Question 1

a.

The task automation systems are:

Navigation management system

Navigation management comprises all radio navigation aid systems. It combines data from all the navigation sources, such as GPS and INS, to provide the best estimate of aircraft position, ground speed etc.

Autopilot

The autopilot relieves the pilot from continuously flying the aircraft. The basic modes of the autopilot are height hold and heading hold functions. More sophisticated autopilot systems can provide very accurate control of aircraft flight path for automatic landing in poor or zero visibility conditions.

Flight management system (FMS)

FMS has enabled safe two crew operations of largest, long range civil jet airliners. Tasks carried out by the FMS include:

- Flight planning.
- Navigation management.
- Engine control.
- Control of aircraft flight path.
- Control of vertical flight profile.
- 4D navigation.
- Minimising fuel consumption.

Engine control and management

It enables the control, management and monitoring of the engines. Many modern engines have a full authority digital engine control system (FADEC), which ensures the engine responds to throttle commands in an optimum manner, and the engine limits in terms of temperatures, speeds and accelerations, are not exceeded. Furthermore, the engine health monitoring systems measure, process and record a wide range of parameters. They give early warnings of:

- Engine performance deterioration.
- Excessive wear.
- Fatigue damage.
- High vibration levels.

House keeping management

House keeping management covers the automation of background tasks essential for safe and efficient operations. These tasks include:

- Fuel management.
- Electrical power supply system management.
- Hydraulic power supply system management.
- Cabin pressurisation and environmental systems.
- Warning systems.

b.

An Air data system provides the following quantities

- Altitude
- calibrated air speed
- vertical speed
- true air speed
- Mach number
- air stream incidence angle

and with the exception of the air stream incidence angle, which measured using an air stream incidence sensor, the air data is computed from measured:

- static pressure
- total pressure
- air temperature

the static and total pressures are measured using a Pitot-Static probe, the temperature is measured using a temperature probe.

c.

i. The Mach number M and the true airspeed V_T are related by $M = \frac{V_T}{A}$,

where A is the speed of sound for $T_s = -34.5^o C$:

$$A = \sqrt{\gamma R_a T_s} = \sqrt{1.4 \times 287.0529 \times 238.5} = 309.6 \text{ m/s}$$

Therefore,

$$M = \frac{V_T}{A} = \frac{155.55}{309.6} = 0.502$$

ii. Since the aircraft is flying in the troposphere region, therefore, the static temperature is related to altitude by:

$$T_s = T_O - L \times H \quad \Rightarrow \quad H = \frac{(T_O - T_S)}{L} = \frac{(288.15 - 238.5)}{6.5 \times 10^{-3}} = 7638\,\text{m}$$

L is the troposphere lapse rate.

iii. The air density is given by:

$$\rho = \frac{P_s}{R_a T_s} = \frac{37650}{287.0529 \times 238.5} = 0.55\,\text{kg/m}^3$$

iv. The calibrated airspeed V_c and the impact pressure are related by:

$$\text{Impact pressure} = P_0\left(\left(1 + \frac{(\gamma - 1)(V_c / A_0)^2}{2}\right)^{\gamma/(\gamma-1)} - 1\right) = P_T - P_s$$

The static pressure P_s and the total pressure P_T are related by:

$$\frac{P_T}{P_s} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{(\gamma-1)}} \Rightarrow P_T = 37.65 \times \left(1 + \frac{(1.4 - 1)}{2} 0.502^2\right)^{\frac{1.4}{(1.4-1)}} = 44.72\,\text{kPa}$$

Therefore, $P_T - P_s$ =7.07 kPa.

Furthermore, at sea level the speed of sound is given by:

$$A_o = \sqrt{\gamma R_a T_o} = \sqrt{1.4 \times 287.0529 \times 288.15} = 340.29\,\text{m/s}$$

Therefore,

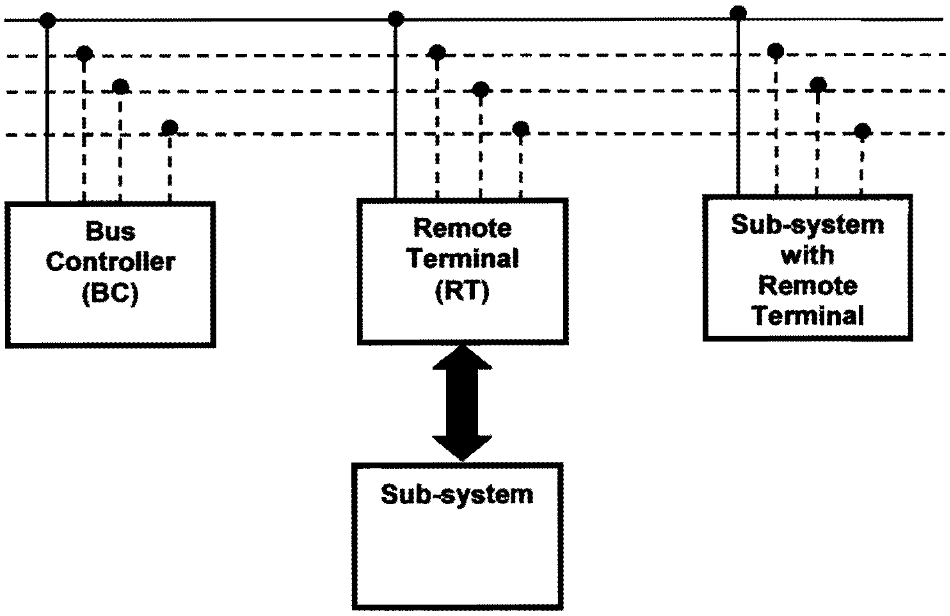
$$\frac{(P_T - P_s)}{P_0} + 1 = \left(1 + 0.2(V_c / A_0)^2\right)^{3.5} \Rightarrow 1 + 0.2(V_c / A_0)^2 = \sqrt[3.5]{\frac{(P_T - P_s)}{P_0} + 1}$$

$$\Rightarrow V_c = A_o\sqrt{\frac{\sqrt[3.5]{\frac{(P_T - P_s)}{P_0} + 1} - 1}{0.2}} = 340.29 \times \sqrt{\frac{\sqrt[3.5]{\frac{(44.72 - 37.65)}{101.325} + 1} - 1}{0.2}} = 106.14$$

m/s = 382.1 km/h.

Question 2

a.



Schematic of MIL STD 1553B bus system

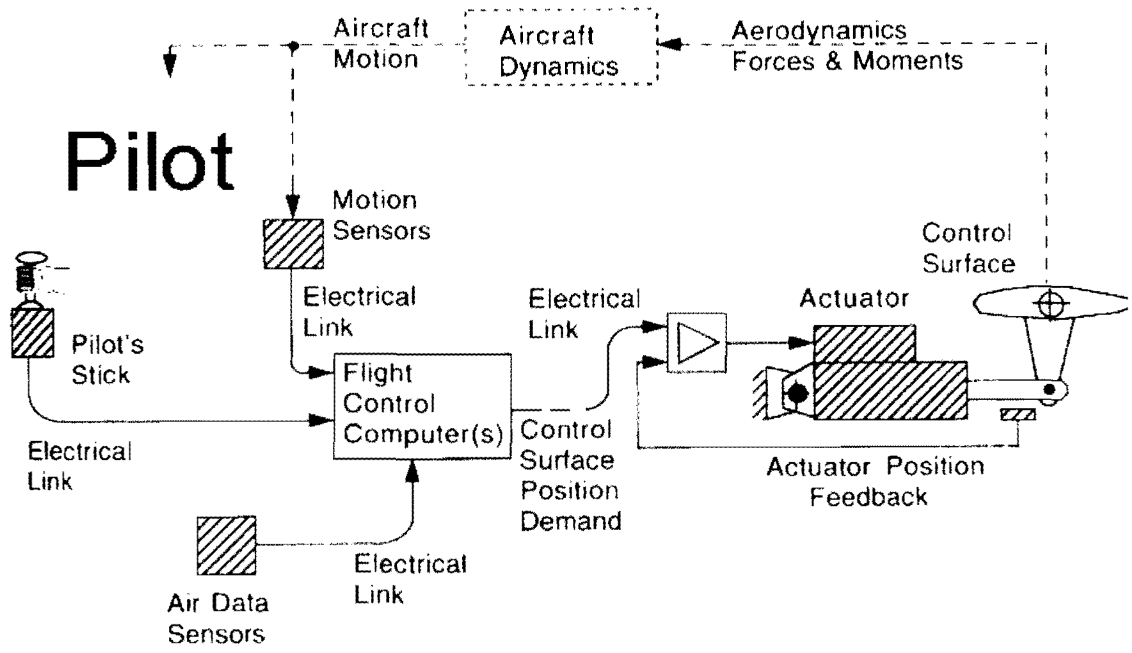
Its main features can be summarised below:

- Each sub-system is connected to the bus through a Remote Terminal (RT).
- Data can only be transmitted from one RT and received by one or more RT(s), following a command from the BC to each RT.
- The bus is formed of a single twisted cable pair with a layer of shielding, with a maximum length of 100m.
- Data is transmitted at 1Mbits/s.
- The technique adopted for data encoding is 'Manchester bi-phase'.
- Number of terminals is limited to 31.

b.

The main advantages of the Fly-By-Wire flight control are summarised as follows:

- Elimination of mechanical control runs.
- Consistent handling over a wide flight envelope.
- Automatic stabilisation.
- Carefree manoeuvring.
- Automatic integration of additional controls.
- Ability to exploit aerodynamically unstable aircraft.



Schematic of the Fly-By-Wire control system

c.

i. Since $n = 3$ and majority voting is adopted, the system is 2-out-of-3 active system, for which the reliability function is given:

$$R(t) = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!}\left[e^{-\lambda k t}\right]\left[1-e^{-\lambda t}\right]^{n-k}$$

where m=2 and n=3, therefore,

$$R(t) = \frac{3!}{2!(3-2)!}e^{-2\lambda t}\left(1-e^{-\lambda t}\right)^{3-2} \qquad (k=2)$$

$$+\frac{3!}{3!(3-3)!}e^{-3\lambda t}\left(1-e^{-\lambda t}\right)^{3-3} \qquad (k=3)$$

$$R(t) = 3e^{-2\lambda t} - 2e^{-3\lambda t}$$

therefore, the probability of failure is given by :

$$F(t = 10\,\text{hours}) = 1 - R(t = 10\,\text{hours}) = 1 - 3e^{\left(-2\times 335\times 10^{-6}\times 10\right)} + 2e^{\left(-3\times 335\times 10^{-6}\times 10\right)}$$

$$F(t = 10\,\text{hours}) = 334.8\times 10^{-7}$$

ii. Since $n = 4$ and non-adaptive majority voting is adopted, the system is 3-out-of-4 active system, for which the reliability function is given:

$$R(t) = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!}\left[e^{-\lambda k t}\right]\left[1-e^{-\lambda t}\right]^{n-k}$$

where m=3 and n=4, therefore,

$$R(t) = \frac{4!}{3!(4-3)!}e^{-3\lambda t}\left(1-e^{-\lambda t}\right)^{4-3} \qquad (k=3)$$

$$+\frac{4!}{4!(4-4)!}e^{-4\lambda t}\left(1-e^{-\lambda t}\right)^{4-4} \qquad (k=4)$$

$$R(t) = 4e^{-3\lambda t} - 3e^{-4\lambda t}$$

therefore, the probability of failure is given by :

$$F(t = 10\,\text{hours}) = 1 - R(t = 10\,\text{hours}) = 1 - 4e^{\left(-3\times 335\times 10^{-6}\times 10\right)} + 3e^{\left(-4\times 335\times 10^{-6}\times 10\right)}$$

$$F(t = 10\,\text{hours}) = 668.1\times 10^{-7}$$

iii. Since $n = 4$ and adaptive majority voting is adopted, the system is 2-out-of-4 active system, for which the reliability function is given:

$$R(t) = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!}\left[e^{-\lambda k t}\right]\left[1-e^{-\lambda t}\right]^{n-k}$$

where m=2 and n=4, therefore,

$$R(t) = \frac{4!}{2!(4-2)!}e^{-2\lambda t}\left(1-e^{-\lambda t}\right)^{4-2} \qquad (k=2)$$

$$+\frac{4!}{3!(4-3)!}e^{-3\lambda t}\left(1-e^{-\lambda t}\right)^{4-3} \qquad (k=3)$$

$$+\frac{4!}{4!(4-4)!}e^{-4\lambda t}\left(1-e^{-\lambda t}\right)^{4-4} \qquad (k=4)$$

$$R(t) = 6e^{-2\lambda t}\left(1-e^{-\lambda t}\right)^2 + 4e^{-3\lambda t} - 3e^{-4\lambda t}$$

therefore, the probability of failure is given by :

$$F(t = 10\,\text{hours}) = 1 - R(t = 10\,\text{hours}) = 1 - 6e^{\left(-2\times 335\times 10^{-6}\times 10\right)}\left(1-e^{\left(-335\times 10^{-6}\times 10\right)}\right)^2$$

$$-4e^{\left(-3\times 335\times 10^{-6}\times 10\right)} + 3e^{\left(-4\times 335\times 10^{-6}\times 10\right)}$$

$$F(t = 10\,\text{hours}) = 149\times 10^{-9}$$

3.

(a) For systems we are concerned with, **System Type** is considered to be given by the number of pure integrators in the forward path of the system, whether they are part of the original open-loop plant, or the controller.

The impact of **System Type** on the steady-state error, e_{ss}, for various inputs, is summarised in the Table below. (A summary in words is sufficient if the students wish).

	e_{ss} for a step input	e_{ss} for a ramp input	e_{ss} for a parabolic input
TYPE 0	1/(1+K)	∞	∞
TYPE 1	0	1/K	∞
TYPE 2	0	0	1/K
TYPE 3	0	0	0

(6)

(b)

(i) Plant transfer function = $\dfrac{0.02}{s(0.2+s)(1+s)}$

First convert to standard form:

$$\frac{0.1}{s(1+5s)(1+s)}$$

Assuming $K_p = 1$, plot the open-loop transfer function on Bode paper using asymptote approximation.

$\frac{0.1}{jw}$ crosses 0dB at 0.1 rad/s and has slope of –20dB/dec. Phase angle is –90° for all frequencies.

$\frac{1}{1+5jw}$ has magnitude 0dB up to the break frequency, ω_b, ($=1/\tau = 0.2$ rad/s) and then has slope of –20dB/dec. The phase angle is 0° up to a frequency $\omega=1/(5\tau)$ (= 0.04 rad/s) and -90° above $\omega = 5/\tau$ (= 1 rad/s). At the break frequency (0.2 rad/s) the phase angle is -45°.

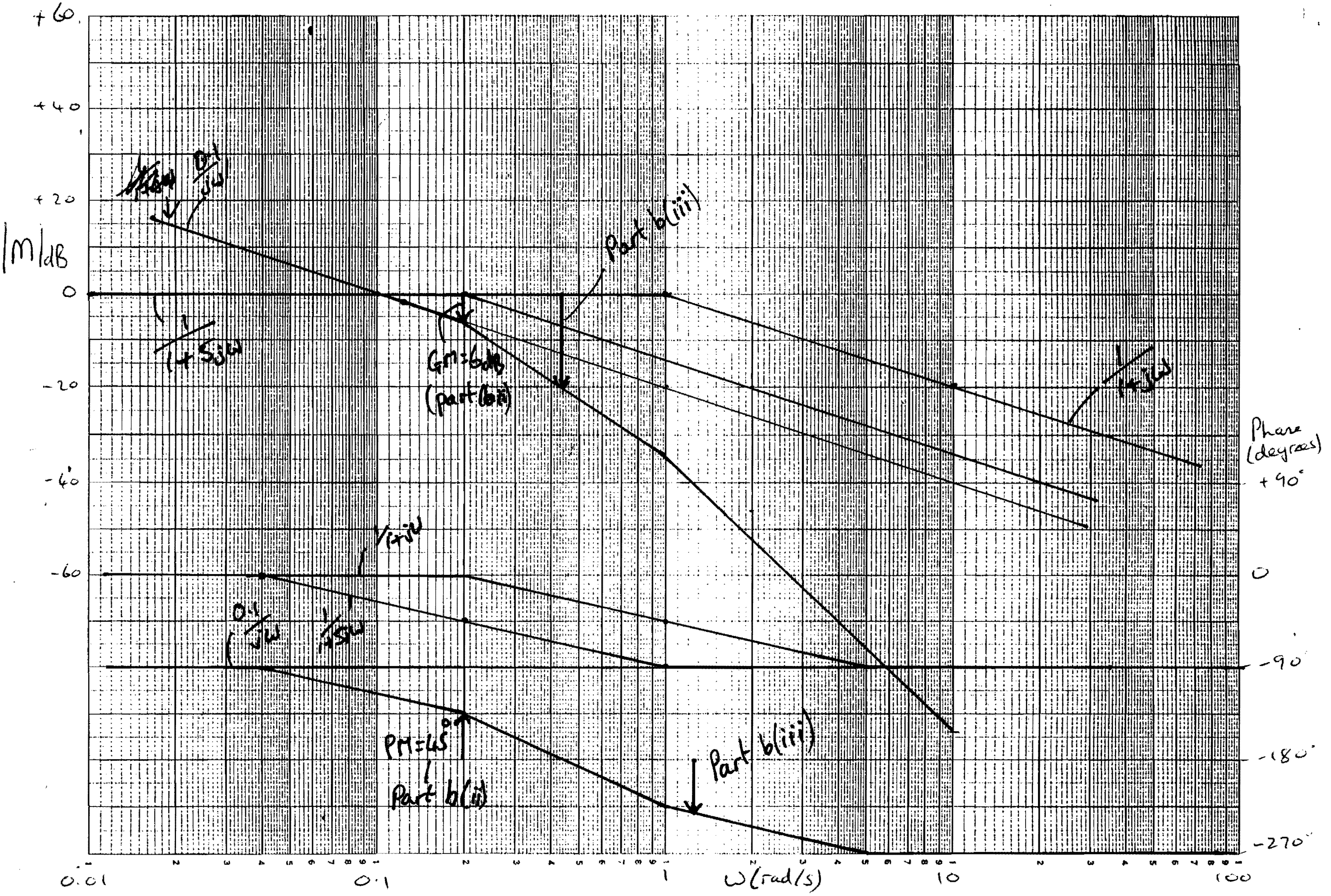
$\frac{1}{1+jw}$ has magnitude 0dB up to the break frequency, ω_b, ($=1/\tau = 1$ rad/s) and then has slope of –20dB/dec. The phase angle is 0° up to a frequency $\omega=1/(5\tau)$ (= 0.2 rad/s) and -90° above $\omega = 5/\tau$ (= 5 rad/s). At the break frequency (1 rad/s) the phase angle is -45°. (6)

These are plotted on the Bode paper provided and the resultant is achieved by summing the 3 components.

(ii) To provide a Phase Margin of 45°, K_p can contribute $\approx$ 6dB i.e. <u>K_p =1.995</u> (4)
The closed-loop bandwidth would be estimated from the resulting 0dB crossing point which is approximately <u>0.2 rad/s</u> (from Bode plot).

(iii) For a value Kp = 100, this has the effect of shifting the magnitude up by +40dB, i.e. when the open-loop plot of just the plant crosses the –40dB line. This produces a Phase Margin of <u>-50°</u> and a Gain Margin of <u>–19dB</u>. Hence the system will be <u>unstable</u>. (4)

Chartwell Graph Data Ref. 5541 Log 4 Cycles x mm, ½ and 1 cm



Q4

The Eigenvalues of the open-loop system may be obtained from:

$$\det(\lambda I - A) = 0$$

$$\therefore \det\left(\lambda I - \begin{pmatrix} -0.32 & 1 \\ -37 & -048 \end{pmatrix}\right) = 0$$

$$\therefore \det\left(\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} -0.32 & 1 \\ -3.7 & -0.48 \end{pmatrix}\right) = 0$$

$$\therefore \det\begin{pmatrix} \lambda + 0.32 & -1 \\ 3.7 & \lambda + 0.48 \end{pmatrix} = 0$$

$$\therefore (\lambda + 0.32)(\lambda + 0.48) - (-3.7) = 0$$

$$\therefore \lambda^2 + 0.8\lambda + 3.8536 = 0$$

Solve for λ_1 and λ_2:

$$\lambda = \frac{-0.8 \pm \sqrt{0.8^2 - 4 \times 3.8536}}{2}$$

$$= \underline{\underline{-0.4 \pm j\,1.922}}$$

Comparing this with the characteristic equation we find:

$$\omega_n^2 = 3.8536 \text{ and } 2\zeta\omega_n = 0.8$$

$$\omega_n = \sqrt{3.8536}$$

$$\therefore \zeta = \frac{0.8}{2 \times \sqrt{3.8536}} = \underline{\underline{0.204}}$$

(5)

This will give very oscilatory dynamics for the rocket.

Q4 (CONTINUED)

(ii) The controllability matrix, $\mathcal{C}$ is given by:

$$\mathcal{C} = (B \,\vdots\, AB) = \left(\begin{matrix} -0.03 \\ -2.8 \end{matrix} \,\vdots\, \begin{pmatrix} -0.32 & 1 \\ -3.7 & -0.48 \end{pmatrix} \begin{pmatrix} -0.03 \\ -2.8 \end{pmatrix} \right)$$

$$= \begin{pmatrix} -0.03 & \vdots & -2.7904 \\ -2.8 & \vdots & 1.455 \end{pmatrix}$$

which is of full rank is the determinant is non-zero

$$|\mathcal{C}| = (-0.03 \times 1.455) - ((-2.8)(-2.7904)) = \underline{\underline{-7.85677}}$$ 3

(iii) Using Ackerman's method.

Step 1: Eigenvalues required are at: $-1.95 \pm j2.49$

Step 2: Form the desired characteristic equation.

$$(s-\lambda_1)(s-\lambda_2) = 0$$

$$\therefore (s + 1.95 - j2.49)(s + 1.95 + j2.49) = 0$$

$$\therefore s^2 + 1.95s + \cancel{j2.49s} + 1.95s + 1.95^2 + j(2.49 \times 1.95) - \cancel{j2.49s} - j(2.49 \times 1.95) + 2.49^2 = 0$$

$$\therefore s^2 + 3.9s + 10 = 0$$

Step 3: Form the matrix $\phi(A) = A^2 + 3.9A + 10I$

$$A^2 = \begin{pmatrix} -0.32 & 1 \\ -3.7 & -0.48 \end{pmatrix} \begin{pmatrix} -0.32 & 1 \\ -3.7 & -0.48 \end{pmatrix} = \begin{pmatrix} -3.5976 & -0.8 \\ 2.96 & -3.496 \end{pmatrix}$$

$$3.9A = \begin{pmatrix} -1.248 & 3.9 \\ -14.43 & -1.872 \end{pmatrix} \qquad 10I = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$

Q4 (CONTINUED)

$$\phi(A) = \begin{pmatrix} -3.5976 - 1.248 + 10 & -0.8 + 3.9 \\ 2.96 - 14.43 & -3.496 - 1.872 + 10 \end{pmatrix}$$

$$= \begin{pmatrix} 5.1544 & 3.1 \\ -11.47 & 4.632 \end{pmatrix}$$

Step 4: Calculate k from:

$$k = \begin{pmatrix} 0 & 1 \end{pmatrix} C^{-1} \phi(A)$$

Now $C = \begin{pmatrix} -0.03 & -2.7904 \\ -2.8 & 1.455 \end{pmatrix}$ $\quad |C| = -7.85677$

$$\text{Adj } C = \begin{pmatrix} 1.455 & 2.8 \\ 2.7904 & -0.03 \end{pmatrix}^T = \begin{pmatrix} 1.455 & 2.7904 \\ 2.8 & -0.03 \end{pmatrix}$$

$$\therefore C^{-1} = \begin{pmatrix} -0.185 & -0.3552 \\ -0.3564 & 0.00382 \end{pmatrix}$$

$$\therefore k = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} -0.185 & -0.3552 \\ -0.3564 & 0.00382 \end{pmatrix} \begin{pmatrix} 5.1544 & 3.1 \\ -11.47 & 4.632 \end{pmatrix}$$

$$= \begin{pmatrix} -0.3564 & 0.00382 \end{pmatrix} \begin{pmatrix} 5.1544 & 3.1 \\ -11.47 & 4.632 \end{pmatrix} = \begin{pmatrix} -1.881 & -1.087 \end{pmatrix}$$

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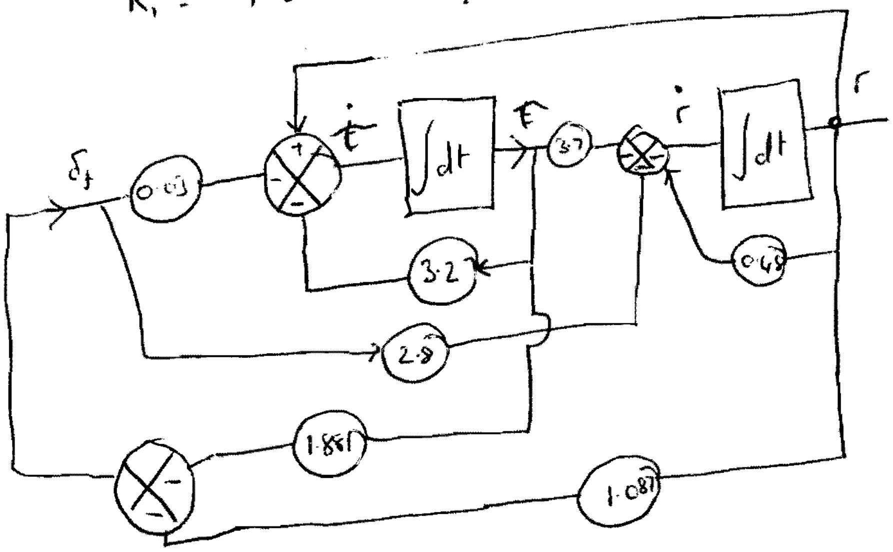
$k_1 = -1.881 \qquad k_2 = -1.087$



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