EEE 117.

Electrical Circuits and Networks.

First half (thats ~~TOZER~~)

- basic ideas of circuits and components and key cct laws.
- d.c. circuits
- a.c. circuits
  - phasor representation
  - complex number representation.
  - power

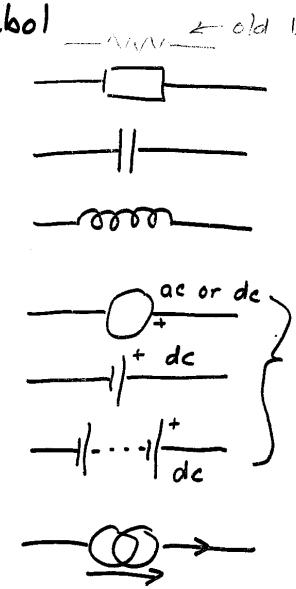
Ken Mitchell

- magnetic circuits
- inductance
- power.
- real + reactive power
- three phase.

Networks

- usually described by circuit diagrams (or schematic diags.) (or ~~sc~~ schematics)
- represents the connectivity of an assembly of electrical/electronic components.

each different component has a symbol



resistor

capacitor

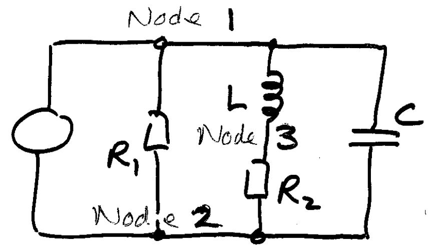
inductor

voltage or e.m.f. sources.

current source.

Components connected together by ideal wires

eg



- a circuit "node" is where two or more components are connected together

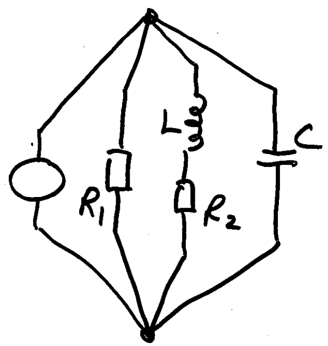
(Node 1 has 4 connections

Node 2 has 4 connections
Node 3 has 2 connections

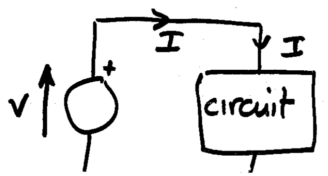
Simple node because it only has 2 connections

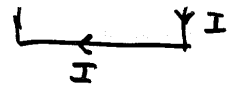
Major nodes because they have 3 or more connections.

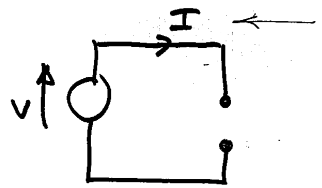
cct can be redrawn to make the nodes more obvious



Current flows around a circuit
— it is driven by "electromotive force" or e.m.f. The emf provides a potential difference which makes the electrons move.

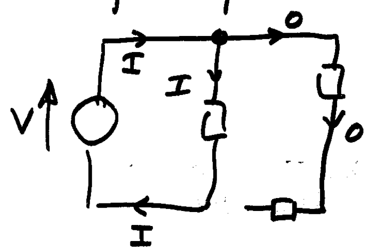


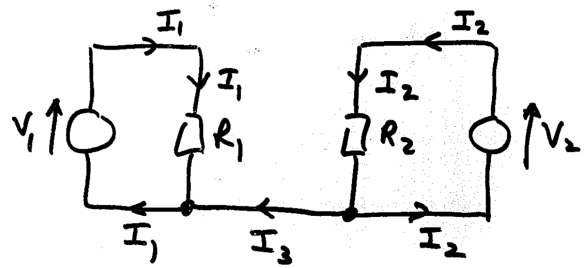




Here, $I = 0$. because the circuit is open.

If part of the circuit is open, no current will flow through the open part



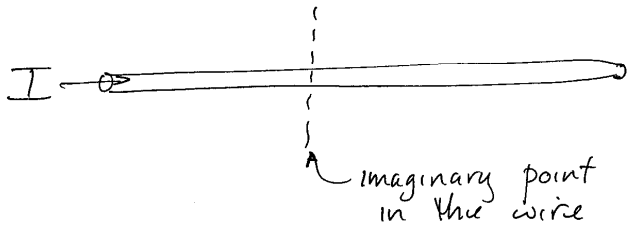


$I_3 = 0.$

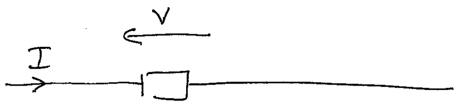
— Objective is to understand how circuits work so that one can be inventive in achieving design aims.

Current

— rate of passage of charge through an imaginary point in a cct



$$I = \frac{dQ}{dt} \text{ at the imaginary point.}$$



Power + Energy:

$$\text{work done} = \text{energy used} = \int_0^t V.I.$$

$$= V.I.t \text{ at dc.}$$

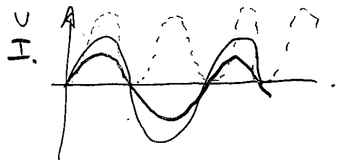
$$\text{power} = \text{rate of energy use}$$

$$= \frac{1}{t}\int_0^t VI. = VI \text{ at dc.}$$

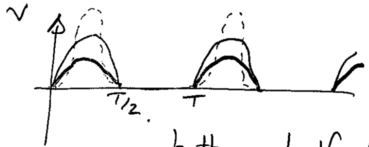
If V + I are functions of time,
ie $V \Rightarrow V(t)$ $I \Rightarrow I(t)$

$$E = \int_0^t V(t)\, I(t)\, dt.$$

$$\text{Power} = \frac{1}{T}\int_0^T V(t)\, I(t)\, dt.$$

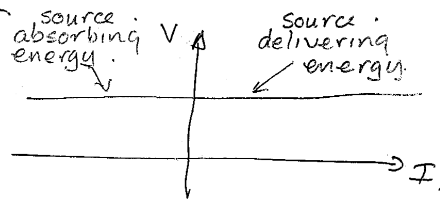


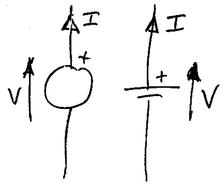
$$P = \frac{1}{T}\int_0^{T/2} V(t) I(t) dt.$$

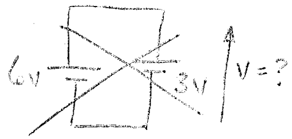


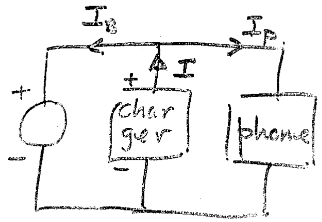
bottom half has been chopped off.

Voltage Sources



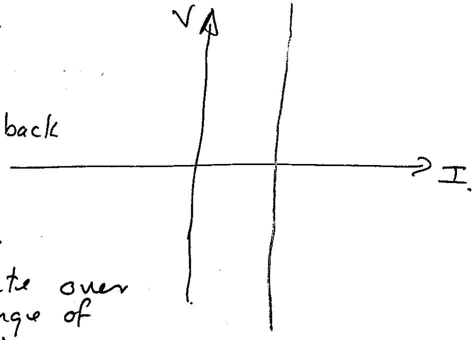


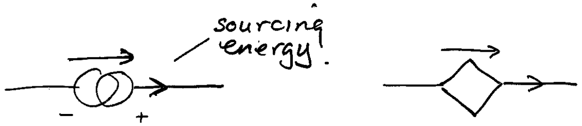


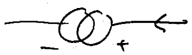


Current Sources

- usually exist as electronic ccts with internal feedback designed to maintain a constant I.
- usually operate over a limited range of terminal voltage.

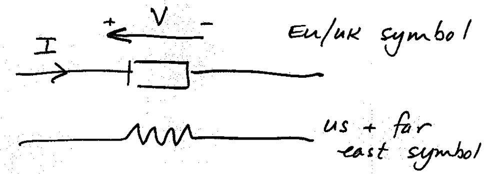






sinking or absorbing energy.

Resistors



$V = IR$

R is an energy sink and it dissipates the energy it sinks

Size (physical) of the resistor determines its ability to dissipate heat.

Power in resistive circuits

$P_{at\ dc} = VI \quad J\,s^{-1}$ or $\boxed{W - \text{Watts}}$

$V = IR$.

$P = VI = V\left(\frac{V}{R}\right) = \frac{V^2}{R}\ W$ (if V = volts)

$\left(I = \frac{V}{R}\right)$

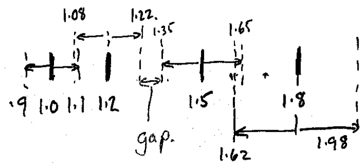
$P = (IR)I = I^2R.\ W$ (if I = amps

Preferred resistor values.

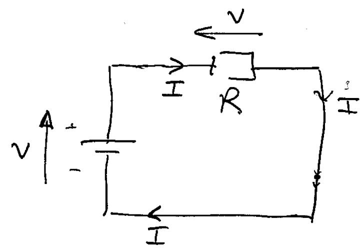
- preferred value series

E6 → 20% tolerance.
E12 → 10% tolerance
E24 → 5% tolerance.

look at E12.



E12 series 1, 1.2, 1.5, 1.8, 2.2, 2.7, 3.3, 3.9, 4.7, 5.6, 6.8, 8.2

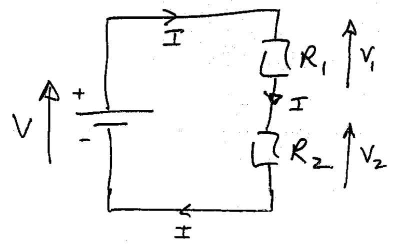


Sum of directed voltages around any closed loop in a cct must be zero.

Current entering a point in the circuit must equal current leaving that point.

→ Kirchoff's laws

If there is more than one resistor in the circuit ...



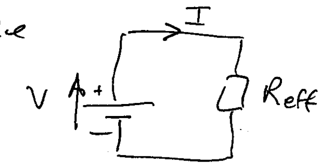
R_1 + R_2 are in series because the same current must flow through them.

$V_1 = IR_1 \qquad V_2 = IR_2$

$$V + (-V_1) + (-V_2) = 0$$

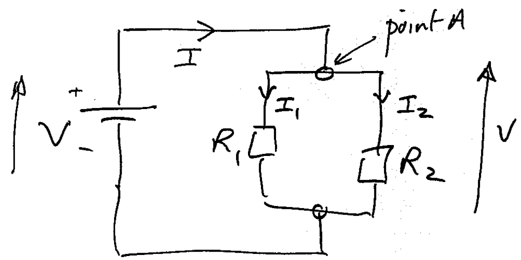
$$\text{or } V = V_1 + V_2 = IR_1 + IR_2$$
$$= I(R_1 + R_2) \quad \text{——} ①$$

want to make



$$R_{eff} = \frac{V}{I} = R_1 + R_2 \quad (\text{from eq } ①)$$

If more than two resistors are connected in series
R_{eff} = sum of values.



$$I_1 = \frac{V}{R_1} \qquad I_2 = \frac{V}{R_2}$$

using Kirchoff's current law.

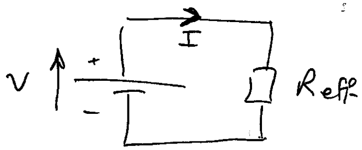
$$I = \underbrace{I_1 + I_2}_{\text{current leaving point A.}}$$

current entering

point A

$$I = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{—(2)}$$

to model this as



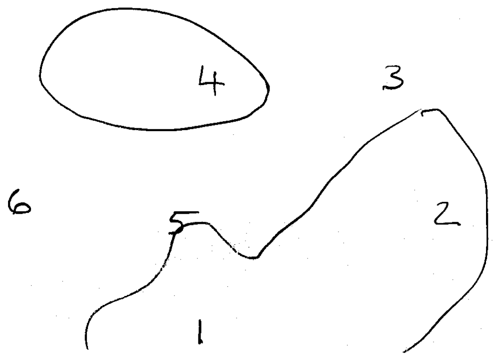
$$R_{eff} = \frac{V}{I} = \frac{V}{V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \qquad \text{(using equ. (2))}$$

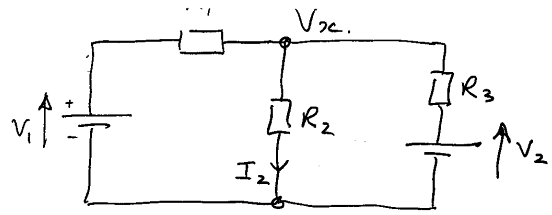
$$\therefore \quad R_{eff} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$\text{or} \quad \frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$$

if there are more than two resistors in parallel

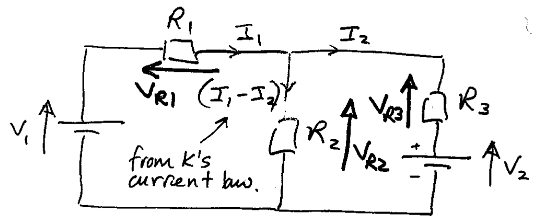
$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$





Methods of solution

(1) Application of Kirchoff's laws.



use Kirchoff's current law at each node to avoid creating redundant variables

use K's voltage law to sum voltages around the two loops.

$$V_1 - V_{R_1} - V_{R_2} = 0 \quad \text{——} \quad \text{loop 1}$$

$$V_2 + V_{R3} - V_{R2} = 0 \quad \text{——} \quad \text{loop 2.}$$

expanding loop 1

$$V_1 - I_1 R_1 - (I_1 - I_2) R_2 = 0 \quad \text{— } \ell 1$$

expanding loop 2

$$V_2 + I_2 R_3 - (I_1 - I_2) R_2 = 0 \quad \text{— } \ell 2$$

collecting terms

collecting terms

$$V_1 - I_1(R_1+R_2) + I_2R_2 = 0 \quad \ell 1.$$

$$V_2 + I_2(R_3+R_2) - I_1R_2 = 0 \quad \ell 2.$$

using the substitution approach.

from $\ell 1$ $\quad I_2 = \dfrac{I_1(R_1+R_2) - V_1}{R_2}.$

$\therefore$ using $\ell 2$ $\quad V_2 + \dfrac{I_1(R_1+R_2)-V_1}{R_2}(R_3+R_2) - I_1R_2 = 0$

$$V_2R_2 + (I_1(R_1+R_2) - V_1)(R_3+R_2) - I_1R_2^2 = 0.$$

$$V_2R_2 + I_1(R_1+R_2)(R_3+R_2) - V_1(R_3+R_2) - I_1R_2^2 = 0.$$

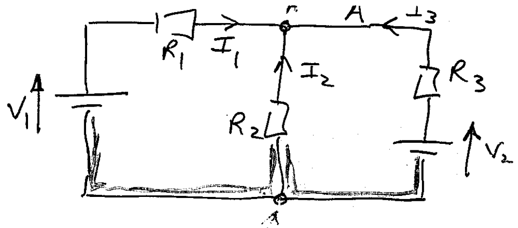
$$V_2R_2 + I_1(R_1R_3 + R_2R_3 + R_2R_1 + \cancel{R_2^2}) - V_1(R_3+R_2) \cancel{- I_1R_2^2} = 0.$$

$$\therefore I_1 = \frac{V_1(R_3+R_2) - V_2R_2}{R_1R_3 + R_2R_3 + R_2R_1}$$

— substitute back in one of the loop equations to find I_2

Nodal Analysis

— aims to find node voltage w.r.t a reference node.



reference node (all the green stuff is reference node.)

let voltage at A w.r.t. reference node be V_A

— label currents entering (or leaving) the node.

— do a current sum at the node

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_1 - V_A}{R_1} + \frac{0 - V_A}{R_2} + \frac{V_2 - V_A}{R_3} = 0$$

collect terms together to end up in this case with.

