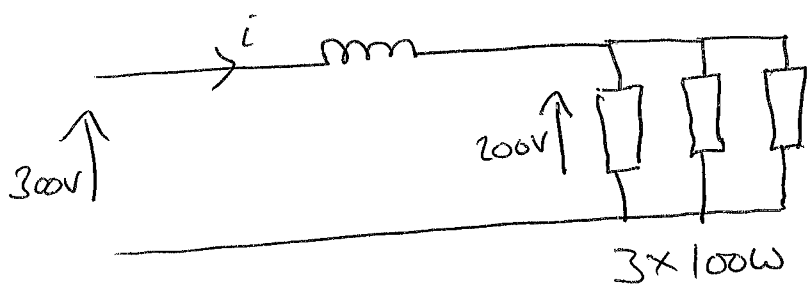
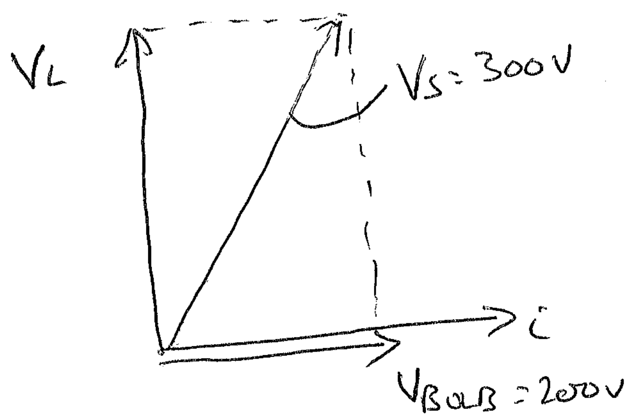
QUESTION 1

(a)(i)



(i)



(ii) The current through each bulb, rated at 100W is:

$$p = vi \Rightarrow i = \frac{p}{V} = \frac{100}{200} = 0.5A_{rms}$$

For 3 bulbs the total current is $3 \times 0.5 = \underline{\underline{1.5A_{rms}}}$

(iii) From the phasor diagram above:

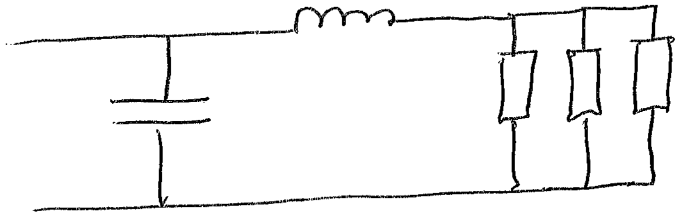
$$V_S^2 = V_B^2 + V_L^2 \Rightarrow V_L = \sqrt{V_S^2 - V_B^2} = \underline{\underline{223.6V_{rms}}}$$

(iv) The value of the inductor can be found from:

$$X_L = \frac{V_L}{I} = \frac{223.6}{1.5} = 149.1\Omega$$

Hence $L = \dfrac{X_L}{2\pi f} = \dfrac{149.1}{2\pi \times 50} = \underline{\underline{0.475H}}$

(v) The capacitor must be placed in parallel with the total load:



Reactive power of inductor is $I^2 X_L = 1.5^2 \times 149.1$
$= 335.5$ VARs

Hence for unity powerfactor the capacitor must cancel these, i.e.

$Q_c = -335.5$ VARs.

Voltage across capacitor $= 300 V_{rms}$

Hence $Q_c = \dfrac{V_c^2}{X_c} \Rightarrow |X_c| = \dfrac{V_c^2}{Q_c} = \dfrac{300^2}{335.5} = 268.3\Omega$

Hence $C = \dfrac{1}{2\pi f X_c} = \dfrac{1}{2\pi . 50 . 268.3} = \underline{\underline{11.86 \mu F}}$

(vi) Peak voltage the capacitor must withstand

$V_{c-pk} = \sqrt{2} \times 300 = \underline{\underline{424.3V}}$

(b) Current to line $= \dfrac{V}{Z} = \dfrac{4000\angle 0°}{(15+j20)} = \underline{\underline{160\angle -53.13° A_{rms}}}$

Powerfactor $= \cos 53.13 = \underline{\underline{0.6 \text{ lagging}}}$

(ii) KVA rating of line = $Vi = 4000 \times 160 = \underline{\underline{640\,KVA}}$

(iii) Reactive power drawn from the supply $= Vi \sin\phi$

$= 4000 \times 160 \times \sin(\cos^{-1} 0.6) = \underline{\underline{512\,KVAR}}$

(c) Lighting load 80kW + 0kVAR

Motor load 360kVA @ 0.7 lag

$P_M = 252\,kW$ $Q_M = 360 \times \sin(\cos^{-1} 0.7)$

$= 257.1\,KVAR$

∴ Total KVA of extended site:

$$S_{new} = \sqrt{(640 \times 0.8 + 80 + 252)^2 + (512 + 257.1)^2}$$

$$= \sqrt{716^2 + 769.1^2} = \underline{\underline{1051\,KVA}}$$

$$P.f. = \frac{716}{1051} = \underline{\underline{0.68 \text{ lagging}}}$$

QUESTION 2

(a)(i) The reluctance of the magnetic circuit is given by:

$$S = \frac{L}{\mu_0 \mu_r A} = \frac{25\times10^{-2}\,\pi}{\mu_0 \times 1000 \times 8\times10^{-4}} = \underline{\underline{7.81\times10^{5}\ H^{-1}}}$$

(ii) The flux in the core is related to the flux density by:

$$\phi = B.A = 1.5 \times 8\times10^{-4} = 1.2\times10^{-3}\ Wb$$

Now since

$$I = \frac{\phi S}{N} = \frac{1.2\times10^{-3} \times 7.81\times10^{5}}{900} = \underline{\underline{1.04\ A}}$$

(iii) The self-inductance of the coil is given by:

$$L = \frac{N^2}{S} = \frac{900^2}{7.81\times10^{5}} = \underline{\underline{1.037H}}$$

(b)(i) After the slot is cut:

$$S_{IRON} = \frac{(0.25\pi - 0.005)}{(\mu_0 \times 1000 \times 8\times10^{-4})} = 7.76\times10^{5}\ H^{-1}$$

$$S_{AIR} = \frac{0.005}{\mu_0 \times 1 \times 8\times10^{-4}} = 4.97\times10^{6}\ H^{-1}$$

$$\therefore S_{TOT} = S_{IRON} + S_{AIR} = 7.76\times10^{5} + 4.97\times10^{6} = 5.75\times10^{6}\ H^{-1}$$

Now the new level of current is:

$$I = \frac{1.2\times10^{-3} \times 5.75\times10^{6}}{900} = \underline{\underline{7.67A}}$$

(ii)

$$L_{NEW} = \frac{900^2}{5.75\times10^{6}} = \underline{\underline{0.141\ H}}$$

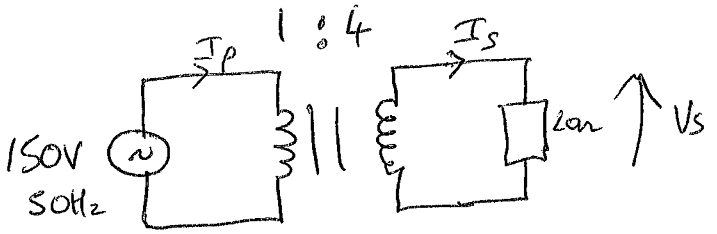
(c) The current flowing in the circuit before the switch is closed is:

$$I = \frac{V}{R} = \frac{10}{5} = 2A$$

Since the switch is opened in 1ms and the current falls to zero in this time then the voltage is given by:

$$|V_L| = L\frac{di}{dt} = \frac{1.037 \times 2}{1\times 10^{-3}} = \underline{\underline{2074\,V}}$$

(d)



(i) Now $\frac{V_p}{V_s} = \frac{N_p}{N_s} \Rightarrow V_s = \frac{N_s V_p}{N_p} = \frac{4}{1} \times 150 = \underline{\underline{600V_{rms}}}$

$$I_s = \frac{V_s}{R} = \frac{600}{20} = \underline{\underline{30A_{rms}}}$$

and since $\frac{I_p}{I_s} = \frac{N_s}{N_p}$ then $I_p = \frac{N_s I_s}{N_p} = \frac{4}{1} \times 30 = \underline{\underline{120A_{rms}}}$

Power dissipated is $I_s^2 R = 30^2 \times 20 = \underline{\underline{18kW}}$

(ii) The secondary winding now comprises a resistance of 18Ω in series with an inductor of 75mH

$$Z_s = R + j2\pi fL = 18 + j2\pi . 50 \times 75\times 10^{-3} = 18 + j23.56\Omega$$
$$= 29.65\angle 52.6^\circ\,\Omega$$

$$I_s = \frac{V_s}{Z_s} = \frac{600\angle 0^\circ}{29.65\angle 52.6^\circ} = \underline{\underline{20.24\angle -52.6^\circ\,A_{rms}}}$$

$$I_p = 4I_s = \underline{\underline{80.96\angle -52.6^\circ\,A_{rms}}}$$

QUESTION 2 (CONTINUED)

Power dissipated in the load $= I_s^2 . R = 20.24^2 \times 18 = \underline{\underline{7.37 kW}}$

(Check $P = V_p I_p \cos\phi = 150 \times 8096 \times \cos 52.6 = 7.37 kW$)

(iii) The input power factor is $\cos 52.6 = \underline{\underline{0.607 \text{ (lagging)}}}$

VA rating $= 150 \times 80.96 = \underline{\underline{12.14 kVA}}$

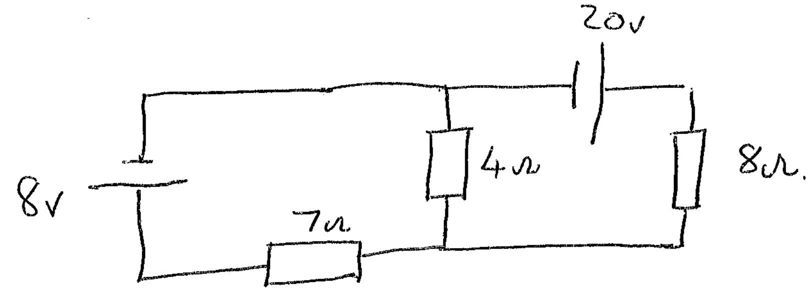
(iv) Since $V_{rms} = 4.44 f N \phi_{max}$

then $N_p = \dfrac{V_{rms}}{4.44 \times 50 \times 4 \times 10^{-3}} = \underline{\underline{169 \text{ TURNS}}}$

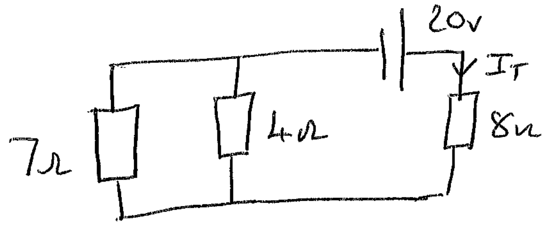
(v) $V_{rms} = 4.44 \times 60 \times 169 \times 4 \times 10^{-3} = \underline{\underline{180 V_{rms}}}$

QUESTION 3

(a)



First consider the 20V source; short out the 8V source.



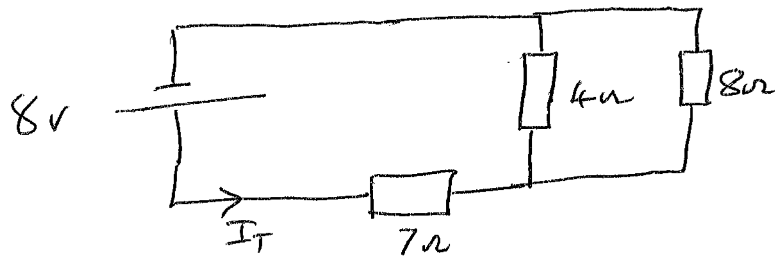
$$R_T = 8 + \frac{1}{\frac{1}{7}+\frac{1}{4}} = 10.545\,\Omega$$

$$\therefore I_T = \frac{20}{10.545} = 1.896\text{A}$$

Therefore the current through the 4Ω resistor from the 20V source is:

$$I_{4\Omega} = \frac{1.896 \times 7}{4+7} = 1.207\text{A} \uparrow$$

Now consider the 8V source; short circuit the 20V source!



$$R_T = 7 + \frac{1}{\frac{1}{4}+\frac{1}{8}} = 9.67\,\Omega$$

$$\therefore I_T = \frac{8}{9.67} = 0.828\text{A}$$

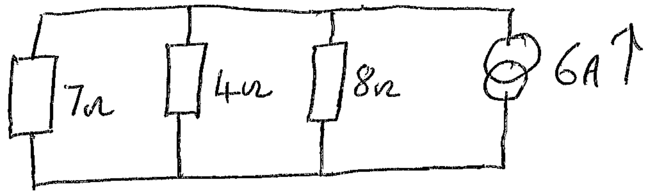
Therefore the current through the 4Ω resistor from the 8V source is $0.828 \times \frac{8}{4+8} = 0.552\text{A} \uparrow$

Therefore the total current through the 4Ω resistor is:

$$I_{4\Omega_{TOT}} = 1.207\uparrow + 0.552\uparrow = \underline{\underline{1.759\text{A}}} \uparrow$$

(b) Using the analysis from part (a) it is only necessary to find the additional contribution from the current source:

Short out the other voltage sources:



$$R_T = \frac{1}{\frac{1}{7}+\frac{1}{4}+\frac{1}{8}}$$

$$= 1.931\,\Omega$$

Voltage across the resistors is therefore:

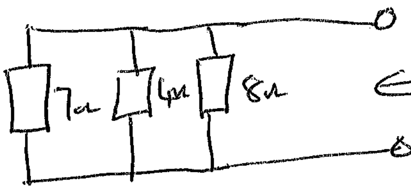
$$V_T = I \times R = 1.931 \times 6 = 11.586\,V \uparrow$$

$$\therefore I_{4\Omega} = \frac{11.586}{4} = 2.897\,A \downarrow$$

Hence the total current in the 4Ω resistor due to all 3 sources is:

$$I_{4\Omega TOT} = 1.759 \uparrow + 2.897 \downarrow = \underline{\underline{1.138 \downarrow A}}$$

(c) To find the Thevenin circuit first short out the voltage sources and find the resistance looking into the terminals.



$$\Leftarrow R_T = \frac{1}{\frac{1}{7}+\frac{1}{4}+\frac{1}{8}} = \underline{\underline{1.931\,\Omega}}$$

The Thevenin voltage is the voltage between A & B without the load connected. In this circuit it is the voltage across the 8Ω resistor. Find the current through the 8Ω resistor using the results from part (a).

Current through the 8Ω resistor due to the 20V source is:

$$I_{8\Omega} = 1.896A \downarrow$$

and the current due to the 8V source is:

$$I_{8\Omega} = \frac{4}{12} \times 0.828 = 0.276A \uparrow$$

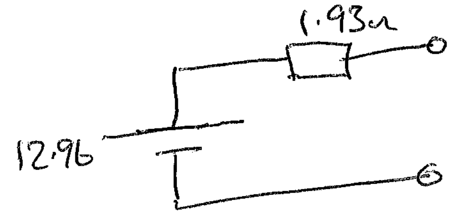
Hence the total current through the 8Ω resistor is:

$$I_{8\Omega TOT} = 1.896 \downarrow + 0.276A \uparrow = \underline{\underline{1.62A \downarrow}}$$

Therefore the voltage across the 8Ω resistor is:

$$1.62 \times 8 = 12.96V \uparrow$$

Therefore the Thevenin circuit appears as:



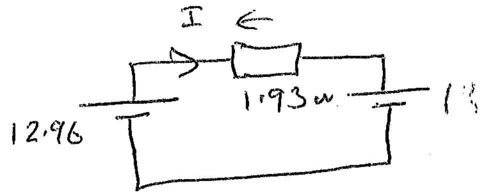
The current through a 6Ω load connected between A and B is:

$$I = \frac{12.96}{(1.93+6)} = 1.634A$$

Hence the power dissipated in the load is

$$P = I^2R = 1.634^2 \times 6 = \underline{\underline{16W}}$$

(d)



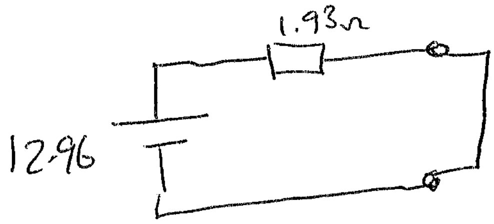
Applying Kirchoff's Law:

$$12.96 - I1.93 - 18 = 0$$

$$\therefore \underline{\underline{I = -2.61A}}$$

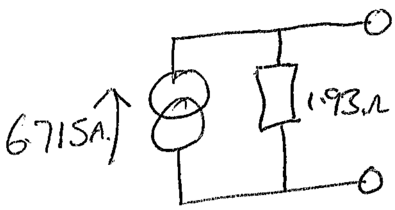
Current is flowing from the rechargeable battery.

(e) For the Norton current short circuit the terminals of the Thevenin equivalent circuit.

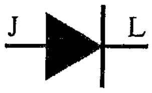


$$I_N = \frac{12.96}{1.93} = \underline{\underline{6.715A}}$$

Hence the Norton equivalent circuit is:

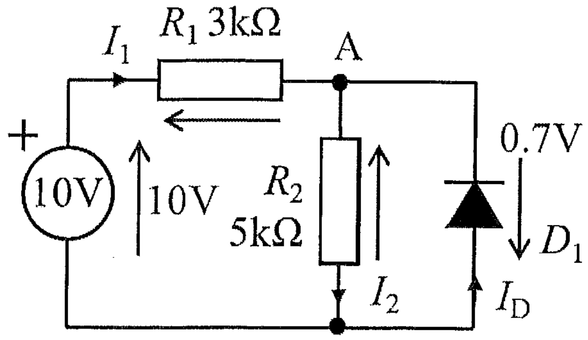


4. **Solutions to question 4**

a. 

J is the ANODE and **L** the CATHODE **(1)**

b. To conduct, **J** (the anode) must be more positive with respect to **L** (the cathode) **(1)**

c. 

By observation it is clear that D_1 is reverse biased and so **NOT conducting.** We can confirm this by calculation. Kirchoff's current law about node A is

$I_2 = I_1 + I_D$ (1)

Using Kirchoff's voltage law on each loop:

$0 = -10V + 3k\Omega . I_1 + 5k\Omega . I_2$ (2)

$0 = -5k\Omega . I_2 - 0.7V$ (3)

From (3) $I_2 = \frac{-0.7V}{5k\Omega} = -140\mu A.$

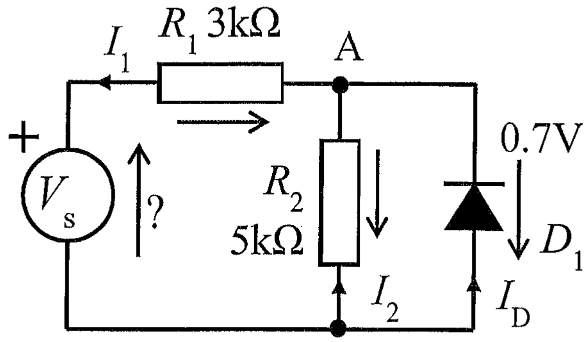
Now using (2) $3k\Omega . I_1 = 10V - 5k\Omega . I_2 = 10V - 5k\Omega . (-140\mu A)$

So

$I_1 = \frac{10V + 0.7V}{3k\Omega} = 3.567mA$ Now substitute these answers into (1)

$I_D = I_2 - I_1 = -140\mu A - 3.567mA = -3.707mA$ Therefor I_D does not flow in direction shown hence D_1 **cannot be conducting** and so must be reverse biased. **(2)**

d. Basic assumptions: diode is conducting $V_D = 0.7V$ but current through, I_D, is zero.



Redraw the current direction with that expected. If diode is conducting must have the same volt drop across both D_1 and $R_2 = 0.7V$ We are assuming $I_D = 0$ therefore $I_2 = I_1$ therefore R_1 & R_2 behave as two resistors in series (sharing same current). Hence, as we know the volt drop across R_2 (0.7V) so we can find that across both R_1 & R_2, which is V_S.

$V_{R2} = \frac{-V_S . R_2}{R_1 + R_2}$ so $V_S = \frac{-V_{R2} . (R_1 + R_2)}{R_2}$

$V_S = \frac{-0.7V . (3k\Omega + 5k\Omega)}{5k\Omega} = -1.12V$ Notice voltage is in opposite direction to arrow on diag. **(5)**

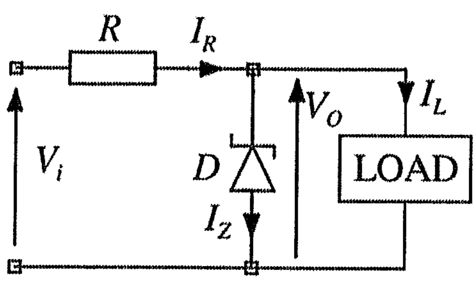
e. 

Figure 4.3

If desired $V_o = 15V$ then Zener voltage V_Z must also be 15V, so a Zener diode with a **(1)** reverse breakdown voltage of 15V is required.

f. For V_o = 15V then the largest value of R allowable will be such that, even when V_i is at its lowest, there will still be sufficient current flowing through this resistor to provide both the maximum possible load current I_L and also at least the minimum required Zener current $I_{Z\,MIN}$ too.

$V_R = V_{i\,MIN} - V_Z$ where $V_{i\,MIN} = 20V - 1V = 19V$ to take into account the ripple

Therefore $V_R = 19V - 15V = 4V$ We also know that $I_R = I_{L\,MAX} + I_{Z\,MIN}$

Where $I_{Z\,MIN}$ = 5mA and $I_{L\,MAX}$ = 100mA so $I_R = 100mA + 5mA = 105mA$

So $R_{MAX} = \frac{V_R}{I_R} = \frac{19V}{105mA} = 38.1\Omega$ **(6)**

g. Without any load connected then all I_R will also flow through I_Z. The largest current will flow when the input voltage is at its highest, that is $V_{i\,MAX} = 20V + 1V = 21V$ so

$V_{R\,MAX} = 21V - 15V = 6V$ As no current flows through the load then I_L = 0 hence

$I_{R\,MAX} = I_{Z\,MAX} = \frac{V_{R\,MAX}}{R} = \frac{6V}{38.1\Omega} = 157.1mA$ **(2)**

h. Maximum dissipated by the Zener diode will be when the maximum current is flowing through the diode. This will be when we have no load connected, as in part g. so

$$P_{Z\,MAX} = V_Z . I_{Z\,MAX} = 15V * 157.1mA = 2.36\,Watts$$ **(2)**

5. <u>Solutions to question 5</u>

a. (i) JFET, (ii) n-p-n BJT, (iii) MOSFET & (iv) p-n-p BJT **(4)**

A – Gate, B – Emitter, C – Gate & D – collector **(4)**

b. i. When the transistor has been on for a long time (the time constant of the resistor and inductor in series is $\tau = L/R$ = 10/50 =0.2 S so 10 S is 50 time constants) then V_L will have fallen to almost zero (the inductor now appears like a short circuit) and so the voltage across R_L can be found as follows

$15V = V_L + V_R + V_{CE}$ where V_L = 0 and $V_{CE\,SAT}$ = 250 mV.

So $V_R = 15V - V_L - V_{CE} = 15V - 0 = 250mV = 14.75V$

The collector current I_C will be the same as the resistor current I_R (diode D_1 being reverse biased and so not conducting). So $I_R = I_C = \frac{V_R}{50\Omega} = \frac{14.75V}{50\Omega} = 295\text{mA}$ **(3)**

ii. To find size of R_2 we need to find the minimum current flowing through it (which also forms I_B) in order to ensure we have the desired collector current I_C. To find the minimum I_B we will use $h_{FE\,MIN}$

$H_{FE\,MIN} = \frac{I_C}{I_{B\,MIN}} = 80$ so $I_{B\,MIN} = \frac{I_C}{H_{FE\,MIN}} = \frac{295mA}{80} = 3.69mA$

When the transistor is in the on-state (input voltage V_1 = 3.3 V) then V_{BE} = 0.7 V so

$$R_{2\,MAX} = \frac{V_{R2}}{I_{B\,MIN}} = \frac{V_1 - V_{BE}}{I_{B\,MIN}} = \frac{3.3V - 0.7V}{3.69mA} = \frac{2.6V}{3.69mA} = 704\Omega$$ **(3)**

iii In order to ensure the transistor is fully turned on, designers often use a resistor 2 or 3 **(1)**

. times smaller, hence ensuring the base current is 2 to 3 times bigger than the minimum that the H_{FE} calculation suggests is necessary thereby over driving the transistor.

iv. The current following in the inductor immediately after the transistor changes to the off-state is the same as that flowing just before (as the inductor will ensure that is so), which was found in part **i.** hence $I_L = 295$ mA **(1)**

v. When the transistor enters the off-state, the inductor back-EMF that attempts to maintain the inductor current makes the lower end of the inductor more positive than the upper, hence diode D1 will be forward biased. D1 has a forward volt drop of 0.7 V therefore the voltage at the collector of the transistor (now off) will be higher than the supply voltage by +0.7 V hence

$$V_{CE} = 15V + 0.7V = 15.7V$$ **(2)**

vi. Maximum current flows in D_1 just after the transistor turns off and the inductor back-EMF tries is trying to maintain the current that existed in the inductor L just before turn-off (as calculated in part **iv.**) so the current in D_1 is the same as $I_L = I_D = 295$ mA. As the forward volt drop of the diode is 0.7 V then power in the diode can be calculated as:

$P_D = V_D * I_D = 0.7 * 295mA = 207mW$ **(2)**

6. Solutions to question 6

a. This configuration is a **non-inverting** amplifier. **(1)**

b. As the inputs of an op-amp are high impedance, we can therefore assume all the current flowing through R_2 must also flow through R_1 as well hence they can be treated as in series. Therefore the v^- input can be found from v_o using the voltage divider principle.

$v^- = v_o \frac{R_1}{R_1+R_2}$

We are also told to assume $v^- = v^+ = v_i$ hence $v_{in} = v_o \frac{R_1}{R_1+R_2}$ So $\frac{v_o}{v_{in}} = \frac{R_1+R_2}{R_1}$ **(3)**

c. Start with $v_o = A_v(v^+ - v^-)$ then create expressions for each of the two op-amp. inputs.

$v^- = v_o \frac{R_1}{R_1+R_2}$ and $v^+ = v_i$ so $v_o = A_v(v^+ - v^-)$ becomes $v_o = A_v\left(v_i - v_o \frac{R_1}{R_1+R_2}\right)$

Rearranging $v_o = A_v\left(v_i - v_o \frac{R_1}{R_1+R_2}\right)$ dividing through by A_v gives $\frac{v_o}{A_v} = v_i - \frac{v_o.R_1}{R_1+R_2}$

collecting terms $v_o\left(\frac{1}{A_v} + \frac{R_1}{R_1+R_2}\right) = v_i$ now put left-hand side over a common denominator $v_o\left(\frac{(R_1+R_2)+A_v.R_1}{A_v.(R_1+R_2)}\right) = v_i$ cross multiply $\frac{v_o}{v_i} = \left(\frac{A_v.(R_1+R_2)}{(R_1+R_2)+A_v.R_1}\right)$ and divide top and bottom by $(R_1 + R_2)$ leaving $\frac{v_o}{v_i} = \frac{1}{\frac{1}{A_v.} + \frac{R_1}{(R_1+R_2)}}$

(6)

d. As $A_v \to \infty$ then v^- is a virtual earth and so we can assume $v^- = 0\ V$ as well.

Using Kirchoff's current law we have $0 = i_f + i_1 + i_2$ re-writing as $i_f = -(i_1 + i_2)$

By ohm's law we know $i_f = \frac{v_o}{R_f}$, $i_1 = \frac{v_1}{R_1}$ and $i_2 = \frac{v_2}{R_2}$ so $\frac{v_o}{R_f} = -\left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right)$ and **(6)** finally multiplying through by R_f then gives $v_o = -\left(v_1 \frac{R_f}{R_1} + v_2 \frac{R_f}{R_2}\right)$

e. The output is made up of two components that can be dealt with separately (an approach that resembles superposition). The DC offset -8V component of the output will be made from the $v_2 = 2\ V$ input so requires a gain of -4.

Therefore $-8V = -\left(v_2 \frac{R_f}{R_2}\right)$ so $-8V = -2V.\frac{R_f}{20k\Omega}$ hence $R_f = \frac{-8V.20k\Omega}{-2} = 80k\Omega$

The sinusoidal component $-6.\sin(\omega t)\ V$ will be made from $v_1 = \sin(\omega t)\ V$ so requires a gain of -6.

$-6.\sin(\omega t) = -\left(v_1 \frac{R_f}{R_1}\right)$ so $-6.\sin(\omega t) = -\left(\sin(\omega t).\frac{80k\Omega}{R_1}\right)$ and so

$R_1 = -\left(\frac{\sin(\omega t).80k\Omega}{-6.\sin(\omega t)}\right)$ and finally $R_1 = \frac{80k\Omega}{6} = 13.333k\Omega$

(4)