EEE101 2009.

Answers.

Q1 (a) K1 - current entering a node must equal the current leaving that node 1

or - the algebraic sum of the currents at a node is zero.

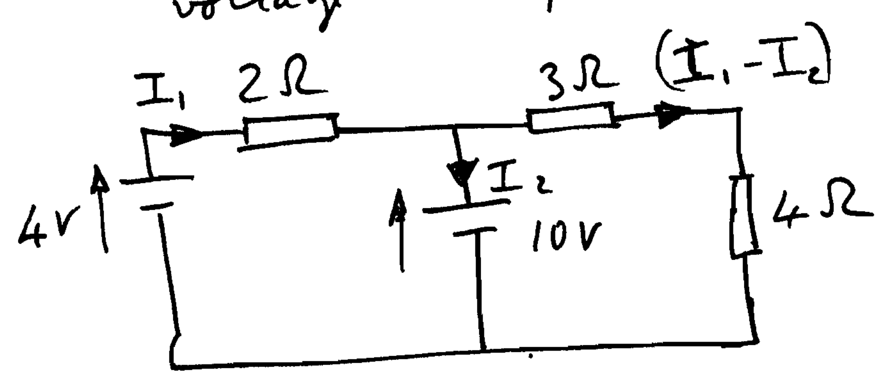
K2 - the algebraic sum of the voltages round a closed loop is zero. 1

Is it reasonable?

K1 - if currents don't balance at a node electrons will continue to pile up or be depleted from the wire. 2

K2 - if K2 is broken this implies the same point in the wire has 2 values of voltage - not possible. 2

(b)



2

Apply K1 at the nodes

K2 LH. loop 2

$$4 - 2I_1 - 10 = 0 \quad \text{—} ①$$

R.H. loop

$$10 - 3(I_1 - I_2) - 4(I_1 - I_2) = 0 \quad \text{—} ② \quad 2$$

2 equations and 2 unknowns hence can solve for I_1 and I_2

Q1 (cont.)

From ① $I_1 = -3\,A$.

Substitute into ②

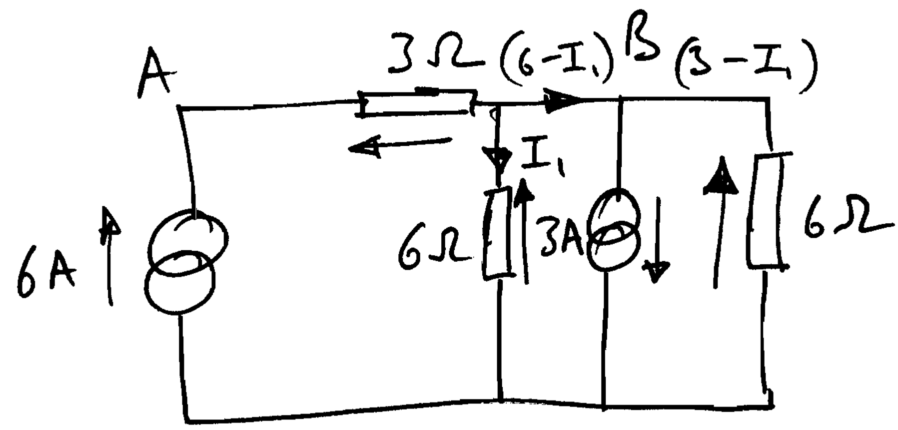
$$10 - 7(-3 - I_2) = 0$$

$$\Rightarrow I_2 = \frac{-31}{7} = \underline{-4.3\,A}$$

Other branch $I_1 - I_2 = -3 + 4.3$

$$= \underline{1.3\,A}.$$

(c)



Apply K1 to get branch currents

Apply K2 to loop bounded by 6Ω resistors (does not include the current sources).

i.e. $6I_1 - 6/(3 - I_1) = 0$

$\therefore 6I_1 - 18 + 6I_1 = 0$

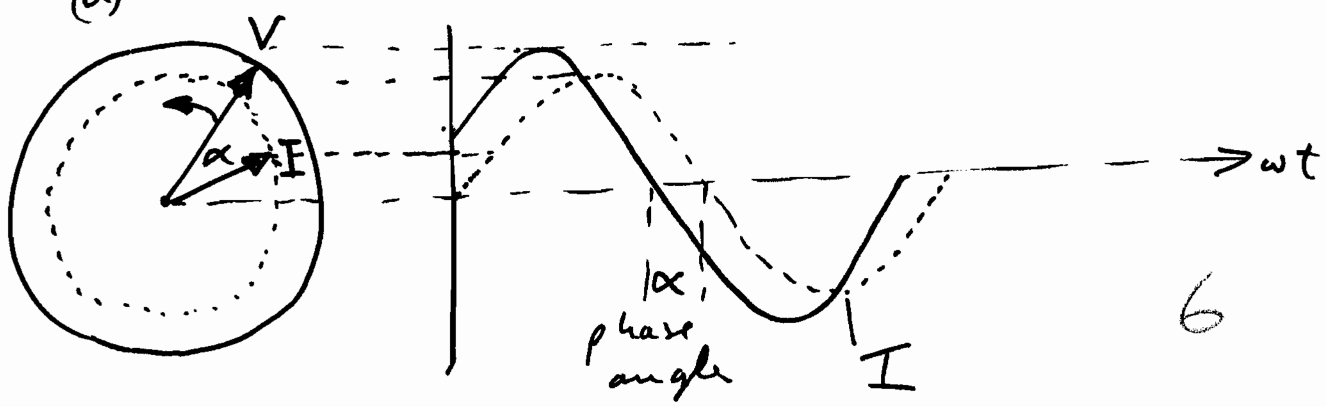
$$\Rightarrow I_1 = \frac{18}{12} = \underline{\underline{1.5\,A}}$$

Voltage at B $= 6(3 - I_1) = \underline{9V}$

Voltage at A $= 9 + (6 \times 3) = \underline{27}$.

Q2

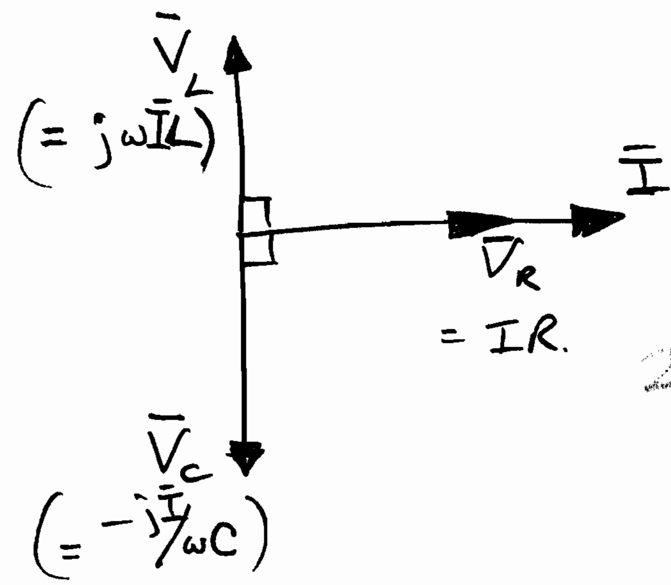
(a)



6

(b)

current is common

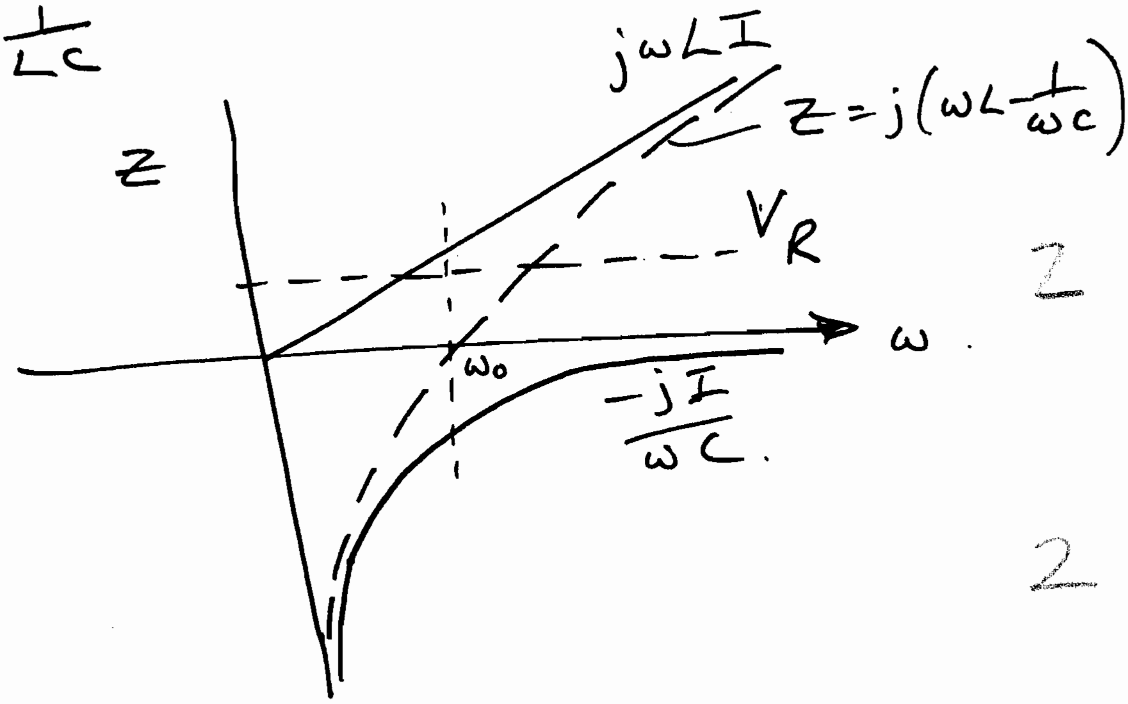


2

Circuit is in resonance when it becomes purely resistive.

i.e. $\omega L = \dfrac{1}{\omega C}$

$\omega = \sqrt{\dfrac{1}{LC}}$

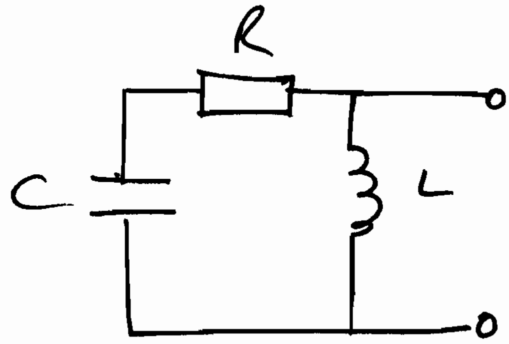


2

2

Q2 (cont.)

(c)



$$Z = j\omega L // (R - j/\omega C)$$ 2

$$= \frac{j\omega L(R - j/\omega C)}{j\omega L + R - j/\omega C}$$

$$= \frac{j\omega LR + \frac{L}{C}}{j(\omega L - \frac{1}{\omega C}) + R}$$

$$= \frac{\left(j\omega LR + \frac{L}{C}\right)\left(-j(\omega L - \frac{1}{\omega C}) + R\right)}{(\omega L - \frac{1}{\omega C})^2 + R^2}$$ 2

Consider numerator only

$$j\omega LR^2 + \omega LR(\omega L - \frac{1}{\omega C}) + \frac{LR}{C} - j\frac{L}{C}(\omega L - \frac{1}{\omega C})$$

At resonance set j terms = 0 2

i.e. $\omega LR^2 - \frac{L}{C}(\omega L - \frac{1}{\omega C}) = 0$

$$\Rightarrow \quad \omega = \sqrt{\frac{1}{LC - C^2R^2}}$$ 2

Q 3 (a) From definition of capacitance

$$Q = Cv$$

Also $\quad i = \frac{dQ}{dt}$ (definition of current)

$$= \frac{d(Cv)}{dt} = C\frac{dv}{dt} \quad (C \text{ is a constant}) \qquad 2$$

To get voltage, separate variables and integrate

$$\int \frac{i}{C}\,dt = \int_{V_0}^{v} dv = v - V_0 \qquad 2$$

where V_0 = initial voltage on capacitor
v = final voltage on capacitor

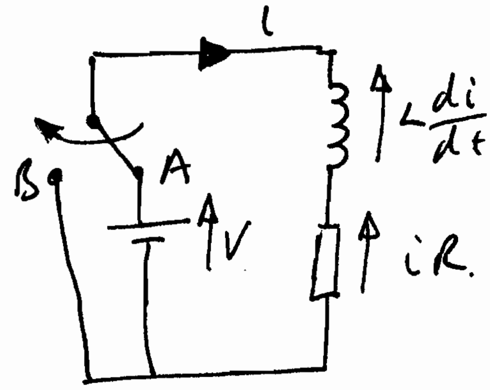
$$\therefore \quad v = V_0 + \frac{1}{C}\int i\,dt. \qquad 2$$

from above, if "i" is a constant then

$$v = V_0 + \frac{i}{C}t$$

i.e. v will increase linearly with time. 2

(b) After switch operation i continues to flow until energy stored in the inductor is all dissipated in R



