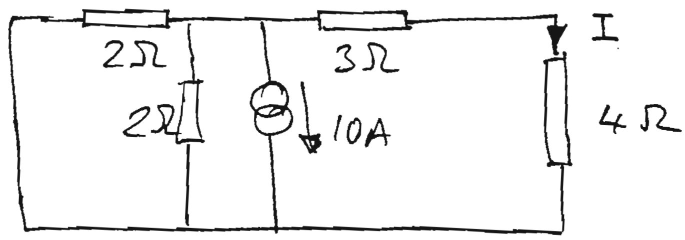
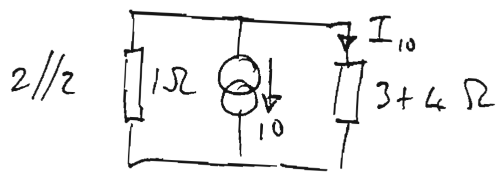
Solutions

1 (a) The response of a linear circuit to several sources acting together is equal to the algebraic sum of the responses due to each source acting individually. 2

(b) <u>10 A source alone</u> (s/c voltage source)

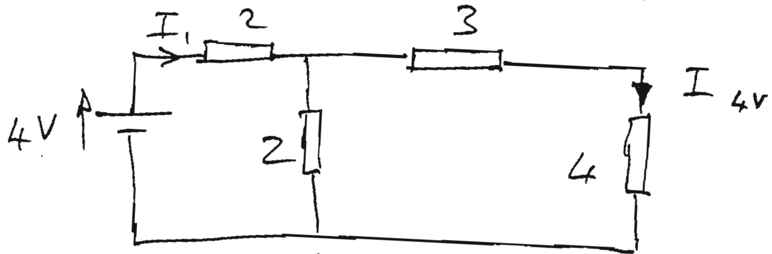


⇓ 1



$$I_{10A} = \frac{1}{1+7} \times 8 = \frac{10}{8} = \underline{-1.25A}$$ (opposite direction to that shown). 2

4V source alone (o/c current source)



1

$$I_1 = \frac{4}{2+2//7} = \frac{4}{2+1.555} = 1.125\ A$$

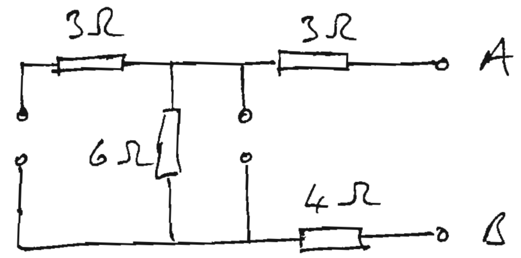
$$I_{4V} = \frac{2}{9} \times 1.125 = 0.25A \qquad 2$$

$$\therefore \quad I = I_{10A} + I_{4V} = -1.25 + 0.25$$

$$= \underline{-1\ A} \qquad 2$$

Q1 (cont.)

(c) To get R_T o/c current sources



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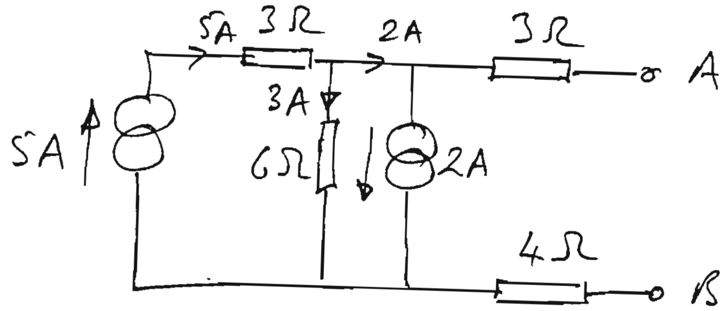
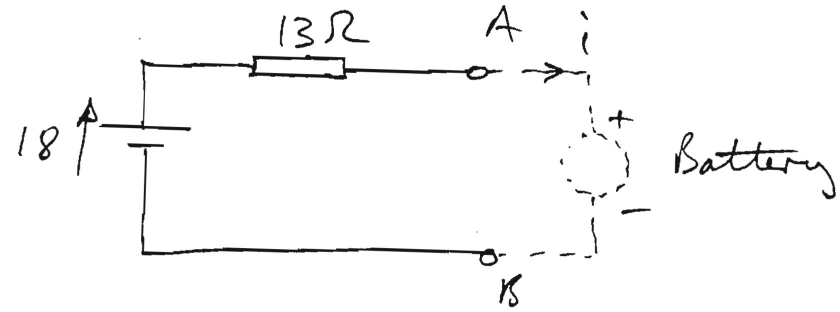
$R_T = 3 + 6 + 4 = 13\ \Omega$

For E_T



$V_{6\Omega} = 3 \times 6 = 18V \equiv$ voltage across AB

hence $E_T = 18\ V$. 4



Battery will always receive power since the supply voltage always exceeds the battery voltage.

