EEE6201 – 2014/2015

Question 1

a.

i. Shunt resistive sensing of currents is simple and inexpensive and therefore used in low cost, low currents applications. Major drawbacks are: low accuracy, power dissipation in the sensing resistor, sensitivity to temperature variation, lack of galvanic isolation. The most commonly used alternative are open- and closed-loop Hall effect sensors. **(2)**

ii. If the maximum peak load currents are $I_{pk} = \pm 35.4\,A$. In order for the voltage on the sensing resistor $R_s I$ to be in the interval $\pm 5V$, the maximum $R_s = 141\ m\Omega$. **(1)**

iii. The measurement of the current has to be synchronized with the PWM. The sensing resistors conducts current only when the bottom switches are conducting. The measurement is therefore normally taken at the center of the PWM pulse of the bottom switch. **(1)**

b.

i. With 720 pulses per revolution, the resolution of the encoder is $\frac{360°\,[deg./rev]}{720\,[pulses/rev]} = \frac{1}{2}\left[\frac{deg}{pulse}\right]$. Counting both up and down transitions, the encoder will produce 9 pulses. Since B is leading A in Fig.2, the direction of rotation is negative and therefore the final position is$-4.5°$. **(2)**

ii. An incremental encoder can only measure relative increments in the angular position. An absolute encoder uses a digital coding to represent absolute angular positions. **(2)**

c.

i. A 12 bit absolute encoder has $2^{12} = 4096$ positions. The angular resolution is therefore $\frac{360°}{4096} = 0.08789° = 1.534 \times 10^{-3} rad$. **(2)**

ii. In order to achieve a resolution of $\frac{1}{10}1° = 0.00174 rad$, the minimum number of bits is $\frac{360°}{2^b} \leq \frac{1}{10}1°$ therefore $b \geq \log_2(360 \times 10) = 11.8$. Therefore $b = 12$. The resolution with $b = 11$ is $\frac{360°}{2^{11}} = 0.176°$ and with $b = 12$ is $\frac{360°}{2^{12}} = 0.088°$. **(2)**

d.

i. The voltages induced in the stator windings as a function of θ are:

$$\begin{aligned} v_{\cos} &= nV_{ex} \sin \omega_{ex} t \cos \theta \\ v_{\sin} &= nV_{ex} \sin \omega_{ex} t \sin \theta \end{aligned}$$

For $\theta = 30°$

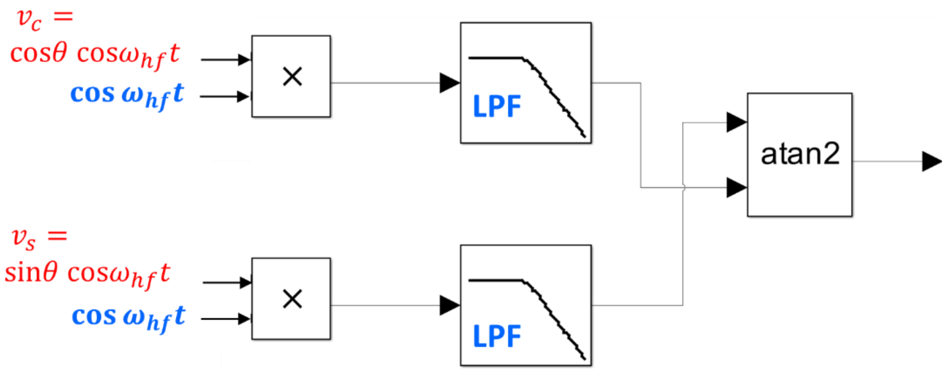
$$\begin{aligned} v_{\cos} &= \frac{\sqrt{3}}{2} nV_{ex} \sin \omega_{ex} t \\ v_{\sin} &= \frac{1}{2} nV_{ex} \sin \omega_{ex} t \end{aligned}$$

For $\theta = 90°$

$$\begin{aligned} v_{\cos} &= 0 \\ v_{\sin} &= nV_{ex}\sin\omega_{ex}t \end{aligned}$$

(2)

ii. A suitable method for extracting angle information from the resolver measurements is illustrated in the Fig. below:



The signals v_{sin} and v_{cos} are multiplied by the "carrier" signal $\cos\omega_{hf}t$ to give

$$\begin{aligned} V_{sin}' &= \left(V\cos\omega_{hf}t\sin\theta\right)\cos\omega_{hf}t = \frac{1}{2}\sin\theta + \frac{1}{2}\sin\theta\cos\omega_{hf}t \\ V_{cos}' &= \left(V\cos\omega_{hf}t\cos\theta\right)\cos\omega_{hf}t = \frac{1}{2}\cos\theta + \frac{1}{2}\cos\theta\cos\omega_{hf}t \end{aligned}$$

