The University Of Sheffield.

Data Provided:
Laplace and z-transforms
Compensator design formulae
Performance criteria mappings
Ziegler-Nichols tuning rules

LEAVE THIS EXAM PAPER ON YOUR DESK.
DO NOT REMOVE IT FROM THE HALL.

DEPARTMENT OF AUTOMATIC CONTROL & SYSTEMS ENGINEERING
Autumn Semester 2016–2017

ACS342 FEEDBACK SYSTEMS DESIGN **2 hours**

Answer THREE questions.

No marks will be awarded for solutions to a fourth question.

Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out.

If more than the required number of questions are attempted, DRAW A LINE THROUGH THE ANSWERS THAT YOU DO NOT WISH TO BE MARKED.

All questions are marked out of 20. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

Registration number from U-Card (9 digits) — to be completed by student

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1. A unity-feedback control system has the open-loop transfer function

$$KG(s) = \frac{K}{s(s+5)(s+15)}$$

a) Sketch the root locus diagram of $KG(s)$. You **do not** need to calculate numerical values for the break-away point and the imaginary axis intersection points.

[6 marks]

b) Find the range of K for which the closed-loop system is stable.

[5 marks]

c) Show that the dominant pole location corresponding to an overshoot of 15% and a 2% settling time of 1 second is

$$s = -4.0 \pm \jmath 6.6$$

[5 marks]

d) Design a phase-lead compensator in order that the compensated root locus passes through the location given in part (c). (You do not need to design the gain, K, in the compensator.)

[4 marks]

2. A first-order system with input $u(t)$ and output $y(t)$ is modelled by the ordinary differential equation

$$T\frac{\mathrm{d}y(t)}{\mathrm{d}t} = Ku(t) - y(t)$$

where the parameter T is the *time constant*.

a) Show that the transfer function of the system is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{sT+1}$$

[3 marks]

b) Write down the order, type number, and locations of the poles and zeros of $G(s)$.

Explain what happens to the pole locations, and the speed of the system response, as (i) T is increased and (ii) K is increased.

[6 marks]

c) Show that the output $y(t)$ in response to a step input $u(t) = A, t \geqslant 0$ is

$$y(t) = KA(1 - e^{-t/T})$$

What is the value of $y(t)$ after one time constant (*i.e.*, at $t = T$)?

[6 marks]

d) Show that the 10%–90% rise time is approximately $2T$ and the 2% settling time is approximately $4T$.

[5 marks]

3. A unity-feedback control system has the open-loop transfer function

$$L(s) = \frac{75}{(s+1)(s+2)(s+10)}$$

a) Sketch the Bode diagram of $L(s)$.

[10 marks]

b) (i) Calculate the position error constant of $L(s)$.

[1 mark]

(ii) Show that the gain crossover frequency is approximately $2.24\,\mathrm{rad\,s^{-1}}$.

[3 marks]

(iii) Estimate, from your Bode diagram, the phase margin of $L(s)$. Is the closed-loop system stable?

[2 marks]

(iv) Using your answers to parts (i)–(iii), estimate the performance characteristics of the closed-loop system, including steady-state error, overshoot and rise time.

[4 marks]

4. a) Define the terms *characteristic equation*, *open-loop transfer function*, and *type number*.

[3 marks]

b) Using block diagram reduction methods, or otherwise, find the transfer function $Y(s)/R(s)$ of the system in Figure 4.1.

[8 marks]

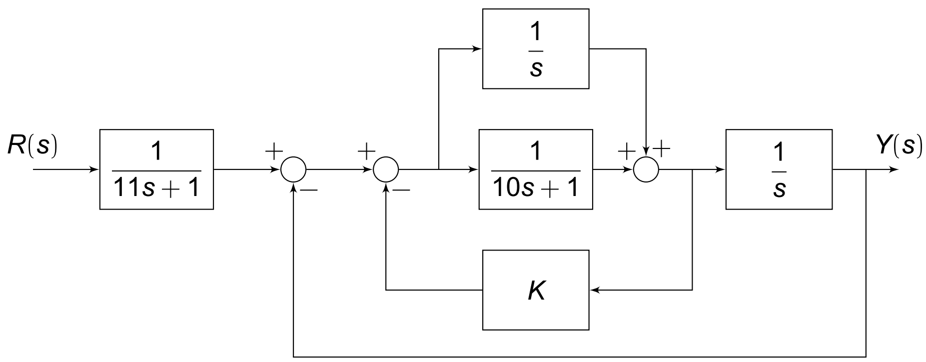

Figure 4.1

c) Explain why a type-0 system under proportional feedback control always exhibits a non-zero steady-state error.

