The University Of Sheffield.

EEE331 Analogue Electronics

10th lecture:

- Active analogue filters
  - passive *LC* filters: only inductors & capacitors: difficult at low *f*
  - active *RC* filters: *R*, *C* & op-amps: thick or hybrid thin-film technol.
  - switched capacitor filters: fully IC-compliant

The University Of Sheffield.

EEE331 Analogue Electronics

2nd order filters: review of the standard from

All 2nd order filters are biquadratic transfer functions of the standard form

$\boxed{T(s) = \text{numerator}/\,(1+s\tau/q+s^2\tau^2)}$

where the **numerator** decides what type of filter we have:

1	= low pass
$s\tau$	= band pass
$s^2\tau^2$	= high pass
$1+s^2\tau^2$	= band stop or notch

and the **denominator** decides the

time constant: $\boxed{\tau=(LC)^{1/2}}$,

pole frequency: $\boxed{\omega_0=1/\tau=(LC)^{-1/2}}$ and

quality factor: $\boxed{q=1/(\omega_0 RC)=\tau/(RC)}$

The University Of Sheffield.

EEE331 Analogue Electronics

Insufficiencies of passive *LC*-filters: geometric problems due to *L*

Consider capacitor:

impedance: $X_C=1/(sC)$, where $s=j\omega$

current: $I=\mathrm{d}Q/\mathrm{d}t=\mathrm{d}(CV)/\mathrm{d}t=C\,\mathrm{d}V/\mathrm{d}t$ is linear to C times rate of voltage change

capacitance: $C=\varepsilon_0\varepsilon_r A/d$, scales linearly with area A and inversely with thickness d, hence is well-behaved and can be easily implemented in CMOS technology

Consider inductor:

impedance: $X_L=sL$

voltage: $V_{ind}=-L\,\mathrm{d}I/\mathrm{d}t$ is linear to L times the rate of current change

inductance: $L=\mu_0\mu_r N^2A/l$ scales quadratic with number of windings N, linearly with area A in cross-section and inversely with length l

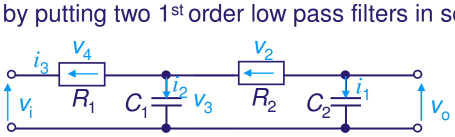
1. problem: **geometry needs to be implemented in 3D,which is a problem for CMOS and indeed any planar thin film layout**
2. problem: **at low frequencies L must be huge for reasonable X_L**

The University Of Sheffield.

EEE331 Analogue Electronics

Insufficiencies of passive *RC*-filters: problems due to limited *q*

Consider combination of pure *RC*-filters, e.g. 2nd order low pass obtained by putting two 1st order low pass filters in series (‘synthesis by factors’)



apply leap-frog ladder step-by-step (always easiest to work out from end!):

$v_o=i_1/(sC_2)$

$v_2= i_1R_2 =v_o sC_2R_2$

$v_3=v_o+v_2 = v_o(1+sC_2R_2)$

$i_2= v_3sC_1= v_osC_1(1+sC_2R_2)$

$i_3=i_1+i_2 = v_o[sC_2+sC_1(1+sC_2R_2)]$

$v_4=i_3R_1 = v_oR_1 [sC_2+sC_1(1+sC_2R_2)]$

$v_i=v_3+v_4 = v_o\{(1+sC_2R_2)+R_1[sC_2+sC_1(1+sC_2R_2)]\}$

$= v_o[1+s(R_2C_2+R_1C_2+R_1C_1)+s^2(R_1R_2C_1C_2)]$

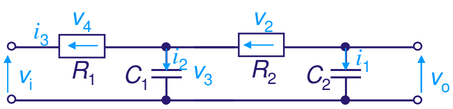
$\boxed{T(s)=v_o/v_i= 1/\{1+s[(R_1+R_2)C_2+R_1C_1]+s^2(R_1C_1R_2C_2)\}}$

The University Of Sheffield.

EEE331 Analogue Electronics

Insufficiencies of passive *RC*-filters: problems due to limited *q*

Consider combination of pure *RC*-filters, e.g. 2nd order low pass obtained by putting two 1st order low pass filters in series ('synthesis by factors')



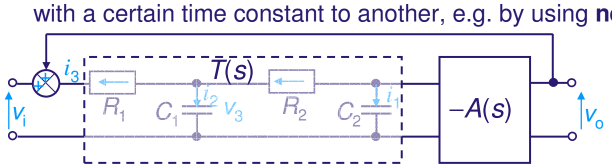
Now consider $R_1+R_2=R$ where $R_1=(1-b)R$, $R_2=bR$ with some constant $0\le b\le 1$:
$T(s)=1/\{1+s[RC_2+(1-b)RC_1]+s^2[b(1-b)R\,C_1\,RC_2]\}$
$\equiv 1/\{1+s(\tau_2+\tau_1/b)+s^2\,\tau_1\tau_2\}$
with $\tau_1=RC_1b(1-b)$, $\tau_2=RC_2$ and $\tau=(\tau_1\tau_2)^{1/2}$
Then this is of the standard form for $\omega_0=1/\tau=(\tau_1\tau_2)^{-1/2}$ and
$\boxed{q=(\tau_1\tau_2)^{1/2}/(\tau_1/b+\tau_2)=[C_1C_2(1-b)]^{1/2}/[C_1(1-b)+C_2]}$
Dividing both numerator and denominator of q by τ_2 and setting $x=\tau_1/\tau_2$ we obtain $q=\sqrt{x}/(x/b+1)$. Differentiating yields:
$dq/dx=[(x/b+1)/(2\sqrt{x})-\sqrt{x}/b][x/b+1]^{-2}=[(x/b+1)-2x/b]/[2\sqrt{x}(x/b+1)]^2=0$ if $x=b$.
As $d^2q/dx^2<0$ for $x=b$, this is a maximum, i.e.: $\boxed{q\le q^{max}=\tfrac12\sqrt{b}\le\tfrac12}$ **is very small.**

The University Of Sheffield.

EEE331 Analogue Electronics

Improving pure *RC*-filters: incorporating amplification+feedback

Most filters will demand $q>>1$ in at least some frequency bands. In order to enhance q, a system is needed where energy is transferred from one part with a certain time constant to another, e.g. by using **negative feedback**:



output voltage: $v_o=(v_i+v_o)\,T(s)\,[-A(s)]$
$v_o[1+T(s)A(s)]=-v_i\,T(s)A(s)$
$v_o/v_i=-T(s)A(s)/[1+T(s)A(s)]=-A(s)/[A(s)+1/T(s)]$
putting in the relationship $T(s)=1/\{1+s(\tau_2+\tau_1/b)+s^2\tau_1\tau_2\}$ then yields
$v_o/v_i=-A/[1+A]\times 1/[1+s(\tau_2+\tau_1/b)/(A+1)+s^2\tau_1\tau_2/(A+1)]$,
i.e. **both ω_0 and q have been multiplied by a common factor of $[A(s)+1]^{1/2}$:**
$\omega_0'=\omega_0[A(s)+1]^{1/2}$, $q'=q[A(s)+1]^{1/2}$. Differentiation and re-writing yields the $A(s)$ dependence as $\Delta\omega_0'/\omega_0'=\Delta q'/q'=\Delta A/[2(A+1)]$, which tells us how sensitive the new centre frequency and the quality factor are to changes in the gain.

EEE331 Analogue Electronics

two different possible technological solutions:

always:

- eliminate L
- adjust q by feedback

then:

1. - adjust/increase gain by restoring some of the energy dissipated by R by
   a) adding an op-amp to obtain an **active *RC*-filter** or
   b) using transistors with voltage controlled output currents as trans-conductance amplifiers (**g_m*C*-filters**)

or

2. replace R by two MOSFET switches and one C in a so-called **switched capacitor filter** (SCF, fully CMOS compliant)

EEE331 Analogue Electronics

Active filters: Sallen-Key filters with single amp.+ passive *RC* circuit

use again leap-frog ladder, starting at output:

$v_1=v_0/A$

$i_1=v_1sC_1=v_osC_1/A$

$v_2=i_1[Rb+1/(sC_1)]$

$\quad =v_osC_1/A\,[Rb+1/(sC_1)]$

And summing currents at v_2 node:

$v_osC_1/A=(v_i-v_2)/[R(1-b)]+(v_o-v_2)sC_2$

Inserting the above expression for v_2 yields:

$T(s)=v_o/v_i=A/\{1+s[\tau_1+(1-A)\tau_2/b]+s^2\tau_1\tau_2\}$

with time constants $\tau_1=C_1R$, $\tau_2=C_2Rb(1-b)$

This is again a 2nd order filter of general form

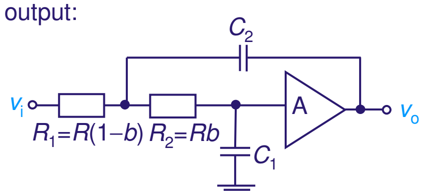
$1/[1+s\tau/q+s^2\tau^2]$ with

$\tau=(\tau_1\tau_2)^{1/2}$

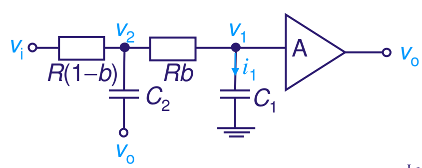
and now

$1/q=[\tau_1+(1-A)\tau_2/b](\tau_1\tau_2)^{-1/2}$

$\quad = (\tau_1/\tau_2)^{1/2}+(1-A)/b\,(\tau_2/\tau_1)^{1/2}$



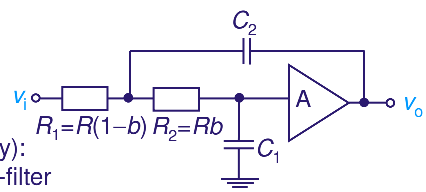
equiv. circuit for negligibly small output resistance of amp. A



EEE331 Analogue Electronics

Active filters: Sallen-Key filters with single amp.+ passive *RC* circuit

$1/q=[\tau_1+(1-A)\tau_2/b](\tau_1\tau_2)^{-1/2}$
$= (\tau_1/\tau_2)^{1/2}+(1-A)/b\,(\tau_2/\tau_1)^{1/2}$



Note for A=1:
$q=(\tau_2/\tau_1)^{1/2}=(C_2(1-b)b/C_1)^{1/2}$

Note for A=1, b=0.5, $C_1=C_2$ (symmetry):
q^{max}=½ as before for the buffered *RC*-filter
Note that changing C_2/C_1 ratio changes q without affecting τ, provided the product C_1C_2=const.

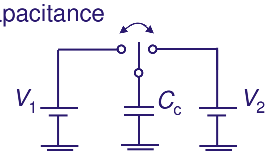
Example: A=1, b=0.5, $C_2=10C_1$:
q=√2.5≈1.58 is modest

q can become quite large if *A* and *b* are modified accordingly ($1/q\to0$), but then it depends sensitively on gain *A*!

EEE331 Analogue Electronics

Switched capacitor filters (SCF): principle

aim: get close tolerance high resistance on an IC without use of *R*.
principle: use MOSFETs as switches plus capacitance
Consider the following network:



switch to the left: $Q_c=C_cV_1$
switch to the right: $Q_c=C_cV_2$
Hence, charge $\Delta Q_c=C_c(V_1-V_2)$ must flow from V_1 to V_2, and when the switch returns to left, ΔQ_c must be replaced. Now assume the switch is moved back and forth (one cycle) f_s times per sec.
Then the current is: $i=\Delta Q_c/\Delta t=f_sC_c(V_1-V_2)$
Thus the effective resistance is: $\boxed{R_c=(V_1-V_2)/i=1/(f_sC_c),}$ i.e. the **combination of switch and capacitor behaves like a resistance,** with $\boxed{1/f_s=R_cC_c}$**, as long as the bandwidth is limited to <½f_s.**

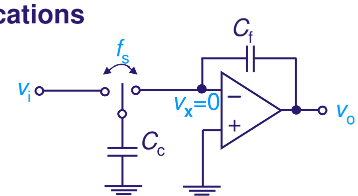
example: C=1pF, f_s=1MHz $\to R_c=1/(10^{-12}\text{F}\times10^6\ 1/\text{s})=10^6\Omega$=1MΩ
This is larger than what would be possible using discrete resistors. For a typical thermal gate oxide: d=100nm, $\varepsilon_r\approx4$. 1pF then requires an area of ~ $3\times10^{-9}\text{m}^2$ ≈50µm×50µm. This can be controlled to within ~1% and R_c only depends on f_s.

The University Of Sheffield.

EEE331 Analogue Electronics

Switched capacitor filters (SCF): applications

switched capacitor **(inverting) integrator**:



charge entering C_f per cycle: $\Delta Q_c=C_c(v_i-v_x)$
charge entering C_f per sec.: $\Delta Q_c f_s=C_c f_s(v_i-v_x)$
charge entering C_f in time δt: $\delta Q = \Delta Q_c f_s\,\delta t$
current: $i=dQ/dt=C_c f_s(v_i-v_x)$
where $v_x=0$ is a virtual earth
Hence, the output voltage is
$\boxed{v_o=-1/C_f \int \delta Q/\delta t\, dt = -1/C_f \int C_c f_s v_i\, dt = -f_s C_c/C_f \int v_i\, dt,}$

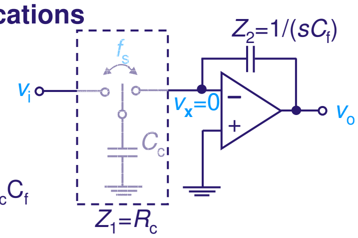
advantage:
The integrator gain is controlled by f_s and the ratio of MOS capacitors C_c/C_f. The latter is given by their areal ratios, which can be controlled highly accurately by the lithography.
disadvantages:
- MOS capacitors are leaky due to thermal electron-hole recombination at Si/SiO_2 interface and in the bulk (and some diffusion of minority carriers to the surface). This limits max. storage time for charge to the 1-2ms range.
- Input node must be low-pass filtered to prevent unwanted down-conversion (aliasing).

The University Of Sheffield.

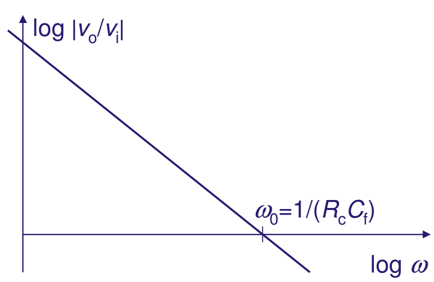
EEE331 Analogue Electronics

Switched capacitor filters (SCF): applications

switched capacitor **(inverting) integrator**:



general treatment of transfer function:
$T(s)=v_o/v_i=-Z_2/Z_1=-1/(sC_fR_c)=-1/(j\omega R_c C_f)$
is a **low pass filter** with time constant $1/\omega_0=R_cC_f$

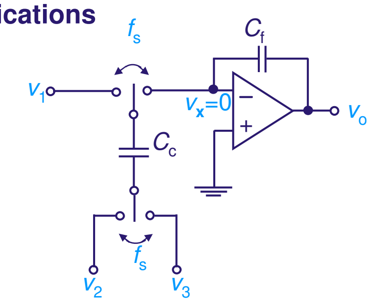


The University Of Sheffield.

EEE331 Analogue Electronics

Switched capacitor filters (SCF): applications

summing and subtracting integrator:



switches to the left: $Q_c=C_c(v_1-v_2)$
switches to the right: $Q_c=C_c(v_x-v_3)$
charge flow: $\Delta Q_c=C_c(v_1-v_2+v_3)$
charge entering C_f in time δt:

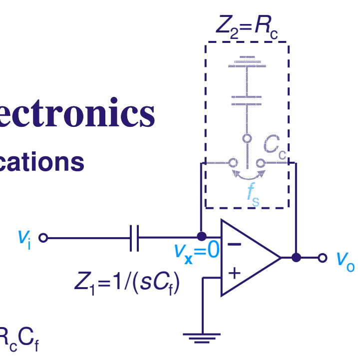
$$\delta Q=\Delta Q_c f_s\,\delta t$$
$$=C_c f_s(v_1-v_2+v_3)\delta t$$

Hence, the output voltage is
$\boxed{v_o=-1/C_f \int \delta Q/\delta t\,\mathrm{d}t=-f_s C_c/C_f \int(v_1-v_2+v_3)\,\mathrm{d}t,}$
NB 1: Input to v_2 with v_3=0 yields an inverting subtracting integrator.
NB 2: Input to v_3 with v_2=0 yields an inverting summing integrator.
NB 3: Input to v_2 with v_1+v_3=0 would yield a non-inverting integrator.

The University Of Sheffield.

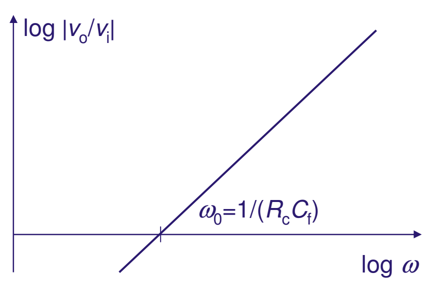
EEE331 Analogue Electronics

Switched capacitor filters (SCF): applications



switched capacitor **(inverting) differentiator**:

general treatment of transfer function:
$T(s)=v_o/v_i=-Z_2/Z_1=-R_c/[1/(sC_f)]=-j\omega R_c C_f$
is a **high pass filter** with time constant $1/\omega_0=R_cC_f$



Note:
current through C_f: $i=\mathrm{d}Q/\mathrm{d}t=C_f\mathrm{d}v_i/\mathrm{d}t$
This directly flows through the feedback resistor R_c to the output:
$\boxed{v_o(t)=-R_cC_f\,\mathrm{d}v_i/\mathrm{d}t}$
is a **differential signal**, i.e.
it suppresses slowly varying signals but amplifies high-f details.
Can also get **band pass filter** using switched capacitors $C_c||C_f$ at both input & output!

The University Of Sheffield.

EEE331 Analogue Electronics

Questionnaire on lecture series

Please tick all appropriate boxes and return anonymously after the last lecture. Thank you!

The lecture content was
- □ demanding but manageable
- □ far too difficult
- □ mostly easy
- □ all familiar

The presentations were
- □ of high standard, combining computer presentations and illustrations on the white board in about the right amount
- □ entertaining and/or interesting
- □ way too fast
- □ too general without enough practical examples
- □ too abstract and/or too mathematical

The lecture scripts were
- □ very good and understandable
- □ of acceptable standard
- □ too poor in quality for revision

The general atmosphere was
- □ relaxed; lecturer answered questions appropriately
- □ sometimes a bit tense; lecturer was not always well prepared
- □ rather stiff; lecturer did not interact enough with the students

Any other comment you may want to make: