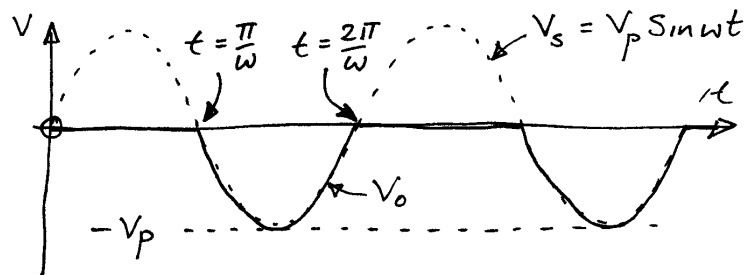


EEE103 / EEE121 / EEE141 Problem Sheet Solutions

Rectifiers and Smoothing

Q1 (i) The ct. is a half wave rectifier with a negative output.



(ii) V_s is always given as an r.m.s. quantity.

$$V_{opk} = -V_{r.m.s.} \sqrt{2} = -9\sqrt{2} = -12.73 \text{ V}.$$

[note that this assumes that the 0.7V diode drop is negligible.]

$$\begin{aligned} \text{(iii)} \quad V_{\text{AVE}} &= \frac{V_{opk}}{\pi} \quad (\text{for a half wave rectified sinusoid}) \\ &= -4.05 \text{ V} \end{aligned}$$

If you don't know that $V_{\text{AVE}} = V_{opk}/\pi$ for a half wave rectified sinusoid, you should know how to work it out. You need to work out the area under the waveform and divide it by the periodic time....

$$\begin{aligned} V_{\text{AVE}} &= \frac{1}{T} \int_{T/2}^T V_p \sin \omega t \, dt = \left(\frac{1}{2\pi/\omega} \right) \int_{\pi/\omega}^{2\pi/\omega} V_p \sin \omega t \, dt \\ &= \frac{\omega}{2\pi} \left[-\frac{V_p}{\omega} \cos \omega t \right]_{\pi/\omega}^{2\pi/\omega} = -\frac{V_p}{2\pi} [1 - (-1)] = -\frac{V_p}{\pi} \end{aligned}$$

note that the result is negative because the area under the curve for the chosen limits is negative. If the integration limit had been π/ω to 0, the answer would have been $+V_p/\pi$.

(iv) The r.m.s (root mean square) value of a sinusoid is $V_p/\sqrt{2}$ so the mean squared value is $V_p^2/2$. A half wave signal will have half the mean squared value of a full sinusoid so the half wave mean squared (ms) value is $V_p^2/4$.

(2)

The r.m.s. value of the waveform of part (i) is therefore $V_{P/2} = \underline{6.36V}$.

If you cannot follow the argument above, you can evaluate:

$$V_{rms} = \left[\frac{\omega}{2\pi} \int_{\pi/\omega}^{2\pi/\omega} V_p^2 \sin^2 \omega t \, dt \right]^{1/2}$$

to get the same answer.

- (v) If you know the r.m.s voltage across a resistor, working out power dissipation in the resistor is straightforward....

$$P_D = \frac{V_{rms}^2}{R} = \frac{6.36^2}{100} = \underline{405 \text{ mW. (0.405W)}}$$

- (vi) If the transformer has an effective series resistance of 10Ω , the voltage across R can be found by potential division of the ideal transformer output voltage...

$$V_{pk} = V_{pk,ideal} \times \frac{R}{10\Omega + R} = \underline{-11.75V}$$

- (vii) The new power dissipated in R is

$$\frac{V_{rms(new)}^2}{R} = \left(\frac{11.75}{2} \right)^2 \cdot \frac{1}{100} = 335 \text{ mW.}$$

You could also have taken the view here that the ideal V_{rms} of part (iv) — $6.36V$ — is acting on R + the 10Ω internal resistance so the power dissipated in this total resistance is

$$\frac{6.36^2}{110\Omega} = 367.7 \text{ mW.}$$

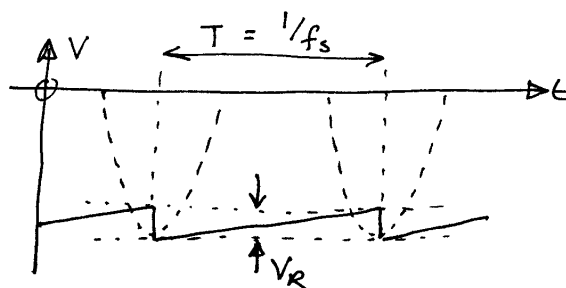
Ten elevenths of this power is being dissipated in R giving $P_D = 335 \text{ mW.}$

(3)

Q2

assume:

- (a) $I_L \approx \text{constant}$
- (b) capacitor charges instantaneously at each -ve half cycle peak.
- (c) $0.7V$ is negligible.



Ripple behaviour governed by $I_L = C \frac{dV_0}{dt}$ and since $I_L = \text{const}$, $\frac{dV_0}{dt}$ is the slope of the straight line, V_R/T

$$I_L \approx \frac{V_{pk}}{R} = C \frac{V_R}{T} = C \frac{V_R}{1/f_s} = CV_R f_s.$$

$$\text{or } C = \frac{V_{pk}}{R V_R f_s} \approx \underline{8500 \mu F} \quad (8.5 \text{ mF})$$

The power now dissipated in R is $\frac{V_{DC}^2}{R}$

$$V_{DC} = V_p - V_R/2 \rightarrow \text{ie, the average value of } V_0.$$

$$= 12.73 - 0.15$$

$$\text{Thus } P_D = \underline{1.58 \text{ W.}}$$

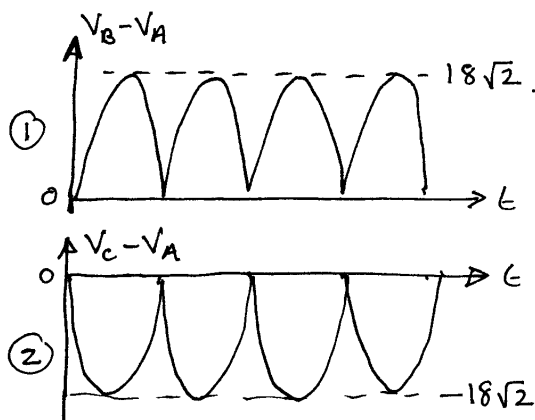
You might legitimately argue that if the $0.7V$ diode drop can be neglected, then so can $V_R/2$ at a mere $0.15V$. Such an argument is fine (in this case) and would give a $P_D = 1.62W$. If the ripple voltage was $3V$, though, ignoring it would be on the border of unreasonable.

Q3 The outputs are as shown

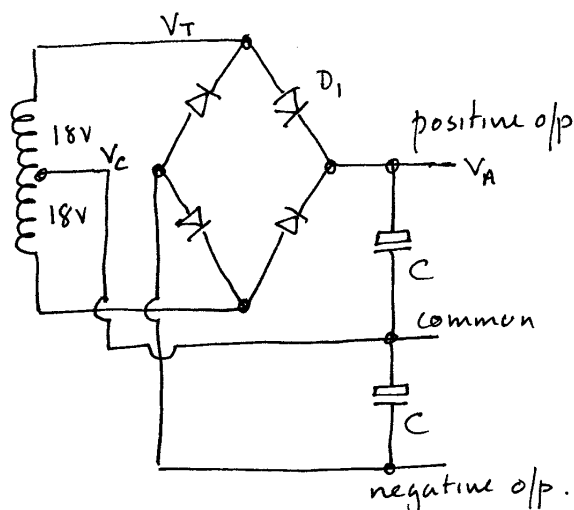
$$(a) (V_B - V_A)_{pk} = \underline{18\sqrt{2}}$$

$$(b) (V_A - V_C)_{pk} = -(-18\sqrt{2}) = \underline{18\sqrt{2}}.$$

$$(c) (V_C - V_B)_{pk} = -18\sqrt{2} - 18\sqrt{2} = \underline{-36\sqrt{2}}$$



Q4 (1)



- (ii) using the same approximations as in Q2 but recognising that I_L is given explicitly and the circuit is full wave (ie the charging interval is $1/2f_s$)

$$I_L = C \frac{dV_o}{dt} = \frac{C V_R}{1/2f_s} = 2f_s C V_R.$$

This gives $C = \frac{I_L}{2f_s V_R} = \frac{2}{2.50 \cdot 1} = \underline{20 \text{ mF}}.$

Since the +ve + -ve outputs have the same load conditions, this value of C applies to both.

- (iii) A variety of answers are acceptable here depending on assumptions made... These must be stated if used. The answer given is based on

$$V_{dc} = V_{pk} - 0.7 - V_R/2 = \underline{24.3 \text{ V}}$$

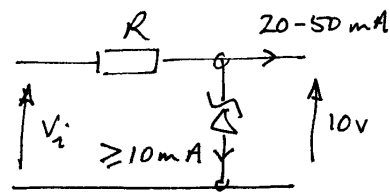
If you choose to neglect the 0.7V or the $V_R/2$ or both — all reasonable approximations in this case, you would get marginally higher answers.

- (iv) Consider D_1 . When V_T is at its +ve peak w.r.t. V_c , D_1 conducts and C is charged to almost $18\sqrt{2}$ V. V_T then falls until at the peak of the next half cycle it is $-18\sqrt{2}$. But V_A is still $\approx +18\sqrt{2}$ so the peak reverse voltage across D_1 is $2 \times 18\sqrt{2} \approx \underline{51 \text{ V}}.$

⑤

Q5. (i) 10V (should be obvious!)

(ii) A range of answers is possible here depending on the assumptions made in determining the output voltage from Q4.