[4 marks]

d) Show that, in a digital negative-feedback control system with reference input $R(z)$, output $Y(z)$, plant $G(z)$ and controller $D(z)$, the closed-loop system has a transfer function $Y(z)/R(z) = T(z)$ if

$$D(z) = \frac{1}{G(z)} \frac{T(z)}{1 - T(z)}$$

Hence, design a controller $D(z)$ in order that a system $G(z) = \dfrac{1 - e^{-T}}{z - e^{-T}}$, with sampling time $T = 0.1\,\text{s}$, has the output

$$y[k] = 0.6065y[k-1] + 0.3935r[k]$$

[5 marks]

Laplace and z-transforms

Time domain	s-domain	z-domain
$f(t)$	$F(s)$	$F(z)$
$f(t-T)$	$e^{-sT}F(s)$	$z^{-1}F(z)$
$\delta(t)$	1	–
1	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{zT}{(z-1)^2}$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
te^{-at}	$\dfrac{1}{(s+a)^2}$	$\dfrac{zTe^{-aT}}{(z-e^{-aT})^2}$
$\sin(\omega t)$	$\dfrac{\omega}{s^2+\omega^2}$	$\dfrac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
$\cos(\omega t)$	$\dfrac{s}{s^2+\omega^2}$	$\dfrac{z^2-z\cos(\omega T)}{z^2-2z\cos(\omega T)+1}$
$e^{-at}\sin(\omega t)$	$\dfrac{\omega}{(s+a)^2+\omega^2}$	$\dfrac{ze^{-aT}\sin(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
$e^{-at}\cos(\omega t)$	$\dfrac{s+a}{(s+a)^2+\omega^2}$	$\dfrac{z^2-ze^{-aT}\cos(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
$f^{(n)}(t)=\dfrac{d^n}{dt^n}f(t)$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{n-1}(0)$	Various forms

Compensator design formulae

Transfer function	$\dfrac{s\alpha\tau+1}{s\tau+1}$ (lead) $\dfrac{s\tau+1}{s\alpha\tau+1}$ (lag)
Maximum phase lead/lag, ϕ_m	$\sin^{-1}\dfrac{\alpha-1}{\alpha+1}$
Centre frequency, ω_m	$\dfrac{1}{\tau\sqrt{\alpha}}$

Performance criteria mappings

2% settling time, T_s	$\dfrac{4}{\zeta\omega_\mathrm{n}}$
10–90% rise time, T_r	$\dfrac{2.16\zeta + 0.6}{\omega_\mathrm{n}}$ for $0.3 \leqslant \zeta \leqslant 0.8$
Percentage overshoot, P.O.	$100\exp\left(\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$ for $0 < \zeta < 1$
Peak time, T_p	$\dfrac{\pi}{\omega_\mathrm{n}\sqrt{1-\zeta^2}}$ for $0 < \zeta < 1$
Peak response, M_p	$1 + \exp\left(\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$ for $0 < \zeta < 1$
Resonant frequency, ω_r	$\omega_\mathrm{n}\sqrt{1-2\zeta^2}$ for $0 < \zeta < \dfrac{1}{\sqrt{2}}$
Resonant peak magnitude, $M_{\mathrm{p}\omega}$	$\dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$ for $0 < \zeta < \dfrac{1}{\sqrt{2}}$
Phase margin, ϕ_pm	100ζ for $0 < \zeta < \dfrac{1}{\sqrt{2}}$
Bandwidth–Rise time	$T_\mathrm{r} = \dfrac{2.2}{\omega_\mathrm{B}}$

Ziegler-Nichols tuning rules

First method (T time constant; L delay time; K process gain)

	K_P	T_I	T_D
P	T/KL	∞	0
PI	$0.9T/KL$	$L/0.3$	0
PID	$1.2T/KL$	$2L$	$0.5L$

Second method (K critical gain; P critical period of oscillation)

	K_P	T_I	T_D
P	$0.5K$	∞	0
PI	$0.45K$	$P/1.2$	0
PID	$0.6K$	$0.5P$	$0.125P$

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