EEE 207 2009-2010 Worked Solutions

Q1(a) Charge neutrality condition

$$n + N_a = p + N_d$$

also $$np = n_i^2$$

$$\therefore \quad n^2 - (N_d - N_a)n - n_i^2 = 0$$

$$n = \frac{N_d - N_a}{2} + \frac{N_d - N_a}{2}\left[1 + \left(\frac{2n_i}{N_d - N_a}\right)^2\right]^{1/2} \quad \text{—— (1)}$$

(i) For n-type extrinsic semiconductor, $N_d - N_a \gg n_i$, so (1) $\Rightarrow$

$$n \simeq N_d - N_a$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d - N_a}$$

(ii) For a compensated near intrinsic case, $n_i \gg N_d - N_a$, so (1) $\Rightarrow$

$$n = n_i + \frac{N_d - N_a}{2} \approx n_i$$

$$p = n_i$$

[No problems here] (7)

(b) $\sigma_i = n_i e(\mu_e + \mu_h)$

$$\frac{1}{5\times10^3} = 2\times10^{-4} = n_i \times 1.6\times10^{-19}(0.12 + 0.05)$$

$$n_i = \frac{2\times10^{-4}}{1.6\times10^{-19}\times0.17} = 7.35\times10^{15}\ m^{-3}$$

when p-doped, σ is 10^4 larger, $\Rightarrow \sigma = 2 = e N_a \mu_h$

(ignore N_d, n_i)

$$2 = 1.6\times10^{-19}\times0.05\times N_a$$

$$N_a = 2.5\times10^{20}\ m^{-3}$$

[No real problems upto here]

In a compensated semiconductor

$$\sigma = e(n\mu_e + p\mu_h) = e\left(\frac{n_i^2\mu_e}{p} + p\mu_h\right)$$

After compensation,

$$\sigma = \frac{1}{7.61\times10^2} = 1.314\times10^{-3} = 1.6\times10^{-19}\left(\frac{(7.35\times10^{15})^2 0.12}{p} + p\,0.05\right)$$

Q1 cont.

$$0.05p^2 - 8.21\times10^{15}p + 6.48\times10^{30} = 0$$

$$p^2 - 1.64\times10^{17}p + 1.29\times10^{32} = 0$$

$$p = \frac{1.64\times10^{17} \pm \sqrt{(1.64\times10^{17})^2 - (4\times1.29\times10^{32})}}{2}$$

[Many got this slightly wrong]

$$= 8.21\times10^{16} \pm 8.13\times10^{16}$$

$$= 8\times10^{14} \text{ or } 1.634\times10^{17}\ m^{-3}$$

Since p must be $> n_i$, $p = 1.634\times10^{17}\ m^{-3}$

$$n = \frac{n_i^2}{p} = \frac{(7.35\times10^{15})^2}{1.634\times10^{17}} = 3.3\times10^{14}\ cm^{-3}$$

For charge neutrality : $p + N_d = n + N_a$

$$N_d = n + N_a - p = 3.3\times10^{14} + 2.5\times10^{20} - 1.634\times10^{17}$$

$$= 2.498\times10^{20}\ m^{-3}$$

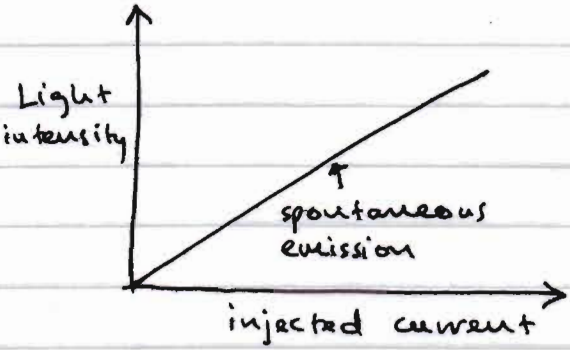
(10)

(c) To achieve the intrinsic resistivity, we need to get back to the $n=p=n_i$ level of carriers. This is very unlikely as it requires a very high degree of precision in the doping of the donors to increase it to $2.5\times10^{20}\ m^{-3}$. (3)

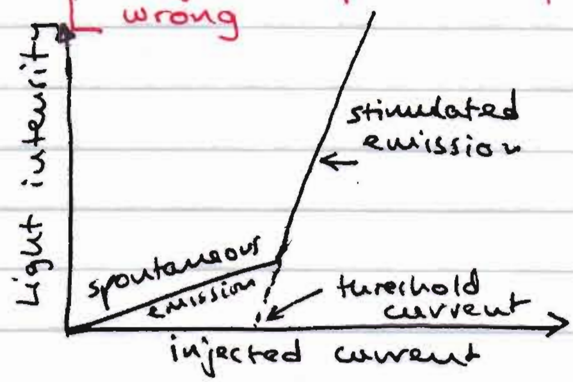
[No problems]

Q 2(a)

[Most got the position of J_{th} wrong]



