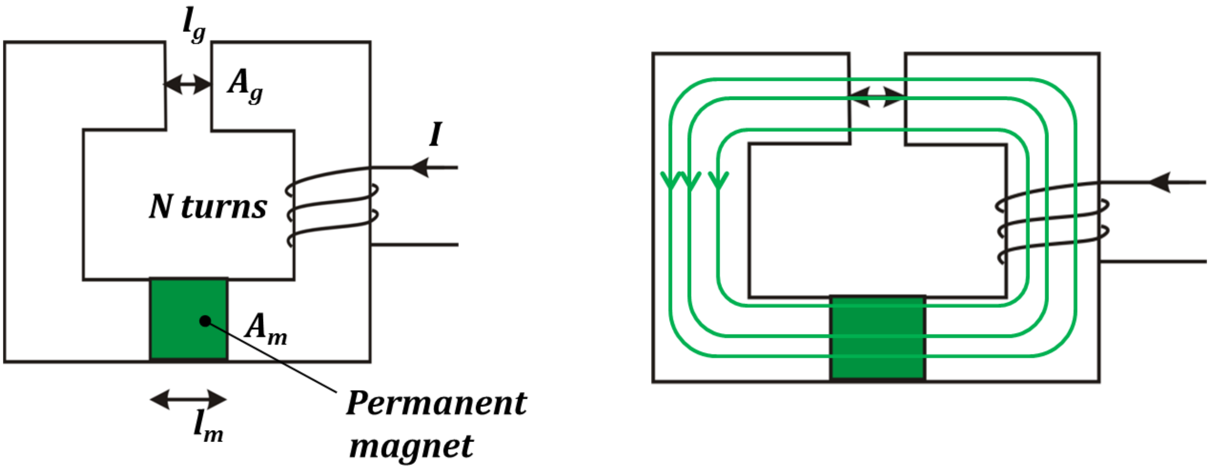
Answers to questions

Answers to question 1:

(a), as shown in the figure, from **Ampere's Law**:

$$\oint Hdl = \sum I$$

$$H_m l_m + H_g l_g = -NI$$



And from **Gauss's Law:**

$$\oint Bds = 0$$

$$B_m A_m = B_g A_g$$

(1)

Demagnetisation characteristic of magnet

$B_m = \mu_0 \mu_r H_m + B_r$ (for linear part)

Therefore, under open-circuit conditions, we have

$$B_g = \frac{A_m}{A_g} B_m = \frac{A_m}{A_g}(\mu_0 \mu_r H_m + B_r) = \frac{A_m}{A_g}\left(-\frac{\boldsymbol{H_g l_g}}{\boldsymbol{l_m}}\mu_0 \mu_r + B_r\right)$$

With $B_g = \mu_0 H_g$, and Replacing H_g using $\frac{B_g}{\mu_0}$, we have:

$$B_g = \frac{A_m}{A_g} B_m = \frac{A_m}{A_g} \frac{B_r}{1 + \mu_r \frac{l_g A_m}{l_m A_g}} = \frac{B_r}{\frac{A_g}{A_m} + \mu_r \frac{l_g}{l_m}}$$

The average air-gap radius is: $R_g = R_i - L_g/2$ = 28.5-0.5=28 mm,

The average magnet radius is: $R_g = R_i - L_g - L_m/2$=28.5-1-1.5=26 mm, therefore,

$$A_g/A_m=R_g/R_m=28/26=1.08.$$

And the peak air-gap flux density is equal to the average flux density B_g = 1.2/(1.08+1/3)=0.84 T. **(3)**

The possible ways to increase the air-gap flux density is to reduce the Ag/Am ratio by using V-shaped IPM machine (flux focusing effect), or reduce the Lg/Lm ratio by increasing permanent magnet thickness and reducing the air-gap length. **(2)**

(b), the slot number is N_s = 12,

The slot pitch is $\tau_s = \frac{2\pi R_i}{N_s} = 14.9mm$, and

$$B_{tooth} = \frac{\tau_s}{t_w} B_g = \frac{14.9}{t_w} \times 0.84 \leq 2$$

Therefore,

$$\frac{14.9}{2} \times 0.84 \leq t_w, \text{and } t_w \geq 6.258\, mm$$

The tooth width cannot be too large, because it will make the slot area too small, and hence compete with the electrical loading. **(4)**

