Data Provided: None

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2013-14 (2.0 hours)

EEE6410 Data Coding for Communications and Storage 6

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. **a.** In the context of baseband synchronous communications, describe the main characteristics that are desirable in a line code. **(3)**

b. You are required to establish a highly reliable synchronous wired connection between two digital systems. However, on installation of the cable, it was found that the bandwidth available is lower than expected. The three line codes with the timing diagrams shown in Figures Q.1.a, Q.1.b, and Q.1.c below were shortlisted as potential candidates for adoption. From the timing diagrams, characterise and contrast the three codes discussing their advantages and drawbacks. Which one of these codes would you select for your application; justify your selection. **(10)**

c. An asynchronous transmission connection is set up with odd parity and an ***m-bit*** data between a Start and a Stop bit. The master clock is set to run at a nominal rate that is 32 times the baud rate with a clock tolerance of **±1%**. Determine a reliable value for ***m*** for this connection, assuming worst case conditions. **(7)**

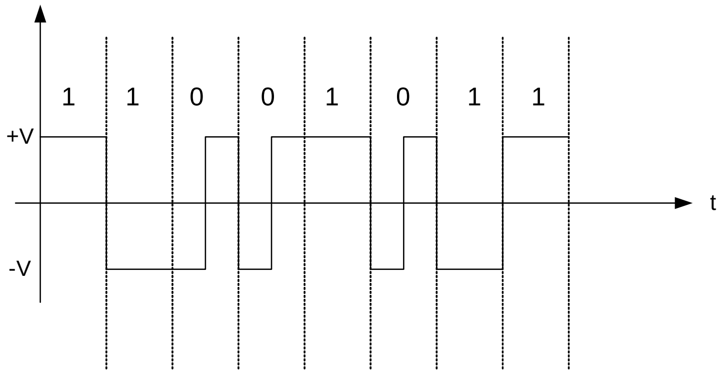

Figure Q.1.a

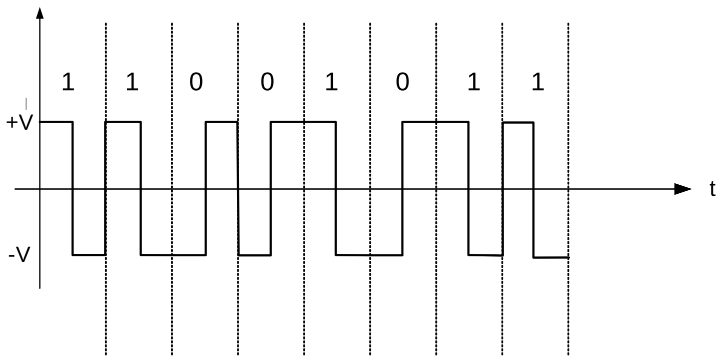

Figure Q.1.b

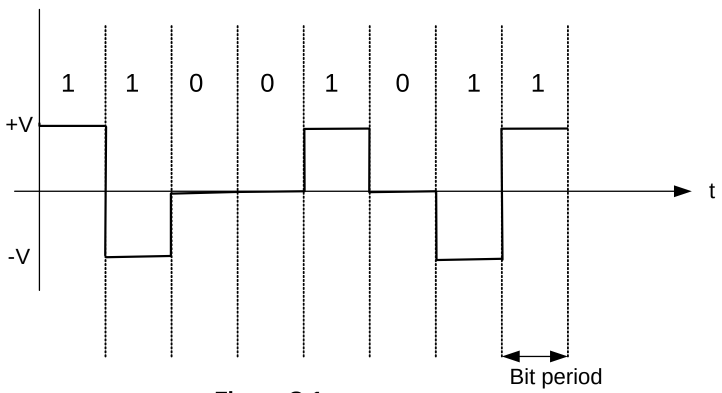

Figure Q.1.c

2. **a.** A 4-bit data input is encoded into a CRC codeword, using the CRC generator $\mathbf{g(x) = x^3+x+1}$, and transmitted over a noisy channel. The codeword is received with 3 errors (shown in bold and underlined) in the data bits as <u>**0**</u>0<u>**11**</u>, the CRC bits being all correct. Perform a CRC check on this received data for error detection and explain your result. **(5)**

b. Draw a circuit to generate all non-zero elements of the Galois Field $\mathbf{GF(2^3)}$ using the primitive polynomial $\mathbf{p(x) = x^3+x^2+1}$. List all of the elements in both binary and polynomial format. **(3)**

c. A (7,3) **RS** (Reed-Solomon) code defined over $\mathbf{GF(2^3)}$ as in (b), is used to encode a 3-symbol message before transmission over a noisy channel.

i) How many errors can be corrected by this code? What form can these errors take? **(2)**

ii) Derive the generator polynomial **g(x)** for this code. **(4)**

iii) After transmission over a noisy channel a message ***m(x)*** encoded using the above ***RS*** code is received with <u>a single symbol</u> error as $r(x)= \alpha^3x^5+ \alpha x^4 +\alpha x^2 + \alpha^6 x + \alpha^6$. Using algebraic decoding, derive the corrected codeword. Note that for a single symbol error correction the error location, X_1, and the error magnitude, Y_1, are given respectively by the equations: **(6)**

$$X_1 = \sigma_1 = \frac{S_2}{S_1} \quad \text{and} \quad Y_1 = \frac{S_1^{\ 2}}{S_2}$$

where S_1 and S_2 are the first 2 syndromes

3. **a.** Compare block codes and convolutional codes, as applied to error correction, giving typical situations where each one might be employed. **(3)**

b. Explain briefly how a block code and a convolutional code can be combined together in a multistage coding configuration. Give the advantage of such approach citing example applications for its use. **(4)**

c. After transmission over a Gaussian channel, a message ***m(x)*** encoded using a (15,7) ***BCH*** code defined over Galois Field $GF(2^4)$ using the primitive polynomial $p(x) = x^4+x+1$, is received with <u>a single bit</u> error as:
$r(x)= x^{14}+x^{11}+x^9+x^2+x$. Using algebraic decoding, derive the corrected codeword. **(6)**

d. An encoded data sequence using the convolutional encoder in Figure Q.3 is received as **001 111 101 010 110 101 111 010 101**. Correct the received sequence assuming that no more than 3 bit errors have occurred. Derive hence the initial input data. **(7)**

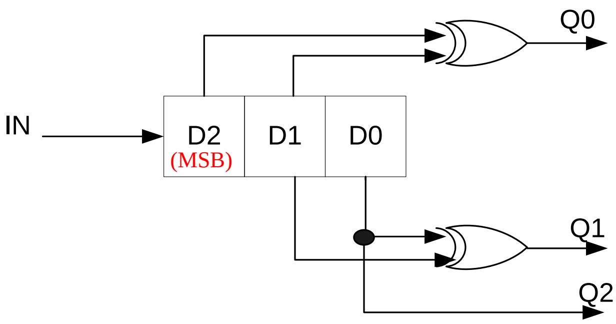

Figure Q.3

4. **a.** A frequency domain (7,3) RS (Reed-Solomon) encoded codeword was received as c'= (α^3, α^2, α, α^6, 1, α^2, α^3); a frequency domain decoding is adopted for the error correction and the spectrum of c' was hence first computed and found to be: C'= (α^4, α^2, α, α, α^5, α, α^2). Using recursive extension, find the complete error spectrum E in this case.
The (7,3) RS code is defined over Galois Field $GF(2^3)$ that is generated by the primitive polynomial $p(x) = x^3+x+1$. **(6)**

b. Give the key attributes of a transform that are attractive for data compression and draw a generic model for a transform-based image compression system explaining briefly how compression is achieved. **(4)**

c. Data representing a straight line feature is to be compressed by half adopting a transform-domain approach. The Discrete Cosine transform (DCT) was applied to an 8-sample block of this data and resulted after quantisation, with a quantisation vector [1, 1, 1, 1, 10, 10, 10, 10], in the compressed block [25, -13, 0, -1, 0, 0, 0, 0].

i) Derive the recovered original 8-sample block of the data and comment on the attributes of the DCT for data compression. **(8)**

ii) What are the potential limitations of the DCT in data compression? **(2)**

The *k-th/n-th DCT/IDCT* pair of an N-sample block input is given by:

$$X_k = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \alpha_k x_n \cos\left[\frac{(2n+1)k\pi}{2N}\right]$$

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$$\alpha_0 = \frac{1}{\sqrt{2}}$$

$$\alpha_k = 1 (k \neq 0)$$

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