EEE408/EEE6022 (2011-2012) Model solutions

1.

a. For a purely inertia load, the torque required to move the payload of inertia J_L is only due to acceleration, and given by: (7)

$$T_L = J_L \frac{d\omega_L}{dt}$$

where $d\omega_L/dt$ is the load acceleration. Since the motor axis and the payload axis are coupled by a gear box with efficiency of η, the input and output powers of the gear box is related by:

$$T_m \omega_m = \frac{1}{\eta} T_L \omega_L$$

where ω_m is the motor angular speed, and T_m is the motor output torque. Hence, the motor output torque is given by:

$$T_m = \frac{a}{\eta} T_L$$

where $a = \omega_L/\omega_m$ is the gear ratio. Further the motor angular acceleration $d\omega_m/dt$ is related to the payload angular acceleration, $d\omega_L/dt$ by:

$$a\, d\omega_m / dt = d\omega_L / dt$$

The torque for accelerating the load that is reflected to the motor axis is given by:

$$T_{mL} = \frac{a}{\eta} T_L = \frac{a}{\eta} J_L \frac{d\omega_L}{dt}$$

Thus when the friction of the motor is neglected, the total motor torque required for accelerating/decelerating the drive system, expressed in terms of $d\omega_L/dt$ is given by:

$$T_{em} = (J_m / a + \frac{a}{\eta} J_L) \frac{d\omega_L}{dt}$$

As can be seen, the equivalent inertia $J_{eq} = (J_m/a + aJ_L/\eta)$ is a function of the gear ratio a, and reaches its minimum when

$$dJ_{eq} / da = -J_m / a^2 + J_L / \eta = 0$$

Thus:

$$a = \sqrt{\frac{\eta J_m}{J_L}}$$

QED.

b.

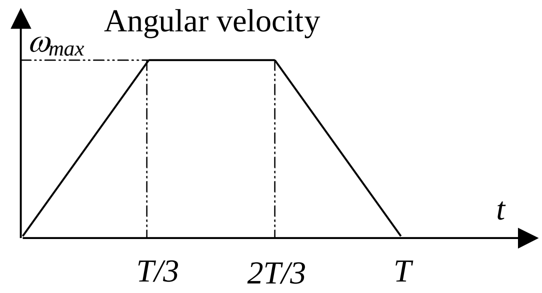
The optimal gear ratio is given by: (8)

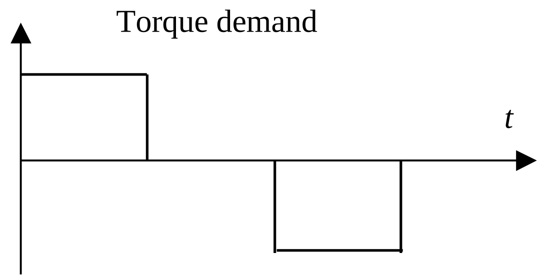
$$a = \sqrt{\eta \frac{J_m}{J_L}} = \sqrt{0{,}95 \frac{0.0005}{0.002}} = 0.487$$

and the combined inertia on the motor axis is:

$$J_{eq} = (J_m + a^2 \ J_L / \eta) = 2J_m = 0.001(kgm^2)$$

The total angular displacement to be rotated by the motor in a period of *T* sec is $\Theta = 2\pi 10 / 0.487 = 41.04\pi \ \ (rad)$. The corresponding trapezoidal velocity profile and the torque demand are shown in the figure below:





From the velocity profile, it can be derived that the maximum speed is related to the angular displacement Θ and the time period *T* by:

$$\omega_{\max} = 3\Theta / 2T$$

and the maximum angular acceleration is

$$\alpha_{\max} = 4.5\Theta/T^2$$

The peak torque requirement is

$$J_{eq}\alpha_{\max} = J_{eq} 4.5\Theta / T^2 = T_{empeak}$$

For a given T_{emmax} = 2 Nm, the time taken to complete the trapezoidal velocity profile is

$$T = \sqrt{J_{eq} 4.5\Theta / T_{em\max}} = \sqrt{0.001 * 4.5 * 41.04 * \pi / 2} = 0.54\,(\text{sec})$$