The low pass filter can be used to extract the low frequency components. The angle can finally be obtained recognizing that:

$$\frac{LPF[V_s']}{LPF[V_c']} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

(6)

Question 2

a. The stator flux linkages equations in the synchronous reference frame (rotating at speed ω_e) are:

$$\begin{aligned}\frac{d\psi_d^s}{dt} &= v_d^s - R_s i_d^s + \omega_e \psi_q^s \\ \frac{d\psi_q^s}{dt} &= v_q^s - R_s i_q^s - \omega_e \psi_d^s\end{aligned}$$

In steady-state conditions $\frac{d}{dt}\psi = 0$ and therefore:

$$\begin{aligned}\psi_q^s &= -\frac{v_d^s - R_s i_d^s}{\omega_e} = -\frac{-324 - 0.3 \times 10}{2\pi 50} = 1.04\ [Vs] \\ \psi_d^s &= \frac{v_q^s - R_s i_q^s}{\omega_e} = \frac{172 - 0.3 \times 50}{2\pi 50} = 0.50\ [Vs]\end{aligned}$$

(2)

b. The stator flux linkages can be expressed in terms of currents as:

$$\begin{aligned}\psi_d^s &= L_s i_d^s + L_m i_d^r \\ \psi_q^s &= L_s i_q^s + L_m i_q^r\end{aligned}$$

hence:

$$\begin{aligned}i_d^r &= \frac{\psi_d^s - L_s i_d^s}{L_m} = \frac{0.500 - 50 \times 10^{-3} \times 10}{40 \times 10^{-3}} = 0.0\ [A] \\ i_q^r &= \frac{\psi_q^s - L_s i_q^s}{L_m} = \frac{1.04 - 50 \times 10^{-3} \times 50}{40 \times 10^{-3}} = -36.48\ [A]\end{aligned}$$

(2)

c. The rotor flux linkages can be expressed in terms of currents as:

$$\begin{aligned}\psi_d^r &= L_r i_d^r + L_m i_d^s = (55 \times 10^{-3} \times 0 + 40 \times 10^{-3} \times 10) = 0.4\ [Vs] \\ \psi_q^r &= L_r i_q^r + L_m i_q^s = (-55 \times 10^{-3} \times 36.5 + 40 \times 10^{-3} \times 50) = 0.0\ [mVs]\end{aligned}$$

Note that the IM is running in FOC, i.e. with $i_d^r = 0, \psi_q^r = 0$

(2)

d. The rotor flux linkages equations in steady state are:

$$\begin{aligned}\frac{d\psi_d^r}{dt} &= -R_r i_d^r + (\omega_e - \omega_r)\,\psi_q^r = 0 \\ \frac{d\psi_q^r}{dt} &= -R_r i_q^r - \underbrace{(\omega_e - \omega_r)}_{\omega_{slip}}\,\psi_d^r = 0\end{aligned}$$

From the second (the first is identically zero in FOC) the slip speed is:

$$\omega_e - \omega_r = -\frac{R_r}{\psi_d^r} i_q^r = \frac{0.25 \times 36.5}{0.4} = 22.82\ [rad/s]$$

The (electrical) rotor speed is therefore:

$$\begin{aligned}\omega_r = \omega_e - \omega_{slip} = 2\pi 50 - 22.82 = 291.34 \left[\frac{rad}{s}\right] = \frac{291.34}{2\pi}\ [Hz] \\ = 46.37\ [Hz]\end{aligned}$$

The mechanical speed in [rpm] is:

$$\omega_m = \frac{\omega_r}{2} \times 60 = \frac{46.36}{2} \times 60 = 1391\ [rpm]$$

(4)

e. The electromechanical torque is given by:

$$T_{em} = \frac{3}{2}\frac{P}{2}\left(\psi_d^s i_q^s - \psi_q^s i_d^s\right) = \frac{3}{2}\frac{4}{2}(0.50 \times 50 - 1.04 \times 10) = 43.8\ [Nm]$$

$$= \frac{3}{2}\frac{P}{2}\left(\psi_q^r i_d^r - \psi_d^r i_q^r\right) = \frac{3}{2}\frac{4}{2}(0.4 \times 36.5) = 43.8\ [Nm]$$

$$= \frac{3}{2}\frac{P}{2} L_m\left(i_d^r i_q^s - i_q^r i_d^s\right) = \frac{3}{2}\frac{4}{2} 40 \times 10^{-3} \cdot (36.5 \times 10) = 43.8\ [Nm]$$

(2)

f. The input electrical power is:

$$P_{elect} = \frac{3}{2}\left(v_d^s i_d^s + v_q^s i_q^s\right) = \frac{3}{2}(-324 \times 10 + 172 * 50) = 8040\ [W]$$

