Modelling of Machines

Section 9

3-phase induction machines

- Dominant machine type for industrial applications between 1kW and 20MW
- Widely used directly from 3-phase mains for essentially constant speed drives
- Increasing use with power electronic converters for variable speed operation

Typical construction of a small to medium (2-100kW) industrial induction motor

Large 20MW induction machine for Type45 naval propulsion

http://www.naval-technology.com/contractors/propulsion/alstom/

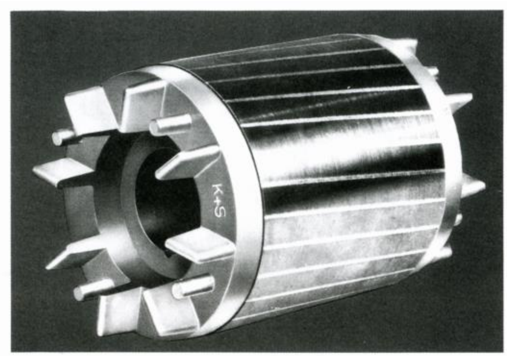
3-phase induction machines

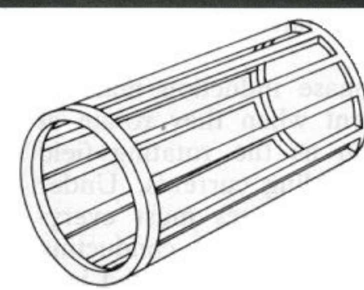
Stator:

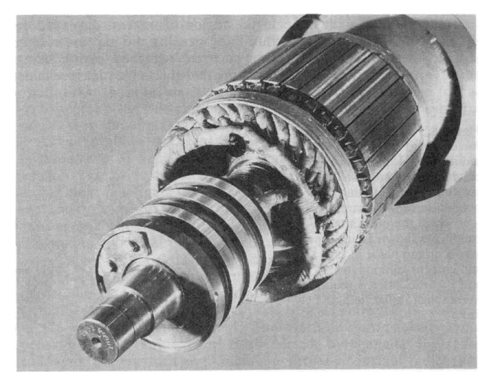
Laminated stator core carries 3-phase winding – when supplied from a constant-voltage constant-frequency 3-phase supply it produces a synchronously rotating, sinusoidally distributed magnetic field in the airgap

Rotor:

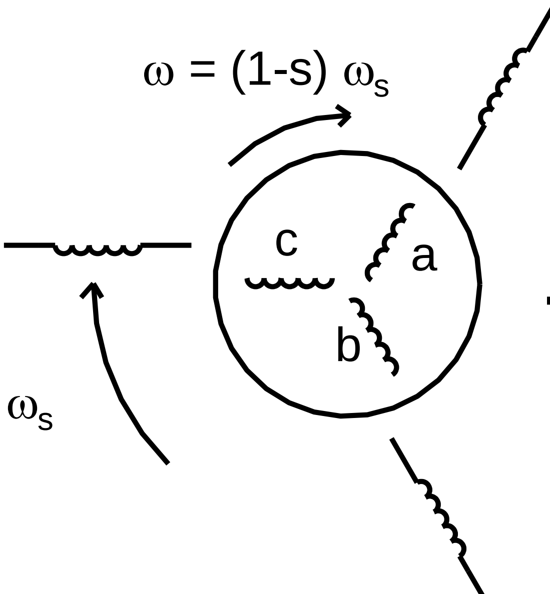
Essentially a short-circuited winding in most cases – can be cast ‘squirrel cage’ or wound rotor field in the airgap



