

QUESTION 1

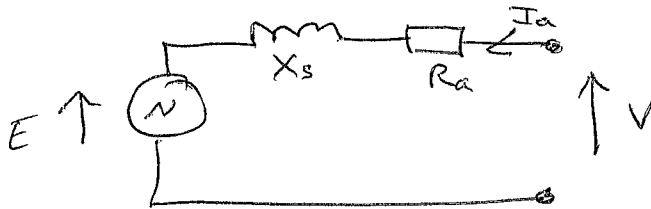
1

(a) An infinite busbar in the context of electrical power systems, is a supply whose voltage and frequency remain constant.

In order to synchronise the machine the following criteria must be met:

- Supply and machine voltages should have the same magnitude, frequency and phase sequence
- Supply and machine voltages should be in phase. (2)

(b) Using the per-phase equivalent circuit:

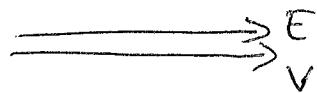


$$E = V - I_a R_a - j I_a X_s$$

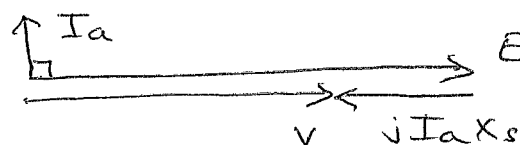
Since, in this case $R = 0$ then:

$$E = V - j I_a X_s$$

(i) Initially the machine is on no-load (no load torque / no shaft power). I_a is therefore zero and E and V are in phase and have the same magnitude. Load angle is zero. (2)



(ii) If the machine excitation is increased then E will increase. There is still no load on the machine so the load angle is still zero.



Current now flows, but the phase angle is 90°

$$\text{Electrical power } VI \cos \phi = 0 \quad (\text{since } \phi = 90^\circ)$$

QUESTION 1 (CONTINUED)

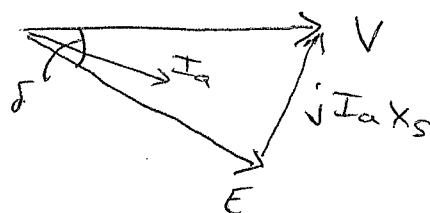
2

$$\text{Mechanical power} = EI \cos \alpha = 0$$

(α = angle between E and $I = 90^\circ$)

No real power flow. I_a leads E and V by 90°
Therefore machine supplies leading VARs.

(iii) E and V have equal magnitude as in (b)(i) however
now a load is applied to the shaft.



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Rotor will slow down until developed electromagnetic torque equals the applied torque. Rotor field lags behind stator field and the load angle, δ , is negative. I_a is slightly lagging.

(c) (i) The base load of the factory is 20 MVA @ 0.75 p.f. lag.

$$P_F = 20 \times 0.75 = 15 \text{ MW}$$

$$Q_F = 20 \times \sin(\cos^{-1} 0.75) = 13.23 \text{ MVAR}$$

With the synchronous machine added the new power is:

$$P_T = P_F + P_{sm} = 15 + 3 = 18 \text{ MW}$$

After the synchronous motor is added the overall p.f. is 0.95 lag

$$VA_T = \frac{P_T}{0.95} = 18.947 \text{ MVA}$$

and the new overall reactive power is

$$Q_T = 18.947 \times \sin(\cos^{-1} 0.95) = 5.916 \text{ MVAR.}$$

QUESTION 1 (CONTINUED)3

Hence the synchronous machine has to supply

$$13.23 - 5.916 = 7.314 \text{ MVAR (leading)}$$

The MVA rating of the machine is then:

$$S_{sm} = \sqrt{P_{sm}^2 + Q_{sm}^2} = \sqrt{3^2 + 7.314^2} = \underline{\underline{7.905 \text{ MVA}}}$$

and its p.f. is $\cos \left(\tan^{-1} \frac{Q_{sm}}{P_{sm}} \right) = \cos \left(\tan^{-1} \frac{7.314}{3} \right) = \underline{\underline{0.379 \text{ leading}}}$

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(ii) The phase voltage is:

$$V_{ph} = \frac{15000}{\sqrt{3}} = 8660.3 \text{ V}$$

and since $P = \sqrt{3} V_L I_L \cos \phi \Rightarrow I_L (= I_{ph}) = \frac{P}{\sqrt{3} \cdot 15000 \cdot 0.379}$
 $= \underline{\underline{304.7 \text{ A}}}$

Now for a synchronous motor:

$$E \angle \delta = V \angle 0^\circ - j I_a X_s = 8660.3 \angle 0^\circ - (304.7 \angle 67.7^\circ \times 10 \angle 90^\circ)$$

$$= \underline{\underline{11533.6 \angle -5.75^\circ}}$$

The load angle is -5.75°

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(iii) Before the load is reduced the phasor diagram is:



Power produced by the machine per phase $= \frac{VE \sin \delta}{X}$

Since the magnitude of V , E and X are unchanged
 Reducing P reduces δ . Power per phase is $2.4/3 = 0.8 \text{ MW}$

$$\sin \delta = \frac{10 \times 0.8 \times 10^6}{8660.3 \times 11533.6} = 0.08$$

$$\therefore \delta = \underline{\underline{-4.59^\circ}}$$

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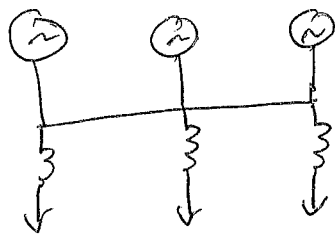
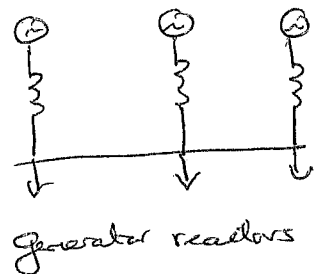
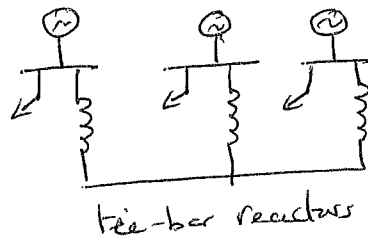
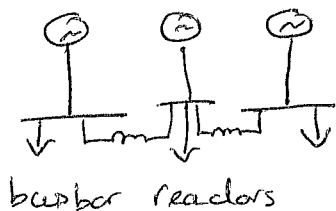
QUESTION 2

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(a) Per unit is the value of any quantity expressed as a ratio or fraction of an arbitrary base value of the same quantity. The rated value is usually selected for the base value. ①

(b) Fault levels may be reduced by:

- inserting fault limiting reactors such as sectionalised busbar reactors, tie-bar reactors, generator reactors and feeder reactors.



Busbar or tie-bar reactors are normally preferred as under normal conditions reactors do not carry full load current.

For generator and feeder reactors the full load current passes through the reactors causing volts drop and losses since reactor has a finite R .

- Sectionalised connections, reduces system interconnection by having normally open isolators and rings, which still enable system security by providing alternative routes once the fault has been isolated.
- Use high voltage DC links (asynchronous links) for interconnecting separate parts of large power systems.

(Power flow controlled by electronic switching $\frac{\$}{\text{ms}}$ hence the fault current can be limited).

⑤

QUESTION 2 (CONTINUED)

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(i) (i)
To get the ratings for the circuit breakers at C and D we first need to find the fault level at C and D.

Choose base of 50 MVA.

Grid infeed - 500 MVA into 1.0 pu reactor, hence:

$$GI_{pu} = \frac{50}{500} \times 1.0 = 0.1 \text{ pu}$$

Generator G1 (0.18 on 60 MVA)

$$= \frac{50}{60} \times 0.18 = 0.15 \text{ pu}$$

Generators G2 and G3 are already given on 50 MVA (0.15 pu)

Transformers T1 and T2:

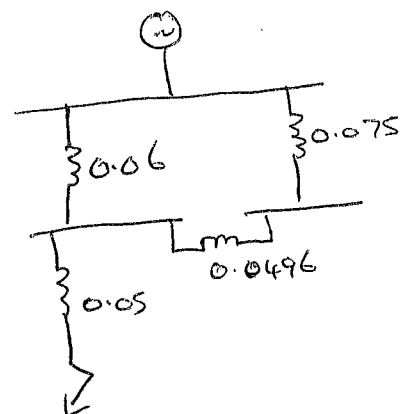
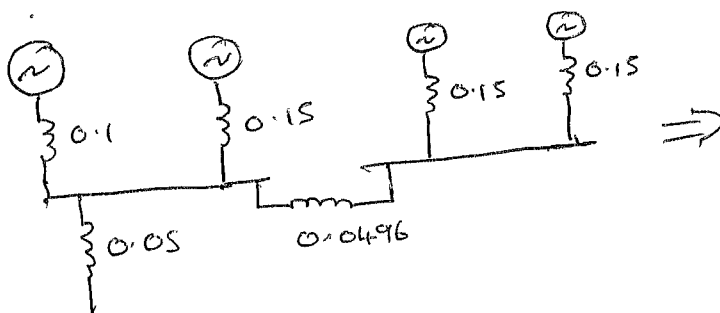
$$= \frac{50}{120} \times 0.12 = 0.05 \text{ pu}$$

For the reactor linking the busbars first calculate the base value:

$$Z_B = \frac{V_B^2}{MVA_B} = \frac{11000^2}{50 \times 10^6} = 2.42 \Omega$$

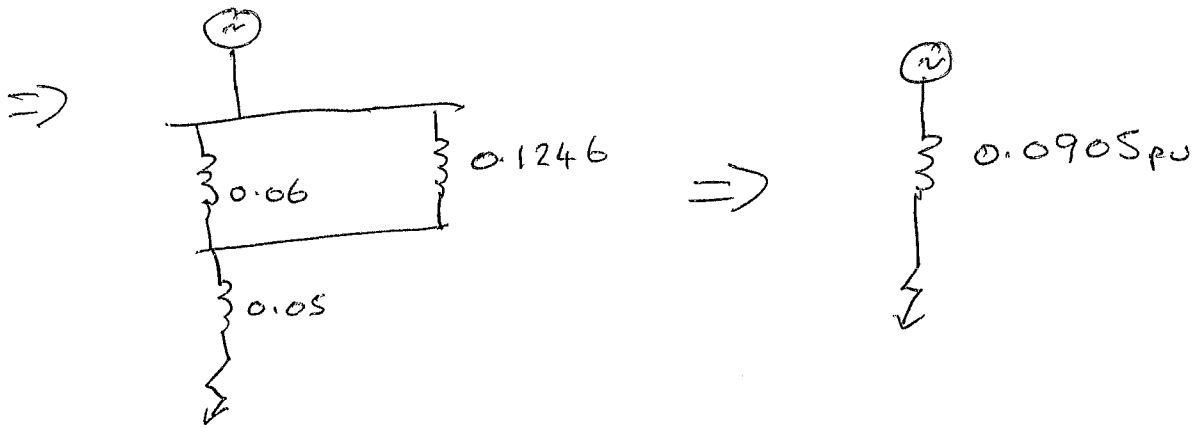
$$\therefore X_{pu} = \frac{0.12}{2.42} = 0.0496 \text{ pu.}$$

For fault at C:



QUESTION 2 (CONTINUED)

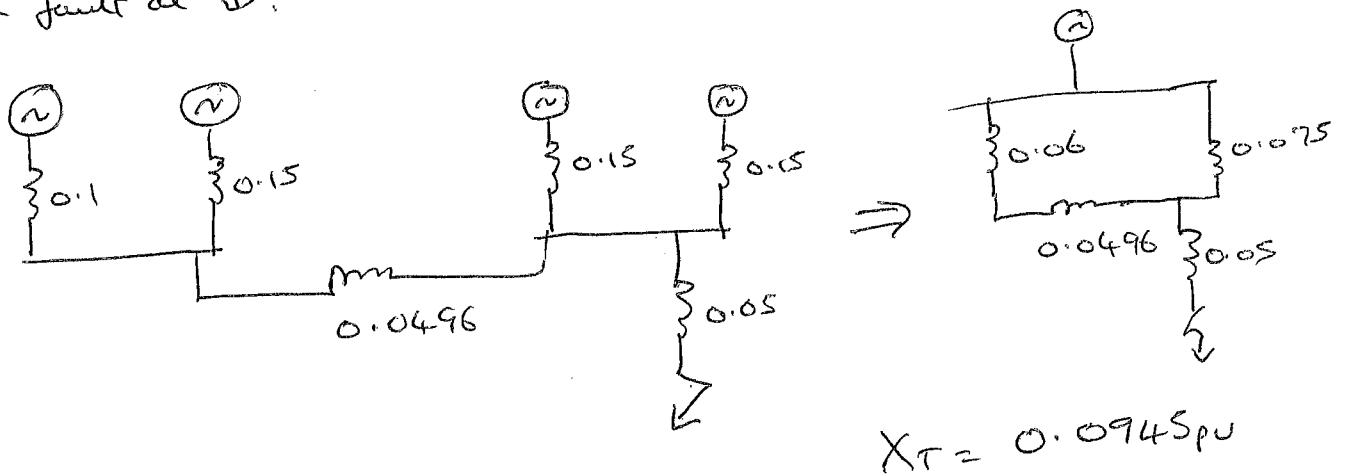
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Therefore the pu fault level at C is $\frac{1}{X_{TPU}} = \frac{1}{0.0905} = 11.05 \text{ pu}$

Hence actual fault level at C is $50 \times 11.05 = \underline{\underline{552.5 \text{ MVA}}}$

For a fault at D:



Therefore pu fault level at D is $\frac{1}{0.0945} = 10.58 \text{ pu}$

Hence actual fault level at D is $50 \times 10.58 = \underline{\underline{529.1 \text{ MVA}}}$ 7

(ii) For a 3-phase fault at D the pu fault current is $\frac{1}{X_{pu}} = 10.58 \text{ pu}$

Referring back to the previous set of diagrams the p.u. fault current in the right hand branch (G2 + G3) is:

$$I_{RHPU} = \frac{10.58 \times (0.06 + 0.0496)}{(0.06 + 0.0496 + 0.075)} = 6.282 \text{ pu}$$

Since G2 and G3 have the same pu reactance then

$$I_{G1pu} = \frac{6.282}{2} = 3.141 \text{ pu}$$

QUESTION 2 (CONTINUED)

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$$\text{Base current at G2} = \frac{50 \times 10^6}{\sqrt{3} \times 11000} = 2.62 \text{ KA}$$

$$\text{Hence actual generator current} = 2.62 \times 3.141 = \underline{\underline{8.23 \text{ KA}}}$$

(3)

(iii) Since faults can occur at any point the maximum current for the reactor will occur when there is a fault at either of the 11KV busbars.

Using previous diagrams and values:

For fault at A



$$\text{Fault level} = \frac{1}{0.0405} = 24.69 \text{ pu}$$

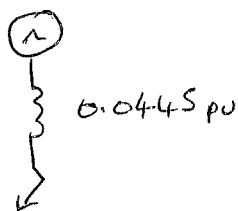
$$\text{or } 1235 \text{ MVA}$$

$$\text{Base current at Busbars / reactor } I_B = \frac{\text{MVA}_B}{\sqrt{3} \cdot 11000} = 2.62 \text{ KA.}$$

$$\text{Hence total fault current at A is } 2.62 \times 24.69 = 64.7 \text{ KA}$$

$$\text{and the proportion through the reactor is } \frac{64.7 \times 0.06}{(0.06 + 0.1246)} = 21.0 \text{ KA}$$

For fault at B:



$$\text{Fault current at B} = \frac{1}{0.0445} \times 2.62 = 58.8 \text{ KA}$$

$$\text{and the proportion through the reactor is } \frac{58.8 \times 0.075}{(0.075 + 0.06 + 0.0496)}$$

$$= \underline{\underline{23.89 \text{ KA}}}$$

Worst case.

(4)

QUESTION 3

8

Using Millman's theorem:

$$V_{N'N} = \frac{\sum YV}{\sum Y} = \frac{Y_A V_{AN} + Y_B V_{BN} + Y_C V_{CN}}{Y_A + Y_B + Y_C}$$

$$Z_A = (10 - j4) \Rightarrow Y_A = 0.0928 \angle 21.8^\circ = 0.086207 + j0.03447$$

$$Z_B = (4 + j6) \Rightarrow Y_B = 0.13868 \angle -56.3^\circ = 0.076923 - j0.115385$$

$$Z_C = (5 + j16) \Rightarrow Y_C = 0.0597 \angle -72.646^\circ = 0.017794 - j0.05694$$

$$\therefore Y_A + Y_B + Y_C = 0.1809 - j0.13786 = 0.2274 \angle -37.3^\circ$$

$$Y_A V_{AN} = 0.0928 \angle 21.8^\circ \times 240 \angle 0^\circ = 22.272 \angle 21.8^\circ$$

$$Y_B V_{BN} = 0.13868 \angle -56.3^\circ \times 240 \angle -120^\circ = 33.2832 \angle -176.3^\circ$$

$$Y_C V_{CN} = 0.0597 \angle -72.646^\circ \times 240 \angle -240^\circ = 14.328 \angle -312.646^\circ$$

$$\therefore V_{N'N} = \frac{22.27 \angle 21.8^\circ + 33.28 \angle -176.3^\circ + 14.33 \angle 47.4^\circ}{0.2274 \angle -37.3^\circ} = \frac{16.94 \angle 99.6^\circ}{0.2274 \angle -37.3^\circ}$$

$$= -54.287 + j50.735 = \underline{\underline{74.3 \angle 136.9^\circ}}$$

Voltage on phase A:

$$V_{AN'} = V_{AN} - V_{N'N} = 240 \angle 0^\circ - 74.3 \angle 136.9^\circ = \frac{298.6 \angle -9.78^\circ V}{= 294.2 - 50.7j}$$

Voltage on phase B:

$$V_{BN'} = V_{BN} - V_{N'N} = 240 \angle -120^\circ - 74.3 \angle 136.9^\circ = \frac{266.8 \angle -104.3^\circ V}{= -65.9 - j258.5}$$

Voltage on phase C:

$$V_{CN'} = V_{CN} - V_{N'N} = 240 \angle -240^\circ - 74.3 \angle 136.9^\circ = \frac{170.3 \angle 112.7^\circ V}{= 259.7 - j258.5}$$

Currents:

$$I_A = \frac{298.6 \angle -9.78^\circ}{10.7703 \angle -21.8^\circ} = \underline{\underline{27.73 \angle 12^\circ A}} = 27.12 + j5.76$$

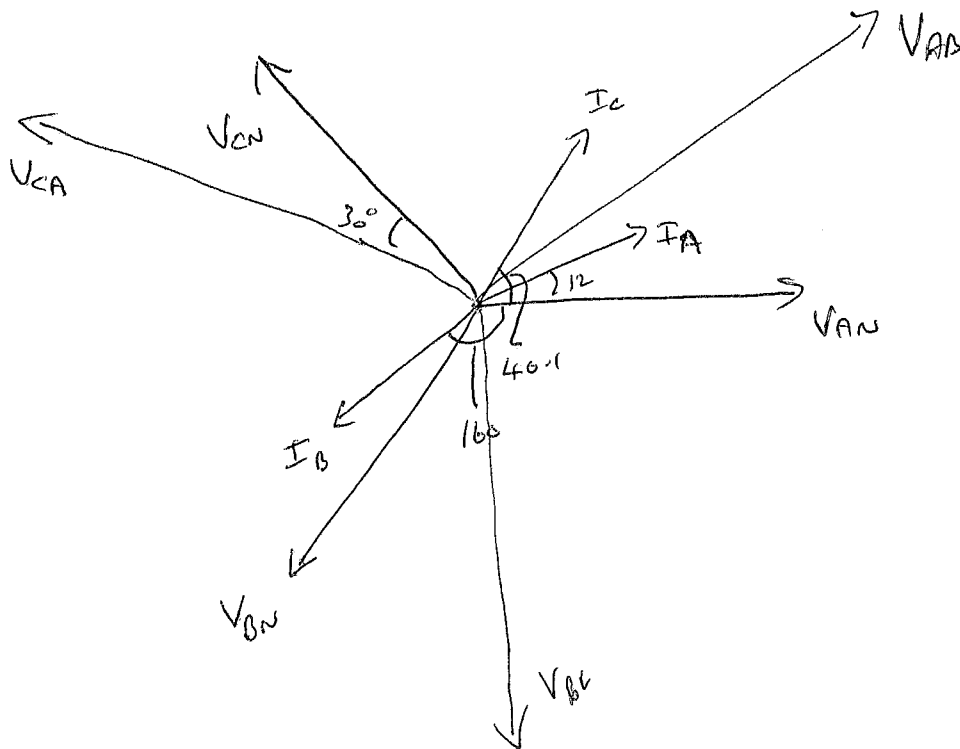
QUESTION 3 (CONTINUED)

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$$I_B = \frac{266.8 \angle -104.3^\circ}{7.211 \angle 56.31^\circ} = \underline{\underline{36.99 \angle -160.61^\circ \text{ A}}} = -34.9 - 12.3j$$

$$I_C = \frac{170.3 \angle 112.7^\circ}{16.763 \angle 72.64^\circ} = \underline{\underline{10.159 \angle 40.1^\circ \text{ A}}} = 7.774 + j6.54 \quad (2)$$

(b)(i)



(3)

(ii) Wattmeter, P_1 , measures $V_{AB} I_A \cos(\text{angle between } V_{AB} \text{ and } I_A)$

$$= 415 \times 27.73 \cos(30 - 12) = \underline{\underline{10.946 \text{ kW}}}$$

Wattmeter, P_2 , measures $V_{CA} I_C \cos(\text{angle between } V_{CA} \text{ and } I_C)$

$$= 415 \times 10.159 \cos(150 - 40.1) = \underline{\underline{-1.435 \text{ kW}}}$$

(2)

$$P_1 + P_2 = P_T = \underline{\underline{9.511 \text{ kW}}}$$

(c)(i) For the correct connection voltage coil of P_2 should be connected to phase B (i.e. V_{CB})

$$\therefore P_{2\text{ new}} = V_{CB} \cdot I_C \cos(90 - 40.1) = \underline{\underline{2.716 \text{ kW}}}$$

$$\therefore P_{T\text{ new}} = \underline{\underline{13.68 \text{ kW}}}$$

(2)

QUESTION 3 (CONTINUED)

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- (ii) check by calculating the I^2R loss in each phase and summing.

$$P_T = I_A^2 R_A + I_B^2 R_B + I_C^2 R_C$$

$$= 27.73^2 \cdot 10 + 36.99^2 \cdot 4 + 10.159^2 \cdot 5$$

$$= \underline{\underline{13.68 \text{ kW}}}$$

①

- (iii) The system is clearly unbalanced and hence there would be a neutral current so the 2 Wattmeter method would give incorrect readings.

②

QUESTION 4

11

(a) Benefits of induction generator (2 of:)

- Does not have to run at synchronous speed
- Easy to connect to grid - no synchronisation needed
- No rotor excitation, brushgear etc. (maintenance)
- Robust and relatively cheap - no power converter.

Drawbacks of induction generator (2 of:)

- No field excitation means no control over generated VARs
- Needs capacitive VAR compensation at terminals using static capacitor bank.
- May need to be pole changing to cope with changing wind speed.

(b)

R_1 - Stator resistance per phase

R_2' - Referred rotor resistance per phase

R_m - Magnetising resistance per phase (represents iron losses)

$R_2' \frac{(1-s)}{s}$ - Represents mechanical input or output power/phase

X_1 - Stator leakage reactance per phase

X_2' - Referred rotor leakage reactance per phase

X_m - Magnetising reactance per phase.

(c) Since the generator is star-connected:

$$V_{ph} = \frac{11000}{\sqrt{3}} = 6351V$$

For an 8-pole machine operating on a 50Hz supply the synchronous speed is:

$$N_{sync} = \frac{60f}{PP} = \frac{3000}{4} = 750 \text{ rpm}$$

Since the turbine is rotating at 870 rpm the per unit slip is:

$$s = \frac{N_s - N_r}{N_s} = \frac{750 - 870}{750} = -0.16$$

QUESTION 4 (CONTINUED)

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$$\text{Hence } R_L = \frac{R_2' (1-s)}{s} = \frac{1.2 (1 - (-0.16))}{-0.16} = -8.7 \Omega$$

$$\begin{aligned} \therefore Z_{\text{LOAD}} &= R_1 + R_2' + \frac{R_2' (1-s)}{s} + jX_1' + jX_2' \\ &= 0.25 + 1.2 - 8.7 + j2.6 + j2.5 = -7.25 + j5.1 \\ &= 8.864 \angle 144.8^\circ \Omega \end{aligned}$$

$$\therefore I_{\text{LOAD}} = \frac{V_{\text{ph}}}{Z_{\text{LOAD}}} = \frac{6351 \angle 0^\circ}{8.864 \angle 144.8^\circ} = 716.5 \angle -144.8^\circ \text{ A}$$
$$= -585.5 - j413$$

Magnetising branch:

$$I_R = \frac{6351 \angle 0^\circ}{280 \angle 0^\circ} = 22.68 \angle 0^\circ \text{ A}$$

$$I_X = \frac{6351 \angle 0^\circ}{65 \angle 90^\circ} = 97.7 \angle -90^\circ \text{ A}$$

$$\therefore I_m = 22.68 - j97.7 \text{ A} = 100.3 \angle -76.9^\circ$$

So the total machine current is:

$$\begin{aligned} I_T &= I_m + I_{\text{LOAD}} = 716.5 \angle -144.8^\circ + 100.3 \angle -76.9^\circ \\ &= 759.9 \angle -137.8^\circ \text{ A} (= -562.9 - j510j) \end{aligned}$$

$$\begin{aligned} \text{Apparent power} &= 3 V_{\text{ph}} I_{\text{ph}}^* = 3 \times 6351 \angle 0^\circ \times 759.9 \angle 137.8^\circ \\ &= 14.478 \angle 137.8^\circ \text{ MVA} \\ &= (-10.73 \text{ MW} + 9.72 \text{ MVAR}) \end{aligned}$$

The real power input to the generator is -10.73 MW
or the electrical power generation is 10.73 MW .

Reactive power = 9.72 MVAR (inductive) load on system.

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QUESTION 4 (CONTINUED)

(13)

(c)(ii) The mechanical power is:

$$\frac{3 R_2' (1-s)}{s} I_{LOAD}^2 = 3 \times 8.7 \times 716.5^2$$
$$= 13.4 \text{ MW}$$

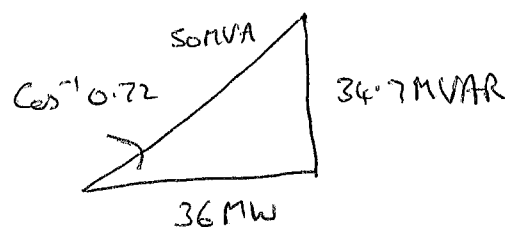
(2)

\therefore the efficiency of the system is $\frac{10.73}{13.4} \times 100 = \underline{\underline{80.1\%}}$

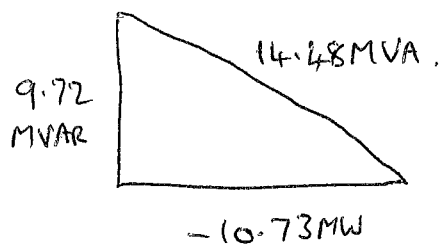
(c)(iii)

Base load of the factory:

$$50 \text{ MVA @ } 0.72 \text{ lagging} = 36 \text{ MW} + 34.7 \text{ MVAR}$$



For the induction generator:



\therefore Total power requirements of the factory with the generator connected: (3)

$$S = (36 - 10.73) + j(34.7 + 9.72) = 25.27 \text{ MW} + 44.42 \text{ MVA}$$
$$= 51.1 \angle 60.36^\circ \text{ (p.f.} = 0.495 \text{)}$$

(iv) The capacitor bank only affects the reactive power:

If overall p.f. = 0.95

$$P = S \cos \phi \Rightarrow S = \frac{P}{\cos \phi} = \frac{25.27}{0.95} = 26.6 \text{ MVA}$$

$$Q_T = S \sin \phi = 26.6 \sin(\cos^{-1} 0.95) = 8.306 \text{ MVAR}$$

Hence the capacitor has to supply $44.42 - 8.306 = 36.114 \text{ MVA}$
or 12.04 MVAR/phase

QUESTION 4 (CONTINUED)

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$$\text{Since } Q_c = \frac{V_{ph}^2}{X} \Rightarrow X = \frac{V_{ph}^2}{Q_c} = \frac{\left(\frac{11000}{\sqrt{3}}\right)^2}{12.04 \times 10^6} = 3.35 \Omega$$

$$\text{and } C = \frac{1}{2\pi f X} = \frac{1}{2\pi \cdot 50 \cdot 3.35}$$

$$= \underline{950 \mu F \text{ per phase}}$$

3

(v) With capacitors connected, but wind turbine not operational the power requirements will be:

$$S_T' = 36 \text{ MW} + 34.7 \text{ MVAR} - 36.114 \text{ MVAR}$$
$$= 36 \text{ MW} - 1.414 \text{ MVAR} = 36.03 \angle -2.25^\circ$$

$$\text{P.f.} = 0.999 \text{ (leading).}$$

1