EEE105 2005-6 Examination Paper -- Solutions

1.a. [Bookwork]

The diode equation is given as $J = J_0\left[\exp\left(\frac{qV}{kT}\right) - 1\right]$ where $J_0 = \frac{qL_e n_p}{\tau_e} + \frac{qL_h p_n}{\tau_h}$

Now the two terms in the equation for the saturation current, J_0, are the electron and hole saturation currents respectively. As the exponential terms will be the same the ratio of the electron and hole currents will be given by the ratio of the electron and hole saturation currents.

Thus we can write: $\frac{J_e}{J_h} = \frac{qL_e n_p}{\tau_e} \cdot \frac{\tau_h}{qL_h p_n}$

Now we can substitute $n_p = \frac{n_i^2}{N_a}$ and similarly $p_n = \frac{n_i^2}{N_d}$.

We can also use the relationship that $L_e = (D_e \tau_e)^{1/2}$ and $L_h = (D_h \tau_h)^{1/2}$

Substituting gives: $\frac{J_e}{J_h} = \frac{D_e}{N_a L_e} \cdot \frac{N_d L_h}{D_h}$

From the Einstein relation we get $D \propto \mu$ and hence $\frac{J_e}{J_h} = \frac{\mu_e N_d}{\mu_h N_a} \frac{L_h}{L_e}$

From the conductivity equation we have $\sigma \propto \mu N$ (where N is the density of donors or acceptors) and hence: $\frac{J_e}{J_h} = \frac{\sigma_n}{\sigma_p} \frac{L_h}{L_e}$ and if $L_e \approx L_h$ then $\frac{J_e}{J_h} \approx \frac{\sigma_n}{\sigma_p}$

1.b. [Bookwork - Bit more advanced]

The Emitter Injection Efficiency is a measure of what ***fraction of the total current*** across the emitter-base junction is due to holes injected into the base (as opposed to electrons injected from the base to the emitter, where they will recombine with holes reducing the hole current).
The Base Transport Factor is a measure of what ***fraction of the injected hole current*** manages to diffuse across the base and into the collector of the device. Some holes will be lost due to recombination with electrons in the base, which is undesirable.

1.c. [Problem]

The emitter injection efficiency is given by $\gamma_E = \frac{J_h}{J_h + J_e} = 1 - \frac{J_e}{J_h}$

(assuming $J_h >> J_e$ by the binomial theorem)

From 1.a. we know that $\frac{J_e}{J_h} \approx \frac{\sigma_n}{\sigma_p} = \frac{N_d \mu_e}{N_a \mu_h}$.

Substituting for the mobilities of Si and the base and emitter doping concentrations we get.

$$\frac{J_e}{J_h} \approx \frac{(7\times10^{23} \bullet 0.12)}{(7\times10^{25} \bullet 0.045)} = 2.7\times10^{-2}$$

Hence $\gamma_E = 0.973$

1.d. [Hidden - Probably quite difficult]

What is being asked is can be get a relationship between the minority carrier diffusion length ratios and the ratios of the conductivities.

We can write $\frac{L_h}{L_e} = \frac{(D_h \tau_h)^{1/2}}{(D_e \tau_e)^{1/2}}$

Now from before we know that $D \propto \mu$

However the minority carrier lifetimes are given by $\tau_h = \frac{1}{Bn}$ and $\tau_e = \frac{1}{Bp}$

We can therefore say that $\frac{\tau_h}{\tau_e} = \frac{p_{(p)}}{n_{(n)}} = \frac{N_a}{N_d}$

Substituting gives $\frac{L_h}{L_e} = \left(\frac{\mu_h N_a}{\mu_e N_d}\right)^{1/2} = \left(\frac{\sigma_p}{\sigma_n}\right)^{1/2}$

From 1.a. we had: $\frac{J_e}{J_h} = \frac{\sigma_n}{\sigma_p}\frac{L_h}{L_e} = \frac{\sigma_n}{\sigma_p} \bullet \left(\frac{\sigma_p}{\sigma_n}\right)^{1/2} = \left(\frac{\sigma_n}{\sigma_p}\right)^{1/2}$

2.a. [Bookwork]

In a conductor carriers are freely moving around the material due to their thermal energy, which translates into kinetic energy. The motion of the carriers is random and as their profile is uniform and there is no field accelerating them there will be no net motion of carriers from one region to another, thus there is no current flow.

In an insulator the electrons are not free to move as there are tied up in the chemical bonds of the material. There may be a very small number of free carriers due to one or two electrons managing to escape from their bonds due to the thermal energy.

2.b. [Bookwork]

"Drift" describes the net motion of carriers in a conductor when an electric field is applied. Carriers are accelerated by the field so there is a net motion in a particular direction. Scattering of the charge carriers occurs causing carriers to lose the velocity gained from the field at regular intervals. This balance of acceleration and scattering gives rise to the drift velocity

"Diffusion" describes the net motion of carriers in a conductor if their density is non-uniform. Carriers are all moving randomly due to their thermal velocity. As the density is higher in one region than another there will be a net motion of carriers from the high concentration region to the low concentration region leading to a diffusion current.

2.c. [Simple Problem]

This is a simple application of the equation $R = \frac{\rho l}{A}$, where all the values are given. The only thing to watch is that 1 $mm^2 = 10^{-6}$ m^2. Hence $R = \frac{1.73 \times 10^{-8} \bullet 1}{10^{-6}} = 1.73 \times 10^{-2}\,\Omega$

2.d. [Applied Bookwork - somewhat obscure]

As temperature increases the carrier concentration in copper remains the same. However, the mobility of the material must decrease due to the increased amount of scattering caused by to the increased amount of vibrations (phonons) in the material. As the mobility decreases and the carrier concentration remains the same so the resistivity of the material must increase.

2.e. [Hidden Problem -- But not too difficult]

In order to solve this problem we need to calculate the resistivity of the Cu at 320^0C, an increase from room temperature of 300^0C.

The resistivity will increase by an amount $\alpha \bullet \delta T = 3.90 \times 10^{-11} \bullet 300 = 1.17 \times 10^{-8}\,\Omega\text{m}$

Thus the new resistivity will be $(1.73+1.17)\times10^{-8} = 2.90\times10^{-8}\,\Omega\text{m}$

2.f. [Hidden]

In order to solve this we need to effectively sum all the resistances in small lengths of the wire with different temperatures together to get the total resistance. Thus we need to integrate:

$R = \frac{1}{A}\int_0^l \rho(x)dx$ (Note that if ρ is constant above we will get the normal resistivity equation.)

As the wire is 1 m long then the end where x = l is given by x = 1 m. Hence $R = \frac{1}{A}\int_0^1 \rho(x)dx$

We need first a function for $\rho(x)$. This is simple to calculate as we know the values of the resistivity at both ends of the wire:

Assuming the room temperature end is at x=0, then $\rho(0) = 1.73\times10^{-8}\,\Omega\text{m}$.

The oven is connected at the end where x=1 m, so $\rho(1) = 2.90\times10^{-8}\,\Omega\text{m}$.

Knowing that the increase in T, and therefore ρ is linear with x we get the equation: $\rho(x) = 1.73\times10^{-8} + 1.17\times10^{-8}x$

Substituting we get $R = \frac{1}{A}\int_0^1 \left(1.73\times10^{-8} + 1.17\times10^{-8}x\right)dx$

This gives $R = \frac{1}{10^{-6}}\left[1.73\times10^{-8}x + 5.85\times10^{-9}x^2\right]_0^1 = 2.32\times10^{-2}\,\Omega$.

[Note that there is a second, less formal way to do this which relies on the fact that the both the relationship for temperature with distance along the wire and the relationship for resistivity change with temperature change are both linear. It is thus possible to calculate an average temperature of the wire and an average resistivity and hence use the standard resistance equation. Students doing this are expected to state these assumptions very clearly in order to obtain full marks].

3.a. [Bookwork]

When a piece of p-type material is brought together with a piece of n-type semiconductor then there will be a concentration gradient of electrons and holes in the junction region. As a result electrons will diffuse into the p-type material and holes into the n-type material, where they will recombine with the majority carriers. However, as the electrons and holes diffuse they will leave behind their donor and acceptor atoms, which are ionised. These atoms cannot move as they are bonded into the crystal. As the diffusion progresses so the number of exposed donors and acceptors increase. This causes an increasing electric field in the junction region which acts to oppose further electron and hole diffusion. Eventually the field is sufficiently strong to set up a drift current that exactly opposes the diffusion current of electrons and holes and the diode reaches equilibrium. The depletion region is the region where all the electrons and holes have diffused and recombined, leaving only their exposed donor and acceptors behind. The layer acts like an insulator as all the free charge carriers have recombined away.

3.b. [Bookwork -- Good students can simplify the proof as follows -- other students may use the assumption that the depletion region thickness for the n-type region is negligible at a later point -- as in the lecture notes]

If the junction is n^+-p then we can ignore the contribution to the depletion region thickness from the n-type side of the junction.

In the depletion region the charge density on the p-type side will be $-qN_a d_p$

Poisson's Equation for this side of the junction will be $\frac{dE}{dx} = -\frac{qN_a}{\varepsilon}$

Thus it is trivial to get $E(x) = -\frac{qN_a x}{\varepsilon} + C$

Now let us assume that the junction is at x=0, that the p-type material is on the left hand side of the junction (where x is negative) and that the E-field is zero outside the depletion region, i.e. for $x < -d_p$.

We have the boundary condition that $E_{(x=-d_p)} = 0$

Thus we can get the equation $E(x) = -\frac{qN_a(x+d_p)}{\varepsilon}$ and that $E(0) = -\frac{qN_a d_p}{\varepsilon}$

Now the built in potential is given by $V_0 = \int E(x)dx$ in the depletion region.

This is simply the area under the $E(x)$ function, which will, ignoring the n-type side, be given by a right angled triangle of height given by the value of $E(0)$ and length given by $-d_p$

This gives $V_0 = \frac{1}{2} \bullet \frac{qN_a d_p}{\varepsilon} \bullet -d_p = \frac{qN_a d_p^2}{2\varepsilon}$

Now $d_j \approx d_p = \sqrt{\frac{2\varepsilon V_0}{qN_a}}$

3.c. [Very simple Bookwork]

Applying reverse or forward bias modifies the built-in potential to make the total potential either larger or smaller, respectively. For reverse bias we replace V_0 with $(V_0 + V)$ -- and the depletion region width becomes larger. For forward bias we replace V_0 with $(V_0 - V)$ -- and the depletion region width will shrink.

3.d. [Problem - Requires an understanding of a JFET to get out.]

For the channel to be pinched off completely we need to apply sufficient gate bias to make the depletion width the same thickness as the thickness of the p-channel.

Thus we need d_j to be equal to a.

Substituting in the equation derived in question 3.b., modified to allow for a reverse gate bias we get

$$(V_0 + V_g) = \frac{qN_a d_p^2}{2\varepsilon} = \frac{1.602\times10^{-19} \bullet 1\times10^{23} \bullet (8\times10^{-7})^2}{2 \bullet 8.85\times10^{-12} \bullet 12} = 48.3\text{V}$$

Allowing for a built-in voltage of 0.7 V then the gate bias must be 47.6 V.

3.e. [Hidden - somewhat tricky I would guess - aimed at enthusiastic students who have read around the subject]

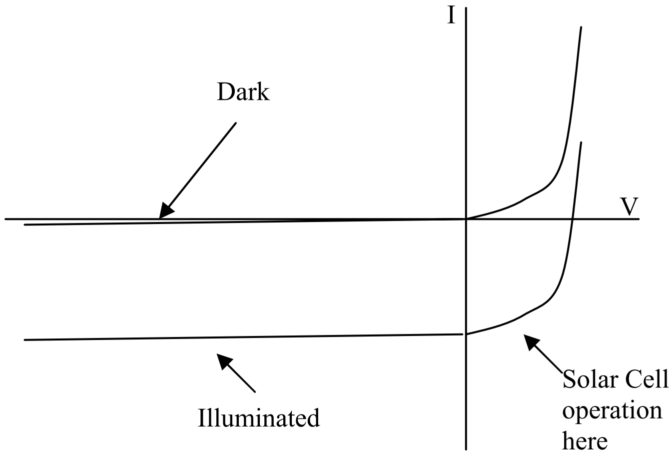
For GaAs we have the same bonding arrangement as for Si except that each Ga atom is bonded to four Arsenic atoms and vice-versa. We still have eight electrons in four bonds: three from the Ga and five from the As.

To dope it we need to consider replacing either a Ga or an As atom with a dopant. Let us consider p-doping. Ga has three outer electrons, so replacing it with a group II element such as Zn will give a hole. For n-doping As has five outer electrons so replacing it with a group VI element such as Se will give an extra electron.

4.a. [Bookwork]

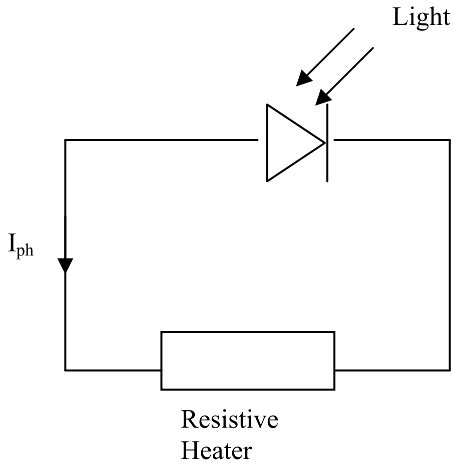
If light is shone onto the junction of a diode and absorbed, electron-hole pairs will be created. The electron-hole pairs will be spatially separated by the electric field in the p-n junction region and the holes will be driven into the p-type material and the electrons into the n-type material. This will lead to a flow of current through the diode. The current will be defined by the electron-hole pair generation rate, and should be independent of the reverse bias voltage.

For the device characteristic the photodiode can be operated under either reverse bias or zero bias. The zero bias condition is also that expected for its operation as a solar-cell.

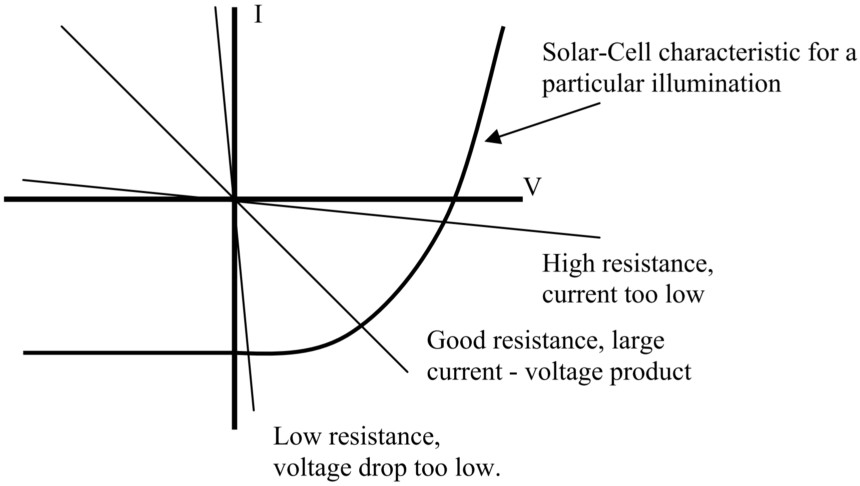


4.b. [Bookwork-slightly applied]

The simplest circuit for using the photodiode is to connect the resistive heater in series with the photodiode. There is no need for any other power supply as the light from the sun will give the power. The current flows in the opposite direction to that for a forward biased p-n junction.



4.c. [Hidden]



The power dissipated by the heater will be given by the product of the voltage across it and the current through it. As in this simple circuit the power supply is the solar-cell so the we wish to arrange the resistance such that the operating point gives the highest possible current voltage product.
If the heater resistance is very low, the voltage drop will be small and the power dissipated will consequently be small and the heating efficiency will be too low. Similarly if the heater resistance is too large then the voltage drop across it will be large and consequently the current will be very low.
We can show this by using load-lines on the I-V characteristic as shown in the figure above:

4.d. [Simple Bookwork]

A minority carrier in a semiconductor is the type of charge carrier that there are less of in the material. For example holes are minority carriers in n-type material
If we inject excess minority carriers into a semiconductor they will recombine with the majority carriers with characteristic average time called the minority carrier lifetime. The injected minority carriers also usually have a concentration gradient and therefore diffuse further into the material before recombining. The characteristic length into the material they reach is the minority carrier diffusion length.

4.e. [Applied Bookwork -- not quite so easy.]

If all the light is absorbed in the p-type material then in order for a minority carrier electron to contribute to the current it must diffuse through the p-type material to the junction and be swept into the p-type material by the internal electric field. If the electron recombines with a hole in the p-type material then the photo-generated electron-hole pair is lost and cannot contribute to the solar-cell output. Thus to get the maximum output we need to minimise the loss of electrons in the p-type material. We can do this by making the p-type layer thin so that the electrons do not need to diffuse so far to the p-n junction. We can also improve things by making the minority carrier diffusion length longer in the p-type material. This can be achieved by reducing the density of the acceptor dopants, which directly increases the minority carrier lifetime and hence the diffusion length. As the solar cell is made of a particular material it is difficult to adjust the materials mobility.