Solutions

Q1(a)

A combination of lumped-constant elements consisting of resistors, capacitors, or inductors can often be used to model a circuit element. A resistor is used to characterize the power or energy lost in a circuit, while a capacitor and an inductor are used to characterize the electric and magnetic energy stored in a circuit respectively. In cases where the circuit dimensions are large with respect to a wavelength, a transmission line model consisting of resistors, capacitors, and inductors will be used to characterize the distributed parameters effect for the circuit elements. Division between lumped and distributed circuit considerations occurs when the dimension of the electronic component is greater than one-tenth of the signal wavelength (λ). **(4 marks)**

Q1(b)

$$\beta\ell = \frac{2\pi}{\lambda}\times 0.1\lambda = 0.628$$

$$\Gamma_{load} = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.34 + j0.56$$ **(1 mark)**

$$Z_{in} = Z_o\frac{Z_L + jZ_o\tan(\beta\ell)}{Z_o + jZ_L\tan(\beta\ell)} = 188 - j100\Omega$$ **(1 mark)**

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = 0.64 - j0.15$$ **(1 mark)**

$$R_L = -20\log\left|\Gamma_{in}\right| = 3.6\text{dB}$$ **(1 mark)**

$$I_L = -10\log(1-\left|\Gamma_{in}\right|^2) = 2.3\text{dB}$$ **(1 mark)**

Q1(c)

The voltage at any point along the line

$$V = V_{inc} + V_{ref}$$ **(1 mark)**

i.e.

$$V = V_{inc}(1 + \Gamma e^{-2j\beta d})$$ **(1 mark)**

$$\left|V\right| = \left|V_{inc}\right|\left|1 + \rho e^{-2j(\beta d - \frac{\theta}{2})}\right|$$ **(1 mark)**

which means

$\left|V_{\max}\right| = \left|V_{inc}\right|\left|1 + \rho\right|$ when $(\beta d - \frac{\theta}{2}) = n\pi$ **(1 mark)**

and

$\left|V_{\min}\right| = \left|V_{inc}\right|\left|1 - \rho\right|$ when $(\beta d - \frac{\theta}{2}) = n\pi + \frac{\pi}{2}$ **(1 mark)**

Therefore, the spacing between $|V_{\max}|$ and $|V_{\min}|$ is $\beta\Delta d = \frac{\pi}{2}$, i.e. $\Delta d = \frac{\lambda}{4}$. **(1 mark)**

Q1(d)

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = 1.5$$ **(1 mark)**

$$|\Gamma| = 0.2$$ **(1 mark)**

i.e. $\Gamma = \pm 0.2$ **(1 mark)**

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$

In the first case $\Gamma = 0.2$, so

$$0.2 = \frac{200 - Z_o}{200 + Z_o}$$

$Z_o = 133.2\ \Omega$ **(1 mark)**

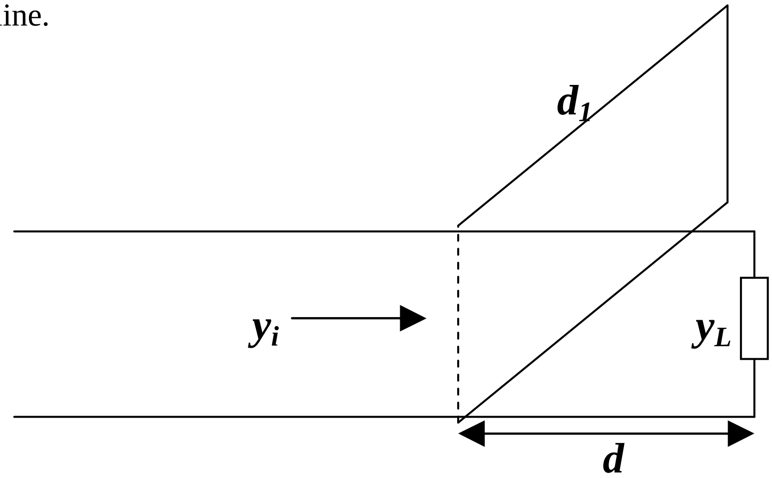
In the second case $\Gamma = -0.2$, so

$$-0.2 = \frac{200 - Z_o}{200 + Z_o}$$

$Z_o = 300\ \Omega$ **(1 mark)**

Q2(a)

