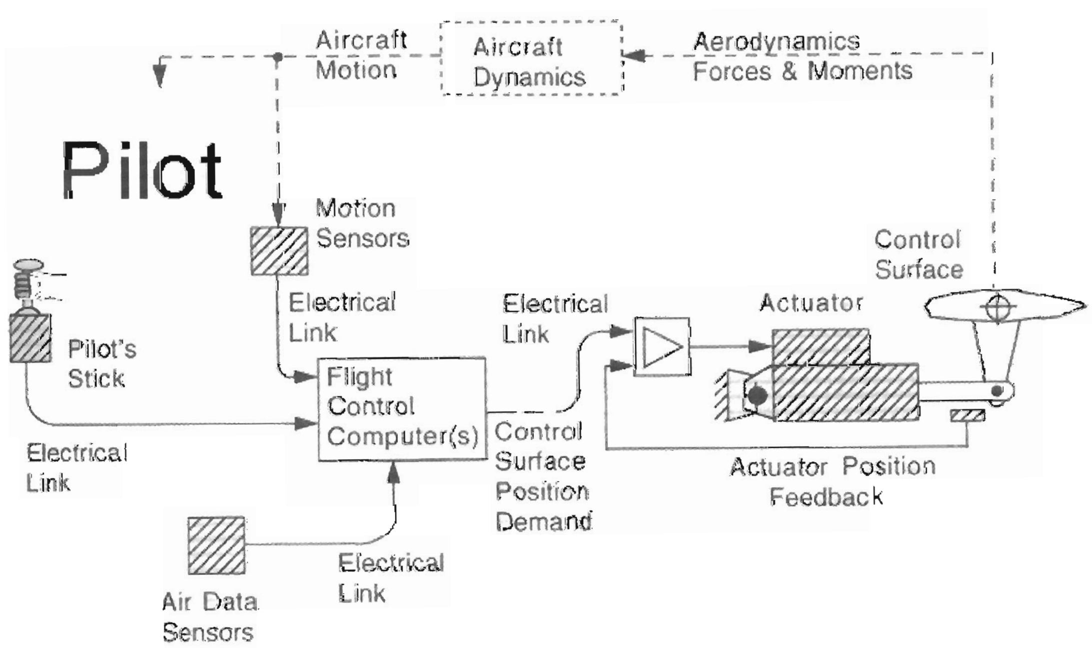
Solution

Question 1

(a)

The main advantages of the Fly-By-Wire flight control are summarised as follows:

- Elimination of mechanical control runs.
- Consistent handling over a wide flight envelope.
- Automatic stabilisation.
- Carefree manoeuvring.
- Automatic integration of additional controls.
- Ability to exploit aerodynamically unstable aircraft.



Schematic of the Fly-By-Wire control system

(b)

i. The Mach number M and the true airspeed V_T are related by $M = \dfrac{V_T}{A}$,

where A is the speed of sound for $T_s = -34.5^o C$:

$$A = \sqrt{\gamma\, R_a\, T_s} = \sqrt{1.4 \times 287.0529 \times 238.5} = 309.6 \text{ m/s}$$

Therefore,

$$M = \frac{V_T}{A} = \frac{155.55}{309.6} = 0.502$$

ii. Since the aircraft is flying in the troposphere region, therefore, the static temperature is related to altitude by:

$$T_s = T_o - L \times H \quad \Rightarrow \quad H = \frac{(T_o - T_s)}{L} = \frac{(288.15 - 238.5)}{6.5 \times 10^{-3}} = 7638\,\text{m}$$

L is the troposphere lapse rate.

iii. The air density is given by:

$$\rho = \frac{P_s}{R_a T_s} = \frac{37650}{287.0529 \times 238.5} = 0.55\,\text{kg/m}^3$$

iv. The calibrated airspeed V_c and the impact pressure are related by:

$$\text{Impact pressure} = P_0\left(\left(1 + \frac{(\gamma - 1)(V_c/A_0)^2}{2}\right)^{\gamma/(\gamma-1)} - 1\right) = P_T - P_s$$

The static pressure P_s and the total pressure P_T are related by:

$$\frac{P_T}{P_s} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{(\gamma-1)}} \Rightarrow P_T = 37.65 \times \left(1 + \frac{(1.4 - 1)}{2} 0.502^2\right)^{\frac{1.4}{(1.4-1)}} = 44.72\,\text{kPa}$$

Therefore, $P_T - P_s$ =7.07 kPa.

Furthermore, at sea level the speed of sound is given by:

$$A_o = \sqrt{\gamma R_a T_o} = \sqrt{1.4 \times 287.0529 \times 288.15} = 340.29\,\text{m/s}$$

Therefore,

$$\frac{(P_T - P_s)}{P_0} + 1 = \left(1 + 0.2(V_c/A_0)^2\right)^{3.5} \Rightarrow 1 + 0.2(V_c/A_0)^2 = \sqrt[3.5]{\frac{(P_T - P_s)}{P_0} + 1}$$

$$\Rightarrow V_c = A_o\sqrt{\frac{3.5\sqrt{\frac{(P_T-P_s)}{P_0}+1}-1}{0.2}} = 340.29\times\sqrt{\frac{3.5\sqrt{\frac{(44.72-37.65)}{101.325}+1}-1}{0.2}} = 106.14 \text{ m/s} =$$

382.1 km/h.

(c)

The impact pressure is given by:

$$Q_c = P_T - P_s = P_0\left(\left(1+\frac{(\gamma-1)}{2}\left(\frac{V_c}{A_0}\right)^2\right)^{\frac{\gamma}{(\gamma-1)}}-1\right)$$

$$P_T = P_0\left(\left(1+\frac{(\gamma-1)}{2}\left(\frac{V_c}{A_0}\right)^2\right)^{\frac{\gamma}{(\gamma-1)}}-1\right)+P_s$$

The speed of sound A_0 at sea level is given by:

$$A_o = \sqrt{\gamma R_a T_o} = \sqrt{1.4\times 287.0529\times 288.15} = 340.3\,\text{m/s}$$

$$P_T = 101.325\times\left(\left(1+\frac{(1.4-1)}{2}\left(\frac{144.44}{340.3}\right)^2\right)^{\frac{1.4}{(1.4-1)}}-1\right)+22.63 = 36\text{kPa}$$

Furthermore, the static pressure P_s and the total pressure P_T are related by:

$$\frac{P_T}{P_s} = \left(1+\frac{(\gamma-1)}{2}M^2\right)^{\frac{\gamma}{(\gamma-1)}} \Rightarrow \frac{2}{(\gamma-1)}\left(\left(\frac{P_T}{P_s}\right)^{\frac{\gamma-1}{\gamma}}-1\right) = M^2$$

$$M = \sqrt{\frac{2}{(\gamma-1)}\left(\left(\frac{P_T}{P_s}\right)^{\frac{\gamma-1}{\gamma}}-1\right)} = \sqrt{\frac{2}{(1.4-1)}\left(\left(\frac{36.18}{22.63}\right)^{\frac{1.4-1}{1.4}}-1\right)} = 0.847$$

Therefore, the speed of sound would be given by:

$$A=\frac{V_T}{M}=\frac{900\times(1000/3600)}{0.847}=295.16\ \text{m/s}$$

And the static air temperature:

$$A=\sqrt{\gamma\,R_a\,T_s}\Rightarrow T_s=\frac{A^2}{R_a\,\gamma}=\frac{295.16^2}{287.053\times1.4}=216.8\,^oK=-56.2^oC$$

Question 2

(a)

A typical RST cycle would consist of :

- Soaking in an environmental chamber at 70°C for a given period.
- Rapidly cooling the equipment to –55°C in 20 minutes and soaking at that temperature for a given period.
- Subjecting the equipment to vibrations during cold and hot soaking periods.

