Q1 (a)

a) $\frac{\pi}{180^\circ} \times 150^\circ = \frac{5\pi}{6}$ or 2.62 radians (1)

b) $-135^\circ \times \frac{\pi}{180^\circ} = -2.36$ or $-3/4\,\pi$ radians (1)

c) $180^\circ/3 = 60^\circ$ (1)

d) $180^\circ \times 3/2 = -270^\circ$ (1)

e) $\frac{360^\circ}{2\pi} \times 2 = 114.6^\circ$ (1)

(b) $i(t) = -20\cos(120\pi t + \pi/2)$ A

a) 20 A (1)

b) 40 A (1)

c) $\omega = 120\pi = 2\pi f$

$\therefore f = \frac{120\pi}{2\pi} = 60$ Hz (1)

d) $t = \frac{1}{f} = \frac{1}{60} = 16.7$ ms (1)

e) $\pi/2$ or 90° (1)

2.1

Q2 a) $2x + 3y = 6$ — ①

$-2x + 3z = 0$ — ②

$x + 2y + 3z = -1$ — ③

Subtract ② from ③

$x + 2y + \cancel{3z} - -2x - \cancel{3z} = -1$

$3x + 2y = -1$

$x = -\frac{2}{3}y - \frac{1}{3}$ — ④ 1

sub value for x back in ①

$2\left(-\frac{2}{3}y - \frac{1}{3}\right) + 3y = 6$

$-\frac{4}{3}y - \frac{2}{3} + 3y = 6$

$-\frac{4}{3}y + 3y = 6 + \frac{2}{3}$

$y\left(\frac{9-4}{3}\right) = \frac{18+2}{3}$

$\frac{5}{\cancel{3}}y = \frac{20}{\cancel{3}}$

$5y = 20$

$\therefore y = \frac{20}{5} = \underline{\underline{4}}$ 1

Sub value for y back into ①

$2x + 3(4) = 6$

$2x + 12 = 6$

$2x = 6 - 12$

$$x = \frac{-6}{2} = \underline{\underline{-3}}$$

sub value for x back into (2)

$$-2(-3) + 3z = 0$$

$$6 + 3z = 0$$

$$3z = -6$$

$$z = \frac{-6}{3} = \underline{\underline{-2}}$$

(1) [illegible]

$$\underline{\underline{x = -3 \quad y = 4 \quad z = -2}}$$

Check by substituting back into eqns (1)-(3)

eqn (1) is $2(-3) + 3(4) = 6$

$$-6 + 12 = 6$$

$$6 = 6 \checkmark$$

eqn. (2) is $-2(-3) + 3(-2) = 0$

$$+6 - 6 = 0 \checkmark$$

eqn (3) is $-3 + 2(4) + 3(-2) = -1$

$$-3 + 8 - 6 = -1$$

$$-1 = -1 \checkmark$$

$\boxed{2 \cdot 3}$

b) $$-4v + I_1(2+3+1) + I_3 = 0$$

$$-4v + I_2(2+3) - I_3 3 = 0$$

$$-3v + I_3(1+1+3) + I_1 - I_2 3 = 0$$

$$6I_1 + I_3 = 4 \quad -① \qquad 1$$

$$5I_2 - 3I_3 = 4 \quad -② \qquad 1$$

$$I_1 - 3I_2 + 5I_3 = 3 \quad -③ \qquad 1$$

$$6I_1 \quad + I_3 = 4$$

$$5I_2 - 3I_3 = 4$$

$$I_1 - 3I_2 + 5I_3 = 3$$

form determinant of left hand side

$$\Delta = \begin{vmatrix} 6 & 0 & 1 \\ 0 & 5 & -3 \\ 1 & -3 & 5 \end{vmatrix}$$

find value use top row

$$6(5\times5 - (-3\times-3)) - 0 + (0-5)$$

$$\Delta = 6(25-9) - 5$$

$$= 6\times16 - 5$$

$$= 96 - 5$$

$$\Delta = \underline{\underline{91}} \qquad 1$$

now form determinant for I_1 by replacing left hand colm

$$\Delta I_1 = \begin{vmatrix} 4 & 0 & 1 \\ 4 & 5 & -3 \\ 3 & -3 & 5 \end{vmatrix}$$ evaluate using top row

$$\Delta_{I_1} = 4\left((5\times5)-(-3\times-3)\right) - 0 + \left((4\times-3)-(5\times3)\right)$$
$$= 4(25-9) + (-12-15)$$
$$= 64 - 27$$
$$\Delta_{I_1} = \underline{\underline{37}}$$ (1)

$$\therefore \quad I_1 = \frac{\Delta I_1}{\Delta} = \frac{37}{91} = 0.407\text{A} \text{ or } +\underline{\underline{407\text{mA}}}$$ (1)

$$\left(\frac{37}{91}\text{A}\right)$$

Similarly for I_2

$$\Delta I_2 = \begin{vmatrix} 6 & 4 & 1 \\ 0 & 4 & -3 \\ 1 & 3 & 5 \end{vmatrix}$$ evaluate by left column

