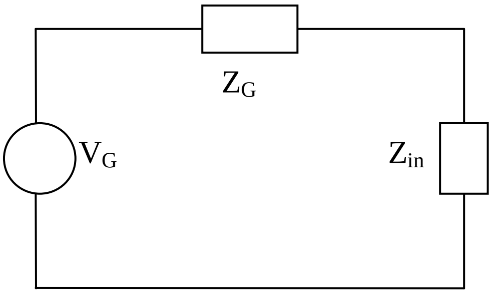
Solutions

Q1(a)

The input section of a transmission line can be represented as



(1 mark)

Conjugate matching can be achieved when $Z_{in} = Z_G^*$ and it can be used to obtain a maximum power transfer. **(2 marks)**

Q1(b)

$\lambda = \frac{c}{f} = 30\text{cm}$ i.e. $\frac{\ell}{\lambda} = 0.1666$ and $\beta\ell = \frac{2\pi}{\lambda} \times 0.1666\lambda = 1.05$ **(1 mark)**

$\Gamma_{load} = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.12 - j0.16$ **(1 mark)**

$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)} = 33.4 - j1.66\Omega$ **(1 mark)**

$VSWR = \frac{1 + |\Gamma_{load}|}{1 - |\Gamma_{load}|} = 1.5$ **(1 mark)**

$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = 0.2 - j0.02$ **(1 mark)**

Q1(c)

Since the load needs to be matched to 75 Ω, then Z_{in} =75Ω

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)}$$

$$75(Z_o + j(50 - j10)\tan(\beta\ell)) = Z_o(50 - j10 + jZ_o \tan(\beta\ell))$$

$75Z_o + j3750\tan(\beta\ell) + 750\tan(\beta\ell)) = 50Z_o - j10Z_o + jZ_o^2 \tan(\beta\ell)$ **(2 marks)**

Equating real and imaginary parts to each other

$75Z_o + 750\tan(\beta\ell) = 50Z_o$ (1) **(0.5 mark)**

$3750\tan(\beta\ell) = -10Z_o + Z_o^2 \tan(\beta\ell)$ (2) **(0.5 mark)**

From equation (1)

$\tan(\beta\ell) = \frac{-Z_o}{30}$ (3) **(1 mark)**

Substitue (3) in (2)

$3450 = Z_o^2$, i.e. $Z_o = 59\Omega$ **(1 mark)**

which gives

$\tan(\beta\ell) = -1.957$ i.e. $\ell = \frac{1}{\beta}\tan^{-1}(-0.316) = 0.325\lambda$ **(1 mark)**

Q1(d)

The reflection coefficient at the load is given by

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$ **(0.5 mark)**

At a distance d from the load towards the generator, the reflection coefficient is

$$\Gamma_{(d)} = \frac{Z_{in(d)} - Z_o}{Z_{in(d)} + Z_o} \qquad (1)$$ **(0.5 mark)**

where

$$Z_{in(d)} = Z_o \frac{Z_L + jZ_o \tan(\beta d)}{Z_o + jZ_L \tan(\beta d)} \qquad (2)$$ **(0.5 mark)**

Substitute equation (2) in (1) gives

$$\Gamma_{(d)} = \frac{Z_L + jZ_o \tan(\beta d) - Z_o - jZ_L \tan(\beta d)}{Z_L + jZ_o \tan(\beta d) + Z_o + jZ_L \tan(\beta d)}$$ **(1 mark)**

Which can be simplified to

$$\Gamma_{(d)} = \frac{Z_L - Z_o - j(Z_L - Z_o)\tan(\beta d)}{Z_L + Z_o + j(Z_L + Z_o)\tan(\beta d)} = \frac{(Z_L - Z_o)1 - j\tan(\beta d)}{(Z_L + Z_o)1 + j\tan(\beta d)}$$ **(1.5 mark)**

i.e.

$$\Gamma_{(d)} = \Gamma \frac{1 - j\tan(\beta d)}{1 + j\tan(\beta d)}$$ **(1 mark)**

Which can be expressed as

$$\Gamma_{(d)} = \Gamma e^{-j2\beta d}$$ **(1 mark)**

Q2(a)

