UNIVERSITY OF SHEFFIELD
Department of Electronic and Electrical Engineering

EEE220 ELECTRIC AND MAGNETIC FIELDS
FORMULA SHEET

ILF/AT/JLW 2006

$\varepsilon_o = 8.854 \times 10^{-12}$ Fm^{-1} charge on electron = -1.6×10^{-19} C

$\mu_o = 4\pi \times 10^{-7}$ Hm^{-1} mass of electron = 9.1×10^{-31} kg

1. ELECTROSTATICS

(a) Coulomb's Law

Force between two point charges, q_1 and q_2 has magnitude $F = \dfrac{q_1 q_2}{4\pi\varepsilon_o R^2}$ in direction along line joining charges. In vector notation $\underline{F} = \dfrac{q_1 q_2}{4\pi\varepsilon_o R^3}\,\underline{R}$ or $\underline{F} = \dfrac{q_1 q_2}{4\pi\varepsilon_o R^2}\,\hat{\underline{R}}$

(b) Electric Field

Defined by $\underline{E} = \dfrac{Q}{4\pi\varepsilon_o R^3}\,\underline{R}$, and then force is $\underline{F} = q\underline{E}$. In electrostatics we want to solve for $\underline{E}$.

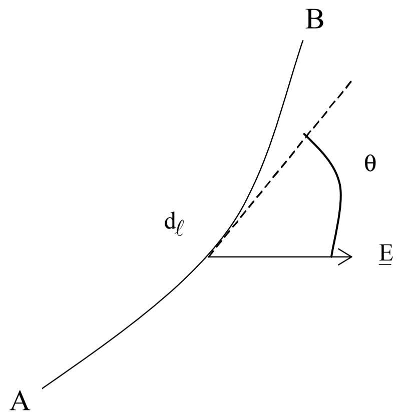
(c) Potential

Work done in moving q_1 from A to B is $W = q_1\left(\phi(A) - \phi(B)\right)$ where ϕ is potential.

Potential due to charge q is $\phi = \dfrac{q}{4\pi\varepsilon_o R}$, and ϕ and $\underline{E}$ are related by

$$\phi(B) - \phi(A) = -\int_A^B \underline{E}\bullet\underline{dl} = -\int_A^B E\cos\theta\, d\ell$$

$$\underline{E} = -\nabla\phi = \left(-\frac{d\phi}{dx},\ -\frac{d\phi}{dy},\ -\frac{d\phi}{dz}\right)$$



(d) **Gauss's Law**

Surface integral of $\underline{E}$ gives $\oint_s E\cos\theta \; da = \dfrac{Q}{\varepsilon_o}$, Q = total charge enclosed by surface S.

(e) **Solving for $\underline{E}$**

Three methods possible.

(i) Use Coulomb's Law, summing all contributions with care about direction.

(ii) Calculate ϕ and then use $\underline{E} = \left(-\dfrac{d\phi}{dx}, -\dfrac{d\phi}{dy}, -\dfrac{d\phi}{dz} \right)$.

(iii) Use Gauss's Law - only works if symmetry can be employed to get $\underline{E}$ outside the integral.

(f) **Important Cases**

(i) Sheet of charge, $\left|\underline{E}\right| = \dfrac{q_s}{2\varepsilon_o}$, q_s is surface density, or charge per unit area.

(ii) Line of charge, $\left|\underline{E}\right| = \dfrac{q_\ell}{2\pi r\varepsilon_o}$, q_ℓ is charge per unit length.

(iii) Sphere of charge Q, $\left|\underline{E}\right| = \dfrac{Q}{4\pi\varepsilon_o r^2}$.

(g) **Capacitance**

Capacitance of two conductors is defined by $C = Q/V$. For parallel plate capacitor $C = \varepsilon \, A/d$, where ε = permittivity of separating medium. Effect of dielectric medium is to increase the capacitance.

(h) **Energy**

Stored energy in capacitor is $\dfrac{1}{2}\,CV^2$. Energy density in electric fields is $\dfrac{1}{2}\,\varepsilon E^2$.

2. MAGNETIC FIELDS

(a) Force between two circuits

Force is given by Ampère's force law, but this is difficult to use. Introduce $\underline{B}$ field, and force in a circuit is $\underline{F} = \oint I \; \underline{dl} \times \underline{B}$.

(b) Biot-Savart Law

$\underline{B}$ field is given by $\underline{B} = \dfrac{\mu_o}{4\pi} \oint \dfrac{I \underline{dl} \times \underline{\hat{r}}}{r^2}$

Analytical results possible only for simple geometries.

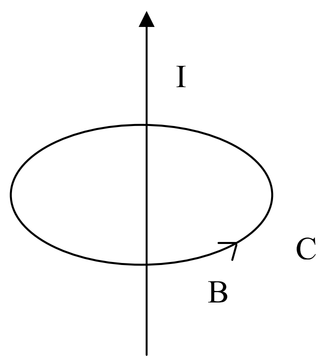
(c) Important cases of $\underline{B}$

(i) Infinitely long straight wire $B = \mu_o I/2\pi r$.

(ii) on axis of circular loop, $B = \mu_o Ia^2 / 2\left(a^2 + d^2\right)^{3/2}$.

(iii) Inside long straight solenoid $B = \mu_o nI$.

(d) Ampère's Law



$$\oint_C \underline{B} \bullet \underline{dl} = \oint_C B \cos\theta \, d\ell = \mu_o I$$

I is the current which threads the path of integration.
Direction given by right-hand rule

(e) Magnetic Flux

Defined by $\Phi = \int B \cos\theta da$, i.e. Φ is given by the integral over area of normal component of $\underline{B}$. For uniform B, $\Phi = BA$, hence B is called magnetic flux density. For a closed surface of integration $\oint B \cos\theta da = 0$, which implies no magnetic poles.

3. MAGNETIC INDUCTION

(a) Faraday's Law

If flux linkages through a circuit change with time, magnitude of emf induced is $\varepsilon = \dfrac{d\Phi}{dt}$.

Polarity of ε given by Lenz's Law, is such as to try to keep Φ constant.

(b) **Self-inductance**

Defined by $\varepsilon = L \dfrac{di}{dt}$ where L depends on geometry of circuit (and also any magnetic materials present). In a circuit L causes current to lag voltage.

Inductance of solenoid $= \dfrac{\mu_o N^2 A}{\ell}$, where N is the total number of turns, A is the cross-sectional area, and ℓ is the length of the solenoid.

(c) **Magnetic Energy**

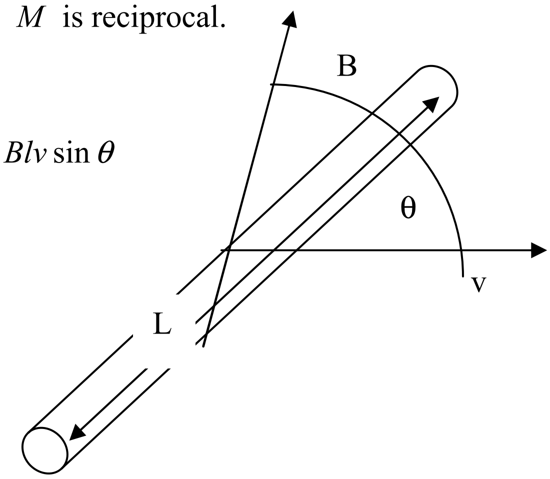
Energy stored in inductance is $\dfrac{1}{2} Li^2$. Energy per unit volume in magnetic fields is $\dfrac{B^2}{2\mu_o}$ or $\dfrac{B^2}{2\mu}$ if magnetic material of permeability μ is present.

(d) **Mutual Inductance**

Current change in one circuit induces emf in nearby circuit $\varepsilon = M \dfrac{di}{dt}$. M is coefficient of mutual inductance, depends on geometry and materials. M is reciprocal.

(e) **EMF induced by Motion**

EMF is generated by conductor moving in B field, $\varepsilon = Blv \sin\theta$



4. **MAGNETIC FORCES**

(a) **Force between parallel wires**

Force per unit length is $f = \mu_o \, I_1 I_2 / 2\pi p$, where p is distance between wires. Like currents attract, unlike repel. The unit of current (Ampere) is defined from this relation.

(b) **Force on Linear Conductor**

$F = BIl \sin\theta$ or in vector notation $\underline{F} = I\underline{l} \times \underline{B}$

(c) **Torque on Current Loop**

$T = NIBA \sin\alpha$
Applications include motor and meter.

(d) **Force on Charged Particle**

$\underline{F} = q\left(\underline{v} \times \underline{B}\right)$ is at right angles to both $\underline{B}$ and $\underline{v}$.
Gives Hall effect and gyration of charges about field lines.