EEE 202

Electromechanical Energy Conversion

Semester 2, 2006/2007 Year

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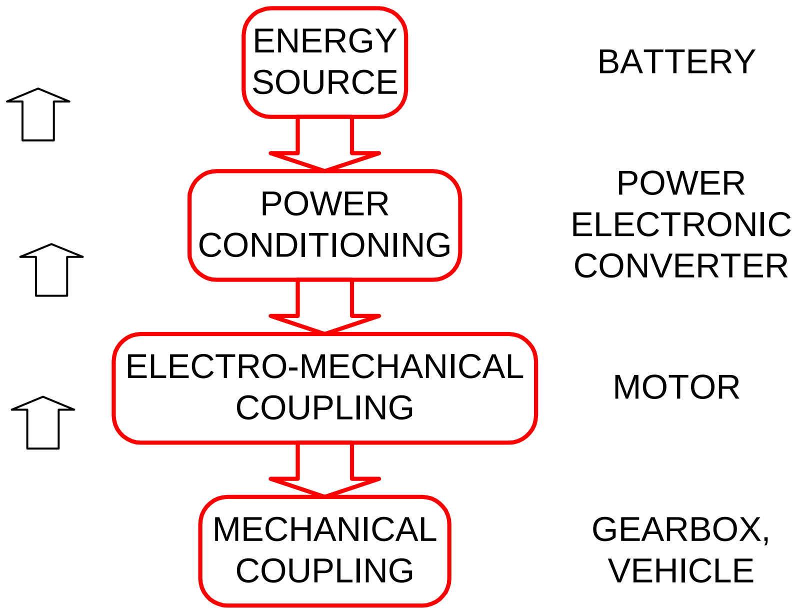
Lectures: 2 Hours/week for 12 weeks
Tutorial: 1 Hour/week for 12 weeks

Examination: 2 Hour exam, answer 3 questions from 4

Typical Electromechanical System



Types of devices considered in the course:

- Voice coil actuator (e.g. loudspeaker)
- Self excited actuators (e.g. relay)
- Brushed D.C. motor (servo drives)
- A.C. Induction motor

Features of any electromechanical system

a) Losses e.g.:

- I^2R in electrical system (Variable losses – copper losses in windings)

- Hysteresis and eddy current losses in iron of the coupling system (Principal core losses – during periodic magnetic reversals e.g. AC supply or direction of rotation)
- Friction losses in mechanical system (Bearings, brushes, air).

b) Energy storage e.g.:

- Energy stored in magnetic fields
- Inertial storage in mechanical system

In general, devices are analysed on the basis of conservation of energy.

Input energy to system = Output energy
+ losses in each component
+/- any change in stored energy within system

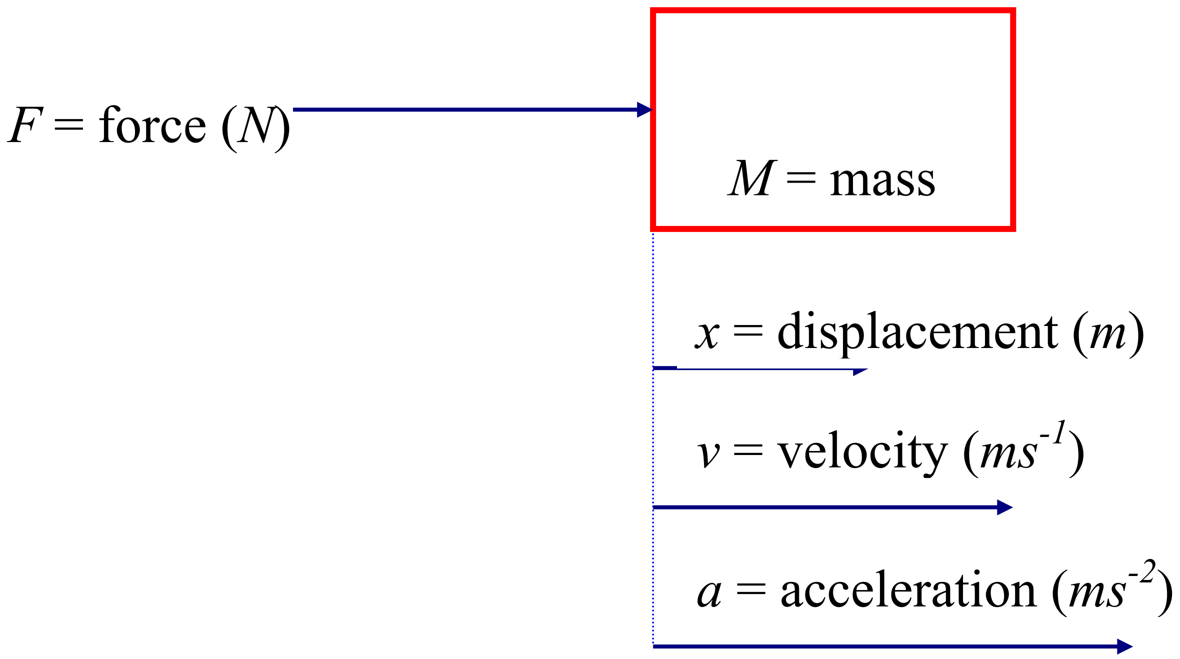
In steady-state the stored energy term can be neglected thus;

Input = Output + Losses

c) All electromechanical systems can operate with bi-directional power flow: *a motor can also act as a generator.*

Basic Tools

1. *Mechanical: Newton's laws of motion*



$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}, \qquad v = \frac{dx}{dt}$$

$$F(N) = Ma = kgms^{-2}$$

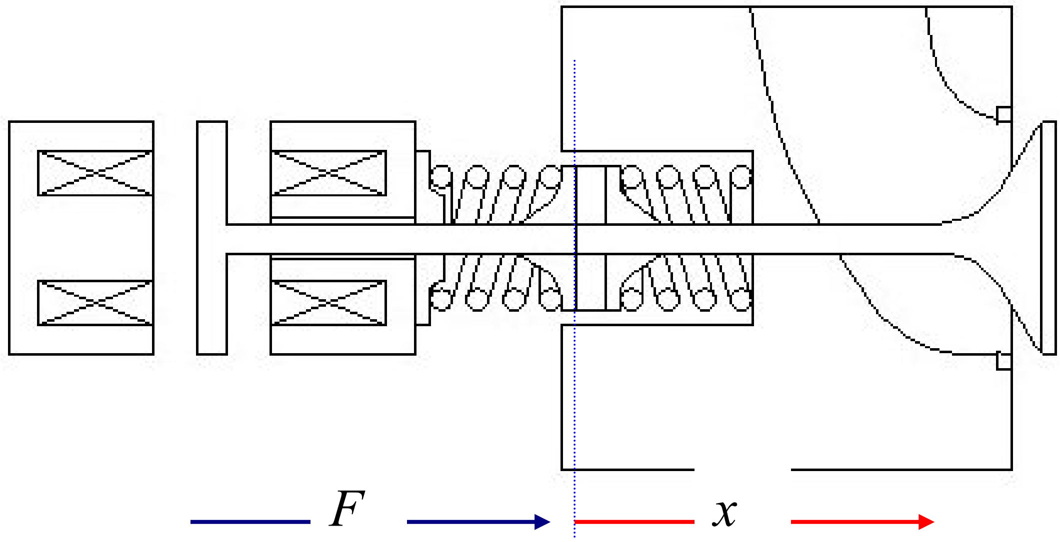
Work (energy) = Force * Distance moved

$$W(J) = \int F.dx$$

Energy Storage

$$Inertia = \frac{1}{2} M v^2$$

Spring:



$$F = \sigma_s x$$

Spring constant

$$\text{Energy stored in spring} = \frac{1}{2} \sigma_s x^2$$

3. *Power dissipation*

Power = Rate of Energy Usage

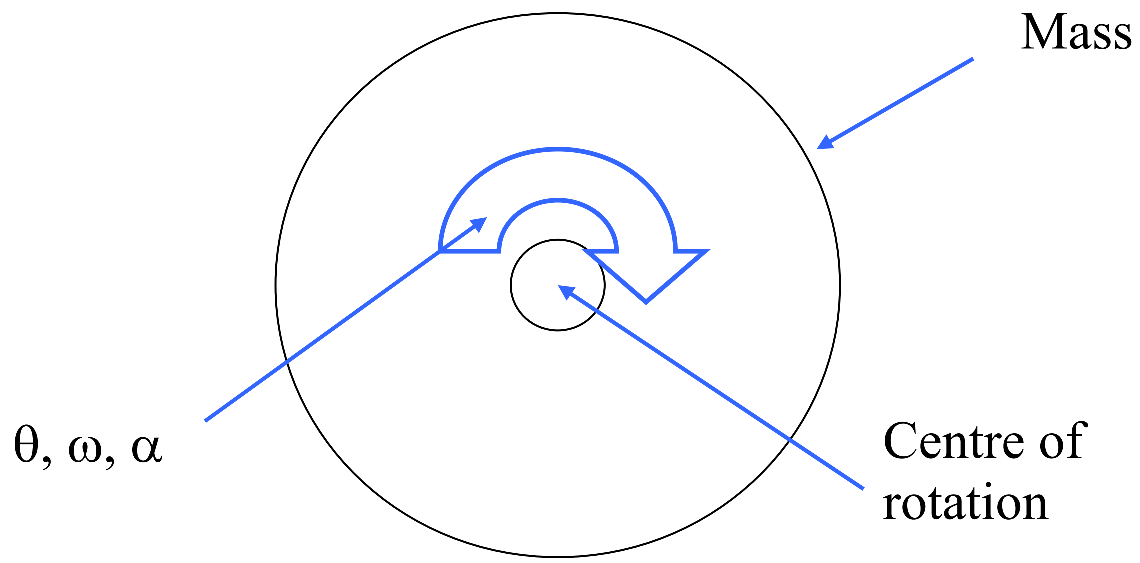
$$P = \frac{dW}{dt}$$

e.g. power associated with a force F and velocity v

$$P = Fv$$

(used to calculate friction losses).

Rotational mechanics



θ = angular displacement (radians)
ω = angular velocity (radians s^{-1})

n.b. rpm to radians s^{-1}

$$1 \cdot radian \cdot s^{-1} = \frac{2\pi}{60} \times 1 \cdot rpm$$

α = angular acceleration

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$T = I\alpha$$

Where;

- T is torque (Nm)
- I is polar inertia (kgm)

Work (energy)

$$W = \int T.d\theta$$

Stored energy in inertia

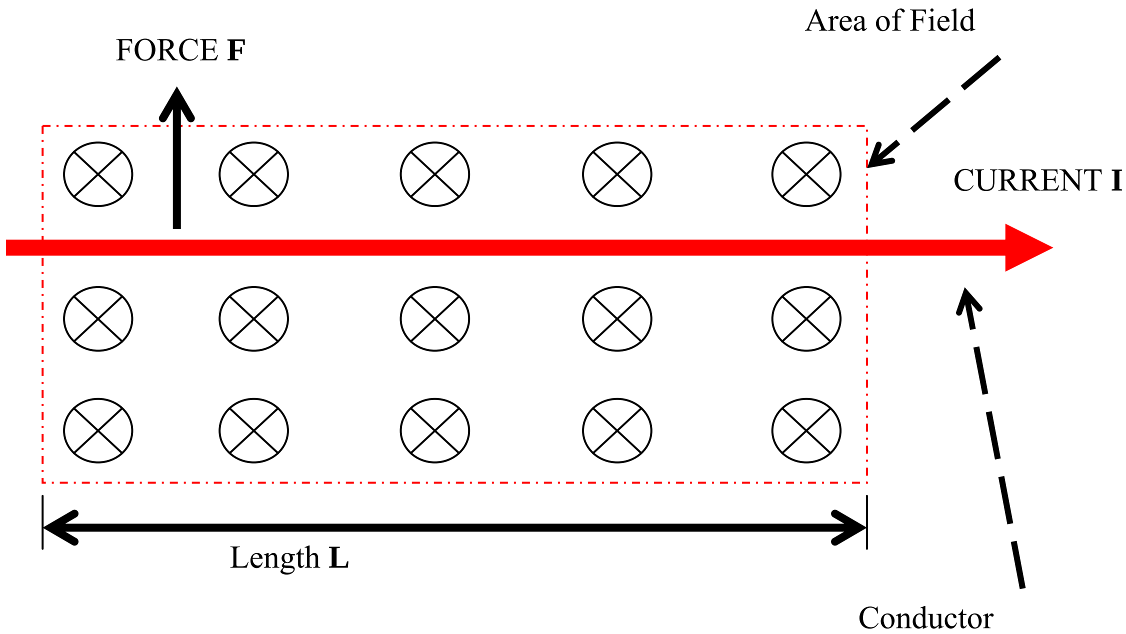
$$W = \frac{1}{2} I\omega^2$$

<u>Electromagnetic Coupling Equations</u>

$F = BIL$ Where B is flux density

Force = Strength of field (flux density B) *current in conductor (I) *length of the conductor which interacts with the fieldDirection of force

i.e the vector relationship is given by Fleming's left hand rule:
Thu**m**b = force or motion (F)
First finger = Field (B)
Se**c**ond finger = Current (I)



Motion of a conductor in a magnetic field will result in an induced voltage (Faraday's observation)

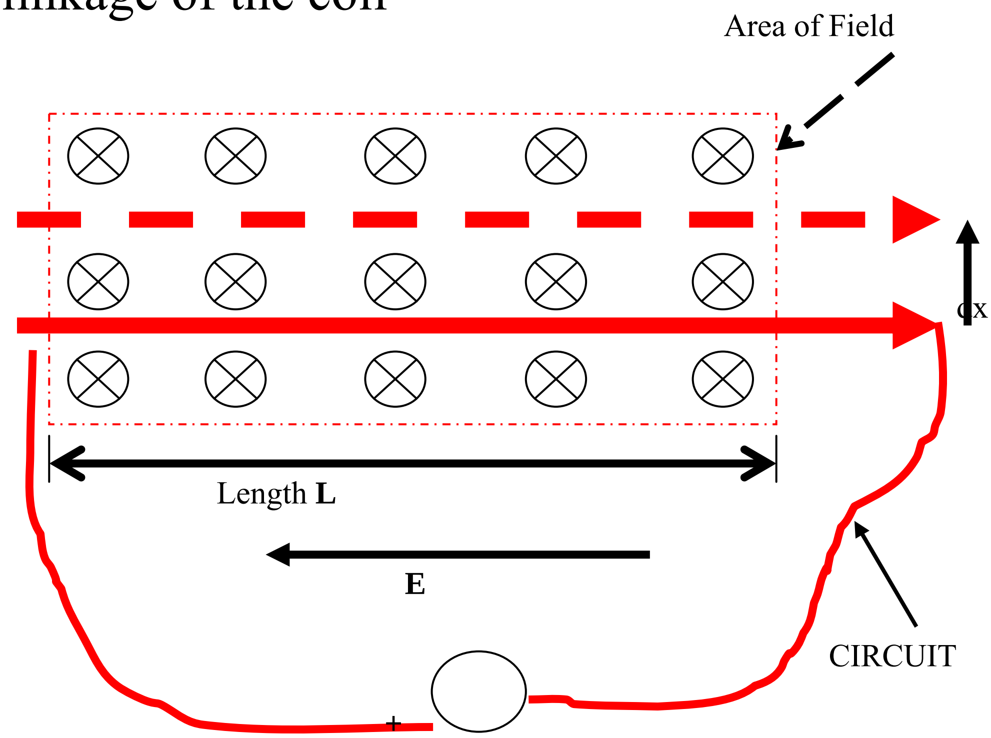
E *is proportional to* rate of change of the flux linking the wire

$$E = -N\frac{d\phi}{dt}$$

where N is the number of turns in the particular coil, and ϕ is the flux coupling a single turn of a coil

$$E = -\frac{d\psi}{dt}$$

where $\psi = N\phi$ which is the total flux linkage of the coil



Consider the movement of a wire from position (1) to (2) over a small displacement *dx*
Over the movement the circuit will 'see' an increase in the flux given by

$$\Delta\phi = BLdx$$

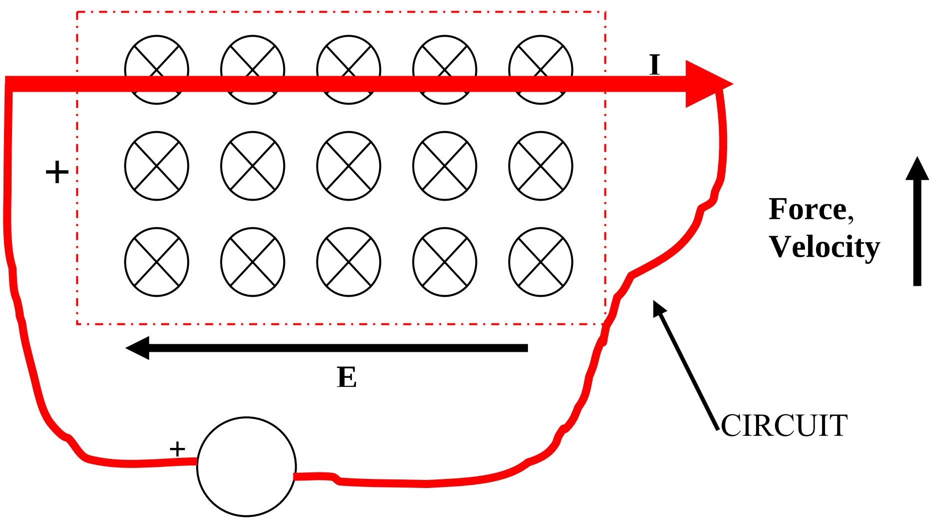
Since there is a change in flux linking the wire and its circuit, then there will be an induced voltage

$$E = -\frac{change\_in\_flux}{time}$$ where *time* =*dt*

$$E = -\frac{BLdx}{dt}$$

$= -BLv$ where *v* is the velocity of the conductor

Direction of the induced voltage is such as to oppose the cause of motion. For example if the motion was caused by a current flowing in the conductor, then the induced voltage as a result of that motion would be in such a direction as to oppose the source of the current



$$V = IR - E$$

where $E = -\frac{d\psi}{dt}$

thus $V = IR + \frac{d\psi}{dt}$

Since there is <u>no change</u> in stored energy in the system:-

> There is stored energy in providing the magnetic field, i.e. the energy in the permanent magnets which produce the field and there is an element of stored

energy due to the self inductance of the wire

$$\frac{1}{2} I^2 L$$

Since neither B nor I are changing then the stored energy is constant

Therefore from conservation of energy, the input electrical energy across the coupling interface must be equal to the output electrical energy.

$$IEdt = Fdx$$

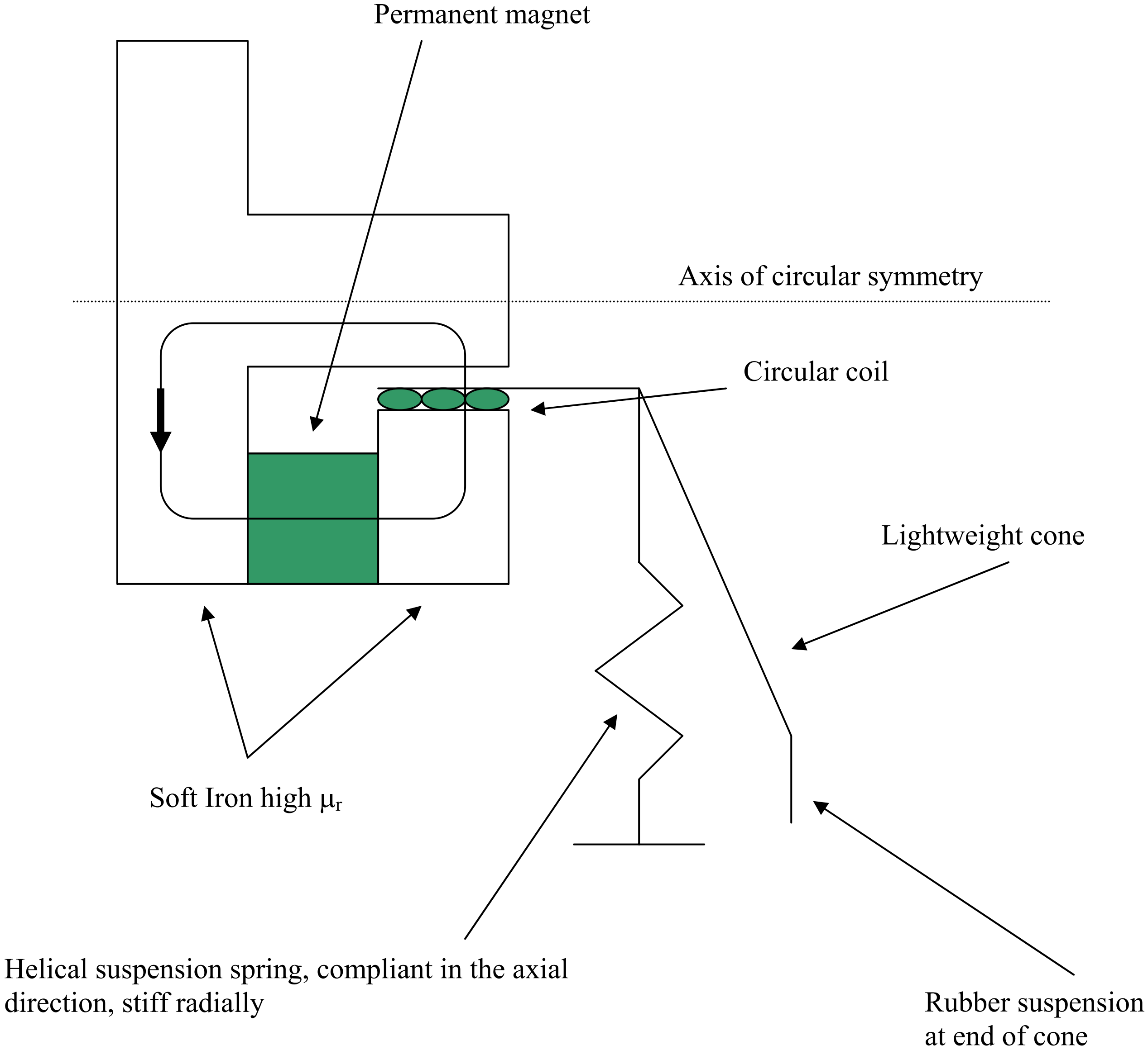
$$IE = F\frac{dx}{dt} = Fv$$

Thus, *current * back emf = output mechanical power*

$$F = BIL$$

$$E = BLv$$

Application: Permanent magnet loudspeaker



Ideally a loudspeaker will have a linear displacement with respect to applied voltage.

Assumptions

1.Current in coil is proportional to the applied voltage, which implies that across the full frequency range, the loudspeaker has a constant impedance (e.g. 8Ω)

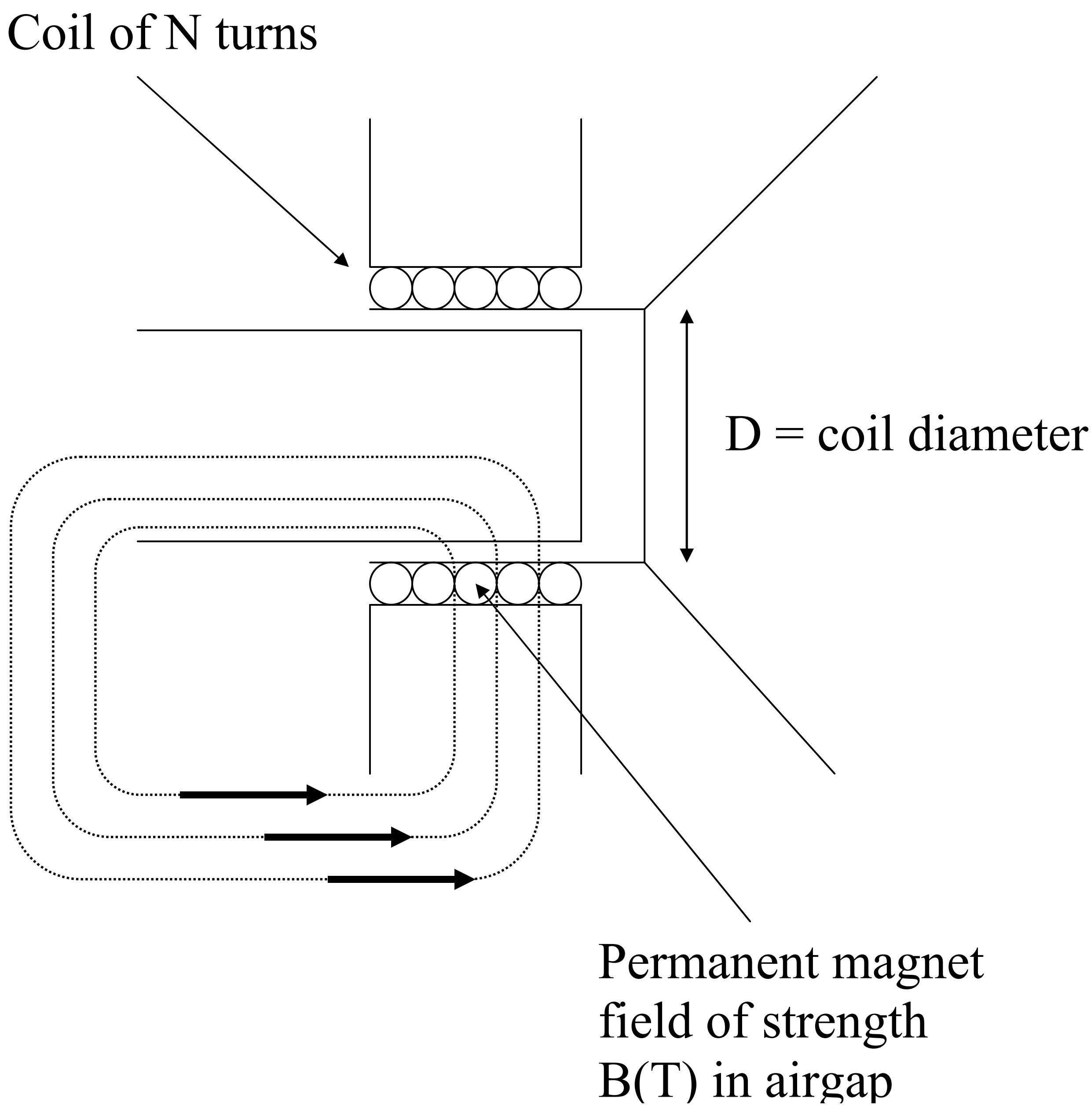
2.Force developed on cone is directly proportional to coil current

$$F = BIL$$

which implies a uniform field across the full displacement of the coil

3.Displacement of the cone is proportional to the applied force i.e. the suspension spring has a uniform force-displacement characteristic

Development of full loudspeaker equivalent circuit will require the use of electrical analogues for the mechanical system.



