Solution to Q1

Using (1.2) in the question,

$$P(\theta)=I_o e^{\frac{j(1-N)}{2}(kd\cos\theta+\gamma)}\left[1+e^{j(kd\cos\theta+\gamma)}+\ldots+e^{j(N-1)(kd\cos\theta+\gamma)}\right] \tag{1.1}$$

Multiplying the LHS of (1.1) by $e^{j(kd\cos\theta+\gamma)}$ yields

$$P(\theta)e^{j(kd\cos\theta+\gamma)}=I_o e^{\frac{j(1-N)}{2}(kd\cos\theta+\gamma)}\left[e^{j(kd\cos\theta+\gamma)}+\ldots+e^{jN(kd\cos\theta+\gamma)}\right] \tag{1.2}$$

and subtracting (1.2) from (1.1) yields

$$P(\theta)(1-e^{j\Psi})=I_o e^{\frac{j(1-N)}{2}\psi}\left[1-e^{jN\psi}\right] \tag{1.3}$$

where

$$\psi=kd\cos\theta+\gamma \tag{1.4}$$

Rewriting (1.3) as

$$P(\theta)e^{j\frac{1}{2}\Psi}(e^{-j\frac{1}{2}\Psi}-e^{j\frac{1}{2}\Psi})=I_o e^{\frac{j(1-N)}{2}\Psi}\left[1-e^{jN\Psi}\right] \tag{1.5}$$

gives

$$P(\theta)(e^{-j\frac{1}{2}\Psi}-e^{j\frac{1}{2}\Psi})=I_o\left[e^{-j\frac{N}{2}\Psi}-e^{\frac{jN}{2}\Psi}\right] \tag{1.6}$$

hence

$$-P(\theta)2j\sin\left(\frac{1}{2}\Psi\right)=-2I_o j\sin\left(\frac{N}{2}\Psi\right) \tag{1.7}$$

so that

$$P(\theta)=I_o\frac{\sin\left(\frac{N}{2}\Psi\right)}{\sin\left(\frac{1}{2}\Psi\right)} \tag{1.8}$$

(b)

(i) For a broadside array, from (1.4)

$$\Psi\big|_{\theta=90^o}=0 \tag{1.9}$$

hence

$$\gamma=0^o \tag{1.10}$$

(ii) For an end-fire array,

$$\Psi\big|_{\theta=0^o} = 0 \qquad (1.11)$$

so

$$\gamma = -kd \qquad (1.12)$$

At 500MHz therefore

$$\gamma = -\frac{360}{.6}\times .3 = -180^o \qquad (1.13)$$

(iii) To fire at 30^o off broadside,

$$\Psi\big|_{\theta=60^o} = 0 \text{ or } \Psi\big|_{\theta=120^o} = 0 \qquad (1.14)$$

so

$$\gamma = \mp\frac{360}{.6}\times .3\times 0.5 = \mp 90^o \qquad (1.15)$$

(c)

As the frequency is varied, the main beam direction will change (assuming γ remains the same) according to

$$\frac{360\times(500\pm 100)\times 10^6\times 0.3}{3\times 10^8}\cos\theta \pm 90 = 0 \qquad (1.16)$$

So, the beam direction will change from

$$\theta = \cos^{-1}\left(\frac{\mp 90\times 3\times 10^8}{360\times 600\times 10^6\times 0.3}\right) = 65.4^o \text{ or } 114.6^o \qquad (1.17a)$$

to

$$\theta = \cos^{-1}\left(\frac{\mp 90\times 3\times 10^8}{360\times 400\times 10^6\times 0.3}\right) = 51.3^o \text{ or } 128.7^o \qquad (1.17b)$$

Thus the beam squint decreases from 38.7^o to 24.6^o off broadside as the frequency increases.

Solution to Q2

(a)

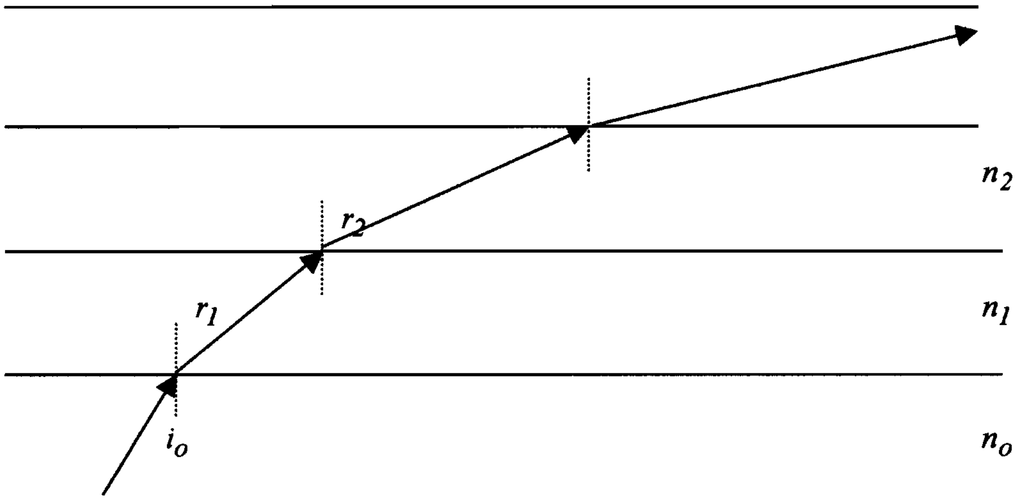


Fig 2.1

Fig 2.1 shows how a wave is progressively refracted by ionospheric layers of differing refractive indices. After the upper layer the wave starts being refracted back down to earth due to the earth's curvature. From Snell's law:

$$n_o \sin i_o = n_1 \sin r_1 = n_2 \sin r_2 = ... \qquad (2.1)$$

but at reflection layer, $r_{ref} = 90^o$, and $n_o \approx 1$ (air) so that

$$\sin i_o \approx n_{ref} \qquad (2.2)$$

The refractive index of this upper layer is given by

$$n_{ref} = \sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} = \sin i_o \qquad (2.3)$$

hence

$$\cos i_o = \frac{\omega_c}{\omega} \qquad (2.4)$$

(ω is the frequency of the radio wave and ω_c the layer *critical frequency*)

(b)

For vertical incidence into the ionosphere,

$$i_o = 0 \qquad (2.5)$$

so

$$\omega = \omega_c \qquad (2.6)$$

Thus, in this case

$$f_c = 9MHz \approx 9\sqrt{N} \quad (2.7)$$

where N denotes the electron density of the reflection layer. Hence

$$N \approx 10^{12} / m^3 \qquad (2.8)$$

The height of the layer is given by the distance travelled by the signal in half of the round trip time,

$$h = \frac{1}{2} \times 3 \times 10^{-3} \times 3 \times 10^{8}\, m = 450km \qquad (2.9)$$

This therefore identifies the layer as F_2.

(c)

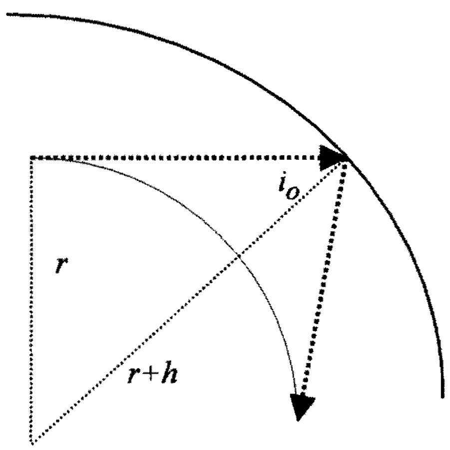


Fig 2.2

With reference to Fig 2.2, we need to determine $\cos i_o$. Hence

$$\sin i_o = \frac{r}{r+h} \qquad (2.10)$$

so

$$\cos i_o = \sqrt{1 - \left(\frac{r}{r+h}\right)^2} = 0.37 \qquad (2.11)$$

and therefore from Eq 2.4

$$f = \frac{9 \times 10^6}{.37} = 24.32MHz \qquad (2.12)$$

(d)
Clearly from Fig 4.2 a zero elevation launch angle produces the maximum value of i_o, and hence the minimum value of $\cos i_o$ and the longest skip distance. A frequency higher than $24.32MHz$ would require a lower value of $\cos i_o$ and thus a greater incidence angle into the ionosphere i_o, and therefore could not be reflected by this

layer. Conversely, lower frequencies down to *9MHz* would require a larger $\cos i_0$ so a non-zero elevation angle giving a smaller i_0 would be required, and such signals would thus be reflected with shorter skip distances. There would be no reflection below *9MHz*.

Solution to Q3

(a)

Radiated power density of an antenna is

$$\underline{P_d} = \frac{1}{2}(\underline{E} \times \underline{H}^*) Wm^{-2} \qquad (3.1).$$

Note that in the far field (3.1) reduces to

$$P_r = \frac{1}{2} E_\theta H_\phi^* \qquad (3.2).$$

Now, since

$$\frac{E_\theta}{H_\phi} = \eta \qquad (3.3)$$

relates the only field components present, then (3.2) reduces to

$$P_r = \frac{1}{2}\frac{|E_\theta|^2}{\eta} Wm^{-2} \qquad (3.4).$$

Note (3.4) is a *power density* in Watts per Square Metre, and so to evaluate the total power radiated by the dipole we must multiply P_r by the area it flows through. The total radiated power over a far field sphere is then given by

$$P = \int_0^{2\pi}\int_0^{\pi} P_r r\, sin(\theta) d\phi r d\theta \text{ W} \qquad (3.5).$$

The field of a $3\lambda/2$ dipole is given in question (3.1) and is independent of ϕ and so we may rewrite (3.5) thus

$$P = 2\pi r^2 \int_0^{\pi} P_r\, sin(\theta) d\theta = \frac{I_o^2 \eta}{4\pi} \int_0^{\pi} \frac{cos^2\left(\frac{3\pi}{2} cos(\theta)\right)}{sin(\theta)} d\theta \text{ W} \qquad (3.6).$$

The value of the integral is given as 1.76 in question (3.2) and therefore the power radiated is

$$P = \frac{377 I_o^2}{4\pi} \times 1.76 = 52.8 I_o^2 \text{ W} \qquad (3.7)$$

Equating this to power flowing through a fictitious radiation resistance gives,

$$\frac{1}{2} I_o^2 R_r = 52.8 I_o^2 \qquad (3.8)$$

giving a radiation resistance of

$$R_r = 105.6\,\Omega \qquad (3.9).$$

(b)

The input impedance of the dipole is given by

$$Z_{in} = R_r + X \qquad (3.10)$$

assuming negligible losses.

(i) Assuming resonance, then $X \approx 0$ and so $Z_{in} \approx 105.6\Omega$ (3.11)

(ii) The effect of the dielectric insulation will be to increase the dipole's electrical length, so that $X \neq 0$. The dipole input impedance will therefore have an inductive component, since its electrical length will now be above the third and below the fourth resonance.

(c)

Since the $3\lambda/4$ monopole only radiates into the half-space above the ground plane, only half the power is radiated for the same excitation current. So from (3.8)

$$\frac{1}{2} I_o^2 R_r = \frac{52.8}{2} I_o^2 \qquad (3.12)$$

so

$$R_r = 52.8\Omega \qquad (3.13).$$

(d)

If the same power is delivered to a $3\lambda/4$ monopole as a $3\lambda/2$ dipole, then since it is radiated into half the space in the former case, the power density must be doubled. Hence the monopole has $3dB$ more directivity than the dipole.

Solution to Q4

(a)

(i) The *area* of a triangle is $\textit{half base} \times \textit{height}$. The height of the triangle A_n^+ in Fig 4.1 is a line from the vertex to the base ℓ_n, which is perpendicular to it. This also represents the normal component of $\underline{\rho}_n^+$ flowing across the edge. Thus,

$$height = \frac{2A_n^+}{\ell_n} = \underline{\rho}_n^+\Big|_{normal} \qquad (4.1)$$

so

$$f_n\Big|_{normal} = \frac{\ell_n}{2A_n^+} \times \underline{\rho}_n^+\Big|_{normal} = \frac{\ell_n}{2A_n^+} \times \frac{2A_n^+}{\ell_n} = 1 \qquad (4.2)$$

Hence the current density normal to the edge is constant, and applying the same argument to triangle A_n^- then it must also be continuous across the edge.

(ii) Current density is related to surface charge density through the continuity relation

$$\nabla \bullet \underline{J} = -j\omega\sigma \qquad (4.3).$$

Thus using the given relation

$$\nabla \bullet f(\rho) = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho f_\rho) \qquad (4.4)$$

on the basis function f_n yields

$$\frac{1}{\rho}\left(f_\rho + \rho\frac{\partial f_\rho}{\partial \rho}\right) = \frac{\ell_n}{2A_n^+} = -j\omega\sigma \qquad (4.5)$$

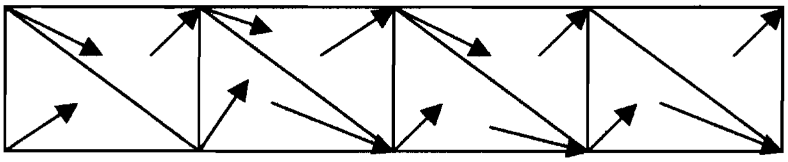
for triangle A_n^+, and

$$-\frac{\ell_n}{2A_n^+} = -j\omega\sigma \qquad (4.6)$$

for triangle A_n^- because of the opposite direction of $\underline{\rho}_n^-$ with respect to the vertex. Hence the charge density is constant within a triangle, and sums to zero over a triangle pair.

(b)

Points to note: No current vectors crossing outside edges. Complimentary current vectors crossing internal edges.



(c)

Impedance matrix elements are associated with internal edges. From (b)

$$\textit{no. of internal edges} = 2 \textit{ x no. of triangle pairs} - 1 \qquad (4.7)$$

For 10 triangle pairs, there are 19 internal edges, and the dipole feed point is across edge 10. If the dipole is excited with $0.1Vm$ in the voltage matrix, this represents a voltage of

$$V_{feed} = \frac{V_{10}}{\ell_{10}} = \frac{1}{.5 \times 10^{-3}} = 200V \qquad (4.8).$$

The current crossing edge 10 is

$$I_{feed} = I_{10} \times \ell_{10} = 5 \times 10^{3} \times .5 \times 10^{-3} = 2.5A \qquad (4.9)$$

Thus

$$Z_{in} = \frac{V_{feed}}{I_{feed}} = \frac{200}{2.5} = 80\Omega \qquad (4.10).$$