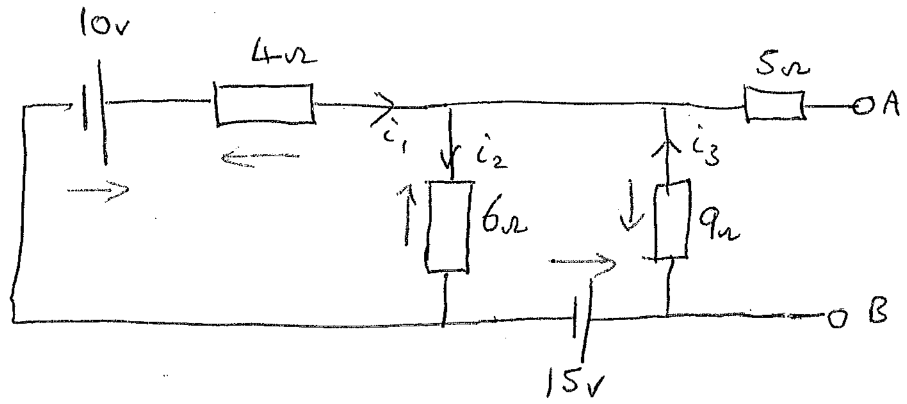
# QUESTION 1

(a) (i)




Clearly current through 5Ω = 0 A.

Using Kirchoff's laws:

$$10 - 4i_1 - 6i_2 = 0 \qquad (1)$$

$$15 - 9i_3 - 6i_2 = 0 \qquad (2)$$

$$i_1 + i_3 = i_2 \qquad (3)$$

Substituting (3) into (1) and (2) gives

$$10 - 4i_1 - 6i_1 - 6i_3 = 0 \Rightarrow 10 - 10i_1 - 6i_3 = 0 \qquad (4)$$

$$15 - 9i_3 - 6i_1 - 6i_3 = 0 \Rightarrow 15 - 15i_3 - 6i_1 = 0 \qquad (5)$$

Multiply (4) by 5 and (5) by 2 gives

$$50 - 50i_1 - 30i_3 = 0$$

$$30 - 12i_1 - 30i_3 = 0$$

Subtracting: $20 - 38i_1 = 0 \Rightarrow i_1 = \frac{20}{38} = \underline{\underline{0.526A}} \rightarrow$

Back substituting into (4):

$$10 - 5.26 = 6i_3 \Rightarrow i_3 = \underline{\underline{0.79A\uparrow}}$$

and into (3):

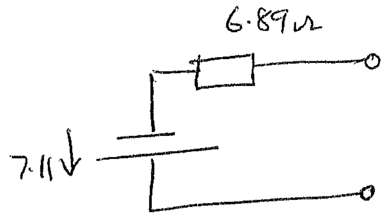
$$i_2 = 0.526 + 0.79 = \underline{\underline{1.316A\downarrow}}$$

(5)

(ii) For Thevenin we require the open circuit voltage between A & B, which is equal to the voltage across the 9Ω resistor:

$$V_{TH} = i_3 \times 9 = 0.79 \times 9 = \underline{\underline{7.11\,V}} \downarrow$$

$$R_{TH} = 5 + \frac{1}{\frac{1}{4} + \frac{1}{6} + \frac{1}{9}} = \underline{\underline{6.89\Omega}}$$




(3)

(b) (i) Since $P = VA\cos\phi \Rightarrow VA = \frac{P}{\cos\phi} = \frac{400}{0.8} = \underline{\underline{500KVA}}$ (1)

and the current is $I = \frac{VA}{V} = \frac{500000}{6600} = \underline{\underline{75.8\,A_{rms}}}$.

