Data Provided: None

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2012-13 (2.0 hours)

EEE6440 Advanced Signal Processing 6

Answer **FOUR** questions (**TWO** questions from **Part A** and **TWO** questions from **Part B**). **No marks will be awarded for solutions to a third question attempted from any of the two sections.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

PART A - Answer only TWO questions from questions 1, 2 and 3.

1. The M-point moving average filter (MAF) operates by averaging a number of points from the input signal x(n) to produce each point in the output signal y(n) as follows:

$$y(n) = \frac{1}{M} \sum_{k=\frac{1-M}{2}}^{\frac{M-1}{2}} x(n+k)$$

Assume M is an odd number.

a. Compute and draw (i) the impulse response, (ii) the step response and (iii) the magnitude of the frequency response of a 5-point MAF. **(5)**

b. The computational complexity of the M-point MAF can be reduced using the recursive implementation. (i) Derive the recursive implementation of the 5-point MAF and (ii) compare its complexity, in terms of number of additions and multiplications, with respect to those for the non-recursive implementation. **(5)**

c. (i) Determine and draw the resulting filter kernel if two 5-point MAFs are cascaded in a system. (ii) Sketch and compare its time-domain and frequency-domain performances with those of the 5-point MAF. **(5)**

2. An input signal $x=(x_0, x_1, x_2, x_3)$ is transformed into $y=(y_0, y_1, y_2, y_3)$ as follows:

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h & h & h & h \\ h & h & -h & -h \\ \sqrt{2}h & -\sqrt{2}h & 0 & 0 \\ 0 & 0 & \sqrt{2}h & -\sqrt{2}h \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a. Write down the basis functions corresponding to the above forward wavelet transform matrix. **(2)**

b. If this set of basis functions forms an orthogonal transform, find the value of *h*. Using your solution, verify the orthogonality of this transform. **(3)**

c. What is the corresponding inverse transform matrix? Verify that your solution provides the perfect reconstruction for the given transform. **(3)**

d. How do you compute the mean of signal *x* using the transform domain coefficients *y*? **(2)**

e. How do you use this transform to remove noise from a measured signal? **(5)**

3. a. Using frequency response diagrams, explain the purpose of using low pass filters in the sampling rate decimators. **(3)**

b. A signal, sampled at 1.024 kHz, is to be decimated using a 2-stage decimator, with decimation rates $M_1=4$ and $M_2=2$, respectively. The signal band of interest extends from 0 to 60 Hz. The overall anti-aliasing digital filtering should satisfy 0.01 dB passband deviation (δ_p) and 80 dB stopband attenuation (δ_s). Estimate the lengths of the low pass filters h_1 and h_2 used for the two decimations, respectively. Note that the filter length N for a low pass filter is approximated as

$N \approx \frac{-10\log(\delta_p\, \delta_s)-13}{14.6(\Delta f)} + 1$, where Δf is the normalised width of transition band.

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c. Estimate the computational complexity of this 2-stage decimator in terms of multiplications per second.

Explain why multistage decimation is more efficient in terms of the computational complexity, compared to a single stage decimation system. **(5)**

PART B - Answer only TWO questions from questions 4, 5 and 6.

4. **a.** Estimate the mean and variance of the following stationary sequence: {1.39, 1.63, 1.87, 2.75, 0.68}. **(2)**

b. Two terms are commonly used to indicate the dependency of a signal at one point in time with the same signal at a different point in time, or more generally for the dependency of one signal upon another. These two terms are "independent" and "uncorrelated". Give a proof to show that statistically independent random processes are uncorrelated. Show all working. **(4)**

c. Zero-mean white Gaussian noise with variance 1 is applied to two filters simultaneously. Filter 1 has transfer function $H_1(z)=2-3z^{-1}$; filter 2 has transfer function $H_2(z)=3-2z^{-2}$. The output of filter 1 is denoted by $y_1(n)$ and the output of filter 2 is denoted by $y_2(n)$.

i) What is the autocorrelation sequence of the output of filter 1? **(3)**

ii) Calculate the cross-correlation sequence $\phi_{y_1 y_2}(m)$ and $\phi_{y_2 y_1}(m)$. **(6)**

5. **a.** Given the correlation sequence $\phi_{xx}(m)$ of a zero-mean random sequence x(n), give the expression of its power spectral density function. Explain why this expression is given the name of "power spectral density".

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b. For an 8-bit A/D converter, what is the dynamic range for a cosine wave input signal? **(3)**

c. **i)** Suppose the length of an FIR (finite impulse response) adaptive filter is N. Its input is denoted by y(n) and the training signal is denoted by x(n). Derive the LMS (least mean square) adaptive algorithm for updating the coefficients of the adaptive filter. **(4)**

ii) The table below shows the input and training signal to a two-tap adaptive filter at sample numbers 14 and 15, where **h**(n) is the vector holding the two taps of the adaptive filter :

Iteration n	y(n)	**h**(n)	x(n)
14	0.25	[1 6]	1.2
15	0.3		-0.2

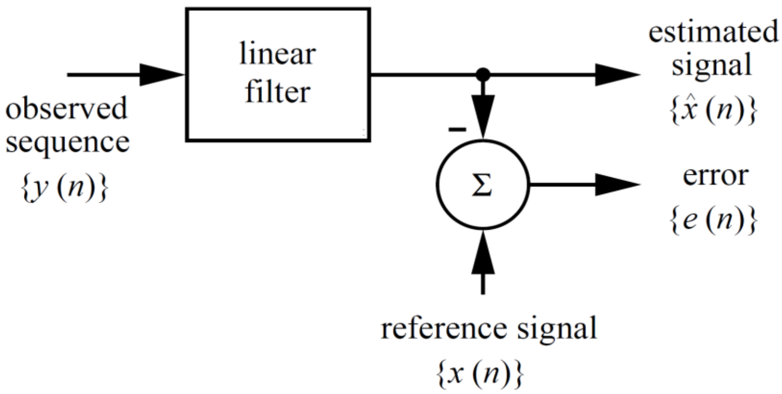
Using the derived LMS algorithm, evaluate **h**(15). The stepsize is fixed at 0.1. **(4)**

6. **a.** Suppose the z-transform of the cross-correlation function between the input x(n) and the output y(n) of a filter is given by $S_{xy}(z)$ and the z-transform of the autocorrelation of the input x(n) is given by $S_{xx}(z)$.

i) Give the relationship between these two z-transforms. **(2)**

ii) Given an unknown linear system with white stationary input x(n) and output y(n), use the above result to show how to measure the impulse response of the system? **(4)**

b. A linear estimator is shown below, where the impulse response of the linear filter is given by h_j, j=0, 1, …, N-1. Derive the Wiener solution for h_j. Show all working.



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