Lecture content

- Trigonometric Fourier Series
  - Example of Fourier Series.
- Conditions for existence of Fourier Series

FS coefficients

As shown in example 3, the square waveform can be expressed as a sum of sinusoids or complex exponentials. We can replace

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$ with $$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos \omega_n t + b_n \sin \omega_n t]$$

where $a_0 = c_0 = \frac{1}{T} \int_{<T>} x(t) dt$ is the d.c term,

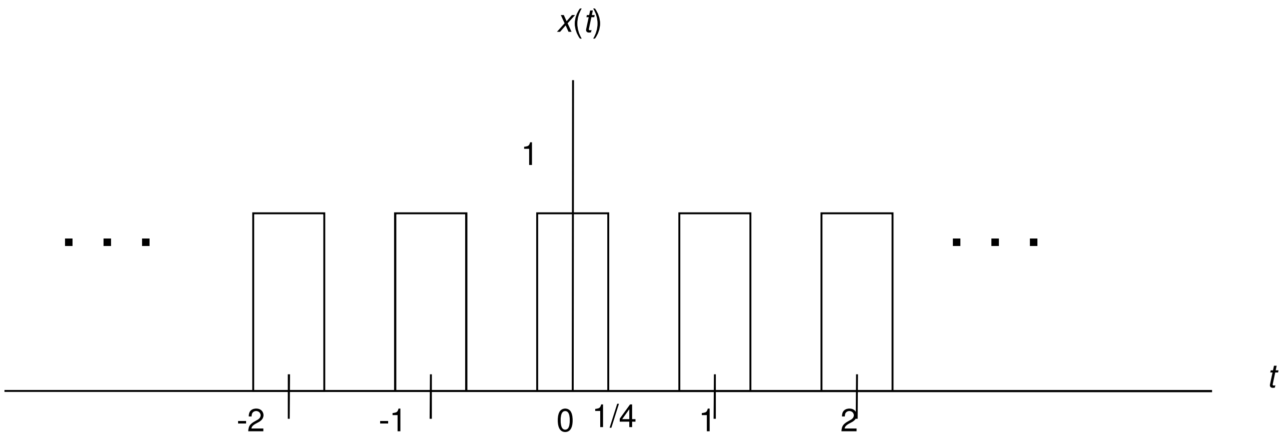
$$a_n = 2\,\mathrm{Re}[c_n] = \frac{2}{T} \int_{<T>} x(t) \cos n\omega_0 t dt$$

$$b_n = -2\,\mathrm{Im}[c_n] = \frac{2}{T} \int_{<T>} x(t) \sin n\omega_0 t dt$$

If $x(t)$ is an even function b_n = 0. If $x(t)$ is an odd function a_o = 0 and a_n = 0.

Example

Consider the periodic square wave $x(t)$ shown in figure 4.3. Find the Fourier Series coefficients for $x(t)$.



Dirichlet conditions

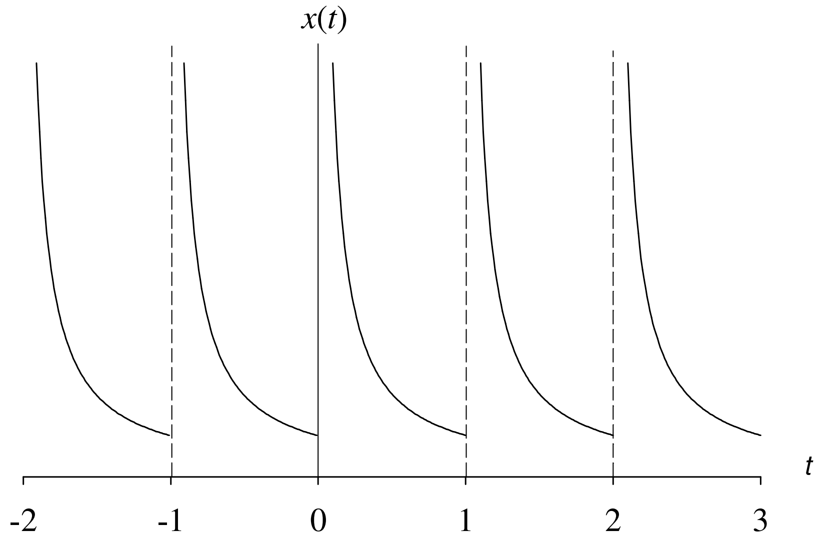
1. $x(t)$ is absolutely integrable over any period.

