Ac (time varying) circuits

— obey the same rules as dc circuits but there are extra things to take into account.

Components

— capacitors — potential energy stores
— inductors.

usually consist of two plates separated by a "dielectric"

governed by a relationship

$$\boxed{I = C\frac{dV}{dt}}$$

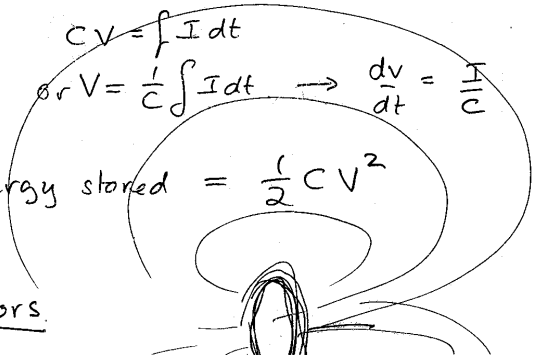
$$Q = CV$$

$$Q = \int I\,dt \longrightarrow I = \frac{dQ}{dt}.$$

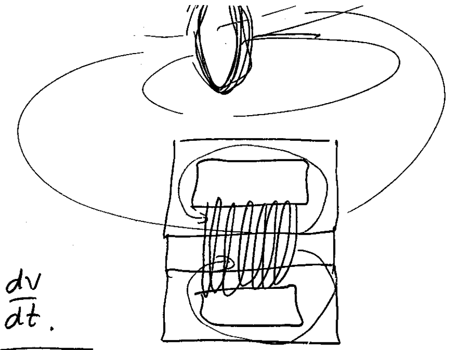
$$CV = \int I\,dt$$

$$\text{or } V = \frac{1}{C}\int I\,dt \longrightarrow \frac{dV}{dt} = \frac{I}{C}$$

$$\text{Energy stored} = \frac{1}{2}CV^2$$



Inductors.

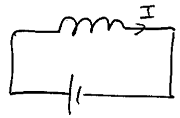


$$I = C\frac{dV}{dt}.$$

$$\boxed{V = L\frac{dI}{dt}}$$

$$I = \frac{1}{L}\int V\,dt.$$

$$V = \frac{1}{C}\int I\,dt.$$



Stored energy in an inductor is $\frac{1}{2}LI^2$

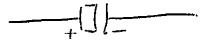
$\frac{1}{2}CV^2$

Symbolic representations

capacitor —||— non-polarised capacitor

plastic dielectrics
mica dielectrics

air —
ceramic —

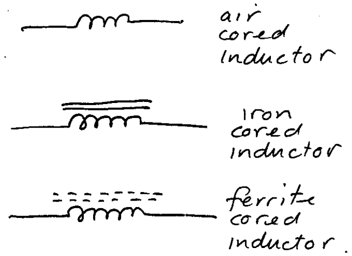


polarised

inductors

$$\boxed{\begin{aligned} I &= C\frac{dV}{dt} \\ V &= L\frac{dI}{dt} \end{aligned}}$$

CIVIL



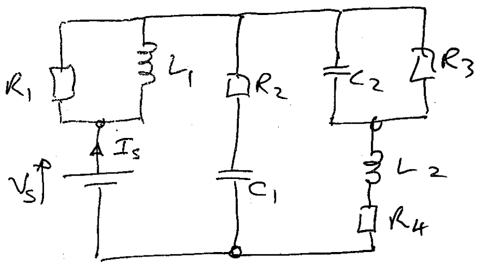
Behaviour of C + L at dc

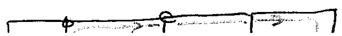
at dc (ie 0Hz)

$\frac{dI}{dt} = 0$ everywhere in the cct

$\frac{dV}{dt} = 0$ everywhere in the cct.

So





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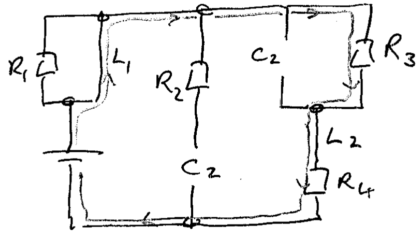
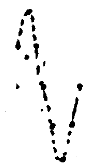
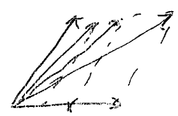


at dc.

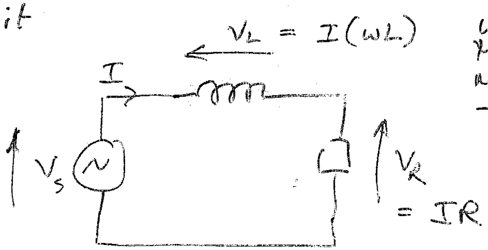
$$I_s = \frac{V_s}{R_3 + R_4}$$

Representing ac voltages + currents.





Applying vector ideas to a circuit



ωL is the inductive reactance.
→ a.c. equivalent of resistance

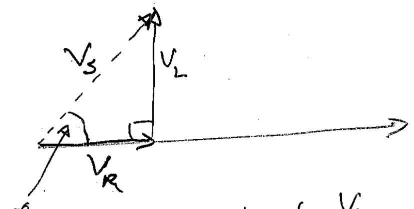
need to express V_L in terms of I

$$V_L = L\frac{dI}{dt}$$

so if I drives L and I is $I_p \sin \omega t$

$$V_L = \underbrace{L\, I_p \omega}_{\text{this is the magnitude of}} \cos \omega t$$

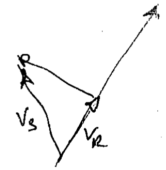
to draw a vector diagram — use I as a reference direction because it is common to all components.

