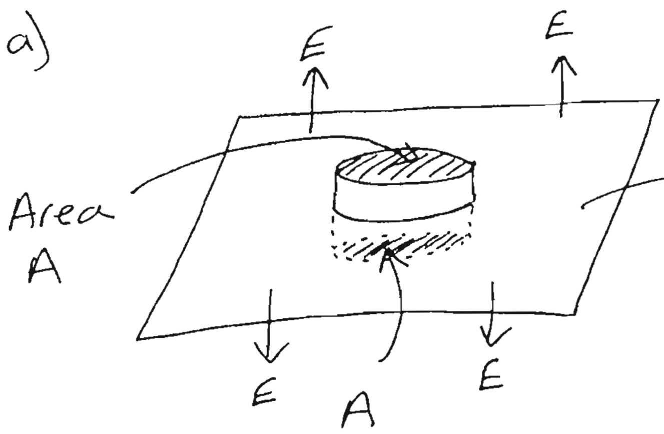
a)



∞ sheet (charge density/unit area $= q_s$)

By symmetry E is perpendicular to sheet and independent of lateral position and has the same value on either side.

Gauss' Law $\oint_S E\,dA = \frac{q_s A}{\varepsilon_0}$ ← enclosed charge

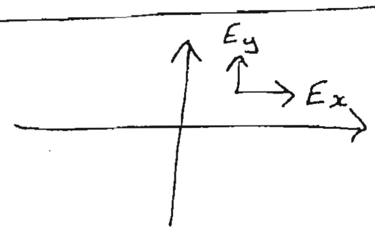
$$\therefore \quad E \times 2A = \frac{q_s A}{\varepsilon_0}$$

contribution from top and bottom surfaces.

$$\rightarrow E = \frac{q_s}{2\varepsilon_0}$$

[5]

b)



$E_x = E_y$: field due to infinite sheet

$$= \frac{q_s}{2\varepsilon_0}$$

$$= \frac{5\times10^{-6}\ \mathrm{C/m^2}}{2\times 8.854\times10^{-12}\ \mathrm{F/m}} = 2.82\times10^{5}\ \mathrm{Vm^{-1}}$$

(or $\mathrm{NC^{-1}}$)

$\Rightarrow$ E-field at:-

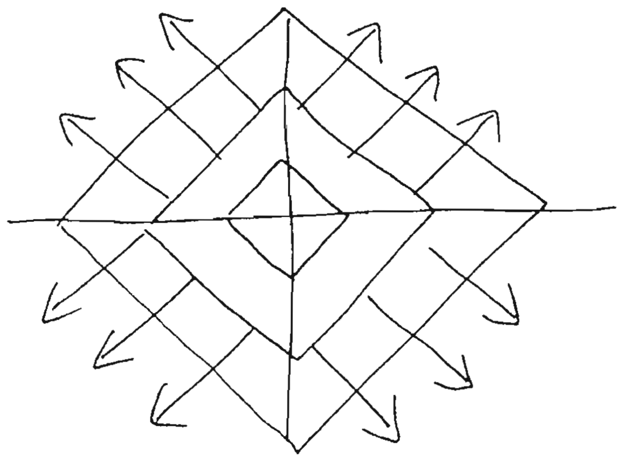
A $(2.82, 2.82, 0) \times 10^5$ Vm^{-1}

B $(2.82, 2.82, 0) \times 10^5$ Vm^{-1}

C $(2.82, -2.82, 0) \times 10^5$ Vm^{-1}

[6]

c)



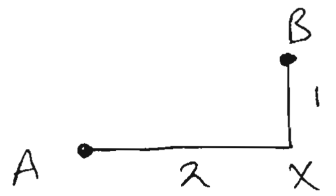
Key:

$\uparrow$ electric field

—— equi-potential surfaces

[3]

d) Split problem up into 2 p.d.s



$$p.d. = \phi(B) - \phi(A) = [\phi(X) - \phi(A)] + [\phi(B) - \phi(X)]$$

$$= -\int_A^X E_x \, dl - \int_X^B E_y \, dl$$

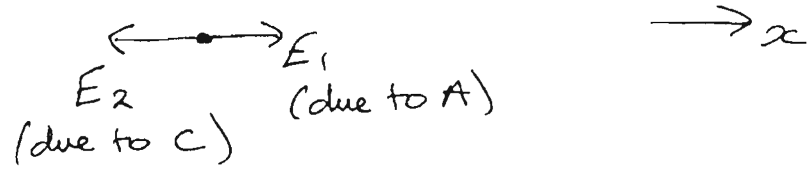
$$= -2E_x - E_y$$

$$= -8.46 \times 10^5 \text{ V}$$

[6]

Q2

a) i) At B, there are 2 E-fields



$$E_1 = \frac{Q}{4\pi \varepsilon_0 R^2} = \frac{2\times10^{-6}}{4\times\pi\times8.854\times10^{-12}\times(1)^2} \quad V/m$$

$$E_2 = \frac{3\times10^{-6}}{4\pi\times8.854\times10^{-12}\times1} \quad V/m$$

$\Rightarrow$ total field (in x-direction) is

$$\underline{E} = \left(\frac{-1\times10^{-6}}{4\pi\times8.854\times10^{-12}}, 0, 0\right) \quad V/m$$

$$\underline{F} = q\,\underline{E}$$
$$= -1\times10^{-6}\times\underline{E}$$
$$= \left(8.99\times10^{-3}, 0, 0\right) \quad N$$

[5]

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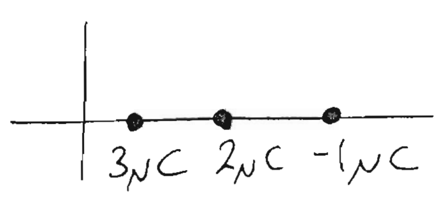
a) ii) Max field at origin when...



$$-E_x = \frac{3\times10^{-6}}{4\pi\varepsilon_0\times(1)^2} + \frac{2\times10^{-6}}{4\pi\varepsilon_0\times(2)^2} - \frac{1\times10^{-6}}{4\pi\varepsilon_0\times(3)^2}$$

$$E_x = (-3.05\times10^4, 0, 0)\ \text{V/m}$$

[5]

b) i) Treat as 3 capacitors in parallel

$$C_1 = \frac{\varepsilon_0\varepsilon_{r1}A_1}{d} \qquad C_2 = \frac{\varepsilon_0\varepsilon_{r2}A_2}{d}$$

$$C_3 = \frac{\varepsilon_0\varepsilon_{r3}A_3}{d}$$

$$C = C_1 + C_2 + C_3 \qquad \left(A_1 = A_2 = A_3 = \frac{ab}{3}\right)$$

$$C = \frac{\varepsilon_0 ab}{3d}(\varepsilon_{r1}+\varepsilon_{r2}+\varepsilon_{r3})$$

[6]

ii) Energy stored $= \frac{1}{2}CV^2$

$$= \frac{\varepsilon_0 ab}{6d}(\varepsilon_{r1}+\varepsilon_{r2}+\varepsilon_{r3})\times V^2$$

$$= \frac{8.854\times10^{-12}\times0.01\times0.005}{6\times0.002}(4+5.5+3)\times12^2$$

$$= 6.64\times10^{-11}\ \text{J}$$

[4]

Q3

a) Ampère's Law says that the line integral of the B-field round a closed path is equal to μ_0 times the current threading the path . . .

$$\oint B\cos\theta \, dl = \mu_0 I$$

Applying this to a circular path around a long wire, then at each point on the path $\theta = 0$, and by symmetry B is constant. θ is the angle between the B-field and the direction of the path.

$$\therefore \quad B \cdot 2\pi r = \mu_0 I$$

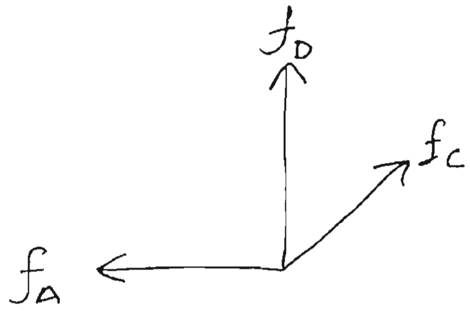
$$\text{or} \quad B = \frac{\mu_0 I}{2\pi r}$$

[6]

b) Force / unit length on parallel current carrying conductors is $f = \dfrac{\mu_0 I_a I_b}{2\pi d}$

if the currents are in the same direction, the force is towards the other conductor

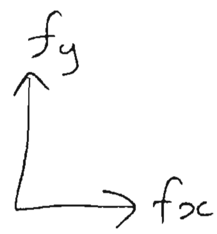
$\therefore$ the forces on B due to A, C, and D are . . .



$$f_A = \frac{\mu_0 I_1 I_2}{2\pi t}$$

$$f_C = \frac{\mu_0 I_1^2}{2\pi t\sqrt{2}}$$

$$f_D = \frac{\mu_0 I_1 I_2}{2\pi t}$$



$$f_x = f_C \cos 45^\circ - f_A$$

$$= \frac{\mu_0 I_1^2}{2\pi t\sqrt{2}} \frac{1}{\sqrt{2}} - \frac{\mu_0 I_1 I_2}{2\pi t}$$

$$= \frac{\mu_0 I_1}{2\pi t}\left(\frac{I_1}{2} - I_2\right)$$

$$f_y = f_D + f_C \cos 45^\circ$$

$$= \frac{\mu_0 I_1 I_2}{2\pi t} + \frac{\mu_0 I_1^2}{2\pi t\sqrt{2}} \frac{1}{\sqrt{2}}$$

$$= \frac{\mu_0 I_1}{2\pi t}\left(I_2 + \frac{I_1}{2}\right)$$

[8]

c) $I_1 = 3A$, $I_2 = 1A$, $t = 20mm$

$$f_x = \frac{4\pi\times10^{-7}\times3}{2\pi\times20\times10^{-3}}(1{\cdot}5-1{\cdot}0) = 1{\cdot}5\times10^{-5}\ Nm^{-1}$$

$$f_y = \frac{4\pi\times10^{-7}\times3}{2\pi\times20\times10^{-3}}(1{\cdot}5+1{\cdot}0) = 7{\cdot}5\times10^{-5}\ Nm^{-1}$$

---

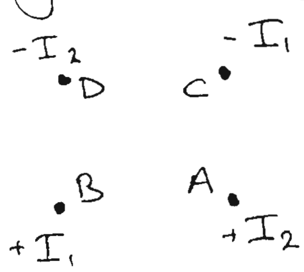
total force is $\sqrt{1{\cdot}5^2+7{\cdot}5^2}\times10^{-5}$

$= 7{\cdot}65\times10^{-5}\ Nm^{-1}$

at an angle of $\tan^{-1}\left(\frac{7{\cdot}5}{1{\cdot}5}\right)$

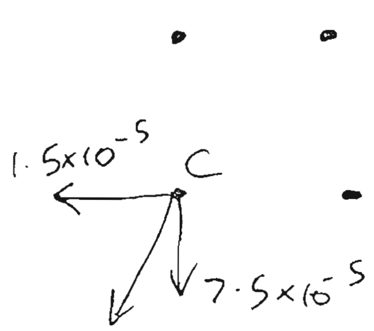
$= 78{\cdot}7^\circ$ to f_x

The force on C can be calculated by turning the diagram through 180° to give



(This is the same as the original but with every current reversed)

Thus, the force on C in this diagram will be the same as the force on B previously



Rotating back, we find that the force on C is equal and opposite to the force on B.

Q4

a) Biot-Savart Law $\underline{B} = \frac{\mu_0 I}{4\pi} \int_L \frac{d\underline{l} \times \hat{\underline{r}}}{r^2}$

For radial sections of circuit $d\underline{l}$ is parallel to $\hat{\underline{r}}$ so no contribution to field.

for arc section $d\underline{l}$ is perpendicular to $\hat{\underline{r}}$

so $|d\underline{l} \times \hat{\underline{r}}| = dl$

and field magnitude is given by $B = \frac{\mu_0 I}{4\pi} \int_L \frac{dl}{r^2}$

or $\frac{\mu_0 I}{4\pi} \int_0^\theta \frac{r\,d\theta}{r^2}$

For arc radius a, $B_a = \frac{\mu_0 I}{4\pi} \int_0^\pi \frac{d\theta}{a} = \frac{\mu_0 I}{4a}$

using RHR, B_a is OUT OF the paper

for arc radius b, $B_b = \frac{\mu_0 I}{4\pi b} \int_0^{\pi/2} d\theta = \frac{\mu_0 I}{8b}$

using RHR, B_b is INTO the paper

$\Rightarrow$ Total field $= B = \frac{\mu_0 I}{4}\left[\frac{1}{a} - \frac{1}{2b}\right]$ [10]

b) The mutual inductance between the two loops is given by

$$M = \frac{\Phi}{I}$$

where Φ is the flux through the second, smaller, loop. Assuming that the B field is uniform over the area of the loop (a good approximation since $A << d^2$)

$$\Phi = BA$$

from formula sheet, B due to one loop is...

$$B = \frac{\mu_0 I a^2}{2(a^2+d^2)^{3/2}}$$

thus, B due to N loops is N times this.

$$\Rightarrow \quad \Phi = \frac{\mu_0 N I A a^2}{2(a^2+d^2)^{3/2}}$$

$$M = \frac{\mu_0 N A a^2}{2(a^2+d^2)^{3/2}}$$

[8]

for $N = 150$, $A = 10^{-4}$ m, $a = 0.2$ m, $d = 0.1$ m

$$M = 3.37 \times 10^{-7} \text{ H}$$

[2]