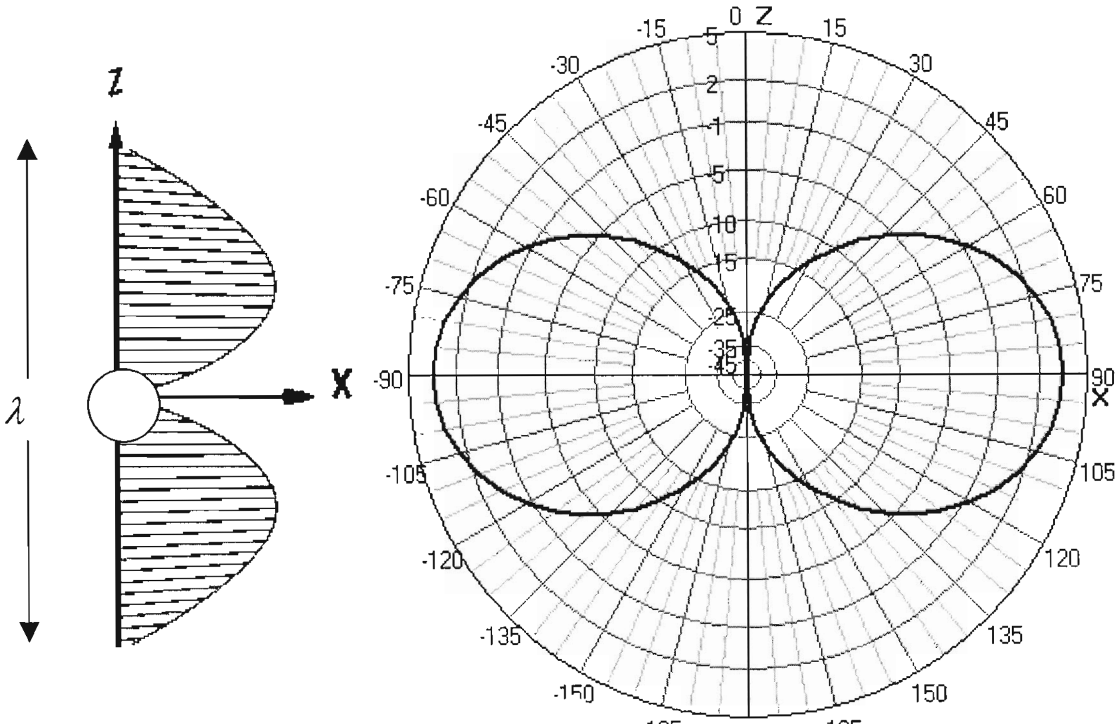
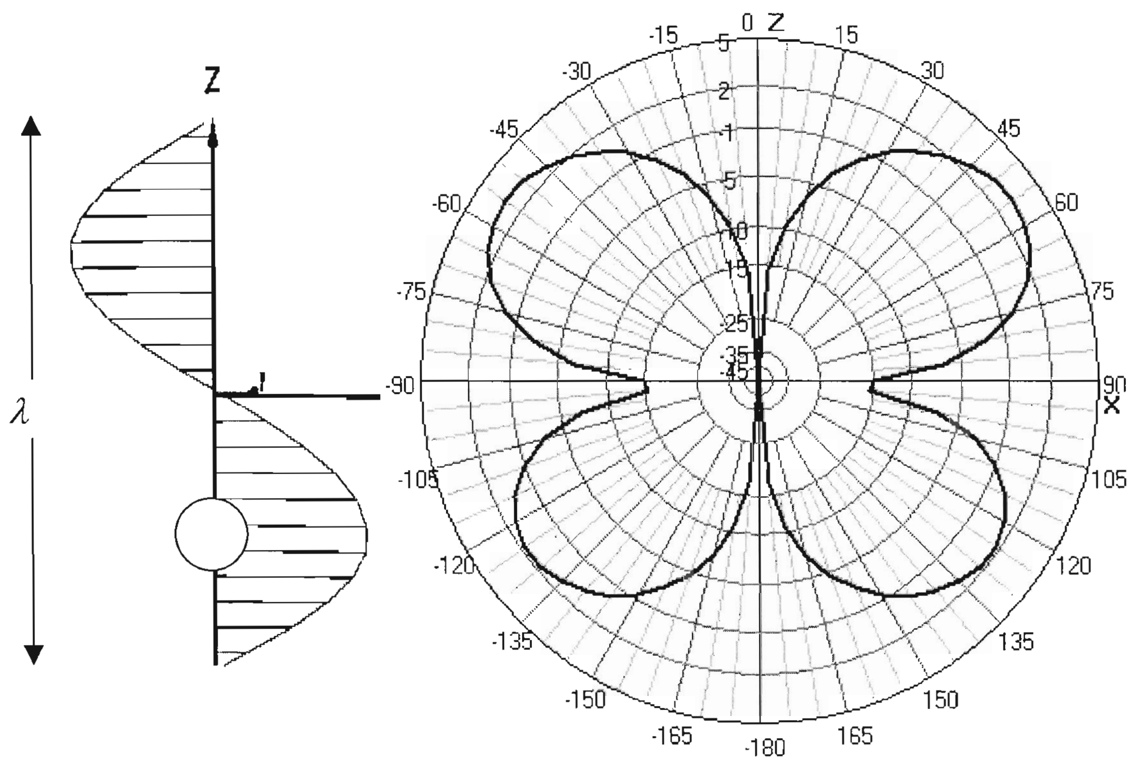
Solution to EEE406/6011 2013 Q1
(a)
(i)



(ii)



(iii) The radiation pattern and current distribution are similar to (ii)

(i) & (iii) $Z_{in} > 1k\Omega$, (ii) $Z_{in} \approx 94\Omega$. (9)

(b)

$$D = \frac{P_r|_{\theta=90^o}}{\frac{P}{4\pi r^2}} \qquad (1),$$

where

$$P_r = \frac{1}{2\eta}|E_\theta|^2 \qquad (2)$$

For the case of the *full wave dipole*, we have the field from the question

$$E_\theta|_{\theta=90^o} = \frac{2\eta I_o}{4\pi r}[2] \qquad (3)$$

so

$$P_r|_{\theta=90^o} = \frac{1}{2\eta}\frac{\eta^2 I_o^2}{\pi^2 r^2} \qquad (4)$$

Now,

$$P = \int_0^{2\pi}\int_0^{\pi} P_r r \sin(\theta) d\phi r d\theta = 2\pi r^2 \int_0^{\pi} P_r \sin(\theta) d\theta \qquad (5)$$

so

$$P = 2\pi r^2 \frac{1}{2\eta}\frac{4\eta^2 I_o^2}{16\pi^2 r^2}\int_0^{\pi}\left(\frac{\cos(\pi\cos(\theta))+1}{\sin(\theta)}\right)^2 \sin(\theta) d\theta \qquad (6)$$

and using the value of the integral given in the question

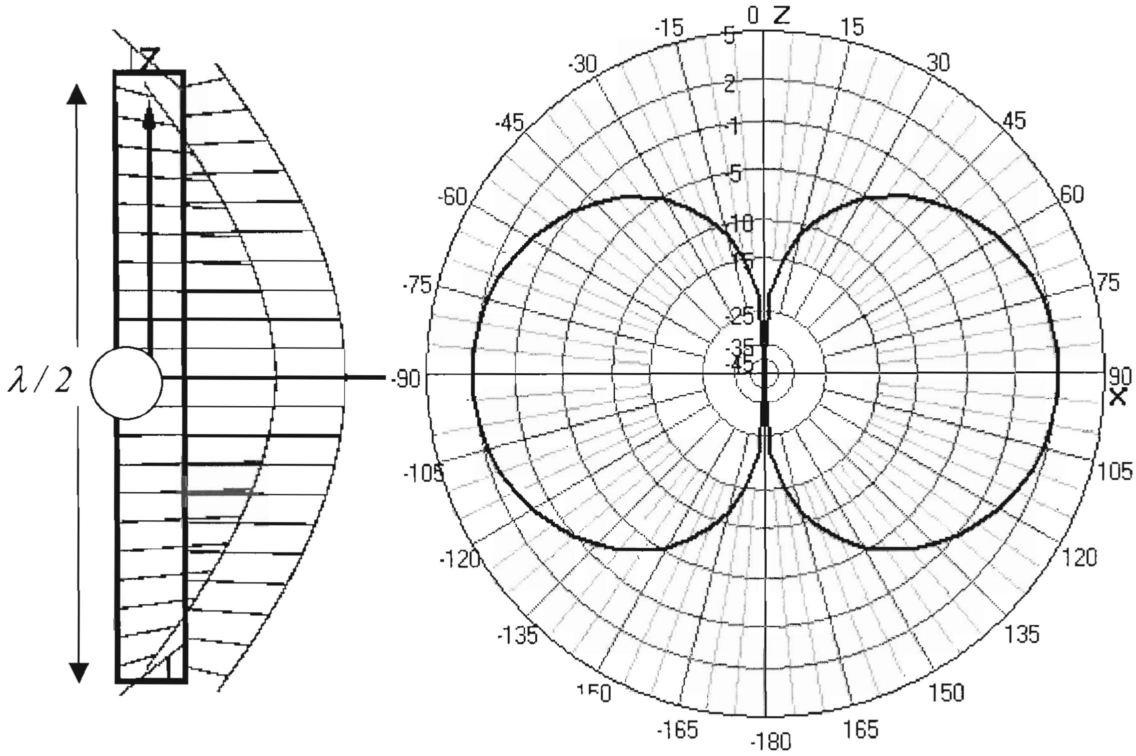
$$P = \frac{\eta I_o^2}{4\pi} \times 3.3 \qquad (7).$$

Thus, substituting (4) and (7) into (1) gives:

$$D = 4\pi r^2 \frac{\eta I_o^2}{2\pi^2 r^2}\frac{4\pi}{3.3\eta I_o^2} = \frac{8}{3.3} = 2.42 = 3.85 dBi$$

(8)

(c)



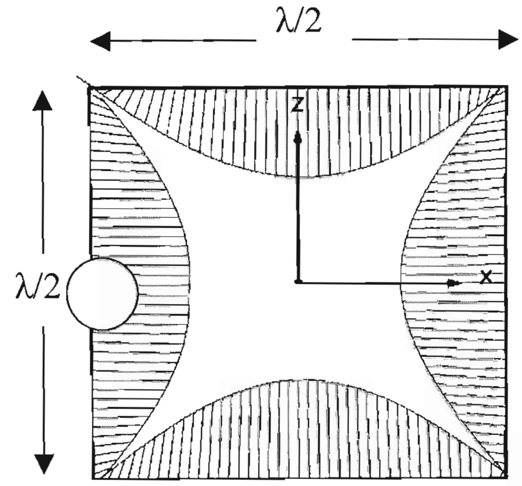
The directivity is that of a half wave dipole, *2.15dBi* and the input impedance about four times that of a dipole $Z_{in} \approx 290\Omega$.

(3)

Solution to EEE406/6011 2013 Q2

(a)

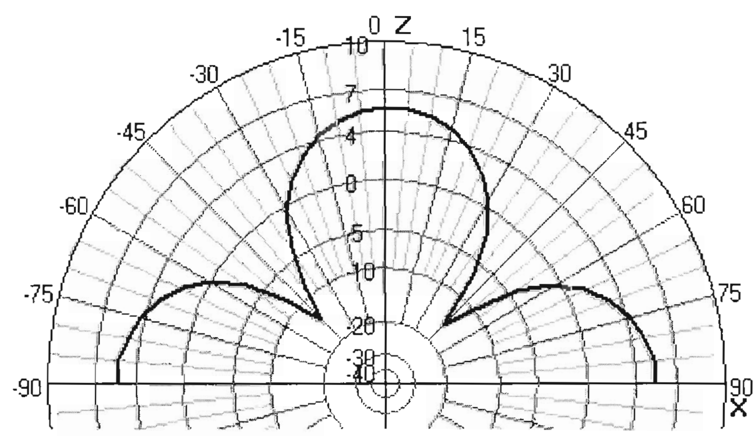
Currents are at $z>0$, image currents at $z<0$.



