Marks **Q 1**

a) Primary objective of a communication system: Ensure received information is approximately equal to the
1. transmitted information for the least energy and bandwidth.

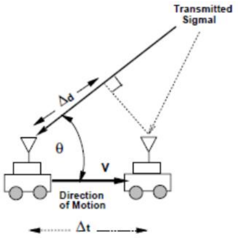
1. Not always possible because noise is always present in a communication system which perturbs the received signal.

1. Multiple-access: allows several users to share a common channel

1. Data detection: Decides which symbol from a finite set at the transmitter has been received

1. **b) Doppler frequency:** Change in received carrier frequency due to relative movement of the transmitter and receiver**.**

Expression for Doppler frequency shift:



$$\frac{\Delta d}{\Delta t \times v} = \cos\theta \Rightarrow \frac{\Delta d}{\Delta t} = v\cos\theta$$

1. But $\Delta\phi = 2\pi\dfrac{\Delta d}{\lambda} \Rightarrow \Delta d = \dfrac{\lambda}{2\pi}\Delta\phi$

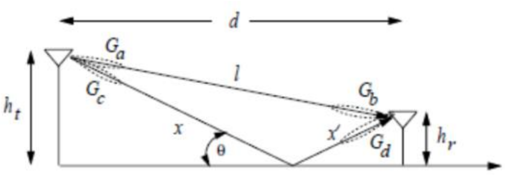
1. $$\therefore \quad f_D = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$$

λ=3e8/2e9=0.15 m, ν=50e3/3600=13.89 m/s and given $f_D = 5$ Hz

1. $\theta = \cos^{-1}(\lambda f_D / \upsilon) = \cos^{-1}(0.15 \times 5/13.89) = \underline{\underline{86.9 \text{ degrees}}}$ #

1. **c) Include ground plane:** Allows the ground reflection to be taken into account to give 2-ray pathloss propagation

Expression for 2-ray pathloss:



1. $$r(t) = \mathrm{Re}\left\{\frac{\lambda}{4\pi}\left[\frac{\sqrt{G_a G_b}\,u(t)e^{-j2\pi l/\lambda}}{l} + \frac{R\sqrt{G_c G_d}\,u(t-\tau)e^{-j2\pi(x+x')/\lambda}}{x+x'}\right]e^{j2\pi f_c t}\right\}$$

1. $$P_r = P_t\left[\frac{\lambda}{4\pi}\right]^2\left|\frac{\sqrt{G_a G_b}}{l} + \frac{R\sqrt{G_c G_d}\,e^{-j\Delta\phi}}{x+x'}\right|^2 \qquad \sqrt{G_a G_b} \approx \sqrt{G_c G_d} \approx \sqrt{G_t G_r}$$

1. 1. $$\Delta\phi = \frac{2\pi(x+x'-l)}{\lambda} \approx \frac{4\pi h_t h_r}{\lambda d} \qquad P_r \approx P_t\left[\frac{\lambda\sqrt{G_t G_r}}{4\pi d}\right]^2\left[\frac{4\pi h_t h_r}{\lambda d}\right]^2 = P_t\left[\frac{\sqrt{G_t G_r}\,h_t h_r}{d^2}\right]^2 \qquad \Rightarrow P_L = \frac{P_t}{P_r} = \frac{d^4}{G_t G_r (h_t h_r)^2}$$

1. Critical distance: $d_c = 4h_t h_r/\lambda = 4\times15\times1.5/0.15 = \underline{\underline{600m}}$ #

1. **d) Log-normal distribution:** If the r.v. y is log-normally distributed, then the r.v. $x = \ln(y)$ is normally distributed.

