Problem Sheet 5 : Solutions

1) $E_g = 0.67\ eV\ (Ge)$, $1.4\ eV\ (GaAs)$

$= e \times 0.67\ J$, $e \times 1.4\ J$

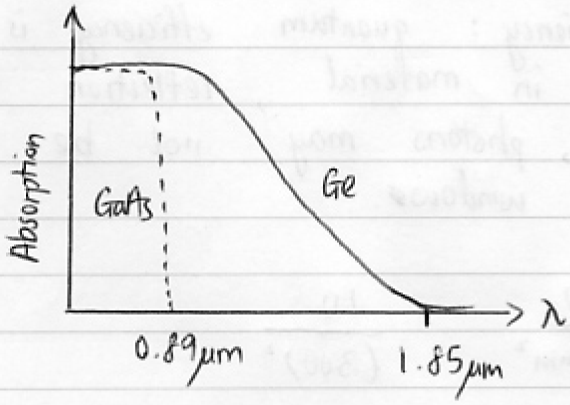
$\therefore \quad \lambda_g = \dfrac{\hbar 2\pi c}{(eE_g)} = \dfrac{hc}{(eE_g)}$ where c = speed of light

$\therefore \quad \lambda_g = \dfrac{1.24}{E_g}$

λ_g in μm. E_g in eV

$\therefore \quad \lambda_g = 1.85\ \mu m\ (Ge)$

$= 0.89 \mu m\ (GaAs)$



2) Lasers have reflective mirrors - usually cleavage plane at each end of the diode "chip". Laser produces large output power only above a threshold current.

LED spectrum is continuous.

$$10\text{ mA} = 10\text{ mC/s}$$

$$= \frac{10 \times 10^{-3}}{e} \quad \text{electrons/s}$$

$$\equiv \frac{10 \times 10^{-3}}{e} E_g \quad \text{J/s produced in photon output.}$$

$$\equiv 14\text{ mW produced internally.}$$

$$\equiv 0.014\text{ mW observed.}$$

$$\therefore \quad \underline{\text{Radiance} = 0.014\text{ mW/mm}^2}$$

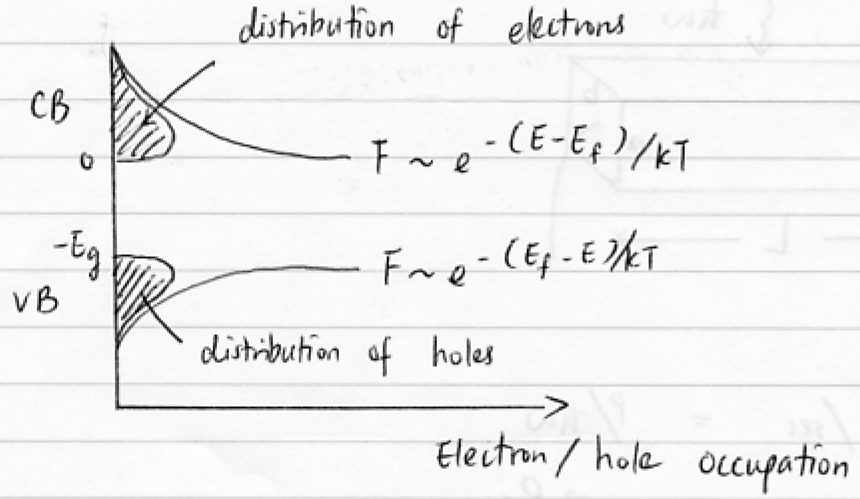
Reasons for inefficiency: quantum efficiency is $\neq 1$, photon absorption in material, reflection from top face of LED, photons may not be directed towards output window.

$$\text{Sunlight} = \frac{10\text{ mW}}{(300)^2\text{ mm}^2} = \frac{10}{(300)^2}$$

$$= \underline{0.11\text{ mW/mm}^2}$$

$$\text{Moonlight} = \frac{5 \times 10^{-5}}{(300)^2} = \underline{0.56\text{ nW/mm}^2}$$

3)

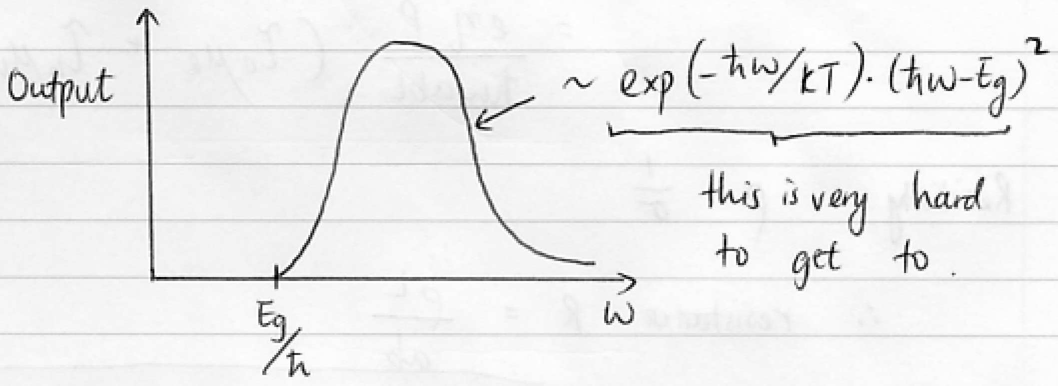


- Transitions allowed from electrons <u>above</u> bottom of CB to holes <u>below</u> top of VB.
  ∴ Photons of greater energy than E_g will be emitted.

Since average electron energy is $\frac{3}{2}kT$ above bottom of CB, and holes have average energy $\frac{3}{2}kT$ below VB edge, average photon energy $= E_g + 3kT$

$$\therefore \text{ average } \omega = \frac{E_g}{\hbar} + \frac{3kT}{\hbar}$$

- Can show in fact that spectrum should be



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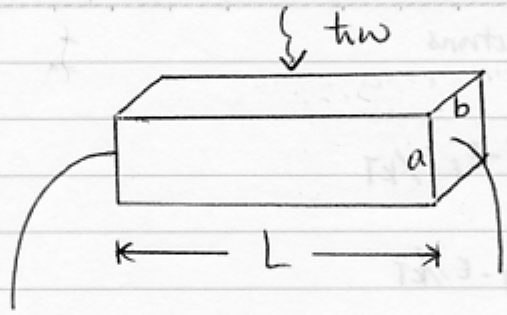
4)



No. of photons / sec $= P/\hbar\omega$

No. of e-h pairs / sec $= \eta P/\hbar\omega$

equilibrium density of e^- per unit volume

$$= n = \frac{\eta P}{\hbar\omega} \times \tau_e \times \frac{1}{abL}$$

and no. of holes / unit volume

$$= p = \frac{\eta P}{\hbar\omega} \times \tau_h \times \frac{1}{abL}$$

Conductivity $\therefore \ \sigma = e(\mu_e n + \mu_h p)$

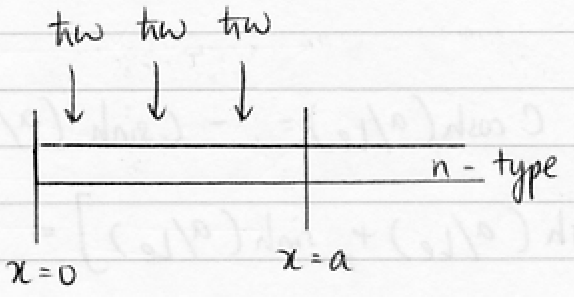
$$= \frac{e\eta P}{\hbar\omega abL}(\tau_e\mu_e + \tau_h\mu_h)$$

Resistivity $\rho = \frac{1}{\sigma}$

$\therefore$ resistance $R = \frac{\rho L}{ab}$

$$\boxed{= \frac{\hbar\omega L^2}{P\eta e(\tau_e\mu_e + \tau_h\mu_h)}}$$

5)



1 - dimensional problem so we must solve 1-D diffusion equation.

$$\frac{d^2(\Delta n)}{dx^2} = \frac{(\Delta n)}{L_e^2} \qquad (x > a) \quad — ①$$

and

$$\frac{d^2(\Delta n)}{dx^2} = \frac{(\Delta n)}{L_e^2} - \frac{g\tau}{L_e^2} \qquad (0 < x < a) \quad — ②$$

- Solution of ① is $\Delta n = A\exp(-x/L_e) + B\exp(x/L_e)$ but $B = 0$ because of boundary condition as $x \to \infty$.

- Solution of ② is $\Delta n = g\tau + C\cosh(x/L_e) + D\sinh(x/L_e)$. The condition $d(\Delta n)/dx = 0$ at $x = 0$ (no carriers injected from $x < 0$) requires $D = 0$.

Matching Δn at $x = a$ requires

$$A\exp(-a/L_e) = g\tau + C\cosh(a/L_e)$$

and matching $d(\Delta n)/dx$ at $x = a$ requires

$$-\frac{A}{\cancel{L_e}}\exp(-a/L_e) = \frac{C}{\cancel{L_e}}\sinh(a/L_e)$$

5) (Cont.)

So. $$g\tau + C\cosh(a/L_e) = -C\sinh(a/L_e)$$

$$\Rightarrow C\left[\cosh(a/L_e) + \sinh(a/L_e)\right] = -g\tau$$

$$\Rightarrow C\left(\exp(a/L_e)\right) = -g\tau$$

$$\Rightarrow C = -g\tau\exp(-a/L_e)$$

Hence, $$A = \frac{g\tau + -g\tau\exp(-a/L_e)\cosh(a/L_e)}{\exp(-a/L_e)}$$

$$= g\tau\left[\exp(a/L_e) - \cosh(a/L_e)\right]$$

$$= g\tau\left[\exp(a/L_e) - \tfrac{1}{2}\exp(a/L_e) - \tfrac{1}{2}\exp(-a/L_e)\right]$$

$$= g\tau\left[\sinh(a/L_e)\right]$$ *

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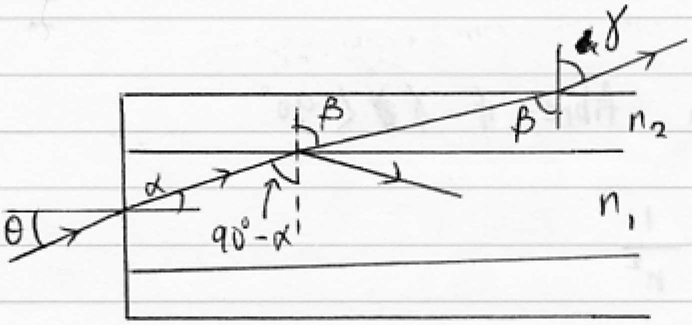

$$\frac{\sin\theta}{\sin\alpha} = n_1$$

$$\frac{\sin(90-\alpha)}{\sin\beta} = \frac{n_2}{n_1}$$

Critical ray if $\beta = 90°$

$\therefore \sin\beta = 1$.

- Thus all energy guided inside core.

$\therefore \cos\alpha = n_2/n_1$

$$\Rightarrow \sin\alpha = \sqrt{1-\cos^2\alpha}$$

$$= \sqrt{1 - n_2^2/n_1^2}$$

$$\boxed{\sin\theta \leq \sqrt{n_1^2 - n_2^2}}$$

[Energy guided in core. If $n_2 > n_1$, some energy propagates in outer surrounding cylinder of glass)]

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6) (Cont.)

Energy lost from fibre if $\gamma < 90^\circ$

But $\dfrac{\sin\beta}{\sin\gamma} = \dfrac{1}{n^2}$

$$\therefore \quad \sin\beta < \frac{1}{n_2}$$

$$\therefore \quad \sin(90-\alpha)\, n_2 < n_2/n_1$$

$$\therefore \quad \sqrt{1-\sin^2\alpha} = \cos\alpha < \frac{1}{n_1}$$

$$\therefore \quad \frac{1}{n_1^2} > 1 - \sin^2\alpha$$

$$\therefore \quad \frac{\sin\theta}{n_1} = \sin\alpha > \sqrt{1 - 1/n_1^2}$$

$$\therefore \quad \boxed{\sin\theta > \sqrt{n_1^2 - 1}}$$ Energy lost from fibre.