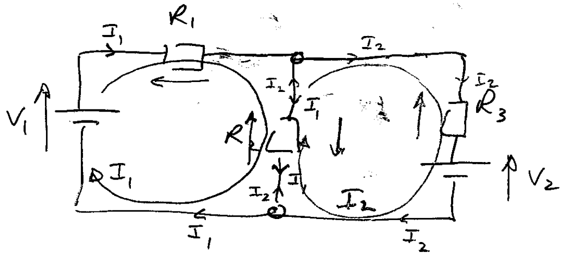
V_A = (a function of V_1, V_2 + R_s).

$$\frac{V_1}{R_1} + \frac{V_2}{R_3} = V_A\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right].$$

$$V_A = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

then all the other cct variables can be found easily once V_A is known.

LOOP ANALYSIS



- define a current flowing ~~=~~ around each loop of the circuit
- add up directed voltage drops around each loop.
- loop 1 $\quad V_1 - I_1R_1 - (I_1 - I_2)R_2 = 0$

  or $\quad V_1 = I_1(R_1 + R_2) - I_2R_2$
- loop 2 $\quad (I_2 - I_1)R_2 + I_2R_3 + V_2 = 0$

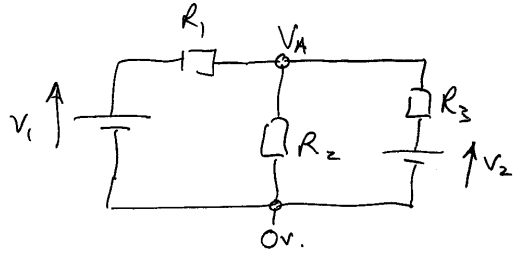
  or $\quad -V_2 = -I_1R_2 + I_2(R_2 + R_3)$

Principle of superposition

The current in any branch of a circuit is linearly proportional to each of the sources driving the circuit

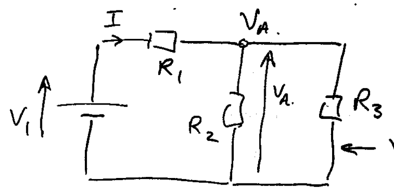
This means that each source can be considered in turn with all the other sources replaced by their internal impedance (0Ω for a voltage source

$\infty\Omega$ (ie open cct) for current source)



eg find V_A using superposition

$V_A\big|_{\text{due to } V_1}$

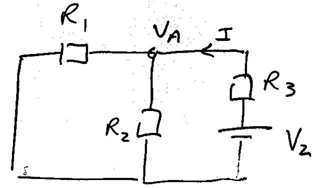


← V_2 replaced by 0Ω.

$$I = \frac{V_1}{R_1 + R_2 \| R_3}$$

$$V_A = I(R_2 \| R_3) = V_1 \frac{R_2 \| R_3}{R_1 + R_2 \| R_3}$$

$V_A\big|_{\text{due to } V_2}$.



$$I = \frac{V_2}{R_3 + (R_1 \| R_2)}$$

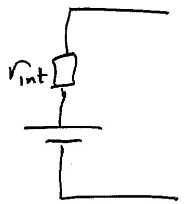
$$V_A = I(R_1 \| R_2) = \frac{V_2 \times (R_1 \| R_2)}{R_3 + (R_1 \| R_2)}$$

$$V_{ATOT} = V_A\Big|_{\text{due to } V_1} + V_A\Big|_{\text{due to } V_2}$$

$$= V_1 \frac{R_2 \| R_3}{R_1 + (R_2 \| R_3)} + V_2 \frac{R_1 \| R_2}{R_3 + (R_1 \| R_2)}$$

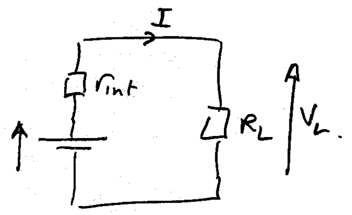
Thevenin and Norton Equivalent Ccts.

Thevenin



this would model a battery.

r_{int} found by



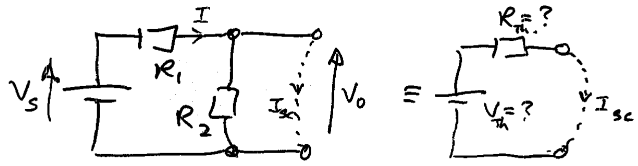
V_{oc} ↑ open cct battery voltage

$$I = \frac{V_{oc}}{r_{int} + R_L}$$

$$V_L = I R_L = \frac{V_{oc} R_L}{r_{int} + R_L}$$

r_{int} is the only unknown.

If I have a cct



what is V_o with no external load?

$$V_o = V_s \frac{R_2}{R_1 + R_2}$$

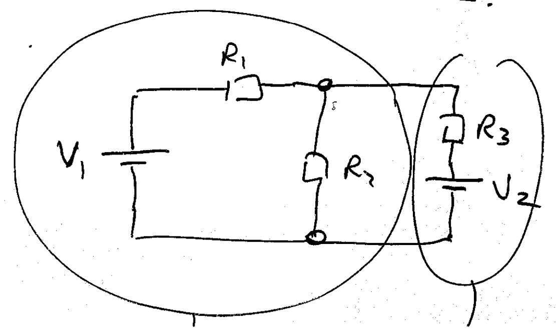
$$I = \frac{V_s}{R_1 + R_2} \qquad V_o = I R_2 = \frac{V_s R_2}{R_1 + R_2}.$$

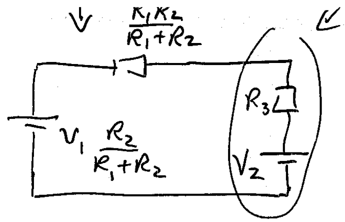
$$V_{Th} = V_o = V_s \frac{R_2}{R_1 + R_2}.$$

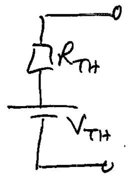
$$I_{sc} = V_s / R_1 \qquad I_{sc} = \frac{V_{TH}}{R_{TH}} = \frac{V_s R_2}{R_{TH}(R_1 + R_2)}$$

$$\therefore \frac{\cancel{V_s}}{R_1} = \frac{\cancel{V_s} R_2}{R_{TH}(R_1 + R_2)}$$

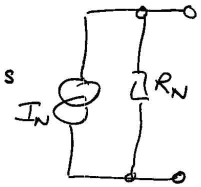
$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2}.$$







can be represented as



Thevenin equivalent cct

Norton equivalent cct.

short cct o/p $I = \frac{V_{Th}}{R_{Th}}$

short cct o/p $I = I_N$

$\therefore \; I_N = \frac{V_{Th}}{R_{Th}}$ for equivalence

open cct o/p $V = V_{Th}$

open cct o/p voltage

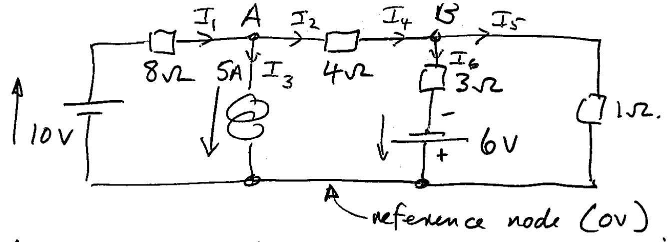
$= I_N R_N$

$= \frac{V_{Th}}{R_{Th}} . R_N$.

$V_{Th} = \frac{V_{Th} R_N}{R_{Th}}$ for equivalence

so $R_{Th} = R_N$

A more complicated cct with a current source.



A nodal approach.

Sum currents at node A

$$\underset{\text{entering}}{I_1} = \underbrace{I_2 + I_3}_{\text{leaving}}.$$

$$\frac{10 - V_A}{8} = \frac{V_A - V_B}{4} + 5 \qquad \text{———} \; (1)$$

Sum currents at node B.

$$I_4 = I_5 + I_6$$

$$\frac{V_A - V_B}{4} = \frac{V_B - 0}{1} + \frac{V_B - (-6)}{3} \qquad \text{———} \; (2)$$

rearrange (1)

$$10 - V_A = 2V_A - 2V_B + 8.5$$
$$= 2V_A - 2V_B + 40$$
$$-30 = 3V_A - 2V_B. \qquad \text{———} \; (1a)$$

rearrange (2)

$$3V_A - 3V_B = 12V_B + 4V_B + 24$$

$$24 = 3V_A - 19V_B \quad \text{———} \quad (2a).$$

$$(-30 = 3V_A - 2V_B)$$
$$+(-24 = -3V_A + 19V_B)$$
$$-54 = 0 + 17V_B$$

$$V_B = -\frac{54}{17} = -3.18\ V.$$

sub V_B in 1a.

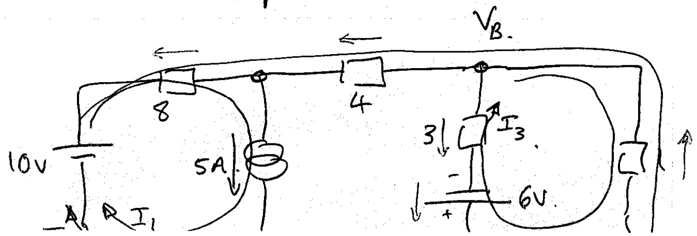
$$-30 = 3V_A - \left(-\frac{108}{17}\right)$$

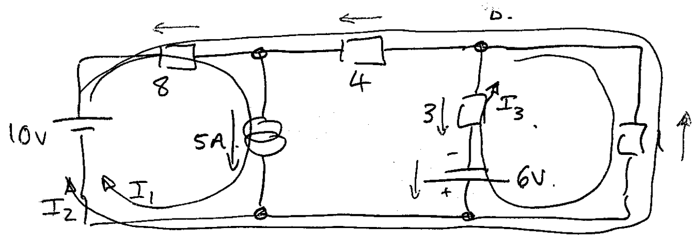
$$-30 - \frac{108}{17} = 3V_A$$

$$V_A = -\frac{510 + 108}{17 \times 3} = -\frac{618}{51}\ V$$
$$= -12.1\ V$$

Check ... using 2a.

$$24 = 3\times(-12.1) - 19\times(-3.18)$$
$$= -36.3 + 60.4$$
$$\approx 24$$





Loop ① $\quad I_1 = 5A$.

loop ② $\quad 10 = 8(I_1 + I_2) + 4I_2 + 1(I_2 + I_3)$

loop ③ $\quad 3I_3 + 6 + (I_2 + I_3)1 = 0$

rearrange loop ②

$$10 = 40 + 13I_2 + I_3.$$
$$-30 = 13I_2 + I_3$$

rearrange loop 3.

