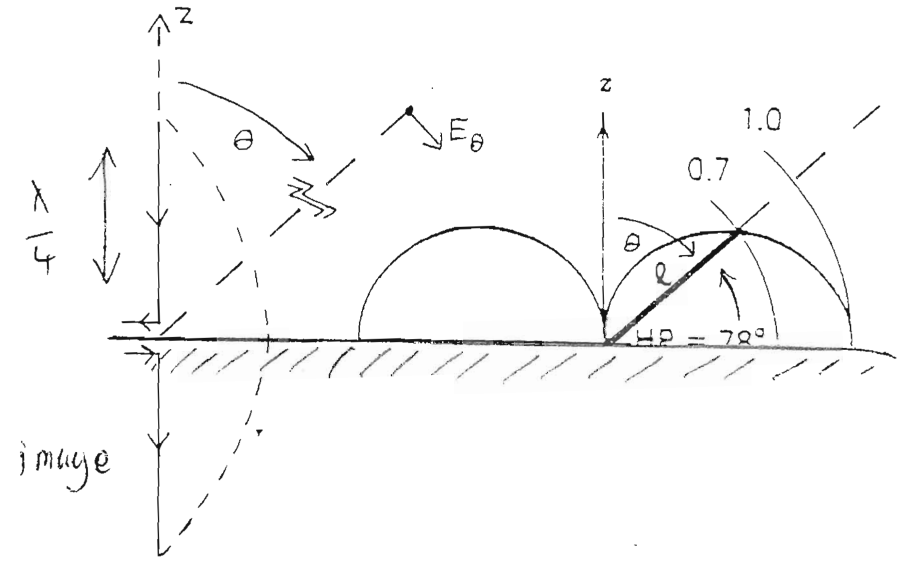
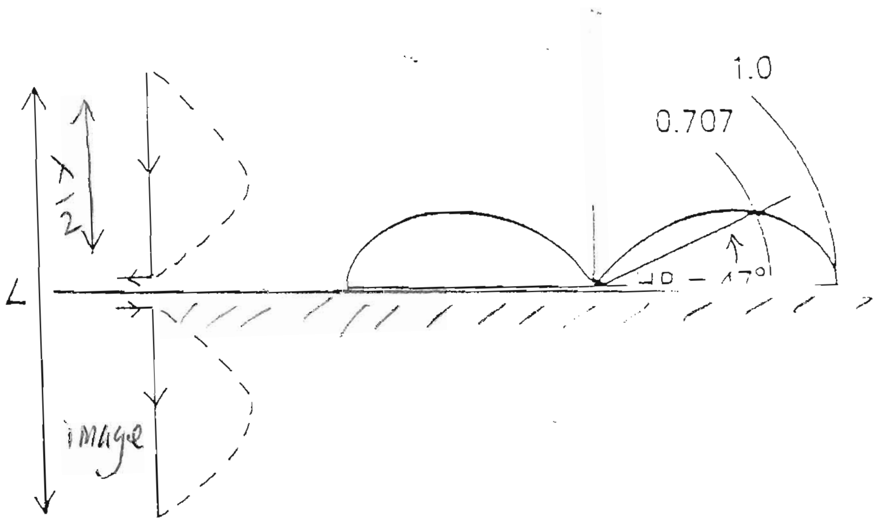
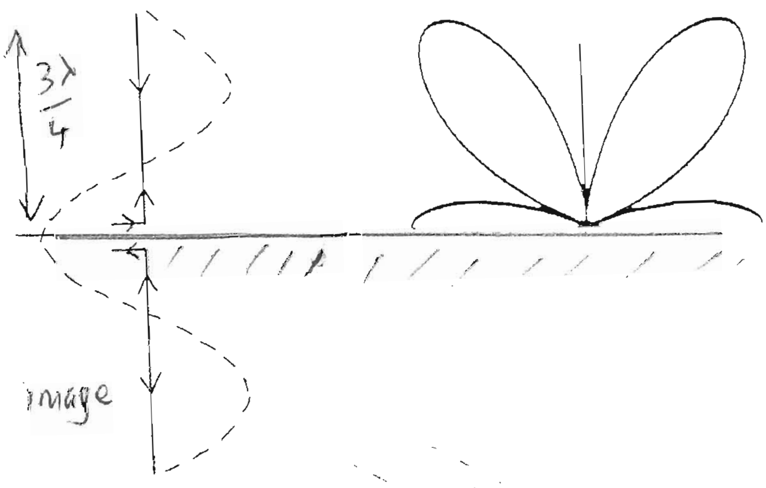
EEE406/6011 solutions

Q1
(a)







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(b)
The radiated power density is given by

$$P_r = \frac{1}{2}\frac{|E_\theta|^2}{\eta} \qquad (1.1)$$

where $|E_\theta|$ is given in the question, so that

$$P_r = \frac{1}{2\eta}\frac{\eta^2 I_o^2}{4\pi^2 r^2}\frac{\cos^2\left(\frac{3\pi}{2}\cos(\theta)\right)}{\sin^2(\theta)} \qquad (1.2)$$

The power radiated into the half space above the ground plane is then

$$P = \int_0^{2\pi}\int_0^{\pi/2} P_r r\sin(\theta) d\phi r d\theta \qquad (1.3)$$

Since the fields are invariant in ϕ, (1.3) reduces to

$$P = 2\pi \int_0^{\pi/2} P_r r^2 \sin(\theta) d\theta \qquad (1.4)$$

Substituting (1.2) into (1.4) then gives

$$P = \frac{\eta I_o^2}{4\pi r^2}\int_0^{\pi/2}\frac{\cos^2\left(\frac{\pi}{2}\cos(\theta)\right)}{\sin(\theta)} r^2 d\theta \qquad (1.5)$$

Substituting the value for the integral given in the question then yields

$$P = \frac{\eta I_o^2}{4\pi}\times 0.88 \qquad (1.6)$$

(i) We now equate this power to that dissipated in a fictitious equivalent circuit 'radiation resistance' R_r thus:

$$\frac{\eta I_o^2}{4\pi}\times 0.88 = \frac{I_o^2}{2}R_r \qquad (1.7)$$

so that

$$R_r = \frac{377}{2\pi}\times 0.88 = 52.8\Omega \qquad (1.8)$$

(ii) Gain (=directivity since no losses) at $\theta = 90^o$ is obtained from (1.2) and (1.3) as

$$G = \frac{P_r|_{\theta=90^o}}{P/4\pi r^2} \qquad (1.9)$$

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hence

$$G = \frac{1}{2\eta}\frac{\eta^2 I_o^2}{4\pi^2 r^2}\times\frac{4\pi}{0.88 I_o^2\eta}\times 4\pi r^2 = 2.27 \equiv 3.57dBi$$

(c) The $\lambda/2$ monopole has the highest low angle ($\theta = 90^o$) gain, but a very high input impedance which is difficult to match. The $3\lambda/4$ monopole provides the best match into 50Ω coax, but the low angle gain is the lowest of the 3 antennas. The $\lambda/4$ monopole is the most compact antenna, whilst still offering a reasonable match into 50Ω cable and a gain higher than the $3\lambda/4$ monopole. 4

Q2
(a)

$$F_x(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_x(x,y,0)e^{j(k_x x+k_y y)}dxdy \quad (2.1)$$

Since the aperture is circular it is more convenient to use polar co-ordinates, where

$$x = \rho cos(\varphi)\ ,\ y = \rho sin(\varphi),\ dxdy = \rho d\rho d\varphi \qquad (2.2)$$

Hence

$$k_x x + k_y y =$$
$$k\,sin(\theta)cos(\phi)\rho cos(\varphi) + k\,sin(\theta)sin(\phi)\rho sin(\varphi) = k\rho\,sin(\theta)cos(\phi-\varphi) \qquad (2.3)$$

and for a circular aperture of radius *a/2* (2.1) becomes

$$F_x = A\int_0^{2\pi}\int_0^{\frac{a}{2}} e^{jk\rho\,sin(\theta)cos(\phi-\varphi)}\rho d\rho d\varphi \qquad (2.4).$$

We need to make a change of variable so that the upper limit of integration in the ρ dimension is *1*, so let

$$\ell = \frac{2}{a}\rho \qquad (2.5)$$

then (2.4) becomes

$$F_x = A\frac{a^2}{4}\int_0^{2\pi}\int_0^{1} e^{jk\frac{a\ell}{2}sin(\theta)cos(\phi-\varphi)}\ell d\ell d\varphi \qquad (2.6).$$

Equating the outer integral in φ to the J_o Bessel function identity in the question with

$$\alpha = k\frac{a\ell}{2}sin(\theta) \qquad (2.7)$$

and

$$\gamma = \phi - \varphi \qquad (2.8)$$

yields

$$F_x = A\frac{a^2}{4}\int_0^{1} 2\pi J_0(k\frac{a\ell}{2}sin(\theta))\ell d\ell \qquad (2.9).$$

Equating this integral to the remaining J_1 identity with

$$u = k\frac{a}{2}sin(\theta) \qquad (2.10)$$

then gives

$$F_x = A\frac{\pi a^2}{2}\frac{J_1(u)}{u} \qquad (2.11).$$

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(b)

(i)
Gain of antenna is given by

$$G = \frac{4\pi}{\lambda^2} \times \text{Aperture Area} \qquad (2.12)$$

where $\lambda = 3cm$. Hence

$$G = \frac{4\pi}{\lambda^2} \times \frac{\pi a^2}{4} = \pi^2 \left(\frac{.45}{.03}\right)^2 = 2220.7 = 33.5dBi \qquad (2.13)$$

(ii)
Interfering satellite is 6.2^o away from main lobe, so we need to find the ratio

$$R = \frac{\left|E_\theta(\theta = 6^o, \phi = 0^o)\right|}{\left|E_\theta(\theta = 0^o, \phi = 0^o)\right|} = \left|\frac{J_1(u_1)}{u_1}\right| / \left|\frac{J_1(u_0)}{u_0}\right| \qquad (2.14)$$

where

$$u_1 = k\frac{a}{2}sin(6^o) = 5.1 \qquad (2.15)$$

$$u_0 = k\frac{a}{2}sin(0^o) = 0 \qquad (2.16).$$

Using Bessel function values given in the question,

$$R = \frac{0.34}{5.1} / 0.5 = 0.13 = -17.5dB$$

Thus the interfering signal is $17.5dB$ below the intended signal.

(iii)

Any circularly polarized signal loses 3dB if received linearly, hence the interference would reduce by this amount.

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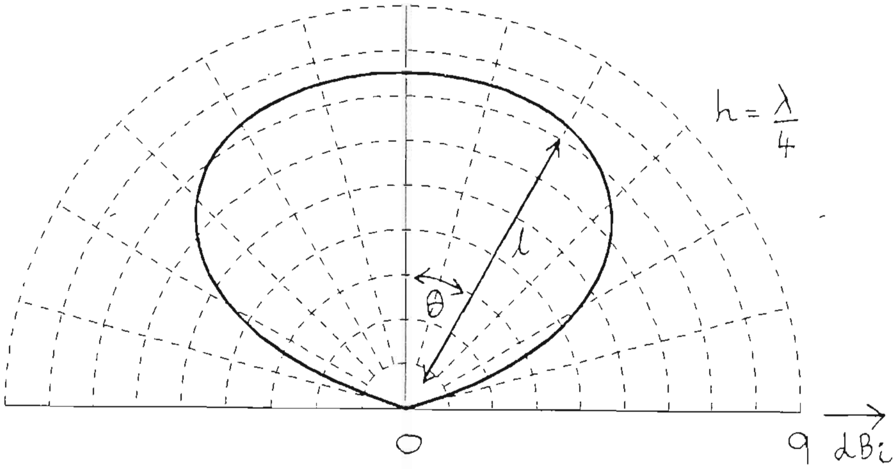
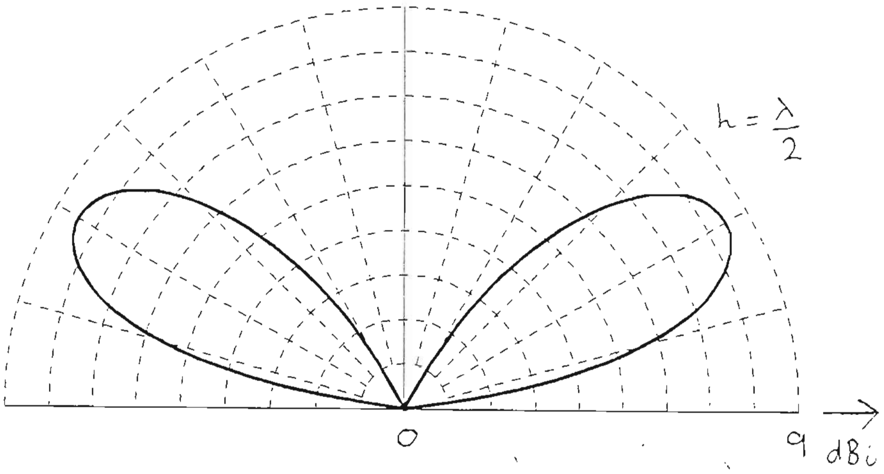
Q3
(a)



(b)



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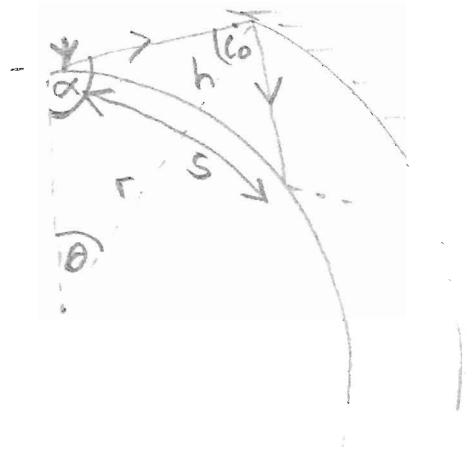

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We need to find θ. The skip distance s is then

$$s = 2r\theta \qquad (3.1)$$

Clearly,

$$\theta = 180 - \alpha - i_o \qquad (3.2)$$

and so the problem resolves into one of finding α and i_o. Now

$$cos(i_o) = \frac{f_c}{f} \quad (3.3)$$

where the critical frequency is given by

$$f_c \approx 9\sqrt{N} = 2.85MHz \qquad (3.4).$$

Then using the sine rule,

$$\frac{sin(180-\alpha)}{(r+h)} = \frac{sin(i_o)}{r} = \frac{\sqrt{1-\frac{f_c^2}{f^2}}}{r} \qquad (3.5)$$

Thus, at $3.57MHz$:

$i_o = 37^o$, $\alpha = 140.25^o$, $\theta = 2.75^o$, and so the skip distance is

$$s = 614.36km \quad (3.6).$$

At $7.14MHz$:

$i_o = 66.47^o$, $\alpha = 103.05^o$, $\theta = 10.48^o$, and so the skip distance is

$$s = 2341.25km \qquad (3.7)$$

(c)

At 3.57MHz, four hops are required to cover the distance, meaning 3 lossy ground reflections. Also, the elevation angle of 50.25^o is 39.75^o from the main lobe peak at 90^o, which will cause further signal attenuation. At 7.14MHz, one hop covers the distance, thus there is no loss due to ground reflections. In addition, the elevation angle of 13.05^o is closer to the main lobe peak at 30^o. Thus the higher frequency signal will be much less attenuated. 4

(d)

The dipoles radiated equally in opposite directions, and therefore it is possible that a more attenuated signal will also be received via the long path, delayed by ~126ms, hence possibly causing an echo. 2

Q4
(a)

The basic steps involved in moment method analysis of an antenna are:

i) Segmentation of the antenna into finite elements, which can be thin wires, surface patches or volumetric segments
ii) Assignment of current basis functions over the segments
iii) Integration of the basis functions over source segments (n) within an EFIE or MPIE type equation to obtain impedance elements Z_{mn} or fields which couple to destination segments (m)
iv) Implementation of E field boundary conditions using antenna excitation V_n via testing or weighting functions over destination segments
v) Solution of M simultaneous equations of the form $\sum_n Z_{mn} I_n = V_m$ for the current basis function amplitudes I_n.

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(b)

(i) Since the wire is thin, $\theta \approx 0$, and hence the field tangential to the mid point of segment 10 arising from the current in segment 2 can be written

$$E_{rn} = Z_{11,2} = \frac{17\Delta z}{r}\left(\frac{1}{kr}cos(kr) - \frac{1}{(kr)^2}sin(kr) - j\left(\frac{1}{kr}sin(kr) + \frac{1}{(kr)^2}cos(kr)\right)\right)$$
(4.1).

At 13.64MHz, $\lambda = 22m$ so the segment lengths are $\Delta z = 1m$, and the distance between the mid points of segments 2 and 11 is then $r = 9m$. Hence

$$kr = \frac{2\pi}{22} \times 9 = \frac{9\pi}{11},\ cos(kr)=-0.841,\ sin(kr)=0.541$$

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and so substituting these values (4.1) becomes

$$Z_{11,2} = -0.773 - j0.157\Omega \quad (4.2).$$

Using such impedance calculations means this is a *point matched* moment method, whereby the E field boundary condition is only forced at the centre of each segment.

(ii) For $Z_{2,11}$, $\theta \approx -\pi$, but E_{r11} now points along $-z$, and so the $cos(\theta) = -1$ term reverses this, meaning the field is in the same direction as for $Z_{11,2}$. Since the distance between source and field point is the same as before, we conclude that

$$Z_{11,2} = Z_{2,11} \quad (4.3).$$

(c)

The impedance matrix

a) Exhibits symmetry about the main diagonal since $Z_{2,1} = Z_{1,2}$ etc.

b) Is Toeplitz since each descending diagonal from left to right is constant.

Thus, the number of different values in the matrix is 11, and only 11 impedances need be calculated therefore.

The self terms $Z_{m,m}$ will have the largest magnitude, since the distance from source to field point is the smallest, ($r \sim$ the wire radius).

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(d)

The input impedance of a half wave dipole at resonance is $\approx R_r = 73.2\Omega$. The power radiated is therefore found from the terminal current I_6 flowing through this resistance,

$$\frac{I_6^2}{2} R_r = 36.6W \qquad (4.4).$$

The excitation voltage amplitude must therefore be

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$$I_6 R_r = 73.2V \quad (4.5).$$