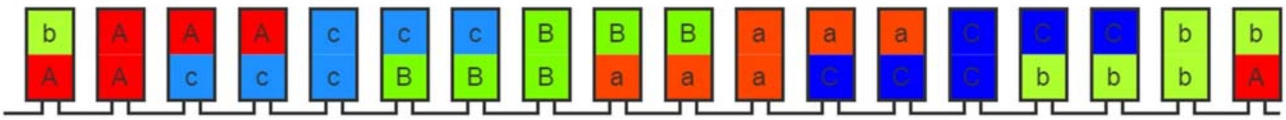
EEE6200 14-15 Solutions

[Commentary on the solutions is provided in italics in parentheses]

1.

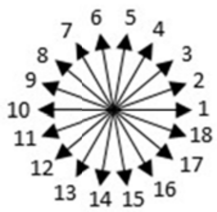
a) The slot diagram should be as follows (although there is some freedom on how candidates denote the orientation of individual coil sides in individual slots)



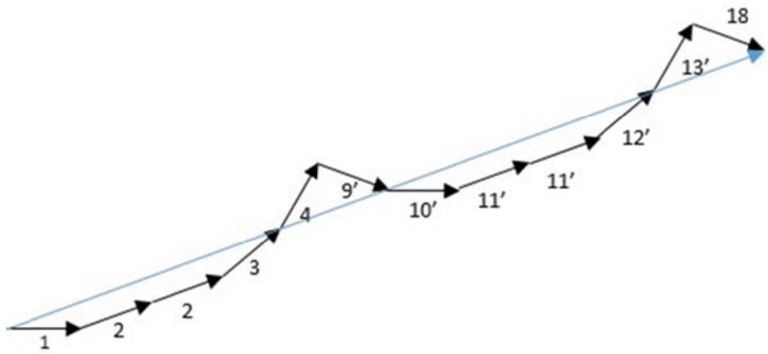
[Naturally variations on starting point are fine providing the sequence is correct. The key point is the 2 slot offset between layers]

(4)

b) There are 18 phasors which make up a full circle



[No need to draw this –simply a useful aid should candidates choose to do so]



[Important to draw to scale to ensure process is understood]

(5)

c.

[Key point here is to recognise the overall winding factor is the product of the distribution factor and the pitching factor and that the latter is at play here]

Taking the distribution factor first, then for the fundamental:

$$k_{d1} = \frac{sin\left(\frac{n\alpha}{2}\right)}{n\,sin\left(\frac{\alpha}{2}\right)}$$

For this winding: n=3 and $\alpha = \frac{\pi}{9}$

(Note n=3 is the number of slots per pole per phase– the short pitching comes in with the pitch factor and so the distribution factor is the same as an 18 slot, 2 pole machine with a fully pitched coil)

$$k_{d1} = \frac{sin\left(\frac{n\alpha}{2}\right)}{n\ sin\left(\frac{\alpha}{2}\right)} = \frac{\sin\left(\frac{3\pi}{18}\right)}{3\sin\left(\frac{\pi}{18}\right)} = 0.960$$

For the 5th harmonic, the distribution factor is given by:

$$k_{d5} = \frac{sin\left(\frac{n5\alpha}{2}\right)}{n\ sin\left(\frac{5\alpha}{2}\right)} = \frac{sin\left(\frac{15\pi}{18}\right)}{3\ sin\left(\frac{5\pi}{18}\right)} = 0.218$$

The pitch factor for the fundamental is given by:

$$k_{p1} = sin\left(\frac{y_s}{y_f}\frac{\pi}{2}\right) = \sin\left(\frac{7}{9}\times\frac{\pi}{2}\right) = 0.940$$

The coil pitch factor for the 5th harmonic is given by:

$$k_{p5} = sin\left(\frac{5y_s}{y_f}\frac{\pi}{2}\right) = \sin\left(\frac{35}{9}\times\frac{\pi}{2}\right) = -\ 0.174$$

The overall winding factor for the fundamental and 5th harmonic are hence given by:

$$k_{w1} = 0.960 \times 0.940 = 0.902$$

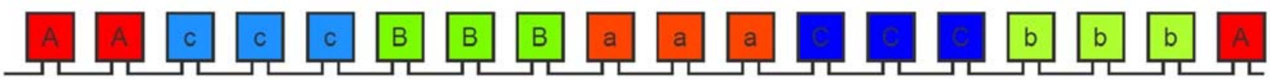
$$k_{w5} = 0.218 \times (-0.174) = -0.038$$

The length of the phase drawn in part (b) should be ~10.8cm whereas the algebraic sum of the lengths is simply 12cm. Hence the winding factor derived from the phasor diagram is 10.8/12=0.90 which is reasonable agreement with the value calculated given the accumulation of minor tolerances in the drawing.

(5)

d)

The slot diagram for the single-layer winding can only be fully pitched and hence the slot diagram is:



(3)

e) [In this case, there is no need to account for the pitching factor and hence the overall winding factor is the same as the distribution factors calculated in part (c) – spotting this rather than recalculating is commendable and will be awarded full marks]

$$k_{w1} = 0.960$$

$$k_{w5} = 0.21$$

The key points to note in terms of machine performance are:

- The slightly higher fundamental factor will yield some dividend in terms of the torque per amp of stator current – but marginal
- The much increased 5th harmonic winding factor will yield harmonics in the induced emf and hence increase rotor induced losses and stator winding losses.

(3)

2.

