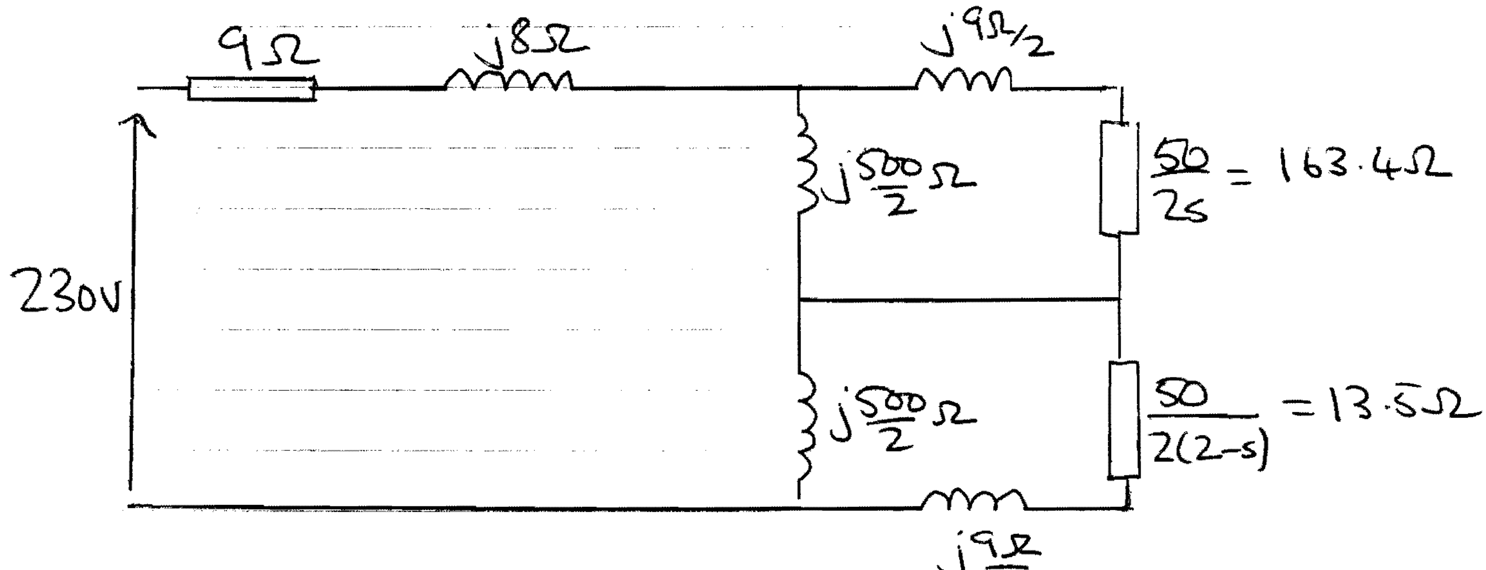
EEE409 / 6120 Modelling of Machines

Tutorial Sheet 5 – Worked Solutions

1. At 1270 rpm, the slip for a four-pole machine is given by:

$$\text{slip} = \frac{1500 - 1270}{1500} = 0.153$$

The equivalent circuit is therefore:



Equivalent impedance of the positive sequence branch:

$$Z_p = \frac{\left(\frac{R_r}{s} + j\frac{X_{lr}}{2}\right) + j\frac{X_m}{2}}{\frac{R_r}{s} + j\frac{X_{lr}}{2} + j\frac{X_m}{2}} = \frac{(163.4 + j4.5) \times j250}{163.4 + j4.5 + j250}$$

$$= \frac{163.5\angle 1.6^\circ \times 250\angle 90^\circ}{302.4\angle 57.3^\circ}$$

$$= \frac{40875\angle 91.6^\circ}{302.4\angle 57.3^\circ} = 135.2\angle 34.3^\circ\ \Omega$$

$= 111.7 + j76.2 \ \Omega$

Similarly for the negative sequence

$$Z_n = \frac{(13.5 + j4.5)(j250)}{13.5 + j4.5 + j250} = \frac{14.2 \angle 18.4^\circ \times 250 \angle 90^\circ}{254.9 \angle 87^\circ}$$

$$= \frac{3550 \angle 108.4^\circ}{254.9 \angle 87^\circ}$$

$$= 13.9 \angle 21.4^\circ$$

$$= 12.9 + j5.1 \ \Omega$$

∴ Total impedance at terminals

$$Z_{TOTAL} = 9 + j8 + 111.7 + j76.2 + 12.9 + j5.1$$

$$= 133.6 + j89.3 \ \Omega = 160.7 \angle 33.8^\circ \ \Omega$$

Input current $= \frac{V}{Z_{TOTAL}} = \frac{230 \angle 0^\circ}{160.7 \angle 33.8^\circ}$

$$= 1.43 \angle -33.8^\circ \text{ A rms}$$

The components of current that flow through the positive and negative sequence resistances are found by first calculating the voltage across the positive and negative sequence branches.

