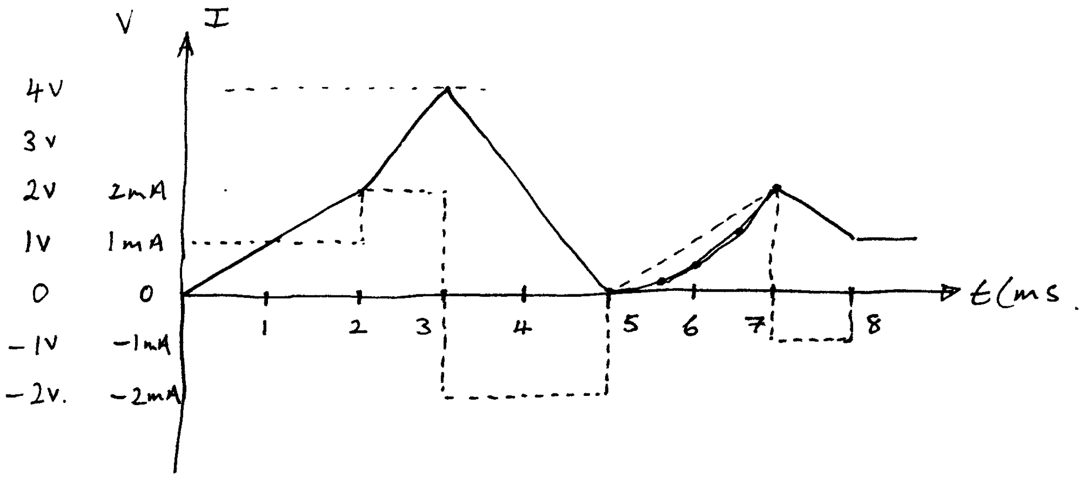
Q1 (i)



In this question, the evolution of charge in the capacitor is the area exposed under the current time graph as t increases. When I is constant, area increases at a constant rate with respect to time. ∴ Charge in and voltage across the capacitor are related by $Q = CV$

or $\Delta Q = C \Delta V$.

The only slightly tricky bit is between $t = 5ms$ + $t = 7ms$. Here the area does not change linearly with time; instead it follows a quadratic relationship. The waveform can be sketched by working out the area between 5ms + 5·5ms (= 0.125μC), between 5ms + 6ms (0.5μC), between 5ms + 6·5ms (= 1.125μC) and between 5ms and 7ms (= 2μC). Alternatively, if you are good at maths, you can say that between 5ms + 7ms $I(t) = t$ (taking 5ms as the time origin).

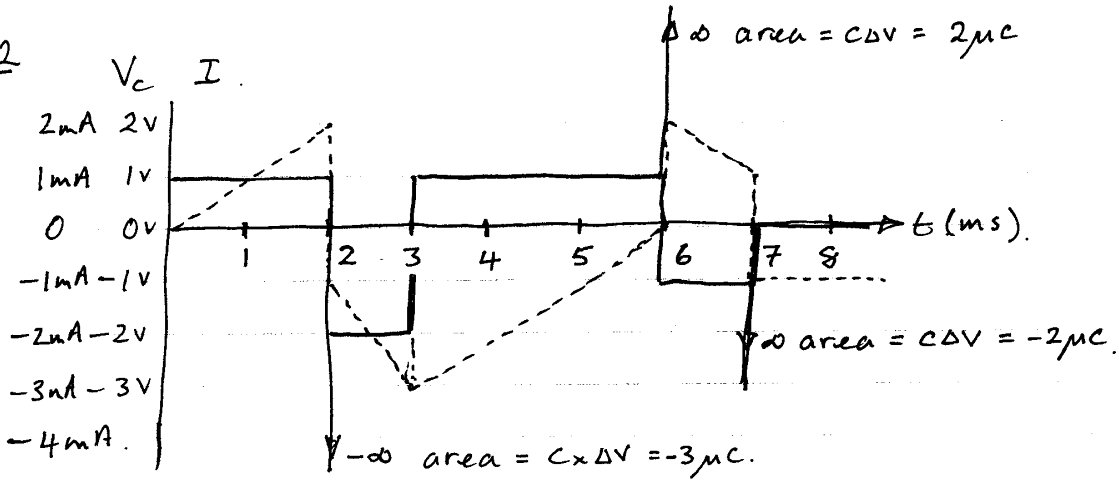
$$\text{so } V_c\Big|_{5ms-7ms} = \frac{1}{C}\int_0^{2ms} t\,dt + V_{initial}$$

$$= \frac{1}{C}\left[\frac{t^2}{2}\right]_0^{2ms} + V_{initial}.$$

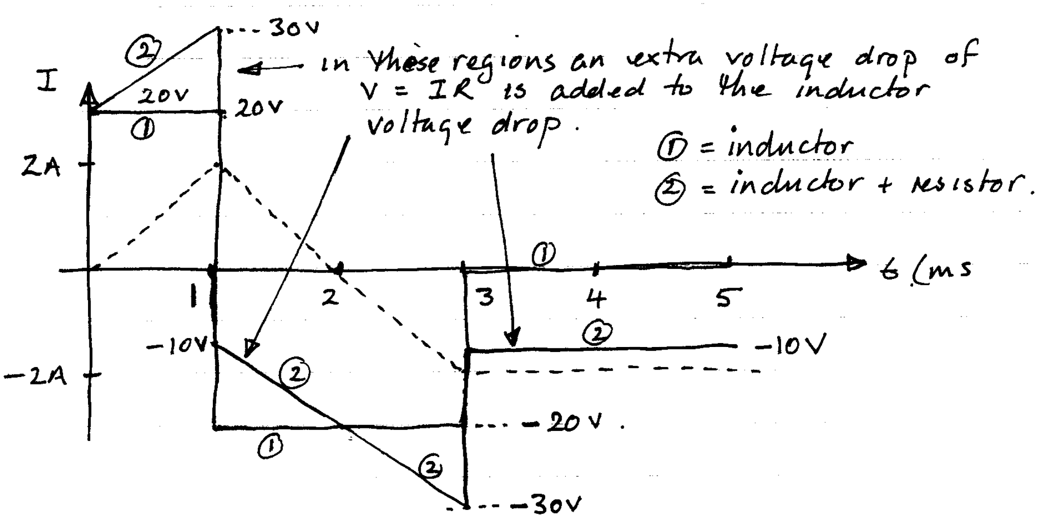
(ii) The total +ve area = 6μC ; total −ve area = 5μC.

net area @ 8ms = 6μC − 5μC = $\underline{\underline{1μC}}$.

Q2



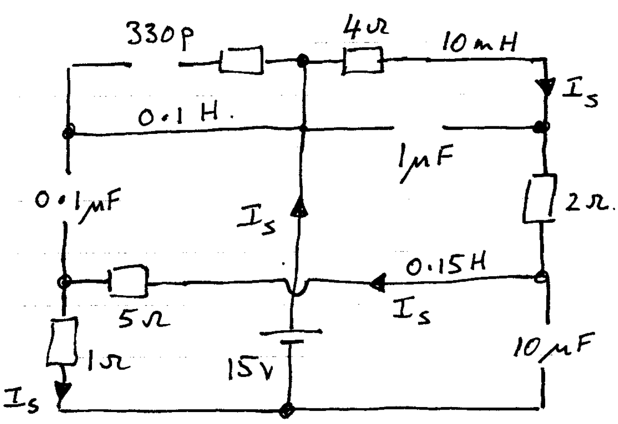
Q3



Q4 The path of I_S is indicated on the diagram. The total resistance of that path is

$4 + 2 + 5 + 1 = 12\,\Omega$

$\therefore I_S = \dfrac{15\text{V}}{12\,\Omega} = \underline{1.25\text{A}}$



To find the stored energies, the voltage across each C must be worked out and the current through each L must be worked out .....

Consider the inductors ....

10mH and 0.15H carry I_s so their stored energies are ....

$$E_{10mH} = \frac{1}{2} \cdot 01 \times 1.25^2 = \underline{\underline{7.8\,mJ}}.$$

$$E_{0.15H} = \frac{1}{2}\, 0.15 \times 1.25^2 = \underline{\underline{117\,mJ}}.$$

The 0.1H inductor carries a current of zero so ...

$$E_{0.10H} = \underline{\underline{0\,J}}.$$

Consider the capacitors ....

The 330pF capacitor has zero volts across it

$$E_{330pF} = \underline{\underline{0\,J}}$$

The $1\mu F$ capacitor has a voltage drop across it of $I_s \times 4\Omega$, ie 5V

$$E_{1\mu F} = \frac{1}{2}\, 10^{-6}.\, 5^2 = \underline{\underline{12.5\,\mu J}}$$

The $0.1\mu F$ capacitor has 15V on its top terminal and $I_s \times 1\Omega$ on its bottom terminal giving a voltage difference of $15 - 1.25 = 13.75V$.

$$E_{0.1\mu F} = \frac{1}{2}\, 0.1\mu F \times 13.75^2 = \underline{\underline{9.45\,\mu J}}.$$

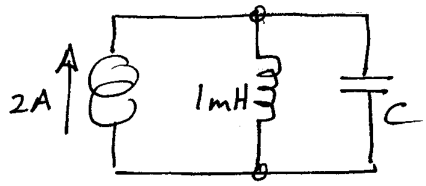
The $10\mu F$ capacitor has across it the voltage generated by I_s flowing through the $5\Omega + 1\Omega$ resistors

$$E_{10\mu F} = \frac{1}{2}\, 10^{-5}\, 7.5^2 = \underline{\underline{281\,\mu J}}.$$

Note that in this circuit the inductors are storing much more energy than the capacitors. If the resistors were all changed to $k\Omega$ instead of Ω, the capacitor energy storage would remain the same but the inductor stored energies would be a factor of 10^6 times smaller than they are here – ie nJ instead of mJ. Inductors make very effective energy stores in low voltage high(ish) current applications.

Q5

(i) Since the 2A is steady and $V_L = L\frac{dI}{dt}$, $\frac{dI}{dt} = 0$ and $\underline{V_L = 0}$



(ii) Energy stored in the inductor is $E_{1mH} = \frac{1}{2} \cdot 10^{-3} \cdot 2^2$

$= \underline{2mJ}$

(iii) There is no voltage across the capacitor so $\underline{E_C = 0\ J}$.

(iv) $E_C = \frac{1}{2} C V^2 = 2mJ$

so for 1nF $V^2 = \frac{2 \times 2mJ}{1nF} = 4 \times 10^6$

$\therefore V_{cmax} = \sqrt{4 \times 10^6} = \underline{2kV}$

for 10nF $V^2 = \frac{2 \times 2mJ}{10nF} = 400 \times 10^3$

$V_{cmax} = \sqrt{400 \times 10^3} = \underline{632\ V}$

for 100nF $V^2 = \frac{2 \times 2mJ}{100nF} = 40 \times 10^3$

$V_{cmax} = \sqrt{40 \times 10^3} = \underline{200\ V}$