

**EEE116 – Multimedia Systems 2007/08**  
**Solutions to tutorial problem sheet 3 (Week 4)**

(Q5) In the process of sampling an analogue signal to convert it to digital formats, the following set of Voltage values were measured. (Read from left to right and top to bottom. We call it Raster scan)

-2	0	1	1	0	2	-1	0	1	2
-1	-1	0	2	0	1	0	0	0	-1
0	1	1	1	0	-1	2	1	-1	0
0	3	0	-1	0	1	0	-1	0	1
-1	0	1	0	3	1	-1	0	-2	0

(i) Fill in the following table

Sample value	-2	-1	0	1	2	3
Number of samples	2	10	20	12	4	2

(ii) Devise a digital code to represent these sample values in digital form.

There are 5 different values.

N bits can represent  $2^N$  different values.

Therefore  $2^N=6$ ;

N=3 (we round upwards, otherwise  $2^2$  can only represent 4 different values)

Therefore we define a 3-bit code as follows:

Sample value	-2	-1	0	1	2	3
Digital code	000	001	010	011	101	111

(Note that you can use any 6 of the 8 possible codes – 000, 001, 010, 011, 100, 101, 110, 111)

(iii) Compute the total number of bits required to represent these samples in digital format.

In the code we designed we used 3 bits per sample and we have 50 samples. Therefore, we require 150 bits altogether.

( bits per sample x number of samples)

(iv) What is the average code length for the digital code devised in (ii).

The type of codes used in this case is called “fixed length” code.

Therefore, the average number of bits per sample is the same as the

code length, 3.

- (v) Now consider a “variable length code” as shown in the following table.

Sample value	-2	-1	0	1	2	3
Digital code	1110	110	0	10	11110	11111

Compute the average code length if the above variable length code is used.

We can create a new table as follows:

Sample value	-2	-1	0	1	2	3
Digital code	1110	110	0	10	11110	11111
Length of codes (bits)	4	3	1	2	5	5
Number of samples (from (i))	2	10	20	12	4	2
Total number of bits (row 3 x row 4)	8	30	20	24	20	10

Total bits required:  $8+30+20+24+20+10 = 112$  bits

The average bits per sample = (total bits)/(total number of samples)  
 $= 112/50$   
 $= 2.24$  bits/sample.

In real life, due to the large amounts of data, it is not feasible to count the numbers of samples for each sample value. The average bits per sample can be computed by estimating their probabilities.

- (vi) How many bits can be saved (in total) by using the variable length code compared to the fixed length code in (ii)?

$150-112 = 38$  bits altogether or 0.76 bits per sample

- (vii) Decode the following bit stream which has been generated using the variable length code in (v). 101111001101110110010101111110

Now use each of the codes in (v) try to separate the codes which match the bits in the bit stream

101111001101110110010101111110 (the code 10 matches)

10 1111001101110110010101111110 (the code 11110 matches)

10 11110 01101110110010101111110 (the code 0 matches)

continuing the matching we can get

10 11110 0 110 1110 110 0 10 10 11111 10

Now we can decode (using the code table)

1 2 0 -1 -2 -1 0 1 1 3 1.

- (viii) If the 8<sup>th</sup> and 9<sup>th</sup> bits in the bit stream in (vii) were corrupted and were unable to recover accurately, what would be the decoded sample values?

8<sup>th</sup> and 9<sup>th</sup> bits recovered inaccurately. Now our data stream is

10111101010110110010101111110

We can start grouping like before

10 11110 10 1110 110 0 10 10 11111 10

Now we can decode (using the code table)

1 2 1 1 -2 -1 0 1 1 3 1. (two symbols have been affected).

(Q6) An information source outputs 5 different symbols {\$, %, &, @, £} with probabilities {0.5, 0.25, 0.125, 0.0625, 0.0625}, respectively. The following table shows 4 different codes (1 fixed length binary code and 3 variable length codes) that can be used for this alphabet.

Symbol	Probability	Code1	Code2`	Code3	Code4
A	1/2	000	1	0	00
B	1/4	001	01	10	01
C	1/8	010	001	110	10
D	1/16	011	0001	1110	11
E	1/16	111	00001	1111	110

- (i) Compute the average code length of the above 4 codes?

Code 1 is a fixed length, Therefore, the average code length = 3 bits/sample

For code 2,

$$\begin{aligned}
 \text{Average code length} &= \sum_{i=1}^5 n_i P_i \\
 &= 1*0.5 + 2*0.25 + 3*0.125 + 4*0.0625 + 5*0.0625 \\
 &= 1.94 \text{ bits/sample}
 \end{aligned}$$

For code 3,

$$\begin{aligned}
 \text{Average code length} &= \sum_{i=1}^5 n_i P_i \\
 &= 1*0.5 + 2*0.25 + 3*0.125 + 4*0.0625 + 4*0.0625
 \end{aligned}$$

$$= 1.88 \text{ bits/sample}$$

For code 4,

$$\begin{aligned} \text{Average code length} &= \sum_{i=1}^5 n_i P_i \\ &= 2*0.5 + 2*0.25 + 2*0.125 + 2*0.0625 + 3*0.0625 \\ &= 2.06 \text{ bits/sample} \end{aligned}$$

- (ii) In an unambiguous code, no codeword is the prefix of another codeword. Which of the above codes are unambiguous?

Code 1 is a fixed length code. Therefore, it is unambiguous.

For the rest, which are variable length codes, you have to check which codes satisfy the prefix test. In an unambiguous code, each of the codewords should not appear as the prefix of any other codeword.

Only code1, code2 and code3 are unambiguous in this case.