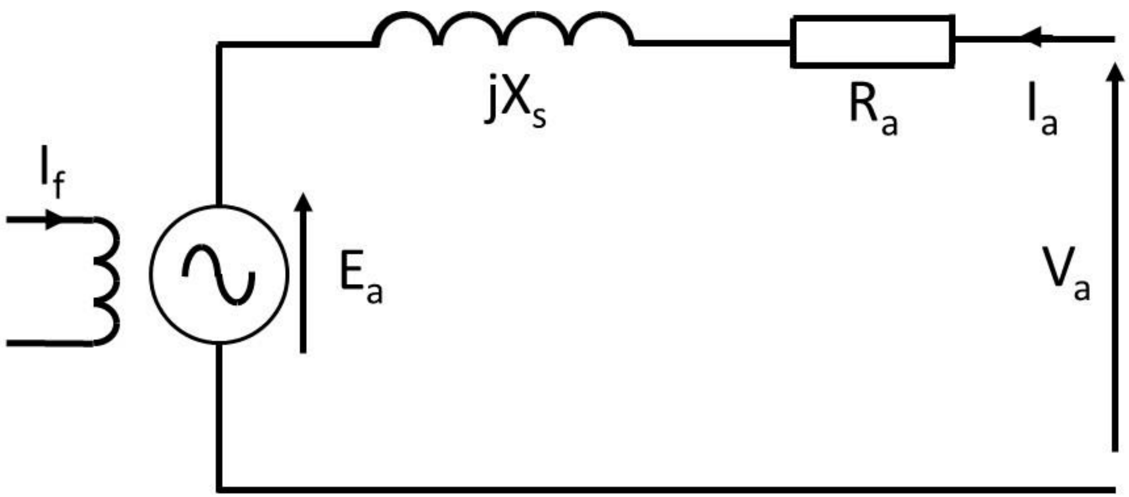
Non-salient per-phase equivalent circuit



I_f – Field coil control current
E_a – Armature nduced emf
X_s – synchronous reactance
R_a – Armature resistance

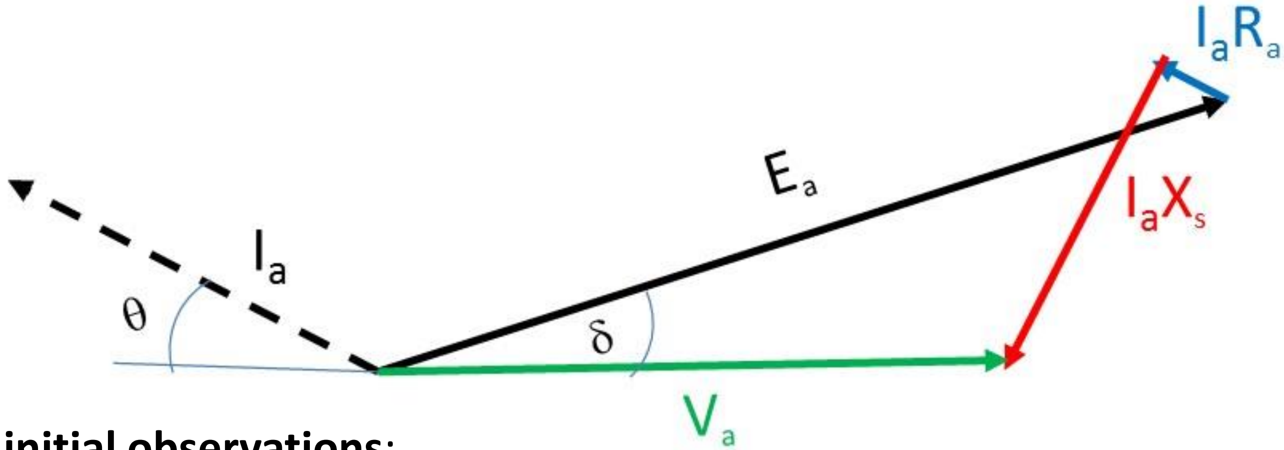
Up the point at which magnetic saturation of the rotor and/or stator core becomes appreciable then recalling that the speed is constant then:

$$E_a = k_e I_f$$

To achieve a constant output voltage, V_a, the magnitude of the excitation current, I_f, must be controlled to accommodate changes in load current I_a

