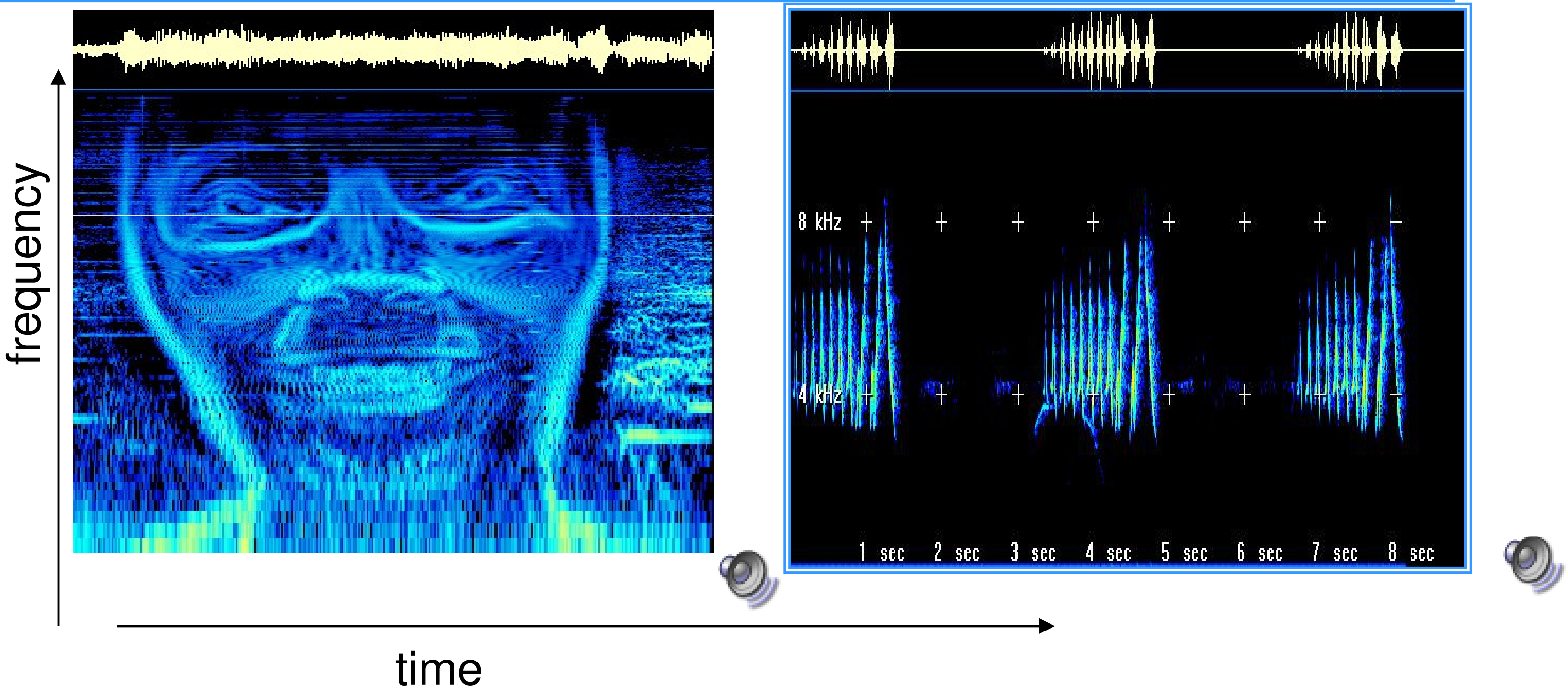
Lecture content

- Fourier series representation of periodic signals
  - A CT periodic signal can be expressed as a sum of harmonically related sinusoids.

