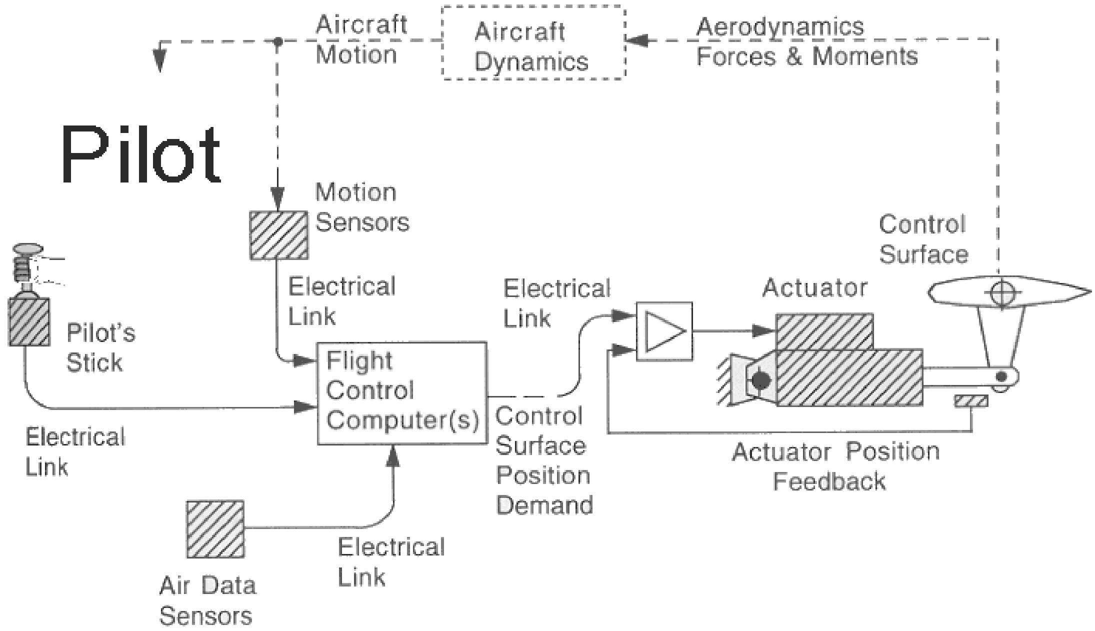
Solution

Question 1

(a)

The main advantages of the Fly-By-Wire flight control are summarised as follows:

- Elimination of mechanical control runs.
- Consistent handling over a wide flight envelope.
- Automatic stabilisation.
- Carefree manoeuvring.
- Automatic integration of additional controls.
- Ability to exploit aerodynamically unstable aircraft.



Schematic of the Fly-By-Wire control system

(b)

i. The Mach number M and the true airspeed V_T are related by $M = \frac{V_T}{A}$,

where A is the speed of sound for $T_S = -34.5^o C$:

$$A = \sqrt{\gamma\, R_a\, T_S} = \sqrt{1.4 \times 287.0529 \times 238.5} = 309.6 \text{ m/s}$$

Therefore,

$$M = \frac{V_T}{A} = \frac{155.55}{309.6} = 0.502$$

Since the aircraft is flying in the troposphere region, therefore, the static temperature is related to altitude by:

$$T_S = T_O - L \times H \quad \Rightarrow \quad H = \frac{(T_O - T_S)}{L} = \frac{(288.15 - 238.5)}{6.5 \times 10^{-3}} = 7638\,\text{m}$$

L is the troposphere lapse rate.

ii. The air density is given by:

$$\rho = \frac{P_s}{R_a\,T_S} = \frac{37650}{287.0529 \times 238.5} = 0.55\,\text{kg/m}^3$$

iv. The calibrated airspeed V_C and the impact pressure are related by:

$$\text{Impact pressure} = P_0\left(\left(1 + \frac{(\gamma - 1)(V_c/A_0)^2}{2}\right)^{\gamma/(\gamma-1)} - 1\right) = P_T - P_S$$

The static pressure P_S and the total pressure P_T are related by:

$$\frac{P_T}{P_S} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{(\gamma-1)}} \Rightarrow P_T = 37.65 \times \left(1 + \frac{(1.4 - 1)}{2} 0.502^2\right)^{\frac{1.4}{(1.4-1)}} = 44.72\,\text{kPa}$$

Therefore, $P_T - P_S$ =7.07 kPa.