$$\Delta_{I_2} = +6\left((4\times5)-(-3\times3)\right) - 0 + \left((4\times-3)-(4)\right)$$
$$= 6(20+9) + (-12-4)$$
$$= 6\times29 - 16$$
$$= 174 - 16$$
$$\Delta_{I_2} = \underline{\underline{158}}$$ (1)

$$\therefore \quad I_2 = \frac{\Delta I_2}{\Delta} = \frac{158}{91} = +\underline{\underline{1.74\text{A}}}$$ (1)

$$\Delta I_3 = \begin{vmatrix} 6 & 0 & 4 \\ 0 & 5 & 4 \\ 1 & -3 & 3 \end{vmatrix}$$ evaluate by ~~left column~~ Top row

$\boxed{2\cdot5}$

$$\Delta_{I_3} = 6((5\times3)-(4\times-3)) - 0 + 4(0-5)$$
$$= 6(15+12) - 20$$
$$= 6\times27 - 20$$
$$= 162 - 20$$
$$\Delta_{I_3} = \underline{\underline{142}}$$ (1)

$$\therefore I_3 = \frac{\Delta_{I_3}}{\Delta} = \frac{142}{91}\,A \sim \underline{\underline{+1.56A}}$$ (1)

So $I_1 = 407\,mA$, $I_2 = 1.74\,A$ & $I_3 = 1.56\,A$.

Check in equations. (1) if checked

$$6(0.407) + 1.56 = \text{(1)}$$
$$2.442 + 1.56 = \underline{\underline{4.002}} \checkmark \simeq 4$$

eqn (2) $5I_2 - 3I_3 = 4$

$$5(1.74) - 3(1.56) = 8.7 - 4.68 = \underline{\underline{4.02}} \checkmark \simeq 4$$

eqn (3) $I_1 - 3I_2 + 5I_3 = 3$

$$0.407 - 3(1.74) + 5(1.56) =$$
$$0.407 - 5.22 + 7.8 = \underline{\underline{2.987}} \simeq 3 \checkmark$$

Q3/ a)(i) $5\cos\omega t - 3\sin\omega t = -3\sin\omega t + 5\cos\omega t$

$$R\sin(\omega t+\alpha) = R\left[\sin\omega t.\cos\alpha + \cos\omega t.\sin\alpha\right]$$

$$= R\cos\alpha\sin\omega t + R\sin\alpha.\cos\omega t$$

$$\therefore\ R\cos\alpha = -3 \quad \text{and} \quad R\sin\alpha = 5$$

$$R = \sqrt{(-3)^2+5^2} = \sqrt{9+25} = \sqrt{34} = \underline{5.83} \quad (1)$$

$$\alpha = \tan^{-1}\left(\frac{R\sin\alpha}{R\cos\alpha}\right) = \tan^{-1}\frac{5}{-3} = -1.03 \text{ radians } (\text{or } -59°)$$

S	A
T	C

Second quadrant has only sine +ve so angle is $\pi - 1.03 = \underline{2.11 \text{ radians}}$ (or $121°$) (2)

check $R\cos\alpha = 5.83\cos(2.11) = -2.99 \simeq -3$ ✓
$R\sin\alpha = 5.83\sin(2.11) = +5$ ✓

So $5\cos\omega t - 3\sin\omega t = \underline{5.83\sin(\omega t + 2.11)}$

(ii) $-4.2\sin\omega t - 3.1\cos\omega t = R\cos(\omega t+\alpha)$

$$R\cos(\omega t+\alpha) = R\left[\cos\omega t.\cos\alpha - \sin\omega t.\sin\alpha\right]$$

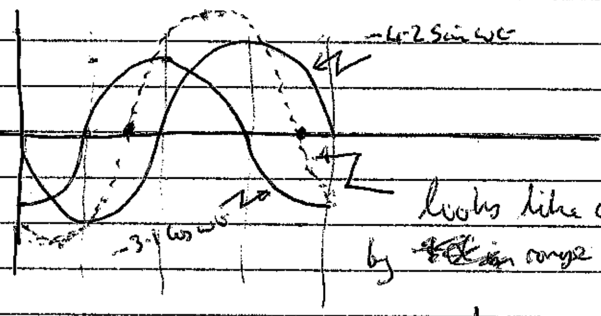
$$= R\cos\alpha.\cos\omega t - R\sin\alpha.\sin\omega t$$

$$R\cos\alpha = -3.1 \ \& \ R\sin\alpha = -(-4.2) = +4.2$$

$$R = \sqrt{3.1^2+4.2^2} = \underline{5.22} \quad \left(\text{or } \frac{\sqrt{109}}{2}\right) \quad (1)$$

$$\alpha = \tan^{-1}\left(\frac{R\sin\alpha}{R\cos\alpha}\right) = \tan^{-1}\left(\frac{4.2}{-3.1}\right) = \underline{-0.93 \text{ radians}} \ (\text{or } -53.6°)$$

sketch wave fun



looks like cosine wave leading by ~~...~~ in range $\pi/2 \rightarrow \pi$

S	A
T	C

Sine is +ve & cos -ve so 2nd quadrant, agrees with ↑

$$\therefore\ \alpha = \pi - 0.93 = \underline{2.21 \text{ radians}} \ (\text{or } 126.4°) \quad (2)$$

