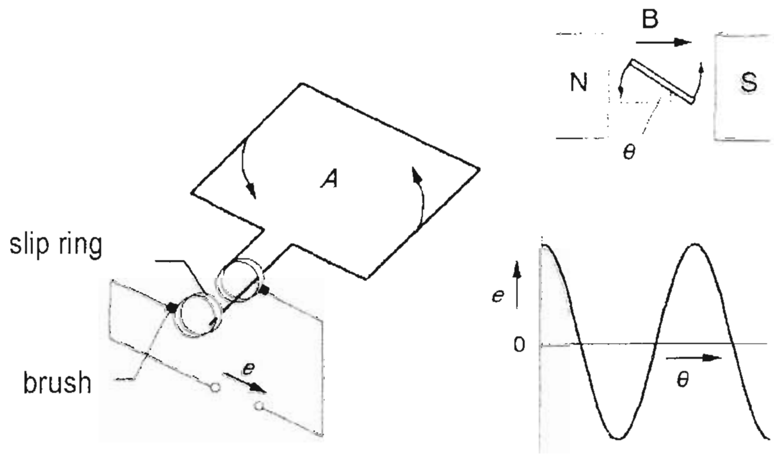
DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2006-2007

Electromechanical Energy Conversion EEE202

Examination Solutions

1a)



If the coil is at an angle, θ, to the field, its area normal to **B** is A sin θ and the e.m.f. is

$$e = d\Phi/dt = BAd(\sin\theta)/dt \text{ by Faraday's law.}$$

But we can write

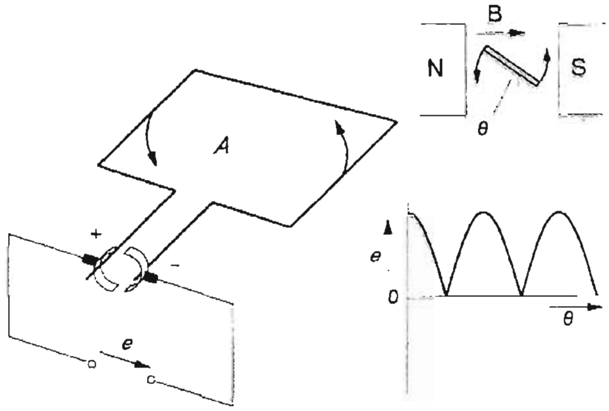
$$\frac{d}{dt} = \frac{d}{d\theta}\frac{d\theta}{dt} = \omega\frac{d}{d\theta}$$

since $d\theta/dt = \omega$, hence

$$e = BAd(\sin\theta)/dt = BA\omega\, d(\sin\theta)/d\theta = BA\omega\cos\theta$$

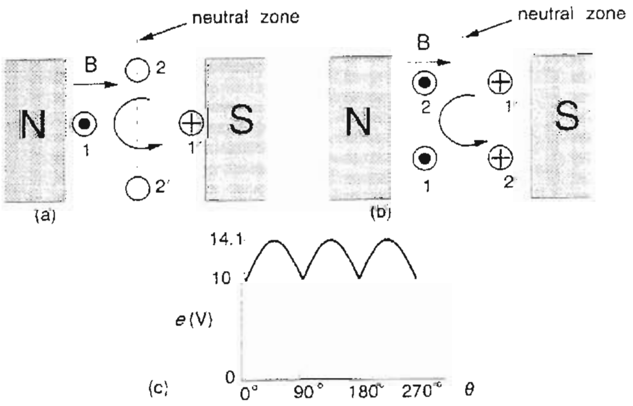
The e.m.f. produced by the rotating coil is sinusoidal, reaching its maximum value of BAω when θ = 0° (when B lies in the plane of the coil) and falling to zero when θ = 90° (when B is normal to the plane of the coil).

