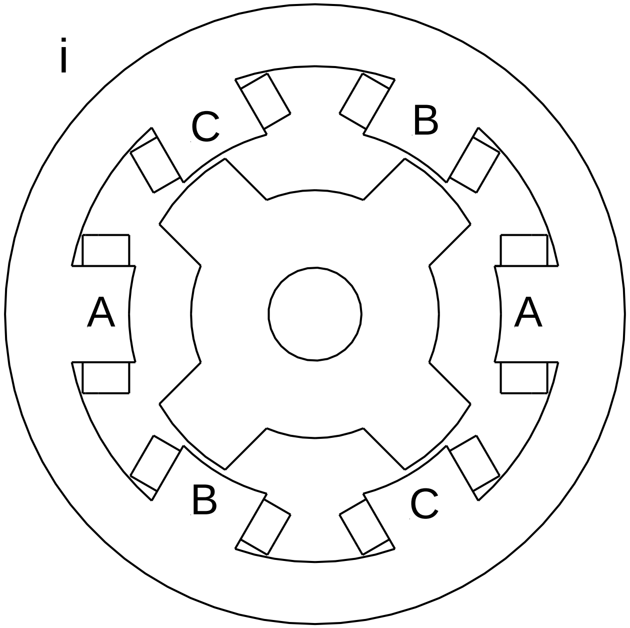
Modelling of machines

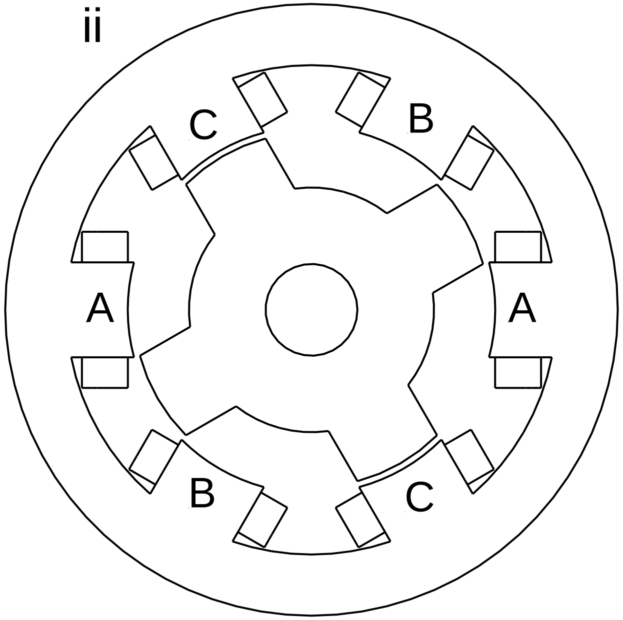
ψ-i modelling – section 2

Alignment definitions

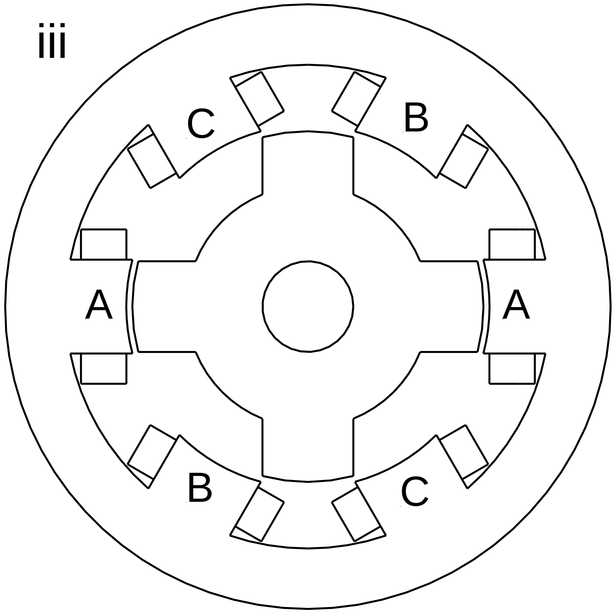
i



ii



iii



Fully un-aligned for phase A, i.e. minimum flux-linkage position

Note: Phase A produces ***no net torque in this position***

Start of normal stroke for phase A
i.e. nominally current is switched into phase A at this angular position