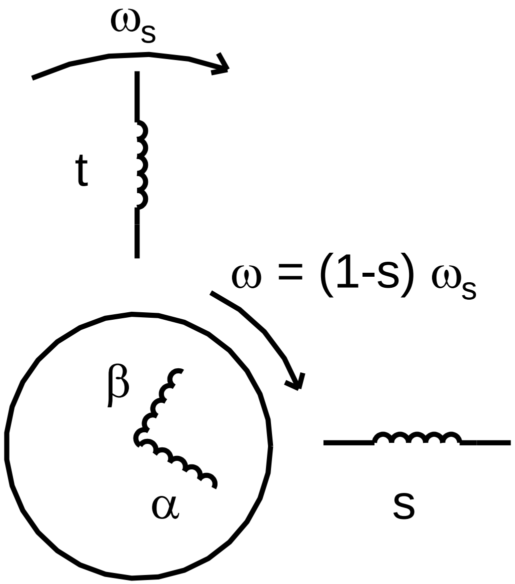




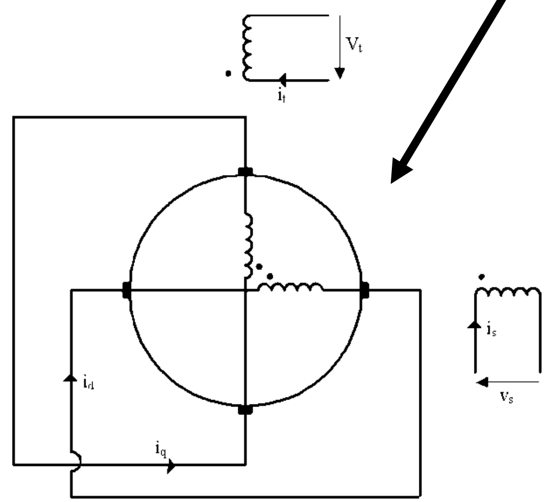
Process for transforming an induction machine to its Kron primitive equivalent:



3 to 2 phase transformation



Commutator transformation



Phase Transformation –

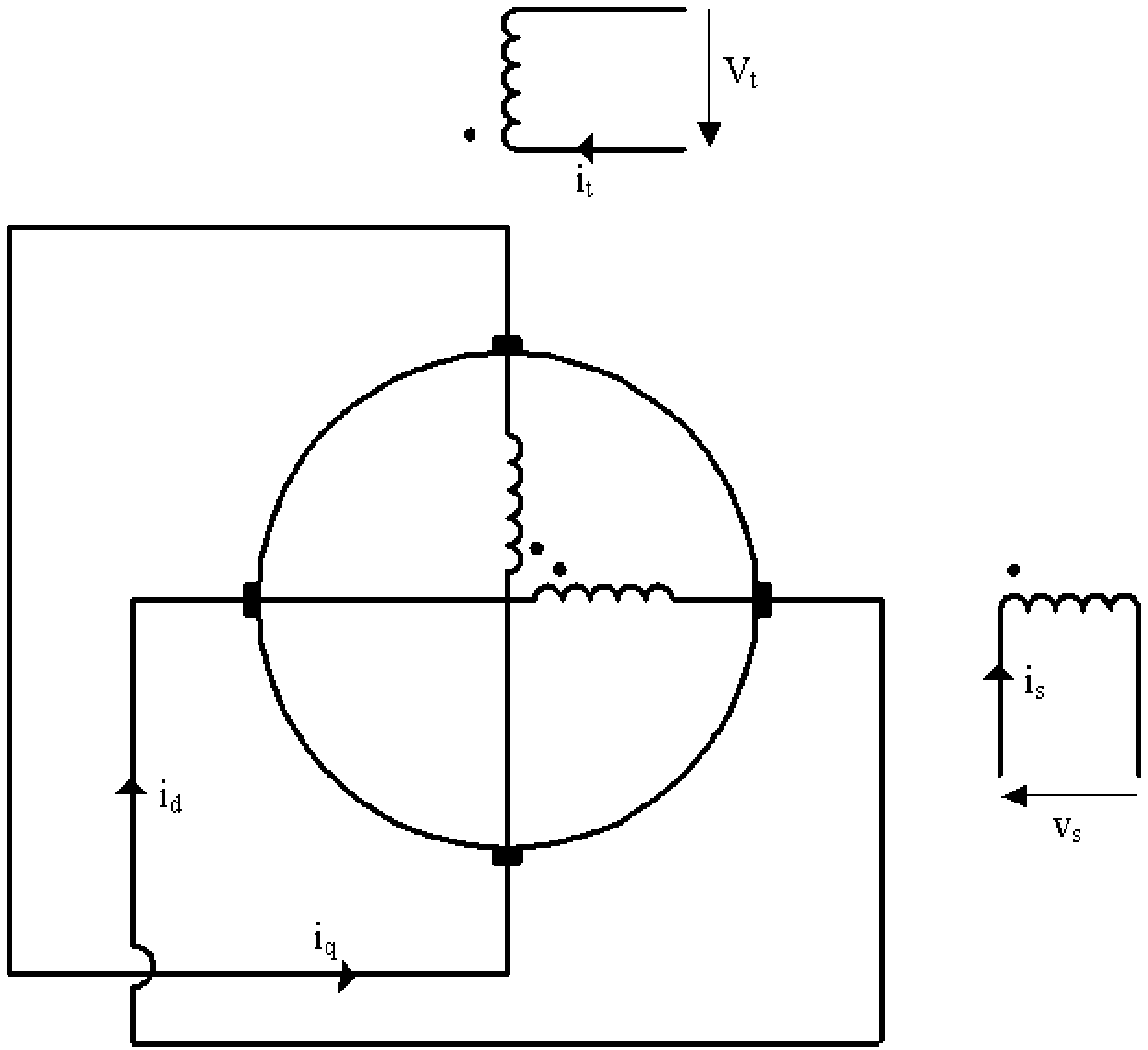
converts the two 3 phase systems of coils to 2 phase equivalents

