# Modulation

# Modulation

The point of communications is that you are trying to transmit useful information from one user to another.

This information could be:-

- Speech
- Video
- Data etc

The problem with the information is the bandwidth it operates over

- Speech (100Hz – 20kHz) although in practice 16kHz is probably the max
- Video (3MHz for SD but higher for HD)

# Modulation

Antenna theory tells us that to transmit information at a frequency, f, requires
an antenna which is typically a quarter of the wavelength you are trying to transmit

For speech the lowest frequency may be 100Hz, which gives an antenna of 3000km

Clearly this is not feasible

Modulation is the means of converting your information signal to a frequency, which can be transmitted sensibly

# Modulation

Consider the equation below

$$V(t) = A(t)\sin\left[\omega_c(t)t + \phi(t)\right]$$

Within this equation we have three possible options for “modulation”

We could use the amplitude A(t)
We could use the frequency $\omega_c$(t)
We could use the phase $\phi$(t)

This is the basis for all modern communications

The aim is to somehow “modulate”the information signal in either
the amplitude, frequency of phase of a high frequency **CARRIER** sine wave

# Definitions

- Baseband
  - is the low frequency information you are trying to transmit
- Modulation
  - is defined as the process by which some characteristic of a carrier is varied in accordance with a modulating wave.
- Demodulation
  - is the way to restore the original baseband signal at the receiver.

# Modulation types

- Analogue modulation
  - Amplitude, frequency and phase
- Digital modulation
  - Amplitude shift keying (ASK)
  - Frequency shift keying (FSK)
  - Phase shift keying (PSK)

# Analogue modulation

- Analogue modulation is not frequently used in modern communications, but there are a few examples.
  - Long wave radio
  - FM radio
  - Analogue TV (not for long in the UK!)

# Amplitude modulation

$$V(t) = A(t)\sin\left[\omega_c(t)t + \phi(t)\right]$$

For amplitude modulation we are assuming the frequency and phase of the signal
Above is constant

$$V(t) = A(t)\sin\left[\omega_c t\right]$$

Note that the sine can be a cosine!

The aim of AM is to modulate our baseband signal, somehow, in the amplitude.

This modulation MUST be linear to simplify reception

# Amplitude modulation

Lets assume that we have a “baseband” signal of the form below

$$A(t) = V_m(t) = V_m \sin[\omega_m t]$$

Lets assume that we have a “carrier” signal of the form below

$$V_c(t) = V_c \sin[\omega_c t]$$

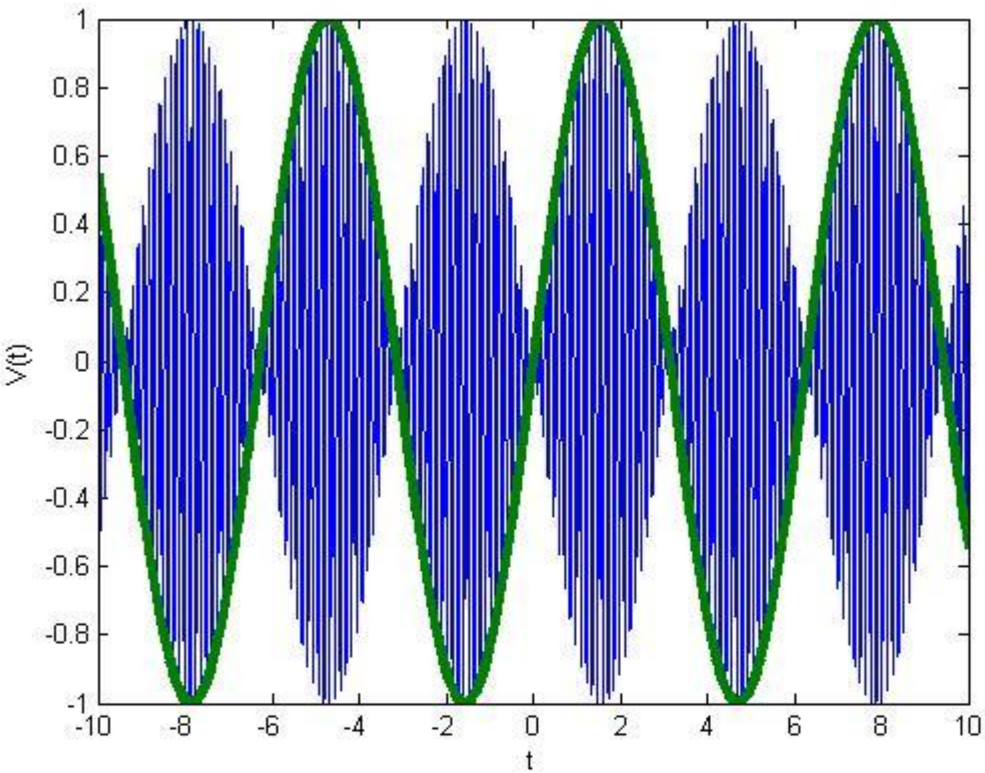
We assume that $\omega_c >> \omega_m$

The subscript “c” denotes carrier
The subscript “m” denotes message

# Amplitude modulation

What happens if we multiply the carrier and message waveform?

$$V_c(t)V_m(t) = V_c V_m \sin[\omega_c t]\sin[\omega_m t]$$




Assume Vc=Vm

The “envelope “of the signal looks like our original message

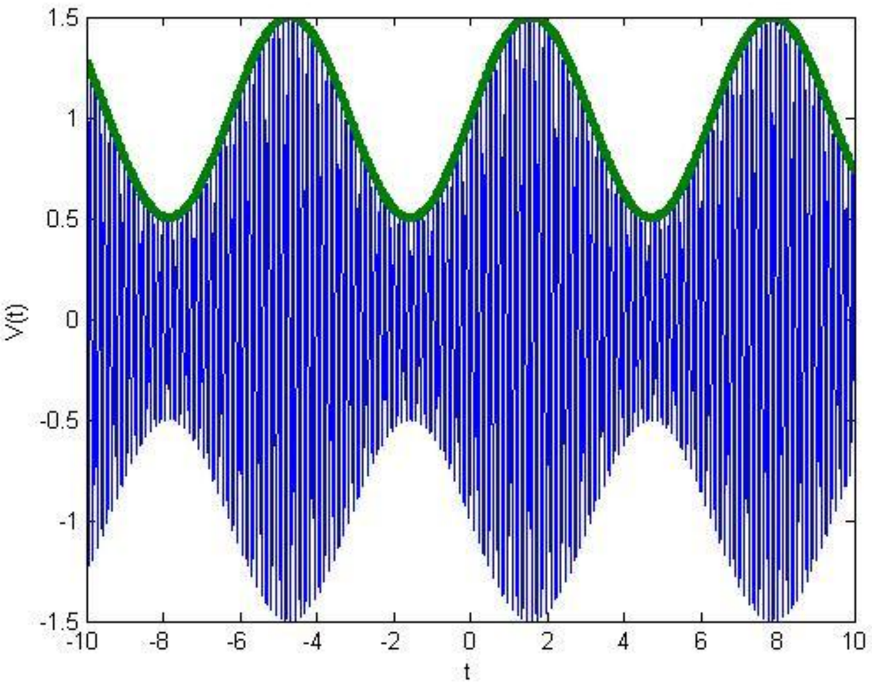
# Amplitude modulation

I will now introduce a new scheme as detailed below

$V_c + V_m \sin[\omega_m t]$ Message is modulated onto the amplitude of the carrier

$[V_c + V_m \sin[\omega_m t]]\sin[\omega_c t]$ The equation above is then multiplied by the carrier sine wave

$V_c[1 + m\sin[\omega_m t]]\sin[\omega_c t]$ The equation is rearranged to introduce the modulation index




"m" is termed the modulation index

$$m = \frac{V_m}{V_c}$$

$$0 \le m \le 1$$

# Amplitude modulation

Lets expand our AM equation

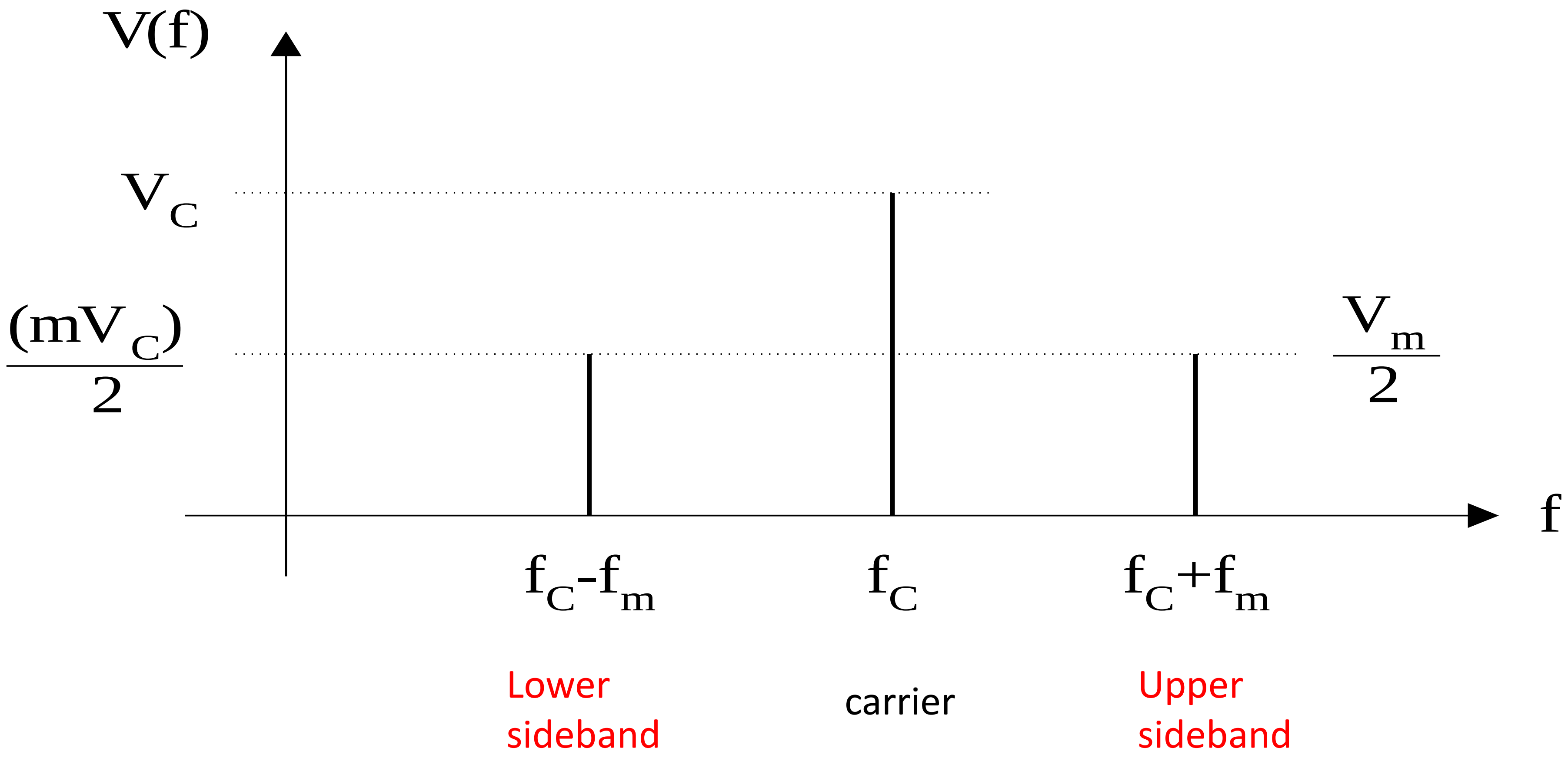
$$V_{AM} = V_c m \sin[\omega_c t]\sin[\omega_m t] + V_c \sin[\omega_c t]$$

$$V_{AM} = V_c \sin[\omega_c t] + mV_c\left[\frac{\cos[(\omega_c - \omega_m)t] - \cos[(\omega_c + \omega_m)t]}{2}\right]$$

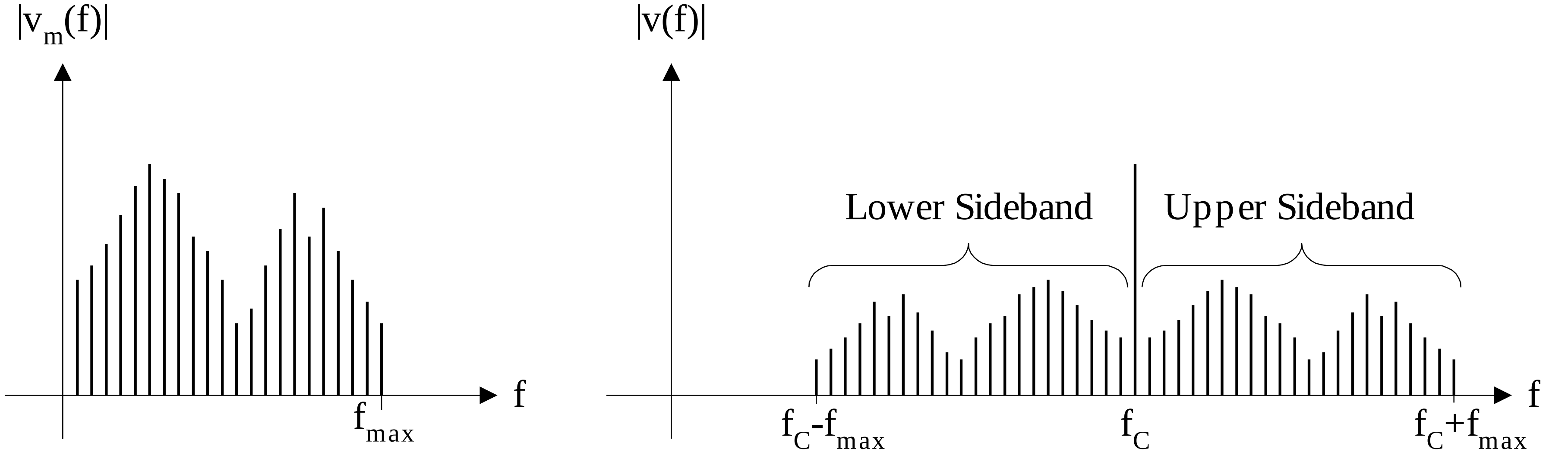
$$V_{AM} = V_c \sin[\omega_c t] + \frac{mV_c}{2}\cos[(\omega_c - \omega_m)t] - \frac{mV_c}{2}\cos[(\omega_c + \omega_m)t]$$

$$(\sin a)(\sin b) = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

# Amplitude Modulation Double Sideband Carrier Present (AM DSB CP)



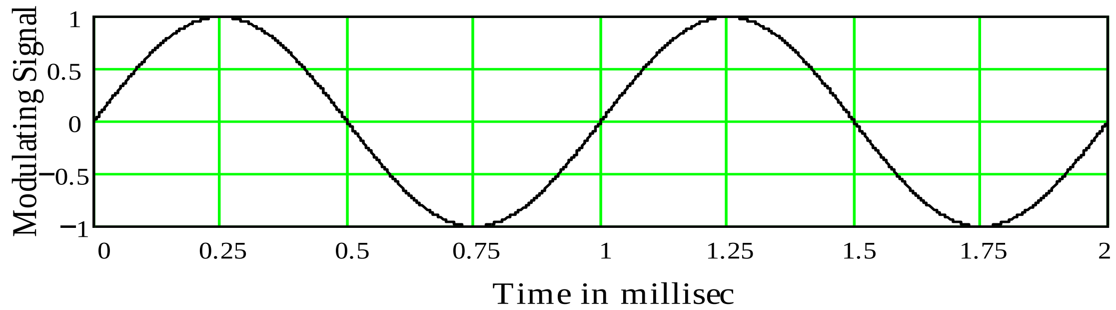
# Complex Signals & Modulation



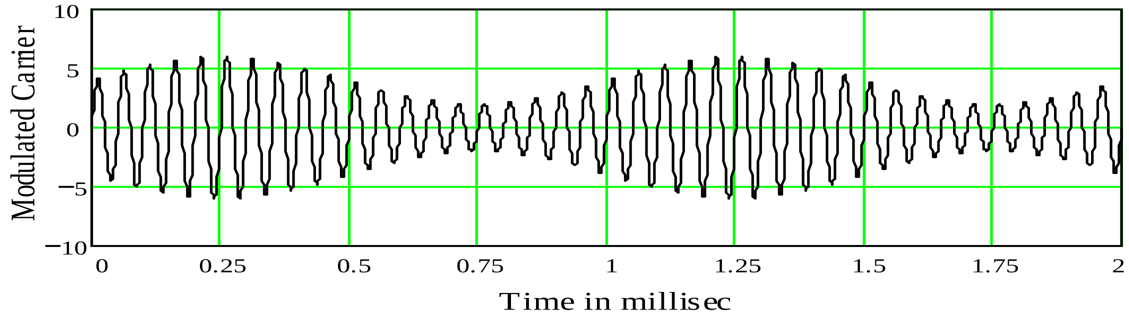

(a) before modulation

(b) after modulation

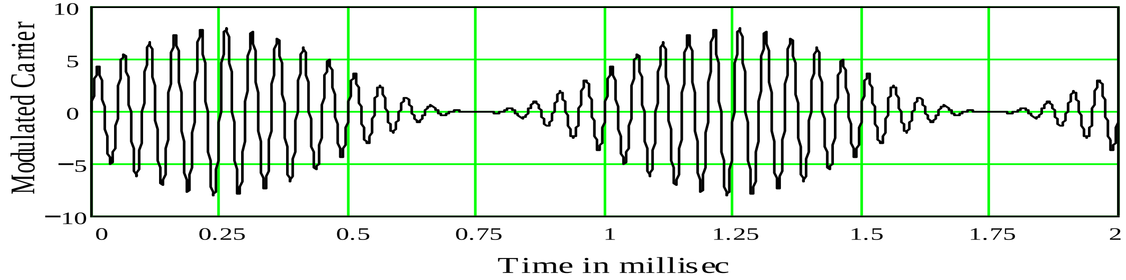
# Effects of Varying Modulation Index
(Show movie!!!)




m = 50%



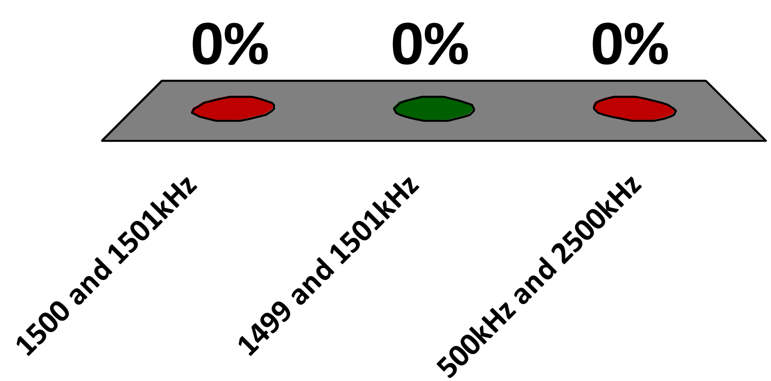

m = 100%



# Phasor diagram of AM

In an AM radio broadcast the “message” has a frequency of 1000Hz and the carrier frequency is 1500kHz. What are the frequencies of the resulting sidebands.

A. 1500 and 1501kHz
B. 1499 and 1501kHz
C. 500kHz and 2500kHz



# More than one sinusoid?

If the input signal has more than one sinusoid then the total modulation index is given below

$$m_t = \sqrt{m_1^2 + m_2^2 .....m_n^2}$$

# Signal Power

Lets start with a little revision

The instantaneous power dissipated in a circuit is given below

$$P(t) = V(t)I(t)$$

Lets assume the voltage and current are given below

$$V(t) = V\sin(\omega t)$$

$$I(t) = I\sin(\omega t)$$

$$P(t) = V(t)I(t) = VI\sin^2(\omega t)$$

$$P(t) = \frac{VI}{2} - \frac{VI}{2}\cos(2\omega t)$$

# Signal Power

The average power is defined as

$$P = \frac{1}{T}\int_{-T/2}^{T/2} P(t)dt$$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} \left[\frac{VI}{2} - \frac{VI}{2}\cos(2\omega t)\right]dt$$

$$P = \frac{VI}{2}$$

For a circuit with resistance, R, we get

$$P = \frac{V^2}{2R}$$

$$P = \frac{I^2}{2}R$$

# Signal Power

If we apply this concept to our AM DSB CP signal, we get

$$V_{AM} = V_c \sin[\omega_c t] + \frac{mV_c}{2}\cos[(\omega_c - \omega_m)t] - \frac{mV_c}{2}\cos[(\omega_c + \omega_m)t]$$

$$P_{AM} = \frac{V_c^{\,2}}{2R} + \frac{\left(\frac{mV_c}{2}\right)^2}{2R} + \frac{\left(\frac{mV_c}{2}\right)^2}{2R}$$

$$P_{AM} = P_c + \frac{m^2}{4}P_c + \frac{m^2}{4}P_c$$

# Signal Power - summary

The power in the carrier is $P_c$

The power in EACH sideband is $\frac{m^2 P_c}{4}$

The power in BOTH sidebands is $\frac{m^2 P_c}{2}$

The total power in an AM DSB CP signal is $P_T = P_c\left(1 + \frac{m^2}{2}\right)$

# Efficiency of AM DSB CP

We know that the modulation index, m, has a maximum value of 1

hence the transmitted power is

$$P_T = P_c\left(1 + \frac{1^2}{2}\right)$$

$$P_T = 1.5P_c$$

The transmission efficiency is defined as

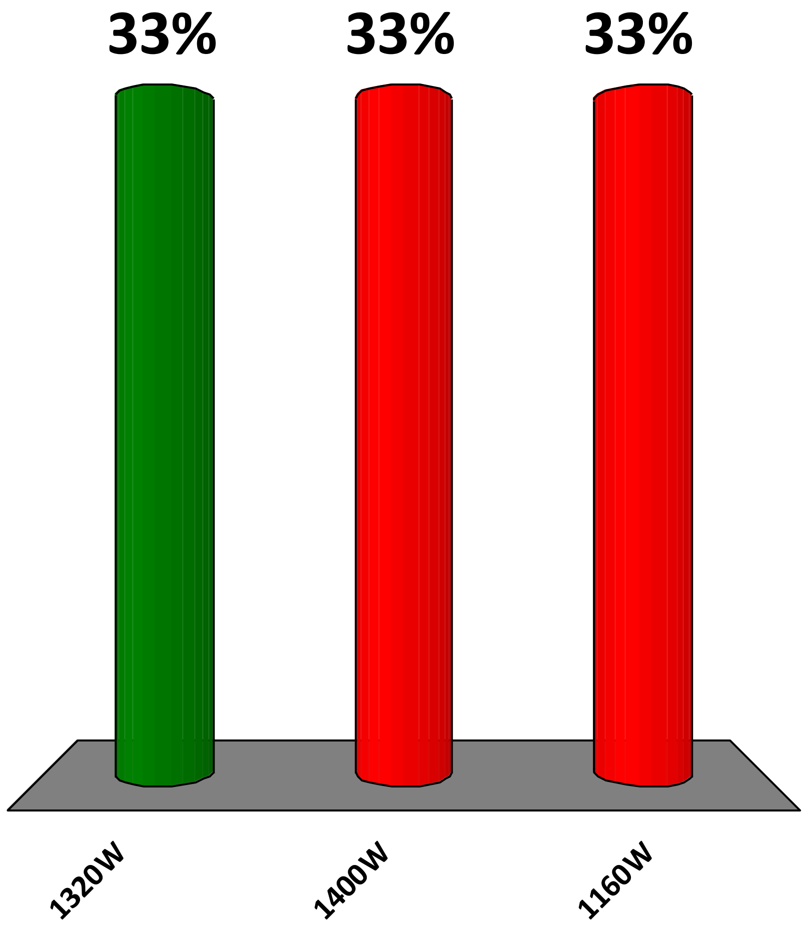
$$efficiency = \frac{\text{Power in the sidebands}}{\text{Total power transmitted}}$$

$$efficiency = \frac{\frac{m^2}{2}P_c}{\left(1 + \frac{m^2}{2}\right)P_c} = \frac{0.5}{1.5} = 0.333$$

In practice the carrier and one sideband
Can be removed to improve efficiency

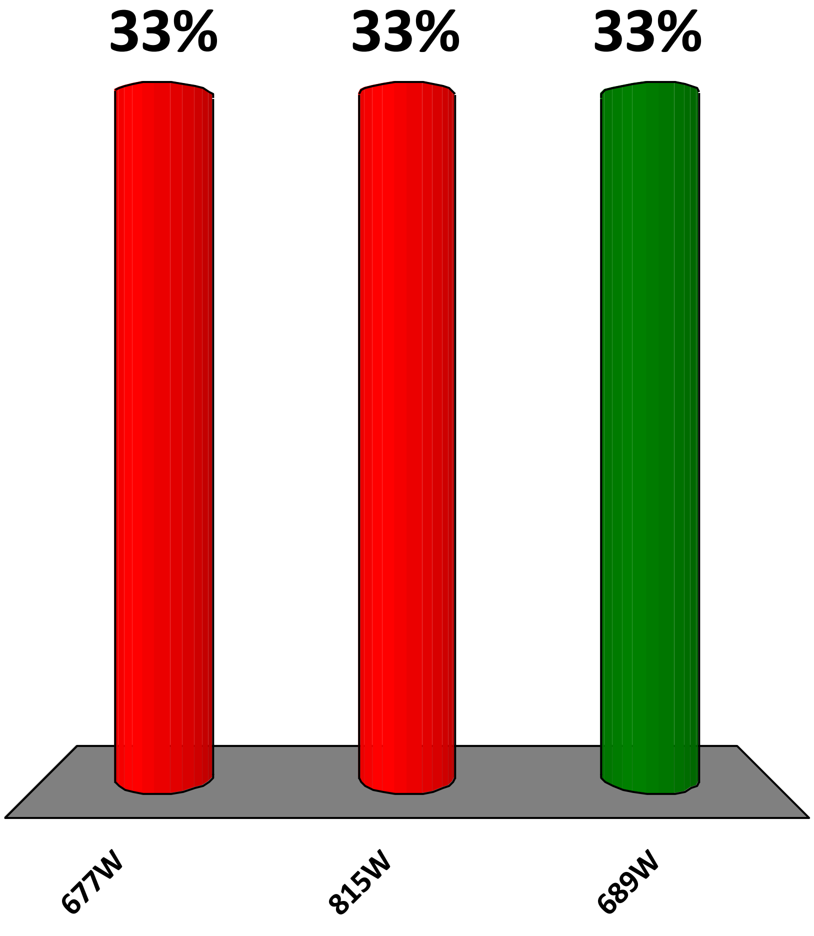
# A carrier of 1000W is modulated with an index of 0.8. What is the total power?

A. 1320W
B. 1400W
C. 1160W



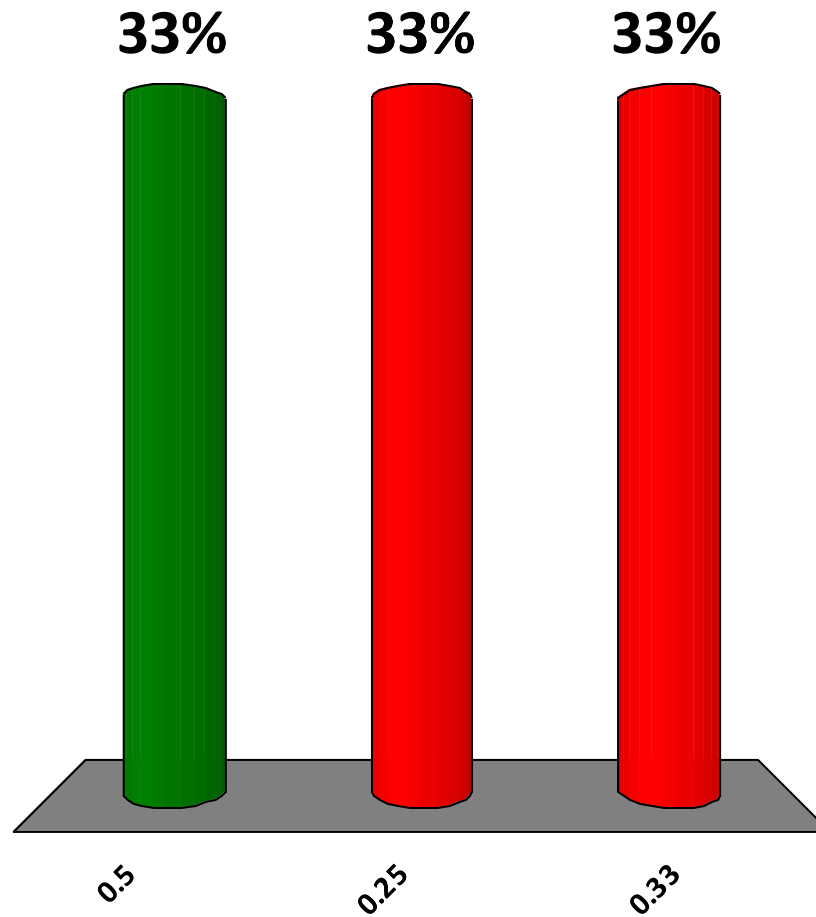
What is the carrier power if the total power is 1000W and the modulation index is 0.95.

A. 677W
B. 815W
C. 689W



A transmitter radiates 9 kW with the carrier unmodulated, and 10.125 kW when the carrier is sinusoidally modulated. Calculate the modulation index.

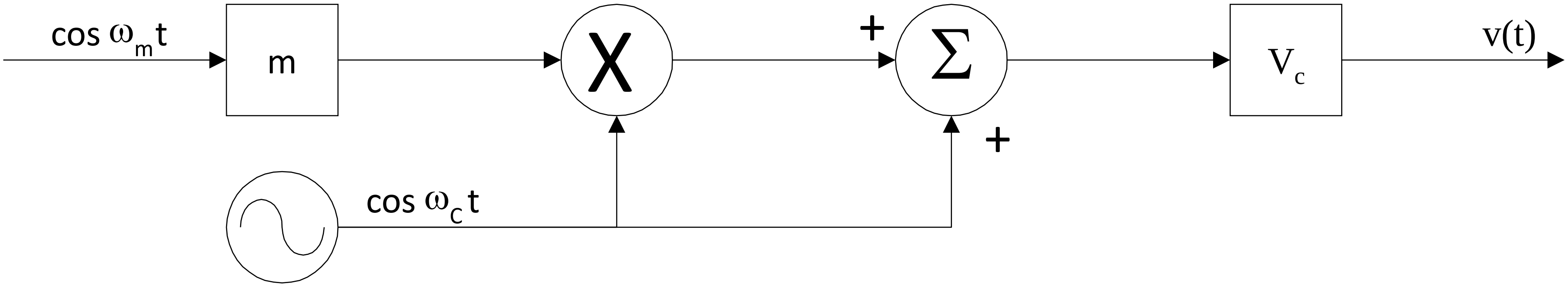
A. 0.5
B. 0.25
C. 0.33



# AM generation

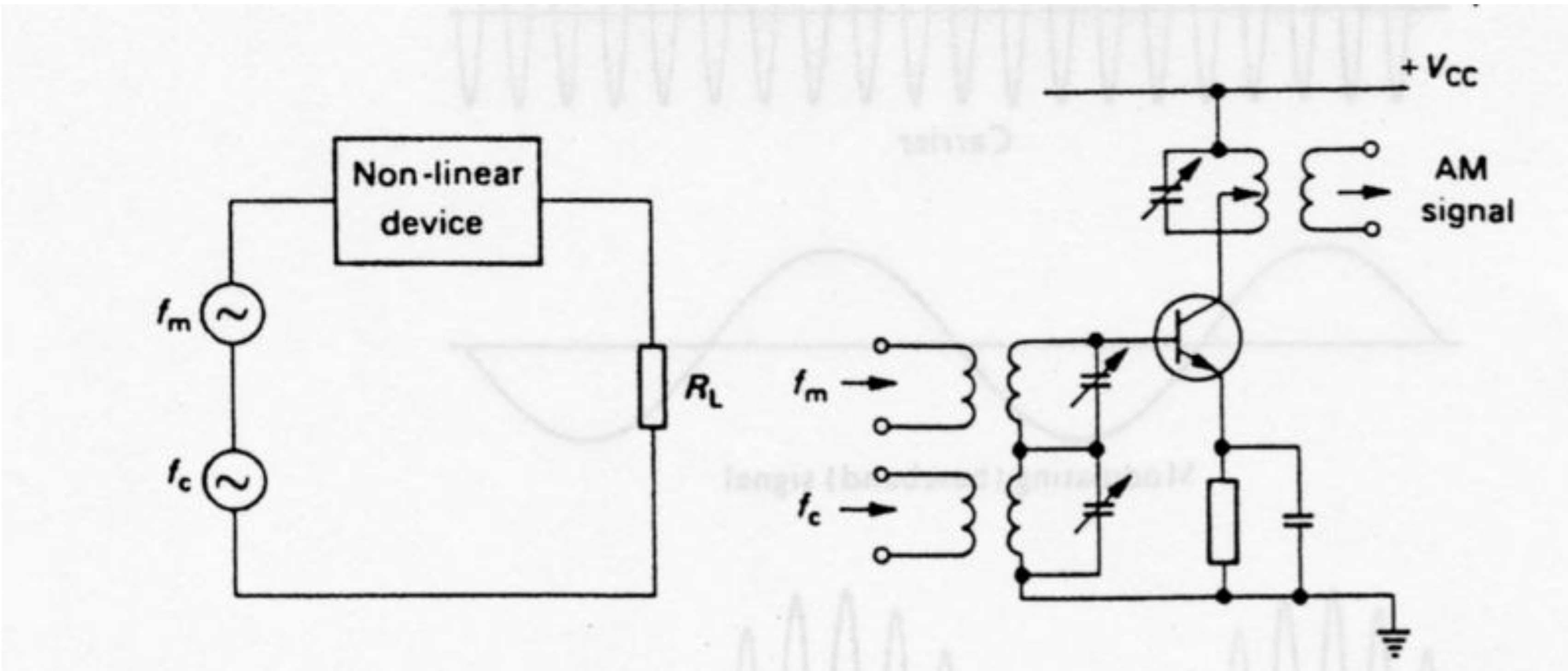
We know all the maths but how do we build a circuit that can achieve AM?

Consider the diagram below




Hence we could generate an AM signal using multiplier and summing circuits

# AM generation




Assume non-linear device characteristic
$i = a + bV + cV^2$

$V$ = input voltage = $V_c \sin \omega_c t + V_s \sin \omega_m t$

$$i = a + b(V_c \sin \omega_c t + V_m \sin \omega_m t) + c(V_c \sin \omega_c t + V_m \sin \omega_m t)^2$$

Expanding the red section above gives

$$i = a + bV_c \sin \omega_c t + bV_m \sin \omega_m t +$$
$$cV_c^2 \sin^2 \omega_c t + cV_m^2 \sin^2 \omega_m t + 2c\, V_c \sin \omega_c t.\ V_m \sin \omega_m t$$

$$\{ \sin A \sin B = ½[\cos(A-B) – \cos(A+B)] \}$$

$$i = a + bV_c \sin \omega_c t + bV_m \sin \omega_m t + cV_c^2 \sin^2 \omega_c t + cV_m^2 \sin^2 \omega_m t + c\, V_c\, V_m \cos(\omega_c – \omega_m)t - c\, V_c\, V_m \cos(\omega_c + \omega_m)t$$

$$\{ 2\sin^2 A = 1-\cos 2A \}$$

Expanding this equation gives

$$
\begin{aligned}
i = a &+ cV_c^2 /2 + cV_s^2 /2 + \\
&bV_m \sin \omega_m t + \\
&bV_c \sin \omega_c t + \\
&c V_c V_m \cos(\omega_c - \omega_m)t - c V_c V_m \cos(\omega_c + \omega_m)t - \\
&cV_m^2 /2 \cos(2 \omega_m t ) - \\
&cV_c^2 /2 \cos(2 \omega_c t )
\end{aligned}
$$

# AM generation

Now remove the unwanted components at $2\omega_c$ and $2\omega_m$ and the DC!

