Exam Question Solutions May 2012

Q1 Charge neutrality condition

(a) $n + N_a = p + N_d$

also $np = n_i^2 = p_i^2$

$$\therefore \frac{p_i^2}{p} + N_a = p + N_d$$

$$p^2 - (N_a - N_d)p - p_i^2 = 0$$

$$p = \frac{N_a - N_d}{2} + \frac{N_a - N_d}{2}\left[1 + \left(\frac{2p_i}{N_a - N_d}\right)^2\right]^{1/2}$$

i) For p-type $N_a - N_d \gg p_i$

$\therefore p = N_a - N_d = N_a$

$$n = \frac{p_i^2}{p} = \frac{p_i^2}{N_a - N_d} = \frac{p_i^2}{N_a} \left(= \frac{n_i^2}{N_a}\right)$$ [2]

Some people did it for n-type rather than p-type

ii) for compensation near intrinsic $p_i \gg N_a - N_d$

$$p = \frac{N_a - N_d}{2} + \frac{N_a - N_d}{2}\left[\frac{2p_i}{N_d - N_a}\right] \simeq p_i$$

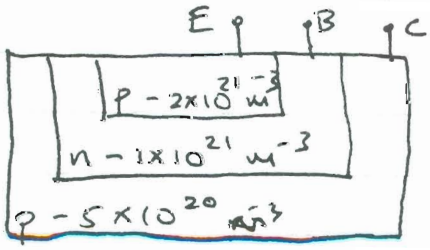
$$n = \frac{p_i^2}{p} = p_i \; (= n_i)$$ [2]

(b) Starting with $N_a = 5\times10^{20}$ i.e. p-type semiconductor

1st diffusion has to be $> 5\times10^{20}$ cm^3 but opposite doping type (n-type)

2nd diffusion has to be > 1st diffusion and p-type

So from choices given 1st diffusion is $1\times10^{21}\,m^{-3}$ with donors and 2nd diffusion is $2\times10^{21}\,m^{-3}$ with acceptors [4



This is a p-n-p transistor

Many people tried to do a n-type base and got confused [2

(c) From (a)

$$p = \frac{N_a - N_d}{2}\left[1 \pm \sqrt{1 + \left(\frac{2p_i}{N_a - N_d}\right)^2}\right]$$

In this semiconductor $|N_d - N_a| \ll n_i$ (or p_i), so

$p_{collector} = 5\times10^{20}\ m^{-3}$

$n_{base} = 10^{21} - 5\times10^{20} = 5\times10^{20}\ m^{-3}$

$p_{emitter} = 2\times10^{21} + 5\times10^{20} - 1\times10^{21} = 1.5\times10^{21}\ m^{-3}$

Relatively few got this simple section fully correct

Minority carrier concentration ~~are~~ in the emitter is

$$n = \frac{(1.3\times10^{16})^2}{1.5\times10^{21}} = 1.12\times10^{11}\ m^{-3}$$

[7]

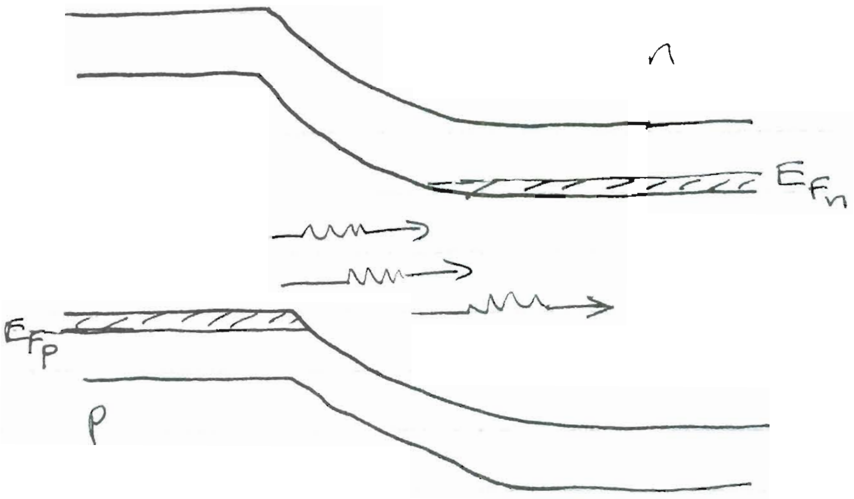
(d) As the temperature increases, the n_i value increases and the minority carrier concentrations increase, making the minority leakage currents in the transistor larger. At high temperatures, the semiconductor can appear to be intrinsic

You can increase the maximum operating temperature by increasing the doping levels in the transistor.

[3]

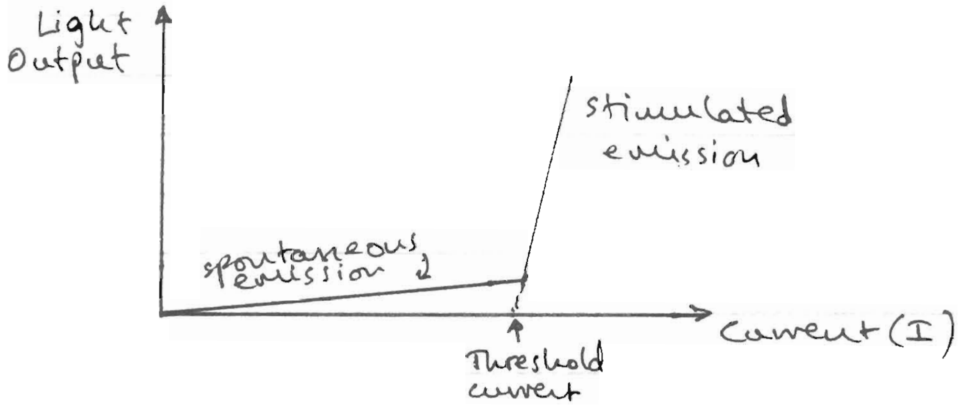
Most people got (d) correct

2 (a)



Heavily doped p an
n doped regions and
forward bias. Electro
and holes diffuse
across the depletion
region and recombi
hence photons are
emitted. Fermi levels
move apart as shown

The difference between the Fermi levels is the voltage under forward bias and population inversion can occur and stimulated emission occurs. High reflectors are needed to form a cavity which reflects the photons and determines the exact lasing wavelength.



[6]

Most people did well in part (a)

(b) Maximum wavelength given by $\frac{1.24}{1.8 eV} = 0.688 \mu m$

Minimum wavelength given by $\frac{1.24}{2.4 eV} = 0.517 \mu m$

InGaP is the QW material (narrower band gap) and InAlP is the barrier material (wider band gap).

No problems with (b) - some did not get the QW material correctly identified. [4]

c We wish to have emission at 650nm ≡ 1.907eV
This is an increase in energy over InGaP of 1.907−1.8 = 107 meV.

Using the expression for the 1st bound energy levels for electrons and holes:

$$\frac{h^2}{8m_e^* m_o L^2} + \frac{h^2}{8m_h^* m_o L^2} = 107\text{ meV}$$

$$\frac{h^2}{8m_o L^2}\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right) = 107\text{ meV}$$

$$\therefore L^2 = \frac{h^2\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right)}{8m_o \times 107 \times 10^{-3} \times 1.6\times10^{-19}}$$

$$= 3.518\times10^{-18} \times 12.54 = 4.412\times10^{-17}$$

$\Rightarrow$ L = 6.64 nm [5]

Many people got this correct – some forgot the e^- so had a huge error

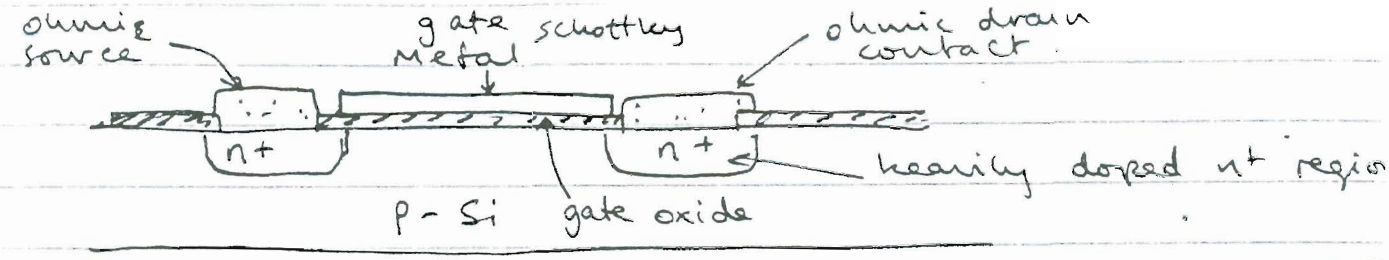
There is an uncertainty of ±0.5nm. This affects the 1st bound level energy as:

$$\Delta E \propto \frac{1}{(L \pm 0.5\text{nm})^2}$$

The accuracy of the emitted wavelength would therefore be higher in structure with a large L as the effect of the ±0.5nm would be relatively small. The opposite is true in smaller L structures and the ±0.5nm would result in a poorer accuracy. Several people calculated this – a description would have sufficed. [3

d At short wavelengths, when L becomes small, the height of the barrier material and tunnelling will affect the shortest wavelength that can be achieved. [2
Quite a few got (d) incorrect despite mentioning tunnelling.

3 (a)



A few did not show the gate edge over the n^+ channel properly [3]

(b) Saturation occurs when $V_d = V_g - V_T$

The drain current is then given by

$$I_{ds} = \frac{\mu_e C_g}{l^2} \frac{V_d^2}{2}$$

Transconductance, $g_m = \left.\frac{\partial I_d}{\partial V_g}\right|_{V_d}$ in saturation region

So $g_m = \frac{\mu_e C_g}{l^2} V_d$ [3]

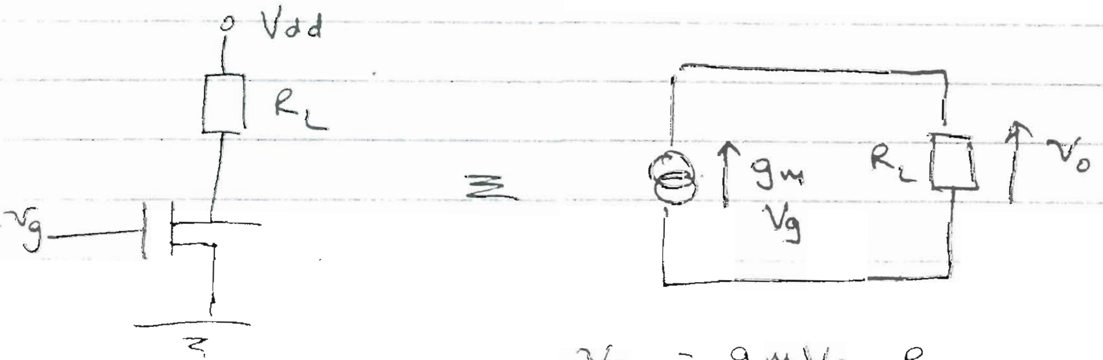
By substituting into above eqns

$$\frac{\mu_e C_g}{l^2} = \frac{g_m}{V_d} = \frac{2 I_{ds}}{V_d^2}$$

so $g_m = \frac{2 I_{ds}}{V_d}$ [2]

Fairly straightforward

(c)



$V_o = g_m V_g \cdot R_L$ [3]

Gain $= \frac{V_o}{V_g} = |A| = g_m \cdot R_L$ [2]

3 cont. $\therefore$ $30 = g_m R_L = \frac{2 I_{ds}}{V_d} \cdot R_L = \frac{2 I_{ds} \cdot R_L}{(V_{dd} - I_{ds} R_L)}$

$$30 = \frac{2 \cdot 30\times10^{-3} R_L}{100 - 30\times10^{-3} R_L}$$

gives $R_L = 3.125\, k\Omega$ 2

Very few got this completely correct – seemed to confuse Vd and 100V supply.

d)

$$V_d = 100 - 30\times10^{-3} \cdot 3.125\, k\Omega = 6.25\, V$$

$$g_m = \frac{2 \cdot I_{ds}}{V_d} = \frac{2\times 30\times10^{-3}}{6.25} = 9.6\, mS$$

Since g_m also $= \frac{\mu_e C_g V_d}{l^2}$

$$l^2 = \frac{\mu_e C_g V_d}{g_m} = \frac{0.13\times10^{-12} \times 6.25}{9.6\times10^{-3}}$$

$l = 9.2\, \mu m$ (gate length)

$$C_g = \frac{\varepsilon_0 \varepsilon_r \, Area}{t_0} = \frac{\varepsilon_0 \varepsilon_r \, l \cdot width}{t_0}$$

length = 0.1 width $\Rightarrow$ width = $92\, \mu m$

$$t_0 = \frac{\varepsilon_0 \varepsilon_r \, l \cdot width}{C_g} = \frac{11.8 \times 8.85\times10^{-12} \times 9.2\mu m \times 92}{10^{-12}}$$

$t_0 = 88\, nm$

Very few managed to do this correctly. [5]

Q4 An electron travelling in a vacuum has energy E,

(a) $E = \frac{1}{2} m v^2 = \frac{p^2}{2m}$ (as momentum $p = mv$)

Differentiating this twice gives:

$$\frac{dE}{dp} = \frac{p}{m}, \quad \frac{d^2E}{dp^2} = \frac{1}{m}$$

In a semiconductor, replace m with an 'effective' mass, m^* to account for lattice interactions

$$m^* = \left(\frac{d^2E}{dp^2}\right)^{-1}$$

[4]

Most managed this with few problems.

(b) E-K relationship is assumed parabolic, so:

$E = A + BK^2$, where A, B are constants

Bandgap at $K=0$ for direct band-gap semiconductor, so

$\Rightarrow A = 1.8\,eV$

$$\frac{dE}{dK} = 0 + 2BK, \quad \frac{d^2E}{dK^2} = 2B, \quad \hbar^2\frac{d^2E}{dp^2} = 2B \quad (p = \hbar k)$$

$$\therefore m^* = \frac{\hbar^2}{2B} = 0.09 \times 9.11 \times 10^{-31}$$

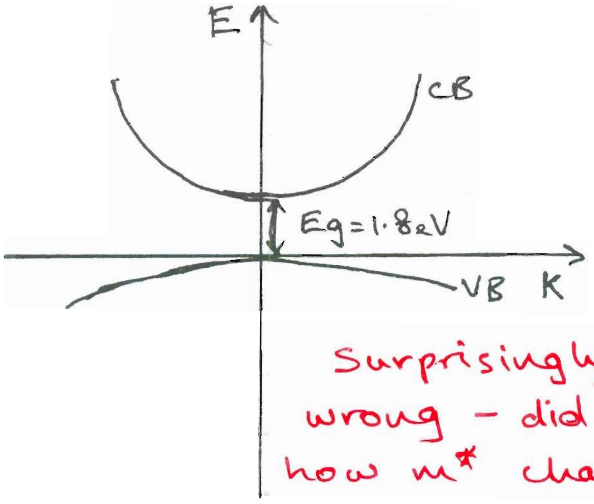
$$B = \left(\frac{6.626 \times 10^{-34}}{2\pi}\right)^2 \times \frac{1}{2} \times \frac{1}{0.09 \times 9.11 \times 10^{-31}} = 0.68 \times 10^{-37}\ J\,m^2$$

$$= 4.25 \times 10^{-19}\ eV$$

$$E = 1.8 + 4.25 \times 10^{-19} k^2\ eV$$

Most got this approximately correct - some did not give units or mixed eV and Jm^2 together

4 b.
cont.



The 8x higher mass for holes results in a much 'flatter' VB structure

Surprisingly many got this wrong – did not seem to know how m* changed with E–K

In reality, the electron effective mass increases as you approach the Brillouin zone edge. The CB energy does not continue to increase at the same rate as you move away from K=0 and eventually becomes constant (electron velocity = 0)

Relatively few got how m* changes correct – several gave the opposite answer [6]

c Heisenberg Uncertainty Principle

$$\Delta p \Delta x \geq \hbar/2$$

This implies that both parameters of a particle, i.e. the momentum and position, cannot be measured simultaneously to an arbitrary high degree of precision.

Quite a few got the exact form wrong, confusing h, ħ etc. [4]

d. de Broglie : $p = mv = \frac{h}{\lambda}$ where h = Plank's const

$\lambda = \frac{h}{mv}$ #

This was apparently easy

The P.E. of electron = K.E.

$$\frac{1}{2}mv^2 = eV$$

4. $\Rightarrow v = \left(\frac{2eV}{m}\right)^{1/2}$

so $\lambda = \frac{h}{(2eVm)^{1/2}} = \frac{1.225\,\text{nm}}{V^{1/2}}$

when $V = 50\,V$, $\lambda = 0.17\,\text{nm}$ so this is the minimum feature size

Relatively few got the correct numerical answer - despite this being a tutorial type question