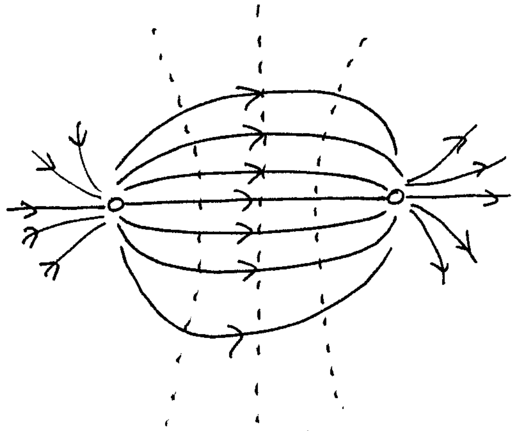
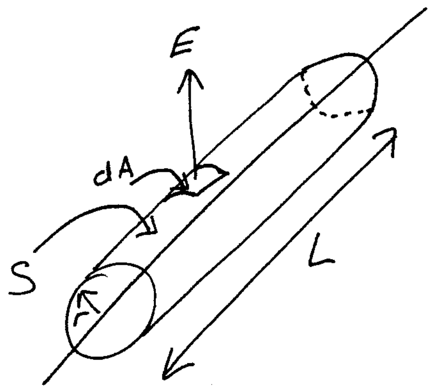
Q1.

a)



[2]

b)

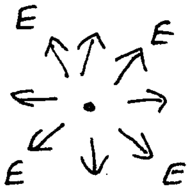


Gauss' Law

$$\oint_S E_{\perp}\, dA = Q/\varepsilon_0$$

Due to symmetry E cannot vary along wire (as ∞) or around wire.

$\therefore$ E-field must point radially outwards



when evaluating $\oint_S E_{\perp}\, dA$, ends of cylinders do not count as dA is parallel to E.

Contribution from curved part of cylinder (S) is:-

$$E_{\perp} \cdot \underbrace{2\pi r L}_{\text{surface area } S} = Q/\varepsilon_0$$

$$\therefore E = \frac{Q}{L} \cdot \frac{1}{2\pi\varepsilon_0 r}$$

$$\underline{E} = \frac{q_\ell}{2\pi\varepsilon_0 r}\,\underline{\hat{r}} \qquad \text{where } q_\ell = \frac{Q}{L} \text{ (charge per unit length)}$$

[6]

c) (i)
$$\underline{E} = \frac{q_{\ell_1}}{2\pi\varepsilon_0 R_1^2}\,\underline{R_1} + \frac{q_{\ell_2}}{2\pi\varepsilon_0 R_2^2}\,\underline{R_2}$$

$$= \frac{3\times10^{-6}}{2\pi\varepsilon_0 (2)^2}(2,0,0) - \frac{3\times10^{-6}}{2\pi\varepsilon_0 (2)^2}(-2,0,0)$$

$$= (5.39, 0, 0) \times 10^4 \text{ N}$$

[2]

(ii)
$$\underline{E} = \frac{q_{\ell_1}}{2\pi\varepsilon_0 R_1^2}\,\underline{R_1} + \frac{q_{\ell_2}}{2\pi\varepsilon_0 R_2^2}\,\underline{R_2}$$

$$= \frac{3\times10^{-6}}{2\pi\varepsilon_0 (\sqrt{8})^2}(-2,-2,0) - \frac{3\times10^{-6}}{2\pi\varepsilon_0 (\sqrt{40})^2}(-6,-2,0)$$

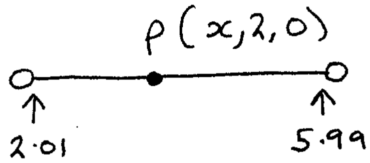
$$= (5.39, -1.08, 0) \times 10^4 \text{ N}$$

[2]

(iii) $\underline{E} = 0$ (no field inside a perfect electric conductor.)

