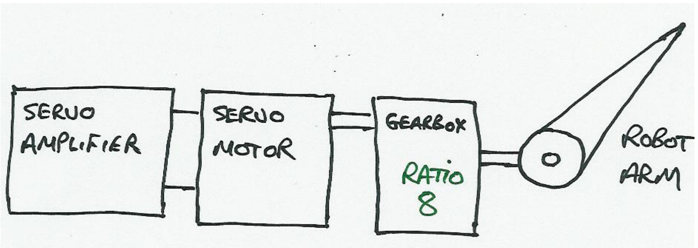
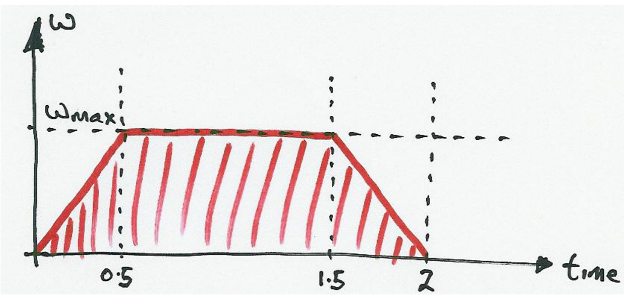
EEE 202 Example Answers – Summer 2011

Question 1

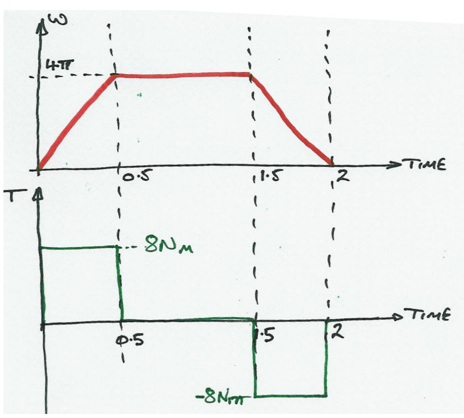
a. If a motor is connected to an inertial load – for example a robot arm as below,

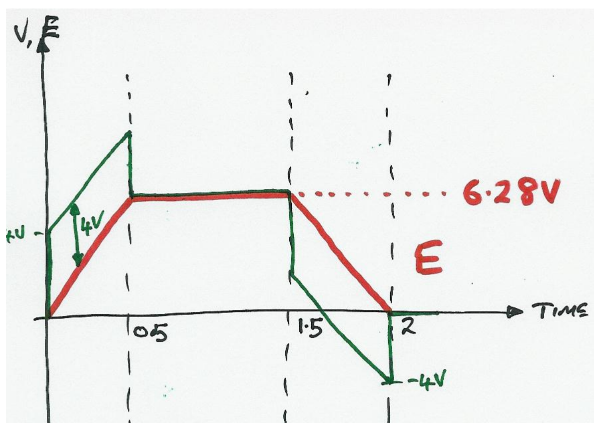


and it is required to move the load from 1 point to another in a certain specified time using a specified velocity profile:



Then the speed of the motor, and hence the back emf is always positive, whilst the torque from the motor, and hence the current needs to be bi-directional:



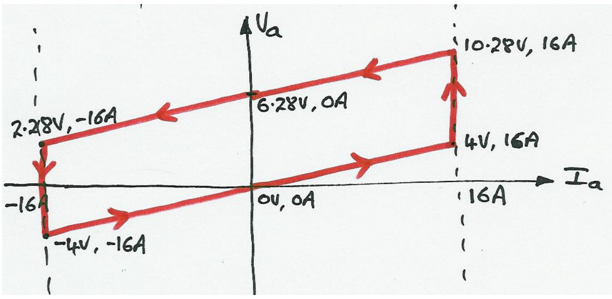


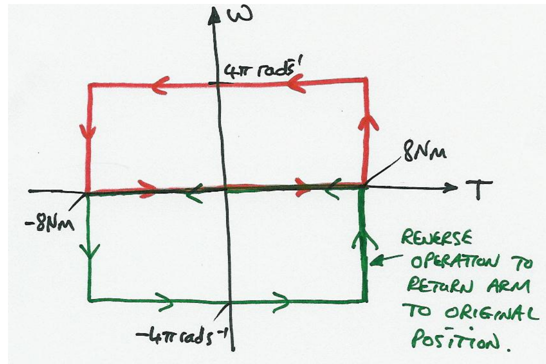
Motor Torque / speed

(Equates to motor I & E)

Motor voltage and back EMF

This leads to the motor operating in two quadrants, whilst the drive which has to overcome the resistive voltage drop of the windings has to operate in 3 quadrants:





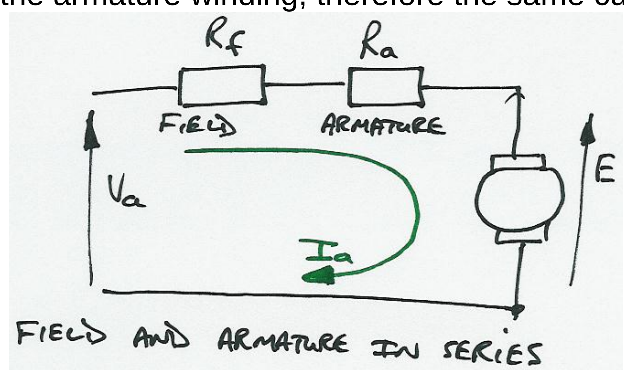
3 Drive operating quadrants

2 Motor operating quadrants

(+ reverse motion)

The diagrams here are from a specific example in the course notes, I will be looking for similar diagrams but without the numbers.

b. Series connected Wound field DC motor – Here field winding connected in series with the armature winding, therefore the same current flows through both windings ($I_a = I_f$).



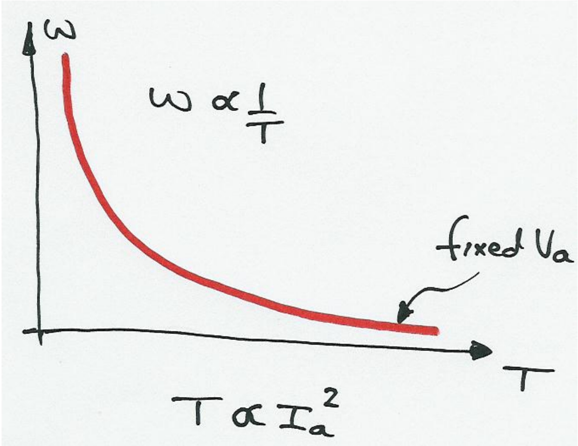
$$I_f = I_a$$

$$\therefore T = MI_f I_a = MI_a^2$$

$$\therefore E = MI_f \omega = MI_a \omega$$

Neglecting voltage drop across R_a and R_f we have:

$$E \approx Supply \;\; V_a$$
$$V_a \approx MI_a\omega$$
$$T = MI_a{}^2$$
$$V_a \approx M\sqrt{\frac{T}{M}}\omega$$
$$\omega \approx \frac{1}{\sqrt{M}\sqrt{T}}V_a$$
$$\therefore \omega \propto \frac{1}{\sqrt{T}} \quad and \quad T \propto I_a{}^2$$



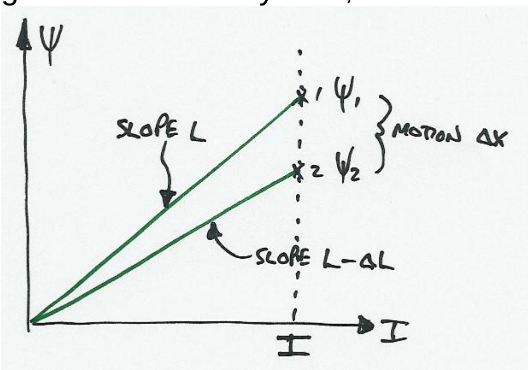
Hence torque is unidirectional and independent on current polarity.

Given the parameters in the question and equating the load torque to the motor torque (which is a function of Ia^2), a simple equation is produced for the motor speed which gives a speed of **10.71 rad/s** and this gives a current of (12-10.71)/1.2 = **1.07A**

c. It would be possible to run the motor from an ac supply, as the motor is series wound and the output torque is proportional to the square of the current, although the current reverses for half of the ac cycle, a unidirectional motor torque is produced. However, the applied voltage would now have to be increased as the winding inductance would dominate the motor impedance.

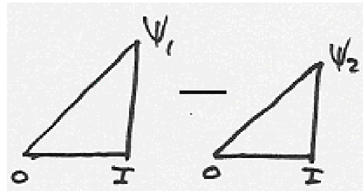
Question 2.

a. Examining the Ψ / I diagram for a linear system,



at ‘1’, Solenoid inductance = L
at ‘2’, Solenoid inductance = L-ΔL (reduces by ΔL)

Therefore the mechanical work performed over the motion Δx is given by the the difference between the two triangles below:-



As

$$W_{co} = \int \psi . dI$$

and the area of the triangle is therefore:

$$W_{co} = \frac{1}{2}\psi . I$$

Therefore, from the area’s, the mechanical work done is:

$$W_{co1} - W_{co2} \quad = \quad \frac{1}{2}\psi_1 I - \frac{1}{2}\psi_2 I$$

As we know, inductance, $L = \frac{\psi}{I}, \quad \psi = L.I$

Therefore the work done becomes:

$$= \frac{1}{2}(L.I).I - \frac{1}{2}((L - \Delta L).I).I$$

$$= \frac{1}{2} I^2 \Delta L$$

And also, as work performed = F.Δx, from this we can get the force, F:

$$F = \frac{1}{2} I^2 \frac{\Delta L}{\Delta x}$$

which, as ΔL and Δx tend to zero, becomes:

$$F = \frac{1}{2} I^2 \frac{dL}{dx}$$

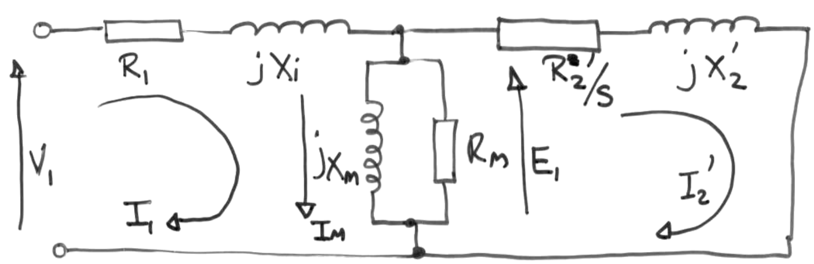
b. From the profile given and the information as to how the inductance of the coil changes with projectile position, the force on the projectile is only positive when it is moving from ‘A’ to ‘B’, the change in inductance with position becomes negative after this point and decelerates the projectile. Therefore the current is only applied as the projectile travels from A to B. It therefore moves 5cm, accelerating from standstill at A to 100m/s at B. From v=u+at and S=ut +0.5at^2 it is possible to calculate the time of the motion as being 1ms, this is the duration of the current pulse.

The amplitude of the current pulse is given from the force equation above, accelerating 25g from standstill. This gives a required current of 2236A, or 2.2kA for 1ms.

c. The maximum stored energy in the coild will be at the peak current (2236A) and the maximum inductance (100μH). This is given from $0.5LI^2$ = 250J. The maximum power dissipated in the coil resistance is given from I^2Rt, and gives 5J for the 1ms current pulse of 2.2kA

Question 3.

FULL STATOR EQUIVALENT CCT.



R_1 = STATOR RESISTANCE PER PHASE

R_2' = REFERRED ROTOR RESISTANCE /ϕ

X_1 = STATOR LEAKAGE REACTANCE /ϕ

X_2' = REFERRED ROTOR LEAKAGE REACTANCE /ϕ

X_m = MAGNETIZING REACTANCE /ϕ

R_m = IRON LOSS RESISTANCE /ϕ

V_1 = RMS SUPPLY PHASE VOLTAGE /ϕ

E_1 = INDUCED STATOR PHASE VOLTAGE

I_2' = REFERRED ROTOR CURRENT

I_m = MAGNETIZING CURRENT

I_1 = STATOR CURRENT.

c. From the locked rotor test results given, R_2' = 0.3Ω, and X_T = 2.6Ω.

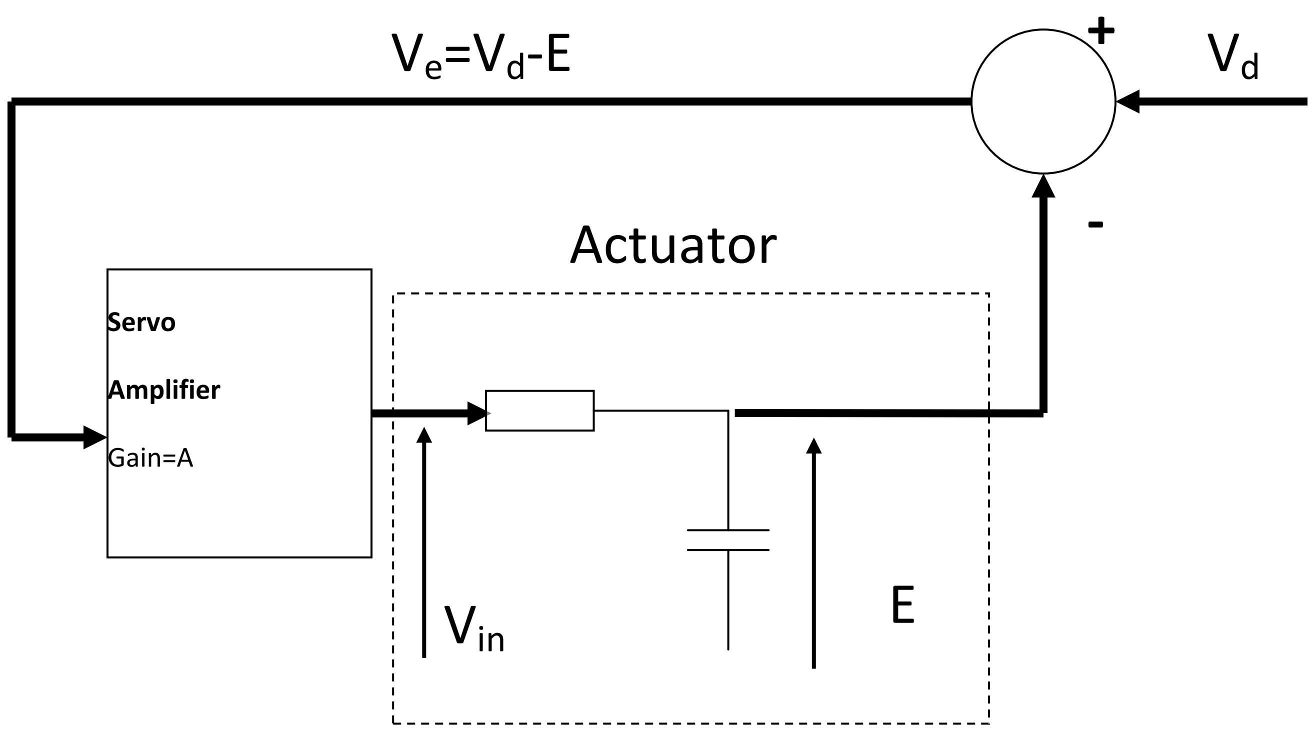
From this and the equation for the peak torque:

$$T_{PULL\text{-}OUT} = \frac{3pV_1^2}{2\pi f_1} \frac{\sqrt{R_1^2+(X_1+X_2')^2}}{\left(R_1+\sqrt{R_1^2+(X_1+X_2')^2}\right)^2+(X_1+X_2')^2}$$

We can work out that the maximum load for a 30% reduction in supply voltage will be 1.43Nm under the worst case voltage conditions.

Question 4.

Given that a simple voice-coil actuator with an assumed purely inertial load may be modelled as a first order system (RC circuit), it may be used in a circuit where it is fed from an amplifier as below:



In this case, $$V_{in} = AV_e = A(V_d - E) \qquad (1)$$

$$V_{in} = iR + E \qquad (2)$$

substitute (1) into (2) for V_{in}…

$$AV_d = iR + (1+A)E$$

$$\frac{A}{1+A}V_d = i\frac{R}{1+A} + E \qquad (3)$$

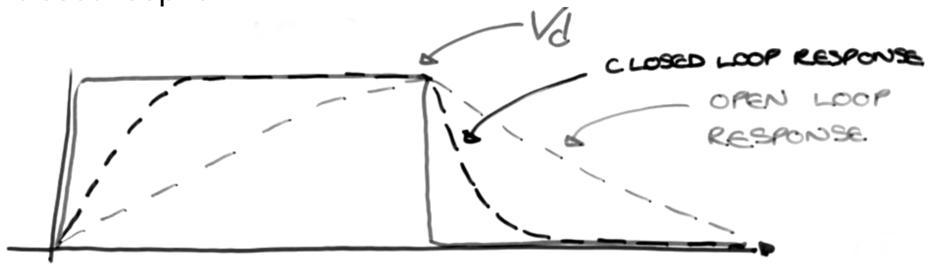
Now….

$$i = C\frac{dE}{dt} \qquad (4)$$

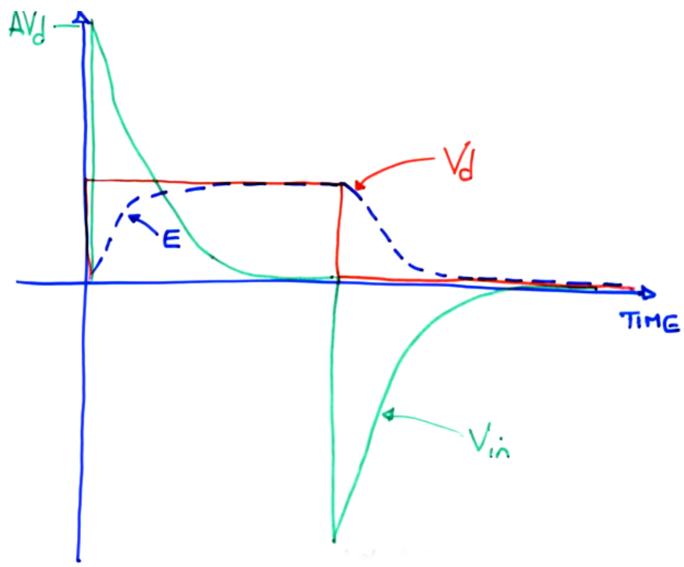
and solving (3) and (4) together for E as a differential equation

$$E = \frac{A}{1+A}V_d\left(1 - e^{\frac{-t}{\left(RC/_{1+A}\right)}}\right)$$ which is the closed loop term.

The time constant of the response has been changed from $\tau = RC$ in open loop to $\tau = RC/(1+A)$ in closed loop form.



The improved response is achieved by driving the actuator with higher voltages than in the open loop case.



b. In the equivalent circuit of the system, the mass and inertia is modelled as an equivalent capacitance. This is given by C = M/K_e^2

From the data given, C = 0.833F.

The open loop time constant is τ = RC, therefore τ = 0.0833sec (83.3ms)

To make the closed loop time constant less than 20ms, the amplifier gain in a closed loop system needs to be greater than 3.17.

c. If the output of the amplifier is required to go above the maximum supply voltage, (42V in this case) due to the action of the amplifier gain on the feedback voltage, the output would be practically limited to the maximum supply voltage (- a small operating voltage drop) and the 'forcing' voltage applied to the actuator would not be as high as would be required to get the response predicted from the system. This is the saturation of the amplifier output, and would slow down the system response.