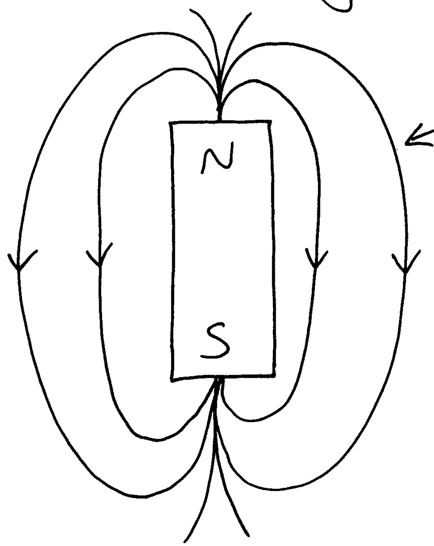
Magnetic Fields



magnetic field due to a bar magnet (2 units)

$\underline{B}$: the magnetic flux density, or B-field. (measured in Tesla, T)

$\underline{H}$: magnetic field intensity (measured in Amps/m)

The relationship is $\underline{B} = \mu \underline{H}$ [c.f. $\underline{D} = \varepsilon \underline{E}$] (electric flux density)

where μ = permeability

[$\mu_0 = 4\pi \times 10^{-7}$ H/m]

Magnetic fields are produced by currents, i.e. moving charges (even in a bar magnet.)

From electrostatics we know that...

$$\underline{F} = q\,\underline{E}$$

- is there an equivalent relationship for magnetic fields?

In magnetostatics, the force acting on a small moving charge q, by a magnetic field, $\underline{B}$, is given by...

$$\boxed{\underline{F} = q\,\underline{V} \times \underline{B}} \quad (N)$$

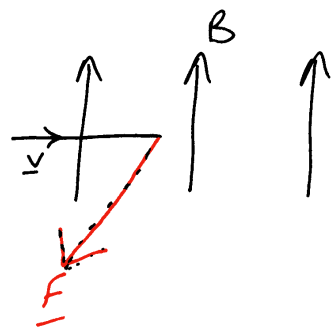
where $\underline{V}$ = velocity of the charged particle

[$F = qV\sin\theta$] where θ is the angle between $\underline{V}$ and $\underline{B}$

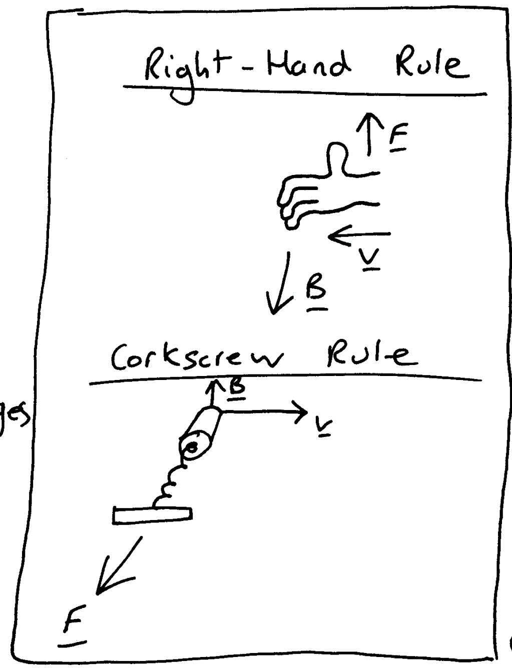
What does this tell us?

1) Only got a force when particle is moving. (charged particle moving ≡ current)

2) Force is at right angles to both $\underline{V}$ and $\underline{B}$ (cross product)



Note: if $q = -ve$, $\underline{F}$ changes direction too (equation takes care of this)



3) If $\underline{v}$ is parallel to $\underline{B}$, no force.

4) No work is done by the field on the particle

→ $dW = \underline{F} \cdot d\underline{x}$ $\quad (dw = F_x dx)$

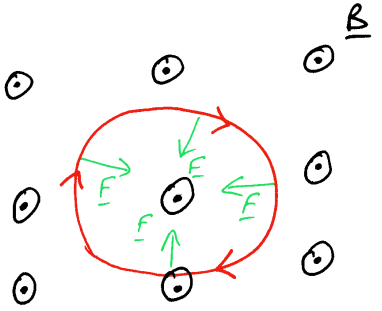
But $\underline{F}$ is always $\perp$ to dx so $\underline{F} \cdot d\underline{x} = 0$

→ Kinetic energy is constant

$|\underline{v}|$ is constant

only the direction of v changes, not magnitude

→ charged particle ends up going round in circles...



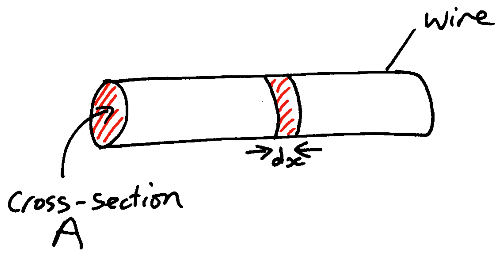
(see cyclotron motion later)

In general, we can describe the total force on a charged particle as

$$\underline{F} = q\underline{E} + q\underline{v} \times \underline{B} \qquad \text{(Lorentz force)}$$

Magnetic Force on a current carrying conductor

- Current is due to charged particles moving along wire.
- Hence, if wire is placed in a magnetic field, it will experience a force.



conductor has:-

n charge carriers / unit volume

q = charge of each particle

v = velocity of charges

Amount of charge in a small volume, dx thick, is

$$dQ = qnA\,dx$$

force acting on this charge is

$$d\underline{F} = dQ\,\underline{v} \times \underline{B} \qquad \text{where } v = \text{velocity of charges}, \; B = \text{mag field}.$$

$$= qnA\,dx\,\underline{v} \times \underline{B}$$

write $\underline{v}\,dx = v\,d\underline{x}$ (as dx is in same direction as $\underline{v}$)

to give

$$d\underline{F} = qnAv\,d\underline{x} \times \underline{B}$$

Now current $i = \frac{dQ}{dt} = qnA\frac{dx}{dt}$

and $\frac{dx}{dt} = v$

So $i = qnAv$

Substituting this into the equation above gives

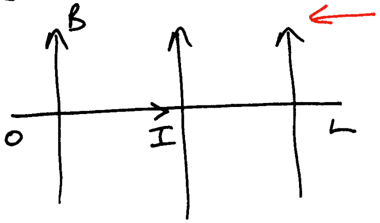
$$\underline{dF} = i\,\underline{dx} \times \underline{B}$$

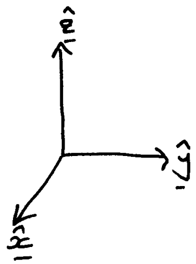
or

$$\boxed{\underline{F} = i\int_L \underline{dl} \times \underline{B}}$$

Example

- Magnetic force on a straight wire of length L





$$\underline{F} = I\int_0^L \underline{dl} \times \underline{B}$$

As $\underline{dl}$ is a straight line and $\underline{B}$ is constant, we can write

$$\underline{F} = I\left(\int_0^L \underline{dl}\right) \times \underline{B}$$

$$= I\,\underline{L} \times \underline{B} \qquad \text{(for a straight wire)}$$

As $\underline{L}$ and $\underline{B}$ are both parallel to the page, the direction of the force is perpendicular to the page (out of the page)

In terms of unit vectors

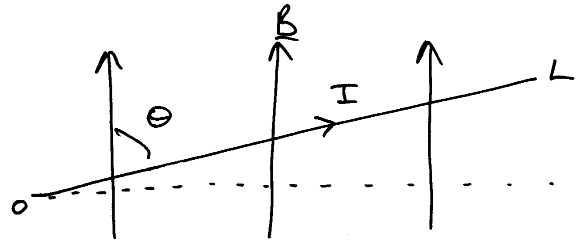
$$\underline{F} = I(L\hat{\underline{y}}) \times (B\,\hat{\underline{z}})$$

$$\underline{F} = ILB\,\hat{\underline{x}}$$

$\hat{\underline{x}} \times \hat{\underline{y}} = \hat{\underline{z}}$

$\hat{\underline{y}} \times \hat{\underline{z}} = \hat{\underline{x}}$

$\hat{\underline{z}} \times \hat{\underline{x}} = \hat{\underline{y}}$



If wire is at an angle θ to B then

$$F = I\underbrace{L\sin\theta}\,B$$

$L\sin\theta$ = component of L $\perp$ to B

Force and Torque on a current carrying loop (motor)

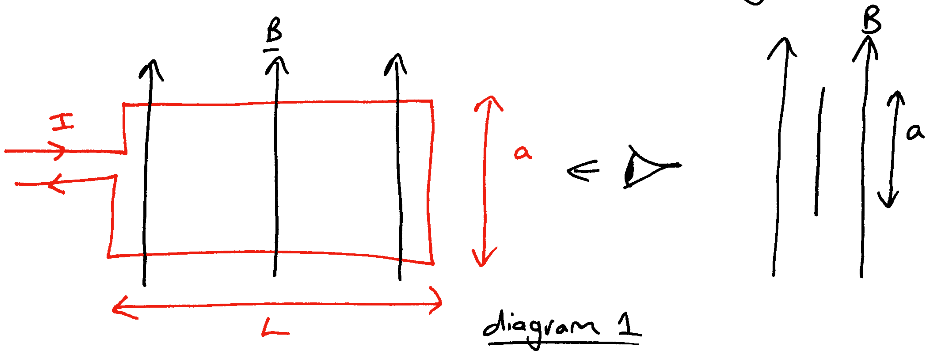


diagram 1

Work out force on each straight line segment of the loop using

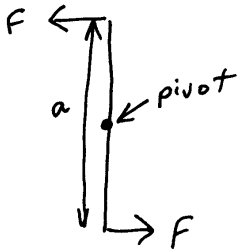
$$\underline{F} = I\underline{L} \times \underline{B}$$

and add together

1) On top wire $F = ILB$ out of paper
2) On bottom wire $F = ILB$ into paper
3) Both sides of loop are parallel to B so no force (only true when vertical.)

→ Net force on loop is zero

But if we pivot the loop on its axis, we get a torque.



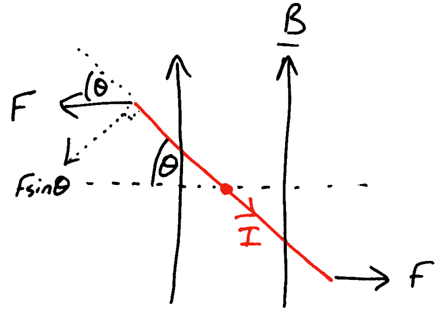
Torque

$$T = F.\frac{a}{2} + F.\frac{a}{2}$$

$$= Fa$$

Hence loop will rotate

Now look at case when loop makes an angle θ

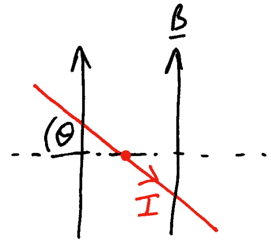


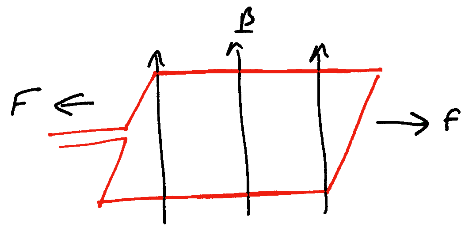
The force on the top and bottom wires (diag 1) are the same, as L is still $\perp$ to B, but the torque is now

$$T = F\sin\theta.\frac{a}{2} + F\sin\theta.\frac{a}{2} = Fa\sin\theta$$

Sides of loop also experience a force given by

$$IaB\cos\theta \quad \text{and} \quad -IaB\cos\theta$$

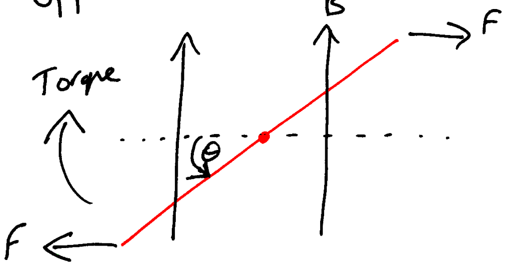




force is out of the paper

Forces are in opposite directions so net force is zero
And, as no pivot, no torque

Loop rotates but when $\theta < 0°$, torque is in opposite direction

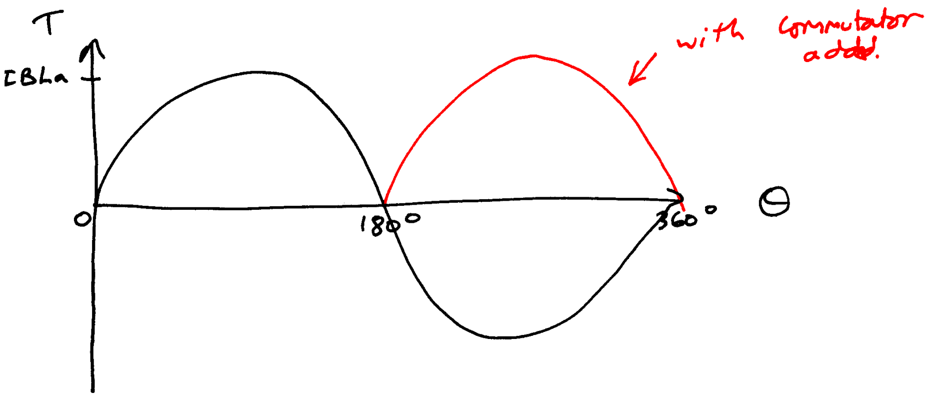


To get continuous rotation, we need a commutator to change direction of current when $\theta = 0°$ and $180°$

$$T = Fa\sin\theta$$

$$= I\underbrace{BLa}_{\text{area of loop}}\sin\theta$$

$$\rightarrow T = IAB\sin\theta$$



for an N turn loop

$$T = NIAB\sin\theta$$

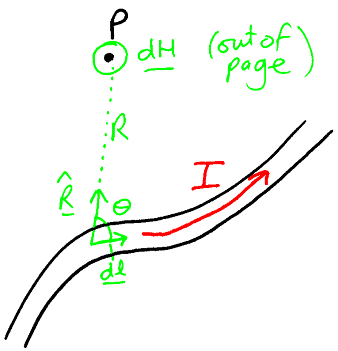
Biot-Savart Law

- derived experimentally
- the magnetic field $\underline{dH}$ generated by a steady current I flowing through a length $\underline{dl}$ is

$$\underline{dH} = \frac{I}{4\pi R^2}\,\underline{dl} \times \underline{\hat{R}} \qquad \text{Amp/m}$$

$\underline{B} = \mu \underline{H}$

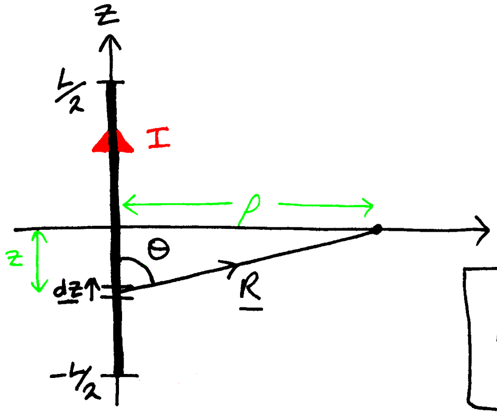
drops off as $\frac{1}{R^2}$ just like electric field

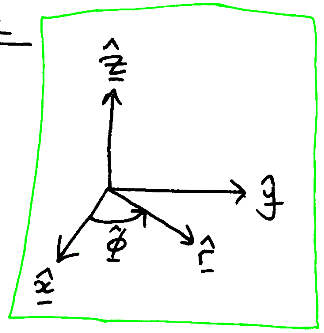


- if wire has total length L, then total H-field is

$$\boxed{\underline{H} = \frac{I}{4\pi}\int_L \frac{\underline{dl} \times \underline{\hat{R}}}{R^2}}$$

Magnetic field from a straight wire





$$\boxed{\underline{H} = \frac{I}{4\pi}\int_L \frac{d\underline{l} \times \hat{\underline{R}}}{R^2}}$$ ← Biot-Savart

$$d\underline{z} \times \hat{\underline{R}} = dz \sin\theta\, \hat{\underline{\phi}}$$

$[\hat{\underline{dz}} \times \hat{\underline{r}} = \hat{\underline{\phi}}]$

Hence $$\underline{H} = \hat{\underline{\phi}}\frac{I}{4\pi}\int_{-L/2}^{L/2}\frac{\sin\theta}{R^2}\,dz$$

From diagram $\sin\theta = \dfrac{\rho}{R}$

and $R^2 = z^2 + \rho^2$

$$\therefore\ \underline{H} = \hat{\underline{\phi}}\frac{I}{4\pi}\int_{-L/2}^{L/2}\frac{\rho}{(z^2+\rho^2)^{3/2}}\,dz$$

Using a standard integral, this gives...

$$\underline{H} = \hat{\underline{\phi}}\frac{I\,L}{2\pi\rho\sqrt{4\rho^2+L^2}}$$

or using $\underline{B} = \mu_0 \underline{H}$

$$\underline{B} = \hat{\underline{\phi}}\frac{\mu_0 I L}{2\pi\rho\sqrt{4\rho^2+L^2}}$$

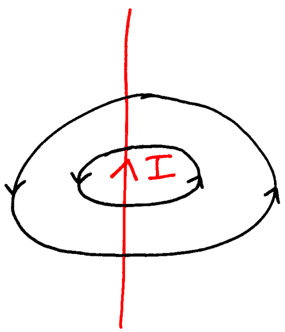
If wire is very long, $L \to \infty$, and $\underline{B}$ becomes... (or $L \gg \rho$)

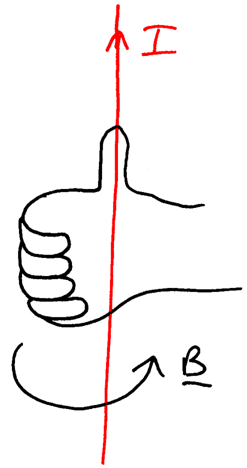
$$\boxed{B = \hat{\underline{\phi}}\frac{\mu_0 I}{2\pi\rho}}$$

given on formula sheet.

different version to the one used previously.

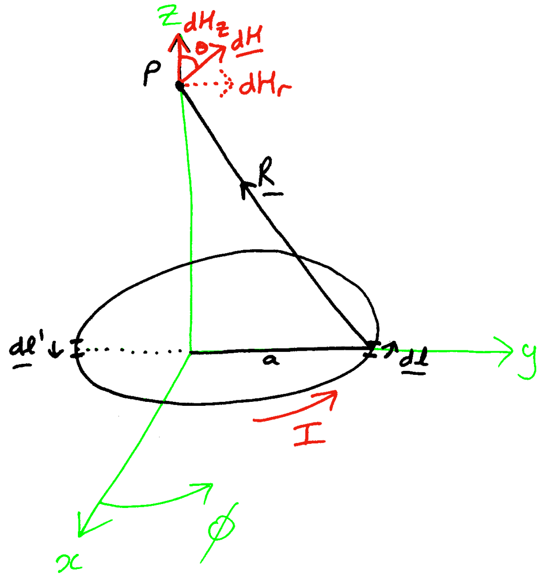
Direction of B given by Right Hand Rule





→ True for direction of $\underline{B}$ from any wire

Magnetic field on the axis of a circular loop



$$\boxed{\underline{H} = \frac{I}{4\pi}\int_L \frac{\underline{dl}\times\hat{\underline{R}}}{R^2}}$$

$\underline{R}$ is perpendicular to $\underline{dl}$ and $R^2 = a^2 + z^2$

magnitude of dH is

$$dH = \frac{I}{4\pi R^2}\left|\underline{dl}\times\hat{\underline{R}}\right| = \frac{I\,dl}{4\pi(a^2+z^2)}$$

direction of $\underline{dH}$ is $\perp$ to both $\underline{dl}$ and $\underline{R}$

$\rightarrow$ $\underline{dH}$ is in the R-z plane and has components dH_z and dH_r

If we consider a small piece of the loop $\underline{dl}'$ which is opposite $\underline{dl}$ we see that the dH_r components cancel but the dH_z components add

Hence the field is in the $\hat{\underline{z}}$ direction only.

$$dH_z = dH\cos\theta = \frac{I\cos\theta}{4\pi(a^2+z^2)}\,dl$$

Total field due to loop is

$$\underline{H} = \hat{\underline{z}}\,\frac{I\cos\theta}{4\pi(a^2+z^2)}\oint dl$$

round loop

$2\pi a$

$$= \hat{\underline{z}}\,\frac{I a\cos\theta}{2(a^2+z^2)}$$

But $\cos\theta = \dfrac{a}{R} = \dfrac{a}{\sqrt{a^2+z^2}}$

to give

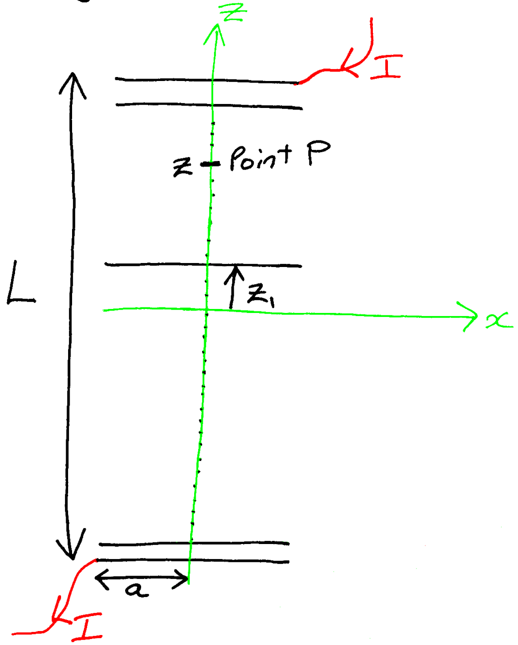
$$\boxed{\underline{H} = \hat{\underline{z}}\,\frac{I a^2}{2(a^2+z^2)^{3/2}}}$$

on formula sheet as $B = \mu_0 \ldots$

At centre of loop $(z=0)$, $\boxed{\underline{H} = \hat{\underline{z}}\,\dfrac{I}{2a}}$

Note: these expressions are only for the field on the axis of the loop. (there is no simple expression for the field off-axis)

Magnetic field of a solenoid



field due to a single loop is

$$\frac{I a^2}{2(a^2+z^2)^{3/2}}$$

Assume solenoid can be modelled as a series of loops of radius a

- Solenoid has n loops (or turns) per unit length

Field at point P on z-axis due to a single loop at z_1 is ...

$$H = \frac{I a^2}{2(a^2 + \underbrace{(z-z_1)^2}_{\text{offset from single loop at } z_1})^{3/2}}$$

In a short length of solenoid dz_1 the number of loops is given by ...

$$n \cdot dz_1$$

which contribute to the field at P

$$dH = \frac{I a^2 n \, dz_1}{2(a^2 + (z-z_1)^2)^{3/2}}$$

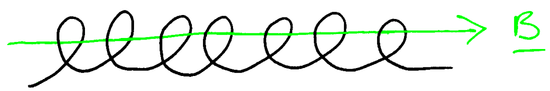
Total field is obtained by integrating over length of solenoid ...

$$H = \int_{-L/2}^{L/2} \frac{I a^2 n \, dz_1}{2(a^2+(z-z_1)^2)^{3/2}}$$

We can solve using a standard integral. The result is a nasty, complex formula (see printed notes) but can be simplified for $L \gg a$ (ie. an infinitely long solenoid) to give ...

$$\boxed{B = \mu_0 n I}$$

Note: B-field is along axis of solenoid



$$\boxed{\underline{B} = \mu_0 \underline{H}}$$

Ampère's Law

In electrostatics potential is given by line integral of E-field from one point to another.

$$V = -\int_{P_1}^{P_2} \underline{E} \cdot d\underline{l}$$

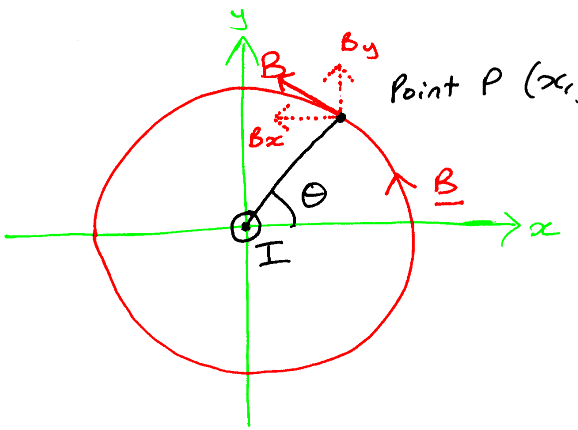
Is there a similar relationship for magnetic fields?

Line integral of magnetic field is given by:—

$$\int_{P_1}^{P_2} \underline{B} \cdot d\underline{l} = \int_{P_1}^{P_2} B\cos\theta \, dl$$

Look at an example – long straight wire

$$\underline{B} = \frac{\mu_0 I}{2\pi r} \hat{\underline{\phi}}$$



In x, y coordinates we have

$$\underline{B} = [B_x, B_y, 0]$$

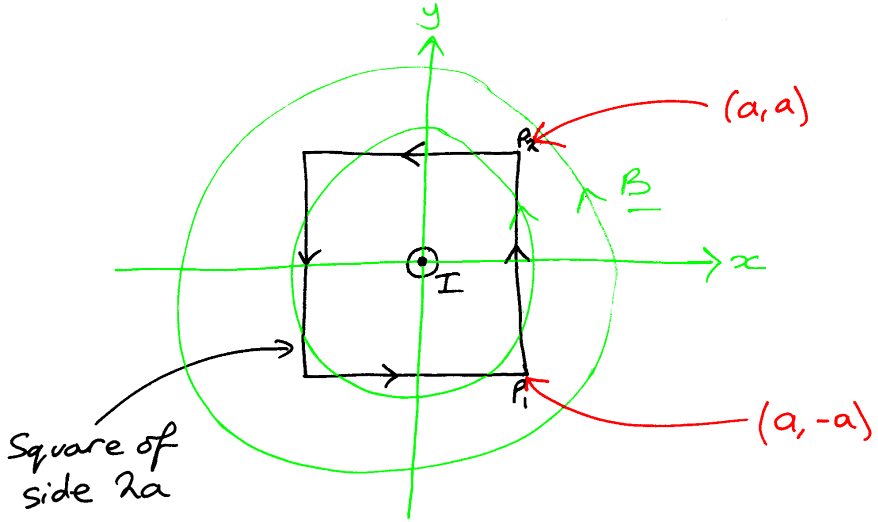
Now $B_x = \underbrace{B}_{\frac{\mu_0 I}{2\pi r}} \underbrace{\sin\theta}_{y/r}$

$$= \frac{\mu_0 I y}{2\pi r^2} \qquad \text{but } r^2 = x^2 + y^2$$

$$\therefore \quad B_x = \frac{\mu_0 I y}{2\pi (x^2 + y^2)}$$

And $$B_y = \frac{\mu_0 I x}{2\pi (x^2 + y^2)}$$

Work out some line integrals...



$$\int_{P_1}^{P_2} \underline{B} \cdot \underline{dl} = \int_{P_1}^{P_2} B_y \cdot dy$$

$$= \int_{P_1}^{P_2} \frac{\mu_0 I x}{2\pi (a^2+y^2)} \, dy$$

$$= \frac{\mu_0 I}{2\pi} \tan^{-1} \frac{y}{a} \Bigg|_{-a}^{a}$$

$$= \frac{\mu_0 I}{2\pi} \Big[\underbrace{\tan^{-1}(1)}_{\pi/4} - \underbrace{\tan^{-1}(-1)}_{-\pi/4} \Big]$$

$$= \frac{\mu_0 I}{4}$$

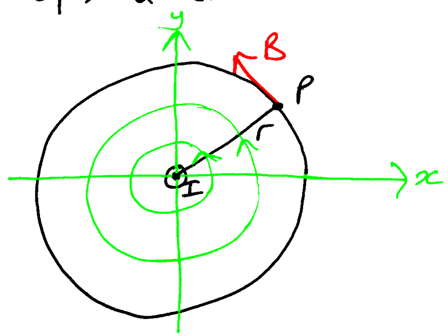
By symmetry we expect all the other sides of the square to give the same result

Hence $\oint \underline{B} \cdot \underline{dl} = 4 \times \frac{\mu_0 I}{4} = \mu_0 I$

around the square

→ independent of size of square

What if the path is not straight?
– perhaps a circle ...



Integrate around circle ...

$$\oint \underline{B} \cdot \underline{dl} = \oint B \cdot dl \quad \text{→ as } \underline{B} \,//\, \text{to } \underline{dl}$$

$$= B \oint dl$$

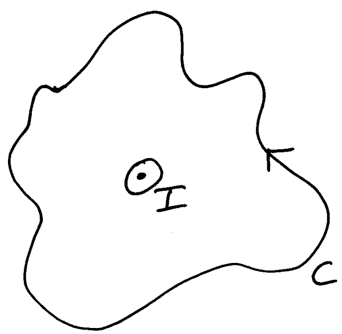
$\frac{\mu_0 I}{2\pi r}$ → circumference $= 2\pi r$

$$= \mu_0 I \qquad \text{(we get the same result!)}$$

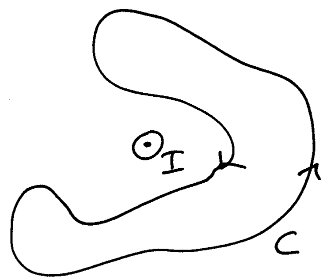
In general, Ampère's Law states that ...

$$\boxed{\oint \underline{B} \cdot \underline{dl} = \mu_0 I} \qquad \boxed{\oint \underline{H} \cdot \underline{dl} = I}$$

where I = current flowing through contour

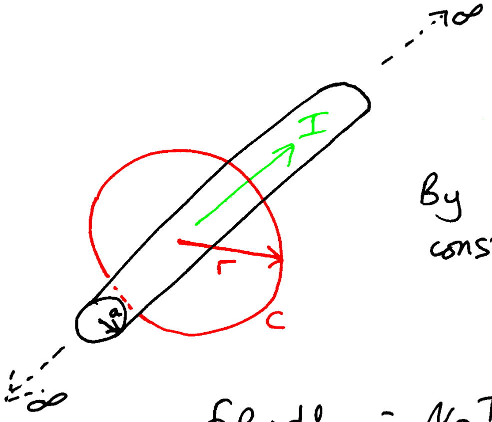


$\oint \underline{B} \cdot \underline{dl} = \mu_0 I$



$\oint \underline{B} \cdot \underline{dl} = 0$ (no current passing through the contour)

Magnetic field due to a long straight wire — using Ampère's Law



By symmetry B-field is constant around contour C

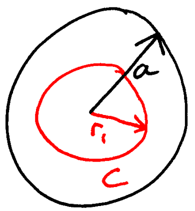
$$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 I$$

$$B \oint_C dl = \mu_0 I$$

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

What about the B-field inside the wire?



Assume current flow is uniform within x-section of the wire

$$I_1 = \left(\frac{\pi r_1^2}{\pi a^2}\right) I$$

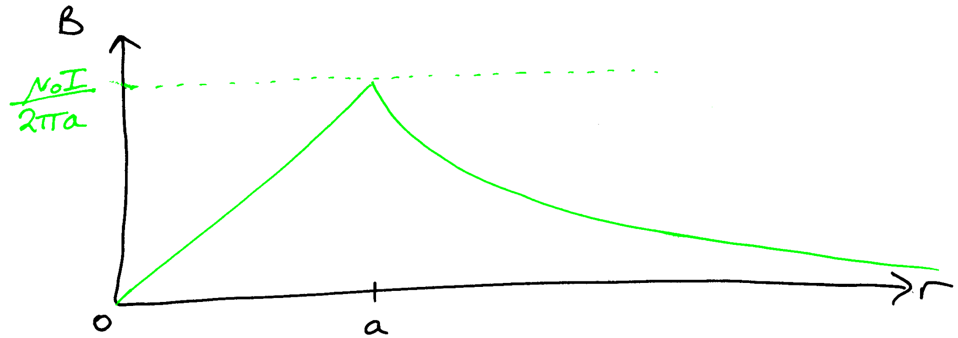
$$= \left(\frac{r_1}{a}\right)^2 I$$

Using Ampère's Law...

$$\oint_C \underline{B} \cdot \underline{dl} = \mu_0 I_1$$

$$B \cdot 2\pi r_1 = \mu_0 \left(\frac{r_1}{a}\right)^2 I$$

$$B = \frac{\mu_0 r_1}{2\pi a^2} I$$



Ampère's Law + Solenoid

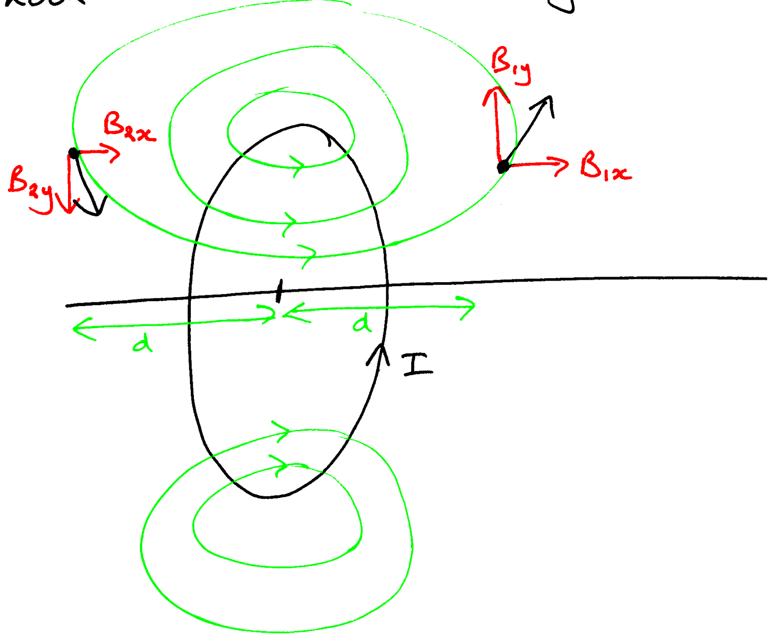
When we looked at solenoid using Biot-Savart Law, we found that the B-field on axis was...

$$B = \mu_0 n I$$

n turns per unit length

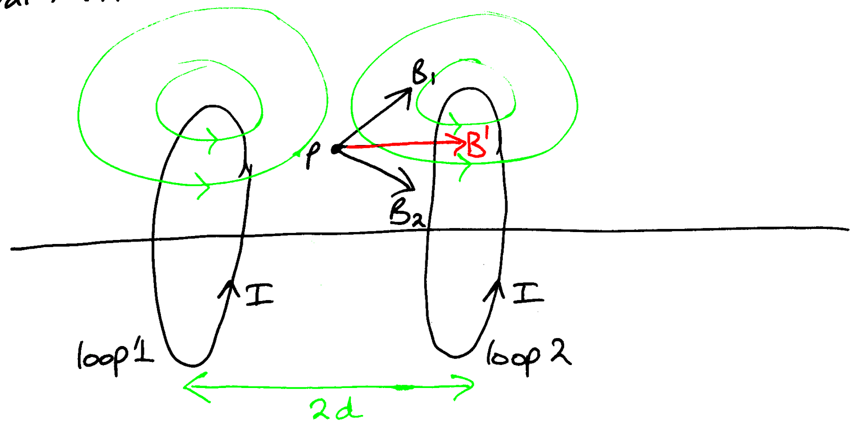
What is B-field off-axis ?

- Assume solenoid consists of an infinite number of circular loops
- Look at B-field from a single loop



B-field at points P_1 and P_2 has same magnitude (using symmetry) but y-components in opposite directions.

Now assume we have 2 loops a distance 2d apart...



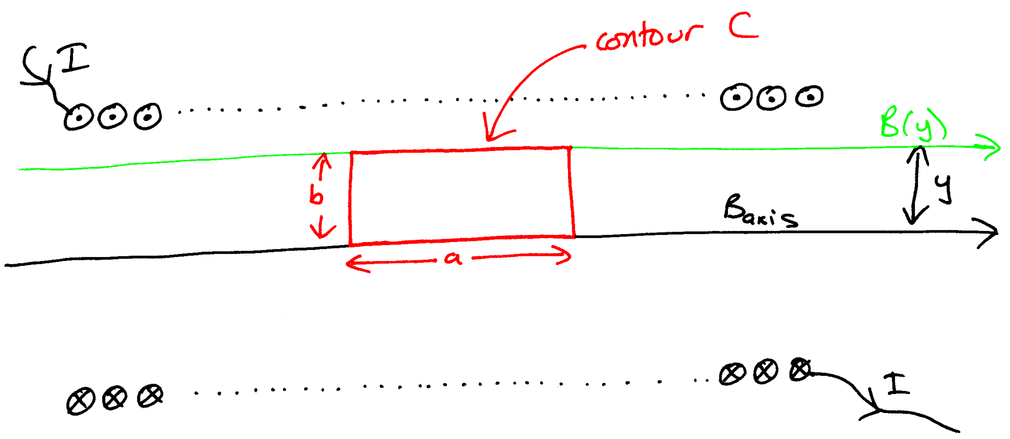
At point P y-components of B_1 and B_2 cancel, but x-components add

→ Resultant B-field (B') is // to axis of loops

→ Hence if solenoid consists of an infinite number of loops, the resulting B-field will be parallel to axis of solenoid.

- But what is its magnitude ?

(- use Ampère's Law)



Apply Ampère's Law around contour C

$$\oint \underline{B} \cdot \underline{dl} = \mu_0 I = 0$$

no current through contour C

$$B_{axis} \times a + 0 \times b - B(y) \times a + 0 \times b = 0$$

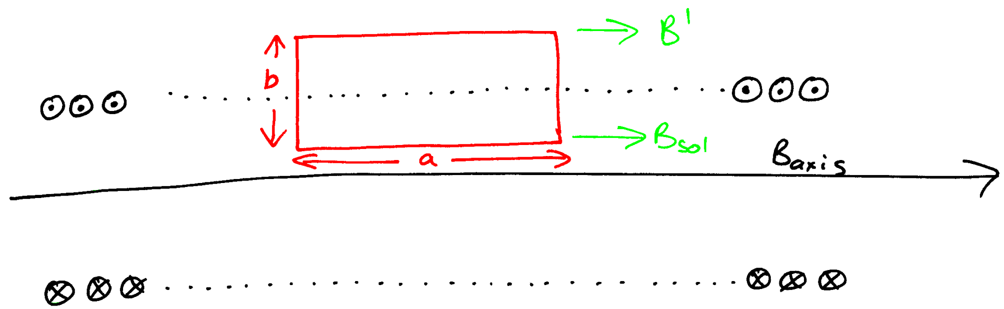
$$B_{axis} = B(y)$$

→ B-field is uniform throughout solenoid

We know that $B_{axis} = \mu_0 n I$

→ Field everywhere within solenoid is $\mu_0 n I$

What is the field outside the solenoid ?



Current through contour $= I n a$

and $\oint B dl = \mu_0 I'$

$$a \cdot B_{sol} - a \cdot B' = \mu_0 I n a$$

But $B_{sol} = \mu_0 n I$

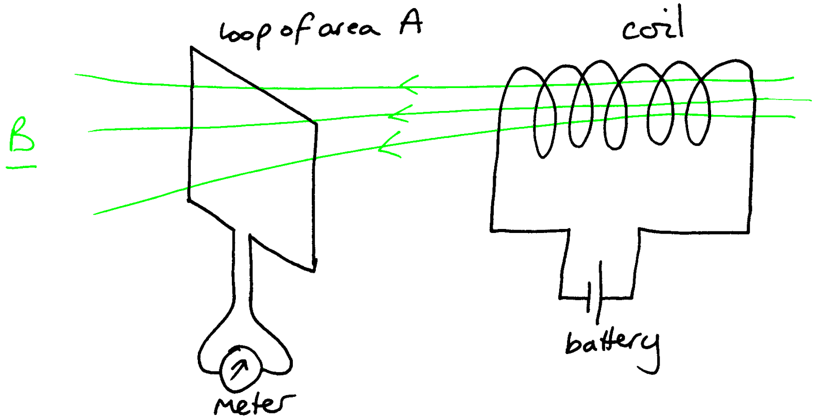
$$\mu_0 n I a - a \cdot B' = \mu_0 I n a$$

$$B' = 0$$

Electromagnetic Induction and Faraday's Law

If a current passing through a wire produces a magnetic field, does a magnetic field produce a current in a wire?

→ A current is induced only if the magnetic field varies with time



Magnetic flux passing through the loop is

$$\phi = B_{\perp} \times A$$

A current flows in the loop (i.e. a voltage is induced) only if the flux changes with time.

- This can happen in two ways...

1) <u>B changes with time</u>
→ current flowing through coil is time varying
→ transformer effect

2) <u>A varies with time</u>
→ relative motion between loop and coil (e.g. loop rotates)
→ motional effect

... or a combination of the two.

In general we can write that voltage induced in the loop is :-

$$V_{emf} = -\frac{d\phi}{dt}$$

electro-motive force

sign will be explained later

If the loop had N turns the total flux linkage $\psi = N\phi$ so :-

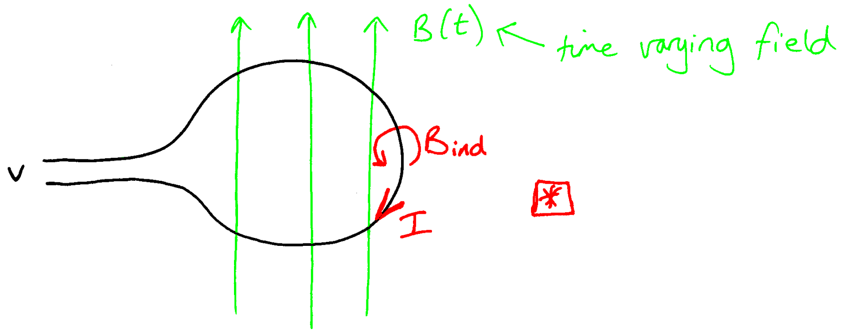
$$V = -N\frac{d\phi}{dt}$$

We can also write $\phi = \int_S \underline{B} \cdot \underline{ds}$ (just as we did in electrostatics with electric flux)

thus, $$V = -N\frac{d}{dt}\int_S \underline{B} \cdot \underline{ds}$$

Stationary loop in a time varying magnetic field

(i.e. case 1 : transformer effect)



Flux passing through loop is $\phi = AB(t)$

From Faraday's Law :-

$$V = -\frac{d\phi}{dt} = -A\frac{dB(t)}{dt}$$

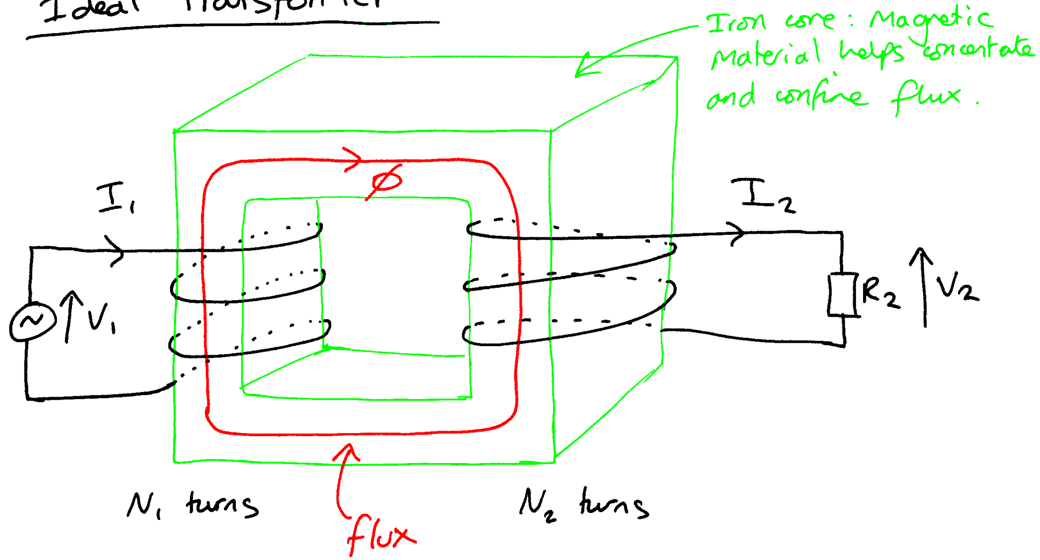
V will cause a current to flow in the loop – but in which direction ?

Lenz's Law : The current in the loop is such to oppose the change of ϕ (flux) that produced it

If $B(t)$ is increasing, i.e. $\frac{dB(t)}{dt} > 0$, then induced current is such that it opposes this increase.

[*] Hence the induced B-field in the loop, Bind is as shown in the diagram and hence the current is as shown.

Ideal transformer



We have :

$$V_1 = -N_1\frac{d\phi}{dt} \quad \text{and} \quad V_2 = -N_2\frac{d\phi}{dt}$$

which gives $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

If the transformer is lossless then :-

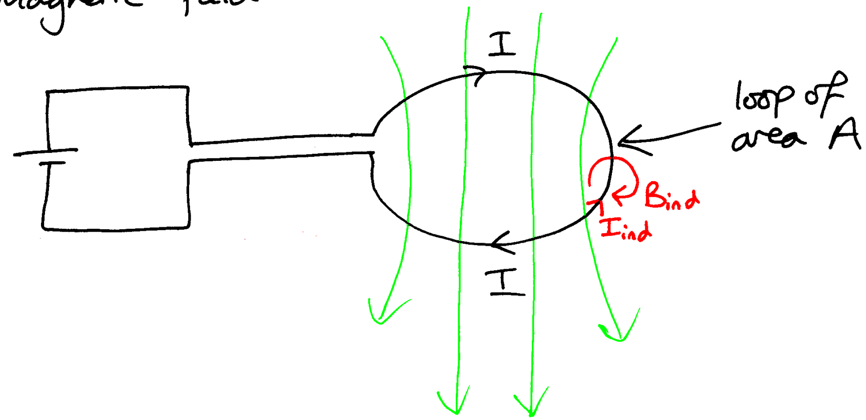
$$I_1 V_1 = I_2 V_2$$ (Power in = Power out)

$$\rightarrow \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Note polarity of V_2 with respect to V_1 depends on direction of secondary winding

Inductance

Connect a battery to our loop so that we generate a magnetic field



$$\text{flux } \phi = \int_A B_\perp \, ds$$

As B is proportional to I, the total flux through A will be proportional to I also.

Hence, we can write $\phi \propto I$

or $\phi = LI$ where the constant L is inductance

Now from Faraday's Law:-

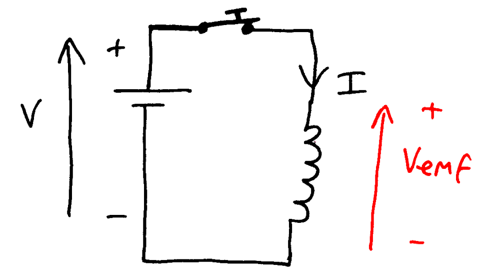
$$V_{emf} = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

So if I is increasing we get Bind and Iind as shown in the diagram resulting in an induced voltage Vemf which opposes the original voltage

This is called the BACK EMF

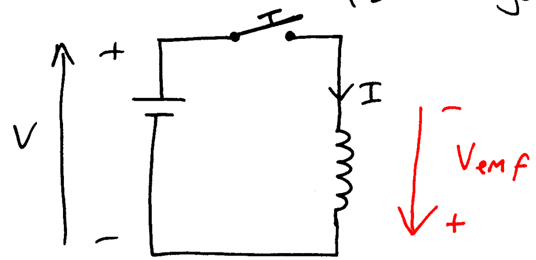
We represent inductors by the symbol —mmm—

example (switch just closed, B-field in coil increasing)



$\frac{dI}{dt}$ +ve

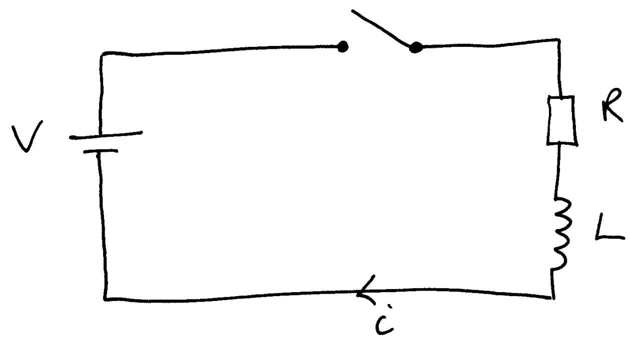
(switch just opened, B-field in coil decreasing)



$\frac{dI}{dt}$ -ve

The back emf due to an inductor always acts to oppose the change in current.

Consider the following circuit

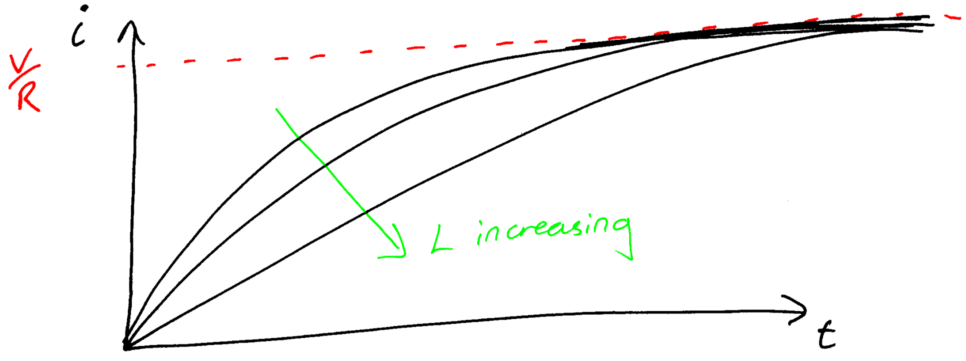


we have:-

$$V = iR + L\frac{di}{dt}$$

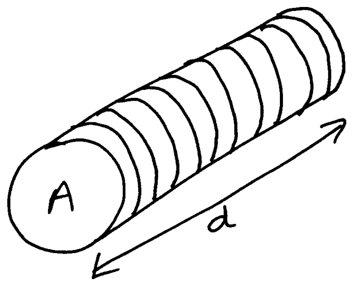
When the switch is first closed $\frac{di}{dt}$ is very large so that most of V is dropped across L and i is small

As i increases more current is dropped across R and $\frac{di}{dt}$ decreases



Inductance of a solenoid

For a long solenoid $B = \mu_0 n I$ [n = turns/unit length]



If x-sectional area is A and length is d, the total flux linking the solenoid is:-

$$\psi = dn\phi = dnAB$$

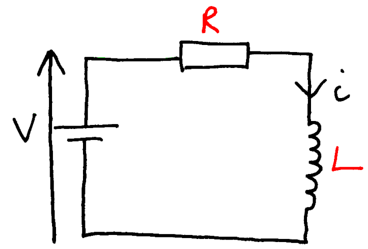
$$\psi = dnA\mu_0 NI$$

$$\psi = \mu_0 n^2 A I d$$

By definition $\psi = Li$

so $L = \mu_0 n^2 A d$

Energy stored in inductance



$$V = iR + L\frac{di}{dt}$$

$$\text{Power} = iV = i^2R + iL\frac{di}{dt}$$

In time dt, energy stored in the inductor is...

$$iL\frac{di}{dt} \cdot dt$$

if $i=0$ at $t=0$ and $i=I$ at $t=T$, then total stored energy is...

$$W = \int_0^I Li\,di = \frac{1}{2}Li^2 \Big|_0^I$$

$$W = \frac{1}{2}LI^2$$

Substituting in $L = \mu_0 n^2 A d$ and using $B = \mu_0 n I$ we get

$$W = \frac{B^2}{2\mu_0} A d$$

or $$W_m = \frac{B^2}{2\mu_0}$$ Energy per unit volume stored in a magnetic field.

Mutual Inductance

Last time we defined inductance as...

$$L = \frac{\Psi}{I}$$

flux linking circuit

current in circuit

and the induced voltage as

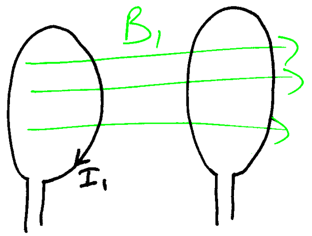
$$V_{emf} = -L\frac{dI}{dt} \quad \left[-\frac{d\Psi}{dt}\right]$$

This was for a single loop → self-inductance

A time varying current in one circuit can induce a voltage (or current) in another circuit

→ described by mutual-inductance

Loop ①
N_1 turns
Area A_1



Loop ②
N_2 turns
Area A_2

Current I_1 in loop ① produces a magnetic field B_1

- note that $B_1 \propto I_1$

Flux linking loop ② is proportional to area of loop ②, number of turns N_2, and B_1

ie $\Psi_{12} \propto B_1 A_2 N_2$

or $\Psi_{12} \propto I_1 A_2 N_2$ [as $B_1 \propto I_1$]

or $\boxed{\Psi_{12} = M I_1}$ $\left[\text{or } M = \frac{\Psi_{12}}{I_1}\right]$

Where M is the mutual inductance between loop ① and loop ②

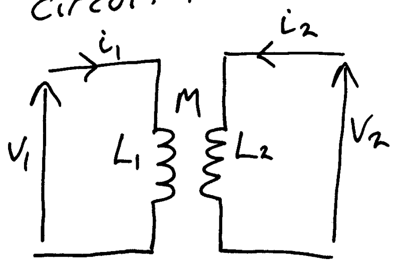
Hence if I_1 changes with time, the flux linking loop ② changes with time and a voltage is induced in loop ②

$$V_2 = \frac{-d\Psi_{12}}{dt} = -M\frac{dI_1}{dt}$$

Also, if we had a time varying current in loop ② it would induce a voltage in loop ①

$$V_1 = -M\frac{dI_2}{dt}$$

We can represent mutual inductances by the following circuit:-



L_1 and L_2 are self-inductances of coils ① + ②

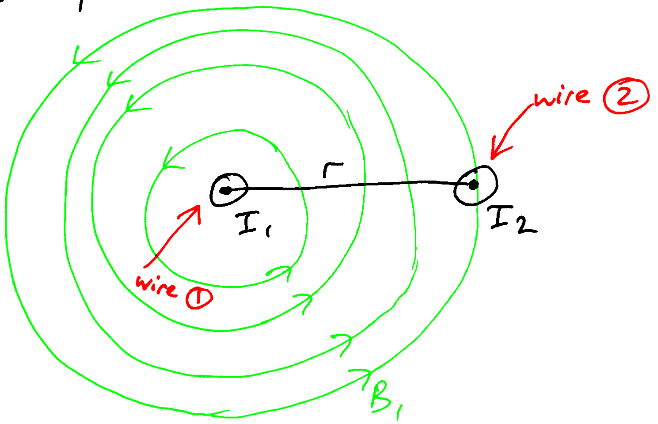
$$\boxed{\begin{aligned} V_1 &= L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ V_2 &= L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \end{aligned}}$$

Force between two parallel wires

From previous work we know that if a current carrying wire is in a B-field it will experience a force

force on a current carrying wire $\underline{F} = I\underline{L} \times \underline{B}$

We know that a current carrying wire produces a B-field. Hence if we place 2 wires next to each other, there will be a force between them.



$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

Wire ① produces B-field B_1. Hence wire ② will experience a force given by :–

$$\underline{F} = I_2 \underline{L} \times \underline{B_1}$$

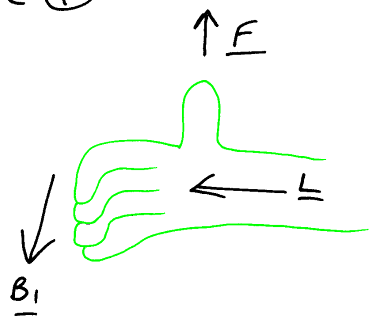
$|\underline{L}|$ = length of wire

$$= I_2 L B_1 \qquad \boxed{\text{as } \underline{L} \perp \underline{B_1}}$$

$$= \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

or force per unit length $= \dfrac{\mu_0 I_1 I_2}{2\pi r}$

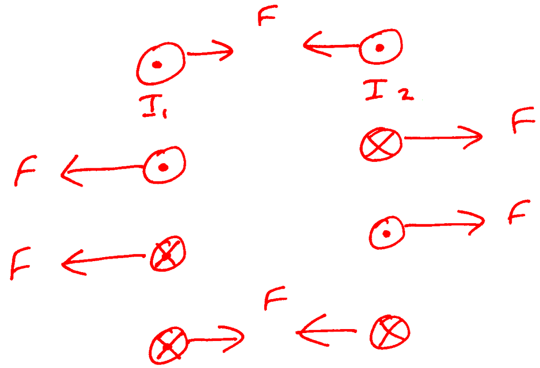
Direction of force is given by RH rule and is along r towards wire ①



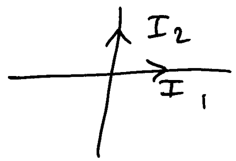
Wire ① also experiences a force due to B-field from wire ②

Hence the 2 wires are attracted to each other.

If direction of current in one of the wires was reversed, the wires would repel each other



Note: if wires at 90°, no force

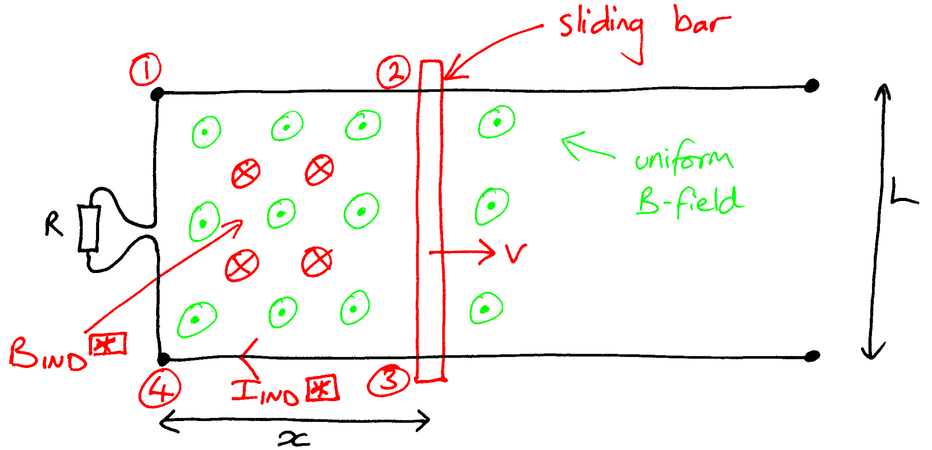


Electromagnetic Generator

From Faraday's law, we know that we get an induced voltage if the flux linking the circuit changes with time.

$$V = -\frac{d\phi}{dt} = \frac{-d\,[BA]}{dt}$$

In previous examples we looked at a time varying B-field. We will now consider what happens if the area of the loop changes with time.



✱ see later.

Circuit consists of a loop defined by points ①, ②, ③ + ④.

Area of loop is Lx

Flux through loop is $B \times$ Area or... assume B ⊥ to area

$$\phi = BLx$$

If the slider moves, the area of the loop changes with time and hence the flux through the circuit changes with time.

→ A voltage is induced

$$V_{emf} = \frac{d\phi}{dt} = BL\frac{dx}{dt}$$

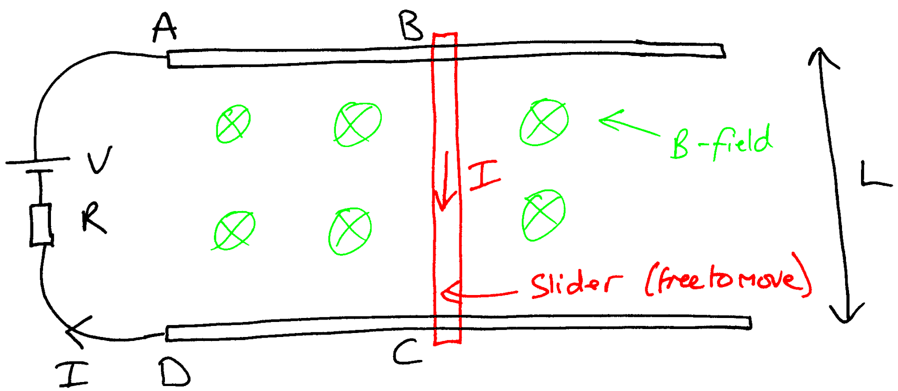
velocity = v

$$= BLv$$

Sign of V_{emf} is such to oppose the change in the flux [Lenz's Law] and gives rise to I_{IND} and B_{IND} as shown in diagram.

Note: $I_{IND} = \dfrac{BLv}{R}$

Linear Motor / Rail gun



Current I flows in the loop ABCD

Force on slider is $\underline{F} = I \underline{L} \times \underline{B}$

$= ILB$ (and direction is to the right)

$\rightarrow$ simple linear motor

However, when the slider moves the flux linking the loop changes with time and so there is a back emf...

$$V_{emf} = BLV_s \quad \text{(see earlier)}$$

V_s ← velocity of slider

This voltage opposes the driving voltage V and reduces the current...

$$\rightarrow \quad I = \frac{V - V_{emf}}{R}$$

As velocity (V_s) increases V_{emf} increases until

$$V = V_{emf}$$

and no current flows.

$\rightarrow$ slider moves at constant velocity

(note: we have ignored friction)