The answer given here uses the instantaneous minimum value of V_o taking diode drop and ripple into account....

$$R_{\max} = \frac{V_{i\min} - V_o}{I_{L\max} + I_{Z\min}} = \frac{(18\sqrt{2} - 0.7 - 1) - 10}{50\text{mA} + 10\text{mA}} = \underline{\underline{230\Omega}}$$

[If you neglected the 0.7 and V_R , $R = 258$.
Ignoring one or the other gives an answer between $230\Omega + 258\Omega$]

(iii) If the load is disconnected, all 60mA will flow through the Zener diode

$$\therefore P_D = 10\text{V} \times 60\text{mA} = \underline{\underline{600\text{mW}}}$$

(iv) This answer will be slightly dependent on your assumptions about V_i and your value of R .

The answer given assumes an average dc V_i of 25V (ie the 0.7V drop is ignored)

$$P_D = \frac{25 \times 25}{230} = \underline{\underline{2.72\text{W}}}$$

(v) Under normal conditions

$$P_{D(\text{zener})} = 10\text{V} \times 40\text{mA} = \underline{\underline{400\text{mW}}}$$

(note that the minimum I_L condition represents the worst case normal operation for the Zener).

$$P_{D\text{res.}} = 15^2/230 = \underline{\underline{980\text{mW}}}$$

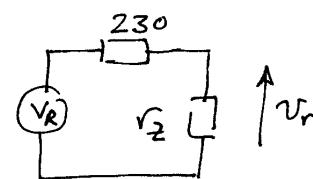
(this value depends on R and V_i assumptions).

$$(vi) \quad V_r = V_R \frac{r_z}{230 + r_z}$$

$$V_r = 10\text{mV}$$

$$V_R = 1\text{V}$$

$$\rightarrow \text{gives } r_{z\max} = \underline{\underline{2.3\Omega}}$$

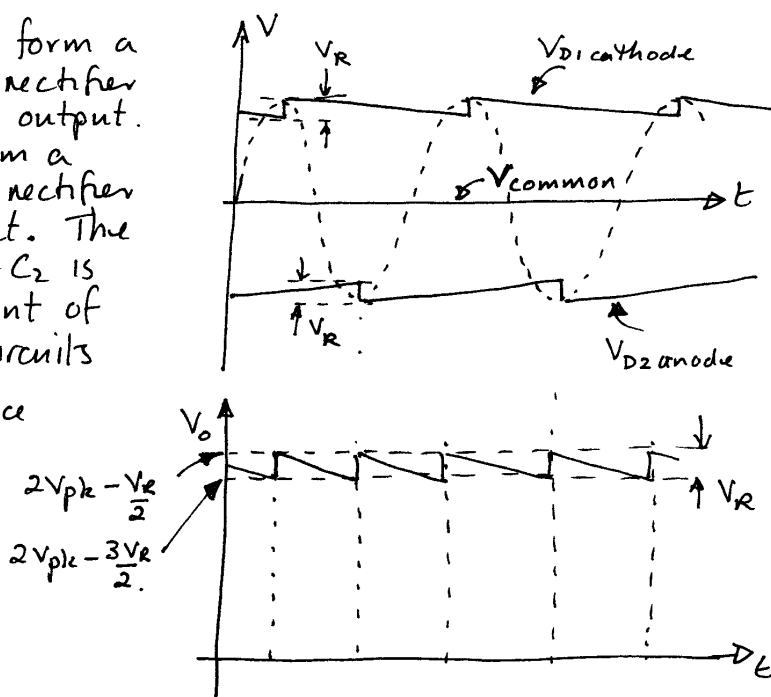


ripple equiv cct.

(6)

Q6 (i) D_1 and C_1 form a half wave rectifier with a +ve output. $D_2 + C_2$ form a half wave rectifier with a -ve output. The junction of $C_1 + C_2$ is the common point of these two h.w. circuits

V_o is the difference in voltage between D_1 cathode and D_2 anode.



(ii) The V_R on V_o is the same in pk-pk value as the V_R on the output of each half wave rectifier.

$$I_L = 0.1A = C \frac{V_R}{T} = C \frac{5}{0.02}$$

$$\text{or } C_1 = C_2 = \frac{0.1 \times 0.02}{5} = \underline{400\mu F}$$

$$\begin{aligned} \text{(iii) Average } V_o &= \frac{2V_{pk} - V_R/2 + (2V_{pk} - 3V_R/2)}{2} \\ &= 2V_{pk} - V_R = \underline{51.6V} \end{aligned}$$

[Here you can ignore the 0.7V but the ripple is significant]

$$\begin{aligned} \text{(iv) When } V_{D1} \text{ cathode changes to } V_{pk}, D_2 \text{ has } -V_{pk} \\ + V_R/2 \text{ on its anode. So worst case PIV} \\ &= 2V_{pk} - V_R/2 \\ &= \underline{54.1V} \end{aligned}$$