Furthermore, at sea level the speed of sound is given by:

$$A_O = \sqrt{\gamma\,R_a\,T_O} = \sqrt{1.4 \times 287.0529 \times 288.15} = 340.29\,\text{m/s}$$

Therefore,

$$\frac{(P_T - P_S)}{P_0} + 1 = \left(1 + 0.2(V_c/A_0)^2\right)^{3.5} \Rightarrow 1 + 0.2(V_c/A_0)^2 = \sqrt[3.5]{\frac{(P_T - P_S)}{P_0} + 1}$$

$$\Rightarrow V_c = A_o\sqrt{\frac{\sqrt[3.5]{\frac{(P_T - P_s)}{P_0}+1}-1}{0.2}} = 340.29\times\sqrt{\frac{\sqrt[3.5]{\frac{(44.72-37.65)}{101.325}+1}-1}{0.2}}$$ =106.14 m/s = 382.1 km/h.

(c)

i. At 15000m, the aircraft is flying in the stratosphere and the temperature is roughly the same as the temperature at the tropopause height of 11000m. The temperature at 11000m is given by:

$$T_s = T_o - L\times H \quad \Rightarrow \quad T_s = 288.15 - 6.5\times10^{-2}\times11000 = 216.65\ {}^oK$$

Therefore, the speed of sound is given by:

$$A = \sqrt{\gamma\, R_a\, T_s} = \sqrt{1.4\times287.0529\times216.65} = 295.07\text{ m/s}$$

ii.

the mach number is given by:

$$M = \frac{V_T}{A} = \frac{333.3}{295.07} = 1.129$$

Therefore, the aircraft is flying at supersonic speed and P_T is given by:

$$P_T = P_s\frac{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{(\gamma-1)}} M^{\frac{2\gamma}{(\gamma-1)}}}{\left(\frac{2\gamma}{(\gamma+1)}M^2 - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\frac{1}{(\gamma-1)}}} = 101.325\times\frac{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{(\gamma-1)}} 1.129^{\frac{2\gamma}{(\gamma-1)}}}{\left(\frac{2\gamma}{(\gamma+1)}1.129^2 - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\frac{1}{(\gamma-1)}}}$$

$= 24.52$ kPa

Question 2

(a)

When adaptive majority voting is adopted, a failed channel is disconnected and is no longer part of the set of valid alternatives. Therefore, the system will function correctly as long as the majority of the remaining channels is functioning. The required majority will decrease following each failure, down to a minimum of 2 channels.

If non-adaptive majority voting is adopted, the majority of the initial channels must function for the system to function. Therefore, for given n independent channels the number of minimum functioning channels are as follows:

$$\left(\frac{n}{2}\right)+1 \quad : \text{ if } n \text{ is even}$$

$$\frac{(n+1)}{2} \quad : \text{if } n \text{ is odd}$$

(b)

i. The maximum drift from the original nominal value to the tolerance limits of IC parameters, which results in failure is constant. Therefore,

$$t^n\, Q(T) = t^n\, Q_o\, e^{(-E_a/kT)} = K$$

where K is a constant and t is the time during which the parameters drift from nominal values to tolerance limits, therefore,

$$t^n = \frac{K}{Q_o} e^{(E_a/kT)} \Rightarrow n\ln(t) = \ln\left(\frac{K}{Q_o}\right) + \frac{E_a}{kT} \Rightarrow \ln(t) = \frac{1}{n}\ln\left(\frac{K}{Q_o}\right) + \frac{E_a}{nkT}$$

since K and Q_o are constants, therefore, we can write

$$\ln(t) = \ln(t_o) + \frac{E_a}{nkT} \Rightarrow t = t_o\, e^{(E_a/nkT)} = MTTF$$

ii. Since the $MTTF_{80}$ of the IC at 100°C (353°K) is 30000 hours, therefore,

$$MTTF_{80} = t_o\, e^{(E_a/nkT)} \Rightarrow t_o = MTTF_{100}\, e^{(-E_a/nkT)}$$
$$= 30000 \times e^{\left(-0.7/8.6\times10^{-5}\times353\right)} = 2.9\times10^{-6} \text{ hours}$$

therefore the $MTTF_{30}$ of the IC at 30°C (303°K) is given by:

$$MTTF_{30} = t_o\, e^{(E_a/nkT)} = 2.9\times10^{-6}\, e^{\left(0.7/8.6\times10^{-5}\times303\right)} = 1.34\times10^{6} \text{ hours}$$

