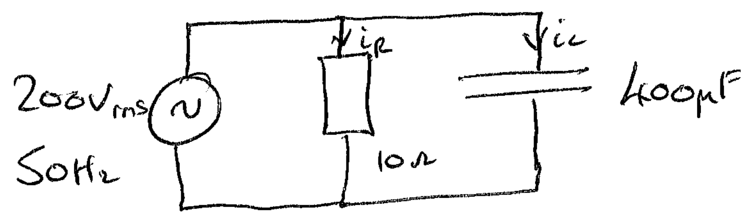
QUESTION 1



(a) (i) This problem can be solved by finding the total impedance first and then finding the current, however it is easier to calculate the current in each branch and sum.

Current through the Resistor, $i_R = \dfrac{V\angle 0^\circ}{R\angle 0^\circ} = \dfrac{200\angle 0^\circ}{10\angle 0^\circ}$

$= 20\angle 0^\circ$ Arms

Current through the Capacitor, $i_C = \dfrac{V\angle 0^\circ}{X_C\angle -90^\circ} = \dfrac{200 \angle +90^\circ}{\frac{1}{2\pi . 50 . 400\times10^{-6}}}$

$= 25.13\angle 90^\circ$ or $j\,25.13$ Arms

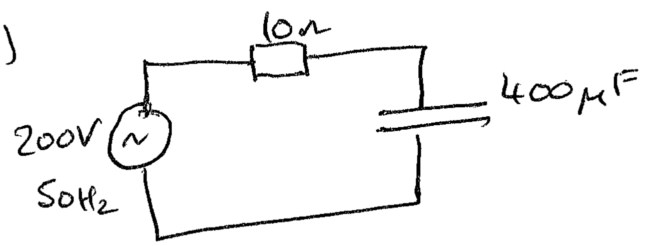
$\therefore$ Total current is $i_R + i_C = 20 + j25.13 = \underline{\underline{32.1\angle 51.5^\circ \text{Arms}}}$

(ii) The real power is $= I^2R = 20^2 \times 10 = 4kW$

or $P = V_T . I_T \cos\phi = 200 \times 32.1 \cos 51.5^\circ = \underline{\underline{4kW}}$

The power factor is $= \cos 51.5^\circ = \underline{\underline{0.623 \text{ leading}}}$

(b) (i)



The impedance is now given by : $Z = R + \dfrac{1}{j2\pi fC}$

$$\therefore Z = 10 - \frac{j}{2\pi . 50 . 400 \times 10^{-6}} = 10 - j7.96 = 12.78\angle -38.5^\circ \,\Omega$$

(ii) The current is then given by, $I = \frac{V}{Z} = \frac{200\angle 0^\circ}{12.78\angle -38.5^\circ}$

$$= 15.65\angle 38.5^\circ \text{ Arms}$$

(iii) Power is $I^2R = 15.65^2 \times 10 = 2450W$

(or $P = VI\cos\phi = 200 \times 15.65 \times \cos 38.5^\circ = 2450W$

(c) (i) For a series resonant circuit:

$$Z = R + j\omega L - \frac{j}{\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= R + j\left(2\pi f L - \frac{1}{2\pi f C}\right)$$

(ii) Condition for resonance is when the imaginary term = 0.

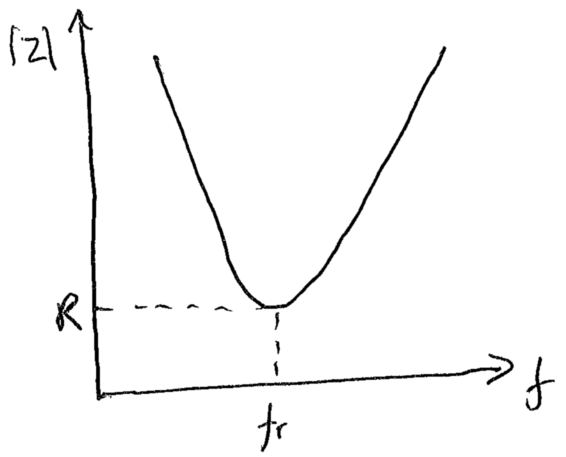
ie when $2\pi f L = \frac{1}{2\pi f C}$

or $f^2 = \frac{1}{4\pi^2 LC}$

$$\therefore f = \frac{1}{2\pi\sqrt{LC}}$$

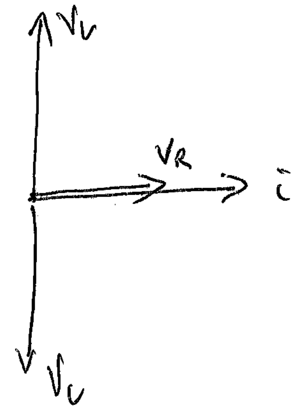
QUESTION 1 (CONTINUED)

(iii)



Z = R at resonance.

(iv)



At resonance $|V_L| = |V_C|$

(v) The value of inductance for maximum current occurs when the circuit resonates at 50Hz

ie. $f = \frac{1}{2\pi\sqrt{LC}}$ or $L = \frac{1}{4\pi^2 f^2 C}$

$= \frac{1}{4\pi^2 \times 50^2 \times 400\times10^{-6}}$

$= \underline{\underline{0.025H \text{ or } 25mH}}$

At resonance Z = R so the current is given by:

$I_{resonance} = \frac{200\angle 0^\circ}{10\angle 0^\circ} = \underline{\underline{20\angle 0^\circ \text{ Arms}}}$

(vi) At resonance the voltage across each component is:

$$|V_L| = I X_L = 20 \times 2\pi \times 50 \times 0.025 = \underline{\underline{158\,V_{rms}}}$$

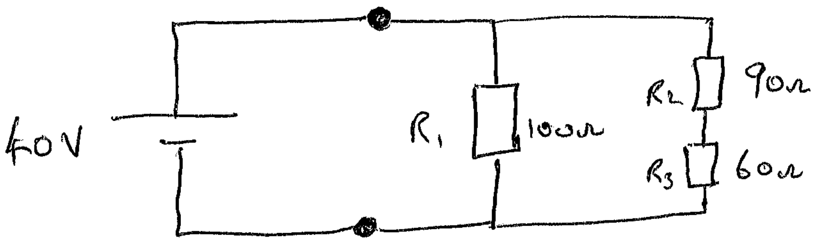
$$|V_C| = I X_C = 20 \times \frac{1}{2\pi \times 50 \times 400 \times 10^{-6}} = \underline{\underline{158\,V_{rms}}}$$

$$Q \text{ factor or magnification factor} = \frac{V_L}{V} \left(\text{or } \frac{V_C}{V}\right)$$

$$= \frac{158}{200} = \underline{\underline{0.79}}$$

(check $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.025}{400 \times 10^{-6}}} = \underline{\underline{0.79}}$

QUESTION 2



(a) (i) . The overall resistance of the load, R_T is given by:

$$\frac{1}{R_T} = \frac{1}{100} + \frac{1}{(90+60)} = \frac{1}{0.0167} \quad \therefore R_T = 60\Omega$$

(ii) The current flowing through resistors R_2 and R_3 is:

$$I_{23} = \frac{V}{R_2+R_3} = \frac{40}{(90+60)} = 0.266\,A$$

(iii) Power dissipated in each resistor:

$$P_{100} = \frac{V^2}{100} = \frac{1600}{100} = 16W.$$

$$P_{90} = I_{23}^2 \cdot R_2 = 0.267^2 \cdot 90 = 6.41\,W$$

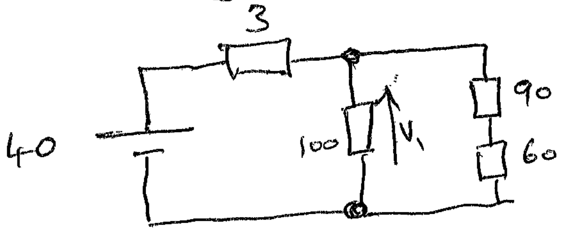
$$P_{60} = I_{23}^2 \cdot R_3 = 0.267^2 \cdot 60 = 4.28W$$

(iv) Total power dissipated in the load

$$P_T = P_{100} + P_{90} + P_{60} = 16 + 6.41 + 4.28 = 26.7W$$

(Alternatively $P_T = \frac{V^2}{R_T} = \frac{40^2}{60} = 26.7W$)

(b) The battery now has an internal resistance of 3Ω:



(i) The total resistance connected to the battery is now 63Ω

$\therefore$ Total current drawn from battery $= \frac{40}{63} = 0.635A$

QUESTION 2 (CONTINUED)

The voltage across R_1 is equal to the voltage across the load:

$$V_1 = V_{Bat} - I.R_{INT} = 40 - (0.635.3) = \underline{\underline{38.1\,V}}$$

(Alternatively $I.60 = 38.1V$)

(ii) The current flowing through R_2 and R_3 is:

$$I_{23}' = \frac{38.1}{150} = 0.254\,A$$

Hence the power dissipated in R_3 is:

$$P_3 = 0.254^2.60 = \underline{\underline{3.87W}}$$

(...) The power dissipated in the battery itself is:

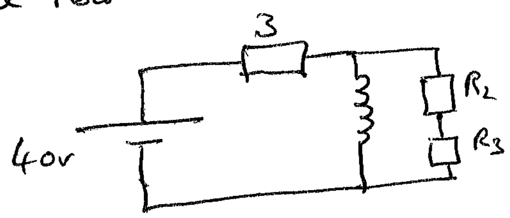
$$P_{INT} = 0.635^2.3 = \underline{\underline{1.21W}}$$

(iii) The efficiency is defined as:

$$\eta = \frac{\text{Useful power}}{\text{Total power}} \times 100\% = \frac{I^2 R_L}{I^2 R_L + I^2 R_{INT}} \times 100\%$$

$$= \frac{60}{60+3} \times 100\% = \underline{\underline{95.2\%}}$$

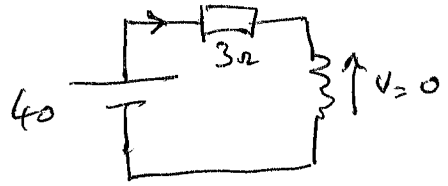
(iv) The circuit now becomes



On DC the inductor appears as a short circuit since $dI/dt = 0$ and hence $V_L = L\,dI/dt = 0$

So there can be no current flowing down the branch containing R_2 and R_3 and hence the power dissipated in R_3 is zero.

However there is still a current flowing through the inductor

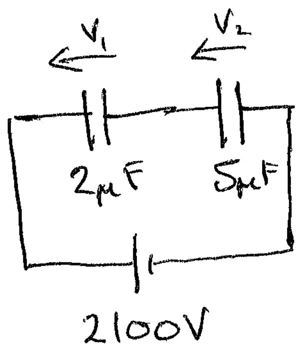


$$I = \frac{40}{3} = \underline{13.3\ A}$$

Hence the energy stored in the inductor $= \frac{1}{2}LI^2 = \frac{1}{2}.0.05 \times 13.3^2$

$$= \underline{4.42\ J}$$

(c)



Since the capacitors are in series:

$$Q = C_1 V_1 = C_2 V_2$$

$$\text{or } 2 \times 10^{-6} V_1 = 5 \times 10^{-6} V_2$$

$$\therefore V_1 = 2.5 V_2$$

Also since the capacitors are in series:

$$V_1 + V_2 = 2100$$

$$\therefore 3.5 V_2 = 2100 \rightarrow V_2 = 600V \text{ and } V_1 = 1500V$$

Clearly Capacitor C_1 is operating above its maximum voltage.
The voltage V_1 must be reduced from 1500V to 1200V
therefore Voltage V_2 must increase to $2100 - 1200 = 900V$

