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The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2015-16 (2.0 hours)

EEE345 Engineering Electromagnetics

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. **a.** State all four Maxwell's equations for electric and magnetic field vectors $\underline{\boldsymbol{E}}$ and $\underline{\boldsymbol{H}}$ in differential form and state what the parameters on the right hand sides mean. **(5)**

b. Consider a damped electromagnetic wave propagating along z-direction of form

$$\underline{\boldsymbol{E}}(z,t) = E_0 \cos(\omega t - kz) \exp(-\alpha z)\, \underline{\boldsymbol{a}}_x \qquad \text{(equation 1) and}$$

$$\underline{\boldsymbol{H}}(z,t) = H_0 \cos(\omega t - kz - \varphi) \exp(-\alpha z)\, \underline{\boldsymbol{a}}_y \qquad \text{(equation 2)}$$

where $\underline{\boldsymbol{a}}_x$ and $\underline{\boldsymbol{a}}_y$ are unit vectors along x- and y-directions, respectively.

(i) Define the Poynting vector.

(ii) Calculate its time averaged value.

(iii) What physical meaning does the phase angle φ have? **(5)**

c. Using Gauss' Law and the following relationship for any vectors $\underline{\boldsymbol{A}}$, $\underline{\boldsymbol{B}}$

$$\text{div}\,(\underline{\boldsymbol{A}} \times \underline{\boldsymbol{B}}) = \underline{\boldsymbol{B}}\ \text{rot}\underline{\boldsymbol{A}} - \underline{\boldsymbol{A}}\ \text{rot}\underline{\boldsymbol{B}} \qquad \text{(equation 3)}$$

integrate the Poynting vector $\underline{\boldsymbol{P}}$ over a closed surface S and interpret the physical meaning of all three terms obtained. This is called Poyntings' theorem. **(10)**

2. **a.** The voltage as a function of position, x, and time, t, along a transmission line can generally be written as a superposition of forward and backward travelling waves in the form:

$$V(x,t)=V_0^+ \exp\,[j(\omega t - k'x)] + V_0^- \exp[j(\omega t + k'x)] \qquad \text{(equation 4)}$$

where ω is the angular frequency and k' is a complex propagation constant. For a lossy transmission line with impedance Z^* per unit length and admittance Y^* per unit length it can be shown that

$$k'^{\,2} = -Z^*Y^* \qquad \text{(equation 5).}$$

(i) Use the definitions of Z^* and Y^* in terms of the standard parameters R^*, G^*, L^* and C^* to derive an exact expression for k' in terms of these parameters.

(ii) Interpret the physical meaning of all three terms you get.

(iii) For the lossless case, calculate the characteristic impedance $Z_0=Z^*/(jk')$.

(iv) For the lossless case, calculate the phase velocity of the signal on the line. **(9)**

b. A 50MHz signal is fed into a computer's printed circuit board that can be described as a lossy transmission line with the characteristics of L^*=1mH/m, C^*=1nF/m, R^*=1Ω/m, G^*=0.001/(Ω m). Use your above solution for $k'^{\,2}$ to calculate over what length the signal can be transferred so that at least 90% of the voltage of the input signal arrives. Compare this to what you would get for 50kHz. **(6)**

c. For a lossless coaxial cable of outer diameter R and inner diameter r one has a capacitance per unit length of

$$C^*=2\pi\varepsilon_0\varepsilon_r/\ln(R/r) \qquad \text{(equation 6)}$$

and an inductance per unit length of

$$L^*=\mu_0\mu_r \ln(R/r)/(2\pi) \qquad \text{(equation 7).}$$

(i) Determine the phase velocity.

(ii) Derive the refractive index n of a dielectric medium wherein the velocity of light is the same as the above and compare to the definition of optics. **(5)**

3. **a.** Consider a plate capacitor of width w, length l, distance d between the plates that is filled with a dielectric of relative permittivity ε_r.

(i) Using Coulomb's Law and Gauss' Law, calculate the magnitude D of the dielectric flux between the plates as a function of the charge Q_{free} on the plates. Give a physical interpretation of your result.

(ii) Perform the same calculation for the magnitude E of the electric field.

(iii) From the above, calculate the magnitude of polarisation in the dielectric as

$$P = D - \varepsilon_0 E \qquad \text{(equation 8)}$$

and give a physical interpretation of your result. **(12)**

b. Consider a p-i-n diode where p- and n-doped regions are 100nm wide and the intrinsic region in-between is 500nm wide.

(i) State Poisson's equations for the one-dimensional case along x-direction.

(ii) Provide a sketch of the x-dependence of the free charge $\rho(x)$, the electric field strength $E(x)$ and the potential $V(x)$ for the above p-i-n diode. Plot the graphs under each other, with the p-region to the left, so the interfaces between the differently doped regions are vertically aligned to each other.

(iii) Compare the built-in voltage qualitatively to that of a p-n diode without intrinsic layer and use the standard model of a plate capacitor to explain your result. **(8)**

4. **a.** Consider light transversing from a medium with refractive index n_1 to another, denser one with refractive index n_2, as sketched in figure 1. Assume the speed of light in medium i is given by $v_i = c/n_i$ where c is the speed of light in vacuum. Derive Snell's Law of refraction under the assumption that the light travels from point A to point C along the fastest possible route.

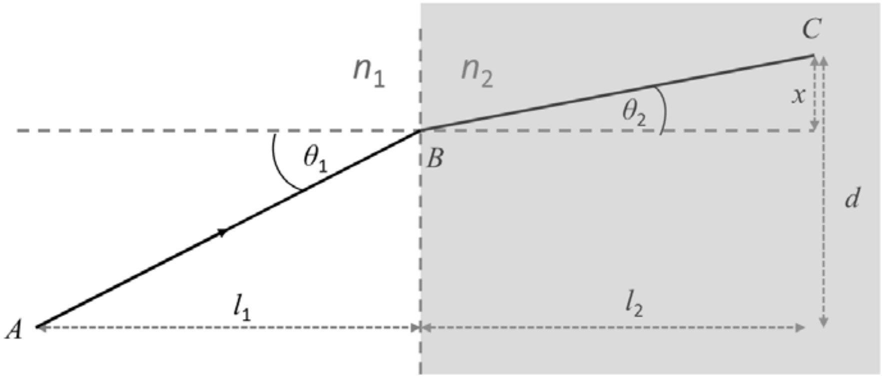


Figure 1 **(6)**

b. Consider the surface of a dielectric material with refractive index $n_1>1$, above which is air ($n_2=1$), as sketched in figure 2 below. Light shines at the interface under an angle θ_1 within the (x,z) plane where the x-direction points to the right and z upwards. Neglect the reflected wave. Assume another wave is generated in point A of the general form

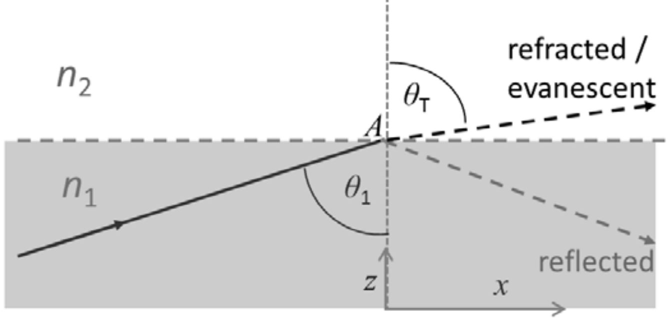
$$\underline{\boldsymbol{E}}_A=\underline{\boldsymbol{E}}_0 \exp \mathrm{j}(\omega t-\underline{\boldsymbol{K}}\cdot\underline{\boldsymbol{r}}) \qquad \text{(equation 9)}$$

where $\underline{\boldsymbol{K}}=(K_x,K_yK_z) = k_T(\sin\theta_T, 0, -\cos\theta_T)$, $\underline{\boldsymbol{r}}=(x,y,z)$ and ω and t have their usual meaning where Snell's law determines the relationship between θ_1 and θ_T.

(i) Calculate $\underline{\boldsymbol{E}}_A$ for the case that $\sin\theta_1 = n_2/n_1$ and explain your result physically.

(ii) Calculate $\underline{\boldsymbol{E}}_A$ for the case that $\sin\theta_1 > n_2/n_1$ and explain your result physically.

(iii)



(iv) Figure 2 **(8)**

c. Consider a coated optical fibre where the inner core material has a larger refractive index (n_2=1.5) than the outer cladding (n_3=1.4). Assume light falls onto the fibre cross-section from outside (air, n_1=1) under an angle θ_1 to the long axis of the fibre. Sketch what happens for sufficiently large θ_1 at the interface between core and cladding and calculate the so-called numerical aperture of the fibre which is defined as the maximal $\sin\theta_1$ value for which light that enters the fibre stays confined within it. **(6)**

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