Richard Tozer E135b

EEE101 "Circuits + Signals".

review of course

- concept of an electrical circuit
- basic circuit elements
  - sources
  - resistors
  - capacitors + inductors

circuit analysis involves working out voltages + currents within networks consisting of sources, resistors, capacitors + inductances.
reactances.

- formal analysis methods
- conventions
- power + energy
- circuit transformations

dealing with sinusoids

- concept of impedance
- impedance of inductor + capacitor.
- a.c. analysis of circuits
  - phase

dealing with phase
- phasor diagrams
- complex numbers

- resonance
- filters

Studying.

- 24 hours of lectures.
- 12 hours of problem classes.

36

100
− 36
12 | 64
5.

Bogart	Electric Circuits	McGraw Hill
Hayt	Engineering Circuit Analysis	"
Paul	Analysis of Linear Circuits	"
EEE101 EEE102 EEE103 } Smith R.J.	Circuits Devices + Systems	Wiley.
Ciletti	Intro to circuit analysis + design	
Floyd	Principles of electric ccts	
Walls	Intro. to cct analysis	
Madhu	Linear circuit analysis	
Nilsson	Electric circuits	
Spence R	Introductory Circuits	Wiley.

Introduction

"Linear" "passive" networks.

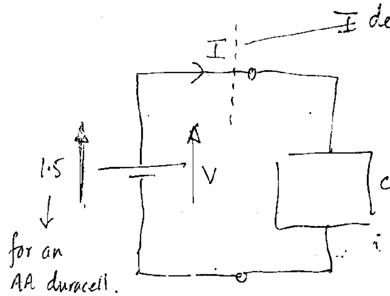
component values are constant

interconnection of electronic circuit elements.

Voltage

— is a driving force
— drives current through a circuit

Consider a source



I defines no. of Coulombs/second that flow through imaginary boundary

If V is changing $I = \frac{dq}{dt}$

component or system of components

In forcing the current through the components the source is ~~do ing~~ doing work on the components
— ie transferring energy to the components.

$$\text{Energy} = \int_0^{t_1} V(t)\, I(t)\, dt \quad \text{Joules (J).}$$

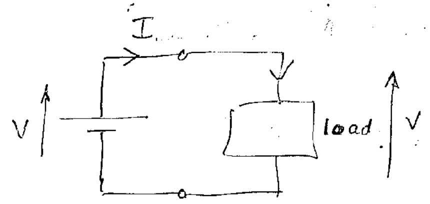
$$= V I t_1 \quad \text{for d.c. quantities}$$

Power is rate of energy dissipation

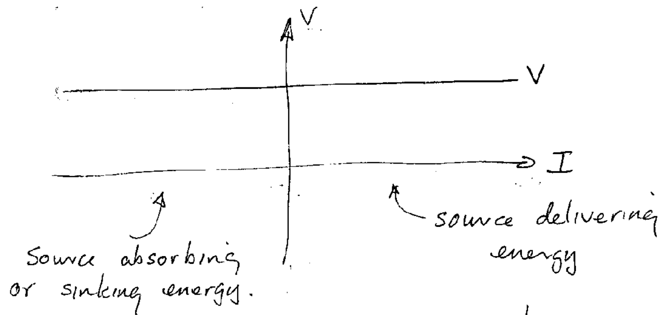
Joules/second ⇒ Watts.

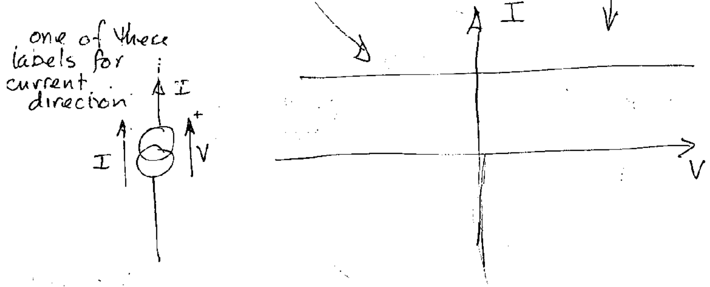
$$\text{Power} = \frac{1}{t_1}\int_0^{t_1} V(t)\, I(t)\, dt$$

for d.c. $P = V.I.$ W.



Source characteristic for voltage source

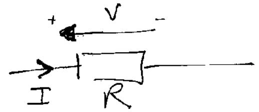




Resistors

— devices that obey Ohm's law

$$R = \text{resistance} = \frac{V}{I}$$



resistors are energy dissipators.

— various technologies

— wire wound for high power (up to several kW)

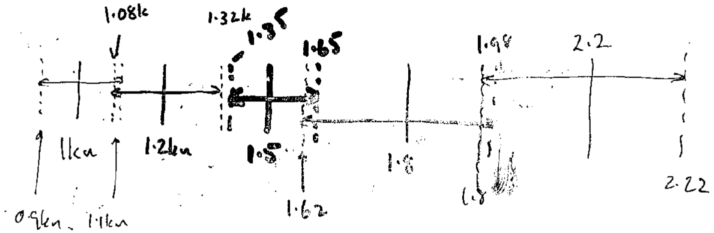
— carbon, metal or oxide film for low to medium power

0.25W to 50W.

— surface mount usually carbon or metal film ... 0.1W to 0.5W (and very fiddly).

Resistors manufactured in a range of different "tolerance series" — idea was that there should be minimal overlap between ~~the~~ the tolerance bands of resistors in a given series.

eg for a 10% series



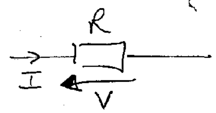
{ 1k 1.2k 1.5k 1.8k 2.2k 2.7k 3.3k 3.9k 4.7k 5.6k 6.8k 8.2k, 10k.

called 10% preferred values.

Power dissipation in resistors.

Power Diss. $= VI$.

Ohm's law $R = \frac{V}{I}$

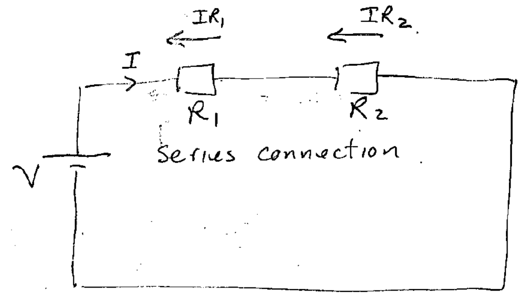


I + V are dc quantities

$\therefore$ Power Diss $= I^2R$ or $\frac{V^2}{R}$

basic relationship.

Resistor Combinations.



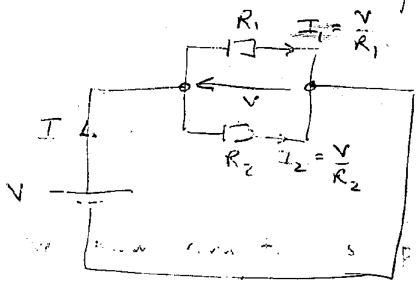
Series connection.

$$V = IR_1 + IR_2 = I(R_1+R_2)$$

$$\text{or } \frac{V}{I} = R_1+R_2$$

Resistors in series add

Other main connection is parallel.



$$I = I_1 + I_2$$
$$= \frac{V}{R_1} + \frac{V}{R_2}$$
$$= V\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

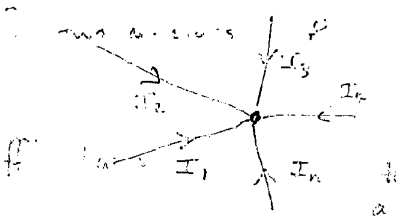
$\therefore$ effective $R = \frac{V}{I} = \frac{V}{V\left[\frac{1}{R_1}+\frac{1}{R_2}\right]} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$

often written as $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

for two resistors $R = \frac{R_1 R_2}{R_1 + R_2} = \frac{3.2}{3+2} \frac{k\Omega^2}{k\Omega}$.

