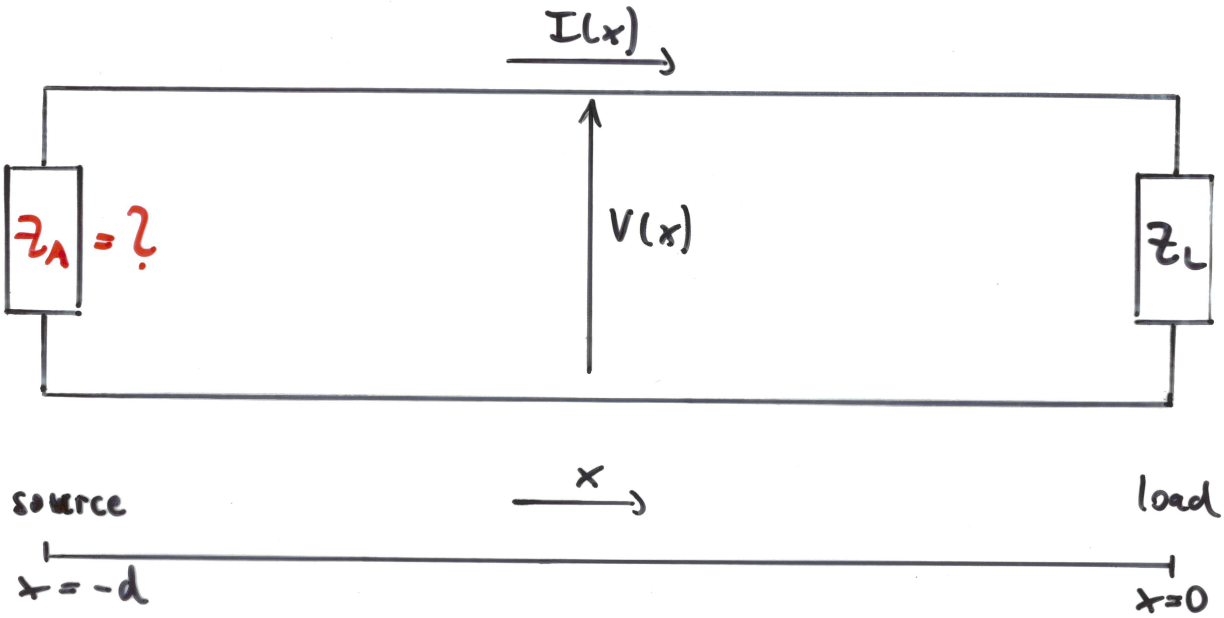
current reflection and apparent impedance at source



$$V_0^+ = Z_0 I_0^+ \quad \& \quad V_0^- = -Z_0 I_0^-$$

$$\Rightarrow \Gamma = \frac{V_0^-}{V_0^+} = -\frac{I_0^-}{I_0^+} \qquad \text{(reversed sign of current SWR !)}$$

$$\text{if } V(x,t) = V_0^+ e^{j\omega t}\left(e^{-j\tilde{k}x} + \Gamma e^{+j\tilde{k}x}\right)$$

$$\text{then } I(x,t) = \frac{V_0^+}{Z_0} e^{j\omega t}\left(e^{-j\tilde{k}x} - \Gamma e^{+j\tilde{k}x}\right)$$

$\Rightarrow$ apparent impedance at source:

$$\boxed{Z_A = \frac{V(-d)}{I(-d)}} = \frac{\cancel{V_0^+}\,\cancel{e^{j\omega t}}\left(e^{+j\tilde{k}d} + \Gamma e^{-j\tilde{k}d}\right)}{\cancel{\frac{V_0^+}{Z_0}}\,\cancel{e^{j\omega t}}\left(e^{+j\tilde{k}d} - \Gamma e^{-j\tilde{k}d}\right)}$$

$$= Z_0 \frac{e^{j\tilde{k}d} + \Gamma e^{-j\tilde{k}d}}{e^{j\tilde{k}d} - \Gamma e^{-j\tilde{k}d}} \qquad \text{with } \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= Z_0 \frac{(Z_L + Z_0) e^{j\tilde{k}d} + (Z_L - Z_0) e^{-j\tilde{k}d}}{(Z_L + Z_0) e^{j\tilde{k}d} - (Z_L - Z_0) e^{-j\tilde{k}d}}$$

$$= Z_0 \frac{Z_L \cos\tilde{k}d + \cancel{j Z_L \sin\tilde{k}d} + \cancel{Z_0 \cos\tilde{k}d} + j Z_0 \sin\tilde{k}d + Z_L \cos\tilde{k}d - \cancel{j Z_L \sin\tilde{k}d} \ldots}{\cancel{Z_L \cos\tilde{k}d} + j Z_L \sin\tilde{k}d + Z_0 \cos\tilde{k}d + \cancel{j Z_0 \sin\tilde{k}d} - \cancel{Z_L \cos\tilde{k}d} + j Z_L \sin\tilde{k}d \ldots}$$

$$\ldots \frac{- \cancel{Z_0 \cos\tilde{k}d} + j Z_0 \sin\tilde{k}d}{+ Z_0 \cos\tilde{k}d - \cancel{j Z_0 \sin\tilde{k}d}}$$

$$= Z_0 \frac{2 Z_L \cos\tilde{k}d + 2j Z_0 \sin\tilde{k}d}{2 Z_0 \cos\tilde{k}d + 2j Z_L \sin\tilde{k}d} \qquad \text{divide numerator and denominator by } 2\cos\tilde{k}d$$

