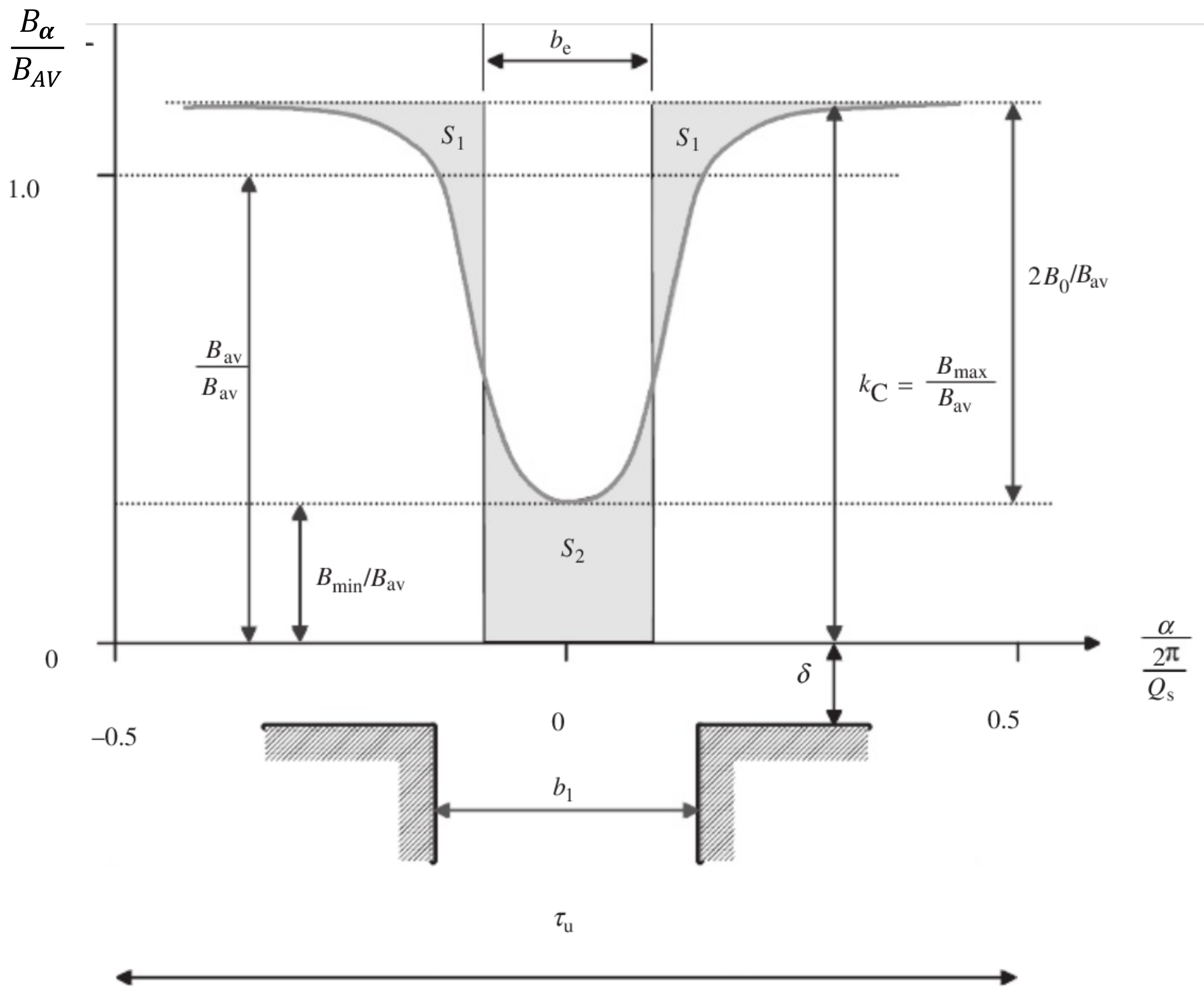
Carter coefficient

- The presence of slots in a core has an inevitable effect on airgap field, both locally and in terms of average flux-linkage.
  - Does this mean we cannot use several of the expressions derived earlier for idealised plain airgaps ?
- Fortunately, there is a means of accounting for slotting in calculating average flux using the so-called ‘Carter Coefficient’
  - Initially developed by FW Carter and first published in 1901 – several developments since
  - Introduces a coefficient c which is used to calculate a modified airgap length lg’

