Data Provided: List of useful formulae at the end of paper

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2012-13 (2.0 hours)

EEE201OR Signals and Systems 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a.

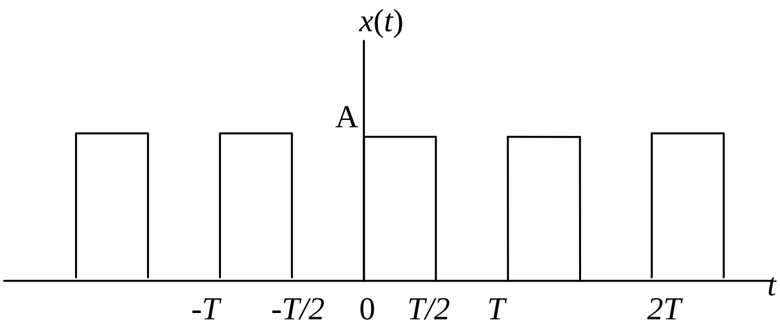

Figure 1.1

Determine the Complex Fourier Series of the signal $x(t)$ shown in Figure 1.1. **(10)**

b. Assume that the output of a simple dc to ac converter, implemented by a 50 Hz switch, is represented by $x(t)$ where $T = 0.02$ s and $A = 1$ V. At the output of this converter all the harmonics, except the fundamental, are removed by a low pass filter. Determine the conversion efficiency, defined as $\dfrac{power \quad out}{power \quad in}$. **(6)**

c. Determine the average power in the signal $y(t) = \cos(2(t+3)) + \cos(10(t+3))$. **(4)**

2. **a.** The response of a Linear Time Invariant (LTI) system, $y(t)$, is shown in Figure 2.1 when subjected to the input signal $x(t)$. Derive $y_1(t)$, the response of this LTI system when the input signal $x_1(t)$ is shown in Figure 2.2.

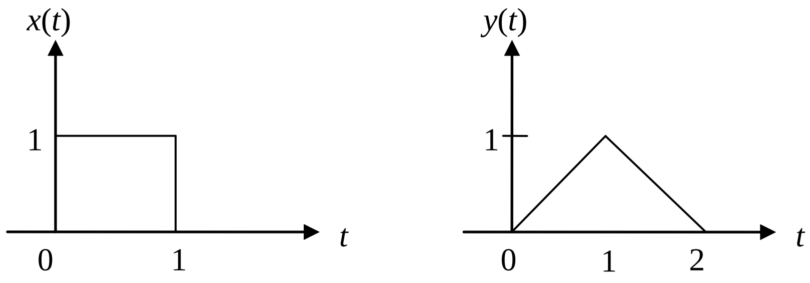

Figure 2.1

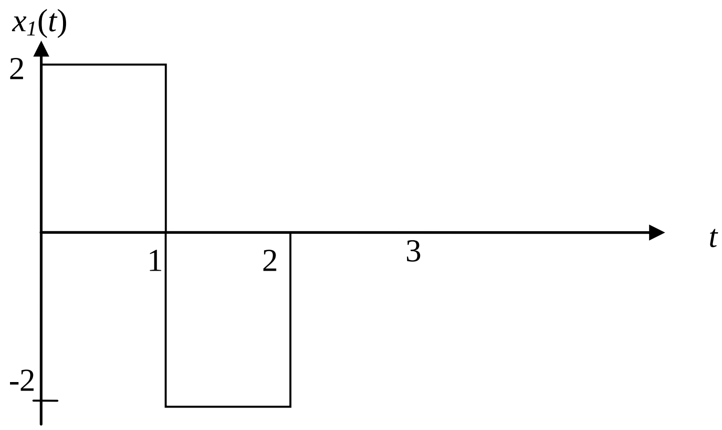

Figure 2.2 **(4)**

b. Derive an equation for the impulse response of a high pass RC circuit. Sketch and label the impulse response. **(8)**

c. Compute the output signal $y(t)$, of a continuous time LTI system whose impulse response and input are given by $h(t) = \exp(-t).\ u(t)$ and $x(t) = \exp(t).\ u(-t)$ respectively. **(8)**

3. a.

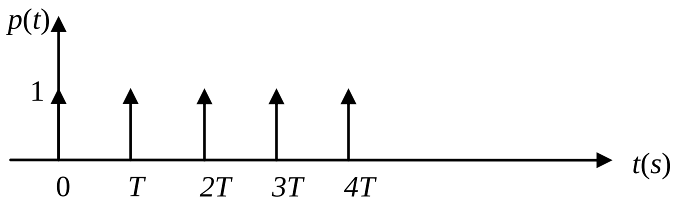

Figure 3.1

Show that the Fourier Transform of $p(t)$ in Figure 3.1 is given by

$$P(\omega) = \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega - n\omega_s)$$

where ω_s is the sampling frequency. **(7)**

b.

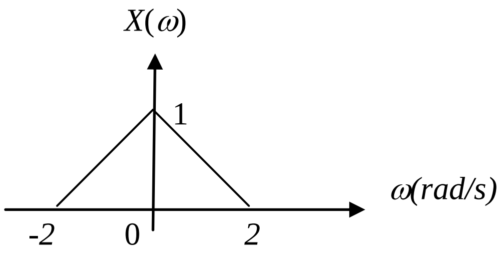

Figure 3.2

Consider a continuous time signal $x(t)$ with the magnitude spectrum shown in Figure 3.2. The signal $x(t)$ is multiplied by the sampling function $p(t)$ in Figure 3.1 to obtain $x_s(t)$, the sampled version of $x(t)$. Assuming T = π, sketch and label

i) the magnitude spectrum $P(\omega)$.

ii) the magnitude spectrum $X_s(\omega)$.

Confirm whether the $x(t)$ can be recovered using a low pass filter. Explain why? **(8)**

c. In a simple modulation scheme called double sideband, the modulated signal is given by $x(t) = [A_o + m(t)]c(t)$, where the condition, $A_o + m(t) > 0$, is used.

i) Obtain the corresponding expression for the $x(t)$ in the frequency domain, $X(\omega)$, assuming that $A_c = 1$.

ii) State a major drawback and an advantage of using this modulation scheme. **(5)**

4. **a.**

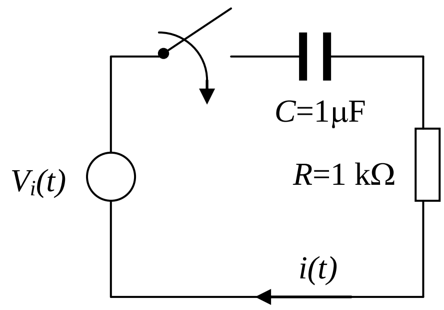

Figure 4.1

Use the Laplace transform to show that the voltage across the capacitor is given by $v_c(t) = A\left(1 - e^{-t/RC}\right)u(t)$ in the RC circuit shown in Figure 4.1, assuming that the initial voltage across the capacitor $v_c(0) = 0$, and $v_i(t) = Au(t)$, where A is a constant. **(10)**

b. Find an expression for the current, $i(t)$, flowing in the circuit. **(3)**

c. Work out the value of the current at time $t = 0$, $i(0)$, and the time taken for the current to decay to 1% of its value at $t = 0$. **(5)**

d. What is the signal frequency range that the circuit will pass without attenuating the signal power by more than 3 dB? **(2)**

CHT

List of useful formulae

$$x[n]=\sum_{k=-\infty}^{\infty}x[k]\delta[n-k] \qquad x(t)=\int_{-\infty}^{\infty}x(\lambda)\delta(t-\lambda)d\lambda$$

$$x(t)*h(t)=\int_{-\infty}^{\infty}x(\lambda)h(t-\lambda)d\lambda \qquad x(t)=\sum_{n=-\infty}^{\infty}c_n e^{jn\omega_o t}$$

$$c_0=\frac{1}{T}\int_{<T>}x(t)dt \qquad c_n=\frac{1}{T}\int_{-T/2}^{T/2}x(t)e^{-jn\omega_o t}dt$$

$$a_n=2\operatorname{Re}[c_n]=\frac{2}{T}\int_{<T>}x(t)\cos n\omega_0 t dt \qquad b_n=-2\operatorname{Im}[c_n]=\frac{2}{T}\int_{<T>}x(t)\sin n\omega_0 t dt$$

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{j\omega t}d\omega \qquad X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$$

$$X(\omega)=2\int_{0}^{\infty}x(t)\cos\omega t dt \qquad X(\omega)=-j2\int_{0}^{\infty}x(t)\sin\omega t dt$$

$$X(s)=\int_{0}^{\infty}x(t)e^{-st}dt \qquad x(t)=\frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty}X(s)e^{st}dt$$

$$\cos(x)\cos(y)=\frac{1}{2}\left[\cos(x-y)+\cos(x+y)\right] \qquad \sin(x)\sin(y)=\frac{1}{2}\left[\cos(x-y)-\cos(x+y)\right]$$

$$\sin(x)\cos(y)=\frac{1}{2}\left[\sin(x-y)+\sin(x+y)\right] \qquad \cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\sin(x)=\frac{e^{jx}-e^{-jx}}{2j} \qquad \cos(x)=\frac{e^{jx}+e^{-jx}}{2}$$

Fourier Transform Pairs

Signal	Fourier Transfrom
$\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$	$2\pi \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_o)$
$e^{j\omega_o t}$	$2\pi\delta(\omega - \omega_o)$
$\cos\omega_o t$	$\pi[\delta(\omega + \omega_o) + \delta(\omega - \omega_o)]$
$\sin\omega_o t$	$j\pi[\delta(\omega + \omega_o) - \delta(\omega - \omega_o)]$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
$\delta(t - t_o)$	$e^{-j\omega t_o}$
$e^{-at}u(t)$, $a > 0$	$\frac{1}{a + j\omega}$
$x(t) = \begin{cases} 1, & \|t\| < \tau \\ 0, & \|t\| > \tau \end{cases}$	$\frac{2\sin\omega\tau}{\omega} = 2\tau \sin c(\omega\tau)$
$\frac{\sin\omega_c t}{\pi t} = \frac{\omega_c}{\pi}\sin c(\omega_c t)$	$X(\omega) = \begin{cases} 1, & \|\omega\| < \omega_c \\ 0, & \|\omega\| > \omega_c \end{cases}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$

Properties of Fourier Transform

Property	Aperiodic signal, $x(t)$	Fourier Transfrom, $X(\omega)$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(\omega) + bX_2(\omega)$
Time Shifting	$x(t - t_o)$	$e^{-j\omega t_o} X(\omega)$
Frequency Shifting	$e^{j\omega_o t} x(t)$	$X(\omega - \omega_o)$
Time Scaling	$x(at)$	$\frac{1}{a} X\left(\frac{\omega}{a}\right)$
Differentiation in Time	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Differentiation in Frequency	$tx(t)$	$j\frac{dX(\omega)}{d\omega}$
Integration in time	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$
Convolution	$x(t)*h(t)$	$X(\omega).H(\omega)$
Multiplication in time	$x(t).h(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\lambda)H(\omega-\lambda)d\lambda$
Parseval's Theorem	$E = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$	
Duality	$x(t) \leftrightarrow X(\omega)$	
	$X(t) \leftrightarrow 2\pi x(-\omega)$	

Laplace Transform pairs

Signal	Transform
Unit step: $u(t)$	$\frac{1}{s}$
Unit impulse: $\delta(t)$	1
Unit ramp: $tu(t)$	$\frac{1}{s^2}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$
$(\cos\omega_o t)u(t)$	$\frac{s}{(s^2+\omega_o{}^2)}$
$(\sin\omega_o t)u(t)$	$\frac{\omega_o}{(s^2+\omega_o{}^2)}$
$(e^{-at}\cos\omega_o t)u(t)$	$\frac{s+a}{((s+a)^2+\omega_o{}^2)}$
$(e^{-at}\sin\omega_o t)u(t)$	$\frac{\omega_o}{((s+a)^2+\omega_o{}^2)}$
$(t\cos\omega_o t)u(t)$	$\frac{s^2-\omega_o{}^2}{(s^2+\omega_o{}^2)^2}$
$(t\sin\omega_o t)u(t)$	$\frac{2\omega_o s}{(s^2+\omega_o{}^2)^2}$

Properties of Laplace Transform

Property	Transform Property
Linearity	$ax_1(t) + bx_2(t) \leftrightarrow aX_1(s) + bX_2(s).$
Time shift	$x(t-t_o)\,u(t-t_o) \leftrightarrow X(s)e^{-st_o}$, $t_o > 0$
Multiplication by a complex exponential	$x(t)e^{s_o t} \leftrightarrow X(s-s_o)$
Time scaling	$x(at) \leftrightarrow X(s/a)/\lvert a\rvert$
Differentiation in time domain	$\dfrac{dx(t)}{dt} \leftrightarrow sX(s) - x(0)$
	$\dfrac{d^2x(t)}{dt^2} \leftrightarrow s^2X(s) - sx(0) - \left.\dfrac{dx(t)}{dt}\right\rvert_{t=0}$
Differentiation in *s* domain	$t^n x(t) \leftrightarrow \dfrac{d^nX(s)}{ds^n}(-1)^n$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \dfrac{1}{s}X(s)$
Convolution in time domain	$x(t)*h(t) \leftrightarrow X(s).H(s)$
Initial value theorem	$x(0) = \lim_{s\to\infty} sX(s)$
Final value theorem	$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$

(if $x(t)$ has a finite value as $t \to \infty$)

Unit step response for 2nd order systems

Damping factor, ζ	Unit step response
>1	$y(t)=\dfrac{k}{p_1 p_2}+k_2 e^{p_1 t}.u(t)+k_3 e^{p_2 t}.u(t)$
1	$y(t)=\dfrac{k}{\omega_n{}^2}\left(1-(1+\omega_n t)e^{-\omega_n t}.u(t)\right)$
$0<\zeta<1$	$y(t)=\dfrac{k}{\omega_n{}^2}\left(1-\dfrac{\omega_n}{\omega_d}e^{-\zeta\omega_n t}\sin(\omega_d t+\theta).u(t)\right)$
0	$y(t)=\dfrac{k}{\omega_n{}^2}\left(1-\cos(\omega_n t).u(t)\right)$