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7th lecture:

- operational amplifiers (Op-Amps), part I
  - single-stage MOS amplifiers
  - MOS cascodes
  - two-stage MOS Op-Amp

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important equations for MOSFETs: a review

This page summarises the most important equations for MOSFETs. They can be sub-divided into materials equations (blue), definitions (red) and conclusions (green):

drain current:

- general square-law behaviour for ***triode region*** with $v_{DS} \leq V_{GS} - V_t$:
  $I_D = \mu_n C_{ox}\, W/L\, [(V_{GS} - V_t) v_{DS} - ½ v_{DS}^2]$
- modified square-law behaviour ***saturation region*** with $v_{DS} > V_{GS} - V_t$:
  $I_D = ½ \mu_n C_{ox}\, W/L\, (V_{GS} - V_t)^2 (1 + V_{DS}/V_A)$

over-voltage: $V_{ov} = V_{GS} - V_t \approx 0.1 \ldots 0.3\text{V}$

body effect on threshold voltage: $V_t = V_{t0} + (2 q_e N_A \varepsilon_{ox})^{1/2} / C_{ox}\, [(2\Phi_F + |V_{SB}|)^{1/2} - (2\Phi_F)^{1/2}]$

channel length modulation parameter: $\lambda = 1/V_A = \Delta L / L\ 1/V_{DS}$

transconductance: $g_m = \partial I_D / \partial V_{GS} = \mu_n C_{ox}\, W/L\, (V_{GS} - V_t) = 2 I_D / V_{ov}$

$g_{mb} = \partial I_D / \partial V_{BS} = (2 q_e N_A \varepsilon_{ox})^{1/2} / \{C_{ox}\, [2(2\Phi_F + |V_{SB}|)^{1/2}\}$

output resistance: $r_o = 1/(\partial I_D / \partial V_{DS}) = V_A / I_D$

intrinsic voltage gain: $A_o = \partial V_{DS} / \partial V_{GS} = g_m r_o = 2 V_A / V_{ov}$

(sign depends on actual circuit!)

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single-stage MOSFET amplifiers

configuration	common source with feedback res. R_S	common gate	source follower
input res. R_i	R_G	$1/g_m$	R_G
output res. R_o	$r_o \| R_D$	$r_o \| R_D$	$r_o \| 1/g_m$
voltage gain G_v	$-R_G/(R_G+R_{sig})$ $\times\, g_m(r_o \| R_D \| R_L)$ $/\,(1+g_m R_S)$	$g_m(r_o \| R_D \| R_L)$ $/\,(1+g_m R_{sig})$	$R_G/(R_G+R_{sig})$ $\times\, g_m(r_o \| R_L)$ $/\,(1+r_o \| R_L)$

note: R_L denotes the load resistance, $R_{S,G,D}$ the resistance of source, gate or drain, R_{sig} the resistance across which the signal is fed into the input terminal.

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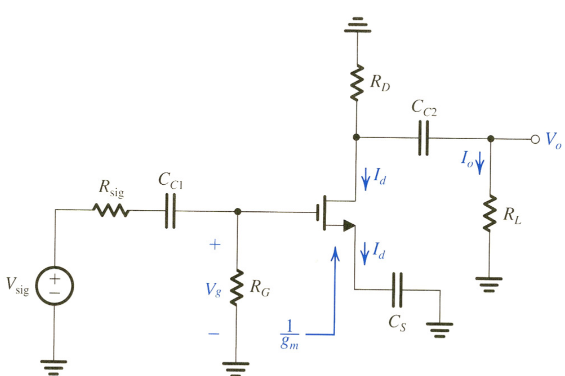
single-stage CS amplifier without r_o: low-frequency gain, I



low-frequency behaviour of CS amplifier, neglecting r_o

voltage at input:

$V_G = V_{sig}\, R_G/[R_G+R_{sig}+1/(j\omega C_{C1})]$

$= V_{sig}\, R_G/(R_G+R_{sig}) \times s/\{s+1/[C_{C1}(R_G+R_{sig})]\}$

is a high-pass filter of form $s/(s+\omega_1)$ with $s=j\omega$ and a pole for a corner frequency of