Maximum motor speed

$$\omega_{\max} = 3\Theta / 2T = 3 * 41.04\pi / 2 / 0.54 = 358.14 (rad / s)$$

The rms torque over the cycle is:

$$T_{emrms} = \sqrt{\frac{1}{T}(T_{em\max}{}^{2} \times \frac{2}{3}T} = \sqrt{\frac{2}{3}}T_{em\max} = 1.633 \text{(Nm)}$$

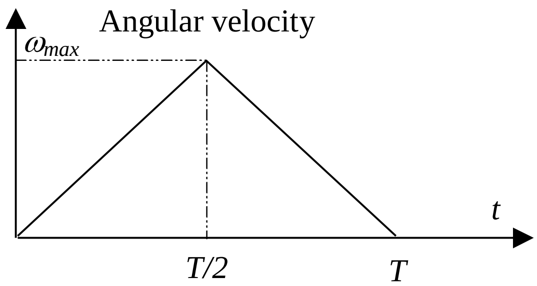
c. For the triangular velocity profile shown in below, the maximum speed is related to the angular distance Θ and the time period T by: (5)

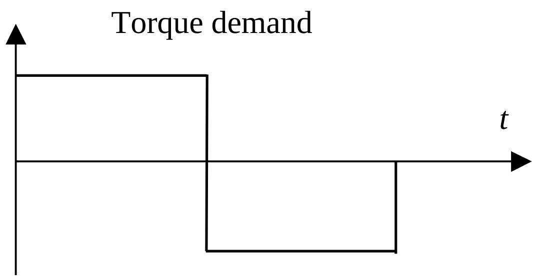
$$\omega_{\max} = 2\Theta / T = 2 * 41.04\pi / 0.54 = 477.52 \text{ (rad/s)}$$

The peak torque requirement is

$$J_{eq}\alpha_{\max} = J_{eq}\omega_{\max} /(T / 2) = 0.001 \times 477.52 \times 2 / 0.54 = 1.77 \text{ (Nm)}$$

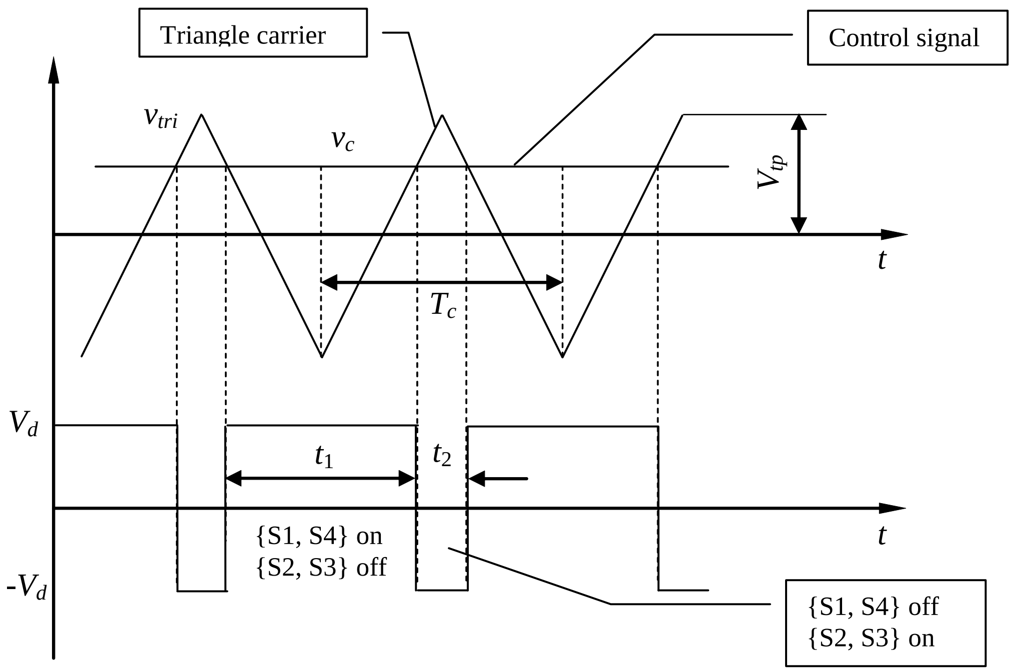
Although the required peak torque of 1.77 (Nm) is less than the available motor peak torque of 2 Nm, the rms torque of the triangle profile is also 1.77 (Nm), i.e., greater than the rated rms torque of 1.65 (Nm). Thus the motor will overheat if this movement is performed continuously. It is therefore not possible to use the triangle velocity profile for the same movement.





2.

a. In the bipolar operation, a triangle carrier signal with period T_c is compared with a control/command signal v_c, as shown in the figure below. The comparison produces a switching signal which controls the operation of all four switches. Switches (S1, S4) and (S3, S2) are controlled in pairs. During period t_1, as shown in the figure, when $v_c > v_{tri}$, (S1, S4) are on, and (S3, S2) off, and the motor terminal voltage is V_d. During period t_2 when $v_c < v_{tri}$, (S1, S4) are off, and (S3, S2) on, and the motor terminal voltage is $-V_d$. Thus the average terminal voltage is dependent upon the (6) duty ratio D defined as t_1/T_c



b. The torque constant of the motor can be determined by: (6)

$k_T = rated\ torque / rated\ current = 10/20 = 0.5$ (Nm/A)

and the back emf constant of the motor is therefore given by

$k_E = k_T = 0.5$ (Vs/rad)

For a payload torque of 5 Nm at a speed of 1500 rpm, the motor current is

$I_a = T_{em} / k_T = 5/0.5 = 10$ (A)

and the motor speed is

$\omega_m = (1500) \times 2\pi / 60 = 157.08$ (rad/s)

Therefore, the motor voltage in steady state is

$V_t = R_a I_a + k_E \omega_m = 10 * 0.4 + 0.5 \times 157.08 = 82.54$ (V)

The average output voltage V_t of the converter over a PWM cycle is related to duty ratio D and DC link voltage V_d by:

$$V_t = (2D-1)V_d$$

The PWM duty ratio is

$$D = \frac{1}{2}\left(\frac{V_t}{V_d} + 1\right) = \frac{1}{2}\left(\frac{82.54}{200} + 1\right) = 0.706$$

The time duration of t_1 is

$$t_1 = D*(T_c) = 0.706/f_c = 0.706/(20\times10^3) = 3.53\times10^{-5}\ (s)$$

At this speed, the motor back-emf is

$$E_a = 0.5*1500*2*\pi/60 = 78.54\ (V)$$

The peak-to-peak current ripple is therefore:

$$\Delta I_p = I_1 - I_0 = (V_d - E_a)/L_a * t_1 = (200 - 78.54)*3.53\times10^{-5}/3.2\times10^{-3}$$
$$= 1.27\ (A)$$

c. The switching of the terminal voltage between V_d and $-V_d$ will inevitable cause ripple in armature current, which not only incurs additional losses, but also produces pulsation torque. In steady state operation, however, the instantaneous motor speed ω_m can be assumed to be constant if there is sufficient inertia, and therefore $e_a(t) = E_a$ = constant. (8)

If the ripple current is primarily determined by the armature inductance L_a and R_a has a negligible effect, the current increases and decreases linearly. At the beginning of period t_1, the armature current I_a is at its minimum value of I_0, Thus

$$I_1 = I_0 + \frac{V_d - E_a}{L_a} t_1 \qquad (1)$$

The average current is $(I_1+I_0)/2$ and the peak-to-peak ripple is

$$\Delta I_{pp} = I_1 - I_0 = \frac{V_d - E_a}{L_a} t_1 \qquad (2)$$

During t_2 period, the terminal voltage is negative, and the current decrease linearly, and reaches I_0 at $t = t_2$, Hence,

$$I_0 = I_1 - \frac{V_d + E_a}{L_a} t_2 \qquad (3)$$

Adding (1) and (3) and solving for E_a yields

$$E_a = (2D-1)V_d \qquad (4)$$

Substitutes (4) into (2) and recognising $t_1 = DT_c$ one obtains

$$\Delta I_{pp} = \frac{2T_c V_d (1-D)D}{L_a}$$

It follows that the peak-to-peak current ripple is a function of duty ratio D, and reaches its maximum when $d(\Delta I_{pp})/dD = 0$, or $D = 1/2$

The maximum ΔI_{PP} is therefore given by:

$$\Delta I_{pp} = \frac{V_d}{2L_a f_c}$$

3.

a. Let the currents and voltages in phases, *a*, *b*, and *c* of the motor be denoted by [i_a i_b i_c] and [v_a v_b v_c] respectively, the input electrical power is given by: (6)

$$P = (i_a v_a + i_b v_b + i_c v_c) = [v_a \quad v_b \quad v_c]\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Represent the *abc* quantities of the voltages and currents in their $\alpha\beta$ components

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = C_{abc\leftarrow\alpha\beta}\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \quad ; \quad \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = C_{abc\leftarrow\alpha\beta}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

$$P = \begin{bmatrix} v_\alpha & v_\beta \end{bmatrix}\left(\left[C_{abc\leftarrow\alpha\beta}\right]^T\right)C_{abc\leftarrow\alpha\beta}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

It can be shown that

$$\left(\left[C_{abc\leftarrow\alpha\beta}\right]^T\right)C_{abc\leftarrow\alpha\beta}$$

$$= \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix} = \frac{3}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Finally

$$P = \begin{bmatrix} v_\alpha & v_\beta \end{bmatrix}\frac{3}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} == \frac{3}{2}\left(v_\alpha i_\alpha + v_\beta i_\beta\right)$$

b(i). From the motor data, the phase open-circuit rms voltage is (3)

E_{rms} =190/$\sqrt{3}$ = 109.7 (V)

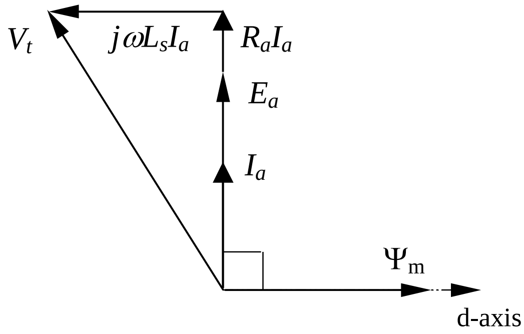
The no-load peak flux linkage of a phase winding produced by rotor permanent magnets is given by:

$$\Psi_m = \sqrt{2}\Psi_{rms} = \sqrt{2}E_{rms} / p\omega = \sqrt{2} * 109.7/3/(2\pi * 2000/60) = 0.247 \text{ (Wb)}$$

The motor torque constant is:

$$K_T = 3p\Psi_m / \sqrt{2} = 1.57 \text{ (Nm/rmsA)}$$

For the maximum torque per Ampere operation, the motor current should be in phase with the motor back-emf. Thus the resulting phase diagram is as follows:



Phasor diagram

b(ii) For a given torque requirement, the motor will reach its maximum torque per Ampere capability when the motor current is in phase with its back-emf. Thus, the rms phase current is (4)

$$I_a = T_{em} / K_T = 30/1.57 = 19.11(\text{A})$$

At 1500 rpm, the synchronous reactance is

$$\omega L_s = (2\pi * 1500/60) * 3 * 5.8 * 10^{-3} = 2.73(\Omega)$$

Induced phase rms back-emf:

E_a = (190/$\sqrt{3}$)*1500/2000 = 82.28 (V)
$R_a I_a$ = 1.05*19.11= 20.07 (V)
$\omega L_a I_a$ = 2.73*19.11 = 52.17(V)

Terminal voltage:

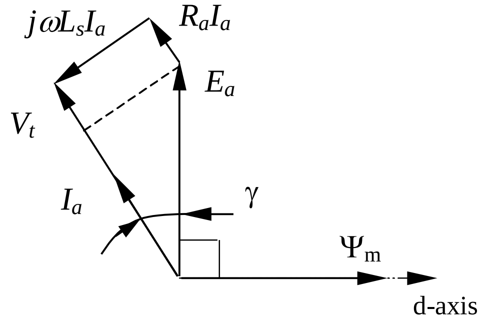
$$V_t = \sqrt{(\omega L_s I_a)^2 + (E_a + R_a I_a)^2} = \sqrt{52.17^2 + (82.28 + 20.07)^2} = 114.88\,(\text{V})$$

Power factor:

$cos\varphi = (E_a + R_a I_a)/V_t$ = 86.29/114.88=0.891

b(iii) (8)

From the phasor diagram at unit power factor operation



the following equation can be derived:

$$E_a \sin\gamma = \omega L_s I_a$$
$$T_{em} = K_T I_a \cos\gamma$$

Solving for I_a and γ gives:

$$\gamma = \frac{1}{2}\sin^{-1}\left(\frac{2\omega L_s T_{em}}{K_T E_a}\right) = \frac{1}{2}\sin^{-1}\left(\frac{2\times 2.73\times 19.11}{1.57\times 82.28}\right) = 26.94\,(\text{deg})$$

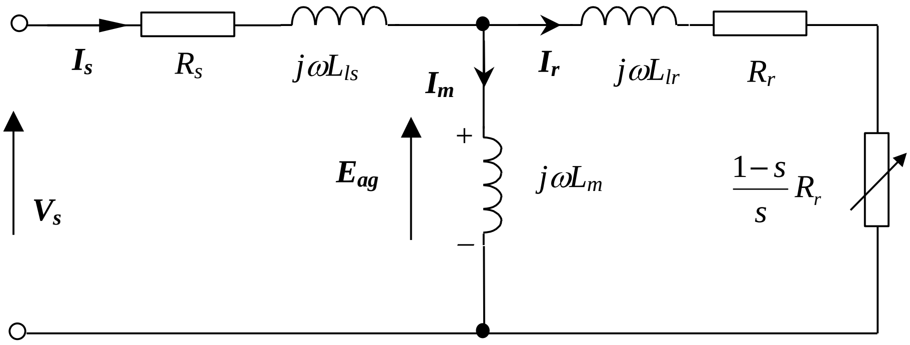
$I_a = T_{em}/(K_T \cos\gamma)$ = 30/1.57/cos 26.94 = 21.43 (A)

Phase voltage

$V_t = R_a I_a + E_a cos\gamma$ = 1.05*21.43+82.28*cos(26.94) = 95.85 (V)

4.

a. Equivalent circuit diagram of the motor operation (2)



R_s --- stator resistance
R_r --- rotor resistance reflected in stator
L_{ls}--- stator leakage inductance
L_{lr}--- rotor leakage inductance reflected in stator
L_m--- magnetising inductance
s --- slip

b. At rated speed of 1450 rpm, the slip s is given by: (9)

s = (1500-1450)/1500 = 0.033

For small values of s, $sR_s << R_r$ and $s\omega L_l << R_r$, and the motor electromagnetic torque is proportional to slip s. Thus, at 50% load torque, the slip s is 0.0167

The rotor speed is 1500*(1-s) = 1475 (rpm)

From the equivalent circuit diagram, the impedance of the rotor branch is

$R_r/s + j\omega L_{lr}$ = 0.55/0.0167+j0.95 = 32.93+ j0.95

The equivalent impedance of the parallel of the magnetising branch and the rotor branch is

$$\frac{(32.9+j0.95)j48.6}{32.9+j(0.95+48.6)} = \frac{1601.3\angle 91.65^0}{59.48\angle 56.42^0} = 26.92\angle 35.23^0 = 21.99 + j15.53$$

Total impedance seen from the stator terminal:

$$0.35 + j1.20 + 21.99 + j15.53 = 22.34 + j16.73 = 27.91\angle 36.82^0$$

Stator current

$$I_s = 240/27.91\angle 36.82^0 = 8.60\angle -36.82^0$$

Power factor

$\cos\varphi = \cos 36.82^0 = 0.80$

Induced air-gap voltage

$$E_{ag} = 240 - (R_s + j\omega L_{ls})I_s = 240 - (1.25\angle 73.74)8.60\angle -36.82^0 = 231.4 - j6.46$$
$$= 231.49\angle -1.6^0$$

Rotor current

$$I_r = E_{ag} /(R_r / s + j\omega L_{lr}) = 231.49\angle -1.6^0 /(32.94\angle 1.65^0) = 7.03\angle -3.25^0$$

Air-gap flux linkage

$$\Psi_{ag} = E_{ag} / 4.44 f = 231.43 / 4.44 / 50 = 1.04 (Wb)$$

The electromagnetic torque

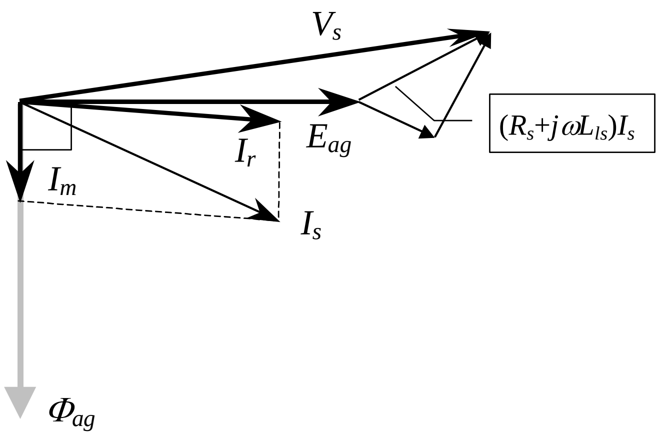
$$T_{em} = \frac{3R_r I_r^2}{s\omega_s} = \frac{3*0.55*7.03^2}{0.0167*157.08} = 31.1(Nm)$$

Efficiency

$$\eta = P_{out} / 3I_s V_s \cos\varphi = 31.1*154.46 /(3*240*8.6*0.8) = 0.97$$

Note the iron loss, friction and windage losses are not represented in the equivalent circuit, and therefore the efficiency is overestimated.

c. With reference to the phase diagram, the rotor position is represented by the magnetising current phasor I_m or the air-gap flux-linkage phasor Ψ_{ag}. The air-gap voltage phasor E_{ag} leads I_m by 90^0. Under the operation condition in 4(b) the motor voltage phasor leads E_{ag} by 1.6^0 (9)

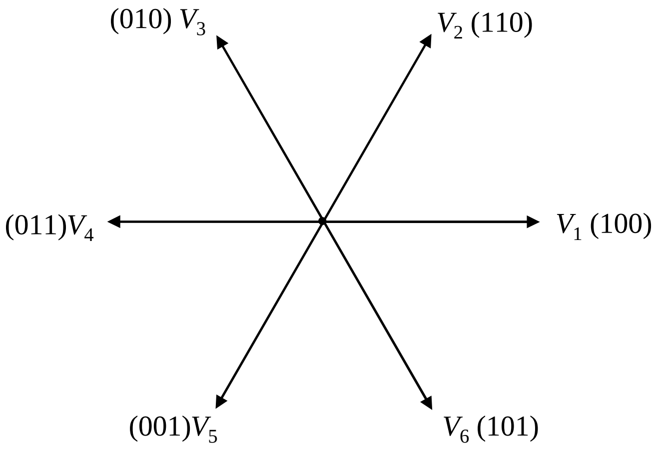


Thus the space voltage vector V_{cs} which needs to be produced by the inverter has a magnitude of $240\sqrt{2} = 339.4$ V and angle of $(91.6^0+55^0) = 146.6^0$

Since the voltage vector lies in the section between 120^0 and 180^0, the two active voltage space vectors, V_3 and V_4 will be used. The required modulation ratio is:

$$m = \frac{\sqrt{3}V_{cs}}{V_d} = \frac{\sqrt{3} * 339.4}{600} = 0.98$$

and the angle between the voltage vector and V_3 is 26.6^0, Thus the time duration for V_3 and V_4 are given respectively by:



$$t_1 = mT\sin(\pi/3 - \delta) = 0.98 * \sin(60^0 - 26.6^0)/2000 = 0.270(ms)$$
$$t_2 = mT\sin\delta = 0.98 * \sin(26.6^0)/2000 = 0.219(ms)$$

The time duration for the two zero vectors are:

$$t_0 = t_7 = 0.5(T - (t_1 + t_2)) = 0.005(ms)$$

The corresponding per-cycle switching sequence waveforms are shown below: