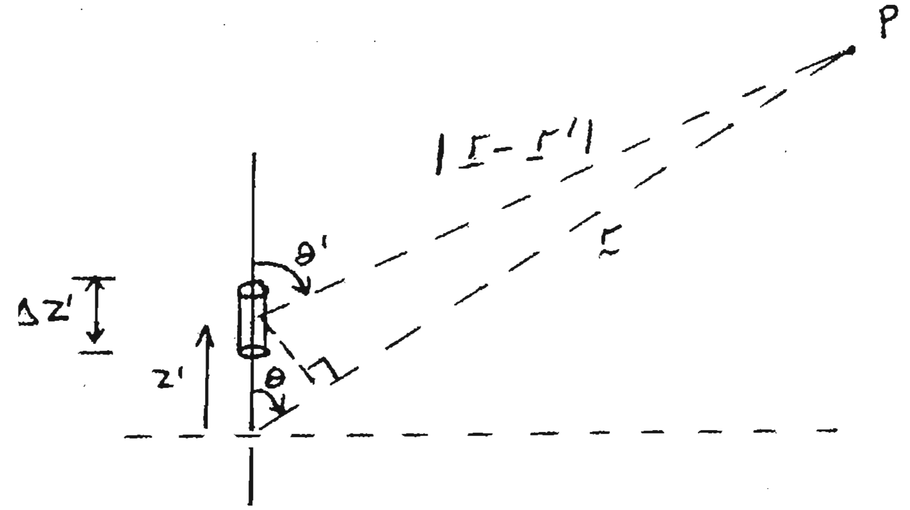
Q1)
(a)



As $r \to \infty$,

i) $\quad \dfrac{1}{|\underline{r}-\underline{r}'|} \to \dfrac{1}{r}$

ii) $\quad |\underline{r}-\underline{r}'| \to r - z'\cos(\theta)$

iii) $\quad \theta' \to \theta$

Hence equation (1.1) can be written

$$\Delta E_\theta = C\frac{e^{-jkr}}{r}\sin(\theta)I(z')e^{jkz'\cos(\theta)}\Delta z' \qquad (1)$$

(b)
The far zone field of the dipole is obtained through integration of Equation (1) over its length, so that

$$E_\theta = C\frac{e^{-jkr}}{r}\sin(\theta)\int_{-d}^{d} I(z')e^{jkz'\cos(\theta)}dz' = C\frac{e^{-jkr}}{r}\sin(\theta)\times Int \qquad (2)$$

Substituting the given current distribution then produces the following integration

$$Int = I_o\int_{-d}^{0}(1+\frac{z'}{d})e^{jkz'\cos(\theta)}dz' + I_o\int_{0}^{d}(1-\frac{z'}{d})e^{jkz'\cos(\theta)}dz' \qquad (3)$$

Now,

$$\int e^{jkz'\cos(\theta)}dz' = \frac{1}{jk\cos(\theta)}e^{jkz'\cos(\theta)} \qquad (4)$$

and

$$\int z' e^{jkz'\cos(\theta)} dz' = z' \frac{e^{jkz'\cos(\theta)}}{jk\cos(\theta)} - \int \frac{e^{jkz'\cos(\theta)}}{jk\cos(\theta)} dz' \quad (5)$$

so

$$\int z' e^{jkz'\cos(\theta)} dz' = z' \frac{e^{jkz'\cos(\theta)}}{jk\cos(\theta)} - \frac{e^{jkz'\cos(\theta)}}{(jk\cos(\theta))^2} \quad (6)$$

Hence,

$$\int_{-d}^{d} e^{jkz'\cos(\theta)} dz' = \frac{1}{jk\cos(\theta)} \times 2j\sin(kd\cos(\theta)) \quad (7)$$

and

$$\frac{1}{d}\int_{-d}^{0} z' e^{jkz'\cos(\theta)} dz' = \frac{1}{jkd\cos(\theta)}\left(-\frac{1}{jk\cos(\theta)} + de^{-jkd\cos(\theta)} + \frac{e^{-jkd\cos(\theta)}}{jk\cos(\theta)}\right) \quad (8)$$

$$\frac{1}{d}\int_{0}^{d} z' e^{jkz'\cos(\theta)} dz' = \frac{1}{jkd\cos(\theta)}\left(de^{jkd\cos(\theta)} - \frac{e^{jkd\cos(\theta)}}{jk\cos(\theta)} + \frac{1}{jk\cos(\theta)}\right) \quad (9)$$

