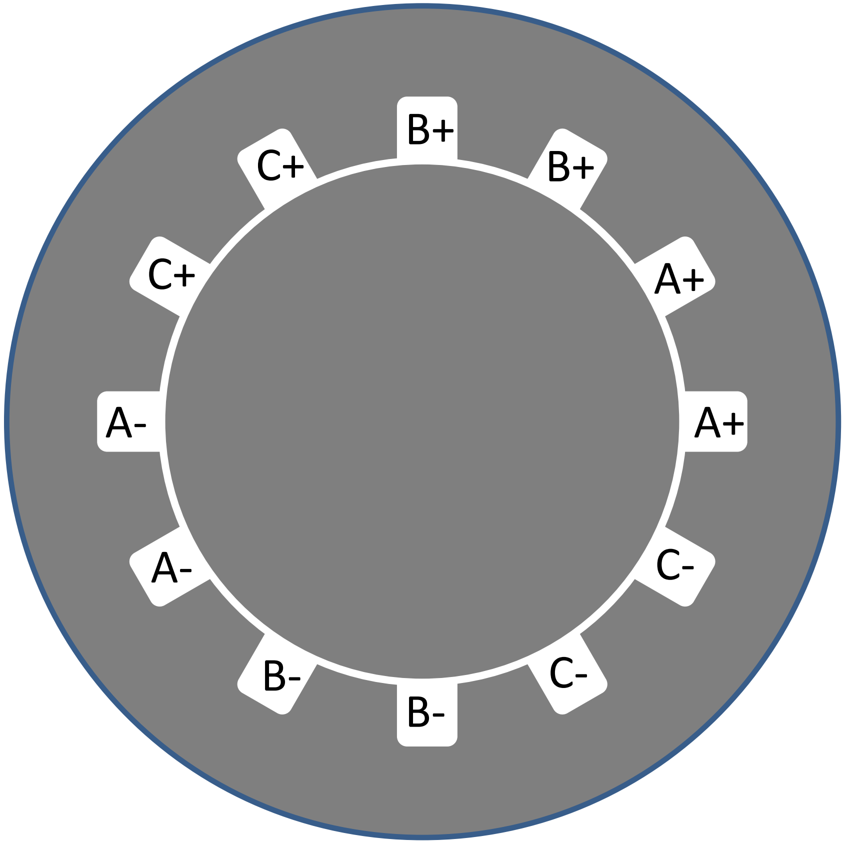
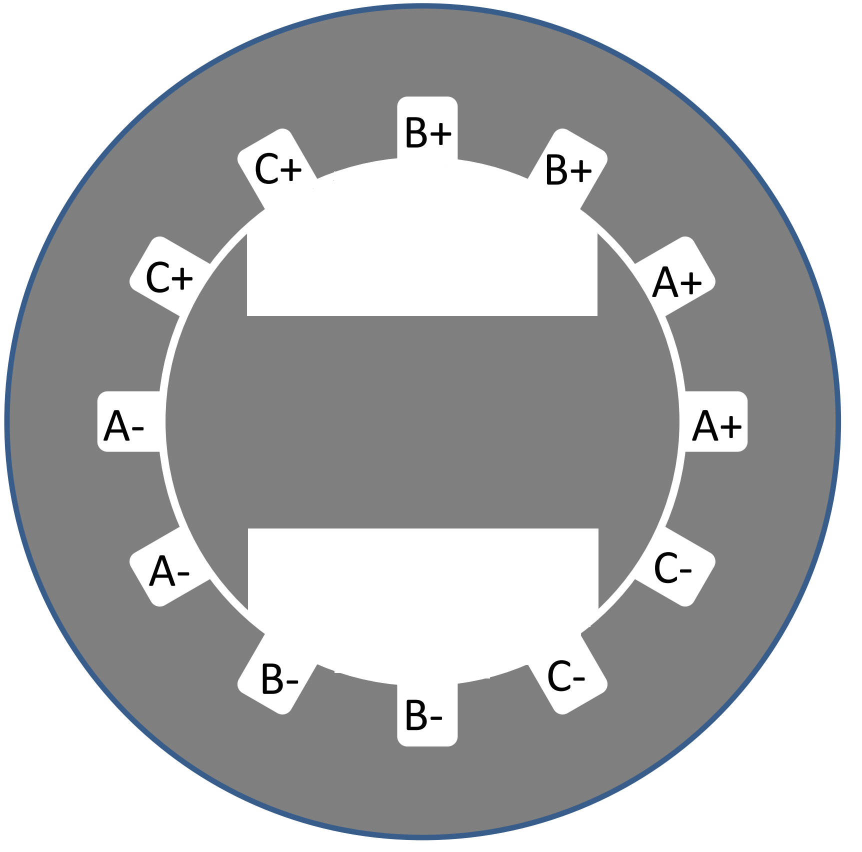
Non-salient and salient rotors



Non-salient

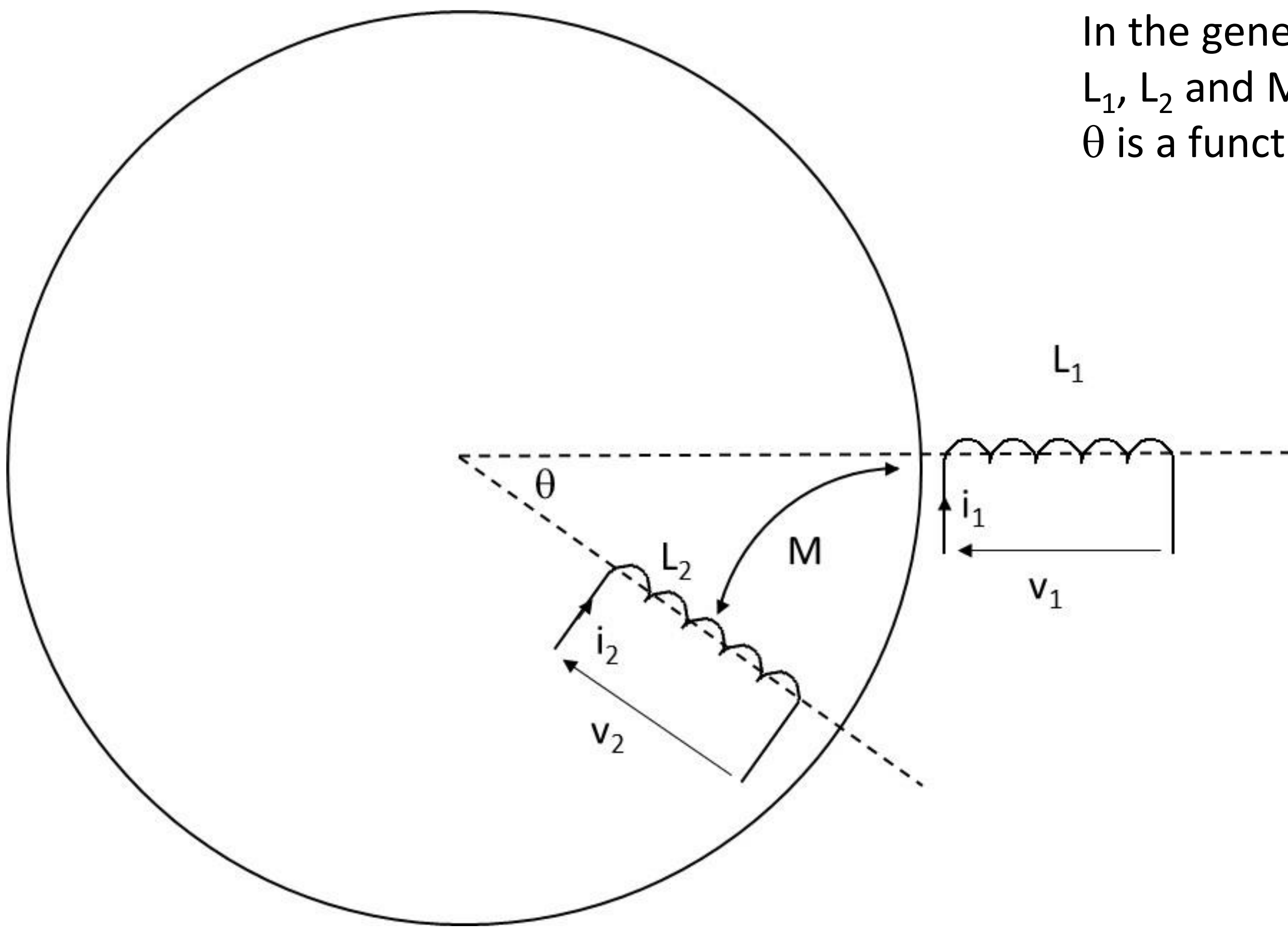


Salient

In the salient rotor arrangement shown, the stator coil inductance is a function of rotor position

Fundamental processes in rotating coils

Consider the general case of two rotating coils, one on the stator and the other on the rotor



In the general case:

L_1, L_2 and M are functions of θ

θ is a function of time

Torque producing mechanism

Change in electrical energy = Change in stored magnetic energy + Change in mech output

$dW_e = dW_f + dW_m$

Consider the energy stored in the magnetic field

$$W_f = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

$$\frac{dW_f}{dt} = \frac{1}{2}\left[i_1^2 \frac{dL_1}{dt} + L_1 \frac{di_1^2}{dt} + i_2^2 \frac{dL_2}{dt} + L_2 \frac{di_2^2}{dt} \right] + M i_1 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt} + i_1 i_2 \frac{dM}{dt}$$

$$dW_f = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 L_1 di_1 + i_2 L_2 di_2 + M i_1 di_2 + M i_2 di_1 + i_1 i_2 dM$$

Torque producing mechanism (cont'd)

$$e_1 = \frac{d}{dt}(i_1 L_1) + \frac{d}{dt}(i_2 M) = \boxed{L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}} + \boxed{\left[i_1 \frac{dL_1}{d\theta} + i_2 \frac{dM}{d\theta} \right] \frac{d\theta}{dt}}$$

Transformer emf Rotational emf

Also,

$$e_2 = \frac{d}{dt}(i_2 L_2) + \frac{d}{dt}(i_1 M) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} + \left[i_2 \frac{dL_2}{d\theta} + i_1 \frac{dM}{d\theta} \right] \frac{d\theta}{dt}$$

Electrical power P_e

$$P_e = e_1 i_1 + e_2 i_2$$

$$P_e = (i_1^2 \frac{dL_1}{d\theta} + 2 i_1 i_2 \frac{dM}{d\theta} + i_2^2 \frac{dL_2}{d\theta}) \frac{d\theta}{dt} + L_1 i_1 \frac{di_1}{dt} + M i_1 \frac{di_2}{dt} + L_2 i_2 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt}$$

$$dW_e = i_1^2 dL_1 + L_1 i_1 di_1 + 2 i_1 i_2 dM + i_1 M di_2 + i_2^2 dL_2 + i_2 L_2 di_2 + i_2 M di_1$$

Torque producing mechanism (cont'd)

From energy balance considerations:

$$dW_m = dW_e - dW_f = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM$$

Hence, the torque is given by:

$$T = \frac{dW_m}{d\theta} = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} i_2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM}{d\theta}$$

Torque which arises from mutual inductance

- By reference to the earlier figure which defined the arrangement of the two coils, it can be seen that the mutual inductance between the two coils is a maximum when $\theta=0^\circ$ and nominally zero (in an idealised machine at least) when $\theta=90^\circ$.
- Between these two values, the nature of the change in mutual inductance is dependant on specific design features, but a useful starting point is to assume that the variation is co-sinusoidal:

$$M = M_{max}\, cos\theta$$

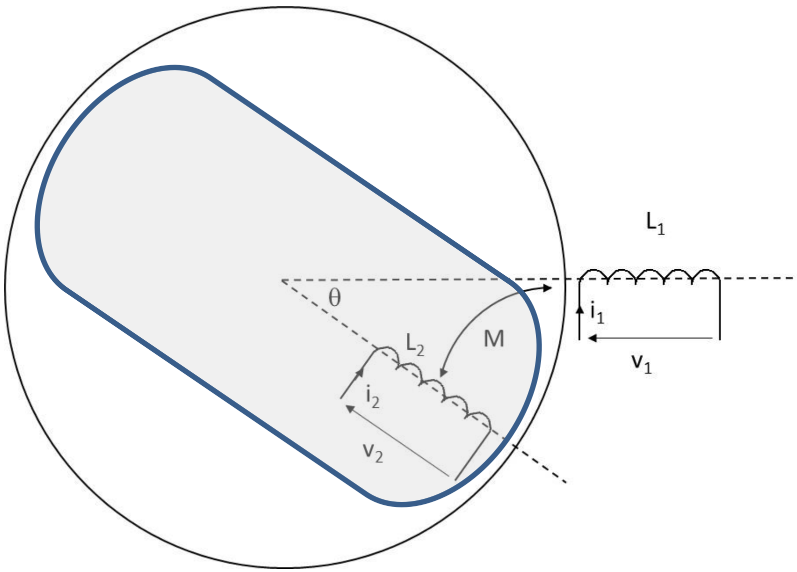
Hence, the term which determines the excitation component of torque is given by:

$$\frac{\partial M}{\partial \theta} = -M_{max}\, sin\theta$$

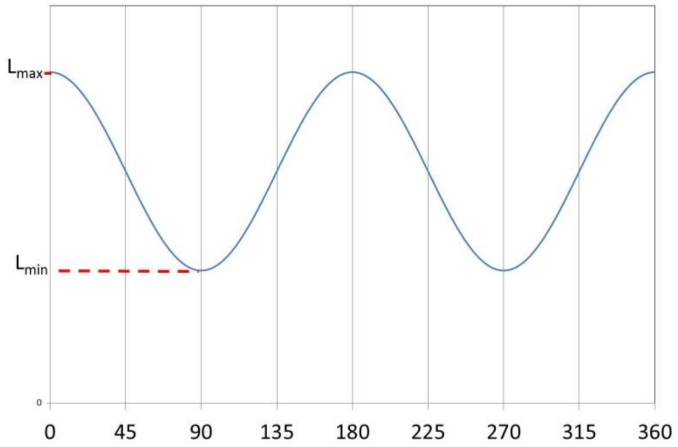
This demonstrates that to maximise the magnitude of the torque produced for a given combination of coil currents, the angle θ should be maintained at $\pm 90^\circ$

Torque due to rotor saliency

- In a machine with rotor saliency, the self-inductance of stator coils is a function of the rotor angular position
- Assume that the variation of inductance for the arrangement of coil shown is as follows:
  - It is a maximum at $\theta = 0°$ and takes a value L_{max}
  - It has a minimum at $\theta = 90°$ and takes a value L_{min} (note not zero)
  - The variation between the minimum and maximum takes the form:



$$L_1(\theta) = \frac{(L_{min} + L_{max})}{2} + \left(\frac{L_{max} - L_{min}}{2}\right) cos2\theta$$



Hence,

$$\frac{\partial L_1}{\partial \theta} = -(L_{max} - L_{min})\ sin2\theta$$

The magnitude of the torque again takes a maximum value at θ at ±45°