The University Of Sheffield.

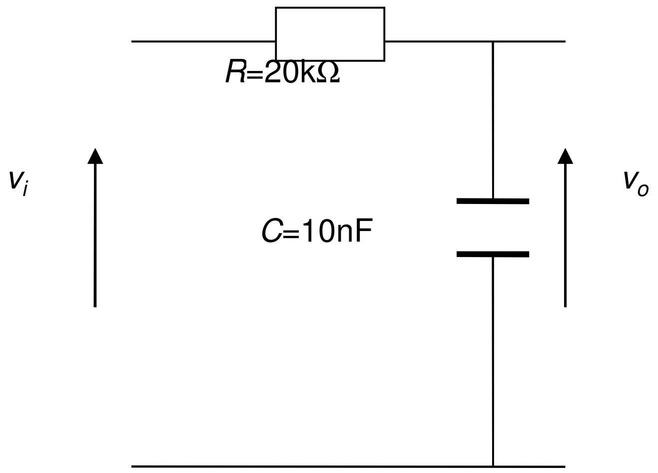
Lecture content

- System analysis via Fourier Series
  - Low pass filter. Obtain the transfer function of the system.
  - Analyse the magnitude and phase of the transfer function at each harmonic frequency.
  - Evaluate and modified the magnitude and phase of each harmonic to obtain the output signal.
- RMS value of a periodic signal
  - $(rms_{tot})^2 = (rms_1)^2 + (rms_3)^2$…..
- Parseval's theorem

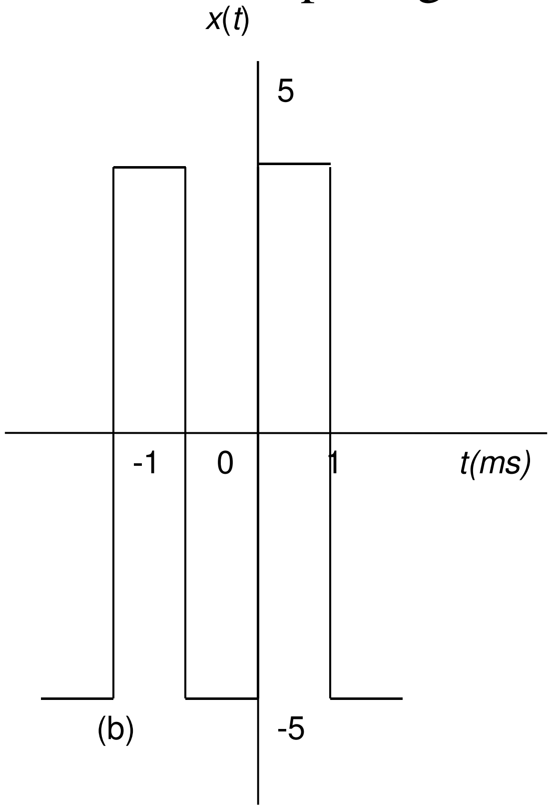
FS example: Low pass filter

Consider a RC low pass filter shown in figure 4.8.

We can show that this RC circuit is a low pass filter by analysing the response of the circuit to the harmonics of a periodic signal. Consider an input signal shown in figure 4.8(b).



(a)

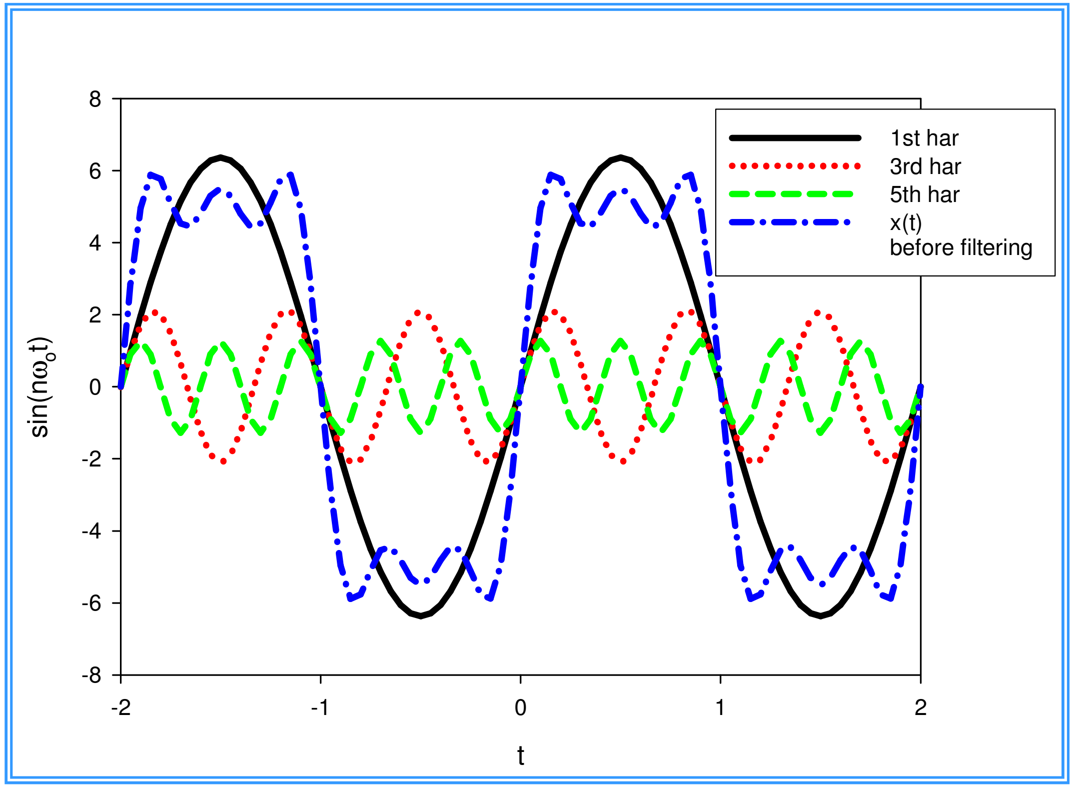


(b)

Low pass filter

Before filtering

$$x(t) = \frac{20}{\pi}\sin 1000\pi t + \frac{20}{3\pi}\sin 3000\pi t + \frac{20}{5\pi}\sin 5000\pi t$$



Before filtering $D_1 = 1$, $D_3 = 1/3$, $D_5 = 1/5, \ldots\ldots D_n = 1/n$.

$\omega_c = 1/RC = 5000$ rad/s and $\omega_o = 2\pi/(2\text{ms}) = 1000\pi$.

The transfer function of the RC circuit is $H(\omega) = \dfrac{1}{1 + j\left(\dfrac{\omega}{\omega_c}\right)}$

After filtering:

At $\omega = \omega_o$, $H(\omega) = \dfrac{1}{1 + j\left(\dfrac{1000\pi}{5000}\right)} = 0.85\angle\text{-}32^{o}$.

The amplitude of the fundamental is reduced to $0.85 \times 20/\pi = 17/\pi$ (15% reduction).

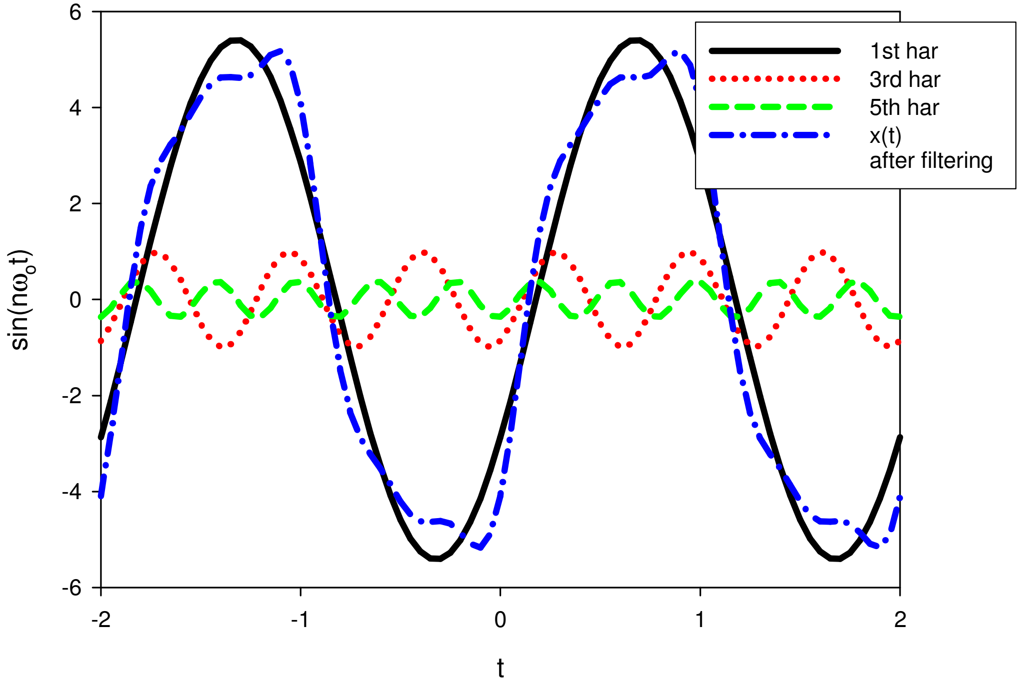
At $\omega = 3\omega_o$, $H(\omega) = \dfrac{1}{1 + j\left(\dfrac{3000\pi}{5000}\right)} = 0.47\angle\text{-}62^{o}$.

The amplitude of the 3rd harmonic is reduced to $0.47 \times 20/3\pi = 9.4/3\pi$ (53% reduction). $D_3 = (9.4/3\pi)/(17/\pi) = 0.184$.

Low pass filter

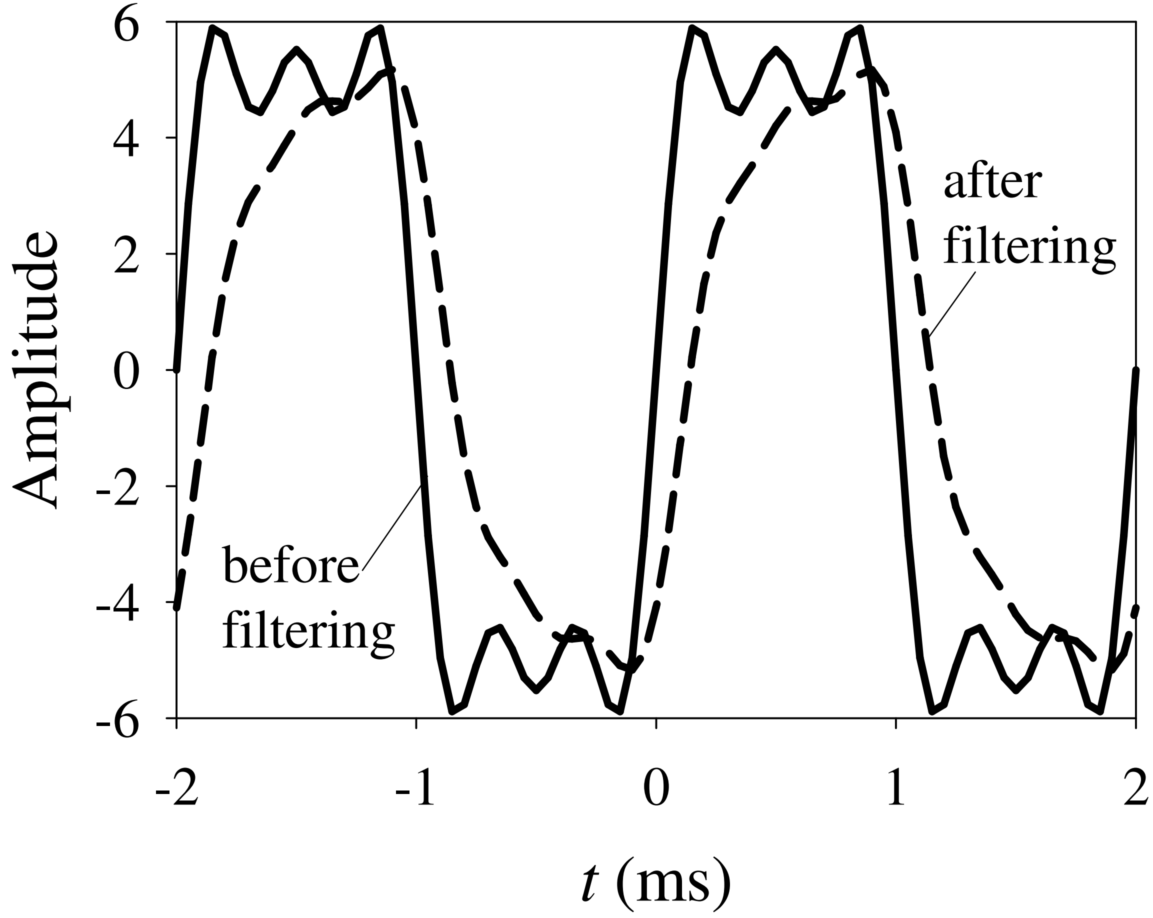
After filtering

$$x_o(t) = \frac{17}{\pi}\sin(1000\pi t - 32\pi/180) + \frac{9.4}{3\pi}\sin(3000\pi t - 62\pi/180) + \frac{6}{5\pi}\sin(5000\pi t - 72\pi/180)$$



Low pass filter

The signal $x(t)$, approximated by including up to the 5th harmonics, before and after filtering are shown below.



Low pass filter

Clearly, the higher frequency harmonics are attenuated more significantly than the fundamental showing that the RC circuit in figure 4.8 behaves like a low pass filter. We find that the harmonic components can be considered separately and the overall response can then be obtained by adding the individual response of each harmonic.

For instance the rms value can be obtained from

$(rms_{tot})^2 = (rms_1)^2 + (rms_2)^2 + (rms_3)^2$ ……….

The response will contain the same frequency components as the input but the amplitude and the phase of each harmonic will be modified differently according to the frequency response of the system.

exercise: Obtain the rms voltage of the signal after filtering in the above example.

Parseval's theorem

Consider a voltage waveform $x(t)$ applied to a 1Ω resistor. The current flowing through the resistor is $x(t)$ and the power is $x(t)^2$. The total energy supplied by x(t) from -∞ to ∞ is

$$E = \int_{-\infty}^{\infty} x(t)^2 \, dt$$

where $x(t)$ is assumed to be real. If $x(t)$ is complex the total energy is given by

$$E = \int_{-\infty}^{\infty} x(t)x^*(t)dt = \int_{-\infty}^{\infty} \left|x(t)\right|^2 dt$$

This equation shows that every periodic signal has $E = \infty$. It is therefore more meaningful to compute the average power over one period as

$$P_{av} = \frac{1}{T}\int_0^T \left|x(t)\right|^2 dt = \sum_{n=-\infty}^{\infty} \left|c_n\right|^2$$

The average power can be calculated if the complex Fourier Series coefficients are known.

Parseval's theorem

Example:
Obtain the average power in the signal shown in figure 4.3, within the frequency range [-7π rad/s, 7π rad/s].

From figure 4.3, $\omega_o = 2\pi$. Therefore only c_m with $m = \pm1$, 0 and ±3 exist within the frequency range [-7π rad/s, 7π rad/s].
$c_o = 1/2$
$|c_1| = |c_{-1}| = 1/\pi$
$|c_3| = |c_{-3}| = 1/3\pi$
The average power within this frequency range is

$$\sum_{-3}^{3} |c_n|^2 =$$