$\boxed{\omega_{p1}=1/\,[C_{C1}(R_G+R_{sig})]}$

drain current:

$I_D = V_G/[1/g_m+1/(j\omega C_S)]$

$= g_m V_G \times s/(s+g_m/C_S)$

is a high-pass filter with corner frequency $\boxed{\omega_{p2}=g_m/C_S}$

voltage at output:

$V_o = I_o R_L$ where

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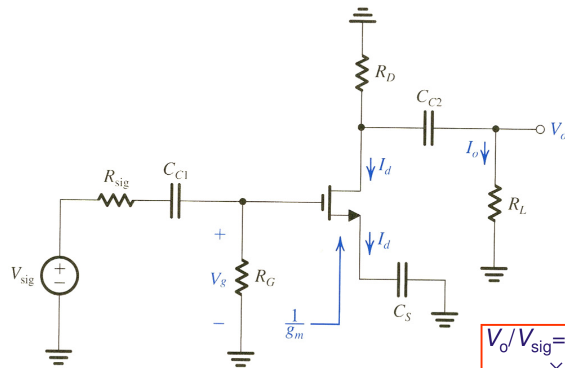
single-stage CS amplifier without r_o: low-frequency gain, II



low-frequency behaviour of CS amplifier, neglecting r_o

the fraction of I_D that flows through R_L is given by:
$I_o = -I_D\, R_D/(R_D+R_L+1/(j\omega C_{C2}))$

insertion yields
$V_o = -I_D\, R_D R_L/(R_D+R_L)$
$\times\, s/\{s+1/[C_{C2}(R_D+R_L)\}$
is another high-pass filter with corner frequency
$\boxed{\omega_{p3} = 1/[C_{C2}(R_D+R_L)\}}$

result for **over-all voltage gain**:

$$\boxed{V_o/V_{sig} = -R_G/(R_G+R_{sig})\,[g_m\,(R_D||R_L)] \times s^3 [1/(s+\omega_{p1})]\,[1/(s+\omega_{p2})]\,[1/(s+\omega_{p3})]}$$

has 3 real low-frequency poles, where 3dB-frequency f_L is given by their maximum, i.e. usually $\boxed{f_L \approx \omega_{p2}/(2\pi)}$

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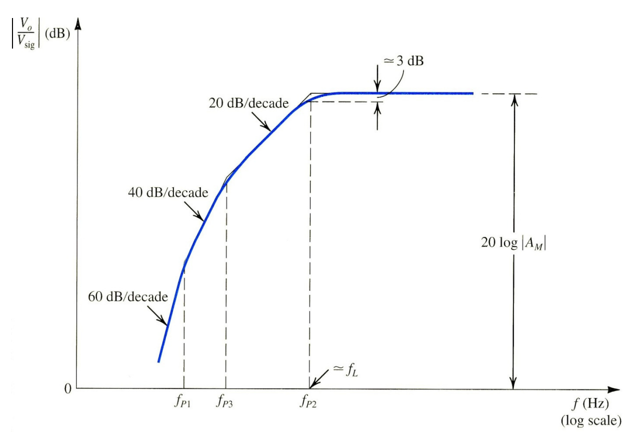
single-stage CS amplifier without r_o: low-frequency gain, III



sketch of low-frequency magnitude response of CS amplifier

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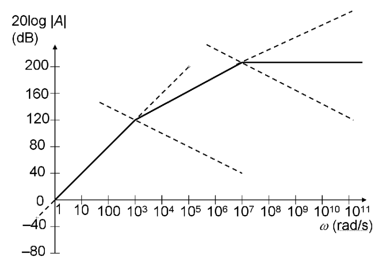
general filter transmission functions

$$T(s)=\frac{s^n}{[1+s/10^a]\,[1+s/10^b]\times\ldots\times[1+s/10^z]}$$



for s=0: zero

(slope=n × +20dB/decade)

for $s=\infty$:

$\rightarrow s^{n-m}$ (where m=# bracketts)

$\rightarrow \begin{cases} \infty \text{ if } n>m \text{ (pole)} \\ 1 \text{ if } n=m \\ 0 \text{ if } m>n \text{ (zero)} \end{cases}$

sketch of typical high pass filter function for n=2, a=3, b=7

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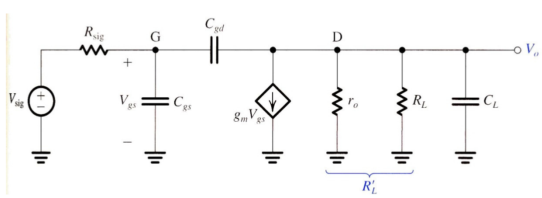
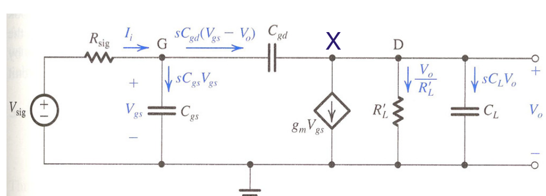
single-stage CS amplifier: high-frequency gain, I



approximation for large R_{sig} and small C_L:
$V_o/V_{sig} \approx -g_m R_L'\, s/(s+\omega_H)$
with upper 3dB-frequency limit $\boxed{f_H \approx 1/(2\pi C_{in} R_{sig})}$
where $C_{in}=C_{GS}+C_{GD}(1+g_m R_L')$
due to the Miller effect.



now perform exact analysis:
consider currents at node X:

$sC_{GD}(V_{GS}-V_o)=$
$g_m V_{GS}+V_o/R_L'+sC_L V_o$

solve for V_{GS}:

$V_{GS}=-V_o/(g_m R_L')$
$\times [1+s(C_L+C_{GD})R_L']$
$/ [1-sC_{GD}/g_m]$

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single-stage CS amplifier: high-frequency gain, II

A loop equation at the input yields: $V_{sig}=I_i R_{sig}+V_{GS}$
where the input current can be calculated from a node equation at the gate G:

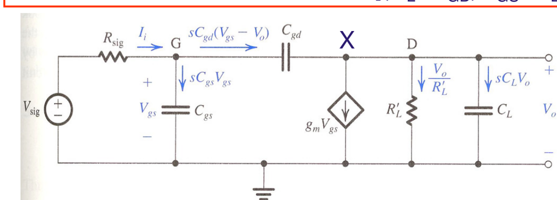
$I_i=sC_{GS}V_{GS} + sC_{GD}(V_{GS}-V_o)$

and can be inserted to obtain

$V_{sig}=V_{GS}[1+s(C_{GS}+C_{GD})R_{sig}]-sC_{GD}R_{sig}V_o$

Inserting the equation for V_{GS} and re-arranging finally yields the voltage gain:

$$\boxed{V_o/V_{sig}=\underbrace{-g_m R_L'}_{\text{DC gain}}(1-sC_{GD}/g_m)\,/\,\{1+s\{[C_{GS}+C_{GD}(1+g_m R_L')]R_{sig}+(C_L+C_{GD})R_L'\}+s^2[(C_L+C_{GD})C_{GS}+C_L C_{GD}]R_{sig}R_L'\}}$$



is a transfer function with 2 zeros in the nominator, namely $s_\infty=\infty$ & $s_Z=g_m/C_{GD}$, where $f_Z=\omega_Z/2\pi$ is very high and so has little effect on the upper bandlimit f_H, and 2 poles in the denominator.

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single-stage CS amplifier: high-frequency gain, III

A loop equation at the input yields: $V_{sig}=I_i R_{sig}+V_{GS}$
where the input current can be calculated from a node equation at the gate G:

$I_i=sC_{GS}V_{GS} + sC_{GD}(V_{GS}-V_o)$

and can be inserted to obtain

$V_{sig}=V_{GS}[1+s(C_{GS}+C_{GD})R_{sig}]-sC_{GD}R_{sig}V_o$

Inserting the equation for V_{GS} and re-arranging finally yields the voltage gain:

$$\boxed{V_o/V_{sig}=\underbrace{-g_m R_L'}_{\text{DC gain}}(1-sC_{GD}/g_m)\,/\,\{1+s[[C_{GS}+C_{GD}(1+g_m R_L')]R_{sig}+(C_L+C_{GD})R_L'\}+s^2[(C_L+C_{GD})C_{GS}+C_L C_{GD}]R_{sig}R_L'\}}$$

Now get the poles by re-writing the denominator D as a product where $\omega_{p2}>>\omega_{p1}$:

$D(s)=(1+s/\omega_{p1})(1+s/\omega_{p2})=1+s(1/\omega_{p1}+1/\omega_{p2})+s^2/(\omega_{p1}\omega_{p2}) \approx 1+s/\omega_{p1}+s^2/(\omega_{p1}\omega_{p2})$

Comparing the coefficients then yields the poles:

$\omega_{p1}\approx 1/\{[C_{GS}+C_{GD}(1+g_m R_L')]R_{sig}+(C_L+C_{GD})R_L'\}$

$\omega_{p2}\approx \{[C_{GS}+C_{GD}(1+g_m R_L')]R_{sig}+(C_L+C_{GD})R_L'\}/\{[(C_L+C_{GD})C_{GS}+C_L C_{GD}]R_{sig}R_L'\}$

Special case of low-resistance signal source ($R_{sig}\to 0$):

$\boxed{V_o/V_{sig}=-g_m R_L'(1-sC_{GD}/g_m)\,/\,\{1+s[(C_L+C_{GD})R_L']\}}$ has only 1 pole (as $\omega_{p2}=\infty$)

$\boxed{\omega_H=\omega_{p1}=1/(C_L+C_{GD})R_L']}$

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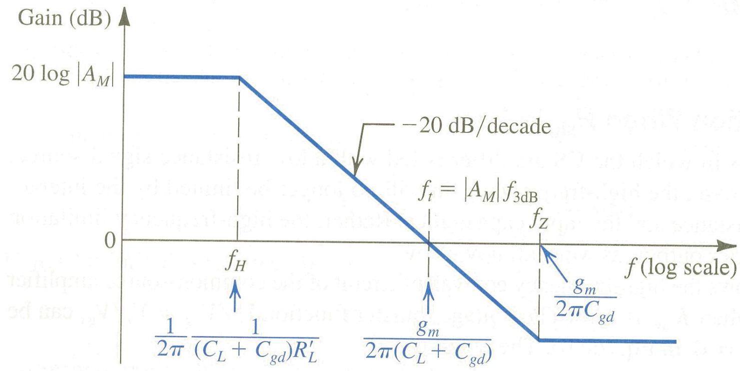
single-stage CS amplifier: high-frequency gain, IV



=upper bandwith f_H

= transition frequency with unity gain
= gain × bandwidth product

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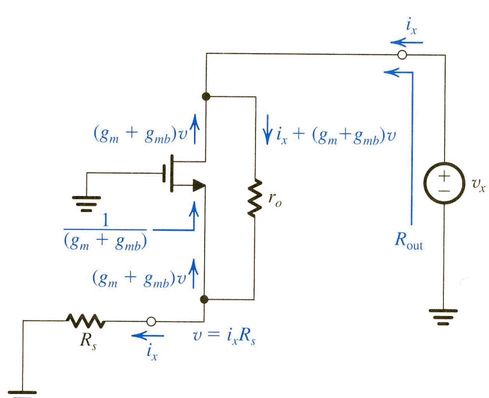
single-stage CG amplifier: output resistance

apply test voltage v_x to output and find current i_x drawn from v_x:

$v_x=[i_x+(g_m+g_{mb})v_S]r_o+v_S$
where $v_S=i_xR_S$,
hence:
$R_{out}= v_x/i_x$
$=r_o+[1+(g_m+g_{mb})r_o]R_S$
$=r_o+A_{vo}R_S$

where A_{vo} is the open-circuit voltage gain, or re-sorted:

$$\boxed{\begin{aligned}R_{out}&=R_S+[1+(g_m+g_{mb})R_S]r_o\\&\approx [1+g_mR_S]r_o\end{aligned}}$$



is enhanced, as in the case of source degeneration of the BJT

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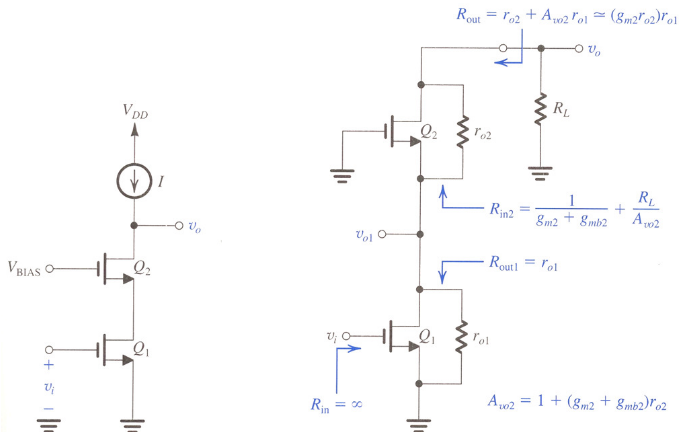
MOS cascode as two-stage amplifier



MOS cascode amplifier (Q_1: CS, Q_2: CG) and small-signal circuit

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MOS cascode as two-stage amplifier: output resistance

consider output voltage:

$v_o = v_{i2} + i r_{o2}$

$= v_{i2} + (g_{m2} + g_{mb2}) v_{i2} r_{o2}$, hence:

$A_{vo2} = v_o / v_{i2} = 1 + (g_{m2} + g_{mb2}) r_{o2}$

Thus input resistance of Q_2 is:

$R_{in2} = (r_{o2} + R_L)/A_{vo2}$

$\approx 1/(g_{m2} + g_{mb2}) + R_L/A_{vo2}$

and total **output resistance** is:

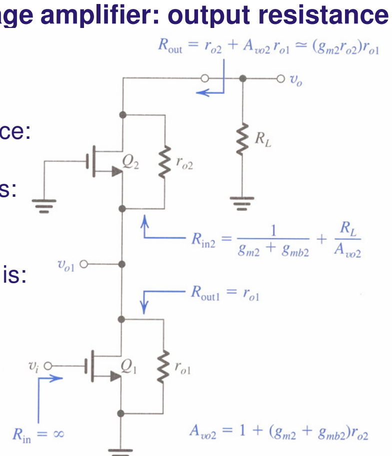
$R_{out} = r_{o2} + A_{vo2} r_{o1}$

$= r_{o2} + [1 + (g_{m2} + g_{mb2}) r_{o2}] r_{o1}$

$\approx r_{o2} + (g_{m2} + g_{mb2}) r_{o2} r_{o1}$

$\approx g_{m2} r_{o2} r_{o1}$

is increased compared to r_{o1} by a factor given by the intrinsic gain $A_{o2} = g_{m2} r_{o2}$ of the second stage



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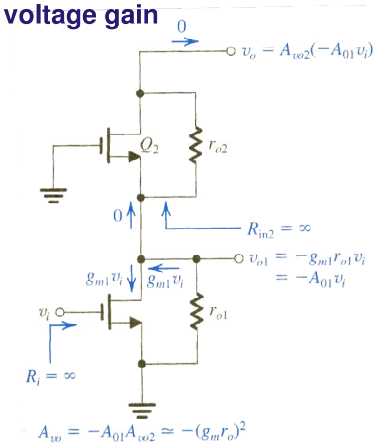
MOS cascode as two-stage amplifier: voltage gain



consider gain of Q_1:
$A_{v1}=v_{o1}/v_i=-g_{m1}r_{o1}=-A_{o1}$
The signal v_{o1} will be further amplified by the open-circuit voltage gain A_{vo2} of Q_2 to get
$v_o=A_{vo2}v_{o1}=-A_{vo2}A_{o1}v_i$
Thus, voltage voltage gain for $R_L\to\infty$ is the product of the two single gains:
$A_{vo}=v_o/v_{in}=-A_{vo2}A_{o1}\approx -A_{o1}A_{o2}$,
which for the usual case of identical transistors becomes
$A_{vo}= -A_o^2 = -(g_m r_o)^2$
The **total voltage gain for finite R_L** is:

$$\boxed{\begin{aligned} A_v &= -A_{o1}A_{o2}\, R_L/(R_L+A_{o2}r_{o2}) \\ &\approx -A_o^2\, R_L/(R_L+A_o r_o) \end{aligned}} = \begin{cases} -A_o^2 & \text{for } R_L=\infty \\ -\tfrac{1}{2}A_o^2 & \text{for } R_L=A_o r_o \\ -A_o & \text{for } R_L=r_o \text{, as for } Q_1 \text{ alone!} \end{cases}$$

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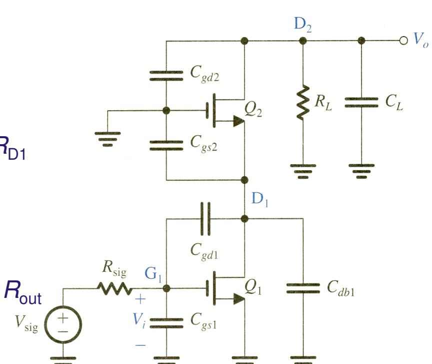
MOS cascode as two-stage amplifier: high-frequency response, I

consider frequency response by including all internal capacitances:



C_{GS1}: sees R_{sig}
C_{GD1}: sees $R_{GD1}=(1+g_{m1}R_{D1})R_{sig}+R_{D1}$
where
$R_{D1}=r_{o1} \,||\, [1/(g_{m2}+g_{mb2})+R_L/A_{vo2}]$
$C_{DB1}+C_{GS2}$: in parallel at node D_1, sees R_{D1}
C_{GD2}: sees R_{D2}
C_L: combines C_{DB2}, input capacitance of following amplifier stage and capacitance of load (if any)
C_L+C_{GD2}: in parallel at node D_2, see $R_L||\, R_{out}$

then get effective time constant:
$\tau_H \approx \sum_x R_x C_x$
$= R_{sig}C_{GS1}+R_{GD1}C_{GD1}+R_{D1}\,(C_{DB1}+C_{GS2})+(R_L||R_{out})(C_L+C_{GD2})$

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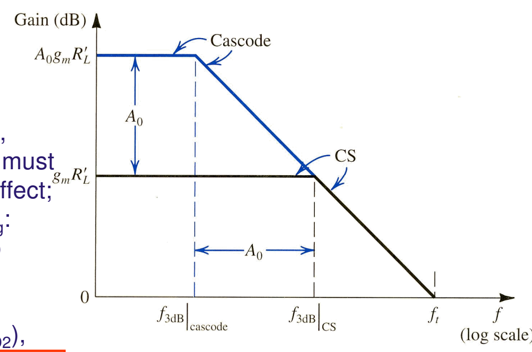
MOS cascode as two-stage amplifier: high-frequency response, II

sort according to resistances:
$\tau_H \approx R_{sig}[C_{GS1}+C_{GD1}(1+g_{m1}R_{D1})]$
$+R_{D1}(C_{GD1}+C_{DB1}+C_{GS2})+$
$+(R_L||R_{out})(C_L+C_{GD2})$
1st term dominates for large R_{sig}, when Q_1 provides most of gain: must lower R_L to compensate Miller effect;
3rd term dominates for small R_{sig}: both Q_1 and Q_2 can be driven to maximise the gain



For R_{sig}=0: $\tau_H \approx (R_L||R_{out})(C_L+C_{GD2})$,
$\boxed{f_H=1/(2\pi\tau_H)=1/[2\pi(R_L||R_{out})(C_L+C_{GD2})]}$ **is of the same form as for the CS single-stage amplifier, but lower by a factor of A_o** because $(R_L||R_{out}) \approx A_o R_L' >> R_L'$.
Note that the **unity-gain frequency $f_t=1/(2\pi)\, g_m/(C_L+C_{GD2})$ remains unchanged!**

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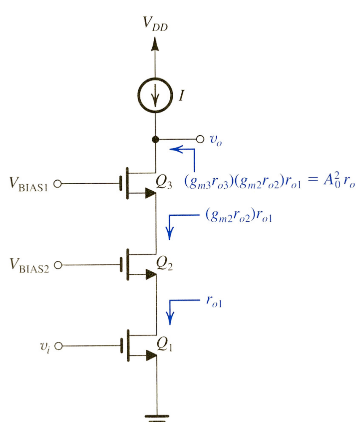
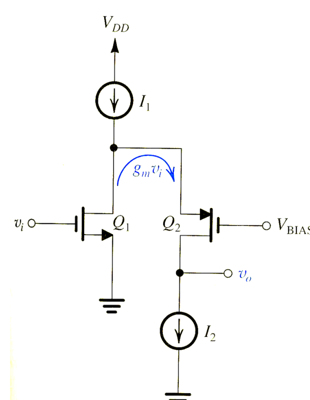
MOS double and folded cascodes



double cascode: increases output resistance even further



folded cascode: uses NMOS & PMOS to avoid large stacks hanging on to single low-voltage power supply

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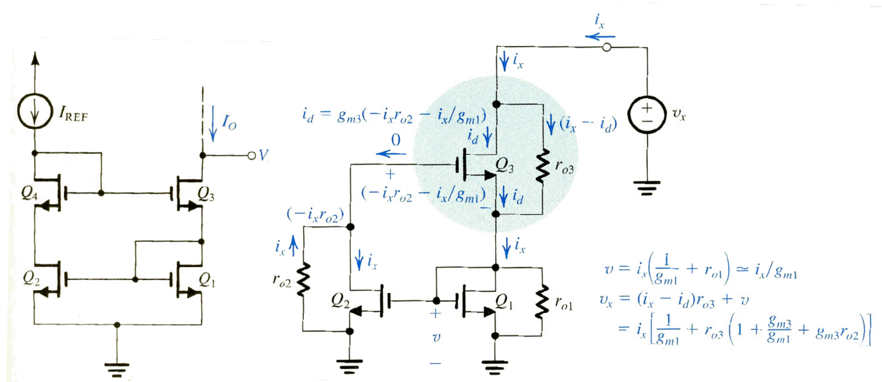
MOS Wilson current mirror



output resistance: $\boxed{R_o = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}]r_{o2} \approx g_{m3}r_{o3}r_{o2}}$

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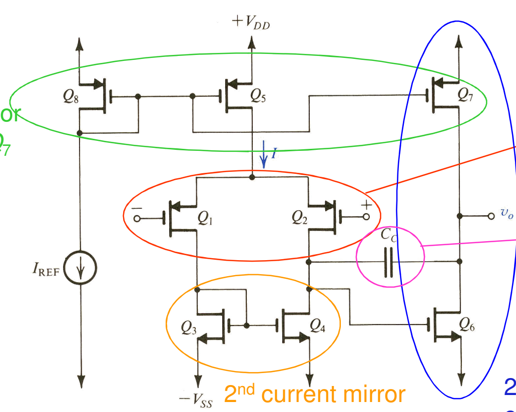
2-stage MOS operational amplifier: voltage gain

current mirror provides bias for output stage Q_7



1. stage: diff. amp. provides voltage gain $A_1 = -g_{m1}(r_{o2} || r_{o4})$

compensation capacitance in neg. feedback loop of Q_6 is Miller-multiplied and reduces corner frequency of output stage

2. stage: class AB output stage provides voltage gain $A_2 = -g_{m6}(r_{o6} || r_{o7})$

2^{nd} current mirror

basic 2-stage CMOS op-amp configuration with over-all open-loop gain $\boxed{A_v = A_1 A_2}$

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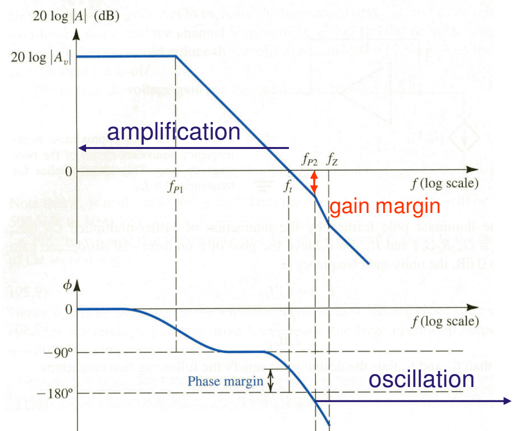
2-stage MOS operational amplifier: frequency behaviour, I



Bode plot of loop gain (top) and phase lag (bottom);
condition for a stable amplifier: 0º>Φ>–180º for |A|>0dB