

EEE331/6037 exam 2012: exam questions and model solutions

1. single BJT circuits

4 points

- a. Derive the small signal voltage gains of an emitter follower and a common emitter without emitter degeneration, as shown below in figure 1, stating the definitions of all variables and approximations used.
Explain why the results differ despite similar connections and despite similar amplitudes of emitter and collector currents.

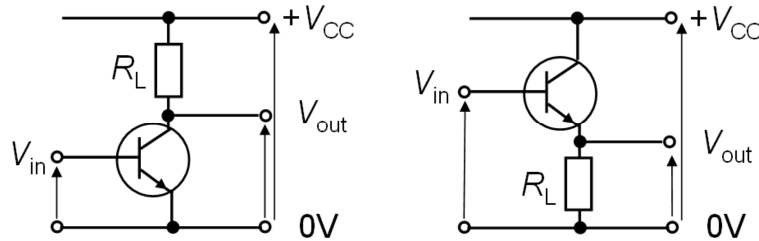


Figure 1: common emitter (CE, left) and emitter follower (EF, right)

Solution:

The circuit layouts are similar: $v_{in} = v_{BE}$ is fed into the base, the supply voltage is V_{CC} on the collector and the emitter is at ground, for the EF via a load R_L and for the CE either directly or via a degeneration resistor R_E (here: $R_E = 0$).

The EF output v_o is connected to the emitter side before the load connected between E and ground.

The CE output v_o is connected to the collector side before the load connected between V_{CC} and C.

With g_m =mutual conductance, R_L =load resistance, β =current gain, r_{XY} =resistance between X and Y terminals of the transistor we get:

Small signal voltage gain for CE:

$$G = v_o / v_{in} = -\beta i_B (r_{CE} \parallel R_L) / v_{BE} = -g_m (r_{CE} \parallel R_L) \approx -g_m R_L \quad (\text{for } r_{CE} \gg R_L)$$

Small signal voltage gain for EF:

The emitter current is $(\beta + 1)(v_{in} - v_o) / r_{BE} = i_E = v_o / R_L$, from which one gets with

$$g_m v_{BE} = \beta i_B:$$

$$G = v_o / v_{in} = (\beta + 1) / [(\beta + 1) + r_{BE} / R_L] = 1 / \{ 1 + r_{BE} / [(\beta + 1) R_L] \} = g_m R_L / [g_m R_L + \beta / (\beta + 1)] \approx 1 \quad (\text{for } \beta \gg 1 \text{ and } g_m R_L \gg 1)$$

This shows the CE circuit acts as an inverting amplifier with large voltage gain, while the EF circuit has hardly any voltage gain at all. The reason for the latter is that here $V_o = v_{BE} + V_{in} \approx V_{in}$ as v_{BE} is usually a small voltage, while in the CE configuration $v_{EB} + V_{in} + v_{BC} + V_o = V_{CC}$, i.e. $V_o = V_{CC} - v_{CE} - V_{in}$ where the sum of $(V_{in} + v_{CE})$ is **subtracted** from the supply voltage, and v_{CE} is larger than v_{BE} .

3 points

- b. The output resistance of a bipolar junction transistor (BJT) with output resistance r_o , current gain β and transconductance g_m in common emitter configuration with emitter degeneration R_E is approximately given by the expression

$$R_o = r_o [1 + \beta g_m R_E / (\beta + g_m R_E)].$$

Interpret this equation by distinguishing three different cases for the size of β relative to $g_m R_E$.

Solution:

The three cases to be considered are

- (i) $\beta \ll g_m R_E$: $R_o = r_o [1 + \beta g_m R_E / (\beta + g_m R_E)] \rightarrow r_o [1 + \beta]$
- (ii) $\beta = g_m R_E$: $R_o = r_o [1 + \beta^2 / (\beta + \beta)] = r_o [1 + \beta/2]$
- (iii) $\beta \gg g_m R_E$: $R_o = r_o [1 + \beta g_m R_E / (\beta + g_m R_E)] \rightarrow r_o [1 + g_m R_E]$

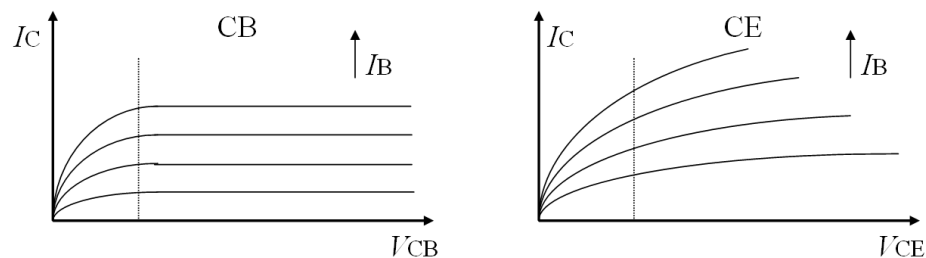
In all cases $R_o \geq r_o$, i.e. the output resistance increases but stays finite, even if $R_E \rightarrow \infty$. For $R_E = 0$ the output resistance is of course not changed ($R_o = r_o$).

