ANSWERS

QUESTION 1

a) Relative appraisal of different clustering algorithms

i) Exclusive clustering

a. Example is k-means clustering
b. Crisp cluster demarcations
c. Easy to implement and computationally efficient
d. Requires knowledge of the number of clusters to begin with
e. Can converge on sub-optimal solutions
f. Can depend on initial conditions
g. Can result in empty clusters

ii) Overlapping clustering

a. Example is Fuzzy membership based techniques
b. Gives a degree of membership of each feature point to each cluster—can provide qualitative measures of membership to clusters
c. Often requires knowledge of number of clusters to begin with
d. Often more robust than k-means and converges to optimal solutions more readily

iii) Hierarchical techniques

a. Do not require knowledge of the number of clusters to begin with.
b. Merges clusters iteratively
c. Agglomerative (bottom-up) techniques more often used
d. Results in dendrograms/hierarchical trees, that can be interpreted in different ways depending on circumstances or information from alternative sources.
e. Does not scale well to large systems—can be computationally inefficient compared to other techniques

b) Clustering K-means

Provide a simple pseudo-code algorithm or flow-chart to show the procedure for K-means clustering.

i) *Initialise N centroids for N data clusters (could be randomly selected)*
ii) *Calculate distance from centroids to data points*
iii) *Group points into N clusters based on minimum distance—from Groupmatrix*
iv) *Calculate new centroids based on N new cluster groups*
v) *If no change in centroids and Group matrix then STOP, otherwise GOTO ii)*

c) A group of 2 satellites have sent back pictures of vehicles on the ground. The 4 data points that are received from the satellites are at :

$$(x,y)= \{ (1.5,1), (2,1), (3, 3), (5, 4.5) \}$$

and are shown plotted in Fig.*** for clarity.

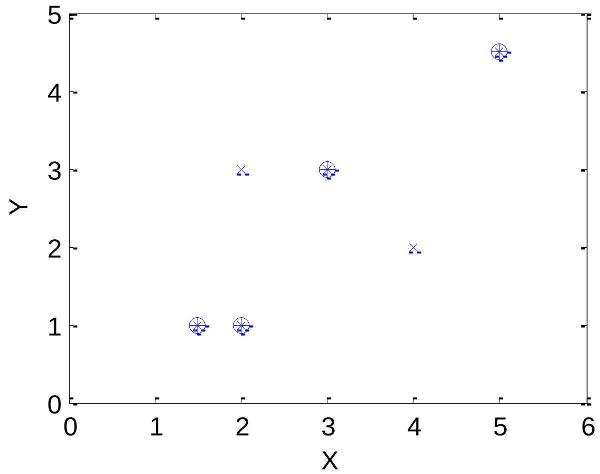


Figure ****

You assume that there are 2 clusters of points and that the 2 centroids (c_1 and c_2) are randomly initialised at $c_1=(4,2)$ $c_2=(2,3)$, and are also shown in Fig.***.

Use the K-means clustering method to identify which data points should be grouped in each of the 2 clusters (note: you should only need a maximum of 3 iterations of the algorithm, possibly less).

Answer
Data points =

1.5000	2.0000	3.0000	5.0000
1.0000	1.0000	3.0000	4.5000

N = 2
c =

4	2
2	3

ITERATION 0

D_dist =

2.6926	2.2361	1.4142	2.6926
2.0616	2.0000	1.0000	3.3541

G =

0	0	0	1
1	1	1	0

ITERATION 1

c =

5.0000	2.1667
4.5000	1.6667

D_dist =

4.9497	4.6098	2.5000	0
0.9428	0.6872	1.5723	4.0069

G =

0	0	0	1
1	1	1	0

There cluster group 1 (c_1) contains only data point 4, and cluster group 2 (c_2) contains data points 1,2 and 3.

QUESTION 2

a)

i) By fusing the data given above, from the 2 sensors, calculate the best estimate, μ_{best} , of the position of the object, and the statistical variance σ_{best}^{2} you expect, just from the given information.

ii) From the results of i) explain why you have more confidence in the position of the object from the 'fused' data, compared to the readings taken individually from each sensor.

Answers

$$\mu_{best} = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\mu s_1 + \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\mu s_2 = \frac{0.2}{0.1+0.2}4 + \frac{0.1}{0.1+0.2}3 = 3.667$$

$$\sigma^2{}_{best} = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2} = 0.0667$$

Notably the variance of the fused data is less that that of the individual sensors, thereby suggesting the more 'confidence' in the results.

b)

To measure the range of a stationary aircraft, data from 2 sensors (S_1 and S_2) is collected at alternate time intervals. The data from S_1 is known to have a variance of $\sigma_1^2 = 0.1$ and a mean $\mu_{s1}=4$, and the data from S_2 a variance of $\sigma_2^2 = 0.2$ and mean $\mu_{s2}=3$.

