EEE337/348: Tutorial 4

1) i) One of the limitations of LED is the internal reflection at the semiconductor-air interface. The refractive index of GaAs is 3.66 while that of the air is 1.0. Calculate the reflection coefficient for normal incidence.
The reflection coefficient is given by

$$R_{reflect} = \left(\frac{n_{semiconductor} - n_{air}}{n_{semiconductor} + n_{air}} \right)^2 = \left(\frac{3.66 - 1}{3.66 + 1} \right)^2 = 0.33$$

Therefore 33% of light is reflected.

ii) Calculate the incident angle for the total internal reflection, which is another loss mechanism in LEDs.

The total internal reflection occurs when the angle of incident is greater than the critical angle

$$\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_{semiconductor}} \right) = \sin^{-1}\left(\frac{1}{3.66} \right) = 15.9^\circ .$$

2) Consider a GaAs pn diode with the following parameters
Electron diffusion coefficient, D_e = 30 cm^2/V-s
Hole diffusion coefficient, D_h = 15 cm^2/V-s
p-doping, N_a = 5x10^{16} cm^{-3}
n-doping, N_d = 5x10^{17} cm^{-3}
Electron minority carrier lifetime, τ_e = 10^{-8} s
Hole minority carrier lifetime, τ_h = 10^{-7} s

i) Calculate the injection efficiency of the GaAs LED, assuming no recombination due to traps.

The injection efficiency is given by (assuming no recombination due to traps, J_{GR} = 0)

$\gamma_{inj} = \frac{J_e}{J_e + J_h}$. Therefore we need to calculate J_e and J_h.

$J_e = \frac{qD_e n_p}{L_e} \exp\left(\frac{qV}{kT} \right)$ and $J_h = \frac{qD_h p_n}{L_h} \exp\left(\frac{qV}{kT} \right)$. Now we need to calculate n_p (minority electron concentration in p-layer) and p_n (minority hole concentration in n-layer).

Assuming and intrinsic concentration is 2x10^6 cm^{-3},

$$n_p = \frac{n_i^2}{N_a} = \frac{(2\times10^6)^2}{5\times10^{16}} = 8\times10^{-5} cm^{-3} \text{ and } p_n = \frac{n_i^2}{N_d} = \frac{(2\times10^6)^2}{5\times10^{17}} = 8\times10^{-6} cm^{-3}$$

Next, we need to calculate the diffusion lengths

$$L_e = \sqrt{D_e \tau_e} = \sqrt{30\times10^{-8}} = 5.47\mu m \text{ and } L_h = \sqrt{D_h \tau_h} = \sqrt{15\times10^{-7}} = 12.25\mu m$$

Using the calculated minority carrier concentrations and diffusion lengths

$$\gamma_{inj} = \frac{\dfrac{qD_e n_p}{L_e}}{\dfrac{qD_e n_p}{L_e} + \dfrac{qD_h p_n}{L_h}} = \frac{\dfrac{30\times8\times10^{-5}}{5.47\times10^{-4}}}{\left(\dfrac{30\times8\times10^{-5}}{5.47\times10^{-4}}\right)+\left(\dfrac{15\times8\times10^{-6}}{12.25\times10^{-4}}\right)} = 0.98 .$$

ii) The electron injected in the p-region, on the top, will be responsible for the photon generation. Calculate the injection current at a forward bias of 1 V. Assume the diode has an area of $1mm^2$.

The current density injected into the p-region is $J_e = \frac{qD_e n_p}{L_e}\exp\left(\frac{qV}{kT}\right)$

$$J_e = \frac{(1.6\times10^{-19}C)\times(30cm^2/s)\times(8\times10^{-5}cm^{-3})}{5.47\times10^{-4}cm}\exp\left(\frac{1}{0.026}\right) = 35mA/cm^2$$

The current is therefore $I_e = AJ_e = 10^{-2}cm^2 \times 35mA/cm^2 = 0.35mA$

iii) If the GaAs LED has a radiative recombination efficiency of 0.5, calculate the photons generated per second.

The photons generated per second are $\frac{I_e}{q}\eta = \frac{0.35\times10^{-3}A\times0.5}{1.6\times10^{-19}C} = 1.09\times10^{15}s^{-1}$

3) Describe a typical structure of a GaN LED that produces high power and narrow emission spectrum.

Due to the lack of high quality and affordable GaN substrate, GaN LED is grown on lattice mismatch substrate such as sapphire. Consequently, thick bulk layer of GaN cannot be grown without high level of crystal defects, which act as non-radiative recombination centers. To overcome this, quantum well LED is adopted. Typically a thin layer of $In_xGa_{1-x}N$ of a few nm is sandwiched between two layers of $In_xGa_{1-x}N$ with a wider bandgap. The composition x is chosen to form a quantum well, so that transitions between the discrete energy levels in the quantum well produces the desired emission wavelength. For instance $In_{0.2}Ga_{0.8}N$ can produce emission wavelength of 450 nm. The discrete energy levels also produces narrow emission spectrum compared to the bulk LED.
To increase the power, multiple quantum wells are incorporated into the active region. Typically waveguiding action, due to the cladding p and n layers is also reduced (for instance by roughening the surface) to increase the output power. The roughened surface can also be fabricated such that it helps to guide the light emission in the vertical direction. The substrate is removed and the LED is mounted on high thermal conductivity sub-mount to improve heat dissipation (since heat reduces the output power). The ohmic contacts are fabricated from the bottom to allow light emission from the top surface.

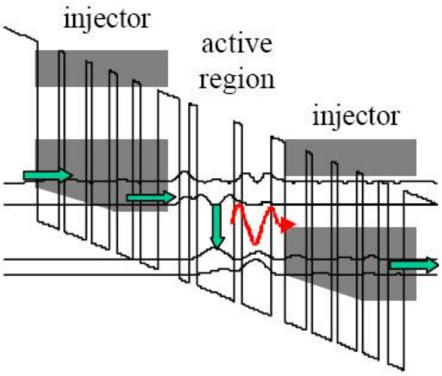
4) Recombination due to traps and Auger process limits the efficiency of a laser. In mature semiconductors such as GaAs and InP, the defect density is low so that recombination due to traps can be kept low. On the other hand the Auger process depends on the intrinsic band structure. What are the material parameters that control the Auger recombination?

Auger recombination depends in the electron and hole concentrations which are dependent on the intrinsic carrier concentration. Since the intrinsic carrier concentration increases exponentially with decreasing semiconductor bandgap and increasing temperature, the Auger rate also increases exponentially with decreasing bandgap and increasing temperature. The effective mass, the parabolicity of the bands in the bandstructure and defect concentration can also influence the Auger rate, but are typically weaker than the effects of bandgap and temperature.

5) Explain why fabrication of room temperature laser is increasingly difficult at infrared wavelengths. Propose how semiconductor infrared laser can be fabricated.

As discussed in question (4) Auger rate increases with decreasing bandgap and increasing temperature. Therefore when the bandgap is small, such as that of InSb, Auger recombination is a dominant non-radiative recombination process that make room temperature infrared laser inefficient. Cooling is used in some bulk infrared laser to reduce the Auger rate but this increases the complexity of the laser packaging and hence the cost of the laser. In addition there is a lack of wide bandgap material that is lattice matched to InSb, such that double heterostructure design to achieve low threshold current is not available.

A much more efficient approach to fabricate the infrared laser is to use thin layers of wider bandgap material, grown into quantum well structures. The quantum cascade laser (QCL) is an important laser for the 3-5 μm, 8-12 μm and even in THz. The band diagram is shown below.



The energy states are dependent on the layer thickness of the quantum well. Note that only electrons are involved in the recombination process. In the injector region, the quantum wells are designed such that under appropriate biasing condition, the energy states are aligned to allow electron to tunnel through the barriers. This enables the electron to move from a low energy state to a higher energy state.

The electron is subsequently injected into the active region containing quantum well with energy state separation that will produce the desired wavelength when the electron make the intra-band transition from the high to low energy states. After the photon emission the electron from the low energy state is transported to the injector region and the process is repeated. Typically the injector and active regions are repeated for 20-100 times to produce a QCL

6) To achieve high radiative recombination, it is necessary to achieve population inversion. Discuss strategies to reduce the threshold current required to achieve population inversion.

The threshold current can be reduced by

i) Ensuring that high concentrations of electrons and holes are confined within the active region, so that they can recombine. The simplest approach is to make use of a double heterostructure laser.

ii) Using a thinner active region since the threshold current is given by

$$J_{th} = \frac{qd_{las}n_{th}}{\tau_r(J_{th})}.$$

iii) Using a quantum well laser. This is a much better approach than the double heterostructure laser. Because of the significantly higher density of states at the discrete energy levels involved in the radiative emission, higher electron and hole concentrations are achieved at low injection current, leading to low threshold current. As the quantum is also very thin, lower threshold current is also achieved as described in (ii).

iv) Using nanostructures such as quantum dots. The higher density of states at a given energy can be exploited to reduce the threshold current.

7) List the advantages of multiple quantum well lasers over bulk lasers.

Key advantages if multiple quantum well lasers are

i) Lower threshold current as discussed in 6(iii).

ii) Narrower emission spectrum.

iii) When combined with selected cavity designs, multi-wavelengths corresponding to the different transitions in the quantum well system, can be produced.

iv) The bandgap of the quantum well and the width of the quantum well can be adjusted to control the emission wavelength.

v) Strained quantum well can be employed to grow either achieve emission using lattice mismatched materials (such as InGaN-GaN) or to reduce non-radiative Auger recombination process (such as InGaAs-GaAs quantum well laser).