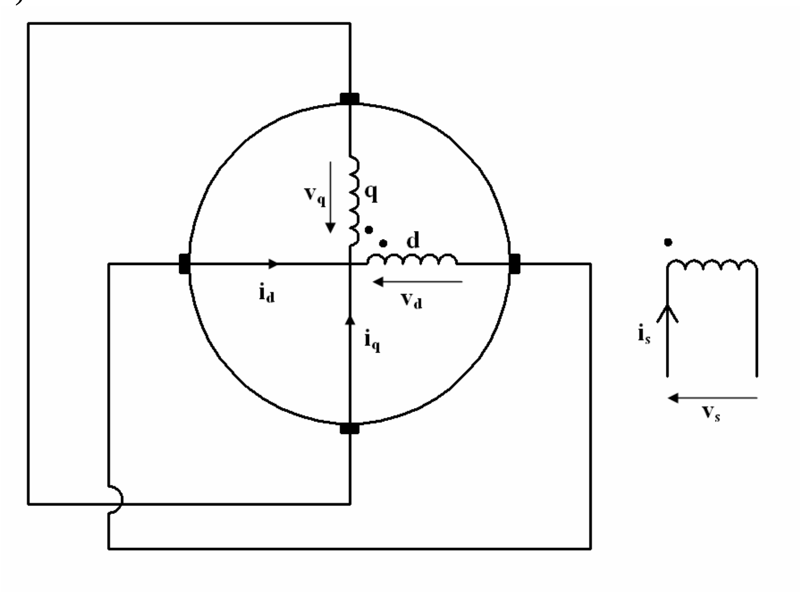
<u>EEE6120 Solutions – 2007</u>

1.

a)



b) The general form of the voltage matrix equations si:

$$\begin{vmatrix} v_s \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_s + L_s p & M_{sd}p & 0 \\ M_{ds}p & R_d+L_dp & -\omega_r L_q \\ \omega_r M_{qs} & \omega_r L_d & R_q+L_qp \end{vmatrix} \begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix}$$

[Note: The use of subscripts 1 for stator and 2 for rotor would be equally correct in the above equations]

For steady-state sinusoidal AC operation, $p=j\omega_s$ and $\omega_r=(1-s)\omega_s$

The short-circuited rotor windings dictate that $V_q=V_d = 0$

Adopting subscripts 1 for stator and 2 for rotor, yields

$$\begin{vmatrix} V_s \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m & 0 \\ jX_m & R_2+jX_2 & -(1-s)X_2 \\ (1-s)X_m & (1-s)X_2 & R_2+jX_2 \end{vmatrix} \begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix}$$

The unbalanced currents on the right hand side can be transformed to symmetrical components:

$$\begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix} = C \begin{vmatrix} Is \\ Ip \\ In \end{vmatrix}$$

Where C is the following transformation matrix:

$$C = 1 / \sqrt{2} \begin{vmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & j & -j \end{vmatrix}$$

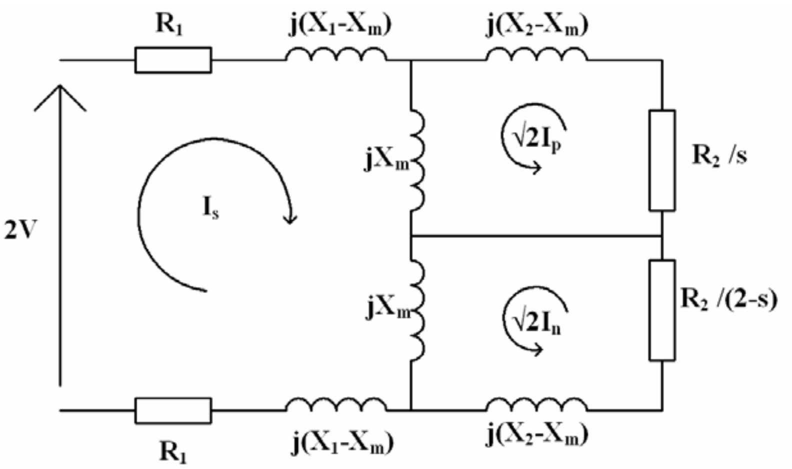
Applying the transformation matrix to the impedance matrix using:

$Z' = C_t^* \, Z \, C$

Following two stages of matrix multiplication, the final result is:

$$\begin{vmatrix} 2V_s \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 2(R_1+jX_1) & jX_m & jX_m \\ jX_m & \dfrac{R_2}{s} + jX_2 & 0 \\ jX_m & 0 & \dfrac{R_2}{(2-s)} + jX_2 \end{vmatrix} \begin{vmatrix} Is \\ \sqrt{2}\, I_p \\ \sqrt{2}\, I_n \end{vmatrix}$$

The equivalent circuit whish satisfies these equations is:

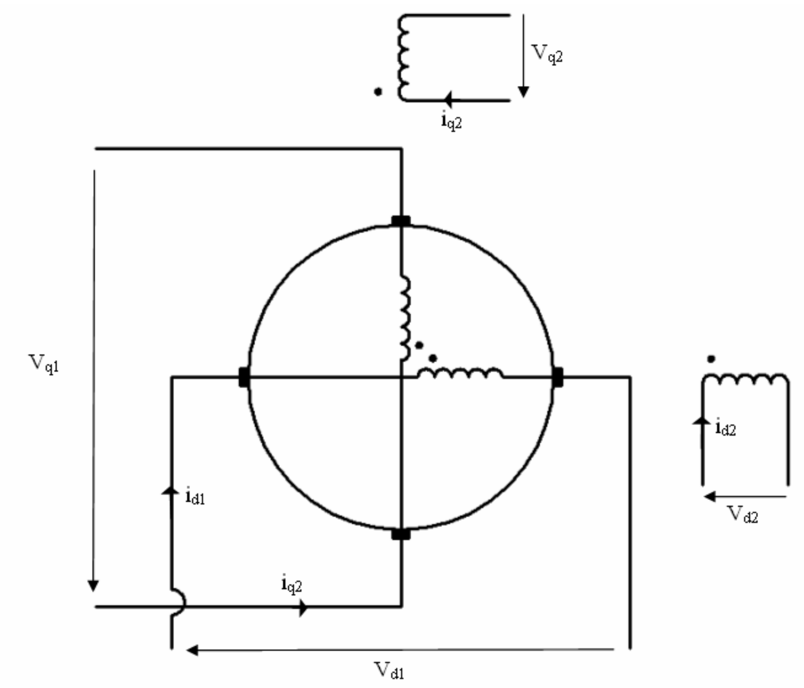


c) Breif description of 3 out of 5:

- Shaded pole motor
- Split phase motor
- Permanent split capacitor motor
- Capacitor start motor
- Dual value capacitor motor

2.

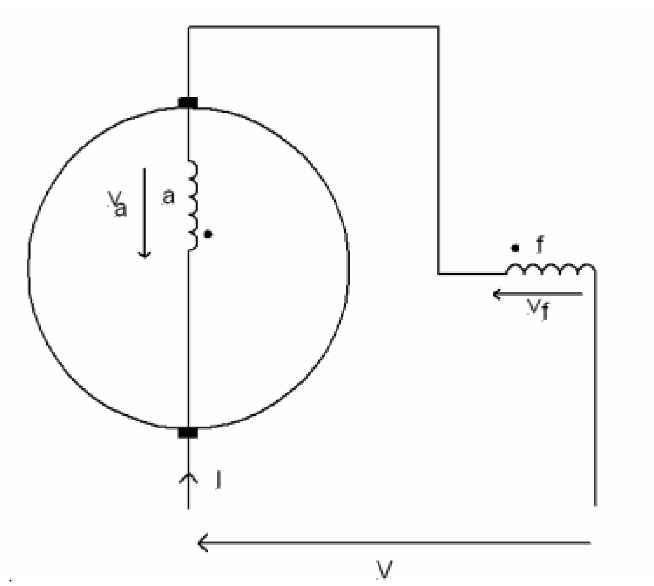
a)



The general form of the voltage matrix equations is given by:

$$\begin{vmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{vmatrix} = \begin{vmatrix} R_{d1}+L_{d1}p & G_{d1q2}\omega_r & M_{d1d2}p & G_{d1q2}\omega \\ G_{q1d1}\omega_r & R_{q1}+L_{q1}p & G_{q1d2}\omega_r & M_{q1q2}p \\ M_{d2d1}p & 0 & R_{d2}+L_{d2}p & 0 \\ 0 & M_{q2q1}p & 0 & R_{q2}+L_{q2}p \end{vmatrix} \begin{vmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{vmatrix}$$

b)



