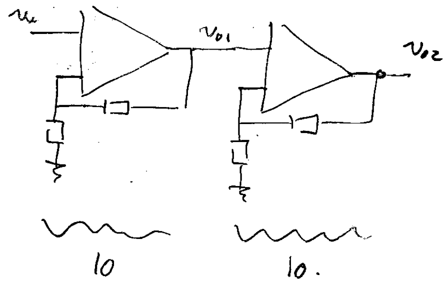
Cascades.

Sometimes it is necessary to use more than one amplifier to achieve a desired gain over a specified BW.

eg suppose a gain of 100 required with a BW of 500kHz.

— Single amp GBP needed is 50MHz.

The gain of 100 could be obtained by using two gains of 10 in series



$$\frac{v_{o1}}{v_i} = \frac{10}{1 + j\,\omega/\omega_0} \qquad \frac{v_{o2}}{v_{o1}} = \frac{10}{1 + j\,\omega/\omega_0}$$

$$= \frac{10}{1 + j\,f/f_0} \qquad = \frac{10}{1 + j\,f/f_0}$$

$$= 10 \times \frac{1}{1 + j\,f/f_0}.$$

at what f does $\left|\frac{1}{1 + j\,f/f_0}\right| = 10^{-1.5/20}$?

$$\left[\frac{1}{1 + (f/f_0)^2}\right]^{1/2} = 10^{-1.5/20}$$

or
$$\frac{1}{1 + (f/f_0)^2} = 10^{-3/20} \approx \frac{1}{\sqrt{2}}$$

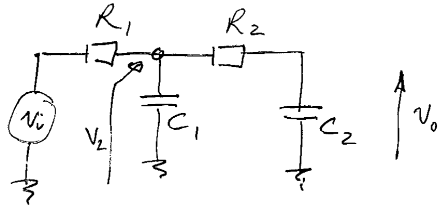
$$\sqrt{2} = 1 + \left(f/f_0\right)^2$$

$$\sqrt{(\sqrt{2} - 1)} = f/f_0 \approx 0.64$$

$$f = 0.64\, f_0.$$

Second Order Circuits

Example of 2nd order cct



$$\frac{V_2}{V_i} = \frac{\dfrac{\frac{1}{sC_1}\left(R_2 + \frac{1}{sC_2}\right)}{\frac{1}{sC_1} + R_2 + \frac{1}{sC_2}}}{R_1 + \dfrac{\frac{1}{sC_1}\left(R_2 + \frac{1}{sC_2}\right)}{\frac{1}{sC_1} + R_2 + \frac{1}{sC_2}}}$$

$$\frac{\frac{1}{sC_1}\left(R_2 + \frac{1}{sC_2}\right)}{\frac{1}{sC_1} + R_2 + \frac{1}{sC_2}} = \frac{R_2 + \frac{1}{sC_2}}{1 + sC_1R_2 + \frac{C_1}{C_2}}$$

$$\frac{1 + sC_2R_2}{sC_2 + s^2C_1C_2R_2 + s\frac{C_1}{C_2}\cdot C_2} = \frac{1 + sC_2R_2}{s(C_1+C_2) + s^2C_1C_2R_2}$$

$$\frac{V_2}{V_i} = \frac{\dfrac{1 + sC_2R_2}{s(C_1+C_2) + s^2C_1C_2R_2}}{R_1 + \dfrac{1 + sC_2R_2}{s(C_1+C_2) + s^2C_1C_2R_2}}$$

$$= \frac{1 + sC_2R_2}{R_1(s(C_1+C_2)) + s^2C_1C_2R_1R_2 + 1 + sC_2R_2}$$

$$= \frac{1 + sC_2R_2}{1 + s(R_1(C_1+C_2) + C_2R_2) + s^2C_1C_2R_1R_2}$$

$$\frac{V_o}{V_2} = \frac{1/sC_2}{R_2 + 1/sC_2} = \frac{1}{1 + sC_2R_2}$$

$$\frac{V_o}{V_i} = \frac{V_o}{V_2} \times \frac{V_2}{V_i} = \frac{\cancel{1 + sC_2R_2}}{1 + s(R_1(C_1+C_2) + C_2R_2) + s^2C_1C_2R_1R_2} \cdot \frac{1}{\cancel{1 + sC_2R_2}}$$