(ii) The reactive power is:

$$Q = 500\sin(\cos^{-1}0.8) = \underline{\underline{300KVAr}}$$ (1)

(c) (i) For the motor load, 360KVA @ 0.7 lagging:

$$P = 360 \times 0.7 = 252\,KW$$

$$Q = 360 \times \sin(\cos^{-1}0.7) = 257\,KVAr$$

For the heaters:

$$P = 100KW \qquad Q = 0KVAr$$

Hence the total real and reactive power is:

$$P_T = 400 + 100 + 252 = 752KW$$

$$Q_T = 300 + 0 + 257 = 557KVAr$$ (5)

$$\therefore KVA = \sqrt{752^2 + 557^2} = \underline{\underline{935.8\,KVA}}$$

(ii) Power factor $= \cos\phi$

$$= \frac{P}{S} = \frac{752}{935.8} = 0.803 \text{ lagging}$$ (1)

(iii) To correct the power factor to unity the capacitor must supply 557 kVAr.

$$\text{Since } \frac{V^2}{X_c} = Q \Rightarrow X_c = \frac{V^2}{Q} = \frac{6600^2}{557000} = 78.2\Omega$$

$$\text{Since } X_c = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_c} = \frac{1}{2\pi . 50 . 78.2} = 40\mu F$$ (3)

(iv) The peak voltage seen by the capacitor is:

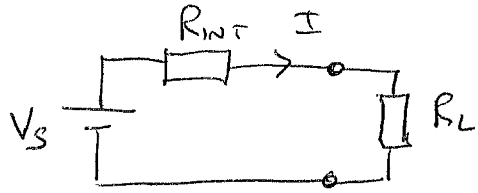
$$V_{c\,peak} = 6600 \times \sqrt{2} = 9334V$$ (1)

and

$$I_{c\,peak} = \frac{9334}{78.2} = 119.4\,A$$

# QUESTION 2

(a) (i)




The current flowing in the load is given by:

$$I = \frac{V_S}{(R_{INT}+R_L)}$$

Therefore the power dissipated in the load is

$$P_L = I^2 R_L = \frac{V_S^2 \cdot R_L}{(R_{INT}+R_L)^2}$$ (1)

(ii) For maximum power in the load, rearrange above expression:

$$P_L = \frac{V_S^2 R_L}{(R_{INT}+R_L)^2} = \frac{V_S^2}{\frac{R_{INT}^2}{R_L} + \frac{2R_{INT} \cdot R_L}{R_L} + \frac{R_L^2}{R_L}}$$

$P_L$ is maximum when the denominator is minimum:

i.e. $$\frac{d}{dR_L}\left(\frac{R_{INT}^2}{R_L} + 2R_{INT} + R_L\right) = 0$$

$$\therefore \frac{-R_{INT}^2}{R_L^2} + 1 = 0 \Rightarrow R_L^2 = R_{INT}^2$$ (3)

or $\underline{\underline{R_L = R_{INT}}}$

(iii) $$\text{Efficiency} = \frac{I^2 R_L}{I^2(R_{INT}+R_L)} \times 100 = \frac{R_L}{R_{INT}+R_L} \times 100\%$$

When $R_{INT} = R_L$ efficiency $= \frac{R_L}{2R_L} \times 100 = \underline{\underline{50\%}}$ (1)

(b) (i) The resistance of the network is:

$$R_T = \frac{1}{\frac{1}{R_1+R_2} + \frac{1}{R_3+R_4}} = \frac{1}{\frac{1}{45}+\frac{1}{90}} = 30\Omega \quad (1)$$

(ii) The current flowing through $R_3$ & $R_4$:

$$I_{34} = \frac{V}{R_3+R_4} = \frac{60}{90} = 0.667A \quad (1)$$

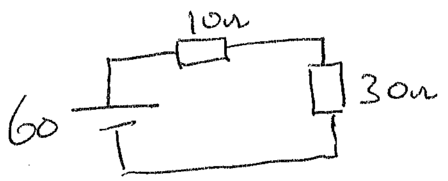
(iii) The power dissipated in $R_3 = I_{34}^2 . R_3 = 0.667^2 . 50 = 22.24W$

The total power dissipated in the network is:

$$P_T = \frac{V^2}{R_T} = \frac{60^2}{30} = 120W \quad (2)$$

(iv) The voltage across $R_2$ is $\frac{30}{45} \times 60 = 40V$ (1)

