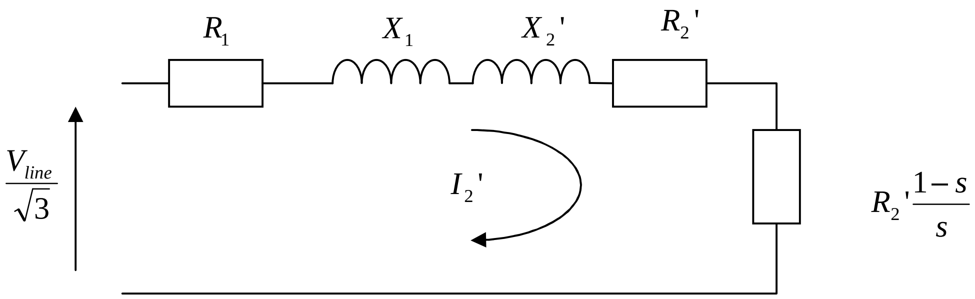
Tutorial Sheet 5 – Induction Motor Solutions

Equivalent circuit of an induction machine:



Question 1

$$Ns = \frac{60f}{p} = \frac{60\times 50}{2} = 1500\,rpm$$

$$Slip\ speed = 1500 - 1455 = 45\,rpm$$

$$Slip, s = \frac{45}{1500} = 0.03$$

a) rated full-load current, I_2'

$$I_2' = \frac{V}{Z} = \frac{V_{phase}}{\sqrt{\left(R_1 + \frac{R_2}{s}\right)^2 + (X_1 + X_2)^2}} = \frac{415/\sqrt{3}}{\sqrt{\left(0.2 + \frac{0.1}{0.03}\right)^2 + (0.9 + 0.7)^2}} = \underline{\underline{61.8A}}$$

b) Torque delivered by the motor

$$T = 3p \times \left(\frac{V}{Z}\right)^2 \times \frac{R_2'}{2\pi fs} = 3\times 2\times \frac{\left(415/\sqrt{3}\right)^2}{\left(0.2 + \frac{0.1}{0.03}\right)^2 + (0.9+0.7)^2} \times \frac{0.1}{2\pi(50)(0.03)} = \underline{\underline{243Nm}}$$

c) output power

$$P_{out} = \frac{I_2'^2\, R_2'(1-s)}{s} = \frac{61.77^2\,(0.1)(1-0.03)}{0.03} = \underline{\underline{12.3kW}}$$

d) power factor, cosθ, will be lagging because of inductive load…

$$P_{in} = I_2'^2\left(R1 + \frac{R_2}{s}\right) = 13.483kW$$

$$>>> P = VI\cos\theta \; then \cos\theta = \frac{P}{VI} = \frac{13.483}{415/\sqrt{3}\,(61.77)} = \underline{\underline{0.91lag}}$$

e) efficiency

$$\frac{P_{out}}{P_{in}} \times 100\% = \frac{12.3}{13.483} \times 100\% = \underline{\underline{91.6\%}}$$

Question 2

The essence of this question involves understanding that you are retaining the same T-w envelope, but using the supply frequency to change the speed. **Note: in the lecture notes the boost deals with a dc-voltage (and hence all impedance = 0), in this question the voltage is ac and hence the frequency change must be taken into account.**

a) The rotor frequency will be the same as in the first part at full-load speed….

$$f_2 = sf_1 = (0.03)(50) = 1.5Hz$$

Half sync speed is 750rpm or 25Hz, to achieve this at full torque in the same envelope the machine must be supplied with 25+1.5 = <u>26.5Hz</u>

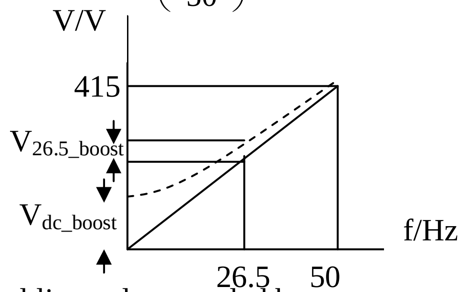
b) V_{phase} is calculated from the current (same current as same torque) and the impedance (different impedance as different frequency and new slip at new speed)

$$s = \frac{N_s - N_r}{N_s} = \frac{26.5 - 25}{26.5} = 0.0566$$

$$V_{phase} = I_2'\left[\left(R_1 + \frac{R_2}{s}\right) - j(X_1 - X_2)\right] = 61.77\left(0.2 + \frac{0.1}{0.0566}\right) - j(0.9 - 0.7)\left(\frac{26.5}{50}\right)$$

$$= 61.77 \times \sqrt{(1.966)^2 + (0.848)^2} = 132.6V$$

$$V_{line} = \sqrt{3}V_{phase} = \underline{\underline{229.6V}}$$



c) Again this is an ac boost…(see graph)
if it was $V_{dc\_boost} = IR_1 = 61.8 \times 0.2 = 12.36V$.
So we know the value is between 0 and 12.36V.
So $V_{line\_old} = 415V$, $V_{line\_new} = 229.6V$ (dashed line), the original line voltage scaled by the frequency (solid line in the graph) = 415*26.5/50. Difference is the boost.

$$V_{boost} = V_{line\_new} - V_{line\_scaled} = V_{line\_new} - V_{line}\frac{f_{new}}{f_{old}} = (229.6) - (415)\left(\frac{26.5}{50}\right) = \underline{\underline{10.05V}}$$

d) same current as same torque….<u>61.8A</u>

e) power factor (lagging: inductive)

$$\tan\theta = \frac{jX}{R} = \frac{0.848}{1.966} \Rightarrow \theta = 23.33°$$

$$\cos\theta = \underline{\underline{0.91 lag}}$$

f) efficiency

$$\frac{P_o}{P_i} = \frac{I_2'^2 R_2 \frac{1-s}{s}}{I_2'^2 \left(R_1 + \frac{R_2'}{s}\right)} = \frac{0.1\frac{1-0.0566}{0.0566}}{\left(0.2 + \frac{0.1}{0.0566}\right)} = \underline{\underline{84.7\%}}$$

Question 3

This time we are increasing the rotor resistance in order to reduce the speed.

$$R_2' = x\Omega$$

$$s = \frac{1500-750}{1500} = 0.5$$

a) Calculate R_2'for Full load torque, half sync speed, rated voltage

$$T = \frac{3p}{2\pi f s} \times \left(\frac{V_{line}}{\sqrt{3}}\right)^2 x \frac{R_2'}{\left(R_1 + \frac{R_2}{s}\right)^2 + \left(X_1 + X_2\right)^2} \Rightarrow \frac{6\pi f s T}{3pV_{line}^{\ 2}} = \frac{x}{\left(0.1 + \frac{x}{0.5}\right)^2 + \left(0.9+0.7\right)^2}$$

$$\frac{3pV_{line}^{\ 2}}{6\pi f s T} x = 0.04 + 0.8x + 4x^2 + 2.56 \Rightarrow 4x^2 + \left(0.8 - \frac{3pV_{line}^{\ 2}}{6\pi f s T}\right) x + 2.6 = 4x^2 - 8.22x + 2.6 = 0$$

$$use\ \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \underline{\underline{0.39\Omega}}$$

b) Rotor current with 1:2 rotor:stator winding ratio

$$I_2' = \frac{N_2}{N_1} I_2 \rightarrow \frac{V_{phase}}{\sqrt{\left(0.1 + \frac{0.39}{0.5}\right)^2 + \left(0.9+0.7\right)^2}} \times \frac{1}{2} = \underline{\underline{63.8A}}$$

c) efficiency, power out is only 2 x copper losses for s = 0.5, so the efficiency will be poor

$$\frac{P_o}{P_i}=\frac{I_2'^2 R_2 \frac{1-s}{s}}{I_2'^2\left(R_1+\frac{R_2'}{s}\right)}=\frac{0.39\frac{1-0.5}{0.5}}{\left(0.2+\frac{0.39}{0.5}\right)}=\underline{\underline{39.7\%}}$$

Question 4

This time we have replaced the wound rotor with a cage rotor and will control the speed with V-I adjustment.

a) new line voltage

$$s=\frac{1500-750}{1500}=0.5$$

$$T_{\frac{1}{2}}=\frac{243}{2}=121.5\,Nm=\frac{3p}{2\pi fs}\times\left(\frac{V_{line}}{\sqrt{3}}\right)^2 x\frac{R_2'}{\left(R_1+\frac{R_2}{s}\right)^2+\left(X_1+X_2\right)^2}$$

$$\Rightarrow 121.5=\frac{6}{2\pi(50)(0.5)}\times\frac{V_{line}{}^2}{3}x\frac{0.3}{\left(0.1+\frac{0.3}{0.5}\right)^2+\left(0.9+0.7\right)^2}$$

$$V_{line}=\sqrt{(121.5)\pi(50)(0.5)\times\frac{\left(0.1+\frac{0.3}{0.5}\right)^2+\left(0.9+0.7\right)^2}{0.3}}=\underline{\underline{319V}}$$

b) new current

$$I_2'=\frac{V}{Z}=\frac{V_{phase}}{\sqrt{\left(R_1+\frac{R_2}{s}\right)^2+\left(X_1+X_2\right)^2}}=\frac{319/\sqrt{3}}{\sqrt{\left(0.2+\frac{0.3}{0.5}\right)^2+\left(0.9+0.7\right)^2}}=\underline{\underline{103A}}$$

c) efficiency

$$\frac{P_o}{P_i}=\frac{I_2'^2 R_2 \frac{1-s}{s}}{I_2'^2\left(R_1+\frac{R_2'}{s}\right)}=\frac{0.3\frac{1-0.5}{0.5}}{\left(0.2+\frac{0.3}{0.5}\right)}=\underline{\underline{37.5\%}}$$