

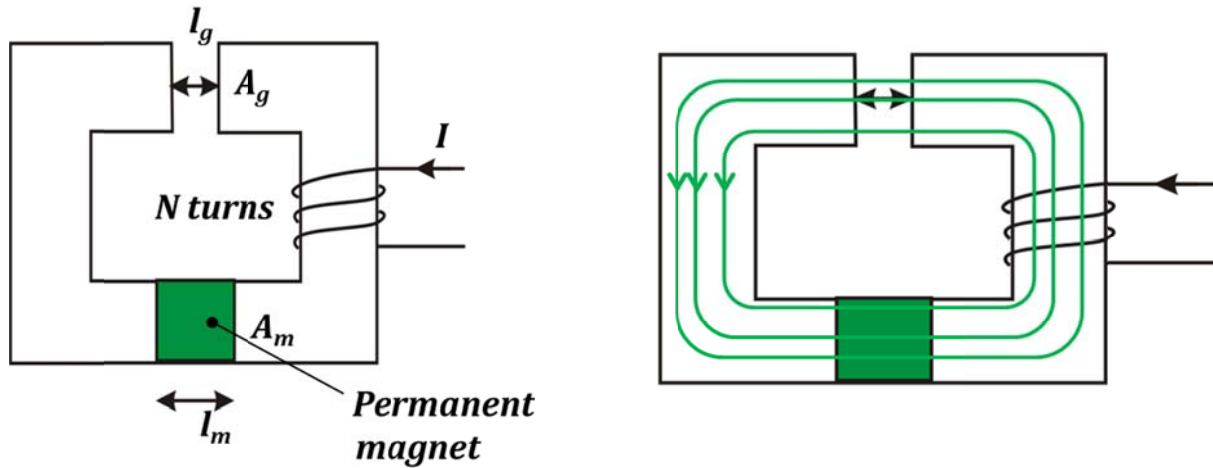
## Answers to questions

### Answers to question 1:

(a), as shown in the figure, from Ampere's Law:

$$\oint H dl = \sum I$$

$$H_m l_m + H_g l_g = -NI$$



And from Gauss's Law:

$$\oint B ds = 0$$

$$B_m A_m = B_g A_g$$

(1)

### Demagnetisation characteristic of magnet

$$B_m = \mu_0 \mu_r H_m + B_r \text{ (for linear part)}$$

Therefore, under open-circuit conditions, we have

$$B_g = \frac{A_m}{A_g} B_m = \frac{A_m}{A_g} (\mu_0 \mu_r H_m + B_r) = \frac{A_m}{A_g} \left( -\frac{H_g l_g}{l_m} \mu_0 \mu_r + B_r \right)$$

With  $B_g = \mu_0 H_g$ , and Replacing  $H_g$  using  $\frac{B_g}{\mu_0}$ , we have:

$$B_g = \frac{A_m}{A_g} B_m = \frac{A_m}{A_g} \frac{B_r}{1 + \mu_r \frac{l_g A_m}{l_m A_g}} = \frac{B_r}{\frac{A_g}{A_m} + \mu_r \frac{l_g}{l_m}}$$

The average air-gap radius is:  $R_g = R_i - L_g/2 = 28.5 - 0.5 = 28$  mm,

The average magnet radius is:  $R_g = R_i - L_g - L_m/2 = 28.5 - 1 - 1.5 = 26$  mm, therefore,

$$A_g/A_m = R_g/R_m = 28/26 = 1.08.$$

And the peak air-gap flux density is equal to the average flux density  $B_g = 1.2/(1.08 + 1/3) = 0.84$  T. (3)

The possible ways to increase the air-gap flux density is to reduce the  $A_g/A_m$  ratio by using V-shaped IPM machine (flux focusing effect), or reduce the  $L_g/L_m$  ratio by increasing permanent magnet thickness and reducing the air-gap length. (2)

(b), the slot number is  $N_s = 12$ ,

The slot pitch is  $\tau_s = \frac{2\pi R_i}{N_s} = 14.9$  mm, and

$$B_{tooth} = \frac{\tau_s}{t_w} B_g = \frac{14.9}{t_w} \times 0.84 \leq 2$$

Therefore,

$$\frac{14.9}{2} \times 0.84 \leq t_w, \text{ and } t_w \geq 6.258 \text{ mm}$$

The tooth width cannot be too large, because it will make the slot area too small, and hence compete with the electrical loading. (4)

