EEE224/227 solutions (2013/14)

Q1 a)

i)

The response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$ (additivity property). **(1 mark)**

The response to $ax_1(t)$ is $by_1(t)$ where a and b are constants (homogeneity property). **(1 mark)**

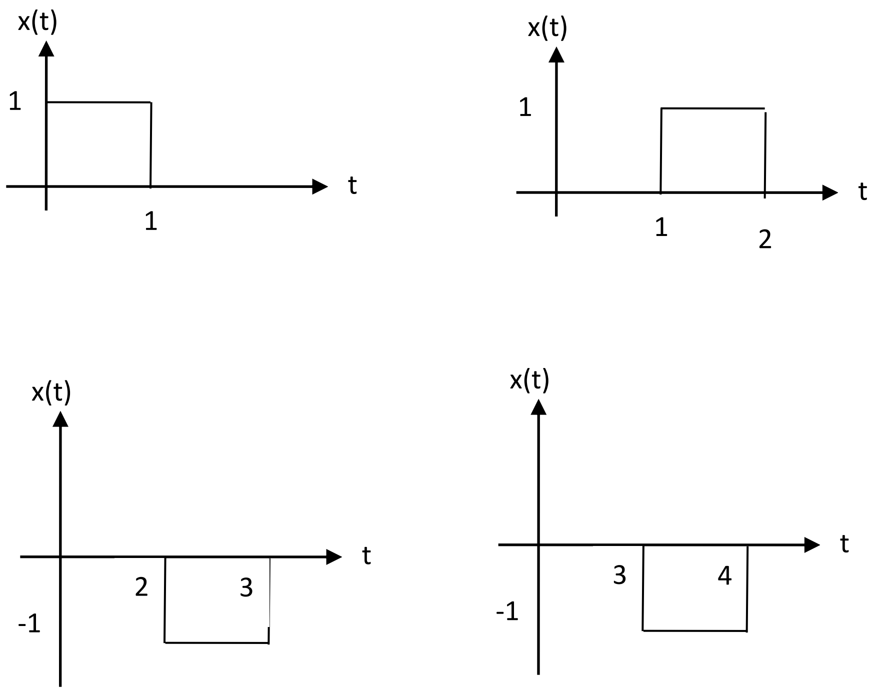
ii)

If the characteristics of a system are independent of time it is said to be ***time invariant.***

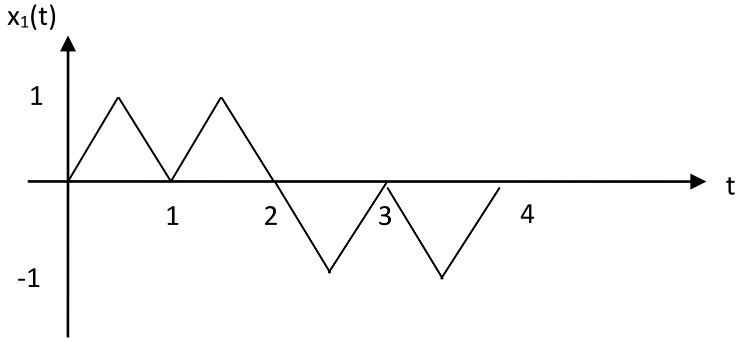
A time shift in the input signal will result in an identical shift in the output signal of a time invariant system. **(1 mark)**

b)

The signal can be thought of as 4 similar versions of the original x(t). **(3 marks)**



Hence the output signal will look like



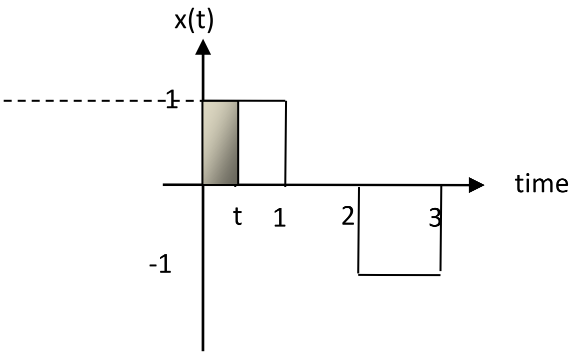
(2 marks)

c)

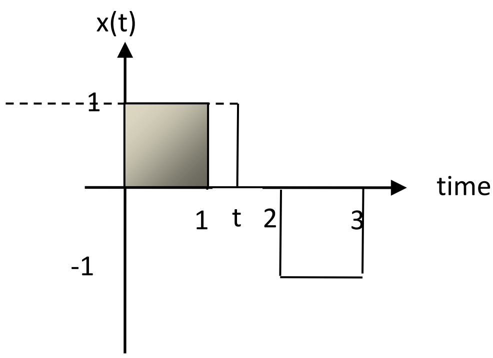
A convolution of h and x is required. Using the graphical method either h or x can be “flipped”. The solution below is for flipping h.

For t<0 y(t)=0

For 0<t<1 the graph shows the overlapping area $y(t)=\int_0^t 1dt = t$

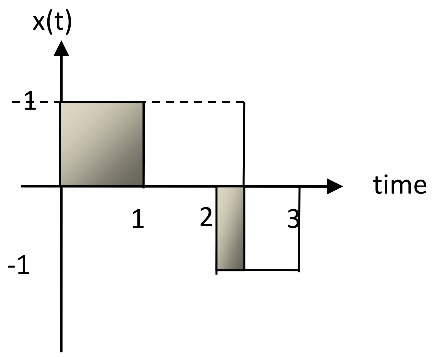


For 1<t<2 the area is constant, hence $y(t)=\int_0^1 1dt = 1$



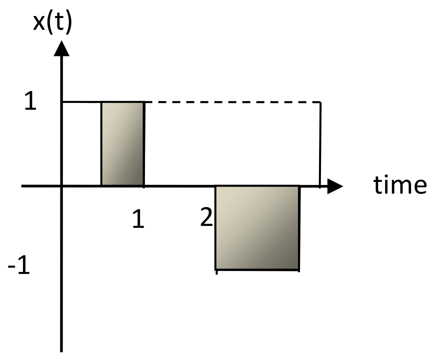
For 2<t<3. In this case the negative area reduces the output. $y(t)=\int_0^1 1dt-\int_2^t 1dt$

$$y(t)=1-(t-2)=3-t$$

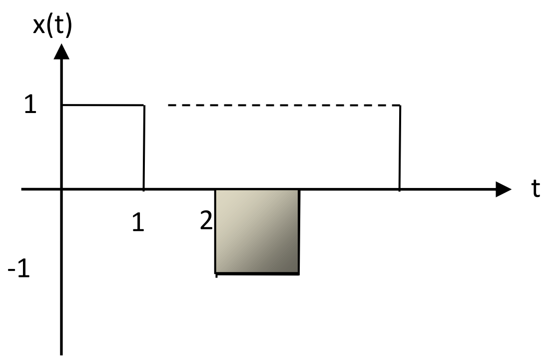


For 3<t<4 $y(t)=\int_{t-3}^{1} 1dt-\int_2^3 1dt$

$$y(t)=1-(t-3)-1=3-t$$

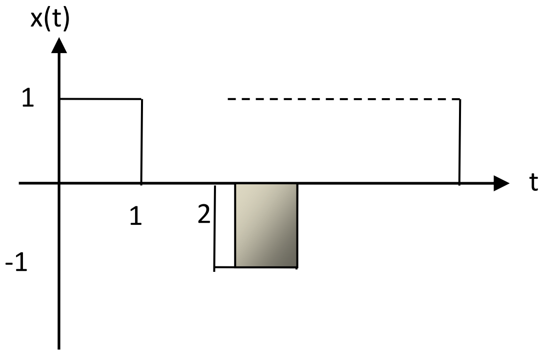


For 4<t<5 $y(t)=\int_2^3 -1dt=-1$



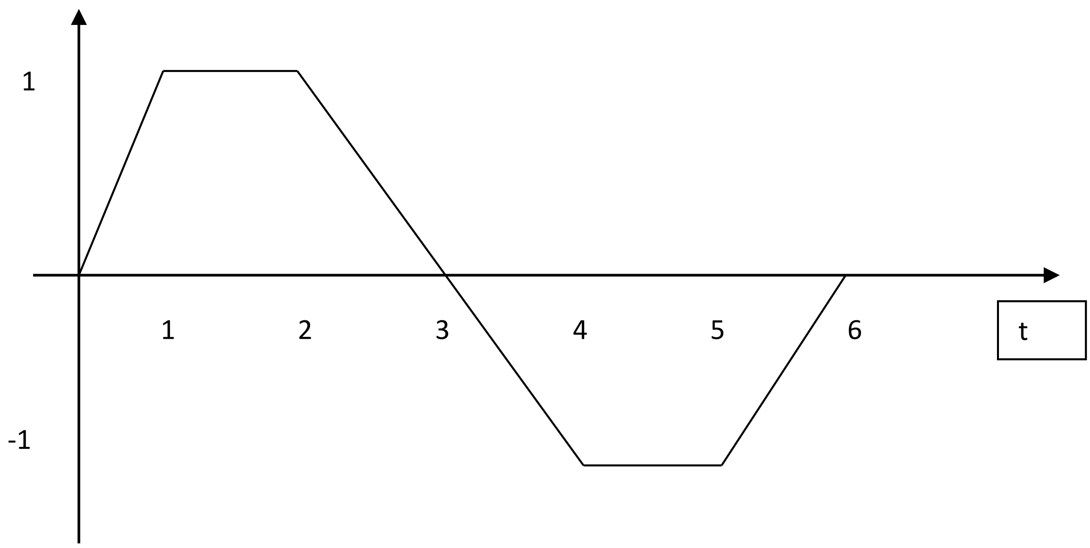
For 5<t<6 $y(t)=\int_{t-3}^{3}-1dt$

$$y(t)=-\left(3-\left(t-3\right)\right)=t-6$$



In summary

$$y(t)=\begin{cases} 0 & t<0 \\ t & 0<t<1 \\ 1 & 1<t<2 \\ 3-t & 2<t<4 \\ -1 & 4<t<5 \\ t-6 & 5<t<6 \\ 0 & t>6 \end{cases}$$



Q2a

Allows for multiplexing

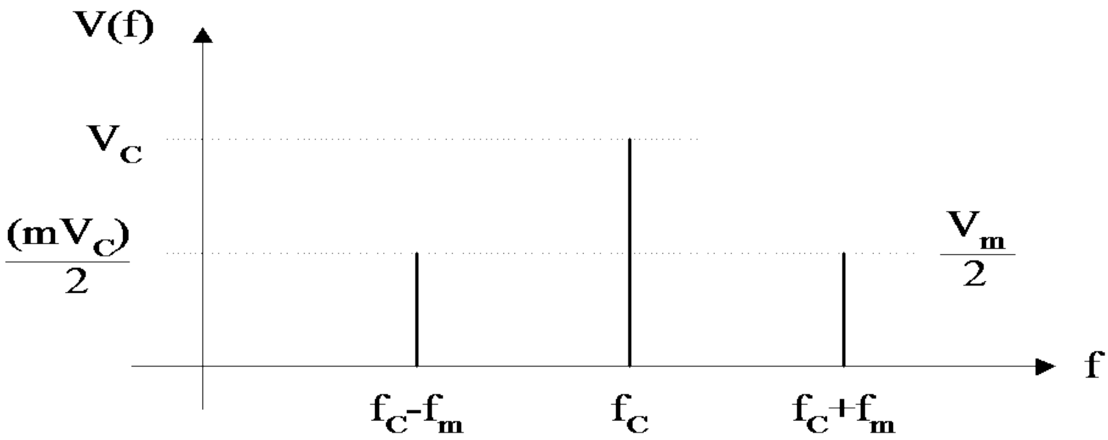
Efficient transmission due to small antennas

Reduces interference, better SNR

Allows use of different carrier frequencies

1 mark each

b)



2 marks for correct frequency components

2 marks for the correct amplitude

c)

To calculate the peak current at 1kHz you need to expand the $0.05V^2$

3 marks

$$V_{AM}^2 = 0.2^2 \sin^2(\omega_c t)(1+0.5\sin(\omega_m t))^2$$
$$V_{AM}^2 = 0.04\sin^2(\omega_c t) + 0.04\sin^2(\omega_c t)\sin(\omega_m t) + 0.01\sin^2(\omega_c t)\sin^2(\omega_m t)$$

The useful part of this to get the w_m is $0.04\sin^2(\omega_c t)\sin(\omega_m t)$ which gives

$0.02\sin(\omega_m t)[1-\cos(2\omega_c t)]$ **1 mark**

Hence the peak current at 1kHz is 0.02*0.05=0.001A=1mA **1 mark**

The useful part of this to get the 2w$_m$ is $0.01\sin^2(\omega_c t)\sin^2(\omega_m t)$ **3 marks**

$$0.01\sin^2(\omega_c t)\sin^2(\omega_m t) = 0.01\left[\frac{1-\cos(2\omega_c t)}{2}\right]\left[\frac{1-\cos(2\omega_m t)}{2}\right]$$
$$= 0.0025\left[1-\cos(2\omega_m t)-\cos(2\omega_c t)+\cos(2\omega_c t)\cos(2\omega_m t)\right]$$

Hence the peak current of the second harmonic is 0.0025*0.05=0.000125A=0.125mA **2 marks**

d.

To retrieve the baseband signal you would need a low pass filter 2 **marks**

Q3a.

$$C = B\log_2(1+S/N)$$

C is the channel capacity in bit/s **1 Mark**
B is the bandwidth in Hz **1 Mark**
S is the signal power in W **1 Mark**
N is the poise power in W **1 Mark**

b.

i

$$C = 40000\log_2(1+100)$$
$$C = 40000*6.65$$
$$C = 266kbit/s$$

2 marks

ii)

If the bandwidth is doubled the noise power also doubles so the SNR is halved **2 marks**

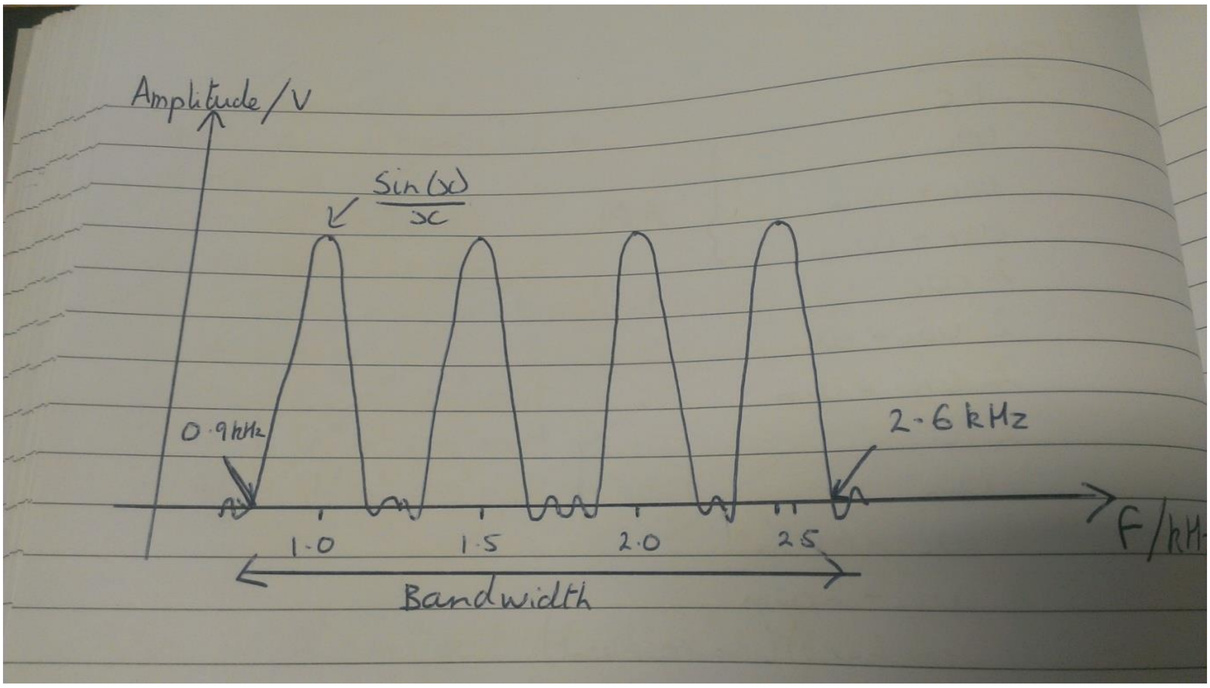
$$C = 80000\log_2(1+50)$$
$$C = 80000*5.67$$
$$C = 454kbit/s$$

2 marks

c.

i) As each symbol carries 2 bits of information the information rate is double the symbol rate =200bit/s **1 mark** ii)

The spectrum will look similar to below.



1 mark for sin(x)/x shape

1 mark for centred at the sub carrier frequencies

1 mark for including the extra 100 Hz for the symbol rate.

1 mark for labelled appropriately

iii)

The difference between the upper carrier and lower carrier is 2.5-1=1.5kHz **1 mark**

The bandwidth of the sin(x)/x is 200Hz **1 mark**

The total bandwidth is 1.5+0.2=1.7kHz **1 mark**

d.

Significant interference between the carriers would begin to occur if the bandwidth of the sin(x)/x cross into each other.

The frequency spacing between each carrier is 500Hz **1 mark**

Hence the maximum bandwidth before the sin(x)/x zero points cross over is 250Hz and hence the maximum symbol rate is 250symbols/s (baud) **1 mark**

Q4a

The reflection coefficient is defined as below

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

i) Short cct =-1 **1 mark**

ii) Open cct = 1 **1 mark**

iii) Matched load =0 **1 mark**

b)

The source reflection coefficient is 0 i.e. matched **1 mark**

The load reflection coefficient is 0.33 **1 mark**

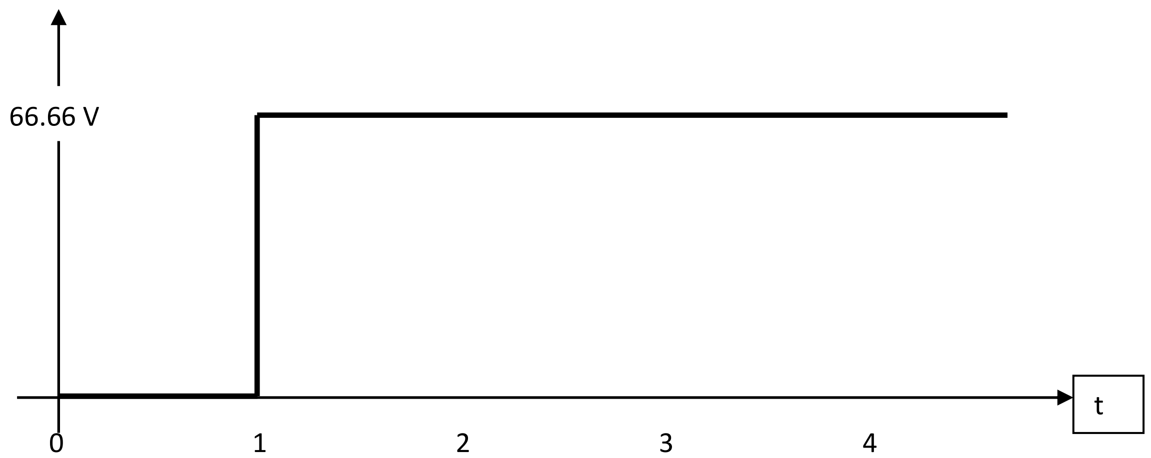
When the switch is closed a forward wave with amplitude 100/2=50V travels along the line. **1 mark**

After 1 micro second there is a reflection causing a backward wave of amplitude 50*0.333=16.66V.

1 mark

The total voltage is then 50+16.66=66.66V **1 mark**

When the backward wave reaches the source there are no more reflections.



2 marks for the graph (full marks will be given if the correct graph is shown)

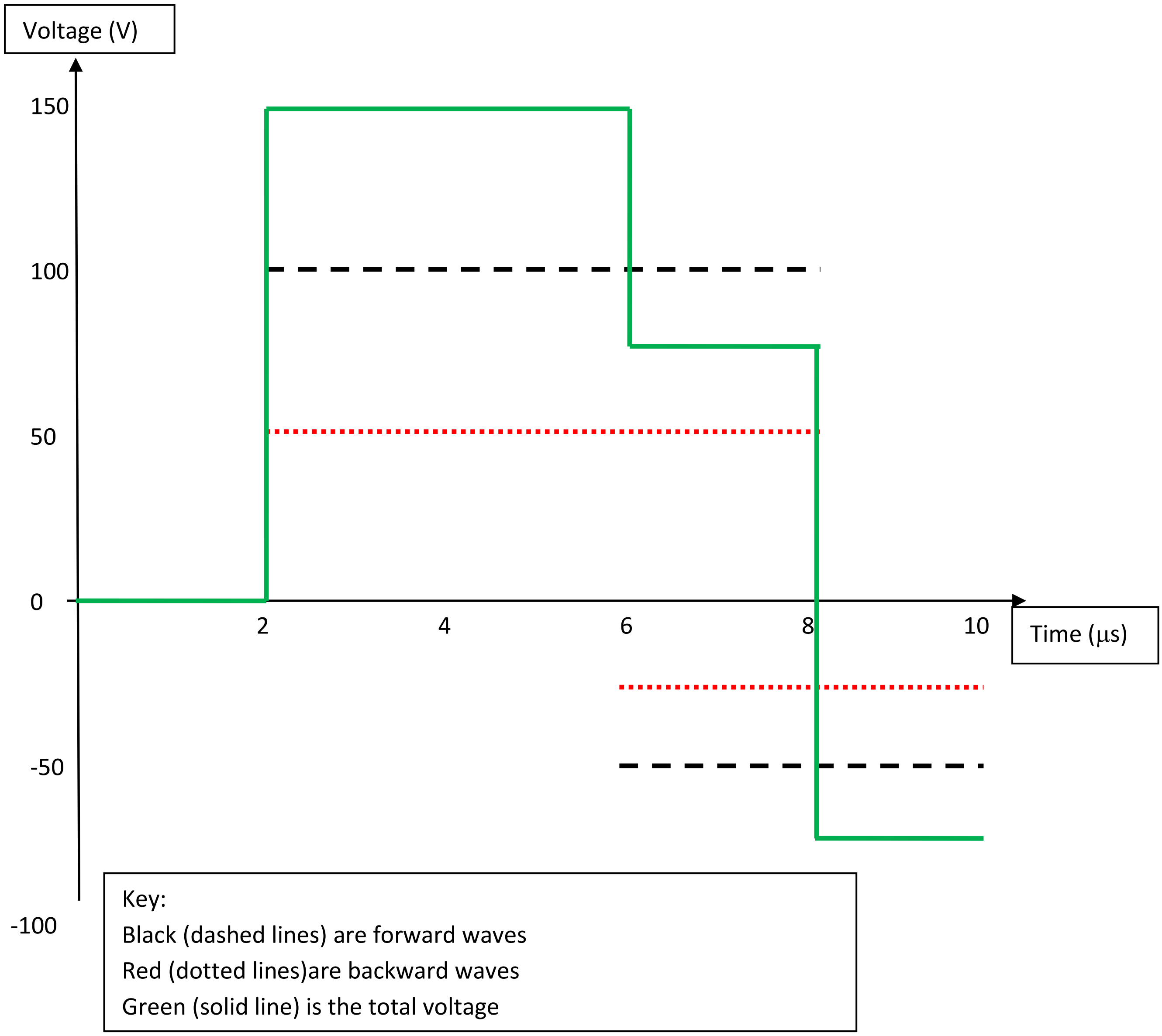
c)

Source reflection coefficient =-1 **1 Mark**

Load reflection coefficient = 0.5 **1 Mark**

time	Source voltage waves (V)	Load voltage waves (V)
0	Vf1=100	0
2	Vf1=100	Vf1=100 Vb1=50 Total=150V
4	Vf1=100 Vb1=50 Vf2=-50	Vf1=100 Vb1=50 Total =150V
6	Vb1=50 Vf2=-50	Vf1=100 Vb1=50 Vf2=-50 Vb2=-25 Total=75V
8	Vb1=50 Vf2=-50 Vb2=-25 Vf3=25	Vf2=-50 Vb2=-25 Total=-75V

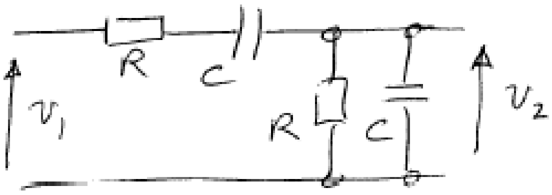
2 marks for each correct value of voltage at the load (i.e. 150V, 75V and -75V)



2 marks for the correct final graph shown in green (full marks will be given if correct final graph is shown)

Q1 (a) The phase shift right around the loop must be 0° or 360° and the total loop gain must be exactly unity.
(one mark for each point)

(b)



$$\frac{v_2}{v_1} = \frac{R /\!/ X_c}{R + X_c + R /\!/ X_c} = \frac{\dfrac{R/j\omega C}{R + 1/j\omega C}}{R + \dfrac{1}{j\omega C} + \dfrac{R/j\omega C}{R + 1/j\omega C}}$$ 1 mark

$$= \frac{\dfrac{R}{1 + j\omega CR}}{\dfrac{\left(R + \frac{1}{j\omega C}\right)(1 + j\omega CR) + R}{1 + j\omega CR}} = \frac{R}{R + \frac{1}{j\omega C} + j\omega CR^2 + R + R}$$

$$= \frac{R}{-\frac{j}{\omega C} + j\omega CR^2 + 3R}$$ 1 mark.

For phase to be 0 or 360°, j terms must vanish. ie, 1 mark

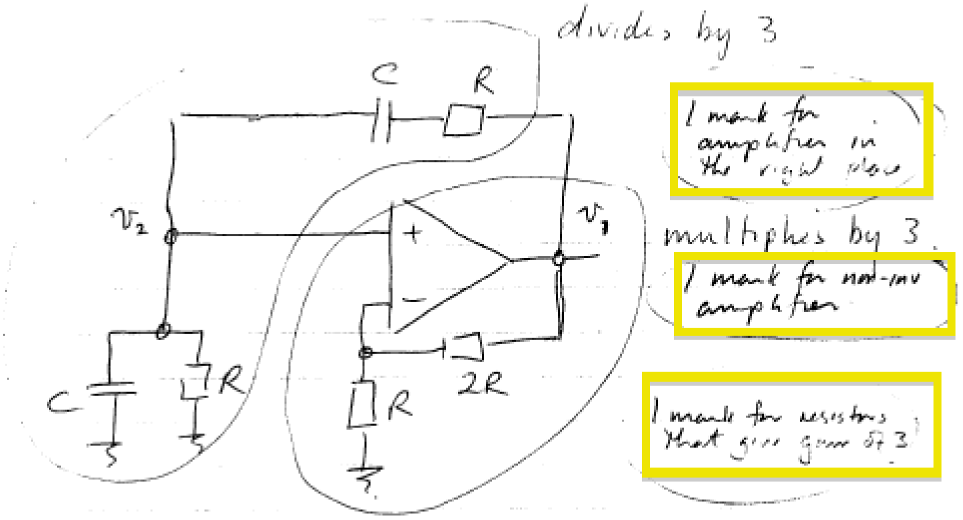
$$\left(\omega CR^2 - \frac{1}{\omega C}\right) = 0$$

or $\omega CR^2 = \frac{1}{\omega C}$ or $\omega^2 = \frac{1}{C^2R^2}$ or $\omega = \frac{1}{CR}$ 1 mark

If j terms $= 0$, $\frac{v_2}{v_1} = \frac{R}{3R} = \frac{1}{3}$ 1 mark

If ω_r and $\frac{v_2}{v_1}$ at ω_r are wrong because of an error in working out $\frac{v_2}{v_1}$, give credit for ω_r + $\frac{v_2}{v_1}$ that are consistent with the $\frac{v_2}{v_1}$ expression derived.

c.



for $f = 100\,\text{kHz}$ CR should be $1.59\,\mu s$ (1 mark)

$100\,k\Omega$ for R would give 15.9 pF for C — this value of C is too small. Circuit parasitic C will be 5 to 10 pF.

1 mark for suitable R
1 mark for suitable C

$10\,k\Omega$ for R would give 159 pF for C — this is about the smallest C one would want with a general purpose op-amps.

$1\,k\Omega$ for R would give 1.59 nF for C — this would be OK.

$100\,\Omega$ for R would load general purpose op-amps too heavily.

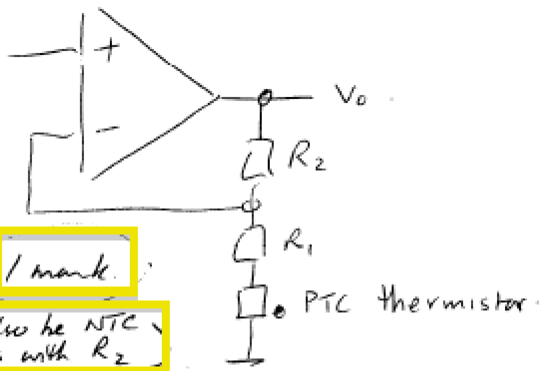
d. Either alter the Rs or the Cs so that values track. It is important that the two Rs and the two Cs have equal values.

1 mark for Rs
1 mark for Cs

e. To obtain a stable output, gain must be exactly unity and phase shift must be exactly zero or 360. Since it is not possible to define a gain exactly some form of feedback is required that reduces gain if the signal gets larger than intended or vice versa. This is commonly achieved using a thermistor (often in the form of a thin filament).

1 mark

1 mark



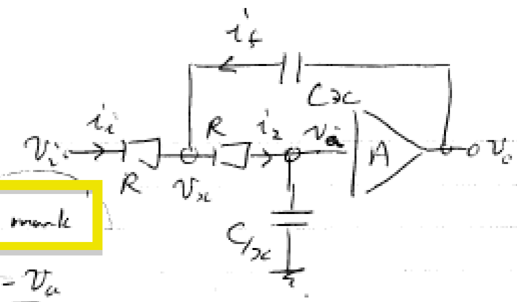
1 mark

(could also be NTC in series with R_2)

if V_o increases, power dissipated in the PTC thermistor increases and gain reduces.

2 marks

Q2 (a). Sum currents at v_x node.



$i_i + i_f = -i_2$ $\boxed{\text{1 mark}}$

$$\frac{v_i - v_x}{R} + \frac{v_o - v_x}{1/sCx} = \frac{v_x - v_a}{R}$$

and $v_a = v_o/A$. $\boxed{\text{1 mark}}$

$$v_i - v_x + v_o sCxR - v_x sCxR = v_x - v_o/A$$

$$v_i + v_o\left[\frac{1}{A} + sCxR\right] = v_x\left[2 + sCxR\right] \quad \boxed{\text{1 mark}} \quad (1)$$

v_a is related to v_x by the potential division between R and C/x

$$v_a = \frac{v_o}{A} = v_x \cdot \frac{1/sC/x}{R + 1/sC/x} = \frac{v_x}{1 + s\frac{C}{x}R} \quad (2) \quad \boxed{\text{1 mark}}$$

putting (2) into (1) to eliminate v_x ...

$$v_i + v_o\left[\frac{1}{A} + sCxR\right] = \frac{v_o\left(1 + \frac{sCR}{x}\right)}{A}\left(2 + sCxR\right)$$

$$v_i = v_o\left[\frac{2}{A} + \frac{sCxR}{A} + \frac{2sCR}{Ax} + \frac{s^2C^2R^2}{A} - \frac{1}{A} - sCxR\right]$$

$$= v_o\left[\frac{1}{A} + \frac{sCR}{A}\left(x + \frac{2}{x} - Ax\right) + \frac{s^2C^2R^2}{A}\right]$$

$$\therefore \frac{v_o}{v_i} = \frac{A}{1 + sCR\left(\frac{2}{x} + x(1-A)\right) + s^2C^2R^2}$$

$\boxed{\text{2 marks (-1 per error)}}$

(b) (i) for stability $\frac{2}{x} + x(1-A) > 0$ $\boxed{\text{2 marks}}$

$$\frac{2}{x} > -x(1-A) \quad \text{or} \quad \frac{2}{x^2} > -(1-A)$$

$$\text{or} \quad \frac{2}{x^2} + 1 > +A$$

$\boxed{\text{2 marks (-1 per error)}}$

b.(ii) by comparison with standard form

$$\omega_n^2 = \frac{1}{C^2R^2} \quad \text{or} \quad \underline{\omega_n = \frac{1}{CR}}$$ 1 mark

$$\frac{1}{\omega_n q} = CR\left(\frac{2}{x} + x(1-A)\right)$$ 1 mark

$$\frac{1}{q} = \omega_n CR\left(\frac{2}{x} + x(1-A)\right) = \left(\frac{2}{x} + x(1-A)\right)$$

$$\underline{q = \frac{1}{\frac{2}{x} + x(1-A)}}$$ 1 mark

c. $A = 1$ so $q = \frac{x}{2}$

for second order factor. normalised $\omega_n = 1$ 1 mark
and $q = 1$ 1 mark

so $\underline{x = 2}$ 1 mark

and $CR = \frac{1}{2 \cdot \pi \cdot 10\text{kHz}}$. or $\underline{C = 1.6\text{nF}}$ 1 mark

for the first order section, corner frequency
= filter cut off frequency 1 mark

$\therefore\ RC = \frac{1}{2\pi \cdot 10\text{kHz}}$ or $C = \underline{1.6\text{nF}}$.

d.