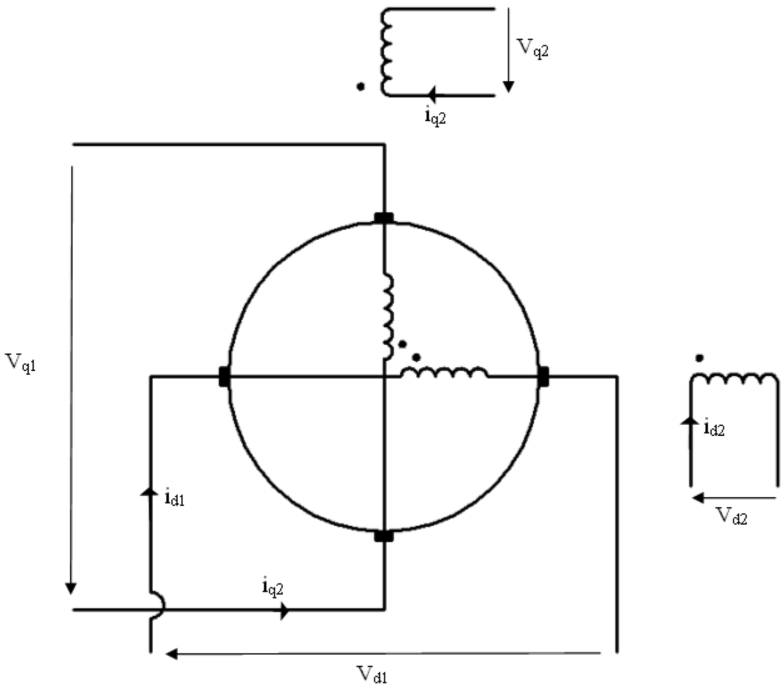
EEE 6120 Modelling of Electrical Machines
2006 Examination Solutions

1.

a)



The general form of the voltage matrix equations is given by:

$$\begin{vmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{vmatrix} = \begin{vmatrix} R_{d1}+L_{d1}p & G_{d1q2}\omega_r & M_{d1d2}p & G_{d1q2}\omega \\ G_{q1d1}\omega_r & R_{q1}+L_{q1}p & G_{q1d2}\omega_r & M_{q1q2}p \\ M_{d2d1}p & 0 & R_{d2}+L_{d2}p & 0 \\ 0 & M_{q2q1}p & 0 & R_{q2}+L_{q2}p \end{vmatrix} \begin{vmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{vmatrix}$$

(5)

b)

On DC:

$$Input\ current = \frac{Input\ power}{Input\ voltage} = \frac{600}{200} = 3A$$

Losses in the machine = $I^2R = 3^2 \times 10 = 90W$

$\therefore$ Output power = Input power – losses = 510W

$\therefore$ Efficiency = 510/600 = 85%

$$\therefore \text{Output torque} = \frac{Output\ power}{rotor angular velocity} = \frac{510}{16000 \times \frac{2\pi}{60}} = 0.304 Nm$$

$$But\ T = MI_{DC}^{\ 2}$$

$$\therefore M = \frac{T}{I^2} = \frac{0.304}{3^2} = 0.0337 H$$

On AC:

At the same load torque condition, the same rms current is drawn as the DC case. Hence, the magnitude of the input current is 3A rms

The voltage equation for AC operation is:

$$V = (R + NX_m + j\omega_s L)\ I$$

where N is the ratio of actual speed to synchronous speed

$$\therefore \left|\frac{V}{I}\right| = |Z| = \sqrt{(R + NX_m)^2 + (\omega_s L)^2}$$

Re-arranging this equation yields:

$$N = \frac{1}{X_m}\left(\sqrt{\left(\frac{V^2}{I^2}\right) - (\omega_s L)^2}\right) - \frac{R}{X_m}$$

For the particular parameters of this motor:

N = 5.0

$\therefore$ Actual speed on AC supply = 5.0 × 3000 = 15,000rpm

$$\text{Power factor on load} = \frac{\mathrm{R + NX_m}}{\sqrt{(\mathrm{R + NX_m})^2 + (\omega_s L)^2}} = 0.82\ \ lagging$$

[Note: It is important to state that the power factor is **lagging]**

For starting torque, $\omega_r = 0$

On DC: $I = \frac{V}{R} = \frac{200}{10} = 20 A$

$$\therefore T = MI^2 = 0.0337 \times 20^2 = 13.48\ \ Nm \text{ on DC}$$

On AC: $$I = \frac{V}{\sqrt{R^2 + (\omega_s L)^2}} = \frac{230}{45.1} = 5.10A$$

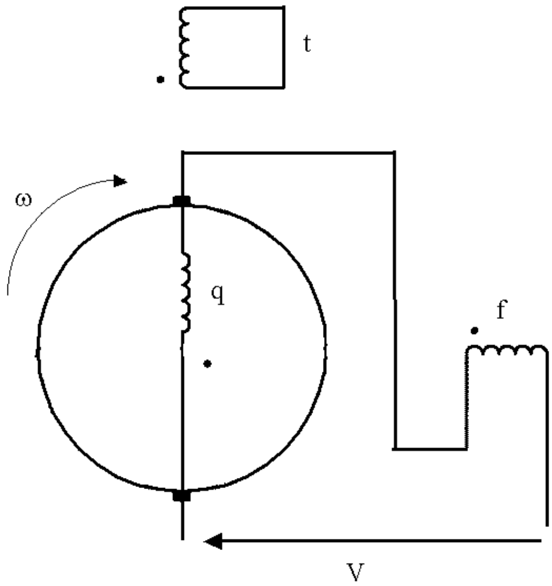
$\therefore T = MI^2 = 0.0337 \times 5.10^2 = 0.88\ Nm$ on AC

Ratio of DC starting torque to AC starting torque = 15.3

(12 – 3 for each of the 4 parts)

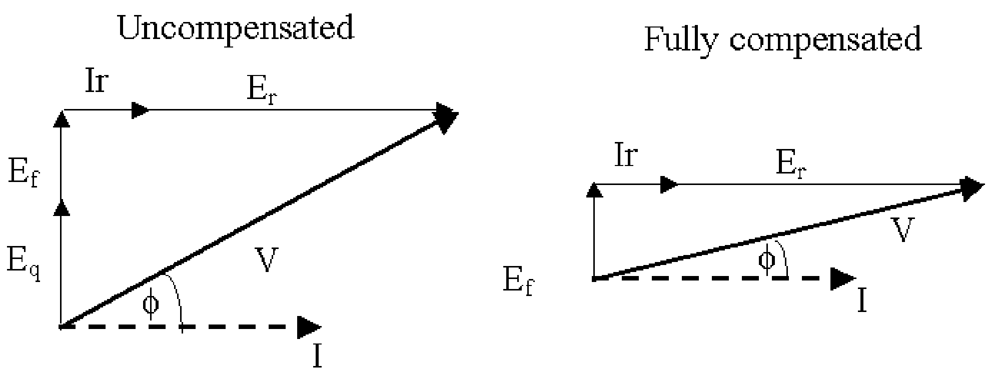
c)

The Kron primitive equivalent of an inductively compensated series universal motor is:



Full compensation is achieved when the q-axis coils have a coupling coefficient of 1 (this contrasts with a conductively coupled machine in which other constraints on the inductance of the compensation coil must be met).

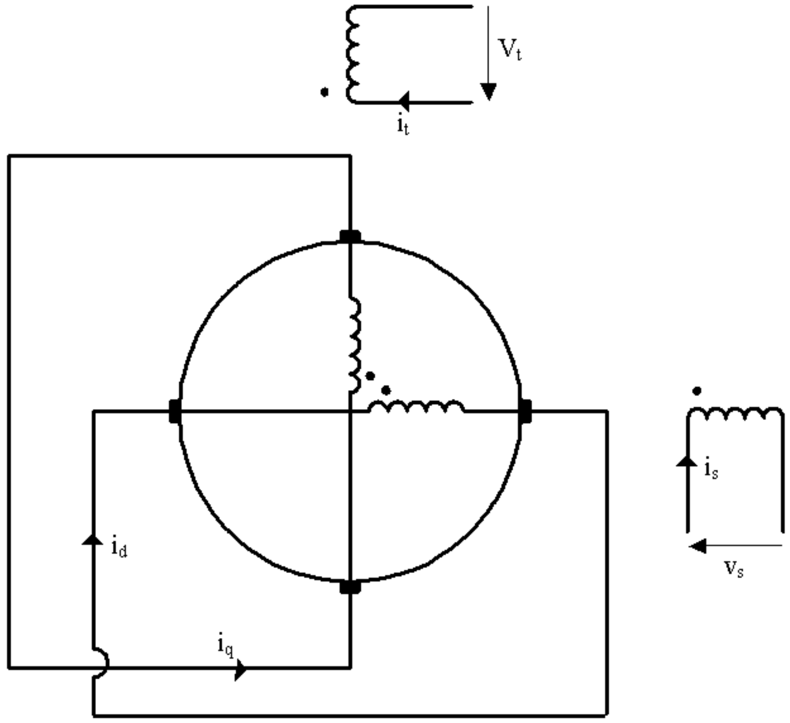
The resulting phasor diagrams are:



(3)

2.

a) The Kron primitive equivalent of a three-phase induction motor is given by:



Adopting subscripts of '1' for the stator and '2' for the rotor, then the general form of the voltage matrix equations is:

$$\begin{vmatrix} v_s \\ v_t \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_1+L_1p & 0 & M_{sd}\,p & 0 \\ 0 & R_1+L_1p & 0 & M_{td}\,p \\ M_{ds}p & -M_{dt}\omega_r & R_2+L_2p & -L_2\omega_r \\ M_{qs}\omega_r & M_{qt}\,p & L_2\omega_r & R_2+L_2p \end{vmatrix} \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix}$$

For steady-state operation for a sinusoidal AC supply:

$p=j\omega_s$ and $\omega_r=(1-s)\,\omega_s$

In addition, the same magnitude of applied to the two stator coils and the two rotor coils, but with a 90^o phase difference

$$\begin{vmatrix} V_s \\ V_t \\ V_d \\ V_q \end{vmatrix} = \begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix} = \begin{vmatrix} I_1 \\ j\,I_1 \\ I_2 \\ j\,I_2 \end{vmatrix}$$

The governing voltage equation is therefore:

$$\begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & 0 & jX_m & 0 \\ 0 & R_1+jX_1 & 0 & jX_m \\ jX_m & -(1-s)X_m & R_2+jX_2 & -(1-s)X_2 \\ (1-s)X_m & jX_m & -(1-s)X_2 & R_2+jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ jI_1 \\ I_2 \\ jI_2 \end{vmatrix}$$

But row 2 is simply row 1 $\times$j and row 4 is simply row 3 $\times$j. Hence the system can be reduced to two matrix equations:

$$\begin{array}{l} \text{Stator} \\ \text{Rotor} \end{array} \quad \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m - j(1-s)X_m & R_2 + jX_2 - j(1-s)X_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix}$$

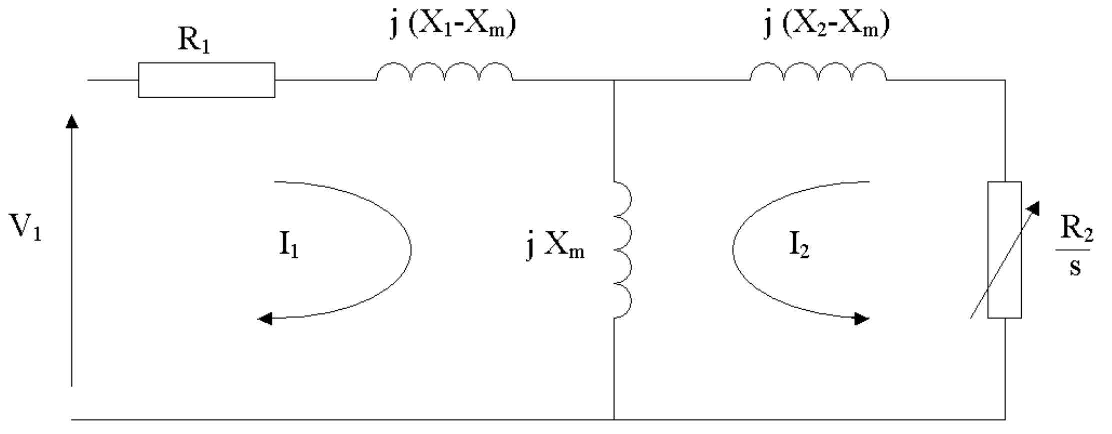
Since the rotor is short circuited, $V_2 = 0$

Substituting for V_2 and dividing the rotor equations by s gives:

$$\begin{array}{l} \text{Stator} \\ \text{Rotor} \end{array} \quad \begin{vmatrix} V_1 \\ 0 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m & R_2/s + jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2' \end{vmatrix}$$

[Note I_2 is transformed to I_2']

An equivalent circuit that satisfies these voltage equations is:



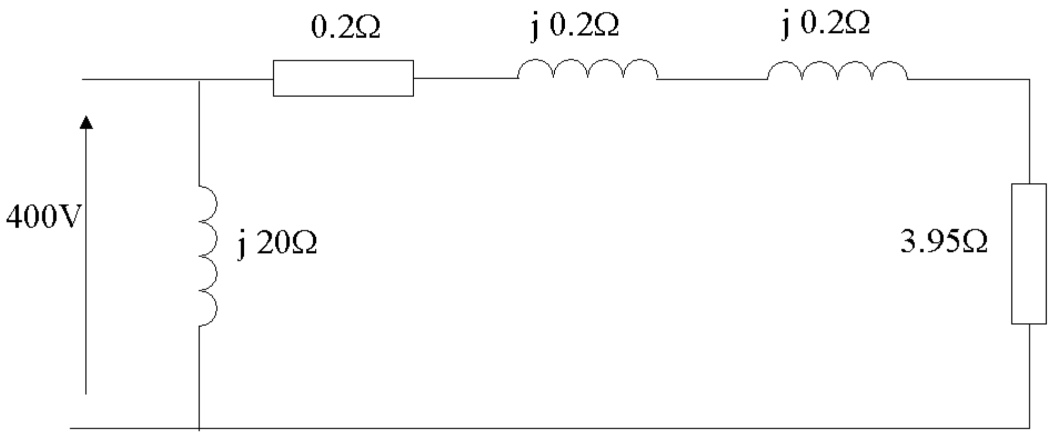
(12)

b)

[This problem can be solved using either the exact or simplified equivalent circuit. The latter involves moving the magnetising branch to the terminals, but is reliant on the magnetising reactance being significantly higher than the other impedances. Providing students recognise this assumption (preferably with some justification based on the values presented in the question) then the use of the simplified equivalent circuit is equally as valid in terms of the marks awarded]

i) The synchronous speed of a 4-pole motor on a 50Hz supply is 1500rpm. At 1462 rpm the slip is therefore 0.025

The approximate equivalent circuit of the machine is therefore given by:



The impedance of the main branch of the circuit is given by:

$$Z_e = (0.2 + 3.95) + j0.4 = 4.17\angle 5.5^o\,\Omega$$

The phase voltage applied to the machine is $\dfrac{400}{\sqrt{3}} = 231\ V$

$$I_1 = \frac{231}{4.17\angle 5.5^o} = 55.4\angle -5.5^o\,A$$

The current in the magnetising branch is given by:

$$I_m = \frac{231}{20\angle 90^o} = 11.5\angle -90^0$$

The net input current is hence given by:

$$I_{ip} = I_1 + I_m = (55.1 - j5.3) - j11.5 = 55.1 - j16.8 = \ 57.6\angle 17.0^o\,A$$

ii) Power factor = cos (17^o) = 0.956

iii) The electromagnetic output power is given by:

$$P_{out} = 3\ |I_1|^2\ \frac{(1-s)R_2^{'}}{s} = 3 \times 55.4^2 \times 3.95 = 35.45kW$$

iv) The copper losses are given by:

$$P_{cu} = 3|I_1|^2 (R_1 + R_2) = 2.76kW$$

v) The net mechanical power, P_{Load} is given by:

$P_{Load} = P_{out} - P_{mech\ loss}$

$\therefore\ P_{Load} = 35.45 - 0.75 = 34.70$ kW

The total input power = $3 \times 231 \times 57.6 \times \cos\ 17^o = 38.17kW$

Neglecting core losses [students are expected to list this assumption then the overall efficiency is given by:

$$Efficiency = \frac{34.70}{38.17} = 90.8\%$$

[As a check, or as an alternative method, the copper loss can also be used to calculate overall efficiency without recourse to the input power]

(8)

3.

a) The mechanical angle through which the rotor rotates between the un-aligned and aligned positions is given by

$$\Delta\theta = \frac{2\pi}{2 \times Number\ of\ rotor\ poles} = \frac{\pi}{6} \qquad \text{(} 45^{o} \text{ mechanical)}$$

Applying the trapezium rule to integrate the area under the fully aligned curve

$$A_{0\to1} = \frac{\Psi_1}{2} = 0.30 \text{ J}$$

$$A_{1\to2} = \frac{\Psi_1 + \Psi_2}{2} = 0.81\text{J}$$

$$A_{2\to3} = \frac{\Psi_2 + \Psi_3}{2} = 1.10\text{J}$$

$$A_{3\to4} = \frac{\Psi_3 + \Psi_4}{2} = 1.21\text{J}$$

$$A_{4\to5} = \frac{\Psi_4 + \Psi_5}{2} = 1.26\text{J}$$

$$A_{0\to5} = A_{0\to1} + A_{1\to2} + A_{2\to3} + A_{3\to4} + A_{4\to5} = 4.68\text{J}$$

The area under the un-aligned curve (which can reasonably regarded as being linear) is simply given by:

$$U_{0\to5} = \frac{5\Psi_5}{2} = 0.5 \text{ J}$$

Hence the change in co-energy is given by:

$$\Delta W' = A_{0\to 5} - U_{0\to 5} = 4.18\text{J}$$

The average torque is therefore given by:

$$T_{AVE} = \frac{\Delta W'}{\Delta \theta} = \frac{4.18}{\pi / 4} = 5.32 Nm$$

(5)

b) From the aligned Ψ-I characteristic it can be seen that the onset of saturation occurs at a flux-linkage of ≈ 1Wb and a current of 2A (an answer based on a slightly different interpretation of saturation is equally acceptable). It is important to note that the flux produced by the pair of coils that constitute a phase (which have a total of N_{ph} turns) crosses 2 diametrically opposite airgaps, each of length l_g.

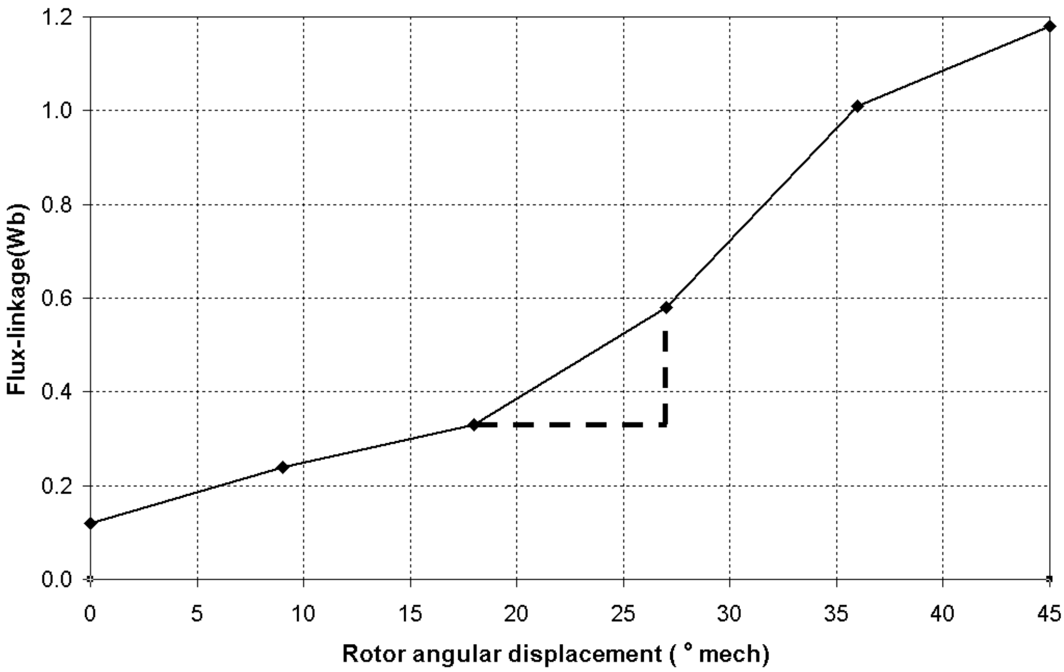
Since $B_g \approx \dfrac{\mu_0 N_{ph} I}{2 l_g}$ prior to saturation then a reasonable estimate of l_g can be obtained from this equation.

$$\therefore l_g = \frac{\mu_0 N_{ph} I}{2 B_g} = \frac{4\pi \times 10^{-7} \times 254 \times 2}{2 \times 1.6} = 0.2 \ mm$$

[a reasonable error band on this value is acceptable given the difficulty in precisely defining the onset of saturation – the method employed is the key factor in determining the marks awarded]

(4)

c) Taking the values of flux-linkage at 3A for the various and re-plotting a graph of flux-linkage versus position yields:



From the graph, the rate of change of flux-linkage with respect to rotor angular displacement around 15^{o} is given to a reasonable approximation by:

$$\left.\frac{d\Psi}{d\theta}\right|_{22.5} \approx \frac{\Psi_{27}-\Psi_{18}}{9\times\frac{\pi}{180}} \approx \frac{0.58-0.33}{9\times\frac{\pi}{180}} \approx 1.59\text{Wb / rad}$$

At 200 rpm, the rate of change of angular displacement is given by:

$$\frac{d\theta}{dt}=\frac{200\times 2\pi}{60}=20.9\text{ rad /s}$$

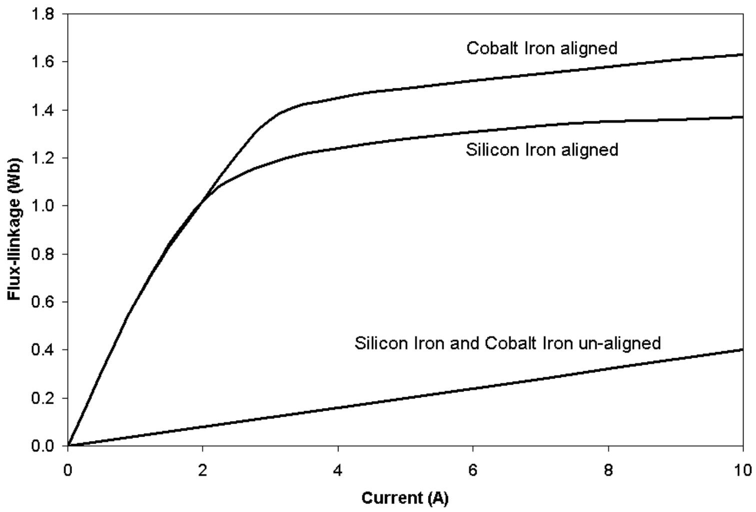
The instantaneous value of the induced emf is hence given by:

$$\frac{d\Psi}{dt}=\frac{d\Psi}{d\theta}\times\frac{d\theta}{dt}=31.8\text{V}$$

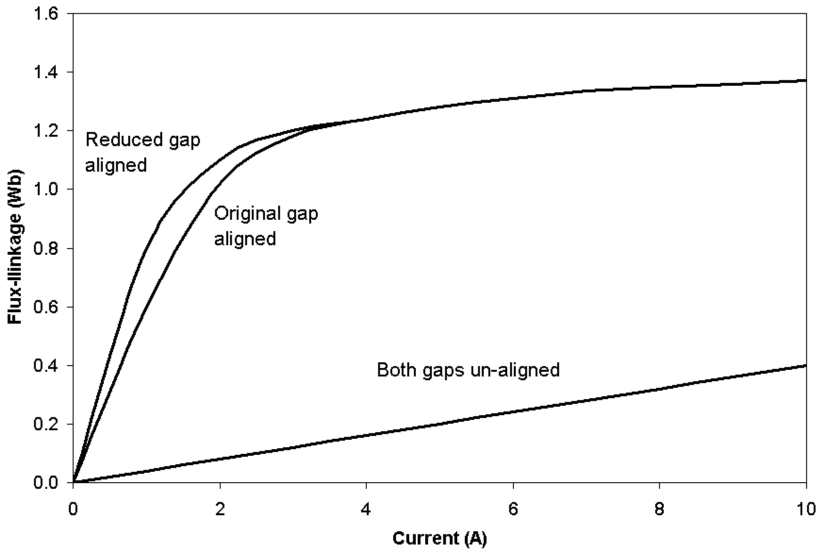
(6)

d) Two design changes that would enhance torque capability are:

i) Substitute the Silicon Iron for Cobalt Iron which has a higher saturation flux density (typically 2.35T as compared to 2.0T). Assuming that the materials have comparable permeabilities in the context of the fact that the net reluctance is dominated by the airgap even in the aligned position, then the two sets of Ψ-I characteristics will have the form:



ii) Reduce the length of the airgap. This will increase the slope of the intial linear part of the aligned characteristic, but willno discernable effect on the saturated regino of the aligned curve. Although a smaller airgap will increase the un-aligned characteristic marginally due to slightly increased leakage flux, it is reasonable to assume that the un-aligned characteristic will remain unchanged. the two sets of Ψ-I characteristics will have the form:



[Sketches which highlight the key features are adequate – there is no need to re-plot the graphs]

[Features such as improved cooling, higher packing factor coils etc although clearly advantageous for enhancing torque density are not the answers being sought in this case – largely since they have no direct bearing on the flux-linkage versus current characteristic but rather influence the degree to which a given level of current can be thermally sustained]

(5)

4.

[An important point to note from the outset of this question is that the machine has 4 rotor poles. This is stated in the explicitly in the text and should be apparent to candidates from the angular range of Figure 4]

a) The inductance can be readily calculated from the additional flux-linkage produced by the current. For the 4 cases :

At 25A:

$$L = \frac{0.05}{25} = 2mH \quad at\ -90^o$$

$$L = \frac{0.05}{25} = 2mH \quad at\ +90^o$$

At 100A:

$$L = \frac{0.20}{100} = 2mH \quad at\ -90^o$$

$$L = \frac{0.144}{25} = 1.44mH \quad at\ +90^o$$

The difference observed at 100A and +90^o is a result of magnetic saturation this is an angular displacement at which the magnet flux and the coil flux add to each other

(3)

b) The flux-linkage characteristics for 0A is a reasonable approximation to a sine-wave [in fact the actual data is generated from a simple sin function]. It is therefore reasonable to assume that the maximum rate of change of flux-linkage will occur at angular displacements around 0^o. From Figure 4, an estimate of the rate of change of flux linkage with rotor position can be made:

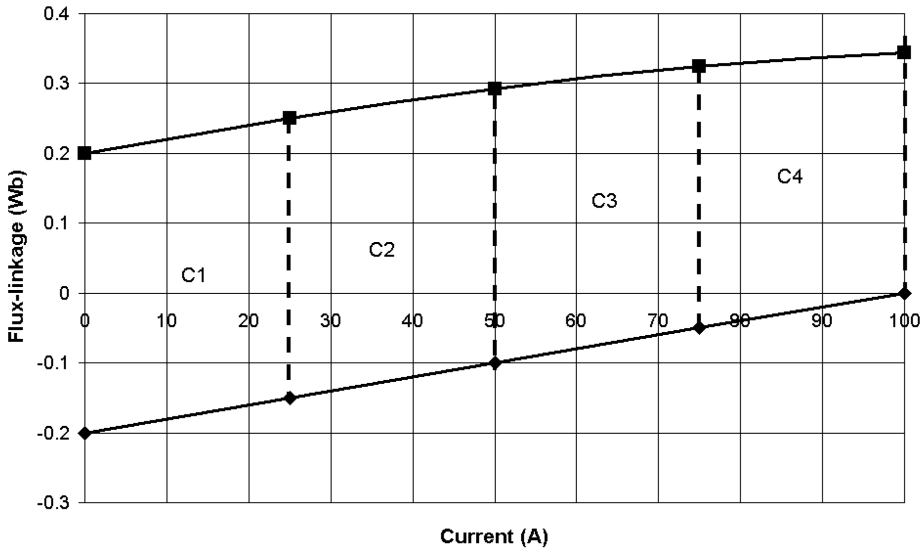
$$\frac{d\Psi}{d\theta} \approx \frac{\Delta\Psi}{\Delta\theta} = \frac{0.07}{10\times\frac{\pi}{180}} = 0.398\ Wb/rad$$ [These calculations have all been performed in terms of mechanical radians]

At 3000rpm

$$\frac{d\theta}{dt} = \frac{3000\times 2\times\pi}{60} = 314\ \ rad/s\ \ \therefore e = \frac{d\Psi}{dt} = \frac{d\Psi}{d\theta}\times\frac{d\theta}{dt} = 125\text{V}$$

(4)

c) In order to estimate the torque for the two currents specified it is necessary to re-plot the data as a flux-linkage versus current characteristic for -90^o and $+90^o$:



The co-energy change can be estimated by trapezoidal integration of the four areas C1 to C4 shown in the graph above. Using this approach:

The change in co-energy for 25A is C1 = 10J

The change in co-energy for 100A is C1+C2+C3+C4 = 10+ 9.9+9.6+9.0=38.5J

Change in rotor angular displacement = $90\times\frac{\pi}{180} = \frac{\pi}{2}$ rads

The torques produced are therefore given by:

At 25A: $T = \frac{dW'}{d\theta} \approx \frac{10\times 2}{\pi} = 6.36\ \ Nm$

At 100A: $T = \frac{dW'}{d\theta} \approx \frac{10\times 2}{\pi} = 24.5Nm$

[An important point here is that the torque per amp is gradually diminishing with onset of magnetic saturation]

(6)

d) There are 6 stator teeth and 3 phases. Hence, each phase consists of two diametrically opposite coils. Assuming that the airgap flux density provides a reasonable estimate of the flux density level in the core as a whole then:

By inspection of the flux-linkage characteristics, the core of the machine appears to be saturating at a flux-linkage of ~0.3Wb, which corresponds to a flux density of ~1.6T.

Considering the case of a rotor angular displacement of 0º (in which no net magnet flux is present). A current of 25A produces a flux-linkage of 0.05Wb, which by equivalence with 1.6T at 0.3Wb, corresponds to a flux density of 0.27T

At this flux density level, then it is reasonable to assume that the rotor and stator cores will be infinitely permeable.

$\therefore$ The total effective magnetic airgap to the coil flux of one phase is:

$$l_{eff} = 2\left(l_g + l_m\right) = 14mm$$

The airgap flux density produced by a given coil mmf is given by:

$$B_g = \frac{\mu_0 NI}{l_{eff}}$$

Re-arranging this equation yields:

$$N = \frac{B_g l_{eff}}{\mu_0 I} = \frac{0.267 \times 14 \times 10^{-3}}{4\pi \times 10^{-7} \times 25} = 120 \text{ turns per phase (i.e. 60 per coil)}$$

[As will all questions in this paper which involve graphical estimates, there is some scope for deviation in the exact answer providing the method is correct]

[It is also important to note assumptions – without these full marks will not be awarded]

(7)

GW Jewell

April 2006