UNIVERSITY OF SHEFFIELD
Department of Electronic and Electrical Engineering

EEE220 ELECTRIC AND MAGNETIC FIELDS

TUTORIAL QUESTION SOLUTIONS
(Part 2)

JLW 2006

Q16 a) From the formula sheet $B = \dfrac{\mu_0 I}{2\pi r}$

On the surface of the wire, $r = 0.5$ mm, so $B = \dfrac{4\pi \times 10^{-7} \times 50}{2\pi \times 0.5 \times 10^{-3}} = 20$ mT

When $r = 10$ mm, $B = \dfrac{4\pi \times 10^{-7} \times 50}{2\pi \times 10 \times 10^{-3}} = 1$ mT

When $r = 0.1$ m, $B = \dfrac{4\pi \times 10^{-7} \times 50}{2\pi \times 0.1} = 0.1$ mT

When $r = 1$ m, $B = \dfrac{4\pi \times 10^{-7} \times 50}{2\pi \times 1} = 10$ μT

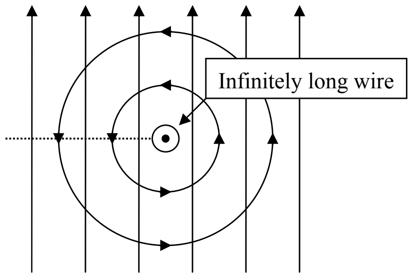
b) From the lecture notes $B = \dfrac{\mu_0 I L}{2\pi r\sqrt{4r^2 + L^2}}$, thus:-

$$B = \frac{4\pi \times 10^{-7} \times 50 \times 1}{2\pi \times 0.1 \times \sqrt{4(0.1)^2 + 1^2}} = 9.806 \times 10^{-5} \text{ T}$$

Percentage error is given by the expression $\dfrac{|\text{calculated - actual}|}{\text{actual}} \times 100\%$

$$= \frac{|0.1 \times 10^{-3} - 9.806 \times 10^{-5}|}{9.806 \times 10^{-5}} \times 100\% = 2\%$$

Q17



By drawing the cross-section of the wire, and showing the two B-fields present, it can be seen that field cancellation can only occur at some point on the dotted line shown.

The field due to the current-carrying wire decreases with increasing distance from the wire according to the equation:-

$$B = \frac{\mu_0 I}{2\pi r}$$

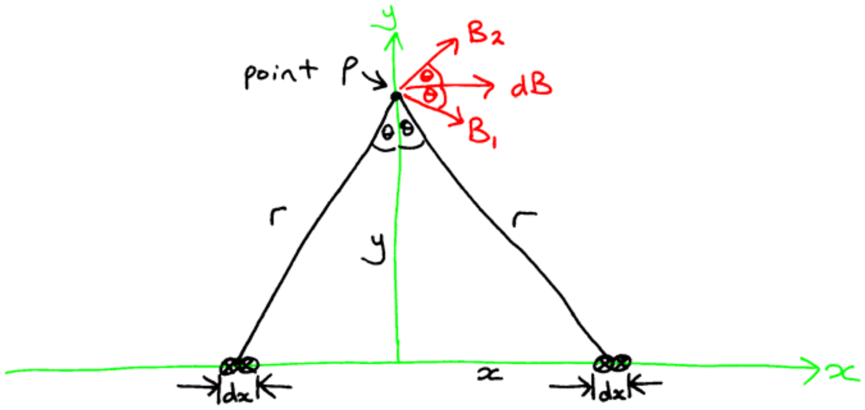
Thus, the fields are equal in magnitude when:-

$$B = \frac{\mu_0 I}{2\pi r} = 5 \text{ mT}$$

$$\frac{\left(4\pi \times 10^{-7}\right) \times 100}{2\pi r} = 5 \text{ mT}$$

$$r = \frac{\left(4\pi \times 10^{-7}\right) \times 100}{2\pi \times \left(5 \times 10^{-3}\right)} = 4 \text{ mm}$$

Q18 First, define a coordinate system and draw a diagram:-



Consider the magnetic field at the point P situated on the y-axis at a height of y. By symmetry the field at this point will be the same as for any point a perpendicular distance y from the wire.

