Data Provided: 1 × Bode paper (Q3)

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2010-2011 (2 hours)

Introduction to Avionics 6

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. **a.** Table 1.1 gives the failure rates and number of the components used in the circuit of an electronic unit.

Component	Failure rate	Number in circuit
Metal film resistance	2×10^{-9} /hour	10
Tantalum capacitor	10×10^{-9} /hour	4
Low-power transistor	5×10^{-9} /hour	5
Electrolytic capacitor	100×10^{-9} /hour	1
Analogue IC	45×10^{-9} /hour	2
Digital IC	7×10^{-9} /hour	2
Soldering joints	0.1×10^{-9} /hour	68
Printed circuit board traces	0.01×10^{-9} /hour	20

Table 1.1 List of components

Assuming that the circuit will fail if any one of its components fails:

i. Calculate the failure rate of the electronic unit. **(2)**
ii. Calculate the mean time to failure *MTTF* of the electronic unit. **(2)**
iii. Calculate the maximum duration of operation when the reliability of the electronic unit may not drop below 0.9999. **(2)**
iv. Calculate the probability that the unit would fail after a 9-minute operation. **(2)**

b. To meet stricter reliability requirements, an n-unit redundancy configuration is used:

i. Calculate the probability of losing the function of the unit during a 1000-hour operation, for n = 2. Failure is detected using monitors, and only 1 unit is active while the other is on stand-by. **(6)**

ii. For n = 3 and when failure is detected using a majority voting scheme, calculate the probability of losing the function of the unit during a 410-hour operation, and the *MTTF* of the 3-unit redundancy configuration. **(6)**

The following may be assumed:

Reliability function for m-out-of-n system (active):

$$R(t)=\sum_{k=m}^{n}\frac{n!}{k!(n-k)!}\left[e^{-\lambda kt}\right]\left[1-e^{-\lambda t}\right]^{n-k}$$

Reliability function for m-out-of-n system (passive):

$$R(t)=e^{-\lambda mt}\sum_{k=m}^{n}\frac{(m\lambda t)^{k-m}}{(k-m)!}$$

In general the mean time to failure is given by: $MTTF=\int_0^{\infty}R(t)dt$

2. An electromechanical actuator consisting of a brushless dc motor, a nut and a ballscrew, figure 2.1, is driving the carbon brakes of an aircraft. The brushless dc motor has a torque constant $k = 0.51$ Nm/A and an equivalent winding resistance $R = 8\,\Omega$. Furthermore, the screw pitch length of the electromechanical actuator is λ = 5.5 mm, and the gear ratio between the nut and the shaft of the brushless motor is 35:1. (i.e. the motor rotates 35 times for 1 revolution of the nut). You may assume that the mechanical transmission of the actuator is loss-less.

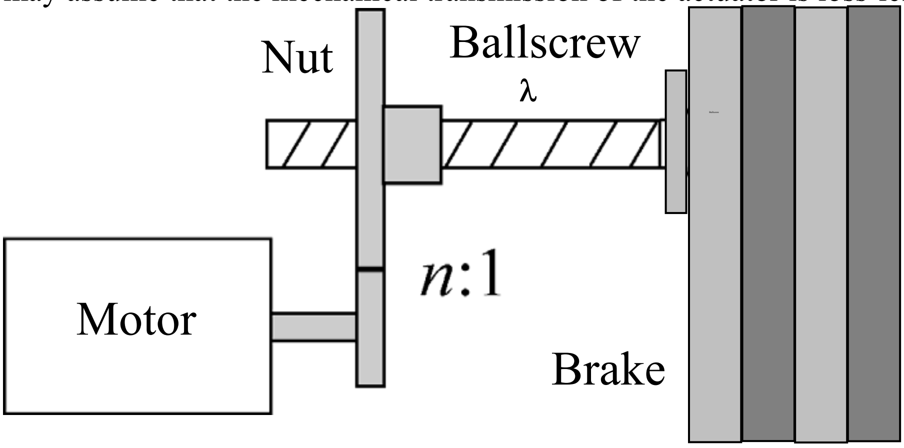


Figure 2.1 Schematic of electromechanical actuator.

a. When the actuator is applying a force $F_a = 30$ kN:

i. Calculate the torque T_m delivered by the brushless dc motor. **(3)**

ii. Calculate the current I, and the copper loss P_c of the brushless dc motor. **(3)**

b. The reaction force from the carbon brakes increases linearly with displacement according to the relationship, $F_r(x) = a\,x$, and $a = 24 \times 10^6$ N/m:

i. Calculate the angular displacement of the brushless dc motor which corresponds to a linear displacement $x = 1.25$ mm. **(2)**

ii. Calculate the energy E delivered by the actuator to the carbon brakes for a displacement $x = 1.25$ mm. **(3)**

c. The brushless dc motor is controlled so as to have a maximum angular displacement θ_m in a time T, following a parabolic velocity profile.

i. Show that for a maximum motor rotational speed Ω_m the angular displacement of the rotor is given by:

$$\theta(t) = 4\,\Omega_m \left(\frac{t^2}{2T} - \frac{t^3}{3T^2} \right)$$ **(4)**

ii. Show that the maximum speed of the motor Ω_m is related to the maximum angular displacement θ_m by:

$$\Omega_m = \frac{3}{2}\frac{\theta_m}{T}$$ **(2)**

(Continued overleaf)

iii. Show that the sum of the torques applied to the rotor of the brushless dc motor, which has an inertia J_r, is given by:

$$T_{tot}(t) = 4 J_r \Omega_m \left(\frac{1}{T} - 2\frac{t}{T^2} \right)$$ **(3)**

<u>The following may be assumed:</u>

For a parabolic velocity profile: $\Omega(t) = 4\Omega_m \left(\frac{t}{T} - \frac{t^2}{T^2} \right)$;

3. Consider the closed-loop system for controlling a robot arm shown in figure 3.1, which incorporates a phase-lead compensation scheme.

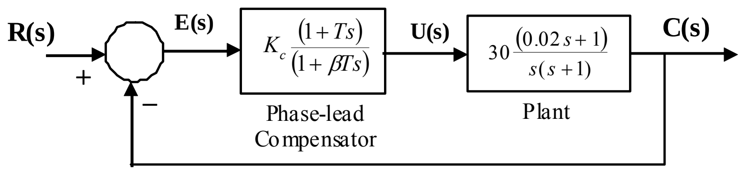


Figure 3.1

a. Calculate the dc gain of the compensator to provide a static velocity (ramp) error constant, K_{ev} of 90s^{-1}. (*Note: show all your calculations*). **(4)**

b. If a value of $K_c = 2$ is selected for the phase-lead compensator:

i. Using asymptotes as an aid, plot the frequency response plot of the transfer function

$$K_c \frac{30\,(0.02s+1)}{s(s+1)}$$

on the Bode paper provided. **(6)**

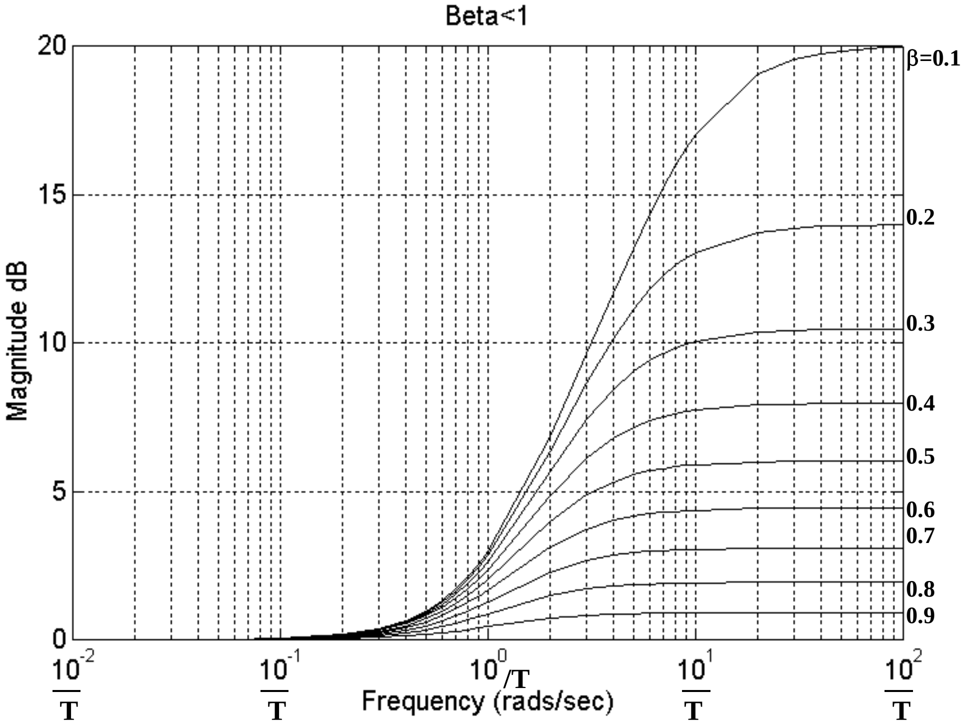
ii. From the Bode plot determine the gain margin and phase margin of the system. **(3)**

iii. Using the normalised phase-lead characteristics provided in figure 3.2, select values of β and T to provide a phase margin of at least 50°. **(4)**

iv. Estimate the closed-loop bandwidth of the system using the values of β and T you have chosen in part **iii**. Justify your answer. **(1)**

v. Determine the actual steady-state error you would expect for a <u>unit ramp input</u> to the closed-loop system. **(2)**

{ENSURE YOUR BODE DIAGRAM IS ATTACHED TO YOUR ANSWER BOOKLET}



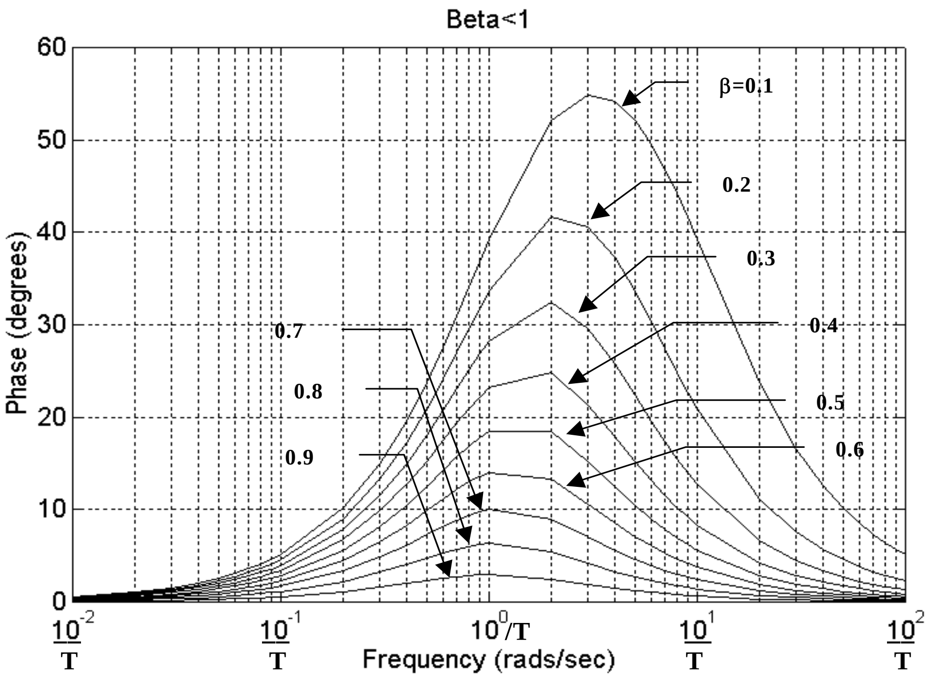


Figure 3.2: Normalised Phase-Lead Characteristics, $K_c\dfrac{(1+Ts)}{(1+\beta Ts)}$, $\beta<1$, $K_c=1$.

4. **a.** Derive the state variable description of the system shown in Figure 4.1 i.e. obtain a model of the form $\dot{x} = Ax + Bu$, $y = Cx$ using the definition of the state-variables shown in the figure.

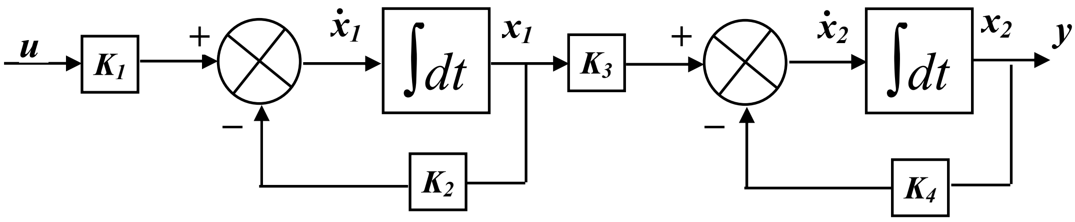


Figure 4.1

(3)

b. The open-loop system of a radio telescope may be described in state-variable form by the equation below.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} u$$

$$= \mathbf{A}x + \mathbf{B}u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \mathbf{C}x$$

Calculate the eigenvalues of **A**. **(4)**

c. By forming the Controllability Matrix C determine whether the system given in part **b**. is controllable. **(3)**

d. Using Ackermann's method, design a state-feedback controller $u = -\boldsymbol{K}x$, such that the resulting closed loop system has Eigenvalues at:

$$\lambda_1 = -6 \qquad \lambda_2 = -10$$

That is, calculate an appropriate state-feedback gain matrix, $\boldsymbol{K}$. **(6)**

e. Sketch the block diagram structure of the resulting closed loop system — include appropriate integrators, states and the controller gain terms. **(4)**

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