[This question involves something of a change around from the normal means by which questions of this type have been posed in that the reactance and the short-circuit current has been given and not the open-circuit characteristic. Understanding that the information provided gives the route to the open-circuit emf versus field excitation current required elsewhere in the question is the key insight]

a) The relationship between the open-circuit emf and the excitation current can be established from the short-circuit at say I_f=40A (any value is fine). At this value:

$$E_{ph} = X_s I_{sc} = 5.5 \times 750 = 4125\ Vrms\ at\ I_f = 40A$$

Hence, the phase open circuit emf per unit of excitation current is 103Vrms per A

The machine is star-connected, and the phase voltage is given by:

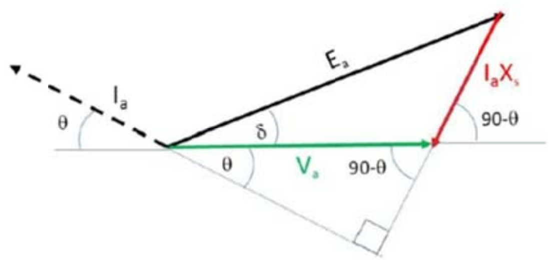
$$V_{ph} = \frac{V_{line-line}}{\sqrt{3}} = \frac{3{,}300}{\sqrt{3}} = 1{,}905V$$

$$P_{ph} = V_{ph} I_{ph} cos\emptyset$$

Adopting the convention that generating power is negative, then:

$$I_{ph} = \frac{P_{ph}}{V_{ph} cos\emptyset} = \frac{-400 \times 10^3}{1905 \times 0.92} = -228A$$

From phasor diagram:



$$E_{ph} \cos\delta = V_{ph} - I_{ph} X_s\ sin\theta$$

$$E_{ph} \sin\delta = -I_{ph} X_s \cos\theta$$

$$\tan\delta = \frac{-I_{ph} X_s \cos\theta}{V_{ph} - I_{ph} X_s\ sin\theta} = \frac{-(-228) \times 5.5 \times 0.92}{1905 - (-228) \times 5.5 \times (0.392)} = 0.481$$

Hence, the load angle $\delta = 25.7°$

Hence,

$$E_{ph} = \frac{I_{ph} X_s \cos\theta}{\sin\delta} = \frac{228 \times 5.5 \times 0.92}{sin(25.7^{\circ})} = \ 2660\, Vrms$$

Using the previously calculated excitation constant this requires a field current of:

$$I_f = \frac{2660}{103} = 25.8A$$

(7- 2 for I_{ph}, 3 for E_{ph} and 2 for I_f)

b) The following assumptions need to be made in order to solve the problem:

- This load change is effectively a step-change in comparison
- The AVR response time is much longer than the transient and hence the excitation remains fixed
- The damping is negligible

At this level of excitation, the variation in the peak power with δ is:

$$P = 3\frac{V_{ph}E_{ph}}{X_s} sin\delta = \ 3 \times \frac{1905 \times 2660}{5.5} sin\delta = 2.77 \times 10^6 sin\delta$$

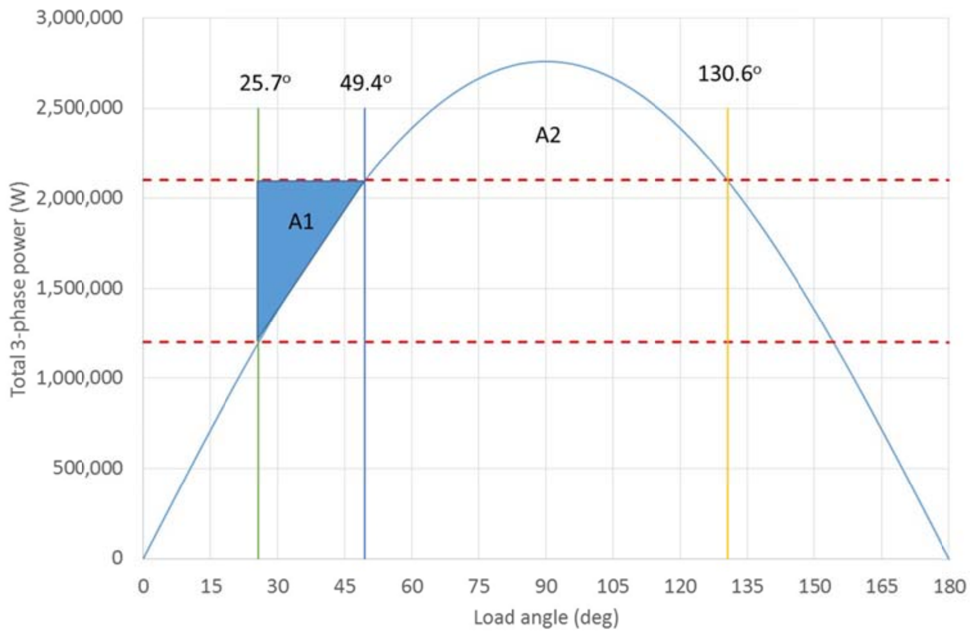
The load angle at the new power of 3.4MW is:

$$sin\delta = \ \frac{X_s P}{3\, V_{ph} E_{ph}} = \frac{5.5 \times 2.1 \times 10^6}{3 \times 1905 \times 2660} = 0.759$$

$$\delta = 49.4°$$

Consider a schematic of the load characteristic for this machine

[No need for candidates to draw this]



The area A_1 corresponds to the deceleration energy (i.e. as the load angle changes from 25.7° (0.449 rads) to 49.4° (0.862 rad) can be established:

$$A_1 = \left(2.1 \times 10^6 \times (0.862 - 0.449)\right) - \left(2.77 \times 10^6 \int_{0.449}^{0.862} \sin\delta\, d\delta\right)$$

$$A_1 = 8.69 \times 10^5 - \left(2.77 \times 10^6 \left[-\cos\delta\ \right]_{0.449}^{0.862}\right)$$

$$A_1 = 1.77 \times 10^5\ Wrad$$

The generator power remains higher than the load power (and hence accelerates the rotor back towards the new steady-state load angle) for angles between 49.4° and maximum excursion of the load angle before the accelerating power changes sign is (180-49.4°) = 130.6°.

