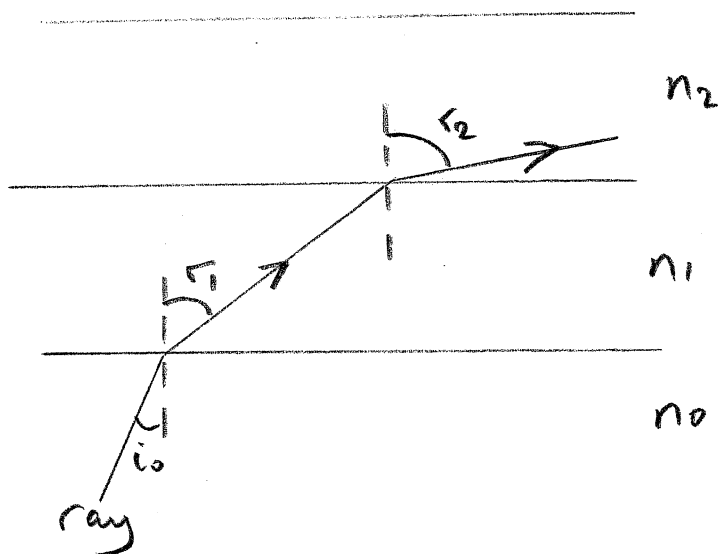


## EEE6223 2015 Solutions

Q1)

(a)



From Snell's law:

$$n_0 \sin i_0 = n_1 \sin r_1 = n_2 \sin r_2 = \dots = n_5 \sin r_5 \text{ (eventually)} \quad (1)$$

but  $r_5 = 90^\circ$  and  $n_0 \approx 1$  (air) so that

$$\sin i_0 \approx n_5 \quad (2)$$

The refractive index of this upper layer is given by

$$n_5 = \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} = \sin i_0 \quad (3)$$

hence

$$\cos i_0 = \frac{\omega_c}{\omega} \quad (4)$$

( $\omega$  is the frequency of the radio wave and  $\omega_c$  the layer critical frequency)

(b)

For vertical incidence into the ionosphere,

$$i_0 = 0 \quad (5)$$

so

$$\omega = \omega_c \quad (6)$$

Thus, in this case

$$f_c = 8\text{MHz} \approx 9\sqrt{N} \quad (7)$$

where  $N$  denotes the electron density of the reflection layer. Hence

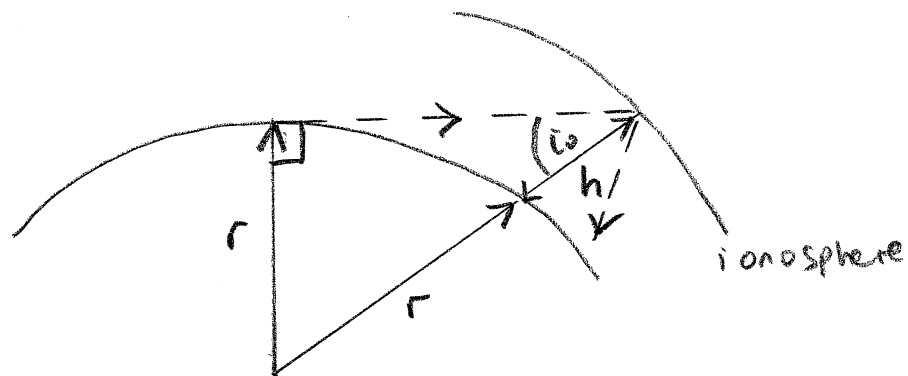
$$N \approx 7.9 \times 10^{11} / \text{m}^3 \quad (8)$$

The height of the layer is given by the distance travelled by the signal in half of the round trip time,

$$h = \frac{1}{2} \times 2.5 \times 10^{-3} \times 3 \times 10^8 \text{ m} = 375 \text{ km} \quad (9)$$

This therefore identifies the layer as  $F_2$ .

