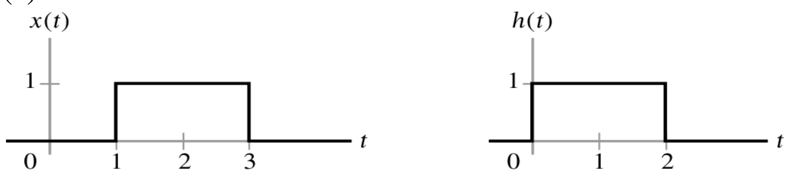
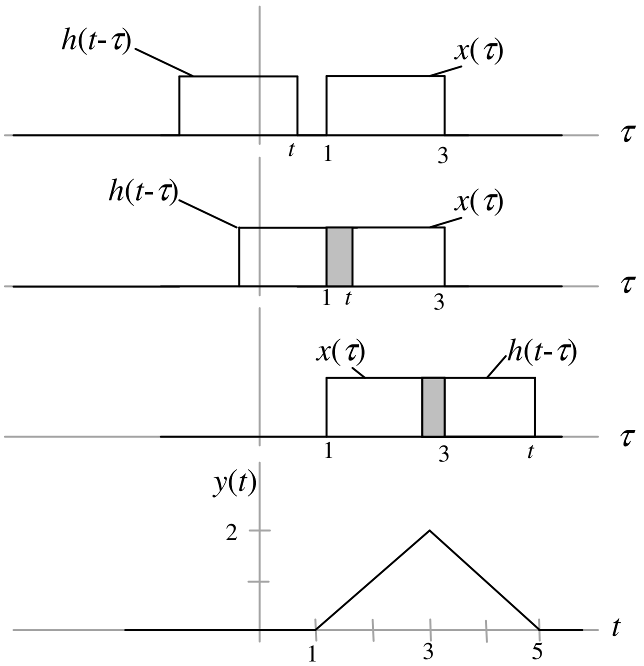
Q1:

(a)



(b)



For t<1, y(t)=0

For 1≤t<3, y(t)=t-1

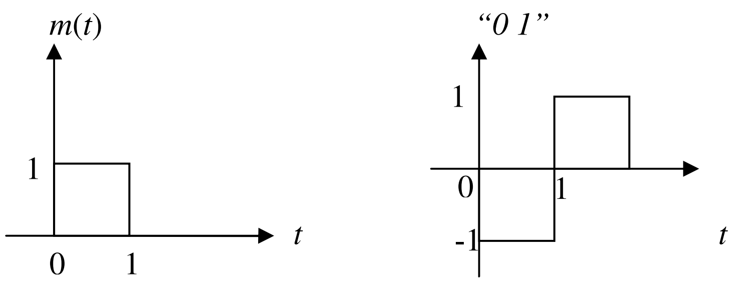
For 3≤t<5, y(t)=3-(t-2)=5-t

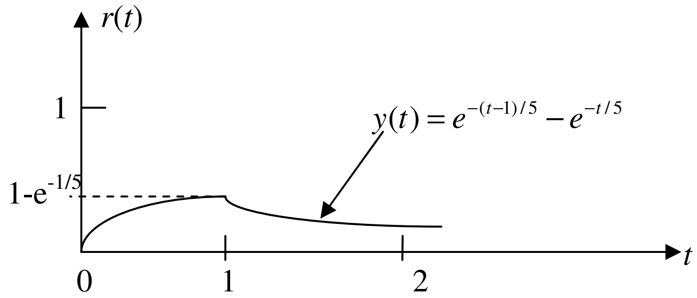
(c) The unit step response is

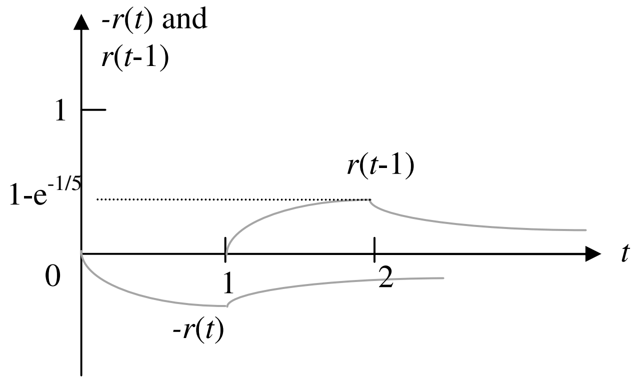
$$= \int_{-\infty}^{\infty} h(\tau).u(t-\tau)d\tau = \int_{-\infty}^{t} h(\tau)d\tau$$

