EEE6220 2014-15 solutions

Q1 (If any mistakes are made following on marks will be awarded for correct methods)

L=100cm

D=0.3cm

i. $$M = 2*L\left(\ln\left(\frac{2L}{D}\right) - 1 + \frac{D}{L}\right)$$ (2 marks)

$$M = 1100nH$$

$$C = \frac{0.0885\pi L}{\cosh^{-1}\left(\frac{D}{d}\right)}$$ (2 marks)

$$C = 15.8pF$$

ii. The voltage and current are both square waves, using the equation given the envelope of the frequency spectrum is

$$V(n) = \frac{2V}{n\pi}$$

$$I(n) = \frac{2V}{n\pi}$$

(3 marks)

$$V(n) = \frac{2*2.5}{\pi} = 1.6V$$

$$I(n) = \frac{2*5}{\pi} = 3.18A$$

(1 mark)

iii. $$V_m = \omega MI(1) = 2*\pi*50*10^3*1100*10^{-9}*3.18$$

$$V_m = 1.1V$$

3 marks

$$V_C = \omega CV(1)*\frac{R_1R_2}{R_1+R_2} = 2*\pi*50*10^3*15.8*10^{-12}*1.6*\frac{10*100*10^3}{10+100*10^3}$$ 3 marks

$$V_C = 79\mu V$$

Almost all of the voltage is dropped across the data logger

iv. The dominant source is inductive 1 mark b.

The frequency of interest (50kHz) is 5 time higher than the cut-off frequency hence the induced voltage will be a maximum of:-

$$V_N = M \frac{R_S}{L_S} I(1)$$ 2 marks

Also, $f_0 = \frac{R_S}{2\pi L_S}$, hence

$$V_N = 2\pi f_0 M I(1)$$
$$V_N = 2\pi * 10 * 10^3 * 1100 * 10^{-9} * 3.18$$ 3 marks
$$V_N = 0.2V$$

Q2

i)

The power density is given by

$$P = \frac{P_t D \cos(\theta)}{4\pi R^2}$$ 1 mark

The distance, R can be found from Pythagoras

$$R = \sqrt{10^2 + 17} = 19.7m$$

The angle can be found from trig

$$\theta = \tan^{-1}\left(\frac{17}{10}\right) = 59 \deg$$

Hence the power density is

$$P = \frac{30 * 10 \cos(59)}{4\pi * 19.7^2} = 31.4W / m^2$$ 2 marks

ii)

The E-field can be found from

$$E = \sqrt{PZ_0} = 3.44V / m$$ 1 mark

iii)

The wavelength is

$$\lambda = \frac{c}{f} = 3m$$ 1 mark

iv)

We need to choose the correct equation from the information given at the end of the paper, which is

$$\frac{V_2}{E} = \frac{Z_2 D\left(Z_0\left[1-\cos(\beta l)\right] + jZ_1 \sin(\beta l)\right)}{(Z_0 Z_1 + Z_0 Z_2)\cos(\beta l) - j\left(Z_0^2 + Z_1 Z_2\right)\sin(\beta l)}$$

1 marks

D=0.02m

d=0.0001m

Z1=10Ω

Z2=100kΩ

L=0.1m

The required calculations needed for the equation above are

$$\beta l = \frac{2\pi}{\lambda} l = 0.209 rad$$

1 mark

The characteristic impedance of the cables is

$$Z_0 = 120 \ln\left(\frac{2*20}{1}\right) = 443\Omega$$

1 mark

Hence, the coupling factor is

$$\frac{V_2}{E} = \frac{100k*0.02\left(443\left[1-\cos(0.209)\right] + jZ_1 \sin(0.209)\right)}{(443*10 + 443*100k)\cos(0.209) - j\left(443^2 + 10*100k\right)\sin(0.209)}$$

$$\frac{V_2}{E} = \frac{9.746 + j2.08}{22.15k - j124.5}$$

3 marks

$$\left|\frac{V_2}{E}\right| = 4.49*10^{-4}$$

Hence the induced voltage is $4.49*10^{-4}*3.44$=1.5mV

1 mark

B

First step is to choose the correct Reflection loss equation.

