Tutorial 4

1. Evaluate the Laplace transform of the signal $x(t) = e^{-t}u(t) + e^{-4t}u(t)$.

2. Verify the following Laplace transform pairs

(i) $\dfrac{dx(t)}{dt} \leftrightarrow sX(s)$

(ii) $-tx(t) \leftrightarrow \dfrac{dX(s)}{ds}$

(iii) $\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \dfrac{X(s)}{s}$.

3. Find the values of $y(t) = 2e^{-2t}u(t) - e^{-t}u(t)$ for $t = 0$ and $t \to \infty$. Verify your answer using the initial and the final value theorems.

4. Compute the impulse response and the step response of a system with transfer function described by $H(s) = \dfrac{3s}{2s^2 + 10s + 12}$.

5. Determine the poles, the natural frequency and the damping factor of systems with the following transfer functions and state the nature of the system response:

(i) $G(s) = \dfrac{0.3}{s^2 + 7s + 10}$

(ii) $G(s) = \dfrac{1}{s^2 + 4s + 13}$

(iii) $G(s) = \dfrac{0.1}{s^2 + 16}$

(iv) $G(s) = \dfrac{15}{s^2 + 6s + 9}$.

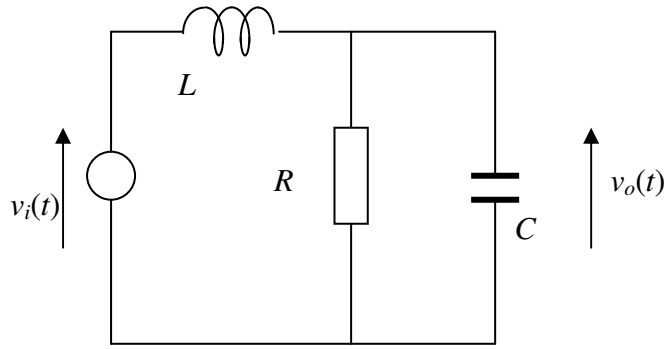
6. Determine the Laplace transforms of the following signals. Sketch the pole-zero plot and region of convergence (if it exists).

(i) $x(t) = e^{-4t}u(t)$ (ii) $x(t) = e^{-t}u(t) + e^{-3t}u(t)$ (iii) $x(t) = e^{-a|t|}$, $a > 0$

7. Find the Laplace transforms of the signal $x(t) = e^{-t}u(t) * u(t)$ and sketch $x(t)$.

8. Determine the initial and the final values of the signal with Laplace transform $X(s) = \dfrac{10s}{s^2 + 10s + 300}$.

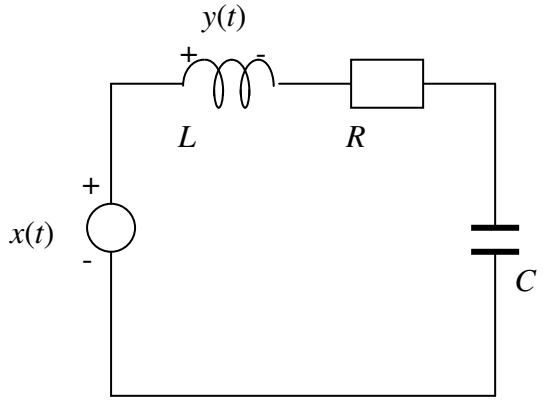
9. Determine the transfer function of the circuit shown below.



(i) If $R = 1\ \Omega$ and $C = 1$ pF calculate the value of L required so that the circuit is critically damped. Sketch $v_o(t)$ if $v_i(t)$ is a unit step function.

(ii) If $R = 50\ \Omega$, $C = 1$ nF and $L = 2.5\mu$H calculate the damping factor and natural oscillating frequency. Sketch and describe $v_o(t)$ if $v_i(t)$ is a unit step function.

10. Consider a system with an input signal $x(t)$ and output signal $y(t)$ shown below.
i) Determine the response of the RLC circuit, $y(t)$, under the conditions R=6, L=2H, C=0.25F and $x(t) = u(t)$ assuming zero initial conditions. ii) Find $y(t)$ when $x(t) = 0$ and the initial conditions are given by $i(0) = 1$A flowing through the inductor and $v(0) = 1$V across the capacitor.



11. Consider a system with a transfer function $H(s) = \dfrac{1}{s+3}$. Find the forced and natural responses of this system if the input signal is given by $x(t) = \exp(-3t)u(t)$ and an initial condition of $y_o(0) = 1$ where $y_o(0)$ is the output signal at $t = 0$.