EEE118 Problem Class Questions – Sheet 2

Permittivity of free space = 8.8×10^{-12} F/m
Electron charge = 1.6×10^{-19} C
Electron mass = 9.1×10^{-31} Kg
For germanium (Ge); $\mu_e = 0.39\ m^2V^{-1}s^{-1}$; $\mu_h = 0.19\ m^2V^{-1}s^{-1}$ $m^*_e = 0.22m_e$, $n_i = 2.5 \times 10^{19}\ m^{-3}$
For silicon (Si); $\mu_e = 0.12\ m^2V^{-1}s^{-1}$; $\mu_h = 0.046\ m^2V^{-1}s^{-1}$ $m_e = 0.97$, $n_i \sim 1.5 \times 10^{16}\ m^{-3}$
Density of atoms in Si = $4.9 \times 10^{28}\ m^{-3}$
Kinetic energy acquired by an electron as is moves through a potential of 1 V = 1 eV

1. The band-gap of silicon at room temperature has an energy of 1.11 eV. What is this in Joules?

(1.76×10^{-19} J)

2. *(second part is difficult, so you should move on if you can't do it)*
In an intrinsic piece of silicon that has been heated up to temperature, T_2, the electron carrier density is $1 \times 10^{18}\ m^{-3}$. The mobility of electrons and holes are $0.05\ m^2V^{-1}s^{-1}$ and $0.02\ m^2V^{-1}s^{-1}$, respectively. What is the conductivity of the material at this temperature? If the temperature (in degrees K) is now reduced to T_1, show that the carrier concentration at the lower temperature is given by (hint: consider the ratios of n_i at the two temperatures, T_1 and T_2):

($1.12 \times 10^{-2}\ (\Omega m)^{-1}$)

$$\therefore n_{i1} = 1x10^{18}\left(\frac{T_1}{T_2}\right)^{3/2} \exp\left(-\frac{W_g}{2k_B}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)\right) .$$

3. **(a)** Calculate the average scattering time of electrons at room temperature in Si and Ge based upon their mobilities and effective masses.

(660 fs, 490 fs)

(b) If these crystals are ultra-pure and defect-free, what is the origin of these scattering events?

4. A rod of heavily p-doped germanium is 6 mm long, l mm wide and 0.5 mm thick. It has an electrical resistance of 120 ohms along its length. Assuming that all the conductivity is due to holes, determine the impurity concentration.

($3.3 \times 10^{21}\ m^{-3}$)

5. A chip of Si is 1 mm x 2 mm in area and 0.1 mm thick. The material has one in every 10^8 atoms replaced by an atom of B
(a) Is the doped material n-type or p-type. Why?
(b) What is the density of majority carriers?
(c) What voltage is required to produce a current of 2mA between the large faces?

($4.9 \times 10^{20}\ m^{-3}$, 28 mV)

EEE118 Sheet 2 Solutions

1. 1 eV is equal to the amount of kinetic energy gained by a free electron when it accelerates through an electric potential difference of one volt. Thus it is 1 volt (1 joule per coulomb) multiplied by the electron charge (1 e, or 1.6×10^{-19} C). Therefore, one electron volt is equal to 1.6×10^{-19} J.

The band-gap of Si at room temperature is therefore $1.11 \times 1.6\times10^{-19}$ J $=1.78\times10^{-19}$ J

2 The conductivity at the higher temperature is given by

$$\sigma = ne(\mu_e + \mu_h) = (1\times10^{18})\times(1.6\times10^{-19})\times(0.05+0.02) = 1.12\times10^{-2}\,\Omega^{-1}\mathrm{m}^{-1}$$

Now the intrinsic carrier concentration, n_i, is given by $C\ (T)^{3/2}\exp\left(-\dfrac{W_g}{2\mathrm{k_B}T}\right)$

Concentration at higher temperature T_2 = 1 x 1018 m^{-3} = $C\ T_2^{3/2}\exp\left(-\dfrac{W_g}{2\mathrm{k_B}T_2}\right)$

Concentration at lower temperature, T_1 = n_{i1} = $C\ T_1^{3/2}\exp\left(-\dfrac{W_g}{2\mathrm{k_B}T_1}\right)$

$$\therefore \frac{n_{i1}}{1x10^{18}} = C\ T_1^{3/2}\exp\left(-\frac{W_g}{2\mathrm{k_B}T_1}\right) \div C\ T_2^{3/2}\exp\left(-\frac{W_g}{2\mathrm{k_B}T_2}\right)$$

Cancelling terms where possible $\therefore \dfrac{n_{i1}}{1x10^{18}} = \left(\dfrac{T_1}{T_2}\right)^{3/2}\exp\left(-\dfrac{W_g}{2\mathrm{k_B}T_1} + \dfrac{W_g}{2k_BT_2}\right)$

Therefore $\therefore n_{i1} = 1x10^{18}\left(\dfrac{T_1}{T_2}\right)^{3/2}\exp\left(-\dfrac{W_g}{2k_B}\left(\dfrac{1}{T_1} - \dfrac{1}{T_2}\right)\right)$

3. The mobility and scattering time are related by

$$\mu = \frac{\mathrm{q}\tau}{\mathrm{m}^*}$$

$$so$$

$$\tau = \frac{\mu m^*}{q}$$

Remembering that m* is the electron mass multiplied by the effective mass factor.

So for Si $$\tau = \frac{\mu m^*}{q} = \frac{0.12 \text{ x } 0.97 \text{ x } 9.1\text{x}10^{-31}}{1.6\text{x}10^{-19}} = 6.6\text{x}10^{-13}\text{s} = 660\text{fs}$$

For Ge $$\tau = \frac{\mu m^*}{q} = \frac{0.39 \text{ x } 0.22 \text{ x } 9.1\text{x}10^{-31}}{1.6\text{x}10^{-19}} = 4.9\text{x}10^{-13}\text{s} = 490\text{fs}$$

(b) The origin of the electron scattering events are disruptions to the perfection of the crystal lattice. This can be through crystal defects such as; ionised impurities, interstitial defects, vacancy defects, and through the disruption of the lattice by lattice vibrations (heat/sound) – these are quantized and are known as "phonons".

If the crystals are ultra-pure and defect-free (i.e. defects and unintentional impurities are essentially zero) then the only major source of electron scattering is with phonons.

4. This material is heavily p-doped so we can assume that the conductivity due to electrons is much less than that due to holes. Let us then estimate the hole concentration:

$$\rho = \frac{RA}{l} = 120x0.5x10^{-6} / 6x10^{-3} = 0.01\Omega\text{m}$$

$$p = \frac{1}{\rho q \mu} = \frac{1}{0.01x1.6x10^{-19}x0.19} = 3.3x10^{21}\text{m}^{-3}$$

5. (a) B is an acceptor in Si as it has 3 outer electrons compared to 4 for silicon, so the Si is doped **p-type**

(b) For silicon, density of atoms = $4.9 \text{ x } 10^{28} \text{ m}^{-3}$

As we have one Si atom in every 10^8 replaced with dopant, and at room temperature each one is assumed to give us a free hole

$p = 4.9 \text{ x } 10^{28} \text{ m}^{-3} \text{ x } 1\text{x}10^{-8} \text{ m}^{-3} = 4.9 \text{ x } 10^{20} \text{ m}^{-3}$

(c) To calculate the voltage we need to calculate the resistance. Since the geometry is given, then provided the resistivity can be calculated, the rest should follow:

$$\rho = \frac{1}{pq\mu} = \frac{1}{4.9x10^{20}x1.6x10^{-19}x0.046} = 0.28\Omega\text{m}$$

$$R = \rho\frac{l}{A} = 0.28\frac{0.1x10^{-3}}{1x10^{-3}x2x10^{-3}} = 14.0\Omega$$

$$V = IR = 2x10^{-3}x14.0 = 0.028\text{V}$$