# Worked Solutions - Tutorial Sheet 1

1. No information on M. Use d.c data to calculate $M_\perp$.

On d.c

$$\begin{vmatrix} V_f \\ V_q \end{vmatrix} = \begin{vmatrix} R_f & 0 \\ \omega_r M & R_q \end{vmatrix} \begin{vmatrix} I_f \\ I_q \end{vmatrix}$$

But $V = V_f + V_q$ & $I = I_f = I_q$ since M/c is a series m/c.

$$V = (R_f + R_q + \omega_r M)\, I$$

$$\omega_r M = \frac{V}{I} - R_f + R_q$$

$$M = \frac{V - IR}{\omega_r} = 1.66\,H.$$

$$P_{i/p} = VI = (R + \omega_r M) I^2$$

$$= \underbrace{I^2 R}_{P_{loss}} + \underbrace{I^2 \omega_r M}_{P_{mech}}$$

$$= P_{diss} + P_{mech}$$

$$P_{mech} = I^2 \omega_r M = T\omega_r$$

$$\therefore T = MI^2 = 0.597\,N$$

On a.c

$$\begin{vmatrix} v_f \\ v_q \end{vmatrix} = \begin{vmatrix} R_f + L_f p & 0 \\ \omega_r M & R_q + L_q p \end{vmatrix} \begin{vmatrix} \tilde{I}_f \\ \tilde{I}_q \end{vmatrix}$$

But again $V = V_f + V_q$ $\quad \tilde{I} = \tilde{I}_f + \tilde{I}_q$

$$V = R\tilde{I} + Lp\tilde{I} + \omega_r M\tilde{I}$$

On sinusoidal supply $p = j\omega$

$$V = R\tilde{I} + j\omega_s L\tilde{I} + \omega_r M\tilde{I}$$

let $N = \dfrac{\text{actual speed}}{\text{synchronous speed}} = \dfrac{\omega_r}{\omega_s}$

$$V = R\tilde{I} + j\omega_s L\tilde{I} + \omega_r \frac{\omega_s}{\omega_s} M\tilde{I}$$

$$= R\tilde{I} + j\omega_s L\tilde{I} + N\omega_s M\tilde{I} \qquad [X_m = \omega_s M]$$

$$= R\tilde{I} + j\omega_s L\tilde{I} + NX_m\tilde{I}$$

$$= (R + NX_m + j\omega_s L)\tilde{I}$$

Power o/p $= Re\{vi\}$

$$= (R + NX_m)\tilde{I}^2$$

$$= P_{diss} + P_{mech.}$$

$$\therefore P_{mech} = N X_m \tilde{I}^2 = T\omega_r$$

$$\therefore T\omega_r = \frac{\omega_r}{\omega_s} \omega_s M \tilde{I}^2$$

$$\therefore T = M\tilde{I}^2$$

Since T is the same as for d.c case then

$$I = \sqrt{\frac{T}{M}} = \underline{\underline{0 \cdot 6A}}$$

[Same as d.c case since in both cases $T = MI^2$ & M is same for both a.c + d.c]

From earlier

$$V = \tilde{I}\,(R + NX_m + j\omega_s L)$$

$$\therefore \left|\frac{V}{\tilde{I}}\right| = \left((R + NX_m)^2 + (\omega_s L)^2\right)^{1/2}$$

$$\therefore (R + NX_m)^2 = \frac{V^2}{\tilde{I}^2} - (\omega_s L)^2$$

$$\therefore N = \frac{1}{X_m}\left(\frac{V^2}{\tilde{I}^2} - (\omega_s L)^2\right)^{1/2} - \frac{R}{X_m} \leftarrow \text{N.B}$$

$$= \frac{1}{\omega_s M}\left(\frac{V^2}{\tilde{I}^2} - (\omega_s L)^2\right)^{1/2} - \frac{R}{X_m}$$

N B

N B

$$= \frac{1}{2\pi f M}\left(\frac{V^2}{\tilde{I}^2} - (2\pi f L)^2\right)^{1/2} - \frac{R}{2\pi f M}$$

$= 0.460$

$$\frac{\omega_r}{\omega_s} = 0.460$$

$$\therefore \quad \omega_r = 144.47 \text{ rad s}^{-1}$$

$$= \underline{\underline{1380}} \text{ r.p.m.}$$

$$\text{p.f} = \frac{R + NX_m}{\sqrt{(R + NX_m)^2 + (\omega_s L)^2}} \qquad X_m = \omega_s M \;\; \text{not} \;\; \omega_s L$$

$$\frac{274.89}{\sqrt{274.89^2 + 188.5^2}}$$

$= 0.825$ lag (since $j$ term is +ve)

[p.f is a function of speed, since N is a function of speed]

Starting torque is the torque produced when $\omega_r = 0$.

For d.c case.

$$I = \frac{V}{R} = 6.57 A$$

$$T = MI^2 = 71.68 \text{ Nm}$$

For a.c case.

$$|I| = \left.\frac{|V|}{|Z|}\right|_{\omega_r = 0}$$

$$= \frac{200}{\sqrt{R^2 + (\omega_s L)^2}}$$

$$= \frac{200}{\sqrt{R^2 + (2\pi f L)^2}}$$

$$= 1.04 \text{ A}$$

$\therefore \quad T = MI^2 = 1.8$ Nm

$\therefore$ Starting torque ratio

$$\frac{\text{a.c starting torque}}{\text{d.c starting torque.}} = \frac{1.8}{71.68} = \frac{1}{39.8}$$

2. $R = 50\,\Omega$
$L = 0.7H$
$240V$
$50Hz$.

For both a.c + d.c operation of a universal motor.

$$T = M_{qf} I^2$$

∴ Since in both cases in question, the load torque remains constant, then as the mutual inductance is the same then the current is the same

$$\underline{\underline{I = 0.6A}}$$

For d.c + a.c operation

$$\begin{vmatrix} V_q \\ V_f \end{vmatrix} = \begin{vmatrix} R_q + L_q p & \omega_r M_{qf} \\ 0 & R_f + L_f \end{vmatrix} \begin{vmatrix} I_q \\ I_f \end{vmatrix}$$

for d.c + a.c operation constraining equations are:

$$V = V_q + V_f \qquad I = I_q = I_f$$

For d.c $p = 0$

$$V = [R_q + R_f + \omega_r M_{qf}] I \qquad \therefore \omega_r = \frac{\frac{V}{I} - (R_q + R_f)}{M_{qf}}$$

[Have got V, I, R & L ∴ need $M_{qf}$
get this from a.c conditions ].

on a.c $p = j\omega_s$

$$V = \left[R_q + R_f + \omega_r M_{qf} + j\omega_s(L_q + L_f)\right] I$$

$$\therefore \frac{V}{I} = R_q + R_f + \omega_r M_{qf} + j\omega_s(L_q + L_f)$$

$$\therefore \left|\frac{V}{I}\right| = \left\{ (R_q + R_f + \omega_r M_{qf})^2 + (\omega_s(L_q + L_f))^2 \right\}^{1/2}$$

But $R_q + R_f = R$ $\quad L_q + L_f = L$

$$\therefore \frac{V^2}{I^2} = (R + \omega_r M_{qf})^2 + (\omega_s L)^2$$

$$\therefore \frac{V^2}{I^2} - (\omega_s L)^2 = (R + \omega_r M_{qf})^2$$

$$\therefore R + \omega_r M_{qf} = \left( \frac{V^2}{I^2} - (\omega_s L)^2 \right)^{1/2}$$

$$\therefore M_{qf} = \frac{1}{\omega_r} \left( \frac{V^2}{I^2} - (\omega_s L)^2 \right)^{1/2} - \frac{R}{\omega_r}$$

Substitute in values given

$$M_{qf} = \frac{284.12}{1500 \times \frac{2\pi}{60}} = 1.81 \text{ H}.$$

$\therefore$ For d.c case of operation

$$\omega_r = \frac{\frac{230}{0.6} - 50}{1.81} = 184.16 \text{ rad s}^{-1}$$

$$= \underline{\underline{1760 \text{ r.p.m}}}$$

[Care not to mix up a.c + d.c data such as applied volts.

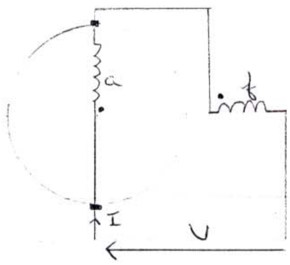
[Similar to question ①
Find M from initial operation data + use in other cases]

3. Torque $= P_p M I^2$ where $P_p$ = pole pairs.

$\therefore$ At 40 NM gross torque

$$I^2 = \frac{T}{P_p M}$$

$$I = \sqrt{\frac{T}{P_p M}} = \sqrt{\frac{40}{2 \times 0.009}} = 47.14 \text{ A.}$$




sice flux axis leads voltage axis.

$$\begin{vmatrix} V_a \\ V_f \end{vmatrix} = \begin{vmatrix} R_a + L_a p & \underset{N.B}{P_p} \omega_r M_{fa} \\ 0 & R_f + L_f p \end{vmatrix} \begin{vmatrix} I_a \\ I_f \end{vmatrix}$$

But $V = V_a + V_f$
$I = I_a = I_f$

$$\therefore V = \left[ (R_a + R_f + P_p \omega_r M_{fa}) + p(L_a + L_f) \right] I$$

But in steady state a.c $p = j\omega_s$

$$\therefore V = \left[ R_a + R_f + P_p \omega_r M_{fa} + j\omega_s (L_a + L_f) \right] I$$

$$\therefore \frac{|V|^2}{|I|^2} = (R_a + R_f + P_p \omega_r M_{fa})^2 + (L_a + L_f)^2 \omega_s^2$$

$$\therefore \omega_r = \frac{\sqrt{\frac{V^2}{I^2} - \omega_s^2 (L_a + L_f)^2} - R_a - R_f}{P_p M_{fa}}$$

$$= \frac{205.70}{2} \text{ rads}^{-1}$$

$$= 102.85 \text{ rads}^{-1}$$

$$\therefore \omega_r = \frac{102.85}{2\pi} \times 60 = 983 \text{ rpm.}$$

[Note importance of pole pairs]

From $V = \left[ (R_a + R_f + P_p \omega_r M_{fa}) + (L_a + L_f)^2 j \omega_s^2 \right] I$

then $P = \mathrm{Re}\{VI\}$

$$= (R_a + R_f + P_p \omega_r M_{fa}) I^2$$

$$= P_{diss} + P_{\substack{elec \\ dp}}$$

$$\therefore P_{\substack{elec \\ dp}} = P_p \omega_r M_{fa} I^2$$

$$= 4.114 \text{ kW}$$

$$= \frac{4.114}{743} \text{ h.p} = 5.53 \text{ hp.}$$

Take care to distinguish betw $\omega_s$ & $\omega_r$

$$\text{Power factor} = \cos\left[\tan^{-1}\left(\frac{\omega_s (L_a + L_f)}{R_a + R_f + P_p \omega_r M_{fa}}\right)\right]$$

$= 0.672$ lagging [Important to include lagging]

[Note importance of pole pairs.
Must be included in matrix with rotational emfs.
Alternative way to do problem would have been to work through theory to get $P_{elec} = P_p \omega_r M_{fa} I^2$

$$T = \frac{P_{elec}}{\omega_r} = P_p M_{fa} I^2$$

i.e. prove $T = P_p M_{fa} I^2$ and not assume at start]