Q1:

a) $a_o = \frac{1}{T}\int_{-T/4}^{T/4} 1dt = \frac{1}{T}[T/4-(-T/4)] = \frac{1}{2}$

Since this is an even function, $b_n = 0$.

$$a_n = \frac{2}{T}\int_{-T/4}^{T/4} \cos\left(\frac{2n\pi t}{T}\right)dt = \frac{2}{\left(\frac{2n\pi}{T}\right)T}\left[\sin\left(\frac{2n\pi t}{T}\right)\right]_{-T/4}^{T/4} = \frac{1}{n\pi}\left(\sin\left(\frac{2n\pi}{T}.\frac{T}{4}\right)-\sin\left(\frac{2n\pi}{T}.\frac{-T}{4}\right)\right)$$

$a_n = \frac{1}{n\pi}\left(2\sin\left(\frac{n\pi}{2}\right)\right)$, $a_n = 0$ when n=even number, $a_n = \frac{2}{n\pi}$ when n=1,5,9….,

$a_n = -\frac{2}{n\pi}$ when n=3,7,11….,

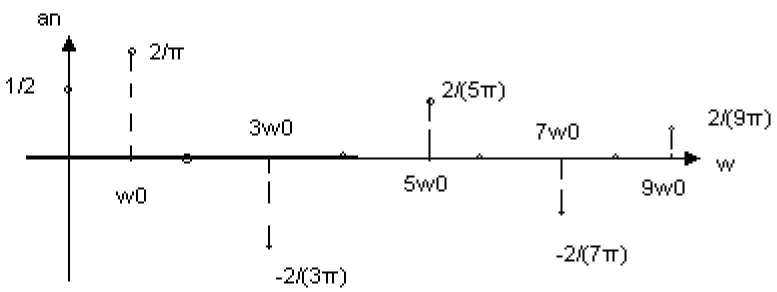
Therefore the Fourier Series representation is given by

$$v(t) = \frac{1}{2}+\frac{2}{\pi}\left[\cos\omega_o t-\frac{1}{3}\cos 3\omega_o t+\frac{1}{5}\cos 5\omega_o t.......\right] \text{ where } \omega_o = \frac{2\pi}{T}$$

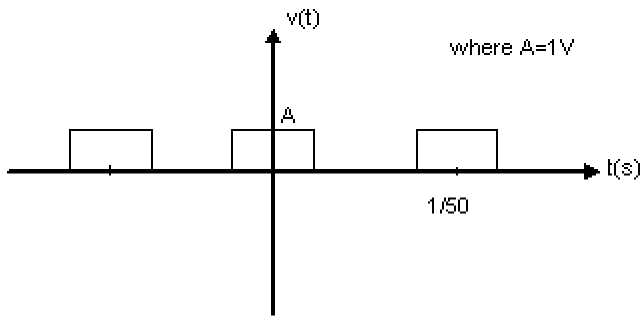
b) From (a) we have:

a_0=1/2, a_1=2/π, a_3=-2/(3π), a_5=2/(5π) a7=-2/(7π) a_9=2/(9 π)

Therefore the amplitude spectrum looks like:



c) The output of the converter is



Within -200Hz to 200 Hz, we have

a_0=A/2, a_1=(2A)/π, a_3=-(2A)/(3π),

we know that the complex Fourier Series coefficients are $|C_0|=|a_0|$ and $|C_n|=|a_n|/2$,

so: $|C_0|$=A/2 and $|C_1|=|C_{-1}|$=A/ π, $|C_{-3}|=|C_3|$=A/(3π)

Using Parseval's theorem, we have

$$Ave.Power = \sum_{n=-3}^{n=3} | Cn |^2 = (\frac{A}{2})^2 + 2(\frac{A}{\pi})^2 + 2(\frac{A}{3\pi})^2 = (\frac{1}{2})^2 + 2(\frac{A}{\pi})^2 + 2(\frac{A}{3\pi})^2$$

$$= (\frac{1}{2})^2 + 2(\frac{1}{\pi})^2 + 2(\frac{1}{3\pi})^2 = 0.475$$

Q2.a) The convolution of 2 continuous time signals, x(t) and h(t) is described by

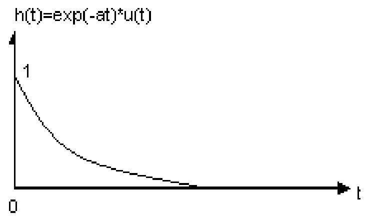
$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau) * x(t-\tau) d\tau$$ where τ is a dummy variable

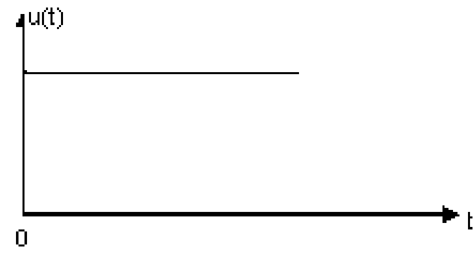
The convolution procedures are as follows:

1) Replace t with τ (the integration variable)
2) Flip h(τ) about the vertical axis to give h(-τ)
3) Shift h(-τ) along τ-axis by t to produce h(t- τ)
4) Multiply x(τ) and h(t- τ) for all values of τ with t fixed
5) Integrated the product x(τ)*h(t- τ) over all τ to produce y(t)

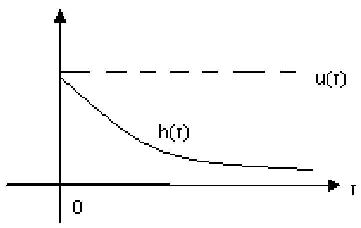
6) Repeat steps (1) to (5) for subsequent time intervals if required

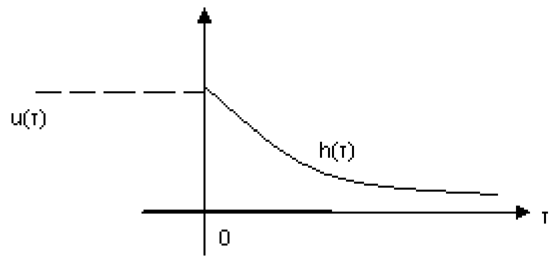
b) The signals are:



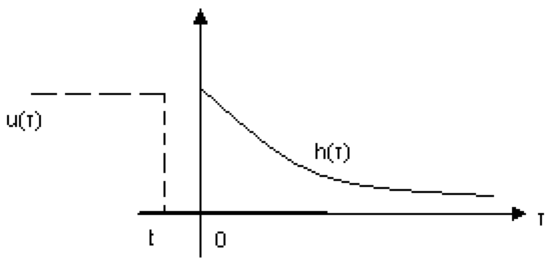


change t to τ

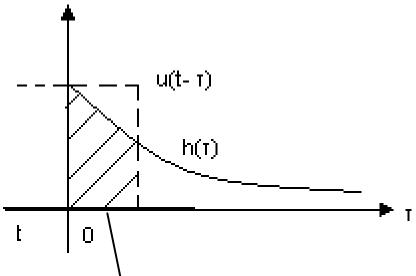




b) for $t<0$



$h(\tau)*x(t-\tau)=0$ so: $y(t)=0$

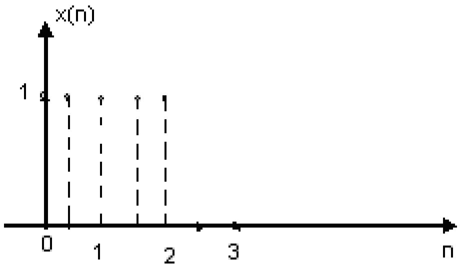


for t>0

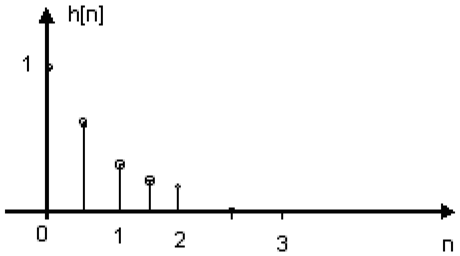
$$y(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau = \int_{-\infty}^{\infty} e^{-a\tau}u(\tau)u(t-\tau)d\tau = \int_{0}^{t} e^{-a\tau}d\tau = \frac{1}{a}\left[1-e^{-a\tau}\right]_0^t = \frac{1}{a}\left[1-e^{-at}\right]u(t)$$

Q2. c) After sampling we have

x[n]= {1 (0≤n≤2); 0 (otherwise)}



h[n]= {e^{-a*n} (0≤ n ≤ 2);   0 (otherwise)}



(assume h[n]=0, for n>2)

To compute the response y[n]=x[n]*h[n]

We can use the convolution table:

k	-2.0	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	2.0	2.5	3.0
x[k]	0	0	0	0	1	1	1	1	1	0	0
h[-k]	e^{-4}	e^{-3}	e^{-2}	e^{-1}	1	0	0	0	0	0	0
h[1-k]	0	e^{-4}	e^{-3}	e^{-2}	e^{-1}	1	0	0	0	0	0
h[2-k]	0	0	e^{-4}	e^{-3}	e^{-2}	e^{-1}	1	0	0	0	0
h[3-k]	0	0	0	e^{-4}	e^{-3}	e^{-2}	e^{-1}	1	0	0	0
h[4-k]	0	0	0	0	e^{-4}	e^{-3}	e^{-2}	e^{-1}	1	0	0
h[5-k]	0	0	0	0	0	e^{-4}	e^{-3}	e^{-2}	e^{-1}	1	0
h[6-k]	0	0	0	0	0	0	e^{-4}	e^{-3}	e^{-2}	e^{-1}	1
h[7-k]	0	0	0	0	0	0	0	e^{-4}	e^{-3}	e^{-2}	e^{-1}
h[8-k]	0	0	0	0	0	0	0	0	e^{-4}	e^{-3}	e^{-2}

So: $y[0]=1$;

$y[1]=e^{-1}+1=1.368$;

$y[2]=e^{-2}+ e^{-}1+1=1.503$;

$y[3]= e^{-3}+ e^{-2}+ e^{-}1+1=1.553$;

$y[4]= e^{-4}+ e^{-3}+ e^{-2}+ e^{-}1+1=1.571$;

$y[5]= e^{-4}+ e^{-3}+ e^{-2}+ e^{-1}=0.571$;

$y[6]= e^{-4}+ e^{-3}+ e^{-2}=0.204$;

$y[7]= e^{-4}+ e^{-3}=0.068$;

$y[8]= e^{-4}=0.018$;

$y[9]=0$

Q3. a) We have:

$$V_i(t) = i(t)R + V_c(t) = i(t)R + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

Taking the Laplace Transform gives:

$V_i(s) = I(s)R + \frac{I(s)}{SC}$ since $V_c(0)=0$

$V_i(t)=Au(t)$, so $V_i(s)=A/s$;

so, we have

$$\frac{A}{s} = I(s)(R + \frac{1}{sC})$$

$$I(s) = \frac{A}{s}.\frac{1}{(R+\frac{1}{sC})} = \frac{A}{R}.\frac{1}{(s+\frac{1}{RC})}$$

Using inverse Laplace Transform:

$$i(t) = \frac{A}{R}e^{-t/(RC)}.u(t)$$

b) When t=0 $i(0)=(A/R)e^0=A/R$

Using the initial value theorem:

$$i(0) = \lim_{s\to\infty} sI(s) = \lim_{s\to\infty}\{s[\frac{A}{s}\ \frac{1}{(R+\frac{1}{sC})}]\} = \lim_{s\to\infty}[\frac{A}{R+1/(sC)}] = A/R$$

c) $$Vc(s) = \frac{I(s)}{sC} = \frac{\frac{A}{R}(\frac{1}{s+1/(RC)})}{sC} = \frac{A}{RC}.(\frac{k1}{s} + \frac{k2}{s+1/(RC)})$$

$$k1 = \frac{1}{s+1/(RC)}|_{s=0} = RC$$

$$k2 = \frac{1}{s}|_{s=-1/(RC)} = -RC$$

$$\therefore Vc(s) = \frac{A}{RC}(\frac{RC}{s} - \frac{RC}{s+1/(RC)}) = A(\frac{1}{s} - \frac{1}{s+1/(RC)})$$

Using inverse Laplace Transform, A=1V, R=10Ω, C=0.1F

$$Vc(t) = (1 - e^{-t})u(t)$$

To find time taken to reach Vc(t)=0.5V

$1-e^{-t}=0.5$ $e^{-t}=0.5$ t=-ln0.5=0.693s

Q4. a) The fourier transform of x(t) is

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt = \int_{-\tau/2}^{\tau/2} e^{-j\omega t}dt = \frac{1}{j\omega}[-e^{-j\omega t}]_{-\tau/2}^{\tau/2}$$

$$= \frac{1}{j\omega}[e^{\frac{j\omega\tau}{2}} - e^{\frac{-j\omega\tau}{2}}] = \frac{\tau \sin(\omega\tau/2)}{(\omega\tau/2)}$$

b) The signal m(t) is given by:

$$m(t) = A[x(t - \frac{3\tau}{2} - x(t - \frac{5\tau}{2})]$$

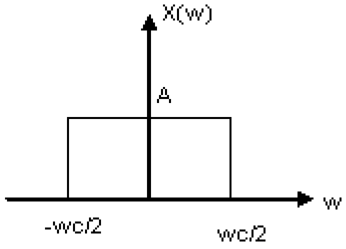
Using the time shift property of FT, $x(t - t_0) \xleftrightarrow{\text{FT}} X(w)e^{-j\omega t_0}$

$$M(\omega) = A[X(\omega)e^{-j\frac{3\omega\tau}{2}} - X(w)e^{-j\frac{5\omega\tau}{2}}]$$

$$= AX(\omega)[e^{-j\frac{3\omega\tau}{2}} - e^{-j\frac{5\omega\tau}{2}}]$$

$$= A\frac{\tau\sin(\frac{\omega\tau}{2})}{\frac{\omega\tau}{2}}[e^{-j\frac{3\omega\tau}{2}} - e^{-j\frac{5\omega\tau}{2}}]$$

c) i) Consider the spectrum:



Using the inverse FT,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{i\omega t}d\omega = \frac{1}{2\pi}\int_{-\omega_c/2}^{+\omega_c/2} Ae^{i\omega t}d\omega$$

$$= \frac{A}{2\pi jt}[e^{i\omega t}]_{-\omega_c/2}^{\omega_c/2} = \frac{A}{j2\pi t}[e^{j\frac{\omega_c t}{2}} - e^{-j\frac{\omega_c t}{2}}]$$

$$= \frac{A}{\pi t}\left(\frac{e^{j\frac{\omega_c t}{2}} - e^{-j\frac{\omega_c t}{2}}}{2j}\right) = \frac{A}{\pi t}\sin\left(\frac{\omega_c t}{2}\right)$$

So, $\mathrm{H}(\omega) = X(\omega - \omega_c)$ $\quad$ $\mathrm{h(t) = x(t)e}^{\frac{\mathrm{j}\omega_c t}{2}} = \frac{A}{\pi t}\sin\left(\frac{\omega_c t}{2}\right)\mathrm{e}^{\frac{\mathrm{j}\omega_c t}{2}}$

ii) Peak amplitude of h(t) is

$$|h(t)| = \left|\frac{A}{\pi t}\sin\left(\frac{\omega_c t}{2}\right)\right| = \left|\frac{A\omega_c}{2\pi}\frac{\sin\left(\frac{\omega_c t}{2}\right)}{\frac{\omega_c t}{2}}\right|$$

about t=0, by using l'Hopital rule

$$\frac{\sin\left(\frac{\omega_c t}{2}\right)}{\frac{\omega_c t}{2}} = 1$$

So: $|h(t)|_{t=0} = \frac{A\omega_c}{2\pi}$

The peak amplitude of x(t) is $\frac{A\omega_c}{2\pi}$.

iii) h(t)=0, when $\sin(\frac{\omega_c t}{2})$ =0, i.e. when $\frac{\omega_c t}{2} = m\pi$, where m is integer

$\omega_c t = 2m\pi \qquad t = \frac{2m\pi}{\omega_c}$

So, first null occurs at $t = \frac{2\pi}{\omega_c}$

Since first null occurs at t=1ms, we have:

$\frac{2\pi}{\omega_c} = 1\times 10^{-3} \qquad \frac{2\pi}{2\pi f_c} = 1\times 10^{-3}$

$f_c = 1000Hz$