The general form of the voltage equations are:

$$\begin{vmatrix} v_a \\ v_f \end{vmatrix} = \begin{vmatrix} R_a + L_a p & \omega_r M \\ 0 & R_f + L_f p \end{vmatrix} \begin{vmatrix} i_a \\ i_f \end{vmatrix}$$

On DC: $p=0$

On AC: $p=j\omega_s$

Constraining equations:

$V = V_a + V_f$

$I = I_a = I_f$

The resulting voltage equations are:

DC operation:

$V = I\,(R_a + R_f + \omega_r M)$

AC operation:

$V = I\,(R_a + R_f + \omega_r M + j(X_a + X_f))$

c) On DC operation, the torque T produced by the machine is given by:

$$T = MI^2$$

Hence,

$$M = \frac{T}{I^2} = \frac{0.25}{1.44^2} = 0.12H$$

The terminal DC voltage is given by:

$$V = I(R + \omega_r M)$$

The above equation can be re-arranged to yield the resistance:

$$R = \frac{V}{I} - \omega_r M = \frac{240}{1.44} - \left(\frac{11000 \times 2\pi}{60}\right) \times 0.12 = 28\Omega$$

The copper losses are therefore given by:

$$P_{cu} = I^2 R = 1.44^2 \times 28 = 56.1W$$

Hence, efficiency is:

$$\eta = \frac{VI - I^2 R}{VI} = \frac{(240 \times 1.44) - 56.1}{(240 \times 1.44)} = 83\%$$

[Could also be derived from:

$$\eta = \frac{T\omega}{VI} = \frac{0.25 \times \left(\frac{11000 \times 2\pi}{60}\right)}{240 \times 1.44} = 83\% \text{]}$$

d) On AC:

For the same torque of 0.25Nm, the rms AC current is 1.44∠-32.9°

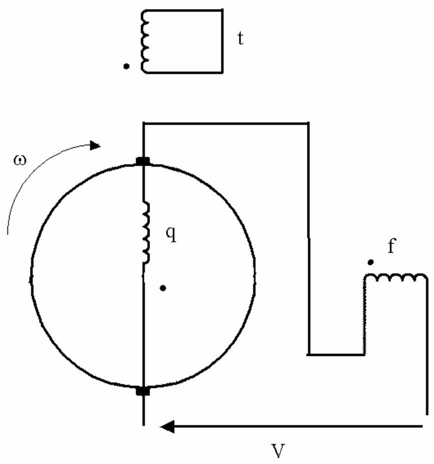
Hence, the impedance Z is given by:

$$Z = \frac{240}{1.44} = 166.7\angle 32.9^o$$

$$\therefore R + \omega_r M = \mathrm{Re}\,al \;\; part\left(166.7\angle 32.9^o\right) = 140\Omega$$

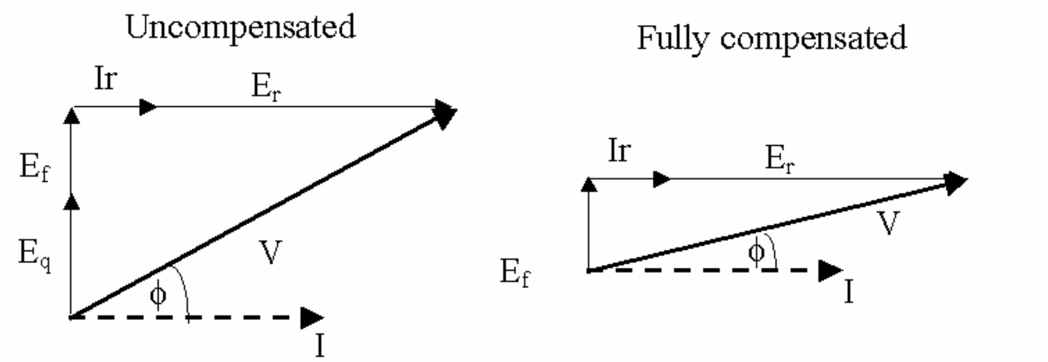
$$\therefore \omega_r = \frac{140 - R}{M} = 933 \;\; rad/s = 8909rpm$$

e) The Kron primitive equivalent of an inductively compensated series universal motor is:



Full compensation is achieved when the q-axis coils have a coupling coefficient of 1 (this contrasts with a conductively coupled machine in which other constraints on the inductance of the compensation coil must be met).