(4)

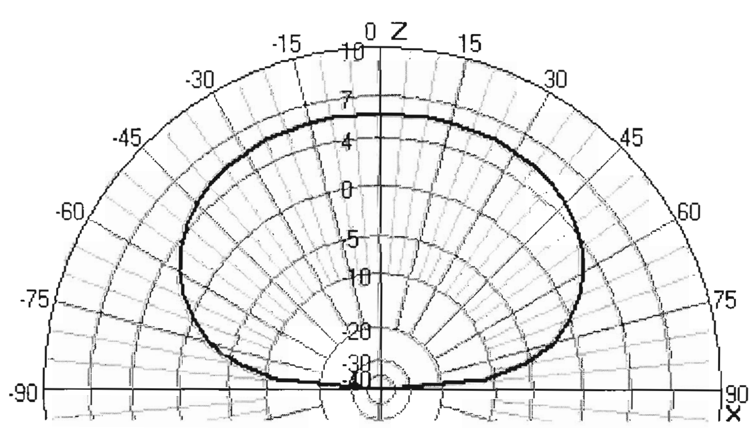
(b)
(i)

$E_\theta(\theta, \phi = 0^o, 180^o)$ radiation pattern



(ii)

$E_\phi(\theta, \phi = 90^o, 270^o)$ radiation pattern



(iii) This is a null field since E_θ is orthogonal to the horizontal side and the fields from the vertical sides cancel.

(5)

(c)

(i) Since the loop and groundplane are perfectly conducting,

$$Re(Z_{in}) = R_r = 118\Omega$$

(ii) Since there are no conductor losses, the radiated power is given by

$$P = Re(VI^*) = V\frac{VR}{R^2 + X^2} = \frac{118}{118^2 + 53^2} \times 10^4 = 70.5W$$

(iii) A series inductance will be required to cancel the capacitive reactance of 53Ω. Thus,

$$\omega L = 53$$

so

$$L = \frac{53}{2\pi \times 14 \times 10^6} = 0.6\mu H$$

(iv) The groundplane resistivity effectively adds to the real part of the input impedance as R_L, thereby dissipating some of the transmitter power as heat. The efficiency is then

$$\eta = \frac{R_r}{R_r + R_L} = \frac{118}{118 + 118} \times 100 = 50\%$$

(v) The radiation resistance of a full loop antenna radiating into full space will be twice that of a half loop, 236Ω, and the directivity 3dB less, $2.8dBi$. (9)

(d)

(i) As an HF loop counterpoised against the earth, the half loop has useful low angle gain for long distance ionospheric hops, but also a vertical lobe which would send power at high angles and be of little use.

(ii) As a GPS antenna mounted on a vehicle roof, the vertical lobe would be of value for satellite communication, and even the low angle lobe may be useful for satellites close to the horizon. However the loop is not circularly polarized therefore reducing received GPS signal strength. (2)

Solution to EEE406/6011 2013 Q3

(a)

(i) Individual dipole radiation pattern for unit current is

$$C\left[\frac{\cos\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin(\theta)}\right]$$

(ii) Array factor is

$$I_o\frac{\sin\left(N\frac{\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)}$$

(iii) Assuming transmitter is matched into R_r of single dipole, with no losses, and all dipoles are fed in parallel, then we equate power either side of matching transformer so

$$I^2R_r = (NI_o)^2\frac{R_r}{N}$$

so

$$I_o = \frac{I}{\sqrt{N}}$$

(iv) We have N dipoles fed in parallel, so

$$Z_{in} \approx \frac{R_r}{N}$$

(5)

(b)

(i)

In the limit $\Psi/2 \rightarrow 0$ at $\theta = 90^o$,

$$I_o\frac{\sin\left(N\frac{\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)} \rightarrow I_oN = I\sqrt{N}$$

so the gain over a single dipole fed the same total current is $I\sqrt{N}/I$,
hence the antenna gain is $10log_{10}(N)+2.15dBi=14.5dBi$

(ii)
Neglecting the dipole pattern modulating effect, the first sidelobe occurs for

$$\frac{Nkd}{2}cos\,\theta=\pm\frac{3\pi}{2}$$

so

$$\theta=cos^{-1}\left(\frac{3\pi}{Nkd}\right)=cos^{-1}\left(\pm\frac{3}{17\times2\times0.6}\right)=81.5^{o}\;or\;98.5^{o}$$

(iii)

The height of this sidelobe is then

$$20log_{10}\left(\frac{I_o\left|sin(\frac{3\pi}{2})\right|}{NI_o\,sin(\frac{3\pi}{2N})}\right)=-13.4dB$$

wrt the main lobe.