$$V_p = I_{in} Z_p = 1.43 \angle -33.8^\circ \times 135.2 \angle 34.3^\circ$$

$$= 193.5 \angle 0.5^\circ \text{ Vrms}$$

$$V_n = I_{in} Z_n = 1.43 \angle -33.8^\circ \times 13.9 \angle 21.4^\circ$$

$$= 19.9 \angle -12.4^\circ \text{ Vrms}$$

Hence, current through positive and negative sequence resistances given by:

$$I_{PR} = \frac{V_p}{j\frac{X_{lr}}{2} + \frac{R_r}{2s}} = \frac{193.5 \angle 0.5^\circ}{163.5 \angle 1.6^\circ} = 1.18 \angle -0.9^\circ \text{ A rms}$$

$$I_{NR} = \frac{V_n}{j\frac{X_{lr}}{2} + \frac{R_r}{2(2-s)}} = \frac{19.9 \angle -12.4^\circ}{14.2 \angle 18.4^\circ} = 1.40 \angle -30.8^\circ \text{ A rms}$$

$$P_{out} = \left(I_{PR}^2 \frac{R_2}{2s} - I_{NR}^2 \frac{R_2'}{2(2-s)} \right) = 227.5 - 26.5$$

$$= 201.5\text{W}$$

$$P_{mech} = P_{out}(1-s) = 170.3\text{W}$$

$$\text{Torque} = \frac{P_{mech}}{\omega_r} = \frac{170.3}{1270 \times \frac{2\pi}{60}} = 1.28 \text{ Nm}$$

