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Lecture content

- Fourier Transform
  - Examples of Fourier transform

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Fourier Transform

1. Obtain the Fourier Transform of the rectangular window function in figure 4.

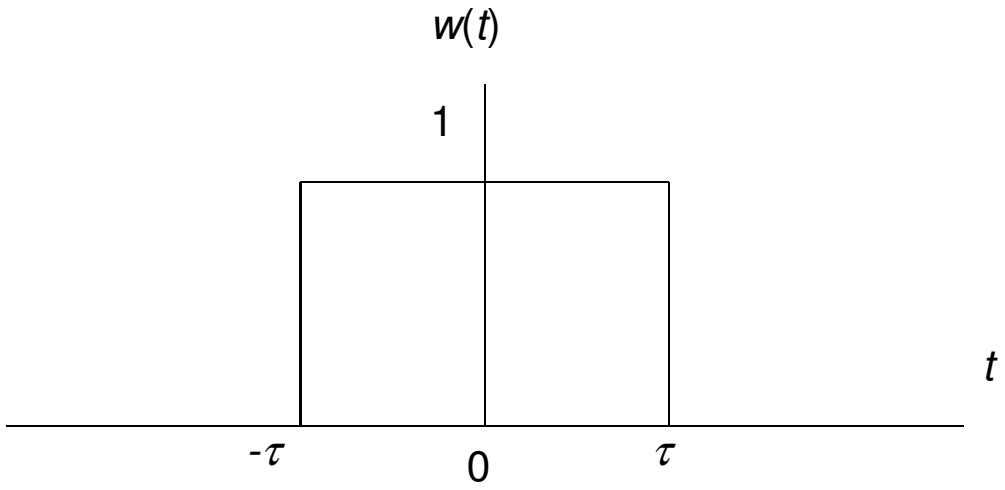

Figure 4: A rectangular window function with a duration of 2τ.

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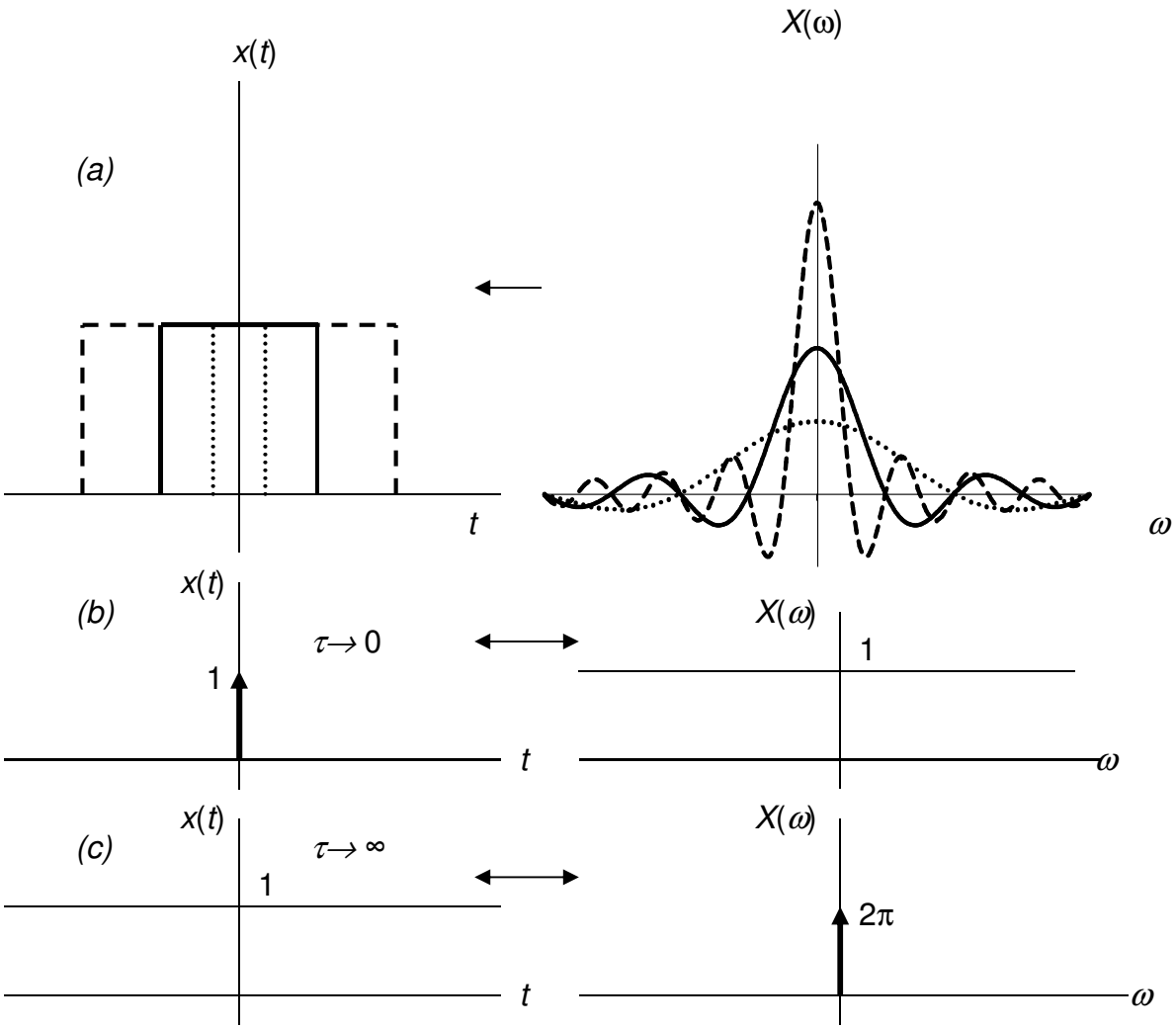
Fourier Transform



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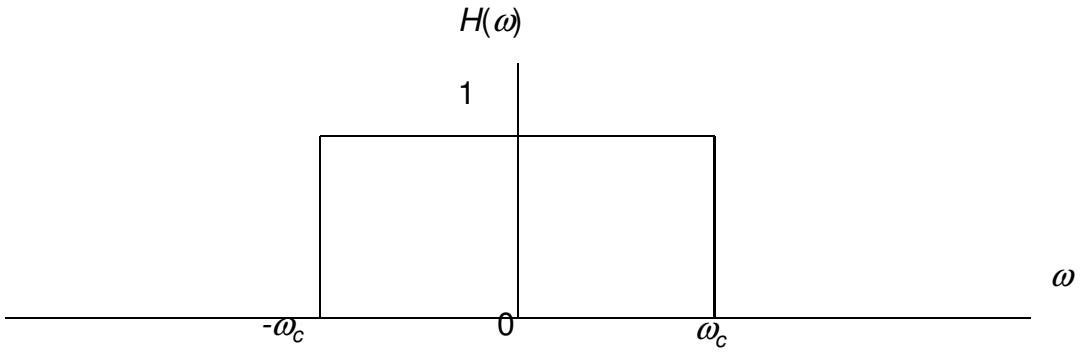
2. Compute the time function that has the magnitude spectrum (the positive half of the spectrum, $0 \leq \omega \leq \omega_c$, is an ideal low pass filter) shown in figure 6.



A rectangular spectrum defined by $H(\omega) = 1$ for $|\omega| \leq \omega_c$ and zero otherwise.

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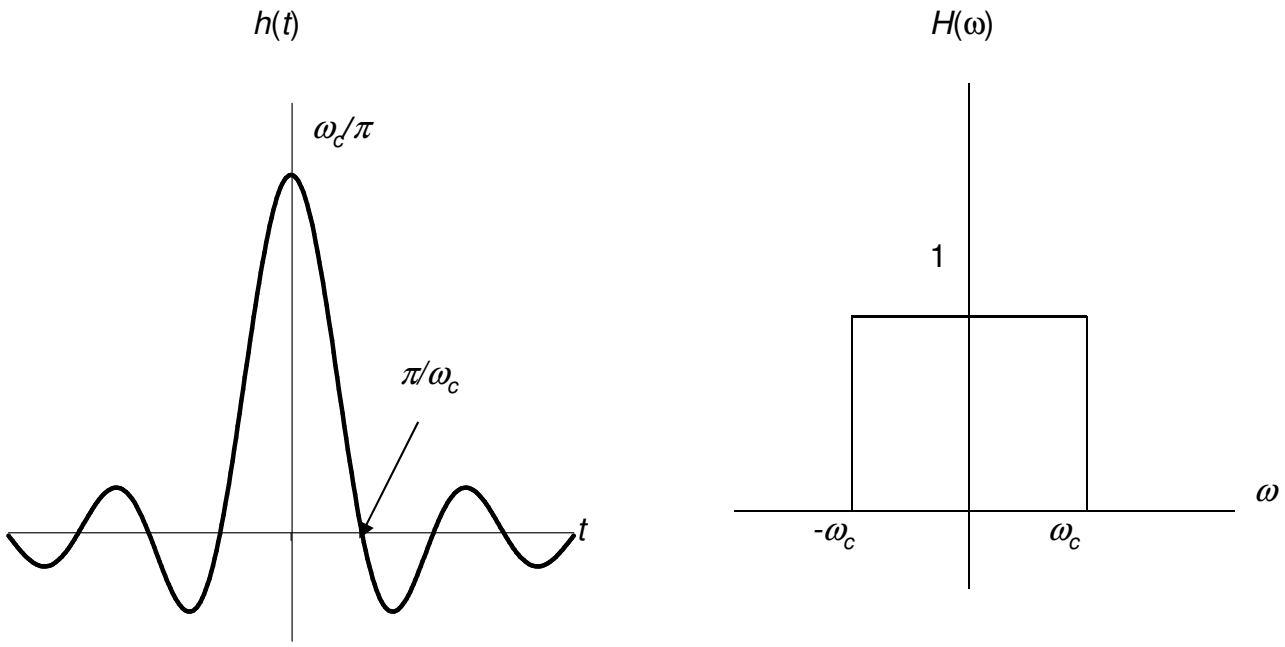

3. Verify the Fourier Transform pair $x(t) = e^{-at}u(t) \leftrightarrow$, $a > 0$.

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Lecture content

- Properties of Fourier Transform
  - Linearity
  - Time Shift
  - Frequency Shift

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Properties of Fourier Transform

Linearity

If $x_1(t) \leftrightarrow X_1(\omega)$ and $x_2(t) \leftrightarrow X_2(\omega)$
Then $ax_1(t) + bx_2(t) \leftrightarrow aX_1(\omega) + bX_2(\omega)$.

Time Shift

If $x(t) \leftrightarrow X(\omega)$ then $x(t- t_o) \leftrightarrow X(\omega)\, e^{-j\omega t_o}$

Example:
Obtain the Fourier Transform of the signal in figure 7 using the time shift property and the Fourier Transform of the signal in figure 4.

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Fourier Transform

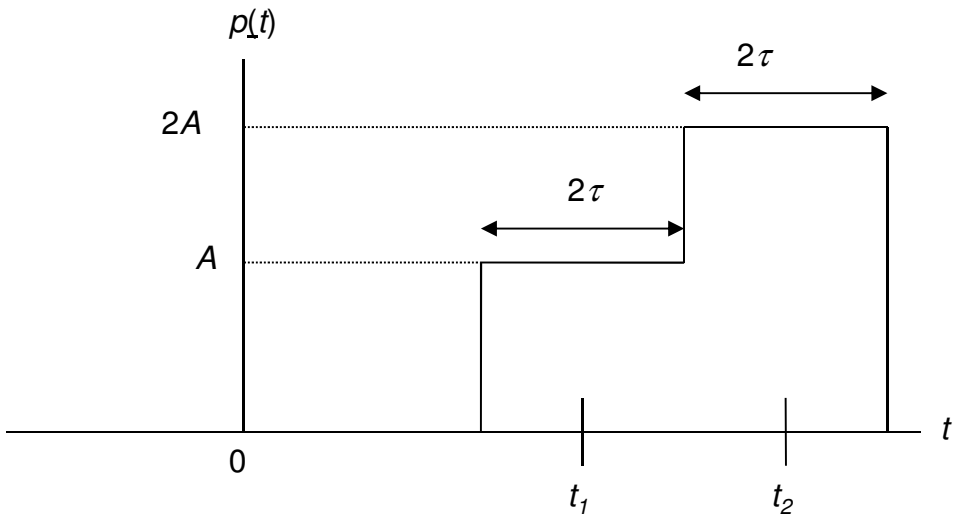

Figure 7: Signal p(t).

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Properties of Fourier Transform

Frequency Shift

If $x(t) \leftrightarrow X(\omega)$ then $x(t)e^{j\omega_o t} \leftrightarrow X(\omega - \omega_o)$

