$$\frac{V_s}{I} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega CR}$$

$$\frac{V_s}{I} = \frac{(R + j\omega L)\left((1-\omega^2 LC) - j\omega CR\right)}{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}$$

looking at j terms in numerator product

$$\cancel{j}\omega L(1-\omega^2 LC) - \cancel{j}\omega CR^2 = 0$$

this must be equated to zero

$$L - \omega^2 L^2 C - CR^2 = 0$$

lets find the ω that will make j terms vanish

$$L - CR^2 = \omega^2 L^2 C$$

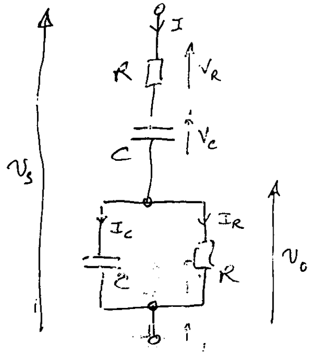
$$\frac{L}{L^2 C} - \frac{CR^2}{L^2 C} = \omega^2$$

$$\frac{1}{LC} - \frac{R^2}{L^2} = \omega^2$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Wein Bridge

look at parallel bit

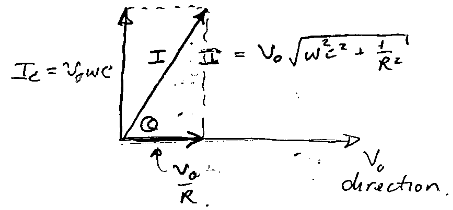


look at parallel bit

$$I = I_c + I_R$$

$$\frac{V_o}{\frac{1}{\omega C}} + \frac{V_o}{R}$$

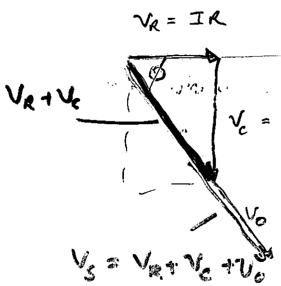
$$V_o \omega C + \frac{V_o}{R}.$$



$$\theta = \tan^{-1}\frac{V_o \omega C}{\frac{V_o}{R}} = \tan^{-1}\omega CR.$$

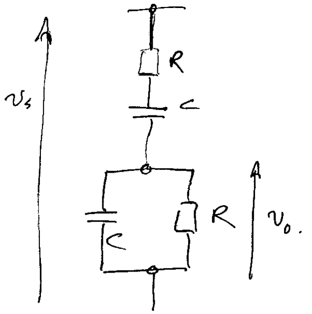
for the whole cct

$$V_o + V_c + V_R = V_s$$



$$\phi = \tan^{-1}\frac{\frac{I}{\omega C}}{IR} = \tan^{-1}\frac{1}{\omega CR}.$$

$$\frac{v_o}{v_s} = \frac{\dfrac{R/j\omega C}{R + 1/j\omega C}}{R + \dfrac{1}{j\omega C} + \dfrac{R/j\omega C}{R + 1/j\omega C}}$$



$$= \frac{\dfrac{R}{1 + j\omega CR}}{R + \dfrac{1}{j\omega C} + \dfrac{R}{1 + j\omega CR}}$$

$$= \frac{\left[\dfrac{R}{\cancel{1 + j\omega CR}}\right]}{\left[\dfrac{\left(R + \frac{1}{j\omega C}\right)(1 + j\omega CR) + R}{\cancel{1 + j\omega CR}}\right]}$$

$$= \frac{R}{R + \dfrac{1}{j\omega C} + j\omega CR^2 + R + R}$$

$$= \frac{R}{3R + j\left(\omega CR^2 - \dfrac{1}{\omega C}\right)}$$

equating j terms to zero

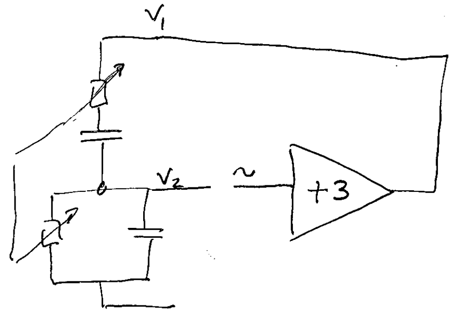
$$j\left(\omega CR^2 - \frac{1}{\omega C}\right) = 0$$

$$\omega CR^2 = \frac{1}{\omega C} \quad \text{or} \quad \omega^2 C^2 R^2 = 1$$

$$\text{or} \quad \omega^2 = \frac{1}{C^2R^2}$$

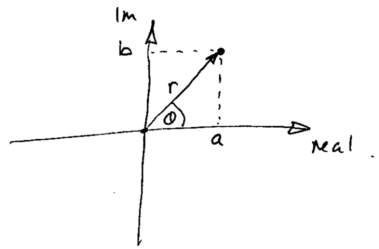
$$\text{or} \quad \omega = \frac{1}{CR}$$

V_1



when phase shift = 0 $\quad \dfrac{V_2}{V_1} = \dfrac{1}{3}$

Polar + Cartesian Representations:



Cartesian approach $\Longrightarrow$ $a + jb$.

