MSc(Eng) Wireless Communication Systems

Module EEE-6431: Broadband Wireless Techniques

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Syllabus Highlights

1. **Introduction - Overview of Broadband Wireless Systems**

2. **Signal Propagation, Pathloss Models and Shadowing**

3. **Statistical Fading Models: Narrowband & Wideband Fading**

4. **Capacity of Wireless Channels**

5. **Multicarrier Modulation**

6. **Spread Spectrum and CDMA**

Section 1 Review

1. Course Basics
2. Course Syllabus
3. The Wireless Vision
4. Technical Challenges
5. Current Wireless Systems
6. Emerging Wireless Systems
7. Spectrum Regulation
8. Standards

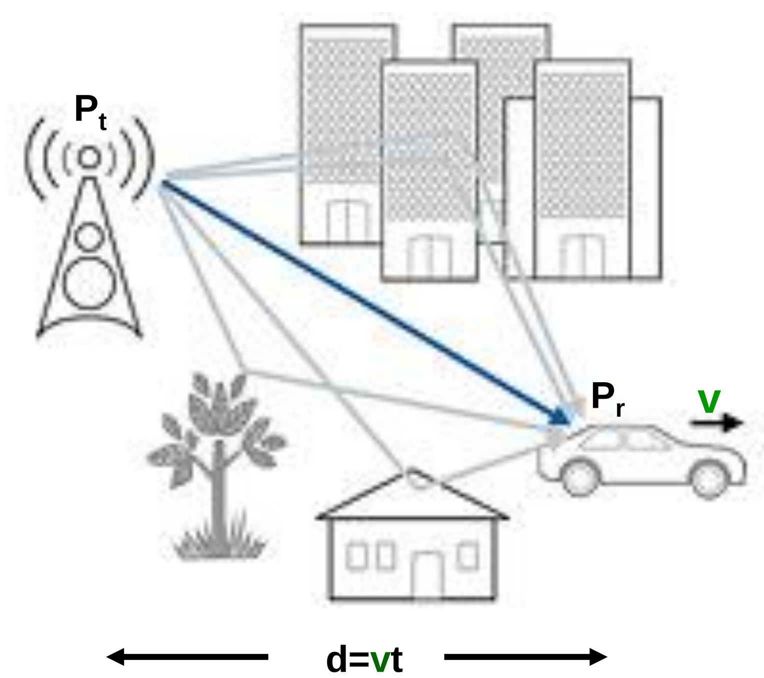
Section 2 Outline

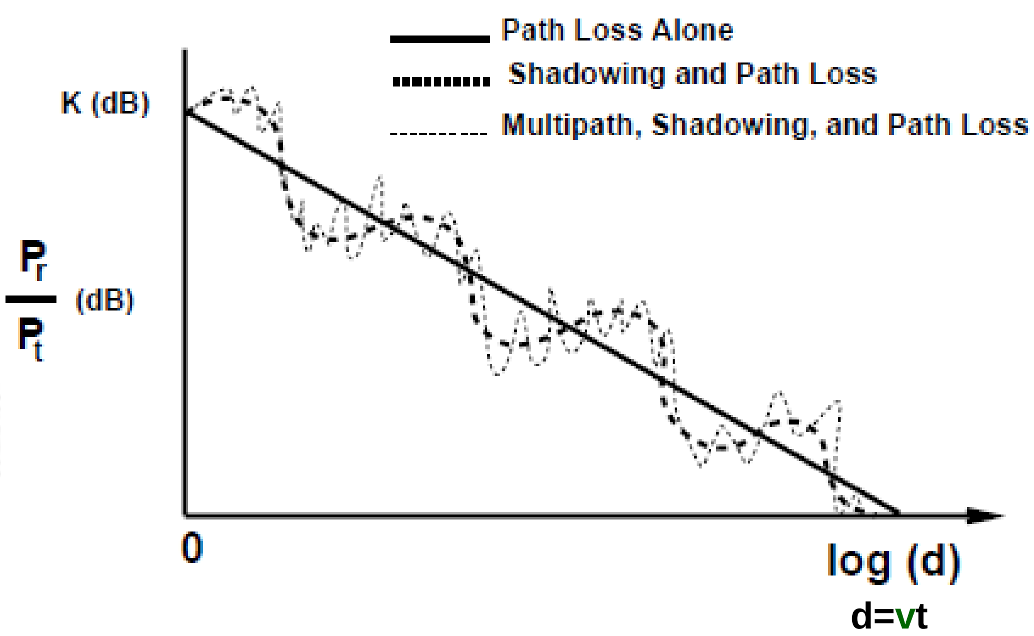
1. Transmitted & received signals
2. Free space pathloss
3. Ray tracing & 2-ray pathloss model
4. Empirical pathloss models – Okumura, Hata, Cost 231, simplified
5. Shadow fading & cell coverage

2. Signal Propagation: Pathloss and Shadowing

Signal propagation characteristics:

- **Path loss: power falls off relative to distance**
- **Shadowing: random fluctuations due to obstructions**
- **Flat and frequency selective fading: caused by multipath**





2. Signal Propagation: Pathloss and Shadowing

Transmitted and Received Signals:

- **Transmitted Signal at carrier frequency f_c and power P_t, is:** $s(t) = Re\{u(t)e^{j2\pi f_c t}\}$

$$\therefore s(t) = Re\{u(t)\}cos(2\pi f_c t) - Im\{u(t)\}sin(2\pi f_c t)$$

$$\therefore s(t) = s_I(t)cos(2\pi f_c t) - s_Q(t)sin(2\pi f_c t)$$

where $u(t) = s_I(t) + js_Q(t)$ is the complex baseband signal of bandwidth B_u and power P_u and $P_t = P_u/2$. $u(t)$ is called the complex envelope of $s(t)$. For simplicity we assume $u(t)$ real for propagation model analysis.

- **Received signal is:** $r(t) = Re\{v(t)e^{j2\pi f_c t}\}$

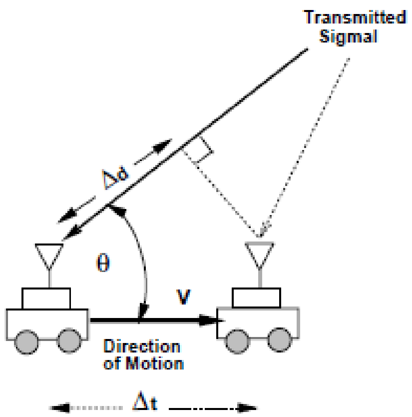
Where $v(t) = u(t) * c(t)$ and $c(t)$ is the equivalent lowpass channel impulse response.

