EEE 225
Exam Run:

Q1 : From 2014 paper Q 3

This Q^n is about anatomy of an opamp...

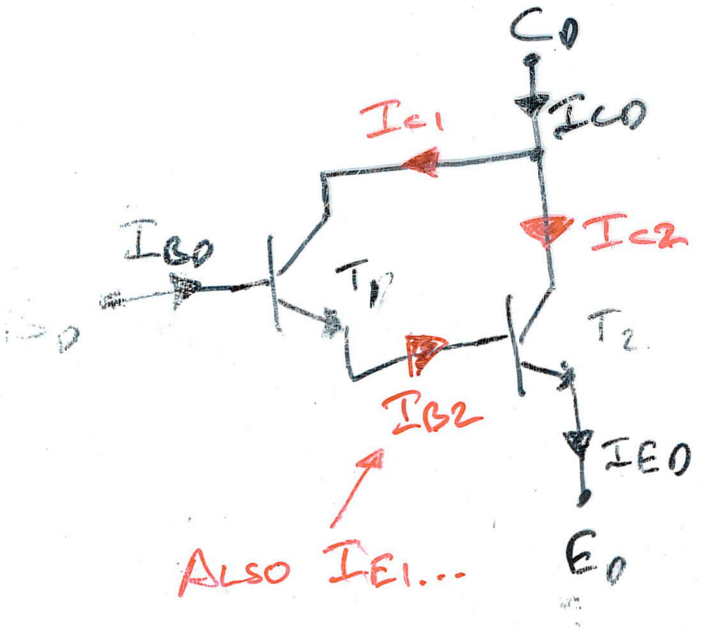
This Q^n is about the Darlington pair

i Show that the h_{FE} of the Darlington pair is given by:

$$h_{FED} = h_{FE1} + h_{FE2} + h_{FE1} \cdot h_{FE2}$$

where h_{FE1} is the current gain of T1 & h_{FE2} " " " " T2.

(3)



Solution:

- Need to find I_{CD} in terms of I_{BD} so that I_{CD}/I_{BD} can be computed...
- Add currents I_{C1}, I_{C2} & I_{B2} to the diagram.
- $I_{C1} = h_{FE1} \cdot I_{BD}$
- $I_{B2} \triangleq I_{E1} = I_{C1} + I_{BD} = h_{FE1} \cdot I_{BD} + I_{BD}$
- $I_{C2} = h_{FE2} \cdot I_{B2}$
- $I_{CD} = I_{C1} + I_{C2}$
- Collect terms and find I_{CD}/I_{BD}...

$$\frac{I_{CD}}{I_{BD}} = \frac{\overbrace{h_{FE1} \cdot I_{BD}}^{I_{C1}} + \overbrace{h_{FE2} \cdot \overbrace{(h_{FE1} \cdot I_{BD} + I_{BD})}^{I_{B2}}}^{I_{C2}}}{I_{BD}}$$

SIMPLIFY...

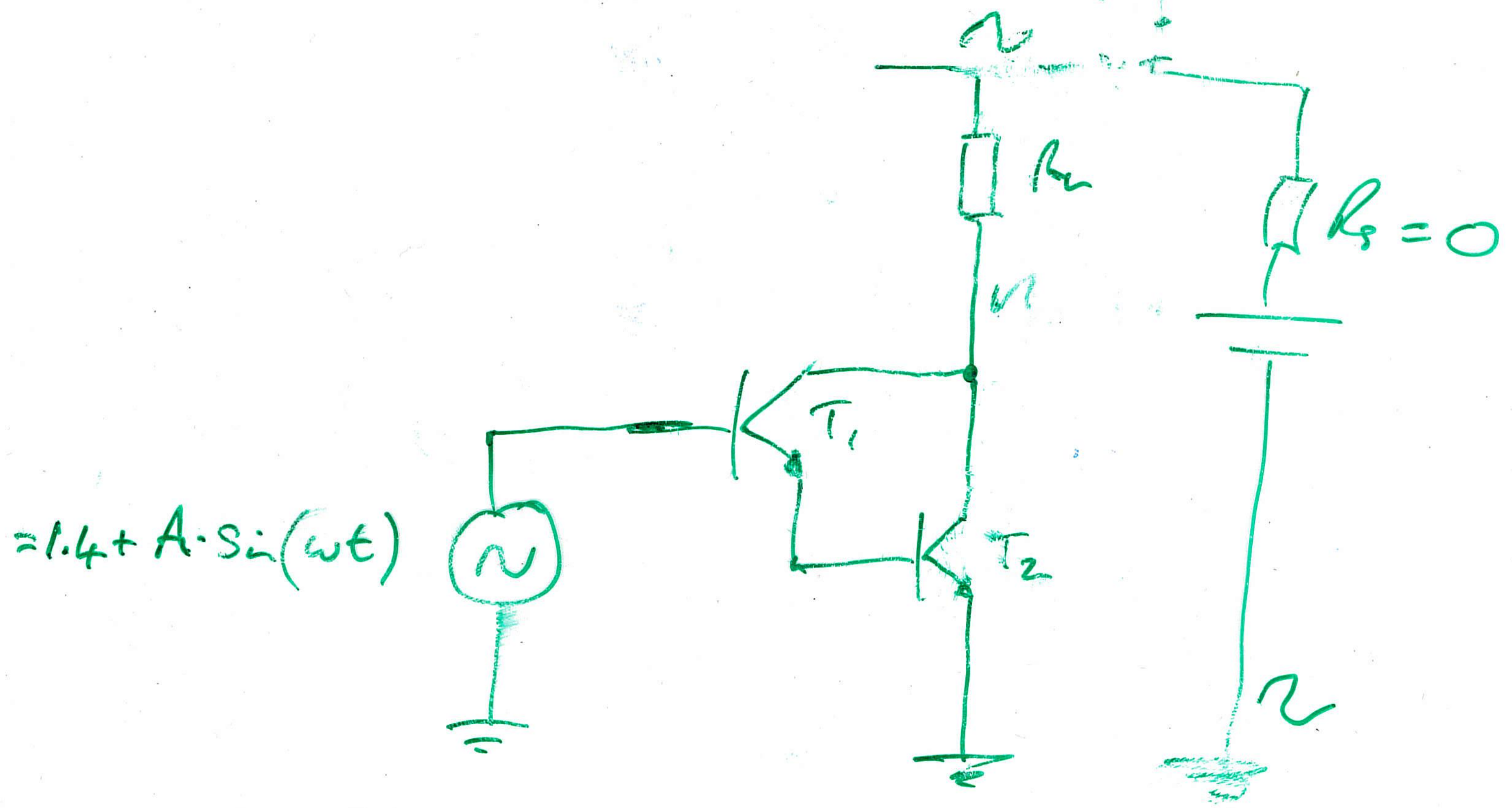
$$\frac{I_{CO}}{I_{BO}} = \frac{h_{FE1}\cdot \cancel{I_{BO}}^{1} + h_{FE2}\cdot\left(h_{FE1}\cdot \cancel{I_{BO}}^{1} + \cancel{I_{BO}}^{1}\right)}{\cancel{I_{BO}}_{1}}$$

$$\therefore \quad \frac{I_{CO}}{I_{BO}} = h_{FE1} + h_{FE2}\cdot h_{FE1} + h_{FE2}$$