$$= \frac{1}{1 + s(R_1C_1 + R_1C_2 + R_2C_2) + s^2C_1C_2R_1R_2}$$

Low pass standard form is

$$\frac{V_o}{V_i} = K \cdot \frac{1}{1 + \frac{s}{\omega_n q} + \frac{s^2}{\omega_n^2}}$$

so for this transfer function

$$\omega_n = \sqrt{\frac{1}{C_1C_2R_1R_2}}$$

$$\frac{1}{\omega_n q} = R_1C_1 + R_1C_2 + R_2C_2$$

$$\frac{1}{q} = \omega_n(R_1C_1 + R_1C_2 + R_2C_2)$$

$$\frac{1}{q} = \omega_n\left(R_1C_1 + R_1C_2 + R_2C_2\right)$$

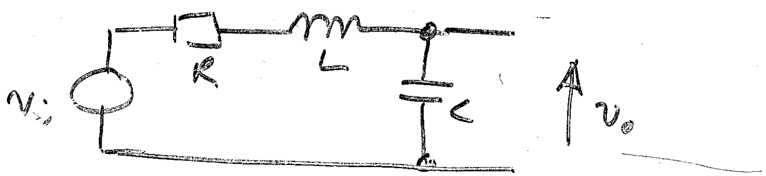
$$= \frac{R_1C_1 + R_1C_2 + R_2C_2}{\sqrt{C_1C_2R_1R_2}}$$

$$= \sqrt{\frac{R_1C_1}{R_2C_2}} + \sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_2C_2}{R_1C_1}}$$

$$\frac{v_o}{v_i} = \frac{1}{1 + \frac{s}{\omega_o q} + \frac{s^2}{\omega_o^2}}$$

$$\frac{1}{1 + \frac{s^2}{\omega_o^2}} \Rightarrow \frac{1}{1 + \frac{(j\omega)^2}{\omega_o^2}} = \frac{1}{1 - \frac{\omega^2}{\omega_o^2}}$$

high pass... $k \cdot \dfrac{s^2/\omega_o^2}{1 + \frac{s}{\omega_o q} + s^2/\omega_o^2}$



$$\frac{v_o}{} = \frac{1/sc}{}$$

$$\frac{v_o}{v_i} = \frac{1/sc}{R + sL + 1/sc}$$

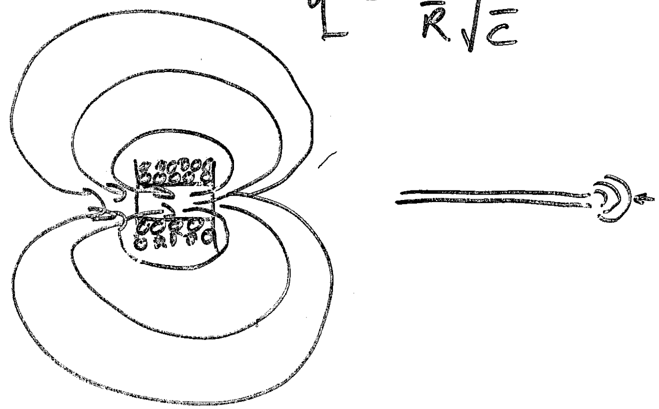
$$= \frac{1}{1 + scR + s^2LC}.$$

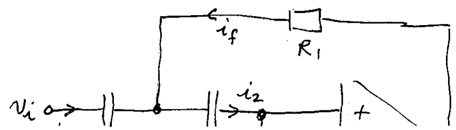
Compare with $\dfrac{1}{1 + \frac{s}{\omega_o q} + \frac{s^2}{\omega_o^2}}$

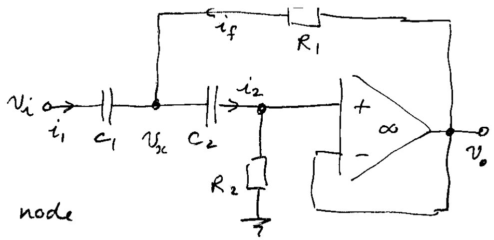
$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$\frac{1}{\omega_o q} = CR \qquad \frac{1}{q} = \frac{CR}{\sqrt{LC}} = R\sqrt{\frac{C}{L}}$$

$$\therefore\ q = \frac{1}{R}\sqrt{\frac{L}{C}}$$







Sum currents at v_x node

$$i_1 + i_f = i_2$$

$$\frac{v_i - v_x}{1/sC_1} + \frac{v_o - v_x}{R_1} = \frac{v_x}{R_2 + \frac{1}{sC_2}}$$

$$v_i sC_1 - v_x sC_1 + \frac{v_o}{R_1} - \frac{v_x}{R_1} = \frac{v_x \cdot sC_2}{R_2 sC_2 + 1}$$

$$v_i sC_1 + \frac{v_o}{R_1} = v_x\left[\frac{sC_2}{1+sC_2R_2} + \frac{1}{R_1} + sC_1\right]$$

$$v_i sC_1R_1 + v_o = v_x\left[\frac{sC_2R_1}{1+sC_2R_2} + 1 + sC_1R_1\right]$$

$$= v_x\left[\frac{sC_2R_1 + (1+sC_2R_2)(1+sC_1R_1)}{1+sC_2R_2}\right]$$

$$= v_x\left[\frac{sC_2R_1 + 1 + s(C_2R_2+C_1R_1) + s^2C_1C_2R_1R_2}{1+sC_2R_2}\right]$$

$$\therefore v_x = \frac{(v_i sC_1R_1 + v_o)(1+sC_2R_2)}{1+s(C_2R_1+C_2R_2+C_1R_1)+s^2C_1C_2R_1R_2}$$

The v^+ node relates v_x to v^+ by potential division and $v^+ = v_o$ since amplifier gain is unity

$$\therefore v^+ = \frac{v_x \cdot R_2}{\dots} = v_o$$

$$\therefore v = \frac{v_x \cdot R_2}{R_2 + 1/sC_2} = v_o$$

or $$\frac{v_x \, sC_2R_2}{1+sC_2R_2} = v_o$$

or $$v_x = \frac{v_o(1+sC_2R_2)}{sC_2R_2}$$

eliminating v_x

$$\frac{v_o(1+\cancel{sC_2R_2})}{sC_2R_2} = \frac{(v_i sC_1R_1 + v_o)\cancel{(1+sC_2R_2)}}{1+s(C_2R_1+C_2R_2+C_1R_1)+s^2C_1C_2R_1R_2}$$

$$v_o\left[1+s(C_2R_1+\cancel{C_2R_2}+C_1R_1)+s^2C_1C_2R_1R_2 - \cancel{sC_2R_2}\right] = v_i s^2C_1C_2R_1R_2$$

$$\frac{v_o}{v_i} = \frac{s^2C_1C_2R_1R_2}{1+s(C_2R_1+C_1R_1)+s^2C_1C_2R_1R_2}.$$

standard HP form $$\frac{s^2/\omega_0^2}{1+\frac{s}{\omega_0 q}+\frac{s^2}{\omega_0^2}}$$

$$\omega_0 = \sqrt{\frac{1}{C_1C_2R_1R_2}}$$

$$\frac{1}{\omega_0 q} = R_1(C_1+C_2)$$

so $$\frac{1}{q} = \omega_0 R_1(C_1+C_2) = \frac{R_1(C_1+C_2)}{\sqrt{R_1R_2C_1C_2}}$$

$$= \sqrt{\frac{R_1}{R_2}}\left[\sqrt{\frac{C_1}{C_2}}+\sqrt{\frac{C_2}{C_1}}\right].$$

is there a relationship ...

is there a relationship between the "C"s that will maximise Q

Let $\sqrt{\frac{C_1}{C_2}} = x$

$$\frac{1}{Q} = \sqrt{\frac{R_1}{R_2}}\left[x + \frac{1}{x}\right]$$

maximising Q is same as minimising $1/Q$
so what x minimises $1/Q$?

$$\frac{d(1/Q)}{dx} = \sqrt{\frac{R_1}{R_2}}\left[1 - \frac{1}{x^2}\right] = 0 \text{ for minimum}$$

ie $\frac{1}{x^2} = 1$ or $x = 1$

ie $\sqrt{\frac{C_1}{C_2}} = 1$ or $C_1 = C_2$.

using this condition....

$$\frac{1}{Q} = \sqrt{\frac{R_1}{R_2}}\left[1 + 1\right] = 2\sqrt{\frac{R_1}{R_2}}$$

or $Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$

and w_0 becomes $\frac{1}{\sqrt{R_1 R_2 C.C.}} = \frac{1}{C\sqrt{R_1 R_2}}$

"C"s are now equal