Checking $R\cos\alpha = 5.22\cos(2.21) = -3.11 \simeq -3.1$ ✓✓

$$R\sin\alpha = 5.22\sin(2.21) = +4 \checkmark$$

$$\text{So } -4.2\sin\omega t - 3.1\cos\omega t = 5.22\cos(\omega t + 2.21)$$

b) $P = 5\sin(\omega t) \cdot 10\sin(\omega t - \pi/2)$

using $\sin A \sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$ 1

So $P = -\frac{50}{2}\left[\cos(\omega t + \omega t - \pi/2) - \cos(\omega t - \omega t + \pi/2)\right]$ 1

$$= -25\left[\cos(2\omega t - \pi/2) - \cancel{\cos(\pi/2)}\right]$$

$$= -25\cos(2\omega t - \pi/2)$$

(~~Cos(π/2)~~ A cosine shifted by $-\pi/2$ = sine)

$P = -25\sin(2\omega t)$ 2

(or shifted by a further π

$P = 25\sin(2\omega t + \pi)$)

Q4(a) (i) Voltage & current are in-phase for a resistor so

peak $V = IR =$

$$V = 17\times10^{-3} \times 1.2\times10^{3} = \underline{20.4\ V}$$

$$\therefore\ \underline{v(t) = 20.4 \sin(120\pi t - 5\pi/6)\ \text{Volts}}$$

2

(ii) $|V_c| = X_c|I_c|$ $\quad I_{pk} = |I_c| = 17\,mA$

$X_c = \frac{1}{\omega C}$ & ω here $= 120\pi$ so

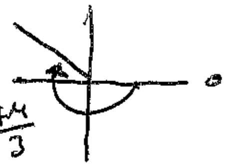
$$X_c = \frac{1}{120\pi \times 3\times10^{-6}} = \underline{884.2\ \Omega}$$

$$\therefore\ |V_c| = 884.2 \times 17\times10^{-3} = \underline{15\ \text{Volts}}$$

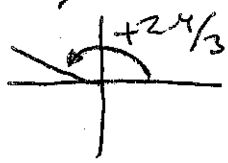
CIVIL $\quad$ V lags I in capacitor by $\pi/2$

$$\therefore\ V_c(t) = 15 \sin(120\pi t - 5\pi/6 - \pi/2)\ \text{Volts}$$

$$= 15 \sin\left(120\pi t - \frac{4\pi}{3}\right)$$



$-\frac{4\pi}{3}$ can also be shown as



$+2\pi/3$

$$2\pi - 4\pi/3 = \frac{2\pi}{3}$$

So $\underline{V_c(t) = 15 \sin(120\pi t + 2\pi/3)\ \text{Volts}}$

2

by integration

$$v_c(t) = \frac{17\times10^{-3}}{C}\int i\,.\,dt$$

$$= \frac{17\times10^{-3}}{C}\int \left(\sin(120\pi t - 5\pi/6)\right).dt$$

not a standard integral so...

let $u = \cancel{\sin}(120\pi t - 5\pi/6)$

$$\frac{du}{dt} = 120\pi \quad \text{so} \quad dt = \frac{du}{120\pi}$$

$$\therefore \quad v(t) = \frac{17\times10^{-3}}{C}\int \sin(u)\cdot\frac{du}{120\pi}$$

$$= \frac{17\times10^{-3}}{120\pi C}\int \sin(u)\cdot du$$

$$= \frac{17\times10^{-3}}{120\pi C}\left[-\cos(u) + c\right]$$

$$= \frac{-17\times10^{-3}}{120\pi C}\cos(120\pi t - 5\pi/6)$$

put $C = 3\times10^{-6}$

$$= \frac{-17\times10^{-3}}{120\times\pi\times3\times10^{-6}}\cos(120\pi t - 5\pi/6)$$

$$= -15\cos(120\pi t - 5\pi/6)$$

(Using $-\cos(x) = \sin(x - \pi/2)$)

$$= +15\sin(120\pi t - 5\pi/6 - \pi/2)$$

$$= +15\sin(120\pi t + 2\pi/3) \text{ Volts}$$

(iii) $L = 4$ Henries

$$|V_L| = X_L|I_L| \qquad X_L = \omega L = 120\pi\times4$$

$$= 480\pi = \underline{1508\ \Omega}$$

$$\therefore \quad |V_L| = 1508\times17\times10^{-3}$$

$$= \underline{25.6\ V}$$

CIVIL voltage leads current in inductor $\pi/2$

$$\therefore \quad V(t) = 25.6\sin(120\pi t - 5\pi/6 + \pi/2)$$

2 | 4.3

$$V(t) = 25.6 \sin\left(120\pi t - \pi/3\right) \text{ Volts}$$

or by differentiation

$$v(t) = L\frac{di(t)}{dt} \qquad i(t) = 17\times10^{-3}\sin\left(120\pi t - 5\pi/6\right)$$

let $u = 120\pi t - 5\pi/6$

so $i(t) = 17\times10^{-3}\sin(u)$

$$\frac{du}{dt} = 120\pi$$

$$\frac{di}{du} = 17\times10^{-3}\cos(u) = 17\times10^{-3}\cos\left(120\pi t - 5\pi/6\right)$$

$$\therefore \frac{di}{du}\times\frac{du}{dt} = \frac{di}{dt} = 120\times\pi\times17\times10^{-3}\cos\left(120\pi t - 5\pi/6\right)$$

$$\therefore v(t) = L\times\frac{di(t)}{dt}$$

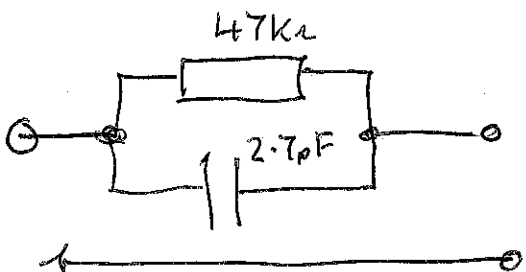
$$= 4\times120\times\pi\times17\times10^{-3}\cos\left(120\pi t - 5\pi/6\right)$$

$$= 25.6\cos\left(120\pi t - 5\pi/6\right)$$

$$= 25.6\sin\left(120\pi t - 5\pi/6 + \pi/2\right)$$

$$= 25.6\sin\left(120\pi t - \pi/3\right) \text{ Volts}$$

(b)



$V = 5\sin(2\pi\, 9\times10^{5}\, t)$

$(= 5,654,867)$

$f = 900\text{ kHz}$

$\therefore \omega = 2\pi f = 2\times\pi\times900\times10^{3}$

4.4

(i) $|I_R| = \left|\frac{V}{R}\right| = \frac{5}{47K\Omega} = \frac{5}{4.7\times10^4} = \underline{106\,\mu A}$

$\therefore\ \cancel{I_R}\ i(t) = \underline{106\ \text{Sin}(2\pi\, 9\times10^5\, t)}\ \mu A$ in phase with V (2)

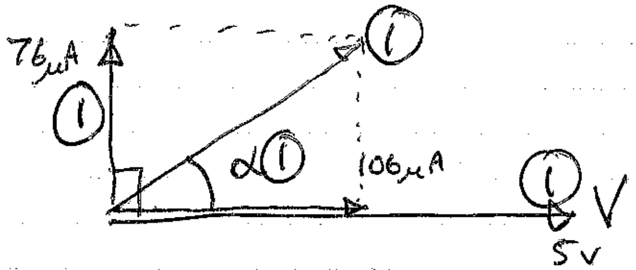
(ii) $|I_R| = \left|\frac{V}{X_C}\right| \qquad X_C = \frac{1}{2\pi f C}$

$\therefore\ |I_R| = \cancel{5}\,5 \times 2\times\pi\times 9\times10^5 \times 2.7\times10^{-12}$

$= \underline{76\,\mu A}$ CIVIL current leads voltage by $\pi/2$

$\therefore\ i_{(t)} = \underline{76\ \text{Sin}(2\pi\, 9\times10^5\, t + \pi/2)}\ \mu A$ or 90° (2)

(iii)



$\Rightarrow$ (4)

(iv) $I_{supply} = \sqrt{I_R^2 + I_C^2} = \sqrt{106^2 + 76^2}$

$= \underline{130\,\mu A}$ (2)

$\alpha = \tan^{-1}\left(\frac{76\mu A}{106\mu A}\right) = +35.6^\circ$ or $+0.62$ radians

$\therefore\ \underline{i(t)_{supply} = 130\ \text{Sin}(2\pi\, 9\times10^5\, t + 35.6^\circ)}$ (2)

Q5

(a)

$$v = E e^{-t/\tau}$$

$$\frac{v}{E} = e^{-t/\tau}$$

$$\ln\left(\frac{v}{E}\right) = \frac{-t}{\tau}$$

$$\therefore \quad t = -\tau \ln\left(\frac{V}{E}\right)$$

2

(b)

$$\cancel{v = 12 e^{-t/\tau}} \quad t = -5\times10^{-3} \ln\left(\frac{1.6}{12}\right)$$