$$\boxed{\int_{<T>} |x(t)|dt < \infty}$$

which ensures that the Fourier Series coefficients will be finite since

$$|c_n| = \frac{1}{T}\int_{<T>} \left|x(t)e^{-jn\omega_o t}\right|dt = \frac{1}{T}\int_{<T>} |x(t)|dt$$

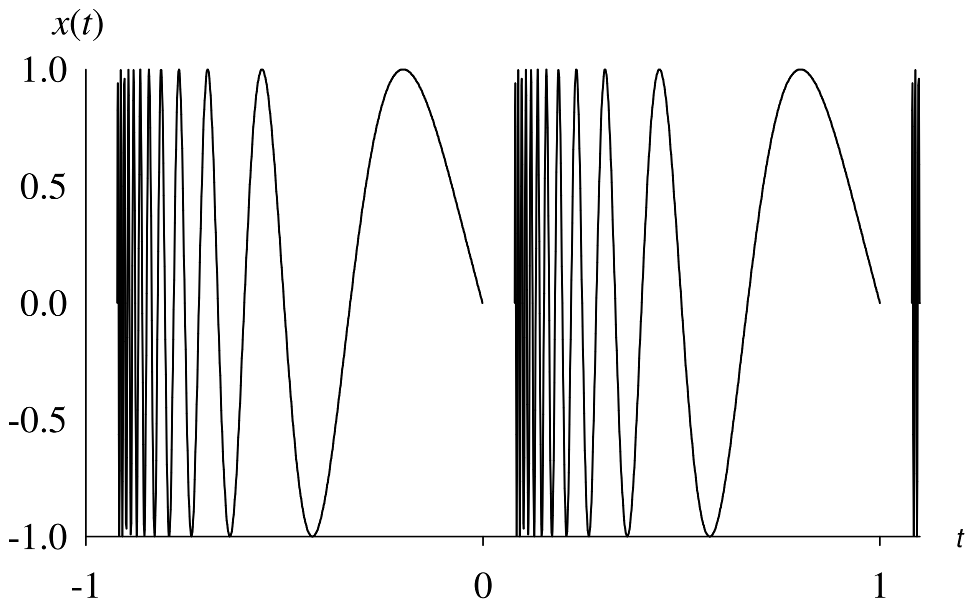
So if $\int_{<T>} |x(t)|dt < \infty \Longrightarrow |c_n| < \infty$



Dirichlet conditions

2. $x(t)$ has a finite number of maxima and minima over any period.

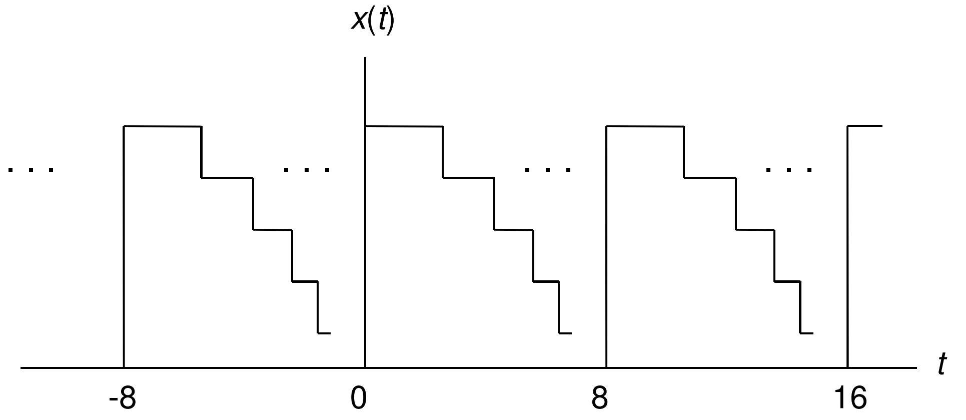
The signal shown below is absolutely integrable but has an infinite number of maxima and minima. $x(t) = sin\ (2\pi/t)$