(c), Pole number  $2p = 10$ , therefore, the pole pitch is

$$\tau_p = \frac{2\pi R_i}{2p} = 17.9 \text{ mm}$$

And we have the peak flux density in the stator yoke such as

$$B_{core} = \frac{B_g \tau_p}{2d_c} = \frac{0.84 \times 17.9}{2d_c} \leq 2$$

Therefore,

$$\frac{0.84 \times 17.9}{4} = 3.76 \text{ mm} \leq d_c \quad (3)$$

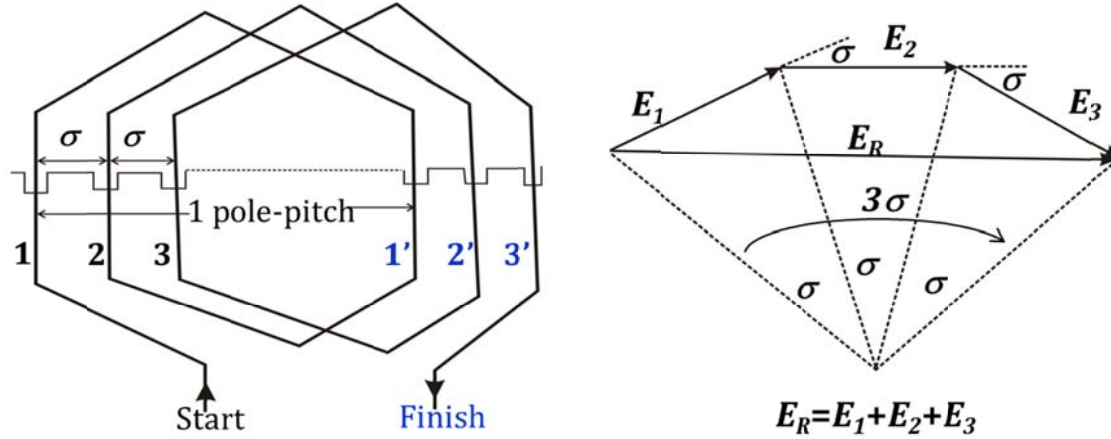
Similar to tooth width, the stator yoke thickness cannot be too large either, because it will also reduce the slot area, and hence compete with the electrical loading Q. (1)

(d), the electrical loading is  $Q = NI/(2\pi R_i) = 1000/(3.14 \times 57) = 5.58$  A/mm = 5580 A/m. The major issues of increasing electrical loading Q are the overheating and magnet demagnetization. (4)

(e), the electromagnetic torque is  $T = \frac{\pi}{2} (2R_{ro})^2 LBQ = \frac{\pi}{2} (0.055)^2 \times 0.05 \times 0.84 \times 5580 = 1.11 \text{ Nm}$ . (2)

### Answers to question 2:

(a) The layout of winding and the EMF vectors of coils are shown:



Assuming we have  $m = 3$  coils per phase, and  $|E_1| = |E_2| = |E_3| = |E_m|$  (all the coils are identical).

Then, from the construction ( $E_m = E_1$ ), we have

$$E_m = 2r \sin \frac{\sigma}{2} \quad \text{and} \quad E_R = 2r \sin \frac{m\sigma}{2}$$

The arithmetic sum of all coil EMFs:  $mE_m = m2r \sin \frac{\sigma}{2}$

However, the vector sum of all coil EMFs:  $E_R = 2r \sin \frac{m\sigma}{2}$

Therefore, the distribution factor for the fundamental is:

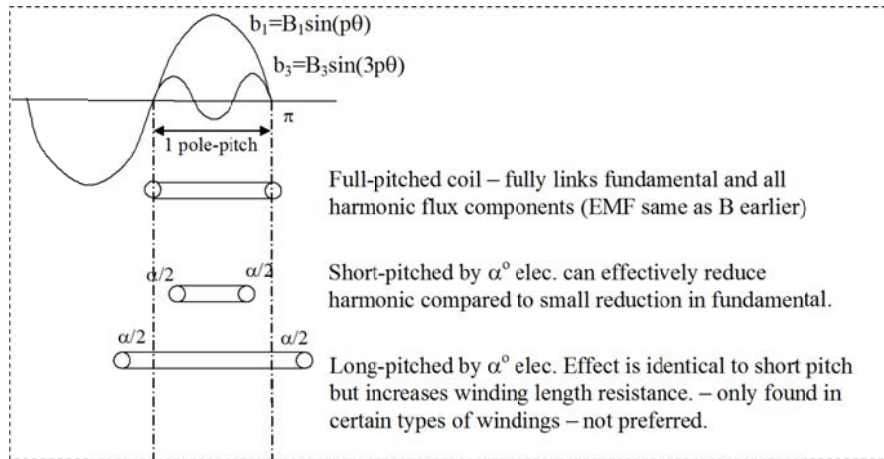
$$k_d = \frac{\text{effective induced emf}}{\text{arithmetic induced emf}} = \frac{E_R}{mE_m} = \frac{\sin \frac{m\sigma}{2}}{m \sin \frac{\sigma}{2}}$$