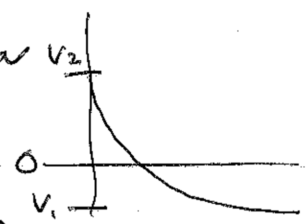
$$= -5\times10^{-3} \times -2.015$$

$$t = 0.01\,s \text{ or } 10\,mS$$

2

(c)

$$\cancel{v = (10v - -5v)\, e^{-t/\tau} - 5v}$$



2

$$v = (V_1 - V_2)\, e^{-t/\tau} + V_2$$

$$v = (10 - -5)\, e^{-25/20} - 5$$

$$v = (10+5)\, e^{-5/4} - 5$$

$$v = 15\, e^{-5/4} - 5$$

$$= 4.3 - 5$$

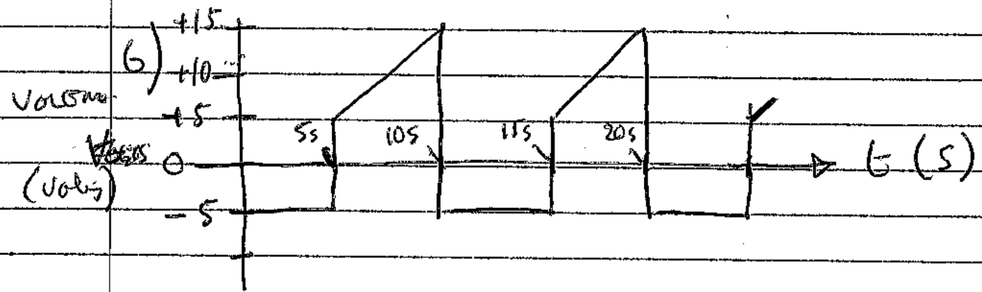
$$= -0.7v$$

2

4

Q6 6.1

a) Mean $V = \frac{1}{T}\int_0^T V(t) \cdot dt$ 2

b) 

Voltage function 4

$\underset{0 \to 5s}{V(t)} = -5 \text{ Volts}$ $\underset{5 \to 10s}{V(t)} = (2t - 5) \text{ Volts}$

$$\text{Mean } V = \frac{1}{T}\left(\int_0^5 -5 \cdot dt + \int_5^{10} (2t-5) \cdot dt\right)$$

$$= \frac{1}{T}\left(\Big[-5t + c\Big]_0^5 + \Big[\frac{2t^2}{2} - 5t + c\Big]_5^{10}\right)$$

$$= \frac{1}{T}\left((-5\times5 - -0) + (10^2 - 50 - 5^2 + 25)\right)$$

$$= \frac{1}{T}\left(-25 + 100 - 50 - \cancel{25} + \cancel{25}\right)$$

$$= \frac{1}{T}(25)$$

$$= \frac{1}{10} \cdot 25 = \underline{\underline{2 \cdot 5 V}}$$ 6

c) $\sqrt{\frac{1}{T}\int_0^T (V(t))^2 \cdot dt}$

$\underset{0 \to 5ms}{V(t)^2} = (-5)^2$ $\underset{5 \to 10ms}{V(t)^2} = (2t-5)^2 = \cancel{4t^2 - 20t + 25}$

$= +25 \text{ Volts}^2$ $= (4t^2 - 20t + 25) \text{ Volts}^2$

Mean Squared value

$$\frac{1}{T}\left(\int_0^5 25 \cdot dt + \int_5^{10} (4t^2 - 20t + 25) \cdot dt\right)$$

$$= \frac{1}{T}\left(\left[25t + c\right]_0^5 + \left[\frac{4t^3}{3} - \frac{\cancel{20}t^2}{\cancel{2}} + 25t + c\right]_5^{10}\right)$$

$$= \frac{1}{T}\left(\left(25\times 5 - 0\right) + \left(\left(\frac{4\times 10^3}{3} - 10\times 10^2 + 25\times 10\right) - \left(\frac{4\times 5^3}{3} - 10\times 5^2 + 25\times 5\right)\right)\right)$$

$$= \frac{1}{T}\left(\cancel{125} + \frac{4}{3}\times 10^3 - 10^3 + 250 - \frac{4}{3}\times 125 + 250 - \cancel{125}\right)$$

$$= \frac{1}{T}\left(\frac{4000}{3} - 1000 + 500 - \frac{500}{3}\right)$$

$$= \frac{1}{T}\left(\frac{3500}{3} - 500\right) = \frac{1}{T}\left(\frac{3500 - 1500}{3}\right)$$

$$= \frac{1}{T}\left(\frac{2000}{3}\right)$$

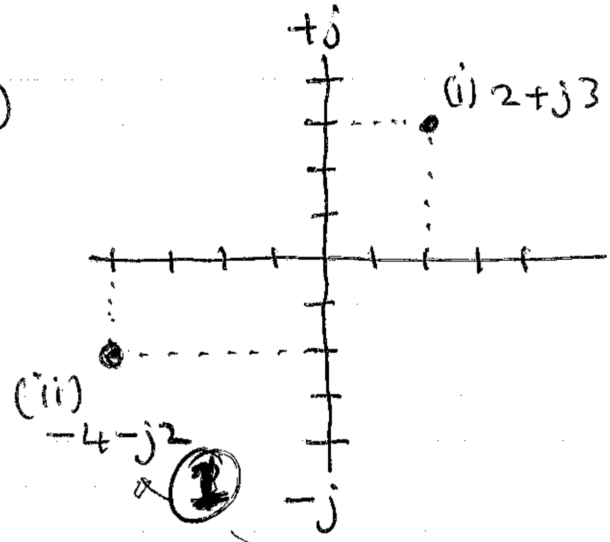
$$= \frac{1}{10}\cdot\frac{2000}{3} = \frac{200\cancel{0}}{3\cancel{0}} = 66.67$$

$$RMS = \sqrt{\text{mean square}} = \sqrt{66.67} = 8.16\ V \text{ or } \frac{10\sqrt{6}}{3}\ V$$

