

Data Provided: Log 3 cycle x linear graph paper.

## DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2011-12 (2.0 hours)

**EEE112 Engineering Applications** 

This paper comprises TWO sections, A and B. You may gain up to 60 MARKS from SECTION A and 40 MARKS from SECTION B. Attempt ALL the questions in SECTION A. Marks will be awarded for your best TWO solutions in SECTION B. Trial answers will be ignored if they are clearly crossed out. A formula sheet is included at the end of the exam paper. The numbers given after each section of a question indicate the relative weighting of that section.

## **SECTION A**

- **1. a.** Convert the following:
  - i) 150° into radians,
  - ii)  $-135^{\circ}$  into radians,
  - iii)  $\pi/3$  radians into degrees,
  - iv)  $-3\pi/2$  radians into degrees,
  - v) 2 radians into degrees.

**b.** A time varying current is described by the equation:

$$i(t) = -20\cos\left(120\pi t + \frac{\pi}{2}\right)Amp.s$$

For this equations write down:

- i) the amplitude,
- ii) the peak-to-peak current,
- iii) the frequency (NOT the angular frequency).
- iv) the period,
- v) the phase shift.

**(5)** 

**(5)** 

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2. a. Solve the following simultaneous equations using the method of <u>substitution</u> (NOT by Cramer's Rule and NOT by Gaussian elimination) to find the values of x, y & z.

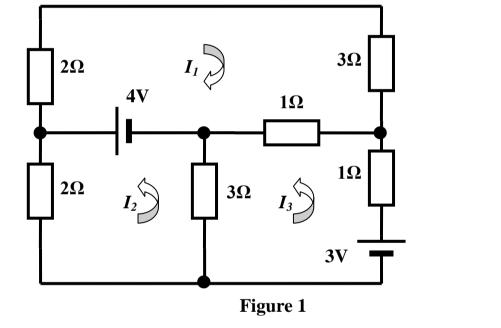
$$2x + 3y = 6$$

$$-2x + 3z = 0$$

$$x + 2y + 3z = -1$$
(3)

b.

- i) For the circuit shown in Figure 1 below, use loop current analysis (by defining closed current loops using Kirchoff's Voltage law) to form 3 equations with 3 unknowns (where the unknowns are the currents  $I_1$ ,  $I_2$  and  $I_3$ ).
- ii) Using <u>Cramer's Rule</u> only (not Gaussian Elimination) solve the simultaneous equations found in part i) to find the values of the currents  $I_1$ ,  $I_2$  and  $I_3$ .



- 3. a.
- i) Find an expression to represent  $5\cos(\omega t) 3\sin(\omega t)$  in the form  $R\sin(\omega t + \alpha)$  representing  $\alpha$  in radians.
- ii) Find an expression to represent  $-4.2\sin(\omega t) 3.1\cos(\omega t)$  in the form  $R\cos(\omega t + \alpha)$  representing  $\alpha$  in radians

**(6)** 

(11)

**b.** A circuit has a voltage across its terminals of  $v(t) = 5\sin(\omega t)$  Volts and a current flowing through it of  $i(t) = 10\sin(\omega t - \frac{\pi}{2})$  Amp.s. Therefore power (as a function of time) in this circuit can be found from the following equation:

$$P(t) = v(t).i(t) = 5\sin(\omega t).10\sin(\omega t - \frac{\pi}{2})$$
 Watts

Using trigonometric identities, show that instantaneous power (as a function of time) in this circuit can also be represented by the equation:

$$P(t) = -25\sin(2\omega t) \text{ Watts}$$
 (4)

- 4. a. A current of  $i(t) = 17 \sin(120\pi t \frac{5\pi}{6})$  mAmp.s passes through each of the following components. Determine the voltage across each component as a function of time.
  - i) A resistor of 1.2 k $\Omega$
  - ii) A capacitor of 3 µF
  - iii) An inductor of 4 H

**(6)** 

- b. A voltage supply of  $v(t) = 5\sin(\omega t)$  Volts with a frequency of 900 kHz is applied across a circuit consisting of a 47 k $\Omega$  resistor connected in parallel with a 2.7 pF capacitor.
  - i) Calculate the current (as a function of time) that flows in the resistor.
  - ii) Calculate the current (as a function of time) that flows in the capacitor.
  - iii) Draw a Phasor Diagram showing the relationship between all the currents calculated in parts i) and ii) and the voltage, all on the same diagram.
  - iv) Calculate the total current taken from the supply. (12)
- **5. a.** The general formula for a voltage decaying exponentially towards zero is:

$$v = E \cdot e^{-\frac{t}{\tau}}$$

where v is the voltage at time t, E is the voltage at time t = 0 and  $\tau$  is the time constant. As t approaches infinity ( $t \to \infty$ ) v approaches 0. Rearrange this equation to make t to be the subject.

**(2)** 

b. For the formula given in part a. above let E = 12 Volts and  $\tau = 5$  mS. Find the value of t when v = 1.6 Volts.

**(2)** 

c. Consider a voltage decaying exponentially, with a time constant  $\tau = 20$  mS, starting at E = +10 Volts and as t approaches infinity  $(t \to \infty)$  v approaches -5 Volts. Find the value of v when t = 25 mS. (4)

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## **SECTION B**

**6.** a. Write down the general expression representing the **mean** of voltage function v(t).

**(2)** 

**b.** A repeating voltage waveform with a period =10 seconds is shown in Figure 2 below:

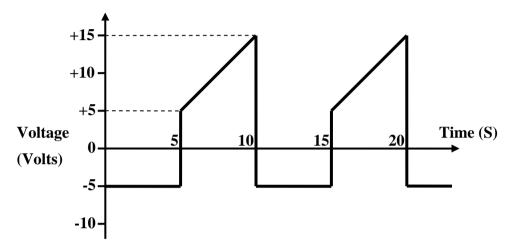


Figure 2

Write down equations to describe this voltage waveform, over one period, as a function of time. (*Hint: write down one function for the period 0 to 5 seconds and another for the period 5 to 10 seconds*). Next, using these equations, find the **mean** voltage of this waveform.

**(10)** 

**c.** Find the **Root Mean Squared** voltage for the waveform shown in part **b.** Figure 2 above. (8)

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- 7. a. Plot the following complex number points, all on the same **ARGAND** diagram:
  - i) 2 + j3

ii) 
$$j(2+j4)$$

**(2)** 

- **b.** Convert the following:
  - i) -3 + j6 into polar  $(r \angle \theta)$  form.
  - ii)  $5 \angle -127^{\circ}$  into rectangular, also called Cartesian, form (Re+iIm).

**(2)** 

**c.** The following two impedances (*Z*) each consist of two components connected in series. One component in each case is a **resistor** the other is a **reactance** (either an **inductor** or a **capacitor**). Determine the value of both components for each of the following two impedances. Assume a frequency of 100 Hz.

i) 
$$Z = (70 + j200) \Omega$$

ii) 
$$Z = 64 \angle -39^{\circ} \Omega$$

**(6)** 

d.

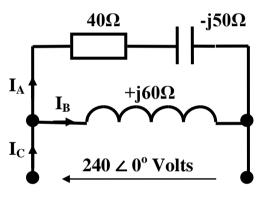


Figure 3

For the circuit shown in Figure 3 above:

- i) Find currents  $I_A$  and  $I_B$ .
- ii) Next find total current  $I_C$ .
- iii) Finally find the total impedance for the whole network.

**(10)** 

8. a. Simplify the following equation:  $3 \log(x) = \log(x^2)$ 

**(2)** 

- b. i) An amplifier has a voltage gain of +40 dB. If  $V_{in}$  = 4 Volts then find  $V_{out}$ .
  - ii) An attenuator has a **power gain** of **-6 dB**. If  $P_{out} = 3$  Watts then find power  $P_{in}$ .

**(4)** 

c. The current  $I_D$  flowing across the forward-biased **P-N** junction of a diode can be described by the equation:

$$I_D = I_S e^{\frac{V_D}{k}}$$

where  $I_s$  and k are **constants** and  $V_D$  is the voltage across the junction.

An engineer performs an experiment using a particular diode and taking a number of measurements of  $I_D$  and  $V_D$ . The values measured are shown in the Table 1 below:

$I_D(mAmp.s)$	$V_D(Volts)$
1	0.53
5	0.57
10	0.59
20	0.61
100	0.65
500	0.69
1000	0.71

Table 1

By plotting the data in Table 1 on 3-cycle Log-Linear graph paper provided show that the behaviour of the diode is consistent with an equation of the form given above.

**(7)** 

**d.** Using the data in Table 1 above, find the values of the **constants**  $I_s$  and k.

**(7)** 

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**9. a.** Given the differential equation:

$$5\frac{dy}{dx} + 2x = 3$$

i) Find the **general solution**.

ii) Find the particular solution given 
$$y = 1\frac{2}{5}$$
 when  $x = 2$ . (5)

**b.** Given the differential equation:

$$\frac{dy}{dx} = 9x^2y$$

Find the general solution and express your answer beginning y = (5)

**c.** Consider the circuit shown in figure 4 below:

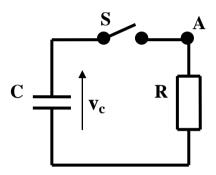


Figure 4

Assume that just before time  $\mathbf{t}=\mathbf{0}$  capacitor  $\mathbf{C}$  has been charged so that its terminal voltage  $\mathbf{v}_c$  is 10 Volts. At time  $\mathbf{t}=\mathbf{0}$  switch  $\mathbf{S}$  is closed and capacitor  $\mathbf{C}$  discharges through resistor  $\mathbf{R}$ . Using Krichoff's current law we can write the following differential equation describing the currents entering and leaving node  $\mathbf{A}$ :

$$C\frac{dv_c}{dt} = -\frac{v_c}{R}$$

Solve the equation above and use the initial conditions given ( $\mathbf{v}_c = \mathbf{10}$  Volts at  $t = \mathbf{0}$ ) to show that an equation for  $\mathbf{v}_c$  can be written in the form shown below:

$$v_c = 10 e^{-\frac{t}{RC}} \tag{6}$$

d. For the circuit described in part c. above and given  $R = 2 \text{ k}\Omega$  and  $C = 25 \mu\text{F}$ , find the value of t when  $v_c$  has fallen to 67 mVolts. (4)

#### FORMULA SHEET

#### **Trig. Identities**

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A \cos B = \cos(A-B) + \cos(A+B)$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\sin^2 \theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2}(1+\cos 2\theta)$$

## **Logarithmic Laws**

$$\log_a x^n = n \log_a x$$

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

# Integration for f(x)

$$\int \sin x = -\cos x + c$$

$$\int \sin k.x = -\frac{1}{k}\cos k.x + c$$

$$\int \cos x = \sin x + c$$

$$\int \cos k.x = \frac{1}{k}\sin k.x + c$$

$$\int \frac{1}{k} = \ln(x) + c$$

### **PLJ**

