Data Provided: None

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2009-2010 (2 hours)

Electric and Magnetic Fields 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. Use Ampere's law to derive expressions for the magnetic field inside *and* outside a circular wire of radius $\boldsymbol{a}$ carrying a uniformly distributed current $\boldsymbol{I}$. Sketch the variation of the field as a function of distance from the centre of the wire.

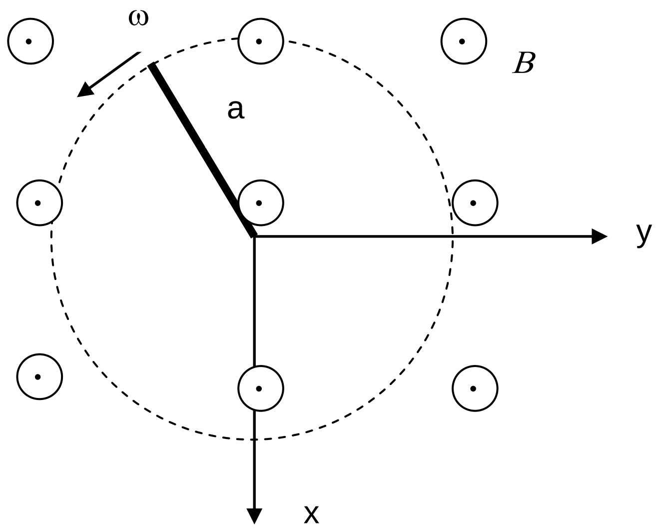
(6)

b. A 1000 turns solenoid is 10cm long, 2cm in diameter and carries a current of 1A. By ignoring end-effects, calculate

i) The magnetic field at the centre of the solenoid;
ii) The self inductance of the solenoid.

(6)

c. Referring to Figure 1, a metal rod of length $\boldsymbol{a}$ rotates about the z-axis at a rate of 180 revolutions per minute If the rotating rod is in a uniform B-field of magnitude $\boldsymbol{B}$, derive an expression for the induced emf along the length of the rod. Evaluate this voltage If $\boldsymbol{B} = \hat{\mathbf{z}}6 \times 10^{-4}$T and $\boldsymbol{a}$=1.0m. Note that the z-axis is into the page.



(8)

Figure 1

2. **a.** Figure 2 shows the cross-section of several current carrying conductors; the magnitudes (in Amps) and directions of the currents are as indicated. Calculate the value of the line integral of the magnetic field, $\boldsymbol{B}$ for each of the four paths A, B, C and D. (6)

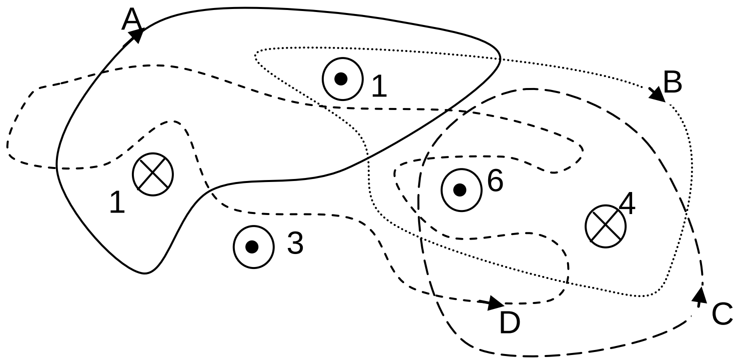


Figure 2

(6)

b. The loop circuit in Fig 3 consists of circular arc segments of radii $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively connected by radial segments all of which have a common origin P. If the loop conducts a current of magnitude $\boldsymbol{I}$, derive an expression for the magnetic field $\boldsymbol{H}$ at point P.

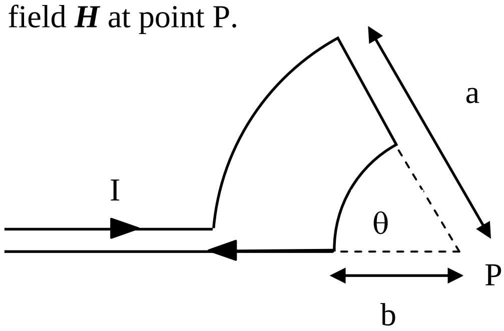


Figure 3

(8)

c. Figure 4 shows the cross-section of three parallel wires each carrying the current indicated. The direction of all three currents is into the page.

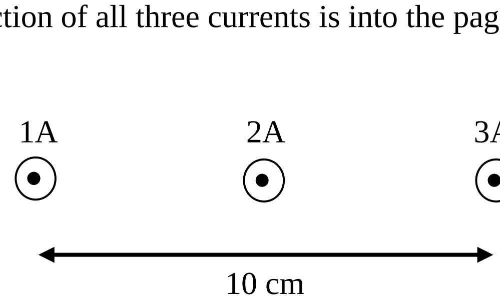


Figure 4

Calculate the forces on each of the wires, assuming the centre wire to be midway between the other two wires. **(8)**

3. **a.** Show that the equation for the electric field due to an infinitely long charged wire is given by:-

$$|\underline{E}| = \frac{q_\ell}{2\pi r \varepsilon_o}$$

State any assumptions which you make. **(6)**

b. Figure 5 shows the cross-section at $z = 0$ of two infinitely long charged wires which are 4m apart. Both wires have a radius of 1cm and a charge per unit length as given in the diagram.

Calculate the electric fields at the following points, giving your answers as vectors.

i. (4,2,0) m

ii. (0,0,0) m

iii. (2,2,0) m

(6)

c.

By integrating the expression for total electric field along a suitable path, find the potential difference between the two wires.

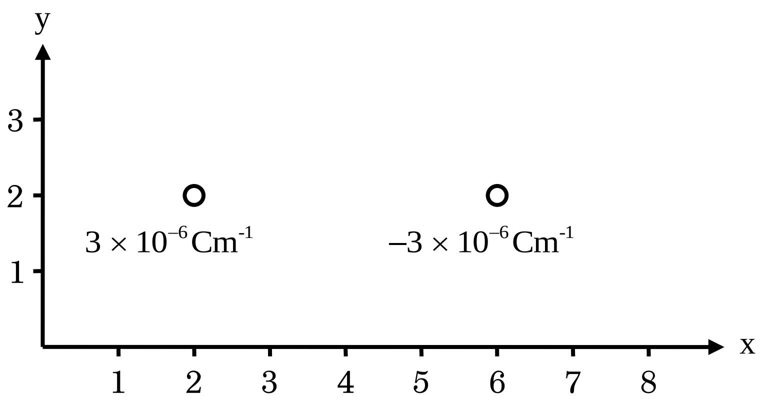


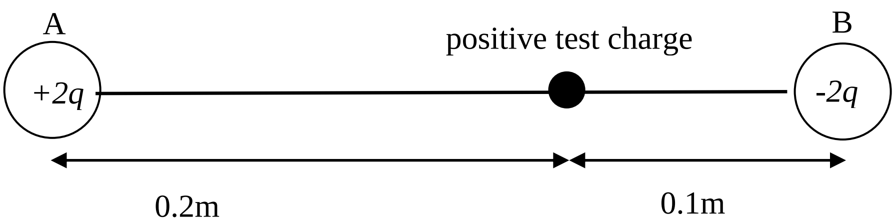
Figure 5 **(8)**

4. **a.** Sketch the electric field lines and equi-potential lines inside a co-axial cable. **(2)**

b. A coaxial cable consists of an inner conductor of radius a and an outer conductor of radius b separated by an insulating material with a relative permittivity of ϵ_r. Derive an expression for the capacitance per unit length of the coaxial cable. State any assumptions that you make. **(6)**

c. If a 10m long, air-spaced coaxial cable has a capacitance of 400pF, and its inner conductor has a radius of 2mm, find the radius of the outer conductor. **(2)**

d. Figure 6 shows two spheres A and B with charges *+2q* and *-2q* respectively. If the force on the positive test charge due to A is ***F***, what is the force on the test charge due to B?



(4)

Figure 6

e. Three small conducting beads carrying charges of *+q*, *-q* and *+q* are arranged on a circular insulating ring as shown in Figure 7. The beads are free to move along the ring without friction and without the effects of gravity. Starting from the initial position shown in Figure 7, describe "what happens next" and draw diagram to illustrate the final equilibrium state of the system.

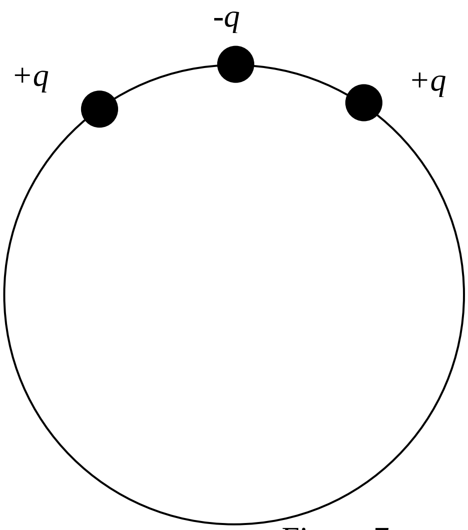


Figure 7

(6)

AT

UNIVERSITY OF SHEFFIELD
Department of Electronic and Electrical Engineering

EEE220 ELECTRIC AND MAGNETIC FIELDS FORMULA SHEET

ILF/AT/JLW 2007

$\varepsilon_o = 8.854 \times 10^{-12}$ Fm^{-1} charge on electron = -1.6×10^{-19} C

$\mu_o = 4\pi \times 10^{-7}$ Hm^{-1} mass of electron = 9.1×10^{-31} kg

1. ELECTROSTATICS

(a) Coulomb's Law

Force between two point charges, q_1 and q_2 has magnitude $F = \dfrac{q_1 q_2}{4\pi\varepsilon_o R^2}$ in direction along line joining charges. In vector notation $\underline{F} = \dfrac{q_1 q_2}{4\pi\varepsilon_o R^3}\underline{R}$ or $\underline{F} = \dfrac{q_1 q_2}{4\pi\varepsilon_o R^2}\underline{\hat{R}}$

(b) Electric Field

Defined by $\underline{E} = \dfrac{Q}{4\pi\varepsilon_o R^3}\underline{R}$, and then force is $\underline{F} = q\underline{E}$. In electrostatics we want to solve for $\underline{E}$.

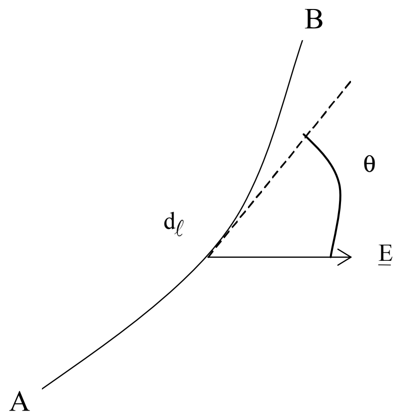
(c) Potential

Work done in moving q_1 from A to B is $W = q_1\left(\phi(A) - \phi(B)\right)$ where ϕ is potential.

Potential due to charge q is $\phi = \dfrac{q}{4\pi\varepsilon_o R}$, and ϕ and $\underline{E}$ are related by

$$\phi(B) - \phi(A) = -\int_A^B \underline{E}\bullet\underline{dl} = -\int_A^B E\cos\theta\, d\ell$$

$$\underline{E} = -\nabla\phi = \left(-\frac{d\phi}{dx}, -\frac{d\phi}{dy}, -\frac{d\phi}{dz}\right)$$



(d) **Gauss's Law**

Surface integral of $\underline{E}$ gives $\oint_s E\cos\theta \; da = \frac{Q}{\varepsilon_o}$, $Q =$ total charge enclosed by surface S.

(e) **Solving for $\underline{E}$**

Three methods possible.

(i) Use Coulomb's Law, summing all contributions with care about direction.

(ii) Calculate ϕ and then use $\underline{E} = \left(-\frac{d\phi}{dx}, -\frac{d\phi}{dy}, -\frac{d\phi}{dz} \right)$.

(iii) Use Gauss's Law - only works if symmetry can be employed to get $\underline{E}$ outside the integral.

(f) **Important Cases**

(i) Sheet of charge, $|\underline{E}| = \frac{q_s}{2\varepsilon_o}$, q_s is surface density, or charge per unit area.

(ii) Line of charge, $|\underline{E}| = \frac{q_\ell}{2\pi r\varepsilon_o}$, q_ℓ is charge per unit length.

(iii) Sphere of charge Q, $|\underline{E}| = \frac{Q}{4\pi\varepsilon_o r^2}$.

(g) **Capacitance**

Capacitance of two conductors is defined by $C = Q/V$. For parallel plate capacitor $C = \varepsilon A/d$, where $\varepsilon =$ permittivity of separating medium. Effect of dielectric medium is to increase the capacitance.

(h) **Energy**

Stored energy in capacitor is $\frac{1}{2} CV^2$. Energy density in electric fields is $\frac{1}{2} \varepsilon E^2$.

2. MAGNETIC FIELDS

(a) Force between two circuits

Force is given by Ampère's force law, but this is difficult to use. Introduce $\underline{B}$ field, and force in a circuit is $\underline{F} = \oint I \ \underline{dl} \times \underline{B}$.

(b) Biot-Savart Law

$\underline{B}$ field is given by $\underline{B} = \frac{\mu_o}{4\pi} \oint \frac{I \underline{dl} \times \hat{\underline{r}}}{r^2}$

Analytical results possible only for simple geometries.

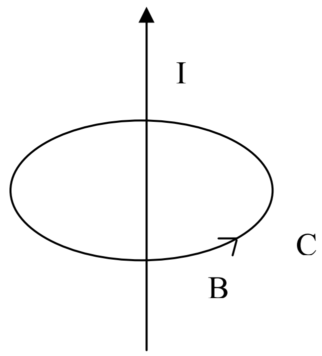
(c) Important cases of $\underline{B}$

(i) Infinitely long straight wire $B = \mu_o I/2\pi r$.

(ii) on axis of circular loop, $B = \mu_o Ia^2 / 2\left(a^2 + d^2\right)^{3/2}$.

(iii) Inside long straight solenoid $B = \mu_o nI$.

(d) Ampère's Law



$$\oint_C \underline{B} \bullet \underline{dl} = \oint_C B \cos\theta d\ell = \mu_o I$$

I is the current which threads the path of integration.
Direction given by right-hand rule

(e) Magnetic Flux

Defined by $\Phi = \int B \cos\theta da$, i.e. Φ is given by the integral over area of normal component of $\underline{B}$. For uniform B, $\Phi = BA$, hence B is called magnetic flux density. For a closed surface of integration $\oint B \cos\theta da = 0$, which implies no magnetic poles.

3. MAGNETIC INDUCTION

(a) Faraday's Law

If flux linkages through a circuit change with time, magnitude of emf induced is $\mathcal{E} = \frac{d\Phi}{dt}$.

Polarity of $\mathcal{E}$ given by Lenz's Law, is such as to try to keep Φ constant.

(b) **Self-inductance**

Defined by $\varepsilon = L \dfrac{di}{dt}$ where L depends on geometry of circuit (and also any magnetic materials present). In a circuit L causes current to lag voltage.

Inductance of solenoid $= \dfrac{\mu_o N^2 A}{\ell}$, where N is the total number of turns, A is the cross-sectional area, and ℓ is the length of the solenoid.

(c) **Magnetic Energy**

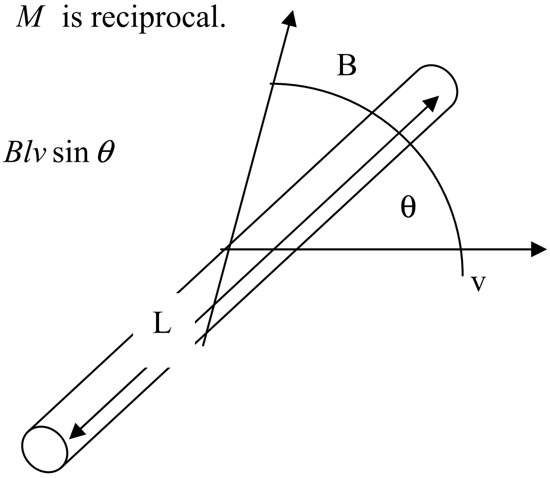
Energy stored in inductance is $\dfrac{1}{2} Li^2$. Energy per unit volume in magnetic fields is $\dfrac{B^2}{2\mu_o}$ or $\dfrac{B^2}{2\mu}$ if magnetic material of permeability μ is present.

(d) **Mutual Inductance**

Current change in one circuit induces emf in nearby circuit $\varepsilon = M \dfrac{di}{dt}$. M is coefficient of mutual inductance, depends on geometry and materials. M is reciprocal.

(e) **EMF induced by Motion**

EMF is generated by conductor moving in B field, $\varepsilon = Blv \sin\theta$



4. **MAGNETIC FORCES**

(a) **Force between parallel wires**

Force per unit length is $f = \mu_o \, I_1 I_2 / 2\pi p$, where p is distance between wires. Like currents attract, unlike repel. The unit of current (Ampere) is defined from this relation.

(b) **Force on Linear Conductor**

$F = BIl \sin\theta$ or in vector notation $\underline{F} = I\underline{l} \times \underline{B}$

(c) **Torque on Current Loop**

$T = NIBA \sin\alpha$

Applications include motor and meter.

(d) **Force on Charged Particle**

$\underline{F} = q\left(\underline{v} \times \underline{B}\right)$ is at right angles to both $\underline{B}$ and $\underline{v}$.

Gives Hall effect and gyration of charges about field lines.