EEE224/227 exam solutions (2015-16)

Q1

a

"A stable system is a system in which the output does not diverge when the input to the system is limited." **2 marks**

b **(1 mark each)**

i not stable

ii stable

iii stable

iv not stable

c

(t<0). y(t)=0 **1 mark**

0<t<1

x(τ)=0.5τ

$y(t) = \int_0^t 0.5\tau.(-1)d\tau = -\frac{t^2}{4}$ **2 marks**

1<t<2

$y(t) = \int_{t-1}^1 0.5\tau.(-1)d\tau = \frac{t^2}{4} - 0.5t$ **2 marks**

2<t<3

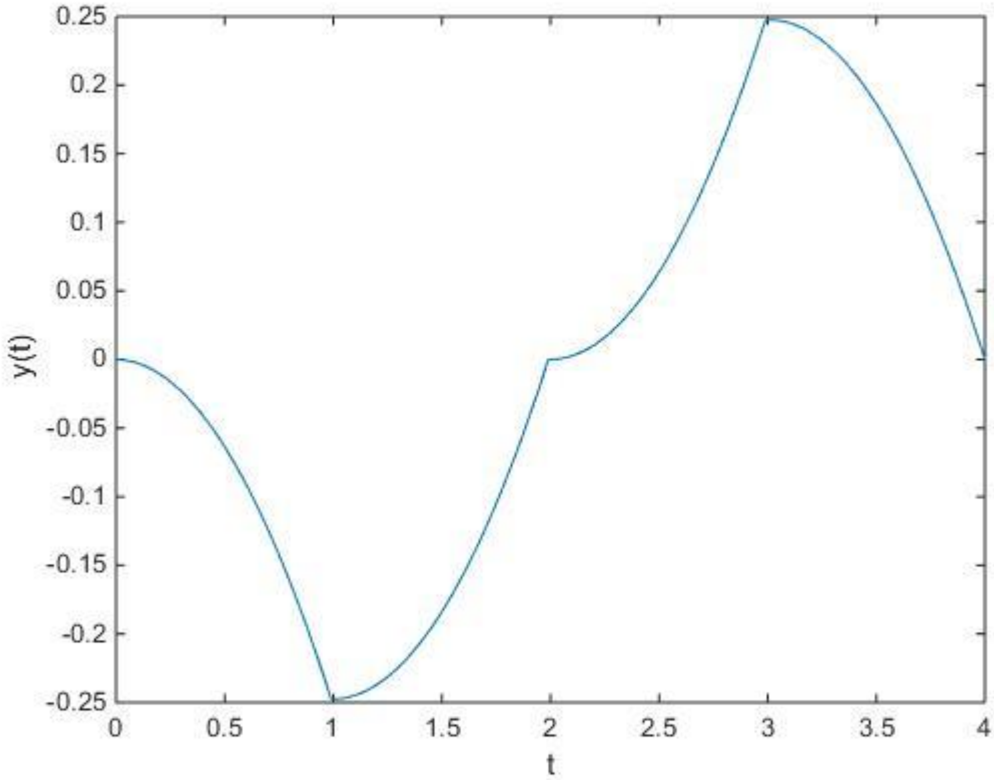
$y(t) = \int_0^{t-2} 0.5\tau.(1)d\tau = \frac{t^2}{4} - t + 1$ **2 marks**

3<t<4

$y(t) = \int_{t-3}^1 0.5\tau.(1)d\tau = \frac{-t^2}{4} + 1.5t - 2$ **2 marks**

t>4: y(t)=0 **1 mark**

d.



Allocate 1 mark for each region on the graph which is correct (including shape and amplitude). If mistakes were made on part c follow these through with no penalty.