Now consider the small part of the magnetic field dB due to the two small regions of wires situated at –x and +x on the x-axis (as shown in the diagram.) This can then be integrated to give the total field.

B1 and B2 are the fields produced by the region on the left and the region on the right respectively. Considering the angles in the diagram, the y-components of these fields will cancel each other, whereas the x-components will be equal and will add.

$$|B_1| = |B_2| = \frac{\mu_0 I}{2\pi r} \times ndx$$

Where $\frac{\mu_0 I}{2\pi r}$ is the B-field from one current carrying wire, and ndx is the number of wires in the small region dx.

$$B_{1x} = B_{2x} = \frac{\mu_0 I}{2\pi r} \times ndx \times \cos\theta$$

$$dB_x = B_{1x} + B_{2x} = \frac{\mu_0 I}{\pi r} \times ndx \times \cos\theta$$

But $\cos\theta = \frac{y}{r}$ and $r = \sqrt{x^2 + y^2}$ so…

$$dB_x = \frac{\mu_0 Inydx}{\pi\left(x^2 + y^2\right)}$$

This is the part of the B-field due to <u>both</u> the region dx on the left and the region dx on the right. To obtain the total field, this expression must be integrated between 0 and ∞.

$$B_x = \frac{\mu_0 Iny}{\pi} \int_0^\infty \frac{dx}{\left(x^2 + y^2\right)}$$

$$= \frac{\mu_0 Iny}{\pi} \left[\frac{1}{y} \tan^{-1} \frac{x}{y} \right]_0^\infty$$

$$= \frac{\mu_0 In}{\pi} \left[\tan^{-1} \frac{x}{y} \right]_0^\infty$$

$$= \frac{\mu_0 In}{\pi} \left[\frac{\pi}{2} - 0 \right]$$

$$= \frac{\mu_0 In}{2}$$

Q19

$$B = \frac{\mu_0 I a^2}{2\left(a^2 + x^2\right)^{3/2}} + \frac{\mu_0 I a^2}{2\left(a^2 + (d-x)^2\right)^{3/2}}$$

$$B = \frac{\mu_0 I}{2a}\left[\frac{1}{\left(1 + \frac{x^2}{a^2}\right)^{3/2}} + \frac{1}{\left(1 + \frac{(a-x)^2}{a^2}\right)^{3/2}}\right]$$

When $I = 10\text{A}$, $d = a = 0.1$ m, $x = 0.5$ m

$$B = \frac{\left(4\pi \times 10^{-7}\right) \times 10 \times 0.1^2}{\left(0.1^2 + 0.05^2\right)^{3/2}} = 8.99 \times 10^{-5} \text{ T}$$

Q20

Ampère's Law states that:-

$$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 I$$

where I is the current which threads the path of integration. Use the right-hand rule to check whether the B-field from each wire is in the same direction as the integration path, negating the currents where necessary.

a) $$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 (5 - 2 - 1) = 2\mu_0 \text{ A}$$

b) $$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 (6 - 5) = \mu_0 \text{ A}$$

c) $$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 (4 - 2 - 6) = -4\mu_0 \text{ A}$$

d) $$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 (-6 - 1) = -7\mu_0 \text{ A}$$

e) $$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 (6 + 1 + 2 - 4 - 5) = 0$$

Q21

Ampère's Law states that:-

$$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 I$$

If we choose the contour to be a circle of radius r centred on the wire, then by symmetry B must be constant around the integration path, and in the same direction as it. Thus :-

$$B \cdot \oint_C dl = \mu_0 I$$

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

When $r = 50$ mm, $B = \dfrac{\left(4\pi \times 10^{-7}\right) \times 10}{2\pi \times \left(50 \times 10^{-3}\right)} = 4 \times 10^{-5}$ T