Fully aligned for phase A

Note:
Angular displacement between fully un-aligned and aligned position is 45° in this 3-phase 6-4 machine, but stroke angle is only 30°

- Unfortunately, there is no consistent approach to the definition of the un-aligned position – could be (i) or (ii) from previous slide
- However, the definition being used in any given textbook, paper or exam questions is either spelt out in the explanation of the characteristic being shown or can be established from the number of phases and the number of rotor and stator teeth

Angular displacement between fully un-aligned and aligned position is given by:

$$= \frac{2\pi}{2 \times Number\,of\;\; rotor\,teeth}$$

Angular displacement of 'normal' stroke:

$$= \frac{2\pi}{Number\,of\;\; phases \;\times\; Number\,of\;\; rotor\,teeth}$$

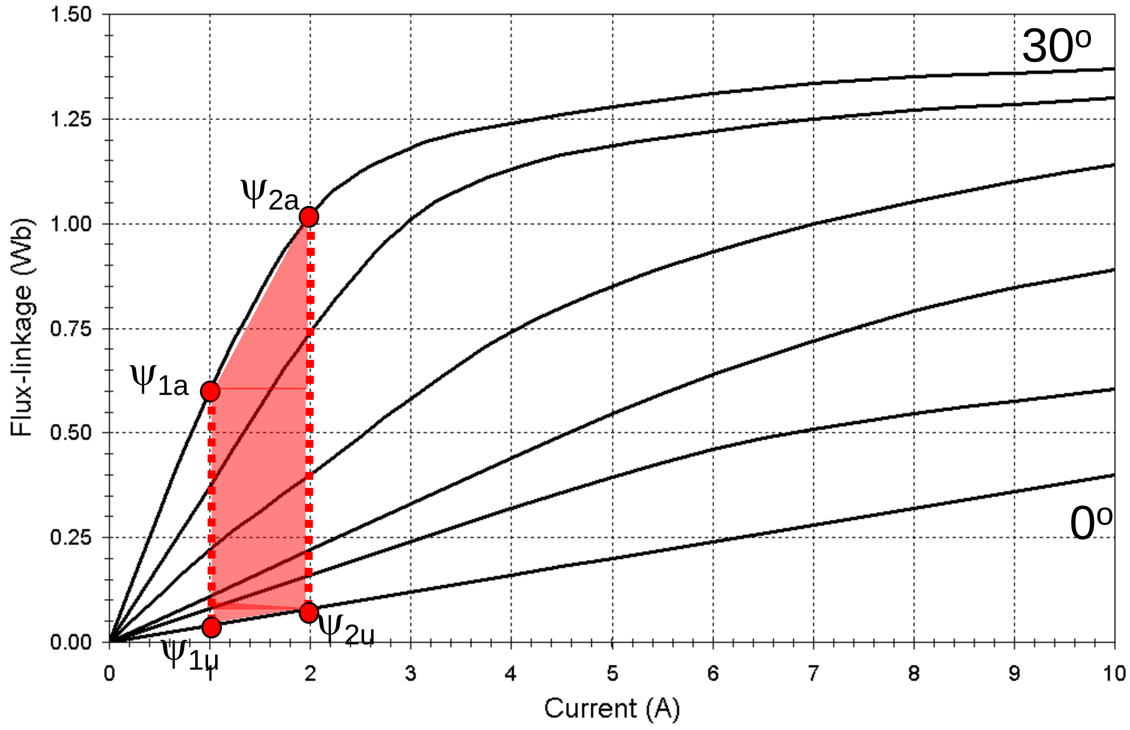
N.B. Examination questions will not be ambiguous in this respect

Calculation of average torque in an SR machine

- SR machines exhibit almost complete de-coupling between phases (at least for conventional combinations of phases / rotor teeth and stator teeth (no magnetic mutual coupling of any significance)
  - each phase can be treated entirely separately and the overall machine performance predicted by appropriate scaling of a single phase contribution
- As described previously, the average torque over one stroke can be calculated from the change in co-energy which in turn can be estimated from the ψ-I diagram
- In order to estimate the change in co-energy, some form of integration method must be used to calculate the areas between the curves

Area integration

Consider a typical ψ-I diagram for a 3-phase SR machine in which curves are presented for the normal stroke range only (30° interval):



