Q1 (i) V_s sees R_1 in series with the parallel combination R_2 and (R_3 in series with R_4)

ie $R_{eff} = R_1 + R_2 // (R_3 + R_4)$.

$= 2 + 10 // 40 = 2 + \frac{400}{50}$

$= \underline{\underline{10\,\Omega}}$.

$\therefore \; I_s = V_s/R_{eff} = \frac{20\,V}{10\,\Omega} = \underline{\underline{2A}}$

(ii) This can be done in two ways. Either

$V_A = V_s - I_s R_1 = 20 - 2.2 = \underline{\underline{16V}}$

or $V_A = I_s \left(R_2 // (R_3 + R_4) \right) = 2.8 = \underline{\underline{16V}}$

(iii) Current sum at node A is

$I_s = I_2 + I_3$.

(iv) Can be done two ways .... (only one needed)

(a) nodal $\quad I_s = I_2 + I_3$

or $\frac{V_s - V_A}{R_1} = \frac{V_A}{R_2} + \frac{V_A}{R_3 + R_4}$

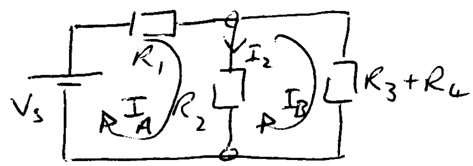
or $\frac{20}{2} = V_A \left[\frac{1}{2} + \frac{1}{10} + \frac{1}{40} \right] = V_A \frac{\cancel{25}\,5}{\cancel{40}\,8}$

or $V_A = \frac{20}{2} \times \frac{8}{5} = 16V$ (as before)

so $I_2 = V_A/R_2 = \underline{\underline{1.6A}}$.

(b) loops ...

two loops, $I_A + I_B$.



$V_s = I_A R_1 + (I_A - I_B) R_2 \quad - (1)$

$0 = (I_B - I_A) R_2 + I_B (R_3 + R_4) \quad - (2)$

(1) gives $20 = I_A.12 - I_B.10$

(2) gives $0 = -I_A.10 + I_B.50$ or $I_A = 5I_B$.

sub (2) in (1) ... $20 = 5I_B.12 - 10I_B$.

$= 60I_B - 10I_B = 50I_B$.

or $I_B = 20/50 = 0.4A$.

$\therefore$ $I_A = 5 \times 0.4 = 2A$ (as before).

$I_2 = I_A - I_B = 2 - 0.4 = \underline{\underline{1.6A}}$.

(v) This can be solved most easily by nodal analysis or superposition ...

(a) nodal Sum currents at A ..

$I_s + 2.5A = I_2 + I_3$

$\frac{20 - V_A}{2\Omega} + 2.5A = \frac{V_A}{10\Omega} + \frac{V_A}{40\Omega}$.

$V_A\left[\frac{1}{2} + \frac{1}{10} + \frac{1}{40}\right] = \frac{20}{2} + 2.5 = 12.5A$.

$V_A = 12.5 \times \frac{40}{25} = 20V$.

Thus, $I_3 = 20/40 = \underline{\underline{0.5A}}$.

$P_{R_1} = I_s^2 R_1 = 0^2 \times R_1 = \underline{\underline{0W}}$

(b) superposition From earlier parts,

$I_s|_{due\ to\ V_s} = 2A$

$I_3|_{due\ to\ V_s} = 0.4A$.

due to current source, $V_A = 2.5 \times 40\|2\|10$

$= 2.5 \times 1.6\Omega = 4V$

$I_s|_{due\ to\ 2.5A} = -\frac{4V}{2\Omega} = -2A$

$I_3|_{due\ to\ 2.5A} = \frac{4V}{40\Omega} = 0.1A$.

$\therefore$ $I_{s\,TOT} = 2 - 2 = 0A$ so $P_{R_1} = \underline{\underline{0W}}$

$I_{3\,TOT} = 0.4A + 0.1A = \underline{\underline{0.5A}}$.

③

Q2 (a) For the circuits to be equivalent, open circuit output voltage from each network must be the same and the short cct output current must also be the same.

cct (i) V_{oc} ....

$$V_{oc} = \frac{V_s \cdot R_2}{R_1 + R_2}$$

cct (ii) V_{oc} ...

$$V_{oc} = V_{Th}$$

$$\therefore \quad V_{Th} = \frac{V_s R_2}{R_1 + R_2}.$$

cct (i) i_{sc} ....

$$i_{sc} = V_s / R_1$$

(since R_2 is short circuited).

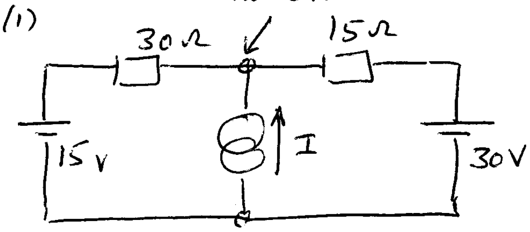
cct (ii) i_{sc} ...

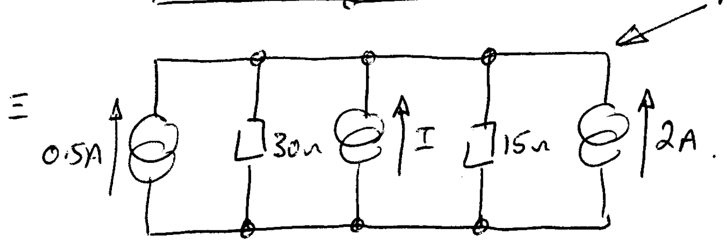
$$i_{sc} = \frac{V_{Th}}{R_{Th}} = \frac{V_s \frac{R_2}{R_1 + R_2}}{R_{Th}}.$$

$$\text{so} \quad V_s/_{R_1} = \frac{V_s \frac{R_2}{R_1 + R_2}}{R_{TH}}.$$

$$\text{or} \quad R_{Th} = \frac{R_1 R_2}{R_1 + R_2} = R_1 // R_2.$$

(b) (i)





(ii) To get V_A of zero we want zero current through the resistors.

Summing current at node A ....

$$0.5 + I + 2 = 0$$

$$\text{or} \quad I = -2.5\,A.$$

(c)(i) When switched changed to position A,

$$V_s = IR + \frac{1}{C}\int I\,dt + \text{const}$$

differentiating with respect to t gives

$$0 = R\frac{dI}{dt} + \frac{1}{C}I \quad \text{or} \quad \frac{dI}{I} = -\frac{dt}{RC}.$$

thus $\int \frac{1}{I}\,dI = -\frac{1}{RC}\int dt + c$

or $\ln I = -\frac{t}{RC} + c$

when $t=0$, $I = V_s/R = I_0$ $\left[\text{since at } t=0,\ V_c = 0 \text{ and all } V_s \text{ appears across } R\right]$

so $c = \ln I_0$

$\therefore \ln I - \ln I_0 = -t/RC$

$\ln \frac{I}{I_0} = -t/RC$ or $I = I_0 e^{-t/RC}$

$$= \underline{\underline{V_s/R\, e^{-t/RC}}}.$$

(ii) $V_c = \frac{1}{C}\int I\,dt + c.$

$$= \frac{1}{C}\int \frac{V_s}{R} e^{-t/RC}\,dt + c.$$

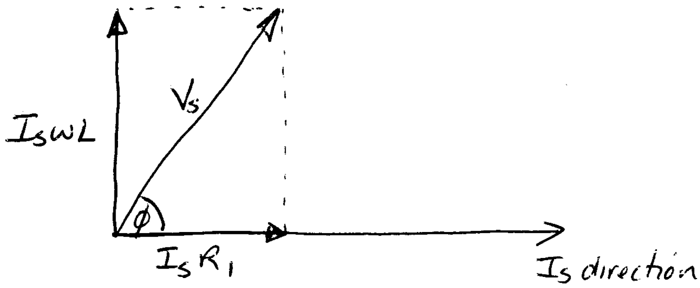
$$= \frac{V_s}{CR}\cdot(-)\frac{1}{\left(\frac{1}{CR}\right)} e^{-t/RC} + c. = -V_s e^{-t/RC} + c.$$

when $t=0$, $V_c = 0$ so $c = V_s$.

and $\underline{\underline{V_c(t) = V_s\left(1 - e^{-t/RC}\right)}}.$

⑤

Q3(a)(i).



The angle between I_s + V_s is

$$\tan^{-1}\frac{\omega L}{R_1} = \tan^{-1}\frac{2.\pi.500.2.5\times10^{-3}}{5}$$

$$= 57.5^\circ.$$

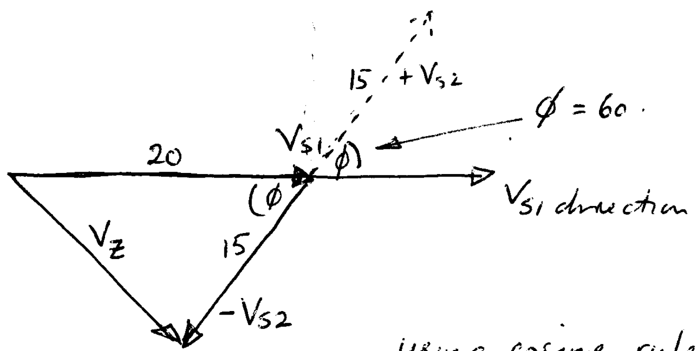
I_s with respect to V_s is $\underline{\underline{-57.5^\circ}}$.

(ii) $$\frac{V_s}{I_s} = j\omega L + R_1 = Z.$$

$$Z = 5 + 2.\pi.500.2.5\times10^{-3} j = \underline{\underline{5 + j7.85}}.$$

$$|Z| = \left[5^2 + 7.85^2\right]^{1/2} = \underline{\underline{9.31\,\Omega}}.$$

(b). $$V_Z = V_{S1} - V_{S2}$$



using cosine rule...

$$V_Z^2 = 20^2 + 15^2 - 2\times20\times15\cos 60 = 400 + 225 - 300$$

$$= 325 \qquad \underline{\underline{|V_Z| \approx 18\text{ V}}}.$$

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(ii) $V_{s1} = 20\angle 0 = \underline{\underline{20 + j0}}$

$V_{s2} = 15\angle 60 = 15\cos 60 + j15\sin 60$

$= \underline{\underline{7.5 + j13}}$

(iii) $I_Z = \dfrac{V_Z}{Z} = \dfrac{20 + j0 - 7.5 - j13}{3 - j4}$

$= \dfrac{12.5 - j13}{3 - j4}$

$= \dfrac{(12.5 - j13)(3 + j4)}{3^2 + 4^2}$

$= \dfrac{37.5 - j39 + j50 + 52}{25}$

$= \dfrac{89.5 + j11}{25} = \underline{\underline{3.58 + j0.44}}$

$\therefore |I_Z| = \left[3.58^2 + 0.44^2\right]^{1/2} = \underline{\underline{3.6A}}$

$\angle I_Z = \tan^{-1}\dfrac{11}{89.5} = \underline{\underline{7^\circ}}$

4(a) (i) A circuit is resonant when its impedance becomes purely real.

(ii) $V_s = I_s\left(j\omega L + \frac{1}{j\omega C} + R\right)$

$$\frac{V_s}{I_s} = Z = j\left(\omega L - \frac{1}{\omega C}\right) + R.$$

At the resonant frequency, the j terms must disappear so

$$j\left(\omega L - \frac{1}{\omega C}\right) = 0$$

$$\text{or } \omega L = \frac{1}{\omega C}$$

$$\text{or } \omega^2 = \frac{1}{LC} \text{ or } \omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

$$\therefore f = \frac{1}{2\pi\sqrt{LC}}$$

(iii) $q = \frac{|V_L|}{|V_R|}$ at resonance (ie when $\omega = \frac{1}{\sqrt{LC}}$).

so $q = \frac{I_s \omega L}{I_s R} = \frac{\omega L}{R} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

(iv) If $f < f_r$ .....

At resonance $X_C = X_L$. Below resonance X_C is increased and X_L is reduced. Since the circuit is a series circuit, X_C (the larger reactance) will dominate and the circuit will look capacitive.

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(b) (i) $V_s = I_s \left(X_C \parallel (R + X_L) \right)$

$$= I_s \frac{\dfrac{R + j\omega L}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}}$$

$$\frac{V_s}{I_s} = Z = \frac{R + j\omega L}{j\omega CR + (j\omega)^2 LC + 1}$$

$$= \frac{R + j\omega L}{(1 - \omega^2 LC) + j\omega CR}$$

(ii). Rationalise to an $a + jb$ form ....

$$\frac{V_s}{I_s} = \frac{(R + j\omega L)\left((1 - \omega^2 LC) - j\omega CR\right)}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

$$= R(1 - \omega^2 LC) + j\omega L(1 - \omega^2 LC) + \omega^2 LCR - j\omega CR^2$$

$$= \underbrace{\left[R(1 - \omega^2 LC) + \omega^2 LCR\right]}_{\text{real}} + j\omega \underbrace{\left[L(1 - \omega^2 LC) - CR^2\right]}_{\text{imaginary}}$$

equate "j" terms to zero ....

$$L(1 - \omega_r^2 LC) = CR^2$$

or $\omega_r^2 L^2 C = L - CR^2$

or $\omega_r^2 = \dfrac{L}{L^2 C} - \dfrac{CR^2}{L^2 C}$

or $\omega_r = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$