MSc(Eng) Wireless Communication Systems

Module EEE-6431: Broadband Wireless Techniques

Contact Details

Module Leader : **Professor Tim O'Farrell**

Room : **Room C37b, Portobello Building**

Email : **T.OFarrell@sheffield.ac.uk**

Tel. : **0114 222 5193**

Syllabus Highlights

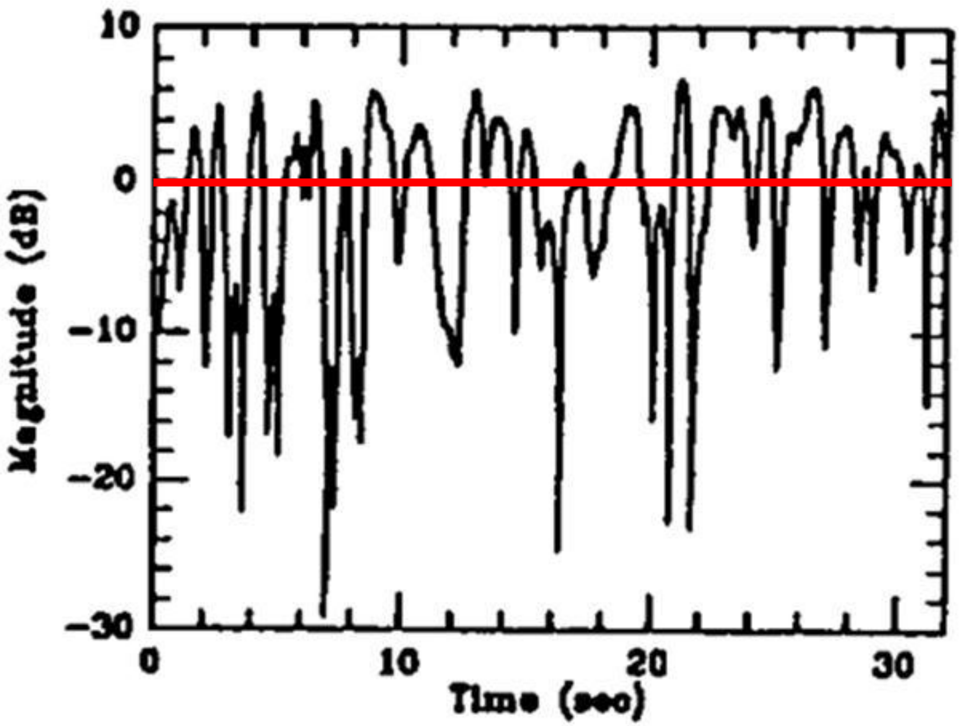
1. **Introduction - Overview of Broadband Wireless Systems**
2. **Signal Propagation, Pathloss Models and Shadowing**
3. **Statistical Fading Models: Narrowband & Wideband Fading**
4. **Capacity of Wireless Channels**
5. **Multicarrier Modulation**
6. **Spread Spectrum and CDMA**

Section 3 Review

1. Statistical fading models
2. Time varying and time invariant channel impulse response
3. Narrowband fading models, Clarks scattering model
4. Rayleigh, Rician & Nagakami fading
5. Outage probability
6. Level Crossing Rate & Average Fade Duration
7. Wideband fading and ISI
8. Time Variant Discrete Channel Impulse Response
9. Mean excess delay and *rms* delay spread
10. Coherence bandwidth & coherence time
11. Classification of small scale fading

Section 4 Outline

1. Capacity in AWGN
2. Capacity of Flat Fading Channels
3. Capacity of Frequency Selective Channels



Observed Fading

4. Capacity of Wireless Channels

Introduction: **Shannon's channel capacity determines the maximum data rates that can be transmitted over (wireless) channels with asymptotically small error probability, assuming no constraints on delay or complexity of the encoder and decoder.**

- **Shannon's coding theorem proved that channel codes exist that could achieve a data rate close to capacity with negligible probability of error;**
- **And any data rate higher than capacity could not be achieved without an error probability bounded away from zero**

In this section we examine the capacity of a *single-user* wireless channel where the transmitter and/or receiver have a single antenna and the channel is either time varying or time invariant. *Only discrete-time channels are considered*.