$$\int_{<T>} |x(t)| dt < \infty$$

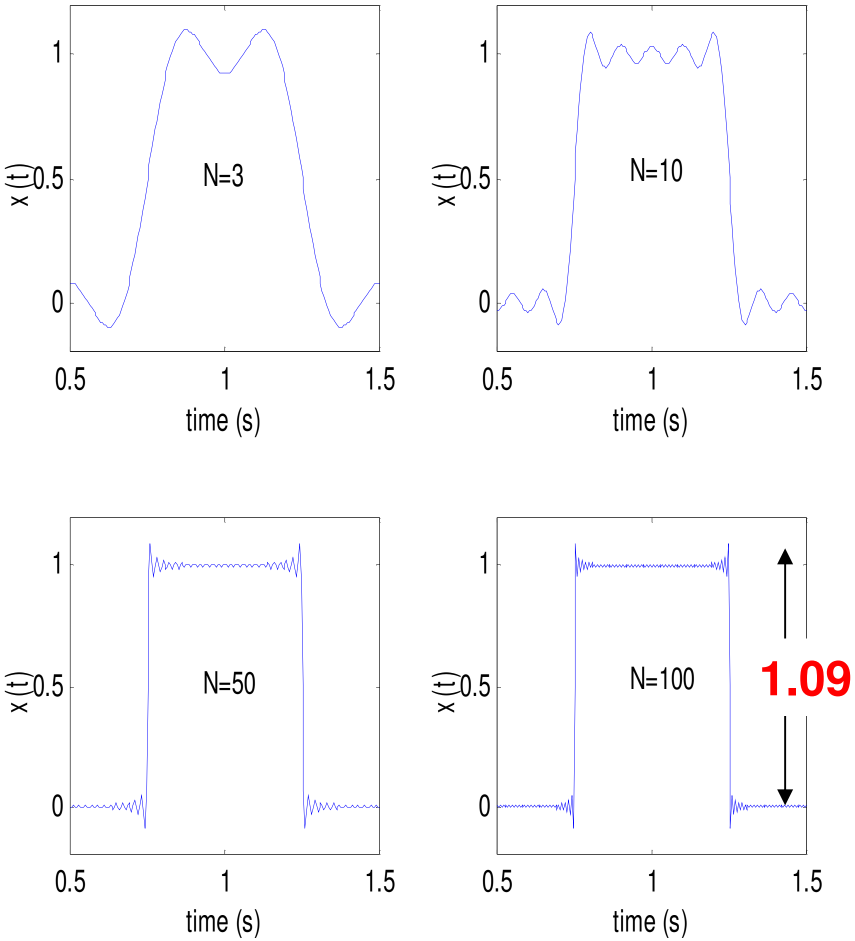
Dirichlet conditions

3. $x(t)$ has a finite number of discontinuities over any period.



$$\int_{<T>} \left| x(t) \right| dt < \infty$$

Gibbs phenomenon



- ripples reduced as the number of components *N* in the Fourier Series representation increases.
- an overshoot of 9% of the height of the discontinuity independent of *N*.
- this behaviour is known as **Gibbs** phenomenon.

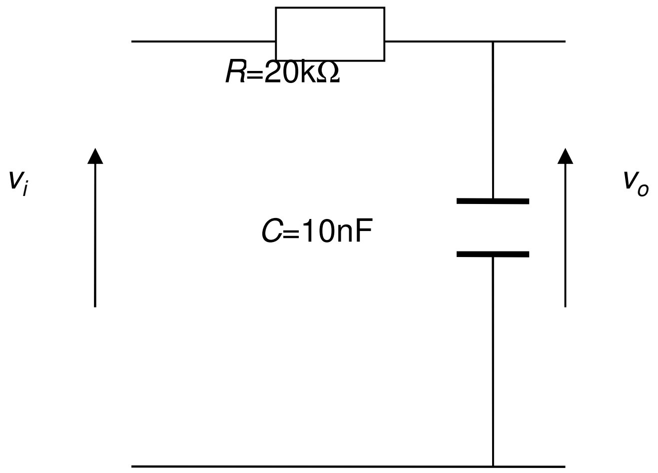
The implication is that the Fourier Series representation of a discontinuous signal, such as the square wave, will in general exhibits high-frequency ripples and overshoot near the discontinuity.

It is therefore necessary to use sufficiently large value of *N* if such approximation is used in practice, so that the total energy in the ripples is insignificant.

FS example

Consider a RC low pass filter shown in figure 4.8.

We can show that this RC circuit is a low pass filter by analysing the response of the circuit to the harmonics of a periodic signal. Consider an input signal shown in figure 4.8(b).



(a)