EEE105
“Electronic Devices”

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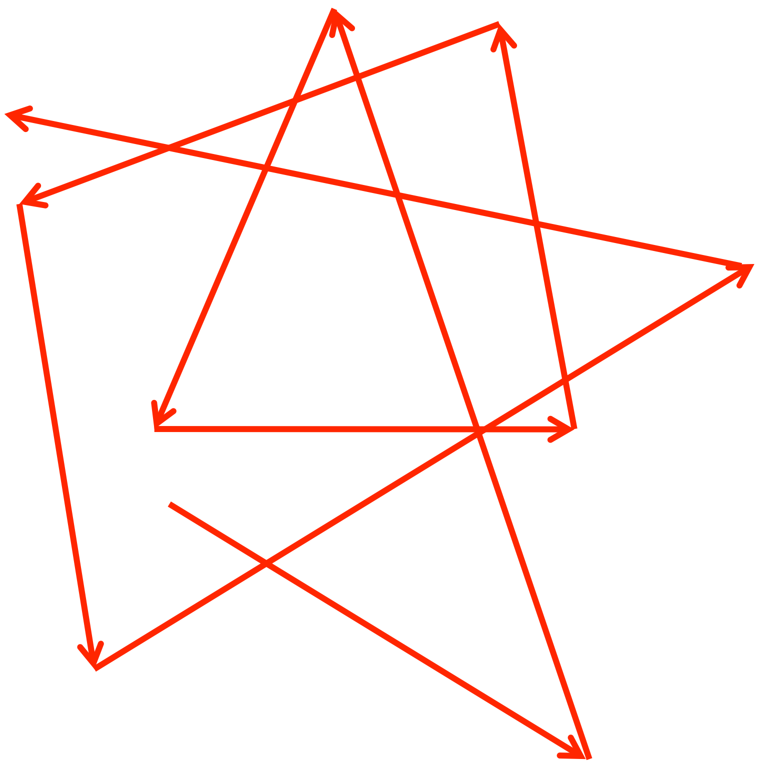
Lecture 4

- Causes of Current
- Effective Mass of Electron (m*)
- Derivation of Drift Velocity, (V_d)
- Mobility, (μ)

Free Electrons in Solids

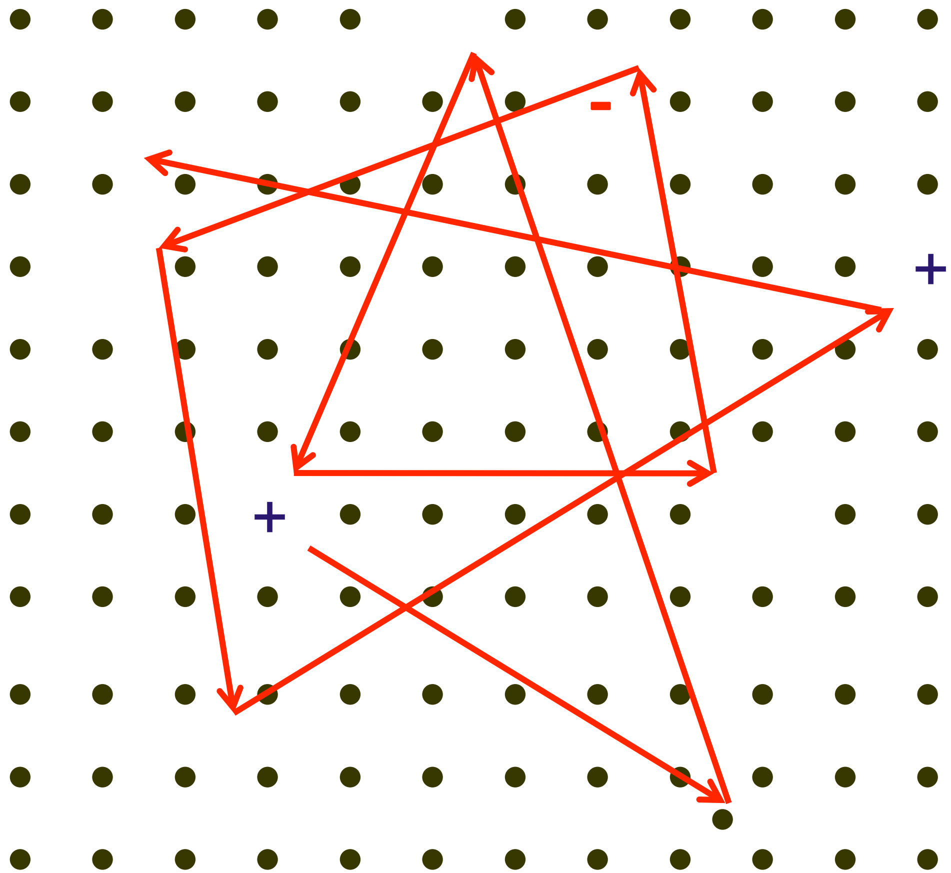


Imagine we can visualise the motion of electrons...

Free electrons have thermal energy and move rapidly around the crystal in a random fashion – from one collision to another
– c.f. Brownian motion

When we observe enough carriers we will see there is no net movement of charge – no net current

What are the collisions?



The collisions are not from the crystal lattice – the allowed states of the electrons are governed by the lattice

Collisions are from imperfections to the crystal lattice;

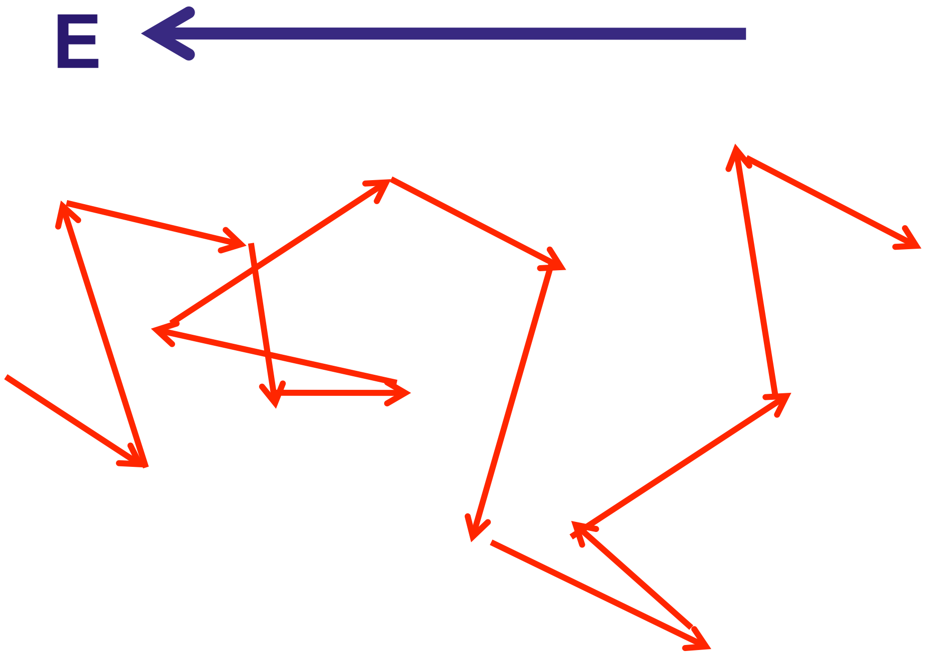
•Ionised impurities
•Interstitial defects
•Vacancy defects
•Phonons

What causes a current?

Three causes of net flow of current

- An electric potential gradient dV/dx (i.e. an E-field)
- An electron density gradient dn/dx
- A temperature gradient dT/dx

Applying an E-field



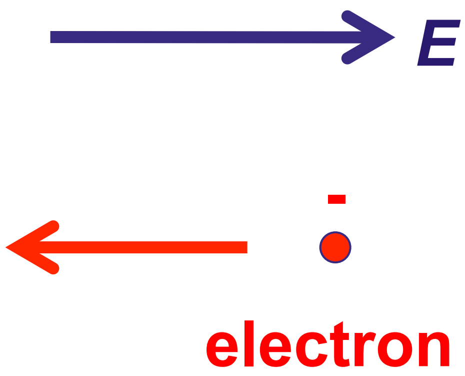
n.b. we have "zoomed out" as our electron is now travelling somehwere!

Scattering still occurs – when we observe many electrons we see there is net motion of charge with a *drift velocity*

Intuitively we can imagine that a semiconductor with a perfect crystal structure may be ideal in making fast efficient devices ....more on this later

Application of E-field

Force = - e E



Electron acceleration?
-In vacuum we use Newton's laws F = mA
Semi-classical laws of motion

Effective Mass in a Crystal

- Complicated in a crystal – non-classical - quantum mechanics, complicated periodic electrostatic potentials
- Much more on this topic later ...(e.g. EEE207)
- Empirically (i.e. various experiments) we know we can assume an “effective mass” for an ensemble of electrons
- Many texts to study this – “willing suspension of disbelief” is minimum required here.......

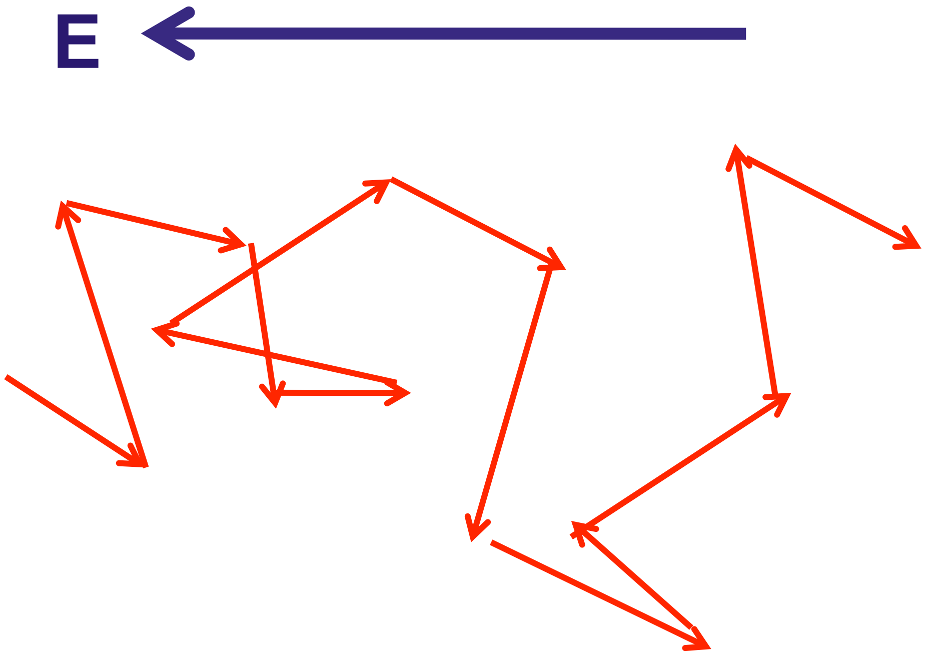
Typical Effective Masses

- Electron effective mass, m* (all multiplied by m_e)

Si	Ge	GaAs	InP	InSb
0.97	0.22	0.07	0.08	0.013

- Effective mass in a semiconductor can be ***lower*** than in vacuum ! -Due to the internal periodic potentials
- Ultimate speed of components and ICs governed by this parameter
- High frequency electronics – need lowest possible m*

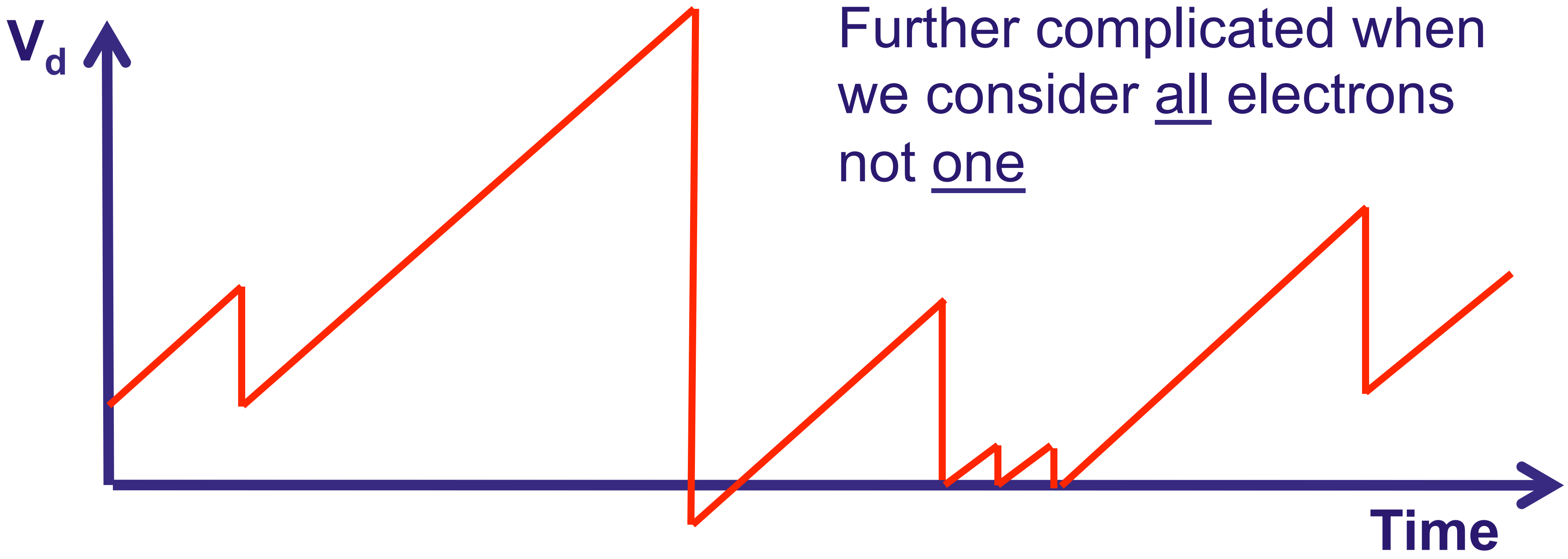
Drift Velocity



- Discussed current in a solid in Lecture 2–
- Electron scatters from imperfections in crystal lattice
- Time between scattering events impacts *drift velocity* of electron
- Need to think about V_d for one/all electrons

Conduction Electron in E-field

- Electron gains velocity as it is accelerated in E-field
- Electron can loose velocity when they are scattered



Further complicated when we consider <u>all</u> electrons not <u>one</u>

Derivation –Terms of Reference

- Statistical mechanic approach – consider ensemble of electrons (c.f. Ideal gas)
- We will calculate the acceleration of electrons and from this the rate of increase in momentum of the electrons
- Consider an average time between scattering events and that these events result in the electron having zero momentum after scattering
- Calculate a rate of change of momentum for the scattering events
- As we have equilibrium these two rates of change of momentum are equal and opposite
- We equate them, re-arrange and can determine an average drift velocity

Derivation (1)

- Solid with free electron concentration, n m^{-3}
- E-field applied, electrons will accelerate $a = \frac{-qE}{m^*}$
- Acceleration so velocity and momentum are changing. Rate of change of momentum with time for each electron

$$\frac{dp_e}{dt} = \frac{d\left(m^* v_d\right)}{dt} = m^* \frac{dv_d}{dt} = m^* a = F = -qE$$

- Rate of change of momentum, p, of all n electrons

$$\left(\frac{dp}{dt}\right)_{drift} = -qEn$$

Derivation (2)

- Electrons will accelerate until scattered
- Chance (or probability) that a particular electron will be scattered in unit time is a number between 0 and 1. For a large number of electrons this fraction <u>will be </u>scattered.

$$\text{Number scattered per unit time} = \frac{n}{\tau}$$

- Where τ is a time constant (= average time between scattering events)
- Assume that on average each scattering event causes the electron to loose all momentum (momentum at this instant = m^*v_d)
- Total change in momentum due to scattering events is;

$$\left(\frac{dp}{dt}\right)_{\text{scatter}} = \frac{\text{number scattering events}}{\text{time}} \times \text{momentum change}$$

The University Of Sheffield.

Derivation (3)

- Where v_d is the average drift velocity of the electron population
- At equilibrium the total momentum change of the population is zero

$$\left(\frac{dp}{dt}\right)_{drift} + \left(\frac{dp}{dt}\right)_{scatter} = 0$$

- Hence

$$-qEn - \frac{n}{\tau} m^* \langle v_d \rangle = 0 \qquad \text{and} \qquad \langle v_d \rangle = -\frac{qE\tau}{m^*}$$

- n.b. Some text books derivations assume regular scattering events and obtain a similar approximate result

Mobility, μ

- Drift velocity given by $\langle v_d \rangle = -\dfrac{q\tau E}{m^*}$
- Important parameter is average time between scattering events, τ
- Governed by impurity concentration, phonons, defects
- Effective mass also important – can simplify to one (easily measureable) material parameter the mobility, μ, to give;

$$\langle v_d \rangle = -\mu E \qquad \text{Where} \qquad \mu = \frac{q\tau}{m^*}$$

Typical Mobilities

- Electron Mobilities at room temperature

Si	Ge
$0.12\ m^2V^{-1}s^{-1}$	$0.39\ m^2V^{-1}s^{-1}$

c.f. Factor 4.4 difference in m* (Ge lower)

>3x mobility for Ge

Summary (1)

- Electrons in a solid behave differently to hose in vacuum – this can be accounted for by modifying the apparent mass of the electron to an effective mass
- Electrons are scattered by random processes within a solid from imperfections of the crystal lattice – defects, impurities, phonons
- Due to random nature of scattering we cannot look at a single electron and extrapolate to the whole population
- In order to derive the relationship between Electric field, E, and drift velocity, v_d, we need to consider the electron population as a whole

Summary (2)

- The derivation here is based on assumptions – the drift velocity depends upon the mean time between scattering events, scattering resulting in complete loss of momentum, being able to use an effective mass
- We can simplify our result by introducing a new (easily measured) parameter, the mobility μ
- The mobility includes the effect of scattering and effective mass to give a single parameter to describe conduction processes and is a measure of how easily electrons can move in a particular solid

