QUESTION 1

(a) (i) The reluctance of the magnetic circuit is given by:

$$S = \frac{L}{\mu_0 \mu_r A} = \frac{25\times10^{-2}\pi}{4\pi\times10^{-7}\times1000\times8\times10^{-4}} = \underline{\underline{7.81\times10^5\,H^{-1}}}$$

(ii) The flux in the core is related to the flux density by:

$$\phi = B.A = 1.5\times8\times10^{-4} = 1.2\times10^{-3}\,Wb$$

Now since:

$$I = \frac{\phi\times S}{N} = \frac{1.2\times10^{-3}\times7.81\times10^5}{900} = \underline{\underline{1.04A}}$$

(iii) The self inductance of the coil:=

$$L = \frac{N^2}{S} = \frac{900^2}{7.81\times10^5} = \underline{\underline{1.037H}}$$

(b) (i) After the slot is cut in the side:

$$S_{NEW} = \frac{(25\times10^{-2}\times\pi) - 0.005}{4\pi\times10^{-7}\times1000\times8\times10^{-4}} + \frac{0.005}{4\pi\times10^{-7}\times8\times10^{-4}}$$

$$= 7.76\times10^5 + 4.97\times10^6 \quad H^{-1}$$

$$= 5.75\times10^6\,H^{-1}$$

Hence the new level of current is given by:

$$I_{NEW} = \frac{\phi S_{NEW}}{N} = \frac{1.2\times10^{-3}\times5.75\times10^6}{900} = \underline{\underline{7.67A}}$$

(ii)

$$L = \frac{N^2}{S_{NEW}} = \frac{900^2}{5.75\times10^6} = \underline{\underline{0.14H}}$$

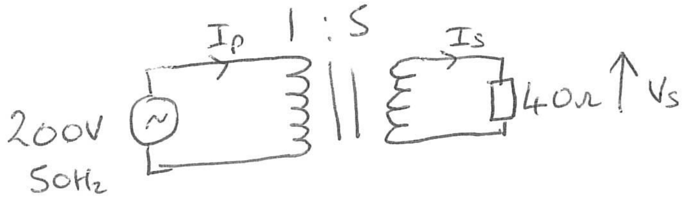
(c) The current flowing in the circuit before the switch is opened is:

$$I = \frac{V}{R} = \frac{10}{5} = 2A$$

Since the switch is opened in 1ms and the current falls to zero in this time then the voltage is given by:

$$|V_L| = L\frac{dI}{dt} = 1.037 \times \frac{2}{1\times10^{-3}} = \underline{\underline{2074V}}$$

(d) (i)



(i) Now $\frac{V_P}{V_S} = \frac{N_P}{N_S} \Rightarrow V_S = \frac{N_S V_P}{N_P} = \frac{5}{1} \times 200 = \underline{\underline{1000V_{rms}}}$

$$I_S = \frac{V_S}{R} = \frac{1000}{40} = 25A_{rms}.$$

and since $\frac{I_P}{I_S} = \frac{N_S}{N_P}$ then $I_P = I_S.\frac{N_S}{N_P} = \frac{25\times5}{1} = \underline{\underline{125A_{rms}}}$

Power dissipated $= I_S^2 R = 25^2 \times 40 = \underline{\underline{25kW}}$

(ii) The secondary now comprises a resistance of 40Ω in series with an inductor of 150mH:

$$Z_S = R + j.2\pi f L = 40 + j\,2\pi \times 50 \times 0.150$$
$$= 40 + j47.1 \equiv 61.8\angle 49.7^\circ\,\Omega$$

$$I_S = \frac{1000\angle 0}{61.8\angle 49.7} = 16.18\angle -49.7A_{rms}$$

$\therefore I_P = I_S . \dfrac{N_S}{N_P} = 16.18\angle -49.7° \times \dfrac{5}{1} = \underline{\underline{80.9\angle -49.7° \text{ Arms}}}$

