1 (a) $\dfrac{\sqrt{9x^4x^{-2}}}{3x^{-3}} = \dfrac{\sqrt{9x^2}}{3x^{-3}} = \dfrac{\cancel{3}x}{\cancel{3}x^{-3}} = \underline{\underline{x^4}}$ 2

(b) $\dfrac{3x+6}{x^2-4}$ Factorize $\dfrac{3\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} = \underline{\underline{\dfrac{3}{(x-2)}}}$ 2

(c) $Z = \dfrac{R.x}{\sqrt{R^2+x^2}}$ $\quad \sqrt{R^2+x^2} = \dfrac{R.x}{Z}$

$$R^2+x^2 = \left(\frac{R.x}{Z}\right)^2 = \frac{R^2x^2}{Z^2}$$

$$x^2 - \frac{R^2x^2}{Z^2} = -R^2$$

$$x^2\left(1-\frac{R^2}{Z^2}\right) = -R^2 \qquad x^2 = \frac{-R^2}{\left(1-\frac{R^2}{Z^2}\right)}$$

$$x = \sqrt{\frac{-R^2}{\left(1-\frac{R^2}{Z^2}\right)}} = \sqrt{\frac{-R^2}{\frac{Z^2-R^2}{Z^2}}}$$

$$\underline{\underline{x = \sqrt{\frac{-R^2Z^2}{Z^2-R^2}}}} \quad \left(\text{or } \underline{\underline{\sqrt{\frac{R^2Z^2}{R^2-Z^2}}}}\right)$$ 2

(d) $y = x^3\cos(x)$ $\quad$ let $y = u.v$ $\therefore$ $u = x^3$ & $\dfrac{du}{dx} = 3x^2$

& $v = \cos(x)$ $\therefore$ $\dfrac{dv}{dx} = -\sin(x)$

$$\frac{dy}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

$$= -x^3.\sin(x) + 3x^2.\cos(x)$$

$$\underline{\underline{= x^2\left(3\cos(x) - x.\sin(x)\right)}}$$ 4

10

2. (a) using ~~Sin(h)~~ $\sin(h+g) = \sin(h)\cos(g) + \cos(h)\sin(g)$

$$\sin(h-g) = \sin(h)\cos(g) - \cos(h)\sin(g)$$

adding these two together gives

$$\sin(h+g) + \sin(h-g) = \sin(h)\cos(g) + \cos(h)\sin(g) + \sin(h)\cos(g) - \cos(h)\sin g)$$

$$= 2\sin(h)\cos(g)$$

$$\therefore \sin(h)\cos(g) = \frac{1}{2}\left(\sin(h+g) + \sin(h-g)\right)$$

2

(b)

$$P = v.i = 240\sin(100\pi t) . 5\cos(100\pi t - 5\pi/6)$$

$$= 1200\sin(100\pi t) . \cos(100\pi t - 5\pi/6)$$

using trig identity $\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\therefore$

$$P = \frac{1200}{2}\left[\sin(100\pi t + 100\pi t - 5\pi/6) + \sin(100\pi t - 100\pi t + 5\pi/6)\right]$$

$$= 600\left[\sin(200\pi t - 5\pi/6) + \sin(5\pi/6)\right]$$

$$= 600\left[\sin(200\pi t - 5\pi/6) + 1/2\right]$$

$$P = 300 + 600\sin(200\pi t - 5\pi/6)$$

4

6

3/3

3.1

(a)

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} f \\ g \\ h \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \\ 1 \end{bmatrix}$$

form augmented matrix

$$\left[\begin{array}{ccc:c} 2 & 1 & -1 & -2 \\ 1 & 2 & 3 & 7 \\ -3 & -2 & 1 & 1 \end{array}\right]$$

1

make new $r_3' = r_3 + 3 \times r_2$

$$\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & 3 & 7 \\ 0 & 4 & 10 & 22 \end{bmatrix}$$

make new $r_2' = 2 \times r_2 - r_1$

$$\begin{bmatrix} 2 & 1 & -1 & -2 \\ 0 & 3 & 7 & 16 \\ 0 & 4 & 10 & 22 \end{bmatrix}$$

make new $r_3'' = r_3' - (r_2' \times 4)/3$

$$\cancel{\begin{bmatrix} 2 & 1 & -1 & -1 \\ 0 & 3 & 7 & 15 \\ 0 & 0 & \frac{2}{3} & \text{[illegible]} \end{bmatrix}} \qquad \begin{bmatrix} 2 & 1 & -1 & -2 \\ 0 & 3 & 7 & 16 \\ 0 & 0 & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

4

~~3~~

re build original matrices

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & 7 \\ 0 & 0 & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} f \\ g \\ h \end{bmatrix} = \begin{bmatrix} -2 \\ 16 \\ \frac{2}{3} \end{bmatrix}$$

3·2

from row 3

$\therefore \quad {}^2/_3 h = {}^2/_3 \quad \therefore \; \underline{\underline{h = 1}}$ (1)

from row 2

$3g + 7h = 16$

$3g + 7 = 16$

$3g = 16 - 7 = 9$

$\underline{\underline{g = 9/3 = 3}}$ (1)

Check using original equations

$2f + g - h = -2$

$-4 + 3 - 1 = -2$

$-2 = -2$ ✓

$f + 2g + 3h = 7$

$-2 + 6 + 3 = 7$

$7 = 7$ ✓

$-3f - 2g + h = 1$

$6 - 6 + 1 = 1$ ✓ (1)

From row 1

$2f + g - h = -2$

$2f + 3 - 1 = -2$

$2f = -2 - 3 + 1$

$2f = -4$

$\therefore \; \underline{\underline{f = -4/2 = -2}}$

6)

$$\begin{vmatrix} 4 & -3 & 2 \\ -2 & 1 & 0 \\ -1 & 0 & 3 \end{vmatrix}$$

Firstly using right hand column

$= 2(1) - 0 + 3(4 - 6)$

$= 2 + 3 \times -2$

$= 2 - 6$

$= \underline{\underline{-4}}$ (4)

~~Alt~~ Alternative solution using top row

$= 4(3) - (-3)(-6) + 2(1)$

$= 12 - 18 + 2$

$= \underline{\underline{-4}}$

12

4 ~~$i_3(t) = 3\sin(\omega t) + 3\cos\omega t$~~ 4·i

$i_3(t) = 3\sin(\omega t) - 2\cos(\omega t) = R\cos(\omega t + \alpha)$

using the trig identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$

let $A = \omega t$ and $B = \alpha$ hence

$\cos(\omega t + \alpha) = \cos(\omega t).\cos(\alpha) - \sin(\omega t).\sin(\alpha)$

Comparing with our original equation so

$R.\cos(\omega t + \alpha) = R(\cos(\omega t).\cos(\alpha) - \sin(\omega t).\sin\alpha)$

$= R\cos(\alpha)\cos(\omega t) - R\sin(\alpha).\sin(\omega t)$

So $R\cos(\alpha) = -2$ and $-R\sin(\alpha) = 3$
or $R\sin(\alpha) = -3$

Square each of above

$R^2\cos^2(\alpha) = 4$ and $R^2\sin^2(\alpha) = 9$

Using trig identity $\sin^2 A + \cos^2 A = 1$ then add above

$4 + 9 = R^2\cos^2(\alpha) + R^2\sin^2(\alpha) = R^2(\cancel{\cos^2(\alpha) + \sin^2(\alpha)})$ 1

$4 + 9 = R^2$

$\therefore R = \sqrt{4+9} = \sqrt{13} = 3.61$ 2

We know that $R\sin(\alpha) = -3$ and $R\cos(\alpha) = -2$

$\therefore \tan(\alpha) = \frac{\cancel{R}\sin(\alpha)}{\cancel{R}\cos(\alpha)} = \frac{-3}{-2}$

$\therefore \alpha = \tan^{-1}\left(\frac{-3}{-2}\right) = 56.3^\circ$ or 0.983 rad

$\sqrt{13}\cos(56.3^\circ) = +2$ ← not correct
$\sqrt{13}\sin(56.3^\circ) = +3$ ← not correct
} must be in wrong quadrant

As $R\sin(\alpha) = -3$ and $R\cos(\alpha) = -2$ we must be in 3rd quadrant.

$\therefore \alpha = 56.3^\circ - 180^\circ = -123.7^\circ$ 4

↑ SIGN -ve

Check ~~RZ Sin~~

$$\sqrt{13}\cos(-123.7^\circ) = -2 \checkmark$$

$$\sqrt{13}\sin(-123.7^\circ) = -3 \checkmark$$

So $i_3(t) = \sqrt{13}\cos(\omega t - 123.7^\circ)$

or $\sqrt{13}\cos(\omega t - 2.16 \text{ radians})$

or 3.61

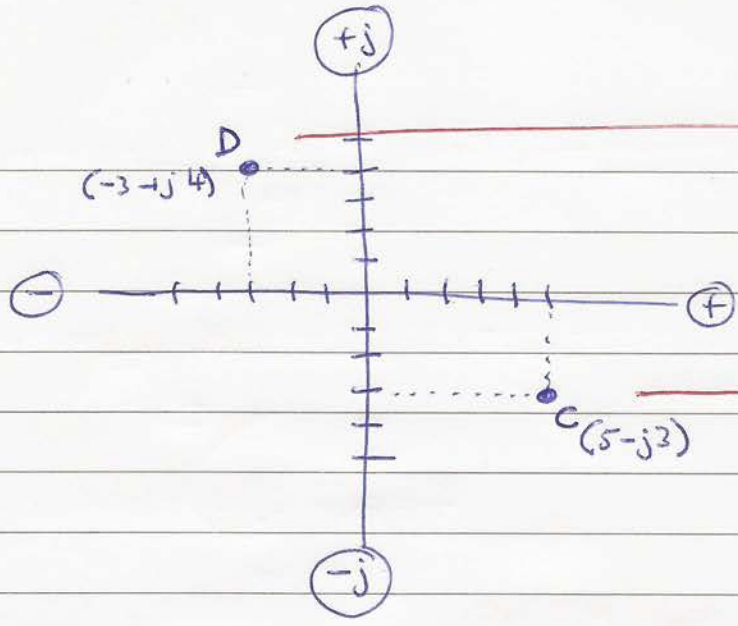
For putting in cosine form (2)

$\boxed{\cancel{4.2}}$ $\boxed{4.2}$

$\boxed{8}$

5/

(a)



1

1

$C = 5 - j3$ modulus $= \sqrt{5^2+3^2} = \sqrt{34} = 5.8$ ½ ½

argument $= \tan^{-1}\left(\frac{-3}{5}\right) = -30.1°$ or -0.54 radians ½

$D = -3+j4$ modulus $= \sqrt{3^2+4^2} = \sqrt{25} = 5$ ½

argument $\tan^{-1}\left(\frac{4}{-3}\right) =$ ~~−0.93 or 53°~~

~~but this is wrong angle~~

~~correct angle is~~

$= -53.1°$ or -0.93 radians

but this is in wrong quadrant

so

$= -53.1° - 180° = +126.9°$ ½

or $-0.93 + \pi = +2.21$ radians

(b) (i) c+d best done in cartesian form

$5 - j3$

$+ -3 + j4$

$2 + j$ $= 2+j$ ½ And $2.24\angle 26.6°$ or 0.46 rad ½

(ii) C×d better done in polar

$5.8\angle -30.1° \times 5\angle 126.9° = 29\angle 126.9 - 30.1$

5.2

$C \cdot D = 29\angle{+96.9^\circ}$ or $\angle 1.69$ radns. ½

in cartesian $= -3.5 + j\,28.8$ ½

(iii) $\frac{C}{D}$ best done in polar

$$\frac{C}{D} = \frac{5.8\angle{-30.1}}{5\angle{+126.9}} = 1.16\angle{-30.1-126.9}$$

$$= 1.16\angle{-157^\circ} \text{ or } \angle{-2.74}$$ 1

and in cartesian form

$= -1.1 - j\,0.5$ 1

(iv) jD multiplying by j do in cartesian

$$j(-3+j4) = -j3 + (j)^2 4 \qquad (j)^2 = -1 \text{ so}$$ 1

$$= -j3 - 4 \text{ or}$$

$$j \cdot D = -4 - j3 \text{ or } 5\angle{-143^\circ} \text{ or } -2.5\text{ rad}$$ 1

Checking, multiplying by j is like rotating 90° clock wise. $D = 5\angle{126.9^\circ + 90^\circ}$

$= 5\angle{+216.9^\circ}$ put in correct ℝ

$= 5\angle{+216.9 - 360^\circ}$

$j \cdot D = 5\angle{-143.1^\circ}$

~~(c)(i) $Z = \frac{V}{I} = \frac{80\angle 0^\circ}{22.36\angle{-26.57}} = 2.24\angle{+26.57^\circ}\ \Omega$~~

(c) (ii) $Z = \frac{V}{I} = \frac{50\angle 0}{2.24\angle -26.57^\circ} = \underline{\underline{22.32\angle +26.57\ \Omega}}$ — 1

or $\underline{\underline{20 + j10\ \Omega}}$ — 1

(ii) $Z = 20 + j10 = R - j\frac{1}{\omega C} + Z_{UNKNOWN}$ — (1)

$R = 20\,\Omega$, $\frac{1}{\omega C} = \cancel{\frac{1}{2\times\pi\times 35.4\mu F}}$

$= \frac{1}{2\times\pi\times F\times C} = \frac{1}{2\times\pi\times 150\times 35.4\times 10^{-6}}$

$= \frac{1}{0.03336} = 30\,\Omega$

$\therefore$ from (1) above

$Z_{UNKNOWN} = (20 + j10) - R + j30$ $\quad -(-j\frac{1}{\omega C})$ — 2

$= 20 + j10 - 20 + j30$

$= (20-20) + j(10+30)$

$Z_{UNKNOWN} = \underline{\underline{0 + j40\,\Omega}}$ or $40\angle +90^\circ\,\Omega$ — 2

$\boxed{16}$

6/

6.1

a) $Z = d\omega^a$

take logs of both sides $\log(Z) = \log(d\omega^a)$ 1

$$\log(Z) = \log(\omega^a) + \log(d)$$

$$\log(Z) = a \log(\omega) + \underbrace{\log(d)}_{\text{offset}}$$ 1

gradient (a) ; offset. 1

b)

gain in dB $= 20 \log_{10}\left(\frac{V_{OUT}}{V_{in}}\right)$ 1

c) Total gain = sum of dB values of each stage

$= +20 + 12 - 6$

$= \underline{+26 \text{ dB}}$ 2

$$\frac{V_{out}}{V_{in}} = 10^{\frac{dB}{20}} = 10^{\frac{26}{20}} = 10^{1.3}$$

$\frac{V_{out}}{V_{in}} = 19.95 \simeq 20$ Volts per Volt 2

8

7) a) i) For time from 0 to 10 seconds. ~~Ge~~ It's a straight line so need to put in form $y = mx + c$ where m is gradient and c is intercept with y axis. So...

$$m = \frac{\Delta v}{\Delta t} = \frac{5 - -10}{10} = 1.5 \text{ V/s} \qquad c = -10 \text{ V}$$

$$\therefore \underline{\underline{\underset{(0 \to 10)}{v(t)} = 1.5t - 10}} \quad — (1)$$

(2)

For time from 10 to 15 seconds...

$$m = \frac{-10 - 5}{5} = -3 \text{ V/s}$$

$$\therefore v(t) = -3t + c. \qquad \text{when } t = 10 \quad v(t) = 5$$

$$\therefore c = v(t) + 3t$$

$$= 5 + 3 \times 10$$

$$c = 35$$

$$\therefore \underline{\underline{\underset{(10 \to 15)}{v(t)} = -3t + 35}} \quad — (2)$$

(2)

(ii)

$$\text{mean} = \frac{1}{T} \int_0^T v(t).dt \qquad T = 15 \text{ s}$$

(1)

$$\text{mean} = \frac{1}{15} \left(\int_0^{10} (1.5t - 10).dt + \int_{10}^{15} (-3t + 35).dt \right)$$

$$\text{mean} = \frac{1}{15} \left(\left[\frac{1.5t^2}{2} - 10t \right]_0^{10} + \left[\frac{-3t^2}{2} + 35t \right]_{10}^{15} \right)$$

$$\text{mean} = \frac{1}{15} \left(\left[\frac{150}{2} - 100 \right] - \left[0 \right] + \left[\frac{-675}{2} + 525 \right] - \left[\frac{-300}{2} + 350 \right] \right)$$

$$\text{mean} = \frac{1}{15}\left(\left(\frac{-50}{2}\right) + \left(\frac{-675+1050}{2}\right) - \left(\frac{-300+700}{2}\right)\right)$$

$$\text{mean} = \frac{1}{15}\left(\left(\frac{-50+375-400}{2}\right)\right)$$

$$\text{mean} = \frac{1}{15}\left(\left(\frac{-75}{2}\right)\right)$$

Note.

$$\text{mean} = \frac{-75}{30} = \frac{-15}{6} = \frac{-5}{2} \text{ or } -2.5 \text{ Volts}$$ (5)

(6)

$$i(t) = 2\cos(\omega t) - 3$$

$$\text{Rms of } i(t) = \sqrt{\frac{1}{T}\int_0^T i(t)^2 \, dt} \quad \text{— (1)}$$ (1)

$$(i(t))^2 = (2\cos(\omega t) - 3)(2\cos(\omega t) - 3)$$

$$(i(t))^2 = 4\cos^2(\omega t) - 6\cos(\omega t) - 6\cos(\omega t) + 9$$ (1)

Use trig substitution $\cos^2(\theta) = \frac{1}{2}(1+\cos(2\theta))$ so ... (1)

$$(i(t))^2 = \frac{4}{2} + \frac{4}{2}\cos(2\omega t) - 12\cos(\omega t) + 9$$

$$(i(t))^2 = 2\cos(2\omega t) - 12\cos(\omega t) + 11$$ (1)

now find mean of this function by integrating

$$\text{mean squared} = \frac{1}{T}\int_0^T (2\cos(2\omega t) - 12\cos(\omega t) + 11) \, dt$$ (1)

$$\text{mean squared} = \frac{1}{T}\left[\frac{2}{2\omega}\sin(2\omega t) - \frac{12}{\omega}\sin(\omega t) + 11t\right]_0^T$$

$$\text{mean squared} = \frac{1}{T}\left[\left[\frac{1}{\omega}\cdot\sin(2\omega T) - \frac{12}{\omega}\sin(\omega T) + 11T\right] - \left[\overset{\sin(0)=0}{0} - 0 + 0\right]\right]$$ 1

Now substitute $T = \frac{2\pi}{\omega}$

$$\text{mean squared} = \frac{\omega}{2\pi}\left[\cancel{\frac{1}{\omega}\cdot\sin\left(\frac{2\cancel{\omega}2\pi}{\cancel{\omega}}\right)} - \frac{12}{\omega}\sin\left(\frac{\cancel{\omega}2\pi}{\cancel{\omega}}\right) + \frac{11.2\pi}{\omega}\right]$$

Sin of 2π & 4π are both zero so

$$\text{mean squared} = \frac{\omega}{2\pi}\left[0 - 0 + \frac{22\pi}{\omega}\right] = \frac{\cancel{\omega}.22.\cancel{\pi}}{2.\cancel{\pi}.\cancel{\omega}}$$

mean squared = 11 1

$\therefore$ RMS $= \sqrt{11}$ 1

(c) Finding the mean of a signal that ~~go~~ goes both positive & negative in one cycle is problematic (~~for~~ for example, mean of $A\sin(\omega t) = 0!$)
Squaring the wave form function produces a function that is ~~never negative~~ all positive (or zero) so finding the mean of that squared function is more useful. To restore the consequences of the squaring, the root of the mean is found.

[FULL MARKS SIMPLY FOR MENTIONING THE POSATIVE & NEGATIVE WAVEFORM ISSUE] 2

20

8.1

8/ a)(i) $R = \frac{V}{I} \therefore I = \frac{V}{R} = \frac{25}{5} = \underline{\underline{5\,A}}$

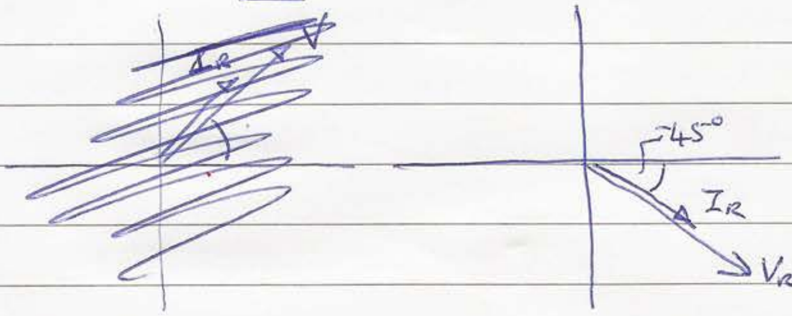
in a resistor V & I are inphase so phase angle = $\underline{\underline{-45^\circ}}$

$\underline{\underline{i(t) = 5\sin(100\pi t - 45^\circ)\ A}}$ — 1

~~(ii)~~ $\cancel{X_c = \frac{V}{I} = \frac{1}{2\pi f c}}$

$\cancel{\therefore I = 2.\pi.f.c.V = 2\times\pi\times 50\times 127\times 10^{-6}}$

$\cancel{= \underline{\underline{1A}}}$ ~~current in a capacitor~~

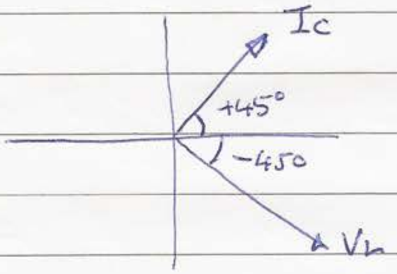


— 1

(ii) $X_c = \frac{V}{I} = \frac{1}{2\pi f C} \quad \therefore I = V.2.\pi.f.C$

$= 25.2.\pi.50.127\times 10^{-6}$

$= \underline{\underline{0.997\,A \simeq 1A}}$



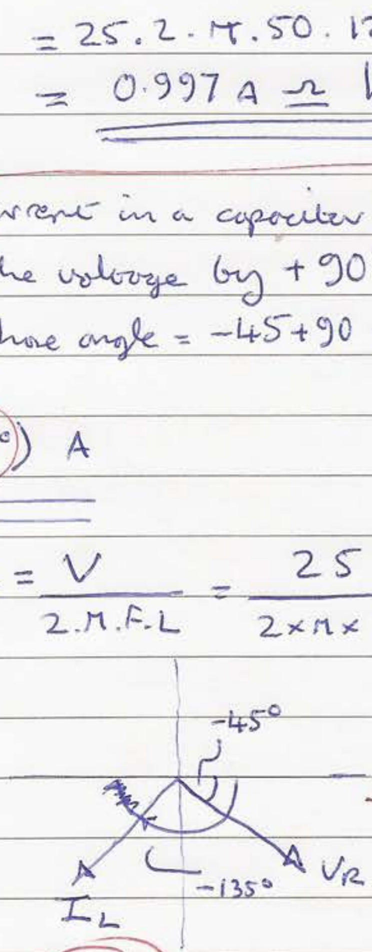
— 1

Current in a capacitor LEADS the voltage by +90° so phase angle = $-45+90 = \underline{\underline{+45^\circ}}$

$\underline{\underline{i(t) = \sin(100\pi t + 45^\circ)\ A}}$ — 1

(iii) $X_L = \frac{V}{I} = 2\pi f L \quad \therefore I = \frac{V}{2.\pi.f.L} = \frac{25}{2\times\pi\times 50\times 8\times 10^{-3}}$

$I = \frac{25}{2.51} = \underline{\underline{9.9\,A}}$



— 1

Current LAGS voltage in an inductor by -90° so phase angle = $-45^\circ - 90^\circ$

$= \underline{\underline{-135^\circ}}$

$\underline{\underline{i(t) = 9.9\sin(100\pi t - 135^\circ)}}$ — 1

8.2

6) (i) Consider series resistor & capacitor. Both experience same, I_A

Impedance of this series combination will be:

$Z = \sqrt{R^2 + X_C^2}$ & magnitude of current will be

$|I_A| = \frac{V_s}{Z}$ $R = 12\,\Omega$ $X_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 265 \times 10^{-6}} = 12\,\Omega$

$\therefore Z = \sqrt{12^2 + 12^2} = \sqrt{288} = 16.97\,\Omega$

$\therefore$ magnitude of $I_A = \frac{325}{16.97} = 19.15\,A$ 1

$\therefore$ magnitude of voltage across resistor $= V_R = I_A R = 19.15 \times 12 =$

$V_R = 234\,V$ 1

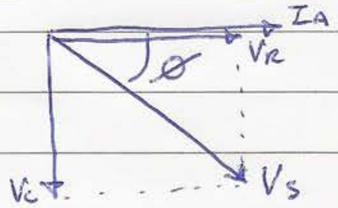
& magnitude of voltage across capacitor $V_C = I_A X_C = 19.15 \times 12$

$V_C = 234\,V$ 1

~~Vc lags by~~ V_R is inphase with I_A 1

V_C is $90°$ lagging I_A 1

(ii) The supply voltage V_s



$\tan \emptyset = \frac{V_C}{V_R}$ $\frac{(OPP)}{(ADJ)}$

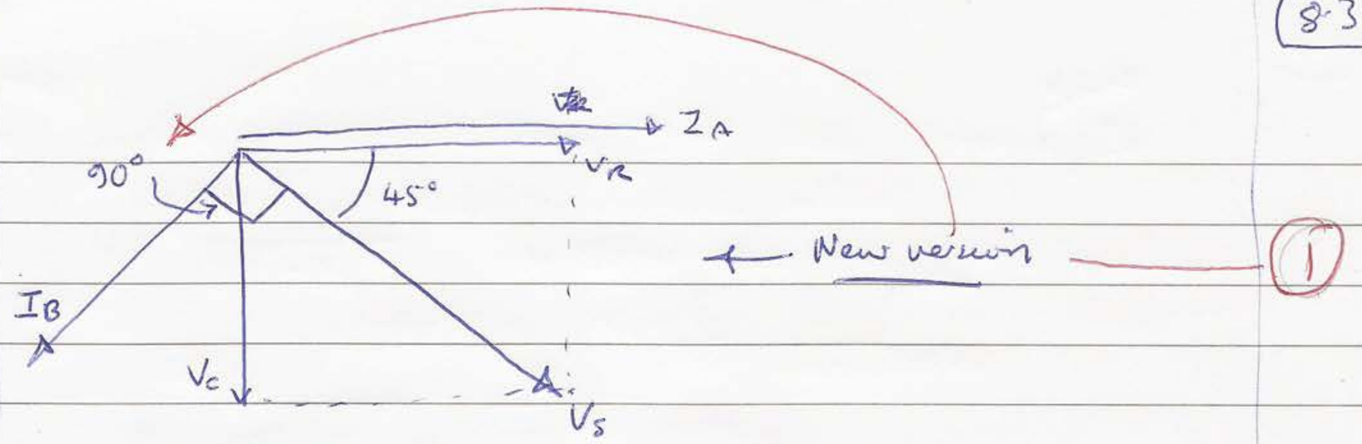
$\therefore \emptyset = \tan^{-1}\left(\frac{234}{234}\right)$

$\emptyset = 45°$ V_s lags I_A by $45°$ 1

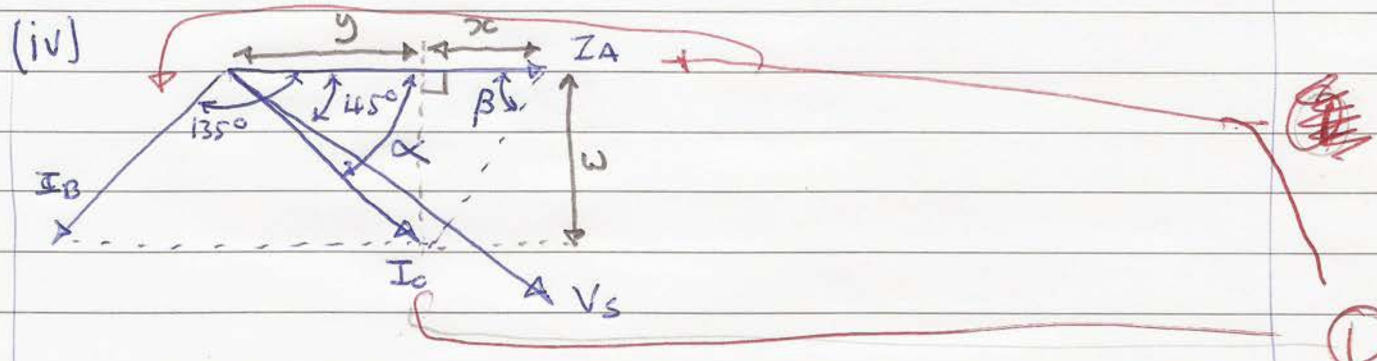
(iii) $I_B = \frac{V_s}{X_L}$ & $X_L = 100\pi \times 76 \times 10^{-3}$

$\therefore I_B = \frac{325}{100 \times \pi \times 76 \times 10^{-3}} = 13.61\,A$ I_B will lag V_s by $90°$ 1

$\therefore I_B$ lags I_A by $135°$ OR   1



(iv)



To find $\dot{I}_C$ ~~project~~ draw I_B from tip of I_A gives I_C

~~[illegible]~~ & so find length of x

$$\cos(\beta) = \frac{x}{I_B} \qquad \beta = 180 - 135^\circ = \underline{\underline{45^\circ}}$$

$$\therefore \quad x = I_B \times \cos(\beta)$$
$$= 13.61 \times \cos(45^\circ)$$
$$x = \underline{\underline{9.62\,A}}$$

$$\therefore \quad \underline{\underline{y}} = I_A - x = 19.15 - 9.62 = \underline{\underline{9.53\,A}}$$

$$\sin(\beta) = \frac{\omega}{I_B}$$
$$\therefore \quad \omega = I_B \sin(\beta)$$
$$= 13.61 \times \sin(45^\circ)$$
$$\omega = \underline{\underline{9.62\,A}}$$

$\therefore$ Phase angle α (angle between I_A & I_C)

$$\alpha = \tan^{-1}\left(\frac{\omega}{y}\right) = \tan^{-1}\left(\frac{9.62}{9.53}\right)$$

$\underline{\underline{\alpha = 45.3^\circ}}$ $\underline{\underline{I_C \text{ lags } I_A}}$ (& ≃ in phase V_S) 2

8.4

∴ ~~Ic~~ Magnitude of I_c can be found.

$$\cos(\alpha) = \frac{y}{I_c}$$

$$\therefore I_c = \frac{y}{\cos(45.3^\circ)}$$

$$= \frac{9.53}{\cos(45.3^\circ)}$$

$$I_c = \underline{\underline{13.5\ A}}$$ $\underline{\underline{45.3^\circ \text{ lagging } I_A \simeq \text{ in phase } V_s}}$

2

20

9.1

9. (a)(i) $\frac{5}{y} \cdot \frac{dy}{dx} = 2x$ rearranging $\frac{1}{y} \cdot dy = \frac{2}{5} x \cdot dx$ 1

now integrate both sides

$$\int \frac{1}{y} \cdot dy = \frac{2}{5} \int x \cdot dx$$ 1

Version (A) (C on RIGHT) | Version (B) (C on LEFT)

$\ln(y) = \frac{\cancel{2} x^2}{5 \cancel{2}} + c$ | $\ln(y) + c = \frac{x^2}{5}$

$\ln(y) = \frac{x^2}{5} + c$ | let $c = \ln(A)$

$y = e^{\left(\frac{x^2}{5} + c\right)}$ | $\ln(y) + \ln(A) = \frac{x^2}{5}$

$y = e^{\left(\frac{x^2}{5}\right)} \cdot e^{c}$ | $\ln(A \cdot y) = \frac{x^2}{5}$

let $e^c = k$ so | $A \cdot y = e^{x^2/5}$

$y = k e^{x^2/5}$ | $y = \frac{1}{A} e^{x^2/5}$ 2

(ii) Given initial conditions $y = 4$ when $x = 0$

$4 = k e^0$ | $4 = \frac{1}{A} e^0$

$k = 4$ | $\frac{1}{A} = 4$

$\therefore y = 4 e^{x^2/5}$ | $\therefore y = 4 e^{x^2/5}$ 2

(b)(i) $V_c = -CR \frac{dV_c}{dt}$ group terms

$\frac{-1}{CR} \cdot dt = \frac{1}{V_c} \cdot dV_c$ integrate both sides 1

9.2

$$-\frac{1}{CR}\int \cancel{E} \cdot dt = \cancel{\int \frac{1}{V_c} \cdot dv} \int \frac{1}{V_c} \cdot dv_c$$ ①

$$\frac{-t}{CR} = \ln(V_c) + C$$ ②

For info, source of original equation
at $t=0$ $V_R = V_c = 0$
Eqs current common to both so
$V_R = iR$ & $i = -C\frac{dv_c}{dt}$
$\therefore \frac{V_R}{R} = -C\frac{dv_c}{dt}$
$\therefore V_R = -CR\frac{dv_c}{dt}$
as $V_R = V_c$
then
$V_c = -CR\frac{dv_c}{dt}$

$$\frac{-t}{CR} - C = \ln(v_c)$$

$$e^{\left(\frac{-t}{CR} - C\right)} = v_c$$

$$e^{-t/CR} \cdot e^{-C} = v_c$$

let $e^{-C} = k$

$$v_c = k \cdot e^{-t/CR} \quad \text{—①}$$ ②

(ii) when $t = 0$, $v_c = E$ rearrange ①

$$E = k \cdot \underbrace{e^{-0/CR}}_{=1}$$

$E = k$ Sub back in equation ① ②

$$\therefore v_c = E \cdot e^{-t/CR}$$ ②

(c) (i)

$$V_c = E\left(1 - e^{-t/RC}\right) \quad \text{so} \quad \frac{V_c}{E} = 1 - e^{-t/RC}$$

$$e^{-t/RC} = 1 - \frac{V_c}{E}$$

$$\frac{-t}{RC} = \ln\left(1 - \frac{V_c}{E}\right) \quad \text{so} \quad \cancel{t = RC \ln\left(1 - \frac{V_c}{E}\right)}$$

$$t = -RC \ln\left(1 - \frac{V_c}{E}\right)$$ ②

9.3

(ii) $t = -4\times10^{3} \times 20\times10^{-6} \times \ln\left(1-\frac{15}{30}\right)$

$= -80\times10^{-3} \times \ln(0.5)$

$= \underline{\underline{55.5\,mS}}$

2

20

10 (a) (i) Amplitude = 50V $\therefore V_{pk-pk}$ = 100V (1)

(ii) phase shift = $+\pi/3$ radians (or $+60°$) (1)

(iii) $\omega = 360\pi = 2\pi f$

$\therefore f = 180\,Hz$ (1)

(iv) $T = \frac{1}{f} = \frac{1}{180} =$ ~~0.00555~~ seconds

$= 0.00556$ or $5.56\,mS$ (1)

(b) Equivalent cosine wave for $\sin(A) = \cos(A - \pi/2)$

$\therefore 50\sin(360\pi t + \pi/3) = 50\cos(360\pi t + \pi/3 - \pi/2)$

$=$ ~~50 cos(360πt~~ $= 50\cos(360\pi t - \pi/6)$ volts (2)

(c) $i(t) = 5\sin(628t - 3\pi/4)$ A $\quad \therefore \omega = 628$ radians/second

Amplitude = 5A

(i) $R = \frac{|V|}{|I|} \therefore |V| = R|I| = 20 \times 5 = 100V$

There is zero phase change for R

$\therefore v(t) = 100\sin(628t - 3\pi/4)$ V (1) (2)

(ii) $X_L = \frac{|V|}{|I|} = \omega L \quad \therefore |V| = \omega L|I|$

$= 628 \times 16 \times 10^{-3} \times 5$

$= 50.2$ Volts

Voltage leads current in an inductor by $\pi/2$ so

$v(t) = 50.2\sin(628t - 3\pi/4 + \pi/2)$

$= 50.2\sin(628t - \pi/4)$ Volts (1) (2)

Alternate solution:

$$v(t) = L\frac{di(t)}{dt} = L\frac{d}{dt}\left(5\sin(628t - 3\pi/4)\right)$$

$$= L \times 5 \times 628 \times \cos\left(628t - 3\pi/4\right)$$

$$= L \times 5 \times 628 \times \sin\left(628t - 3\pi/4 + \pi/2\right)$$

$$= L \times 5 \times 628 \times \sin\left(628t - \pi/4\right)$$

$$= L \times 3140 \times \sin\left(628t - \pi/4\right)$$

$$= 16\times10^{-3} \times 3140 \times \sin\left(628t - \pi/4\right)$$

$$v(t) = 50.2\sin\left(628t - \pi/4\right)\ \text{Volts}$$

(iii)

$$X_C = \frac{|V|}{|I|} = \frac{1}{\omega C} \qquad \therefore |V| = X_C|I| = \frac{|I|}{\omega C}$$

$$|V| = \frac{5}{628\times 53\times10^{-6}} = \frac{5}{33.3\times10^{-3}} = 150.2\ \text{Volts}$$

In a capacitor Voltage lags Current by $\pi/2$ therefore

$$v(t) = 150.2\sin\left(628t - 3\pi/4 - \pi/2\right)$$

$$= 150.2\sin\left(628t - 5\pi/4\right)$$

$$= 150.2\sin\left(628t + 3\pi/4\right)\ \text{Volts}$$

① ②

alternative solution

$i(t) = C \frac{dv(t)}{dt} \quad \therefore \quad v = \frac{1}{C}\int i(t) \cdot dt$

So $v(t) = \frac{5}{C}\int \sin(628t - 3\pi/4) \cdot dt$

$= \frac{-5}{628C}\cos(628t - 3\pi/4)$

$= \frac{-5}{628C}\sin(628t - 3\pi/4 + \pi/2)$

$= \frac{-5}{628C}\sin(628t - \pi/4)$

$= \frac{+5}{628C}\sin(628t - \pi/4 + \pi)$

$= \frac{5}{628C}\sin(628t + 3\pi/4)$

$= \frac{5}{628 \times 53 \times 10^{-6}}\sin(628t + 3\pi/4)$

$v(t) = 150.2 \sin(628t + 3\pi/4)$ Volts

(d) $\omega = 2\pi F = 48\pi$ (or 150.8) radians per second,
If the waveforms are sinusoidal and have the same frequency ~~then~~ but exactly 180° (π radians) out of phase with one another then they will actually result in a waveform of the same frequency but with an amplitude that is the difference of the amplitudes of the two source waveforms

$\therefore 16\sin(48\pi t) + 19\sin(48\pi t - \pi)$ — 2

$= -3\sin(48\pi t)$ $\boxed{\text{or}}$ $3\sin(48\pi t - \pi)$ — 3

20