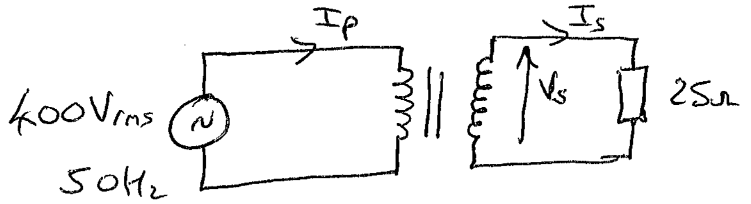
$$\therefore C_1 V_1' = C_2' V_2'$$

$$2 \times 10^{-6} \times 1200 = C_2' \times 900$$

$$\therefore C_2' = 2.67 \times 10^{-6} = \underline{2.67\mu F}$$

The value of C_2 needs to be reduced from $5\mu F$ to $2.67\mu F$

QUESTION 3



(a)(i) Since $\frac{V_p}{V_s} = \frac{N_p}{N_s} \Rightarrow V_s = \frac{V_p \cdot N_s}{N_p} = \frac{400 \times 1}{8} = \underline{\underline{50V_{rms}}}$

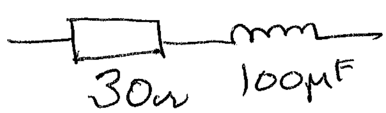
Now $I_s = \frac{V_s}{R_s} = \frac{50}{25} = \underline{\underline{2A_{rms}}}$

and since $\frac{I_p}{I_s} = \frac{N_s}{N_p} \Rightarrow I_p = \frac{I_s \cdot N_s}{N_p} = \frac{2 \times 1}{8} = \underline{\underline{0.25A_{rms}}}$

The power dissipated in the load is $I^2R = 2^2 \times 25$
$= \underline{\underline{100W}}$

(Alternatively $P_{IN} = V_p \cdot I_p = 400 \times 0.25 = 100W$)

(ii) The load now comprises:



$\therefore Z = 30 - \frac{j}{2\pi \times 50 \times 100 \times 10^{-6}} = 30 - j31.83\Omega$
$= 43.74\angle{-46.7^\circ}\ \Omega$

The secondary voltage remains unchanged at $50V_{rms}$.

$I_s = \frac{50\angle 0^\circ}{43.74\angle{-46.7^\circ}} = \underline{\underline{1.143\angle 46.7^\circ A_{rms}}}$

$I_p = \frac{1.143\angle 46.7^\circ \times 1}{8} = \underline{\underline{0.143\angle 46.7^\circ A_{rms}}}$.

Power dissipated in the load, $P = 1.143^2 \times 30 = \underline{\underline{39.2W}}$

(iii) The input power factor is $\cos(46.7^\circ) = 0.686$ (leading)

and the $VA = V \times I = 400 \times 0.143 = 57.2\ VA$

(iv) Since $V_p = 4.44\, f\, N_p\, \phi_{max}$

$400 = 4.44 \times 50 \times N_p \times 0.004 \Rightarrow N_p = 450$ TURNS

$V_s = 4.44\, f\, N_s\, \phi_{max} = 4.44 \times 50 \times N_s \times 0.004$

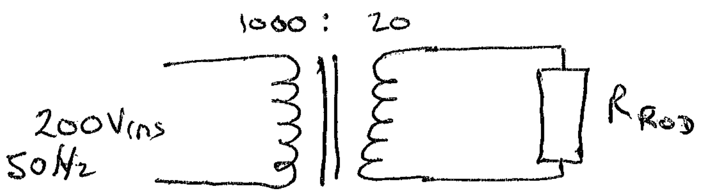
$\Rightarrow N_s = 56$ TURNS

(v). If the frequency now increases to 60Hz:

$V_p = 4.44 \times 60 \times 450 \times 0.004$

$= 479\ V_{rms}$

(b)



(i) First calculate the resistance of the rod at both temperatures

$$R_0 = \frac{\rho L}{A} \quad \text{and} \quad A = \frac{\pi d^2}{4}$$

Since $L = 0.8m$ $\quad d = 10mm$ $\quad \rho = 8.33 \times 10^{-8}\ \Omega m$

$$R_0 = \frac{8.33 \times 10^{-8} \times 0.8 \times 4}{\pi \times (0.01)^2} = 8.485 \times 10^{-4}\ \Omega$$

Now at 30°C

$$R_{30} = R_0(1 + \alpha T) = 8.485 \times 10^{-4} \times (1 + 6 \times 10^{-3} \times 30)$$

$$= 1.001 \times 10^{-3}\ \Omega$$

QUESTION 3 (CONTINUED)

$$R_{700} = 8.485\times10^{-4} \times \left(1 + 6\times10^{-3}\times 700\right) = 4.4122\times10^{-3}\,\Omega$$

The transformer secondary voltage is:

$$V_S = \frac{N_S}{N_P} \cdot V_P = \frac{20\times 200}{1000} = 4V_{rms}$$

At 30°C the power dissipated is $\frac{V_S^2}{R_{30}} = \frac{4^2}{1.001\times10^{-3}} = \underline{\underline{15.98kW}}$

At 700°C the power dissipated is $\frac{4^2}{4.4122\times10^{-3}} = \underline{\underline{3.63kW}}$

(ii) Since the load is purely resistive the power factor is unity.

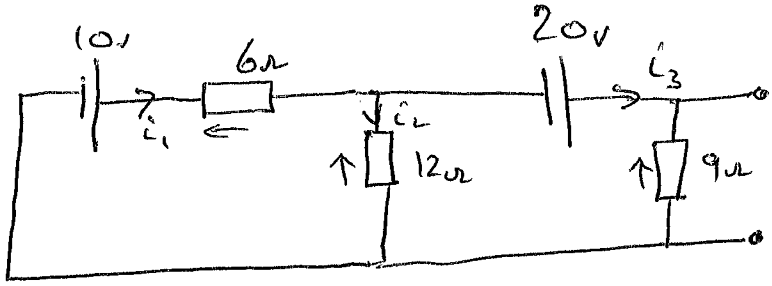
$$P_{30} = V_P \times I_{P30} \Rightarrow I_{P30} = \frac{15980}{200} = \underline{\underline{79.9A_{rms}}}$$

$$P_{700} = V_P \times I_{P700} \Rightarrow I_{P700} = \frac{3630}{200} = \underline{\underline{18.15A_{rms}}}$$

(iii) If the transformer has an efficiency of 95% then if the output power is 15.98kW the input power is:

$$P_{IN} = \frac{15980}{0.95} = \underline{\underline{16.82kW}}$$

QUESTION 4



(a) (i) Find the Thevenin voltage (ie the open-circuit voltage which in this circuit is the voltage across the 9Ω resistor)

First find the current through the 9Ω resistor (i_3):

$$10 - 6i_1 - 12i_2 = 0 \quad (1)$$

$$12i_2 + 20 - 9i_3 = 0 \quad (2)$$

$$i_1 = i_2 + i_3 \quad (3)$$

Substitute for i_1 in equation (1) using (3):

$$10 - 6i_2 - 6i_3 - 12i_2 = 0 \quad (4)$$

$$\therefore 10 - 18i_2 - 6i_3 = 0$$

Multiply (4) by 2 and (2) by 3 and add:

$$20 - 36i_2 - 12i_3 = 0$$

$$60 + 36i_2 - 27i_3 = 0$$

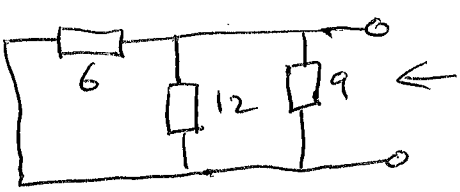
$$80 \qquad -39i_3 = 0$$

$$\therefore i_3 = 2.051\text{ A}$$

Hence the voltage across 9Ω resistor is:

$$V_T = 9 \times 2.051 = 18.46\text{V}$$

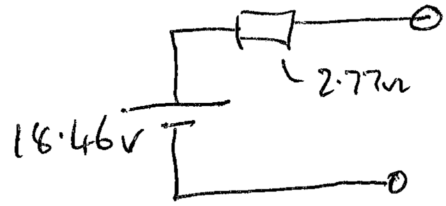
The Thevenin resistance is:



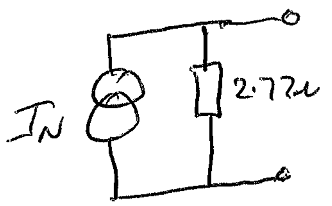
$$R_T = \frac{1}{\frac{1}{6} + \frac{1}{12} + \frac{1}{9}} = 2.77\Omega$$

QUESTION 4 (CONTINUED)

Hence the Thevenin equivalent circuit is:



(ii) The Norton Equivalent circuit is:



$$I_N = \frac{E_T}{R_T} = \frac{18.46}{2.77} = \underline{\underline{6.66A}}$$

(b) (i) Factory takes 800kW @ 0.8 p.f lag.

Since $P = VA\cos\phi \Rightarrow VA = \frac{P}{\cos\phi} = \frac{800}{0.8} = \underline{\underline{1000KVA}}$

(ii) Reactive power $Q = VA\sin\phi$

$$= 1000 \times \left(\sqrt{1-0.8^2}\right) = \underline{\underline{600KVAr}}$$

(iii) Current drawn from the supply:

$$I = \frac{VA}{V} = \frac{1000\times10^3}{11\times10^3} = \underline{\underline{90.9A_{rms}}}$$

(c) Process heaters $P = 100kW$ $Q = 0KVAr$

Motor load 300kVA @ 0.75 lagging

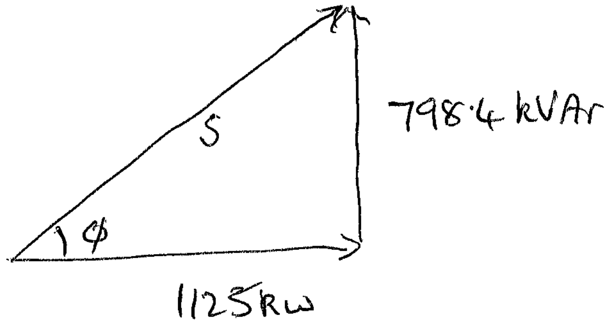
$P = VA\cos\phi = 300 \times 0.75 = 225kW$

$Q = VA\sin\phi = 300 \times \left(\sqrt{1-0.75^2}\right) = 198.4kVAr$

$\therefore$ The total factory load is:

$P = 800 + 100 + 225 = 1125kW$

$Q = 600 + 0 + 198.4 = 798.4kW$



(i) New kVA rating, $S = \sqrt{1125^2 + 798.4^2} = \underline{\underline{1379.5 \text{ kVA}}}$

(ii) $\phi = \tan^{-1} \dfrac{798.4}{1125} = 35.4°$

$\therefore \underline{\underline{\cos\phi = 0.816 \text{ lagging}}}$

(iii) To correct the p.f. to unity the capacitor must supply 798.4 kVAr

Since $Q_c = \dfrac{V_c^2}{X_c} \implies X_c = \dfrac{V_c^2}{Q_c} = \dfrac{11000^2}{798.4 \times 10^3} = 151.5\,\Omega$

and $X_c = \dfrac{1}{2\pi f C} \implies C = \dfrac{1}{2\pi f X_c} = \dfrac{1}{2\pi \times 50 \times 151.5}$

$= \underline{\underline{21 \mu F}}$