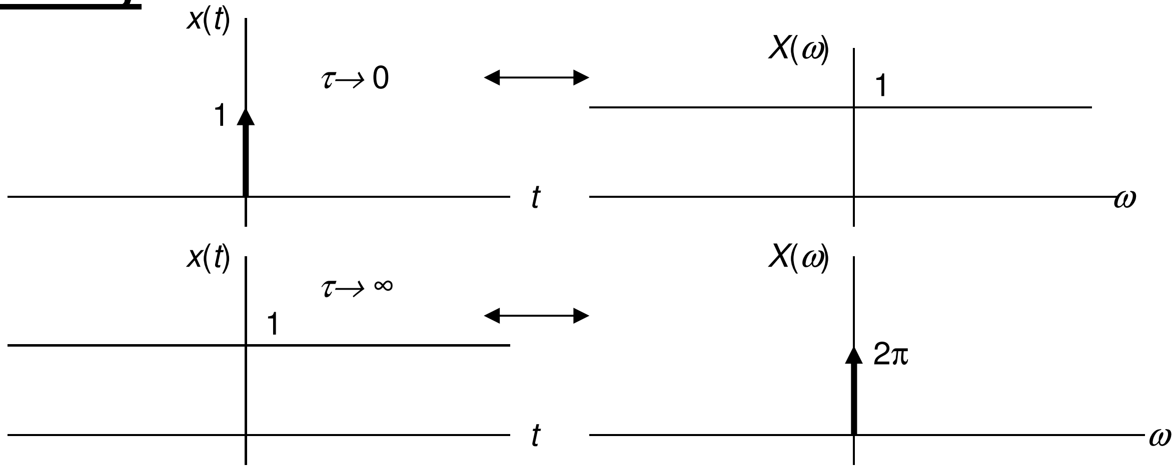
Lecture content

- Properties of Fourier Transform
  - Duality
  - Convolution
  - Multiplication
  - Parseval's Theorem

Properties of Fourier Transform

Duality

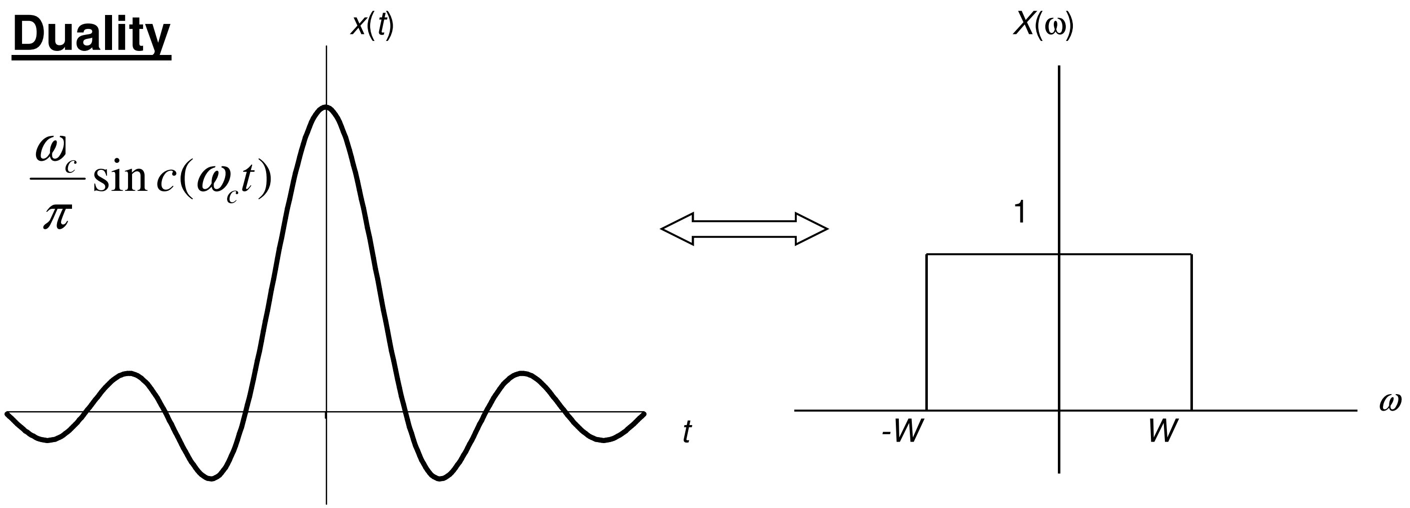


$$x(t) \leftrightarrow X(\omega)$$

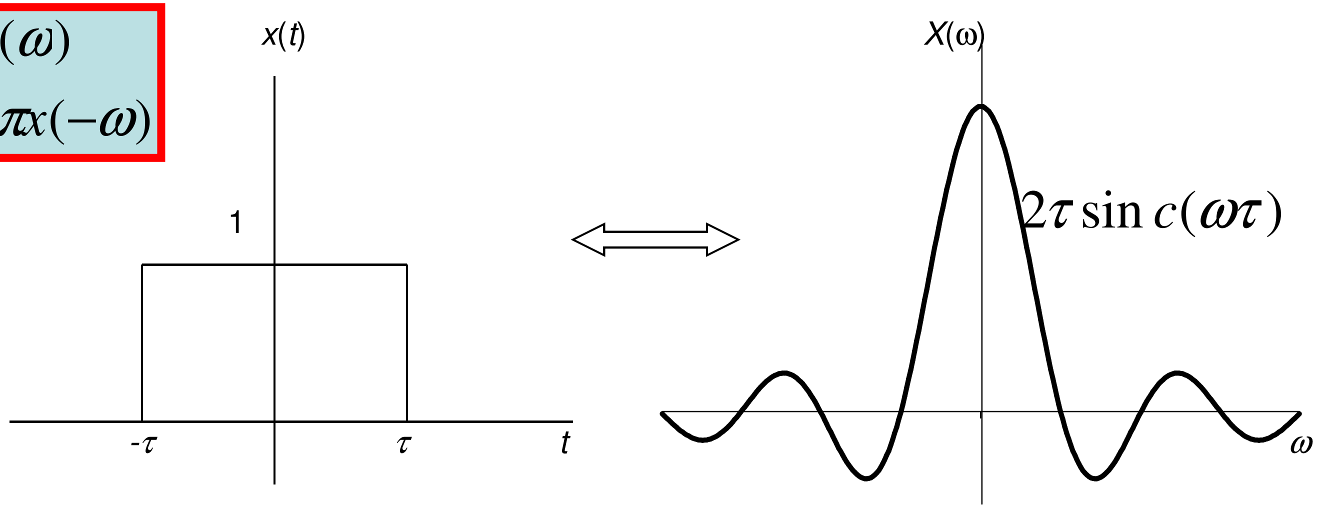
$$X(t) \leftrightarrow 2\pi x(-\omega)$$

Properties of Fourier Transform

Duality



$$\boxed{\begin{array}{l} x(t) \leftrightarrow X(\omega) \\ X(t) \leftrightarrow 2\pi x(-\omega) \end{array}}$$



Properties of Fourier Transform

Convolution

$x(t)*h(t) \leftrightarrow X(\omega).H(\omega)$. Convolution in time domain is equivalent to multiplication in frequency domain.

$$x(t)*h(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$$

Properties of Fourier Transform

Convolution

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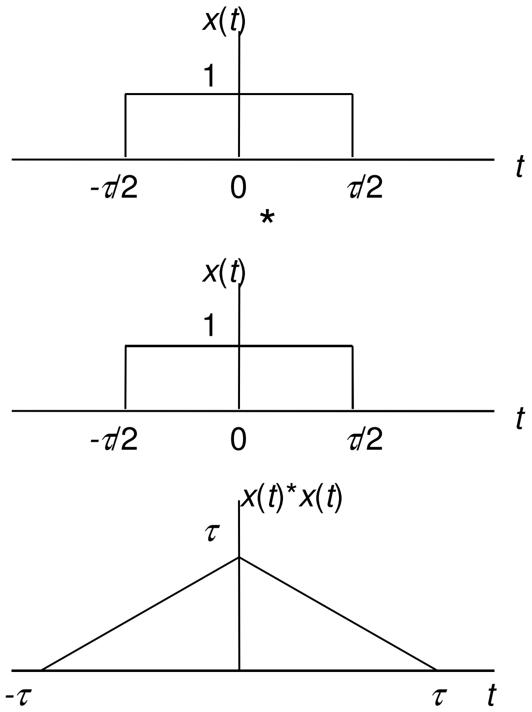

$$\leftrightarrow \quad \tau\frac{\sin(\omega\tau/2)}{(\omega\tau/2)} = \tau\sin c(\omega\tau/2)$$

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$$\leftrightarrow \quad \left[\tau\frac{\sin(\omega\tau/2)}{(\omega\tau/2)}\right]^2 = \tau^2\sin c^2(\omega\tau/2)$$

Properties of Fourier Transform

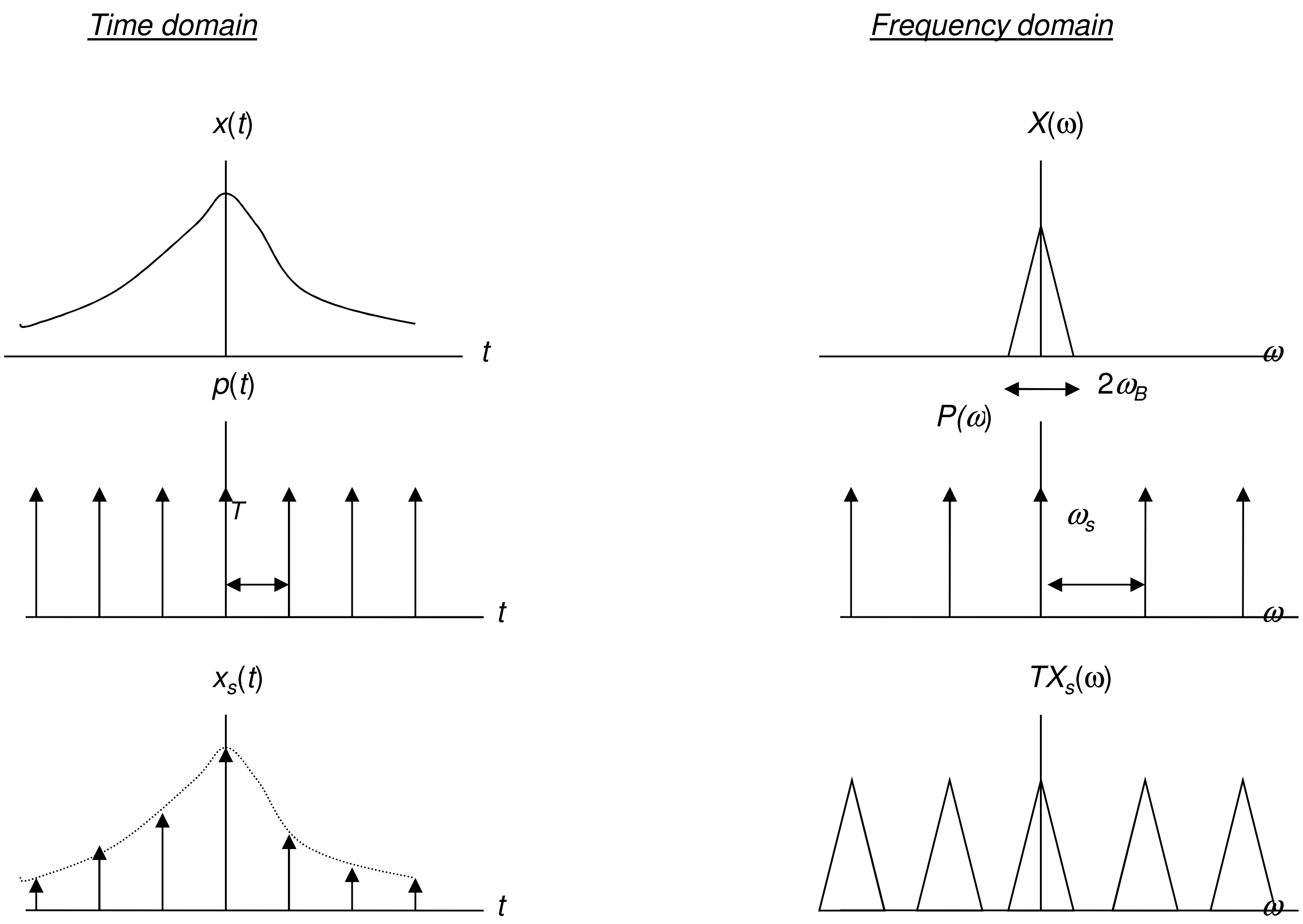
Multiplication

$$x(t).h(t) \leftrightarrow \frac{1}{2\pi}\left(X(\omega) * H(\omega)\right)$$

$$x(t).h(t) \leftrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\lambda)H(\omega-\lambda)d\lambda$$

Multiplication in time domain is equivalent to convolution in frequency domain.

Nyquist Sampling theorem



Properties of Fourier Transform

<u>Parseval's Theorem</u>

Total energy of a signal $x(t) = E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$

The energy contained within the frequency range $[\omega_1, \omega_2]$ is therefore given by

$$E = \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} |X(\omega)|^2 \, d\omega$$

Parseval's theorem

Example:
Find the energy contained within the frequency range [0,2 rad/s] for the signal $x(t) = e^{-t}.u(t)$.