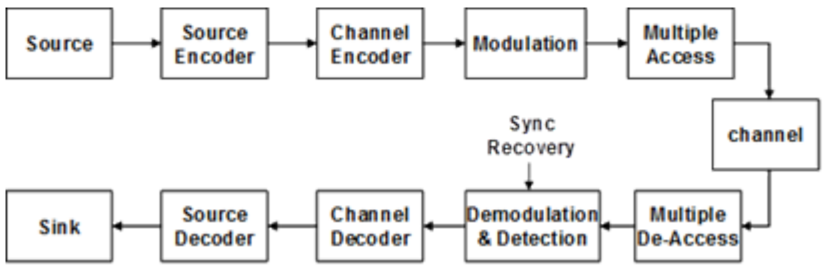
Marks **Q 1**

a) Schematic block diagram of generalised digital wireless communication system:

4. 

1. **i. path loss:** introduces attenuation of the signal as a function of range and/or frequency.

1. **ii. bandwidth limitation:** introduces signal distortion and intersymbol interference (ISI).

2. **b) Free-space pathloss -** When a signal travels through free space to a Rx located at distance *d* from the Tx; i.e. no boundary conditions. There are no obstructions between the Tx and Rx and the signal propagates along a straight line between the two giving a line-of-sight (LOS) channel.

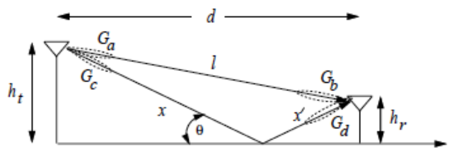
Find P_r in dBW: Isotropic Tx and Rx antenna means $G_t = G_r = 1$, fc = 2100MHz, d = 3km

1. $P_t(\text{dBW}) = 10\log(40) = 16.02$ dBW

1. $P_L(\text{dB}) = 32.44+20\log(f_{MHz}) +20\log(d_{Km}) = 32.44+20\log(2100) +20\log(3) = 108.43$ dB

1. $P_r(\text{dBW}) = P_t(\text{dBW}) - P_L(\text{dB}) = 16.02-108.43 = \underline{-92.41 \text{ dBW}}$ #

2. **c) 2-Ray Path Loss Model:** Differs from free space path loss model by including a ground reflection. The received signal consists of two components: the LOS component or ray, which is just the Tx signal propagating through free space, and a reflected component or ray, which is the Tx signal reflected off the ground. There is no dependence on frequency and a $1/d^4$ dependency on range.

1. 

2. $P_L(\text{dB}) = 40\log(d) - 20\log(h_t) - 20\log(h_r) - 10\log(G_t) - 10\log(G_r) = 40\log(3000) - 20\log(15) - 20\log(1.5) - 10\log(1) - 10\log(1) = 112.04$ dB

2. $P_t(\text{dBW}) = P_r(\text{dBW}) + P_L(\text{dB}) = -92.41 + 112.04 = 19.63 \text{ dBW} \equiv \underline{91.94 \text{ watts}}$#

i.e. over twice the transmit power is required compared with part(b).

2. **d) Shawdow Fading:** A signal transmitted through a wireless channel will typically experience random variation due to blockage from objects in the signal path, giving rise to random variations of the received power at a given distance.

$P_t = 40\text{W} \equiv 16.02$ dBW, $P_{min} = -128$ dBW, $s_{dB} = 8$ dB

2. $$P_{out}(\text{-128dBW, 3km}) = \text{Prob}(P_r(3km) < \text{-}128dBW) = Q\left(\frac{|P_{\min} - P_r|}{\sigma_{dB}}\right)$$

1. $$P_r = P_t - P_L = 16.02 - 112.04 = -96.02dBW$$

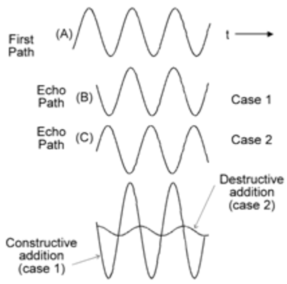
(USE GRAPHICAL Q-FUNCTION)

2. $$P_{out} = Q\left(\frac{|-128-(-96.02)|}{8}\right) = Q(4) = \underline{3.2\times10^{-5} \quad \text{or } 0.0032\% \text{ \#}}$$

[25]

Marks **Q 2**

a) Fading due to multipath propagation:



Differential time delays introduce fixed relative phase shifts between component waves causing constructive and destructive addition at one instant of time or space.

