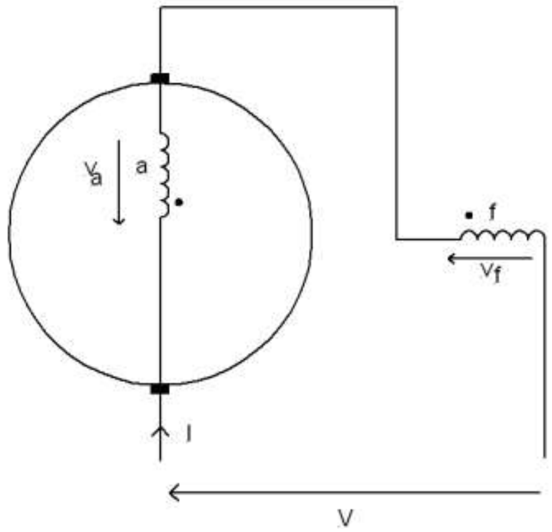
EEE6120 – Solutions for 2010/11 session examinations

[Notes in italics within square parenthesis are intended to provide background context to the question and/or to give further details of the methodology expected].

1.

a)



[½ mark deducted for each error in figure in terms of conventions]

The general form of the voltage equations are:

$$\begin{vmatrix} v_a \\ v_f \end{vmatrix} = \begin{vmatrix} R_a + L_a p & \omega_r M \\ 0 & R_f + L_f p \end{vmatrix} \begin{vmatrix} i_a \\ i_f \end{vmatrix}$$

On DC: $p=0$

On AC: $p=j\omega_s$

Constraining equations:

$V = V_a + V_f$

$I = I_a = I_f$

The resulting voltage equation for operation from a sinusoidal AC supply is:

AC operation:

$V = I\,(R_a + R_f + \omega_r M + j(X_a+X_f))$

(3)

b) *[This calculation phase of this question takes a very different route from questions all previous papers in which the machine parameters were generally supplied and the candidate asked to calculate various aspects of performance. This question starts with the specification of the resistance and some performance data. The key to the first two parts of the question is to use the copper loss and torque data to gradually unlock the missing terms in the AC voltage equation for this machine,* *thereby allowing the self reactance of the machine and ultimately the self inductance of the machine to be calculated].*

The magnitude of the current can be derived from the specified copper losses:

$$|I_{rms}| = \sqrt{\frac{P_{cu}}{R}} = \sqrt{\frac{323}{3.9}} = 9.10A$$

From the mechanical output power information provided during high-speed spin, the motor torque is given by:

$$P_{mech} = T\omega = 0.51 \times \frac{17200 \times 2\pi}{60} = 920W$$

The efficiency is given by:

$$Efficiency = \frac{Power\ output}{Power\ output + losses} = \frac{920}{920 + 323} = 74.0\%$$

When connected to the mains supply, the voltage equation is:

$V= [R_a + R_f + \omega_r M + j\omega_s (L_a+L_f)] I$

The impedance is therefore given by:

$$|Z| = \left|\frac{V}{I}\right| = \sqrt{(R + \omega_r M)^2 + X^2}$$

Hence, the reactance is given by:

$$X = \sqrt{\left|\frac{V}{I}\right|^2 - (R + \omega_r M)^2}$$

In order to calculate the total reactance between the terminals it is necessary to obtain a value of M. This can be derived from:

$$T = MI^2$$

Hence,

$$M = \frac{T}{I^2} = \frac{0.51}{9.1^2} = 0.00617H$$

Having calculated M, the reactance is given by:

$$X = \sqrt{\left(\frac{230}{9.10}\right)^2 - \left(3.9 + \left(\frac{17200 \times 2\pi \times 0.00617}{60}\right)\right)^2} = 20.3\Omega$$

And hence the self-inductance is given by:

$$L = \frac{X}{2\pi f} = 0.0647H$$

Power factor is given by:

$$\cos\emptyset = \frac{Re\{Z\}}{|Z|} = \frac{(R + \omega_r M)}{\left|\frac{V}{I}\right|} = 0.594\ lagging$$

[Note: Important to include in the answer that power factor is lagging – otherwise 1 mark is deducted for an incomplete answer]

c)

i) At the reduced voltage of 92Vrms, the magnitude of the current drawn is:

$$I = \sqrt{\frac{T}{M}} = \sqrt{\frac{0.1}{0.00617}} = 4.02Arms$$

The magnitude of the effective impedance is therefore given by:

$$|Z| = \left|\frac{V}{I}\right| = \frac{92}{4.02} = 22.8\Omega$$

Hence,

$$\omega_r = \frac{1}{M}\left(\sqrt{Z^2 - X^2} - R\right) = \frac{1}{0.00617}\left(\sqrt{22.8^2 - 20.3^2} - 3.9)\right) = 1040\ mech\ rad/s$$

This corresponds to a mechanical speed of 9931 rpm for the motor and 710 rpm for the drum

[1 mark deducted if the answer is left at 9931rpm (i.e. motor speed) and not scaled appropriately for the drum]

ii) Power factor is given by:

$$\cos\emptyset = \frac{Re\{Z\}}{|Z|} = \frac{(R + \omega_r M)}{\left|\frac{V}{I}\right|} = \frac{(3.9 + 1040 \times 0.00617)}{22.8} = 0.452\ lagging$$

[Such a poor power factor is only tolerable because of the low power operating point]

iii) Mechanical output power during the wash cycle is given by:

$$P_{mech} = T\omega_r = 0.1 \times 1040 = 104W$$

Copper losses are given by:

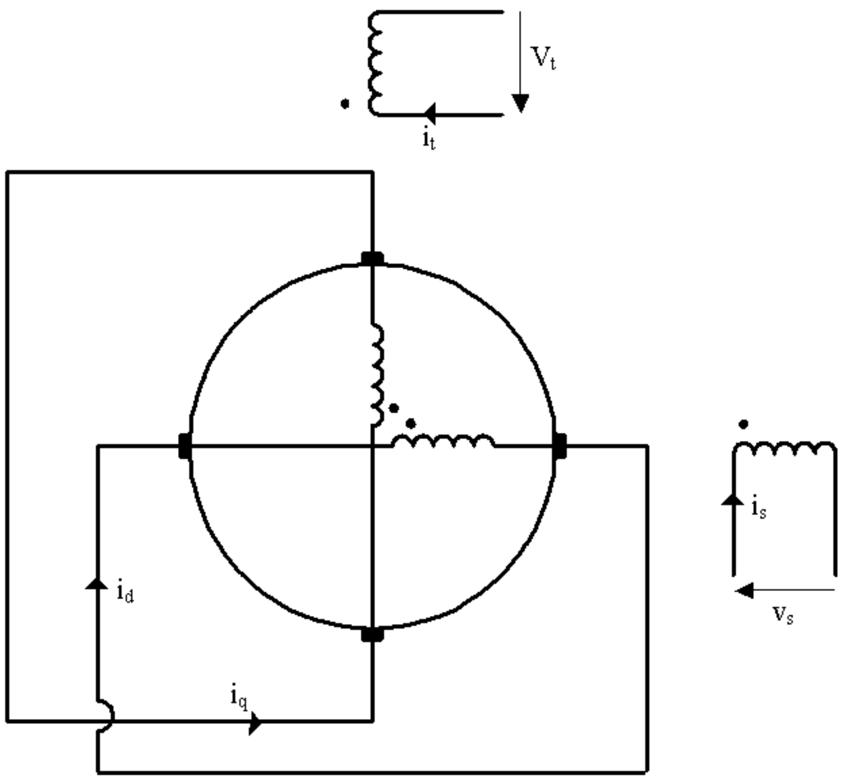
$P_{cu} = I^2R = 4.02^2 \times 3.9 = 63.0W$

$$Efficiency = \frac{P_{mech}}{P_{mech} + P_{cu}} = \frac{104}{104 + 63.0} = 62.2\%$$

[Although this is low, it is not unreasonable for a low power, low torque operating point – noting that the magnitude of the losses are much lower than the high-speed and high torque point and hence the poor efficiency has no implications on sizing of the machine]

2.

a) The Kron primitive equivalent of a three-phase induction motor is given by:



Adopting subscripts of '1' for the stator and '2' for the rotor, then the general form of the voltage matrix equations is:

$$\begin{vmatrix} v_s \\ v_t \\ v_d \\ v_q \end{vmatrix} = \begin{vmatrix} R_1+L_1p & 0 & M_{sd}\,p & 0 \\ 0 & R_1+L_1p & 0 & M_{td}\,p \\ M_{ds}p & -M_{dt}\omega_r & R_2+L_2p & -L_2\omega_r \\ M_{qs}\omega_r & M_{qt}\,p & L_2\omega_r & R_2+L_2p \end{vmatrix} \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix}$$

For steady-state operation for a sinusoidal AC supply:
$p=j\omega_s$ and $\omega_r=(1-s)\,\omega_s$

In addition, the same magnitude of applied to the two stator coils and the two rotor coils, but with a 90^o phase difference

$$\begin{vmatrix} V_s \\ V_t \\ V_d \\ V_q \end{vmatrix} = \begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} i_s \\ i_t \\ i_d \\ i_q \end{vmatrix} = \begin{vmatrix} I_1 \\ j\,I_1 \\ I_2 \\ j\,I_2 \end{vmatrix}$$

The governing voltage equation is therefore:

$$\begin{vmatrix} V_1 \\ jV_1 \\ V_2 \\ jV_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & 0 & jX_m & 0 \\ 0 & R_1+jX_1 & 0 & jX_m \\ jX_m & -(1-s)X_m & R_2+jX_2 & -(1-s)X_2 \\ (1-s)X_m & jX_m & -(1-s)X_2 & R_2+jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ jI_1 \\ I_2 \\ jI_2 \end{vmatrix}$$

But row 2 is simply row 1 $\times$j and row 4 is simply row 3 $\times$j. Hence the system can be reduced to two matrix equations:

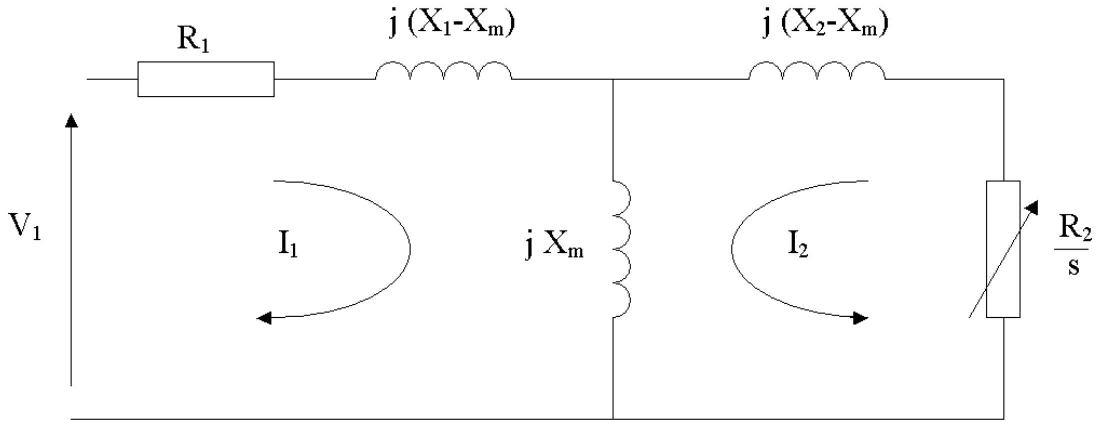
$$\begin{array}{l} \text{Stator} \\ \text{Rotor} \end{array} \quad \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m- j(1-s)X_m & R_2+ jX_2- j(1-s)X_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix}$$

Since the rotor is short circuited, V_2 =0
Substituting for V_2 and dividing the rotor equations by s gives:

$$\begin{array}{l} \text{Stator} \\ \text{Rotor} \end{array} \quad \begin{vmatrix} V_1 \\ 0 \end{vmatrix} = \begin{vmatrix} R_1+jX_1 & jX_m \\ jX_m & R_2/s+ jX_2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2' \end{vmatrix}$$

[Note I_2 is transformed to I_2']

An equivalent circuit that satisfies these voltage equations is:



(8)

b) ***[****This problem can be solved using either the exact or simplified equivalent circuit. The latter involves moving the magnetising branch to the terminals, but is reliant on the magnetising reactance being significantly higher than the other impedances. The clue that this is indeed a reasonable assumption in this case contained in the question which states 'This magnetising current is small in comparison with the total input current drawn at rated load'. Providing students recognise this* *assumption (preferably with some justification based on the values presented in the question) then the use of the simplified equivalent circuit is equally as valid in terms of the marks awarded.]*

[The addition of a resistor across the terminals to model core loss, although routine in most equivalent circuits of induction machines, does not naturally flow from the kron primitive analysis (which is based on no core loss being represented). Hence the rather laboured mention of this is deemed to be reasonable given that in many senses part b flows on from part a.]

Since the magnetising current is much smaller than the rated current it is reasonable to move the magnetising branch to the terminals of the machine.

As the machine is star-connected, the phase voltage of the machine is given by:

$$V_{ph} = \frac{3300}{\sqrt{3}} = 1905V\ rms$$

The real component of the no-load current is given by:

$$Real\ \{I_{no\ load}\} = 20\cos(85^{o}) = 1.74A$$

and similarly the imaginary part is given by:

$$Imaginary\ \{I_{no\ load}\} = 20\sin(85^{o}) = 19.9A$$

i) The magnetising reactance is given by:

$$X_m = \frac{V_{ph}}{Im\{I_{no\ load}\}} = \frac{1905}{19.9} = 95.6\Omega$$

(2)

ii) and similarly, the effective core loss resistance is given by:

$$R_{core} = \frac{V_{ph}}{Re\{I_{no\ load}\}} = \frac{1905}{1.74} = 1093\Omega$$

(2)

iii) The total core loss is given by:

$$P_{core} = 3 \times Re\{I_{no\ load}\}^2 \times R_{core} = 3 \times 1.74^2 \times 1093 = 9.96kW$$

(1)

iv) At rated load the current into the main branch of the network is

$I_{mb} = I_{in-}I_{noload} = 400\angle -27° - 20\angle -85° = 354.7 - j161.7 = 389.8\ \angle$-24.5°A

[Is it not an unreasonable approximation to neglect the relatively small influence of the no-load current on the main branch current – although it is strictly not negligible for the particular combination of parameters being considered. Hence, providing this assumption is explicitly noted by the candidate then full marks will be awarded. However, if there is no such reference to an approximation is made, then it is reasonable to infer that they have not appreciated that the no-load current should be considered (if only to be neglected) and a proportion of the marks will be deducted]

The impedance of the main branch is therefore:

$$Z_{mb} = \frac{V_{ph}}{I_{mb}} = \frac{1905\angle 0°}{389.8\angle -24.5°} = 4.89\angle 24.5°\Omega$$

Real component of Z_{mb} is given by:

$$Re\{Z_{mb}\} = 4.89\cos(24.5°) = 4.44\Omega$$

Hence $\frac{R_2^{'}}{s}$ is given by:

$$\frac{R_2^{'}}{s} = Re\{Z_{mb}\} - R_1 = 4.44 - 0.09 = 4.35\Omega$$

Hence the slip is given by:

$$s = \frac{R_2^{'}}{4.35} = \frac{0.16}{4.35} = 0.0367$$

Therefore speed at rated load is given by:

$$rated\ speed = (1-s)\omega_s = (1-0.0367)\times 3000 = 2890\ rpm$$

(4)

iv) The mechanical output power is given by:

$$P_{mech} = 3\times {I_{mb}}^2 \frac{R_2'(1-s)}{s} = 389.8^2 \times \frac{0.16(1-0.0367)}{0.0367} = 1.91MW$$

Hence, torque is given by:

$$T = \frac{P_{mech}}{\omega_r} = \frac{1.91\times 10^6}{\frac{2980}{60}\times 2\pi} = 6320Nm$$

(3)

3.

a) Applying the trapezium rule to integrate the area under the fully aligned curve (i.e. the curve at an angular displacement of 30°) for currents up to 5A yields:

$$A_{0\to 2.5} = \frac{2.5\times\Psi_{2.5}}{2} = 0.15J$$

$$A_{2.5\to 5} = \frac{2.5(\Psi_{2.5}+\Psi_5)}{2} = 0.41J$$

Hence the total area under the curve up to 5A is:

$$A_{0\to 5} = A_{0\to 2.5} + A_{2.5\to 5} = 0.56J$$

The area under the un-aligned curve (which can reasonably regarded as being linear) is simply given by:

$$U_{0\to5} = \frac{5\Psi_5}{2} = 0.04\text{J}$$

Hence the change in co-energy is given by:

$$\Delta W' = A_{0\to5} - U_{0\to5} = 0.52\text{J}$$

The average torque for 30A is therefore given by:

$$T_{AVE} = \frac{\Delta W'}{\Delta\theta} = \frac{0.52}{\pi/6} = 1.0\text{Nm}$$

Repeating the same process from 5A to 15A yields:

$$A_{5\to7.5} = \frac{2.5(\Psi_5 + \Psi_{7.5})}{2} = 0.55\text{J}$$

$$A_{7.5\to10} = \frac{2.5(\Psi_{7.5} + \Psi_{10})}{2} = 0.61\text{J}$$

$$A_{10\to12.5} = \frac{2.5(\Psi_{10} + \Psi_{12.5})}{2} = 0.63\text{J}$$

$$A_{0\to12.5} = A_{0\to2.5} + A_{2.5\to5} + A_{5\to7.5} + A_{7.5\to10} + A_{10\to12.5} = 2.34\text{J}$$

$$U_{0\to12.5} = \frac{12.5\Psi_{12.5}}{2} = 0.25\text{J}$$

Hence the change in co-energy is given by:

$$\Delta W' = A_{0\to12.5} - U_{0\to12.5} = 2.09\text{J}$$

The average torque for 30A is therefore given by:

$$T_{AVE} = \frac{\Delta W'}{\Delta\theta} = \frac{2.09}{\pi/6} = 1.86\text{Nm}$$

(7)

b) From the aligned Ψ-I characteristic it can be seen that the onset of saturation occurs at a current of ~4A (an answer based on a slightly different interpretation of saturation is equally acceptable).

It is important to note that the flux produced by 2 coils that constitute a phase crosses 2 airgaps, each of length l_g. Hence, prior to saturation then a reasonable estimate of N_c can be obtained from this equation.

$$N_c = \frac{B_g l_g}{\mu_0 I} = \frac{1.5\times0.5\times10^{-3}}{4\pi\times10^{-7}\times4.0} = 149 \text{ turns}$$

[Reasonable error on this acceptable in light of interpretation of saturation occurring at 4A]

(3)

c) *[The key to this question is identifying that the maximum value of inductance is obtained in the fully aligned position and at modest current levels, i.e. those below the onset of saturation, while the minimum value corresponds to the unaligned position (at any current given the absence of saturation). This neglects the very minor issue of the reversible region of a typical iron B-H characteristic which has not been covered in this course – this would tend to suggest using enough current to get the iron beyond its reversible region. The conditions for maximum and minimum inductance have not been covered in the notes (although the maximum value alone was asked for in the 2009 examination), and so this is probing a thorough understanding of what the flux-linkage versus current characteristics represents. The calculation itself is almost trivial so the majority of the marks will be awarded for identifying reasonable combinations of rotor angle and current for both conditions]*.

The maximum value of absolute inductance (i.e.Ψ/ i) is achieved in the aligned position at low current.

[Providing the machine does not saturate then any value of current can be used, but it is good practice, to use a value which is sufficiently high to read off to a reasonable degree of precision.]

Hence in the aligned position (30^o) and a current of 2A, the maximum value of phase self-inductance is given by:

$$L_{max} = \frac{\Psi}{I} = \frac{0.10}{2} = 0.050H$$

The minimum value of inductance is the un-aligned position (0°) and at a current of 12.5A (any value up to 25A would be fine, but 12.5A simply sits on a convenient value of Ψ to read-off the graph). Hence:

$$L_{min} = \frac{\Psi}{I} = \frac{0.04}{12.5} = 0.0032H$$

[As with all questions which draw on information derived from a graph, a reasonable tolerance will be accepted]

(3)

d.

[As indicated in the hint in the question, this problem can be solved by plotting a flux-linkage characteristics and identifying the steepest part of the slope. However, whereas a solution that adopts this approach will be awarded full marks if correctly followed through, there is no need to take this rather laboured approach. The maximum rate of change of flux-linkage at 12.5A occurs, by inspection of Figure 3, between rotor angular displacements of 18° and 24°.]

Between 18° and 24°, the rate of change of flux-linkage with respect to rotor angular displacement is given by:

$\frac{d\Psi}{d\theta} \approx \frac{\Delta\Psi}{\Delta\theta} = \frac{0.24-0.16}{6\times\frac{\pi}{180}} =$ 0.764 Wb/rad

At 3000rpm

$\frac{d\theta}{dt} = \frac{3000\times2\times\pi}{60}$ = 314 rad/s

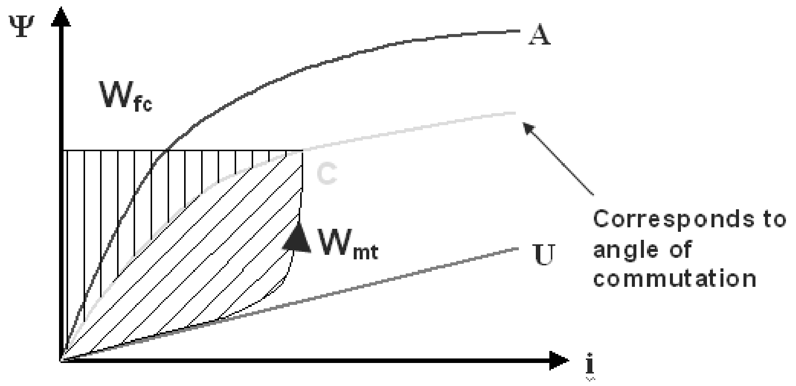
Hence, the peak emf when operating with a current of 12.5A is given by:

$$e = \frac{d\Psi}{d\theta} \times \frac{d\theta}{dt} = 0.764 \times 314 = 240V$$

(3)

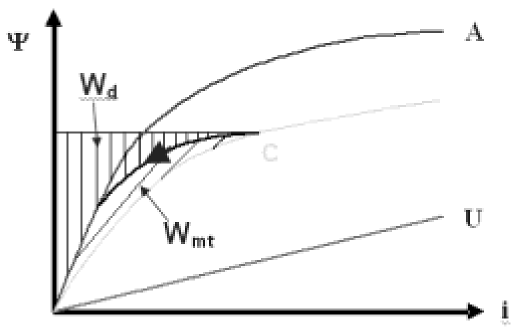
e) In motoring mode, the two dynamic ψ-I characteristics are:

Up to the instant of commutation:



W_{mt} – Mechanical output work
W_{fc} – Magnetic stored energy
Total energy accumulated from supply = W_{fc} + W_{mt}

Following commutation:



W_{mt} – mechanical output work (J)
W_d –Energy returned to supply

[In marking this section, particular emphasis will be placed on precise definitions and identification of the various energy changes]

(3)

f) Exact form of trajectory depends on:

Rotational speed
Rotor and load inertia
Magnitude of applied voltage
Commutation angles

Commutating prior to alignment to allow the current to decay by the time full alignment is reached – otherwise braking torque would be produced which would reduce the net torque produced

(1)

4)

a) *[There is a hint (possibly generous) on how to solve this problem in that the reference to the Silicon Iron saturating at 1.6T is specifically mentioned – as opposed to being part of the pre-amble. Since this question has not be posed in previous examination papers, this on balance seems a reasonable steer to provide for an opening section]*.

From Figure 4b, it is apparent that saturation begins at a net coil flux of 0.02 Wb *[there is a reasonable tolerance band on this given the difficulty in unequivocally identifying the onset of saturation]*

Hence, 0.02Wb of coil flux-linkage corresponds to 1.6T in the core.

At 0°, there is no net permanent magnet flux and hence the flux-linkage is entirely due to the coil. Taking the case of 2.0A [*any value will do but this provides the best option in terms of reading off a value*] the flux-linkage is 0.013Wb.

Hence, with no permanent magnet flux and a 2A coil current, the core flux density is approximately given by:

$$B_g = \frac{0.13}{0.02} \times 1.6 = 1.04T$$

For the rotor geometry shown and the details provided in the question, the total effective magnetic airgap is 7mm (i.e. 2x3mm thick magnets and 2x0.5mm mechanical airgaps).

Hence, the number of turns is given by:

$$N = \frac{B_g l_{g\,eff}}{\mu_0 I} = \frac{1.04 \times 7 \times 10^{-3}}{4\pi \times 10^{-7} \times 2} = 2896\ turns$$

(6)

b) *[The more typical form of question asked in previous years was based on simply calculating the emf. Although the heart of the calculation is similar in this case, it is phrased, and must be solved, rather differently]*.

[The flux-linkage characteristics for 0A is a reasonable approximation to a sine-wave [in fact the actual data is generated from a simple sin function]. It is therefore reasonable to assume

that the maximum rate of change of flux-linkage will occur at angular displacements around 0^o. [It is not necessary to identify this with a sine wave, just to recognise visually that the maximum rate of change will occur around 0^o].

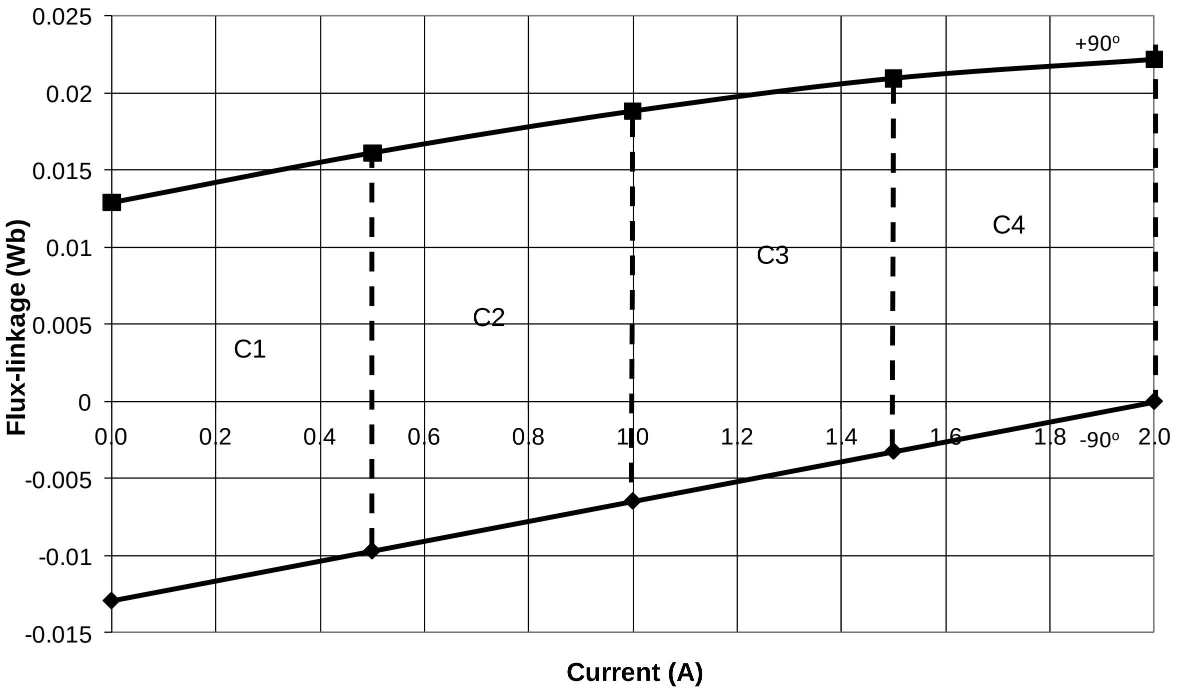
From Figure 4b around 0^o, an estimate of the rate of change of flux linkage with rotor position can be made:

$$\frac{d\Psi}{d\theta} \approx \frac{\Delta\Psi}{\Delta\theta} = \frac{0.44}{20\times\frac{\pi}{180}} = 0.0128 \text{ Wb/rad}$$

$$\frac{d\theta}{dt} = \frac{e}{\frac{d\Psi}{d\theta}} = \frac{8}{0.0128} = 623\ rad/s \leq 5950 \text{ rpm}$$

(4)

c) In order to estimate the torque for the two currents specified it is necessary to re-plot the data as a flux-linkage versus current characteristic for –90° and + 90°:



The co-energy change can be estimated by trapezoidal integration of the four areas C1 to C4 shown in the graph above. Using this approach:

The change in co-energy for 0. 5A is C1 = 0.0129J

The change in co-energy for 2.0A is C1+C2+C3+C4 = 0.0129+ .0128 + 0.0124 + 0.0116

=0.0496J

[Again, the values used above are taken from the actual data in the spreadsheet used to generate the initial characteristics, whereas candidates will be reading off values from the figure in the paper. Hence, there will be a reasonable tolerance on the exact answer]

Change in rotor angular displacement = $180 \times \dfrac{\pi}{180} = \pi$ rads

The torques produced are therefore given by:

At 0.5A: $T = \dfrac{dW'}{d\theta} \approx \dfrac{0.0129}{\pi} = 4.1 \times 10^{-3}\, Nm$

At 2.0A: $T = \dfrac{dW'}{d\theta} \approx \dfrac{0.0496}{\pi} = 15.8 \times 10^{-3}\, Nm$

[An important point here is that the torque per amp is gradually diminishing with onset of magnetic saturation]

(7)

d) The various inductances can be calculated from the additional flux-linkage produced by the current. For the 4 cases :

At 0.5A:

$$L = \frac{0.0032}{0.5} = 6.4mH \quad at \ -90^o$$

$$L = \frac{0.0032}{0.5} = 6.4mH \quad at \ +90^o$$

At 2.0A:

$$\mathrm{L} = \frac{0.0129}{2.0} = 6.4\mathrm{mH} \quad \mathrm{at} \ -90^o$$

$$\mathrm{L} = \frac{0.0093}{2.0} = 4.6\mathrm{mH} \quad \mathrm{at} \ +90^o$$

The difference observed at 2A and +90° is a result of magnetic saturation this is the worst case angular displacement at which the magnet flux and the coil flux add to each other.

(3)

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