A simple, and usually adequate means of summing the areas is by first order trapezium integration

Consider the co-energy change represented by the shaded area:

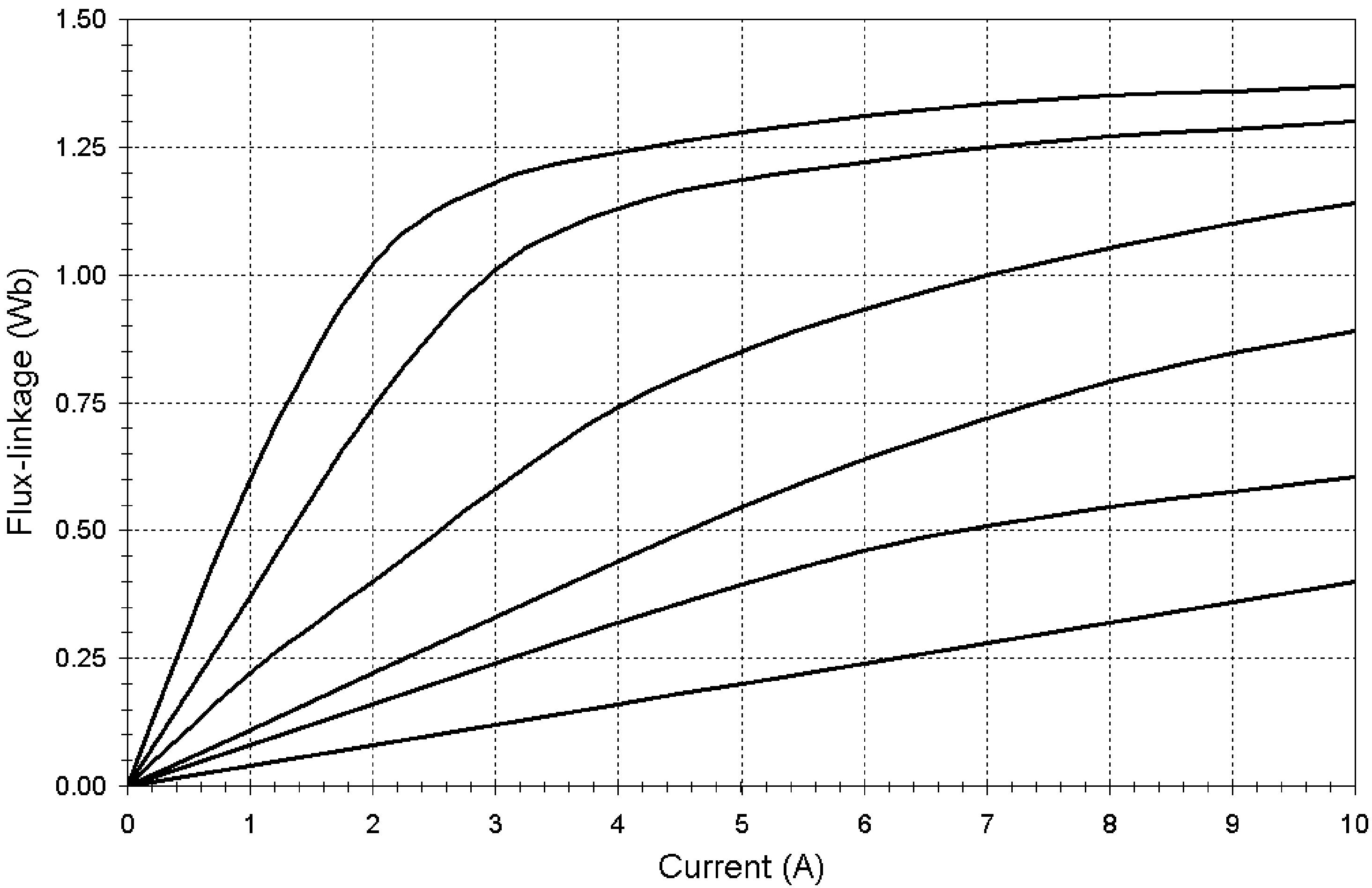
The area of this can be reasonably approximated by a trapezium of area:

$$Area = \left(\frac{\left(\psi_{1a} - \psi_{1u}\right) + \left(\psi_{2a} - \psi_{2u}\right)}{2} \right) \Delta I$$

