EEE6203/EEE6210 (2014-2015) Model solutions

1.

(a) The table and work piece moves back and forth in air at relatively low speeds, and therefore the aerodynamic drag is negligible. The force required to move them is purely due to acceleration, and given by: (8)

$$F = (m_T + m_w)\frac{dv}{dt}$$

where d*v/dt* is the acceleration. For a given linear velocity *v* and transmission efficiency, η, the input power to the transmission is:

$$T_L\omega_m = \frac{1}{\eta}Fv = \frac{1}{\eta}(m_T + m_w)\frac{dv}{dt}v$$

where ω_m is the angular velocity of the motor or lead-screw, and T_L is load torque required for the acceleration and is obtained by:

$$T_L = \frac{1}{\eta}(m_T + m_w)\frac{dv}{dt}(v/\omega_m) = \frac{1}{\eta}(m_T + m_w)\frac{dv}{dt}\cdot a$$

where $a = v/\omega_m$ is the transmission gear ratio, which is equal to the leads-crew pitch length *s* divided by 2π, i.e.,

$$a = v/\omega_m = s/2\pi$$

The motor torque required for the acceleration is therefore obtained by:

$$T_{em} = (J_m + J_s)\frac{d\omega_m}{dt} + T_L = (J_m + J_s)\frac{d\omega_m}{dt} + \frac{1}{\eta}(m_T + m_w)\frac{dv}{dt}a$$

which can be expressed in terms of d*v/dt*:

$$T_{em} = \frac{1}{a}\frac{dv}{dt}\left[(J_m + J_s) + \frac{1}{\eta}(m_T + m_w)a^2\right]$$

As can be seen, the motor torque is a function of the gear ratio *a*, and reaches its minimum when

$$dT_{em}/da = -(J_m + J_s)/a^2 + \frac{1}{\eta}(m_T + m_w) = 0$$

Thus:

$$a = \sqrt{\frac{(J_m + J_s)\eta}{(m_T + m_w)}} \text{ or } s = 2\pi a = 2\pi\sqrt{\frac{\eta(J_m + J_s)}{(m_T + m_w)}}$$

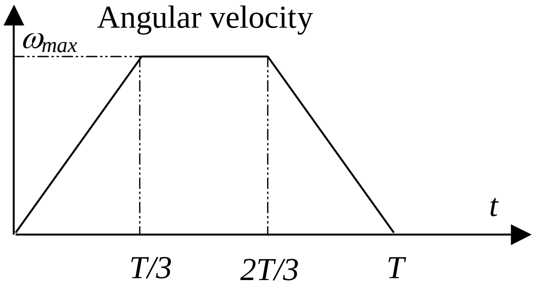
(b) The optimal pitch length of the lead-screw is given by: (2)

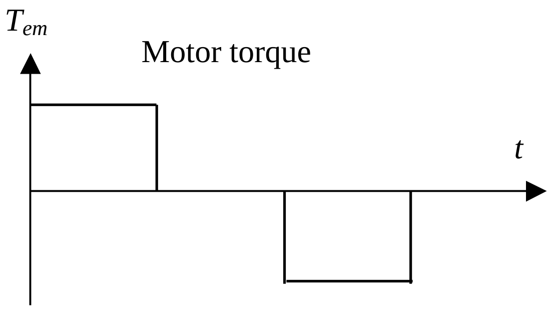
$$s = 2\pi a = 2\pi\sqrt{\frac{\eta(J_m + J_s)}{(m_T + m_w)}} = 2\pi\sqrt{\frac{0.7\times(1.2+2.8)\times 10^{-3}}{500}} = 14.87\times10^{-3}\text{(m)}$$

and the combined inertia on the motor axis is:

$$J_{eq} = (J_m + J_s) + \frac{a^2}{\eta}(m_T + m_w) = (1.2+2.8)\times10^{-3} + (\frac{14.87\times10^{-3}}{2\pi})^2 \times 500/0.7$$
$$= 8.0\times10^{-3}(kgm^2)$$

