Tutorial 5

1. Prove mathematically that convolution is
(i) a commutative operation, i.e, $x(t)*h(t) = h(t)*x(t)$.
(ii) an associative operation, i.e, $(x(t)*h(t))*g(t) = x(t)*(h(t)*g(t))$.
(iii) a distributive operation, i.e, $x(t)*(h(t) + g(t)) = x(t)*h(t) + x(t)*g(t)$.

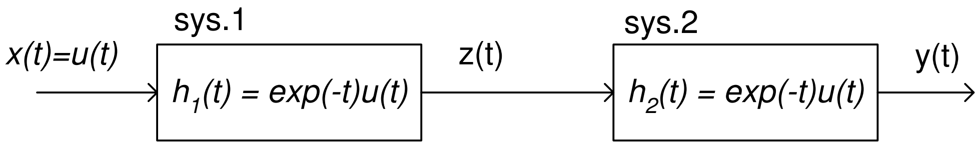
2. An RC high-pass circuit has a step response $g(t)=u(t)\exp(-t/RC)$. Sketch and derive an equation for the impulse response

3. A system has an impulse response $h(t)=\exp(-t)u(t)$. Find the step response of this system.

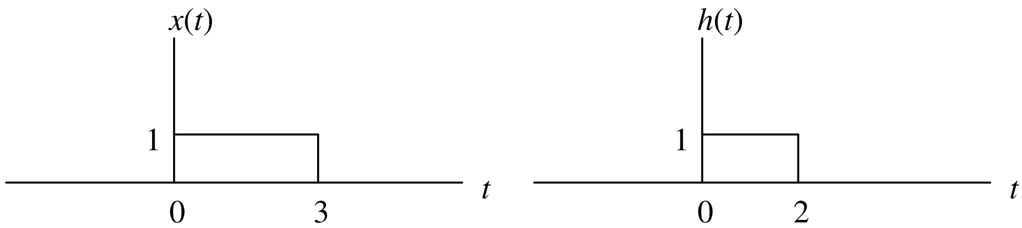
4. Compute and sketch $y[n]=x[n]*z[n]$ where:
$x[n] = 1,-1,2$ for $n = 0,1,2$
$z[n] = 1,2,3,-1$ for $n = -1,0,1,2$
assume that each signal is zero elsewhere

5. The impulse response of a system is given by $h[n]=-\delta[n-1] + \delta[n]$. By considering the input signal $x[n] = u[n-7]$, show that the system acts as an edge detector.

6. Find the output $y(t)$ for the system shown below when a unit-step input, $u(t)$ is applied.



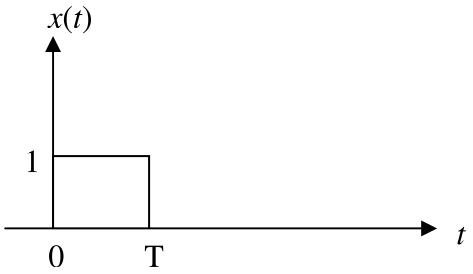
7. Consider the signals $x(t)$ and $h(t)$ shown below. Compute $y(t) = x(t)*h(t)$ using (i) the graphical method (ii) the analytical method and write down the analytical expressions for $y(t)$.



8. Consider a signal $y[n] = 3x[n] + x[n-2]$. Obtain the impulse response and evaluate the response of the system to an input

$$x_1[n] = \begin{cases} 1 & n=0 \\ 1 & n=1 \\ 2 & n=2 \\ 0 & otherwise \end{cases}.$$

9. The impulse response of the RC circuit shown below is given by $h(t) = \frac{1}{RC} e^{-t/RC} u(t)$. Derive the expression for the response of the circuit to the signal $p(t)$ shown below. Sketch and label the response signal.



10. Consider an LTI digital communication system, in which a bit "1" is represented by $p(t)$ in Q.13 and a bit "0" is represented by $-p(t)$. Evaluate the response of the circuit for a sequence "110" for cases where $T = 1/RC$ and $T = 1/(5RC)$. Hence comment how the intersymbol interference (ISI) of this digital communication system is affected by T.
[You may assume $T = 1$s]