EEE6081 (EEE421) Visual Information Engineering (VIE)

Topic 02: Revision – Background knowledge

- Part 2: Signal Processing Preliminaries
  - Discrete time signals & systems
  - Convolution
  - Impulse & Frequency response
  - Filters (low pass and high pass)
  - Transforms

- Background reading: Digital Signal Processing (Proakis / Manolakis)
  Chapters 1 and 2.
  (Or Introduction and Discrete time systems and signals chapters on any DSP text book)

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Discrete time signals

- A discrete time signal x(n) is a function of an independent variable that is an integer.

- We can assume that x(n) is defined for all integer values of n for $-\infty < n < \infty$

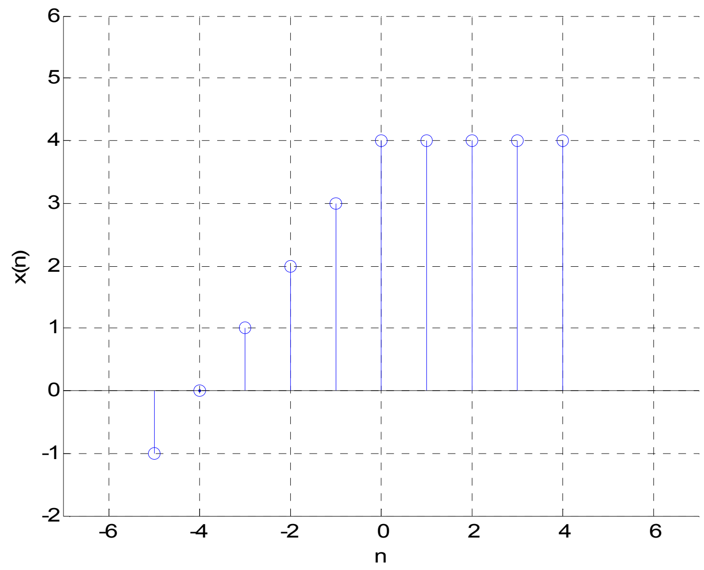
- We refer x(n) as the n^{th} sample of the signal.
- $x(n) \equiv x_a(nT)$, where x_a is the analogue signal, T is the sampling interval and n is the sampling index.

- Commonly used signals:
  - Unit impulse function ---------- ?
  - Unit step signal ------------ ?

Discrete time signals

- Simple manipulations of discrete time signals
- What is x(n)?

  x(n) =



- Time shifting ---- x(n-k)

  x(n-3)?

  x(n+2)?

- Folding ----- x(-n)

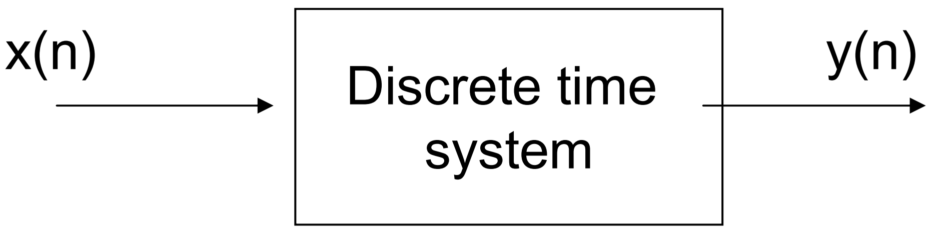
- Time scaling ---- x(mn)

  x(2n)?

Discrete time systems

- A discrete time system is an operation or a set of operations performed on a discrete time input signal x(n) to produce the discrete time output signal y(n).

- We can also say x(n) is transformed to y(n) by the system.



$$y(n) \equiv T[x(n)]$$

- The output when the input is the impulse function is called the impulse response of a system . $h(n,k) = T[\delta(n-k)]$

Discrete time systems

Time (shift or translation) invariant systems

- A system is called time invariant if its input-output characteristics do not change with time.
- That means for a system $x(n) \rightarrow y(n)$

  $x(n-k) \rightarrow y(n-k)$, for every input signal $x(n)$ and every time shift k.

- How to check? Check whether the shifted output ($y(n-k)$) is the same as the output computed using the shifted input ($T[x(n-k)]$).
- Determine the following are time invariant or not
  - $y(n)=x(n)-x(n-1)$
  - $y(n)=nx(n)$
  - $y(n)=x(-n)$
  - $y(n)=x(2n)$
  - $y(n)=x(n)\cos(wn)$

Linear systems

- A system is called linear if it satisfies the superposition principle.
- The response of the system to a weighted sum of signals is the same as the corresponding weighted sum of the responses of the system to each of the individual input signals.

- $T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$
- This is due to scaling and additive properties of a linear system.
- Determine the following are linear or non-linear
  - $y(n)=nx(n)$
  - $y(n)=x(n^2)$
  - $y(n)=x^2(n)$
  - $y(n)=x(2n)$

Causal systems

- A system is called causal if the output of the system at any time [y(n)] depends only on the present [x(n)] and past inputs [x(n-1), x(n-2),….], but not the future inputs [x(n+1), x(n+2),….].

- Otherwise the system is called non-causal.

- What are the practical implications?

Interconnection of systems

- Determine the combined system (T) of two systems (T_1 and T_2) interconnected:

(a) in cascade or

(b) in parallel

- For cascade interconnections, is the order of performance (T_1 followed by T_2 or T_2 followed by T_1) important?

Convolution

Response of a linear time invariant (LTI) system to an arbitrary input x(n).

We know: $y(n) = T[x(n)]$

$h(n) = T[\delta(n)]$

An arbitrary signal x(n) can be expressed as a sum of weighted impulses: $x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$

Now we can write the output y(n):

$$y(n) = T[x(n)] = T\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)T\left[\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$= x * h$$

Convolution

Response of a linear time invariant (LTI) system to an arbitrary input x(n).

Steps:

1) folding: fold h(k) about k=0 to get h(-k)
2) Shifting: shift h(-k) by n_0 to the right if n is positive to get $h(n_0-k)$
3) Multiplication: multiply x(k) by $h(n_0-k)$ to get the product sequence
4) Summation: sum all the values of the product sequence.

Repeat the above steps 2 to 4 for all n.

Convolution

Computation by hand

A good way to compute h*x is to arrange it as an ordinary multiplication. But don't carry digits from one column to the other.

e.g., consider {x(0), x(1), x(2)} & {h(0), h(1), h(2)}

		x(2)	x(1)	x(0)
		h(2)	h(1)	h(0)
		(2)	(1)	(0)
	(3)	(2)	(1)	
(4)	(3)	(2)		
y(4)	y(3)	y(2)	y(1)	y(0)

Compute the convolution for x(n)={4,2,3} and h(n)={2,5,1}

Convolution

Convolution of x(n) by h(n) in time domain becomes multiplication of X by H in frequency domain, where X & H are the Fourier transform of h.

$$X\left(e^{j\omega}\right)=\sum_{-\infty}^{\infty} x(n)e^{-jn\omega} = X(\omega)$$

$$H\left(e^{j\omega}\right)=\sum_{-\infty}^{\infty} h(n)e^{-jn\omega} = H(\omega)$$

$$Y(\omega) = H(\omega)X(\omega)$$

Similarly in the z-transfom domain

$$X(z) = \sum_{-\infty}^{\infty} x(n)z^{-n}$$

$$H(z) = \sum_{-\infty}^{\infty} h(n)z^{-n}$$

$$Y(z) = H(z)X(z)$$

Filters

A filter is a linear time-invariant operator.

It acts on input signal x and the output signal y is the convolution sum of x with the fixed vector h, which is the impulse response of the system.

The values of the vector h are known as the filter coefficients. E.g., h(0), h(1),……

Low pass filters & High pass filters (later in detail)

Transforms

A transform of a signal is a new representation of that signal.

Consider the following system of equations:

$y(0) = x(0) + x(1)$

$y(1) = x(0) - x(1)$

$y(2) = x(2) + x(3)$

$y(3) = x(2) - x(3)$

This transforms x into y using a 4-point transform.

In other words the signal is represented in y-domain using the linear combinations of signal components in the x domain.

What is the inverse transform for the above transform?

Transforms

Three types of transforms:

1. Lossless transforms (orthogonal)
2. Invertible transforms (biorthogonal)
3. Lossy transforms (non-invertible)

(next lecture)

Matrix form of convolution

What is the purpose of a transform?

Conditions for lossless & invertible transforms.

How to use a 1D transform on an image (2D signal)?

Homework: MATLAB

- Exercise 2:
  - Create the time axis values for 10 cycles with 512 data points using t=linspace(0,10, 512);
  - Consider the signal x=3sin(5t)-6cos(9t)
  - Plot x
  - Add random noise n to obtain a noisy signal y=x+n
  - Consider you are using a 3 point moving average filter. What is “h” for this filter?
  - Use convolution to find the cleaned signal “z”
  - Check the size of the output z
  - Plot all x, y and z in the SAME figure

  - Think of an alternative approach for de-noising using the Fourier Transform and implement it using MATLAB