

# **Topic 23**

## **23 Some issues in semiconductor laser diodes**

23.1 Introduction

23.2 Semiconductor laser rate equations

Carrier density rate equation

Photon density rate equation

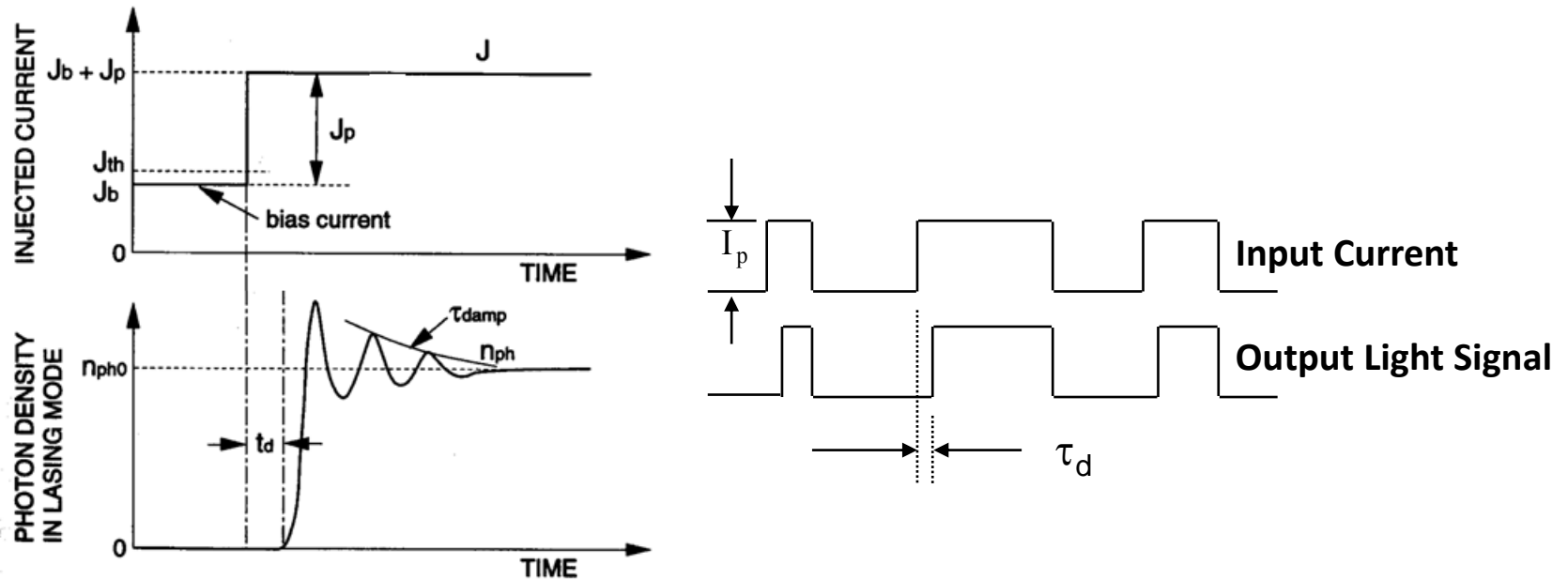
23.3 Laser turn on delay

23.4 Dynamic response above threshold

23.5 Frequency modulation response

23.6 Chirp

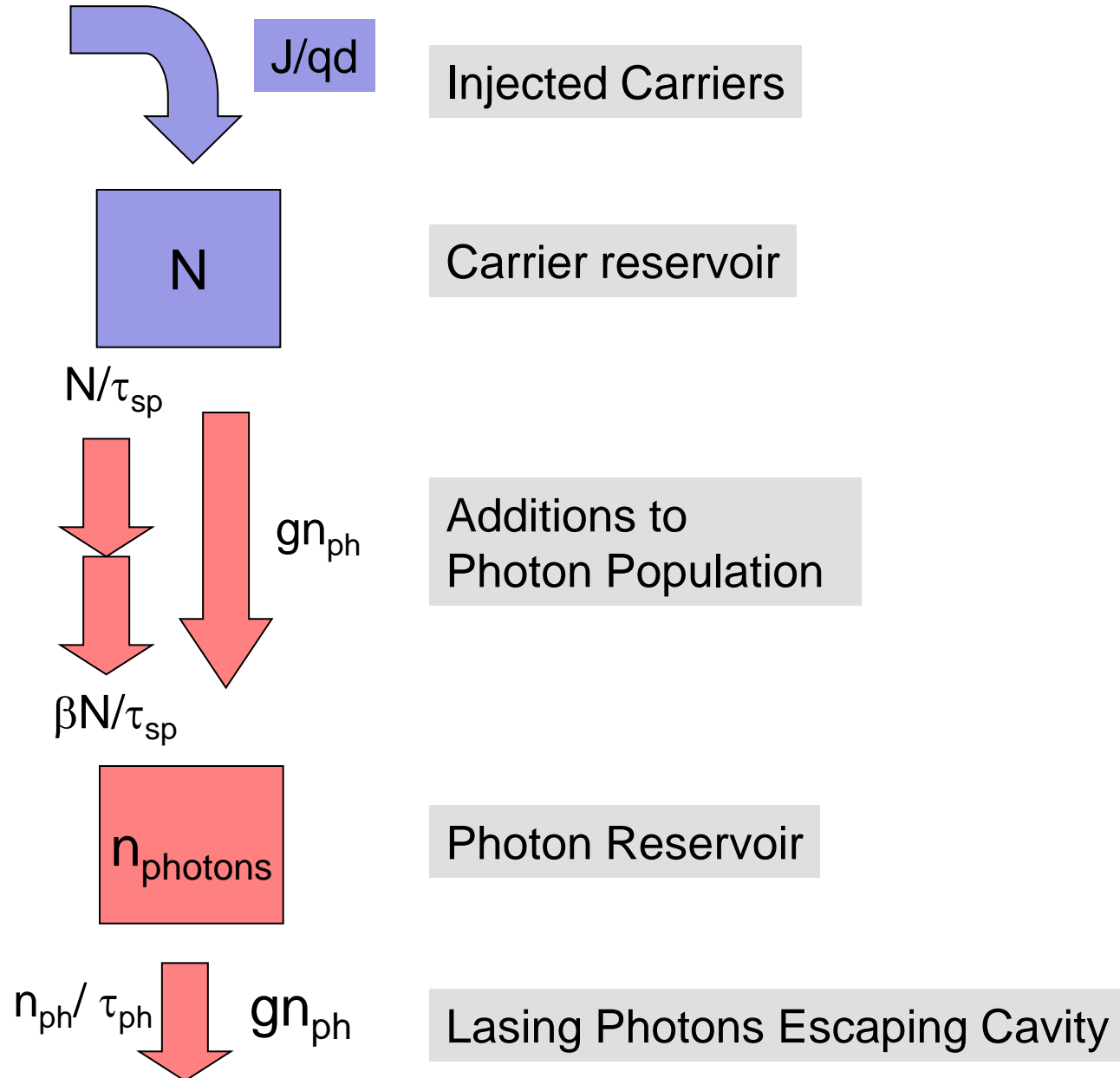
# Introduction



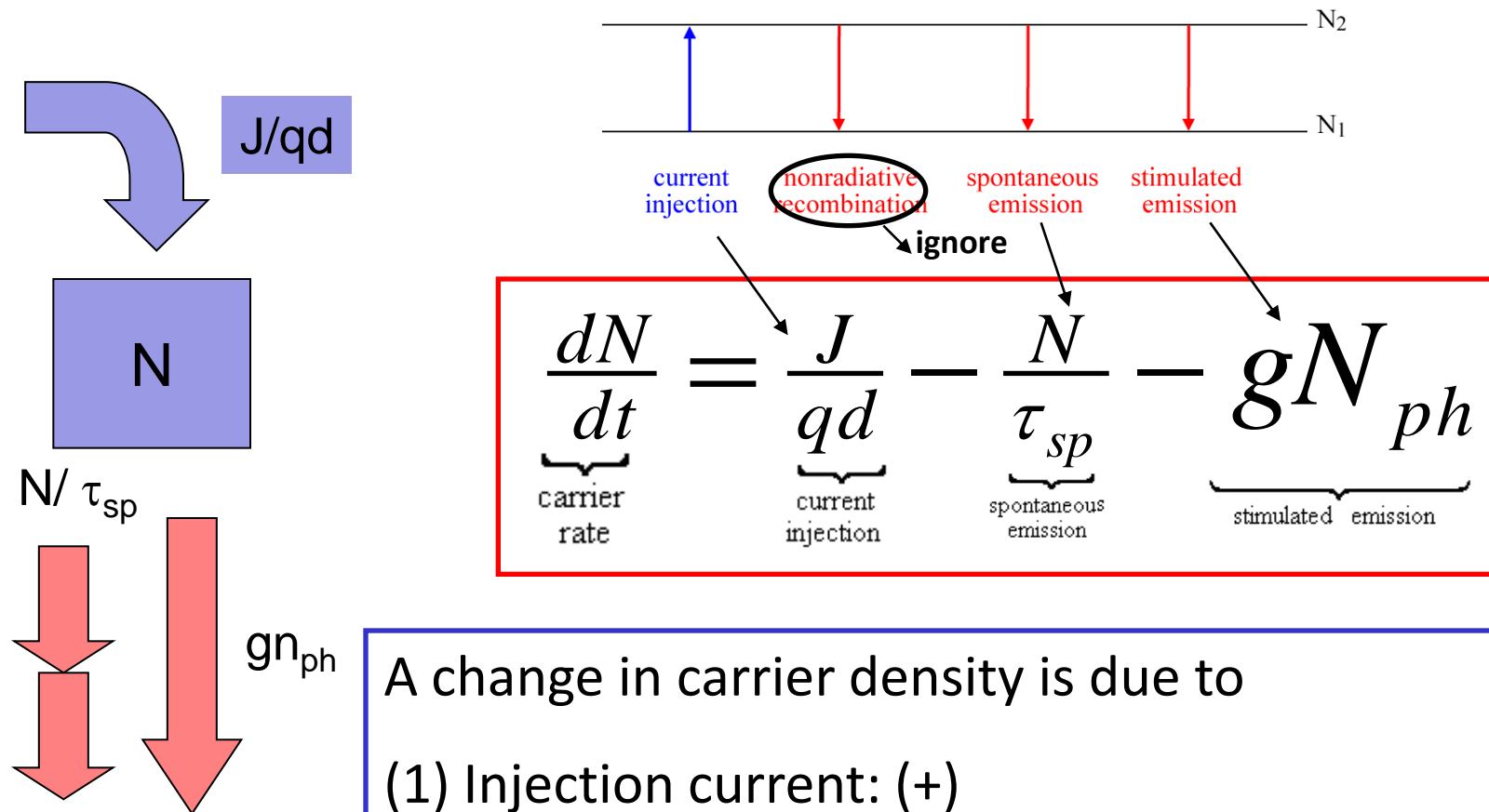
For practical applications, we can often observe:

- (1) Laser turn-on delay
- (2) Relaxation Oscillation

# Schematic Process of Electrical Injection LDs



# Carrier Density Rate Equation



A change in carrier density is due to

(1) Injection current: (+)

**add carriers**

(2) spontaneous and stimulated recombination: (-)

**remove carriers**

(3) non-radiative recombination is ignored

# Photon Density Rate Equation

$$\underbrace{\frac{dN_{ph}}{dt}}_{\text{Photon rate}} = - \left( \underbrace{gN_{ph}}_{\text{stimulated emission}} + \underbrace{\beta \frac{N}{\tau_{sp}}}_{\text{spontaneous emission}} \right) + \underbrace{\frac{N_{ph}}{\tau_{ph}}}_{\text{loss of photons}}$$

## 3 channels for a change in photon density

- (1) Stimulated recombination: (-)
- (2) Spontaneous recombination (-)
- (3) Optical loss in gain region (scattering, absorption, etc): (+)

Introducing photon lifetime ( $\tau_{ph}$ ) to describe optical loss:

$\beta_{sp}$ : percentage of spontaneous emission ( $\sim 10E^{-5}$ )