8

Q7

(a)



(1) (1)

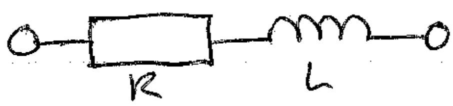
ii) $j(-2+j4) = -2j + j.j.4 = \underline{\underline{-4-j2}}$

(b) ~~Convert the following~~

(i) $-3+j6 = \underline{\underline{6.7\angle 117^\circ}}$ (or $6.7\angle 2.04$ radians) $(3\sqrt{5})$ (1)

(ii) $5\angle -127^\circ = \underline{\underline{-3-j4}}$ (1)

(c) (i)

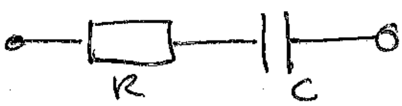


$\underline{\underline{R = 70\,\Omega}}$ $X_L = 200$ (1)

$X_L = 2\pi f L$ & $f = 100Hz$

So $L = \dfrac{X_L}{2\times\pi\times 100} = \dfrac{200}{200\pi} = \underline{\underline{318\,mH}}$ (2)

(ii)



~~R = 40Ω X~~

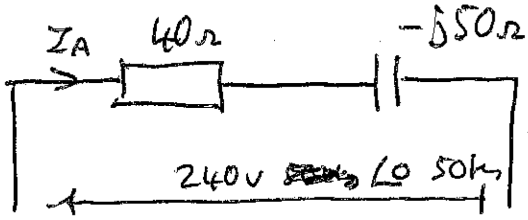
$Z_T = 64\angle -39^\circ = 49.7 - j\,40.3\ \Omega$

$\therefore R = \underline{\underline{49.7\,\Omega}}\ (\simeq 50\,\Omega)$ (1)

$X_C = 40.3 = \dfrac{1}{2\pi f C}$ $\therefore$ $C = \dfrac{1}{2\times\pi\times 100\times X_C}$

$C = \dfrac{1}{2\times\pi\times 100\times 40.3} = \dfrac{1}{8060\pi} = \underline{\underline{39.5\,\mu F}}$ (2)

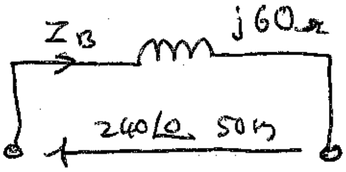
(d)



$$Z_A = 40 - j50\,\Omega = 64\angle{-51.3^\circ}$$

$$I_A = \frac{V}{Z_A} = \frac{240\angle 0}{64\angle{-51.3^\circ}} = \frac{240}{64}\angle 0 - -51.3^\circ$$

$$= \underline{\underline{3.75\angle{+51.3}}} \text{ Ampere} \quad (\text{or } 2.34 + j2.92\text{ A}) \quad 2$$



$$Z_B = +j60\,\Omega \text{ or } 60\angle 90^\circ$$

$$I_B = \frac{V}{Z_B} = \frac{240\angle 0}{60\angle 90^\circ} = \underline{\underline{4\angle{-90^\circ}}}\text{ A}$$

$$(\text{or} = \underline{\underline{-j4\text{ A}}}) \quad 2$$

$$\therefore I_C = I_A + I_B = 2.34 + j2.92 - j4$$

$$= \underline{2.34 - j1.08\text{ A}} \quad 2$$

$$(\text{or} = 2.58\angle{-24.8^\circ}\text{ A})$$

$$\therefore Z_T = \frac{V}{I_C} = \frac{240\angle 0}{2.58\angle{-24.8}} = \underline{\underline{93.02\angle{+24.8}\,\Omega}} \quad 4$$

$$(\text{or} = \underline{\underline{(84.4 + j39)\,\Omega}})$$

OR can find Z_T from impedances

$$Z_T = \frac{(40 - j50)\cdot j60}{40 - j50 + j60} = \frac{64\angle{-51.3^\circ} \times 60\angle{+90^\circ}}{40 + j10}$$

$\boxed{7.3}$

$$Z_T = \frac{3840 \angle 38.7^\circ}{41.2 \angle 14^\circ}$$

$$= \underline{\underline{93.2 \angle +24.7^\circ}} \ \Omega$$

$$\left(\text{or} \quad \underline{\underline{84.7 + j\,38.9}} \quad \Omega \right)$$

Q8 8.1

(a) $3\log x - \log x^2$

$= 3\log x - 2\log x$

$= \underline{\underline{\log x}}$ (2)

(b) (i) $V_{gain} \text{ in dB} = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right)$

