EEE224/227 2014-15 exam solutions

Q1

Part a

Causality is a system where the output depends on present or previous inputs 1 mark

A system where the output ONLY depends on the present inputs is memoryless. 1 mark

. **(0.5 marks for memory correct, 0.5 marks for causal correct)**

i) No memory, causal

ii) Memory, non causal

iii) Memory, causal

Part b.

$$y(t) = t^3 * t * t$$

$$y(t) = \left(t^3 * t\right) * t$$

$$\left(t^3 * t\right) = \int_0^t \tau^3 (t-\tau) d\tau$$ 1 mark

$$\left(t^3 * t\right) = t\int_0^t \tau^3 d\tau - \int_0^t \tau^4 d\tau$$

$$\left(t^3 * t\right) = \frac{t^5}{20}$$ 1 mark

$$y(t) = \frac{t^5}{20} * t = \frac{1}{20}\int_0^t \tau^5 (t-\tau) d\tau = \frac{1}{20}\left[t\int_0^t \tau^5 d\tau - \int_0^t \tau^6 d\tau\right]$$ 1 mark

$$y(t) = \frac{t^7}{840}$$ 1 mark

Part c

$$a_0 = \frac{1}{T}\int_{-0.1T}^{0.9T} f(t)dt = \frac{1}{T}\left[\int_{-0.1T}^{0.1T} Adt - \int_{0.1T}^{0.9T} Adt\right]$$ 1 Mark

$$a_0 = \frac{A}{T}\left[0.2T - 0.8T\right] = -0.6A$$

The function is even hence bn=0. 1 mark

$$a_n = 2 * \frac{2}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)dt$$

$$a_n = \frac{4A}{T}\left[\int_0^{0.1T}\cos(n\omega_0 t)dt - \int_{0.1T}^{0.5T}\cos(n\omega_0 t)dt\right]$$

2 marks

$$a_n = \frac{4A}{n\omega_0 T}\left[\sin(0.2n\pi) - (\sin(0.5n\pi) - \sin(0.2n\pi))\right]$$

$$a_n = \frac{4A}{n\pi}\sin(0.2n\pi)$$

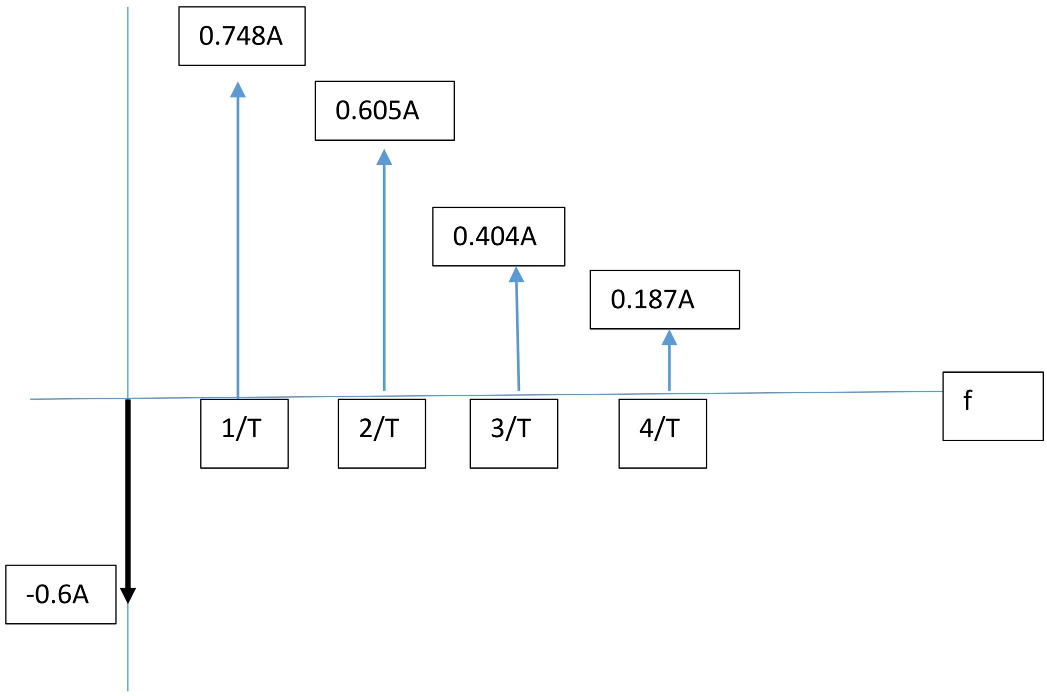
2 marks

Hence,

$$f(t) = -0.6A + \frac{4A}{\pi}\sum_{n=1}^{\infty}\frac{\sin(0.2n\pi)}{n}\cos(n\omega_0 t)$$

1 mark

0.5 marks for correct amplitudes, 0.5 marks for correct frequency of each component (no marks for DC component, just shown for completeness)



Q2

a **1 mark each**

Allows for multiplexing

Efficient antenna size

Improves S/N

Allows users to have different carrier frequencies

b

$$C = B\log_2\left(1 + S/N\right)$$
$$C = 20\log_2(1 + 1000)$$ 3 marks
$$C = 199 Mbit/s$$

$$C = \frac{B}{2}\log_2\left(1 + S/2N\right)$$
$$C = 40\log_2(1 + 500)$$ 3 marks
$$C = 359 Mbit/s$$

c

i) sampling frequency =2*10=20kHz (1 mark)
ii) 8=2^N, hence N=3 (2 marks)
iii) Data rate = 3*20= 60 bit/s (2 marks)

At t=0 the voltage is 5V. Quantisation level 6 : PCM =101 (1 mark)

At t=50 the voltage is 4.3V. Quantisation level 5: PCM=100 (1 mark)

At t=100 the voltage is 3V. Quantisation level 4: PCM= 011 (1 mark)

At t=150 the voltage is 2.6V. Quantisation level 4: PCM=011 (1 mark)

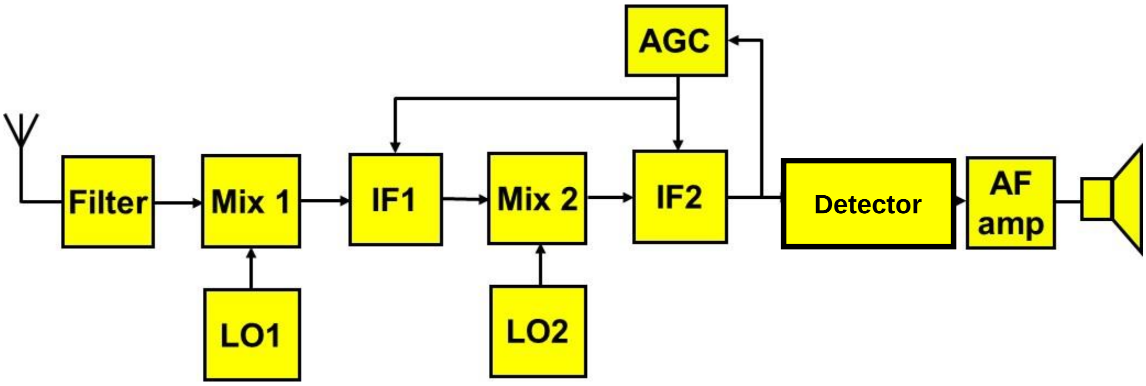
At t=200 the voltage is 1.8V. Quantisation level 3: PCM 010 (1 mark)

Hence total PCM code is

101 100 011 011 010

Q3

a



1 mark is awarded for each of the elements below, 1 mark for correct diagram

- A bandpass filter reduces the effect of image frequencies
- Mixer down covers f_{RF} to the 1st IF
- IF amplifier to required levels
- 2nd mixer coverts to final IF
- AGC adjusts the gain to keep the carrier levels constant
- Detector demodulates the signal to receive the baseband information

b

Firstly the parameters we need are below.

$$F_{RF} = 420MHz$$
$$F_{IF1} = 60MHz$$
$$F_{IF2} = 0.455MHz$$
$$F_{LO1} > F_{RF}$$
$$F_{LO2} < F_{IF1}$$

i) $F_{LO1} - 420 = 60$
$F_{LO1} = 480MHz$ 1 mark

ii) $60 - F_{LO2} = 0.455$
$F_{LO2} = 59.545MHz$ 1 mark

iii) $F_{image1} = 480 + 60 = 540MHz$ 1 mark

iv) $F_{image1} = 59.545 - 0.455 = 59.09MHz$ 1 mark

$$F_1 \to 480 - F_1 = 59.09$$
$$F_1 = 420.91$$

v) 2 marks

$$F_2 \to F_2 - 480 = 59.09$$
$$F_2 = 539.09MHz$$

Part C

i) 1 mark

$$F_{RF} - 100 = 0.455$$
$$F_{RF} = 100.455MHz$$

$$100 - F_{image} = 0.455$$
$$F_{image} = 99.545MHz$$

ii) $IFRR = \sqrt{1+Q^2x^2}$ 3 marks

$$x = \frac{F_{image}}{F_{RF}} - \frac{F_{RF}}{F_{image}}$$
$$x = \frac{99.545}{100.455} - \frac{100.455}{99.545} = -0.0182$$
$$IFRR = 1.235 = 1.8dB$$

part D

Could improve by increasing IF or Q factor, practical issues would suggest IF is easier.

Question 4

a. (10 marks)

$$\frac{v_o}{v_i} = \frac{\dfrac{\dfrac{R}{j\,\omega C_2}}{R+\dfrac{1}{j\,\omega C_2}}}{\dfrac{1}{j\,\omega C_1}+\dfrac{\dfrac{R}{j\,\omega C_2}}{R+\dfrac{1}{j\,\omega C_2}}}$$

(2 marks)

$$= \frac{\dfrac{R}{1+j\,\omega C_2 R}}{\dfrac{1}{j\,\omega C_1}+\dfrac{R}{1+j\,\omega C_2 R}} = \frac{j\,\omega C_1 R}{1+j\,\omega C_2 R+j\,\omega C_1 R}$$

(2 marks)

$$= \frac{j\,\omega C_1 R}{1+j\,\omega (C_1+C_2)R} = \frac{C_1}{C_1+C_2}\,\frac{j\,\omega (C_1+C_2)R}{1+j\,\omega (C_1+C_2)R} = k\,\frac{j\dfrac{\omega}{\omega_0}}{1+j\dfrac{\omega}{\omega_0}}$$

(2 marks)

where $\omega_0 = 1/R\,(C_1+C_2)$ and $k = C_1/(C_1+C_2)$. (2 marks)

This is a high pass circuit. (2 marks)

b. (10 marks)

$$\frac{v_o}{v_i} = \frac{R_1+\dfrac{R_2\left(R_3+\dfrac{1}{sC}\right)}{R_2+R_3+\dfrac{1}{sC}}}{R_1}$$

$$= \frac{R_1R_2+R_1R_3+\dfrac{R_1}{sC}+R_2R_3+\dfrac{R_2}{sC}}{R_1R_2+R_1R_3+\dfrac{R_1}{sC}}$$

$$= \frac{R_1+R_2+sC\,(R_1R_2+R_1R_3+R_2R_3)}{R_1(sC\,(R_2+R_3)+1)}$$

(2 marks)

$$= \frac{R_1+R_2}{R_1}\,\frac{1+sC\left(\dfrac{R_1R_2+R_1R_3+R_2R_3}{R_1+R_2}\right)}{1+sC\,(R_2+R_3)}$$

(2 marks)

$$\equiv k_L\,\frac{1+j\dfrac{\omega}{\omega_1}}{1+j\dfrac{\omega}{\omega_0}}$$

$$\omega_0 = \text{pole frequency} = \frac{1}{C\,(R_2+R_3)} = 2\,\pi\,10$$

$$\omega_1 = \text{zero frequency} = \frac{R_1+R_2}{C\,(R_1R_2+R_1R_3+R_2R_3)} = 2\,\pi\,500$$

$$k_H = \text{high frequency gain} = \frac{R_1R_2+R_1R_3+R_2R_3}{R_1\,(R_2+R_3)} = 10$$

(3 marks)

Solving the above three equations, we obtain

$R_2 = 4.99\text{M}\Omega.$

$R_3 = 91.6\text{k}\Omega$

$C = 3.13\text{nF}$

(3 marks)

Question 5

a. (8 marks)

For a unit step input, the output is given by

$Y(s) = H(s)U(s) = \frac{s/3}{(s+1)(s+2)}\frac{1}{s} = \frac{1/3}{(s+1)(s+2)} = \frac{k_1}{s+1} + \frac{k_2}{s+2}$ (2 marks)

Using partial fraction expansion, we have

$k_1 = \frac{1/3}{(s+1)(s+2)}(s+1)\Big|_{s=-1} = \frac{1/3}{s+2}\Big|_{s=-1} = \frac{1}{3}$

$k_2 = \frac{1/3}{(s+1)(s+2)}(s+2)\Big|_{s=-2} = \frac{1/3}{s+1}\Big|_{s=-2} = -\frac{1}{3}$ (2 marks)

Alternatively,

$$H(s) = \frac{1/3}{(s+1)(s+2)} = \frac{k_1(s+2)+k_2(s+1)}{(s+1)(s+2)}$$

$$\frac{1}{3} = (k_1+k_2)s + 2k_1 + k_2$$

$k_1 + k_2 = 0$

$2k_1 + k_2 = 1/3$

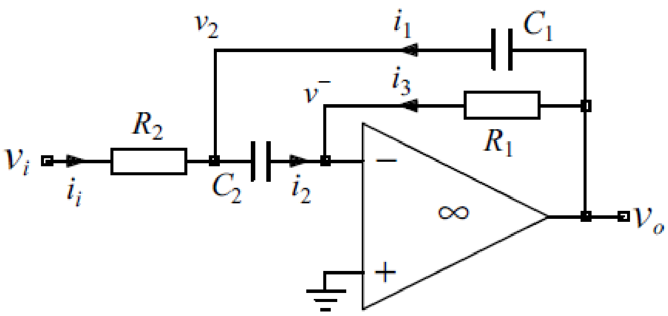
$k_1 = 1/3$, and $k_2 = -1/3$

$Y(s) = \frac{1/3}{s+1} - \frac{1/3}{s+2}$ (2 marks)

Therefore, the unit step response in time domain is given by

$y(t) = \frac{1}{3}e^{-t}u(t) - \frac{1}{3}e^{-2t}u(t).$ (2 marks)

b. (12 marks)



i)

Begin by summing currents at the v_2 node:

$i_i + i_1 = i_2$ or,

$$\frac{v_i - v_2}{R_2} + (v_o - v_2)sC_1 = v_2 sC_2$$ (2 marks)

$v^{-} = 0$ (1 mark)

$$i_2 + i_3 = 0 = v_2 s C_2 + \frac{v_o}{R_1}$$

(1 marks)

v_2 can be eliminated from these two equations to give,

$$v_2 = \frac{v_i + v_o\, s\, C_1 R_2}{1 + s\,(C_1 + C_2) R_2} = -\frac{v_o}{s\, C_2 R_1} \text{ which can be developed as follows:}$$

$$v_i\, s\, C_2 R_1 + v_o\, s^2\, C_1 R_2 C_2 R_1 = -\, v_o\,(1 + s\,(C_1 + C_2) R_2\,)$$

$$v_i\, s\, C_2 R_1 = -\, v_o\,(1 + s\,(C_1 + C_2) R_2 + s^2 C_1 R_2 C_2 R_1\,)$$

(1 mark)

$$\frac{v_o}{v_i} = \frac{-\,s\, C_2 R_1}{1 + s\,(C_1 + C_2) R_2 + s^2 C_1 R_2 C_2 R_1} = \frac{C_2 R_1}{(C_1 + C_2) R_2}\,\frac{-\,s\,(C_1 + C_2) R_2}{1 + s\,(C_1 + C_2) R_2 + s^2 C_1 R_2 C_2 R_1}$$

(1 marks)

The response is a band-pass response with a standard form

$$\frac{v_o}{v_i} = k\,\frac{\dfrac{s}{\omega_0 q}}{1 + \dfrac{s}{\omega_0 q} + \dfrac{s^2}{\omega_0^2}}.$$

(2 marks)

ii)

If $C_1 = C_2 = C$,

$$\omega_0^2 = \frac{1}{C^2 R_1 R_2},\ \frac{1}{\omega_0 q} = 2CR_2 \text{ and therefore } q = \frac{1}{2}\sqrt{\frac{R_1}{R_2}}$$

For a q of 3, $R_1/R_2 = 36$

(4 marks)

Question 6

a. (3 marks)

Since the cable is very long, we can assume that there is not reflection. (1 marks)

For a unidirectional wave with no reflection, $Z_0 = V/I$ at all points along the transmission line, hence the probe adds a load of 46Ω to the circuit. (2 marks)

b. (9 marks)

The reflection coefficients at the load and the source are both 1/3. (2 marks)

The voltage of the first forward wave is 144×50/150 = 48V. (1 marks)

Each reflection will produce a wave with amplitude reduced by 1/3.

t=1us Vf1=48V Vtotal=48+48/3=64V

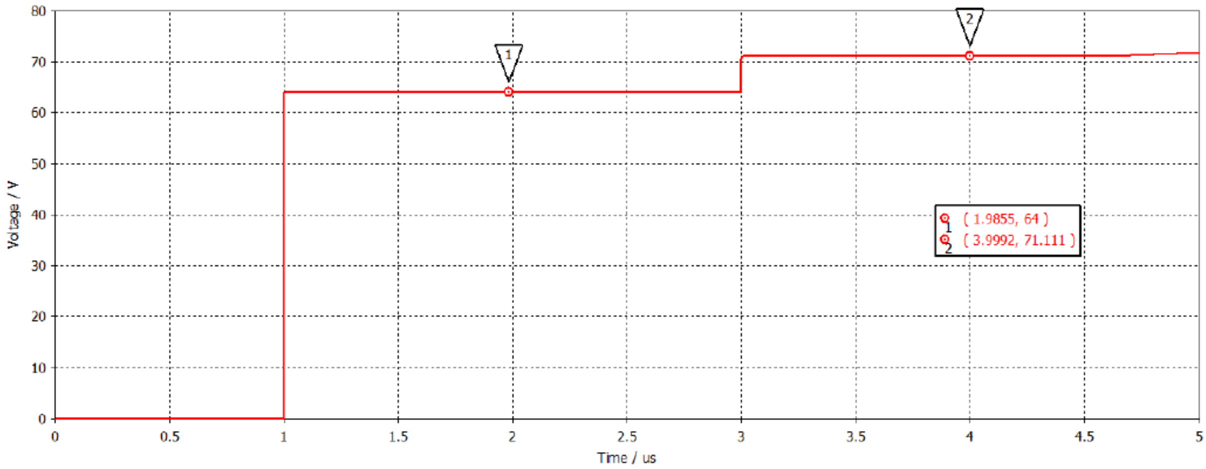
Vb1=48/3 (2 marks)

t=3us Vf1=48V Vtotal =48+48/3+48/9+48/27=71.1V

Vb1=48/3

Vf2=Vb1/3=48/9

Vb2=Vf2/3=48/27 (2 marks)



(2 mark)

c. (8 marks)

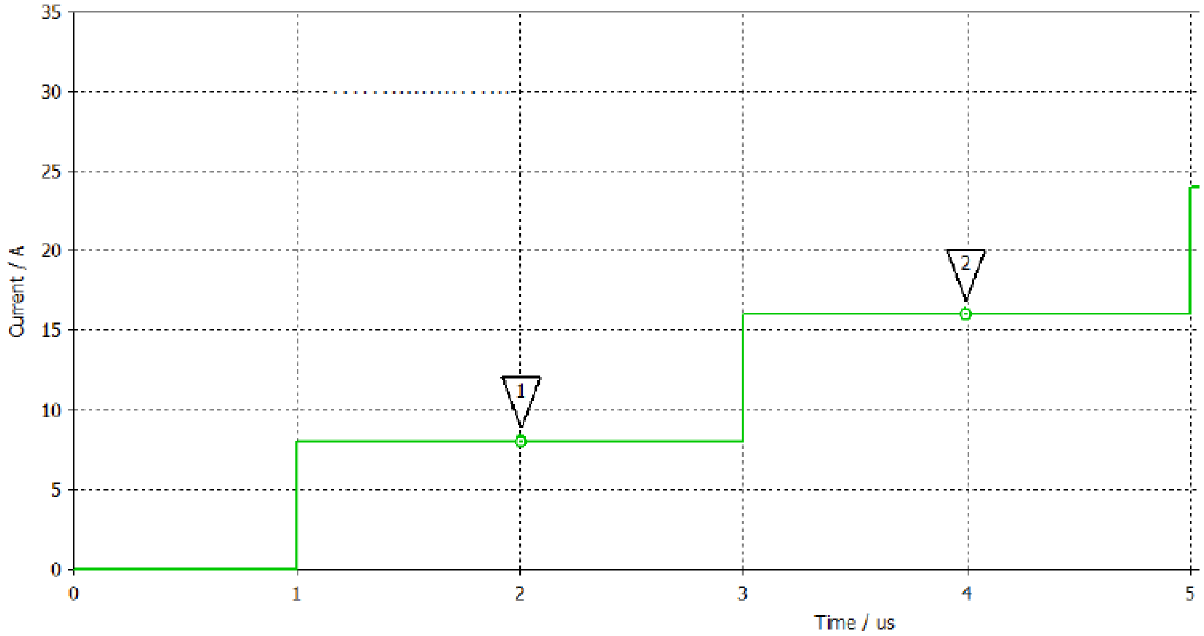
The source and load reflection coefficients are both −1. (2 marks)

Just after the switch is closed, the current is $I = V/Z$ = 100/25 = 4A. (1 marks)

After 1μs, the forward wave reflects and the current in the short circuit is

$$I = I_f - I_r = 4 - -4 = 8\text{A}. \qquad \text{(2 marks)}$$

Every 2μs after this an additional 8A is added to the current in the short circuit. (1 marks)



(2 marks)