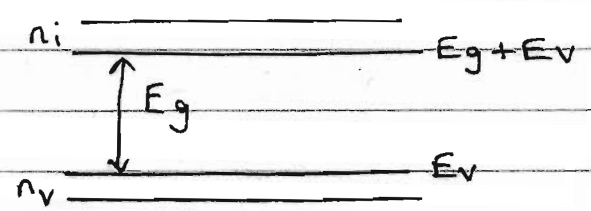
Semiconductors for Electronics and Devices EEE 207
Spring 2006 – Worked Solutions

1(a)



Assume that width of levels in CB and VB is small c.f. E_g i.e. all levels are identical

$$n_v = \text{no. of } e^- \text{ in VB} = P(E_v).N_{tot} = \frac{N_{tot}}{1 + \exp[(E_v - E_F)/kT]}$$

$$n_i = \text{no. of } e^- \text{ in CB} = P(E_v + E_g).N_{tot} = \frac{N_{tot}}{1 + \exp[(E_v + E_g - E_F)/kT]}$$

Now $N_{tot} = n_v + n_i$, so

$$N_{tot} = \frac{N_{tot}}{1 + [\exp(E_v - E_F)/kT]} + \frac{N_{tot}}{1 + \exp[(E_v + E_g - E_F)/kT]}$$

rearranging this gives:

$$\exp[(2E_v - 2E_F + E_g)/kT] = 1$$

$$\therefore \quad 2E_v - 2E_F + E_g = 0$$

$$E_F = E_v + E_g/2$$

$$\therefore \quad n_i = \frac{N_{tot}}{1 + \exp[(E_v + E_g - E_v - E_g/2)/kT]} = \frac{N_{tot}}{1 + \exp(E_g/2kT)}$$

$$\simeq N_{tot} \exp(-E_g/2kT) \quad \text{as } E_g \gg kT \text{ normally}$$

$$\therefore \quad n_i \propto \exp(A/T) \quad \text{where} \quad A = \frac{-E_g}{2k}$$

2

6

Most people knew that $A = \frac{-E_g}{2k}$ and simply stated it. The question required you to show how this value was derived

1 (b) Resistance $\alpha \dfrac{1}{\sigma} = \dfrac{1}{n_i e(\mu_e + \mu_h)}$ 1

$$\frac{25\ \Omega}{2{\cdot}5\ k\Omega} = \frac{n_i \text{ at } 20C}{n_i \text{ at } 100C} = \frac{\exp - E_g/2k.293}{\exp - E_g/2k.373}$$ 1

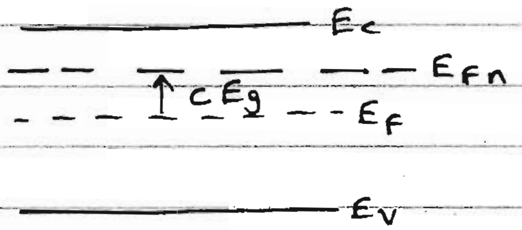
$$0{\cdot}01 = \exp\left[-\frac{E_g \times 1{\cdot}6\times10^{-19}}{2\times1{\cdot}38\times10^{-23}} \cdot \left(\frac{1}{293} - \frac{1}{373}\right)\right]$$

5 $0{\cdot}01 = \exp(-E_g \times 4{\cdot}243)$

$E_g = 1{\cdot}085\ eV$ (or $1{\cdot}736\times10^{-19}$ J) 3

Several people confused the 1% value of 25 Ω and hence got the wrong numerical answer

1 (c)



E_{Fn} moves towards E_c as it is n-doped.

$$n \;\alpha\; P(E_g) \;\alpha\; \exp\frac{-(E_g - E_F)}{kT} \;\alpha\; \exp\frac{-[E_g - (E_g/2 + cE_g)]}{kT}$$

$$\alpha\; \exp\frac{-(E_g/2 - cE_g)}{kT} \;\alpha\; \exp\frac{-E_g}{2kT}(1-2c)$$ 2

Ratio of e^- in doped : intrinsic is

$$\frac{n}{n_i} = \frac{\exp - E_g/2kT \cdot (1-2c)}{\exp - E_g/2kT} = \exp \frac{E_g}{\cancel{2}kT} . \cancel{2}c$$

5 $$= \exp\left(\frac{cE_g}{kT}\right)$$ 3

Many people were broadly aware of what was required but very few managed to get the final correct answer

