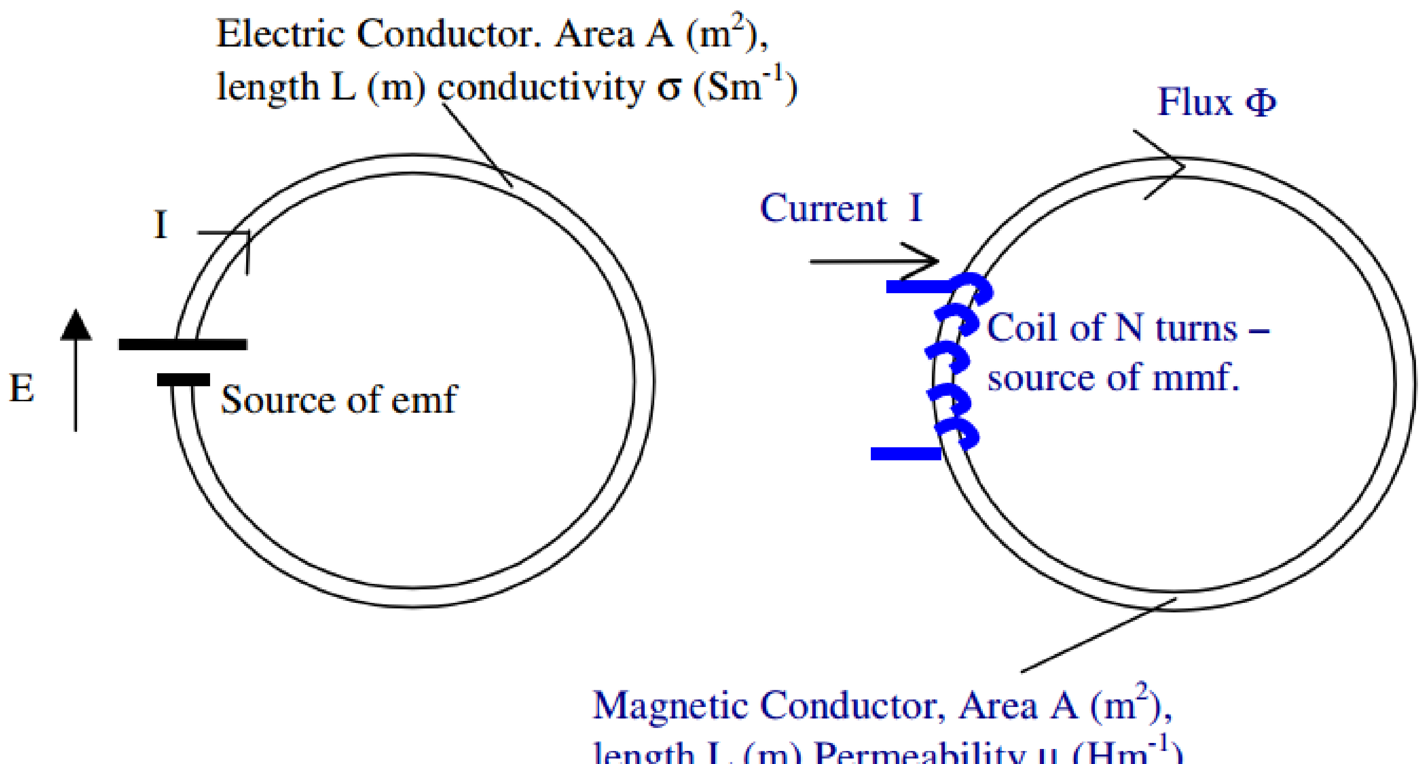
Magnetic circuits

- Convenient method (when it is applicable) for reducing magnetic ‘field’ problems to much simpler circuit type problems
- Relies on having dominant and well understood flux paths in the device
- Can be useful for 1^{st} order analysis of some aspects of electrical machines – but not all.

Electrical and magnetic circuit analogy

Electric circuit			**Magnetic circuit**		
Voltage (V)	V	V	Magneto-motive force	F	A or *A.turns*
Current (I)	I	A	Flux	ϕ	Wb
Resistance	R|	Ω	Reluctance	S	H
Conductivity	σ	Sm^{-1}	Permeability	μ	Hm^{-1}



Electric Circuit

For a circuit made up of a number of elements we have:

$$I = \frac{E}{\Sigma R} = \frac{\text{emf}}{\text{cct resistance}} \quad \frac{(V)}{(\Omega)} \quad A$$

where for each element: $$R = \frac{L}{\sigma A} \quad \Omega$$

Magnetic Circuit

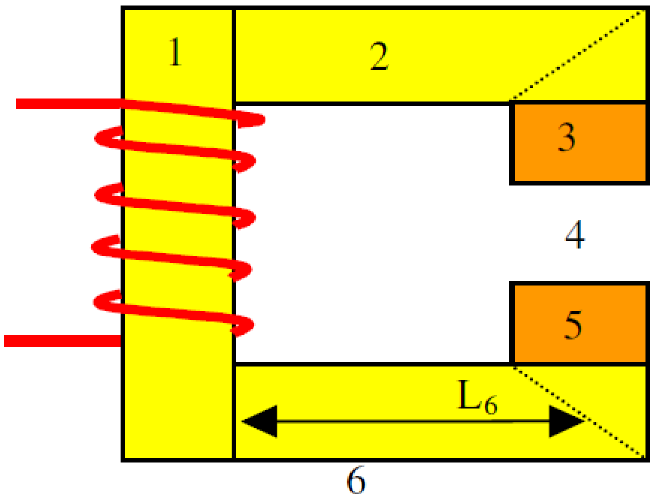
By analogy, the magnetic circuit quantities are related by:

$$\Phi = \frac{F}{\Sigma S} = \frac{\text{mmf}}{\text{cct reluctance}} \quad Wb$$

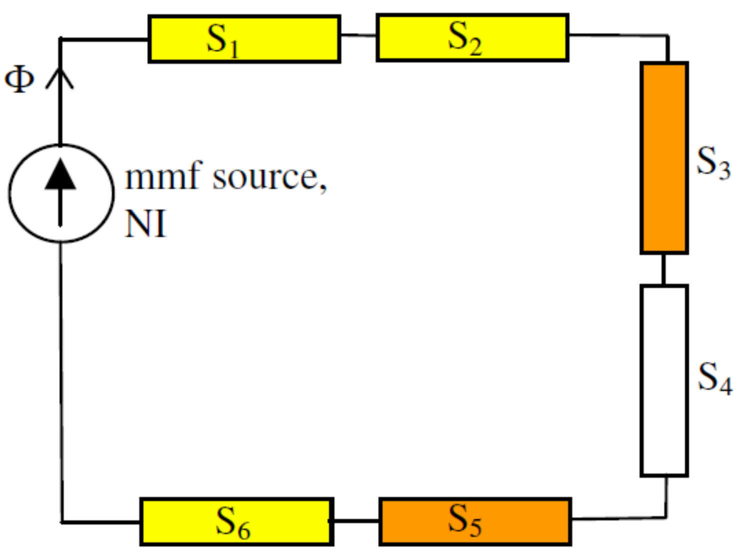
where Φ = flux in Webers (Wb)

$$S = \frac{L}{\mu A} \quad H^{-1} \quad (\text{Henry}^{-1})$$

Simple magnetic circuit



Each section having a length, L_n, area, A_n, and permeability, μ_n.