By using the similar approach, the distribution factor for the nth harmonic is:

$$k_{dn} = \frac{\sin \frac{mn\sigma}{2}}{m \sin \frac{n\sigma}{2}}$$

(2)

The pitch factor then can be calculated based on the following graph:



$k_p$  is defined as: 
$$\frac{\text{effective EMF}}{\text{EMF of full – pitch coil}} \propto \frac{\text{effective flux linkage}}{\text{flux linkage of full pitch coil}} = \frac{\Psi_s}{\Psi_F}$$

For a short pitch coil:

$$\Psi_s = \int_{\alpha/2}^{\pi-\alpha/2} \hat{B} \sin \theta d\theta = 2\hat{B} \cos \frac{\alpha}{2}$$

And for full pitch coil:

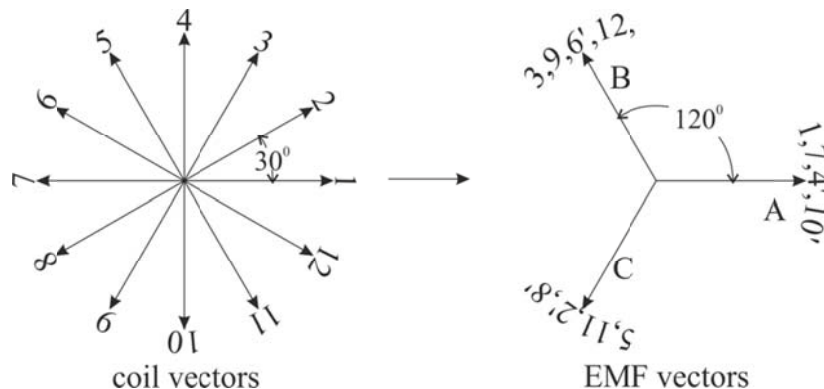
$$\Psi_F = \int_0^{\pi} \hat{B} \sin \theta d\theta = 2\hat{B}$$

Therefore, the pitch factor is:

$$k_p = \frac{\Psi_s}{\Psi_F} = \frac{2\hat{B} \cos \frac{\alpha}{2}}{2\hat{B}} = \cos \frac{\alpha}{2}$$

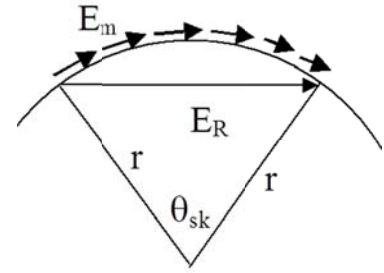
Similarly, the pitch factor for long pitch is:  $k_p = \cos \frac{\alpha}{2}$  (2)

For a 12-slot/4-pole surface mounted permanent magnet machine which has double layer, distributed winding, there are 12 coils allow us to establish a 3-phase winding structure. This means each phase will only have 4 coils. It is important that the coils span 3 slot pitch in order to achieve a maximum pitch factor 1. The coil vector and coil EMF vector of this machine are the same and shown in the following graph: (2)



(b), If the skew angle is  $\theta_{sk}$  and the winding consists of  $m$  element as shown in the following graph, then we have:

$$k_{sk} = \frac{\text{vector sum } E_R}{\text{arithmetic sum } mE_m} = \frac{\text{chord of circle}}{\text{arc of circle}} = \frac{2r \sin \frac{\theta_{sk}}{2}}{r\theta_{sk}}$$



Finally, the skew factor can be calculated by:

$$k_{sk} = \frac{\sin \frac{\theta_{sk}}{2}}{\frac{\theta_{sk}}{2}}$$

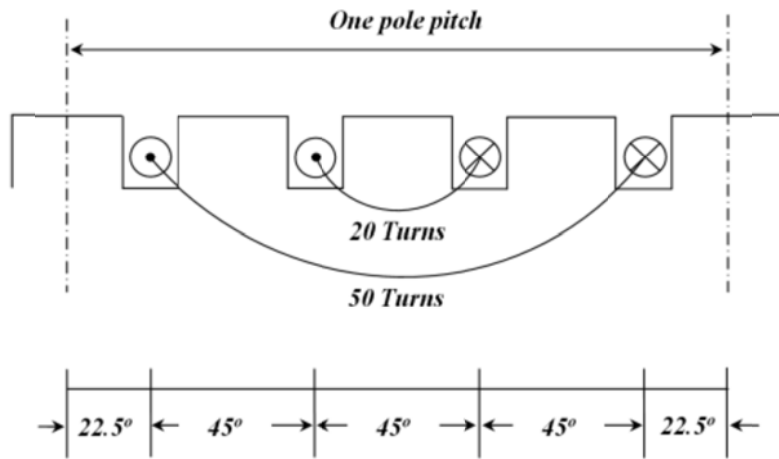
As for distribution factor, the skew factor for  $n$ th harmonic is:

$$k_{skn} = \frac{\sin \frac{n\theta_{sk}}{2}}{\frac{n\theta_{sk}}{2}}$$

The winding skew is a very effective approach to reduce higher harmonics. However, it reduces the fundamental as well. Moreover, due to its complex structure, it will also increase the manufacturing difficulty. (4)