[2]

d)



At point P:-

$|\underline{E}|$ due to wire on LHS

$$= \frac{q_\ell}{2\pi\varepsilon_0 r} = \frac{3\times10^{-6}}{2\pi\varepsilon_0(x-2)} \quad \text{to the right}$$

$|\underline{E}|$ due to wire on RHS

$$= \frac{q_\ell}{2\pi\varepsilon_0 r} = \frac{-3\times10^{-6}}{2\pi\varepsilon_0(6-x)} \quad \text{to the left}$$

$$\therefore \text{total } E_x = \frac{3\times10^{-6}}{2\pi\varepsilon_0(x-2)} - \frac{-3\times10^{-6}}{2\pi\varepsilon_0(6-x)}$$

$$= \frac{3\times10^{-6}}{2\pi\varepsilon_0}\left[\frac{1}{x-2} + \frac{1}{6-x}\right]$$

$$p.d. = \int_{2.01}^{5.99} E_x \, dx$$

$$= \frac{3\times10^{-6}}{2\pi\varepsilon_0}\left[\int_{2.01}^{5.99}\frac{1}{x-2}\,dx + \int_{2.01}^{5.99}\frac{1}{6-x}\,dx\right]$$

$$= \frac{3\times10^{-6}}{2\pi\varepsilon_0}\Big[\ln(x-2) - \ln(6-x)\Big]_{2.01}^{5.99}$$

$$= \frac{3\times10^{-6}}{2\pi\varepsilon_0}\Big[\ln(3.99) - \ln(0.01) - \ln(0.01) + \ln(3.99)\Big]$$

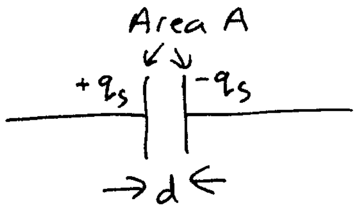
$$= 646 \text{ kV}.$$

[6]

Q2.

a) Assume field between the two plates is the same as that between two infinite sheets

$$\therefore \quad E_x = \frac{q_s}{2\varepsilon_0} + \frac{q_s}{2\varepsilon_0} = \frac{q_s}{\varepsilon_0}$$



$$p.d. = -\int_0^d E_x \, dx$$

$$= \frac{q_s d}{\varepsilon_0}$$

Let $q_s = \dfrac{Q}{A}$, thus $p.d. = \dfrac{Qd}{A\varepsilon_0}$

But $C = \dfrac{Q}{V} = \dfrac{A\varepsilon_0}{d}$ [6]

b) (i) $C = \dfrac{A\varepsilon_0}{d} = \dfrac{1\times10^{-2} \times 8.854\times10^{-12}}{1\times10^{-3}}$

$= 88.5\ pF$ [2]

(ii) $C = \dfrac{Q}{V} \Rightarrow Q = CV$

$= 88.5\times10^{-12} \times 12$

$= 1.06\times10^{-9}\ C$ [2]

(iii) energy $= \frac{1}{2} C V^2$

$$= \frac{1}{2} \times 88.5 \times 10^{-12} \times 12^2$$

$$= 6.37 \times 10^{-10} \text{ J}$$ [2]

(iv) P.T.O. [2]

c) Voltage must remain the same, so Q changes.

$$Q_{new} = C_{new} \times V$$

$$= \frac{\varepsilon_0 A}{d_{new}} \times V$$

$$= \frac{8.854 \times 10^{-12} \times 1 \times 10^{-2}}{0.5 \times 10^{-3}} \times 12$$

$$= 2.12 \times 10^{-9}$$

$$\Delta Q = Q_{new} - Q_{old}$$

$$= 2.12 \times 10^{-9} - 1.06 \times 10^{-9}$$

$$= 1.06 \times 10^{-9} \text{ C}$$

$i = \frac{dQ}{dt}$, thus average current $= \frac{\Delta Q}{t}$

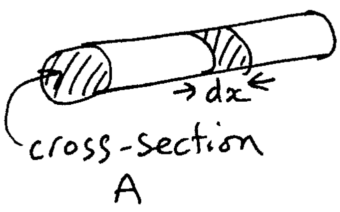
$$= \frac{1.06 \times 10^{-9}}{1 \times 10^{-3}} = 1.06 \text{ nA}$$

Charge increases on the capacitor, thus current is flowing anti-clockwise around the circuit.