The capacity of a wireless channel depends on what is known about the time-varying channel at the transmitter and the receiver.

Capacity of AWGN Channels: **Consider a discrete-time AWGN channel with input/output relationship $y[i] = x[i] + n[i]$, where $x[i]$ is the channel input at time i, $y[i]$ is the channel output, and $n[i]$ is a white Gaussian noise random process. Assume a channel bandwidth $B = 1/T_s$ and transmit power *P*, where T_s is the symbol period.**

The channel SNR (the power in $x[i]$ divided by the power in $n[i]$, is constant and given by $\gamma = P/N_0 B$, where N_0 is the noise 1-sides PSD (and $N_0/2$ is the noise 2-sides PSD). The capacity of this channel in bit/s is given by:

$$C = B\log_2(1+\gamma)$$

4. Capacity of Wireless Channels

Capacity of AWGN Channels Contd:

Example: Consider a wireless channel where power fall-off with distance follows the pathloss formula $P_r(d) = P_t(d_0/d)^3$ for d_0 = 10 m. Assume the channel has bandwidth B = 30 KHz and AWGN with 1-sided PSD of $N_0 = 10^{-9}$ W/Hz. For a transmit power of 1W, find the capacity of this channel for a transmit-receive distance of 100 m and 1 km.

Solution:

The received SNR is $\gamma = \frac{P_r(100m)}{N_0 B} = \frac{(0.1)^3}{10^{-9}\times 30\times 10^3} = 33 \equiv 15dB$ for d = 100m.

And $\gamma = \frac{P_r(1000m)}{N_0 B} = \frac{(0.01)^3}{10^{-9}\times 30\times 10^3} = 0.033 \equiv -15dB$ for d = 1000 m.

The corresponding capacities are:

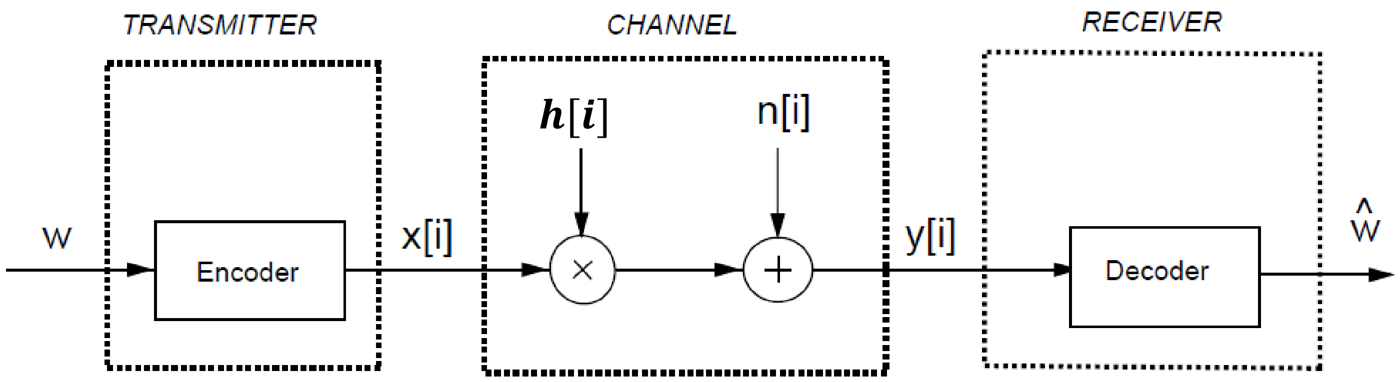
$C = Blog_2(1+\gamma) = 30000 \times log_2(1+33) = 152.6$ kbit/s for d = 100 m

$C = Blog_2(1+\gamma) = 30000 \times log_2(1+0.033) = 1.4$ kbit/s for d = 1000 m.

