Tutorial Sheet – No 1 Answers

1 For a general complex number, A + jB, the magnitude and phase angle for polar conversion are given by:

$$Mag = \sqrt{A^2 + B^2} \quad \text{and} \quad \phi = \tan^{-1}\frac{B}{A}$$

applying these to the examples:

(i) $Mag = \sqrt{\sqrt{3}^2 + 1^2} = 2$ and $\phi = \tan^{-1}\frac{1}{\sqrt{3}} = 30^\circ$ i.e. **2∠30°**

(ii) $Mag = \sqrt{(-3)^2 + 5^2} = 5.83$ and $\phi = \tan^{-1}\frac{5}{-3} = 121^\circ$ i.e. **5.83∠121°**

Note: When obtaining the phase angle it is useful to draw an argand diagram to ensure the angle is in the correct quadrant.

Performing polar to complex calculations:

$$A = Mag\cos\phi \quad \text{and} \quad B = Mag\sin\phi \text{ so}$$

applying these to the examples:

(i) $A = 2\cos 30^\circ = \sqrt{3}$ and $B = 2\sin 30^\circ = 1$ i.e. **√3 + j1**

(ii) $A = 5.83\cos 121^\circ = -3$ and $B = 5.83\sin 121^\circ = 5$ i.e. **-3 + j5**

2 In this question you should try and use the conversion buttons on your calculator. If your calculator does not have this facility use the equations from question 1.

(i) $Mag = \sqrt{3^2 + 3^2} = 4.24$ and $\phi = \tan^{-1}\frac{3}{3} = 45^\circ$ i.e. **4.24∠45°**

(ii) $Mag = \sqrt{2^2 + \left(2\sqrt{3}\right)^2} = 4$ and $\phi = \tan^{-1}\frac{2\sqrt{3}}{2} = 60^\circ$ i.e. **4∠60°**

(iii) $Mag = \sqrt{(-3)^2 + (-4)^2} = 5$ and $\phi = \tan^{-1}\frac{-4}{-3} = -126.9^\circ$ i.e. **5∠-126.9°**

and vice-versa:

(i) $A = 4.24\cos 45^\circ = 3$ and $B = 4.24\sin 45^\circ = 3$ i.e. **3 + j3**

(ii) $A = 4\cos 60^\circ = 2$ and $B = 4\sin 60^\circ = 2\sqrt{3}$ i.e. **2 + j2√3**

(iii) $A = 5\cos\left(-126.9^\circ\right) = -3$ and $B = 5\sin(-126.9)^\circ = -4$ i.e. **– 3 – j4**

3 For addition and subtraction of polar quantities convert to complex first. For multiplication and division of complex numbers convert to polar first.

$$3\angle 22^\circ = 2.78 + j1.12 \quad \text{and} \quad 4\angle 112^\circ = -1.5 + j3.71$$

(i) summing complex numbers we get **1.28 + j4.83**

$Mag = \sqrt{1.28^2 + 4.83^2} = 5$ and $\phi = \tan^{-1}\frac{4.83}{1.28} = 75.1^\circ$ i.e. **5∠75.1°**

(ii) multiplication use polar form:

$$3\angle 22^\circ \times 4\angle 112^\circ = 3\times 4\angle\left(22^\circ + 112^\circ\right) = \mathbf{12\angle 134^\circ}$$

$A = 12\cos 134^\circ = -8.34$ and $B = 12\sin 134^\circ = 8.63$ i.e. **– 8.34 + j8.63**

(iii) division use polar form:

$$3\angle 22^\circ \div 4\angle 112^\circ = 3\div 4\angle\left(22^\circ - 112^\circ\right) = \mathbf{0.75\angle -90^\circ}$$

$A = 0.75\cos -90^\circ = 0$ and $B = 0.75\sin -90^\circ = -0.75$ i.e. **0 – j0.75**

$$1+j2 = 2.24\angle 63.4^\circ \quad \text{and} \quad 3+j4 = 5\angle 53.1^\circ$$

(iv) summing complex numbers we get **4 + j6**

$$Mag = \sqrt{4^2+6^2} = 7.21 \quad \text{and} \quad \phi = \tan^{-1}\frac{6}{4} = 56.3^\circ \quad \text{i.e. } \mathbf{7.21\angle 56.3^\circ}$$

(v) dividing complex numbers use polar form:

$$2.24\angle 63.4^\circ \div 5\angle 53.1^\circ = 2.24 \div 5\angle\left(63.4^\circ - 53.1^\circ\right) = \mathbf{0.45\angle 10.3^\circ}$$

$$A = 0.45\cos 10.3^\circ = 0.44 \quad \text{and} \quad B = 0.45\sin 10.3^\circ = 0.08 \quad \text{i.e. } \mathbf{0.44 + j0.08}$$

4 (a) Calculate the impedance in polar form:

$$Z = 50 + j50 = 70.7\angle 45^\circ \ \Omega$$

It is normal to take the voltage as reference:

$$I = \frac{V}{Z} = \frac{40\angle 0^\circ}{70.7\angle 45^\circ} = \mathbf{0.57\angle -45^\circ\ A}$$

(b) Same method as (a) but first we need to obtain the reactance of the inductor at 5kHz.

$$Z = R + j2\pi fL = 50 + j2\pi\times 5000\times 0.002 = 50 + j62.8 = 80.3\angle 51.5^\circ\ \Omega$$

and:

$$I = \frac{V}{Z} = \frac{40\angle 0^\circ}{80.3\angle 51.5^\circ} = \mathbf{0.5\angle -51.5^\circ\ A}$$

(c) First obtain the capacitive reactance and obtain the impedance:

$$Z = R - \frac{j}{2\pi fC} = 50 - \frac{j}{2\pi\times 5000\times 0.2\times 10^{-6}} = 50 - j159 = 166.7\angle -72.5^\circ\ \Omega$$

and:

$$I = \frac{V}{Z} = \frac{40\angle 0^\circ}{166.7\angle -72.5^\circ} = \mathbf{0.24\angle 72.5^\circ\ A}$$

(d) The total impedance is:

$$Z = R + j2\pi fL - \frac{j}{2\pi fC} = 50 + j50 - j50 = 50 + j0 = 50\angle 0^\circ\ \Omega$$

and:

$$I = \frac{V}{Z} = \frac{40\angle 0^\circ}{50\angle 0^\circ} = \mathbf{0.8\angle 0^\circ A}$$

5 (a) In this part, since the supply voltage appears across each limb it is easiest to calculate the current in each limb and sum them. Use the impedance values from the last question.

$$Z_1 = R_1 + j2\pi fL = 50 + j2\pi\times 5000\times 0.002 = 50 + j62.8 = 80.3\angle 51.5^\circ\ \Omega$$

and:

$$I_1 = \frac{V}{Z} = \frac{40\angle 0^\circ}{80.3\angle 51.5^\circ} = \mathbf{0.5\angle -51.5^\circ = 0.31 - j0.39\ A}$$

$$Z_2 = R_2 - \frac{j}{2\pi fC} = 50 - \frac{j}{2\pi\times 5000\times 0.2\times 10^{-6}} = 50 - j159 = 166.7\angle -72.5^\circ\ \Omega$$

and:

$$I = \frac{V}{Z} = \frac{40\angle 0^\circ}{166.7\angle -72.5^\circ} = \mathbf{0.24\angle 72.5^\circ = 0.07 + j0.23\ A}$$

Summing these complex currents we get:

$$I_{TOT} = 0.31 - j0.39 + 0.07 + j0.23 = 0.38 - j0.16 = \mathbf{0.41\angle -22^\circ\ A}$$

(b) In this case we do not know the voltage across each limb as there is a voltage drop across the series resistor. We therefore have to calculate the total impedance before the current can be found.

The total impedance is given by:

$$Z_{TOT} = R + \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = R + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Depending on your calculator will determine the best method to use to find the impedance. Usually use polar form for multiplication and division and complex form for addition and subtraction.

$$Z_{TOT} = R + \frac{Z_1 Z_2}{Z_1 + Z_2} = 50 + \frac{80.3\angle 51.5^\circ \times 166.7\angle -72.5^\circ}{50 + j62.8 + 50 - j159} = 50 + \frac{13386\angle -21^\circ}{100 - j96.2}$$

$$Z_{TOT} = 50 + \frac{13386\angle -21^\circ}{138.8\angle -43.9^\circ} = 50 + 96.4\angle 22.9^\circ = 50 + 88.8 + j37.5 = 138.8 + j37.5$$

$$Z_{TOT} = 143.8\angle 15.1^\circ\ \Omega$$

and:

$$I_{TOT} = \frac{V}{Z_{TOT}} = \frac{40\angle 0^\circ}{143.8\angle 15.1^\circ} = \mathbf{0.28\angle -15.1^\circ\ A}$$