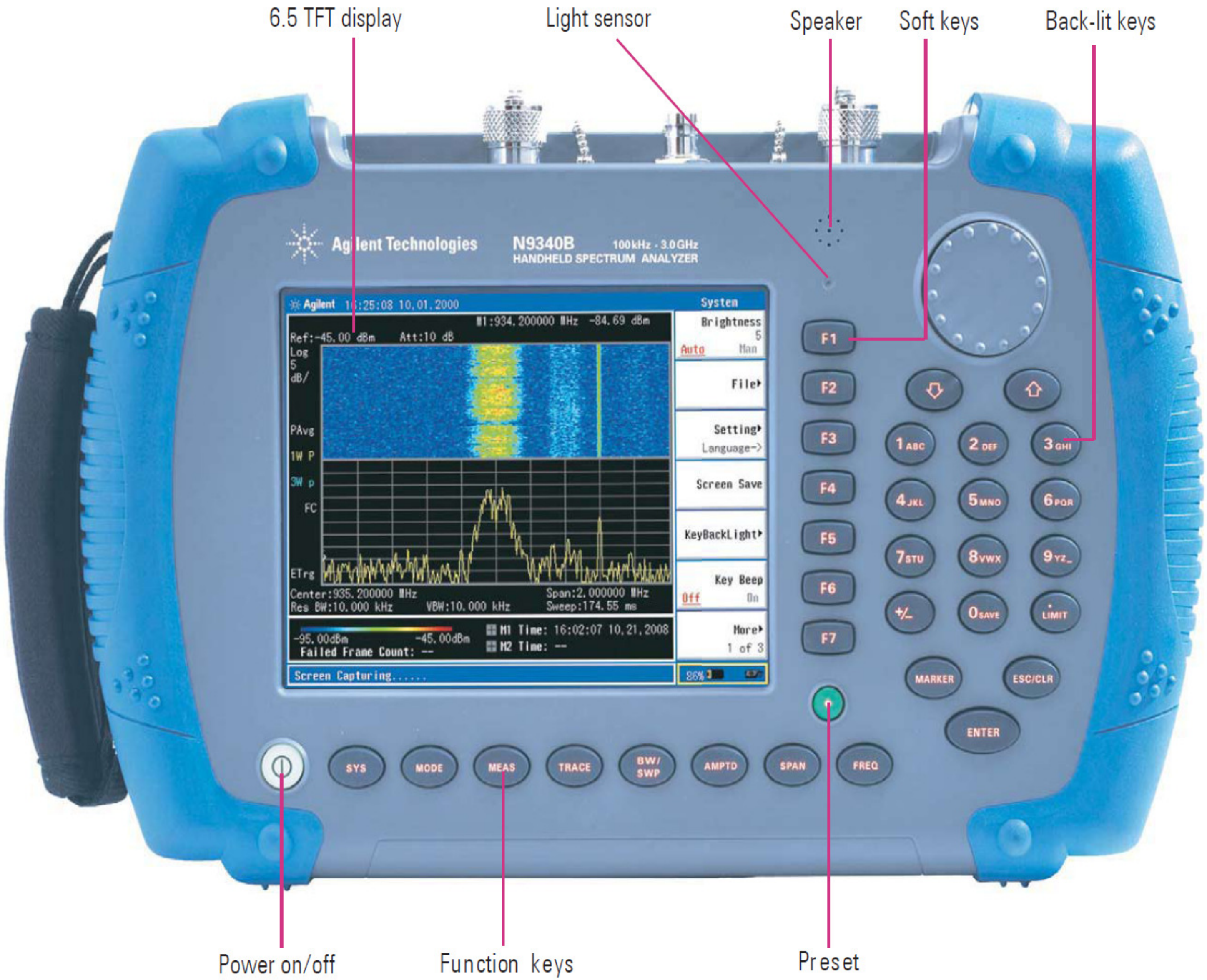
Applet:
http://www.fourier-series.com/fourierseries2/flash_programs/four_freqs/index.html

Signals: Spectrogram

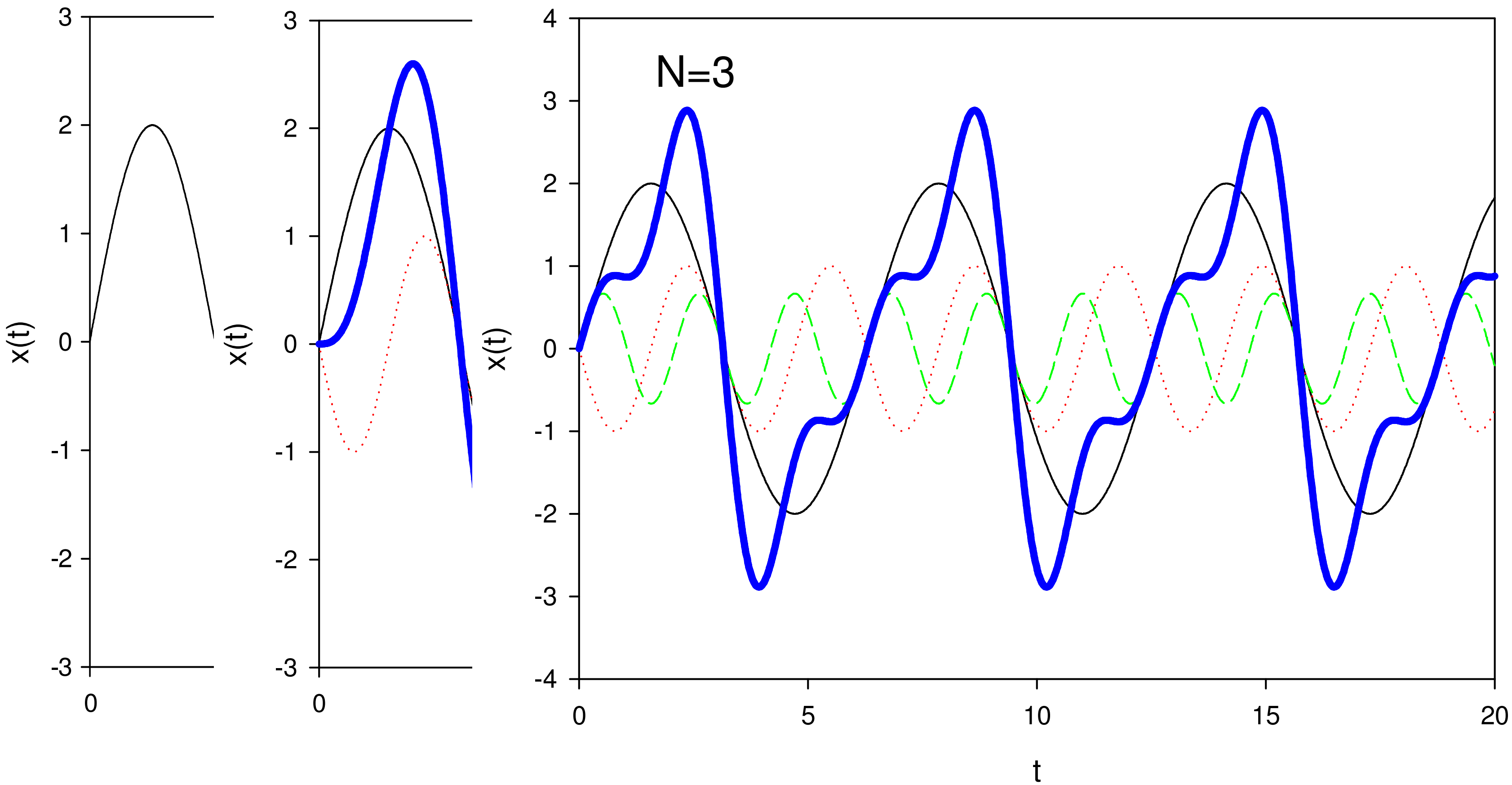
2) Audio signals such as speech waveform or music. Signal processing can be developed to characterise the speech signals in terms of their frequency spectrum. (Spectrogram)



The University Of Sheffield.



Periodic sawtooth waveform

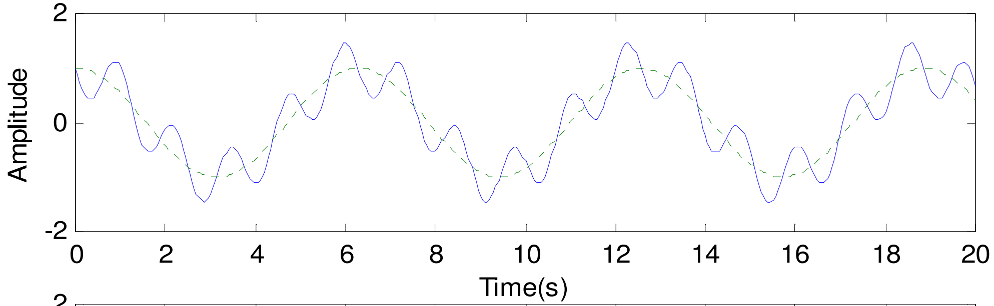


$$x(t) = \sum_{n=1}^{N} \frac{2}{n} (-1)^{n+1} \sin(nt)$$

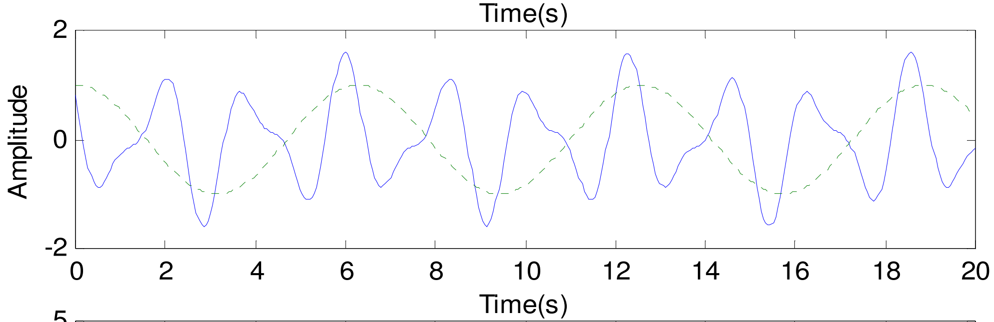
Fourier Series

$$x(t) = A_1 \cos t + A_2 \cos(3t + \pi / 4) + A_3 \cos(5t + \pi / 2)$$

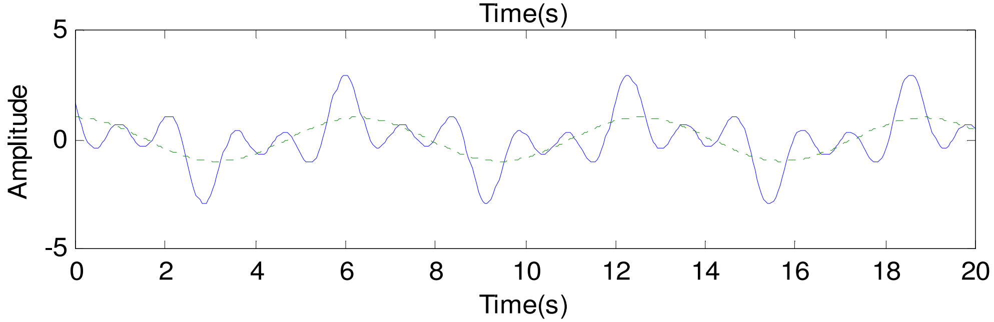
fundamental 3rd harmonic 5th harmonic



$A_1 = 1,\ A_2 = 0,\ A_3 = 0.5.$



$A_1 = 0.1,\ A_2 = 1,\ A_3 = 0.5.$

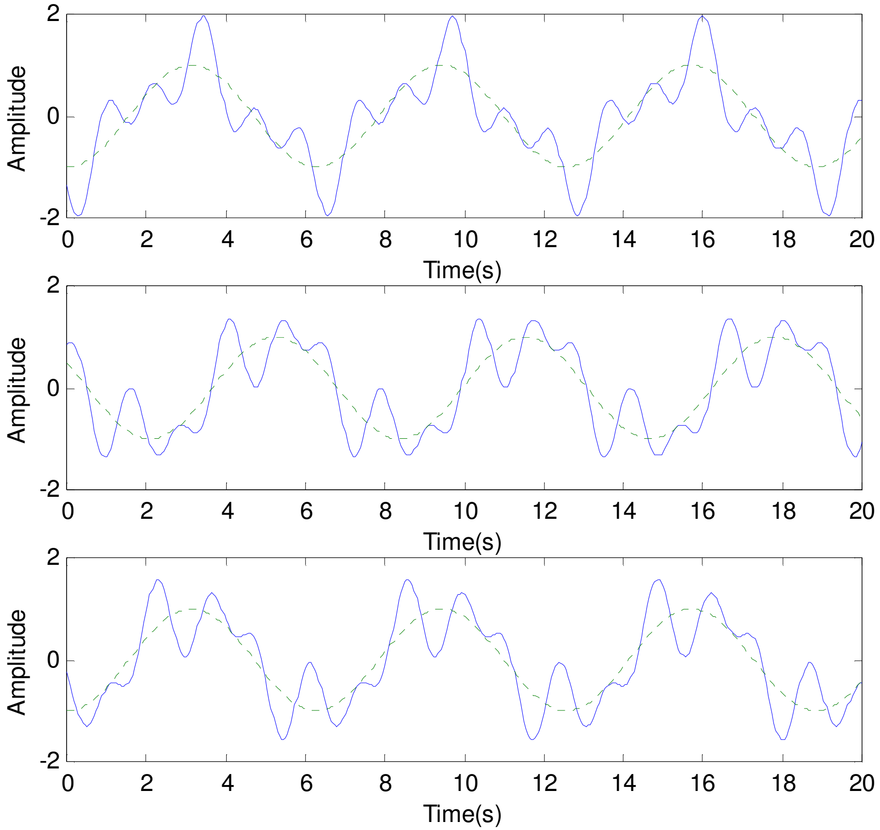


$A_1 = 1,\ A_2 = 1,\ A_3 = 1.$

Fourier Series

$$x(t) = A_1 \cos t + A_2 \cos(3t + \pi/4) + A_3 \cos(5t + \pi/2)$$

$A_1 = 1$, $A_2 = A_3 = 0.5$, the frequencies are 1, 3 and 5 rad/s and the phases are p_1, p_2, and p_3.