# Steady State Solution

- Steady state requires: both carrier density and photon density have to remain unchanged

$$\frac{dN}{dt} = 0 \quad \frac{dN_{ph}}{dt} = 0$$

- From Carrier density rate equation and photon density rate equation:

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_{sp}} - gN_{ph} = 0$$

$$\frac{dN_{ph}}{dt} = -(gN_{ph} + \beta \frac{N}{\tau_{sp}}) + \frac{N_{ph}}{\tau_{ph}} = 0$$

$$\Rightarrow J = qd \left[ \frac{N}{\tau_{sp}} + gN_{ph} \right]$$

$$\Rightarrow gN_{ph} = \frac{N_{ph}}{\tau_{ph}} - \beta \frac{N}{\tau_{sp}}$$

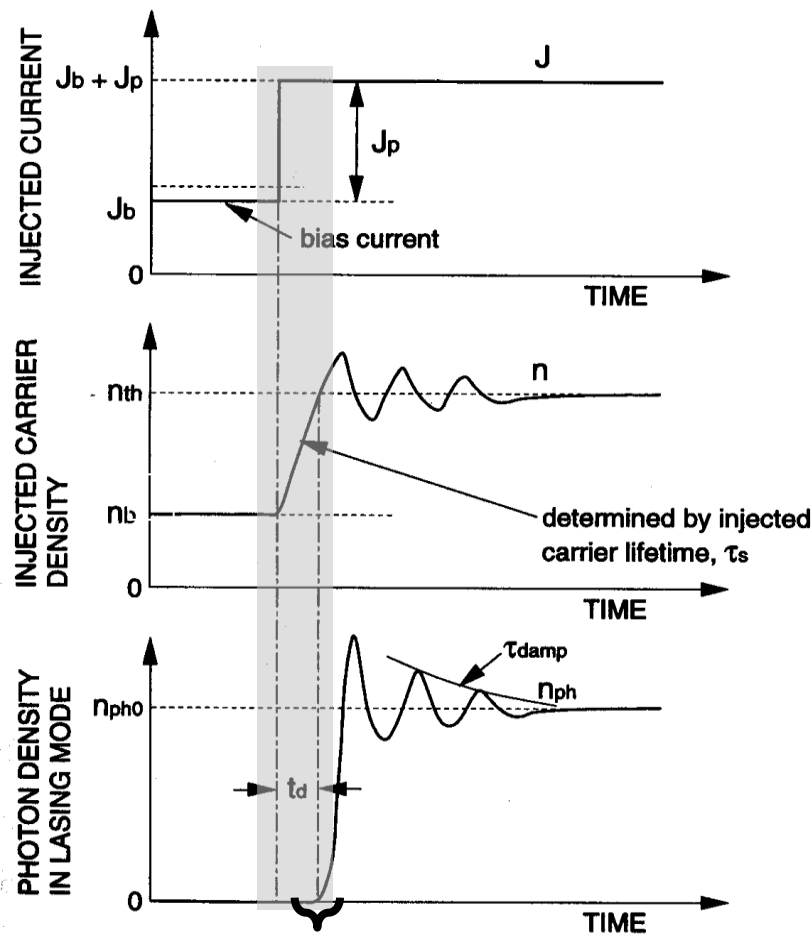
- (1) At threshold: Ignore effect of stimulated recombination of carriers

$$J_{th} = \frac{qdN_{th}}{\tau_{sp}}$$

- (2) Above threshold: Ignore spontaneous emission into lasing mode

$$g = \frac{1}{\tau_{ph}}$$

# Laser Turn on Delay



Laser Turn on

- Initial carrier density (Before onset of stimulated emission)

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau_{sp}}$$

- Rate of increase ( $dN/dt$ ) is positive : cause an increase in carrier density

