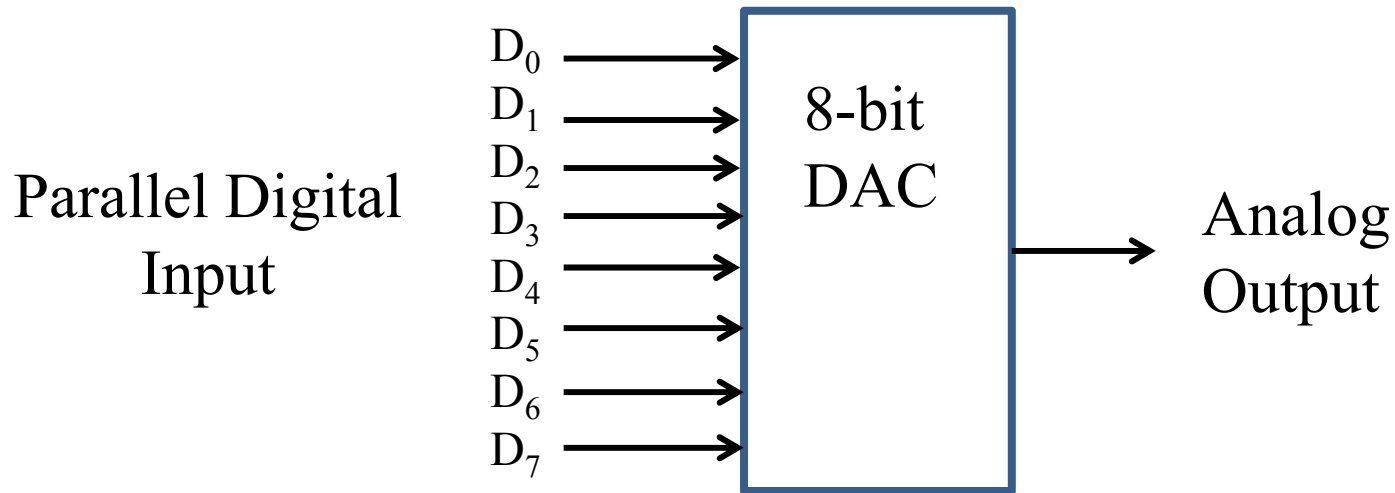


Digital-to-Analog Conversion (DAC)

- Binary Weighted Input
- R-2R Ladder

Resolution



Consider an 8-bit input, 0 (00000000) to 255 (11111111)

An 8-bit converter will have 255 steps.

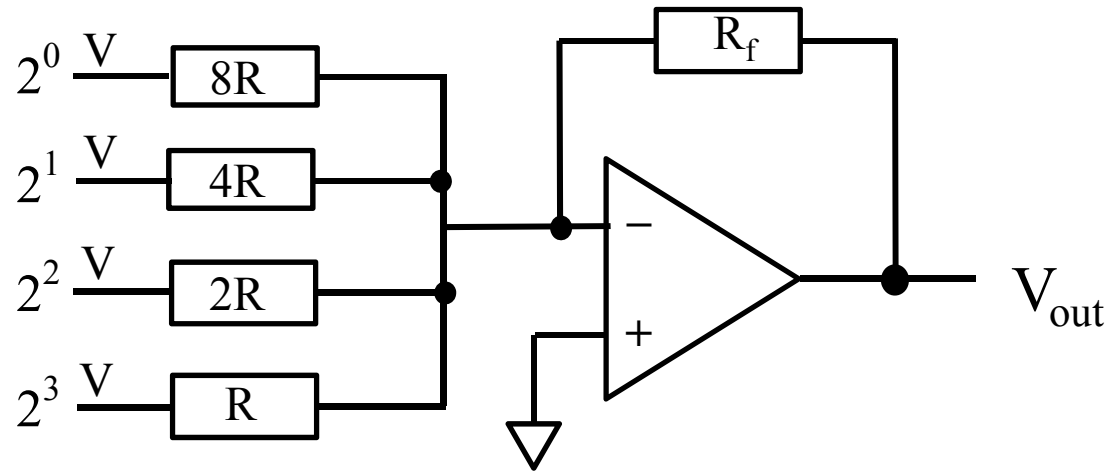
$$\text{Resolution} = 1/(2^n - 1) \times 100 = 1/255 \times 100 = 0.392\%$$

$$\text{For a 5V converter, step size} = 5/255 = 0.019 \text{ V}$$

$$\text{For an input of 137 (10001001), output voltage} = 137 \times 0.019 = 2.603 \text{ V}$$

Binary Weighted Input DAC

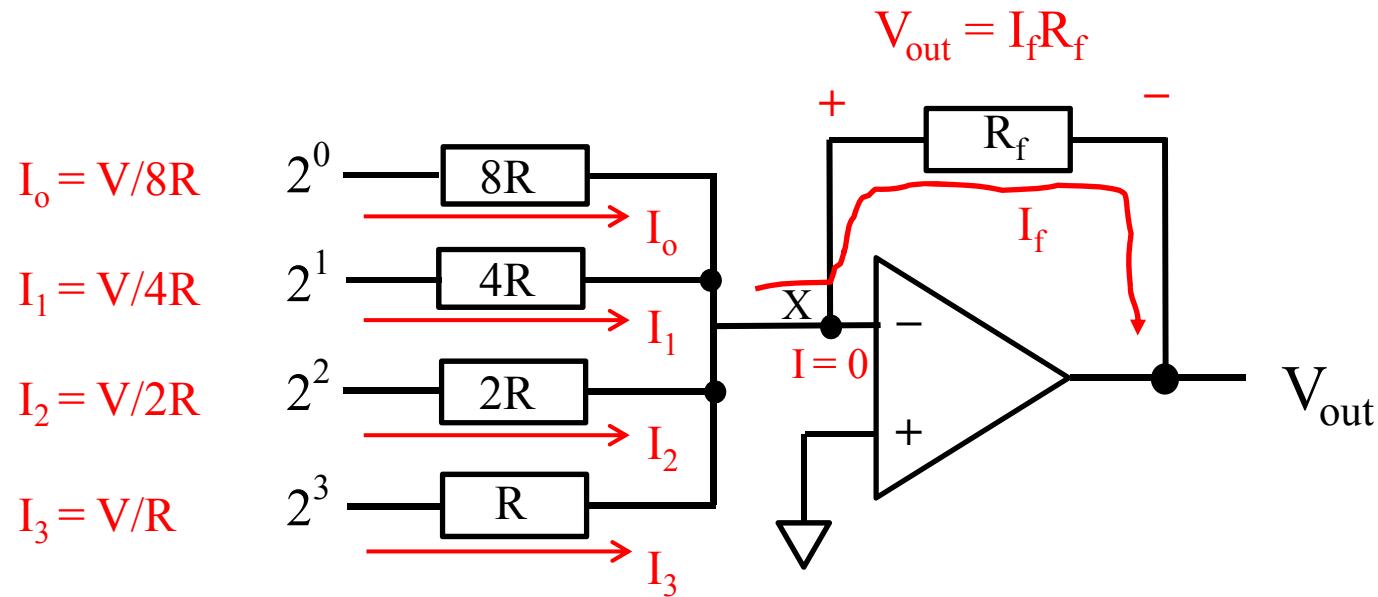
Consider a 4-bit binary conversion:



The resistor network has values corresponding to the binary weights of the input word as shown.

There is practically no current flowing into the inverting input of the op-amp which is a virtual ground (0V).

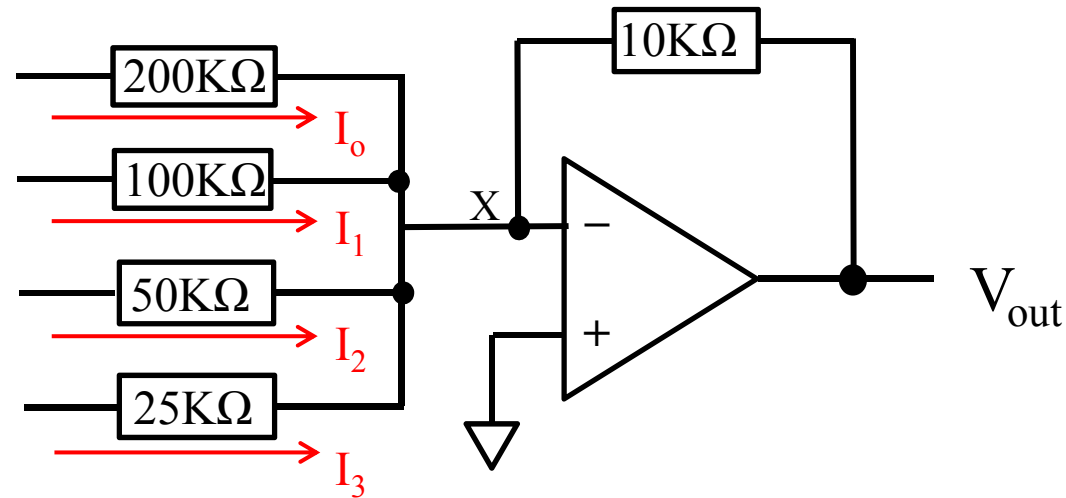
The resistor values are inversely proportional to the binary weightings of the inputs.



The input currents will thus be proportional to the binary weights. Due to the virtual ground at point X, the currents can be calculated as shown.

The sum of the input currents will flow through R_f and hence the output voltage will be proportional to the sum of the binary weights.

For the resistor values shown, calculate the output voltage for an input of (i) 1111 (ii) 1001. (Assume the input voltages be 5V)



Currents flow in each resistor:

$$I_0 = 5V/200K\Omega = 0.025mA$$

$$I_1 = 5V/100K\Omega = 0.05mA$$

$$I_2 = 5V/50K\Omega = 0.1mA$$

$$I_3 = 5V/25K\Omega = 0.2mA$$

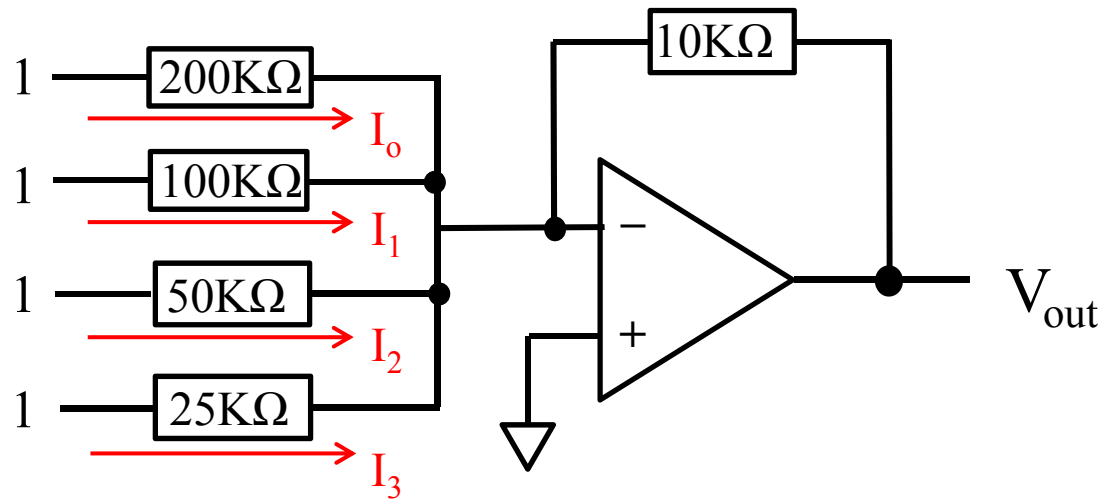
Voltage drop contribution across R_f

$$V_{out(D0)} = 10K\Omega (-0.025mA) = -0.25V$$

$$V_{out(D1)} = 10K\Omega (-0.05mA) = -0.5V$$

$$V_{out(D2)} = 10K\Omega (-0.1mA) = -1V$$

$$V_{out(D3)} = 10K\Omega (-0.2mA) = -2V$$



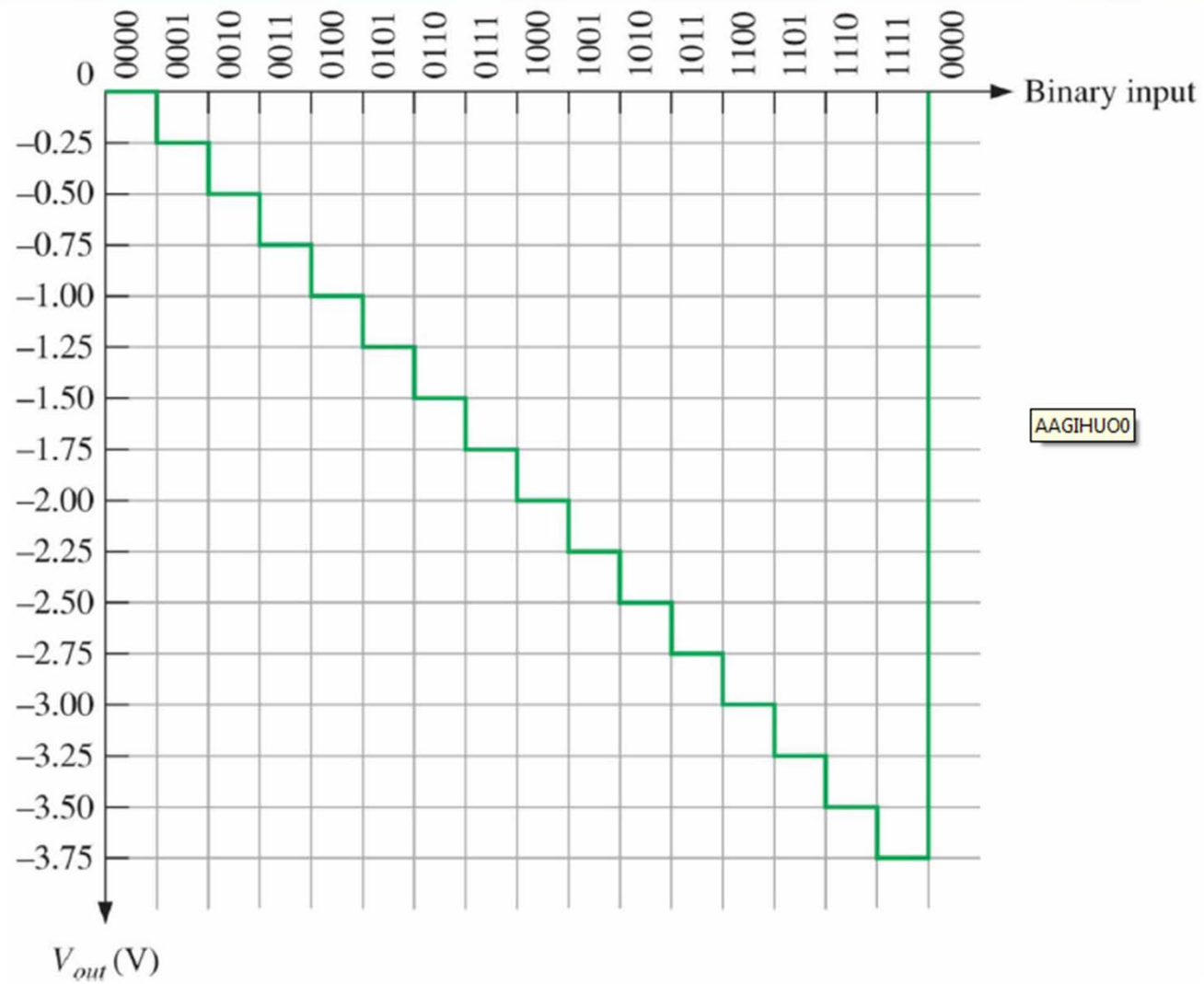
(i) 1111

Voltage drop across $R_f = V_{\text{out}(D0)} + V_{\text{out}(D1)} + V_{\text{out}(D2)} + V_{\text{out}(D1)}$

$$V_{\text{out}} = -0.25V - 0.5V - 1V - 2V = -3.75V$$

(ii) 1001 Voltage drop across $R_f = V_{\text{out}(D0)} + V_{\text{out}(D1)}$

$$V_{\text{out}} = -0.25V - 2V = -2.25V$$



Disadvantage:

The disadvantage of this method is the number of different resistor values required.

For a 12 bit converter, 12 resistors in the range R to $2048R$ would be required.

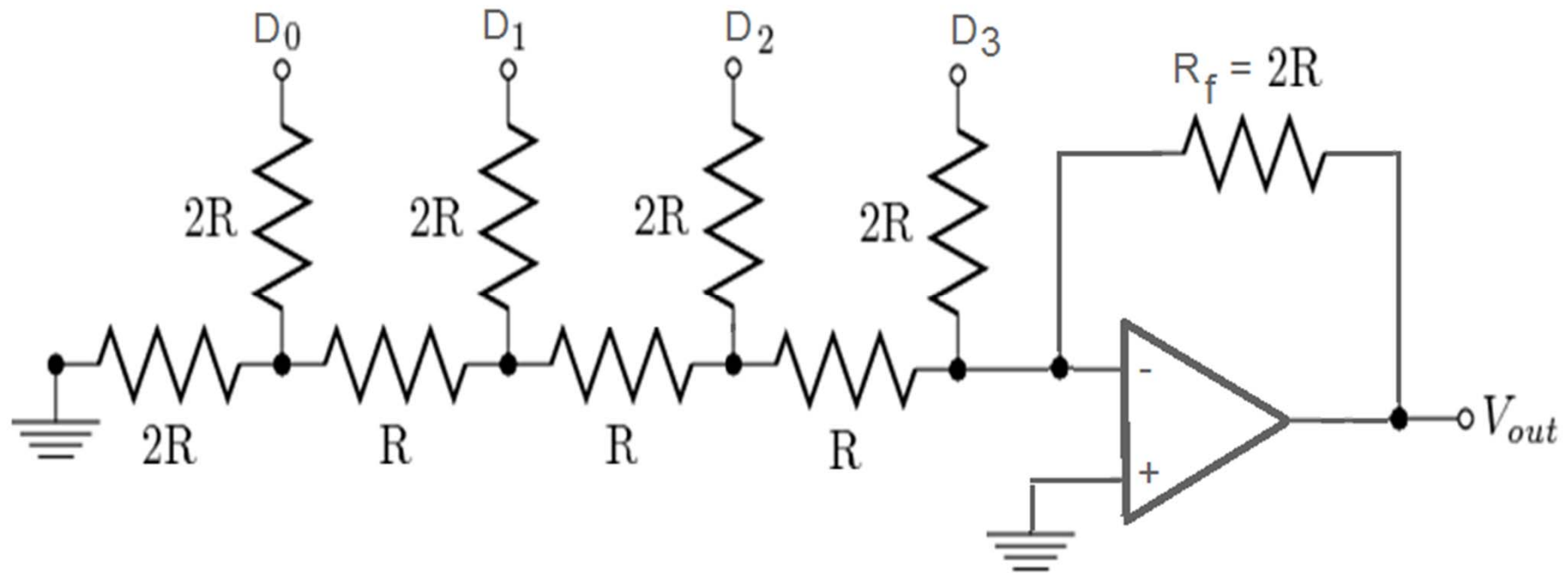
Tolerance required would be 1 part in 4095 (0.0244%)

It is difficult to mass produce these resistors within the required tolerance.

An $R/2R$ Ladder DAC can be used to overcome this as it only uses two resistor values.

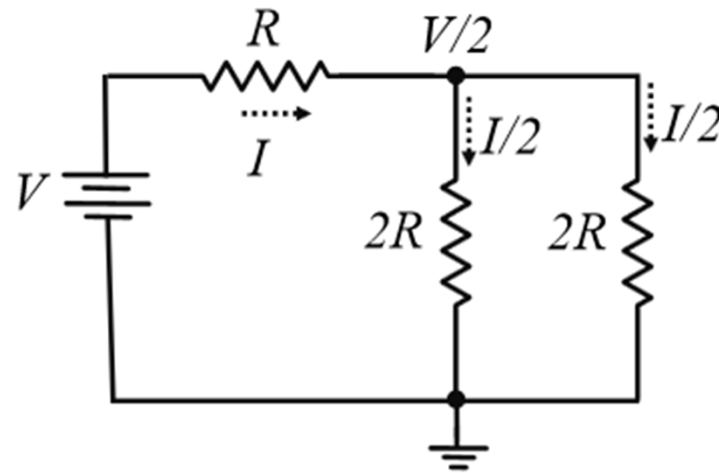
R/2R Ladder

Assume an input voltage level of 5V

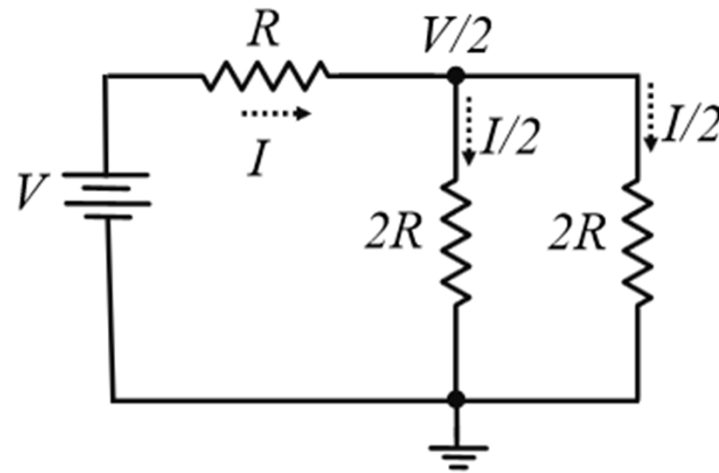


An equivalent circuit will be formed to find the contribution of each input voltage to the current in R_f

The circuit works on the principle that the voltage and current are both divided by two using the network below.



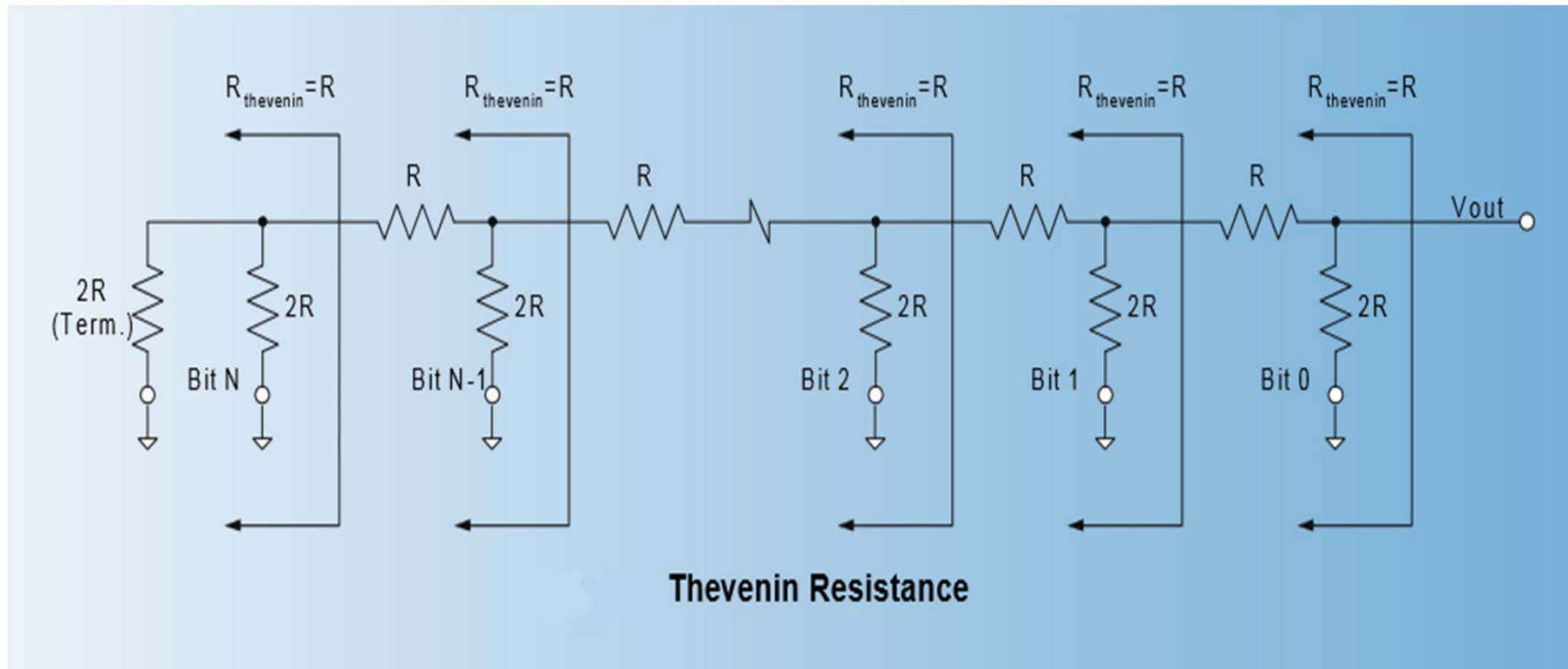
The circuit works on the principle that the voltage and current are both divided by two using the network below.



Parallel combination of $2R$ resistors:

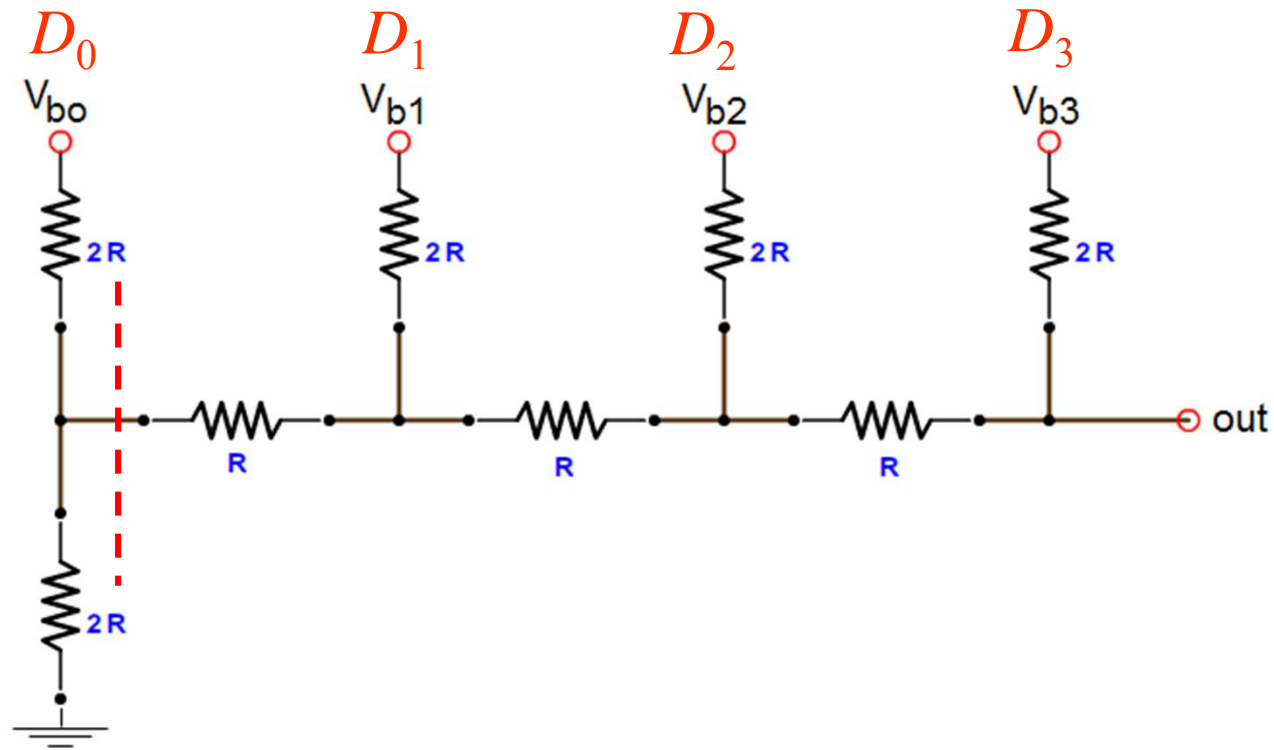
$$\frac{2R \times 2R}{2R + 2R} = \frac{4R^2}{4R} = R$$

Thevenin resistance is found by short circuiting all independent voltage sources.



