EEE 345 : Engineering Electromagnetics

year : 2015

lecturer : Dr. T. Walther

office : Mappin Building, 1st floor, E150d

lecture times : Thursdays, 15-16, LT #10
Fridays, 12-13, LT #6

available for questions on an individual basis :
Mondays, 10-12, office

Lecture dates :

February : 12, 13, ~~19~~, 20, 26, 27
March : 5, 6, 12, 13, 19, ~~20~~
April : 16, 17, 23, 24, 30
May : 1, 7, 8, 14, 15, 21, ~~22~~

i.e. 20 lectures + 1 hour exam prep.

exam : 2hrs, answer 3 out of 4 questions

content : Maxwell's equations and applications to

1. electrostatics → solid state electronic devices
2. magnetostatics → electric cables, eddy currents
3. propagating waves & standing waves

↓ planar waves, spherical waves, dipole fields and antennas, reflection, transmission & diffraction i.e. optics, wave attenuation (dispersion)

↓ transmission lines, coaxial cables

Field equations for materials:

flux		field
$\underline{D}$	$= \varepsilon_0 \varepsilon_r$	$\underline{E}$
$\underline{B}$	$= \mu_0 \mu_r$	$\underline{H}$
$\underline{j}$	$= \sigma$	$\underline{E}$ (Ohm's Law)

units: $[D] = C/m^2$ $\quad [E] = V/m$

$[B] = T$ (or Wb/m^2) $\quad [H] = A/m$

$[j] = A/m^2$

$\varepsilon_0 = 8.8542 \cdot 10^{-12}$ F/m permittivity

$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am permeability

$$c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

Helmholtz's theorem:

A vector field is completely specified by its div and rot components (to be shown later).

$\downarrow$

Maxwell's equations (in differential form, 1873, after O. Heaviside):

$\mathrm{div}\, \underline{D} = \rho_{free}$ (Coulomb's Law)	$\mathrm{div}\, \underline{B} = 0$ (no magnetic monopoles)
$\mathrm{rot}\, \underline{E} = -\frac{\partial \underline{B}}{\partial t}$ (Faraday's induction Law)	$\mathrm{rot}\, \underline{H} = \underline{j} + \frac{\partial \underline{D}}{\partial t}$ (Ampere's Law plus charge conservation)

1. electrostatics: $\underline{B} = \text{const.}$

$$\Downarrow$$

$$\frac{\partial B}{\partial t} = 0$$

$$\Downarrow$$

$$\operatorname{div} \underline{E} = \frac{\rho}{\varepsilon_0} \quad , \quad \operatorname{rot} \underline{E} = 0$$

→ oscilloscopes, inkjet printers, photocopiers, LCDs, solid state devices (pn-diodes, transistors)

2. magnetostatics: $\underline{D} = \text{const}$

$$\Downarrow$$

$$\frac{\partial D}{\partial t} = 0$$

$$\Downarrow$$

$$\operatorname{div} \underline{H} = 0 \quad , \quad \operatorname{rot} \underline{H} = \underline{j}$$

→ electric cables / transmission lines, eddy currents

3. electromagnetics: coupling of $\underline{E}$ and $\underline{H}$ fields

$$\Downarrow$$

electromagnetic waves

→ coaxial cables, propagation of waves, reflection, transmission, diffraction, attenuation, light optics and lenses, dipole fields, antennas

for understanding: need vector algebra + vector calculus

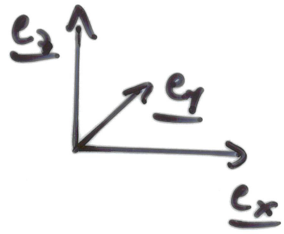
vector algebra

use Cartesian coordinates

let $\underline{A} = A_x \underline{e_x} + A_y \underline{e_y} + A_z \underline{e_z} = (A_x, A_y, A_z)$

$\underline{B} = B_x \underline{e_x} + B_y \underline{e_y} + B_z \underline{e_z} = (B_x, B_y, B_z)$

be vectors with unity vectors $\underline{e_x}$, $\underline{e_y}$ and $\underline{e_z}$ defining an orthonormal basis like so:



dot (scalar) product:

$$\underline{A} \cdot \underline{B} = A_x B_x + A_y B_y + A_z B_z = |\underline{A}| \cdot |\underline{B}| \cdot \cos \sphericalangle(\underline{A}, \underline{B})$$

cross (vector) product:

$$\underline{A} \times \underline{B} = (A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x)$$

$$= \begin{vmatrix} \underline{e_x} & \underline{e_y} & \underline{e_z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \sum_{i,j,k} \varepsilon_{ijk}\, \underline{e_i}\, \underline{A_j}\, \underline{B_k}$$

where the Levi-Civita-symbol is defined by

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{, if } i,j,k \text{ is an even permutation of } 1,2,3 \\ -1 & \text{, } \quad \text{odd} \\ 0 & \text{; otherwise} \end{cases}$$

note: $|\underline{A} \times \underline{B}| = |\underline{A}| \cdot |\underline{B}| \cdot \sin \sphericalangle(\underline{A}, B)$

$$\det(\underline{A}) = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} = \sum_{i,j,k} \varepsilon_{ijk} A_{1i} A_{2j} A_{3k}$$

properties:

$$\underline{A} \times (\underline{B} \times \underline{C}) = (\underline{A} \cdot \underline{C}) \cdot \underline{B} - (\underline{A} \cdot \underline{B}) \cdot \underline{C}$$

$$\underline{A} \cdot (\underline{B} \times \underline{C}) = \underline{B} \cdot (\underline{C} \times \underline{A}) = \sum_{i,j,k} \varepsilon_{ijk} A_i B_j C_k$$

$$(\underline{A} \times \underline{B}) \cdot (\underline{C} \times \underline{D}) = (\underline{A} \cdot \underline{C}) \cdot (\underline{B} \cdot \underline{D}) - (\underline{B} \cdot \underline{C}) \cdot (\underline{A} \cdot \underline{D})$$

vector calculus

uses operators

gradient:

$$\text{grad} = \underline{\nabla} = \frac{\partial}{\partial x}\underline{e_x} + \frac{\partial}{\partial y}\underline{e_y} + \frac{\partial}{\partial z}\underline{e_z} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

is a vector pointing along the direction of max. change

note: a vector field that results from the gradient operator is called conservative because $\oint \underline{\nabla} A \, d\underline{r} = 0$

divergence:

$$\text{div } \underline{A} = \underline{\nabla} \cdot \underline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \lim_{V \to 0} \frac{1}{V} \iint_S \underline{A} \, d\underline{S}$$

is a scalar that gives the net amount of flow into / out of a surface S

(=0, if no sources or sinks are present)

rotation:

$$\text{rot } \underline{A} = \underline{\nabla} \times \underline{A} = \begin{vmatrix} \underline{e_x} & \underline{e_y} & \underline{e_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

is a vector with i-th component is $\varepsilon_{ijk} \frac{\partial}{\partial x_j} A_k$

Gauß's theorem:

$$\iiint_V \text{div } \underline{A} \, dV = \oiint \underline{A} \, d\underline{s}$$

sources enclosed in volume

flux through closed surface

Stoke's theorem:

$$\iint_S \text{rot } \underline{A} \, d\underline{s} = \oint \underline{A} \, d\underline{r}$$

border line of surface S

some important properties of grad, div, rot

$$\operatorname{div} \operatorname{grad} A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} = \nabla^2$$

Laplace-operator (scalar !)

$$\begin{aligned}\operatorname{div} \operatorname{rot} \underline{A} = & \frac{\partial}{\partial x}\left(\cancel{\frac{\partial A_z}{\partial y}} - \cancel{\frac{\partial A_y}{\partial z}}\right) \\ & + \frac{\partial}{\partial y}\left(\cancel{\frac{\partial A_x}{\partial z}} - \cancel{\frac{\partial A_z}{\partial x}}\right) \\ & + \frac{\partial}{\partial z}\left(\cancel{\frac{\partial A_y}{\partial x}} - \cancel{\frac{\partial A_x}{\partial y}}\right) = 0\end{aligned}$$

$$\operatorname{rot} \operatorname{grad} \underline{A} = \begin{vmatrix} \underline{e_x} & \underline{e_y} & \underline{e_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial A}{\partial x} & \frac{\partial A}{\partial y} & \frac{\partial A}{\partial z} \end{vmatrix} = (0,0,0) = \underline{0}$$

$$\begin{aligned} & \underline{e_x}\left(\frac{\partial}{\partial y}\frac{\partial A}{\partial z} - \frac{\partial}{\partial z}\frac{\partial A}{\partial y}\right) = 0 \\ + & \underline{e_y}\left(\frac{\partial}{\partial z}\frac{\partial A}{\partial y} - \frac{\partial}{\partial y}\frac{\partial A}{\partial z}\right) = 0 \\ + & \underline{e_z}\left(\frac{\partial}{\partial x}\frac{\partial A}{\partial y} - \frac{\partial}{\partial y}\frac{\partial A}{\partial x}\right) = 0 \end{aligned}$$

$$\operatorname{rot} \operatorname{rot} \underline{A} = \begin{vmatrix} \underline{e_x} & \underline{e_y} & \underline{e_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} & \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} & \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{vmatrix}$$

$$= \begin{pmatrix} \frac{\partial^2 A_y}{\partial y \partial x} - \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_z}{\partial y \partial z} - \frac{\partial^2 A_x}{\partial z^2} - \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial x^2} \\ -\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_x}{\partial x \partial y} + \frac{\partial^2 A_z}{\partial z \partial y} - \frac{\partial^2 A_y}{\partial z^2} - \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial y^2} \\ \frac{\partial^2 A_x}{\partial x \partial z} - \frac{\partial^2 A_z}{\partial x^2} - \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_y}{\partial z \partial y} - \frac{\partial^2 A_z}{\partial z^2} + \frac{\partial^2 A_z}{\partial z^2} \end{pmatrix}$$

$$= \operatorname{grad} \operatorname{div} - \nabla^2$$

(again, a vector !)