- **Doppler Shift: when the transmitter or receiver is moving, the received signal will have a Doppler shift given by:** $f_D = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}cos(\theta)$

$$\frac{\Delta d}{\Delta t \times v} = \cos\theta \Rightarrow \frac{\Delta d}{\Delta t} = v\cos\theta$$

$$\text{But } \Delta\phi = 2\pi\frac{\Delta d}{\lambda} \Rightarrow \Delta d = \frac{\lambda}{2\pi}\Delta\phi$$

$$\therefore \quad f_D = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$$



2. Signal Propagation: Pathloss and Shadowing

Pathloss Definition: We define the linear pathloss of the channel as the ratio of transmit power to receive power:

$$\boxed{\begin{aligned} P_L &= \frac{P_t}{P_r} \\ P_L(dB) &= 10\log_{10}\left(\frac{P_t}{P_r}\right) \quad dB \end{aligned}}$$

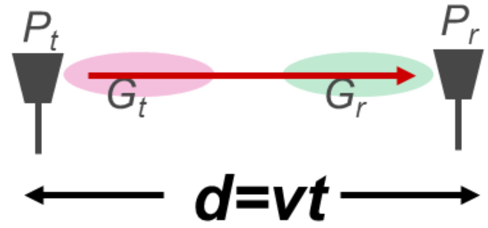
Pathloss Modelling:

- Maxwell's equations - Complex and impractical
- Free space path loss model - Too simple
- Ray tracing models - Requires site-specific information
- Empirical Models - Don't always generalize to other environments
- Simplified power fall-off models - Main characteristics: good for high-level analysis

Free Space Pathloss Model: Consider a signal transmitted through free space to a receiver located at distance *d* from the transmitter. There are no obstructions between the Tx and Rx and the signal propagates along a straight line between the two giving a line-of-sight (LOS) channel. Free-space pathloss introduces a complex scale factor resulting in the received signal –

2. Signal Propagation: Pathloss and Shadowing

Free Space Pathloss Model Contd:



***EIRP* = P_tG_t**

$$r(t) = Re\left\{\frac{\lambda\sqrt{G_tG_r}e^{-j2\pi d/\lambda}}{4\pi d}u(t)e^{j2\pi f_c t}\right\}$$

G_t and G_r are Tx and Rx antenna gains, and $e^{-j2\pi d/\lambda}$ is the phase shift due to distance *d*.

The absolute power in the transmitted signal *s*(*t*) is P_t, so the received signal power P_r is:

$$P_r = P_t\left[\frac{\sqrt{G_tG_r}\lambda}{4\pi d}\right]^2 = P_t\left[\frac{\sqrt{G_tG_r}c}{4\pi df}\right]^2$$

Free-space propagation obeys an inverse square-law with distance d, so that the received power falls by 6 dB when the range is doubled. Also, the path loss increases with the square of the transmission frequency so that losses increase by 6 dB if the frequency is doubled.

The free space channel pathloss is given by:

$$P_L(dB) = 10\log\frac{P_t}{P_r} = 20\log\left[\frac{4\pi df}{\sqrt{G_tG_r}c}\right] = k + 20\log d + 20\log f - 10\log G_t - 10\log G_r$$

$$k = 20\log\frac{4\pi}{c} = 20\log\frac{4\pi}{3\times10^8} = -147.56$$

The basic passloss P_{LB} for an isotropic antenna is obtained from the above expression when G_T = 1 and G_R = 1, then:

$$P_{LB} = 32.44 + 20\log f_{\mathrm{MHz}} + 20\log d_{\mathrm{km}}$$

2. Signal Propagation: Pathloss and Shadowing

Free Space Pathloss Model Contd: Example: If a transmitter produces 50 W of power, express the transmit power in units of (a) dBm, and (b) dBW. If 50 W is applied to a unity gain Tx antenna with a 900 MHz carrier frequency, find the received power in dBm at a free space distance of 100 m from the antenna assuming the Rx antenna gain is also unity. What is the received power at 10 km?

Solution: Given P_t = 50 W, carrier frequency f_c = 900 MHz:

Transmitter power in dBm –

$$P_t\,(\mathrm{dBm}) = 10\log\{P_t(\mathrm{W})/(1\mathrm{mW})\} = 10\log\{50 \times 10^3\} = 47.0\ \mathrm{dBm}$$

Transmitter power in dBW –

$$P_t\,(\mathrm{dBW}) = 10\log\{P_t(\mathrm{W})/(1\mathrm{W})\} = 10\log\{50\} = 17.0\ \mathrm{dBW}$$

As G_t = 1 and G_r = 1, the basic pathloss for d = 100 m is

$$\boxed{P_{LB} = 32.44 + 20\log(900) + 20\log(0.1) = 71.5\ \mathrm{dB}}$$

$$\boxed{\begin{aligned} P_r(dBm) &= P_t(dBm) - P_{LB} \\ &= 47.0 - 71.5 \\ &= -24.5\ \mathrm{dBm} \quad (3.55\ \mu\mathrm{W}) \end{aligned}}$$

For d = 10 km:

$$P_{LB} = 32.44 + 20\log(900) + 20\log(10) = 111.5\ \mathrm{dB}$$

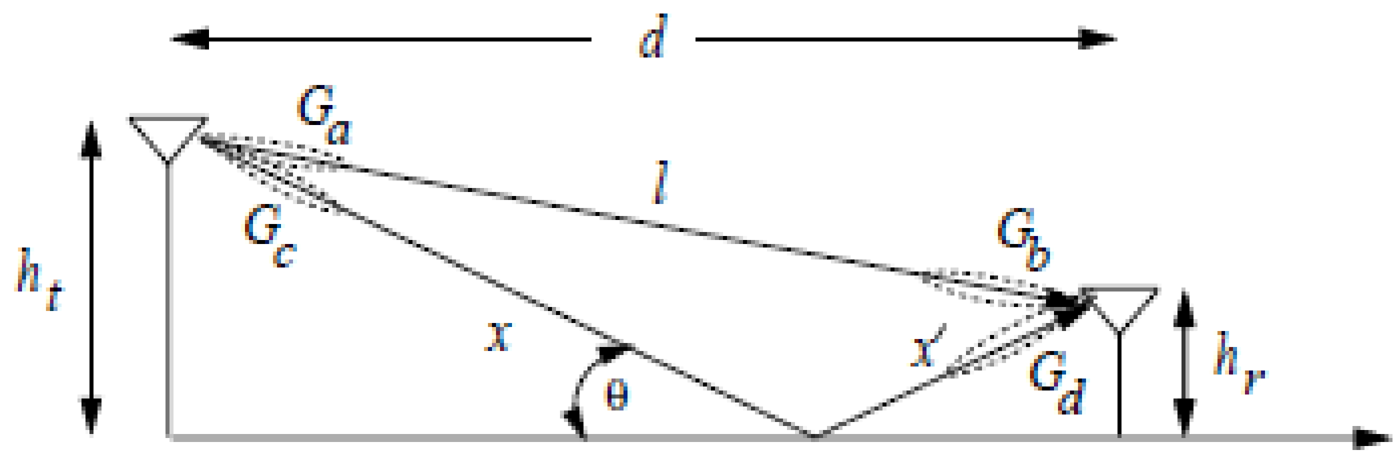
$$\boxed{\begin{aligned} P_r(dBm) &= P_t(dBm) - P_{LB} \\ &= 47.0 - 111.5 \\ &= -64.5\ \mathrm{dBm} \quad (0.355\ \mathrm{nW}) \end{aligned}}$$

2. Signal Propagation: Pathloss and Shadowing

Ray Tracing Pathloss Models:

- Represent wavefronts as simple particles
- Geometry determines received signal from each signal component
- Typically includes reflected rays, can also include scattered and diffracted rays
- Requires site parameters – Geometry, Dielectric properties

Two-Ray Model - The two-ray model is used when a single ground reflection dominates the multipath effect. The received signal consists of two components: the LOS component or ray, which is just the Tx signal propagating through free space, and a reflected component or ray, which is the Tx signal reflected off the ground.



2. Signal Propagation: Pathloss and Shadowing

Two-Path Model Contd - The received signal for the two-ray model is

$$r(t) = \mathrm{Re}\left\{ \frac{\lambda}{4\pi} \left[\frac{\sqrt{G_a G_b}\, u(t) e^{-j2\pi l/\lambda}}{l} + \frac{R\sqrt{G_c G_d}\, u(t-\tau) e^{-j2\pi(x+x')/\lambda}}{x+x'} \right] e^{j2\pi f_c t} \right\}$$

If the transmitted signal is narrowband relative to the *Delay Spread, i.e.* $\tau = \frac{x+x'-l}{c} \ll (B_u)^{-1}$, then $u(t) \approx u(t-\tau)$ and the received power for narrowband transmission is:

$$P_r = P_t \left[\frac{\lambda}{4\pi} \right]^2 \left| \frac{\sqrt{G_a G_b}}{l} + \frac{R\sqrt{G_c G_d}\, e^{-j\Delta\phi}}{x+x'} \right|^2$$

The phase difference between the 2 rays is approximated by: $\Delta\phi = \frac{2\pi(x+x'-l)}{\lambda} \approx \frac{4\pi h_t h_r}{\lambda d}$

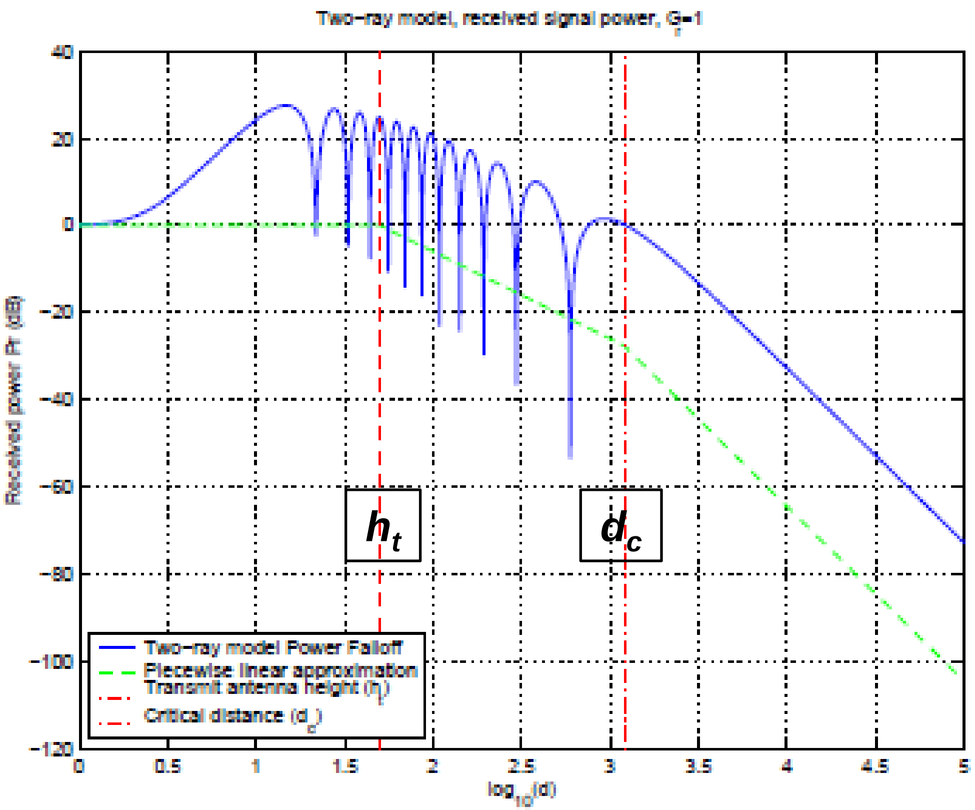
$$\text{since} \quad (x+x'-l) = \sqrt{(h_t+h_r)^2 + d^2} - \sqrt{(h_t-h_r)^2 + d^2} \quad and \quad d >> (h_t+h_r)$$

For asymptotically large *d* we have (*x* + *x'*) ≈ *l* ≈ *d*, *R* ≈ -1, and $\sqrt{G_a G_b} \approx \sqrt{G_c G_d} = \sqrt{G_t G_r}$ and the Rx power is given by:

$$P_r \approx P_t \left[\frac{\lambda\sqrt{G_t G_r}}{4\pi d} \right]^2 \left[\frac{4\pi h_t h_r}{\lambda d} \right]^2 = P_t \left[\frac{\sqrt{G_t G_r}\, h_t h_r}{d^2} \right]^2$$

2. Signal Propagation: Pathloss and Shadowing

Two-Path Model Contd – **The Rx Power versus Distance for the Two-Ray Model is shown**



For small distances ($d < h_t$) the two rays add constructively and the path loss is roughly flat.

For distances between h_t and a certain critical distance d_c, the signal experiences constructive and destructive interference of the two rays, resulting in a sequence of maxima and minima.

At critical distance $d_c = 4h_t h_r/\lambda$ (when $\Delta\phi = \pi$), the final maximum is reached, after which the signal power falls off proportionally to d^{-4}.

The pathloss above the critical distance follows a d^4 law and is independent of frequency:

$$P_L = \frac{P_t}{P_r} = \frac{d^4}{G_t G_r h_t^2 h_r^2} \qquad P_L(dB) = 40\log(d) - 20\log(h_t) - 20\log(h_r) - 10\log(G_t) - 10\log(G_r)$$

2. Signal Propagation: Pathloss and Shadowing

Two-Ray Model Contd – Example: In the downlink of a 900 MHz cellular system, a mobile station (MS) is located 5 km from a base station (BS). If the transmit power is 40W, find the received power in dBm at the MS using the 2-ray model assuming $G_t = G_r = 2$, $h_t = 20$ m and $h_r = 1.5$ m. What is the critical distance for this deployment?

Solution:

Tx power P_T in dBm: $P_t(dBm) = 10log\left(\frac{40}{10^{-3}}\right) = 46\ dBm$

Path-loss at 5 km: $P_L(dB) = 40\log 5000 - 20\log 20 - 20\log 1.5 - 10\log 2 - 10\log 2 = 112.4\ \text{dB}$

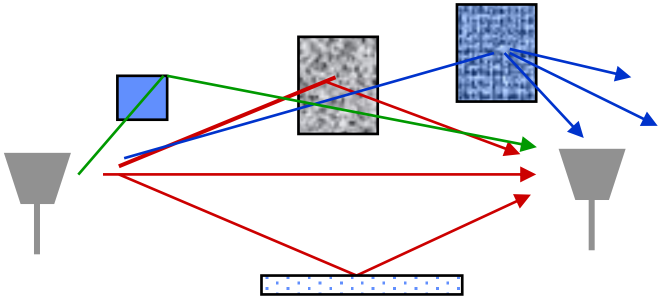
Rx power P_r in dBm: $P_r(dBm) = P_t(dBm) - P_L(dB) = 46 - 112.4 = -66.4\ dBm\ (229\text{pW})$

The Critical Distance d_c is: $= \frac{4h_t h_r}{\lambda} = \frac{4\times 20\times 1.5}{3\times 10^8/900\times 10^6} = 360\text{m}$

Compare the above values with the free space propagation path model.

General Ray Tracing:

- **Models all signal components (Reflection, Scattering & Diffraction)**
- **Requires detailed geometry and dielectric properties of site (similar to Maxwell, but easier maths)**
- **Computer packages often used**



2. Signal Propagation: Pathloss and Shadowing

Empirical Pathloss Models: Several different models available depending on parameters. Empirical models are typically used in the computer simulation of cellular systems.

Okumura model

- Empirically based (site/freq specific)
- Awkward (uses graphs)

Hata model

- Analytical approximation to Okumura model

Cost 231 Model:

- Extends Hata model to higher frequency (2 GHz)

Simplified pathloss model:

- Used for general trade-off analysis of systems

Further details follow -

2. Signal Propagation: Pathloss and Shadowing

Empirical Pathloss Models Contd: Okumura model – based on a set of curves giving median attenuation relative to free space of signal propagation in irregular terrain taken from measurements in Tokyo. Basic expression for the pathloss is:

$$P_L(d)\text{ dB} = L(f_c, d) + A_{mu}(f_c, d) - G(h_t) - G(h_r) - G_{AREA}$$

Applicable for: distances of 1-100 Km, frequencies of 150-1500 MHz, base station heights of 30-100m.

- **$L(f_c, d)$ = free space path loss at distance d and carrier frequency f_c**
- **$A_{mu}(f_c, d)$ = median attenuation on top of free space path loss for all environments**
- **$G(h_t)$ = base station antenna height gain factor**
- **$G(h_r)$ = mobile antenna height gain factor**
- **G_{AREA} = gain due to the type of environment.**

$A_{mu}(f_c, d)$ and G_{AREA} obtained from Okumura's empirical plots

$$G(h_t) = 20\log_{10}(h_t/200),\ 30m < ht < 1000m$$

$$G(h_r) = \begin{cases} 10\log_{10}(h_r/3) & h_r \le 3m \\ 20\log_{10}(h_r/3) & 3m < h_r < 10m \end{cases}$$

2. Signal Propagation: Pathloss and Shadowing

Empirical Pathloss Models Contd: Hata Model – is an empirical formulation of the graphical pathloss data provided by Okumura. Also applicable for frequencies of 150-1500 MHz and *d* > 1km. The standard closed form expression for the Hata pathloss is:

$$P_{L,urban}(d)\text{ dB} = 69.55+26.16\log_{10}(f_c)-13.82\log_{10}(h_t)-a(h_r)+(44.9-6.55\log_{10}(h_t))\log_{10}(d)$$

$a(h_r)$ = correction factor for the mobile antenna height based on the coverage area size.

For small to medium sized cities - $a(h_r) = (1.1\log_{10}(f_c) - 0.7)h_r - (1.56\log_{10}(f_c) - 0.8)$ dB

For larger cities at frequencies f_c > 300 MHz - $a(h_r) = 3.2(\log_{10}(11.75h_r))^2 - 4.97$ dB

Corrections to the urban model are made for suburban and rural propagation as follows:

$$P_{L,suburban}(d)\text{ dB} = P_{L,urban}(d) - 2[\log_{10}(f_c/28)]^2 - 5.4$$

$$P_{L,rural}(d)\text{ dB} = P_{L,urban}(d) - 4.78[\log_{10}(f_c)]^2 + 18.33\log_{10}(f_c) - K$$

Where *K* = 35.94 (countryside) to 40.94 (desert)!!!

2. Signal Propagation: Pathloss and Shadowing

Empirical Pathloss Models Contd: Cost 231 Model – The Hata model was extended by the European cooperative for scientific and technical research (EURO-COST) to 2 GHz:

$$P_{L,urban}(d)\ \text{dB} = 46.3+33.9\log_{10}(f_c)-13.82\log_{10}(h_t)-a(h_r)+(44.9-6.55\log_{10}(h_t))\log_{10}(d)+C_M$$

The correction factor $a(h_r)$ is the same as in the Hata model.

C_M = 0 dB for medium sized cities & suburbs

C_M = 3 dB for metropolitan sized areas

The following parameter ranges apply to Cost 231 – 1.5GHz < f_c < 2 GHz, 30m < h_t < 200 m, 1m < h_r < 10 m, and 1Km < d < 20 Km.

Simplified Pathloss Model - The complexity of signal propagation makes it difficult to obtain a single model that characterizes path loss accurately across a range of different environments.

For general trade-off analysis of various system designs a simple model that captures the essence of signal propagation is:

$$P_r = P_t K\left[\frac{d_0}{d}\right]^{\gamma}$$

2. Signal Propagation: Pathloss and Shadowing

Empirical Pathloss Models Contd: Simplified Pathloss Formula –The dB attenuation is:

$$P_r \text{ dBm} = P_t \text{ dBm} + K \text{ dB} - 10\gamma \log_{10}\left[\frac{d}{d_0}\right]$$

K = dimensionless constant that depends on –

- **antenna characteristics and**
- **Average channel attenuation,**

d_0 = antenna far field reference distance (1-10 m indoors and 10-100 m outdoors

γ is the path loss exponent

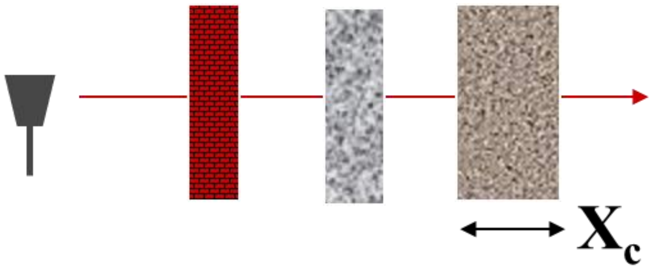
The values for K, d_0, and γ are set to approximate either an analytical or empirical model. When approximating empirical measurements, K is often determined by the free space path loss at distance d_0 assuming omnidirectional antennas are used - $K\ dB = 20\log_{10}\frac{\lambda}{4\pi d_0}$

A table summarizing γ values for different indoor and outdoor environments and antenna heights at 900 MHz and 1.9 GHz is opposite.

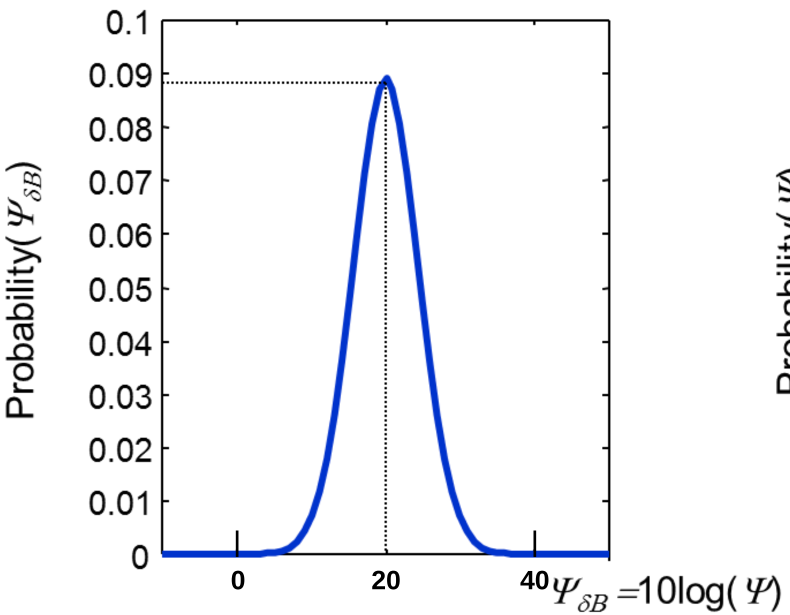
Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

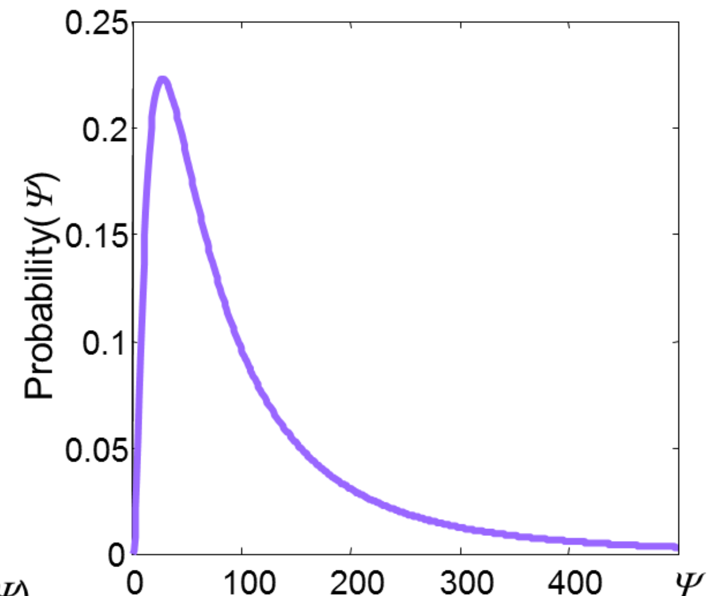
2. Signal Propagation: Pathloss and Shadowing

Shadow Fading: **A signal transmitted through a wireless channel will typically experience random variation due to blockage from objects in the signal path, giving rise to random variations of the received power at a given distance.**



Random due to random # and type of obstructions and obeys a *log-normal distribution.* The ratio of transmit-to-receive power $\psi = P_t/P_r$ is random with a log-normal distribution –





$$p(\psi_{dB}) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{dB}}} \exp\left[-\frac{(\psi_{dB} - \mu_{\psi_{dB}})^2}{2\sigma_{\psi_{dB}}^2}\right]$$

$$p(\psi) = \frac{\xi}{\psi\sqrt{2\pi}\sigma_{\psi_{dB}}} \exp\left[-\frac{(10\log_{10}\psi - \mu_{\psi_{dB}})^2}{2\sigma_{\psi_{dB}}^2}\right]$$

2. Signal Propagation: Pathloss and Shadowing

Shadow Fading: The key statistical parameters of the log-normal distribution are

$$\psi > 0, \quad \xi = 10/\ln 10$$

$$\mu_{\psi_{dB}} = \text{mean of } "\psi_{dB} = 10\log_{10}\psi"$$

$$\sigma_{\psi_{dB}} = \text{standard deviation of } \psi_{dB}$$

$$A(\delta) = \sigma^2_{\psi_{dB}} \exp(-\delta / X_c)$$

$$\mu_\psi = E[\psi] = \exp\left[\frac{\mu_{\psi_{dB}}}{\xi} + \frac{\sigma^2_{\psi_{dB}}}{2\xi^2}\right]$$

$$10\log_{10}\mu_\psi = \mu_{\psi_{dB}} + \frac{\sigma^2_{\psi_{dB}}}{2\xi}$$

***A*(δ) = the autocorrelation between shadow fading at two points separated by distance δ.**
The decorrelation distance X_c is the distance at which the autocorrelation equals 1/*e* of its maximum value and indicates that shadow variation are of the order X_c.

Proof of lognormal distribution:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-(x-m)^2}{2\sigma^2}\right), \; x = \ln(y) \Rightarrow p(\ln(y)) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln(y)-m)^2}{2\sigma^2}\right)$$

Since the probability of *y* lying between *y* and *y* + *dy* must equal the probability of x=ln(y) lying between *x*=ln(*y*) and (*x*+d*x*)=ln(*y* + *dy*) then

$$p(y)\,dy = p(x)\,dx = p(\ln(y))\,d(\ln(y))$$

$$p(y) = p(\ln(y))\frac{d(\ln(y))}{dy} = \frac{1}{y}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-(\ln(y)-m)^2}{2\sigma^2}\right)$$

2. Signal Propagation: Pathloss and Shadowing

Combined Path Loss and Shadow Fading: Path loss and shadowing can be superimposed to model power fall-off versus distance.

$$P_r(dB) = P_t(dB) - \psi_{dB}$$

In the combined model, mean dB path loss $\mu_{\psi_{dB}}$ is equal to the path loss $P_L(dB)$. Then the (random) shadow fading, denoted by $\tilde{\psi}_{dB}$, is regarded as normally distributed with mean 0 dB and the same standard deviation $\sigma_{\psi_{dB}}$ dB.

$$P_r(dB) = P_t(dB) - P_L(dB) - \tilde{\psi}_{dB}$$

Outage Probability Under Path Loss & Shadowing: In wireless systems there is a target minimum received power level P_{min} below which performance becomes unacceptable. With shadowing the received power at a given distance from the transmitter is *log-normally* distributed with some probability of falling below P_{min}.

We define outage probability $P_{out}(P_{min}, d)$ under path loss and shadowing to be the probability that the average received power at a given distance d, $P_r(d)$, falls below P_{min}:

$$P_{out}\left(P_{\min}, d\right) = \Pr\{P_r(d) < P_{\min}\}$$

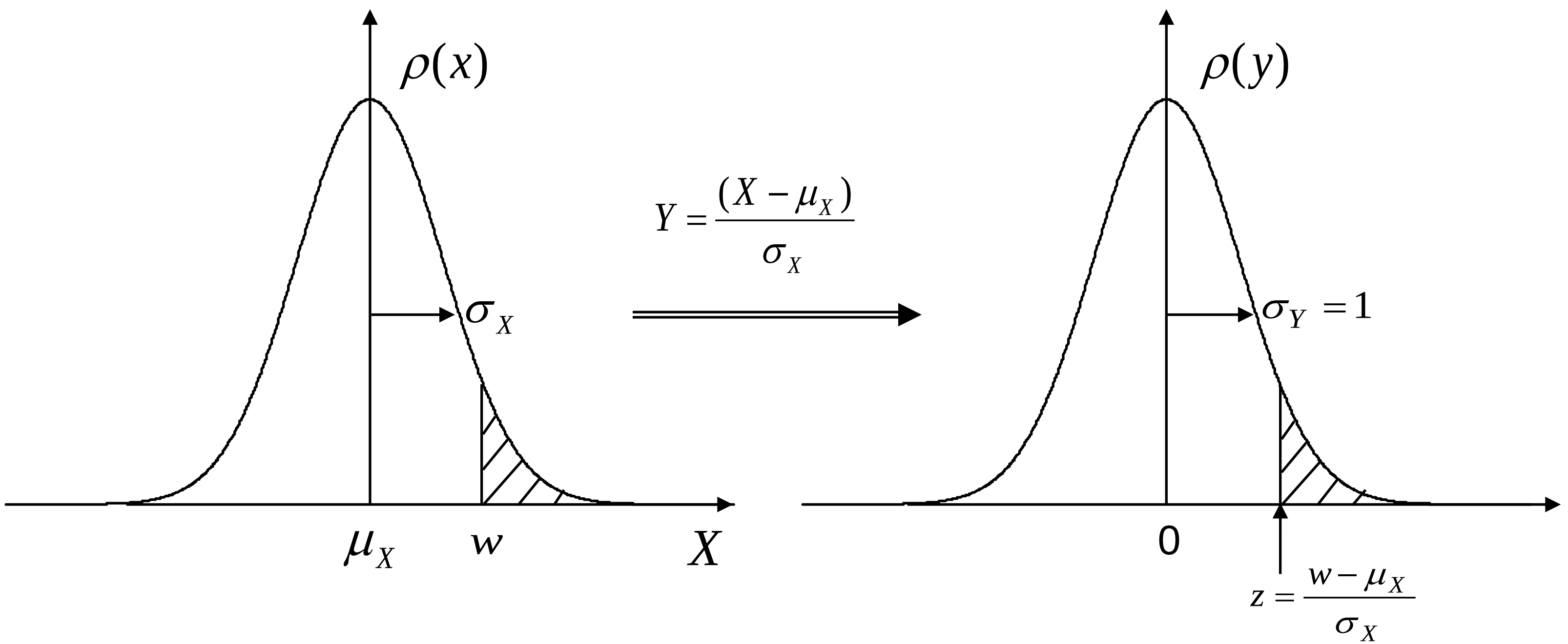
All powers are in dBs

$$\Pr\{P_r(d) < P_{\min}\} = 1 - Q\left(\frac{P_{\min} - P_r}{\sigma_{\psi_{dB}}}\right) = 1 - Q\left(\frac{P_{\min} - (P_t - P_L)}{\sigma_{\psi_{dB}}}\right)$$

2. Signal Propagation: Pathloss and Shadowing

Outage Probability Under Path Loss & Shadowing: **The Q-function is defined as**

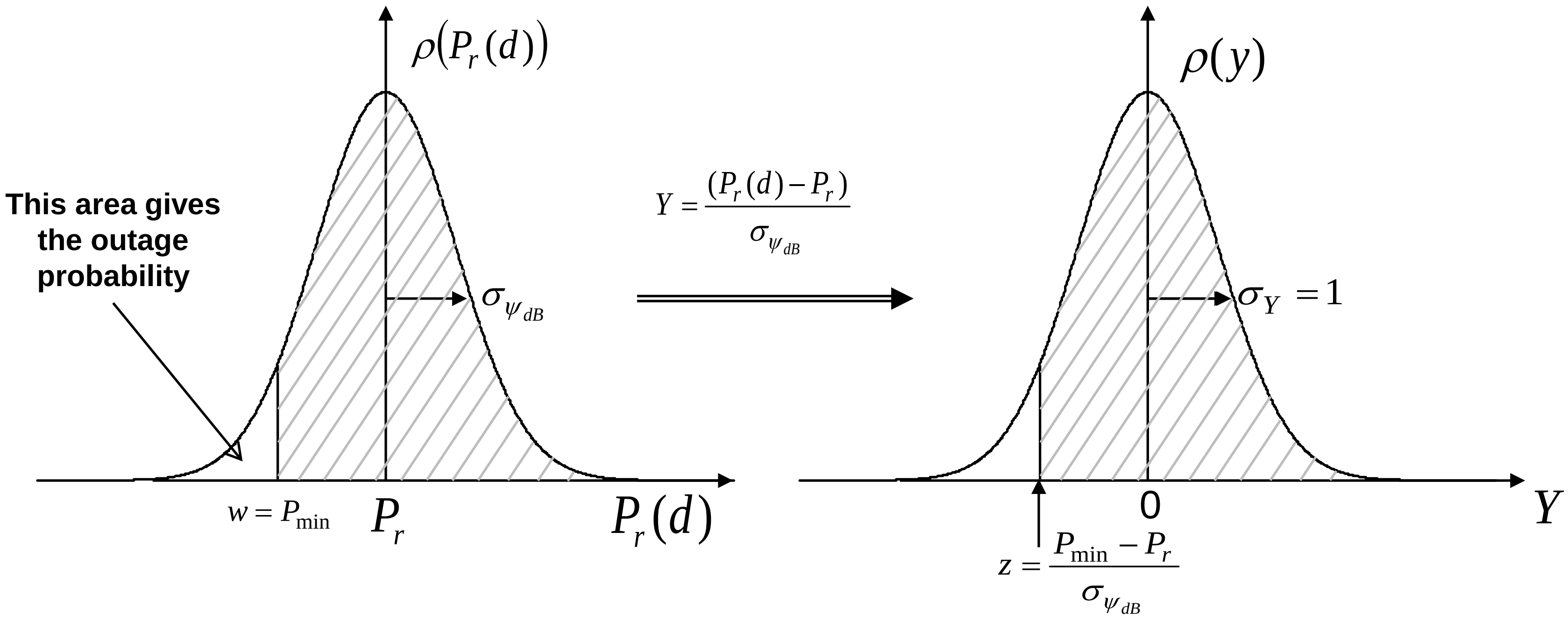
$$\Pr\{X > w\} = \Pr\{Y > z\} = Q(z) == \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy = \frac{1}{2} erfc\left(\frac{z}{\sqrt{2}}\right)$$



$$\Pr\{X > w\} = Shaded\ Area = Q\left(\frac{w-\mu_X}{\sigma_X}\right)$$

2. Signal Propagation: Pathloss and Shadowing

Outage Probability Under Path Loss & Shadowing: **In the case of shadowing the relevant conditions are shown below with the implicit assumption that $P_{min} < P_r$**



$$\boxed{\Pr\{P_r(d) > P_{\min}\} = Shaded\ \ Area = Q\left(\frac{P_{\min} - P_r}{\sigma_{\psi_{dB}}}\right)}$$

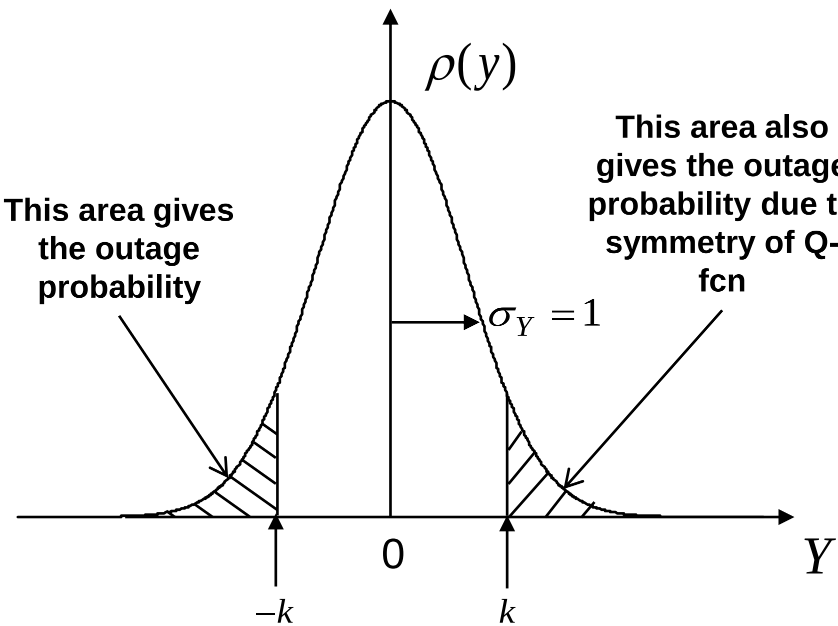
$$\boxed{\Pr\{P_r(d) < P_{\min}\} = 1 - Q\left(\frac{P_{\min} - P_r}{\sigma_{\psi_{dB}}}\right) = 1 - Q\left(\frac{P_{\min} - (P_t - P_L)}{\sigma_{\psi_{dB}}}\right)}$$

2. Signal Propagation: Pathloss and Shadowing

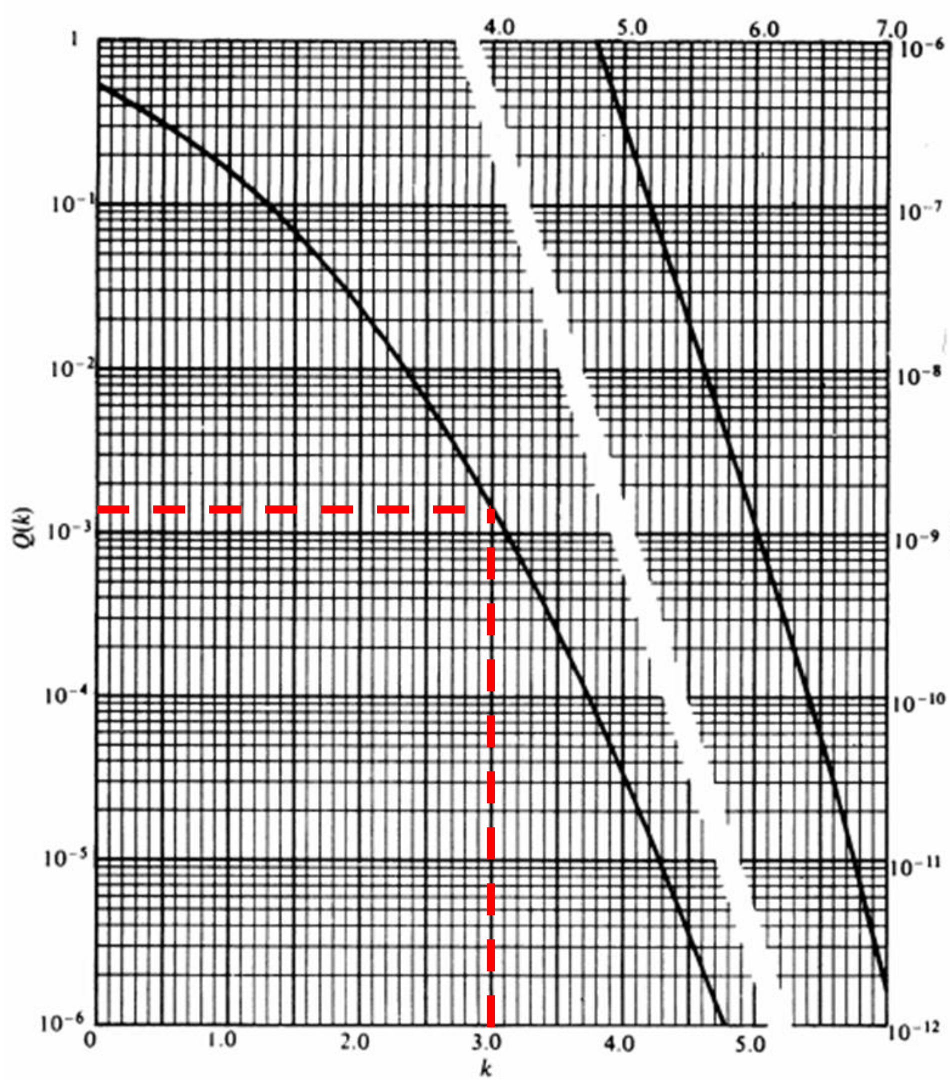
Outage Probability Under Path Loss & Shadowing: Graphical Q-function

While the computation of the Q-fcn can be done in a software package like MATLAB, more typically the graphical Q-fcn is used. However, the graphical Q-fcn is only defined over positive arguments > 0

To calculate the outage probability we use the symmetry of the Q-fcn thus -



$$Q(k=3) = 0.0012 \quad \#$$



2. Signal Propagation: Pathloss and Shadowing

Outage Probability Under Path Loss & Shadowing: Example - Find the outage probability at 150m for a channel based on the simplified path loss model at 900MHz with a propagation path loss exponent $\gamma = 3.71$. Assume that $d_0 = 1$ m, K is determined by the free space path gain formula at this d_0, the variance of shadow fading on this path is $(\sigma_{\psi_{dB}})^2 = 13.29$ dB², the transmit power $P_t = 10$ mW (10 dBm) and the minimum power requirement $P_{min} = -110.5$ dBm.

Solution: From the simplified path loss model $P_r = P_t + K_{dB} - 10\gamma \log_{10}(d/d_0) \quad dB\ scale$

From the free-space path loss we obtain:

$$K_{dB} = 20\log_{10}\left(\frac{\lambda}{4\pi d_0}\right) = 20\log_{10}\left(\frac{c}{4\pi f_c d_0}\right) = 20\log_{10}\left(\frac{3\times10^8}{4\pi\times900\times10^6\times1}\right) = -31.53$$

$$\begin{aligned}
P_{out}(P_{\min} = -110.5dBm, 150m) &= \Pr\{P_r(150m) < -110.5dBm\} \\
&= 1 - Q\left(\frac{P_{\min} - (P_t + k_{dB} - 10\gamma\log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right) \\
&= 1 - Q\left(\frac{-110.5 - (10 - 31.53 - 37.1\log_{10}(150))}{\sqrt{13.29}}\right) \\
&= \underline{\underline{1 - Q(-2.26)}}
\end{aligned}$$

2. Signal Propagation: Pathloss and Shadowing

Outage Probability Under Path Loss & Shadowing: Graphical Q-function

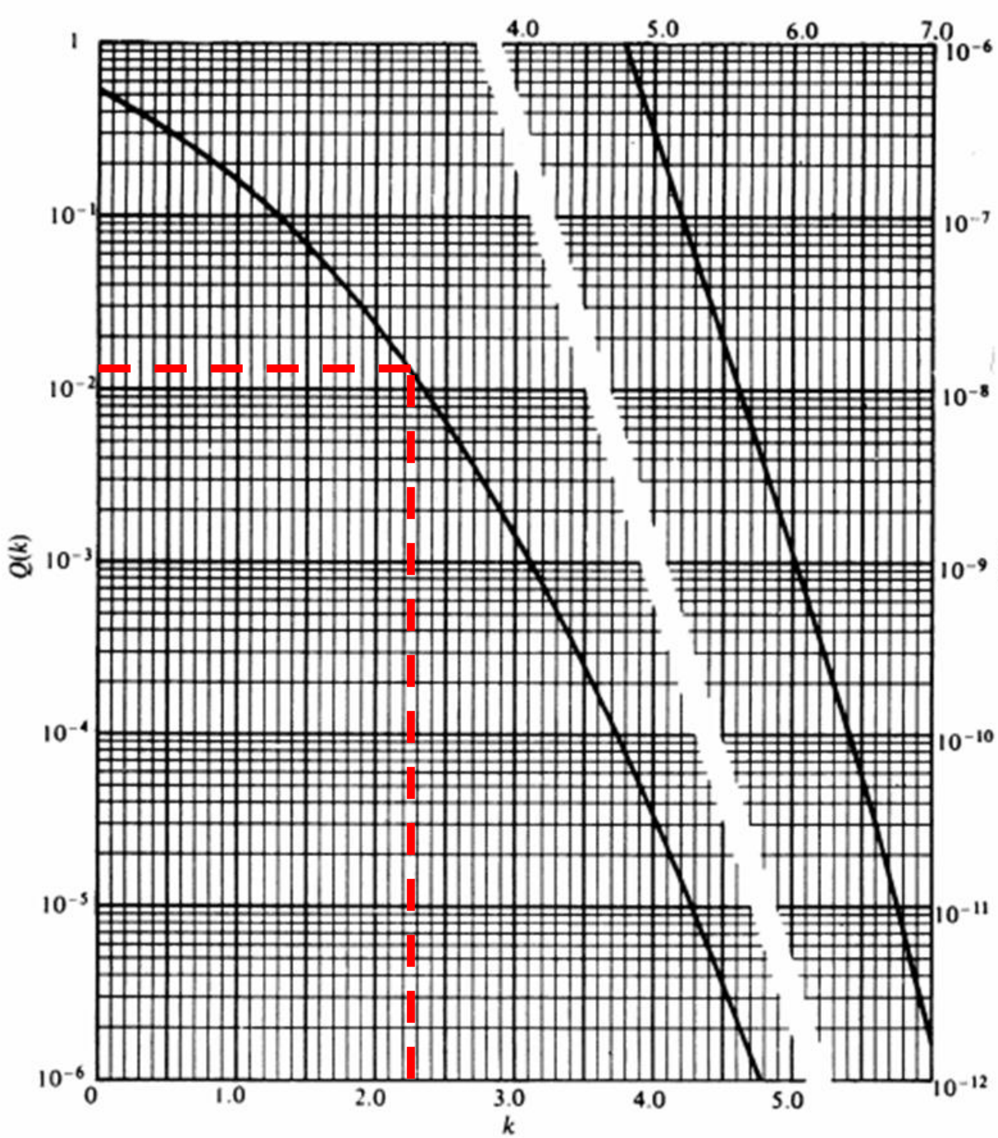
Using the graphical Q-fcn we need only look up:

$$P_{out} = Q(2.26) = \underline{\underline{0.0119 \quad or \quad 1.19\%}} \quad \#$$

An outage probabilities of 1% is a typical target in wireless system designs.

NB if the argument was already positive i.e. $P_r < P_{min}$, we would calculate the outage probability as given by the full expression. For example, if

$$\begin{aligned} P_{out} &= 1 - Q(2.26) \\ &= 1 - 0.0119 \\ &= 0.9881 \text{ or } 98.81\% \end{aligned}$$



2. Signal Propagation: Pathloss and Shadowing

Cell Coverage Area: **The cell coverage area in a cellular system is defined as the expected percentage of area within a cell that has received power above a given minimum.**

The transmit power at the base station is designed for an average received power at the cell boundary of $P_{min} = P_R$, averaged over the shadowing variations.

Shadowing will cause some locations within the cell to have $P_x > P_R$ and other locations where $P_x < P_R$, where x is the distance to BS.

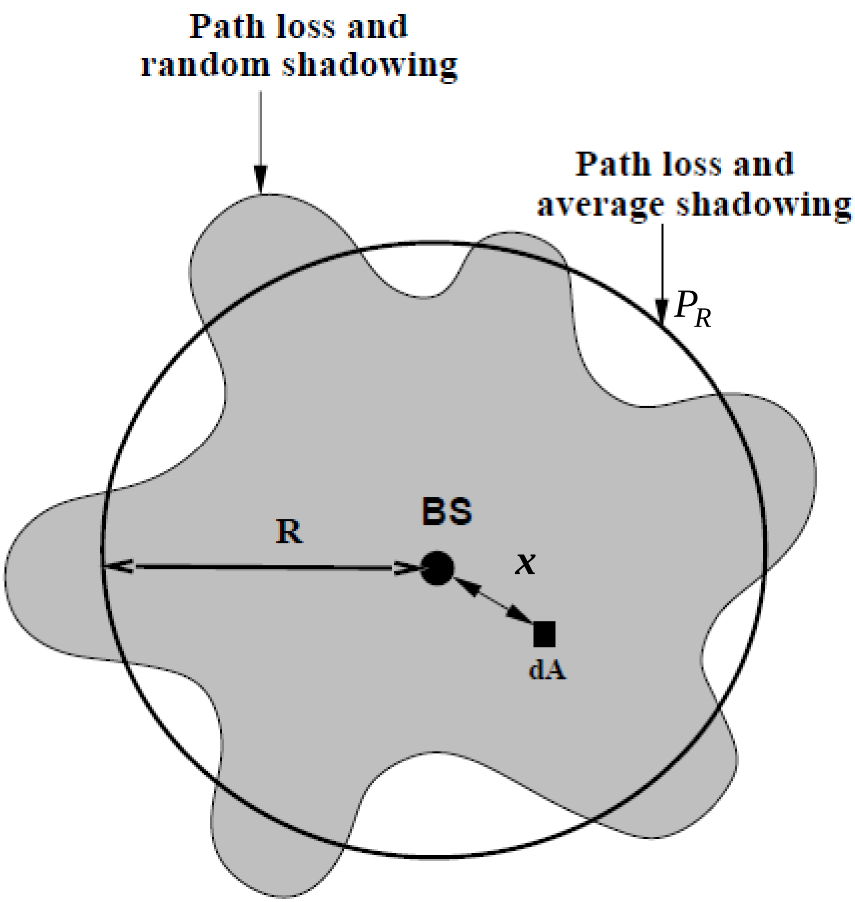
The total area A within the cell where $P_x > P_R$ is obtained by integrating over all incremental areas dA where P_R is exceeded. We define the %age area that meets $P_x > P_R$ as the ratio -

$$C = \frac{1}{\pi R^2} \int_{cell\ area} P_x dA = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^{R} P_x x dx d\theta$$

$$\& \{P_x > P_R\} \text{ at each } dA \text{ included in the integration}$$

The outage probability of the cell is the %age area in the cell that does not meet $P_x > P_R$.

$$\boxed{P_{out}^{cell} = 1 - C}$$



2. Signal Propagation: Pathloss and Shadowing

Summary & Main Points:

- **Path loss models simplify Maxwell's equations**
- **Models vary in complexity and accuracy**
- **Power fall-off with distance is proportional to d^2 in free space, d^4 in 2-ray model**
- **Empirical models used in system simulations**
- **Main characteristics of path loss captured in simple model $P_r = P_t K(d_0/d)^{\gamma}$**
- **Random attenuation due to shadowing modelled as log-normal (empirical parameters)**
- **Shadowing decorrelates over decorrelation distance X_c**
- **Combined path loss and shadowing leads to outage and amoeba-like cell shapes**
- **Cellular coverage area dictates the percentage of locations within a cell that are not in outage**
- **Path loss and shadowing parameters are obtained from empirical measurements**
- **Statistical models used for random environments**