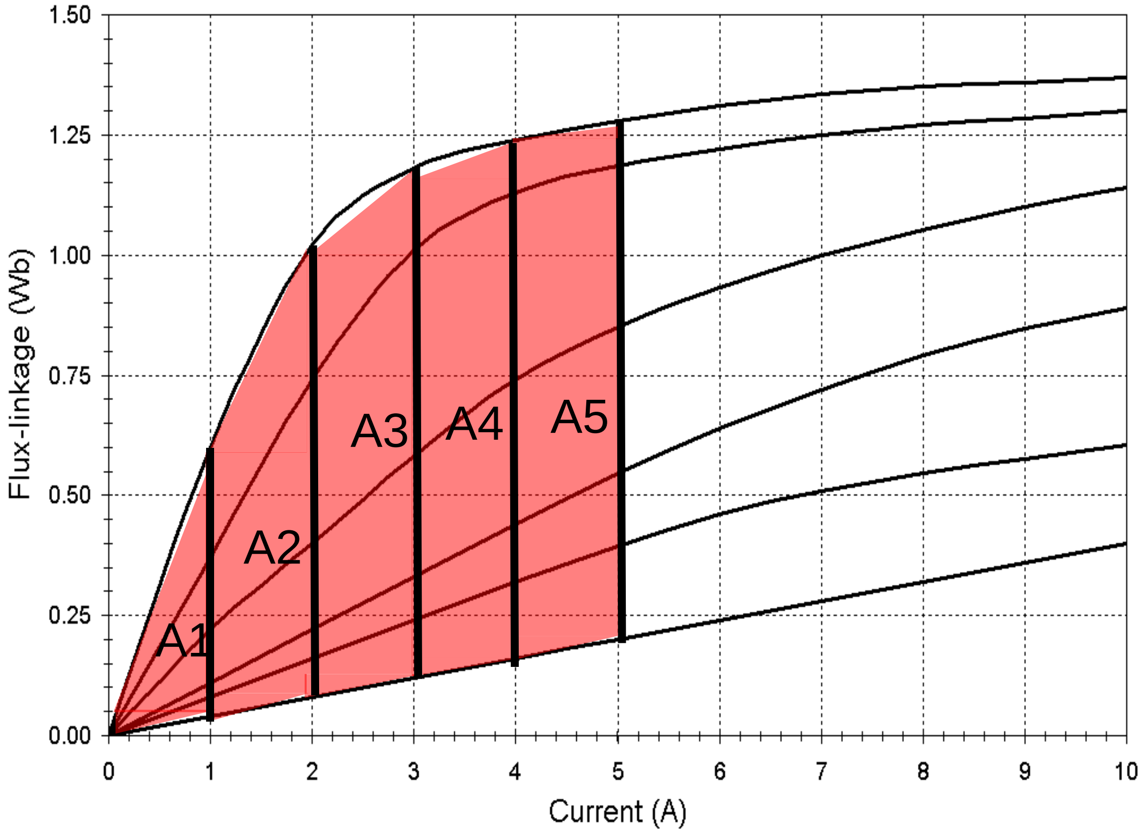
This can be applied to all intervals of current up to the current of interest – Note 1st interval from zero forms a triangle, but the formulae can be used without modification

Full torque calculation example

- Consider a 3-phase, 6/4 machine with the phase flux-linkage versus current characteristics shown below (correspond to the 30° stroke only and not from full un-alignment to fully aligned)



Supposing we wish to calculate the average torque produced for a current of 5A



Applying trapezium integration to each area in turn yields: a total area between the curves of 4.18J

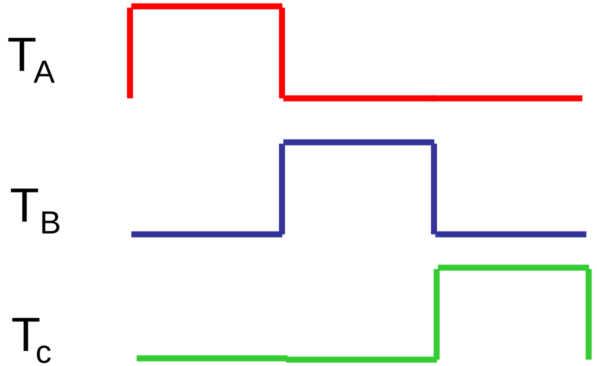
The corresponding torque is given by:

$$T_{AVE} = \frac{\Delta W'}{\Delta \theta} = \frac{4.18}{\pi / 6} = 7.98Nm$$

Note: There is a slightly quicker method which can be used if the un-aligned curve is a good approximation to a linear characteristic – see solution to Q3 in 2006

Total torque of a machine

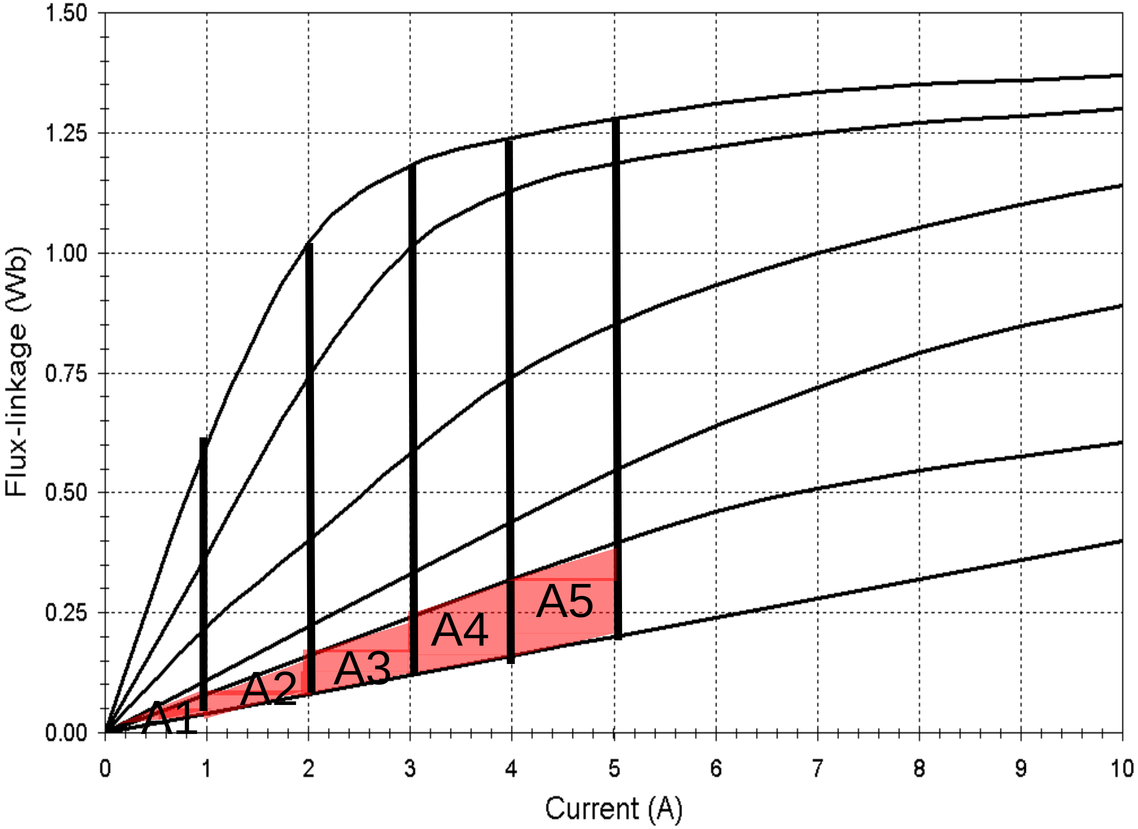
- The calculations performed up to this point have been based on an isolated single phase of the machine (reasonable assumption given the low mutual coupling between phases)
- To estimate the total machine torque from the per phase torque, it is important to consider the commutation pattern, i.e. the number of phases which are carrying current during a given interval
- In the case of most conventional 1,2,3,4 and 5 phase machines (all those of interest in this course) then each phase is fired sequentially, e.g. A-B-C-A-B-C in a 3-phase machine.
- Each of the torque pulses then make up a pseudo continuous torque output. Taking the example below (which shows idealised torque pulses), it can be shown that the net torque over one revolution has the same value as the torque produced by one phase during it's stroke – NOT 3 x phase torque !



In some control strategies (not considered in this course) some overlap of phases is employed to smooth out the overall torque waveform – particularly avoiding dips around the phase changes.

Torque variation over a stroke

- **The method shown previously for calculating average torque can be extended to estimate the variation of torque over one stroke – strictly speaking this is a sequence of averages over sub-divisions of the stroke rather than true instantaneous torques (which would require many more separate curves to approximate)**
- **For the example shown in the previous slide the average torque for each increment of angle can be calculated using the same method – taking care to use the correct angular displacement in the denominator**



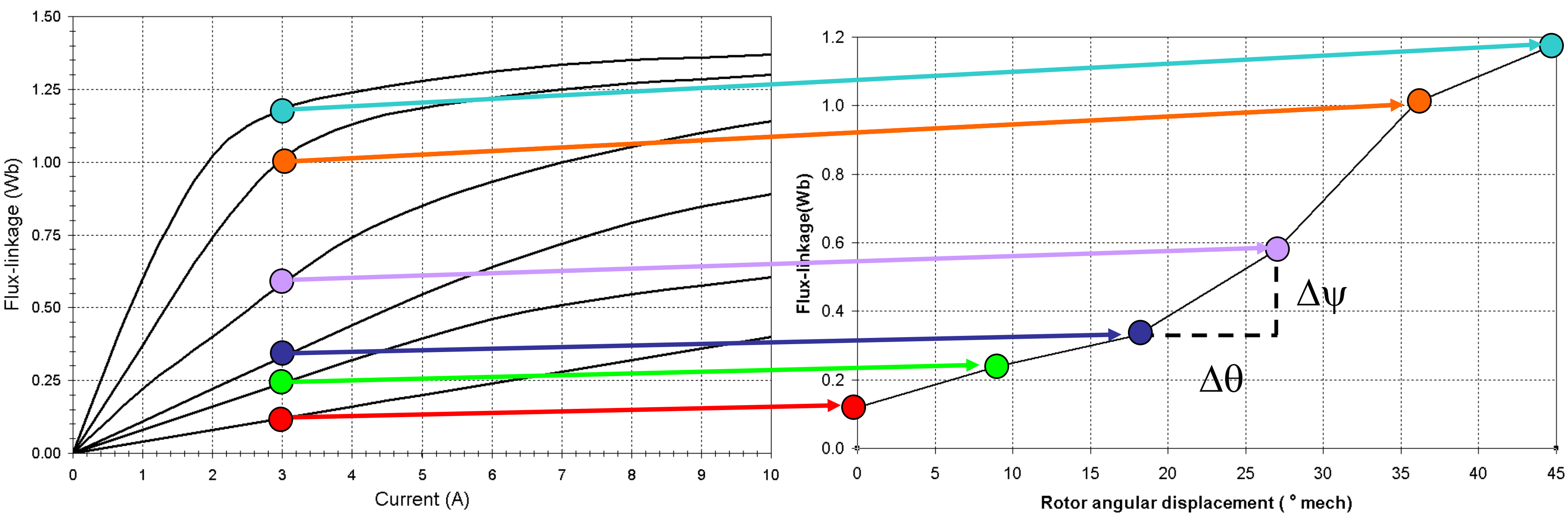
This process can be repeated for each interval of angular displacement to calculate a series of small interval average torques over one stroke

Calculation of induced emf

- A set of ψ-i curves for different rotor angular displacements can be used to calculate induced emf (often called 'back emf' in the winding).
- However, the data needs to be transferred to ψ-θ curves
- Consider a 3-phase 6-4 machine in which the characteristic refers to the full 45º for fully un-aligned to aligned (different to previous case which referred to 30º). The transformation is performed as shown for the particular case of 3A of current – further curves could be transformed using the same method

The rate of change of flux linkage with rotor angular displacement over any interval can be calculated as

$$\frac{d\psi}{d\theta} \approx \frac{\Delta\psi}{\Delta\theta}$$



For the example shown on the previous slide, the emf over the interval 18º to 27º at 200rpm can be calculated as follows:

- From the graph, the rate of change of flux-linkage with respect to rotor angular displacement around 15º is given to a reasonable approximation by:

$$\left.\frac{d\Psi}{d\theta}\right|_{22.5} \approx \frac{\Psi_{27}-\Psi_{18}}{9\times\dfrac{\pi}{180}} \approx \frac{0.58-0.33}{9\times\dfrac{\pi}{180}} \approx 1.59Wb/rad$$

At 200 rpm, the rate of change of angular displacement is given by:

$$\frac{d\theta}{dt}=\frac{200\times 2\pi}{60}=20.9rad/s$$

The instantaneous value of the induced emf is hence given by:

$$\frac{d\Psi}{dt}=\frac{d\Psi}{d\theta}\times\frac{d\theta}{dt}=31.8V$$

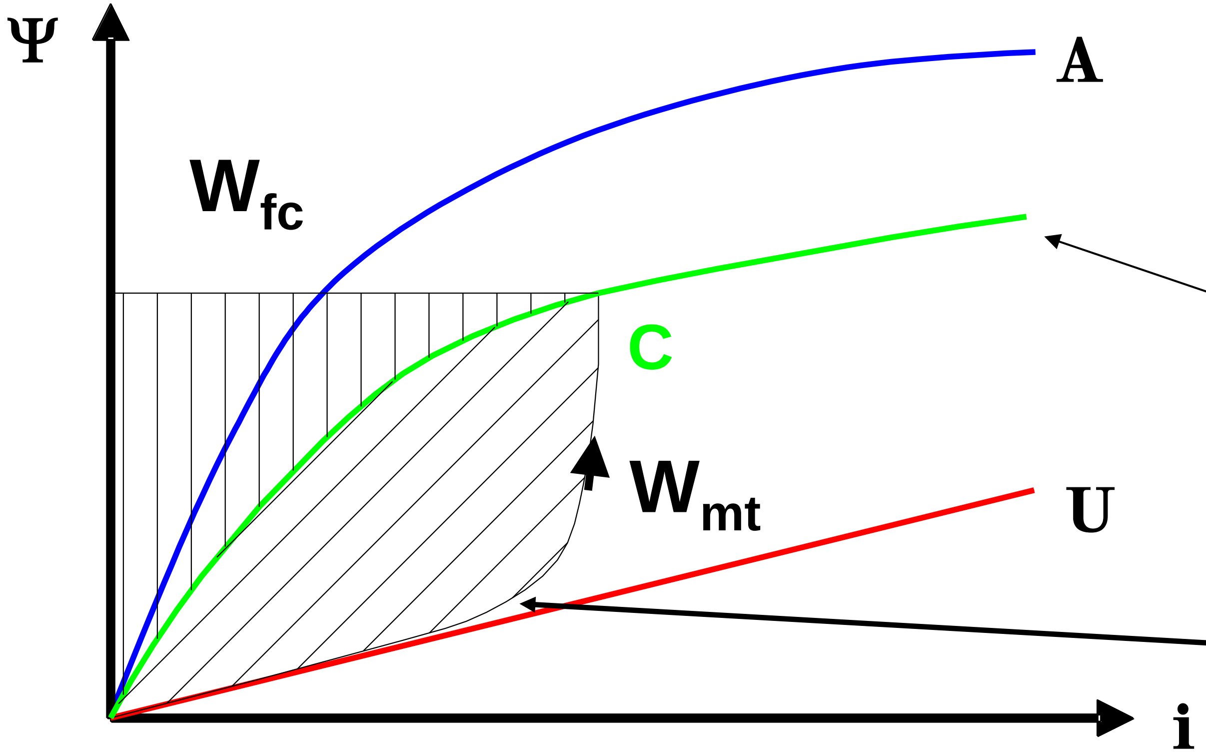
Dynamic operation

- Previous torque calculation based on constant current throughout stroke
- Most practical machines operate from fixed DC link voltage
- Current has finite rise time and fall time (significance of which increases with rotor speed and pole number)
- Must avoid producing negative braking torque
- Common practice to turn-off applied voltage (commutate) before the fully aligned position
- Unable to exploit full static torque capability under dynamic conditions

Dynamic Ψ/i up to commutation

Often termed 'fluxing' period

Motoring



Corresponds to angle of commutation

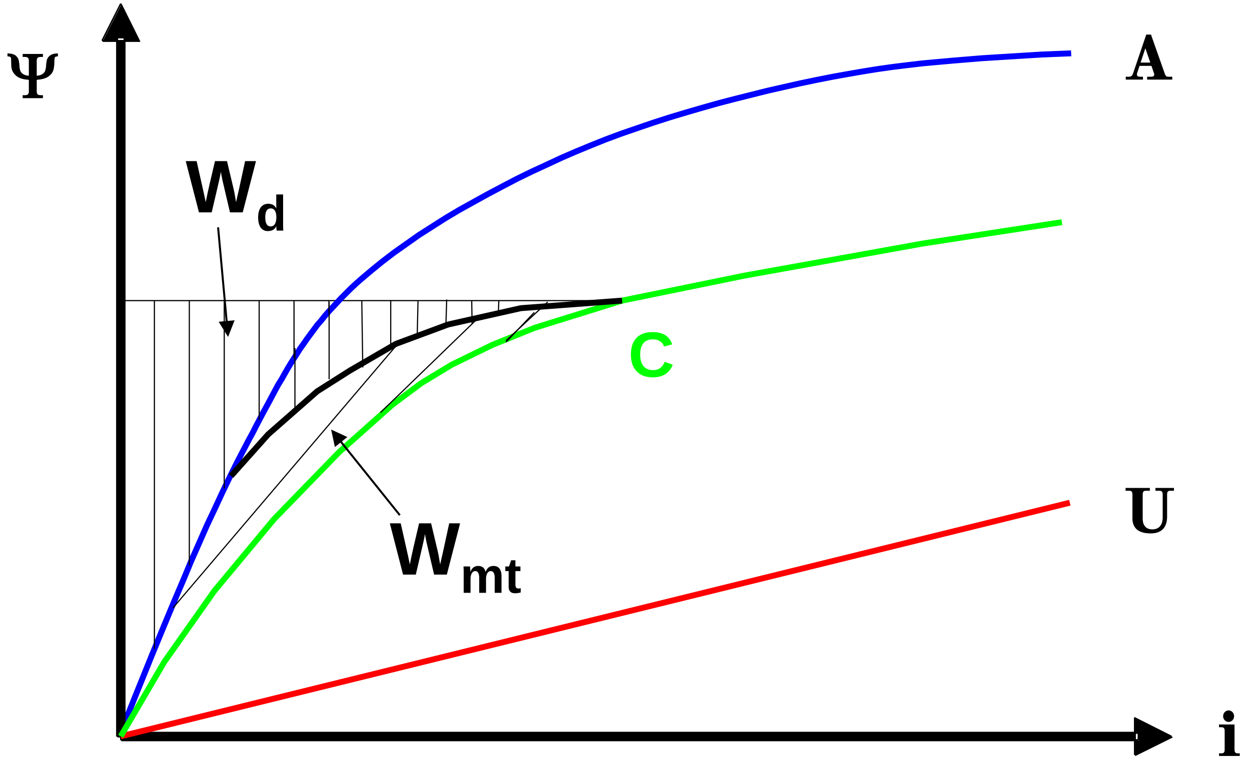
Rotor has moved before demand current is reached

W_{mt} – Mechanical output work
W_{fc} – Magnetic stored energy
Total energy accumulated from supply = W_{fc} + W_{mt}

Dynamic Ψ/i after commutation

Often termed 'de-fluxing' period

Motoring



W_{mt} – mechanical output work (J)
W_d –Energy returned to supply

Dynamic operation summary

Exact form of trajectory depends on rotational speed, rotor and load inertia, magnitude of applied voltage, commutation angles

Always lees torque generated than would be calculated from static Ψ-I characteristic

Limited utilisation of power drawn from supply

Example shown:

~65% of energy drawn from supply is converted to mechanical output – remaining 35% returned to supply (assuming no losses)

Utilisation of power not to be confused with efficiency (which is usually 85%+ depending on design)

Limited utilisation requires large energy storage in converter (often cited drawback of SR machines)

Dynamic operation is very complex – has implications on control system