Solution

Question 1

(a)

Avionic systems are usually subdivided into 5 groups, which are:

- Systems interfacing directly with the pilot.
- Aircraft state sensors
- Navigation systems.
- External world sensors.
- Task automation systems.

(b)

i. Since the aircraft is flying in the troposphere region, therefore, the static temperature is related to altitude by:

$$T_S = T_O - L \times H \quad \Rightarrow \quad H = \frac{(T_O - T_S)}{L} = \frac{(288.15 - 255)}{6.5 \times 10^{-3}} = 5104\,\text{m}$$

L is the troposphere lapse rate.

The speed of sound A for $T_S = 255^o K$ is given by:

$$A = \sqrt{\gamma\, R_a\, T_S} = \sqrt{1.4 \times 287.0529 \times 255} = 320.12 \text{ m/s}$$

Therefore,

$$V_T = M \times A = 0.6 \times 320.12 = 192.07\,\text{m/s} = 691.48\,\text{km/h}$$

ii. The impact pressure is given by:

$$P_T - P_S = P_0\left(\left(1 + \frac{(\gamma - 1)(V_c / A_0)^2}{2}\right)^{\gamma/(\gamma-1)} - 1\right)$$

$$= 101.325 \times \left(\left(1 + \frac{(1.4 - 1)(138.88/340.3)^2}{2}\right)^{1.4/(1.4-1)} - 1\right) = 12.313\,\text{kPa}$$

Furthermore,

$$\frac{P_T}{P_s} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{(\gamma - 1)}}$$

$$= \left(1 + \frac{(1.4 - 1)}{2} \times 0.6^2\right)^{\frac{1.4}{(1.4-1)}} = 1.275$$

Therefore,

$$P_T - P_s = 1.275 \times P_s - P_s = 12.313 \Rightarrow P_s = 44.77\,\text{kPa}$$

And

$$P_T = 1.275 \times P_s = 57.08\,\text{kPa}$$

(c)

i. The air temperature T_s and Mach number M:

In the troposphere region the temperature reduces linearly with altitude, until 11000m, after which the temperature remains fairly constant until 20000m. Therefore, the temperature is the same as the one at the tropopause height of 11000m, and is given by:

$$T_s = T_o - L \times H = 288.15 - 6.5 \times 10^{-3} \times 11000 = 216.65\,^oK$$

The speed of sound is then given by:

$$A = \sqrt{\gamma\, R_a\, T_s} = \sqrt{1.4 \times 287.0529 \times 216.65} = 295 \text{ m/s}$$

And the mach number M is given by:

$$M = \frac{V_T}{A} = \frac{250}{295} = 0.847$$

ii. The impact pressure:

$$\frac{P_T}{P_s} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \Rightarrow P_T = P_s\left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$= 18.82 \times \left(1 + \frac{(1.4 - 1)}{2} 0.847^2\right)^{\frac{1.4}{1.4 - 1}} = 30.08\,\text{kPa}$$

Therefore,

$$Q_c = P_T - P_s = 30.08 - 18.82 = 11.26\,\text{kPa}$$

iii. The calibrated airspeed V_c and the impact pressure are related by:

$$\text{Impact pressure} = P_0\left(\left(1 + \frac{(\gamma - 1)(V_c/A_0)^2}{2}\right)^{\gamma/(\gamma - 1)} - 1\right) = P_T - P_s$$

Furthermore, at sea level the speed of sound is given by:

$$A_o = \sqrt{\gamma\, R_a\, T_o} = \sqrt{1.4 \times 287.0529 \times 288.15} = 340.29\,\text{m/s}$$

Therefore,

$$\frac{(P_T - P_s)}{P_0} + 1 = \left(1 + 0.2(V_c/A_0)^2\right)^{3.5} \Rightarrow 1 + 0.2(V_c/A_0)^2 = \sqrt[3.5]{\frac{(P_T - P_s)}{P_0} + 1}$$

$$\Rightarrow V_c = A_o\sqrt{\frac{\sqrt[3.5]{\frac{(P_T - P_s)}{P_0} + 1} - 1}{0.2}} = 340.29 \times \sqrt{\frac{\sqrt[3.5]{\frac{(30.08 - 18.82)}{101.325} + 1} - 1}{0.2}}$$ =133.03 m/s = 478.91 km/h.

Question 2

(a)

i. When each channel is equipped with a monitor, the number of failures which can be tolerated is 3

ii. If non-adaptive majority voting is adopted, the majority of the initial channels must function for the system to function. Therefore, for given *4* independent channels the number failures which can be tolerated is 1:

iii. When adaptive majority voting is adopted, a failed channel is disconnected and is no longer part of the set of valid alternatives. Therefore, the system will function correctly as long as 2 channels are still functioning. Therefore, the maximum number of failures which can be tolerated is 2.

(b)

i. Since if one of the circuits fail the electronic unit fails, therefore, the failure rate of the signal conditioning unit is given by:

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3$$

$$\lambda_1 = 2\times10^{-9}\times10 + 10\times10^{-9}\times4 + 5\times10^{-9}\times5 + 100\times10^{-9}\times1$$
$$+45\times10^{-9}\times2 + 7\times10^{-9}\times2 = 289\times10^{-9}\ /\text{hour}\quad\text{for circuit 1}$$

$$\lambda_2 = 2\times10^{-9}\times12 + 10\times10^{-9}\times5 + 5\times10^{-9}\times4 + 100\times10^{-9}\times2$$
$$+45\times10^{-9}\times3 + 7\times10^{-9}\times2 = 443\times10^{-9}\ /\text{hour}\quad\text{for circuit 2}$$

$$\lambda_3 = 2\times10^{-9}\times15 + 10\times10^{-9}\times4 + 5\times10^{-9}\times3$$
$$+45\times10^{-9}\times1 + 7\times10^{-9}\times3 = 151\times10^{-9}\ /\text{hour}\quad\text{for circuit 3}$$

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 883\times10^{-9}\ /\text{hour}$$

and the mean time to failure is given by:

$$MTTF = \frac{1}{\lambda} = 1.13\times10^{6}\ \text{hours}\ \ \text{for the electronic unit.}$$

ii. The probability of survival is given by the reliability function $R(t) = e^{-\lambda t}$. Therefore, the probability of failure during a 15-hour operation is given by:

$$F(t = 15\,\text{hours}) = 1 - e^{\left(-883\times10^{-9}\times15\right)} = 132\times10^{-7}$$

iii. Since majority voting is adopted, the system is an *m*-out-of-*n* system with *m*=2 and *n=3*. Therefore, the reliability function will be given by:

$$R(t)=\sum_{k=m}^{n}\frac{n!}{k!(n-k)!}\left[e^{-\lambda kt}\right]\left[1-e^{-\lambda t}\right]^{n-k}$$

where m=2 and n=3, therefore,

$$R(t)=\sum_{k=2}^{3}\frac{3!}{k!(3-k)!}\left[e^{-\lambda kt}\right]\left[1-e^{-\lambda t}\right]^{3-k}$$

$$\Rightarrow R(t)=3e^{(-2\lambda t)}\left(1-e^{(-\lambda t)}\right)^{3-2} \qquad (k=2)$$

$$+e^{(-3\lambda t)}\left(1-e^{(-\lambda t)}\right)^{3-3} \qquad (k=3)$$

$$\Rightarrow R(t)=3e^{(-2\lambda t)}-2e^{(-3\lambda t)}$$

Therefore, the probability of failure is given by:

$$F(t)=1-R(t)=1-3e^{(-2\lambda t)}+2e^{(-3\lambda t)}$$

and for a 15-hour operation:

$$F(t=15)=1-3e^{\left(-2\times 883\times 10^{-9}\times 15\right)}+2e^{\left(-3\times 883\times 10^{-9}\times 15\right)}$$
$$=526\times 10^{-12}$$

iv. The mean time to failure is generally given by:

$$MTTF=\int_0^{\infty}R(t)dt=\int_0^{\infty}\left(3e^{(-2\lambda t)}-2e^{(-3\lambda t)}\right)dt$$

$$=\left[-\frac{3}{2\lambda}e^{(-2\lambda t)}+\frac{2}{3\lambda}e^{(-3\lambda t)}\right]_0^{\infty}$$

$$=\frac{3}{2\lambda}-\frac{2}{3\lambda}=\frac{5}{6\lambda}=0.94\times 10^{6}\text{ hours}$$

v. The system is an m-out-of-n passive system with m=1 and n=*2*. Therefore, the reliability function will be given by:

$$R(t)=e^{-\lambda mt}\sum_{k=m}^{n}\frac{(m\lambda t)^{k-m}}{(k-m)!}$$

where m=1 and n=2, therefore,

$$R(t)=e^{-\lambda t}\sum_{k=1}^{n=2}\frac{(\lambda t)^{k-1}}{(k-1)!}$$

$$\Rightarrow R(t)=e^{-\lambda t}\frac{(\lambda t)^{1-1}}{(1-1)!} \qquad (k=1)$$

$$+e^{-\lambda t}\frac{(\lambda t)^{2-1}}{(2-1)!} \qquad (k=2)$$

$$\Rightarrow R(t)=e^{(-\lambda t)}(1+\lambda t)$$

therefore, the probability of failure is given by:

$$F(t)=1-R(t)=1-e^{(-\lambda t)}(1+\lambda t)$$

and for a 15-hour operation:

$$F(t=15)=1-R(t)=$$
$$=1-e^{\left(-883\times10^{-9}\times15\right)}\left(1+883\times10^{-9}\times15\right)$$
$$=877\times10^{-13}$$

vi. The hazard rate is given by:

$$z(t)=\frac{f(t)}{R(t)}, \text{ where } f(t)=\frac{dF(t)}{dt}=-\frac{dR(t)}{dt}=\lambda^2 te^{-\lambda t}$$

Therefore,

$$z(t)=\frac{f(t)}{R(t)}=\frac{\lambda^2 te^{-\lambda t}}{e^{-\lambda t}(1+\lambda t)}=\frac{\lambda^2 t}{(1+\lambda t)}$$

And when $t\to\infty$ then $z(t)\to\lambda$, therefore, is always $z(t)\le\lambda$.

Question 3

(a)

i. The torque delivered by the brushless servo motor is given by:

$$T_m = \frac{\lambda}{2\pi n} a x = \frac{5.5\times 10^{-3}}{2\times\pi\times 35}\times 30000 = 0.75\,\mathrm{Nm}$$

ii. The armature current and the torque are related by:

$$T_m = k I \quad \Rightarrow \quad I = \frac{T_m}{k} = \frac{0.75}{0.51} = 1.47\ \mathrm{A}$$

And the copper loss of the motor:

$$P_c = R\times I^2 = 8\times 1.47^2 = 17.28\ \mathrm{W}$$

iii. the efficiency is given by:

$$\eta = \frac{F_a\times v}{F_a\times v + I^2 R} = \frac{30000\times 2.0\times 10^{-3}}{30000\times 2.0\times 10^{-3} + 1.47^2\times 8} = 77.6\%$$

(b)

i. For a parabolic velocity profile, the angular displacement is given by:

$$\theta(t) = \int_0^t v(u)du = \int_0^t 4\Omega_m\left(\frac{u}{T} - \frac{u^2}{T^2}\right)du$$

$$= 4\Omega_m\left(\frac{u^2}{2T} - \frac{u^3}{3T}\right)_0^t = 4\Omega_m\left(\frac{t^2}{2T} - \frac{t^3}{3T}\right)$$

ii. The maximum speed of the brushless servo motor is given by:

$$\theta(t) = 4\Omega_m\left(\frac{t^2}{2T} - \frac{t^3}{3T^2}\right)$$

$$\Rightarrow \theta_m = \theta(T) = 4\Omega_m\left(\frac{T^2}{2T} - \frac{T^3}{3T^2}\right) = 4\Omega_m T\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$\Rightarrow \Omega_m = \frac{3}{2}\frac{\theta_m}{T} = \frac{3}{2}\times\frac{63}{0.3} = 315\,\mathrm{rad/s}$$

iii. The energy is given by:

$$E = \int_0^T T_m \times \Omega(t)dt = \int_0^T 4T_m\Omega_m\left(\frac{t}{T} - \frac{t^2}{T^2}\right)dt = 4\Omega_m T_m\left(\frac{t^2}{2T} - \frac{t^3}{3T^2}\right)_0^T$$

$$= 4\Omega_m T_m\left(\frac{T^2}{2T} - \frac{T^3}{3T^2}\right) = 4\Omega_m T_m\left(\frac{T}{2} - \frac{T}{3}\right) = \frac{2}{3}\Omega_m T_m T$$

iv. The some of the torques is given by:

$$\sum Torques = T_{tot}(t) = J_m \frac{d\Omega}{dt}$$

And since $\Omega(t) = 4\Omega_m\left(\frac{t}{T} - \frac{t^2}{T^2}\right) \Rightarrow \frac{d\Omega(t)}{dt} = 4\Omega_m\left(\frac{1}{T} - 2\frac{t}{T^2}\right)$**,** therefore**,**

$$T_{tot}(t) = 4J_m\,\Omega_m\left(\frac{1}{T} - 2\frac{t}{T^2}\right)$$

$$\sum Torques = T_{tot}(t) = J_m \frac{d\Omega}{dt}$$

And since $\Omega(t) = 4\Omega_m\left(\frac{t}{T} - \frac{t^2}{T^2}\right) \Rightarrow \frac{d\Omega(t)}{dt} = 4\Omega_m\left(\frac{1}{T} - 2\frac{t}{T^2}\right)$**,** therefore**,**

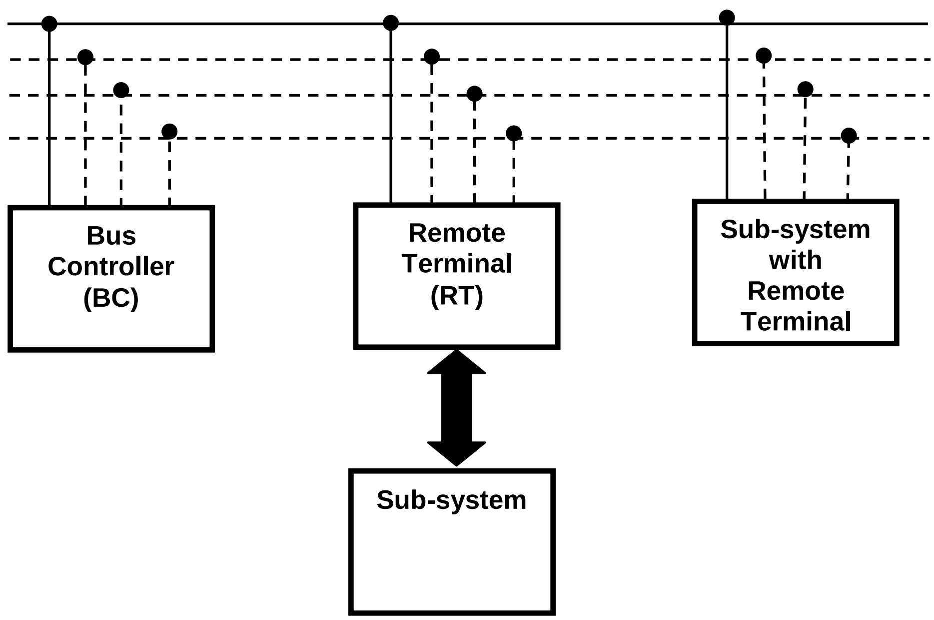
$$T_{tot}(t) = 4J_m\,\Omega_m\left(\frac{1}{T} - 2\frac{t}{T^2}\right)$$

The maximum torque occurs at t=0, therefore,

$$T_{tot}(t) = 4J_m\,\Omega_m\left(\frac{1}{T} - 2\frac{t}{T^2}\right) = 4\times 2.45\times 10^{-4}\times 315\times\left(\frac{1}{0.3} - 2\frac{0}{0.3^2}\right) = 1.03\,\text{Nm}$$

Question 4

(a)



Schematic of MIL STD 1553B bus system

Its main features can be summarised below:

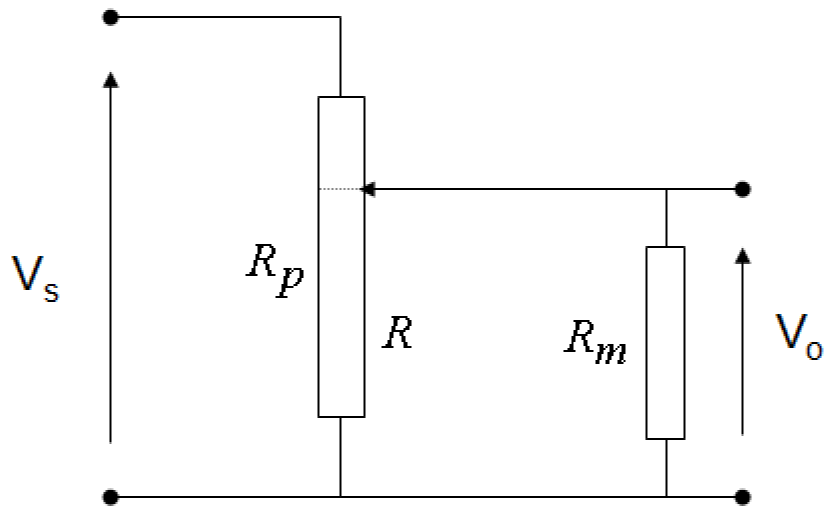
- Each sub-system is connected to the bus through a Remote Terminal (RT).
- Data can only be transmitted from one RT and received by one or more RT(s), following a command from the BC to each RT.
- The bus is formed of a single twisted cable pair with a layer of shielding, with a maximum length of 100m.
- Data is transmitted at 1Mbits/s.
- The technique adopted for data encoding is ‘Manchester bi-phase’.
- Number of terminals is limited to 31.

b)

i. The maximum voltage, which can be applied, is limited by the maximum power, which can be dissipated by the potentiometer, therefore,

$$P_{\max} = \frac{V_{s\max}^2}{R_p} \Rightarrow V_{s\max} = \sqrt{P_{\max} \times R_p} = \sqrt{0.020 \times 20000} = 20 \ \text{V}$$

ii. the output voltage of the potentiometer is given by:



Circuit of potentiometer

$$\left.\begin{aligned} V_o &= \frac{(R_m // R)}{(R_m // R) + (R_p - R)} V_s \\ R &= a R_p \end{aligned}\right\} \Rightarrow V_o = \frac{a}{\left(1 + a(1-a)\dfrac{R_p}{R_m}\right)} V_s$$

For x=10mm:

$$V_o = \frac{a}{\left(1 + a(1-a)\dfrac{R_p}{R_m}\right)} V_s = \frac{0.25}{\left(1 + 0.25 \times (0.75)\dfrac{20}{100}\right)} \times 20 = 4.82 \, \text{V}$$

For x=20mm:

$$V_o = \frac{a}{\left(1 + a(1-a)\dfrac{R_p}{R_m}\right)} V_s = \frac{0.5}{\left(1 + 0.5 \times (0.5)\dfrac{20}{100}\right)} \times 20 = 9.52 \, \text{V}$$

iii. For a potentiometer the maximum error occurs when $a = 0.5$, therefore,

$$Error(\%) = 100 \times \frac{a(1-a)\dfrac{R_p}{R_m}}{1 + a(1-a)\dfrac{R_p}{R_m}} \leq 0.25$$

$$\Rightarrow a(1-a)\frac{R_p}{R_m} \leq 0.0025 \times \left(1 + a(1-a)\frac{R_p}{R_m}\right)$$

$$\Rightarrow a(1-a)\frac{R_p}{R_m} \times (1 - 0.0025) \leq 0.0025$$

$$\Rightarrow \frac{R_p}{R_m} \leq \frac{0.0025}{0.9975 \times a(1-a)}$$

$$\Rightarrow R_m \geq \frac{0.9975 \times a(1-a) \times R_p}{0.0025} = \frac{0.9975 \times 0.5 \times (1-0.5) \times 20000}{0.0025}$$

$$\Rightarrow R_m \geq 1.995\,\mathrm{M\Omega}$$

(c)

i. The truth table of the logic circuit is as follows:

Position (mm)	*ADC output*					
	b_2	b_1	b_0	Red	Yellow	Green
0	0	0	0	1	1	0
5	0	0	1	1	1	0
10	0	1	0	0	1	0
15	0	1	1	0	0	1
20	1	0	0	0	0	1
25	1	0	1	0	1	0
30	1	1	0	1	1	0
35	1	1	1	1	1	0

ii. The SOP expressions for the logic function Red:

$$\begin{aligned} Re\,d &= \overline{b_2}\,\overline{b_1}\,\overline{b_0} + \overline{b_2}\,\overline{b_1}\,b_0 + b_2\,b_1\,\overline{b_0} + b_2\,b_1 b_0 \\ &= \overline{b_2}\,\overline{b_1}\left(\overline{b_0} + b_0\right) + b_2\,b_1\left(\overline{b_0} + b_0\right) \\ &= \overline{b_2}\,\overline{b_1} + b_2\,b_1 \end{aligned}$$

iii. The POS expressions for the logic function Yellow:

$$
\begin{aligned}
Yellow &= \left(b_2 + \overline{b_1} + \overline{b_0}\right)\left(\overline{b_2} + b_1 + b_0\right) \\
&= b_2\overline{b_2} + b_2 b_1 + b_2 b_0 + \overline{b_1}\,\overline{b_2} + \overline{b_1} b_1 + \overline{b_1} b_0 + \overline{b_0}\,\overline{b_2} + \overline{b_0} b_1 + \overline{b_0} b_0 \\
&= b_2 b_1 + b_2 b_0 + \overline{b_1}\,\overline{b_2} + \overline{b_1} b_0 + \overline{b_0}\,\overline{b_2} + \overline{b_0} b_1 \\
&= b_2\left(b_1 + b_0\right) + \overline{b_1}\left(\overline{b_2} + b_0\right) + \overline{b_0}\left(\overline{b_2} + b_1\right)
\end{aligned}
$$