[6]

Q3.

a) - Current is due to moving charged particles



Conductor has: -

n charge carriers / unit volume

q = charge of each particle

v = velocity of charges

Amount of charge in a small volume, dx thick, :-

$$dQ = qnA\,dx$$

Force acting on this charge is: -

$$d\underline{F} = dQ\,\underline{v} \times \underline{B} = qnA\,dx\,\underline{v} \times \underline{B}$$

Writing $\underline{v}\,dx$ as $v\,\underline{dx}$ gives: -

$$d\underline{F} = qnAv\,\underline{dx} \times \underline{B}$$

Current $i = \frac{dQ}{dt} = qnA\frac{dx}{dt}$

and $\frac{dx}{dt} = v$

so $i = qnAv$

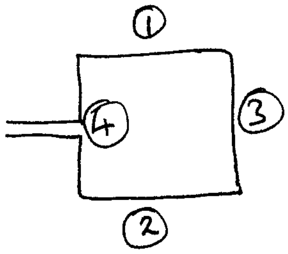
Substituting this into the equation above gives: -

$$d\underline{F} = i\,\underline{dx} \times \underline{B}$$

or $$\underline{F} = i\int_L \underline{dl} \times \underline{B}$$

[6]

b)
(i)



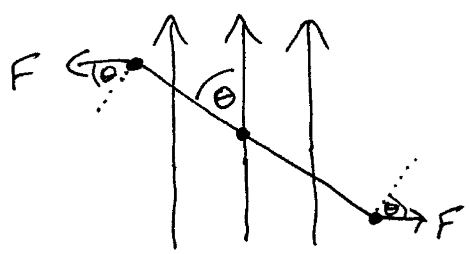
$F_{①} = ILB$
$= 0.1 \times 0.05 \times 0.5$
$= 2.5 \times 10^{-3}$ N (out of the page)

$F_{②} = ILB$
$= 2.5 \times 10^{-3}$ N (in to the page)

$F_{③} = 0$ (as wire is parallel to the field.)

[4]

(ii) Looking at the loop from the RHS...



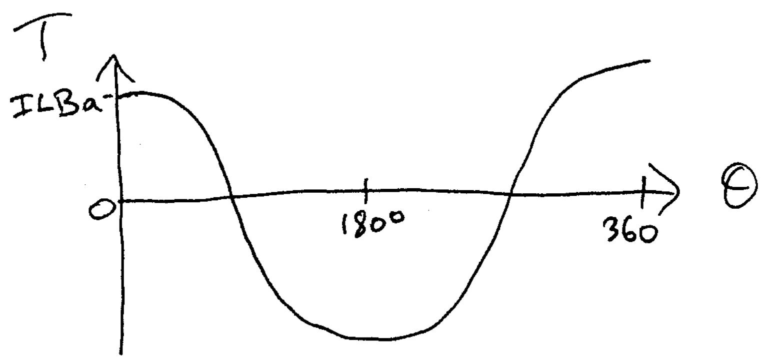
Torque = Force × distance from pivot

$\Rightarrow T = F\cos\theta \times \frac{a}{2} + F\cos\theta \times \frac{a}{2}$

$= Fa\cos\theta$

(Note that sides ③ and ④ will experience a force when $\theta > 0$, but no torque.)

$T = ILBa\cos\theta$



[6]

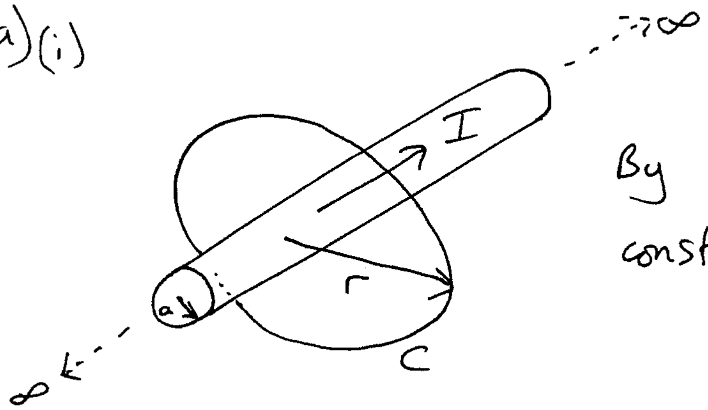
(iii)

- for $90° < \theta < 270°$, torque is in opposite direction.
- a solution is to add a commutator to change the direction of the current every $180°$
- when $\theta = 90°, 270°$, torque is zero which would prevent the motor from starting from one of these positions.
- a solution is to add a second loop at $90°$ to the first

[4]

Q4.

a)(i)



By symmetry B-field is constant around contour C.

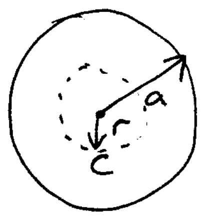
$$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 I$$

$$B \oint_C dl = \mu_0 I$$

$$B \cdot \underbrace{2\pi r}_{\text{circumference of loop.}} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$ [5]

(ii)



Assume current flow is uniform within cross-section of wire

Current within contour C $=$ total current $\times \dfrac{\text{area of C}}{\text{total area}}$

$$I' = I\left(\frac{\pi r^2}{\pi a^2}\right)$$

$$= \frac{I r^2}{a^2}$$

Using Ampère's Law :-

$$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 I'$$

$$B \cdot 2\pi r = \frac{\mu_0 I r^2}{a^2}$$

$$B = \frac{\mu_0 r}{2\pi a^2} I$$ [5]

b) (i)

$$f = \frac{\mu_0 I_1 I_2}{2\pi r}$$

for 1A wire :-

$$f_{2A} = \frac{4\pi \times 10^{-7} \times 1 \times 2}{2\pi \times 0.05} = 8 \times 10^{-6} N \quad \text{(to the right)}$$

$$f_{3A} = \frac{4\pi \times 10^{-7} \times 1 \times 3}{2\pi \times 0.1} = 6 \times 10^{-6} N \quad \text{(to the right)}$$

$\Rightarrow$ total $f = 14 \times 10^{-6} N$ (to the right)

[2]

for 2A wire: -

$$f_{1A} = \frac{4\pi \times 10^{-7} \times 1 \times 2}{2\pi \times 0.05} = 8 \times 10^{-6} N \quad \text{(to the left)}$$

$$f_{3A} = \frac{4\pi \times 10^{-7} \times 2 \times 3}{2\pi \times 0.05} = 24 \times 10^{-6} N \quad \text{(to the right)}$$

$\Rightarrow$ total f = $16 \times 10^{-6} N$ (to the right) [2]

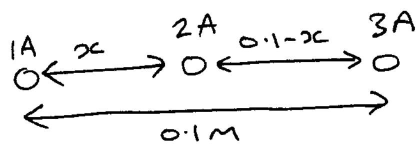
for 3A wire: -

$f_{1A} = 6 \times 10^{-6} N$ (to the left)

$f_{2A} = 24 \times 10^{-6} N$ (to the left)

$\Rightarrow$ total f = $30 \times 10^{-6} N$ (to the left) [2]

(ii)



$$f_{1A} = f_{3A}$$

$$\frac{4\pi \times 10^{-7} \times 1 \times 2}{2\pi \times (x)} = \frac{4\pi \times 10^{-7} \times 2 \times 3}{2\pi \times (0.1 - x)}$$

$$2(0.1 - x) = 6(x)$$

$$8x = 0.2$$

$$x = 0.025$$

$\Rightarrow$ 2A wire should be 2.5 cm from 1A wire

[4]