A single open, or short, circuited transmission line stub whose length d_1 may be varied between 0 and 0.25λ, and whose position along a transmission line (d) is adjustable over a range of 0.5λ will match Z_L to the main line. **(1 mark)**



(2 marks)

The main problem is to choose d_1, d such that $y_{in}=1$, i.e. matched to the line. The line distance d is chosen such that y_L is transformed to some point lying on the unit conductance circle, i.e. in the absence of stub $y_{in}=1 \pm jb$. **(1 mark)**

An open circuit, or short circuit, stub then placed across the main line will cancel the $\pm jb$ of the transformed load, leaving the matched condition i.e. $y_{in}=1\pm j b0$. **(1 mark)**

Q2(b)

f=300 MHz λ=1 m d=15 cm= 0.15λ **(1 mark)**

αd=0.15x18=2.7 dB Nepers=8.686dB

αd =0.31 nepers 2αd=0.62 nepers **(1 mark)**

Draw point A at z=1.4+j0.4, then move 0.15λaround chart *towards generator*, i.e. point B on the chart (1.03-j0.5). **(1 mark)**

The radius at point B must be reduced by a factor of $e^{-2\alpha d}$, i.e. 0.54=54% of the radius for the lossless case. This is shown as point C on the chart. **(1 mark)**

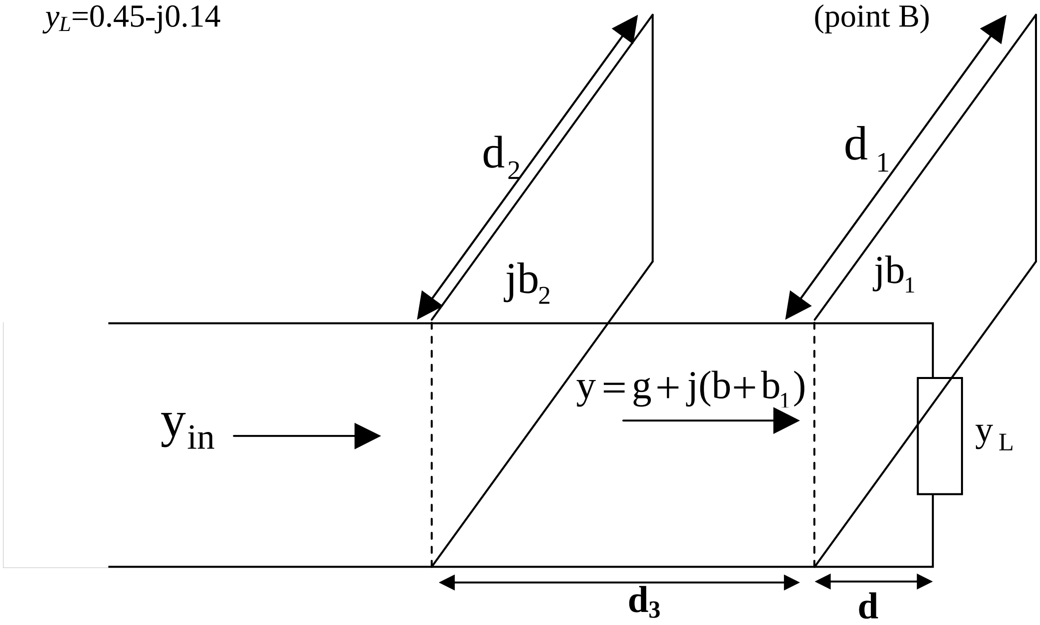
Input impedance at this point is (1.07-j0.25)×50=53.5-j12.5 Ω. **(1 mark)**

<u>Q2(c)</u>

$$\frac{Z_L}{Z_0} = \frac{(100 + j30)}{50} = 2 + j0.6$$ (point A) **(1 mark)**

i.e.

y_L=0.45-j0.14 (point B) **(1 mark)**



Step 1
Rotate the unit g circle *Towards Load*, by a distance of d_3=0.125λ. **(1 mark)**

