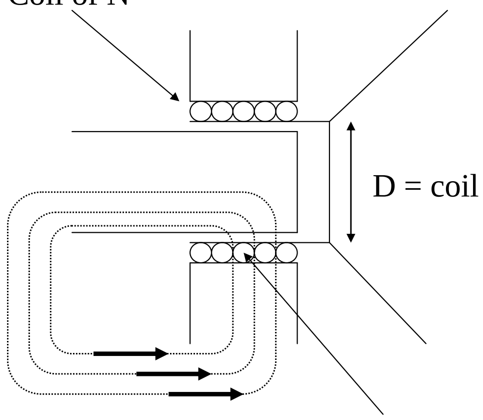
EEE202 - June 2009 Answers

Qu 1.
a.



<u>Assumptions</u>

1. Current in coil is proportional to the applied voltage, which implies that across the full frequency range, the loudspeaker has a constant impedance (e.g. 8Ω)

2. Force developed on cone is directly proportional to coil current

$$F = BIL$$

which implies a uniform field across the full displacement of the coil

3. Displacement of the cone is proportional to the applied force i.e. the suspension spring has a uniform force-displacement characteristic

Length of interaction of coil with the magnetic field = coil circumference = πD

Thus;

$$F = BI\pi DN$$ where N is the number of series turns

$$F = (BN\pi D)I$$

b.

<u>Mechanical system</u>

<u>Mass of cone and the air that it moves</u>

$$F = Ma = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$

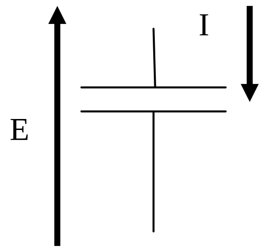
Substituting for F and v

$$K_eI = M\frac{d\left(E/K_e\right)}{dt}$$

$$I = \frac{M}{K_e^2}\frac{dE}{dt}$$

Thus the electrical equivalent can be given as

$$I = C\frac{dV}{dt}$$ i.e, mass is represented as a capacitance



$$M \big/ K_e^2$$

Spring

$$F = \sigma_s x$$

spring compliance

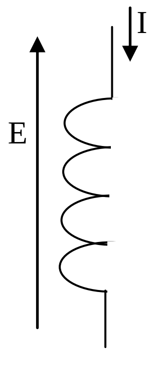
$$K_e I = \sigma_s \int v.dt = \sigma_s \int \left(\frac{E}{K_e}\right).dt$$

$$I = \frac{\sigma_s}{K_e^2}\int E.dt \quad \text{or....} \; E = \frac{K_e^2}{\sigma_s}\frac{dI}{dt}$$

Giving the electrical equivalent

$$V = L\frac{dI}{dt}$$

$$I = \frac{1}{L}\int V.dt$$



$$\frac{K_e^2}{\sigma_s}$$

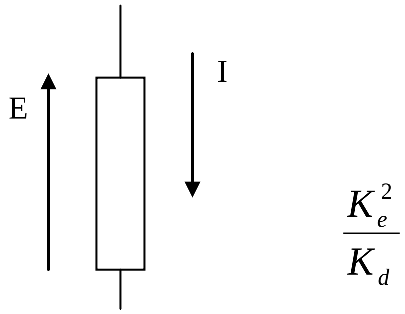
Mechanical loss and damping

$F = K_d v$ where K_d is the damping coefficient

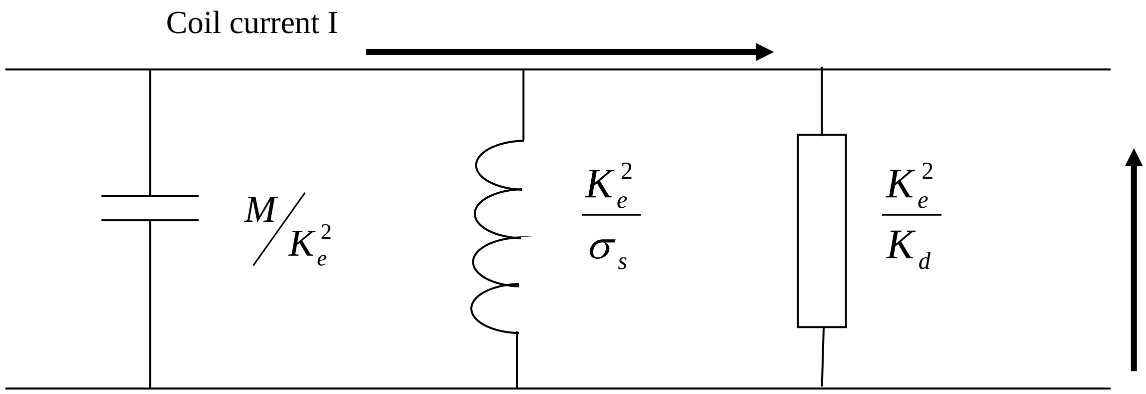
$$K_e I = K_d \left(\frac{E}{K_e} \right)$$

$$\frac{K_e^2}{K_d} I = E$$

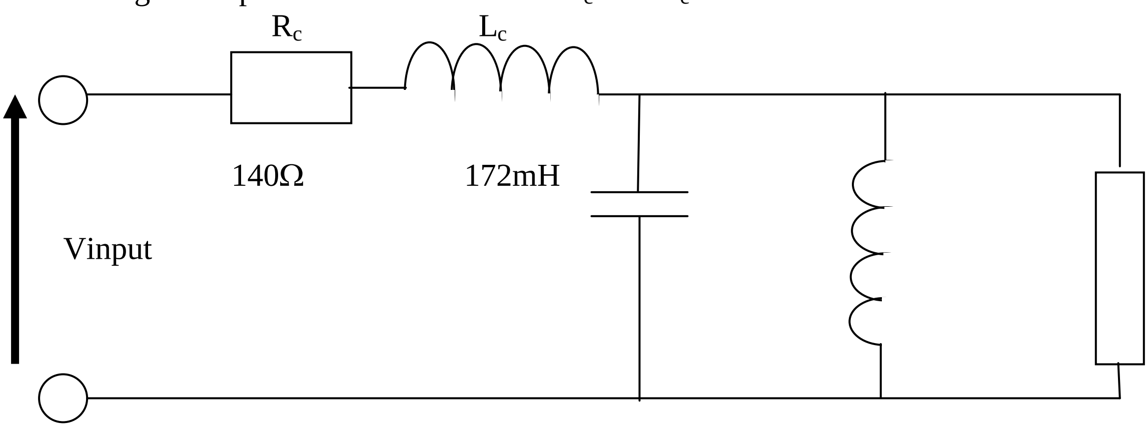
which is analogous to $V = IR$ thus



In forming the complete electrical analogue, the connection of the components needs to be considered, since force is proportional to current



adding the impedance of the coil itself R_c and L_c



c. Given that the dynamics are dominated by the mass, the equivalent circuit becomes a series resonant circuit with a capacitance of:
$6\times10^{-3}/(10.05)^2 = 59.4\mu F$

The resonant frequency is therefore $1/2\pi(LC)^{0.5}$ = 49.8Hz close to 50Hz.

d. At 50Hz, the system is in resonance, and the series LC impedance cancels out. The only impedance is therefore the resistance of the coil. The current is therefore given by Ohms law.

$$I = V/R = 86mA$$

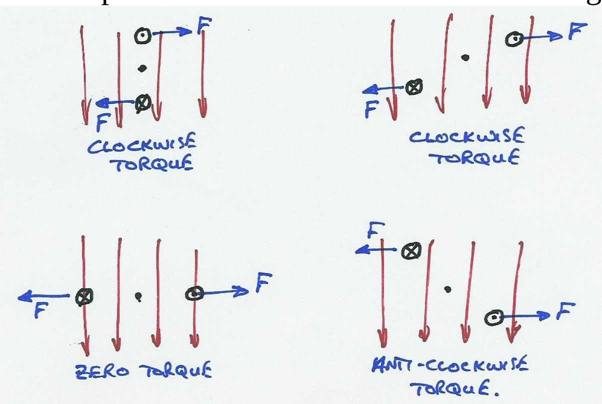
Qu 2.
a. Rating parameters:

T_{stall} – Continuous torque that the motor can output at zero speed. This rating relates to the ability of the motor to dissipate the I^2R loss in the armature. With the armature stationary, this is the worst condition with regard to cooling – The airgap between the armature and stator acts as a thermal barrier, and with the armature stationary, there is no air movement in this gap to assist cooling.

n_{max} – Maximum operating speed, limited by mechanical constraints, also the commutator action is speed limited.

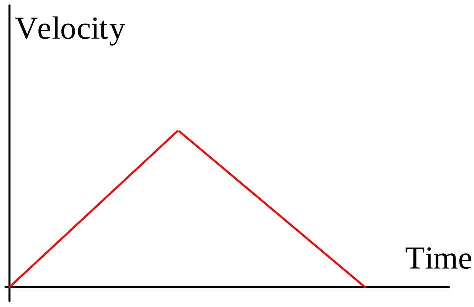
T_{max} – Maximum peak torque available form the machine – 5 to 10 times the continuous rating. Limited by commutator action at high armature currents. Also, at high armature currents, the armature reaction field may demagnetise the permanent magnets. The field from the permanent magnets is augmented at one side of the stator pole, and decreased at the other side of the stator pole.

b.
If we consider a number of positions of a coil as it rotates in a magnetic field:



The action of the commutator as it rotates is to reverses the currents in the motor armature winding (rotor) as it passes through the position of zero torque. The reversal of the winding connection is achieved using a mechanical switch arrangement consisting of copper segments acting with carbon brushes.

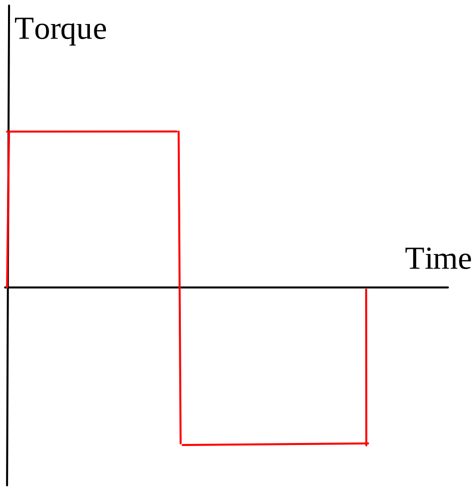
c. Triangular velocity profile required.



The peak velocity is given from area under graph = $150^o = 0.5 \times \omega_{max} \times 1sec$

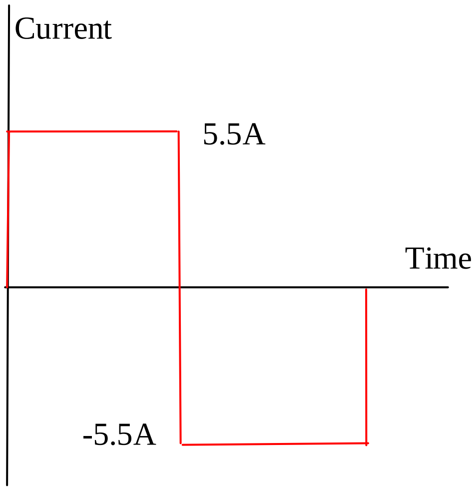
$\therefore \omega_{max} = 300^o/sec = 5.24\ rads^{-1}$

Given an acceleration of $5.24/0.5 = 10.48 rads^{-2}$, and T=Ja, then $T = 0.5 \times 10.48 = 5.24Nm$, the torque profile being:

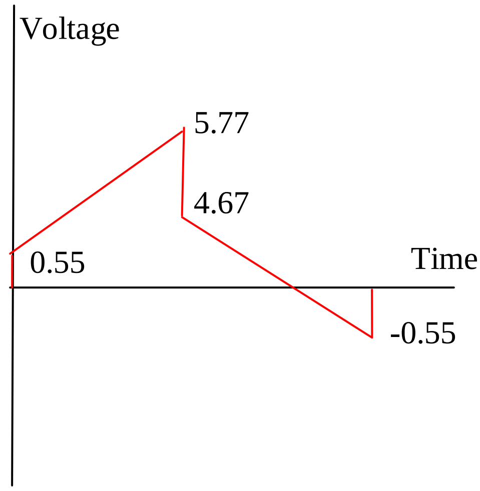


With a motor constant of $1000rpm = 1000 \times 2\pi / 60\ rads^{-1} = 104.72 rads^{-1} = 100V$
$\therefore K_e = 100/104.72 = 0.955$.

For $T = 5.24Nm$, $I_{max} = 5.24/0.955 = 5.5A$



$V = E + IR = K_e \, . \, \omega \; + 0.1 \times 5.5 = 5.77V$

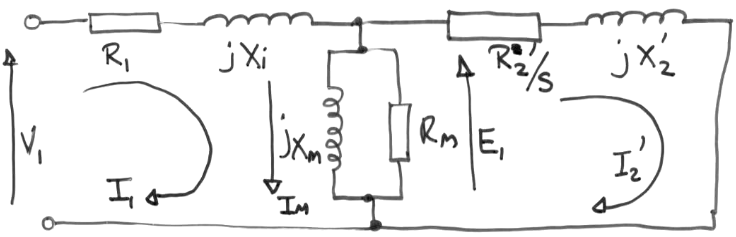


d. The drive needs to operate in 4 quadrants as the arm needs to move back to the start.

Qu. 3.

a.

FULL STATOR EQUIVALENT CCT.



R_1 = STATOR RESISTANCE PER PHASE

R_2' = REFERRED ROTOR RESISTANCE $/\phi$

X_1 = STATOR LEAKAGE REACTANCE $/\phi$

X_2' = REFERRED ROTOR LEAKAGE REACTANCE $/\phi$

X_m = MAGNETIZING REACTANCE $/\phi$

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R_m = IRON LOSS RESISTANCE $/\phi$

V_1 = RMS SUPPLY PHASE VOLTAGE $/\phi$

E_1 = INDUCED STATOR PHASE VOLTAGE

I_2' = REFERRED ROTOR CURRENT

I_m = MAGNETIZING CURRENT

I_1 = STATOR CURRENT.

b.

Starting torque:

$$T_{START} = \frac{3PV_1^2}{2\pi f_1} \frac{R_2'}{(R_1+R_2')^2+(X_1+X_2')^2}$$

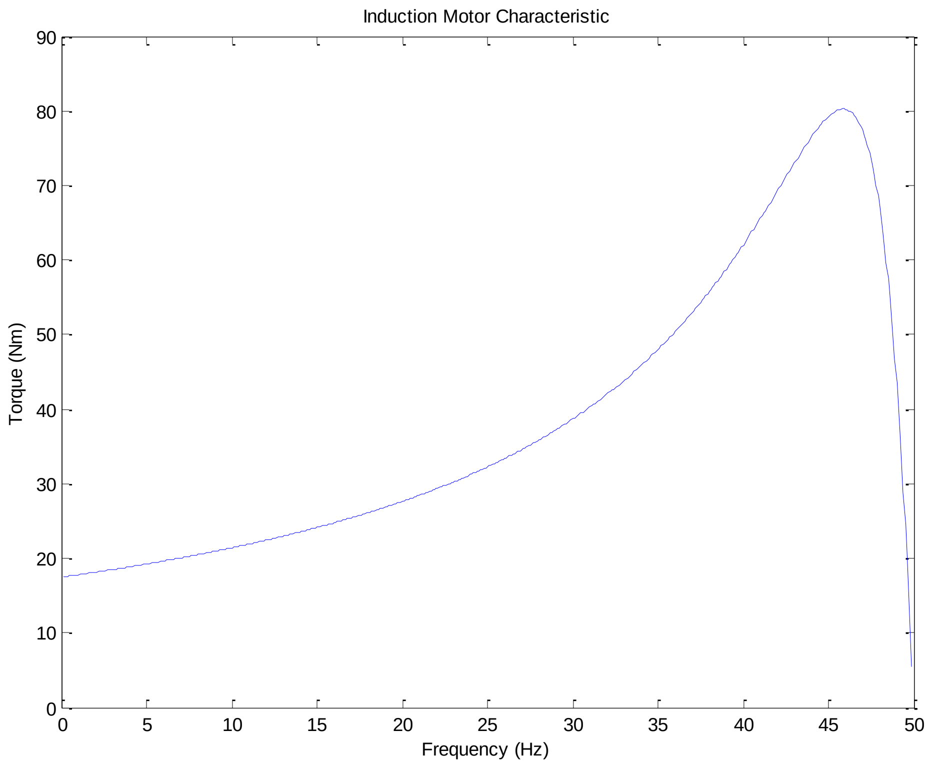
$R_2^{/} = 0.2\Omega$, $X_2^{/} = 2mH$. P=1

$T_{Start} = 17.5Nm$

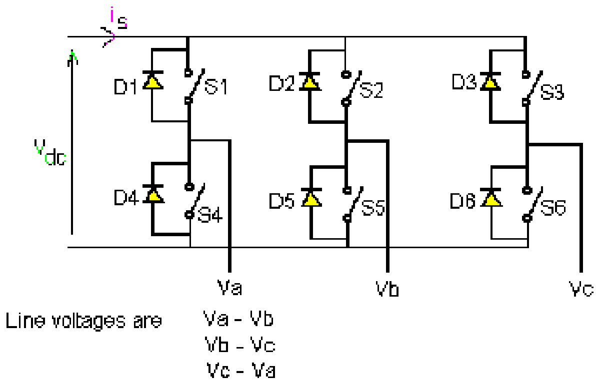
Pull-out torque:

$$T_{PULL\text{-}OUT} = \frac{3pV_1^2}{2\pi f_1} \frac{\sqrt{R_1^2+(X_1+X_2')^2}}{\left(R_1+\sqrt{R_1^2+(X_1+X_2')^2}\right)^2+(X_1+X_2')^2}$$

Therefore peak pull-out torque = 80.25Nm



c.



Qu. 4

$$\text{m.m.f} = \text{Reluctance} \times \text{flux}$$

$$\text{N.I} = \text{Reluctance of airgap} \times \phi$$

$$\therefore N.I = \frac{x}{\mu_o A} \times \phi$$

Therefore

$$\phi = \frac{N.I}{\frac{x}{\mu_o A}} = \frac{NI\mu_o A}{x}$$

Hence, inductance,

$$L = \frac{\varphi}{I} \quad = \quad \frac{N\phi}{I}$$

so:

$$L = \frac{N^2 \mu_o A}{x}$$

and the force on the iron core is given from:

$$F = \frac{1}{2} I^2 \frac{dl}{dx}$$

$$\therefore F = \frac{1}{2} I^2 \left(-\frac{N^2 \mu_o A}{x^2} \right)$$

$$\therefore F = -\frac{1}{2} . \frac{I^2 N^2 \mu_o A}{x^2}$$

The '-' sign indicating the force is towards the main body of the actuator, pulling the core into the system

b. Using the above equations, to close the relay I>1A, to open the relay, I<0.6A

c. The two levels are not equal as the reluctance of the magnetic circuit is different at both positions, hence the force generated by the current in the coil is different. This is desirable to introduce some form of 'hysteresis' into the operation of the device. Once the relay starts to close, the gap gets less, and therefore less current is required to overcome the spring. This implies the device will never be in a position which is 'undecided' it will either be open or closed.