1b) At the point in the rotation of the coil when the current in brush A would reverse, the brush breaks contact with one split ring and makes contact with the other. Thus brush A is in contact only with the conductor moving down through the magnetic field on the left and must therefore be connected to the positive terminal of the generator. The coil e.m.f. is made up of a positive e.m.f. from the arm of the coil which passes in front of the N pole and an equal negative e.m.f. from the coil which passes a S pole. The total e.m.f. of the generator, though unidirectional, is sinusoidal and varies in magnitude from zero to a maximum of BAω



The large ripple of this rectified sinewave could be reduced by employing more coils connected in series, each with its own commutator segment. For example, a 2-coil, 2-pole generator, which we shall suppose gives an e.m.f. of 10 V in each coil when it passes the centre of a pole face. When the coils are in the position shown in figure a, the e.m.f. from coil 1 is 10 V while that from coil 2 is zero and the combined e.m.f. will be 10 V. Coil 2 gives no e.m.f. as it is in the magnetically neutral plane, or in the *neutral zone* of the generator.

On rotating a further 10°, the e.m.f. of coil 1 will be 10 sin 80°, while that of coil 2 will be 10 sin 10°, for a total e.m.f. of 10(sin80° + sin 10°) or 11.6 V. Clearly, the maximum e.m.f. will be obtained from the pair of coils when they are in the position shown in figure b, which is 10(sin45° + sin45°) = 14.1 V. The peak-to-peak ripple has been reduced to 29% compared to 100% with one coil.



1c) The assumption has been made that the machine has no losses, and so consequently disregards-

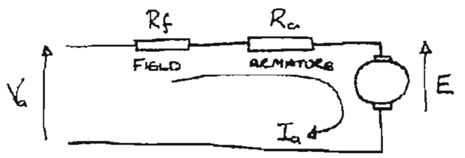
- Copper losses (i^2r losses in the windings)

- Iron Losses (due to eddy currents)
- Windage (due to air resistance against the rotor)
- Frictional / bearing losses

2a

SERIES D.C. MOTOR.

(5x)



FIELD CONNECTED IN SERIES WITH THE ARMATURE.

$$\Rightarrow I_f \; I_a$$

$$T = M I_f I_a = M I_a^2$$

$$E = M I_f \omega = M I_a \omega$$

NEGLECTING THE VOLTAGE DROP ACROSS $R_a + R_f$

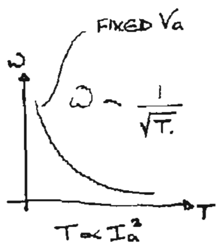
$$E \simeq \text{SUPPLY } V_a$$

$$V_a \simeq M I_a \omega$$

$$T = M I_a^2$$

$$V_a \simeq M\sqrt{T/M}\;\omega$$

$$\omega \approx \frac{1}{\sqrt{M}\sqrt{T}} V_a$$



HENCE TORQUE IS UNI DIRECTIONAL AND INDEPENDENT OF CURRENT POLARITY. MACHINE WILL HENCE OPERATE WITH AN ALTERNATING A.C. SUPPLY.

A.C. VERSION IS CALLED THE UNIVERSAL MOTOR – COMMON IN DOMESTIC APPLIANCES e.g. WASHING MACHINES.

2b Maximum no-load speed:

$$E = V - I_a R_a$$

However at no-load, torque and hence current are zero. Max no-load speed occurs when the applied voltage equals the back emf;

$$E = V = 200V$$

Therefore max no-load speed is

$$\omega = \frac{E}{\psi_f} = \frac{200V}{0.4775} = 418.8 rad/s$$

2c Maximum torque at 500rpm.

Converting speed to radians...

$$\omega = \frac{2\pi f}{60} = 52.4 rads^{-1}$$

At this speed the back emf is,

$$E = 0.4775 \times 52.4 = 25V$$

Now;

$$\psi_f = \frac{E}{\omega} = \frac{25}{52.4} = 0.4775$$

With a maximum armature current of 10A, then the maximum torque is

$$T = 10 \times 0.477 = 4.8Nm$$

2d We know that $T=4.8Nm$ @ $10A$. Now;

$$E = V - R_a I_a = 200 - (3 \times 10) = 170V$$

And;

$$\psi_f = 0.477$$

Therefore max speed when providing 4.8Nm is:

$$\omega_{max} = \frac{E}{\psi_f} = \frac{170}{0.4775} = 356 rad/s$$

3a

Synchronous speed $= \frac{60f}{p} = \frac{60 \times 50}{2} = 1500rpm$

Slip$= 0.04 = \frac{1500 - rotor_speed}{1500} \therefore rotor_speed = 1440rpm$

3b

$$per\_unit\_slip = \frac{1500 - 600}{1500} = 0.6$$

hence, rotor frequency $= 0.6 \times 50 = 30Hz$

3c

Input power to rotor $= 40 - 1.5 = 38.5kW$
Rotor I^2R loss / $38.5 = 0.04$
Therefore, Rotor I^2R loss $= 1.54kW$

Finally, mechanical power developed by the rotor $= 38.5 - 1.54 = 36.96kW$

3d

Output power of motor $= 36.96 - 0.8 = 36.16kW$

Efficiency of the motor $= 36.16/40 = 0.904p.u. = 90.4\%$

3e

METHOD OF DETERMINING OUTPUT TORQUE

(OUTPUT POWER PER PHASE $= I_2'^2 R_2' \frac{(1-s)}{s}$

TOTAL OUTPUT POWER $= 3 I_2'^2 R_2' \frac{(1-s)}{s}$

3 PHASES

$$= \left(\frac{2\pi N}{60}\right) T = \left(\frac{2\pi N_s}{60}\right)(1-s) T$$

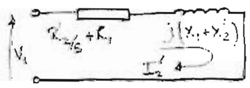
$N = (1-s)N_s$

$$T = \left(\frac{60}{2\pi N_s}\right) 3 I_2'^2 \frac{R_2'}{s}$$

ANGULAR SYNCHRONOUS SPEED

$$\frac{2\pi N_s}{60} = \frac{2\pi f_1}{P}$$

$$T = \frac{3P}{2\pi f_1} \; I_2'^2 \frac{R_2'}{s}$$

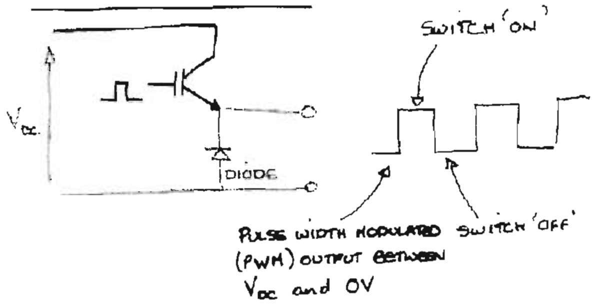


$$I_2' = \frac{V_1}{\sqrt{\left(\frac{R_2'}{s} + R_1\right)^2 + (X_1 + X_2')^2}}$$

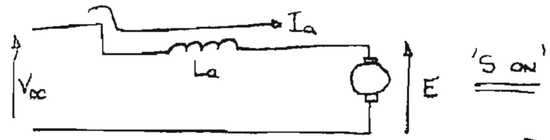
$$T = \frac{3P}{2\pi f_1} \; \frac{V_1^2 \, R_2'/s}{\left(R_2'/s + R_1\right)^2 + (X_1 + X_2')^2}$$

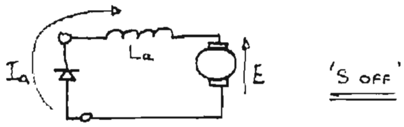
4a

BASIC 'CHOPPER' CIRCUIT



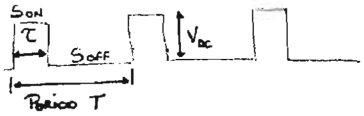
TYPICALLY THE OUTPUT IS SWITCHED AT MODERATE TO HIGH FREQUENCIES (2 kHz – 20 kHz). AT THESE FREQUENCIES THE INDUCTANCE OF THE MOTOR ARMATURE WINDING IS SIGNIFICANT.





ONCE A CURRENT IN THE ARMATURE INDUCTANCE HAS BEEN ESTABLISHED, THERE WILL BE A STORED ENERGY WITHIN THAT INDUCTANCE.
THE ACTION OF THE DIODE IS TO MAINTAIN A PATH FOR THIS STORED ENERGY BY ALLOWING THE ARMATURE CURRENT TO BE 'FLYWHEELED' AROUND THE CCT.

IN CONTINUOUS MODE OF OPERATION THE CCT IS SWITCHED AT A CONSTANT FREQ. AND WIDTH OR PERIOD OF THE SWITCH IS VARIED.



FREQ = $1/T$ AVERAGE OUTPUT VOLTAGE = $\frac{\tau}{T} V_{DC}$

4b

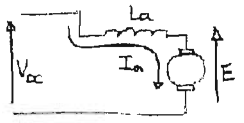
τ = S_{on} PERIOD

T = REPETITION PERIOD.

ASSUMING MOTOR BACK EMF REMAINS CONSTANT OVER SWITCHING PERIOD (SWITCHING RATE OF ELECTRONICS >> MOTOR TIME CONSTANT) AND NEGLECTING ARMATURE RESISTANCE.

τ

ON

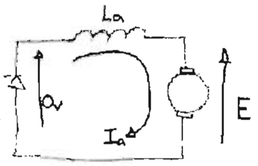


$$(V_{DC} - E) = L_a \frac{dI_a}{dt}$$

$$\frac{dI_a}{dt} = \frac{V_{dc} - E}{L_a}$$

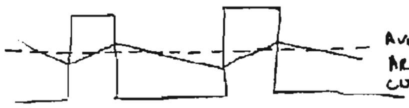
OFF

$T-\tau$



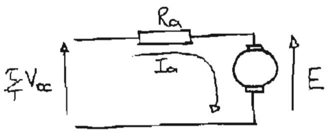
$$0 - E = L_a \frac{dI_a}{dt}$$

$$\frac{dI_a}{dt} = \frac{-E}{L_a}$$



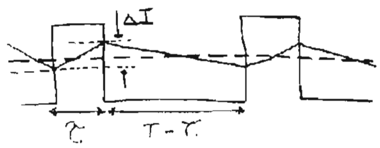
(47)

AVERAGE ARMATURE CURRENT CAN BE FOUND BY CONSIDERING THE AVERAGE VOLTAGE SUPPLIED TO THE MACHINE AND THE AVERAGE VOLTAGE DROP ACROSS THE ARMATURE RESISTANCE.



$$I_a \text{ average} = \frac{\frac{\tau}{T} V_{DC} - E}{R_a}$$

THE RIPPLE CURRENT ON TOP OF THE AVERAGE VALUE CAN BE FOUND FROM THE ABOVE.



$$-E = L_a \frac{dI}{dt} \quad \text{(OFF)}$$

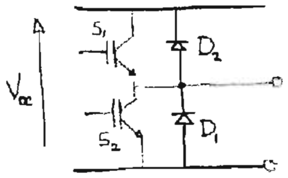
$$-E = L_a \frac{-\Delta I}{(T-\tau)}$$

PEAK TO PEAK RIPPLE $\Delta I = \dfrac{E(T-\tau)}{L_a}$

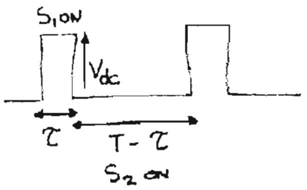
(48)

4c

2 QUADRANT CHOPPER

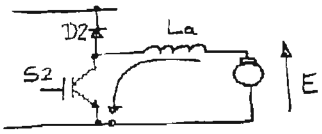


S_1 AND S_2 SWITCHED IN OPPOSITION
ie. S_1 ON S_2 OFF
S_1 OFF S_2 ON



$$V_{average} = \frac{\tau}{T} V_{dc}$$

CONSIDER THE SECOND QUADRANT ACTION OF S2 AND D2.



ASSUME MOTOR IS RUNNING AND GENERATING A BACK EMF. WHEN S_2 IS TURNED ON, THE ARMATURE IS SHORT - CIRCUITED AND A CURRENT WILL BUILD UP.

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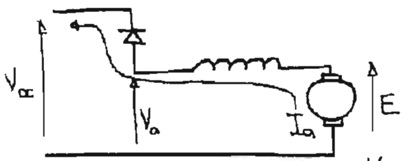
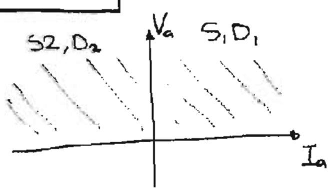
WHEN S_2 IS TURNED OFF, THIS REGENERATIVE CURRENT IS RETURNED TO THE SUPPLY VIA D_2





NOTE: THIS IS ONLY USEFUL IF THE DC SUPPLY IS CAPABLE OF ABSORBING THE REGEN. ENERGY.