(c), Pole number 2p = 10, therefore, the pole pitch is

$$\tau_p = \frac{2\pi R_i}{2p} = 17.9mm$$

And we have the peak flux density in the stator yoke such as

$$B_{core} = \frac{B_g \tau_p}{2d_c} = \frac{0.84 \times 17.9}{2d_c} \leq 2$$

Therefore,

$\frac{0.84 \times 17.9}{4} = 3.76mm \leq d_c$ **(3)**

Similar to tooth width, the stator yoke thickness cannot be too large either, because it will also reduce the slot area, and hence compete with the electrical loading Q. **(1)**

(d), the electrical loading is Q = NI/(2π*R_i)=1000/(3.14*57)=5.58 A/mm = 5580A/m. The major issues of increasing electrical loading Q are the overheating and magnet demagnetization. **(3)**

(e), the electromagnetic torque is $T = \frac{\pi}{2}(2R_{ro})^2 LBQ = \frac{\pi}{2}(0.055)^2 \times 0.05 \times 0.84 \times 5580 = 1.11\ Nm$. **(3)**

Answers to question 2:

(a), From the data given, current loading $Q = \frac{N_s A_s J K_p}{\pi D} = \frac{15\times30\times8\times0.5}{\pi\times30} = 19.1 \times 10^3 A/m$

At ***P* = 600W @ 5000rpm**, torque $T = P/\Omega$, [Ω in rad/s], then

$$T = \frac{600}{\frac{5000 \times 2\pi}{60}} = 1.15\ Nm$$

Hence, B loading required can be calculated from $T = \frac{\pi}{2} D^2 LBQ$

Giving B = 0.53 T (average) **(2)**

Since the B field is around and concentrated under the magnets, then

$B_m = B \times \frac{\pi}{\alpha} = 0.53 \times \frac{180}{140} = 0.68\ T$ **(2)**

B_m can be obtained either graphically using expression $B_m = \frac{B_r}{\left(1+\mu_r \frac{A_m l_g}{A_g l_m}\right)}$

Giving $l_m = \frac{B_m \mu_r l_g}{(B_r - B_m)} \frac{A_m}{A_g} = 1.8\ mm = 1.8 \times 10^{-3}\ m$ **(2)**

(b), the possible ways to increase air-gap flux density are as following:

- Use better magnet material that has higher remanence,
- When l_m is small, the increase in l_m leads to important increase in B_g,
- When l_m is big, B_g is almost constant,
- Increase A_m can further boost the B_g. **(3)**

(c), **Either:**

Check that magnet will not be demagnetised by the general relationship

$H_m = -\frac{NI\beta}{l_m(\beta+\mu_r)} - \frac{B_r}{\mu_0(\beta+\mu_r)}$ with $\beta = \frac{l_m A_g}{l_g A_m} = \frac{1.8\times180}{1\times140} = 2.31$ **(3)**

where (NI) is given by

$$NI = \frac{Q'\pi D}{4p} \times \frac{\alpha}{\pi}$$

Q'=overload×Q=3×19.1×10^3 A/m, then we have

$H_m = -\frac{(3\times19.1\times10^3)\times\pi\times30\times10^{-3}}{8}\times\frac{140}{180}\times\frac{2.31}{1.8\times10^{-3}\times(2.31+1.1)}-\frac{1.0}{4\pi\times10^{-7}\times(2.31+1.1\times1)}\approx-431\times 10^3$ (A/m) **(2)**

i.e. $H_m < H_{lim}$ $\therefore$ magnet will not be irreversibly demagnetized. **(1)**

<u>Or using:</u>

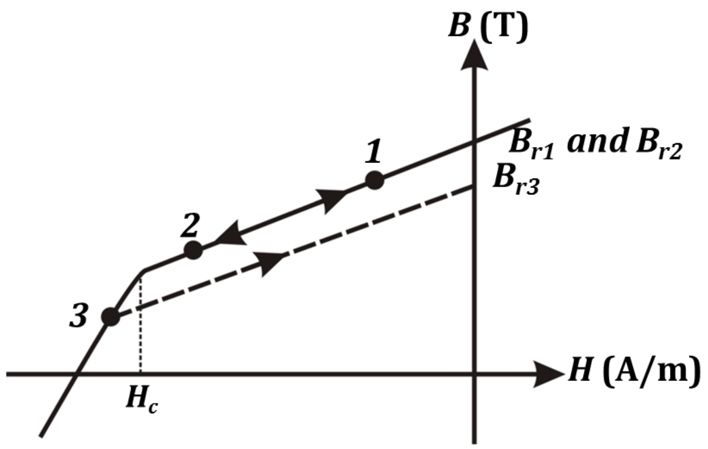
$l_{m(\text{lim})} = -\frac{NI}{H_{lim}} - \frac{B_r}{\mu_0 H_{lim}} l_g \frac{A_m}{A_g} - \mu_r l_g \frac{A_m}{A_g}$ **(3)**

Where $NI = \frac{Q'\pi D}{4p} \times \frac{\alpha}{\pi} = 700 \times 10^3 A$

which gives a minimum magnet thickness $l_{m(\text{lim})} = 0.998\, mm$, therefore **(2)**

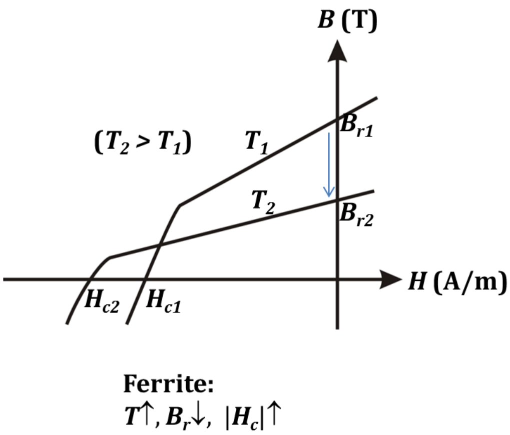
The magnets are OK since l_m = 1.8mm > 0.998 mm for required B loading. **(1)**

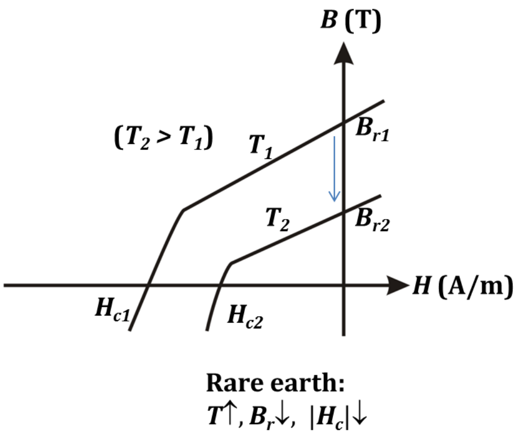
(d), The reversible and irreversible demagnetization curves are shown:



With the increase in demagnetizing field (H), the flux density in permanent magnet B decreases. However, if H does not exceed the H_c such as between points 1 and 2, the demagnetization of magnets can be recovered when $-H$ decrease. However, if the working point of magnet is beyond the knee point ($H < H_c$) such as at point 3, then, when $-H$ decrease, the demagnetization cannot be recovered and the magnet remanence reduce from $B_{r1} = B_{r2}$ to B_{r3}. **(3)**

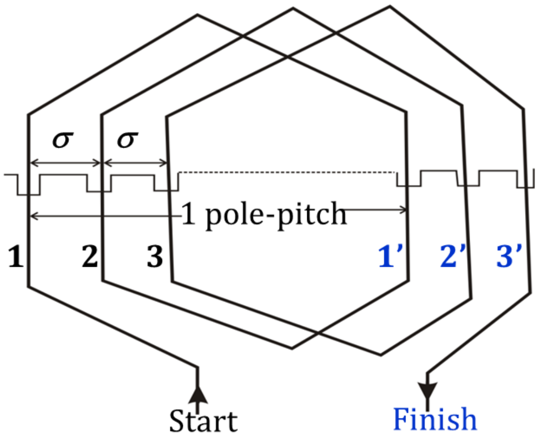
(e), The temperature rise will normally aggravate the irreversible demagnetization, for ferrite, it will decrease the magnet remenance but will increase the H_c. However, for rare earth such as NdFeB, the temperature reduces not only Br but also H_c, making the irreversible demagnetization much worse. **(2)**

