Data Provided: List of useful formulae

Fourier Transform Pairs

Laplace Transform Pairs

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2009-2010 (2 hours)

Signals and Systems EEE201

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a.

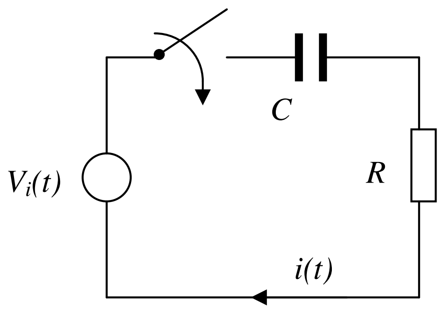

Figure Q1.1

Use the Laplace transform to show that the voltage across the capacitor is given by $v_c(t) = A\left(1 - e^{-t/RC}\right)u(t)$ in the RC circuit shown in Figure Q1.1, assuming that the initial voltage across the capacitor $v_c(0) = 0$, and $v_i(t) = Au(t)$, where A is a constant. **(10)**

b. Find an expression for the current, $i(t)$, flowing in the circuit. **(3)**

c. Work out the value of the current at time $t = 0$, $i(0)$, and the time taken for the current to decay to 10% of its value at $t = 0$. **(5)**

d. What is the signal frequency range that the circuit will pass without attenuating the signal power by more than 3dB? **(2)**

2. **a.** Prove that the impulse response of a simple RC low pass circuit is given by $h(t) = \frac{1}{RC} e^{-t/RC} u(t)$ where R is the resistance and C is the capacitance. **(4)**

b. Suppose that in a digital communication channel, the bit "1" is represented by the signal $p(t)$, shown in Figure Q2.1. If $p(t)$ is the input signal to the RC circuit and $y(t)$ is the output signal, use convolution to show that the output signal is given by

$$y(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-t/RC} & 0 < t < T \\ e^{-(t-T)/RC} - e^{-t/RC} & t \geq T \end{cases}.$$

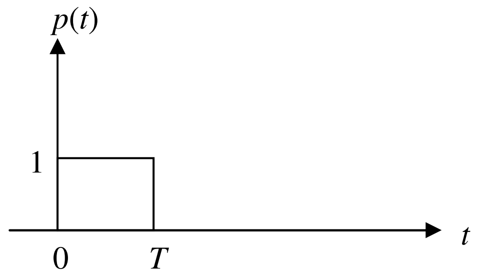

Figure Q2.1 **(10)**

c. Assuming that T = 1s and RC = 0.1s, sketch and label the response of the circuit to the single bit of "1". **(3)**

d. Sketch and label the response of the same circuit to two successive bits of "1". **(3)**

3. **a.** Verify that the complex Fourier Series representation of the sampling function $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$ shown in Figure Q3.1 is given by $p(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\omega_s t}$, where ω_s is the sampling frequency in rad/s.

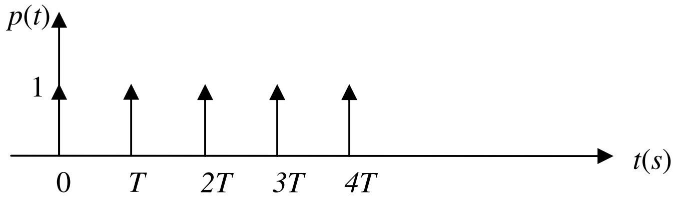

Figure Q3.1 **(5)**

b. Show that the Fourier Transform of the signal $p(t)$ in part (a) is given by $P(\omega) = \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$. **(2)**

3 c.

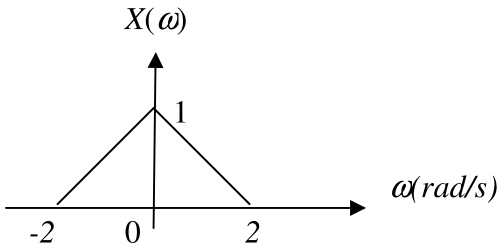

Figure Q3.2

Consider a continuous time signal $x(t)$ with the magnitude spectrum shown in Figure Q3.2. The signal $x(t)$ is multiplied by the sampling function $p(t)$ in Figure Q3.1 to obtain $x_s(t)$, the sampled version of $x(t)$. Assuming ω_s= 2 rad/s, sketch and label

i) the magnitude spectrum $P(\omega)$.

ii) the magnitude spectrum $X_s(\omega)$.

Confirm whether the $x(t)$ can be recovered using a low pass filter. Explain why? **(6)**

d. The modulated signal in a double sideband amplitude modulation scheme is given by $x(t) = [A_o + \cos(\omega_m t)]\cos(\omega_c t)$, where $A_o + \cos(\omega_m t) > 0$. Here ω_m and ω_c are the frequencies for the modulating and carrier signals, respectively while A_o is a constant. To demodulate the signal an envelope detector depicted in Figure Q3.3 can be used.

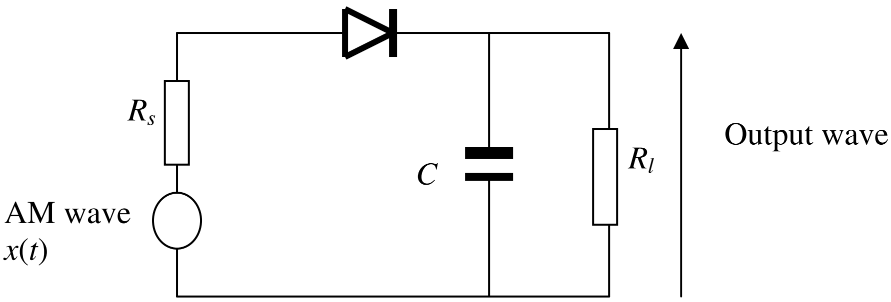

Figure Q3.3

The capacitance voltage during charging is described by

$v_c(t) = A_c[1 - \exp(-t/R_sC)]$,

while $v_d(t) = A_d \exp(-t/R_lC)$ described the capacitance voltage during discharging. Assuming C = 0.01μF, $\omega_c = 2\pi \times 10^5$ rad/s and $\omega_m = 0.01\omega_c$, suggest suitable values for R_s and R_l. **(7)**

4. **a.** Consider a continuous time system described by a transfer function

$$H(s) = \frac{4(s+2)}{s^2 + 16s + 8}.$$

i) Find the poles and zeros. **(4)**

ii) Sketch and label the magnitude response of the system. Determine whether this system is lowpass or highpass. **(4)**

iii) Find the natural oscillating frequency and the damping factor of this system. **(2)**

b. Sketch the unit step response of the system. Describe how this unit step response changes with time and confirm whether the system is stable? **(4)**

c. Find the system response $y(t)$ when the input is $x(t) = e^{-2t}u(t)$. **(6)**

CHT