(b)

i. The maximum drift from the original nominal value to the tolerance limits of IC parameters, which results in failure is constant. Therefore,

$$t^n\,Q(T)=t^n\,Q_o\,e^{(-E_a/kT)}=K$$

where K is a constant and t is the time during which the parameters drift from nominal values to tolerance limits, therefore,

$$t^n=\frac{K}{Q_o}e^{(E_a/kT)}\Rightarrow n\ln(t)=\ln\left(\frac{K}{Q_o}\right)+\frac{E_a}{kT}\Rightarrow \ln(t)=\frac{1}{n}\ln\left(\frac{K}{Q_o}\right)+\frac{E_a}{nkT}$$

since K and Q_o are constants, therefore, we can write

$$\ln(t)=\ln(t_o)+\frac{E_a}{nkT}\Rightarrow t=t_o\,e^{(E_a/nkT)}=MTTF$$

ii. Since the $MTTF_{85}$ of the IC at 85°C (353°K) is 24000 hours, therefore,

$$MTTF_{85}=t_o\,e^{(E_a/nkT)}\Rightarrow t_o=MTTF_{100}\,e^{(-E_a/nkT)}$$

$$=24000\times e^{\left(-1/8.6\times10^{-5}\times358\right)}=1.88\times10^{-10}\ \text{hours}$$

Therefore, the $MTTF_{30}$ of the IC at 30°C (303°K) is given by:

$$MTTF_{30} = t_o e^{(E_a/nkT)} = 1.88\times10^{-10}\, e^{\left(1/8.6\times10^{-5}\times303\right)} = 8.72\times10^{6} \text{ hours}$$

(c)

i. Since there are 4 generators and only 3 are required to produce 270kW, the system is 3-out-of-4 active system, for which the reliability function is given:

$$R(t) = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!}\left[e^{-\lambda kt}\right]\left[1-e^{-\lambda t}\right]^{n-k}$$

where m=3 and n=4, therefore,

$$R(t) = \frac{4!}{3!(4-3)!} e^{-3\lambda t}\left(1-e^{-\lambda t}\right)^{4-3} \qquad (k=3)$$

$$+\frac{4!}{4!(4-4)!} e^{-4\lambda t}\left(1-e^{-\lambda t}\right)^{4-4} \qquad (k=4)$$

$+$

$$R(t) = 4e^{-3\lambda t} - 3e^{-4\lambda t}$$

therefore, the probability of failure is given by :

$$F(t=10\,\text{hours}) = 1 - R(t=10\text{ hours}) = 1 - 4e^{\left(-3\times1\times10^{-4}\times10\right)} + 3e^{\left(-4\times1\times10^{-4}\times10\right)}$$
$$F(t=10\,\text{hours}) = 6.0\times10^{-6}$$

ii. Since there are 4 generators and only 2 are required to produce 180kW, the system is 2-out-of-4 active system, for which the reliability function is given:

$$R(t) = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!}\left[e^{-\lambda kt}\right]\left[1-e^{-\lambda t}\right]^{n-k}$$

where m=2 and n=4, therefore,

$$R(t)=\frac{4!}{2!(4-2)!}e^{-2\lambda t}\left(1-e^{-\lambda t}\right)^{4-2} \qquad (k=2)$$

$$+\frac{4!}{3!(4-3)!}e^{-3\lambda t}\left(1-e^{-\lambda t}\right)^{4-3} \qquad (k=3)$$

$$+\frac{4!}{4!(4-4)!}e^{-4\lambda t}\left(1-e^{-\lambda t}\right)^{4-4} \qquad (k=4)$$

$$R(t)=6e^{-2\lambda t}\left(1-e^{-\lambda t}\right)^{2}+4e^{-3\lambda t}-3e^{-4\lambda t}$$

therefore, the probability of failure is given by :

$$F(t=10\,\text{hours})=1-R(t=10\,\text{hours})=1-6e^{\left(-2\times1\times10^{-4}\times10\right)}\left(1-e^{\left(-1\times10^{-4}\times10\right)}\right)^{2}$$

$$-4e^{\left(-3\times1\times10^{-4}\times10\right)}+3e^{\left(-4\times1\times10^{-4}\times10\right)}$$

$$F(t=10\,\text{hours})=4.0\times10^{-9}$$

iii. The MTTF of the capability of producing 180kW of electrical power is given by:

$$\text{MTTF}=\int_0^\infty R(t)dt=\int_0^\infty (6e^{-2\lambda t}\left(1-e^{-\lambda t}\right)^{2}+4e^{-3\lambda t}-3e^{-4\lambda t})\,dt$$

$$=\int_0^\infty\left[6e^{-2\lambda t}\left(1+e^{-2\lambda t}-2e^{-\lambda t}\right)+4e^{-3\lambda t}-3e^{-4\lambda t}\right]dt$$

$$=\int_0^\infty\left[6e^{-2\lambda t}+6e^{-4\lambda t}-12e^{-3\lambda t}+4e^{-3\lambda t}-3e^{-4\lambda t}\right]dt$$

$$=\int_0^\infty\left[6e^{-2\lambda t}-8e^{-3\lambda t}+3e^{-4\lambda t}\right]dt$$

$$=\left[\frac{-3}{\lambda}e^{-2\lambda t}+\frac{8}{3\lambda}e^{-3\lambda t}-\frac{3}{4\lambda}3e^{-4\lambda t}\right]_0^\infty=\frac{1}{\lambda}\left(3-\frac{8}{3}+\frac{3}{4}\right)$$

$$=\frac{1}{1.0\times10^{-4}}\times\left(3-\frac{8}{3}+\frac{3}{4}\right)=10833\,\text{hours}$$

The MTTF of the capability of producing 180kW of electrical power is given by:

$$MTTF = \int_0^\infty R(t)\,dt = \int_0^\infty \left(4e^{-3\lambda t} - 3e^{-4\lambda t}\right)dt$$

$$= \left[-\frac{4}{3\lambda}e^{-3\lambda t} + \frac{3}{4\lambda}e^{-4\lambda t}\right]_0^\infty = \left(\frac{4}{3\lambda} - \frac{3}{4\lambda}\right)$$

$$= \frac{1}{\lambda}\left(\frac{4}{3} - \frac{3}{4}\right) = 5833\,\text{hours}$$

Question 3

(a)

i. The load inertia referred to the shaft is given by:

$$J_r = \left(\frac{\lambda}{2\pi}\right)^2 \times \frac{m}{n^2} = \left(\frac{10\times10^{-3}}{2\pi}\right)^2 \times \frac{35000}{10^2} = 8.86\times10^{-4}\,\text{kg.m}^2$$

ii. At no-load the motor torque T_m and the angular acceleration α_m are related by:

$$(J_m + J_r)\left(\frac{2\pi n}{\lambda}\right)\alpha_m = T_m$$

$$\Rightarrow T_m = \left(1.5\times10^{-3} + 8.86\times10^{-4}\right)\times\left(\frac{2\times\pi\times10}{10\times10^{-3}}\right)\times1.0 = 15.0\text{ Nm}$$

iii. The armature current and the torque are related by:

$$T_m = k\,I \quad\Rightarrow\quad I = \frac{T_m}{k} = \frac{15.0}{0.15} = 100\text{ A}$$

and the copper loss of the motor:

$$P_c = R \times I^2 = 18\times10^{-3}\times100^2 = 180\text{ W}$$

i. When the copper loss is considered, the efficiency of the motor is give by:

$$\eta = \frac{T_m \Omega_m}{T_m \Omega_m + R I^2} = \frac{T_m \Omega_m}{T_m \Omega_m + R\left(\frac{T_m}{k}\right)^2} = \frac{T_m \Omega_m}{T_m \Omega_m + \frac{R}{k^2} T_m^2}$$

$$\Rightarrow \eta = \frac{1}{1 + \frac{R}{k^2}\frac{T_m}{\Omega_m}}$$

(b)

i. For a sinusoidal velocity profile, the linear displacement is given by:

$$x(t) = \int_0^t v(u)\,du = \int_0^t V_m \sin\left(\frac{\pi}{T} u\right) du$$

$$= \frac{V_m T}{\pi}\left(-\cos\left(\frac{\pi}{T} u\right)\right)_0^t = \frac{V_m T}{\pi}\left(1 - \cos\left(\frac{\pi}{T} t\right)\right)$$

ii. The maximum speed of the actuator is given by:

$$x(t) = \frac{V_m T}{\pi}\left(1 - \cos\left(\frac{\pi}{T} t\right)\right)$$

$$\Rightarrow x_m = x(T) = \frac{V_m T}{\pi}\left(1 - \cos\left(\frac{\pi}{T} T\right)\right) = \frac{2 V_m T}{\pi}$$

$$\Rightarrow V_m = \frac{x_m \pi}{2T} = \frac{5\times 10^{-3} \times \pi}{2 \times 1} = 7.85\times 10^{-3}\ \text{m/s}$$

iii. The some of the forces is given by:

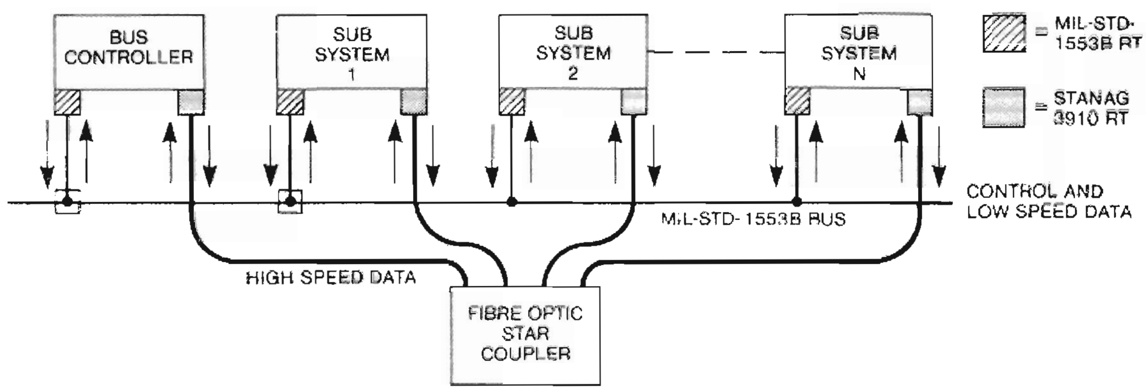
$$\sum Forces = F_{tot}(t) = m\frac{dv}{dt}$$

And since $v(t) = V_m \sin\left(\frac{\pi}{T} t\right) \Rightarrow \frac{dv(t)}{dt} = \frac{\pi}{T} V_m \cos\left(\frac{\pi}{T} t\right)$, therefore,

$$F_{tot}(t) = \frac{\pi}{T} V_m\ m\ \cos\left(\frac{\pi}{T} t\right) = \frac{\pi}{1} \times 1 \times 35000\ \cos(0) = 110\text{kN}$$

Question 4

(a)



STANAG 3910 data bus

Its main features could be summarised as follows:

- 20 Mbits/s message transfer over a fibre optic network.
- MIL STD 1553B (STANAG 3838) controlling protocol.
- Maximum number of sub-systems 31.
- Maximum message transfer 4096 words.
- Central bus control.
- Data encoding – Manchester bi-phase.

(b)

i. For a potentiometer loaded by a resistance R_m, the maximum error occurs when $a = \theta/\theta_{\max}$ =0.5, therefore,

$$Error\,(\%) = \frac{a(1-a)\frac{R_p}{R_m}}{1+a(1-a)\frac{R_p}{R_m}} \times 100 = \frac{0.5\times(1-0.5)\times\frac{10000}{90000}}{1+0.5\times(1-0.5)\frac{10000}{90000}} \times 100 = 2.7\%$$

ii. The resolution of the ADC should $\geq$ the required resolution of 1/1600. Therefore,

$$\frac{1}{2^n} \geq \frac{1}{1600} \Rightarrow 2^n \geq 1600 \Rightarrow n \geq \text{int}\left(\frac{\ln 1600}{\ln 2}\right) + 1 = 11$$

iii. To ensure that the position waveform is well-represented by the ADC output, the sampling frequency of the ADC should be at least 10 times larger than the frequency of the highest significant harmonic of the periodic position profile, $\frac{\theta_0}{3}\sin\left(\frac{10\pi}{T}t\right)$.

$f_s \geq 50/T = 50$ Hz.

(c)

i. The truth table of the logic circuit is as follows:

Position (degrees)	ADC output				LEDs		
	b_3	b_2	b_1	b_0	Red	Yellow	Green
0	0	0	0	0	1	0	0
10	0	0	0	1	1	0	0
20	0	0	1	0	0	1	0
30	0	0	1	1	0	1	0
40	0	1	0	0	0	1	0
50	0	1	0	1	0	1	0
60	0	1	1	0	0	1	0
70	0	1	1	1	0	0	1
80	1	0	0	0	0	0	1
90	1	0	0	1	0	1	0
100	1	0	1	0	0	1	0
110	1	0	1	1	0	1	0
120	1	1	0	0	0	1	0
130	1	1	0	1	0	1	0
140	1	1	1	0	1	0	0
150	1	1	1	1	1	0	0

ii. The SOP expressions for the logic functions Green and Yellow:

$$\text{Green} = \overline{b_3}\, b_2\, b_1\, b_0 + b_3\, \overline{b_2}\, \overline{b_1}\, \overline{b_0}$$

$$\text{Red} = \bar{b}_3\,\bar{b}_2\,\bar{b}_1\,\bar{b}_0 + \bar{b}_3\,\bar{b}_2\,\bar{b}_1\,b_0 + b_3\,b_2\,b_1\,\overline{b_0} + b_3\,b_2\,b_1\,b_0$$

iii.

$$\begin{aligned}\text{Red} &= \bar{b}_3\,\bar{b}_2\,\bar{b}_1\,\bar{b}_0 + \bar{b}_3\,\bar{b}_2\,\bar{b}_1\,b_0 + b_3\,b_2\,b_1\,\overline{b_0} + b_3\,b_2\,b_1\,b_0 \\ &= \bar{b}_3\,\bar{b}_2\,\bar{b}_1\overbrace{\left(\bar{b}_0 + b_0\right)}^{1} + b_3\,b_2\,b_1\overbrace{\left(\bar{b}_0 + b_0\right)}^{1} \\ &= \bar{b}_3\,\bar{b}_2\,\bar{b}_1 + b_3\,b_2\,b_1\end{aligned}$$

iv. The POS expression for the logic function Yellow:

$$\text{Yellow} = \left(b_3 + b_2 + b_1 + b_0\right)\left(b_3 + b_2 + b_1 + \overline{b_0}\right)\left(b_3 + \bar{b}_2 + \bar{b}_1 + b_0\right)\left(\bar{b}_3 + b_2 + b_1 + b_0\right)$$
$$\left(\bar{b}_3 + \bar{b}_2 + \bar{b}_1 + b_0\right)\left(\bar{b}_3 + \bar{b}_2 + \bar{b}_1 + \bar{b}_0\right)$$