The far field can region can be approximated by $\frac{\lambda}{2\pi} = \frac{3}{2\pi} = 0.5m$, hence the shield is in the far field

So the reflection loss equation is

$$R = 168 - 10.\log_{10}\left(\left(\frac{\mu_r}{\sigma_r}\right).f\right)$$

1 mark

The reflection loss from the aluminium shield is

$$R = 168 - 10.\log_{10}\left(\left(\frac{1}{0.6}\right).100*10^6\right) = 85.8dB$$

2 marks

Hence we need at least 14.2dB from absorption to meet the SE requirement

$$A = 8.69\frac{d}{\delta}$$

$$\delta = \frac{1}{\sqrt{\pi\mu\sigma f}} = 8.5\mu m$$

2 marks

$$d > 8.69*8.5\mu = 73.8\mu m$$

Solutions

Q3(a)

The Smith chart can be used to solve transmission line problems by using the following steps

- Normalise load impedance and find reflection coefficient at load
- Rotate reflection coefficient
- Record normalised input impedance
- De-normalise input impedance **(2 marks)**

Q3(b)

$$\lambda = \frac{c}{f} = 10\text{cm}$$

Therefore

$$\beta\ell = \frac{2\pi}{\lambda} \times 3 = 0.1884$$ **(1 mark)**

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)} = (27 + j8.9)\Omega$$ **(1 mark)**

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = -0.28 + j0.148$$ **(1 mark)**

$$\text{VSWR} = \frac{1 + |\Gamma_{in}|}{1 - |\Gamma_{in}|} = 1.93$$ **(1 mark)**

Q3(c)

$$Z_o = \sqrt{Z_{in} Z_L} = \sqrt{75 \times 40} = 55\Omega$$ **(1 mark)**

The input impedance is given by

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)}$$

The frequency dependence comes from the $(\beta\ell)$ term, which can be written in terms of f/f_o as

$$\beta\ell = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_o}{4}\right) = \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4f_o}\right) = \frac{\pi f}{2 f_o}$$ **(1 mark)**

i.e.

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\left(\frac{\pi f}{2 f_o}\right)}{Z_o + jZ_L \tan\left(\frac{\pi f}{2 f_o}\right)}$$

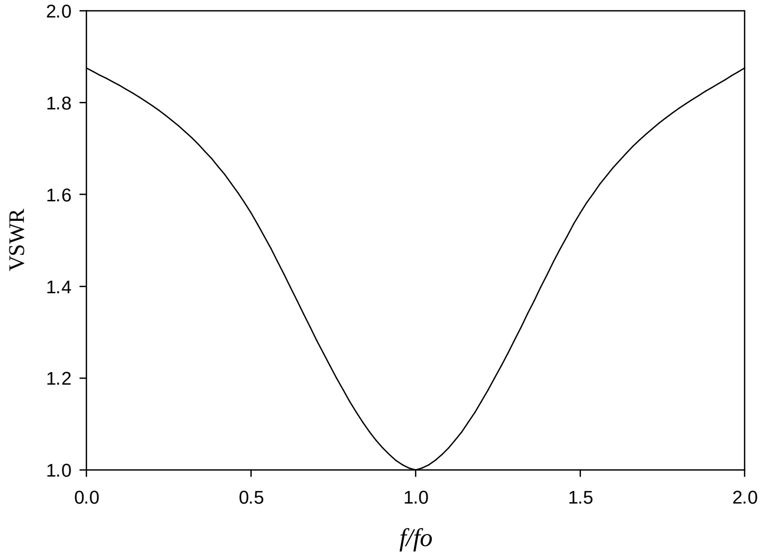
The input reflection coefficient and VSWR can be calculated as

$$\left|\Gamma_{\text{in}}\right| = \left|\frac{Z_{\text{in}} - Z_{\text{o}}}{Z_{\text{in}} + Z_{\text{o}}}\right| \qquad \text{VSWR} = \frac{1+\left|\Gamma_{in}\right|}{1-\left|\Gamma_{in}\right|}$$

(1 mark)

f/f_o	Z_{in}	VSWR
0	40+j0.	1.875
0.5	52+j16.7	1.56
1	75+j0.	1
1.5	52-j16.7	1.56
2	40+j0.	1.875

(2 marks)



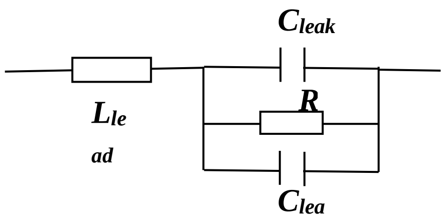
(1 mark)

