Q1.

(a) $c_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_o t}dt = \frac{1}{T}\int_0^{T/2} Ae^{-jn\omega_o t}dt = -\frac{A}{jn\omega_o T}e^{-jn\omega_o t}\Big|_0^{T/2}$

$$c_n = \frac{A}{jn\left(\frac{2\pi}{T}\right)T}\left[1-e^{-jn\omega_o T/2}\right] = \frac{A}{j2n\pi}\left[1-e^{-jn(2\pi/T)T/2}\right] = \frac{A}{j2n\pi}\left[1-e^{-jn\pi}\right]$$

$$c_n = \begin{cases} 0 & n = even \\ \frac{A}{jn\pi} & n = odd \end{cases}.$$

$$c_n = \begin{cases} 0 & n = even \\ -j\frac{A}{n\pi} & n = odd \end{cases}$$

Note that $e^{-jn\pi} = \cos(n\pi) - j\sin(n\pi) = 1$ for n = even and $e^{-jn\pi} = \cos(n\pi) - j\sin(n\pi) = -1$ for n = odd.

The dc component,

$$c_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{T}\int_0^{T/2} Adt = \frac{A}{T}t\Big|_0^{T/2} = \frac{A}{2}.$$

For n>0 $\angle c_n = \tan^{-1}(-\infty) = -\frac{\pi}{2}$

For n<0 $\angle c_n = \tan^{-1}(\infty) = \frac{\pi}{2}$

Finally we have,

$$x(t) = \sum_{n=-\infty}^{\infty}|c_n|e^{j(n\omega_o t+\angle c_n)} = c_0 + \sum_{n=1}^{\infty}|c_n|e^{j(n\omega_o t+\angle c_n)} + \sum_{n=-\infty}^{-1}|c_n|e^{j(n\omega_o t+\angle c_n)}.$$

Since $c_n = 0$ when n = even number, we have

$$x(t) = \frac{A}{2} + \sum_{m=1}^{\infty}\left|\frac{A}{(2m-1)\pi}\right|e^{j((2m-1)\omega_o t-\pi/2)} + \sum_{m=-\infty}^{-1}\left|\frac{A}{(2m+1)\pi}\right|e^{j((2m+1)\omega_o t+\pi/2)}.$$

(b) After low pass filtering $v(t) = \frac{1}{2} + \frac{1}{\pi}e^{j(\omega_o t-\pi/2)} + \frac{1}{\pi}e^{-j(\omega_o t-\pi/2)}$.

The input signal is a dc at 1 V. The power can be calculated using Parsevals' theorem.

Therefore the conversion efficiency is $\frac{power\quad out}{power\quad in} = \frac{\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{\pi}\right)^2}{1^2} = \frac{1}{4} + \frac{2}{\pi^2}$

(c) The average power $= \frac{1}{T}\int_{<T>}|y(t)|^2 dt$.

However we can also use the Parseval's theorem to find the average power.

$$y(t) = \frac{e^{j2(t-3)} + e^{-j2(t-3)}}{2} + \frac{e^{j10(t-3)} + e^{-j10(t-3)}}{2}$$

$$= \frac{e^{j2t}e^{-j6}}{2} + \frac{e^{-j2t}e^{j6}}{2} + \frac{e^{j10t}e^{-j30}}{2} + \frac{e^{-j10t}e^{j30}}{2}.$$

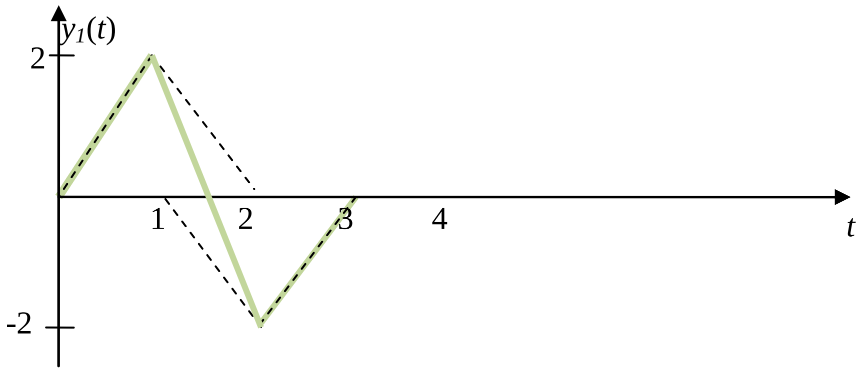
Parseval's theorem states:

The average power $P_{ave} = \sum_{n=-\infty}^{\infty} |c_n|^2$.

Assuming ω_o =1, we have, $|c_{-10}| = |c_{10}| = \frac{1}{2}$ and $|c_{-2}| = |c_2| = \frac{1}{2}$, since $|e^{j30}| = |e^{-j30}| = 1$ and $|e^{j6}| = |e^{-j6}| = 1$. Therefore, $P_{ave} = |c_{-10}|^2 + |c_{-2}|^2 + |c_2|^2 + |c_{10}|^2 = 4 \times \left(\frac{1}{2}\right)^2 = 1$.

Q2.

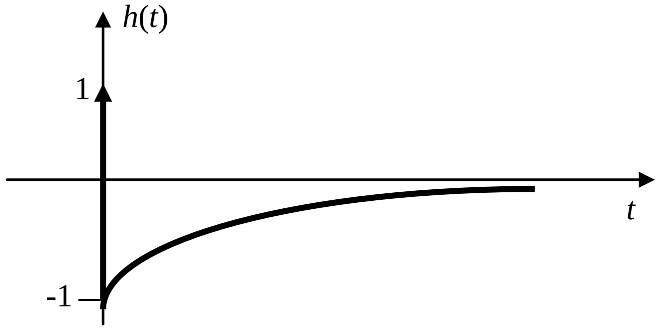
(a) $x_1(t)=2x(t)-2x(t-1)$. Therefore the output is 2y(t)-2y(t-1)



(b) The transfer function is given by

$$H(s) = \frac{R}{R+1/sC} = \frac{sRC}{1+sRC} = \frac{1+sRC}{1+sRC} - \frac{1}{1+sRC} = 1 - \frac{1}{1+sRC}.$$

The impulse response is therefore δ(t) – exp(-t/RC).u(t).



(c) At t <0, $y(t) = x(t) * h(t) = \int_{-\infty}^{t} e^{\tau} e^{-(t-\tau)} d\tau = \int_{-\infty}^{t} e^{-t+2\tau} d\tau = e^{-t}\left[\frac{e^{2\tau}}{2}\right]_{-\infty}^{t} = \frac{1}{2}e^{t}$

At t > 0, $y(t) = x(t) * h(t) = \int_{-\infty}^{0} e^{\tau} e^{-(t-\tau)} d\tau = \int_{-\infty}^{0} e^{-t+2\tau} d\tau = e^{-t}\left[\frac{e^{2\tau}}{2}\right]_{-\infty}^{0} = \frac{1}{2}e^{-t}$.

Therefore we have

$$y(t) = \begin{cases} \frac{1}{2}e^{t} & t < 0 \\ \frac{1}{2}e^{-t} & t > 0 \end{cases}$$

Q3 First we need to work out P(ω)
The period = T.

The Fourier series coefficient = $C_n = \frac{1}{T}\int_{-T}^{T}\delta(t)e^{-jn\omega_s t}dt$ where $\omega_s = \frac{2\pi}{T}$.

$$= \frac{1}{T}e^{-jn\omega_s(0)} = \frac{1}{T}$$

Therefore the complex Fourier series is

$$p(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_s t} = \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{jn\omega_s t}$$

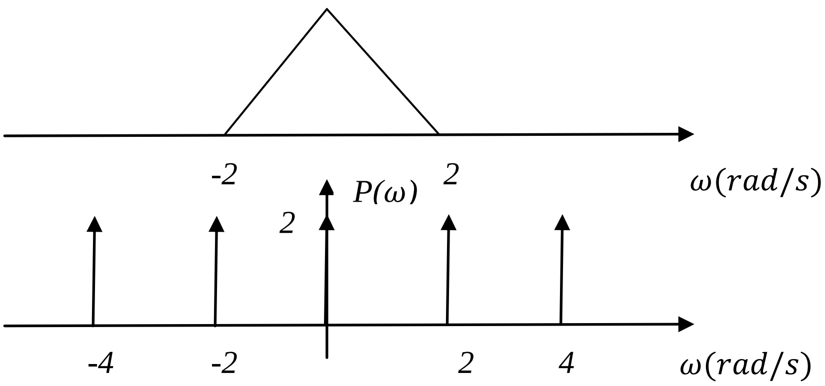
The FT of $e^{j\omega_s t}$ is $2\pi\delta(\omega - \omega_s)$.

Therefore the FT of *p(t)* is

$$P(\omega) = \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega - n\omega_s) = 2\sum_{n=-\infty}^{\infty}\delta(\omega - 2n)$$ since

ωs = 2π/π = 2.

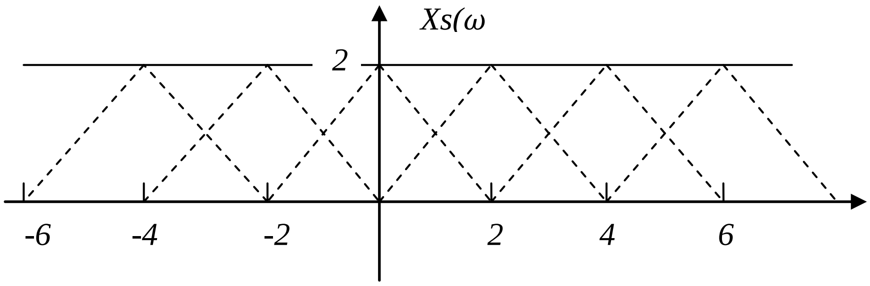
(b)
We have



if $\omega_s = 2\,rad/s,$

Let $X_S(\omega) = X(\omega) * P(\omega)$ since $x_s(t) = x(t)\cdot p(t)$.

Therefore we have



Low pass filtering will not recover the signal.

Since $\omega_s = 2\,rad/s$ equals to the largest frequency present in $X(\omega)$, the Nyquist sampling theorem has not been satisfied. Hence severe aliasing leading to a constant of 2.

(c) i)Let Ac=1. $m(t) = A_m\cos(\omega_m t)$ $c(t) = \cos(\omega_c t)$ $x(t) = (A_o + m(t))c(t)$

,

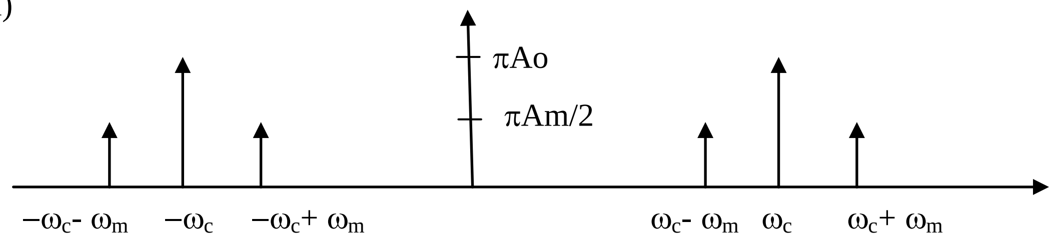
$$x(t) = (A_o + A_m\cos(\omega_m t))\cos(\omega_c t) = A_o\cos(\omega_c t) + A_m\cos(\omega_c t)\cos(\omega_m t)$$

$$= A_o\cos(\omega_c t) + \frac{A_m}{2}\left[\cos(\omega_m - \omega_c)t + \cos(\omega_m + \omega_c)t\right]$$

Therefore

$$X(\omega) = \pi A_0[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)] + \frac{\pi A_m}{2}[\delta(\omega+\omega_m-\omega_c) + \delta(\omega-\omega_m-\omega_c) + \delta(\omega+\omega_m+\omega_c) + \delta(\omega-\omega_m+\omega_c)]$$

ii)



Drawback: This modulation scheme requires transmission of the carrier signal and have higher power consumption.
Advantage: Signal can be recovered without the value of ω_m.

Q4

(a) We know that $V_i(t) = V_c(t) + i(t)R$ and $i(t) = C \cdot \frac{dV_c(t)}{dt}$

Therefore $V_i(t) = V_c(t) + RCdV_c(t)/dt$

Taking the Laplace Transform gives

$V_i(s) = V_c(s) + RCsV_c(s) = (1+RCs)V_c(s)$

Since $V_i(t) = A \cdot u(t)$, we have $V_i(s) = A/s$.

Therefore $A/s = V_c(s)(1+RCs)$

$$V_c(s) = \frac{A}{s(1+sRC)} = \frac{A}{RC} \cdot \frac{1}{s(s+\frac{1}{RC})} = \frac{A_1}{s} + \frac{A_2}{(s+\frac{1}{RC})}$$

$$A_1 = \left(\frac{A}{RC} \cdot \frac{1}{\left(s+\frac{1}{RC}\right)}\right)|_{s=0} = A$$

$$A_2 = \left(\frac{A}{RC} \cdot \frac{1}{s}\right)|_{s=-1/RC} = -A$$

Therefore $V_c(s) = A \cdot (\frac{1}{s} - \frac{1}{\left(s+\frac{1}{RC}\right)})$

Taking the reverse Laplace Transform

$$V_c(t) = A(1-e^{-t/RC}) \cdot u(t)$$

b) $i(t) = C \cdot \frac{dV_c(t)}{dt} = C \cdot \frac{d}{dt}\left[A\left(1-e^{-t/RC}\right)\right] = \frac{AC}{RC}e^{-t/RC}$

Since the signal *u(t)=0 for t<0,* $i(t) = \frac{A}{R}e^{-t/RC} \cdot u(t)$.

Or $i(t) = C \cdot \frac{dV_c(t)}{dt}$ then $V_c(t) = \frac{1}{C}\int_0^{\tau} i(t)dt$

$V_c(s) = \frac{I(s)}{sC}$ assuming zero initial condition.

$$I(s) = sC\left[\frac{A}{RC} \cdot \frac{1}{s\left(s+\frac{1}{RC}\right)}\right] = \frac{A}{R} \cdot \frac{1}{\left(s+\frac{1}{RC}\right)}]$$

$i(t) = \frac{A}{R}e^{-t/RC} \cdot u(t)$.

c) At *t=0*, $\mathrm{i}(0) = \frac{\mathrm{A}}{\mathrm{R}} \cdot \mathrm{e}^0 = \frac{\mathrm{A}}{\mathrm{R}}$.

For *i(t)=0.01A/R*, $\mathrm{i}(\mathrm{t}) = \frac{\mathrm{A}}{\mathrm{R}}\mathrm{e}^{-\mathrm{t}/\mathrm{RC}} = 0.01\frac{\mathrm{A}}{\mathrm{R}}$

$$\mathrm{e}^{-\mathrm{t}/\mathrm{RC}} = 0.01$$

$$-t/RC = ln(0.01)$$

$$-t = RCln(0.01) = 4.6ms \ .$$

d) The cutoff frequency $= \frac{1}{2\pi\mathrm{RC}} = \frac{1}{2\pi\cdot 0.001} = 159$ the circuit will allow frequencies >160Hz to pass without significant attenuation.