Step 2
Move from point B to intersect the new, rotated, unit circle at point C. The movement should be on the corresponding conductance circle, since the stub does not alter the real part of the admittance. **(1 mark)**

Step 3
The admittance at point C is y_C=0.45+j0.16, compare it with that at B y_L=0.45-j0.14shows that stub 1 has provided j0.3, i.e. b_1=0.3 **(2 marks)**

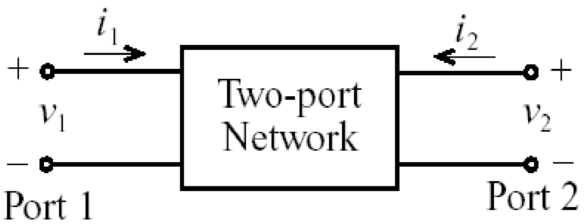
Step 4
For an o.c. stub, this means d_1=0.046λ, i.e. the distance from D to E on the chart. **(1 mark)**

Step 5
Move a distance d_3=0.125λ along the line from the 1st stub position to the 2nd stub position (from point C to F). **(1 mark)**

Step 6
At point F, the admittance is y_F=1+j0.82i.e. stub 2 must provide b_2=–j0.82 to reach the matched condition. **(1 mark)**

Step 7
For a s.c. stub, this means d_2=(0.39-0.25)λ=0.14λ, i.e. the distance from G to H on the chart. **(1 mark)**

Q3(a)



(1 mark)

The transmission matrix, or the ABCD matrix, of a two-port circuit relates the output terminal voltage and current (V_2, I_2) to the input voltage and (V_1, I_1) current as

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

(1 mark)

In matrix form

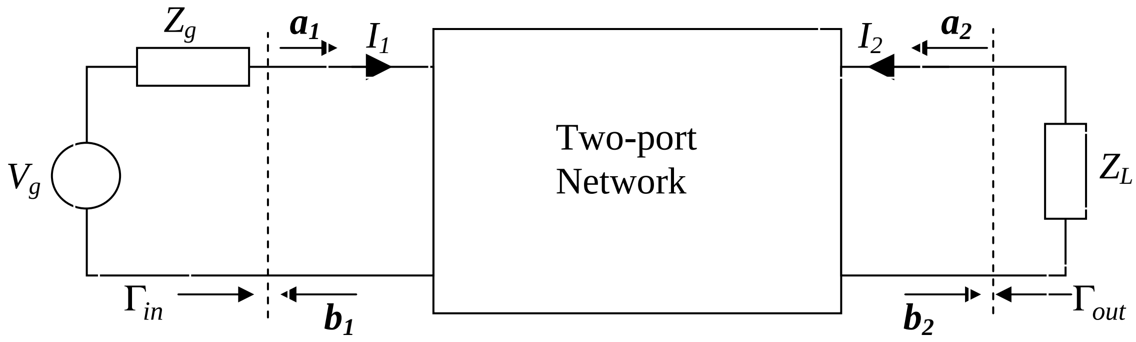
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

(1 mark)

(V_1, I_1) and (V_2, I_2) are the actual voltages and currents (*not normalized*), and they are continuous at the boundaries of the two ports. This means the output of one two-port circuit can be considered as the input of another two-port circuit. This makes the ABCD representation useful when cascading two-port networks. **(1 mark)**

Q3(b)

For the following two ports network



(1 mark)

The S_{11} parameter can be determined if the output port is terminated with a matched load, i.e. Γ_L=0

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$ **(1 mark)**

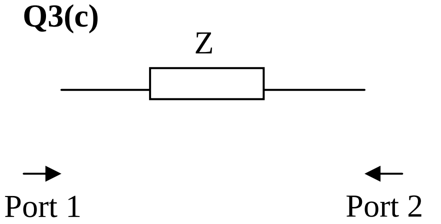
While the input reflection coefficient is given by

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$ **(1 mark)**

From these equations it can be seen that $S_{11}= \Gamma_{in}$ when the output port is terminated by a matched load.

