

## Solutions

1(a) In a semiconductor and insulator the bonding is normally fairly similar. Essentially all the outer electrons in the material are tied up in the bonds between the atoms in the material. In order for electrons to conduct they need to escape from the bonds and they need to gain energy to do this. In a semiconductor the required energy gain is not too big and hence a small fraction of the electrons are able to escape and conduct electricity at any time. In an insulator the required energy gain is much larger and hence very few if any electrons can escape their bond at any instance in time and hence the conductivity is much lower.

1(b) Doping is the process where impurities are deliberately added to a semiconductor. These impurities either have one more or one less electron in their outer bonding state than for the semiconducting material. For example Si has four outer electrons, so it can be doped using an element with five or three outer electrons. If the doping element has more outer electrons then each dopant atom gives one extra electron which cannot fit in a bond and is hence free to wander around the material. Similarly a dopant with less outer electrons leaves a hole in a bond and this hole can also wander around the material. In both cases the density of charge carriers in the material is increased and hence the conductivity is also increased.

$$1(c) \sigma = nq\mu_e + pq\mu_h = n_i q (\mu_e + \mu_h)$$

$$\sigma = 1.45 \times 10^{16} \times 1.6 \times 10^{-19} (0.12 + 0.045)$$

$$\sigma = 3.8 \times 10^{-2} \Omega^{-1} \text{m}^{-1}$$

1(d) Arsenic is a donor in Si so at room temperature we can expect the electron density to be equal to the doping concentration so  $n = 1 \times 10^{21} \text{ m}^{-3}$ .

$$n_i^2 = pn$$

$$\text{so } p = \frac{n_i^2}{n}$$

$$p = \frac{(1.45 \times 10^{16})^2}{1 \times 10^{21}}$$

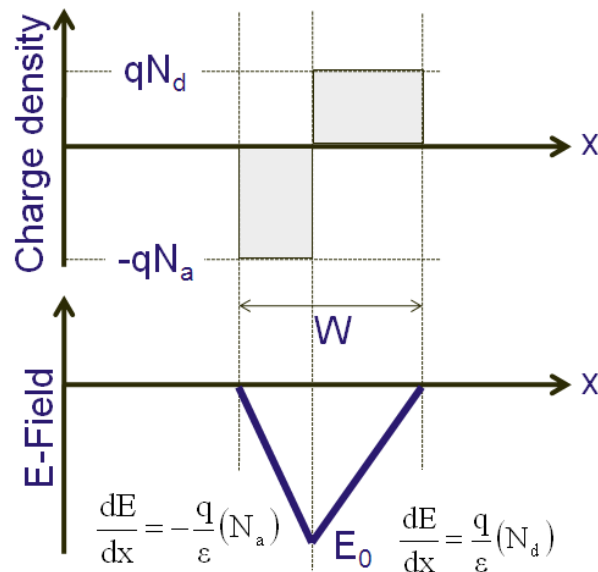
$$p = 2.1 \times 10^{11} \text{ m}^{-3}$$

$$1(e) \quad \tau = \frac{\mu m^*}{q} = \frac{0.12 \times 0.97 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}} = 6.6 \times 10^{-13} \text{ s} = 660 \text{ fs}$$

- i) With increasing doping concentration the density of ionized donor atoms will increase which we would expect to result in more frequent scattering events, reducing this average scattering time.
- ii) With increasing temperature the density of phonons will increase, which will tend to increase the probability of phonon scattering, again reducing the average scattering time.

2(a) When a piece of p-type material is brought together with a piece of n-type semiconductor then there will be a concentration gradient of electrons and holes in the junction region. As a result electrons will diffuse into the p-type material and holes into the n-type material, where they will recombine with the majority carriers. However, as the electrons and holes diffuse they will leave behind their donor and acceptor atoms, which are ionised. These atoms cannot move as they are bonded into the crystal. As the diffusion progresses so the number of exposed donors and acceptors increase. This causes an increasing electric field in the junction region which acts to oppose further electron and hole diffusion. Eventually the field is sufficiently strong to set up a drift current that exactly opposes the diffusion current of electrons and holes and the diode reaches equilibrium. The depletion region is the region where all the electrons and holes have diffused and recombined, leaving only their exposed donor and acceptors behind. The layer acts like an insulator as all the free charge carriers have recombined away.

2(b)



Can determine  $E_0$  by integration of

$$\frac{dE}{dx} = -\frac{q}{\epsilon}(N_a) \quad \text{or} \quad \frac{dE}{dx} = \frac{q}{\epsilon}(N_d)$$

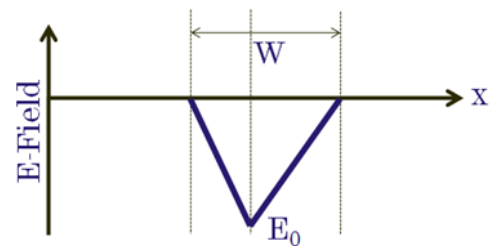
With correct limits e.g.

$$\int_0^{E_0} dE = -\frac{q}{\epsilon} N_a \int_{-x_{p0}}^0 dx \quad \text{for} \quad -x_{p0} < x < 0$$

$$\int_{E_0}^0 dE = \frac{q}{\epsilon} N_d \int_0^{x_{n0}} dx \quad \text{for} \quad 0 < x < x_{n0}$$

yielding

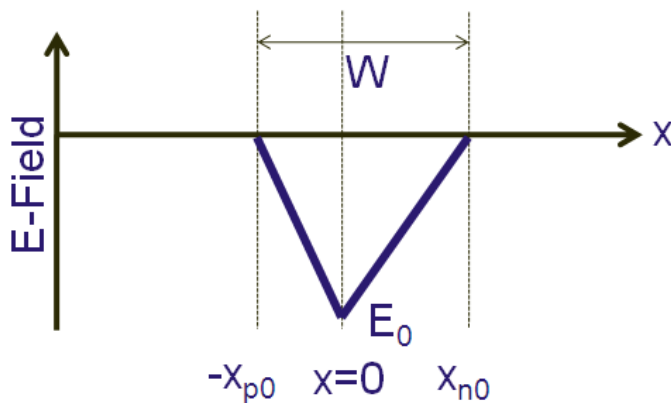
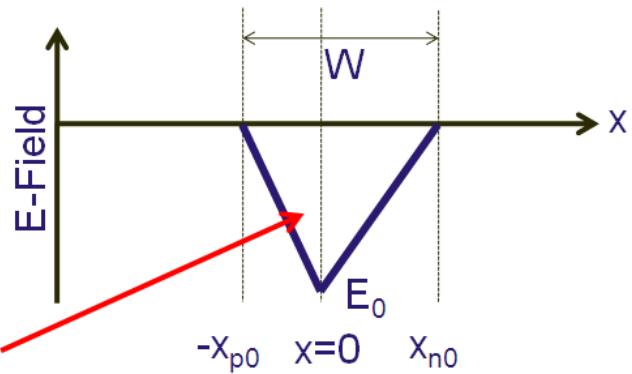
$$E_0 = -\frac{q}{\epsilon} N_d x_{n0} = -\frac{q}{\epsilon} N_a x_{p0}$$



$$E = -\frac{dV}{dx} \text{ or alternatively,}$$

$$-V_0 = \int_{-x_{p0}}^{x_{n0}} E(x) dx$$

So  $V_0$  is area of this triangle



$$V_0 = -\frac{1}{2} E_0 W$$

e.g.

$$V_0 = \frac{1}{2} \frac{q}{\epsilon} N_d x_{n0} W$$

$$x_{n0} N_d = x_{p0} N_a$$

$$V_0 = \frac{1}{2} \frac{q}{\epsilon} N_a x_{p0} W$$

remembering  $x_{p0} N_a = x_{n0} N_d$

and  $W = x_{p0} + x_{n0}$

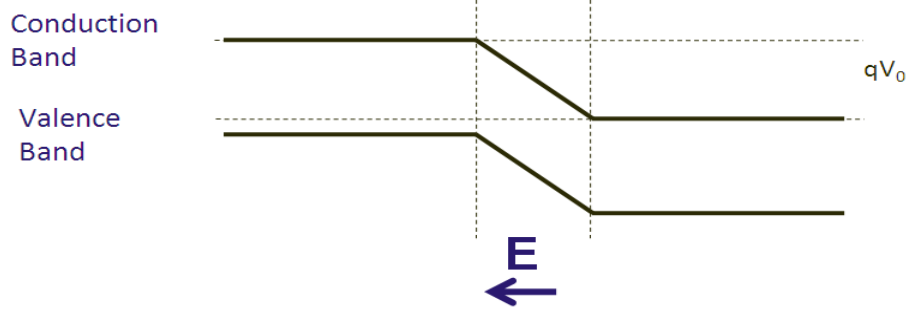
so  $(W - x_{n0}) N_a = x_{n0} N_d$

$$x_{n0} = \frac{W N_a}{(N_d + N_a)}$$

From, e.g.  $V_0 = \frac{1}{2} \frac{q}{\epsilon} N_d x_{n0} W = \frac{1}{2} \frac{q}{\epsilon} \frac{N_a N_d}{N_a + N_d} W^2$

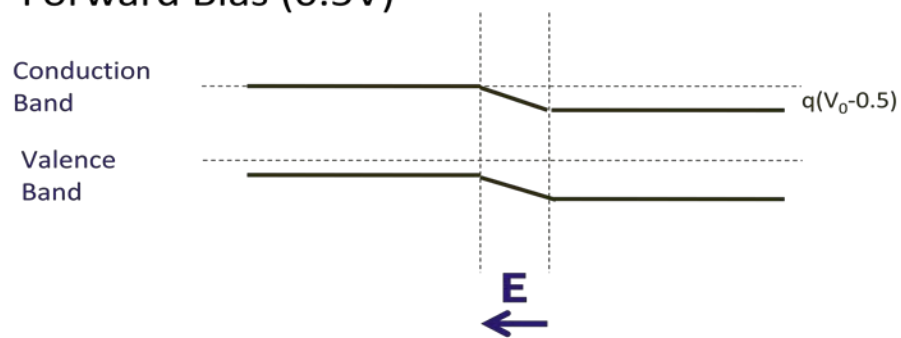
$$W = \left[ \frac{2V_0 \epsilon}{q} \left( \frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} = \left[ \frac{2\epsilon(V_0 - V_f)}{q} \left( \frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}$$

## Zero Bias



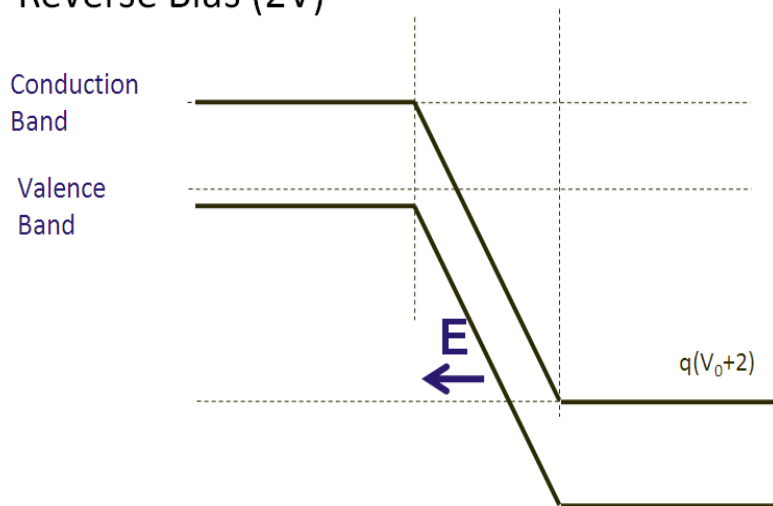
2(c)

## Forward Bias (0.5V)

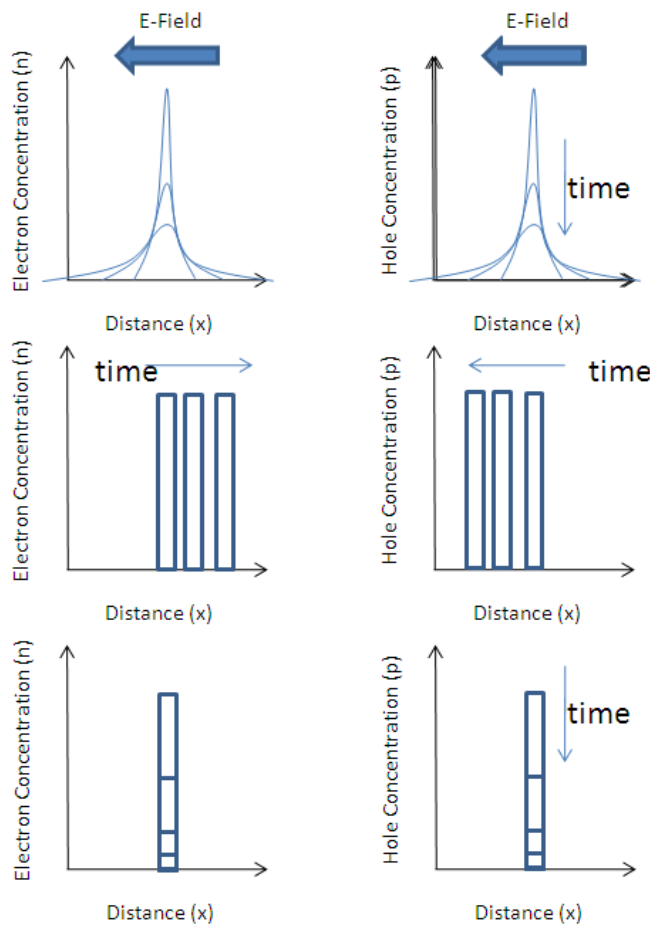


Depletion region reduces in width

## Reverse Bias (2V)



Depletion region increases in width



3) (a) In the figures the behaviour of the electrons is on the left and the holes on the right in each case. As specified in the exam question the direction of the electric field is from right to left.

#### i) Diffusion

Diffusion will cause the electrons and holes to spread out to both sides equally with time as indicated below. There may be some difference in the rate of diffusion of the electrons compared to that of the holes depending on the relative carrier mobilities. As there is no recombination the area under the curve should remain the same with time and the number of carriers does not change.

#### ii) Drift

Drift will move the electrons to the right in the opposite direction to the field, while holes will move to the left with the electric field. In the absence of Diffusion we would expect the carrier profile to remain unchanged.

#### iii) Recombination

As electrons and holes recombine the number of both the electrons and holes will decrease exponentially as the recombination rate is a function of the minority carrier density. The overall shape of the profile will remain unchanged and it will not move with respect

to  $x$  as there is no diffusion or drift involved.

$$(b) \quad J_e^{\text{total}}(x) = J_e^{\text{drift}} + J_e^{\text{diffusion}} = q\mu_e E_x n + qD_e \frac{dn}{dx}$$

$$E_x = -\frac{D_e}{n\mu_e} \frac{dn}{dx} \quad D_{e,h} = \frac{k_B T \mu_{e,h}}{q}$$

$$\text{So} \quad E_x = -\frac{K_B T}{q} \frac{dn/dx}{n}$$

$$\text{We are told} \quad n = 10^{20}(1 + Ax)$$

$$\frac{dn}{dx} = 10^{20} A$$

so

$$E_x = -\frac{K_B T}{q} \frac{A}{1 + Ax}$$

The origin of this field is that as the free electrons diffuse, space charge (ionized donors) is revealed. The electric field generated is due to this space charge.

4) (a)

i) In order to operate as a transistor one p-n junction must act to inject minority carriers of one type into the other p-n junction.

ii) Base transport factor is the fraction of injected current into the base which successfully transits the base to the collector.

ii) Base transport factor can be increased by reducing the width of the base and reducing the doping concentration of the base.

iv) Current gain is obtained by introducing current to the base. Due to the rapid transit time of the emitter current in the base, and the need to maintain charge neutrality in the base a large emitter current is required. The collector current is therefore an amplified version of the base current.

v) The Early effect (Base narrowing, or base modulation) is due to the base being reduced in thickness as the reverse bias between base and collector is increased (due to an increase in depletion region width). This results in the observation of an increased base transport factor and increased current amplification factor.

(b) i)

In the device we need to know what fraction of the excess holes injected into the base will travel the 1  $\mu\text{m}$  across the base without recombining.

$$\hat{p} = \hat{p}_0 \exp\left(\frac{-x}{L_h}\right)$$

Hence:

$$\frac{\hat{p}(x)}{\hat{p}_0} = \exp\left(\frac{-x}{L_h}\right)$$

Substituting we get

$$\frac{\hat{p}(x)}{\hat{p}_0} = \exp\left(\frac{-0.7}{15}\right) = 0.954$$

ii)

This part requires us to remember that the ratio of electron to hole current is approximately given by the ratio of conductivities in the material.

We can calculate the conductivity of the emitter and base material using

$$\sigma_p = pq\mu_h = 1.2 \times 10^{26} \times 1.6 \times 10^{-19} \times 0.045 = 8.64 \times 10^5 \Omega^{-1}\text{m}^{-1}$$

$$\sigma_n = nq\mu_e = 1 \times 10^{24} \times 1.6 \times 10^{-19} \times 0.12 = 1.92 \times 10^4$$

Thus the electron current will 2.2% the size of the hole current. As the hole current is much larger we can say that the electron current will be 2.2% of the total current and hence the hole current will be 97.8% of the total current

The emitter injection efficiency is thus 0.978.