Kirchoff's Laws

— Kirchoff's current law

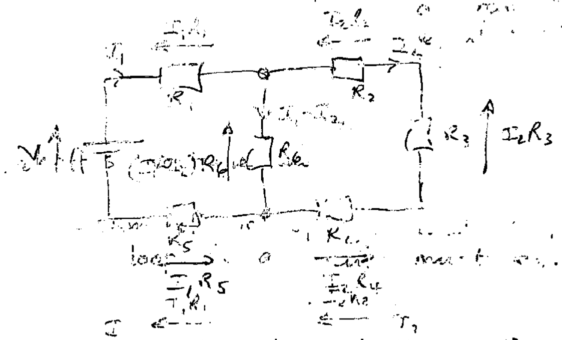


sum of currents entering the node must equal zero.

total current entering a circuit node must equal total leaving it.

— Kirchoff's voltage law

— sum of voltages around any closed loop in a circuit must equal zero.



$V - I_1 R_1 - (I_1 - I_2) R_6 - I_1 R_5 = 0$

$(I_1 - I_2) R_6 - I_2 R_2 - I_2 R_3 - I_2 R_4 = 0$

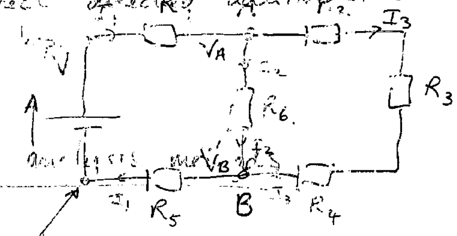
— main thing to be careful of here is correct application of convention and correct directed addition of voltages around the loops.

$(I_1 - I_2) R_6 - I_2 R_2 - I_2 R_3 - I_2 R_4 = 0$

Formal analysis methods

Nodal analysis

correct directed addition of volts



at node A $I_1 = I_2 + I_3$

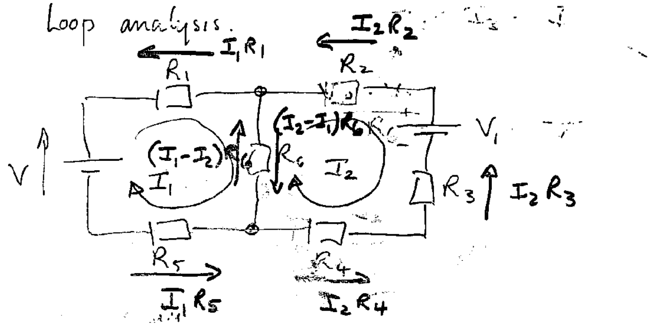
$$\frac{V-V_A}{R_1} = \frac{V_A-V_B}{R_6} + \frac{V_A-V_B}{R_2+R_3+R_4}$$

at node B

$$I_2 + I_2 = I_1$$

at node $\frac{V_A-V_B}{R_6} + \frac{V_A-V_B}{R_2+R_3+R_4} = \frac{V_B-0}{R_5}$

Mesh analysis or Maxwell's current loops or Loop analysis.



loop 1 $\quad I_1R_1 + (I_1-I_2)R_6 + I_1R_5 - V = 0$

loop 2 $\quad I_2R_2 + V_1 + I_2R_3 + I_2R_4 + (I_2-I_1)R_6 = 0$

next step

$$I_1(R_1+R_6+R_5) - I_2R_6 = V$$

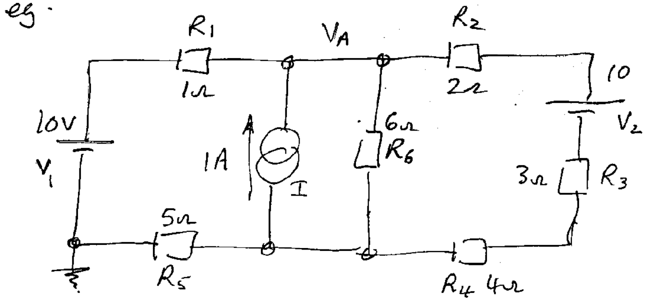
$$I_2(R_2+R_3+R_4+R_6) - I_1R_6 = -V_1$$

V_1 + V would be known.
— unknowns are I_1 + I_2 can be found by simultaneous solution of these two equations.

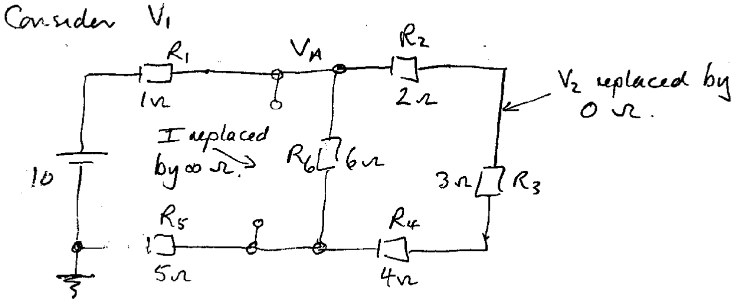
The principle of superposition.

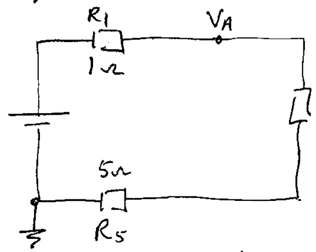
— The response of a linear network to a number of inputs is equal to the sum of the responses to each input applied alone in turn to the network.

eg.

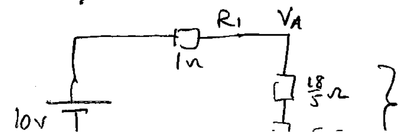


Consider V_1

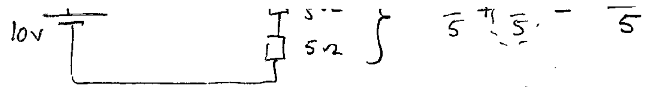




$$9 \| 6 = \frac{9\times 6}{9+6} = \frac{\cancel{54}}{15} = \frac{18}{5}\,\Omega.$$

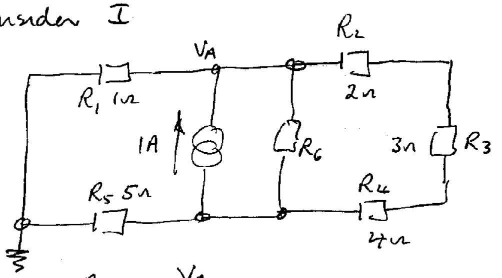


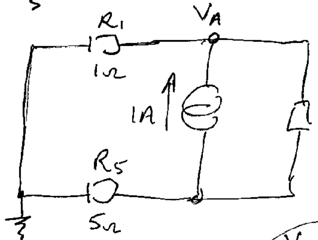
$$\frac{18}{5} + \frac{25}{5} = \frac{43}{5}$$



