

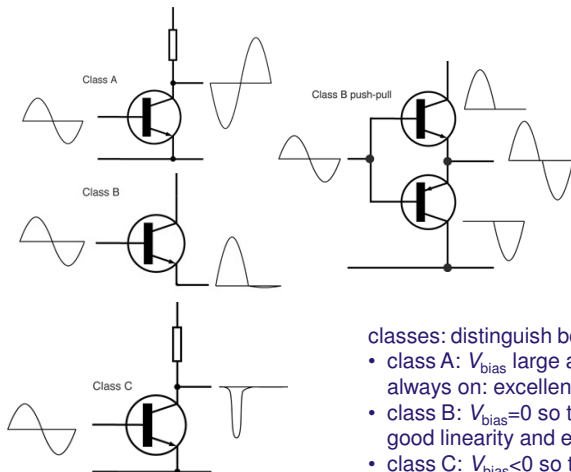
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6th lecture:

- output stages (2nd part)
- operational amplifiers (introduction)

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output stages: class types



aims:

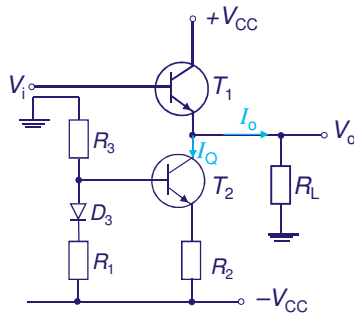
1. deliver power into the load
2. distort signal as little as possible
3. should not limit frequency response
4. low output impedance to ensure output voltage remains unaffected by load
5. low power consumption

classes: distinguish between different bias modes

- class A: V_{bias} large and positive so that BJT is always on: excellent linearity but poor efficiency
- class B: $V_{bias}=0$ so that BJT on only for one half: good linearity and efficiency
- class C: $V_{bias}<0$ so that BJT is just on near peak: poor linearity but excellent efficiency

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emitter follower as typical class A output stage



emitter follower T_1 biased with constant quiescent current I_Q supplied by T_2

remark on amplifier gain:

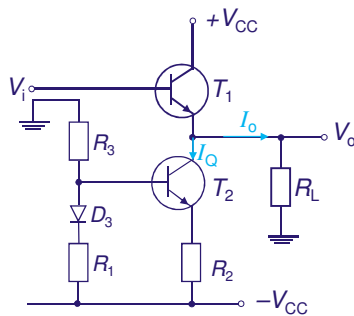
Remember the emitter follower T_1 , where the load resistor is connected between emitter and ground and the collector is connected to the supply voltage, has **no voltage gain**:

$$\begin{aligned} G &= V_o / V_i \\ &= R_L (1 + \beta) / [r_{BE} + R_L (1 + \beta)] \\ &\quad \text{with } g_m v_{BE} = \beta i_B \text{ and } r_{BE} = v_{BE} / i_B \\ &= g_m R_L / [g_m R_L + \beta / (\beta + 1)] \\ &\approx 1 \end{aligned}$$

It can amplify current, however, and therefore produce power gain!

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emitter follower as typical class A output stage



emitter follower T_1 biased with constant quiescent current I_Q supplied by T_2

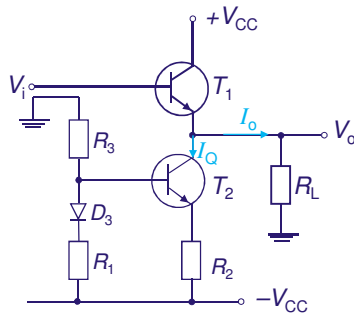
consider large signals for

transfer characteristic:

- Consider voltages:
 $V_i = V_{BE1} + V_o$
 with
 $V_{BE1} = kT/q_e \ln (I_{C1}/I_S)$
 for saturation current I_S if T_1 is in forward-active region and $R_L \ll r_{BE}$
- Consider currents:
 $I_{C1} \approx I_{E1} = I_Q + V_o/R_L$
 if T_2 is also in forward-active region and $\beta_1 \gg 1$
- Substitute I_{C1} into V_i expression:
 $V_i = kT/q_e \ln [(I_Q + V_o/R_L)/I_S] + V_o$
 relates V_o and V_i if both transistors are in forward-active region

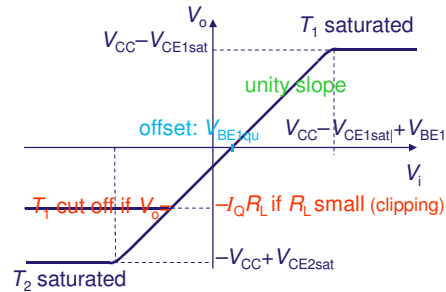
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emitter follower as typical class A output stage



emitter follower T_1 biased with constant quiescent current I_Q supplied by T_2

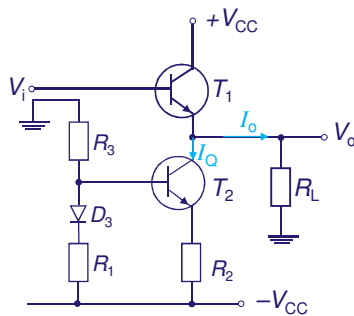
transfer characteristic:



$V_i = kT/q_e \ln [(I_Q + V_o/R_L)/I_S] + V_o$
relates V_o and V_i if both transistors are in forward-active region

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emitter follower as typical class A output stage



emitter follower T_1 biased with quiescent current I_Q supplied by T_2

power output and efficiency:

- consider sinusoidal signal $V_i = V_i^{\max} \sin \omega t$
- average output power delivered to R_L for large load:

$$P_L = \langle V_o \rangle \langle I_o \rangle = \frac{1}{2} V_o^{\max} I_o^{\max}$$

where for equal saturation of T_1 and T_2
 $V_o^{\max} = V_{CC} - V_{CEsat}$ and $I_o^{\max} = V_o^{\max}/R_L = I_Q$
just before clipping, hence

$$P_L^{\max} = \frac{1}{2} (V_{CC} - V_{CEsat}) I_Q$$

- power drawn from power supply:

$$P_{sup} = V_{CC} (I_o + I_Q) \text{ where } \langle I_o \rangle = I_Q, \text{ hence}$$

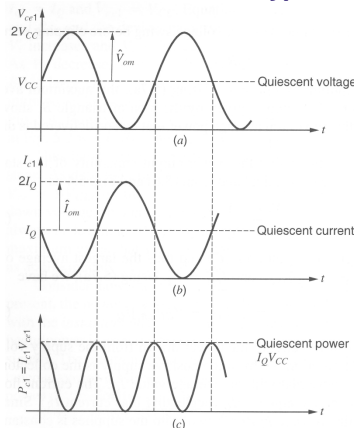
$$P_{sup} = 2V_{CC} I_Q$$

- power conversion efficiency:

$$\eta_A = P_L / P_{sup} = \frac{1}{4} (1 - V_{CEsat}/V_{CC}) \leq 1/4 = 25\%$$

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emitter follower as typical class A output stage



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instantaneous power dissipation:

consider sinusoidal signals of V_{CE1} and I_{C1} , and neglect V_{CEsat} , i.e.

$$V_{CE1} = V_{CC}(1 + \sin \omega t)$$

$$I_{C1} = I_Q (1 - \sin \omega t)$$

instantaneous power dissipation then is

$$P_{C1} = V_{CE1} I_{C1} = V_{CC} I_Q \cos^2 \omega t = \frac{1}{2} V_{CC} I_Q (1 + \cos 2\omega t)$$

has the following properties:

(i) time averaged value is $\frac{1}{2} V_{CC} I_Q$ as before

(ii) minima occur at voltage extrema

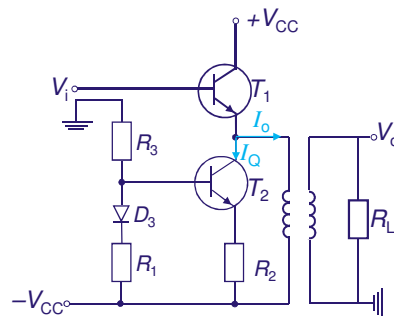
$$(I_{C1} = 0 \text{ or } V_{CE1} = 0)$$

(iii) maxima occur at quiescent operation

($I_{C1} = I_Q$), i.e. the class A amplifier draws max. power when it does not amplify anything!

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transformer coupled class A output stage



emitter follower T_1 biased by T_2 but now transformer coupled to the load

NB: The 25% max. efficiency rule also holds for a single emitter follower T_1 without T_2 :

I_{C1} still swings from 0 to $2I_Q$, i.e. $I_{C1}^{max} = \pm I_Q$, but

V_{C1} only swings from 0 to V_{CC} , i.e. $V_{C1}^{max} = \pm \frac{1}{2} V_{CC}$:

$$\eta_{direct} \leq P_{L,ac} / P_{dc} = (\frac{1}{2} V_{C1}^{max} I_{C1}^{max}) / (V_{CC} I_Q) = \frac{1}{4} = 25\%$$

The reason for the low efficiency is the **quiescent current I_Q passing through the load all the time.**

This can be avoided by using a transformer for coupling of the load. As DC bias components are not transferred by the transformer, only the time-varying I_o but not the constant I_Q needs to be considered in the term for P_{sub} , halving the load to $P_{sup} = V_{CC} I_Q$, thus doubling efficiency to:

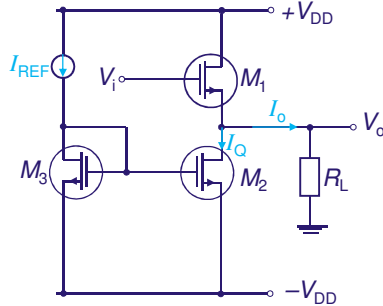
$$\eta_{A+transformer} \leq P_L / P_{sup} = \frac{1}{2} = 50\%$$

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source follower as class A output stage



source follower M_1 biased by current mirror M_2 & M_3

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consider large voltage signals for **transfer**:

$$V_i = V_o + V_{GS1} = V_o + V_{t1} + V_{ov1}$$

problems:

(i) **body effect changes threshold voltage**:

$$V_{t1} = V_{t0} + \gamma[(2\Phi_F + V_{SB})^{1/2} - (2\Phi_F)^{1/2}]$$

with $\gamma = t_{ox}/\epsilon_{SiO_2} (2q_e \epsilon_0 N_A)^{1/2} \approx 0.5V^{0.5}$ in Si for permittivity ϵ , doping density N_A , Fermi level $\Phi_F = kT/q_e \ln(N_A/n_i)$, source-body voltage $V_{SB} = V_o + V_{DD}$

(ii) **overdrive voltage** $V_{ov} = V_{GS} - V_{t0}$ is not constant but depends on drain current and **increases with temperature**:

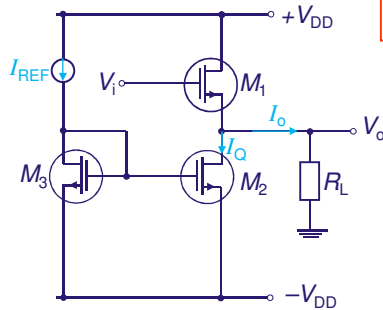
$$V_{ov1} = [2I_D/(\mu_n C_{ox} (W/L))]^{1/2} \text{ with } \mu_n = \text{mobility, } C_{ox} = \epsilon_{SiO_2} \epsilon_0 / t_{ox}, I_D = I_o + V_o/R_L$$

(for detailed derivation: Grey, Hurst, Lewis, Meyer: Analysis and design of analog ICs, Wiley, New York, 4th ed., 2001, chapter 1.5)

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source follower as class A output stage

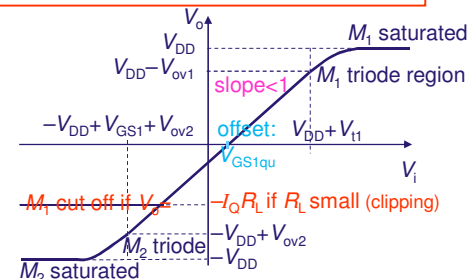


source follower M_1 biased by current mirror M_2 & M_3

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Hence complete transfer function:

$$V_i = V_o + V_{t0} + \gamma[(2\Phi_F + V_o + V_{DD})^{1/2} - (2\Phi_F)^{1/2}] + [2(I_o + V_o/R_L)/(\mu_n C_{ox} (W/L)_1)]^{1/2}$$



$$\text{offset: } V_{GS1qu} = V_{t0} + \gamma[(2\Phi_F + V_{DD})^{1/2} - (2\Phi_F)^{1/2}] + [2(I_o + V_o/R_L)/(\mu_n C_{ox} (W/L)_1)]^{1/2}$$

$$\text{slope: } v_o/v_i = g_m R_L / [1 + (g_m + g_{mb}) R_L] \text{ for } r_o \rightarrow \infty \approx 1/(1 + g_{mb}/g_m) \approx 0.8 \text{ for } g_{mb}/g_m = 0.2, R_L \rightarrow \infty$$

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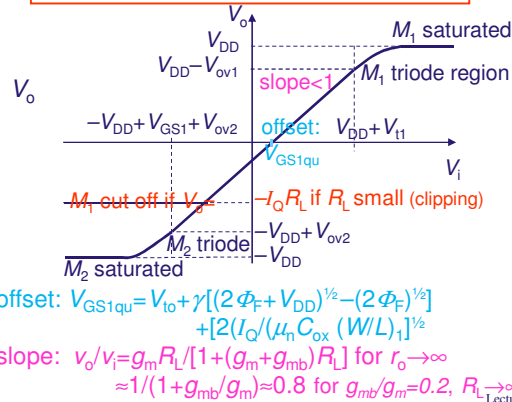
distortion in the source follower as class A output stage

Hence complete transfer function:

$$V_i = V_o + V_{to} + \gamma[(2\phi_F + V_o + V_{DD})^{1/2} - (2\phi_F)^{1/2}] + [2(I_Q + V_o/R_L)/(\mu_n C_{ox} (W/L)_1)]^{1/2}$$

main problem:

$g_{mb}/g_m = \gamma/[2(2\phi_F + V_{SB})^{1/2}]$ is a not a constant ratio of 0.1-0.3 but depends on $V_{SB} = V_o + V_{DD}$. Therefore the slope changes as V_o changes, even when M_1 operates in the active region. This causes significant distortion.



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distortion in the source follower as class A output stage

derivation of quantitative body effect

drain current:

with effective channel length L_{eff} :

this yields:

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_t)^2$$

$$L_{eff} = L - \Delta L \text{ which depends on } V_{DS} \text{ at pinch-off}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

with channel length modulation parameter

$$\lambda = 1/L_{eff} (\partial \Delta L / \partial V_{DS})$$

definition of g_{mb} :

$$g_{mb} = \partial I_D / \partial V_{BS}$$

$$= -\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t) (1 + \lambda V_{DS}) \partial V_t / \partial V_{BS}$$

where the threshold voltage is

first derivate:

insertion into above equation:

$$V_t = V_{to} + \gamma[(2\phi_F + V_{SB})^{1/2} - (2\phi_F)^{1/2}]$$

$$\partial V_t / \partial V_{BS} = -\gamma/[2(\phi_F + V_{SB})^{1/2}]$$

$$g_{mb} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t) (1 + \lambda V_{DS}) \gamma/[2(\phi_F + V_{SB})^{1/2}]$$

$$= \underbrace{\partial I_D / \partial V_{GS}}_{= g_m} \underbrace{\gamma/[2(\phi_F + V_{SB})^{1/2}]}_{\approx 1} \underbrace{1}_{g_{mb}/g_m}$$

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distortion in the source follower as class A output stage

derivation of higher harmonic distortions (not needed for exam):

use transfer function $V_i=f(V_o)$, for $R_L \rightarrow \infty$ for simplicity:

$$V_i = V_o + V_{i0} + \gamma[(2\Phi_F + V_o + V_{DD})^{1/2} - (2\Phi_F)^{1/2}] + V_{ov1}$$

principle: Taylor series expansion of $V_i=f(V_o) = \sum_{n=0, \dots, \infty} \frac{1}{n!} (\partial V_i / \partial V_o) |_{V_o=V_x} (V_o - V_x)^n$
 $= \sum_{n=0, \dots, \infty} b_n V_o^n$

around some DC value $V_x = V_o - V_o$

differentiating with respect to V_o yields:

$$f'(V_o) = 1 + \frac{1}{2}\gamma(V_o + V_{DD} + 2\Phi_F)^{-1/2}$$

$$f''(V_o) = -\frac{1}{4}\gamma(V_o + V_{DD} + 2\Phi_F)^{-3/2}$$

$$f'''(V_o) = \frac{3}{8}\gamma(V_o + V_{DD} + 2\Phi_F)^{-5/2}$$

This yields the coefficients:

$$b_0 = f(V_o = V_x) = V_x + V_{i0} + \gamma[(V_x + V_{DD} + 2\Phi_F)^{1/2} - (2\Phi_F)^{1/2}] + V_{ov1} = \text{constant DC input}$$

$$b_1 = f'(V_o = V_x) = 1 + \frac{1}{2}\gamma(V_x + V_{DD} + 2\Phi_F)^{-1/2}$$

$$b_2 = f''(V_o = V_x) = -\frac{1}{8}\gamma(V_x + V_{DD} + 2\Phi_F)^{-3/2}$$

$$b_3 = f'''(V_o = V_x) = \frac{1}{16}\gamma(V_x + V_{DD} + 2\Phi_F)^{-5/2}$$

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distortion in the source follower as class A output stage

derivation of higher harmonic distortions (not needed for exam):

To find the distortions we will have to re-arrange $v_i=f(v_o) = \sum_{n=1, \dots, \infty} b_n v_o^n$ into some form

$$v_o = \sum_{n=1, \dots, \infty} a_n v_i^n$$

Substituting this into the above gives:

$$v_i = b_1(a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \dots) + b_2(a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \dots)^2 + b_3(a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \dots)^3 + \dots$$

$$= b_1 a_1 v_i + (b_1 a_2 + b_2 a_1^2) v_i^2 + (b_1 a_3 + 2b_2 a_1 a_2 + b_3 a_1^3) v_i^3 + \dots \text{ (sorted acc. to powers of } v_i)$$

$$\begin{array}{ccc} \underbrace{}_{=1} & \underbrace{}_{=0} & \underbrace{}_{=0} \\ \downarrow & \downarrow & \downarrow \\ a_1 = 1/b_1 & \rightarrow a_2 = -b_2/b_1^3 & \rightarrow a_3 = 2b_2^2/b_1^5 - b_3/b_1^4 \end{array}$$

Inserting the expressions for the b_i yields finally:

$$a_1 = 1/[1 + \frac{1}{2}\gamma(V_x + V_{DD} + 2\Phi_F)^{-1/2}]$$

$$a_2 = \gamma/8 (V_x + V_{DD} + 2\Phi_F)^{-3/2} / [1 + \frac{1}{2}\gamma(V_x + V_{DD} + 2\Phi_F)^{-1/2}]^3$$

$$a_3 = -\gamma/16 (V_x + V_{DD} + 2\Phi_F)^{-5/2} / [1 + \frac{1}{2}\gamma(V_x + V_{DD} + 2\Phi_F)^{-1/2}]^5$$

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distortion in the source follower as class A output stage

derivation of higher harmonic distortions (not needed for exam):

To find the **harmonic distortions** use

$$v_i = u_i \sin \omega t$$

with amplitudes u_i and insert this into the equation for v_o to get

$$v_o = a_1 u_i \sin \omega t + a_2 u_i^2 \sin^2 \omega t + a_3 u_i^3 \sin^3 \omega t + \dots$$

$$= a_1 u_i \sin \omega t + \frac{1}{2} a_2 u_i^2 (1 - \cos 2\omega t) + \frac{1}{4} a_3 u_i^3 (3 \sin \omega t - \sin 3\omega t) + \dots$$

fundamental frequency, ω higher harmonic frequencies, $N\omega$, not present in input

Definition:

x^{th} harmonic distortion = ratio of amplitude of output frequency component $x\omega$ to amplitude of fundamental frequency component

second harmonic distortion $HD_2 = \frac{1}{2} a_2 u_i / a_1$

$$= \frac{\gamma (V_x + V_{DD} + 2\phi_F)^{-3/2} u_i / \{16[1 + \gamma/2(V_x + V_{DD} + 2\phi_F)^{-1/2}]^2\}}{\approx \gamma (V_x + V_{DD} + 2\phi_F)^{-3/2} u_i / 16 \text{ for small } \gamma}$$

can be reduced by increasing the DC output V_x and is proportional to the signal amplitude u_i and γ

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distortion in the source follower as class A output stage

derivation of higher harmonic distortions (not needed for exam):

third harmonic distortion $HD_3 = \frac{1}{4} a_3 u_i^2 / a_1$

$$= \frac{-\gamma (V_x + V_{DD} + 2\phi_F)^{-5/2} u_i^2 / \{64[1 + \gamma/2(V_x + V_{DD} + 2\phi_F)^{-1/2}]^4\}}{\approx -\gamma (V_x + V_{DD} + 2\phi_F)^{-5/2} u_i^2 / 64 \text{ for small } \gamma}$$

can be reduced by increasing the DC output V_x & is proportional to square of signal amplitude u_i^2

application example:

peak sinusoidal input voltage $u_i = 1\text{V}$ (no DC component: $V_i = 0$), $V_{DD} = 2.5\text{V}$, $\phi_F = 0.3\text{V}$, $V_{t0} = 0.7\text{V}$, $I_Q = 1\text{mA}$, $R_L = \infty$, $(W/L)_1 = 1000$, $\mu_n C_{ox} = 200\mu\text{A/V}^2$, $\gamma = 0.5\text{V}^{1/2}$

First determine $V_{ov1} = [2I_Q / (\mu_n C_{ox} (W/L)_1)]^{1/2} = 0.1\text{V}$

Now get the DC output voltage $V_x = V_i - V_{t0} - \gamma[(V_x + V_{DD} + 2\phi_F)^{1/2} - (2\phi_F)^{1/2}] - V_{ov1}$

Rearrange to get a quadratic equation that can be solved for V_x :

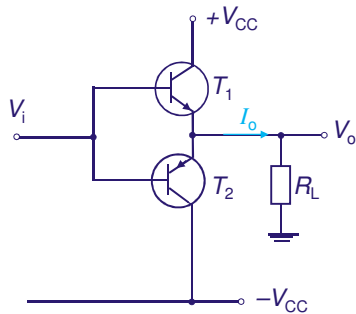
$$(V_x + V_{DD} + 2\phi_F) + \gamma(V_x + V_{DD} + 2\phi_F)^{1/2} - V_i + V_{ov1} + V_{t0} - \gamma(2\phi_F)^{1/2} - V_{DD} - 2\phi_F = 0$$

$$V_x = -V_{DD} - 2\phi_F \pm \{-1/2\gamma + [1/4\gamma^2 + V_i - V_{ov1} - V_{t0} + \gamma(2\phi_F)^{1/2} + V_{DD} + 2\phi_F]^{1/2}\}^2 = -1.1168\text{V}$$

Thus: $a_0 = 0$, $a_1 = 0.8492$, $a_2 = 0.0137$, $a_3 = 0.0025$, $HD_2 = 0.008 = 0.8\%$, $HD_3 = 0.0007 = 0.07\%$

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class B output stages



simple complementary output stage with pnp & npn BJTs acting as emitter followers

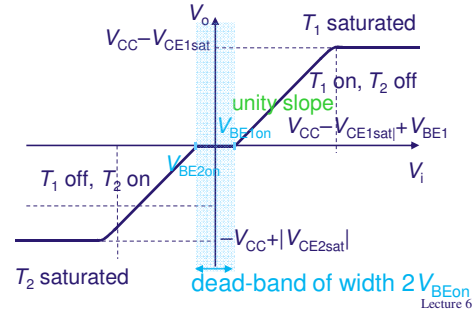
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principle: use 2 active transistors, each of which conducts at alternate half cycles (**push-pull configuration**)

advantages:

1. higher efficiency, thus also
2. lower heating effects

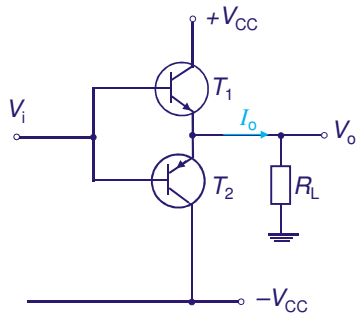
transfer characteristic:



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class B output stages

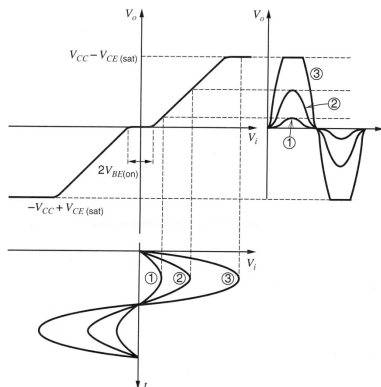


simple complementary output stage with pnp & npn BJTs acting as emitter followers

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transfer characteristic:

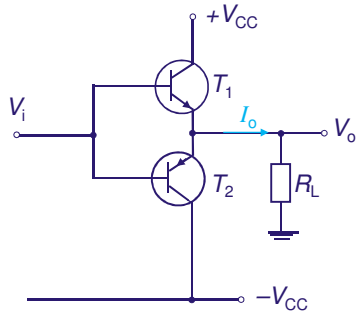
The dead band of width $2V_{BE(on)}$ results in **cross-over distortions of small signals** (also shown is clipping of large signals)



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class B output stages



simple complementary output stage with pnp & npn BJTs acting as emitter followers

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power output and efficiency:

Neglect cross-over distortion and consider case of output sinusoid of peak amplitude V_o^{\max} .

average load power: $P_L = \frac{1}{2} (V_o^{\max})^2 / R_L$

The currents drawn from each supply consist of half-sine waves of peak amplitudes V_o^{\max} / R_L . Because of $\langle \sin x \rangle = 1/\pi \int_0^\pi \sin x dx = -1/\pi \cos x \big|_0^\pi = 2/\pi$ the average current from **each** power supply will be $\frac{1}{2} 2 V_o^{\max} / (\pi R_L)$, hence total power drawn from supply is $P_S = 2 P_{S\pm} = 2 V_o^{\max} V_{CC} / (\pi R_L)$

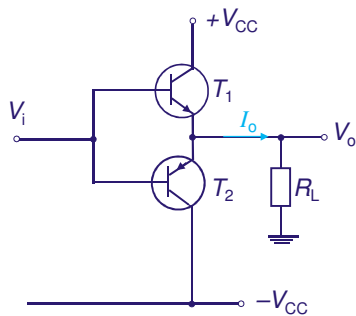
Thus the efficiency is

$$\begin{aligned} \eta_B &= P_L / P_S = \frac{1}{4} \pi V_o^{\max} / V_{CC} \\ &\leq \frac{1}{4} \pi (V_{CC} - V_{CEsat}) / V_{CC} \\ &\leq \frac{1}{4} \pi \\ &= 78.54\% \leq 1 \end{aligned}$$

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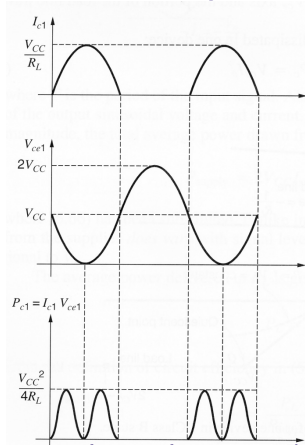
class B output stages



simple complementary output stage with pnp & npn BJTs acting as emitter followers

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instantaneous power dissipation:



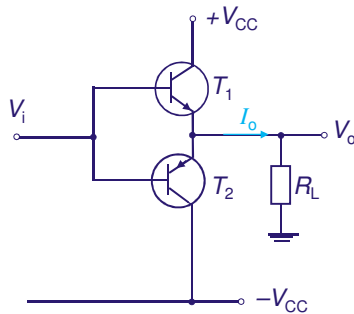
waveforms of current, voltage and power dissipation at max. output of T_1

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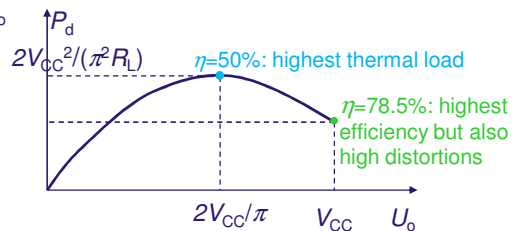
class B output stages

average power dissipation:



simple complementary output stage with pnp & npn BJTs acting as emitter followers

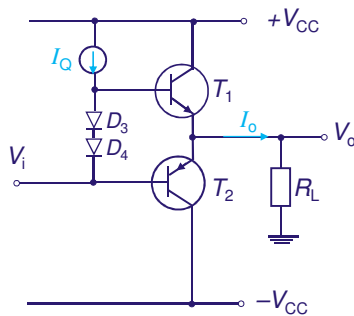
average dissipated power of both transistors (each gets half of it):
 $P_d = P_S - P_L = 2U_o V_{CC} / (\pi R_L) - U_o^2 / (2R_L)$
 has a maximum when plotted as function of output amplitude $U_o \leq V_{CC}$:



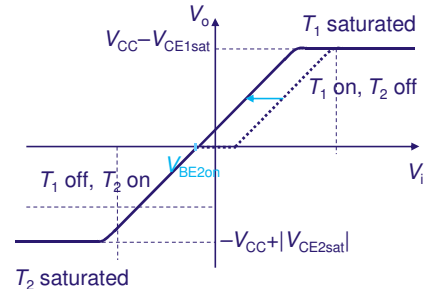
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class AB output stages

principle: as class B (push-pull) but additional biasing such that each transistor conducts a small quiescent current for $V_i = 0$, e.g. here T_1 is biased by diodes about $2V_{BE2on}$. In the transfer characteristic **cross-over distortion is minimised** as the dead-band has been eliminated.

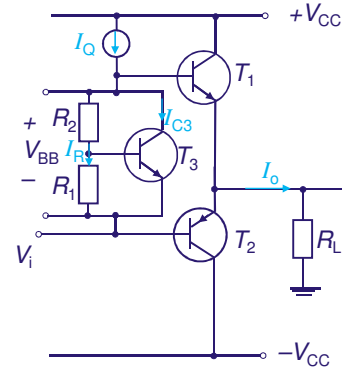


complementary output stage with pnp & npn BJTs and biasing diodes



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class AB output stages



complementary output stage with pnp & npn BJTs and bias by V_{BE} multiplier

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Biased can alternatively be performed by a third transistor T_3 and two resistors R_1 and R_2 in a configuration known as **V_{BE} multiplier**:
 $I_R = V_{BE3}/R_1$, if base current I_{B3} negligible
 $V_{BB} = I_R(R_1 + R_2) = V_{BE3}(1 + R_2/R_1)$
 then forms an adjustable bias

further biasing alternatives:

- emitter follower as unity-gain buffer for T_2 (output stage of OpAmp 709: Meyer, p. 370)
- Darlington pair instead of diode pair (output stage of OpAmp 741: Meyer, p. 372)

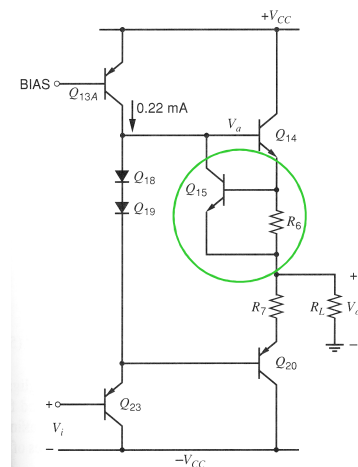
general problem with BJTs:

no high-power substrate pnp BJTs: use

- all-npn BJTs plus diodes or
- create quasi-complementary pnp using lateral pnp and high-power npn BJT
- BICMOS combines BJTs and MOSFETs

Lecture 6

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schematic of the output stage of the 741 OpAmp where Q_{14} & Q_{20} operate as class AB

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overload protection

problem: if base currents get above a certain threshold then high-gain stages without protection can create destructively high currents

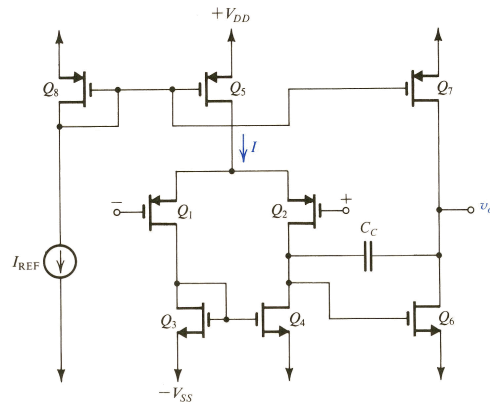
aim: overload protection in IC output stages against self-destruction in case of short-circuited output

implementation: divert current from base of transistor in question (here: Q_{14}) by using a resistor (here: R_6) that switches another transistor on (here: Q_{15}) only if appreciable current flows across it

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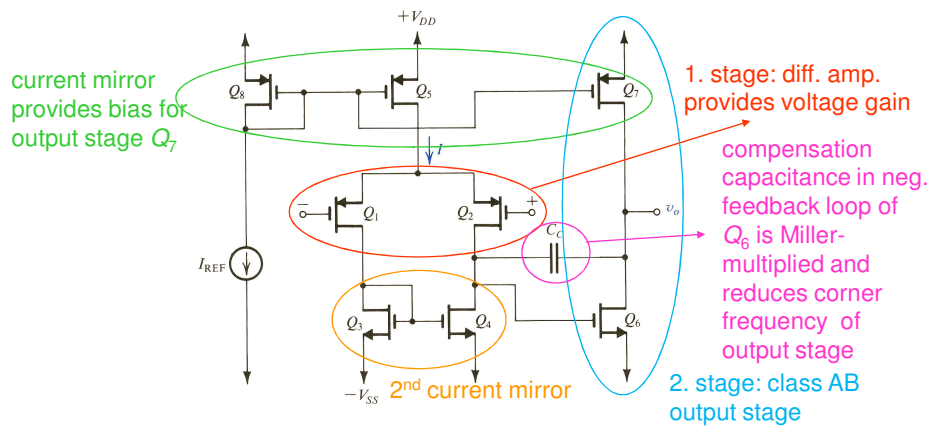
operational amplifiers (Op Amps): introduction



basic 2-stage CMOS op-amp configuration

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operational amplifiers (Op Amps): introduction



basic 2-stage CMOS op-amp configuration