

EEE116 – Multimedia Systems 2007/08

Tutorial solution sheet 2 (Week 3)

Q3) Estimate the maximum uncorrupted data rate, in bits/s, for the following communication link:

Bandwidth = 12 MHz

Signal power = 100 mW

Noise power = 20 μ W

From the Shannon-Hartley law, we know how to compute the maximum channel capacity for a communication link.

The data rate R is

$$R \leq W \log_2(1+S/N)$$

Where, W=12 MHz signal bandwidth

S=100mW signal power

N= 20 μ W noise power

Therefore, the upper limit

$$\begin{aligned} R_{\max} &= 12 \times 10^6 \log_2(1 + 100 \times 10^{-3} / 20 \times 10^{-6}) \\ &= 1.4745 \times 10^8 \text{ bits/sec} \\ &\text{(now divide by } 10^{20} \text{ to convert into M bits/sec)} \\ &= 140.6 \text{ M bits/sec} \end{aligned}$$

What is the signal-to-noise power ratio in dBs?

$$\begin{aligned} \text{Signal-to-noise ratio (SNR)} &= 10 \log_{10}(S/N) \\ &= 10 \log_{10}(100 \times 10^{-3} / 20 \times 10^{-6}) \\ &= 37 \text{ dB.} \end{aligned}$$

As an engineer, you are asked to improve this link – which is best (i) to double the bandwidth, or (ii) to double the signal power?

From the Shannon-Hartley law, we can compute the data rate R as

$$R \leq W \log_2(1+S/N)$$

$$\text{Let, } R_{\max} = C = W \log_2(1+S/N) \text{ -----(1)}$$

(i) If we double the bandwidth $W_{\text{new}} = 2W$

Using (1) we can say the new data rate $C_{(i)} = 2W \log_2(1+S/N)$

(ii) If we double the signal power $S_{\text{new}} = 2S$

Using (1) we can write

$$C_{(ii)} = W \log_2(1 + (2S/N))$$

Now check $C_{(i)} - C_{(ii)}$

$$\begin{aligned}
 &= 2W \log_2 \left(1 + \frac{S}{N} \right) - W \log_2 \left(1 + \frac{2S}{N} \right) \\
 &= W \log_2 \left(1 + \frac{S}{N} \right)^2 - W \log_2 \left(1 + \frac{2S}{N} \right) \\
 &= W \log_2 \left[\frac{\left(1 + \frac{S}{N} \right)^2}{\left(1 + \frac{2S}{N} \right)} \right] \\
 &= W \log_2 \left[\frac{1 + \frac{2S}{N} + \left(\frac{S}{N} \right)^2}{\left(1 + \frac{2S}{N} \right)} \right] \\
 &= W \log_2 \left[1 + \frac{\left(\frac{S}{N} \right)^2}{\left(1 + \frac{2S}{N} \right)} \right] \\
 &= W \log_2 (1 + x)
 \end{aligned}$$

In the above expression, $x > 0$. (Because $\text{SNR} > 0$)

Therefore, $(1+x) > 1$.

Therefore we can say $(C_{(i)} - C_{(ii)}) > 0$.

Therefore the method (i) is the better way to improve the data rate.

- Q4) You have a 160 G Byte hard disc – full of important data – that needs to be transferred to a computer in London. Assume that the distance between London and Sheffield is 250km and the effective speed of light for cases a., b. and c. is $3 \times 10^8 \text{ ms}^{-1}$. For b. and c. assume that you can send one bit of information per period. Estimate the time it will take to transfer all this data, if you use:

We know the total time to transfer is

= Transmit time + propagation time + network queuing delays

Transmit time = Data size / capacity of the link

The data size in all cases is $D = 8 \times 160 \times 2^{30}$ bits

Transmit time has to be computed for the three scenarios separately.

$$\begin{aligned}\text{Propagation time} &= \text{distance} / \text{propagation speed} \\ &= 250 \times 10^3 / (3 \times 10^8) \\ &= 83.3 \times 10^{-3} \text{ s.}\end{aligned}$$

No information on queuing time was given. So we assume no delays due to network processes.

- a. A conventional phone line with a normal modem attached (assume that the modem can send 20 Kbits/s),

$$\begin{aligned}\text{Transmit time} &= \text{Data size} / \text{capacity of the link} \\ &= (8 \times 160 \times 2^{30} \text{ bits}) / (20 \times 2^{10} \text{ bits/s}) \\ &= 776 \text{ days}\end{aligned}$$

In this case, the total time is totally dependent on the transmit time.

- b. A high speed computer network link with a bandwidth of 10 MHz,

1 bit of information per period.

Therefore, the link capacity for 10MHz is 10×10^6 bits/s

$$\begin{aligned}\text{Transmit time} &= \text{Data size} / \text{capacity of the link} \\ &= (8 \times 160 \times 2^{30} \text{ bits}) / (10 \times 10^6 \text{ bits/s}) \\ &= 1.59 \text{ days}\end{aligned}$$

In this case also, the total time is totally dependent on the transmit time.

- c. A direct optical link using light at a wavelength of $0.7 \mu\text{m}$.

We have to compute the Carrier signal frequency considering the speed of light.

$$\text{Frequency } F = (\text{speed of light}) / (\text{wave length}) = (3 \times 10^8) / (0.7 \times 10^{-6})$$

1 bit of information per period. Therefore, the link capacity for F Hz is F bits/s

$$\begin{aligned}\text{Transmit time} &= \text{Data size} / \text{capacity of the link} \\ &= (8 \times 160 \times 2^{30} \text{ bits}) / ((3 \times 10^8) / (0.7 \times 10^{-6}) \text{ bits/s}) \\ &= 3.2 \times 10^{-3} \text{ s}\end{aligned}$$

$$\begin{aligned}\text{In this case, the total time is Transmit time+ Propagation time} \\ &= (3.2 + 83.3) \times 10^{-3} \text{ s} \\ &= 85.5 \times 10^{-3} \text{ s}\end{aligned}$$

- d. The train?

Around 2 hours and 45 minutes (if you are lucky) ☺