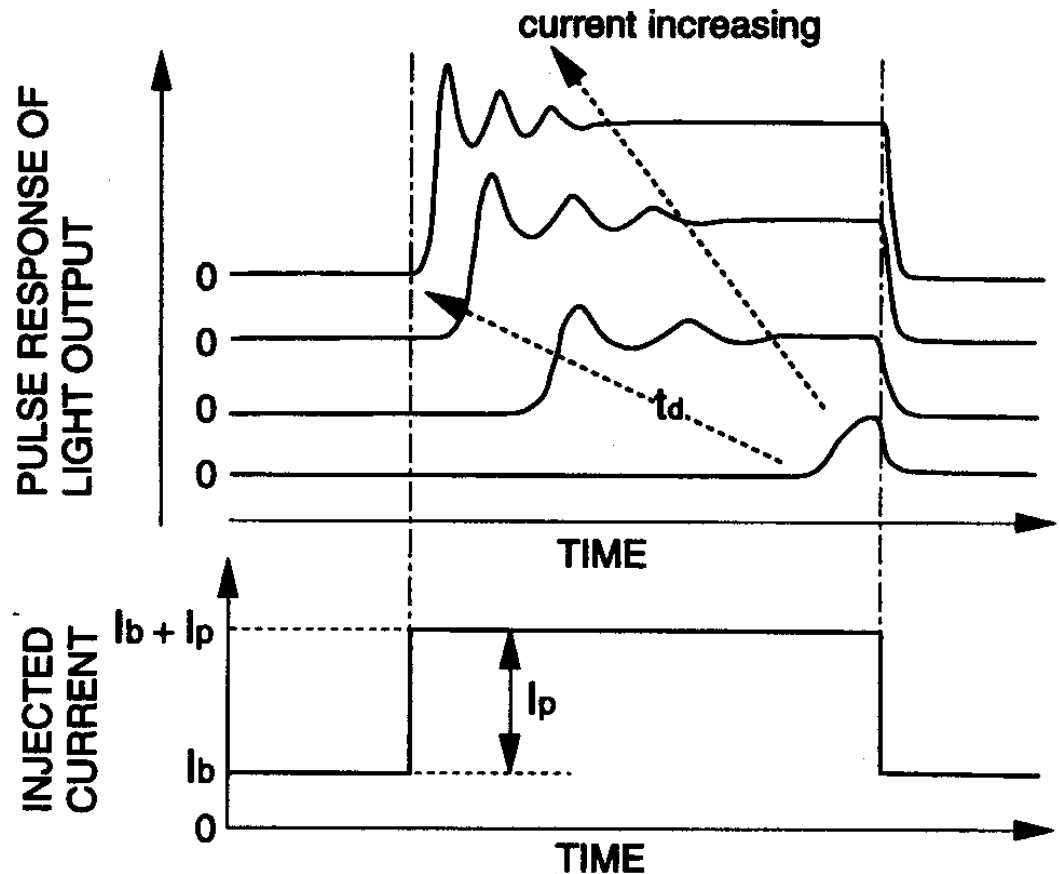
- When  $N \geq N_{th}$ : lasing starts

- Time delay,  $t_d$  given by:

$$t_d = \tau_{sp} \ln \left[ \frac{J}{J - J_{th}} \right]$$

# Laser Turn on Delay – increasing current

$$t_d = \tau_{sp} \ln \left[ \frac{J}{J - J_{th}} \right]$$



Constantly add extra  $I_{bias}$  - reduces delay time

- Use a low threshold laser and make  $I_p$  large
- Bias the laser at or above threshold

