Q1

$v_1 = 212 \sin(\omega t + 45) \Rightarrow \mathbf{212\angle 45}$

$v_2 = 141 \sin(\omega t - 90) \Rightarrow \mathbf{141\angle -90}$

$v_3 = 127 \cos(\omega t + \pi/6) = 127 \sin(\omega t + \pi/6 + \pi/2) \Rightarrow \mathbf{127\angle 120}$

$v_4 = 85 \cos(\omega t - 45) = 85 \sin(\omega t - 45 + 90) \Rightarrow \mathbf{85\angle 45}$

$v_5 = 141 \sin(\omega t + 180) \Rightarrow \mathbf{141\angle 180}$

$v_6 = 100 \cos(\omega t - \pi/3) = 100 \sin(\omega t - \pi/3 + \pi/2) \Rightarrow \mathbf{100\angle 30}$

notice that in each case the ωt is understood but not explicitly stated. π radians $= 180°$

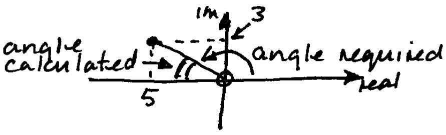
Q2 $(2-j2)$ $r = \sqrt{2^2+2^2} = \sqrt{8} = 2.83$ $\phi = \tan^{-1}\frac{-2}{2} = -45°$

$\therefore$ $\mathbf{2.83\angle -45}$

$(3+j8)$ $r = \sqrt{3^2+8^2} = \sqrt{73} = 8.54$ $\phi = \tan^{-1}\frac{8}{3} = 69.4°$

$\therefore$ $\mathbf{8.54\angle 69.4}$

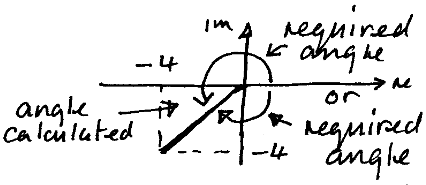
$(-5+j3)$ $r = \sqrt{5^2+3^2} = \sqrt{34} = 5.83$ $\phi = \tan^{-1}\frac{3}{-5} = -31$



This phase shift is in the -ve real area so angle w.r.t. positive real $= 180° + (-31°) = 149°$

$\therefore$ $\mathbf{5.83\angle 149}$

$(-4-j4)$ $r = 4\sqrt{1+1} = 5.66$ $\phi = \tan^{-1}\frac{-4}{-4} = 45°$



This phase shift is in the -ve real area so angle w.r.t. positive real $= 180 + 45 = 225°$ or $-180 + 45 = -135°$

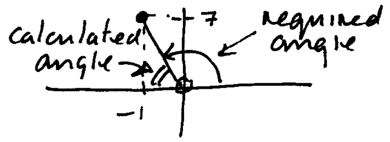
$\therefore$ $\mathbf{5.66\angle 225}$ or $\mathbf{5.66\angle -135}$

$(2-j2)(3+j8) = 22 + j10$ $r = \sqrt{584} = 24.2$ $\phi = 24.4°$

$\therefore$ $\mathbf{24.2\angle 24.4}$

[check using first two.. $(2.83\angle -45) \times (8.54\angle 69.4)$
$= 2.83 \times 8.54 \angle(-45 + 69.4) = 24.2\angle 24.4$]

$(-5+j3) - (-4-j4) = -1+j7 \quad r = 7.1 \quad \phi = \tan^{-1}\frac{7}{-1} = -81.9$



This angle is in the negative real area so angle required $= 180 + (-81.9)$
$= 98.1$

$\therefore\ \mathbf{7.1\angle 98.1}$

Q3

$6\angle 45 = 6\cos 45 + 6j\sin 45 = \mathbf{4.2 + j4.2}$

$50\angle -170 = 50\cos(-170) + 50j\sin(-170) = \mathbf{-49.2 - j8.7}$

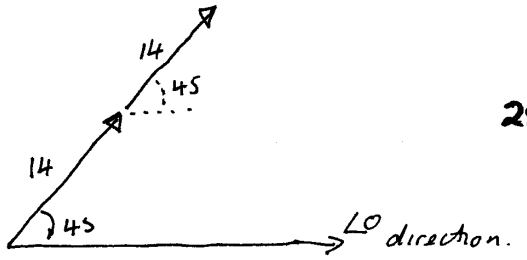
$4\angle 105 = 4\cos 105 + 4j\sin 105 = \mathbf{-1 + j3.9}$

$3\angle -90 = 3\cos(-90) + 3j\sin(-90) = \mathbf{0 - 3j}$

$(5\angle -30)(6\angle 120) = 30\angle 90 = \mathbf{0 + j30}$

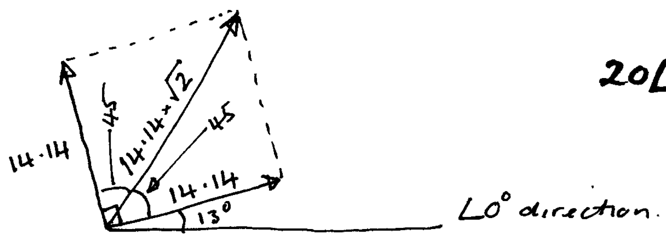
$3\angle 15 + 3\angle 135 + 3\angle -105 = 2.90 + j0.78 - 2.12 + 2.12j$
$- 0.78 - 2.9j = \mathbf{0 + j0}$

Q4 (i)



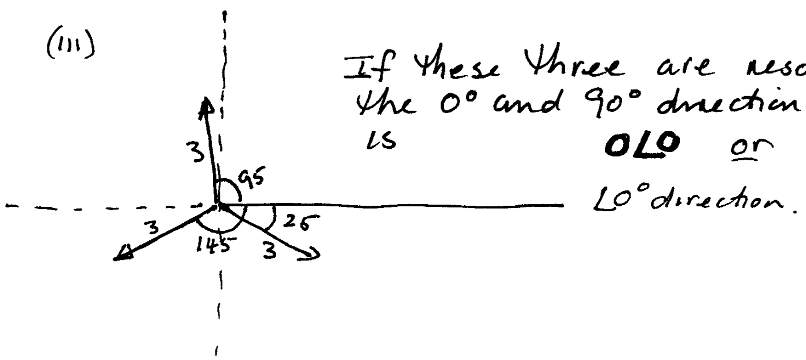
$\mathbf{28\angle 45}$

(ii)



$\mathbf{20\angle 58}$

(iii)



If these three are resolved along the 0° and 90° directions the answer is $\mathbf{0\angle 0}$ or $0 + j0$

Q5 (i) The impedance of the components is

$$\frac{V}{I} = \frac{280\angle 150}{11\angle 140} = \frac{280}{11}\angle 10 = 25{\cdot}46\angle 10$$

$$25{\cdot}46\angle 10 = 25{\cdot}1 + j4{\cdot}42$$

Since the phase of V. w.r.t. I is positive the circuit is inductive and since the phase is less than 90° there must be resistance involved

So the series combination is $\underline{L + R}$.

(ii) The impedance of a series L − R combination is

$$Z = R + j\omega L$$

and this must be equal to the impedance calculated from the given V + I

$$\therefore R + j\omega L = 25{\cdot}1 + j4{\cdot}42.$$

$$\therefore \underline{R = 25{\cdot}1\,\Omega}$$

$$\omega L = 4{\cdot}42 \quad \text{or} \quad L = \frac{4{\cdot}42}{2.\pi.f} = \frac{4{\cdot}42}{800}$$

$$= \underline{5{\cdot}5\,mH}.$$

(iii) The peak value of current is 11 A and this flows through R

$$\therefore P_{DISS} = \frac{I_P^2}{2} \times R = \frac{121}{2} \times 25{\cdot}1 = \underline{1{\cdot}52\,kW}.$$

mean squared value

Q6

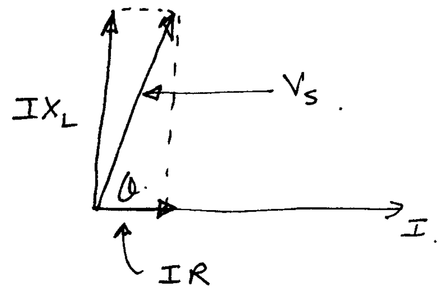
$$|Z| = \frac{230}{10} = \sqrt{2^2 + \omega^2 L^2}$$

$$\left(\frac{230}{10}\right)^2 = 529 = 4 + \omega^2 L^2$$

$$L^2 = \frac{529-4}{\omega^2} = \frac{525}{(2.\pi.50)^2} = 5{\cdot}32 \times 10^{-3}$$

$$\therefore \underline{L = 73\,mH}.$$

$$\theta = \tan^{-1} \frac{X_L}{R}.$$
$$= \tan^{-1} \frac{22.9}{2} = 85^\circ$$



$\therefore$ I lags V_s by 85° or
the phase of I wrt V = $\underline{\underline{-85^\circ}}$

Q7 $$|Z| = 110 = \sqrt{47^2 + \frac{1}{\omega^2 C^2}} = \sqrt{47^2 + X_c^2}$$

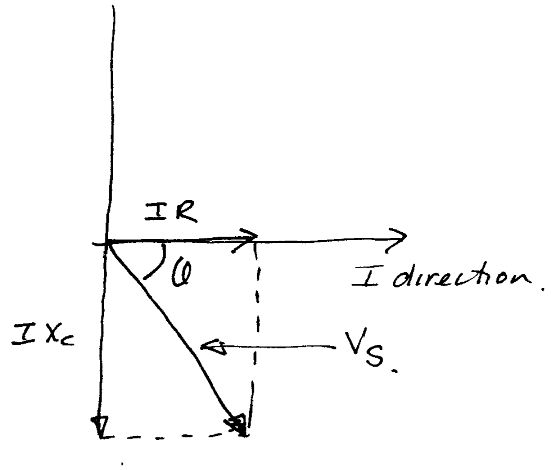
$$110^2 - 47^2 = X_c^2 = 9.89 \times 10^3$$

$$\therefore X_c = \underline{\underline{99.5\ \Omega}}.$$

$$X_c = \frac{1}{2\pi f C} \quad \therefore \quad C = \frac{1}{2.\pi.f.X_c} = \underline{\underline{16\mu F}}.$$

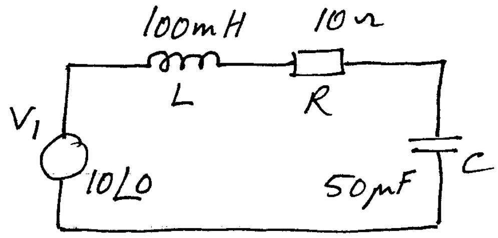
$$\theta = \tan^{-1} \frac{X_c}{R} = \tan^{-1} \frac{99.5}{47}$$
$$= 64.7^\circ$$

In this case the current leads the voltage so the phase of I w.r.t. V is $\underline{\underline{64.7^\circ}}$



Q8 at 50 Hz $X_L = j2\pi f L = j31.4$

$X_C = \frac{1}{j2\pi f C} = -j63.7$



at 150 Hz $X_L = j94.2$

$X_C = -j21.2$.

(i) for 50Hz, $10 + j0 = I[j31.4 + 10 - j63.7]$

$\therefore I = \frac{10\angle 0}{10 - j32.3} = \frac{10\angle 0}{33.8\angle -72.8} = \underline{0.3\angle 73°}$

$V_C = IX_C = -j63.7\,(0.3\angle 73)$
$= (63.7\angle -90)(0.3\angle 73)$
$= \underline{18.8\angle -17}$

(ii) for 150 Hz $10 + j0 = I[j94.2 + 10 - j21.2]$
$= I[10 + j73] = I[73.7\angle 82]$

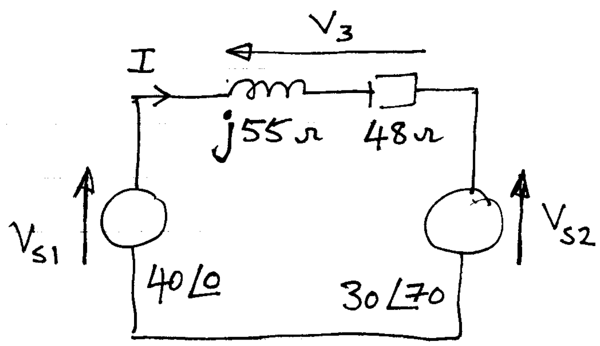
$\therefore I = \frac{10\angle 0}{73.7\angle 82} = \underline{0.14\angle -82}$.

$V_C = IX_C = (0.14\angle -82)(0 - j21.2)$
$= (0.14\angle -82)(21.2\angle -90)$
$= \underline{2.9\angle -172}$

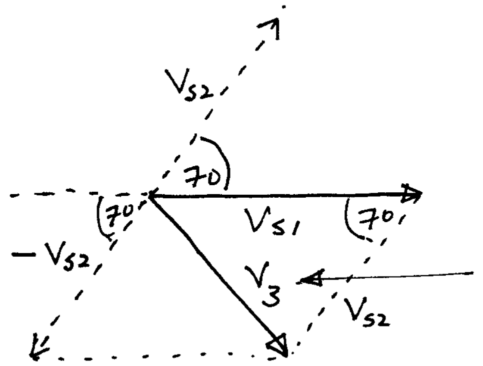
Q9

$V_{S1} = V_{S2} + V_3$

or $V_{S1} - V_{S2} = V_3$



we need a phasor diagram to represent this vector equation ...



$V_3 = V_{S1} + (-V_{S2})$ as required.

using the cosine rule ...

$$V_3^2 = V_{S1}^2 + V_{S2}^2 - 2V_{S1}V_{S2}\cos 70.$$

where all V's are the moduli of the appropriate quantity

$$\therefore \quad V_3^2 = 1600 + 900 - 2 \times 1200 \times 0.342$$

$$= 2500 - 821 = 1679 \text{ V}^2$$

$$\therefore \underline{\underline{V_3 = 41 \text{ V}}}$$

(ii) $40\angle 0 = 40 + j0$

$30\angle 70 = 10.3 + j28.2.$

(iii) $$I = \frac{40\angle 0 - 30\angle 70}{Z} = \frac{40 + j0 - 10.3 - j28.2}{48 + j55}$$

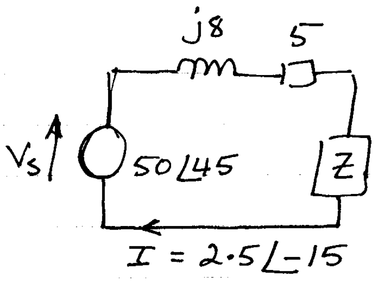
$$= \frac{29.7 - j28.2}{48 + j55} = \frac{(29.7 - j28.2)(48 - j55)}{48^2 + 55^2}$$

$$= \underline{\underline{-0.024 - j0.561}}$$

$$\equiv \underline{\underline{0.56\angle -92.4}}$$

All phases are measured with respect to V_{S1}.

Q10 (i) Total Z seen by source is



$$Z_T = \frac{V_s}{I} = \frac{50\angle 45}{2.5\angle -15}$$

$$= 20\angle 60$$

Z_T is also equal to the sum of impedances in the series circuit

$$Z_T = j8 + 5 + Z$$

$$\therefore\ 20\angle 60 = j8 + 5 + Z = 10 + j17.3$$

$$\therefore\ Z = 10 + j17.3 - j8 - 5$$

$$= \underline{\underline{5 + j9.3}} \equiv \underline{\underline{10.6\angle 61.7}}.$$

(ii) Z has a real part + +j term so the circuit is inductive and could consist of a resistance

$R = 5\Omega$

in series with an inductance

$X_L = j9.3\ \Omega$.

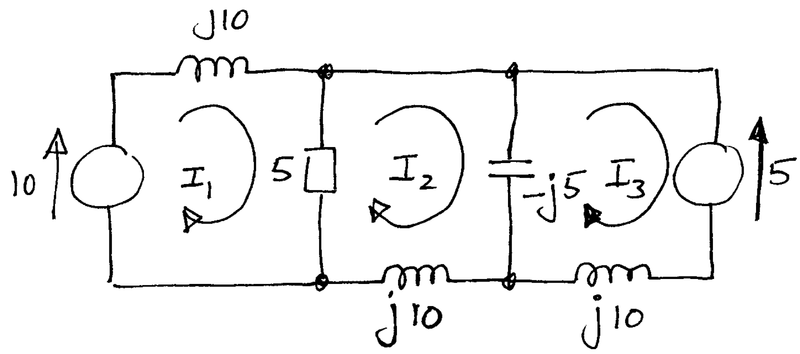
(iii) If the source phase is modified to $50\angle 0$, all the other phases of voltages + currents are reduced by 45° so

$I = 2.5\angle -60$

But Z depends only on the components and hence remains unchanged at

$Z = 5 + j9.3$.

Q11



loop I_1:
$$10 = j10 I_1 + 5(I_1 - I_2)$$
$$2 = j2 I_1 + I_1 - I_2$$
$$\therefore \; 2 = I_1(1 + j2) - I_2 \quad \text{——— (1)}$$

loop I_2
$$5(I_2 - I_1) + j10 I_2 - j5(I_2 - I_3) = 0$$
$$I_2 - I_1 + 2j I_2 - j I_2 + j I_3 = 0$$
$$I_2(1 + j) + j I_3 - I_1 = 0 \quad \text{——— (2)}$$

loop I_3
$$5 + j10 I_3 - j5(I_3 - I_2) = 0$$
$$1 + j2 I_3 - j I_3 + j I_2 = 0$$
$$1 + j I_3 + j I_2 = 0 \quad \text{——— (3)}$$

sub (3) into (2)
$$I_2(1 + j) - (1 + j I_2) - I_1 = 0$$
$$I_2 + \cancel{j I_2} - 1 - \cancel{j I_2} - I_1 = 0$$
$$I_2 - I_1 - 1 = 0 \quad \text{——— (4)}$$

sub (4) into (1)
$$2 = I_1(1 + 2j) - I_1 - 1$$
$$2 = \cancel{I_1} + 2j I_1 - \cancel{I_1} - 1$$
$$3 = 2j I_1 \quad \text{or} \quad \underline{I_1 = -1.5j}$$

sub I_1 into (1)
$$2 = -1.5j(1 + 2j) - I_2$$
$$2 = -1.5j + 3 - I_2$$

$$-1 + 1.5j = -I_2 \quad \text{or} \quad \underline{\underline{I_2 = 1 - 1.5j}}$$

sub I_2 into ③.

$$1 + jI_3 + j(1 - 1.5j) = 0$$

$$1 + jI_3 + j + 1.5 = 0.$$

$$\frac{2.5 + j}{j} = -I_3 = -j2.5 + 1$$

$$\text{or} \quad I_3 = \underline{\underline{-1 + j2.5}}$$

$$\begin{aligned}\text{Power delivered to cct} &= 10 \times \text{real } I_1 + 5 \times \text{real}(-I_3) \\ &= 0 + 5 \\ &= \underline{\underline{5W}}.\end{aligned}$$