When charged to 15V

Current $i = \frac{18-15}{13} = \underline{0.23A}$ 2

Q2 (a) Capacitor energy stored in electric field
Inductor energy store in magnetic field. 2

Capacitor energy established by charging from a voltage source through a resistor. 2
Inductor energy established by setting up a current through it.

Energy recovered by:-
Discharging capacitor through a resistor
Removing source of current in the inductor and allow the current to dissipate through a resistive load. 2

(b) Instantaneous power $p = vi = L i \frac{di}{dt}$ 1

Energy absorbed in interval t

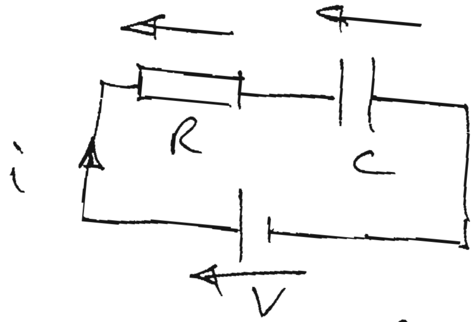
$$= \int_0^t p\,dt' = L \int_0^t i \frac{di}{\cancel{dt'}} \cdot \cancel{dt'}$$ 1

$$= L \int_0^I i\,di$$ where I is the current flowing at time t

$$= \underline{\tfrac{1}{2} L I^2}$$ 2

Q 2 (cont.)

(c)



1

Apply K2

$$V - iR - \frac{1}{C}\int i\,dt = 0 \qquad 1$$

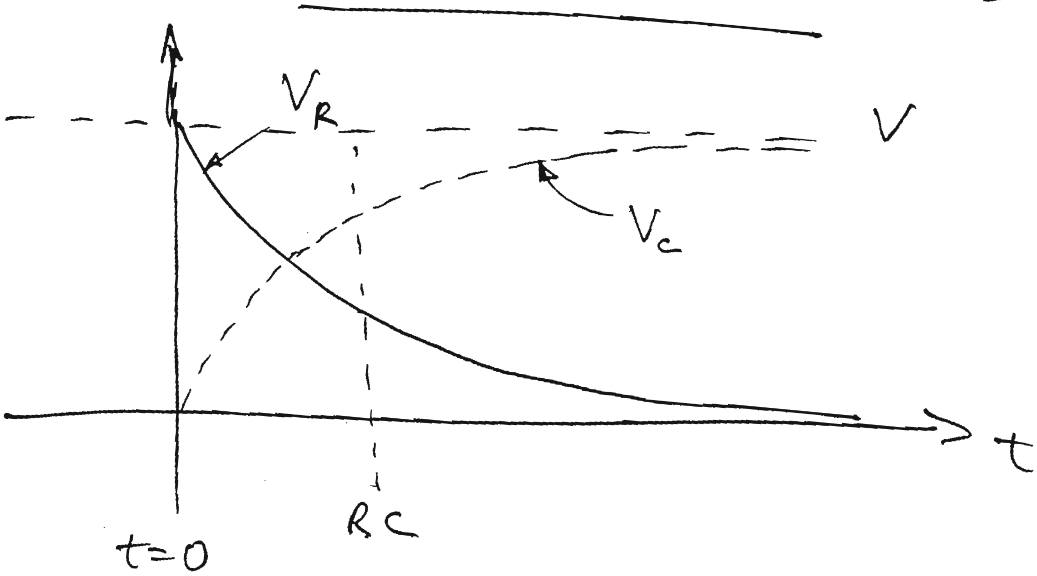
$$\therefore \; -R\frac{di}{dt} = i/C$$

$$\therefore \int \frac{di}{i} = -\frac{1}{RC}\int dt$$

$$\therefore \ln i = -\frac{1}{RC}t + A \qquad 1$$

at $t = 0$ $\quad i = \frac{V}{R} \quad \therefore A = \ln V/R$ $\quad$ 1

$$\therefore \quad \underline{i = V/R \; e^{-t/RC}} \qquad 2$$



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Q3

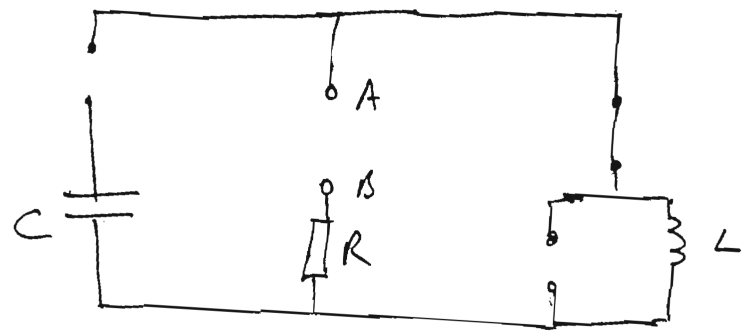
(a) Thevenin:- an active network having 2 accessible terminals, A and B, behaves, as far as the load is concerned, as if the network contained a single voltage source, E_T, and a series internal impedence, Z. (5) 2

$Z_T \equiv$ impedence across AB with current sources O/C and voltage sources S/C.

$E_T \equiv$ voltage across AB with load disconnected 2

(b)

Z_T O/C current source, S/C voltage source



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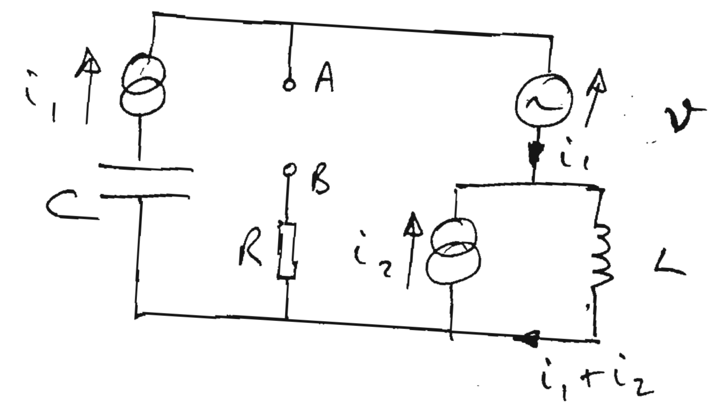
$$Z_T = \underline{R + j\omega L}$$



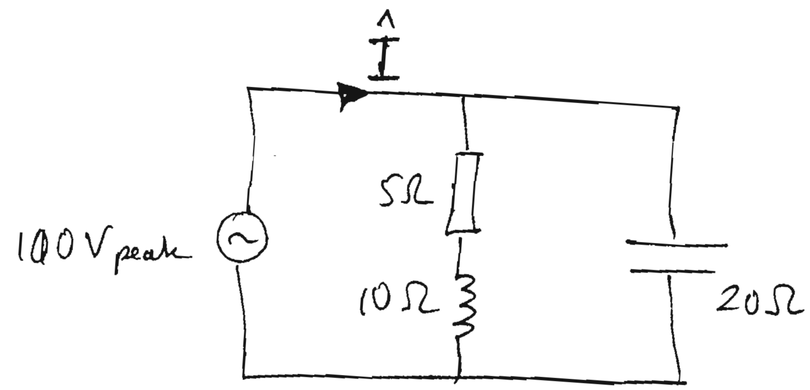
From inspection

$$\underline{E_T = v + j\omega L(i_1 + i_2)}$$

3

Q3 (cont.)

(c)



$$Z = (5 + j10) // -j20$$

$$= \frac{(5 + j10)(-j20)}{5 + j10 - j20} = \frac{(2 - j)100}{5 - j10}$$

rationalize $$= \frac{100(2-j)(5+j10)}{5^2 + 10^2}$$

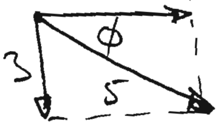
$$= \frac{100(10 - j5 + j20 + 10)}{125}$$

$$= \underline{16 + j12}$$ 4

$$\hat{I} = \frac{100}{16 + j12} = \frac{100(16 - j12)}{16^2 + 12^2} = \underline{\underline{4 - j3}}$$

2

Phase angle



$$|\hat{I}| = 5$$

$$\phi = \tan^{-1} 3/4 = \underline{-36.9°}$$

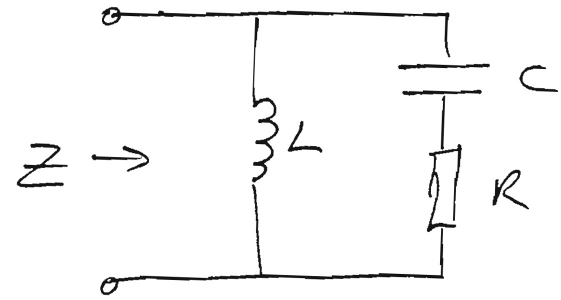
4

EEE101

Q4 (a) Resonance occurs at a non-zero frequency where the imaginary part of the circuit is zero ie. Z becomes real. 2

(b)

$$Z = j\omega L // \left(R - \frac{j}{\omega C}\right)$$



2

$$= \frac{j\omega L\left(R - j/\omega C\right)}{j\left(\omega L - \frac{1}{\omega C}\right) + R}$$

$$= \frac{j\omega L\left(R - j/\omega C\right)\left(R - j\left(\omega L - \frac{1}{\omega C}\right)\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Numerator

$$= \left(j\omega LR + \frac{L}{C}\right)\left(R - j\left(\omega L - \frac{1}{\omega C}\right)\right) \qquad 2$$

$$= j\omega LR^2 + \frac{RL}{C} + \omega LR\left(\omega L - \frac{1}{\omega C}\right) - j\frac{L}{C}\left(\omega L - \frac{1}{\omega C}\right)$$

Imaginary part.

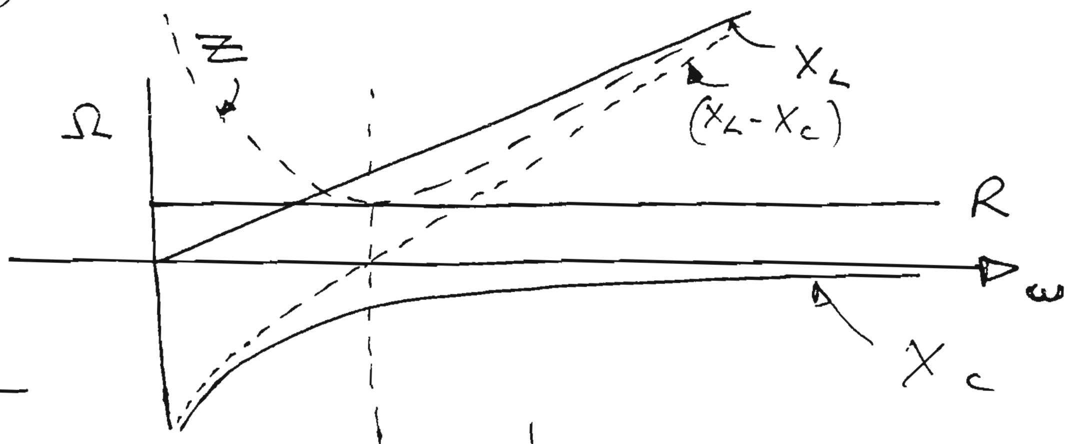
$$-\omega^2 LR^2 + \frac{L^2\omega^2}{C} - \frac{L}{\cancel{\omega} C^2} = 0$$

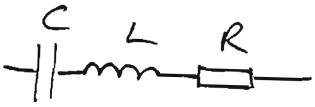
$$\therefore \quad \omega = \sqrt{\frac{L/C^2}{L^2/C - LR^2}}$$

$$= \sqrt{\frac{1}{LC - C^2R^2}} \qquad 2$$

Q4 (cont.)

(c)





$$\omega_r = \frac{1}{\sqrt{LC}}$$

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$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$
$$= R + (X_L - X_c)$$

(d) At resonance (30kHz) $Z = R$

$$\therefore R = \frac{V}{I} = \frac{5}{20\times10^{-3}} = \underline{250\,\Omega}$$ 2

now $$Q = \frac{f_r}{\Delta f} = \frac{30}{3} = 10 = \frac{\omega_0 L}{R}$$

$$\therefore L = \frac{10\times 250}{2\pi\times3\times10^{4}} = \underline{13.27\text{ mH.}}$$ 2