Where $S_1 = \dfrac{L_1}{\mu_1 A_1}$ etc

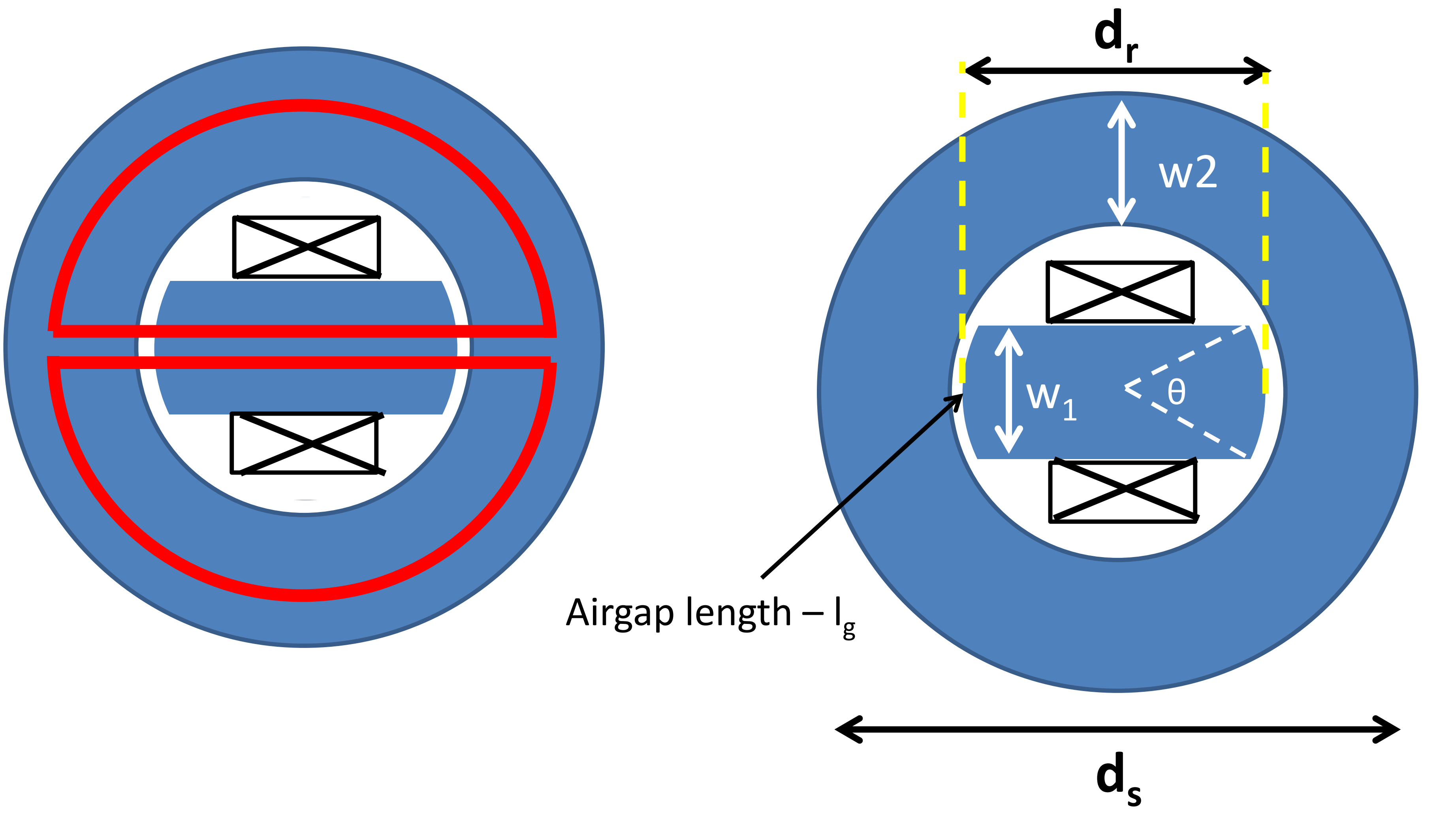
and flux

$$\Phi = \frac{NI}{S_1 + S_2 + S_3 + S_4 + S_5 + S_6}$$

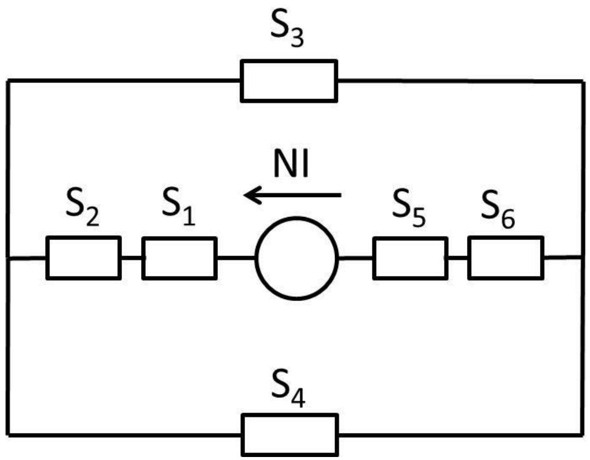
$$= \frac{NI}{S_T}$$

or $$\Phi = \frac{NI}{\Sigma S}$$

Simplified representation of the field winding of a salient pole AC synchronous machine



Simplified magnetic circuit



Some dimensions used in reluctance calculations based on approximations

Element	Symbols	Length	Cross-sectional Area	Reluctance
Rotor core	S1 and S5	$\frac{d_r}{2}$	$w_1 L$	$\frac{d_r}{2 w_1 L \mu_{core1}}$
Airgap	S2 and S6	l_g	$\frac{d_r \theta}{2} L$	$\frac{l_g}{w_1 L \mu_0}$
Stator core	S3 and S4	$\sim \frac{1}{2} \times \frac{\pi(d_s + d_r)}{2}$	$w_2 L$	$\frac{\pi(d_s + d_r)}{4 w_2 L \mu_{core2}}$

Example

- Put in some representative dimensions and properties for a medium sized (several kW) machine
- d_s = 250mm; d_r = 150mm, l_g=0.5mm, w_1=100mm, w_2= 75mm, L = 250mm which gives θ = 1.458 rad
- $\mu_{core1} = \mu_{core2} = 10{,}000\mu_0$

	Reluctance (note use of SI units)
S1 and S5	$\frac{0.15}{2 \times 0.1 \times 0.25 \times 10000 \times 4\pi \times 10^{-7}} = 239\ A/Wb$
S2 and S6	$\frac{2 \times 0.0005}{0.15 \times 1.458 \times 0.25 \times 4\pi \times 10^{-7}} = 14{,}544\ A/Wb$
S3 and S4	$\frac{\pi(0.25 + 0.15)}{4 \times 0.075 \times 0.25 \times 10000 \times 4\pi \times 10^{-7}} = 1{,}333\ A/Wb$

Note: Dominated by reluctance of the airgap when permeability of the cores is high

The total effective reluctance is 30,253 A/Wb

For a current density of $1A/mm^2$ in the entire coil area this gives a total mmf (NI) of 900 A.turns

Flux in magnetic circuit = 0.297Wb

Approximate airgap flux density = Flux /area of pole

= 1.09T

Given a number of turns, the inductance can be calculated from N^2/S

Also possible to estimate airgap flux density from Ampere's Law assuming that the reluctance of the iron can be neglected from

$$B_g = \frac{\mu_0 NI}{2l_g} = \frac{4\pi \times 10^{-7} \times 900}{0.001} = 1.13T$$

Check with finite element analysis