Solutions

Q1.a

The Insertion Loss (*IL*) is the ratio of transmitted power P_t to the incident power P_{in}^{+}

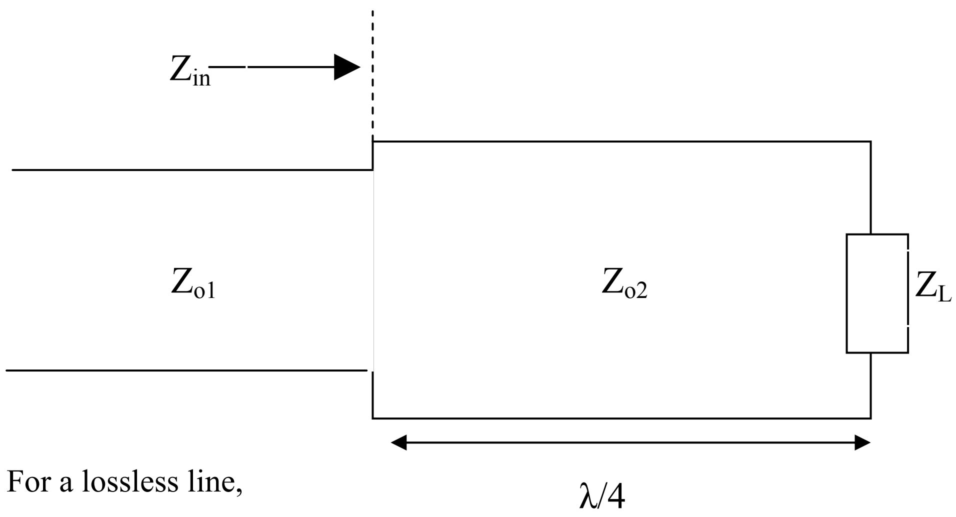
$$IL = -10\log\left(\frac{P_t}{P_{in}^{+}}\right) = -10\log\left(\frac{P_{in}^{+} - P_{in}^{-}}{P_{in}^{+}}\right) = -10\log(1 - |\Gamma_{in}|^2) \qquad \textbf{(1 mark)}$$

The fraction of the incident power that is absorbed by the load, is missing from the signal returned to the generator. This "*loss*" is called the return loss (*RL*)

$$RL = -10\log\left(\frac{P_{in}^{-}}{P_{in}^{+}}\right) = -10\log|\Gamma_{in}|^2 = -20\log|\Gamma_{in}| \qquad \textbf{(1 mark)}$$

Q1.b

Usually used as an intermediate matching between two transmission lines of different real characteristic impedances. For matching over a narrow band of frequency *f* (±5%), a single section is adequate but 2 or more sections are required for broadband matching. **(1 mark)**



(2 marks)

For a lossless line,

$$Z_{o2_{\frac{\lambda}{4}}} = \sqrt{Z_{o1} Z_L}$$

for real Z_{o1}, Z_{o2} and Z_L. **(1 mark)**

Q1.c

$$\Gamma_{load} = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(50 + j80) - 50}{(50 + j80) + 50} = 0.39 - j0.49 \qquad \textbf{(1 mark)}$$

$$\beta = \frac{2\pi}{\lambda} = 0.628 \qquad \textbf{(1 mark)}$$

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)} = 11.7 - j6.3\Omega \qquad \textbf{(1 mark)}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = -0.6 - j0.17 \qquad \textbf{(1 mark)}$$

$RL = -20\log|\Gamma_{in}| = -4.1dB$ **(1 mark)**

$IL = -10log(1\text{-}|\Gamma_{in}|^2) = -2.1dB$ **(1 mark)**

Q1.d

The distance between successive minima is

$$d = \frac{\lambda}{2}$$

i.e.

$\lambda = 2d = 4.2cm$ **(1 mark)**

$\beta = \frac{2\pi}{\lambda} = 1.5$ **(1 mark)**

The reflection coefficient is given by

$\Gamma = \rho e^{j\theta}$ **(1 mark)**

where

$\rho = \frac{VSWR-1}{VSWR+1} = 0.5$ **(1 mark)**

At the 1st voltage minima, we have

$$0.9\beta - \frac{\theta}{2} = \pi$$

i.e.

$\theta = 2\pi + 1.8\beta = 9.58$ **(1 mark)**

Therefore at 0.9cm from the load

$\Gamma_{(0.9cm)} = 0.5e^{j9.58}$ **(1 mark)**

The reflection coefficient at the load can be obtained as

$\Gamma_L = \Gamma_{(0.9cm)} e^{2j\beta\times 0.9} = 0.5e^{j12.28} = 0.48 - j0.14$ **(1 mark)**

The load impedance can be calculated as

$Z_L = Z_o \frac{1-\Gamma_L}{1+\Gamma_L} = 17 + j6.4$ **(1 mark)**

Q2.a

Impedance representation in the smith chart

$z = r + jx = \frac{1+\Gamma_{(d)}}{1-\Gamma_{(d)}}$ **(1 mark)**

represented as

$y = \frac{1}{z} = \frac{1-\Gamma_{(d)}}{1+\Gamma_{(d)}} = \frac{1+\Gamma_{(d)}e^{-j\pi}}{1-\Gamma_{(d)}e^{-j\pi}}$ **(1 mark)**

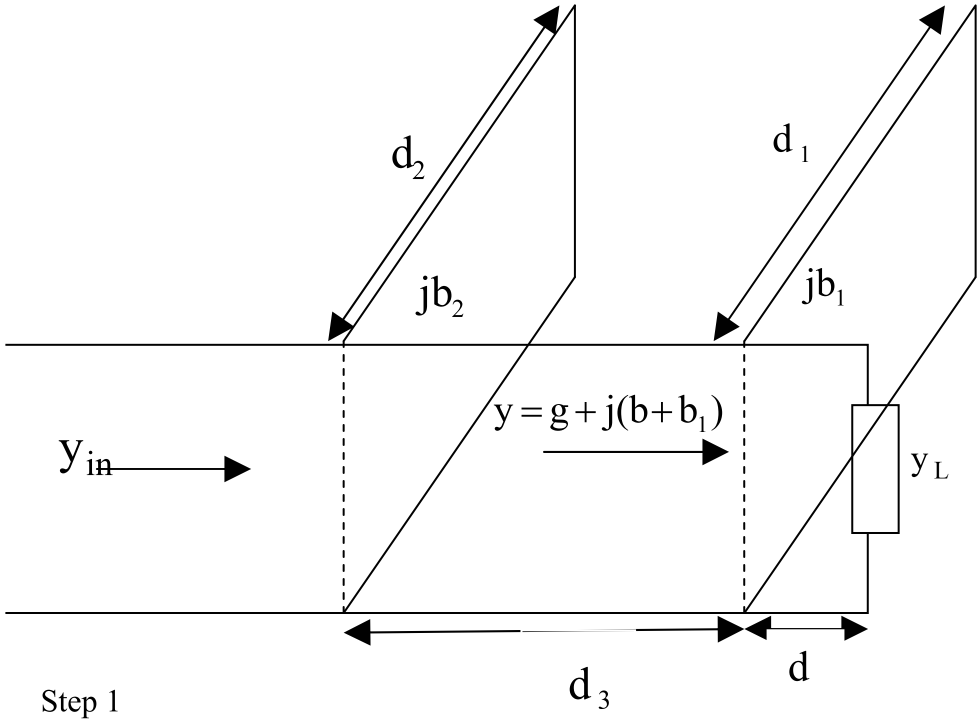
This shows that the Smith chart can be used to find transformation of admittance as well as impedance. This can be achieved be rotating an arc with a radius of $\Gamma_{(d)}$ through an angle of 180° ($\equiv 0.25\lambda$). **(2 marks)**

Q2.b

$z_L = \frac{(150 + j30)}{50} = 3 + j0.6$ (point A) **(1 mark)**

i.e.

y_L=0.31-j0.07 (point B) **(1 mark)**



Step 1

Rotate the unit g circle *Towards Load*, by a distance of d_3=0.125λ. **(1 mark)**

Step 2

Move from point B to intersect the new, rotated, unit circle at point C. The movement should be on the corresponding conductance circle, since the stub does not alter the real part of the admittance. **(1 mark)**

Step 3

The admittance at point C is

y_C=0.31+j0.27

compare it with that at B

y_L=0.31-j0.07

shows that stub 1 has provided j0.34, i.e. b_1=0.34. **(2 marks)**

Step 4

For an o.c. stub, this means d_1=0.052λ, i.e. the distance from D to E on the chart. **(1 mark)**

Step 5

Move a distance d_3=0.125λ along the line from the 1st stub position to the 2nd stub position (from point C to F). **(1 mark)**

Step 6

At point F, the admittance is y_F=1+j1.3, i.e. stub 2 must provide -j1.3 (b_2=-1.3) to reach the matched condition. **(1 mark)**

Step 7
For a s.c. stub, this means d_2=(0.354-0.25)λ=0.104λ, i.e. the distance from G to H on the chart. **(1 mark)**

Q2.c

$$z_L = \frac{(150 + j30)}{50} = 3 + j0.6$$ (point A) **(1 mark)**

Moving a distance of 0.1λ towards generator take us to point A′ on the Smith chart
z_L=0.9-j1.14 (point A′) **(1 mark)**

i.e.
y_L=0.42+j0.55 (point B) **(1 mark)**

Step 1
Rotate the unit g circle *Towards Load*, by a distance of d_3=0.125λ. **(1 mark)**

Step 2
Move from point B to intersect the new, rotated, unit circle at point C. The movement should be on the corresponding conductance circle, since the stub does not alter the real part of the admittance. **(1 mark)**

Step 3
The admittance at point C is
y_C=0.42+j1.85
compare it with that at B
y_L=0.42+j0.55
shows that stub 1 has provided j1.3, i.e. b_1=1.3 **(1 mark)**

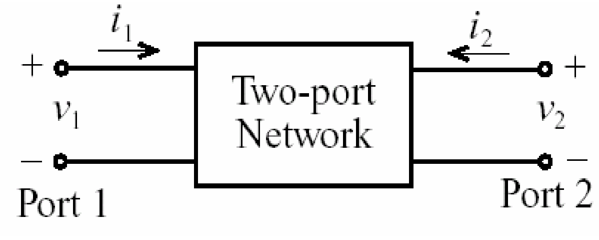
Step 4
For an o.c. stub, this means d_1=0.146λ, i.e. the distance from D to E on the chart. **(1 mark)**

Step 5
Move a distance d_3=0.125λ along the line from the 1st stub position to the 2nd stub position (from point C to F). **(1 mark)**

Step 6
At point F, the admittance is y_F=1-j2.8, i.e. stub 2 must provide j2.8 (b_2=2.8) to reach the matched condition. **(1 mark)**

Step 7
For an o.c. stub, this means d_2=0.196λ, i.e. the distance from D to G on the chart. **(1 mark)**

Q3.a



(1 mark)

The transmission matrix, or the ABCD matrix, of a two-port circuit relates the output terminal voltage and current (V_2, I_2) to the input voltage and (V_1, I_1) current as

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$ **(1 mark)**

In matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$ **(1 mark)**

(V_1, I_1) and (V_2, I_2) are the actual voltages and currents (*not normalized*), and they are continuous at the boundaries of the two ports. This means the output of one two-port circuit can be considered as the input of another two-port circuit. This makes the ABCD representation useful when cascading two-port networks. **(1 mark)**

Q3.b

Available Power Gain

$$G_A = \frac{P_{avn}}{P_{avs}}$$ **(1 mark)**

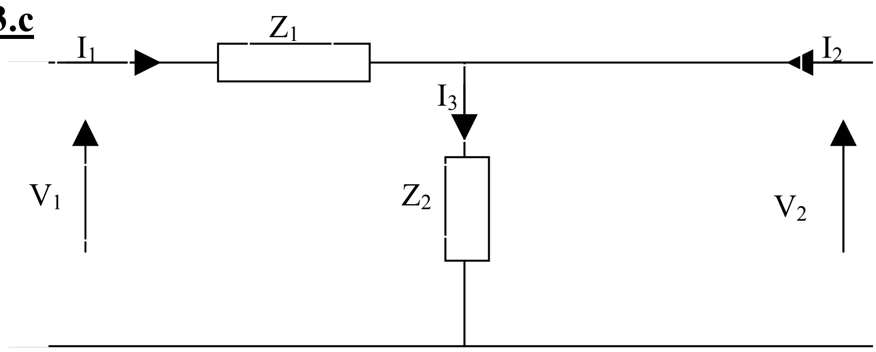
It is the ratio between power available from the network and the power available from the source. In most of the cases this gain is independent of Z_L. Gain of some active circuits is a function of Z_L. **(1 mark)**

Transducer Power Gain

$$G_T = \frac{P_L}{P_{avs}}$$ **(1 mark)**

It is the ratio between power available at the load and the power available from the source. This gain depends on both Z_g and Z_L. **(1 mark)**

Q3.c



$V_1 = I_1 Z_1 + I_1 Z_2 + I_2 Z_2$ **(1 mark)**

$\therefore \quad I_1 = \dfrac{V_1 - I_2 Z_2}{Z_1 + Z_2}$ **(1 mark)**

$V_2 = I_2 Z_2 + I_1 Z_2$ **(1 mark)**

i.e.

$V_2(Z_1 + Z_2) = I_2 Z_1 Z_2 + V_1 Z_2$

Hence

$V_1 = V_2(1 + \dfrac{Z_1}{Z_2}) - I_2 Z_1$ **(1 mark)**

$I_1 = \dfrac{V_2}{Z_2} - I_2$ **(1 mark)**

In matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{Z_1}{Z_2} & Z_1 \\ \dfrac{1}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

i.e.

$A = 1 + \dfrac{Z_1}{Z_2}, B = Z_1, C = \dfrac{1}{Z_2}, D = 1$ **(1 mark)**

Q3.d

Assume Z_1=82.2Ω and Z_2=14Ω. To calculate S_{11}, the impedance Z_{in1} is required when the network is terminated with Z_o,

$$Z_{in1} = \left[\frac{Z_2\left(Z_1 + \dfrac{Z_2 Z_o}{Z_2 + Z_o}\right)}{Z_1 + Z_2 + \dfrac{Z_2 Z_o}{Z_2 + Z_o}} \right]$$

$Z_{in1} = Z_2 \dfrac{Z_1 Z_o + Z_1 Z_2 + Z_2 Z_o}{Z_2 Z_o + (Z_2 + Z_o)(Z_2 + Z_1)} = 12.17\Omega$ **(1 mark)**

$S_{11} = \Gamma_{in} = \dfrac{Z_{in1} - Z_o}{Z_{in1} + Z_o} = 0.6$ **(1 mark)**

$S_{21} = \left.\dfrac{b_2}{a_1}\right|_{a_2 = 0}$

For the 1st port

$V_1 = \sqrt{Z_o}(a_1 + b_1)$

$V_2 = \sqrt{Z_o} b_2$

Therefore

$$\frac{V_1}{V_2}=\frac{(a_1+b_1)}{b_2}=\frac{(1+\frac{b_1}{a_1})}{\frac{b_2}{a_1}}=\frac{1+S_{11}}{S_{21}}$$ **(1 mark)**

i.e.

$$S_{21}=\frac{V_2}{V_1}(1+S_{11})$$

$$V_2=V_1\frac{Z_p}{(Z_p+Z_1)}$$ **(1 mark)**

where

$$Z_p=\frac{Z_oZ_2}{(Z_o+Z_2)}$$

Hence

$$\frac{V_2}{V_1}=\frac{Z_oZ_2}{(Z_oZ_2+Z_1Z_2+Z_oZ_1)}$$ **(1 mark)**

Therefore

$$S_{21}=\frac{2Z_oZ_2^2}{2Z_1Z_oZ_2+2Z_oZ_2^2+2Z_2Z_o^2+Z_1Z_2^2+Z_1Z_o^2}=0.046$$ **(1 mark)**

Similarly it can be shown that S_{12}=0.046 and S_{22}=0.6

Q4.a

(i) *Conditional Stability;* the network is potentially unstable and may oscillate for certain combinations of load and source impedance values. **(1 mark)**

(ii) *Unconditional Stability*; the network is unconditionally stable for any combinations of source and load impedance values. **(1 mark)**

Q4.b

Generally transistors presents a significant impedance mismatch, so matching will be achieved over a narrow frequency bandwidth. When bandwidth is an issue, then we design for a gain less than the maximum, imperfect matching, to improve bandwidth. **(1 mark)**

Sometimes it is required to design an amplifier with a specific gain, other than the maximum. *Constant gain circles* will be used to facilitate gain design. **(1 mark)**

Q4.c

The power gains can be calculated as follows:

First, find the source and load reflection coefficients

$$\Gamma_L=\frac{Z_L-Z_o}{Z_L+Z_o}=0.333$$ **(0.5 mark)**

$$\Gamma_s=\frac{Z_s-Z_o}{Z_s+Z_o}=0.2$$ **(0.5 mark)**

Next the input and output reflection coefficients are computed via

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.33 - j0.62$$ **(1 mark)**

$$\Gamma_{out} = S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.42 - j0.58$$ **(1 mark)**

Which can be used to calculate:

Transducer power gain

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2|1-\Gamma_s\Gamma_{in}|^2} = 6.47$$ **(1 mark)**

Available power gain

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{|1 - S_{11}\Gamma_s|^2\left(1-|\Gamma_{out}|^2\right)} = 11.6$$ **(1 mark)**

Operating power gain

$$G = \frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2\left(1-|\Gamma_{in}|^2\right)} = 15.4$$ **(1 mark)**

Q4.d

Since $S_{12} = 0.0$, $S_{11} < 1$ and $S_{22} < 1$, then the transistor is unconditionally stable.

(1 mark)

The matching sections gain can be calculated as

$$G_{S_{\max}} = \frac{1}{1-|S_{11}|^2} = 2.29 = 3.6dB$$ **(1 mark)**

$$G_{L_{\max}} = \frac{1}{1-|S_{22}|^2} = 1.59 = 1.9dB$$ **(1 mark)**

Since

$$G_o = |S_{21}|^2 = 6.25 = 8.0dB$$ **(1 mark)**

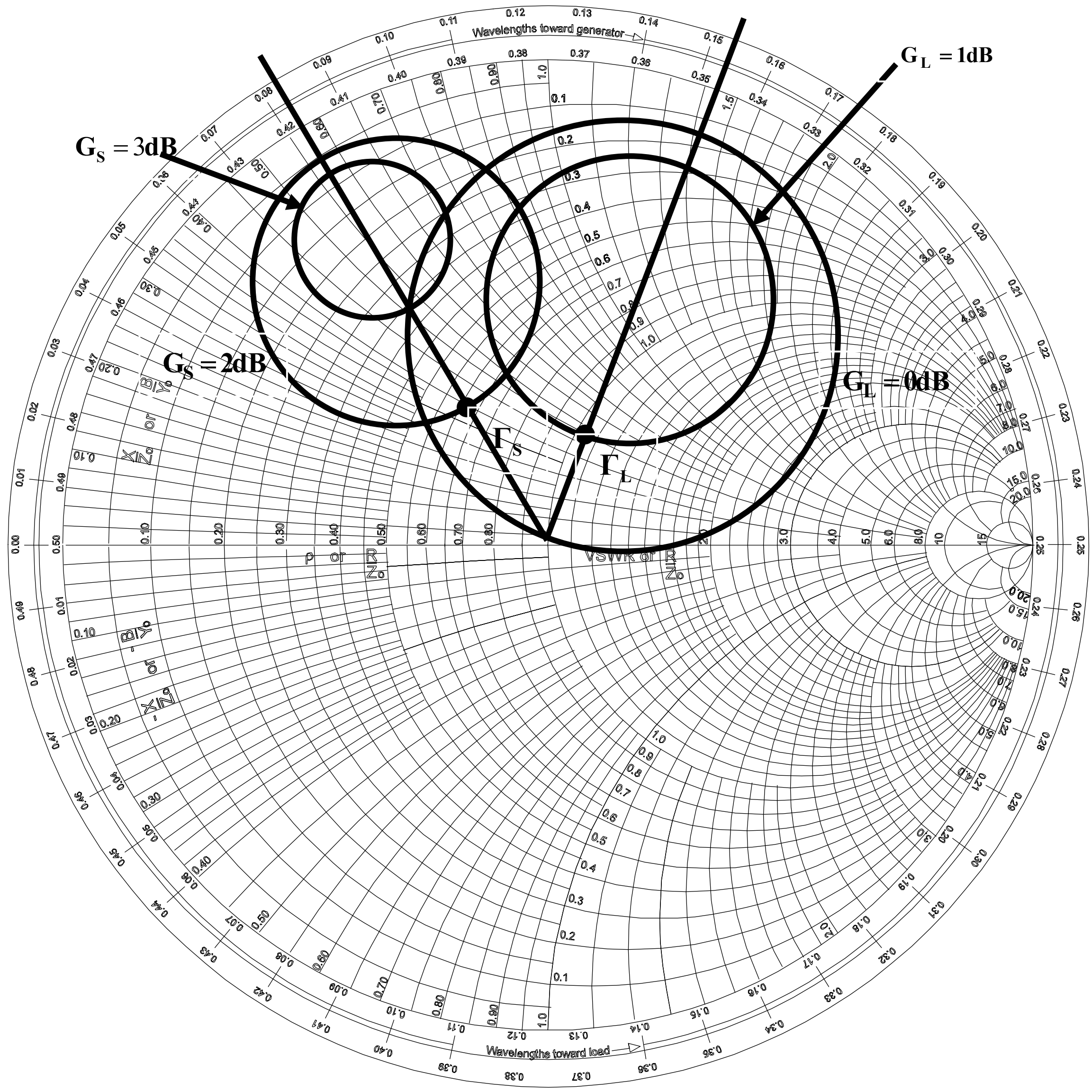
The overall gain

$$G_{T\max} = 3.6 + 8 + 1.9 = 13.5dB$$

This means there is a 2.5dB gain higher than the design requirements.

Equations (52), (53) and (54) may be used to get

G_s=3dB	g_s=0.88	$C_S = 0.706\angle 120°$	r_s=0.166
G_s=2dB	g_s=0.69	$C_S = 0.627\angle 120°$	r_s=0.294
G_L=1dB	g_L=0.81	$C_L = 0.52\angle 70°$	r_L=0.3
G_L=0dB	g_L=0.64	$C_L = 0.44\angle 70°$	r_L=0.44



(4 marks,
1 mark for plotting each circle)

The constant gain circles are plotted on the Smith chart as shown in the figure. For an overall gain of 11dB, we will choose G_s=2dB and G_L=1dB. We select Γ_S and Γ_L along these circles to minimise the distance from the centre of the chart. This will put Γ_S and Γ_L along the radial lines at 120° and 70° respectively. Thus Γ_S== $0.33\angle 120°$ and Γ_L== $0.22\angle 70°$. **(2 marks)**