EEE 207 May 2005 Worked Solutions

(a) Compensation doping – doping with the opposite dopant type to what was originally there. Used to reduce effective doping level or to change doping type.

Many got this only partly correct.

Charge neutrality condition

$$n + N_a = p + N_d$$

$$np = n_i^2$$

~25% of students got N_A + N_D confused

$$\therefore \; n^2 - (N_d - N_a)n - n_i^2 = 0$$

$$n = \frac{N_d - N_a}{2} + \frac{N_d - N_a}{2}\left[1 + \left(\frac{2n_i}{N_d - N_a}\right)^2\right]^{1/2}$$

If $N_d - N_a \gg n_i$

$$n = N_d - N_a$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d - N_a}$$

If $n_i \gg N_d - N_a$

$$n = n_i + \frac{N_d - N_a}{2} \approx n_i = p$$

) $n = 10^{12}\ m^{-3}$, $N_a = 10^{21}\ m^{-3}$, $n_i = 2\times10^{16}\ m^{-3}$

$n \ll n_i$ so p-type and $N_a - N_d \gg n_i$

$$\therefore \; p = N_a - N_d$$

$$n = \frac{n_i^2}{N_a - N_d}, \quad 10^{12} = \frac{4\times10^{32}}{10^{21} - N_d}$$

$$N_d = 6\times10^{20}\ m^{-3}$$

Most students got this correct. Mistakes from (a) resulted in some wrong values for (ii)

$$p = \frac{n_i^2}{n} = \frac{4\times10^{32}}{10^{12}} = 4\times10^{20}\ m^{-3}$$

$$\sigma = e(p\mu_h + \underbrace{n\mu_e}_{\text{small}}) = 1.6\times10^{-19}\times4\times10^{20}\times0.05 = 3.2\ Sm^{-1}$$

:) $N_d - N_a = (5.02 - 5.01) \times 10^{19} = 10^{17}\ m^{-3}$

This is close to n_i, so must use full expression for n.

Majority $n = \frac{10^{17}}{2} + \frac{10^{17}}{2}\left[1 + \left(\frac{2 \times 10^{16}}{10^{17}}\right)^2\right]^{1/2} = 1.039 \times 10^{17}\ m^{-3}$

minority $p = \frac{n_i^2}{n} = \frac{4 \times 10^{32}}{1.039 \times 10^{17}} = 3.85 \times 10^{15}\ m^{-3}$

$\sigma = e(p\mu_h + n\mu_e) = 1.6 \times 10^{-19}(0.05 \times 3.85 \times 10^{15} + 1.039 \times 10^{17} \times 0.13)$

Most got this correct.

$= 2.19\ mS\ m^{-1}$

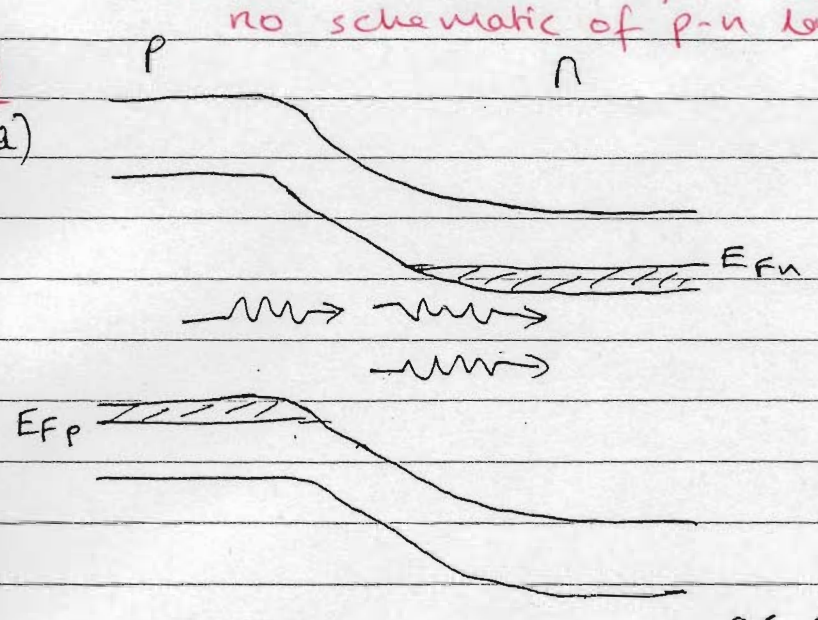
Comment: The compensation in the second sample results in a significantly reduced majority carrier, hence lower σ.

Most got this partly correct but few got full marks.

Very few got full marks for 2(a). Not all the information asked for was given, e.g. no band diagram, no schematic of p-n laser, no Fermi-level etc.

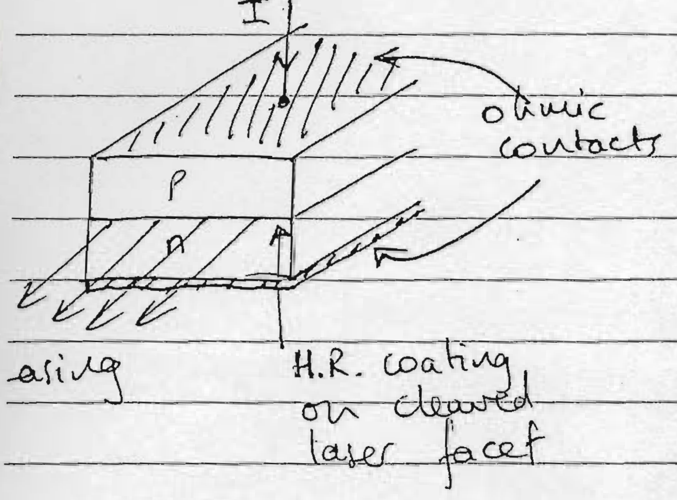
2

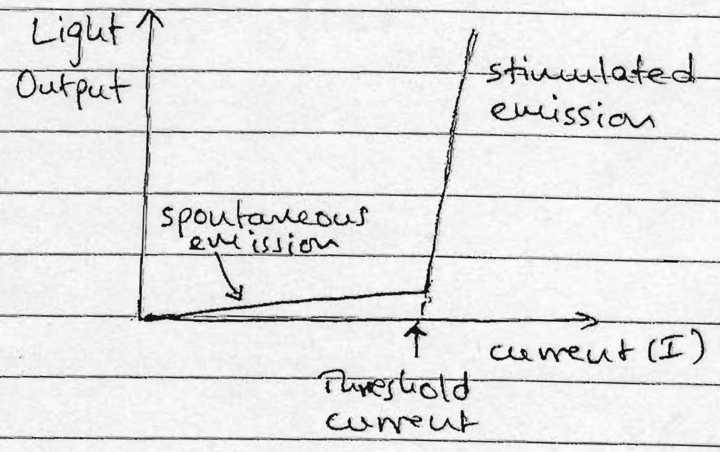
(a)



Heavily doped p and n-regions under forward bias. Electrons and holes diffuse across the depletion region and recombination, hence photons are emitted. Fermi levels move apart as shown - difference is the voltage.

Under large forward bias, population inversion occurs and stimulated emission can occur. High reflectors are needed to form a cavity which reflects the photons and determines the exact lasing wavelength.





b) Maximum lasing λ determined by bulk InGaAs band gap

$$\lambda_{max} = \frac{1.24}{0.75} = 1.65\,\mu m$$

Minimum lasing λ determined by InP barrier band gap

$$\lambda_{max} = \frac{1.24}{1.35} = 0.918\,\mu m$$

No problems here

c) Energy corresponding to $1{\cdot}55\mu m = 0{\cdot}8eV$
Bulk band-gap of InGaAs $= 0{\cdot}75eV$, so quantisation should increase the band gap by 50 meV.

Using expression for 1st bound energy levels for electron and holes;

$$\frac{h^2}{8m_e^* m_0 L^2} + \frac{h^2}{8m_h^* m_0 L^2} = 50\,meV$$

$$\frac{h^2}{8m_0 L^2}\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right) = 50\,meV$$

$$L^2 = \frac{h^2}{8m_0} \cdot \frac{1}{50\times10^{-3}\times1{\cdot}6\times10^{-19}} \cdot \left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right)$$

$$= 2{\cdot}05\times10^{-16}$$

$\therefore L = 14{\cdot}3\,nm$

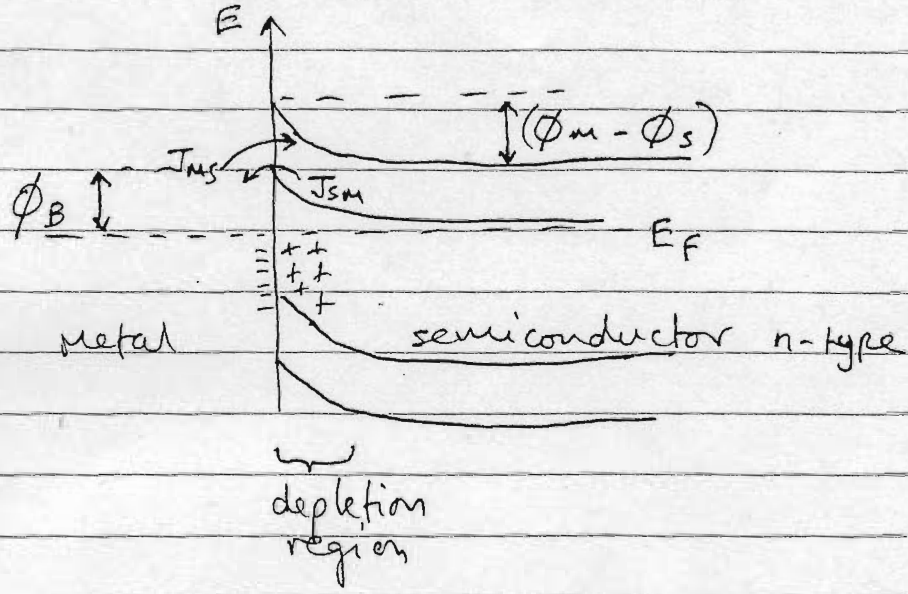
Most got part of this correct but few full marks.

(d) As the operating wavelength becomes increasingly shorter, carriers may escape thermionically due to temperature effects. Also as the bound levels rise in energy, they may start to tunnel through the finite InP barriers.

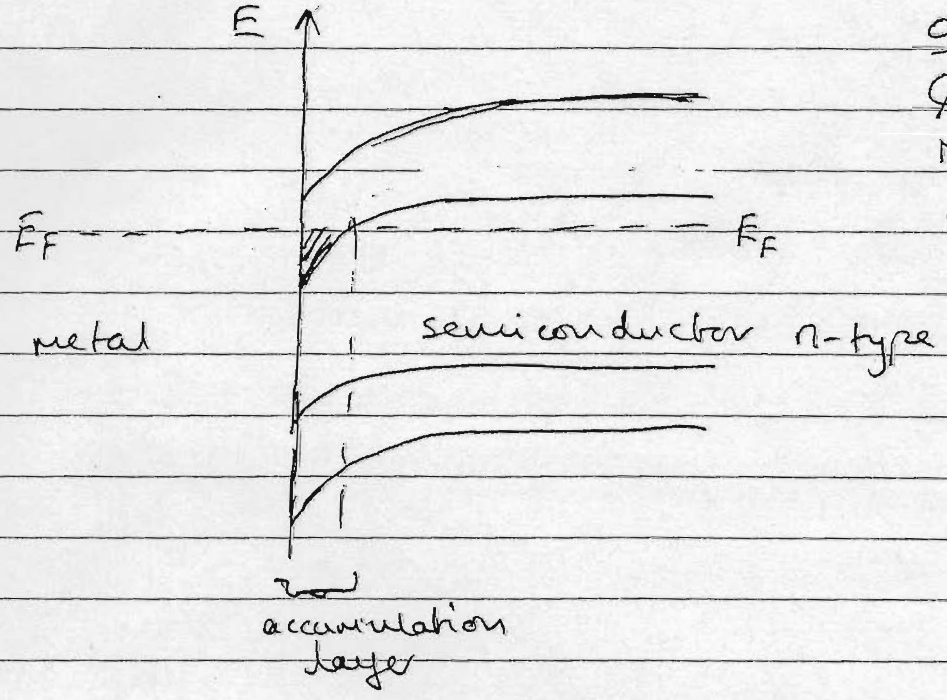
A reasonable attempt but not quite answering in full.

Mostly 3(a) was O.K. but there were bits of information missing which was specifically asked for.

(a)



<u>Schottky contact</u>
rectifies
$\phi_m > \phi_s$



<u>ohmic contact</u>
$\phi_s > \phi_m$
No barrier to impede carrier flow in either direction.

(b) <u>Schottky contact</u>: Forward bias - semiconductor potential rises relative to metal. J_{SM} increases dramatically as barrier $(\phi_m - \phi_s)$ is reduced. Minority current J_{MS} remains constant in opposite direction.

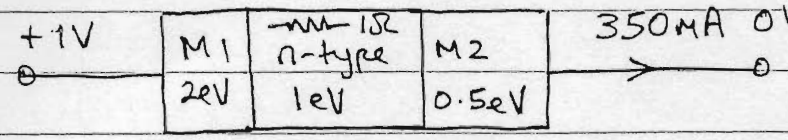
Reverse bias - semiconductor potential decreases. The barrier $(\phi_m - \phi_s)$ increases and J_{SM} becomes very small. J_{MS} remains constant as ϕ_B is unaffected.

