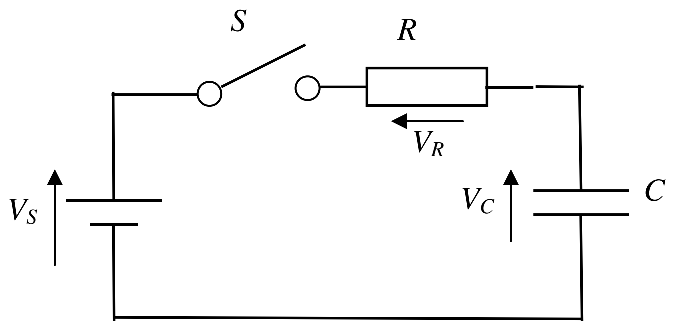
# Transient Circuits

So far we have limited our studies to circuits which have currents and voltages which are constant. Now let us look at circuits in which currents and voltages are changing with time.

## Charging a capacitor through a resistor

Consider the circuit:




Before the switch, *S*, is closed there is no voltage across the resistor, *R*, or capacitor, *C*, and the current is zero. When the switch is closed $V_S$ appears across *R* and *C*. ***The voltage across the capacitor cannot change instantaneously.***

$$V_S = V_R + V_C \text{ at all times}$$

substituting for $V_R$ and $V_C$ gives:

$$V_S = I \times R + \frac{1}{C}\int_0^t I\,dt$$

Differentiating this equation with respect to time:

$$\frac{dV_S}{dt} = \frac{dI}{dt}R + \frac{I}{C}$$

Now since $V_S$ is constant (battery voltage):

$$\frac{dI}{dt}R + \frac{I}{C} = 0$$

or

$$\frac{dI}{dt} = -\frac{1}{RC}I$$

Rearranging:

$$\frac{dI}{I} = -\frac{1}{RC}dt$$

and integrating:

$$\ln(I) = -\frac{1}{RC}t + A$$

Now when $t = 0_+$ (immediately after the switch is closed) the capacitor is in its initial (uncharged) state and hence $V_C = 0$. Let the current immediately after the switch is closed be $I_0$, so:

$$V_S = V_R + V_C = (I_0 \times R) + 0$$

or

$$I_0 = \frac{V_S}{R}$$

therefore:

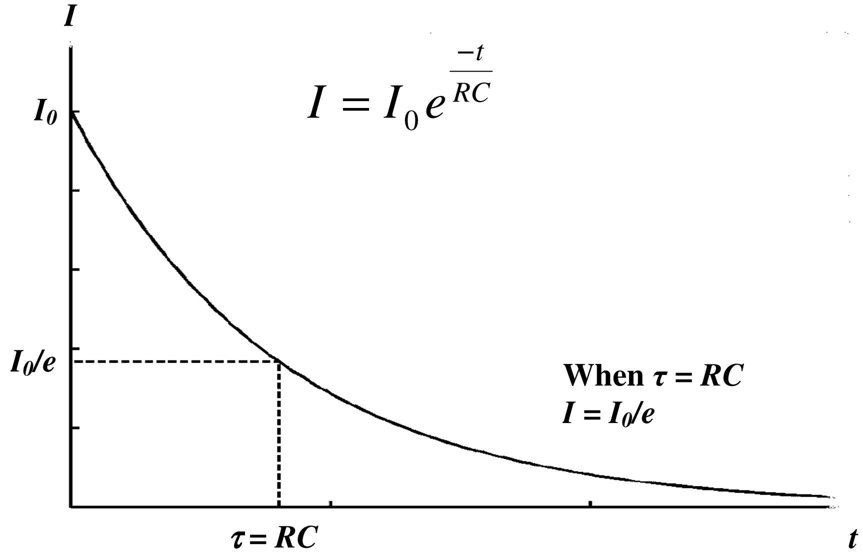
$$A = \ln(I_0)$$

Substituting for the constant of integration, A, gives:

$$\ln(I) - \ln(I_0) = -\frac{1}{RC}t$$

Rearranging:

$$I = I_0 e^{\frac{-t}{RC}}$$




$\tau$ is called the time constant and is a measure of how quickly a circuit can respond to change. It is only dependent on $R$ and $C$ (not $V_S$). The voltage across the resistor is:

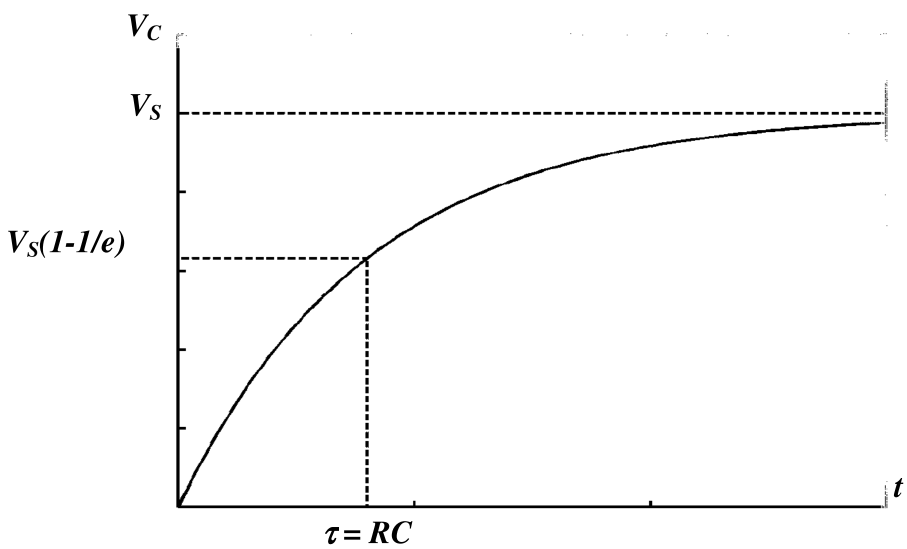
$$V_R = I \times R$$

$$V_R = RI_0 e^{\frac{-t}{RC}} = V_S e^{\frac{-t}{RC}}$$

and the voltage across the capacitor is:

$$V_C = \frac{1}{C}\int_0^t I\,dt = \frac{1}{C}\int_0^t I_0 e^{\frac{-t}{RC}}\,dt = -\frac{RC}{C} I_0 \left[ e^{\frac{-t}{RC}} \right]_0^t$$

$$V_C = -I_0 R \left( e^{\frac{-t}{RC}} - 1 \right) = I_0 R \left( 1 - e^{\frac{-t}{RC}} \right) = V_S \left( 1 - e^{\frac{-t}{RC}} \right)$$




Voltage is shared between $R$ and $C$. I.e. as $V_C$ increases $V_R$ decreases.

Example

A 10μF capacitor is connected in series with a 0.6MΩ resistor across a 100V d.c. supply. Calculate (i) the time constant of the circuit, (ii) the initial current that will flow when the supply is connected (assume the capacitor is initially discharged), (iii) the time taken for the voltage across the capacitor to reach 80V, and (iv) the current and voltage across the capacitor 6s after it is connected to the supply.

(i) The time constant is:

$$\tau = RC = 0.6\times10^{6}\times10\times10^{-6} = 6 \text{ s}$$

(ii) Initially the capacitor is discharged and at the moment the supply is connected the voltage across it, $V_C$, will be zero. Therefore the whole of the supply voltage is dropped across the resistor.

$$V_S = V_R + V_C = I_0 R + 0$$

$$I_0 = \frac{100}{0.6\times10^{6}} = 167 \ \mu\text{A}$$

(iii) Using the derived results the voltage across the capacitor is given by:

$$V_C = V_S\left(1 - e^{-\frac{t}{RC}}\right)$$

so:

$$80 = 100\left(1 - e^{-\frac{t}{6}}\right)$$

or:

$$e^{-\frac{t}{6}} = 0.2$$

which gives a value of t = 9.66 s.

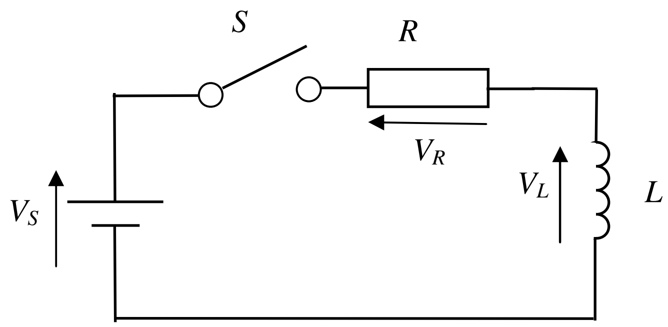
(iv) Using the same expression as in part (iii), the voltage across the capacitor after 6 s is:

$$V_C = V_S\left(1 - e^{-\frac{t}{RC}}\right) = 100\left(1 - e^{-\frac{6}{6}}\right) = 63.2 \text{ V}$$

and the current will be:

$$I = I_0 e^{-\frac{t}{RC}} = 167\times10^{-6}\times e^{-\frac{6}{6}} = 61.4 \ \mu\text{A}$$

**Current growth in an inductor.**




Before the switch, $S$, is closed there is no voltage across the resistor or inductor and the current is zero. When the switch is closed $V_S$ appears across $R$ and $L$.

$$V_S = V_R + V_L \text{ at all times}$$

substituting for $V_R$ and $V_L$ gives:

$$V_S = IR + L\frac{dI}{dt}$$

therefore:

$$\frac{V_S}{R} - I = \frac{L}{R}\frac{dI}{dt}$$

Now at steady-state (final value):

$$\frac{dI}{dt} = 0$$

hence, if $I_0$ is the final current:

$$I_0 = \frac{V_S}{R}$$

therefore:

$$I_0 - I = \frac{L}{R}\frac{dI}{dt}$$

or

$$\frac{R}{L}dt = \frac{dI}{I_0 - I}$$

integrating:

$$\frac{R}{L}t = -\ln(I_0 - I) + A$$

At the instant of closing the switch, $t = 0$, $I = 0$ and:

$$A = \ln(I_0)$$

thus:

$$\frac{R}{L}t = -\ln(I_0 - I) + \ln(I_0)$$

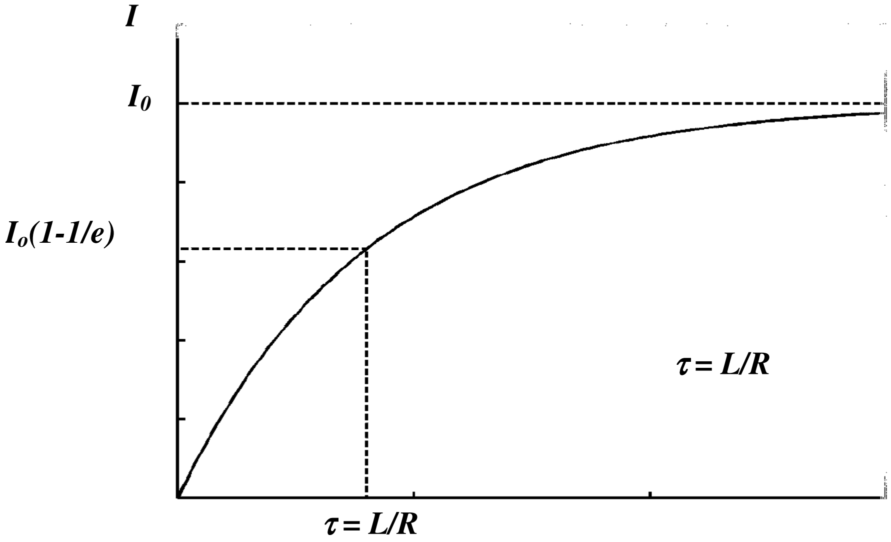
$$-\frac{R}{L}t = \ln(I_0 - I) - \ln(I_0) = \ln\left(\frac{I_0 - I}{I_0}\right)$$

so:

$$e^{-\frac{R}{L}t} = \left(\frac{I_0 - I}{I_0}\right)$$

Rearranging:

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$




Now the voltage across the inductor, $V_L$ is given by:

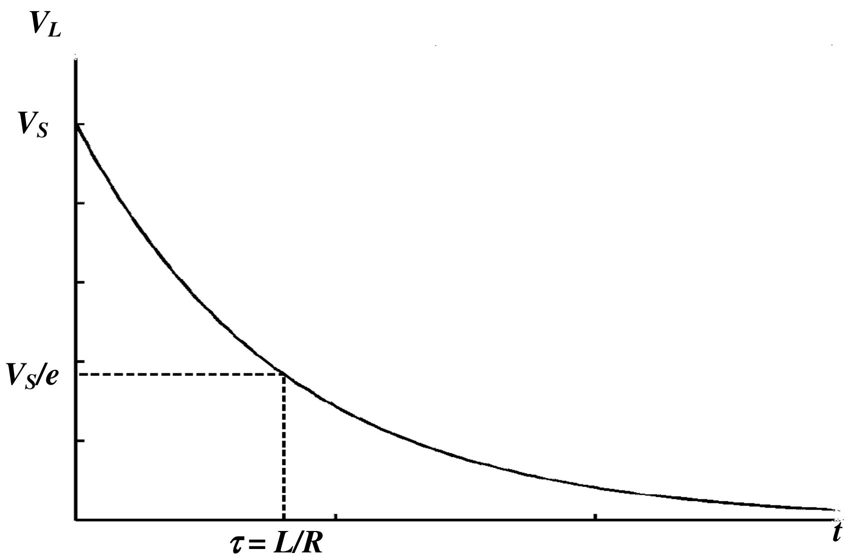
$$V_L = L\frac{dI}{dt} = L\frac{d}{dt}\left(I_0\left(1 - e^{-\frac{R}{L}t}\right)\right)$$

$$V_L = L\frac{d}{dt}\left(-I_0 e^{-\frac{R}{L}t}\right)$$

$$V_L = -L\left(\frac{-R}{L}\right)I_0 e^{-\frac{R}{L}t}$$

$$V_L = RI_0 e^{-\frac{R}{L}t}$$

$$V_L = V_S\, e^{-\frac{R}{L}t}$$




Voltage is shared between $R$ and $L$. I.e. as $V_L$ decreases $V_R$ increases.

Example

A coil having a resistance of 4Ω and an inductance of 2H is switched across a 20V d.c. supply. Calculate (i) the time constant, (ii) the final value of the current, and (iii) the current and voltages across both the resistor and capacitor 1s after the switch is closed.

(i) The time constant is:

$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5 \text{ s}$$

(ii) The final value of current is when the circuit has reached steady state and there is no further change in the value of the current and $V_L$ is zero. Therefore the whole of the supply voltage is dropped across the resistor.

$$V_S = V_R + V_L = I_0 R + 0$$

$$I_0 = \frac{20}{4} = 5 \text{ A}$$

(iii) Using the derived results the current flowing round the circuit is:

$$I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

so:

$$I = 5\left(1 - e^{-\frac{1}{0.5}}\right) = 4.323 \text{ A}$$

The voltage across the resistor is given by:

$$V_R = I \times R = 4.323 \times 4 = 17.29 \text{ V}$$

and the voltage across the inductor must be:

$$V_L = V_S - V_R = 20 - 17.29 = 2.71 \text{ V}$$

Alternatively we could have used:

$$V_L = V_S e^{-\frac{Rt}{L}} = 20e^{-\frac{1}{0.5}} = 2.71 \text{ V}$$