(iv) From the question we know that

$$0.082=\frac{\Psi}{2}=\frac{kd}{2}cos(\theta)=0.6\pi\,cos(\theta)$$

hence

$$\theta=87.5^{o}$$

so 3dB beamwidth is

$$2\times(90-87.5)=5^{o}$$

(9)

(c)
For a beam squint to $\theta=30^{o}$,

$$kd\,cos(30)+\gamma=0$$

so

$$\gamma=-2\pi\times0.6\times0.87=-3.28rads\text{ or }-187.9^{o}$$

However, at this angle the dipole pattern is only 40% of its $\theta = 90^o$ value, thus reducing the array gain by ~*8dB*. (4)

(d)

If two 17 element co-linear arrays were positioned side by side at a distance of $\lambda / 2$, the total directivity would increase by *3dB* to *17.5dBi*, assuming correct matching into the TX/RX. This extra directivity would come from pattern focussing in the H plane rather than the E plane. (2)

Solution to EEE406/6011 2013 Q4

(a)

$$e\underline{E} = m\frac{d\underline{V}}{dt} \quad (1)$$

Assuming a Cartesian component of velocity and field for simplicity, let the velocity be time harmonic as

$$V = V_o\,cos(\omega t) \qquad (2)$$

From equation (1) therefore

$$eE = -m\omega V_o\,sin(\omega t) = m\omega V_o\,cos(\omega t + 90^o) \qquad (3).$$

Thus the electric field leads the velocity by 90^o, and

$$E = -E_o \sin(\omega t) \quad (4)$$

where

$$E_o = \frac{m\omega V_o}{e} \qquad (5)$$

so

$$V_o = \frac{eE_o}{m\omega} \quad (6)$$

and hence

$$V = \frac{eE_o}{m\omega} cos(\omega t) \quad (7).$$

The plasma conduction current density is given by

$$J = NeV \qquad (8)$$

so that from (7)

$$J = \frac{Ne^2}{m\omega} E_o\,cos(\omega t) \qquad (9).$$

From (4) the free space displacement current density is

$$\frac{\partial D}{\partial t} = -\varepsilon_o \omega E_o\,cos(\omega t) \quad (10)$$

and hence the conduction and displacement currents are 180^o out of phase. Since

$$\nabla \times \underline{H} = J + \frac{\partial D}{\partial t} \quad (11)$$

the currents cancel completely and produce zero magnetic field when from (9) and (10)

$$\nabla \times \underline{H} = \left(\frac{Ne^2}{m\omega} - \varepsilon_o \omega \right) cos(\omega t) = 0 \qquad (12)$$

i.e. when

$$\frac{Ne^2}{m\omega} = \varepsilon_o \omega \qquad (13)$$

and hence

$$\omega^2 = \frac{Ne^2}{m\varepsilon_o} = \omega_c^2 = (2\pi f_c)^2 \qquad (14)$$

is the critical frequency. (9)

(b)

The reflection height h is given by

$$h = \frac{ct}{2}$$

where t is the echo time and $c = 3 \times 10^8 m/s$. Also, from the question,

$$N = \left(\frac{2\pi f_c}{56} \right)^2$$

Hence

(i)

$$N = \left(\frac{2\pi \times 2.8 \times 10^6}{56} \right)^2 = 9.9 \times 10^{10} / m^3$$

and

$$h = \frac{3 \times 10^8 \times .8 \times 10^{-3}}{2} = 120km$$

This is the E layer.

(ii)

$$N = \left(\frac{2\pi \times 6.3 \times 10^6}{56} \right)^2 = 5 \times 10^{11} / m^3$$

and

$$h = \frac{3 \times 10^8 \times 1.3 \times 10^{-3}}{2} = 195km$$

This is the F1 layer.
(iii)

$$N = \left(\frac{2\pi \times 8.9 \times 10^6}{56} \right)^2 = 10^{12} / m^3$$

and

$$h = \frac{3 \times 10^8 \times 2 \times 10^{-3}}{2} = 300km$$

(9)

This is the F2 layer.

(c)
During the day, the E, F1 and F2 layers are present. At night, the F1 and F2 layers combine into a single F layer, and the E layer is not present. (2)