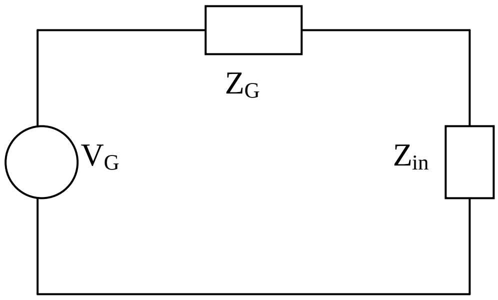
Solutions

Q1(a)

The input section of a transmission line can be represented as



(1 mark)

Conjugate matching can be achieved when $Z_{in} = Z_G^*$ and it can be used to obtain a maximum power transfer. **(2 marks)**

Q1(b)

$$\beta\ell = \frac{2\pi}{\lambda} \times 0.15\lambda = 0.942$$ **(1 mark)**

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)} = 29 - j9\Omega$$ **(1 mark)**

$$\Gamma_{load} = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(70 - j30) - 50}{(70 + j30) + 50} = 0.2 - j0.2$$ **(1 mark)**

$$VSWR = \frac{1 - |\Gamma_{load}|}{1 + |\Gamma_{load}|} = 1.8$$ **(1 mark)**

Q1(c)

The voltages and currents at the load are given by

$$V_{inc\ load} + V_{ref\ load} = V_L \quad (1)$$ **(1 mark)**

$$\frac{V_{inc\ load}}{Z_o} - \frac{V_{ref\ load}}{Z_o} = \frac{V_L}{Z_L} \quad (2)$$ **(1 mark)**

Equation (2) can be rewritten as

$$\frac{Z_L}{Z_o} V_{inc\ load} \left(1 - \frac{V_{ref\ load}}{V_{inc\ load}}\right) = V_L$$ **(1 mark)**

Substitute for V_L from eq. (1) gives

$$\frac{Z_L}{Z_o}\left(1 - \frac{V_{ref\ load}}{V_{inc\ load}}\right) = \left(1 + \frac{V_{ref\ load}}{V_{inc\ load}}\right) \quad (3)$$ **(1 mark)**

Since the reflection coefficient is defined as

$$\Gamma = \frac{V_{ref\ load}}{V_{inc_{load}}}$$ **(1 mark)**

Then equation 3 can be expressed as

$$\frac{Z_L}{Z_o}(1-\Gamma)=(1+\Gamma)$$ **(1 mark)**

which can be re-arranged to obtain

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$ **(1 mark)**

<u>Q1(d)</u>

For a quarter length transformer

$$Z_{in} = \frac{Z_o^2}{Z_L}$$ **(1 mark)**

i.e.

$$Z_{in} = \frac{(100)^2}{210} = 47.6\Omega$$ **(1 mark)**

Since this is a lossless line then the power at the sending end equals the power at the receiving end, **(1 mark)**

i.e

$$\frac{V_{in}^2}{Z_{in}} = \frac{V_L^2}{Z_L}$$ **(1 mark)**

which gives

$$V_{in}^2 = \frac{Z_{in}}{Z_L}V_L^2 = \frac{47}{210}\times(80)^2$$ **(1 mark)**

i.e.

$$V_{in} = 37.9V$$ **(1 mark)**

<u>Q2(a)</u>

A basic transmission line problem is that of determining one of load impedance, line length, and input impedance from knowledge of the remaining two. The Smith chart can be used to solve such problems by using the following steps

- Normalise load impedance and find reflection coefficient at load **(1 mark)**
- Rotate reflection coefficient **(1 mark)**
- Record normalised input impedance **(1 mark)**
- De-normalise input impedance **(1 mark)**

<u>Q2(b)</u>

f=*300* MHz λ=1 m

d=20. cm= 0.2λ **(1 mark)**

α*d*=0.2x15=3. dB

Nepers=8.686dB

α*d* =0.345 nepers 2α*d*=0.69 nepers **(1 mark)**

Draw point A at r=2.5, then move 0.2λaround chart *towards generator*, i.e. point B on the chart (0.43-j0.27). **(2 marks)**

The radius at point B must be reduced by a factor of $e^{-2\alpha d}$, i.e. 0.5=50% of the radius for the lossless case. This is shown as point C on the chart.

(1 mark)

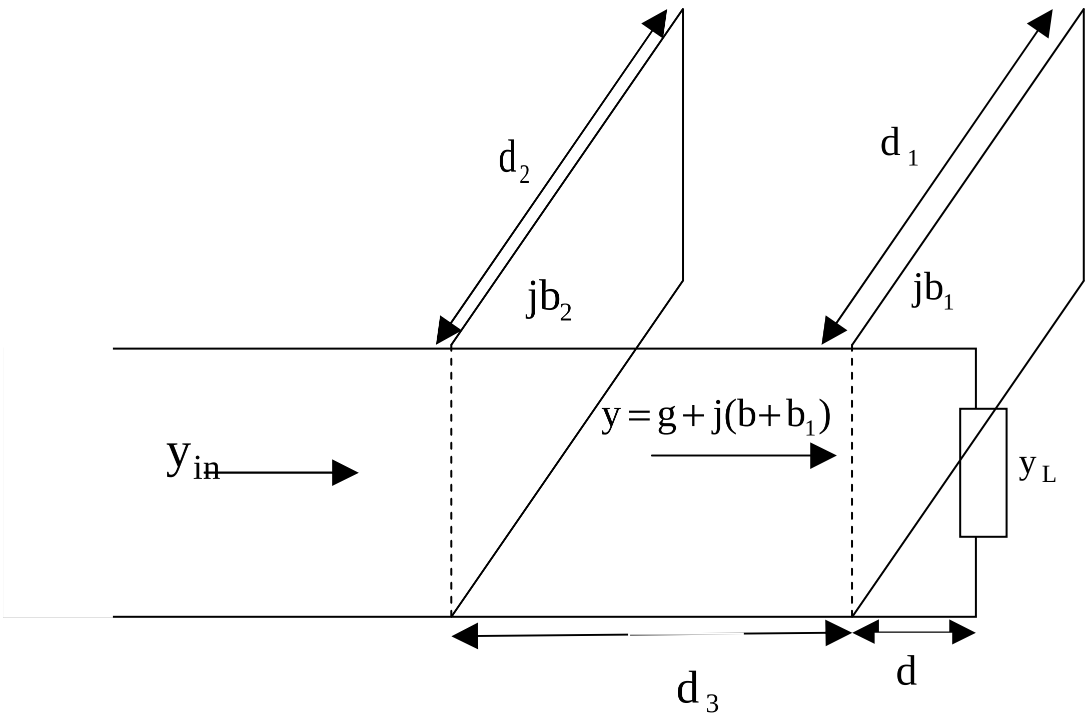
VSWR on the line at this point is found by rotating an arc radius (OC) to the real axis (point D) and read the value VSWR=r=1.6 **(1 mark)**

<u>Q2(c)</u>

$z_L = \frac{(110 + j50)}{50} = 2.2 + j1.$ (point A) **(1 mark)**

i.e.

y_L=0.38-j0.17 (point B) **(1 mark)**



Step 1
Rotate the unit g circle *Towards Load*, by a distance of d_3=0.125λ. **(1 mark)**

Step 2
Move from point B to intersect the new, rotated, unit circle at point C. The movement should be on the corresponding conductance circle, since the stub does not alter the real part of the admittance. **(1 mark)**

Step 3

The admittance at point C is

y_C=0.38+0.22

compare it with that at B

y_L=0.31-j0.17

shows that stub 1 has provided j0.39i.e. b_1=0.39 **(2 marks)**

Step 4

For an o.c. stub, this means d_1=0.059λ, i.e. the distance from D to E on the chart.

(1 mark)

Step 5

Move a distance d_3=0.125λ along the line from the 1st stub position to the 2nd stub position (from point C to F). **(1 mark)**

Step 6

At point F, the admittance is y_F=1+j1.05i.e. stub 2 must provide -j1.05, *i.e.* b_2=-1.05) to reach the matched condition. **(1 mark)**

Step 7

For a s.c. stub, this means d_2=(0.371-0.25)λ=0.121λ, i.e. the distance from G to H on the chart. **(1 mark)**

<u>Q3(a)</u>

<u>Rule 1</u>(Series Rule)
Two series paths are equivalent to one single path whose transmission coefficient is the multiplication of the coefficients of the original paths. **(1 mark)**

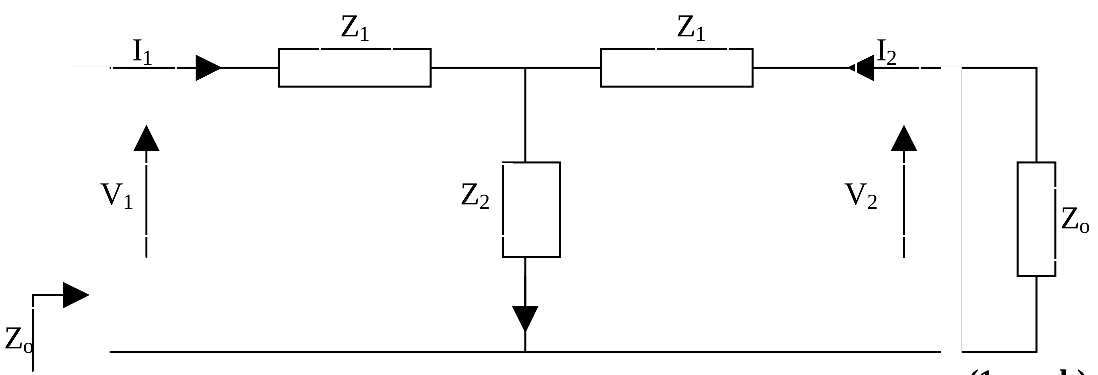
<u>Rule 2</u>(Parallel Rule)
Two parallel paths are equivalent to one single path whose transmission coefficient is the sum of the coefficients of the original paths. **(1 mark)**

<u>Rule 3</u>(Loop Rule)
A feed back loop may be eliminated by dividing the input coefficient by 1 minus the coefficient around the loop, which is the product of other coefficients according to rule 1. **(1 mark)**

<u>Rule 4</u>(Splitting Rule)
A node may be splitted into two separate nodes. **(1 mark)**

<u>Q3(b)</u>

Transmission line has an equivalent symmetrical TEE, or PI, circuit as shown below



(1 mark)

$$Z_o = Z_1 + \frac{Z_2(Z_1 + Z_o)}{Z_1 + Z_2 + Z_o}$$ **(1 mark)**

$$I_1 = -I_2 \frac{(Z_1 + Z_o + Z_2)}{Z_2}$$ **(1 mark)**

From the transmission line theory

$$I_\ell = I_{in} e^{-\gamma\ell} \qquad \text{i.e.} \quad I_2 = -I_1 e^{-\gamma\ell}$$ **(1 mark)**

Therefore

$$e^{\gamma\ell} = \frac{(Z_1 + Z_o + Z_2)}{Z_2}$$ **(1 mark)**

$$Z_o = Z_1 + (Z_1 + Z_o)e^{-\gamma\ell}$$

which gives

$$Z_o(1 - e^{-\gamma\ell}) = Z_1(1 + e^{-\gamma\ell})$$

$$Z_1 = Z_o \tanh(\frac{\gamma\ell}{2})$$ **(1 mark)**

and

$$Z_2 = \frac{Z_o}{\sinh(\gamma\ell)}$$ **(1 mark)**

Using the ABCD parameters of a TEE circuit, we get

$$A = D = 1 + \tanh(\frac{\gamma\ell}{2})\sinh(\gamma\ell) = \cosh(\gamma\ell),$$

$$B = Zo \tanh(\frac{\gamma\ell}{2})(1 + \cosh(\gamma\ell)) = Z_o \sinh(\gamma\ell)$$

$$C = \frac{\sinh(\gamma\ell)}{Z_o}$$ **(1 mark)**

<u>Q3(c)</u>

To calculate S_{11}, the impedance Z_{in1} is required when the network is terminated with Z_o, where $Z_o = Z_{o1} = Z_{o2}$

$$Z_{in1} = \left[\frac{Z_2\left(Z_1 + \frac{Z_2 Z_o}{Z_2 + Z_o}\right)}{Z_1 + Z_2 + \frac{Z_2 Z_o}{Z_2 + Z_o}} \right]$$

$$Z_{in1} = Z_2 \frac{Z_1 Z_o + Z_1 Z_2 + Z_2 Z_o}{Z_2 Z_o + (Z_2 + Z_o)(Z_2 + Z_1)}$$ **(1 mark)**

$$S_{11} = \Gamma_{in} = \frac{Z_{in1} - Z_o}{Z_{in1} + Z_o}$$

then

$$S_{11} = \frac{Z_1 Z_2^2 - 2Z_2 Z_o^2 - Z_1 Z_o^2}{2Z_1 Z_o Z_2 + 2Z_o Z_2^2 + 2Z_2 Z_o^2 + Z_1 Z_2^2 + Z_1 Z_o^2}$$ **(1 mark)**

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

For the 1st port

$$V_1 = \sqrt{Z_o}(a_1 + b_1)$$
$$V_2 = \sqrt{Z_o} b_2$$ **(1 mark)**

Therefore

$$\frac{V_1}{V_2} = \frac{(a_1 + b_1)}{b_2} = \frac{(1 + \frac{b_1}{a_1})}{\frac{b_2}{a_1}} = \frac{1 + S_{11}}{S_{21}}$$

i.e.

$$S_{21} = \frac{V_2}{V_1}(1 + S_{11})$$

$$V_2 = V_1 \frac{Z_p}{(Z_p + Z_1)}$$ **(1 mark)**

where

$$Z_p = \frac{Z_o Z_2}{(Z_o + Z_2)}$$

Hence

$$\frac{V_2}{V_1} = \frac{Z_o Z_2}{(Z_o Z_2 + Z_1 Z_2 + Z_o Z_1)}$$ **(1 mark)**

$$1 + S_{11} = \frac{2Z_2(Z_1 Z_o + Z_o Z_2 + Z_1 Z_2)}{2Z_1 Z_o Z_2 + 2Z_o Z_2^2 + 2Z_2 Z_o^2 + Z_1 Z_2^2 + Z_1 Z_o^2}$$

Therefore

$$S_{21} = \frac{2Z_o Z_2^2}{2Z_1 Z_o Z_2 + 2Z_o Z_2^2 + 2Z_2 Z_o^2 + Z_1 Z_2^2 + Z_1 Z_o^2}$$ **(0.5 mark)**

Similarly

$$S_{12} = \frac{2Z_o Z_2^2}{2Z_1 Z_o Z_2 + 2Z_o Z_2^2 + 2Z_2 Z_o^2 + Z_1 Z_2^2 + Z_1 Z_o^2}$$ **(0.5 mark)**

Finally

$$S_{22} = \frac{Z_{in2} - Z_o}{Z_{in2} + Z_o} \qquad Z_{in2} = \left[\frac{Z_2\left(Z_1 + \frac{Z_2 Z_o}{Z_2 + Z_o}\right)}{Z_1 + Z_2 + \frac{Z_2 Z_o}{Z_2 + Z_o}}\right]$$

$$Z_{in2} = Z_2 \frac{Z_1 Z_o + Z_1 Z_2 + Z_2 Z_o}{Z_2 Z_o + (Z_2 + Z_o)(Z_2 + Z_1)}$$ **(1 mark)**

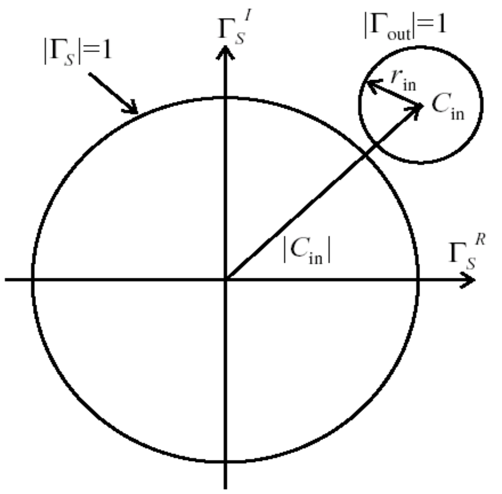
then

$$S_{22} = \frac{Z_1 Z_2^2 - 2Z_2 Z_o^2 - Z_1 Z_o^2}{2Z_1 Z_o Z_2 + 2Z_o Z_2^2 + 2Z_2 Z_o^2 + Z_1 Z_2^2 + Z_1 Z_o^2}$$ **(1 mark)**

Q4(a)

***Unconditional Stability*; the network is unconditionally stable for any combinations of source and load impedance values. This is possible in two cases **(2 marks)**

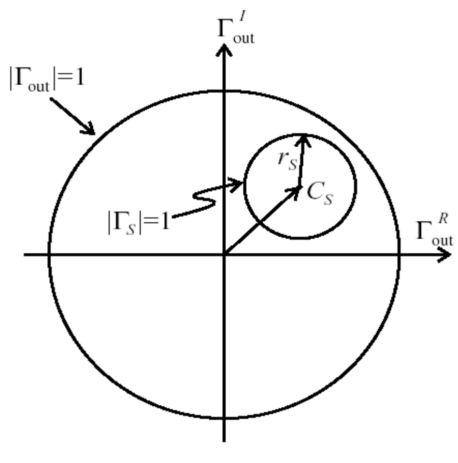
(a) Stability circle is completely outside the Smith chart.



(a) $|\Gamma_{out}|$ = 1 circle must reside outside

(1 mark)

(b) Stability circle totally enclose the Smith chart



(b) $|\Gamma_S|$ = 1 circle must reside inside

(1 mark)

Q4(b)

$\Delta = |S_{11}S_{22} - S_{12}S_{21}| = 1.58$ **(1 mark)**

$$K = \frac{1+|\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|} = 1.15$$ **(1 mark)**

So the transistor is potentially unstable.

Calculate the centre and radius of each stability circle as

$$r_{in} = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| = 0.54$$ **(1 mark)**

$$C_{in} = \frac{\left(S_{11} - \Delta S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} = 0.21\angle 52^\circ$$ **(1 mark)**

$$r_{out} = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| = 0.54$$ **(1 mark)**

$$C_{out} = \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} == 0.21\angle 27^\circ$$ **(1 mark)**

The input and output stability circles then can be plotted on the Smith chart as shown in the figure. **(2 marks)**

Q4(c)

First we test for stability

$$K = \frac{1+|\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|} = 1.195$$ **(1 mark)**

$\Delta = |S_{11}S_{22} - S_{12}S_{21}| = 0.488$ **(1 mark)**

Since K>1 and $|\Delta| < 1$ then the transistor is unconditional stable at 4 GHz. Therefore there is no need t o plot the stability circles.

To achieve a maximum gain, a conjugate matching is required, i.e.:

$\Gamma_{in} = \Gamma_s^*$ and $\Gamma_{out} = \Gamma_L^*$ **(1 mark)** i.e.

$$\Gamma_s = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|}}{2C_1} = 0.872\angle 123^\circ$$ **(1 mark)**

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|}}{2C_2} = 0.876\angle 61^\circ$$ **(1 mark)**

The effective gains can be calculated as

$$G_o = |S_{21}|^2 = 6.76 = 8.30\,dB$$ **(0.5 mark)**

$$G_s = \frac{(1 - |\Gamma_s|^2)}{|1 - \Gamma_s \Gamma_{in}|^2} = 4.17 = 6.20\,dB$$ **(1 mark)**

$$G_L = \frac{(1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2} = 1.67 = 2.22\,dB$$ **(1 mark)**

The overall gain

$$G_{T\max} = 6.20 + 8.30 + 2.22 = 16.7\,dB$$ **(0.5 mark)**