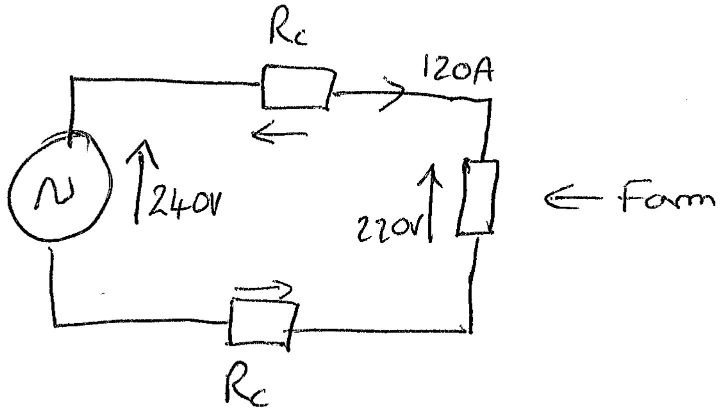
Q1



Rc is the resistance of the cable.

Applying Kirchoff's law around the loop.

$$240 - 120R_c - 220 - 120R_c = 0$$

$$\therefore \quad 20 = 240R_c$$

$$\therefore R_c = 0.0833\,\Omega$$

Since $R_c = \dfrac{\rho L}{A}$ then $A = \dfrac{\rho L}{R_c}$

$$\therefore A_{min} = \frac{1.8\times10^{-8}\times 800}{0.0833} = 1.728\times10^{-4}\,m^2 = \underline{\underline{17.3\,mm^2}}$$

If $A < 17.3\,mm^2$ R_c will be $> 0.0833\,\Omega$ and there will be a greater voltage drop.

(ii) Power consumed by the farm $= 220\times120 = \underline{\underline{26.4\,kW}}$

Power provided by the generator $= 240\times120 = \underline{\underline{28.8\,kW}}$

Hence the efficiency is $\dfrac{26.4\times10^3}{28.8\times10^3}\times100 = \underline{\underline{91.7\%}}$

(Check: Losses in cables $= 2I^2R_c = 2.120^2.0.0833 = 2.4\,kW$

$26.4 + 2.4 = 28.8\,kW$)

(b)(i) The factory draws a power of 100kW at 0.85 p.f. lag.

$$P = S\cos\phi \Rightarrow S = \frac{P}{\cos\phi} = \frac{100\times10^3}{0.85} = \underline{\underline{117.65\,\text{KVA}}}$$

(ii) Reactive power is given by:

$$Q = S\sin\phi = 117.65 \times \sin(\cos^{-1}0.85) = \underline{\underline{61.98\,\text{kVAR}}}$$

(iii) Current drawn from the supply:-

$$S = VI \Rightarrow I = \frac{S}{V} = \frac{117.65\times10^3}{800} = \underline{\underline{147\,A_{rms}}}$$

(C)(i) After the factory is enlarged the total load is as follows:

	P	Q
Original	100000	61980
Heaters	20000	0
Motor	84000	112000
	204000W	173980 VAR

$\therefore$ New KVA rating of the site is $\sqrt{P^2+Q^2} = \sqrt{204^2+173.98^2}$

$= \underline{\underline{268\,\text{KVA}}}$

(ii) The new power factor is $\frac{204}{268} = \underline{\underline{0.761\text{ lagging}}}$

$\left(\text{or } \tan^{-1}\left(\frac{173.98}{204}\right) = 40.46^\circ \Rightarrow \cos 40.46 = 0.761\right)$

(iii) To correct the power factor to unity the capacitor must supply 173.98kVAR leading.

The current drawn by the capacitor:

$$I_c = \frac{173.98\times10^3}{800} = 217.5\,\text{Arms}.$$

QUESTION 1 (CONTINUED)

$$X_C = \frac{V_C}{I_C} = \frac{800}{217.5} = 3.68\Omega.$$

Now $X_C = \dfrac{1}{2\pi f C}$ $\Rightarrow$ $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{100\pi . 3.68}$

