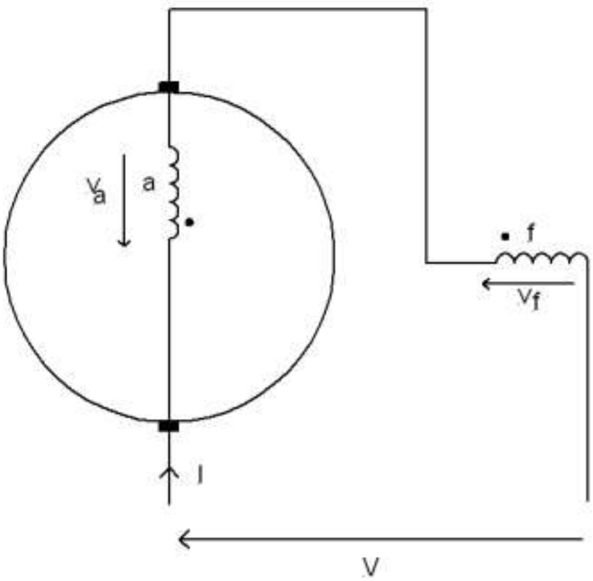
EEE 6120 Modelling of Electrical Machines
2009 Examination Solutions

[Notes in italics within square parenthesis are intended to provide background context to the question and/or to give further details of the methodology expected].

1.

a)



a)

The general form of the voltage equations are:

$$\begin{vmatrix} v_a \\ v_f \end{vmatrix} = \begin{vmatrix} R_a + L_a p & \omega_r M \\ 0 & R_f + L_f p \end{vmatrix} \begin{vmatrix} i_a \\ i_f \end{vmatrix}$$

On DC: $p=0$

On AC: $p=j\omega_s$

Constraining equations:

$V = V_a + V_f$

$I = I_a = I_f$

The resulting voltage equations are:

DC operation:

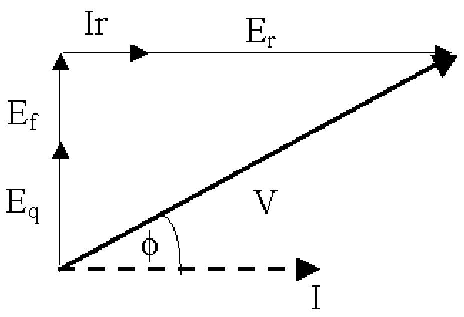
$V = I (R_a + R_f + \omega_r M)$

AC operation:

$V = I (R_a + R_f + \omega_r M + j(X_a+X_f))$

(3)

b) The phasor diagram AC operation:



Where:

$r=R_f+R_a$

$E_f = jXI_f$

$E_q = jXI_q$

$Er = I\omega_r M$

(1)

c) On an AC supply:

$$T = M I_{RMS}{}^2$$

From the operating point of 1.2Nm at 7.2Arms, then:

$$\therefore M = \frac{T}{I^2} = \frac{0.65}{7.2^2} = 12.5\times10^{-3} H$$

From the phasor diagram in part (b):

$$V\cos\phi = I(r + \omega_r M)$$

$$\therefore r = \frac{V\cos\phi}{I} - \omega_r M = \frac{230\times0.78}{7.2} - (17000\times\frac{2\pi}{60}\times0.0125) = 2.6\Omega$$

Similarly, from the phasor diagram:

$$V\sin\phi = I(X_f + X_q) = IX$$

$$\therefore X = \frac{V\sin\phi}{I} = \frac{230\times0.626}{7.2} = 20\Omega$$

The copper loss is given by:

Copper loss = I_{RMS}^2 r = $7.2^2 \times 2.6 = 134.5$W

(6)

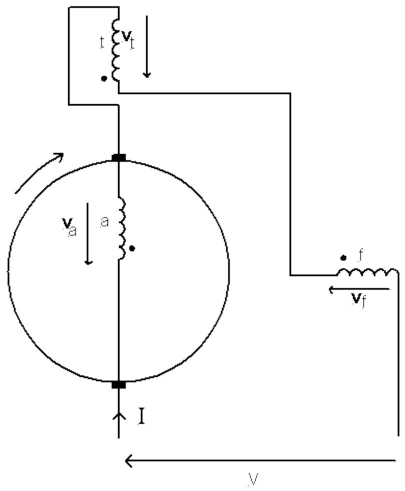
d) For starting torque, $\omega_r = 0$

On AC: $$I = \frac{V}{\sqrt{r^2 + X^2}} = \frac{230}{\sqrt{2.6^2 + 20^2}} = \frac{230}{20.2} = 11.4A$$

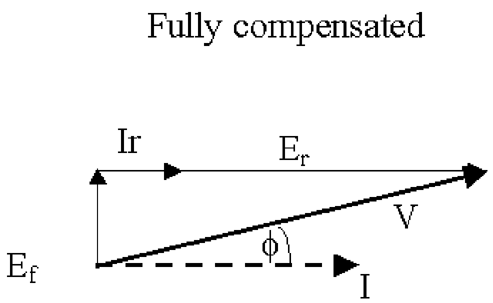
$$\therefore T = MI^2 = 0.0125 \times 11.4^2 = 1.62 \text{ Nm}$$

(2)

e) The Kron primitive equivalent of an conductively compensated series universal motor is:



The phasor diagram for a fully compensated machine is:



(3)

f) The reactance of the field coil is given by:

$X = 2\pi fL = 2\pi \times 50 \times 44 \times 10^{-3} = 13.8\Omega$

The total resistance r = 2.6+0.8 = 3.4 Ω

At 17,000rpm:

$$\omega_r M = 17000 \times \frac{2\pi}{60} \times 0.0125 = 22.3\Omega$$

The current drawn at 17,000rpm is:

$$\mathrm{I} = \frac{\mathrm{V}}{\sqrt{(\mathrm{r} + \omega_r \mathrm{M})^2 + \mathrm{X_f}^2}} = \frac{230}{\sqrt{25.7^2 + 13.8^2}} = \frac{230}{29.2} = 7.88\mathrm{A}$$

Hence,

$$I(\omega_r M + r) = 7.88(22.3 + 3.4) = 202.5\, V$$

[Could also have progressed this via: $E_f = IX_f = 7.88 \times 13.8 = 108.8V$*]*

The power factor is therefore given by:

$$cos\theta = \frac{202.5}{230} = 0.88\, lagging$$

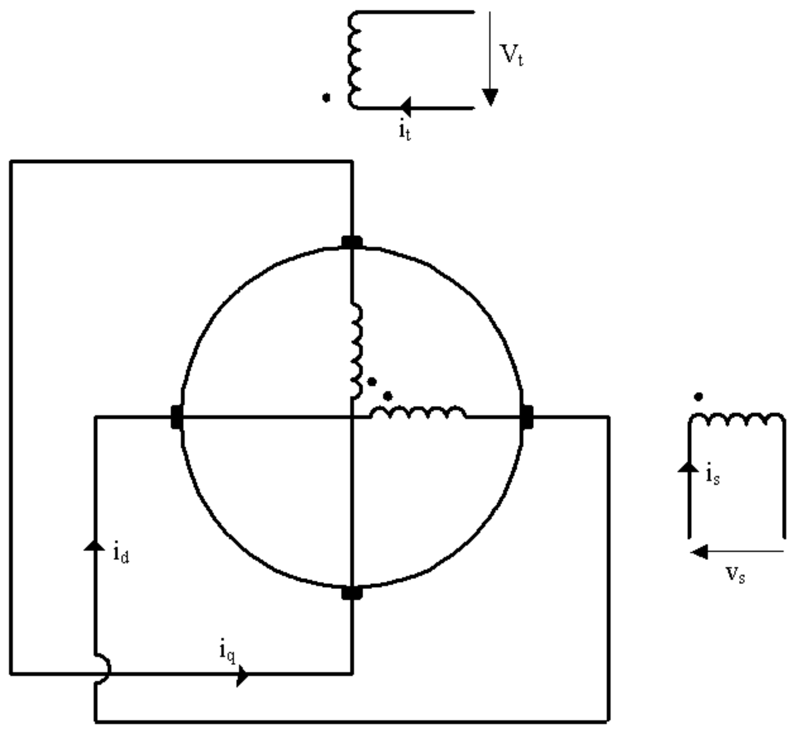
[Important to note that the power factor is lagging. As expected the power factor is improved compared to the non compensated machine value of 0.78 lagging- useful check]

Mechanicaloutput torqe = MI^2 = 0.0125×7.88^2 = 0.78Nm *[which as expected is higher that the 0.65Nm for the uncompensated machine]*

(5)

2.

a) The Kron primitive equivalent of a three-phase induction motor is given by:



Adopting subscripts of ‘1’ for the stator and ‘2’ for the rotor, then the general form of the voltage matrix equations is:

$$\begin{vmatrix} v_s \\ v_t \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_1+L_1p & 0 & M_{sd}\,p & 0 \\ 0 & R_1+L_1p & 0 & M_{td}\,p \\ M_{ds}p & -M_{dt}\omega_r & R_2+L_2p & -L_2\omega_r \\ M_{qs}\omega_r & M_{qt}\,p & L_2\omega_r & R_2+L_2p \end{vmatrix} \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix}$$

For steady-state operation for a sinusoidal AC supply:

$p=j\omega_s$ and $\omega_r=(1\text{-}s)\ \omega_s$

In addition, the same magnitude of applied to the two stator coils and the two rotor coils, but with a 90^o phase difference

$$\begin{vmatrix} V_s \\ V_t \\ V_d \\ V_q \end{vmatrix} = \begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix} = \begin{vmatrix} I_1 \\ j\,I_1 \\ I_2 \\ j\,I_2 \end{vmatrix}$$

The governing voltage equation is therefore:

$$\begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & 0 & jX_m & 0 \\ 0 & R_1+jX_1 & 0 & jX_m \\ jX_m & -(1\text{-}s)X_m & R_2+jX_2 & -(1\text{-}s)X_2 \\ (1\text{-}s)X_m & jX_m & -(1\text{-}s)X_2 & R_2+jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ jI_1 \\ I_2 \\ jI_2 \end{vmatrix}$$

But row 2 is simply row 1 $\times$j and row 4 is simply row 3 $\times$j. Hence the system can be reduced to two matrix equations:

$$\begin{array}{l} \text{Stator} \\ \text{Rotor} \end{array} \quad \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m\text{- } j(1\text{-}s)X_m & R_2+\ jX_2\text{- } j(1\text{-}s)X_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix}$$

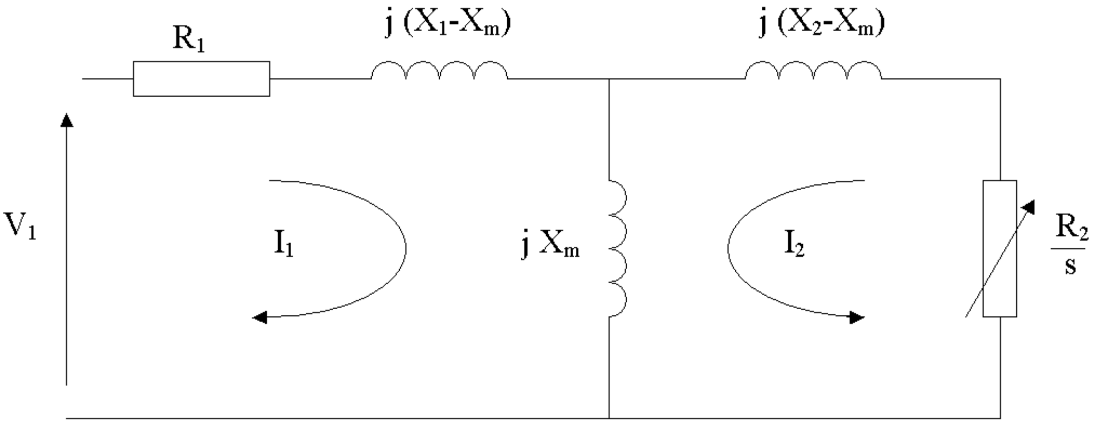
Since the rotor is short circuited, $V_2=0$

Substituting for V_2 and dividing the rotor equations by s gives:

$$\begin{array}{l} \text{Stator} \\ \text{Rotor} \end{array} \quad \begin{vmatrix} V_1 \\ 0 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m & R_2/s+\ jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2' \end{vmatrix}$$

[Note I_2 is transformed to I_2']

An equivalent circuit that satisfies these voltage equations is:



(9)

b)

[Note 1: *This question differs significantly from those used previously to test the student's knowledge in the application of the equivalent circuit derived in part (a) (e.g. Q2 in 2006). In previous questions, the full set of equivalent circuit parameters were given in the question and the students were required to use these in the equivalent circuit to calculate various aspects of performance. In this case, two of the machine parameters must be calculated from performance for two operating conditions, i.e. no-load and a very onerous maximum load condition. Having calculated these, the questions returns to a more conventional format in that these parameters are deployed to calculate various aspects of performance.]*

[Note 2: *This problem can be solved using either the exact or simplified equivalent circuit. The latter involves moving the magnetising branch to the terminals, but is reliant on the magnetising reactance being significantly higher than the other impedances. The clue that this is indeed a reasonable assumption in this case contained in the question which states 'This magnetising current is small in comparison with the total input current drawn at rated load'. Providing students recognise this assumption (preferably with some justification based on the values presented in the question) then the use of the simplified equivalent circuit is equally as valid in terms of the marks awarded.]*

For a star-connected machine, phase current = line current

$$V_{ph} = \frac{415}{\sqrt{3}} = 240V$$

b) The magnetising reactance can be derived directly from the magnetising current and the terminal voltage for the simplified per-phase equivalent circuit in which the magnetising branch is moved to the terminals:

$$Z = \frac{V_{ph}}{I_{ph}} = \frac{240\angle 0°}{15\angle -90°} = 16\angle 90°\Omega$$

The referred rotor resistance R_2' can be derived from the start-of-mix conditions. When operating at this point, the total mechanical output power is given by:

$$P_{out} = 3\left|I_1\right|^2 \frac{(1-s)R_2'}{s}$$

Rearranging gives:

$$R_2' = \frac{sP_{out}}{3|I_1|^2(1-s)} = \frac{sT\omega_{mech}}{3|I_1|^2(1-s)}$$

The synchronous speed of a 2-pole motor on a 50Hz supply is 3000rpm. Hence, at 2850rpm:

$$s = \frac{(3000-2850)}{3000} = 0.05$$

$$\therefore R_2' = \frac{0.05\times 101.5\times \frac{2850\times 2\pi}{60}}{3\times 46.1^2\times (1-0.05)} = 0.25\Omega$$

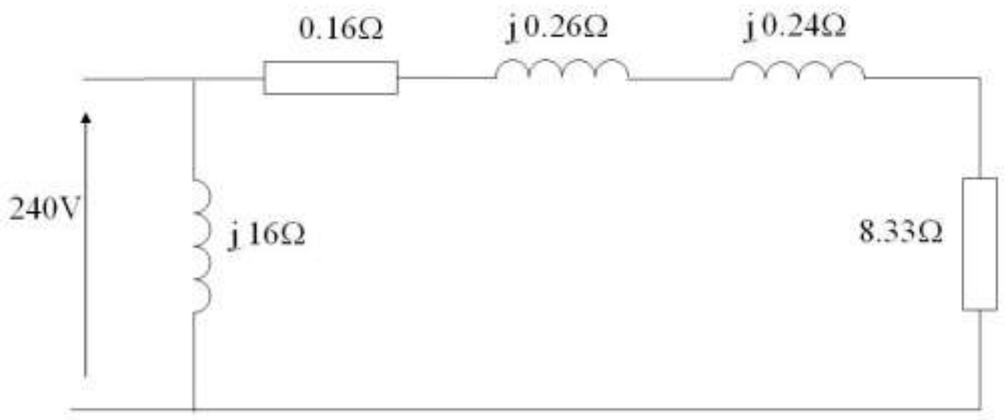
(3)

c) For the end-of-mix condition, the slip, s, is given by:

$$s = \frac{(3000-2910)}{3000} = 0.03$$

[***Note***: It is likely that some candidates will miss this change is slip and plough ahead with 0.05– it is a key point to probe since slip is a function of the operating point at not a fixed machine parameters]

The per-phase equivalent circuit is:



The total impedance of the main branch is:

$$Z_e = (0.16 + 8.33) + j0.5 \;\; = 8.50\angle 3.4^{o}\,\Omega$$

The phase current in the main branch is

$$I_1 = \frac{240\angle 0^o}{8.50\angle 3.4^o} = 28.2\angle -3.4^o\,A$$

The total input current at the end-of-mix operating point is hence given by:

$$I_{ip} = I_1 + I_m = 28.17 - j1.67 - j15 = 28.17 - j16.67$$

$$= \; 32.7\angle -30.6^o\,A$$

The power factor is cos (-30.6°) = <u>0.861 lagging</u>

[Note: mark deducted for not specifying 'lagging' or 'lag']

The net mechanical power, P_{out} is given by:

$$P_{out} = 3\,|I_1|^2\,\frac{(1-s)R_2^{'}}{s} = 3\times 28.2^2 \times \frac{0.97\times 0.25}{0.03} = 19.28kW$$

Hence, output torque is given by:

$$T = \frac{P_{out}}{\omega_{mech}} = \frac{19284}{\frac{2910}{60}\times 2\pi} = 62.3Nm$$

The total input power = $3\times 240\times 32.7\times \cos\ -30.6° = 20.28kW$

Neglecting core losses *[students are expected to note this assumption]* then the overall efficiency is given by:

$$Efficiency\ = \frac{19.28}{20.28} = 95\%$$

[As a check, or as an alternative method, the copper loss can also be used to calculate overall efficiency without recourse to the input power]

(8)

3.

a) Applying the trapezium rule to integrate the area under the fully aligned curve (i.e. the curve at an angular displacement of 10°) for currents up to 30A yields:

$$A_{0\to 10} = \frac{10\Psi_{10}}{2} = 3.0\text{ J}$$

$$A_{10\to 20} = \frac{10(\Psi_{10} + \Psi_{20})}{2} = 8.1\text{J}$$

$$A_{20\to 30} = \frac{10(\Psi_{20} + \Psi_{30})}{2} = 11.0\text{J}$$

Hence the total area under the curve up to 30A is:

$$A_{0\to 30} = A_{0\to 10} + A_{10\to 20} + A_{20\to 30} = 22.1J$$

The area under the un-aligned curve (which can reasonably regarded as being linear) is simply given by:

$$U_{0\to 30} = \frac{30\Psi_{30}}{2} = 1.8J$$

Hence the change in co-energy is given by:

$$\Delta W^{'} = A_{0\to 30} - U_{0\to 30} = 20.3\mathrm{J}$$

The average torque for 30A is therefore given by:

$$\mathrm{T_{AVE}} = \frac{\Delta \mathrm{W}^{'}}{\Delta\theta} = \frac{20.3}{\pi/18} = 116\mathrm{Nm}$$

Repeating the same process from 30A to 50A yields:

$$\mathrm{A}_{30\to 40} = \frac{10(\Psi_{30} + \Psi_{40})}{2} = 12.1\mathrm{J}$$

$$\mathrm{A}_{40\to 50} = \frac{10(\Psi_{40} + \Psi_{50})}{2} = 12.6\mathrm{J}$$

$$\mathrm{A}_{0\to 30} = \mathrm{A}_{0\to 10} + \mathrm{A}_{10\to 20} + \mathrm{A}_{20\to 30} + \mathrm{A}_{30\to 40} + \mathrm{A}_{40\to 50} = 46.8\mathrm{J}$$

$$\mathrm{U}_{0\to 30} = \frac{50\Psi_{50}}{2} = 5.0\mathrm{J}$$

Hence the change in co-energy is given by:

$$\Delta \mathrm{W}^{'} = \mathrm{A}_{0\to 50} - \mathrm{U}_{0\to 50} = 41.8\mathrm{J}$$

The average torque for 30A is therefore given by:

$$\mathrm{T_{AVE}} = \frac{\Delta \mathrm{W}^{'}}{\Delta\theta} = \frac{41.8}{\pi/18} = 239\mathrm{Nm}$$

(6)

b) From the aligned Ψ-I characteristic it can be seen that the onset of saturation occurs at a flux current of 11.5A (an answer based on a slightly different interpretation of saturation is equally acceptable). It is important to note that the flux produced by 6 coils that constitute a phase crosses 6 airgaps, each of length l_g. Hence, if the number of turns on one coil is N_c

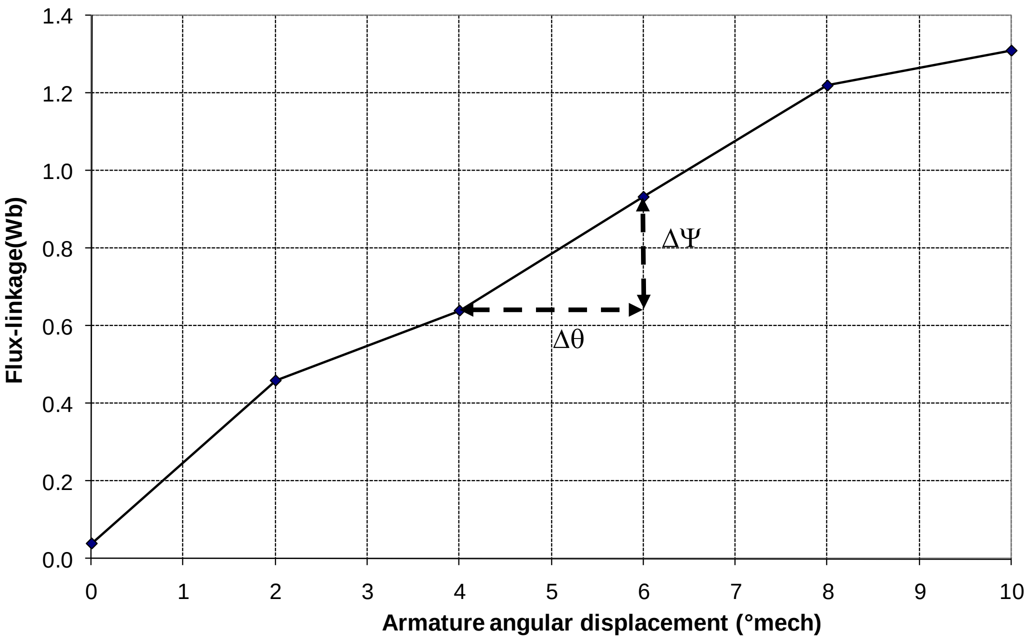
Since $\mathrm{B_g} \approx \frac{\mu_0 \mathrm{N_c I}}{\mathrm{l_g}}$ prior to saturation then a reasonable estimate of l_g can be obtained from this equation.

$$\therefore \mathrm{l_g} = \frac{\mu_0 \mathrm{N_c I}}{\mathrm{B_g}} = \frac{4\pi\times 10^{-7}\times 52\times 11.5}{1.5} = 0.50\ \mathrm{mm}$$

[a reasonable error band on this value is acceptable given the difficulty in precisely defining the onset of saturation – the method employed is the key factor in determining the marks awarded]

(3)

c) Taking the values of flux-linkage at 60A for the each angular displacement and re-plotting a graph of flux-linkage versus angular displacement yields:



From the graph, the rate of change of flux-linkage with respect to rotor angular displacement around 5^o is given to a reasonable approximation by:

$$\left.\frac{\mathrm{d}\Psi}{\mathrm{d}\theta}\right|_5 \approx \frac{\Psi_6 - \Psi_4}{2\times\dfrac{\pi}{180}} \approx \frac{0.93-0.64}{2\times\dfrac{\pi}{180}} \approx 8.31\,\mathrm{Wb\,/\,rad}$$

At 300 rpm, the rate of change of angular displacement is given by:

$$\frac{\mathrm{d}\theta}{\mathrm{dt}} = \frac{300\times 2\pi}{60} = 31.4\ \mathrm{rad\,/s}$$

The instantaneous value of the induced emf is hence given by:

$$\frac{d\Psi}{dt} = \frac{d\Psi}{d\theta}\times\frac{d\theta}{dt} = 261\mathrm{V}$$

[Note: The question does not explicitly ask for the students to plot a flux-linkage versus angular displacement characteristic – although this is the method introduced and applied in the lecture notes. Should they choose to simply read off the values of flux-linkage at both 4^o and 6^o from the flux-linkage versus current characteristic at 60A and then proceed, then this will be awarded full marks. Indeed this 'shortcut' demonstrates a good understanding]

(4)

d) *[The key to this question is identifying that the maximum value of inductance is obtained in the fully aligned position and at modest current levels, i.e. those below the onset of saturation. This neglects the very minor issue of the reversible region of a typical iron B-H characteristic which has not been covered in this course – this would tend to suggest using enough current to get the iron beyond its reversible region. The conditions for maximum inductance have not been covered in the notes or previous examination papers, and so this is probing a thorough understanding of what the flux-linkage versus current characteristics represents. The calculation itself is almost trivial so the majority of the marks will be awarded for identifying a reasonable combination of rotor angle and current].*

The maximum value of absolute inductance (i.e. Ψ/i) is achieved in the aligned position at low current. Providing the machine does not saturate then any value of current can be used, but it is good practice, to use a value which is sufficiently high to read off Ψ to a reasonable degree of precision. Hence in the aligned position (10^o) and a current of 8A, the phase self-inductance is given by:

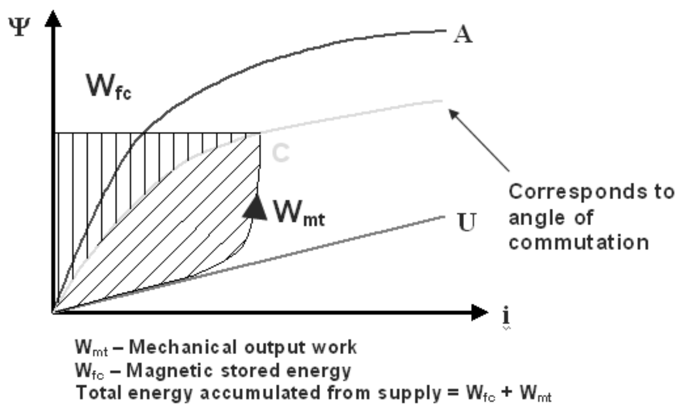
$$L = \frac{\psi}{\mathrm{I}} = \frac{0.5}{8} = 62.5 \times 10^{-3} H$$

[As with all questions which draw on information derived from a graph, a reasonable tolerance will be accepted]

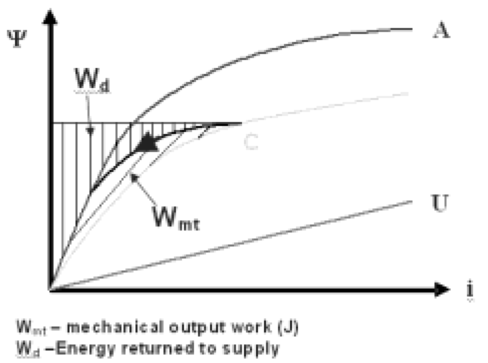
(3)

e) In motoring mode, the two dynamic ψ-I characteristics are:

Up to the instant of commutation:



Following commutation:



[In marking this section, particular emphasis will be placed on precise definitions and identification of the various energy changes]

Exact form of trajectory depends on:

- Rotational speed
- Rotor and load inertia
- Magnitude of applied voltage
- Commutation angles

(4 – 3 for diagrams, 1 for complete list)

4.

b) The flux-linkage characteristics for 0A is a reasonable approximation to a sine-wave [in fact the actual data is generated from a simple sin function]. It is therefore reasonable to assume that the maximum rate of change of flux-linkage will occur at angular displacements around 0^{o}. [*It is not necessary to identify this with a sine wave, just to recognise visually that the maximum rate of change will occur around 0^{o}*]. From Figure 4b, an estimate of the rate of change of flux linkage with rotor position can be made:

$$\frac{\mathrm{d}\Psi}{\mathrm{d}\theta} \approx \frac{\Delta\Psi}{\Delta\theta} = \frac{0.007}{20\times\frac{\pi}{180}} = 0.020 \text{ Wb/rad}$$

[These calculations have all been performed in terms of mechanical radians]

At 3200rpm

$$\frac{\mathrm{d}\theta}{\mathrm{dt}} = \frac{3200\times 2\times\pi}{60} = 335 \text{ rad/s} \quad \therefore \mathrm{e} = \frac{\mathrm{d}\Psi}{\mathrm{dt}} = \frac{\mathrm{d}\Psi}{\mathrm{d}\theta}\times\frac{\mathrm{d}\theta}{\mathrm{dt}} = 6.7\mathrm{V}$$

An alternative is to note that the variation in flux-linkage can approximated as:

$\Psi = 0.02\sin(\theta)$

Hence $\frac{d\Psi}{d\theta} = 0.02\cos(\theta)$

This has a peak value of 0.02 Wb/rad (as before from graphical interpolation)

(4)

b) From Figure 4b, it is apparent that saturation begins at a net coil flux of 0.03 Wb *[there is a reasonable tolerance band on this given the difficulty in unequivocally identifying the onset of saturation]*

Hence, 0.03Wb of coil flux-linkage corresponds to 1.6T in the core

At 0°, there is no net permanent magnet flux and hence the flux-linkage is entirely due to the coil. Taking the case of 1A, the flux-linkage is 0.020Wb.

Hence, with no permanent magnet flux and a 1A coil current, the core flux density is approximately given by:

$B = \frac{0.02}{0.03}\times 1.6 = 1.07\mathrm{T}$

The total effective magnetic airgap is:

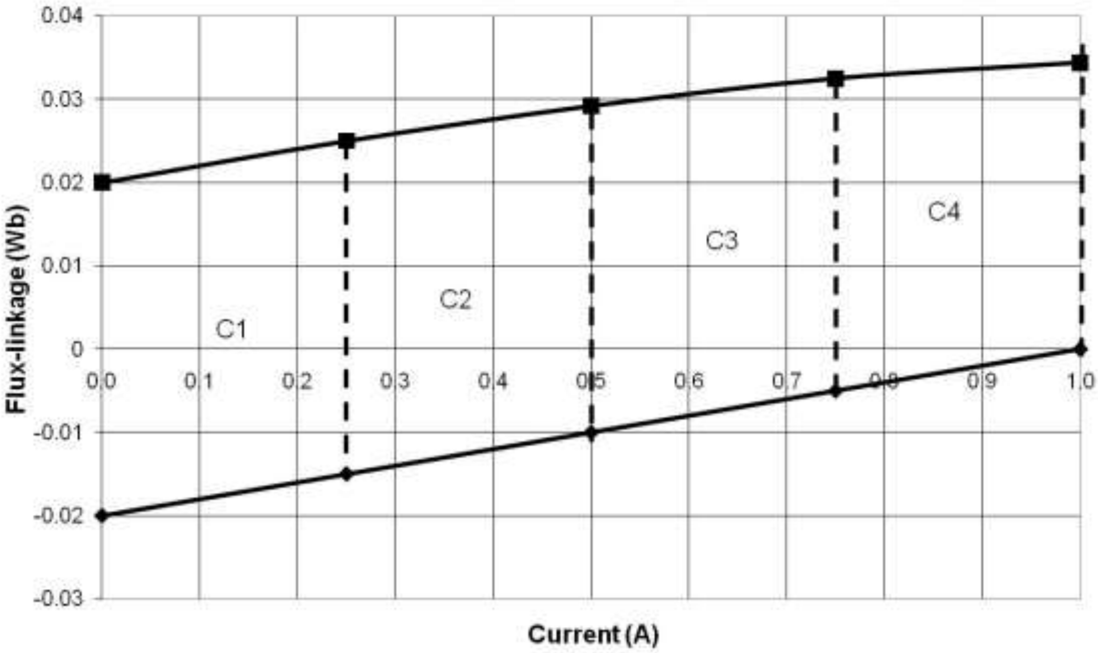
$l_{g-eff} = \frac{\mu_0 NI}{B} = \frac{4\pi\times10^{-7}\times4000\times1.0}{1.07}$ =4.7mm

But this consists of 4mm total thickness of magnet and two airgaps.

Hence, the net airgap (i.e. clearance between rotor magnet surface and stator core) is 0.35mm

(5)

c) In order to estimate the torque for the two currents specified it is necessary to re-plot the data as a flux-linkage versus current characteristic for –90^o and + 90^o:



The co-energy change can be estimated by trapezoidal integration of the four areas C1 to C4 shown in the graph above. Using this approach:

The change in co-energy for 0.25A is C1 = 0.010J

The change in co-energy for 100A is C1+C2+C3+C4 = 0.01+ 0.0099+0.0096+0.0090=0.0385J

Change in rotor angular displacement = $180\times\frac{\pi}{180} = \pi$ rads

The torques produced are therefore given by:

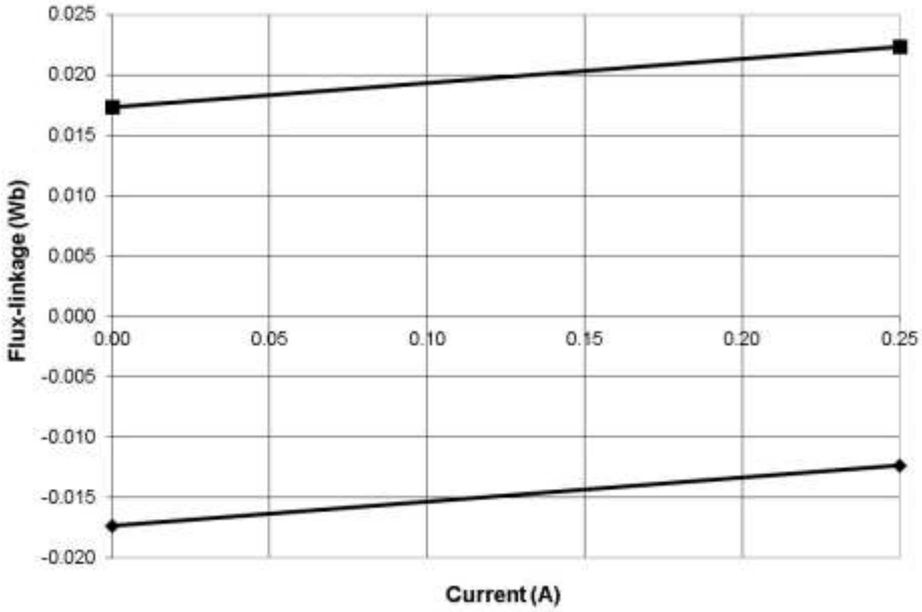
At 0.25A: $T = \frac{dW'}{d\theta} \approx \frac{0.01}{\pi} = 3.18\ \times10^{-3}\,Nm$

At 100A: $T = \frac{dW'}{d\theta} \approx \frac{0.0385}{\pi} = 12.25\times10^{-3}\,Nm$

[An important point here is that the torque per amp is gradually diminishing with onset of magnetic saturation]

(7)

d) *[The same process as that for the 180^o conduction interval can be applied to produce corresponding Ψ/i characteristics. However, since the question only asks for the 0.25A case, there are only 4 points to plot. Since the question does not ask the candidate to necessarily plot a characteristic, it is perfectly acceptable to plot these points on a new characteristic, add them to the answer to part (c) or simply take down the values from the Ψ/θ characteristic]*



The area between these curves is:

$$Area = 0.25 \times \frac{(0.0173+0.0173)+(0.0223+0.0123)}{2} = 0.00866 \text{ J}$$

Change in rotor angular displacement = $120 \times \frac{\pi}{180} = \frac{2\pi}{3}$ rads

The torques produced are therefore given by:

At 0.25A: $T = \frac{dW'}{d\theta} \approx \frac{0.00866 \times 3}{2\pi} = 4.13 \times 10^{-3} Nm$

However, this is the torque over the central 120^o only, hence the average torque is:

$$T_{ave} = \frac{4.13 \times 10^{-3} \times 2}{3} = 2.76 \times 10^{-3} \; Nm$$

(4)

GW Jewell

April 2009