Data Provided: None

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2011-12 (2.0 hours)

EEE6440 Advanced Signal Processing

Answer **FOUR** questions (**TWO** questions from **Part A** and **TWO** questions from **Part B**). **No marks will be awarded for solutions to a third question attempted from any of the two sections.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

PART A - Answer only TWO questions from questions 1, 2 and 3.

1. The M-point moving average filter (MAF) operates by averaging a number of points from the input signal x(n) to produce each point in the output signal y(n) as follows:

$$y(n) = \frac{1}{M} \sum_{k=\frac{1-M}{2}}^{\frac{M-1}{2}} x(n+k)$$

Assume M is an odd number.

a. Find the impulse response, h(n), for the 3-point MAF, clearly indicating the position of n=0. **(1)**

b. Show the step response and comment on its time-domain performance **(2)**

c. Determine and draw the magnitude of the frequency response, H(ω). Comment on its frequency domain performance. **(3)**

d. The computational complexity of the M-point MAF can be reduced using the recursive implementation. Derive an expression for the recursive implementation of the M-point MAF and compare its complexity, in terms of number of additions and multiplications, with respect to those for the non-recursive implementation. **(4)**

e. Determine and draw the resulting filter kernel if the 3-point MAF is applied on a signal in 2 passes. Sketch and compare its time-domain and frequency-domain performances with those of the 3-point MAF and L-point MAF, where L is the length of the new filter kernel. **(5)**

2. An input signal $x=(x_0, x_1, x_2, x_3)$ is transformed into $y=(y_0, y_1, y_2, y_3)$ using a type of wavelet transform. The first level of decomposition, in matrix form, is shown in the following equation.

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h & h & 0 & 0 \\ 0 & 0 & h & h \\ h & -h & 0 & 0 \\ 0 & 0 & h & -h \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a. Write down the basis functions corresponding to the above forward wavelet transform matrix. **(2)**

b. If this set of basis functions forms an orthogonal transform, find the value of h. Using your solution, verify the orthogonality of this transform. **(3)**

c. What are the low pass and the high pass filter kernels used in the corresponding filter bank implementation of this wavelet transform? **(1)**

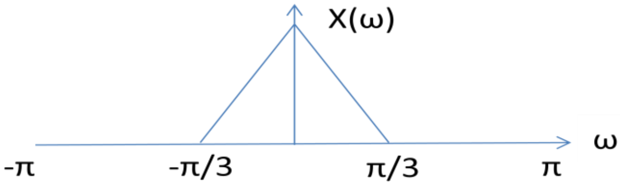
d. What is the corresponding inverse transform matrix? Verify that your solution provides the perfect reconstruction for the given transform. **(3)**

e. What is the transform matrix corresponding to the second level of transform to obtain a dyadic decomposition? **(2)**

f. How do you compute the mean of signal x using the transform domain coefficients y? **(2)**

g. Explain how you extend this transform matrix to obtain the 1-level dyadic wavelet decomposition of a signal containing 128 data points. **(2)**

3. a. Consider the signal $x(n) = \{2, 3, 4, 3, 2, 4, 1, 2, 1, 2, 1\}$ for $-5 \leq n \leq 5$ and the magnitude of its Fourier transform as shown below.



If the signal x(n) is sampled to get y(n) as

$$y(n) = \begin{cases} x(n), & n = 0, \pm 2, \pm 4, \ldots \\ 0, & n = \pm 1, \pm 3, \ldots. \end{cases}$$

i. Compute and sketch signal y(n)

ii. Giving explanations, sketch the magnitude of the Fourier transform of the sampled signal y(n).

iii. Does this sampling system require an anti-aliasing and/or an anti-imaging filter? If so, what are the transition bandwidth(s)? **(5)**

b. A signal, sampled at 2.048 kHz, is to be decimated by a factor of 32 to yield a signal at a sampling frequency of 64 Hz. The signal band of interest extends from 0 to 32 Hz. The overall anti-aliasing digital filtering should satisfy 0.01 dB passband deviation (δ_p) and 80 dB stopband attenuation (δ_s).

It is suggested to use a 2-stage decimator, with decimation rates M_1=16 and M_2=2, for the above mentioned multi-rate system.

(i) Estimate the lengths of the anti-aliasing filters h_1 and h_2 used for the two decimations, respectively.

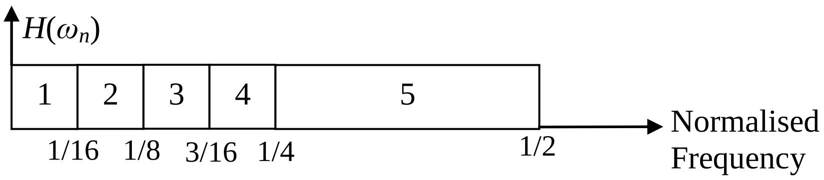
Note that the filter length N for a low pass filter is approximated as

$N \approx \frac{-10\log(\delta_p\,\delta_s)-13}{14.6(\Delta f)} + 1$, where Δf is the normalised width of transition band.

(ii) Estimate the computational complexity of this 2-stage decimator in terms of multiplications per second.

(iii) Explain why multistage decimation is more efficient in terms of the computational complexity, compared to a single stage decimation system (for example using an M=32 decimator in this problem). **(8)**

c. A speech signal originally sampled at 8 kHz is decomposed into 5 frequency subbands as defined below:



If qauantization of 5 bits/sample in the first subband, 4 bits/sample in the next two subbands, 2 bits/sample in the fourth subband and 1 bit/sample in the 5th subband are used, what will be the output data rate of the subband coded speech signal? **(2)**

PART B - Answer only TWO questions from questions 4, 5 and 6.

4. **a.** Derive the relationship between the variance and the mean of a random process x(n) and show all working. **(3)**

b. Zero-mean white Gaussian noise with variance 1 is applied to two filters simultaneously. Filter 1 has transfer function $H_1(z)=1-3z^{-1}$; filter 2 has transfer function $H_2(z)=1-2z^{-2}$. The output of filter 1 is denoted by $y_1(n)$ and the output of filter 2 is denoted by $y_2(n)$.

i) What is the autocorrelation sequence of the output of filter 1? **(2)**

ii) Calculate the cross-correlation sequence $\phi_{y_1 y_2}(m)$ and $\phi_{y_2 y_1}(m)$. **(4)**

c.

i) For a 20-bit A/D converter, what is the dynamic range for a cosine wave input signal? **(3)**

ii) What is the dynamic range for a uniformly distributed random input signal? (Note that such an input signal has a uniform probability density function). **(3)**

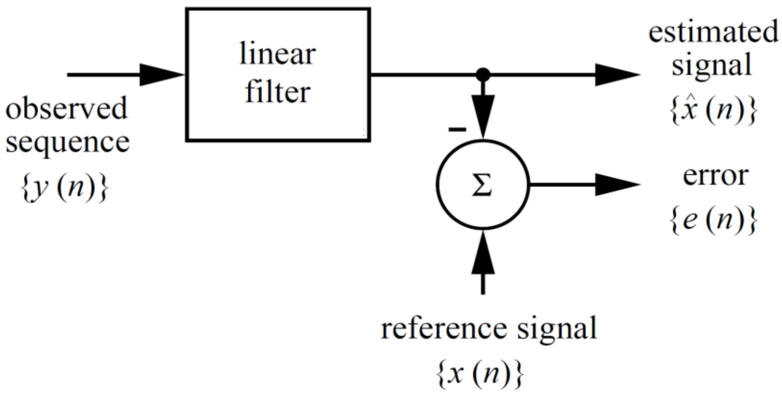
5. **a.** Suppose the z-transform $S_{yy}(z)$ of the autocorrelation function of a correlated sequence y(n) is given by

$$S_{yy}(z)=(z-1/2)(z-3)(z^{-1}-1/2)(z^{-1}-3)$$

Design a filter U(z) whose output will be white when passing y(n) through it. List all of the possible choices for such a filter. **(4)**

Which one is the minimum-phase whitening filter for y(n)? **(1)**

b. A linear estimator is shown below, where the impulse response of the linear filter is given by h_j, j=0, 1, …, N-1. Derive the Wiener solution for h_j. Show all working.



(10)

6. **a.** Suppose the z-transform of the cross-correlation function between the input x(n) and the output y(n) of a filter is given by $S_{xy}(z)$ and the z-transform of the autocorrelation of the input x(n) is given by $S_{xx}(z)$.

i) Give the relationship between these two z-transforms. **(2)**

ii) Given an unknown linear system with white stationary input x(n) and output y(n), use the above result to show how to measure the impulse response of the system? **(5)**

b. **i)** Suppose the length of an FIR (finite impulse response) adaptive filter is N. Its input is denoted by y(n) and the training signal is denoted by x(n). Derive the LMS (least mean square) adaptive algorithm for updating the coefficients of the adaptive filter. **(4)**

ii) The table below shows the input and training signal to a two-tap adaptive filter at sample numbers 3 and 4, where **h**(n) is the vector holding the two taps of the adaptive filter :

Iteration *n*	y(*n*)	**h**(*n*)	x(*n*)
3	0.25	[1 3]	1.03
4	0.5		-0.27

Using the derived LMS algorithm, evaluate **h**(4). The stepsize is fixed at 0.1. **(4)**

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