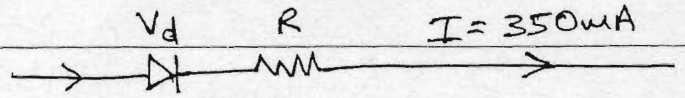
Most got 3(b) correct

Ohmic contact: Energy bands bend downwards at metal-semi. interface, so no barrier to impede carrier flow. Under 'forward bias', as semiconductor potential rises, large J_{sm} will flow. Under 'reverse' bias, large J_{ms} will flow.

c)

Very few got 3(c) correct





Due to the values of ϕ's given, we know that 350mA flows when M1 junction is forward biassed.

For perfect diode $I = I_o\left(e^{\frac{eV}{kT}} - 1\right)$

Diode voltage, $V_d = V_{app} - IR$

$$\therefore\ I = I_o\left(\exp\left[\frac{e(V_{app} - IR)}{kT}\right] - 1\right)$$

$$0.35 = I_o\left(\exp\left[\frac{e(1 - 0.35)}{kT}\right] - 1\right) = I_o \times 7.2 \times 10^{10}$$

$$I_o = 4.86 \times 10^{-12} \quad \left(\text{assume } \frac{kT}{e} = 26\,\text{meV}\right)$$

When polarity is reversed, M2 junction remains ohmic but M1 junction is 'off' - only minority saturation current flows. Ignore voltage drop across 'R' as current is very small, so current flow is:

$$I = I_o = -4.86\,\text{pA}$$

- (a) Heisenberg Uncertainty Principle

Reasonable attempt

$$\Delta p \, \Delta x \geq \hbar/2$$

or

$$\Delta E \, \Delta t \geq \hbar/2$$

Both parameters of a particle, e.g. momentum and position cannot be measured simultaneously to an arbitrarily high degree of precision.

No problems here

(b) de Broglie : $p = mv = \frac{h}{\lambda}$, h = Planck's constant

p = momentum , m = mass , v = velocity , λ = wavelength

To be able to resolve 0.2nm, the wavelength has to be ≤ 0.2 nm.

If we equate K.E. to P.E.

$$\frac{1}{2} m v^2 = eV \implies v = \left(\frac{2eV}{m}\right)^{1/2}$$

From de Broglie,

$$mv = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mv} = \frac{h}{(2eVm)^{1/2}} = \frac{1.225\,\text{nm}}{V^{1/2}}$$

$$V = \left(\frac{1.225 \times 10^{-9}}{0.2 \times 10^{-9}}\right)^2 = 37.5\,\text{V}$$

Very few managed to get this bit correct.

(c) $E = E_g + AK^2 + BK^4$

$$m^* = \left(\frac{d^2E}{dp^2}\right)^{-1}$$

$$p = \hbar K$$

$$m^* = \hbar^2\left(\frac{d^2E}{dK^2}\right)^{-1}$$

This was apparently no problem

$$\frac{dE}{dK} = 2AK + 4BK^3 \quad , \quad \frac{d^2E}{dK^2} = 2A + 12BK^2$$

At $K = 0$, $m^* = \hbar^2 (2A + 0)^{-1} = \left[\frac{6.626\times10^{-34}}{2\pi}\right]^2 \left[2\times10^{-38}\right]^{-}$

$$= 5.56\times10^{-31}$$
$$= 0.61\, m_0$$

However few managed to get as far as this

Need to determine K at Brillouin zone, when $v_g = 0$

$$v_g = \frac{dE}{dp} = \frac{1}{\hbar}\cdot\frac{dE}{dK} = \frac{1}{\hbar}(2AK + 4BK^3) = 0$$

$$AK + 2BK^3 = 0 \implies K = \left[\frac{A}{-2B}\right]^{1/2}$$

(B is negative so sq. root is ok)

m^* @ $K = \left[\frac{A}{-2B}\right]^{1/2}$

$$m^* = \hbar^2 \frac{1}{2A + 12BK^2} = \hbar^2 \frac{1}{2A + 12B\left(\frac{A}{-2B}\right)} = \frac{\hbar^2}{-4A}$$

$$= \left(\frac{6.626\times10^{-34}}{2\pi}\right)^2 \cdot \frac{1}{(-4\times10^{-38})} = -2.78\times10^{-31} = -0.305\, m_0$$

Very few got this last bit correct.