(c) The internal resistance is now $10\Omega$:




Therefore the total current flowing reduces to $\frac{60}{40} = 1.5A$

and hence the voltage across each branch of the load will be

$$30 \times 1.5 = 45V$$

(or alternatively $60 - 10 \times 1.5 = 45V$)

The voltage across $R_1$ is:

$$V_{R1} = \frac{15}{45} \times 45 = 15V \quad (3)$$

## QUESTION 2 (CONTINUED)

(ii) The current through $R_3$ is:

$$I_{34} = \frac{45}{90} = 0.5A$$

Hence power dissipated in $R_3$

$$P_{R3} = I_{34}^2 . R_3 = 0.5^2 . 50 = \underline{\underline{12.5W}}$$

Power dissipated in the internal resistance is:

$$P_{INT} = 1.5^2 \times 10 = \underline{\underline{22.5W}}$$

(3)

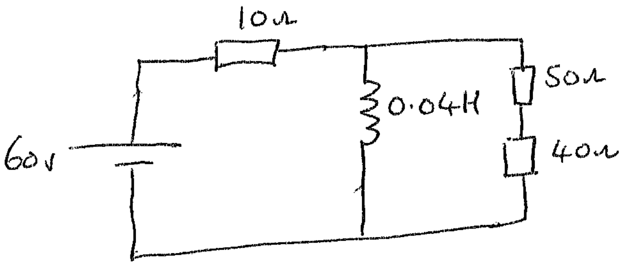
(iii) The total power drawn from the source is

$$P_T = V . I = 60 \times 1.5 = 90W$$

$$\therefore \text{ Efficiency} = \frac{90 - 22.5}{90} \times 100 = \underline{\underline{75\%}}$$

(1)

(iv) The circuit now becomes:




On DC the inductor appears as a short circuit since $\frac{dI}{dt} = 0$ and hence $V_L = L\frac{dI}{dt} = 0$ so there can be no current flowing down the branch containing $R_3$ and $R_4$ and hence the power dissipated in $R_3$ is zero

(3)

The current is then $I = \frac{60}{10} = 6A$

Hence the energy stored in the inductor $= \frac{1}{2}LI^2 = \frac{1}{2} . 0.04 . 6^2$

$= \underline{\underline{0.72J}}$

## QUESTION 3

(a) Since the secondary impedance $Z_2 = \frac{V_2}{I_2}$ and

given that $\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$

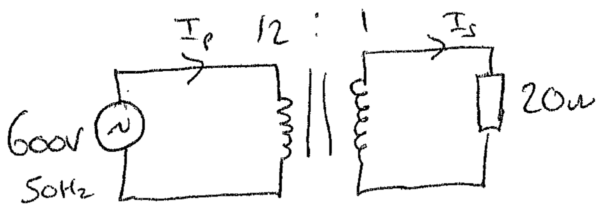
then $Z_2 = \frac{V_2}{I_2} = \frac{V_1 N_2}{N_1} \times \frac{N_2}{N_1 I_1} = \left(\frac{N_2}{N_1}\right)^2 \cdot \frac{V_1}{I_1}$

Therefore the reflected impedance at the primary side

$$Z_1 = \frac{V_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$$

(2)

(b)




Since $\frac{V_p}{V_s} = \frac{N_p}{N_s} \Rightarrow V_s = \frac{V_p . N_s}{N_p} = \frac{600 . 1}{12} = \underline{\underline{50V}}$

Now $I_s = \frac{V_s}{R_s} = \frac{50}{20} = \underline{\underline{2.5 \text{ Arms}}}$

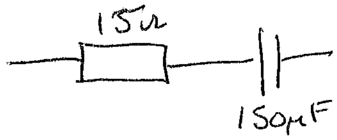
and since $\frac{I_p}{I_s} = \frac{N_s}{N_p} \Rightarrow I_p = \frac{I_s . N_s}{N_p} = \frac{2.5 \times 1}{12} = \underline{\underline{0.208 \text{ Arms}}}$

