List of useful formulae

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$x(t) = \int_{-\infty}^{\infty} x(\lambda)\delta(t-\lambda)d\lambda$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$

$$c_0 = \frac{1}{T}\int_{<T>} x(t)dt$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jn\omega_o t}dt$$

$$a_n = 2\,\mathrm{Re}[c_n] = \frac{2}{T}\int_{<T>} x(t)\cos n\omega_0 t dt$$

$$b_n = -2\,\mathrm{Im}[c_n] = \frac{2}{T}\int_{<T>} x(t)\sin n\omega_0 t dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$X(\omega) = 2\int_{0}^{\infty} x(t)\cos\omega t dt$$

$$X(\omega) = -j2\int_{0}^{\infty} x(t)\sin\omega t dt$$

$$X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$$

$$x(t) = \frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}dt$$

Unit step response for 2nd order systems

Damping factor, ζ	Unit step response
>1	$y(t) = \frac{k}{p_1 p_2} + k_2 e^{p_1 t}.u(t) + k_3 e^{p_2 t}.u(t)$
1	$y(t) = \frac{k}{\omega_n^{\ 2}}\left(1-(1+\omega_n t)e^{-\omega_n t}.u(t)\right)$
$0 < \zeta < 1$	$y(t) = \frac{k}{\omega_n^{\ 2}}\left(1-\frac{\omega_n}{\omega_d}e^{-\zeta\omega_n t}\sin(\omega_d t+\theta).u(t)\right)$
0	$y(t) = \frac{k}{\omega_n^{\ 2}}\left(1-\cos(\omega_n t).u(t)\right)$

Fourier Transform Pairs

Signal	Fourier Transfrom
$\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$	$2\pi \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_o)$
$e^{j\omega_o t}$	$2\pi\delta(\omega - \omega_o)$
$\cos\omega_o t$	$\pi[\delta(\omega + \omega_o) + \delta(\omega - \omega_o)]$
$\sin\omega_o t$	$j\pi[\delta(\omega + \omega_o) - \delta(\omega - \omega_o)]$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
$\delta(t - t_o)$	$e^{-j\omega t_o}$
$e^{-at}u(t),\ a > 0$	$\frac{1}{a + j\omega}$
$x(t) = \begin{cases} 1, & \lvert t\rvert < \tau \\ 0, & \lvert t\rvert > \tau \end{cases}$	$\frac{2\sin\omega\tau}{\omega} = 2\tau Sa(\omega\tau)$
$\frac{\sin\omega_c t}{\pi t} = \frac{\omega_c}{\pi} Sa(\omega_c t)$	$X(\omega) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \lvert\omega\rvert > \omega_c \end{cases}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$

Properties of Laplace Transform

Property	Transform Property
Linearity	$ax_1(t) + bx_2(t) \leftrightarrow aX_1(s) + bX_2(s)$.
Time shift	$x(t- t_o)\, u(t- t_o) \leftrightarrow X(s)e^{-st_o}$,$t_o > 0$
Multiplication by a complex exponential	$x(t)e^{s_o t} \leftrightarrow X(s-s_o)$
Time scaling	$x(at) \leftrightarrow X(s/a)/\lvert a\rvert$
Differentiation in time domain	$\frac{dx(t)}{dt} \leftrightarrow sX(s) - x(0)$
	$\frac{d^2x(t)}{dt^2} \leftrightarrow s^2X(s) - sx(0) - \left.\frac{dx(t)}{dt}\right\rvert_{t=0}$
Differentiation in s domain	$t^n x(t) \leftrightarrow \frac{d^n X(s)}{ds^n}(-1)^n$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{1}{s}X(s)$
Convolution in time domain	$x(t)*h(t) \leftrightarrow X(s).H(s)$
Initial value theorem	$x(0) = \lim_{s\to\infty} sX(s)$
Final value theorem	$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$

(if $x(t)$ has a finite value as $t \to \infty$)

Laplace Transform pairs

Signal	Transform
Unit step: $u(t)$	$\frac{1}{s}$
Unit impulse: $\delta(t)$	1
Unit ramp: $tu(t)$	$\frac{1}{s^2}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$
$(\cos\omega_o t)u(t)$	$\frac{s}{(s^2+\omega_o{}^2)}$
$(\sin\omega_o t)u(t)$	$\frac{\omega_o}{(s^2+\omega_o{}^2)}$
$(e^{-at}\cos\omega_o t)u(t)$	$\frac{s+a}{((s+a)^2+\omega_o{}^2)}$
$(e^{-at}\sin\omega_o t)u(t)$	$\frac{\omega_o}{((s+a)^2+\omega_o{}^2)}$
$(t\cos\omega_o t)u(t)$	$\frac{s^2-\omega_o{}^2}{(s^2+\omega_o{}^2)^2}$
$(t\sin\omega_o t)u(t)$	$\frac{2\omega_o s}{(s^2+\omega_o{}^2)^2}$