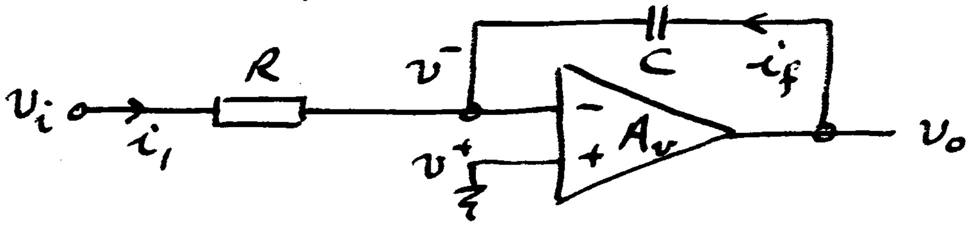
Frequency dependent feedback in op-amp circuits

- many applications exist where different frequencies must be amplified by different amounts.... eg, sound system tone adjustment or equalisation; filtering in instrumentation systems; .... etc

- many possible circuits but simplest and one of the most useful is.....

(i) The Integrator – Can be looked at from either a time domain or a frequency domain point of view....

time domain analysis – first write down time domain equations assuming as usual that the op-amp draws no input current.....



$$\frac{v_i - v^-}{R} = i_1 \qquad (1)$$

$$v_o - v^- = \frac{1}{C}\int i_f \, dt \qquad (2)$$

$$i_f + i_1 = 0 \qquad (3)$$

$$v_o = A_v(v^+ - v^-) \qquad (4)$$

Combining ① + ③ to eliminate i_1 and substituting the resulting expression for i_f into ② gives:

$$v_o - v^- = -\frac{1}{C}\int \frac{v_i - v^-}{R} \cdot dt \qquad (5)$$

Since $v^+ = 0$, ④ can be rewritten $v^- = -v_o/A_v$ and using this, ⑤ becomes:

$$v_o\left[1 + \frac{1}{A_v}\right] = -\frac{1}{CR}\int\left(v_i + \frac{v_o}{A_v}\right) dt$$

If $A_v \Rightarrow \infty$, $1 \gg \frac{1}{A_v}$ and $v_i \gg v_o/A_v$, in other words if v^- is a virtual earth,

$$v_o = -\frac{1}{CR}\int v_i \, dt$$

— ie, if A_v is large, the output voltage is the integral of the input voltage multiplied by $\frac{1}{CR}$, the "integrator gain". The "−" sign indicates phase inversion as usual.
— This circuit was the backbone of analogue computers because a system of integrators could be used to solve complicated differential equations.

__frequency domain analysis__ — this time write down the frequency domain relationships again assuming no op-amp input current is drawn .....

$$\frac{v_i - v^-}{R} = i_1 \qquad (6)$$

$$(v_o - v^-)sC = i_f \qquad (7)$$

$$i_f + i_1 = 0 \qquad (8)$$

$$v_o = A_v(v^+ - v^-) \quad \text{or} \quad v^- = -v_o/A_v \qquad (9)$$

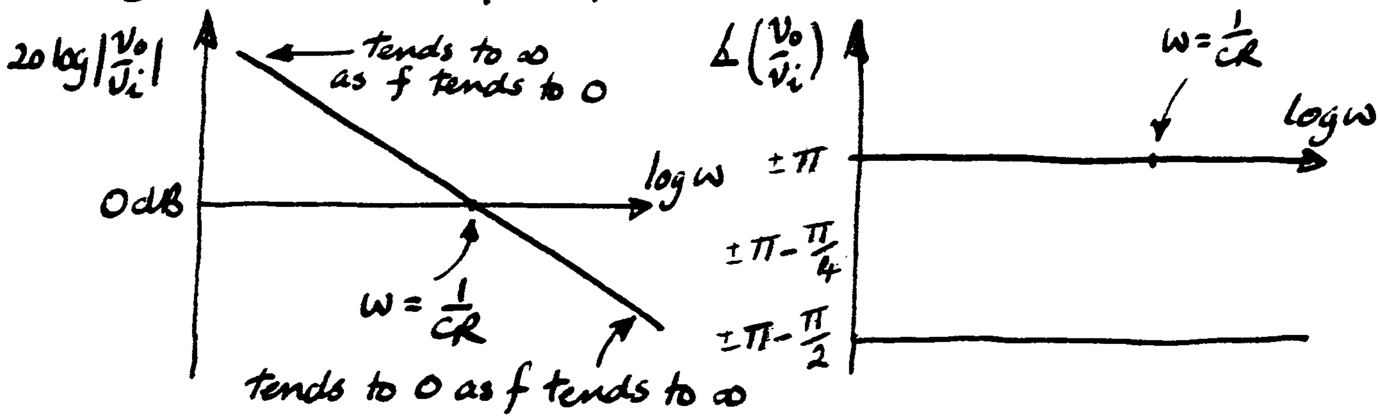
The same substitution strategy as before leads to:

$$\frac{v_i + v_o/A_v}{R} = -(v_o + v_o/A_v)sC$$

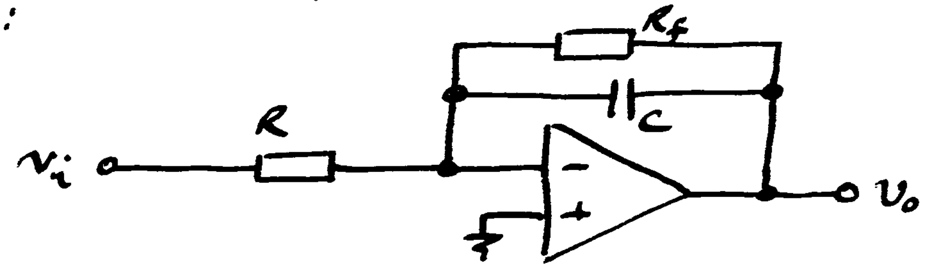
$$\text{or} \quad \frac{v_o}{v_i} = -\frac{1}{sCR(1+\frac{1}{A_v}) + \frac{1}{A_v}} \approx -\frac{1}{sCR} \quad \text{if } A_v \gg 1.$$

Note that "s" has been used here in place of "jω". If fact "jω" is a special case of "s" and the two are the same in the absence of any transient effects.

Integrator frequency response:

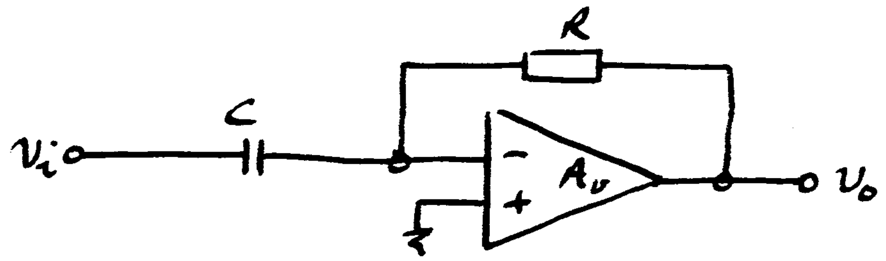


Note that a simple integrator circuit such as that shown will not work as an isolated circuit. Remember that all op-amps require some input current and all op-amps suffer from offset problems. Without d.c. feedback there is no mechanism for defining d.c. conditions so the circuit is usually modified in some way to provide either continuous or occasional d.c. feedback. One solution is shown below:



R_f provides d.c. feedback but also raises the lowest frequency at which the circuit behaves like an integrator — one can see at a glance that at 0Hz the circuit gain is $-R_f/R$ rather than the $-\infty$ that it should be. In practice $1/2\pi R_f C$ must be made much smaller than the lowest frequency of interest.

(ii) The Differentiator



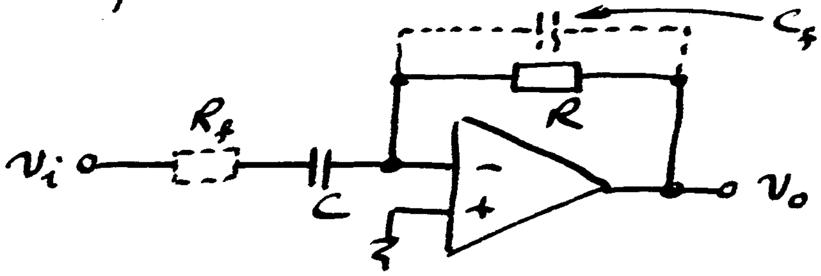
if op-amp draws no input current and $A_v \Rightarrow \infty$, a similar process as for the integrator leads to:

$$V_o = -CR\frac{dV_i}{dt} \quad \text{(time domain)}$$

$$\frac{V_o}{V_i} = -sCR \quad \text{(frequency domain)}.$$

— not often used in practice because the R + C interact with the first order frequency dependence of A_v, $A_v = \dfrac{A_0}{1+s\tau_0}$ to give a second order system that is underdamped over almost all the useful bandwidth of the amplifier.

– Where use of a differentiator is essential, the underdamped behaviour can be controlled by the addition of a suitable value resistor in series with C or the addition of a suitable value capacitor in parallel with R.

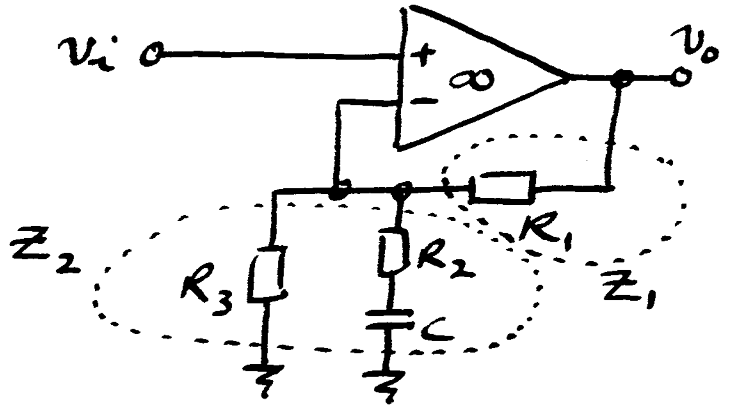


As with the integrator, the addition of these extra component(s) limits the range of frequencies over which the circuit functions as a differentiator.

(iii) <u>Pole Zero Circuits</u> (– also called lead-lag or lag-lead circuits.).

– used extensively for equalisation in audio and other fields
– used as phase compensation or correction in feedback circuits
– many different forms of the circuit are used but analysis approach similar in each case ...

one example ....



This is a non-inverting amplifier connection so

$$\frac{v_o}{v_i} = \frac{Z_1 + Z_2}{Z_2}$$

$$Z_1 = R_1$$

$$Z_2 = R_3 \,//\, (R_2 + 1/sC)$$

$$Z_2 = \frac{R_3(R_2 + 1/sC)}{R_3 + R_2 + 1/sC} = \frac{R_3(1+sCR_2)}{1+sC(R_2+R_3)}.$$

$$\therefore \frac{v_o}{v_i} = \frac{Z_1+Z_2}{Z_2} = \frac{R_1 + \dfrac{R_3(1+sCR_2)}{(R_2+R_3)sC+1}}{\dfrac{R_3(1+sCR_2)}{1+sC(R_2+R_3)}}$$

$$= \frac{R_1(1+sC(R_2+R_3)) + R_3(1+sCR_2)}{R_3(1+sCR_2)}$$

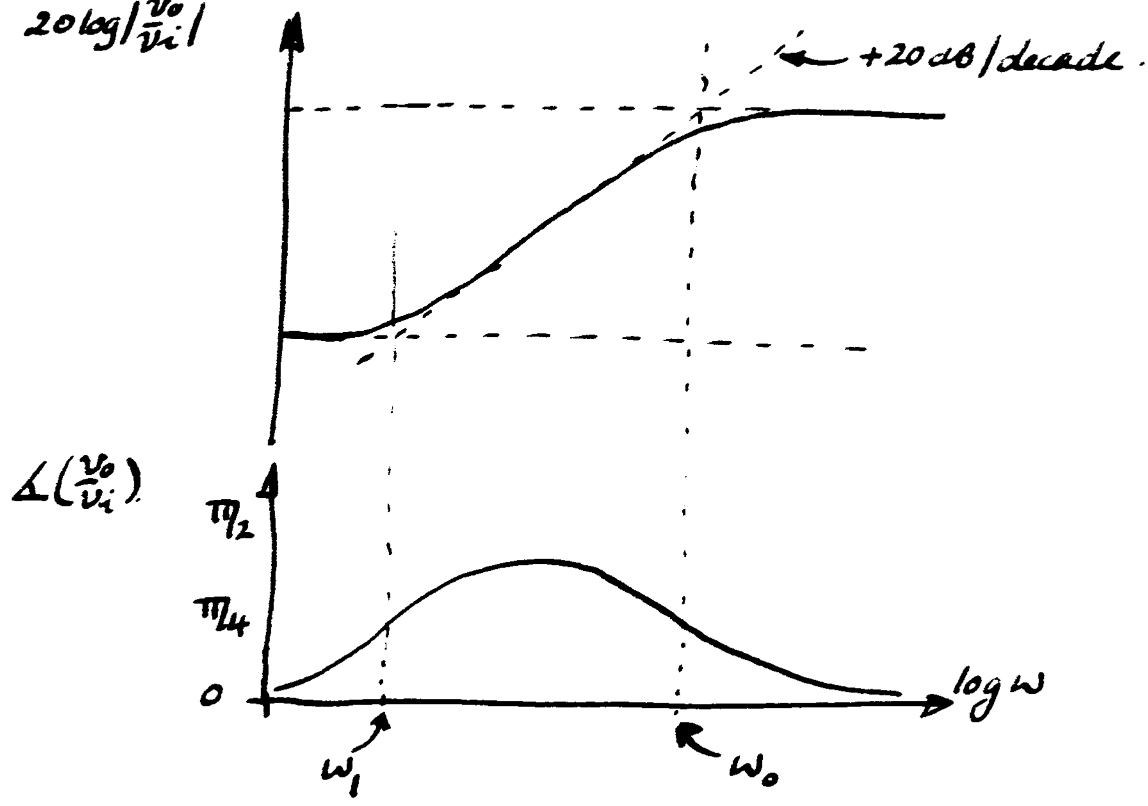
$$= \frac{R_1+R_3+sC(R_1R_3+R_1R_2+R_2R_3)}{R_3(1+sCR_2)}$$

$$= \frac{R_1+R_3}{R_3} \cdot \frac{1+sC\left[\dfrac{R_1R_2+R_1R_3+R_2R_3}{R_1+R_3}\right]}{1+sCR_2} \equiv k \cdot \frac{1+j\omega/\omega_1}{1+j\omega/\omega_0}$$

where $k = \dfrac{R_1+R_3}{R_3}$, $\omega_1 = \dfrac{R_1+R_3}{C(R_1R_2+R_1R_3+R_2R_3)}$

and $\omega_0 = 1/CR_2$

In this case the high frequency gain must be greater than the low frequency gain since at high frequencies C approaches a short circuit and $R_2 \| R_3 < R_3$ so the response will be:



Intrinsic Frequency Response of Op-amp

- most op-amps can be represented by a first order transfer function:

$$v_o = A_v(v^+ - v^-) \quad \text{where} \quad A_v = \frac{A_o}{1 + j\,\omega/\omega_o}$$

A_o is the open loop d.c. gain (typ 10^4 to 10^7 V/V)

ω_o is the open loop corner frequency (typ 60 to 600 rads/sec or ≈ 10 to 100 Hz)

- the first order response is engineered by manufacturers because it makes the op-amps easy to use.

[In fact, without this deliberate imposition of first order behaviour, an op-amp would be at least a third order circuit with serious implications for ease of use. The penalty for forcing a first order behaviour on the op-amp is a significant loss of gain at high frequencies]

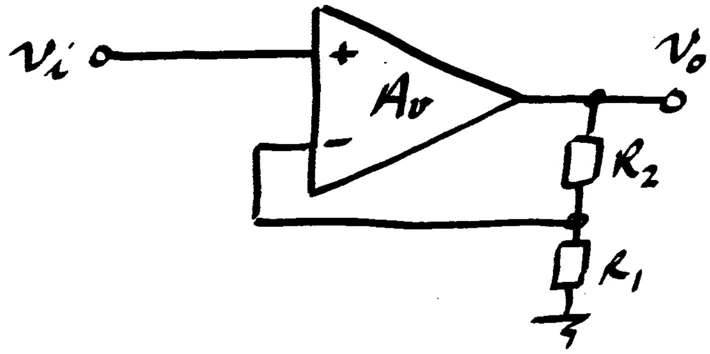
- frequency domain op-amp performance is usually specified by manufacturers in the form of "gain-bandwidth product", GBP, or "unity gain frequency". Both these terms mean the same thing.

- GBP is simply the product of the open loop d.c. gain and the open loop corner frequency

$$\text{ie} \quad GBP = A_o\omega_o \ \text{rads/sec}$$
$$= A_o f_o \ \text{Hz} \quad (\omega_o = 2\pi f_o).$$

- GBP enables a user instantly to predict the frequency dependence of a circuit <u>due to the op-amp</u> .....

consider the non-inverting amplifier...



$$A_v = \frac{A_0}{1 + j^{\omega}/\omega_0}$$

The question of interest is, how is the gain-bandwidth product of the non-inverting amplifier circuit affected by the GBP of the op-amp used?

The analysis follows the usual lines....

$$v_o = A_v(v^+ - v^-) = A_v\left(v_i - \frac{v_o R_1}{R_1 + R_2}\right)$$

rearranging to give the gain,

$$v_o/v_i = \frac{1}{\frac{1}{A_v} + \frac{R_1}{R_1 + R_2}}.$$

now, using the first order form of A_v,

$$\frac{v_o}{v_i} = \frac{1}{\frac{1 + j^{\omega}/\omega_0}{A_0} + \frac{R_1}{R_1 + R_2}} = \frac{A_0}{1 + j^{\omega}/\omega_0 + \frac{A_0 R_1}{R_1 + R_2}}$$

$$= \frac{A_0}{1 + \frac{A_0 R_1}{R_1 + R_2}} \cdot \frac{1}{1 + j^{\omega}/\omega_0\left(1 + \frac{A_0 R_1}{R_1 + R_2}\right)}$$

$$\equiv \frac{A_0'}{1 + j^{\omega}/\omega_0'} \quad \text{where } A_0' = A_0 \Big/ \left(1 + \frac{A_0 R_1}{R_1 + R_2}\right)$$

$$\omega_0' = \omega_0\left(1 + \frac{A_0 R_1}{R_1 + R_2}\right)$$

[Note that if $A_0 \Rightarrow \infty$...

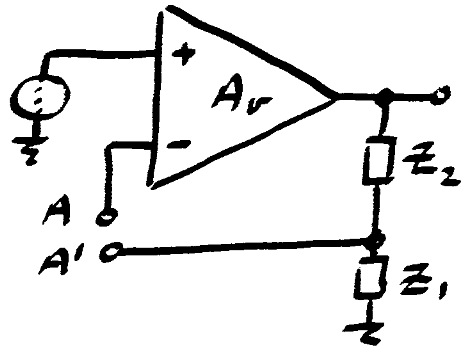
$A_0' = \dfrac{A_0}{1 + \frac{A_0 R_1}{R_1 + R_2}} \approx \dfrac{A_0}{\frac{A_0 R_1}{R_1 + R_2}} = \dfrac{R_1 + R_2}{R_1}$ as expected for a non-inv amp.]

the GBP of the non-inverting amplifier circuit is then

$$GBP = A_0' \omega_0' = \frac{A_0}{1+\frac{A_0 R_1}{R_1+R_2}} \cdot \omega_0\left(1+\frac{A_0 R_1}{R_1+R_2}\right) = \underline{\underline{A_0 \omega_0}}$$

This is an important result because it shows that the G.B.P. is a property only of the op-amp; it is independent of the non-inverting circuit gain. Thus if required circuit gain and bandwidth are known, the GBP necessary in the op-amp can be calculated.

– real reason why manufacturers go to the trouble to produce first order op-amps is the certainty of stability under normal (ie resistive) feedback conditions. For an op-amp to be unstable, the loop phase shift must be zero or 360°. In other words, 'if a signal injected at point A travels around the loop and appears at A' with the same amplitude + phase as it started with at A, the system will be unstable.'



If Z_2 + Z_1 are resistors and the op-amp has a first order frequency response, the largest phase shift possible between A + A' is −270° (or +90° depending upon whether a signal inversion is regarded as +180° or −180°) and the circuit is unconditionally stable.

– very easy to use
– don't need to know much about electronics to succeed with op-amp circuits.

Large Signal B.W. Limit – Slew Rate

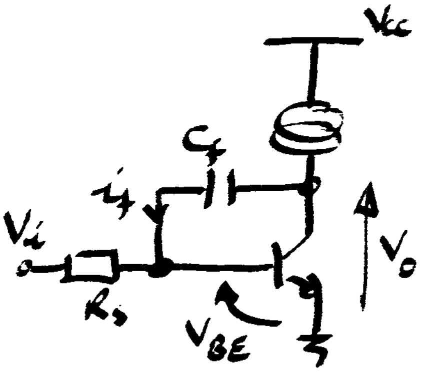
- GBP is a linear effect, ie circuit gain may change with frequency but it does not change with amplitude

- "Slew Rate" is a non-linear effect. When slew rate limiting is active, circuit gain is a function of signal amplitude and harmonic distortion is introduced – ie sinewave input gives a different shaped output.

- "Slew Rate" is the name given to the maximum rate of change of output voltage that can be supported by the op-amp.

- quoted by manufacturers as slew rate and given in units of V/μs.

- slew rate problems are always tackled by identifying the maximum $\frac{dV}{dt}$ in a particular signal and equating this value to slew rate.



The diagram opposite is a simplified version of an op-amp gain stage. C_f is the capacitor responsible for both GBP and slew rate effects.

If V_o rises, a current i_f given by $i_f = C\frac{dV_o}{dt}$ will flow through C_f. Most of this i_f will flow through R_s so if the minimum value of V_i is 0V, the maximum i_f must be approximately $0.7/R_s$. i_fs bigger than this will generate more than 0.7v across R_s and so will tend to turn the transistor on more, and hence reduce the rate of rise of V_o. In effect a dynamic equilibrium is set up for the duration of the change in V_o. The maximum $\frac{dV_o}{dt}$ that can be supported by the circuit is then given by

$$\frac{0.7}{R_s} = C_f\frac{dV_o}{dt}.$$

This is exactly the same sort of $\frac{dV}{dt}$ limiting as that you will encounter in EEE205 in the context of power mosfet switches.