When $r = 10$ mm, $B = \dfrac{\left(4\pi \times 10^{-7}\right) \times 10}{2\pi \times \left(10 \times 10^{-3}\right)} = 2 \times 10^{-4}$ T

When $r = 2$ mm, $B = \dfrac{\left(4\pi \times 10^{-7}\right) \times 10}{2\pi \times \left(2 \times 10^{-3}\right)} = 1 \times 10^{-3}$ T

For the final part where the field is to be calculated inside the wire, we must only consider the current which flows through the path of integration. If we assume current flow to be uniform inside the wire, we can calculate I', the current flowing through the contour, as follows :-

$$I' = I\left(\frac{\pi r^2}{\pi a^2}\right) \text{ where } a \text{ is the radius of the wire}$$

$$I' = 10 \times \left(\frac{\pi\left(1 \times 10^{-3}\right)^2}{\pi\left(2 \times 10^{-3}\right)^2}\right) = 2.5 \text{ A}$$

$$B = \frac{\mu_0 I'}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\right) \times 2.5}{2\pi \times \left(1 \times 10^{-3}\right)} = 5 \times 10^{-4} \text{ T}$$

Q22 To calculate the B field at different radii from the centre we use Ampère's Law:-

$$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 I$$

An appropriate contour for the integration is a circle of radius r.

For each of the different regions we must calculate the current flowing through the contour.

(To avoid confusion, define I' to be the current flowing through the contour, and I to be the total current referred to in the question.)

a)

$$I' = I\left(\frac{\pi r^2}{\pi a^2}\right)$$

$$B = \frac{\mu_0 I'}{2\pi r} = \frac{\mu_0 I r}{2\pi a^2}$$

b)

$$I' = I$$

$$B = \frac{\mu_0 I'}{2\pi r} = \frac{\mu_0 I}{2\pi r}$$

c)

$$I' = I - I\left(\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}\right)$$

$$I' = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

$$I' = I\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$$

$$B = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$$

d)

$$I' = I - I = 0$$

$$B = 0$$

Q23 By symmetry the field must be parallel to the surface occupied by the wires, and be equal and opposite on the two sides of the wires. If the field is B at a distance $\frac{h}{2}$ from the wires, then we have:-

$$Bg + 0h + Bg + 0h = \mu_0 n g I$$

$$B = \frac{\mu_0 n I}{2}$$

Q24 The field a distance r from wire M is:-

$$B = \frac{\mu_0 I}{2\pi r}$$

Therefore the flux through the circuit R is:-

$$\Phi = \int_{z_1}^{z_2}\int_{r_1}^{r_2} \frac{\mu_0 I}{2\pi r}\, dr dz$$

$$\Phi = (z_2 - z_1)\frac{\mu_0 I}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$$

$$\varepsilon = \frac{d\Phi}{dt} = (z_2 - z_1)\frac{\mu_0}{2\pi}\ln\left(\frac{r_2}{r_1}\right) \times \frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{d}{dt}(I_0 \sin \omega t) = I_0 \omega \cos \omega t$$

$$\frac{dI}{dt} = (4 \times \sqrt{2}) \times (2\pi \times 50) \times \cos \omega t$$

Considering only the rms value:-

$$\frac{dI}{dt} = (4) \times (2\pi \times 50) = 400\pi \text{ As}^{-1} \text{ rms}$$

$$\varepsilon = (80 \times 10^{-3})\frac{4\pi \times 10^{-7}}{2\pi}\ln\left(\frac{60 \times 10^{-3}}{20 \times 10^{-3}}\right) \times 400\pi$$

$$\varepsilon = 22\ \mu\text{V}$$

Q25 Defining x to be the length of the loop inside the field, then the flux through the loop is given by:-

