EEE118 Problem Class Questions – Sheet 1

Fundamental Constants

Boltzman Constant, $k_B = 1.381 \times 10^{-23}$ JK^{-1}
Charge on Electron, $e = 1.6 \times 10^{-19}$ C
Mass of the Electron, $m_e = 9.11 \times 10^{-31}$ kg
Planck's Constant, $h = 6.626 \times 10^{-34}$ Js
Speed of Light in a Vacuum, $c = 3\times10^{8}$ ms^{-1}
Mass of a Proton, $m_p = 1.673 \times 10^{-27}$ kg
Permittivity of Free Space, $\varepsilon_0=8.85 \times 10^{-12}$ Fm^{-1}
Band gap energy for Si, W_g = 1.1eV = $1.1 \times 1.6 \times 10^{-19}$ J

1. A metal wire has a conductivity of 6.7×10^{7} $\Omega^{-1}m^{-1}$ at a specified temperature. If the electric field is 100 V/m calculate the average drift velocity of electrons, assuming there are 10^{29} free conduction electrons per cubic metre. Calculate also the mobility and mean free time between collisions. Use the free electron mass.

   (0.42 m/s, 4.2×10^{-3} m^2/Vs, 2.38×10^{-14} s)

2. A sample of Si has a resistivity of 4.3×10^{-3} Ωm at room temperature and the free electron density is 1.2×10^{22} m^{-3}. Calculate the mobility from this data. What is the mean time between collisions (τ) appropriate to this mobility? Assume $m_e^*=0.98m_e$.

   (0.12 m^2/Vs, 6.7×10^{-13} s)

3. A parallel plate capacitor has a spacing of 10 µm with air between the plates. The area of the capacitor is $1\times10^{-5}m^2$.

(a) If a dielectric of relative permittivity ε_r =10 is placed between the plates what should the new spacing be to leave the capacitance unchanged?

(100 µm)

(b) Calculate the capacitance.

(8.8 pF)

(c) The dielectric in the capacitor has a breakdown field of $50MVm^{-1}$. Calculate the maximum voltage which may be applied to the capacitor.

(5kV)

Solutions to Sheet 1

1. We know the electric field and want to know drift velocity. But these are proportional to each other with mobility as the coefficient of proportionality. So first we need to calculate mobility. The conductivity and the carrier concentration are known, so the mobility can be easily found from:

$$\sigma = nq\mu$$
$$\therefore \mu = \frac{\sigma}{nq} = \frac{6.7x10^{-7}}{10^{29}x1.6x10^{-19}} = 4.2x10^{-3}m^2/Vs$$

Note that it will be often necessary to manipulate this equation. Later we will look at using this equation when there is more than one type of carrier (both electrons and holes).

Returning to this question, we get drift velocity by

$$v = \mu E = 4.2x10^{-3}x10^2 = 0.42m/s$$

To get the mean time between collisions we need to remember the physical origin of mobility and

$$\mu = \frac{q\tau}{m^*}$$
$$\therefore \tau = \frac{4.2x10^{-3}x9.1x10^{-31}}{1.6x10^{-19}} = 2.38x10^{-14}s$$

2. First we are asked to calculate the mobility.

$$\mu = \frac{1}{nq\rho}$$

where ρ = 4.3x10^{-3} Ωm and n = 1.2x10^{22} m^{-3} (given in the question) and q is the charge on the electron. Substituting gives

μ = 0.12 m^{2}V^{-1}s^{-1}.

The second part is to calculate the mean time between collisions (or the mean scattering time). [Recall from the notes that electrons accelerate in the electric field until they hit something and scatter. τ is the average time between these scattering events.]

$$\mu = \frac{\tau q}{m_e^*}$$

Now we know that this time is directly related to the mobility via

This equation can be simply rewritten to solve for τ. The mobility was calculated above and the effective mass is given as 0.98m$_e$.

Substituting in we can obtain **τ = 6.7x10^{-13} s (or 0.67 ps)**

Note that the value of time between collisions is an extremely small value, as we might expect if the electrons are 'rattling around' inside the material.

3. (a). Call the situation with air "state 1", and with dielectric "state 2". i.e in state 1 we have a relative permittivity ε_{r1}, and plate separation d_1. (A is constant in both cases). As we aim to keep capacitance constant, we can equate capacitance in state 1 and state 2.

$$C = \frac{\varepsilon_0 \varepsilon_{r1} A}{d_1} = \frac{\varepsilon_0 \varepsilon_{r2} A}{d_2}$$

giving –

$$d_2 = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} d_1 = \frac{10}{1} \text{ x } 1\text{x}10^{-5}$$

$$d_2 = 100\ \mu\text{m}$$

So the plate separation with dielectric needs to be 100 µm.

(b)

$$C = \frac{\varepsilon_0 \varepsilon_{r2} A}{d_2}$$

So –

$$C = \frac{8.8\text{x}10^{-12} \text{ x } 10.1\text{x}10^{-5}}{1\text{x}10^{-4}}$$

$$C = 8.8\text{x}10^{-12}\,\text{F} = 8.8\,\text{pF}$$

(c) E=V/d so Vmax = EBreakdown x d_2

= $50\text{x}10^{6}$ Vm^{-1} x 10^{-4}m = 5kV