Power dissipated in the load $= I_S^2 . R = 16.18^2 \times 40 = \underline{\underline{10.47\text{ kW}}}$

(check $P = V_P I_P \cos\phi = 200 \times 80.9 \times \cos 49.7 = 10.47\text{ kW}$)

(iii) The input power-factor $= \cos(49.7) = \underline{\underline{0.647 \text{ lagging}}}$

VA rating $= 200 \times 80.9 = \underline{\underline{16.18\text{ kVA}}}$

(iv) Since $V_{rms} = 4.44 f N \phi_{MAX}$

then $N_P = \dfrac{V_{Prms}}{4.44 f . \phi_{MAX}} = \dfrac{200}{4.44 \times 50 \times 5 \times 10^{-3}} = \underline{\underline{180 \text{ Turns}}}$

(v) $V_{rms} = 4.44 f N_P \phi_{max} = 4.44 \times 60 \times 180 \times 5 \times 10^{-3} = \underline{\underline{239.7 V_{rms}}}$

QUESTION 2

(a) (i) The impedance of the circuit is:

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

At resonance the imaginary terms cancel and the impedance becomes purely resistive.

Therefore

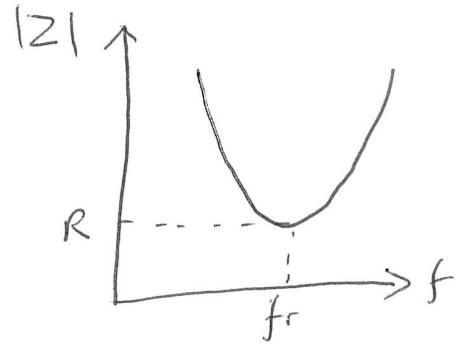
$$\omega L - \frac{1}{\omega C} = 0 \quad \text{at resonance}$$

$$\therefore \quad \omega_R L = \frac{1}{\omega_R C} \quad \Rightarrow \quad \omega_R^2 = \frac{1}{LC}$$

Since $\omega_R = 2\pi f_R$ then:

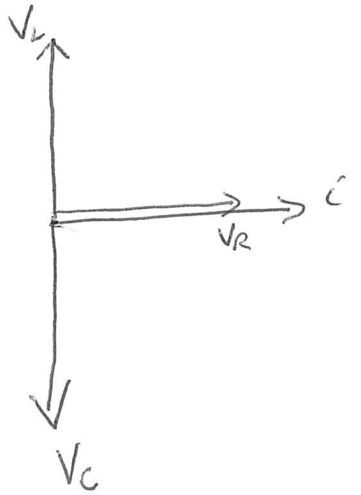
$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

(ii)



$|Z| = R$ at resonance.

(iii)



$|V_L| = |V_C|$

and $|V_S| = |V_R|$ at resonance.

(iv) $Q = \left|\frac{V_L}{V_R}\right|$ at resonance.

$$Q = \frac{I_S \omega L}{I_S R} = \frac{\omega L}{R}$$

However at resonance $\omega_R = \frac{1}{\sqrt{LC}}$

$$\therefore \quad Q = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

(v) Using $Q = 6{\cdot}86$, $f_R = 23200\,\text{Hz}$ and $R = 100\,\Omega$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow 6{\cdot}86 = \frac{1}{100}\sqrt{\frac{L}{C}}$$

$$\therefore \quad 686^2 = \frac{L}{C} \Rightarrow 470596C = L \quad (1)$$

Now $f_R = 23200 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow LC = 4{\cdot}706 \times 10^{-11}$ (2)

Substituting for L in (2) from (1)

$$470596C^2 = 4{\cdot}706 \times 10^{-11} \Rightarrow C = 0{\cdot}01\,\mu F$$

Then back substituting:

$$L = 4{\cdot}7\,mH$$

(These are the values used in the AC Circuits lab)