Q2

a **1 mark each**

Allows for multiplexing
Efficient antenna size
Improves S/N
Allows users to have different carrier frequencies

b

I average power = $\frac{5^2}{2*50} = 0.25W$ **2 marks**

Ii Modulation index by inspection = 4.4 **2 marks**

Iii max frequency deviation=βf_m=4.4*2M=8.8MHz **2 marks**

Iv From tables $J_0(4.4)$=-0.342, hence power =0.25*$(0.342)^2$=0.029W **2 marks**

c

i

For an FM signal with modulation index of 2.1 the Bessel functions are

$J_0=0.167$

$J1=0.568$

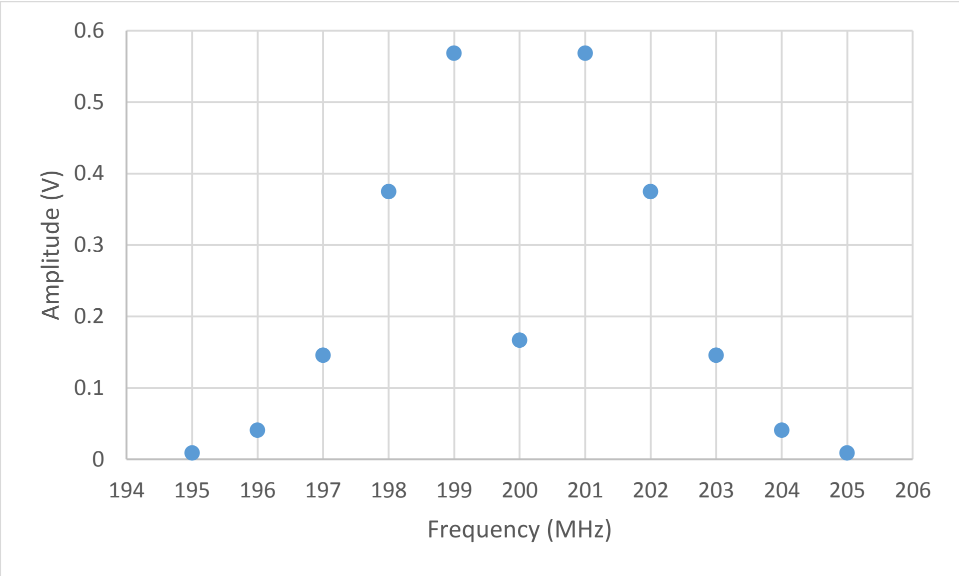
$J2=0.375$

$J3=0.145$

$J4=0.040$

$J5=0.009$

Hence graph would look like below



0.5 marks for correct amplitude

0.5 mark for correct frequency

Subtract half the total marks if both sidebands are not included

ii

Only the first four sidebands have an amplitude greater than 0.01 hence the bandwidth is

B=2*4*1MHz=8MHz **2 marks**

Q3

a

$C = Blog_2\left(1 + \frac{S}{N}\right)$ **1 mark**

B is the bandwidth **1 mark**

S is the signal power **1 mark**

N is the noise power **1 mark**

b i

$C = Blog_2\left(1 + \frac{S}{N}\right)$

$C = 20klog_2(1 + 1000) = 199kbit/s$ **2 marks**

ii

If B is halved the noise power is also halved **2 marks**

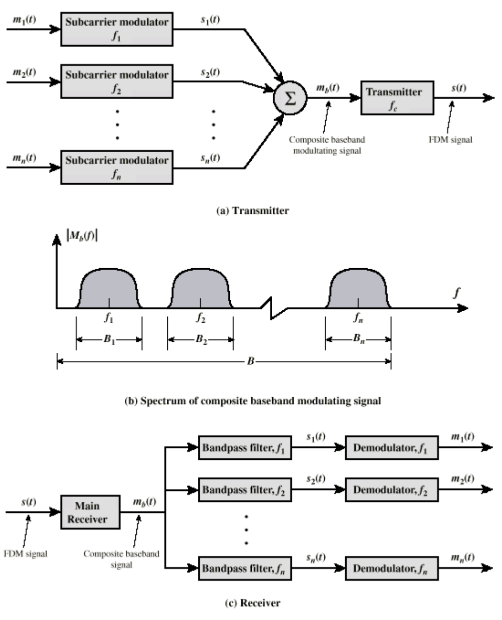
$C = 0.5Blog_2\left(1 + \frac{S}{0.5N}\right)$

$C = 10klog_2(1 + 2000) = 110kbit/s$ **2 marks**

c

Multiplexing is a technique for sending many signals simultaneously over the same transmission medium, e.g. cable, optical fibre, radio link. **1 mark**

A diagram similar to below, not including the frequency spectrum. **2 marks for Tx, 2 marks for Rx**



(a) Transmitter

(b) Spectrum of composite baseband modulating signal

(c) Receiver

d

Using SSB-SC, each channel occupies 20 + 4 = 24 kHz.

Hence number of channels = $45 \times 10^6 / 24 \times 10^3$ = **1875**

3 marks

e

TDM with PCM will take up much more bandwidth but with a better quality signal (S/N).

2 marks

Question 4

a. i) (4 marks)

Two conditions are required to sustain oscillations:

1) The phase shift around the closed feedback loop must be $N \times 360°$, where $N = 0, 1, 2, \ldots$ **(2 marks)**
2) The loop gain (i.e., the voltage gain around the closed feedback loop) must be 1 (unity). **(2 marks)**

a. ii) (2 marks)

An oscillator requires no input (other than dc power).

b. (7 marks)

i) $K \equiv 1 + \dfrac{R_2}{R_1} = 3 \quad \Rightarrow \quad R_1 = \dfrac{1}{2} R_2 = 50\text{k}\Omega$

(3 marks)

ii) It will be a Wien-bridge oscillator.

(1 marks)

iii) $f_0 = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 10^3 \times 0.015 \times 10^{-6}} = 10.61$ kHz

(3 marks)

c. (7 marks)

i) $\dfrac{R_2}{R} = 29 \Rightarrow R_2 = 29R = 136.3\text{k}\Omega$

(3 marks)

ii) It will be a phase-shift oscillator.

(1 marks)

iii) $f_0 = \dfrac{1}{2\pi\sqrt{6}RC} = \dfrac{1}{2\pi\sqrt{6} \times 4700 \times 0.02 \times 10^{-6}} = 691.2\text{Hz}$

(3 marks)

Question 5

a. (4 marks)

$$\frac{dy(t)}{dt} + \frac{1}{RC} \cdot y(t) = \frac{1}{RC} \cdot x(t)$$

Taking the Laplace transform,

$$sY(s) + \frac{1}{RC}Y(s) = \frac{1}{RC}X(s)$$

(1 mark)

$$(s + \frac{1}{RC})Y(s) = \frac{1}{RC}X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{RC} \cdot \frac{1}{s+1/RC}$$

(1 mark)

Taking the inverse Laplace transform gives,

$$h(t) = \frac{1}{RC}e^{-t/RC} \cdot u(t)$$

(2 marks)

b. i) (2 marks)

Voltage standing wave ratio (VSWR) is the ratio between the maximum and minimum AC voltages along a transmission line.

VSWR is a measure of impedance matching of a load to the characteristic impedance of a transmission line.

b. ii) (2 marks)

To minimize losses along the transmission line, the impedances of both source and load need to be equal to (matched with) the characteristic impedance.

c. (12 marks)

The initial forward wave has a voltage of:

$$V_{f1} = V\frac{Z_0}{Z_0 + 150} = 100\frac{50}{50+150} = 25V$$

(2 marks)