2. When the receiver is moving there is a continuous change in electrical length of every propagation path and thus the relative phases change with spatial location. Therefore, a moving receiver experiences time varying fades.

3. In narrowband fading the small excess delay results in the resolution of a single fading path at the receiver where as in wideband fading the large excess delay results in the resolution of two or more paths at the receiver.

b) Probability of outage:

1. Envelope is Rayleigh distributed $p_R(r) = \frac{2r}{\overline{P}_r}\exp\left[\frac{-r^2}{\overline{P}_r}\right]$

1. Power distribution obtained by substituting $z = r^2$

2. Then $dz = 2rdr$ and $p(r)dr = p(z)dz$ which gives $p(z) = p(r)\frac{dr}{dz} = \frac{p(r)}{2r} = \frac{1}{\overline{P}_r}\exp\left(-\frac{z}{\overline{P}_r}\right)$

1. Given $\overline{P}_r = 0\ dBm \equiv 1\ mW$ and $P_{\min} = -10\ dBm \equiv 0.1\ mW$, hence the outage probability is given by

2. $P_{out} = \Pr\{P_r < P_{\min}\} = \Pr\{P_r < 0.1\ mW\} = \int_0^{0.1} e^{-z}\,dz = \left[-e^{-z}\right]_0^{0.1}$

1. $= 1 - e^{-0.1} = \underline{0.095 \quad or \quad 9.5\%\ \#}$

c) Average fade duration:

2. Average fade duration indicates the number of bits or symbols affected by a deep fade. The longer the fade, the greater the burst error leading to irrecoverable data packets.

1. $\bar{t}_R = \frac{\exp(\rho^2) - 1}{\rho f_D\sqrt{2\pi}}$, where $\rho = \sqrt{P_R / \overline{P}_r}$ and P_R = target power level whereas $\overline{P}_r$ = average power level.

2. From the question, $P_R = \overline{P}_r/10 \Rightarrow \rho = 0.3162$

2. Require $\bar{t}_R = \frac{\exp(0.1) - 1}{0.3162 \cdot f_D \cdot \sqrt{2\pi}} \le 0.01s$ which gives $f_D \ge \frac{\exp(0.1) - 1}{0.3162 \cdot 0.01 \cdot \sqrt{2\pi}} = \underline{13.3\ Hz\ \#}$

d) Fading rate:

2. From first principles $\bar{t}_r = \frac{\text{Average time spent faded}}{\text{Average number of fades}} = \frac{P(r<R)}{L_R} \Rightarrow L_R = \frac{P(r<R)}{L_R}$

1. From Part2(b) $P(r<R) = 1 - e^{-\rho^2}$

1. Hence $L_R = \frac{1 - e^{-\rho^2}}{\bar{t}_R} = \frac{1 - e^{-0.1}}{0.01} = \underline{\underline{9.5 \text{ fades/s}\ \#}}$

1. **Comment:** the fading rate and maximum Doppler shift are highly correlated.

[25]

Marks | **Q 3** |

a) Time invariant discrete CIR:

1. $L =$ the number of significant multipaths contributing to dispersion;

2. h_l = *l-th* complex path gain denotes the magnitude & phase variation in received signal envelope.

2. τ_l = *l-th* path delay denotes the amount of delay due to multipath propagation.

b) Dispersive channel

$ExcessDelay\mu s$	$PathPowerdBr$	$PathPowerLinear$
0	−3	0.5
1	0	1
5	0	1
10	−3	0.5

$$\bar{\tau} = \frac{0.5(0)+1.0(1)+1.0(5)+0.5(10)}{0.5+1+1+0.5} = \underline{3.67\mu s} \quad \#$$

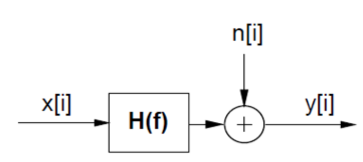
2. $$\overline{\tau^2} = \frac{0.5(0)^2+1.0(1)^2+1.0(5)^2+0.5(10)^2}{0.5+1+1+0.5} = 25.33\times10^{-12}s^2$$

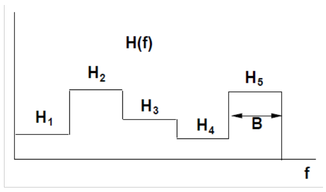
3. $$\tau_{rms} = \sqrt{\overline{\tau^2}-\bar{\tau}^2} = \underline{3.45\mu s} \quad \#$$

2. 90% coherence bandwidth is given by $B_c = \dfrac{1}{50\tau_{rms}} = \dfrac{1}{50\times3.45\times10^{-6}} = \underline{5.8\ kHz} \quad \#$

c) Time invariant block frequency selective channel -

2.

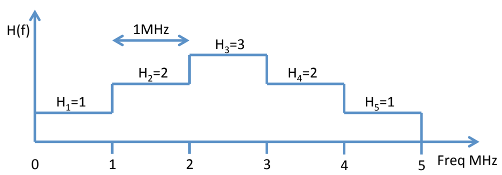




2. Frequency band can be divided into subchannels of bandwidth B with $H(f) = H_i$ constant over each sub-channel as a function of time. The block freq selective channel thus consists of a set of AWGN channels in parallel.

2. This type of channel benefits a broadband wireless comms system as $H(f)$ can be known at the transmitter as well as the receiver enabling an optimum power allocation scheme, called water filling, to be used.

d) Shannon capacity and power allocation: Transmit power constraint P = 5 mW and 1-sided $N_0 = 10^{-9}$ WHz^{-1}.



Find

1. $$\gamma_i = \frac{|H_i|^2 P}{N_0 B}\ \forall i \Rightarrow \gamma_1 = \frac{|1|^2\times5mW}{10^{-9}\times10^6} = 5,$$

1. $$\gamma_2 = 20, \gamma_3 = 45, \gamma_4 = 20, \gamma_5 = 5$$

1. The cut-off SNR must satisfy the Lagrangian – $\sum_i\left(\dfrac{1}{\gamma_0}-\dfrac{1}{\gamma_i}\right) = 1 \Rightarrow \dfrac{5}{\gamma_0} = 1+\sum_i\dfrac{1}{\gamma_i} = 1+\dfrac{2}{5}+\dfrac{2}{20}+\dfrac{1}{45} = 1.522$

1. $\therefore \gamma_0 = 5/1.52 = 3.29, \quad \therefore \gamma_0 < \gamma_i \quad \forall i$ means all subchannels can be allocated power.

Therefore the Shannon Capacity is given by

1. $$C = \sum_{i=1}^{5} B\log_2(\gamma_i/\gamma_0) = 10^6[2\cdot\log_2(5/3.29)+2\cdot\log_2(20/3.29)+\log_2(45/3.29)] = \underline{10.2\ Mbit/s} \quad \#$$

And the power allocations are given by

1. $$\frac{P_i}{P} = \frac{1}{\gamma_0}-\frac{1}{\gamma_i} \quad \gamma_i \ge \gamma_0, \quad P_1 = P_5 = \left(\frac{1}{3.29}-\frac{1}{5}\right)\times5\ mW = 0.52\ mW,$$

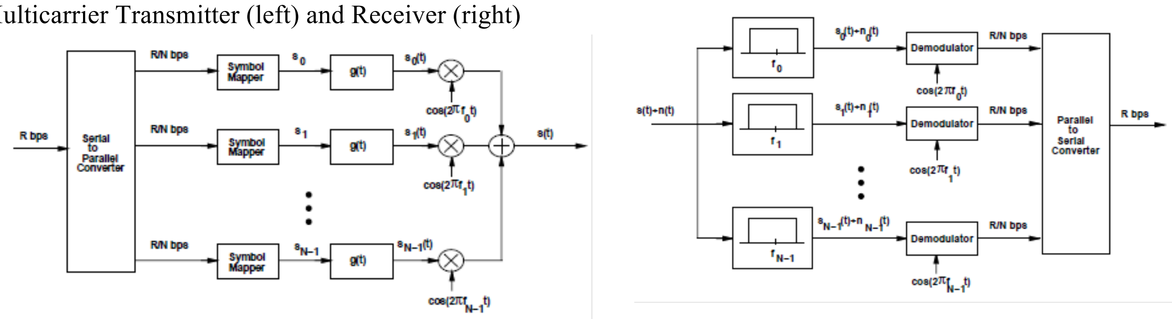
$$P_2 = P_4 = \left(\frac{1}{3.29}-\frac{1}{20}\right)\times5\ mW = 1.27\ mW, \quad P_3 = \left(\frac{1}{3.29}-\frac{1}{45}\right)\times5\ mW = 1.41\ mW$$

1. Note that $P_1 + P_2 + P_3 + P_4 + P_5$ = 5 mW as required.

[25]

Marks Q 4

4. **a)** Multicarrier Transmitter (left) and Receiver (right)



2. Multicarrier modulation divides the data stream into multiple sub-streams that are transmitted simultaneously over different orthogonal sub-channels each of different carrier frequency. In a wideband frequency selective channel, the sub-channel bandwidth is chosen to be less than or equal to the coherence bandwidth B_c. This ensures that each sub-channel does not experience significant ISI.

b) The *rms* delay spread $T_m = 1/B_c$

3. **i.** By using $T_n = 10T_m = 10/B_c = 10/31.25\times10^3 = 0.32\ ms$ ensures flat fading on the individual sub-channels.

3. **ii.** Total Bandwidth $B = \dfrac{n+\beta+\varepsilon}{T_n} = \dfrac{64+1+0.1}{0.00032} = \dfrac{65.1}{0.00032} = \underline{203\ kHz\quad \#}$

c) Insufficiently long CP:

2. There will be ISI between OFDM symbols, which may result in an irreducible bit error rate.

3. The CIR consists of three taps corresponding to three paths. There are two excess delay taps at 1 and 2 s, therefore a CP length of 2 is sufficient to avoid ISI between OFDM symbols.

d) For "all ones" QPSK transmission:

2. i.

$$\mathbf{X} = \frac{1}{\sqrt{2}}\cdot\begin{bmatrix}1+j\\1+j\\1+j\\1+j\end{bmatrix} \Rightarrow \mathbf{x} = \mathbf{M}\cdot\mathbf{X} = \frac{1}{\sqrt{2}}\cdot\frac{1}{2}\cdot\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}\cdot\begin{bmatrix}1+j\\1+j\\1+j\\1+j\end{bmatrix} = \begin{bmatrix}\sqrt{2}+\sqrt{2}i\\0\\0\\0\end{bmatrix},$$

1. Hence the transmitted OFDM symbol with CP is

$$\underline{\begin{bmatrix}0\\0\\\sqrt{2}+\sqrt{2}j\\0\\0\\0\end{bmatrix} = \sqrt{2}\begin{bmatrix}0\\0\\1+j\\0\\0\\0\end{bmatrix}\quad \#}$$

ii. Received vector after CP removal is:

1.

$$\mathbf{r} = j\times\begin{bmatrix}0\\0\\\sqrt{2}+\sqrt{2}j\\0\\0\\0\\\bullet\\\bullet\end{bmatrix} + 1\times\begin{bmatrix}\bullet\\0\\0\\\sqrt{2}+\sqrt{2}j\\0\\0\\0\\\bullet\end{bmatrix} - 0.5j\times\begin{bmatrix}\bullet\\\bullet\\0\\0\\\sqrt{2}+\sqrt{2}j\\0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\-\sqrt{2}+\sqrt{2}j\\\sqrt{2}+\sqrt{2}j\\1/\sqrt{2}-j/\sqrt{2}\\0\\0\\0\end{bmatrix} \Rightarrow \underline{\underline{\mathbf{x}_r = \begin{bmatrix}-\sqrt{2}+\sqrt{2}j\\\sqrt{2}+\sqrt{2}j\\1/\sqrt{2}-j/\sqrt{2}\\0\end{bmatrix}}}\quad \#$$

and discarding convolution tail.

	iii. Take FFT of $\mathbf{x}_r$
1.	$\Rightarrow \mathbf{X}_r = \mathbf{M}^{*T} \cdot \mathbf{x}_r = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} -\sqrt{2} + \sqrt{2}j \\ \sqrt{2} + \sqrt{2}j \\ 1/\sqrt{2} - j/\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} +0.3536 + 1.0607j \\ -0.3536 + 0.3536j \\ -1.0607 - 0.3536j \\ -1.7678 + 1.7678j \end{bmatrix}$
	Take FFT of
1.	$\mathbf{h} \Rightarrow \mathbf{H} = \mathbf{M}^{*T} \cdot \mathbf{h} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} j \\ 1 \\ -0.5j \\ 0 \end{bmatrix} = \begin{bmatrix} +1.0 + 0.5j \\ +0.0 + 0.5j \\ -1.0 + 0.5j \\ +0.0 + 2.5j \end{bmatrix}$
1.	The QPSK symbols after ZF equalisation are: $\mathbf{S} = \mathbf{X_r}/\mathbf{H} = \begin{bmatrix} 1+j & 1+j & 1+j & 1+j \end{bmatrix}^T / \sqrt{2}$
1.	All four symbols are successfully recovered by the zero forcing equaliser without degenerate channel effects, so the receiver would not encounter any difficulties.
[25]	End