University of Sheffield

Department of Electronic and Electrical Engineering

First Year Laboratory

Passive Networks – First and Second Order Ciruits

AIMS

The aims of the experiment are:

- to determine the transient response and frequency response of first order LR and RC networks and second order LCR circuits
- to interpret data in graphical form
- to write a technical report
- Practice soldering

NOTE: YOUR ARE REQUIRED TO DO A SIGNIFICANT AMOUNT OF PREPARATION BEFORE ATTENDING THIS LAB – REFER TO SECTIONS 2.6 AND 2.9.

1. INTRODUCTION

- 1st-order circuits form the building blocks for filter and pulse shaping circuits. In this laboratory you will be performing experiments on *RL* and *RC* circuits to investigate the transient and frequency domain response of these circuits. You have been supplied with an inductor, a capacitor and two resistors, together with a piece of matrix board, which you will use to form first order circuits.
- 2nd-order circuits exist in many forms but you will be examining the behaviour of *LCR* circuits. The key difference between 1st and 2nd order behaviour is that 2nd order circuits can exhibit resonant behaviour and this part of the lab is centred around the observation of resonant behaviour.
- To simplify your understanding of the circuits involved in the Passive Networks laboratory this sheet has been divided into three parts. The first (section 2) deals with the theoretical aspect of work and defines some tasks that **must** be completed before the practical session (see sections 2.5 and 2.9). The second part (section 3) concentrates on the measurements that you must make in the laboratory. The final part (section 4) gives you advice on how to present your results in the form of a standard scientific report of experimental work together with things that you should have in mind when making comparisons between expectations (theoretical predictions) and experiment.

2. INTRODUCTION TO 1st-ORDER NETWORKS

Introduction

The behaviour of frequency dependent circuits is usually described by the nature of an "output" or "response" to a particular "input" or "stimulus". In figure 1, the stimulus or input would be provided by the signal generator and would be described algebraically as V_{in}. The response or output can be deduced for any current or voltage variable in the circuit; in the case of figure 1 the current I in the circuit or the voltage V_{out} across the resistor are the most obvious choices.

There are two types of response; "time" or "transient" responses and "frequency" responses. Sometimes people talk of time domains and frequency domains which is simply another way of expressing which variables are of interest.

Transient behaviour describes the way the output voltage or current variable changes as a function of time in response to an input stimulus that is a function of time. The stimulus most commonly used in transient measurement and analysis is a "step" input which, as its name suggests is an instantaneous change in the input from one value to another. The step is usually assumed to take place at t=0 and no changes occur for t>0 or t<0.

Frequency domain behaviour assumes that the stimulus is sinusoidal and of constant amplitude. The variable of interest here is the angular frequency, ω. The frequency response has two parts; an amplitude response and a phase response. The amplitude response is the ratio of the amplitude of the output variable to that of the input variable, ie., in the case of figure 1, V_{out}/V_{in} or I/V_{in} , as a function of frequency. The phase response describes how the phase of the output varies with respect to that of the input as a function of ω. In this experiment the phase response will not be investigated.

2.1. R-L circuit frequency response

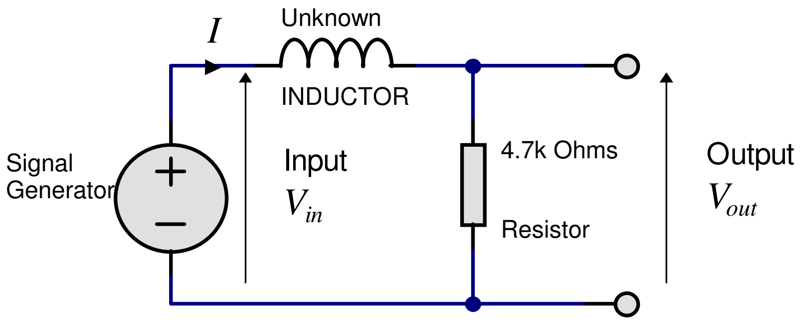

Figure 1. Series *L-R* Circuit

The impedance of the *L-R* combination in the circuit of Figure 1, is

$$Z=\sqrt{R^2+(\omega L)^2} \tag{1}$$

If a sinusoidal voltage of constant amplitude and varying frequency is applied to the input of this circuit, the current flowing, and hence the output voltage, will be a function of frequency. At very low frequencies where $R >> \omega L$, the current will be relatively independent of frequency and given by $I \approx V/R$. At very high frequencies where $\omega L >> R$, the current will be inversely proportional to frequency and given by $I \approx V/\omega L$. At one particular frequency, often called the "corner" frequency or the "3dB" frequency, $R = \omega L$, and the current flowing in the circuit will be reduced by a factor of √2, or 3dB, from its low frequency value. The corner frequency, which is usually given the symbol ω_c or ω_0, is a key parameter of *L-R* (and *C-R*) circuits; it is the frequency at which the power delivered to the resistance will be reduced to 50% of the power delivered as ω approaches zero. The corner frequency may be found from:

$$2\pi f_c = \omega_c = \frac{R}{L} \qquad \text{or} \qquad R = \omega_c L$$

2.2. *RL* circuit time constant

This investigation aims to determine by measurement the time constant of the response and hence of the circuit and compare this to the values expected from theoretical predictions, given an inductance value of 4.7mH, and a resistor of 4.7kΩ.

When a step increase in voltage from 0 to *V* is applied to a series *L-R* circuit, as shown in figure 1, the current flowing in the circuit will be of the form:

$$i = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

After sufficient time the current reaches a steady-state value of *V/R*. If the voltage is now decreased to zero as a step, the current will be of the form:

$$i = \frac{V}{R}\left(e^{-\frac{Rt}{L}}\right)$$

If the current waveform is observed on an oscilloscope, the time-constant may be found from the exponential curve (e.g. Figure 2). Some oscilloscopes will measure the 90% to 10% fall times for you. The time taken for the current to fall between 90% and 10% of the initial value is given by 2.2τ, where the time-constant τ is

$$\tau = \frac{L}{R}\ s$$

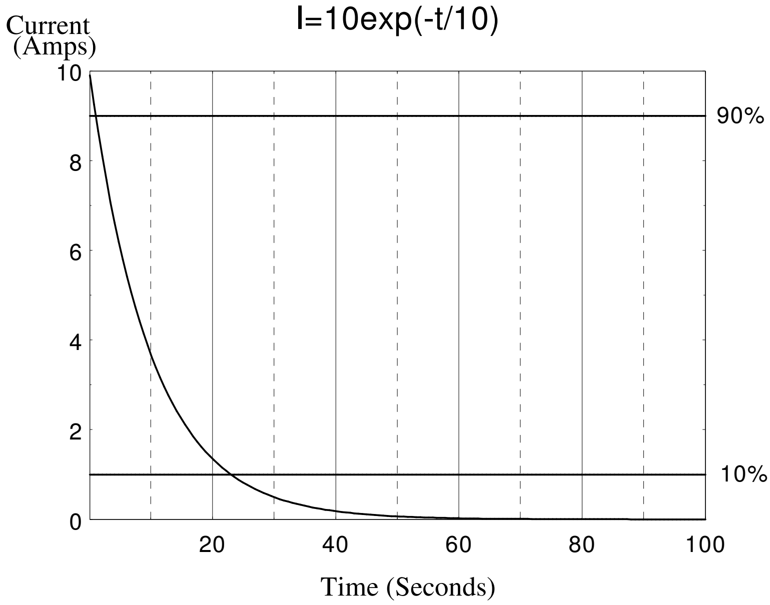

Figure 2 Example of Exponential Decay

As the output voltage across the resistor in figure 1 will be proportional to the current flowing through it (Ohm's Law), we can monitor the current in the circuit by observing the voltage across the resistor.

2.3. *RC* circuit time constant

Having investigated the characteristics of a first order inductor / resistor circuit, a capacitor / resistor circuit will now be examined. The inductor is now replaced with a 2.2nF capacitor, forming the circuit shown in Fig. 3.

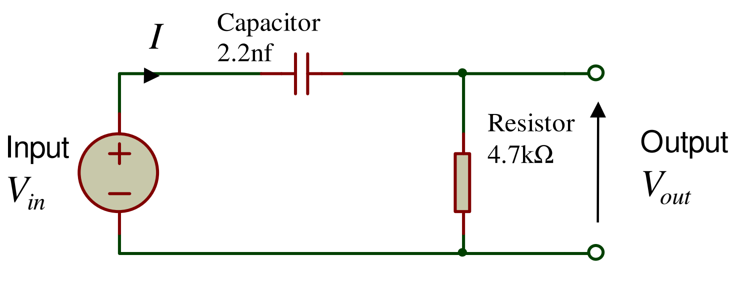

Figure 3. Series *C*-*R* circuit.

When a step increase in voltage from 0 to *V* is applied to a series *C*-*R* circuit, the current flowing in the circuit will be of the form:

$$i = \frac{V}{R}\left(e^{-\frac{t}{RC}}\right)$$

Initially, the current rises to a value of *V/R*, and falls back to zero as the capacitor is charged. If the voltage is now decreased to zero as a step, the current will be of the form:

$$i = \frac{-V}{R}\left(e^{-\frac{t}{RC}}\right)$$

If the current waveform is observed on an oscilloscope, the time-constant may be found from the exponential curve (e.g. Figure 2 above). The time taken for the current to fall between 90% and 10% of the initial value is now given by 2.2τ, where the time-constant τ is

$$\tau = RC \quad s$$

As the output voltage across the resistor in figure 3 will be proportional to the current flowing through it (Ohm's Law), we can monitor the current in the circuit by observing the voltage across the resistor.

2.4. *RC* circuit frequency response

We have examined the frequency domain and time domain responses of a *L-R* first order circuit, and a time domain response of a *C-R* first order circuit, we now need to consider the frequency domain response of the *C-R* circuit examined previously.

For the circuit of figure 3, the series impedance of the *C-R* combination is

$$Z=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$$

If a sinusoidal voltage of constant amplitude and varying frequency is applied at the input to this circuit, the current flowing, and hence the output voltage, will be a function of frequency. At very low frequencies, $R \ll 1/\omega C$, the current will be proportional to frequency, given approximately by $I \approx V\omega C$, whereas at very high frequencies, $1/\omega C \ll R$, and the current will be relatively independent of frequency. At one frequency, ω_c, $R = 1/\omega C$, and the current flowing in the circuit will be reduced by a factor of $\sqrt{2}$, or 3dB when compared to the current flowing at high frequency. At this 'corner frequency', the power delivered to the resistance will be reduced by 50% when compared to the value at $\omega=\infty$. The corner frequency may be found from

$$2\pi f_c = \omega_c = \frac{1}{RC}$$

2.5. First Order tasks to complete before the Lab

- Work out the theoretical frequency responses for both the *R-L* and *R-C* in using R=4.7kΩ, L=4.7mH and C=2.2nF, over the same frequency range that you will use in the experiment (See sections 3.1 and 3.4). Calculations should be recorded in your lab book and graphs plotted on paper with a logarithmic horizontal scale and a vertical (dB) scale.
- Work out the time constants of both the *R-L* and *R-C* circuits and record these in your lab book.
- ***You will perform experiments on both*** *R-L* and *R-C* ***circuits in the lab (see sections 3.1 to 3.4) and you will be required to compare your predicted graphs and calculations with those derived from measured values.***

2.6. Introduction to Second Order circuits

In this laboratory exercise you will be investigating the phenomena of resonance using 2nd-order *RLC* series and parallel resonant circuits.

Resonance is a key phenomenon in electronic and electrical engineering. It occurs when a "resonant circuit" is excited by a frequency equal to its "natural frequency" – a frequency to which the circuit responds to in a particularly marked way. Electrical engineers exploit the properties of resonance extensively in the areas of communications, signal processing, power management and many others. In the late 19th century and early 20th century Nicola Tesla put these ideas into action to create a resonating circuit capable of producing over 100kV at several hundred kilohertz. Tesla coils are still in use today and have been employed to create the lightning special effects seen in the Terminator® movies.

2.7. Series resonant circuit.

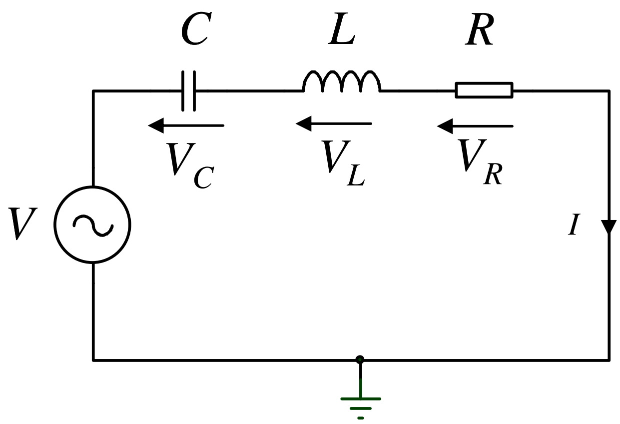

Figure 4. Series resonant circuit.

Consider the circuit of figure 4. The circuit impedance is given by:

$$Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

So

$$I=\frac{V}{Z}=\frac{V}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

And

$$\frac{V_R}{V}=\frac{IR}{V}=\frac{R}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

The origin of these relationships will become clearer during the 'Electrical Circuits and Networks' course (EEE117), but for the purposes of this experiement it is sufficient to use these relationships.

Notice that if $\omega L = \frac{1}{\omega C}$, the expression for Z reduces to $Z = R$.

An expression for the resonant frequency can be derived by equating the magnitudes of V_L and V_C, and setting the frequency $\omega=\omega_r$ (ω_r is the angular resonant frequency measured in radians per second):

$$|V_L| = |V_C|$$

$$I\omega_r L = \frac{I}{\omega_r C}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$

Although at resonance the voltage contruibutions of V_L and V_C effectively cancel with each other to leave $V_R=V$, V_L and V_C can be very large. At resonance these voltages are given by:

$$|V_L| = \omega_r L I_r = \frac{\omega_r L V}{R} = QV$$

$$|V_C| = \frac{1}{\omega_r C} I_r = \frac{V}{\omega_r C R} = QV$$

Where Q is called the magnification factor or **quality factor** (or Q-factor). Q-factor is a measure of the magnification of the voltage developed across L and C at resonance compared with the input voltage. The term quality factor originated at the beginning of the radio era when it was realised that high Q circuits were better able to discriminate between radio stations that had similar carrier frequencies. High Q meant high quality. Since we know the resonant frequency, it is a simple matter to show that for the series circuit,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

The **bandwidth**, Δf, of the circuit is defined as the difference between the frequencies at which the power dissipated is half that of the power dissipated at resonance. These frequencies, termed f_1 and f_2 here, are known as the half power or *-3dB* points. For the series resonant circuit, this corrersponds to the point where the current flowing in the circuit is related to that at resonance (I_r) by:

$$I = \frac{I_r}{\sqrt{2}} \quad or \quad |Z| = Z_r\sqrt{2}$$

It can be shown that Q is related to bandwidth by

$$Q = \frac{f_r}{\Delta f} \quad where\ \Delta f = f_2 - f_1$$

2.8. Parallel resonant circuit.

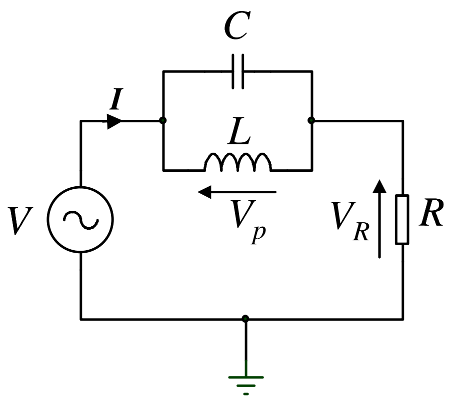

Figure 5. Parallel resonant circuit.

For the circuit in figure 5,

$$Z = \sqrt{R^2 + \left(\frac{L}{C}\right)^2 \left(\frac{1}{\omega L - 1/\omega C}\right)^2}$$

Notice that if $\omega L = 1/\omega C$ the second term within the square root goes to infinity and, therefore, so does Z_o. This means that I=0 at resonance so V_r=0 at resonance.

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\frac{L}{C}\right)^2 \left(\frac{1}{\omega L - 1/\omega C}\right)^2}}$$

And

$$\frac{V_r}{V} = \frac{IR}{V} = \frac{R}{\sqrt{R^2 + \left(\frac{L}{C}\right)^2 \left(\frac{1}{\omega L - 1/\omega C}\right)^2}}$$

2.9. Second order tasks to complete before the lab.

- Work out and plot gain (i.e. V_r/V) in dB as a function of log(frequency) over a range of 5kHz to 50kHz for the series circuit with values of L=4.7mH, C=0.01µF, R=100Ω and L=4.7mH, C=0.01µF, R=1.2kΩ on the same axes. Calculations should be recorded in your lab book and graphs plotted on paper with a logarithmic horizontal scale and a vertical (dB) scale.
- Repeat the above procedure for the parallel circuit.
- Evaluate Q for both circuits and both component sets.

You will perform experiments on both series and parallel circuits in the lab (see sections 3.5 and 3.6) and you will be required to compare your predicted graphs with those derived from measured values.

3. EXPERIMENTAL STUDY

3.1. Test 1 –Frequency response of the *RL* circuit

Assemble the simple circuit shown in figure 1 on the matrix board provided. Push the component leads through the matrix board and bend them at the back, where the component leads cross, they may be soldered together to form a solid electrical connection, if in doubt, please consult a demonstrator. The connections to the wave generator (Wave Gen) may be made by 'croc clips' to your circuit, the wave generator is then set to give a 1V (peak) sinusoidal signal (V_{in}). The voltage across the resistor (V_{out}) is measured as the frequency is varied from 1kHz to 500kHz (keeping V_{in} constant), and ratio of V_{out}/V_{in} is plotted against frequency on log-linear graph paper. The corner frequency can then be read at the point where V_{out}/V_{in} falls to 70% of the low frequency (d.c.) value. Measurements should be made at multiples of 1, 2, 5 & 10 in every decade of interest (i.e 1kHz, 2kHz, 5kHz, 10kHz, 20kHz……), and plotted on log/lin graph paper provided. Before beginning the experiment, ensure your probes are correctly calibrated.

The value of the inductance can be calculated from the corner frequency, and checked against the value given by the coloured bands on the body of the inductor.

3.2. Test 2 –Time constant measurement using the *RL* circuit

Using the same circuit as Test 1, figure 1, set the wave generator (Wave Gen) may now be set to give a 5V, square wave output at 10kHz. The time taken for the current to travel between 90% and 10% of the peak value, may then be measured from the oscilloscope. Push the **Trigger level** control knob to set the level at 50% and adjust the **Horizontal** and **Vertical** scale to display a single cycle on the screen. Press the **Measure (Meas)** button and from the menu scroll down and select **Type: Fall Time** (press select knob to confirm selection and note the value of the fall time.

The time-constant of the circuit may be derived from this measurement, and the value of the inductance can thus be calculated, and checked against the value quoted on the inductor body. The value of the resistor should be measured carefully to ensure minimum errors in the inductor value obtained.

3.3. Test 3 –Time constant measurement using the *RC* circuit

Assemble the simple circuit shown in figure 3 on the matrix board provided. As before, push the component leads through the matrix board and bend them at the back, where the component leads cross, they may be soldered together to form a solid electrical connection, if in doubt, please consult a demonstrator. The connections to the waveform generator, as with the *L-R* circuit of figure 1, may be made by 'croc clips' to your circuit. Set the waveform generator to give a 5V, square wave output at 10kHz. The time taken for the current to travel between 90% and 10% of the peak value, may then be measured from the oscilloscope Push the **Trigger level** control knob to set the level at 50% and adjust the **Horizontal** and **Vertical** scale to display a single cycle on the screen. Press the **Measure (Meas)** button and from the menu scroll down and select **Type: Fall Time** (press select knob to confirm selection) and note the value of the fall time. Is this what you expect to see? Now adjust the **Horizontal** and **Vertical** scale to display only the positive portion of the waveform and repeat the measurement.

The time-constant of the circuit may be derived from this measurement, and the value of the capacitance can thus be calculated, and checked against the value quoted on the capacitor body. The value of the resistor should be measured carefully to ensure minimum errors in the capacitor value obtained.

3.4. Test 4 –Frequency response of the *RC* circuit

Given the simple circuit built above, shown in figure 3, the waveform generator is now set to give a 1V (peak) sinusoidal signal (V_{in}). The voltage across the resistor (V_{out}) is measured as the frequency is varied from 1kHz to 500kHz (keeping V_{in} constant), and ratio of V_{out}/V_{in} is plotted against frequency on log-linear graph paper. The corner frequency can then be read at the point where V_{out}/V_{in} is 70% of the high frequency value. Measurements should be made at multiples of 1, 2, 5 & 10 in every decade of interest (i.e 1kHz, 2kHz, 5kHz, 10kHz, 20kHz…….), and plotted on log/lin graph paper provided.

The value of the capacitance can be calculated from the corner frequency, and checked against the value given on the capacitor body (2.2nF).

3.5. AC Measurements on the Series Resonant Circuit.

This experiment will illustrate the behaviour of a simple series resonant circuit, so called because its elements are connected in series.

Procedure:

1. Connect the circuit shown in Fig. 4 above using the component values R=100Ω, L=4.7mH and C=0.01µF. The sinusoidal voltage source V represents the signal generator. Be careful of where you place your ground connections!
2. Set up the signal generator to apply a sinusoidal voltage of 1V RMS. You should use the DMM to accurately measure its magnitude.
3. Keeping the magnitude of V constant (adjusting the signal generator as necessary to maintain a constant amplitude), vary the frequency of the sinusoidal voltage between 5kHz and 50kHz. You have the freedom to choose your own measurement intervals so ensure that you have taken enough readings to accurately represent the circuit's behaviour.
   - Measure and record V_R at each frequency.
4. Use the DMM to measure the values of V_R, V_L and V_C at resonance. Also, make a note of how you determined resonance and the resonant frequency itself.
5. Now using R=1.2kΩ repeat steps 3 and 4.

By the end of this experiment you should have recorded measurements of V_R and frequency for R=100Ω and R=1.2kΩ. You should also have measurements of V_R, V_L and V_C at resonance for both resistors.

Evaluation and assessment:

1. Using your measurements from step 3 of the procedure described above:
   - **i)** For R=100Ω and R=1.2kΩ, plot a graph of V_R/V against frequency on the same axes as your calculated responses.
   - **ii)** Compare the shapes of your plots and comment on the relationships between them.

iii) Estimate Q using the $f/\Delta f$ and your plot of experimental results. Compare this estimate with your theoretical expectation.

3.6. AC measurements of the Parallel Resonant Circuit

Procedure:

1. Set up the circuit shown in Fig. 5 above, using R=100Ω, L=4.7mH and C=0.01µF.
2. Setup the signal generator to apply a sinusoidal voltage of 1V RMS – use the DMM.
3. Keeping the magnitude of V constant, vary the frequency between 5kHz and 50kHz. Again, you have the freedom to choose your own measurement intervals but be mindful of circuit behaviour.
   i) Measure and record the magnitude of V_R.
4. Now using R=1.2kΩ repeat step 3.

By the end of this experiment you should have recorded measurements of V_R and frequency for both R=100Ω and R=1.2kΩ.

Evaluation and assessment:

1. Plot V_R against frequency for both values of R on the same axes as your theoretical curves.
2. Comment on the shape of your graphs and on the agreement between theory and measurement.

4. Report.

In keeping with scientific convention, you should write in the third person past tense; do not use 'I' or "we". You do not need to reproduce the theory included in this labsheet but you should describe briefly the aim(s) of the experiment before continuing to describe the experimental procedure that you used, including the way the experiment was set up. Remember that your experimental procedure should be sufficiently detailed for someone to repeat your experiment simply by reading your account. Results are usually presented in graphical form and for experiments where there is a theoretical expectation, the theoretical result and measured results should be plotted on the same axes to aid comparison. In this piece of work, we want you to draw the graphs by hand; frequency response magnitude should be plotted on paper with a logarithmic horizontal scale and a vertical (dB) scale. You should comment on differences between theoretical and measured result to draw the reader's attention to any important differences and, if you can, explain why the differences might exist. Always consider discrepancies in the context of experimental error and ask yourself whether experimental error could be large enough to account for what you see. You need to be quantitative in these considerations. For all calculations and graphs compare theoretical and experimental results. If any discrepancies exist, explain why and comment on possible errors. Include all results in graphical form hand plotted on log-linear graph paper. You should support all your predicted shapes and values from sections 2.5 and 2.9 by appropriate experiemental evidence.

5. Bibliography.

[1] ‘Engineering Circuit Analysis’ - William H. Hayt Jnr & Jack E. Kemmerly. Published by McGraw-Hill.

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