(c), In general, for  $N$  coils linking the  $n^{\text{th}}$  harmonic, the winding factor for concentric winding is

$$K_{wn} = \frac{\sum_{i=1}^N N_i \cos\left(n\frac{\alpha_i}{2}\right)}{\sum_{i=1}^N N_i} \quad (2)$$



$$N1 = 50, N2 = 20, \alpha_1 = \pi/4, \alpha_2 = 3\pi/4$$

For the fundamental:

$$K_w = \frac{50 \cos \frac{\pi}{8} + 20 \cos \frac{3\pi}{8}}{50 + 20} = \frac{53.848}{70} = 0.77 \quad (2)$$

(d), compared to the fully pitched distributed winding, the short-pitched concentrated winding often has shorter end-winding, therefore, smaller overall axial length and less copper

losses due to lower resistance. However, the short-pitched concentrated windings could have lower pitch factor for fundamental EMF, and hence lower output torque. (3)

(e), In general, the main advantage of single layer winding is:

- Higher winding factor (higher distribution factor and the same pitch factor),
  - Higher self inductance but lower mutual inductance (less coil number but high number of turns/coil to have the same number of turns/phase and hence the same phase EMF level),
  - Physical, electromagnetic and thermal separations between coils, and hence much higher fault tolerance capability, very desirable for aerospace applications,
  - Higher saturation level due to more flux concentrate on each stator tooth with windings, leading to lower torque at high phase currents. (3)
- full mark will be given when at least three advantages will be listed)**

### Answers to question 3:

(a), The flux density in airgap can be first calculated such as

$$B_g = \frac{B_r}{1 + \mu_r \frac{l_g}{l_m}} \quad \text{Assuming } A_m = A_g \text{ for rectangular flux density waveform.} \quad (2)$$

Then the flux per pole can be calculated using

$$\Phi = B_g \times \left(\frac{\pi DL}{2p}\right) \times \left(\frac{\alpha}{\pi}\right) = \frac{1.0}{1 + 1.1 \frac{0.8}{5}} \times \left(\frac{\pi \times 60 \times 10^{-3} \times 40 \times 10^{-3}}{2 \times 1}\right) \times \left(\frac{110}{180}\right) = 1.96 \text{ mWb} \quad (3)$$

(b), The total number of conductor is  $Z_{\text{total}} = 1532$ ,  $a = 2$ ,  $V_{\text{dc}} = 100\text{V}$ , then each path has a conductor number:

$$Z = Z_{\text{total}}/2 = 766$$

$$K = \frac{E}{\omega_r} = \frac{Zp\Phi}{\pi} = \frac{766 \times 1 \times 1.96 \times 10^{-3}}{\pi} = 0.478 (\text{Nm/A or V/rad s}^{-1}) \quad (3)$$

Under no-load condition,  $I = 0$  and  $V = E$ , we can have

$$\omega_{NL} = \frac{V}{K} = \frac{100}{0.478} = 209 \text{ rad/s} \quad (2)$$

(c),  $R = 3\Omega$ ,  $V_{\text{dc}} = 100\text{V}$ ,  $E = 0$ ,  $I = (V - E)/R = V/R$ , the stall-torque can be calculated by

$$T_{\text{stall}} = K \frac{V}{R} = 0.478 \frac{100}{3} = 15.9 \text{ Nm} \quad (2)$$

From Figure, we know that the  $H_{\text{lim}}$  is 650 kA/m, and  $I_{\text{stall}} = 100/3\text{A}$ , then

$$NI = \left( \frac{Q\pi D}{4p} \right) \frac{\alpha}{\pi} \quad \text{and} \quad Q = \frac{Z_{total} I / a}{\pi D}$$

We can obtain

$$NI = \frac{Z_{total}}{a} \frac{I}{4p} \frac{\alpha}{\pi} = \frac{1532}{2} \frac{100/3}{4 \times 1} \frac{110}{180} = 3901.2 \text{ ATurns}$$

Then

$$H_m = -\frac{NI}{l_m + \mu_r l_g} - \frac{B_r l_g}{\mu_0 (l_m + \mu_r l_g)} = -771.7 \text{ KA/m}$$

