**Data Provided: None**

The University Of Sheffield.

**DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING**

**Spring Semester 2014-15 (3.0 hours)**

**EEE6221 Data Coding Techniques for Communications and Storage**

Answer **FOUR** questions. **No marks will be awarded for solutions to a fifth or sixth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

**1.** a. In the context of base-band synchronous communications, describe the key parameters that need to be considered when devising or evaluating a particular line coding scheme. **2**

b. Figure Q.1 illustrates the timing diagrams corresponding to an all zero sequence and an all 1 sequence encoded using a particular line code. Using this code encode:

i) The sequence: **011100**

ii) The sequence: **011110**

From this, characterise the code giving its advantages and drawbacks. Suggest how you may modify this code to overcome the drawbacks identified. **8**

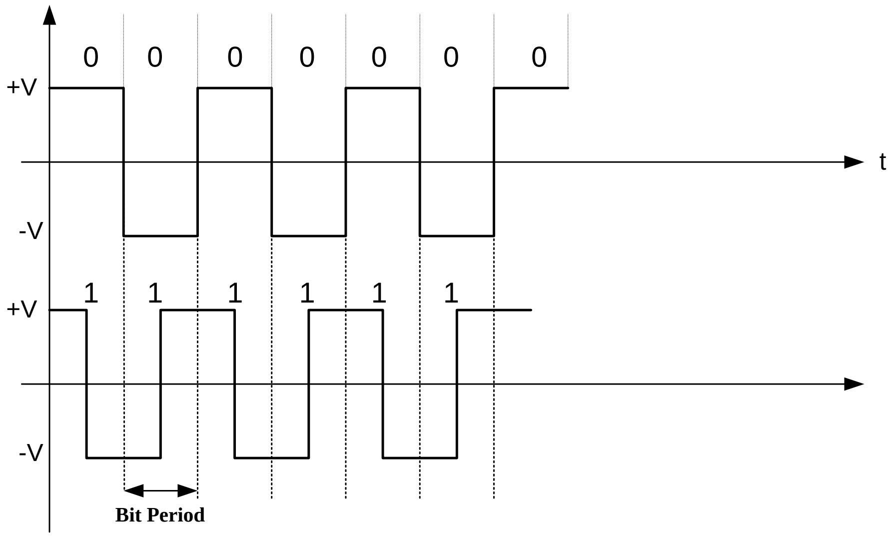


**Figure Q.1**

c. An asynchronous transmission connection is set up with odd parity and an ***m***-bit data between a Start and a Stop bit. The master clock is set to run at a nominal rate that is 16 times the baud rate.

i) What are the worst case conditions that will affect the reliability of this connection? **2**

ii) Find the minimum tolerance that the receiver and transmitter master clocks must have for reliable operation under worst case conditions for $m = 8, 16$ and $32$ bits. Discuss your results. **6**

d. Explain briefly how a data scrambler would be useful in line coding. What are the advantages and drawbacks of employing a data scrambler for line coding. **2**

**2.** **a.** A 4-bit message $(m_3m_2m_1m_0) = (1000)$ is encoded into a 7-bit codeword by adding to it 3 Cyclic Redundancy Check (**CRC**) bits $(c_2\ c_1\ c_0)$ in the following order $(m_3m_2m_1m_0\ c_2\ c_1\ c_0)$. The **CRC** bits are generated using the generator polynomial $g(x) = x^3+x+1$.

i) Draw a circuit diagram for the **CRC** generatorand hence, using the diagram, derive the resulting codeword. How many clock cycles are required to generate this codeword. Check that your resulting codeword thus derived is correct by calculation. **6**

ii) The encoded message above is transmitted over a noisy channel and received with a single bit error in one of the 4 message bits. The **CRC** checking yielded a *(110)* remainder. Determine the error pattern that has corrupted the message in this case and comment on the limitation of **CRC** for error correction. **6**

**b.** The CRC's shortcomings for error correction are overcome by adopting the more powerful BCH codes. A *(15,7)* primitive ***BCH*** code defined over $GF(2^4)$ using the primitive polynomial $p(x) = x^4+ x+ 1$, was used to encode a 7 bit message into a 15-bit codeword. After transmission over a noisy channel the codeword is received with correctable errors in the last four Least Significant Bits (LSBs), as $r(x)=x^{10}+x^9+x^8+x^6+x^2+x+1$. On algebraic decoding, the error locator polynomial was found to be: $\sigma(x)= x^2 + \alpha^4x + \alpha$

i) How many errors can this code correct? **2**

ii) Find the correct codeword. **6**

**3.** **a.** Contrast briefly the algebraic and the non-algebraic approaches to decoding of cyclic block codes. **3**

**b.** Sketch a generic circuit for a **Meggitt** decoder and explain briefly how it works in the case of a single error correction. **5**

**c.** Explain how you would correct 2 errors using **Meggitt** decoding. What are the implications for multiple error correction. **3**

**d.** A message $m(x)$ encoded using a *(15,7)* binary cyclic code is received with <u>2- bit</u> errors in the first 4 most significant bits as: $r(x)= x^{14}+x^{13}+x^{12}+x^{11}+x^9+x^7+x^2+x$. Using **Meggitt** decoding, derive the corrected codeword given that the syndrome corresponding to the 2-bit error pattern is $S= x^7+x^4+x^3+x^2$. Show all your workings and validate your result. Assume that the *(15,7)* code is generated using the generator $g(x) = x^8+x^7+x^6+x^4+1$. **9**

**4.** **a.** Draw a circuit to generate all non zero elements of the Galois Field $\boldsymbol{GF(2^3)}$ using the primitive polynomial $\boldsymbol{p(x) = x^3+x+1}$. List all of the elements in both binary and polynomial format. **4**

**b.** Before transmission, a 3-symbol message is ***RS*** (Reed-Solomon) encoded so that any 2-symbol errors can be corrected.

i) Determine the required number of parity check symbols to be added to the message to enable this error correction capability; hence define the resulting ***RS*** code. What form can these errors take? **2**

ii) Using your defined ***RS*** code, derive the corresponding generator polynomial $\boldsymbol{g(x)}$ and hence find the **RS** codeword for the message $\boldsymbol{u(x) = x^2 + \alpha^3 x + 1}$. Validate your result. **7**

**c.** After transmission over a noisy channel, a message encoded using the above ***RS*** code is received with <u>one</u> symbol error as $\boldsymbol{r(x)=\alpha^5x^6+x^5+\alpha^2x^4+\alpha^5x^3+\alpha x^2+x+1}$. Using algebraic decoding, derive the corrected codeword given that for a single symbol error correction the error location, $\boldsymbol{X_1}$, and the error magnitude, $\boldsymbol{Y_1}$, are given respectively by the equations: **7**

$$X_1 = \sigma_1 = \frac{S_2}{S_1} \quad \text{and} \quad Y_1 = \frac{S_1^{\;2}}{S_2}$$

where $S_1$ and $S_2$ are the first 2 syndromes.

**5.** **a.** Contrast briefly **DPCM** and Vector Quantisation techniques for image data compression explaining the advantages and drawbacks of each. **4**

**b.** The **DCT** is used in practice in image and video compression.

i) List its main attractive properties for data compression **2**

ii) Explain briefly how compression is achieved in **DCT**-based compression. **1**

iii) Explain briefly potential limitations of **DCT**-based compression **1**

**c.** Determine the **DCT** of the two-dimensional data given below explaining any shortcuts made. **7**

$$\begin{bmatrix} 4 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 1 \end{bmatrix}$$

**d.** The following quantisation matrix is used to compress the **DCT** transformed data in c.:

$$\begin{bmatrix} 1 & 1 & 10 & 10 \\ 1 & 1 & 10 & 10 \\ 1 & 1 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{bmatrix}$$

i) Determine the compression ratio in this case **3**

ii) Comment on the quality of the resulting compressed data **2**

The ***k-th/n-th DCT/IDCT*** pair of an ***N***-sample block input is given by:

$$X_k = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \alpha_k x_n \cos\left[\frac{(2n+1)k\pi}{2N}\right]$$

$$x_n = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} \alpha_k X_k \cos\left[\frac{(2n+1)k\pi}{2N}\right]$$

$$\alpha_0 = \frac{1}{\sqrt{2}}$$

$$\alpha_k = 1 (k \neq 0)$$

**6.** **a.** Compare block codes and convolutional codes, as applied to error correction, giving typical situations where each one might be employed. **3**

**b.** A ***(7,3) RS*** code defined over ***GF(2^3)***, that is generated using irreducible polynomial ***p(x) = x^3 + x+1***, is used to encode the message ***m(x) =*** $\alpha x^2$. Using the frequency domain encoding approach, derive the ***RS*** codeword for the message ***m(x)***. **5**

**c.** The trellis in Figure Q.6 below is derived from a convolutional encoder.

i) Draw a circuit for the corresponding convolutional encoder **3**

ii) Encode the data sequence ***010110110*** **2**

iii) Correct the received sequence ***00 10 01 11 00 11 10 01 00*** assuming that **no more than 3 bit** errors have occurred. Derive hence the initial input data.

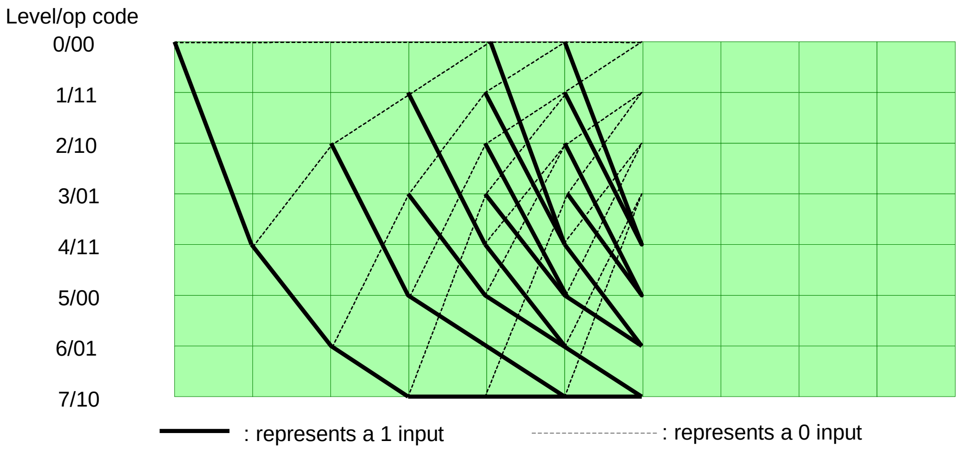


**Figure Q.6**

**MB/SK**