Q.E.D

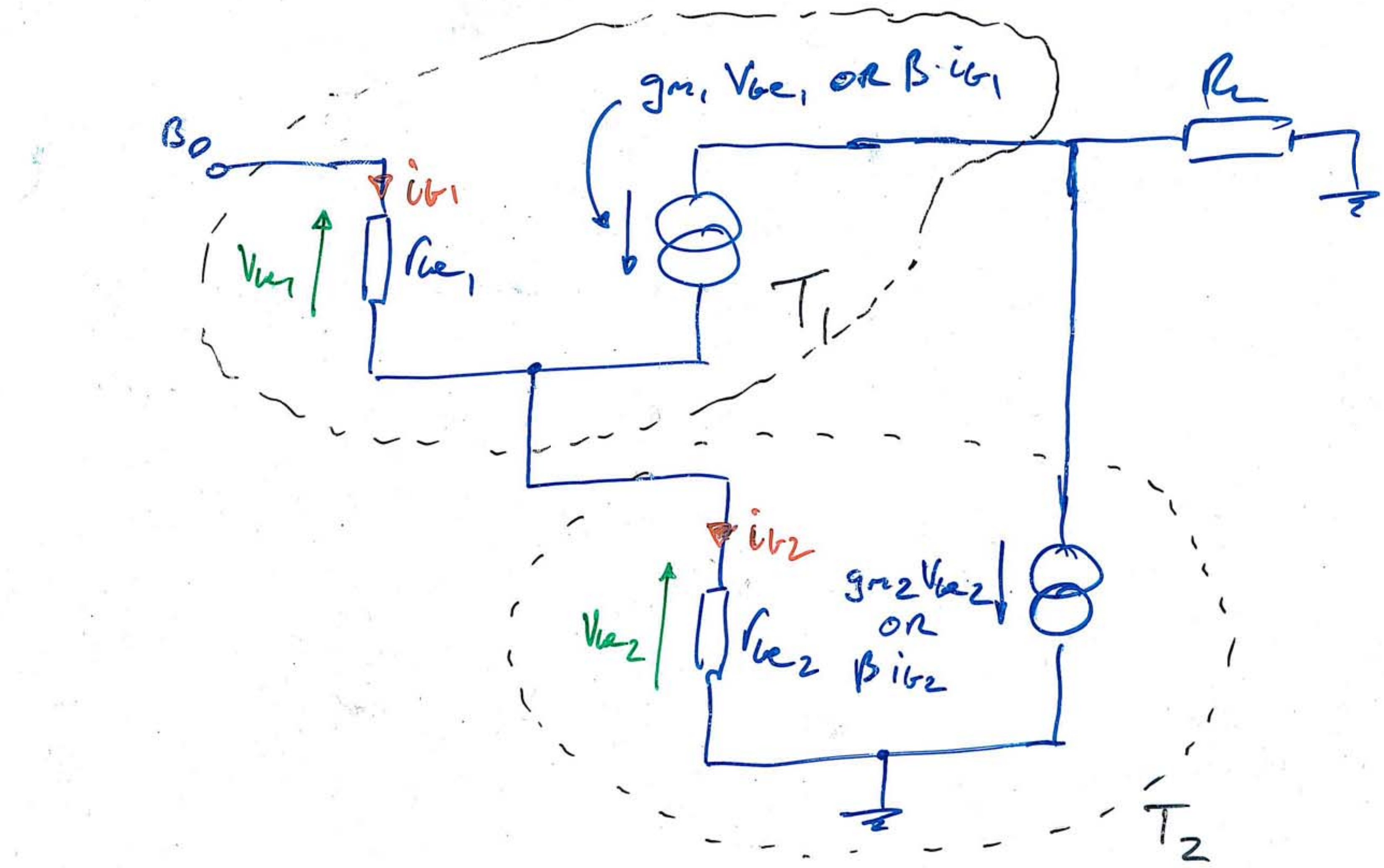
ii) THE DARLINGTON IS USED TO MAKE A COMMON EMITTER AMPLIFIER... E_D IS CONNECTED TO GROUND, B_D TO A SIGNAL SOURCE, C_D IS CONNECTED TO A LOAD RESISTOR R_L, THE OTHER END OF WHICH IS CONNECTED TO A POWER SUPPLY.

WHAT DOES THIS LOOK LIKE?



... DRAW A SMALL SIGNAL EQUIVALENT CCT ASSUMING r_{ce} OF T_1 & T_2 HAS NEGLIGIBLE EFFECT ON PERFORMANCE.

SMALL SIGNAL MODEL... (4)



iii) BOTH T_1 & T_2 HAVE $h_{FE} = 100$ & $\beta = 200$
THE BIAS CIRCUIT DEFINES $I_{c2} = 1\,mA$.
WHAT IS THE INPUT RESISTANCE? (5)

$$g_m = \frac{eI_c}{KT} \; ; \; r_{be} = \beta / g_m \qquad \frac{KT}{e} = 0.026\,V$$

SOLUTION:

- COLLECTOR CURRENT OF T_1 IS MUCH LESS THAN T_2. ASSUME THAT $I_{E1} \simeq I_{c1}$...