Stability will be achieved if the area below the curve from 49.4° (0.862 rad) to 130.6° (2.28 rad) is greater than A1:

$$A_2 = 2.77 \times 10^6 \int_{0.862}^{2.28} sin\delta\, d\delta - 2.1 \times 10^6 \times (2.28 - 0.862)$$

$$A_2 = 2.77 \times 10^6 (-\cos 2.28 + \cos 0.862) - 2.\times 10^6$$

$$A_2 = 6.27 \times 10^5 Wrad$$

Since $A_2 > A_1$, then the system is stable

(5-1 for assumptions)

c) The equal area criteria can be applied to establish the excursion of the angle (assuming that there is no damping)

$$A_1 remains\ the\ same\ at\ 1.77 \times 10^5\ Wrad$$

The accelerating area A_2 is given by:

$$A_2 = 2.77 \times 10^6 \int_{0.862}^{\delta_2} sin\delta\, d\delta - 2.1 \times 10^6 \times (\delta_2 - 0.862)$$

Performing the integration yields:

$$A_2 = 2.77 \times 10^6 \left(cos(0.862) - cos(\delta_2)\right) - 2.1 \times 10^6 \times (\delta_2 - 0.862)$$

[There is no closed form solution for this equation (something the students are familiar with) and hence an iterative approach must be adopted to solve for δ_2]

75° seems a reasonable starting estimate (although clearly any value in terms of getting the iterative process moving)

At 75° (1.309 rad)

$$A_2 = 2.77 \times 10^6 \left(cos(0.862) - cos(1.309)\right) - 2.1 \times 10^6 \times (1.309 - 0.862) = 1.47 \times 10^5\ Wrad$$

Need to increase slightly. Try 78° (1.361):

$$A_2 = 2.77 \times 10^6 \left(cos(0.862) - cos(1.361)\right) - 2.1 \times 10^6 \times (1.361 - 0.862) = 1.78 \times 10^5\ Wrad$$

Which is almost equal to A1 and it is reasonable to stop the iterative process at this step *[values of 77° and 79° would be accepted]*

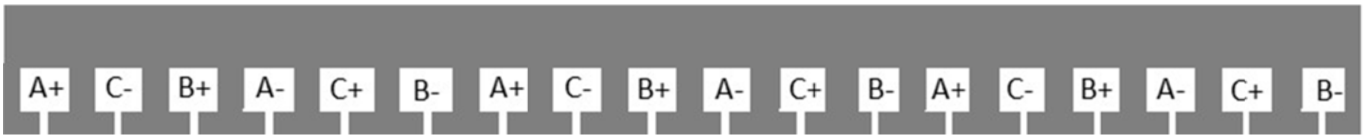
(5)

d) The most common features which are included to damp oscillations are damper windings or damper bars. When the rotor is in steady state, there is negligible current flowing in these components and hence no loss. However, when the rotor moves relative to the stator field an emf is induced in these damper windings/bars which causes a current to flow. The resulting energy loss removes a proportion of the accelerating and decelerating energy on every oscillation, causing the oscillation to rapidly dampen down.

(3)

3.

a) [*There is only one option for this integral slot winding – starting point is not critical providing sequence is correct*]



(3)

b) In order to calculate the airgap component of flux, it is necessary to incorporate the influence of the slot opening by way of Carter coefficients.

The stator pole pitch is given by:

$$\tau_{us} = \frac{\pi \times 0.25}{18} = 43.6mm$$

The rotor pole pitch is given by:

$$\tau_{ur} = \frac{\pi \times 0.248}{6} = 130.0mm$$

The stator slot opening is given by:

$$w_{ss} = \frac{\pi \times 0.25 \times (1 - 0.85)}{18} = 6.54mm$$

And similarly for the rotor slot

$$w_{rs} = \frac{\pi \times 0.248 \times (1 - 0.92)}{6} = 10.4mm$$

The same procedure can be used to calculate the stator and rotor tooth widths:

$$w_{st} = \frac{\pi \times 0.25 \times 0.85}{18} = 37.1mm$$

$$w_{rt} = \frac{\pi \times 0.248 \times 0.92}{6} = 119.5mm$$

The Carter coefficient which accounts for stator slotting can now be calculated:

For the stator:

Slot width = b_1 = 6.5mm and unmodified airgap = δ = 1mm:

$$\kappa = \frac{\frac{b_1}{\delta}}{5 + \frac{b_1}{\delta}} = \frac{\frac{0.0065}{0.001}}{5 + \frac{0.0065}{0.001}} = 0.567$$

$$k_{cs} = \frac{\tau_{us}}{\tau_{us} - \kappa b_1} = \frac{0.0436}{0.0436 - 0.567 \times 0.0065} = 1.09$$

And similarly for the rotor:

Slot width = b_1 = 10.4mm and unmodified airgap = δ = 1mm:

$$\kappa = \frac{\frac{b_1}{\delta}}{5 + \frac{b_1}{\delta}} = \frac{\frac{0.0104}{0.001}}{5 + \frac{0.0104}{0.001}} = 0.675$$

$$k_{cr} = \frac{\tau_{ur}}{\tau_{ur} - \kappa b_1} = \frac{0.13}{0.13 - 0.675 \times 0.0104} = 1.06$$

Hence, the effective magnetic airgap is given by:

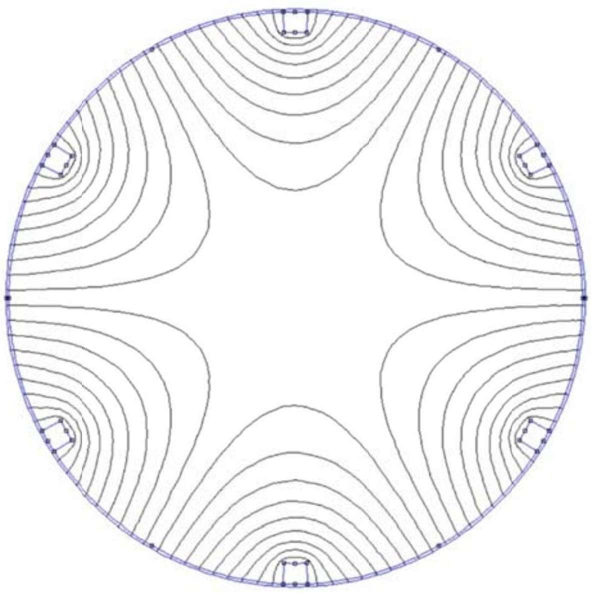
$$l_g' = l_g k_{cs} k_{cr} = 1 \times 1.09 \times 1.06 = 1.16mm$$