(2)

g. The output mechanical power is:

$$P_{mech} = T_{em}\omega_m = 43.8 \times \left(\frac{1390}{60} \times 2\pi\right) = 6375\ [W]$$

(2)

h. The stator and rotor losses are:

$$P_s = \frac{3}{2} R_s \left((i_d^s)^2 + \left(i_q^s\right)^2\right) = \frac{3}{2} \times 0.3 \times (10^2 + 50^2) = 1170\ [W]$$

$$P_s = \frac{3}{2} R_r \left((i_d^r)^2 + \left(i_q^r\right)^2\right) = \frac{3}{2} \times 0.25 \times (36.48^2) = 499\ [W]$$

(2)

i. The efficiency is:

$$\eta = \frac{P_{mech}}{P_{elect}} = 79.4\%$$

(2)

Question 3

a. From Fig.4, the period of one electrical revolution is $3ms$ and therefore the electrical frequency is:

$$f_{elec} = \frac{1}{3} \times 10^3 = f_{mech} \times \frac{P}{2}$$

where P is the number of magnetic poles. The mechanical frequency f_{mech} at $10000\ rpm$ is:

$$f_{mech} = \frac{10000}{60}$$

Therefore the number of poles P is:

$$P = 2 \times \frac{f_{elec}}{f_{mech}} = 2 \times \frac{\frac{1}{3} \times 10^3}{10000/60} = 4$$

(2)

b. The steady state voltage equations in the synchronous reference frame are:

$$\begin{aligned} v_d &= Ri_d - \omega L_q i_q \\ v_q &= Ri_q + \omega L_d i_d + \omega \psi_m \end{aligned}$$

In open circuit conditions $i_d = i_q = 0$, the back-EMF is on the q −axis only:

$$v_q = \omega \psi_m$$

Therefore the magnet flux is:

$$\psi_m = \frac{v_q}{\omega} = \frac{\frac{250}{\sqrt{3}}}{2\pi \times \frac{10000}{60} \times 2} = 0.069\ [Vs]$$

(2)

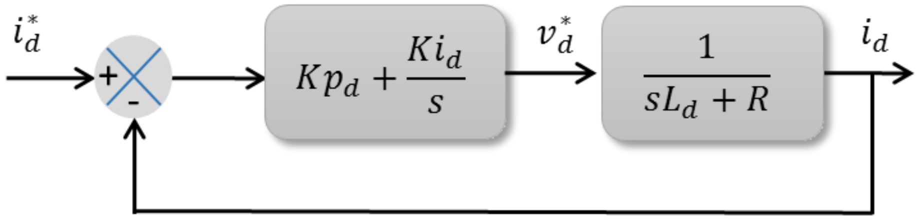
c. The inductances can be calculated from the steady-state voltage equations as:

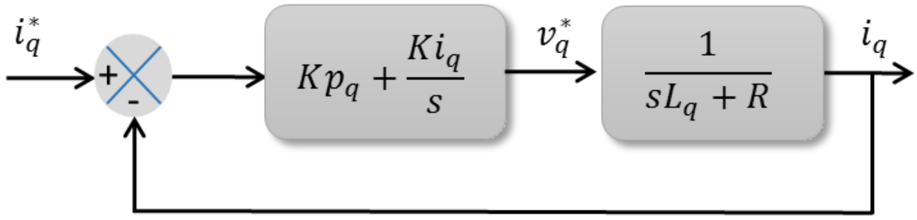
$$L_d = -\frac{v_d - Ri_d}{\omega i_q} = -\frac{-214 + 0.1 \times 50}{2\pi \frac{10000}{60} 2 \times 100} = 1mH$$

$$L_q = \frac{v_q - Ri_q - \omega \psi_m}{\omega i_d} = \frac{102 - 0.1 \times 100 - 2\pi \frac{10000}{60} 2 \times 0.069}{-2\pi \frac{10000}{60} 2 \times 50} = 500\mu H$$

(3)

d. Assuming that ideal compensation cancels dq −axis cross-coupling, the current control loops become:





The closed loop transfer functions are:

$$\frac{i_{d,q}}{i_{d,q}^*} = \frac{\left(Kp + \frac{Ki}{s}\right) \cdot \frac{1}{sL_{d,q} + R}}{1 + \left(Kp + \frac{Ki}{s}\right) \cdot \frac{1}{sL_{d,q} + R}} = \frac{\left(sKp_{d,q}/Ki_{d,q} + 1\right) \cdot \frac{Ki_{d,q}/R}{sL_{d,q}/R + 1}}{s + \left(sKp_{d,q}/Ki_{d,q} + 1\right) \cdot \frac{Ki_{d,q}/R}{sL_{d,q}/R + 1}}$$

