Data Provided: Formula Sheet

The University of Sheffield

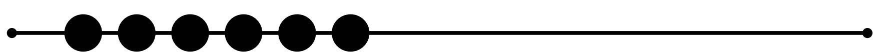
DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2005-2006 (2 hours)

Electric and Magnetic Fields 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a.



In the figure above, six small beads of equal positive charge are free to move without the effects of gravity or friction along a straight wire (the wire has end-stops to prevent the beads dropping off). Sketch the approximate bead distribution that corresponds to an equilibrium state. If the wire is now bent to form a perfect closed circle, how would the bead distribution change? Explain all your reasoning. **(6)**

b. A hollow conducting sphere of radius 0.5m is located at the origin, and carries a total charge of -4×10^{-8} C.

i. Calculate the magnitude of the electric field at the points (0.25, 0, 0) m and (2.5, 0, 0) m. **(4)**

An identical sphere, carrying a charge of $+16 \times 10^{-8}$ C is now positioned at the point (y, 0, 0) m.

ii. Find the value of y such that a point charge situated at the point (2.5, 0, 0) m would experience zero force. **(4)**

iii. Calculate the potential difference between the two spheres **(6)**

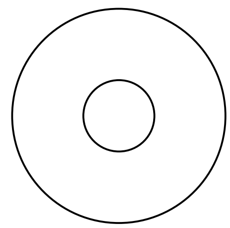
2. **a.** **i.** Using Gauss' Law, show that the electric field due to an infinite line of charge is given by the following expression, and define the symbols used:-

$$\mathbf{E} = \frac{q_\ell}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$$

(6)

ii. An infinitely long wire parallel to the y-axis passes through the point (0, 0, 1), and carries a charge per unit length of $+7 \times 10^{-6}$ Cm^{-1}. Calculate the electric field at the point (2, 1, 0). **(4)**

b.



The figure above shows the cross-section of a coaxial cable consisting of two hollow conducting cylinders. The inner cylinder has a radius of *a*, and is negatively charged; the outer has a radius of *b* and is positively charged.

i. Redraw the diagram above, showing the field lines and lines of equipotential. **(2)**

ii. Show that the capacitance per unit length is given by the expression:-

$$C_\ell = \frac{2\pi\varepsilon_0}{\ln\left(\dfrac{b}{a}\right)}$$

(6)

iii. If a 10m long coaxial cable has a capacitance of 400pF, and its inner conductor has a radius of 2mm, find the radius of the outer conductor. **(2)**

3. **a.** Use the Biot-Savart Law to show that the magnitude of the magnetic flux density at a radial distance r from the centre of a current carrying wire is given by the following expression, where L is the length of the wire, and I is the current.

$$|\mathbf{B}| = \frac{\mu_0 I L}{2\pi r\sqrt{4r^2 + L^2}}$$

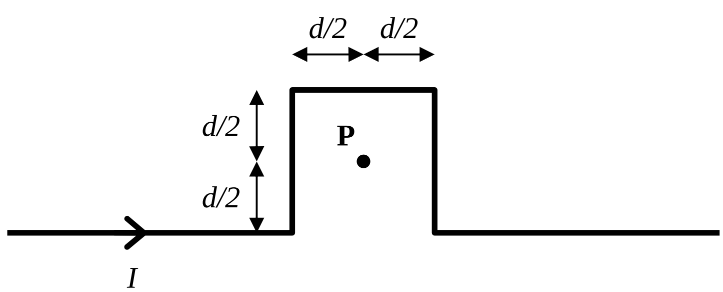
You may need to use the standard integral:-

$$\int \frac{dx}{\left(ax^2 + bx + c\right)^{3/2}} = \frac{2\left(2ax + b\right)}{\left(4ac - b^2\right)\sqrt{ax^2 + bx + c}}$$

(8)

b. Use the above expression to find the magnetic flux density at the centre of a 1m square circuit carrying a current of 5A **(4)**

c.



The figure above shows a detour in an otherwise infinitely long wire carrying a current I. Use superposition to derive an expression for the flux density at the point P. Evaluate this expression assuming I = 400A and d = 0.1m, and state the direction of the field. **(8)**

4. **a.** The magnetic flux density along the axis of an infinitely long solenoid is given by the expression $B = \mu_0 nI$ where n is the number of turns per unit length, and I is the current flowing. Using Ampère's Law, describe in detail how you would find expressions for the field off-axis, both inside and outside the solenoid. **(10)**

b. A 2000 turn solenoid is 15cm long, 3cm in diameter and carries a current of 1A. Calculate:

i. the magnetic field at the centre of the solenoid **(2)**

ii. the self-inductance of the solenoid **(2)**

c. A 12V battery is now connected, via a series resistor of 10Ω, to the solenoid described in (b).

i. Calculate the energy stored in the solenoid when the circuit reaches steady-state. **(2)**

ii. What will happen to this energy when the battery is disconnected? When might this be a problem, and what could be done to prevent it? **(4)**

JW / JBW