Worked Solutions to Problems in Tutorial Sheet 3

1. (i) $\mu_r = 1$, $\rho = 1.72 \times 10^{-8}\Omega m$

$$\therefore \sigma = \frac{1}{1.72x10^{-8}} = 58.1x10^6\, Sm$$

$$\therefore \delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{314.2x4\pi x10^{-7}\, x58.1x10^6}} = 9.34mm$$

(ii) $\mu_r = 2000$, $\rho = 2.8 \times 10^{-7}\Omega m$

$\therefore \sigma = 3.57 \times 10^6$ Sm

$$\therefore \delta = \sqrt{\frac{2}{314.2x2000x4\pi x10^{-7}\, x3.57x10^6}} = 0.84mm$$

NOTE: Even though mild steel is ~16 times less conducting than copper, the skin effect is significantly more pronounced due to its high relative permeability.

2. $J_x = J_s\, e^{\alpha y} = \alpha H_s e^{\alpha y}$

Eddy current loss $P_e = \dfrac{H_s^2}{2\sigma\delta}\, w/m^2$

Since skin depth δ << radius of conductor, thick plate model is a reasonable representation of the circular conductor of radius a

$\therefore$ From Amperes Law, $H_s = \dfrac{I}{2\pi a} \quad A/m$

$\therefore$ Loss/unit area $= \left(\dfrac{I}{2\pi a}\right)^2 \dfrac{1}{2\sigma\delta} \quad w/m^2$

Loss/m length of conductor = $\left(\dfrac{I}{2\pi a}\right)^2 \dfrac{1}{2\sigma\delta} 2\pi a \quad w/m$

where I is the peak ac current

$\therefore$ Loss/m length of conductor = $\dfrac{1}{2} I^2 R_{ac} \quad w/m$

$$\therefore R_{ac} = \frac{1}{2\pi a\sigma\delta} \quad \Omega/m$$

For $f = 20 \text{ x } 10^3$ Hz

$a = 2 \text{ x } 10^{-3}$ m

$$\sigma = \frac{1}{1.72x10^8}$$

$R_{ac} = 2.9 \text{ x } 10^{-3}\ \Omega/\text{m}$

$$R_{dc} = \frac{1}{\sigma\pi a^2} = 1.37x10^{-3}\Omega / m$$

$$\therefore \frac{R_{ac}}{R_{dc}} = 2.11$$