  OR IF YOU WANT TO INCLUDE IT

  $$I_{E1} = I_{c1} + I_{B1} = I_{c1}\left(1 + \frac{1}{h_{FE1}}\right)$$

- CALCULATE g_{m1} & g_{m2} AND FROM THESE COMPUTE r_{be1} & r_{be2}.

$$g_{m1} = \frac{eI_{c1}}{KT} = \frac{1.6 \times 10^{-19} \cdot 10 \times 10^{-6}}{1.38 \times 10^{-23} \cdot 301.45}$$

$$= 385 \times 10^{-6}\ A/V$$

$$g_{m2} = \frac{e I_{C2}}{KT} = \frac{1.6\times10^{-19} \cdot 1\times10^{-3}}{1.38\times10^{-23} \cdot 301.45}$$

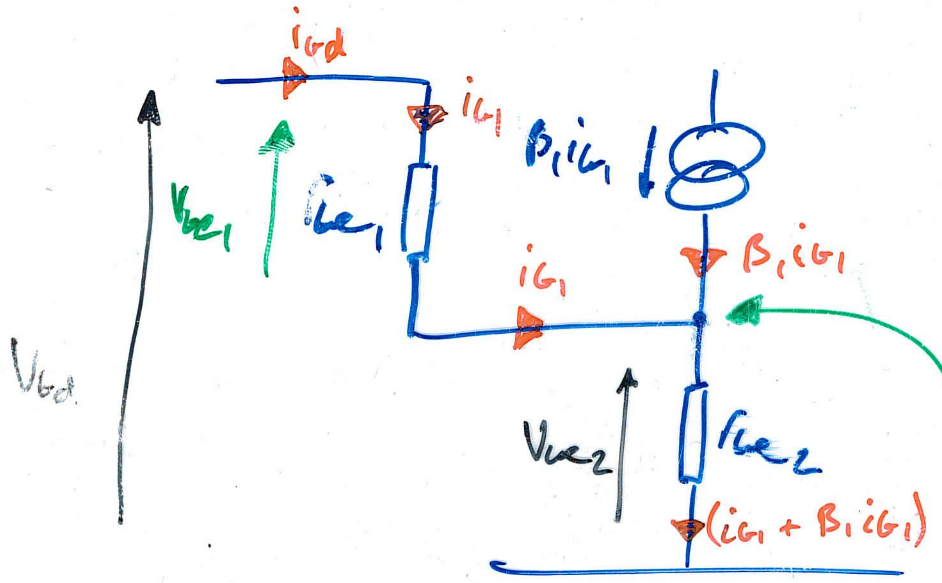
$$= \underline{38.5\ mA/V.}$$

$$r_{be1} = \frac{B_1}{g_{m1}} = \frac{200}{385\ \mu A/V} = \underline{520\,k\Omega}$$

$$r_{be2} = \frac{B_2}{g_{m2}} = \frac{200}{38.5\ mA/V} = \underline{5.2\,k\Omega.}$$

THIS IS QUIESCENT T_2 BASE CURRENT IF $h_{FE2} = 100\ldots$ $I_{B2} = I_{E1}$ AND ASSUME $I_{C1} = I_{E1}$.

NOW HOW ABOUT THE INPUT RESISTANCE?



$$r_i = \frac{V_{bd}}{i_{bd}} = \frac{V_{bd}}{i_{b1}}$$

$$V_{bd} = \underbrace{V_{be1}}_{i_{b1} \cdot r_{be1}} + \underbrace{V_{be2}}_{i_{b1}(1+B_1) r_{be2}}$$

GOT THIS BY SUMMING I AT THE EMITTER OF T_1

NOW ONLY NEED TO DO SOME MATHS...

$$V_{bd} = i_{b1} \cdot r_{be1} + i_{b1}(1+B_1) r_{be2}$$

$$V_{bd} = i_{b1}\left(r_{be1} + (1+B_1) r_{be2}\right)$$

$$\frac{V_{bd}}{i_{b1}} = \left(r_{be1} + (1+B_1) r_{be2}\right)$$

$$= 520\,k\Omega + (1 + 200) \cdot 5.2\,k\Omega$$

$$= \underline{1.565\ M\Omega}$$

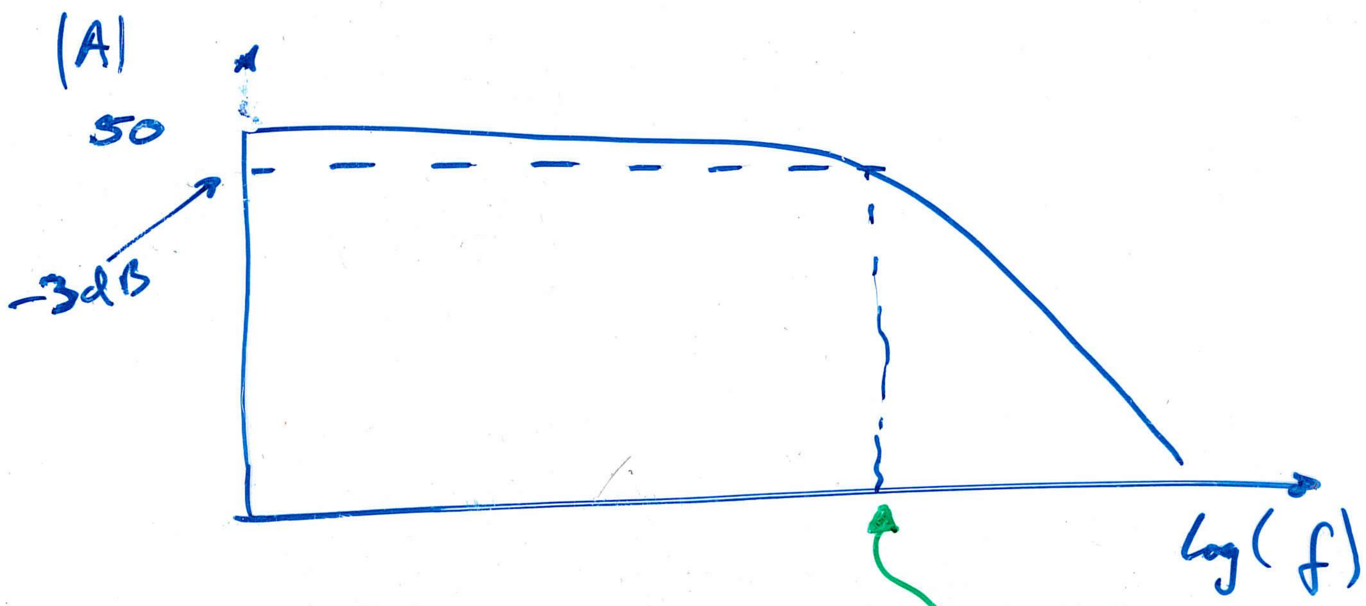
THERE ARE USUALLY DC CONDITIONS PARTS AND SMALL SIGNAL CALCULATION PARTS TO THIS SORT OF QUESTION.

EEE 225 2014 Q 36

A NON-INVERTING OPAMP AMPLIFIER IS MADE USING AN OPAMP WITH GBP = 40MHz. THE CLOSED LOOP GAIN IS 50.

i) IS A -3dB POINT OF 2MHz POSSIBLE? (1)

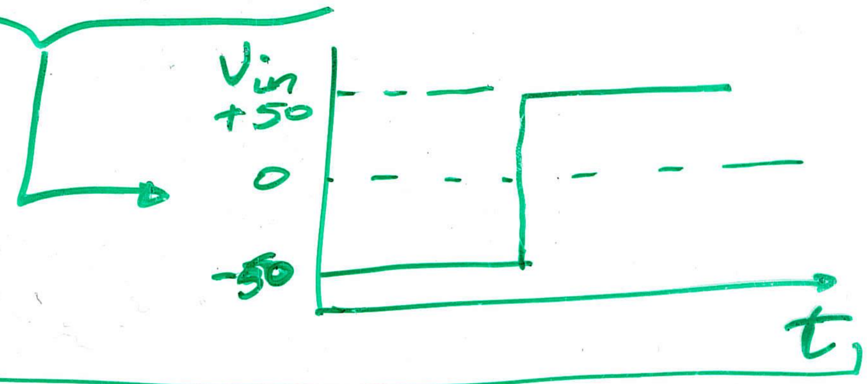
GAIN = 50 AT LOW FREQUENCIES...



MAX -3dB BANDWIDTH $= \dfrac{GBP}{GAIN} = \dfrac{40 \times 10^6}{50}$

$= \underline{800\,kHz}$ SO NO 2MHz IS NOT POSSIBLE.

ii) WHAT IS THE RISE TIME IN RESPONSE TO A STEP INPUT OF -50mV → +50mV.

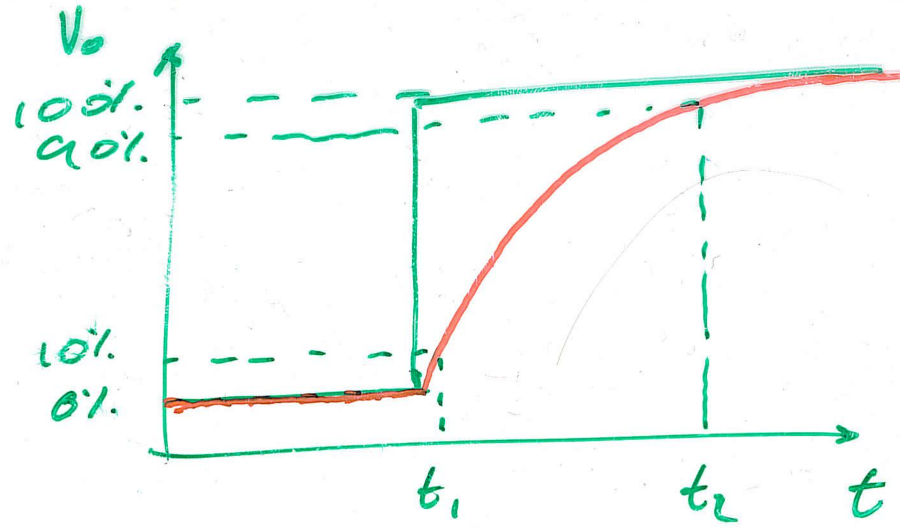


THIS IS PADDING AND IS NOT NEEDED TO ANSWER THE QUESTION...

- $\tau = \dfrac{1}{\omega_{-3dB}} = \dfrac{1}{2\pi f_{-3dB}}$
- $t_R = 2.2\tau$

FOR A 1ST ORDER SYSTEM...

RISE TIME FROM 10% TO 90% OF TOTAL OUTPUT.



$V_o = V_{MAX}(1 - e^{-t/\tau})$

$t_R = t_2 - t_1$

SOME EASY (ISH) MATHS:

FROM PART i

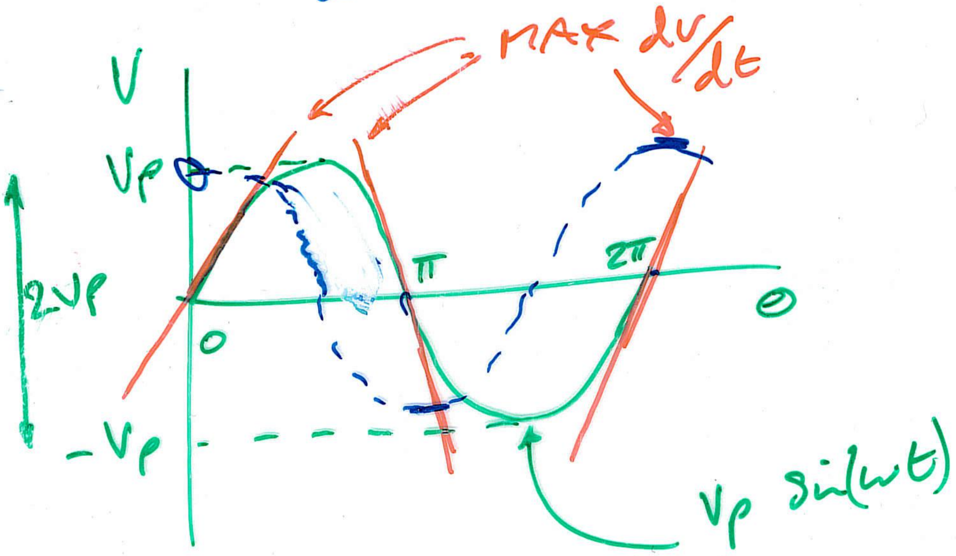
$$\omega_{-3dB} = 2\pi \cdot 800\times10^{3} = 5.026\times10^{6}\ \text{rads}^{-1}$$

$$\tau = \frac{1}{5.02\ \text{Mrads}^{-1}} = \underline{198.94\ nS}$$

$$t_R = 2.2\tau = \underline{437.67\ nS}$$

iii) IF THE AMP. SLEW RATE IS 50 V/μS WHAT IS THE MAX PEAK - PEAK VOLTAGE THAT CAN BE SUPPORTED AT 2MHz ?