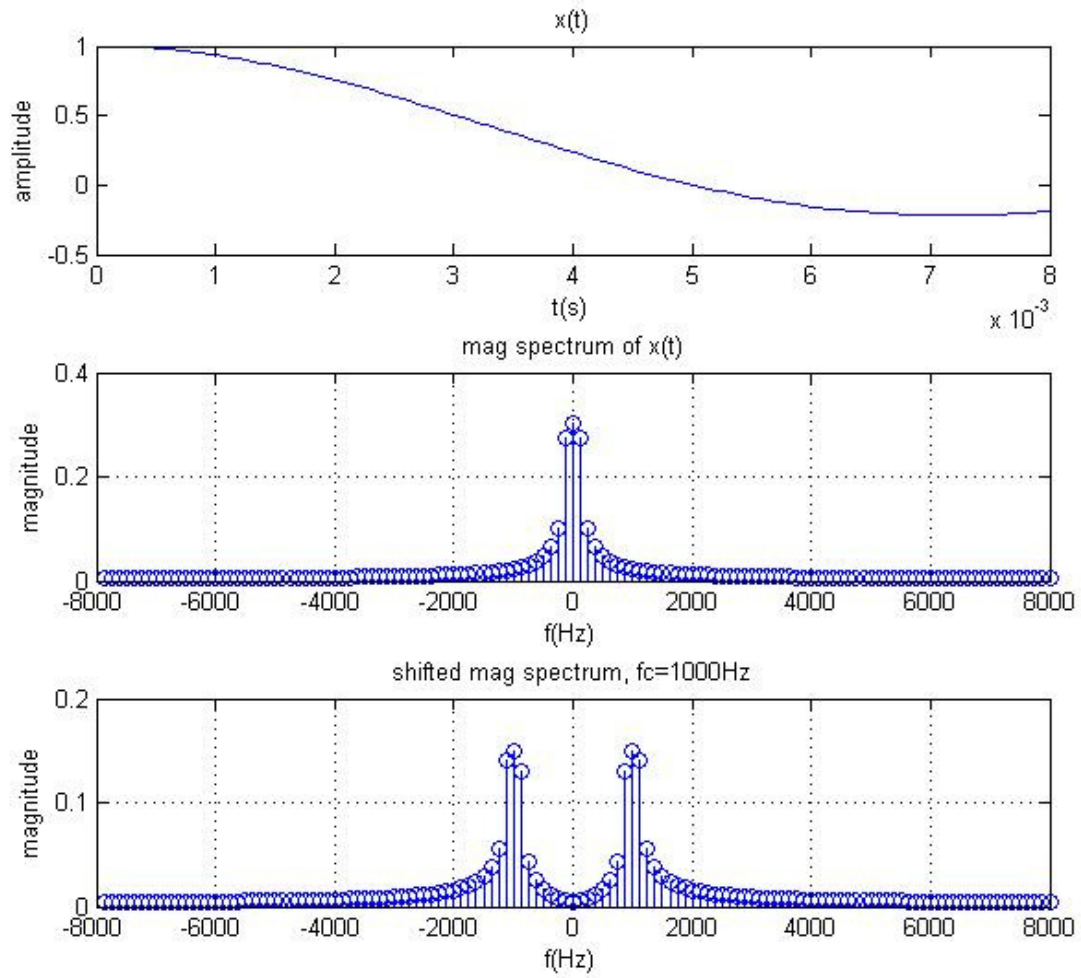
The frequency spectrum of $x(t)$ has been shifted to ω_o. If $x(t)$ is multiplied by a sinusoidal signal we have,

$$x(t)\cos\omega_o t \leftrightarrow \frac{1}{2}[X(\omega+\omega_o)+X(\omega-\omega_o)]$$

and

$$x(t)\sin\omega_o t \leftrightarrow \frac{j}{2}[X(\omega+\omega_o)-X(\omega-\omega_o)].$$

Frequency Shift




```
function FT_freq_shift(f,fc)
A=1;
fs=16000;
n=128;  %number of points
t=[1/fs:1/fs:n/fs];
x=A*(sin(2*pi*f*t))./(2*pi*f*t);    %generate a sinc function
y1=cos(2*pi*fc*t);
y=x.*y1;

%generate magnitude spectrum
k=[0:64 -63:-1];
Y=fft(y)/n;
mag_sig=abs(Y);
X=fft(x)/n;
mag_x=abs(X);
fa=(fs/n)*k;

%plot graphs
subplot(3,1,1),plot(t,x);
str1=['x(t)'];
title(str1);
xlabel('t(s)');
ylabel('amplitude');

subplot(3,1,2),stem(fa,mag_x);
str2=['mag spectrum of x(t)'];
title(str2);
grid;
xlabel('f(Hz)');
ylabel('magnitude');

subplot(3,1,3),stem(fa,mag_sig);   %phase in radian
str3=['shifted mag spectrum, fc=',num2str(fc),'Hz'];
title(str3);
grid;
xlabel('f(Hz)');
ylabel('magnitude');
```