EEE105 Tutorial Question Set 2
Solutions

1. Poisson's equation says

$$\frac{d^2V}{dx^2} = -\rho/\varepsilon$$

Remembering that dV/dx = -E. So Poisson's equation above is equal to -dE/dx

We need to know ρ the charge density in the sheet. We know the total amount of charge = Q in the left hand plate and the area = A and thickness = t, so ρ = Q/At. So the rate of change of field within the sheet is

$$\frac{dE}{dx} = \frac{Q}{At\varepsilon}$$

Q, A, t, ε are constant, therefore E equals this constant times x. So the *total change* in field across the sheet is the value of dE/dx multiplied by the sheet thickness. (Alternatively one can integrate the equation with respect to x over the limits from 0 to t and obtain the same result). So if the field on the left hand side of the plate is zero, (this is our so-called "boundary condition") that on the right must be

$$E = \frac{Q}{A\varepsilon}$$

2. This is just the same sort of problem as question 1. In this case we know that the field on the right hand side of the left hand plate is the result obtained in question 1. We then apply Poisson's equation between the plates where the charge density is zero, and therefore as dE/dx=0, the field, E must be constant between the plates. We have the boundary condition that at the right hand side of the left hand plate E=Q/Aε. Hence for all points between the plates:

$$E = \frac{Q}{A\varepsilon}$$

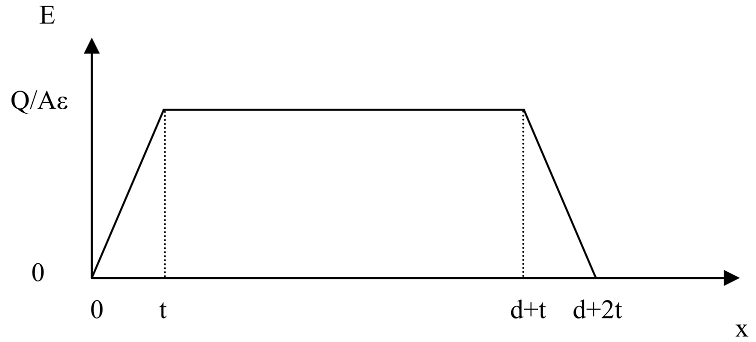
A key point to note as an aside here is that the value of E derived in question 1 is *independent of distance from the plate* in a charge free region. This is true for any charged surface which is infinite and will be properly derived in EEE220 next year. For the assumption to hold all we need is that the dimensions of the plate giving its area are much larger than distance we are away from it.

3. To do this we need to calculate the difference in electric field in passing through the right hand plate. The simple way to do this is by considering the answer in question 1 the difference in field here must be -Q/Aε as the plate is identical in all respects except the charge present on it.

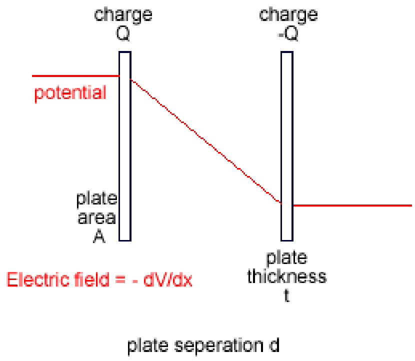
More formally we can reuse Poisson's Equation again and integrate with respect to x across the limits (d+t) to (d+2t) and obtain the same result.

Now we need a boundary condition. From question 2 the field on the the left hand side of the right hand plate must be Q/Aε. Hence using the difference in field calculated here, the field on the right hand side of the right hand plate must be 0.

Plotting the field against distance x as we pass through the plates gives:



4. If the electric field is E, then recall dV/dx=-E, therefore the *difference* in potential will be E times the plate separation, i.e. Ed =Qd/Aε. We are being a bit cavalier with the signs here, so we need to be sure we understand hat the left hand plate will be at the higher voltage, as the potential must be falling with x between the plates. Graphically the variation in potential will roughly look as follows (assuming t<<d):



5. These questions were designed to make you familiar with Poisson's equation, but since it should be obvious that what we have been looking at a parallel plate capacitor, we might as well show how easy it is to derive the relation for capacitance of such a capacitor.
C=Q/V (by our definition of capacitance). We calculated V in part (5) so

$$C = \frac{Q}{V} = \frac{Q}{Qd/\varepsilon A} = \frac{\varepsilon A}{d}$$