Q1 (i) $q = f_0/\Delta f = \dfrac{1.59\,\text{kHz}}{199\,\text{Hz}} = \underline{\underline{8.0}}$.

(ii) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{L.\,100\text{nF}}} = 1.59\,\text{kHz}$

or $L = \dfrac{1}{10^{-7} \times (2\pi\, 1.59\,\text{kHz})^2}$

$= \underline{\underline{100\,\text{mH}}}$

(iii) $q = \dfrac{1}{R_T}\sqrt{\dfrac{L}{C}} = \dfrac{1}{R_T}\sqrt{\dfrac{100\,\text{mH}}{100\,\text{nF}}} = \dfrac{10^3}{R_T} = 8$

$\therefore R_T = R + R_L = 100 + R_L = 125\,\Omega$

$\therefore \underline{\underline{R_L = 25\,\Omega}}$.

(iv) Voltage measured at V_R would be

$\dfrac{V_s R}{R+R_L} = \dfrac{V_s\, 100}{100+25} = \dfrac{4V_s}{5} = \underline{\underline{0.8\,V_s}}$

Voltage measured at V_C would be.

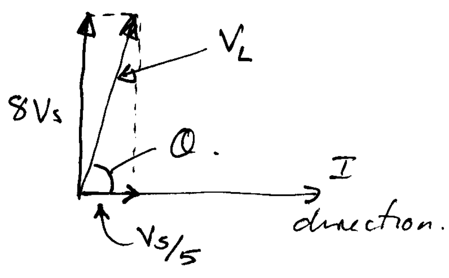
$|V_C|$ at res $= q V_s = \underline{\underline{8V_s}}$.

(v). $|\text{Voltage across ideal bit of } L| = 8V_s$.

Voltage across $R_L = V_s \cdot \dfrac{R_L}{R+R_L} = \dfrac{V_s \cdot 1}{5}$

$|V_L| = \sqrt{(8V_s)^2 + (V_s/5)^2}$

$\approx \underline{\underline{8V_s}}$.



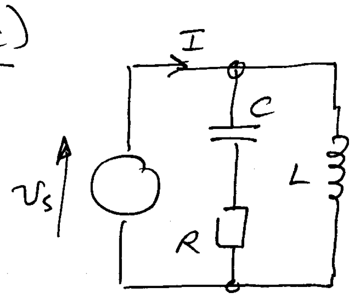
$\angle V_L = \tan^{-1} \dfrac{8V_s}{V_s/5} = \tan^{-1} 40$

$= \underline{\underline{+88.6^\circ}}$

Q2

$$Z = \frac{V_s}{I} = \frac{j\omega L\,(R + 1/j\omega C)}{j\omega L + R + 1/j\omega C}$$

$$= \frac{j\omega L\,(1 + j\omega CR)}{1 + j\omega CR - \omega^2 LC}$$



Resonance occurs when Z is real – ie j terms vanish ....

$$\underbrace{\frac{(-\omega^2 LCR + j\omega L)((1-\omega^2 LC) - j\omega CR)}{}}_{\text{real}}$$

extracting j terms ....

$$j\omega\left(CR\,LCR\,\omega^2 + L(1-\omega^2 LC)\right) = 0$$

$$\therefore\ C^2R^2\omega^2 L + L - \omega^2 L^2 C = 0$$

$$\omega^2(C^2R^2L - L^2C) = -L$$

$$\text{or}\quad \omega^2(L^2C - C^2R^2L) = L$$

$$\omega^2 = \frac{L}{L^2C - C^2R^2L} = \frac{1}{LC - C^2R^2}$$

$$\text{or}\quad \omega = \frac{1}{\sqrt{LC - C^2R^2}} \quad \text{or} \quad f = \frac{1}{2\pi}\cdot\frac{1}{\sqrt{LC - C^2R^2}}$$

Q3. The impedance of the network is

$$Z = \frac{(R_c + 1/j\omega C)(R_L + j\omega L)}{R_c + 1/j\omega C + R_L + j\omega L}$$

$$= \frac{R_cR_L + \frac{R_L}{j\omega C} + R_c j\omega L + \frac{L}{C}}{R_c + R_L + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Q3 cont..
$$= \frac{\left(R_C R_L + \frac{L}{C}\right) + j\left(\omega L R_C - \frac{R_L}{\omega C}\right)}{(R_C + R_L) + j\left(\omega L - \frac{1}{\omega C}\right)}.$$

We can make the imaginary parts of the numerator + denominator the same, except for a factor, if we make $R_C = R_L = R$. Z then becomes

$$Z = \frac{\left(R^2 + \frac{L}{C}\right) + Rj\left(\omega L - \frac{1}{\omega C}\right)}{2R + j\left(\omega L - \frac{1}{\omega C}\right)}.$$

(This is the argument needed by the real experts.... ... the experts start from here .....)

We also need to take R out of the real part of the numerator and if we do this we get.

$$Z = R.\frac{\left(R + {}^{L}/_{CR}\right) + j\left(\omega L - {}^{1}/_{\omega C}\right)}{2R + j\left(\omega L - {}^{1}/_{\omega C}\right)}$$

and the complex numerator and denominator can be made to cancel if the real parts are the same (the imaginary parts are already the same), ie

$$\text{if} \quad R + \frac{L}{CR} = 2R$$

$$\text{or} \quad \frac{L}{CR} = R \quad \text{or} \quad \underline{\underline{R = \sqrt{\frac{L}{C}}}}.$$

This will give

$$Z = R.\frac{\left(R + {}^{R^2}/_{R}\right) + j\left(\omega L - {}^{1}/_{\omega C}\right)}{2R + j\left(\omega L - {}^{1}/_{\omega C}\right)} = R$$

= purely real = resonant.

Q4 (i) at $t = 0^-$, $\underline{I = 0}$ since $\frac{dv}{dt}$ must be zero.

$t = 0^+$, $I = \frac{10 - V_c}{2k\Omega} = \frac{10 - 0}{2k\Omega} = \underline{5mA}$ since at $t = 0^+$ there has not been any time to allow charge to build up in C.

$t \Rightarrow \infty$ $\underline{I = 0}$ since, once again, $\frac{dv}{dt}$ must be zero.

(ii) at $t = 0^-$, $\underline{V_C = 0}$ since $V_S = 0$ and there is no voltage drop across R (since $I = 0$).

$t = 0^+$, $\underline{V_c = 0}$ since there has been insufficient time for the charge in C to change.

$t \Rightarrow \infty$, $\underline{V_c \Rightarrow 10V}$ since all transient effects will have settled down, $\frac{dV_c}{dt} = 0$ so $I_C = 0$ so no voltage drop across R

Q5 (i) I_L at $t = 0^- \ldots = \frac{3V}{2k\Omega} = \underline{1.5mA}$. Since at $t = 0^-$ the circuit is at a steady state – ie all transient effects have died away, $I_c = 0$ and $V_L = 0$ and so all V_s appears across $2k\Omega$.

at $t = 0^+$ $\underline{I_L = 1.5mA}$. Since $I = \frac{1}{L}\int V\,dt$ I must be continuous over an infinitesimally small time interval unless V can be infinitely big for an infinitely small time.

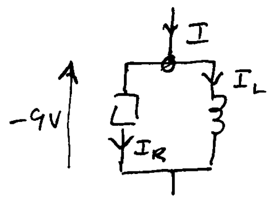
at $t \Rightarrow \infty$, $I_L = -\frac{6V}{2k\Omega} = \underline{-3mA}$. Again, all the transient effects have settled at $t \Rightarrow \infty$, L looks like a short circuit and all of V_s appears across the $2k\Omega$.

(ii) at $t = 0^-$, $I = I_L = \underline{1.5mA}$ since $V_L = 0$, all of I must flow through L

Q5 cont. at $t = 0^+$ $I = 1.5\,mA - \frac{9V}{1k\Omega} = \underline{\underline{-7.5\,mA}}$.

This is the trickiest one so far. On the transient the voltage across C remains unchanged – ie LHS is 3V +ve w.r.t. RHS. But the LHS voltage changes from +3 to −6 and if the voltage across C remains unchanged the RHS must show the same change – ie from 0V to −9V. I_L at $t=0^+$ is 1.5mA and $I = I_L + I_R = 1.5\,mA + \frac{-9}{1k\Omega}$.

Notice that V_L can change without changing I_L over small timescales.



at $t \Rightarrow \infty$ all transients will have settled and we essentially have a d.c. problem with $V_s = -6$

$\therefore$ $I_L = I$ as $t \Rightarrow \infty$ $= \frac{-6}{2k\Omega} = \underline{\underline{-3\,mA}}$.

(ii) at $t=0^-$ $\underline{\underline{V_L = 0\,V}}$; the problem is a d.c. one.

at $t = 0^+$ $\underline{\underline{V_L = -9V}}$ as described for the $t=0^+$ part of (ii).

at $t \Rightarrow \infty$ $\underline{\underline{V_L = 0\,V}}$ because the problem is once again a d.c. problem – all transients have settled.

Q6 (i) at $t = 0^-$ the problem is a dc problem, $V_L = 0$ and $I = -6 \times \frac{2k/\!/2k}{2k + 2k/\!/2k} \cdot \frac{1}{2k} = \underline{\underline{-1\,mA}}$.

at $t = 0^+$ L maintains the level of current present at $t = 0^-$, ie $I = -1\,mA$.

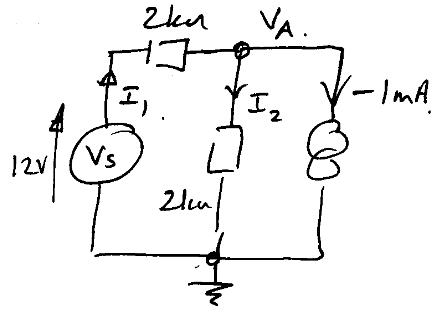
at $t \Rightarrow \infty$ the problem is once more dc

$I = +12 \frac{2k/\!/2k}{2k + 2k/\!/2k} \cdot \frac{1}{2k} = \underline{\underline{+2\,mA}}$

Q6 cont... (ii) V_R at $t=0^-$ is $-6 \times \frac{2k\Omega // 2k\Omega}{2k\Omega + 2k\Omega // 2k\Omega} = \underline{\underline{-2V}}$.

at $t=0^+$ L behaves like a current source that maintains the $t=0^-$ value of I

summing currents at V_A node at $t=0^+$



$$\frac{12 - V_A}{2k\Omega} = \frac{V_A}{2k\Omega} + (-1mA)$$

$$\frac{12}{2k\Omega} + 1mA = \frac{2V_A}{2k\Omega}$$

$$6mA + 1mA = \frac{V_A}{1k\Omega}$$

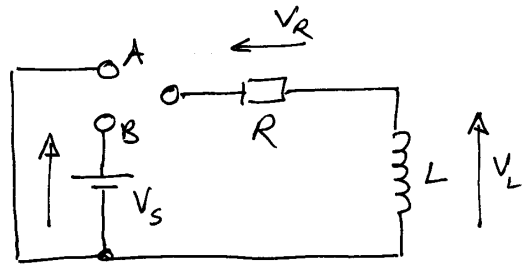
$$V_A = 7mA \times 1k\Omega = \underline{\underline{7V}} = V_R$$

at $t \Rightarrow \infty$ the problem reverts to a dc one and

$$V_R = 12 \times \frac{2k\Omega // 2k\Omega}{2k\Omega + 2k\Omega // 2k\Omega}$$

$$= \underline{\underline{4V}}$$

<u>Q7</u>. When switch switched to position A after having been in position B for a long time...



$$V_R + V_L = 0$$

$$IR + L\frac{dI}{dt} = 0 \quad \text{or} \quad \frac{dI}{I} = -\frac{R}{L}dt$$

integrating both sides gives

$$\ln I = -\frac{R}{L}t + C$$

$$\underline{\text{or}} \quad I = e^{\left(-\frac{Rt}{L} + C\right)} \equiv Ae^{-\frac{Rt}{L}}$$

Q7 cont... when $t = 0$ $I = \frac{V_s}{R}$

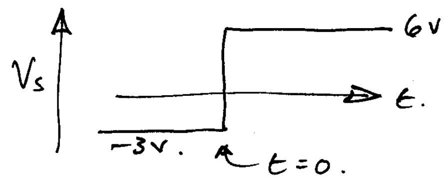
$\therefore$ $A = \frac{V_s}{R}$ and

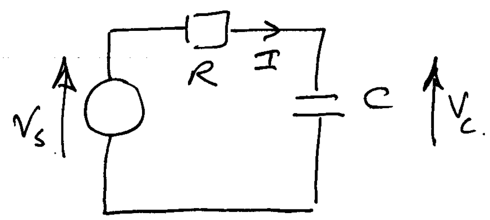
$$\underline{\underline{I = \frac{V_s}{R} e^{-t/(L/R)}}}$$

$$V_L = L\frac{dI}{dt} = L\frac{V_s}{R}\left(-\frac{1}{L/R}\right)e^{-t/(L/R)}$$

$$\underline{\underline{= -V_s e^{-t/(L/R)}}}$$

Q8





$$V_s = V_R + V_c = IR + \frac{1}{C}\int I\,dt + V_{c(0)}$$

to get this equation in differential form, differentiate both sides...

this is the initial condition for V_c to which the integrated current adds.

$$0 = R\frac{dI}{dt} + \frac{I}{C} + 0$$

or $\frac{dI}{I} = -\frac{dt}{RC}$

integrating both sides

$$\ln I = -t/RC + C$$

$$\therefore \quad I = e^{(-t/RC + C)}$$

$$\equiv A e^{-t/RC}.$$

when $t = 0^+$, $I = \frac{6 - V_{c(0)}}{R} = \frac{6-(-3)}{R} = \frac{9}{R} = A$.

$$\text{so} \quad \underline{\underline{I = \frac{9}{R}e^{-t/CR}}}$$

Q8 cont... The question actually asks you to solve the problem by developing an equation in V_c

$$V_c(t) + RC\frac{dV_c(t)}{dt} = V_s$$

V_c. $\qquad V_R = IR$, $\quad I = C\frac{dV(t)}{dt}$

or $-\frac{dt}{RC} = \frac{dV_c(t)}{(V(t) - V_s)}$ or $C - \frac{t}{RC} = \ln\left(V(t) - V_s\right)$.

or $e^{-t/RC} \cdot e^{C} = V(t) - V_s = Ae^{-t/RC}$

When $t = 0^+$ $V_c(0^+) = -3$ and $V_s = 6$

$\therefore A = -9$.

$$V_c(t) - 6 = -9e^{-t/RC}$$

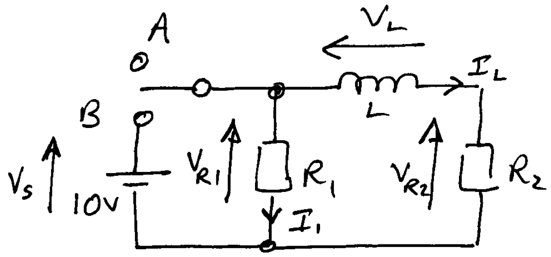
or $V_c(t) = 6 - 9e^{-t/RC} = \underline{\underline{9(1 - e^{-t/RC}) - 3}}$.

$I_c(t)$ can be derived from this result...

$$I_c(t) = C\frac{dV_c(t)}{dt} = C(-1)\left(-\frac{1}{RC}\right)9e^{-t/RC}$$

$$= \frac{9}{R}e^{-t/RC}.$$

which agrees with the direct solution of $I_c(t)$.



Q9 When switch is moved to A,

$$I_1 = -I_L$$

and

$$V_{R_1} = V_L + V_{R2}$$

$$-I_L(t) R_1 = L\frac{dI_L(t)}{dt} + I_L(t) R_2.$$

or $$-I_L(t)\left[R_1 + R_2\right] = L\frac{dI_L(t)}{dt}$$

or $$-\frac{R_1+R_2}{L}\,dt = \frac{dI_L(t)}{I_L(t)}$$

integrating both sides gives.....

$$-\frac{R_1+R_2}{L}t + C = \ln I_L(t).$$

or $$A e^{-\frac{R_1+R_2}{L}t} = I_L(t).$$

when $t=0$, $I_L(t) = V_s/R_2 = \frac{10}{R_2}$.

$$\therefore \quad \frac{10}{R_2} e^{-\frac{R_1+R_2}{L}t} = I_L(t).$$

$$V_{R_1} = -I_L(t).R_1 = -\frac{10R_1}{R_2} e^{-\frac{R_1+R_2}{L}t}$$

peak value when $t=0$ because this is the biggest value of $e^{-\frac{R_1+R_2}{L}t}$

$$\therefore \text{peak } V_{R_1} = -10\frac{R_1}{R_2} = \underline{-50V}.$$

When switch goes back to position B the circuit is governed by....

$$V_s = V_L + V_{R2}$$

$$= L\frac{dI_L(t)}{dt} + I_L(t) R_2$$

$$\frac{V_s}{R_2} = \frac{L}{R_2}\frac{dI_L(t)}{dt} + I_L(t).$$

$$-\left(I_L(t) - \frac{V_s}{R_2}\right) = \frac{L}{R_2}\frac{dI_L(t)}{dt}$$

$$-\frac{R_2}{L}dt = \frac{dI_L(t)}{I_L(t) - V_s/R_2}$$

integrating both sides ....

$$-\frac{R_2}{L}t + C = \ln\left(I_L(t) - V_s/R_2\right).$$

$$Ae^{-\frac{R_2}{L}t} = I_L(t) - V_s/R_2.$$

when $t = 0$ $I_L(t) = 0$ so $A = -V_s/R_2$.

$$\underline{\underline{I_L(t) = \frac{V_s}{R_2}\left(1 - e^{-\frac{R_2}{L}t}\right)}}.$$

Notice that the time constant for "charging" the inductor with current, L/R_2, is longer than that for discharging, $L/(R_1+R_2)$.

The fact that current takes time to rise is important in electromechanical devices such as motors and solenoids because force tends to be proportional to current. There is thus a time delay between switching on a relay by driving a voltage across its coil and the relay switch contacts operating.