

Magnetic field and Magnetic circuits: - *Background*

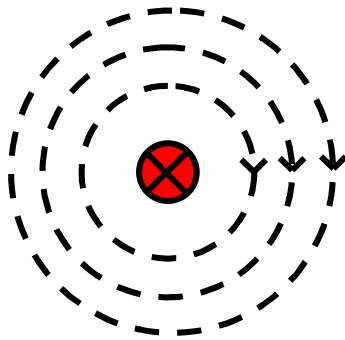
A 'field' describes a state of space in which there is a force on a body. i.e. the field can only be detected by it's effect.

e.g.

- Gravitational field – Force of attraction between masses
- Electric Field – Force on a charged particle
- Magnetic Field – Force on a moving charge. e.g. a current carrying conductor etc.

Pre 1820 – Electricity and Magnetism treated more or less independently

1820 – Oersted and Ampère discovered that the magnetic field is always associated with an electric current, and that a conductor carrying an electric current had a force acting on it when it was in a magnetic field.



Magnetic field of a straight conductor – represented by concentric circles for equipotentials

1831 – Faraday deduced and proved experimentally that when a magnetic field linking an electric circuit changed, there is a transient induced emf (and hence current flow).

Thus the electric and magnetic effects are always inter-related

Whilst all electric and magnetic phenomena of interest in electrical engineering can be explained in terms of the forces between charges (either stationary – electric field, or moving – magnetic field), we find it much easier to solve many problems by the use of electric circuits and magnetic circuits rather than the field.

The existence of the magnetic field associated with the electric circuit may or may not be what the electric circuit designer wants. Sometimes it is essential and useful, other times it is a pain in the circuit!

Examples of systems which utilise the magnetic field

- Machines – motors, generators, actuators, loudspeakers, instruments
- Transformers – power, measuring, matching
- Communication systems – H-field antenna, ferrite rod
- Inductor – Filters, tuning etc
- Electron beam devices
- μ wave devices – microwave ovens etc.

Examples of systems where the effects of the magnetic field are minimised

- Screening to eliminate pickup or crosstalk
- Special windings (bifilar) to minimise inductance
- Careful PCB layout to minimise ‘stray’ inductance
- Careful design and ‘clamping’ of wires to reduce effects of unwanted forces associated with current flow – high current levels.

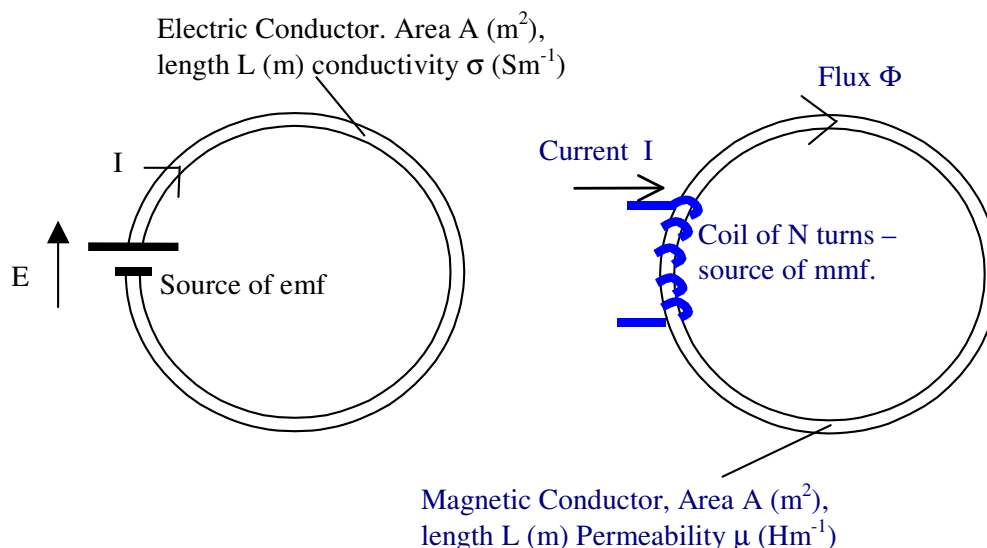
Sources of magnetic fields

All sources may be related to an equivalent current flow or charge movement

- Currents in circuits – magnetic field surrounds the conductors
- Permanent Magnets – Fields associated with electron motion in atomic structures
- High frequency ‘displacement’ currents in radiated fields (antennas)

Magnetic Circuits

In many practical devices the magnetic field is confined to a well-defined circuit that can be thought of as analogous to the more familiar electric circuit. **This analogy must be used with care!**



Electric Circuit

For a circuit made up of a number of elements we have:

$$I = \frac{E}{\Sigma R} = \frac{\text{emf}}{\text{cct resistance}} \frac{(\text{V})}{(\Omega)} \text{ A}$$

where for each element: $R = \frac{L}{\sigma A} \Omega$

Magnetic Circuit

By analogy, the magnetic circuit quantities are related by:

$$\Phi = \frac{F}{\Sigma S} = \frac{\text{mmf}}{\text{cct reluctance}} \quad \text{Wb}$$

where Φ = flux in Webers (Wb)

$$S = \frac{L}{\mu A} \quad \text{H}^{-1} \quad (\text{Henry}^{-1})$$

F = magneto-motive force (mmf) = NI Amp (turns)

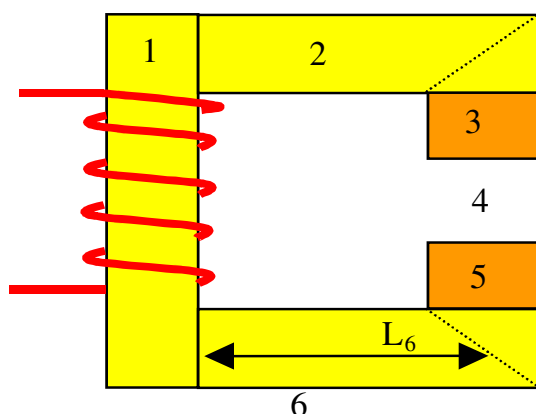
μ = permeability (Hm^{-1})

Provided the magnetic field is confined to a path of known area A , length L , and permeability μ , the above method is very useful. – Note that whilst the current, I , can be associated with the ‘flow of charge’, the concept of flux Φ flowing around a circuit is only a useful concept. In practice, there is no physical flow involved. We can however say that the flux Φ carries the effect (the field) of the mmf. Because air is not a ‘magnetic insulator’, unlike electric circuits where the current can reasonably be assumed to be confined to the wires, in the magnetic circuits of some devices, significant amounts of flux ‘leaks’ or spreads into unwanted sections of the device.

Calculation of a ‘lumped’ circuit reluctance model for linear materials

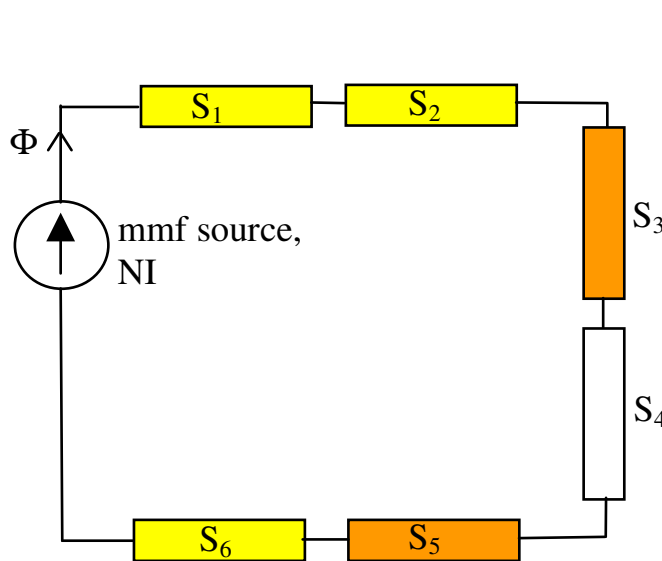
For circuits where each section can be defined in terms of sections of known length, L , area, A , and permeability, μ , then a ‘lumped’ reluctance model can be constructed.

Example 1 Series magnetic circuit:



Each section having a length, L_n , area, A_n , and permeability, μ_n .

For this circuit we introduce the concept of an ‘effective’ or ‘mean’ path length, L_1 , L_2 , etc, and areas A_1 , A_2 , etc. Clearly for section 1, there are longer and shorter paths through the section, and the area is not constant on the corners. However, the errors involved are very small in practice for most situations. Hence we can construct our model.



Where $S_1 = \frac{L_1}{\mu_1 A_1}$ etc

and flux

$$\Phi = \frac{NI}{S_1 + S_2 + S_3 + S_4 + S_5 + S_6}$$

$$= \frac{NI}{S_T}$$

or $\Phi = \frac{NI}{\Sigma S}$

- note that in the series circuit, the flux is continuous, and the same flux passes through all sections.

Example 2 Parallel magnetic circuit:

By the same analogy to electric circuits, we may analyse parallel magnetic circuits.

$$\frac{I}{S_T} = \frac{I}{S_1} + \frac{I}{S_2} + \frac{I}{S_3} \dots$$

Magnetisation Curves and Magnetic Materials

For a material of length L, area A, and permeability μ , from the previous equations we have:

Flux = mmf / reluctance

$$\Phi = \frac{NI \mu A}{L}$$

This can be arranged into a different form to give:

$$\left(\frac{\Phi}{A} \right) = \mu \left(\frac{NI}{L} \right)$$

or

$$B = \mu H$$

There is a unique relationship for each material known as it's B/H curve, or magnetisation characteristic, independent on circuit dimensions.

where: $\mathbf{B} = \Phi / \mathbf{A}$ is the **Flux Density** in **Tesla (T)** (Wbm^{-2})

and $\mathbf{H} = \mathbf{NI} / \mathbf{L}$ is the **Magnetising Force** or
Magnetic field Strength (Am^{-1})

Note: H is the cause, B is the effect

Permeability of Materials

Air (free space) – the permeability is defined as

$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

μ_0 is the primary magnetic constant, and has been assigned the above value for use in the S I unit system.

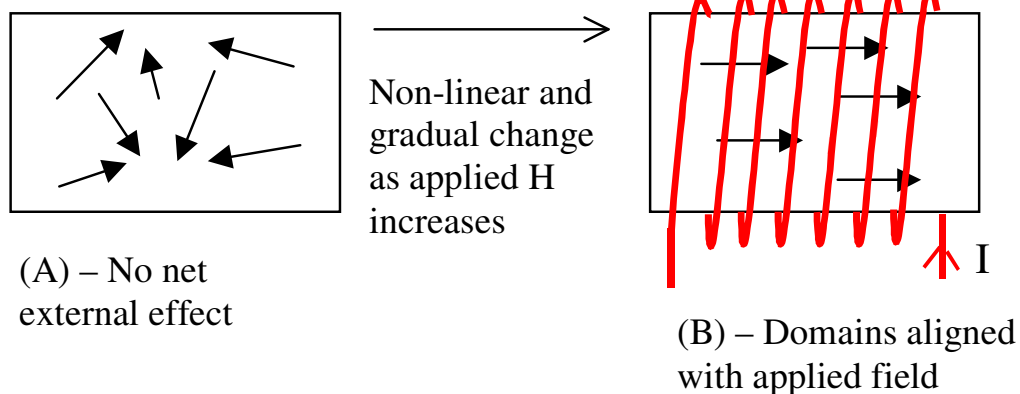
‘Non magnetic’ Materials

On a molecular or atomic scale, materials have electron motion, which is equivalent to an atomic current. These atomic currents can be associated with both electron orbital motion about the nucleus, and the electrons spinning about their own axes. Both can give rise to magnetic fields and both can be affected by an external magnetic field. However, for most substances, these effects are either zero or so small that for most purposes such materials are classed as ‘non magnetic’ [Diamagnetic if $\mu < \mu_0$, Paramagnetic if $\mu > \mu_0$]

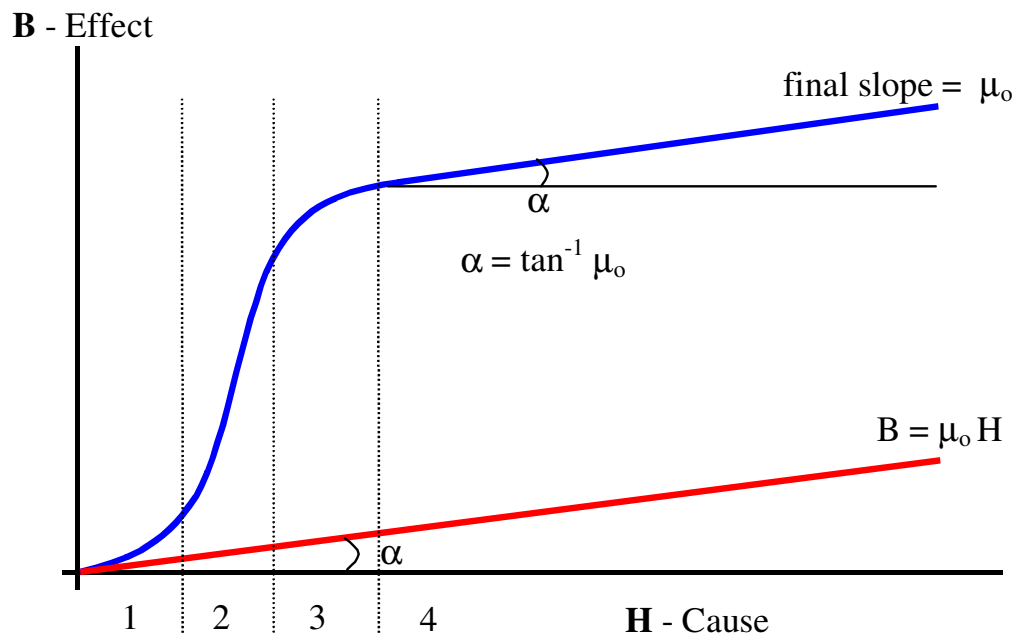
For these materials $\mathbf{B} = \mu_0 \mathbf{H}$

Magnetic Materials - Ferromagnetism

In a few elements, IRON, NICKEL, COBALT, and some of their alloys, the atoms are arranged such that their electron currents supplement each other and give a net magnetic moment. Within a specimen of such materials, the structure is usually divided up into **magnetic domains** of microscopic size within which the atomic magnets are aligned. However, the specimen as a whole may be unmagnetised due to the random or ‘regular’ orientation of the domains, but which can be magnetised (domains aligned) by an external applied field.



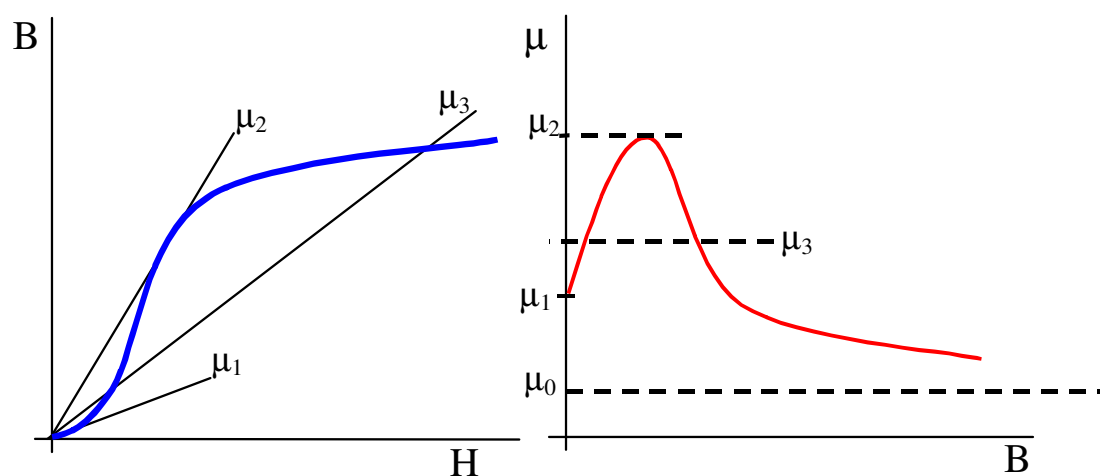
The process from A to B is non-linear and leads to a non-linear relationship known as the B/H curve. The effect of the domain alignment is that the B value obtained for a given H is much greater than that for air or a non-magnetic material.



Magnetisation curve or B/H curve

1. Initial growth of closely aligned domains
2. Rapid reversal of domains
3. final rotation of domains
4. No further contribution from domains - saturation

The alignment process is dependant on the strength of the applied magnetic field, H . Thus the B/H relationship is a curve rather than linear. For these materials, μ varies with the applied field and hence the flux density in the circuit.



By careful design a magnetic circuit can be operated with a high value of μ . (Other definitions of μ based on incremental slope, dB/dH , are also used for small signal models).

Saturation

A material is said to be saturated when complete alignment of the magnetic domains is achieved. This point coincides with the slope of the B/H curve having a value of μ_0 .
NOTE: beyond this point B continues to increase but at a rate determined by:

$$B = \mu_0 H$$

as with non-magnetic materials.

Relative permeability

The general relationship $B = \mu \cdot H$ is often re-written as:

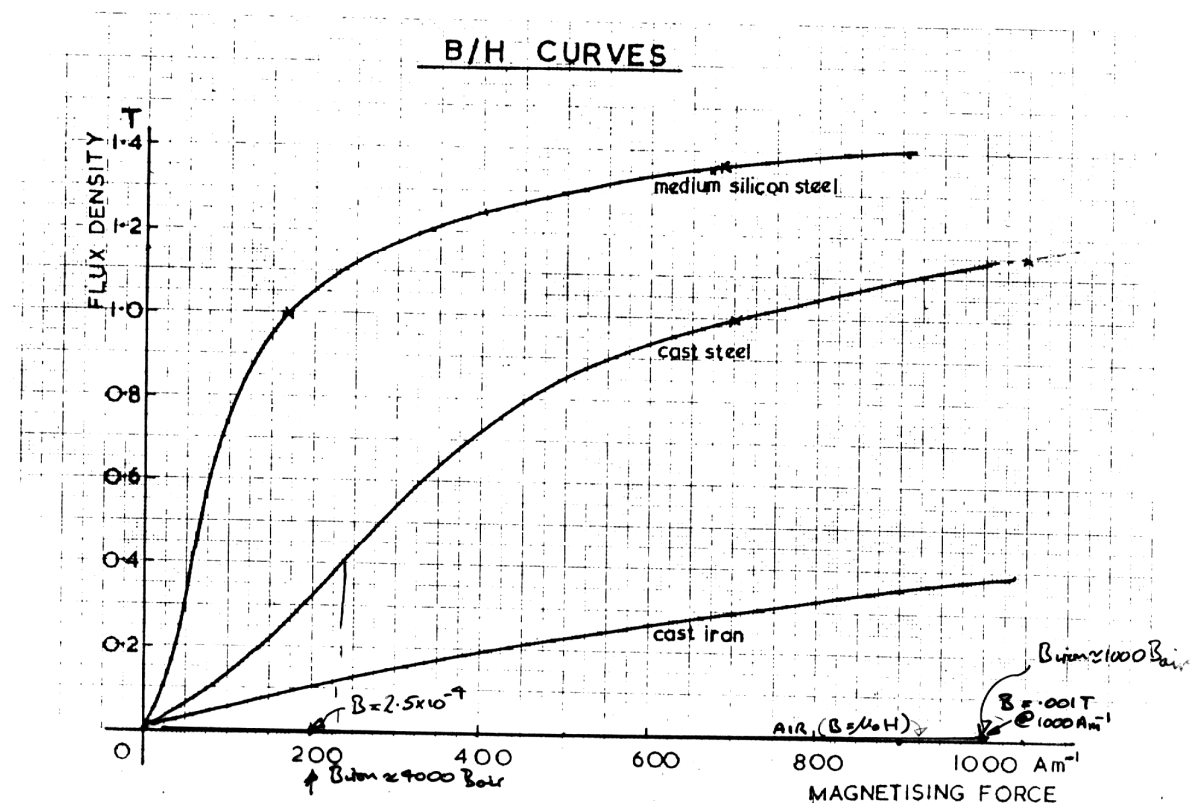
$$B = \mu_r \mu_0 H$$

where:

$$\mu_r = \mu / \mu_0 \text{ (dimensionless)}$$

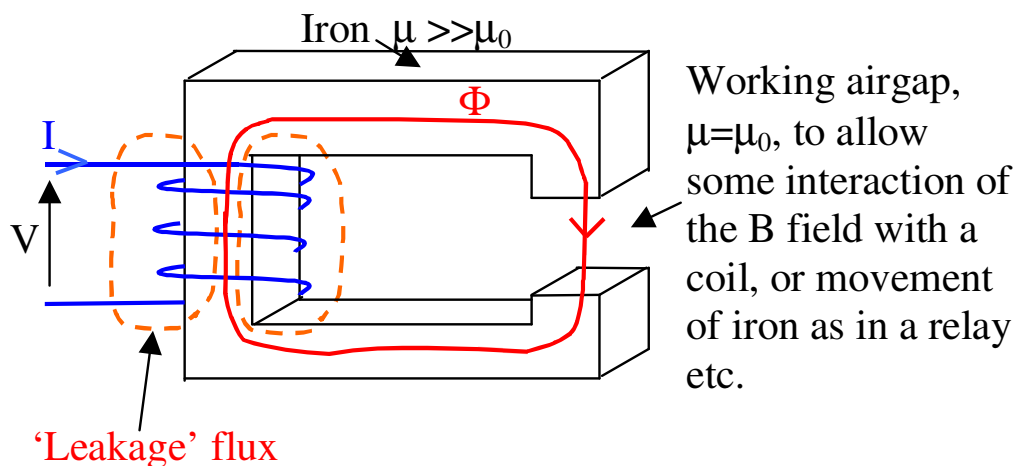
and is effectively a measure of the 'magnification' of the B field which can be achieved if the magnetic material is used in a circuit. For typical materials, μ_r varies over the range 50 to 2000, although special materials can be produced with a μ_r of over 500,000! (suitable only for limited application).

[For curve given, at 1000A/m, for air $B=0.001T$!].



Circuit Analysis

This approach is particularly useful if the magnetic field is confined to paths of mainly simple geometry. The designer who wants to increase the magnetic flux for a given **mmf** is clearly going to use the multiplication available by incorporating iron or other ferromagnetic material in the circuit. In many cases the iron essentially forms the whole of the circuit, e.g. transformer and inductors, whereas in others, the iron acts to guide the flux into a 'working airgap' in the device. The latter is almost invariably the case in electromechanical devices (motors, actuators etc).



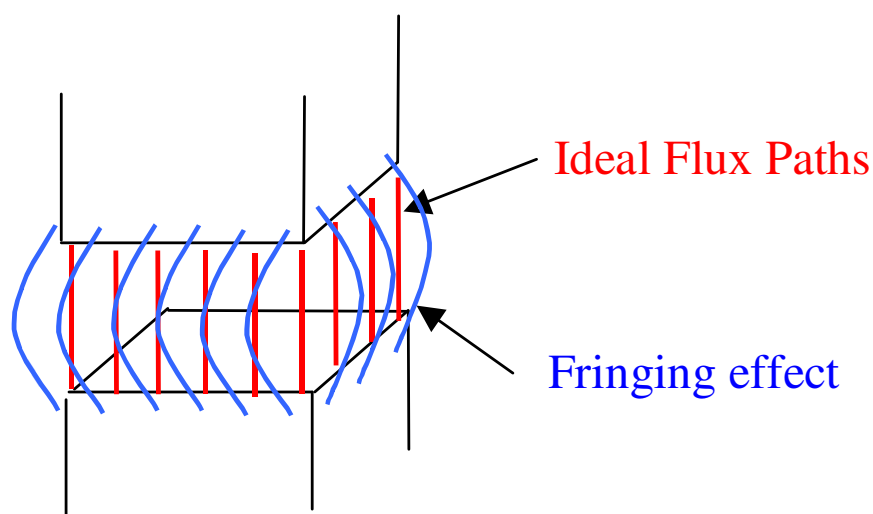
Fringing and Leakage fields

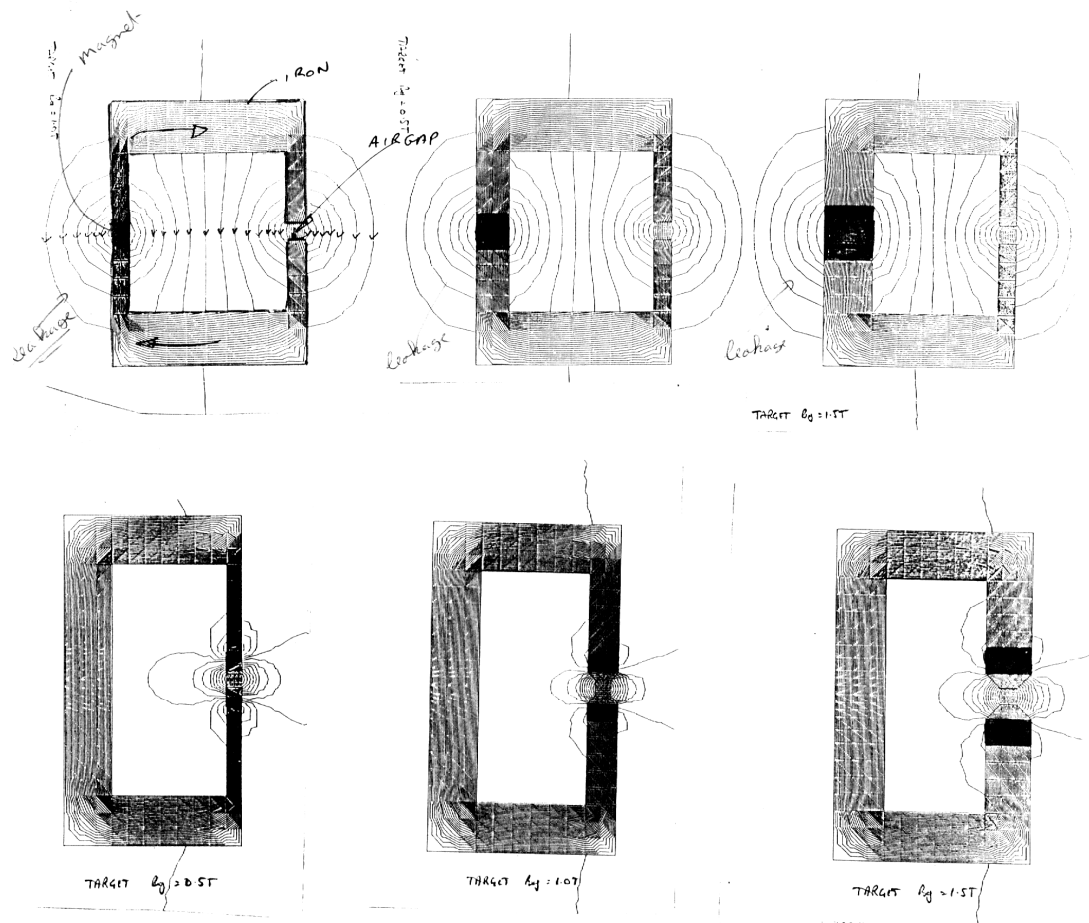
Whereas the electric current in an electric circuit can be reasonably confined to the conducting wires, the analogous effect of flux being confined to the iron is not such a reasonable assumption. This is due to the fact that:

$$\frac{\sigma_{\text{air}}}{\sigma_{\text{copper}}} \ll \frac{\mu_{\text{air}}}{\mu_{\text{iron}}}$$

i.e. air is not a good 'magnetic insulator'

Hence, in the circuit above, the flux would not only be produced in the working airgap, but also in the other paths shown. The flux will also tend to spread out in the working airgap – known as 'fringing effects'.





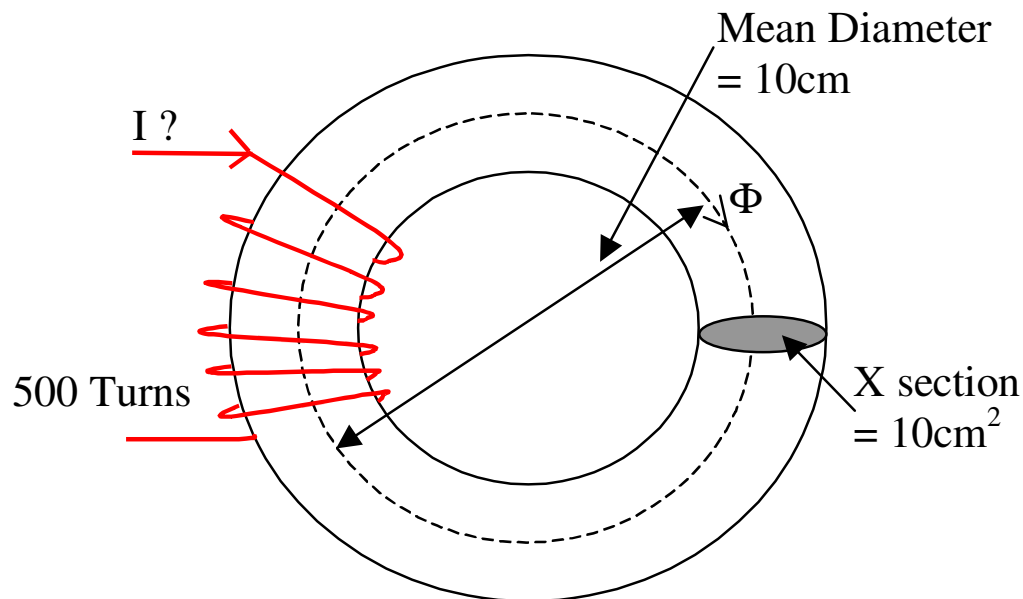
Solution of Typical Problems:

(In some cases the examples given are non-practical but demonstrate the principles)
Generally, this method of solving problems gives a useful 'first order' calculation

Example1

A mild steel ring has a cross-sectional area of 10cm^2 and a mean diameter of 10cm . A coil of 500 turns is wound on the ring. Calculate the current required to give a total flux of 1.25mWb in the ring.

- If the ring is of a specified relative permeability of 1560.
- (More realistically), if a B/H curve of the material is provided.



a) μ_r specified as 1560

$$\Phi = 1.25\text{mWb} = 1.25 \times 10^{-3} \text{ Wb},$$

$$\text{Area } A = 10\text{cm}^2 = 10 \times 10^{-4} \text{ m}^2$$

GET UNITS INTO S.I. UNITS

$$\text{Now, } B = \Phi / A = 1.25 \times 10^{-3} / 10^{-3} = 1.25 \text{ T}$$

In this problem the cross-section is uniform, and since Φ is continuous, then B is the same throughout. Also as the circuit only contains 1 material, then the relative permeability is the same throughout, therefore we can say:

$$B = \mu_r \mu_0 \cdot H$$

$$\text{where } H = 1.25 / (4\pi \times 10^{-7} \times 1560) = 6.4 \times 10^2 \text{ Am}^{-1}$$

and total mmf required:

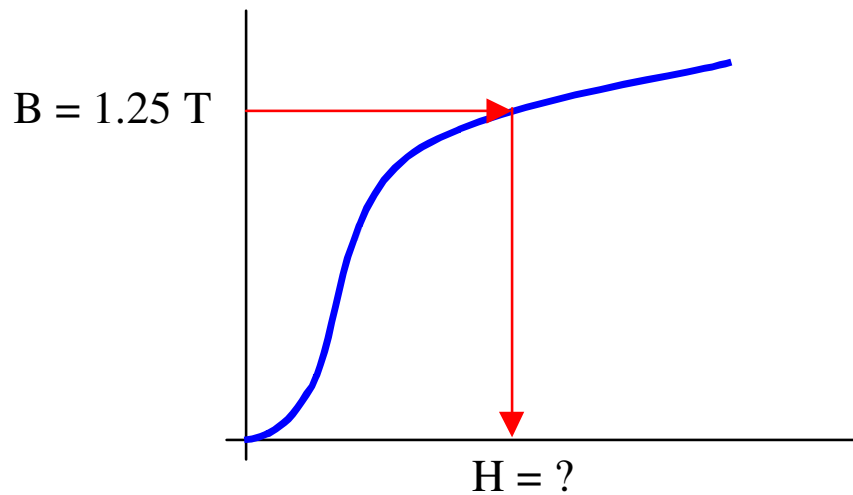
$$F = H \times L = H \times \pi D = H \times \pi \times 10 \times 10^{-2} = 201 \text{ A (turns)}$$

$$\text{As } F = NI = 201$$

$$I = 201 / 500 = 0.4 \text{ A}$$

b) If B/H curve of material is provided

Calculate B as before, now use B/H curve to read H directly from the graph at this value of B.

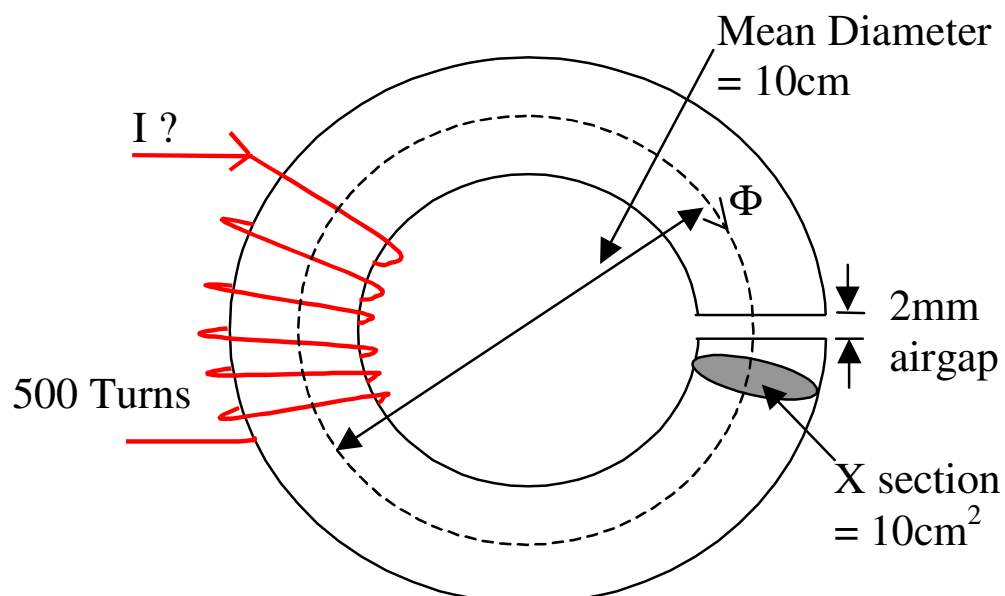


Now, given the value of H from the graph, calculate $F = H \times L$ as before.

Note: do not use the curve to give μ , as it varies over the range of B, and you don't know it accurately. Also, it takes too long!

Example 2

Same as above, but with an airgap of 2mm cut through the ring.



Now the circuit consists of 2 parts (iron and air), but the flux (Φ) is still continuous, and the cross-section (neglecting fringing) is still the same.

Can use either of two methods:

- (i) $F = \Sigma H \times L$ where $H_{\text{iron}} \neq H_{\text{air}}$ and $L_{\text{iron}} \neq L_{\text{air}}$
- (ii) $\Phi = NI / \Sigma S$ where $S = S_{\text{iron}} + S_{\text{air}}$

Using (i), then as before: $B_{\text{iron}} = B_{\text{air}} = 1.25 \text{ T}$

$$H_{\text{air}} = B_{\text{air}} / \mu_0 = 9.947 \times 10^5 \text{ Am}^{-1}$$

$$H_{\text{iron}} = B_{\text{iron}} / \mu_0 \mu_r = 6.376 \times 10^2 \text{ Am}^{-1} \text{ (as before)}$$

(note $H_{\text{air}} \gg H_{\text{iron}}$)

$$\text{Now } F = H_{\text{iron}} L_{\text{iron}} + H_{\text{air}} L_{\text{air}}$$

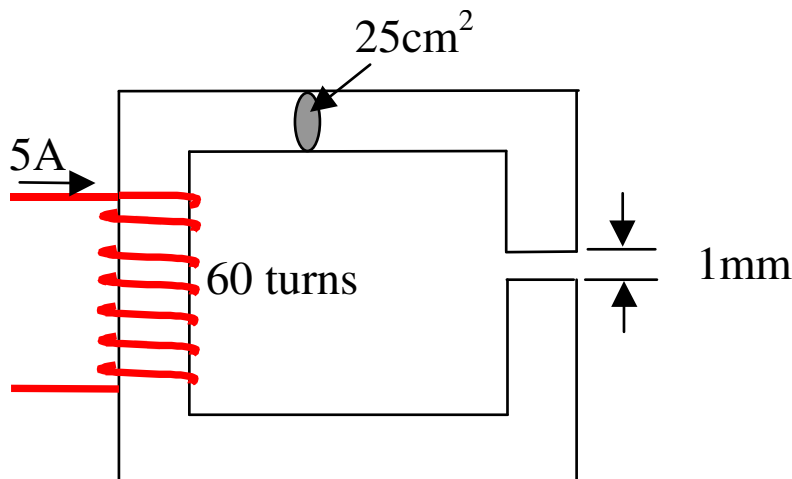
$$\therefore F = 199 + 1989$$

$$\therefore F_T = 2188 = NI$$

$$\therefore I = 2201 / 500 = 4.38 \text{ A}$$

Note how 2mm airgap dominates this solution and if H_{iron} had been ignored then the current, $I = 4 \text{ A}$, which would only be 10% out. i.e. In first order calculations, it is often possible to ignore the iron and assume all the *mmf* appears across the airgap.

Example 3



The diagram shows a magnetic circuit of constant cross-sectional area (25 cm^2), with a mean path length of 15cm in the iron, and a 1mm airgap. The 60 turn coil carries 5A, calculate the flux produced:

- if μ_r is known to be 1000
- if a B/H curve is provided

Since B is unknown at this stage, and the subdivision of the total *mmf* between the iron and the air is also unknown, then use:

$$\Phi = \frac{F_{\text{Total}}}{S_{\text{Total}}} = \frac{60 \times 5}{(S_{\text{iron}} + S_{\text{air}})}$$

As this is a series circuit, and we have the individual path lengths, L:

$$S_{\text{Total}} = \frac{L_{\text{iron}}}{\mu_r \mu_0 A_{\text{iron}}} + \frac{L_{\text{air}}}{\mu_r \mu_0 A_{\text{air}}}$$

$$\therefore S_{\text{Total}} = \frac{15 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 25 \times 10^{-4}} + \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 25 \times 10^{-4}} \quad \text{H}^{-1}$$

$$\therefore S_{\text{Total}} = 0.48 \times 10^5 + 3.18 \times 10^5 \quad \text{H}^{-1}$$

$$\therefore S_{\text{Total}} = 3.66 \times 10^5 \quad \text{H}^{-1}$$

$$\Phi = \frac{60 \times 5}{3.66 \times 10^5} = 82 \text{mWb} \quad (\text{or } B = 0.33 \text{ T})$$

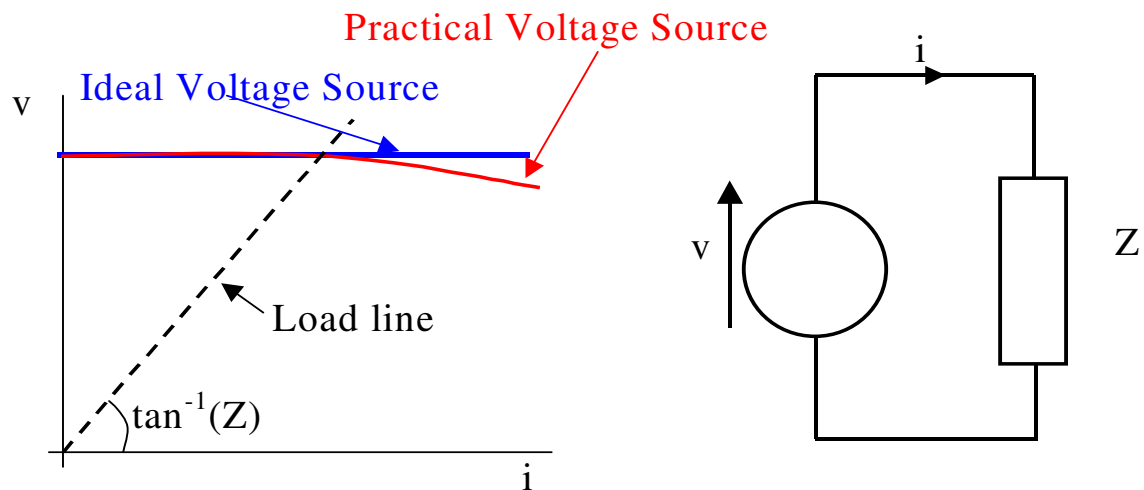
Solution if B/H curve is provided:

In this case there is no analytical solution and only a graphical or numerical (iterative) solution is possible. – not included in the course!

The Effects of Time Variation of Flux / Flux Density, Voltage and Current Sources

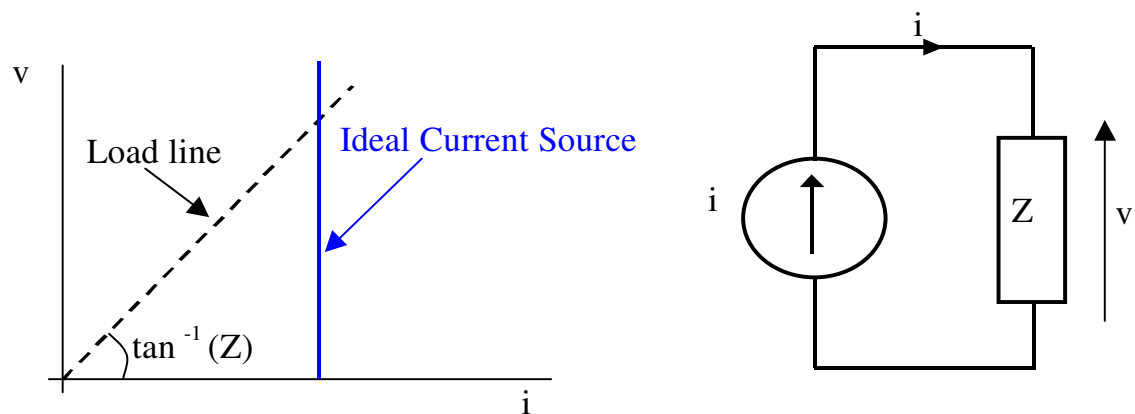
The previous analysis shows the direct links between current, I ($mmf = NI$) and Flux Φ . However the majority of electrical supplies are specified in terms of a voltage source rather than a current source. – How do we link the magnetics to the electrics in such a case ?

Voltage Source:



Current determined by load impedance: $i = v/Z$

Current Source:



Voltage determined by load impedance: $v = i.Z$

1. Steady-State dc excitation

With no time variation the only impedance is resistive i.e.

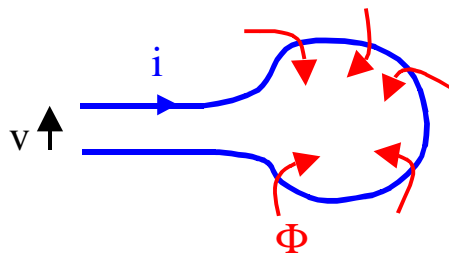
$$I = V / R$$

Hence the link between the supply voltage, V , and the current, I , is directly via the circuit resistance (Ohm's Law) and:

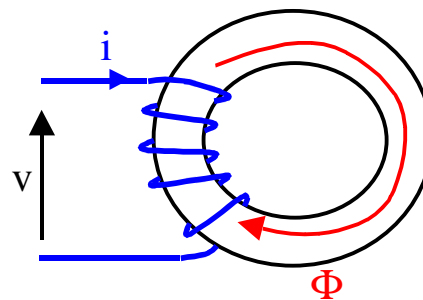
$$\Phi = N.I / S$$

as before.

2. Time Varying Excitation



Single turn circuit



Multi-turn circuit

In either case, in general we must write:

$$v = i.R + \frac{d\varphi}{dt}$$

where v , i , and φ are all instantaneous and R is the electric circuit resistance.

We can re-write this as:

$$v = i.R + e$$

where:

$$e = \frac{d\varphi}{dt}$$

is the induced *emf* due to the changes in the flux linkage with the current or coil

and φ is called the FLUX LINKAGE = $N\Phi$

i.e N is the number of turns linked by the flux Φ

Self Inductance

From previous notes we can also write:

$$\Phi = \frac{Ni}{S} \text{ Wb}$$

and assuming all of the N turns are linked by the flux, Φ , we say that the coil has flux linkages:

$$\varphi = N\Phi = \frac{N^2 i}{S} \text{ Wb (turns)}$$

Hence if φ is time varying, then by Faradays Law:

$$e = \frac{d\varphi}{dt} = \frac{d}{dt} \left(\frac{N^2 i}{S} \right) \text{ Volts}$$

if the current, i , is the only time-varying quantity then:

$$e = \left(\frac{N^2}{S} \right) \frac{di}{dt} \quad \text{or} \quad e = L \frac{di}{dt}$$

in electrical circuit models. i.e. the **SELF INDUCTANCE**, L , models the effect of the magnetic field surrounding an electric circuit, and in general:

$$v = i.R + L \frac{di}{dt}$$

where the inductance L can be calculated from the magnetic circuit properties, given that S is the total magnetic reluctance surrounding the N turn coil:

$$L = \frac{N^2}{S}$$

N.B. The inductance, L , is determined by the dimensions and properties of the magnetic circuit and is not a function of the current (unless the magnetic circuit saturates). i.e. circuit inductance L can be minimised by careful design if necessary.

Note that since:

$$\varphi = Li = \frac{N^2 i}{S}$$

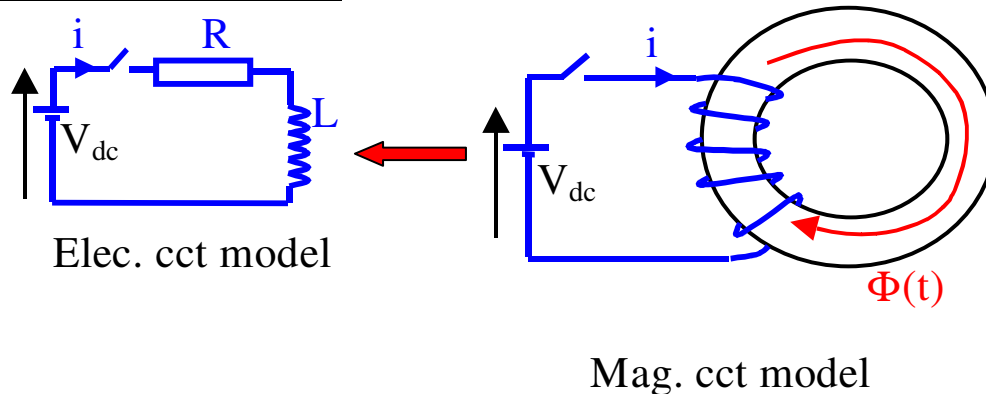
This leads to another definition:

$$L = \frac{\varphi}{i}$$

Or, the inductance may be expressed as the flux linkage per amp of current in the circuit.

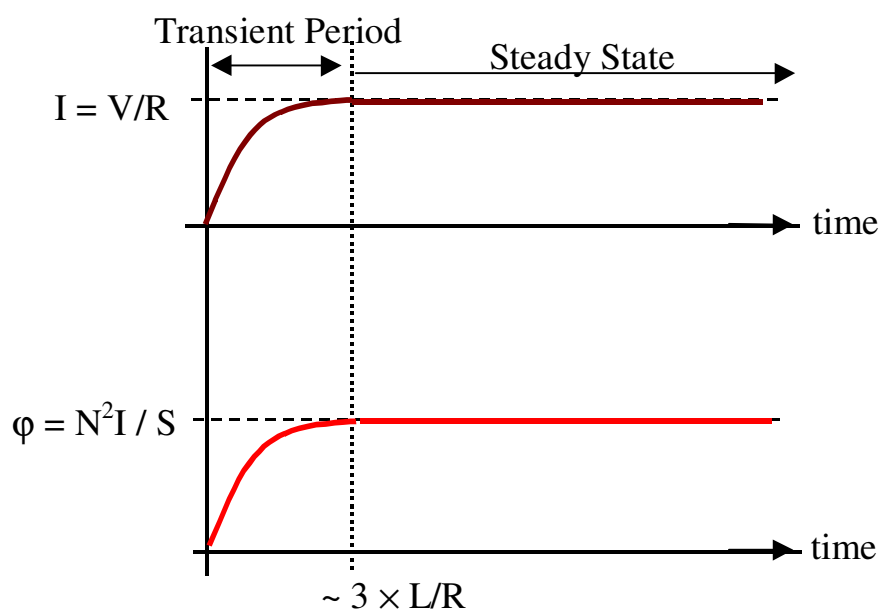
3. Inductive Electric Circuit Models with Time Variation

(i) Switched dc excitation



For this case in general we write:-

$$V_{dc} = i.R + \frac{d\phi}{dt} \quad \text{OR} \quad V_{dc} = i.R + L \frac{di}{dt}$$



Stage (i) – There is a transient period during which the current rises and $di/dt \neq 0$ and:

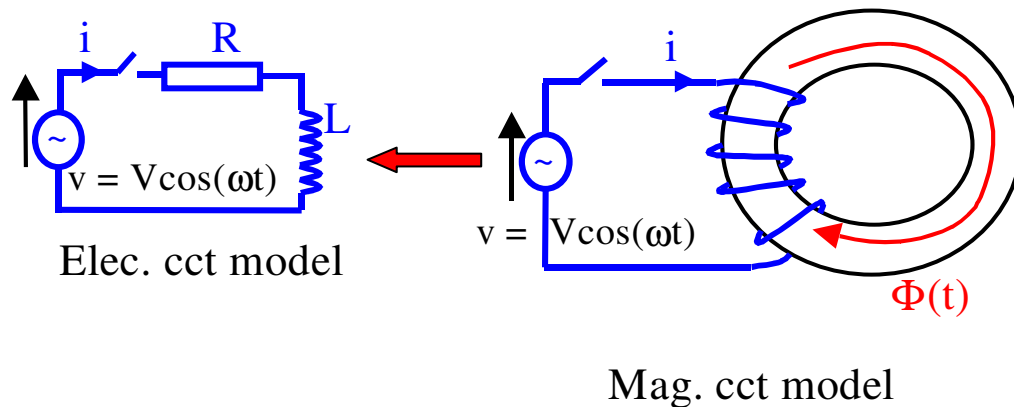
$$\phi = N\Phi = \frac{N^2 i}{S}$$

also rises.

Stage (ii) – In the steady-state, $di/dt = 0$:

$$I_{ss} = \frac{V_{dc}}{R} \quad \text{and} \quad \Phi_{ss} = \frac{N i}{S}, \quad \phi_{ss} = \frac{N^2 i}{S} \quad \text{etc.}$$

Stage (i) lasts for about $3 \times L/R$ where (L/R) is called the **TIME CONSTANT**. After this time, the current, I , is determined entirely by the coil resistance.

(ii) Sinusoidal ac excitation

in general:

$$V_{dc} = i.R + \frac{d\phi}{dt} \quad \text{OR} \quad V_{dc} = i.R + L \frac{di}{dt}$$

At initial switch on there will be a **transient** period as before, but the exact shape will depend on the point in the ac cycle at which the switch was closed.

Steady-state – Even here, v and i are not time independent. If v and i are sinusoidally time varying then we can also write the general expression in the phasor form:-

$$V = R.I + j\omega L.I \quad \text{OR} \quad V = (R + jX).I$$

where V and I are either peak or RMS values for the voltage and the current.

We can also write the impedance in polar form:

$$(R + jX) = Z \angle \phi$$

where we write:

$$|Z| = \sqrt{R^2 + X^2}$$

and:

$$\phi = \tan^{-1}\left(\frac{X}{R}\right)$$

In this case the link between the magnitude of the current flowing in the electric circuit and the ac voltage supply is:

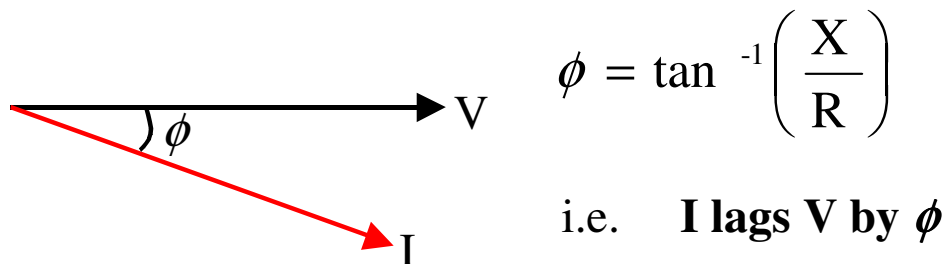
$$|I| = \frac{|V|}{|Z|}$$

where I and V are **either** peak or RMS values, where the effect of the sinusoidal time varying magnetic field surrounding the circuit is reflected in the impedance term Z (for dc, $Z=R$ in steady-state).

If the above current of I amps flowing in a circuit of N turns, then the for a peak voltage V applied:

$$I = \frac{V}{Z}, \quad \Phi = \frac{NI}{S}, \quad B = \frac{\Phi}{A} \quad \text{etc}$$

note also that the effect of the surrounding magnetic field caused the current and voltage to be out of phase by ϕ where:-



$$\phi = \tan^{-1} \left(\frac{X}{R} \right)$$

i.e. **I lags V by ϕ**

Special case of $R \ll \omega L$ or $R \ll X$

This is often true of devices in which the magnetic circuit is an essential part of the device (e.g. transformers, inductors, machines etc.) For many such devices we can simplify the analysis by assuming $R \ll \omega L$, giving only an induced *emf* term

$$v = \frac{d\phi}{dt} = \frac{d(N\Phi)}{dt} \quad \text{Faraday's Law}$$

and if v is sinusoidal, then the flux will vary sinusoidally also. i.e if Φ is of the form:

$$\Phi \sin(\omega t) \quad \text{then} \quad v = \frac{d(N\Phi \sin(\omega t))}{dt}$$

therefore:

$$v = \omega N \Phi \cos(\omega t)$$

i.e. $\pi/2$ out of phase with Φ which we can write as:

$$v = V \cos(\omega t)$$

where:

$$V = \omega N \Phi$$

and:

$$V_{\text{rms}} = \frac{\omega N \Phi}{\sqrt{2}} = \frac{2\pi f N \Phi}{\sqrt{2}} = 4.44f N \Phi$$

i.e. in such devices, where R is negligible ($\ll \omega L$) it is possible to estimate the peak flux, Φ , and hence the peak flux density, $B = \Phi/\text{Area}$, directly from the applied voltage and we do not need to go through the procedure of calculating the impedance and the current (Z and I) and hence the flux Φ etc.

Example 1

A 1000 turn coil is wound on a magnetic circuit of cross-sectional area 25cm^2 . The coil has a resistance of 10Ω and the magnetic circuit has a reluctance of $5 \times 10^5 \text{ H}^{-1}$. Calculate the steady-state peak coil current and the corresponding peak flux density in the magnetic circuit.

- (a) if the coil is supplied from a 100V dc source
 (b) if the coil is supplied from a 100V(rms) sinusoidal ac supply at 50Hz.

(a) On dc:

the steady-state current is given by

$$I = V/R = 100/10 = 10\text{A}$$

hence the peak flux $\Phi = NI/S = 10 \times 1000 / 5 \times 10^5 = 20\text{mWb}$.

$$\text{and } B_{\text{max}} = \Phi/A = 20 \times 10^{-3} / 25 \times 10^{-4} = 8\text{T} !!!$$

note: This is not a practical possibility since most magnetic materials saturate at $B < 2\text{T}$. In practice, the magnetic circuit would saturate leading to a reduction in μ and an increase in the reluctance such that B would be limited to somewhere in the region of 1-2 T.

(b) On ac:

we must now include the effect of time varying flux linkage produced by the coil and the corresponding induced *emf*. i.e. now the steady-state current is obtained from:

$$V = (R + jX) \cdot I \quad \text{or} \quad |I| = \frac{|V|}{|Z|}$$

where:

$$|Z| = \sqrt{R^2 + X^2}$$

But first we need to calculate the inductance, L, from the magnetic circuit. From $L = N^2/S$, i.e.

$$L = 1000^2 / 5 \times 10^5 = 2\text{H}$$

$$\text{And hence, } \omega L = 2\pi fL = 100\pi \times 2 = 628.3\Omega$$

Therefore:

$$\begin{aligned} |I| &= \frac{V}{Z} = \frac{100}{\sqrt{10^2 + 628.3^2}} \\ &= 0.16\text{A} - \text{rms since } V = 100\text{V (rms)} \end{aligned}$$

from this we get that the peak current: $I = \sqrt{2} \cdot I_{\text{rms}} = 0.23\text{A}$

and hence the peak flux: $\Phi = NI/S$,

and peak flux density: $B = \Phi/\text{Area} = 0.18\text{T}$

Note that in this ac circuit model, $R \ll \omega L$. If we had approached this problem from this point of view, we could have used the relationship (see earlier)

$$V_{\text{rms}} = \frac{2\pi f N \Phi}{\sqrt{2}} \quad \text{or} \quad V_{\text{rms}} = 4.44 f N \Phi$$

which becomes:

$$V_{\text{rms}} = 4.44 \times f \times N \times B \times \text{Area}$$

this therefore gives the flux density $B = 0.18\text{T}$ directly.

The Effect of Frequency on the Size of Magnetic Devices.

From the expression:

$$V_{\text{rms}} = 4.44 \times f \times N \times B \times \text{Area}$$

and recalling that for magnetic materials the flux density, B , is limited by the material properties ($B_{\text{max}} \leq 1\text{--}2\text{T}$ for iron, or $B_{\text{max}} \leq 0.3\text{T}$ for ferrite ‘pot’ cores), then it can be seen that to sustain a certain voltage, V , across the winding of a device we have a choice of balancing it from the product

$$f \times N \times A \quad (B_{\text{max}} \text{ limited by material})$$

Increasing N requires more turns on the coil – Increasing the overall device size

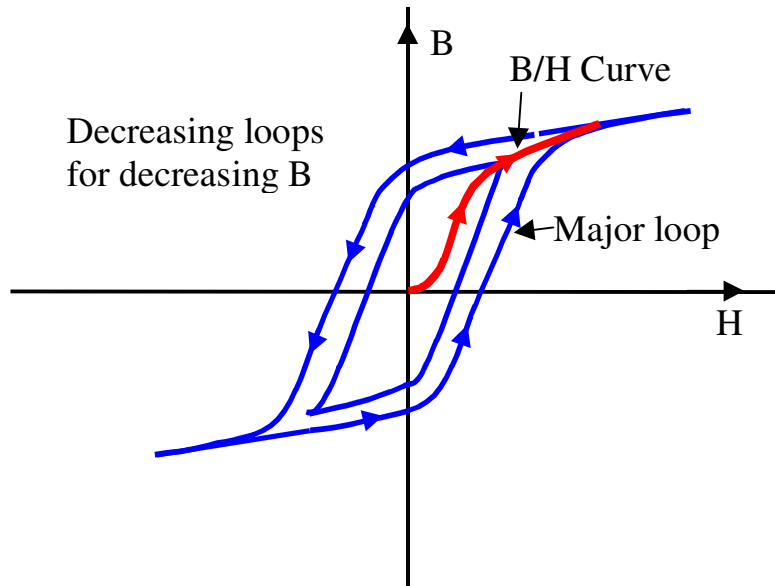
Increasing A requires a larger cross-section of the magnetic core – Increasing the overall device size

Increasing f can be used to reduce both N and A and therefore **reduce** the overall device size

This is common practice in a wide range of devices – transformers, motors, generators etc. In aircraft, a frequency of 400Hz has been used for many years in recognition of this fact. Increasing the frequency also reduces the size of smoothing inductors required in electronic circuits.

Losses in Magnetic Circuits

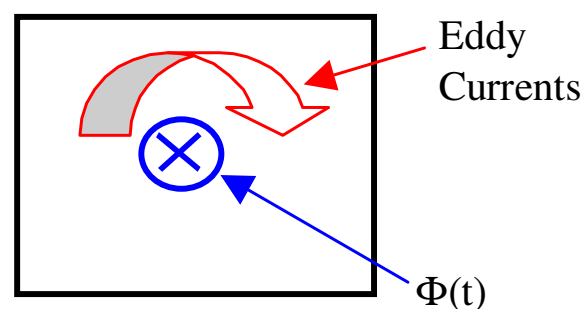
From a size point of view (see earlier notes about the advantages of using a higher frequency on the dimensions of magnetic devices), an increase in frequency seems very beneficial. However, there are losses in the magnetic material, which can put a limit on the frequency that can be used. These losses are usually lumped together and called 'Iron losses'. They consist of two basic loss mechanisms:



- i) **Hysteresis loss**
All magnetic materials have a B/H loop, which characterises the relationship between the flux density in the material, B, and the *mmf*, H. The B/H curve given earlier in the notes is only relevant at steady-state (dc). On ac the material is subjected to an alternating B and H and the material cycles around the loop. Up to a limiting value of flux density, B, the loop area increases with increasing B. the loss is proportional to the loop area per cycle. Hence loss, $P_h \propto \text{loop area and frequency (no of times the loop is traversed per second)}$.

$$P_h \propto B \text{ and } f$$

- ii) **Eddy Current loss**
If an alternating field passes through a material a voltage is induced around a given path (any material).



$$e = \frac{d\phi}{dt} = \frac{d(NBA)}{dt}$$

i.e $e \propto B, A \text{ and } f$

around the loop this causes eddy currents to flow,

$$i = v/R \quad (R = \text{resistance of material in loop})$$

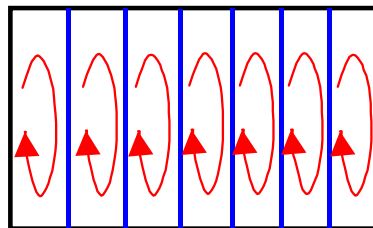
$$P_e = \oint i^2 R = \oint \frac{e^2}{R}$$

hence:

$$P_e \propto B^2, f^2, A^2, R$$

i.e. The eddy current power loss, P_e , is proportional to the square of the frequency.

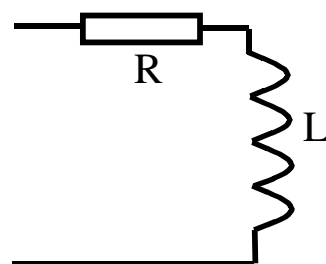
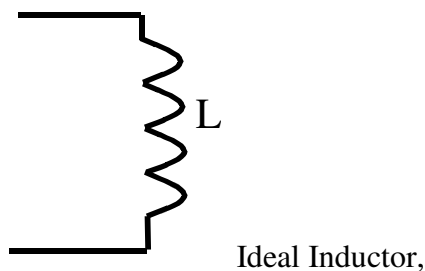
The eddy current loss may be reduced with the use of Laminations (0.5mm at 50Hz, 0.1mm at 1kHz). This reduces the area of the material per loop. Laminations of less than 0.1mm thick are difficult to produce and handle etc. Beyond 1kHz we use ferrite 'pot' cores, but now the maximum available flux density (B_{\max}) is now 0.3T as compared to 1.2T for silicon steel.



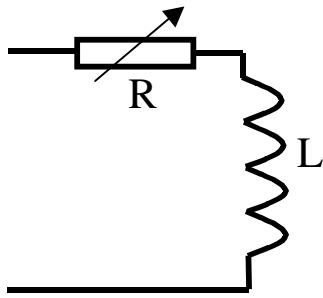
Modelling of Losses in Equivalent Circuit of Devices

To simplify the calculation of device behaviour we often use equivalent circuit models where the model application is chosen to suit the model application and accuracy required. E.g. inductor models:

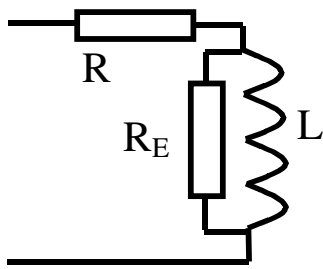
Note. R_E is a fictitious resistor, it simply gives a measure of the iron loss, $= E^2/R$, which gives a reasonable estimate of the watts lost due to eddy and hysteresis loss in the device.



Model allowing for finite coil resistance (copper loss can now be calculated)



Model allowing for changes in R with frequency (Skin effect)



Model including loss in the iron core (Iron loss)