The University Of Sheffield.

Data Provided:

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m,

$\mu_0 = 4\pi \times 10^{-7}$ H/m,

Formulae for Vector Differential Operations

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2008-2009 (2 hours)

Applied Electromagnetics 6

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. Figure 1 represents an elemental length of a transmission line.

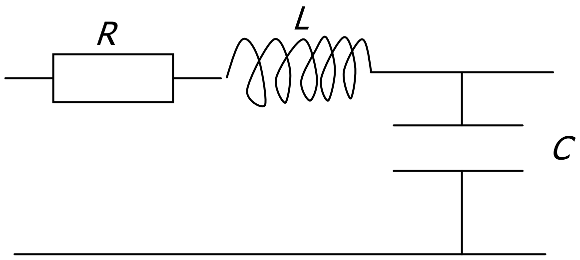

Fig. 1

R, L and C are the resistivity, inductance and capacitance of the line per unit length respectively.

a) By examining the voltage (V) and current (I) differences over a length of Δx of this line in time Δt, show that

$$-\frac{\partial V}{\partial x} = RI + L\frac{\partial I}{\partial t} \qquad \text{equation 1}$$

and

$$-\frac{\partial I}{\partial x} = C\frac{\partial V}{\partial t} \qquad \text{equation 2}$$

(3)

b) Show that when R is negligibly small, then

$$\frac{\partial^2 V}{\partial t^2} = v^2 \frac{\partial^2 V}{\partial x^2}\,. \qquad \text{equation 3}$$

(3)

c) Show that

$$V(x,t) = 3e^{j(\omega t - \beta x)} + 3e^{j(\omega t + \beta x)} \qquad \text{equation 4}$$

where ω and β are real constants, is a solution of equation 3. Explain what this function physically represents and sketch the magnitude of the time-independent amplitude of the voltage disturbance as a function of *x*, labelling the *x* axis scale in terms of β.

(5)

d) What is the voltage standing wave ratio of *V(x,t)* in equation 4.

(2)

e) How is the quantity β affected if the resistance per unit length of the line is small but finite? Sketch how this would affect the amplitude of the first term on the RHS of equation 4 as a function of *x*. **(3)**

f) Consider the terminated transmission line in Fig. 2.

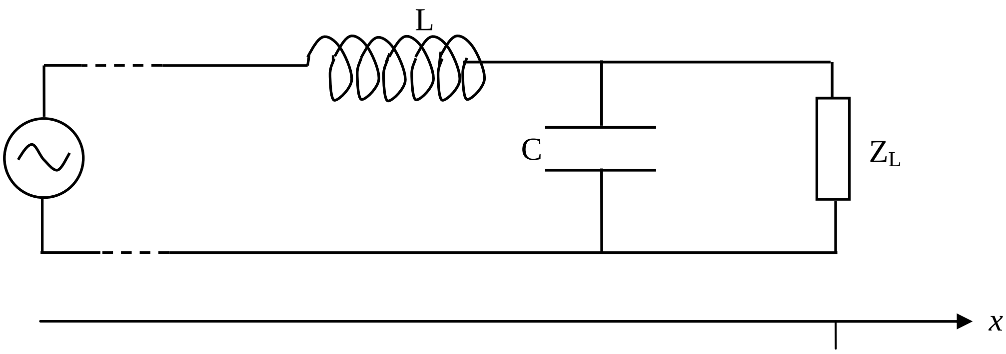

Fig. 2

A sinusoidal voltage of frequency 3GHz is applied to the line at some distance to left of the diagram. The line is terminated at *x=0* with an impedance of Z_L. Given that v = 2x10[8]m/s, and C = 80pF/m, find the value of Z_L which will result in a there only being a wave travelling in the positive x direction. Sketch the amplitude of the *current* as a function of *x*, near *x=0*, in the cases when Z_L =0 and $Z_L = \infty$. **(4)**

2. A transmission line consists of two conducting parallel plates of width *w* separated by a distance *b*, as shown in Fig. 3

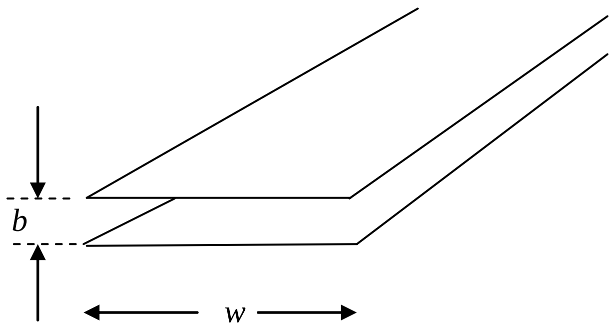

Fig. 3

A periodic voltage of the form $V = V_0 cos\omega t$ is applied across the plates at one end of the line, causing time-dependent electromagnetic fields to propagate down the line. The relative permeability and permittivity of space between the plates is μ_r and ε_r respectively. Assume that $w>>b$, so that non-parallel components of the magnetic and electric fields at the edges of the plates can be ignored.

a) By considering the voltage across and the current down a short length of the line *a*, derive expressions in terms of μ_r and ε_r for

i) The capacitance, *C*, per unit length of the line.

ii) The inductance, *L*, per unit length of the line.

iii) The characteristic impedance, Z_0, of the line.

iv) The phase velocity of the wave that propagates down the line.

(8)

If *b* and *w* are made very large, the electric and magnetic fields in free space between the plates can be described by

$$\boldsymbol{E}(x,t) = E_0\, \hat{\boldsymbol{y}}\, cos(\omega t - \beta x)$$

$$\boldsymbol{H}(x,t) = H_0\, \hat{\boldsymbol{z}}\, cos(\omega t - \beta x)$$

b) On a diagram similar to the one above, indicate the relationship of the *x, y* and *z* axes. If the frequency of the source is 1GHz, draw the relationship between the vector components of ***E*** and ***H*** as a function of *x*. **(4)**

c) Such a wave travels through a medium with $\mu_r = \varepsilon_r = 1$ before it encounters a planar surface of a second medium lying normal to the *x-axis* at $x=0$. For $x>0$, $\mu_r=1$, $\varepsilon_r = 1.25$, by equating the continuity of components of ***E*** and ***H*** at the interface, determine the electric field amplitude of the transmitted wave relative to the incident wave. **(6)**

d) Calculate the energy per unit area incident upon the surface described above if the average value of electric field of the wave travelling in the positive *x* direction is 2.3×10^{-7} V/m. **(2)**

3. **a.** The electric scalar potential, V, in a region of free space ($\varepsilon_0 = 8.85\times10^{-12}$ F/m) is given by:

$$V = x^2 + y + z^3 \text{ (V)}$$

(i) Calculate the electric field strength and charge density at the point (x, y, z) = (1.0, 0.5, 0.2) m.

(ii) Determine the total electric flux through the surface of the cube, as shown in Fig. 4, and the total charge inside the cube.

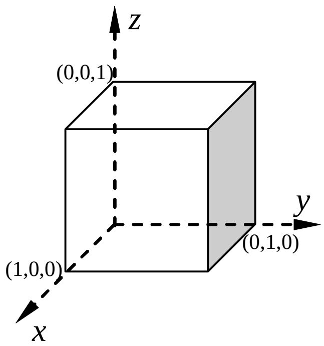

Fig. 4

(6)

b. Using the conservative property of electrostatic field strength $\vec{E}$, $\oint_C \vec{E}\cdot d\vec{l} = 0$, and Gauss's Law, $\oiint_S \vec{D}\cdot d\vec{S} = Q$, show that

(i) the boundary condition at the interface between a conductor and dielectric material, as shown in Fig. 5, is given by:

$$E_t = 0$$

(ii) the charge density, σ, at the conductor surface is given by:

$$\sigma = D_n$$

where E_t and D_n and are the tangential component of the electric field strength and the normal component of the electric flux density, respectively, in the dielectric region at the interface.

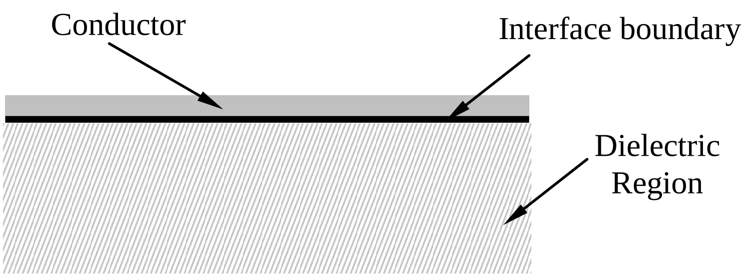

Fig. 5

(6)

c. Two coaxial conducting cylinders are located at r = 0.005 m and r = 0.012 m. The region between the cylinders is filled with a homogeneous perfect dielectric. If the inner cylinder is at 100 V and the outer at 0 V, by solving Laplace's equation, find

(i) the location of the 20 V equipotential surface,
(ii) the maximum electrical field strength E_{rmax},
(iii) the permittivity of the dielectric if the charge per unit length on the inner cylinder is 0.03 μC/m,
(iv) the capacitance per unit length.

Laplace operator in cylindrical co-ordinate systems is given by:

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

(8)

4. a. The vector magnetic potential $\boldsymbol{A}$ in a region of free space ($\mu_0 = 4\pi\times10^{-7}$ H/m) is given by:

$$\boldsymbol{A} = (0.3xyz)\,\boldsymbol{e_x} + (0.4xy)\,\boldsymbol{e_y} + (0.3)\,\boldsymbol{e_z} \quad \text{(Wb/m)}$$

Calculate the magnetic flux density $\boldsymbol{B}$ and current density $\boldsymbol{J}$ at the point (1, 3, 9)m. **(5)**

b. Figure 6 shows a schematic representation of the track section of a levitated rapid transport system. It consists of a sheet of infinitely permeable iron ($\mu = \infty$) on which there is a surface current sheet $J\sin(px)$ A/m. Starting from the appropriate governing magneto-static field equation, derive expressions for the x and y components of flux density in the air region above the track.

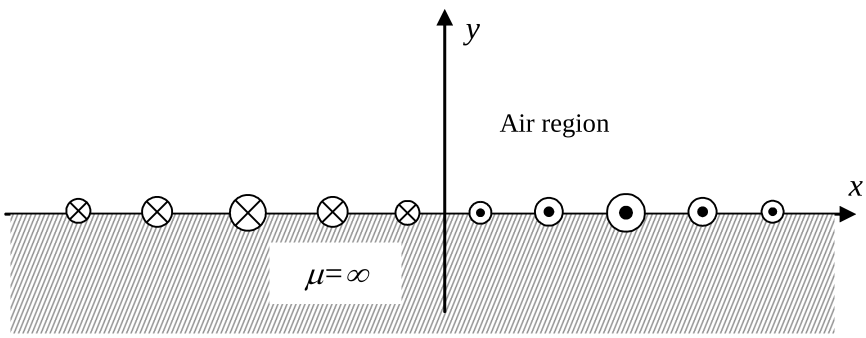

Fig. 6 **(6)**

c. Explain what is meant by "skin effect" in relation to time-varying electromagnetic fields, and indicate how the skin depth depends on the properties of a material. **(3)**

d. A circular copper conductor has a diameter of 4mm and carries a current at a frequency of 20kHz. Estimate the per unit length ac resistance of the conductor if the resistivity of copper is $1.7\text{x}10^{-8}$ Ωm. **(6)**

Vector differential operations

Let Φ be a scalar function and ***D***, ***H*** and ***A*** be vector functions.

Cartesian Co-ordinates (*x, y, z*)

$$\nabla\Phi = \frac{\partial\Phi}{\partial x}e_x + \frac{\partial\Phi}{\partial y}e_y + \frac{\partial\Phi}{\partial z}e_z$$

$$\nabla \bullet \boldsymbol{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla \times \boldsymbol{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)e_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)e_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)e_z$$

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}$$

$$\nabla^2\boldsymbol{A} = \nabla^2 A_x e_x + \nabla^2 A_Y e_y + \nabla^2 A_z e_z$$

Cylindrical Co-ordinates (*r, θ, z*)

$$\nabla\Phi = \frac{\partial\Phi}{\partial r}e_r + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}e_\theta + \frac{\partial\Phi}{\partial z}e_z$$

$$\nabla \bullet \boldsymbol{D} = \frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{1}{r}\frac{\partial D_\theta}{\partial\theta} + \frac{\partial D_z}{\partial z}$$

$$\nabla \times \boldsymbol{H} = \left[\frac{1}{r}\frac{\partial H_z}{\partial\theta} - \frac{\partial H_\theta}{\partial z}\right]e_r + \left[\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r}\right]e_\theta + \left[\frac{1}{r}\frac{\partial(rH_\theta)}{\partial r} - \frac{1}{r}\frac{\partial H_r}{\partial\theta}\right]e_z$$

$$\nabla^2\Phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2} + \frac{\partial^2\Phi}{\partial z^2} = \frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2} + \frac{\partial^2\Phi}{\partial z^2}$$

$$\nabla^2\boldsymbol{A} = \left(\nabla^2 A_r - \frac{2}{r^2}\frac{\partial A_\theta}{\partial\theta} - \frac{A_r}{r^2}\right)e_r + \left(\nabla^2 A_\theta + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{A_\theta}{r^2}\right)e_\theta + \left(\nabla^2 A_z\right)e_z$$

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