(c) (6)





For the trapezoidal velocity profile, the maximum angular velocity of the motor is given by

$$\omega_{\max} = \frac{3}{a}\frac{d}{2T} = \frac{2\pi\times3}{14.87\times10^{-3}\times2\times3} = 211.27\text{ (rad/s)}$$

and the maximum motor acceleration is given by :

$$a_{\max} = \frac{4.5}{a}\frac{d}{T^2} = \frac{2\pi\times4.5}{14.87\times10^{-3}\times3^2} = 211.27\text{ (rad/s}^2\text{)}$$

The motor torque is:

$$T_{em} = J_{eq}a_{\max} = 8.0\times10^{-3}\times211.27 = 1.69\text{ (Nm)}$$

and the motor current is

$$I_a = T_{em} / K_T = 1.69 / 0.9 = 1.88 (A)$$

The maximum motor voltage occurs when the motor speed reaches 211.27 rad/s, and neglecting the inductance effect, is given by

$$V_t = K_E \omega + R_a I_a$$

Since in the ISO system, the motor back-emf constant, K_E, is equal to the torque constant K_T. Thus,

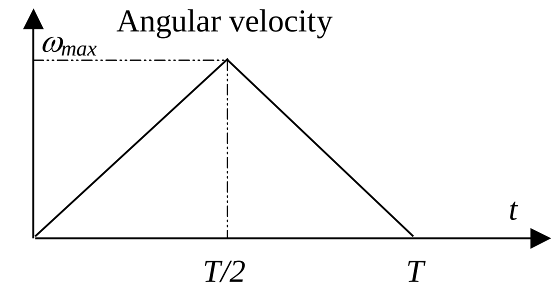
$$V_t = K_E \omega + R_a I_a = 0.9 \times 211.27 + 0.5 \times 1.88 = 191.08 \text{ (V)}$$

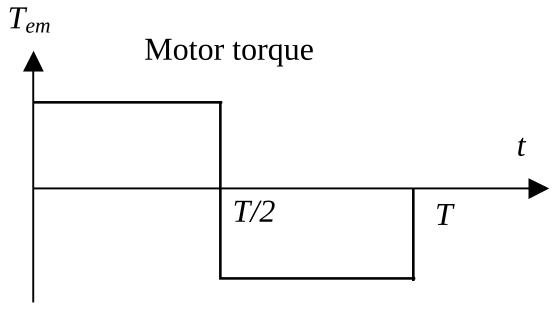
The rms torque of the motor is given by

$$T_{rms} = \sqrt{\frac{2}{3}(1.69)^2} = 1.38 \text{ (Nm)}$$

which is less than the rated rms torque. Thus the motor would not overheat when the movement were executed repeatedly.

(d) (4)





For the triangle velocity profile, the maximum motor acceleration is given by:

$$a_{\max} = \frac{4}{a}\frac{d}{T^2} = \frac{2\pi \times 4}{14.87 \times 10^{-3} \times 3^2} = 187.8 \text{ (rad/s}^2\text{)}$$