$p_1 = \pi,\ p_2 = 3\pi/4,\ p_3 = \pi/2$

$p_1 = \pi/3,\ p_2 = \pi/4,\ p_3 = 3\pi/2$

$p_1 = \pi,\ p_2 = 0,\ p_3 = \pi/3$

Fourier Series Representation

In fact we can express a CT periodic signal, $x(t)$ as a sum of sinusoids

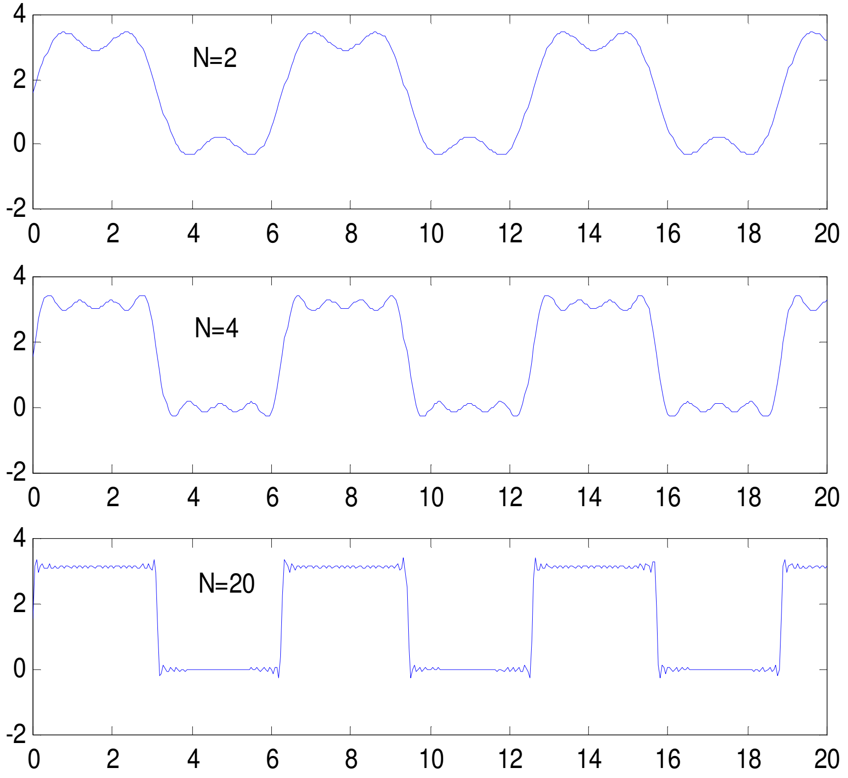
$$x(t) = \sum_{k=1}^{N} A_k \sin(\omega_k t + \theta_k)$$

where N is a positive integer, A_k is the amplitude, ω_k is the frequency in rad/s and θ_k is the phase angle. This is the ***Fourier Series*** representation of the periodic signal $x(t)$. We can approximate any periodic signal by using the Fourier Series and the converse is true, any periodic signal may be broken down into a series of sinusoidal components that are harmonically related.