$= 8.65 \times 10^{-4}$ F

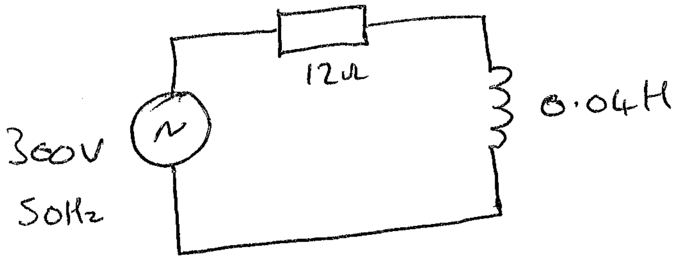
or 865 μF

(iv) The peak voltage the capacitor must withstand is:

$$V_{PK} = \sqrt{2}\, V_{rms} = \sqrt{2} \times 800 = \underline{\underline{1131 \text{ Volts}}}$$

QUESTION 2

(a)



(i) Impedance of the circuit:

$$Z = R + j\omega L = R + j2\pi f L = 12 + j2\pi.50.0.04$$
$$= 12 + j12.57$$
$$= \underline{\underline{17.4\angle 46.3^\circ \,\Omega}}$$

(ii) The current flowing in the coil is given by:

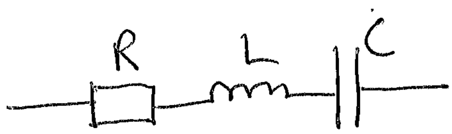
$$I = \frac{V}{Z} = \frac{300\angle 0^\circ}{17.4\angle 46.3^\circ} = \underline{\underline{17.24\angle -46.3^\circ}} \text{ Arms}$$

(iii) The real power drawn from the supply is:

$$P = I^2R = 17.24^2 \times 12 = \underline{\underline{3566.6\,W}}$$

(or alternatively $P = VI\cos\phi = 300 \times 17.24 \times \cos 46.3 \simeq 3570W$)

(b) (i)



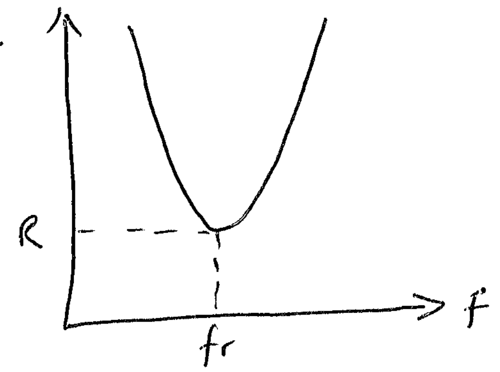
$$Z = R + j\omega L - \frac{j}{\omega C} = \underline{\underline{R + j\left(\omega L - \frac{1}{\omega C}\right)}}$$

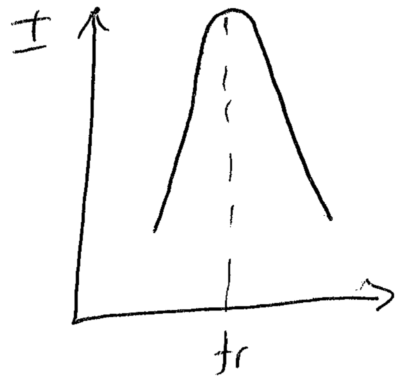
Circuit is resonant when the imaginary term is zero.

$$\therefore Z = R \quad \text{and} \quad \omega L = \frac{1}{\omega C}$$

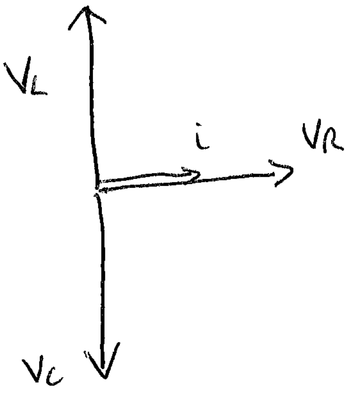
Hence $\omega_R^2 = \frac{1}{LC} \Rightarrow \omega_R = \frac{1}{\sqrt{LC}}$ or $\underline{\underline{f_R = \frac{1}{2\pi\sqrt{LC}}}}$

(ii)





(iii)



(iv) For maximum current the circuit must be at resonance i.e. $f_r = 50\text{Hz}$.

Since $f_r = \dfrac{1}{2\pi\sqrt{LC}} \Rightarrow C = \dfrac{1}{4\pi^2 f_r^2 L}$

$$\therefore C = \frac{1}{4 \cdot \pi^2 \cdot 50^2 \cdot 0.04} = \underline{\underline{253\mu F}}$$

Since at resonance $Z = R$ then the current will be

$$I = \frac{300}{12} = 25A$$

and the power dissipated will be $I^2R = 25^2 \times 12 = \underline{\underline{7500W}}$

QUESTION 2 (CONTINUED) 6

(c) If the frequency increases to 60Hz

then $jX_L = j\,2\pi.60.\,0{\cdot}04 = j15{\cdot}08\,\Omega$

and $-jX_c = \dfrac{-j}{2\pi.60.253\times10^{-6}} = -j\,10{\cdot}48\,\Omega$

$\therefore\ Z = 12 + j15{\cdot}08 - j\,10{\cdot}48 = 12 + j4{\cdot}6\,\Omega$

$= \underline{\underline{12{\cdot}85\angle 20{\cdot}97^\circ\,\Omega}}$

$I = \dfrac{300\angle 0^\circ}{12{\cdot}85\angle 20{\cdot}97^\circ} = \underline{\underline{23{\cdot}35\angle -20{\cdot}97^\circ}}$ Arms

$|V_R| = I.R = 23{\cdot}35\times 12 = 280{\cdot}2$ Vrms.

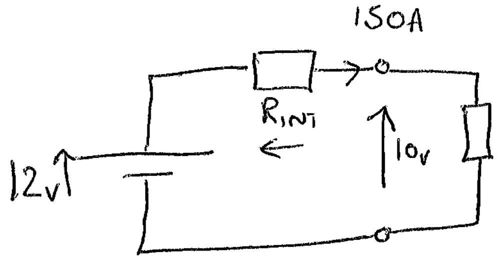
$|V_L| = I.X_L = 23{\cdot}35\times 15{\cdot}08 = 352{\cdot}1$ Vrms

$\therefore$ Voltage across the electromagnet $= \sqrt{V_R^2 + V_L^2}$

$= \sqrt{280{\cdot}2^2 + 352{\cdot}1^2} = \underline{\underline{450\text{ Vrms.}}}$

QUESTION 3

(a)



Applying Kirchoff's law:

$$12 - 150R_{INT} - 10 = 0$$

$$\therefore R_{INT} = \frac{2}{150} = \underline{\underline{0.0133\,\Omega}}$$

Power dissipated within the battery $= I^2 \times R_{INT} = 150^2 \times 0.01333$

$= \underline{\underline{299.925\,W}}$

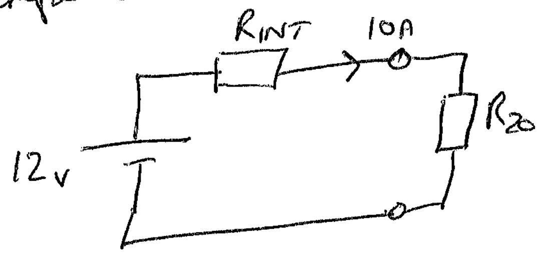
$P_{output} = V_{OUT} \times I = 10 \times 150 = 1500W$

$P_{BATT} = V_{BATT} \times I = 12 \times 150 = 1800W$

Hence efficiency $= \frac{P_{OUT}}{P_{BATT}} = \frac{1500}{1800} \times 100\% = \underline{\underline{83.3\%}}$

(Check $P_{out} + P_{LOSS} = 1500 + 299.9 \simeq 1800W$).

(b) First calculate the resistance at 20°C and at the final temperature:



$R_{INT} + R_{20} = \frac{V}{I} = \frac{12}{10} = 1.2\,\Omega$

Since $R_{INT} = 0.01333$

then $R_{20} = 1.1867\,\Omega$

Similarly at the final temperature

$$R_{INT} + R_F = \frac{12}{7} = 1.714\,\Omega$$

Hence $R_F = 1.714 - 0.01333 = 1.7007\,\Omega$

Now $$\frac{R_F}{R_{20}} = \frac{\cancel{R_0}}{\cancel{R_0}} \frac{(1+\alpha_0 . T_F)}{(1+\alpha_0 . 20)}$$

$$\therefore \frac{1.7007}{1.1867} = \frac{(1+12.5\times10^{-3}. T_F)}{(1+12.5\times10^{-3}. 20)}$$

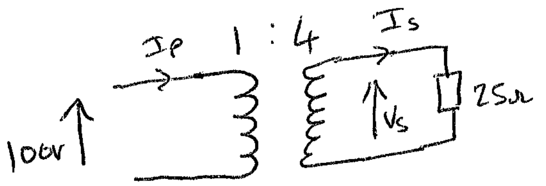
$$\therefore 1.433 = \frac{(1+12.5\times10^{-3} T_F)}{1.25}$$

$$\therefore T_F = \underline{\underline{63.3°C}}$$

(ii) At the final temperature Power $= I^2 . R_F = 7^2 \times 1.7007$
$= \underline{\underline{83.33W}}$

Hence the efficiency $= \frac{83.33}{12\times7} \times 100\% = \underline{\underline{99.2\%}}$

(c)



Since $\frac{V_P}{V_S} = \frac{N_P}{N_S} \Rightarrow V_S = \frac{N_S . V_P}{N_P} = \frac{4\times100}{1} = \underline{\underline{400V}}$

$I_S = \frac{400}{25} = 16$ Arms.

Now $\frac{I_P}{I_S} = \frac{N_S}{N_P} \Rightarrow I_P = \frac{I_S . N_S}{N_P} = \frac{16\times4}{1} = \underline{\underline{64 \text{ Arms}}}$

Power dissipated $= I^2R = 16^2 . 25 = \underline{\underline{6.4kW}}$

(ii) The secondary impedance is now:

$$Z_s = R + j.2\pi f L = 30 + j.2\pi.50.0.1$$

$$= 30 + j31.4 = 43.4\angle 46.3^\circ\ \Omega$$

$$I_s = \frac{400\angle 0^\circ}{43.4\angle 46.3^\circ} = 9.217\angle -46.3^\circ \text{ Arms}$$

$$\therefore I_p = 4 \times 9.217\angle -46.3^\circ = \underline{\underline{36.87\angle -46.3^\circ}}$$

Power dissipated in load $= I_s^2 . R_s = 9.217^2 . 30 = \underline{\underline{2.55\text{kW}}}$.

(Check $P = V_p I_p \cos\phi = 100 \times 36.87 \times \cos 46.3 = 2.55\text{kW}$)

(iii) The input power factor would be $\cos 46.3^\circ = \underline{\underline{0.69 \text{ lag}}}$

The VA rating $= 100 \times 36.87 = \underline{\underline{3678\text{ VA}}}$.

(iv). Since $V_{rms} = 4.44 f N \phi_{max}$

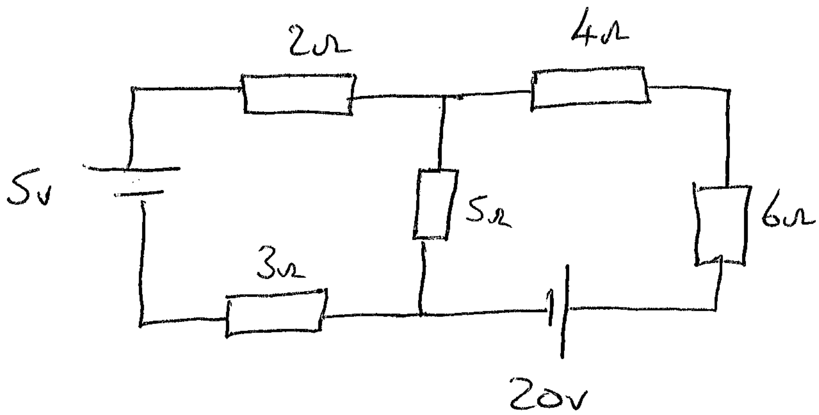
then $$N_p = \frac{V_{prms}}{4.44 f . \phi_{max}} = \frac{100}{4.44 \times 50 \times 4 \times 10^{-3}}$$