The power dissipated in the load $= I_s^2 R_s = 2.5^2 \times 20$

$= \underline{\underline{125W}}$

(3)

(ii) The load now comprises:




$$\therefore Z = 15 - \frac{j}{2\pi \times 50 \times 150 \times 10^{-6}} = 15 - j21.22 = 25.98\angle -54.7^\circ$$

The secondary voltage remains unchanged at $\underline{\underline{50\,V_{rms}}}$

$$I_S = \frac{50\angle 0^\circ}{25.98\angle -54.7^\circ} = \underline{\underline{1.925\angle 54.7^\circ \text{ Arms}}}$$

$$I_P = \frac{1.925\angle 54.7^\circ}{12} = \underline{\underline{0.16\angle 54.7^\circ \text{ Arms}}}$$

4

Power dissipated in the load is, $P = I_S^2 R_L = 1.925^2 \times 15 = \underline{\underline{55.6\,W}}$

(iii) The input power factor is $\cos 54.7^\circ = \underline{\underline{0.58 \text{ leading}}}$

The VA rating is $600 \times 0.16 = \underline{\underline{96\,VA}}$

(or $50 \times 1.925 = 96VA$)

2

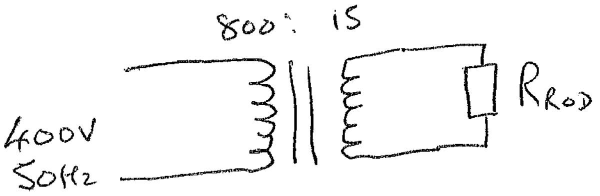
(iv) Since $V_P = 4.44 f N \phi_{MAX}$

$600 = 4.44 \times 50 \times N_P \times 5 \times 10^{-3} \Rightarrow N_P = \underline{\underline{541 \text{ TURNS}}}$

and $50 = 4.44 \times 50 \times N_S \times 5 \times 10^{-3} \Rightarrow N_S = \underline{\underline{45 \text{ TURNS}}}$

2 1

c.




(v) $V = 4.44 \times 60 \times 541 \times 5 \times 10^{-3} = \underline{\underline{720.6\,V_{rms}}}$

1

First calculate the resistance of the rod at both temperatures:

$$R_0 = \frac{\rho L}{A} = \frac{8.33 \times 10^{-8} \times 0.6 \times 4}{\pi \times 0.008^2} = 9.94 \times 10^{-4}\,\Omega$$

Now at $40^\circ C$

$$R_{40} = R_0(1+\alpha_0 T) = 9.94\times10^{-4}(1+6\times10^{-3}\times40)$$

$$= 1.23\times10^{-3}\,\Omega$$

$$R_{650} = R_0(1+\alpha_0 T) = 9.94\times10^{-4}(1+6\times10^{-3}\times650)$$

$$= 4.87\times10^{-3}\,\Omega$$

The secondary voltage of the transformer is:

$$V_S = \frac{400}{800}\times 15 = 7.5\,V_{rms}$$

At $40^\circ C$ the power dissipated is:

$$P_{40} = \frac{V^2}{R_{40}} = \frac{7.5^2}{1.23\times10^{-3}} = \underline{\underline{45.7\,kW}}$$

At $650^\circ C$ the power dissipated is:

5

$$P_{650} = \frac{7.5^2}{4.87\times10^{-3}} = \underline{\underline{11.5\,kW}}$$

(ii) Since the load is purely resistive the power factor is unity

$$P_{40} = V_p . I_{P40} \Rightarrow I_{P40} = \frac{45700}{400} = \underline{\underline{114.25\,A_{rms}}}$$

1

$$P_{650} = V_p I_{p650} \Rightarrow I_{p650} = \frac{11500}{400} = \underline{\underline{28.75\,A_{rms}}}$$