(5)

c) Applying Ampere's Law around each rotor slot, The flux density produced in this modified airgap with 1500 A.turns per slot of the stator is given by:

$$B_g = \frac{\mu_0 NI}{2l_g} = \frac{4\pi \times 10^{-7} \times 1500}{2.32 \times 10^{-3}} = 0.812T$$

[As an aside – and something that is clearly not expected of the candidates, this can be checked by FE analysis. The field distribution is shown below and yields an average flux density of 0.80T]



(3)

d) The peak flux linkage can be established by integrating the airgap flux density produced by the field winding over one coil pitch for a rotor angular orientation in which the entire stator coil is subjected to only one rotor pole. Hence:

$$\Psi_{max} = \frac{100 \times 3 \times 0.812 \times \pi \times 0.25 \times 0.3}{6} = 9.56\, Wb.turns$$

(factor of 3 is to account for 3 coils per phase)

[Note: Need to include N to get full marks as the question asks for flux-linkage not flux]

(3)

e) In order to calculate the stator phase inductance, it is necessary to calculate the airgap flux density using a defined value of current in the series connected conductors. Any value can be used to calculate inductance, but 1A in the 100 conductors per slot is as good as any.

The main airgap flux density produced by the coil at each pole is given by:

$$B_g = \frac{\mu_0 NI}{2l_g} = \frac{4\pi \times 10^{-7} \times 100}{2.32 \times 10^{-3}} = 54.2mT$$

(note flux needs to cross 2 airgaps; also note that the expression is where the number of turns come in)

The resulting flux-linkage can be calculated by integrating across 1 coil pitch (i.e. 1/6 of stator periphery) and along the length of the machine. The integration is trivial since the airgap flux density is constant and hence:

$$\phi_c = B_g \frac{\pi \times 0.25}{6} \times 0.3 = 2.13mWb$$

Hence the airgap flux component of the inductance for one coil is given by:

$$L_{coil} = \frac{N\phi_c}{I} = \frac{100 \times 1.033 \times 10^{-3}}{1} = 0.213\ H$$

But since this is a 6 pole machine with 3 coils per phase, the overall airgap flux component of inductance is:

$$L_{phase} = 3 \times L_{coil} = 0.639H$$

(4)

f) Other components of flux which could be added in:

- Cross-slot leakage flux
- Leakage flux between tooth tips (which could be included in cross-slot leakage)
- Airgap inter-pole leakage
- End-winding flux

(2)

4.

a) The torque produced by a synchronous reluctance machine is given by:

$$T = \frac{3}{2}p\left(\psi_d i_q - \psi_q i_d\right)$$

Where

$$\psi_d = L_d i_d$$

$$\psi_q = L_q i_q$$

Hence:

$$T = \frac{3}{2}p\left(L_d i_d i_q - L_q i_q i_d\right)$$

$$T = \frac{3}{2}p\ i_d i_q\left(L_d - L_q\right)$$

but $i_d = i_s cos\delta$ and

$$i_q = i_s sin\delta$$

Hence,

$$i_d i_q = i_s^2 cos\delta\ sin\delta = \frac{1}{2} i_s^2 sin(2\delta)$$

Substituting into the torque expression for $i_d i_q$ yields:

$$T = \frac{3}{4}p\left(L_d - L_q\right) i_s^2 sin(2\delta)$$

This has a maximum value when $sin(2\delta) = 1$ and hence when $\delta = 45^o$

(5)

b) For the operating condition specified, it is necessary to start by establishing i_d and i_q

$$i_d = i_s cos\delta = 15 \times cos(30°) = 13.0A$$

$$i_q = i_s sin\delta = 15 \times sin(30°) = 7.5A$$

From Figure 4.1, $L_d = 0.043H$

[Any reasonable estimate of this value from the graph is fine]

Hence, substituting for the various values in the torque equation yields:

$$T = \frac{3}{4}p\left(L_d - L_q\right) i_s^2 sin(2\delta) = \frac{3}{4} \times 2 \times (0.043 - 0.028) \times 15^2 \times sin(60°) = 4.38Nm$$

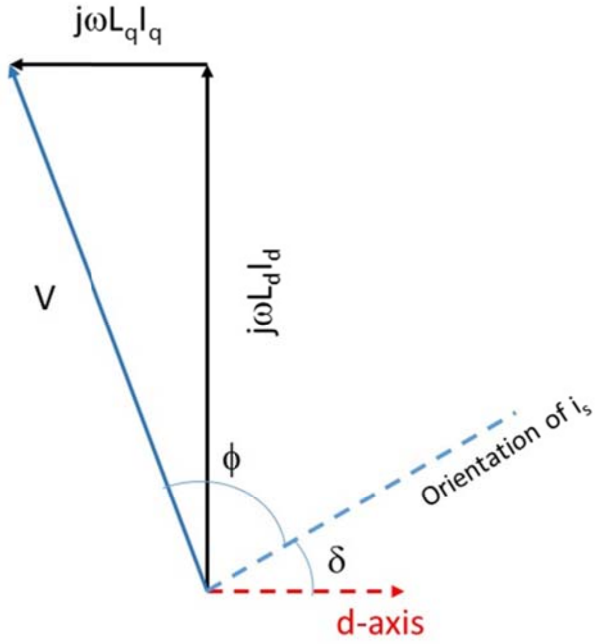
(5)

c) At 1500rpm, the electrical frequency is 50Hz and hence:

$j\omega L_d I_d = j2\pi \times 50 \times 0.043 \times 13 = 176\,Vrms$

$j\omega L_q I_q = j2\pi \times 50 \times 0.028 \times 7.5 = 66.0Vrms$

Take 20V per cm as a scale for the phasor diagram (or 10V:cm would also be fine) leads to the following phasor diagram



[Important to draw to reasonable scale- not least since it provides a useful check on the next set of calculations]

(4)

d) The magnitude of the voltage can be established from:

$$V = \sqrt{(\omega L_d I_d)^2 + \left(\omega L_q I_q\right)^2} = 188V$$

Which can be checked against the scaled phasor diagram

The power factor is given by:

$$Power\ factor = \cos\theta = \frac{\left(\frac{L_d}{L_q} - 1\right)}{\left(\frac{L_d}{L_q} + 1\right)}$$

The inductance ratio at this current is 0.043/0.028 =1.53

This leads to a power factor of 0.211 lagging *[Important to indicate lagging]*

[This is poor, which is a common feature of this type of machine and is, in this case, a particular consequence of the low inductance ratio when saturation sets – some acceptance of this and for students not to panic with such a low value is a key test of understanding]

This can also be checked from phasor diagram by measuring angle of 78-80° (cos 78°=0.20)

(3)

e) The maximum value of torque occurs when $\delta = 45°$. Hence for the new value of the net current phase of 10A rms, the d and q axis currents are given by:

$i_d = i_s cos\delta = 10 \times cos(45°) = 7.07A$

$i_q = i_s sin\delta = 10 \times sin(45°) = 7.07A$

From Figure 4.1, $L_d = 0.065H$

The power factor is given by:

$$Power\ factor = \cos\theta = \frac{\left(\frac{L_d}{L_q} - 1\right)}{\left(\frac{L_d}{L_q} + 1\right)}$$

The inductance ratio at this current is 0.065/0.028 =2.32

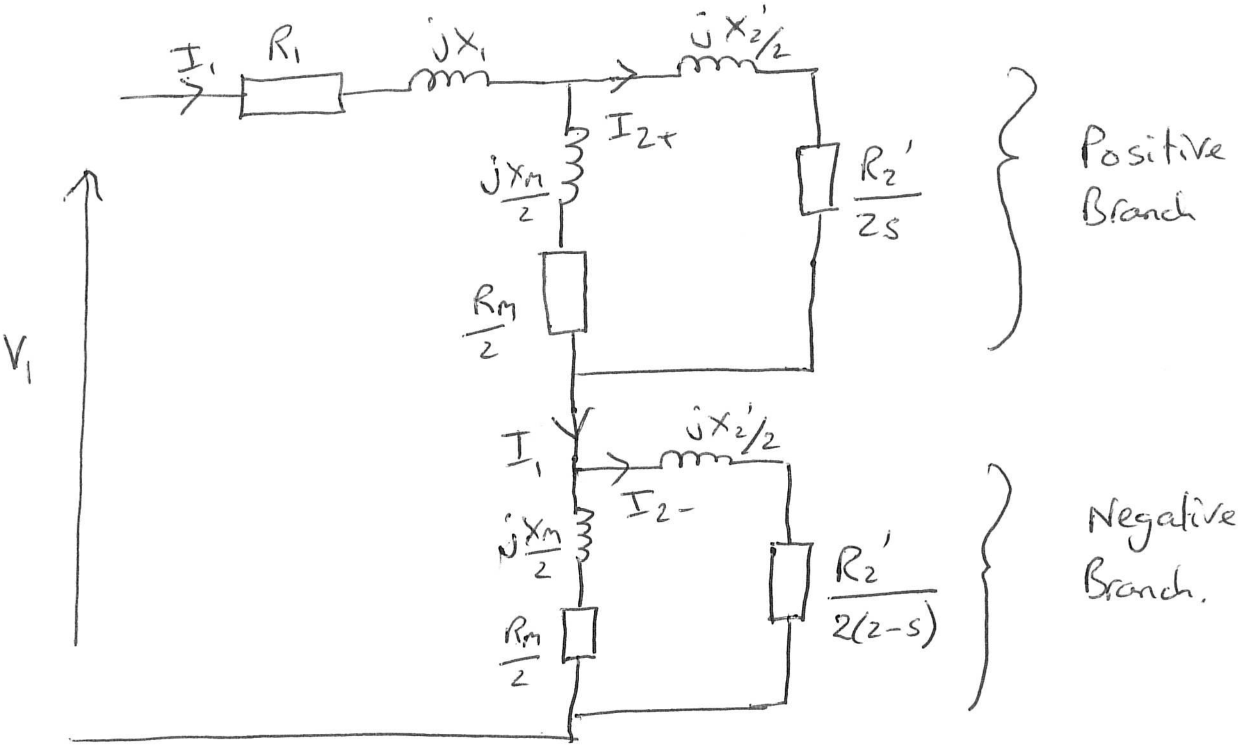
This leads to a power factor of 0.40 lagging *[Important to indicate lagging]*

This improvement is a result of the increased inductance ratio since L_d is not subjected to significant magnetic saturation at 7.07A.

(3)

QUESTION 5

(a)



R_1 = Stator winding resistance

jX_1 = stator leakage reactance

jX_2 = Referred rotor leakage reactance

R_2' = Referred rotor resistance

jX_m = Magnetising reactance

R_m = Core loss resistance

2

(b) For the locked rotor test, $s = 1$, and the referred value value of the rotor impedance will be much smaller than the impedance of the magnetising branch in both the positive and negative circuits:

$$\therefore Z_{LR} = \frac{V_{LR}}{I_{LR}} = R_1 + jX_1 + \frac{R_2'}{2} + j\frac{X_2'}{2} + \frac{R_2'}{2} + j\frac{X_2'}{2}$$

$$= R_1 + R_2' + j(X_1 + X_2')$$

From test data

$$Z_{LR} = \frac{75\angle 0^\circ}{2.1\angle -\cos^{-1} 0.618} = 35.71\angle 51.8^\circ = 22 + j28\ \Omega$$

QUESTION 5 (CONTINUED)

$\therefore R_1 + R_2' = 22\Omega$ but $R_1 = 10\Omega$ (given) hence $\underline{\underline{R_2' = 12\Omega}}$

also $X_1 + X_2' = 28\Omega$, but $X_1 = X_2'$ (given) hence $X_2' = X_1 = \underline{\underline{14\Omega}}$

From the no-load conditions slip $\Rightarrow 0$ and the magnetising impedance of the positive circuit $\left(\frac{R_m}{2} + j\frac{X_m}{2}\right)$ will be much lower than the impedance of the positive rotor branch $\left(\frac{R_2'}{2s} + j\frac{X_2'}{2}\right)$ which tends to an infinite value and hence appears as an open circuit, however for the negative circuit $\left(\frac{R_2'}{2(2-s)} + j\frac{X_2'}{2}\right)$ tends to $\left(\frac{R_2'}{4} + j\frac{X_2'}{2}\right)$ as s tends to zero. This means here the impedance of the negative rotor branch is much smaller than the magnetising branch, and here it is the magnetising branch which is assumed open circuit

$$\therefore Z_{NL} = \frac{V_{NL}}{I_{NL}} = R_1 + jX_1 + \frac{R_m}{2} + j\frac{X_m}{2} + \frac{R_2'}{4} + j\frac{X_2'}{2}$$

$$= \left(R_1 + \frac{R_2'}{4} + \frac{R_m}{2}\right) + j\left(X_1 + \frac{X_2'}{2} + \frac{X_m}{2}\right)$$

Now $Z_{NL} = \dfrac{220\angle 0^\circ}{0.584\angle -\cos^{-1}0.167} = 376.7\angle 80.4^\circ\,\Omega \equiv 62.8 + j371.4\,\Omega$

$\therefore \dfrac{R_m}{2} = 62.8 - \dfrac{R_2'}{4} - R_1 = 62.8 - 3 - 10 \simeq 50\Omega$

$\therefore \underline{\underline{R_m = 100\Omega}}$

5

$\therefore \dfrac{X_m}{2} = 371.4 - \dfrac{X_2'}{2} - X_1 = 371.4 - 7 - 14 \simeq 350\Omega$

$\therefore \underline{\underline{X_m = 700\Omega}}$

QUESTION 5 (CONTINUED)

(c)(i) To calculate the input current first calculate the total impedance at the speed of operation:

$$\text{Slip, } s = \frac{3000-2850}{3000} = 0.05$$

Then $\dfrac{R_2'}{2s} \rightarrow \dfrac{12}{0.1} = 120\,\Omega$

and $\dfrac{R_2'}{2(2-s)} = \dfrac{12}{2(1.95)} = 3.08\,\Omega$

Therefore the impedance of the positive circuit rotor branch is:

$$Z_{2+} = \frac{R_2'}{2s} + j\frac{X_2'}{2} = 120 + j7 = 120.2\angle 3.34^\circ\,\Omega$$

and for the negative circuit rotor branch:

$$Z_{2-} = \frac{R_2'}{2(2-s)} + j\frac{X_2'}{2} = 3.08 + j7 = 7.65\angle 66.25^\circ\,\Omega$$

and the magnetising branches:

$$Z_{M+} = Z_{M-} = 50 + j350 = 353.6\angle 81.9^\circ\,\Omega$$

$$\therefore Z_T = R_1 + jX_1 + \left[(Z_{2+})^{-1} + (Z_{M+})^{-1}\right]^{-1} + \left[(Z_{2-})^{-1} + (Z_{M-})^{-1}\right]^{-1}$$

$$= 10 + j14 + 100.6 + j37.95 + 2.98 + j6.88 = 113.58 + j58.83\,\Omega$$

$$\therefore I_1 = \frac{220\angle 0}{113.58 + j58.83} = \underline{\underline{1.72\angle -27.4^\circ\text{ A}}}$$

4

(ii) To calculate the torque first find current through positive and negative rotor branches:

$$I_{2+} = I_1 \times \frac{Z_{M+}}{Z_{2+} + Z_{M+}} = 1.72\angle -27.4^\circ \times \frac{353.6\angle 81.9^\circ}{(353.6\angle 81.9^\circ + 120.2\angle 3.34^\circ)}$$

$$= 1.54\angle -10.1^\circ\text{ A}$$

QUESTION 5 (CONTINUED)

$$I_{2-} = I_1 \times \frac{Z_{m-}}{Z_{m-}+Z_{2-}} = 1.72\angle -27.4 \times \frac{353.6\angle 81.9^\circ}{(353.6\angle 81.9^\circ + 7.65\angle 66.25^\circ)}$$

$$= 1.68\angle -27.1$$

For a 50Hz machine running at 2850 rpm the rotor speed w_r is

$$w_r = \frac{2\pi f}{p}(1-s) = \frac{2\pi . 50}{1} . 0.95 = 298.5 \text{ rad/s}$$

Forward (positive) torque:

$$T_f = (I_{2+})^2 \frac{R_2'}{2s} \frac{(1-s)}{w_R} = 1.54^2 . 120 . \frac{0.95}{298.5} = 0.906 \text{ Nm}$$

Reverse (negative) torque:

$$T_b = (I_{2-})^2 . \frac{R_2'}{2(2-s)} \frac{(1-s)}{w_R} = 1.68^2 . 3.08 . \frac{0.95}{298.5} = 0.0277 \text{ Nm}$$

Hence nett torque is $0.906 - 0.0277 = \underline{\underline{0.878 \text{ Nm}}}$ 3

(iii) Losses = Input power − output power

$= V_1 . I_1 \cos\phi - T . w_r = 220 . 1.72 . \cos(27.4) - 0.878 . 298.5$

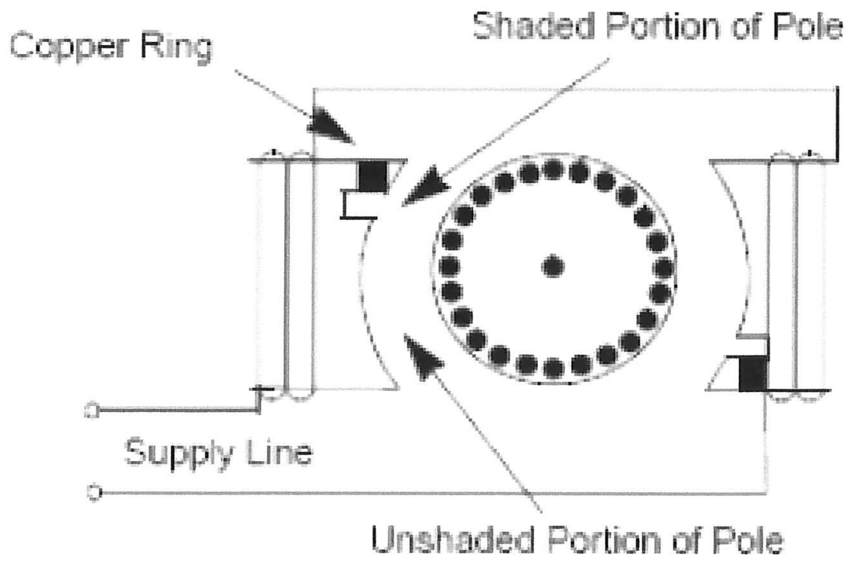
$= \underline{\underline{73.9 W}}$ 1

(iv) Efficiency $= \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{262.1}{335.9} \times 100 = \underline{\underline{78\%}}$ 1

QUESTION 5 (CONTINUED)

Shaded-pole motor (Taken from course notes)

The shaded-pole motor has salient poles carrying the stator winding. One portion of each pole is surrounded by a short-circuited turn of copper called a shading coil or shading ring. Induced currents in the shading ring cause the flux in the shaded portion of the pole to lag in phase behind the flux in the unshaded portion. Hence the stator has in effect two axes of magnetisation with resultant mmfs, which are displaced in phase, acting along these axes. An elliptical rotational field is thus produced.



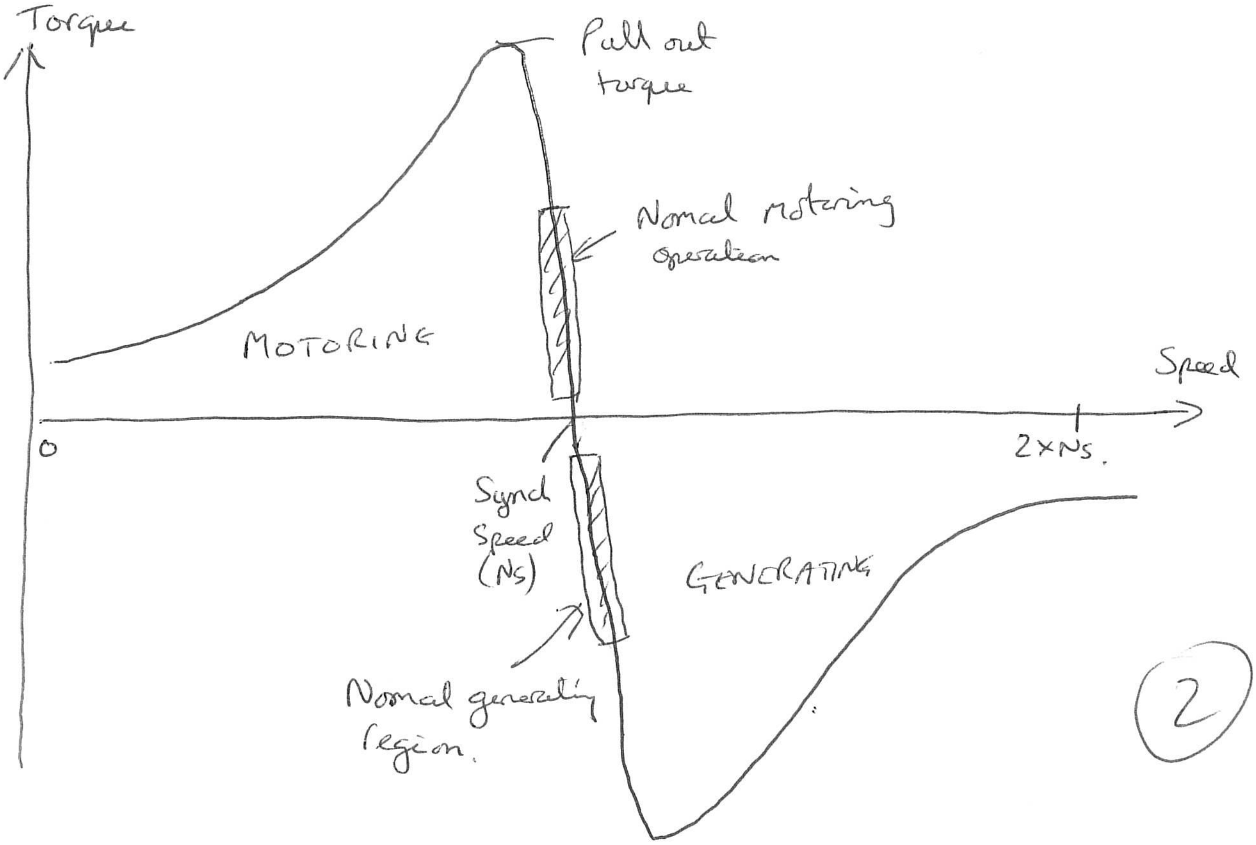
Benefits: (One of the following): Simple construction; Reliable; Cheap.

Drawbacks: (One of the following): Low starting torque; Low efficiency.

4

QUESTION 6

(a) Torque speed characteristic for an induction machine.



(2)

(b) (i) Assume simplified equivalent circuit where magnetising branch r-
moved to terminals of machine.

For star connection $V_{ph} = \frac{V_{Line}}{\sqrt{3}} = \frac{3300}{\sqrt{3}} = 1950\,V$

The magnetising reactance can be found:

$$X_m = \frac{1905\angle 0^\circ}{15.0\angle -90^\circ} = \underline{\underline{127\angle 90^\circ\ \Omega}}$$

(2)

(Note purely reactive so no iron losses).

(ii) For the speed of 960rpm, find the slip.

Synchronous speed $N_s = \frac{60 \times f}{\text{Pole pairs}} = \frac{60 \times 50}{3} = 1000\text{rpm}$

Hence slip $= \frac{N_s - N_R}{N_s} = \frac{1000 - 960}{1000} = 0.04$

QUESTION 6 (CONTINUED)

The output power is given by :-

$$P_{out} = 3\,I_2'^2\,\frac{R_2'(1-s)}{s} = 3\times 163^2 . \frac{R_2'(0.96)}{0.04}$$

But:

$$P_{out} = T.\,\omega_R = 8575\times\frac{960\times 2\pi}{60} = 862.05\,kW$$

Equating the above 2

$$1913\times 10^3 R_2 = 862.05\times 10^3$$

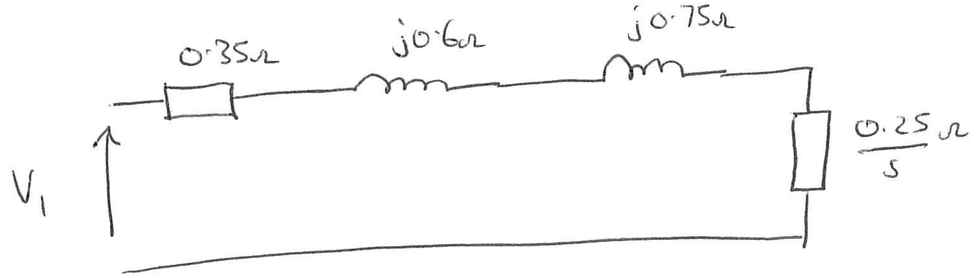
$$\therefore\ \underline{\underline{R_2 = 0.45\,\Omega}}$$

(iii) Losses $= 3I_2'^2(R_1+R_2) = 3\times 163^2\times 0.8 = 63.76\,kW$

Hence efficiency $= \dfrac{P_{out}}{P_{out}+P_{loss}}\times 100 = \dfrac{862.05}{(862.05+63.76)}\times 100\,\%$ 1

$$\underline{\underline{= 93.1\,\%}}$$

(c) With the new rotor the circuit becomes:



The output power is :

$$P_{out} = 3I_2'^2\,\frac{R_2'\cancel{(1-s)}}{s} = T.\,\omega_r = T.\,\omega_s\cancel{(1-s)}$$

But

$$|I_2'| = \frac{V_1}{\sqrt{\left(R_1+\frac{R_2'}{s}\right)^2+\left(X_1+X_2'\right)^2}}$$

QUESTION 6 (CONTINUED)

Substituting for I_2'

$$\frac{3V_1^2}{\left[\left(R_1+\frac{R_2'}{s}\right)^2+\left(X_1+X_2'\right)^2\right]} \cdot \frac{R_2'}{s} = T_1 \omega_s$$

or $$s\left[\left(R_1+\frac{R_2'}{s}\right)^2+\left(X_1+X_2'\right)^2\right] = \frac{3V_1^2 R_2'}{T_1 \omega_s}$$

Substituting values:

$$s\left[\left(0.35+\frac{0.25}{s}\right)^2+1.35^2\right] = \frac{3\times 1905^2 \times 0.25}{8575\times 104.7}$$

$$\therefore\ 0.1225s + 0.175 + \frac{0.0625}{s} + 1.8225s = 3.032$$

Gathering terms and multiplying by s:

$$1.945s^2 - 2.857s + 0.0625 = 0$$

$$s = \frac{2.857 \pm \sqrt{2.857^2 - 4\times 1.945\times 0.0625}}{2\times 1.945}$$

$$= \frac{2.857 \pm 2.771}{3.89} \Rightarrow s = 1.447 \text{ or } \underline{0.0221}$$

(5)

Hence speed is:

$$N_R = 1000(1-0.0221) = \underline{\underline{977.8 \text{ rpm}}}$$

(Some students may in fact realise that $\frac{R_2'}{s}$ has to be constant and new slip can be found that way – much easier!)

(ii) Input phase current:

$$I_2' = \frac{1905\angle 0^\circ}{\left(0.35+\frac{0.25}{0.0221}\right)+j1.35} = \frac{1905\angle 0^\circ}{11.74\angle 6.6^\circ} = \underline{\underline{162.3\angle -6.6^\circ \text{ A}}}$$

(1)

QUESTION 6 (CONTINUED)

(iii) Total losses in the motor:

$$P_{LOSS} = 3 \times I_2'^2 . (R_1 + R_2) = 3 \times 162.3^2 \times 0.6 = \underline{47.41 \text{ kW}}$$

(1)

(iv) $$\text{Efficiency} = \frac{P_{out}}{P_{out} + P_{LOSS}} \times 100\%$$

Now $$P_{out} = 8575 \times 977.8 \times \frac{2\pi}{60} = 878.04 \text{ kW}$$

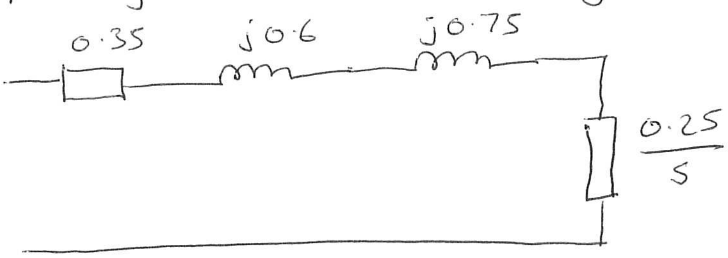
(check: $P_{IN} = \sqrt{3}\, V_L I \cos\phi = \sqrt{3}.3300.162.3 \cos 66° = 924.52 \text{ kW}$)

$P_{out} + P_{LOSS} = 925.45$ kW.

$$\text{Efficiency} = \frac{878.04}{878.04 + 47.41} \times 100 = 94.8\%$$

(1)

(d) When operating as an induction generator:



Slip is:

$$\frac{N_S - N_R}{N_S} = \frac{1000 - 1020}{1000} = -0.02$$

Hence $$\frac{R_2'}{s} = \frac{0.25}{-0.02} = -12.5\,\Omega \text{ (Note negative)}$$

$$\therefore Z_2 = (0.35 - 12.5) + j1.35 = -12.15 + j1.35 = 12.22\angle 173.7°$$

$$\therefore I_2' = \frac{190.5\angle 0°}{12.22\angle 173.7°} = 155.8\angle -173.7°$$

NB.

Now apparent power, $S = 3\, V_{ph} I_{ph}^* = 3 \times 190.5\angle 0° . 155.8\angle +173.7$
$= -885.1 \text{kW} + 97.7 \text{kVAr}$

(Provides $\underline{885.1 \text{ kW}}$ of real power but draws $\underline{97.7 \text{ kVAr}}$ of reactive power)

(5)