$$-6 = I_2 + 4I_3$$

multiply loop 2 by (−4) and add to loop 3

$$120 = -52I_2 - 4I_3.$$
$$-6 = I_2 + 4I_3$$
$$114 = -51I_2 + 0$$

$$I_2 = -\frac{114}{51} = -2.24\ A$$

sub. in loop 3

$$-6 = -\frac{114}{51} + 4I_2$$

$$-6 = -\frac{114}{51} + 4I_3$$

$$\frac{-306 + 114}{51 \times 4} = I_3 = \frac{-192}{4 \times 51} = -0.941\,A$$

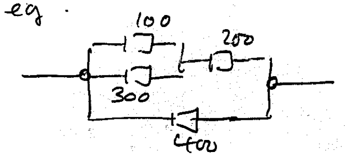
$$V_B = (I_2 + I_3)\,1$$
$$= (-2.24 - 0.94)\,1$$
$$= -3.18\,V.$$

A brief review of homework.

main points

— Not showing working.

eg.



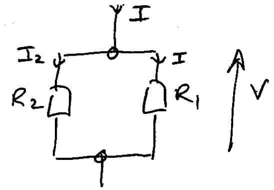
163 Ω.

$$R_{eff} = 400 \,//\, (200 + (100 \,//\, 300))$$
$$= 163.$$

— Not finding voltage across or current the resistor of interest to calculate power.

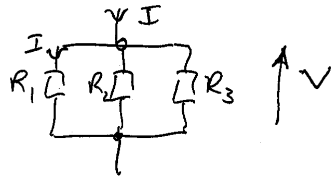
— Abusing current sharing rules.

$$V = I(R_1 \| R_2)$$
$$= I\,\frac{R_1R_2}{R_1+R_2}.$$



$$I_2 = \frac{V}{R_2} = I\,\frac{R_1R_2}{R_1+R_2} \times \frac{1}{R_2}$$
$$= I\,\frac{R_1}{R_1+R_2}$$

$$V = I\,R_1 \| (R_2 \| R_3)$$
$$= I\,\frac{R_1(R_2 \| R_3)}{R_1 + (R_2 \| R_3)}$$



$$I_1 = \frac{V}{R_1} = I\,\frac{\cancel{R_1}(R_2 \| R_3)}{R_1 + (R_2 \| R_3)} \times \frac{1}{\cancel{R_1}}$$
$$= I\,\frac{R_2 \| R_3}{R_1 + (R_2 \| R_3)}$$
$$= I\,\frac{R_2R_3}{R_1R_3 + R_1R_2 + R_2R_3}$$

* — rounding

.03 A → 30 mA or 30×10^{-3} A.

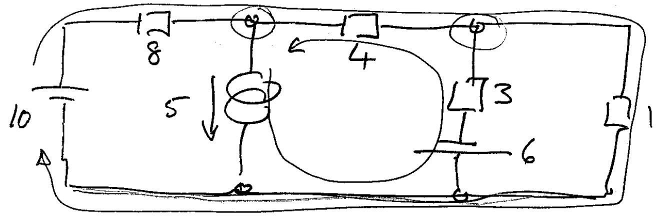
1894 V 1.89×10^3 V or 1.89 kV

3 μA = .000003 A not a good idea

3 significant figures is sufficient

If the answer is 2 it should be written as 2.00

— must draw a circuit if you are going to use algebraic variables like I_1, I_2, V_3, etc.



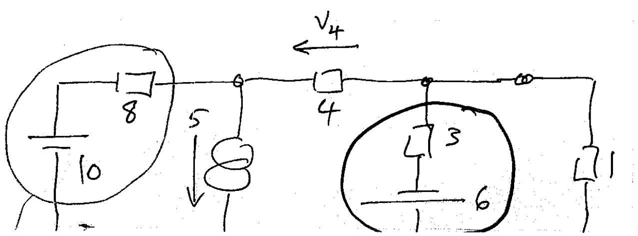
— need as many equations as there are simple closed loops (3 in this case).

number of nodes, $n = 3$.

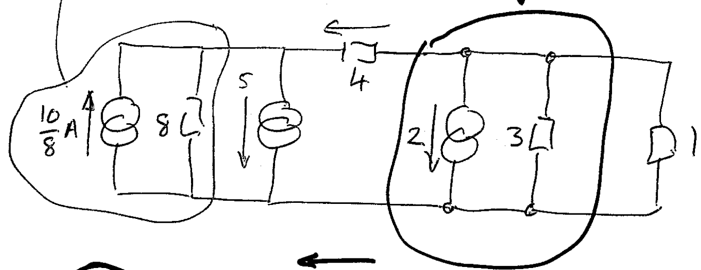
number of branches, $b = 5$

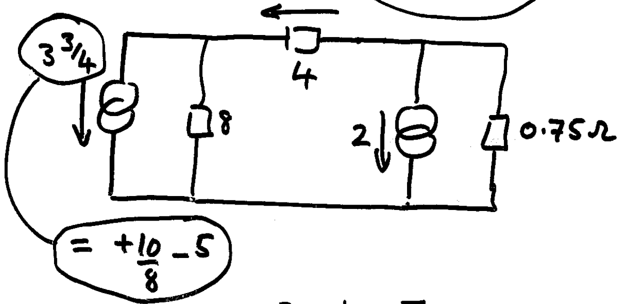
minimum number of loops required

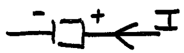
$$= (b - n) + 1$$
$$= (5 - 3) + 1 = 3.$$

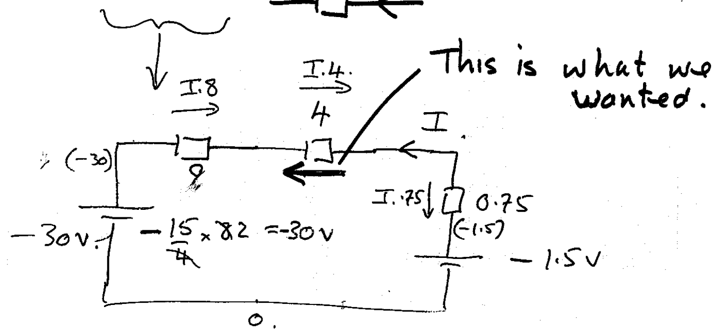


what is V_4 ?









$$-1.5 - I\,0.75 - I4 - I8 = -30.$$

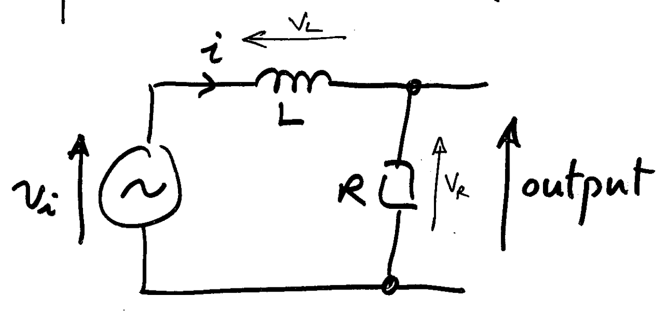
$$28.5 = I\ 12.75$$

$$I = 28.5$$

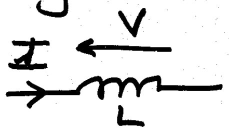
$I = \frac{28.5}{12.75}$

$V_{4\Omega} = -I.4 = -\frac{28.5}{12.75} \times 4$.

A quick background to the passive circuits lab.

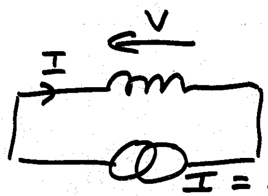


inductor governed by



$$V = L\frac{dI}{dt}.$$

let I be a sinusoidal forced current



angular frequency

$I = I_p \sin \omega t.$

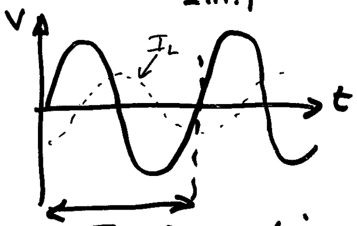
dI ... $= 2\pi f$

$I = I_p \sin \omega t.$ ($\omega = 2.\pi.f$)

$$V = L\frac{dI}{dt}$$

$$V = L\frac{d(I_p \sin \omega t)}{dt}$$

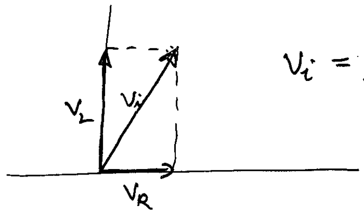
$$= L I_p \omega \cos \omega t$$



T = periodic time $= \frac{1}{f}$

f = cyclic frequency.

I lags V by 90°



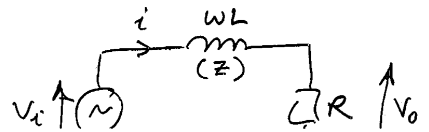
$$V_i = \sqrt{V_L^2 + V_R^2}$$

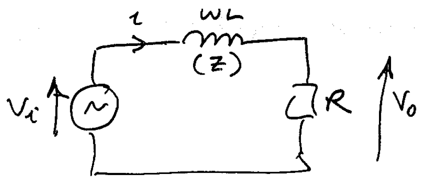
for a sinusoid the "impedance" of an inductor is

$$Z_L = j\omega L = X_L$$

inductive impedance

inductive reactance.





$$i = \frac{v_i}{Z_L + R} = \frac{v_i}{\sqrt{Z_L^2 + R^2}}.$$

$$= \frac{v_i}{\sqrt{\omega^2 L^2 + R^2}}.$$

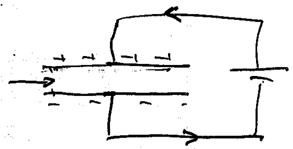
$$\frac{v}{i} = Z_{cct} = \sqrt{\omega^2 L^2 + R^2}$$

$$V_o = iR = v_i \frac{R}{\sqrt{\omega^2 L^2 + R^2}}$$

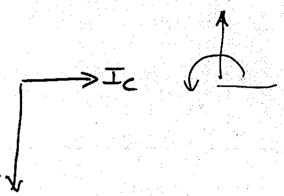
---

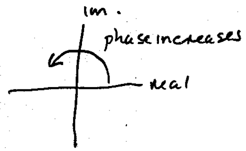
circuits containing C

$$I = C\frac{dV}{dt}.$$

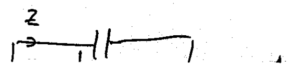


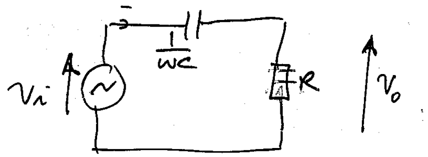
for C





Capacitive reactance $= \frac{1}{\omega C}.$



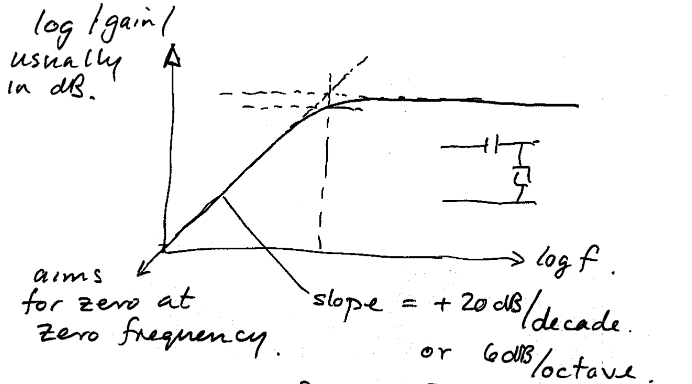


$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

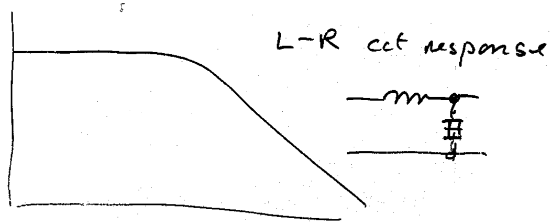
$$V_o = i_z R$$

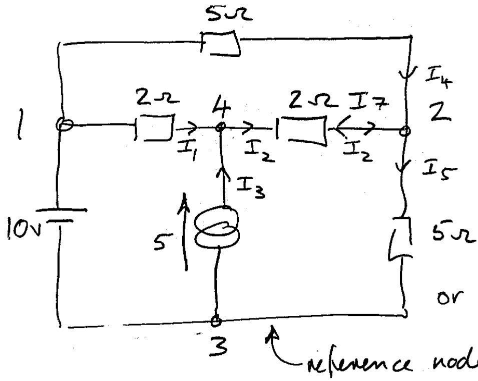
$$i_z = \frac{v_i}{Z}.$$

$$V_o = \frac{V_i}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \cdot R. = \frac{V_i R \, \omega C}{\sqrt{1 + R^2\omega^2C^2}}$$



$$dB = 20 \log [cct\ gain].$$





$$I_4 = I_7 + I_5$$

$$\frac{10-V_2}{5} = \frac{V_2-V_4}{2} + \frac{V_2}{5}$$

$$\frac{10-V_2}{5} - \frac{(V_2-V_4)}{2} = \frac{V_2}{5}$$

or $$\frac{10-V_2}{5} + \frac{V_4-V_2}{2} = \frac{V_2}{5}$$

$V_{node}\ 1 = 10v$ by inspection.

— need node equations to find V_4 + V_2

Sum currents at node 4: $I_1 + I_3 = I_2$

$$\frac{10-V_4}{2} + 5 = \frac{V_4 - V_2}{2}$$

or $\cancel{10} - V_4 + \cancel{10} = V_4 - V_2$

or $20 = 2V_4 - V_2$ ——— ①

Sum currents at node 2: $I_4 + I_2 = I_5$

$$\frac{10-V_2}{5} + \frac{V_4 - V_2}{2} = \frac{V_2}{5}$$

$$20 - 2V_2 + 5V_4 - 5V_2 = 2V_2$$

$$20 = -5V_4 + 9V_2 \text{ ——— ②}$$

Sub V_2 from eqn. ① into ②

$$20 = -5V_4 + 9\left(2V_4 - 20\right)$$

$$20 = 13V_4 - 180$$

or $$V_4 = \frac{200}{13} = 15.4\ V.$$

Using ① $$20 = \frac{2 \times 200}{13} - V_2.$$

$$260 = 400 - 13V_2$$

$$-140 = -13V_2$$

$$V_2 = \frac{140}{13} = 10 \text{ and a bit.}$$

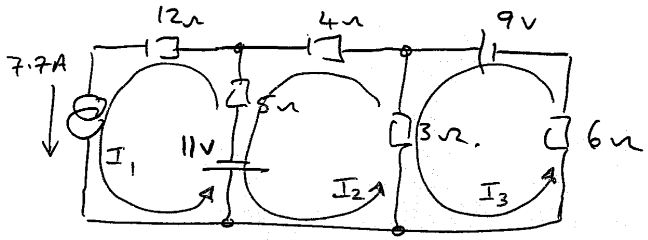
I_1 is the current through top 5Ω from left to right. (I_4 in the cct diag above.)

$$I_1 (= I_4) = \frac{10 - V_2}{5}$$

$$= \frac{10 - \frac{140}{13}}{5}$$

$$= \frac{130 - 140}{5.13} = -\frac{2}{13}$$

$$= -0.154\ A.$$



$I_1 = 7.7A$

Sum voltages around loop I_2.

$$11 - (I_2 - I_3)3 - I_2 4 - (I_2 - I_1)5$$

or $$11 - 12I_2 + 3I_3 + 38.5 = 0.$$

or $11 - 12I_2 + 3I_3 + 38.5 = 0.$

Sum voltages around loop I_3

$$9 - (I_3 - I_2)3 - I_3 6 = 0$$

$$9 - 9I_3 + 3I_2 = 0$$

$$3 - 3I_3 + I_2 = 0$$