(c)

i. If option 1 is adopted only one generator is responsible for delivering the maximum power of 30 kW. Therefore, the reliability function is given by:

$$R(t) = e^{(-\lambda t)}$$

and the failure distribution is given by,

$$F(t) = 1 - R(t) = 1 - e^{(-\lambda t)}$$

and the probability of losing the capability of generating the maximum power is:

$$F(t = 10\,\text{hours}) = 1 - R(t) = 1 - e^{\left(-1.0\times10^{-4}\times10\right)} = 1.0\times10^{-3}$$

If option 2 is adopted two identical generators are required to generate the maximum power of 30 kW, and the failure of one generator would result in a loss of the capability of generating the maximum power of 30 kW. Therefore, the system is a series system with a failure rate of $2\times\lambda$. The probability of losing the capability of generating the maximum power is then given by:

$$F(t = 10\,\text{hours}) = 1 - e^{(-2\lambda t)} = 1 - e^{\left(-2\times1.0\times10^{-4}\times10\right)} = 2.0\times10^{-3}$$

If option 3 is adopted three identical generators are required to generate the maximum power of 30 kW, and the failure of one generator would result in a loss of the capability of generating the maximum power of 30 kW. Therefore, the system is a series system with a failure rate of $3\times\lambda$. The probability of losing the capability of generating the maximum power is then given by:

$$F(t = 10\,\text{hours}) = 1 - e^{(-3\lambda t)} = 1 - e^{\left(-3\times1.0\times10^{-4}\times10\right)} = 3.0\times10^{-3}$$

ii. If option 2 is adopted two identical generators are required to generate the maximum power of 20 kW, and the failure of one generator would result in a loss of the capability of generating the maximum power of 20 kW. Therefore, the system is a series system with a failure rate of $2\times\lambda$. The probability of losing the capability of generating the maximum power is then given by:

$$F(t = 10\,\text{hours}) = 1 - e^{(-2\lambda t)} = 1 - e^{\left(-2\times1.0\times10^{-4}\times10\right)} = 2.0\times10^{-3}$$

If option 3 is adopted, only two generators will be required to generate the electrical power of 20 kW and a failure of one generator can be tolerated. Therefore, the system is an *m*-out-of-*n* system with *m*=2 and *n=3*. Since, the generators are sharing the power generation at all times (active system), therefore, the reliability function will be given by:

$$R(t) = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!} \left[e^{-\lambda k t}\right]\left[1 - e^{-\lambda t}\right]^{n-k}$$

where *m*=2 and *n*=3, therefore,

$$R(t) = \sum_{k=2}^{3} \frac{3!}{k!(3-k)!} \left[e^{-\lambda k t}\right]\left[1 - e^{-\lambda t}\right]^{3-k}$$

$$\Rightarrow R(t) = 3e^{(-2\lambda t)}\left(1 - e^{(-\lambda t)}\right)^{3-2} \qquad (k = 2)$$

$$+ e^{(-3\lambda t)}\left(1 - e^{(-\lambda t)}\right)^{3-3} \qquad (k = 3)$$

$$\Rightarrow R(t) = 3e^{(-2\lambda t)} - 2e^{(-3\lambda t)}$$

therefore, the probability of failure is given by:

$$F(t) = 1 - R(t) = 1 - 3e^{(-2\lambda t)} + 2e^{(-3\lambda t)}$$

and for a 10-hour flight:

$$F(t = 10\,\text{hours}) = 1 - 3e^{\left(-2\times1.0\times10^{-4}\times10\right)} + 2e^{\left(-3\times1.0\times10^{-4}\times10\right)} = 3.0\times10^{-6}$$

iii. The MTTF is given by:

$$MTTF = \int_0^{\infty} R(t)dt = \int_0^{\infty}\left(3e^{(-2\lambda t)} - 2e^{(-3\lambda t)}\right)dt$$

$$= \frac{-3}{2\lambda}\left[e^{(-2\lambda t)}\right]_0^{\infty} - \frac{-2}{3\lambda}\left[e^{(-3\lambda t)}\right]_0^{\infty}$$

$$= \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{9}{6\lambda} - \frac{4}{6\lambda} = \frac{5}{6\lambda}$$