The resulting phasor diagrams are:



3.

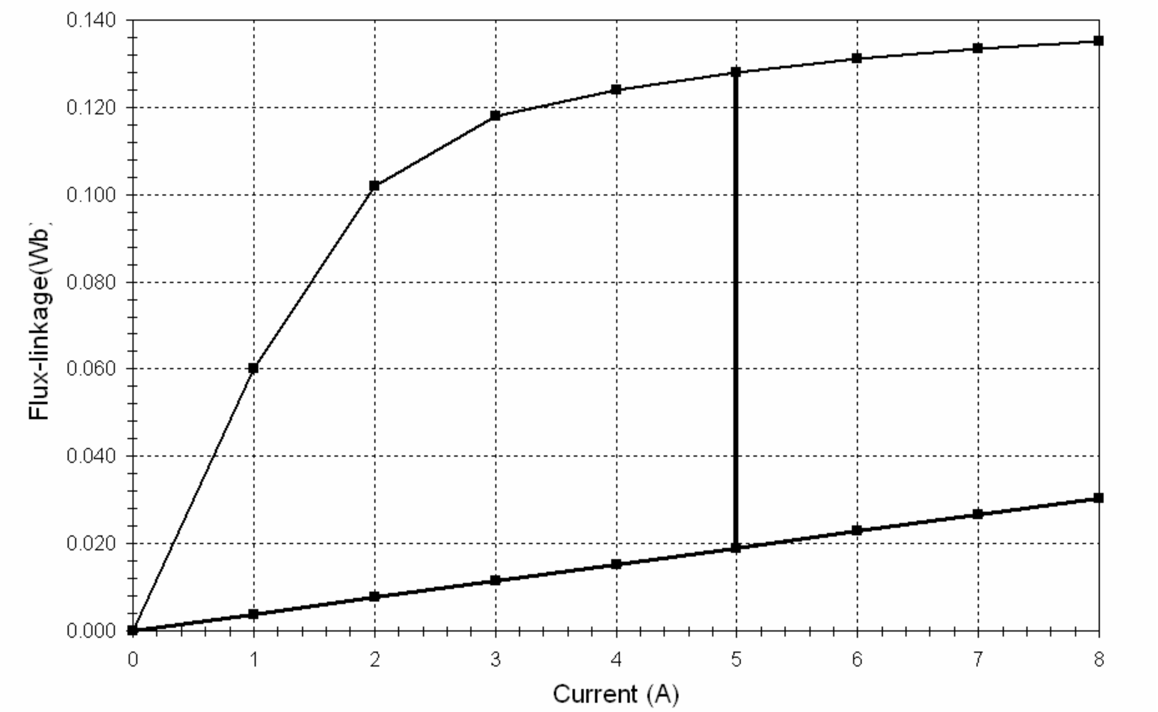
a) From Figure 3, the rate of change of flux linkage has a maximum value between 17.5° and 20°. The rate of change is given by:

$$\frac{d\psi}{d\theta} = \frac{113\times10^{-3} - 74\times10^{-3}}{2.5\times\frac{\pi}{180}} = 0.894 Wb/radian$$

At 3000rpm

$$\frac{d\psi}{dt} = \frac{d\psi}{d\theta}\times\frac{d\theta}{dt} = 0.894\times\frac{3000\times2\pi}{60} = 281V$$

b) Flux-linkage versus current characteristic is:



Applying the trapezium rule to integrate the area under the 22.5° curve:

$$A_1 = \frac{\Psi_1}{2} = \frac{60\times10^{-3}}{2} \qquad = 30\times10^{-3}\text{ J}$$

$$A_2 = \frac{\Psi_1+\Psi_2}{2} = \frac{60\times10^{-3}+0.102}{2} \qquad = 81\times10^{-3}\text{ J}$$

$$A_3 = \frac{\Psi_2+\Psi_3}{2} = \frac{0.102+0.118}{2} \qquad =110\times10^{-3}\text{ J}$$

$$A_4 = \frac{\Psi_3+\Psi_4}{2} = \frac{0.118+0.124}{2} \qquad =121\times10^{-3}\text{ J}$$

$$A_5 = \frac{\Psi_4 + \Psi_5}{2} = \frac{0.124 + 0.128}{2} \qquad =126\times10^{-3}\ \text{J}$$

$$Total\ \ area = A_1 + A_2 + A_3 + A_4 + A_5 = 468\times10^{-3}\ \ \text{J}$$

The area under the 7.50° curve (which can reasonably regarded as being linear) is simply given by:

$$U_5 = \frac{5\Psi_5}{2} = \frac{5\times19\times10^{-3}}{2} = 47\times10^{-3}\ \text{J}$$

