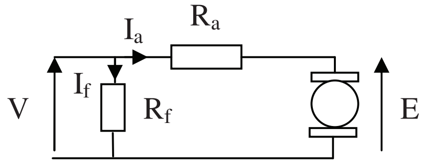
Tutorial Sheet 3 – DC Motors

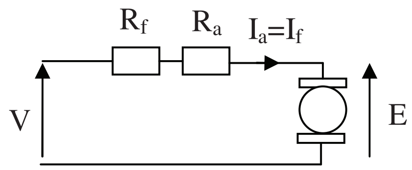
3.1 Terms

n_{max}: no load speed rpm
ω_{max}: no load speed rad s^{-1}
T_{max}: maximum (peak) torque
ψ_f: back EMF (V / rpm or V rad^{-1} s)
M: mutual inductance (flux linkage per amp)

Shunt motor



Series Motor



3.2 Equations

$$E = V - I_a R_a = \omega\psi_f = \omega M I_f$$

$$\psi_f = k_e = k_t$$

$$1\ rpm = \frac{2\pi}{60} rad\ s^{-1}$$

$$T = \psi_f I_a = M I_f I_a = \frac{E I_a}{\omega} \quad ....with\ \psi_f\ in\, rad\ s^{-1}$$

$$series\, motor...\ T = M I_a^{\ 2}$$

$$\theta = \frac{N}{t}$$

$$J_{total} = J_1 + J_2 = J_1 + \left(\frac{1}{R^2}\right) J_2'(in\, kgm^2)$$

$$T = J\theta$$

$$Energy\, disipated, E = I^2 Rt$$

$$Power\, dissipated, P = \frac{\sum E}{t}$$

$$P_{in} = I^2 R = VI_{total}, \quad P_{out} = T\omega,\ P_{loss} = I_a^{\ 2} R + I_f V$$

$$Efficency, \phi = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}}$$

$$\theta = \int \omega . dt$$

Tutorial Sheet 3: DC Motor Solutions

Q1

i)
a) Maximum no load speed. At no load (no torque) $I_a = 0$

$$E = V - I_a R_a ....I_a = 0$$
$$E = V = 200V$$
$$\omega_{rpm} = \frac{E}{\psi_f} = \frac{200}{\frac{50}{1000}} = 4000rpm$$

b) Maximum torque at low speeds. "Low speed" implies maximum torque envelope not limited by "high speed" drop off. The motor is quoted in rpm and needs converting into rad/s. Use maximum current to calculate torque.

$$E = \psi_{f\,(rpm)}\omega_{rpm} = \frac{50}{1000}\omega_{rpm} = 0.05\omega_{rpm}$$
$$\omega_{rad} = \frac{2\pi\omega_{rpm}}{60}$$
$$\psi_{f\,(rad)} = \frac{E}{\omega_{rad}} = \frac{0.05\omega_{rpm}}{\frac{2\pi\omega_{rpm}}{60}} = \frac{60\times 0.05}{2\pi} = \frac{3}{2\pi}$$
$$T = \psi_{f\,(rad)} A = \frac{3}{2\pi}\times 10 = \frac{15}{\pi} = 4.8Nm$$

c) Now we need to calculate the high speed drop off.

$$E = V - I_a R_a = 200 - (10\times 3) = 170V$$
$$\psi_{f\,(rpm)} = \frac{50}{1000} V/rpm$$
$$\omega_{rpm\_\max} = \frac{E}{\psi_{f\,(rpm)}} = \frac{170\times 1000}{50} = 3400rpm$$

ii) Calculate I and V to produce 1500rpm at 2Nm. Watch out for rpm and rad/s again.

From pervious question:

$$\frac{E}{w_{rad}} = \psi_{f\,(rad)} = \frac{T}{I_a} = \frac{3}{2\pi}$$

Convert 1500rpm to rad/s:

$$1500rpm \rightarrow \omega_{rad} = \frac{2\pi \times 1500}{60} = 50\pi$$

Substitute:

$$\frac{E}{50\pi} = \frac{2}{I_a} = \frac{3}{2\pi}$$

$$I_a = \frac{2\times 2\pi}{3} = 4.19A$$

$$E = \frac{3\times 50\pi}{2\pi} = 75V \text{..........} V = E + I_a R_a = 75 + (3\times 4.19) = 87.6V$$

Q2

a)
i) Load torque

$$E = V - I_a R_a = 500 - (2.4\times 4) = 490V$$

$$\omega_{rad} = \frac{2\pi}{60}\times 750 = (25\pi) rad/s$$

$$I_f M = \frac{E}{\omega_{rad}}$$

$$T = I_a I_f M = 4\times \frac{490}{25\pi} = 25Nm$$

ii) Efficiency

$$P_{out} = \omega_{rad} \times T = 25\pi \times 25 = 1.963kW \text{.....} (625\pi)kW$$

$$P_{in} = VI = V(I_a + I_f) = 500 * (4 + 0.33) = 2.165kW$$

$$Efficiency, \eta = \frac{P_o}{P_i} = \frac{625\pi}{2.165} = 90.7\%$$

b) New armature current

Original M:

$$T = I_a I_f M \rightarrow M = \frac{T}{I_a I_f} = \frac{25}{4 \times 0.33} = 18.94$$

I_f = 120mA, calculate I_a

$$I_a = \frac{T}{MI_f} = \frac{25}{18.94 \times 0.12} = 11A$$

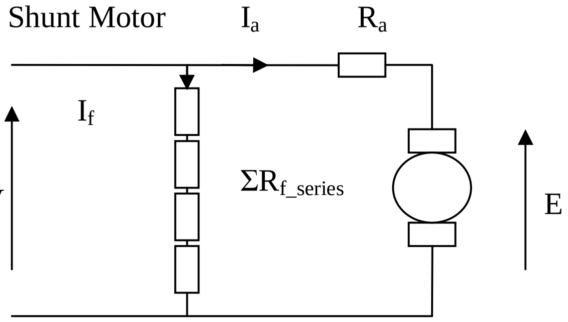
New speed:

$$E = V - I_a R_a = 500 - (11 \times 2.5) = 472.5V$$

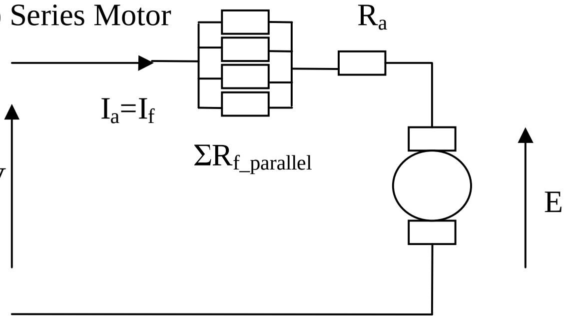
$$\omega_{rad} = \frac{E}{I_f M} = \frac{472.5}{0.12 \times 18.94} = 207.8933 rad/s = 1985 rpm$$

Q3

3a) Shunt Motor



3b) Series Motor



a) Shunt Motor
Load torque:

$$E = V - I_a R_a = 100 - (20 \times .25) = 95V$$

$$750 rpm = 25\pi \, rad/s$$

$$I_f M = \frac{E}{\omega_{rad}} \rightarrow M = \frac{E}{\omega_{rad} I_f} = \frac{95}{25\pi \times 5} = \frac{0.76}{\pi}$$

$$T = I_a I_f M = 20 \times 5 \times \frac{0.76}{\pi} = 24.2Nm$$

b) 1-field series motor ($I_f = I_a$), n-field series motor ($I_f = I_a/n$)
Each field 'sees' I_a/n when it is split n ways
Same T, Same M, $I_f = I_a/4$,
Calculate I_a *and* R_{total}....

$$T = I_a I_f M = I_a \frac{I_a}{4} M \rightarrow I_a = \sqrt{\frac{4T}{M}} = \sqrt{\frac{4\pi(24.2)}{0.76}} = 20A$$

$$R_{total} = R_a + \sum R_{f\_parallel} .... from\, Q3a: R_a = 0.25\Omega$$

$$from\ part\ (a)...\ field ...V = 100V, I = 5A$$

$$\sum R_{f\_series} = \frac{V}{I} = \frac{100}{5} = 20\Omega .... R_f = \frac{20}{4} = 5\Omega \rightarrow \sum R_{f\_parallel} = \frac{1}{4/5} = 1.25\Omega$$

$$E = V - I_a R_{total} = 125 - (20 \times (0.25 + 1.25) = 95V$$

$$E = \omega_{rad} I_f M = \frac{\omega_{rad} I_a M}{4} \rightarrow \omega_{rad} = \frac{4E}{I_a M} = \frac{4 \times 95\pi}{20 \times 0.76} = 25\pi \, rad/s = 750 rpm$$