Tutorial Sheet – No 2 Answers

1 Since:

$$F = N \times I = \phi \times S = B \times A \times \frac{l}{\mu_0 \mu_r A} = \frac{Bl}{\mu_0 \mu_r}$$

then:

$$N = \frac{Bl}{\mu_0 \mu_r} \times \frac{1}{I} = \frac{0.28 \times \pi \times 20 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 1000} \times \frac{1}{2.8}$$ = **50 Turns**

The self-inductance is given by:

$$L = \frac{N^2}{S} = \frac{N^2 \mu_0 \mu_r A}{l} = \frac{50^2 \times 4 \times \pi \times 10^{-7} \times 1000 \times 10 \times 10^{-4}}{\pi \times 20 \times 10^{-2}}$$ = **5mH**

2 Using the formula derived in the previous question:

$$L = \frac{N^2}{S} = \frac{N^2 \mu_0 \mu_r A}{l} = \frac{250^2 \times 4 \times \pi \times 10^{-7} \times 1000 \times 10 \times 10^{-4}}{\pi \times 10 \times 10^{-2}}$$ = **0.25H**

3 (a) Using

$$S_{TOT} = S_{Iron} + S_{Air}$$

$$S_{TOT} = \frac{L_{Iron}}{\mu_o \mu_r A} + \frac{L_{Air}}{\mu_o A} = \frac{1}{4\pi \times 10^{-7} \times 10^{3} \times 10^{-6}} \left(\frac{200 \times 10^{-3}}{4000} + \frac{1 \times 10^{-3}}{1} \right)$$ = **8.4 × 10^5 H^{-1}**

(b) Since:

$$L = \frac{N^2}{S}$$

then

$$N = \sqrt{SL} = \sqrt{2 \times 8.4 \times 10^5}$$ = **1296 turns**

(c) At 50Hz the reactance of the coil will be:

$$X_L = 2\pi f L = 2\pi \times 50 \times 2$$ = 628 Ω

and the current flowing through the coil will be:

$$I_L = \frac{V}{X_L} = \frac{500}{628}$$ = **0.79 A_{rms}**

Now since:

$$\phi = \frac{NI}{S} \text{ and } \phi = BA$$

then the peak flux density will be:

$$B_{\max} = \sqrt{2} \times \frac{NI_L}{SA} = \sqrt{2} \times \frac{1296 \times 0.79}{8.4 \times 10^5 \times 10^3 \times 10^{-6}}$$ = **1.73T**

alternatively using:

$$V_{rms} = 4.44 f N \phi_{\max}$$

or:

$$B_{\max} = \frac{V_{rms}}{4.44 fNA} = \frac{500}{4.44 \times 50 \times 1296 \times 10^3 \times 10^{-6}}$$ = 1.73T

(d) If the coil now has a resistance of 500Ω the impedance of the coil is:

$$Z = \sqrt{R^2 + X^2} = \sqrt{500^2 + 628^2} = 802.7\Omega$$

and the new current will be:

$$I_L = \frac{V}{X_L} = \frac{500}{802.7} = 0.62\text{A}_{rms}$$

and the new peak flux density will be:

$$B_{\max} = 1.73 \times \frac{0.62}{0.79} = \mathbf{1.36T}$$

Note: if you wanted to use the formula:

$$B_{\max} = \frac{V_{rms}}{4.44\, fNA}$$

you would have to first find the voltage across the inductor which is then used in the above formula.

4 Using

$$S_{TOT} = S_{Iron} + S_{Air}$$

$$S_{TOT} = \frac{L_{Iron}}{\mu_o \mu_r A_{Iron}} + \frac{L_{Air}}{\mu_o A_{Air}}$$

$$= \frac{1}{4\pi \times 10^{-7}} \left(\frac{\pi \times 35 \times 10^{-2}}{700 \times 2.4 \times 10^{-4}} + \frac{1.2 \times 10^{-2}}{12 \times 10^{-4}} \right) = \mathbf{13.16 \times 10^6\ H^{-1}}$$

Now:

$$\phi = \frac{NI}{S} \text{ and } \phi = BA$$

so:

$$I = \frac{BAS}{N} = \frac{0.25 \times 12 \times 10^{-4} \times 13.16 \times 10^6}{300} = 13.16\text{ A}$$

Since flux is continuous:

$$B_{Iron} = B_{Air} \frac{A_{Air}}{A_{Iron}} = 0.25 \times \frac{12}{2.4} = \mathbf{1.25T}$$

5 (a) Using

$$S_{TOT} = S_{Iron} + S_{Air}$$

$$S_{TOT} = \frac{L_{Iron}}{\mu_o \mu_r A_{Iron}} + \frac{L_{Air}}{\mu_o A_{Air}}$$

$$= \frac{1}{4\pi \times 10^{-7}} \left(\frac{500 \times 10^{-3}}{300 \times 200 \times 10^{-6}} + \frac{1 \times 10^{-3}}{200 \times 10^{-6}} \right) = \mathbf{1.06 \times 10^7\ H^{-1}}$$

(b) The flux is given by:

$$\phi = \frac{NI}{S} = \frac{100 \times 2}{1.06 \times 10^7} = 1.89 \times 10^{-5}\text{ Wb}$$

and the flux density in the airgap is:

$$B_{Gap} = \frac{\phi}{A} = \frac{1.89 \times 10^{-5}}{200 \times 10^{-6}} = \mathbf{94.3mT}$$

(c) The forces between the faces can be found using:

$$F = \frac{B^2 A}{2\mu_0} = \frac{0.0943^2 \times 200 \times 10^{-6}}{2 \times 4 \times \pi \times 10^{-7}}$$ = **0.71N**

(d) Since the peak force will occur at the peak value of flux, we can use:

$$V_{rms} = 4.44\, fN\phi_{\max} = 4.44 \times 1000 \times 100 \times 1.89 \times 10^{-5}$$ = **8.4V$_{rms}$**

6 Looking at the equation:

$$V_{rms} = 4.44\, fN\phi_{\max}$$

it can be seen that for a given voltage the flux will decrease as the frequency increases. Therefore the highest flux will occur at the lowest operational frequency, in this case 20Hz.

$$\phi_{\max} = \frac{V_{rms}}{4.44\, fN} = \frac{200}{4.44 \times 20 \times 200}$$ = 0.01126 Wb

The required area to give a flux density of 1.5T can now be found:

$$A = \frac{\phi}{B} = \frac{0.01126}{1.5}$$ = **75cm^2**

At the highest operational frequency of 1000Hz the flux density will be 20/1000 or 0.02 of its value at 20Hz, i.e. 1.5 × 0.02 = **0.03 T**

7 The inductance of the coil may be calculated from:

$$L = \frac{N^2}{S} = \frac{500^2}{4 \times 10^5}$$ = **0.625H**

and the reactance at a frequency of 50Hz is:

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.625$$ = 196.3Ω

The overall impedance of the motor is thus:

$$Z = 50 + j196.3$$ = 202.6∠75.7°

and the current may be calculated as:

$$I = \frac{V}{Z} = \frac{240}{202.6}$$ = 1.185∠-75.7° A$_{rms}$

The peak flux will occur at the peak of the current waveform:

$$I_{PK} = \sqrt{2} I_{rms} = \sqrt{2} \times 1.185$$ = 1.676A

and this will be the required dc current. Under steady-state dc conditions there will be no voltage appearing across the inductance and the resistance will limit the current. Hence the required supply voltage may be found:

$$V_{DC} = I_{DC} \times R = 1.676 \times 50$$ = **83.8 V**

8 The first thing to note in this question is that there are 2 main flux paths, as indicated on the diagram. Certain assumptions have to be made in this problem:

- there is no leakage flux (all flux leaving the end of the plunger crosses the airgap horizontally)
- there is no airgap between the core and the plunger at the right hand side
- an approximation has to be made as to the length of the two flux paths in the iron core. Both are assumed to be of length 15+10/2 = 20cm since we are not given any further details of the dimensions of the plunger, other than its area
- The area of the airgap is assumed equal to that of the plunger

Firstly the reluctance of the circuit needs to be calculated for both cases of the plunger being fully out (as shown in the figure) and fully in (when there is no airgap beween the end of the plunger and the iron core at the left hand side).

$$S_{Out} = S_{Core} + S_{Pl\_Out} + S_{Air}$$

$$S_{In} = S_{Core} + S_{Pl\_In}$$

However the reluctance of the core is that of the upper and lower limbs in parallel. Since both limbs are identical, the total reluctance of the core will be equal to half the reluctance of a single limb:

$$S_{Core} = \frac{1}{2} \times \frac{l_{Core}}{\mu_0 \mu_r A_{core}} = \frac{1}{2} \times \frac{20 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 500 \times 10 \times 10^{-4}} = 1.59 \times 10^5 \text{ H}^{-1}$$

Now calculate the reluctance of the remaining components:

$$S_{Pl\_Out} = \frac{l_{Pl\_Out}}{\mu_0 \mu_r A_{Pl}} = \frac{11 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 500 \times 5 \times 10^{-4}} = 3.5 \times 10^5 \text{ H}^{-1}$$

$$S_{Pl\_In} = \frac{l_{Pl\_In}}{\mu_0 \mu_r A_{Pl}} = \frac{13 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 500 \times 5 \times 10^{-4}} = 4.14 \times 10^5 \text{ H}^{-1}$$

$$S_{Air} = \frac{l_{Air}}{\mu_0 A_{Pl}} = \frac{2 \times 10^{-2}}{4 \times \pi \times 10^{-7} \times 5 \times 10^{-4}} = 3.18 \times 10^7 \text{ H}^{-1}$$

Combining these elements:

$$S_{Out} = 1.59 \times 10^5 + 3.5 \times 10^5 + 3.18 \times 10^7 = 3.23 \times 10^7 \text{ H}^{-1}$$

$$S_{In} = 1.59 \times 10^5 + 4.14 \times 10^5 = 5.73 \times 10^5 \text{ H}^{-1}$$

The inductance can now be calculated for each position from:

$$L = \frac{N^2}{S}$$

so:

$$L_{Out} = \frac{N^2}{S_{Out}} = \frac{1000^2}{3.23 \times 10^7} = 30.95\text{mH}$$

$$L_{In} = \frac{N^2}{S_{In}} = \frac{1000^2}{5.73 \times 10^5} = 1.745\text{H}$$

The impedance can then be calculated for each position from:

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

so:

$$|Z_{Out}| = \sqrt{120^2 + (2\pi \times 50 \times 30.95 \times 10^{-3})^2} = 120.4\Omega$$

$$|Z_{In}| = \sqrt{120^2 + (2\pi \times 50 \times 1.745)^2} = 561.2\Omega$$

Now the current in each position may be obtained:

$$|I_{Out}| = \frac{V}{|Z_{Out}|} = \frac{240}{120.4} = 1.99\text{A}_{\text{rms}}$$

$$|I_{In}| = \frac{V}{|Z_{In}|} = \frac{240}{561.2} = 0.427\text{A}_{\text{rms}}$$

The power dissipated in the 120Ω resistance can be calculated as:

$$P_{Out} = I_{Out}^2 \times R = 1.99^2 \times 120 = 475.2\text{W}$$

$$P_{In} = I_{In}^2 \times R = 0.427^2 \times 120 = 21.88\text{W}$$

Therefore the ratio of the powers is:

$$ratio = \frac{P_{Out}}{P_{In}} = \frac{475.2}{21.88} = \mathbf{21.7}$$