Since  $|H_m| < |H_{lim}|$ , the magnet will be demagnetized, especially the magnet tips. (3)

(d), The torque density is limited by the magnetic loading B and the electrical loading Q. The magnetic loading is limited by the magnetic saturation in the tooth and back-iron, as well as the iron losses, including hysteresis and eddy current, which will limit the magnetic loading depending on the speed. It is also limited by the permanent magnet materials in permanent magnet machines. The electrical loading is primarily limited by the copper loss, and the demagnetisation withstand in the permanent magnet machines. (2)

(e), The ideal back-EMF and current waveforms for Brushless DC (BLDC) machines are trapezoidal back-EMF, at least with  $>120$  deg. elec. flat top and rectangular current waveform, while those for brushless AC (BLAC) machines are both sinusoidal. If they are non-ideal, the average torque will be reduced and the torque ripples will be increased. (3)

#### **Answers to question 4:**

(a), neglecting the magnetic saturation, the airgap field strength H and flux density can be calculated from the MMF F by:

$$H(\theta) = \frac{F(\theta)}{L_g}, \quad (L_g = \text{effective gap length}) \quad \text{and} \quad B(\theta) = \mu_0 H(\theta)$$

For single-phase winding excitation:

Assuming excitation winding carries a peak ac current of  $\sqrt{2}I_c$  at a frequency of  $\omega = 2\pi f$ , then for  $n^{th}$  harmonic:

$$|F_n| = \frac{4H}{n\pi} k_{wn} \quad \text{where} \quad H = \frac{Ni(t)}{2p} \quad \text{and} \quad i(t) = \sqrt{2}I_c \sin \omega t \quad \text{for } n = 1, 3, 5, 7, \dots$$

N is the total number of turns,  $2p$  is the number of poles, and  $K_{wn}$  is the winding factor. The resultant time & space content of the winding. & the resultant time & space content of the winding MMF is:

$$\begin{aligned} F(\theta, t) &= [F_1 \sin \theta + \dots + F_n \sin n\theta] \sin \omega t \\ &= \frac{F_1}{2} [\cos(\theta - \omega t) - \cos(\theta + \omega t)] + \dots + \frac{F_n}{2} [\cos(n\theta - \omega t) - \cos(n\theta + \omega t)] \end{aligned} \quad (5)$$

For three-phase winding excitation:

If 3-phase windings displaced in space by  $\frac{n2\pi}{3}$  for the  $n^{th}$  harmonic and in time by  $\frac{2\pi}{3}$ , then

$$F_a = [F_1 \sin \theta + \dots + F_n \sin n\theta] \sin \omega t$$

$$F_b = \left[ F_1 \sin \left( \theta - \frac{2\pi}{3} \right) + \dots + F_n \sin n \left( \theta - \frac{2\pi}{3} \right) \right] \sin \left( \omega t - \frac{2\pi}{3} \right)$$

$$F_c = \left[ F_1 \sin \left( \theta - \frac{4\pi}{3} \right) + \dots + F_n \sin n \left( \theta - \frac{4\pi}{3} \right) \right] \sin \left( \omega t - \frac{4\pi}{3} \right)$$

Using same expressions as for 1-phase example, giving a resultant field:

$$F_R = F_a + F_b + F_c$$

And

$$F_R = \frac{3}{2} [F_1 \cos(\theta - \omega t) + F_5 \cos(5\theta + \omega t) + F_7 \cos(7\theta - \omega t) + F_{11} \cos(11\theta + \omega t) + \dots] \quad (3)$$

(b), for single-phase excitation, The terms  $\cos(n\theta - \omega t)$  describe the components rotating forwards at  $\omega/n$  rad/s, and the terms  $\cos(n\theta + \omega t)$  describes the components rotating backward at  $\omega/n$  rad/s. A single phase winding produces a complete set of field components with forward & backward fields of the same amplitude. Hence, no starting torque will be produced in a single phase machine. (2)

For three-phase excitation,

- A balanced 3-phase winding produces only one rotating field component for each harmonic (e.g. no backward fundamental field),
- The resultant field is  $\frac{3}{2}(\text{not } 3) \times$  amplitude of 1-phase winding field,
- No resultant triplen harmonics produced (i.e., 3, 9, 15,...) etc. Hence, no need to design these out,
- $n = 7, 13, \dots$  harmonics are forward rotating,
- $n = 5, 11, \dots$  harmonics are backward rotating. (2)

(c), The possible ways to minimize harmonics in airgap field produced single-phase winding is using winding skew, using different winding arrangements such as concentric winding, short-pitched windings, etc. (3)

(d), For single-phase winding:

From Part a, the peak fundamental mmf/pole is  $F = \left( \frac{N_T}{2p} k_{w1} \right) \times \sqrt{2} I_m \times \frac{4}{\pi}$

This produces a peak field strength H in the air-gap:



$$\hat{H} = \frac{F}{g_e}$$

And a peak fundamental flux density:

$$\hat{B} = \mu_0 H = \mu_0 \frac{F}{g_e} = \left( \frac{\mu_0}{g_e} \right) \times \frac{2\sqrt{2}N_T k_{wl} I_m}{\pi p}$$

The fundamental flux /pole:

$$\Phi = \frac{1}{\pi} \int_0^\pi \hat{B} \sin \theta d\theta \times \frac{\pi DL}{2p} = \frac{2}{\pi} \hat{B} \times \frac{\pi DL}{2p}$$

Hence

$$L_m = \frac{N\Phi}{\hat{I}} = 2p \times \left( \frac{N_T k_w}{2p} \right) \times \frac{\hat{B} DL / p}{\sqrt{2} I_m}$$

Where  $\hat{B} = \left( \frac{\mu_0}{g_e} \right) \times \frac{2\sqrt{2}N_T k_{wl} I_m}{\pi p}$ , i.e.

$$L_m = \frac{2\mu_0 (N_T k_w)^2 DL}{\pi p^2 g_e} \quad (3)$$

For 3-phase winding:

The resultant MMF of 3 phases =  $\frac{3}{2} \times MMF(1\phi)$ , where MMF(1 $\phi$ ) is the single phase MMF.

$$\hat{F}_{3\phi} = \frac{3}{2} F_{1\phi}, \quad \hat{H}_{3\phi} = \frac{3}{2} H_{1\phi} \text{ and } \hat{B}_{3\phi} = \frac{3}{2} B_{1\phi}$$

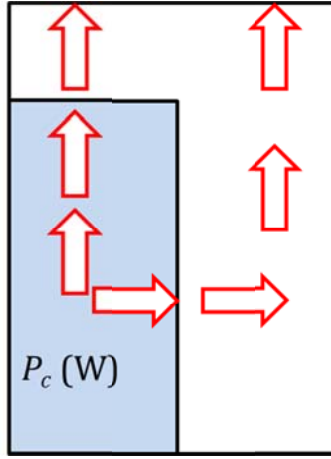
Giving  $B_{ave3\phi} = \frac{3}{2} \times B_{ave1\phi}$ ,  $\hat{\Phi}_{3\phi} = \frac{3}{2} \Phi_{1\phi}$

Therefore,

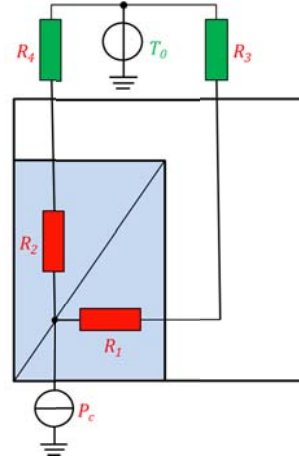
$$L_{m3\phi} = \frac{3}{2} L_{m1\phi} = \frac{3\mu_0 (N_T k_{wl})^2 DL}{\pi p^2 g_e} \quad (2)$$

### **Answers to question 5:**

(a), To establish the lumped parameter model, it is important to identify the heat flow path. Assuming the iron losses negligible, then the heat flow paths are shown in the Figure 1.



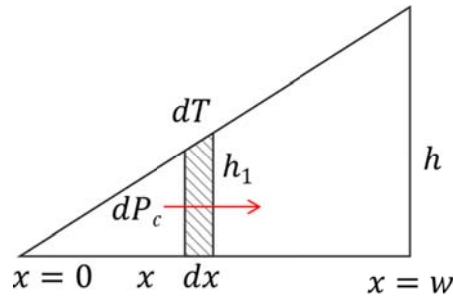
**Figure 1**



**Figure 2**

And the lumped parameter circuit model can be established as in the Figure 2. (4)

$R_I$  can be calculated as follows:



Consider an elemental strip length  $dx$  at position  $x$ , the copper loss is:

$$dP_c = \frac{h_1 x}{hw} P$$

where  $P$  is the total copper loss in the triangle,  $h_1$  is the height of the strip.