1. 1. Use: $P_{out} = Q\left(\dfrac{P_r - P_{\min}}{\sigma_{\psi_{db}}}\right) \qquad 0.05 = Q\left(\dfrac{P_r - (-102)}{8}\right) \Rightarrow P_r = 1.645\times8 - 102 = -88.84dBm$

2. $P_t = P_r + P_L \quad and \quad P_L = (5000)^4/(1\times1\times15^2\times1.5^2) = 1.235e12 \quad or \quad 121dB$

1. $P_t = -88.84 + 121 = 32dBm \quad or \quad \underline{\underline{1.63mW}}$ #

[20]

Marks **Q 2**

1. **a) Difference between narrow- and wideband MP fading:** In narrowband fading the excess delay is much less than the data symbol duration; in wideband fading the excess delay is large compared with the data symbol duration.

Expressions for I and Q components: For narrowband fading we assume an unmodulated carrier such that

1. $$r(t) = \mathrm{Re}\left\{\left(\sum_{l=0}^{L(t)-1} \alpha_l(t) e^{-j\phi_l(t)}\right) e^{j2\pi f_c t}\right\} = r_I(t)\cos(2\pi f_c t) - r_Q(t)\sin(2\pi f_c t)$$

2. $$r_I(t) = \sum_{l=0}^{L(t)-1} \alpha_l(t)\cos(\phi_l(t)) \qquad r_Q(t) = \sum_{l=0}^{L(t)-1} \alpha_l(t)\sin(\phi_l(t))$$

$$\phi_l(t) = 2\pi f_c t \tau_l(t) - \phi_{D_l} - \phi_0 \quad \text{incorporates the phase off-set } \phi_0$$

1. For large $L(t)$ the Central Limit theorem means that both $r_I(t)$ and $r_I(t)$ can be modelled as independent $N(0, \sigma^2)$ Gaussian random processes.

1. **b)** The autocorrelation of $r_I(t)$ is given by: $A_{r_I}(t, t+\tau) = E[r_I(t) r_I(t+\tau)]$

1. $$E[r_I(t) r_I(t+\tau)] = \sum_l E[\alpha_l^2] E[\cos(\phi_l(t))\cos(\phi_l(t+\tau))] = \frac{1}{2}\sum_l E[\alpha_l^2] E[\cos(2\pi f_{D_l}\tau)] = \frac{1}{2}\sum_l E[\alpha_l^2] E\left[\cos\left(2\pi \frac{\upsilon\tau}{\lambda}\cos(\theta_l)\right)\right]$$

1. For Clarke's uniform scattering model, the channel has many densely packed scatterers as a function of arrival angle giving L multipath components with angle of arrival $\boldsymbol{\theta_l = l\Delta\theta}$ where $\boldsymbol{\Delta\theta = 2\pi/L}$ and each multipath component has equal power such that $\boldsymbol{E[\alpha_l^2] = 2P_r/L}$. Then

$$A_{r_I}(\tau) = \frac{P_r}{L}\sum_{l=1}^{L}\cos\left(2\pi\frac{\upsilon\tau}{\lambda}\cos(l\Delta\theta)\right) = \frac{P_r}{2\pi}\sum_{l=1}^{L}\cos\left(2\pi\frac{\upsilon\tau}{\lambda}\cos(l\Delta\theta)\right)\Delta\theta \quad \text{when substituting } L = 2\pi/\Delta\theta \text{ gives}$$

1. $$\text{In the limit as } L \to \infty, \Delta\theta \to 0, \quad A_{r_I}(\tau) = \frac{P_r}{2\pi}\int \cos\left(2\pi\frac{\upsilon\tau}{\lambda}\cos(\theta)\right) d\theta$$

1. Significance: Bessel function of the first kind $P_r J_0(2\pi f_D \tau)$ which at the first null, $\boldsymbol{f_D \tau = 0.4}$ defines the coherence time

1. **c)** The *pdf* of the received signal envelope is Rayleigh: $\rho_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} = \frac{2r}{\overline{P}_r} e^{-r^2/\overline{P}_r}$

1. The average Rx Power $\overline{P}_r = \sum_l E[\alpha_l^2] = \overline{r^2} = 2\sigma^2$. Where $2\sigma^2$ is the Rayleigh *pdf* variance.