$$\phi = B\ell x$$

The induced emf is then:-

$$\varepsilon = \frac{d\phi}{dt} = \frac{d}{dt}\left(B\ell x\right)$$

$$\varepsilon = B\ell\frac{dx}{dt} = B\ell v$$

$$\varepsilon = 1\times\left(100\times10^{-3}\right)\times10$$

$$\varepsilon = 1\ \text{V}$$

Q26 From the formula sheet:-

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi\rho}$$

Thus:-

$$\frac{F}{\ell} = \frac{4\pi\times10^{-7}\times1\times1}{2\pi\left(2\times10^{-3}\right)}$$

$$\frac{F}{\ell} = 1\times10^{-4}\ \text{Nm}^{-1}$$

The currents are in opposite directions so the force will be repulsive.

Q27 a) From the formula sheet:-

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi\rho}$$

Thus:-

$$\frac{F}{\ell} = \frac{4\pi\times10^{-7}\times1000\times1000}{2\pi\times1}$$

$$\frac{F}{\ell} = 0.2\ \text{Nm}^{-1}$$

b) $$F = BI\ell\sin\theta$$

Assuming $\theta = 90°$ then the force per unit length is…

$$\frac{F}{\ell} = 10^{-4}\times1000$$

$$\frac{F}{\ell} = 0.1\ \text{Nm}^{-1}$$

c) Assume the cable has a diameter of 25mm and is made of aluminium thus having a density ρ of 2.7×10^3 kg m^{-3}

$$F = mg = \frac{\pi d^2\ell}{4}\times\rho\times g$$

$$\frac{F}{\ell} = \frac{\pi\left(25\times10^{-3}\right)^2}{4}\times\left(2.7\times10^3\right)\times9.81$$

$$\frac{F}{\ell} = 13\ \text{Nm}^{-1}$$

Thus, the magnetic forces calculated in parts (a) and (b) are very small compared to the force due to gravity.

Q28 a) We start by calculating the induced emf:-

$$\varepsilon = B\ell v$$

The current in circuit is then given by:-

$$I = \frac{V}{R} = \frac{B\ell v}{R}$$

Thus, the Power into resistor R is:-

$$P = I^2 R = \frac{B^2\ell^2 v^2}{R}$$

b) The rate of doing mechanical work, P_m, can be calculated by differentiating the expression for work done:-

$$P_m = \frac{d}{dt}\left(BI\ell \times \text{distance}\right) = BI\ell \frac{d}{dt}\left(\text{distance}\right)$$

$$P_m = BI\ell v$$

c) Substituting the expression for I into the above expression gives:-

$$P_m = BI\ell v = B\ell v\left(\frac{B\ell v}{R}\right)$$

$$P_m = \frac{B^2\ell^2 v^2}{R}$$

Thus P_m is equal to P

Q29 The Hall effect is not currently on the syllabus, so this question can be safely ignored.

Q30 The force on a charged particle moving in a magnetic field is given by:-

$$\underline{F} = q\left(\underline{v} \times \underline{B}\right)$$

If we assume the electron (which has a charge of 1.6×10^{-19} C) to be travelling at right angles to the field then:-

$$F = \left(1.6 \times 10^{-19}\right) \times \left(4 \times 10^{7}\right) \times 10^{-4}$$

$$F = 6.4 \times 10^{-16} \text{ N}$$

From the formula sheet, the mass of an electron is 9.1×10^{-31} kg, thus the force due to gravity is:-

$$F_g = mg = \left(9.1 \times 10^{-31}\right) \times 9.81$$

$$F_g = 8.9 \times 10^{-30} \text{ N}$$

Clearly, this is negligible when compared to the force due to the magnetic field.

To find the magnitude of an electric field which would produce the same force, we can use the equation:-

$$F = qE$$

$$E = \frac{F}{q} = \frac{6.4 \times 10^{-16}}{1.6 \times 10^{-19}} = 4 \times 10^{3} \text{ Vm}^{-1}$$