Data Provided: Properties of Fourier Transform and List of useful formulae

The University of Sheffield

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2005-2006 (2 hours)

Signals and Systems 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. i) Describe the characteristics of a linear system and a time invariant system. **(3)**

ii) The response of a linear time invariant system, $y(t)$, is shown in figure Q.1.1 when subjected to an input signal $x(t)$. Derive the response of this linear time invariant system to the input signal $x_1(t)$ shown in figure Q.1.2.

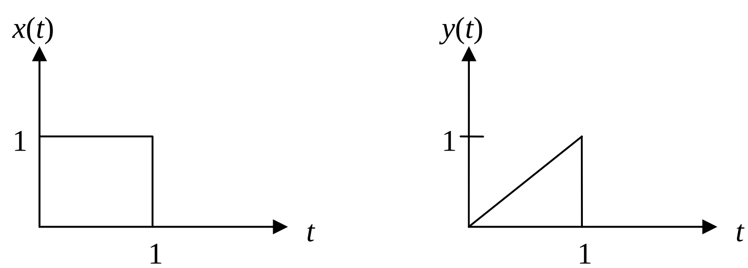

Figure Q.1.1

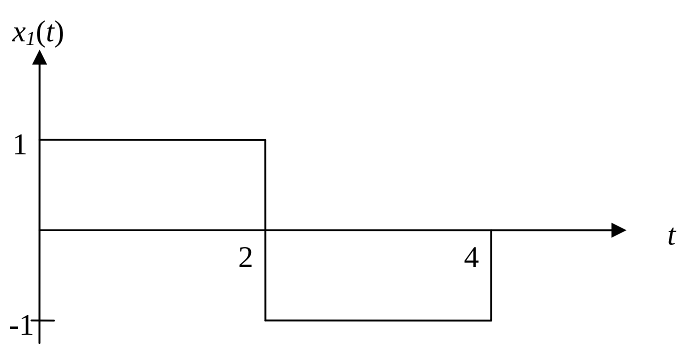

Figure Q.1.2 **(5)**

b. The impulse response of a linear time-invariant system, $h(t)$ is shown in figure Q.1.3. Use the graphical convolution technique to compute the response of the system, $y(t)$, when subjected to the input $x(t)$ shown in figure Q.1.3. Sketch and label $y(t)$.

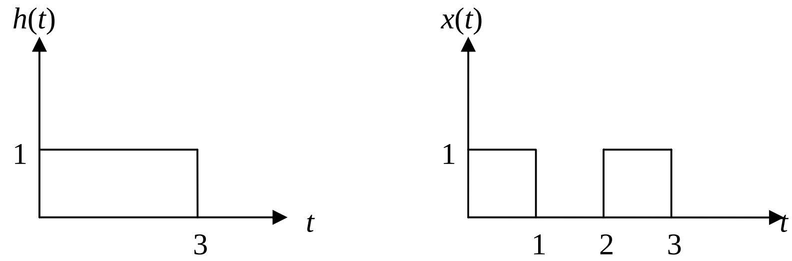

Figure Q.1.3 **(12)**

2. **a.**

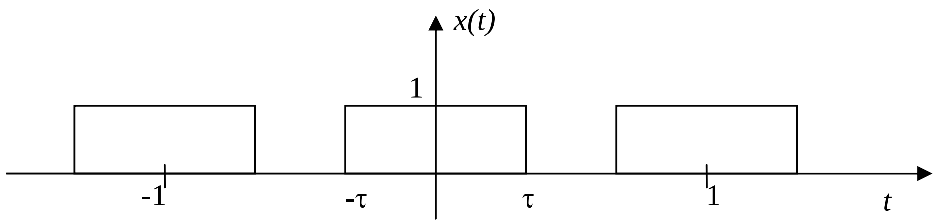

Figure Q.2.1

Show that the trigonometric Fourier series representation of the signal $x(t)$ shown in figure Q.2.1 is given by $x(t) = 2\tau + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin(2n\pi\tau)\cos(2n\pi t)$. **(6)**

b. Write down an expression for a signal $v(t)$ by computing the first three nonzero harmonics of the expression in part (a), when $\tau = 1/4$. **(2)**

c. The signal $v(t)$ from part (b) is applied to a low pass filter with a transfer function $H(\omega) = \frac{1}{1 + j\omega/\omega_c}$, where ω_c is the cut off frequency of the filter. Obtain an expression for $v(t)$ after low pass filtering if $\omega_c = \pi$ rad/s. **(12)**

3. **a.** **i)** From first principles show that the Fourier Transform $W(\omega)$ of the rectangular pulse $w(t)$ shown in figure Q.3.1 is given by $W(\omega) = \tau \dfrac{\sin(\omega\tau/2)}{(\omega\tau/2)}$.

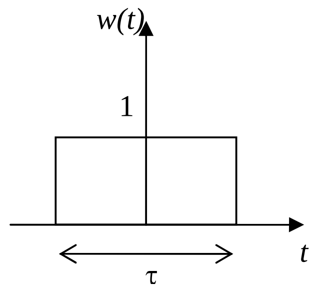

Figure Q.3.1 **(5)**

ii) Identify the peak amplitude of the spectrum $W(\omega)$ and the frequencies where the harmonics have zero amplitude. **(5)**

b. **i)** Sketch and label a signal $g(t) = \dfrac{d}{dt}\left(m(t)\right)$, where $m(t)$ is shown in figure Q.3.2.

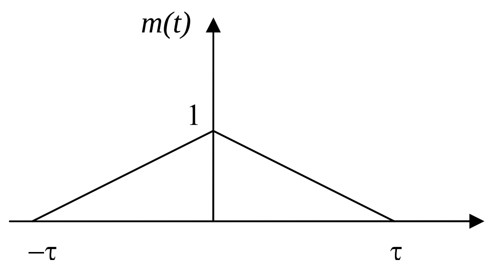

Figure Q.3.2 **(2)**

ii) Derive the Fourier Transform of $m(t)$ using the information in part a(i) and your answer in part b(i) together with the linearity, time shift and integration properties of the Fourier Transform. **(8)**

4. **a.** Use the transform impedance to prove that the transfer function of the RLC circuit shown in figure Q.4.1, assuming zero initial conditions, is given by $H(s) = \dfrac{1/LC}{s^2 + sR/L + 1/LC}$.

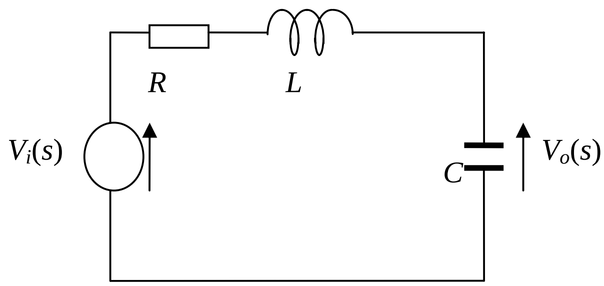

Figure Q.4.1 **(4)**

b. Obtain the damping factor, ζ, and the natural oscillating frequency, ω_n, of the circuit shown in figure Q.4.1 if R = 25Ω, L = 10mH and C = 100μF. **(6)**

c. Obtain the poles for the circuit above. Sketch and label the pole-zero plot. **(5)**

d. Describe and sketch the unit step response of the circuit above. **(5)**

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