EEE105 Tutorial Questions & Review Topics – W7

Fundamental Constants

Boltzman Constant, $k = 1.381 \times 10^{-23}$ JK^{-1} $= 8.62 \times 10^{-5}$ eVK^{-1}
Charge on Electron, $q = 1.602 \times 10^{-19}$ C

Material Parameters

Band-gap of silicon (Si) = 1.1 eV
Band-gap of germanium (Ge) = 0.66eV

1(a)The intrinsic free carrier concentration for Si at 300K is 1.5×10^{16} m^{-3}. From this derive the constant of proportionality in the equation for n_i.

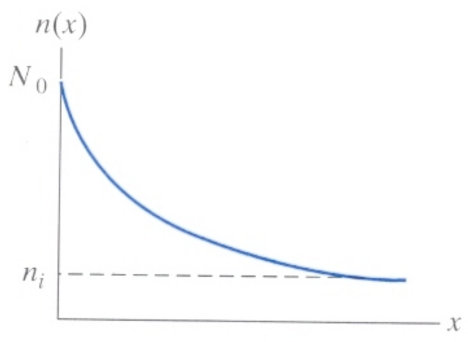
(b) Using this number, calculate the free carrier concentration of intrinsic silicon at 250K and 350K.

(c) Using the same constant, calculate the intrinsic carrier concentration for germanium at 250K, 300K and 350K.

(d) How might you expect the majority and minority carrier concentration of heavily n-doped silicon to change over this temperature range? Describe your assumptions.

(e) Which of these three materials may be most suitable to be used in a sensor to measure temperature over this temperature range? Describe your assumptions.

2) A silicon sample is doped with donors from one side such that $N_d = N_0 \exp(-ax)$. See Figure. The sample is at room temperature where all donors may be expected to be ionized.



(a) Find an expression for E(x) at equilibrium over the range for which $N_d >> n_i$

(b) Evaluate E(x) when $a = 1 \times 10^6$ m^{-1} (i.e. $a = 1$ $(\mu m)^{-1}$)

(c) Sketch a band diagram and indicate the direction of E

Solutions

1(a) This question is asking you to calculate the value of the constant C in the equation

$$n_i = CT^{3/2}\exp\left(-\frac{E_g}{2kT}\right)$$

Re-arranging to get

$$\mathrm{C} = \frac{\mathrm{n_i}}{\mathrm{T}^{3/2}}\exp\left(\frac{\mathrm{E_g}}{2\mathrm{kT}}\right) = \frac{1.5\mathrm{x}10^{16}}{300^{3/2}}\exp\left(\frac{1.1}{2\mathrm{x}300\mathrm{x}8.62\mathrm{x}10^{-5}}\right)$$

$$C = \frac{1.5\mathrm{x}10^{16}}{5196}\exp\left(\frac{1.1}{0.052}\right)$$

$$C = 2.9\mathrm{x}10^{12}\,\mathrm{x}\,1.7\mathrm{x}10^{9} = 5\mathrm{x}10^{21}$$

(n.b. Take care here not to mix up eV and Joules)

1(b). Intrinsic carrier concentration for Si at;

250K

$$\mathrm{n_i} = \mathrm{CT}^{3/2}\exp\left(-\frac{\mathrm{E_g}}{2\mathrm{kT}}\right) = 5\mathrm{x}10^{21}\mathrm{x}250^{3/2}\exp\left(-\frac{1.1}{2\mathrm{x}8.62\mathrm{x}10^{-5}\mathrm{x}250}\right) = 1.6\mathrm{x}10^{14}\,\mathrm{m}^{-3}$$

350K

$$\mathrm{n_i} = \mathrm{CT}^{3/2}\exp\left(-\frac{\mathrm{E_g}}{2\mathrm{kT}}\right) = 5\mathrm{x}10^{21}\mathrm{x}350^{3/2}\exp\left(-\frac{1.1}{2\mathrm{x}8.62\mathrm{x}10^{-5}\mathrm{x}350}\right) = 4\mathrm{x}10^{17}\,\mathrm{m}^{-3}$$

1(c). Germanium at;

250K

$$\mathrm{n_i} = \mathrm{CT}^{3/2}\exp\left(-\frac{\mathrm{E_g}}{2\mathrm{kT}}\right) = 5\mathrm{x}10^{21}\mathrm{x}250^{3/2}\exp\left(-\frac{0.66}{2\mathrm{x}8.62\mathrm{x}10^{-5}\mathrm{x}250}\right) = 4.4\mathrm{x}10^{18}\,\mathrm{m}^{-3}$$

300K

$$\mathrm{n_i} = \mathrm{CT}^{3/2}\exp\left(-\frac{\mathrm{E_g}}{2\mathrm{kT}}\right) = 5\mathrm{x}10^{21}\mathrm{x}300^{3/2}\exp\left(-\frac{0.66}{2\mathrm{x}8.62\mathrm{x}10^{-5}\mathrm{x}300}\right) = 7.4\mathrm{x}10^{19}\,\mathrm{m}^{-3}$$

350K

$$\mathrm{n_i} = \mathrm{CT}^{3/2}\exp\left(-\frac{\mathrm{E_g}}{2\mathrm{kT}}\right) = 5\mathrm{x}10^{21}\mathrm{x}350^{3/2}\exp\left(-\frac{0.66}{2\mathrm{x}8.62\mathrm{x}10^{-5}\mathrm{x}350}\right) = 5.8\mathrm{x}10^{20}\,\mathrm{m}^{-3}$$

1(d). For n-doped silicon over this temperature range we might expect the majority carrier concentration to be equal. This is due to the energy required to free carriers bound to the donor atom to be ~5 meV. The thermal energy at 250K = kT ~22meV and at 350K is ~30meV. Both are >>5meV resulting in essentially all the donors being ionized, giving a free carrier per donor atom.

For the minority carriers

$$p = \frac{n_i^2}{n}$$

And n_i is a function of temperature – it increases exponentially with T. The minority carrier density will therefore increase exponentially with temperature.

1(e). Here we need to think about how conductivity will change with temperature. We would like the material with the biggest change in conductivity to give the best sensitivity in measuring temperature. We will assume mobility is constant over this temperature range (we aren't given any information about this in the question!).

The doped Si can be ruled out as if majority carrier density is fixed then conductivity will be constant. Whilst minority carrier density is changing, the minority carrier does not contribute significantly to conduction.

The intrinsic silicon has a >3 orders of magnitude change in carrier density from 250K to 350K. The germanium has a higher conductivity than the intrinsic Si, but the *change* in conductivity is less than two orders of magnitude from 250K to 350K. The intrinsic Si seems our best choice for this sensor at this temperature range.

2(a) From notes
$$E_x = -\frac{\mathrm{D_e}}{n\mu_e}\frac{\mathrm{dn}}{\mathrm{dx}}$$

We can substitute for D
$$\mathrm{D_{e,h}} = \frac{\mathrm{k_B T \mu_{e,h}}}{\mathrm{q}}$$

and rearrange to give
$$E_x = -\frac{\mathrm{K_B T}}{q}\frac{\mathrm{dn/dx}}{\mathrm{n}}$$

We are given the information that $n = N_d = N_0 \exp(-ax)$. So $dn/dx = -aN_0\exp(-ax)$

Substituting these
$$E_x = -\frac{\mathrm{K_B T}}{q}\frac{\mathrm{dn/dx}}{\mathrm{n}} = -\frac{\mathrm{K_B T}}{q}\frac{\mathrm{(-a)N_0}\exp(-ax)}{\mathrm{N_0}\exp(-ax)} = a\frac{\mathrm{K_B T}}{q}$$

(n.b. please note the need to choose this "exotic" doping profile to allow us to cancel the spatially varying term and give a constant electic field)

2(b) Insert values into equation above – you need to define room temperature! I'll use 300K

$$\mathrm{E_x} = \mathrm{a}\frac{\mathrm{K_B T}}{\mathrm{q}} = 1\mathrm{x}10^6\,\frac{1.38\mathrm{x}10^{-23}\mathrm{x}300}{1.6\mathrm{x}10^{-19}} = 2.59\mathrm{x}10^4\ \mathrm{Vm^{-1}}$$

2(c)