Data Provided: List of Useful Equations (attached at the end of the paper)

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2015-16 (2.0 hours)

EEE218 Electric Circuits 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1.

a. A coil of 900 turns is wound on a toroidal iron core having of mean diameter 25cm and cross-sectional area 8cm^2 as shown in Figure 1.1. The core has a relative permeability, $\mu_r = 1000$. (***Assume the permeability of free space, $\mu_0 = 4 \times \pi \times 10^{-7}$ H/m.***)

(i) Calculate the reluctance of the magnetic circuit. **(2)**

(ii) What is the coil current required to establish a flux density of 1.5T in the core? **(2)**

(iii) Calculate the self inductance of the coil. **(1)**

b. If a 5mm wide slot is now cut in the iron toroid to form an airgap as shown in Figure 1.2, find:

(i) The new level of current required to maintain the previous flux density of 1.5T. **(3)**

(ii) The new value of the self inductance of the coil. **(1)**

c. The coil wound on the toroidal iron core from part **(a)** is connected to a 10V d.c. supply through a switch which is initially closed. The switch is then opened within 1ms. If the wire forming the coil has a resistance of 5Ω, calculate the magnitude of the peak voltage which appears across the coil after the switch is opened. **(2)**

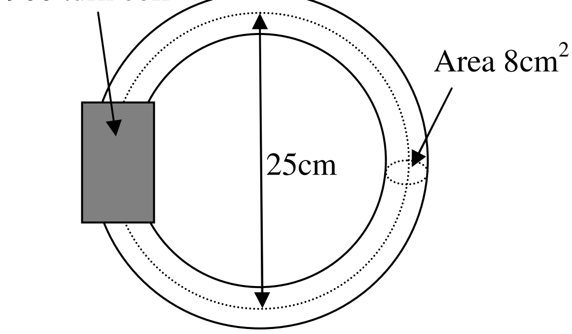


Figure 1.1

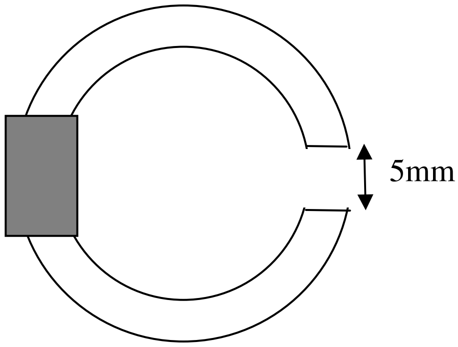


Figure 1.2

d. An ideal transformer has a turns ratio of 1:5 (primary:secondary) and an input voltage of 200V_{rms} at 50Hz.

(i) Calculate the secondary voltage, the current in the primary winding and the power dissipated if a resistive load of 40Ω is connected across the secondary. **(3)**

(ii) Calculate the current in the primary winding and the power dissipated if the load across the secondary now comprises a resistance of 40Ω in series with an inductance of 150mH. **(3)**

(iii) For the case described in part **(ii)** above, what would be the input power factor and the required VA rating of the transformer? **(1)**

(iv) If the maximum core flux of the transformer is 5mWb calculate the actual number of turns on the primary winding. **(1)**

(v) If the transformer were to be operated in a country where the supply frequency is 60Hz, what is the maximum permissible supply voltage without the maximum core flux of 5mWb being exceeded? **(1)**

2.

a. An inductance, L, capacitance, C, and resistance, R, are connected to form a series resonant circuit as shown in Figure 2.1.

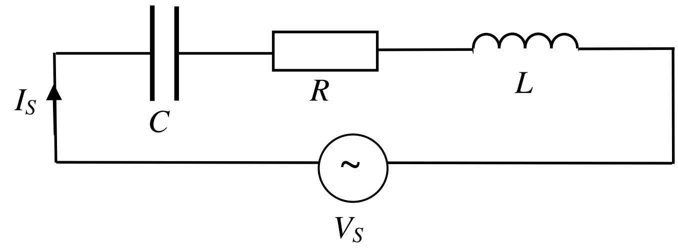


Figure 2.1

(i) Work out the impedance of the circuit shown in Figure 2.1, state the condition for resonance, and hence show that the resonant frequency of the circuit, f_r, *is:*

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

(4)

(ii) Sketch the variation of the magnitude of the total impedance, Z, with frequency, f, over the frequency range $f << f_{resonant}$ to $f >> f_{resonant}$. **(2)**

(iii) Sketch a phasor diagram for the circuit at resonance showing the relative direction of the voltages across each component. Take the current phasor as reference. **(2)**

(iv) The Q factor for this circuit is $Q = |V_L|/|V_R|$ at resonance, where V_L and V_R are the voltages across the inductor and resistor respectively. Express Q in terms of the circuit parameters. **(2)**

(v) Given that f_r = 23.2kHz, Q = 6.86 and R = 100Ω, calculate suitable values for L and C. **(3)**

b.

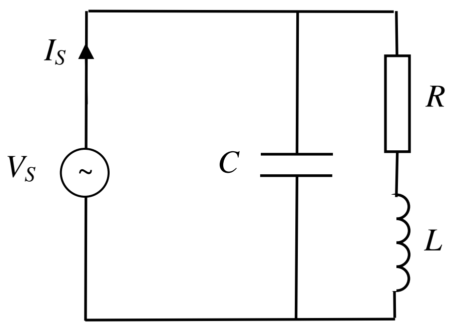


Figure 2.2

b. (i) Show that the impedance of the circuit shown in Figure 2.2 is:

$$Z = \frac{V_S}{I_S} = \frac{R + j\omega L}{1 + j\omega CR - \omega^2 LC}$$

(3)

(ii) Using the impedance given in part **b.(i)** above, derive an expression for the angular resonant frequency, ω_r of the circuit. **(4)**

3.

a. An electric heater is connected to a 400V d.c. supply and initially draws a current of 12A. After the heater has been in use for a long period of time and reached its steady-state temperature the current has dropped to 8A. If the heater was initially at 15°C, calculate the final temperature of the heater. You may assume the following: **(4)**

Temp. coefficient = $\alpha_0 = 6.8 \times 10^{-3}$ /°C
$R_T = R_0\,(1 + \alpha_0 T)$
R_T = Resistance at temperature T°C
R_0 = Resistance at temperature 0°C

b. The heater of part (**a**) is now allowed to cool down to 15°C and moved to a new location 120m away from the supply. It is connected to the supply by a cable having conductors with a cross-sectional area of 2.5mm^2.

(i) If the resistivity of the cable conductor material is $1.725\times10^{-8}\Omega$m calculate the resistance of one conductor of the cable. **(2)**

(ii) Calculate the new current drawn by the heater when it is first switched on (i.e. at 15°C) and the power dissipated by the heater at this temperature. (*Hint: remember that there is both a supply and return conductor in the circuit*). **(3)**

(iii) Assuming any power dissipated in the cable is waste energy, calculate the system efficiency at 15°C. **(1)**

(iv) The cable is now upgraded to ensure that the voltage across the heater at 15°C is at least 390V. Calculate the minimum cross-sectional area required for the new cable to ensure this requirement is met. **(3)**

c. A piece of electrical equipment with an effective impedance of (6 + j8)Ω is connected to a 600V_{rms} 50Hz sinusoidal supply.

(i) Calculate the magnitude and phase of the current drawn from the supply. **(1)**

(ii) What is the kVA rating of the equipment? **(1)**

(iii) Calculate the reactive power drawn from the supply in kVAr **(1)**

(iv) It is decided to use a capacitor to correct the overall power-factor to 0.98 lagging. By means of a suitable diagram, explain where the capacitor should be connected and calculate its required value in Farads. **(4)**

4.

a. For the network shown in Figure 4.1, find the value of the current through the 4Ω resistance using the method of superposition. Indicate the direction of the current. **(6)**

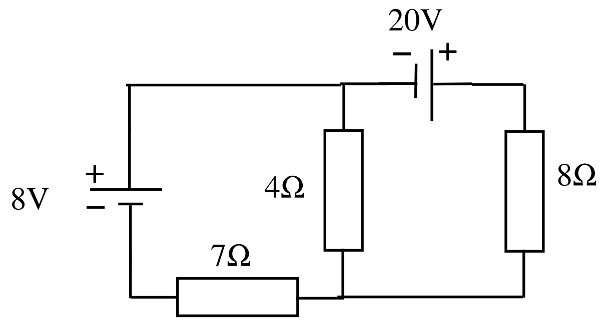


Figure 4.1

b. A constant current source of 4A is added to the network in parallel with the 8Ω resistor, as shown in Figure 4.2. Calculate the new current through the 4Ω resistor. **(4)**

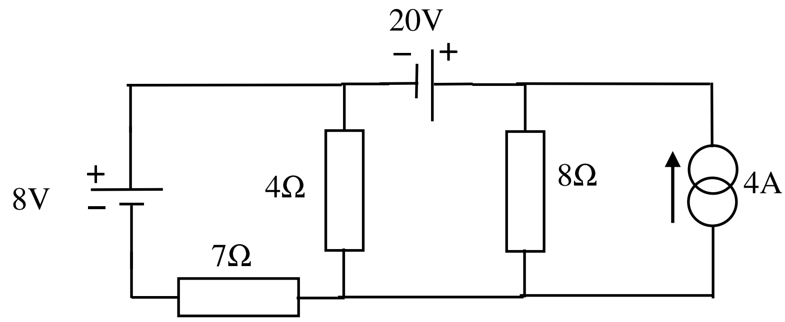


Figure 4.2

c. The network of Figure 4.1 is to be used as a source for a load resistor of 5Ω, which is connected between **A** and **B**, as shown in Figure 4.3. Derive the Thevenin equivalent circuit for the source and hence calculate the power dissipated in the load resistor. **(6)**

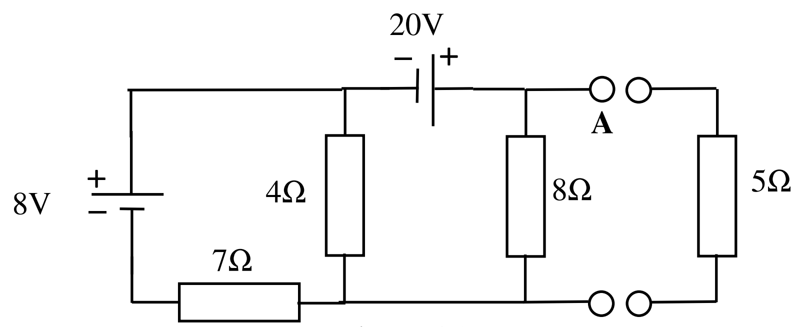


Figure 4.3

d. A rechargeable battery of nominal voltage of 12V is now connected between **A** and **B** (positive terminal connected to **A**) of Figure 4.3 in place of the 5Ω load resistor. Determine whether this battery will receive or deliver power and find the magnitude of the current passing through it when it is fully charged (12V). Assume that the battery has a negligible internal resistance. **(2)**

e. Calculate the Norton equivalent circuit for the source network shown in Figure 4.3 (i.e. not including the load resistor). **(2)**

KM / MH

USEFUL EQUATIONS – EEE140 / EEE218

Electric Circuits

Resistance (*R*) – units Ohms (Ω)

Resistors in series $$R_{TOT} = R_1 + R_2 + R_3 + \cdots R_n$$

Resistors in parallel $$\frac{1}{R_{TOT}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \frac{1}{R_n}$$

Resistance (Ohms law) $$R = \frac{V}{I}$$

Resistance $$R = \frac{\rho l}{A} = \frac{l}{\sigma A}$$

(*l=length, m; A = cross-sectional area, m^2; ρ = resistivity, Ω m; σ = conductivity, S/m*)

Temperature dependence of resistors $$R_{T1} = R_0(1 + \alpha_0 T_1)$$

α_0 = temperature coefficient of resistance
R_0 = Resistance (Ω) at 0℃
R_{T1} = Resistance (Ω) at T_1℃
T_1 = Temperature in ℃
For temperatures T_1 and T_2 use ratio

$$\frac{R_{T1}}{R_{T2}} = \frac{(1 + \alpha_0 T_1)}{(1 + \alpha_0 T_2)}$$

Voltage across a resistor $$V_R = IR$$

Power dissipated in a resistor $$P = I^2 R = \frac{V^2}{R} = V \cdot I$$

Energy dissipated in a resistor $$E = I^2 Rt = \frac{V^2 t}{R} = V \cdot I \cdot t$$

Capacitance (*C*) – units Farads (F)

Charge (*Q*) $$Q = I \cdot t$$ (Constant current , I)

$$Q = \int_0^t i(t)dt$$ (Time varying current, $i(t)$)

$$Q = CV$$ (Capacitance × Voltage)

Capacitors in series $$\frac{1}{C_{TOT}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_n}$$

Capacitors in parallel $$C_{TOT} = C_1 + C_2 + C_3 + \cdots C_n$$

Voltage across a capacitor $$V_c = \frac{Q}{C}$$

Energy stored in a capacitor $$E = \frac{1}{2} CV^2$$

Inductance (*L*) – units Henrys (H)

Inductors in series $$L_{TOT} = L_1 + L_2 + L_3 + \cdots L_n$$

Inductors in parallel $$\frac{1}{L_{TOT}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots \frac{1}{L_n}$$

Voltage across an inductor $$V_L = L\frac{dI}{dt}$$

Energy stored in an inductor $$E = \frac{1}{2} LI^2$$

A.C. Circuits

Power dissipated in a resistance $P = I_{rms}^2 R = \frac{V_{rms}^2}{R}$

For other circuits having capacitance and/or inductance, there is a phase shift, ϕ, between the current and voltage waveforms.

Real Power (*P*) (*Watts*) $P = V_{rms} I_{rms} \times \cos\phi$

Reactive Power (*Q*) (*VARs*) $Q = V_{rms} I_{rms} \times \sin\phi$

Power factor (p.f. or cos ϕ) $0 < \cos\phi < 1$

Power = Energy s^{-1} $Watts(W) = Js^{-1}$

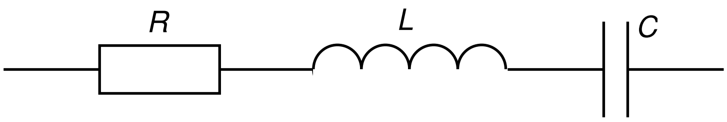
Capacitive reactance $X_C = \frac{1}{j\omega C} = \frac{1}{j2\pi f C} = \frac{-j}{\omega C} = \frac{-j}{2\pi f C}$

(ω = electrical frequency in rad/s (= $2\pi f$); f is the electrical frequency in Hz)

Inductive reactance $X_L = j\omega L = 2\pi f L$

(ω = electrical frequency in rad/s (= $2\pi f$); f is the electrical frequency in Hz)

Series Resonant Circuit

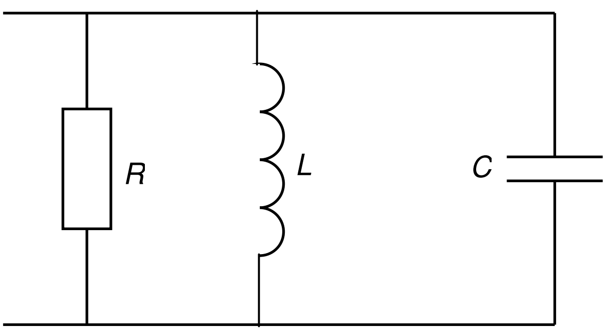


At resonance $X_C = X_L$ and $Z = R$

Frequency for resonance $\omega_r = \frac{1}{\sqrt{LC}}$ or $f_r = \frac{1}{2\pi\sqrt{LC}}$

Q factor $Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Parallel Resonant Circuit



At resonance $X_C = X_L$ and $Z = R$

Frequency for resonance $\omega_r = \frac{1}{\sqrt{LC}}$ or $f_r = \frac{1}{2\pi\sqrt{LC}}$

Q factor $Q = \frac{R}{\omega_r L} = \omega_r CR = R\sqrt{\frac{C}{L}}$

Transient Circuits

Current growth in an inductive circuit containing inductance and resistance:
Instantaneous current $i = I_0(1 - e^{-t/\tau})$ where $\tau = L/R$

Current decay in an inductive circuit containing inductance and resistance:
Instantaneous current $i = I_0 e^{-t/\tau}$ where $\tau = L/R$

Charging a capacitor through a resistor:
Instantaneous voltage $v = V_0(1 - e^{-t/\tau})$ where $\tau = RC$
Instantaneous current $i = I_0 e^{-t/\tau}$ where $\tau = RC$

Disharging a capacitor through a resistor:
Instantaneous voltage $v = V_0 e^{-t/\tau}$ where $\tau = RC$
Instantaneous current $i = -I_0 e^{-t/\tau}$ where $\tau = RC$

Magnetic Circuits

Reluctance (S) – units H^{-1} $S = \dfrac{l}{\mu_0 \mu r A}$

(l=length, m; A = cross-sectional area, m^2; μ_0 = permeability of free space (Hm^{-1}); μ_r = relative permeability)

Inductance (L) $L = \dfrac{N^2}{S}$

(*N = number of turns on the coil*)

Flux density (B) – units Tesla (T) $B = \mu_0 \mu_r H = \dfrac{\phi}{A}$

(H = magnetic field strength(A/m); φ = flux (Wb))

MagnetoMotive Force – MMF $F = H \cdot l = N \cdot I = \phi S$

Induced EMF (E) – units Volts (V) $E = N \dfrac{d\phi}{dt}$

Transformers (ideal)

Voltage ratio $\dfrac{V_{in}}{V_{out}} = \dfrac{N_1}{N_2} = turns \quad ratio$

Current ratio $\dfrac{I_{in}}{I_{out}} = \dfrac{N_2}{N_1} = \dfrac{1}{turns \quad ratio}$

Impedance ratio $\dfrac{Z_{in}}{Z_{out}} = \left(\dfrac{N_1}{N_2}\right)^2 = (turns \quad ratio)^2$

V_{in} = voltage across primary winding; *I_{in} = current through primary winding;*
V_{out} = voltage across secondary winding; *I_{out} = current through secondary winding;*

Induced voltage $V_{rms} = 4.44f \cdot N \cdot \phi_{max}$

(ϕ_{MAX} = maximum flux in the transformer core (Wb); f = frequency (Hz))

Mechanics

Mechanical Power (W) $P_{mech} = \omega_{mech} \cdot T$

(T = torque (Nm); ω_{mech} = rotational speed (rad/s))

Rotational speed (rad/s) $\omega_{mech} = \frac{2\pi}{60} \cdot n$

(n = speed in revs per minute)

Torque $T = F \cdot r$

(F = force(Nm); r = radius (m))

Gearbox (no losses) $\omega_{in} T_{in} = \omega_{out} T_{out}$

DC motors

Force on a current carrying conductor $F = B \cdot I \cdot l$

(B = flux density (T); I = current (A); l = length (m))

DC motor armature voltage $V_A = E_A + I_A R_A$

(E_A = induced emf (V); I_A = armature current (A); R_A = armature resistance (Ω))

Induced voltage, E_A is proportional to the speed of rotation, ω (rad/s), and the flux, ϕ (Wb).
Torque, T is proportional to the armature current, I (A) and the flux, ϕ (Wb).

For a wound field machine:

Induced emf (V) $E_A = K I_F \omega$

Torque (Nm) $T = K I_F I_A$

(I_F = field current (A); K = constant)

For a permanent magnet machine:

Induced emf (V) $E_A = K_E \cdot \omega$

Torque (Nm) $T = K_T \cdot I_A$

(K_E (V/rad/s) and K_T (Nm/A) are constants with the same numerical values)