UNIVERSITY OF SHEFFIELD
Department of Electronic and Electrical Engineering

# EEE220 ELECTRIC AND MAGNETIC FIELDS

# TUTORIAL QUESTION SOLUTIONS (Part 1)

*JLW 2006*

**Q1** Using Coulomb's Law, $F = \dfrac{q_1 q_2}{4\pi\varepsilon_0 R^2}$, and $\varepsilon_0 = 8.854 \times 10^{-12}$ Fm$^{-1}$

$$F = \frac{1 \times 1}{4 \times 3.142 \times 8.854 \times 10^{-12} \times (1)^2}$$

$$F = 8.99 \times 10^{9}\ N$$

Using $F = mg$ with $g = 9.81$ Nkg$^{-1}$ we have

$$m = \frac{8.99 \times 10^9}{9.81} = 9.16 \times 10^8 \text{ kg} = 9.16 \times 10^5 \text{ tonnes}$$

(1 tonne = $10^3$ kg)

**Q2 a)** The electric field is given by $\mathbf{E} = \dfrac{Q}{4\pi\varepsilon_0 R^3}\mathbf{R}$

$$R = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$$

$$\mathbf{E} = \frac{3 \times 10^{-6}}{4 \times 3.142 \times 8.854 \times 10^{-12} \times \left(\sqrt{14}\right)^3}(\ 3\ ,\ 2\ ,\ 1\ )$$

$$\mathbf{E} = 5.15 \times 10^2 \cdot (\ 3\ ,\ 2\ ,\ 1\ )$$

$$\mathbf{E} = \left(\ 1.54 \times 10^3\ ,\ 1.03 \times 10^3\ ,\ 5.15 \times 10^2\ \right) \text{ Vm}^{-1}$$

**b)**

$$\mathbf{E} = \frac{-3 \times 10^{-6}}{4 \times 3.142 \times 8.854 \times 10^{-12} \times \left(\sqrt{14}\right)^3}(\ 3\ ,\ 2\ ,\ 1\ )$$

$$\mathbf{E} = -5.15 \times 10^2 \cdot (\ 3\ ,\ 2\ ,\ 1\ )$$

$$\mathbf{E} = \left(\ -1.54 \times 10^3\ ,\ -1.03 \times 10^3\ ,\ -5.15 \times 10^2\ \right) \text{ Vm}^{-1}$$

**c)**

$$\mathbf{E} = \frac{3 \times 10^{-6}}{4 \times 3.142 \times 8.854 \times 10^{-12} \times \left(\sqrt{14}\right)^3}(\ 1\ ,\ 2\ ,\ 3\ )$$

$$\mathbf{E} = 5.15 \times 10^2 \cdot (\ 1\ ,\ 2\ ,\ 3\ )$$

$$\mathbf{E} = \left(\ 5.15 \times 10^2\ ,\ 1.03 \times 10^3\ ,\ 1.54 \times 10^3\ \right) \text{ Vm}^{-1}$$

**d)**

$$\mathbf{E} = \frac{3 \times 10^{-6}}{4 \times 3.142 \times 8.854 \times 10^{-12} \times \left(\sqrt{14}\right)^3}(\ -1\ ,\ -2\ ,\ -3\ )$$

$$\mathbf{E} = 5.15 \times 10^2 \cdot (\ -1\ ,\ -2\ ,\ -3\ )$$

$$\mathbf{E} = \left(\ -5.15 \times 10^2\ ,\ -1.03 \times 10^3\ ,\ -1.54 \times 10^3\ \right) \text{ Vm}^{-1}$$

**e)** Yes, the magnitudes of the fields in parts (a) – (d) are all equal.

Looking at the equation for the Electric Field, and taking only the magnitude of the vector, we see that…

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$

This is dependent only on $R$, which is the same, $\sqrt{14}$, in all the above cases.

Thus the magnitudes will all be the same – only the direction of the Electric Field vector will vary.