Commutator transformation –

converts a 2 phase system of coils into a pseudo-stationary winding

Involves several transformation matrices and intermediate calculation steps

Kron primitive equivalent



Adopting subscripts of ‘1’ for the stator and ‘2’ for the rotor, then the general form of the voltage matrix equations is:

$$\begin{vmatrix} v_s \\ v_t \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_1+L_1p & 0 & M_{sd}\,p & 0 \\ 0 & R_1+L_1p & 0 & M_{td}\,p \\ M_{ds}p & -M_{dt}\omega_r & R_2+L_2p & -L_2\omega_r \\ M_{qs}\omega_r & M_{qt}\,p & L_2\omega_r & R_2+L_2p \end{vmatrix} \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix}$$

For steady-state operation for a sinusoidal AC supply:
$p=j\omega_s$ and $\omega_r=(1-s)\,\omega_s$

In addition, the same magnitude of applied to the two stator coils and the two rotor coils, but with a 90^o phase difference

$$\begin{vmatrix} V_s \\ V_t \\ V_d \\ V_q \end{vmatrix} = \begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix} = \begin{vmatrix} I_1 \\ j\,I_1 \\ I_2 \\ j\,I_2 \end{vmatrix}$$

The governing voltage equation is therefore:

$$\begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & 0 & jX_m & 0 \\ 0 & R_1+jX_1 & 0 & jX_m \\ jX_m & -(1-s)X_m & R_2+jX_2 & -(1-s)X_2 \\ (1-s)X_m & jX_m & (1-s)X_2 & R_2+jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ jI_1 \\ I_2 \\ jI_2 \end{vmatrix}$$

But row 2 is simply row 1 $\times$ j and row 4 is simply row 3 $\times$ j. Hence the system can be reduced to two matrix equations:

$$
\begin{array}{l} \text{Stator} \\ \text{Rotor} \end{array}
\quad
\begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m - j(1-s)X_m & R_2 + jX_2 - j(1-s)X_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix}
$$

Since the rotor is short circuited, $V_2 = 0$

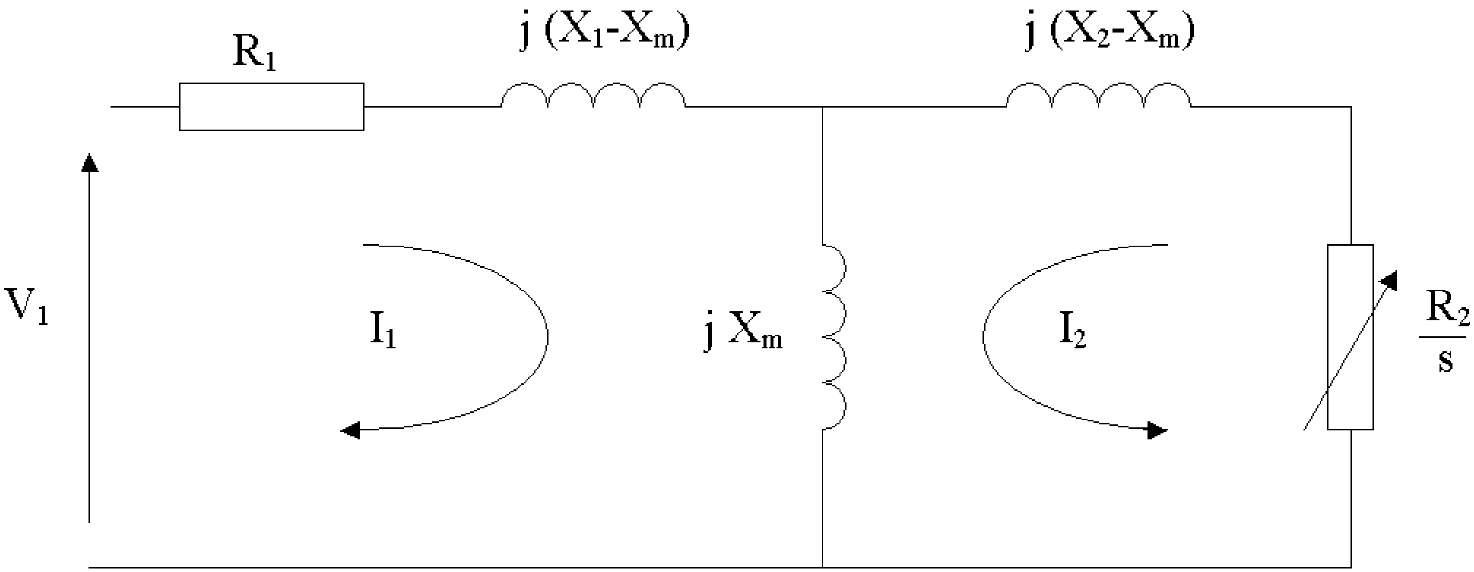
Substituting for V_2 and dividing the rotor equations by s gives:

$$
\begin{vmatrix} V_1 \\ 0 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m & R_2/s + jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2/s \end{vmatrix}
$$

