EEE118 Problem Class Questions – Sheet 3

Fundamental Constants
Boltzman Constant, $k = 1.381\text{x}10^{-23}$ JK^{-1} = 8.62×10^{-5} eVK^{-1}
Charge on Electron, $e = 1.602\text{x}10^{-19}$ C
Data for germanium
Hole mobility, μ_h=0.19
Electron mobilityμ_e=0.39 $m^2V^{-1}s^{-1}$
Band-gap of germanium (Ge) = 0.66eV
Data for silicon
Hole mobility μ_h=0.046 $m^2V^{-1}s^{-1}$.
Electron mobility μ_e =0.12 $m^2V^{-1}s^{-1}$
Band-gap of silicon (Si) = 1.1 eV

1(a) The intrinsic free carrier concentration for Si at 300K is 1.5 x 10^{16} m^{-3}. From this, derive the constant of proportionality in the equation for n_i. ($5\text{x}10^{21}$ $m^{-3}K^{-3/2}$)

(b) Using this number, calculate the free carrier concentration of intrinsic silicon at 250K and 350K. ($1.6\text{x}10^{14}$ m^{-3}, $4\text{x}10^{17}$ m^{-3})

(c) Using the same constant, calculate the intrinsic carrier concentration for germanium at 250K, 300K and 350K. (4.4x 10^{18} m^{-3}, 7.4x 10^{19} m^{-3} 5.8x 10^{20} m^{-3})

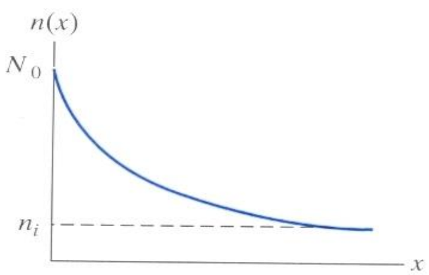
(d) How might you expect the majority and minority carrier concentration of heavily n-doped silicon to change over this temperature range? State your assumptions.

(e) Which of these three materials may be most suitable to be used in a sensor to measure temperature over this temperature range? State your assumptions.

2). A bar of intrinsic germanium, 2 mm in length, has $2.5\text{x}10^{19}$ free electrons per m^3. A voltage of 1 V is applied across its length.

a. Calculate the conductivity of the material. (2.3 $\Omega^{-1}m^{-1}$)
b. Find the net drift current density. (1160 Am^{-2})
c. What fraction of the drift current is due to electrons? (67%)
d. What are the drift velocities of the electrons and holes? (195 ms^{-1}, 95 ms^{-1})

3) A silicon sample is doped with donors from one side such that $N_d= N_0\exp(-ax)$ (see Figure). The sample is at room temperature where all donors may be expected to be ionized.



(a) Find an expression for the resultant built-in electric field $E(x)$ at equilibrium over the range for which $N_d >> n_i$

(b) Evaluate $E(x)$ when $a=1\text{x}10^6$ m^{-1} (i.e. a=1 $(\mu m)^{-1}$) ($2.6\text{x}10^4$ Vm^{-1})

(c) Indicate the direction of E on the figure opposite

Solutions Sheet 3

1(a) This question is asking you to calculate the value of the constant C in the equation

$$n_i = CT^{3/2}\exp\left(-\frac{W_g}{2kT}\right)$$

Re-arranging to get

$$C = \frac{n_i}{T^{3/2}}\exp\left(\frac{W_g}{2kT}\right) = \frac{1.5\text{x}10^{16}}{300^{3/2}}\exp\left(\frac{1.1}{2\text{x}300\text{x}8.62\,\text{x}10^{-5}}\right)$$

$$C = \frac{1.5\text{x}10^{16}}{5196}\exp\left(\frac{1.1}{0.052}\right)$$

$$C = 2.9\text{x}10^{12}\,\text{x}\,1.7\text{x}10^{9} = 5\text{x}10^{21}m^{-3}K^{-3/2}$$

(Note: take care here not to mix up eV and Joules)

1(b). Intrinsic carrier concentration for Si at;

250K

$$n_i = CT^{3/2}\exp\left(-\frac{E_g}{2kT}\right) = 5\text{x}10^{21}\text{x}250^{3/2}\exp\left(-\frac{1.1}{2\text{x}8.62\text{x}10^{-5}\text{x}250}\right) = 1.6\text{x}10^{14}\text{m}^{-3}$$

350K

$$n_i = CT^{3/2}\exp\left(-\frac{E_g}{2kT}\right) = 5\text{x}10^{21}\text{x}350^{3/2}\exp\left(-\frac{1.1}{2\text{x}8.62\text{x}10^{-5}\text{x}350}\right) = 4\text{x}10^{17}\text{m}^{-3}$$

1(c). Germanium at;

250K

$$n_i = CT^{3/2}\exp\left(-\frac{E_g}{2kT}\right) = 5\text{x}10^{21}\text{x}250^{3/2}\exp\left(-\frac{0.66}{2\text{x}8.62\text{x}10^{-5}\text{x}250}\right) = 4.4\text{x}10^{18}\text{m}^{-3}$$

300K

$$n_i = CT^{3/2}\exp\left(-\frac{E_g}{2kT}\right) = 5\text{x}10^{21}\text{x}300^{3/2}\exp\left(-\frac{0.66}{2\text{x}8.62\text{x}10^{-5}\text{x}300}\right) = 7.4\text{x}10^{19}\text{m}^{-3}$$

350K

$$n_i = CT^{3/2}\exp\left(-\frac{E_g}{2kT}\right) = 5\text{x}10^{21}\text{x}350^{3/2}\exp\left(-\frac{0.66}{2\text{x}8.62\text{x}10^{-5}\text{x}350}\right) = 5.8\text{x}10^{20}\text{m}^{-3}$$

1(d). For n-doped silicon over this temperature range we might expect the majority carrier concentration to be unchanged. This is due to the energy required to free carriers bound to the donor atom to be ~5 meV. The thermal energy at 250K = kT ~22meV and at 350K is

~30meV. Both are >>5meV resulting in essentially all the donors being ionized, giving one free carrier per donor atom.

For the minority carriers

$$p = \frac{n_i^2}{n}$$

And n_i is a function of temperature – it increases exponentially with T. The minority carrier hole density will therefore increase with temperature. The thermally generated electrons will also increase exponentially with temperature but since they are a small fraction of the electrons due to doping, the increase due to these can be ignored.

1(e). Here we need to think about how conductivity will change with temperature. We would like the material with the biggest change in conductivity to give the best sensitivity in measuring temperature. We will assume mobility is constant over this temperature range.

The doped Si can be ruled out since, if majority carrier density is essentially constant with temperature, then conductivity will be constant. Whilst minority carrier density is changing, the minority carrier does not contribute significantly to conduction.

The intrinsic silicon has a >3 orders of magnitude change in carrier density from 250K to 350K. The germanium has a higher conductivity than the intrinsic Si, but the *change* in conductivity is less than two orders of magnitude from 250K to 350K. The intrinsic Si seems our best choice for this sensor at this temperature range.

2. Applied field E = V/L = $1/2\text{x}10^{-3}$ = 500 V/m

a) The conductivity of a semiconductor is given by;

$$\sigma = \mathrm{nq}\mu_e + \mathrm{pq}\mu_h = \mathrm{n_i q}(\mu_e + \mu_h)$$
$$\sigma = 2.5\text{x}10^{19} \text{ x } 1.6\text{x}10^{-19} \text{x} (0.39 + 0.19)$$
$$\sigma = 2.3\ \Omega^{-1}\mathrm{m}^{-1}$$

b) The electron current density is given by:

$$\mathrm{J} = \sigma\mathrm{E}$$
$$\text{so, inserting from above}$$
$$\mathrm{J} = \mathrm{n_i qE}(\mu_e + \mu_h)$$
$$\therefore \mathrm{J} = 2.5\text{x}10^{19}\text{x}1.6\text{x}10^{-19}\text{x}500\text{x}(0.39 + 0.19) = 1160\mathrm{A/m}^2$$

c) The total drift current can be split into an electron and hole component;

$$\mathrm{J_{Drift}} = J_{Drift}^{electron} + J_{Drift}^{hole}$$

$$\mathrm{J_{Drift}} = \mathrm{nqE}\mu_e + \mathrm{pqE}\mu_h$$

Since we have an intrinsic semiconductor -

$$\mathrm{n = p = n_i}$$

$$\mathrm{J_{Drift}} = \mathrm{n_i qE}(\mu_e + \mu_h)$$

The fraction of the drift current due to electrons is therefore given by

$$\frac{J_{Drift}^{electron}}{J_{Drift}} = \frac{n_i qE\mu_e}{n_i qE\left(\mu_e + \mu_h\right)}$$

So

$$\frac{J_{Drift}^{electron}}{J_{Drift}} = \frac{\mu_e}{\mu_e + \mu_h}$$

$$\frac{J_{Drift}^{electron}}{J_{Drift}} = \frac{0.39}{0.58} = 67\%$$

67% of the drift current is due to electrons in this case. Note that this is a special case where the electron and hole populations are equal. This result does not hold if these carrier densities are not equal e.g. in a doped semiconductor.

d) For the drift velocity we can examine electrons and holes separately;

For electrons

$$v_d = -\mu E = 0.39 \text{ x } 5x10^2 = 195ms^{-1}$$

For holes

$$v_d = -\mu E = 0.19 \text{ x } 5x10^2 = 95ms^{-1}$$

3 (a) From notes, built-in field $E_x = -\frac{D_e}{n\mu_e}\frac{dn}{dx}$

We can substitute for D $D_{e,h} = \frac{k_B T\mu_{e,h}}{q}$

and rearrange to give $E_x = -\frac{k_B T}{q}\frac{dn/dx}{n}$

We are given the information that n= N_d= N_0 exp(-ax). So dn/dx = -aN_0exp(-ax)

Substituting these $E_x = -\frac{k_B T}{q}\frac{dn/dx}{n} = -\frac{k_B T}{q}\frac{(-a)N_0\exp(-ax)}{N_0\exp(-ax)} = a\frac{k_B T}{q}$

(Note the need to choose this “exotic” doping profile to allow us to cancel the spatially varying term and give a constant electric field)

3 (b) Insert values into equation above – you need to define room temperature! I’ll use 300K

$$E_x = a\frac{k_B T}{q} = 1x10^6\frac{1.38x10^{-23}x300}{1.6x10^{-19}} = 2.59x10^4 \text{ Vm}^{-1}$$

2(c)