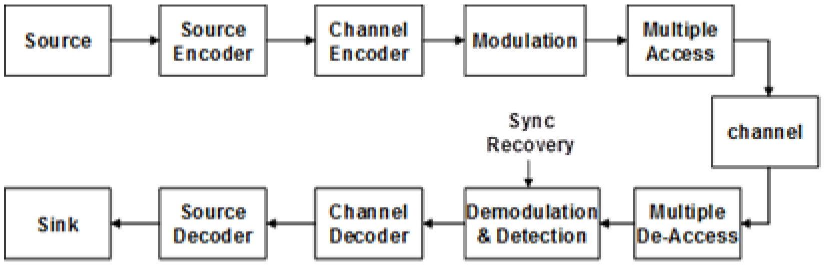
Marks **Q 1**

a) Main functional blocks of a digital wireless communication system:

3. 

1. **i. Channel coding:** Adds controlled redundancy to allow detection or correction of symbol errors at the receiver.

1. **ii. Modulation:** Translates baseband message symbols to a waveform suitable for transmission over a physical medium.

2. **b) Free space pathloss** - The received power follows an inverse quadratic law in both range (d) and frequency (f). That is $P_r \propto 1/d^2$ and $P_r \propto 1/f^2$.

1. For isotropic Tx and Rx antennas (i.e. unity gain), the free space pathloss is given by:

1. $P_L(dB) = 32.44+20\log(f_{MHz})+20\log(d_{Km}) = 32.44+20\log(2000)+20\log(5) = 112.44$ dB

1. Power at Tx is $P_{tx}(dB) = P_L(dB) +10-105 = 112.44-95 = 17.44$ dBm ≡ **55.5 mW** #

2. **c) Model differences:** The 2-Ray pathloss model theoretically does not depend on frequency and exhibits a $1/d^4$ dependency at ranges above the critical distance. For ranges below the critical distance, the 2-ray model exhibits peaks and troughs in the pathloss due to non-grazing incidence of the ground wave.

Isotropic Tx and Rx antennas have unity gain. Hence 2-ray pathloss is given by

3. $P_L(dB) = 40\log(d) - 20\log(h_t) - 20\log(h_r) = 40\log(5000) - 20\log(20) - 20\log(2) = 115.92$ dB

Power above the noise floor = $P_r - (-105) = P_t - P_L + 105 = 17.44 - 115.92 + 105 = 6.62 \leq 10$ dB

Therefore mobile will not achieve reliable communication #

2. **d) Shawdow Fading:** Is the random variation in received signal strength caused by attenuation due to blockages within the line-of-sight between the transmitter and receiver.

$P_t = 55.5$ mW ≡ 17.44 dBm, $P_r = 17.44 - 115.92 = -98.48$ dBm, $P_{min} = 10 - 105 = -95$ dBm, $\sigma_{dB} = 10$ dB

1. $$P_{out}(-95\text{dBm}, 5\text{km}) = \text{Prob}(P_r(5\text{km}) < -95\text{dBm}) = 1 - Q\left(\frac{P_{min} - P_r}{\sigma_{dB}}\right)$$

(USE GRAPHICAL Q-FUNCTION)

1. $$P_{out} = 1 - Q\left(\frac{-95-(-98.48)}{10}\right) = 1 - Q(0.348) = \underline{1 - 0.364 = 0.636 \quad \text{or} \sim 64\,\%} \ \#$$

1. **Comment:** The outage due to shadow fading is high at 64 %. This is a very high outage rate and demonstrates how severely the quality of service is affected when the received signal strength falls below the 10 dB SNR margin.

[20]

Marks **Q 2**

a) Constructive and destructive interference in mobile radio channel:

1. Differential time delays introduce fixed relative phase shifts between component waves causing constructive and destructive addition at one instant of time or space.

1. When the receiver is moving there is a continuous change in electrical length of every propagation path and thus the relative phases change with spatial location. A moving receiver experiences time varying fades.

Envelope is Rayleigh distributed $p_R(r) = \frac{2r}{\overline{P}_r}\exp\left[-\frac{r^2}{\overline{P}_r}\right]$

1. Power distribution obtained by substituting $z = r^2$, then $dz = 2rdr$ and $p(r)dr = p(z)dz$ which gives

2. $p(z) = p(r)\frac{dr}{dz} = \frac{p(r)}{2r} = \underline{\frac{1}{\overline{P}_r}\exp\left(-\frac{z}{\overline{P}_r}\right)}$ #

1. **b)** The system will experience unacceptable bit errors when $P_R \le \overline{P}_r/5$

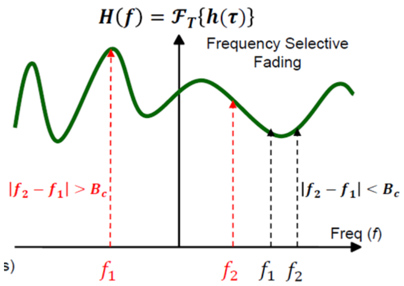
1. $\Pr\{P_r < P_R\} = \int_0^{P_R}\frac{1}{\overline{P}_r}e^{-z/\overline{P}_r}dz = \frac{1}{\overline{P}_r}\left[-\overline{P}_r e^{-z/\overline{P}_r}\right]_0^{P_R} = 1 - e^{-P_R/\overline{P}_r}$

2. $= 1 - e^{-1/5} = \underline{0.18\ (18\%)}$ #

1. In Rician fading there is a fixed line-of-sight component in the received signal which if large compared to the scattered multipath components results in less fading. Therefore, the probability of error will be less for same received power.

c) Coherence B/W:

2. The coherence b/w B_c is the minimum frequency separation for which the channel frequency response is independent. That is the autocorrelation between two frequencies f_1 and f_2 is zero or in the case of $B_c(50\%)$ the ACR(f_1, f_2) = 0.5. For ACR(f_1, f_2) = 0.5, $B_c(50\%) = 1/5\tau_{rms}$, therefore need to calculate τ_{rms} for the channel given:



ExcessDelay μs	PathPowerdBr	PathPowerLinear
0	0	1
1	−3	0.5
2	0	1
4	−3	0.5
8	−6	0.25

2. $$\overline{\tau} = \frac{1(0)+0.5(1)+1(2)+0.5(4)+0.25(8)}{1+0.5+1+0.5+0.25} = 2\mu s$$

2. $$\overline{\tau^2} = \frac{1(0^2)+0.5(1^2)+1(2^2)+0.5(4^2)+0.25(8^2)}{1+0.5+1+0.5+0.25} = 8.77\ (\mu s)^2$$

$$\tau_{rms} = \sqrt{\overline{\tau^2} - \overline{\tau}^2} = 2.184\ \mu s$$

$$B_c(50\%) = \frac{1}{5\cdot\tau_{rms}} = \frac{1}{5\times 2.184\times 10^{-6}} = \underline{91.6\ kHz} \quad \#$$

2. **d)** Coherence time is a statistical measure of the time duration over which the mobile channel impulse response does not change. That is, the time over which the received signal envelope does not change.

The 50% coherence time is given by $T_c = \frac{9}{16\pi f_D}$ where f_D is the maximum Doppler shift.

2. $$f_D = \frac{\upsilon}{\lambda} = \frac{50000/3600}{3\times 10^8/2\times 10^9} = \frac{13.89}{0.15} = 92.6\ Hz \quad and \quad T_c = \frac{9}{16\times\pi\times 92.6} = \underline{1.93\ ms} \quad \#$$

[20]

a) Optimum Power Adaptation: Since the $\boldsymbol{g}[\boldsymbol{i}]$ are time varying, the optimum power adapation policy to maximise the capacity is a "water filling" formula in time. That is, when channel conditions are good then transmit with more power. 3.

b) Ergodic Capacity of narrowband channel –

1. Given: average transmit power constraint P = 200 mW, N_0= 10^{-9} W/Hz and Channel Bandwidth B = 20 MHz
$P/N_0B = (0.2)/(10^{-9}\times20\times10^6) = 10$
The channel has 3 possible received SNRs:
1. $\gamma_1 = \boldsymbol{g}[1]\, P/N_0B = 0.2\times 10 = 2$
$\gamma_2 = \boldsymbol{g}[2]\, P/N_0B = 1.5\times 10 = 15$
$\gamma_3 = \boldsymbol{g}[3]\, P/N_0B = 0.9\times 10 = 9$

Given P(γ_1) = 0.2, P(γ_2) = 0.5, and P(γ_3) = 0.3

2. $$\sum_{\gamma_i\geq\gamma_o}\left(\frac{1}{\gamma_o}-\frac{1}{\gamma_i}\right)\rho(\gamma_i)=1 \quad\Rightarrow\quad \frac{1}{\gamma_o}=1+\sum_{i=1}^{3}\frac{\rho(\gamma_i)}{\gamma_i}=1+\frac{0.2}{2}+\frac{0.5}{15}+\frac{0.3}{9}=1.1667 \quad \therefore\gamma_o=1/1.1667=0.86$$

1. Since $\gamma_o<\gamma_i \quad \forall i$, then all instances of the channel can be used to send data bits and the capacity is given by:

2. $$C=\sum_1^3 B\log_2\left(\frac{\gamma_i}{\gamma_o}\right)\rho(\gamma_i)=20\times10^6\left[0.2\log_2\left(\frac{2}{0.86}\right)+0.5\log_2\left(\frac{15}{0.86}\right)+0.3\log_2\left(\frac{9}{0.86}\right)\right]=\underline{66.54}\ Mbit/s\ \#$$

c) Other capacities

i. Ergodic Capacity:

2. $$C=\sum_1^3 B\log_2(1+\gamma_i)\rho(\gamma_i)=20\times10^6\left[0.2\log_2(3)+0.5\log_2(16)+0.3\log_2(10)\right]=\underline{66.27}\ Mbit/s\ \#$$

ii. Shannon Capacity in AWGN:

2. $$\bar{\gamma}=\sum_1^3\gamma_i\rho(\gamma_i)=0.2\times2+0.5\times15+0.3\times9=10.6 \quad\Rightarrow\quad C=B\log_2(1+\bar{\gamma})=20\times10^6\times\log_2(11.6)=\underline{70.72}\ Mbit/s\ \#$$

2. Comment: The AWGN capacity gives an upper bound that is significantly higher than the Ergodic capacity and the Water-filling capacity. However, the latter two are almost equal with Water-filling giving the optimum performance.

d) Optimum Power Allocations

2. $$P(\gamma_i)=\left(\frac{1}{\gamma_o}-\frac{1}{\gamma_i}\right)\times P \quad\Rightarrow$$

2. $$P(\gamma_1)=\left(\frac{7}{6}-\frac{1}{2}\right)\times0.2=0.1333\,W \quad P(\gamma_2)=\left(\frac{7}{6}-\frac{1}{15}\right)\times0.2=0.22\,W \quad P(\gamma_3)=\left(\frac{7}{6}-\frac{1}{9}\right)\times0.2=0.2111\,W$$

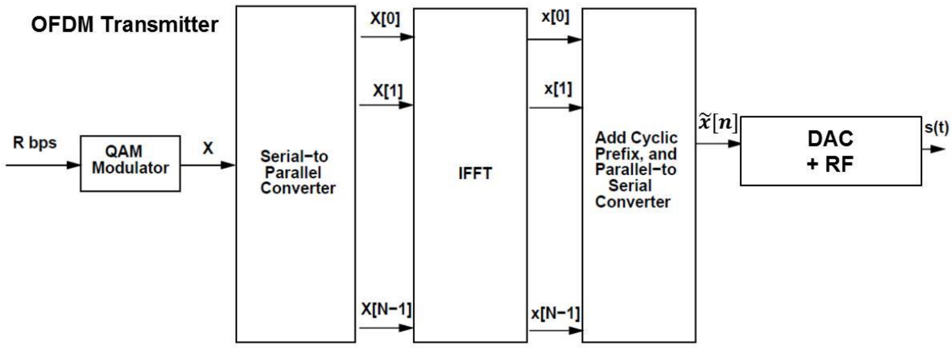
Check: $P=0.1333\times0.2+0.22\times0.5+0.2111\times0.3=0.2 \quad QED$

[20]

Marks **Q 4**

a) MC Techniques: Used to reduce ISI. The number of substreams or subchannels are selected in order to make the
1. symbol time on each substream greater than the delay spread of the broadband wireless channel. For this condition, the substream bandwidth is less than the channel coherence bandwidth.

2.



The purpose of the cyclical prefix (CP) is:

1. i.) to eliminate ISI between OFDM data blocks as the CP is discarded at the receiver; and

1. ii.) the CP in a digital sampling system converts a linear channel convolution into a circular channel convolution which means that each OFDM subcarrier can be equalised by a single tap filter in the frequency domain.

b) Main feature of 802.11a PHY:

- N =64 subcarriers, though only 48 are used to carry data (12 are nulled and 4 are used for pilot symbols).
- The cyclic prefix uses $\mu = 16$ samples.
- 2. Total number of samples per OFDM symbol = 64 + 16 = 80 samples
- The same channel code and modulation order is used on all subcarriers.
- Possible code rates are ½, 2/3 and ¾.
- Possible modulation orders are BPSK, QPSK, 16QAM and 64QAM.
- Each OFDM channel has a 20 MHz bandwidth.

Duration of an OFDM block in 802.11a is: (64+16)/20MHz = 4 μs

The data rate R = (# data subcarrier x Code rate x bits per modulation symbol)/ 4 μs

3.

1. Code rate = ½, QPSK modulation gives R = (48 x 0.5 x 2)/ 4 μs = 12 Mbit/s
2. Code rate = ½, 16 QAM modulation gives R = (48 x 0.5 x 4)/ 4 μs = 24 Mbit/s
3. Code rate = 3/4, 64QAM modulation gives R = (48 x 0.75 x 6)/ 4 μs = 54 Mbit/s

c) Minimise ISI: A spreading sequence should have low aperiodic autocorrelation sidelobes in order to reduce ISI. This
1. property is expressed by the Merit Factor given below. The larger the Merit factor the greater the ISI reduction.
Merit factor:

$$F = \frac{|C_a(0)|^2}{2\sum_{\tau=1}^{N-1}|C_a(\tau)|^2}, \quad C_a(\tau) = \sum_{n=0}^{N-1-\tau} a_n a_{n+\tau}$$

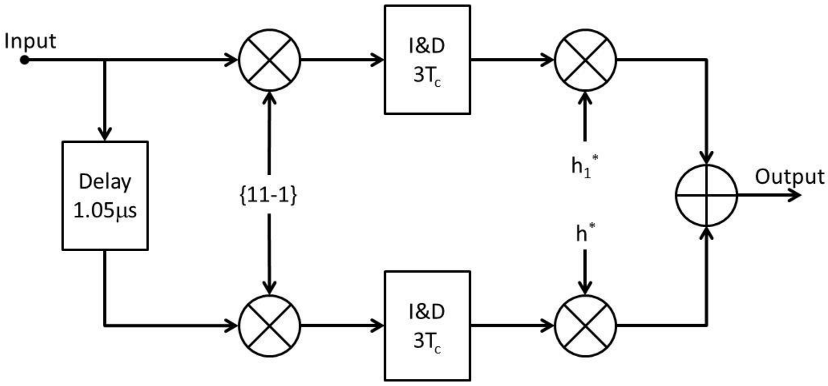
A suitable length 3 binary bipolar spreading sequence must come from the 2^3 = 8 binary tuples {111, 11-1, 1-11, 1-1-1, -111, -11-1, -1-11, -1-1-1}. However, there are only two unique sequences, the others being formed from cyclical shifts or
2. inversions of {111, 11-1}. Of these two sequences only {11-1} can be a spreading sequences. There are three possible initial phases of {11-1} giving 3 possible aperiodic ACFs and associated Merit Factors as follows:

$C_a(\tau)$ of $[+1+1-1]$ = -1 0 3 0 -1 $\Rightarrow F = 3^2/2\times(0^2+1^2) = 4.5$

$C_a(\tau)$ of $[+1-1+1]$ = 1 -2 3 -2 1 $\Rightarrow F = 3^2/2\times(2^2+1^2) = 0.9$

1. $C_a(\tau)$ of $[-1+1+1]$ = -1 0 3 0 -1 $\Rightarrow F = 3^2/2\times(0^2+1^2) = 4.5$

There are two best sequences with identical properties: <u>{11-1, -111}</u> #

	Q4 Contd.
1.	**d) RAKE receiver:** A RAKE receiver provides a method of diversity combining which involves collecting the signal energy in each significant multipath component of a frequency selective channel. Provided each multipath is at least delayed by one chip duration T_c then the receiver can resolve individual multipaths.
2.	A suitable RAKE receiver for the channel impulse response is: Since $T_c = \frac{1}{R_c} = \frac{1}{1\times10^6} = 1\mu s$, then multipath h_1 can be resolved from h_2 and h_3. However, multipaths h_2 and h_3 cannot be resolved. Instead, h_2 and h_3 will be unresolved as one complex path $h = h_2 + h_3$ with a mean delay (1+1.1)/2 = 1.05 μs. Then a suitable RAKE receiver will be:
2.	
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