- NEED TO REMEMBER (OR DERIVE) THAT MAX $\frac{dv}{dt}$ OF A SINE IS $V_p \cdot \omega$



$$V_p \sin(\omega t) \rightarrow \frac{d(V_p \sin(\omega t))}{dt}$$

$$= V_p \cdot \omega \cdot \cos(\omega t)\Big|_{max}$$

$\cos(0) = 1 \ldots$

$\therefore$ $V_p \cdot \omega$ IS MAX $\frac{dv}{dt}$ OF $V_p \sin(\omega t)$.

$$V_p \cdot \omega = 50\times10^{6}$$

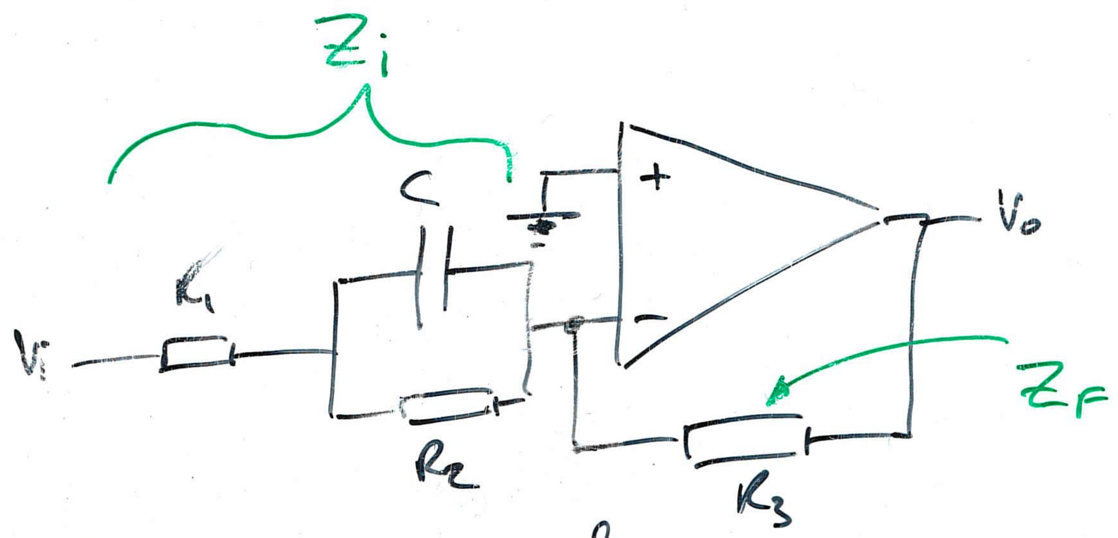
ω: $2\,MHz \cdot 2\pi$

$50\,V/\mu S$ IN V/S ...

$$V_p = \frac{50\times10^{6}}{\ldots} = \underline{3.9788\,V}$$

THERE IS ALMOST ALWAYS SOME QUESTION THAT USES THE FIRST ORDER APPROXIMATIONS.

EEE 225 2014 PAPER Q4.



a i) WRITE DOWN THE H.F. & L.F. GAINS IN TERMS OF R_1, R_2, C & R_3.

- AT L.F. $X_C \to \infty\,\Omega$. SO $Z_i = R_1 + \left(\frac{1}{sC} // R_2\right)$ $\to \infty \therefore R_2$.
- GAIN $= \frac{-Z_F}{Z_i} = \frac{-R_3}{R_1 + R_2}$

- AT H.F $X_C \to 0\,\Omega$ SO $Z_i = R_1 + 0$
- GAIN $\frac{-Z_F}{Z_i} = \frac{-R_3}{R_1}$ $\frac{1}{sC} = 0$ SO R_2 IS S/C.

ii) IF $f_0 = 50Hz$, $f_1 = 1000Hz$ & $K = 10$ FIND R_3/R_1, R_2/R_1 AND $C \cdot R_2$ (THE TIME CONSTANT) SUGGEST COMPONENT VALUES.

$$\frac{V_o}{V_i} = -K \cdot \frac{1 + jf/f_0}{1 + jf/f_0}, \quad K = \frac{R_3}{R_1 + R_2}, \quad f_0 = \frac{1}{2\pi C R_2}$$

$$f_1 = \frac{R_1 + R_2}{2\pi C_1 R_1 R_2}$$

- MANIPULATE THE GIVEN EQUATIONS TO YIELD THE RATIOS REQUIRED.
- TACKLE R_2/R_1 FIRST. DERIVE f_1/f_0:

$$\frac{f_1}{f_0} = \frac{\dfrac{R_1 + R_2}{\cancel{2\pi C_1 R_1 R_2}}}{\dfrac{1}{\cancel{2\pi C R_2}}} = \frac{R_1 + R_2}{R_1} = \frac{R_1}{R_1} + \frac{R_2}{R_1}$$

$$\frac{f_1}{f_0} = 1 + \frac{R_2}{R_1} = \frac{1000}{50} = 1 + \frac{R_2}{R_1}$$

$$\frac{1000}{50} = \frac{100}{5} = 20\ldots$$

$$\therefore\ 20 - 1 = \frac{R_2}{R_1} \quad \& \quad 19 \cdot R_1 = R_2$$

$$\underbrace{\text{LET } R_2 = \underline{100\,k\Omega}}_{\text{THIS IS A CHOICE!}} \quad \text{So } R_1 = \underline{5.26\,k\Omega}$$

- NOW DO R_3/R_1 :

$$K = \frac{R_3}{R_1 + R_2} \quad \div R_1 \quad \frac{R_3/R_1}{R_1/R_1 + R_2/R_1} = \frac{R_3/R_1}{1 + R_2/R_1}$$

