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The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2013-14 (2.0 hours)

EEE345 Engineering Electromagnetics

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. Using both Maxwell equations for the rotation operators of the electrical and magnetic fields, the materials equations relating corresponding fluxes and fields, and the mathematical identity **rot rot** $\underline{\boldsymbol{E}}$ = **grad** div $\underline{\boldsymbol{E}} - \nabla^2 \underline{\boldsymbol{E}}$ show that in vacuum the electric field vector $\underline{\boldsymbol{E}}$ obeys a wave equation. **(10)**

b. Use Maxwell's equation for the rotation of the magnetic field, together with Ohm's Law and complex expressions for both the dielectric constant $\boldsymbol{\varepsilon}_r$ and a planar wave of form $\underline{\boldsymbol{E}} = \underline{\boldsymbol{E}}_0 \exp(j\omega t)$ to derive an expression for $\boldsymbol{\varepsilon}_r$. Interpret the imaginary part of $\boldsymbol{\varepsilon}_r$ physically: what does it mean? **(6)**

c. Using Maxwell's modification of Ampere's Law calculate the divergence of the current density and interpret the result in terms of changes of the electrical charge. **(4)**

2. **a.** Sketch and annotate a short elementary length dx of a lossy transmission line where the only resistive component to be considered is the Ohmic resistance R^* per unit length along the line.
Show that the propagation constant k' for a fixed frequency source ω is approximately given by the expression

$$k' = \omega\,(L^*C^*)^{1/2}\,[1 - j\,R^*/(2\omega L^*)]$$

where ω is the angular frequency, L^* the inductance per unit length and C^* the capacitance per unit length.

(9)

b. A 50Hz signal is fed into the lossy transmission line with the characteristic given in Question 2a above which has C*=1nF/m, L*=1mH/m, R*=1Ω/m. Over what length can the signal be transferred so that at the end of the cable at least 95% of the voltage of the input signal arrives?

(4)

c. A 30cm short coaxial cable with inner and outer cable diameters of 0.5mm and 3mm, respectively, and a non-magnetic dielectric with a relative permittivity (dielectric constant) of ε_r=2 is to be used for high frequency measurements.

Write down equations for and calculate:

i) its capacity,

ii) its inductance,

iii) its approximate real-valued impedance in the lossless case and

iv) the voltage reflection coefficient for Ohmic loads of Z_L=50 or Z_L=75Ω.

Which of the two loads would be the better termination choice and why?

(7)

3. **a.** The electric potential in a region of free space may be given as:

$$V(x,y,z)= (x^3+2y^3+2z^2) \times 100\text{V}.$$

(i) Determine whether the expression above satisfies the Laplace equation.

(ii) Calculate the electric field strength E and the charge density ρ at the point (x,y,z)=(1,2,3)m for a permittivity of ε_0=8.8542×10^{-12} As/(Vm).

(6)

b. Show that the function:

$$V(x)= (2ax-x^2)\, \rho_{\text{free}}/(2\varepsilon_0\, \varepsilon_r)$$

solves the 1-dimensional Poisson equation for a semiconducting pn-junction of total depletion layer width 2a along the x-direction.

(7)

c. The potential of a static electric dipole consisting of a pair of two charges $-q$ and $+q$ is given by the equation

$$V(\underline{\boldsymbol{r}})= \underline{\boldsymbol{p}}\ \underline{\boldsymbol{r}}\ /(4\pi\varepsilon_0 r^3)$$

where $r=|\underline{\boldsymbol{r}}|$ is the distance from charge $+q$ and $\underline{\boldsymbol{p}} = q\,\underline{\boldsymbol{ds}}$ is defined as the dipole moment where the vector $\underline{\boldsymbol{ds}}$ points from $-q$ to $+q$.

Provide a sketch of the dipole geometry and calculate its electric field vector, using the identity grad($\underline{\boldsymbol{r}}^{n}$)= $n r^{n-1} \underline{\boldsymbol{e}}_r$ where $\underline{\boldsymbol{e}}_r = \underline{\boldsymbol{r}}/r$ is the radial unity vector pointing outwards. Compare the electric field along and perpendicular to the dipole axis.

(7)

4. **a.** Indicate which of the following $f(x,t)$ functions (where x= spatial coordinate, t=time, a,b,c=constants, h=any function) represent travelling or standing waves?

Explain your answers.

(i) $f(x,t) = \sin(4xt+a)$

(ii) $f(x,t) = b\cos(2x+t^2)$

(iii) $f(x,t) = \exp j(3at-bx)$

(iv) $f(x,t) = \sin(4x)\exp(-3x)$

(v) $f(x,t) = [g(bt-x)]^2$

(vi) $f(x,t) = g(at+x^2)$

(5)

b. Show explicitly by double differentiation that the function

$$f(r,t)=[\exp j(\omega t-kr)]/r$$

fulfils the wave equation, using the mathematical operator identity

$$\nabla^2_{\mathrm{r}} = 1/r^2\,[\partial/\partial r\,(r^2\,\partial/\partial r)]$$

for the radial component of the second derivative ∇^2 in spherical coordinates.

What is the physical meaning of $f(r,t)$ if $\underline{\boldsymbol{r}}$ is the usual radial vector with $r=|\underline{\boldsymbol{r}}|$?

(7)

c. For an oscillating electric dipole $\underline{\boldsymbol{p}}$ the magnetic flux in the far field at position $\underline{\boldsymbol{r}}$ is given by the equation

$$\underline{\boldsymbol{B}}_{\mathrm{f}} \approx \mu_0\,(\underline{\ddot{\boldsymbol{p}}} \times \underline{\boldsymbol{e}}_{\mathrm{r}}) / (4\pi cr)$$

where $\underline{\ddot{\boldsymbol{p}}} = \partial^2\underline{\boldsymbol{p}}/\partial t^2$ and $\underline{\boldsymbol{e}}_{\mathrm{r}}=\underline{\boldsymbol{r}}/r$ is the radial unity vector.

Using the additional relationships

$$\underline{\boldsymbol{E}}_{\mathrm{f}}=c\underline{\boldsymbol{B}}_{\mathrm{f}}\times\underline{\boldsymbol{e}}_{\mathrm{r}}$$

and

$$\underline{\boldsymbol{B}}_{\mathrm{f}}=\underline{\boldsymbol{e}}_{\mathrm{r}}\times \underline{\boldsymbol{E}}_{\mathrm{f}}/c$$

between the electrical field and the magnetic flux in the far field, calculate the Poynting vector.

Express the result in terms of the angle θ between $\underline{\boldsymbol{p}}$ and $\underline{\boldsymbol{r}}$ and interpret the result physically.

(8)

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