List of useful formulae

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$f(t) = a_0 + \sum_{n=1}^{N}\left[a_n \cos\left(\frac{2n\pi t}{T}\right) + b_n \sin\left(\frac{2n\pi t}{T}\right)\right]$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$X(\omega) = 2\int_{0}^{\infty} x(t)\cos\omega t dt$$

$$X(\omega) = -j2\int_{0}^{\infty} x(t)\sin\omega t dt$$

$$X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$$

$$x(t) = \frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}dt$$

$$\cos(x)\cos(y) = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\sin(x)\sin(y) = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

$$\sin(x)\cos(y) = \frac{1}{2}\left[\sin(x-y) + \sin(x+y)\right]$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

Fourier Transform Pairs

Signal	Fourier Transfrom
$e^{j\omega_o t}$	$2\pi\delta(\omega - \omega_o)$
$\cos\omega_o t$	$\pi[\delta(\omega + \omega_o) + \delta(\omega - \omega_o)]$
$\sin\omega_o t$	$j\pi[\delta(\omega + \omega_o) - \delta(\omega - \omega_o)]$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$u(t)$	$\dfrac{1}{j\omega} + \pi\delta(\omega)$
$\delta(t - t_o)$	$e^{-j\omega t_o}$
$e^{-at}u(t),\ a > 0$	$\dfrac{1}{a + j\omega}$
$x(t) = \begin{cases} 1, & \lvert t\rvert < \tau \\ 0, & \lvert t\rvert > \tau \end{cases}$	$\dfrac{2\sin\omega\tau}{\omega} = 2\tau\sin c(\omega\tau)$
$\dfrac{\sin\omega_c t}{\pi t} = \dfrac{\omega_c}{\pi}\sin c(\omega_c t)$	$X(\omega) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \lvert\omega\rvert > \omega_c \end{cases}$
$\sum_{n=-\infty}^{\infty}\delta(t - nT)$	$\dfrac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \dfrac{2\pi k}{T}\right)$

Properties of Fourier Transform

Property	Aperiodic signal, $x(t)$	Fourier Transfrom, $X(\omega)$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(\omega) + bX_2(\omega)$
Time Shifting	$x(t - t_o)$	$e^{-j\omega t_o} X(\omega)$
Frequency Shifting	$e^{j\omega_o t} x(t)$	$X(\omega - \omega_o)$
Time Scaling	$x(at)$	$\frac{1}{a} X\left(\frac{\omega}{a}\right)$
Differentiation in Time	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Differentiation in Frequency	$tx(t)$	$j\frac{dX(\omega)}{d\omega}$
Integration in time	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$
Convolution	$x(t)*h(t)$	$X(\omega).H(\omega)$
Multiplication in time	$x(t).h(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\lambda)H(\omega-\lambda)d\lambda$
Parseval's Theorem	$E = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$	

Properties of Laplace Transform

Property	Transform Property
Linearity	$ax_1(t) + bx_2(t) \leftrightarrow aX_1(\mathrm{s}) + bX_2(\mathrm{s})$.
Time shift	$x(t- t_o)\, u(t- t_o) \leftrightarrow X(s)e^{-st_o}$, $t_o > 0$
Multiplication by a complex exponential	$x(t)e^{s_o t} \leftrightarrow X(s\text{-}s_o)$
Time scaling	$x(at) \leftrightarrow X(s/a)/\|a\|$
Differentiation in time domain	$\dfrac{dx(t)}{dt} \leftrightarrow sX(\mathrm{s}) - x(0)$
	$\dfrac{d^2x(t)}{dt^2} \leftrightarrow s^2X(\mathrm{s}) - sx(0) - \left.\dfrac{dx(t)}{dt}\right\|_{t=0}$
Differentiation in s domain	$t^n x(t) \leftrightarrow \dfrac{d^nX(s)}{ds^n}(-1)^n$
Integration	$\displaystyle\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{1}{s}X(s)$
Convolution in time domain	$x(t)*h(t) \leftrightarrow X(s).H(s)$
Initial value theorem	$x(0) = \lim\limits_{s\to\infty} sX(s)$
Final value theorem	$\lim\limits_{t\to\infty} x(t) = \lim\limits_{s\to 0} sX(s)$

(if $x(t)$ has a finite value as $t \to\infty$)

Laplace Transform pairs

Signal	Transform
Unit step: $u(t)$	$\frac{1}{s}$
Unit impulse: $\delta(t)$	1
Unit ramp: $tu(t)$	$\frac{1}{s^2}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$
$(\cos\omega_o t)u(t)$	$\frac{s}{(s^2+\omega_o^2)}$
$(\sin\omega_o t)u(t)$	$\frac{\omega_o}{(s^2+\omega_o^2)}$
$(e^{-at}\cos\omega_o t)u(t)$	$\frac{s+a}{((s+a)^2+\omega_o^2)}$
$(e^{-at}\sin\omega_o t)u(t)$	$\frac{\omega_o}{((s+a)^2+\omega_o^2)}$
$(t\cos\omega_o t)u(t)$	$\frac{s^2-\omega_o^2}{(s^2+\omega_o^2)^2}$
$(t\sin\omega_o t)u(t)$	$\frac{2\omega_o s}{(s^2+\omega_o^2)^2}$