$$19$$

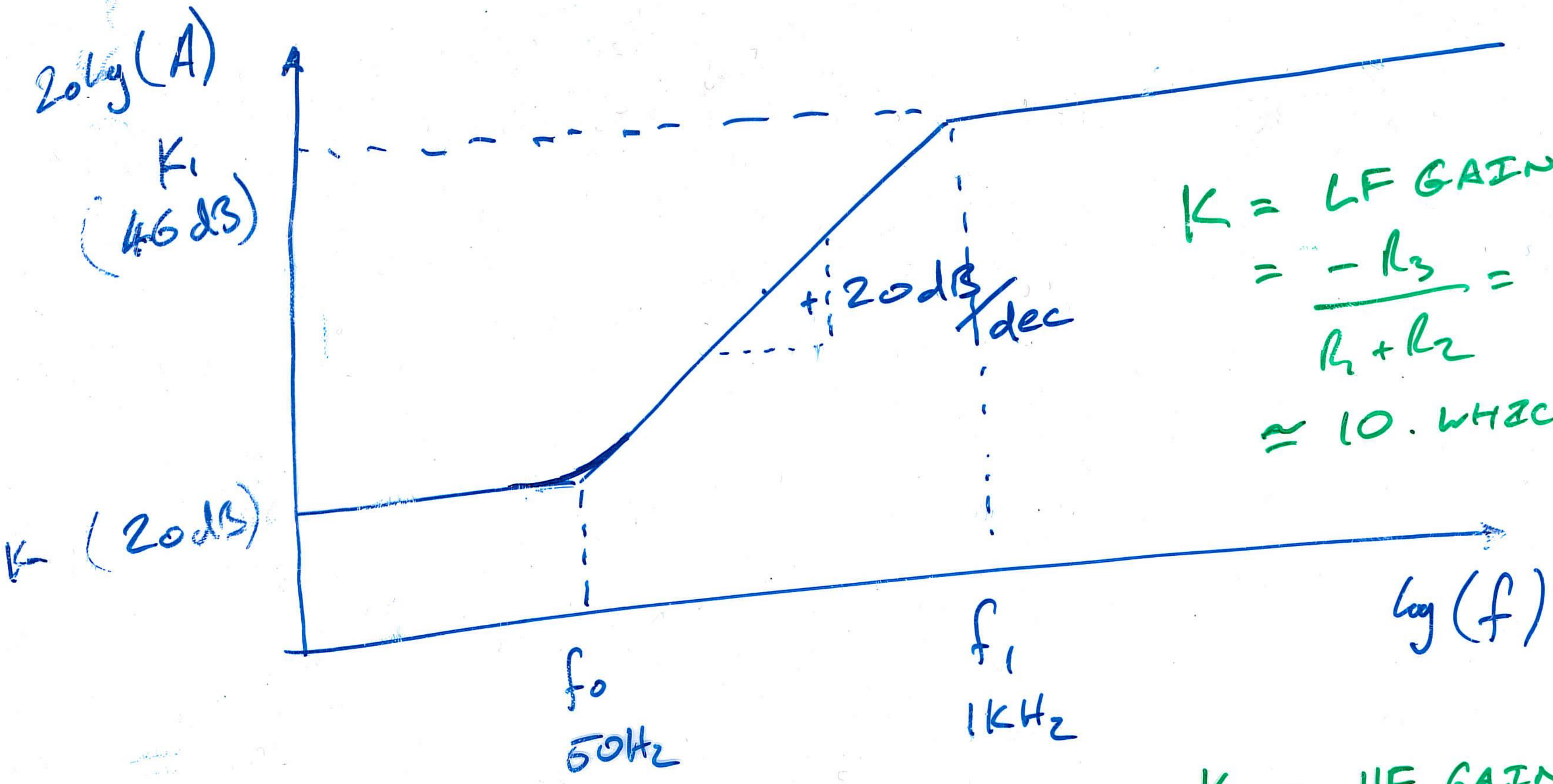
$$\therefore\ K = \frac{R_3/R_1}{20} \quad \& \quad K = 10 \quad \text{So:}$$

$$20 \cdot 10 = \frac{R_3}{R_1} = \underline{200}. \qquad R_1 = 5.26K \text{ So}$$

$$R_3 = \underline{1.052\,M\Omega} \qquad (200 \cdot R_1)$$

ONE CANT CHOOSE VALUES LESS THAN ABOUT $1\,k\Omega$ OR MORE THAN $1\,M\Omega$. $< 1\,k\Omega$ AND THE OPAMP WON'T BE ABLE TO SUPPLY THE NECESSARY CURRENT. $> 1\,M\Omega$ AND INSUFFICIENT BIASING CURRENT WILL BE AVAILABLE TO THE INVERTING AND/OR NON-INVERTING INPUTS.

iii) SKETCH A BODE PLOT OF THE POLE-ZERO CIRCUIT...



K = LF GAIN

$= \frac{-R_3}{R_1 + R_2} = \frac{-1.052M\Omega}{5.26K + 100K}$

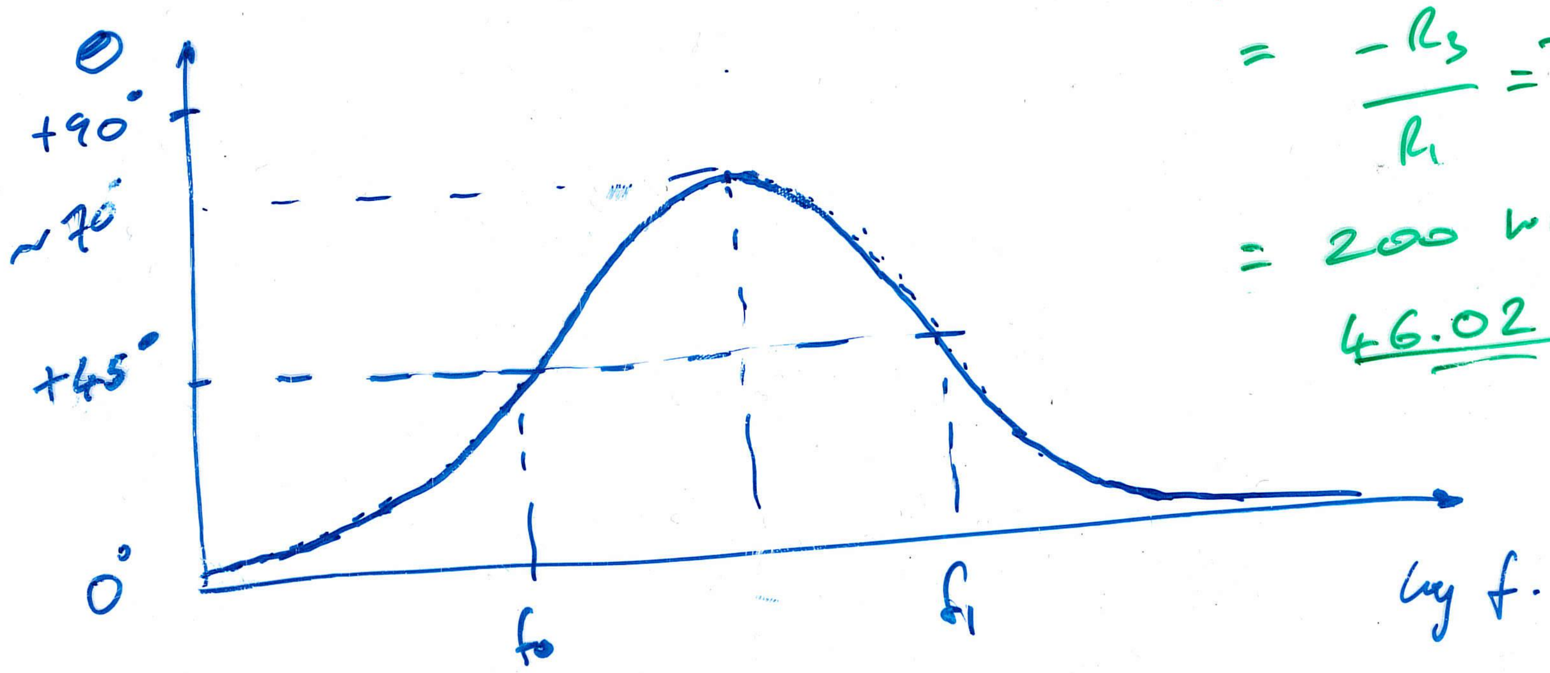
$\simeq$ 10. WHICH IS 20 dBV.

K1 = HF GAIN

$= \frac{-R_3}{R_1} = \frac{-1.052M\Omega}{5.26K\Omega}$

= 200 WHICH IS

46.02 dBV



- A ZERO GIVES +20dB/DEC IN MAGNITUDE AND +90° PHASE (LEADING)
- A POLE GIVES −20dB/DEC IN MAGNITUDE AND −90° PHASE (LAGGING).

> IT WOULD BE WISE TO ENSURE YOU'RE CONFIDENT AT SKETCHING THE MAGNITUDE AND PHASE PLOTS FOR POLE-ZERO CIRCUITS. IT USUALLY COMES UP ON EXAMS.

4(i) DEFINE SIGNAL TO NOISE RATIO. WHY IS IT USELESS AS A MEASURE OF SYSTEM PERFORMANCE?

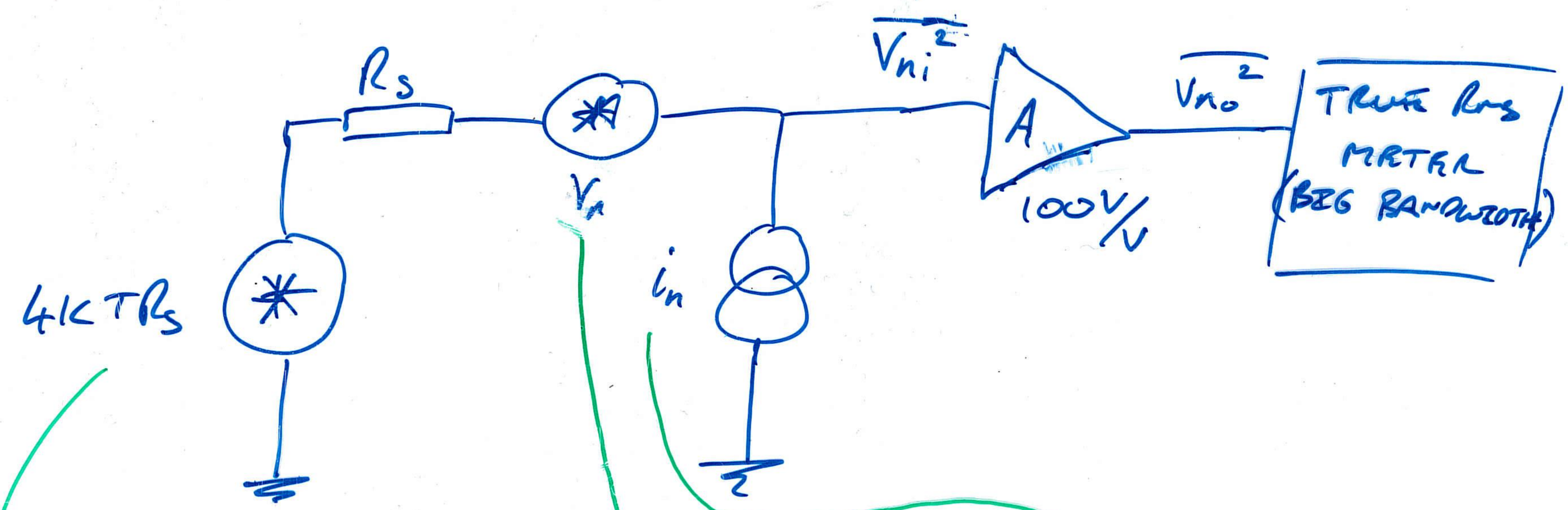
- SNR IS THE RATIO OF THE MEAN SQUARED SIGNAL AND THE MEAN SQUARED NOISE.
- SNR TELLS THE RELATIVE CORRUPTION OF THE SIGNAL AT THE POINT IN THE CIRCUIT FOR WHICH IT IS CALCULATED BUT SINCE THE ACTUAL VALUE OF SIGNAL AND/OR NOISE ARE NOT KNOWN IT CAN'T BE RELATED TO A SYSTEM'S SPECIFICATIONS WITHOUT EXTRA INFO.

ii) AN AMPLIFIER WITH $r_{in} = \infty\,\Omega$ & NOISE EQUIVALENT BANDWIDTH OF 10KHz & GAIN = 100 HAS A INPUT NOISE VOLTAGE GENERATOR OF $8\,nV/\sqrt{Hz}$ & INPUT NOISE CURRENT GENERATOR OF $3\,pA/\sqrt{Hz}$. THE AMPLIFIER IS DRIVEN BY A THÉVENIN SOURCE WHO'S $R_s = 1.8K\Omega$.

- CALCULATE THE RMS NOISE AT THE OUTPUT WHEN NO SIGNAL IS PRESENT.

SOLUTION

- USE STD. AMPLIFIER NOISE MODEL.



- DO SUPERPOSITION OF THE SOURCES OR JUST REMEMBER...

$$\overline{V_{ni}^2} = 4KTR_s + \overline{V_n^2} + \overline{i_n^2}R_s^2 \quad V^2/Hz$$

$\overline{V_{ni}^2}$ IS THE MEAN SQUARED NOISE AT THE INPUT

$\overline{V_{no}^2}$ " " " " " " " OUTPUT.

THE METER READS THE ROOT OF THE MEAN SQUARED NOISE (RMS)

$$V_m = \sqrt{\overline{V_{no}^2} \cdot BW}$$

(10KHz IN THE QUESTION)

So $\overline{V_{ni}^2} = 4KT \cdot 1.8\times10^3 + (8\times10^{-9})^2 + (3\times10^{-12})^2 \cdot (1.8\times10^3)^2$

$= 2.99\times10^{-17} + 64\times10^{-18} + 9\times10^{-24} \cdot 3.24\times10^6$

$= 1.2306\times10^{-16}\ V^2/Hz$ @ THE INPUT

$\overline{V_{no}^2} = A^2 \cdot \overline{V_{ni}^2} = 100^2 \cdot 1.2306\times10^{-16}$

IN THE Q^{n}

$= 1.2306\times10^{-12}\ V^2/Hz$ @ THE OUTPUT

$V_m = \sqrt{\overline{V_{no}^2} \cdot BW} = (1.2306\times10^{-12} \cdot 10\times10^3)^{1/2}$

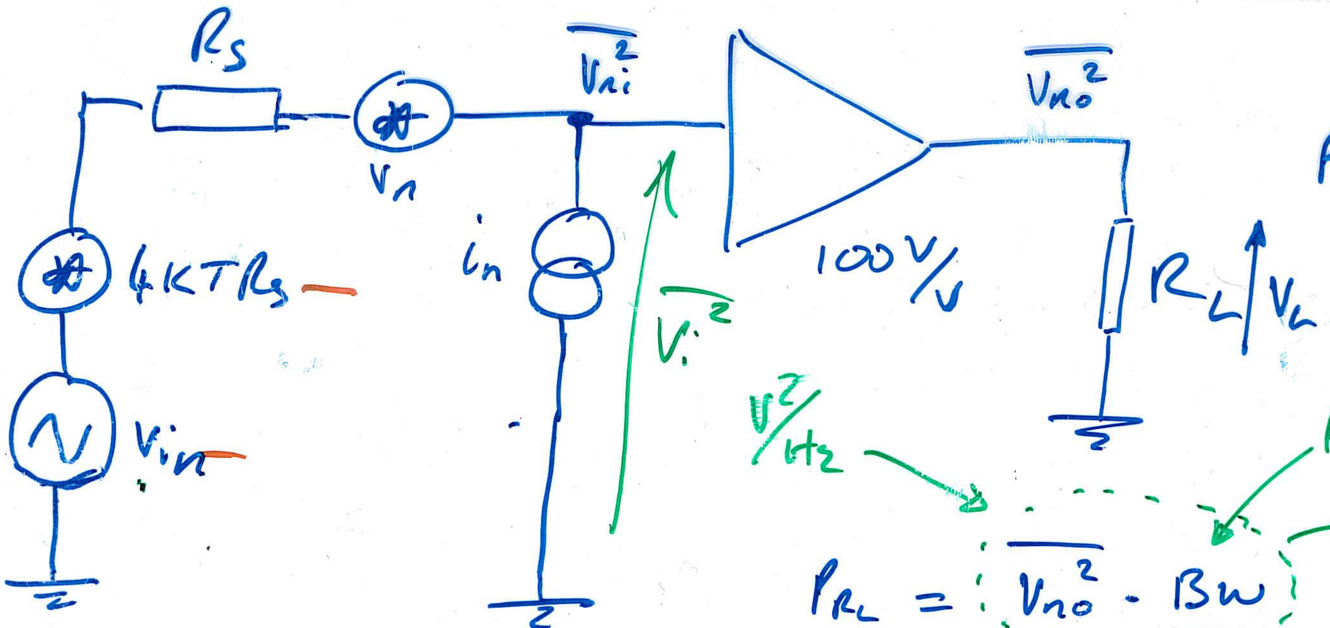
$= \underline{110.93\mu V}$ RMS

iii) WHAT AMPLITUDE OF INPUT SIGNAL WILL GIVE AN SNR OF UNITY AT THE OUTPUT?

ONE WAY TO APPROACH THIS IS BY THINKING ABOUT THE POWER THE NOISE DISSIPATES.

$$\boxed{P = V^2/R}$$

- TO GET AN SNR OF UNITY SIGNAL POWER AT THE OUTPUT AND NOISE POWER AT THE OUTPUT MUST BE EQUAL.
- DEFINE A LOAD RESISTANCE R_L



$$P_{R_L} = \frac{V_L^2}{R_L}$$

V^2/Hz — Hz

$$P_{R_L} = \frac{\overline{V_{no}^2} \cdot BW}{R_L}$$

Ω

THIS IS ALREADY VOLTS² SO NO NEED TO SQUARE AGAIN

$$\therefore \frac{V_L^2}{R_L} = \frac{\overline{V_{no}^2} \cdot B_W}{R_L}$$

THE R_L CANCELS ...

$$V_L^2 = \overline{V_{no}^2} \cdot BW$$

$$(110.93 \times 10^{-6})^2 \; V^2$$

$$V = 110.93 \times 10^{-6} \; V$$

$\div$ BY GAIN $\rightarrow$ 1·1093 μV RMS

$$V_P = \sqrt{2} \cdot V_{RMS} \text{ (FOR SINE)}$$

$$= \underline{1.568 \; \mu V}$$

NOISE ALWAYS SHOWS UP ON THE EXAM. THE "TRICK" OF THINKING ABOUT POWER IS ALWAYS HELPFUL BECAUSE IF YOU'RE WORKING IN V^2 PER UNIT BANDWIDTH YOU'RE ALREADY WORKING WITH POWER!!

EXAM FORMAT

- SECTION A
  - ANSWER FIVE OF NINE
  - EACH WORTH 8 MARKS
  - 3 ON DIGITAL, 3 ON ANALOGUE
    3 ON SEMICONDUCTORS.
  - WORTH 40% OF THE TOTAL.
    (5 × 8 = 40 MARKS).
- SECTION B
  - ANSWER ALL QUESTIONS
  - EACH WORTH 20 MARKS. (3 × 20 = 60 MARKS)
  - 1 ON DIGITAL, 1 ON ANALOGUE
    1 ON SEMICONDUCTORS.

THE TOTAL TIME AVAILABLE IS 3 HOURS.

- PLAN TO SPEND:- JUST OVER 1 HOUR ON PART A
  - " " 1h 30 " PART B
  - 20 MINUTES CHECKING EVERYTHING

  (i.e. ABOUT 1.6 MINUTES/MARK.)

FORMULA GIVEN ON THE SCRIPT:

$$g_m = \frac{eI_c}{KT}, \quad r_{be} = \frac{\beta}{g_m}, \quad h_{FE} = \frac{I_c}{I_B}$$

$$\frac{KT}{e} = 0.026\ V$$

PHYSICAL CONSTANTS GIVEN ON THE SCRIPT.

$e = 1.6 \times 10^{-19}$ C

$m_o = 9.11 \times 10^{-31}$ kg

$c = 2.998$ ms^{-1}

$h = 6.626 \times 10^{-34}$ Js

$k = 1.381 \times 10^{-23}$ JK^{-1}

$0°C = 273.15$ K.

$\varepsilon_o = 8.854 \times 10^{-12}$ Fm^{-1}

$\mu_o = 4\pi \times 10^{-7}$ Hm^{-1}