Tutorial Sheet – No 4

(Energy and Power)

Hint : ***In questions 1 and 3, for the quantities to be calculated, initially derive the relevant expressions as general functions of time t (i.e. only substitute for the given value of t at the end)***

1 A circuit consists of a resistance of 200Ω in parallel with a capacitance of 1000μF. The circuit is supplied by a voltage source $v(t) = 200t$ Volts, where t is in seconds from the time of application. At time t = 0.5s calculate:

(a) the instantaneous power dissipated in the resistor
(b) the instantaneous power supplied to the capacitor
(c) the power supplied by the source
(d) the total energy supplied by the source from t = 0 to t = 0.5s
(e) the energy stored in the circuit at t = 0.5s

(50W; 20W; 70W; 13.3J; 5J)

2 A circuit consists of a resistance of 15Ω in series with an inductance of 10H. If the circuit is excited by a sinusoidal voltage source of peak value 100V and frequency 2Hz calculate the magnitude and phase of the rms current in the circuit, its power factor, and the values of the VA, W and VAR of the circuit.

($0.56\angle -83.2°A_{rms}$; 0.118lagging; 39.6VA; 4.67W; 39.3VAR)

3 The same circuit from question 2 is now excited by a current source of $i(t) = 5t$ A, where t is in seconds from the time the current is applied, calculate the following at t = 2s:

(a) the instantaneous power in the resistance R
(b) the instantaneous power input to the inductor
(c) the total energy input to the circuit in the time interval t = 0 to t = 2s
(d) the energy stored in the circuit.

(1500W; 500W; 1500J; 500J)

4 The total load on an $11kV_{rms}$, 50 Hz, single-phase supply consists of:

- an electromagnet, which draws $9A_{rms}$ at 0.2 power factor lagging
- a general load of 60 kVA, operating at 0.6 power factor leading
- a 90% efficient motor, which is delivering a mechanical output power of 100kW and is operating at 0.85 power factor lagging

(a) For each load component calculate the VA, Watts, and VAR demand and input current.

*(Electromagnet 99kVA, 19.8kW, 97kVAR, $9\angle -78.5°A_{rms}$;
general load 60kVA, 36kW, -48kVAR, $5.45\angle 53.1°A_{rms}$;
motor 130.7 kVA, 111kW, 68.9kVAR, $11.88\angle -31.8°A_{rms}$)*

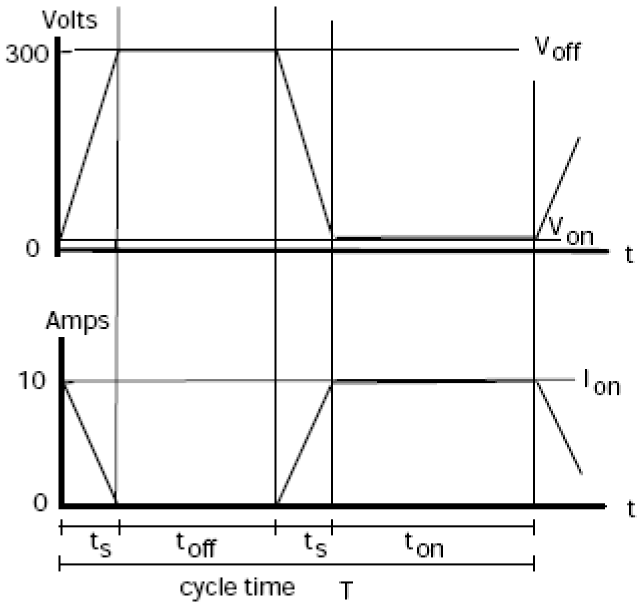
(b) Calculate the resultant VA of the total load and its overall power-factor. Also calculate the total load current and its phase angle.

(Total = 204.3kVA @ 0.82pf lag; $18.6\angle -35.2°A_{rms}$)

(c) Calculate the capacitance required to improve the overall load power-factor to 0.95 lagging.

(1.66μF)

5 The figure shows one cycle of the voltage across, and the current through, a power-electronic switching device, operating at a frequency of 5kHz. The switch has equal on and off periods t_{on} and t_{off} and the two switching events both take a time of t_s = 2μs.



(a) Neglecting the small 'V_{on}' volts drop in the switch, derive an expression for the instantaneous v_t and i_t during the first switching event of length t_s.

$$(v = V_{off}\frac{t}{t_s}; \quad i = I_{on}\left(1 - \frac{t}{t_s}\right))$$

(b) Hence derive an expression for p, the instantaneous power loss during one switching event, and also its peak value.

$$(p = vi = V_{off}I_{on}\left(\frac{t}{t_s} - \frac{t^2}{t_s^2}\right); \quad p_{\max} = \frac{1}{4}V_{off}I_{on}; \quad P_{max} = 750W)$$

(c) What temperature rise would the semiconductor have if it had a negligible thermal time-constant and a thermal resistance of 5°C/W ?

(ΔT =3750°C -not for long !!!)

(d) Show that the average power loss per cycle is given by:

$$p_{av} = \frac{1}{3}\frac{t_s}{T}V_{off}I_{on}$$

(e) Hence calculate a more realistic temperature rise assuming a thermal time-constant >>200ms

(P_{av} = 10W; ΔT = 50°C)

***Note:** In practice thermal time-constants of components are usually >> cycle times. Also, whilst the peak instantaneous power loss is independent of the switching time, the average is proportional to t_s/T – hence it is important to make the switching time as short as possible when designing such circuits.*