Data Provided: List of Useful Equations (attached at the end of the paper)

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2014-15 (2.0 hours)

EEE218 Electric Circuits 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1.

a. A small hydro electric generator, which may be assumed to be a constant $240V_{rms}$ supply, provides power to a farm via underground cables. The farm draws a current of $120A_{rms}$ at a unity power-factor and the distance between generator and the farm is 800m.

(i) If the voltage at the farm must not be less than $220V_{rms}$ estimate the minimum cross-sectional area of the cables. Assume the resistivity of the copper conductors is $1.8\times10^{-8}\Omega$m. (*Hint: remember that there is both a supply and return conductor in the circuit*). **(4)**

(ii) Calculate the power consumed by the farm, the power supplied by the generator and the power dissipated in the cables. Hence calculate the efficiency of the system. **(3)**

b. An existing factory is connected to a $800V_{rms}$, 50Hz supply and consumes 100kW of power at a 0.85 power-factor lagging.

(i) Calculate the kVA rating of the factory. **(1)**

(ii) Calculate the reactive power in kVAr. **(1)**

(iii) Calculate the magnitude of the current drawn from the supply. **(1)**

c. The existing factory is enlarged and the following additional loads are added:

- Process heaters rated at 20kW (assume these are purely resistive)
- A motor load of 140kVA at 0.6 power-factor lagging.

(i) Calculate the new kVA rating of the factory. **(4)**

(ii) Calculate the new overall power-factor and state whether it is lagging or leading. **(2)**

(iii) It is decided to connect a capacitor in parallel with all the other loads (i.e factory, process heaters and the motor) to correct the power-factor to unity. Find kVAr rating of the capacitor, the magnitude of the current drawn by the capacitor, and its value in Farads. **(3)**

(iv) What is the peak voltage the capacitor must withstand under normal operating conditions. **(1)**

2.

a. The coil of an electromagnet can be modelled as a resistance, R = 12Ω in series with an inductance, L = 0.04H. The electromagnet is connected to a $300V_{rms}$, 50Hz sinusoidal supply, V_S, as shown in Figure 2.1.

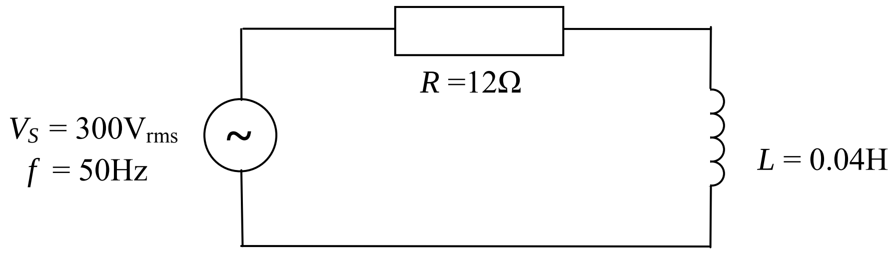


Figure 2.1

(i) Calculate the magnitude and phase angle of the impedance of the electromagnet. **(2)**

(ii) Hence calculate the magnitude and phase of the current flowing in the coil. **(1)**

(iii) What is the power drawn from the supply and the power-factor of the circuit? **(2)**

b. A capacitor, C, is then placed in series with the coil of the electromagnet to form a series resonant circuit.

(i) Write down an expression for the total impedance, Z, of the circuit in terms of R, L, and C, and state the condition for resonance. Write down an expression for the resonant frequency (in Hz) of the circuit in terms of its components. **(3)**

(ii) Sketch the variations of the magnitude of the total impedance, Z, and the current, I, with frequency, f, over the frequency range $f << f_{resonant}$ to $f >> f_{resonant}$. **(2)**

(iii) Sketch a voltage phasor diagram for the circuit at resonance showing the relative direction of the voltages across each component. (*Actual values of voltage are not required*). **(2)**

(iv) What value of capacitance is required to give a maximum current flow in the coil of the electromagnet and what is the power dissipated under these conditions? **(3)**

c. If the frequency of the supply increases to 60Hz whilst the voltage remains at $300V_{rms}$ calculate the new voltage across the coil of the electromagnet. (Assume the capacitor is still connected in the circuit and has the value of capacitance calculated in part **b (iv)**). **(5)**

3.

a. A 12V car battery is connected to the starter motor. When the motor is running it draws a current of 150A and the voltage across the terminals of the battery drops to 10V. Calculate the internal resistance of the battery, the power dissipated within the battery and the overall system efficiency. **(3)**

b. The same battery is then used to power a heating element (starter motor no longer connected). When this is first switched on the current flowing is 10A. After a long period of time the current has dropped to 7A and remains at this level.