$$= \underline{\underline{113 \text{ TURNS}}}$$

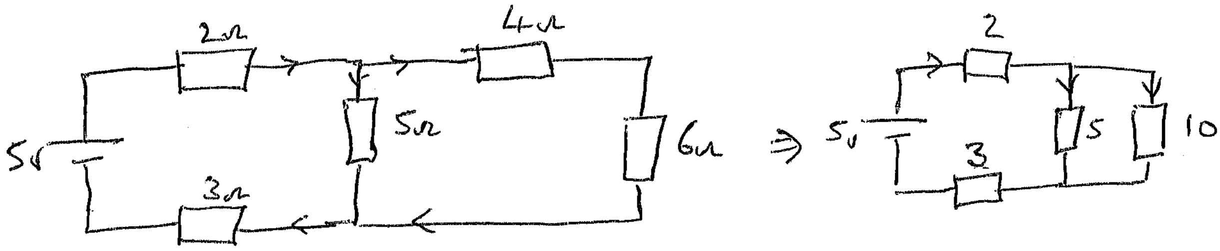
(v) If the frequency is 60 Hz

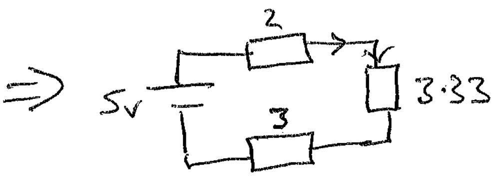
$$V_{rms\,max} = 4.44 \times 60 \times 113 \times 4 \times 10^{-3} = \underline{\underline{120\text{ Vrms}}}$$

QUESTION 4



Consider the 5V supply – short out the 20V battery.



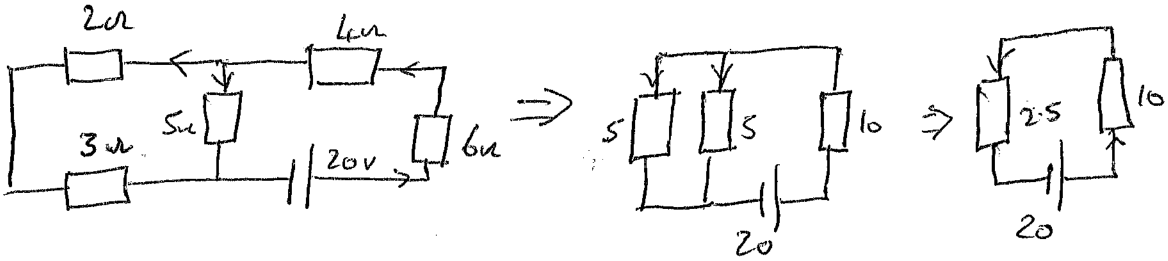


$$\therefore \quad I_T = \frac{5}{2+3+3.33} = 0.600 \text{ A}$$

And hence current through 5Ω resistor is:

$$I_5 = I_T \cdot \frac{10}{5+10} = 0.6 \times \frac{10}{15} = 0.4 \text{ A} \downarrow$$

Now consider the 20V supply short the 5V supply:



$$I_{T'} = \frac{20}{12.5} = 1.6 \text{ A}$$

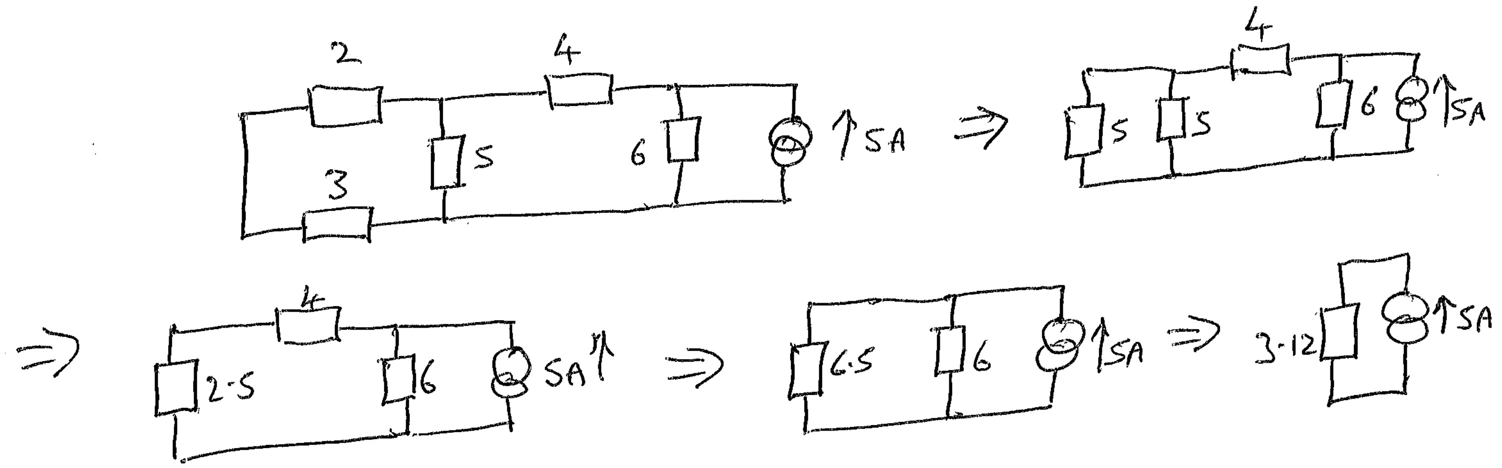
Current through the 5Ω resistor:

$$I_S' = I_T' = \frac{5}{5+5} \times 1.6 = 0.8A \downarrow$$

By superposition the total current through the 5Ω resistor is:

$$I_{STOT} = 0.4\downarrow + 0.8\downarrow = \underline{\underline{1.2A\downarrow}}$$

(b) We can use the analysis of part (a) and now need to consider the effect of the 5A current source alone:



∴ Voltage across 3.12Ω = I.R = 15.6V. Hence current through the 6.5Ω resistor is 15.6/6.5 = 2.4. This is the same current flowing through the 2.5Ω. Hence the current flowing through the 5Ω resistor is half this = 1.2A↓

By superposition the total current flowing through the 5Ω resistor is now:

$$I_{STOT} = \underset{\text{(from (a))}}{1.2\downarrow} + 1.2\downarrow = \underline{\underline{2.4A\downarrow}}$$

(c) For Thevenin we need the open-circuit voltage, i.e. the voltage across the 6Ω resistor:

Using the working from part (a).

For 5V source the current through the 6Ω resistor (10Ω branch in network)

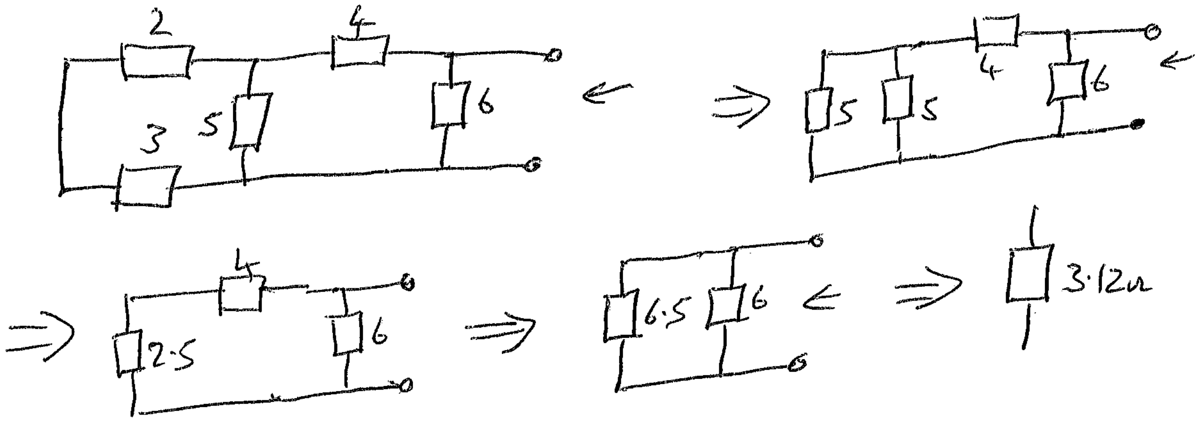
$$I_{61} = I_T \times \frac{5}{5+10} = 0.6 \times \frac{5}{15} = 0.2A \downarrow$$

For the 20V source the current through the 6Ω resistor is equal to $I_T' = 1.6A \uparrow$.

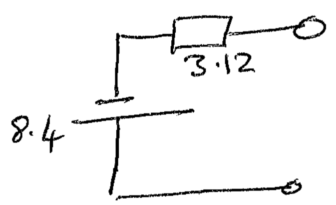
∴ Total current through the 6Ω resistor is 1.4A ↑.

Hence the Thevenin voltage is $1.4 \times 6 = 8.4V \downarrow$

Resistance network is:



Hence Thevenin circuit is:



When load is connected $I = \frac{8.4}{(10+3.12)} = 0.64A \uparrow$

Hence power in the load is $I^2 R_L = 0.64^2 \cdot 10 = \underline{\underline{4.096W}}$

(d) The Noton circuit can be found directly from the thevenin circuit.

$$I_N = \frac{E_T}{R_T} = \frac{8.4}{3.12} = \underline{\underline{2.69A}}$$

Hence Norton circuit is: