EE 207 Worked Solutions Spring Session 2006-07

1

) Charge neutrality condition :

$$n + N_a = p + N_d$$

lso $np = n_i^2$ (2)

$$\therefore \quad n^2 - (N_d - N_a)\, n - n_i^2 = 0$$

$$n = \frac{N_d - N_a}{2} + \frac{N_d - N_a}{2}\left[1 + \left(\frac{2n_i}{N_d - N_a}\right)^2\right]^{1/2}$$

) For n-type extrinsic semiconductor, $N_d - N_a \gg n_i$, so above is

$$n = N_d - N_a \doteq N_d$$

since $p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d - N_a} = \frac{n_i^2}{N_d}$ (2)

) For near compensated case, $n_i \gg N_d - N_a$, so above is

$$n \approx n_i + \frac{N_d - N_a}{2} \doteq n_i,$$

$$p = \frac{n_i^2}{n_i} = n_i$$ (2)

[6]

b) Intrinsic conductivity, $\sigma_i = \frac{1}{\rho_i} = n_i e\,(\mu_e + \mu_h)$

$$\frac{1}{5\times10^{3}} = 2\times10^{-4} = n_i \times 1{\cdot}6\times10^{-19}\,(0{\cdot}12 + 0{\cdot}05) = n_i \times {\cdot}272\times10^{-19}$$

$$n_i = \frac{2\times10^{-4}}{0{\cdot}272\times10^{-19}} = 7{\cdot}35\times10^{15}\ m^{-3}$$ (3)

With initial n-doping only, $\rho$ is $10^{-4}\rho_i$ * $= 10^{-4}\times5\times10^{3} = 0{\cdot}5$

since n-doped, ignore $n_i$, so

$$\frac{1}{\rho} \approx e N_d \mu_e \Rightarrow \frac{1}{0{\cdot}5} = 2 = 1{\cdot}6\times10^{-19}\times N_d \times 0{\cdot}12$$

$$\therefore\ N_d = 1{\cdot}04\times10^{20}\ m^{-3}$$ (3) (2)?

* Doping can only reduce resistivity c.f. intrinsic value.

After compensation, $\rho = 7.61 \times 10^{2}\ \Omega\text{-m}$

For compensated case,

$$\frac{1}{\rho} = \sigma = e(n\mu_e + p\mu_h) = e\left(n\mu_e + \frac{n_i^2}{n}\mu_h\right)$$

$$\frac{1}{7.61\times10^{2}} = 1.314\times10^{-3} = 1.6\times10^{-19}\left[n\times0.12 + \left(\frac{7.35\times10^{15}}{n^2}\right)^2\times0.05\right]$$

$$n^2 - 6.84\times10^{16}\,n + 2.25\times10^{31} = 0$$

$$n = \frac{6.84\times10^{16} \pm \sqrt{(6.84\times10^{16})^2 - 4\times2.25\times10^{31}}}{2}$$

$n = 6.81\times10^{16}$ or $3.2\times10^{14}$

$n$ must be $> n_i$, so $n = 6.81\times10^{16}\ m^{-3}$ lot of work

$$p = \frac{n_i^2}{n} = \frac{(7.35\times10^{15})^2}{6.81\times10^{16}} = 7.93\times10^{14}\ m^{-3}$$ (2) (3)?

more marks for these workings? Swap?

We need Na: From charge neutrality,

$N_a = p + N_d - n = 7.93\times10^{14} + 1.04\times10^{20} - 6.81\times10^{16}$

$= 1.039\times10^{20}\ m^{-3}$ (2)

[10]

c) To get back original resistivity requires slightly more acceptor doping. The doping control required for this to happen makes this impossible in practice.

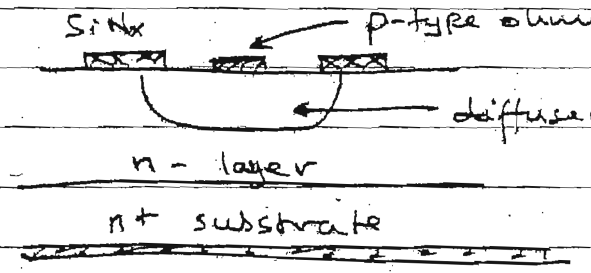
is ~~this right~~?

[4]

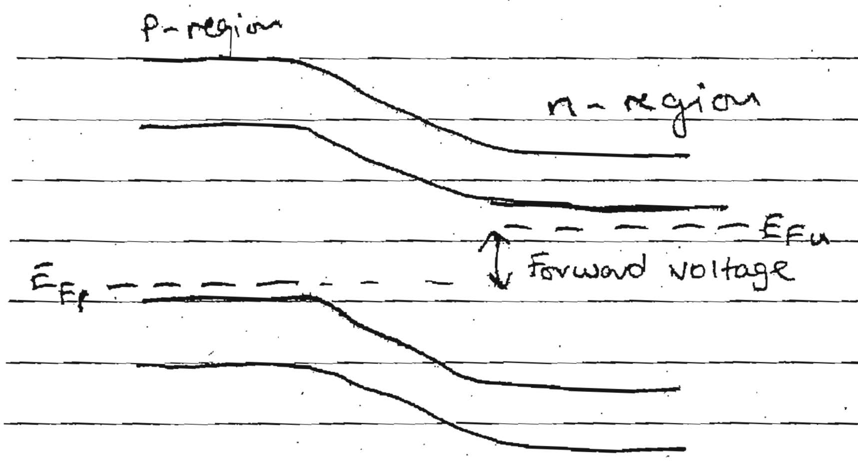
2

Q2

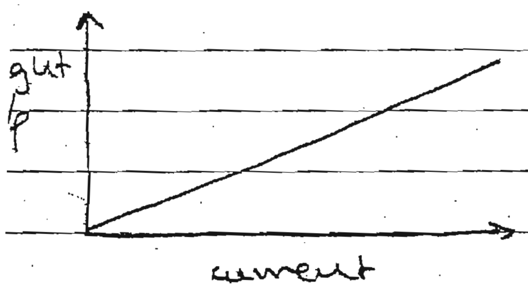
a)




LED cross-section (3)




Band diagram under LED emission (3)




light output increases ~ linearly (2) with injection current. In a laser, there will be a region of very small increase initially, before a sudden increase at the threshold condition

[8]

b) The range of wavelengths is given by the bulk GaAs band-gap to that of the AlGaAs barrier, i.e.

$$\lambda_{max} = \frac{1.24}{1.42} = 0.87\ \mu m$$

$$\lambda_{min} = \frac{1.24}{1.85} = 0.67\ \mu m$$

[3]

c) Operating wavelength = 850nm $\equiv \frac{1.24}{0.85} = 1.46\,eV$

$$E_n = \frac{n^2 h^2}{8 m^* L^2} \quad ; \; n = 1, \; m_e^* = 0.063 m_o, \; m_h^* = 0.48 m_o$$

Total band-gap $= 1.42 + E_{1elect.} + E_{1hole}$ (3)

$$E_{1elect} = \frac{(6.63\times10^{-34})^2}{8\times0.063\times9.11\times10^{-31}\times1.6\times10^{-19}} \cdot \frac{1}{L^2} \text{ eV}$$

$$E_{1hole} = \frac{(6.63\times10^{-34})^2}{8\times0.48\times9.11\times10^{-31}\times1.6\times10^{-19}} \cdot \frac{1}{L^2} \text{ eV}$$

$$1.46 = 1.42 + \frac{6.768\times10^{-18}}{L^2}$$ (3)

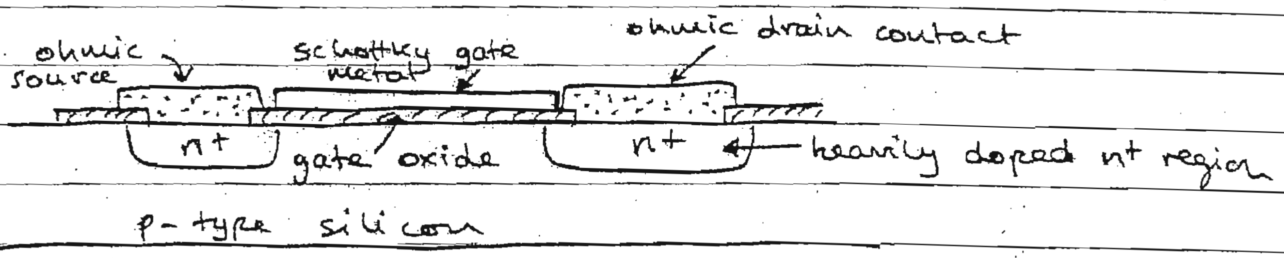
$L = 1.3\times10^{-8}$ m or 13 nm [6]

d) In reality, as the QW dimension reduces, the onset of tunnelling through the finite barriers will limit the minimum wavelength.

[3]

Q3

(a)




[4]

b) $I_d = \frac{\mu_e C_g}{l^2}\left[V_g - V_T - \frac{V_d}{2}\right] V_d$

Saturation occurs when $V_d = V_g - V_T$, ∴ $I_{ds}$ is

$= \frac{\mu_e C_g}{l^2} \frac{V_d^2}{2}$ (2)

$g_m = \left.\frac{\partial I_d}{\partial V_g}\right|_{V_d}$ in saturation region, so differentiate above

∴ $g_m = \frac{\mu_e C_g V_d}{l^2}$ (2)

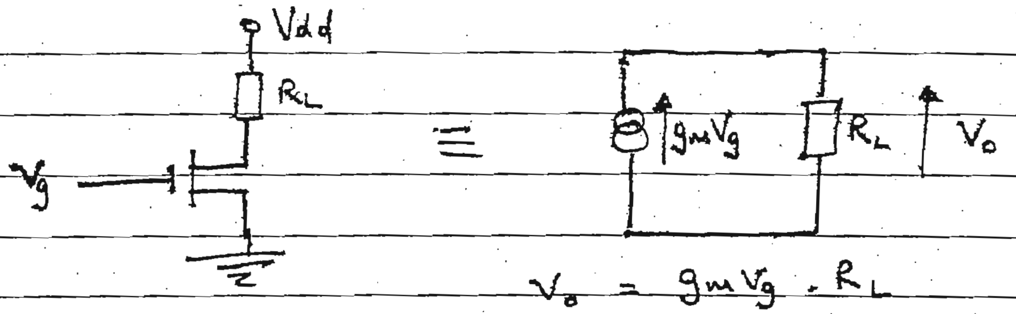
Rearrange and substitute into expression of $I_{ds}$

$\frac{\mu_e C_g}{l^2} = \frac{g_m}{V_d} = \frac{2 I_{ds}}{V_d^2}$

∴ $g_m = \frac{2 I_{ds}}{V_d}$ (1)

[5]

c)




$V_o = g_m V_g \cdot R_L$

Gain $= \frac{V_o}{V_g} = |A| = g_m R_L$ (2)

∴ $30 = g_m R_L = \frac{2 I_{ds} \cdot R_L}{V_d} = \frac{2 I_{ds} \cdot R_L}{(V_{dd} - I_{ds} R_L)}$ (3)

out. $30 = \dfrac{2 \times 30 \times 10^{-3} R_L}{100 - 30 \times 10^{-3} R_L}$

gives $R_L = 3.125\ k\Omega$ (2)

[7]

d) $g_m = 9.6\,mS = \dfrac{\mu_e C_g V_d}{l^2}$

$l^2 = \dfrac{\mu_e C_g V_d}{g_m} = \dfrac{0.13 \times 10^{-12} \times 6.25}{9.6 \times 10^{-3}}$

$l = 9.2\,\mu m$ (gate length)

$C_g = \dfrac{\varepsilon_0 \varepsilon_r . Area}{t_0} = \dfrac{\varepsilon_0 \varepsilon_r . l . width}{t_0}$

length = 0.1 width $\Longrightarrow$ width = $92\,\mu m$

$t_0 = \dfrac{\varepsilon_0 \varepsilon_r . l . width}{C_g} = \dfrac{11.8 \times 8.85 \times 10^{-12} \times 9.2\,\mu m \times 92\,\mu m}{10^{-12}}$

$t = 88\,nm$

[4]

Q4

a) $E = hf$ , $c = f\lambda$

$= \frac{hc}{\lambda}$

$\lambda = \frac{hc}{E}$ where E is in joules

To get E in eV, divide numerator + denominator by 'e'

$\therefore \lambda = \left(\frac{hc}{e}\right) / (E/e)$

$\therefore L = \frac{hc}{e} = 1 \cdot 24$ μm eV

[4]

b) de Broglie : $p = mv = \frac{h}{\lambda}$ (3)

h = Planck's constant, m = mass, v = velocity, λ = wavelength

Equate K.E. to P.E

$\frac{1}{2}mv^2 = eV \implies v = \left(\frac{2eV}{m}\right)^{1/2}$

From de Broglie, $mv = \frac{h}{\lambda}$

(3)

$\lambda = \frac{h}{mv} = \frac{h}{(2eVm)^{1/2}} = \frac{1 \cdot 225\,\text{nm}}{V^{1/2}} = \frac{1 \cdot 225}{6}$ nm

$= 0 \cdot 204$ nm, which is the minimum resolvable size.

[6]

(c) $E = E_g + AK^2 + BK^4$

$m^* = \left(\frac{d^2E}{dp^2}\right)^{-1}$ , $p = \hbar K$

$m^* = \hbar^2 \left(\frac{d^2E}{dK^2}\right)^{-1}$ (2)

cont. $\frac{dE}{dK} = 2AK + 4BK^3$, $\frac{d^2E}{dK^2} = 2A + 12BK^2$

) At $K=0$, $m^* = \hbar^2(2A+0)^{-1} = \left[\frac{6.626\times10^{-34}}{2\pi}\right]^2\left[2\times10^{-38}\right]^{-1}$

$= 5.56\times10^{-31}$

$= 0.61\,m_o$ (3)

) At Brillouin zone edge, $v_g = 0$

$v_g = \frac{dE}{dp} = \frac{1}{\hbar}\cdot\frac{dE}{dK} = \frac{1}{\hbar}(2AK + 4BK^3) = 0$

$AK + 2BK^3 = 0 \Rightarrow K = \left[\frac{A}{-2B}\right]^{1/2}$ (B is -ive so $\sqrt{\ }$ ok)

Need $m^*$ @ $K = \left[\frac{A}{-2B}\right]^{+1/2}$

$m^* = \frac{\hbar^2}{2A + 12BK^2} = \frac{\hbar^2}{2A + 12B\left(\frac{A}{-2B}\right)} = \frac{\hbar^2}{-4A}$

$= \left(\frac{6.626\times10^{-34}}{2\pi}\right)^2\cdot\left(\frac{1}{-4\times10^{-38}}\right) = -2.78\times10^{-31}$ (5)

$= -0.305\,M_o$

[10]

Bit tricky?