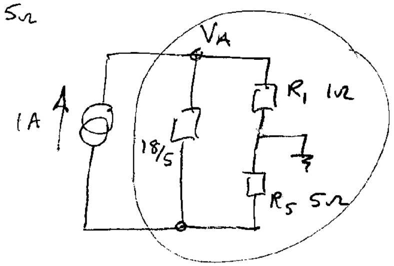
$$V_A\Big|_{\text{due to } V_1} = 10 \times \frac{43/5}{1 + 43/5} = 10 \times \frac{43}{48}.$$

now consider I





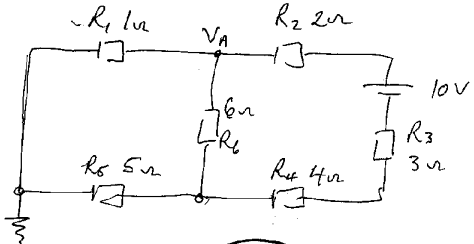
$9//6 = \frac{18}{5}\,\Omega.$

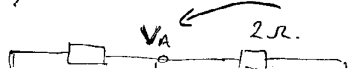


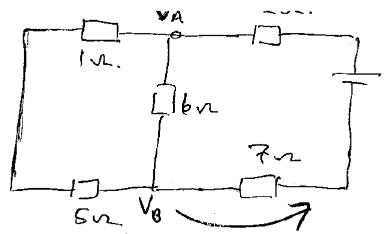
$$R_T = 18/5 \,//\, (1+5)$$
$$= \frac{18/5 \times 6}{18/5 + 6}$$
$$= \frac{108}{48} = \text{voltage across } R_T$$

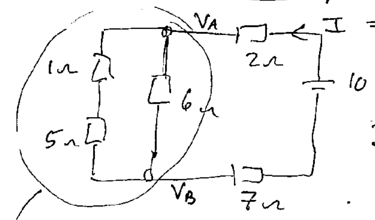
$$V_{R1} = V_A = V_{RT} \cdot \frac{1}{6}.$$

Partial cct for V_2 ...









$\equiv \frac{6\times 6}{6+6} = 3.$

$$I = \frac{10}{2+7+3} = \frac{10}{12}\ \text{A}.$$

I will generate a $V_A - V_B$ of $I\,3\Omega$

$$= \frac{10}{12} \times 3$$
$$= \frac{10}{4}\ \text{V}$$
$$= 2.5\text{V}.$$

V_A – ref voltage

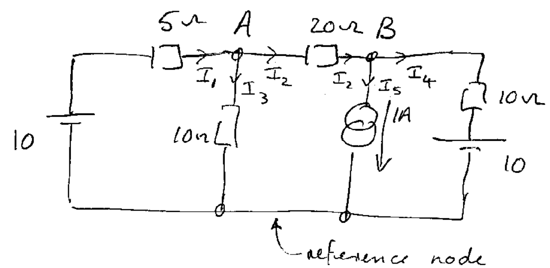
$$= V_A = \frac{2.5 \times 1}{1+5} = \frac{2.5}{6}\ \text{V} = \frac{5}{12}\ \text{V} = \frac{20}{48}$$

$$V_A \text{ total} = V_A\Big|_{\text{due to } V_1} + V_A\Big|_{\text{due to } I} + V_A\Big|_{\text{due to } V_2}$$

$$= \frac{430}{48} + \frac{108}{48}\cdot\frac{1}{6} + \frac{20}{48} = \frac{171}{48}\ \text{V}$$

a different answer.

A nodal analysis example



$$I_1 = I_2 + I_3$$

$$\frac{10-V_A}{5\Omega} = \frac{V_A-V_B}{20\Omega} + \frac{V_A-0}{10\Omega} \quad \text{———— (1)}$$

$$I_2 = I_4 + I_5$$

$$\frac{V_A-V_B}{20} = \frac{V_B-10}{10\Omega} + 1A \quad \text{———— (2)}$$

modifying (1)

$$40 - 4V_A = V_A - V_B + 2V_A$$

or $40 = 7V_A - V_B$ ———— (3)

modifying (2)

$$V_A - V_B = 2V_B - \cancel{20} + \cancel{20}$$

or $V_A = 3V_B$ ———— (4)

(4) into (3)

$$40 = 7(3V_B) - V_B = 21V_B - V_B = 20V_B$$

$$V_B = \frac{40}{20} = 2V$$

$$\therefore V_A = 3V_B = 6V.$$

Check ....

$$I_1 = \frac{10-V_A}{5} = \frac{10-6}{5} = 0.8A$$

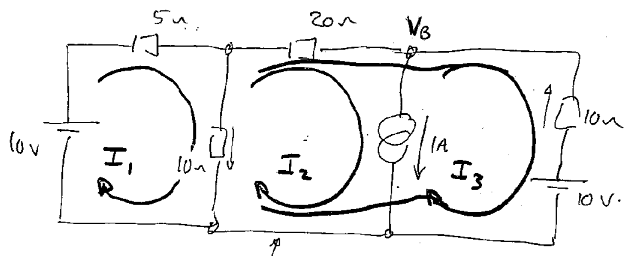
$$I_3 = \frac{V_A}{10} = \frac{6}{10} = 0.6A$$

$$I_2 = \frac{V_A-V_B}{20} = \frac{6-2}{20} = \frac{4}{20} = 0.2A$$

$I_1 = I_2 + I_3$ (first node equation)

$0.8 = 0.2 + 0.6$

loop analysis



reference node.

$10 = 5I_1 + 10(I_1 - I_2)$ ———— (1)

$0 = 10(I_2 - I_1) + I_2 . 20 + V_B$ ———— (2)

$10 + I_3 10\Omega = V_B$ ———— (3)

$I_2 - I_3 = 1A.$ ———— (4)

from 4+3

$$10 + (I_2 - 1)10 = V_B.$$

$$10 I_2 = V_B$$

eliminating V_B from (2)

$$0 = 10(I_2 - I_1) + I_2 20 + 10 I_2$$

$$= 10I_2 + 20I_2 + 10I_2 - 10I_1$$

$$= 40I_2 - 10I_1$$

or $I_1 = 4I_2$.

4.1 2kΩ 5mA 5V 10mA

4.2	0.75kΩ	−3.33mA	10V	−2.5mA
4.3	1.67kΩ	−4mA	6V	−6.7mA
4.4	0.67kΩ	5mA	5V	3.33mA
4.5	2.5kΩ	−2mA	2V	−5mA
4.6	0.67kΩ	−5mA	5V	−3.33mA
4.7	2.38kΩ	2.4mA	2.8V	5.7mA
4.8	1.33kΩ	2.5mA	2.5V	3.33mA
4.9	0.79kΩ	4mA	2.0V	3.16mA

Sheet 2 Q7

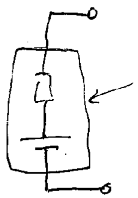
Q7 → 0.53A

Q8 → 1.91A

Q9 → 7.65V 3.53A

Q10 → −0.588A 3Ω 16.57W.

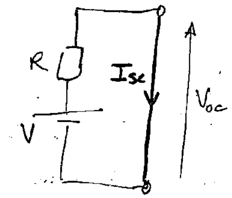
Thevenin Equivalent Circuits



this is a standard model for a battery and is a Thevenin equivalent representation of what is inside

... useful for modelling non electrical systems like batteries and certain aspects of humans + animals

useful also as a circuit transformation tool in conventional electronic circuits.

