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The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2014-15 (2.0 hours)

EEE309 Introduction to Digital System Processing

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. **a.** i) In the context of a discrete-time system, explain the concepts of causality, stability, linearity and time invariance. (6 marks)

ii) A sequence is said to be the eigenfunction of a linear time invariant (LTI) system, when given such a sequence at its input, its output is a simple scaled version of the same sequence. Determine whether the sequence $x[n]=4^n$ is the eigenfunction of an LTI system. Explain your answer. (4 marks)

(10)

b. i) Determine whether the following two signals are periodic. If the signal is periodic, state its period: (4 marks)

1) $x[n] = e^{j2n\pi/3}$ 2) $x[n] = \cos(n\pi/\sqrt{2})$

ii) Suppose the above two sequences are obtained by sampling two continuous-time signals, respectively. What extra information do you need to recover the original continuous-time signal? (3 marks)

iii) Draw the block diagram of an ideal system for recovering the original continuous-time signals. (3 marks) **(10)**

2. **a.** Given the spectral coefficients of a filter, *H(k)*, which are symmetrical about k=0, the original impulse response h[n], can be reconstituted using the following equation, where N is the total number of coefficients:

$$h[n] = \frac{1}{N}\sum_{k=-(N-1)/2}^{(N-1)/2} H(k)e^{j2\pi nk/N} = \frac{1}{N}\left(H(0) + 2\sum_{k=1}^{(N-1)/2} H(k)\cos(2\pi nk/N) \right)$$

From this you are going to design a lowpass FIR filter with $N=5$ coefficients with a passband range of 0.5kHz at a sampling frequency $f_s=2$kHz.

i) Use the frequency sampling method to calculate the FIR filter coefficients. (4 marks)

ii) Sketch the structure of the filter using unit-delay elements. (2 marks)

iii) Derive the linear constant coefficient difference equation of the filter.

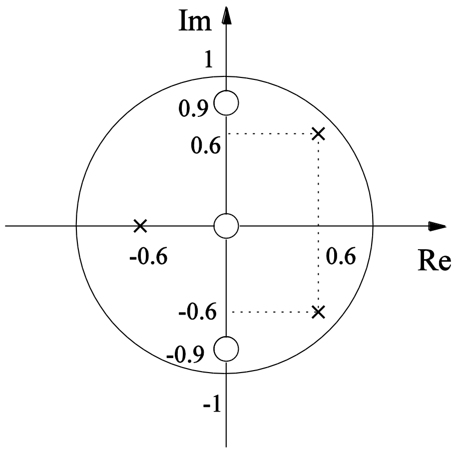
(2 marks)

(8)

b. For a causal stable LTI system to also have a causal stable inverse, what kind of characteristics should such a system possess? Explain your answer.

(6)

c. i) Derive the transfer function (z-transform) for a causal IIR filter which has the z-plane pole-zero plot shown in the following figure, where there are 3 poles and 3 zeros. Specify its region of convergence. (4 marks)



ii) Find the transfer function (z-transform) of its inverse system and specify its region of convergence. (2 marks)

(6)

3. **a.** Calculate the Discrete Fourier Transform (DFT) of the discrete series x[n]={1, 2, 2, 1}.

(4)

b. Consider a sequence $x_1[n]$ whose length is L points (nonzero for n=0, 1, …, L-1) and a sequence $x_2[n]$ whose length is P (nonzero for n=0, 1, …, P-1). A linear convolution of these two sequences will generate a third sequence $x_3[n]$. Describe the process involved in calculating this linear convolution using DFT.

(6)

c. A lowpass digital filter is to be designed for a digital signal processing system and the first-order analogue filter given in the following equation is used as a prototype, where ω_b is the filter cutoff frequency.

$$H(s) = \frac{\omega_b}{s + \omega_b}$$

i. Design the digital filter using the Impulse Invariant method if ω_b=50rad/sec and the filter is implemented at a sampling frequency of 80Hz. (5 marks)

ii. Given the same sampling frequency of 80 Hz, design the digital filter using the Bilinear Transform method and make sure that the resultant digital filter has a normalised cutoff frequency corresponding to 50rad/sec. (5 marks)

(10)

4. **a.** What is the output y[n] of an LTI system with impulse response h[n], n=-∞, …, 0, …, +∞, given the sinusoidal input $x[n]=A\cos(\Omega_0 n+\varphi)$? How about if h[n] is real-valued?

(8)

b. The impulse response of a causal LTI system can be described by the following equation:

$$h[n] = \left(\frac{1}{2}\right)^n \quad \text{for} \quad n \geq 0$$

i) Obtain the linear constant coefficient difference equation describing its behaviour and draw a block diagram of the system. (4 marks)

ii) Find its frequency response and then calculate its output given the following input sequence: (4 marks)

$$x[n]=10+2\sin(n\pi/2)$$

(8)

c. Give a comparison between IIR and FIR filters in terms of their relative advantages and disadvantages. **(4)**

WL/JROD