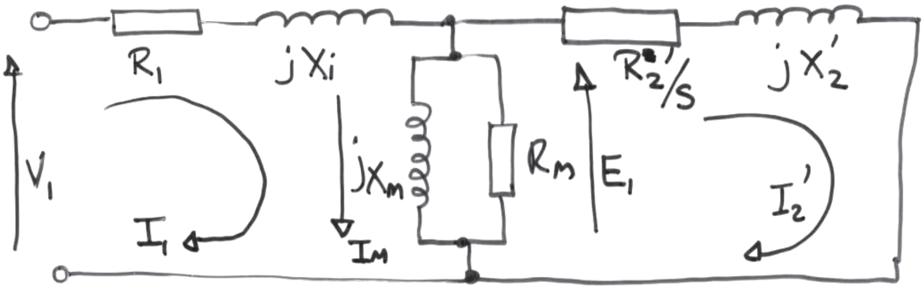
Answers to EEE223 – Summer 2015.

Qu 1.

a.

FULL STATOR EQUIVALENT CCT.



i).

Here,

R_1 = STATOR RESISTANCE PER PHASE

R_2' = REFERRED ROTOR RESISTANCE /ϕ

X_1 = STATOR LEAKAGE REACTANCE /ϕ

X_2' = REFERRED ROTOR LEAKAGE REACTANCE /ϕ

X_m = MAGNETIZING REACTANCE /ϕ

R_m = IRON LOSS RESISTANCE /ϕ

V_1 = RMS SUPPLY PHASE VOLTAGE /ϕ

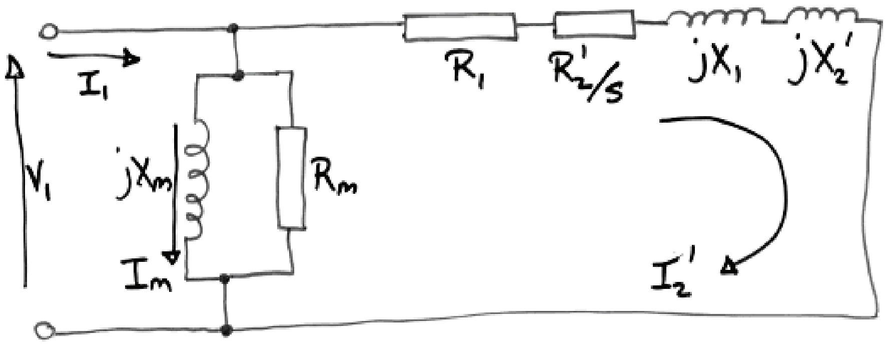
E_1 = INDUCED STATOR PHASE VOLTAGE

I_2' = REFERRED ROTOR CURRENT

I_m = MAGNETIZING CURRENT

I_1 = STATOR CURRENT.

ii). If we assume that the back emf is approximately equal to the supply voltage, the approximate equivalent circuit is valid:



b. From the approximate equivalent circuit, power transferred to the rotor is given by I^2R_2', but I^2R_2'/s is the loss in the rotor winding therefore $I^2R_2'(1-s)/s$ is the power available at the output, therefore

$$\text{OUTPUT POWER PER PHASE} = I_2'^2 R_2' \frac{(1-s)}{s}$$

$$\text{TOTAL OUTPUT POWER} = 3I_2'^2 R_2' \frac{\cancel{(1-s)}}{s}$$

3 PHASES

$$= \left(\frac{2\pi N}{60}\right)T = \left(\frac{2\pi N_s}{60}\right)\cancel{(1-s)}\,T$$

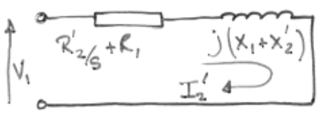
$$N = (1-s)N_s$$

$$T = \left(\frac{60}{2\pi N_s}\right) 3 I_2'^2 \frac{R_2'}{s}$$

$$\frac{1}{\text{ANGULAR SYNCHRONOUS SPEED}}$$

$$\frac{2\pi N_s}{60} = \frac{2\pi f_1}{p}$$

$$T = \frac{3p}{2\pi f_1} \quad I_2'^2 \frac{R_2'}{s}$$



$$I_2' = \frac{V_1}{\sqrt{\left(\frac{R_2'}{s} + R_1\right)^2 + (X_1 + X_2')^2}}$$

$$T = \frac{3p}{2\pi f_1} \quad \frac{V_1^2 \; R_2'/s}{\left(R_2'/s + R_1\right)^2 + (X_1 + X_2')^2}$$

c. From a locked rotor test the parameters of the machine can be determined. The power per phase is one third of the total input power, therefore the total resistance can be found.
$\mathbf{R_1+R_2' = P_{Phase} / I^2 = 0.041667\Omega}$. Given $R_1 = 0.02\Omega$ then $\mathbf{R_2' = 0.02167\Omega}$.

Now 50V/40A = Z, therefore $\mathbf{(X_1+X_2') = 1.25\Omega}$ From this the maximum pull-out torque can be calculated from:

$$T_{PULL\text{-}OUT} = \frac{3pV_1^2}{2\pi f_1} \; \frac{\sqrt{R_1^2 + (X_1 + X_2')^2}}{\left(R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2}\right)^2 + (X_1 + X_2')^2}$$

The peak pull-out torque is therefore **215Nm.**

Qu 2.

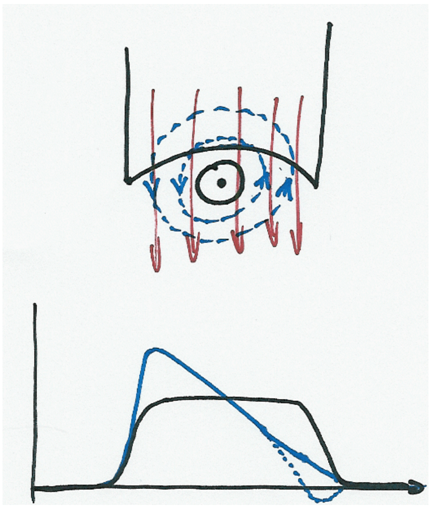
a. Rating parameters:

T_{stall} – Continuous torque that the motor can output at zero speed. This rating relates to the ability of the motor to dissipate the I^2R loss in the armature. With the armature stationary, this is the worst condition with regard to cooling – The airgap between the armature and stator acts as a thermal barrier, and with the armature stationary, there is no air movement in this gap to assist cooling.

n_{max} – Maximum operating speed, limited by mechanical constraints, also the commutator action is speed limited.

T_{max} – Maximum peak torque available form the machine – 5 to 10 times the continuous rating. Limited by commutator action at high armature currents. Also, at high armature currents, the armature reaction field may demagnetise the permanent magnets. The field from the permanent magnets is augmented at one side of the stator pole, and decreased at the other side of the stator pole.

b.

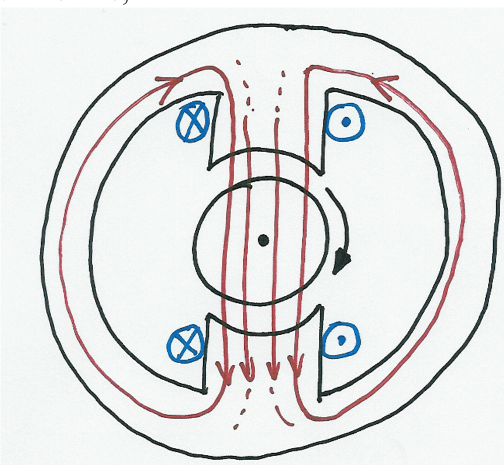


If the total flux through the stator pole is examined across the magnet pole surface, the effect of the armature reaction field may be seen. If the flux through the magnet pole goes negative, some de-magnetisation of the permanent magnet may occur.

c.(i) Given the connection of the 4 field windings in series across a 200V supply, the total field winding resistance is 200V / 7A = 28.57Ω total, or 7.14Ω per field winding.
For a field current of 7A, the machine constant is $E = M \times I_f \times \omega$, and therefore for an armature current of 28A through a resistance of 0.1Ω, the back-emf = 200-2.8V = 197.2V. The machine constant, M, is therefore $(197.2 \times 60) / (7 \times 2 \times \pi \times 3000) = 0.09$.
The load torque therefore becomes, $T = 0.09 \times 7 \times 28 =$ **17.64Nm**.

(ii) The four field coils now connected in parallel as a series connected motor gives a total winding resistance of $0.1\Omega + (7.14\Omega/4) =$ **1.885Ω**
Given the load torque remains the same, and the machine constant won't change, the current through the armature winding and the 4 parallel connected field windings will be 28A, which equates to the rated 7A in each field winding. The supply voltage is now 250V, and given that $V = IR + E$, the back emf can be calculated to be, $E = 197.22V$. This equates to a new motor speed of **313 rad/sec** or **2989 rpm**, a reduction in speed from the 3000 rpm previously.

d. For a conventional permanent magnet brushed DC machine, $T = \psi_f \times I_a$
ψ_f depends on the permanent magnet field, the dimensions of the machine and the winding turns, i.e. is **constant for a given machine**. Similarly, induced voltage across the armature winding, $E = \psi_f \times \omega$
For a wound field machine,



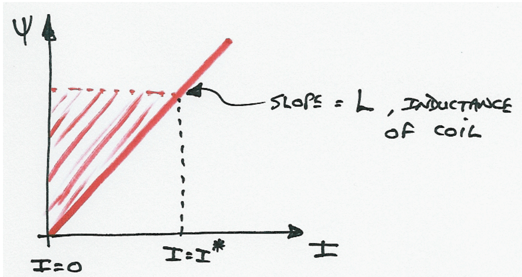
Here, the excitation field depends on the amplitude of the current in the separate field winding, $\psi_f = M \times I_f$ Where M = mutual inductance between field and armature winding **(N.B. Mutual inductance = mutual flux linkage per amp)**
Therefore

$$T = M \times I_f \times I_a$$
$$E = M \times I_f \times \omega$$

Now if the field winding is connected in series with the armature winding, the toque is proportional to the square of the current, therefore if the current is AC, the torque is only unidirectional – Universal Motor. The main factor determining the choice of voltage if the machine is now operated on AC is the winding inductance, as this does not influence the level of the current on a DC supply, but forms a major part of the machine impedance when supplied as a universal machine on AC.

Qu 3.

a. If we consider a linear system in which there is no saturation, the slope of the linear curve is the circuit inductance, $\psi = LI$.



A linear system implies a constant inductance, i.e. a constant slope. As stored magnetic energy:

$$W = \int_0^{I^*} I.d\psi$$

And since **ψ = LI**, and as L is constant, **dψ = L.dI** therefore the stored energy:

$$W = \int_0^{I^*} I.L.dI$$

$$= L.\int_0^{I^*} I.dI$$

$$= L.\left[\frac{1}{2}I^2\right]_0^{I^*}$$

$$= \frac{1}{2}LI^{*2}$$

This is the standard equation for energy in an inductor, and only applies to NON-SATURATED linear systems.

b. For a simple relay, $F = -\frac{1}{2}.\frac{I^2N^2\mu_o A}{x^2}$

Therefore with N=1200, A=100mm^2 = 100×10^{-6}m^2, and X=5mm, the required current to close the device is when F=1Nm = the spring force.

Therefore **I = 0.53A**

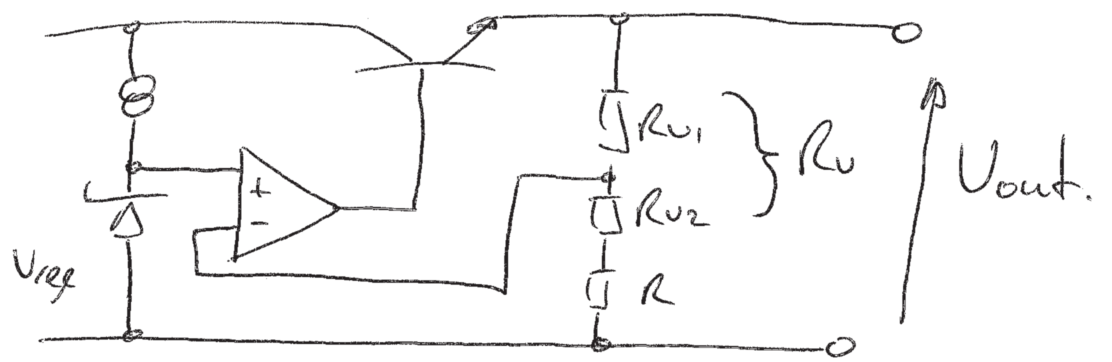
The current for the relay to re-open will be given by the current which produces a force of 1Nm with an airgap length of 2mm.

Therefore the re-opening current **I = 0.21A**

c. (i) The current levels in part b are not equal because the current required to produce a given force is proportional to the length of the airgap. When the relay is open, the airgap is 5mm, and the current required to overcome the spring force is produced by a current of 0.53A, whereas when the armature moves to give an airgap length of 2mm, the force required to allow spring force to open the relay only requires a current of 0.21A in the coil.

(ii) The advantage gained by this differing current level in the relay is the that once the relay is operated, current flows in the coil sufficient to overcome the force of the spring and the armature starts to move. The movement closes the airgap and therefore lowers the current requirement to overcome the4 spring force. This gives certainty to the movement of the armature, as once the relay starts to close the movement will continue until fully closed. The current in the coil will have to fall significantly before the relay is allowed to open. Similarly, once the relay starts to open, the movement will continue. This leads to a system which does not contain any ambiguous states.

Q4 a).



$A_V = \infty$, $V^+ = V_{ref}$, $V^- = \dfrac{R_{U2} + R}{R_{U1} + R_{U2} + R} \times V_{out}$.

$\therefore \quad V_{out} = \dfrac{R_{U1} + R_{U2} + R}{R_{U2} + R} V_{ref}$.

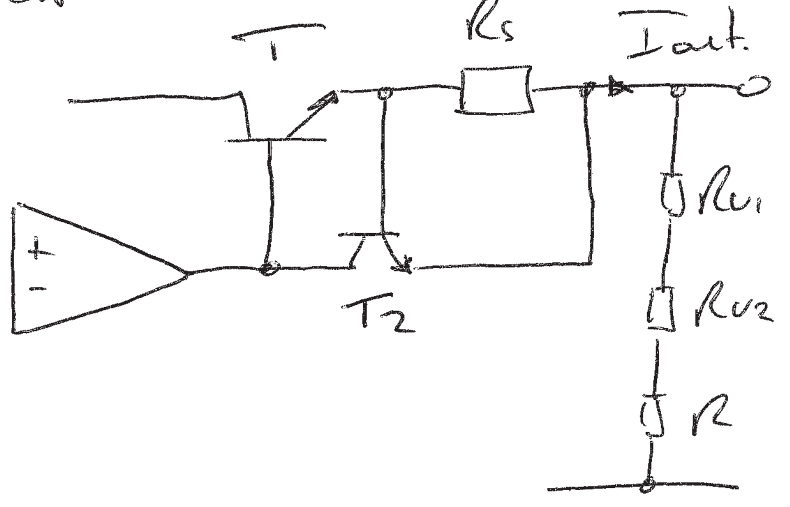
if x is the wiper position.

$R_{U1} = R_U x$, $R_{U2} = R_U(1-x)$.

$x = 0 \quad V_{out} = V_{out\,min} = \underline{\underline{1{\cdot}25V}}$

$x = 1 \quad V_{out} = V_{out\,max} = \underline{\underline{10{\cdot}21V}}$

b) current limit.



I_{out} causes a voltage $R_S I_{out}$ to be developed between base-emitter terminals of T_2. When $R_S I_{out}$ exceeds 0·7V T_2 turns on reducing the drive to T reducing the current

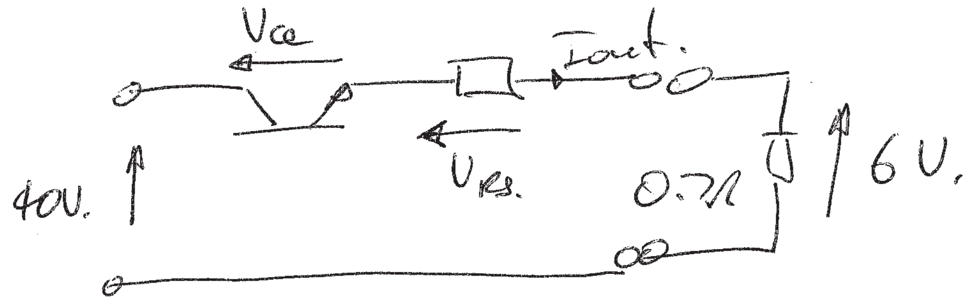
Q4 b) cont.

$I_{o\,lim} R_s = 0.7$

$$R_s = \frac{0.7}{I_{o\,lim}} = 0.07\Omega.$$

c) Efficiency calculation.