$$l_g' = k_c \, l_g$$

  - The carter coefficient is a function of the ratio of the airgap to the slot opening can be calculated from so-called ‘conformal’ or ‘Schwartz-Christophel’ transformations
  - Originally developed and still largely drawn from rectilinear representations of slots

Calculation of Carter coefficient



Source: DESIGN OF ROTATINGELECTRICALMACHINES - Juha Pyrhonen, Tapani Jokinen, Valeria Hrabovcova, John Wiley and Sons Ltd, ISBN 978-0-470-69516-6

Actual flux density variation shows a dip in the vicinity of the slot

One widely used method is based on identifying an equivalent slot width b_e over which a step change gives the same flux as the actual geometry

From this consider $S_1+S_1 = S_2$

The modified slot opening is given by:

$$b_e = \kappa b_1$$

Where:

$$\kappa = \frac{2}{\pi}\left(tan^{-1}\left(\frac{b_1}{2\delta}\right) - \frac{2\delta}{b_1} ln\sqrt{1 + \left(\frac{b_1}{2\delta}\right)^2}\right)$$

Although this expression can be calculated directly, a reasonable approximation is given by the simpler expression:

$$\kappa = \frac{\frac{b_1}{\delta}}{5 + \frac{b_1}{\delta}}$$

Having calculated this factor, then the Carter coefficient k_c is given by:

$$k_c = \frac{\tau_u}{\tau_u - b_e} = \frac{\tau_u}{\tau_u - \kappa b_1}$$

The mathematical derivation which underpins the calculation of κ is based on conformal mapping, the details of which are beyond the scope of this course. A useful reference for general interest is the original 1901 paper by FW Carter, a copy which is available at:

http://perso.uclouvain.be/ernest.matagne/ELEC2311/SEM04/CARTER01.PDF

- The conformal mapping also allows to estimate the extent of the flux density variation in the vicinity of the slot:

$$\frac{B_{max}}{B_{av}} = k_c$$

$$\frac{B_0}{B_{max}} = \frac{(B_{max} - B_{min})}{2B_{max}} = \frac{1 + u^2 - 2u}{2(1 + u^2)}$$

$$\frac{B_{min}}{B_{max}} = \frac{2u}{1 + u^2}$$

Where *u* is given by

$$u = \frac{b_1}{2\delta} + \sqrt{1 + \left(\frac{b_1}{2\delta}\right)^2}$$

Finally, it is worth noting that if both the rotor and stator are slotted, then an effective aigrap length can be derived by a consideration of both sets of slots separately and the application of two Carter coefficients to adjust the airgap

Example

The diameter of the stator bore of an electrical machine is 230mm. The stator has 36 slots and the slot openings are 5mm wide. The airgap of the machine is 0.75mm. Calculate the Carter coefficient and hence the modified airgap length.

From the information in the question:

$$b_1 = 5mm \qquad \delta = 0.75mm$$

Hence, $$\kappa = \frac{\frac{b_1}{\delta}}{5 + \frac{b_1}{\delta}} = \frac{\frac{0.005}{0.00075}}{5 + \frac{0.005}{0.00075}} = 0.571$$

(would have been OK in mm since the terms are ratios but alwys wise to stick to SI)

The slot pitch is given by:

$$\tau_u = \frac{\pi \times 0.23}{36} = 0.020m$$

$$k_c = \frac{0.020}{0.020 - 0.571 \times 0.005} = 1.167$$

Hence, the effective airgap is:

$$l'_g = k_c\, l_g = 1.167 \times 0.75 = 0.875mm$$