Data Provided: None

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2014-15 (2.0 hours)

EEE6209 Advanced Signal Procsessing

Answer **FOUR** questions (**TWO** questions from **Part A** and **TWO** questions from **Part B**). **No marks will be awarded for solutions to a third question attempted from any of the two sections.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

PART A - Answer only TWO questions from questions 1, 2 and 3.

1. Consider the filter h(n) with values {1/5, 1/5, 1/5, 1/5, 1/5} for n= -2, -1, 0, 1, 2, respectively.

a. Compute and draw time-domain and frequency-domain performances of the filter h(n). **(3)**

b. Determine and draw the impulse response of the resulting filter kernel, p(n), if two h(n) filters are cascaded in a system. **(2)**

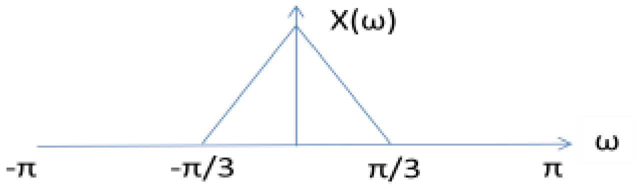
c. Sketch time-domain and frequency-domain performances of p(n) and compare them with those of h(n). **(3)**

d. A signal x(n) is filtered with p(n) to get the new signal s(n). Then the final output signal y(n) is computed by subtracting the signal s(n) from the original signal x(n). Draw a system block diagram to show this operation and derive the impulse response of the resulting filter r(n). **(2)**

e. Sketch time-domain and frequency-domain performances of r(n). **(2)**

f. Derive the recursive implementation of h(n) and compare its complexity, in terms of number of additions and multiplications, with respect to those for the non-recursive implementation. **(3)**

2. **a.** Consider the signal $x(n) = \{a, b, c, d, e, f, g, h, i, j, k\}$ for $-5 \leq n \leq 5$ and the magnitude of its Fourier transform as shown below.



If the signal x(n) is sampled to get y(n) as

$$y(n) = \begin{cases} x(n), & n = 0, \pm 2, \pm 4, \ldots \\ 0, & n = \pm 1, \pm 3, \ldots. \end{cases}$$

i. Compute signal y(n). (1 mark)

ii. Showing all the intermediate steps, sketch the magnitude of the Fourier transform of the sampled signal y(n). (2 marks)

iii. Does this sampling system require an anti-aliasing and/or an anti-imaging filter? If so, specify the pass band edge and stop band edge frequencies of the filter(s)? (2 marks) **(5)**

b. A signal, sampled at 2.048 kHz, is to be decimated by a factor of 32 to yield a signal at a sampling frequency of 64 Hz. The signal band of interest extends from 0 to 20 Hz. The overall anti-aliasing digital filtering should satisfy 0.01 dB passband deviation (δ_p) and 80 dB stopband attenuation (δ_s).

It is suggested to use a 2-stage decimator, with decimation rates M_1=8 and M_2=4, for the above mentioned multi-rate system.

Estimate the lengths of the anti-aliasing filters h_1 and h_2 used for the two decimations, respectively.

Note that the filter length N for a low pass filter is approximated as

$N \approx \frac{-10\log(\delta_p\,\delta_s)-13}{14.6(\Delta f)} + 1$, where Δf is the normalised width of transition band. **(7)**

c. Estimate the computational complexity of this 2-stage decimator in terms of multiplications per second. Explain why multistage decimation is more efficient in terms of the computational complexity, compared to a single stage decimation system. **(3)**

3. An orthogonal 2-channel filter bank of length 2 filters with coefficients [p, q] and [r, s] is shown in the following matrix equation:

$$\begin{bmatrix} y_0 \\ y_1 \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix},$$

where [x_0, x_1] and [y_0, y_1] are the input and output data vectors consisting of 2 data points.

a. Considering the constraints for wavelet filters, find the filter coefficients, p, q, r and s. **(4)**

b. Derive the forward wavelet transform matrix (T1) corresponding to the first level of decomposition for an input signal, A, consisting of 8 data elements (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8). **(2)**

c. Derive the corresponding inverse transform matrix of the forward wavelet transform matrix (T1) in question 3.b. **(3)**

d. Derive the transform matrix (T2) corresponding to the second level of decomposition in a 2-level dyadic wavelet decomposition scheme and show how such a decomposition scheme is realised using T1 and T2 transform matrices? **(3)**

e. How do you use the wavelet transform in this question to remove noise from a measured signal? **(3)**

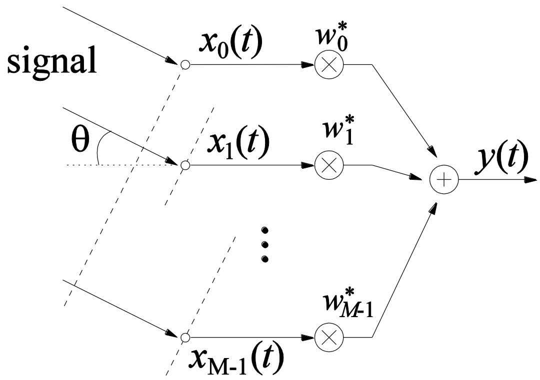
PART B - Answer only TWO questions from questions 4, 5 and 6.

4. **a.** i) Estimate the mean, mean-square and variance of the following stationary sequence: {1, 2, 4, 3, 5}. (3 marks)

ii) Derive the relationship of the three averages (mean, mean-square and variance) and verify it using the above estimated results. (3 marks) **(6)**

b. For a 10-bit A/D converter, what is the dynamic range for a cosine wave input signal? **(3)**

c. Derive in detail the beam response of a narrowband uniform linear array with *M* sensors (shown below) and an adjacent sensor spacing of *d*. Simplify the result assuming that *d* is half of the wavelength of the impinging signals?



(6)

5. **a.** Suppose the z-transform $S_{yy}(z)$ of the autocorrelation function of a correlated sequence y(n) is given by

$$S_{yy}(z)=(z-1/2)(z-3)(z^{-1}-1/2)(z^{-1}-3)$$

i) Design a filter U(z) whose output will be white when passing y(n) through it. List all of the possible choices for such a filter. (4 marks)

ii) Which choice for U(z) is the minimum-phase whitening filter for y(n)? (2 marks)

(6)

b. A zero-mean white Gaussian noise with variance 1 is applied to a filter with a transfer function $H_1(z)=2-3z^{-1}$. Calculate the autocorrelation sequence of its output. **(4)**

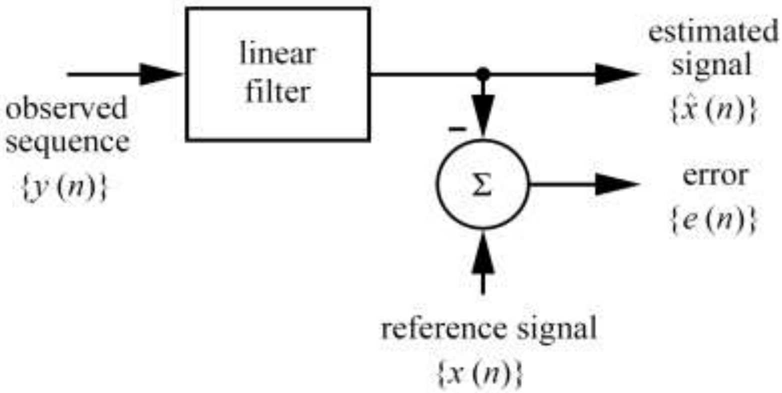
c. The table below shows the input and training signal to a two-tap adaptive filter at sample numbers 10 and 11, where **h**(n) is the vector holding the two taps of the adaptive filter :

Iteration *n*	y(*n*)	**h**(*n*)	x(*n*)
10	0.25	[1 6]	1.2
11	0.3		-0.2

Using the LMS algorithm, evaluate **h**(11). The stepsize is fixed at 0.1. **(5)**

6. **a.** Two terms are commonly used to indicate the dependency of a signal at one time instant with the same signal at a different time instant, or more generally for the dependency of one signal upon another. These two terms are "independent" and "uncorrelated". Give a proof to show that statistically independent random processes are uncorrelated. Show all working. **(4)**

b. i) A linear estimator is shown below, where the impulse response of the linear filter is given by h_j, j=0, 1, …, N-1. Derive the Wiener solution for h_j. Show all working. (9 marks)



ii) Give the update equation of the Method of Steepest Descent and explain briefly how it works. (2 marks) **(11)**

GCKA/WL/KLF