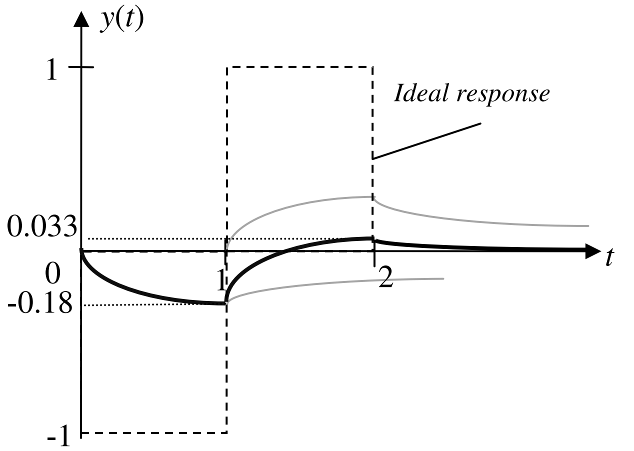
$$= \int_{-\infty}^{t} \frac{1}{RC} e^{-\tau/RC} u(\tau)d\tau = \frac{1}{RC}\int_{0}^{t} e^{-\tau/RC} d\tau = \left[-e^{-\tau/RC}\right]_0^t = 1 - e^{-t/RC}, t \geq 0.$$

(d)









The output $y(t)$ is severely distorted. The maximum values are -0.18 and 0.033, significantly smaller than the values of -1 and 1 expected for the bits 0 and 1 respectively. Therefore in a practical system it will be very difficult to recover the sequence "0 1". The RC time constant should be much less than the bit duration to minimize this distortion.

Q2:

a) $a_o = \frac{1}{T}\int_{-T/4}^{T/4} 1 dt = \frac{1}{T}[T/4 - (-T/4)] = \frac{1}{2}$

Since this is an even function, $b_n = 0$.

$$a_n = \frac{2}{T}\int_{-T/4}^{T/4} \cos\left(\frac{2n\pi t}{T}\right) dt = \frac{2}{\left(\frac{2n\pi}{T}\right)T}\left[\sin\left(\frac{2n\pi t}{T}\right)\right]_{-T/4}^{T/4} = \frac{1}{n\pi}\left(\sin\left(\frac{2n\pi}{T}.\frac{T}{4}\right) - \sin\left(\frac{2n\pi}{T}.\frac{-T}{4}\right)\right)$$

$a_n = \frac{1}{n\pi}\left(2\sin\left(\frac{n\pi}{2}\right)\right)$, $a_n = 0$ when n=even number, $a_n = \frac{2}{n\pi}$ when n=1,5,9….,

$a_n = -\frac{2}{n\pi}$ when n=3,7,11….,

Therefore the Fourier Series representation is given by

$v(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos\omega_o t - \frac{1}{3}\cos 3\omega_o t + \frac{1}{5}\cos 5\omega_o t.......\right]$ where $\omega_o = \frac{2\pi}{T}$

b) The transfer function of the RC circuit is

$H(j\omega) = \frac{1/RC}{1/RC + j\omega}$.

After filtering the amplitude of the nth harmonic becomes

$\left|\frac{1/RC}{1/RC + jn\omega_o}\right| \frac{1}{n\pi}\left(2\sin\left(\frac{n\pi}{2}\right)\right) = \left|\frac{10}{10 + j2n\pi}\right| \frac{1}{n\pi}\left(2\sin\left(\frac{n\pi}{2}\right)\right)$.