Note the significant decrease in capacity at farther distances, due to the path loss exponent of 3, which greatly reduces received power as distance increases.

4. Capacity of Wireless Channels

Capacity of Flat Fading Channels: Consider a discrete-time channel with *stationary and ergodic* time-varying channel gain $|h[i]| = \sqrt{g[i]} > 0$, and AWGN $n[i]$.



- **The channel power gain $g[i] = |h[i]|^2$ follows a known distribution $\rho[g]$ (e.g. exponential for Rayleigh) - $g[i]$ is called the *Channel Side Information* (CSI);**
- **$g[i]$ is independent of the channel input, can change at each time i as an i.i.d. random variable or with some correlation over time;**
- **In a block fading channel $g[i]$ is constant over some block length (period) *T* after which $g[i]$ changes to a new i.i.d. value governed by distribution $\rho[g]$**

The instantaneous and mean SNRs are given by:

$$\text{Instantaneous SNR}: \gamma[i] = g[i] \times \frac{\overline{P}}{N_0 B} \quad 0 \le \gamma[i] \le \infty, \quad \text{Average SNR}: \overline{\gamma} = \overline{g} \times \frac{\overline{P}}{N_0 B}$$

Where $\overline{P}$ denotes the average transmit power (NB the pathloss is included in $g[i]$)

4. Capacity of Wireless Channels

Capacity of Flat Fading Channels Contd: The capacity of this channel depends on knowing $g[i]$ at the transmitter and receiver:

- ***Channel Distribution Information (CDI)*** **:** Only the distribution $\rho[g]$ of $g[i]$ is known at the transmitter and the receiver;
- ***Receiver CSI*** : The value of $g[i]$ is known to the receiver at time i and both the transmitter and receiver know the distribution $\rho[g]$ of $g[i]$;
- ***Transmitter and Receiver CSI*** : The value of $g[i]$ is known to the transmitter and receiver at time i and both know the distribution $\rho[g]$ of $g[i]$.

Transmitter and receiver CSI allow the transmitter to adapt both its power and rate to the channel gain $g[i]$ at time i, leading to the highest capacity of the three case – *i.e. the more that is known about the channel the greater the capacity.*

Since $\gamma[i] = g[i] \times constant$ then $\gamma[i]$ takes the distribution of $g[i]$.

Capacity of Flat Fading Channels – Only $\rho[g]$ Known: The capacity is determined by $\rho[g]$ and hence $\rho[\gamma]$. This remains an open problem for almost all channel distributions. The Rayleigh case is known but difficult to apply in practice.

Capacity of Flat Fading Channels – CSI at the Receiver: i.e. $g[i]$ is known to the receiver at time i and $\rho[g]$ is known at the transmitter and receiver. This case gives two channel capacity definitions:

- **Shannon Capacity** (also called *Ergodic Capacity*) – the channel transmission rate is constant as the transmitter cannot adapt rate with $g[i]$, so poor channel states typically reduce the ergodic capacity;

4. Capacity of Wireless Channels

Capacity of Flat Fading Channels – CSI at the Receiver Contd:

The Shannon or Ergodic capacity is given by the probabilistic average:

$$C = \int_0^\infty B\log_2(1+\gamma)\rho(\gamma)d\gamma$$

Where γ is the instantaneous SNR. [*Aside - the capacity-achieving code must be sufficiently long so that a received codeword is affected by all possible fading states*]

If $\bar{\gamma}$ is the average SNR on the channel, then by Jensen's inequality we can write:

$$C = \mathrm{E}\left[B\log_2(1+\gamma)\right] = \int_0^\infty B\log_2(1+\gamma)\rho(\gamma)d\gamma \le B\log_2(1+\mathrm{E}[\gamma]) = B\log_2(1+\bar{\gamma})$$

Thus we see that the Shannon capacity of a fading channel with receiver CSI only is less than the Shannon capacity of an AWGN channel with the same average SNR.

Example: Consider a flat-fading channel with i.i.d. channel gain $g[i]$ which can take on three possible values:

- $g_1 = (0.05)^2$ **with prob.** $p_1 = 0.1$, $g_2 = (0.5)^2$ **with prob.** $p_2 = 0.5$, $g_3 = (1.0)^2$ **with prob.** $p_3 = 0.4$

The transmit power is 10 mW, the 1-sided noise PSD is $N_0 = 10^{-9}$ W/Hz, and the channel bandwidth is 30 kHz. Assume the receiver has knowledge of the instantaneous value of $g[i]$ but the transmitter does not. Find the Shannon capacity of this channel and compare with the capacity of an AWGN channel with the same average SNR.

4. Capacity of Wireless Channels

Solution: **The channel has 3 possible received SNRs:**

$\boldsymbol{\gamma_1 = g_1 P_t / N_0 B = (0.0025 \times 0.01)/(30 \times 10^3 \times 10^{-9}) = 0.83\ (-0.8\ \mathrm{dB})}$

$\boldsymbol{\gamma_2 = g_2 P_t / N_0 B = (0.25 \times 0.01)/(30 \times 10^3 \times 10^{-9}) = 83.33\ (19.2\ \mathrm{dB})}$

$\boldsymbol{\gamma_3 = g_3 P_t / N_0 B = (1.0 \times 0.01)/(30 \times 10^3 \times 10^{-9}) = 333.33\ (25.2\ \mathrm{dB})}$

Given $p(\gamma_1) = 0.1$, $p(\gamma_2) = 0.5$, and $p(\gamma_3) = 0.4$, the Shannon capacity is given by

$$C = \sum_i B \log_2(1+\gamma_i) \rho(\gamma_i)$$
$$= 30 \times 10^3 \left[0.1 \log_2(1.83) + 0.5 \log_2(84.33) + 0.4 \log_2(334.33)\right] = 199.26\ \text{kbit/s}$$

The average SNR is $\boldsymbol{\bar{\gamma} = 0.1 \times 0.83 + 0.5 \times 83.33 + 0.4 \times 333.33 = 175.08\ (22.43\ \mathrm{dB})}$

The AWGN capacity is $\boldsymbol{C = B \log_2(1 + 175.08) = 223.8\ \mathrm{kbit/s}}$

Note that this rate is about 25 kbit/s larger than that of the flat-fading channel with receiver CSI and the same average SNR.

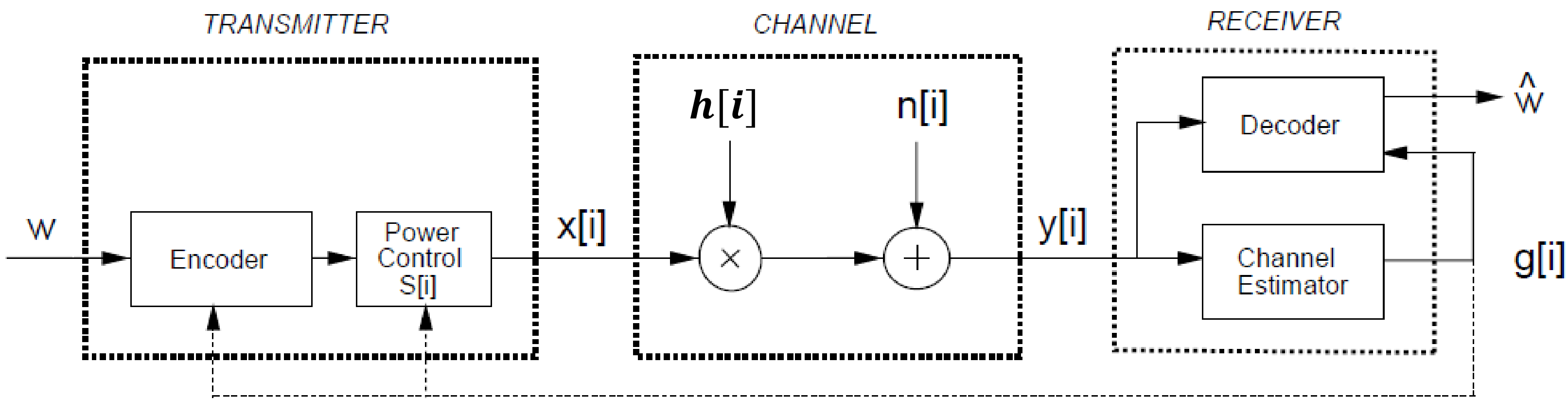
Capacity with Outage: **The transmitter fixes a minimum received SNR $\boldsymbol{\gamma_{min}}$ and encodes for a data rate $\boldsymbol{C = B\log_2(1 + \gamma_{min})}$. The probability of outage is $\boldsymbol{p_{out} = p(\gamma < \gamma_{min})}$. The average rate correctly received over many transmission bursts is:**