(i) LED

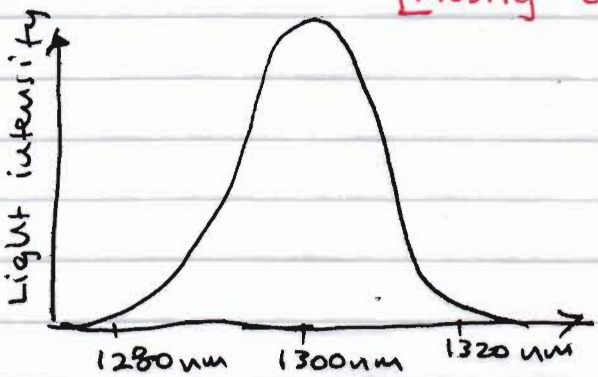


(ii) Laser

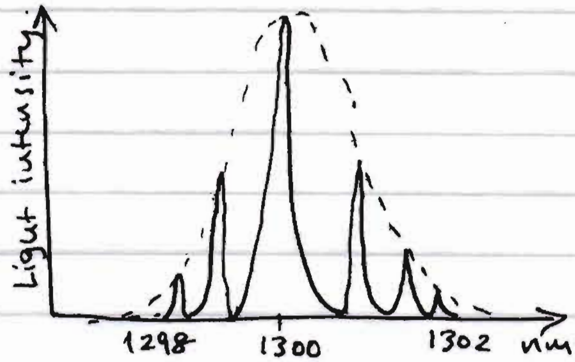
(4)

(b)

[Mostly o.k.]



spontaneous emission mode below J_{th}



stimulated emission mode above J_{th} (4)

(c) 14.5 nm QW of InGaAs with InP barriers gives 1.550 nm emission

$\frac{1.24}{1.55}$ = 0.8 eV emission from QW. Bulk is 0.75 eV, so effect of 14.5 nm of quantisation is 0.8 − 0.75 eV = 50 meV

$$\therefore \frac{h^2}{8 m_e^* L^2 m_0} + \frac{h^2}{8 m_h^* L^2 m_0} = 0.05 \times 1.6 \times 10^{-19} \text{ J}$$

$$\frac{h^2}{8 m_0 L^2}\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right) = 0.05 \times 1.6 \times 10^{-19} \quad \text{where } L = 14.5\text{nm}$$

$$\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right) = \frac{8 m_0 L^2 \, 0.05 \times 1.6 \times 10^{-19}}{h^2} \quad \text{— (1)}$$

Well width required for 1300 nm?

1300 nm ≡ 954 meV, so quantisation is 0.954 − 0.75 = 0.204 eV

$$\frac{h^2}{8m_0L^2}\left(\frac{1}{m_e^*}+\frac{1}{m_h^*}\right) = 0.204 \times 1.6 \times 10^{-19}$$

subs ① into above for $\left(\frac{1}{m_e^*}+\frac{1}{m_h^*}\right)$

$$\frac{h^2}{8m_0 \; 0.204\times 1.6\times 10^{-19}} \cdot \frac{8m_0}{h^2}(14.5\times 10^{-9})^2 \times 0.05 \times 1.6\times 10^{-19} = L^2$$

[mostly o.k.]

$$L = \sqrt{\frac{0.05}{0.204}} \times 14.5\times 10^{-9} = 7.18\,\text{nm}$$

is well width for 1300nm emission

(10)

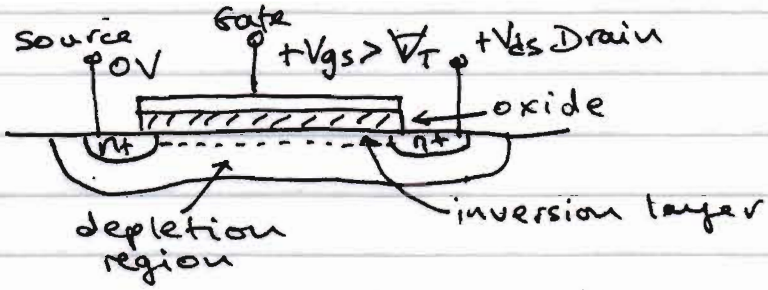
(d) Maximum wavelength is when InGaAs width is very wide, i.e. bulk, so is $\frac{1.24}{0.75} = 1.65\,\mu m$

Minimum wavelength occurs when QW is very narrow, and the band-gap of the barrier material will act as the limit, so is $\frac{1.24}{1.35} = 0.920\,\mu m$

[Very easy]

(2)

Q 3(a)



[Mostly O.K.]

(4)

(b) Unsaturated drain current is

(i) $$I_d = \frac{\mu_e C_g}{l^2}\left[V_{gs} - V_T - \frac{V_{ds}}{2}\right]V_{ds}$$

Saturation occurs when I_d is maximum – this occurs when $\partial I_d / \partial V_{ds} = 0$ ← [several did not say this!]

$$\frac{\partial I_d}{\partial V_{ds}} = \frac{\mu_e C_g}{l^2}\left[V_{gs} - V_T - V_{ds}\right] = 0$$

Maximum I_{ds} occurs when $V_{gs} - V_T - V_{ds} = 0$

Saturation occurs $V_{ds} \geqslant V_{gs} - V_T$

Substituting this into equation for I_d gives

$$I_{ds} = \frac{\mu_e C_g}{l^2}\frac{(V_{gs} - V_T)^2}{2} \quad \text{or} \quad \frac{\mu_e C_g}{l^2}\frac{V_{ds}^2}{2}$$

[This was O.K.]

(ii) $$g_m = \left.\frac{\partial I_{ds}}{\partial V_{gs}}\right|_{\text{constant } V_{ds}}$$

$$\frac{\partial I_{ds}}{\partial V_{gs}} = \frac{\mu_e C_g}{l^2}(V_{gs} - V_T)$$

[Mostly easy]

Real g_m is lower than this because

i) we ignore the source and drain parasitic resistances

ii) channel mobility is reduced due to interface scattering

[very few got this]

(10)

Q3 (cont)

(c) Gate and drain are connected, so $V_{gs} = V_{ds}$

We showed earlier that

$$I_d = \frac{\mu_e C_g}{l^2} \frac{(V_{gs} - V_T)^2}{2}$$, so can rewrite as

$$I_d = \frac{\mu_e C_g}{l^2} \frac{(V_{ds} - V_T)^2}{2}$$

Since current flows when $V_{ds} = 2.5\,V$, this implies $V_T = 2.5\,V$

$$\therefore \quad I_d = \frac{\mu_e C_g}{l^2} \frac{(V_{ds} - 2.5)^2}{2}$$

1 mA flows when $V_{ds} = 4\,V$

$$10^{-3} = \frac{\mu_e C_g}{l^2} \frac{(4 - 2.5)^2}{2} \implies \frac{\mu_e C_g}{l^2} = 0.89 \times 10^{-3}\ A\,V^{-2}$$

When $V_{ds} = 5\,V$

$$I_d = 0.89 \times 10^{-3} \frac{(5 - 2.5)^2}{2} = 2.77\ mA$$

(6)

[Many got V_T, but could not get the final correct answer]

4(a) $E = E_g + AK^2 - BK^4$

(i) $m^* = \left(\frac{d^2E}{dp^2}\right)^{-1}$, $p = \hbar K$

$$m^* = \hbar^2 \left(\frac{d^2E}{dK^2}\right)^{-1}$$

$$\frac{dE}{dK} = 2AK - 4BK^3 \quad , \quad \frac{d^2E}{dK^2} = 2A - 12BK^2$$

At $K = 0$, $m^* = \hbar^2 (2A)^{-1} = 0.122\, m_o$

$$\therefore A = \frac{\hbar^2}{2 \times 0.122 \times 9.11 \times 10^{-31}} = \left[\frac{6.626 \times 10^{-34}}{2\pi}\right]^2 \frac{1}{2 \times 0.122 \times 9.11 \times 10^{-31}}$$

$$= 5 \times 10^{-38}\ \mathrm{J\,m^{-2}}$$

[Most people got this]

(ii) At Brillouin zone edge, $v_g = 0$

$$v_g = \frac{dE}{dp} = \frac{1}{\hbar}\frac{dE}{dK} = \frac{1}{\hbar}\left(2AK - 4BK^3\right) = 0$$

$$2AK = 4BK^3 \implies K = \left[\frac{A}{2B}\right]^{1/2} \text{ at zone edge}$$

[This seems easy for most.]

(iii) This is a direct band-gap semiconductor, as the minimum energy occurs at $K = 0$

[This last part should have been very easy but almost 40% got it wrong!] (10)

8(b) The Heisenberg Uncertainty Principle states that the position and momentum of a particle cannot be simultaneously measured with arbitrarily high precision.

$$\Delta x \, \Delta p \geq \frac{h}{4\pi}$$

[Most got some marks but not all gave the correct expression]

There is a minimum for the product of uncertainties of these two measurements.

(4)

(c) mass $= 1.67 \times 10^{-27}$ kg

$1/10$ velocity of light $= 3 \times 10^{7}\ m\,s^{-1}$

1% uncertainty in velocity $= 0.01 \times 3 \times 10^{7} = 3 \times 10^{5}\ m\,s^{-1}$

Δ momentum $= \Delta V \times$ mass $= 3 \times 10^{5} \times 1.67 \times 10^{-27}$

$\Delta p = 5 \times 10^{-22}$

[Only a few got this part correct.]

From Heisenberg, $\Delta x \, \Delta p \geq \frac{h}{4\pi}$

$$\Delta x \geq \frac{h}{4\pi \Delta p} = \frac{6.626 \times 10^{-34}}{4\pi \times 5 \times 10^{-22}} \approx 10^{-13}\ m$$

$\therefore$ Uncertainty in proton position is $\pm\ 10^{-13}$ m.

(5)