Question 3

(a)

i. The armature current and the torque are related by:

$$T_m = k\,I \quad \Rightarrow \quad I = \frac{T_m}{k} = \frac{15.0}{0.16} = 93.75\,\mathrm{A}$$

and the copper loss of the motor:

$$P_c = R \times I^2 = 20 \times 10^{-3} \times 93.75^2 = 175.8\ \mathrm{W}$$

ii. When the copper loss is considered, the efficiency of the motor is give by:

$$\eta = \frac{T_m\,\Omega_m}{T_m\,\Omega_m + R\,I^2} = \frac{T_m\,\Omega_m}{T_m\,\Omega_m + R\left(\frac{T_m}{k}\right)^2} = \frac{T_m\,\Omega_m}{T_m\,\Omega_m + \frac{R}{k^2}T_m^2}$$

$$\Rightarrow \eta = \frac{1}{1 + \frac{R}{k^2}\frac{T_m}{\Omega_m}}$$

iii. When copper loss and friction torque are considered, the efficiency of the motor is given by:

$$\eta = \frac{\left(T_m\,\Omega_m - T_f\,\Omega_m\right)}{\left(T_m\,\Omega_m - T_f\,\Omega_m\right) + R\,I^2} = \frac{\left(T_m\,\Omega_m - T_f\,\Omega_m\right)}{\left(T_m\,\Omega_m - T_f\,\Omega_m\right) + R\left(\frac{T_m}{k}\right)^2}$$

$$= \frac{\left(T_m\,\Omega_m - T_f\,\Omega_m\right)}{\left(T_m\,\Omega_m - T_f\,\Omega_m\right) + \frac{R}{k^2}T_m^2}$$

$$\Rightarrow \eta = \frac{\left(1 - \frac{T_f}{T_m}\right)}{\left(1 - \frac{T_f}{T_m}\right) + \frac{R}{k^2}\frac{T_m}{\Omega_m}}$$

(b)

i. The load inertia referred to the shaft is given by:

$$J_r = \left(\frac{\lambda}{2\pi}\right)^2 \times \frac{m}{n^2} = \left(\frac{10\times10^{-3}}{2\pi}\right)^2 \times \frac{35000}{10^2} = 8.86\times10^{-4}\,\mathrm{kg.m^2}$$

ii. The time T required to travel a certain distance following a parabolic velocity profile and the maximum torque delivered by the motor are related by:

$$T_p = 6\left(J_m + J_r\right)\frac{\theta_m}{T^2}$$

therefore,

$$T = \sqrt{6\left(J_m + J_r\right)\theta_m / T_p}$$

where,

$$\theta_m = \frac{2\pi n}{\lambda} x_m = \frac{2\pi\times10}{10\times10^{-3}}\times25\times10^{-3} = 157.05 \text{ radians}$$

therefore,

$$T = \sqrt{6\left(J_m + J_r\right)\theta_m / T_p} = \sqrt{6\times\left(1.5\times10^{-3} + 8.86\times10^{-4}\right)\times157.05/20} = 0.332\,\mathrm{s}$$

iii. The time T required to travel a certain distance following a trapezoidal velocity profile and the maximum torque delivered by the motor are related by:

$$T_p = \left(J_m + J_r\right)\frac{\theta_m}{\tau\left(T-\tau\right)}$$

for a triangular velocity profile $\tau = \frac{T}{2}$, therefore,

$$T_p = 4\left(J_m + J_r\right)\theta_m / T^2$$

$$\Rightarrow T = \sqrt{4\left(J_m + J_r\right)\theta_m / T_p} = \sqrt{4\times\left(1.5\times10^{-3} + 8.86\times10^{-4}\right)\times157.05/20} = 0.274\,\mathrm{s}$$

iv. When following a triangular velocity profile, the magnitude of the torque is constant, being positive in the time interval [0, T/2], and negative in the time interval [T/2, T]. Therefore, the copper loss is given by:

$$P_c = R\,I^2 = R\left(\frac{T_p}{k}\right)^2 = 20\times10^{-3}\times\left(\frac{20}{0.16}\right)^2 = 312.5\ \text{W}$$

And the energy is given by:

$$E_c = P_c \times T = 312.5\times0.247 = 77.18\,\text{Joules}$$

<u>Question 4</u>

(a)

i. the error of the potentiometer is given by:

$$Error(\%) = 100\times\frac{V_o\left(R_p=\infty\right)-V_o\left(R_p\neq\infty\right)}{V_o\left(R_p=\infty\right)}$$

$$= 100\times\frac{aV_s - \dfrac{a}{\left(1+a\left(1-a\right)\dfrac{R_p}{R_m}\right)}V_s}{aV_s}$$

$$= 100\times\left(1-\frac{1}{\left(1+a\left(1-a\right)\dfrac{R_p}{R_m}\right)}\right) = 100\times\left(\frac{\left(1+a\left(1-a\right)\dfrac{R_p}{R_m}-1\right)}{\left(1+a\left(1-a\right)\dfrac{R_p}{R_m}\right)}\right)$$

$$= 100\times\frac{a\left(1-a\right)\dfrac{R_p}{R_m}}{1+a\left(1-a\right)\dfrac{R_p}{R_m}}$$

For a potentiometer loaded by a resistance R_m, the maximum error occurs when $a=\theta/\theta_{\max}$ =0.5, therefore,

$$Error\,(\%) = \frac{a\left(1-a\right)\dfrac{R_p}{R_m}}{1+a\left(1-a\right)\dfrac{R_p}{R_m}}\times100 = \frac{0.5\times\left(1-0.5\right)\times\dfrac{10000}{100000}}{1+0.5\times\left(1-0.5\right)\dfrac{10000}{100000}}\times100 = 2.44\,\%$$

ii. To ensure that the position waveform is well-represented by the ADC output, the sampling frequency of the ADC should be at least 10 times larger than the frequency of the highest significant harmonic of the periodic position profile, $\frac{\theta_0}{5}\sin(6\pi t)$.

$$f_s \geq 10 \times \frac{6}{2\times\pi} = 9.55 \text{ Hz}$$

(b)

i. The truth table of the logic circuit is as follows:

Position (degrees)	ADC output				LEDs		
	b_3	b_2	b_1	b_0	Red	Yellow	Green
0	0	0	0	0	1	0	0
10	0	0	0	1	0	1	0
20	0	0	1	0	0	1	0
30	0	0	1	1	0	1	0
40	0	1	0	0	0	1	0
50	0	1	0	1	0	1	0
60	0	1	1	0	0	1	1
70	0	1	1	1	0	0	1
80	1	0	0	0	0	0	1
90	1	0	0	1	0	1	1
100	1	0	1	0	0	1	0
110	1	0	1	1	0	1	0
120	1	1	0	0	0	1	0
130	1	1	0	1	0	1	0
140	1	1	1	0	0	1	0
150	1	1	1	1	1	0	0

ii. The SOP expressions for the logic functions Green and Yellow:

$$\text{Green} = \overline{b_3}\, b_2\, b_1\, \overline{b_0} + \overline{b_3}\, b_2\, b_1\, b_0 + b_3\, \overline{b_2}\, \overline{b_1}\, \overline{b_0} + b_3\, \overline{b_2}\, \overline{b_1}\, b_0$$

$$\text{Red} = \bar{b}_3\,\bar{b}_2\,\bar{b}_1\,\bar{b}_0 + b_3\,b_2\,b_1\,b_0$$

$$\text{Green} = \bar{b}_3\,b_2\,b_1\,\bar{b}_0 + \bar{b}_3\,b_2\,b_1\,b_0 + b_3\,\bar{b}_2\,\bar{b}_1\,\bar{b}_0 + b_3\,\bar{b}_2\,\bar{b}_1\,b_0$$
$$= \bar{b}_3\,b_2\,b_1\left(\bar{b}_0 + b_0\right) + b_3\,\bar{b}_2\,\bar{b}_1\left(\bar{b}_0 + b_0\right) = \bar{b}_3\,b_2\,b_1 + b_3\,\bar{b}_2\,\bar{b}_1$$

iii. The POS expression for the logic function Yellow:

$$\text{Yellow} = \left(b_3 + b_2 + b_1 + b_0\right)\left(b_3 + \bar{b}_2 + \bar{b}_1 + \bar{b}_0\right)\left(\bar{b}_3 + b_2 + b_1 + b_0\right)\left(\bar{b}_3 + \bar{b}_2 + \bar{b}_1 + \bar{b}_0\right)$$