(i) Calculate the final temperature of the heating element if initially it was at an ambient temperature of 20°C. You may assume the following: **(6)**

Temp. coefficient = $\alpha_0 = 12.5 \times 10^{-3}$ /°C
$R_T = R_0\,(1+\alpha_0 T)$
R_T = Resistance at temperature T°C
R_0 = Resistance at temperature 0°C

(ii) Calculate the power dissipated in the heating element at the final temperature, and hence calculate the system efficiency at this temperature. **(2)**

c. An ideal transformer has a turns ratio of 1:4 (primary:secondary) and an input voltage of $100V_{rms}$ at 50Hz.

(i) Calculate the secondary voltage, the current in the primary winding and the power dissipated if a resistive load of 25Ω is connected across the secondary. **(3)**

(ii) Calculate the current in the primary winding and the power dissipated if the load across the secondary now comprises a resistance of 30Ω in series with an inductance of 100mH. **(3)**

(iii) For the case described in part **(ii)** above, what would be the input power factor and the required VA rating of the transformer? **(1)**

(iv) If the maximum core flux of the transformer is 4mWb calculate the actual number of turns on the primary winding. **(1)**

(v) If the transformer were to be operated in a country where the supply frequency is 60Hz, what is the maximum permissible supply voltage without the maximum core flux of 4mWb being exceeded? **(1)**

4.

a. For the network shown in Figure 4.1, find the value of the current through the 5Ω resistance using the method of superposition. Indicate the direction of the current. **(6)**

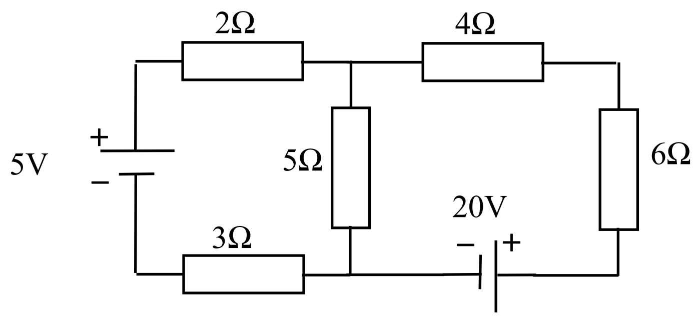


Figure 4.1

b. A constant current source of 5A is added to the network in parallel with the 6Ω resistor, as shown in Figure 4.2. Calculate the new current through the 5Ω resistor. **(5)**

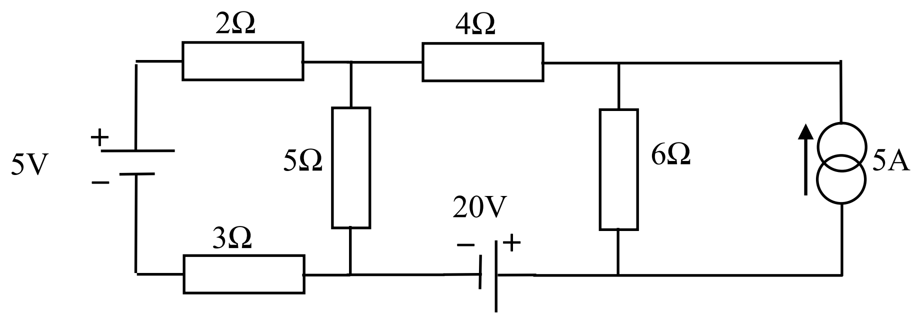


Figure 4.2

c. The network of Figure 4.1 is to be used as a source for a load resistor of 10Ω, as shown in Figure 4.3. Derive the Thevenin equivalent circuit for the source and hence calculate the power dissipated in the load resistor. **(6)**

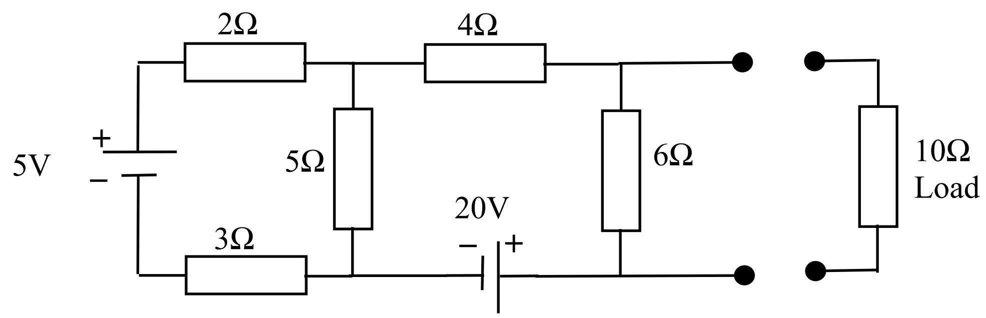


Figure 4.3

d. Calculate the Norton equivalent circuit for the source network shown in Figure 4.3 (i.e. not including the load resistor). **(3)**

