

EEE105 Tutorial Questions & Review Topics – Sheet 2

Fundamental Constants and Materials Data

Constants

Permittivity of free space = 8.8×10^{-12} F/m

Electron charge = 1.6×10^{-19} C

Electron mass = 9.1×10^{-31} Kg

Materials – Room Temperature

For silicon (Si);

Mobility = $0.12 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$

Electron effective mass = 0.97

For germanium (Ge);

Mobility = $0.39 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$

Electron effective mass = 0.22

Question 1

A parallel plate capacitor has a spacing $10 \mu\text{m}$ with air between them. The area of the capacitor is $1 \times 10^{-5} \text{ m}^2$.

- (a). If a dielectric plate of relative permittivity $\epsilon_r = 10$ is placed between the plates what should the new spacing be to leave the capacitance unchanged?
- (b). Calculate the capacitance?
- (c). The dielectric in the capacitor in question 1 has a breakdown field of 50 MVm^{-1} . Calculate the maximum voltage which may be applied to the capacitor.
- (d) The capacitor is connected in series with a 50Ω resistor. Calculate the cut-off frequency for this filter.

Question 2

- (a). Calculate the average scattering time of electrons at room temperature in Si and Ge based upon their mobilities and effective masses.
- (b). If these crystals are ultra-pure, what is the origin of these scattering events?

Review Topics – Keywords

Dielectric. Polarization. Capacitance. RC time constant.

Derivation of drift velocity and mobility of a conductor. Impurities, Defects & Phonons.

Solutions

1(a). Call the situation with air “state 1”, and with dielectric “state 2”. i.e in state 1 we have a relative permittivity ϵ_{r1} , and plate separation d_1 . (A is constant in both cases). As we aim to keep capacitance constant, we can equate capacitance in state 1 and state 2.

$$C = \frac{\epsilon_0 \epsilon_{r1} A}{d_1} = \frac{\epsilon_0 \epsilon_{r2} A}{d_2}$$

giving –

$$d_2 = \frac{\epsilon_{r2}}{\epsilon_{r1}} d_1 = \frac{10}{1} \times 1 \times 10^{-5}$$

$$d_2 = 100 \mu\text{m}$$

So the plate separation with dielectric needs to be 100 μm .

(b)

$$C = \frac{\epsilon_0 \epsilon_{r2} A}{d_2}$$

So –

$$C = \frac{8.8 \times 10^{-12} \times 10.1 \times 10^{-5}}{1 \times 10^{-4}}$$

$$C = 8.8 \times 10^{-12} \text{ F} = 8.8 \text{ pF}$$

(c) $E = V/d$ so

$$\begin{aligned} V_{\text{max}} &= E_{\text{Breakdown}} \times d_2 \\ &= 50 \times 10^6 \text{ Vm}^{-1} \times 10^{-4} \text{ m} = 5 \text{ kV} \end{aligned}$$

(d) The cut-off frequency is given by the RC time-constant;

$$f_c = \frac{1}{2\pi RC}$$

so

$$f_c = \frac{1}{2\pi \times 50 \times 8.8 \times 10^{-12}} = 3.6 \times 10^8 \text{ Hz} = 360 \text{ MHz}$$

2. The mobility and scattering time are related by;

$$\mu = \frac{q\tau}{m^*}$$

so

$$\tau = \frac{\mu m^*}{q}$$

Remembering that m^* is the electron mass multiplied by the effective mass....

So for Si

$$\tau = \frac{\mu m^*}{q} = \frac{0.12 \times 0.97 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}} = 6.6 \times 10^{-13} \text{ s} = 660 \text{ fs}$$

And for Ge

$$\tau = \frac{\mu m^*}{q} = \frac{0.39 \times 0.22 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}} = 4.9 \times 10^{-13} \text{ s} = 490 \text{ fs}$$

(b) The origin of the electron scattering events are disruptions to the perfection of the crystal lattice. This can be through crystal defects such as; ionised impurities, interstitial defects, vacancy defects, and through the disruption of the lattice by lattice vibrations (heat/sound) – these are quantized and are known as “phonons”.

If the crystals are ultra-pure (i.e. defects and unintentional impurities are essentially zero) then the only major source of electron scattering is with phonons.