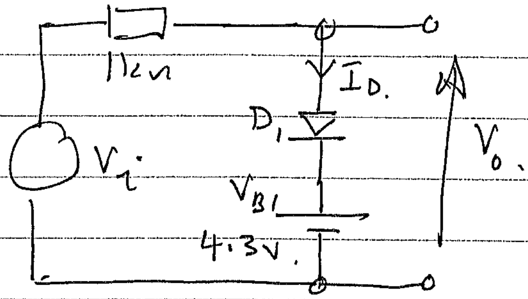
(iii) If the transformer has an efficiency of 96% then if the output is 45.7 kW the input power is:

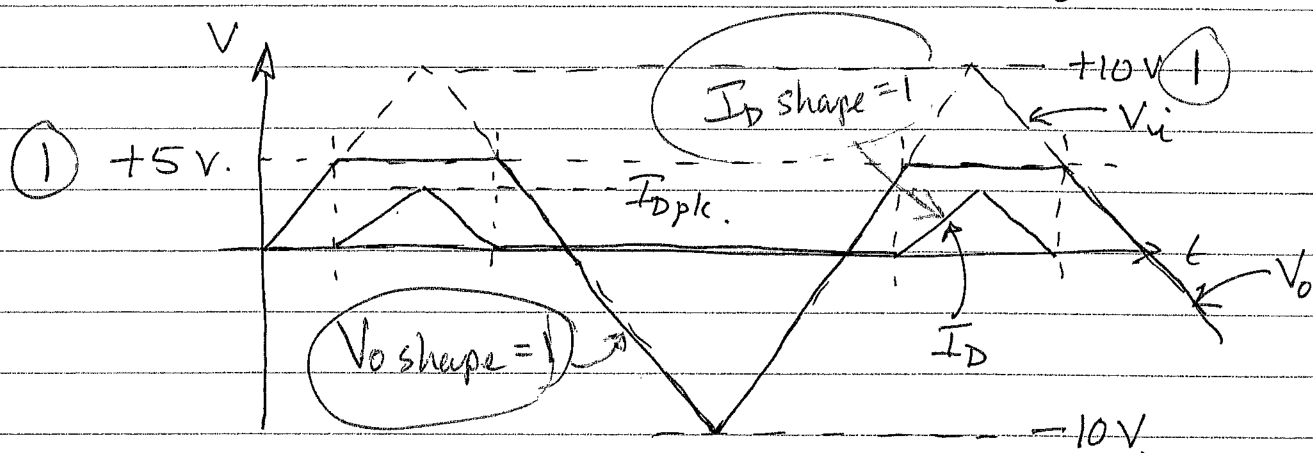
1

$$P_{IN} = \frac{45.7}{0.96} = \underline{\underline{47.6\,kW}}$$

Q4 a

$D_1$ will be on the point of conduction when $V_i = 5V$ ($= 4.3 + 0.7$). $D_1$ will conduct for $V_i > 5V$.



