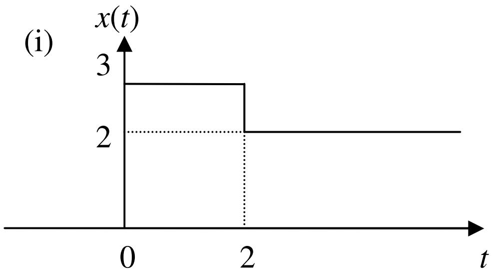
Tutorial 1: Solutions

1. How is the unit step function $u(t)$ related to (i) $\delta(t)$ and (ii) ramp function $r(t)$?

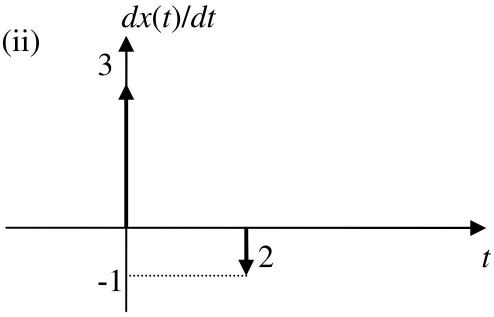
$$\text{(i) } u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau \text{ or } \delta(t) = \frac{du(t)}{dt}. \quad \text{(ii) } r(t) = \int_{-\infty}^{t} u(\tau)d\tau = \int_{0}^{t} u(\tau)d\tau.$$

2. For a signal $x(t) = 3u(t) - u(t\text{-}2)$, sketch and label

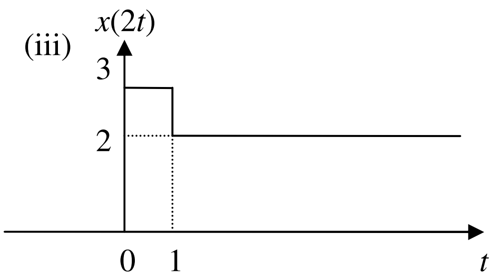
(i)



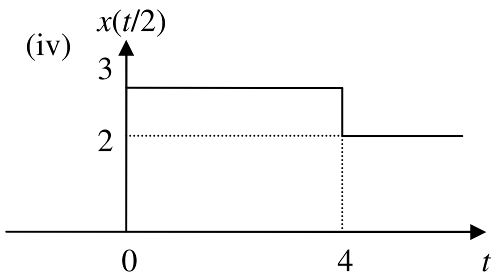
(ii)



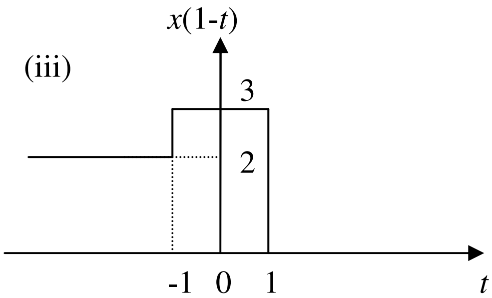
(iii)



(iv)

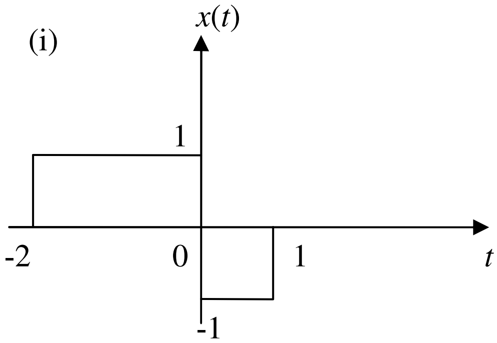


(iii)

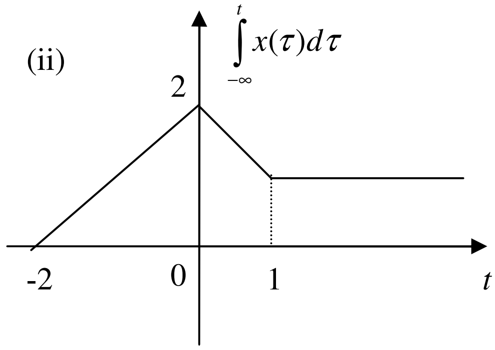


3. For $x(t) = u(t+2) - 2u(t) + u(t-1)$, sketch and label

(i)

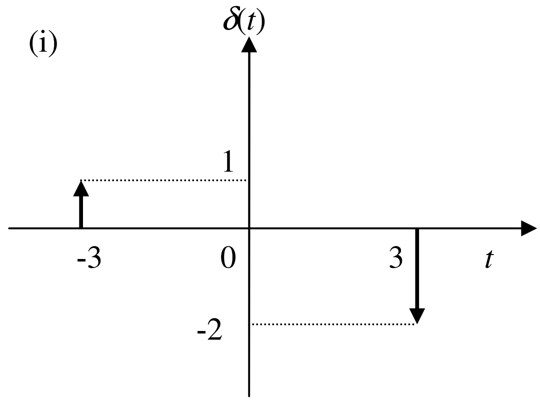


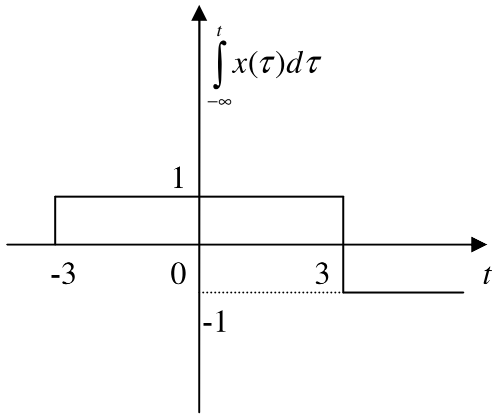
(ii)



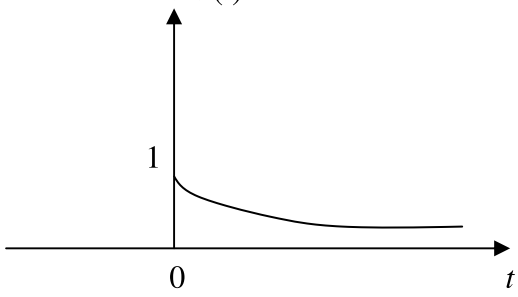
4. For $x(t) = \delta(t+3) - 2\delta(t-3)$, sketch and label

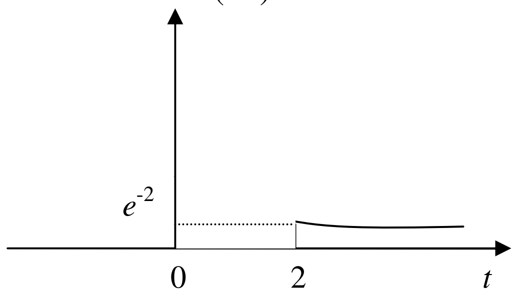
(i)

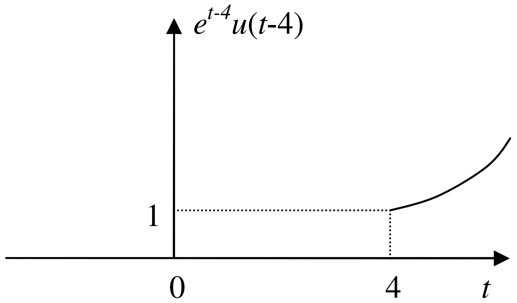


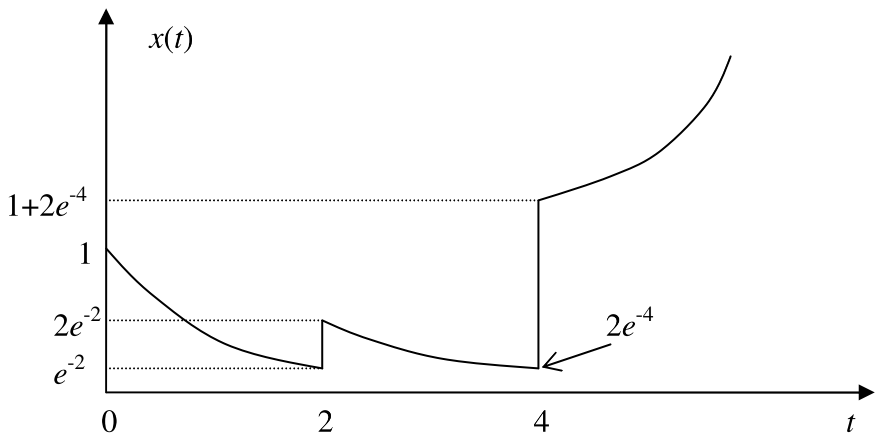


5. Sketch and label $x(t) = e^{-t}u(t) + e^{-t}u(t\text{-}2) + e^{t\text{-}4}u(t\text{-}4)$.

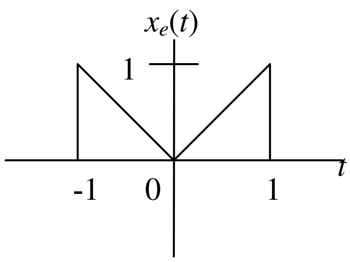


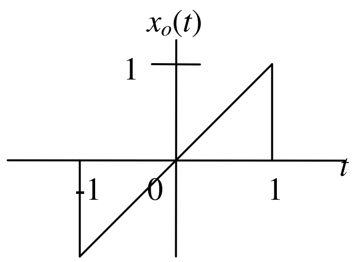




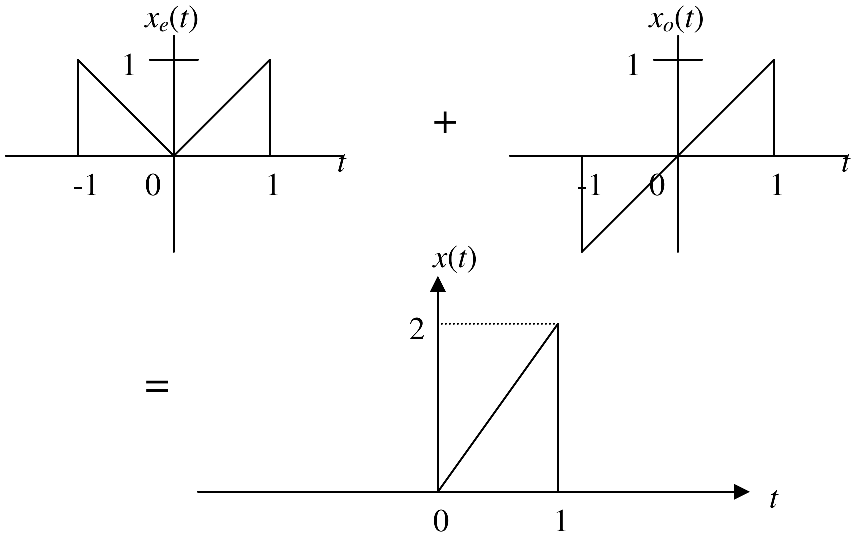


6. Find the signal that has an even and an odd component shown below.





We know that $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ and $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$. Therefore we have $x(t) = x_e(t) + x_o(t)$.



7. Consider a sinusoidal signal $x(t) = A\cos(\omega t)$. Determine the average value, the average power and the root mean square of $x(t)$.

The average value is given by $\frac{1}{T}\int_{-T/2}^{T/2} x(t)dt = \frac{1}{T}\int_{-T/2}^{T/2} A\cos(\omega t)dt = \frac{A}{\omega T}[\sin\omega t]_{-T/2}^{T/2}$

$$= \frac{A}{\omega T}\left[\sin\left(\frac{\omega T}{2}\right) - \sin\left(-\frac{\omega T}{2}\right)\right] = \frac{A}{\pi}\sin\pi = 0.$$

The average power is given by

$$\frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt = \frac{1}{T}\int_{-T/2}^{T/2} A^2\cos^2(\omega t)dt = \frac{A^2}{2T}\int_{-T/2}^{T/2} 1 + \cos(2\omega t)dt$$

$$= \frac{A^2}{2T}\left[\left(\frac{T}{2}\right) - \left(-\frac{T}{2}\right)\right] + \frac{A^2}{4\omega T}\left[\sin\left(\frac{2\omega T}{2}\right) - \sin\left(-\frac{2\omega T}{2}\right)\right] = \frac{A^2}{2} + \frac{A^2}{8\pi}[\sin(2\pi) + \sin(2\pi)] = \frac{A^2}{2}$$

The root mean square is $\frac{A}{\sqrt{2}}$.

8. Are the following systems with or without memory, causal of noncausal?

(i) $y(t) = 2u(t)$: without memory, causal
(ii) y(t) = sin($u(t)$) : without memory, causal
(iii) $y(t) = \sin(u(t+1))$: with memory, noncausal
(iv) y(t) = $e^{t-2}u(t-2)$: with memory, causal

9. Is the system represented by $y(t) = 1/x(t)$ linear and time-invariant?

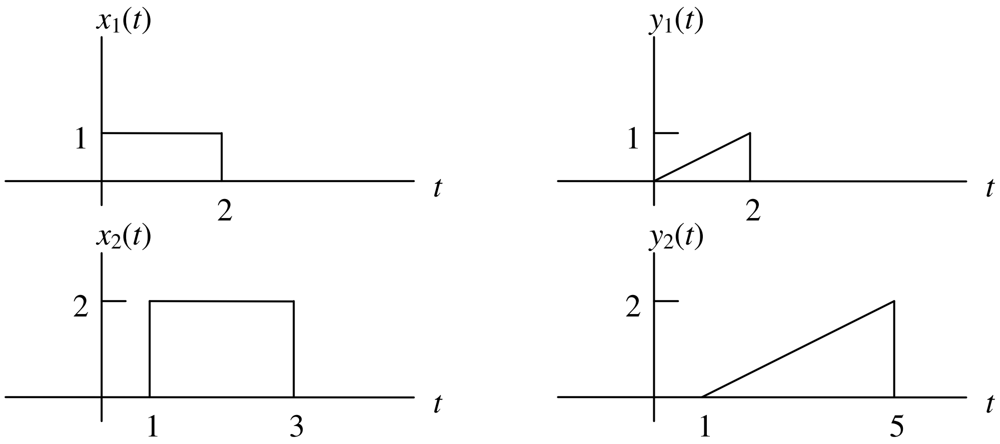
The system output-input is described by $y(t) = 1/x(t)$.
If the input is $x_1(t)$ then the output will be $y_1(t) = 1/x_1(t)$.
If the input is $x_2(t)$ then the output will be $y_2(t) = 1/x_2(t)$.

However if the input is $ax_1(t) + bx_2(t)$ then the output will be

$$\frac{1}{ax_1(t)+bx_2(t)} \neq ay_1(t)+by_2(t).$$ Therefore the system is nonlinear.

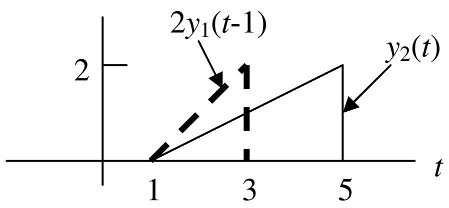
If the input is $x(t-t_o)$ then the output will be $y(t-t_o) = 1/x(t-t_o)$. Hence the system is time invariant.

10. Consider a linear system with an input-output pairs shown below.

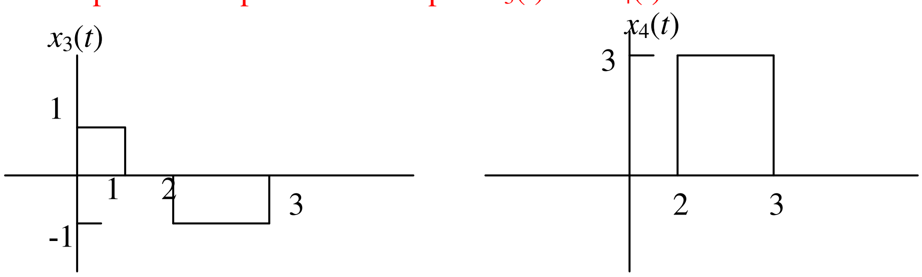


Is the system time invariant?

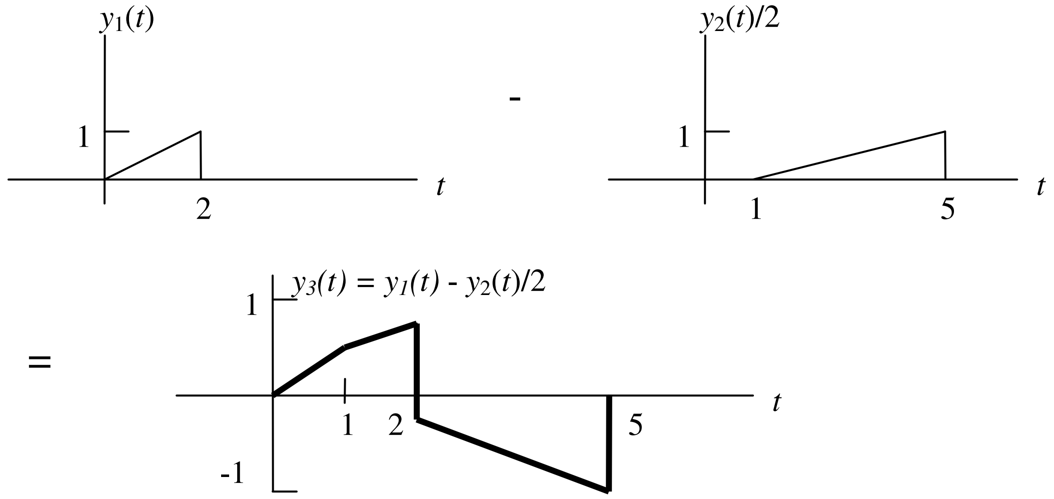
By inspection $x_2(t) = 2x_1(t-1)$ but $y_2(t) \neq 2y_1(t-1)$. Therefore the system is time varying.



Can we compute the response to the inputs $x_3(t)$ and $x_4(t)$?



The system is linear and $x_3(t) = x_1(t) - x_2(t)/2$. So we can compute the response to $x_3(t)$ as follows

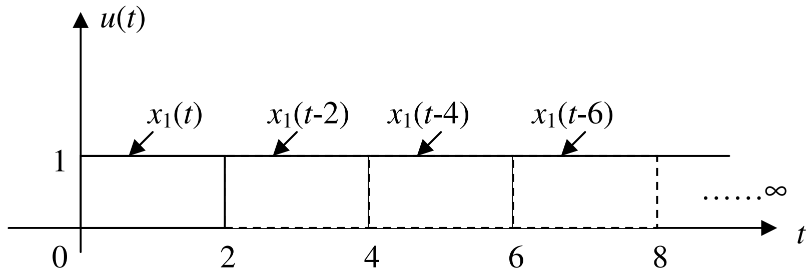


However the response of $x_4(t)$ cannot be computed since the system is time varying.

(ii) If $y_1(t)$ is the response of another system that is linear time-invariant when the input is $x_1(t)$ find the response to the unit step function.

If the input is $x(t\text{-}t_o)$ the response will be $y(t\text{-}t_o)$ since the system is linear time-invariant. A step function can be constructed by

$$u(t) = \sum_{n=1}^{\infty} x_1(t-2n).$$



The output is therefore $y(t) = \sum_{n=1}^{\infty} y_1(t-2n)$