Question 1

(a)

i. Since the circuit will fail, if one of the components fails, its failure rate is given by:

$$\lambda = 2\times10^{-9}\times10 + 10\times10^{-9}\times4 + 5\times10^{-9}\times5 + 100\times10^{-9}\times1$$
$$+45\times10^{-9}\times2 + 7\times10^{-9}\times2 + 0.1\times10^{-9}\times68 + 0.01\times10^{-9}\times20 = 296\times10^{-9}\ \text{/ hour}$$

ii. Therefore the mean time to failures of the circuit is:

$$MTTF = \frac{1}{\lambda} = 3.38\times10^{6}\ \text{hours}$$

iii. The maximum duration of operation is given by:

$$R(t) = e^{-\lambda t} \geq 0.9999 \quad \Rightarrow \quad t \leq \frac{-\ln(0.9999)}{\lambda} = 337.85\ \text{hours}$$

iv. The probability of failure is given by the failure distribution $F(t) = 1 - R(t) = 1 - e^{-\lambda t}$. Therefore, the probability of failure during a 9-minute (0.15 hour) operation is given by:

$$F(t = 0.15\text{hours}) = 1 - R(t = 0.15\,\text{hours}) = 1 - e^{\left(-296\times10^{-9}\times0.15\right)} = 4.44\times10^{-8}$$

(b)

i. The system is an *m*-out-of-*n* passive system with *m*=1 and *n*=*2*. Therefore, the reliability function will be given by:

$$R(t) = e^{-\lambda mt}\sum_{k=m}^{n}\frac{(m\lambda t)^{k-m}}{(k-m)!}$$

where *m*=1 and *n*=2, therefore,

$$R(t) = e^{-\lambda t} \sum_{k=1}^{n=2} \frac{(\lambda t)^{k-1}}{(k-1)!}$$

$$\Rightarrow R(t) = e^{-\lambda t} \frac{(\lambda t)^{1-1}}{(1-1)!} \qquad (k=1)$$

$$+ e^{-\lambda t} \frac{(\lambda t)^{2-1}}{(2-1)!} \qquad (k=2)$$

$$\Rightarrow R(t) = e^{(-\lambda t)} (1 + \lambda t)$$

therefore, the probability of failure is given by:

$$F(t) = 1 - R(t) = 1 - e^{(-\lambda t)} (1 + \lambda t)$$

and for a 1000-hour operation:

$$F(t = 1000) = 1 - R(t) =$$
$$= 1 - e^{\left(-296 \times 10^{-9} \times 1000\right)} \left(1 + 296 \times 10^{-9} \times 1000\right)$$
$$= 4.38 \times 10^{-8}$$

ii.

ii.i. Since majority voting is adopted, the system is an *m*-out-of-*n* system with *m*=2 and *n=3*. Therefore, the reliability function will be given by:

$$R(t) = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!} \left[e^{-\lambda k t}\right]\left[1 - e^{-\lambda t}\right]^{n-k}$$

where *m*=2 and *n*=3, therefore,

$$R(t) = \sum_{k=2}^{3} \frac{3!}{k!(3-k)!} \left[e^{-\lambda k t}\right]\left[1 - e^{-\lambda t}\right]^{3-k}$$

$$\Rightarrow R(t) = 3 e^{(-2\lambda t)} \left(1 - e^{(-\lambda t)}\right)^{3-2} \qquad (k=2)$$

$$+ e^{(-3\lambda t)} \left(1 - e^{(-\lambda t)}\right)^{3-3} \qquad (k=3)$$

$$\Rightarrow R(t) = 3 e^{(-2\lambda t)} - 2 e^{(-3\lambda t)}$$

Therefore, the probability of failure is given by:

$F(t) = 1 - R(t) = 1 - 3e^{(-2\lambda t)} + 2e^{(-3\lambda t)}$

and for a 410-hour operation:

$$F(t = 410) = 1 - 3e^{\left(-2\times 296\times 10^{-9}\times 410\right)} + 2e^{\left(-3\times 296\times 10^{-9}\times 410\right)}$$
$$= 4.39\times 10^{-8}$$

ii.ii. The mean time to failure is generally given by:

$$MTTF = \int_0^\infty R(t)dt = \int_0^\infty \left(3e^{(-2\lambda t)} - 2e^{(-3\lambda t)}\right)dt$$
$$= \left[-\frac{3}{2\lambda}e^{(-2\lambda t)} + \frac{2}{3\lambda}e^{(-3\lambda t)}\right]_0^\infty$$
$$= \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{5}{6\lambda} = 2.81\times 10^6 \text{ hours}$$

Question 2

(a)

i. The torque delivered by the brushless servo motor is given by:

$$T_m = \frac{\lambda}{2\pi n}F_a = \frac{5.5\times 10^{-3}}{2\times\pi\times 35}\times 30000 = 0.75\,\text{Nm}$$

ii. The armature current and the torque are related by:

$$T_m = k\,I \quad \Rightarrow \quad I = \frac{T_m}{k} = \frac{0.75}{0.51} = 1.47\text{ A}$$

And the copper loss of the motor:

$$P_c = R\times I^2 = 8\times 1.47^2 = 17.28\ \text{W}$$

(b)

i.
the angular displacement which corresponds to x=1.25mm is given by:

$$\theta = \frac{2\times\pi}{\lambda}\times n = \frac{2\times\pi}{0.0055}\times 35 = 40000\,\text{radians}$$

ii. The energy delivered by the actuator:

$$E = \int_0^{x_m} F_a(x)dx = \int_0^{x_m} a\,x\,dx = \left[\frac{1}{2}a\,x^2\right]_0^{x_m} = \frac{1}{2}a\,x_m^2$$

$$E = \frac{1}{2}\times 24\times 10^6 \times \left(1.25\times 10^{-3}\right)^2 = 18.75\,\text{Joules}$$

(c)

i. For a parabolic velocity profile, the angular displacement is given by:

$$\theta(t) = \int_0^t v(u)du = \int_0^t 4\Omega_m\left(\frac{u}{T}-\frac{u^2}{T^2}\right)du$$

$$= 4\Omega_m\left(\frac{u^2}{2T}-\frac{u^3}{3T}\right)_0^t = 4\Omega_m\left(\frac{t^2}{2T}-\frac{t^3}{3T}\right)$$

ii. The maximum speed of the brushless servo motor is given by:

$$\theta(t) = 4\Omega_m\left(\frac{t^2}{2T}-\frac{t^3}{3T^2}\right)$$

$$\Rightarrow \theta_m = \theta(T) = 4\Omega_m\left(\frac{T^2}{2T}-\frac{T^3}{3T^2}\right) = 4\Omega_m\,T\left(\frac{1}{2}-\frac{1}{3}\right)$$

$$\Rightarrow \Omega_m = \frac{3}{2}\frac{\theta_m}{T}$$

iii. The some of the torques is given by:

$$\sum Torques = T_{tot}(t) = J_r\,\frac{d\Omega}{dt}$$

And since $\Omega(t) = 4\Omega_m\left(\frac{t}{T}-\frac{t^2}{T^2}\right) \Rightarrow \frac{d\Omega(t)}{dt} = 4\Omega_m\left(\frac{1}{T}-2\frac{t}{T^2}\right)$**,** therefore**,**

$$T_{tot}(t) = 4J_r\,\Omega_m\left(\frac{1}{T}-2\frac{t}{T^2}\right)$$