(1 mark)

Q3(c)

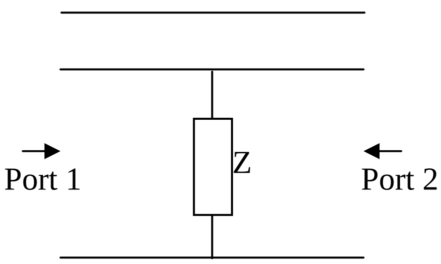


V_1=V_2+ I_1Z

I_1=-I_2

i.e. V_1= V_2-I_2Z **(1 mark)**

i.e. A=1, B=Z, C=0 and D=1 **(1 mark)**



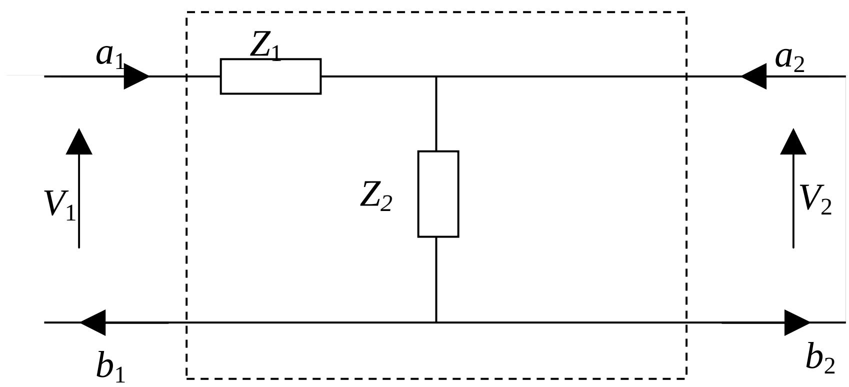
$$V_1 = V_2$$

$$I_1 + I_2 = \frac{V_1}{Z} = \frac{V_2}{Z}$$

i.e. $I_1 = \frac{V_2}{Z} - I_2$ **(1 mark)**

i.e. A=1, B=0, C=Z^{-1} and D=1 **(1 mark)**

Q3(d)



To calculate S_{11}, the network needs to be terminated with Z_0, where $Z_0 = Z_{01} = Z_{02}$

$$Z_{in1} = Z_1 + \left[\frac{Z_o Z_2}{Z_2 + Z_o} \right]$$

$$Z_{in1} = \frac{Z_1 Z_2 + Z_1 Z_o + Z_2 Z_o}{Z_2 + Z_o}$$ **(1 mark)**

$$S_{11} = \Gamma_{in} = \frac{Z_{in1} - Z_o}{Z_{in1} + Z_o}$$

then

$$S_{11} = \frac{Z_1 Z_2 + Z_1 Z_o - Z_o^2}{Z_1 Z_2 + Z_o (Z_1 + 2Z_2) + Z_o^2}$$ **(1 mark)**

Since

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2 = 0}$$

then from equation (31)

$$V_1 = \sqrt{Z_o}(a_1 + b_1)$$

$$V_2 = \sqrt{Z_o} b_2$$ **(1 mark)**

Therefore

$$\frac{V_1}{V_2} = \frac{(a_1 + b_1)}{b_2} = \frac{(1 + \frac{b_1}{a_1})}{\frac{b_2}{a_1}} = \frac{1 + S_{11}}{S_{21}}$$ **(1 mark)**

i.e.

$$S_{21} = \frac{V_2}{V_1}(1 + S_{11})$$

$$\frac{V_2}{V_1} = \frac{Z_2 Z_o}{Z_o (Z_2 + Z_1) + Z_1 Z_2}$$

$$1 + S_{11} = \frac{2(Z_1 Z_2 + (Z_1 + Z_2) Z_o)}{Z_1 Z_2 + Z_o (Z_1 + 2Z_2) + Z_o^2}$$ **(1 mark)**

Therefore

$$S_{21} = \frac{2 Z_o Z_2}{Z_1 Z_2 + Z_o (Z_1 + 2Z_2) + Z_o^2}$$ **(1 mark)**

Similarly

$$S_{12} = \frac{2Z_o Z_2}{Z_1 Z_3 + Z_o(Z_1 + 2Z_2) + Z_o^2}$$ **(1 mark)**

Finally

$$S_{22} = \frac{Z_{in2} - Z_o}{Z_{in2} + Z_o}$$

$$Z_{in2} = \left[\frac{Z_2(Z_1 + Z_o)}{Z_2 + Z_1 + Z_o}\right]$$

$$Z_{in2} = \frac{Z_2 Z_1 + Z_2 Z_o}{Z_2 + Z_1 + Z_o}$$

i.e. $$S_{22} = \frac{Z_1 Z_2 - Z_1 Z_o - Z_o^2}{Z_1 Z_2 + Z_o(Z_1 + 2Z_2) + Z_o^2}$$ **(1 mark)**

Q4(a)

When S_{12} cannot be neglected, the input impedance will change with the load reflection coefficient. Similarly, the output impedance will change as a function of the source reflection coefficient. In these situations design methods based on the operating, or available, power gains will be introduced.

•Operating Power Gain

When a matching is required at the input side of the amplifier, the operating power gain approach should be used. We plot the constant operating gain circles and find the best Γ_L, assuming that $\Gamma_s = \Gamma_{in}^*$, which satisfies our gain requirements. **(2 marks)**

•Available Power Gain

When a matching is required at the output side of the amplifier, the available power gain approach should be used instead of the operating power gain approach. We plot the constant available gain circles and find the best Γ_S, assuming that $\Gamma_L = \Gamma_{out}^*$, which satisfies our gain requirements. **(2 marks)**

Q4(b)

For a matched 3dB attenuator

$\Gamma_s = 0 \quad \Gamma_L = 0$

Therefore

$\Gamma_{in} = 0 \quad \Gamma_{out} = 0$

$$G_A = \frac{|S_{21}|^2(1 - |\Gamma_s|^2)}{|1 - S_{11}\Gamma_s|^2\left(1 - |\Gamma_{out}|^2\right)} = |S_{21}|^2 = 0.5$$ **(1 mark)**

$$G_T = \frac{|S_{21}|^2(1 - |\Gamma_s|^2)(1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2|1 - \Gamma_s\Gamma_{in}|^2} = |S_{21}|^2 = 0.5$$ **(1 mark)**

$$G = \frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-S_{22}\Gamma_L|^2\left(1-|\Gamma_{in}|^2\right)} = |S_{21}|^2 = 0.5$$ **(1 mark)**

For $Z_L = 25\Omega$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{25-50}{25+50} = -\frac{1}{3}$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = -\frac{1}{6}$$

$$\Gamma_s = 0 \quad \Gamma_{out} = 0$$

$$G_A = |S_{21}|^2 = 0.5$$ **(1 mark)**

$$G_T = |S_{21}|^2(1-|\Gamma_L|^2) = 0.444$$ **(1 mark)**

$$G = \frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{\left(1-|\Gamma_{in}|^2\right)} = 0.457$$ **(1 mark)**

Q4(c)

$$N = \frac{NF - NF_{min}}{4R_N/Z_o}|1+\Gamma_{opt}|^2 = \frac{1.58\text{-}1.445}{80/50}|1+0.62\angle 100^\circ|^2 = 0.0986$$ **(1 mark)**

$$C_{NF} = \frac{\Gamma_{opt}}{(N+1)} = 0.56\angle 100^\circ$$

$$r_{NF} = \frac{\sqrt{N(N+1-|\Gamma_{opt}|^2)}}{(N+1)} = 0.24$$ **(1 mark)**

Next we calculate data for several input section constant gain circles

G_S	g_S	C_S	R_S
1.0dB	0.805	$0.52\angle 60^\circ$	0.3
1.5dB	0.904	$0.56\angle 60^\circ$	0.205
1.7dB	0.946	$0.58\angle 60^\circ$	0.15

(2 marks)

The noise figure and constant gain circles are plotted on the Smith chart and the G_S =1.7dB circle just intersects the noise figure circle, and any higher gain will results in a worse noise figure.

(4 marks, 1 for each circle)

From the Smith chart the optimum solution is then Γ_s=0.53$\angle$75°and NF =2.dB. **(2 marks)**