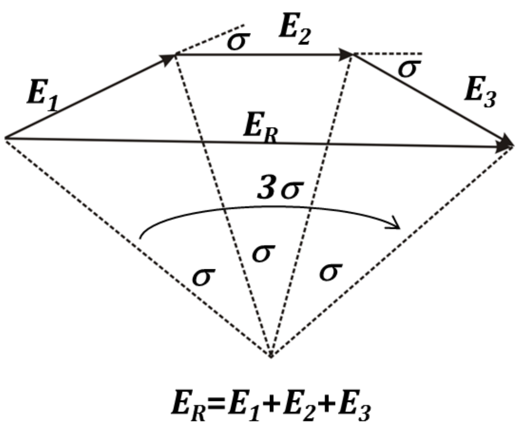




Answers to question 3:

(a), The layout of winding and the EMF vectors of coils are shown:





(1)

Assuming we have m = 3 coils per phase, and $|E_1| = |E_2| = |E_3| = |E_m|$ (all the coils are identical).

Then, from the construction (***E****m* = ***E****1*), we have

$$E_m = 2r\sin\frac{\sigma}{2} \quad \text{and} \quad E_R = 2r\sin\frac{m\sigma}{2}$$

The arithmetic sum of all coil EMFs: $mE_m = m2r\sin\frac{\sigma}{2}$

However, the vector sum of all coil EMFs: $E_R = 2r\sin\frac{m\sigma}{2}$ **(1)**

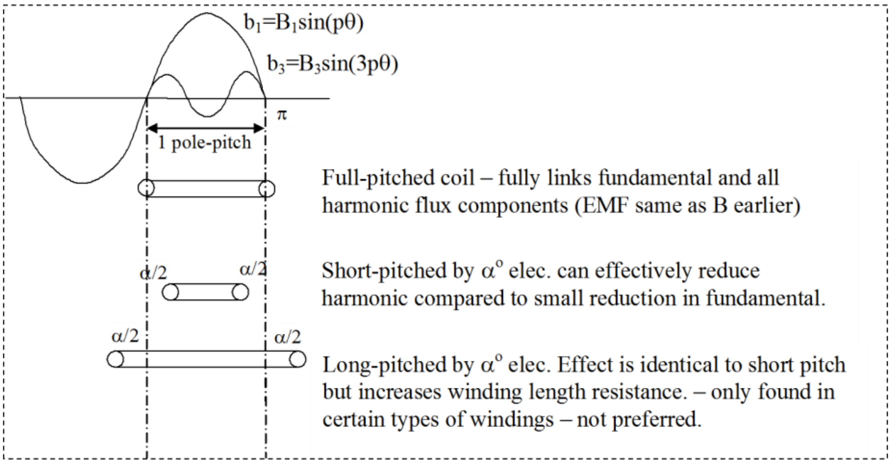
Therefore, the distribution factor for the fundamental is:

$$\boxed{k_d = \frac{\textit{effective induced emf}}{\textit{arithmetic induced emf}} = \frac{E_R}{mE_m} = \frac{\sin\frac{m\sigma}{2}}{m\sin\frac{\sigma}{2}}}$$

By using the similar approach, the distribution factor for the nth harmonic is:

$$\boxed{k_{dn} = \frac{\sin\frac{mn\sigma}{2}}{m\sin\frac{n\sigma}{2}}} \qquad \textbf{(1)}$$

The pitch factor then can be calculated based on the following graph:



k_p is defined as: $\dfrac{effective\ \ EMF}{EMF\ of\ \ full-pitch\ coil} \propto \dfrac{effective\ \ flux\ linkage}{flux\ linkage\ of\ \ full\ \ pitch\ coil} = \dfrac{\Psi_s}{\Psi_F}$

For a short pitch coil:

$$\Psi_s = \int_{\alpha/2}^{\pi-\alpha/2} \hat{B}\sin\theta d\theta = 2\hat{B}\cos\frac{\alpha}{2} \qquad \textbf{(1)}$$

And for full pitch coil:

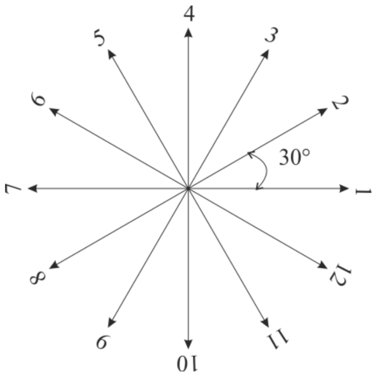
$$\Psi_F = \int_0^{\pi} \hat{B}\sin\theta d\theta = 2\hat{B} \qquad \textbf{(1)}$$

Therefore, the pitch factor is:

$$\boxed{k_p = \frac{\Psi_s}{\Psi_F} = \frac{2\hat{B}\cos\frac{\alpha}{2}}{2\hat{B}} = \cos\frac{\alpha}{2}}$$

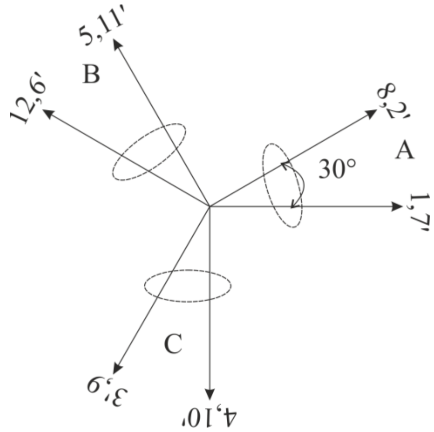
Similarly, the pitch factor for long pitch is: $k_p = \cos\dfrac{\alpha}{2}$ **(1)**

(b), For a 12-slot/14-pole double layer permanent magnet machine which has non-overlapping concentrated winding, there are 12 coils allow us to establish a 3-phase winding structure. This means each phase will only have 4 coils. The coil vectors are shown in the following graph:



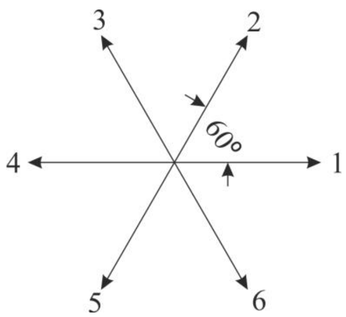
(1)

Therefore, the coil connection (some EMF vectors have been reversed to achieve the highest distribution factor) for a maximum distribution factor should be as:



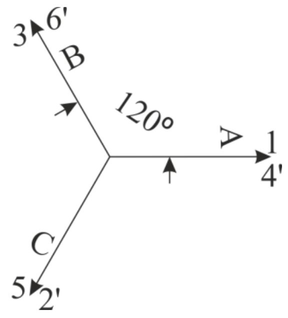
(3)

(c), For a 12-slot/14-pole alternate teeth wound permanent magnet machine which has non-overlapping concentrated winding, there are 6 coils allow us to establish a 3-phase winding structure. This means each phase will only have 2 coils. The coil vector and coil EMF vector of this machine are the same and shown in the following graph:



(1)

Therefore, the coil connection for a maximum distribution factor should be as:



(2)

(d), **Single layer windings over double layer windings:**

- Higher winding factor, **(1)**
- Higher self inductance but lower mutual inductance and hence higher fault tolerance capability, **(1)**
- Physical separation between coils
- Higher saturation level, lower torque at high phase currents **(1)**
- Potentially lower power factor **(1)**

(e), the concentrated winding often refers to the winding that is wound around only one single stator tooth, so two sides of the coil span one slot pitch, leading to short end-winding, but its MMF often contains rich harmonics. **(1.5)**

However, the distributed winding often refers to the winding that has a coil span of a few slot pitches, leading to longer end-windings. This can effectively reduce MMF harmonics and also achieve high winding factor (sometimes a winding factor of 1 can be achieved). **(1.5)**

<u>Answers to question 4:</u>

(a), Neglect mmf drop in the iron and assume that all fluxes cross the airgap and coil regions radially.