7 points

- c. Sketch the common base and the common emitter output characteristic of a typical npn BJT for a set of four different base currents of I_B , $2I_B$, $3I_B$ and $4I_B$, e.g. $20\mu A$, $40\mu A$, $60\mu A$ and $80\mu A$. Neglect reverse active and breakthrough regions but show and label forward-active and saturation regions. Pay attention to and comment on the gradients of the curves, their separations and their lengths.

Solution:

Common base (CB) and common emitter (CE) output curves plot the collector current I_C vs. voltage V_{CB} or V_{CE} , respectively. These look like the following:



The steep rises left of the dotted lines mark the saturation regions, the flatter (nearly constant) regions right of them the forward-active regions.

CB curves are horizontal (as $I_C \approx I_E = \text{const.}$ for given I_B), while the CE curves show a significant slope which increases with I_B (due to increasing V_{CE} increasing the reverse bias, thus extending the CE region, decreasing the base width, which in turn increases I_C : the resistor shows a decreased resistance). The CB curves are equidistantly spaced and of same length; the CE curves become denser, steeper and shorter with increasing I_B (as the increased I_C leads to a voltage drop across the load so V_{CE} cannot reach the full supply voltage any more).

6 points

- d. Assume the collector current I_C of a BJT is approximately given by the equation $I_C = A/Q_b \exp [qV_{BE}/(kT)]$ where A is a constant, Q_b the areal density of doping atoms in the base, V_{BE} the base-emitter voltage, q the elementary charge, k Planck's constant and T absolute temperature.

Express Q_b in terms of base width w and doping (volume) density n .

From this derive an expression for the output resistance R_o .

From your expression obtained comment on the dependence of the base width on V_{CE} .

Solution:

$Q_b = wn$ from above definitions. Then the slope of the CE emitter output curve is

$$\begin{aligned} 1/R_0 &= I_C/V_A = \partial I_C / \partial V_{CE} \\ &= (\partial I_C / \partial Q_b) \times (\partial Q_b / \partial V_{CE}) \\ &= (-I_C/Q_b) \times (\partial Q_b / \partial V_{CE}) \\ &= -I_C/(wn) \times (n \partial w / \partial V_{CE}) \\ &= -I_C/w \times \partial w / \partial V_{CE} \end{aligned}$$

Inversion of both sides yields $R_0 = -w/I_C \partial V_{CE} / \partial w$, which must be a positive value. This means $\partial w / \partial V_{CE} < 0$, i.e. the finite output resistance of a forward active BJT is due to the base width shrinking with increasing V_{CE} .

Remarks: All questions are new.

2. Multiple BJT circuits

8 points

- a. The circuit shown below in figure 2 is called a cascode pair. The signal currents at various points are indicated, where i_{b1} is the base current to T_1 and β_1, β_2 are the small signal current gains of both transistors T_1 and T_2 . From this it can be seen that the small signal current gain of the cascode is $\beta_2\beta_1/(\beta_2+1) \approx \beta_1$.

Name the configurations of both transistors and describe their functions.

Calculate the approximate small signal voltage gain in terms of resistances and β_1, β_2 .

Calculate the approximate output resistance of the cascode pair if the output resistances of the individual transistors are r_{o1}, r_{o2} , using the relationship $R_o = r_{CE}[1 + \beta g_m R_E / (\beta + g_m R_E)]$ for a common emitter configuration with emitter degeneration R_E , output resistance R_o , collector-emitter resistance r_{CE} , small signal current gain β and transconductance g_m .

Consider the case $g_{m2}r_{o1} \gg \beta_2 \gg 1$.

Interpret your result in terms of noise and high-frequency transfer.

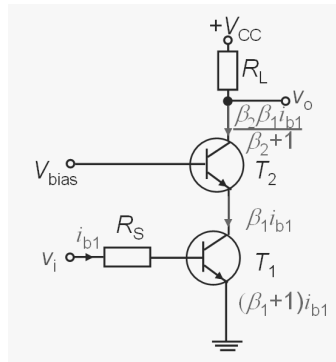


Figure 2

Solution:

T_1 is a common emitter stage and as thus provides current and voltage gain.

T_2 is common base stage which provides no gain but increases the output resistance.

Summing the voltages from input to ground yields $v_i = R_S i_{b1} + r_{BE1} (\beta_1 + 1) i_{b1}$.

The output voltage is $v_o = -i_o R_L = -\beta_1 \beta_2 / (\beta_2 + 1) R_L i_{b1}$.

The voltage gain is then $v_o / v_i = -\beta_1 \beta_2 / (\beta_2 + 1) R_L / [R_S + (\beta_1 + 1) r_{BE1}] \approx -R_L / (r_{BE1} + R_S / \beta_1)$ (for $\beta_1, \beta_2 \gg 1$) and will depend on choice of the external resistors as well as β_1 .

The output resistance can be calculated by injecting a test current into the output and shortening the input to ground. With $v_i = 0$, T_1 becomes inactive and behaves like an emitter degeneration with $R_E = r_{o1}$ hanging on the emitter of transistor T_2 which itself is then in CE configuration. Using the equation provided in question 1b then yields with $R_E = r_{o1}$ and $r_o = r_{o2}$ a total output resistance of

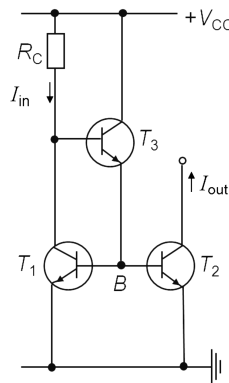
$$R_o = r_{o2} [1 + \beta_2 g_{m2} r_{o1} / (\beta_2 + g_{m2} r_{o1})].$$

For $g_{m2} r_{o1} \gg \beta_2 \gg 1$ this leads to $R_o \rightarrow \beta_2 r_{o2}$, i.e. a significantly increased output resistance. This will reduce the sensitivity of the voltage gain to ripples on the voltage power supply and also reduce capacitive feedback at high frequencies. Both will be beneficial for high-frequency transfer.

8 points

- b. (i) Draw a circuit diagram of a current mirror with transistor T_1 connected to the resistor with the incoming current, output transistor T_2 and transistor T_3 providing base current compensation to both T_1 and T_2 .
- (ii) Assuming that the base currents to transistors T_1 and T_2 are equal, prove that the ratio of output to input current for the general case that all three transistors have different individual small signal current gains of β_i ($i=1,2,3$) is given by $I_{out}/I_{in}=(\beta_2\beta_3+\beta_2)/(\beta_1\beta_3+\beta_1+2)$. Neglect the Early effect.
- (iii) Assume all transistor current gains are $\beta_i=100$ at room temperature for $i=1,2,3$. A typical temperature dependence of the small signal current gain β of a BJT may be given by $\partial\beta/(\beta\partial T)=0.007\text{ K}^{-1}$. Compare the cases where only individual transistors or pairs or all three transistors are heated from room temperature (20°C) to 90°C . Interpret your results, describing where temperature compensation is most relevant and how that may be achieved in practice.

Solution:



Starting at point B, the base currents into T_1 and T_2 are I_B . Hence, $2I_B$ flows into the emitter of T_3 . This yields at the base of T_3 a current of $2I_B/(\beta_3+1)$. The sum of this plus the collector current of T_1 , which is $\beta_1 I_B$, must flow through R_C , so

$$(I) \quad I_{in} = \beta_1 I_B + 2I_B/(\beta_3+1)$$

The output current is the collector current of T_2 , which is

$$(II) \quad I_{out} = \beta_2 I_B$$

The ratio is thus

$$(III) \quad I_{out}/I_{in} = \beta_2 / [\beta_1 + 2/(\beta_3+1)] = (\beta_2\beta_3 + \beta_2) / (\beta_1\beta_3 + \beta_1 + 2)$$

As this derivation was identical to a question in the 2010 exam, this deduction gives only 1.5 points, as does the circuit diagram; the temperature consideration, on the other hand, has been completely new.

At room temperature, $I_{out}/I_{in} = (100^2 + 100) / (100^2 + 102) = 0.999802$.

For the $\Delta T = 70\text{K}$ temperature increase, β will go up by $\Delta\beta/\beta = 0.49 = 49\%$. This means to substitute for each heated transistor the original value of $\beta_i = 100$ by $1.49\beta_i = 149$ in the above equation and evaluate the corresponding effect.

$$(IV) \text{ heating only } T_1: I_{out}/I_{in} = (100^2 + 100) / (149 \cdot 100 + 151) = 0.671$$

$$(V) \text{ heating only } T_2: I_{out}/I_{in} = (149 \cdot 100 + 149) / (100^2 + 102) = 1.490$$

$$(VI) \text{ heating } T_1 \text{ \& } T_2: I_{out}/I_{in} = (149 \cdot 100 + 149) / (149 \cdot 100 + 151) = 0.999867$$

$$(VI) \text{ heating only } T_3: I_{out}/I_{in} = (100 \cdot 149 + 100) / (100 \cdot 149 + 102) = 0.999867$$

$$(VII) \text{ heating all transistors: } I_{out}/I_{in} = (149^2 + 149) / (149^2 + 151) = 0.999911$$

Only when T_1 or T_2 are heated separately will the current ratio deviate from unity.

In order to prevent this they should be mounted as close together as possible,

either on top or back-to-back so there is no temperature gradient between them. All other heating effects are negligible, i.e. a current mirror is very temperature stable. Heating the whole setup to 90°C here only changes the output by 0.1‰ or 100ppm relative to room temperature.

4 points

- c. Explain how a class C amplifier is biased.
What are the benefits and drawbacks of this compared to a class A amplifier?
Give one example where a class C amplifier may be used.

Solution:

Class C amplifiers are negatively biased ($V_{\text{bias}} < 0$) so that they only switch on when the input voltage exceeds a certain threshold (given by $|V_{\text{bias}}| + V_{\text{BE}}$).

As a result they are very energy efficient (as they stay off for small signals below this threshold) but have very high distortions.

They may be used whenever signal distortions are irrelevant and power consumption is a relevant aspect, e.g. battery powered alarm systems, pre-amps feeding into digital systems where signal heights need to be only above/below a threshold etc.

Remarks: Old questions are the first part of 2a (voltage gain of cascade, new: output resistance) and 2b (current ratio of 3 transistor current mirror, new: temperature dependence). Q2c is new, but a very basic one.

3. MOSFETs

8 points

- a. The general output characteristic for the drain current of a MOSFET may be described by the equation $I_D = \mu_n C_{ox} W/L [(V_{GS}-V_{to}) V_{DS} - \frac{1}{2}V_{DS}^2]$.
- Define all parameters on the right hand side of the drain current equation.
 - Derive the approximate relationships for triode region and saturation region of the output characteristic.
 - Sketch the output characteristic, neglecting the Early effect. State where the transition from one to the other region occurs in terms of the overdrive voltage.
 - Describe one typical application for each of both regions.

Solution:

The parameters are

μ_n : carrier mobility,

C_{ox} : specific gate capacitance per unit area,

W : gate width,

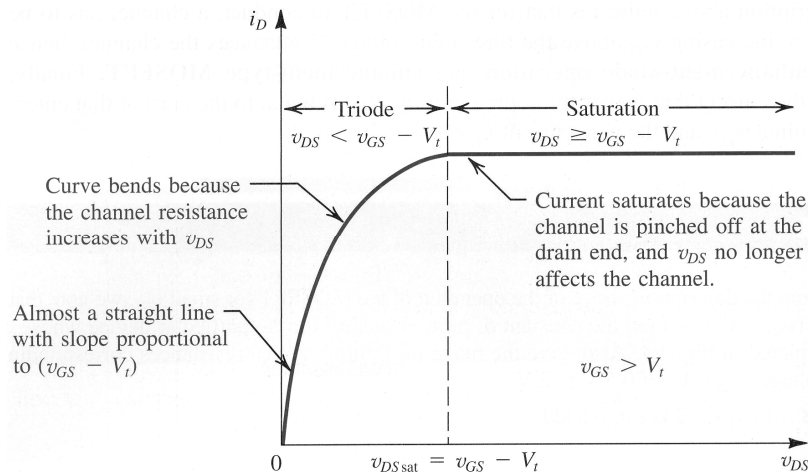
L : gate length,

V_{GS} : gate-source voltage,

V_{to} : turn-on voltage,

V_{DS} : drain-source voltage.

Triode region: $V_{DS} \ll (V_{GS}-V_{to})=V_{ov}$. Then the quadratic term in V_{DS} can be neglected, and $I_D \approx \mu_n C_{ox} W/L V_{ov} V_{DS}$. This linear triode region extends up to $V_{DS} \leq V_{GS}-V_{to}$. For $V_{DS} \geq V_{GS}-V_{to}=V_{ov}$ we get $I_D = \frac{1}{2} \mu_n C_{ox} W/L V_{ov}^2$ and this will stay constant even for larger values of V_{DS} , if the Early effect is negligible. The MOSFET is then fully switched on with maximum drain current.



The overdrive voltage $V_{ov} = V_{GS}-V_{to}$ describes the transition between both regions. In the triode region, for small voltages V_{DS} , I_D is proportional to V_{DS} , so the MOSFET behaves like an ohmic resistor, of resistance $\partial V_{DS}/\partial I_D \approx L/(\mu_n C_{ox} W V_{ov})$. This is useful for switches (using a threshold current) or biasing. In the saturation region, I_D stays constant while V_{DS} increases, so the resistance is infinite. As I_D is approximately proportional to the square of V_{GS} , the transistor behaves like an amplifier.

5 points

- b. Assume an active MOSFET amplifier with an output characteristic, as given above in question 3a, has a transition frequency given by $f_t = g_m / (2\pi C_{\text{total}})$ where g_m is the transconductance and C_{total} the capacitance of the whole device, which can be considered as a plate capacitor of area A and thickness d .

Calculate g_m as function of overdrive voltage for the saturation region.

Eliminate all current dependencies to determine what device and materials parameters influence f_t .

Derive a design criterion for optimal high-frequency transfer.

Solution:

Square-model for drain current:

$$i_D = \frac{1}{2} \mu_n C_{\text{ox}} W/L (V_{\text{GS}} - V_{\text{to}})^2$$

Definition of transconductance:

$$g_m = \partial i_D / \partial V_{\text{GS}} = 2i_D / (V_{\text{GS}} - V_{\text{to}}) = 2i_D / V_{\text{ov}}$$

Inserting into above formula for transition frequency with $C_{\text{total}} = C_{\text{GS}} + C_{\text{GD}}$:

$$f_t = g_m / [2\pi (C_{\text{GS}} + C_{\text{GD}})]$$

$$= i_D / [\pi V_{\text{ov}} (C_{\text{GS}} + C_{\text{GD}})]$$

$$= \frac{1}{2} \mu_n C_{\text{ox}} W V_{\text{ov}} / [\pi L (C_{\text{GS}} + C_{\text{GD}})]$$

Now insert $C_{\text{ox}} = \epsilon_0 \epsilon_r / d$ (specific capacitance!) and $C_{\text{GS}} + C_{\text{GD}} = \epsilon_0 \epsilon_r A / d$ where $A = WL$.

$f_t = \frac{1}{2} \mu_n V_{\text{ov}} / (\pi L^2)$ will be large for given V_{ov} , if

i) μ_n is high (high carrier mobility)

ii) L is small.

The main design criterion will thus be to make the channel as short as possible, and a minor criterion will be to choose a semiconductor with a high mobility.

7 points

- c. Identify the configurations of MOSFETs in the following circuit diagram shown in figure 3, and briefly describe their functions and that of the complete circuit.

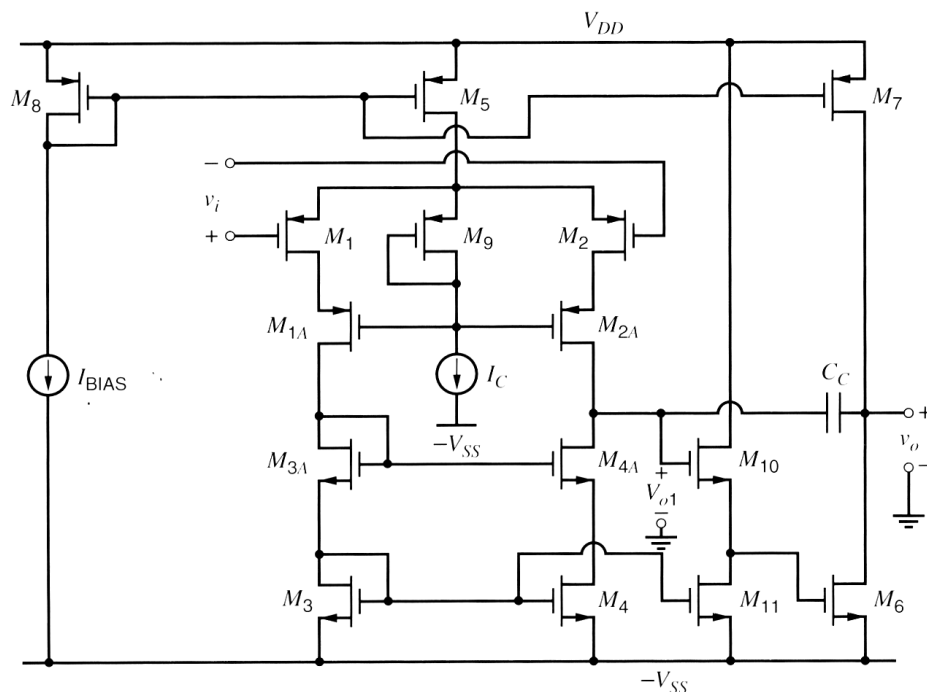


Figure 3

Solution:

This is a 2-stage operational amplifier with a rather complicated first stage. The combinations of transistors $M1$ & $M1A$ (and also the pairs $M2$ & $M2A$, $M3$ & $M3A$ and $M4$ & $M4A$) form cascodes to enhance the output resistance compared to single transistors. $M1$ & $M2$ form the differential amplifier for the input, and the current mirror $M3$ & $M4$ converts the signal to single ended output. The transistors $M5$, $M8$ and $M9$ provide bias ($M5$, $M7$ & $M8$ form another current mirror). $M10$ & $M11$ are dc level shifters (by V_{GS10} and V_{GS3} , respectively), and $M6$ (n-channel) & $M7$ (p-channel) form the class AB output stage of this 2-stage-amplifier.

Remarks: Q3a (i)-(iii) is similar to a question in 2010, (iv) & (v) are new. Q3b is a mixture of questions asked in 2009 and 2011. Q3c is new.

4. Filters

8 points

- a. Figure 4 may be considered a small signal equivalent circuit of a MOSFET as common source amplifier with input to the gate via a current i_s through the resistor R_1 and a shunt capacitance C_2 on the output. v_o is the output voltage at the drain.

- Calculate the complex transfer function v_o/i_s .
- What physical meaning does this quantity have?
- State order and type of the transfer function, its zeros and calculate the approximate poles.
- How do the pole frequencies depend on capacity C_{GD} ?

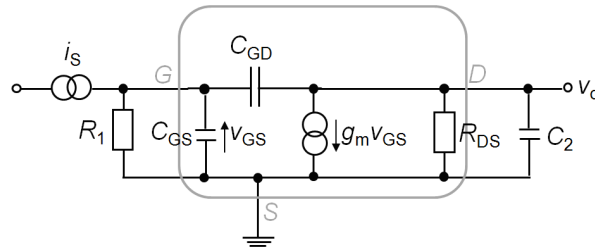


Figure 4

Solution:

(i)

Splitting the circuit in the middle and applying Kirchhoff's current laws on either side gives with $s=j\omega$

$$\text{for the left half: } i_s + v_{GS}/R_1 + v_{GS} s C_{GS} + (v_{GS} - v_o) s C_{GD} = 0 \quad (i)$$

$$\text{for the right half: } (v_o - v_{GS}) s C_{GD} + g_m v_{GS} + v_o/R_{DS} + v_o s C_2 = 0 \quad (ii)$$

These two equations contain as variables i_s , v_o and v_{GS} , the latter of which needs to be eliminated.

$$\text{from (i): } v_{GS} = [-i_s + v_o s C_{GD}] / [1/R_1 + s C_{GS} + s C_{GD}]$$

$$\text{from (ii): } v_{GS} = v_o [s C_{GD} + 1/R_{DS} + s C_2] / [s C_{GD} - g_m]$$

Equating both sides and multiplication with both denominators yields:

$$[-i_s + v_o s C_{GD}] \times [s C_{GD} - g_m] = v_o [s C_{GD} + 1/R_{DS} + s C_2] \times [1/R_1 + s C_{GS} + s C_{GD}]$$

This finally yields

$$v_o/i_s = (s C_{GD} - g_m) R_1 R_{DS} /$$

$$\{ 1 + s [R_{DS}(C_2 + C_{GD}) + R_1(C_{GD} + C_{GS}) + g_m R_1 R_{DS} C_{GD}] + s^2 R_1 R_{DS} (C_2 C_{GS} + C_2 C_{GD} + C_{GS} C_{GD}) \}$$

(ii)

Because of Ohm's law, the ratio v_o/i_s is a resistance and can be interpreted as the output resistance of the circuit.

(iii)

This is a 2nd order transmission function of a low-pass filter with

$v_o/i_s = -g_m R_1 R_{DS}$ (CS as an inverting amp.!) for $s=0$ and $v_o/i_s \rightarrow 0$ for $\lim s \rightarrow \infty$.

The zeros are given for $s = g_m/C_{GD}$ (real and positive) and also for $s = \infty$.

The two poles are a bit tricky to evaluate because of the form of the denominator.

If it were written in the usual form with two poles p_1 and p_2 , then

$$\text{Denominator} = (s - p_1)(s - p_2) = s^2 - s p_1 - s p_2 + p_1 p_2 = p_1 p_2 (1 - s/p_1 - s/p_2 + s^2/(p_1 p_2))$$

$$\approx p_1 p_2 (1 - s/p_1 + s^2/(p_1 p_2)) \text{ if } p_1 \text{ is the dominant pole, i.e. } p_1 \ll p_2. \text{ Under this}$$

assumption one can obtain the poles by comparison of the pre-factors:

$$p_1 = -1/[R_{DS}(C_2 + C_{GD}) + R_1(C_{GD} + C_{GS}) + g_m R_1 R_{DS} C_{GD}] \approx -1/[g_m R_1 R_{DS} C_{GD}] \text{ for large } g_m$$

and hence at higher frequencies:

$$p_2 \approx -[g_m R_1 R_{DS} C_{GD}] / [R_1 R_{DS} (C_2 C_{GS} + C_2 C_{GD} + C_{GS} C_{GD})]$$

$$= -(g_m C_{GD}) / (C_2 C_{GS} + C_2 C_{GD} + C_{GS} C_{GD})$$

(iv)

As p_1 has C_{GD} only in its denominator, increasing C_{GD} will move $|p_1|$ to lower frequencies. As p_2 has C_{GD} linearly in both in numerator and denominator, increasing C_{GD} will increase $|p_2|$ and move it asymptotically towards a value of $g_m / (C_2 + C_{GS})$. The poles thus split apart with increasing C_{GD} .

6 points

- b. A Chebychev filter is given by the transfer function $T(s) = [1 + \epsilon^2 C_n^2(\omega/\omega_0)]^{-1/2}$ with the Chebychev polynomial of first kind given as $C_n(x) = \cos(n \arccos x)$, if $0 \leq x \leq 1$ and $C_n(x) = \cosh(n \operatorname{arccosh} x)$, if $x > 1$.

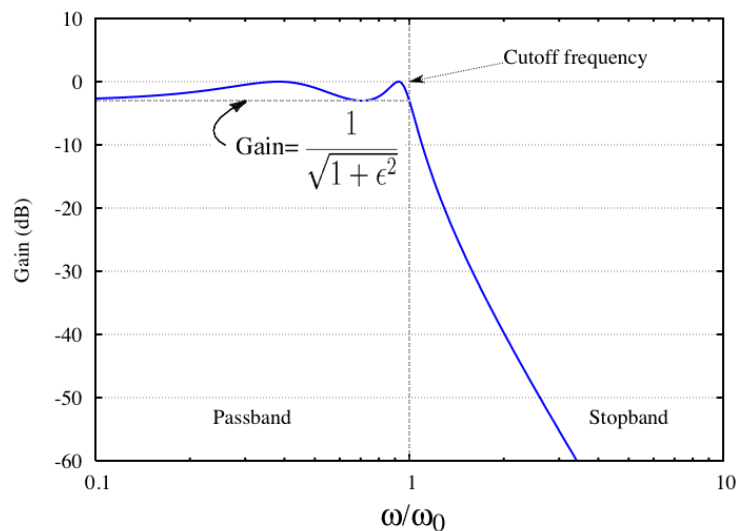
Use the properties of the cosine function and the relationship $\cosh x = \frac{1}{2}(e^x + e^{-x})$ to describe *qualitatively* what this filter looks like. Provide a sketch of a typical response.

Determine the relationship between the parameter ϵ and remaining ripple amplitude of $\pm\gamma$.

What value of ϵ would be needed to keep the ripple within the pass-band to within $\pm 3\text{dB}$?

Solution:

For $\omega = 0 \dots \omega_0$ the arcos function falls from $\pi/2$ to 0 (inverse of the cos), then the cos adds n oscillations starting from 0 at $\omega = 0$ to 1 at $\omega = \omega_0$. These ripples are damped by a factor $[1 + \epsilon^2]^{-1/2}$. For $\omega > \omega_0$ the arccosh function yields a slowly increasing function (like \ln) which the cosh turns into a rapidly increasing value (as the exponential factor dominates). As the expression stands (with the square) in the denominator this in effect describes a damping by an exponentially decaying function. As the exponential function is the fastest growing function, $T(\omega > \omega_0) \approx [1 + \epsilon^2 [0.5 \exp(\omega/\omega_0)]^2]^{-1/2} \approx 2/\epsilon \exp(-\omega/\omega_0)$ decays maximally quickly. Both functions, oscillations below ω_0 and exponential decay above ω_0 , match at this frequency so the filter function is continuous and continuously differentiable. So the filter looks something like this (for $n = \epsilon = 1$) :



The polynomial C_n can assume values between 0 and 1, so the amplitude of the ripples are determined by the term $(1+\varepsilon^2)^{-1/2}$. For ripples of $\pm\gamma$ dB amplitude we would thus have to set

$$-\gamma = 20 \log (1+\varepsilon^2)^{-0.5} = -10 \log (1+\varepsilon^2)$$

$$\text{This yields } \varepsilon = (10^{0.1\gamma} - 1)^{0.5}$$

So, for $\gamma=3$ we get $\varepsilon=0.9976 \approx 1$, which agrees with $T(s, C_n=1, \varepsilon=1) = 1/\sqrt{2}$ because $20 \log (1/\sqrt{2}) = -3.010... \approx -3$ (dB).

6 points

- c. Find the zeros and poles and sketch the Bode plot of the magnitude of the transfer

$$\text{function } T(s) = \frac{s^2 - 100}{\left(1 + s/10^2\right)\left(1 + s/10^4\right)\left(1 + s/10^6\right)}.$$

Name the order and the type of the filter.

What is the transition frequency of unity gain?

Solution:

zeros: $s=10$ and $s=\infty$

poles: $s=-10^2$, $s=-10^4$ and $s=-10^6$

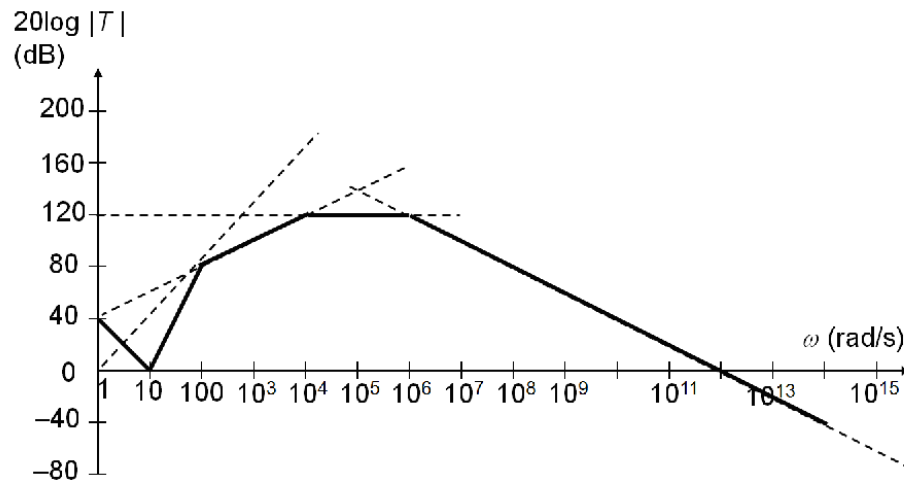
The Bode plot can be obtained from the multiplicative superposition of several curves.

(i) $T(s)=s^2$ should be a straight line with a steep slope of +40dB/decade, however, our curve must go through zero at $\omega=10$ rad/s and is actually mirrored for the range $\omega=0...10$ rad/s where $T<0$ as we only consider $|T|$. For the range above this the curve will approximate back to the straight line from the origin to (100rad/s, 80dB).

(ii) $T(s)=1/(1+s/10^2)$ gives a line of slope -20dB/decade intersecting at $\omega=10^2$, thereby reducing the slope from $\omega=10^2$ onwards to +20dB/decade.

(iii) $T(s)=1/(1+s/10^4)$ gives a line of slope -20dB/decade intersecting at $\omega=10^4$, thereby eliminating the slope from $\omega=10^4$ onwards. The maximum reached should be 120dB (in reality only ~118dB)

(iii) $T(s)=1/(1+s/10^6)$ gives a line of slope -20dB/decade intersecting at $\omega=10^6$, bringing the slope from $\omega=10^6$ onwards to -20dB/decade.



This can be described as a 3rd order band-pass filter, with an additional low-pass with signal inversion below the zero at 10 rad/s.
Unity gain is reached for $f_t = \omega/(2\pi) = 10^{12}/2\pi = 159$ GHz.

Remarks: Q4a and Q4b are new. Q4c is similar to an earlier questions, but with an additional twist due to the offset in the numerator.