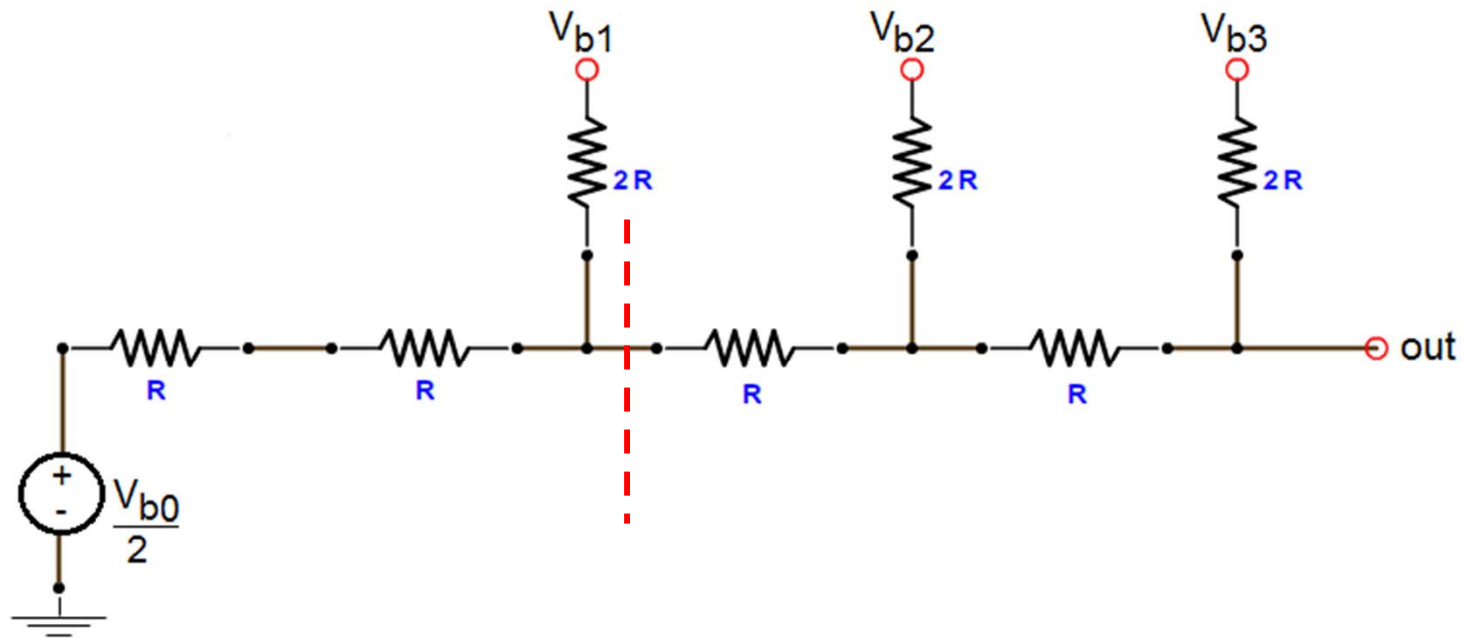
If we break the circuit at any of the points shown, the Thevenin resistance looking towards the LSB position is always R .

Contribution to output by V_{b0}



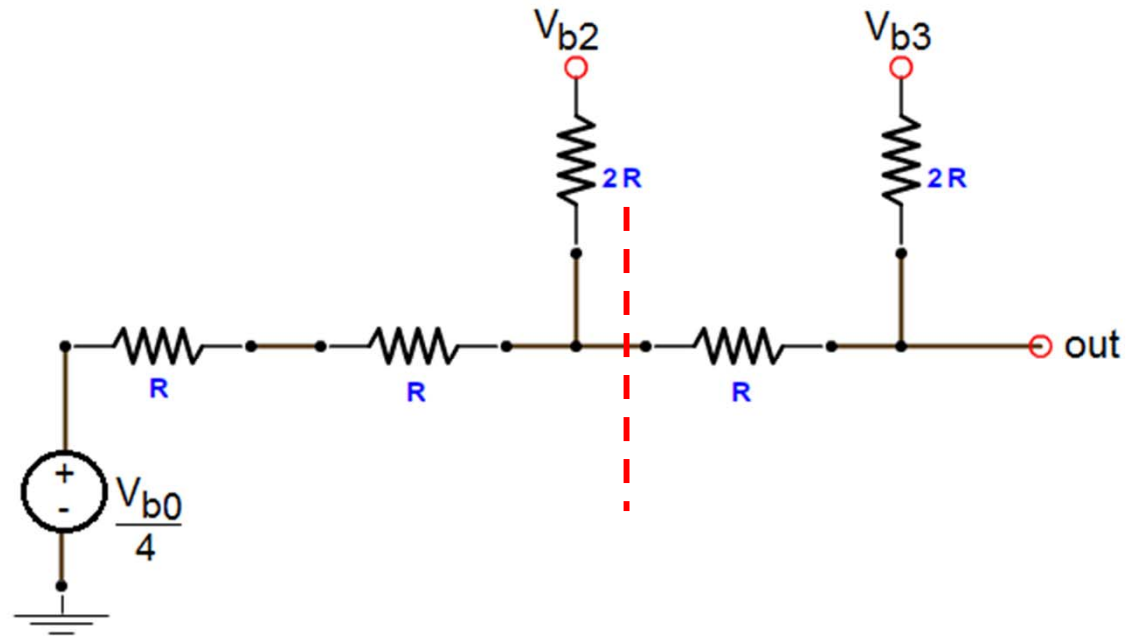
Replace the circuit to the left of the cut line with its Thevenin equivalent.

Contribution to output by V_{b0}



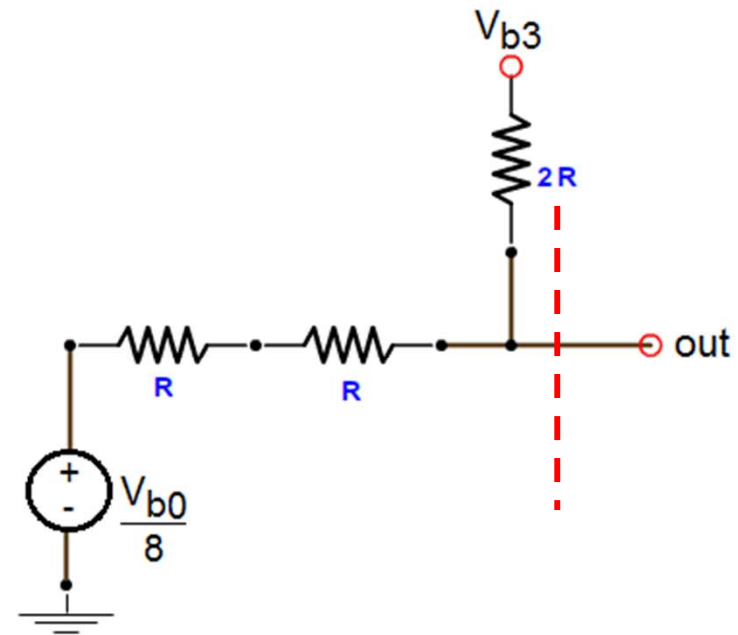
Repeat for the next cut line.

Contribution to output by V_{b0}



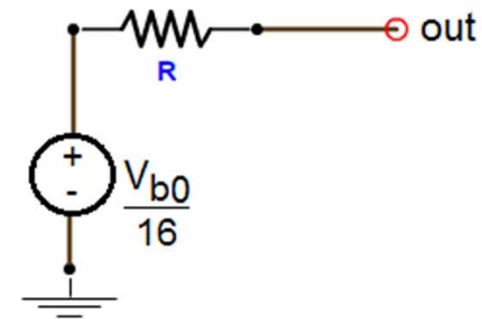
Repeat for the next cut line.

Contribution to output by V_{b0}



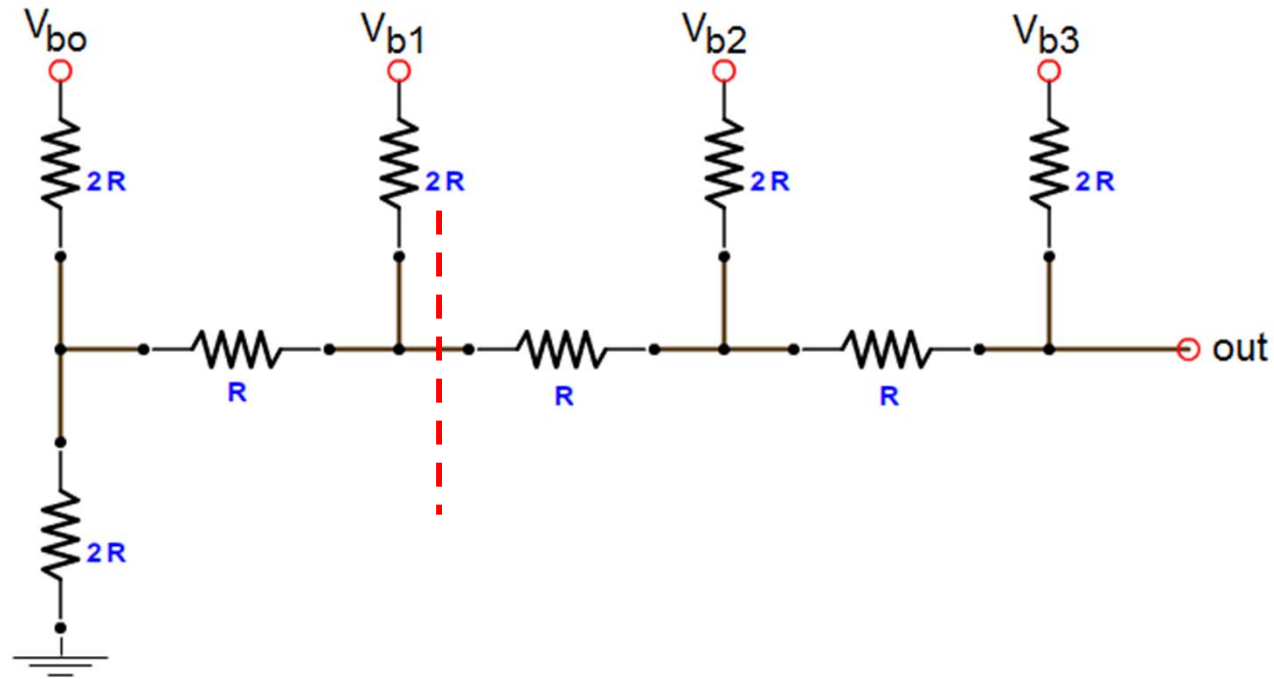
Repeat for the next cut line.

Contribution to output by V_{b0}



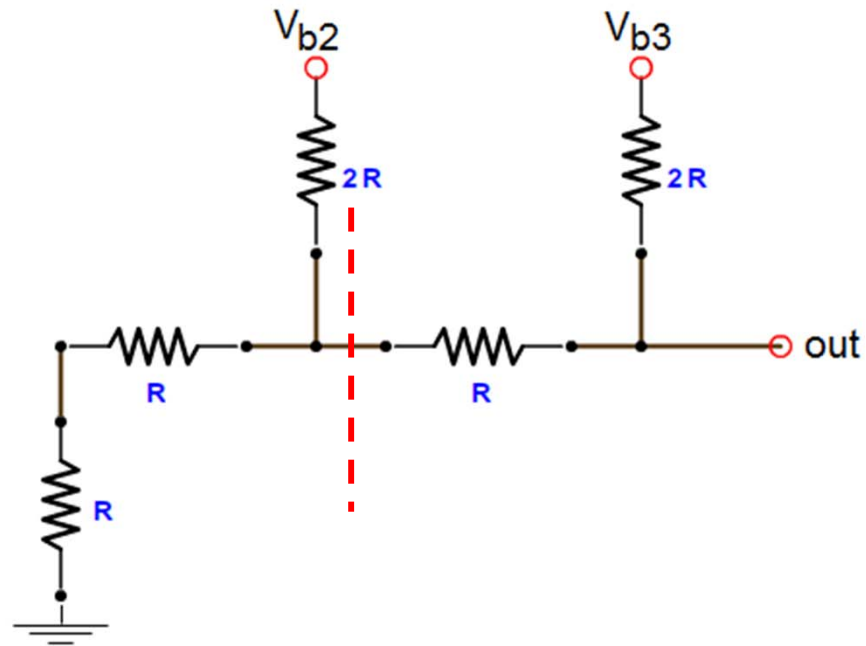
Contribution to the output by V_{b0} is $1/16$ of the HIGH level input voltage.