KM / MH

USEFUL EQUATIONS – EEE140 / EEE218

Electric Circuits

Resistance (*R*) – units Ohms (Ω)

Resistors in series $R_{TOT} = R_1 + R_2 + R_3 + \cdots R_n$

Resistors in parallel $\frac{1}{R_{TOT}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \frac{1}{R_n}$

Resistance (Ohms law) $R = \frac{V}{I}$

Resistance $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$

(*l=length, m; A = cross-sectional area, m^2; ρ = resistivity, Ω m; σ = conductivity, S/m*)

Temperature dependence of resistors $R_{T1} = R_0(1+\alpha_0 T_1)$

α_0 = *temperature coefficient of resistance*
R_0 = *Resistance (Ω) at 0°C*
R_{T1} = *Resistance (Ω) at T_1°C*
T_1 = *Temperature in °C*
For temperatures T_1 and T_2 use ratio

$$\frac{R_{T1}}{R_{T2}} = \frac{(1+\alpha_0 T_1)}{(1+\alpha_0 T_2)}$$

Voltage across a resistor $V_R = IR$

Power dissipated in a resistor $P = I^2 R = \frac{V^2}{R} = V \cdot I$

Energy dissipated in a resistor $E = I^2 Rt = \frac{V^2 t}{R} = V \cdot I \cdot t$

Capacitance (*C*) – units Farads (F)

Charge (*Q*) $Q = I \cdot t$ (Constant current , I)

$Q = \int_0^t i(t)dt$ (Time varying current, *i(t)*)

$Q = CV$ (Capacitance × Voltage)

Capacitors in series $\frac{1}{C_{TOT}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_n}$

Capacitors in parallel $C_{TOT} = C_1 + C_2 + C_3 + \cdots C_n$

Voltage across a capacitor $V_c = \frac{Q}{C}$

Energy stored in a capacitor $E = \frac{1}{2} CV^2$

Inductance (*L*) – units Henrys (H)

Inductors in series $L_{TOT} = L_1 + L_2 + L_3 + \cdots L_n$

Inductors in parallel $\frac{1}{L_{TOT}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots \frac{1}{L_n}$

Voltage across an inductor $V_L = L\frac{dI}{dt}$

Energy stored in an inductor $E = \frac{1}{2} LI^2$

A.C. Circuits

Power dissipated in a resistance $P = I_{rms}^2 R = \dfrac{V_{rms}^2}{R}$

For other circuits having capacitance and/or inductance, there is a phase shift, ϕ, between the current and voltage waveforms.

Real Power (*P*) (*Watts*) $P = V_{rms} I_{rms} \times \cos\phi$

Reactive Power (*Q*) (*VARs*) $Q = V_{rms} I_{rms} \times \sin\phi$

Power factor (p.f. or cos ϕ) $0 < \cos\phi < 1$

Power = Energy s^{-1} $Watts(W) = Js^{-1}$

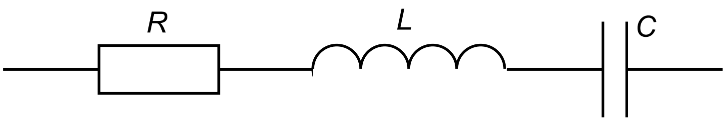
Capacitive reactance $X_C = \dfrac{1}{j\omega C} = \dfrac{1}{j2\pi f\, C} = \dfrac{-j}{\omega C} = \dfrac{-j}{2\pi f\, C}$

(ω = electrical frequency in rad/s (= $2\pi f$); f is the electrical frequency in Hz)

Inductive reactance $X_L = j\omega L = 2\pi f L$

(ω = electrical frequency in rad/s (= $2\pi f$); f is the electrical frequency in Hz)

Series Resonant Circuit

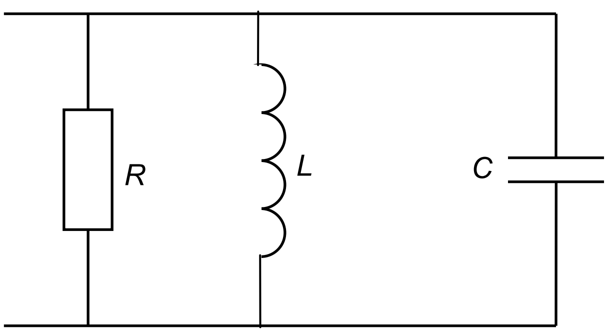


At resonance $X_C = X_L$ and $Z = R$

Frequency for resonance $\omega_r = \dfrac{1}{\sqrt{LC}}$ or $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

Q factor $Q = \dfrac{\omega_r L}{R} = \dfrac{1}{\omega_r CR} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

Parallel Resonant Circuit



At resonance $X_C = X_L$ and $Z = R$

Frequency for resonance $\omega_r = \dfrac{1}{\sqrt{LC}}$ or $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

Q factor $Q = \dfrac{R}{\omega_r L} = \omega_r CR = R\sqrt{\dfrac{C}{L}}$

Transient Circuits

Current growth in an inductive circuit containing inductance and resistance:

Instantaneous current $i = I_0(1 - e^{-t/\tau})$ where $\tau = L/R$

Current decay in an inductive circuit containing inductance and resistance:

Instantaneous current $i = I_0 e^{-t/\tau}$ where $\tau = L/R$

Charging a capacitor through a resistor:

Instantaneous voltage $v = V_0(1 - e^{-t/\tau})$ where $\tau = RC$

Instantaneous current $i = I_0 e^{-t/\tau}$ where $\tau = RC$

Disharging a capacitor through a resistor:

Instantaneous voltage $v = V_0 e^{-t/\tau}$ where $\tau = RC$

Instantaneous current $i = -I_0 e^{-t/\tau}$ where $\tau = RC$

Magnetic Circuits

Reluctance (S) – units H^{-1} $$S = \frac{l}{\mu_0 \mu r A}$$

(*l=length, m; A = cross-sectional area, m^2; μ_0 = permeability of free space (Hm^{-1}); μ_r = relative permeability)*

Inductance (L) $$L = \frac{N^2}{S}$$

(*N = number of turns on the coil*)

Flux density (B) – units Tesla (T) $$B = \mu_0 \mu_r H = \frac{\phi}{A}$$

(H = magnetic field strength(A/m); φ = flux (Wb))

MagnetoMotive Force – MMF $$F = H \cdot l = N \cdot I = \phi S$$

Induced EMF (E) – units Volts (V) $$E = N\frac{d\phi}{dt}$$

Transformers (ideal)

Voltage ratio $$\frac{V_{in}}{V_{out}} = \frac{N_1}{N_2} = turns \quad ratio$$

Current ratio $$\frac{I_{in}}{I_{out}} = \frac{N_2}{N_1} = \frac{1}{turns \quad ratio}$$

Impedance ratio $$\frac{Z_{in}}{Z_{out}} = \left(\frac{N_1}{N_2}\right)^2 = (turns \quad ratio)^2$$

V_{in} = voltage across primary winding; *I_{in} = current through primary winding;*
V_{out} = voltage across secondary winding; *I_{out} = current through secondary winding;*

Induced voltage $$\mathrm{V_{rms}} = 4.44\mathrm{f} \cdot \mathrm{N} \cdot \phi_{\max}$$

(ϕ_{MAX} = maximum flux in the transformer core (Wb); f = frequency (Hz))

Mechanics

Mechanical Power (W) $P_{mech} = \omega_{mech} \cdot T$

(T = torque (Nm); ω_{mech} = rotational speed (rad/s))

Rotational speed (rad/s) $\omega_{mech} = \frac{2\pi}{60} \cdot n$

(n = speed in revs per minute)

Torque $T = F \cdot r$

(F = force(Nm); r = radius (m))

Gearbox (no losses) $\omega_{in} T_{in} = \omega_{out} T_{out}$

DC motors

Force on a current carrying conductor $F = B \cdot I \cdot l$

(B = flux density (T); I = current (A); l = length (m))

DC motor armature voltage $V_A = E_A + I_A R_A$

(E_A = induced emf (V); I_A = armature current (A); R_A = armature resistance (Ω))

Induced voltage, E_A is proportional to the speed of rotation, ω (rad/s), and the flux, ϕ (Wb).
Torque, T is proportional to the armature current, I (A) and the flux, ϕ (Wb).

For a wound field machine:

Induced emf (V) $E_A = K I_F \omega$

Torque (Nm) $T = K I_F I_A$

(I_F = field current (A); K = constant)

For a permanent magnet machine:

Induced emf (V) $E_A = K_E \cdot \omega$

Torque (Nm) $T = K_T \cdot I_A$

(K_E (V/rad/s) and K_T (Nm/A) are constants with the same numerical values)