Answers for EEE6140 2008-2009

Q1: Answer

(a) When the upper and low cages are considered independently. The important issues include that for upper cage, the airspace and low cage will not contribute to the permeance, while for lower cage the airspace and upper cage are equivalent to air. Then procedure is similar to that given in the lecture.

The answers are:

For upper cage $\lambda_{upper\,cage} = \dfrac{b_0}{h_0} + \dfrac{b_1}{3h_1}$

For lowe cage $\lambda_{lower\,cage} = \dfrac{b_0}{h_0} + \dfrac{b_1}{h_1} + \dfrac{b_2}{h_2} + \dfrac{b_3}{3h_3}$

(b) In this case, the influence of lower cage on the upper cage should be considered.

Assuming that the current density is uniform, J, and the total current in the upper and lower cages is I, the current density is then given by

$$J = \frac{I}{b_1h_1 + b_3h_3}$$

The current in the low cage conductor is

$$I_{low\,cage} = Jb_3h_3 = I\,\frac{b_3h_3}{b_1h_1 + b_3h_3}$$

For section b_1h_1: Assuming a strip of width dx, at a distance of x from the bottom of upper cage, the mmf below the strip is proportional to $Jb_1x + I_{low\,cage} = \dfrac{Ib_1x}{b_1h_1 + b_3h_3} + \dfrac{Ib_3h_3}{b_1h_1 + b_3h_3}$

The flux through the strip is $\dfrac{Jb_1x + I_{low\,cage}}{\dfrac{b_1}{\mu_0 dx}} = I\left(\dfrac{b_1x}{b_1h_1 + b_3h_3} + \dfrac{b_3h_3}{b_1h_1 + b_3h_3}\right)\dfrac{\mu_0 dx}{b_1}$

the flux-linkage is then $I\left(\dfrac{b_1x}{b_1h_1 + b_3h_3} + \dfrac{b_3h_3}{b_1h_1 + b_3h_3}\right)^2 \dfrac{\mu_0 dx}{b_1}$

The flux-linkages for section b_1h_1 is

$$I\int_0^{h_1}\left(\frac{b_1x}{b_1h_1 + b_3h_3} + \frac{b_3h_3}{b_1h_1 + b_3h_3}\right)^2 \frac{\mu_0 dx}{b_1} = \mu_0 I\,\frac{b_1h_1 + b_3h_3}{b_1^2}\,\frac{1}{3}\left(1 - \left(\frac{b_3h_3}{b_1h_1 + b_3h_3}\right)\right)$$

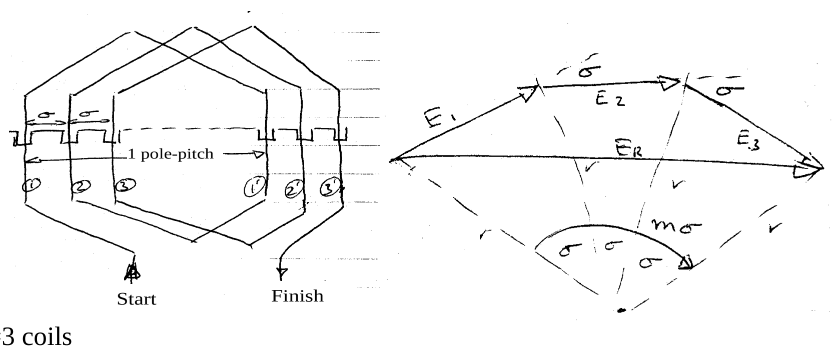
Therefore the specific permeance is $\dfrac{1}{3}\,\dfrac{b_1h_1 + b_3h_3}{b_1^2}\left(1 - \left(\dfrac{b_3h_3}{b_1h_1 + b_3h_3}\right)\right)$

(c) The upper cage is for starting. Due to eddy current skin effect, the current will flow in the upper cage during starting. Consequently, the starting rotor reisistance is high and the starting torque is high while the starting current is low. The low cage is for normal operation. With large cross-section, its resistance is low and consequently the copper loss is low.

Q2: Answer

(a)

(i) Winding Distribution – distribution factor k_d



Phasor diagram for m=3 coils

Note $|E_1| = |E_2| = |E_3| = |E_m|$ (all coils identical)

From the construction ($E_m = E_1$)

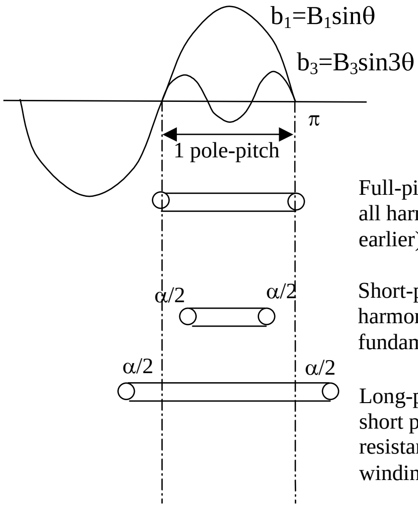
$E_m = 2r\sin\frac{\sigma}{2}$ & $E_R = 2r\sin\frac{m\sigma}{2}$

Hence distribution factor $k_d = \frac{effective\ induced\ emf}{arithmetic..............} = \frac{E_R}{mE_m} = \frac{\sin\frac{m\sigma}{2}}{m\sin\frac{\sigma}{2}}$

For the nth harmonic field the distribution factor is

$$k_{dn} = \frac{\sin\frac{mn\sigma}{2}}{m\sin\frac{n\sigma}{2}}$$

(ii) Coil pitch (or span) factor (k_p)



Full-pitched coil – fully links fundamental and all harmonic flux components (emf same as B earlier)

Short-pitched by α° elec. can effectively reduce harmonic compared to small reduction in fundamental.

Long-pitched by α° elec. Effect is identical to short pitch but increases winding length resistance. – only found in certain types of windings – not preferred.

k_p is defined as $\frac{effective\ emf}{emf\ of\ full-pitch\ coil} \propto \frac{effective\ flux\ linkage}{flux\ linkage\ of\ full\ pitch\ coil} = \frac{\Psi_s}{\Psi_F}$

For a short pitch coil $\Psi_s = \int_{\alpha/2}^{\pi-\alpha/2} \hat{B}\sin\theta d\theta = 2\hat{B}\cos\frac{\alpha}{2}$ & for a full pitch $\Psi_F = \int_0^{\pi} \hat{B}\sin\theta d\theta = 2\hat{B}$

$$k_p = \frac{\Psi_s}{\Psi_F} = \frac{2\hat{B}\cos\frac{\alpha}{2}}{2\hat{B}} = \cos\frac{\alpha}{2}$$

For the nth harmonic, the short or long pitch angle is $n\alpha$, $k_{pn} = \cos\frac{n\alpha}{2}$

(b). Single-phase excitation

Assume excitation winding carries a peak ac current of $\sqrt{2}I_c$ at frequency $\omega = 2\pi f$. Then for nth harmonic:

$|F_n| = \frac{4h}{n\pi}k_{wn}$, where $h = \frac{Ni}{2p}$ and $i = \sqrt{2}I_c\sin\omega t$

& the resultant time & space content of the winding mmf is:

$$F(\theta,t) = [F_1\sin\theta + ..+ ...+ F_n\sin n\theta]\sin\omega t = \frac{F_1}{2}[\cos(\theta-\omega t) - \cos(\theta+\omega t)] + ...+...+ \frac{F_n}{2}[\cos(n\theta-\omega t) - \cos(n\theta+\omega t)]$$

Consider a term such as $\cos(n\theta-\omega t)$ where the peak occurs at

$\cos(n\theta-\omega t) = 1$ or when $(n\theta-\omega t) = 0$ i.e. $n\theta = \omega t$ or $\theta = \frac{\omega t}{n}$

This represents a field component rotating at speed $\frac{d\theta}{dt} = \frac{\omega}{n}$ rad s^{-1}

and describes a component rotating **forwards** (+ve θ) at $\frac{\omega}{n}$ rad s^{-1}

Similarly terms such as $\cos(n\theta+\omega t)$ describes **backward** rotating field at $\frac{\omega}{n}$ rad s^{-1}

Clearly, a true single phase each harmonic excitation produces a complete set of field components with **forward** & **backward** fields of the same amplitude. Hence, no starting torque unless m/c can accelerate in less than ½ cycle

(c) 3-phase excitation

3-phase windings displaced in space by $\frac{n2\pi}{3}$ for the nth harmonic & in time phase by $\frac{2\pi}{3}$

$$F_a = [F_1 \sin\theta + .. + ... + F_n \sin n\theta]\sin\omega t$$

$$F_b = \left[F_1 \sin\left(\theta - \frac{2\pi}{3}\right) + .. + ... + F_n \sin n\left(\theta - \frac{2\pi}{3}\right)\right]\sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$F_c = \left[F_1 \sin\left(\theta - \frac{4\pi}{3}\right) + .. + ... + F_n \sin n\left(\theta - \frac{4\pi}{3}\right)\right]\sin\left(\omega t - \frac{4\pi}{3}\right)$$

Using same expressions as for 1-phase example, giving a resultant field of

$$F_R = F_a + F_b + F_c$$

$$F_R = \frac{3}{2}[F_1 \cos(\theta - \omega t) + F_5 \cos(5\theta + \omega t) + F_7 \cos(7\theta - \omega t) + F_{11} \cos(11\theta + \omega t) + ...]$$

Note:

(i) A balanced 3-phase winding produces only one rotating field component for each harmonic (e.g. no backward fundamental field)

(ii) The resultant field is $\frac{3}{2}\times$ amplitude of 1-phase winding field

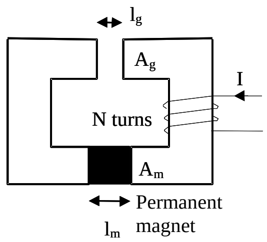
(iii) No resultant triplen harmonics produced (i.e., 3, 9, 15,…) etc. Hence, no need to design these out

(iv) n=7,13,etc harmonics are forward rotating

(v) n=5,11, etc ………………backward rotating

Q3: Answer

The problem can be simplified by utilising the symmetry. In this case, replacing A_g by $A_g' = \frac{1}{2} A_g$ into a single coil and single magnet circuit (which is given below).



(a) (b) (c)

From Ampere's law:

$$\oint Hdl = \sum I \qquad (1) \quad \text{and} \quad H_m l_m + H_g l_g = -NI \qquad (2)$$

From Gauss's law:

$$\oint Bds = 0 \qquad (3) \quad \text{and} \quad B_m A_m = B_g A_g \qquad (4)$$

While demagnetisation characteristic -magnet internal characteristic

$$B_m = \mu_0 \mu_r H_m + B_r \quad \text{(for linear part)} \qquad (5)$$

Load line -external circuit characteristic

From (1) & (2)

$$B_m A_m = B_g A_g = -\mu_0 \frac{A_g}{l_g}(H_m l_m + NI)$$

therefore

$$B_m = -\mu_0 \frac{A_g}{l_g A_m}(H_m l_m + NI) = -\mu_0 \frac{l_m A_g}{l_g A_m}\left(H_m + \frac{NI}{l_m}\right) = -\mu_0 \beta\left(H_m + \frac{NI}{l_m}\right) \qquad (6)$$

where $\beta = \frac{l_m A_g}{l_g A_m}$

when NI=0, $B_m = -\mu_0 \frac{l_m A_g}{l_g A_m} H_m = -\mu_0 \beta H_m$

Magnet working point - the crossing point of load line and demagnetization curve

$B_m = \mu_0 \mu_r H_m + B_r$ (7)

$B_m = -\mu_0 \beta \left(H_m + \frac{NI}{l_m} \right)$ (8)

Therefore $\mu_0 \mu_r H_m + B_r = -\mu_0 \beta \left(H_m + \frac{NI}{l_m} \right)$

$$H_m = -\frac{NI\beta}{l_m(\beta + \mu_r)} - \frac{B_r}{\mu_0(\beta + \mu_r)} \quad (9)$$

where $\beta = \frac{l_m A_g}{l_g A_m}$

and is an expression for the value of H_m in the presence of a demagnetisation field NI.

Airgap flux density due to magnet - Open-circuit airgap flux density

From (5) & (6) or (9), the magnet working point (B_m, H_m)

$B_m = \mu_0 \mu_r H_m + B_r$

$H_m = -\frac{NI\beta}{l_m(\beta + \mu_r)} - \frac{B_r}{\mu_0(\beta + \mu_r)}$

$$B_m = -\frac{\mu_0 \mu_r NI\beta}{l_m(\beta + \mu_r)} + \frac{B_r \beta}{\beta + \mu_r} \quad (10)$$

When NI=0, $B_m \equiv B_{m(oc)}$

$$B_m = \frac{B_r \beta}{\beta + \mu_r} = \frac{B_r}{1 + \mu_r \frac{l_g A_m}{l_m A_g}} \quad (11)$$

The airgap flux density

$$B_g = \frac{A_m}{A_g} B_m = \frac{B_r}{\frac{A_g}{A_m} + \mu_r \frac{l_g}{l_m}} \quad (12)$$

Minimum magnet length to avoid demagnetisation

To avoid demagnetisation $|H_m| < |H_{\lim}|$

From (9):

$$-\frac{NI\beta}{l_m(\beta + \mu_r)} - \frac{B_r}{\mu_0(\beta + \mu_r)} = H_{\lim} \quad (13)$$

Or directly from

$\mu_0 \mu_r H_m + B_r = -\mu_0 \beta \left(H_m + \frac{NI}{l_m} \right)$

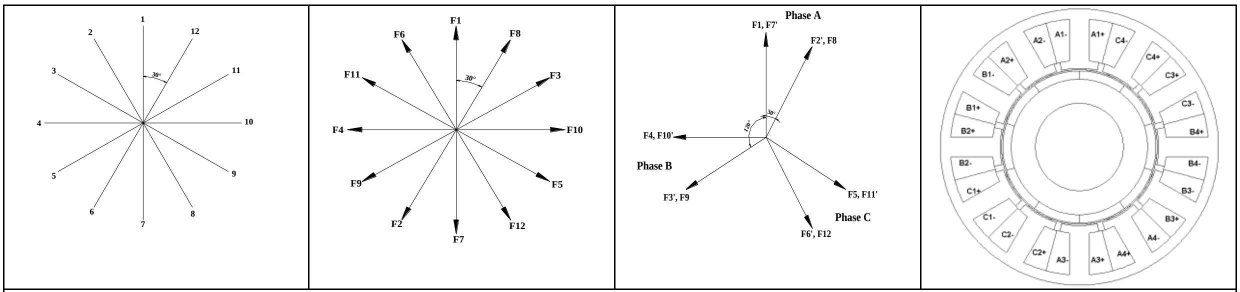
$$l_{m(\lim)} = -\frac{NI}{H_{\lim}} - \frac{B_r}{\mu_0 H_{\lim}} \times \left(l_g \frac{A_m}{A_g} \right) - \mu_r l_g \left(\frac{A_m}{A_g} \right) \quad (14)$$

where $H_{\lim} < 0$. $l_{m(\lim)}$ is the minimum length of magnet required to withstand the external mmf *NI*.

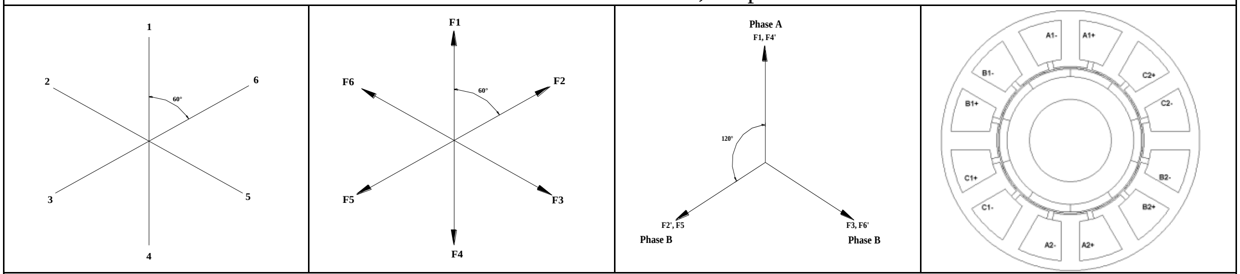
Q4: Answers

(a)

(a) stator coils (mech. deg.)	(b) mmf vectors for each coil (elec. deg.)	(c) selection of coils for each phase based on mmf vectors	(d) typical winding arrangement



All teeth wound 12-slot, 10-pole



Alternate teeth wound 12-slot, 10-pole

(b) Derivation is not given here and the final results are shown below:

	K_{pn}	K_{dn}	K_{dpn}
All teeth wound	$\sin\left(\dfrac{np\pi}{N_s}\right)$	$\sin\left(\dfrac{np\pi}{N_s}\right)$	$\sin^2\left(\dfrac{np\pi}{N_s}\right)$
Alternate teeth wound	$\sin\left(\dfrac{np\pi}{N_s}\right)$	1	$\sin\left(\dfrac{np\pi}{N_s}\right)$

(c) All teeth wound machine: more sinusoidal back-emf, less harmonics, two coil sides share the same slot
Alternate teeth wound machine: higher fundamental due to higher winding factor, one coil side in one slot, less sinusoidal.