The University of Sheffield
Department of Electronic and Electrical Engineering

EEE101 Problem Sheet Solutions

dc Circuit Analysis

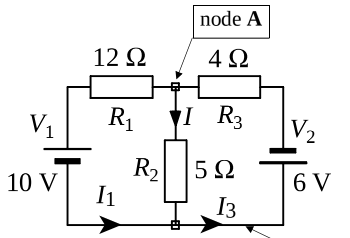

Figure 1

Q1 For the circuit of figure 1 find I using any method you like. What is the power dissipation in R_1?

(i) Nodal analysis . . .

Choose node **B** as 0 V reference point and sum currents at node **A . . .**

$$I_3 = I_1 + I \quad \text{or} \quad \frac{-V_2 - V_A}{R_3} = \frac{V_A - V_1}{R_1} + \frac{V_A}{R_2}$$

or $-\frac{V_2}{R_3} + \frac{V_1}{R_1} = V_A\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$. Using $V_1 = 10$ V and $V_2 = 6$ V gives $V_A = -\frac{5}{4}$ V

Thus $I = \frac{V_A}{R_2} = -$ **0.25 A** and $P_{R1} = \frac{V_{R1}^2}{R_1} = \frac{(V_1 - V_A)^2}{R_1} =$ **10.5 W.**

(ii) Loop analysis . . .

Choose two loops - say V_1, R_1, R_2 and R_2, R_3, V_2 and call the circulating currents I_{L1} and I_{L2} respective ly. Let I_{L1} circulate in a clockwise direction and I_{L2} in an anticlockwise one (you could choose different directions) . . .

loop 1; $10 = I_{L1}R_1 + (I_{L1} + I_{L2})R_2$ or $10 = (R_1 + R_2)I_{L1} + R_2 I_{L2}$ or $10 = 17I_{L1} + 5I_{L2}$

loop 2; $6 + I_{L2}R_3 + (I_{L2} + I_{L1})R_2 = 0$ or $6 + I_{L2}(R_3 + R_2) + I_{L1}R_2 = 0$ or $-6 = 5I_{L1} + 9I_{L2}$

Solve these two equations to get $I_{L1} = \frac{15}{16}$ A and $I_{L2} = -\frac{19}{16}$ A. I is then given by $I = I_{L1} + I_{L2}$.

(iii) Superposition . . .

Easiest approach is to find the voltage across R_2 due to the sources V_1 and V_2 and then calculate I

$$V_{R2} \text{ due to 10V is } V_{R2} = 10\frac{R_2//R_3}{R_1 + R_2//R_3} = 10\frac{20/9}{12 + 20/9} = \frac{200}{128} = \frac{25}{16} \text{ V}$$

$$V_{R2} \text{ due to -6V is } V_{R2} = -6\frac{R_1//R_2}{R_3 + R_1//R_2} = -6\frac{60/17}{4 + 60/17} = -\frac{360}{128} = -\frac{45}{16} \text{ V}$$

Thus $V_{R2TOT} = \frac{25}{16} - \frac{45}{16} = -\frac{20}{16} = -1.25$ V which gives the same I as the other two methods.

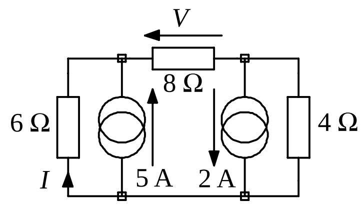

Figure 2

Q2 Using any method you like, find the values of I and V in figure 2.

It is easiest here to sum voltages around the outer loop to find I . . .

$$6I + 8(I + 5) + 4(I + 5 - 2) = 0 \quad \text{or} \quad 18I + 52 = 0$$

Thus $I = -$ **2.89 A** and $V = 8(-2.89 + 5) =$ **16.89 V**

It would also be easy to use Norton to Thevenin transformations on the 6 Ω and 5 A and the 4 Ω and 2 A combinations to find V. V will give the current through the 8 Ω so I can easily be found.

Q3 In figure 3, I is initially 1 A. Use nodal analysis to find I_S and V_R. What value of I is necessary to give $V_R = -4$ V?

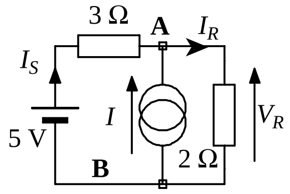

Figure 3

Use node **B** as the reference potential. Sum currents at node **A** . . .

$$I_S + I = I_R \quad \text{or} \quad \frac{5 - V_A}{3} + 1 = \frac{V_A}{2} \quad \text{or} \quad \boldsymbol{V_A\ (= V_R) = 3.2\ \mathrm{V}.}$$

Thus $I_R = V_R/2 = 1.6$ A and since $I_S + I = I_R$, $\boldsymbol{I_S} = \mathbf{0.6\ A}$

The current required to give $V_R = -4$ V is easy to work out because that condition means that all node voltages are known so currents can be easily worked out . . .

$$I_S + I = I_R \quad \text{or} \quad \frac{5 - (-4)}{3} + I = \frac{(-4)}{2} \quad \text{or} \quad \boldsymbol{I = -5\ \mathrm{A}}$$

Q4 For the circuit of figure 4, use nodal analysis and superposition to find I_1 and the potential difference $V_4 - V_3$, $V_{4\text{-}3}$. What is the power dissipation in R_4?

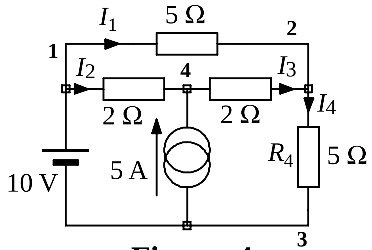

Figure 4

(i) Nodal analysis

Take node **3** as the 0 V reference. The node voltages V_2 and V_4 are unknowns. Summing currents . . .

at node 2; $\quad I_1 + I_3 = I_4 \quad \text{or} \quad \dfrac{10 - V_2}{5} + \dfrac{V_4 - V_2}{2} = \dfrac{V_2}{5} \quad \text{or} \quad 20 = 9V_2 - 5V_4$

at node 4; $\quad I_2 + 5 = I_3 \quad \text{or} \quad \dfrac{10 - V_4}{2} + 5 = \dfrac{V_4 - V_2}{2} \quad \text{or} \quad 20 = -V_2 + 2V_4$

solving these equations yields $\boldsymbol{V_4} = 200/13 = \mathbf{15.39\ V}$ and $V_2 = 140/13 = 10.77$ V.

$$P_{R4} = \frac{V_2^{\ 2}}{R_4} = \frac{10.77^2}{5} = \mathbf{23.2\ W}$$

$$I_1 = \frac{10 - \dfrac{140}{13}}{5} = -\frac{2}{13} = \mathbf{-\,0.15\ A}$$

(ii) Superposition

V_4 due to 10V (replace 5 A source with an open circuit) . . .

Current from 10V source, $I_S = \dfrac{10}{5//(2+2) + 5} = \dfrac{90}{65} = \dfrac{18}{13}$ A. So $V_{R4} = \dfrac{18}{13}5 = \dfrac{90}{13}$ V. The voltage at node **4** is 10 V minus the voltage across the left hand 2 Ω resistor. The difference between 10 V and V_{R4} is shared equally between the two 2 Ω resistors so $V_{4(10V)} = 10 - \dfrac{10 - \dfrac{90}{13}}{2} = \dfrac{110}{13}$ V.

V_4 due to 5A (replace 10V with a short circuit) . . .

The current source sees a parallel combination of the left hand 2 W and right hand 2 Ω in series with the two 5 Ω resistors in parallel. Thus $V_{4(5A)} = 5 \times 2//(2+5//5) = 5\dfrac{2 \times 4.5}{2+4.5} = 5 \times \dfrac{9}{6.5} = \dfrac{90}{13}$ V.

$V_{4TOT} = 110/13 + 90/13 = 200/13 = \mathbf{15.39\ V}$

I_1 due to 10 V is that portion of the total I_S that flows through the 5 Ω arm of the parallel combination of the two 2 Ω resistors and the top 5 Ω resistor

$$I_{1(10V)} = \frac{18}{13} \times \frac{4}{9} = \frac{8}{13}\ \text{A.}$$

I_1 due to the 5A source is the current flowing through one of the two parallel 5 Ω resistors. This will be half the current that flows down the arm containing the 5 Ω resistors and will be negative.   Using current splitting,

$$I_{1(5A)} = -5 \times \frac{2}{6.5} \times \frac{1}{2} = -\frac{10}{13}\,\text{A}.$$

Thus $\boldsymbol{I_{1TOT}}$ = 8/13 – 10/13 = –2/13 = – **0.15 A**

Q5 Use loop analysis and superposition to find I_2 and I_4 in the circuit of figure 5a. State with brief reasoning which component could be replaced by a short circuit without affecting either of these currents.

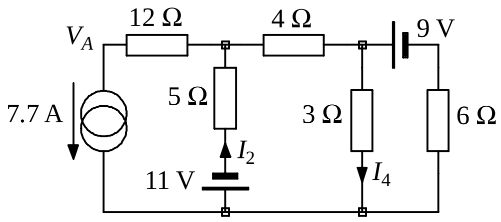

Figure 5a

(i) loop analysis

Choose three current loops. The choice here is three counter-clockwise loops; I_A throught 7.7A, 11V, 5 Ω and 12 Ω; I_B through 3 Ω, 4 Ω, 5 Ω and 11V; I_C through 3 Ω, 6 Ω and 9V. Many other choices are possible. The thing to remember here is that the objective of the loop method is to leave you with the minimum number of unknowns neccessary to solve the circuit. In loop A it is clear that I_A = 7.7A so there is no need to investigate loop A further. (But if you did investigate further, you would need to define a voltage drop V_A across the current source in order to complete the sum of voltage drops around loop A.)

For loop B; $11 = 3(I_B - I_C) + 4I_B + 5(I_B - I_A) = 12I_B - 3I_C - 5 \times 7.7$ or $49.5 = 12I_B - 3I_C$

For loop C; $9 = 3(I_C - I_B) + 6I_C = 9I_C - 3I_B$

There are many approaches that can be used to solve this pair of equations. Here we shall multiply the loop C equation by 4 and add it to the loop B equation to eliminate I_B . . .

$49.5 + 36 = 0 + (36-3)I_C$ or $85.5 = 33I_C$ or $I_C = 85.5/33 = 2.591$ A

Substituting in the loop C equation, $3 = 3I_C - I_B = \frac{85.5}{11} - I_B$ and so $I_B = 52.5/11 = 4.773$ A

Then $\boldsymbol{I_2} = I_A - I_B = 7.7 - 4.77\mathbf{3} = \mathbf{2.93}$ A and $I_4 = I_C - I_B = 28.5/11 - 52.5/11 = -24/11 = -\mathbf{2.18\,A}$

(ii) superposition

The important issue here is to make sure the partial circuit is interpreted correctly in each case.

(a) I_2 and I_4 due to the 7.7 A source - replace 11V and 9 V with short circuits as shown in figure 5b . . .

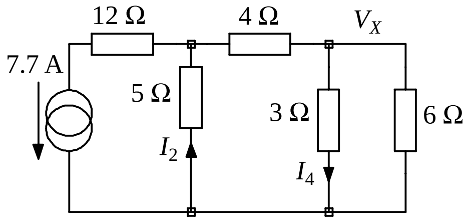

Figure 5b

The partial circuit consists of two parallel paths, one of 5 Ω through which I_2 flows and one through the series parallel combination of (6 Ω // 3 Ω) in series with 4 Ω, which join forces at the top of 5 Ω to return the 7.7 A through the 12 Ω to the source.

The combined resistance of (6 Ω // 3 Ω) in series with 4 Ω is 6 Ω so I_2 is that fraction of 7.7 A that takes the 5 Ω route, ie

$$I_{2(7.7\text{A})} = 7.7 \times \frac{6}{11} = 4.2\ \text{A}$$

I_4 is the division of (7.7 - I_2) between 3 Ω and 6 Ω, ie

$$I_{4(7.7\text{A})} = -(7.7 - 4.2) \times \frac{6}{9}\ \text{A} = -\frac{7}{3}\ \text{A}.$$ Note the "–" sign.

