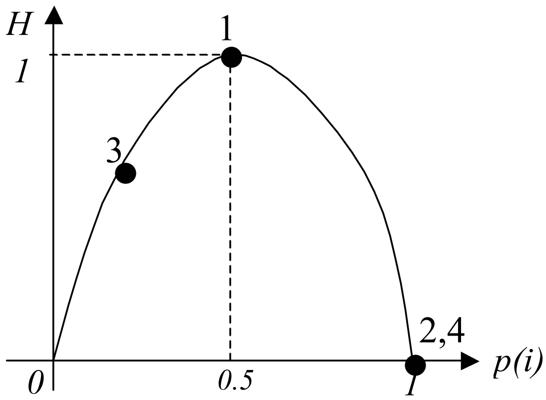
EEE 317 Tutorial answers – Source and Channel encoding

1) $$H = -\sum_{i=1}^{N} p(i)\log_2\{p(i)\}$$

2)

0	**0**	**0**	
0	**0**	**1**	
0	**1**	**0**	
0	**1**	**1**	
1	**1**	**1**	**0**
1	**1**	**0**	**1**
1	**1**	**1**	**0**
1	**1**	**1**	**1**



The above left figure shows the possible codewords with the shaded areas indicating the possible codewords as we transmit the successive bits, 1,1,0,1. Taking each bit in turn,

- The first bit is a one. There are 8 equally likely possible messages, and half of them start with a 1, so $H = 1$ for the first bit.
- Since we received a 1 first time, the probability of the next bit being a 1 is 100% as all of the messages that start with 1 have 1 as a second digit also, therefore $H = 0$.
- We now receive a 0, which is one of four equally likely possibilities, hence $H = -(0.25*\log_2(0.25)-(0.75*\log_2(0.75)=0.81$.
  Since we know with certainty that the next codeword will be 1 at this stage then the amount of information contained is 0.

3) $$P_\varepsilon = P_{(0|1)} \times P_{(1)} + P_{(1|0)} \times P_{(0)}$$

4) There are two types of error, burst and random. Burst errors occur with great intensity over a short period of time, for example errors caused by EM interference due to lightning strikes. Random errors occur at random and are usually characterised by a mean rate of occurrence, a good example of a mechanism that causes random errors is shot noise in the solid-state devices in the transmitter and receiver.

5) The aim of Huffman coding is to ensure that the most often occurring messages have the shortest codes, thus increasing the effective data rate of the channel.

6)

$p(b) = 0.42^{0}$	$p(b) = 0.42^{0}$	$p(b) = 0.42^{0}$	$p(b) = 0.42^{0}$	$p(decfa) = 0.58^{1}$
$p(d) = 0.25^{10}$	$p(d) = 0.25^{10}$	$p(d) = 0.25^{10}$	$p(ecfa) = 0.33^{11}$	$p(b) = 0.42^{0}$
$p(e) = 0.23^{111}$	$p(e) = 0.23^{111}$	$p(e) = 0.23^{111}$	$p(d) = 0.25^{10}$	
$p(c) = 0.05^{1101}$	$p(c) = 0.05^{1101}$	$p(cfa) = 0.1^{110}$		
$p(f) = 0.03^{11001}$	$p(fa) = 0.05^{1100}$			
$p(a) = 0.02^{11000}$				

- The superscript numbers show the Huffman code working from right to left and the arrows show how the codes have been combined as we move from left to right.

7) From the notes, *p(n errors in an N bit word) = p(n,N)* $= \binom{N}{n} P_E{}^{n}\left(1-P_E\right)^{(N-n)}$

a) So if $P_E = 10^{-9}$, $N = 12$ then,

$$p(1,12) = = \binom{12}{1} 10^{-9}\left(1-10^{-9}\right)^{11} = 1.2\times10^{-8}$$

therefore, the average number of codewords transmitted before a 1 bit error is suffered is $(1.2\times10^{-8})^{-1} = 83\times10^{6}$.

b) similarly,

$$p(2,12) = = \binom{12}{2}\left(10^{-9}\right)^{2}\left(1-10^{-9}\right)^{10} = 6.6\times10^{-17}$$

therefore, the average number of codewords transmitted before a 2 bit error is suffered is $(6.6\times10^{-17})^{-1} = 1.5\times10^{16}$.

8) The mean information contained within each of the letters is as follows

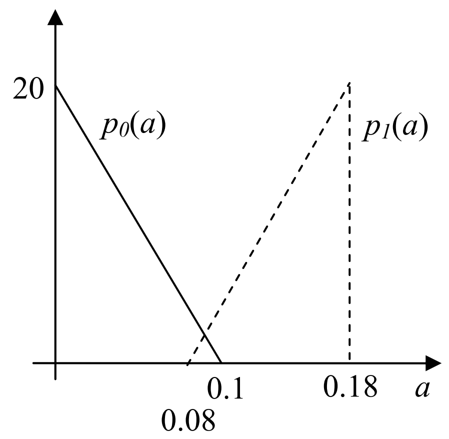
$$H_{a,e,t,s} = -p(a,e,t,s)\log_2\{p(a,e,t,s)\}$$
$$H_{a,e,t,s} = -0.065\log_2\{0.065\} = 0.256$$
$$H_{r,n,m,l,d,g} = -0.05\log_2\{0.05\} = 0.216$$
$$H_{o,u,h,c,i,p,b} = -0.04\log_2\{0.04\} = 0.186$$
$$H_{q,w,y,j,k,z,x,v} = -0.02\log_2\{0.02\} = 0.113$$

To find the average information contained in a letter, we must now add up all of the contributions,

$$H_{total} = (0.256\times4)+(0.216\times6)+(0.186\times7)+(0.113\times8) = 4.526\,\text{bits}$$

(well done if you noticed that the letter f was missing)

9) These, rather strange looking, pdfs have a simple functional form which can be sketched as follows.



The optimum point for the decision threshold is where the two pdfs cross, which in this case is $a_m = 0.09$. Finding the probabilities of a false 1 and a false 0 is relatively straightforward.

$$p(false\_1) = \int_{a_m}^{\infty} p_0(a)da = \frac{0.01 \times 2}{2} = 0.01$$

$$p(false\_0) = \int_{-\infty}^{a_m} p_1(a)da = \frac{0.01 \times 2}{2} = 0.01$$

We need not integrate these integrals 'formally' in this case as the shape of the pdf is triangular and consequently we can use 0.5*base*height of the overlapping area. Further, since the pdfs are symmetrical about a_m, *p*(*false 1*) = *p(false 0)* and so we need have only calculated one.

From the notes,

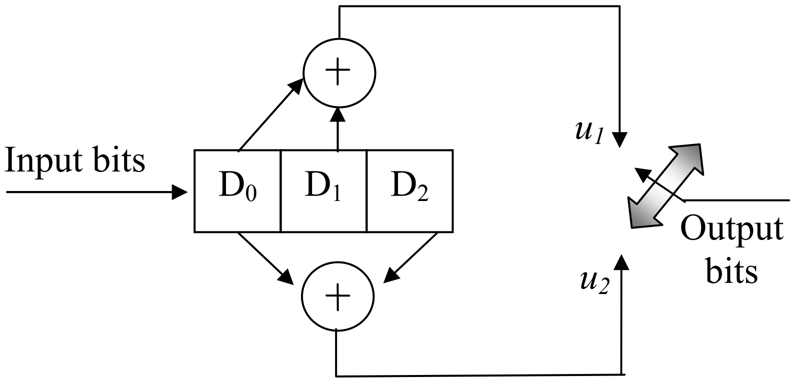
$$P_E = \sum_{n=0}^{N} p(n)r(n)$$

$$P_E = p(1)p(false\_1) + p(0)p(false\_0)$$

$$P_E = 0.6 \times 0.01 + 0.4 \times 0.01 = 0.01$$

10) Following the notes… This shift register has three blocks, and therefore there are $2^{3-1} = 4$ states (remember we do not count the first shift register when determining the number of states since it is the number in this shift register that determines the next state). These states are, D_1D_2 = 00, 01, 10 and 11.

To draw the state diagram we need to determine what transitions will occur when we input a 1 or 0 for each state. So, looking at the circuit diagram….



Initial state D_1D_2	Input D_0	Output u_1u_2	Next state
00	0	00	00
00	1	11	10
01	0	01	00
01	1	10	10
10	0	10	01
10	1	01	11
11	0	11	01
11	1	00	11

Hence the state diagram is as follows,