Polar approach. $\Longrightarrow$ $r\angle\theta$

($re^{j\theta}$

If I want to add two quantities together cartesian is more convenient.

$$(a+jb)+(c+jd) = (a+c)+j(b+d).$$

If I want to divide two quantities

$$\frac{r_1\angle\theta_1}{r_2\angle\theta_2} \equiv \frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}}$$

but $\frac{1}{e^{j\theta_2}} = e^{-j\theta_2}$

$$\frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}} = \frac{r_1}{r_2} e^{j\theta_1} e^{-j\theta_2}$$

$$= \frac{r_1}{r_2} e^{j(\theta_1-\theta_2)}$$

$$\equiv \frac{r_1}{r_2}\angle(\theta_1-\theta_2)$$

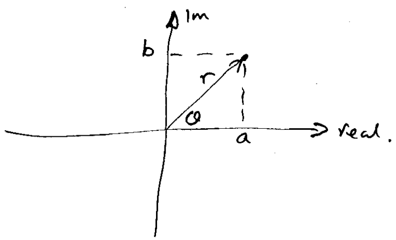
complacated.

$$\frac{a+jb}{c+jd} = \frac{(a+jb)(c-jd)}{c^2+d^2} = \frac{ac+bd+j(bc-ad)}{c^2+d^2}$$

to switch between polar + cartesian

$a = r\cos\theta \qquad b = r\sin\theta$

$r^2 = a^2+b^2 \qquad \theta = \tan^{-1}\frac{b}{a}$

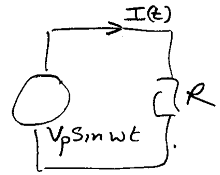


Power with a.c

Power integral $P = \frac{1}{T}\int_0^T V(t)\, I(t)\, dt.$

— here $V(t) = V_p \sin\omega t$

$$P = \frac{1}{T}\int_0^T V_p \sin\omega t\, \frac{V_p}{R}.\sin\omega t\, dt$$



$$I(t) = \frac{V_p \sin\omega t}{R}$$

$$[\equiv I_p \sin\omega t]$$

$$= \frac{1}{R}\cdot\frac{V_p^2}{T}\int_0^T \sin^2\omega t\, dt.$$

$= \frac{1}{R} \times$ mean squared voltage.

$$= \frac{1}{R}\,\frac{V_p^2}{T}\int_0^T \frac{1-\cos 2\omega t}{2}\, dt.$$

$$= \frac{1}{R}\,\frac{V_p^2}{T}\left[\frac{t}{2} - \frac{1}{4\omega}\sin 2\omega t\right]_0^T$$

$\omega = 2\pi f \qquad f = \frac{1}{T}$

$\omega = \frac{2\pi}{T}$ or $T = \frac{2\pi}{\omega}.$

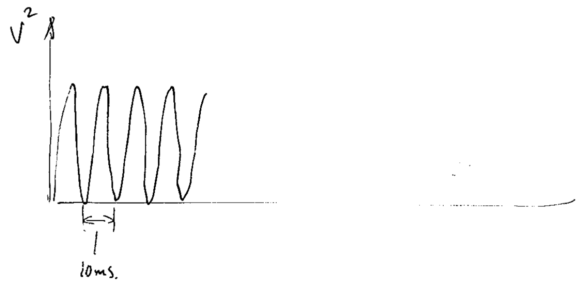
$$= \frac{1}{R}\,\frac{V_p^2}{T}\left[\frac{2\pi}{2\omega} - \cancel{\frac{1}{4\omega}\sin \omega 2\frac{2\pi}{\omega}} - 0 - 0\right]$$

$$= \frac{1}{R}\, V_p^2\, \frac{\cancel{\omega}}{\cancel{2\pi}}\left[\frac{\cancel{2\pi}}{2\cancel{\omega}}\right]$$

$V_p^2 \quad 1$

$$= \frac{V_p^2}{2} \cdot \frac{1}{R}.$$

$$= \frac{V_{rms}^2}{R} \qquad V_{rms} = \sqrt{\frac{V_p^2}{2}} = \frac{V_p}{\sqrt{2}}$$ for a sinusoid.



What if $V = V_p \sin \omega t$

$I = I_p \sin(\omega t + \phi)$

$$\frac{V_p}{I_p} = |Z|$$

Power integral becomes

$$P = \frac{1}{T}\int_0^T V_p \sin \omega t \; I_p \sin(\omega t + \phi) \; dt.$$

$$= \frac{V_p I_p}{T}\int_0^T \sin \omega t \, . \, \sin(\omega t + \phi) \; dt$$

$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

$$= \frac{V_p I_p}{2T}\int_0^T \Big[\cos\big(\omega t - (\omega t + \phi)\big) - \cos(\omega t + \omega t + \phi)\Big]$$

answer is $P = V_{rms} I_{rms} \cos\phi$

$$= \frac{V_p I_p}{2} \cos\phi$$

$$= \frac{V_{rms}^2}{|Z|} \cos\phi$$

power factor

Power dissipation with more than one source.

with 2 dc sources.

$$V = V_{DC1} + V_{DC2}.$$

$$P \propto \overline{V^2} \text{ ie } \propto \overline{(V_{DC1} + V_{DC2})^2}$$

$$\propto \overline{V_{DC1}^2} + \overline{2V_{DC1}V_{DC2}} + \overline{V_{DC2}^2}$$

with 1 dc source + 1 ac source

$$V = V_{DC1} + V_p \sin \omega t$$

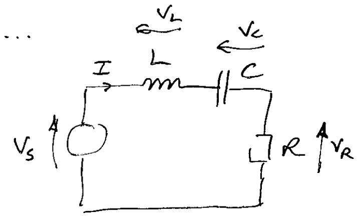
$$P \propto \overline{(V_{DC1} + V_p \sin \omega t)^2}$$

$$P \propto \overline{V_{DC1}^2} + \underbrace{\overline{2 V_{DC1} V_p \sin \omega t}}_{=0} + \overline{V_p^2 \sin^2 \omega t}$$

Resonance

— occurs when a reactive network has a purely resistive behaviour — ie at a frequency where j terms disappear.

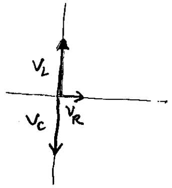
Series resonant ccts ...



$$Z = \frac{V_S}{I} = j\omega L + \frac{1}{j\omega C} + R$$

$$= \frac{-\omega^2 LC + 1 + j\omega CR}{j\omega C}$$

multiply by $\frac{j}{j}$



$$= \frac{j(1-\omega^2 LC) - \omega CR}{-\omega C}.$$

j terms vanish when $j(1-\omega^2 LC) = 0$

ie $\omega^2 = \frac{1}{LC}$

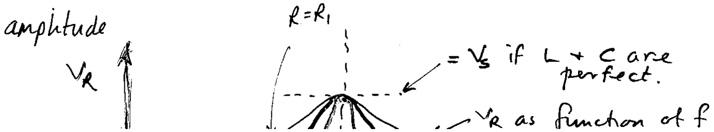
$$\frac{|V_L|}{V_R} = \frac{|V_C|}{V_R} \rightarrow \frac{I \times \frac{1}{\omega C}}{IR} = \frac{1}{\omega CR} = \frac{\sqrt{LC}}{CR} = \sqrt{\frac{L}{C}} \cdot \frac{1}{R}$$

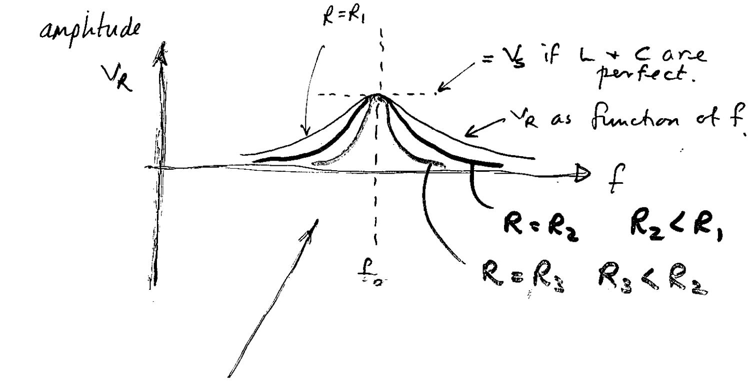
$$\frac{I\,\omega L}{I\,R} = \frac{\omega L}{R}$$

this is the resonant angular frequency, $\omega_0 = \frac{1}{\sqrt{LC}}$

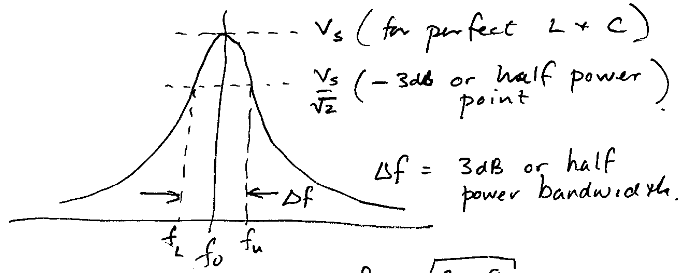
$$= \frac{L}{\sqrt{LC}\,R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

describes the "voltage magnification" or "q" factor.



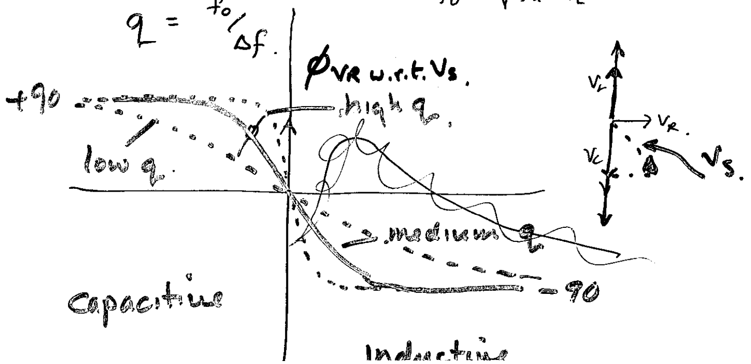


width of the response is related to "q" factor and hence to R → smaller R leads to bigger q leads to narrower shape of frequency response

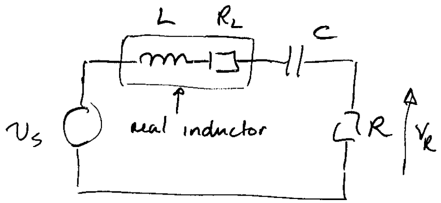


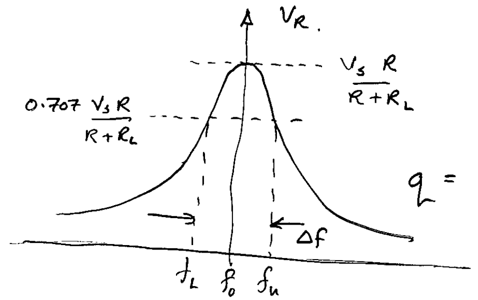
$$f_0 = \sqrt{f_u \times f_L}$$

$$q = f_0/\Delta f.$$



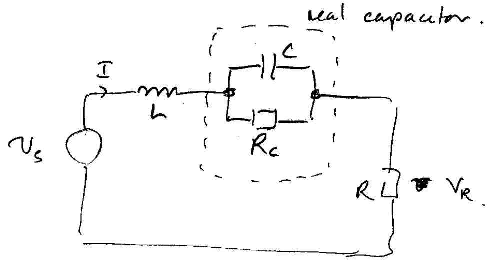
— Inductors usually have some series resistance.





$$q = f_0 / \Delta f$$

Capacitors main problem is parallel resistance — mostly caused by dielectric losses



$$Z = \frac{v_s}{I} = jwL + \frac{R_c/jwc}{\ldots} + R$$

$$Z = \frac{v_s}{I} = jwL + \frac{R_c/jwc}{R_c + 1/jwc} + R$$

$$= jwL + \frac{R_c}{1 + jwcR_c} + R$$

$$= \frac{jwL - w^2LcR_c + R_c + R + jwcRR_c}{1 + jwcR_c}$$

$$= \frac{jw(L + cRR_c) + (R + R_c - w^2LcR_c)}{1 + jwcR_c}$$

$$= \frac{\left[(R + R_c - w^2LcR_c) + jw(L + cRR_c)\right](1 - jwcR_c)}{1 + w^2c^2R_c^2}$$

$$\cancel{jw}\left(L + cRR_c - cR_c(R + R_c - w^2LcR_c)\right) = 0$$

to make j terms disappear

$$L + cRR_c = cR_c(R + R_c - w^2LcR_c)$$

$$L + \cancel{cRR_c} = \cancel{cRR_c} + cR_c^2 - w^2Lc^2R_c^2$$

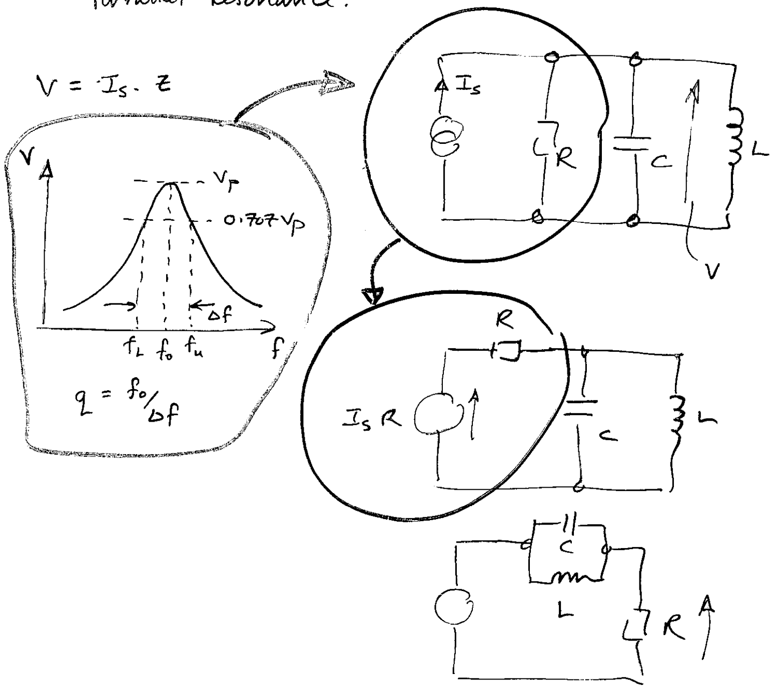
$$\therefore \quad w^2Lc^2R_c^2 = cR_c^2 - L$$

$$\therefore \quad w^2 = \frac{\cancel{c}\cancel{R_c^2}}{Lc^{\cancel{2}}\cancel{R_c^2}} - \frac{\cancel{L}}{\cancel{L}c^2R_c^2} = \frac{1}{Lc} - \frac{1}{c^2R_c^2}$$

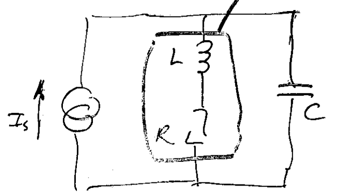
— its the process that matter

Parallel Resonance.

$V = I_s \cdot Z$



real L.

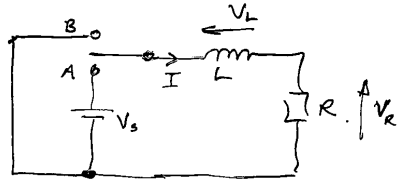


$$w_o^2 = \frac{1}{LC} - \frac{R^2}{L^2}.$$

Transient behaviour of circuits

Transient behaviour of circuits.

Lets assume that switch has been in position A for a long time



– at $t=0$ we switch it to B. How does I change with time for $t>0$.

$$V_L = L\frac{dI}{dt} \qquad V_R = IR$$

$$V_B = V_L + V_R$$

$$\text{or } 0 = L\frac{dI}{dt} + IR.$$

$$-IR = L\frac{dI}{dt}$$

$$-\frac{R}{L}dt = \frac{dI}{I}$$

$$-\frac{R}{L}t + C = \ln I$$

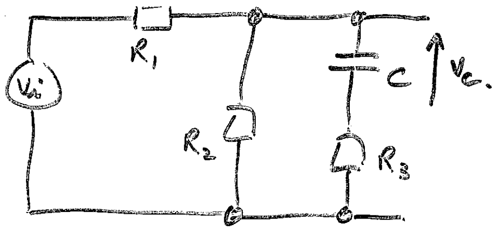
$$e^{(-\frac{R}{L}t + C)} = I$$

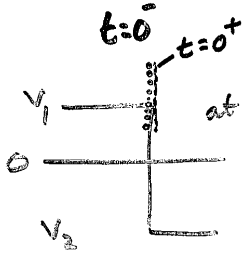
$$e^{-\frac{R}{L}t} \cdot e^{C} =$$

$$A e^{-\frac{R}{L}t} = I$$

$$\text{when } t=0 \quad I = V_s/R$$

$$\therefore \quad I = \frac{V_s}{R} e^{-\frac{Rt}{L}} = \frac{V_s}{R} e^{-t/(\frac{L}{R})}$$

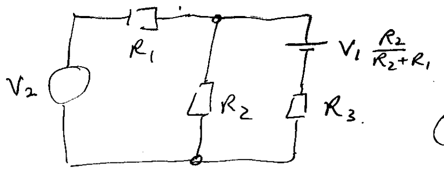




at $t=0^-$ $V_c = V_1 \frac{R_2}{R_2+R_1}$ since $I_c = 0$

at $t=0^+$ $V_c = V_1 \frac{R_2}{R_2+R_1}$ since C will not allow its terminal voltage to change in zero time.

equivalent circuit at $t = 0^+$ is.



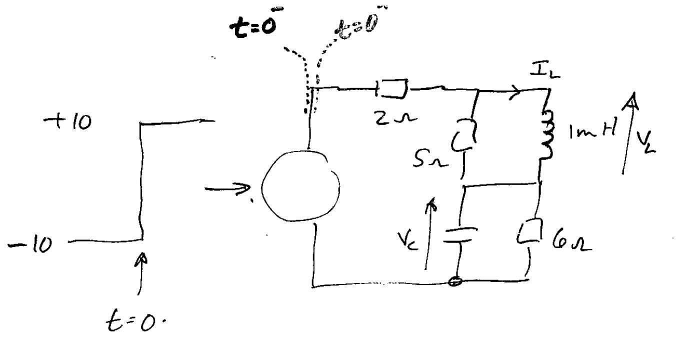
can be solved.

(try it if you like)

What happens at $t \Rightarrow \infty$?

— same situation as at $t=0^-$ except that the cct is driven by V_2.

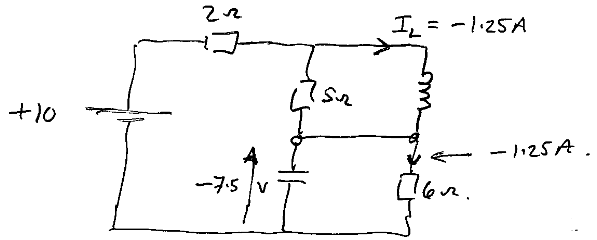
$$V_c = V_2 \frac{R_2}{R_1+R_2}.$$

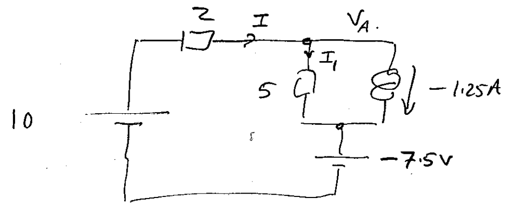


at $t=0^-$ $V_L = 0$, $I_L = -\frac{10}{8}$

$= -1.25\ A$.

$V_c = -7.5\ V$. (= voltage developed across the 6Ω.)





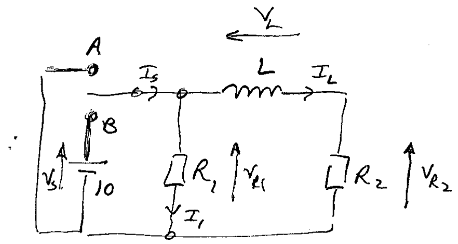
$I = I_1 + (-1.25A)$

$$\frac{10 - V_A}{2} = \frac{V_A - (-7.5)}{5} + (-1.25)$$

$$+1.25 + \frac{10}{2} - \frac{7.5}{5} = \frac{V_A}{5} + \frac{V_A}{2} = \frac{7V_A}{10}$$

$$1.25 + 5 - 1.5 =$$

$$4.75 = \frac{7V_A}{10} \qquad V_A = \frac{47.5}{7} \approx 6.8\ \text{v}.$$



Assume switch has been in position A for a long time → then suddenly switched to B

$$V_s = V_L + V_{R2}$$

$$V_s = L\frac{dI_L}{dt} + I_L R_2$$

$$\frac{V_s}{L} = \frac{dI_L}{dt} + I_L \frac{R_2}{L}$$

$$\frac{V_s}{L} - \frac{I_L R_2}{L} = \frac{dI_L}{dt}$$

$$dt = \frac{dI_L}{\frac{1}{L}(V_s - I_L R_2)}$$

$$dt = \frac{dI_L}{\frac{R_2}{L}\left(\frac{V_s}{R_2} - I_L\right)}$$

$$-\frac{R_2}{L}dt = \frac{dI_L}{I_L - \frac{V_s}{R_2}}$$

$$+c \quad -\frac{R_2}{L}t = \ln\left(I_L - \frac{V_s}{R_2}\right)$$

$$e^{\left(c - \frac{R_2}{L}t\right)} = I_L - \frac{V_s}{R_2}$$

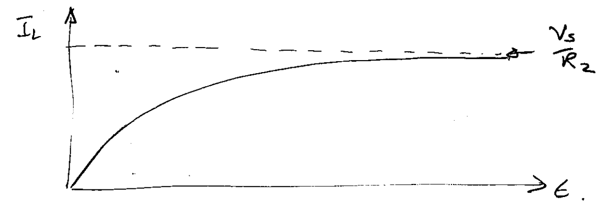
$$Ae^{-\frac{R_2}{L}t} = I_L - \frac{V_s}{R_2}$$

when $t = 0^+$ $I_L = 0$

$$\therefore A = 0 - \frac{V_s}{R_2}$$

$$\therefore -\frac{V_s}{R_2}e^{-\frac{R_2}{L}t} = I_L - \frac{V_s}{R_2}$$

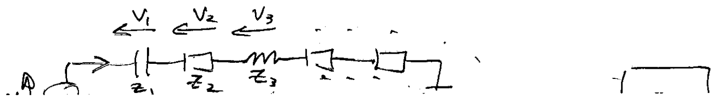
$$\frac{V_s}{R_2}\left(1 - e^{-\frac{R_2}{L}t}\right) = I_L$$



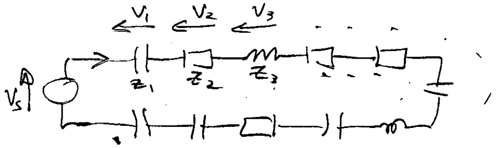
Key integral...

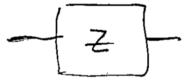
$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx).$$

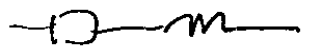
Admittance Conductance + Susceptance.



Admittance Conductance + Susceptance.



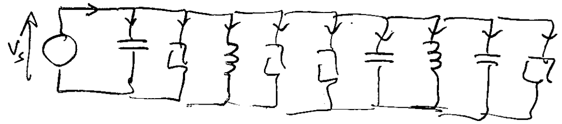




$Z = \frac{V}{I}$ $\quad V_s = I Z$

$a + jb.$

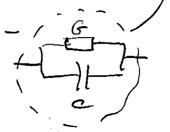
$Z = Z_1 + Z_2 + Z_3$



$Y = \frac{I}{V} \left(= \frac{1}{Z} \right) = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots$ $\quad a + jb.$

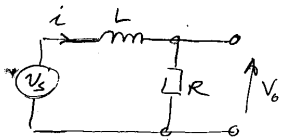
$= Y_1 + Y_2 + Y_3 + \cdots$

$\left(= j\omega C + \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{R_4} \right.$



Filter circuits

$i = \frac{v_s}{Z}$



$v_o = R i = \frac{v_s R}{Z}$

$Z = j\omega L + R$

$v_o = v_s \cdot \frac{R}{R + j\omega L}$

so $\frac{v_o}{v_s} = \frac{R}{R + j\omega L} = \frac{R/R}{R/R + j\omega L/R} = \frac{1}{1 + j\omega L/R}$

$L/$ = time constant

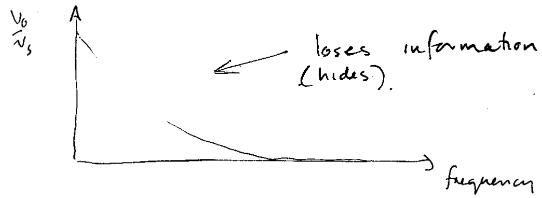
L/R = time constant

Let $\omega_0 = \frac{1}{(L/R)}$.

frequency domain constant

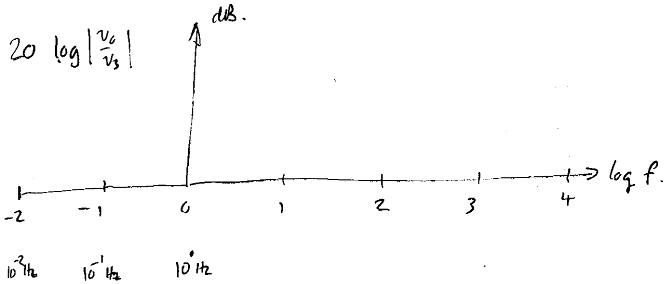
related to time constant.

so $\frac{v_o}{v_s} = \frac{1}{1 + j\,\omega/\omega_0} = \frac{1}{1 + j\,f/f_0}$.



loses information (hides).

usually plot



$\log \left| \frac{v_o}{v_s} \right|$ usually expressed as dB or decibels

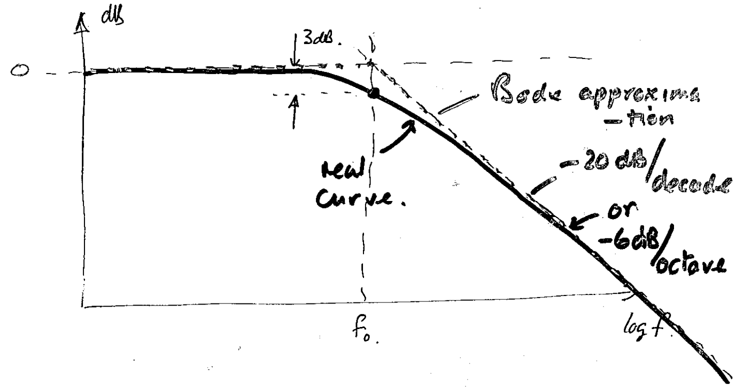
$\boxed{\text{gain in dB} = 20 \log \left| \frac{v_o}{v_s} \right|}$ $\left[= 10 \log \frac{P_o}{P_i} \right]$

$\left[\text{or } 20 \log \left[\frac{v_o}{\ldots} \right] \right]$ or $10 \log \frac{P_o}{P_i}$

$$\left[\text{or } 20 \log\left[\frac{v_o}{v_{ref}}\right]\right] \quad \text{or} \quad 10 \log \frac{P_o}{P_{ref}}$$

$$dBv = 20 \log \left|\frac{v}{1\,V\,rms}\right|$$

$$\frac{v_o}{v_i} = \frac{1}{1 + j\,f/f_o}$$



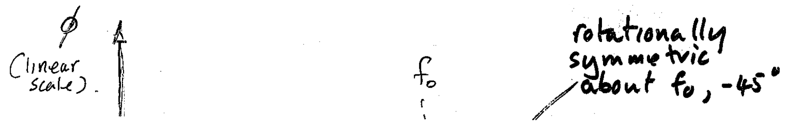
$$\left|\frac{v_o}{v_i}\right| = \left|\frac{1}{1+j\,f/f_o}\right| = \left[\frac{1}{1+(f/f_o)^2}\right]^{1/2}$$

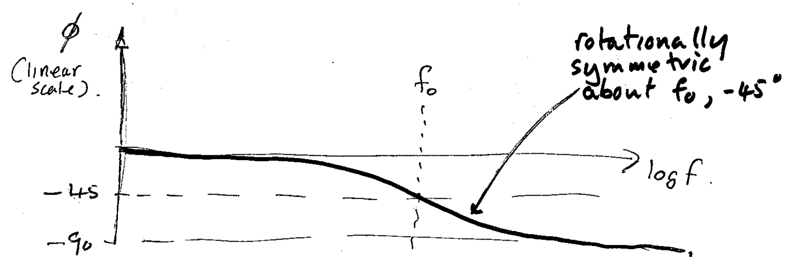
if $f \ll f_o$ $\quad \left|\frac{v_o}{v_i}\right| \Rightarrow 1$ as $f \to 0$.

if $f = f_o$ $\quad \left|\frac{v_o}{v_i}\right| = \left[\frac{1}{2}\right]^{1/2} = \frac{1}{\sqrt{2}}$.

$20 \log \frac{1}{\sqrt{2}} \approx -3dB$.

if $f \gg f_o$ $\quad \left|\frac{v_o}{v_i}\right| \Rightarrow \frac{f_o}{f}$ as $f \Rightarrow \infty$





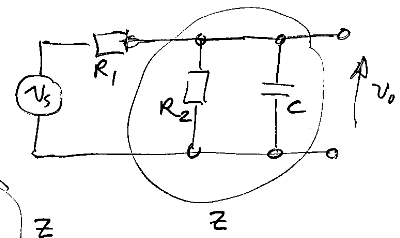
$\phi\left[\frac{1}{1+j\,\cancel{\omega/\omega_o}\,f/f_o}\right]$ is $-\tan^{-1} f/f_o$

if $f \ll f_o$ $\quad \phi = -\tan^{-1}(\underbrace{\text{a small number}}_{\text{ie } \ll 1}) \Rightarrow 0$ from below

if $f = f_o$ $\quad \phi = -\tan^{-1} 1 = -45$

if $f \gg f_o$ $\quad \phi = -\tan^{-1}(\underbrace{\text{a large no.}}_{\text{ie } \gg 1}) \Rightarrow -90$ from above

Another first order low pass



$$v_o = \frac{\underbrace{\dfrac{R_2 \times \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}}}_{Z}}{R_1 + \underbrace{\dfrac{R_2 \times \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}}}_{Z}}$$

$$= \frac{\dfrac{R_2}{1+j\omega CR_2}}{R_1 + \dfrac{R_2}{1+j\omega CR_2}} = \frac{R_2}{(1+j\omega CR_2)R_1 + R_2}$$

$$= \frac{R_2}{R_1 + j\omega C R_2 R_1 + R_2} = \frac{R_2}{(R_1+R_2) + j\omega C R_1 R_2}$$

need to force this to unity.

$$= \frac{R_2}{(R_1+R_2)\left(1 + j\omega \frac{C R_1 R_2}{R_1+R_2}\right)}$$

$$= \underbrace{\frac{R_2}{R_1+R_2}}_{k} \cdot \frac{1}{1 + j\,\omega/\omega_0}$$

$$\omega_0 = \frac{1}{\left(\frac{C R_1 R_2}{R_1+R_2}\right)}$$

$$20 \log \left| k \cdot \frac{1}{1+j\,\omega/\omega_0} \right|$$

$$= 20 \log k + 20 \log \left| \frac{1}{1+j\,\omega/\omega_0} \right|$$