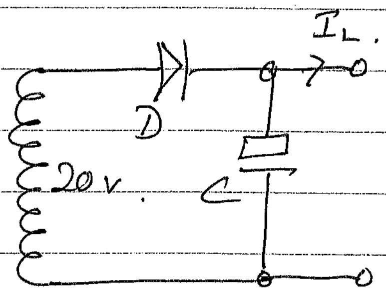




$$I_{DPK} = \frac{10V - 5V}{1k\Omega} = \underline{5mA} \quad 1$$

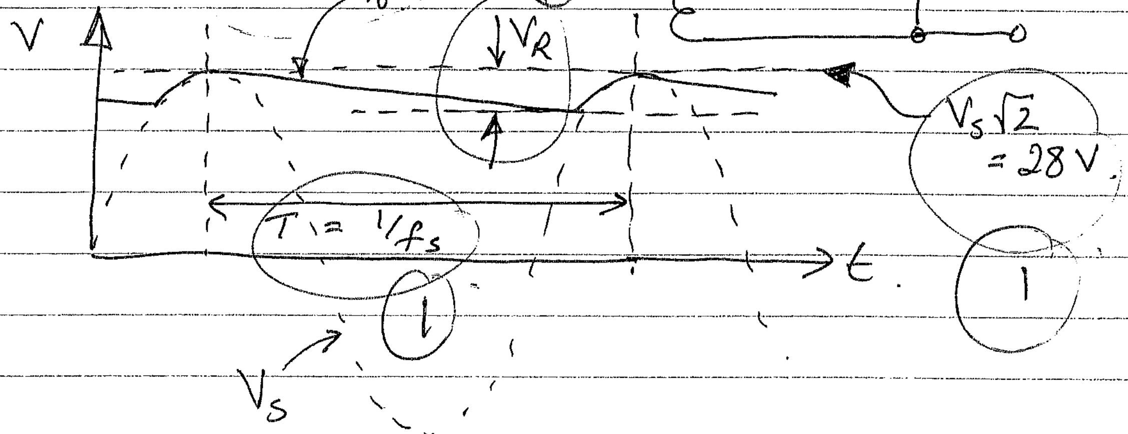
If $D_2$ & $V_{B2}$ are now added, $D_2$ will be on point of conduction when $V_i = V_{B2} - 0.7$ and will conduct for $V_i < V_{B2} - 0.7$. If the point of conduction of $D_2$ is required to be $-1V$ 1

$$-1 = V_{B2} - 0.7 \quad \text{or} \quad \underline{V_{B2} = -0.3\,V} \quad 1$$

b (i) The cct is a half-wave rectifier. 1

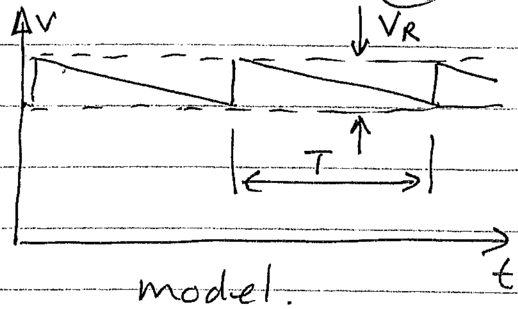



(ii)



(iii) assume that C charges instantaneously 1
assume that $I_L$ remains constant... 1

