EEE6212 Lecture 16 “LASERs”

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Outline

- Interference and Coherence
- Spatial and Temporal Coherence
- Revisit the Two-Level system
- Spontaneous Emission
- Stimulated Emission and Lasing (needs population inversion)
- Feedback (needs a resonating cavity)
- Gain
- Semiconductor Materials
- Types of Semiconductor LASERs

Interference and Amplitude Modulation

Take two waves of frequencies ω_1, ω_2

$$E_1 = E_0 e^{j\omega_1 t}, \qquad E_2 = E_0 e^{j\omega_2 t}$$

Get $E_1 + E_2 = E_0[e^{j\omega_1 t} + e^{j\omega_2 t}]$

Trick:

$$\omega_1 = \frac{\omega_1+\omega_2}{2} + \frac{\omega_1-\omega_2}{2}, \qquad \omega_2 = \frac{\omega_1+\omega_2}{2} - \frac{\omega_1-\omega_2}{2}$$

So $E_1 + E_2 =$

$$2E_0 \cos\left(\frac{(\omega_1 - \omega_2)t}{2}\right) e^{j\left(\frac{\omega_1+\omega_2}{2}\right)t}$$

Slow amplitude modulation x fast oscillation

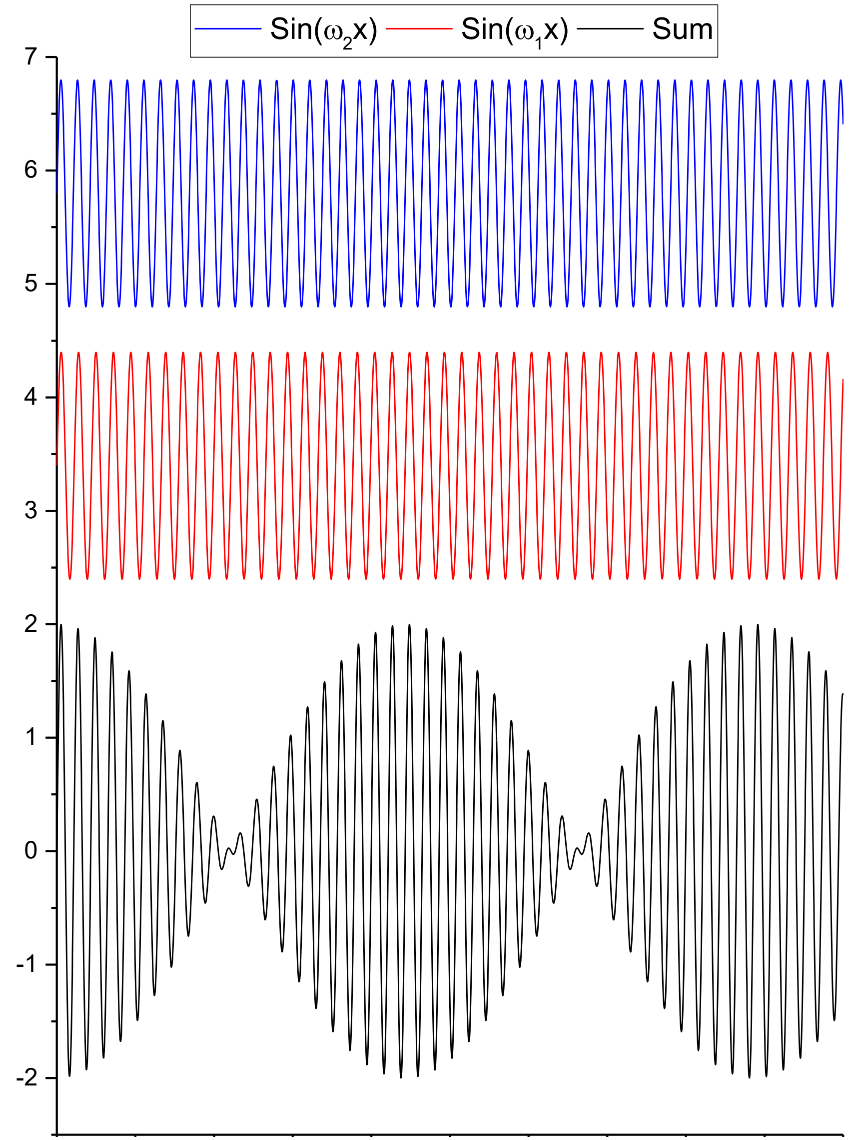
Intensity for a slow detector

$$\langle E^2 \rangle = 4E_0 \cos^2[(\omega_1 - \omega_2)\, t/2]$$

Consider source as monofrequent,

with $\Delta\omega = (\omega_1 - \omega_2)/2$ if measurement time is

$\tau \approx \frac{\pi}{\Delta\omega}$ ➔ Temporal coherence



Coherence

Coherence: Our ability to predict the phase of the oscillation at a later time if we know the phase now. Usually described as a necessary condition to observe interference.

Temporal Coherence Time

(two waves of slightly different frequencies travelling along the same path)

$$\tau \approx \frac{\pi}{\Delta\omega}$$

A monofrequent wave without any broadening $\Delta\omega$ would be perfectly temporally coherent

Similarly,

Spatial Coherence Length

(Phase relationship between waves travelling side by side through space)

$$x \approx \frac{\pi}{\Delta k}$$

A wave of fixed wavevector $\underline{k}$, $\left(\Delta\underline{k} = 0\right)$ would be perfectly spatially coherent

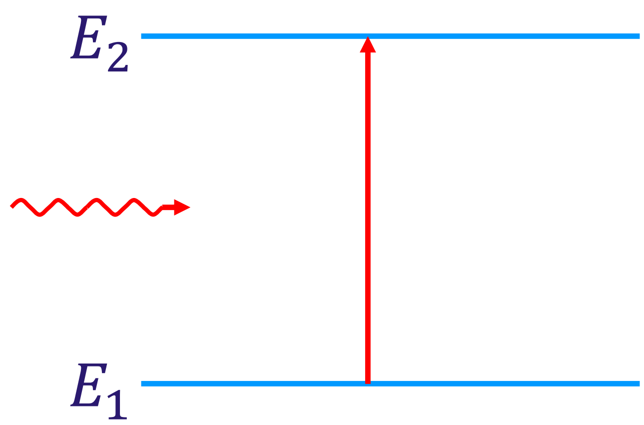
Often this is written in terms of angles: $\Delta\Omega = \lambda^2/\Delta A$, where ΔA is the area of the source.

A perfectly spatially coherent wave would come from a single, perfect point $(\Delta A = 0)$

<u>Note that in practice no light source is either perfectly coherent or incoherent.</u>

Absorption

Consider transitions between **two** discrete energy levels E_1 and E_2 that are occupied by N_1 and N_2 electrons respectively



Probability for absorption of a photon by electron in the ground state E_1

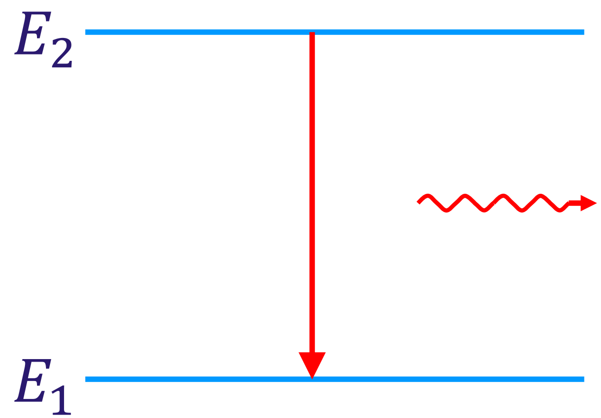
$$\frac{dN_1}{dt} = -B_{12}N_1\rho_{12}$$

With Einstein coefficient B_{12} for induced absorption and "spectral energy density" ρ_{12} of the radiation

The equation is known as a "rate equation"

Spontaneous Emission

Consider transitions between **two** discrete energy levels E_1 and E_2 that are occupied by N_1 and N_2 electrons respectively



Probability for spontaneous emission of a photon by electron in the excited state E_2

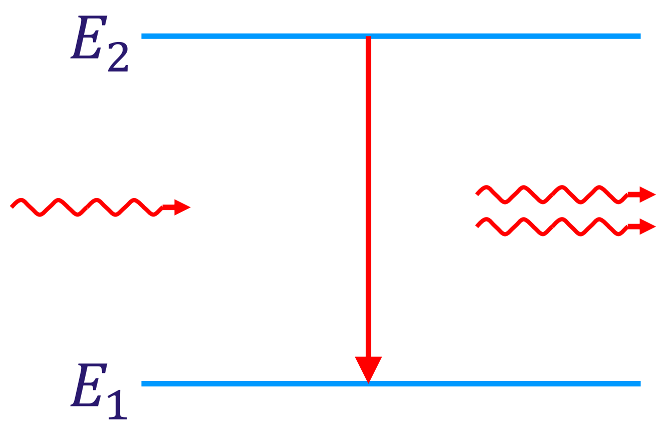
$$\frac{dN_2}{dt} = -A_{21}N_2$$

With Einstein coefficient $A_{21} = 1/\tau_{21}$ for spontaneous emission.

This spontaneous emission is isotropic and can therefore often be neglected.

Stimulated Emission

Consider transitions between **two** discrete energy levels E_1 and E_2 that are occupied by N_1 and N_2 electrons respectively



Probability for stimulated emission of a photon by electron in the excited state E_2 caused by an external radiation field

$$\frac{dN_2}{dt} = -B_{21} N_2 \rho_{21}$$

With Einstein coefficient B_{21} for stimulated emission and "spectral energy density" ρ_{21} of the radiation

Photon generation rate in a 2-level system

Einstein could show that

$$B_{21} = B_{12}$$

(If the degeneracies of the two states are equal)

and that

$$\frac{A_{21}}{B_{12}} = (energy) \times (spectral\ mode\ density) = hf \times \frac{8\pi f^2}{c^3} = \frac{8\pi h f^3}{c^3}$$

The density of states N_i, in *thermodynamic equilibrium* obeys the Fermi-Dirac (or at elevated temperatures, the Boltzmann) statistics

$$N_i \propto e^{-E_i/kT}$$

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT}$$

Note for $E_2 > E_1$ at any given temperature, that $N_2 < N_1$ *always*

Define $\Delta N = N_2 - N_1$ hence $\Delta N < 0$

The number of photons generated per length unit dz in time $dt = \frac{dz}{c}$

$$dQ = (dN_2 - dN_1)dt = B_{21}N_2\rho_{21}\frac{dz}{c} - B_{12}N_1\rho_{21}\frac{dz}{c}$$

$$\frac{dQ}{dz} = \frac{B_{21}\Delta N\rho_{21}}{c} < 0$$

This means it is impossible to get more photons out by amplification in thermodynamic equilibrium.

The physical reason is that the ratio of the probabilities for absorption to stimulated emission is always ≥ 1

3-level systems

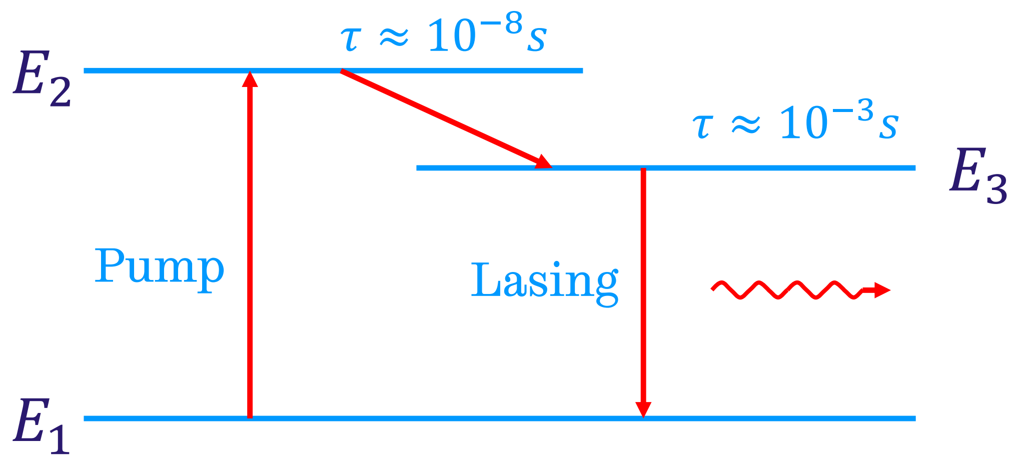
$\frac{dQ}{dz}$ will only be > 0 if $\Delta N > 0$ i.e. we need **population inversion**

For this we need (at least) a 3rd long lived (metastable) energy level E_3

$$N_1 + N_2 + N_3 = N$$

But $N_2 \approx 0$, because this state decays rapidly

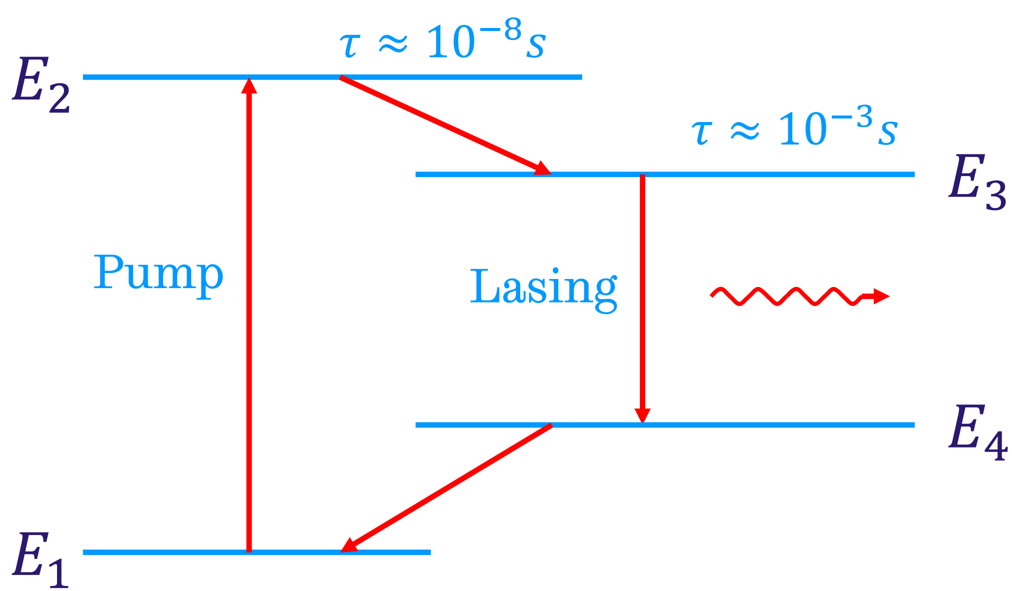
Hence for inversion we need $N_3 > N/2$

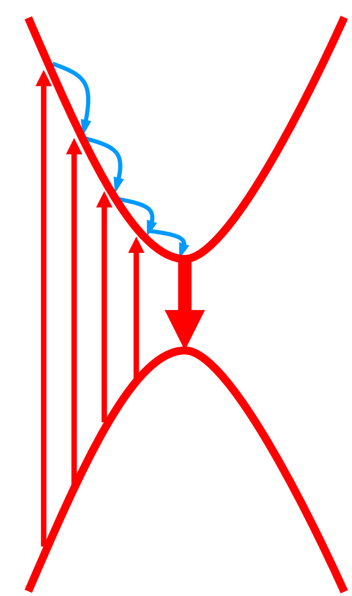


4-level systems

If a fourth level E_4 between E_3 and E_1 is involved and lasing occurs for the transition from $E_3 \rightarrow E_4$ then inversion will occur **automatically** if E_4 is not normally thermally occupied.

All semiconductor ***interband*** LASERs are actually 4-level systems: Pumping occurs from anywhere in the valence band to anywhere in the conduction band. i.e. $E_3 = E_c, E_4 = E_v$





Feedback Theory: Loop Gain and Total Gain

Signal: x_s

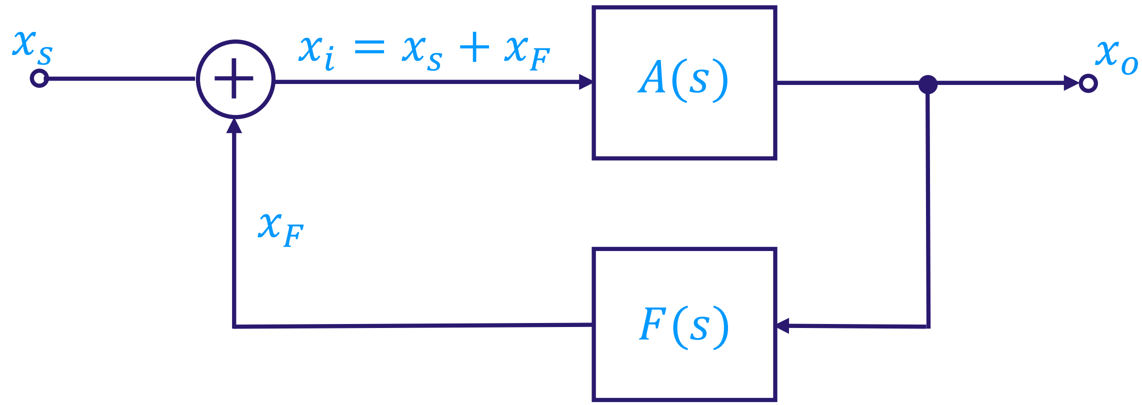
Input: $x_i = x_s + x_F$

Output: $x_o = A(s)x_i$

Feedback: $x_F = F(s)x_o$

Loop Gain: AF

Total Gain: $\frac{A}{1-AF}$



Consider a simple amplifier where x_s can be either a voltage or a current signal to be amplified. $A(s)$ denotes the frequency dependent amplification factor and $F(s)$ the feedback. Then we have

$$x_i = x_s + x_F$$
$$x_o = A(s)x_i \Rightarrow x_o = A(s)[x_s + F(s)x_o]$$
$$x_F = F(s)x_o$$

Loop Gain and Total Gain

Hence gain

$$G(s) = \frac{x_o}{x_s} = \frac{A(s)}{1 - \underline{A(s)F(s)}}$$

Loop gain

For large loop gain $A(s)F(s) \gg 1$

$$\Rightarrow G \approx \frac{A}{AF} = \frac{1}{F(s)}$$

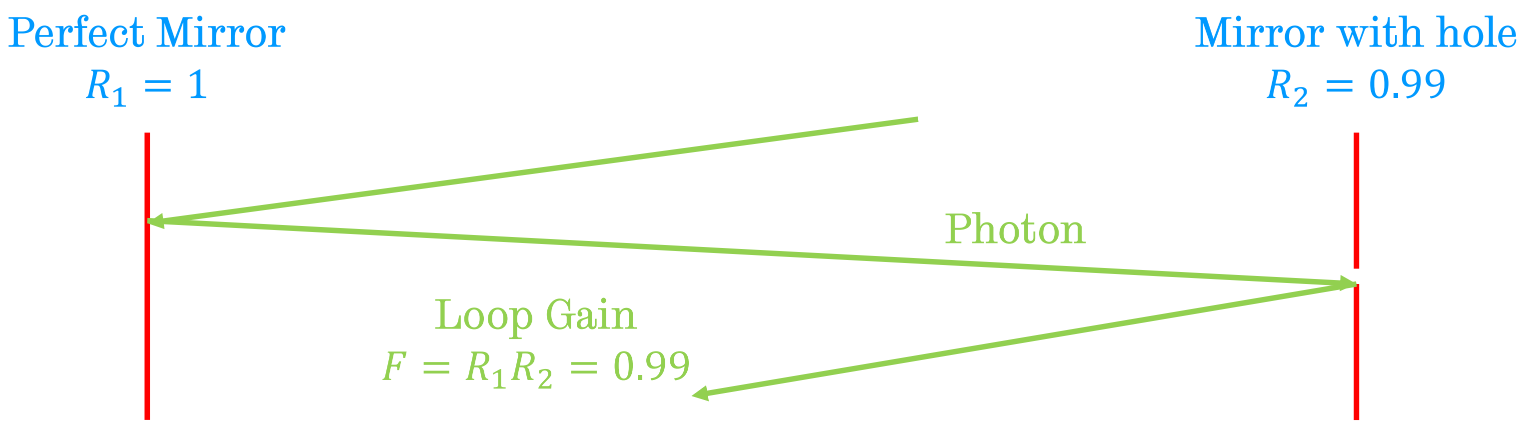
So the gain depends on the feedback rather than the actual amplifier

For lasers if $AF \rightarrow +1$ ie. The loop gain reaches unity then $G \rightarrow \infty$

Feedback in a simple laser

If a nearby photon with energy $\sim E_g$ causes recombination by stimulated emission, this will generate another photon of the *same frequency*, travelling in the *same direction*, with the *same polarisation* as the first photon.

This means that stimulated emission causes gain in an optical wave (of the correct wavelength) in the injection region, and the gain increases as the number of electrons and holes injected across the junction increases.



Feedback in a light amplifier consists of partially transmissive mirrors that create a resonant cavity. In the example above R describes the reflectivity of the mirrors. Only 1% of the light reaching the right hand mirror escapes the cavity.

Round Trip Gain

Thus if we had an initial intensity I_0 then after one "round trip" if our laser material of length L with gain G_A then the intensity becomes

$$I = R_1 \times R_2 \times G_A^2 \times I_0$$

If we take losses into account and assume they occur uniformly along the cavity then we can define a loss coefficient α and an absorption factor k

$$k = e^{-2\alpha L}$$

Factor of two as we cover a distance $2L$

And we can include this in our expression above

$$I = R_1 \times R_2 \times G_A^2 \times I_0 \times k$$

And define a round trip gain

$$G = \frac{I}{I_0} = R_1 \times R_2 \times G_A^2 \times k$$

Threshold

If we assume gain occurs uniformly along the cavity length we can define a gain coefficient γ and assume the active medium gain G_A is also distributed evenly

$$G_A = e^{\gamma L}$$

Substituting into our expression for the round trip gain gives

$$G = R_1 \times R_2 \times e^{2(\gamma - \alpha)L}$$

When $G > 1$ the intensity will increase with each pass through the laser medium

When $G < 1$ the intensity will decrease with each pass through the laser medium

Thus there is a threshold condition $G_{th} = 1$ which describes the onset of lasing

Typical Laser Characteristic

Below threshold:

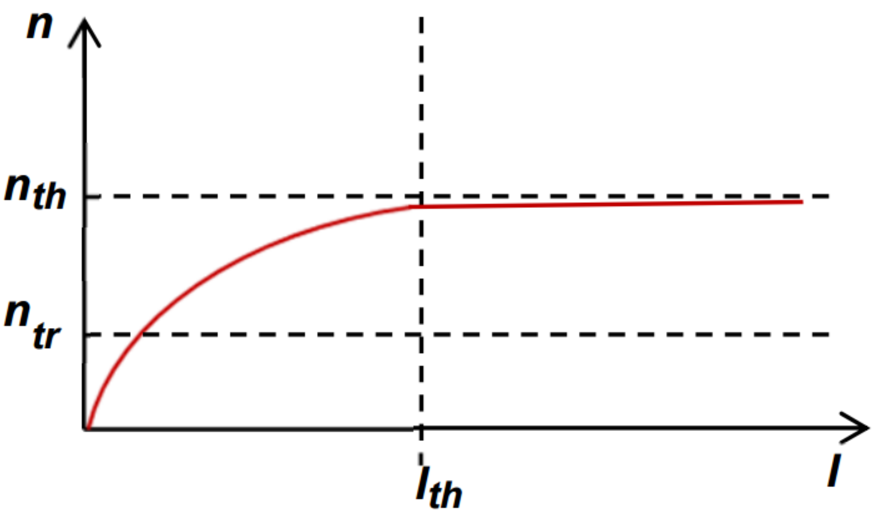
Spontaneous emission dominates (behaves like an LED i.e. largely incoherent)

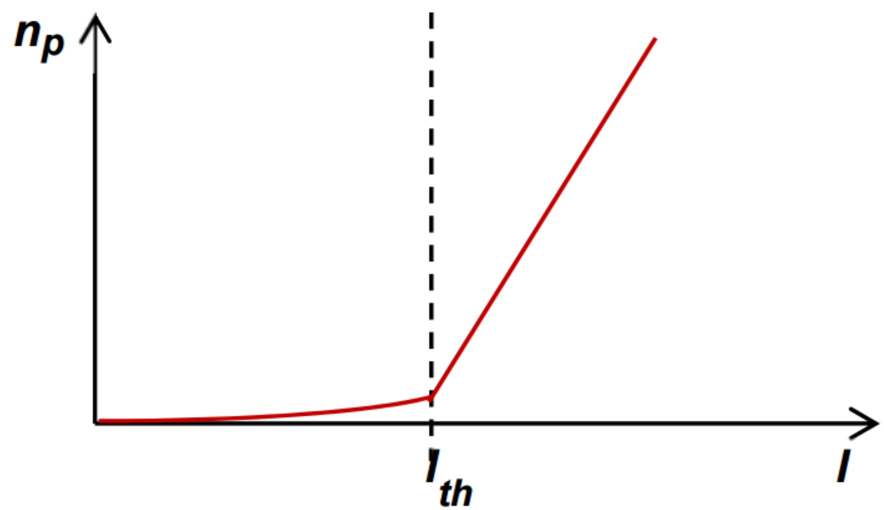
Above threshold:

Stimulated emission dominates → Lasing (i.e. largely coherent)

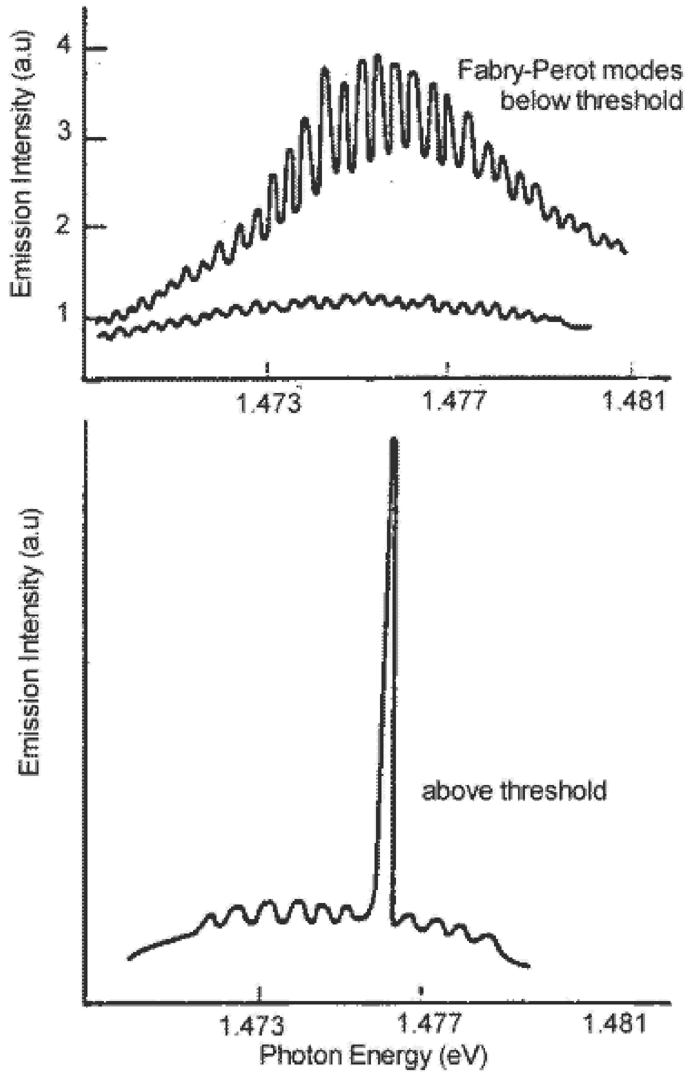
Transparency:

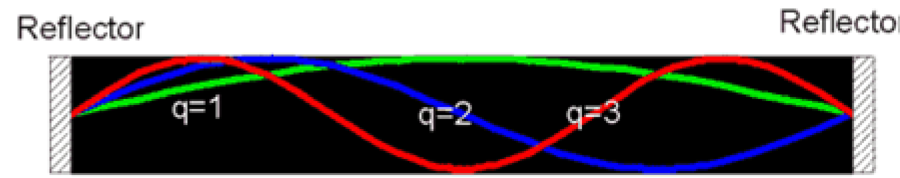
Spontaneous + Stimulated emission = Absorption





Fabry Perot Laser Modes





The optical cavity can support a number of “modes”

Each of which is an integer number of wavelengths

The cavity could be $\sim 1cm$ in length

⇒ many closely spaced modes are supported by the cavity

If the emission is broad in energy then a number of these modes are excited below threshold

Above threshold one laser mode “wins out”

Varying temperature changes the length of the cavity and hence the emission wavelength by changing the lasing mode

In practice it is more complex (the bandgap and refractive index also shifts with temperature, current induced heating, mode “hopping” etc.)

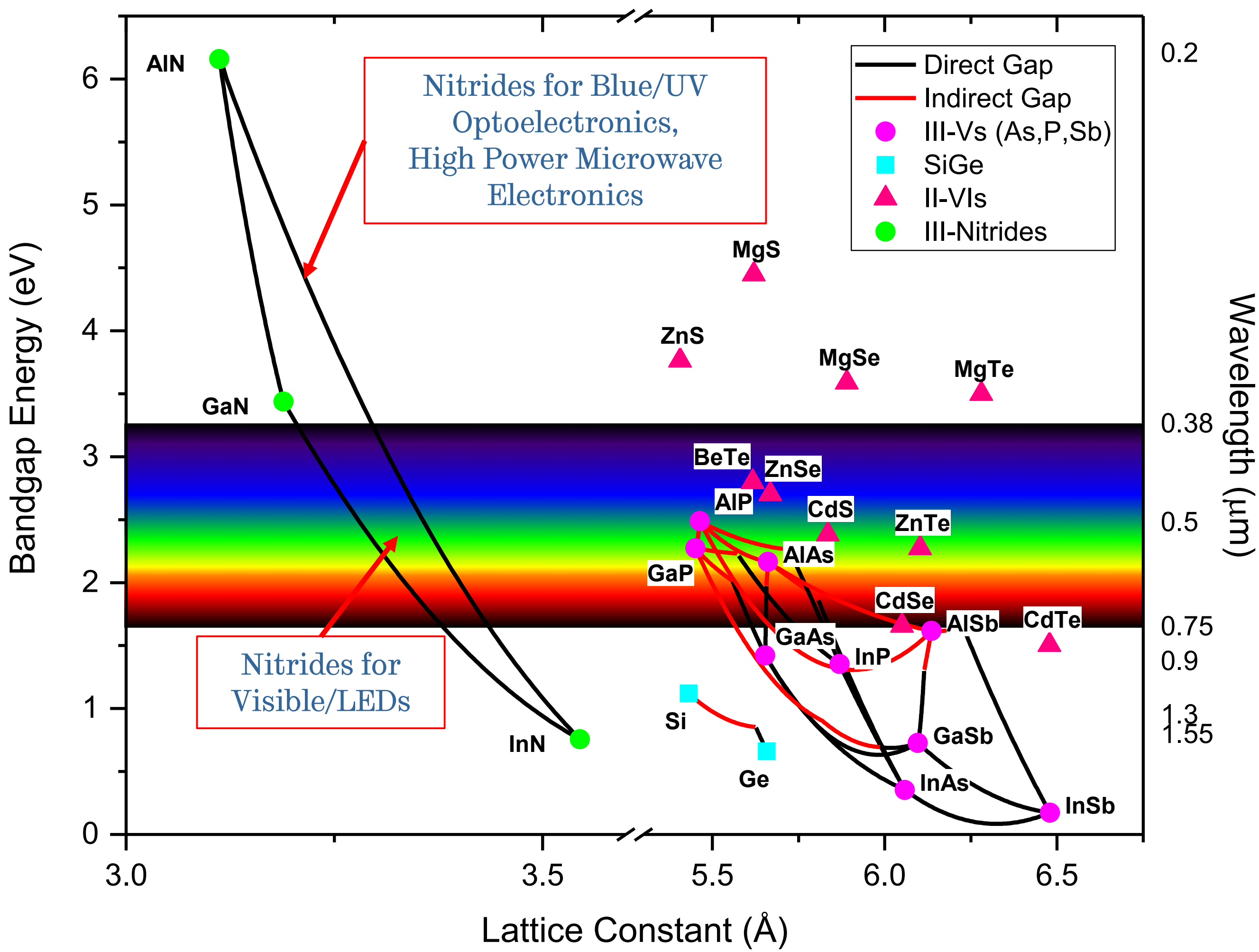
http://britneyspears.ac/physics/fplasers/fplasers.htm

Choice of materials

The spontaneous and stimulated emission processes are vastly more efficient in the direct band-gap semiconductors than in indirect band-gap ones

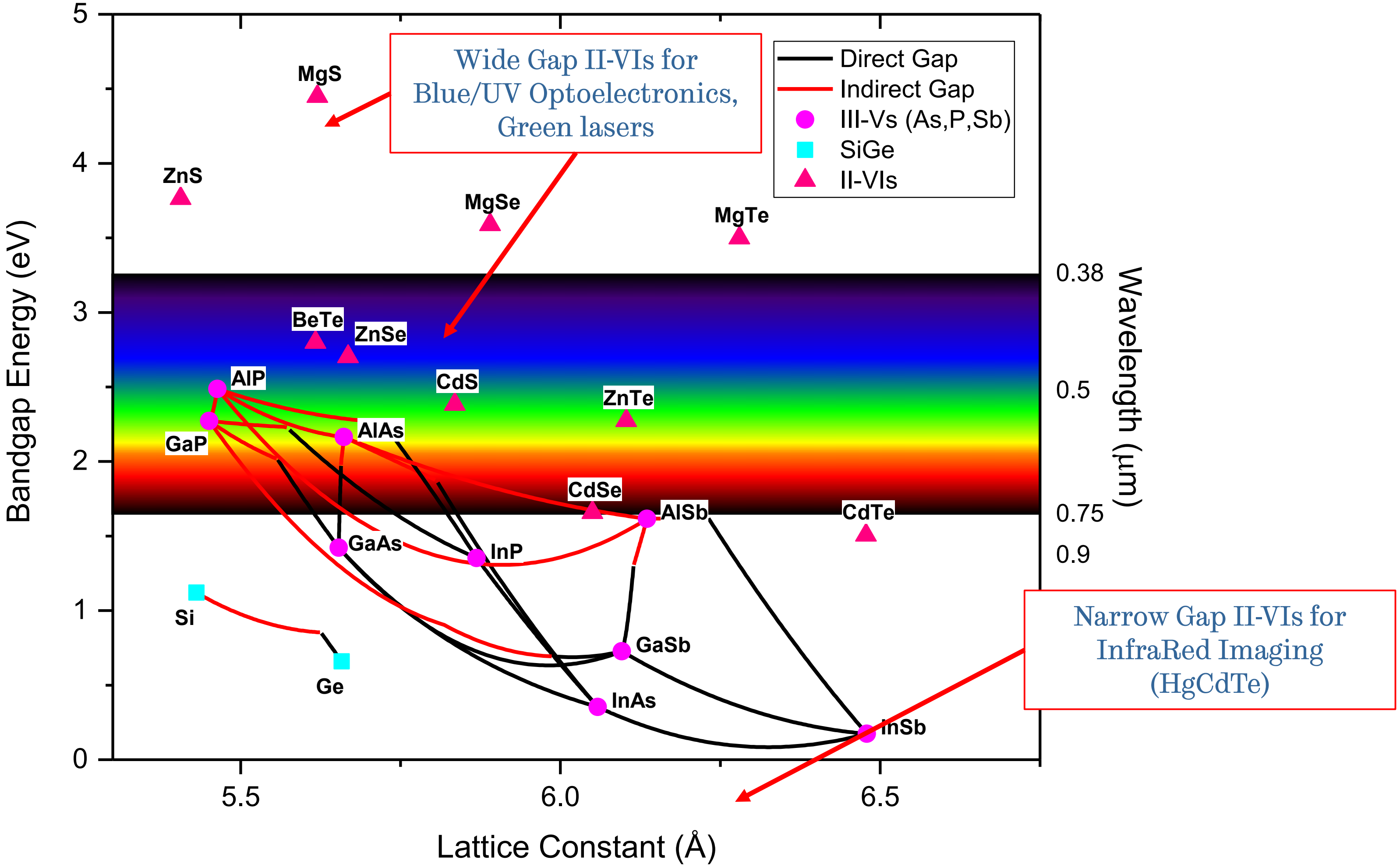
Therefore materials such as GaAs, InGaAs and GaN rather than Silicon are commonly used for laser diodes

Semiconductor Materials



Data from "Semiconductor Band Parameters" (Kyle Montgomery) http://www.kmontgomery.net/resources/semiconductor-band-parameters
Colors in eV by Vojtěch Hála and Gringer - Wikimedia Commons - https://commons.wikimedia.org/wiki/File:Colors_in_eV.svg
http://www.sandia.gov/mstc/services/images/III-V-epitaxial-growth.gif

Semiconductor Materials



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http://www.sandia.gov/mstc/services/images/III-V-epitaxial-growth.gif

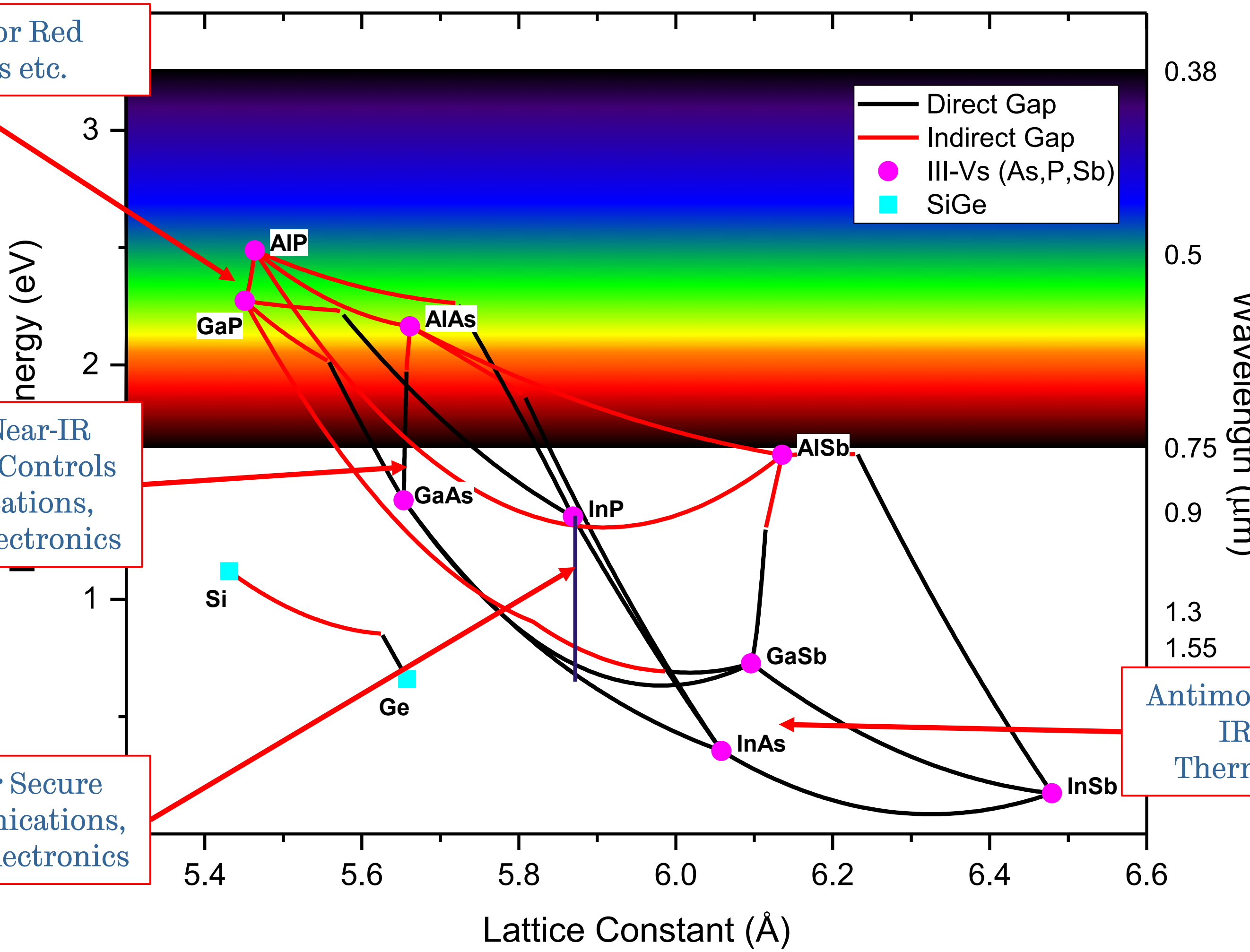
Semiconductor Materials

Phosphides for Red Lasers, LEDs etc.

Arsenides for Near-IR Optical Sensing, Controls and Communications, Radhard Microelectronics

Phosphides for Secure Optical Communications, Radhard Microelectronics

Antimonides for Far/Mid-IR Sensors and Thermophotovoltaics



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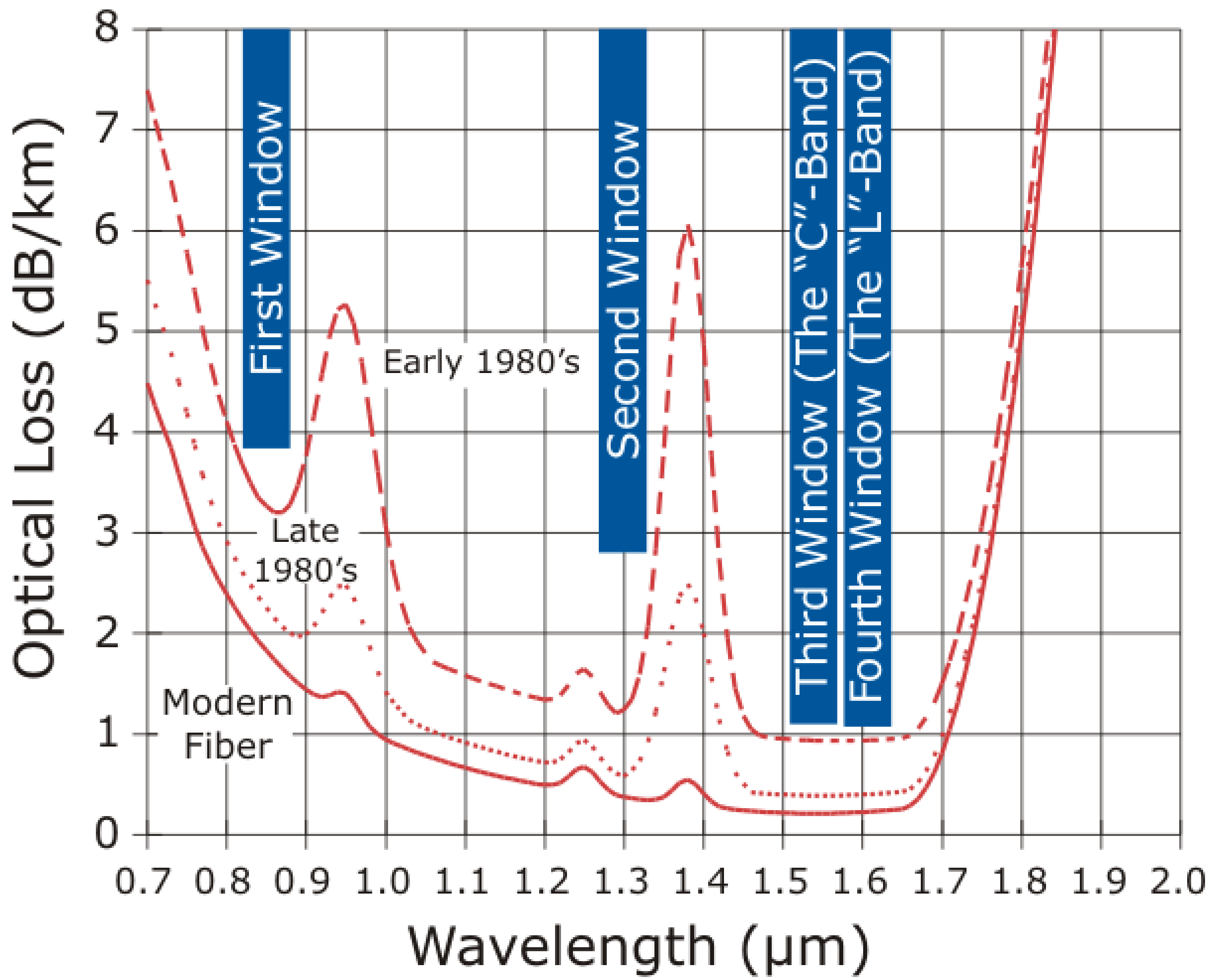
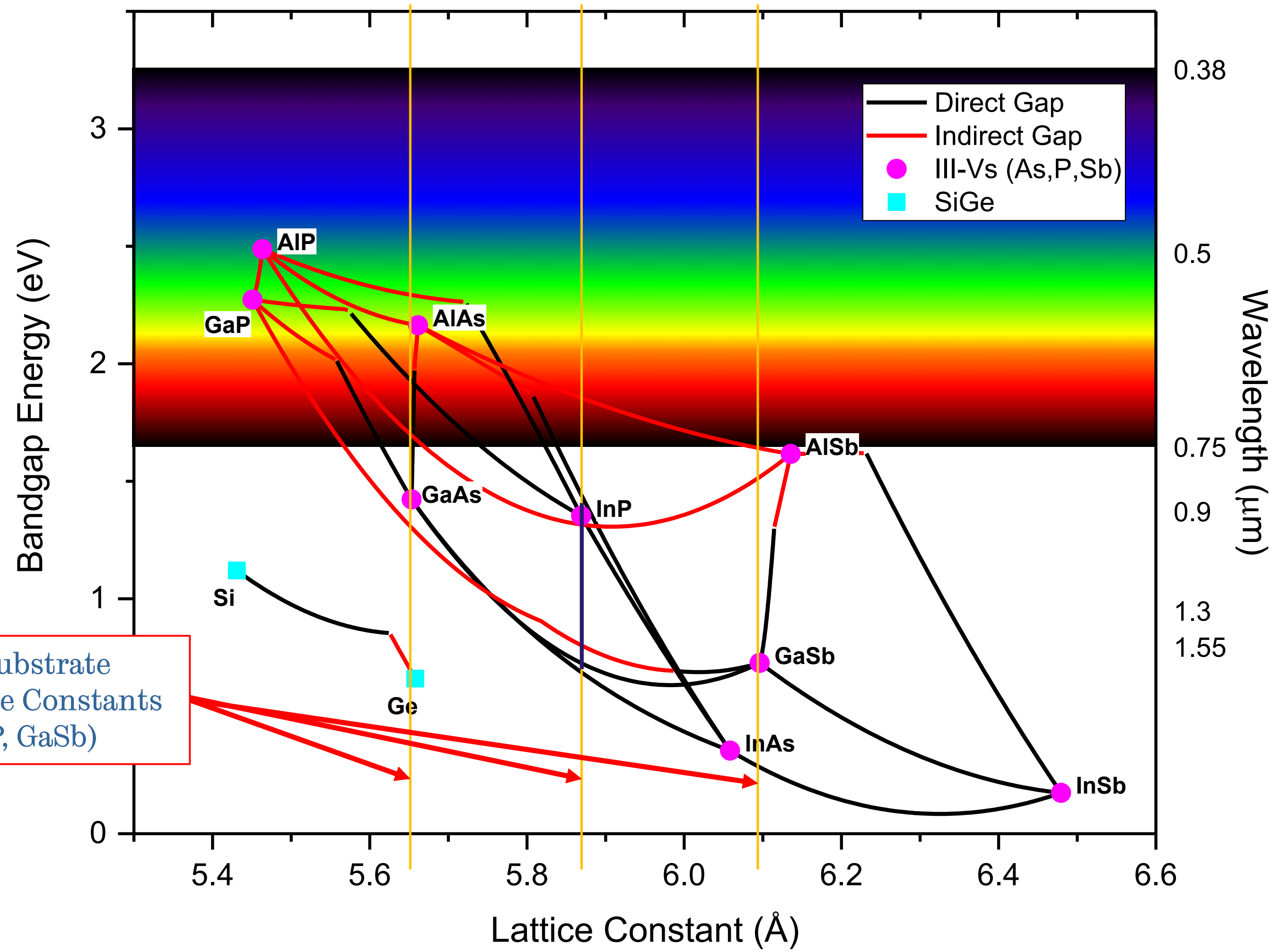
http://www.sandia.gov/mstc/services/images/III-V-epitaxial-growth.gif

Optical Fibre Absorption Windows



http://www.olson-technology.com/mr_fiber/fiber-history.htm

Semiconductor Materials



Common Substrate Material Lattice Constants (GaAs, InP, GaSb)

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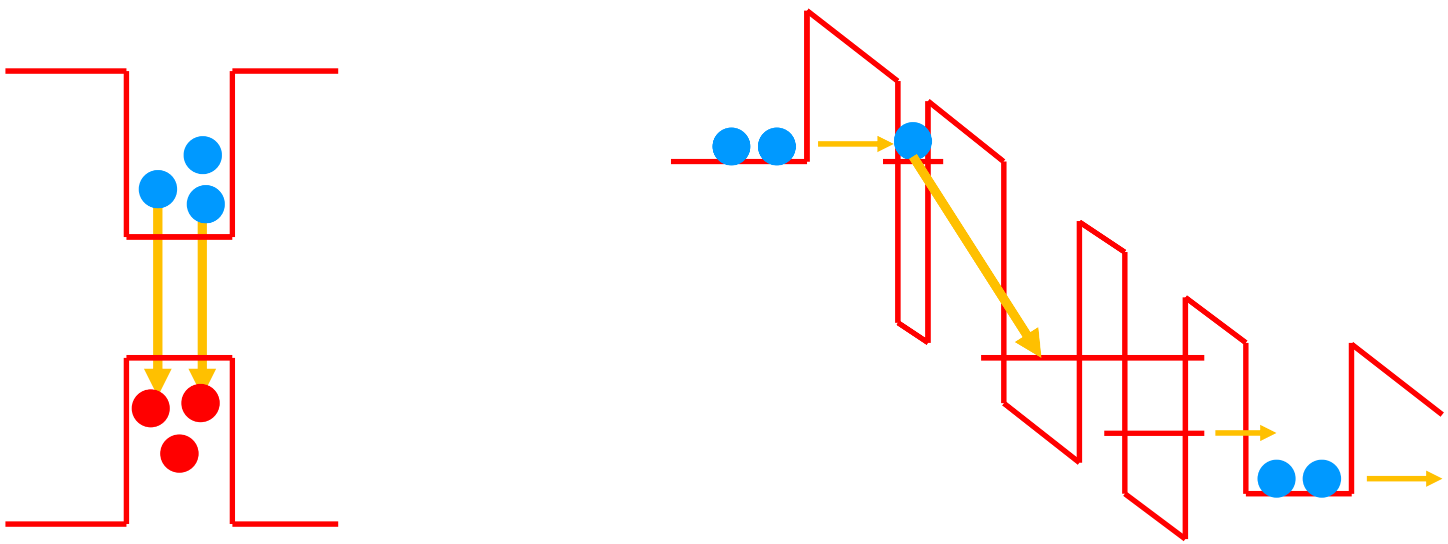
http://www.sandia.gov/mstc/services/images/III-V-epitaxial-growth.gif

Types of Semiconductor laser

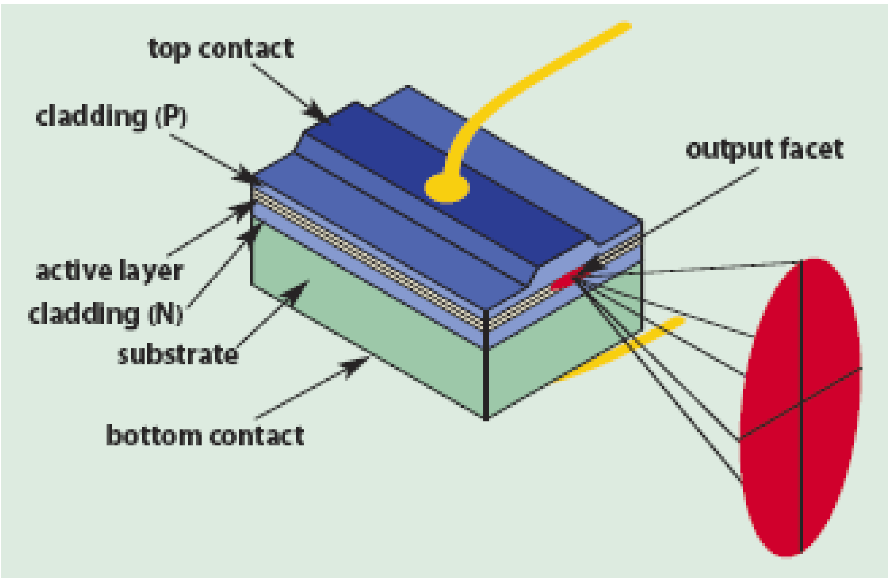
There are two main types of semiconductor laser

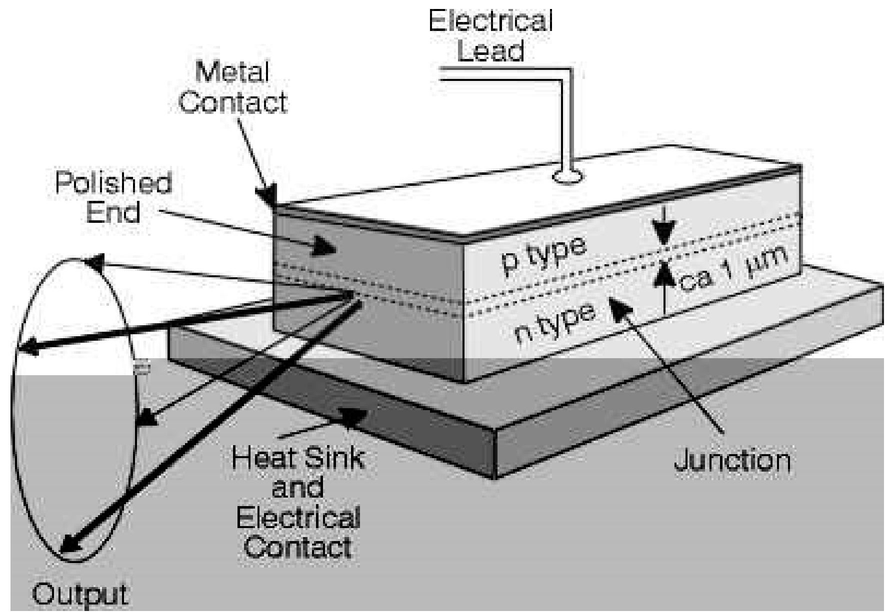
The interband laser (left) involves recombination of electrons in the conduction band with holes in the valence band and is a bipolar device



The intersubband laser (right) involves the transition of electrons only from one state to another in the conduction band (or holes only from one state to another in the valence band) and is a unipolar device. This is often known as a Quantum Cascade Laser (QCL)

Edge emitting lasers





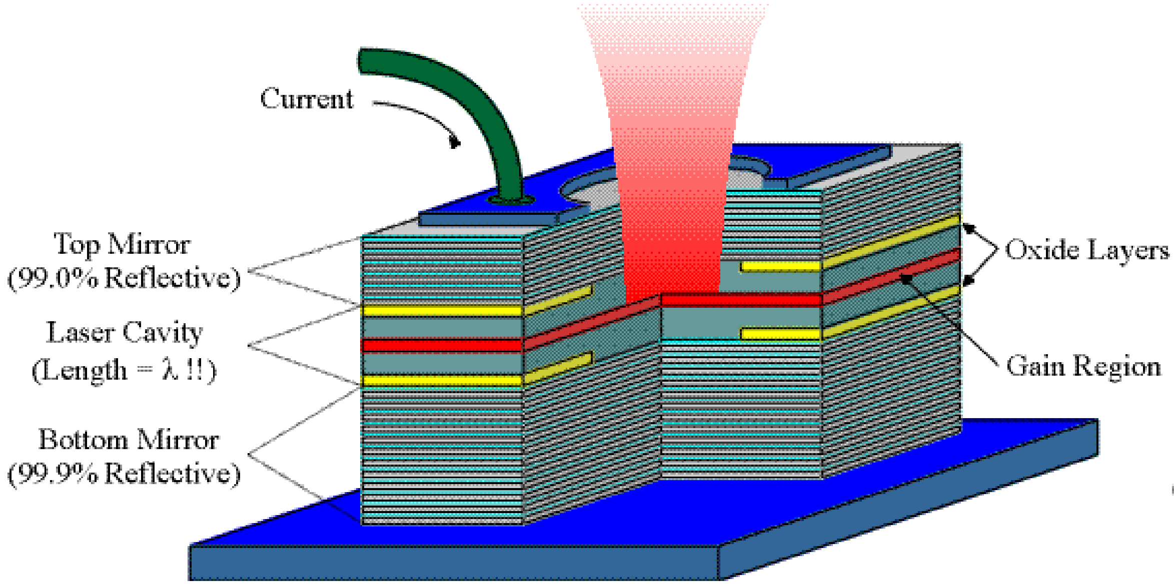
Inject carriers into a forward biased p-n diode

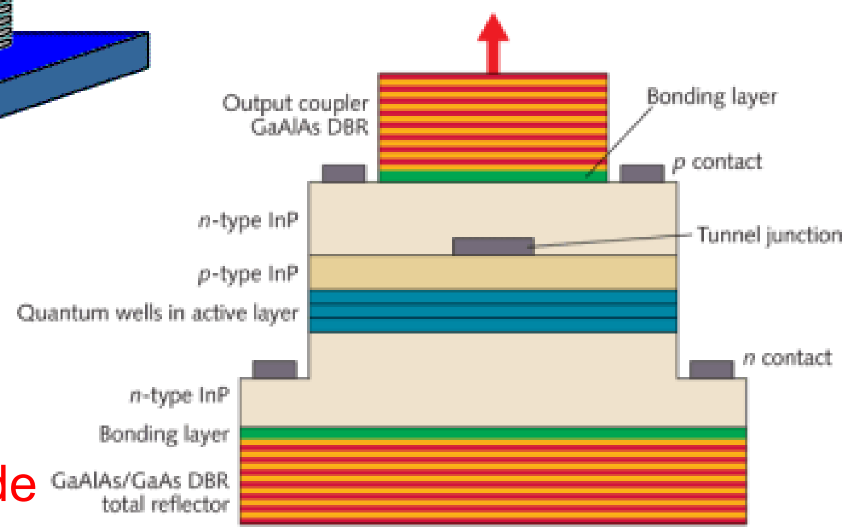
Induce interband transitions from E_c to E_v

Use a cleaved facet as a partially reflective mirror

2nd facet is metal coated to provide a “perfect” mirror

VCSEL





Vertical Cavity Surface Emitting LASER

Inject carriers into a Quantum Well p-n diode

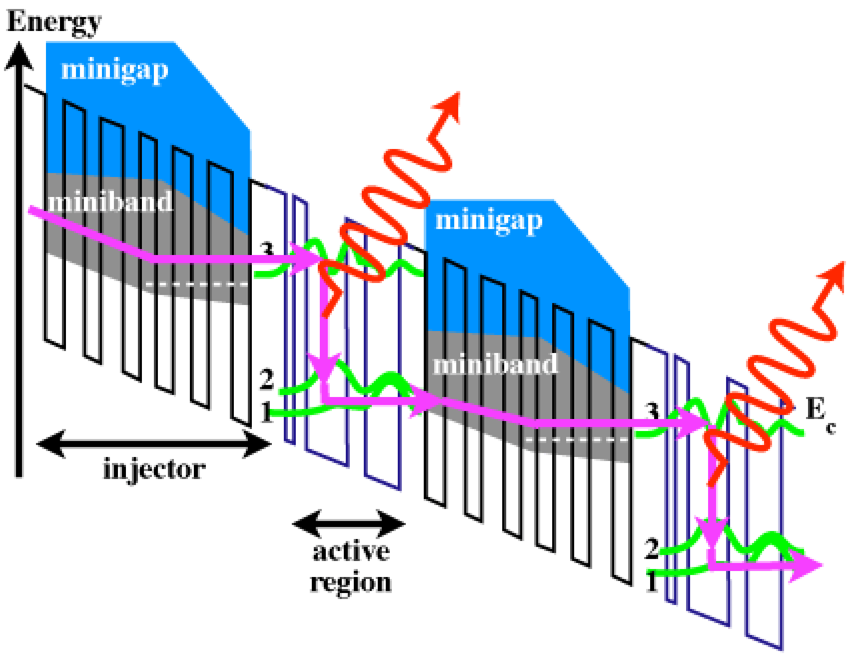
Induce interband transitions from E_{e1} to E_{h1}

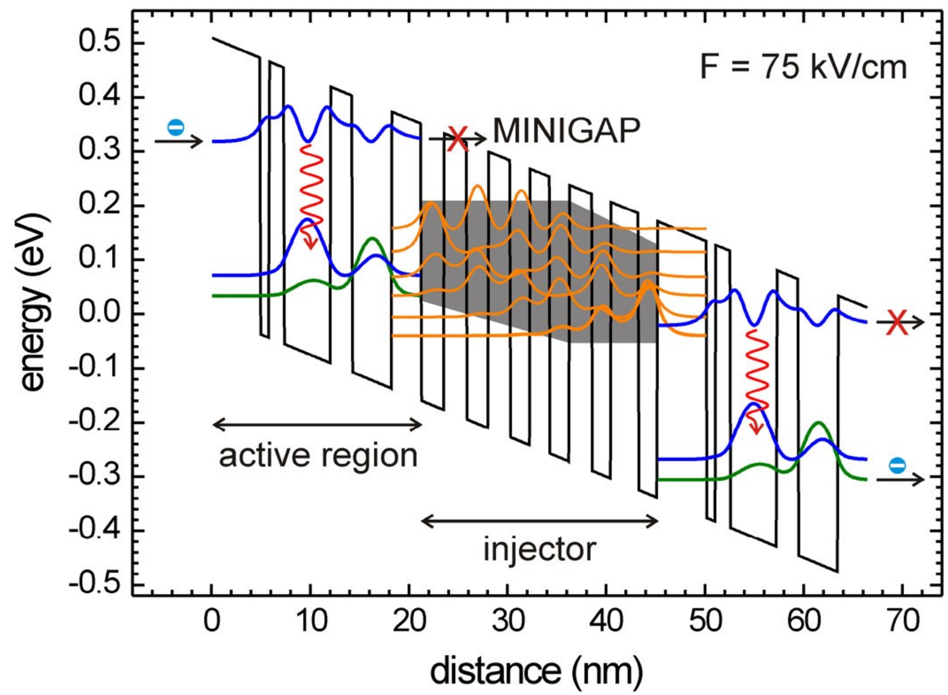
Use semiconductor multilayers as Distributed Bragg Reflector (DBR) mirrors

The upper mirror is less reflective and has an aperture for the output beam

EPFL/Laser Focus World
CNR, Italy

Quantum Cascade Lasers





First demonstrated by Bell Labs in 1994

Inject electrons into a Multiple Quantum Well (MQW) with **very** thin barriers

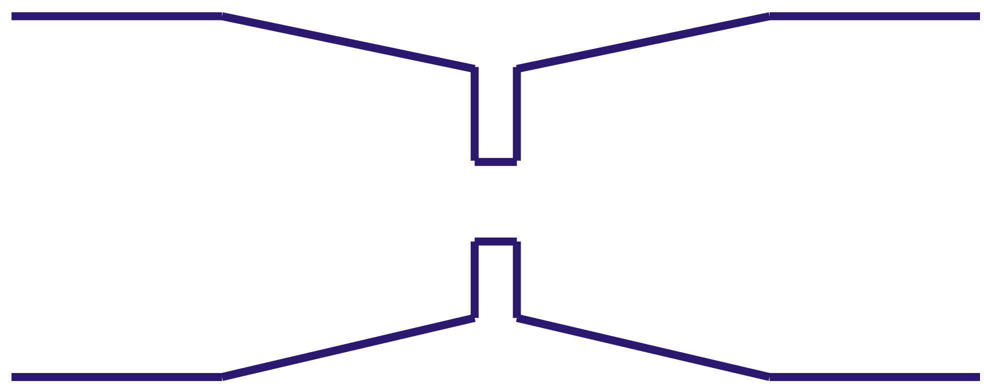
Wavefunctions in the adjacent wells overlap

Each electron moving through the structure can produce many low energy photons by intersubband transitions (c.f. a waterfall)

Crucially it allows emission at energies smaller than the bandgap of any of the materials involved. Applications from IR→THz

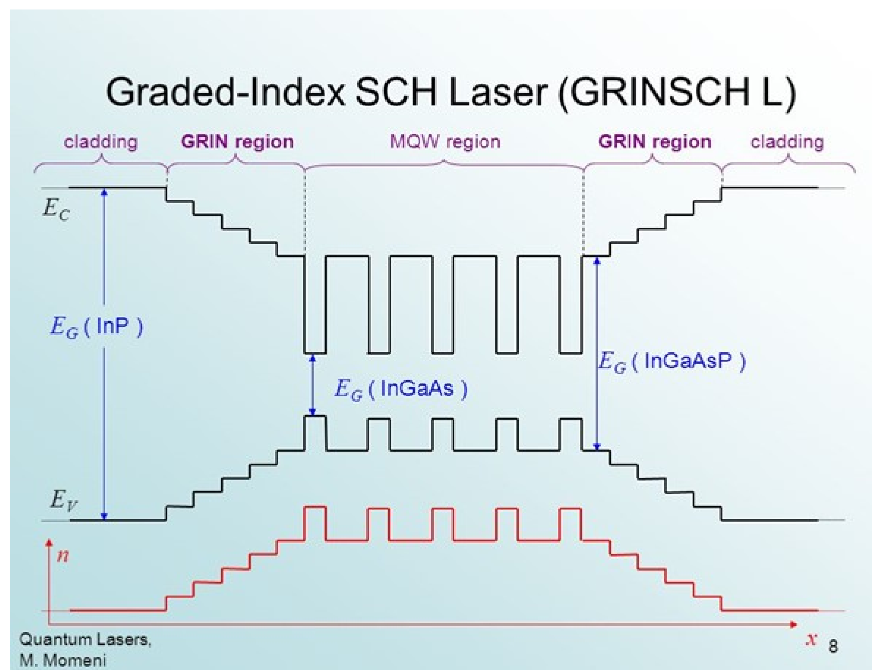
Bandstructure Engineering

By grading the alloy composition it is possible to create a pseudo electric field

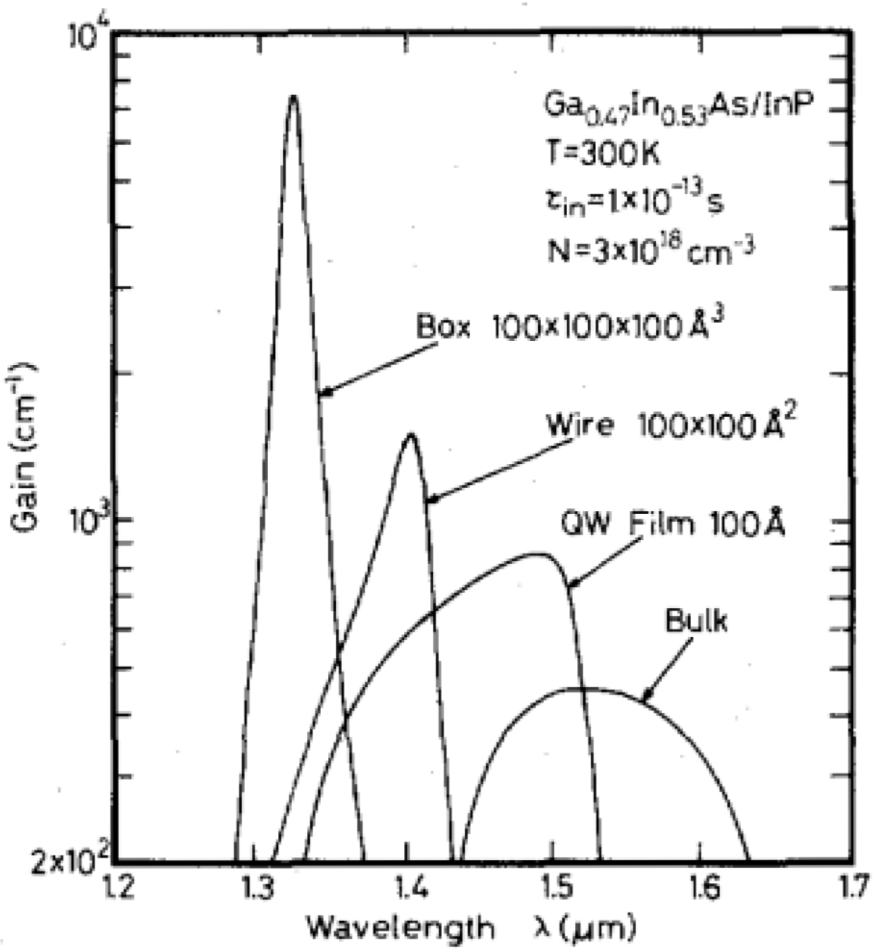


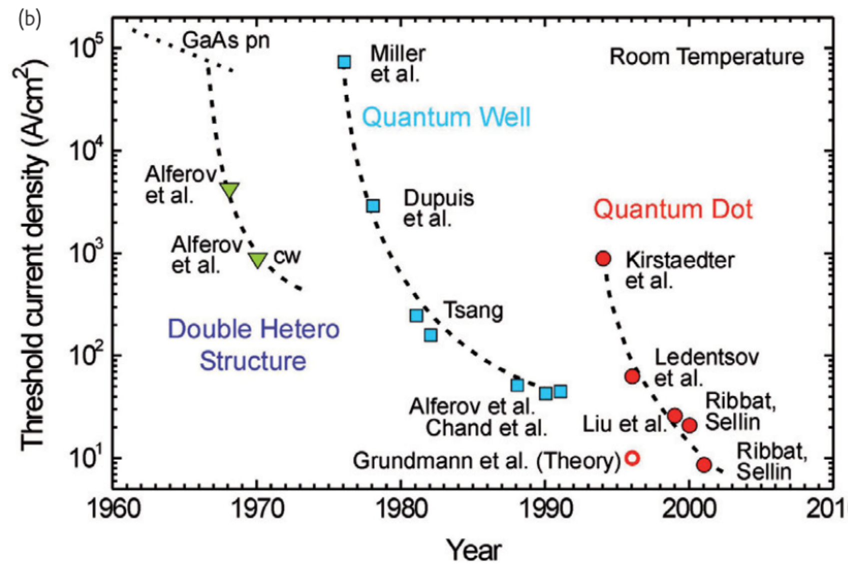
Drives both electrons and holes into the active region which enhances recombination

E.g. Graded Index Separate Confinement Heterostructure (GRIN-SCH) laser



Reducing Dimensions





Quantum confinement results in a concentration of both the emission and gain in a narrower range of wavelengths

The cost is reduction in the total volume of active material meaning that stacked QD structures are required (strain issues could arise)

Reduction in threshold current for lasing → more efficient laser devices

Asada et al. IEEE J Quantum Electronics (1986)
Bimberg et al, Materials Today (2011)

Summary

- Interference and Coherence
- Spatial and Temporal Coherence
- Revisited the Two-Level system
- Spontaneous Emission
- Stimulated Emission and Lasing (needs population inversion)
- Feedback (needs a resonating cavity)
- Gain
- Semiconductor Materials and Wavelengths of Technological Interest
- Looked at various types of Semiconductor LASERs
- Discussed bandstructure engineering and relevance of quantum confinement to latest generation of laser devices