Data Provided: Laplace Transform pairs and List of useful formulae

The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Autumn Semester 2007-2008 (2 hours)

Signals and Systems 2

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. a. The impulse response of a simple RC circuit is given by $h(t) = \frac{1}{RC} e^{-t/RC} u(t)$ where *R* is the resistance and *C* is the capacitance. Show that the circuit response, $y(t)$, when subjected to an input signal $p(t)$, shown in Figure Q1.1, is given by

$$y(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-t/RC} & 0 < t < T \\ e^{-(t-T)/RC} - e^{-t/RC} & t \geq T \end{cases} .$$

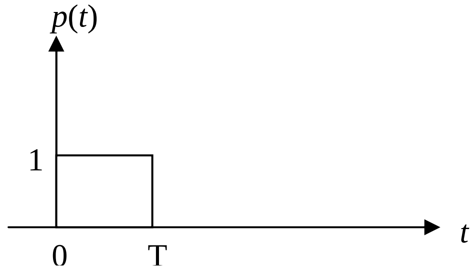

Figure Q.1.1

Sketch and label $y(t)$. **(10)**

b. Consider a Linear Time Invariant digital communication system, in which a bit "1" is represented by $p(t)$ in Figure Q.1.1 and a bit "0" is represented by $-p(t)$.

i) Assuming T = 1s and RC = 1s, sketch and label a sequence "1 0". **(2)**

ii) Sketch and label the response of the circuit to the sequence "1 0". **(5)**

iii) Comment on how the intersymbol interference (ISI) of this digital communication system is affected by the value of *RC*. **(3)**

2. **a.** i) Derive the Fourier Transform, $X(\omega)$, of the signal $x(t)$ shown in Figure Q.2.1 below.

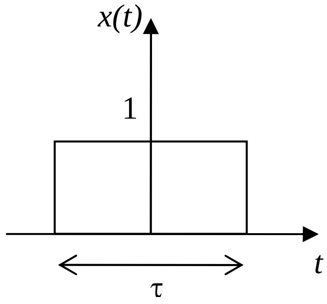

Figure Q.2.1 **(5)**

ii) Sketch and label clearly the maximum magnitude of $X(\omega)$ and the frequencies at which the magnitude of $X(\omega) = 0$. **(5)**

b. In an amplitude modulation system the amplitude of a carrier signal $x_c(t)=\cos(\omega_c t)$, where ω_c is the carrier frequency, is modulated by a signal $x(t)$ in figure Q.2.1. Write down the magnitude spectrum, $X_{mod}(\omega$, of the modulated signal. **(3)**

c. i) Assuming that the spectrum $X_{mod}(\omega)$ at frequencies $|\ \omega\ | > 2\pi/\tau$ is negligible, sketch and label $X_{mod}(\omega)$. What is the bandwidth of the low pass filter required to demodulate $X_{mod}(\omega)$ using a synchronous demodulator? **(7)**

3. **a.**

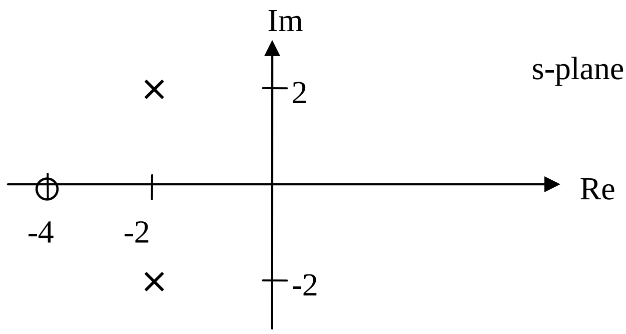

Figure Q.3.1

Figure Q.3.1 shows the pole-zero plot of a continuous time system. The corresponding transfer function can be expressed as $H(s) = \dfrac{N(s)}{D(s)}$. Obtain the polynomial functions $N(s)$ and $D(s)$. **(5)**

b. For the system shown in Fig.Q.3.1, obtain i) the damping factor, ii) the natural frequency and iii) the oscillation frequency of the unit step response. **(6)**

c. Sketch the transient component of the unit step response and describe the changes expected when the natural frequency of the system is doubled and the damping factor remains constant. **(5)**

d. Find the system response $y(t)$ when the input is $x(t) = e^{-4t}u(t)$. **(4)**

4. **a.** Show that the Trigonometric Fourier Series coefficients of the signal $p(t)$ shown in Figure Q.4.1 is given by $b_n = \begin{cases} 0 & n = even \\ \dfrac{4A}{n\pi} & n = odd \end{cases}$.

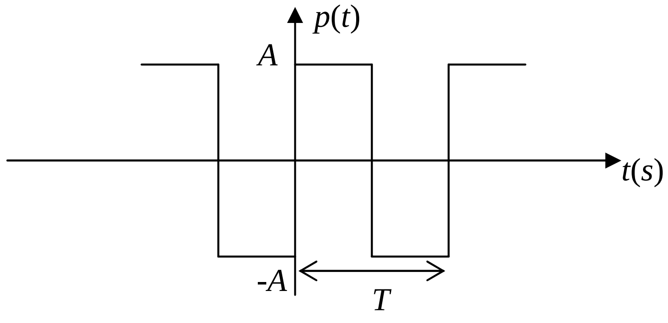

Figure Q.4.1 **(7)**

b. Write down an expression for a voltage waveform described by $p(t)$, assuming that $p(t)$ can be approximated by the first three non-zero harmonics in its Fourier Series when A = 1 and T = 1s. **(3)**

c. Using the transform impedance write down the transfer function of the circuit shown in Figure Q.4.2.

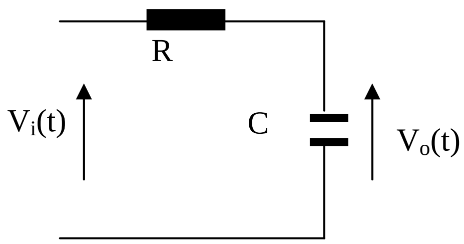

Figure Q.4.2 **(2)**

d. Consider that the signal in part (b) is contaminated with high frequency noise. The RC circuit in Figure Q.4.2 can be used to attenuate the noise.

i) Obtain the value of capacitance required so that after filtering the amplitude of the largest harmonic in $p(t)$ is at least 69% of its value before filtering. (Assume R=3.3 kΩ.) **(4)**

ii) Calculate the amplitudes of the first two harmonics in $p(t)$and the r.m.s value of $p(t)$ after filtering. **(4)**

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