Q3 a)

$$\mathbf{R} = (5,5,0) - (2,2,0) = (3,3,0)$$

$$R = \sqrt{3^2 + 3^2 + 0} = \sqrt{18}$$

$$\mathbf{E} = \frac{10^{-5}}{4 \times 3.142 \times 8.854 \times 10^{-12} \times (18)^{3/2}} \cdot (3,3,0)$$

$$\mathbf{E} = \left(3.53 \times 10^{3},\ 3.53 \times 10^{3},\ 0\right)\ \mathrm{Vm^{-1}}$$

b)

$$\mathbf{R} = (5,5,0) - (7,1,0) = (-2,4,0)$$

$$R = \sqrt{(-2)^2 + 4^2 + 0} = \sqrt{20}$$

$$\mathbf{E} = \frac{-2 \times 10^{-5}}{4 \times 3.142 \times 8.854 \times 10^{-12} \times (20)^{3/2}} \cdot (-2,4,0)$$

$$\mathbf{E} = \left(4.02 \times 10^{3},\ -8.04 \times 10^{3},\ 0\right)\ \mathrm{Vm^{-1}}$$

c)

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$\mathbf{E} = \left(3.53 \times 10^{3},\ 3.53 \times 10^{3},\ 0\right) + \left(4.02 \times 10^{3},\ -8.04 \times 10^{3},\ 0\right)$$

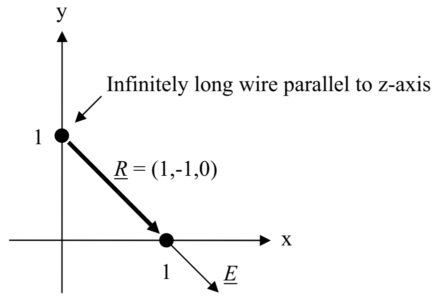
$$\mathbf{E} = \left(7.55 \times 10^{3},\ -4.51 \times 10^{3},\ 0\right)\ \mathrm{Vm^{-1}}$$

d) Use $\mathbf{F} = q\mathbf{E}$

$$\mathbf{F} = 10^{-6} \cdot \left(7.55 \times 10^{3},\ -4.51 \times 10^{3},\ 0\right)$$

$$\mathbf{F} = \left(7.55 \times 10^{-3},\ -4.51 \times 10^{-3},\ 0\right)\ \mathrm{N}$$

Q4 a)




$$\mathbf{E} = \frac{q_\ell}{2\pi\varepsilon_0 \times R} \times \frac{\mathbf{R}}{|\mathbf{R}|}$$

$$R = \sqrt{1^2 + (-1)^2 + 0^2}$$

$$\mathbf{E} = \frac{3 \times 10^{-6}}{2 \times 3.142 \times 8.854 \times 10^{-12} \times \sqrt{2}} \times \frac{(1,\ -1,\ 0)}{\sqrt{2}}$$

$$\mathbf{E} = \left(2.70 \times 10^{4},\ -2.70 \times 10^{4},\ 0\right)\ \mathrm{Vm^{-1}}$$

b)

$$\mathbf{R} = (1,\ 1,\ 0)$$

$$q_\ell = 1 \times 10^{-6}$$

$$\mathbf{E_2} = \left(8.99 \times 10^{3},\ 8.99 \times 10^{3},\ 0\right)$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E_2}$$

$$\mathbf{E} = \left(2.70 \times 10^{4},\ -2.70 \times 10^{4},\ 0\right) + \left(8.99 \times 10^{3},\ 8.99 \times 10^{3},\ 0\right)$$

$$\mathbf{E} = \left(3.60 \times 10^{4},\ -1.80 \times 10^{4},\ 0\right)$$

c)

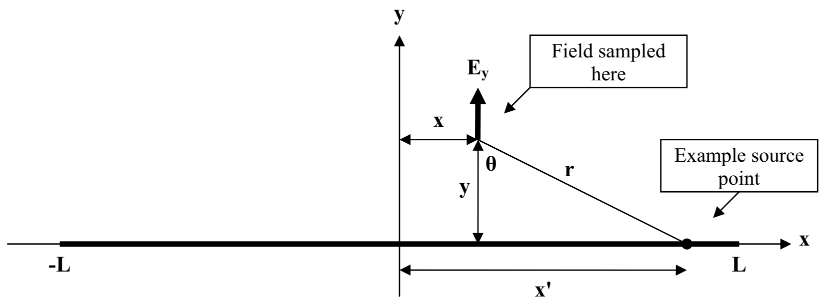
$$\mathbf{F} = q\mathbf{E}$$

$$\mathbf{F} = 1 \times 10^{-7} \times \left(3.60 \times 10^{4},\ -1.80 \times 10^{4},\ 0\right)$$

$$\mathbf{F} = \left(3.60 \times 10^{-3},\ -1.80 \times 10^{-3},\ 0\right)\ \mathrm{N}$$

Q5 There are two routes to answering this question – one is conceptually easier, the other is mathematically easier (making more use of the derivation in the lecture notes) – either method is acceptable so choose the one you are most comfortable with.

Either way, you should always start by drawing the diagram…




You will notice that the question states that your derivation must be valid for **any** field point given by the coordinates $(x, y)$. This is slightly different from the derivation we did in lectures – there we only calculated the field for a point on the y-axis, i.e $(0, y)$. In the lectures we used an infinitely long wire, in which case it does not matter where along the wire we take the origin to be, so the derivation became valid anywhere is space. In this question, the wire is of finite length, so we must explicitly include in the calculations the observation point – where we observe the field. (see diagram above)

An integration along the wire will be required, and for this we need an integration variable. Previously, we used $x$, but $x$ has already been used, so we can choose $x'$ instead.

The horizontal distance between the source and observation points is now $(x' - x)$ instead of $x$.

The first method of tackling this problem is to take the derivation from the lectures…

$$E_y = \int_0^{\infty} \left[ \frac{q_\ell \cos\theta}{4\pi\varepsilon_0 r^2} \right] dx$$

$$E_y = \frac{q_\ell}{4\pi\varepsilon_0} \int_0^{\infty} \left[ \frac{\cos\theta}{r^2} \right] dx$$

But $\cos\theta = \frac{y}{r}$ (SOHCAHTOA)

$$E_y = \frac{q_\ell y}{4\pi\varepsilon_0} \int_0^{\infty} \left[ \frac{1}{r^3} \right] dx$$

And $r^2 = x^2 + y^2$ so…

$$E_y = \frac{q_\ell y}{4\pi\varepsilon_0} \int_0^{\infty} \left[ \frac{1}{\left(x^2 + y^2\right)^{3/2}} \right] dx$$

From a table of standard integrals…

$$\int \frac{dx}{\left(ax^2 + bx + c\right)^{3/2}} = \frac{2(2ax + b)}{\left(4ac - b^2\right)\sqrt{ax^2 + bx + c}}$$

Using $a = 1,\ b = 0,\ c = y^2$

$$E_y = \frac{q_\ell y}{4\pi\varepsilon_0} \left[ \frac{4x}{4y^2\sqrt{x^2 + y^2}} \right]_0^{\text{inf}}$$

$$E_y = \frac{q_\ell}{4\pi\varepsilon_0 y} \left[ \frac{x}{\sqrt{x^2 + y^2}} \right]_0^{\text{inf}}$$

… then replace the integration variable $dx$ by $dx'$, substitute in the new limits, -L and L, and use $(x' - x)$ instead of $x$. Thus the integration becomes…

$$E_y = \frac{q_\ell y}{4\pi\varepsilon_0} \int_{-L}^{L} \left[ \frac{1}{\left(\left(x' - x\right)^2 + y^2\right)^{3/2}} \right] dx'$$

This can be solved using the same standard integral as before (after expanding out the $(x' - x)^2$ term) giving…

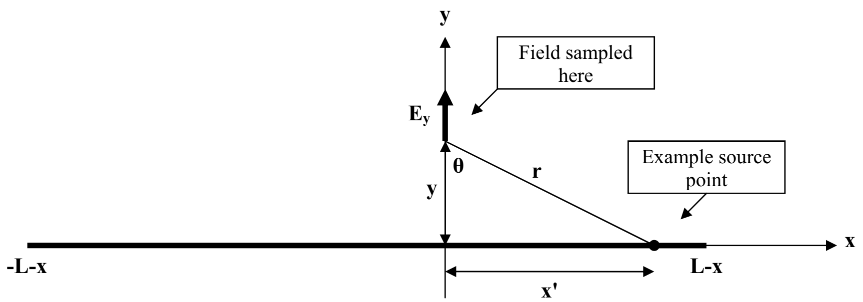
$$E_y = \frac{q_\ell y}{4\pi\varepsilon_0} \left[ \frac{1}{y^2} \cdot \frac{(x' - x)}{\left(\left(x' - x\right)^2 + y^2\right)^{3/2}} \right]_{-L}^{L}$$

Substitute in the limits and we obtain…

$$E_y = \frac{q_\ell}{4\pi\varepsilon_0 y} \left[ \frac{(L - x)}{\sqrt{(L - x)^2 + y^2}} - \frac{(-L - x)}{\sqrt{(-L - x)^2 + y^2}} \right]$$

which, after tidying, gives the answer on the tutorial sheet.

The second method of tackling this problem (which should involve a little less maths) is to realise that instead of moving the field point from the y-axis to some point $(x, y)$, you would get the same net effect by moving the wire the same distance in the opposite direction, and leaving the field point on the y-axis. Compare the diagram below with the previous one…




…all we have done is to shift everything x units to the left, putting the field point back on the y-axis. This will allow us to use exactly the same derivation and integration as we did in the lectures, but using the limits –**L–x** and **L–x** instead of **–L** and **L.** Thus we obtain…

$$E_y = \frac{q_\ell}{4\pi\varepsilon_0 y}\left[\frac{x'}{\sqrt{x'^2 + y^2}}\right]_{-L-x}^{L-x}$$

which gives…

$$E_y = \frac{q_\ell}{4\pi\varepsilon_0 y}\left[\frac{(L-x)}{\sqrt{(L-x)^2 + y^2}} - \frac{(-L-x)}{\sqrt{(-L-x)^2 + y^2}}\right]$$

**Important Note:** Regardless of which route you use to solve it, because the wire is not infinitely long $E_x$ will not always be zero as was the case with the infinitely long wire. $E_x$ will only be zero if we observe the field on the plane perpendicular to the wire, which passes through its midpoint, thus we have not calculated a full solution for the Electric Field!

Q6 The Electric field due to an infinitely long wire is given by $|\mathbf{E}| = \dfrac{q_\ell}{2\pi r\varepsilon_0}$

$$= \frac{1 \times 10^{-5}}{2 \times \pi \times 1 \times 8.854 \times 10^{-12}}$$

$$= 1.798 \times 10^5 \ \mathrm{Vm^{-1}}$$

The Electric field due to a wire of length $2L$ is given by:-

$$E_y = \frac{q_\ell}{4\pi\varepsilon_0 y}\left[\frac{L + x}{\left[(L+x)^2 + y^2\right]^{1/2}} + \frac{L - x}{\left[(L-x)^2 + y^2\right]^{1/2}}\right]$$

$$= \frac{1 \times 10^{-5}}{4 \times \pi \times 8.854 \times 10^{-12} \times 1}\left[\frac{5 + 0}{\left[(5+0)^2 + 1^2\right]^{1/2}} + \frac{5 - 0}{\left[(5-0)^2 + 1^2\right]^{1/2}}\right]$$

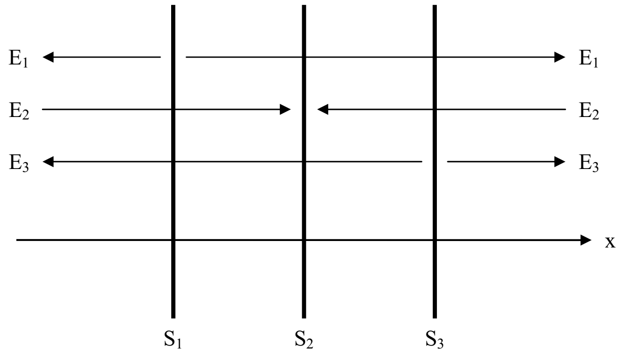
$$= 1.763 \times 10^5 \ \mathrm{Vm^{-1}}$$

The percentage error is given by the expression $\dfrac{|\text{calculated - actual}|}{\text{actual}} \times 100\%$

$$= \frac{|1.798 - 1.763|}{1.763} \times 100\%$$

$$= 2\%$$

Q7 a)




$$|\mathbf{E}_1| = \frac{q_s}{2\varepsilon_0} = \frac{1 \times 10^{-7}}{2 \times 8.854 \times 10^{-12}} = 5.65 \times 10^3 \text{ Vm}^{-1}$$

$$|\mathbf{E}_2| = \frac{q_s}{2\varepsilon_0} = \frac{2 \times 10^{-7}}{2 \times 8.854 \times 10^{-12}} = 11.29 \times 10^3 \text{ Vm}^{-1}$$

$$|\mathbf{E}_3| = \frac{q_s}{2\varepsilon_0} = \frac{3 \times 10^{-7}}{2 \times 8.854 \times 10^{-12}} = 16.94 \times 10^3 \text{ Vm}^{-1}$$

| **Region** | $E_x$ | |
|---|---|---|
| Left of $S_1$ | $E_x = -\lvert E_1\rvert + \lvert E_2\rvert - \lvert E_3\rvert$ | $= -11.3 \times 10^3$ Vm$^{-1}$ |
| Between $S_1$ and $S_2$ | $E_x = +\lvert E_1\rvert + \lvert E_2\rvert - \lvert E_3\rvert$ | $= 0$ Vm$^{-1}$ |
| Between $S_2$ and $S_3$ | $E_x = +\lvert E_1\rvert - \lvert E_2\rvert - \lvert E_3\rvert$ | $= -22.6 \times 10^3$ Vm$^{-1}$ |
| Right of $S_3$ | $E_x = +\lvert E_1\rvert - \lvert E_2\rvert + \lvert E_3\rvert$ | $= 11.3 \times 10^3$ Vm$^{-1}$ |

b) $E_y = E_z = 0$

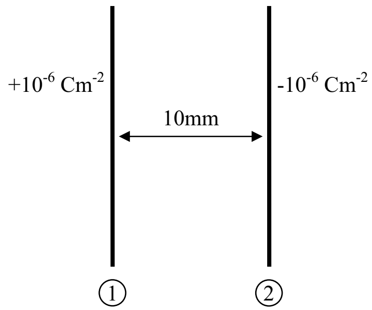
Between $S_1$ and $S_2$, $E_x = 0$ thus potential is also zero

Between $S_2$ and $S_3$, $E_x = -22.6 \times 10^3$ Vm$^{-1}$

$$\text{p.d.} = \phi_2 - \phi_1 = \int_{S_2}^{S_3} E_x dx$$

$$\text{p.d.} = 22.6 \times 10^3 \times 10 \times 10^{-3} = 226 \text{ V}$$

Q8




$$\text{Field due to sheet (1)} = \frac{q_\ell}{2\varepsilon_0} = \frac{10^{-6}}{2 \times 8.854 \times 10^{-12}}$$

$$= 5.65 \times 10^4 \text{ Vm}^{-1}$$

(Note that this is not dependent on the separation distance between the sheets)

Force per unit area $=$ charge per unit area $\times$ electric field [c.f. $F = qE$]

$$= 10^{-6} \times 5.65 \times 10^4$$

$$= 5.65 \times 10^{-2} \text{ Nm}^{-2}$$

Sheets are of opposite polarity; therefore force between them is attractive.

Q9 a)

$$\phi_B = \frac{q_1}{4\pi\varepsilon_0 R} = \frac{10^{-3}}{4\times\pi\times 8.854\times 10^{-12}\times 0.5} = 1.798\times 10^7 \text{ V}$$

$$\phi_C = \frac{q_1}{4\pi\varepsilon_0 R} = \frac{10^{-3}}{4\times\pi\times 8.854\times 10^{-12}\times 0.75} = 1.198\times 10^7 \text{ V}$$

$$W = q_2\left(\phi_B - \phi_C\right)$$

$$= 10^{-5}\times\left(1.798 - 1.198\right)\times 10^7$$

$$= 60 \text{ J}$$

b) Points C and D are the same distance from the charge $q_1$ at A, thus $\phi_D = \phi_C$ and the calculations are the same as above:-

$$W = q_2\left(\phi_B - \phi_D\right)$$

$$= 10^{-5}\times\left(1.798 - 1.198\right)\times 10^7$$

$$= 60 \text{ J}$$

c) Work done is independent of the path taken, thus the calculations are again the same as above:-

$$W = q_2\left(\phi_B - \phi_D\right)$$

$$= 10^{-5}\times\left(1.798 - 1.198\right)\times 10^7$$

$$= 60 \text{ J}$$

d) Work done $=$ force $\times$ distance moved **in the direction of the force**

In this case, the force which is exerted by the field is in the same direction as the charge $q_2$ is moved. Thus, it is the field (as opposed to any external forces) which does the work.

e)

$$W = q_2\left(\phi_B - \phi_C\right)$$

$$= -10^{-5}\times\left(1.798 - 1.198\right)\times 10^7$$

$$= -60$$

We usually state work done as a positive value, so taking the magnitude of this:-

$$W = 60 \text{ J}$$

(The negative sign reminds us to consider whether or not the charge has moved in the direction of the force exerted by the field, and thus determine which field/force did the work.)

f) In this case, the charge $q_2$ has moved in the opposite direction to the direction of the force which the field exerts on it. Therefore, an external force must have done the work.

Q10

$$\mathbf{E} = -\nabla\phi$$

$$= \left(\frac{-d\phi}{dx}, \frac{-d\phi}{dy}, \frac{-d\phi}{dz}\right)$$

$$= \left(-4x\ ,\ -3z\ ,\ -3y\right)$$

At the point $(1,2,3)$ $\quad \mathbf{E} = \left(-4(1)\ ,\ -3(3)\ ,\ -3(2)\right)$

$$= \left(-4\ ,\ -9\ ,\ -6\right)\ \text{Vm}^{-1}$$

At the point $(-1,-1,-1)$ $\quad \mathbf{E} = \left(-4(-1)\ ,\ -3(-1)\ ,\ -3(-1)\right)$

$$= \left(4\ ,\ 3\ ,\ 3\right)\ \text{Vm}^{-1}$$

Q11 a)

$$\phi = \frac{q}{4\pi\varepsilon_0 R}$$

$$\phi_{total} = \frac{q_1}{4\pi\varepsilon_0\sqrt{x^2 + (y-a)^2 + z^2}} - \frac{q_1}{4\pi\varepsilon_0\sqrt{x^2 + (y+a)^2 + z^2}}$$

b) Take the $1/R$ part of the first term, expand the $(y-a)^2$ term, and divide the expression under the square root by $(x^2 + y^2 + z^2)$…

$$\frac{1}{\sqrt{x^2 + (y-a)^2 + z^2}} = \frac{1}{\sqrt{(x^2+y^2+z^2)\left(1 - \frac{2ay - a^2}{x^2+y^2+z^2}\right)}}$$

$$= \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \left(1 - \frac{2ay - a^2}{x^2+y^2+z^2}\right)^{-1/2}$$

The $a^2$ term can be neglected, and $\left(1 - \frac{2ay - a^2}{x^2+y^2+z^2}\right)^{-1/2}$ expanded using $(1+u)^n \approx 1 + nu$

$$\frac{1}{\sqrt{x^2 + (y-a)^2 + z^2}} = \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \left(1 + \frac{ay}{x^2+y^2+z^2}\right)$$

Taking the $1/R$ part of the second term, and repeating the same steps, we obtain…

$$\frac{1}{\sqrt{x^2 + (y+a)^2 + z^2}} = \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \left(1 - \frac{ay}{x^2+y^2+z^2}\right)$$

Thus:-

$$\phi = \frac{q_1}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \left[\left(1 + \frac{ay}{x^2+y^2+z^2}\right) - \left(1 - \frac{ay}{x^2+y^2+z^2}\right)\right]$$

$$\phi = \frac{q_1}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \left[\frac{2ay}{x^2+y^2+z^2}\right]$$

$$\phi = \frac{q_1 ay}{2\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$$

c)

$$\mathbf{E} = -\nabla\phi = \left(-\frac{\partial\phi}{\partial x}, -\frac{\partial\phi}{\partial y}, -\frac{\partial\phi}{\partial z}\right)$$

To solve for $E_x$, use the chain rule, and let $R^2 = x^2 + y^2 + z^2$

$$E_x = -\frac{\partial\phi}{\partial x} = -\frac{\partial\phi}{\partial R^2}\cdot\frac{\partial R^2}{\partial x}$$

$$= -\left(\frac{q_1 ay}{2\pi\varepsilon_0}\right)\cdot\left(-\frac{3}{2}\right)\cdot(x^2+y^2+x^2)^{-5/2}\cdot 2x$$

$$= \frac{q_1 a}{2\pi\varepsilon_0}\cdot\frac{3xy}{(x^2+y^2+x^2)^{5/2}}$$

To solve for $E_y$, use the quotient rule, $\frac{d}{dy}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$, and let $u = y$, $v = (R^2)^{3/2}$

$$E_y = -\frac{\partial\phi}{\partial y} = -\left(\frac{q_1 a}{2\pi\varepsilon_0}\right)\left[\frac{(R^2)^{3/2} - y\left(3/2\right)(R^2)^{1/2}\cdot 2y}{(R^2)^{6/2}}\right]$$

$$= \frac{q_1 a}{2\pi\varepsilon_0}\left[\frac{3y^2}{(x^2+y^2+z^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2)^{3/2}}\right]$$

To solve for $E_z$, the solution is very similar to that for $E_x$

$$E_z = -\frac{\partial\phi}{\partial z} = -\frac{\partial\phi}{\partial R^2}\cdot\frac{\partial R^2}{\partial z}$$

$$= -\left(\frac{q_1 ay}{2\pi\varepsilon_0}\right)\cdot\left(-\frac{3}{2}\right)\cdot(x^2+y^2+x^2)^{-5/2}\cdot 2z$$

$$= \frac{q_1 a}{2\pi\varepsilon_0}\cdot\frac{3zy}{(x^2+y^2+x^2)^{5/2}}$$

Q12 a) Using the equation for the electric field due to a line source, $E_r = \frac{q_\ell}{2\pi r \varepsilon_0}\hat{\mathbf{r}}$, and applying superposition, the field on the x-axis between the two wires is given by:-

$$E_x = \frac{q_\ell}{2\pi x \varepsilon_0} + \frac{q_\ell}{2\pi(h-x)\varepsilon_0}$$

$$= \frac{q_\ell}{2\pi\varepsilon_0}\left[\frac{1}{x} + \frac{1}{h-x}\right]$$

b)

$$V = -\int_a^{h-a}\left(\frac{q_\ell}{2\pi\varepsilon_0}\left[\frac{1}{x} + \frac{1}{h-x}\right]\right)dx$$

$$= -\frac{q_\ell}{2\pi\varepsilon_0}\left[\int_a^{h-a}\left(\frac{1}{x}\right)dx + \int_a^{h-a}\left(\frac{1}{h-x}\right)dx\right]$$

$$= -\frac{q_\ell}{2\pi\varepsilon_0}\Big[\ln(x) - \ln(h-x)\Big]_a^{h-a}$$

$$= -\frac{q_\ell}{2\pi\varepsilon_0}\Big[\ln(h-a) - \ln(a) - \ln(a) + \ln(h-a)\Big]$$

$$= -\frac{q_\ell}{\pi\varepsilon_0}\ln\left(\frac{h-a}{a}\right)$$

(The negative sign indicates that the potential decreases as you move from the wire on the left to the wire on the right, i.e. the wire on the right is at a lower potential.)

c) Adapting the equation $C = \frac{Q}{V}$ we get $C_\ell = \frac{q_\ell}{V}$, where $C_\ell$ is the capacitance per unit length, and $q_\ell$ is the charge per unit length, thus:-

$$C_\ell = \frac{q_\ell}{V}$$

$$= \frac{q_\ell}{\frac{q_\ell}{\pi\varepsilon_0}\ln\left(\frac{h-a}{a}\right)}$$

$$= \frac{\pi\varepsilon_0}{\ln\left(\frac{h-a}{a}\right)}$$

Q13 a) Merging the equations $Q = \frac{C}{V}$, and $C = \frac{\varepsilon_0\varepsilon_r A}{d}$, we obtain:-

$$Q = \frac{\varepsilon_0\varepsilon_r A}{d}\cdot V$$

or $$\frac{Q}{A} = \frac{\varepsilon_0\varepsilon_r}{d}\cdot V$$

Thus, substituting in the values from the question, we find:-

$$\frac{Q}{A} = \frac{8.854\times10^{-12}\times 8}{1\times10^{-5}}\cdot 100$$

$$= 7.08\times10^{-4}\ Cm^{-1}$$

b) The field can be found using the equation $E = \frac{V}{d}$, thus:-

$$E = \frac{V}{d} = \frac{100}{1\times10^{-5}}$$

$$= 10^7\ Vm^{-1}$$

Q14 a) The charge enclosed by S is calculated by multiplying the charge per unit length by the length enclosed by S:-

$$= \frac{Q}{L}\times\ell$$

$$= \frac{Q\ell}{L}$$

b) Flux out of S $= \dfrac{\text{total charge enclosed}}{\varepsilon_0}$

$$= \frac{Q\ell}{L\varepsilon_0}$$

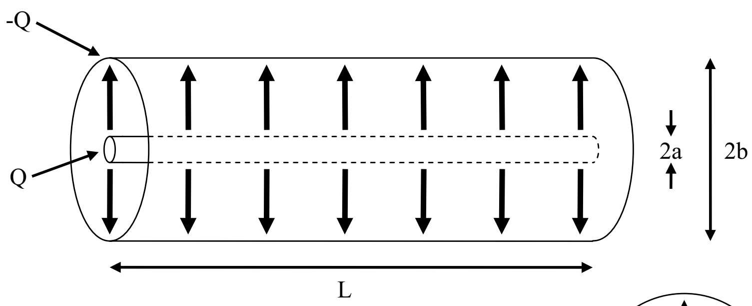
c) Total flux $= \frac{Q\ell}{L\varepsilon_0} = \oint_S E\cos\theta\, da$

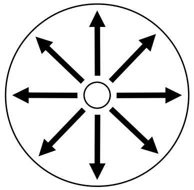
Due to symmetry, the E-field is radial, so the field is perpendicular to the surface S, and $\cos\theta = 1$, thus:-

$$\frac{Q\ell}{L\varepsilon_0} = E\oint_S da = E\times 2\pi r\ell$$

$$E = \frac{Q}{2\pi r L\varepsilon_0}$$

Q15 a)



Field is radially outwards

b) The charge on the outer conductor does not contribute to the field inside it so the field in the region concerned is that due to the charged cylinder in the centre.

$$E = \frac{Q_\ell}{2\pi r \varepsilon_0} = \frac{Q}{2\pi r L \varepsilon_0}$$

c) Using the equation $V = -\int_{r=a}^{r=b} \mathbf{E} \bullet \mathbf{dl}$ and remembering that p.d. is independent of path taken, we can choose to direct $\mathbf{dl}$ radially (i.e. parallel to the field.)

$$V = -\int_a^b \frac{Q}{2\pi r L \varepsilon_0} dr$$

$$= -\frac{Q}{2\pi L \varepsilon_0} \int_a^b \frac{1}{r} dr$$

$$= -\frac{Q}{2\pi L \varepsilon_0} \left[\ln r\right]_a^b$$

$$= -\frac{Q}{2\pi L \varepsilon_0} \ln\left(\frac{b}{a}\right)$$

Note: The negative sign indicates that the outer cylinder is negative with respect to the inner (remember that we integrated from the inner to the outer.) When stating a potential difference, we would usually ignore the sign and just state its magnitude, thus:-

$$V = \frac{Q}{2\pi L \varepsilon_0} \ln\left(\frac{b}{a}\right)$$

d) $$C = \frac{Q}{V} = \frac{2\pi L \varepsilon_0}{\ln\left(\frac{b}{a}\right)}$$