EEE408/EEE6022 (2012-2013) Model solutions

1.

(a) When friction and the inertia of other components in the system are negligible, the torque required to move the tape is purely due to acceleration, and given by: (7)

$$T = (J_m + J_c)\frac{d\omega}{dt}$$

where ω is the angular acceleration. However, the linear acceleration of the tape, dv/dt, is related to the angular acceleration, $d\omega/dt$, of the motor by:

$$dv / dt = Rd\omega / dt$$

where R is the radius of the capstan. Thus the total motor torque required for accelerating/decelerating the drive system, expressed in terms of dv/dt is given by:

$$T_{em} = (J_m + J_c)\frac{1}{R}\frac{dv}{dt}$$

Substituting Jc using $Jc = kR^3$ results in:

$$T_{em} = (J_m + kR^3)\frac{1}{R}\frac{dv}{dt}$$

As can be seen, the equivalent inertia $J_{eq} = (J_m + kR^3)\frac{1}{R}$ is a function of the radius R, and reaches its minimum when

$$dJ_{eq} / dR = -J_m / R^2 + 2kR = 0$$

Thus:

$$R = \sqrt[3]{\frac{J_m}{2k}}$$

QED.

(b) The optimal capstan radius is given by: (2)

$$R = \sqrt[3]{\frac{J_m}{2k}}$$

and the combined inertia on the motor axis is:

$$J_{eq} = (J_m + kR^3) = J_m + k\frac{J_m}{2k} = \frac{3}{2}J_m = 1.8 \times 10^{-3}\ (kgm^2)$$

and the capstan inertia is

$$J_c = \frac{1}{2} J_m = 0.6 \times 10^{-3}\ (kgm^2)$$

(c) The angular acceleration is given by (5)

$$\frac{d\omega}{dt} = \frac{1}{R}\frac{dv}{dt} = \frac{1}{0.1} \times \frac{1.0}{10 \times 10^{-3}} = 1000\,(\text{rad/s}^2)$$

The required acceleration torque is:

$$T_{em} = (J_m + J_c)\frac{d\omega}{dt} = 1.8 \times 10^{-3} \times 1000 = 1.8\ (\text{Nm})$$

The armature current is therefore given by:

$$I_a = T_{em} / K_T = 1.8 / 0.9 = 2.0\ (A)$$

The maximum motor speed is

$$\omega = \frac{v}{R} = \frac{1.0}{0.1} = 10\ (rad/s)$$

The maximum motor voltage occurs when the motor speed reaches 10 rad/s, and neglecting the inductance effect, is given by

$$V_t = K_E\omega + R_a I_a$$

Since in the ISO system, the motor back-emf constant, K_E, is equal to the torque constant K_T. Thus,

$$V_t = K_E\omega + R_a I_a = 0.9 \times 10 + 0.5 \times 2 = 10\ (\text{V})$$

(d) The total angular distance to be rotated by the motor in every 0.015 seconds is (6)

$$\Theta = \frac{s}{R} = 0.02 / 0.1 = 0.2\ \ (rad)$$

The corresponding trapezoidal velocity profile, and the torque demand is shown in the figure below.

From this velocity profile, it can be shown that the maximum motor speed is related to the angular distance Θ and the time period T by:

$$\omega_{\max} = 3\Theta / 2T = 3 \times 0.2 / 2 / 0.015 = 20\ (\text{rad/s})$$

and the maximum angular acceleration is

$$\alpha_{\max} = \omega\text{max}/(T/3) = 20/0.005 = 4000\ (\text{rad/s}^2)$$

The peak torque requirement is

$J_{eq}a_{\max} = 1.8 \times 10^{-3} * 4000 = 7.2$ (Nm)

Peak current rating: 7.2/0.9 = 8 (A)

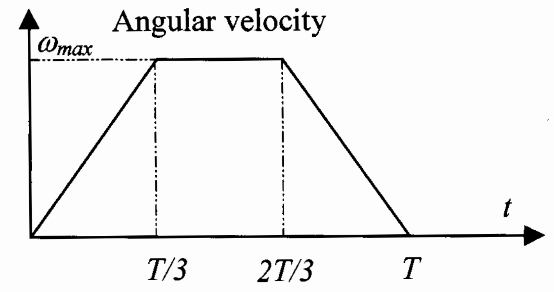
Peak voltage rating:

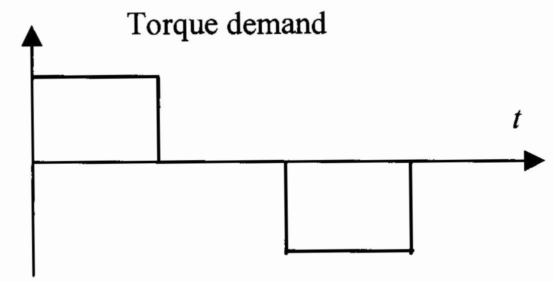
$V_t = K_E\omega_{max} + R_a I_{max} = 0.9 \times 20 + 0.5 \times 8 = 22$ (V)

The rms torque is

$\sqrt{2 * 7.2^2 / 3} = 5.88$ (Nm)

which is less than the rated rms torque of 6.0 (Nm). Thus the motor would not overheat.





2.

a. The torque constant of the motor can be determined by:

$k_T = rated\,torque\,/\,rated\,current = 5/10 = 0.5$ (Nm/A)

and the back emf constant of the motor is therefore given by

$k_E = k_T = 0.5$ (Vs/rad)

For a payload torque of 5 Nm at a speed of 1500 rpm with a gear ratio of 0.5, the motor current is

$I_a = aT_{em} / k_T = 0.5 \times 5 / 0.5 = 5$ (A)

and the motor speed is related to the gear ratio, a, by

$\omega_m = \omega / a = (1500/0.5) \times 2\pi / 60 = 314.16$

Therefore, the motor voltage in steady state is

$V_t = R_a I_a + k_E \omega_m = 5 * 0.4 + 0.5 \times 314.16 = 159.08$ (V)

If the load torque is increased to 10 Nm, the motor current increases accordingly to 0.5*10 Nm/0.5(Nm/A) = 10(A). The additional voltage required in steady state in order to maintain the same payload speed at1500rpm is therefore

$\Delta V_t = R_a \Delta I_a = (10-5) * 0.4 = 2.0$ (V)

b. At 1500rpm load speed and 5Nm load torque, the motor current is 5(A) and the motor voltage is (6)

$$Vt = R_a I_a + k_E \omega = 0.4*5+0.5*3000*2*\pi/60 = 159.8\text{(V)}$$

The PWM duty ratio is

$$D = 2t_1/(T_c/2) = V_t/V_{dc} = 159.8/200 = 0.8$$

The time duration of $2t_1$ is

$$2t_1 = D*(T_c/2) = 0.8/(2f_c) = 0.8/2/20\times10^3 = 2.0\times10^{-5} \text{ (s)}$$

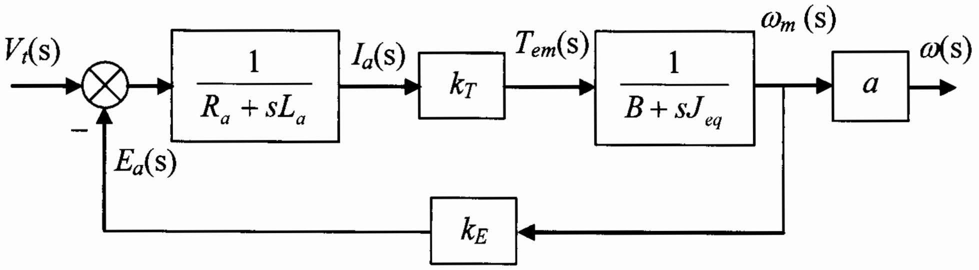
At this speed, the motor back-emf is

$$E_a = 0.5*3000*2*\pi/60 = 157.08 \text{ (V)}$$

The peak-to-peak current ripple is therefore:

$$\Delta I_p = I_1 - I_0 = (V_d - E_a)/L_a *2t_1 = (200 - 157.08)*2.0\times10^{-5}/3.2\times10^{-3}$$
$$= 0.27 \text{ (A)}$$

c. The transfer function block diagram between the motor voltage, $v_t(s)$, and the payload speed, ω(s) is shown below: (3)



The parameters of various blocks are given as follows:

Gear ratio $a = 0.5$; $k_T = k_E = 0.5$; $B = 0.0$; armature resistance $R_a = 0.4\Omega$ armature inductance $L_a = 3.2$ mH. The total moment of inertia reflected to the motor axis is

$J_{eq} = J_m + a^2 J_L$ = 4.0x10^{-3} +0.25x2.5x10^{-3} = 4.625x10^{-3} (kgm^2)

From the above block diagram, the transfer function between the motor voltage and load speed can be derived:

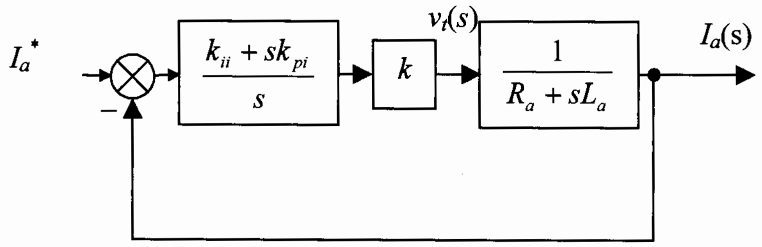
$$G(s) = \left.\frac{\omega(s)}{v_t(s)}\right| = \frac{ak_T}{(R_a + sL_a)sJ_{eq} + k_T k_E} = \frac{a}{k_E} \frac{1}{\left(\frac{L_a J_{eq}}{k_T k_E} s^2 + \frac{R_a J_{eq}}{k_T k_E} s + 1\right)}$$

The electrical and mechanical time constants of the system are given respectively by:

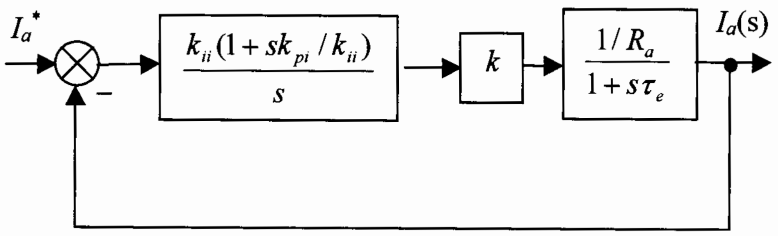
Mechnical time constant $\tau_m = \frac{R_a J_{eq}}{k_T k_E} = \frac{0.4 \times 4.625 \times 10^{-3}}{0.5 \times 0.5} = 0.0296(s)$

Electrical time constant $\tau_e = \frac{L_a}{R_a} = \frac{3.2 \times 10^{-3}}{0.4} = 8.0\,(\text{ms})$

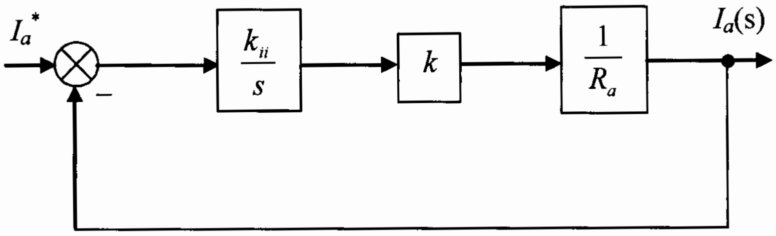
d. The block diagram of the PI current control loop, neglecting the effect of back-emf, is shown below: (4)



where k_{ii} and k_{pi} are the integral and proportional gains of the current PI control loop , and k is the gain of the H-bridge converter. The above diagram may be redrawn in the following form:



Use pole-zero cancelling, $k_{pi}/k_{ii} = \tau_e$, the resulting transfer function block diagram becomes:



The closed loop current transfer function is:

$$\frac{I_a^*(s)}{I_a(s)} = \frac{\dfrac{k_{ii}k}{R_a s}}{1+\dfrac{k_{ii}k}{R_a s}} = \frac{k_{ii}k}{R_a s + k_{ii}k} = \frac{1}{\dfrac{R_a}{k_{ii}k}s+1}$$

To achieve the desired time constant of 1.0ms

$R_a/(k_{ii}k) = 0.001$ or $k_{ii} = R_a/(0.001*20) = 20.0$, $k_{pi} = k_{ii}\tau_e = 20*0.0080 = 0.16$

3.

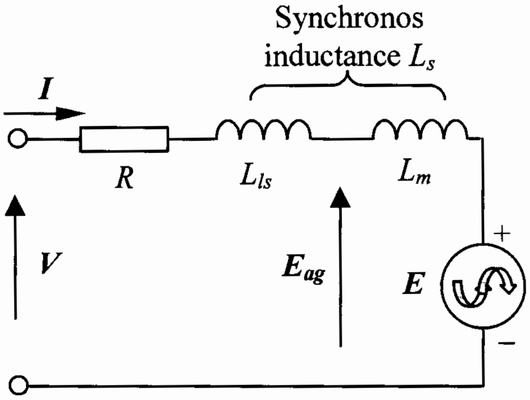
a. (2 mark)
From the motor data, the no-load peak flux linkage is produced by the permanent magnets and is given by:

$$\Psi_m = \sqrt{2}\Psi_{rms} = \sqrt{2}E_{rms}/\omega = \sqrt{2}*25/(2\pi*2*1000/60) = 0.169 \text{ (Wb)}$$

The motor torque constant is:

$$K_T = 3p\Psi_m/\sqrt{2} = 0.716 \text{ (Nm/A)}$$

b. (6 marks)
Equivalent circuit diagram:



where R, L_s and E are the winding phase resistance, synchronous inductance and no-load back-emf, respectively. L_s is the sum of the winding leakage inductance L_{ls} and the magnetising inductance L_m. V and I are the phase voltage and current, respectively.

From the equivalent circuit, the electrical power which is converted into mechanical power can be evaluated by:

$$P = 3IEcos(\theta)$$

Where θ is the angle between the current I and the back-emf E. Since E leads the no-load flux linkage Ψ_m by 90^0,

$$\theta = 90 - \delta,$$

where δ is the angle between I and Ψ_m. Thus,

$$P = 3IEcos(90 - \delta) = 3IEsin(\delta)$$

If the speed of rotation is ω_m, the electromagnetic torque is therefore given by:

$$T_{em} = P/\omega_m = \frac{3IE}{\omega_m} sin(\delta)$$

The no-load back-emf, E is related to the no-load peak flux linkage Ψ_m by:

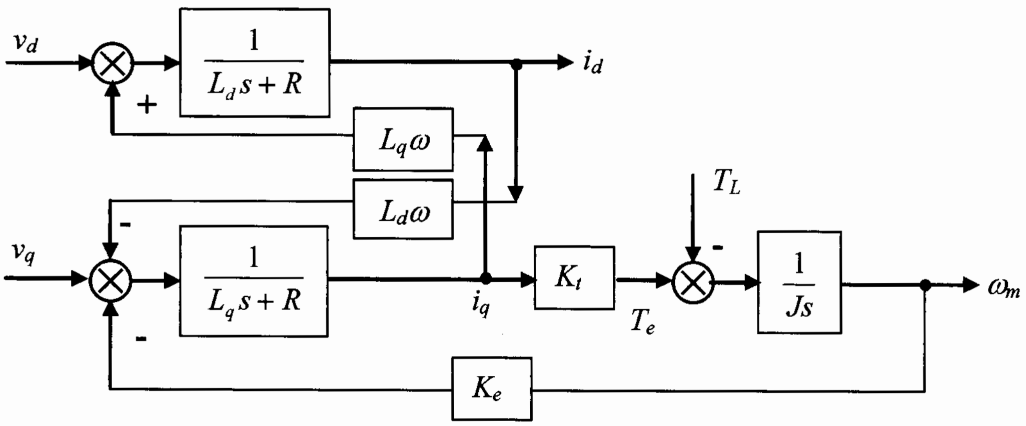
$$E = p\omega_m \Psi_m/\sqrt{2}$$

where p is the number of pole-pairs. Substituting E into the torque equation yields:

$$T_{em} = \frac{3pI\,\Psi_m}{\sqrt{2}} sin(\delta)$$

From the torque expression, the torque reaches its maximum for a given current when $\delta = 90^0$. Thus, to achieve maximum torque per Ampere operation for high efficiency, the current I should be controlled to be perpendicular to the no-load flux linkage.

c. (4 marks)
The transfer function block diagram of the motor in dq reference frame is



$L_d = L_q = L_s = 3/2*4.8 = 7.2$ (mH), $R = R_a = 4.2\Omega$,

$K_t = 3p\Psi_m/2 = 0.506$, $K_e = p\Psi_m = 0.338$, $J = 0.005$ kgm^2

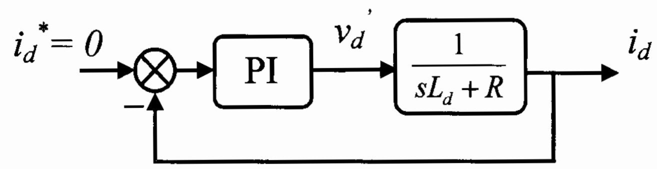
d. (8 marks)
To compensate for the coupling terms in the dq axis currents, two new control inputs, v_d' and v_q' are used and given by:

$$v'_d = v_d + \omega L_q i_q$$
$$v'_q = v_q - \omega L_d i_d$$

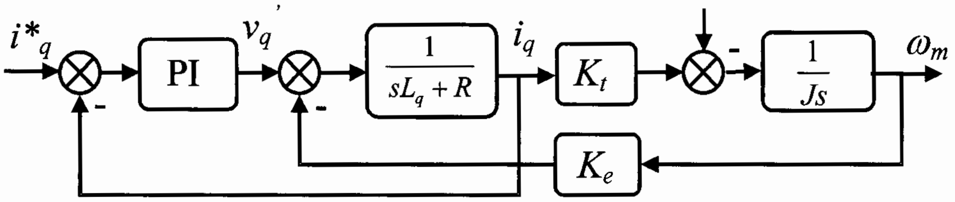
The resultant d, q axis current dynamics are now governed by:

$$L_d \frac{di_d}{dt} + Ri_d = v'_d$$
$$L_q \frac{di_q}{dt} + Ri_q = v'_q - K_e \omega_m$$

and the corresponding PI control loops are shown below:



d-axis current control loop



q-axis current control loop

Since the current response is much faster than the speed response, the effect of back emf on the q axis current may be neglected in the controller design.

If the PI current controller takes the form of

$$\frac{k_{ii} + k_{pi}s}{s}$$

and using pole-zero cancelling:

$$k_{pi}/k_{ii} = L_d/R$$

The closed loop transfer function for both d, and q axis currents is:

$$\frac{i^*_{d,q}(s)}{i_{d,q}(s)} = \frac{\frac{k_{ii}}{R\,s}}{1 + \frac{k_{ii}}{R\,s}} = \frac{k_{ii}}{R\,s + k_{ii}} = \frac{1}{\frac{R}{k_{ii}}s + 1}$$

To achieve the desired time constant of 1.0ms

$$R/(k_{ii}) = 0.001 \quad \text{or} \quad k_{ii} = R/(0.001) = 4.2/0.001 = 4200$$

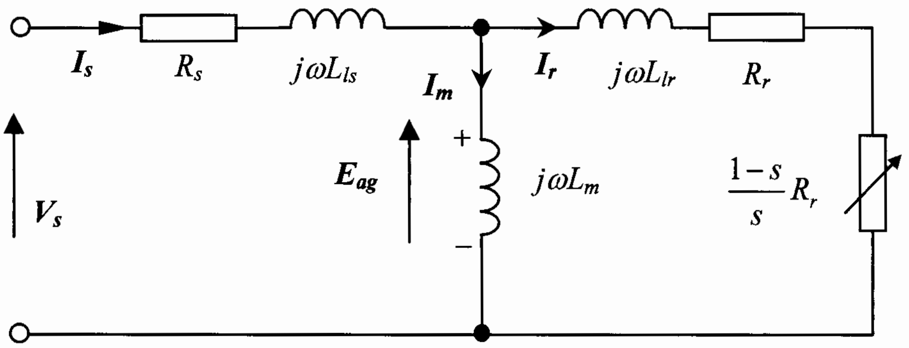
$$k_{pi} = k_{ii}\tau_e = 4200*0.0072/4.2 = 7.2$$

Note the gains k_{pi} and k_{ii} include the dc gain of a power electronics converter. Finally, the actual control outputs for v_d and v_q are :

$$v_d = v'_d - \omega L_q i_q$$
$$v_q = v'_q + \omega L_d i_d$$

4.

a. Equivalent circuit diagram of the motor operation (2)



R_s --- stator resistance
R_r --- rotor resistance reflected in stator
L_{ls}--- stator leakage inductance
L_{lr}--- rotor leakage inductance reflected in stator
L_m--- magnetising inductance
s --- slip

b. In induction machines, the rotor speed, ω_r, is related to the synchronous speed ω_s and the slip s by: (8)

$$\omega_r = \omega_s(1 - s)$$

The slip s is usually very small, or << 1.0 and the synchronous speed is related to the number of pole pairs, p, and the supply frequency f, by:

$$\omega_s = 2\pi f/p$$

Thus

$$\omega_r = \frac{2\pi f}{p}(1 - s)$$

By varying the supply frequency, the rotor speed can be controlled. However, the induced air-gap voltage, E_{ag} is related to the air-gap flux linkage, Ψ_{ag} and supply frequency by:

$$E_{ag} = 2\pi f \Psi_{ag} / \sqrt{2} = \sqrt{2}\pi f \Psi_{ag}$$

and the torque is proportional to the air-gap flux linkage. When the rotor speed or frequency is relatively high, the voltage drop across the winding resistance and stator leakage inductance is very small. Hence, the air-gap voltage E_{ag} is approximately equal to the phase voltage V_s :

$$V_s \approx E_{ag} = 2\pi f \Psi_{ag} / \sqrt{2} = \sqrt{2}\pi f \Psi_{ag}$$

or

$$\Psi_{ag} \approx \frac{V_s}{\sqrt{2}\pi f} \propto \frac{V_s}{f}$$

Thus, by maintaining a constant voltage to frequency ratio, the air-gap flux linkage is kept approximately constant, and the motor torque capability is not affected when the rotor speed is varied.

At low speeds, however, the voltage drop across the stator resistance becomes significant, and consequently, the air-gap voltage differs from the input phase voltage by:

$$E_{ag} \approx V_s - R_s I_s$$

the air-gap flux linkage is influenced by the frequency in the following manner

$$\Psi_{ag} \propto E_{ag}/f \approx (V_s/f - R_s I_s/f) = \text{constant} - R_s I_s/f.$$

Thus, by maintaining a constant V_s to f ratio, the air-gap flux linkage decreases with decrease in frequency, and the motor torque capability reduces and the performance deteriorates

c. At rated speed of 1450 rpm, the slip s is given by: (9)

s = (1500-1450)/1500 = 0.033

For small values of s, $sR_s << R_r$ and $s\omega L_l << R_r$, and the motor electromagnetic torque is proportional to slip s. Thus, at 50% load torque, the slip s is 0.0167

The rotor speed is 1500*(1-s) = 1475 (rpm)

From the equivalent circuit diagram, the impedance of the rotor branch is

$R_r/s + j\omega L_{lr}$ = 0.55/0.0167+j0.95 = 32.93+ j0.95

The equivalent impedance of the parallel of the magnetising branch and the rotor branch is

$$\frac{(32.9 + j0.95)j48.6}{32.9 + j(0.95 + 48.6)} = \frac{1601.3\angle 91.65^0}{59.48\angle 56.42^0} = 26.92\angle 35.23^0 = 21.99 + j15.53$$

Total impedance seen from the stator terminal:

$$0.35 + j1.20 + 21.99 + j15.53 = 22.34 + j16.73 = 27.91\angle 36.82^0$$

Stator current

$$I_s = 240 / 27.91\angle 36.82^0 = 8.60\angle -36.82^0$$

Power factor

$$\cos\varphi = \cos 36.82^0 = 0.80$$

Induced air-gap voltage

$$E_{ag} = 240 - (R_s + j\omega L_{ls})I_s = 240 - (1.25\angle 73.74)8.60\angle -36.82^0 = 231.4 - j6.46$$
$$= 231.49\angle -1.6^0$$

Rotor current

$$I_r = E_{ag} /(R_r / s + j\omega L_{lr}) = 231.49\angle -1.6^0 /(32.94\angle 1.65^0) = 7.03\angle -3.25^0$$

Air-gap flux linkage

$$\Psi_{ag} = E_{ag} / 4.44 f = 231.43 / 4.44 / 50 = 1.04\,(Wb)$$

The electromagnetic torque

$$T_{em} = \frac{3R_r I_r^2}{s\omega_s} = \frac{3 * 0.55 * 7.03^2}{0.0167 * 157.08} = 31.1\,(Nm)$$

Efficiency

$$\eta = P_{out} / 3I_s V_s \cos\varphi = 31.1 * 154.46 /(3 * 240 * 8.6 * 0.8) = 0.97$$

Note the iron loss, friction and windage losses are not represented in the equivalent circuit, and therefore the efficiency is overestimated.