Data Provided: Formula Sheet

The University of Sheffield

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2012-13 (2 hours)

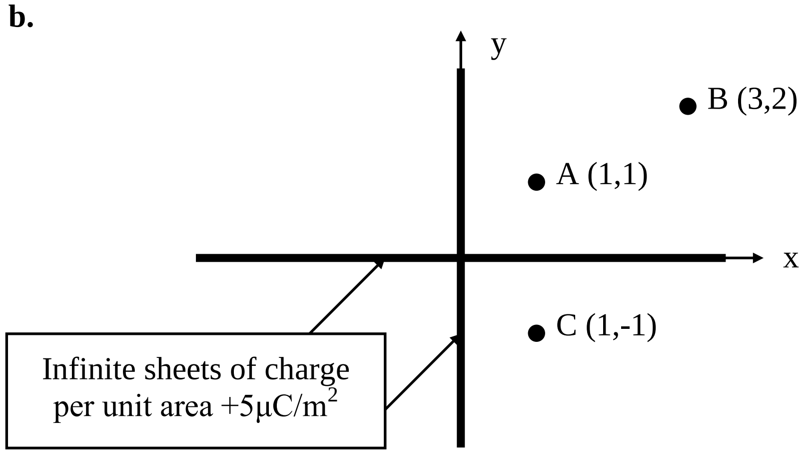
Electric and Magnetic Fields 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. **a.** Using Gauss' law, show that the electric field resulting from an infinite plane sheet carrying a charge q_s per unit area has a magnitude

$E = \dfrac{q_s}{2\varepsilon_0}$ and is directed away from the sheet. **(5)**

b.

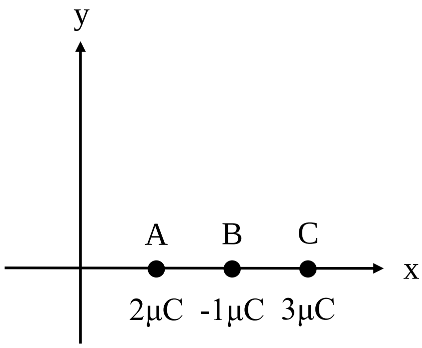


Two such infinite plane sheets with $q_s = 5\mu C/m^2$ are placed perpendicular to each other in the planes x = 0 and y = 0, as in the diagram above. Calculate the x- and y-components of the electric fields at the points A, B, and C. **(6)**

c. Redraw the above diagram, sketching in electric field lines and surfaces of equi-potential. **(3)**

d. Calculate the potential difference between points A and B. **(6)**

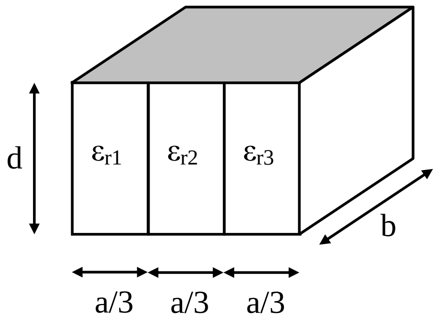
2. a.



i. Three point charges are placed at coordinates (1,0,0), (2,0,0), and (3,0,0) as shown in the diagram above, where the unit of length is 1m. Calculate the force exerted on charge B due to charges A and C. **(5)**

ii. Rearrange the charges (by swapping their positions) to produce the largest magnitude of electric field at the origin. What is the magnitude and direction of this field? **(5)**

b.



i. A parallel plate capacitor consists of two rectangular plates of area $a \times b$ separated by a distance d and contains three materials with different dielectric constants, ε_{r1}, ε_{r2}, ε_{r3}, as shown in the figure above. Ignoring fringing fields, derive an expression for the total capacitance of this arrangement. **(6)**

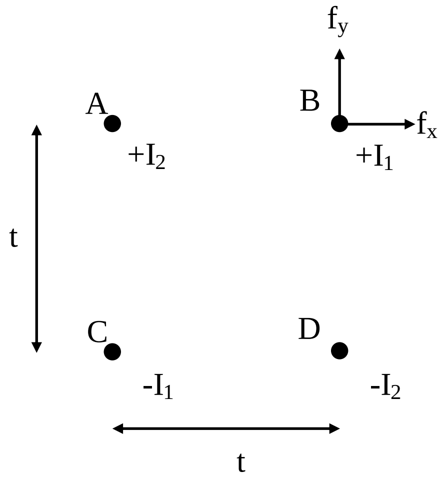
ii. If the capacitor is charged to 12V, calculate the energy stored in the capacitor when $a = 0.01\,\text{m}$, $b = 0.005\,\text{m}$, $d = 0.002\,\text{m}$, $\varepsilon_{r1} = 4$, $\varepsilon_{r2} = 5.5$, $\varepsilon_{r3} = 3$. **(4)**

3. **a.** Using Ampère's law, show that the magnitude of the B-field at a radial distance r from an infinitely long straight wire carrying current I is

$$B = \frac{\mu_0 I}{2\pi r}.$$

(6)

b.



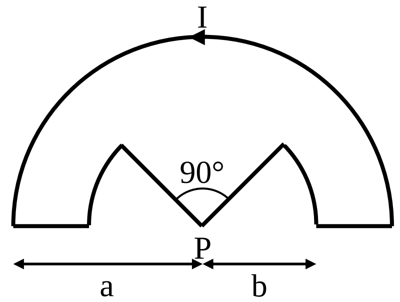
The figure above shows the cross-section of four infinitely long, straight, parallel wires, A, B, C and D, placed at the corners of a square of side t. The wires carry currents as shown, flowing into the plane of the figure. Show that the components f_x and f_y of the force per unit length acting on wire B are given by

$$f_x = \frac{\mu_0 I_1}{2\pi t}\left(\frac{I_1}{2} - I_2\right) \text{ and } f_y = \frac{\mu_0 I_1}{2\pi t}\left(\frac{I_1}{2} + I_2\right).$$

(8)

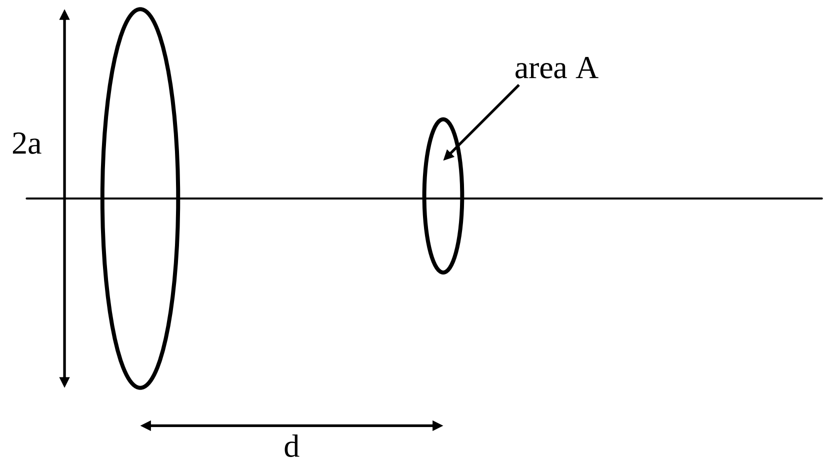
c. If t = 20mm, I_1 = 3A and I_2 = 1A, calculate the magnitude and direction of the force per unit length acting on wire B. Using symmetry arguments calculate the force acting on wire C. **(6)**

4. a.



The diagram above shows a loop circuit consisting of circular arcs and straight radial lines. The arcs have a common centre, P and radii a and b. The circuit carries a current I in the direction shown. Use the Biot-Savart rule to find an expression for the B-field at the point P. **(10)**

b.



The diagram above shows two circular loops of wire positioned a distance d apart. The loop on the left has N turns of radius a. The smaller loop on the right has only one turn of area A. Assuming the loops are parallel and that $A \ll d^2$, deduce an expression for the mutual inductance between the two loops. Evaluate this inductance for a = 0.2m, N = 150, d = 0.1m and $A = 10^{-4}m^2$. **(10)**