$$C_{out} = (1 - p_{out}) \times B \log_2(1 + \gamma_{\min})$$

since data is only correctly received on $\boldsymbol{(1 - p_{out})}$ transmissions.

4. Capacity of Wireless Channels

CSI at Transmitter and Receiver: When both the transmitter and receiver have CSI, the transmitter can adapt its transmission strategy relative to the CSI. It is possible to derive the Shannon capacity assuming optimal power and rate adaptation relative to the CSI.



The Tx only sends bits on channels it knows can be decoded. In this case the Shannon capacity is derived assuming optimal power and rate adaptation.

Shannon Capacity: The capacity of a fading channel with TX & Rx CSI is given by –

$$C = \int_0^\infty B \log_2(1+\gamma)\rho(\gamma)d\gamma \quad \text{Constrained by} \quad \int_0^\infty P(\gamma)\rho(\gamma)d\gamma \le \overline{P}$$

Where $P(\gamma)$ is the instantaneous Tx power subject to an average power constraint $\overline{P}$.

The capacity C is maximised when the power is optimally distributed over time as

4. Capacity of Wireless Channels

CSI at Transmitter and Receiver Condt: Shannon Capacity -

$$C = \max_{P(\gamma):\int_0^\infty P(\gamma)\rho(\gamma)d\gamma=\bar{P}} \int_0^\infty B\log_2\left(1+\frac{\gamma\cdot P(\gamma)}{\bar{P}}\right)\rho(\gamma)d\gamma$$

The method of Lagrangian is used to find the optimum power allocation $P(\gamma)$, $P(\gamma) > 0$, that maximises C -

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \dfrac{1}{\gamma_0} - \dfrac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases}$$

The channel is only used at time i if $\gamma_0 \leq \gamma[i] < \infty$ for some cut-off $SNR_0 = \gamma_0$ and C is

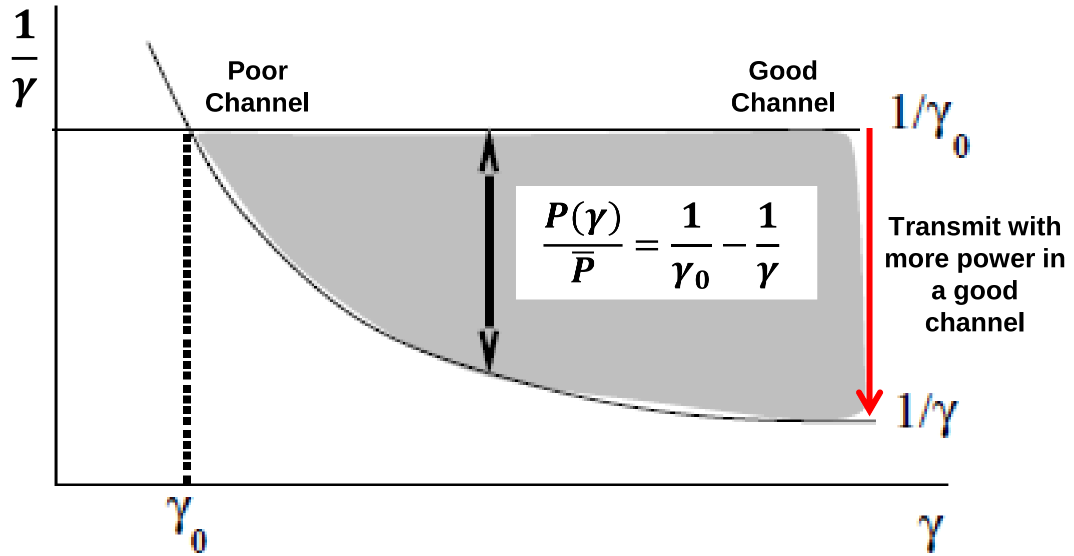
$$C = \int_{\gamma_0}^\infty B\log_2\left(1+\gamma\times\left[\frac{1}{\gamma_0}-\frac{1}{\gamma}\right]\right)\rho(\gamma)d\gamma = \int_{\gamma_0}^\infty B\log_2\left(\frac{\gamma}{\gamma_0}\right)\rho(\gamma)d\gamma$$

Rearranging the average power constraint when equality is achieved gives:

$$\int_0^\infty \frac{P(\gamma)}{\bar{P}}\rho(\gamma)d\gamma = 1 \quad \text{and } \gamma_0 \text{ must satisfy} \quad \int_{\gamma_0}^\infty \left(\frac{1}{\gamma_0}-\frac{1}{\gamma}\right)\rho(\gamma)d\gamma = 1$$

4. Capacity of Wireless Channels

CSI at Transmitter and Receiver Contd: Shannon Capacity - Since γ is time-varying, the power adaptation policy to maximise C is a "water-filling" formula in time, as illustrated –



The logic behind water-filling is to take advantage of good channel conditions: when channel conditions are good (γ large) more power and a higher data rate is sent over the channel. As channel quality degrades (γ small) less power and rate are sent over the channel. If the instantaneous channel SNR falls below the cut-off value $1/\gamma_0$, the channel is not used. Adaptive modulation and coding techniques used in HSPA, LTE and WiFi follow this principle.

4. Capacity of Wireless Channels

CSI at Transmitter and Receiver Condt: Example - Assume the same channel as in the previous example, with a bandwidth of 30 kHz and three possible received SNRs: γ_1 = 0.8333 with $p(\gamma_1)$ = 0.1, γ_2 = 83.33 with $p(\gamma_2)$ = 0.5, and γ_3 = 333.33 with $p(\gamma_3)$ = 0.4. Find the Ergodic capacity of this channel assuming both transmitter and receiver have instantaneous CSI.

Solution: The optimum power allocation is water-filling, therefore we need to find γ_0 that satisfies the discrete version of

$$\sum_{\gamma_i \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i} \right) \rho(\gamma_i) = 1$$

Use all of the channel states to obtain γ_0 by assuming $\gamma_0 \leq min\{\gamma_i\}$ and establish if the cut-off value is less that the minimum SNR.

$$\sum_{i=1}^{3} \frac{\rho(\gamma_i)}{\gamma_0} - \sum_{i=1}^{3} \frac{\rho(\gamma_i)}{\gamma_i} = 1 \quad \Rightarrow \frac{1}{\gamma_0} = 1 + \sum_{i=1}^{3} \frac{\rho(\gamma_i)}{\gamma_i} = 1 + \left(\frac{0.1}{0.8333} + \frac{0.5}{83.33} + \frac{0.4}{333.33} \right) = 1.13 \quad \Rightarrow \quad \gamma_0 = 1/1.13 = 0.885 > 0.8333 = \gamma_1$$

Since this value of γ_0 is greater than the SNR in the weakest channel, it is inconsistent as the channel should only be used for SNRs above the cut-off value.

Hence, we redo the calculation assuming that the weakest channel is not used, giving -

$$\sum_{i=2}^{3} \frac{\rho(\gamma_i)}{\gamma_0} - \sum_{i=2}^{3} \frac{\rho(\gamma_i)}{\gamma_i} = 1 \quad \Rightarrow \frac{0.9}{\gamma_0} = 1 + \sum_{i=2}^{3} \frac{\rho(\gamma_i)}{\gamma_i} = 1 + \left(\frac{0.5}{83.33} + \frac{0.4}{333.33} \right) = 1.0072$$

$$\Rightarrow \quad \gamma_0 = 0.9/1.0072 = 0.894 \quad \Rightarrow \quad \gamma_1 < \gamma_0 \leq \gamma_2$$

4. Capacity of Wireless Channels

CSI at Transmitter and Receiver Condt: Example - Contd

The capacity of the channel becomes -

$$C = \sum_{i=2}^{3} B \log_2\left(\frac{\gamma_i}{\gamma_0}\right)\rho(\gamma_i)$$

$$= 30\times10^3\left(0.5\log_2\left(\frac{83.33}{0.89}\right)+0.4\log_2\left(\frac{333.33}{0.89}\right)\right) = 200.82 \text{ kbit/s}$$

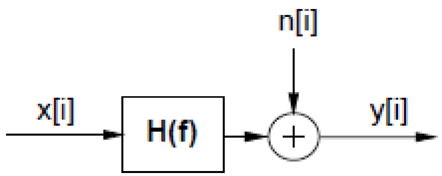
Compared to the results of the previous example, the capacity is only slightly higher than for the case of receiver CSI only, and is still significantly below that of an AWGN channel with the same average SNR.

It is a useful exercise to determine $\bar{P}$, $P(\gamma_1), and\ P(\gamma_2)$ for this example.

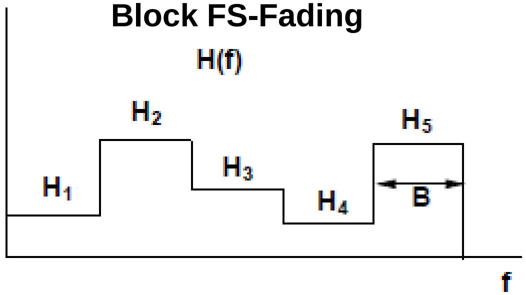
4. Capacity of Wireless Channels

Capacity of Frequency Selective (FS) Fading Channels: **This capacity analysis is similar to that of a flat fading channel with the time axis replaced by the frequency axis.**

Time Invariant Channels: **Consider a time-invariant channel with frequency response $H(f)$ and a total transmit power constraint P. When the channel is time-invariant it is reasonable to assume that the CSI is known at the transmitter and the receiver.**



Time-Invariant FS-Fading Channel



Assume that $H(f)$ is *Block Fading* where by frequency is divided into subchannels of bandwidth *B* and $H(f) = H_j$ is constant over each subchannel.

The FS fading channel is modelled as a set of independent parallel AWGN channels each of SNR $|H_j|^2 P_j / N_0 B$ on the *j-th* channel, where P_j is the power allocated on the *j-th* channel subject to the constraint $\sum_j P_j \leq P$.

The capacity is the sum of the rates on each channel with power optimally allocated/ channel -

$$C = \sum_{\max P_j : \sum_j P_j \leq P} B \log_2 \left(1 + \frac{|H_j|^2 P_j}{N_0 B} \right)$$

4. Capacity of Wireless Channels

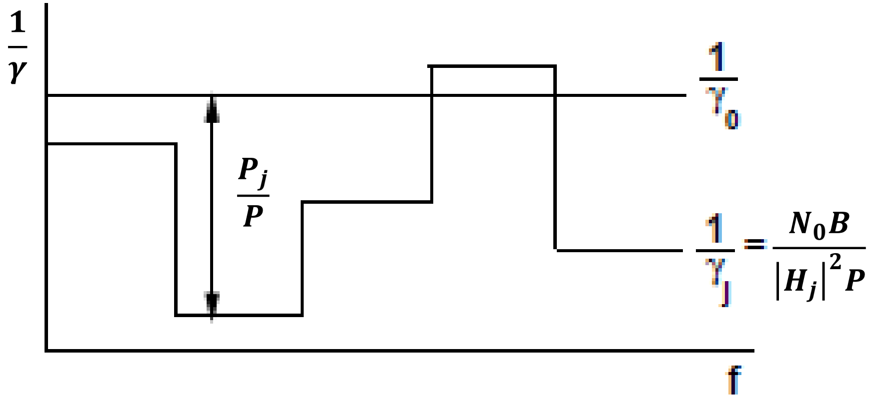
Capacity of Frequency Selective (FS) Fading Channels Contd: Similar to the flat fading case, the same Lagrange technique gives -

$$\frac{P_j}{P} = \begin{cases} \dfrac{1}{\gamma_0} - \dfrac{1}{\gamma_j} & \gamma_j \geq \gamma_0 \\ 0 & \gamma_j < \gamma_0 \end{cases}$$

Where γ_0 is the cut-off SNR and $\gamma_j = |H_j|^2 P/N_0 B$ is the SNR of the *j-th* channel when allocated the entire power budget P. Then γ_0 must satisfy –

$$\sum_j P_j = P \quad \Rightarrow \quad \sum_j \frac{P_j}{P} = 1 \quad \Rightarrow \quad \sum_j \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_j} \right) = 1$$

The optimum power allocation and Capacity are -



$$C = \sum_{j:\gamma_j \geq \gamma_0} B \log_2 \left(\frac{\gamma_j}{\gamma_0} \right)$$

This capacity is achieved by sending at different rates and powers over each subchannel. Multicarrier modulation uses the same technique in adaptive loading.

4. Capacity of Wireless Channels

Capacity of Frequency Selective (FS) Fading Channels Contd: Example – Consider a time-invariant frequency-selective block fading channel consisting of three subchannels of bandwidth B = 1 MHz.

The frequency response associated with each channel is $H_1 = 1$, $H_2 = 2$ and $H_3 = 3$.

The transmit power constraint is P = 10 mW and the noise PSD is $N_0 = 10^{-9}$ W/Hz.

Find the Shannon capacity of this channel

Solution - First find $\gamma_j = |H_j|^2 P / N_0 B$ for each subchannel, which gives $\gamma_1 = 10$, $\gamma_2 = 40$ and $\gamma_3 = 90$.

When all subchannels are allocated power, we have –

$$\sum_{j=1}^{3}\left(\frac{1}{\gamma_0}-\frac{1}{\gamma_j}\right)=1 \quad\Rightarrow\quad \frac{3}{\gamma_0}=1+\sum_{j=1}^{3}\left(\frac{1}{\gamma_j}\right)=1.14 \quad\Rightarrow\quad \gamma_0=2.64<\gamma_j \ \forall j$$

Since the cut-off γ_0 is less than γ_j for all j, the assumption that all subchannels are allocated power is consistent, so this is the correct cut-off value.

The corresponding capacity is given by –

$$C=\sum_{j=1}^{3} B\log_2\left(\frac{\gamma_i}{\gamma_0}\right)=10^6\left(\log_2\left(\frac{10}{2.64}\right)+\log_2\left(\frac{40}{2.64}\right)+\log_2\left(\frac{90}{2.64}\right)\right)=10.93\ \text{Mbit/s}$$

A useful exercise is to find the optimal power allocation that achieves this capacity.

Summary & Main Points:

- **Shannon's channel capacity determines maximum data rates on channels with asymptotically small error probability.**
- **The capacity of a wireless channel depends what is known about the time-varying channel at the transmitter and the receiver**
- **The capacity of flat fading channels when only $\rho[\gamma]$ is known are largely unsolved.**
- **For a flat fading channel when CSI at receiver only is known, the Shannon or Ergodic capacity is given by a probabilistic average.**
- **For a flat fading channel when CSI at transmitter and receiver is known, the power adaptation policy to maximise Shannon capacity is a "water-filling" formula in time**
- **For a frequency selective channel when CSI at the transmitter and receiver is known, the power adaptation policy to maximise Shannon capacity is a "water-filling" formula in frequency.**