At $t<0 \quad i = I_0 = \frac{V}{R}$ 1

$t>0$ K2 gives $L\frac{di}{dt} + iR = 0$ 2

Q 3 (cont.)

separate variables and integrate

$$\int \frac{1}{i}\, di = -\frac{R}{L}\int dt$$

$$\Rightarrow \quad -\frac{L}{R}\ln i = t + A$$

to get A $\quad i = I_0$ at $t = 0$

$$\therefore \quad -\frac{L}{R}\ln I_0 = A$$

$$\therefore \quad -\frac{L}{R}\left[\ln i - \ln I_0\right] = t$$

$$\Rightarrow \quad \underline{i(t) = I_0 e^{-Rt/L}} \qquad 3$$

(c) current in inductor $\bar{I}_L = \frac{\bar{V}}{j8} = \frac{20}{j8} = \underline{-j\,2.5}$ A

current in capacitor $\bar{I}_c = \frac{20}{5-j12} = \frac{20(5+j12)}{5^2+12^2}$ $\quad 2$

$$= \underline{0.6 + j\,1.44}$$

$\therefore$ Total current from source $= \bar{I}_L + \bar{I}_c$

$$= -j\,2.5 + 0.6 + j\,1.44$$

$$= \underline{0.6 - j\,1.06} \text{ A} \qquad 2$$

Magnitude $|I_L + I_c| = \sqrt{0.6^2 + 1.06^2}$

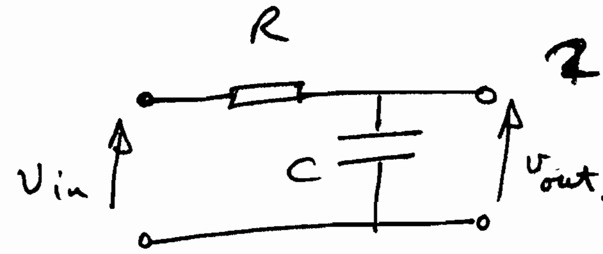
$$= \underline{1.22 \text{ A}} \qquad 2$$

Q 4 (a) Voltage ratio in dB $= 20 \log_{10}\left(\frac{V_2}{V_1}\right)$ 1

Power ratio in dB $= 10 \log_{10}\left(\frac{P_2}{P_1}\right)$ 1

dB is a log scale which allows large variations in ratio values to be illustrated graphically. 2

(b) low pass filter 2



$$\frac{v_{out}}{v_{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega CR} \qquad 2$$

cut-off frequency $\omega_0 = \frac{1}{CR}$ 2

$$\therefore \frac{v_{out}}{v_{in}} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

At $\omega = \omega_0$ $\left|\frac{v_{out}}{v_{in}}\right| = \left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}}$

At $\omega \gg \omega_0$ $\left|\frac{v_{out}}{v_{in}}\right| = \frac{\omega_0}{\omega}$ i.e. $\propto \frac{1}{\omega}$ 2

i.e. ratio will reduce according to $\frac{1}{\omega}$ (not sharp)

For $\omega \gg \omega_0$ $20 \log_{10}\left(\frac{\omega_0}{\omega}\right)$ at ω

and $20 \log_{10} \frac{\omega_0}{10\omega}$ at 10ω i.e. decade increase in frequency

$\therefore$ Difference in dB $= dB_{10\omega} - dB_{\omega}$

$$= 20 \log_{10} \frac{\omega_0/10\omega}{\omega_0/\omega} = 20 \log_{10} \frac{1}{10}$$

$$= -20\,dB \qquad 4$$

i.e. 20 dB/decade "roll-off"

(c) $Q = R\sqrt{\frac{C}{L}} = 5 \implies C = \frac{L}{100}$

$f_R = \frac{1}{2\pi\sqrt{LC}} = 3\times10^4\ Hz$

$\therefore LC = \left(\frac{1}{2\pi\times3\times10^4}\right)^2 = 2.8\times10^{-11}$

$\therefore 100C^2 = 2.8\times10^{-11}$

$\implies C = \sqrt{\frac{2.8\times10^{-11}}{100}} = \underline{0.53\mu F}$ 2

$\therefore L = 100C = \underline{53\mu H}$

2