Thus, $\frac{Int}{I_o} = Equations\left((7)+(8)-(9)\right) =$

$$\frac{1}{j\alpha}\left(j2d\sin(\alpha) - \frac{d}{j\alpha} + de^{-j\alpha} + \frac{de^{-j\alpha}}{j\alpha} - de^{j\alpha} + \frac{de^{j\alpha}}{j\alpha} - \frac{d}{j\alpha}\right) \quad (10)$$

$$= \frac{1}{j\alpha}\left(\frac{2d\cos(\alpha)}{j\alpha} - \frac{2d}{j\alpha}\right) = \frac{2d}{\alpha^2}(1-\cos(\alpha)) \quad (11)$$

where $\alpha = kd\cos(\theta)$. Multiplying the numerator and denominator of Equation (11) by $(1+\cos(\alpha))$ then yields

$$Int = I_o \frac{2d}{\alpha^2}\frac{\sin^2(\alpha)}{1+\cos(\alpha)} \quad (12)$$

Substitution of Equation (12) into (2) then yields Equation (1.2) in the question, where

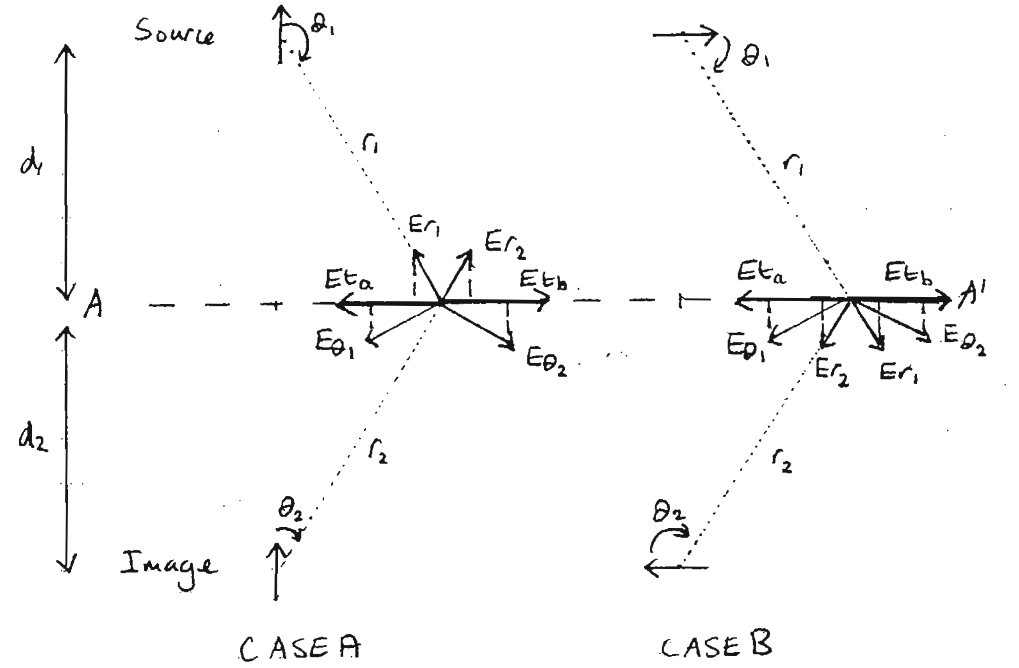
$$Sa(\alpha) = \frac{\sin(\alpha)}{\alpha} \quad (13).$$

(c)

When $\theta = 90^o$, $\alpha = 0$ and $Sa(\alpha) = 1$, hence Equation (1.2) becomes

$$E_\theta\big|_{\theta=90^o} = 2CI_o \frac{e^{-jkr}}{r} d \quad (14)$$

Q2)
(a)



The fields of an elemental dipole antenna behave as

$$E_\theta = C(r)sin(\theta), E_r = D(r)cos(\theta) \quad (1)$$

The boundary conditions at a perfect groundplane require zero tangential E field, and therefore in both *Case A* and *B* the dipole image must be orientated so that

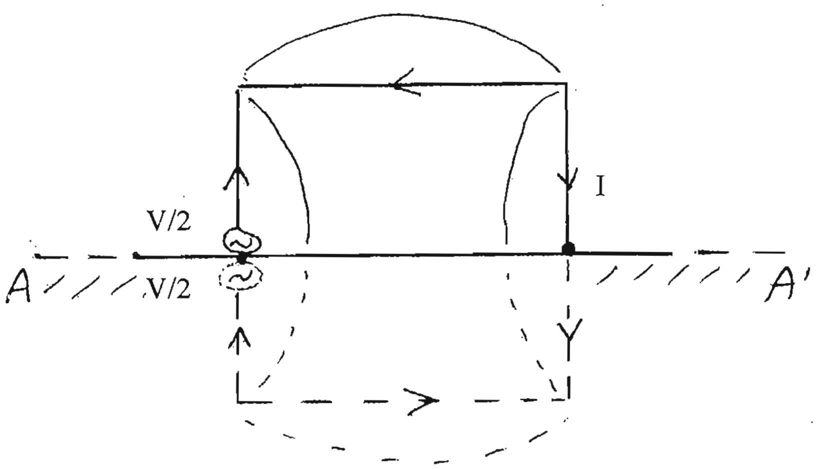
$$E_{ta} = E_{tb} \qquad (2)$$

This forces the image orientations to be as shown, and also requires that

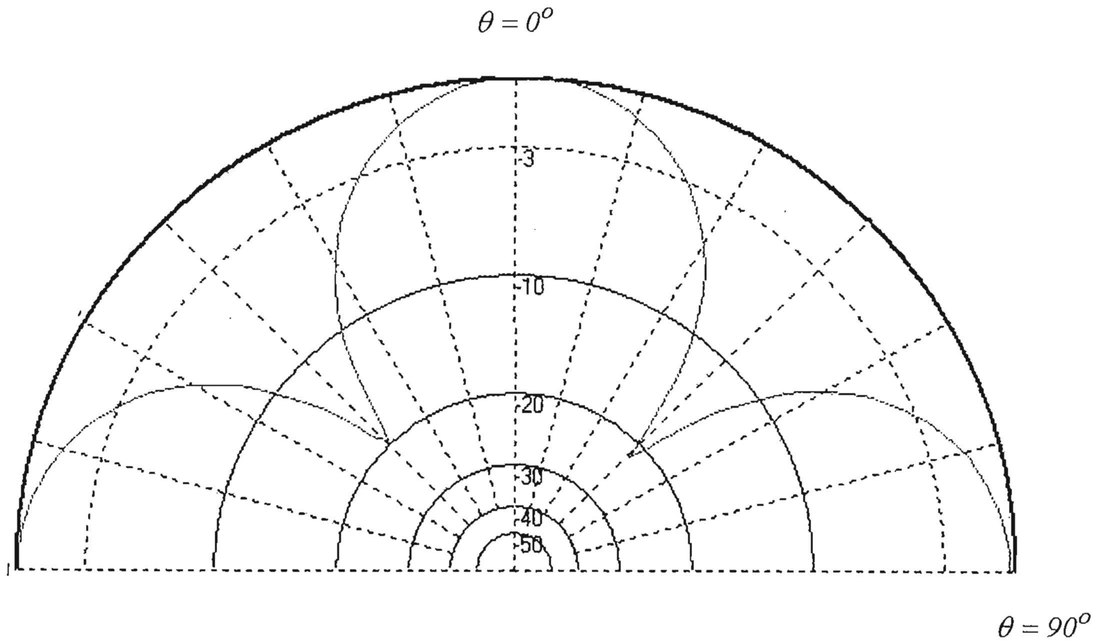
$$d_1 = d_2 \qquad (3)$$

to ensure equal source and image field amplitudes at the groundplane.

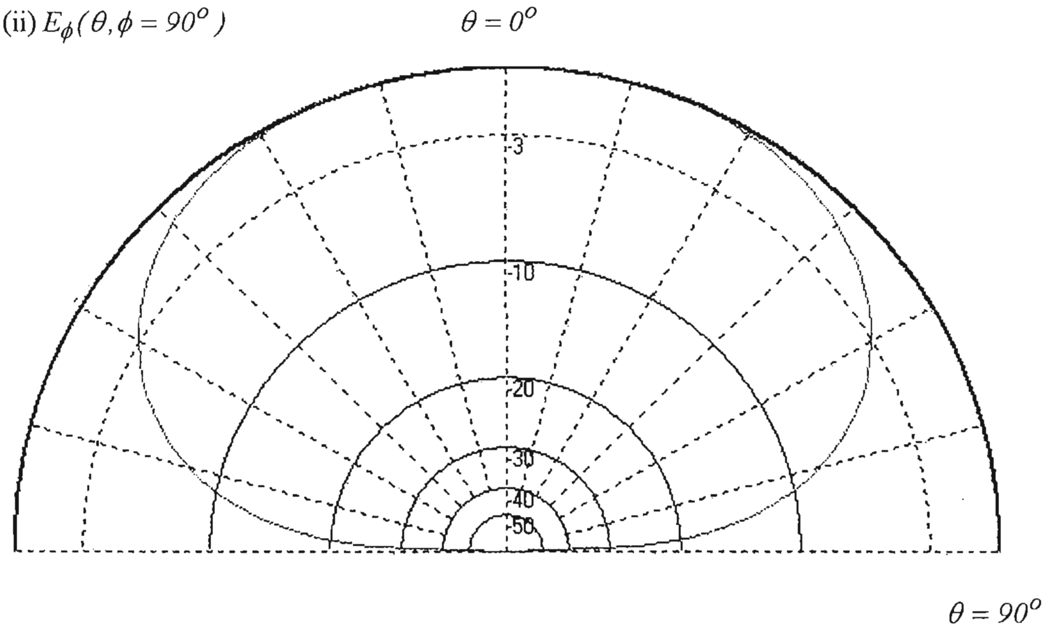
(b)



(i) $E_\theta(\theta, \phi = 0^o)$



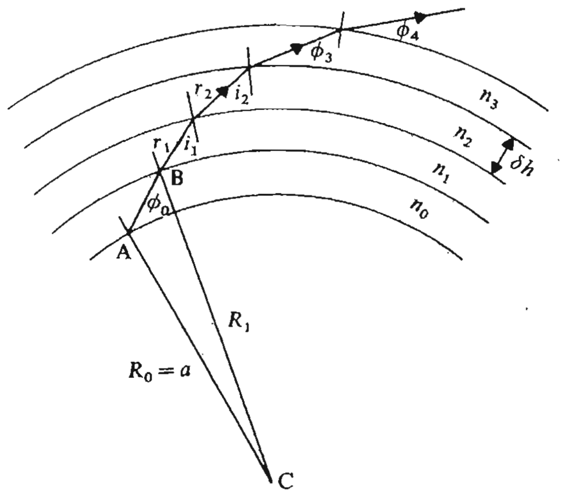
(ii) $E_\phi(\theta, \phi = 90^o)$



(c)

The half loop only radiates the supplied transmitter power into a hemispherical half space above the groundplane, instead of radiating it into all of space like the whole loop. Thus if the same transmit power is used in both cases, double the power is radiated by the half loop into half the space, giving the half loop a gain of 3dB over the whole loop. Thus the whole loop gain will be *2.7dBi*. Regarding the input impedance, the half loop driving point voltage between the loop and groundplane is half that across the terminals of a full loop, thus for the same antenna current the input impedance is halved. Hence the full loop input impedance is $234 - j86\Omega$.

Q3)
(a)



With respect to the above figure, the radius of the earth $AC = R_0 = a$ and subsequent layers at $BC = R_1 = a + \delta h$ for example represent tropospheric layers. Also, i_j denotes angle of incidence of wave propagating in layer j with refractive index n_j onto layer $j+1$, r_j is angle of refraction in layer n_j and ϕ_j is the complementary angle to r_j, so that $r_j = 90 - \phi_j$. From Snell's law:

$$n_0 \sin i_0 = n_1 \sin r_1 = n_1 \cos\phi_1,\ n_1 \sin i_1 = n_2 \sin r_2 = n_2 cos\phi_2 \quad etc... \qquad (1)$$

and from the sine rule:

$$\frac{R_0}{\sin i_0} = \frac{R_1}{\sin(180 - r_0)} = \frac{R_1}{\cos\phi_0},\ \frac{R_1}{\sin i_1} = \frac{R_2}{\sin(180 - r_1)} = \frac{R_2}{\cos\phi_1} \quad etc... \qquad (2)$$

so that

$$n_0 R_0 \cos\phi_0 = n_1 R_1 \cos\phi_1 = n_2 R_2 \cos\phi_2 \quad etc... \qquad (3)$$

Thus at any given height h where the refractive index is n

$$a n_0 \cos\phi_0 = (a+h) n \cos\phi \qquad (4)$$

so (4) can then be re-written for a 'planar' earth as

$$n_0 \cos\phi_0 = n^* \cos\phi \qquad (5)$$

where the modified refractive index is

$$n^* = \frac{(a+h)}{a} n \approx n + \frac{h}{a} \quad (6)$$

Thus n^* can be used in a 'flat earth' model and takes account of the earth's curvature.

(b)

From (6)

$$\frac{\partial n^*}{\partial h} = \frac{\partial n}{\partial h} + \frac{1}{a} \quad (7)$$

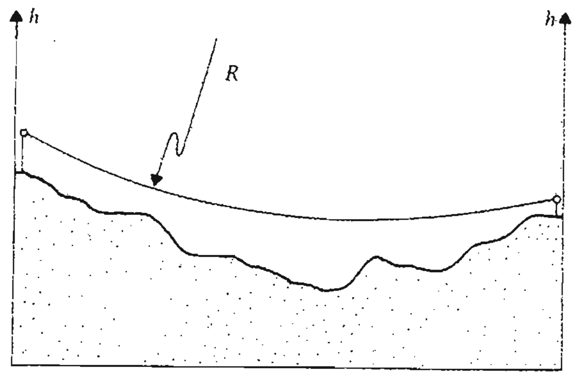
so that

$$\frac{\partial n^*}{\partial h} = -39 \times 10^{-6} + \frac{1}{6370} = 118 \times 10^{-6}\, km^{-1} \quad (8)$$

Since the radius of curvature R of the propagation path is given by

$$\frac{1}{R} \approx -\frac{\partial n^*}{\partial h} \qquad (9)$$

the waves propagate with an upward curvature of radius of $R \approx 8475km$ with respect to the 'flat' earth's surface, as shown below.



(c)

If $\frac{dn^*}{dh} = 0$ then from (9) the radius of curvature is infinite, meaning the waves propagate 'in a straight line' at their launch height parallel to the 'flat' earth's surface. In reality this means the propagation path follows the curvature of the earth at the fixed launch height, meaning that communication distances are significantly increased. Such propagation could occur as a result of a *surface duct* being produced due to anomalous weather conditions causing a temperature inversion.

Q4)
(a)

If $L = \lambda$ then

$$\frac{kL}{2} = \pi \qquad (1)$$

so that Equation (4.1) becomes

$$|E_\theta| = \frac{2\eta I_o}{4\pi r}\left[\frac{cos(\pi cos(\theta)) - \cos(\pi)}{\sin(\theta)}\right] = \frac{2\eta I_o}{4\pi r}\left[\frac{cos(\pi cos(\theta)) + 1}{\sin(\theta)}\right] \qquad (2)$$

Now the gain is given by

$$G = \frac{P_r|_{\theta=90^o}}{\frac{P}{4\pi r^2}} \qquad (3)$$

where P_r denotes radiated power density and P the total radiated power. Now,

$$P_r = \frac{1}{2}\frac{|E_\theta|^2}{\eta} Wm^{-2} \qquad (4)$$

and

$$P = \int_0^{2\pi}\int_0^{\pi} P_r r \sin(\theta) d\phi r d\theta \qquad (5)$$

so that

$$P = 2\pi r^2 \int_0^{\pi} P_r \sin(\theta) d\theta = \frac{I_o^2 \eta}{4\pi}\int_0^{\pi}\frac{(\cos(\pi\cos(\theta)) + 1)^2}{\sin(\theta)} d\theta \qquad (6)$$

Hence from Equation (4.2)

$$P = \frac{I_o^2 \eta}{4\pi} \times 3.318 \qquad (7)$$

Also from Equation (4.1) and (4)

$$P_r|_{\theta=90^o} = \frac{\eta I_o^2}{2\pi^2 r^2} \qquad (8)$$

Thus, from (3)

$$G = \frac{8}{3.318} = 2.41 \qquad (9)$$

so

$$10 log_{10} G = 3.8dBi \qquad (10)$$

(b)

Advantage: The gain of a full wave dipole is *1.7dB* higher than that of a half wave dipole.

Disadvantage: The input impedance of a full wave dipole is very high since it's fed at a current minimum, which makes matching into a transmission line difficult. A half wave dipole however has an input impedance $\sim 73\Omega$ which is commensurate with the characteristic impedance of coaxial cable. A full wave dipole is also twice the length of a half wave dipole of course.

(c)

The effective aperture of an antenna is given by

$$A_e = \frac{\lambda^2}{4\pi} G \qquad (11)$$

So from (9),

$$A_e = \frac{0.75^2}{4\pi} \times 2.41 = 0.1 m^2 \qquad (12)$$

The power received by the antenna P is then

$$P = A_e \times P_d \qquad (13)$$

where P_d is the power density of the incident plane wave. Here, from (4)

$$P_d = \frac{1}{2\eta} \times (55 \times 10^{-3})^2 = 4\mu W / m^2 \qquad (14)$$

Thus,

$$P = 0.4\mu W \qquad (15)$$