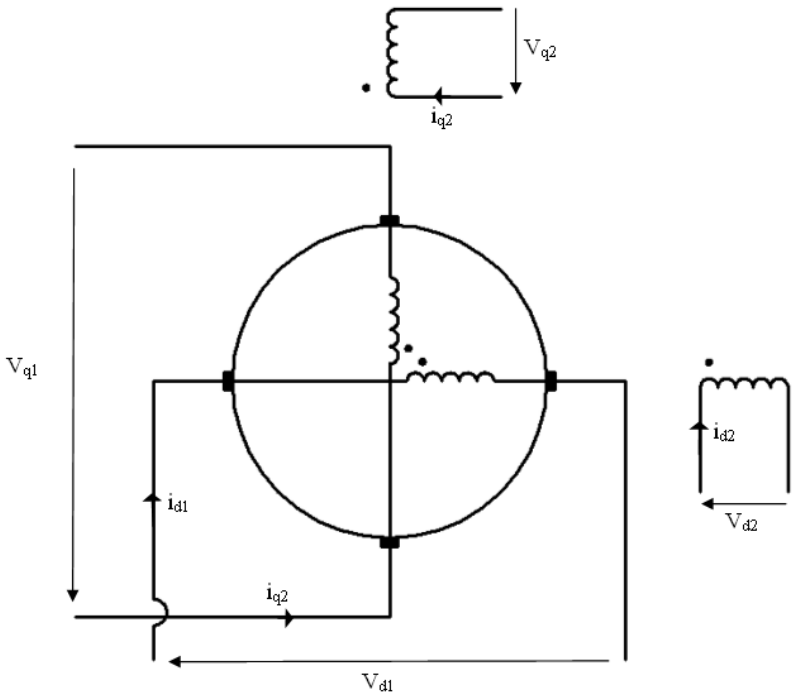
EEE6120 – 2012/13 session
Worked Solutions

1.

a)



The general form of the voltage matrix equations is given by:

$$\begin{vmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{vmatrix} = \begin{vmatrix} R_{d1}+L_{d1}p & G_{d1q2}\omega_r & M_{d1d2}p & G_{d1q2}\omega \\ G_{q1d1}\omega_r & R_{q1}+L_{q1}p & G_{q1d2}\omega_r & M_{q1q2}p \\ M_{d2d1}p & 0 & R_{d2}+L_{d2}p & 0 \\ 0 & M_{q2q1}p & 0 & R_{q2}+L_{q2}p \end{vmatrix} \begin{vmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{vmatrix}$$

b) On DC operation, the torque T produced by the machine is given by:

$$T = MI^2$$

Hence,

$$M = \frac{T}{I^2} = \frac{0.18}{1.30^2} = 0.106H$$

The terminal DC voltage is given by:

$$V = I(R + \omega_r M)$$

The above equation can be re-arranged to yield the resistance:

$$R = \frac{V}{I} - \omega_r M = \frac{240}{1.30} - \left(\frac{12000 \times 2\pi}{60}\right) \times 0.106 = 51.4\Omega$$

The copper losses are therefore given by:

$$P_{cu} = I^2 R = 1.30^2 \times 51.4 = 86.9W$$

Hence, efficiency is:

$$\eta = \frac{VI - I^2 R}{VI} = \frac{(240 \times 1.30) - 86.9}{(240 \times 1.30)} = 72.1\%$$

[Could also be derived from: $\eta = \frac{T\omega}{VI} = \frac{0.18 \times \left(\frac{12000 \times 2\pi}{60}\right)}{240 \times 1.30} = 72.1\%$]

c) On AC:

For the same torque of 0.18Nm, the AC current is 1.30 Arms at a power factor of 0.76 lagging

Hence, the current is 1.30 $\angle$-40.5° Arms

Hence, the impedance Z is given by:

$$Z = \frac{240}{1.30\angle -40.5^o} = 184.6\angle 40.5^o$$

$$\therefore R + \omega_r M = \mathrm{Re}\,al\ \ part\left(184.6\angle 40.5^o\right) = 140.3\Omega$$

$$\therefore \omega_r = \frac{140.3 - R}{M} = \frac{140.3 - 51.4}{0.106} = 839\ \ rad/s = 8008rpm$$

which, as expected, is lower than on the same DC voltage

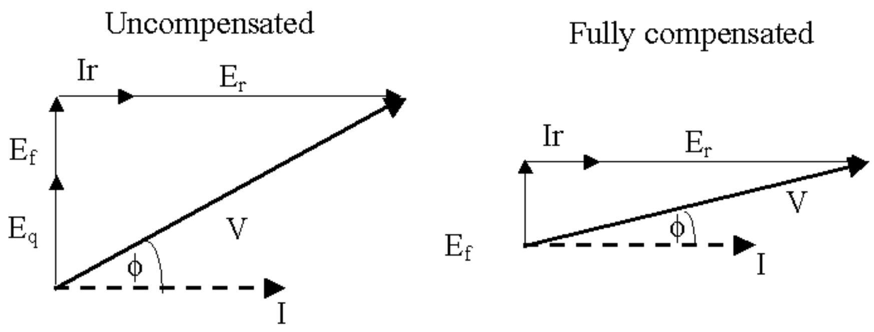
$$P_{mech} = \frac{8008 \times 2\pi}{60} \times 0.18 = 151W$$

$$P_{in} = VI\ cos\phi = 240 \times 1.3 \times 0.76 = 237W$$

$$Efficiency = \frac{P_{mech}}{P_{in}} = \frac{151}{237} = 63.8\%$$

[Could also get from:
$P_{loss} = I^2R = 1.3^2 \times 50.8 = 85.8W(as\ before\ since\ the\ torque\ is\ the\ same)$
$Efficiency = \frac{P_{mech}}{P_{mech}+P_{loss}} = \frac{151}{151+85.8} = 63.8\%]$

d) The phasor diagrams for the uncompensated and compensated motors are:



e) The key assumption is that the coupling coefficient for the two coils on the q-axis is 1.0. If this condition is satisfied then the inductive compensating coil automatically compensates in full for the jX_aI voltage drop that would occur in an uncompensated machine. For this condition and noting that field and armature reactance are equal, then the overall series reactive volts drop caused by the two coils of the original uncompensated machine is halved.

From the previous uncompensated machine:

$$Eq + Ef\ =\ 240\sin(40.5^o) = 156V$$

Hence, in the compensated machine, E_f=78V

Same torque of 0.18Nm and hence magnitude of current remains at 1.30Arms

Power factor = $cos\left(sin^{-1}\left(\frac{78}{240}\right)\right) =$ **<u>0.945 lagging</u>** [*Note: Important to state lagging*]

$$Z = \frac{240}{1.30\angle -19.0^o} = 184.6\angle 19.0^o$$

$$\therefore R + \omega_r M = \mathrm{Re}al\ \ part\left(184.6\angle 19.0^o\right) = 175\Omega$$

$$\therefore \omega_r = \frac{175-50}{0.106} = \ \ rad/s = \underline{\mathbf{11{,}260rpm}}$$

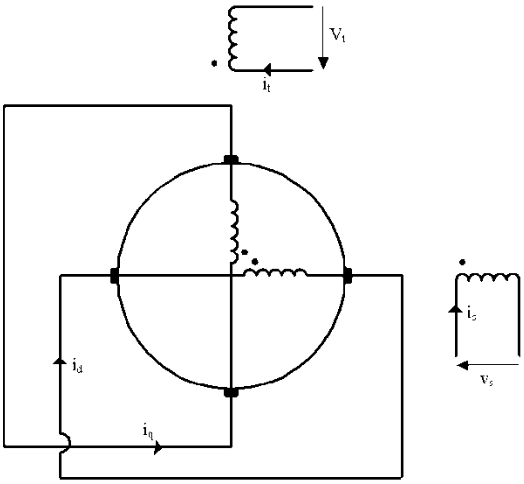
which as expected is higher than the uncompensated machine but lower than on DC.

Output power = $T_{em}\omega r = 0.18 \times 11260 \times \frac{2\pi}{60} = 212W$

$$Efficiency = \frac{212}{212 + 85.8} \times 100\% = 71.2\%$$

2.

a) The Kron primitive equivalent of a three-phase induction motor is:



[Important to get various conventions correct – marks lost for incorrect or ambiguous labelling]

b)

[*This is rather different to past questions in that it only asks for the very first stage of the analysis that leads to the derivation of an equivalent circuit – with a consequent significant reduction in marks available. Should students not adhere to the question posed and progress the derivation beyond this first step than no marks will deducted. The eagle-eyed will see that this section yields the same matrix a Q1 part (a)*]

$$\begin{vmatrix} v_s \\ v_t \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_s+L_sp & 0 & M_{sd}\,p & 0 \\ 0 & R_t+L_tp & 0 & M_{tq}\,p \\ M_{ds}p & -M_{dt}\omega_r & R_d+L_dp & -L_q\omega_r \\ M_{qs}\omega_r & M_{qt}\,p & L_d\omega_r & R_q+L_qp \end{vmatrix} \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix}$$

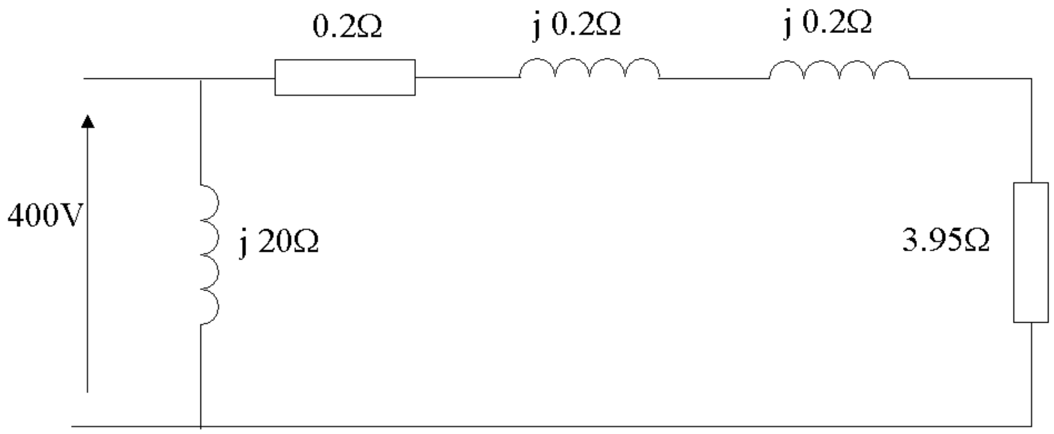
b)

[*This problem can be solved using either the exact or simplified equivalent circuit. The latter involves moving the magnetising branch to the terminals, but is reliant on the magnetising reactance being significantly higher than the other impedances. Providing students recognise this assumption then the use of the simplified equivalent circuit is equally as valid in terms of the marks awarded*]

i) The synchronous speed of a 4-pole motor on a 50Hz supply is 1500rpm. At 1462 rpm the slip is therefore 0.025.

The approximate equivalent circuit of the machine is therefore given by:

NEED to change reactance in drawing



The impedance of the main branch of the circuit is given by:

$$Z_e = (0.2 + 3.95) + j0.7 \; = 4.21\angle 9.6^o \Omega$$

The phase voltage applied to the machine is $\dfrac{400}{\sqrt{3}} = 231\ V$

$$I_1 = \frac{231}{4.21\angle 5.5^o} = 54.6\angle -9.5^o\, A \text{ rms}$$

The current in the magnetising branch is given by:

$$I_m = \frac{231}{20\angle 90^o} = 11.5\angle -90^0$$

The net input current is hence given by:

$$I_{ip} = I_1 + I_m = (53.9 - j9.1) - j11.5 = 53.9 - j20.6 = \; 57.7\angle -20.9^o\, A \text{ rms}$$

Hence magnitude of current = **57.7 Arms**

ii) Power factor = cos (-20.9^{o}) = **0.934 lagging**

iii) The electromagnetic output power is given by:

$$P_{em} = 3\,|I_1|^2 \frac{(1-s)R_2^{'}}{s} = 3 \times 54.6^2 \times 3.85 = \; 34.43\text{kW}$$

[Note: Factor of 3 for 3 phases and that current is main branch current and not total current]

And hence the electromagnetic output torque is given by:

$$T_{em} = \frac{P_{em}}{\omega_{r-mech}} = \frac{34.43\times 10^3}{1462\times\frac{2\pi}{60}} = \mathbf{225Nm}$$

iv) The copper losses are given by:

$$P_{cu} = 3|I_1|^2 (R_1 + R_2^{'}) = \; \mathbf{2.68kW}$$

[again note factor of 3 and that current is main branch current and not total current]

b) On no-load the only components of current drawn are due to no-load iron loss and the magnetizing current. (based on simplified equivalent circuit – this being hinted at in the question).

The real component of the no-load current when the machine is operating on no-load is given by:

$Re\langle 12.1\angle -72.6^{o}\rangle = 12.1 \times cos(-72.6^{o})$=3.61 Arms

Hence, iron loss resistor is given by:

$$R_{fe} = \frac{V_{ph}}{Real\ componet\ of\ no\text{-}load\ current} = \frac{231}{3.61} = \underline{\mathbf{64.2\ \Omega}}$$

Iron loss is given by either:

$$P_{fe} = 3 \times \frac{V_{ph}^2}{R_{fe}} = 3 \times \frac{231^2}{64} = \underline{\mathbf{2.50kW}}$$

or

$$P_{fe} = 3 \times 3.61^2 \times 64.2 = \underline{\mathbf{2.50kW}}$$

c) At 1462 rpm:

Copper loss = 2.68kW from part (a)

Iron loss = 2.50kW from part (b)

Electromagnetic output power = 34.43kW from part (a)

[Note: It is not correct to base an efficiency calculation on any value of electrical input power from part (a) as this does not include the effect of iron loss]

$$Efficiency = \frac{P_{em}}{P_{em} + P_{cu} + P_{fe}} = \frac{34.43}{34.43 + 2.68 + 2.50} \times 100\% = 86.9\%$$

3.

a) Applying the trapezium rule to integrate the area under the fully aligned curve (i.e. the curve at an angular displacement of 30°) for currents up to 5A yields:

$$A_{0\to 2.5} = \frac{2.5 \times \Psi_{2.5}}{2} = 0.15J$$

$$A_{2.5\to 5} = \frac{2.5(\Psi_{2.5} + \Psi_5)}{2} = 0.41J$$

Hence the total area under the curve up to 5A is:

$$A_{0\to 5} = A_{0\to 2.5} + A_{2.5\to 5} = 0.56J$$

The area under the un-aligned curve (which can reasonably be regarded as being linear) is simply given by:

$$U_{0\to 5} = \frac{5\Psi_5}{2} = 0.04\text{J}$$

Hence the change in co-energy is given by:

$$\Delta W' = A_{0\to 5} - U_{0\to 5} = 0.52\text{J}$$

The average torque for 5A is therefore given by:

$$T_{AVE} = \frac{\Delta W'}{\Delta\theta} = \frac{0.52}{\pi/6} = 1.0\text{Nm}$$

Repeating the same process from 5A to 15A yields:

$$A_{5\to 7.5} = \frac{2.5(\Psi_5 + \Psi_{7.5})}{2} = 0.55\text{J}$$

$$A_{7.5\to 10} = \frac{2.5(\Psi_{7.5} + \Psi_{10})}{2} = 0.61\text{J}$$

$$A_{0\to 10.0} = A_{0\to 2.5} + A_{2.5\to 5} + A_{5\to 7.5} + A_{7.5\to 10} = 1.72J$$

$$U_{0\to 12.5} = \frac{10.0\times\Psi_{10u}}{2} = 0.16J$$

Hence the change in co-energy is given by:

$$\Delta W' = A_{0\to 10} - U_{0\to 10} = 1.56J$$

The average torque for 10A is therefore given by:

$$T_{AVE} = \frac{\Delta W'}{\Delta\theta} = \frac{1.56}{\pi/6} = 2.98Nm$$

b. From the aligned Ψ-I characteristic it can be seen that the onset of saturation occurs at a current of ~4.5A (an answer based on a slightly different interpretation of saturation is equally acceptable).

It is important to note that the flux produced by 2 coils that constitute a phase crosses 2 airgaps, each of length l_g.

Assuming the permeability of the stator and rotor core is >> μ_0 up the onset of saturation, then:

$$l_g = \frac{\mu_0 N_{ph} I}{2B_g} = \frac{\mu_0 N_c I}{B_g} = \frac{4\pi\times 10^{-7}\times 120\times 4.5}{1.5} 0.45mm$$

[Reasonable tolerance on this solution is acceptable in light of interpretation of saturation occurring at 4.5A]

C

[As noted in the question, although some previous questions have asked candidates to plot a flux-linkage versus current characteristic as a pre-cursor to calculating some emf values, in this case, the information requested can be obtained straightforwardly from the Ψ-I characteristic]

A rotor angular displacement mid-way through the stroke = 15^o

An estimate of the rate of change of flux-linkage at 15^o can be obtained from the flux-linkage at 12^o and 18^o. At 17.5A, the rate of change of flux-linkage with respect to angular displacement is given by:

$$\frac{d\psi}{d\theta} \approx \frac{\Delta\psi}{\Delta\theta} = \frac{\psi_{18} - \psi_{12}}{6 \times \frac{\pi}{180}} = \frac{0.20 - 0.145}{0.105} = 0.525\ Wb/rad$$

The rate of change of angular displacement with time at 4000rpm is given by:

$$\frac{d\theta}{dt} = \frac{4000}{60} \times 2\pi = 419\ rad/s$$

Hence, an estimate of the emf can be obtained from

$$\frac{d\psi}{dt} = \frac{d\psi}{d\theta} \times \frac{d\theta}{dt} = 0.525 \times 419 = \underline{\mathbf{220V}}$$

d.

[The influence of a change in airgap on the nature and shape of the Ψ-I characteristic have been discussed in lectures, but quantitative assessments have not featured in the lectures nor in previous tutorials or examination papers – this is therefore a challenging test of a candidates in-depth understanding of Ψ-I behaviour].

[There is no issue of carry-through of an error in airgap from part (b) as the key point is that it is halved and not its absolute value].

If there is no significant saturation in the stator and rotor cores up to a flux linkage of 0.20Wb, then the slope of the Ψ-I characteristic will double.

With original gap, slope is 0.20/4 = 0.05 Wb/A

With reduced airgap, slope = 0.10 Wb/A

Hence, the flux linkage at 2A with the reduced airgap is 0.20Wb

$$A_{0\to 2} = \frac{2.0 \times 0.20}{2} = 0.20J$$

The area under the un-aligned curve (which can reasonably regarded as being linear) is simply given by:

$$U_{0\to 2} = \frac{2\times 0.008}{2} = 0.008J$$

Hence the change in co-energy is given by:

$$\Delta W' = A_{0\to 2} - U_{0\to 2} = 0.192J$$

The average torque for 2A with the reduced airgap is therefore given by:

$$T_{AVE} = \frac{\Delta W'}{\Delta\theta} = \frac{0.192}{\pi/6} = 0.366Nm$$

[*again some tolerance on this is acceptable*]

4.

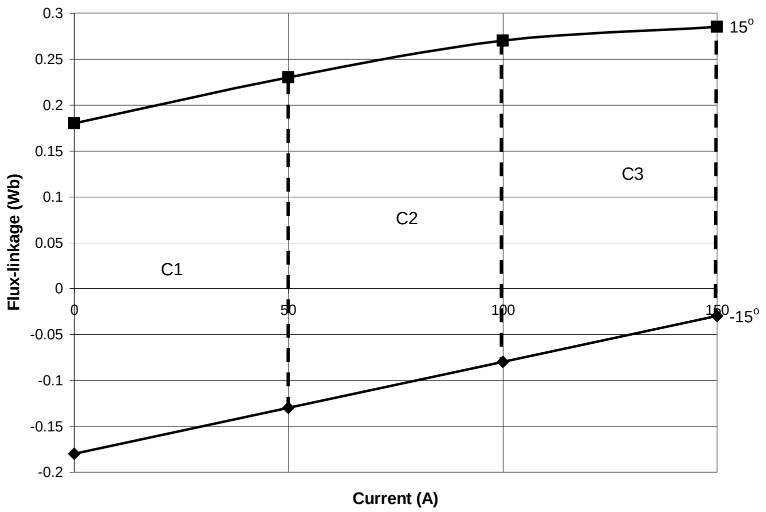
a) The flat-top of the nominally trapezoidal emf corresponds with the linear region of the flux-linkage versus rotor angular displacement characteristics. Although the slope can be taken at any point along the linear section, the region around an angular displacement of 0° mechanical is as reliable an estimate as any. The slope of the characteristic is:

$$\frac{d\psi}{d\theta} \approx \frac{0.06+0.06}{10\times\frac{\pi}{180}} = 0.686\ Wb/rad(mech)$$

Hence, at 6000rpm, the induced emf is given by:

$$e = \frac{d\Psi}{dt} = \frac{d\Psi}{d\theta}\times\frac{d\theta}{dt} = 0.686\times\frac{6000\times 2\pi}{60} = \underline{\mathbf{431V}}$$

b) In brushless DC mode, currents are driven into a phase between rotor angular displacements of -60° to +60° (elec) which corresponds to -15° to +15° on the characteristics of this 8 pole machine. Re-plotting the key date points as a ψ-I curves yields:



The co-energy changes C_1, C_2 and C_3 are given by:

$$C_1 = \frac{(0.36+0.36)}{2} \times 50 = 18.0J$$
$$C_2 = \frac{(0.36+0.35)}{2} \times 50 = 17.75J$$
$$C_3 = \frac{(0.35+0.315)}{2} \times 50 = 16.62J$$

Average torque during 120° commutation interval for one phase at 50A:

$$T_{ave}\big|_{50A} = \frac{C_1}{\Delta\theta} = \frac{18}{30 \times \frac{\pi}{180}} = 34.4Nm$$

And similarly at 150A:

$$T_{ave}\big|_{150A} = \frac{C_1 + C_2 + C_3}{\Delta\theta} = \frac{52.4}{30 \times \frac{\pi}{180}} = 100.0Nm$$

(i.e. less than 3×50A value due to saturation)

[*The average torque produced by the machine is 2 × values calculated above (comes from 3 × 2/3 utilisation) rather than 3 times value above (common mistake) – 2 marks deducted for this error*]

Hence, average machine torque at 50A is **68.8Nm** and at 150A is **200Nm**

c) The various inductances can be calculated from the additional flux-linkage produced by the current. For the 4 cases :

At 50A:

$$L = \frac{0.05}{50} = 1mH \quad at \ -22.5^o$$
$$L = \frac{0.05}{50} = 1mH \quad at \ +22.5^o$$

At 150A:

$$L = \frac{0.15}{150} = 1mH \quad at \ -22.5^o$$
$$L = \frac{0.088}{150} = 0.59mH \quad at \ +22.5^o$$

The difference observed at 150A and +22.5^o is a result of magnetic saturation this is the worst case angular displacement at which the magnet flux and the coil flux add to each other

d)

[*This is the first example in examinations on this course in which the number of turns on the coils is calculated from the magnet thickness – the few instances of similar questions have posed the problem the other way around. Moreover, rather than specifying a saturation flux density for the core material, it is necessary for candidates the recognise that Silicon Iron starts to show appreciable saturate at ~1.6T – although there is clearly a need to accept a degree of reasonable tolerance on this*].

The exact nature of the flux-paths in a multi-phase brushless DC machine is complex. However, a reasonable assumption in terms of calculating the total effective magnetic airgap (i.e. magnet length + airgap length) is to assume that each coil produces the mmf across a single combined airgap and magnet length. This stems from the fact that each 'closed loop' of main coil flux path passes across 2 airgaps, through 2 magnets and is 'driven' by 2 coils.

Assuming that the airgap flux density provides a reasonable estimate of the flux density level in the core as a whole then by inspection of the flux-linkage characteristics, the core of the machine appears to be saturating at a flux-linkage of ~0.27Wb, which corresponds to a flux density of ~1.6T [*see note about tolerance*]

Considering the case of a rotor angular displacement of 0º (in which no net magnet flux is present). A current of 50A produces a flux-linkage of 0.05Wb, which by equivalence with 1.6T at 0.27Wb, corresponds to a flux density of 0.30 T

At this flux density level, then it is reasonable to assume that the rotor and stator cores will be infinitely permeable. The airgap flux density produced by a given coil mmf is given by:

$$B_g = \frac{\mu_0 NI}{l_{eff}}$$

The effective airgap length is 6mm (1mm gap + 5mm magnet)

Re-arranging this equation yields:

$$N = \frac{B_g l_{eff}}{\mu_0 I} = \frac{0.3 \times 6 \times 10^{-3}}{4\pi \times 10^{-7} \times 50} = 29 \;\; turns \;\; per\, coil$$

But each phase consists of 4 coils and hence the number of series turns per phase = **<u>108</u>**