$V_{in} = 40V$, $V_{out} = 6V$, $R_L = 0.7\Omega$.



$$I_{out} = \frac{V_{out}}{R_L} = 8.57A.$$

$V_{Rs} = I_{out} R_s = 8.57 \times 0.07 = 0.6V.$

$\therefore V_{ce} = V_{in} - V_{Rs} - V_{out} = 40 - 0.6 - 6 = 33.4V.$

Power in transistor

$P_T = V_{ce} \times I_{out} = 33.4 \times 8.57 = 286.24W.$

Power in resistor

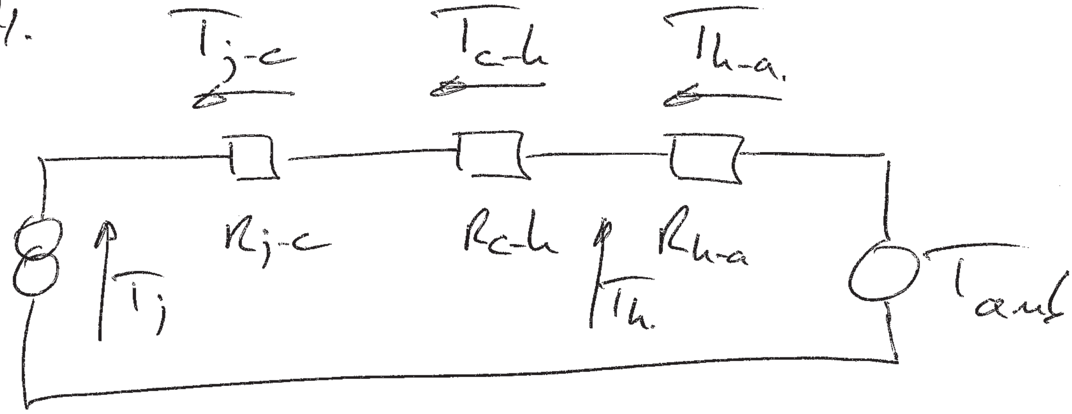
$P_{Rs} = I_{out}{}^2 R_s = 8.57^2 \times 0.07 = 5.14W.$

Power in load / output power

$$P_{RL} = \frac{V_{out}{}^2}{R_L} = \frac{6^2}{0.7} = 51.43W.$$

$$\text{Efficiency } \eta = \frac{P_{out}}{P_{in}} = \frac{P_{RL}}{P_{RL} + P_T + P_{Rs}} \times 100\% = 15\%.$$

Q4d).



$P_T = V_T I_{out} = (40 - 0.7 - 6) \times 10 = 333W.$

$T_{j-c} = 333 \times 0.1 = 33.3°C$

$T_{c-h} = 333 \times 0.08 = 26.6°C$

Need to ensure $T_h < 40°C$ and $T_j < 100°C$

Two options: i) start with T_j

ii) start with T_h.