If $I_2`$ (transformed value) = I_2/s then:

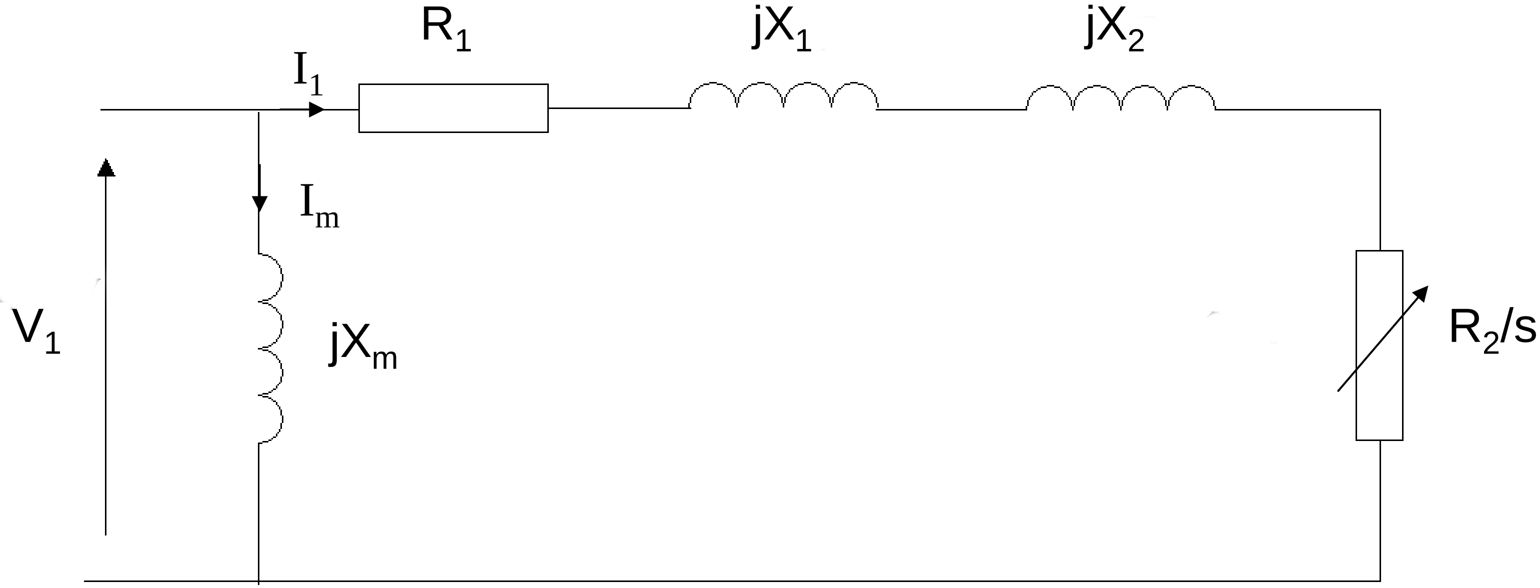
$$
\begin{vmatrix} V_1 \\ 0 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m & R_2/s + jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2` \end{vmatrix}
$$

An equivalent circuit that satisfies these voltage equations is:



(which is the well known per-phase equivalent circuit for three-phase induction machines)

This equivalent can be used to predict machine performance. However, in many cases, the equivalent circuit can be simplified by moving the magnetising branch (represented by X_m) to the terminals. This simplification is reliant on Xm being >> X1 and X2.
[In an exam, you need to justify this rather than just strarting with the simplified equivalent circuit]



This is the per phase equivalent circuit – hence V_1 is a phase voltage

This equivalent circuit can be used to predit several aspects of performance

Copper loss is given by: $P_{cu} = 3|I_1|^2 (R_1 + R_2)$

Electromagnetic output power is given by: $P_{out} = 3\ |I_1|^2\ \dfrac{(1-s)R_2^{'}}{s}$

Input power = P_{cu} + P_{out}

Input current = I_1 + I_m

Several good examples in past paper solutions, e.g. Q2 in 2006

Iron loss in induction machines

- One of the underlying assumptions in Universal machine theory is that iron losses are not included in the analysis
- An approximation to account iron loss can be added into the equivalent circuit
- Use the same approach as that which is widely used for transformers, i.e. add in an additional resistance across the terminals which dissipates a power equivalent to the core loss

Value of Rm is given by:

$$R_m = \frac{(Rms\ phase\ voltage)^2}{Core\ loss\ per\ phase}$$