(b) (i) The impedance of the RL branch is:

$$Z_{RL} = R + j\omega L$$

but this is in parallel with the capacitor:

$$\therefore \frac{1}{Z} = \frac{1}{Z_{RL}} + \frac{1}{1/j\omega C} = \frac{1}{R+j\omega L} + j\omega C = \frac{1 + j\omega CR - \omega^2 LC}{R + j\omega L}$$

$$\therefore Z = \frac{R + j\omega L}{1 + j\omega CR - \omega^2 LC}$$

(ii) Rationalise the above formula to the form $a + jb$ by multiplying through by the complex conjugate of the denominator:

$$Z = \frac{(R+j\omega L)(1-\omega^2 LC + j\omega CR)}{(1-\omega^2 LC)^2 - (\omega CR)^2}$$

The denominator is now real so we need to set the imaginary terms in the numerator to zero. Multiplying out the numerator:

$$R - \omega^2 RLC - j\omega CR^2 + j\omega L - j\omega^3 L^2 C + \omega^2 LCR$$

Setting the imaginary terms to zero:

$$-\omega CR^2 + \omega L - \omega^3 L^2 C = 0$$

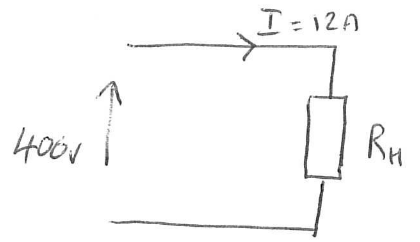
$$\therefore \omega^2 L^2 C = L - CR^2$$

$$\therefore \omega^2 = \frac{L - CR^2}{L^2 C}$$

$$\therefore \omega = \sqrt{\frac{L - CR^2}{L^2 C}} \equiv \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

ESTION 3

(a)



Initially at 15°C the heater draws a current of 12A

Hence the resistance at 15°C is;

$$R_{15} = \frac{400}{12} = 33.33\,\Omega$$

After it has reached the final (steady-state) temperature its resistance is:

$$R_F = \frac{400}{8} = 50\,\Omega$$

Now since $\dfrac{R_F}{R_{15}} = \dfrac{\cancel{R_0}}{\cancel{R_0}} \cdot \dfrac{(1+\alpha_0 \theta_F)}{(1+\alpha_0 . 15)}$

Then

$$\frac{50}{33.33} = \frac{(1+6.8\times10^{-3}\theta_F)}{(1+6.8\times10^{-3}.15)} = \frac{1+6.8\times10^{-3}\theta_F}{1.102}$$

$$\therefore 6.8\times10^{-3}\theta_F = \left(\frac{50}{33.33}\times1.102\right) - 1 = 0.653$$

$$\therefore \theta_F = 96°C$$

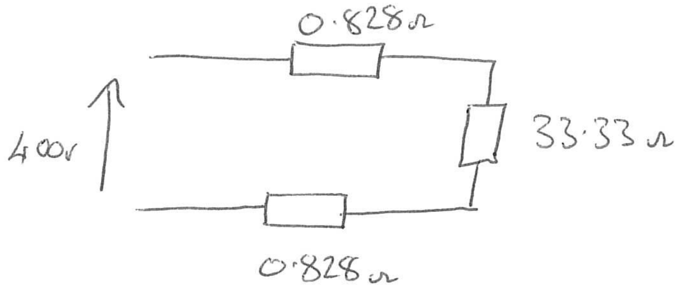
(b) (i) The length of one conductor is 120m

Cross-sectional area of the conductor is $2.5\times10^{-6}\,m^2$

$\rho = 1.725\times10^{-8}\,\Omega m$

$$R_{cond} = \frac{\rho L}{A} = \frac{1.725\times10^{-8}\times120}{2.5\times10^{-6}} = 0.828\,\Omega$$

(ii)