T_h: $T_{h-a} = T_h - T_{amb} = 40 - 25 = 15°C$

check T_j

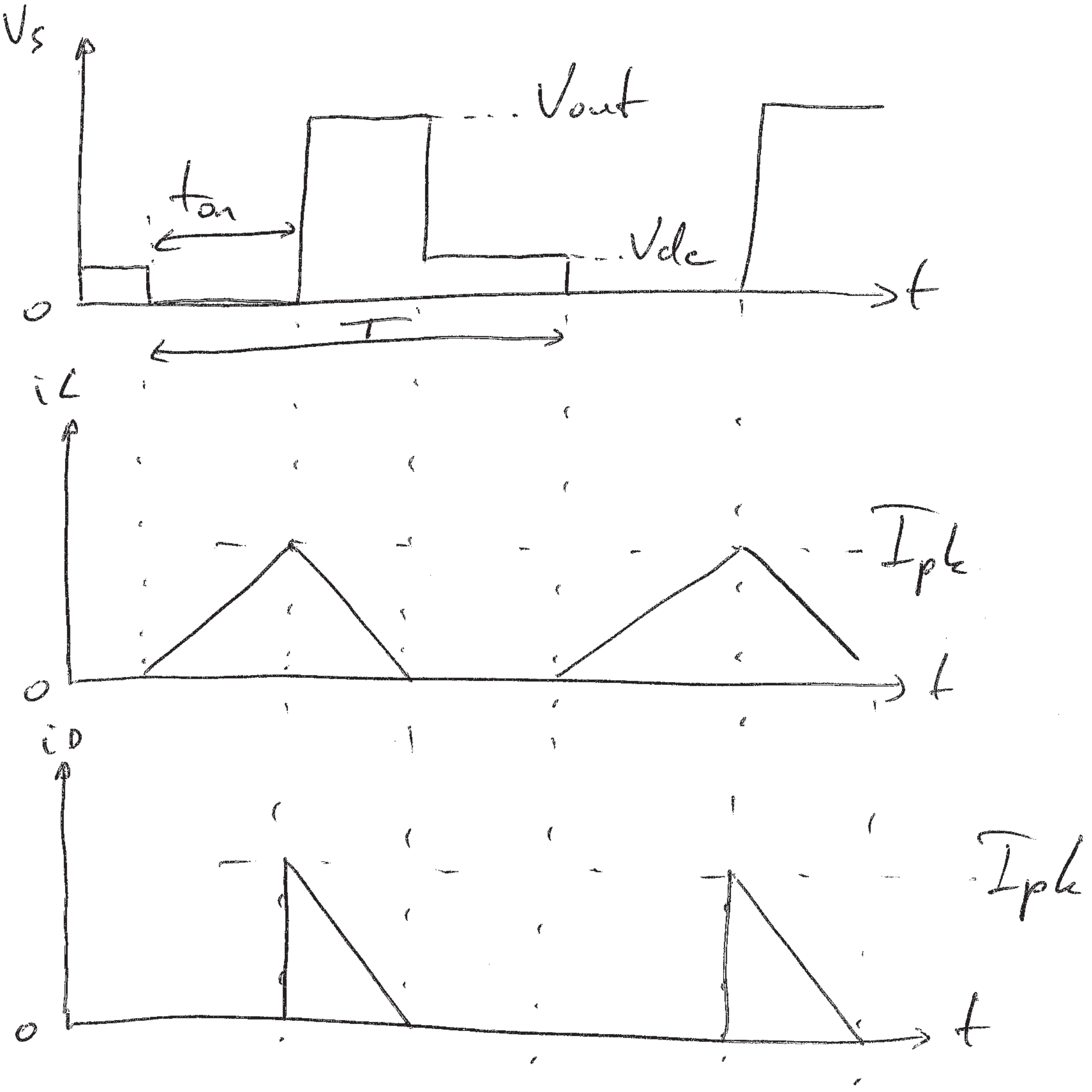
$T_j = T_{amb} + T_{ha} + T_{ch} + T_{j-c}$

$= 25 + 15 + 26.6 + 33.3 = 99.9°C$

Just works!

$R_{h-a} = \frac{T_{h-a}}{P_T} = \frac{15}{333} = \underline{\underline{0.045\ °C/W.}}$

5 a)



b) Expression for Vout.

Peak energy stored inductor is given by

$$E_L = \frac{1}{2} L I_{pk}^2 \quad , \quad I_{pk} = \frac{V_{dc}}{L} t_{on}.$$

and is consumed by the load every cycle

$$E_{RL} = P_{out} \cdot T = \frac{V_{out}^2}{R_L} T.$$

$E_L = E_{RL}$ so,

$$V_{out} = \frac{1}{\sqrt{2}} \sqrt{\frac{R_L}{TL}} \, V_{dc} \, t_{on}$$

5c) $t_{on} = 5\mu s$ $\quad T = \frac{1}{f} = \frac{1}{125,000}$ $\quad V_{out} = 200V$

$V_{dc} = 5V.$

$$V_{out} = \frac{1}{\sqrt{2}} \sqrt{\frac{R_L}{TL}}\, V_{dc}\, t_{on}$$

$$L = \frac{R_L}{2T} \frac{V_{dc}^2 t_{on}^2}{V_{out}^2} > \underline{\underline{27\mu H.}}$$

d). $I_{pk} = \frac{V_{dc}}{L} t_{on} = \underline{\underline{2.22A.}}$

$E_L = \frac{1}{2} L I_{pk}^2 = \underline{\underline{66.67\mu J.}}$

e) Vout at boundary of continuous current flow

During t_{on}

$V_{dc} = \frac{L \Delta I}{t_{on}}$

During t_{off} $\quad t_{off} = T - t_{on}$

$V_{dc} - V_{out} = \frac{L \times -\Delta I}{T - t_{on}}.$

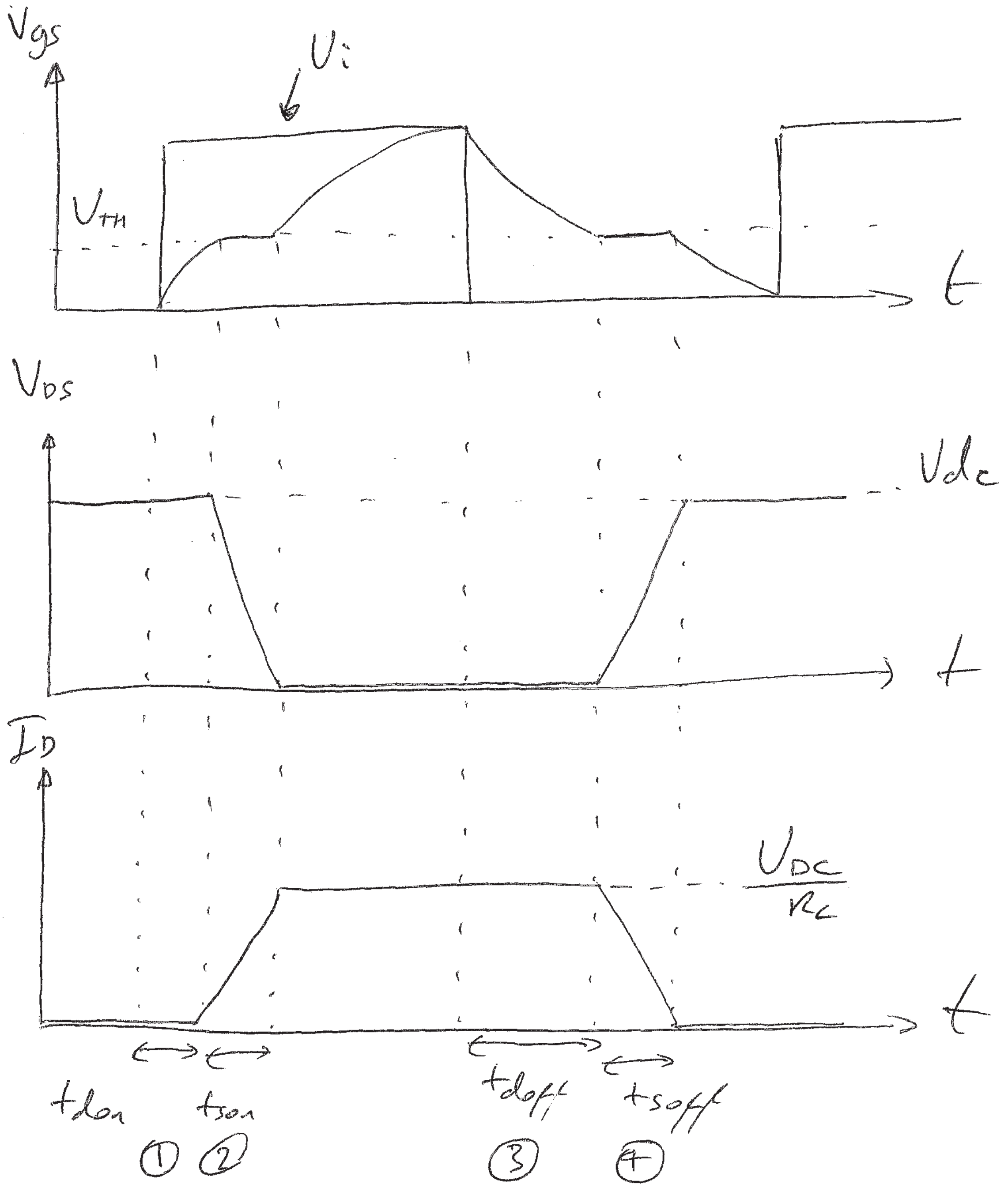
$\Delta I = \frac{V_{dc} t_{on}}{L}$

$-\Delta I = \frac{(V_{dc} - V_{out})(T - t_{on})}{L}.$

$-V_{dc} t_{on} = (V_{dc} - V_{out})(T - t_{on})$

$V_{out} = V_{dc} \frac{T}{T - t_{on}} = \underline{\underline{17.85V.}}$

6 a)



① V_i is applied and V_{gs} charges exponentially aiming for V_i

② $V_{gs} = V_{TH}$ MOSFET SWITCHES ON. V_{DS} falls and I_D rises linearly.

③ V_i is set to 0V so V_{gs} discharges exponentially aiming for 0V.

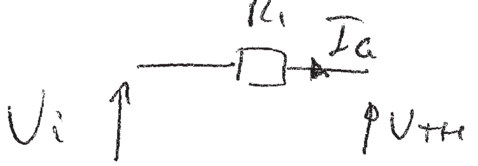
④ $V_{gs} = V_{TH}$ MOSFET switches off. V_{DS} rises and I_D falls linearly

6.b) Turn on delay

$$V_{gs}(t) = V_i(1 - e^{-\frac{t}{\tau}})$$ $\tau = (C_{DG} + C_{GS})R_1$, $t = t_{don}$, $V_{gs} = V$

$$t_{don} = -\tau \ln\left[1 - \frac{V_{TH}}{V_i}\right] = \underline{\underline{626.6\,ns}}$$

Turn on switching time.



$$I_G = \frac{V_i - V_{TH}}{R_1} = 4.7\,mA.$$

$$I_G = C_{DG}\frac{dV}{dt} = C_{DG}\frac{V_{DD}}{t_{son}} = \frac{C_{DG}V_{DC}}{t_{son}}$$

$$t_{son} = \frac{C_{DG}V_{DD}}{I_G} = \underline{\underline{1.35\,\mu s}}$$

c) To ensure one phase is fully turned off before the other phase is turned on.

$t_{off} < t_{don}$ where t_{off} is the total time turn-off inc. delay and switching

$t_{doff} + t_{soff} < t_{don}$.

t_{doff} : $V_{TH} = V_i e^{-\frac{t_{doff}}{\tau_{off}}}$ $\tau_{off} = R_{off}(C_{DG} + C_{GS})$

$R_{off} = R_1 || R_2$

$$t_{doff} = -\tau_{off} \ln\left(\frac{V_{TH}}{V_i}\right)$$

t_{soff} : $t_{soff} = \frac{C_{DG}V_{dc}}{V_i - V_{TH}} R_{off}$.

6 c) cont.

$t_{doff} + t_{soff} < t_{don}$.

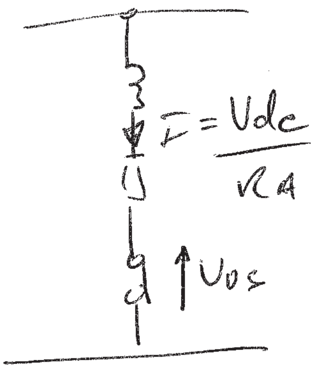
$$R_{off}\left[\frac{C_{DG} V_{dc}}{V_i - V_{TH}} - (C_{DG} + C_{GS}) \ln\left(\frac{V_{TH}}{V_i}\right)\right] < 626.6\,ns$$

$R_{off} < 396\,\Omega$

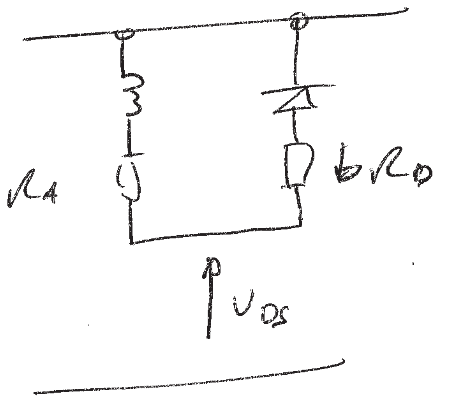
$R_{off} = R_1 \| R_2$

$$R_2 = \frac{R_{off} R_1}{R_1 - R_{off}} < \underline{\underline{539\,\Omega}}$$

d) when T_1 is on



when T_1 is off



Maximum switch/inductor current is $I = \frac{V_{dc}}{R_A} = 1.2A$.

which produces a voltage drop IR_D across R_D.

$V_{DSLIM} = V_{DC} + IR_D$.

$$\therefore \cancel{R_D = \frac{V_{DSLIM} - V_{DC}}{R_D}}$$

$$R_D = \frac{V_{DSLIM} - V_{DC}}{I} = \underline{\underline{20\,\Omega}}$$