Impedance representation in the smith chart

$$z = r + jx = \frac{1 + \Gamma_{(d)}}{1 - \Gamma_{(d)}}$$ **(1 mark)**

represented as

$$y = \frac{1}{z} = \frac{1 - \Gamma_{(d)}}{1 + \Gamma_{(d)}} = \frac{1 + \Gamma_{(d)} e^{-j\pi}}{1 - \Gamma_{(d)} e^{-j\pi}}$$ **(1 mark)**

This shows that the Smith chart can be used to find transformation of admittance as well as impedance. This can be achieved by rotating an arc with a radius of $\Gamma_{(d)}$ through an angle of 180° (≡0.25λ). **(2 marks)**

Q2(b)

f=*300* MHz λ=1 m

d=10cm= 0.1λ i.e. α*d*=0.1∗15=1.5 dB

Nepers=8.686dB

αd=0.17 nepers $2\alpha d$=0.34 nepers **(1.5 mark)**

Draw point A at z=4+j0, then move 0.1λ around chart *towards generator*, i.e. point B on the chart **(1.5 mark)**

The radius at point B (OB) must be reduced by a factor of $e^{-2\alpha d}$, i.e. 0.34=34% of the radius for the lossless case. This is shown as point C on the chart (0.88+j0.25). **(1.5 mark)**

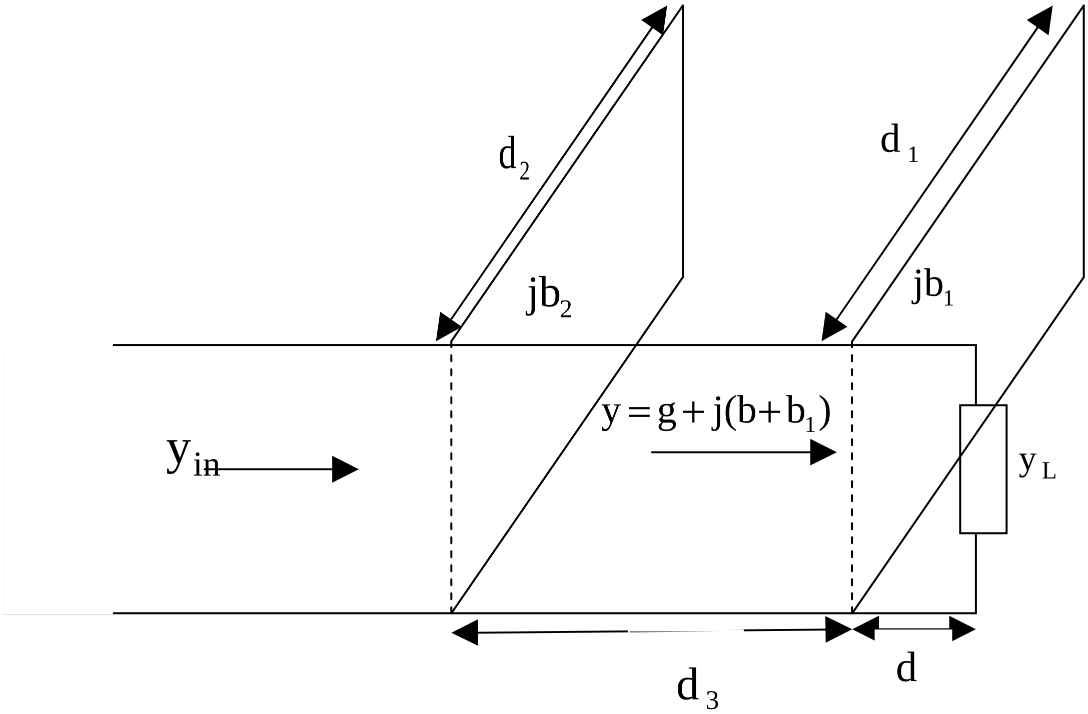
VSWR on the line at this point is found by rotating an arc radius (OC) to the real axis (point D) and read the value VSWR=r=1.35. **(1.5 mark)**

Q2(c)

$$z_L = \frac{(140 - j110)}{50} = 2.8 - j2.2$$ (point A) **(1 mark)**

i.e.

y_L=0.22+j0.17 (point B) **(1 mark)**



Step 1

Rotate the unit g circle *Towards Load*, by a distance of d_3=0.125λ. **(1 mark)**

Step 2

Move from point B to intersect the new, rotated, unit circle at point C. The movement should be on the corresponding conductance circle, since the stub does not alter the real part of the admittance. **(1 mark)**

Step 3

The admittance at point C is

y_C=0.22+j0.38

compare it with that at B

y_L=0.22+j0.17

shows that stub 1 has provided j0.18 i.e. b_1=0.18 **(2 marks)**

Step 4

For an o.c. stub, this means d_1=0.028λ, i.e. the distance from D to E on the chart.

(1 mark)

Step 5

Move a distance d_3=0.125λ along the line from the 1st stub position to the 2nd stub position (from point C to F). **(1 mark)**

Step 6

At point F, the admittance is y_F=1+j1.75i.e. stub 2 must provide -j1.75, *i.e.* b_2=-1.75) to reach the matched condition. **(1 mark)**

Step 7

For a s.c. stub, this means d_2=(0.333-0.25)λ=0.083λ, i.e. the distance from G to H on the chart. **(1 mark)**

<u>Q3(a)</u>

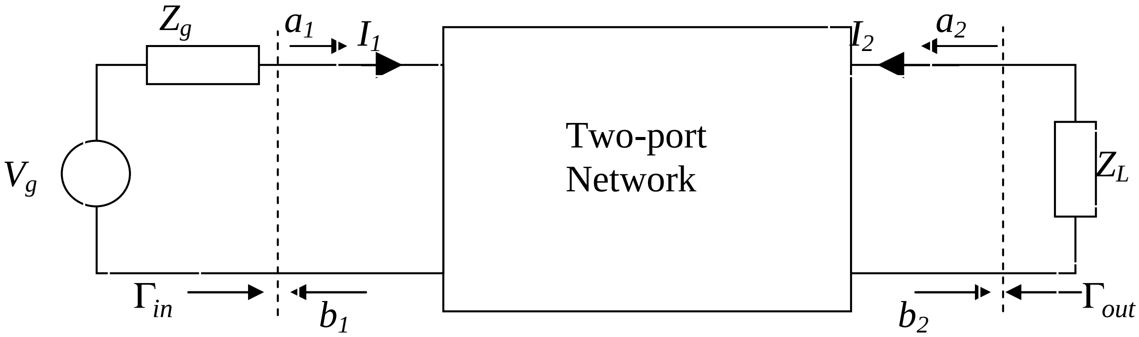
The basic circuit approximation becomes progressively less good, for a number of reasons. As frequency increases wavelength decreases, i.e. it becomes comparable to circuit dimensions. Then circuit components (R, L and C) start to deviate in their electric response from the ideal behaviour. **(2 marks)**

The energy stored in reactive components is held in the space around the components, and different components can have "fields" which overlap. Wires are also reactive components, which store energy. The division of the circuit into separate reactive "components" interconnected by non-reactive "wires" is only an approximation; it is useful in that often it relates quite closely to how the circuit is constructed topographically, but is less good in describing how it behaves electromagnetically.

(2 marks)

<u>Q3(b)</u>

For the following two ports network



(1 mark)

The S_{11} parameter can be determined if the output port is terminated with a matched load, i.e. Γ_L=0

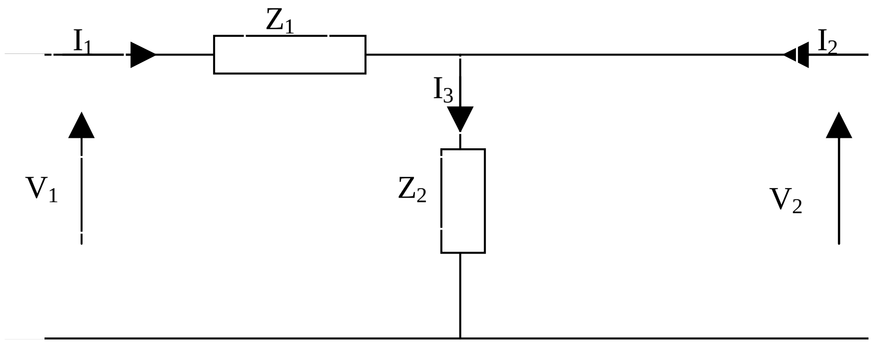
$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$ **(1 mark)**

While the input reflection coefficient is given by

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$ **(1 mark)**

From these equations it can be seen that $S_{11} = \Gamma_{in}$ when the output port is terminated by a matched load. **(1 mark)**

Q3(c)



$$V_1 = I_1 Z_1 + I_1 Z_2 + I_2 Z_2$$ **(0.5 mark)**

$$\therefore \quad I_1 = \frac{V_1 - I_2 Z_2}{Z_1 + Z_2}$$ **(0.5 mark)**

$$V_2 = I_2 Z_2 + I_1 Z_2$$ **(0.5 mark)**

i.e.

$$V_2(Z_1 + Z_2) = I_2 Z_1 Z_2 + V_1 Z_2$$

Hence

$$V_1 = V_2(1 + \frac{Z_1}{Z_2}) - I_2 Z_1$$ **(0.5 mark)**

$$I_1 = \frac{V_2}{Z_2} - I_2$$ **(0.5 mark)**

In matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$ **(1 mark)**

i.e.

$$A = 1 + \frac{Z_1}{Z_2}, B = Z_1, C = \frac{1}{Z_2}, D = 1$$ **(0.5 mark)**

Q3(d)

To calculate S_{11}, the impedance Z_{in1} is required when the network is terminated with Z_o, where $Z_o = Z_{o1} = Z_{o2}$

$$Z_{in1} = \left[\frac{Z_2 \left(2Z_1 + \frac{Z_2 Z_o}{Z_2 + Z_o} \right)}{2Z_1 + Z_2 + \frac{Z_2 Z_o}{Z_2 + Z_o}} \right]$$

$$Z_{in1} = Z_2 \frac{2Z_1 Z_o + 2Z_1 Z_2 + Z_2 Z_o}{Z_2 Z_o + (Z_2 + Z_o)(Z_2 + 2Z_1)}$$ **(1 mark)**

$$S_{11} = \Gamma_{in} = \frac{Z_{in1} - Z_o}{Z_{in1} + Z_o}$$

then

$$S_{11} = \frac{Z_1 Z_2^2 - Z_2 Z_o^2 - Z_1 Z_o^2}{2Z_1 Z_o Z_2 + Z_o Z_2^2 + Z_2 Z_o^2 + Z_1 Z_2^2 + Z_1 Z_o^2}$$ **(1 mark)**

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2 = 0}$$

For the 1st port

$$V_1 = \sqrt{Z_o}(a_1 + b_1)$$

$$V_2 = \sqrt{Z_o} b_2$$ **(1 mark)**

Therefore

$$\frac{V_1}{V_2} = \frac{(a_1 + b_1)}{b_2} = \frac{(1 + \frac{b_1}{a_1})}{\frac{b_2}{a_1}} = \frac{1 + S_{11}}{S_{21}}$$

i.e.

$$S_{21} = \frac{V_2}{V_1}(1 + S_{11})$$

$$V_2 = V_1 \frac{Z_p}{(Z_p + 2Z_1)}$$ **(1 mark)**

where

$$Z_p = \frac{Z_o Z_2}{(Z_o + Z_2)}$$

Hence

$$\frac{V_2}{V_1} = \frac{Z_o Z_2}{(Z_o Z_2 + 2Z_1 Z_2 + 2Z_o Z_1)}$$ **(1 mark)**

$$1 + S_{11} = \frac{Z_2 \left(2Z_1 Z_o + Z_o Z_2 + 2Z_1 Z_2 \right)}{4Z_1 Z_o Z_2 + Z_o Z_2^2 + Z_2 Z_o^2 + Z_1 Z_2^2 + 2Z_1 Z_o^2}$$

Therefore

$$S_{21} = \frac{Z_o Z_2^2}{4Z_1 Z_o Z_2 + 2Z_o Z_2^2 + 2Z_2 Z_o^2 + Z_1 Z_2^2 + 2Z_1 Z_o^2}$$ **(0.5 mark)**

Similarly

$$S_{12} = \frac{Z_o Z_2^2}{4Z_1 Z_o Z_2 + 2Z_o Z_2^2 + 2Z_2 Z_o^2 + Z_1 Z_2^2 + 2Z_1 Z_o^2}$$ **(0.5 mark)**

Finally

$$S_{22} = \frac{Z_{in2} - Z_o}{Z_{in2} + Z_o} \qquad Z_{in2} = \left[\frac{Z_2 \left(2Z_1 + \frac{Z_2 Z_o}{Z_2 + Z_o} \right)}{2Z_1 + Z_2 + \frac{Z_2 Z_o}{Z_2 + Z_o}} \right]$$

$$Z_{in2} = Z_2 \frac{2Z_1 Z_o + 2Z_1 Z_2 + Z_2 Z_o}{Z_2 Z_o + (Z_2 + Z_o)(Z_2 + 2Z_1)}$$ **(1 mark)**

then

$$S_{22} = \frac{2Z_1 Z_2^2 - 2Z_2 Z_o^2 - 2Z_1 Z_o^2}{4Z_1 Z_o Z_2 + 2Z_o Z_2^2 + 2Z_2 Z_o^2 + 2Z_1 Z_2^2 + 2Z_1 Z_o^2}$$ **(1 mark)**

Q4.a

(i) *Conditional Stability;* the network is potentially unstable and may oscillate for certain combinations of load and source impedance values. **(1 mark)**

(ii) *Unconditional Stability*; the network is unconditionally stable for any combinations of source and load impedance values. **(1 mark)**

Q4.b

Generally a transistor presents a significant impedance mismatch, so matching will be achieved over a narrow frequency bandwidth. When bandwidth is an issue, then we design for a gain less than the maximum, imperfect matching, to improve bandwidth. **(1 mark)**

Sometimes it is required to design an amplifier with a specific gain, other than the maximum. *Constant gain circles* will be used to facilitate gain design. **(1 mark)**

Q4.c

For this transistor ***K***=1.18 and $|\Delta| = 0.56$**,** i.e. it is unconditionally stable. **(1 mark)**

Since a match is required at the input port, then the operating gain circles will be employed.

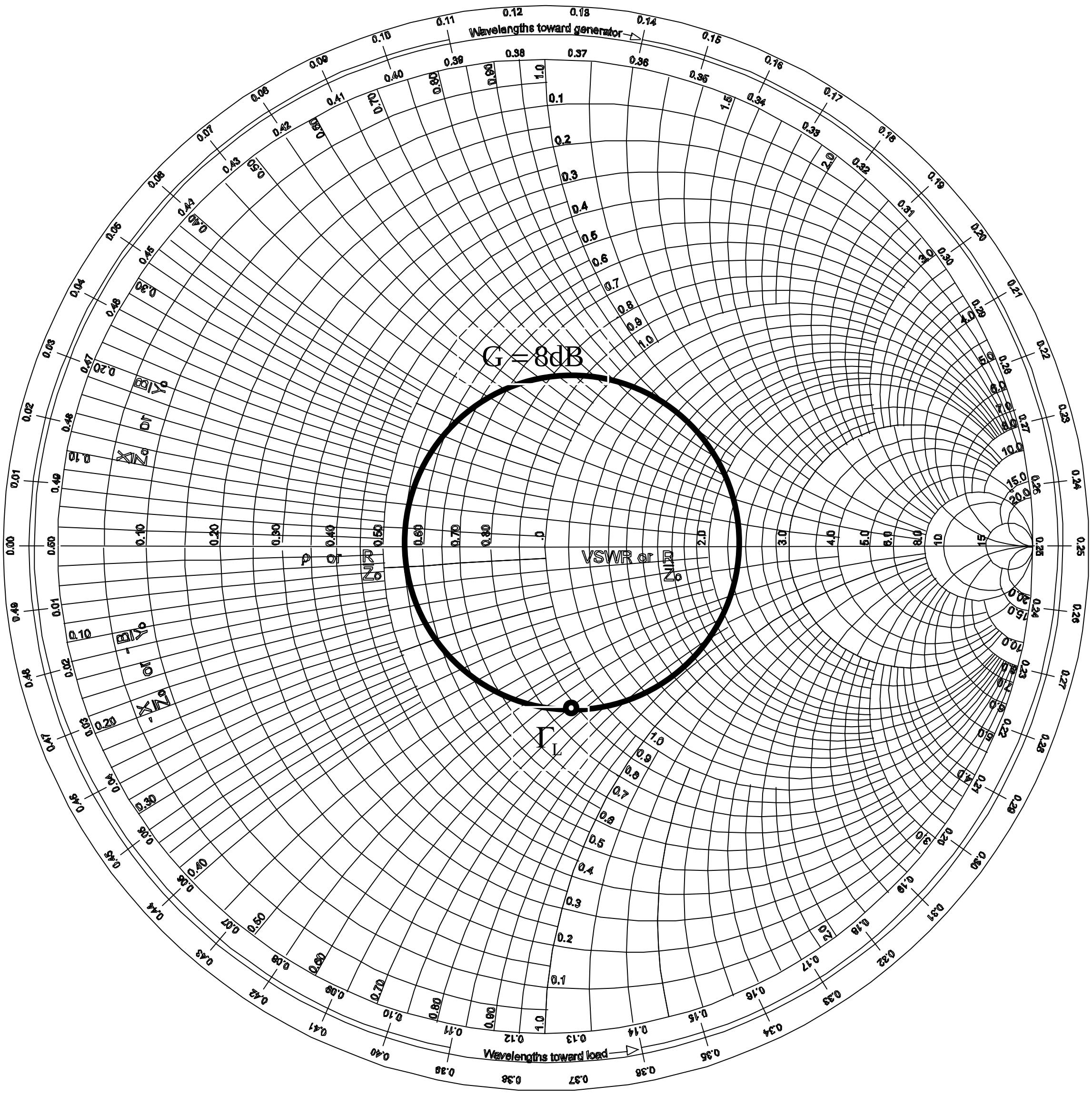
$$g_o = \frac{G}{|S_{21}|^2} = 1.0095$$ **(1 mark)**

This will give the radius and centre of the operating gain circle as $r_{g_o} = 0.35$ and

$C_{g_o} = 0.11\angle 69^\circ$. **(1 mark)**

The operating gain circle is shown in the following figure. **(1 mark)**

To simplify the output matching circuit, we pick Γ_L at the intersection of the constant gain circle with the *r*=1 circle, i.e. $\Gamma_L = 0.26\angle -75^\circ$. **(2 marks)**



Q4.d

$S_{11} = 0.61\angle -170^\circ$, $S_{21} = 2.24\angle 32^\circ$, $S_{12} = 0$ and $S_{22} = 0.72\angle -83^\circ$

i.e.

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = \infty$$

and

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| = 0.137$$

So the transistor is unconditionally stable. **(1 mark)**

The matching sections gain can be calculated from

$$G_{S_{max}} = \frac{1}{1-|S_{11}|^2} = 1.59 = 2.dB$$ **(0.5 mark)**

$$G_{L_{max}} = \frac{1}{1-|S_{22}|^2} = 2.08 = 3.18dB$$ **(0.5 mark)**

The maximum gain of the unmatched transistor

$G_o = |S_{21}|^2 = 7.0\text{dB}$ **(0.5 mark)**

The overall gain

$G_{T\max} = 3.18 + 7.0 + 2 = 12.18\text{dB}$

This means there is a 2.18dB gain higher than the design requirements. **(1.5 mark)**

The required gain circles can be plotted using the following set of equations

$$C_S = \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}$$

$$r_S = \frac{\sqrt{1-g_S}\,(1-|S_{11}|^2)}{1-(1-g_S)|S_{11}|^2}$$

$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2}$$

$$r_L = \frac{\sqrt{1-g_L}\,(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}$$

$$g_S = \frac{G_S}{G_{S\max}}$$

$$g_L = \frac{G_L}{G_{L\max}}$$

which results in

$G_S = 0.5\text{dB}$	$g_S = 0.71$	$C_S = 0.485\angle 170^\circ$	$r_S = 0.38$
$G_S = 1.5\text{dB}$	$g_S = 0.89$	$C_S = 0.59\angle 170^\circ$	$r_S = 0.23$
$G_L = 1.5\text{dB}$	$g_L = 0.68$	$C_L = 0.59\angle 83^\circ$	$r_L = 0.33$
$G_L = 2.5\text{dB}$	$g_L = 0.85$	$C_L = 0.66\angle 83^\circ$	$r_L = 0.2$

The constant gain circles are plotted on the Smith chart as shown in the figure.

(4 marks, 1 mark for each circle)

For an overall gain of 10dB, we will choose $G_S = 1.5\text{dB}$ and $G_L = 1.5\text{dB}$. We select Γ_S and Γ_L along these circles to minimise the distance from the centre of the chart. This will put Γ_S and Γ_L along the radial lines at 170° and 83° respectively. Thus $\Gamma_S = 0.33\angle 170^\circ$ and $\Gamma_L = 0.27\angle 83^\circ$. **(2 marks)**