$\therefore \quad 40 = 20\log_{10}\left(\frac{V_{out}}{4}\right)$

$\cancel{\frac{40}{20}}^{2} = \log_{10}\left(\frac{V_{out}}{4}\right)$

$10^2 = \frac{V_{out}}{4}$

$\therefore \quad V_{out} = 4\times 10^2 = \underline{\underline{400V}}$ (2)

(ii) $\text{Power in dB} = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$

$\therefore \quad -6 = 10\log_{10}\left(\frac{3}{P_{in}}\right)$

$-0.6 = \log_{10}\left(\frac{3}{P_{in}}\right)$

$10^{-0.6} = \cancel{3}\frac{3}{P_{in}}$ (2)

$\therefore \quad P_{in} = \frac{3}{10^{-0.6}} = 3\times 10^{0.6} = \underline{\underline{11.9 \text{ Watts}}}$

(c) $I_D = I_s e^{V_D/k}$

~~See graph~~

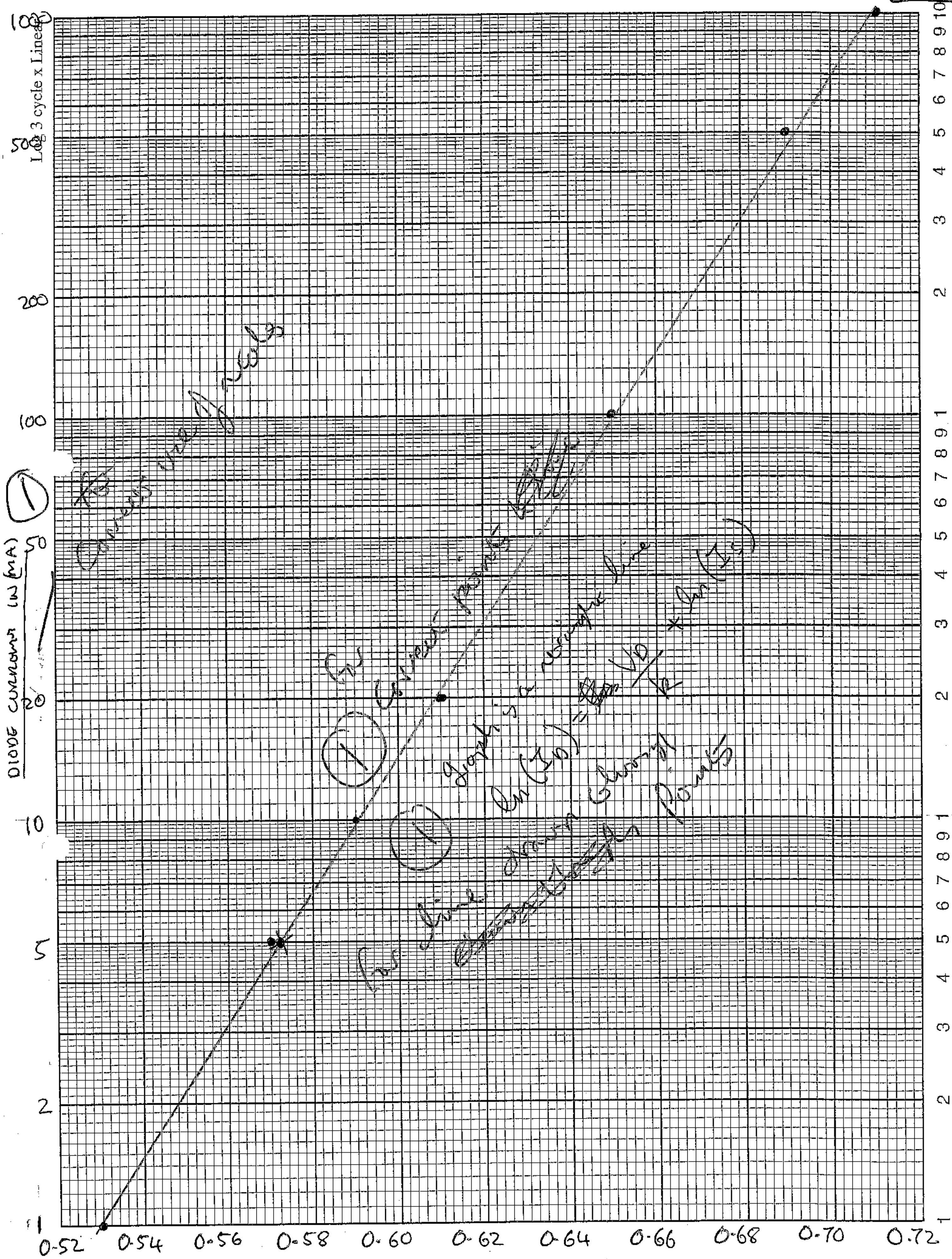
$\ln(I_D) = \ln(I_s) + \frac{V_D}{k}$

or $\ln(I_D) = \frac{V_D}{k} + \ln(I_s)$ (3)

equivalent to straight line formula $\quad y = mx + c \quad$ where $m = 1/k$

See graph – straight line ✓

8.2



DIODE VOLTAGE IN (VOLTS)

1 for correct use of scales

8.3

(d) gradient of ~~log graph~~ straight line on log graph is $\frac{1}{k} = \frac{\Delta \ln(I_D)}{\Delta V_D}$ 2

~~lets use $\frac{\ln(1000) - \ln(10^{-3})}{0.74}$~~

lets use $\frac{\ln(1000) - \ln(1)}{0.71 - 0.53} = \frac{1}{k}$

$= \frac{6.9 - 0}{0.18}$

$\frac{1}{k} = 38.33 \quad \left(\text{or } \frac{115}{3}\right)$

$\therefore k = 0.026 \quad \left(\text{or } \frac{3}{115}\right)$

[if working in amps
$\frac{\ln(1) - \ln(10^{-3})}{0.71 - 0.53} = \frac{1}{k}$
$= \frac{6.9}{0.18}$
$= 38.37 \quad \therefore k = 0.026$] 2

To find I_S lets substitute k into the ~~equa~~ equation $I_D = I_S e^{\frac{V_D}{k}}$

lets use $I_D = 1\,mA$ & $V_D = 0.53V$ & $k = 0.026$

$\therefore \cancel{I_D}\ 1 = I_S e^{\frac{0.53}{0.026}}$

$I_S = \frac{1}{e^{0.53/k}}$ or $e^{-0.53/0.026} = 1.4 \times 10^{-9}\,mA$ 3

$\left(\text{or } = 1.4 \times 10^{-12}\,A\right)$

Check at other end

let $I_D = 1A$ & $V_D = 0.71V$

$\therefore 1 = I_S e^{\frac{0.71}{0.026}}$

$\therefore I_S = \frac{1}{e^{\frac{0.71}{0.026}}} = e^{-\frac{0.71}{0.026}} = 1.38 \times 10^{-12}\,A$

$\left(\text{or } 1.38 \times 10^{-9}\,mA\right)$

Q9 (a) $5\frac{dy}{dx} + 2x = 3$ 9.1

re arrange $5\frac{dy}{dx} = 3 - 2x$

$\frac{dy}{dx} = \frac{3-2x}{5} = \frac{3}{5} - \frac{2x}{5}$

$dy = \left(\frac{3}{5} - \frac{2x}{5}\right) \cdot dx$

integrate both sides $y = \int\left(\frac{3}{5} - \frac{2x}{5}\right) \cdot dx$

$y = \frac{3x}{5} - \frac{2x^2}{2.5} + C$

General Solution $y = \frac{3x}{5} - \frac{x^2}{5} + C$ $\left(\text{or } \frac{3x - x^2}{5} + c\right)$ (2)

Particular Solution where $y = 1\frac{2}{5}$ & $x = 2$

$1\frac{2}{5} = \frac{3(2)}{5} - \frac{(2)^2}{5} + c$

$\frac{7}{5} = \frac{6}{5} - \frac{4}{5} + c$

$\frac{7 - 6 + 4}{5} = c$

$\frac{5}{5} = c \quad \therefore \quad c = 1$

Particular Solution is $y = \frac{3x}{5} - \frac{x^2}{5} + 1$ (3)

(b) $\frac{dy}{dx} = 9x^2 \cdot y$

$\frac{1}{y} \cdot dy = 9x^2 \cdot dx$

integrate both sides $\int \frac{1}{y} \cdot dy = \int 9x^2 \cdot dx$

$$\ln(y) = \frac{9x^3}{3} + c = 3x^3 + c$$ 2

raise both to power of e $$y = e^{3x^3 + c}$$ 3

(c) $$C \frac{dv_c}{dt} = \frac{-v_c}{R}$$

rearrange
$$\frac{1}{v_c} \cdot dv_c = \frac{-1}{RC} \cdot dt$$

integrate both sides

$$\cancel{\ln(v_c) = \frac{-1}{RC}}$$

$$\int \frac{1}{v_c} \cdot dv_c = \frac{-1}{RC} \int \cdot dt$$

$$\ln(v_c) = \frac{-t}{RC} + c$$ 2

raise both sides to power of e

$$v_c = e^{-t/RC + c}$$ 2

$$v_c = e^{-t/RC} \cdot e^c$$

using initial conditions $t = 0$ when $v_c = 10v$

$$10 = e^{-0/RC} \cdot e^c$$

anything to power of 0 = 1

$$10 = e^c$$

$$\therefore \quad v_c = 10 e^{-t/RC}$$ 2

9.3

(d) $\frac{V_c}{10} = e^{-t/RC}$

$\ln\left(\frac{V_c}{10}\right) = -t/RC$

$-RC\ln\left(\frac{V_c}{10}\right) = t$ 2

~~Then~~ Put in values

$\cancel{-RC\ln\left(\frac{10}{0.067}\right)}$

$-2\times10^3 \times 2.5\times10^{-5} \times \ln\left(\frac{0.067}{10}\right) = t$ 2

$t = 0.25\,s$ (or 250 mS)