Length of interaction of coil with the magnetic field = coil circumference = πD

Thus;

$F = BI\pi DN$ where N is the number of series turns

$$F = (BN\pi D)I$$

or

$F = K_e I$ where K_e is the electromagnetic or force constant of the system

The induced voltage in the coil as a result of the motion is given as

$E = K_e v$ where v is the velocity of motion

And finally

$EI = Fv$ which gives the power balance across the electromechanical interface

<u>Mechanical system</u>

Mass of cone and the air that it moves

$$F = Ma = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$

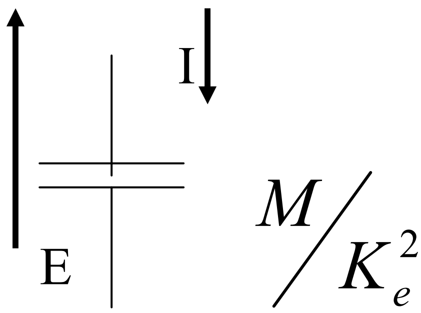
Substituting for F and v

$$K_e I = M \frac{d\left(E/K_e\right)}{dt}$$

$$I = \frac{M}{K_e^2}\frac{dE}{dt}$$

Thus the electrical equivalent can be given as

$$I = C\frac{dV}{dt}$$ i.e, mass is represented as a capacitance



<u>Spring</u>

$$F = \sigma_s x$$

spring compliance

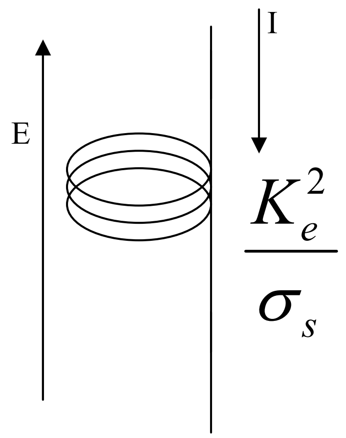
$$K_e I = \sigma_s \int v.dt = \sigma_s \int \left(\frac{E}{K_e}\right).dt$$

$$I = \frac{\sigma_s}{K_e^2}\int E.dt \quad \text{or….} \quad E = \frac{K_e^2}{\sigma_s}\frac{dI}{dt}$$

Giving the electrical equivalent

$$V = L\frac{dI}{dt}$$

$$I = \frac{1}{L}\int V.dt$$



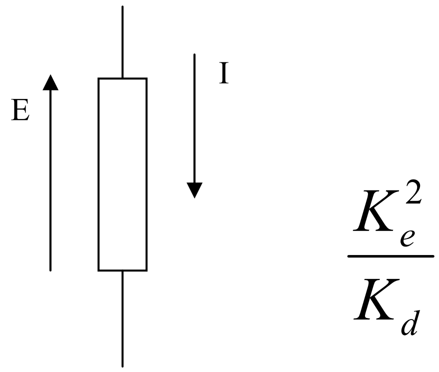
<u>Mechanical loss and damping</u>

$F = K_d v$ where K_d is the damping coefficient

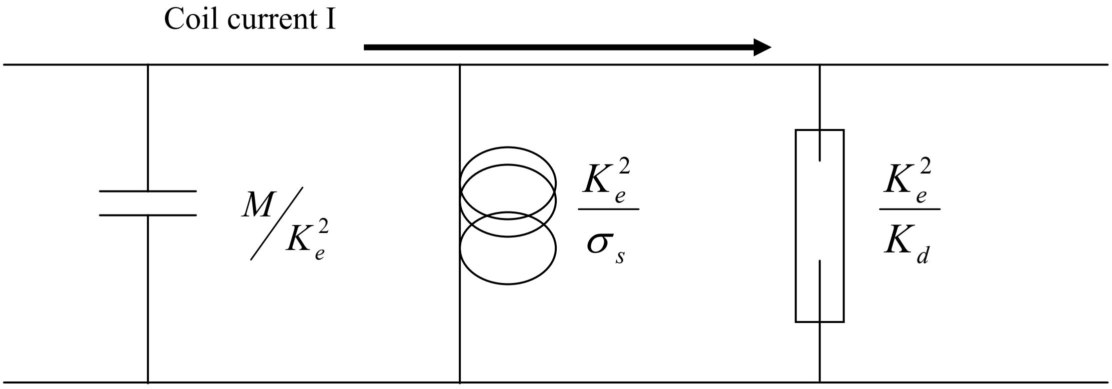
$$K_e I = K_d\left(\frac{E}{K_e}\right)$$

$$\frac{K_e^2}{K_d} I = E$$

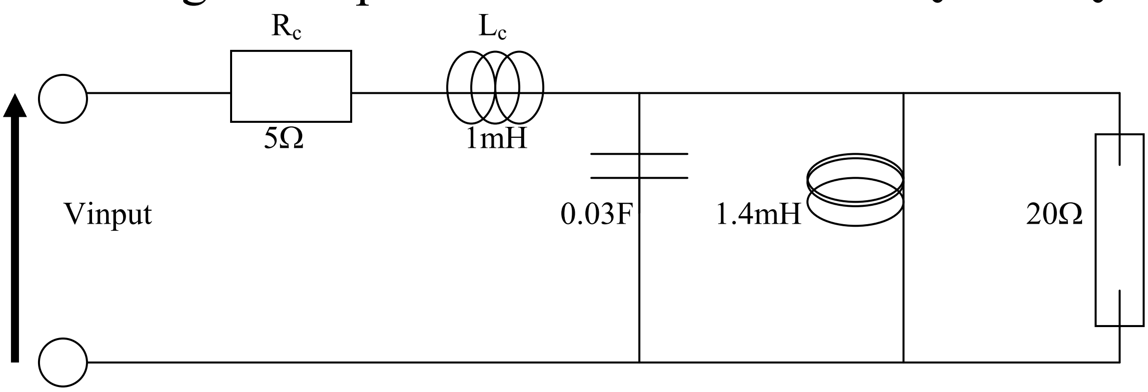
which is analogous to $V = IR$ thus



In forming the complete electrical analogue, the connection of the components needs to be considered, since force is proportional to current

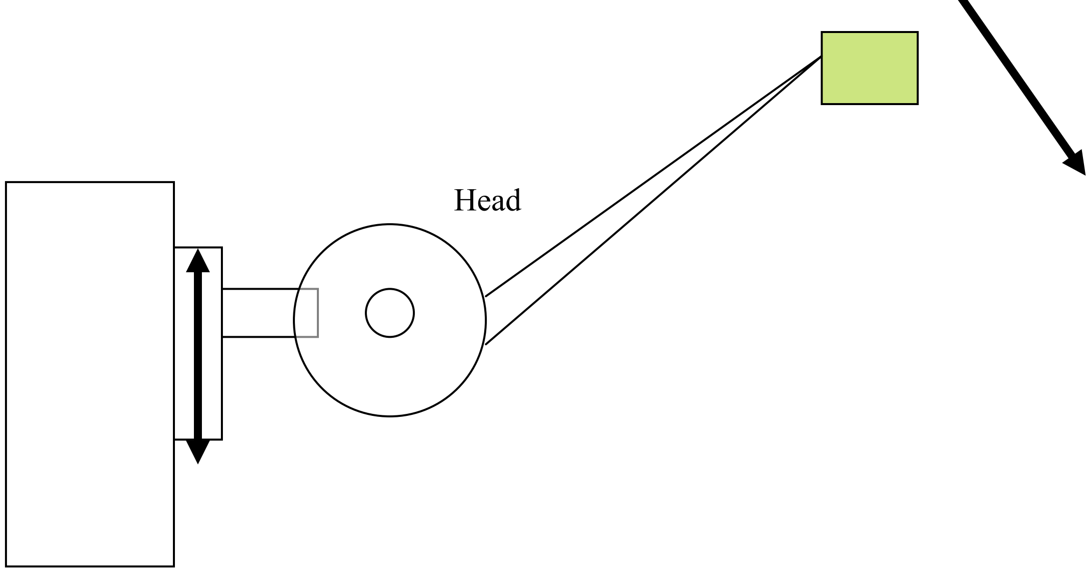


adding the impedance of the coil itself R_c and L_c



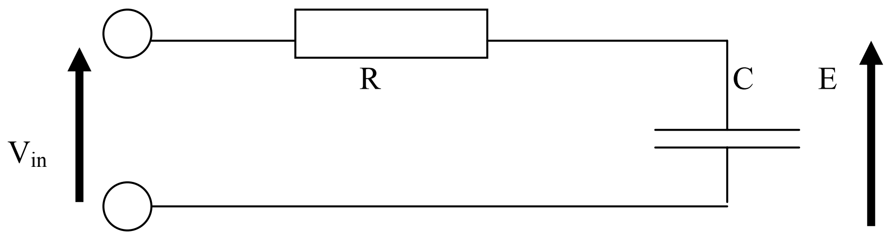
$$f = \frac{1}{2\pi\sqrt{LC}} = 25Hz$$

Application – hard disc read/write head actuator



Assume: zero damping, no spring, zero winding self-inductance

System equivalent circuit reduces to:



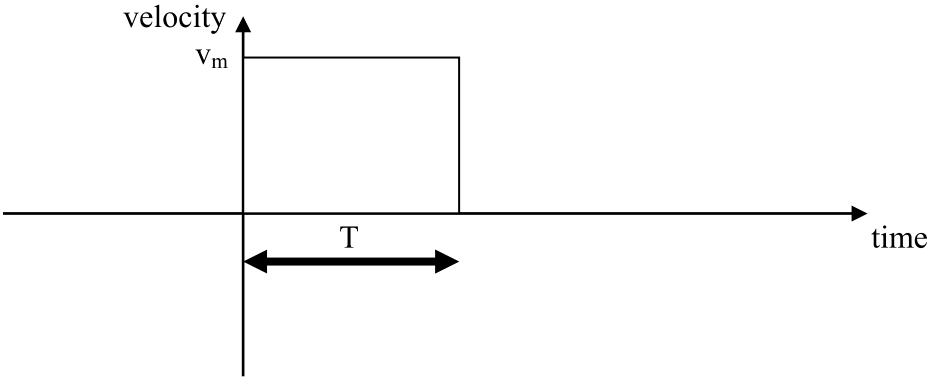
Where R=resistance of actuator winding

C=equivalent of the mechanical inertia of the system

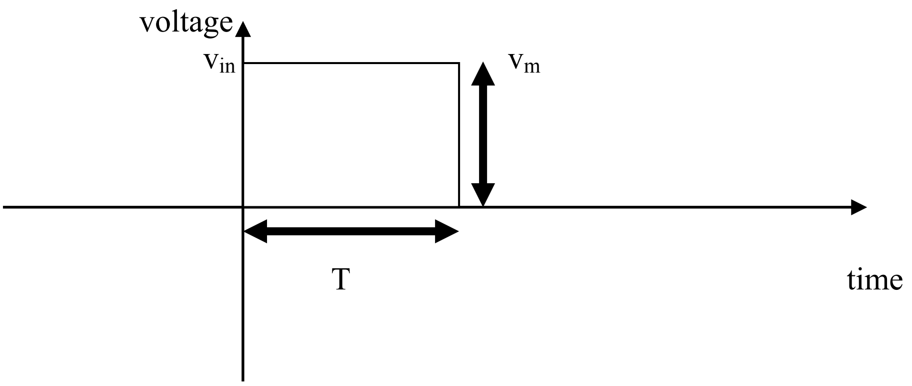
V_{in}=applied voltage to actuator coil

E=induced emf across coil=$K_e v$

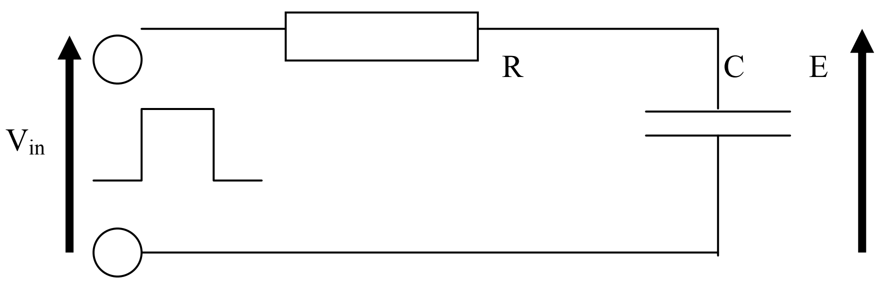
We wish to move the head a fixed distance across the disc



Distance travelled = $v_m T$
To achieve this, an input voltage or pulse is applied to the input of the actuator



Find the transfer function of the system



$$V_{in} = iR + E$$

$$i = C\frac{dE}{dt}$$

$$V_{in} = CR\frac{dE}{dt} + E$$

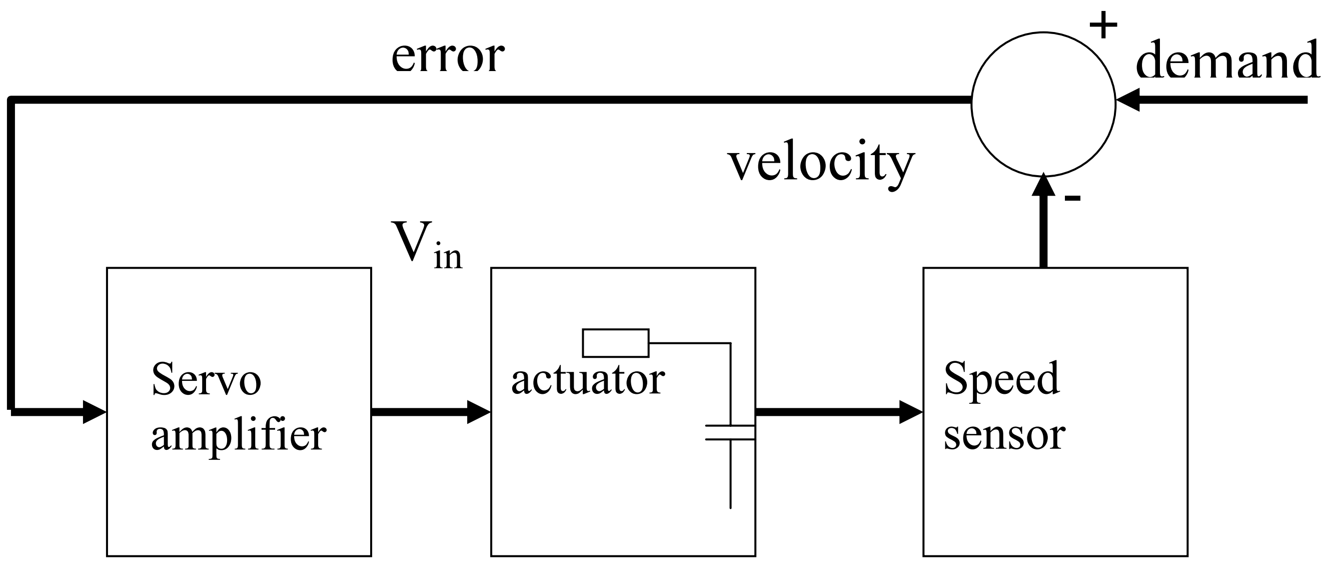
at t=0 i=0, $V_{in}=v_m$

$$E = v_m\left(1 - e^{t/RC}\right)$$

where RC is the time constant of the system. Once we know E, we can derive the velocity

$$v = E/K_e$$

Actuation control system



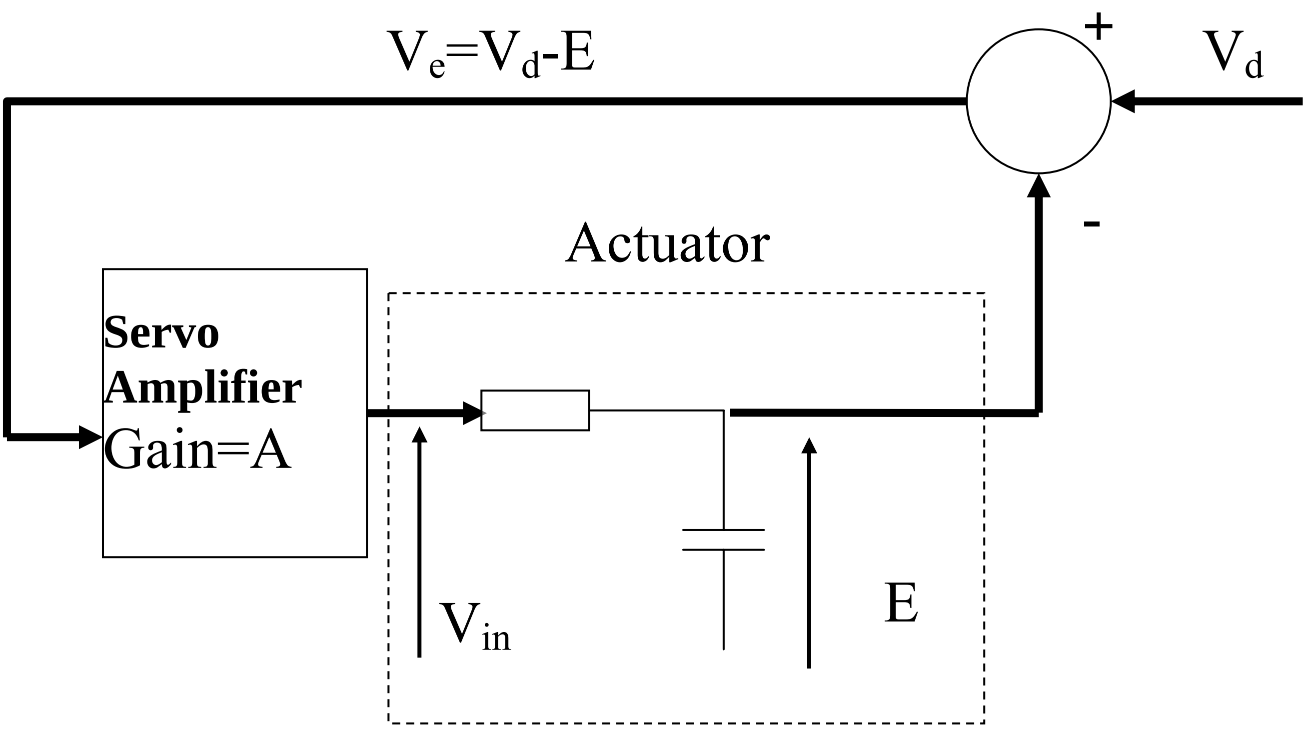
Power amplifier at fixed gain A

In a closed loop system, the input to the servo amplifier is generated by subtracting the feedback signal from the demand signal

Demand = required velocity profile
Feedback = the actual motion of the actuator
Error = demand-feedback

The error signal is used to drive the servo amplifier.

We assume that the feedback velocity sensor is set up such that its output is equal to the actuator EMF. (i.e. sensor constant is identical to the actuator constant).



$$V_{in} = AV_e = A(V_d - E) \quad (1)$$

$$V_{in} = iR + E \quad (2)$$

substitute (1) into (2) for V_{in}…

$$A V_d = iR + (1 + A)E$$

$$\frac{A}{1+A} V_d = i \frac{R}{1+A} + E \qquad (3)$$

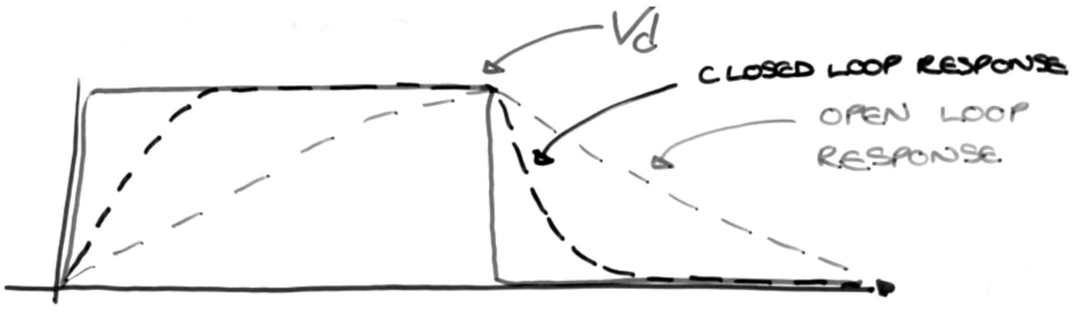
Now….

$$i = C \frac{dE}{dt} \qquad (4)$$

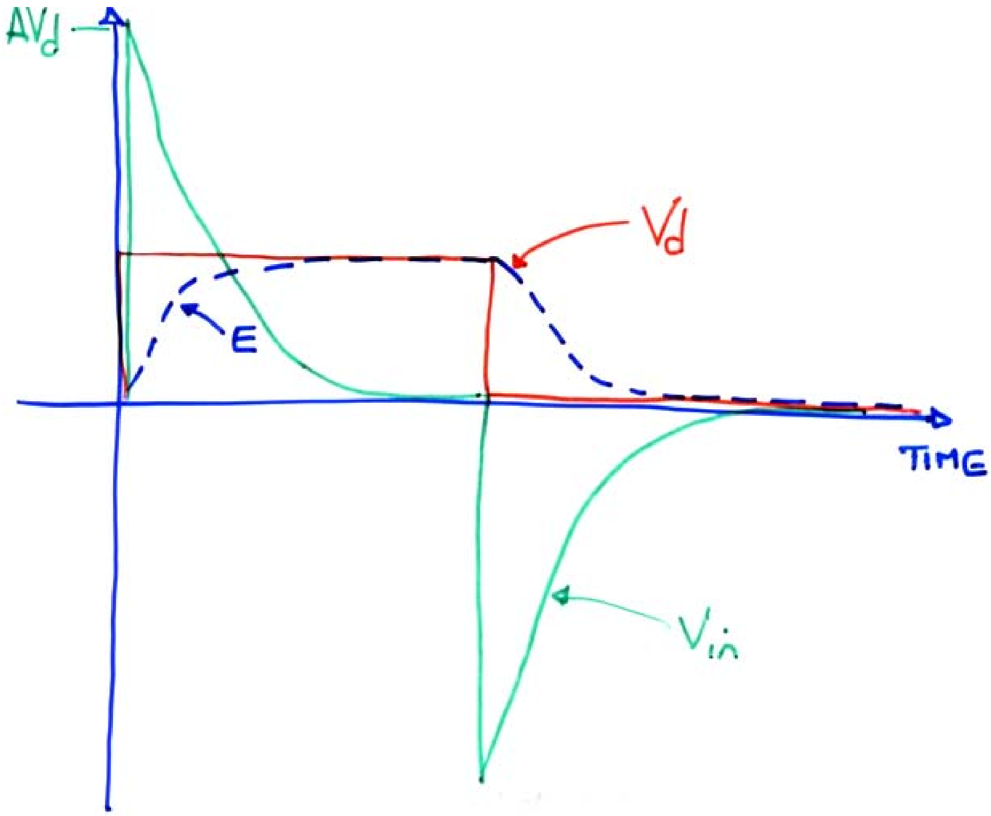
and solving (3) and (4) together for E as a differential equation

$$E = \frac{A}{1+A} V_d \left(1 - e^{\frac{-t}{\left(RC/_{1+A}\right)}} \right)$$ which is the closed loop term.

The time constant of the response has been changed from *RC* in open loop to *RC/(1+A)* in closed loop form.



The improved response is achieved by driving the actuator with higher voltages than in the open loop case.



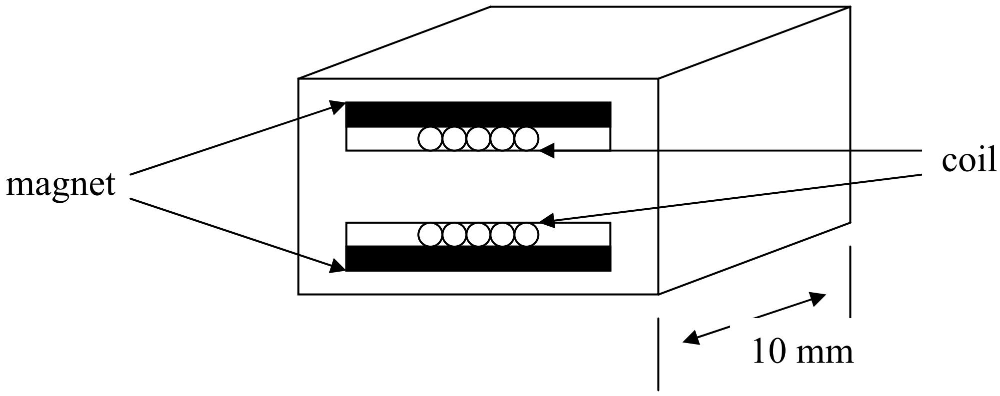
Initially when the output is at rest, i.e. $E=0$, then as soon as the step demand V_d is applied, the amplified output will rise to AV_d.
i.e. the actuator is driven hard with a large positive input voltage

$$V_{in} = AV_d - E$$

Similarly, when the output has reached the required speed, ie when $E=V_d$, and we wish to stop the motion, then at the step demand $V_d=0$, then $V_d= -AE$. A large negative voltage is applied across the actuator which will pull a negative current and bring the mechanical system quickly to a standstill.

Tutorial Sheet 1.
Voice Coil Actuators

(1.) The principal components and dimensions of a computer hard disc read/write 'voice-coil' actuator are given in figure 1. Two permanent magnets provide a uniform field of 0.6T across the two central slots. A coil which has 150 turns, a resistance of 10 Ω and a self-inductance of 24 mH is located in the slots to provide linear motion.



Find:

(a) The force constant (Force per Ampere of coil current) of the actuator.
(b) The voltage constant (Volts per unit of coil velocity)
(c) The electrical capacitor equivalent of the actuator system given the read/write mechanism has a total effective mass of 3 grams.

(2.) Derive an expression for the impedance of an RLC circuit at a supply frequency of f Hz. Show that the capacitor voltage V_c is given by:

$$V_c = \frac{V_{in}}{\sqrt{\left(1-(2\pi f)^2 LC\right)^2 + (2\pi fRC)^2}} \quad \text{where } V_{in} = \textit{input supply voltage}$$

(3.) The actuator is supplied from a 5V rms, 50Hz voltage source causing the coil to oscillate.

Determine:

(a) The rms current in the coil
(b) The peak velocity of the coil
(c) The maximum peak to peak movement of the coil.

Voice coil actuator – solutions

(1) a: K_e = BLN = 0.6 x 0.02 x 150 = 1.8 Nm/A
b: K_v = BLN = 0.6 x 0.02 x 150 = 1.8 V/m/s

c: $C = \frac{M}{k_e^2} = \frac{(3\times10^{-3})}{(1.8)^2} = 926\mu F$

(2) For a series RLC circuit

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Therefore

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Since $\omega = 2\pi f$, then

$$|Z|\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$

Current in circuit

$$I = \frac{V_{in}}{|Z|}$$

Finally, voltage across the capacitor

$$V_c = \frac{V_{in}}{|Z|} \times \frac{1}{\omega C} = \frac{V_{in}}{\sqrt{(\omega CR)^2 + (1-\omega^2 LC)^2}}$$

(3) a: rms current in coil

$$|I| = \frac{V_{in}}{|Z|} = \frac{5}{\sqrt{10^2 + \left(2\pi 50(24\times10^{-3}) - \frac{1}{(2\pi 50(926\times10^{-6}))}\right)^2}} = 0.48A$$

b: Peak velocity of the coil

$$v_c = \frac{V_{cpk}}{k_v}$$

$$V_{cpk} = \frac{5\sqrt{2}}{\sqrt{(1-(2\pi 50)^2(24\times10^{-3})(926\times10^{-6}))^2 + ((2\pi 50)(10)(926\times10^{-6}))^2}} = 2.25V$$

Therefore

$$V_{pk} = \frac{2.25}{1.8} = 1.25ms^{-1}$$

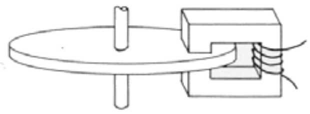
c: 50Hz sine wave has peak magnitude 1.25ms^{-1} and period 0.02s. Therefore the peak to peak displacement is given by

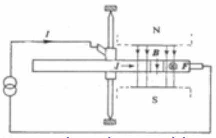
$$x_{pk-pk} = \int_0^{0.02} (1.25)\sin(2\pi 50t).dt = \left[-\frac{1.25}{2\pi 50t}\cos(2\pi 50t)\right]_0^{0.02} = 8mm$$

Early Electrical Machine Inventions

- At this time, the only permanent magnet materials were lodestone and various iron carbon alloys (cast iron), but the principles of electromagnets were well known

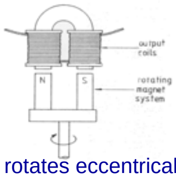
1831: Faraday's disc dynamo – to demonstrate laws of magnetic induction





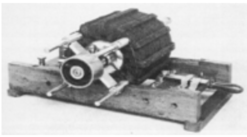
1831: Pixii's ac generator – has most elements of a modern alternator, except that the working surface was a plane rather than a cylinder

A commutator was subsequently added to obtain dc output



1841: Wheatstone's eccentric rotor motor – rotor rotates eccentrically inside bore of a ring of electro-magnets as they are switched on and off sequentially

1888: Tesla's induction machine



1890: 1st 3-phase public supply, W. London

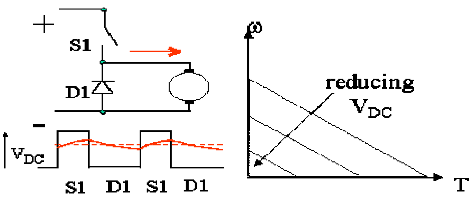
500HP AC Generator - 1893

- Commutator machines, induction machines and wound-field synchronous machines remain the dominant technologies. However, with the development of power electronics, permanent magnet brushless machines and switched reluctance machines are now the preferred technologies for an increasing variety of applications.

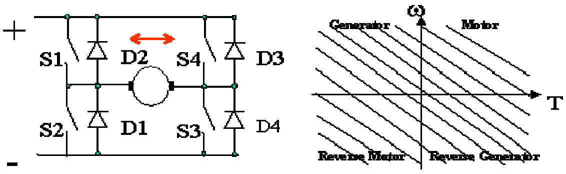
- Of course, power electronics also enables the operating characteristics of traditional electrical machine technologies to be controlled.

e.g. brushed DC drive

1-Quadrant 'Chopper'



4-Quadrant drive

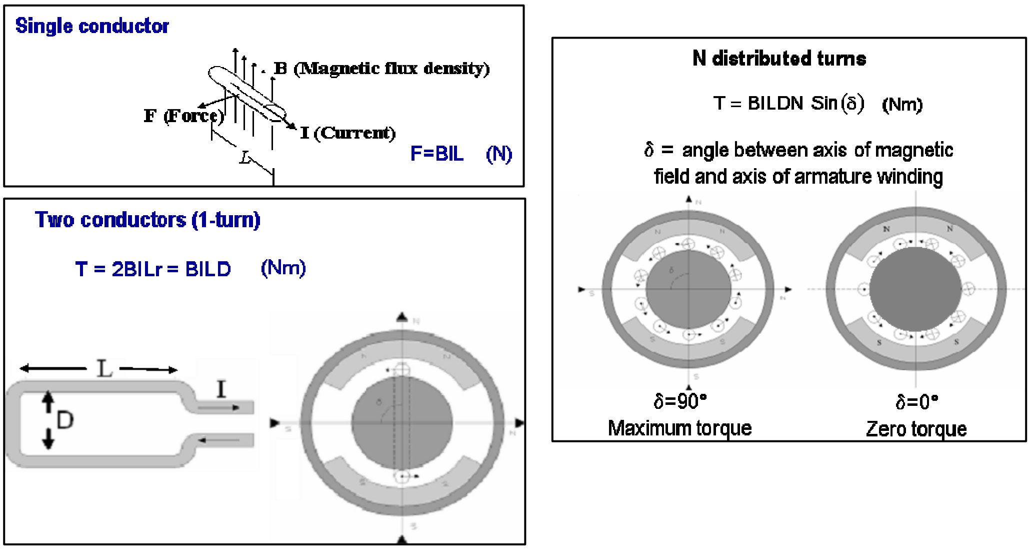


Motoring S1,S3 and D1,D3

Braking S2,S4 and D2,D4

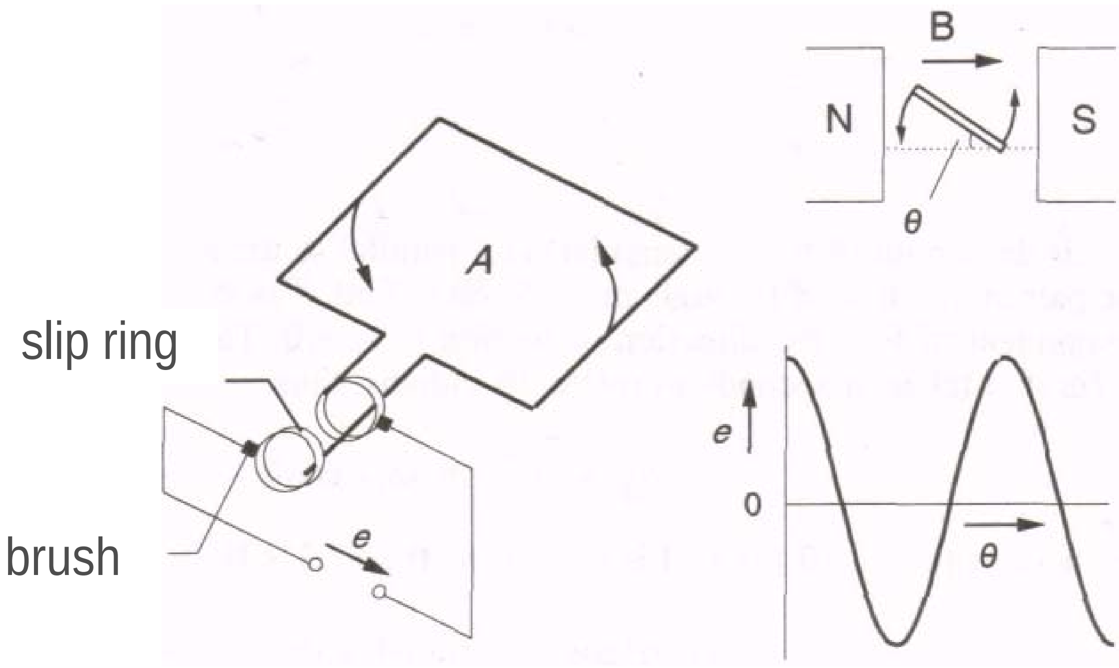
Torque Production Mechanisms

- Excitation torque - due to the interaction of current-carrying conductor with magnetic field



A prototype dc generator

Consider the single rectangular loop of conducting wire in, which is of area *A* and is being rotated in a uniform magnetic field whose flux density is B. In order to supply current to an external load, the ends of the coil are connected to *slip rings* that are contacted by *brushes,* which in turn are connected to copper wires going to the generator's terminals and thence to the load. Brushes are actually spring-loaded blocks of graphite that form a low-resistance contact with the slip ring, while the slip rings are made from brass or hardened copper. Graphite is soft and produces relatively little wear in the slip rings, besides having other advantages.



If the coil is at an angle, θ, to the field, its area normal to **B** is A sin θ and the e.m.f.

is

$e = d\Phi/dt = BA\,d(\sin\theta)/dt$ by Faraday's law.

But we can write

$$\frac{d}{dt} = \frac{d}{d\theta}\frac{d\theta}{dt} = \omega\frac{d}{d\theta}$$

since $d\theta/dt = \omega$, whence

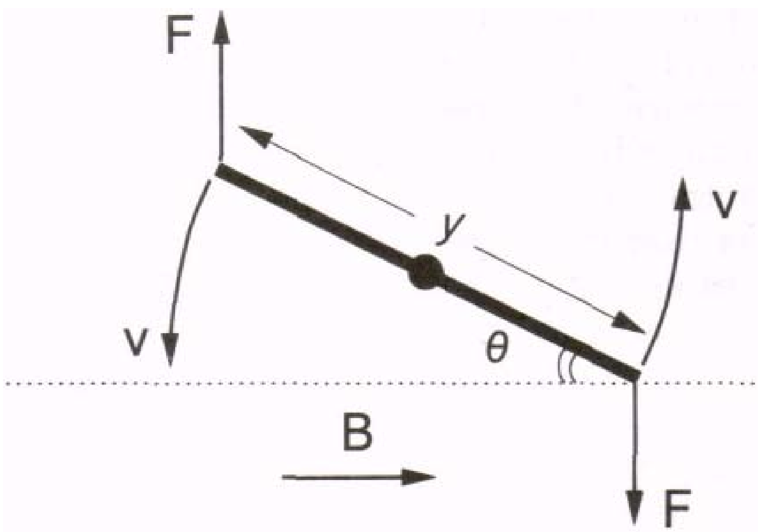
$$e = BAd(\sin\theta)/dt = BA\omega d(\sin\theta)/d\theta = BA\omega\cos\theta$$

The e.m.f. produced by the rotating coil is sinusoidal, reaching its maximum value of BAω when θ = 0° (when B lies in the plane of the coil) and falling to zero when θ = 90° (when B is normal to the plane of the coil). Hence this simple machine is an AC generator.

13.1.1 Energy conversion

We have seen that the electrical power developed in a conductor in motion relative to a magnetic field is derived from the work done against the Lorentz force. Suppose the terminals of the coil in figure 13.1 are connected to a resistance, *R,* while the coil itself is of negligible resistance. The instantaneous current flowing is *i* and the instantaneous electrical power generated, p_e, is *ei,* or e^2/R. The mechanical power can be calculated with reference to the following figure

A rectangular coil rotating in a uniform magnetic field. The Lorentz force, F, *has a component F* cos θ *along the direction of motion*



If the length of the (rectangular) coil parallel to the field is y, the work done to rotate the pair of conductors through an angle $\delta\theta$ is $T\delta\theta$. T is the torque, which is $yF\cos\theta$ as the component of F in the direction of motion is $y\cos\theta$. The rate of working is $p_m = T\,\delta\theta/\delta t = T\omega$ if it takes δt seconds to rotate $\delta\theta$ radians, thus

$$p_m = T\omega = \omega yF\cos\theta$$

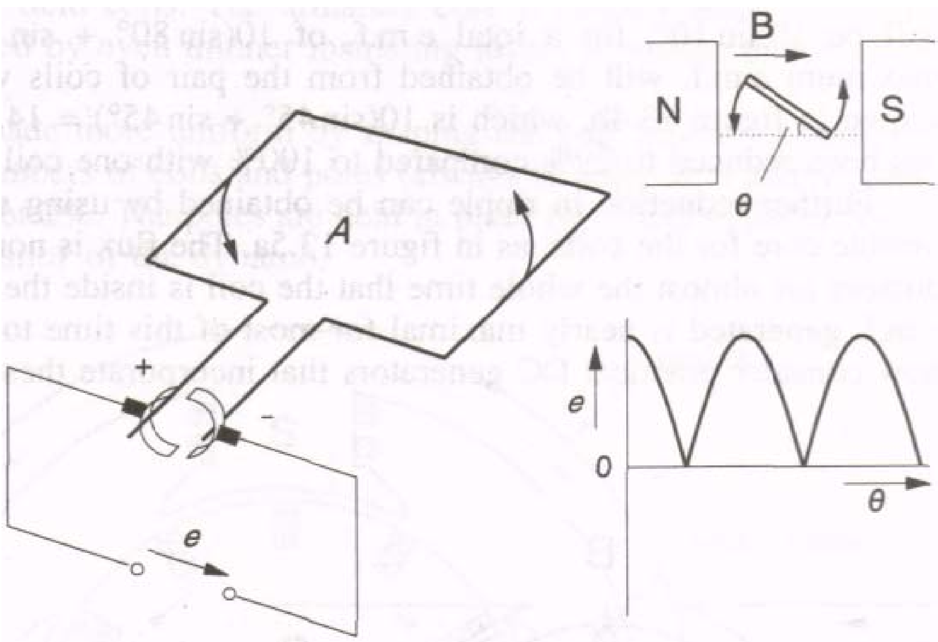
Then substituting ilB for F (**l** is normal to **B** so $|iL\times B| = ilB$) gives

$$p_m = \omega yiLB\cos\theta = i\omega BA\cos\theta = ei = Pe$$

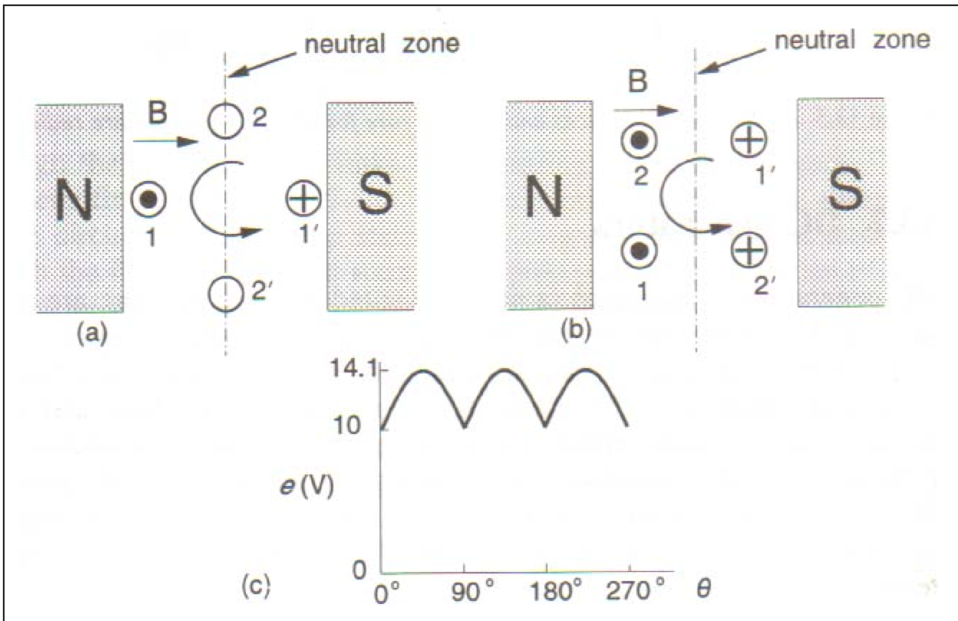
since $yl = A$ and $e = BA\omega\cos\theta$. Thus the mechanical power put in is equal to the electrical power out, ideally.

Commutation In order to produce DC from the coil and magnet, it is necessary to provide *commutation* in the form of split slip-rings.

A conducting coil rotating in a uniform magnetic field produces DC when furnished with a slit-ring commutator. The terminal e.m.f. is a rectified sine-wave



At the point in the rotation of the coil when the current in brush A would reverse, the brush breaks contact with one split ring and makes contact with the other. Thus brush A is in contact only with the conductor moving down through the magnetic field on the left and must therefore be connected to the positive terminal of the generator. The coil e.m.f. is made up of a positive e.m.f. from the arm of the coil which passes in front of the N pole and an equal negative e.m.f. from the coil which passes a S pole. The total e.m.f. of the generator, though unidirectional, is sinusoidal and varies in magnitude from zero to a maximum of BAω



A series-connected, 2-coil generator (a) Coils in position for minimum e.m.f. (b) Coils in position for maximum e.m.f. (c) Resultant e.m.f.

The large ripple of this rectified sinewave could be reduced by employing more coils connected in series, each with its own commutator segment. Consider, for example, a 2-coil, 2-pole generator, which we shall suppose gives an e.m.f. of 10 V in each coil when it passes the centre of a pole face. When the coils are in the position shown in figure a, the e.m.f. from coil 1 is 10 V while that from coil 2 is zero and the combined e.m.f. will be 10 V. Coil 2 gives no e.m.f. as it is in the magnetically neutral plane, or in the *neutral zone* of the generator.

On rotating a further 10°, the e.m.f. of coil 1 will be 10 sin 80°, while that of coil 2
will be 10 sin 10°, for a total e.m.f. of 10(sin80° + sin 10°) or 11.6 V. Clearly, the
maximum e.m.f. will be obtained from the pair of coils when they are in the position
shown in figure b, which is 10(sin45° + sin45°) = 14.1 V. The peak-to-peak ripple has been reduced to 29% compared to 100% with one coil.

DC Motors

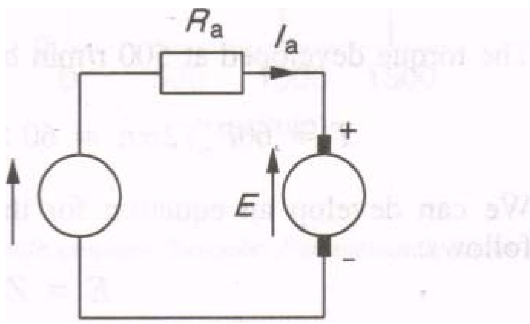
A DC generator can be run as a DC motor simply by applying a voltage to its terminals. DC generators have almost entirely been superseded by AC generators and semiconductor rectifiers, but DC motors are still valued for their wide range of speed and torque, accompanied by high overall efficiency. Traction is often performed by DC motors and many special applications such as driving conveyer belts and lifts in mines and quarries employ DC motors too. There are essentially three types of DC motor according to the manner of field excitation: (1) shunt-wound, including permanent-magnet and separately-excited motors, (2) series-wound and (3) compound-wound. Each type has distinctive torque-speed characteristics.

Back emf

As the armature starts to rotate in the magnetic field of the pole pieces, the armature conductors generate an e.m.f., *E,* which is in opposition to the applied voltage, *V,* and is therefore called the *back e.m.f.*. The armature current, I_a, flows through the armature winding of resistance R_a, and so by Kirchhoffs voltage law

$$V = E + I_a R_a$$

Therefore the current drawn by the motor is $I_a = (V - E)/R_a$. The electrical power that is converted to



mechanical work is EI_a.