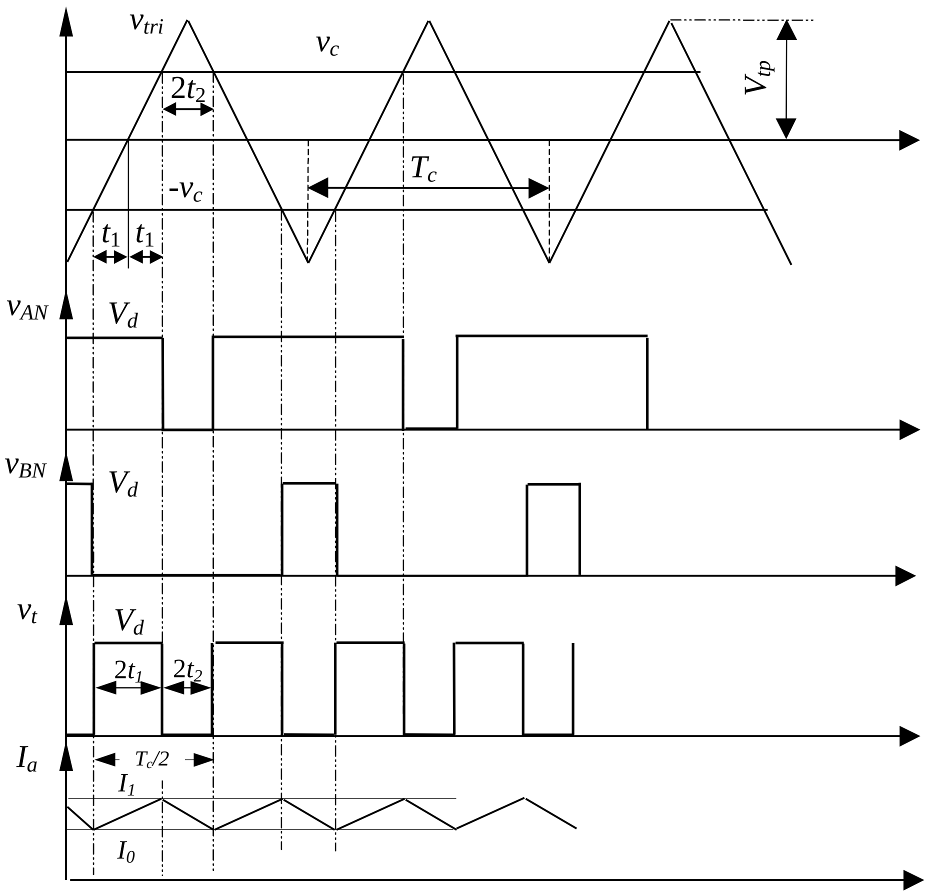
The peak motor torque is:

$$T_{em} = J_{eq} a_{\max} = 8.0 \times 10^{-3} \times 187.8 = 1.5 \text{ (Nm)}$$

The rms torque of the triangle velocity profile is the same as the peak torque, which is greater than the rated rms torque of the motor. Hence, the movement cannot be repeated continuously, otherwise the motor would be overheating, and its lifetime reduced.

2.

a. In the unipolar operation, a triangle carrier signal v_{tri} with a period T_c is compared with both the positive command signal v_c, and the negative command signal $-v_c$, as shown in the figure below. The comparison with v_c produces switching signals for switches S1 and S2 whilst the comparison with $-v_c$ generates switching signals for switches S3 and S4. During the period when $v_c > v_{tri}$, S1 is on and S2 is off, and the motor voltage at terminal *A* with respect to the negative supply is V_d. Conversely, during the period when $v_c < v_{tri}$, S1 is off and S2 is on, and the motor voltage at terminal *A* is 0. Similarly, during the period when $-v_c > v_{tri}$, S3 is on and S4 is off, and the motor voltage at terminal *B* with respect to the negative supply is V_d whilst during the period when $-v_c < v_{tri}$, S3 is off and S4 is on, and the motor voltage at terminal *B* is 0. The resulting voltage waveforms at terminals *A* and *B* are shown in the figure below. Thus the net voltage, v_t, applied to the motor armature is $V_{AN} - V_{BN}$, also shown in the figure. (6)



b. With reference to the above waveforms, the average voltage output of the converter is given by: (3)

$$v_o = 2*2t_1V_d/T_c$$

From the triangle waveform, $\dfrac{V_{tp}}{T_c/4} = \dfrac{v_c}{t_1}$ or $t_1 = v_c/V_{tp}*T_c/4$, Thus

$$v_o = (V_d/V_{tp})\, v_c = k\, v_c$$

where V_{tp} is the peak voltage of the triangular carry signal, and the dc gain of the converter is $k = V_d/V_{tp}$.

c. The variation of the terminal voltage between V_d and 0 due to switching will inevitably cause ripples in the armature current, which not only incurs additional losses, but also produces pulsation torque. In steady state operation, however, the instantaneous motor speed ω_m can be assumed to be constant if there is sufficient inertia, and therefore $e_a(t) = E_a$ = constant. (8)

If the ripple current is primarily determined by the armature inductance L_a and R_a has a negligible effect, the current increases and decreases linearly, as illustrated in the above diagram. At the beginning of period $2t_1$, the armature current I_a is at its minimum value of I_0, and increases linearly. At $t = 2t_1$, it reaches the maximum value of I_1. Thus:

$$I_1 = I_0 + (V_d - E_a)/L_a *2t_1 \qquad (1)$$

During $2t_2$ period, the terminal voltage is 0, and the current decreases linearly under the influence of back-emf E_a, and reaches I_0 at $t = 2t_2$, Hence,

$$I_0 = I_1 – E_a/L_a *2t_2 \qquad (2)$$

The peak-to-peak current ripple is

$$\Delta I_p = I_1 - I_0 = (V_d - E_a)/L_a *2t_1$$

(1) + (2) and solving for E_a yields

$$(2t_1 + 2t_2)E_a = 2V_d\, t_1$$

since $(2t_1 + 2t_2) = T_c/2$

$$E_a = 4V_d t_1/T_c$$

Thus

$$\Delta I_p = V_d(1 -4t_1/T_c)/L_a *2t_1 = (V_d/L_a)(2t_1-8t_1^2/T_c)$$

The maximum peak ripple occurs at

$$\frac{d\Delta I_p}{dt_1} = 0,\ \ i.e.,\ \ t_1 = T_c/8$$

and

$$\Delta I_{pmax} = T_c V_d / 8L_a \quad \text{or} \quad \Delta I_{pp} = \frac{V_d}{8L_a f_c}$$

d. If the voltage source is replaced by a diode rectifier, the energy during the braking operation will not be absorbed by the supply as the current in the diodes cannot be reversed. Consequently, the electric energy converted from the kinetic energy of the drive system during braking operation can only be stored in the DC link capacitor, and results in significant increase in the DC-link voltage. If the DC link voltage becomes too high, catastrophic failures of the capacitor and power electronic device may occurs, leading to complete failure of the drive. (3)

3.

a. From the motor data, the phase open-circuit rms voltage is (4)

$$E_{rms} = 190/\sqrt{3} = 109.7 \text{ (V)}$$

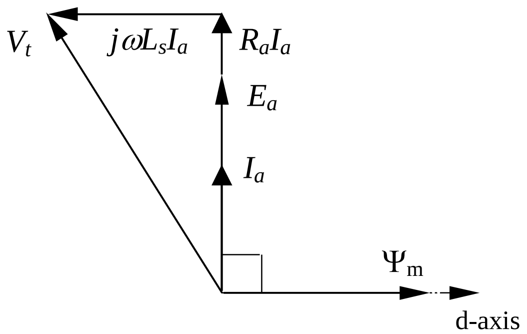
The no-load peak flux linkage of a phase winding produced by rotor permanent magnets is given by:

$$\Psi_m = \sqrt{2}\Psi_{rms} = \sqrt{2}E_{rms} / p\omega = \sqrt{2} * 109.7/3/(2\pi * 2000/60) = 0.247 \text{ (Wb)}$$

The motor torque constant is:

$$K_T = 3p\Psi_m / \sqrt{2} = 1.57 \text{ (Nm/A)}$$

For the maximum torque per Ampere operation, the motor current will be in phase with the motor back-emf. Thus the resulting phase diagram is as follows:



Phasor diagram

b. For a given torque requirement, the motor current will be at minimum when it is in phase with the back-emf. Under this condition, the motor torque is related to the current by: (6)

$T_{em} = K_T I_a \sin 90$. Thus, the rms phase current is

$I_a = T_{em} / K_T = 6/1.57 = 3.82\,(\mathrm{A})$

At 1500 rpm, the synchronous reactance is

$\omega L_s = (2\pi * 1500/60) * 3 * 5.7 * 10^{-3} = 2.69\,(\Omega)$

Induced phase rms back-emf:

$E_a = (190/\sqrt{3})*1500/2000 = 82.28$ (V)
$R_aI_a = 1.01*3.82 = 3.86$ (V)
$\omega L_aI_a = 2.69*3.82 = 10.28$(V)

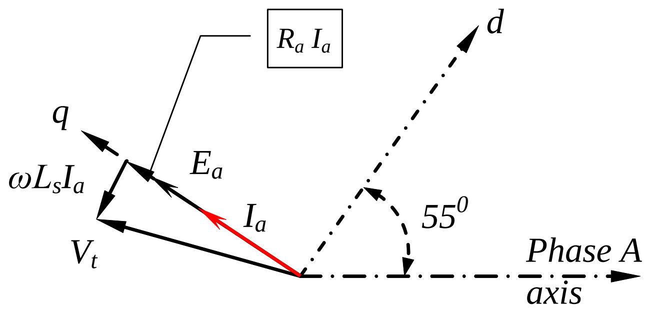
Terminal voltage:

$V_t = \sqrt{(\omega L_s I_a)^2 + (E_a + R_a I_a)^2} = \sqrt{10.28^2 + (82.28 + 3.86)^2} = 86.75\,(\mathrm{V})$

Power factor:

$\cos\varphi = (E_a + R_aI_a)/V_t = 86.14/86.75 = 0.993$

c. With reference to the voltage and current space vector diagram below, the rotor position is represented by the d-axis. The back E_{ag} leads the *d-axis* by 90^0. Under the operation condition in 3(b) the motor voltage space vector leads E_a by 6.8^0 (10)

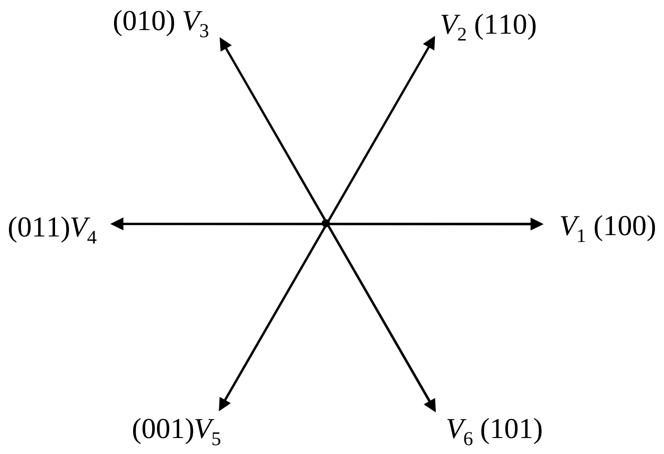


Thus the space voltage vector V_{cs} which needs to be produced by the inverter has a magnitude of $86.75\sqrt{2} = 122.7$ V and angle of $(96.8^0 + 55^0) = 151.8^0$

Since the voltage vector lies in the section between 120^0 and 180^0, the two active voltage space vectors, V_3 and V_4 will be used. The required modulation ratio is:

$$m = \frac{\sqrt{3}V_{cs}}{V_d} = \frac{\sqrt{3}*122.7}{600} = 0.3542$$

and the angle between the voltage vector and V_3 is 31.8^0, Thus the time duration for V_3 and V_4 are given respectively by:

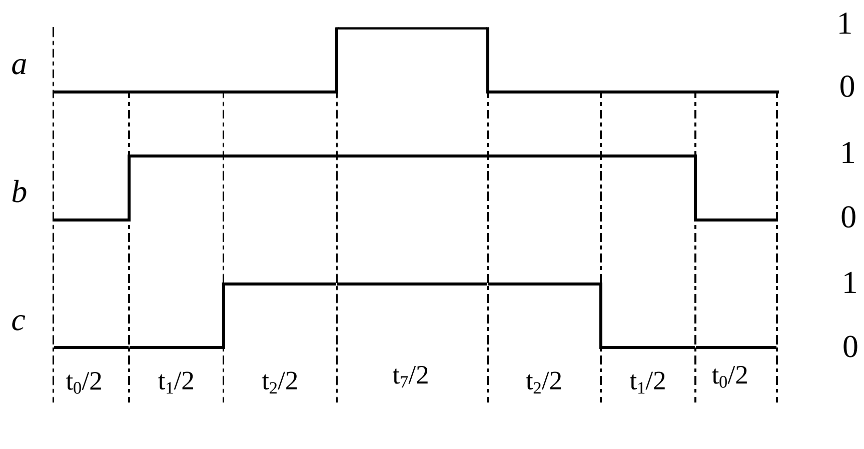


$$t_1 = mT\sin(\pi/3-\delta) = 0.3542*\sin(60^0-31.8^0)/10000 = 16.7\,(\mu s)$$
$$t_2 = mT\sin\delta = 0.3542*\sin(31.8^0)/10000 = 18.7\,(\mu s)$$

The time duration for the two zero vectors are:

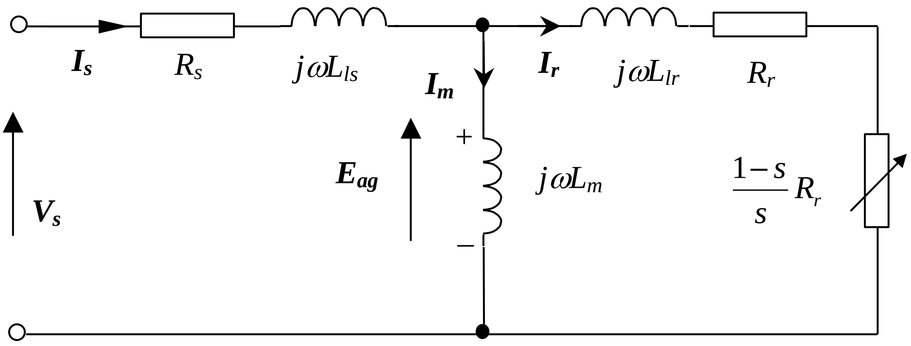
$$t_0 = t_7 = 0.5(T-(t_1+t_2)) = 23.3(\mu s)$$

The corresponding per-cycle switching sequence waveforms are shown below:

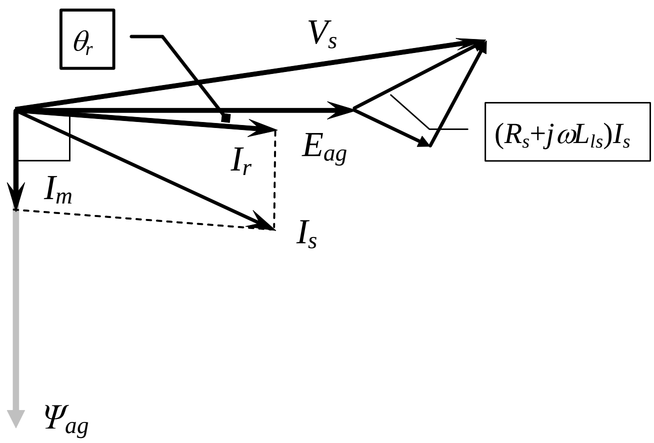


4.

a. Equivalent circuit diagram of the motor operation (5)



R_s --- stator resistance
R_r --- rotor resistance reflected in stator
L_{ls}--- stator leakage inductance
L_{lr}--- rotor leakage inductance reflected in stator
L_m--- magnetising inductance
s --- slip
ω --- angular frequency of the supply



Phasor diagram

b. The electromagnetic power transferred through the airgap is given by: (6)

$$P_{em} = 3E_{ag}I_r \cos\theta_r$$

where θ_r is the angle between the air-gap induced voltage E_{ag} and the rotor current I_r. Further from the phasor diagram, E_{ag} leads the air-gap flux linkage Ψ_{ag} by 90^0, and while I_r lags E_{ag} by θ_r. Hence, the angle between Ψ_{ag} and I_r is given by:

$$\delta = 90 - \theta_r$$

The air-gap flux linkage is related to the induced E_{ag} by

$$E_{ag} = \omega \Psi_{ag} / \sqrt{2}$$

The electromagnetic torque is equal to the air-gap power P_{em} divided by the synchronous speed ω_s, i.e.,

$$T_{em} = P_{em} / \omega_s = 3 E_{ag} I_r \cos\theta_r = 3 \frac{\omega \Psi_{ag}}{\omega_s \sqrt{2}} I_r \cos(90 - \delta)$$

$$\because \omega / \omega_s = p \ ; \cos(90 - \theta_r) = \sin\delta$$

$$\therefore T_{em} = 3 \frac{p \Psi_{ag}}{\sqrt{2}} I_r \sin\delta$$

c. At rated speed of 1450 rpm, the slip s is given by: (9)

s = (1500-1450)/1500 = 0.033

For small values of s, $sR_s << R_r$ and $s\omega L_l << R_r$, and the motor electromagnetic torque is proportional to slip s. Thus, at 50% load torque, the slip s is 0.0167

The rotor speed is 1500*(1-s) = 1475 (rpm)
From the equivalent circuit diagram, the impedance of the rotor branch is

$R_r/s + j\omega L_{lr}$ = 0.55/0.0167+j0.95 = 32.93+ j0.95

The equivalent impedance of the parallel of the magnetising branch and the rotor branch is

$$\frac{(32.9 + j0.95) j48.6}{32.9 + j(0.95 + 48.6)} = \frac{1601.3\angle 91.65^0}{59.48\angle 56.42^0} = 26.92\angle 35.23^0 = 21.99 + j15.53$$

Total impedance seen from the stator terminal:

$$0.35 + j1.20 + 21.99 + j15.53 = 22.34 + j16.73 = 27.91\angle 36.82^0$$

Stator current

$$I_s = 240 / 27.91\angle 36.82^0 = 8.60\angle -36.82^0$$

Power factor

$$\cos\varphi = \cos 36.82^0 = 0.80$$

Induced air-gap voltage

$$E_{ag} = 240 - (R_s + j\omega L_{ls})I_s = 240 - (1.25\angle 73.74)8.60\angle -36.82^0 = 231.4 - j6.46$$
$$= 231.49\angle -1.6^0$$

Rotor current

$$I_r = E_{ag} / (R_r / s + j\omega L_{lr}) = 231.49\angle -1.6^0 / (32.94\angle 1.65^0) = 7.03\angle -3.25^0$$

Air-gap flux linkage

$$\Psi_{ag} = E_{ag} / 4.44 f = 231.43 / 4.44 / 50 = 1.04 (Wb)$$

The electromagnetic torque

$$T_{em} = \frac{3R_r I_r^2}{s\omega_s} = \frac{3 * 0.55 * 7.03^2}{0.0167 * 157.08} = 31.1 (Nm)$$

Efficiency

$$\eta = P_{out} / 3I_s V_s \cos\varphi = 31.1 * 154.46 / (3 * 240 * 8.6 * 0.8) = 0.97$$

Note the iron loss, friction and windage losses are not represented in the equivalent circuit, and therefore the efficiency is overestimated.