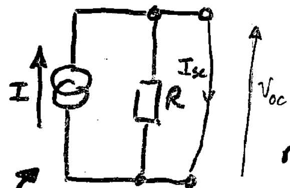


for no load connected

$V_{oc} = V$ since no current flows

for a short circuited output

$I_{sc} = \frac{V}{R}$



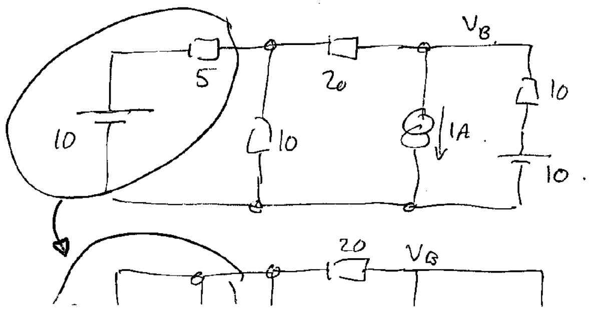
$V_{oc} = IR$ $\quad I_{sc} = I$

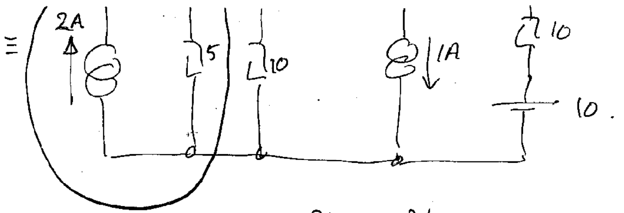
note that if $I = V/R$.

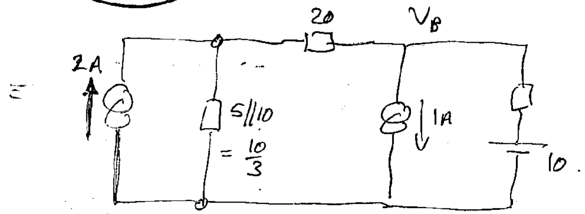
$V_{oc} = V \quad I_{sc} = V/R$.

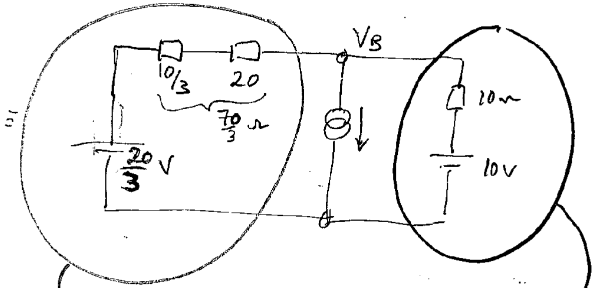
This is a Norton equivalent cct.

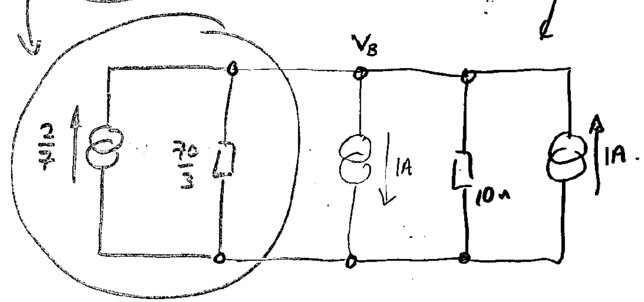
Application to circuit problems...

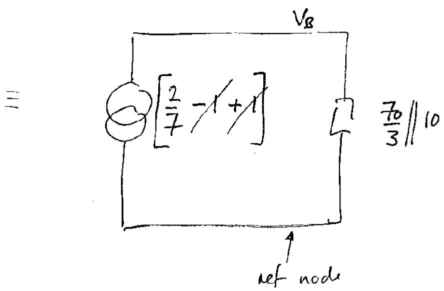




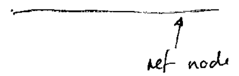






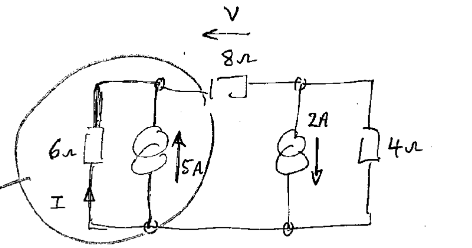


$$\frac{70}{3} \parallel 10 = \frac{700/3}{\frac{70}{3} + 10} = \frac{700/3}{100/3} = 7\ \Omega$$

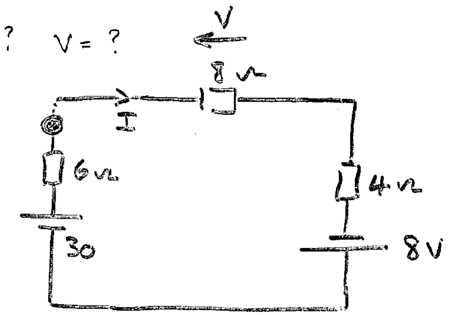


$$\frac{100}{3} = 7\ \Omega.$$

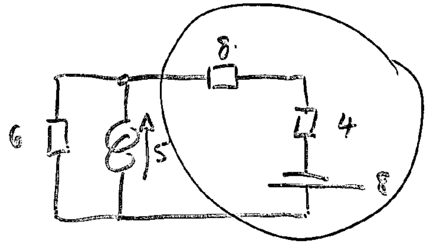
$$V_B = \frac{2}{7} \times 7 = 2V.$$

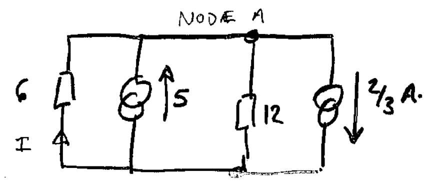


I = ? V = ?



$$I = \frac{38}{18}\ A \qquad \therefore\ V = \frac{38}{\cancel{18}\ 9} \times \cancel{8}\ 4$$



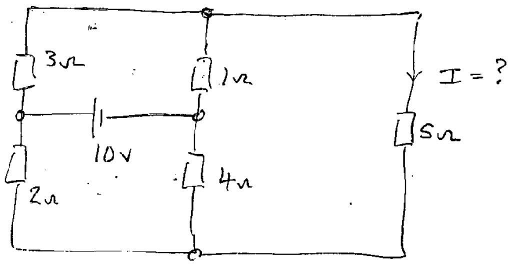


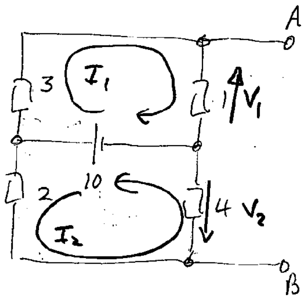
Current entering node A $= 5 - 2/3$
$= 4\frac{1}{3}$ A.

That current flows through $6\Omega \parallel 12\Omega$
$= \cancel{\frac{72}{18}} \cdot \cancel{\frac{12}{3}}\ 4\Omega$.

So $V_A = 4\frac{1}{3} \times 4$
$= 17\frac{1}{3}$ V.

$$I = \frac{0 - V_A}{6} = \frac{0 - 17\frac{1}{3}}{6} = -2.89\text{ A}.$$





$$I_1 = \frac{10}{4} = 2.5\text{A}.$$

$$V_1 = 2.5\text{V}.$$

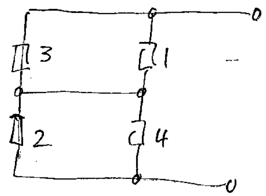
$$I_2 = \frac{10}{6} = \frac{5}{3}\text{A}$$

$$V_2 = \frac{5}{3} \times 4\text{ V} \quad \frac{20}{3}\text{V}.$$

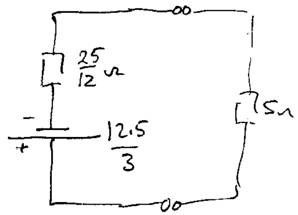
B

$$V_2 = \frac{5}{3} \times 4\text{ V} \quad \frac{20}{3}\text{V}.$$

$$V_A - V_B = 2.5\text{V} - \frac{20}{3}\text{V}.$$

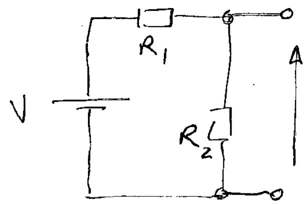


$R = ? = 1\parallel 3 + 2\parallel 4$
$= \frac{3}{4} + \frac{8}{6}$
$= \frac{3}{4} + \frac{4}{3} = \frac{9}{12} + \frac{16}{12}$
$= \frac{25}{12}\ \Omega$.

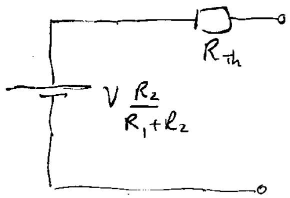


$2.5 - \frac{20}{3} = \frac{7.5 - 20}{3}$
$= -\frac{12.5}{3}$

Thevenin and potential dividers

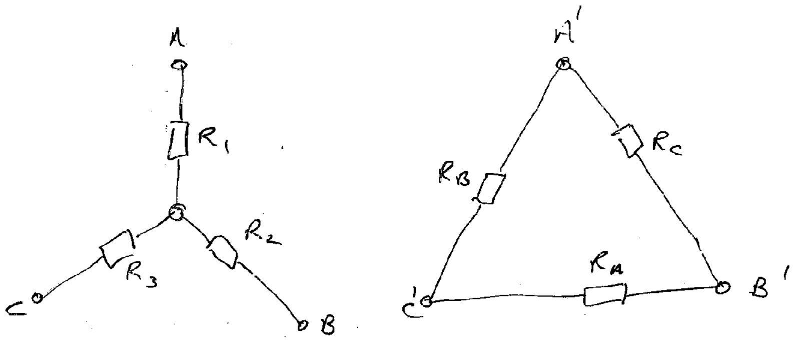


$$V_o = \frac{VR_2}{R_1 + R_2}.$$

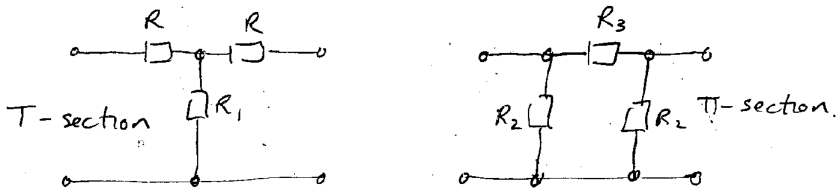


$$R_{TH} = R_1 \parallel R_2$$

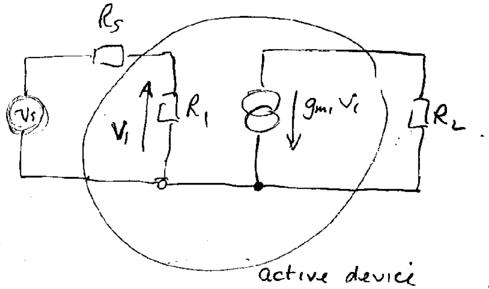
Star – Delta transformations.



power system.



HF transmission lines, attenuators + filters.



— bipolar ...
— field effect transistor
— thermionic vaccuum tube (valve).

$\frac{1}{2}CV^2$ $\quad \frac{1}{2}\cdot 2500 \cdot 6.25 \quad \frac{15000 \text{ ish}}{2} = 7500$ J.

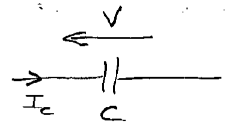
$\frac{1000^2 \cdot 0.5\times 10^{-6}}{2}$ $\quad .25$ J.

$400 \times 300 \quad \frac{\cancel{120000} \cdot 22\times\cancel{10^{-6}}}{2}$

$\frac{2.4}{2}$ 1.2 J.

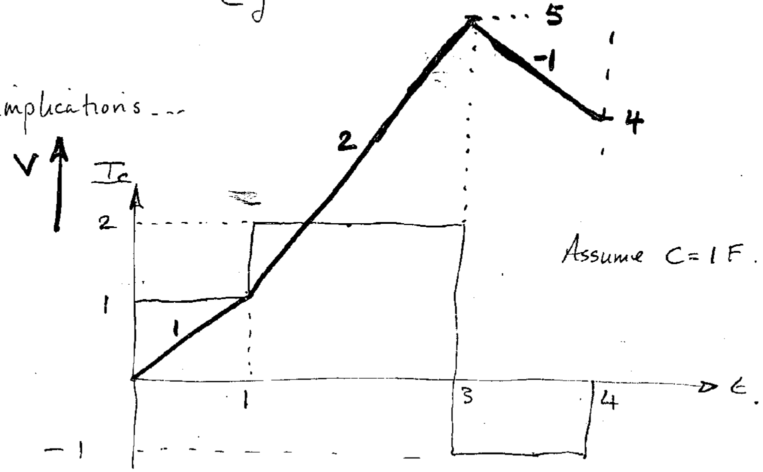
Capacitors

$$I_c = C\frac{dV}{dt}$$

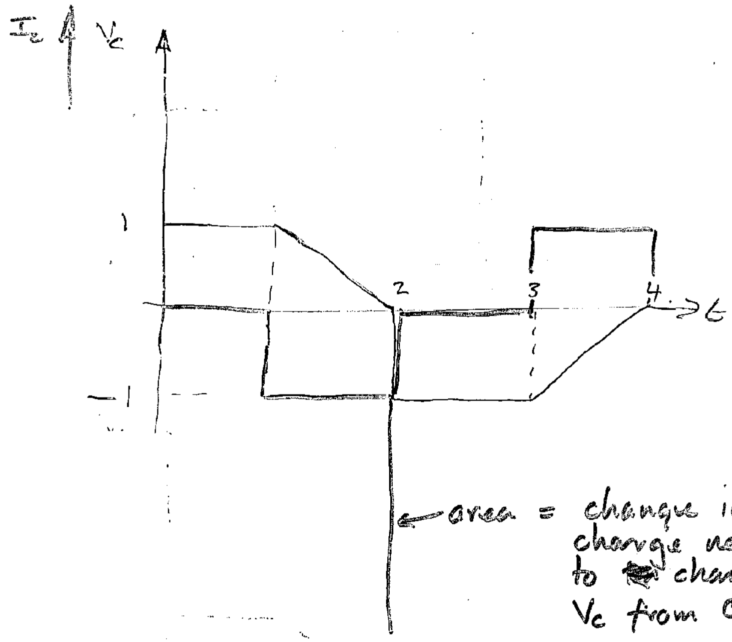


or $V = \frac{1}{C}\int I_c \, dt + \text{const.}$

implications ...