$$I_L = C\frac{dV}{dt} = C\frac{V_R}{T} \quad 1$$



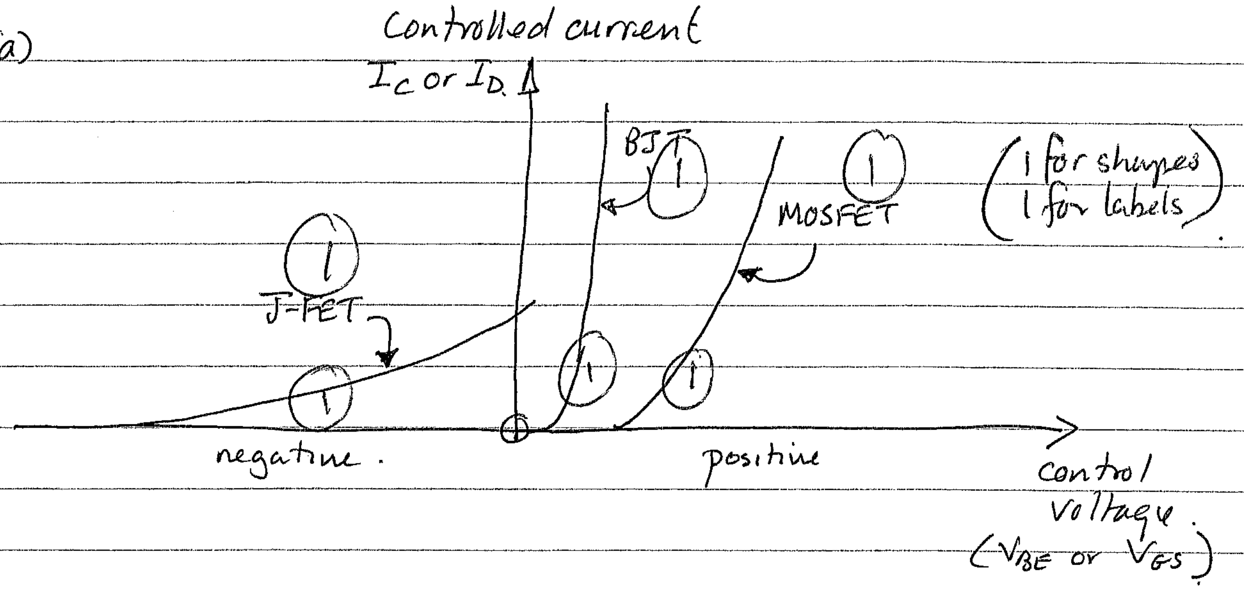

The longer T is, the bigger will be $V_R$ for a given C so we must choose the lowest possible input frequency as the basis for T. 1

$$\therefore I_L = 0.15 = C\frac{2}{1/380} \quad 1$$

$$\text{or } C = \frac{0.15}{2 \times 380} = \underline{\underline{197\,\mu F}} \quad 1$$

(iv) The energy supplied to the load during the discharge period is replaced over a short interval near the peaks of the charging half cycles (+ve half cycles in this case). 1
This kind of very peaky current demand creates a significantly higher loss in the generation and distribution system than a sinusoidal current demand with equivalent load power. 1

Q5 (a)

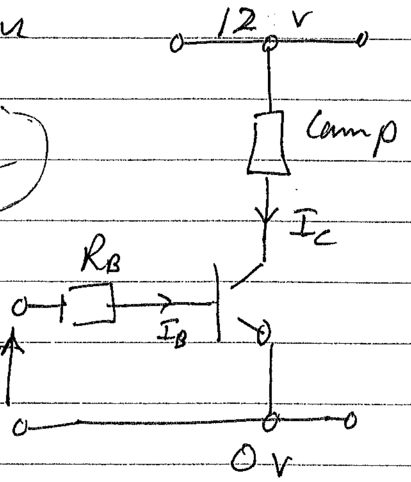



(1 for shapes, 1 for labels).

(b)(i) cold lamp resistance $= 0{\cdot}6\,\Omega$

$\therefore$ Initial current

$$= \frac{12}{0{\cdot}6} = \underline{\underline{20A}}. \quad 2$$




(ii) under normal running conditions the lamp dissipates 50W with a 12V supply. 1

$$\therefore \; 50 = {12^2}/{R_{lamp(hot)}} \qquad R_{lamp(hot)} = \underline{\underline{2{\cdot}88\ \Omega}}. \quad 1$$

$$\text{normal running current} = \frac{50W}{12V} = \underline{\underline{4{\cdot}17\,A}} \quad 1$$

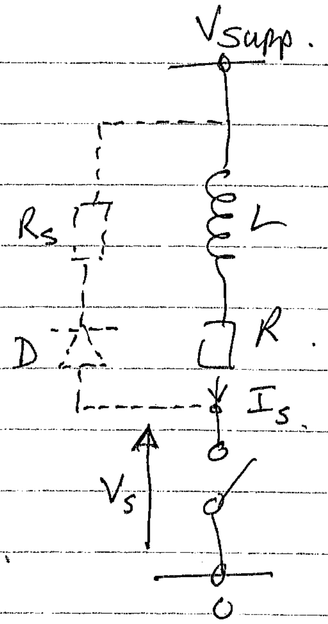
(iii) $$\text{worst case } I_B \text{ required} = \frac{\text{worst case } I_C}{h_{FE}}. \quad 1.$$

$$= \frac{20A}{h_{FE}} = 0{\cdot}2A. \quad 1$$

$$\text{Voltage across } R_B = 5 - 0{\cdot}7 \quad 1$$

$$\therefore R_B = \frac{5-0{\cdot}7}{I_{B\,max}} = \frac{5-0{\cdot}7}{0{\cdot}2A} = \underline{\underline{21{\cdot}5\ \Omega}} \quad 1 .$$

c The problem with inductive loads is that they store energy. 1



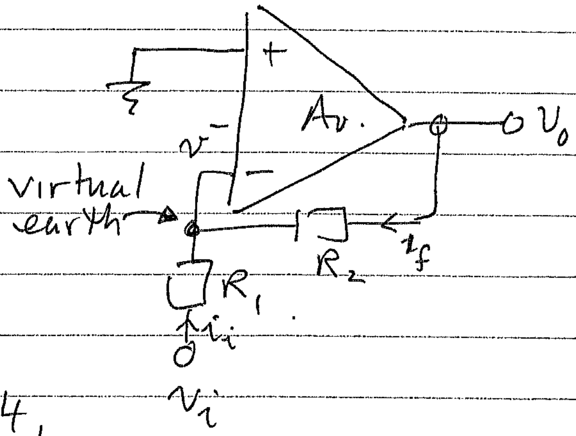

When the switch switches off the inductive stored energy tries to keep the current flowing so $V_s$ 1 rapidly rises and, if not controlled, will rise to levels that will cause 1 damage.

One way of limiting the rise of $V_s$ is to provide a path for the inductive current — D + $R_s$ will 1 provide such a path in the circuit above. $R_s$ is not essential but D is ... when $V_s$ rises to $V_{supp}$ + 0.7 D conducts and provides a path for the inductor current. The inductive stored 1 energy is then dissipated in R and D (and $R_s$ if it is present).

Q6. a

analogue (1)
low output resistance (1)
high voltage gain (1)
high input impedance (1)
differential input. (1)

b (i) inverting (1)




(ii) the op-amp's inverting input is the virtual earth node. (1)

to get a |gain| of 24,
$R_2/R_1 = \underline{24}$. (1)

$v_o$ is $\underline{\pm 180^\circ}$ w.r.t. $v_i$ (1)

(iii) If $A_v$ is finite ... sum currents at $v^-$ node

$$i_i + i_f = 0 \quad (1)$$

$$\frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2} = 0$$

$$\text{or } v^- = \frac{v_i R_2}{R_1 + R_2} + \frac{v_o R_1}{R_1 + R_2}. \quad (1)$$

using the op-amp equation ...

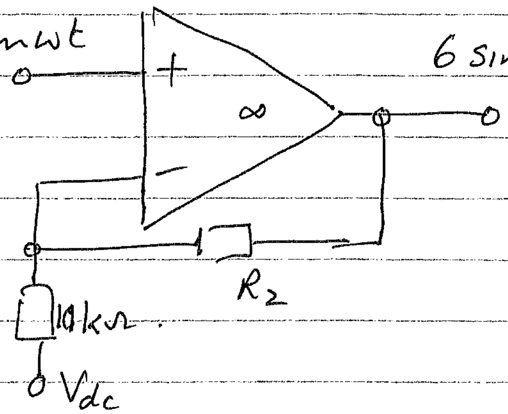
$$v_o = A_v(v^+ - v^-) = -A_v v^- \text{ (since } v^+ = 0) \quad (1)$$

$$\text{or } \quad \frac{v_o}{A_v} = v^- = -\frac{v_i R_2}{R_1 + R_2} - \frac{v_o R_1}{R_1 + R_2} \quad (1)$$

$$\text{or } \quad v_o\left[\frac{1}{A_v} + \frac{R_1}{R_1 + R_2}\right] = -\frac{v_i R_2}{R_1 + R_2}$$

or $$\frac{v_o}{v_i} = -\frac{R_2/(R_1+R_2)}{1/A_v + R_1/(R_1+R_2)}.$$ 1

c




The a.c. component must be operated on by a non-inverting gain 1 of 4.

$$4 = \frac{10k\Omega + R_2}{10k\Omega}.$$

$\therefore\ \underline{R_2 = 30k\Omega}.$ 1

The total dc component at the output must be zero ....

$$V_o\Big|_{\text{due to } 1.5V \text{ on } v^+} = 4 \times 1.5 = 6V.$$ 1

$$V_o\Big|_{\text{due to } V_{dc}} = -\frac{R_2}{10k\Omega} \times V_{dc} = -3V_{dc}.$$ 1

Total must = 0

so $6V + (-3V_{dc}) = 0$ 1

or $\underline{V_{DC} = 2V}$. 1