The source and load reflection coefficients are given by

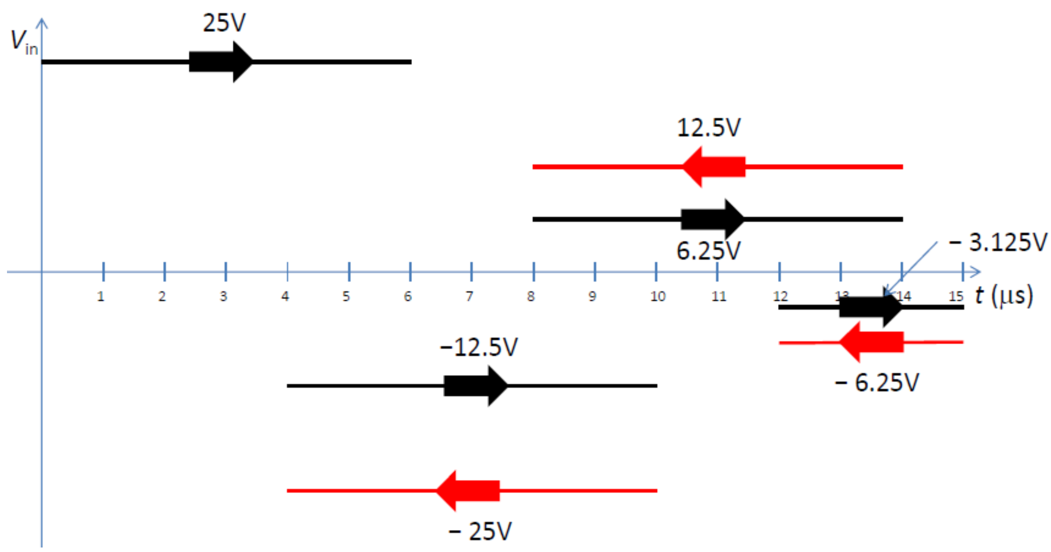
$$\rho_S = \frac{Z_{source} - Z_0}{Z_{source} + Z_0} = \frac{150-50}{150+50} = 0.5$$

(2 marks)

$$\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0-50}{0+50} = -1$$

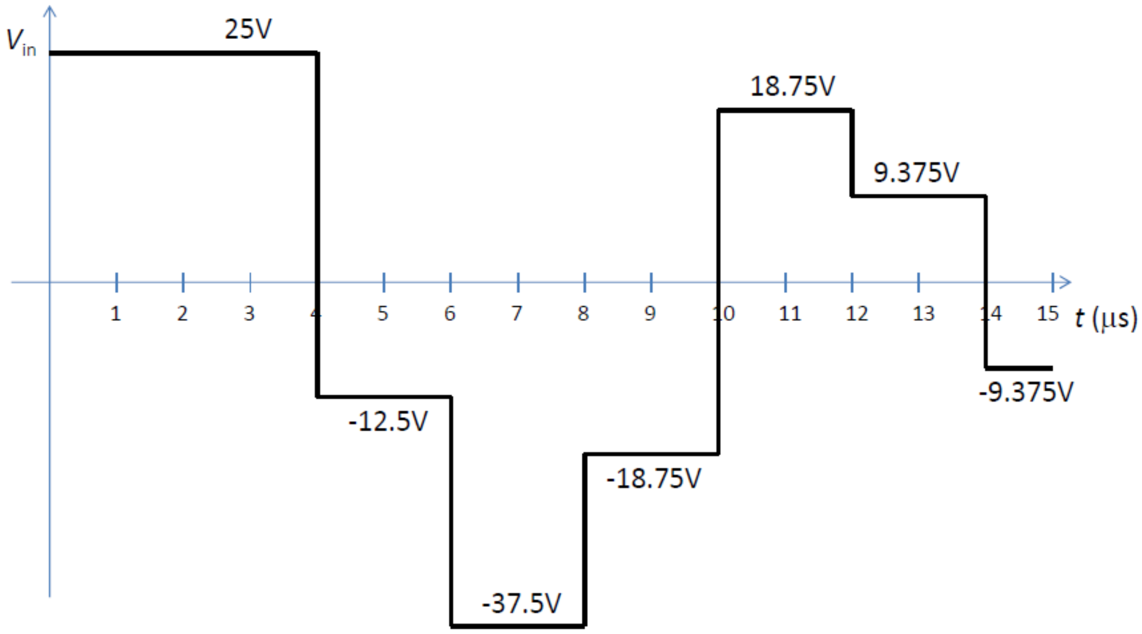
(2 marks)

(2 marks)



(2 marks)

Time (μs)	V_{f1}	V_{f2}	V_{f3}	V_{f4}	V_{b1}	V_{b2}	V_{b3}	Total voltage (V)
0-4	25	0	0	0	0	0	0	**25**
4-6	25	-12.5	0	0	-25	0	0	**-12.5**
6-8	0	-12.5	0	0	-25	0	0	**-37.5**
8-10	0	-12.5	6.25	0	-25	12.5	0	**-18.75**
10-12	0	0	6.25	0	0	12.5	0	**18.75**
12-14	0	0	6.25	-3.125	0	12.5	-6.25	**9.375**
14-16	0	0	0	-3.125	0	0	-6.25	**-9.375**



(2 marks)

Question 6

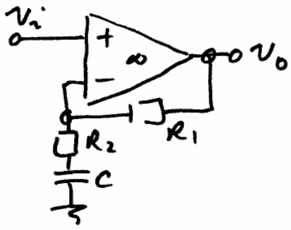
a. i) (4 marks)

l.f. gain ; $X_c \Rightarrow \infty$

$\therefore \frac{v_o}{v_i} \Rightarrow 1$

h.f. gain ; $X_c \Rightarrow 0$

$\therefore \frac{v_o}{v_i} \Rightarrow \frac{R_1 + R_2}{R_2}$



a. ii) (5 marks)

$$\frac{v_o}{v_i} = \frac{Z_1 + Z_2}{Z_2} = \frac{R_1 + R_2 + 1/j\omega C}{R_2 + 1/j\omega C}$$

(2 marks)

$$= \frac{1 + j\omega C(R_1 + R_2)}{1 + j\omega C R_2} \equiv k \cdot \frac{1 + j\,\omega/\omega_1}{1 + j\,\omega/\omega_0}$$

where $k = 1$, $f_1 = \frac{1}{2\pi C(R_1 + R_2)}$, $f_0 = \frac{1}{2\pi C R_2}$.

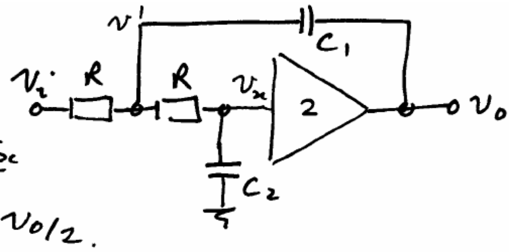
(3 marks)

a. iii) (2 marks)

The four terms are “first order”, “active”, “analogue”, and “pole zero”.

b. i) (6 marks)

sum currents at v' node ....



$$\frac{v_o - v'}{1/sC_1} + \frac{v_i - v'}{R} = \frac{v' - v_x}{R}$$

since gain is 2, $v_x = v_o/2$.

so $v_o sC_1 R - v' sC_1 R + v_i - v' = v' - v_o/2$

or $$v' = \frac{2v_i + v_o(1 + 2sC_1R)}{2(2 + sC_1R)}.$$

(2 marks)

also $v_x = v' \frac{1/sC_2}{R + 1/sC_2}$ or $v' = \frac{v_o}{2}(1 + sC_2R)$

(2 mark)

$$\frac{v_o}{\cancel{2}}(1+sC_2R) = \frac{2v_i + v_o(1+2sC_1R)}{\cancel{2}(2+sC_1R)}$$

$$v_o\left[(1+sC_2R)(2+sC_1R) - (1+2sC_1R)\right] = 2v_i$$

$$v_o\left[1 + 2sC_2R - sC_1R + s^2C_1C_2R^2\right] = 2v_i$$

$$\text{or} \quad \frac{v_o}{v_i} = \frac{2}{1+s(2C_2R - C_1R) + s^2C_1C_2R^2}$$

(2 marks)

b. ii) (3 marks)

$k = 2$

(1 mark)

$$\omega_o^2 = \frac{1}{C_1C_2R^2} \quad \text{or} \quad \omega_o = \frac{1}{R\sqrt{C_1C_2}}$$

(1 mark)

$$\frac{1}{\omega_o q} = R(2C_2 - C_1)$$

$$\frac{1}{q} = \omega_o R(2C_2 - C_1) = \frac{\cancel{R}(2C_2 - C_1)}{\cancel{R}\sqrt{C_1C_2}}$$

(1 mark)