A.C. MACHINES – INDUCTION MOTOR

(74)

ENERGY USEAGE = 50 TWHr in U.K.

50×10^{12} Whr OF ENERGY.

IS CONSUMED BY INDUSTRY VIA INDUCTION MOTORS TO GIVE CONTROLLED ROTARY MOTION.

$\Rightarrow$ PUMPS, COMPRESSORS
PACKING AND PROCESS MACHINERY
PRINTING + ROLLING ACTIVITIES

EFFICIENCIES > 90%

N.B. A 0.5% IMPROVEMENT IN EFFICIENCY OF THE INDUCTION MOTOR WOULD SAVE

$$\frac{0.5}{100} \times 50 \times 10^{12} \text{ Whr OF ENERGY}$$

$$2.5 \times 10^{11} \text{ Whr} \qquad 1 \text{ kWhr}$$

$$2.5 \times 10^{8} \text{ KWhr} \Rightarrow \qquad \approx 3p$$

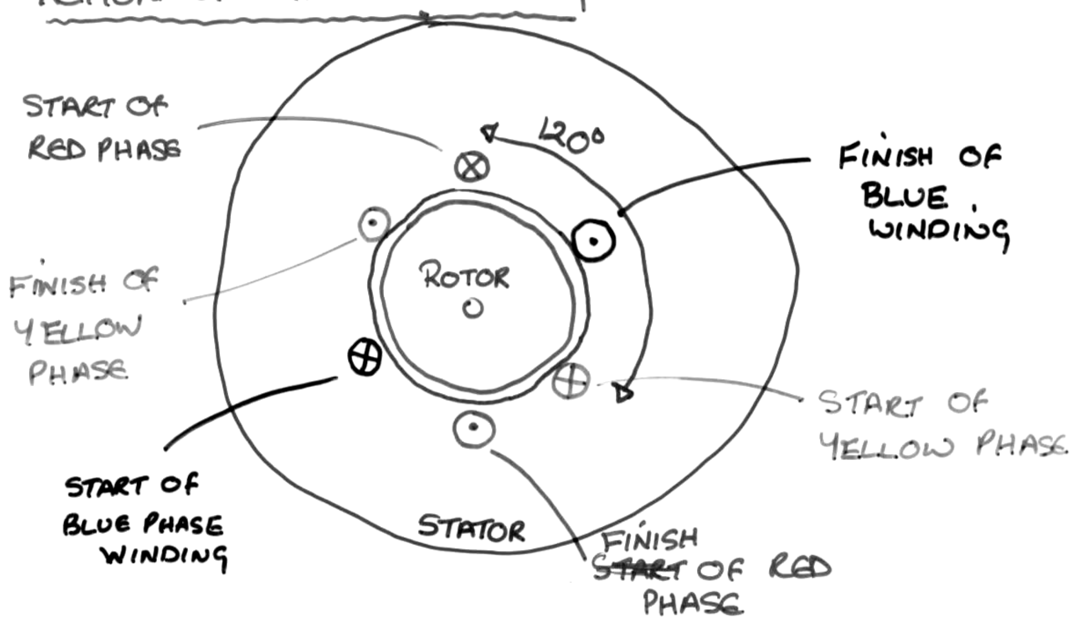
$$£7.5M$$

KEY COMPONENTS:

- SOFT IRON STATOR CONSISTING OF A STACK OF SLOTTED LAMINATIONS

- 3 PHASE WINDING DISTRIBUTED AMONG SLOTS WITHIN THE STATOR LAMINATIONS.
- SOFT IRON ROTOR, AGAIN CONSISTING OF A STACK OF SLOTTED LAMINATIONS
- DISTRIBUTED WINDING ON THE ROTOR WHICH IS SHORT CIRUITED. MAJORITY OF ROTORS ARE IN THE FORM OF A 'CAGE' WINDING WHERE THE ROTOR WINDING IS FORMED BY EXTRUDING ALUMINIUM INTO THE ROTOR SLOTS AND FORMING A SHORT CIRCUIT VIA END RINGS.

ACTION OF STATOR WINDING

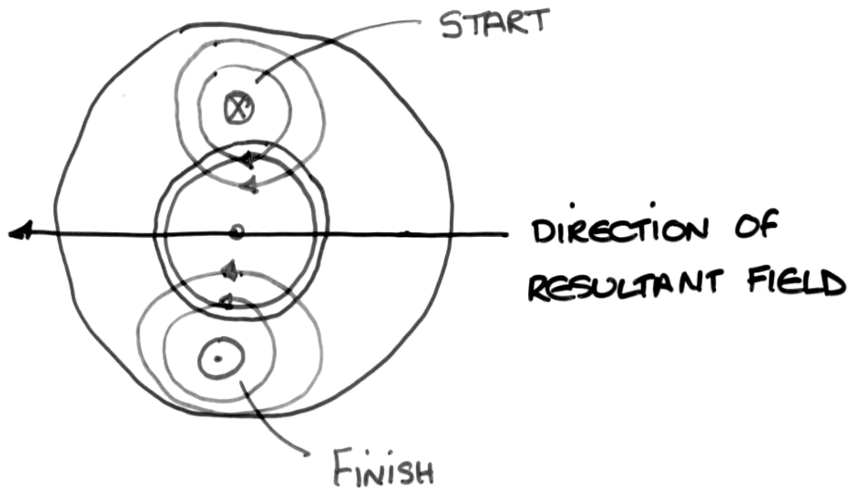


START OF WINDING IMPLIES THE CURRENT FLOW INTO THE WINDING WILL FLOW INTO THE PAPER, AND A +VE

CURRENT FLOW WILL FLOW OUT OF THE PAPER AT THE FINISH OF THE WINDING.

BASIC 2 POLE WINDING CONSISTS OF 3 IDENTICAL COILS - ONE FOR EACH ~~<S>~~ PHASE - DISPLACED 120° MECHANICAL w.r.t. EACH OTHER.

CONSIDER THE +VE CURRENT FLOW IN THE RED PHASE WINDING ONLY.



THE DIRECTION OF THE FIELD WILL BE 90° TO THE PLANE OF THE COIL.

$$B_r = K N I_r$$

FIELD STRENGTH

COIL AMPERE TURNS.

CONSTANT WHICH RELATES TO THE DIMENSIONS OF THE IRON CCT OF THE MACHINE

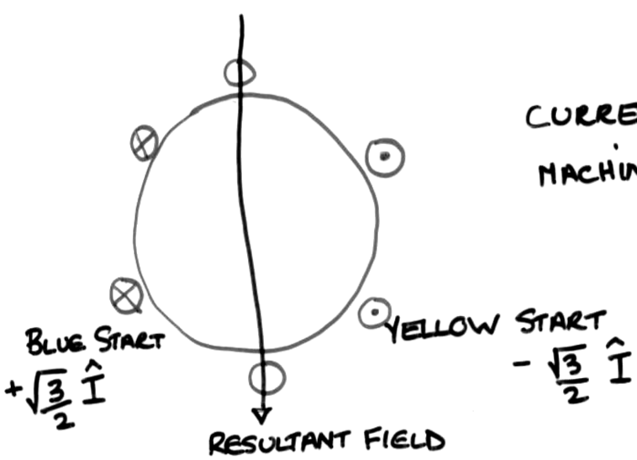
CONSIDER A 3 PHASE SUPPLY TO THE MACHINE SUCH THAT;

$$I_r = \hat{I} \sin \omega_1 t$$
$$I_y = \hat{I} \sin (\omega_1 t - 120^\circ)$$
$$I_b = \hat{I} \sin (\omega_1 t - 240^\circ)$$

ω_1 = STATOR SUPPLY FREQUENCY
$= 2\pi f_1$ (f_1 IN Hz)

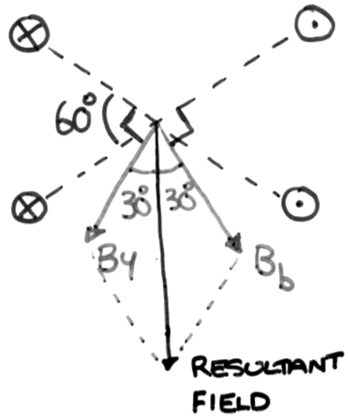
CONSIDER TIME $t = 0$

$$I_r = 0$$
$$I_y = \hat{I} \sin (-120^\circ) = -\frac{\sqrt{3}}{2} \hat{I}$$
$$I_b = \hat{I} \sin (-240^\circ) = +\frac{\sqrt{3}}{2} \hat{I}$$



CURRENT PATTERN IN MACHINE @ $t = 0$

TO FIND RESULTANT FIELD, CONSIDER THE TWO CONTRIBUTIONS OF Y + B WINDINGS, ADD THE VECTORS



I_y

$|B_y| = kN\left(\frac{\sqrt{3}}{2}\hat{I}\right)$

$|B_b| = kN\left(\frac{\sqrt{3}}{2}\hat{I}\right)$

I_b

RESULTANT FIELD STRENGTH $= |B_y|\cos 30^\circ + |B_b|\cos 30^\circ$

$= |B_y|\sqrt{3}/2 + |B_b|\sqrt{3}/2$

$= kN\left(\sqrt{3}/2\,\hat{I}\right)\sqrt{3}/2 + kN\left(\sqrt{3}/2\,\hat{I}\right)\sqrt{3}/2$

$= kN\hat{I}\,3/4 + kN\hat{I}\,3/4$

$= 3/2\,kN\hat{I}$

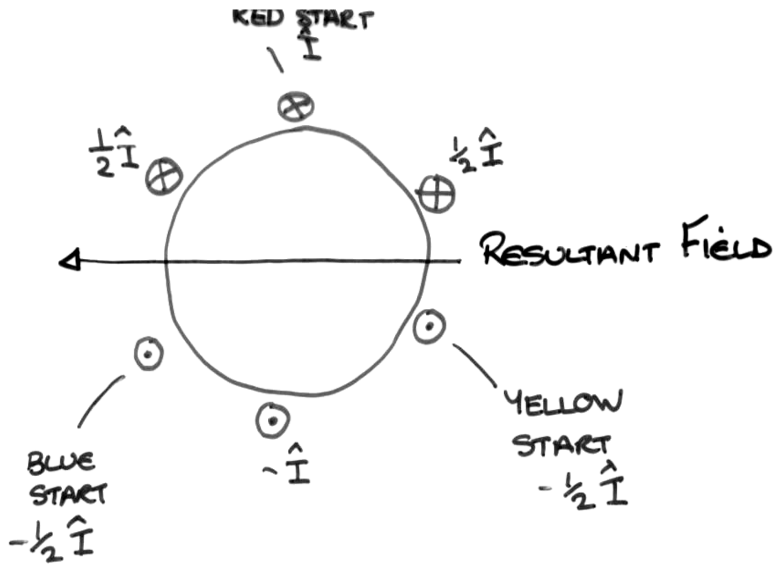
I.e AT $t=0$ WE HAVE A RESULTANT FIELD VERTICALLY DOWNWARDS WITH STRENGTH, $3/2\,kN\hat{I}$

CONSIDER $\omega, t = 90^\circ$ $\quad t = 90^\circ/\omega$

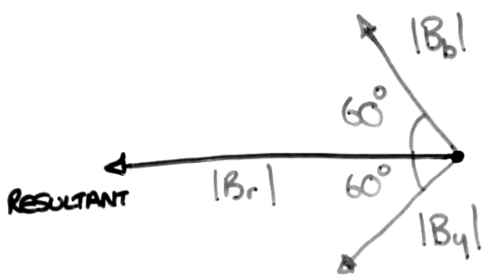
$I_r = \hat{I}\sin(90^\circ) = \hat{I}$

$I_y = \hat{I}\sin(90^\circ - 120^\circ) = -\frac{1}{2}\hat{I}$

$I_b = \hat{I}\sin(90^\circ - 240^\circ) = -\frac{1}{2}\hat{I}$



CONSIDER CONTRIBUTIONS FROM EACH OF THE RED, YELLOW AND BLUE COILS AS BEFORE



$$|B_r| = kN\hat{I}$$
$$|B_y| = kN\hat{I}/2$$
$$|B_b| = kN\hat{I}/2$$

<u>RESULTANT AMPLITUDE</u>

$$= |B_r| + |B_y|\cos 60^\circ + |B_b|\cos 60^\circ$$
$$= kN\hat{I} + kN\frac{\hat{I}}{2} \times \frac{1}{2} + kN\frac{\hat{I}}{2} \times \frac{1}{2}$$
$$= \frac{3}{2}kN\hat{I}$$

∴ WE CONCLUDE

- THE FIELD AMPLITUDE IS CONSTANT FOR A GIVEN MAGNITUDE OF SUPPLY CURRENT
- THE FIELD DIRECTION ROTATES AROUND THE MACHINE OVER THE SUPPLY CYCLE.

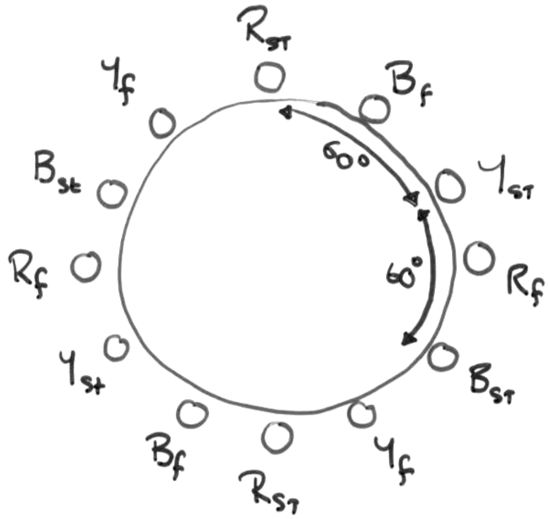
IN THE CASE OF A TWO POLE WINDING, THE SPEED OF ROTATION OF THE FIELD IS EQUAL TO THE SUPPLY FREQUENCY.

SYNCHRONOUS SPEED (N_s) ≡ FIELD SPEED OF ROTATION

2 POLE WINDING

$$
\begin{aligned}
N_s &= \omega_1 \ (\text{RADIANS } s^{-1}) \\
&= f_1 \ (\text{Hz}) \\
&= 60 f_1 \ (\text{rpm})
\end{aligned}
$$

4 POLE WINDING



FOR THE 4 POLE WINDING, SYNCHRONOUS SPEED IS

$$N_s = \frac{f_1 \times 60}{2} \text{ rpm.}$$

IN THE GENERAL CASE FOR A WINDING OF 'P' POLE PAIRS

$$N_s = \frac{60 f_i}{P} \text{ rpm}$$

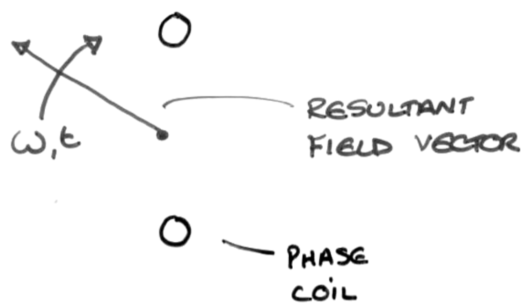
4 pole ⇒ 2 POLE PAIRS $P = 2$ (1500 rpm)

6 pole ⇒ 3 POLE PAIRS $P = 3$ (1000 rpm)

2 pole ⇒ 1 POLE PAIR $P = 1$ (3000 rpm)

IN PRACTICE A POLYPHASE WINDING WILL CONSIST OF MANY COIL SIDES DISTRIBUTED AROUND THE STATOR AIRGAP.

THIS IS DONE FOR REASONS OF A UNIFORM HEAT DISTRIBUTION IN THE MACHINE AND TO IMPROVE THE WAVEFORM OF THE MACHINE FIELD.



A PHASE COIL WILL 'SEE' AN ALTERNATING FLUX LINKAGE AS THE FIELD IN THE MACHINE ROTATES PAST IT

i.e. FLUX LINKAGE OF THE RED WINDING FOR EXAMPLE

$$\hat{\phi}_M N_1 \sin \omega_1 t$$

PEAK RESULTANT FLUX

NUMBER OF TURNS

VARIATION OF FLUX LINKAGE AT SUPPLY FREQUENCY.

HENCE, INDUCED VOLTAGE IN THE WINDING (FARADAY) OF MAGNITUDE

$$-N_i \frac{d\phi}{dt}$$

$$= -N_i \frac{d}{dt}\left(\hat{\phi}_M \sin \omega_1 t\right)$$

$$= -N_i \hat{\phi}_M \omega_1 \cos \omega_1 t$$

MAGNITUDE OF INDUCED VOLTAGE, PEAK

$$= N_i \hat{\phi}_M \omega_1 \qquad \omega_1 = 2\pi f_1$$

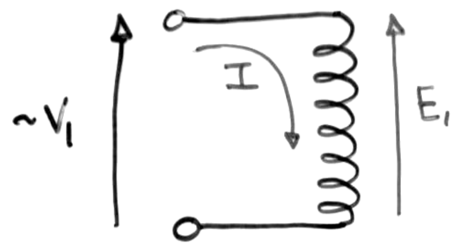
RMS VALUE OF THE INDUCED VOLTAGE

$$E_1 = \frac{\text{PEAK}}{\sqrt{2}} = \frac{N_1 \hat{\phi}_M \omega_1}{\sqrt{2}}$$

$$= \frac{N_1 \hat{\phi}_M 2\pi f_1}{\sqrt{2}}$$

$$E_1 = \underbrace{\sqrt{2}\,\pi}_{4\cdot 44}\ \underbrace{f_1}_{\text{SUPPLY FREQUENCY}}\ \underbrace{N_1}_{\text{WINDING TERMS}}\ \underbrace{\hat{\phi}_M}_{\text{PEAK FLUX IN MACHINE}}$$

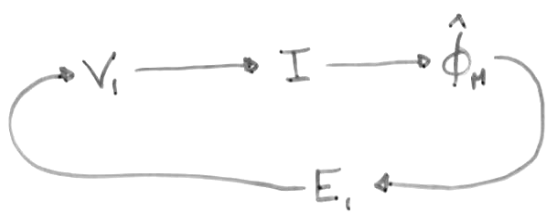
CONSIDER THE CONNECTION OF THE STATOR WINDING TO A 3 PHASE VOLTAGE SUPPLY, OF MAGNITUDE V_1 (RMS / PHASE)



NEGLECTING THE RESISTANCE OF THE STATOR WINDING, IT FOLLOWS THAT,

$$V_1 \equiv E_1$$

THE MACHINE THEREFORE CAN BE VIEWED TO OPERATE IN A 'CLOSED LOOP' SITUATION



V_1 CAUSES CURRENT I TO FLOW

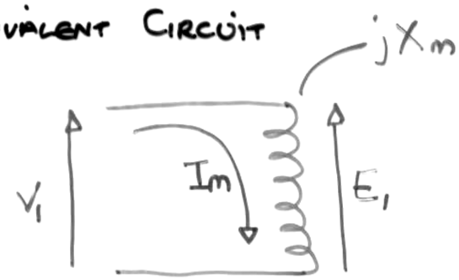
I RESULTS IN A FIELD $\hat{\phi}_M$ WITHIN THE MACHINE

$\hat{\phi}_M$ CAUSES AN INDUCED VOLTAGE E_1 AND CCT DICTATES THAT $V_1 \equiv E_1$ (NEGLECTING WINDING RESISTANCE)

THEREFORE IT IS THE SUPPLY VOLTAGE WHICH DEFINES THE MAGNITUDE OF THE FIELD IN THE MACHINE

$$V_1 \equiv E_1 = 4{\cdot}44\, f_1 N_1 \hat{\phi}_M$$

EQUIVALENT CIRCUIT



THIS EFFECT IS EQIVALENT TO AN INDUCTANCE

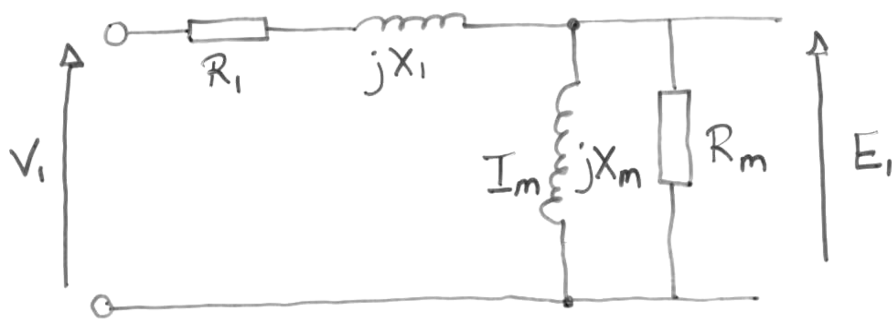
WHERE $X_m = 2\pi f_1 L_m = E_1 / I_m$

X_m = MAGNETIZING REACTANCE

L_m = MAGNETIZING INDUCTANCE

I_m = MAGNETIZING CURRENT WHICH IS THE CURRENT REQUIRED TO ESTABLISH THE FIELD WITHIN THE MACHINE

STATOR PER-PHASE EQUIVALENT CIRCUIT.



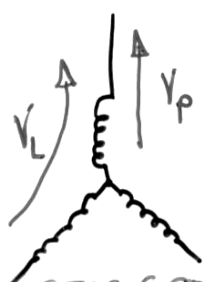
R_1 = STATOR WINDING RESISTANCE PER PHASE

X_1 = STATOR LEAKAGE REACTANCE WHICH MODELS LEAKAGE COMPONENTS OF FIELD, i.e. THE COMPONENTS OF THE STATOR FIELD THAT DO NOT CROSS THE AIRGAP AND LINK THE ROTOR. e.g. FIELDS CREATED BY ENDWINDINGS

X_m = MAGNETIZING REACTANCE WHICH MODELS THE USEFUL COMPONENT OF FIELD WITHIN THE MACHINE

R_m = IRON LOSSES IN THE MACHINE, EDDY CURRENT AND HYSTERESIS LOSS.

<u>PER</u> PHASE i.e. V, E_1, I_m ARE PHASE QUANTITIES (RMS)



STAR CONNECTION

$$V_p \text{ (PHASE)} = \frac{V_L \text{ (LINE)}}{\sqrt{3}}$$

$$I_P = I_L$$

DELTA CONNECTION

$$V_p = V_L$$
$$I_P = I_L / \sqrt{3}$$

TOTAL POWER DISSIPATED ON OUTPUT $\equiv 3\times$ POWER/PHASE

TORQUE PRODUCTION + SLIP

- THE ROTOR CONSISTS OF A POLYPHASE WINDING WHICH IS SHORT CIRCUITED.
- THE STATOR, WHEN SUPPLIED FROM A 3ϕ SUPPLY AT FREQUENCY f_1 CREATES A ROTATING FIELD WITH SPEED N_S (SYNCHRONOUS SPEED)

$$N_S = \frac{60 f_1}{P} \quad (rpm)$$

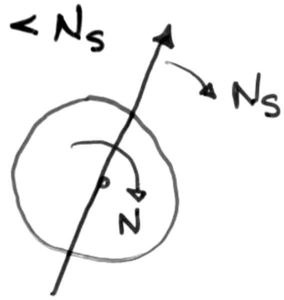
CONSIDER THE ROTOR INITIALLY AT REST. THE STATOR FIELD WILL PASS THE ROTOR AT SPEED N_S, THEREFORE A VOLTAGE WILL BE INDUCED IN THE ROTOR WINDINGS, AND THIS WILL HAVE A FREQUENCY OF F_1

HENCE, A CURRENT WILL BE INDUCED IN THE ROTOR WINDING THROUGH THE SHORT CIRCUIT.

THE INTERACTION BETWEEN THE STATOR FIELD AND THE INDUCED ROTOR CURRENTS RESULTS IN A TORQUE.

NOW CONSIDER A ROTOR SPEED N rpm

$N < N_S$



N = ROTOR SPEED.

TO AN OBSERVER ON THE ROTOR, THE STATOR FIELD WILL PASS AT A SPEED $N_S - N$.

DEFINE SLIP $s = \frac{N_S - N}{N_S}$

THE INDUCED VOLTAGES AND CURRENTS IN THE ROTOR WILL NOW OCCUR AT A LOWER FREQUENCY

$$f_2 = s f_1$$

SLIP × SUPPLY FREQUENCY

ROTOR FREQUENCY OF INDUCED VOLTS / CURRENTS

SHOULD THE ROTOR REACH SYNCHRONOUS SPEED N_S, THEN THERE WILL BE NO RELATIVE MOTION BETWEEN THE ROTOR AND THE STATOR FIELD. I.E THE ROTOR WINDING WILL SEE NO CHANGE IN FLUX LINKAGE, THEREFORE THERE IS NO LONGER AN INDUCED VOLTAGE, ZERO CURRENT + HENCE ZERO TORQUE

THEREFORE, IN PRACTICE THE ROTOR WILL ALWAYS OPERATE AT A SPEED LOWER THAN N.S i.e WILL ALWAYS SLIP W.R.T. SYNCHRONOUS SPEED OF THE STATOR FIELD, AS THIS WILL LEAD TO AN INDUCED ROTOR VOLTAGE, A CURRENT THROUGH THE SHORT CIRCUITED WINDING + TORQUE.

FROM NEWTON'S LAW THE TORQUE PRODUCED BY THE STATOR ROTATING FIELD MUST BE IDENTICAL TO THE TORQUE WHICH APPEARS ON THE ROTOR.

HOWEVER, SINCE THERE IS A DIFFERENCE IN SPEED BETWEEN STATOR + ROTOR (SLIP) THERE MUST BE A POWER LOSS ACROSS THE INTERFACE WHICH MANIFESTS ITSELF AS OHMIC I^2R LOSS IN THE ROTOR.

- TO OBTAIN TORQUE, WE NEED SLIP, ELSE THERE WILL BE NO INDUCED VOLTS OR CURRENT ON THE ROTOR.

- SLIP IMPLIES LOSS WHICH MANIFESTS ITSELF AS ROTOR I^2R LOSS.

CONSIDER THE POWER BALANCE IN THE STEADY - STATE BETWEEN THE STATOR + ROTOR.

STATOR DELIVERS POWER

$$= \underbrace{\left(\frac{2\pi N_s}{60}\right)}_{\text{ANGULAR SPEED OF SYNCHRONOUS TORQUE}} \underbrace{T}_{\text{TORQUE}}$$

N_s in rpm

MECHANICAL OUTPUT FROM THE ROTOR

$$= \underbrace{\left(\frac{2\pi N}{60}\right)}_{\text{ANGULAR SPEED OF ROTOR}} T$$

N in rpm

ROTOR LOSS = DIFFERENCE BETWEEN THESE TWO.

$$= \left(\frac{2\pi N_s}{60}\right) T - \left(\frac{2\pi N}{60}\right) T$$

$$= \frac{2\pi N_s}{60} \frac{(N_s - N)}{N_s} T$$

$$S = \frac{N_s - N}{N_s}$$

$$= \left(\frac{2\pi N_s}{60} T\right) \times S$$

= POWER DELIVERED BY STATOR × SLIP

MECHANICAL OUTPUT OF ROTOR

= POWER DELIVERED BY STATOR $\times (1-S)$

SINCE EFFICIENCY = $\frac{\text{OUTPUT}}{\text{INPUT}}$ POWER

THE EFFICIENCY OF THE ELECTROMECHANICAL CONVERSION PROCESS

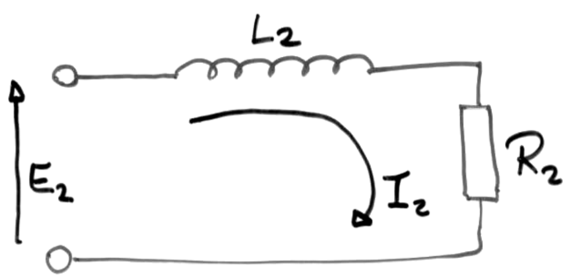
$$= (1-s) \times 100\%$$

TYPICALLY AN INDUSTRIAL INDUCTION MOTOR WILL OPERATE AT SLIPS OF $s = 0{\cdot}01$ TO $0{\cdot}04$

ROTOR EQUIVALENT CIRCUIT

SHORT CIRCUITED WINDING

PER-PHASE EQUIVALENT



R_2 = ROTOR RESISTANCE

L_2 = ROTOR LEAKAGE INDUCTANCE

(REPRESENTS FIELDS INDUCED BY THE ROTOR CURRENT THAT DO NOT COUPLE THE STATOR)

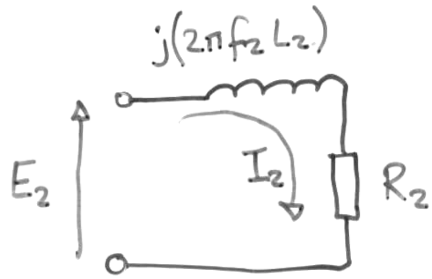
INDUCED VOLTAGE E_2 AND CURRENT I_2 AND A ROTOR FREQUENCY f_2

$$f_2 = s f_1$$

RECALL: $E_1 = \sqrt{2}\pi N_1 f_1 \hat{\Phi}_M$ PEAK MAGNETIZING FLUX

$$E_2 = \sqrt{2}\pi N_2 f_2 \hat{\Phi}_m$$

N_2 = ROTOR EQUIVALENT TURNS PER PHASE



$$I_2 = \frac{E_2}{\sqrt{(2\pi f_2 L_2)^2 + R_2^2}}$$

HENCE ROTOR LOSS PER PHASE = $I_2^2 R_2$

$$= \frac{E_2^2 R_2}{(2\pi f_2 L_2)^2 + R_2^2}$$

MECHANICAL OUTPUT POWER = $(1 - s)$ × STATOR POWER

ROTOR LOSS = s × STATOR POWER

$$\text{Rotor loss} = s \times \left(\frac{2\pi N_s}{60}\right) T$$

$$T = \frac{\text{Rotor Loss}}{s\left(\frac{2\pi N_s}{60}\right)}$$

$$= \frac{3 I_2^2 R_2}{s\left(\frac{2\pi N_s}{60}\right)}$$

$$= \frac{3}{s\left(\frac{2\pi N_s}{60}\right)} \cdot \frac{E_2^2 R_2}{(2\pi f_2 L_2)^2 + R_2^2}$$

$$N_s = \frac{60 f_1}{p}, \qquad E_2 = \sqrt{2}\pi N_2 f_2 \hat{\phi}_M$$

$$T = \frac{3}{s\left(\frac{\cancel{2}\pi f_1}{p}\right)} \cdot \frac{\cancel{2}\pi^{\cancel{2}} N_2^2 f_2^2 \hat{\phi}_M^2 R_2}{(2\pi f_2 L_2)^2 + R_2^2}$$

$$f_2 = s f_1$$

$$T = \frac{3p}{\cancel{f_2}} \cdot \frac{\pi N_2^2 f_2^{\cancel{2}} \hat{\phi}_M^2 R_2}{(2\pi f_2 L_2)^2 + R_2^2}$$

$$\boxed{T = 3p\pi N_2^2 \frac{f_2 \hat{\phi}_M^2 R_2}{(2\pi f_2 L_2)^2 + R_2^2}}$$

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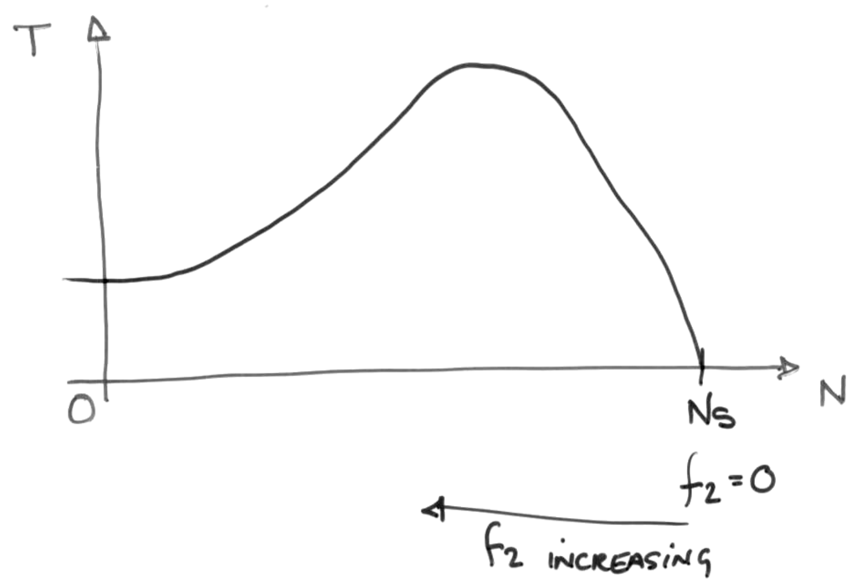
UNDER NORMAL OPERATION FROM A CONSTANT VOLTAGE AND FREQUENCY SUPPLY TO THE STATOR, $\hat{\phi}_m$ IS DEFINED ALSO, SINCE p, N_2, R_2, L_2 ARE MACHINE SPECIFIC.

$T =$ FUNCTION OF f_2 ALONE

$$T \propto \frac{f_2}{(2\pi f_2 L_2)^2 + R_2^2}$$



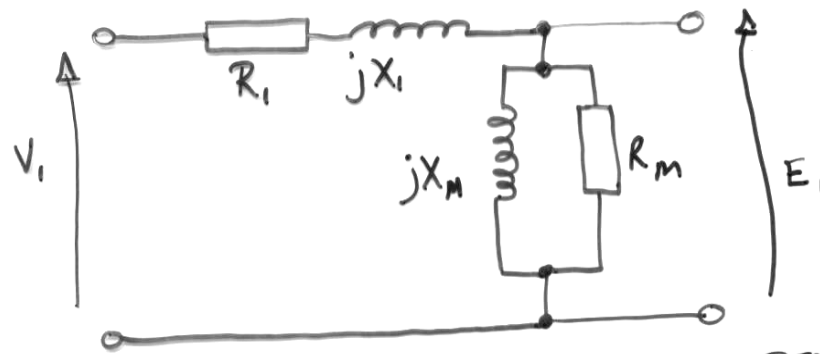
$f_2 = s f_1$

ALSO ROTOR SPEED

$$N = (1-s)N_s$$
$$= \cancel{N_s}\ (1-s)\left(\frac{60 f_1}{p}\right)$$

$$N = N_s - \frac{s\,60 f_1}{p} = N_s - \frac{60 f_2}{p}$$

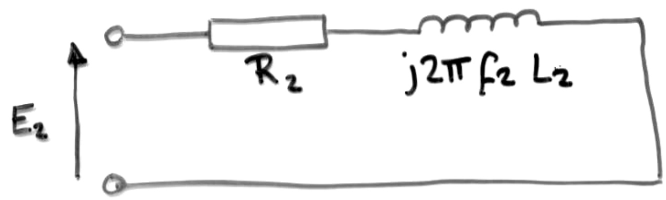
CLASSICAL INDUCTION MOTOR EQUIVALENT CCT.



STATOR EQUIVALENT CCT.

- PER PHASE
- EVALUATED AT SUPPLY FREQUENCY f_1 (50Hz)

$$E_1 = \sqrt{2}\,\pi\, N_1 f_1 \hat{\phi}_m$$



ROTOR EQUIVALENT CCT.

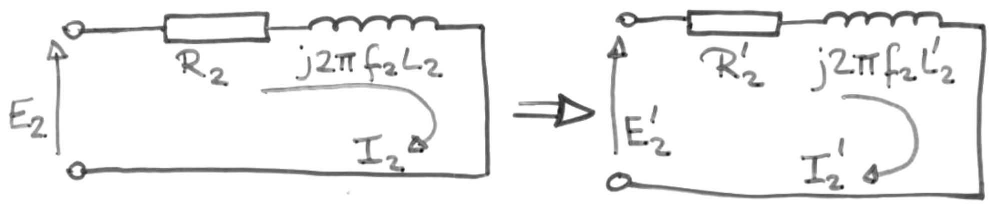
- PER PHASE
- CALCULATED AT INDUCED ROTOR FREQUENCY

$$f_2 = s f_1$$

$$E_2 = \sqrt{2}\,\pi\, N_2 f_2 \hat{\phi}_M$$

TO REFER THE ROTOR EQUIVALENT CCT. TO THE STATOR $E_2 \Rightarrow E_1$ REQUIRES TWO OPERATIONS

a) ADJUST THE ROTOR TURNS FROM N_2 (THE ACTUAL VALUE) TO N_1 (EQUIVALENT STATOR TURNS)



THE REFERRED VALUE OF ROTOR INDUCED VOLTAGE

$$E_2' = \frac{N'}{N^2} E_2 \qquad (1)$$

A 'CONSTANT POWER' REFERRAL IS IMPOSED

i.e. LOSS BEFORE = LOSS AFTER REFERRAL

$$I_2^2 R_2 = I_2'^2 R_2' \qquad (3)$$

SIMILARLY $E_2 I_2 = E_2' I_2'$ (2)

FROM (2) AND (1)

$$I_2' = \frac{N_2}{N_1} I_2$$

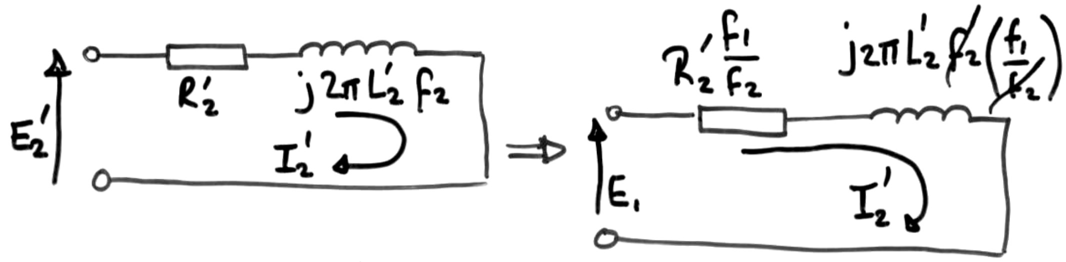
FROM (3) AND ABOVE

$$R_2' = R_2 \left(\frac{N_1}{N_2}\right)^2$$

AND ALSO $L_2' = L_2 \left(\frac{N_1}{N_2}\right)^2$

NOW $E_2' = \frac{N_1}{N_2} E_2 = \frac{N_1}{\cancel{N_2}} \sqrt{2}\pi \cancel{N_2} f_2 \hat{\phi}_M$

b) CHANGE THE ROTOR FREQUENCY FROM f_2 TO f_1

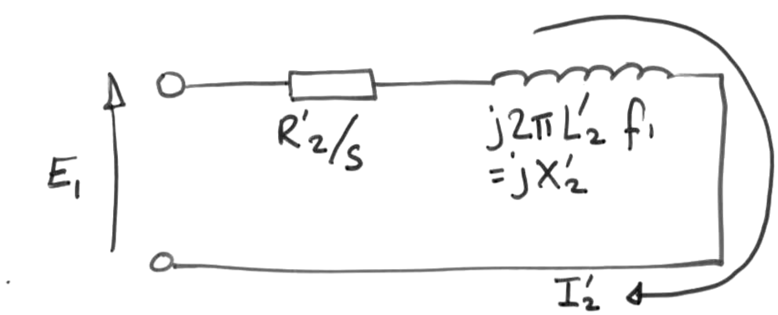


$E_2' = \sqrt{2}\pi N_1 f_2 \hat{\phi}_M$ $\qquad\qquad$ $E_1 = \sqrt{2}\pi N_1 f_1 \hat{\phi}_M$

THE REFERRED ROTOR EQUIVALENT CIRCUIT ON L.H.S. IS MULTIPLIED THROUGH BY f_1/f_2

$$\frac{f_1}{f_2} E_2' = E_1$$

$$\frac{f_1}{f_2} = \frac{1}{S} \qquad f_2 = sf_1$$



(8)

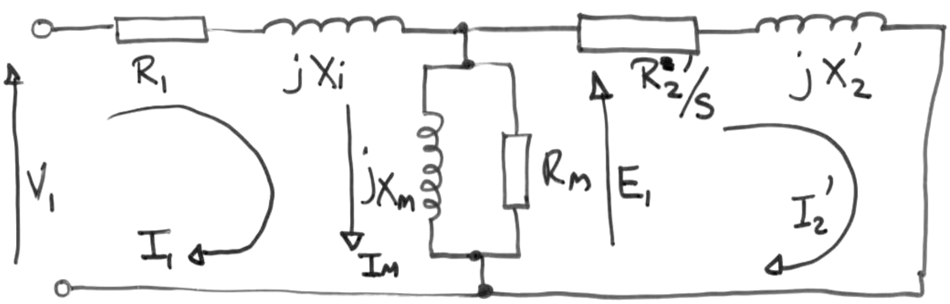
NOTE/ RESISTANCE IS NOW R_2'/s AND LEAKAGE REACTANCE IS THE EQUIVALENT REACTANCE OF L_2' EVALUATED AT THE STATOR SUPPLY FREQUENCY.

$$X_2' = 2\pi f_1 L_2'$$

NOTE NEW POWER LOSS IN ROTOR CIRCUIT

$$= I_2'^2 \frac{R_2'}{s} = \underbrace{I_2'^2 R_2'}_{\text{COPPER LOSS IN ROTOR PER PHASE}} + \underbrace{I_2'^2 R_2 \frac{(1-s)}{s}}_{\text{MECHANICAL OUTPUT FROM MACHINE PER PHASE}}$$

FULL STATOR EQUIVALENT CCT.



R_1 = STATOR RESISTANCE PER PHASE

R_2' = REFERRED ROTOR RESISTANCE /ϕ

X_1 = STATOR LEAKAGE REACTANCE /ϕ

X_2' = REFERRED ROTOR LEAKAGE REACTANCE /ϕ

X_m = MAGNETIZING REACTANCE /ϕ

R_m = IRON LOSS RESISTANCE / ϕ

V_1 = RMS SUPPLY PHASE VOLTAGE / ϕ

E_1 = INDUCED STATOR PHASE VOLTAGE

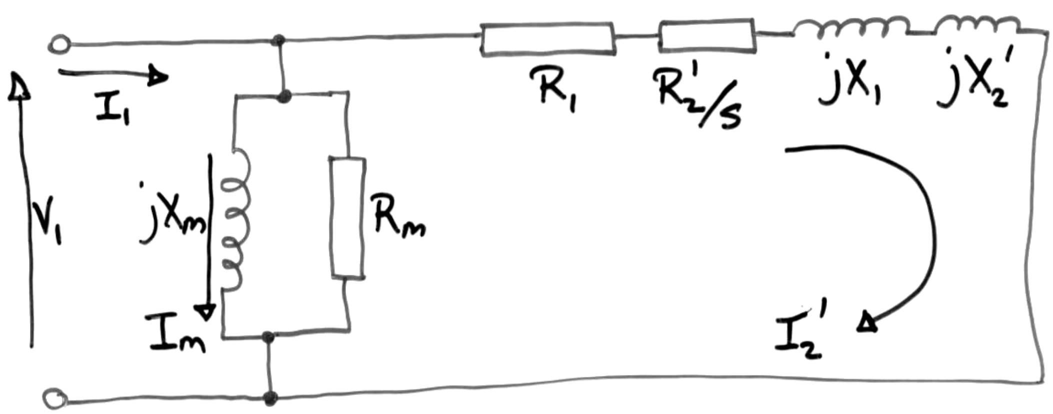
I_2' = REFERRED ROTOR CURRENT

I_m = MAGNETIZING CURRENT

I_1 = STATOR CURRENT.

APPROXIMATE EQUIVALENT CIRCUIT

ASSUME $E_1 = V_1$ ACCURACY 1-2%
FOR A TYPICAL MACHINE



MUCH SIMPLER TO ANALYSE, AS MAGNETIZING BRANCH CAN BE CONSIDERED SEPARATELY TO THE ROTOR BRANCH.

METHOD OF DETERMINING OUTPUT TORQUE

$$\text{OUTPUT POWER PER PHASE} = I_2'^2 R_2' \frac{(1-s)}{s}$$

$$\text{TOTAL OUTPUT POWER} = \underset{\text{3 PHASES}}{3} I_2'^2 R_2' \frac{\cancel{(1-s)}}{s}$$

$$= \left(\frac{2\pi N}{60}\right) T = \left(\frac{2\pi N_s}{60}\right) \cancel{(1-s)}\, T$$

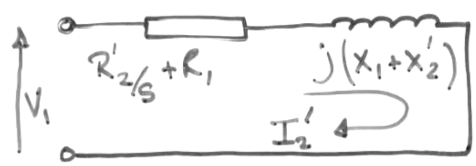
$$N = (1-s) N_s$$

$$T = \left(\frac{60}{2\pi N_s}\right) 3 I_2'^2 \frac{R_2'}{s}$$

$$\frac{1}{\text{ANGULAR SYNCHRONOUS SPEED}}$$

$$\frac{2\pi N_s}{60} = \frac{2\pi f_1}{P}$$

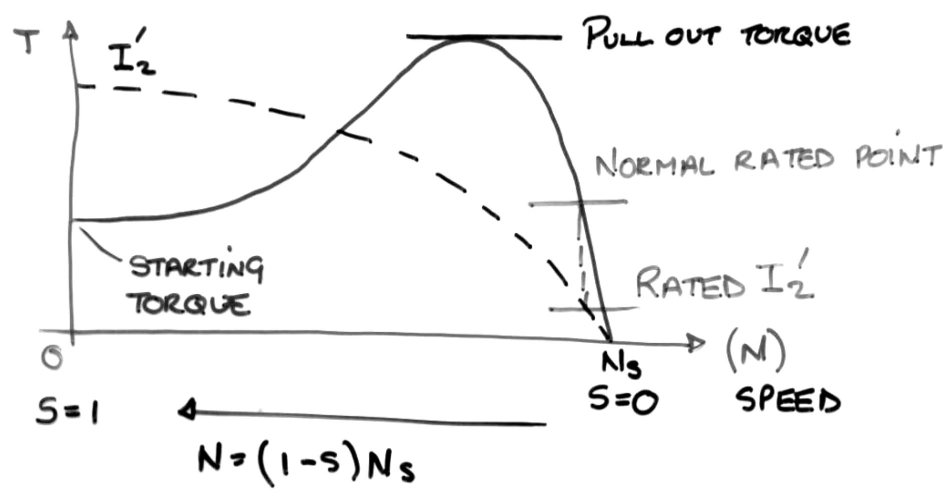
$$T = \frac{3P}{2\pi f_1} \quad I_2'^2 \frac{R_2'}{s}$$



$$I_2' = \frac{V_1}{\sqrt{\left(\frac{R_2'}{s} + R_1\right)^2 + (X_1 + X_2')^2}}$$

$$T = \frac{3P}{2\pi f_1} \frac{V_1^2 \; R_2'/s}{\left(R_2'/s + R_1\right)^2 + \left(X_1 + X_2'\right)^2}$$

N.B. $p, f_1, V_1, R_1, R_2', X_1, X_2'$ ARE CONSTANT FOR A GIVEN MACHINE. T IS A FUNCTION OF $1/S$ ALONE.



NB STARTING $I_2' \simeq 5\times$ RATED VALUE. IF LOAD TORQUE IS INCREASED BEYOND THE PULL-OUT TORQE, MACHINE WILL STALL.

TYPICALLY PULL OUT TORQUE 3 → 5 × FULL LOAD RATED TORQUE

a) STARTING TORQUE $S = 1$

$$T_{START} = \frac{3PV_1^2}{2\pi f_1} \frac{R_2'}{(R_1 + R_2')^2 + (X_1 + X_2')^2}$$

NOTE: STARTING TORQUE WILL INCREASE WITH ROTOR RESISTANCE R_2'

'02

b) PEAK PULL-OUT TORQUE

$$T = \frac{3pV_1^2}{2\pi f_1} \frac{R_2'/s}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2}$$

$$= \frac{3pV_1^2}{2\pi f_1} \frac{R_2'}{\left[s(R_1 + R_2'/s)^2 + s(X_1 + X_2')^2\right]}$$

TORQUE WILL BE A MAXIMUM WHEN $\min\left[s(R_1 + R_2'/s)^2 + s(X_1 + X_2')^2\right]$

i.e. $\frac{\partial}{\partial s} = 0 \Rightarrow$ MINIMUM

$$s(R_1^2 + 2R_1R_2'/s + R_2'^2/s^2) + s(X_1 + X_2')^2$$

$$\frac{\partial}{\partial s}\left(sR_1^2 + 2R_1R_2' + R_2'/s + R_2'^2/s + s(X_1 + X_2')^2\right)$$

$$R_1^2 + 0 - R_2'^2/s^2 + (X_1 + X_2')^2 = 0$$

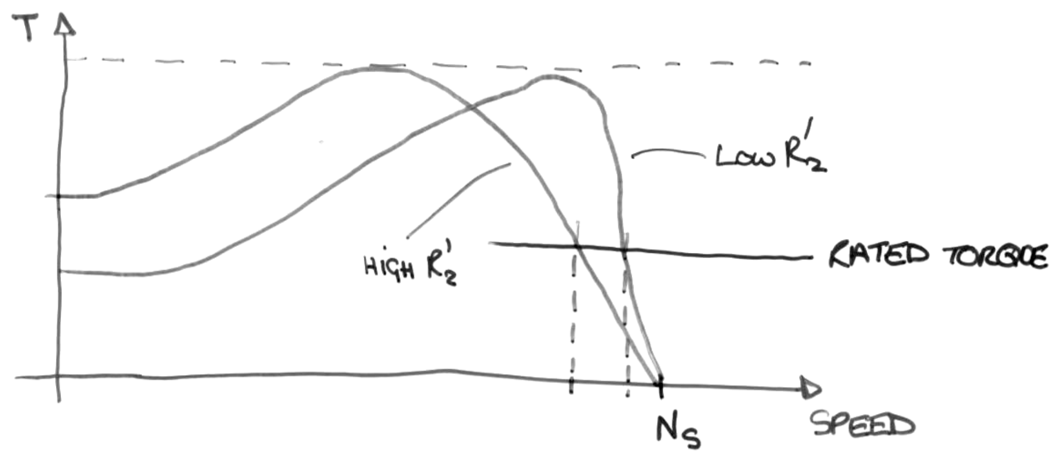
$$R_2'^2/s^2 = R_1^2 + (X_1 + X_2')^2$$

$$R_2'/s = \sqrt{R_1^2 + (X_1 \times X_2')^2}$$

SUBS IN TORQUE EXPRESSION

$$T_{PULL-OUT} = \frac{3pV_1^2}{2\pi f_1} \frac{\sqrt{R_1^2 + (X_1 + X_2')^2}}{\left(R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2}\right)^2 + (X_1 + X_2')^2}$$

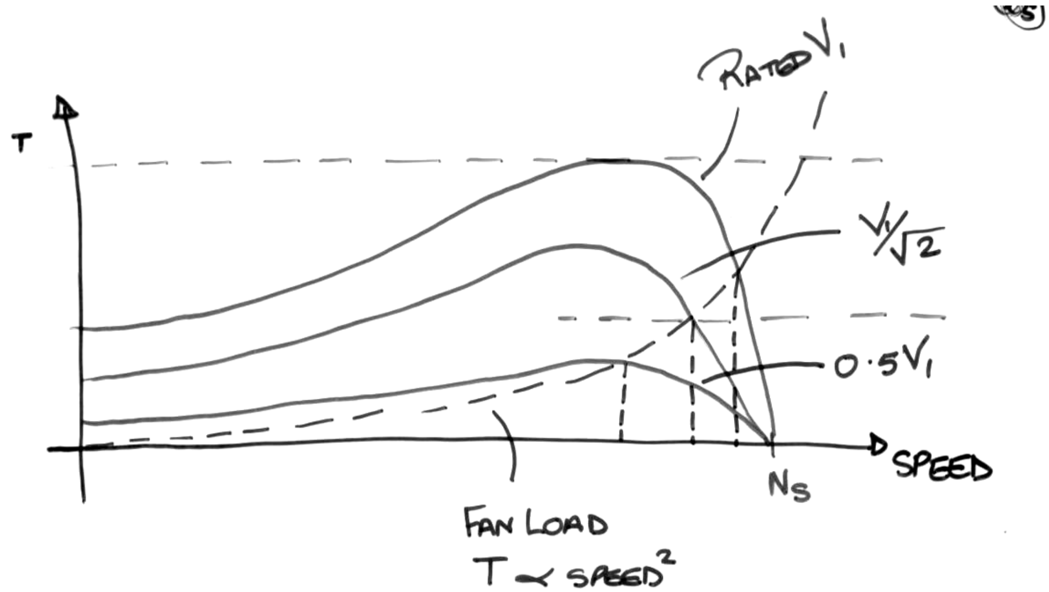
NOTE// $T_{PULL-OUT}$ IS INDEPENDENT OF R_2', THE ROTOR RESISTANCE. ALSO, FOR FIXED R_1, X_1 + X_2' THE VALUE OF SLIP S AT WHICH THE PULL-OUT TORQUE OCCURS $\propto R_2'$ SINCE R_2'/S IS FIXED



A POSSIBLE METHOD OF CONTROLLING THE SPEED OF AN INDUCTION MOTOR IS TO INCREASE THE ROTOR RESISTANCE

$\Rightarrow$ ACCESS TO ROTOR WINDING I.E. WOUND ROTOR MACHINE WITH SLIP RINGS (EXPENSIVE)

$\Rightarrow$ INEFFICIENT SINCE THIS LEADS TO LOSSES



SUITED TO SPEED CONTROL OF A FAN OR COMPRESSOR WHERE $T \propto SPEED^2$

NOT SUITED TO A CONSTANT TORQUE LOAD SINCE PEAK TORQUE MAY FALL BELOW THE LOAD TORQUE.

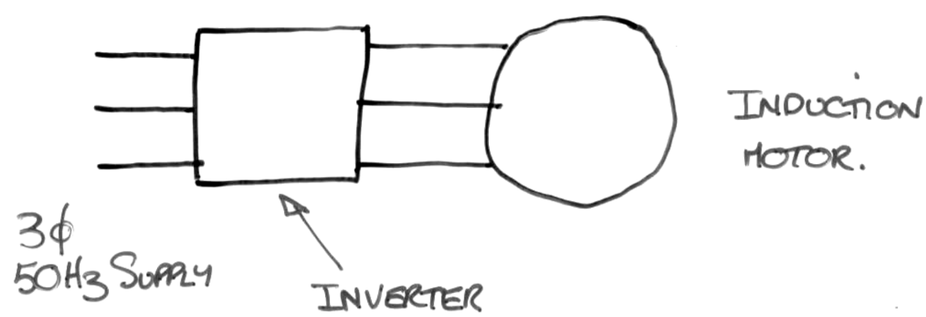
B) VARY THE SYNCHRONOUS SPEED OF THE MACHINE

1) CHANGE THE WINDING NUMBER OF POLES

2 POLE	~~RE~~ $P = 1$	N_S = 3000 RPM
4 POLE	$P = 2$	= 1500 RPM
6 POLE	$P = 3$	= 1000 RPM
8 POLE	$P = 4$	= 750 RPM.

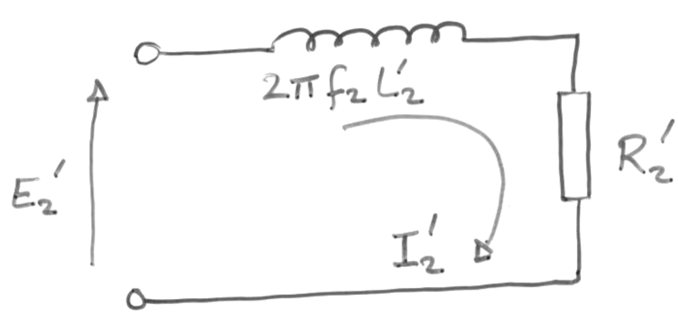
GIVES DISCRETE SPEED CHANGES BUT EXPENSIVE.

VARIABLE FREQ. CONTROL



A VARIABLE V_1, f_1 SUPPLY IS CREATED USING POWER ELECTRONICS. INVERTERS ARE EXPENSIVE $\propto$ 2-3x THE COST OF THE MOTOR ITSELF.

MACHINE IS OPERATED AT RATED FLUX $\hat{\phi}_M$ WHICH GIVES BEST UTILISATION OF THE MACHINE, BEST TORQUE/AMP.



$$E_2' = \sqrt{2}\pi N_2 f_2 \hat{\phi}_M$$

$$\& \; E_1 = \sqrt{2}\pi N_1 f_1 \hat{\phi}_M$$

$$I_2' = \frac{E_2'}{\sqrt{(2\pi f_2 L_2')^2 + (R_2'^2)}}$$

IF $\hat{\phi}_M$ TO BE KEPT CONSTANT

$$\frac{E_1}{f_1} = \text{CONST.}$$

$$\text{ROTOR LOSS / PHASE} = I_2'^2 R_2'$$

$$= \frac{E_2'^2 R_2'}{(2\pi f_2 L_2')^2 + R_2'^2}$$

RECAP

$$\text{OUTPUT POWER} = (1-s) \text{ INPUT POWER}$$

$$= \frac{(1-s)}{s} \text{ ROTOR LOSS}$$

$$\text{ROTOR LOSS} = s \times \text{INPUT POWER}_{\text{ROTOR}}$$

$$\text{OUTPUT POWER} = \frac{2\pi N'}{60} T \qquad N' = (1-s)N_s$$

$$= \frac{2\pi N_s}{60}(1-s)\, T$$

$$= \left(\frac{1-s}{s}\right) \text{ ROTOR LOSS.}$$

$$T = \frac{1}{\left[\frac{2\pi N_s}{60}\right]} \times \frac{1}{s} \times \text{ROTOR LOSS}$$

$$\frac{N_s}{60} = \frac{f_1}{p}, \quad f_2 = s f_1$$

$$T = \frac{P}{2\pi f_2} \times \text{ROTOR LOSS}$$

$$\text{ROTOR LOSS} = 3 \times I_2'^2 R_2'$$

$$= \frac{3 \times E_2'^2 R_2'}{(2\pi f_2 L_2')^2 + R_2'^2}$$

$$E_2' = \sqrt{2}\pi N_1 f_2 \hat{\phi}_m$$

$$T = 3\pi p N_1^2 (\hat{\phi}_m)^2 \frac{f_2 R_2'}{(2\pi f_2 L_2')^2 + R_2'^2}$$

UNDER VARIABLE FREQUENCY CONTROL

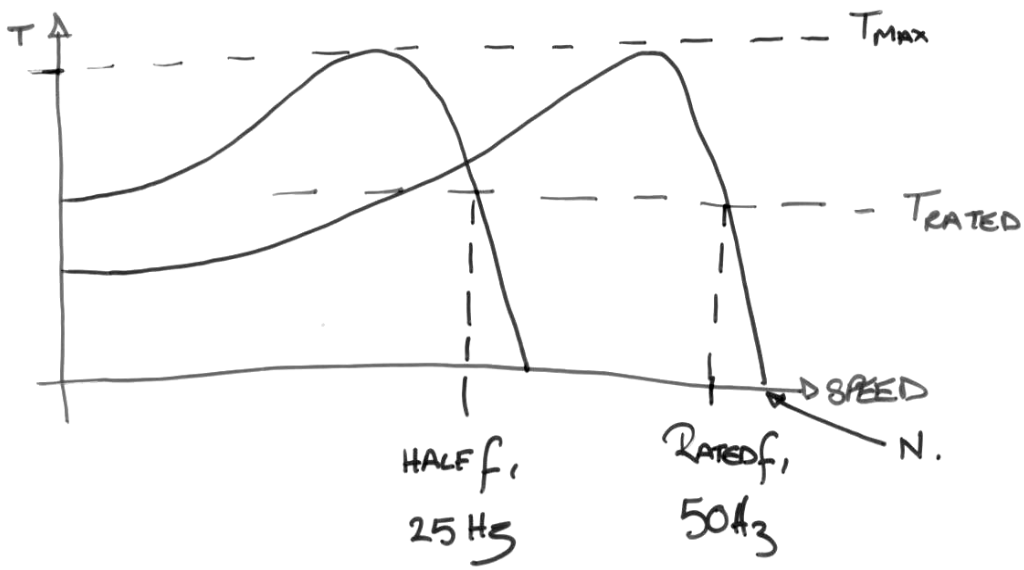
$\hat{\phi}_m$ IS KEPT CONSTANT

I.E FOR A GIVEN VALUE OF TORQUE

f_2 WILL BE THE SAME

I_2' WILL BE THE SAME.

f_1 IS VARIED, V_1 IS VARIED TO KEEP ϕ_M CONSTANT, AS $N_s = \frac{f_1 60}{P}$ RPM



<u>Ex</u> 4 POLE MACHINE HAS A RATED SPEED OF 1470 RPM.

@ 50 Hz $N_s = 1500$ RPM

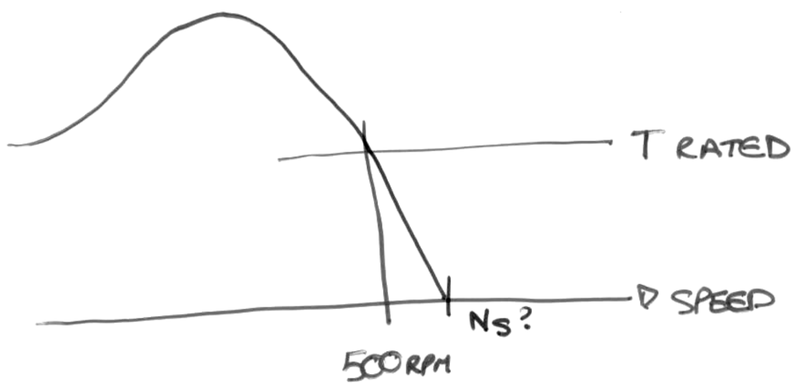
$$S \text{ @ } 50\,Hz = \frac{1500 - 1470}{1500}$$

$$= 0.02$$

$$\text{RATED } f_2 = 0.02 \times 50 = 1\,Hz$$

(110)

? VALUE OF SUPPLY FREQUENCY TO GIVE RATED TORQUE AT 500 RPM



f_2 IS UNCHANGED SINCE $\hat{\phi}_M$ IS CONST.

I.E. f_2 REMAINS AT 1 Hz

FOR A 4 POLE MACHINE, 1 Hz $\rightarrow$ 30 RPM

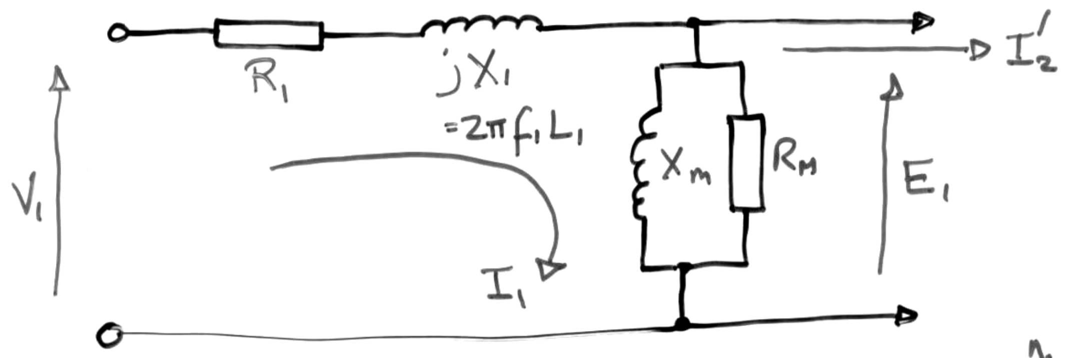
NOW $N_S = 500 + 30$ RPM

$= 530$ RPM

NEW $f_1 = \dfrac{530}{1500} \times 50\,\text{Hz}$

$= \underline{17.7\ \text{Hz}}$

$V_1 - f_1$ RELATIONSHIP TO GIVE CONSTANT FLUX $\hat{\phi}_M$



$E_1 = \sqrt{2}\,\pi N_1 f_1 \hat{\phi}_M$

AS f DROPS X_M BECOMES LOWER $\rightarrow$ LOWER V_1

$E_1 \propto f_1$ AT CONSTANT $\hat{\phi}_M$

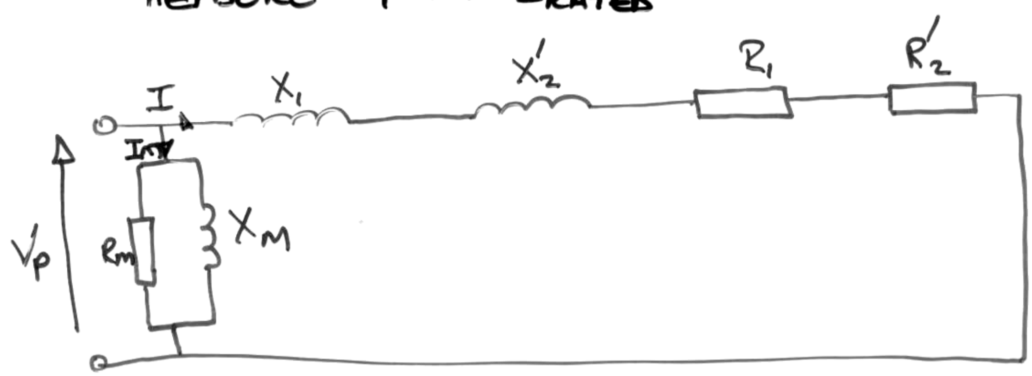
VOLTAGE BOOST TO OVERCOME $I_1 R_1$ VOLTAGE DROP.

(12)

LOCKED ROTOR TEST

REDUCED VOLTAGE APPLIED TO INDUCTION MOTOR WITH ROTOR LOCKED TO PREVENT ROTATION.

MEASURE V_1 AT I_{RATED}



USUALLY $I_m \ll I$ as $Z_m \gg Z$

$\therefore$ IGNORE THE MAGNETIZING BRANCH

MEASURE INPUT POWER

$$\Rightarrow R_1 + R_2' = \frac{P_{LR}}{3I^2}$$

R_1 MEASURED

$\therefore R_2'$ CALCULATED

ALSO $\dfrac{V_p}{I} = \sqrt{X_T^2 + R_T^2}$ WHERE $R_T = (R_1 + R_2')$

MAY THEN FIND X_T', CAN THEN USE GIVEN EQUATIONS TO FIND MAX PULL-OUT TORQUE AT GIVEN VOLTAGE FOR MAX LOAD