The thermal resistance of this strip is:  $dR = \frac{dx}{\lambda h_1 L}$

where  $\lambda$  is the equivalent conductivity and  $L$  is the axial length

So the temperature rise within this strip is:  $dT = dR \times dP_c$

Total temperature rise within this triangle is:

$$\theta = \int_0^w dT = \int_0^w dR \times dP_c = \int_0^w \frac{dx}{\lambda h_1 L} \times \frac{h_1 x}{hw} P$$

Therefore,  $\theta = P \times \frac{w}{2\lambda hL}$ , and hence the thermal resistance

$$R_1 = \frac{w}{2\lambda hL} = \frac{0.01}{2 \times 0.2 \times 0.02 \times 0.05} = 25K/W$$

And

$$R_2 = \frac{h}{2\lambda wL} = \frac{0.02}{2 \times 0.2 \times 0.01 \times 0.05} = 100K/W$$

The thermal resistances due to convection and radiation  $R_3$  and  $R_4$  are:

$$R_3 = \frac{1}{hwL} = \frac{1}{12 \times 0.01 \times 0.05} = 166.7K/W$$

And

$$R_4 = \frac{1}{hwL} = \frac{1}{12 \times 0.01 \times 0.05} = 166.7K/W$$

So the resultant thermal resistance of the heat flow path is:

$$R_{th} = \frac{(R_1 + R_3)(R_2 + R_4)}{(R_1 + R_3) + (R_2 + R_4)} = \frac{(25 + 166.7)(100 + 166.7)}{25 + 100 + 166.7 + 166.7} = 111.5K/W$$

And the peak winding temperature rise is:

$$\theta = R_{th} \times P_c = 111.5 \times P_c \leq 80K$$

Therefore,

$$P_c \leq 0.71$$

And the resultant copper loss is

$$P \leq 2 \times 12 \times 0.71 = 17.2W \quad (4)$$

(b), when auxiliary tooth has been added in the middle of each stator slot, the main heat flow path has been changed. Due to the symmetry, half slot can be used for analysis such as shown in Figure 3 and the lumped parameter circuit model is shown in Figure 4. Here, for simplicity, it has assumed that the thermal resistances  $R_l = R_3$ .

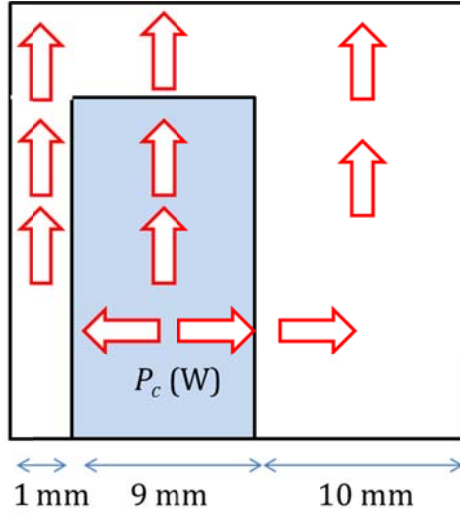


Figure 3

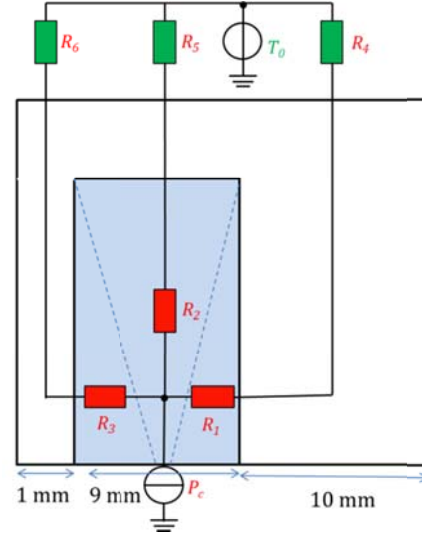


Figure 4

(4)

The calculation of thermal resistance  $R_1$  is the same as for the thermal resistance  $R_1$  in 5.a, while the calculation of  $R_2$  is slightly different, here,  $R_2$  can be regarded as the resultant thermal resistances 2 identical thermal resistances that are connected in parallel. The calculation of these two resistances is the same as that for  $R_2$  in 5.a. Therefore,

$$R_1 = R_3 = \frac{w}{2\lambda hL} = \frac{0.009/2}{2 \times 0.2 \times 0.02 \times 0.05} = 11.25K/W$$

And

$$R_2 = \frac{1}{2} \frac{h}{2\lambda wL} = \frac{1}{2} \frac{0.02}{2 \times 0.2 \times (0.009/2) \times 0.05} = 111.1K/W$$