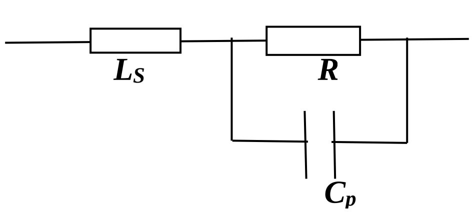
Q3(d)

At higher frequencies, resistors and capacitors have inductance due to the leads, and energy stored in the magnetic field. In addition, there is a parasitic capacitance both internally and due to the proximity with the surrounding structures. **(1 mark)**

The equivalent circuit of a resistor include a series inductance in the order of 5 to 10 nH as well as a parallel capacitance up to few pico Farads. Therefore, the equivalent impedance is given by

$$Z_{resisor}(f) = \frac{X_L\left(X_C + jR\right)}{\left(R - jX_C\right)}$$



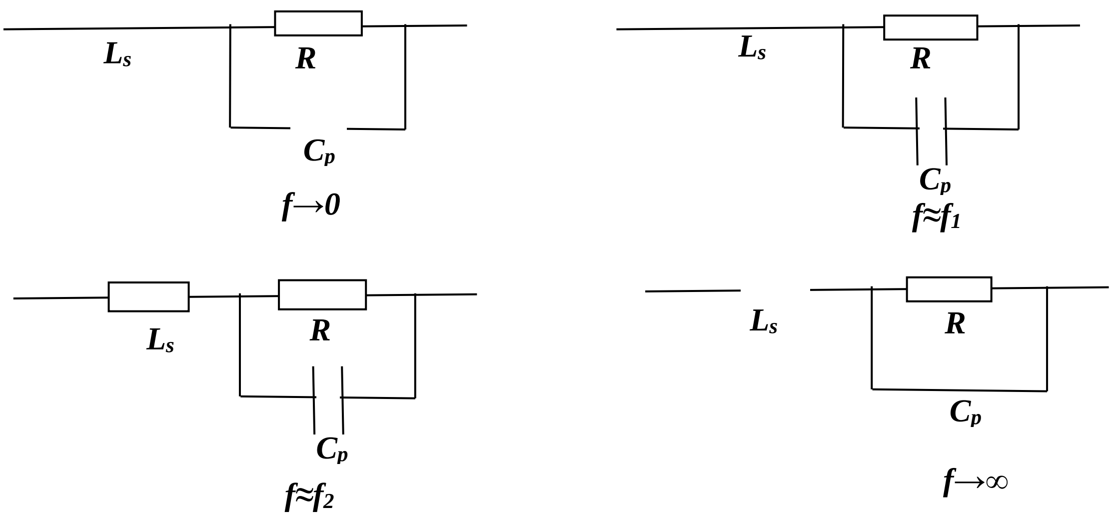


(2 marks)

At low frequencies $Z_{resisor}(f) \approx R$ since the inductance and capacitance can be represented as short and open circuits, respectively. However, as the frequency increases, the capacitive reactance reduces and at a frequency of f_1 it becomes equal to R, i.e. $f_1 = \dfrac{1}{2\pi R C_p}$. **(1 mark)**

Further frequency increment, results in a higher current through the capacitor compared to that in the resistor. For higher frequencies, the impedance reduces until the resonance is achieved at a frequency of $f_0 = \dfrac{1}{2\pi\sqrt{L_S C_p}}$. **(1 mark)**

After this frequency the lead inductance dominates until it approaches infinite while the capacitive reactance approaches zero, thus the resistor impedance can be modelled as an open circuit. **(1 mark)**



(2 marks)

Q4(a)

There are two basic components of any SFD:

Nodes; each port in the network has two nodes a_n and b_n. The node a_n represents a wave entering port n, while node b_n represents a wave reflected from port n. **(1 mark)**

Branches; each branch represents a direct path between a-node and b-node. For each branch there is an associated S parameter or reflection coefficient. **(1 mark)**

Q4(b)

