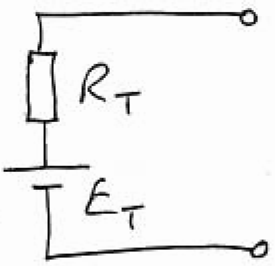
EEE101 Circuits & Signals

AUTUMN 06/07 Exam Solutions

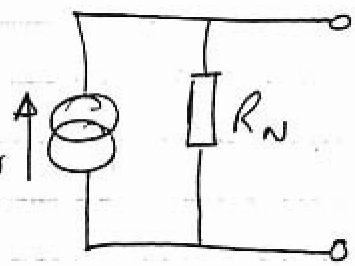
Q1 (a) Thevenin



R_T – Thevenin resistance, obtained by s/c voltage sources and o/c current sources and then calculating equivalent resistance between the terminals. 1 1

E_T – Thevenin voltage, equal to the voltage appearing across the terminals with no load connected 1

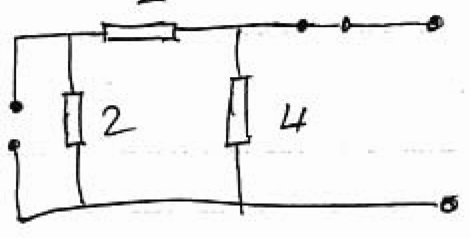
Norton



R_N – Norton resistance, obtained as above 1 1

i_N – Norton current source, equal to the current flowing in a short circuit placed across the terminals. 1

(b) Find R_T



$$R_T = (2+2) // 4 = \frac{4 \times 4}{4+4}$$

$$= 2\,\Omega \quad 2$$

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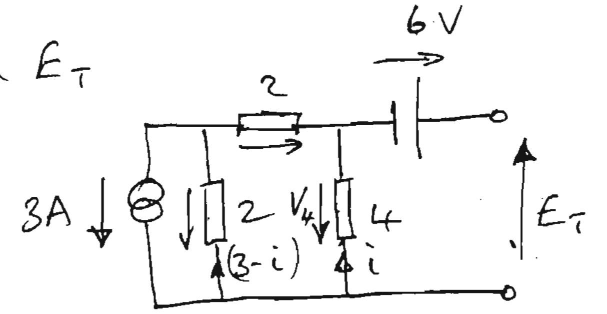
Find E_T



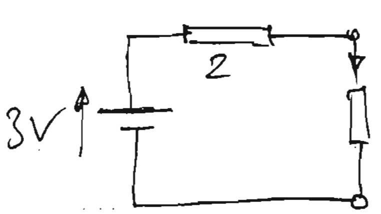
$$(4+2)i - 2(3-i) = 0$$

$$\Rightarrow i = 0.75\,A$$

$$V_4 = 4 \times 0.75 = 3V$$

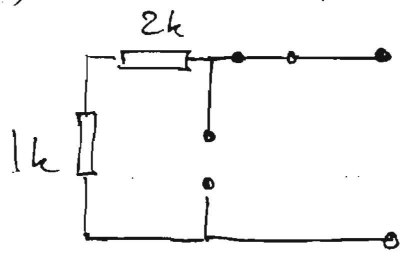
$$\therefore E_T = 6V - 3V = \underline{3V} \qquad 3$$

Power in 3Ω resistor $= i^2 R$



$$i = \frac{3}{5} \qquad = \underline{1.08\,W} \qquad 2$$

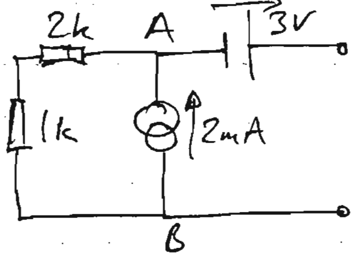
(c) Find R_T



$$R_T = 2k + 1k = \underline{3k\Omega} \qquad 2$$

Find V_T

2mA flows through 1k + 2k resistors



$$\therefore V_{AB} = 2 \times 10^{-3} \times 3 \times 10^{3} = 6V$$

$$\therefore V_T = V_{AB} + 3V = 9V. \qquad 3$$

For $5k\Omega$ load $I = \frac{9}{8 \times 10^3} = 1.13 \times 10^{-3} A$ $\therefore$ Power $= I^2R$ $= \underline{6.33\,mW}$