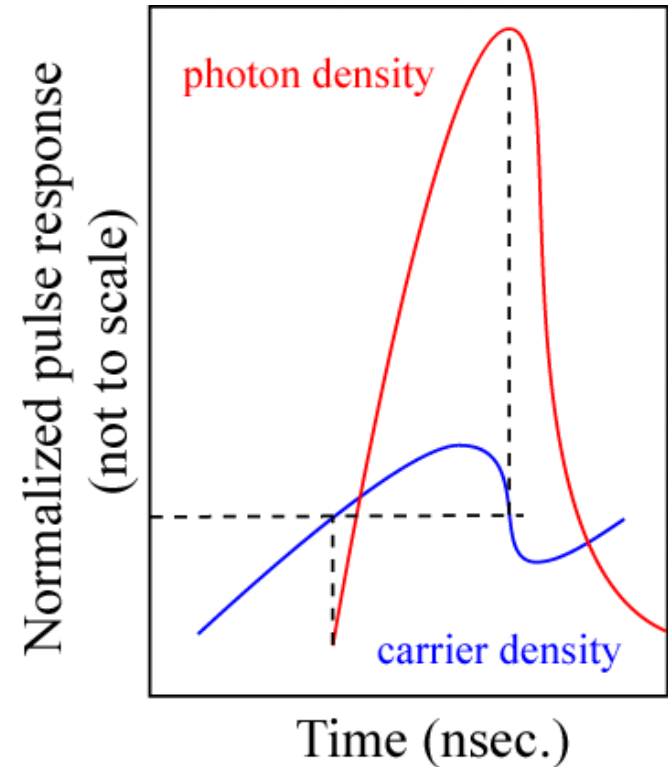


# Above Threshold

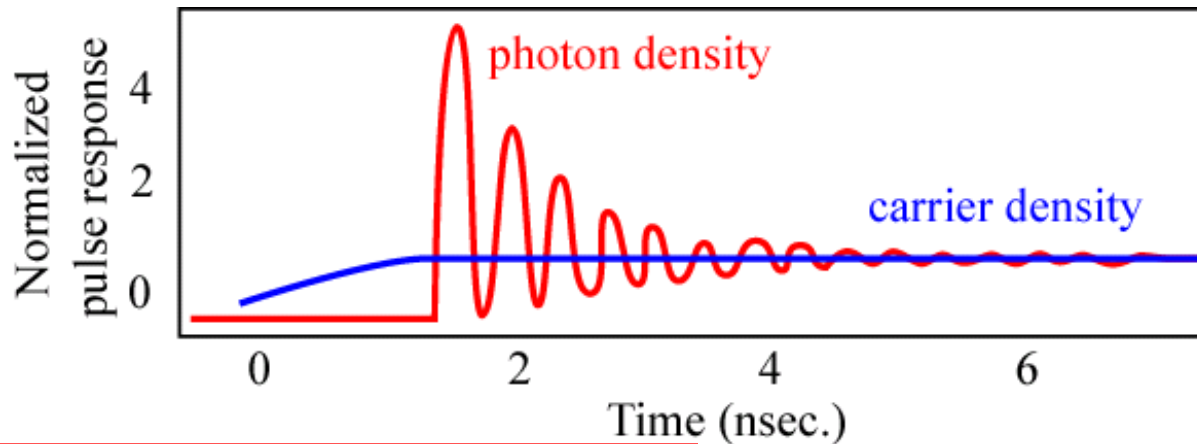
$$\underbrace{\frac{dN}{dt}}_{\text{carrier rate}} = \underbrace{\frac{J}{qd}}_{\text{current injection}} - \underbrace{\frac{N}{\tau_{sp}}}_{\text{spontaneous emission}} - \underbrace{gN_{ph}}_{\text{stimulated emission}}$$

$$\underbrace{\frac{dN_{ph}}{dt}}_{\text{Photon rate}} = -\left( \underbrace{gN_{ph}}_{\text{stimulated emission}} + \underbrace{\beta \frac{N}{\tau_{sp}}}_{\text{spontaneous s emission}} \right) + \underbrace{\frac{N_{ph}}{\tau_{ph}}}_{\text{loss of photons}}$$

- When  $N > N_{th}$ :
  - (1) The increase in  $N_{ph}$  causes
    - decrease in the  $dN/dt$  because the stimulated emission term is negative
  - (2) When  $N_{ph}$  reaches a certain value  $dN/dt$  becomes negative
    - $N$  starts to decrease



# Relaxation Oscillations (RO)



$$\underbrace{\frac{dN}{dt}}_{\text{carrier rate}} = \underbrace{\frac{J}{qd}}_{\text{current injection}} - \underbrace{\frac{N}{\tau_{sp}}}_{\text{spontaneous emission}} - \underbrace{gN_{ph}}_{\text{stimulated emission}}$$

$$\underbrace{\frac{dN_{ph}}{dt}}_{\text{Photon rate}} = -\left( \underbrace{gN_{ph}}_{\text{stimulated emission}} + \underbrace{\beta \frac{N}{\tau_{sp}}}_{\text{spontaneous emission}} \right) + \underbrace{\frac{N_{ph}}{\tau_{ph}}}_{\text{loss of photons}}$$

- When  $N$  drops below  $N_{th}$ 
  - $N_{ph}$  drops quickly, and then  $N$  starts increasing again
  - The process repeats itself as a damped oscillation
- After repeating it several times, a steady state can be achieved, and both carrier density and photon density are stable

# RO Frequency

$$\underbrace{\frac{dN}{dt}}_{\text{carrier rate}} = \underbrace{\frac{J}{qd}}_{\text{current injection}} - \underbrace{\frac{N}{\tau_{sp}}}_{\text{spontaneous emission}} - \underbrace{gN_{ph}}_{\text{stimulated emission}}$$

$$\underbrace{\frac{dN_{ph}}{dt}}_{\text{Photon rate}} = \underbrace{gN_{ph}}_{\text{stimulated emission}} + \underbrace{\beta \frac{N}{\tau_{sp}}}_{\text{spontaneous emission}} - \underbrace{\frac{N_{ph}}{\tau_{ph}}}_{\text{loss of photons}}$$

- $I = I_0 + I_1 e^{j\omega t}$ ;  $N = N_0 + N_1 e^{j\omega t}$ ;  $N_{\text{photon}} = N_{\text{photon}0} + N_{\text{photon}1} e^{j\omega t}$
- Substitute, remove d.c. components satisfying steady state conditions

$$f = \frac{1}{2\pi} \frac{1}{(\tau_{sp} \tau_{ph})^{1/2}} \left( \frac{J}{J_{th}} - 1 \right)^{1/2}$$

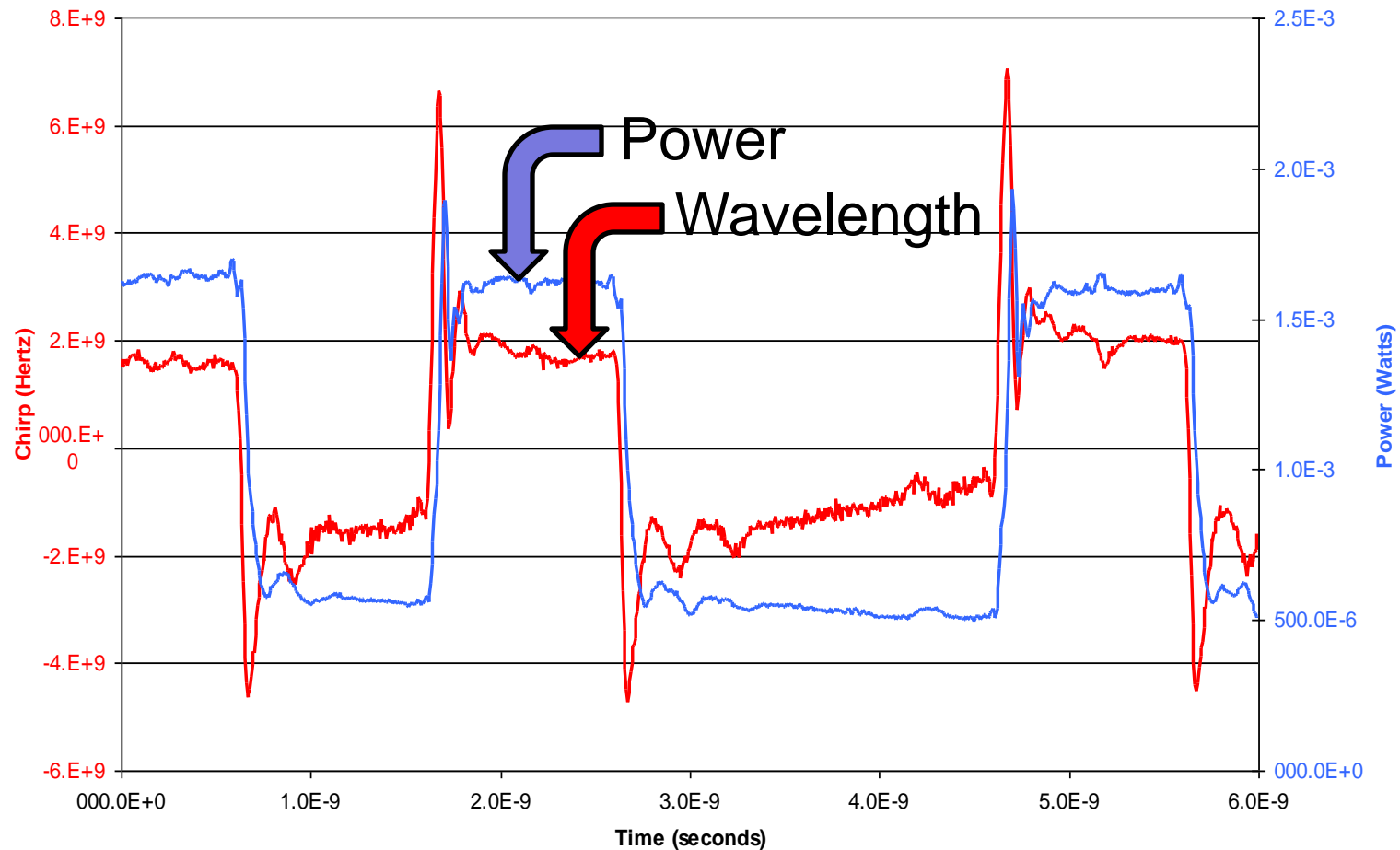
- Relaxation Oscillations: refers to attempt to return to steady state
- **RO Frequency** -determines the maximum modulation rate of laser diode
- **Turn injected current up** – might destroy the facet if it is too high
- **Decrease photon lifetime** – shorter optical cavity but  $J_{th}$  goes up as mirror loss increases

