EEE105 - Electronic Devices
Lecture 9

Disturbing the Equilibrium: Excess Minority Carrier Recombination

(CAL: semic(g))

So far we have only considered the case where carriers are thermally generated. However we can introduce *excess* carriers by two methods:

In both these cases the minority carrier density will increase dramatically and thus R will increase. The thermal generation rate on the other hand will not change, thus $R>G$.

Let us consider a volume of p-type material, where some concentration, δn, extra electrons are injected in a short time (e.g. due to a light pulse) where the pulse ends at a time t=0.

The total electron concentration in the p-type material will be

where

These extra electrons will modify the recombination rate of carriers in the material. There will be extra (additional) recombination due to the hole concentration *and* the extra electrons. As there is extra recombination the concentration of the extra electrons must fall after the injection pulse has finished. The rate of change of the electron concentration will depend on the difference between the rate of generation of carriers and their rate of recombination:

$$\frac{dn}{dt} = G - R = G - \left(Bn_p p + B\delta np\right)$$

Now from before we have $G = Bn_p p$, based on the equilibrium situation, and hence we can write:

$$\frac{dn}{dt} = \frac{d(\delta n)}{dt} = -B\delta np$$

[Note that in this example we are assuming that the holes created by the light pulse (which will be of the same density as the electrons created in the light pulse) will have a lower concentration that the hole concentration in the p-type material. i.e. $n_p << \delta n << p$. This means that $p + \delta n \approx p$.]

This relation should look familiar to you as it is another

Now we can integrate this equation, and set a boundary condition that at t=0 then $\delta n = \delta n_0$.

We will get the solution that $\boxed{\delta n(t) = \delta n_0 \exp\left(-t/\tau_e\right)}$ where $\boxed{\tau_e = \frac{1}{Bp}}$

You should check for yourself by substitution that this equation is valid

This equation means that if excess electrons are introduced and then their supply turned off at time t=0 then the excess electrons will recombine exponentially with time until the excess electron concentration is zero and the overall concentration of electrons is the minority carrier equilibrium value given by the thermal generation and recombination process (described in lecture 8).

We call τ_e the ***minority carrier lifetime.***

Note (1): τ_e decreases as p increases as there are more holes to recombine with the excess electrons more quickly.

Note (2): τ_e is the average time excess electrons exist in a semiconductor. We will meet this concept in both p-n junction diodes and bipolar junction transistors.

Disturbing the Equilibrium: Minority Carrier Diffusion Length

(CAL: semic(h))

We have just seen that if we introduce excess minority carriers into a material they will recombine with a time constant τ_e. We now want to look at the motion of the excess minority carriers during the period up until they recombine. To do this let us consider the following situation:

Let us introduce minority carriers along one edge of the material. Let use assume that, once again the material is p-type and hence the minority carriers will be electrons.

In this case there is clearly a concentration gradient of the minority carriers from the edge, where the injection makes it high into the bulk of the material where at a large distance away from the edge the minority carrier concentration will be the equilibrium value (that is the concentration of extra electrons will be zero).

As there is a concentration gradient there will be ***diffusion*** of the carriers away from the edge of the material and into the bulk. This diffusions will occur until the excess minority carriers (the electrons in this case) are lost through recombination.

If the electrons are diffusing away from the surface there will be a ***diffusion current***, J_e, towards this edge of the material.

Let us look this time at a steady state situation: in this case we will as assume that there is continuous replacement of the excess minority carriers on the edge of the sample, and we will define this edge as $x = 0$. Thus at $x = 0$ we will have δn equal to some value δn_0 for all time.

Now as the excess electrons diffuse further and further into the material the value of $\delta n(x)$ will decrease due to recombination. It can be found (although we will not prove this formally here) that $\delta n(x)$ will reduce exponentially with x.

This gives use the relationship:

$$\delta n(x) = \delta n_0 \exp\left(-\frac{x}{L_e}\right)$$

where L_e is the ***minority carrier diffusion length*** for the electrons, where $L_e = (D_e \tau_e)^{1/2}$.

D_e is the areal spead of diffusion (in $m^2 s^{-1}$) and τ_e the minority carrier recombination time (in s).

Excess minority carrier holes in n-type material.

We can use the same analogy as above for minority carrier holes in n-type material to obtain the hole minority carrier lifetime and hole minority carrier diffusion length:

Application of these equations

These equations are very important in considering p-n junctions: electrons injected into the p-type material will be used up recombining with holes, and vice-versa. As they are used up more charge carriers will be injected into the p-type material driven by a forward bias.

Thus the recombination of minority carriers in a p-n junction leads to the current flow when the junction (or diode) is forward biased.

Example

A pulse of light is shone on a sample of p-type silicon. After 100 ns the excess minority carrier density fell to 0.01% of its original value. What fraction of the minority carriers will be able to diffuse through the sample if it is 10 μm thick. [Note for Si μ_e = 0.12 m^{2}V^{-1}s^{-1}, μ_h = 0.045 m^{2}V^{-1}s^{-1}]

Answer

In order to estimate this we will need to get the minority carrier diffusion length of the p-Si. We can calculate this if we know the minority carrier lifetime and the diffusion coefficient.

For the minority carrier lifetime we can use: $n(t) = n_0 \exp\left(-{}^{t}\!/_{\tau_e}\right)$

We can calculate the diffusion coefficient from the Einstein Relation: $D_e = \frac{kT}{q}\mu_e$ (Note: that the electrons are the minority carriers here, the information provided in the question about the hole mobility is superfluous.)

We now have both τ_e and D_e and therefore we can calculate L_e :

Finally we can hence calculate the fraction of electrons that can diffuse through 10 μm of our p-type silicon:

Key Points to Remember:

1. In equilibrium the generation rate and recombination rate are equal. We can use this relationship to see that we would expect a certain density of free charge carriers in intrinsic material.
    a. The same relationship also allows us to quantify the density of minority carriers in extrinsic (or doped) material.
2. We can increase the density of minority carriers by injecting them into the material
3. If a pulse of minority carriers is injected into the material the concentration of excess minority carriers will decay away exponentially with time.
4. Excess minority carriers will recombine with a characteristic time constant, τ_e, called the minority carrier lifetime.
5. If we inject minority carriers into a material the carriers will diffuse into the material, recombining as they go.
    a. This will lead to an exponential decay in excess minority carrier density with distance from the surface where they are being injected.
6. There will be a characteristic length constant, L_e, describing the average distance into the material the carriers reach before recombining.