This process is referred to as regulation, and is often performed by a so-called AVR (**A**utomatic **V**oltage **R**egulator)

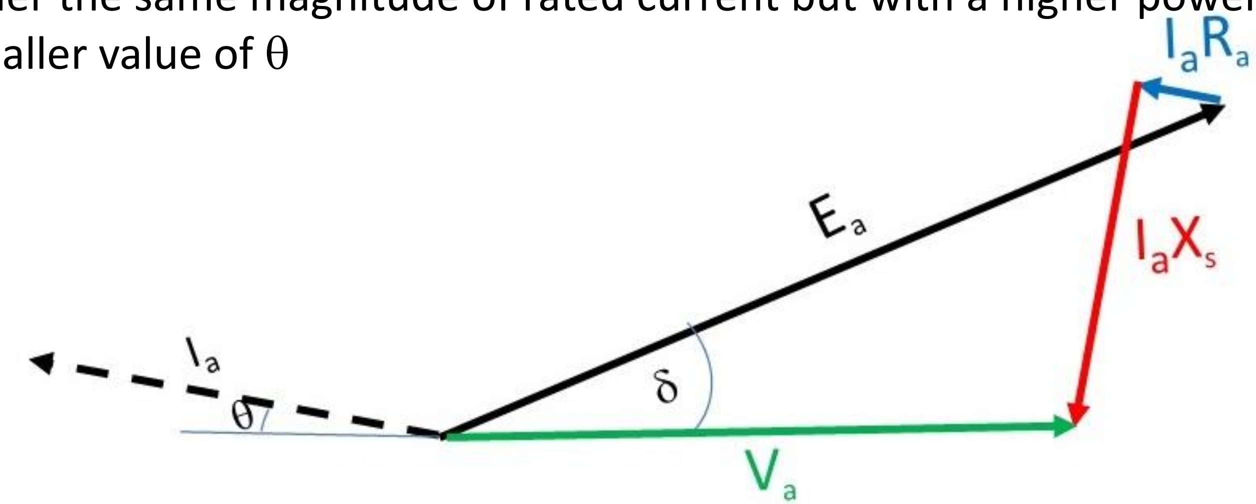
Phasor diagram in generating mode: Non-salient machine



Some initial observations:

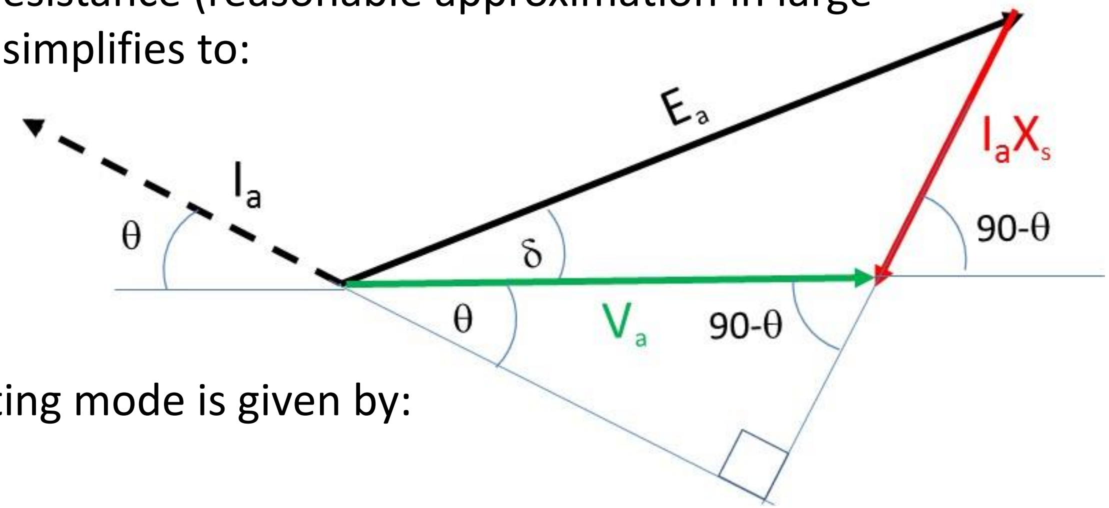
- E_a is larger than V_a in generating mode to accommodate voltage drops within the generator
- The magnitude of E_a required to produce rated voltage (and by implication the I_f required) depends on both the magnitude and phase of the load current.

Consider the same magnitude of rated current but with a higher power factor load, i.e. smaller value of θ



Calculation of real and reactive power – non-salient machine

Neglecting the voltage drop due to resistance (reasonable approximation in large machines) then the phasor diagram simplifies to:



The real power per phase in generating mode is given by:

$$P = V_a I_a \cos\theta$$

Resolving along a direction orthogonal to the axis of V_a:

$$E_a cos(90 - \delta) + I_a X_s \cos\theta = 0$$

Which can be re-written as:

$$I_a \cos\theta = -\frac{E_a \sin\delta}{X_s}$$

Substituting back into the expression for the real power yields:

$$P = -\frac{V_a E_a \sin\delta}{X_s}$$

A similar procedure can be applied to the reactive power per phase

$$Q = V_a I_a \sin\theta$$

Resolving along the axis of V_a

$$V_a = E_a \cos\delta + I_a X_s cos(90 - \theta) = E_a \cos\delta + I_a X_s \sin\theta$$

Hence:

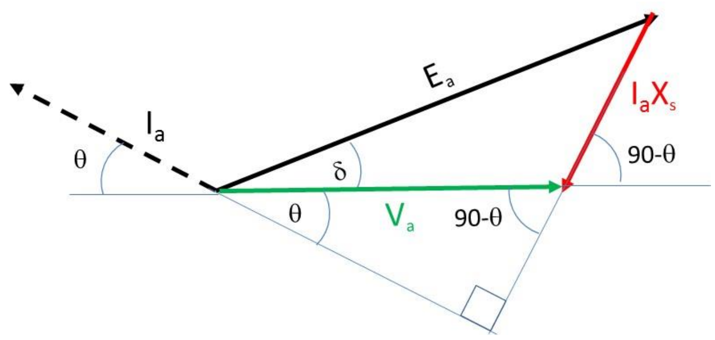
$$I_a \sin\theta = \frac{V_a - E_a \cos\delta}{X_s}$$

Substituting back into the expression for Q yields:

$$Q = \frac{V_a^2 - V_a E_a \cos\delta}{X_s}$$

Convention on load angle and power

- The load angle, δ, is defined as the angle between the terminal voltage and the emf. The convention of this angle is important in terms the sign of the machine power.
- Angles on a phasor diagram, by convention, increase in an anti-clockwise direction (as they do in a normal Cartesian x-y system)
- Looking again at the phasor diagram in generating mode.



- The angle between V (starting phasor) and E (end phasor) is positive since it involves an anti-clockwise sweep.

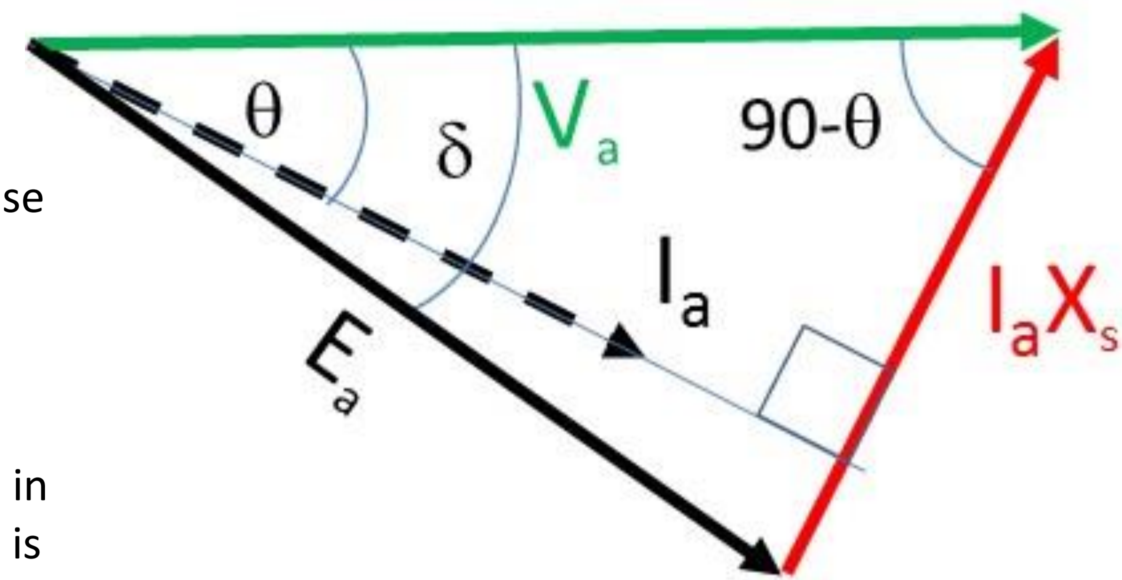
- For positive δ then $\sin\delta$ is positive and hence the power is negative since:

$$P = -\frac{V_a E_a \sin\delta}{X_s}$$

This is consistent with the convention for positive V_a and I_a in the equivalent circuit

Phasor diagram in motoring mode – non-salient machine

Phasor diagram drawn for same magnitude of V and I and same lagging power factor as was the case in generating mode



Applying the same logic as before, then δ angle between V and E is negative (i.e. we move from V to E in a clockwise direction). Hence sin δ is also negative and therefore from the power equation:

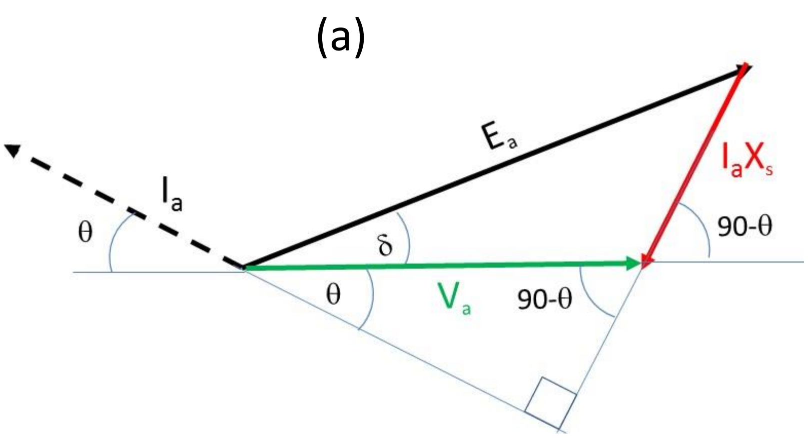
$$P = -\frac{V_a E_a \sin\delta}{X_s}$$

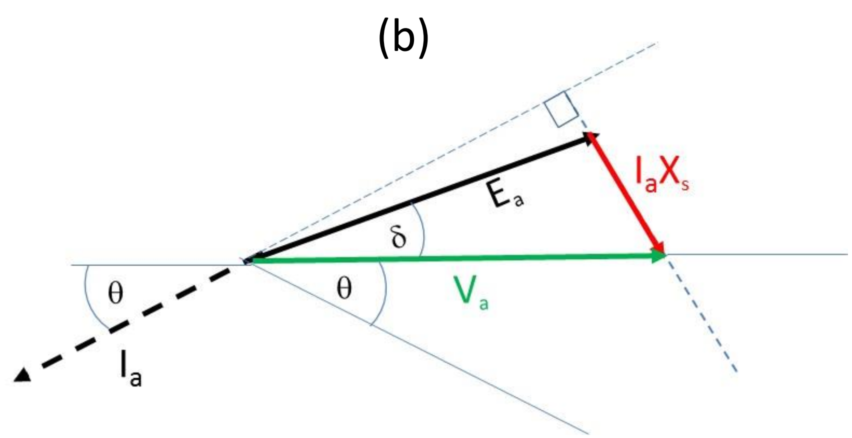
P is positive, i.e. motoring power according to our convention

Under- and over- excited operation

If we have a voltage stiff grid with multiple generators (tending towards to a so-called infinite bus) then the excitation can be used to control the import or export reactive power to the grid. We can therefore, at least in principle, set the field current such that the machine is under-excited (import VAR) or over-excited (export VAR).

Consider the same phasor diagram (a) alongside a phasor diagram with the same V_a and magnitude of I_a, but with an equal but opposite power factor (b). According to our definitions, then (a) is over-excited and (b) is under excited.





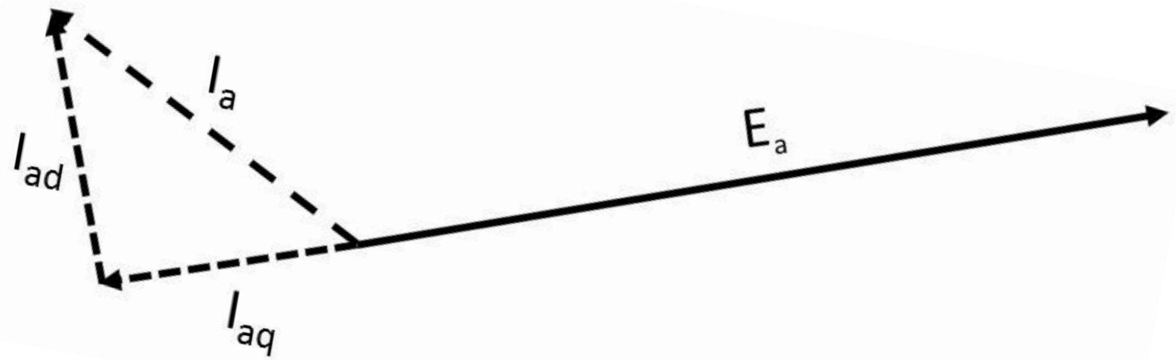
This approach controls the power factor of the generator and NOT the output voltage (which is set and maintained by the voltage stiff grid).

Whereas this provides, at least in principle, a means of controlling the power factor of the generator on a stiff grid, in practice, large generators are operated in AVR mode not power factor mode – improves stability of the network.

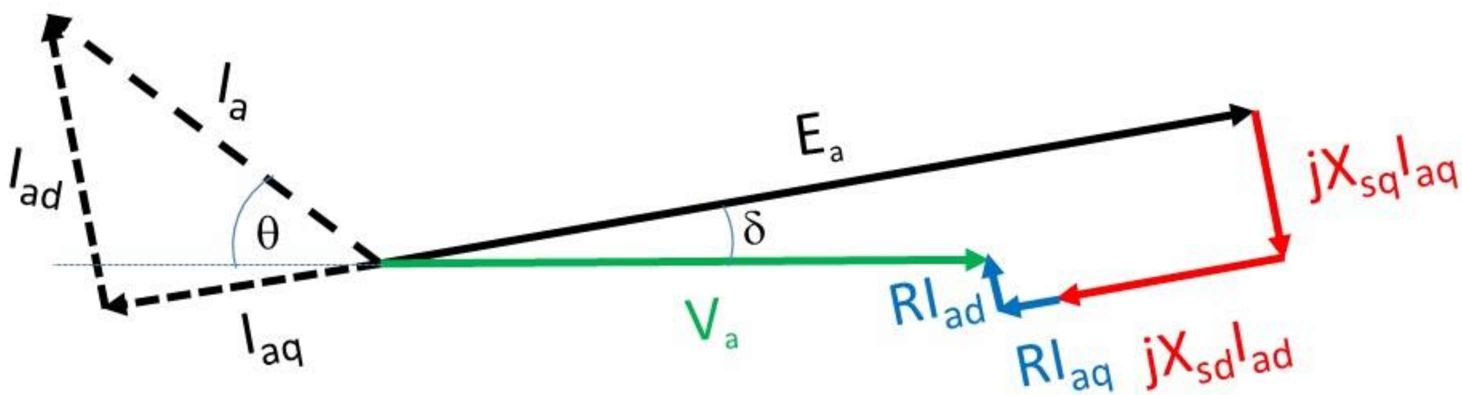
Phasor diagram in generating mode: Salient machine

In a salient machine, the reactance of the stator winding is a function of rotor position
This can be conveniently expressed in terms of values along the d and q axis: X_d and X_q

The armature current can be resolved into d and q axis components by noting that the q-axis is aligned with the induced emf phasor E_a.

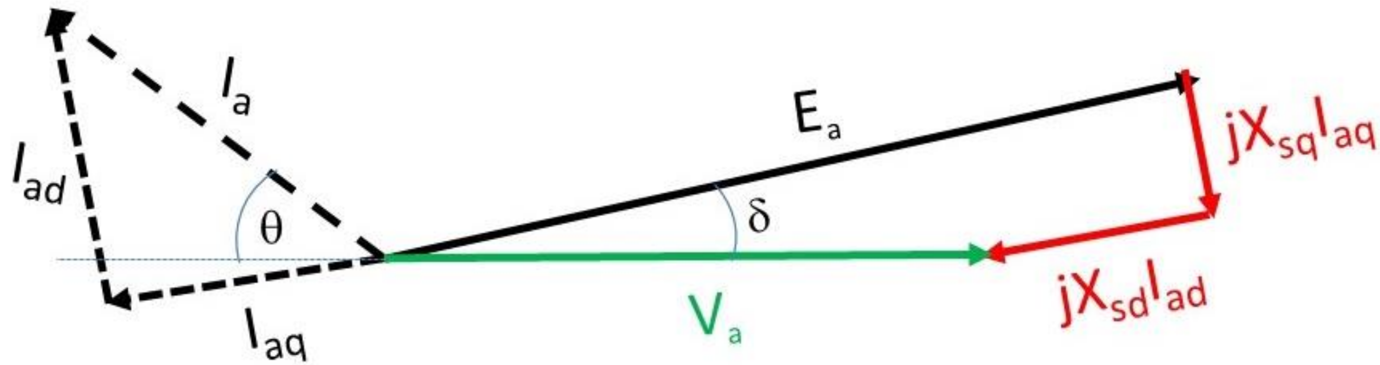


The phasor diagram for a salient machine operating in generating mode

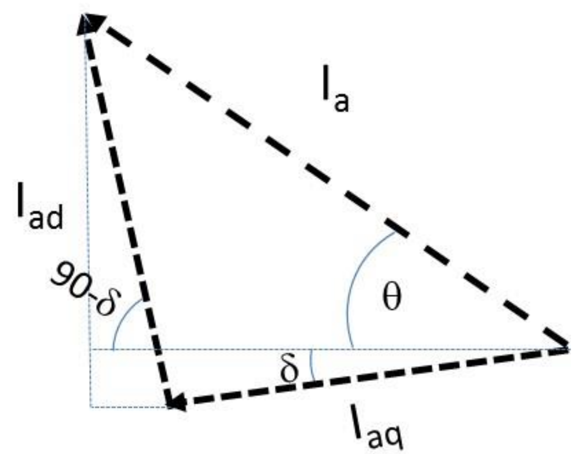


Calculation of real and reactive power – salient pole

Neglecting the voltage drop due to resistance (reasonable approximation in large machines) then the phasor diagram simplifies to:



$$P = V_a I_a \cos\theta$$



$$I_a \cos\theta \; = I_{aq}\cos\delta + I_{ad}\cos(90-\delta)$$

$$= I_{aq}\cos\delta + I_{ad}\;\sin\delta$$

Resolving voltage phasors in orthogonal direction to E_a

$$X_{sq}I_{aq} + V_a\sin\delta = 0$$

$$X_{sq}I_{aq} = -V_a\sin\delta$$

Resolving voltage phasors parallel to direction to E_a

$$E_a + X_{sd} I_{ad} - V_a \cos\delta = 0$$

$$X_{sd} I_{ad} = V_a cos\delta \ - E_a$$

Hence,

$$I_{aq} = \frac{-V_a \sin\delta}{X_{sq}} \quad \text{and} \quad I_{ad} = \frac{(V_a cos\delta - E_a)}{X_{sd}}$$

Substituting back into the expression for $I_a\,cos\theta$

$$I_a \cos\theta \ = -\frac{V_a \sin\delta}{X_{sq}} \cos\delta + \frac{(V_a cos\delta - E_a)}{X_{sd}} \sin\delta$$

Hence, the real power is given by:

$$P = V_a I_a \cos\theta = V_a \left[\frac{-V_a \sin\delta}{X_{sq}} \cos\delta + \frac{(V_a cos\delta - E_a)}{X_{sd}} \sin\delta \right]$$

$$= -\frac{V_a E_a}{X_{sd}} sin\delta - \frac{V_a^2 sin\delta \ cos\delta}{X_{sq}} + \frac{V_a^2 cos\delta \ sin\delta}{X_{sd}}$$

Making use of the trigonometric identity:

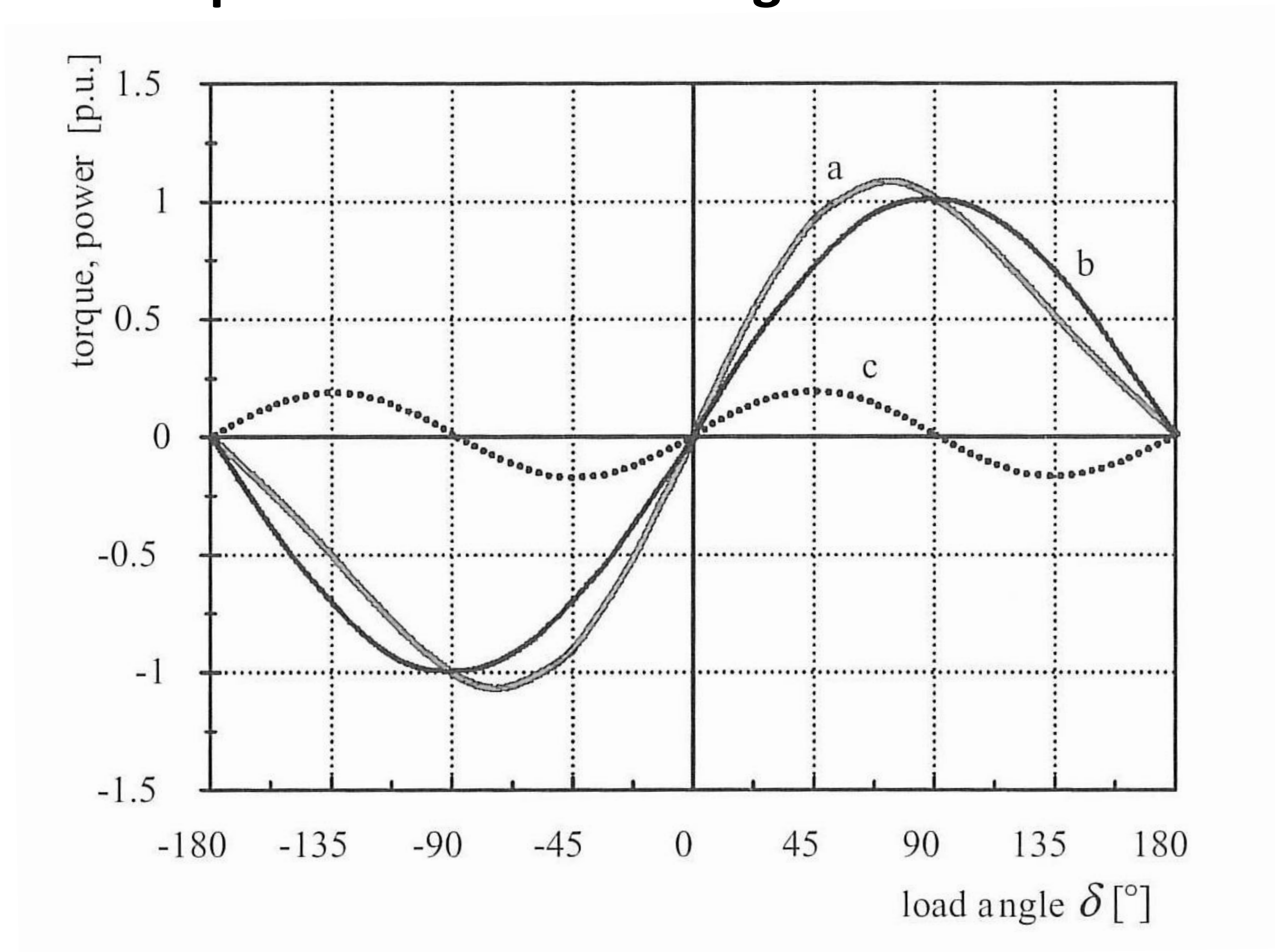
$\sin 2\delta = 2 \ sin\delta \ cos\delta$

$$P = -\frac{V_a E_a}{X_{sd}} sin\delta - \frac{V_a^2 sin2\delta}{2} \left(\frac{1}{X_{sq}} - \frac{1}{X_{sd}} \right)$$

This can be tidied up to:

$$P = -\frac{V_a E_a}{X_{sd}} sin\delta - \frac{V_a^2 (X_{sd} - X_{sq}) sin2\delta}{2 X_{sq} X_{sd}}$$

Variation of power versus load angle



(a) Salient pole generator
(b) Non-salient
(c) Contribution from saliency