For $25k\Omega$ load $I = \frac{9}{28 \times 10^3} = 0.32\,mA$ $\therefore$ Power $= 2.58\,W$

$\therefore$ Power range $\underline{2.58 \rightarrow 6.33\,mW}$. 2

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Q2

(a) Inductor – energy stored as a magnetic field – extracted by interruption of current to produce spark or arrange to disconnect power supply and allow current to dissipate through resistor. 2

Capacitor – energy stored as an electric field – connect to resistor and discharging current produces heat. 2

Energy Stored in Inductor

Absorbed power $p = vi = Li\frac{di}{dt}$

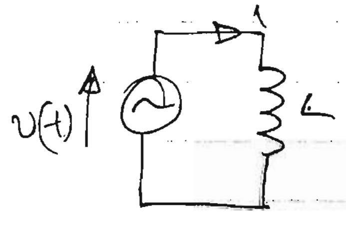
Energy absorbed in a time interval t

$$\mathcal{E}_L = \int_0^t p\,dt' = L\int_0^t i\frac{di}{dt'}\,dt'$$

$$= L\int_0^I i\,di = \underline{\frac{1}{2}LI^2} \qquad 4$$

(where I is current at t and $I=0$ at $t=0$).

(b)



$\omega L = 300$

$$\hat{I} = \frac{\hat{V}}{\omega L}$$

$= \underline{0.033\ A}$ 1

$i = 0.033\sin(100t + 90°)$ (lags voltage by 90°) 1

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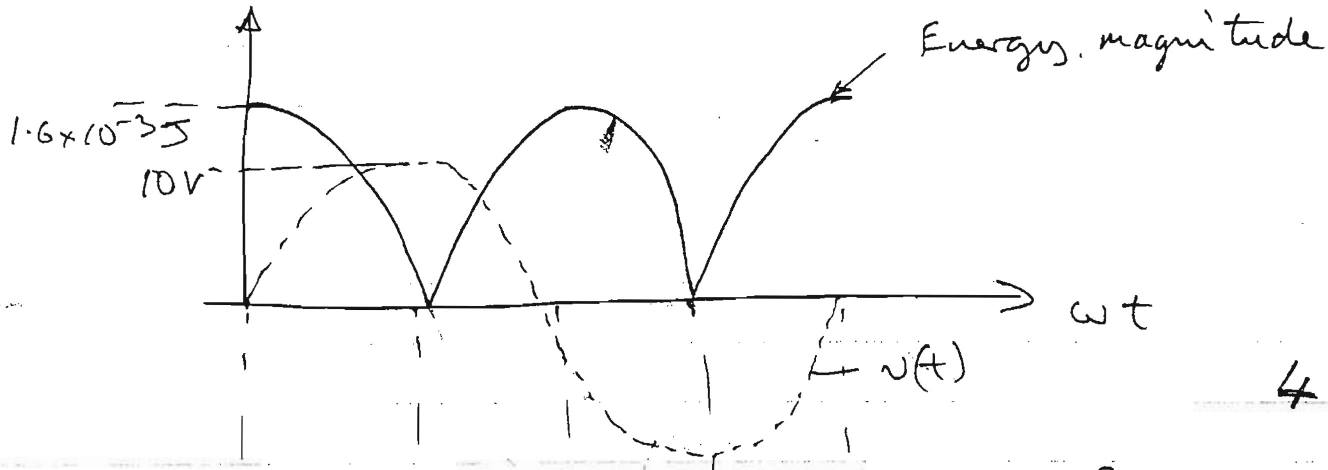
Stored energy $= \frac{1}{2} L i^2$

$= 1\cdot5 \times (0\cdot033)^2 \sin^2(100t + 90°)$

$= 1\cdot6 \times 10^{-3} \sin^2(100t + 90°)$ 1

Peak stored energy $= \underline{1\cdot6 \times 10^{-3} \text{ J.}}$ 1



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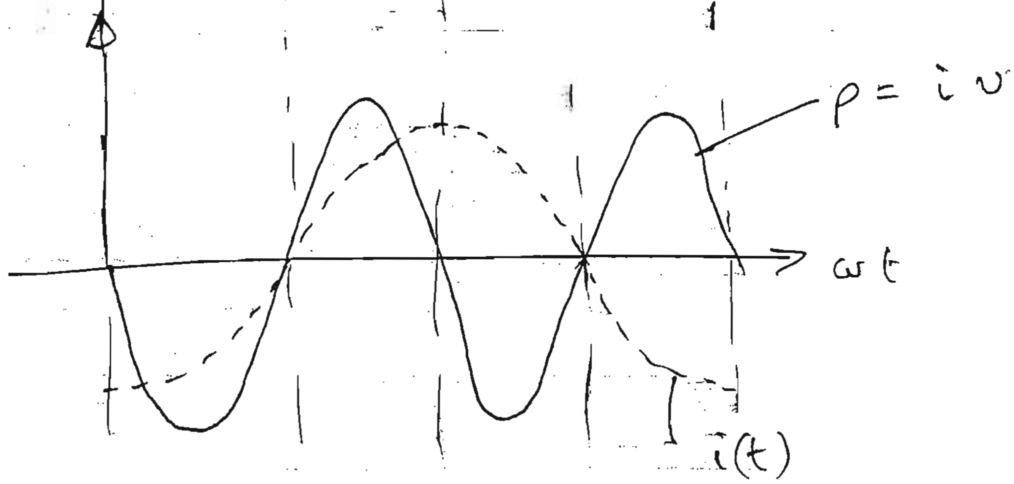
i.e. energy follows current since $E \propto i^2$



power = i × v 4

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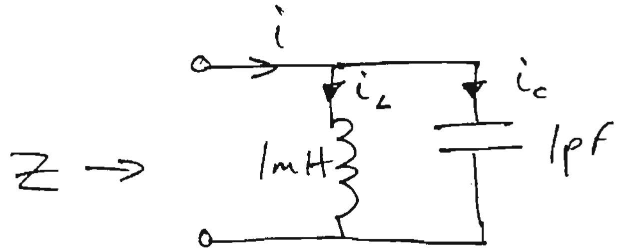
Q.3

(a)



$$Z = \frac{j\omega L \times \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{\frac{j\omega L}{\omega C}}{\frac{1}{\omega C} - \omega L}$$ 1

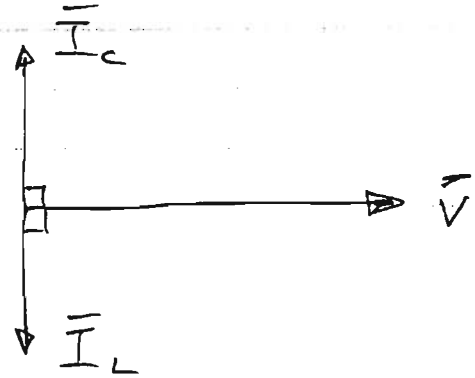
Z is infinite when $\frac{1}{\omega C} = \omega L$ 1

i.e. $\underline{\omega = \frac{1}{\sqrt{CL}}} = \frac{1}{\sqrt{10^{-6} \times 1}} = \underline{1\,kHz}$ 1

$$\bar{I}_L = \frac{\bar{V}}{j\omega L} = \frac{-j\bar{V}}{\omega L}$$ 1

$$\bar{I}_c = \frac{\bar{V}}{\frac{1}{j\omega C}} = j\omega C\bar{V}$$ 1

$$\therefore \bar{I} = \bar{I}_L + \bar{I}_c = j\bar{V}\left(\omega C - \frac{1}{\omega L}\right)$$ 1



i.e. zero current drawn (infinite impedance) when $|\bar{I}_c| = |\bar{I}_L|$

i.e. $\omega C = \frac{1}{\omega L}$.

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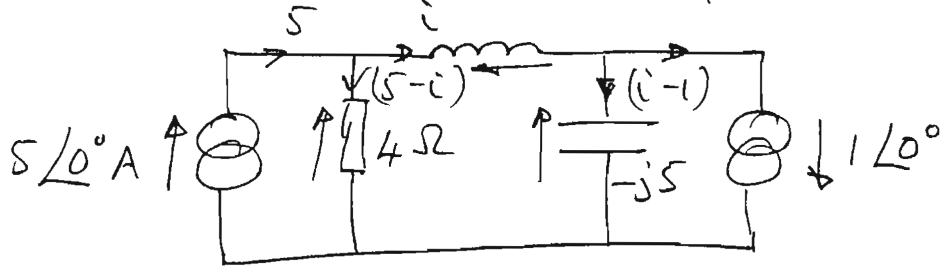
(b)



KVL round inner loop $-4(5-i)+j2i+(-j5(i-1))=0$ 2

$$\therefore \quad i = \frac{20-j5}{4-j3} = \frac{(20-j5)(4+j3)}{16+9}$$

$$= \underline{3{\cdot}8 + j1{\cdot}6 \text{ A}}$$ 2

Magnitude of $i = \sqrt{3{\cdot}8^2 + 1{\cdot}6^2} = 4{\cdot}12$A $\quad \phi_L = \tan^{-1}\left(\frac{1{\cdot}6}{3{\cdot}8}\right) = 22{\cdot}83^\circ$

$\therefore \quad \underline{i = 4{\cdot}12\angle 22{\cdot}83^\circ}$ (relative to current source) 2

Current in capacitor $= i-1 = 2{\cdot}8 + j1{\cdot}6$ A

magnitude $= \sqrt{2{\cdot}8^2+1{\cdot}6^2} = 3{\cdot}22$ A

$\phi_c = \tan^{-1}\left(\frac{1{\cdot}6}{2{\cdot}8}\right) = 29{\cdot}74^\circ$

$\therefore$ current in capacitor $= \underline{3{\cdot}22\angle 29{\cdot}74^\circ}$ 2

$\bar{V}_c = (i-1)(-j5) = 8 - j14$; $|\bar{V}_c| = \sqrt{8^2+14^2} = 16{\cdot}12$ V

$\phi_{V_c} = \tan^{-1}\left(\frac{-14}{8}\right) = -60{\cdot}25^\circ \quad \therefore \quad \underline{\bar{V}_c = 16{\cdot}12\angle -60{\cdot}25^\circ}$ 2

Phase angle between current and voltage in capacitor from above 2

$= 29{\cdot}74 + 60{\cdot}25 \simeq \underline{90^\circ}$ as expected.

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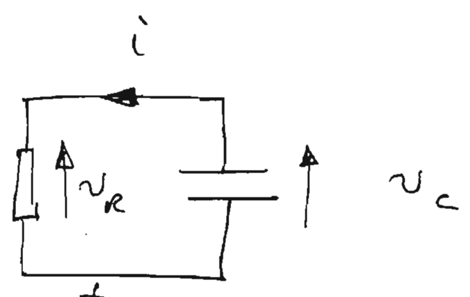


Q4 (a)

$v_c = V_0$ at $t = 0$

$\therefore i = V_0/R$ at $t \geq 0$ 1

$\therefore \quad v_c - v_R = 0 = V_0 - \frac{1}{C}\int_0^t i\,dt' - iR$ 1

differentiate $\quad i/C + R\frac{di}{dt'} = 0$ 1

separate variables + integrate

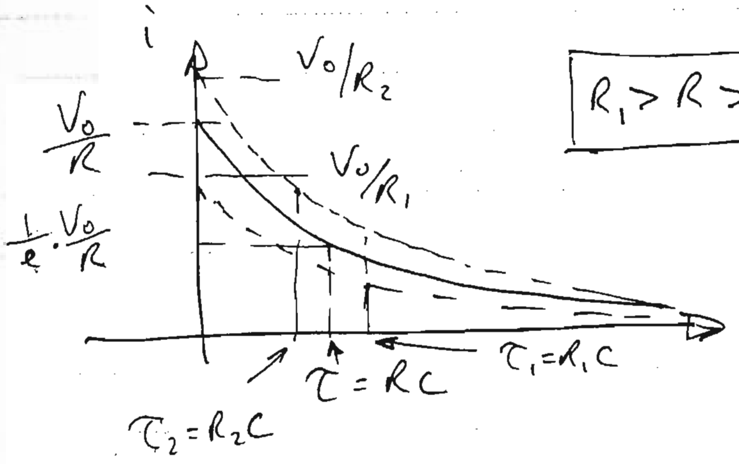
$$RC\int \frac{di}{i} = \int -dt'$$

$$\Rightarrow RC \ln i = -t + A \qquad \text{— (1)}$$

when $t = 0$, $i = \frac{V_0}{R} \implies A = RC \ln \frac{V_0}{R}$

subst. into (1) $\quad RC \ln\left(i/{V_0/R}\right) = -t$

$$\therefore \quad \underline{i = \frac{V_0}{R} \exp -t/CR}$$ 3



$\boxed{R_1 > R > R_2}$

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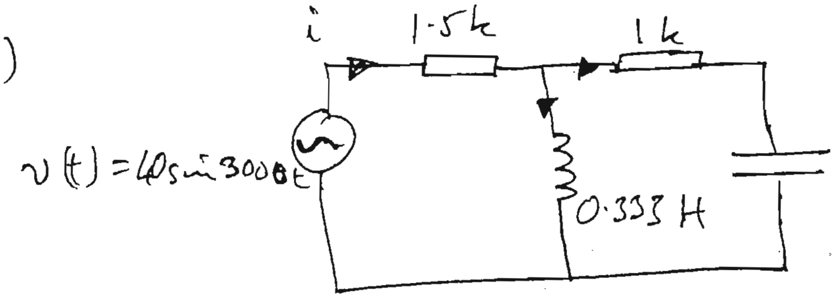
Q4 (b)



$\omega L = 3000 \times 0{\cdot}333$
$= 1000\,\Omega$.

$\frac{1}{\omega C} = \frac{1}{3000 \times 0{\cdot}667 \times 10^{-6}}$
$= 2000\,\Omega$ 1

total impedance $Z = 1{\cdot}5 + \frac{j(1-j2)}{j + (1+j2)}$ kΩ 1

$$= 1{\cdot}5 + \frac{j+2}{1-j}$$

$$= 1{\cdot}5 + \frac{(2+j)(1+j)}{1^2 + 1^2}$$

$$= 1{\cdot}5 + \frac{2 - 1 + j1 + j2}{2}$$

$$= \underline{2 + j1{\cdot}5 \text{ k}\Omega}$$ 2

$|Z| = \sqrt{2^2 + 1{\cdot}5^2} = 2{\cdot}5$

$\phi = \tan^{-1}\left(\frac{1{\cdot}5}{2}\right) = 36{\cdot}87^\circ$

current i $\quad \bar{I} = \frac{\bar{V}}{Z} = \frac{40\angle 0^\circ}{2{\cdot}5\angle 36{\cdot}87^\circ}$

$= 1{\cdot}6\angle -36{\cdot}87^\circ$ mA 2

or $\underline{i(t) = 1{\cdot}6 \sin(3000t - 36{\cdot}87^\circ) \text{ mA}}$

Current is lagging voltage hence inductive in character. 2

As ω is increased the capacitor impedance would decrease and tend to take more current resulting in the overall impedance becoming more capacitive. 2