(c)



We need to determine  $\cos i_o$ . Hence

$$\sin i_o = \frac{r}{r+h} \quad (10)$$

so

$$\cos i_o = \sqrt{1 - \left(\frac{r}{r+h}\right)^2} = 0.3379 \quad (11)$$

and therefore from (3.4)

$$f = \frac{8}{.3379} = 23.68 \text{ MHz} \quad (12)$$

A zero elevation launch angle produces the maximum value of  $i_o$ , and hence the minimum value of  $\cos i_o$  and the longest skip distance. A frequency higher than 23.68 MHz would require a lower value of  $\cos i_o$  and thus a greater incidence angle into the ionosphere  $i_o$ , and therefore could not be reflected by this layer. Conversely, lower frequencies down to 8 MHz would require a larger  $\cos i_o$  so a non-zero elevation angle giving a smaller  $i_o$  would be required and such signals would thus be reflected with shorter skip distances. There would be no reflection below 8 MHz.

Q2)

(a)

- (i) The dipole radiation pattern is the first term in question

$$C \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \quad (1) \quad \textcircled{1}$$

- (ii) The array factor is the second term

$$I \frac{\sin(\frac{Nkd}{2} \cos \theta)}{\sin(\frac{kd}{2} \cos \theta)} \quad (2) \quad \textcircled{1}$$

(iii)

$$I = \frac{I_o}{\sqrt{N}} \quad (3) \quad \textcircled{1}$$

- (iv) Neglecting mutual coupling

$$Z_{in} \approx \frac{73}{N} \Omega \quad (4) \quad \textcircled{2}$$

(b)

- (i) The antenna gain is

$$10 \log_{10} N = 10.4 \text{ dBd} = 12.6 \text{ dBi} \quad (5) \quad \textcircled{2}$$

- (ii) Assuming the dipole pattern changes much more slowly than the array factor, at the first sidelobe

$$\frac{Nkd}{2} \cos \theta = \pm \frac{3\pi}{2} \quad (6) \quad \textcircled{1}$$

so substituting values

$$\cos \theta = \pm \frac{3\pi}{11} \times \frac{\lambda}{2\pi} \times \frac{1}{0.7\lambda} = \pm 0.195 \quad (7) \quad \textcircled{1}$$

so first sidelobe position is at

$$\theta = 78.8^\circ \text{ or } 101.2^\circ \quad (8) \quad \textcircled{2}$$

Height of first sidelobe with respect to main lobe is

$$20 \log_{10} \left( \frac{I \left| \sin\left(\frac{3\pi}{2}\right) \right|}{\sin\left(\frac{3\pi}{2N}\right)} / (NI) \right) = -13.2 \text{ dB} \quad (9) \quad \textcircled{2}$$

(iii) Given in question

$$\frac{\sin(11 \times 0.127)}{11 \times \sin(0.127)} = 0.71 \quad (10).$$

This defines the  $-3\text{dB}$  point on the radiation pattern, and so we have

$$\frac{kd}{2} \cos \theta = 0.127 \quad (11) \quad \textcircled{2}$$

and therefore the 3dB beamwidth is given by

$$2 \times (90^\circ - \theta) = 6.6^\circ \quad (12). \quad \textcircled{2}$$

(c)

The main beam could be electronically steered by feeding each dipole element with a differential phase shift. This could be achieved by having programmable phase shifters in the transmission lines feeding the elements. An asymmetric sidelobe distribution could be achieved by having non-uniform element spacings. This may be useful for stealth purposes. \textcircled{2}

\textcircled{1}

Q3)

(a)

If  $L = \lambda$  then

$$\frac{kL}{2} = \pi \quad (1) \quad \textcircled{1}$$

so that Equation (1) becomes

$$|E_\theta| = \frac{2\eta I_o}{4\pi r} \left[ \frac{\cos(\pi \cos(\theta)) - \cos(\pi)}{\sin(\theta)} \right] = \frac{2\eta I_o}{4\pi r} \left[ \frac{\cos(\pi \cos(\theta)) + 1}{\sin(\theta)} \right] \quad (2) \quad \textcircled{1}$$

Now the gain is given by

$$G = \frac{P_r|_{\theta=90^\circ}}{\frac{P}{4\pi r^2}} \quad (3) \quad \textcircled{1}$$

where  $P_r$  denotes radiated power density and  $P$  the total radiated power. Now,

$$P_r = \frac{1}{2} \frac{|E_\theta|^2}{\eta} \text{ Wm}^{-2} \quad (4) \quad \textcircled{1}$$

and

$$P = \int_0^{2\pi} \int_0^\pi P_r r \sin(\theta) d\phi r d\theta \quad (5) \quad \textcircled{2}$$

so that

$$P = 2\pi r^2 \int_0^\pi P_r \sin(\theta) d\theta = \frac{I_o^2 \eta}{4\pi} \int_0^\pi \frac{(\cos(\pi \cos(\theta)) + 1)^2}{\sin(\theta)} d\theta \quad (6) \quad \textcircled{2}$$

Hence from Equation (2)

$$P = \frac{I_o^2 \eta}{4\pi} \times 3.318 \quad (7) \quad \textcircled{1}$$

Also from Equation (1) and (4)

$$P_r|_{\theta=90^\circ} = \frac{\eta I_o^2}{2\pi^2 r^2} \quad (8) \quad \textcircled{1}$$

Thus, from (3)

$$G = \frac{8}{3.318} = 2.41 \quad (9) \quad \textcircled{1}$$

so

$$10 \log_{10} G = 3.8 \text{ dB} \quad (10) \quad \textcircled{1}$$

(b)

*Advantage:* The gain of a full wave dipole is  $1.7 \text{ dB}$  higher than that of a half wave dipole.

*Disadvantage:* The input impedance of a full wave dipole is very high since it's fed at a current minimum, which makes matching into a transmission line difficult. A half wave dipole however has an input impedance  $\sim 73 \Omega$  which is commensurate with the characteristic impedance of coaxial cable. A full wave dipole is also twice the length of a half wave dipole of course. \textcircled{2}

(c)

The effective aperture of an antenna is given by

$$A_e = \frac{\lambda^2}{4\pi} G \quad (11) \quad \textcircled{1}$$

So from (9),

$$A_e = \frac{0.375^2}{4\pi} \times 2.41 = 0.027 \text{ m}^2 \quad (12) \quad \textcircled{1}$$

The power received by the antenna  $P$  is then

$$P = A_e \times P_d \quad (13) \quad \textcircled{1}$$

where  $P_d$  is the power density of the incident plane wave. So

$$P_d = \frac{I}{2\eta} \times (75 \times 10^{-3})^2 = 7.5 \mu\text{W} / \text{m}^2 \quad (14) \quad \textcircled{1}$$

Thus,

$$P = 0.2 \mu\text{W} \quad (15)$$

(d)

In (11) the aperture is proportional to the wavelength squared. At  $1600 \text{ MHz}$  the wavelength is halved, so the absorbed power in (13) is a quarter i.e.  $P = 0.05 \mu\text{W}$ . \textcircled{2}

Q4)

(a) As shown in Fig.1

(b)

The radiated power density is given by

$$P_r = \frac{1}{2} \frac{|E_\theta|^2}{\eta} \quad (1) \quad \textcircled{1}$$

where  $|E_\theta|$  is given in the question, so that

$$P_r = \frac{1}{2\eta} \frac{\eta^2 I_o^2}{4\pi^2 r^2} \frac{\cos^2\left(\frac{\pi}{2} \cos(\theta)\right)}{\sin^2(\theta)} \quad (2) \quad \textcircled{1}$$

The power radiated into the half space above the ground plane is then

$$P = \int_0^{2\pi} \int_0^{\pi/2} P_r r \sin(\theta) d\phi d\theta \quad (3) \quad \textcircled{2}$$

Since the fields are invariant in  $\phi$ , (3) reduces to

$$P = 2\pi \int_0^{\pi/2} P_r r^2 \sin(\theta) d\theta \quad (4) \quad \textcircled{1}$$

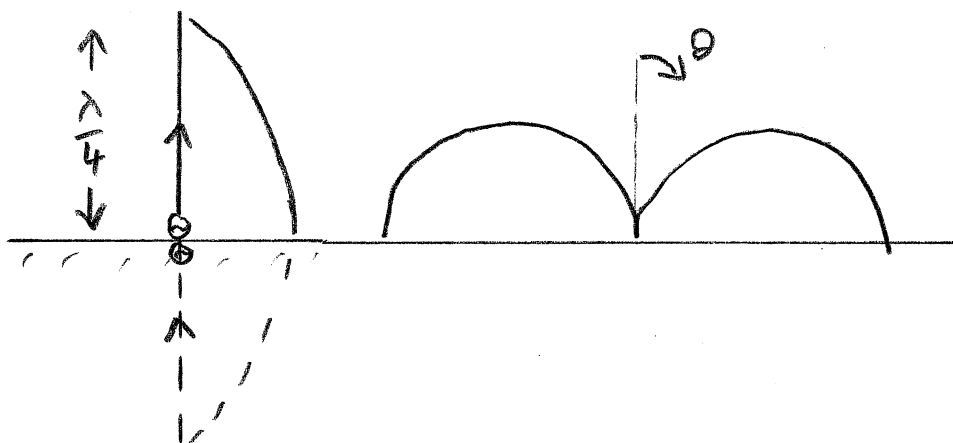
Substituting (2) into (4) then gives

$$P = \frac{\eta I_o^2}{4\pi^2} \int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2} \cos(\theta)\right)}{\sin(\theta)} r^2 d\theta \quad (5) \quad \textcircled{1}$$

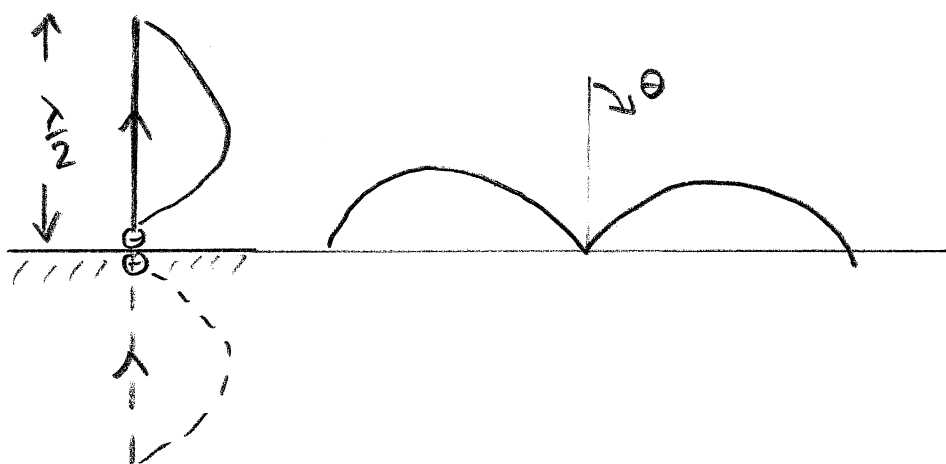
Substituting the value for the integral given in the question then yields

$$P = \frac{\eta I_o^2}{4\pi} \times 0.61 \quad (6) \quad \textcircled{1}$$

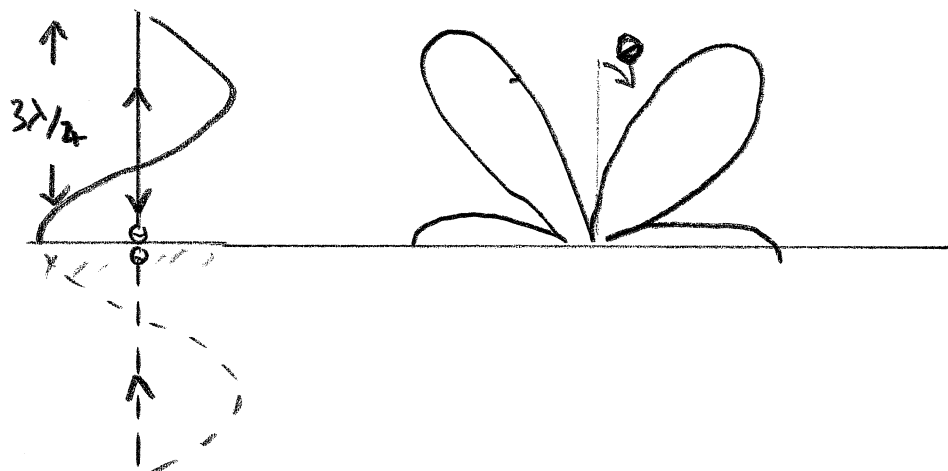
(i) We now equate this power to that dissipated in a fictitious equivalent circuit 'radiation resistance'  $R_r$  thus:



②



②



②

Fig 1.



$$\frac{\eta I_o^2}{4\pi} \times 0.61 = \frac{I_o^2}{2} R_r \quad (7) \quad (1)$$

so that

$$(i) R_r = \frac{377}{2\pi} \times 0.61 = 36.6 \Omega \quad (8) \quad (1)$$

(ii) Gain (=directivity since no losses) at  $\theta = 90^\circ$  is obtained from (2) and (3) as

$$G = \frac{P_r|_{\theta=90^\circ}}{P / 4\pi r^2} \quad (9) \quad (1)$$

hence

$$(ii) G = \frac{1}{2\eta} \frac{\eta^2 I_o^2}{4\pi^2 r^2} \times \frac{4\pi}{0.61 I_o^2 \eta} \times 4\pi r^2 = 3.28 \equiv 5.2 dBi \quad (10) \quad (2)$$

(c) The half wave dipole has (i)  $R_r = 73.2 \Omega$  and  $G = 5.16 - 3 dBi = 2.2 dBi$ . In other words the radiation resistance is double whilst the gain is half that of the quarter wave monopole. This is because

(i) Constant supplied current: For the same terminal current  $I_o$  twice the power is now radiated (over a far field sphere instead of a hemisphere) so  $R_r$  in (7) must be doubled. (2)

(ii) Constant radiated power: The same radiated power is now distributed over a sphere instead of a hemisphere, so the power density must be half that of a monopole, so the gain in (10) is halved.

**Q5)**

a) *Discuss the orbits available to satellite system designers and typical uses for each. (6)*

LEO - Low Earth Orbit. Altitude 500-1,500km. Suitable for mobile satellite phones. Small path losses.

MEO – Medium Earth Orbit. Altitude 5,000-15,000km. Suitable for weather and sensing applications.

GEO – Geostationary Earth Orbit. Altitude 36,000km. Suitable for TV broadcast, fixed communications links. High path losses.

**6 Marks**

b) *What advantages do satellite communications have over terrestrial systems and what are the key challenges faced by designers?*

**Main advantages**

- Extended capability of existing terrestrial cellular systems
- Coverage in remote areas
- Large coverage area
- Seamless service to subscriber in any part of the world

**Challenges**

- Propagation delay
- High signal attenuation with path loss ~200 dB
- Low spectral efficiency
- High costs, >\$100M
- Power limited
- Reliability , requiring a lifetime of >10 years in a harsh environment

**6 marks**

c) *Explain the terms: Noise power; Noise factor; Noise figure.*

- (i) Noise power is expressed in Watts or Watts/Hz but it is more convenient in system design to relate it to a fictitious noise temperature T through the formula  $P=kTB$  Wats.

Where

k is the Boltzmann constant =  $1.3807 \times 10^{-23}$

T is the ambient temperature (K)

B is the system bandwidth (Hz)

- (ii) Noise factor F is defined by

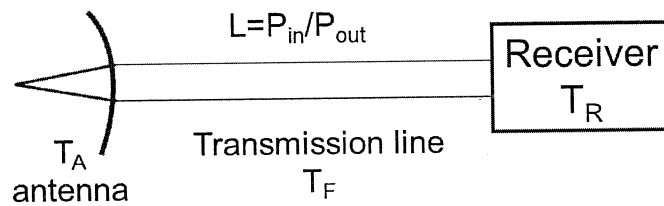
$$F = \frac{C_{in}/N_{in}}{C_{out}/N_{out}} = \frac{\text{output noise power}}{\text{input noise power}}$$

Where C is the carrier power and N is the noise power.

- (iii) Noise figure =  $10 \log (F)$

**3 marks**

- a) An antenna with a noise temperature of 105K is connected to a receiver using a cable with a loss of 2 dB. If the receiver noise figure is 2dB what is the overall noise temperature of the system? Assume the cable temperature is 290 K.



Noise temperature at the **receiver** is

$$T'_S = \frac{T_A}{L} + (1 - \frac{1}{L})T_0 + T_R$$

As

$$T_0 = T_P$$

Since noise factor for lossy line = loss  $L$  and noise temperature referred to the output is

$$T_{eout} = (1 - 1/L)T_F$$

Now  $L = 2\text{dB} = 1.585$

For the receiver

$$F = 1 + T_e/T_0 = 1.585$$

And

$$T_R = 170 \text{ K}$$

So

$$T'_S = 105/1.585 + 0.585 \cdot 290/1.585 + 170 = 66.2 + 107 + 170 = 343.2 \text{ K}$$

**5 marks**

Q6)

a) Describe the challenges facing a communications satellite from launch to life in orbit and how these are alleviated in spacecraft design.

- Satellite components need to be specially “hardened”
- Circuits which work on the ground will fail very rapidly in space
- Temperature is also a problem as the temperature gradient across a satellite can be up to 200C and therefore satellites use electric heaters to keep circuits and other vital parts warmed up. They also need to control the temperature carefully.
- Antennas need to be heat distortion resistant
- Corrosion
- Need to withstand the high levels of G forces and vibrations which occur during launch.
- The environment is a vacuum.

6 marks

b) A satellite receiver operating at 12 GHz has a noise figure of 2 dB.  
If it is directly connected to an antenna/pre-amplifier with a gain of 7 dB and a noise temperature of 100 K, estimate the overall system noise figure.

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots \dots \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}}$$

$$F_n = 1 + T_{en} / T_0$$

Antenna amplifier gain  $G_1 = 7\text{dB} = 5$

And

$$F_1 = 1 + 100/290 = 1.345$$

Receiver

$$F_2 = 2\text{dB} = 1.585$$

Hence

$$F = 1.345 + (1.585 - 1)/5 = 1.462 = 1.65 \text{ dB}$$

If the antenna/pre-amplifier is now connected to the receiver via a 5m length of coaxial cable with an attenuation of 3dB, estimate the new system noise figure.

Add lossy cable

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

Antenna amplifier gain

$$G_1 = 5 \text{ and } F_1 = 1.345$$

Receiver

$$F_3 = 2\text{dB} = 1.585$$

Lossy cable

$$F_2 = 3\text{dB} = 2 \quad \text{and } G_2 = -3\text{dB} = 0.5$$

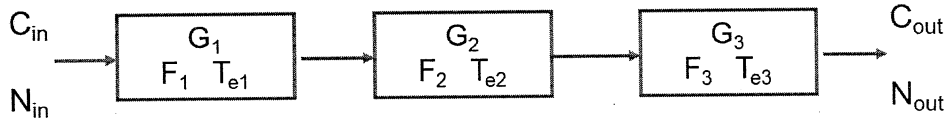
Hence

$$F = 1.345 + (2 - 1)/5 + (1.585 - 1)/(5 * 0.5) = 1.8$$

$$\therefore F = 2.5 \text{ dB}$$

7 marks

- c) Derive an expression for the noise figure of a cascade of three amplifier stages in a receiver. Comment on the design of the first amplifier (Pre-amplifier) that would be suitable for use in a satellite receiver.



Noise generated by each amplifier stage multiplied by gain of next stage and succeeding stages.

**3 stage amplifier:**

$$C_{out} = C_{in}G_1G_2G_3$$

$$N_{in} = kT_0B, \quad T_0 = 290K$$

$$N_{out} = kT_0BG_1G_2G_3 + kT_{e1}BG_1G_2G_3 + kT_{e2}BG_2G_3 + kT_{e3}BG_3$$

Now

$$F_n = 1 + T_{en} / T_0$$

**Then**

$$N_{out} = N_{in}G_1G_2G_3 + N_{in}(F_1 - 1)G_1G_2G_3 + N_{in}(F_2 - 1)G_2G_3 + N_{in}(F_3 - 1)G_3$$

$$F = (C_{in}/N_{in}) / (C_{out}/N_{out})$$

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \dots \dots \frac{F_n - 1}{G_1G_2 \dots G_{n-1}}$$

Hence it is an advantage to have a first amplifier stage with a low noise factor  $F_1$  and a high gain  $G_1$

7 marks