The University of Sheffield
Department of Electronic and Electrical Engineering

EEE117 Problem Sheet Solution Guide

dc Circuit Analysis

Q1 For the circuit of figure 1 find I using any method you like. What is the power dissipation in R_1?

(i) Nodal analysis . . .

First identify the major nodes; there are two in this circuit, node **A** and node **B**. Choose one of them, say node **B,** as the 0 V reference point and sum currents at node **A**

Besides the reference node, there is only one major node in this circuit so the current sum at **A** is all that is needed to find V_A, the voltage of node **A** with respect to the reference (0 V) node. The result of this process gives

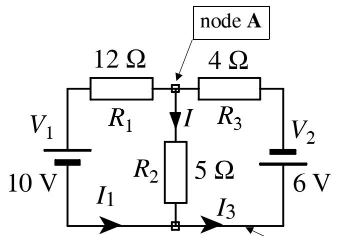

Figure 1

$$V_A = \left(\frac{V_1}{R_1} - \frac{V_2}{R_3}\right)\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right).$$

Once V_A is known it is easy to work out every current and voltage difference in the circuit and hence the power dissipated or sourced by each circuit element. **ANS: $V_A = -5/4$ V, $I = -0.25$ A and $P_{R1} = 10.5$ W.**

(ii) Loop analysis . . .

Choose two loops - say V_1, R_1, R_2 and R_2, R_3, V_2 and call the circulating currents I_{L1} and I_{L2} respective ly. Let I_{L1} circulate in a clockwise direction and I_{L2} in an anticlockwise one (you could choose different directions). After simplification you will get

loop 1; $10 = 17I_{L1} + 5I_{L2}$

loop 2; $-6 = 5I_{L1} + 9I_{L2}$

Solve these two equations to get $I_{L1} = \frac{15}{16}$ A and $I_{L2} = -\frac{19}{16}$ A. I is then given by $I = I_{L1} + I_{L2}$.

[If you choose I_{L1} and / or I_{L2} to be flowing in the opposite direction, the sign of I_{L1} and / or I_{L2} will change in the two loop equations above but the magnitudes of the coefficients will be the same.]

(iii) Superposition . . .

Easiest approach is to find the voltage across R_2 due to the sources V_1 and V_2 and then calculate I

V_{R2} due to 10V is $V_{R2} = 10\,\dfrac{R_2//R_3}{R_1 + R_2//R_3}$ V

V_{R2} due to -6V is $V_{R2} = -6\,\dfrac{R_1//R_2}{R_3 + R_1//R_2}$ V

V_{R2TOT} is the sum of these two contributions which gives $V_{R2TOT} = -1.25$ V. This results in the same I as the other two methods.

Q2 Using any method you like, find the values of I and V in figure 2. I_R

It is easiest here to sum voltages around the outer loop to find I . . .

$$6I + 8(I + 5) + 4(I + 5 - 2) = 0$$

This leads to a value for I and once I is known it is easy to work out circuit voltage differences.

ANS: $I = -2.89$ A and $V = 16.89$ V

It would also be easy to use Norton to Thevenin transformations on the 6 Ω and 5 A and the 4 Ω and 2 A combinations to find V. V will give the current through the 8 Ω so I can easily be found. **Try it!**

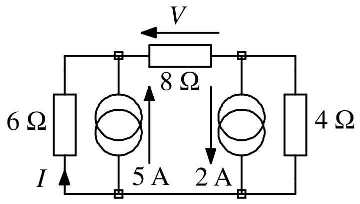

Figure 2

Q3 In figure 3, I is initially 1 A. Use nodal analysis to find I_S and V_R. What value of I is necessary to give $V_R = -4$ V?

Use node **B** as the 0 V reference potential and sum currents at node **A**. The resulting equation gives V_A directly as **3.2 V**. Once V_A is known, Ohm's law can be used to find that I_S = **0.6 A**

Since the answer to "what current will make $V_R = -4$ V?" is the same as the answer to "what current will make $V_A = -4$ V?" (since $V_R = V_A$) the current sum at node **A** can be used to solve this problem. In this case V_A is known and I is an unknown. The answer is **− 5 A**

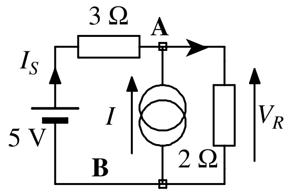

Figure 3

Q4 For the circuit of figure 4, use nodal analysis and superposition to find I_1 and the potential difference $V_4 - V_3$, $V_{4\text{-}3}$. What is the power dissipation in R_4?

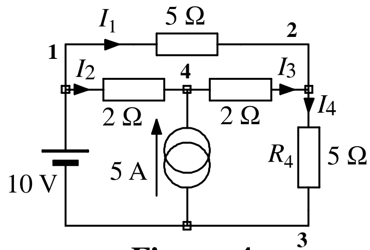

Figure 4

(i) Nodal analysis

There are four major nodes in this circuit. If node **3** is used as the 0 V reference, node 1 must have a voltage, V_1, of 10 V because of the 10 V source. This leaves node voltages V_2 and V_4 as unknowns. Summing currents at the V_2 and V_4 nodes in turn leads to

at node 2; $\quad 20 = 9V_2 - 5V_4$

at node 4; $\quad 20 = -V_2 + 2V_4$

solving these equations yields $\boldsymbol{V_4}$ = **15.39 V** and $V_2 = 10.77$ V.

$$P_{R4} = \frac{V_2{}^2}{R_4} = \frac{10.77^2}{5} = \mathbf{23.2\ W}$$

Once V_2 and V_4 are known all the currents, including I_1, can be calculated with ease. Answer = **− 0.15 A**

(ii) Superposition

V_4 due to 10V (replace 5 A source with an open circuit) . . .

Current from 10V source is 10/effective circuit resistance. The effective resistance can be worked out by using the rules that describe series and parallel resistor connections. The voltage at node **2** is this current times R_4. Once V_2 is known, working out I_1 is straightforward. To find V_4, notice that 10V and V_2 are connected together by two 2 Ω resistors in series. V_4 must be exactly half way between 10 V and V_2.

The difference between 10 V and V_{R4} is shared equally between the two 2 Ω resistors so

$$V_{4(10V)} = 10 - \frac{10 - \frac{90}{13}}{2} = \frac{110}{13}\text{ V.}$$

V_4 due to 5A (replace 10V with a short circuit) . . .

The current source sees a parallel combination of the left hand 2 Ω in parallel with a series combination of right hand 2 Ω and two 5 Ω resistors in parallel. Thus $V_{4(5A)} = 5 \times 2//(2+5//5) = \frac{90}{13}$ V.

$V_{4TOT} = 110/13 + 90/13 = 200/13 =$ **15.39 V**

I_1 due to 10 V is that portion of the total I_S that flows through the 5 Ω arm of the parallel combination of the two 2 Ω resistors and the top 5 Ω resistor

$$I_{1(10V)} = \frac{18}{13} \times \frac{4}{9} = \frac{8}{13} \text{ A.}$$

I_1 due to the 5A source is the current flowing through one of the two parallel 5 Ω resistors. This will be half the current that flows down the arm containing the 5 Ω resistors and will be negative. Using current splitting,

$$I_{1(5A)} = -5 \times \frac{2}{6.5} \times \frac{1}{2} = -\frac{10}{13} \text{ A.}$$

Thus $\boldsymbol{I_{1TOT}} = 8/13 - 10/13 = -2/13 = -$ **0.15 A**

Q5 Use loop analysis and superposition to find I_2 and I_4 in the circuit of figure 5a. State with brief reasoning which component could be replaced by a short circuit without affecting either of these currents.

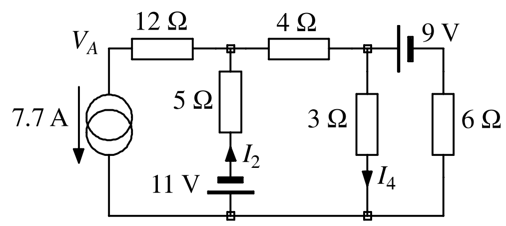

Figure 5a

(i) loop analysis

Choose three current loops. The choice here is three counter-clockwise loops; I_A throught 7.7A, 11V, 5 Ω and 12 Ω; I_B through 3 Ω, 4 Ω, 5 Ω and 11V; I_C through 3 Ω, 6 Ω and 9V. Many other choices are possible. The thing to remember here is that the objective of the loop method is to leave you with the minimum number of unknowns neccessary to solve the circuit. In loop A it is clear that $I_A = 7.7$A so there is no need to investigate loop A further. (Remember that the goal of the loop method is to find I_A, I_B and I_C). After simplification the loop equations are

For loop B; $49.5 = 12I_B - 3I_C$

For loop C; $9 = 9I_C - 3I_B$

There are many approaches that can be used to solve this pair of equations. Here we shall multiply the loop C equation by 4 and add it to the loop B equation to eliminate I_B . . .

$$49.5 + 36 = 0 + (36-3)I_C \text{ or } 85.5 = 33I_C \text{ or } I_C = 85.5/33 = 2.591 \text{ A}$$

Substituting in the loop C equation, $3 = 3I_C - I_B = \frac{85.5}{11} - I_B$ and so $I_B = 52.5/11 = 4.773$ A

Then $\boldsymbol{I_2} = I_A - I_B = 7.7 - 4.773 =$ **2.93** A and $I_4 = I_C - I_B = 28.5/11 - 52.5/11 = -24/11 = -$ **2.18 A**

(ii) superposition

The important issue here is to make sure the partial circuit is interpreted correctly in each case.

(a) I_2 and I_4 due to the 7.7 A source - replace 11V and 9 V with short circuits as shown in figure 5b . . .

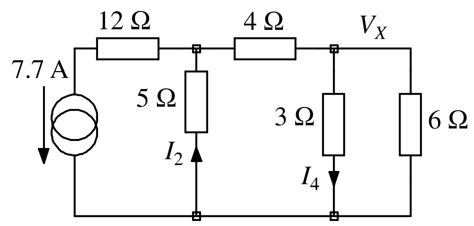

Figure 5b

The partial circuit consists of two parallel paths, one of 5 Ω through which I_2 flows and one through the series parallel combination of (6 Ω // 3 Ω) in series with 4 Ω, which join forces at the top of 5 Ω to return the 7.7 A through the 12 Ω to the source.

The combined resistance of (6 Ω // 3 Ω) in series with 4 Ω is 6 Ω so I_2 is that fraction of 7.7 A that takes the 5 Ω route, ie

$$I_{2(7.7A)} = 7.7 \times \frac{6}{11} = 4.2 \text{ A}$$

I_4 is the division of (7.7 - I_2) between 3 Ω and 6 Ω, ie

$$I_{4(7.7A)} = -(7.7-4.2) \times \frac{6}{9} \text{ A} = -\frac{7}{3} \text{ A}.$$ Note the "−" sign.

(b) I_2 and I_4 due to the 11 V source - replace 7.7 A with an open circuit and 9 V with a short circuit as shown in figure 5c . . .

The 11V source sees 9 Ω (5 Ω + 4 Ω) in series with the parallel combination 3 Ω // 6 Ω (= 2 Ω). Thus

$I_{2(11V)} = -\frac{11 \text{ V}}{11\ \Omega} = -1\text{A}$ and I_4 is that part of I_2 that flows through the 3 Ω resistor, ie, $I_{4(11V)} = I_{2(11V)} \frac{6}{3+6} = -\frac{2}{3}$ A

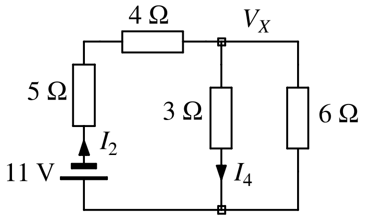

Figure 5c

(c) I_2 and I_4 due to the 9 V source - replace 7.7 A with an open circuit and 11V with a short circuit as shown in figure 5d . . .

Here the 9 V source sees (4 + 5)//3 Ω in series with 6 Ω, a total of 33/4 Ω. Thus the total current driven by the 9 V source is 9/(33/4) = 36/33 = 12/11 A. This current divides down the two parallel arms to give I_2 and I_4,

$I_{2(9V)} = -\frac{12}{11} \times \frac{3}{3+9} \text{ A} = -\frac{3}{11}$ A and $I_{4(9V)} = \frac{12}{11} \times \frac{9}{12} \text{ A} = \frac{9}{11}$ A

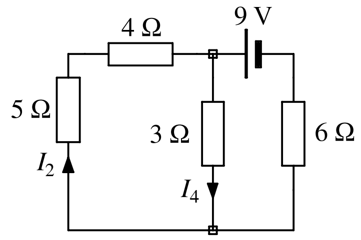

Figure 5d

(d) To get the overall solution, add the contributions due to each source:

$$\boldsymbol{I_2} = I_{2(7.7A)} + I_{2(11V)} + I_{2(9V)} = (4.2 - 1 - 3/11) \text{ A} = \mathbf{2.93\ A}$$

$$\boldsymbol{I_4} = I_{4(7.7A)} + I_{4(11V)} + I_{4(9V)} = (-7/3 - 2/3 + 9/11) \text{ A} = \mathbf{-2.18\ A}$$

Q6 Find $V_2 - V_3$, $V_{2\text{-}3}$, in the circuit of figure 6 using any method you like.

The key to solving this problem is the recognition that although node **5** is common to the left hand loop and the right hand loop, it is the ***only*** connection between those loops. Thus, I_2 must equal zero.

Since node **5** is the only common node it makes sense to use it as the reference potential and to evaluate V_2 and V_3 with respect to node **5**.

Left hand loop . . .

The 3 A source drives current around the loop and in doing so creates a voltage drop of 15V across the 5 Ω resistor with its positive end at node **2**.

$$(V_2 - V_5) = (V_2 - V_6) + (V_6 - V_5) = 15 - 11 = 4 \text{ V}$$

Right hand loop . . .

In the right hand loop, I_3 = 9 V / (3 Ω + 6 Ω) = 1 A. The voltage at node **3** with respect to node **5** is the voltage developed across the 3 Ω resistor by I_3, i.e., 3 V. So

$$(V_3 - V_5) = 3 \text{ V}$$

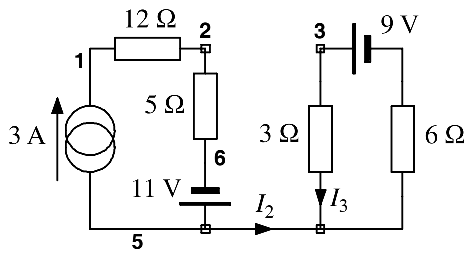

Figure 6

So $(V_2 - V_3) = (V_2 - V_5) - (V_3 - V_5) = 4 - 3 = \mathbf{1\ V}$

Q7 - 10 In these questions you are asked to use a method to find a circuit parameter. You should then check whether your answer is right either by using your answer to evaluate other voltages and currents in the circuit and checking that these obey basic laws - i.e., the two Kirchoff laws - or by using a second method.

Q11 This is a hard question because it requires you to interpret partial circuits of awkward shape. The numerical answers are given on the sheet and some of the superposition partial solutions are also given. Again, you can check that your answers are self consistent.