Lecture 4

- Current in a Solid
  - Carrier (electron) Density
  - Conductivity
  - Ohm’s Law
- Worked Example

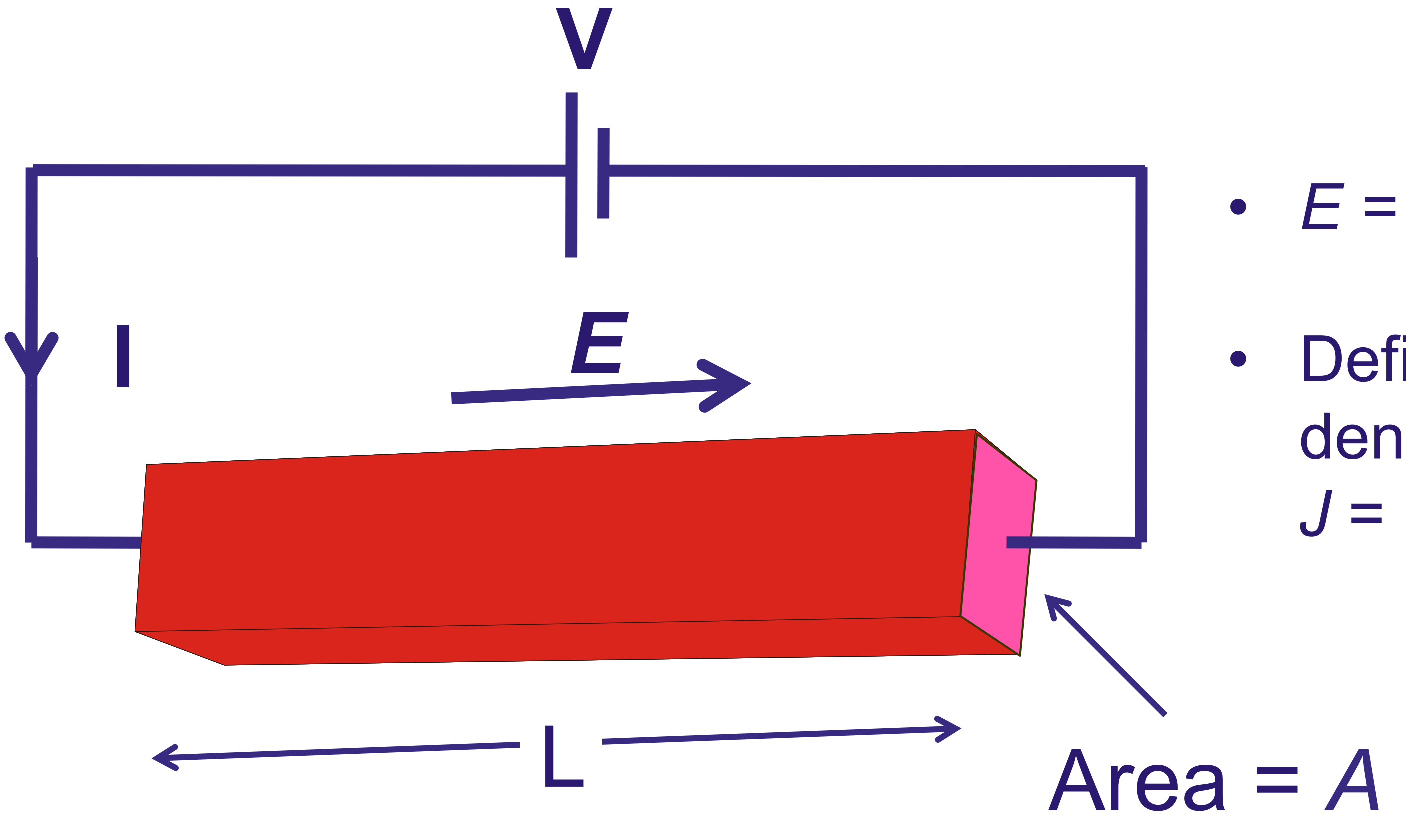
Current in a Solid

Three causes of current

1. An electric potential gradient dV/dx (i.e. an ***E***-field)
2. An electron density gradient dn/dx
3. A temperature gradient dT/dx

We will first look at 1 – applying a voltage difference or ***E***-field

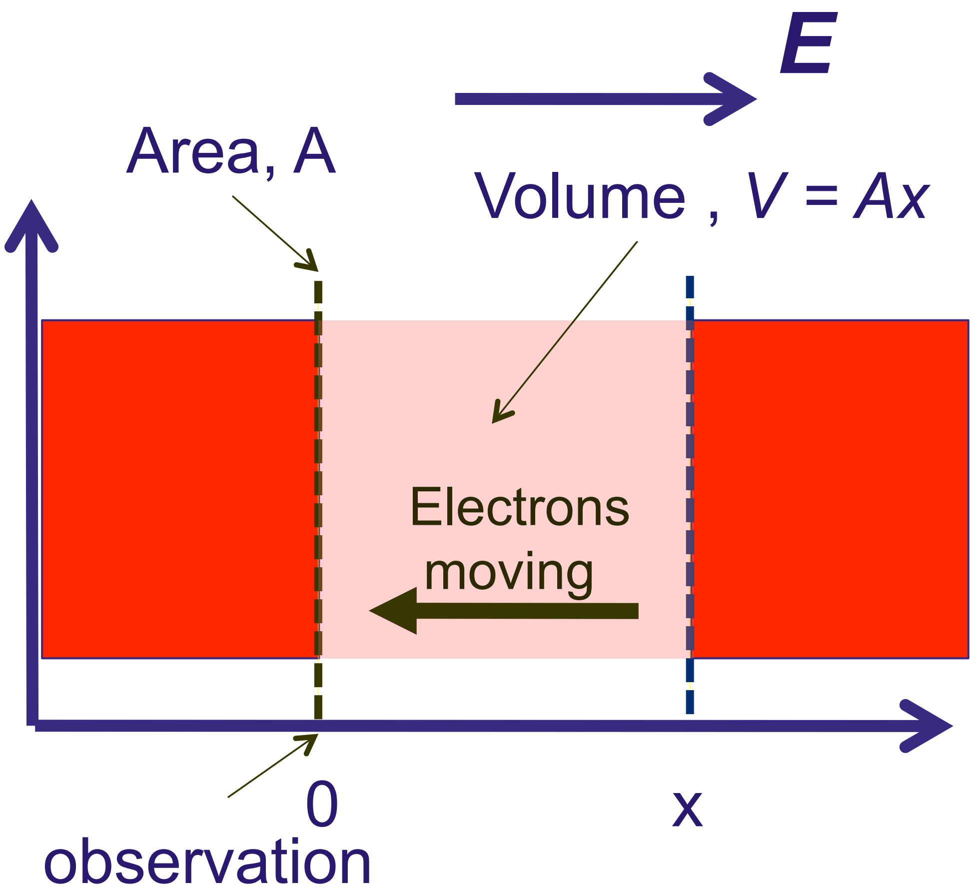
Solid with Free Electrons



- $E = V/L$
- Define a current density, $J = I/A$ (Am^{-2})

Note the direction of E

Longitudinal Slice



Average drift velocity, $\boldsymbol{v}_d$, for electrons with density, n m^{-3}.

In time t, all electrons in light coloured region will move past observation point

$\rightarrow \quad x = \boldsymbol{v}_d t$

Number of electrons in this volume is

$n V = n A x = n A v_d t$

- Remember we are interested in Current.
- Definition of Current – charge flow per second.
- Charge on electron = -e (e = 1.6×10^{-19} C) so in unit time (per second) the amount of charge flowing past our observation point is:

$$\text{current, } I = -n\,A\,e\,v_d$$

- The current density (current per unit area) is given by;

$$\boxed{J = I/A = -n\,e\,v_d}$$

- NOTE: *J* and *I* are both in opposite direction to electron flow
- Sometimes drift velocity written as just *v*
- Sometimes charge on electron written as e or q

Eliminate v_d

- The previous equation is only useful if we know the drift velocity (we generally don't), which we have derived as

$$v_d = - \mu E$$

- Giving the more useful form $\boxed{J = ne\mu E}$
- So the current density in our solid depends upon
  - Carrier concentration – i.e. how many electrons are there
  - ***E***-field – magnitude of dV/dx
  - Mobility - how easy the carriers can move
  - The charge on an electron (fixed)

Ohm's Law

We can now derive a familiar and important equation:

$J = ne\mu E$ can be simplified to $\boxed{J = \sigma E}$

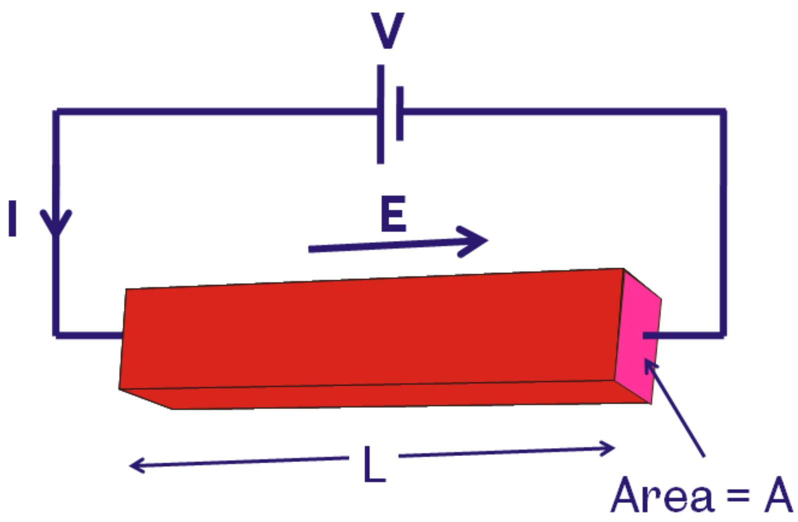
Where the conductivity $\sigma = ne\mu$

Conductivity is inverse of *resistivity* $\rho = \dfrac{1}{\sigma}$

This is the general form of Ohm's law *I* = *V/R*

(J becomes I, E becomes V, σ *becomes1/R)*

Ohm's Law (2)



$$J = \frac{I}{A} \qquad E = \frac{V}{L} \qquad J = \sigma E$$

combining

$$\frac{I}{A} = \frac{\sigma V}{L}$$

$$I = \frac{\sigma A V}{L}$$

This is just Ohm's law ($I = V/R$), hence $\boxed{R = \frac{L}{\sigma A} = \rho L/A}$

i.e. R depends on ***both*** geometry and conductivity ($ne\mu$)

Drift Velocity Example

A 2cm long Si rod with cross sectional area of 5mm^2 has a voltage of 10V applied across its length, giving a current of 3 mA. The rod is known to have a uniform density of free electrons and temperature throughout its length.

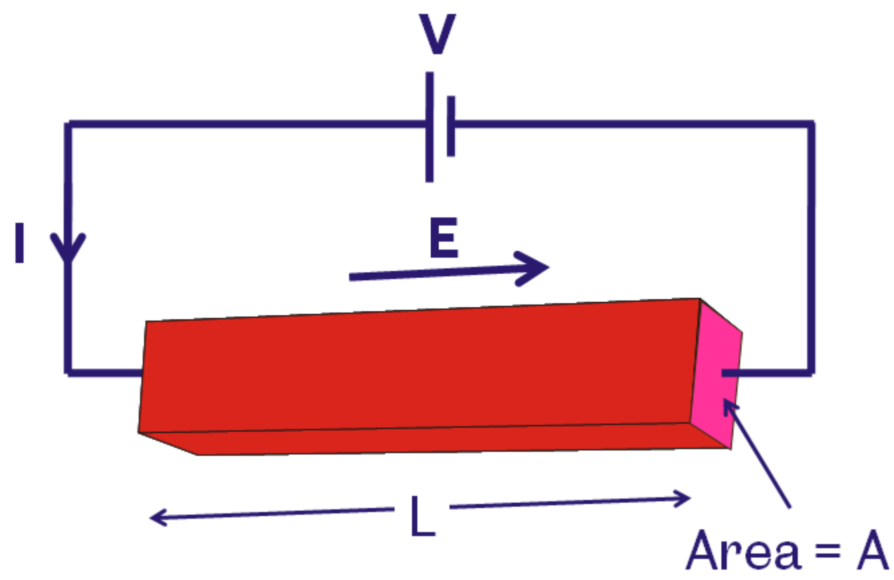
a) What is the average time between collisions in the material?

b) What is the average drift velocity of the electrons in the rod?

c) What is the concentration of the electrons in the material?

Given that μ = 1200 cm^{2}V^{-1}s^{-1}; m^* = 0.98m$_e$ (m$_e$ = 9.11x10^{-31} Kg); e = 1.6x10^{-19} C

Visualization, Unit Conversion



L = 2x10^{-2} m
A = 5x10^{-6} m^2
V = 10 V
I = 3x10^{-3} A

μ = 0.12 m^{2}V^{-1}s^{-1}
m^* = 0.98m$_e$
m$_e$ = 9.11x10^{-31} Kg

a) Determine τ

Definition of mobility $$\mu = \frac{e\tau}{m^*}$$

Rearranging $$\tau = \frac{\mu m^*}{e}$$

$$\tau = \frac{0.12 \times 0.98 \times 9.11 \times 10^{-31}}{1.6 \times 10^{-19}} = 6.7 \times 10^{-13}\,\mathrm{s}$$

b) Determine v_d

Drift velocity and E-field related by $v_d = -\mu E$

E-field? 10V over 2cm length $E = \frac{V}{L} = \frac{10}{0.02} = 500\,\mathrm{Vm^{-1}}$

Hence drift velocity, $v_d = -\mu E = -0.12 \mathrm{x} 500 = -60\ \mathrm{ms^{-1}}$

c) Determine n

Current and voltage give value of resistance of rod

$$R = \frac{V}{I} = \frac{10}{3\text{x}10^{-3}} = 3.3\text{x}10^{3}\,\Omega$$

Dimensions are known - can calculate the conductivity

$$R = \frac{L}{\sigma A} \qquad \sigma = \frac{L}{RA} = \frac{0.02}{3.33\text{x}10^{3}\text{ x }5\text{x}10^{-6}} = 1.2\ \Omega^{-1}\text{m}^{-1}$$

Know relation between conductivity and carrier density

$$n = \frac{\sigma}{\text{e}\mu} = \frac{1.2}{1.6\text{x}10^{-19}\text{ x }0.12} = 6.25\text{x}10^{19}\ \text{m}^{-3}$$

Summary

- Current density is current per unit area flowing in a material
- Current density is increased with increasing carrier density, increased mobility, and increased ***E***-field.
- Conductivity is the reciprocal of resistivity
- The current density is the product of the ***E***-field and conductivity of the material