Solutions

Q1.a

For a lossless line

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)}$$ **(1 mark)**

Since $\ell = \frac{\lambda}{4}$ and $\beta = \frac{2\pi}{\lambda}$, then

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\frac{\pi}{2})}{Z_o + jZ_L \tan(\frac{\pi}{2})}$$ **(1 mark)**

which means

$$Z_{in} = \frac{Z_o^2}{Z_L}$$ **(1 mark)**

Re-arrange the above equation to get characteristic impedance of a lossless 0.25λ line as

$$Z_o = \sqrt{Z_{in} Z_L}$$ **(1 mark)**

Q1.b

The charectrestic impedance of a lossless transmission line is given by

$$Z_o = \sqrt{\frac{L}{C}}$$ **(1 mark)**

Therefore the inductance can be calculated as

$$L = Z_o^2 C = 2500 \times 67 \times 10^{-12} = 0.17 \mu H/m$$ **(1 mark)**

and the phase constant is

$$\beta = \omega\sqrt{LC} = \frac{\omega}{v_p}$$ **(1 mark)**

which means

$$v_p = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{\left(67 \times 0.17 \times 10^{-18}\right)}} = 0.88 \times 10^8 \text{ m/s}$$ **(2 marks)**

Q1.c

$$\beta\ell = \frac{2\pi}{\lambda} \times 0.1\lambda = 0.628$$ **(1 mark)**

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)} = 86.4 + j40\Omega$$ **(1 mark)**

$$\Gamma_{load} = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(50 + j20) - 75}{(50 + j20) + 75} = 0.17 + j0.19$$ **(1 mark)**

$$\Gamma_{(d=0.05\lambda)} = \Gamma_{load} e^{-2j\beta j} = -0.03 + j0.25$$ **(1 mark)**

$$\mathrm{VSWR} = \frac{1+|\Gamma_{\mathrm{load}}|}{1-|\Gamma_{\mathrm{load}}|} = 1.68$$ **(1 mark)**

Q1.d

For a lossless line

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)}$$

When the line is terminated by a load impednce of Z_L=0, i.e. short circuited line, the input impednace is given by

$$Z_{inSC} = jZ_o \tan(\beta\ell)$$ **(1 mark)**

which is a reactive impedance that can be made equivalent to a lumped capacitor or indictor by adjusting the length ℓ. An inductive reactance can be obtained using $0 \le \ell \le 0.25\lambda$, while for $0.25\lambda \le \ell \le 0.5\lambda$ a capacitive reactance can be achieved. **(2 marks)**

When the line is terminated by a load impednce of $Z_L=\infty$, i.e. open circuited line, the input impednace is given by

$$Z_{inOC} = -jZ_o \cot(\beta\ell)$$ **(1 mark)**

Again, this is a reactive impedance that can be made equivalent to a lumped capacitor or indictor by adjusting the length ℓ. A capacitive reactance can be obtained using $0 \le \ell \le 0.25\lambda$, while for $0.25\lambda \le \ell \le 0.5\lambda$ an inductive reactance can be achieved. **(2 marks)**

Q2.a

Smith chart can be used to analyse lossy lines by considering that the reflection coefficient at a distance d from the load is given by

$$\Gamma_{(d)} = \Gamma e^{-2\gamma d}$$ **(1 mark)**

i.e.

$$\Gamma_{(d)} = \Gamma e^{-2\alpha d} e^{-2j\beta d}$$ **(1 mark)**

As we move from the load towards the generator $\Gamma e^{-2\alpha d}$ continuously decreases. Hence the radius of the locus in the Smith chart, Γ, no longer moves around in a circle. Instead it spirals in towards the centre of the chart z=(1+j0), i.e. a long lossy line appears to be matched irrespective of its termination. On the Smith chart, move along a constant radius arc through the angle $2\beta d$ and then move radially inwards to $\Gamma e^{-2\alpha d}$. **(2 marks)**

Q2.b

At a frequency of 1 GHz, the free space wavelength can be calculated as

$$\lambda = \frac{c}{f} = \frac{3\times10^{10}}{1\times10^{9}} = 30 \text{ cm}$$ **(1 mark)**

The guided wavelength is

$\lambda_g = 0.77\lambda = 23.1$ cm **(1 mark)**

Since the line length is given as 5 cm, then

$\frac{\ell}{\lambda_g} = 0.216$ **(1 mark)**

The normalized load impedance is

$z_L = \frac{40 + j35}{50} = 0.8 + j0.7$ (point A on the Smith Chart) **(1 mark)**

Then the normalised input impedance can be determined by moving a distance of $0.216\lambda_g$ (from point A to point B on the chart) as

$z_{in} = 0.95 - j0.8$ **(1 mark)**

De-normalised input impedance is

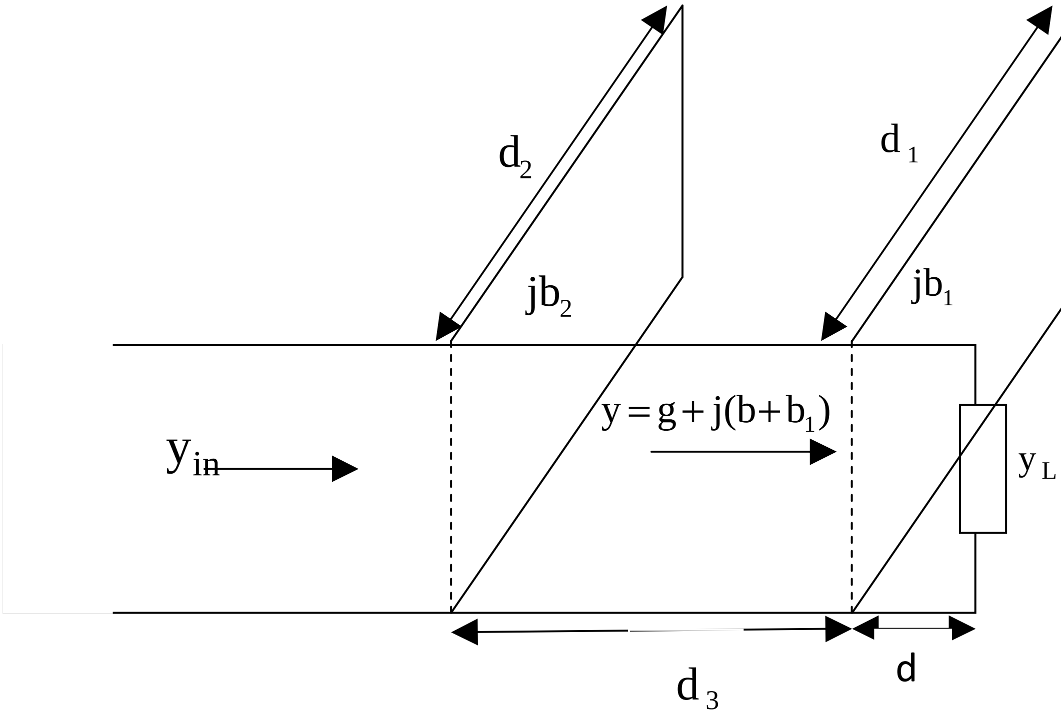
$Z_{in} = 50 \times (0.95 - j0.8) = (47.5 - j40)\Omega$ **(1 mark)**

Q2.c

$z_L = \frac{(150 - j50)}{50} = 3 - j1.$ (point A) **(1 mark)**

i.e.

y_L=0.3+j0.1 (point B) **(1 mark)**



Step 1

Rotate the unit g circle *Towards Load*, by a distance of d_3=0.125λ. **(1 mark)**

Step 2

Move from point B to intersect the new, rotated, unit circle at point C. The movement should be on the corresponding conductance circle, since the stub does not alter the real part of the admittance.

(1 mark)

Step 3
The admittance at point C is
y_C=0.3+0.275
compare it with that at B
y_L=0.3+j0.1
shows that stub 1 has provided j0.175i.e. b_1=0.175 **(2 marks)**
Step 4
For an o.c. stub, this means d_1=0.03λ, i.e. the distance from D to E on the chart.

(1 mark)

Step 5
Move a distance d_3=0.125λ along the line from the 1st stub position to the 2nd stub position (from point C to F). **(1 mark)**

Step 6
At point F, the admittance is y_F=1+j1.35i.e. stub 2 must provide -j1.35, *i.e.* b_2=-1.35) to reach the matched condition. **(1 mark)**

Step 7
For a s.c. stub, this means d_2=(0.352-0.25)λ=0.102λ, i.e. the distance from G to H on the chart.

(1 mark)

Q3.a

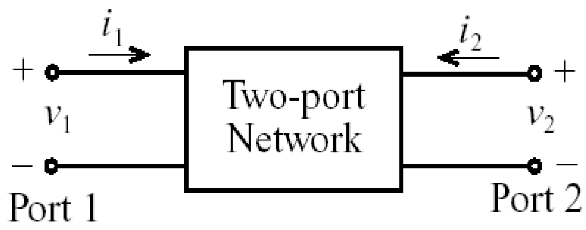
Available Power Gain is the ratio between power available from the network and the power available from the source. In most of the cases this gain is independent of Z_L. Gain of some active circuits is a function of Z_L. The available power gain can be calculated as $G_A = \frac{P_{avn}}{P_{avs}}$ **(2 marks)**

Transducer Power Gain is the ratio between power available at the load and the power available from the source. This gain depends on both Z_g and Z_L and it can be calculated as $G_T = \frac{P_L}{P_{avs}}$

(2 marks)

Q3.b

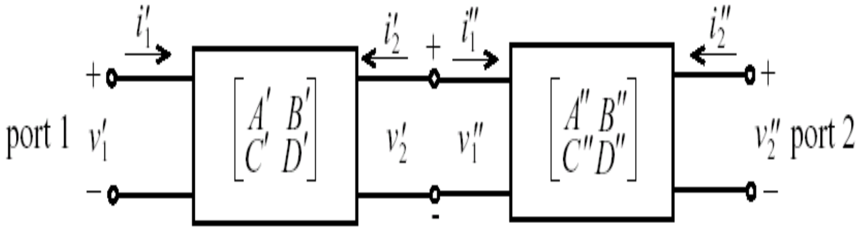
The transmission matrix, or the ABCD matrix, of a two-port circuit relates the output terminal voltage and current (v_2, i_2) to the input voltage and current (v_1, i_1)



(1 mark)

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$
In matrix form
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$ **(1 mark)**

(v_1, i_1) and (v_2, i_2) are the actual voltages and currents (*not normalised*), and they are continuous at the boundaries of the two ports. This means the output of one two-port circuit can be considered as the input of another two-port circuit. This makes the ABCD representation useful when cascading two-port networks as shown below



(1 mark)

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} V_2'' \\ -I_2'' \end{bmatrix}$$

which may be expressed as

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = T'T'' \begin{bmatrix} V_2'' \\ -I_2'' \end{bmatrix}$$ **(1 mark)**

Q3.c

The voltage and current at each port are given by

$$V_n = V_{ni} + V_{nr}$$
$$I_n = \frac{1}{Z_{0n}}(V_{ni} - V_{nr})$$

(1 mark)

Vni and Vnr can be determined by adding, and subtracting, those equations, respectively, to give

$$V_{ni} = \frac{1}{2}(V_n + Z_{0n}I_n)$$
$$V_{nr} = \frac{1}{2}(V_n - Z_{0n}I_n)$$

(1 mark)

which can be used to calculate the required voltages as

$V_{1i} = 7.25\angle 9.9°$, $V_{1r} = 3.1\angle -23.68°$ **(1 mark)**

and

V_{2i} =9.4.∠-78.5°, V_{2r} =3.35.∠56° **(1 mark)**

Q3.d

The scattering parameters S_{11} is calculated when port is terminated with $Z_L=Z_{02}=50\Omega$ as

$$S_{11} = \Gamma_{in} = \frac{Z_{in1} - Z_{o1}}{Z_{in1} + Z_{o1}}$$ **(1 mark)**

$$Z_{in1} = 14 + \left[\frac{82.2(14+50)}{82.2+14+50}\right] = 50\Omega$$ **(1 mark)**

i.e. $S_{11}=0$. **(1 mark)**

and

$$S_{21} = \frac{2Z_o Z_3}{Z_1(Z_1 + 2Z_3) + 2Z_o(Z_1 + Z_3) + Z_o^2}$$

(1 mark)

$$= \frac{8242}{14(14+164.48) + 100(14+82.24) + (2500)} = 0.5636$$

In the same way it can be shown that

$S_{22}=0$.

$S_{12}=0.5636$

(1 mark)

Q3.e

The input and output powers are

$$P_{in} = \frac{|V_{i1}|^2}{2Z_{o1}}$$

$$P_{out} = \frac{|V_{r2}|^2}{2Z_{o2}}$$

(1 mark)

Then

$$\frac{P_{out}}{P_{in}} = \frac{|V_{r2}|^2}{|V_{i1}|^2} = \frac{|S_{21}V_{i1}|^2}{|V_{i1}|^2} = |S_{21}|^2 = 0.318 = -5\text{dB}$$ **(1 mark)**

This means that the above circuit is a passive circuit, which provides 5dB attenuation. **(1 mark)**

Q4.a

The following conditions must be satisfied for unconditional stability to be achieved

$$|\Gamma_{in}| = \left|S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right| < 1$$

and

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| < 1$$

(1 mark)

This means that there will be a range of values for Γ_s and Γ_L where the amplifier will be stable. Finding the range of Γ_s and Γ_L can be done by plotting the input and output *stability circles* on the Smith chart. Stability circles define the boundaries between stable and potentially unstable regions of Γ_s and Γ_L. **(2 marks)**

Q4.b

Generally transistors presents a significant impedance mismatch, so matching will be achieved over a narrow frequency bandwidth. When bandwidth is an issue, then we design for a gain less than the maximum, imperfect matching, to improve bandwidth. **(2 marks)**

Sometimes it is required to design an amplifier with a specific gain, other than the maximum. Constant gain circles will be used to facilitate design for a specific gain. **(2 marks)**

Q4.c

$$\Delta = |S_{11}S_{22} - S_{12}S_{21}| = 0.69$$ **(1 mark)**

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|} = 0.415$$ **(1 mark)**

Since $K<1$ and $|\Delta| < 1$ then the transistor is potentially unstable. **(1 mark)**

Q4.d

Since S_{12} = 0.0, S_{11} <1 and S_{22}<1, then the transistor is unconditionally stable. **(1 mark)**

The matching sections gain can be calculated as

$$G_{S_{max}} = \frac{1}{1 - |S_{11}|^2} = 1.74 = 2.4\text{dB}$$ **(1 mark)**

$$G_{L_{max}} = \frac{1}{1 - |S_{22}|^2} = 2 = 3\text{dB}$$ **(1 mark)**

Since

$$G_o = |S_{21}|^2 = 5.76 = 7.6\text{dB}$$ **(1 mark)**

The overall gain

$$G_{Tmax} = 2.4 + 3. + 7.6 = 13\text{dB}$$

This means there is a 3 dB gain higher than the design requirements.

Equations (52), (53) and (54) may be used to get

G_s=1dB	g_s=0.725	$C_S = 0.538\angle 140°$	r_s=0.34
G_s=0dB	g_s=0.575	$C_S = 0.46\angle 140°$	r_s=0.46
G_L=3dB	g_L=1.	$C_L = 0.7\angle 83°$	r_L=0.
G_L=2dB	g_L=0.8	$C_L = 0.63\angle 83°$	r_L=0.25

Plot the constant gain circles on the Smith Chart.

(4 marks,1 mark each circle)

For an overall gain of 11dB, we will choose G_L=2dB and G_s=1dB. We select Γ_S and Γ_L along these circles to minimise the distance from the centre of the chart. This will put Γ_S and Γ_L along the radial lines at 140° and 83° respectively. Thus Γ_S== $0.2\angle 140°$ and Γ_L== 0.39 $\angle 83°$.

(2 marks)