Consider the following input signal

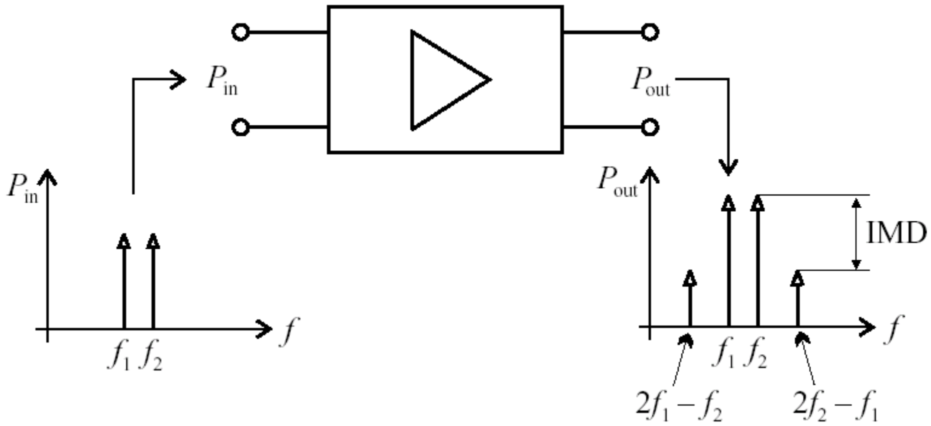
$$\mathrm{v}_i = V_0(\cos\omega_1 t + \cos\omega_2 t)$$

Substituting in

$v_o = a_0 + a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \cdots\cdots$

gives six third-order inter-modulation products, four of them will be outside the pass band of the amplifier, but two products will be located near the original input signals. **(2 marks)**

This is shown in the following diagram;



(1 mark)

For an amplifier these products should be as small as possible. The difference between the desired and undesired power levels (in dBm) is the IMD (in dB)

$IMD(dB) = P_{out}(f_2)(dBm) - P_{out}(2f_2 - f_1)(dBm)$ **(1 mark)**

<u>Q4(c)</u>

The scattering parameter S_{11} is calculated when port is terminated with $Z_L=Z_{o2}=50\Omega$ as

$$S_{11} = \Gamma_{in} = \frac{Z_{in1} - Z_{o1}}{Z_{in1} + Z_{o1}}$$

$$Z_{in1} = 15 + \left[\frac{75.6(15+50)}{75.6+15+50}\right] = 50\Omega$$

i.e. $S_{11}=0$ **(1.5 mark)**

and

$$S_{21} = \frac{2Z_0Z_3}{Z_1(Z_1+2Z_3)+2Z_0(Z_1+Z_3)+Z_0^2}$$

$$= \frac{75.6\times100}{15(15+151.2)+100(15+75.6)+(2500)} = 0.53$$ **(1 mark)**

In the same way it can be shown that $S_{22}=0$.and $S_{12}=0.53$ **(0.5 mark)**

The input and output powers are

$$P_{in} = \frac{|V_{i1}|^2}{2Z_{o1}} \qquad P_{out} = \frac{|V_{r2}|^2}{2Z_{o2}}$$ **(1 mark)**

then

$$\frac{P_{out}}{P_{in}} = \frac{|V_{r2}|^2}{|V_{i1}|^2} = \frac{|S_{21}V_{i1}|^2}{|V_{i1}|^2} = |S_{21}|^2 = 0.28 = -5.5\text{dB}$$ **(1 mark)**

This means that the above circuit is a passive circuit, which provides 5.5dB attenuation. **(1 mark)**

<u>Q4(d)</u>

$$N = \frac{NF - NF_{min}}{4R_N / Z_o}\left|1+\Gamma_{opt}\right|^2 = \frac{1.58\text{-}1.445}{80/50}\left|1+0.62\angle 100^\circ\right|^2 = 0.0986$$

$$C_{NF} = \frac{\Gamma_{opt}}{(N+1)} = 0.56\angle 100^\circ$$

$$r_{NF} = \frac{\sqrt{N(N+1-|\Gamma_{opt}|^2)}}{(N+1)} = 0.24$$ **(1.5 mark)**

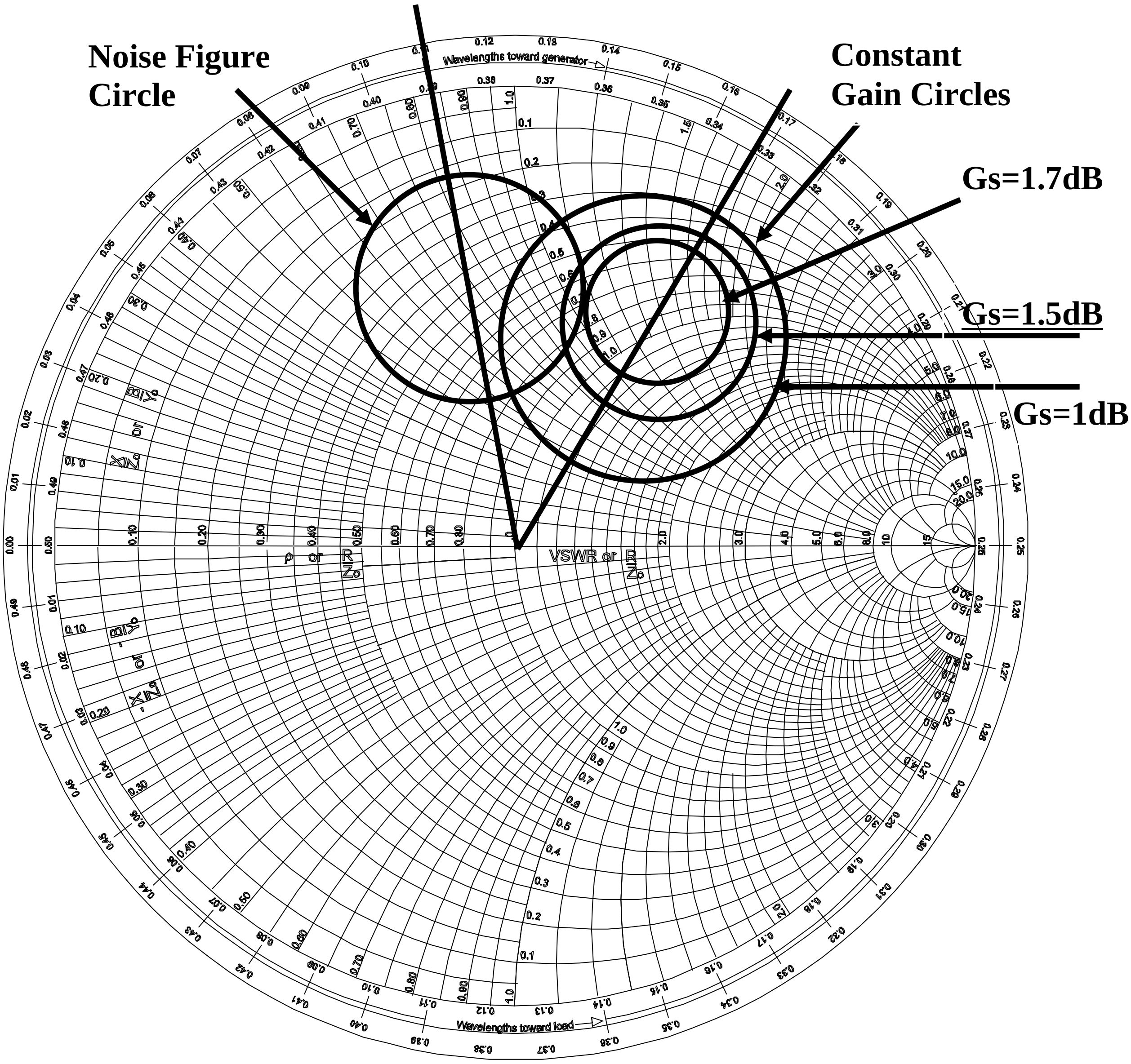
Next we calculate data for several input section constant gain circles

G_S	g_S	C_S	r_S
1.0dB	0.805	$0.52\angle 60°$	0.3
1.5dB	0.904	$0.56\angle 60°$	0.205
1.7dB	0.946	$0.58\angle 60°$	0.15

(2 marks)

The noise figure and constant gain circles are plotted on the Smith chart and the G_S =1.7dB circle just intersects the noise figure circle, and any higher gain will results in a worse noise figure. **(4 marks, one for each circle)**

From the Smith chart the optimum solution is then Γ_s=0.53∠75°and NF =2.dB. **(0.5 mark)**



Noise Figure
Circle
Constant
Gain Circles
Gs=1.7dB
Gs=1.5dB
Gs=1dB
Wavelengths toward generator
Wavelengths toward load
VSWR or R/Zo