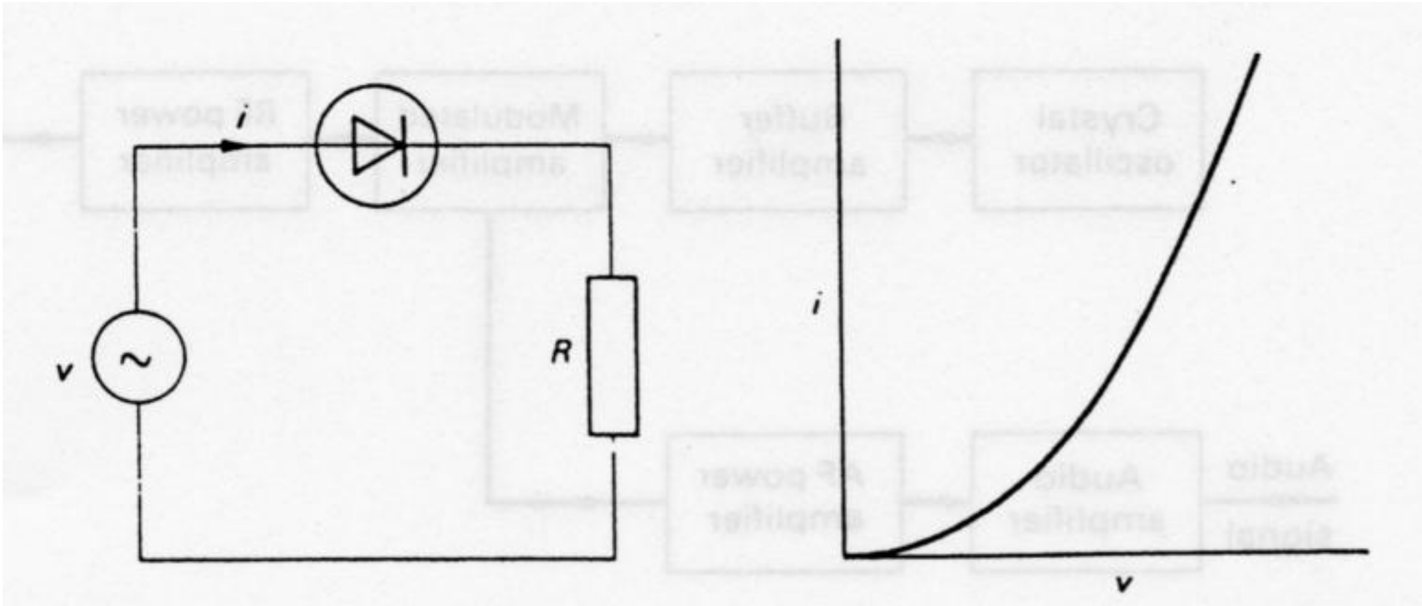
A filter (tuned load) in the amplifier output can select wanted components near $\omega_c$. (Acts as a band pass filter)

$$i = bV_c \sin \omega_c t + cV_c V_m \left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

# AM DSB CP demodulation

- We will consider 2 methods to demodulate a AM DSB CP signal
  - Non-linear (very similar to non-linear generation)
  - Envelope detection

# Square law detector




$i = aV + bV^2$ where $V = V_c(1 + m \sin \omega_m t)\sin \omega_c t$

$i = aV_c(1 + m \sin \omega_m t)\sin \omega_c t + bV_c^2(1 + m \sin \omega_m t)^2 \sin^2 \omega_c t$

$i = aV_c \sin \omega_c t + aV_c\, m \sin \omega_m t \sin \omega_c t +$

$bV_c^2 \sin^2 \omega_c t\, (1 + 2m \sin \omega_m t + m^2 \sin^2 \omega_m t\,)$

# Square law detector

Expanding this fully gives

$i = bV_c^2/2 + bm^2V_c^2/4 +$

$bmV_c^2 \sin(\omega_m t) -$

$bm^2V_c^2/4 \cos(2\omega_m t) +$

$aV_c \sin \omega_c t +$

$aV_c \, m/2 \cos(\omega_c - \omega_m)t + aV_c \, m/2 \cos(\omega_c + \omega_m)t -$

$bV_c^2/2 \, (2m\sin(\omega_m t)\cos(2\,\omega_c t) - \cos(2\,\omega_c t) - m^2\sin^2(\omega_m t)\cos(2\,\omega_c t))$

Component at $\omega_m$ has amplitude $bV_c^2\, m$ $(\sin \omega_s t)$

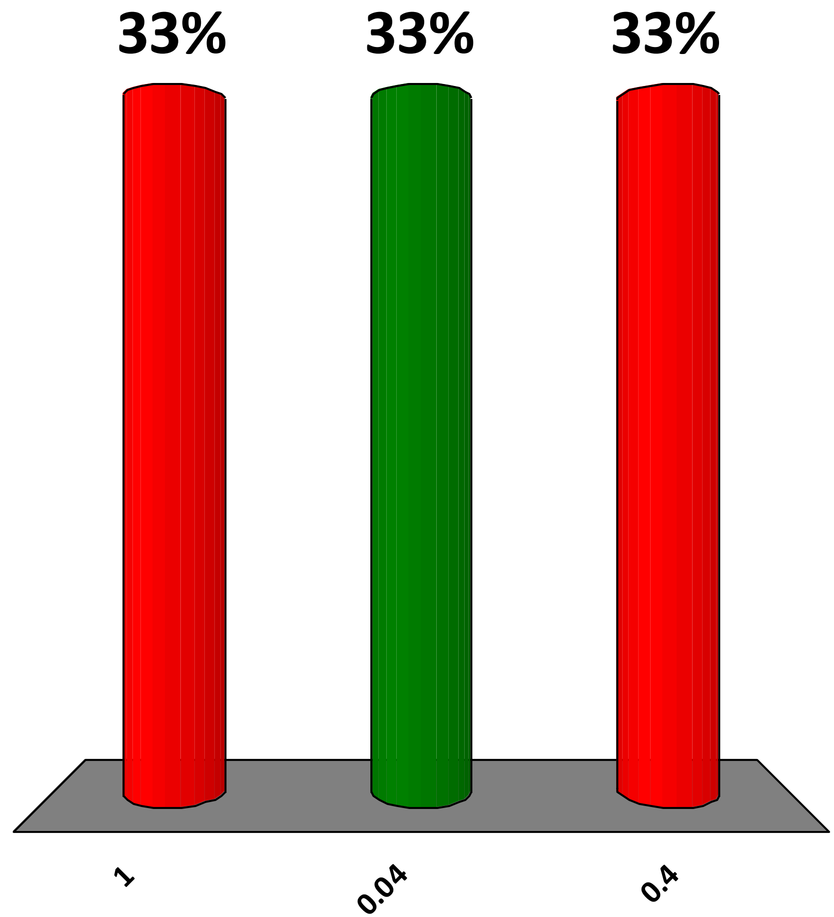
Component at $2\omega_m$ has amplitude $0.25\, bV_c^2\, m^2$ $(\cos 2\omega_s t)$

$$\frac{\text{Second harmonic term}}{\text{Fundamental term}} = \frac{0.25\, m^2}{m} = \frac{m}{4}$$

- So must keep *m* small to avoid harmonic distortion (0.3)

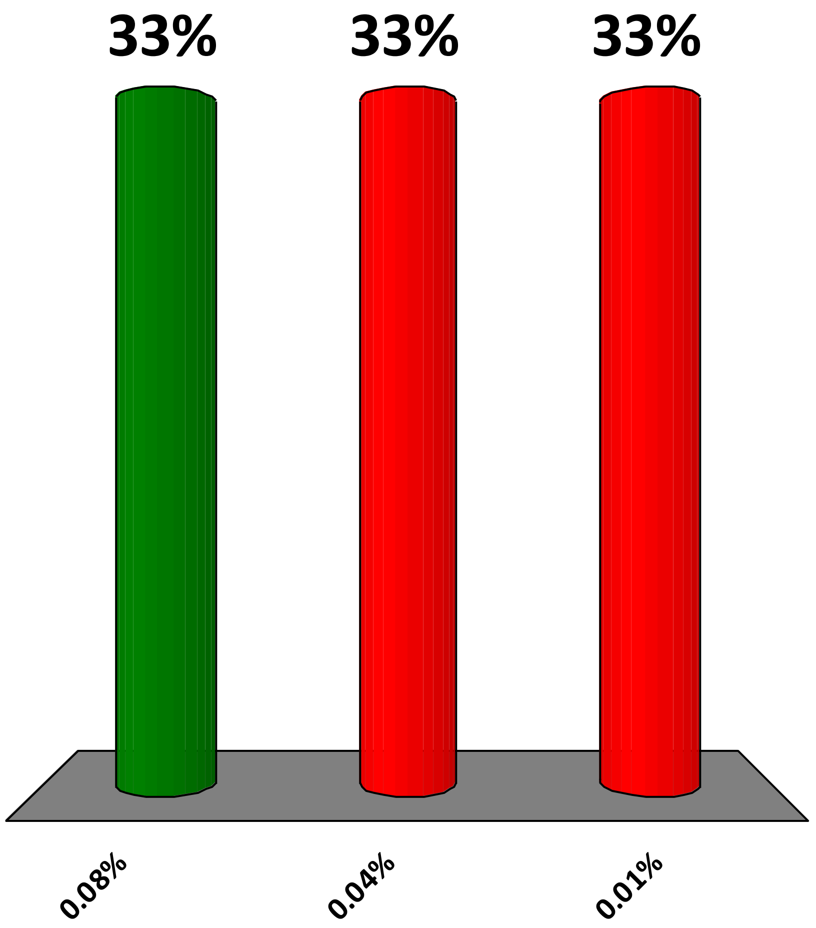
# For the 2nd harmonic to be less than 1%, what is the maximum modulation index?

A. 1
B. 0.04
C. 0.4



# What would be the transmission efficiency using this modulation index?

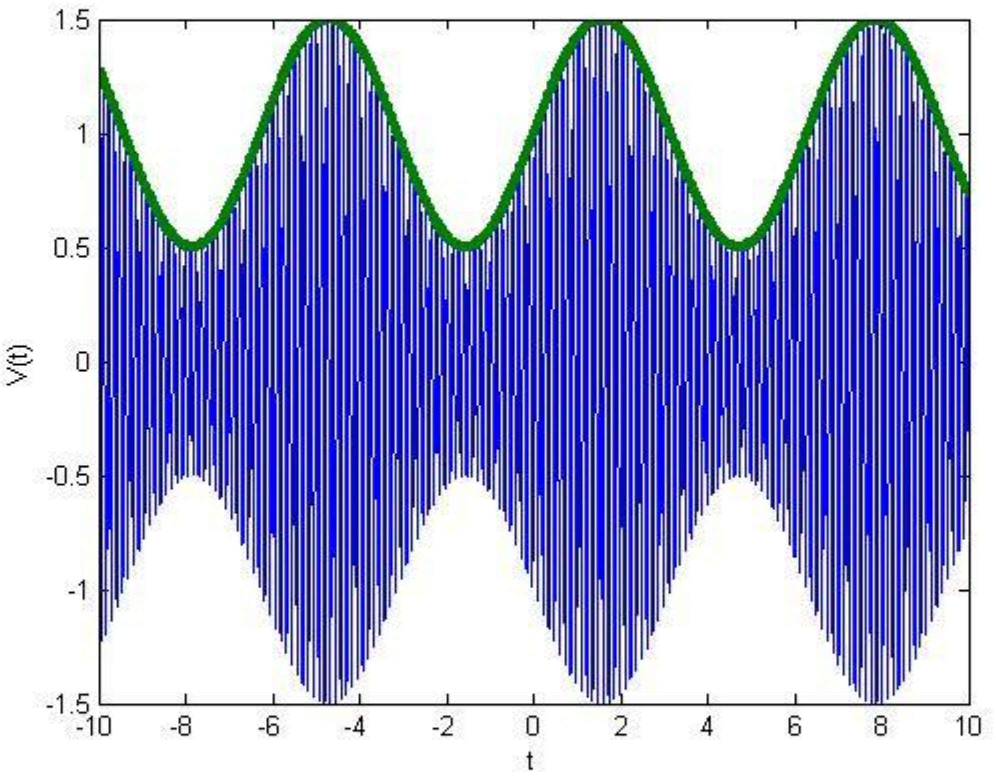
A. 0.08%
B. 0.04%
C. 0.01%

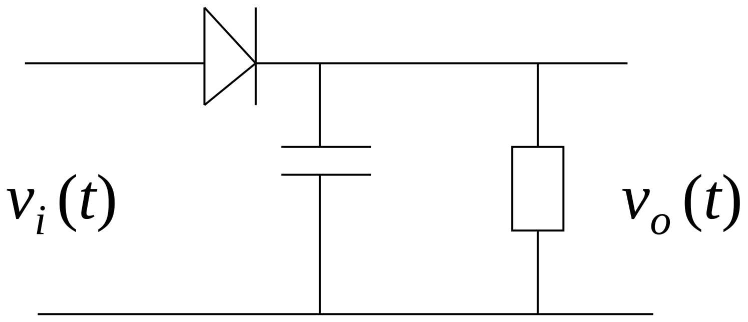


# Envelope detector

We want to design a circuit that can “track” the envelope of the signal below

To do this we need a circuit with a FAST charge time and SLOW discharge time

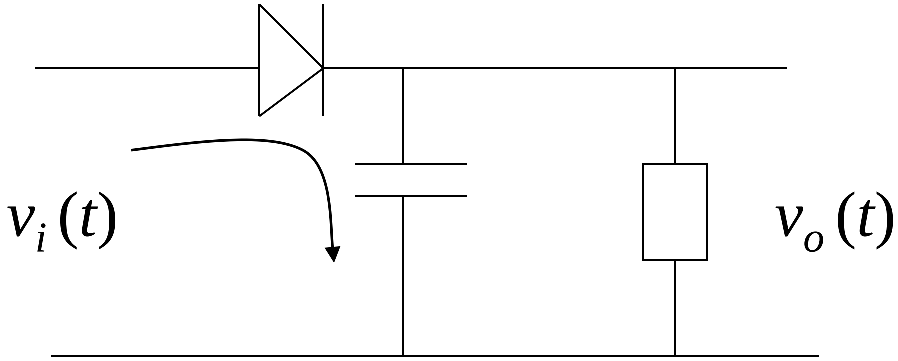


# Envelope detector

<u>Operation</u>

In the positive cycle of $v_i(t)$, the diode is ON.

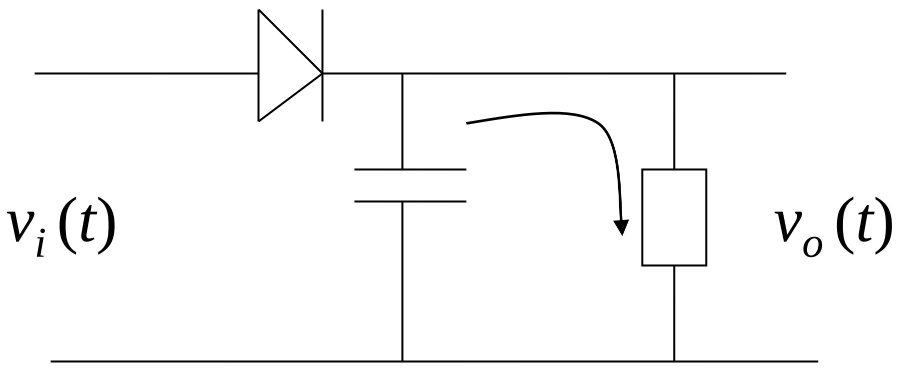



**If the signal is very small the diode will not be turned on!**

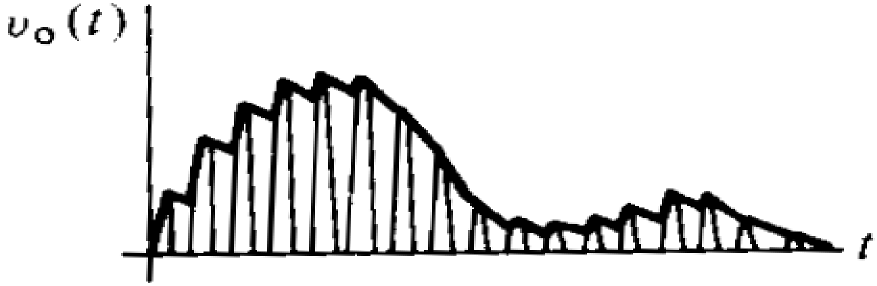
# Envelope detector

Operation

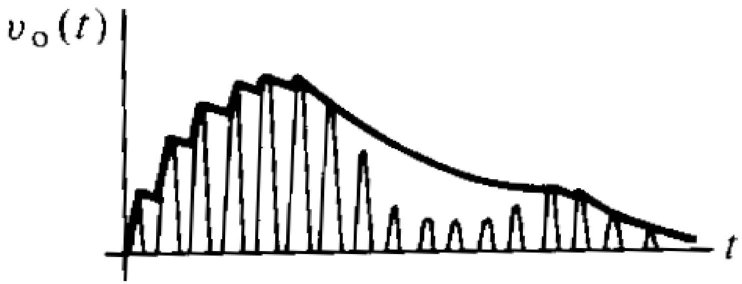
In the negative cycle of $v_i\,(t)$, the diode is OFF



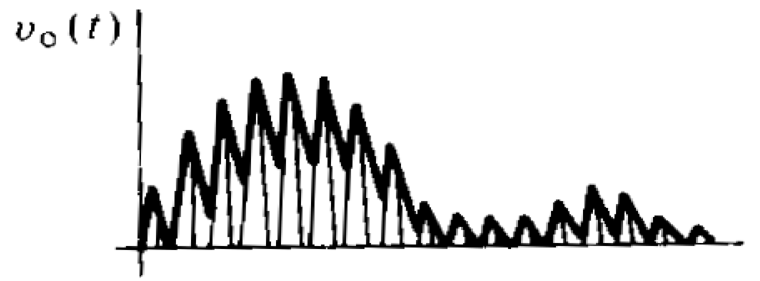
# Effect of incorrect choice of RC




Correct $RC$




$RC$ too large




$RC$ too small

RC too large
$\rightarrow$ missing signal $\because$ can't follow the AM envelope

RC too small
$\rightarrow$ poor smoothing efect
$\rightarrow$ high frequency generation

# Optimum value of RC

It can be shown that the optimum RC value is

$$CR \leq \frac{\sqrt{1-m^2}}{m\omega_m}$$

In practice can meet this for low values of m = 0.3 to 0.4 for music broadcasting to ensure minimum distortion.

Choose R >> diode forward resistance, C>> diode capacitance.

# AM DSB CP summary

So far we have learned

- the maths behind AM DSB CP
- How to calculate modulation index and power
- Circuits that generate AM DSB CP
- Circuits that demodulate AM DSB CP

We also learned that AM DSB CP is not power efficient so what other forms of AM are there?

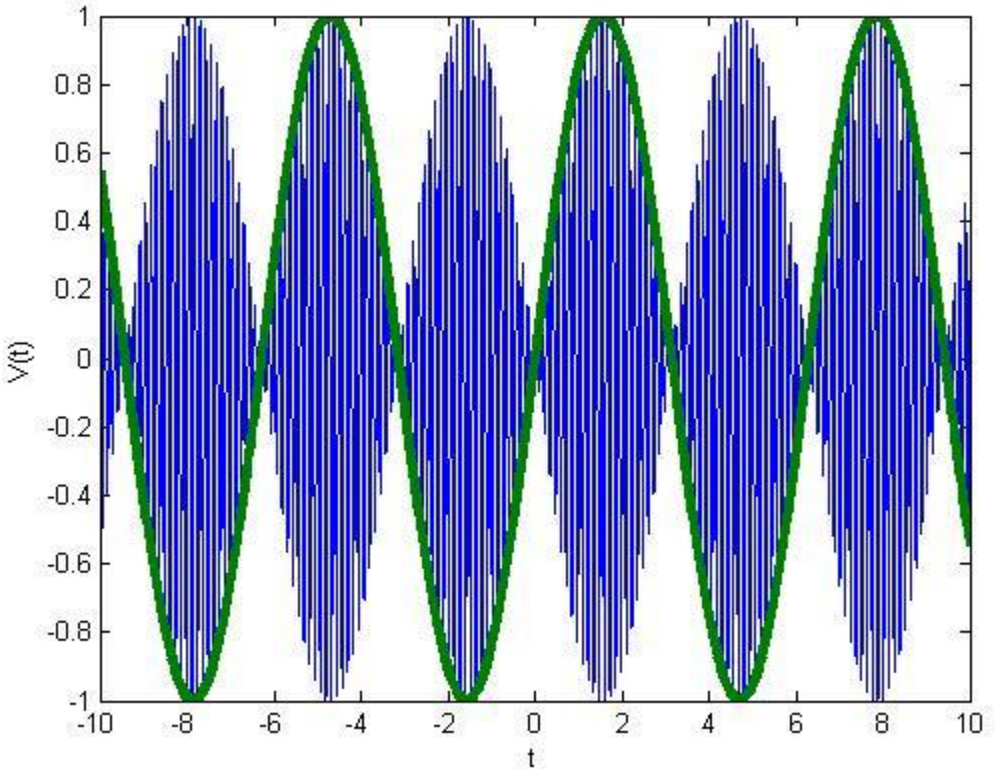
- Suppressed carrier AM
- Single sideband AM

# Suppressed carrier

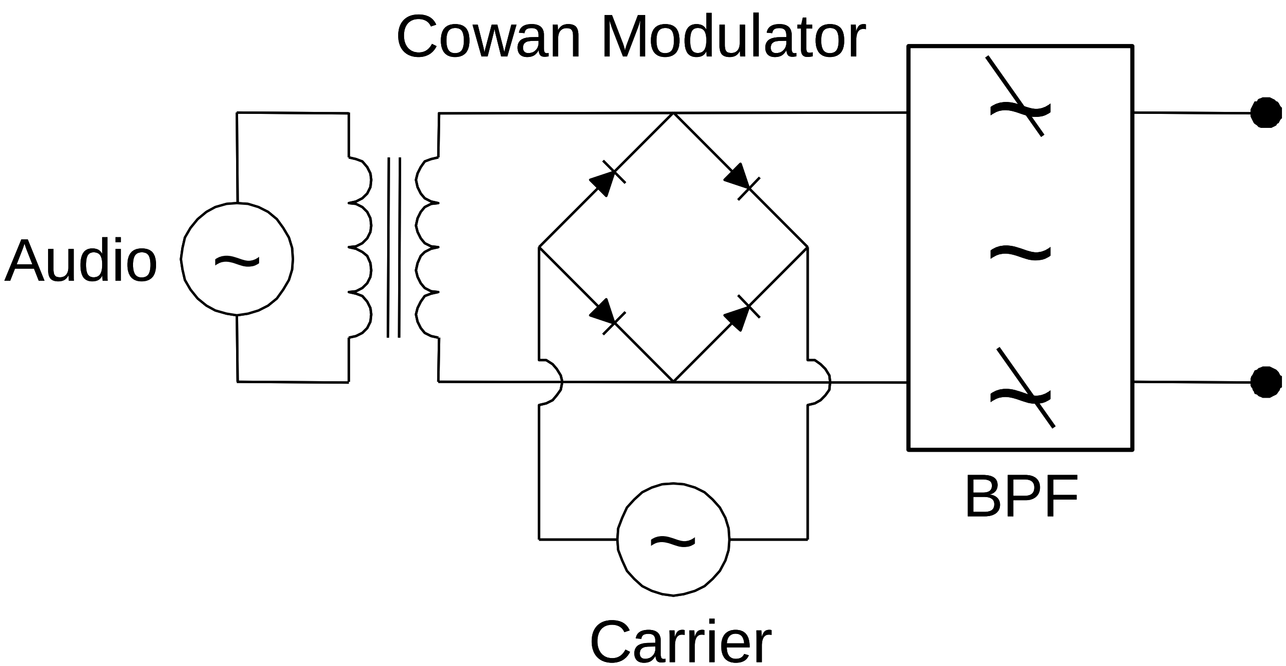
- Full-carrier AM is simple but not efficient
- Removing the carrier before power amplification allows full transmitter power to be applied to the sidebands
- Removing the carrier from a fully modulated AM system results in a double-sideband suppressed-carrier transmission **DSBSC**
- The carrier is removed using a band stop filter (pilot carrier -26dB)
- The carrier must be reinserted at the demodulator at exactly the same frequency

# DSBSC

$V_{DSBSC} = k\mathit{m}\,[\cos(\omega_c + \omega_s)t + \cos(\omega_c - \omega_s)t\,]$



# DSBSC generation




In this circuit the “Carrier” is a periodic square wave

# Cowan Modulator

Need carrier amplitude > modulating amplitude to turn diodes on
If A > B  diodes on -  modulation signal is s/c
If A < B  diodes off -  modulation signal is o/c

Modulation is switched on/off at carrier frequency
Switching signal is a square wave given by (from the Fourier series!!)
$v_{sw} = 0.5 + 2/\pi\ (\sin \omega_c t + 1/3\ \sin 3\omega_c t + \ldots\ldots)$

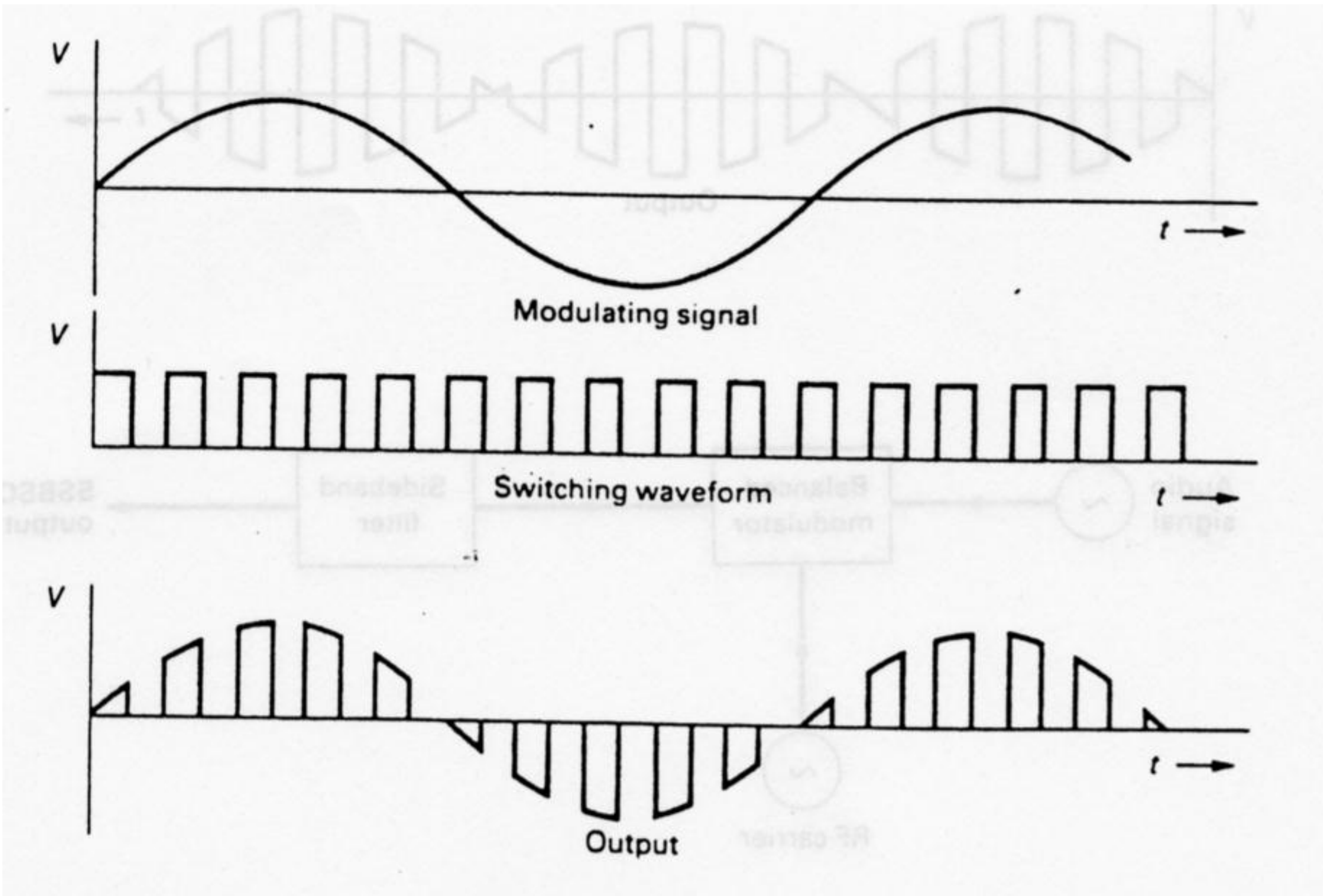
$$v_o = k v_{sw} V_s \sin(\omega_m t)$$

$$v_o = k V_s \sin(\omega_m t)\left[0.5 + \frac{2}{\pi}\left(\sin(\omega_c t) + \frac{1}{3}\sin(3\omega_c t) + \ldots\right)\right]$$

$$v_o = 0.5 k V_s \sin(\omega_m t) + \frac{k V_s}{\pi}\left[\cos((\omega_c - \omega_m)t) - \cos((\omega_c + \omega_m)t) + \frac{2}{3}\ldots\ldots\right]$$

Filter out second term i.e. DSBSC

# Cowan Modulator



# AM DSBSC demodulator

Demodulation of this signal is relatively simple. Consider the DSBSC equation below

$$V_{sc} = km\sin(\omega_c t)\sin(\omega_m t)$$

If we multiply this by the carrier then we get

$$v = km\sin(\omega_m t)\sin^2(\omega_c t)$$

$$v = km\sin(\omega_m t)\left[\frac{1-\cos(2\omega_c t)}{2}\right]$$

$$v = 0.5km\sin(\omega_m t) - 0.5km\cos(2\omega_c t)\sin(\omega_m t)$$

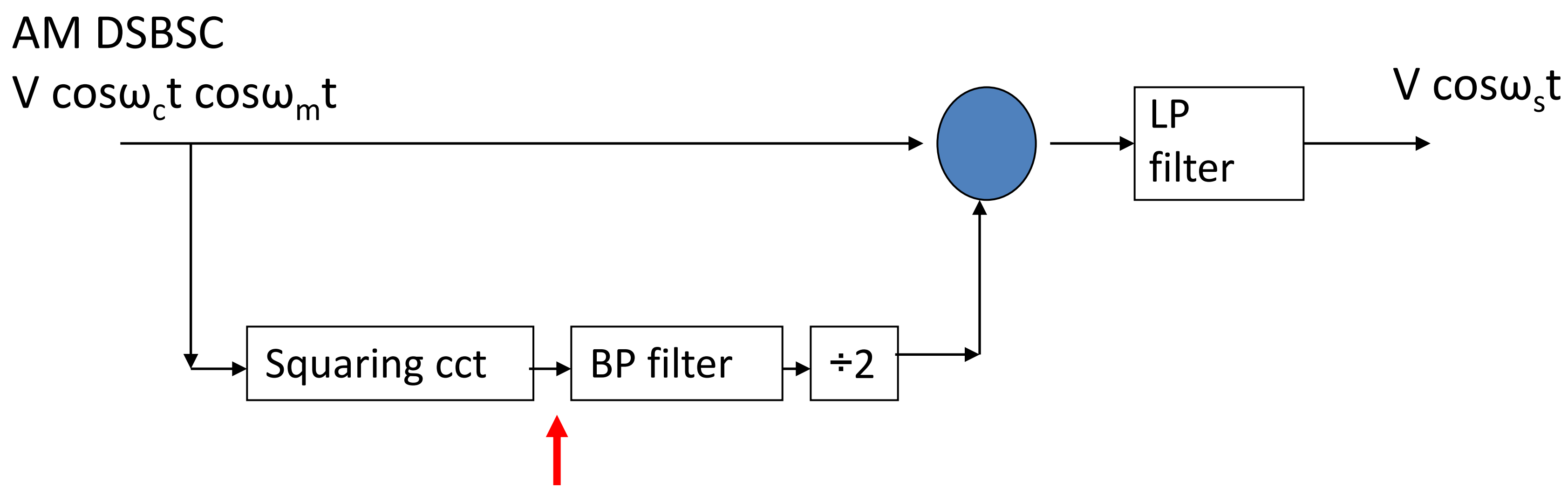
If we use a low pass filter then we can obtain our message

# AM DSBSC demodulator

- You have to generate the carrier signal in the receiver
- If the carrier is not in phase with the message then you will get errors

# Generating carrier

The carrier can be generated form the DSBSC signal as shown below




$V^2/4[1 + 0.5\cos(2\omega_c+2\omega_m)t + 0.5\cos(2\omega_c-2\omega_m)t + \cos 2\omega_c t + \cos\omega_m t]$

Select $\cos 2\omega_c t$ using band pass filter and divide by 2 to get $\cos\omega_c t$

# Single sideband (SSB)

Conventional DSB AM systems have disadvantages:

Two thirds or more of the total transmitted power is in the carrier which has no signal information;
The two sidebands of an AM signal are mirror images of one another;
As a result, one of the sidebands is redundant.

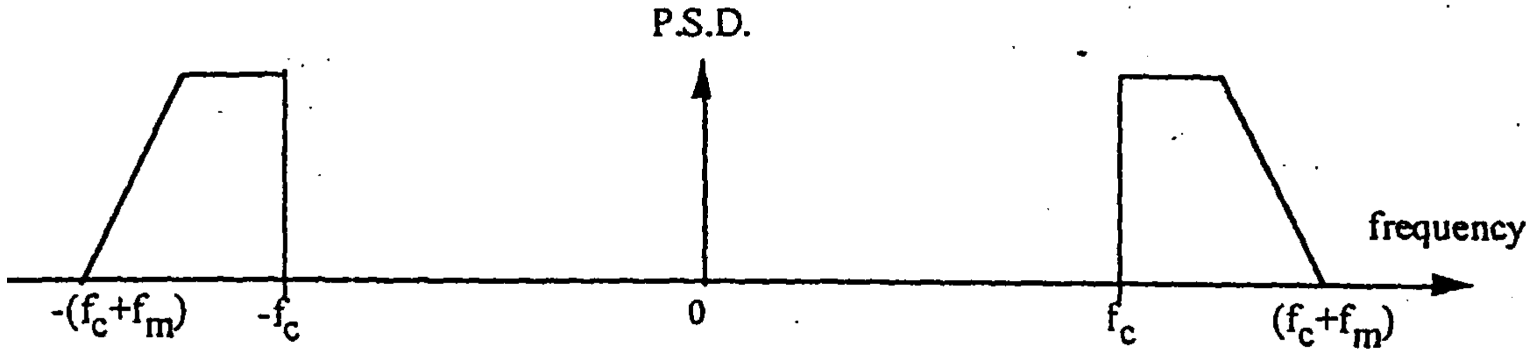
Using *single-sideband suppressed-carrier* transmission results in improved efficiency and reduced bandwidth and therefore twice as many signals may be transmitted in the same spectrum allotment
Typically, a 3dB improvement in signal-to-noise ratio is achieved as a result of SSBSC (due to bandwidth reduction)

# Single sideband (SSB)

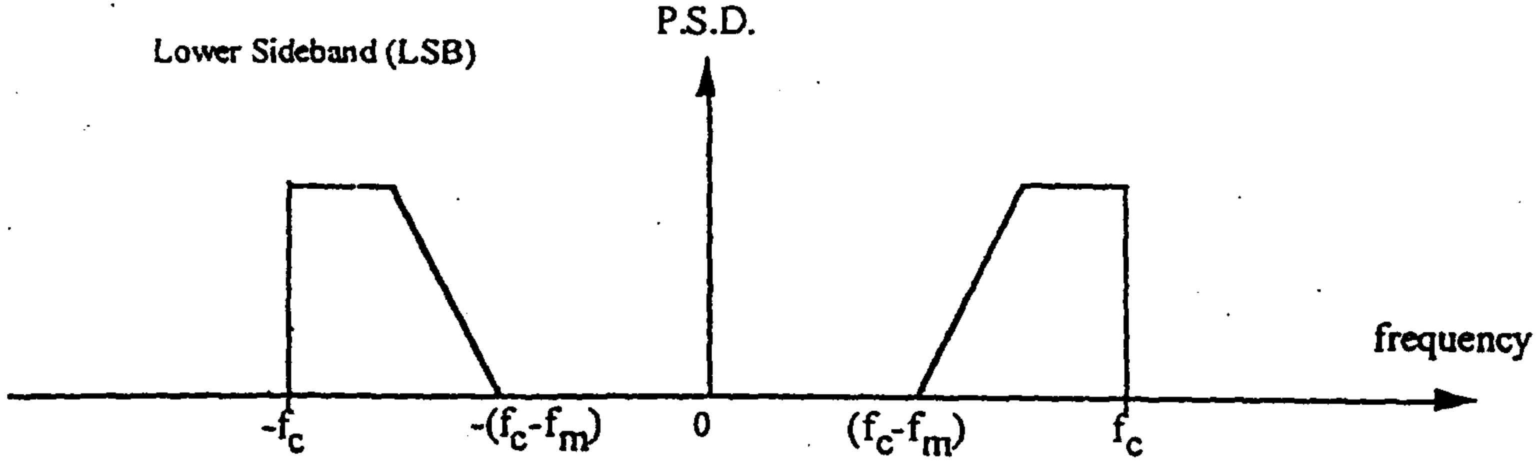
- SSB modulation is a suppressed carrier modulation, therefore it must be demodulated using synchronous demodulator, as for DSB-SC. Since synchronous demodulation is required, SSB is not used for commercial broadcasting, but it is very popular in applications where the channel bandwidth or the available power are limited, such as: <u>mobile radio communications</u>, <u>military communications</u>, <u>maritime mobile communications</u> (i.e. ships) and <u>amateur radio communications</u>.
- SSB is also used in telephone systems to combine many voice channels into single band limited channel. This is called Frequency Division Multiplexing (FDM).

# SSB Spectrum

upper sideband(USB)



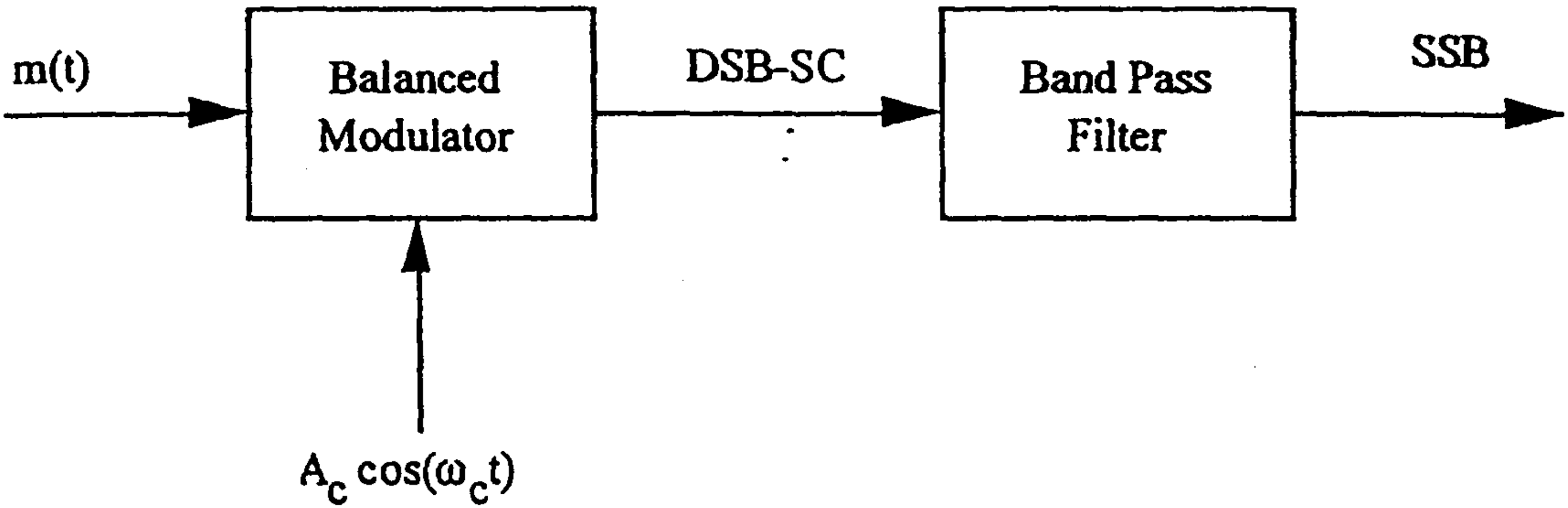

lower sideband (LSB)



# SSB generation

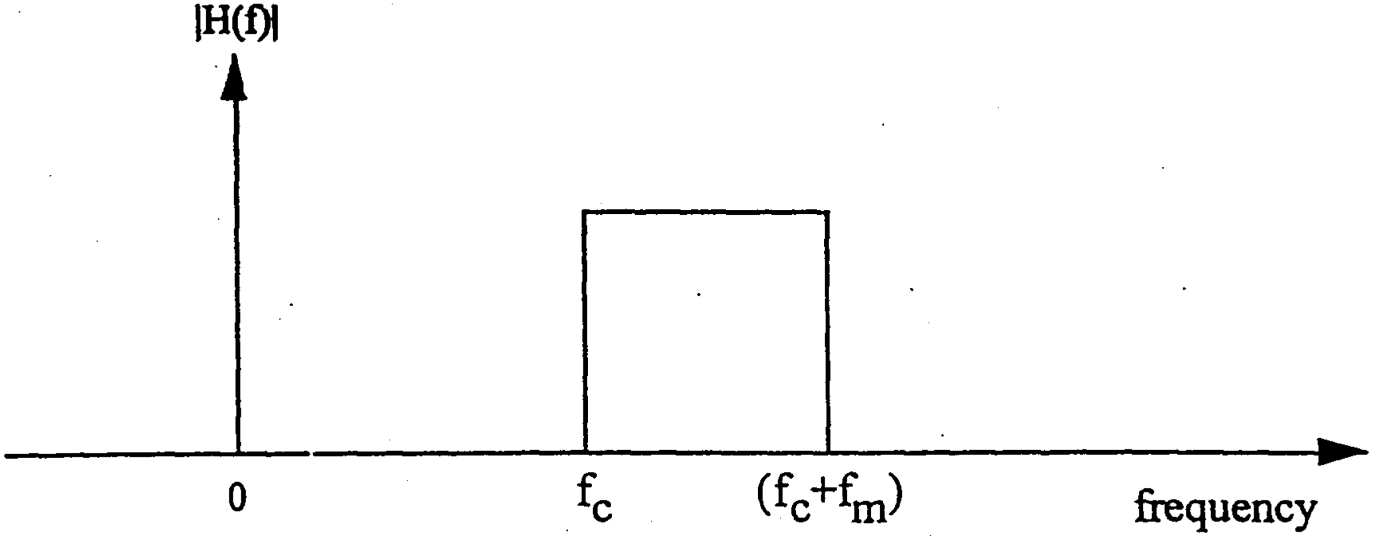
- From the frequency domain description of SSB we can see that the SSB signal is simply a DSB-SC signal with one sideband removed
- This leads to an obvious method of generating SSB by generating a DSB-SC signal and then using a bandpass filter to select the desired sideband, this is called the **Frequency Discrimination Method**, or the **Filter Method** of SSB generation:

# SSB generation



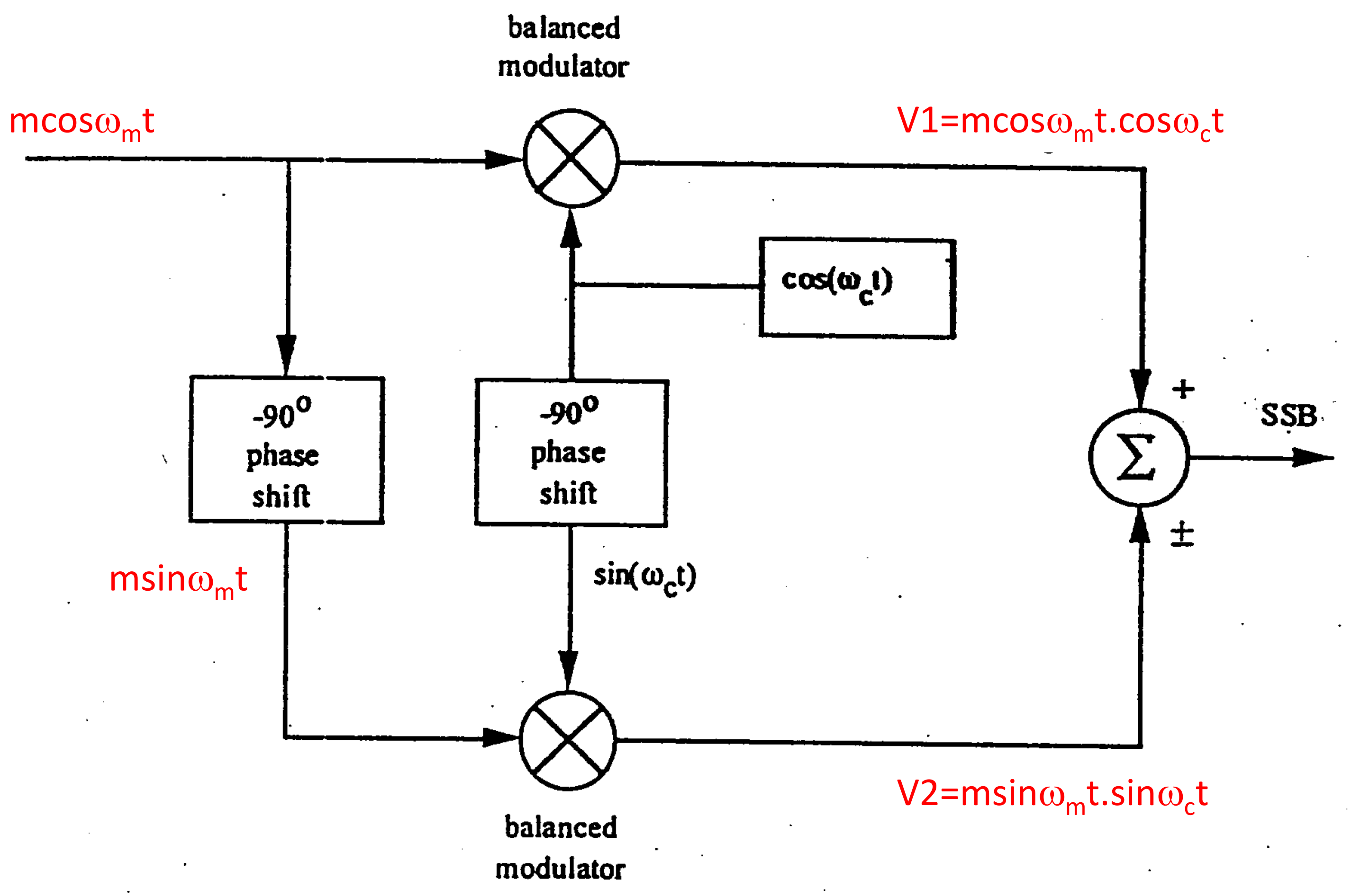
# SSB generation

For example, for USB we require a band pass filter with following frequency response:




***Problem with getting a very sharp filter response***

# SSB generation – phasing method



# SSB phasing method

Idea is to generate 2xDSBSC signals with phase shift between them so that 1 SB adds constructively while other SB is cancelled

$V_1 = 0.5mV_c\,[\cos(\omega_c - \omega_s)t - \cos(\omega_c + \omega_s)t\,]$

$V_2 = 0.5mV_c\,[\cos(\omega_c - \omega_s)t + \cos(\omega_c + \omega_s)t\,]$

Output $V_0 = V_1 + V_2 = mV_c \cos(\omega_c - \omega_s)t$ - LSB

No sharp filter needed

# Demodulation of SSB signals – synchronous detector as per DSBSC demodulator

$v_o = 0.5km V_c \cos(\omega_c - \omega_m)t \,.\, \sin\omega_c\, t = SSB_{lsb}$

$= 0.25kV_m\, [\sin(2\omega_c - \omega_m)t + \sin\omega_m\, t\, ]$

After LPF **$o/p = 0.25kV_m \sin\omega_m\, t$**

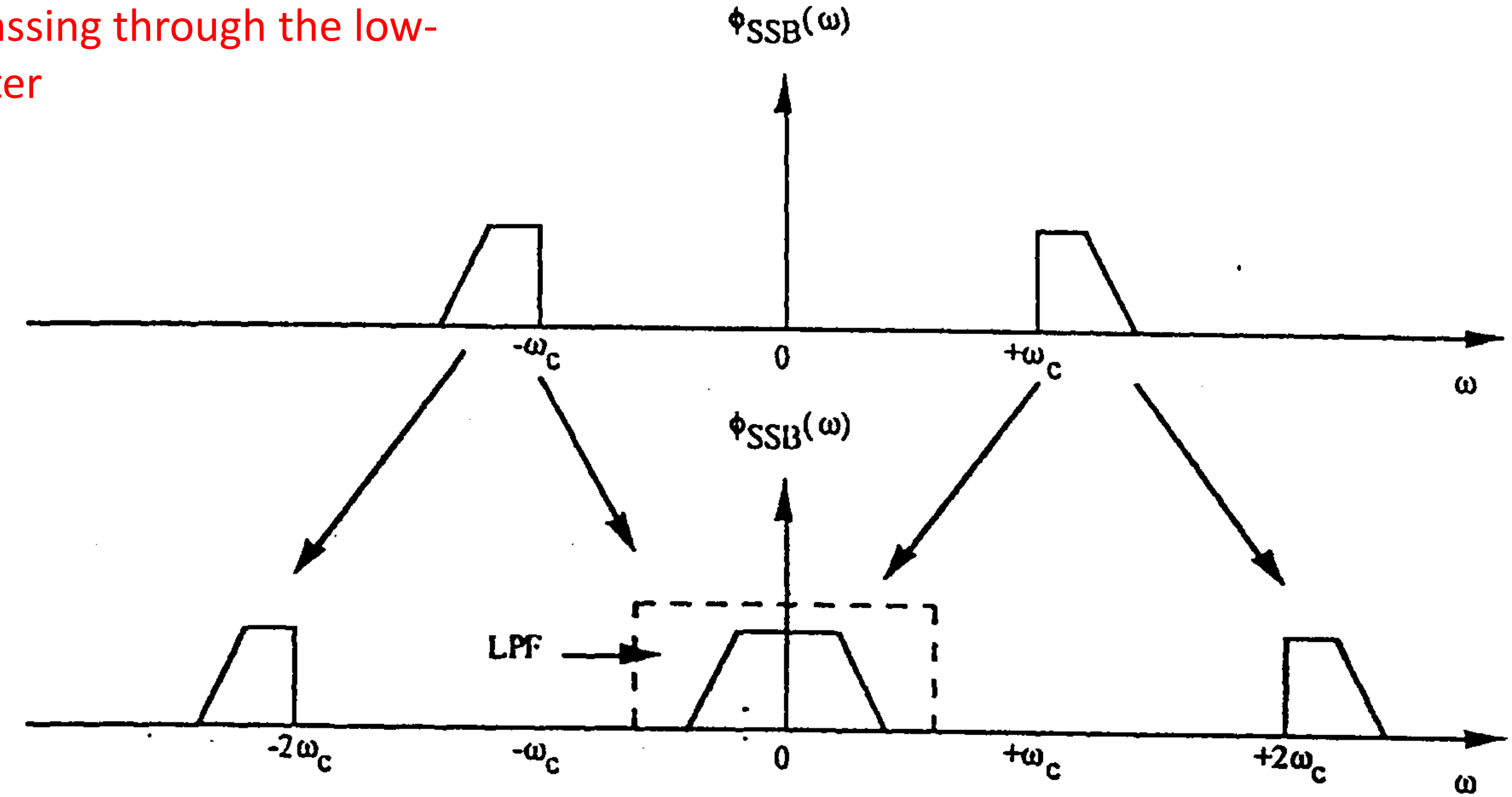
*which is the modulating signal*

*Again phase errors in carrier will cause reduced or zero o/p*

In frequency spectrum sense, multiplication of the SSB signals by $\sin\omega_c t$ translates half of each spectral density up in the frequency by $\omega_c$ rad/s and half down by the same amount as shown next

# *Demodulation of SSB signals*

The portion shifted up to a frequency $2\omega_c$ can be filtered out with a low-pass filter. Hence, the demodulated signal after passing through the low-pass filter



# AM SSB Full carrier (SSBFC)

Carrier is transmitted at full power with only one of the sidebands. In this only half as much bandwidth will be required

**The power relations will be as follows:**

Power in carrier = $P_c$

Power in lower sideband = 0

Power in upper sideband = $\frac{m^2}{4} P_c$

Total power $P_T = P_c\left(1 + \frac{m^2}{4}\right)$

# AM Single sideband suppressed carrier (SSBSC)

In this this the carrier is totally removed together with one of the sidebands. Only half the bandwidth is required.

**Power Relations**

The sideband power will constitute 100% of the total transmitted power.

Power in carrier, $P_c = 0$

Power in lower sideband = 0

Power in upper sideband = $P_T$

# AM SSB Reduced carrier (SSBRC)

In this one sideband is removed and the carrier reduced to about 10% of the unmodulated amplitude.
The carrier will have to be reinserted at reduced amplitude for the purpose of demodulation

**Power relations**

Power in carrier, $P_{cnew} = \frac{(0.1V)^2}{2} = \frac{0.01V^2}{2} = 0.01P_c$

Power in lower sideband = 0

Note the Pc is the original carrier power

Power in upper sideband = $\frac{m^2}{4}P_c$

Total power = $P_T = 0.01P_c + \frac{m^2}{4}P_c$

# Comparison of SSB to DSB AM

**Advantages of SSB**

*Bandwidth conservation*: Only half the bandwidth is required

*Power conservation*: Only one sideband with carrier removed or suppressed. Hence total transmitted power will be less. This allows smaller transmitters to be used.

*Selective fading*: In double sideband, the two sidebands may experience different impairments as they propagate along different paths in the medium. This could result in carrier phase shift. This cannot happen if only one sideband is transmitted.

*Noise Reduction*: Thermal noise is reduced to half, because the bandwidth is also half.

# Comparison of SSB to DSB AM

## Disadvantages

*Complex receivers*

*Tuning Difficulties*: More difficult to tune than conventional AM receivers. More expensive tuning circuits can be used.
A pilot carrier can be transmitted to aid demodulation at receiver.

**Examples:** A double sideband AM radio transmitter gives a power output of 5 kW when the carrier is modulated to a depth of 95%. A speech signal is then used to modulate the carrier with a depth of 20% and the carrier and one sideband are suppressed. Find the output power in the other sideband.

## **Solution**

Power in DSB signal $$P_T = P_c\left(1+\frac{m^2}{2}\right)$$

$P_T$ = 5000 W, m = 0.95 hence $\underline{P_C = 3445.3\ \text{W}}$

For a SSB signal power in sideband $$P_T = \frac{m^2}{4}P_c$$

Now $P_C$ = 3445.3 W, m = 0.2 hence $\underline{\mathbf{P_T = 34.45\ W}}$

# Summary

- DSB-SC is power efficient but sensitive to phase error
- DSB-SC can be demodulated by simple envelop detector  but is power inefficient.
- SSB is bandwidth efficient but need complicated receiver.

# Summary

**AM Advantages**

- Low bandwidth
- Easy to modulate, demodulate signal

**AM Disadvantages**

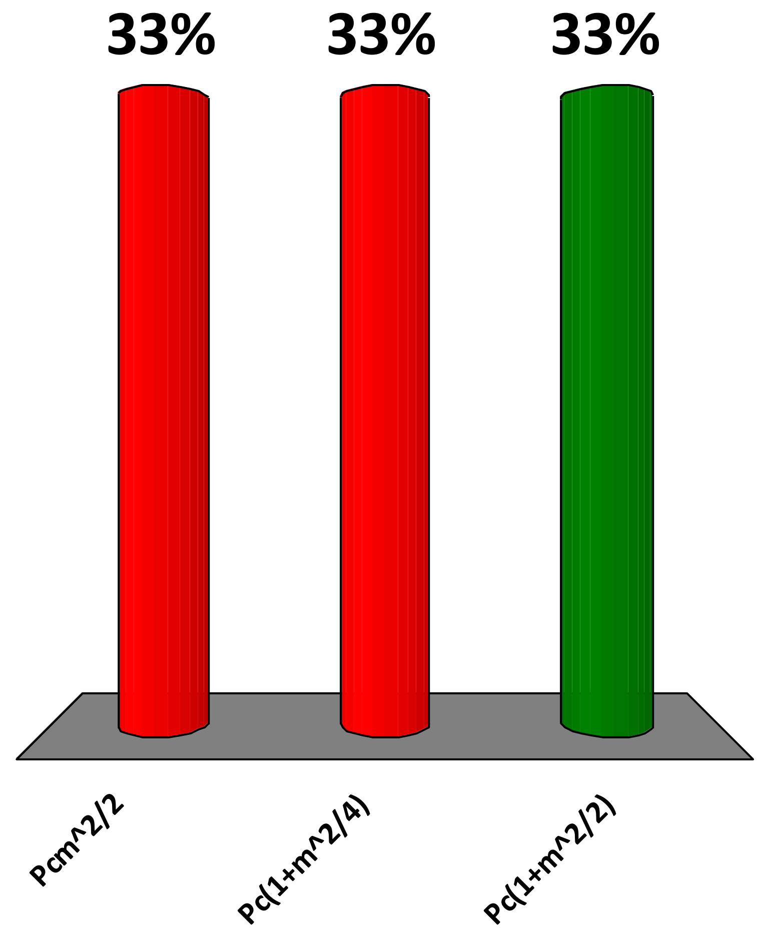
- Fading
- Noise greatly affects amplitude of received signal
- Hard to lock frequency of receiver local oscillator to carrier frequency (esp. in S.C. systems)

# What have you learnt?

- Amplitude Modulation – basic mathematics
- Spectra
- DSB, DSB-SC, SSB principles
- AM Modulation circuits
- AM Demodulation circuits

# What is the power in an AM DSB CP signal?

A. Pcm^2/2
B. Pc(1+m^2/4)
C. Pc(1+m^2/2)



# What circuit could you use to generate AM DSB

A. Envelope detector
B. Cowan modulator
C. Balanced modulator