# Chirp (1)

- Definition: the modulation of a single mode laser diodes can cause a dynamic shift of the peak wavelength emitted from the device
- Formation mechanisms:
  - (1) variation carrier density with time
  - (2) Change in carrier density leads to a change in refractive index of active region
  - (3) This results in a change in the (frequency) lasing wavelength
- Only important for single mode lasers – DFBs, WDM systems, etc

- Chirping gets worse at high frequencies
- Relaxation oscillations will produce large  $dp/dt$  which leads to large chirping
- Damping of relaxation oscillations will reduce chirp

## Chirp (2)



Measurement of DFB wavelength as a function of time

- Change in wavelength is due to a change in refractive index

# Summary (1)

- For modulation of a laser diode – set up rate equations for carrier and photon reservoirs
- Solution of rate equations below and above threshold – get turn on delay for modulation from below to above threshold – this is why lasers are modulated *above* threshold
- For modulation above threshold – the maximum modulation rate is determined by the characteristic time it takes for carrier and photon reservoirs to relax to their equilibrium values – the Relaxation Oscillation Frequency

## Summary (2)

- A number of strategies exist for increasing the maximum modulation rate (increase photon density, decrease photon lifetime, etc)
- Spontaneous emission into lasing mode - due to relaxation of carrier and photon populations to equilibrium values get noise peak at RO frequency
- Due to change in cavity temperature and carrier density during modulation, refractive index of cavity and hence emission energy (linewidth) changes under modulation (chirp) - deleterious for DWDM, dispersion limited system

# Tutorial Questions

T23.1 (Hard! Needs integration skill!) The parasitic recombination rate per unit volume for carriers of density  $n$  in the active region of a laser is given by  $R(n) = n/\tau$ . Show that the current  $I_{th}$  needed to maintain threshold carrier concentration  $n_{th}$  in an active region of volume  $V$  is given by  $I_{th} = eVn_{th}/\tau$ . If the current is stepped from a constant value  $I = I_{th}/2$  to  $I = 2I_{th}$  show that the time delay to lasing is given by  $t_d = 0.405\tau$ .

If the carrier recombination is dominated by spontaneous emission at a rate per unit volume given by  $R(n) = Bn^2$  show that  $I_{th} = eVB(n_{th})^2$  and  $t_d = 0.235/(Bn_{th})$ .

T23.2 The maximum direct modulation rate of a laser is governed by the relaxation oscillation. Describe factors which determine this parameter and strategies for maximising the data transmission rate. Comment on the practicalities of your suggestions.