To obtain the best estimate of the aircraft's range, the data from the two sensors is fused using a minimum variance estimator (simple Kalman filter). A flowchart showing the structure of the estimator is given in Fig.2.1. Some data from sensors S_1 and S_2, and the output from the estimator, are given in Table 2.1. Complete Table 2.1 by calculating the output of the estimator for $k = 3, 4, 5$.

Figure 2.1 Flowchart of minimum variance estimator

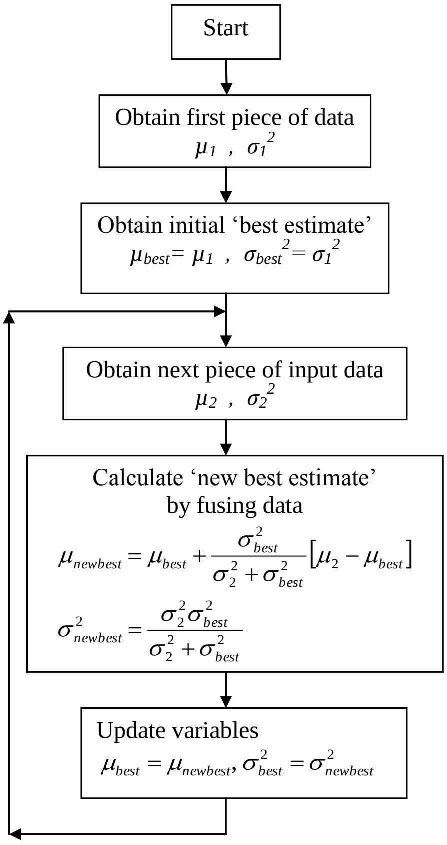


Table 2.1

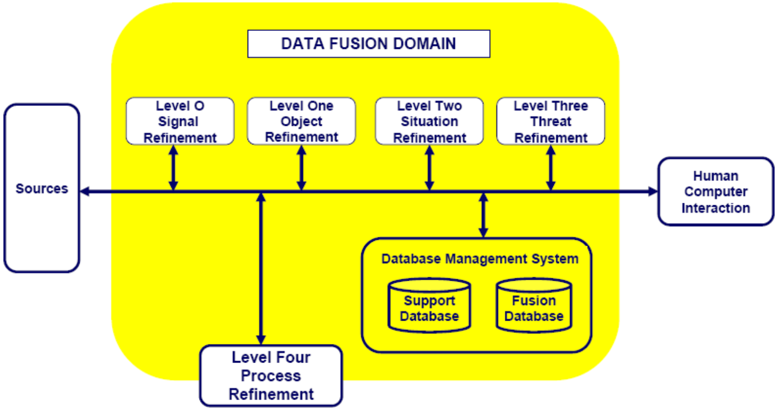
k	S_1	σ_1^2	S_2	σ_2^2	$\boldsymbol{\mu_{best}}$	$\boldsymbol{\sigma_{best}}$
0	4.1893	0.1			**4.1893**	**0.1**
1			3.2677	0.2	**3.8821**	**0.0667**
2	4.3292	0.1			**4.0609**	**0.0400**
3			3.4656	0.2	**?**	**?**
4	3.8561	0.1			**?**	**?**
5			2.7964	0.2	**?**	**?**
6	4.0332	0.1			**3.8496**	**0.0182**
7			3.0470	0.2	**3.7827**	**0.0167**
8	3.9524	0.1			**3.8069**	**0.0143**

Answer (from kalmanfusion_exam.m)

k	S_1	σ_1^2	S_2	σ_2^2	μ_{best}	σ_{best}
0	4.1893	0.1			4.1893	0.1
1			3.2677	0.2	3.8821	0.0667
2	4.3292	0.1			4.0609	0.0400
3			3.4656	0.2	**3.9617**	**0.0333**
4	3.8561	0.1			**3.9353**	**0.0250**
5			2.7964	0.2	**3.8088**	**0.0222**
6	4.0332	0.1			3.8496	0.0182
7			3.0470	0.2	3.7827	0.0167
8	3.9524	0.1			3.8069	0.0143

QUESTION 3

a)



The JDL model (1987-91) and the draft revised model (1997)
Level 0—*Source/signal refinement: Pre-processing, filtering, alignment in time, normalisation and unit conversions—usually sensor specific.*
Level 1—*Object Refinement: Combines data to provide a more cohesive 'picture' of an objects state (eg position and velocity) or identity. Often uses statistical evaluation or probability analysis, or dynamic models. For example, identifying and monitoring multiple aircraft by fusion of data from multiple sensors—observation-to-track association, continuous state estimation (e.g. kinematics) and discrete state estimation.*
Level 2—*Situation Refinement: Uses level 1 data to try and develop a description of relationships among identified objects to give a meaning or interpretation of the situation. This is includes determining where squadrons of aircraft may be heading, their numbers and classification. Example techniques include object clustering and relational analysis using neural networks, pattern recognition, fuzzy logic.*
Level 3—*Threat Refinement or consequence prediction. Takes data from levels 1 and 2 and makes an assessment about future events eg. predicting time to complete failure of a machine, or trying to assess what an opponent may do and the consequences, intent estimation, [event prediction], consequence prediction, susceptibility and vulnerability assessment, multi-perspective assessment (from neutral, friendly and aggressor stand-points). Usually involves neural networks, case-based reasoning or expert systems.*
Level 4—*Process Refinement. Considered as a 'meta-process'. System performance management—often change sensor-network fusion behaviour characteristics based on multi-objective optimisations, adaptive search and processing (an element of resource management), linear and quadratic programming, Fuzzy membership function etc.*
Level 5—*Cognitive refinement—Human computer interaction—only recently introduced. Gesture recognition, menu and display interaction, search-engines, visual clues etc.*

b)
Maximum likelihood decision making.
i)Combine the sensor decision matrices for the 2 sensors as follows:

$$\begin{array}{cc} & \begin{array}{cccc} m_{11}\,m_{21} & m_{11}\,m_{22} & m_{12}\,m_{21} & m_{12}\,m_{22} \end{array} \\ \begin{array}{c} Enemy \\ Friend \end{array} & \begin{bmatrix} 0.9\times 0.9=0.81 & 0.9\times 0.1=0.09 & 0.1\times 0.9=0.09 & 0.1\times 0.1=0.01 \\ 0.2\times 0.2=0.04 & 0.2\times 0.8=0.16 & 0.8\times 0.2=0.16 & 0.8\times 0.8=0.64 \end{bmatrix} \end{array}$$

ii) For each row, add up the elements that we regard as contributing to a positive identification:

$$\begin{array}{c} Enemy \\ Friend \end{array} \begin{bmatrix} 0.81+0.09+0.09=0.99 & 0.01 \\ 0.04 & 0.16+0.16+0.64=0.96 \end{bmatrix}$$

This means that the network will statistically identify an Enemy correctly 99% of the time, and will identify a Friend correctly 96% of the time.

QUESTION 4

a) Give 4 advantages of using multi-sensor data-fusion systems, along with example applications where the advantages can be obtained.

i) *Use of several identical sensors. Eg. radar tracking a moving object can be benefit from multiple sensor information—improvement can be by a factor of √N, where N is the number of sensors.*

ii) *Relative placement of multiple sensors to improve observations. Eg. two angular sensors at the bow and stern of a ship can be combined to give accurate position and velocity through triangulation.*

iii) *Use of different sensors to provide a combined integrated 'picture'. Eg. Tracking a moving object such as an aircraft. Radar can provide Range information whilst Infra-red imaging can provide angular direction.*

iv) *Sensor reliability. Eg. Can determine if a sensor in a multi-sensor network is faulty—conflicting information—outliers.*

b) What are the 3 different types of Sensor Network commonly encountered in data-fusion systems, and their attributes ? Give clear example applications of each type.

i) *Complementary Networks: They do not depend on each other directly but the data they provide can give a more complete picture of the environment. Complementary data can often be fused by simply extending the limits of the sensors eg. overlapping photographs.*

ii) *Competitive Networks: They provide independent measurements of the same information. This can provide increased reliability and accuracy. Note: since competitive networks involve a degree of redundancy, care must be taken to accommodate inconsistencies. When used properly this type of network can improve system robustness and reduce the effects of uncertainty and erroneous measurements.*

iii) *Co-operative Networks: Can provide independent measurements that when combined provide information that is not available from any single sensor alone. Eg. Radar/IR sensor problem.*

c)

A multi-sensor network consists of $k=5$ similar sensors ($S_1,\ldots,S_5$) to measure the range of a ship. In order to monitor the validity of the sensor readings, you take $\bar{n}=17$ readings from each sensor and apply Cochran's method to determine if any outliers are present, and significant. The variance of the sensor measurements is given in Table ***.

	S_1	S_2	S_3	S_4	S_5
σ^2	0.04	0.2	0.05	0.055	0.045

Table 4.1

Use Cochran's method and the 'significance table' (Fig.4.1) with $\alpha=0.05$ show that the data from S_2 should be considered with caution.

Level of significance $\alpha = 0.05$

k \ ν_x	1	2	3	4	5	6	7	8	9	10	16	36	144
2	0.9985	0.9750	0.9392	0.9057	0.8772	0.8534	0.8332	0.8159	0.8010	0.7880	0.7341	0.6602	0.5813
3	0.9669	0.8709	0.7977	0.7457	0.7071	0.6771	0.6530	0.6333	0.6167	0.6025	0.5466	0.4748	0.4031
4	0.9065	0.7679	0.6841	0.6287	0.5895	0.5598	0.5365	0.5175	0.5017	0.4884	0.4366	0.3720	0.3093
5	0.8412	0.6838	0.5981	0.5441	0.5065	0.4783	0.4564	0.4387	0.4241	0.4118	0.3645	0.3066	0.2513
6	0.7808	0.6161	0.5321	0.4803	0.4447	0.4184	0.3980	0.3817	0.3682	0.3568	0.3135	0.2612	0.2119
7	0.7271	0.5612	0.4800	0.4307	0.3974	0.3726	0.3535	0.3384	0.3259	0.3154	0.2756	0.2278	0.1833
8	0.6798	0.5157	0.4377	0.3910	0.3595	0.3362	0.3185	0.3043	0.2926	0.2829	0.2462	0.2022	0.1616
9	0.6385	0.4775	0.4027	0.3584	0.3286	0.3067	0.2901	0.2768	0.2659	0.2568	0.2226	0.1820	0.1446
10	0.6020	0.4450	0.3733	0.3311	0.3029	0.2823	0.2666	0.2541	0.2439	0.2353	0.2032	0.1655	0.1308
12	0.5410	0.3924	0.3264	0.2880	0.2624	0.2439	0.2299	0.2187	0.2098	0.2020	0.1737	0.1403	0.1100
15	0.4709	0.3346	0.2758	0.2419	0.2195	0.2034	0.1911	0.1815	0.1736	0.1671	0.1429	0.1144	0.0889
20	0.3894	0.2705	0.2205	0.1921	0.1735	0.1602	0.1501	0.1422	0.1357	0.1303	0.1108	0.0879	0.0675
24	0.3434	0.2354	0.1907	0.1656	0.1493	0.1374	0.1286	0.1216	0.1160	0.1113	0.0942	0.0743	0.0567
30	0.2929	0.1980	0.1593	0.1377	0.1237	0.1137	0.1061	0.1002	0.0958	0.0921	0.0771	0.0604	0.0457
40	0.2370	0.1576	0.1259	0.1082	0.0968	0.0887	0.0827	0.0780	0.0745	0.0713	0.0595	0.0462	0.0347
60	0.1737	0.1131	0.0895	0.0765	0.0682	0.0623	0.0583	0.0552	0.0520	0.0497	0.0411	0.0316	0.0234
120	0.0998	0.0632	0.0495	0.0419	0.0371	0.0337	0.0312	0.0292	0.0279	0.0266	0.0218	0.0165	0.0120
∞	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 4.1

If you ignore the data from S2, does the data from the remaining sensors in the network subsequently appear statistically robust i.e. are there any further outliers ?

Answers:

$$C = \frac{\max\left\{\sigma_{si}^2\right\}}{\sum_{i=1}^{5}\sigma_{si}^2} = \frac{0.2}{0.39} = 0.513$$

From the tables, with k=5 and $v_x = \bar{n} - 1 = 16$ *gives a confidence value of 0.3645, which is less that C, implying that* S_2 *is an outlier and the data from it should be treated with caution.*

If we ignore S_2*, then:*

$$C = \frac{\max\left\{\sigma_{si}^2\right\}}{\sum_{i=1}^{4}\sigma_{si}^2} = \frac{0.055}{0.19} = 0.289$$

From the tables, with k=4 and $v_x = \bar{n} - 1 = 16$ *gives a confidence value of 0.4366, which is greater than C, implying that the remaining sensors are statistically co-incident and the network is ok.*