List of useful formulae

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$x(t) = \int_{-\infty}^{\infty} x(\lambda)\delta(t-\lambda)d\lambda$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$

$$c_0 = \frac{1}{T}\int_{<T>} x(t)dt$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jn\omega_o t}dt$$

$$a_n = 2\,\mathrm{Re}[c_n] = \frac{2}{T}\int_{<T>} x(t)\cos n\omega_0 t dt$$

$$b_n = -2\,\mathrm{Im}[c_n] = \frac{2}{T}\int_{<T>} x(t)\sin n\omega_0 t dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$X(\omega) = 2\int_{0}^{\infty} x(t)\cos\omega t dt$$

$$X(\omega) = -j2\int_{0}^{\infty} x(t)\sin\omega t dt$$

$$X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$$

$$x(t) = \frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}dt$$

$$\cos(x)\cos(y) = \frac{1}{2}\left[\cos(x-y)+\cos(x+y)\right]$$

$$\sin(x)\sin(y) = \frac{1}{2}\left[\cos(x-y)-\cos(x+y)\right]$$

$$\sin(x)\cos(y) = \frac{1}{2}\left[\sin(x-y)+\sin(x+y)\right]$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

Fourier Transform Pairs

Signal	Fourier Transfrom
$\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$	$2\pi\sum_{n=-\infty}^{\infty} c_n\delta(\omega-n\omega_o)$
$e^{j\omega_o t}$	$2\pi\delta(\omega - \omega_o)$
$\cos\omega_o t$	$\pi[\delta(\omega+\omega_o) + \delta(\omega-\omega_o)]$
$\sin\omega_o t$	$j\pi[\delta(\omega+\omega_o) - \delta(\omega-\omega_o)]$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
$\delta(t-t_o)$	$e^{-j\omega t_o}$

$e^{-at}u(t),\ a>0$	$\dfrac{1}{a+j\omega}$
$x(t)=\begin{cases}1, & \lvert t\rvert<\tau\\ 0, & \lvert t\rvert>\tau\end{cases}$	$\dfrac{2\sin\omega\tau}{\omega}=2\tau\sin c(\omega\tau)$
$\dfrac{\sin\omega_c t}{\pi t}=\dfrac{\omega_c}{\pi}\sin c(\omega_c t)$	$X(\omega)=\begin{cases}1, & \lvert\omega\rvert<\omega_c\\ 0, & \lvert\omega\rvert>\omega_c\end{cases}$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\dfrac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\dfrac{2\pi k}{T}\right)$

<u>Properties of Fourier Transform</u>

Property	Aperiodic signal, $x(t)$	Fourier Transfrom, $X(\omega)$
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(\omega)+bX_2(\omega)$
Time Shifting	$x(t-t_o)$	$e^{-j\omega t_o}X(\omega)$
Frequency Shifting	$e^{j\omega_o t}x(t)$	$X(\omega-\omega_o)$
Time Scaling	$x(at)$	$\dfrac{1}{a}X\left(\dfrac{\omega}{a}\right)$
Differentiation in Time	$\dfrac{dx(t)}{dt}$	$j\omega X(\omega)$
Differentiation in Frequency	$tx(t)$	$j\dfrac{dX(\omega)}{d\omega}$
Integration in time	$\int_{-\infty}^{t}x(\tau)d\tau$	$\dfrac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$
Convolution	$x(t)*h(t)$	$X(\omega).H(\omega)$
Multiplication in time	$x(t).h(t)$	$\dfrac{1}{2\pi}\int_{-\infty}^{\infty}X(\lambda)H(\omega-\lambda)d\lambda$
Parseval's Theorem	$E=\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt=\dfrac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2d\omega$	

Properties of Laplace Transform

Property	Transform Property
Linearity	$ax_1(t) + bx_2(t) \leftrightarrow aX_1(\mathrm{s}) + bX_2(\mathrm{s})$.
Time shift	$x(t-t_o)\,u(t-t_o) \leftrightarrow X(s)e^{-st_o}$,$t_o > 0$
Multiplication by a complex exponential	$x(t)e^{s_o t} \leftrightarrow X(s-s_o)$
Time scaling	$x(at) \leftrightarrow X(s/a)/\lvert a\rvert$
Differentiation in time domain	$\dfrac{dx(t)}{dt} \leftrightarrow sX(\mathrm{s}) - x(0)$
	$\dfrac{d^2x(t)}{dt^2} \leftrightarrow s^2X(\mathrm{s}) - sx(0) - \left.\dfrac{dx(t)}{dt}\right\rvert_{t=0}$
Differentiation in *s* domain	$t^n x(t) \leftrightarrow \dfrac{d^nX(s)}{ds^n}(-1)^n$
Integration	$\displaystyle\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{1}{s}X(s)$
Convolution in time domain	$x(t)*h(t) \leftrightarrow X(s).H(s)$
Initial value theorem	$x(0) = \lim\limits_{s\to\infty} sX(s)$
Final value theorem	$\lim\limits_{t\to\infty} x(t) = \lim\limits_{s\to 0} sX(s)$
(if $x(t)$ has a finite value as $t \to\infty$)	

Laplace Transform pairs

Signal	Transform
Unit step: $u(t)$	$\dfrac{1}{s}$
Unit impulse: $\delta(t)$	1
Unit ramp: $tu(t)$	$\dfrac{1}{s^2}$
$e^{-at}u(t)$	$\dfrac{1}{s+a}$
$t^n e^{-at}u(t)$	$\dfrac{n!}{(s+a)^{n+1}}$

$(\cos\omega_o t)u(t)$	$\frac{s}{\left(s^2+\omega_o{}^2\right)}$
$(\sin\omega_o t)u(t)$	$\frac{\omega_o}{\left(s^2+\omega_o{}^2\right)}$
$(e^{-at}\cos\omega_o t)u(t)$	$\frac{s+a}{\left((s+a)^2+\omega_o{}^2\right)}$
$(e^{-at}\sin\omega_o t)u(t)$	$\frac{\omega_o}{\left((s+a)^2+\omega_o{}^2\right)}$
$(t\cos\omega_o t)u(t)$	$\frac{s^2-\omega_o{}^2}{\left(s^2+\omega_o{}^2\right)^2}$
$(t\sin\omega_o t)u(t)$	$\frac{2\omega_o s}{\left(s^2+\omega_o{}^2\right)^2}$

Unit step response for 2nd order systems

Damping factor, ζ	Unit step response
>1	$y(t)=\frac{k}{p_1 p_2}+k_2 e^{p_1 t}.u(t)+k_3 e^{p_2 t}.u(t)$
1	$y(t)=\frac{k}{\omega_n{}^2}\left(1-(1+\omega_n t)e^{-\omega_n t}.u(t)\right)$
$0<\zeta<1$	$y(t)=\frac{k}{\omega_n{}^2}\left(1-\frac{\omega_n}{\omega_d}e^{-\zeta\omega_n t}\sin(\omega_d t+\theta).u(t)\right)$
0	$y(t)=\frac{k}{\omega_n{}^2}\left(1-\cos(\omega_n t).u(t)\right)$