

The Heisenberg Uncertainty Principle

"Those who are not shocked when they first come across quantum theory cannot possibly have understood it."

-Niels Bhor

The **position** and **momentum** of a particle cannot be simultaneously measured with arbitrarily high precision. There is a minimum for the product of uncertainties of these two measurements.

$$\Delta x \Delta p \geq h/4\pi$$

This is not a statement about the inaccuracy of measurement instruments, nor a reflection on the quality of experimental methods; it arises from the wave properties inherent in the quantum mechanical description of nature. Even with perfect instruments and techniques, the uncertainty is inherent in the nature of things.

Let us imagine that we wish to measure the position and momentum of a particular particle. To 'see' the particle, we need to shine some light of wavelength λ on it. There is a limit to the resolving power of the ability to see the particle given by the wavelength λ . This means that there is always an uncertainty in the position of the particle

$$\Delta x \sim \lambda$$

This results from considering the light as a wave.

Now consider the light as a particle (photon).

The photon has momentum as we have noted earlier.

When the photon 'hits' the particle, it gives up some or all of its momentum to the particle. We don't know how much is transferred as we don't measure the photon's properties after the interaction.

There is now some uncertainty in the momentum of the particle, given by

$$\Delta p \sim h/\lambda$$

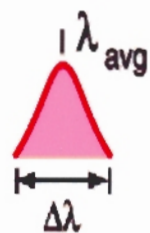
Combining these two equations we obtain

$$\Delta x \Delta p \sim h$$

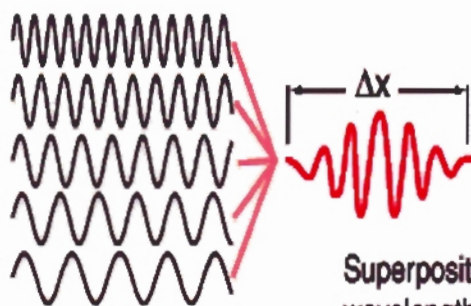
Note that this is *independent* of wavelength and says that in principle, there is a limit to how accurately one can simultaneously measure the position and momentum of a particle.

If we try and measure the position more accurately by **decreasing** the wavelength, the uncertainty in the momentum of the particle **increases**. The more refined treatment by Heisenberg results in the proper expression.

A continuous distribution of wavelengths can produce a localized "wave packet".



$$p = \frac{h}{\lambda}$$



Each different wavelength represents a different value of momentum according to the DeBroglie relationship.

Superposition of different wavelengths is necessary to localize the position. A wider spread of wavelengths contributes to a smaller Δx .

$$\Delta x \Delta p > \frac{h}{2}$$

Let us look at this another way:

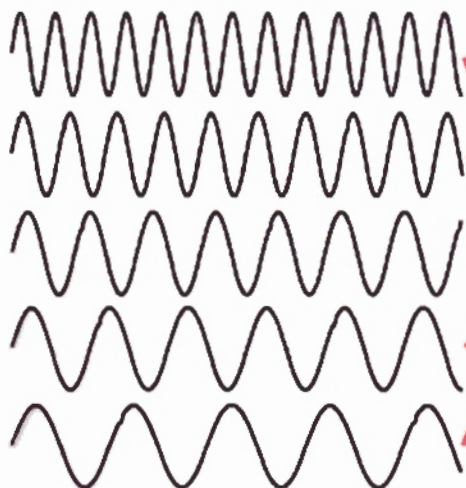
Precisely determined momentum



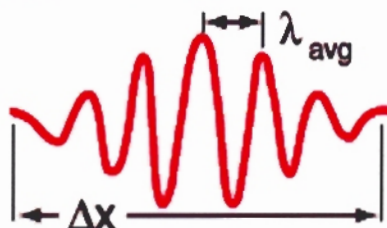
A sine wave of wavelength λ implies that the momentum p is precisely known: But the wavefunction and the probability of finding the particle $\psi^*\psi$ is spread over all of space.

$$p = \frac{h}{\lambda}$$

p precise
 x unknown



Adding several waves of different wavelength together will produce an interference pattern which begins to localize the wave.



but that process spreads the momentum values and makes it more uncertain. This is an inherent and inescapable increase in the uncertainty Δp when Δx is decreases.

$$\Delta x \Delta p > \frac{\hbar}{2}$$