Solutions

Q1a

The fraction of the incident power that is absorbed by the load, is missing from the signal returned to the generator. This "*loss*" is called the return loss (*RL*) **(1 mark)**

$$\mathrm{RL} = -10\log\left(\frac{\mathrm{P}_{in}^{-}}{\mathrm{P}_{in}^{+}}\right) = -10\log|\Gamma_{in}|^2 = -20\log|\Gamma_{in}|$$ **(1 mark)**

Q1b

$$\lambda = \frac{c}{\mathrm{f}} = 10\mathrm{cm}$$

Therefore

$$\beta\ell = \frac{2\pi}{\lambda}\times 1 = 0.628$$ **(1 mark)**

$$Z_{in} = Z_o\frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} = (62 - j35)\Omega$$ **(1 mark)**

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = 0.19\text{-}j0.25 = 0.31\angle\text{-}52.7^\circ$$ **(1 mark)**

$$\mathrm{VSWR} = \frac{1+|\Gamma_{\mathrm{in}}|}{1-|\Gamma_{\mathrm{in}}|} = 1.9$$ **(1 mark)**

$$IL = 10\log\left(1-|\Gamma_{in}|^2\right) = -0.45\mathrm{dB}$$ **(1 mark)**

Q1c

For a lossless line

$$Z_{in} = Z_o\frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)}$$

When the line is terminated by a load impednce of Z_L=0, i.e. short circuited line, the input impedance is given by

$$Z_{inSC} = jZ_o\tan(\beta\ell)$$ **(1 mark)**

which is a reactive impedance that can be made equivalent to a lumped capacitor or indictor by adjusting the length ℓ. An inductive reactance can be obtained using $0 \le \ell \le 0.25\lambda$, while for $0.25\lambda \le \ell \le 0.5\lambda$ a capacitive reactance can be achieved. **(2 marks)**

When the line is terminated by a load impednce of $Z_L=\infty$, i.e. open circuited line, the input impedance is given by

$$Z_{inOC} = -jZ_o\cot(\beta\ell)$$ **(1 mark)**

Again, this is a reactive impedance that can be made equivalent to a lumped capacitor or indictor by adjusting the length ℓ. A capacitive reactance can be obtained using $0 \le \ell \le 0.25\lambda$ while for $0.25\lambda \le \ell \le 0.5\lambda$ an inductive reactance can be achieved. **(2 marks)**

Q1d

The voltage at any point along the line is a superposition of the incident and reflected voltage components, i.e.

$$V = V_{inc} + V_{ref}$$ **(1 mark)**

Therefore the voltage at a distance *d* from the load is given by

$$V = V_{inc}(1 + \Gamma e^{-2j\beta d})$$ **(1 mark)**

$$|V| = |V_{inc}|\left|1+\rho e^{-2j(\beta d - \frac{\theta}{2})}\right|$$

In which ρ and θ represent the magnitude and phase of the reflection coefficient at the load, **(1 mark)** which means

$|V_{\max}| = |V_{inc}||1+\rho|$ when $(\beta d - \frac{\theta}{2}) = n\pi$ **(1 mark)**

and

$|V_{\min}| = |V_{inc}||1-\rho|$ when $(\beta d - \frac{\theta}{2}) = n\pi + \frac{\pi}{2}$ **(1 mark)**

Since the voltage standing wave ration is defined as

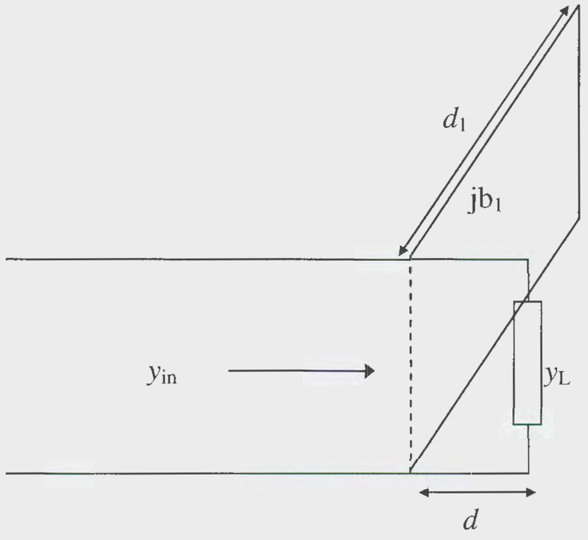
$$\text{VSWR} = \frac{|V_{\max}|}{|V_{\min}|}$$ **(1 mark)**

Therefore

$$\text{VSWR} = \frac{1+|\rho|}{1-|\rho|}$$ **(1 mark)**

Q2a

A single open, or short, circuited transmission line stub whose length d_1 may be varied between 0 and 0.25λ, and whose position along a transmission line, d, is adjustable over a range of 0.5λ will match Z_L to the main line. **(1 mark)**



(1 mark)

The main problem is to choose d_1 and d such that y_{in}=1, i.e. matched to the line. The distance d is chosen such that y_L is transformed to some point lying on the unit conductance circle, i.e. in the absence of stub $y_{in}=1\pm jb$. **(2 marks)**

Q2b

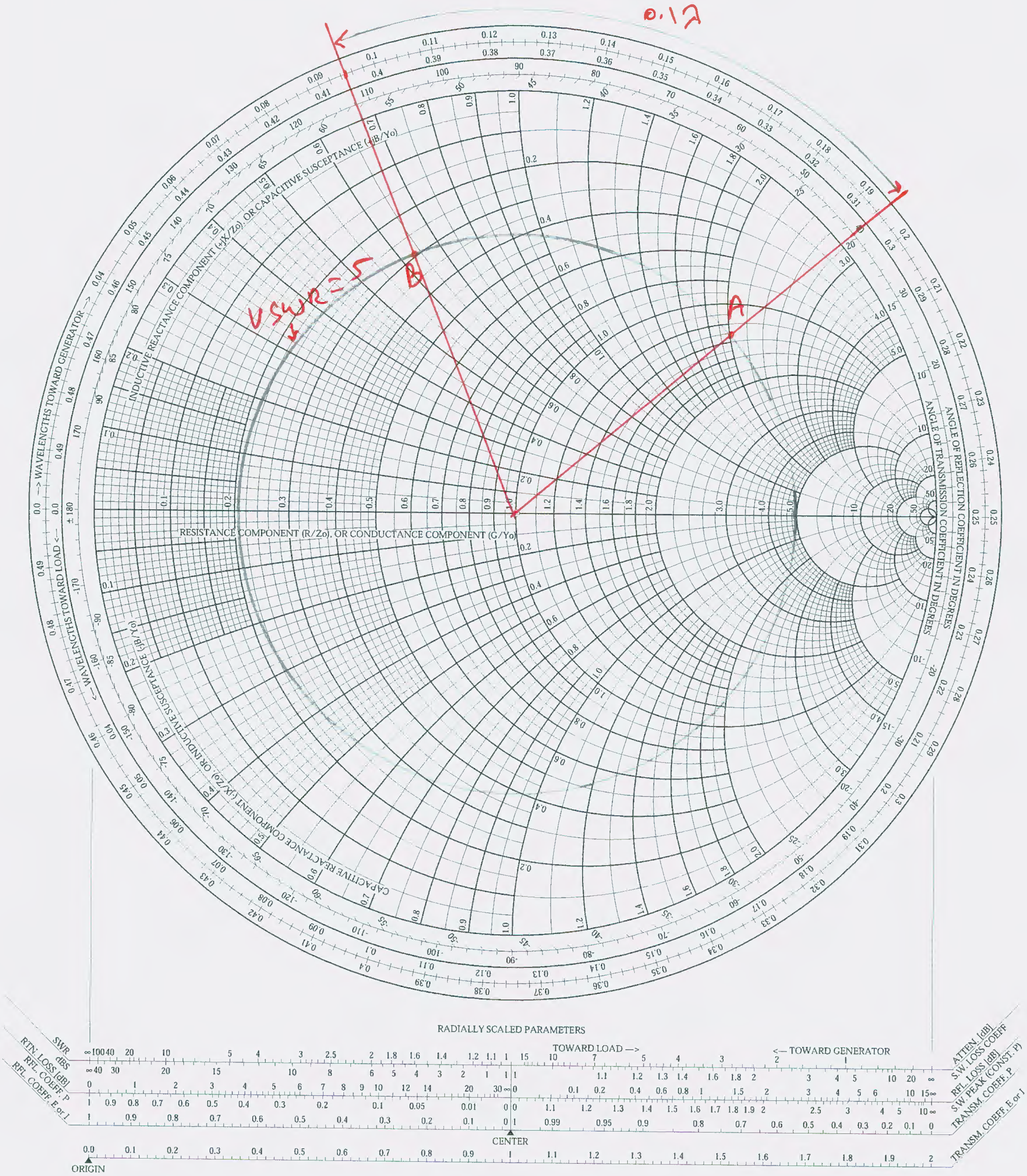
For a VSWR of 5, the VSWR circle can be plotted on the Smith chart. **(1 mark)**
Since the angle of input reflection coefficient is 40°, point A represents the normalized input impedance, i.e. z_{in}= 1.3+j2 **(1 mark)**
which means Z_{in}=(65+j100)Ω **(1 mark)**

To find the load impedance, move a distance of 0.1λ, *towards load*, from point A to B. **(1 mark)**
The impedance at B z_L=0.3+j0.63 **(1 mark)**
which means Z_L=15+31.5 **(1 mark)**

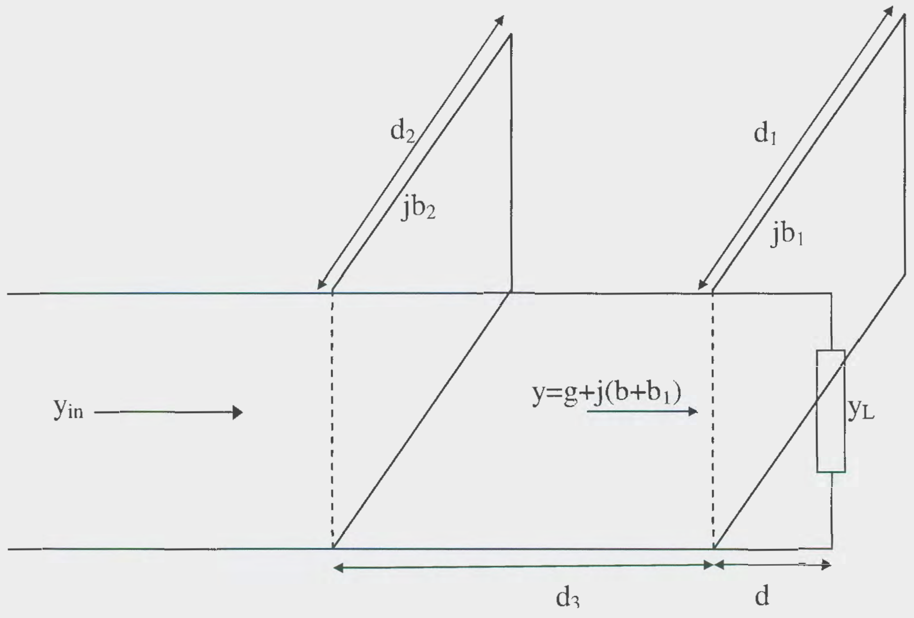
2.2b

The Complete Smith Chart

Black Magic Design



Q2c



$z_L = \frac{(100+j40)}{50} = 2 + j0.8$ (point A) **(1 mark)**

i.e. y_L=0.43-j0.17 (point B) **(1 mark)**

Step 1

Rotate the unit g circle *Towards Load*, by a distance of d_3=0.125λ. **(1 mark)**

Step 2

Move from point B to intersect the new, rotated, unit circle at point C. The movement should be on the corresponding conductance circle, since the stub does not alter the real part of the admittance. **(1 mark)**

Step 3

The admittance at point C is

y_C=0.43+j0.18 **(1 mark)**

compare it with that at B

y_L=0.43-j0.17

shows that stub 1 has provided j0.35, i.e. b_1=0.35 **(1 mark)**

Step 4

For an o.c. stub, this means d_1=0.054λ, i.e. the distance from D to E on the chart. **(1 mark)**

Step 5

Move a distance d_3=0.125λ along the line from the 1st stub position to the 2nd stub position (from point C to F). **(1 mark)**

Step 6

At point F, the admittance is y_F=1+j0.9, i.e. stub 2 must provide - j0.9 (b_2=-0.9) to reach the matched condition. **(1 mark)**

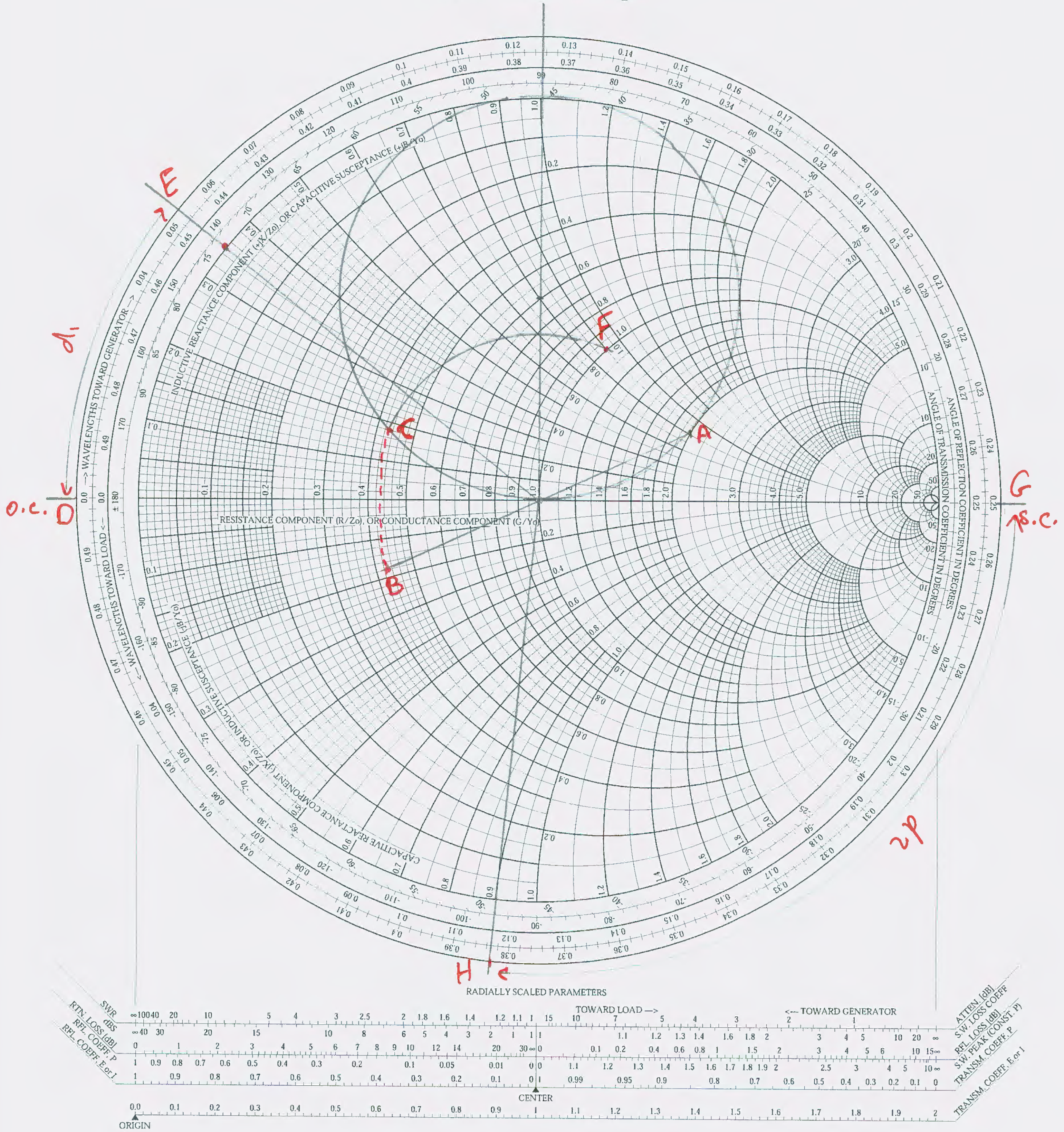
Step 7

For an o.c. stub, this means d_2=0.133λ, i.e. the distance from G to H on the chart. **(1 mark)**

Q2C

The Complete Smith Chart

Black Magic Design



Q3a

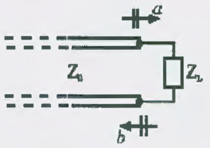
SFD can be used for a graphical representation of the interaction between the incident and scattered waves in a microwave network. **(1 mark)**

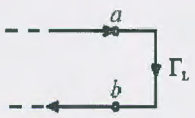
There are two basic components of any SFD:

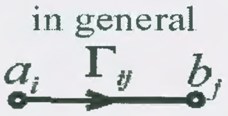
Nodes; each port in the network has two nodes a_n and b_n. The node a_n represents a wave entering port n, while node b_n represents a wave reflected from port n.

Branches; each branch represents a direct path between a-node and b-node. For each branch there is an associated S parameter or reflection coefficient. **(1 mark)**

An example of the SFD of a one-port network is shown below







(2 marks)

Q3b

Available Power Gain

$$G_A = \frac{P_{avn}}{P_{avs}}$$ **(1 mark)**

It is the ratio between power available from the network and the power available from the source. In most of the cases this gain is independent of Z_L. Gain of some active circuits is a function of Z_L. **(1 mark)**

Transducer Power Gain

$$G_T = \frac{P_L}{P_{avs}}$$ **(1 mark)**

It is the ratio between power available at the load and the power available from the source. This gain depends on both Z_g and Z_L. **(1 mark)**

Q3c

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$ **(1 mark)**

For the given network

$$V_1 = V_2 - 75\, I_2$$ **(1 mark)**
$$I_1 = -\, I_2$$ **(1 mark)**

which gives

A=1, B=75, C=0, D=1 **(1 mark)**

Q3d

To calculate S_{11}, the impedance Z_{in1} is required when the network is terminated with Z_o, where $Z_o = Z_{o1} = Z_{o2}$

$$Z_{in1} = \frac{Z_1(Z_2 + Z_o)}{Z_1 + Z_2 + Z_o}$$ **(1 mark)**

$$S_{11} = \Gamma_{in} = \frac{Z_{in1} - Z_o}{Z_{in1} + Z_o}$$

i.e.

$$S_{11} = \frac{Z_1 Z_2 - Z_o(Z_2 + Z_o)}{Z_1 Z_2 + Z_o(Z_2 + Z_o) + 2Z_o Z_1}$$ **(1 mark)**

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

For the 1st port

$$V_1 = \sqrt{Z_o}(a_1 + b_1)$$

$$V_2 = \sqrt{Z_o} b_2$$ **(1 mark)**

Therefore

$$\frac{V_1}{V_2} = \frac{(a_1 + b_1)}{b_2} = \frac{(1 + \frac{b_1}{a_1})}{\frac{b_2}{a_1}} = \frac{1 + S_{11}}{S_{21}}$$

i.e.

$$S_{21} = \frac{V_2}{V_1}(1 + S_{11})$$ **(1 mark)**

Since

$$V_2 = V_1 \frac{Z_o}{(Z_o + Z_2)}$$

then

$$\frac{V_2}{V_1} = \frac{Z_o}{(Z_o + Z_2)}$$ **(1 mark)**

$$1 + S_{11} = \frac{2Z_1(Z_o + Z_2)}{Z_1 Z_2 + Z_o(Z_2 + Z_o) + 2Z_o Z_1}$$

Therefore

$$S_{21} = \frac{2Z_o Z_1}{Z_1 Z_2 + Z_o(Z_2 + Z_o) + 2Z_o Z_1}$$ **(1 mark)**

Similarly

$$Z_{in2} = Z_2 + \left(\frac{Z_1 Z_o}{Z_1 + Z_o}\right)$$

then

$$S_{22} = \frac{Z_{in2} - Z_o}{Z_{in2} + Z_o} = \frac{Z_1 Z_2 + Z_o(Z_2 - Z_o)}{(Z_1 Z_2 + Z_o(Z_o + Z_2)) + 2Z_1 Z_o}$$ **(1 mark)**

and

$$S_{12} = \frac{2Z_o Z_1}{Z_1 Z_2 + Z_o(Z_2 + Z_o) + 2Z_o Z_1}$$ **(1 mark)**

Q4a

Generally transistors presents a significant impedance mismatch, so matching will be achieved over a narrow frequency bandwidth. When bandwidth is an issue, then we design for a gain less than the maximum, imperfect matching, to improve bandwidth. **(2 marks)**

Sometimes it is required to design an amplifier with a specific gain, other than the maximum. Constant gain circles will be used to facilitate design for a specific gain. **(2 marks)**

Q4b

The reflections coefficients can be calculated as

$$\Gamma_s = \frac{Z_g - Z_o}{Z_g + Z_o} = \frac{20-50}{20+50} = -0.429$$ **(0.5 mark)**

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{30-50}{30+50} = -0.25$$ **(0.5 mark)**

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.455\angle 150^\circ$$ **(1 mark)**

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = 0.408\angle -151^\circ$$ **(1 mark)**

Therefore the required gains are

$$G = \frac{|S_{21}|^2\left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2\left(1 - |\Gamma_{in}|^2\right)} = 5.94$$ **(1 mark)**

$$G_A = \frac{|S_{21}|^2(1 - |\Gamma_s|^2)}{|1 - S_{11}\Gamma_s|^2\left(1 - |\Gamma_{out}|^2\right)} = 5.85$$ **(1 mark)**

$$G_T = \frac{|S_{21}|^2(1 - |\Gamma_s|^2)(1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2|1 - \Gamma_s\Gamma_{in}|^2} = 5.49$$ **(1 mark)**

Q4c

$$N = \frac{NF - NF_{min}}{4R_N/Z_o}\left|1 + \Gamma_{opt}\right|^2 = \frac{1.778\text{-}1.58}{80/50}\left|1 + 0.62\angle 100^\circ\right|^2 = 0.145$$ **(1 mark)**

$$C_{NF} = \frac{\Gamma_{opt}}{(N+1)} = 0.541\angle 100^\circ$$ **(1 mark)**

$$r_{NF} = \frac{\sqrt{N(N + 1 - |\Gamma_{opt}|^2)}}{(N+1)} = 0.29$$ **(1 mark)**

Next we calculate data for several input section constant gain circles

G_S	g_S	C_S	R_S
1.5dB	0.905	$0.56\angle 60^\circ$	0.204
1.7dB	0.948	$0.58\angle 60^\circ$	0.149
1.8dB	0.970	$0.59\angle 60^\circ$	0.112

The noise figure and constant gain circles are plotted on the Smith chart **(4 marks, 1 for each circle)** and the G_S =1.8dB circle just intersects the noise figure circle, and any higher gain will results in a worse noise figure. **(1 mark)**

From the Smith chart the optimum solution is then $\Gamma_S = 0.55\angle 70^\circ$ when NF =2.5dB. **(2 marks)**