Total resistance of the circuit is: $R_T = 33.33 + 0.828 + 0.828$

$= 34.99\Omega$

Hence the new current is

$$I_{NEW} = \frac{400}{34.99} = \underline{\underline{11.43A}}$$

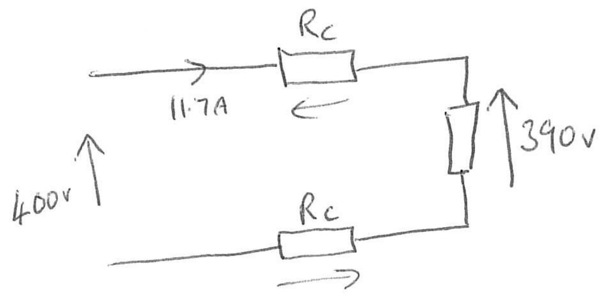
Power dissipated in the heater $= I^2R = 11.43^2 \times 33.33 = \underline{\underline{4.35kW}}$

(iii) Total power input $= VI = 400 \times 11.43 = 4.57kW$

Hence efficiency $= \frac{4350}{4570} \times 100 = \underline{\underline{95.2\%}}$

(iv) If the voltage across the heater is fixed at 390V the the current flowing will be

$$I = \frac{390}{33.33} = 11.7A$$



Applying K's law

$400 - 11.7R_c - 390 - 11.7R_c = 0$

$\therefore R_c = 0.427\Omega$

Hence the minimum cross-section is given by:

$$A = \frac{\rho L}{R_c} = \frac{1.725 \times 10^{-8} \times 120}{0.427} = \underline{\underline{4.85mm^2}}$$

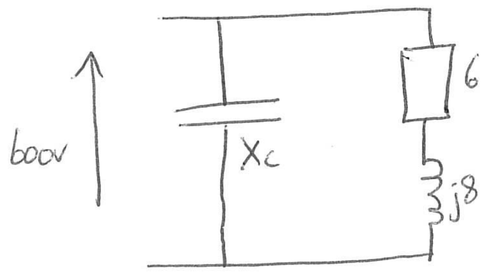
(c)(i) $Z = 6 + j8\,\Omega = 10\angle 53.13^\circ\,\Omega$

Hence current $= \dfrac{600\angle 0^\circ}{10\angle 53.13^\circ} = \underline{\underline{60\angle -53.13^\circ\ A_{rms}}}$

(ii) KVA rating $VA = 600 \times 60 = \underline{\underline{36\,kVA}}$

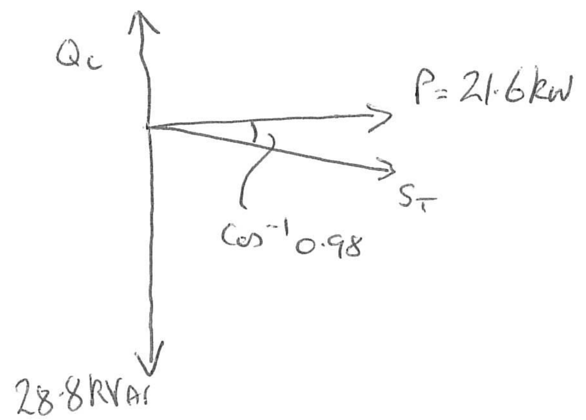
(iii) The reactive power $= I^2 X = 60^2 \times 8 = \underline{\underline{28.8\,kVAr}}$

(iv)



Capacitor must be connected in parallel with the load so that the voltage across the equipment remains at 600V.

When the capacitor is connected the real power remains constant.



The KVArs after the capacitor is added $= 21.6 \times \tan(\cos^{-1} 0.98)$
$= 4.386\,kVAr$

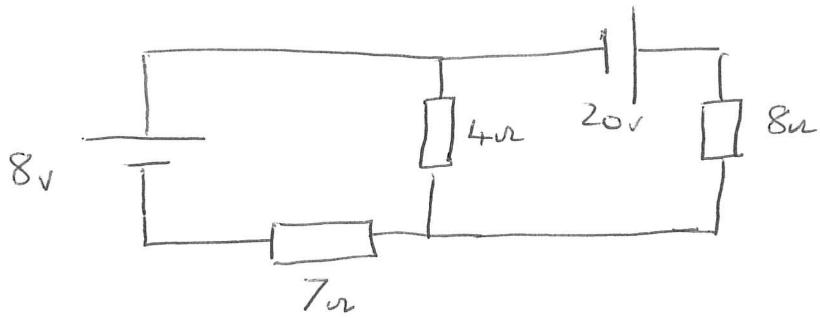
Hence the capacitor supplies $28.8 - 4.386 = 24.4\,kVAr = Q_c$

The reactance of the capacitor $X_c = \dfrac{V^2}{Q_c} = \dfrac{600^2}{24400} = 14.75\,\Omega$

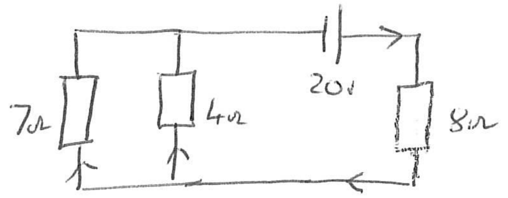
Hence $C = \dfrac{1}{2\pi f X_c} = \dfrac{1}{2\pi . 50 . 14.75} = \underline{\underline{216\,\mu F}}$

QUESTION 4

(a)



First consider the 20v battery; short out the 8v battery:



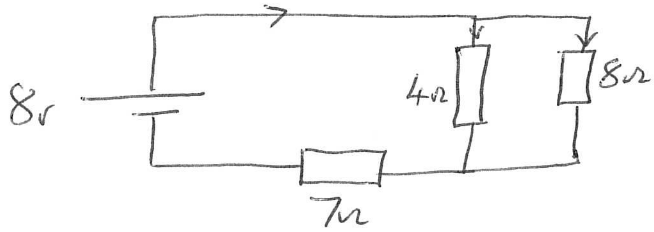
$$R_T = 8 + \frac{1}{\frac{1}{7}+\frac{1}{4}} = 10.545\Omega$$

$$\therefore I_T = \frac{20}{10.545} = 1.896A$$

Therefore the current through the 4Ω resistor from the 20v source is:

$$I_{4\Omega} = \frac{1.896 \times 7}{4+7} = \underline{\underline{1.207A\uparrow}}$$

Now consider the 8v battery; Short out the 20v battery.



$$R_T = 7 + \frac{1}{\frac{1}{4}+\frac{1}{8}} = 9.67\Omega$$

$$\therefore I_T = \frac{8}{9.67} = 0.828A$$

Therefore the current through the 4Ω resistor from the 8v source is:

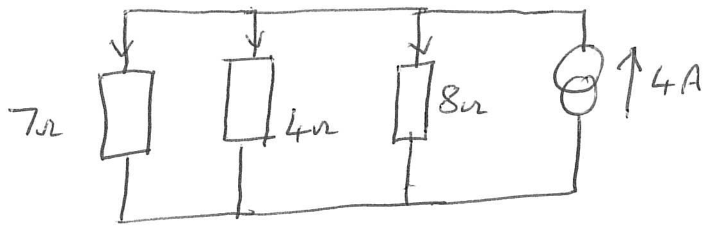
$$I_{4\Omega} = 0.828 \times \frac{8}{4+8} = \underline{\underline{0.552A\downarrow}}$$

Therefore the total current through the 4Ω resistor from both sources is:

$$I_{4\Omega T} = 1.207\uparrow + 0.552A\downarrow = 1.207\uparrow - 0.552\uparrow$$

$$= \underline{\underline{0.655A\uparrow}}$$

(b) Using the analysis from part (a) it is only necessary to find the additional contribution from the current source: Short out the other voltage sources



$$R_T = \frac{1}{\frac{1}{7} + \frac{1}{4} + \frac{1}{8}} = \underline{1.931\,\Omega}$$

Voltage across the resistors is therefore

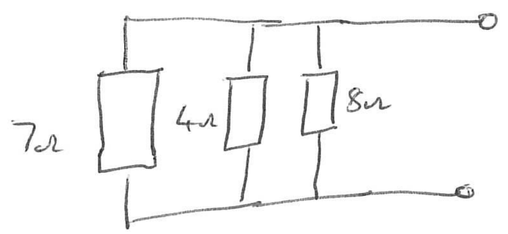
$$V_T = I \times R = 1.931 \times 4 = 7.724\,V \uparrow$$

$$\therefore I_{4\Omega} = \frac{7.724}{4} = 1.931\,A \downarrow$$

Hence the total current in the 4Ω due to all 3 sources is:

$$I_{4\Omega T} = 1.931 \downarrow + 0.655 \uparrow = \underline{\underline{1.276\,A \downarrow}}$$

(c) To find the Thevenin circuit first short out the voltage sources and find the resistance looking into the terminals:



$$\leftarrow R_T = \frac{1}{\frac{1}{7} + \frac{1}{4} + \frac{1}{8}} = \underline{\underline{1.931\,\Omega}}$$

The Thevenin voltage is the voltage between A and B without the load connected. In this circuit it is the voltage across the 8Ω resistor. First find the current through the 8Ω resistor using results from part (a)

Current through the 8Ω resistor due to 20V source is

$$I_{8\Omega} = 1.896\,A \downarrow$$

QUESTION 4 (CONTINUED)

and the current due to the 8v source is:

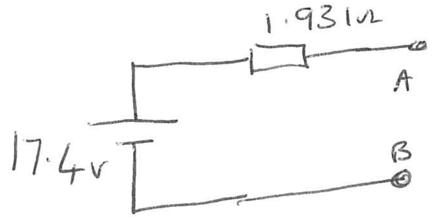
$$I_{8\Omega} = \frac{4}{12} \times 0.828 = 0.276\text{ A}\downarrow$$

Hence the total current through the 8Ω resistor is:

$$I_{8\Omega T} = 1.896 + 0.276 = \underline{\underline{2.172}}\downarrow$$

∴ the voltage across the 8Ω resistor is $2.172 \times 8 = \underline{\underline{17.4\text{V}}}\uparrow$

Therefore the Thevenin circuit appears as:

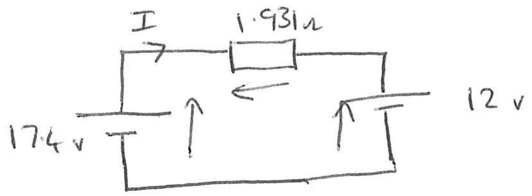


Current through a 5Ω load connected between A + B is

$$I = \frac{17.4}{5 + 1.931} = \underline{\underline{2.51\text{A}}}$$

Hence the power dissipated in the load is $= I^2R = 2.51^2 \times 5$
$= \underline{\underline{31.5\text{W}}}$

(d) When the rechargeable battery is connected the circuit becomes



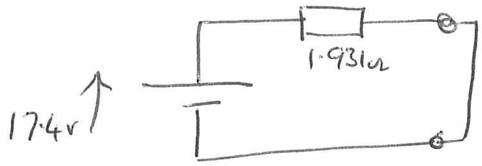
Applying Kirchoff's law:

$$17.4 - 1.931I - 12 = 0$$

$$\therefore \underline{\underline{I = 2.79\text{A}}}$$

Current is flowing into the battery so it is charging.

(e) For the Norton current short circuit the terminals of the Thevenin equivalent circuit:



$$I_N = \frac{17.4}{1.931} = \underline{\underline{9A}}$$

Hence the Norton equivalent circuit is: