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The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2015-16 (2.0 hours)

EEE309 Introduction to Digital System Processing

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. i) In the context of a discrete-time system, explain the concepts of causality, stability, linearity and time invariance. (5 marks)

ii) Determine whether the following system is (a) causal, (b) stable, (c) linear time-invariant: (3 marks)

$$y[n] = \sum_{k=-1}^{6} x[n-k]$$

(8)

b. The following is a linear time-invariant (LTI) system (Figure 1) with an input x[n] and an output y[n]. It consists of three sub-systems with impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$, respectively.

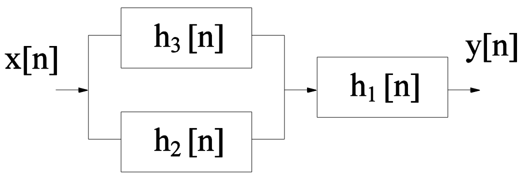

Figure 1

Suppose their impulse responses are given by

$$h_1[n] = h_2[n] = \begin{cases} 1 & n = 0,1 \\ 0 & otherwise \end{cases} \quad \text{and} \quad h_3[n] = \begin{cases} 1 & n = 1,2 \\ 0 & otherwise \end{cases}$$

i) Calculate the impulse response of the whole LTI system. (6 marks)

ii) State the gain of the system for $\Omega = 0$ and $\Omega=\pi$. (2 marks) **(8)**

c. State the Nyquist sampling theorem and determine the minimum sampling frequency required for sampling the following continuous-time signal x(t)

$$x(t)=\cos(30\pi t)+ \cos(50\pi t)+\cos(100\pi t)$$

(4)

2. **a.** i) Give the expressions for the unit sample sequence and the unit step sequence. (2 marks)

ii) We can express the unit step sequence in terms of the unit sample sequence in two different ways. Give these two expressions. (2 marks)

(4)

b. For a particular linear discrete-time filtering system, its output y[n] for each time index n is given by the average of its inputs at n and $n-1$.

i) Obtain the linear constant coefficient difference equation describing the behaviour of the filter. (2 marks)
ii) Determine the z-transform H(z) for this system and sketch the associated pole-zero plot. (4 marks)
iii) Is this system a minimum phase system? Explain your answer. (3 marks)

(9)

c. i) As far as possible, derive the transfer function for an IIR filter which has the z-plane pole-zero plot shown in the following (Figure 2), where there are 2 poles and 2 zeros (3 marks).

ii) Sketch the frequency response of the filter (no details needed) — Does it possess a lowpass, highpass, bandpass or bandstop characteristic (4 marks)?

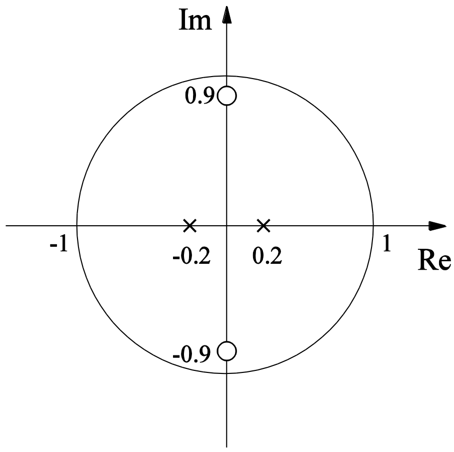

Figure 2

(7)

3. **a.** The impulse response h[n] of an LTI discrete-time system is given by

$$h[n]= \delta[n]+3\delta[n-1]-\delta[n-2].$$

Use z-transforms to calculate the output y[n] of the system given the input signal

$$x[n]=\delta[n]+3\delta[n-1]-\delta[n-2]+3\delta[n-3].$$

(5)

b. Give the expressions for the Discrete Fourier Transform (DFT) and Inverse DFT, and calculate the DFT of the discrete series x[n]={0.5, 1, 1, 0.5}.

(6)

c. Consider a sequence $x_1[n]$ whose length is L (nonzero for n=0, 1, …, L-1) and a sequence $x_2[n]$ whose length is P (nonzero for n=0, 1, …, P-1). A linear convolution of these two sequences will generate a third sequence $x_3[n]$. Describe the process involved in calculating this linear convolution using DFT.

(5)

d. A lowpass digital filter is to be designed and the first order lowpass filter given in the following equation is used as a prototype, where ω_b is the filter cutoff frequency.

$$H(s)=\frac{\omega_b}{s+\omega_b}$$

Design the digital filter using the Impulse Invariance method if ω_b=5rad/sec and the filter is implemented at a sampling frequency of 8Hz. (4 marks)

(4)

4. **a.** A sequence is said to be the eigenfunction of a linear time invariant (LTI) system, when given such a sequence at its input, its output is a simple scaled version of the same sequence. Determine whether the sequence $x[n]=\alpha^n$ (α is a nonzero constant) is the eigenfunction of an LTI system. Explain your answer.

(4)

b. Consider the system function

$$H(z)=\frac{1+2z^{-1}}{1-1.5z^{-1}+0.9z^{-2}}$$

Give its direct form I and direct form II implementation structures.

(4)

c. Given the spectral coefficients of a filter, *H(k)*, which are symmetrical about *k*=0, the original impulse response *h*[n] can be reconstituted using the following equation, where *N* is the total number of coefficients:

$$h[n]=\frac{1}{N}\sum_{k=-(N-1)/2}^{(N-1)/2}H(k)e^{j2\pi nk/N}=\frac{1}{N}\left(H(0)+2\sum_{k=1}^{(N-1)/2}H(k)\cos(2\pi nk/N)\right)$$

From this you are going to design a **highpass** FIR filter with $N=5$ coefficients with a passband range between *0.5*kHz and 1kHz at a sampling frequency $f_s=2$kHz.

Use the frequency sampling method to calculate the FIR filter coefficients (6 marks).

(6)

d. Suppose $X_1(z)$ is the z-transform of the sequence $x_1[n]$ and $X_2(z)$ is the z-transform of the sequence $x_2[n]$. Then we have the following property:

$$x_1[n]*x_2[n]\xleftrightarrow{z-transform}X_1(z)X_2(z)$$

where * denote the convolution operation. Derive the above result.

(6)

WL/JROD