MEC 316 Tutorial Sheet 2 Solutions

1) Each cell generates current current I_1, I_2

Total $I = I_1 + I_2 = I_{d1} - I_{L1} + I_{d2} - I_{L2}$

Voltages across both cells have to be identical for parallel cell configuration, so:

$$I = I_{01}\left[\exp(eV/kT) - 1\right] - I_{L1} + I_{02}\left[\exp(eV/kT) - 1\right] - I_{L2}$$

$$= (I_{01} + I_{02})\left[\exp(eV/kT) - 1\right] - (I_{L1} + I_{L2})$$

Under open circuit condition, $I = 0$ and exponent term $\gg 1$,

$$\therefore \quad \exp(eV_{oc}/kT) \approx \frac{I_L}{I_0}$$

$$V_{oc} \approx \frac{kT}{e} \ln\left(\frac{I_L}{I_0}\right)$$

Under short circuit, $V = 0$, $I_L = -(I_{L1} + I_{L2}) = -2.8\,A$

Need I_{01} and I_{02}

Cell 1 : $0.6 \approx \frac{kT}{e} \ln\left(\frac{1.5}{I_{01}}\right) \Longrightarrow I_{01} = 5.66 \times 10^{-11}\,A$

Cell 2 : $1.2 \approx \frac{kT}{e} \ln\left(\frac{1.3}{I_{02}}\right) \Longrightarrow I_{02} = 1.85 \times 10^{-21}\,A$

$$\therefore \quad V_{oc} \approx \frac{kT}{e} \ln\left(\frac{2.8}{5.66 \times 10^{-11}}\right) = 0.615\,V$$

Equation for array is : $I = 5.66 \times 10^{-11}\left[\exp(eV/kT) - 1\right] - 2.8$

V(V) :	0.1	0.2	0.3	0.4	0.45	0.5	~~0.53~~ 0.53	0.55
I(A) :	2.8	2.8	2.8	2.8	2.79	2.77	2.71	2.60
Power(W) :	0.28	0.56	0.84	1.12	1.26	1.39	1.44	1.43

Maximum power = 1.44 W

2) $I = I_0 [\exp(eV/kT) - 1] - I_L$

I = current through resistor $= -\frac{V}{R} = -\frac{0.5}{2} = -0.25\,A$

substituting and rearranging equation gives:

$I_L = 2\times10^{-10} [\exp(e\,0.5/kT) - 1] + 0.25 = 0.347\,A$

$V_{oc} = \frac{kT}{e} \ln\left(\frac{I_L}{I_0} + 1\right) \approx \frac{kT}{e} \ln\left(\frac{I_L}{I_0}\right) = 0.025 \times 21.27$

$= 0.532\,V$

Power $= V.I. = 0.5 \times 0.25 = 0.125\,W$

$V_{oc} \times I_{sc} = 0.532 \times 0.347 = 0.184\,W$

Fill factor $= \frac{\text{Power}}{V_{oc} \times I_{sc}} = \frac{0.125}{0.184} = 0.68$

This is a reasonable fill factor, so $2\,\Omega$ load is close to optimum

When the temperature decreases, I_0 decreases, hence V_{oc} would increase. The minority carrier diffusion length would also increase slightly, so I_L would increase. The higher V and larger I will result in more power being developed by the cell.

3) Power $= V \times I = V I_o [\exp(eV/kT) - 1] - V I_L$

Maximum in power occurs when $\frac{dP}{dV} = 0$ and $V = V_m$

so we get,

$$\left(1 + \frac{eV_m}{kT}\right) \times \exp(eV_m/kT) = \frac{I_L}{I_o} + 1$$

since $V_m \gg kT/e$ and $\frac{I_L}{I_o} \gg 1$ and $I_L = I_{sc}$

$$\ln\left(\frac{eV_m}{kT}\right) \approx \ln\left(\frac{I_{sc}}{I_o}\right) - \frac{eV_m}{kT}$$

$I_o = 10^{-10}$ A and $I_{sc} = 50$ mA, assume $\frac{kT}{e} = 25$ mV

$$\ln\left(\frac{V_m}{0.025}\right) - \left(\frac{V_m}{0.025}\right) \approx \ln\left(\frac{50 \times 10^{-3}}{10^{-10}}\right) = 20.03$$

substituting different values for V_m in L.H.S. of above gives 20.89 for $V_m = 0.45$, so good enough

$$I_{max} = I_o [\exp(e.\,0.45/kT) - 1] - I_L = 43.44 \text{ mA}$$

Max. Power $= V_m \times I_{max} = 19.5$ mW

Cell develops maximum power when optimum R (load) is chosen, so $R = \frac{V_m}{I_{max}} = 0.45/43.44 \times 10^{-3} = 10.36\ \Omega$

$$V_{oc} \approx \frac{kT}{e} \ln\left(\frac{I_L}{I_o}\right) = 0.025 \times \ln(5 \times 10^8) = 0.5\text{ V}$$

$I_{sc} \times V_{oc} = 0.025$ W

$\therefore$ Fill factor $= \frac{19.5}{25} = 0.78$ i.e. a good high value.

4) $V_{oc} = \frac{kT}{e} \ln\left(\frac{I_L}{I_o} + 1\right)$, $\frac{kT}{e} = 0.025V$ at RT, $V_{oc} = 0.5V$, $I_o = 2\times10^{-10}$ A

substituting into above gives, $I_L = 97mA$

Load current, $I = I_d - I_L = I_o[\exp(eV/kT) - 1] - I_L$

I at $V = 0.45V$ given by

$I = 2\times10^{-10}[\exp(0.45/0.025) - 1] - 0.097 = 84$ mA

Power $= VI = 0.45 \times 0.084 = 37.8$ mW

Efficiency $= \frac{P_{out}}{P_{in}} \times 100\%$

P_{in} = area of cell × 1kW

Eff. $= \frac{37.8 \times 10^{-3}}{(25\times10^{-3})^2 \times 1000} = 6\%$

5) For cells in parallel equations, see Question 1.

First need I_o for each cell.

Si cell : $V_{oc} = 0.7 = \frac{kT}{e} \ln\left(\frac{1.4}{I_o}\right) \Rightarrow I_o = 9.68\times10^{-13}$ A

AlGaAs cell : $V_{oc} = 1.0 = \frac{kT}{e} \ln\left(\frac{1.3}{I_o}\right) \Rightarrow I_o = 5.52\times10^{-18}$ A

For array under short circuit conditions,

$V = 0$, $I_{sc} = -1.4 - 1.3 = -2.7A$

Under open circuit conditions, $I = 0$

$2.7 = 5.52\times10^{-18}[\exp(eV_{oc}/kT) - 1] + 9.68\times10^{-13}[\exp(eV_{oc}/kT) - 1]$

$2.7 \approx 9.68\times10^{-13}[\exp(eV_{oc}/kT) - 1]$

$V_{oc} \approx 0.716$ V

I-V of array is therefore

$$I = [9.68\times10^{-13} + 5.52\times10^{-18}]\,[\exp(eV/kT)-1] - 2.7\ \text{A}$$

Power = $V \times I$

V (V) :	0	0.2	0.4	0.55	0.6	0.65	0.7
I (A) :	2.7	2.7	2.7	2.7	2.68	2.51	1.3
Power (W) :	0	0.54	1.08	1.485	1.608	1.631	0.91

Maximum power is ~ 1.631 W

$$\text{Fill factor} = \frac{\text{Max. Power}}{V_{oc}\times I_{sc}} = \frac{1.631}{0.716\times 2.7} = 0.84$$

6) See question 3 for first part.

$I_{sc} = 100\,\text{mA}$, $I_o = 10^{-9}\,\text{A}$

$$\left(1+\frac{eV_m}{kT}\right)\exp\left(\frac{eV_m}{kT}\right) = 1 + \frac{I_{sc}}{I_o}$$

substitute values for V_m until above is approx. true

$V_m = 0.4$ gives L.H.S. $\simeq$ R.H.S.

for max. power, we need max. current,

$$I_{max} = 10^{-9}\,[\exp(0.4/0.025)-1] - 0.1\,\text{A} = 91\,\text{mA}$$

$$P = I_{max}\times V_{max} = 91\,\text{mA}\times 0.4 = 36.4\,\text{mW}$$

Need V_{oc} for fill factor ; $V_{oc} = \frac{kT}{e}\ln\left(\frac{I_{sc}}{I_o}\right) = 0.025\times 18.42$

$$= 0.46\,\text{V}$$

$$\text{Fill factor} = \frac{I_{max}\times V_{max}}{I_{sc}\times V_{oc}}$$

$$= \frac{36.4\,\text{mW}}{0.1\times 0.46\,\text{W}} = 0.79$$

7) For cells in parallel, see Question 1.

$$I = [I_{01} + I_{02}]\,[\exp(eV/kT) - 1] - (I_{L1} + I_{L2})$$

$$I = [4\times10^{-11} + 5\times10^{-10}]\,[\exp(V/0.025) - 1] - (0.25 + 0.1)$$

$$= [5.4\times10^{-10}]\,[\exp(\tfrac{V}{0.025}) - 1] - 0.35 \text{ A}$$

V (V):	0	0·2	0·4	0·42	0·44	0·46	0·48
I (I):	0·35	0·35	0·345	0·339	0·326	0·297	0·232
P (W):	0	0·07	0·138	0·142	0·143	0·137	0·115

Maximum power ~ 143 mW

This occurs at 0·44V when a current of 0·326A flows, so optimum load resistor is:

$$R = \frac{V}{I} = \frac{0.44}{0.326} = 1.35\ \Omega$$

$$V_{oc} \text{ for array} = \frac{kT}{e}\ln\left(\frac{I_{L1}+I_{L2}}{I_{01}+I_{02}}\right) = 0.025\ln\left(\frac{0.35}{5.4\times10^{-10}}\right)$$

$$= 0.507\text{ V}$$

$$\text{Fill factor} = \frac{V_{max}\times I_{max}}{V_{oc}\times I_{sc}} = \frac{143\text{ mW}}{0.507\times0.35} = 0.805$$