$$\boxed{= Z_0 \frac{Z_L + j Z_0 \tan\tilde{k}d}{Z_0 + j Z_L \tan\tilde{k}d}} \qquad \text{transforms } Z_L \text{ into } Z_A$$

consider some special cases at load end:

i) $d=0 \Rightarrow \tan \tilde{k}d = \tan 0° = 0 \rightarrow Z_A = Z_L$

(consistency)

ii) $d = \lambda/4 \rightarrow \tan \tilde{k}d = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right) = \tan \pi/2 = \infty$

$$\Rightarrow Z_A = \frac{Z_0^2}{Z_L}$$

iii) $d = \lambda/2 \rightarrow \tan \tilde{k}d = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\right) = \tan \pi = 0$

$$\Rightarrow Z_A = Z_L$$

iv) short-cut with $Z_L = 0$

$$\Rightarrow Z_A = j\, Z_0 \tan \tilde{k}d = j\omega L_A$$ purely inductive

v) open line with $Z_L = \infty$

$$\Rightarrow Z_A = \frac{Z_0}{j \tan \tilde{k}d} = \frac{1}{j\omega C_A}$$ purely capacitive

application examples:

a) choice of Z_0 to match load and source

given: $Z_L = 100\,\Omega$, $Z_A = 25\,\Omega$, use line of length $d = \lambda/4$

calculate Z_0

At $d = \lambda/4$ see (ii) above: $Z_A = \frac{Z_0^2}{Z_L}$

$\Rightarrow Z_0 = \sqrt{Z_A \cdot Z_L} = \sqrt{2500}\,\Omega = 50\,\Omega$ (geometrical mean!)

b) choice of line length to get purely inductive or purely capacitive load

given: $Z_0 = 50\,\Omega$, $f = 2\,\mathrm{GHz}$, $v = 2/3 \cdot c$, $L_A = 10\,\mathrm{nH}$

calculate d

Short-circuit line and use iv) above; i.e.

$j Z_0 \tan \tilde{k}d = Z_A = j\omega L_A$

with $\tilde{k} = \frac{2\pi}{\lambda}$ and $v = \lambda \cdot f \Rightarrow \tilde{k} = \frac{2\pi f}{v}$ and $\omega = 2\pi f$

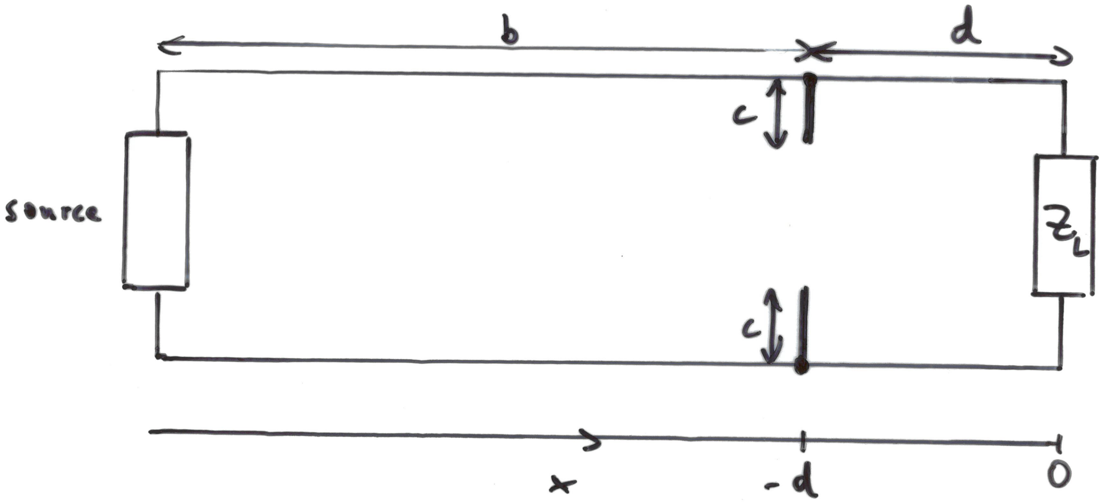
$\Rightarrow \tan \tilde{k}d = \frac{2\pi f L_A}{Z_0} = 2.5133 = \tan 68.3°$

$\rightarrow \tilde{k}d = 68.3° = 1.192$

$\Rightarrow d = \frac{1.192}{\tilde{k}} = \frac{1.192}{2\pi}\cdot\lambda = \frac{1.192}{2\pi f} v = 0.019\,\mathrm{m} = 1.9\,\mathrm{cm}$

c) use of tuning stubs to reduce Z_A

By adding another transmission line, Z_0 can be increased to match Z_L. If Z_0 needs to be reduced, short unterminated „stubs" are added in parallel to tune the transmission line impedance.



total apparent admittance at $x = -d$

$$\frac{1}{Z} = \frac{1}{Z_{line}} + \frac{1}{Z_{stub}}$$

For the stub, the line appears open with $Z_L = \infty$, hence

$$Z_{stub} = \frac{Z_0}{j \tan \tilde{k}c}$$

$$\rightarrow \quad \frac{1}{Z} = \frac{1}{Z_A(-d)} + \frac{j \tan \tilde{k}c}{Z_0}$$

gives modified Z_A at position $x = -d-b$ of source.