Contribution to output by V_{b2}



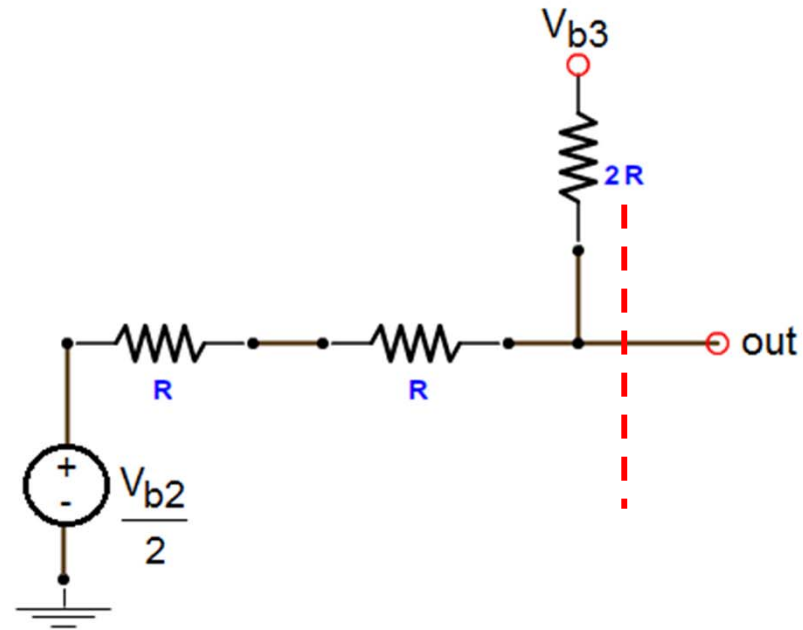
Replace the circuit to the left of the cut line with its Thevenin equivalent.

Contribution to output by V_{b2}



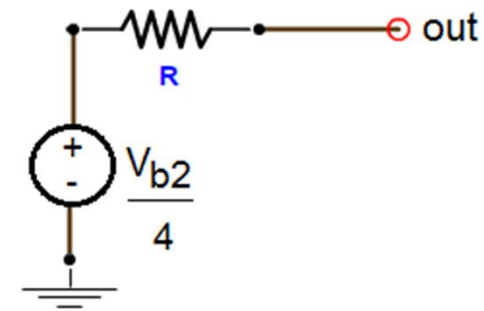
Replace the circuit to the left of the cut line with its Thevenin equivalent.

Contribution to output by V_{b2}



Replace the circuit to the left of the cut line with its Thevenin equivalent.

Contribution to output by V_{b2}



Contribution to the output by V_{b2} is $1/4$ of the HIGH level input voltage.

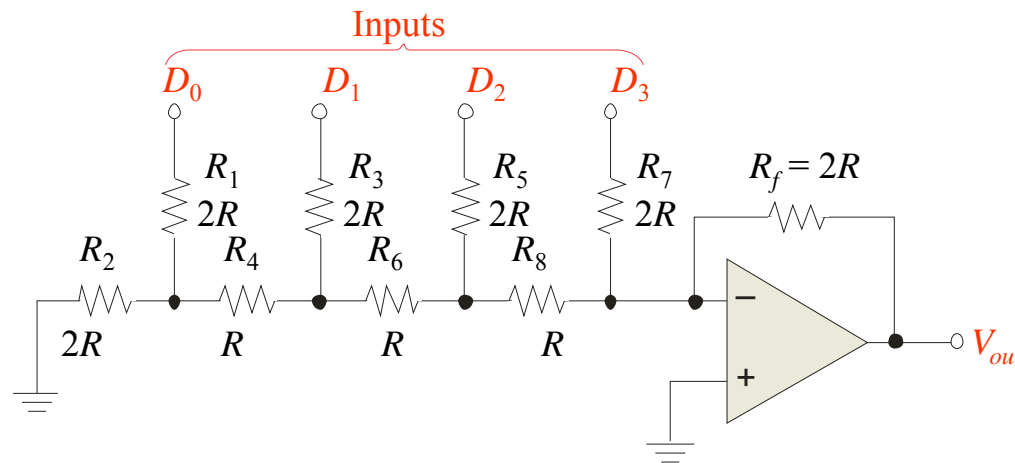
By calculating a Thevenin equivalent circuit for each input, it can be shown that the output is proportional to the binary weight of inputs that are HIGH.

Each input that is HIGH contributes to the output: $V_{out} = -\frac{V_S}{2^{n-i}}$

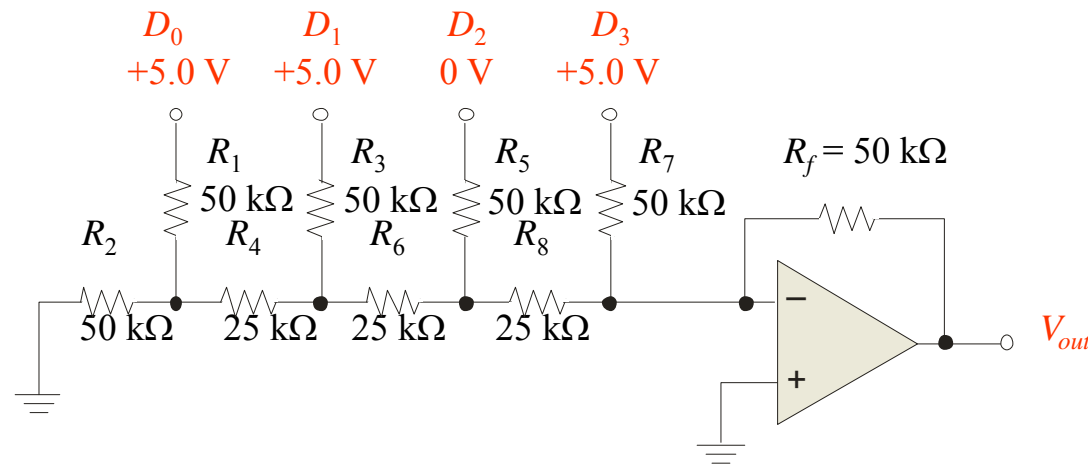
where V_S = input HIGH level voltage

n = number of bits

i = bit number



An R - $2R$ ladder has a binary input of 1011. If a HIGH = +5.0 V and a LOW = 0 V, what is V_{out} ?



Apply $V_{out} = -\frac{V_S}{2^{n-i}}$ to all inputs that are HIGH, then sum the results.

$$V_{out}(D_0) = -\frac{5\text{ V}}{2^{4-0}} = -0.3125\text{ V} \quad V_{out}(D_1) = -\frac{5\text{ V}}{2^{4-1}} = -0.625\text{ V}$$

$$V_{out}(D_3) = -\frac{5\text{ V}}{2^{4-3}} = -2.5\text{ V} \quad \text{Applying superposition, } V_{out} = -3.43\text{ V}$$