The thermal resistances due to convection and radiation  $R_4$ ,  $R_5$  and  $R_6$  are:

$$R_4 = \frac{1}{hwL} = \frac{1}{12 \times 0.01 \times 0.05} = 166.7K/W$$

$$R_5 = \frac{1}{hwL} = \frac{1}{12 \times 0.009 \times 0.05} = 185.2K/W$$

$$R_6 = \frac{1}{hwL} = \frac{1}{12 \times 0.001 \times 0.05} = 1667K/W$$

So the resultant thermal resistance is

$$\begin{aligned} R_{th} &= \frac{\frac{(R_1 + R_4)(R_2 + R_5)}{(R_1 + R_4) + (R_2 + R_5)}(R_3 + R_6)}{\frac{(R_1 + R_4)(R_2 + R_5)}{(R_1 + R_4) + (R_2 + R_5)} + R_3 + R_6} \\ &= \frac{\frac{(11.25 + 166.7)(111.1 + 185.2)}{11.25 + 166.7 + 111.1 + 185.2}(11.25 + 1667)}{\frac{(11.25 + 166.7)(111.1 + 185.2)}{11.25 + 166.7 + 111.1 + 185.2} + 11.25 + 1667} = 104.3K/W \end{aligned}$$

And the peak winding temperature rise is:

$$\theta = R_{th} \times P_c = 104.3 \times P_c \leq 80K$$

Therefore,

$$P_c \leq 0.77$$

And the resultant copper loss is

$$P \leq 2 \times 12 \times 0.77 = 18.4W \quad (4)$$

(c), the possible ways to improve the cooling of electrical machines are

- Rotor mounted fans to increase the internal air circulation within electrical machines,
  - Additional fins on the outer surface of the frame, this can effectively increase the exchange surface for convection, and reduce the thermal resistance due to thermal convection,
  - Instead of using natural convection on the outer surface of frame, the forced convection can be used,
  - The outer frame can be painted in black to increase thermal radiation. (4,
- at least three methods should be given to achieve full mark)**

#### **Answers to question 6:**

(a) The cross-sectional area of 1 turn is:  $\pi \frac{d^2}{4} = 0.785mm^2$

The cross-sectional area of the coil cross-section =  $8 \times 70 = 560mm^2$

$$Packing\ factor = \frac{430 \times 0.785}{560} = 0.60$$

(2)

(b) From the demagnetisation characteristic:

$$B_r \approx 1.28T$$

$$H_{cn} \approx -970\text{ kA/m}$$

Relative recoil permeability is given by:

$$\mu_r = \frac{B_r}{H_{cn}\mu_0} = \frac{1.28}{970000 \times 4\pi \times 10^{-7}} = 1.05$$

**(2-no marks for simply guessing 1.05 which is a typical value hence the emphasis on showing calculations)**

(c) Assuming:

That the iron is infinitely permeable

The magnetic field in the airgap and the coil is one-dimensional then:

$$B_g = \frac{B_r}{\left(1 + \mu_r \frac{l_g}{l_m}\right)} = \frac{1.1}{\left(1 + \frac{1.05 \times 0.09}{0.01}\right)} = 0.658T \quad (3)$$

(d) Assuming that the magnet only produces usefully oriented flux in the region immediately below the magnet, then calculating the force from BIL for a current of 1A for both halves of the coil yields:

$$F = 0.658 \times 1 \times 2 \times 430 \times 0.04 \times \frac{0.045}{0.075} = 13.6N/A$$

**(3 – 1 for assumption)**

(e) From the demagnetisation characteristic at 150°C, the demagnetisation limit on H in the magnet is ~530kA/m ( any reasonable estimate around this point is fine).

The mmf which can be applied (recalling the need to use Br for 150°C of 1.14T)

$$I = -\frac{1}{N} \left( \frac{(B_r + \mu_r \mu_0 H_{lim}) l_g}{\mu_0} + l_m H_{lim} \right) \\ = -\frac{1}{430} \left( \frac{(1.14 + 1.05 \times 4\pi \times 10^{-7} \times (-530000)) \times 9 \times 10^{-3}}{4\pi \times 10^{-7}} + 0.01 \times (-530000) \right) \quad (4)$$

The force produced by this current will be modified by the temperature of the permanent magnets, specifically its influence on the remanence. For the reduced remanence of 1.14T, the force per amp can be simply scaled (alternatively, the airgap flux density can be recalculated from scratch).

$$\text{Force per unit current} = \frac{1.14}{1.28} \times 13.6 = 12.1 N/A$$

Hence, the force for the maximum current of 25.6A is 310N.

**(2)**

(f) Under state-state DC conditions, the current in the coil to produce 50N is:

$$\text{Current} = \frac{50}{12.1} = 4.13A$$

(1)

At this current the resistive voltage drop is:

$$IR = 4.13 \times 2.5 = 10.3 \text{ V}$$

(1)

The induced emf is therefore given by:

$$E = V - IR = 24 - 10.3 = 13.7 \text{ V}$$

(1)

The emf constant of 20.1 V per m/s can be obtained from the force constant of 20.1 N/A

Therefore, the maximum speed of motion is:

$$v = \frac{13.7}{20.1} 1.13 \text{ m/s}$$

(1)