1(d) $\frac{\sigma_n}{\sigma_i} = \frac{n e \mu_e}{n_i e(\mu_e + \mu_h)} = 10^6$

$\therefore \frac{n}{n_i} = 10^6 \frac{\mu_e + \mu_h}{\mu_e} = 10^6 \frac{(0.13 + 0.05)}{0.13} = 1.38 \times 10^6$

From part (c), $\frac{n}{n_i} = 1.38 \times 10^6 = \exp \frac{c E_g}{kT}$ 2

$1.38 \times 10^6 = \exp. \frac{c \times 1.085 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 293}$

4

gives $c \simeq 0.33$ 2

Change in $E_F = c E_g = 0.33 \times 1.085 = 0.358\,eV$

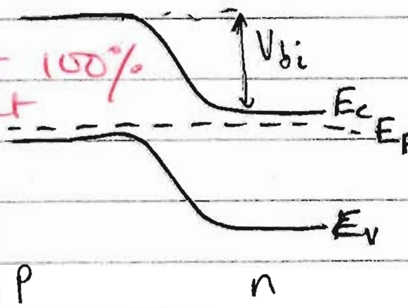
(This level is $1.085 - 0.358\,eV = 0.184\,eV$ below CB.)

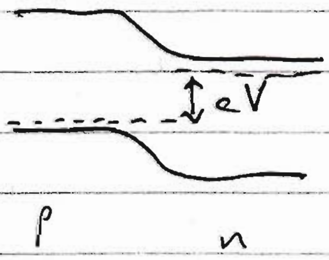
2

Most got the fact this this involved a ratio of σ correct. However there were many silly errors e.g. confusing °c and °k.

2(a) 2 2

Almost 100% correct





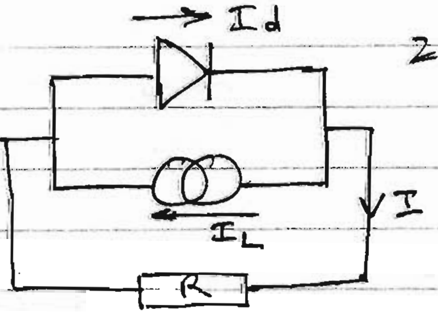
About 10% got the levels going the other way!

6 dark under illumination

Maximum voltage $\equiv V_{bi} \approx E_g$ 2 Mostly correct.

(b) 2

4



I is negative under illumination. When maximum voltage of junction is across R, I = 0, so no power. There is an optimum voltage that maximises the IV (power) product

Most people got this partly correct – several did not show the direction of current flow 2

2(c) Longest λ determined by $E_g = 0.75\,eV \equiv 1.65\,\mu m$

Virtually 100% correct

2

When InGaAs = 10 nm, quantisation occurs and E_g effectively increases.

$$E_n = \frac{n^2 h^2}{8 m L^2}$$

$n = 1$, $h = 6.63 \times 10^{-34}$, $m_e = 0.04\, m_o$, $m_h = 0.45\, m_o$ and $L = 10^{-8}\, m$.

$$E_{1e} = \frac{(6.63 \times 10^{-34})^2}{8 \times 0.04 \times 9.11 \times 10^{-31} \times 10^{-16}} = 94\ meV$$ 1

$$E_{1h} = \frac{(6.63 \times 10^{-34})^2}{8 \times 0.45 \times 9.11 \times 10^{-31} \times 10^{-16}} = 8.3\ meV$$ 1

6

Total effective band-gap = 0.75 + 94 meV + 8.3 meV = 0.852 eV

$\therefore$ max. λ now $= \frac{1.24}{0.852} = 1.455\,\mu m$ 2

Very few got this correct although were aware of what to do

(d) As InGaAs width reduces, the $e^- + h$ levels increase until they reach the top of the barrier and it saturates there.

$$\lambda_{max}\ \text{now} = \frac{1.24}{1.35} = 0.918\,\mu m$$ 4

4

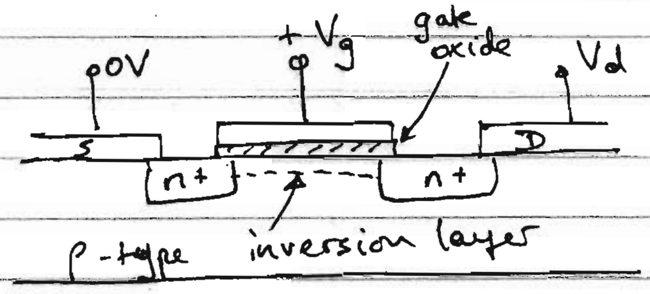
(As the InGaAs width becomes very thin, there is little absorption, hence small photocurrent. Absorption then mainly due to InP barriers).

Also OK

Several people gave the correct numerical answer but not the correct reason for it.

Very easy question generally

3(a)



3

At small $+V_g$, repel holes and form depletion under gate.
At large $+V_g$, minority $\bar{e}$ from S+D form an inversion layer under gate oxide.
6 As voltage is applied between D+S, ohmic conduction in this inversion layer or channel.
As $+V_g \uparrow$, $I_d \uparrow$ — enhancement type MOST 3

Virtually 100% correct

(b) The expression given holds only for $V_g - V_x > V_T$ at all x, otherwise no channel formed. Limit at drain end when $V_x = V_d$.

$\therefore$ saturation occurs when $V_g - V_T = V_d$ 1

substitute this into equation given:

$$I_{ds} = \frac{\mu_e C_g}{l^2} \frac{(V_g - V_T)^2}{2} = \frac{\mu_e C_g}{l^2} \frac{V_d^2}{2}$$ 1

1

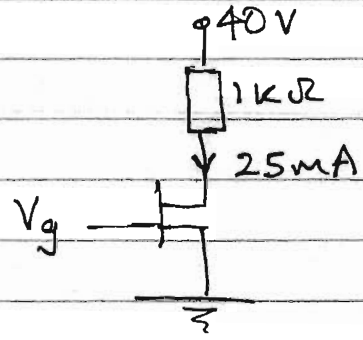
Transconductance, $g_m = \left.\frac{\partial I_d}{\partial V_g}\right|_{V_d}$ in saturated region

Differentiating above:

5 $$g_m = \frac{\mu_e C_g (V_g - V_T)}{l^2} = \frac{\mu_e C_g V_d}{l^2}$$ 2

A large number got this fully correct - a few 'missed out' some bits

3(c)



$V_d = 40 - 25\text{mA} \times 1\text{k}\Omega = 15\text{V}$

Very few realised that Vd is actually 15 and ~~not~~ 40V 2

From previous equations,

$$C_g = \frac{2 I_{ds} l^2}{\mu_e V_d^2}$$

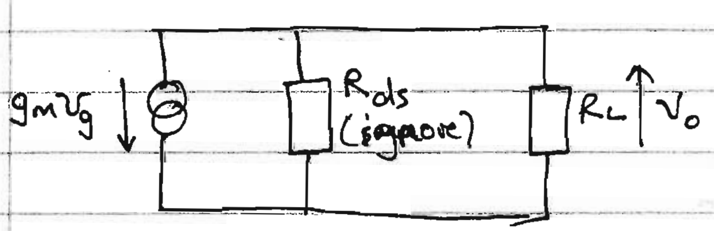
4 2

$$= \frac{2 \times 25 \times 10^{-3} \times (25 \times 10^{-6})^2}{0.15 \times 15 \times 15} = 0.926\text{pF}$$

(d) $$g_m = \frac{\mu_e C_g V_d}{l^2} = \frac{0.15 \times 0.926 \times 10^{-12} \times 15}{(25 \times 10^{-6})^2} = 3.33\text{mS}$$

2

small signal gain :



$v_o = g_m v_g R_L$

$$\text{Gain} = \frac{v_o}{v_g} = g_m R_L$$

$= 3.33 \times 10^{-3} \times 10^3$

$= 3.33$

5 3

Many knew what the small signal gain was but did not get the correct value due to the earlier error in Cg.

4(a) Force = rate of change in momentum

$$= \frac{dp}{dt} = \hbar\frac{dk}{dt} \quad \text{as } p = \hbar k$$ 1

$$\text{Force} = \frac{dE}{dx} = \frac{dE}{dk}\cdot\frac{dk}{dt}\cdot\frac{dt}{dx}$$

$$\therefore \hbar = \frac{dE}{dk}\cdot\frac{dt}{dx} = \frac{dE}{dk}\Big/\text{velocity}$$ 1

$$\text{Acceleration} = \frac{d(\text{velocity})}{dt} = \frac{1}{\hbar}\,\frac{d^2E}{dk^2}\cdot\frac{dk}{dt}$$

$$= \frac{1}{\hbar^2}\cdot\frac{d^2E}{dk^2}\cdot\text{Force}$$ 2

Force = mass × acc., so from above

6

$$\text{Mass} = m_e^* = \hbar^2\Big/\left(d^2E/dk^2\right)$$ 2

Mostly correct but not derived properly in all cases

(b) E-k relationship is assumed parabolic, so

$E = A + Bk^2$, where A and B are constants. 1

Band gap at $k=0$ for direct-gap semiconductor, so

$A = 1.42\,\text{eV}$ 1

$$\frac{dE}{dk} = 2Bk \quad\text{and}\quad \frac{d^2E}{dk^2} = 2B, \text{ so}$$

$$m_e^* = 0.06\times 9.11\times10^{-31} = \left(\frac{6.626\times10^{-34}}{2\pi}\right)^2\times\frac{1}{2B}$$ 2

Rearranging: $B = 1.017\times10^{-37}\ \text{J m}^2$

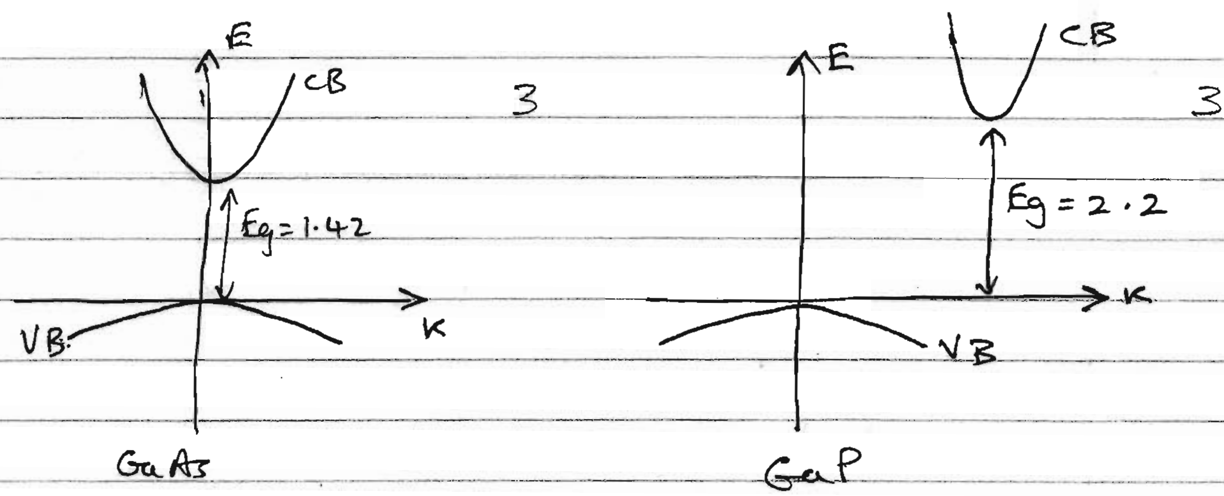
$= 6.35\times10^{-19}\ \text{eV m}^2$

6

$$\therefore E = 1.42 + 6.35\times10^{-19}k^2\ \text{eV}$$ 2

Very few got even part way through this question correctly. It was relatively straightforward.

4(c)



Recombination with photon emission possible in GaAs as K=0 transitions can occur. 1

8 Recombination in indirect gap material requires interaction with phonon – much less likely event, so weak photon emission. 1

The drawings were of a variable quality but most people seemed to have the correct idea. The explanations were generally good.

Data Provided: You may need to use the following physical constants:

Charge on electron:	-1.602×10^{-19} C
Free electron rest mass:	$m_0 = 9.110\times10^{-31}$ kg
Speed of light in vacuum	$c = 2.998\times10^{8}$ m s^{-1}
Planck's constant:	$h = 6.626\times10^{-34}$ Js
Boltzmann's constant:	$k = 1.381\times10^{-23}$ JK^{-1}
Melting point of ice:	$0°C = 273.2$ K
Permittivity of free space:	$\varepsilon_0 = 8.854\times10^{-12}$ Fm^{-1}
Permeability of free space:	$\mu_0 = 4\pi\times10^{-7}$ Hm^{-1}

The University of Sheffield

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2005-2006 (2 hours)

Semiconductors for Electronics and Devices 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

requires demonstration – many just showed what A was

1. a. Show that the temperature dependence of the intrinsic carrier density of a semiconductor, n_i, is of the form:

Most knew what A was

$$n_i \propto \exp\left(\frac{A}{T}\right)$$

where T is the temperature and A is a constant, related to the band gap, to be determined. State clearly any assumptions that you make. – no assumptions given by anyone **(6)**

(b) was not too difficult

b. A thermistor made from intrinsic silicon has a resistance of 2.5kΩ at 20°C dropping to 1% of this resistance at 100°C. a few people think 1% = 1/1000!

interpreted as by

Estimate the band gap energy, E_g, of the material, assuming that the mobilities and E_g do not change significantly over the temperature range. **(5)**

c. Supposing that the intrinsic silicon is lightly doped n-type, such that E_F, the Fermi level moves from its intrinsic position by an energy cE_g, where c is a constant, derive an expression for the ratio of the electron density in the doped semiconductor to that in the original intrinsic material. **(5)**

Many people knew that that it moved up, but the 'by' confused some people. A lot of people got the sign wrong, so could not get rid of the Eg term.

Most got the fact that this involved a ratio correct but simple errors (e.g. C° instead K) resulted in few correct answers.

d. In a particular case, the light n-doping changes the conductivity of the silicon at 20°C by a factor of 10^6. Estimate the corresponding change in the Fermi level, assuming that μ_e=0.13m^2 V^{-1} s^{-1}, μ_h=0.05m^2 V^{-1} s^{-1}. **(4)**

2. a. Sketch the energy band diagram of a semiconductor p-n junction when no dc bias is applied under the following conditions:

(i) in the dark, and virtually all correct

(ii) under illumination by short wavelength light. mostly correct – a few exceptions

Show clearly the conduction band, valence band and Fermi level in both cases. What is the maximum voltage such a junction can theoretically deliver to an external load? – many got E_g or equivalent **(6)**

b. Draw the equivalent circuit of such a junction under illumination when it is connected to a resistive load as in a photovoltaic or solar cell. Show clearly the direction of conventional current flow. Mostly correct but direction of I not always shown

Explain qualitatively why the load is chosen such that less than the maximum voltage of the junction is dropped across it. Very few got this correct **(4)**

c. The active region of the p-n junction comprises a narrow band gap semiconductor, InGaAs (E_g=0.75eV), surrounded by a wider band gap semiconductor InP (E_g=1.35eV).

What is the longest wavelength of light that could cause a photocurrent to flow in this semiconductor junction? Many got this correct

The thickness of the InGaAs layer is now known to be 10nm. How does this change the answer above? (Assume that in a quantum well with infinitely high and wide barriers the bound energy levels are given by $E_n=n^2h^2/8mL^2$, where the terms have their usual meaning, and that the electron and hole effective masses in InGaAs are $0.04m_0$ and $0.45m_0$ respectively). Many started this correctly but got lost towards the end **(6)**

d. As the width of the InGaAs layer is reduced further, what in reality becomes the longest wavelength that causes a photocurrent in this junction and why? **(4)**

Surprisingly few got this correct – should have been very easy.

Vast majority got full marks here

3. a. Sketch and discuss briefly the conduction mechanisms in an induced channel Metal Oxide Semiconductor Transistor (MOST). (6)

b. Assuming that the I-V characteristics in the unsaturated region are given by:-

$$I_d = \frac{\mu_e C_g}{l^2}\left[V_g - V_T - \frac{V_d}{2}\right]V_d$$

This was apparently quite straightforward and there were few problems

where the symbols have their usual meaning, state the condition for current saturation to occur. From this, derive expressions for the drain current and transconductance in the saturated region. (5)

Many/most got the voltage wrong consequently the numerical sections following were wrong

c. A particular n-channel MOST is operated via a 1kΩ load resistor from a 40V supply rail and biased such that the saturated drain current is 25mA.

Assuming a channel length of 25μm and an electron mobility of $0.15\text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$, estimate the gate capacitance. (4)

d. Find the device transconductance under the conditions described in part **(c)** above and estimate the small signal voltage gain in the common emitter mode. (5)

First part of (d) was straightford numerical calculation. A reasonable number of people got the last part correct.

4. a. Show that from the energy versus momentum relationship, the conduction band effective mass of an electron can be represented by:-

$$m^* = \hbar^2\left(\frac{d^2E}{dk^2}\right)^{-1}$$

Most got this about right, although the derivation was not always good.

where the terms have their usual meaning. (Remember that momentum, $p=\hbar k$) (6)

Very few got this even partly correct - first part is obvious

b. GaAs has a direct band gap energy of 1.42eV and at the centre of the first Brillouin zone, the conduction band can be assumed to be parabolic. If we assume that the effective mass of an electron at the zone centre is $0.06m_0$, deduce an expression for how the electron energy varies with the electron wavenumber, k. (6)

c. GaAs (E_g=1.42eV) is a direct band gap semiconductor while GaP (E_g=2.2eV) is an indirect band gap semiconductor. Assuming that the hole effective mass is about five times that of the electron effective mass in both materials, sketch the approximate form of the electron and hole energy versus wavenumber for both materials, labelling the conduction band, valence band and band gap.

From this diagram, explain why one material can be used to make lasers but not the other. (8)

Most people got this correct but some people did not mention lattice vibrations or phonons.

JPRD / CHT