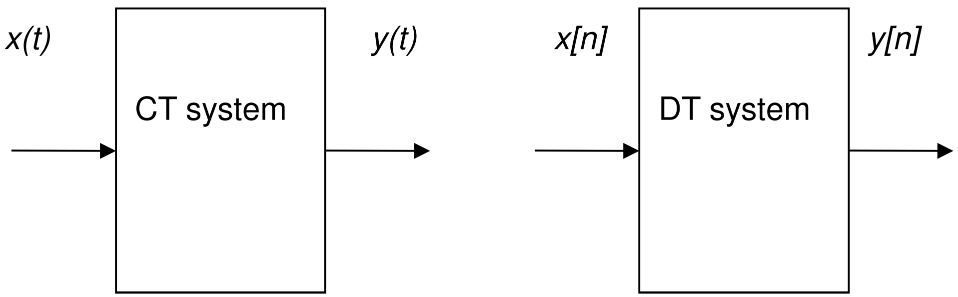
Lecture content

- Definition of system
- Basic system properties
  - Memory
  - Causality
  - Stability
  - Linearity
  - Time Invariance

Systems

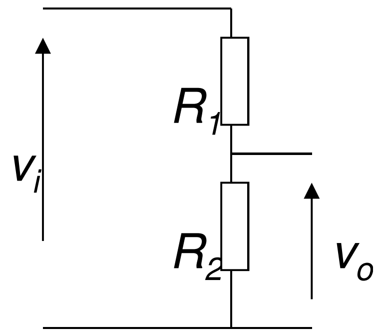
A system can be thought of as a process of transforming an input signal from one form to another as an output signal. Examples: Hifi speakers, amplifiers and filters.

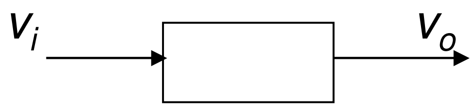
We will treat a system as a BLACK BOX with at least one input and one output (SISO).



Systems

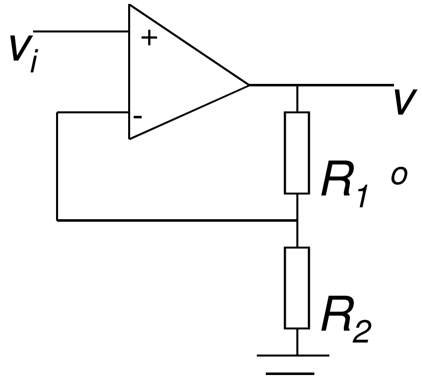
Consider a potential divider depicted below. The output v_o is given by $v_0 = \dfrac{R_2 v_i}{R_1 + R_2}$.

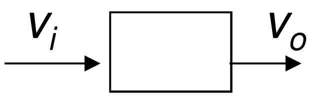




$$\frac{R_2}{R_1 + R_2}$$

An op-amp can be used to form a scaling system

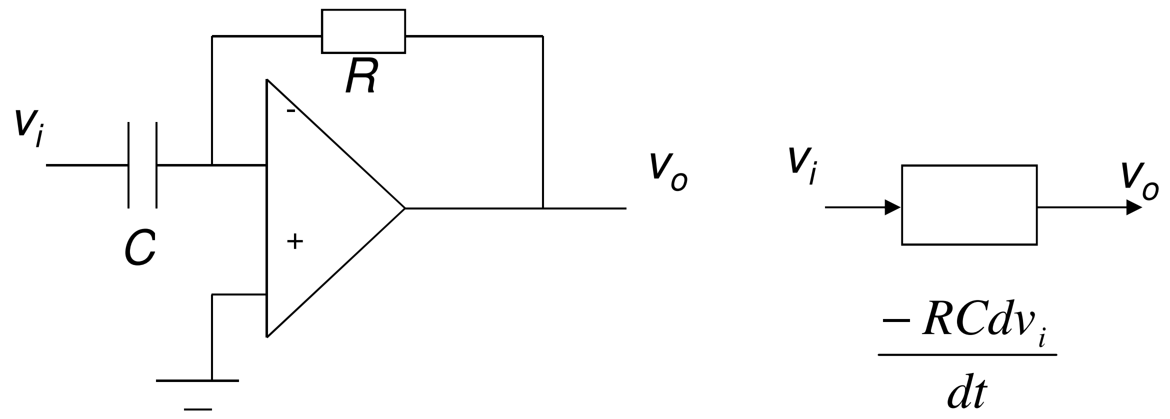




$$1 + \frac{R_1}{R_2}$$

Systems

An op-amp can also be used to form a differentiation system.



We can use mathematical equations of systems to describe the relationships between input and output signals.

Basic system properties: Memory

A system is said to be ***memoryless*** if its output $y(t_o)$ **depends** only **on the input** $x(t)$, applied **at $t = t_o$**. $y(t_o)$ is independent of the input applied before and after $t = t_o$.

$y[n] = x[n] - 3x[n]$ and $v_o(t) = \dfrac{R_2}{R_1 + R_2} v_i(t)$ are memoryless.

If the output value depends on past input or future input, the system is said to have ***memory***. Examples of system with memory are:

1) Unit time delay $y(t) = u(t-1)$.
2) Voltage across a capacitor $V_c(t) = \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} i(\tau)\,d\tau$
3) An accumulator output $y[n] = \displaystyle\sum_{k=-\infty}^{n} p[k]$

$y(t) = u(t+1)$ is also said to have memory because the output at time t depends on input at a future time $t+1$.

Basic system properties: Causality

A system is ***causal*** if its output at current time **depends only on past** and **current inputs** but **is independent of future input**.

For instance the integrator system is causal or ***non-anticipatory*** because $V_c(t)$ does not depend on future input.

The unit-time advance system is non-causal since its output $y(t)$ depends on future input $u(t+1)$. **In practice all memoryless systems are causal.**

Causality is not an important issue in applications in which time is not the variable such as in image processing. Here an algorithm is developed to make use of all the information surrounding a pixel to improve the quality of the image.

Basic system properties: Stability

A ***stable*** system is a system in which the **output does not diverge when the input to the system is bounded** (i.e if its magnitude does not grow indefinitely).

For example a system described by $y_1(t) = tx(t)$ is unstable.

When the input $x(t) = 1$ is bounded, $y_1(t) = t$ in unbounded.

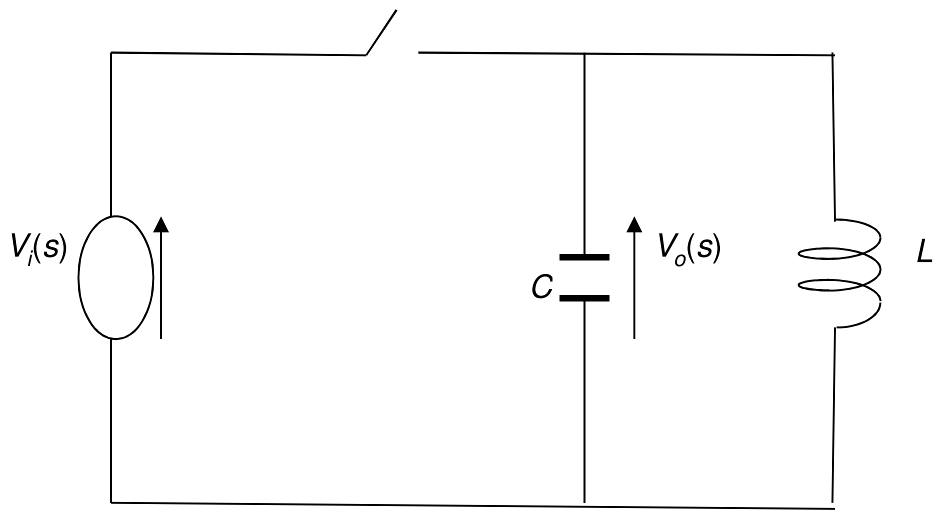
A system $y_2(t) = \cos(x(t))$ is stable since the output is bounded when the input $x(t)$ is bounded.

Unstable system

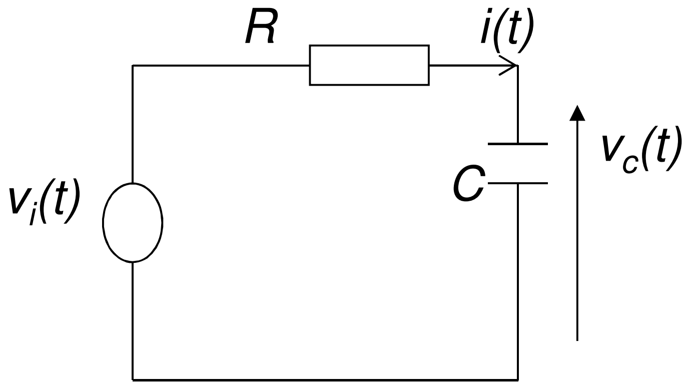
Photos of the Bridge collapsing
The following images and captions were taken from the report:
Smith, Doug,
A Case Study and Analysis of the Tacoma Narrows Bridge Failure
99.497 Engineering Project,
Department of Mechanical Engineering, Carleton University, Ottawa, Canada, March 29, 1974.
Supervised by Professor G. Kardos.

http://youtu.be/hBxQCvVykRE

Oscillator



Basic system properties: Initial state



$$i(t) = C\frac{dv_c(t)}{dt}$$

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$t = -\infty$ implies that the voltage depends on *i(t)* from the time the circuit was first built to the time *t*. It is virtually impossible to find the value *i(-∞)*. However if we know the ***initial state*** at $t = t_o$ or the value of $v_c(t_o)$ then we can evaluate the voltage as

$$v_c(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v_c(t_o)$$

Basic system properties: Linearity

A system is linear if

1) The response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$ (additivity property).

2) The response to $ax_1(t)$ is $ay_1(t)$ where a is a constant (homogeneity property).

These two properties can be combined into:

- **CT: $ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$,**
- **DT: $ax_1[t] + bx_2[t] \rightarrow ay_1[t] + by_2[t]$.**

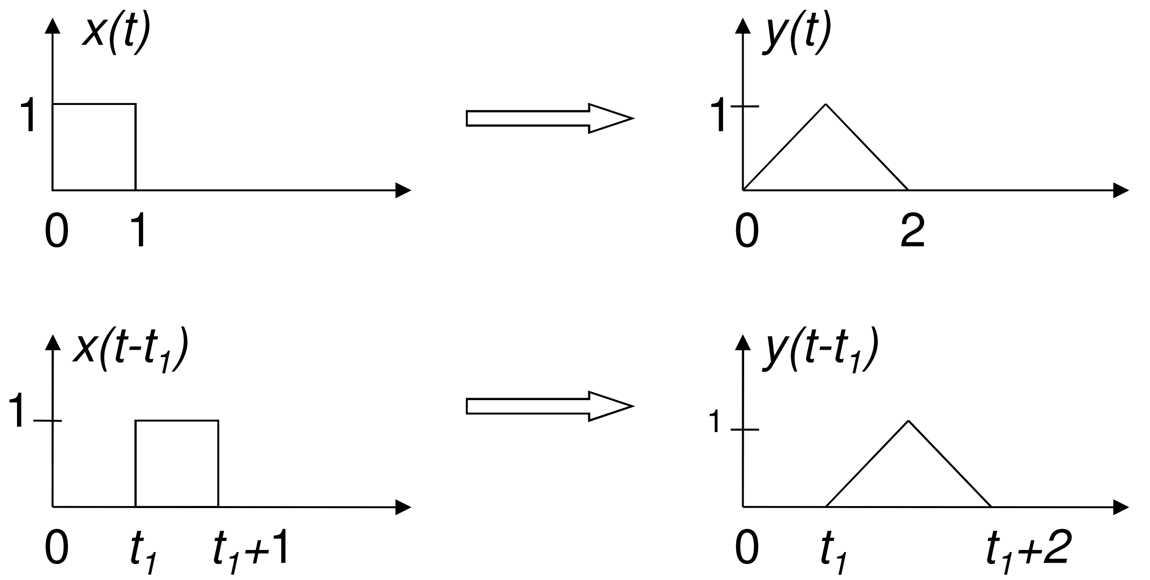
Linearity

1. Determine whether the system $y(t) = K\dfrac{dx(t)}{dt}$ is linear.
2. Show that the system $y(t) = e^{x(t)}$ is non-linear.
3. Determine whether $y(t) = 3x(t) + 4$ is a linear system.

Basic system properties: Time Invariant

If the characteristics of a system are independent of time it is said to be ***time invariant.*** The RC low pass circuit is an example of time invariant system since R and C are constant over time.

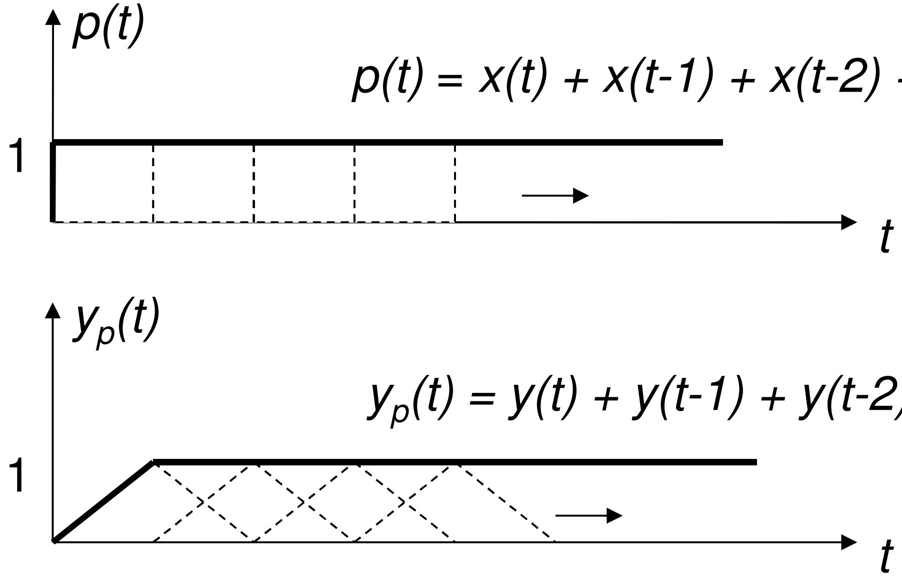
A time shift in the input signal will result in an identical shift in the output signal of a time invariant system.



Linear Time Invariant (LTI)

A system that is linear and time-invariant is therefore known as a ***Linear Time Invariant*** (LTI) system.

If we know the input-output ($x(t)$-$y(t)$) pair of an LTI system, then we can compute the response of the system to any signal that can be constructed from $x(t)$.



$$p(t) = x(t) + x(t-1) + x(t-2) + x(t-3) + \ldots\ldots\ldots$$

$$y_p(t) = y(t) + y(t-1) + y(t-2) + y(t-3) + \ldots\ldots\ldots.$$