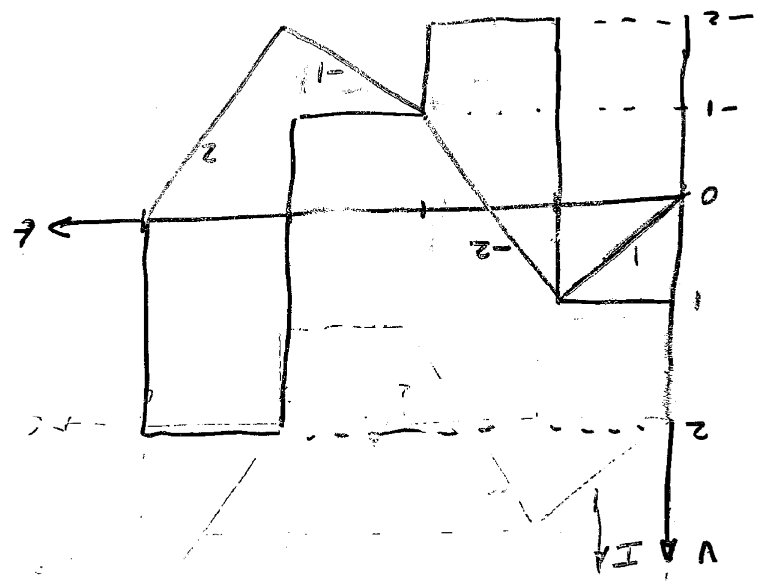


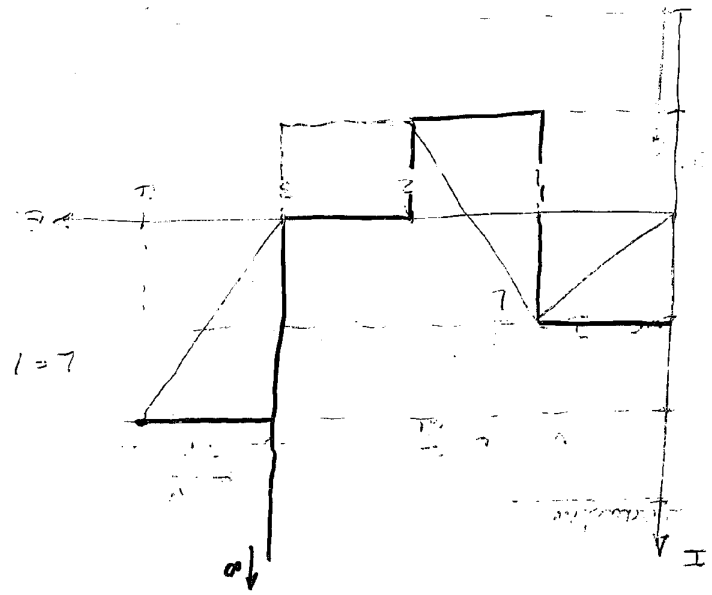
Assume $C = 1$ F.



← area = change in charge necessary to change V_c from 0V to −1V.

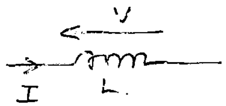
($\Delta Q = C\Delta V$).



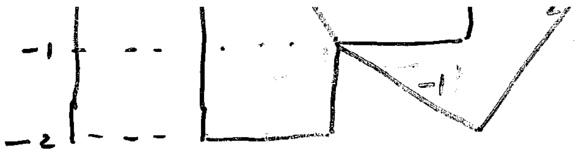


<u>Inductance</u>

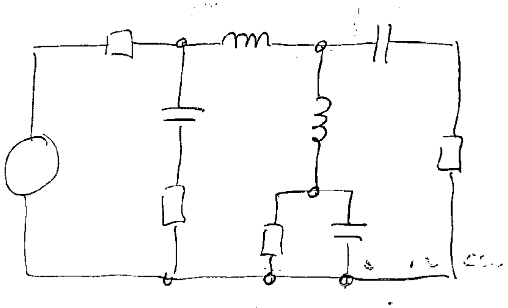
$$V = L\frac{dI}{dt}$$



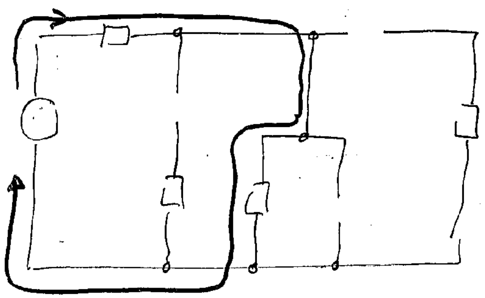
or $$I = \frac{1}{L}\int V\,dt + \text{const.}$$



At dc, inductance = short cct
capacitance = open cct.

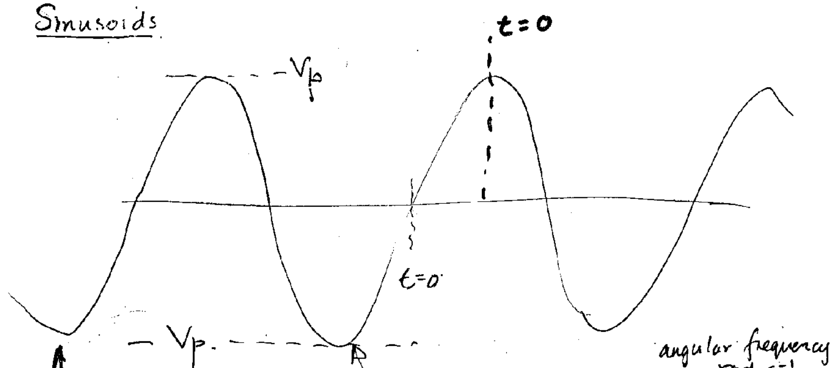


If the source is d.c.



What about a.c. + capacitors + inductors?

Sinusoids



$V(t) = V_p \cos \omega t.$

$V(t) = V_p \sin \omega t$

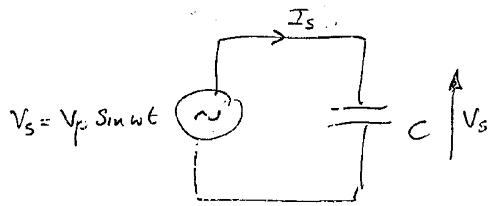
amplitude

angular frequency rad s^{-1}

$\omega = 2.\pi.f$

cyclic frequency (Hz) $= \frac{1}{\text{periodic time}}$.

Capacitors + Sinusoids



$$I_s = C\frac{dV_s}{dt} = C\frac{d(V_p \sin \omega t)}{dt}.$$

$$= C\omega V_p \cos \omega t = C\omega V_p \sin\left(\omega t + \frac{\pi}{2}\right).$$

$= C\omega V_p \cos\omega t = C\omega V_p \sin(\omega t + \frac{\pi}{2})$.

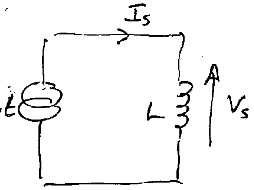
Impedance of the capacitance is

$$|Z_c| = \left|\frac{V_s}{I_s}\right| = \frac{V_p}{C\omega V_p} = \frac{1}{\omega C}$$

and V_s lags I_s by $\frac{\pi}{2}$ radians or 90°

For Inductors

$I_s = I_p \sin\omega t$



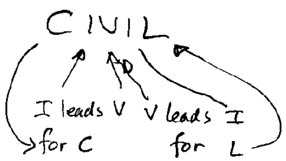
$$V_s = L\frac{dI_s}{dt} = L\frac{d(I_p \sin\omega t)}{dt}$$

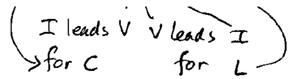
$$= \omega L I_p \cos\omega t = \omega L I_p \sin(\omega t + \pi/2)$$

$$|Z_L| = \left|\frac{V_s}{I_s}\right| = \frac{I_p \omega L}{I_p} = \omega L$$

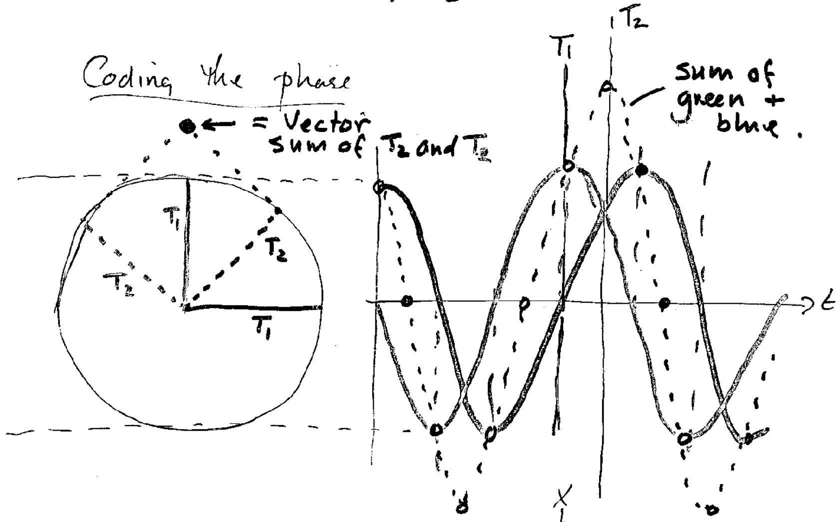
and V_s leads I_s by $\pi/2$ radians or 90°.

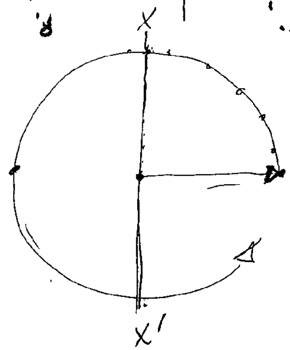
To help remember what lags what in inductors + capacitors



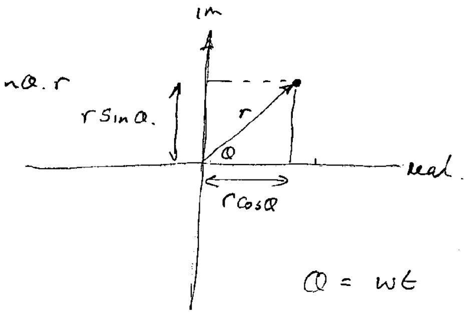


Coding the phase





$$re^{j\theta} = r\cos\theta + j\sin\theta . r$$

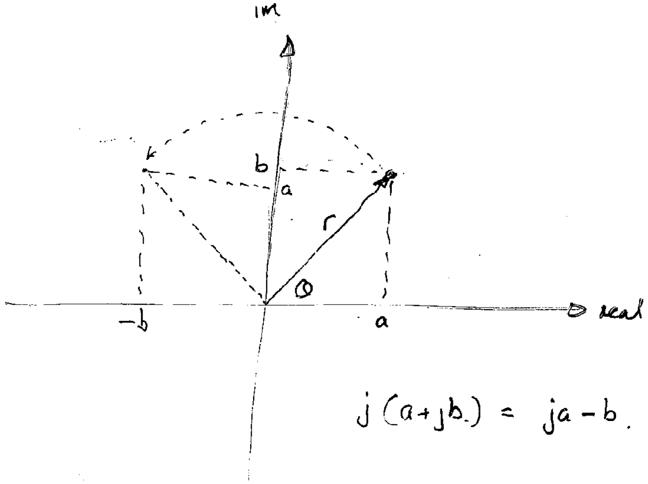


$\theta = \omega t$

in our case.

$\theta = \omega t$

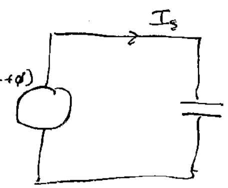
in our case.



$$j(a+jb) = ja - b.$$

Consider a capacitance ...

$V_s = V_p e^{j(\omega t+\phi)}$



$$I_s = C\frac{dV_s}{dt} = C\frac{d\left(V_p e^{j(\omega t+\phi)}\right)}{dt}.$$

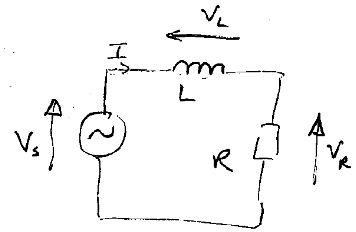
$$= C V_p j\omega\, e^{j(\omega t+\phi)}.$$

$$Z_c = \frac{V_s}{I_s} = \frac{V_p \cancel{e^{j(\omega t+\phi)}}}{C V_p j\omega \cancel{e^{j(\omega t+\phi)}}}$$

$$= \frac{1}{j\omega C}$$

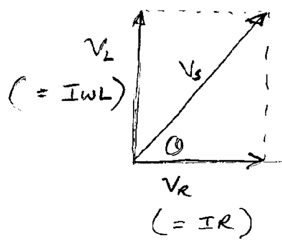
Some phasor & complex examinations of simple circuits

$$\bar{V}_R + \bar{V}_L = \bar{V}_s$$



using a phasor diagram ..

use current as a reference direction because current is common to both elements.



$$V_s^2 = I^2R^2 + I^2\omega^2L^2$$

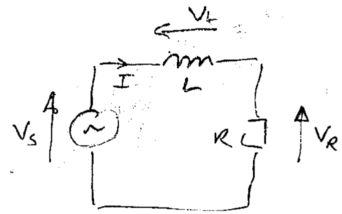
$$\frac{V_s^2}{I^2} = R^2 + \omega^2L^2$$

$$\frac{V_s}{I} = |Z| = \sqrt{R^2+\omega^2L^2}$$

I direction

$$\theta = \tan^{-1}\frac{\cancel{I}\omega L}{\cancel{I}R}$$

using "j" notation.

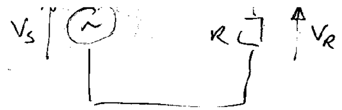


$$V_L = I.Z_L = I.j\omega L$$

$$V_R = I.R$$

$$V_s = V_L + V_R = I.j\omega L + IR$$

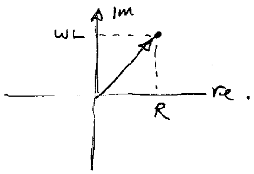
$V_L = I.Z_L = I.j\omega L$



$V_R = I.R$

$V_s = V_L + V_R = Ij\omega L + IR$

$= I(R + j\omega L)$



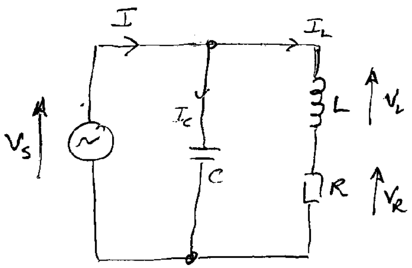
so $\frac{V_s}{I} = R + j\omega L \equiv Z$

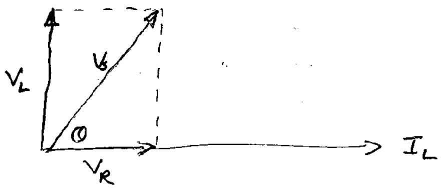
$\theta = \tan^{-1}\frac{Im}{re} = \tan^{-1}\frac{\omega L}{R}$

$|Z| = \sqrt{R^2 + \omega^2 L^2}$

phasor ...