(b) I_2 and I_4 due to the 11 V source - replace 7.7 A with an open circuit and 9 V with a short circuit as shown in figure 5c . . .

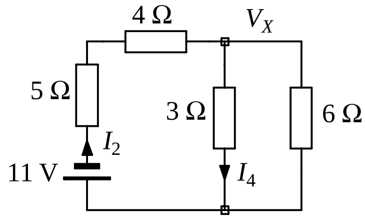

Figure 5c

The 11V source sees 9 Ω (5 Ω + 4 Ω) in series with the parallel combination 3 Ω // 6 Ω (= 2 Ω). Thus

$I_{2(11V)} = -\frac{11\ \text{V}}{11\ \Omega} = -1\text{A}$ and I_4 is that part of I_2 that flows through the 3 Ω resistor, ie,

$I_{4(11V)} = I_{2(11V)} \frac{6}{3+6} = -\frac{2}{3}\ \text{A}$

(c) I_2 and I_4 due to the 9 V source - replace 7.7 A with an open circuit and 11V with a short circuit as shown in figure 5d . . .

Here the 9 V source sees (4 + 5)//3 Ω in series with 6 Ω, a total of 33/4 Ω. Thus the total current driven by the 9 V source is 9/(33/4) = 36/33 = 12/11 A. This current divides down the two parallel arms to give I_2 and I_4,

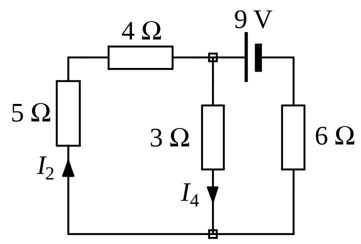

Figure 5d

$I_{2(9V)} = -\frac{12}{11} \times \frac{3}{3+9}\ \text{A} = -\frac{3}{11}\ \text{A}$ and $I_{4(9V)} = \frac{12}{11} \times \frac{9}{12}\ \text{A} = \frac{9}{11}\ \text{A}$

Thus $\boldsymbol{I_2} = I_{2(7.7A)} + I_{2(11V)} + I_{2(9V)} = (4.2 - 1 - 3/11)\ \text{A} = \mathbf{2.93\ A}$

and

$\boldsymbol{I_4} = I_{4(7.7A)} + I_{4(11V)} + I_{4(9V)} = (-7/3 - 2/3 + 9/11)\ \text{A} = \mathbf{-2.18\ A}$

Q6 Find $V_2 - V_3$, $V_{2\text{-}3}$, in the circuit of figure 6 using any method you like.

The key to solving this problem is the recognition that although node **5** is common to the left hand loop and the right hand loop, it is the ***only*** connection between those loops. Thus, I_2 must equal zero.

Since node **5** is the only common node it makes sense to use it as the reference potential and to evaluate V_2 and V_3 with respect to node **5**.

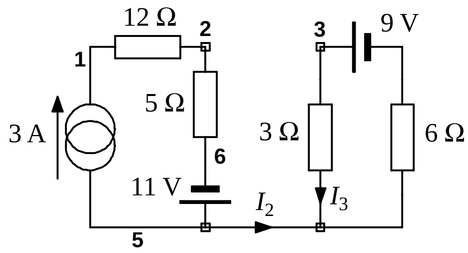

Figure 6

Left hand loop . . .

The 3 A source drives current around the loop and in doing so creates a voltage drop of 15V across the 5 Ω resistor with its positive end at node **3**.

$$(V_2 - V_5) = (V_2 - V_6) + (V_6 - V_5) = 15 - 11 = 4\ \text{V}$$

Right hand loop . . .

In the right hand loop, $I_3 = 9\ \text{V} / (3\ \Omega + 6\ \Omega) = 1\ \text{A}$. The voltage at node **3** with respect to node **5** is the voltage developed across the 3 Ω resistor by I_3, i.e., 3 V. So

$$(V_3 - V_5) = 3\ \text{V}$$

So $(V_2 - V_3) = (V_2 - V_5) - (V_3 - V_5) = 4 - 3 = \mathbf{1\ V}$