Tutorial Sheet – No 4 Answers

1 (a) The instantaneous power dissipated in the resistor is given by:

$$P_R = I_R^2 R$$

and:

$$I_R = \frac{V}{R} = \frac{200t}{R}$$

therefore:

$$P_R = \left(\frac{200t}{R}\right)^2 R = \frac{40000t^2}{R} = 200t^2$$

At $t = 0.5$s:

$$P_R = 200 \times 0.5^2 = \mathbf{50W}$$

(b) The instantaneous power supplied to the capacitor is given by:

$$\mathrm{P_C = VI_C = VC\frac{dV}{dt} = 200t \times C \times \frac{d}{dt}(200t) = 40000t \times 1000 \times 10^{-6} = 40t}$$

At $t = 0.5$s:

$$P_C = 40 \times 0.5 = \mathbf{20W}$$

(c) Instantaneous power supplied by the source:

$$P_S = P_R + P_C = 50 + 20 = \mathbf{70W}$$

(d) Energy supplied by the source from $t = 0$ to $t = 0.5$s:

$$E_S = \int_0^{0.5} (P_R + P_C)dt = \int_0^{0.5} \left(200t^2 + 40t\right)dt = \left[\frac{200t^3}{3} + 20t^2\right]_0^{0.5} = \mathbf{13.3J}$$

(d) The energy stored in the circuit is that stored in the capacitor:

$$E_C = \int_0^{0.5} (P_C)dt = \int_0^{0.5} (40t)dt = \left[20t^2\right]_0^{0.5} = \mathbf{5J}$$

2 First calculate the reactance of the inductor at the supply frequency of 2Hz:

$$X_L = 2\pi fL = 2\pi \times 2 \times 10 = 125.67\ \Omega$$

Therefore the impedance of the series combination of R and L may be expresses as:

$$Z_T = R + jX_L = 15 + j125.67 = 126.6\angle 83.2^\circ\ \Omega$$

Since the supply is sinusoidal the rms voltage may be obtained from:

$$V_{rms} = \frac{V_{pk}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71\mathrm{V_{rms}}$$

The current flowing in the circuit is:

$$I_{rms} = \frac{V_{rms}}{Z_T} = \frac{70.71\angle 0^\circ}{126.6\angle 83.2^\circ} = \mathbf{0.56\angle -83.2^\circ}\ \mathrm{A_{rms}}$$

Since the circuit is inductive (and the current is lagging behind the voltage) the power factor is lagging:

$$pf = \cos\phi = \cos 83.2 = \mathbf{0.118\ lagging}$$

The VA rating is simply the product of the rms voltage and current:

$$VA = V_{rms} I_{rms} = 70.71 \times 0.56 = \mathbf{39.6\ VA}$$

The real and imaginary powers can then be found as:

$$P = V_{rms} I_{rms} \cos\phi = 70.71 \times 0.56 \times 0.118 = \mathbf{4.67\ W}$$

$$Q = V_{rms} I_{rms} \sin\phi = 70.71 \times 0.56 \times \sqrt{1 - 0.118^2} = \mathbf{39.3\ VAr}$$

3 (a) The instantaneous power dissipated in the resistor is given by:

$$P_R = I^2 R$$

but:

$$I = 5t$$

therefore:

$$P_R = (5t)^2 R = 375t^2$$

At $t = 2$s:

$$P_R = 375 \times 2^2 = \mathbf{1500W}$$

(b) The instantaneous power supplied to the inductor is given by:

$$P_L = V_L I = L\frac{dI}{dt} I = 10 \times \frac{d}{dt}(5t) \times 5t = 250t$$

At $t = 2$s:

$$P_L = 250 \times 2 = \mathbf{500W}$$

(c) The energy input to the circuit from $t = 0$ to $t = 2$s:

$$E_S = \int_0^2 (P_R + P_L)dt = \int_0^2 (375t^2 + 250t)dt = \left[125t^3 + 125t^2\right]_0^2 = \mathbf{1500J}$$

(e) The energy stored in the circuit is that stored in the inductor:

$$E_L = \int_0^2 (P_L)dt = \int_0^2 (250t)dt = \left[125t^2\right]_0^2 = \mathbf{500J}$$

4 (a) First calculate the VA, W, VAr and current for each load using:

$$VA = V_{rms} I_{rms} \qquad W = V_{rms} I_{rms} \cos\phi \qquad VAr = V_{rms} I_{rms} \sin\phi$$

Electromagnet:

$$I_{EM} = 9\angle -\cos^{-1}(0.2) = \mathbf{9\angle -78.5° \, A_{rms}}$$

$$VA_{EM} = 11000 \times 9 = \mathbf{99kVA}$$

$$W_{EM} = 11000 \times 9 \times 0.2 = \mathbf{19.8kW}$$

$$VAr_{EM} = 11000 \times 9 \times \sqrt{1 - 0.2^2} = \mathbf{97kVAr}$$

General load:

$$VA_{GL} = \mathbf{60kVA}$$

$$W_{GL} = VA_{GL} \times \cos\phi = 60 \times 0.6 = \mathbf{36kW}$$

$$VAr_{GL} = VA_{GL} \times \sin\phi = -60 \times \sqrt{1 - 0.6^2} = \mathbf{-48kVAr}$$

Note the minus sign since the load is capacitive (leading power factor). The current is obtained from the VA rating:

$$I_{GL} = \frac{VA_{GL}}{V} = \frac{60}{11}\angle \cos^{-1}(0.6) = \mathbf{5.45\angle 53.1° \, A_{rms}}$$

Motor:

Since we are given the output power and the efficiency we can calculate the input power:

$$W_M = \frac{P_{Mout}}{\eta} = \frac{100}{0.9} = \mathbf{111kW}$$

$$VA_M = \frac{W_M}{pf} = \frac{111}{0.85} = \mathbf{130.7kVA}$$

$$VAr_M = VA_M \times \sin\phi = 111 \times \sqrt{1 - 0.85^2} = \mathbf{68.9kVAr}$$

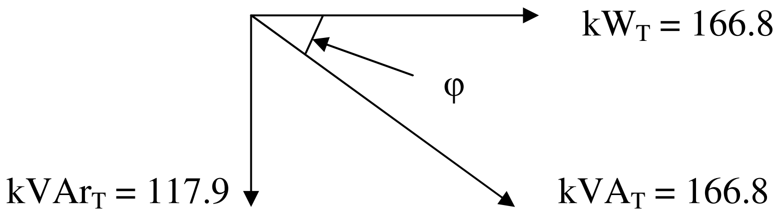
$$I_M = \frac{VA_M}{V} = \frac{130.7}{11}\angle -\cos^{-1}(0.85) = \mathbf{11.88\angle -31.8° \, A_{rms}}$$

(b) Note that we cannot simply add up the component Vas to obtain the total as the quantities are vectors. Add up the real and imaginary components first.
Total real power:

$$W_T = W_T + W_{GL} + W_M = 19.8 + 36 + 111 = 166.8\text{kW}$$

Total imaginary power:

$$VAr_T = VAr_T + VAr_{GL} + VAr_M = 97 - 48 + 68.9 = 117.9\text{kVAr}$$



$$VA_T = \sqrt{W_T^2 + VAr_T^2} = \sqrt{166.8^2 + 117.9^2} = \textbf{204.3 kVA}$$

From the diagram the phase angle can be calculated as:

$$\phi = \tan^{-1}\left(\frac{117.9}{166.8}\right) = 35.2^\circ \quad lagging$$

and the power factor is then:

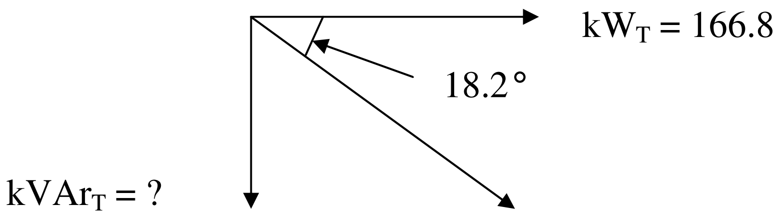
$$pf = \cos\phi = \cos 35.2^\circ = \textbf{0.82 lagging}$$

and the total load current is:

$$I_T = \frac{VA_T}{V}\angle -\cos^{-1}(0.82) = \mathbf{18.6\angle -35.2^\circ\ A_{rms}}$$

(c) The capacitor which is added will not affect the overall real power but will provide negative VAr to offset the net positive VAr from the three loads. The phase angle is:

$$\phi = \cos^{-1} 0.95 = 18.2^\circ$$



From the diagram it can be seen that the new overall VAr is:

$$kVAr_T = 166.8 \times \tan 18.2^\circ = 54.84kVAr$$

and hence the capacitor must provide 117.9 – 54.84 = 63kVAr. Since:

$$VAr_C = \frac{V^2}{X_C} \quad \text{and} \quad X_C = \frac{1}{2\pi fC}$$

then:

$$C = \frac{VAr_C}{V^2} \times \frac{1}{2\pi f} = \frac{63000}{11000^2} \times \frac{1}{2\pi \times 50} = \mathbf{1.66\mu F}$$

5 (a) From the voltage graph:

$$gradient = \frac{V_{OFF}}{t_S}$$

hence:

$$v(t) = \frac{V_{OFF}}{t_S}t + C$$

apply known conditions to determine the constant C. At $t = 0$ $v = 0$ so $C = 0$ then:

$$v(t) = \frac{V_{OFF}}{t_S}t$$

(b) From the current graph:

$$gradient = -\frac{I_{ON}}{t_S}$$

hence:

$$i(t) = -\frac{I_{ON}}{t_S}t + C1$$

apply known conditions to determine the constant $C1$. At $t = t_S$ $I = 0$ so $C1 = I_{on}$ then:

$$i(t) = -\frac{I_{ON}}{t_S}t + I_{ON} = I_{ON}\left(1 - \frac{t}{t_S}\right)$$

(b) The instantaneous power is given by:

$$p = v(t)i(t) = V_{OFF}\frac{t}{t_S} \times I_{ON}\left(1 - \frac{t}{t_S}\right) = V_{OFF}I_{ON}\left(\frac{t}{t_S} - \frac{t^2}{t_S^2}\right)$$

To find the maximum power set $\frac{dp}{dt} = 0$:

$$\frac{dp}{dt} = V_{OFF}I_{ON}\frac{d}{dt}\left(\frac{t}{t_S} - \frac{t^2}{t_S^2}\right) = V_{OFF}I_{ON}\left(\frac{1}{t_S} - \frac{2t}{t_S^2}\right) = 0$$

giving the condition:

$$t = \frac{t_S}{2}$$

substituting back gives:

$$p_{\max} = V_{OFF}I_{ON}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{4}V_{OFF}I_{ON}$$

hence for the given voltages and currents:

$$p_{\max} = \frac{1}{4} \times 300 \times 10 = \mathbf{750W}$$

(c) Temperature rises by 5℃ per Watt, therefore for 750W the temperature rise will be:

$$\Delta T = 750 \times 5 = \mathbf{3750℃}$$

(d) Energy loss only occurs during the switching events as at other times either v or I are zero. Since there are two switching events per cycle the energy dissipated is:

$$E = 2\int_0^{t_S} p\,dt = 2\int_0^{t_S}\left(V_{OFF}I_{ON}\left(\frac{t}{t_S} - \frac{t^2}{t_S^2}\right)\right)dt = \left[2V_{OFF}I_{ON}\left(\frac{t^2}{2t_S} - \frac{t^3}{3t_S^2}\right)\right]_0^{t_S} = V_{OFF}I_{ON}\frac{t_S}{3}$$

The average power is the energy dissipated over a complete cycle:

$$p_{av} = \frac{E}{T} = \frac{1}{T} V_{OFF} I_{ON} \frac{t_S}{3}$$

(e) Substituting values in the above equation gives:

$$p_{av} \frac{1}{2\times 10^{-4}} \times 300 \times 10 \times \frac{2\times 10^{-6}}{3} = \mathbf{10W}$$

And the temperature rise is:

$$\Delta T = 10 \times 5 = \mathbf{50^{\circ}C}$$