1. A BER of 5% with a single error event per fade corresponds to an outage rate of 5%. For a Rayleigh *pdf*

2. $$P(R \le r) = 1 - e^{-P_R/\overline{P}_r} = 0.05 \quad \Rightarrow \quad \frac{P_R}{\overline{P}_r} = \ln(1-0.05) = 0.0513 \quad or \quad \underline{\underline{P_R = \overline{P}_r / 19.49}} \quad \#$$

1. **d)** λ=3e8/1.8e9=0.17 m, ν=35e3/3600=9.72 m/s gives the maximum Doppler shift f_D = ν/λ = 58.33 Hz

1. From Part 2(c), ρ = sqrt($\frac{P_R}{\overline{P}_r}$) = sqrt (0.051) = 0.2265

1. R_B = 0.2265*58.33*sqrt (2π)/(exp(0.0513) - 1) =629.14 bit/s

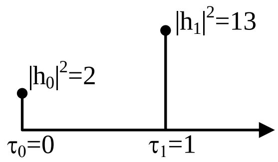
2. Fading rate = $R_B * P_{out}$ = R_B * Pr{$R \le r$} = 629.14 * 0.05 = **31.457 fades/sec #**

[20]

Marks **Q 3**

a) Power delay profile:

1.
1. $h_0 = 1 - j \quad \Rightarrow \quad |h_0|^2 = h_0 h_0^* = (1-j)(1+j) = 2 \quad and$
1. $h_1 = 3 + 2j \quad \Rightarrow \quad |h_1|^2 = h_1 h_1^* = (3+2j)(3-2j) = 13$



b) Mean and *rms* delay spread: –

1.
1. $\bar{\tau} = \frac{\sum_{i=0}^{1} \tau_i |h_i|^2}{\sum_{i=0}^{1} |h_i|^2} = \frac{0 \times 2 + 1 \times 13}{2 + 13} = \frac{13}{15} = \underline{\underline{0.867\,\text{sec}}} \quad \# \quad \overline{\tau^2} = \frac{\sum_{i=0}^{1} \tau_i^2 |h_i|^2}{\sum_{i=0}^{1} |h_i|^2} = \frac{0^2 \times 2 + 1^2 \times 13}{2 + 13} = \frac{13}{15} = 0.867\,\text{sec}^2$

2. $\therefore \tau_{rms} = \sqrt{\overline{\tau^2} - \bar{\tau}^2} = \sqrt{0.867 - 0.867^2} = \underline{\underline{0.34\,\text{sec}}} \quad \#$

1. Coherence bandwidth B_c is the least frequency above which spectral components are uncorrelated.

2. $B_c(50\%) = \frac{1}{5\tau_{rms}} = \frac{1}{5 \times 0.34} = \underline{\underline{0.588\text{ Hz}}} \quad \#$

c) 2-Block Frequency Selective Channel: -

The 2-block FS channel is obtained from the 2-point FFT of $h(\tau)$ as:

1. $H[i] = \frac{1}{\sqrt{2}} \sum_{n=0}^{1} h[n] e^{-j2\pi n i/2} \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1-j \\ 3+2j \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 4+j \\ -2-3j \end{bmatrix}$

1. $\text{H}[0] = (4+\text{j})/\sqrt{2} \quad and \quad \text{H}[1] = (-2-3\text{j})/\sqrt{2}$

$verification: \quad |H[0]|^2 + |H[1]|^2 = 15$

Condition for optimum capacity is $\frac{P_i}{P} = \begin{cases} \frac{1}{\gamma_m} - \frac{1}{\gamma_i} & \gamma_i \geq \gamma_m \\ 0 & \gamma_i < \gamma_m \end{cases}$

B = subchannel b/w = $B_u/2$ = $1/2T_s$ =0.5 Hz; Given P = 1 W and N_0 = 1 W/Hz

1. $\gamma_0 = |\text{H}[0]|^2 \frac{P}{N_0 B} = \frac{17}{2} \times \frac{1}{1 \times 0.5} = 17 \quad and \quad \gamma_1 = |\text{H}[1]|^2 \frac{P}{N_0 B} = \frac{13}{2} \times \frac{1}{1 \times 0.5} = 13$

1. $\sum_{i=0}^{1} \left(\frac{1}{\gamma_m} - \frac{1}{\gamma_i} \right) = 1 \quad \Rightarrow \quad \frac{2}{\gamma_m} = 1 + \sum_{i=0}^{1} \frac{1}{\gamma_i} = 1 + \frac{1}{17} + \frac{1}{13} = 1.136 \quad \therefore \gamma_m = 2/1.136 = 1.76$

Since $\gamma_m < \gamma_i \quad \forall i$, then all instances of the channel can be used to send data bits and the capacity is given by:

2. $C = \sum_{i=0}^{1} B \log_2\left(\frac{\gamma_i}{\gamma_m} \right) = 0.5 \times \left[\log_2\left(\frac{17}{1.76} \right) + \log_2\left(\frac{13}{1.76} \right) \right] = \underline{\underline{3.08}}\, bit/s\ \#$

The optimum power allocations are:

1. $P_0 = 1 \times \left(\frac{1}{1.76} - \frac{1}{17} \right) = 0.509 \quad and \quad P_1 = 1 \times \left(\frac{1}{1.76} - \frac{1}{13} \right) = 0.492 \quad \therefore$ almost equal power so channel is not very FS

Q3-Contd

d) Shannon Capacity in AWGN: -

1. Use the average SNR for a b/w of 1 Hz as given by $\bar{\gamma} = \frac{17/2 + 13/2}{2} = 7.5$

1. $C = B_u \log_2(1+\bar{\gamma}) = 1 \times \log_2(1+7.5) = \underline{\underline{3.09\, bit/s}}$ #

1. Would expect the Shannon capacity in AWGN to be slightly greater than the optimum capacity in the FS channel as the latter is not significantly frequency selective while the AWGN Shannon bound defines the maximum capacity possible.

[20]

Marks **Q 4**

a) The purpose of the cyclical prefix (CP) is:

1. i.) to eliminate ISI between OFDM data blocks as the CP is discarded at the receiver and because the CP in a digital sampling system converts a linear channel convolution into a circular channel convolution which means that each OFDM subcarrier can be equalised by a single tap filter in the frequency domain.

1. ii.) Formed by pre-pending the last *n* OFDM symbol samples to the beginning of the OFDM symbol.

1. For the channel given, as there is just one excess delay tap at time $t = T_s = 1$ s, then CP = 1 is sufficient.

b) For "all-ones" BPSK modulation:

2. i. $\mathbf{X}=\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \Rightarrow \mathbf{x}=\mathbf{M}\cdot\mathbf{X}=\frac{1}{2}\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\\1\end{bmatrix}=\begin{bmatrix}2\\0\\0\\0\end{bmatrix}$, hence the transmitted OFDM symbol with CP is $\begin{bmatrix}0\\2\\0\\0\\0\end{bmatrix}$

2. ii. Received vector $\mathbf{r}=1\times\begin{bmatrix}0\\2\\0\\0\\0\\\bullet\end{bmatrix}+j\times\begin{bmatrix}\bullet\\0\\2\\0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\2\\2j\\0\\0\\0\end{bmatrix}\Rightarrow \mathbf{x}_r=\begin{bmatrix}2\\2j\\0\\0\end{bmatrix}$ after CP removal and discarding convolution tail

1. iii. Take FFT of $\mathbf{x}_r \Rightarrow \mathbf{X}_r=\mathbf{M}^{*T}\cdot\mathbf{x}_r=\frac{1}{2}\begin{bmatrix}1&1&1&1\\1&-j&-1&j\\1&-1&1&-1\\1&j&-1&-j\end{bmatrix}\cdot\begin{bmatrix}2\\2j\\0\\0\end{bmatrix}=\begin{bmatrix}1+j\\2\\1-j\\0\end{bmatrix}$

1. Take FFT of $\mathbf{h}\Rightarrow\mathbf{H}=\mathbf{M}^{*T}\cdot\mathbf{h}=\frac{1}{2}\begin{bmatrix}1&1&1&1\\1&-j&-1&j\\1&-1&1&-1\\1&j&-1&-j\end{bmatrix}\cdot\begin{bmatrix}1\\j\\0\\0\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1+j\\2\\1-j\\0\end{bmatrix}$

The BPSK symbols after ZF equalisation are: $\mathbf{S}=\mathbf{X}_r/\mathbf{H}=\begin{bmatrix}2 & 2 & 2 & 0/0\end{bmatrix}^T$

1. While the first 3 bits are successfully recovered, the fourth bit is indeterminate as 0/0. In a real system with AWGN this would result in noise amplification and 50% chance of a bit error. A better equaliser for OFDM would be $\mathbf{S}=\mathbf{H}^{*T}\mathbf{X}_r$.

1. **c) {1 1 1 -1}:** The aperiodic ACF of {+++-} is equal to {-1 0 1 0 4 0 1 0 -1} [students to show calculation method based on aperiod ACF equation below]. This spreading sequence has low aperiodic autocorrelation sidelobes which help to reduce ISI due to the large excess delay of a frequency selective channel. The low sidelobe energy leads to a high Merit Factor.

1. Aperiodic ACF $C_a(\tau)=\sum_{n=0}^{N-1-\tau}a_n a_{n+\tau}$ Merit Factor $F=\dfrac{|C_a(0)|^2}{2\sum_{\tau=1}^{N-1}|C_a(\tau)|^2}$,

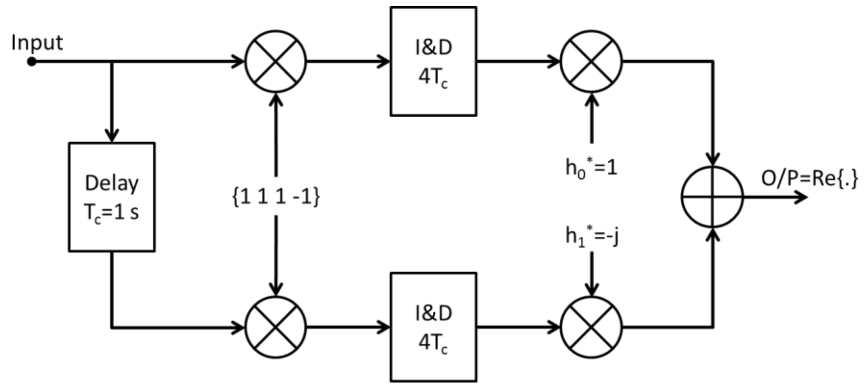
1. $C_a(\tau)$ of $[+1+1+1-1]=$ -10104010-1 $\Rightarrow \mathrm{F}=4^2/2\times(0^2+1^2+0^2+1^2)=4$

Q4 Contd.

2. **d) RAKE receiver:** A RAKE receiver provides a method of diversity combining which involves collecting the signal energy in each significant multipath component of a frequency selective channel. Provided each multipath is at least delayed by one chip duration T_c then the receiver can resolve individual multipath components.

A suitable RAKE receiver for the channel impulse response $h(\tau) = \delta(\tau) + j\delta(\tau - 1)$ is shown:

2.



1. Received vector $\mathbf{r} = 1 \times \begin{bmatrix} +1 \\ +1 \\ +1 \\ -1 \\ \bullet \end{bmatrix} + j \times \begin{bmatrix} \bullet \\ +1 \\ +1 \\ +1 \\ -1 \end{bmatrix} = \begin{bmatrix} +1 \\ +1+j \\ +1+j \\ -1+j \\ +0-j \end{bmatrix}$ forms the RAKE input.

The peak magnitude of the output is:

2.

$$|O/P| = \left| \mathrm{Re}\left\{ 1 \times \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} +1 \\ +1+j \\ +1+j \\ -1+j \end{bmatrix} - j \times \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} +1+j \\ +1+j \\ -1+j \\ +0-j \end{bmatrix} \right\} \right| = \left| \mathrm{Re}\{(4+j) + (4-j)\} \right| = 8$$

[20]