<u>Opening section</u>

mmf across opening=total slot mmf=NI, where N=no of conductors, I=current per conductor

$\therefore$ flux across opening/unit length of machine

$$\Phi = \frac{mmf}{\frac{g_1}{\mu_0 h_1}} \text{ /unit length } = NI\mu_0\left(\frac{h_1}{g_1}\right)$$

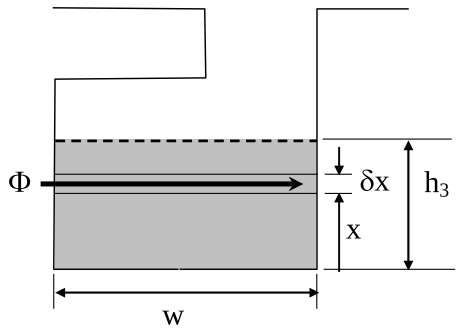
& flux linkage /unit length of machine ψ = Φ × No of conductor linked= $N^2 I\mu_0\left(\frac{h_1}{g_1}\right)$

Inductance/unit length $\quad L = \frac{\Psi}{I} = N^2\mu_0\left(\frac{h_1}{g_1}\right)$ **(1)**

<u>Unwound section (above conductors)</u>

Similarly, inductance/unit lenght $L = N^2 \mu_0 \left(\frac{h_2}{w} \right)$ **(1)**

Wound section



The mmf is distributed throughout the section & we need to integrate across the depth h_3.

Consider elemental strip depth δ_x at x from bottom of winding:

Mmf available below strip = $NI \times \frac{x}{h_3}$

Flux Φ through strip

$$= \frac{mmf}{reluctance} = NI \frac{x}{h_3} \Big/ \frac{w}{\mu_0 \delta_x}$$ /unit length

& flux linkage = $N^2 I \frac{x^2}{h_3^2} \frac{\mu_0 \delta_x}{w}$

Hence, effective flux linkages for total section = $\frac{N^2 I \mu_0}{h_3^2 w} \int_0^{h_3} x^2 dx = N^2 I \mu_0 \left(\frac{h_3}{3w} \right)$

And inductance= $N^2 \mu_0 \left(\frac{h_3}{3w} \right)$ **(3)**

Total inductance will be of the sum of above inductance components

Inductance/unit length of machine= $N^2 \mu_0 \left[\frac{h_1}{g_1} + \frac{h_2}{w} + \frac{1}{3} \frac{h_3}{w} \right]$ **(1)**

(b), Similar to the case in (a), the inductance/unit length of machine can be derived (the procedure may not be given by utilising the derived results in (a))

$N^2 \mu_0 \left[\frac{h_1}{g_2} + \frac{1}{3} \frac{(h_2 + h_3)}{w} \right]$ **(3)**

Therefore, the difference in the winding inductance is

$$N^2\mu_0\left[\frac{h_1}{g_1}+\frac{h_2}{w}+\frac{1}{3}\frac{h_3}{w}\right]-N^2\mu_0\left[\frac{h_1}{g_2}+\frac{1}{3}\frac{(h_2+h_3)}{w}\right] \quad \textbf{(1)}$$

(c), Open slots are often used for large machines, they have the following pros and cons:

- Easy for winding (by machine),
- Compact winding,
- Higher packing factor(0.7),
- High PM eddy current loss
- High conductor eddy current losses
- High cogging torque **(3)**

However, semi-close slot is often used for small and medium size machines, they have the following pros and cons:

- Difficult for winding
- Lower packing factor (0.3-0.4)
- Low PM and conductor eddy current losses
- Low cogging torque
- Better magnetic performances **(3)**

(d), the winding leakage inductance can be increased by adopting semi-closed slot with larger slot depth. Moreover, the increase in number of turns can increase significantly the winding inductances. **(4)**