Examples of Fourier Series Representation

1. The square waveform shown below can be represented as

$$x(t) = \frac{\pi}{2} + \sum_{n=0}^{N} \frac{2}{2n+1} \sin\left((2n+1)t\right)$$

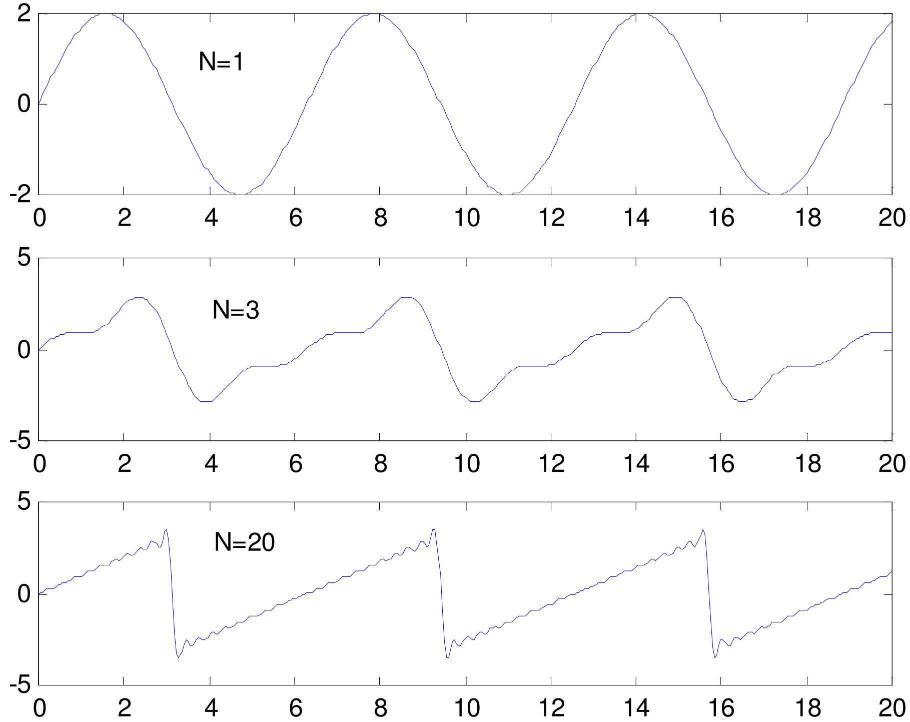



```
function generate_squarewave(H)
%generate squarewave
%number of harmonics = H

t=0:20/400:20;
w0=1; %w0 = 1rad/s
x=0;
for N=[0:1:H]
   x1=sin((2*N+1)*w0*t)/(2*N+1);
   x=x+x1;
end;
y=pi/2+2*x;

plot(t,y);
xlabel ('t(s)'), ylabel('y(t)');
```

Examples of Fourier Series Representation

2. The sawtooth waveform shown below can be represented as

$$x(t) = \sum_{n=1}^{N} \frac{2}{n} (-1)^{n+1} \sin(nt)$$

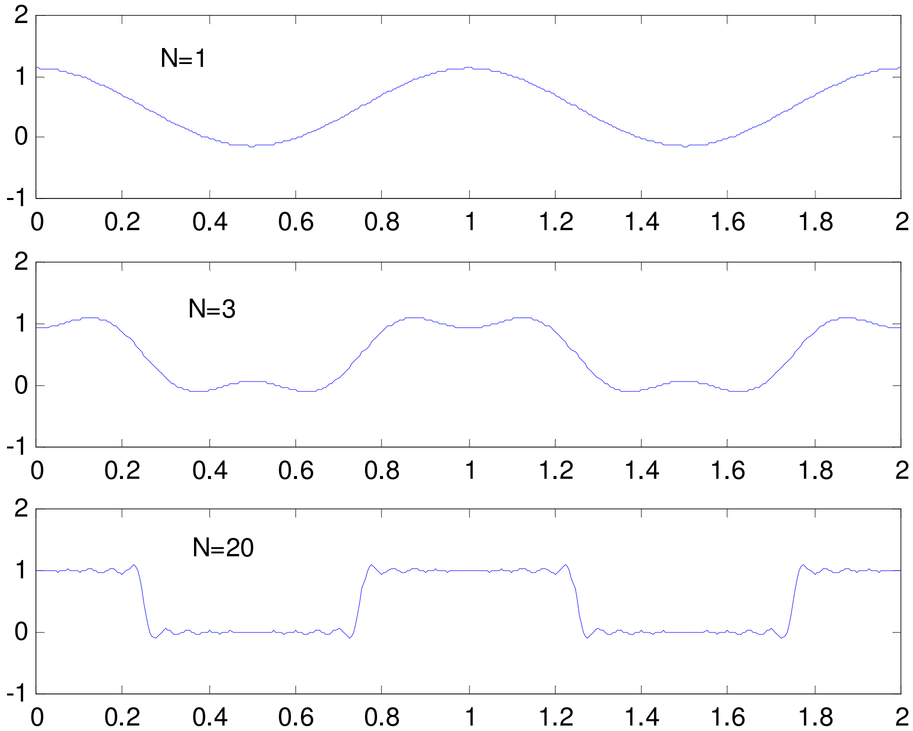



```
function generate_sawtooth(H)
%generate sawtooth
%number of harmonics = H
t=0:20/400:20;
w0=1;  %w0=1rad/s
x=0;
for N=[1:1:H]
   x1=(2*power(-1,N+1)/N)*sin(N*w0*t);
   x=x+x1;
end;
y=x;

plot(t,y);
xlabel ('t(s)'), ylabel('y(t)');
```

Complex Fourier Series Representation

3. The square waveform shown below can also be represented as.

$$x(t)=\frac{1}{2}+\sum_{n=1}^{N}\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos(2\pi t)=\frac{1}{2}+\sum_{n=1}^{N}\frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)\left(e^{j2n\pi t}+e^{-j2n\pi t}\right)$$




```
%generate square wave
t=0:2/400:2;
H=1;          %number of harmonics
w0=2*pi;
x=0;

for N=[1:1:H]
   an=sin(N*w0*0.25)/(N*pi);
   %x=x+2*an*cos(N*w0*t);
   x=x+an*(exp(j*N*w0*t)+exp(-j*N*w0*t));
end;
y=1/2+x;
plot(t,y);
```

FS coefficients

Consider a periodic signal that can be represented by

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t}}$$

where ω_o is the fundamental frequency of a periodic exponential and c_k is the amplitude of harmonics, known as the **Complex Fourier Series Coefficients**.

Multiplying both sides by $e^{-jn\omega_o t}$ gives

$$x(t)e^{-jn\omega_o t} = \sum_{k=-\infty}^{\infty} c_k e^{j(k-n)\omega_o t}$$

Integrating both sides from 0 to $T = \omega_o/2\pi$, we have

$$\int_0^T x(t)e^{-jn\omega_o t}dt = \int_0^T \sum_{k=-\infty}^{\infty} c_k e^{j(k-n)\omega_o t}dt$$

Interchanging the order of integration and summation, we have

$$\int_0^T x(t)e^{-jn\omega_o t}dt = \sum_{k=-\infty}^{\infty} c_k \int_0^T e^{j(k-n)\omega_o t}dt$$

The University Of Sheffield.

FS coefficients

We know that $e^{j(k\text{-}n)\omega o t} = \cos(k\text{-}n)\omega_o t + \mathrm{j}\sin(k\text{-}n)\omega_o t$

$$For\ k \neq n,\ \int_0^T e^{j(k-n)\omega_o t} dt = 0\ since\ \int_0^T \cos(k-n)\omega_o t dt = 0\ \ and\ \int_0^T \sin(k-n)\omega_o t dt = 0$$

therefore

$$c_k \int_0^T e^{j(k-n)\omega_o t} dt = 0$$

$$For\ k = n,\ \ c_n \int_0^T e^{j(k-n)\omega_o t} dt = c_n T\ \ since\ \ \int_0^T e^0 dt = T$$

$$\boxed{c_n = \frac{1}{T} \int_{<T>} x(t) e^{-jn\omega_o t} dt}$$

FS coefficients

The coefficients c_n represents the amplitude of the nth harmonic of the periodic signal $x(t)$. Sometimes, there is a constant or d.c component in the signal $x(t)$ given by

$$\boxed{c_0 = \frac{1}{T}\int_{<T>} x(t)dt}$$

Example

Consider the periodic square wave $x(t)$ shown in figure 4.3. Find the Fourier Series coefficients for $x(t)$.