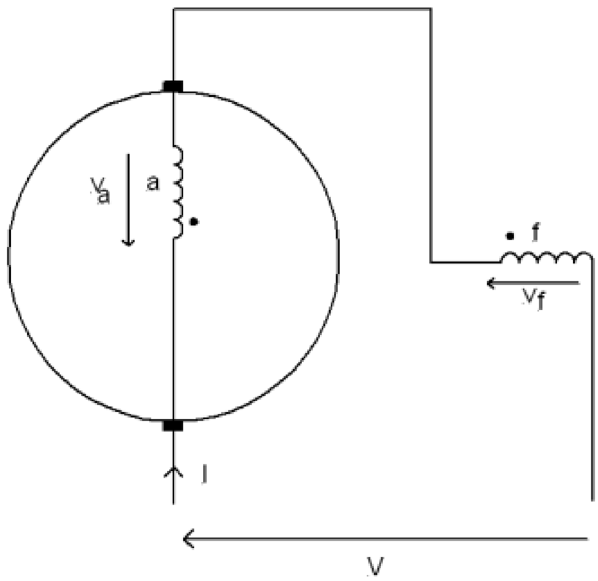
EEE 409 Modelling of Electrical Machines
2010 Examination Solutions

1.

a)



a) .

The general form of the voltage equations are:

$$\left|\begin{array}{c} v_a \\ v_f \end{array}\right| \begin{array}{c} = \\ = \end{array} \left|\begin{array}{cc} R_a + L_a p & \omega_r M \\ 0 & R_f + L_f p \end{array}\right| \left|\begin{array}{c} i_a \\ i_f \end{array}\right|$$

On DC: $p=0$

On AC: $p=j\omega_s$

Constraining equations:

$V = V_a + V_f$

$I = I_a = I_f$

The resulting voltage equations are:

DC operation:

$V = I\ (R_a + R_f + \omega_r M)$

AC operation:

$V = I\ (R_a + R_f + \omega_r M + j(X_a+X_f))$

(3)

b) **On DC:**

$$Input\ current = \frac{Input\ power}{Input\ voltage} = \frac{600}{200} = 3A$$

Losses in the machine = $I^2R = 3^2 \times 12 = 108W$

$\therefore$ Output power = Input power – losses = 492W

$$\therefore \text{Output torque} = \frac{\text{Output power}}{\text{rotor angular velocity}} = \frac{492}{15650 \times \frac{2\pi}{60}} = 0.30\text{Nm}$$

$$But\ T = MI_{DC}^{\ 2}$$

$$\therefore \text{M} = \frac{\text{T}}{\text{I}^2} = \frac{0.30}{3^2} = 0.033\text{H}$$

On AC:

At the same load torque condition, the same rms current is drawn as the DC case. Hence, the magnitude of the input current is 3A rms

The voltage equation for AC operation when operating a multiple N of the synchronous speed (substituting with terminal quantities R and L) is:

$$V = (R + NX_m + j\omega_s L)\ I$$

where N is the ratio of actual speed to synchronous speed

$$\therefore \left|\frac{V}{I}\right| = |Z| = \sqrt{\left((R + NX_m)^2 + (\omega_s L)^2\right)}$$

Re-arranging this equation yields:

$$N = \frac{1}{X_m}\left(\sqrt{\left(\frac{V^2}{I^2}\right) - (\omega_s L)^2}\right) - \frac{R}{X_m}$$

For the particular parameters of this motor:

N = 4.90

$\therefore$ Actual speed on AC supply = 4.90 × 3000 = 14,700rpm

$$\text{Power factor on load} = \frac{\text{R} + \text{NX}_\text{m}}{\sqrt{\left((\text{R} + \text{NX}_\text{m})^2 + (\omega_s L)^2\right)}} = 0.82\ lagging$$

[Note: It is important to state that the power factor is **lagging]**

For starting torque, $\omega_r = 0$

On DC: $\text{I} = \frac{\text{V}}{\text{R}} = \frac{200}{12} = 16.67\text{A}$

$\therefore$ T = MI^2 = 0.033×16.67^2 = 9.25 Nm on DC

On AC: $I = \dfrac{V}{\sqrt{R^2 + (\omega_s L)^2}} = \dfrac{230}{45.6} = 5.04A$

$\therefore$ T = MI^2 = 0.0333×5.04^2 = 0.846 Nm on AC

Ratio of DC starting torque to AC starting torque = 10.9

(9)

c) The torque when operating at 15,650rpm on a 200V DC supply is 0.30Nm. For the inductively compensated motor on a sinusoidal AC supply at the same load torque condition, the same rms current of 3A is drawn (as was the case for DC operation).

With ideal compensation, the q-axis rotor reactance is completely cancelled by the compensating coil and hence effective series inductance presented at the terminals is the 0.07H. Hence, the speed ratio (actual : synchronous) of the compensated coil is:

$$N = \frac{1}{X_m}\left(\sqrt{\left(\frac{V^2}{I^2}\right) - (\omega_s L)^2}\right) - \frac{R}{X_m}$$

$$N = \frac{1}{10.4}\left(\sqrt{\left(\frac{230^2}{3^2}\right) - (22.0)^2}\right) - \frac{12}{10.4} = 5.93$$

$\therefore$ Actual speed on AC supply = 5.93 × 3000 = 17,780rpm

[Note: This is higher than the DC case, but it is important to note that the DC case was for 200V not 230V – for equal voltage you would expect the DC case to run faster]

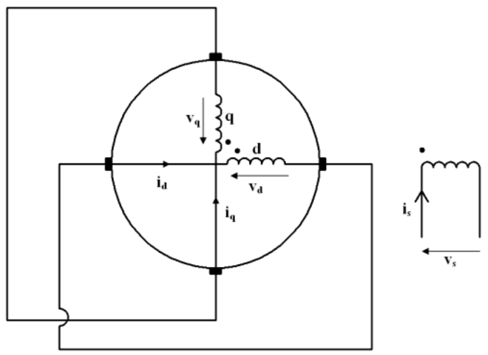
$$\text{Power factor on load} = \frac{\mathrm{R} + \mathrm{NX_m}}{\sqrt{(\mathrm{R} + \mathrm{NX_m})^2 + (\omega_s L)^2}} = \frac{12 + (5.92 \times 10.4)}{\sqrt{(12 + (5.92 \times 10.4))^2 + (22.0)^2}} = 0.96 \textit{ lagging}$$

(i.e power factor improves as expected)

(5)

2.

a)



b) The general form of the voltage matrix equations is:

$$\begin{vmatrix} v_s \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_s + L_s p & M_{sd}p & 0 \\ M_{ds}p & R_d+L_d p & -\omega_r L_q \\ \omega_r M_{qs} & \omega_r L_d & R_q+L_q p \end{vmatrix} \begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix}$$

[Note: The use of subscripts 1 for stator and 2 for rotor would be equally correct in the above equations]

For steady-state sinusoidal AC operation, $p=j\omega_s$ and $\omega_r=(1\text{-}s)\omega_s$

The short-circuited rotor windings dictate that $V_q=V_d = 0$

Adopting subscripts 1 for stator and 2 for rotor, yields

$$\begin{vmatrix} V_s \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m & 0 \\ jX_m & R_2+jX_2 & -(1\text{-}s)X_2 \\ (1\text{-}s)X_m & (1\text{-}s)X_2 & R_2+jX_2 \end{vmatrix} \begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix}$$

The unbalanced currents on the right hand side can be transformed to symmetrical components:

$$\begin{vmatrix} i_s \\ i_d \\ i_q \end{vmatrix} = C \begin{vmatrix} Is \\ Ip \\ In \end{vmatrix}$$

Where C is the following transformation matrix:

$$C = 1 / \sqrt{2} \begin{vmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & j & -j \end{vmatrix}$$

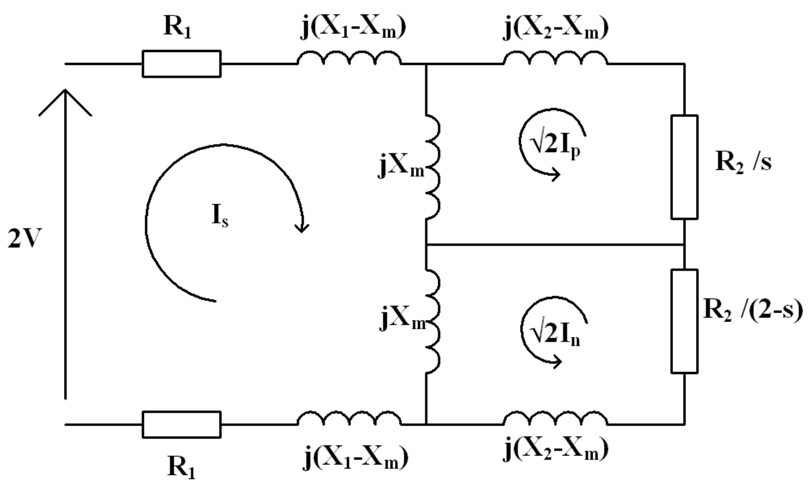
Applying the transformation matrix to the impedance matrix using:

$Z' = C_t^* \, Z \, C$

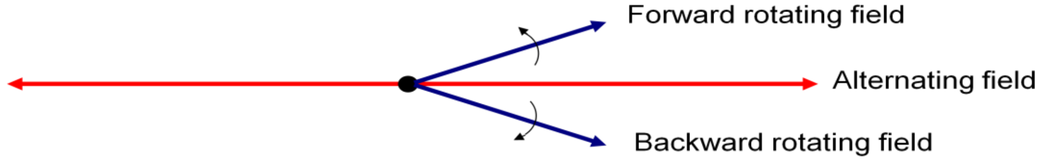
Following two stages of matrix multiplication *[which candidates may well perform during the course of the derivation rather than recalling from memory]*, the final result is:

$$\begin{vmatrix} 2V_s \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 2(R_1+jX_1) & jX_m & jX_m \\ jX_m & \dfrac{R_2}{s} + jX_2 & 0 \\ jX_m & 0 & \dfrac{R_2}{(2-s)} + jX_2 \end{vmatrix} \begin{vmatrix} I_s \\ \sqrt{2}\, I_p \\ \sqrt{2}\, I_n \end{vmatrix}$$

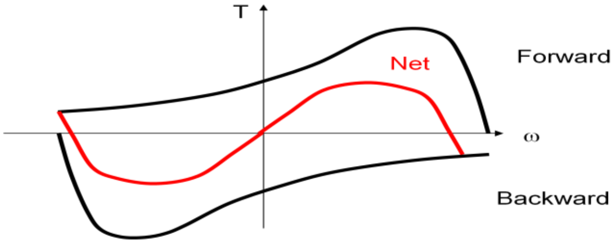
The equivalent circuit whish satisfies these equations is:



c) The alternating field produced by the sole d-axis coil can be resolved into contra-rotating forward and backward fields, each of which produces a torque speed curve which is broadly similar to that of a three-phase induction machine.



The net torque can be considered as the sum of these two curves as shown below. As would be expected the curves are symmetrical and hence cancel at zero speed, i.e. no net torque at zero speed.



d) Brief description of 3 out of 5 of the possible methods:

- Shaded pole motor
- Split phase motor
- Permanent split capacitor motor
- Capacitor start motor
- Dual value capacitor motor

Key points in each case are the
- A schematic of the physical configuration
- Basic principles and roles of additional components
- A reasonably sketched torque-speed curve which illustrates key operating points

3.

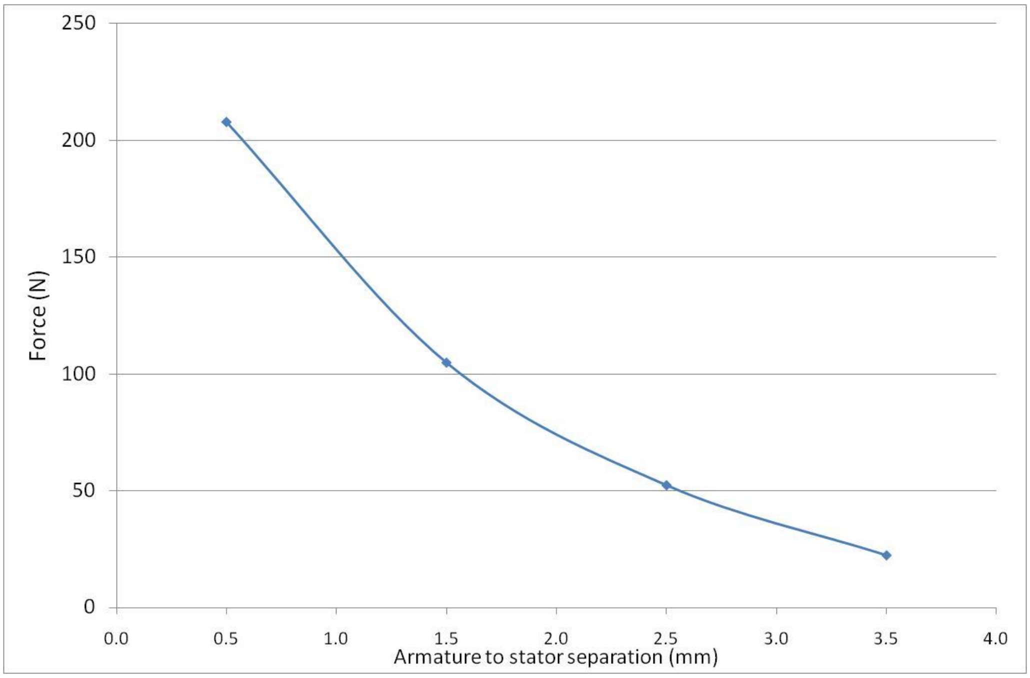
[*This questions differs in two key respects from previous questions on pure reluctance devices, viz. it deals with a normal force device, the question is seeking a variation in force with displacement (as compared to the more normal average force over a stroke).]*

a. *[The key to this question is to recognise that the best(and arguably only) way that this can be achieved is by estimating the force variation as a series of averages for the steps between intermediate angles, which one could quite reasonably specify as the value at the mid-point of the intermediate step. This is something that has been discussed in lectures. Another useful 'trick' is that for a current of 3A, several of the characteristics can be regarded, to a good approximation, as linear. Hence the co-energy change up to 3A can be obtained from a single calculation of the area of a triangle rather than the more laborious trapezoidal integration of each interval – not that the latter is incorrect. Identifying this time-saving step will require some initiative on the part of the candidate]*

Separation (mm)	**Area under curve to 3A (J)**
4	0.03×3/2 = 0.045
3	0.45×3/2 =0.0675
2	0.08×3/2 =0.12
1	0.15×3/2 =0.225
0 – need to perform some trapezium integration because of non-linearity	0.11×1/2 + 1× ((0.20+0.11)/2) + 1×((0.246+0.20)/2)=0.433

Interval	Change in co-energy (J)	Average force over 1mm interval (N)
4-3	0.0675-0.045 =0.0225	22.5
3-2	0.12-0.675 =0.0525	52.5
2-1	0.225-0.12=0.105	105
1-0	0.433-0.225 =0.208	208

Plotting this on graph paper yields:



(10)

b). *[This has not been covered in lecture explicitly and requires some insight from the students to draw on knowledge from previous courses]*

The force produced by a normal force variable airgap actuator is given to a very good approximation by:

$$F = \frac{\boldsymbol{B}^2 A}{2\mu_0}$$

where:

B – strictly the normal component of airgap flux density, but to a very good approximation this is equal to the magnitude of airgap the flux density in such a device
A – pole face area
μ_0 – permeability of free space

Hence, the force is proportional to the square of flux density.

Assuming that the degree of saturation in the core is sufficiently low that the effective magnetic circuit reluctance remains dominated by the airgap, then to a reasonable approximation the airgap flux density is given by:

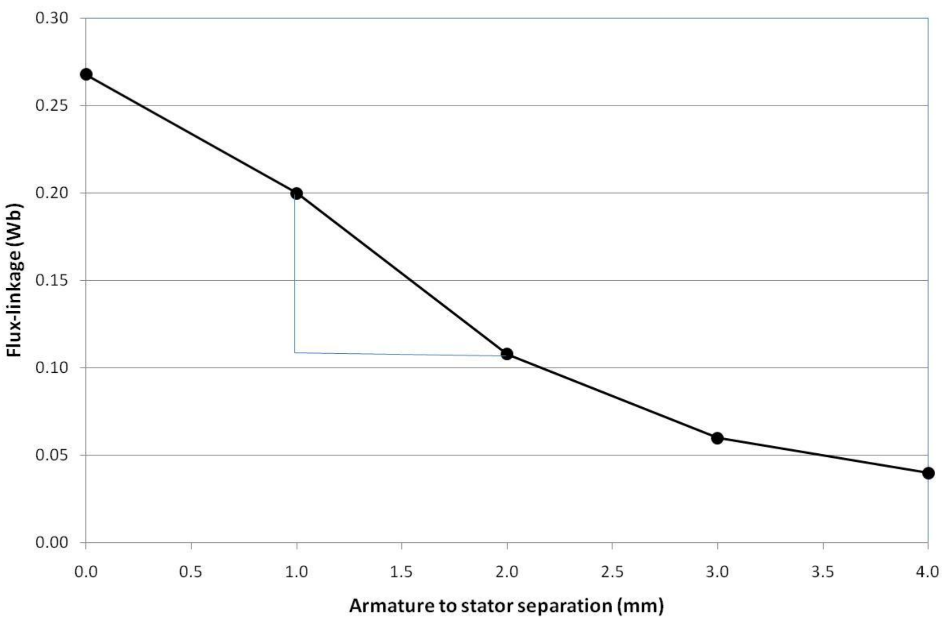
$$B = \frac{\mu_0 N I}{l_g}$$

Where
N – number of turns on the coil
I – coil current
l_g – effective magnetic airgap length

Hence, the force would be expected to be inversely proportional to the square of the effective magnetic airgap, i.e. the separation between the armature and the stator, at least before the onset of appreciable saturation. This relationship is indeed observed in the predicted force-displacement characteristic, although there is some departure from this relationship at the lowest values of separation because of the onset of magnetic saturation (which is clearly evident in the original flux-linkage versus current characteristic provided).

(4)

c) Taking the data for 4A and re-plotting as flux-linkage versus armature to stator separation yields:



By inspection, the maximum rate of change of flux-linkage with respect to a change in the separation between armature and stator occurs for separation between 1mm and 2mm. Over this interval, the rate of change can be estimated as:

$$\frac{d\psi}{dz} = \frac{0.2 - 0.108}{0.001} = 92\, Wb/m$$

At 0.6m/s the induced voltage is given by:

$$e = \frac{d\psi}{dz} \times \frac{dz}{dt} = 92 \times 0.6 = 55.2V$$

[As will all questions which are based on reading value from graphs, there is a reasonable margin of tolerance on the exact numerical answer]

4.

a) The inductance can be readily calculated from the additional flux-linkage produced by the current over and above any bias from the permanent. For the 4 cases :

At 25A:

$$\mathrm{L} = \frac{0.05}{25} = 2\mathrm{mH} \quad \text{at } 0^{\mathrm{o}}$$

$$\mathrm{L} = \frac{0.05}{25} = 2\mathrm{mH} \quad \text{at } +45^{\mathrm{o}}$$

At 100A:

$$L = \frac{0.20}{100} = 2mH \quad at\ 0^{o}$$

$$L = \frac{0.14}{100} = 1.4mH \quad at\ +45^{o}$$

The difference observed at 100A and $+45^{\mathrm{o}}$ is a result of magnetic saturation this is an angular displacement at which the magnet flux and the coil flux add to each other.

(3)

b) The flux-linkage characteristics for 0A is a reasonable approximation to a sine-wave [in fact the actual data is generated from a simple sin function]. It is therefore reasonable to assume that the maximum rate of change of flux-linkage will occur at angular displacements around 0^{o} [*This should also be apparent by inspection*]. From Figure 4, an estimate of the rate of change of flux linkage with rotor position can be made:

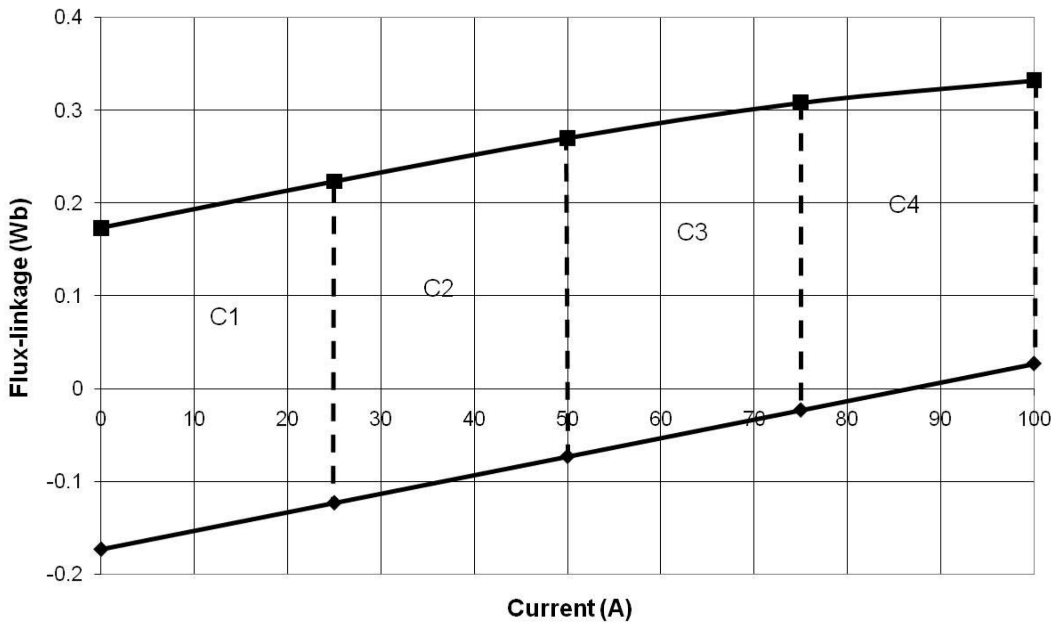
$$\frac{\mathrm{d}\Psi}{\mathrm{d}\theta} \approx \frac{\Delta\Psi}{\Delta\theta} = \frac{0.07}{10 \times \frac{\pi}{180}} = 0.40 \ \mathrm{Wb/rad}$$

At 5500rpm

$$\frac{\mathrm{d}\theta}{\mathrm{dt}} = \frac{5500 \times 2 \times \pi}{60} = 576 \ \mathrm{rad/s} \ \therefore \mathrm{e} = \frac{\mathrm{d}\Psi}{\mathrm{dt}} = \frac{\mathrm{d}\Psi}{\mathrm{d}\theta} \times \frac{\mathrm{d}\theta}{\mathrm{dt}} = 230\mathrm{V}$$

(4)

c) In order to estimate the torque for the two currents specified it is necessary to re-plot the data as a flux-linkage versus current characteristic for -30^{o} and $+30^{\mathrm{o}}$:



The co-energy change can be estimated by trapezoidal integration of the four areas C1 to C4 shown in the graph above. Using this approach:

The change in co-energy for 25A is C1 = 8.66J

The change in co-energy for 100A is C1+C2+C3+C4 = 8.66+8.62+8.43+7.96=33.67J

Change in rotor angular displacement = $60 \times \frac{\pi}{180} = \frac{\pi}{3}$ rads

The torques produced are therefore given by:

At 25A: $T = \frac{dW'}{d\theta} \approx \frac{8.66}{\pi/3} = 8.27\ Nm$

At 100A: $T = \frac{dW'}{d\theta} \approx \frac{33.67}{\pi/3} = 32.14 Nm$

[An important point here is that the torque per amp is diminishing slightly with onset of magnetic saturation at 100A]

(6)

d) There are 6 stator teeth and 3 phases. Hence, each phase consists of two diametrically opposite coils. Assuming that the airgap flux density provides a reasonable estimate of the flux density level in the core as a whole then:

By inspection of the flux-linkage characteristics, the core of the machine appears to be saturating at a flux-linkage of ~0.3Wb, which corresponds to a flux density of ~1.6T.

Considering the case of a rotor angular displacement of 0º (in which no net magnet flux is present). A current of 25A produces a flux-linkage of 0.05Wb, which by equivalence with 1.6T at 0.3Wb, corresponds to a flux density of 0.27T

At this flux density level, then it is reasonable to assume that the rotor and stator cores will be infinitely permeable.

$\therefore$ The total effective magnetic airgap to the coil flux of one phase is:

$$l_{eff} = 2(l_g + l_m) = 14mm$$

The airgap flux density produced by a given coil mmf is given by:

$$B_g = \frac{\mu_0 NI}{l_{eff}}$$

Re-arranging this equation yields:

$$N = \frac{B_g l_{eff}}{\mu_0 I} = \frac{0.267 \times 14 \times 10^{-3}}{4\pi \times 10^{-7} \times 25} = 120 \text{ turns per phase (i.e. 60 per coil)}$$

[As will all questions in this paper which involve graphical estimates, there is some scope for deviation in the exact answer providing the method is correct]

[It is also important to note assumptions – without these full marks will not be awarded]

(7)