# EEE118: Electronic Devices and Circuits

## Lecture II

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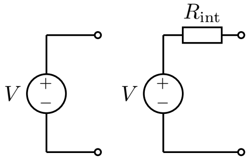
# Last Lecture: Review

- Stated the Aims and Objectives of the course
  How electronic devices (diodes, transistors et al.) work in circuits
- Introduced some Circuit Terminology (Voltage, Current, Node, Branch)
- Introduced Engineering Units
  units use powers of three. 100 nA, 1 uA, 10 uA, 100 uA, 1 mA, 10 mA etc.
- Discussed Passive Components, their physical construction (Resistors, Capacitors & Inductors), relative price and performance.
- Considered the relationship between current and voltage in inductors, capacitors and resistors.

# Outline

# Voltage and Current Sources

An ideal voltage source is a two terminal circuit element supplying a fixed voltage and having zero internal resistance. A real voltage source can only supply a finite current and behaves as an ideal source with a resistance in series. It has non-zero internal resistance *in series* with the ideal voltage source.

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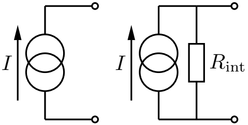
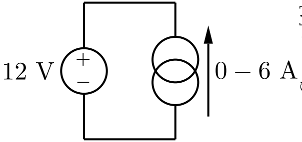
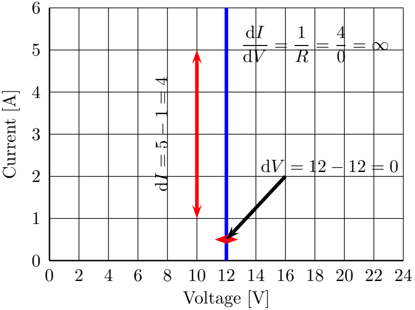
An ideal current source is a two terminal circuit element supplying a fixed current and having infinite internal resistance. A real current source can only supply the specified current over a range of terminal voltages. It has a finite internal resistance *in parallel* with the ideal current source.



# Voltage Source Internal Resistance

What is the internal resistance of a perfect voltage source? Force a known current into a perfect voltage source and observe the change in voltage, then use Ohm's law to find the internal resistance.

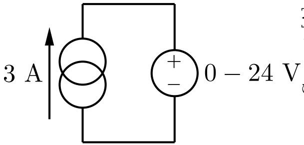


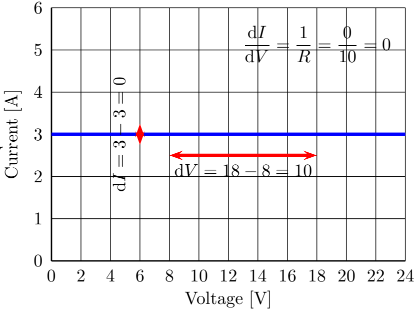




$\frac{1}{\infty} = 0\ \Omega$, so the internal resistance of a perfect voltage source is zero.

# Current Source Internal Resistance

What is the internal resistance of a perfect current source? Force a known voltage across a perfect current source and observe the change in current, then use Ohm's law to find the internal resistance.



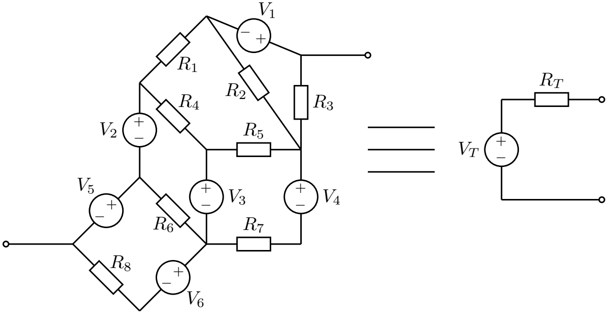




$\frac{?}{0} = \infty\ \Omega$, so the internal resistance of a perfect current source is infinite.

# Thévenin

## Theorem

*Any network of resistance elements and energy sources can be replace by a series combination of an ideal voltage source $V_T$ and a resistance $R_T$ where $V_T$ is the open-circuit voltage of the circuit and $R_T$ is the ratio of the open circuit voltage to the short circuit current.*



# Thévenin Method

- Find $V_T$ by measurement or calculation of the voltage across the nodes of interest without anything connected (open-circuit)
- Find by measurement or calculation the current ($I_{sc}$) that flows when the nodes of interest are connected together (short-circuit).
- Divide $V_T$ by $I_{sc}$ to yield $R_T$.

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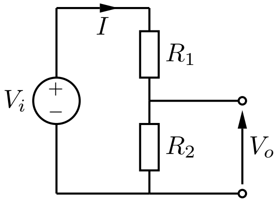
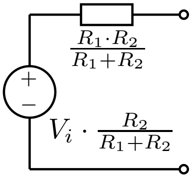
For example,




$$V_T = V_o = V_i \cdot \frac{R_2}{R_1 + R_2}$$

$$I_{sc} = \frac{V_i}{R_1}$$

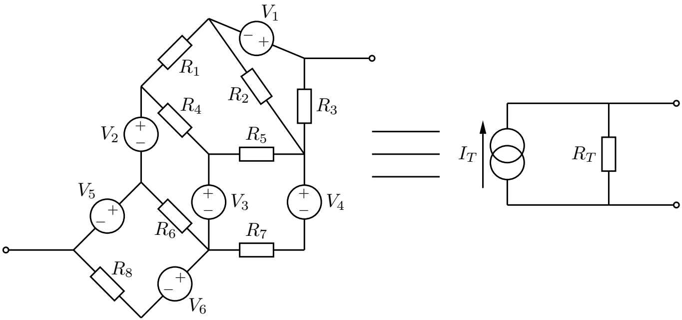
$$R_T = \frac{R_1\, R_2}{R_1 + R_2}$$



# Norton

## Theorem

*Any network of resistance elements and energy sources can be replace by a parallel combination of an ideal current source $I_T$ and a resistance $R_T$ where $I_T$ is the shot-circuit current of the circuit and $R_T$ is the ratio of the open circuit voltage to the short circuit current.*



# Norton Method

- Find $I_N$ by measurement or calculation the current that flows from one node to the other when they are short-circuit (connected together)
- Find by measurement or calculation the voltage ($V_{oc}$) that appears across the nodes of interest when nothing is connected between them (open-circuit)
- Divide $V_{oc}$ by $I_N$ to yield $R_N$.

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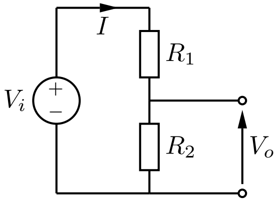
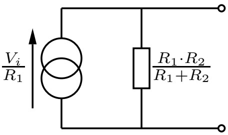
For example,




$$I_N = \frac{V_i}{R_1}$$

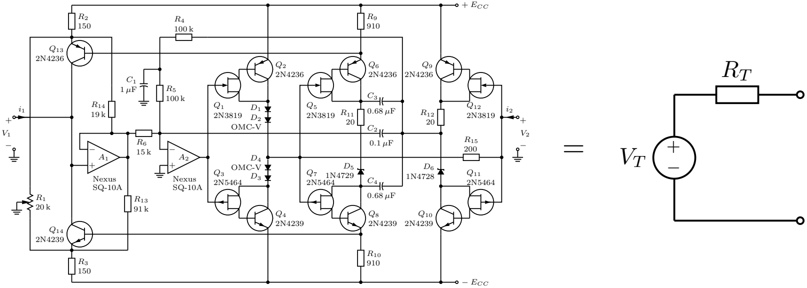
$$V_{oc} = V_i \cdot \frac{R_2}{R_1 + R_2}$$

$$R_N = \frac{R_1\, R_2}{R_1 + R_2}$$



# Source Transformations Summary

Active and passive circuits can be treated as a "black box" and thought of in terms of their Thévenin equivalent voltage and series resistance or Norton equivalent current and parallel resistance.



# Superposition

**Theorem**

*If a circuit consists of linear components (or components that can be considered linear over a small range of voltage and current), the combined effect of several energy sources on the circuit is equal to the sum of the effects of each source acting alone.*

The theorem implies that the sources should be considered independently, but does not say what to do with the ones we are not considering!

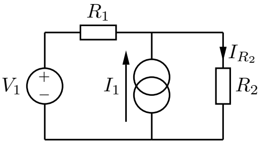
Consider the internal resistance of perfect voltage and current sources (look back at the earlier slides).

Current sources are replaced by an infinite resistance open circuit.

Voltage sources are replaced by zero resistance short circuit.

# Superposition Example

Find the contribution of each source to the current flowing in $R_2$.




Both Sources

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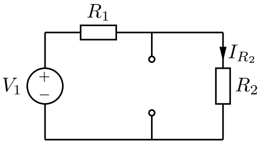
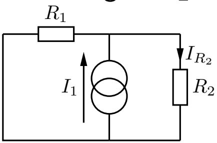



Voltage Source

$I_{R_2} = V_1 \cdot \frac{1}{R_1+R_2}$




Current Source

$I_{R_2} = I_1 \cdot \frac{R_1}{R_1+R_2}$

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Also by inspection the two expressions for $I_2$ have current flowing in the same direction so they are summed to yield,

$$I_{R_2} = V_1 \cdot \frac{1}{R_1 + R_2} + I_1 \cdot \frac{R_1}{R_1 + R_2}$$

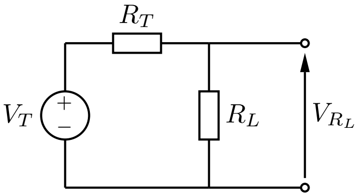
See Smith, R. J., and Dorf, R. C., *Circuits Devices and Systems* 5th ed., Wiley, 1992, pp. 56, dd. 621.3

# Power Transfer

Consider an imperfect voltage source, where the internal resistance is not zero. Is there an optimum resistance to transfer the maximum power from the source into the circuit?

Two methods,

1. Trial and error with example numbers
2. Mathematical derivation

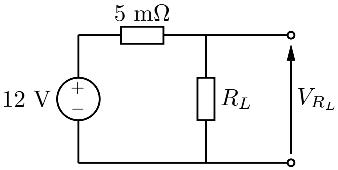


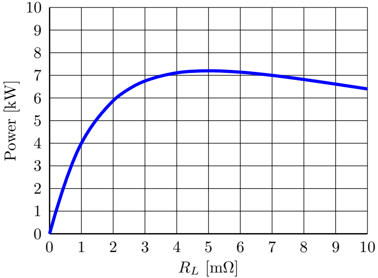

$$P = I\,V \text{ and } P = \frac{V^2}{R} \text{ and } P = I^2\,R$$

# Trial and Error

Let $R_L$ be,

1. $2.5\ \text{m}\Omega$
   $V_{R_L} = 12 \cdot \frac{2.5}{2.5+5}\ \text{m}\Omega = 4\ \text{V}.$
   $P = \frac{V^2}{R} = \frac{4^2}{2.5\times10^{-3}} = 6.4\ \text{kW}$
2. $5\ \text{m}\Omega$
   $V_{R_L} = 12 \cdot \frac{5}{5+5}\ \text{m}\Omega = 6\ \text{V}.$
   $P = \frac{V^2}{R} = \frac{6^2}{5\times10^{-3}} = 7.2\ \text{kW}$
3. $7.5\ \text{m}\Omega$
   $V_{R_L} = 12 \cdot \frac{7.5}{7.5+5}\ \text{m}\Omega = 7.2\ \text{V}.$
   $P = \frac{V^2}{R} = \frac{7.2^2}{7.5\times10^{-3}} = 6.9\ \text{kW}$




The maximum power transfer seems to occur when $R_L = R_T$.
A more rigorous approach is desirable however.

# Derivation of Maximum Power Transfer Condition

$$P_{R_L} = \frac{V_{R_L}^2}{R_L} \quad V_{R_L} = \frac{V_T\,R_L}{R_L + R_T}$$

Substituting,

$$P_{R_L} = \frac{V_T^2\,R_L}{(R_L + R_T)^2}$$

Differentiating with respect to $R_L$,

$$\frac{\mathrm{d}P_{R_L}}{\mathrm{d}R_L} = \frac{V_T^2}{(R_L + R_T)^2} - \frac{2\,V_T^2\,R_L}{(R_L + R_T)^3}$$

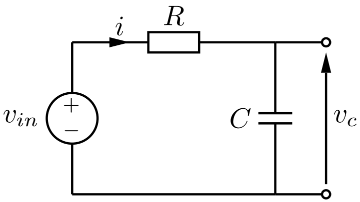
Set equal to zero (to find the turning point) and solve for $R_L$,

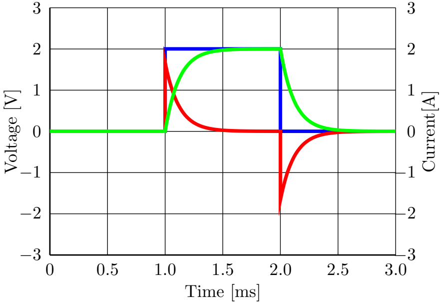
$$R_L = R_T$$

# Low Pass RC

Consider this RC Circuit. The differential relationship $i = C\,\frac{\mathrm{d}v}{\mathrm{d}t}$ is used and the circuit has exponential solution

$$v_c = v_{in}\left(1 - \exp\left(\frac{-t}{R\,C}\right)\right).$$

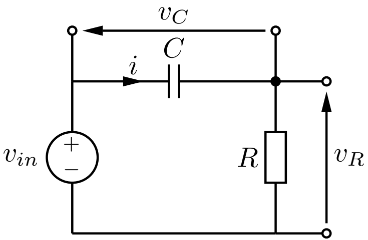


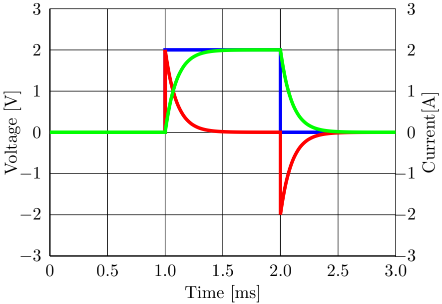




Red: Current ($i$), Blue: Input Voltage ($v_{in}$), Green: Capacitor Voltage ($v_c$).

# High Pass RC

Consider a high pass RC circuit. Similar graph?




Red: Current ($i$), Blue: Input Voltage ($v_{in}$),
Green: Capacitor Voltage ($v_c$).

# Time Domain RC Analysis

This analysis holds for all input signals (and is not examinable)

$$v_{in} = v_c + v_r \quad \text{and} \quad i = \frac{v_r}{R} \quad \text{and} \quad i = C\frac{\mathrm{d}v_c}{\mathrm{d}t}$$

combining the current expressions and isolating for $v_r$,

$$v_c(t) = \frac{1}{R\,C}\int v_r(t)\,\mathrm{d}t. \tag{1}$$

Re-writing the loop equation and inserting (1),

$$v_{in}(t) = \frac{1}{R\,C}\int v_r(t)\,\mathrm{d}t. + v_r(t) \tag{2}$$

Differentiating both sides,

$$\frac{\mathrm{d}}{\mathrm{d}t}(v_{in}) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{R\,C}\int v_r\,\mathrm{d}t. + v_r\right) \tag{3}$$

Re-writing the differential using prime notation $\left(v_r\prime = \frac{\mathrm{d}v_r}{\mathrm{d}t}\right)$

$$v_{in}\prime = \frac{1}{R\,C}v_r + v_r\prime \tag{4}$$

Setting $=\ 0$ and isolating for $v_r\prime$

$$v_r\prime = -\frac{1}{R\,C}v_r \tag{5}$$

A solution to this differential equation is (from a table of ODEs)

$$v_r = A\,\exp\left(\frac{-t}{R\,C}\right) \tag{6}$$

This is the transient term. The steady state term asks what happens as time $\rightarrow \infty$.

The steady state term, depends on the type of input signal (sinusoidal, unit step etc.). In this example a unit step has been used. Since the capacitor looks like on open circuit to DC signals, $v_c \rightarrow v_{in}$ as $t \rightarrow \infty$ and so $v_r \rightarrow 0$.

Consider the steady state term for the low pass circuit.
(ans: $v_c \rightarrow v_{in}$)

Why is an exponential a logical place to look for a solution to a differential equation that sets the derivative of a variable equal to that variable? (i.e. $v_r\prime = -\frac{1}{RC} v_r$)? (Hint: Think about what happens when you differentiate an exponential...)

See, Smith, R. J., and Dorf, R. C., *loc. cit.*, pp. 110 – 115. For a similar example.

# Review

- Considered perfect and imperfect voltage and current sources
- Showed that perfect current sources have infinite parallel resistance and that voltage sources have zero series resistance.
- Introduced the Thévanin and Norton theorems of source transformation. And gave a simple example of each.
- Introduced the Superposition theorem and gave a simple example.
- Considered the conditions required for maximum power transfer from a Thévanin source ($\mathrm{R}_L = R_T$). Could you derive for Norton on your own?
- Described what happens to the current and voltage in a simple RC network excited by a *unit step* in the time domain.
- Showed some analysis of the differential equation from which the exponential relationship is derived (not examinable)