The desired closed loop transfer function has bandwidth $\omega_{cc} = 1/\tau$ and is:

$$\frac{i_{d,q}}{i_{d,q}^*} = \frac{\omega_{cc}}{s + \omega_{cc}} = \frac{1/\tau}{s + 1/\tau}$$

Using pole-zero cancellation the desired response can be obtained with:

$$\frac{Kp_{d,q}}{Ki_{d,q}} = \frac{L_{d,q}}{R}$$

and:

$$\frac{Ki_{d,q}}{R} = \frac{1}{\tau}$$

Therefore:

$$Ki_d = Ki_q = \frac{R}{\tau} = \frac{0.1}{2 \times 10^{-3}} = 50$$

$$Kp_d = \frac{L_d}{\tau} = \frac{500 \times 10^{-6}}{2 \times 10^{-3}} = 0.25$$

$$Kp_q = \frac{L_q}{\tau} = \frac{1 \times 10^{-3}}{2 \times 10^{-3}} = 0.5$$

(5)

e. The constraint on the maximum stator current can be expressed as

$$I_{max} = \sqrt{i_d^2 + i_q^2} = 200A$$

Under this constraint, the torque is given by:

$$T = \frac{3}{2}\frac{P}{2}\left[\psi_m\sqrt{200^2 - i_d^2} + \left(L_d - L_q\right)i_d\sqrt{200^2 - i_d^2}\right]$$

The maximum torque can be obtained by calculating the derivative of T as a function of i_d and equating to zero:

$$\frac{dT}{di_d} = 0 = -\frac{\psi_m i_d}{\sqrt{200^2 - i_d^2}} + \left(L_d - L_q\right)\left(\sqrt{200^2 - i_d^2} - \frac{i_d^2}{\sqrt{200^2 - i_d^2}}\right)$$

which can be simplified to:

$$-\psi_m i_d + \left(L_d - L_q\right)(200^2 - 2i_d^2) = 0$$

Substituting numerical values results in:

$$10^{-3}i_d^2 - 0.069i_d - 20 = 0$$

Which can be solved to give:

$$i_d = \frac{0.069 - \sqrt{0.069^2 + 4 \times 20 \times 10^{-3}}}{2 \times 10^{-3}} = -111\ A$$

Then:

$$i_q = \sqrt{200^2 - 111^2} = 166.4\ A$$

The maximum torque in these conditions is:

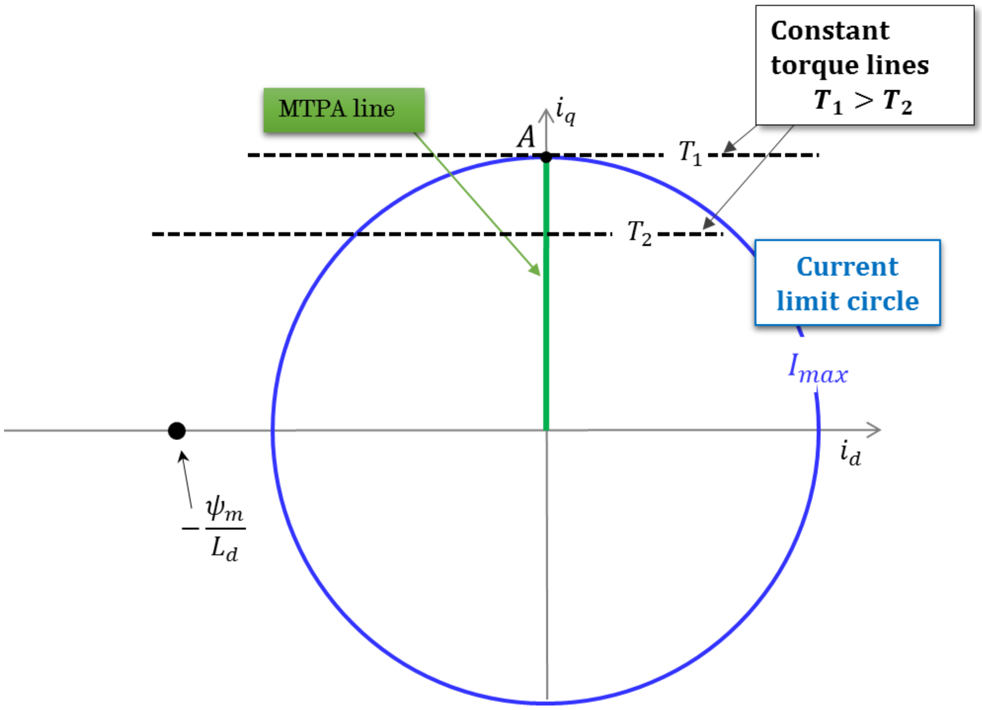
$$T = \frac{3}{2}\frac{P}{2}\left[\psi_m i_q + \left(L_d - L_q\right)i_d i_q\right]$$

$$= \frac{3}{2}\frac{4}{2}[0.069 \times 166.4 + 500 \times 10^{-6} \times 111 \times 166.4] = 62.15\ Nm$$

(8)

Question 4

a. The operating region satisfying the current constraint is the interior of the circle in the following figure. The MTPA line is a straight line with $i_d = 0$.



(2)

b. Since the machine is non salient, there is no reluctance torque contribution and the maximum torque is obtained with $i_d = 0, i_q = I_{max}$:

$$T_{max} = \frac{3}{2}\frac{P}{2}\psi_m I_{max} = \frac{3}{2}\frac{6}{2} \times 0.08 \times 75 = 27\ Nm$$

(2)

c. The maximum phase voltage an inverter can produce under sine-triangle modulation in the linear region is:

$$V_{max} = \frac{V_{dc}}{2} = \frac{100}{2} = 50\ V.$$

In steady-state conditions and neglecting resistive voltage drops:

$$\begin{aligned} v_d &= -\omega L i_q \\ v_q &= \omega\psi_m + \omega L i_d \end{aligned}$$

The maximum speed ω_b at which maximum torque can be generated can be calculated setting $i_d = 0, i_q = I_{max}$ and stator voltage equal to V_{max} i.e.:

$$\begin{aligned} v_d &= -\omega_b L I_{max} \\ v_q &= \omega_b \psi_m \end{aligned}$$

and

$$\sqrt{v_d^2 + v_q^2} = V_{max}$$

Therefore:

$$\omega_b = \frac{V_{max}}{\sqrt{(LI_{max})^2 + \psi_m^2}} = \frac{50}{\sqrt{(800 \times 10^{-6} \times 75)^2 + (0.08)^2}} = 500\ [\frac{rad}{s}]$$

In [*rpm*] the mechanical speed is

$$\omega_{b,rpm} = 500 \times \frac{1}{3} \times \frac{60}{2\pi} = 1591\ [rpm]$$

(5)

d. Above base speed, the machine must be operated under field weakening i.e. with $i_d < 0$ in order to counteract the back-EMF. Maximum field weakening is obtained in short-circuit conditions when $i_d = i_{sc}$ completely cancels out the magnet flux ($v_d = v_q = 0$):

$$\psi_m = -Li_{sc}$$

The short circuit current is then:

$$i_{sc} = -\frac{\psi_m}{L} = -\frac{0.08}{800 \times 10^{-6}} = -100\ A$$

The short circuit current is greater, in absolute value, than the maximum available current, i.e. $|i_{sc}| > I_{max}$ and therefore complete field weakening is not possible but is only possible up to a maximum speed ω_{max}. At maximum field weakening

$$\begin{aligned} i_d &= -I_{max} \\ i_q &= 0 \end{aligned}$$

Then:

$$V_{max} = \omega_{max}(\psi_m - LI_{max})$$

Hence:

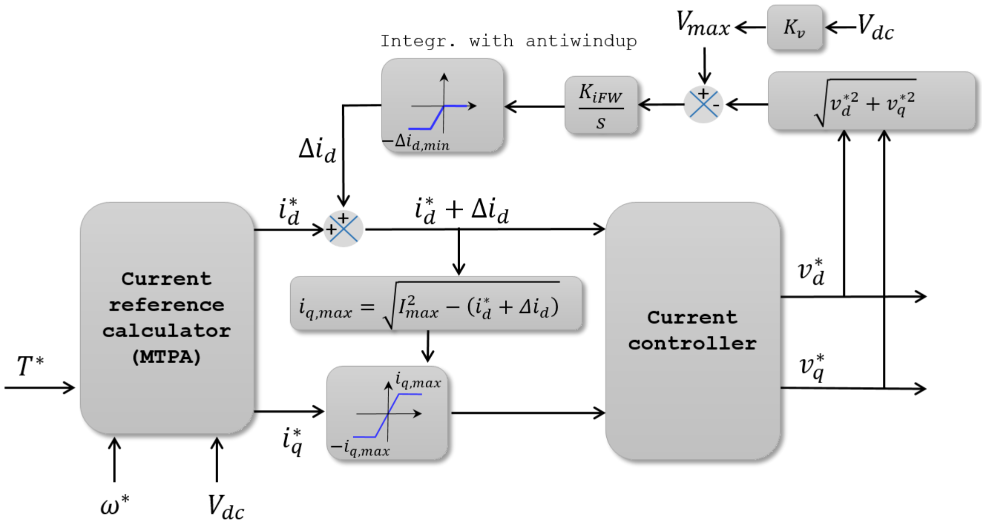
$$\omega_{max} = \frac{50}{0.08 - 800 \times 10^{-6} \times 75} = 2500\ [\frac{rad}{s}]$$

The maximum mechanical speed in [*rpm*] is:

$$\frac{2500}{2\pi} \times \frac{1}{3} \times 60 = 7958\ [rpm]$$

(5)

e. A suitable algorithm for closed loop field oriented control is shown in the figure below.
The voltages requested from the current control are compared with the voltage limitation. A voltage request over the maximum generates a negative $-\Delta i_d$ request to the current control which weakens the flux.

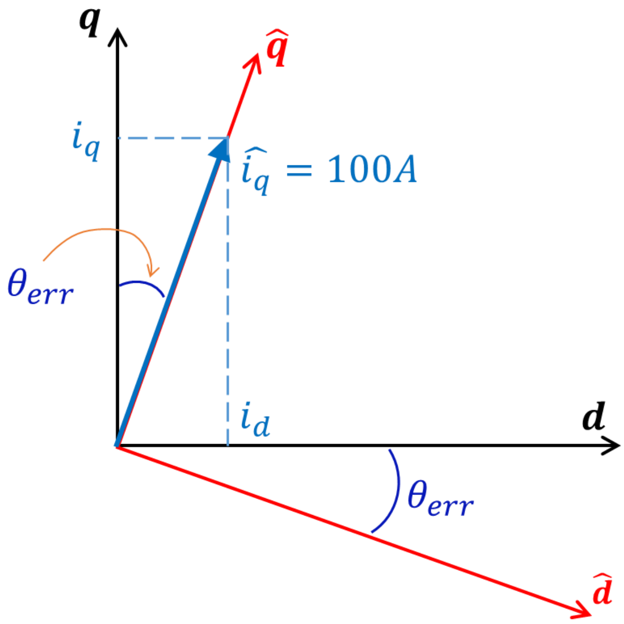


V_{max}
K_v
V_{dc}
Integr. with antiwindup
$-\Delta i_{d,min}$
$\frac{K_{iFW}}{s}$
$\sqrt{v_d^{*2} + v_q^{*2}}$
Δi_d
i_d^*
$i_d^* + \Delta i_d$
Current reference calculator (MTPA)
$i_{q,max} = \sqrt{I_{max}^2 - (i_d^* + \Delta i_d)}$
$i_{q,max}$
$-i_{q,max}$
Current controller
v_d^*
v_q^*
T^*
i_q^*
ω^*
V_{dc}

(6)

Question 5

a. The vector diagram in the Fig. below shows the actual dq and the estimated $\hat{d}\,\hat{q}$ reference frames, where θ_{err}, i.e. the estimation error, is the angle between the two frames



The synchronous current controller operates with an erroneous angle estimation resulting in $\widehat{i_q} = 100\ A$. The actual currents in the motor are however:

$$i_d = \widehat{i_q}\cos\theta_{err} = 100\cos\frac{13.3\pi}{180} = 97.3\ A$$
$$i_q = \widehat{i_q}\sin\theta_{err} = 100\sin\frac{13.3\pi}{180} = 23\ A$$

(5)

b. The torque in a PMSM is

$$T = \frac{3}{2}\frac{P}{2}\left[\psi_m i_q + \left(L_d - L_q\right)i_d i_q\right]$$
$$= \frac{3}{2}\frac{4}{2}\left[0.1 \times 97.36 - 10^{-3} \times 23 \times 97.3\right] = 22.48\ Nm$$

The expected torque without angle error is

$$T = \frac{3}{2}\frac{P}{2}\psi_m\widehat{i_q} = \frac{3}{2}\frac{4}{2} \times 0.1 \times 100 = 30\ Nm$$

The angle error results in a reduction of 7.5 Nm, i.e. a 25% reduction

(3)

c.

i. The blocks $\frac{\tau}{\tau s+1}$ are first order transfer functions which approximates the behaviour of an ideal integrator. An integrator is difficult to use in practical applications due to its tendency to drift when offsets are presents and its sensitivity to initial conditions.

(3)

ii. Signals (1) and (2) are the estimated stator flux linkages:

(1): $\widehat{\psi_d} = \int\left[\left(\widehat{v_d} - R\,\widehat{\iota_d}\right) + \widehat{\omega_e}\,\widehat{\psi_q}\right]dt$

(2): $\widehat{\psi_q} = \int\left[\left(\widehat{v_q} - R\,\widehat{\iota_q}\right) - \widehat{\omega_e}\,\widehat{\psi_d}\right]dt$

Signal (3) is the error signal generated by calculating the error between the q −axis flux linkage $\widehat{\psi_q}$ estimated from voltage integration and the q −axis flux linkage $\psi_q = L_q\,\widehat{\iota_q}$ which is the reference model:

(3) $\epsilon = \widehat{\psi_q} - L_q\,\widehat{\iota_q}$

Assuming steady-state conditions, the error input to the tracking controller is zero, therefore

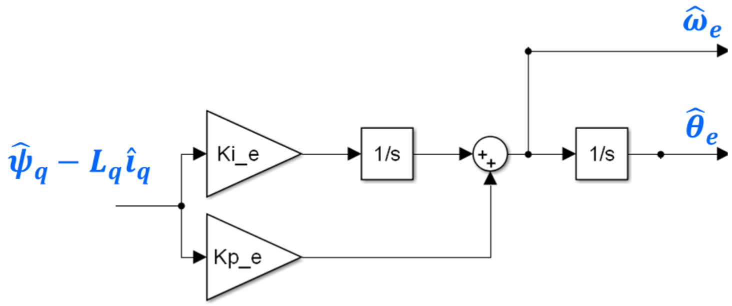
(3) $\epsilon = 0$

(2) $\widehat{\psi_q} = L_q\widehat{\iota_q} = 2 \times 10^{-3} \times 100 = 0.2\ Vs$

(1) $\widehat{\psi_d} = L_d\widehat{\iota_d} + \psi_m = \psi_m = 0.1\ Vs$

(6)

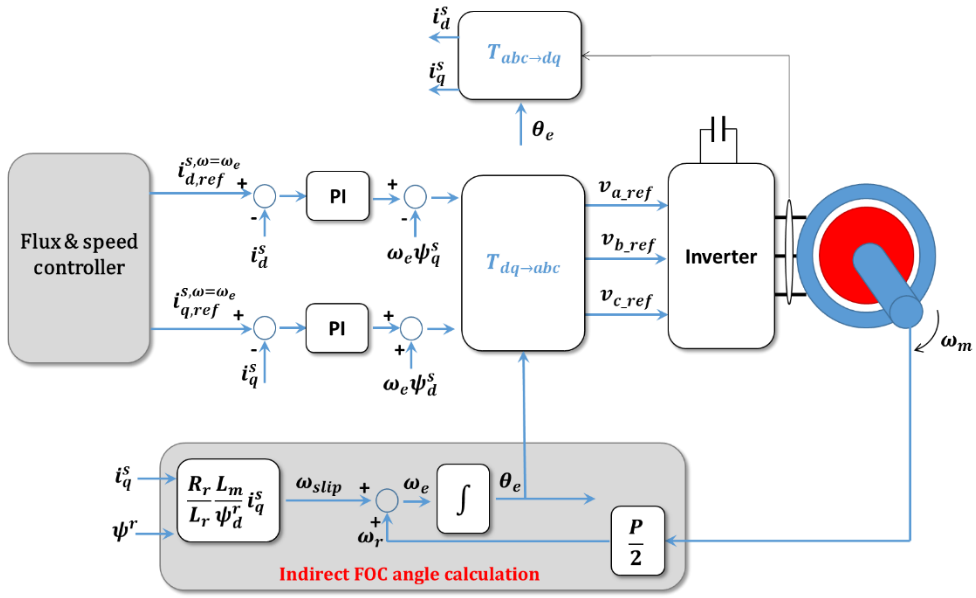
iii. The simplest tracking controller is cascaded PI+integrator as illustrated in fig. below. In steady state conditions the error input to the PI is zero and the estimator should produce an estimate of the speed whose integration gives an estimation of the angle.



(3)

Question 6

a. A block diagram illustrating the operation of indirect FOC is shown in the fig. below:



The controller operates in the synchronous reference frame, with the angle orientation generated by integrating the electrical speed $\omega_e = \omega_r + \omega_{slip}$. The rotor speed ω_r is obtained by measuring (e.g. with an encoder) the mechanical shaft speed. The slip speed ω_{slip} must be calculated from flux and current as:

$$\omega_{slip} = \frac{R_r}{L_r}\frac{L_m}{\psi_d^r}i_q^s$$

(5)

b. The slip frequency is:

$$\omega_{slip} = \frac{R_r}{L_r}\frac{L_m}{\psi_d^r}i_q^s = \frac{0.15}{25\times10^{-3}}\frac{15\times10^{-3}}{0.3}\times100$$

$$= \frac{R_r}{L_r}\frac{i_q^s}{i_d^s} = \frac{0.15}{25\times10^{-3}}\frac{100}{20} = 30\ [\frac{rad}{s}]$$

$$\omega_e = \omega_r + \omega_{slip} = \frac{1500}{60}2\pi\times2+20 = 334.1\ [\frac{rad}{s}]$$

(4)

c. The steady-state stator voltages are:

$$v_d^s = R_s i_d^s - \omega_e\psi_q^s$$
$$v_q^s = R_s i_q^s + \omega_e\psi_d^s$$

where:

$$\omega_e = \omega_r + \omega_{slip} = \frac{1500}{60}2\pi\times2+20 = 334.1\ [\frac{rad}{s}]$$

$$
\begin{aligned}
v_d^s &= 0.2 \times 20 - 334.1 \times 0.125 = -37.8\,V \\
v_q^s &= 0.2 \times 100 + 334.1 \times 0.4 = 153.6\,V
\end{aligned}
$$

(3)

d. Assuming ideal dq- cross-coupling compensation, the stator flux dynamics are described by:

$$
\begin{aligned}
\frac{d\psi_d^s}{dt} &= v_d^s - R_s i_d^s \\
\frac{d\psi_q^s}{dt} &= v_q^s - R_s i_q^s
\end{aligned}
$$

Where:

$$
\begin{aligned}
\psi_d^s &= L_s i_d^s + L_m i_d^r \overset{i_d^r = 0 \text{ in } FOC}{\triangleq} L_s i_d^s \\
\psi_q^s &= L_s i_q^s + L_m i_q^r
\end{aligned}
$$

The rotor current i_q^r can be calculated from the q-axis rotor flux linkage equation which is zero in FOC:

$$
\psi_q^r = 0 = L_r i_q^r + L_m i_q^s \Rightarrow i_q^r = -\frac{L_m}{L_r} i_q^s
$$

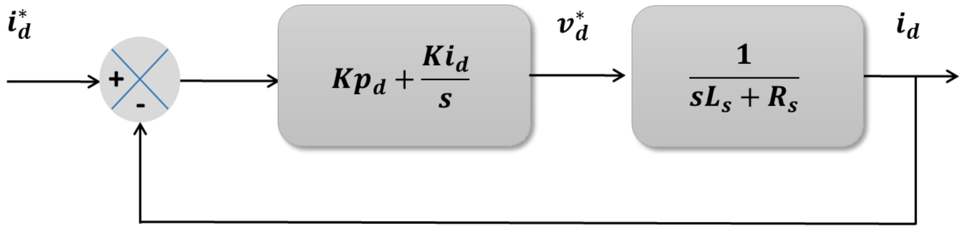
Therefore:

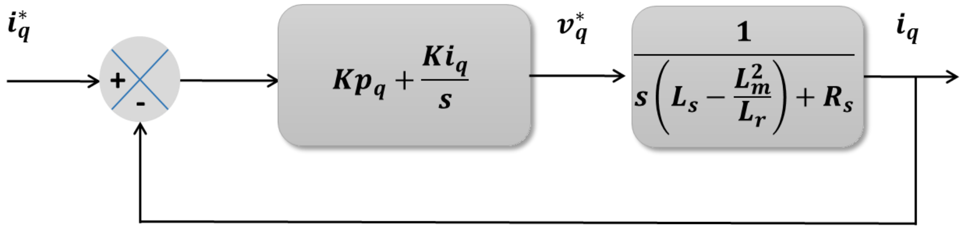
$$
\psi_q^s = \left(L_s - \frac{L_m^2}{L_r}\right) i_q^s
$$

The stator flux dynamic equations can now be written in terms of stator currents as:

$$
\begin{aligned}
L_s \frac{di_d^s}{dt} &= v_d^s - R_s i_d^s \\
\left(L_s - \frac{L_m^2}{L_r}\right) \frac{di_q^s}{dt} &= v_q^s - R_s i_q^s
\end{aligned}
$$

The PI control loops are therefore:





The desired closed loop transfer function has bandwidth $\omega_{cc} = 1/\tau$ and is:

$$\frac{i_{d,q}}{i_{d,q}^*} = \frac{\omega_{cc}}{s + \omega_{cc}} = \frac{1/\tau}{s + 1/\tau}$$

Using pole-zero cancellation the desired response can be obtained with:

$$\frac{Kp_{d,q}}{Ki_{d,q}} = \frac{L_{d,q}}{R_s}$$

and:

$$\frac{Ki_{d,q}}{R} = \frac{1}{\tau}$$

Therefore:

$$Ki_d = Ki_q = \frac{R_s}{\tau} = \frac{0.2}{5 \times 10^{-3}} = 40$$

$$Kp_d = \frac{L_s}{\tau} = \frac{20 \times 10^{-3}}{5 \times 10^{-3}} = 4$$

$$Kp_q = \frac{L_s - \frac{L_m^2}{L_r}}{\tau} = \frac{20 \times 10^{-3} - \frac{15^2}{25} \times 10^{-3}}{5 \times 10^{-3}} = 2.2$$

(8)