Hence the change in co-energy is given by:

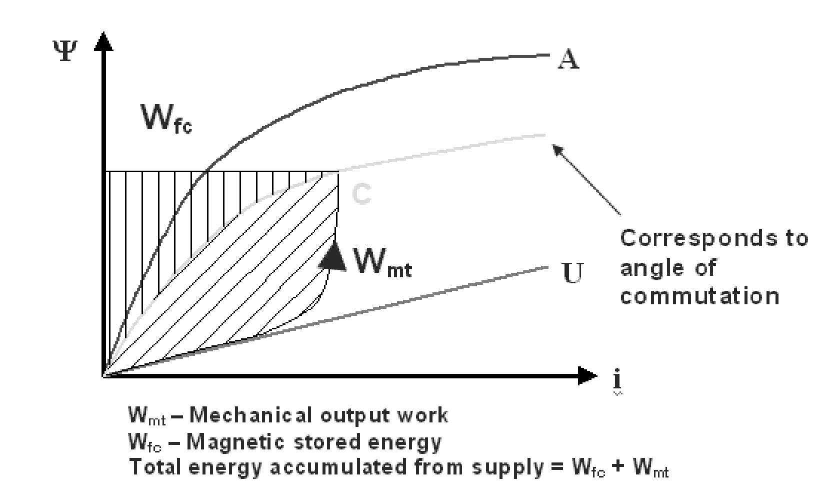
$$\Delta W' = A_{0\to5} - U_{0\to5} = 421\times10^{-3}\ \text{J}$$

The average torque is therefore given by:

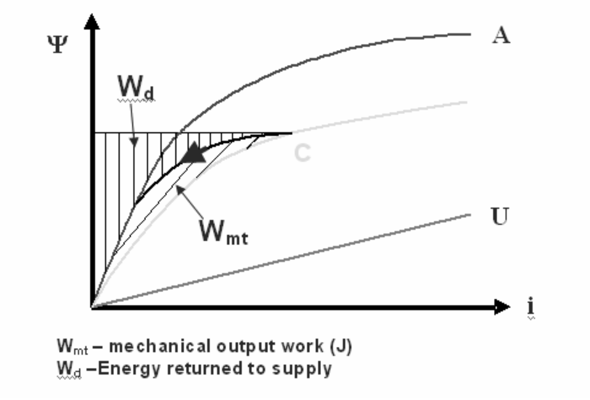
$$T_{AVE} = \frac{\Delta W'}{\Delta\theta} = \frac{421\times10^{-3}}{15\times\frac{\pi}{180}} = 1.61Nm$$

c) In motoring mode, the two dynamic ψ-I characteristics are:

Up to the instant of commutation:



Following commutation:



[In marking this section, particular emphasis will be placed on precise definitions and identification of the various energy changes]

d) Exact form of trajectory depends on:

Rotational speed
Rotor and load inertia
Magnitude of applied voltage
Commutation angles

Commutating prior to alignment to allow the current to decay by the time full alignment is reached – otherwise braking torque would be produced which would reduce the net torque produced

4.

a) By neglecting the inductive and resistive voltage drop, the maximum velocity is achieved when the induced back emf is equal to the applied voltage. An estimate of the rate of change of flux-linkage with linear displacement can be derived from Figure 4:

$$\frac{d\Psi}{dx} = \frac{0.16}{4\times10^{-3}} = 40 \;\; Wb/m$$

The maximum velocity is hence given by:

$$\frac{dx}{dt} = \frac{e}{\frac{d\Psi}{dx}} = \frac{V}{\frac{d\Psi}{dx}} = \frac{28}{40} = 0.7ms^{-1}$$

b) The inductance at 0mm is given by:

$$L_0 = \frac{0.05-0}{1} = 0.05H \text{ at 1A}$$

$$L_0 = \frac{0.25 - 0.20}{1} = 0.05H \text{ at 5A}$$

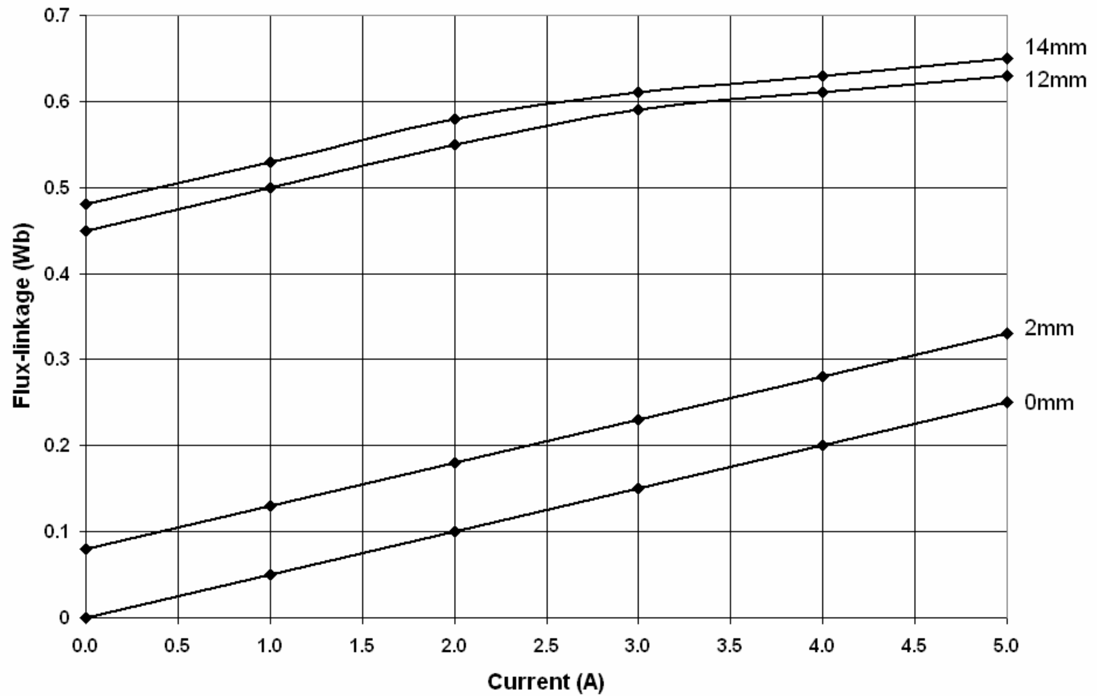
The inductance at 14mm is given by:

$$L_{14} = \frac{0.53 - 0.48}{1} = 0.05H \text{ at 1A}$$

$$L_{14} = \frac{0.65 - 0.63}{1} = 0.02H \text{ at 5A}$$

The reduction in inductance can be attributed to the onset of magnetic saturation in the actuator core(s), as 14mm displacement and 5A current correspond to the highest flux conditions encountered in the device.

c) The Ψ-I characteristics at these four linear displacements are:



d) Assuming that the characteristics are linear for displacements of 0mm and 2mm, then the change in co-energy can be calculated from a single trapezoidal integration: The resulting change in co-energy for an excursion between 0mm and 2mm is given by:

$$\Delta W' = 5 \times \left(\left(\frac{\Psi_{2mm|5A} + \Psi_{2mm|0A}}{2} \right) - \left(\frac{\Psi_{0mm|5A} + \Psi_{0mm|0A}}{2} \right) \right)$$

$$= 5\times\left(\left(\frac{0.33+0.08}{2}\right)-\left(\frac{0.25+0}{2}\right)\right)=0.4\text{J}$$

$$F = \frac{\Delta W^{'}}{\Delta x} = \frac{0.4}{2\times10^{-3}} = 200N$$

e) For the excursion between 12mm and 14mm, it is necessary to apply trapezoidal integration for each current interval:

$$\Delta W^{'} = 0.03 + 0.03 + 0.025 + 0.02 + 0.02 = 0.125J$$

$$F = \frac{\Delta W^{'}}{\Delta x} = \frac{0.125}{2\times10^{-3}} = 62.5N$$

The reduction in force is due to two factors:

- The onset of magnetic saturation in the actuator core(s)
- The roll-off in the rate of change of flux-linkage for strokes beyond the design stroke of the actuator