USEFUL EQUATIONS – EEE123

Electric Circuits

Resistance (*R*) – units Ohms (Ω)

Resistors in series $R_{TOT} = R_1 + R_2 + R_3 + \cdots R_n$

Resistors in parallel $\dfrac{1}{R_{TOT}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots \dfrac{1}{R_n}$

Resistance (Ohms law) $R = \dfrac{V}{I}$

Resistance $R = \dfrac{\rho l}{A} = \dfrac{l}{\sigma A}$

(*l=length, m; A = cross-sectional area, m^2; ρ = resistivity, Ω m; σ = conductivity, S/m*)

Temperature dependence of resistors $R_{T1} = R_0(1 + \alpha_0 T_1)$

α_0 = temperature coefficient of resistance
R_0 = Resistance (Ω) at 0 °C
R_{T1} = Resistance (Ω) at T_1 °C
T_1 = Temperature in °C
For temperatures T_1 and T_2 use ratio

$$\frac{R_{T1}}{R_{T2}} = \frac{(1 + \alpha_0 T_1)}{(1 + \alpha_0 T_2)}$$

Voltage across a resistor $V_R = IR$

Power dissipated in a resistor $P = I^2 R = \dfrac{V^2}{R} = V \cdot I$

Energy dissipated in a resistor $E = I^2 Rt = \dfrac{V^2 t}{R} = V \cdot I \cdot t$

Capacitance (*C*) – units Farads (F)

Charge (*Q*) $Q = I \cdot t$ (Constant current , I)

$Q = \int_0^t i(t)dt$ (Time varying current, $i(t)$)

$Q = CV$ (Capacitance × Voltage)

Capacitors in series $\dfrac{1}{C_{TOT}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \cdots \dfrac{1}{C_n}$

Capacitors in parallel $C_{TOT} = C_1 + C_2 + C_3 + \cdots C_n$

Voltage across a capacitor $V_c = \dfrac{Q}{C}$

Energy stored in a capacitor $E = \dfrac{1}{2}CV^2$

Inductance (*L*) – units Henrys (H)

Inductors in series $L_{TOT} = L_1 + L_2 + L_3 + \cdots L_n$

Inductors in parallel $\dfrac{1}{L_{TOT}} = \dfrac{1}{L_1} + \dfrac{1}{L_2} + \dfrac{1}{L_3} + \cdots \dfrac{1}{L_n}$

Voltage across an inductor $V_L = L\dfrac{dI}{dt}$

Energy stored in an inductor $E = \dfrac{1}{2}LI^2$

A.C. Circuits

Power dissipated in a resistance $P = I_{rms}^2 R = \frac{V_{rms}^2}{R}$

For other circuits having capacitance and/or inductance, there is a phase shift, ϕ, between the current and voltage waveforms.

Real Power (P) (*Watts*) $P = V_{rms} I_{rms} \times \cos\phi$

Reactive Power (Q) (*VARs*) $Q = V_{rms} I_{rms} \times \sin\phi$

Power factor (p.f. or cos ϕ) $0 < \cos\phi < 1$

Power = Energy s^{-1} $Watts(W) = Js^{-1}$

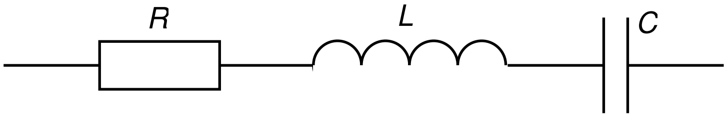
Capacitive reactance $X_C = \frac{1}{j\omega C} = \frac{1}{j2\pi f C} = \frac{-j}{\omega C} = \frac{-j}{2\pi f C}$

(ω = electrical frequency in rad/s (= $2\pi f$); f is the electrical frequency in Hz)

Inductive reactance $X_L = j\omega L = 2\pi fL$

(ω = electrical frequency in rad/s (= $2\pi f$); f is the electrical frequency in Hz)

Series Resonant Circuit

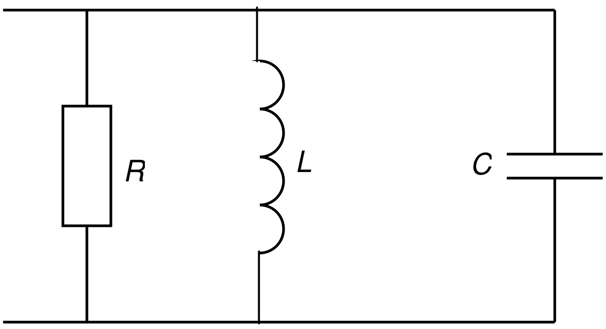


At resonance $X_C = X_L$ and $Z = R$

Frequency for resonance $\omega_r = \frac{1}{\sqrt{LC}}$ or $f_r = \frac{1}{2\pi\sqrt{LC}}$

Q factor $Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Parallel Resonant Circuit



At resonance $X_C = X_L$ and $Z = R$

Frequency for resonance $\omega_r = \frac{1}{\sqrt{LC}}$ or $f_r = \frac{1}{2\pi\sqrt{LC}}$

Q factor $Q = \frac{R}{\omega_r L} = \omega_r CR = R\sqrt{\frac{C}{L}}$

Transient Circuits

Current growth in an inductive circuit containing inductance and resistance:

Instantaneous current $i = I_0(1-e^{-t/\tau})$ where $\tau = L/R$

Current decay in an inductive circuit containing inductance and resistance:

Instantaneous current $i = I_0 e^{-t/\tau}$ where $\tau = L/R$

Charging a capacitor through a resistor:

Instantaneous voltage $v = V_0(1-e^{-t/\tau})$ where $\tau = RC$

Instantaneous current $i = I_0 e^{-t/\tau}$ where $\tau = RC$

Disharging a capacitor through a resistor:

Instantaneous voltage $v = V_0 e^{-t/\tau}$ where $\tau = RC$

Instantaneous current $i = -I_0 e^{-t/\tau}$ where $\tau = RC$

Magnetic Circuits

Reluctance (*S*) – units H^{-1} $S = \dfrac{l}{\mu_0 \mu_r A}$

(*l=length, m; A = cross-sectional area, m^2; μ_0 = permeability of free space (Hm^{-1}); μ_r = relative permeability)*

Inductance (L) $L = \dfrac{N^2}{S}$

(*N = number of turns on the coil*)

Flux density (*B*) – units Tesla (T) $B = \mu_0 \mu_r H = \dfrac{\phi}{A}$

(H = magnetic field strength(A/m); φ = flux (Wb))

MagnetoMotive Force – MMF $F = H \cdot l = N \cdot I = \phi S$

Induced EMF (*E*) – units Volts (V) $E = N\dfrac{d\phi}{dt}$

Transformers (ideal)

Voltage ratio $\dfrac{V_{in}}{V_{out}} = \dfrac{N_1}{N_2} = turns \quad ratio$

Current ratio $\dfrac{I_{in}}{I_{out}} = \dfrac{N_2}{N_1} = \dfrac{1}{turns \quad ratio}$

Impedance ratio $\dfrac{Z_{in}}{Z_{out}} = \left(\dfrac{N_1}{N_2}\right)^2 = (turns \quad ratio)^2$

V_{in} = voltage across primary winding; *I_{in} = current through primary winding;*
V_{out} = voltage across secondary winding; *I_{out} = current through secondary winding;*

Induced voltage $V_{rms} = 4.44f \cdot N \cdot \phi_{max}$

(ϕ_{MAX} = maximum flux in the transformer core (Wb); f = frequency (Hz))

Mechanics

Mechanical Power (W) $P_{mech} = \omega_{mech} \cdot T$

(T = torque (Nm); ω_{mech} = rotational speed (rad/s))

Rotational speed (rad/s) $\omega_{mech} = \frac{2\pi}{60} \cdot n$

(n = speed in revs per minute)

Torque $T = F \cdot r$

(F = force(Nm); r = radius (m))

Gearbox (no losses) $\omega_{in} T_{in} = \omega_{out} T_{out}$

DC motors

Force on a current carrying conductor $F = B \cdot I \cdot l$

(B = flux density (T); I = current (A); l = length (m))

DC motor armature voltage $V_A = E_A + I_A R_A$

(E_A = induced emf (V); I_A = armature current (A); R_A = armature resistance (Ω))

Induced voltage, E_A is proportional to the speed of rotation, ω (rad/s), and the flux, ϕ (Wb).
Torque, T is proportional to the armature current, I (A) and the flux, ϕ (Wb).

For a wound field machine:

Induced emf (V) $E_A = K I_F \omega$

Torque (Nm) $T = K I_F I_A$

(I_F = field current (A); K = constant)

For a permanent magnet machine:

Induced emf (V) $E_A = K_E \cdot \omega$

Torque (Nm) $T = K_T \cdot I_A$

(K_E (V/rad/s) and K_T (Nm/A) are constants with the same numerical values)

Electronic Circuits

$$g_m = \frac{eI_C}{kT} \qquad r_{be} = \frac{\beta}{g_m} \qquad h_{FE} = \frac{I_C}{I_B} \qquad \beta = \frac{\Delta I_C}{\Delta I_B} = \frac{i_c}{i_b} \qquad \tau = RC$$

$$I = C\frac{dV}{dt} \qquad \omega = 2\pi f \qquad V(t) = (V_{START} - V_{FINISH})\exp\left(\frac{-t}{\tau}\right) + V_{FINISH}$$

$V_{AVE} = \frac{V_P}{\pi}$ for a half-wave rectified sinusoid $\qquad$ $V_{rms} = \frac{V_P}{\sqrt{2}}$ for a sinusoid

$$v_0 = A_v\left(v^+ - v^-\right) \qquad \frac{kT}{e} = 0.026\,\text{V}$$

All the symbols have their usual meanings

Standard Unit Multipliers

$p = \times 10^{-12}$, $n = \times 10^{-9}$, $\mu = \times 10^{-6}$, $m = \times 10^{-3}$, $k = \times 10^{3}$, $M = \times 10^{6}$, $G = \times 10^{9}$