Therefore the output can be expressed as

$y(t) = \frac{1}{2} + \sum_{n=1}^{\infty}\left|\frac{10}{10 + j2n\pi}\right| \frac{1}{n\pi}\left(2\sin\left(\frac{n\pi}{2}\right)\right)\cos(2n\pi t)$

c) Within -210Hz to 210 Hz, we have

a_0=1/2, a_1=(2)/π, a_3=-(2)/(3π),

we know that the complex Fourier Series coefficients are $|C_0|=|a_0|$ and $|C_n|=|a_n|/2$,

so: $|C_0|=1/2$ and $|C_1|=|C_{-1}|=1/\pi$, $|C_{-3}|=|C_3|=1/(3\pi)$

Using Parseval's theorem, we have

$$Ave.Power = \sum_{n=-3}^{n=3}|Cn|^2 = (\frac{1}{2})^2 + 2(\frac{1}{\pi})^2 + 2(\frac{1}{3\pi})^2 = (\frac{1}{2})^2 + 2(\frac{1}{\pi})^2 + 2(\frac{1}{3\pi})^2$$

$$= (\frac{1}{2})^2 + 2(\frac{1}{\pi})^2 + 2(\frac{1}{3\pi})^2 = 0.475$$

Q3. a) $\frac{dy(t)}{dt} + \frac{1}{RC} \cdot y(t) = \frac{1}{RC} \cdot x(t)$

Taking the Laplace transform,

$sY(s) + \frac{1}{RC} Y(s) = \frac{1}{RC} X(s)$

$(s + \frac{1}{RC}) Y(s) = \frac{1}{RC} X(s)$

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{RC} \cdot \frac{1}{s+1/RC}$

Taking the inverse Laplace transform gives,

$h(t) = \frac{1}{RC} e^{-t/RC} \cdot u(t)$

b) $\frac{dy(t)}{dt} + \frac{1}{RC} \cdot y(t) = \frac{1}{RC} \cdot \frac{dx(t)}{dt}$

Taking the Laplace transform,

$sY(s) + \frac{1}{RC} Y(s) = \frac{1}{RC} sX(s)$

$H(s) = \frac{s/RC}{s+1/RC} = \frac{1}{RC} \cdot \frac{s}{s+1/RC}$

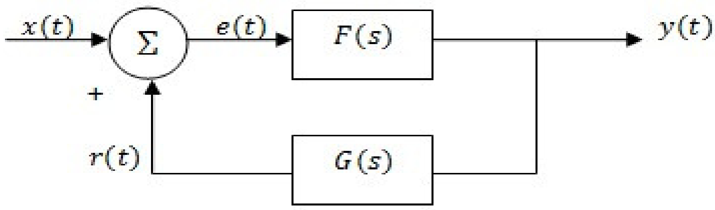
We can rewrite this as

$H(s) = \frac{1}{RC} \cdot \left[\frac{s + \frac{1}{RC} - \frac{1}{RC}}{s + \frac{1}{RC}} \right] = \frac{1}{RC} \left[1 - \frac{\frac{1}{RC}}{S + \frac{1}{RC}} \right]$

Taking the inverse Laplace transform gives,

$h(t) = \frac{1}{RC} \delta(t) - \frac{1}{(RC)^2} e^{-t/RC} \cdot u(t)$

c)



Let the signal-transform pairs be

$x(t) \longleftrightarrow X(s)$ $\quad e(t) \longleftrightarrow E(s)$ $\quad r(t) \longleftrightarrow R(s)$

$y(t) \longleftrightarrow Y(s)$

$Y(s) = E(s) \cdot F(s)$ (1)

$R(s) = Y(s) \cdot G(s)$ (2)

Since $e(t) = x(t) + r(t)$

$E(s) = X(s) + R(s)$ (3)

Sub. (3) into (1): $Y(s) = [X(s) + R(s)]F(s)$

$= [X(s) + Y(s) \cdot G(s)]F(s)$

$Y(s)[1 - F(s)G(s)] = X(s)F(s)$

$H(s) = \frac{F(s)}{1-F(s)G(s)}$

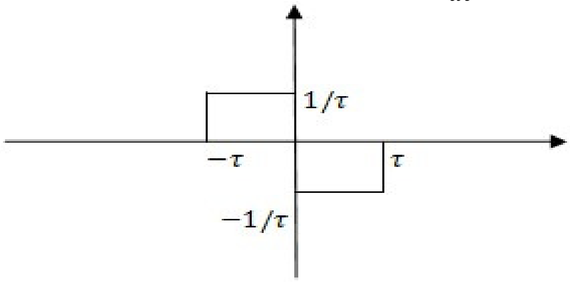
d) $H(s) = \frac{1/sC}{R+sL+1/sC} = \frac{1}{sRC+LCs^2+1} = \frac{1/LC}{s^2+Rs/L+1/LC}$

Since $L = 0.5, C = 0.4, R = 1,\ LC = 0.2, \frac{R}{L} = 2$, thus

$H(s) = \frac{1/0.2}{s^2+2s+1/0.2} = \frac{5}{s^2+2s+5}$

So $\omega_n = \sqrt{5} = 2.236 \text{ rad/s}$ $\qquad 2\xi\omega_n = 2 \quad \xi = \frac{1}{\sqrt{5}} = 0.45$

Q4. a) Differentiate $m(t)$ with respect to t gives $g(t) = \frac{dm(t)}{dt}$



$$G(\omega) = \tau\left(\frac{1}{\tau}\right)\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}e^{\frac{j\omega\tau}{2}} - \tau\left(\frac{1}{\tau}\right)\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}e^{\frac{-j\omega\tau}{2}}$$

$$G(\omega) = \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}\left[e^{\frac{j\omega\tau}{2}} - e^{-\frac{j\omega\tau}{2}}\right] = \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}\cdot j2\sin\left(\frac{\omega\tau}{2}\right) = j\omega\tau\left[\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}\right]^2$$

$$m(t) = \int_{-\infty}^{t} g(\tau)d\tau$$

$$M(\omega) = \frac{\mathrm{G}(\omega)}{j\omega} + \pi\mathrm{G}(0)\delta(\omega) = \frac{\mathrm{G}(\omega)}{j\omega} \quad \text{Since } \mathrm{G}(0) = 0$$

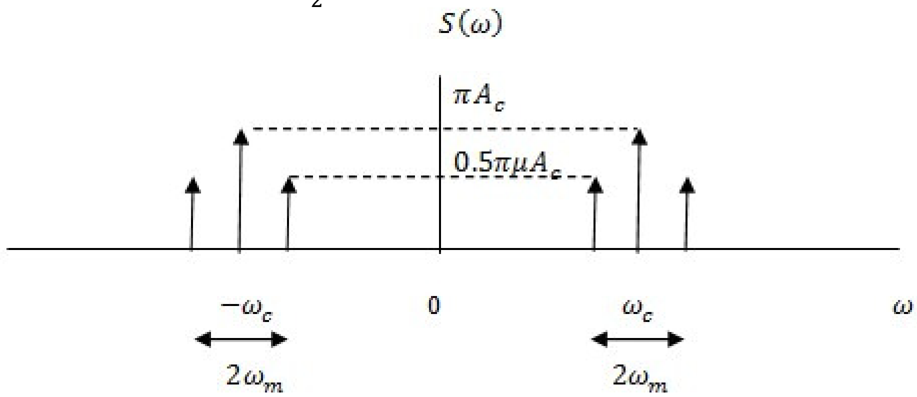
Therefore $M(\omega) = \tau\left[\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}\right]^2$

b) i)

$$\mathrm{s(t)} = \mathrm{A_c}\cos(\omega_c t) + \mu\,\mathrm{A_c}\cos(\omega_c t)\cos(\omega_m t)$$
$$= \mathrm{A_c}\cos(\omega_c t) + \mu\frac{\mathrm{A_c}}{2}[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)\,t]$$

Taking the Fourier transform,

$$S(\omega) = \pi A_c[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$$
$$+\pi\mu\frac{\mathrm{A_c}}{2}[\delta(\omega - \omega_c - \omega_m) + \delta\,(\omega + \omega_c + \omega_m)]$$
$$+\pi\mu\frac{\mathrm{A_c}}{2}[\delta(\omega - \omega_c + \omega_m) + \delta\,(\omega + \omega_c - \omega_m)]$$



ii) The average power of a cosine signal is $\frac{A^2}{2}$ if A is the amplitude

Therefore the average power in the carrier signal is $\frac{{A_c}^2}{2}$, the average power in the side bands is $\frac{1}{2}\left(\frac{\mu A_c}{2}\right)^2 \times 2 = \frac{\mu^2 {A_c}^2}{4}$ (since there are 2 sidebands).

The ratio of the average power in the sidebands to the total average power is

$$\frac{\frac{\mu^2 {A_c}^2}{4}}{\frac{\mu^2 {A_c}^2}{4} + \frac{{A_c}^2}{2}} = \frac{\mu^2}{\mu^2 + 2}$$