$$\text{Input power} = V_{in} I_{in} \cos\phi$$

$$= 230 \times 1.43 \times \cos(-33.8^\circ)$$

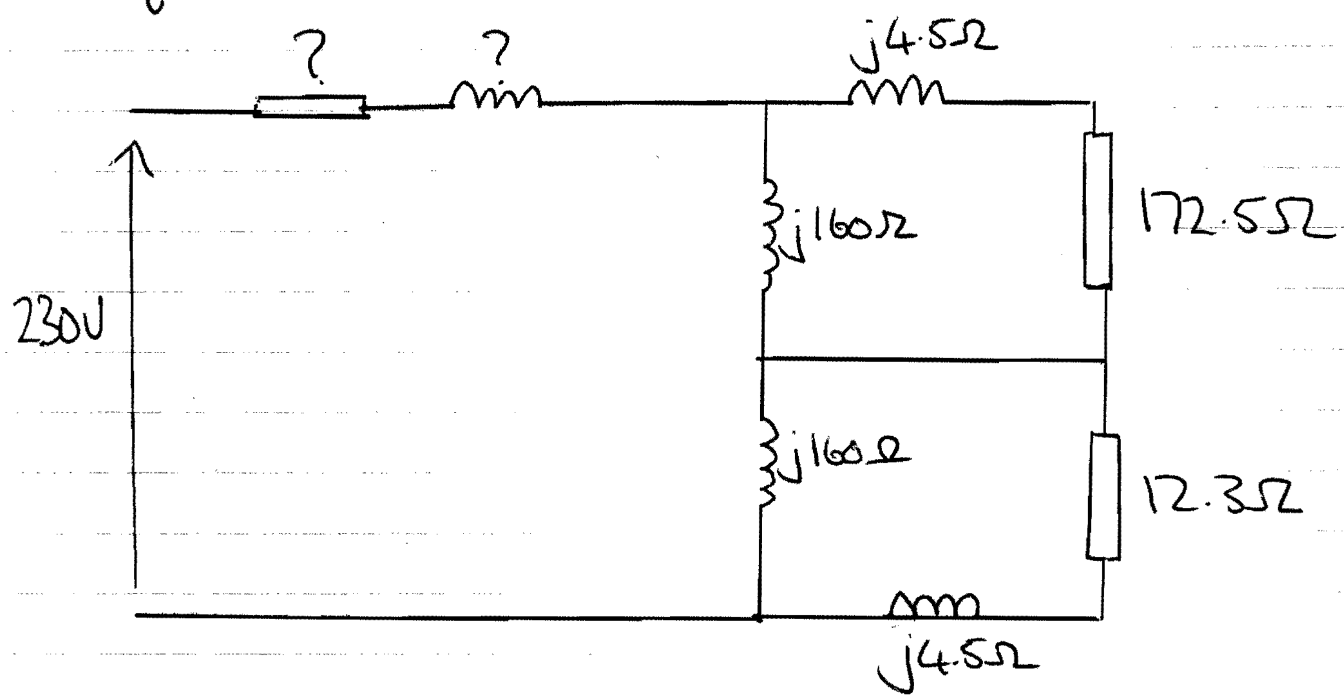
$$= 273\text{W}$$

$$\text{Efficiency} = \frac{\text{Output Power}}{\text{Input Power}} = \frac{170.3}{273} = \underline{\underline{62.4\%}}$$

2 For a 4-pole machine, the slip is given by:

$$s = \frac{1500-1300}{1500} = 0.133$$

Equivalent circuit



$$Z_p = \frac{(172.5+j4.5)\times j160}{172.5+j4.5+j160}$$

$$= \frac{172.6\angle 1.5^\circ \times 160\angle 90^\circ}{238.4\angle 43.6^\circ}$$

$$= \frac{27520\angle 91.5^\circ}{238.4\angle 43.6^\circ}$$

$$= 115.4\angle 47.9^\circ\ \Omega$$

$$= 77.4 + j85.6\,\Omega$$

$$Z_r = \frac{(12.3 + j4.5) \times j160}{12.3 + j160 + j4.5} = \frac{13.1\angle 20.1^\circ \times 160\angle 90^\circ}{12.3 + j164.5}$$

$$= 12.7\angle 25.4^\circ\,\Omega$$

$$= 11.4 + j5.45\,\Omega$$

Current drawn at the specified operating point:

$$I = 1.72\angle -45.6^\circ$$ sign comes from lagging

$$\therefore Z_{TOTAL} = \frac{230\angle 0^\circ}{1.72\angle -45.6^\circ} = 133\angle 45.6^\circ\,\Omega$$

$$= 93.1 + j95.0\,\Omega$$

The stator impedance Z_{stator} can be calculated from

$$Z_{TOTAL} = Z_{stator} + Z_p + Z_r$$

$$\Rightarrow Z_{stator} = Z_{TOTAL} - Z_p - Z_r$$

$$= 93.1 + j95.0 - 11.4 - j5.45 - 77.4 - j85.6$$

$$= 4.3 + j3.95\,\Omega$$

$\therefore$ Stator resistance $= \underline{\underline{4.3\,\Omega}}$

Stator leakage reactance $= \underline{\underline{j3.95\,\Omega}}$

In order to calculate the output power it is necessary to calculate the currents through the positive and negative sequence resistences.

$$V_p = I_{in} Z_p = 1.72 \angle{-45.6^\circ} \times 115.4 \angle{47.9^\circ}$$

$$= 198.5 \angle{2.3^\circ} \text{ Vrms}$$

$$V_n = I_{in} Z_n = 1.72 \angle{-45.6^\circ} \times 12.7 \angle{25.4^\circ}$$

$$= 21.8 \angle{-20.2^\circ} \text{ Vrms}$$

$$\Rightarrow I_{RP} = \frac{V_p}{\cancel{Z}172.5 + j4.5} = \frac{V_p}{172.6\angle{1.5^\circ}} = \frac{198.5 \angle{2.3^\circ}}{172.6 \angle{1.5^\circ}}$$

$$= 1.15 \angle{0.8^\circ}$$

$$I_{RN} = \frac{V_n}{12.3 + j4.5} = \frac{21.8 \angle{-20.2^\circ}}{13.1 \angle{20.1^\circ}} = 1.66 \angle{-40.3^\circ} \text{ Arms}$$

$$\Rightarrow P_{out} = \left(I_{RP}^2 \frac{R_r}{2s} - I_{RN}^2 \frac{R_r}{2(2-s)} \right)$$

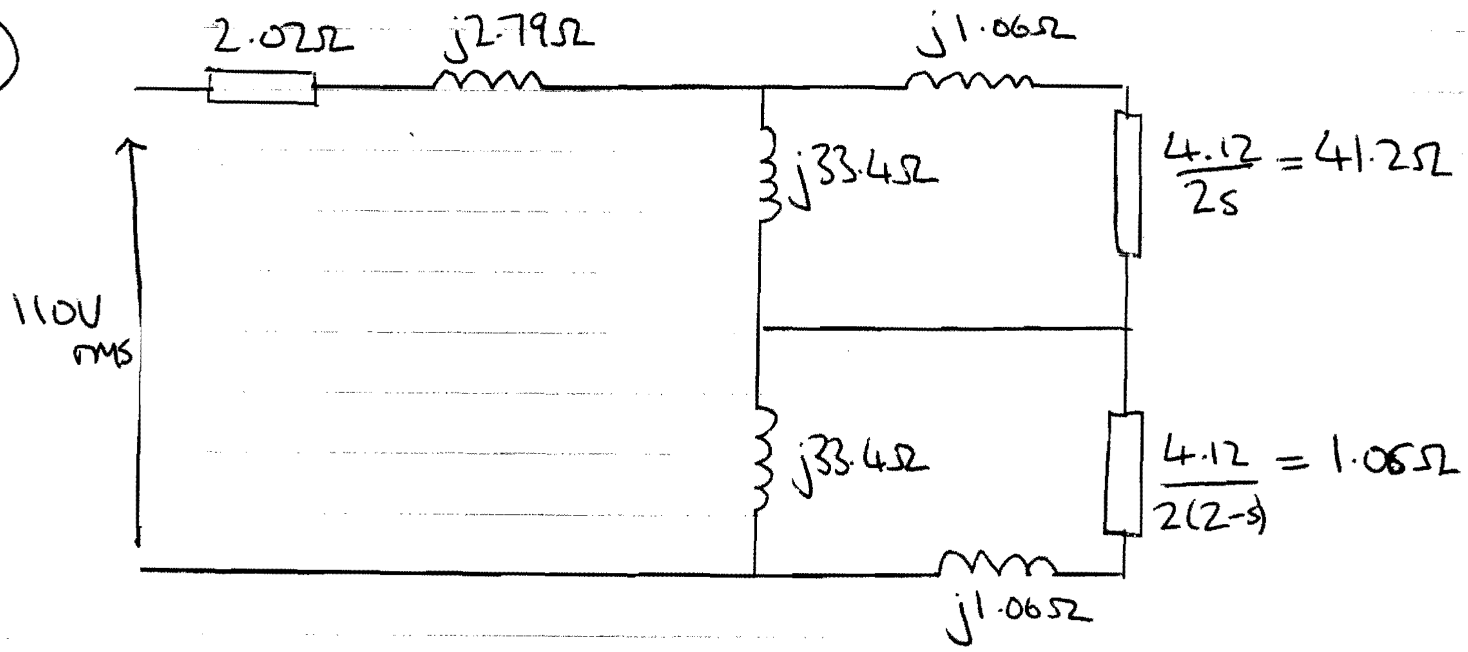
$$= (1.15^2 \times 172.5 - 1.66^2 \times 12.3)$$

$$= (228.1 - 33.9) = 194.2\text{W}$$

$$P_{mech} = P_{out}(1-s) = 194.2(1-0.133)$$

$$= \underline{\underline{168.4\text{W}}}$$

(3)



$$Z_p = \frac{(41.2 + j1.06) \times j33.4}{41.2 + j1.06 + j33.4} = \frac{41.2 \angle 1.47^\circ \times 33.4 \angle 90^\circ}{41.2 + j34.46}$$

$$= \frac{1376 \angle 91.47^\circ}{53.7 \angle 39.9^\circ}$$

$$= 25.6 \angle 51.6^\circ \ \Omega$$

$$Z_n = \frac{(1.06 + j1.06) \times j33.4}{1.06 + j1.06 + j33.4} = \frac{1.5 \angle 45^\circ \times 33.4 \angle 90^\circ}{1.06 + j34.46}$$

$$= \frac{50.1 \angle 135^\circ}{34.5 \angle 88.2^\circ}$$

$$= 1.45 \angle 46.8^\circ \ \Omega.$$

$$Z_{TOTAL} = 2.02 + j2.79 + 25.6 \angle 51.6^\circ + 1.45 \angle 46.8^\circ$$

$$= 2.02 + j2.79 + 15.9 + j20.1 + 0.99 + j1.06$$

$$= 18.9 + j24.0 \ \Omega$$

$$= 30.5 \angle 51.8^\circ$$

(The question is slightly unclear in terms of whether the power being asked for is total output (mechanical + losses). and so the answer above would be OK)

- The mechanical output power is

$$P_{mech} = 193.8\,(1-0.05) = \underline{\underline{184.1\ W}}$$

The speed can be calculated from the slip

$$\omega_r = \omega_s(1-s) = 1500\,(1-0.05)$$

$$= 1425 \text{ rpm.}$$

$$\text{Torque} = \frac{P_{mech}}{\omega_r} = \frac{184.1}{1425 \times \frac{2\pi}{60}} = 1.23\ \text{NM}$$

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