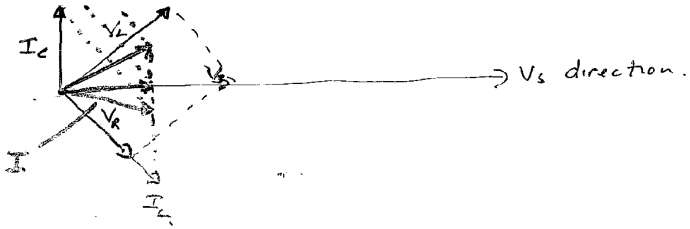
consider the series arm





$V_s^2 = V_L^2 + V_R^2 = I_L^2\omega^2L^2 + I_L^2R^2$

$I_L = \frac{V_s}{\sqrt{\omega^2L^2 + R^2}}$ $\theta = \tan^{-1}\frac{\omega L}{R}$



j notation

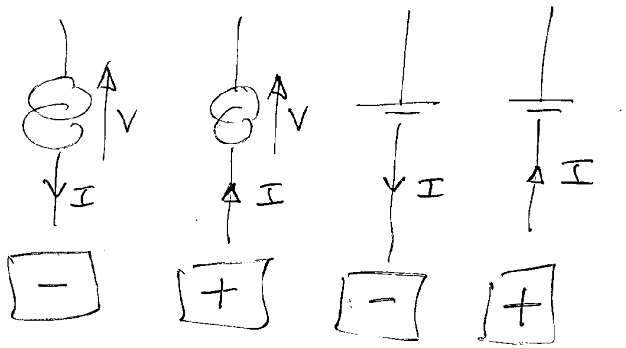
$\frac{V_s}{I} = \frac{1}{j\omega C} \| (R + j\omega L)$

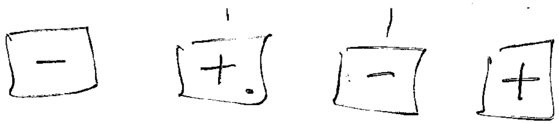
$= \frac{\frac{1}{j\omega C}\cdot(R + j\omega L)}{\frac{1}{j\omega C} + R + j\omega L}$

$= \frac{R + j\omega L}{1 - \omega^2 LC + j\omega CR}$

to find a condition where V_s and I are in the same phase — look for a condition that will make "j" terms disappear.

Q1

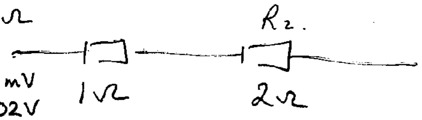




Q2

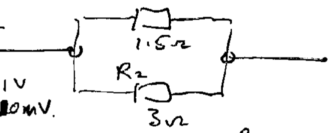
$R_T = 3\Omega$

$V_{R2} = 20mV$ or $.02V$



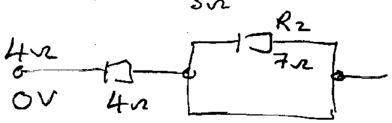
$R_T = 1\Omega$

$V_{R2} = .01V$ or $10mV$



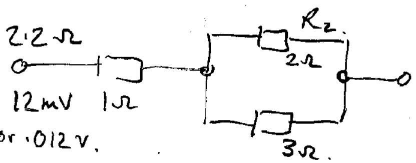
$R_T = 4\Omega$

$V_{R2} = 0V$



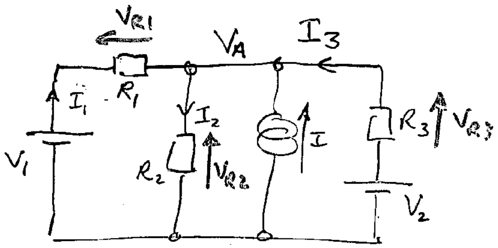
$R_T = 2.2\Omega$

$V_{R2} = 12mV$ or $.012V$



Q3.

R_3 is associated with convention error

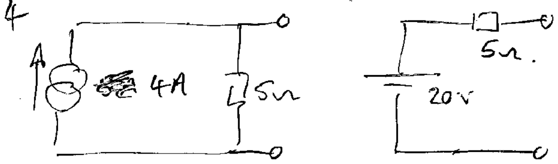


$$I_1 + I_3 + I = I_2$$

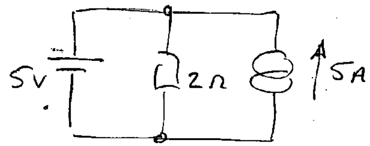
$$\frac{V_1 - V_A}{R_1} + \frac{V_2 - V_A}{R_3} + I = \frac{V_A}{R_2}$$

$$\frac{V_1 - V_A}{R_1} + \frac{V_2 - V_A}{R_3} + I = \frac{V_A}{R_2}$$

Q4

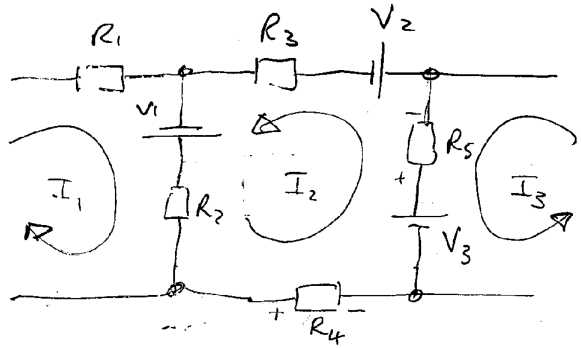


Q5



$$P_R = \frac{5^2}{2} = \frac{25}{2} = 12.5\ W.$$

Q6.

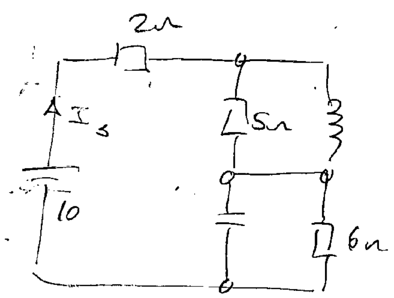


$$I_2R_4 - V_3 + R_5(I_2 - I_3) + V_2 + R_3I_2 - V_1 + R_2(I_2 + I_1) = 0$$

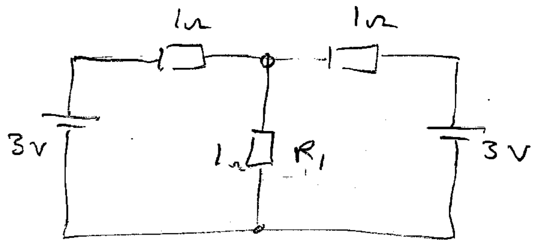
Q7

$$I_s = \frac{10V}{2\Omega + 6\Omega}$$

$$= \frac{10}{8} = 1.25A.$$



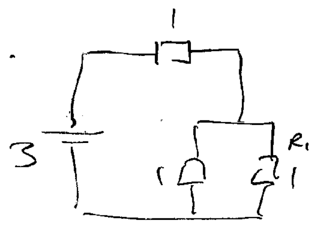
Q8.



Ans = 4W

eg by superposition...

due to 3V (left.)



$V_{R1} = 1V.$

Since 3V (right) cct is identical it also contributes 1V.

$\therefore \; V_{R1\,TOT} = 2V.$