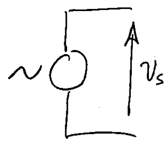


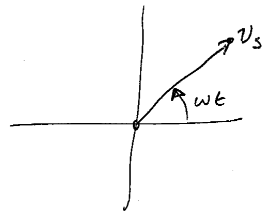
ϕ — $\phi = \tan^{-1} \frac{V_L}{V_R} = \tan^{-1}\left(\frac{\omega L}{R}\right)$

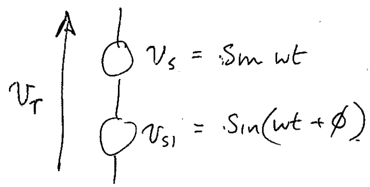
ϕ — $\phi = \tan^{-1} \frac{V_L}{V_R} = \tan^{-1}\left(\frac{\omega L}{R}\right)$

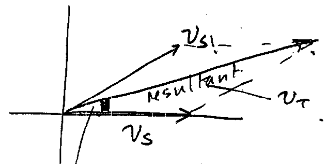
$$V_S^2 = V_L^2 + V_R^2$$
$$= I^2\omega^2L^2 + I^2R^2$$
$$= I^2(\omega^2L^2 + R^2)$$
$$\frac{V_S}{I} = |Z| = \sqrt{\omega^2L^2 + R^2}$$







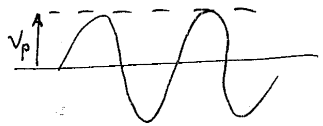




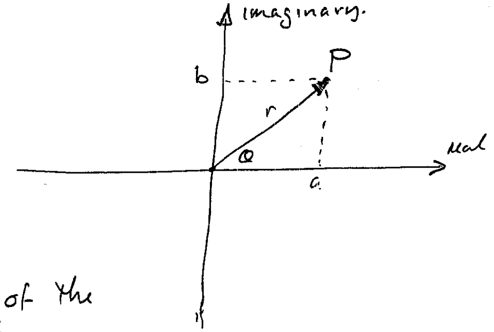
resultant phase.

for a sinusoid $V = V_p \sin(\omega t + \phi)$.

amplitude



Complex number representation of a.c.



coordinates of the point P are

$a + jb \Rightarrow$ Cartesian representation

$r = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1} b/a$

if $\theta = \omega t$, P would rotate in an anti-clockwise direction at a rate of ω rad s^{-1}

an alternative expression of the position of P is $re^{j\theta} \Rightarrow$ polar representation

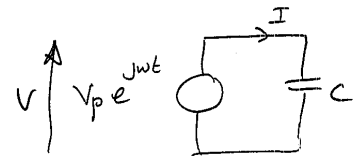
Complex numbers are easy to manipulate if the appropriate form is used.

→ addition and subtraction are best done in cartesian $(a + jb)$ form

→ multiplication & division are best done in polar form.

The rotating ac phasor is

$$V_p e^{j\omega t}.$$



$$I = C\frac{dV}{dt} = CV_p \omega j e^{j\omega t}$$

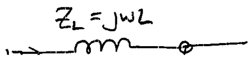
$$Z_c = \frac{V}{I} = \frac{\cancel{V_p e^{j\omega t}}}{jC\cancel{V_p}\omega \cancel{e^{j\omega t}}}$$

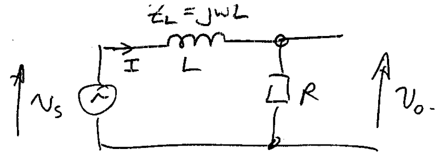
$$= \frac{1}{j\omega C} = -\frac{j}{\omega C} = -jX_c$$

Similarly for an inductor

$$Z_L = j\omega L. = jX_L.$$

Using "j" on circuits



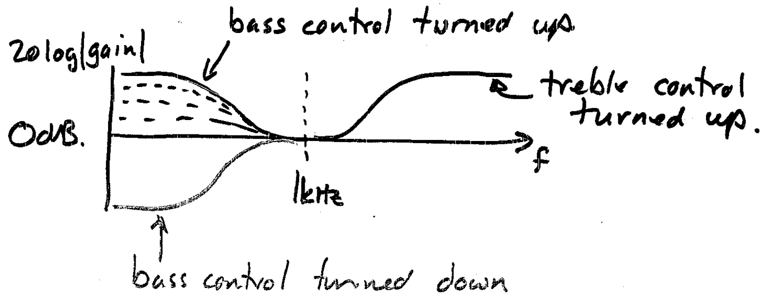


$$I = \frac{v_s}{Z_L + R} = \frac{v_s}{jwL + R}.$$

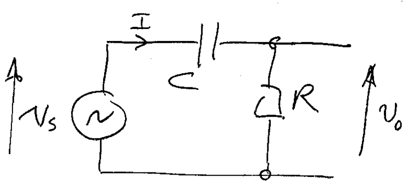
$$\therefore v_o = IR = \frac{v_s R}{jwL + R} = \frac{v_s}{1 + jw^L/_R}.$$

$$\text{ie } \frac{v_o}{v_i} = \frac{1}{1 + jw^L/_R}.$$

$$\left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{1^2 + (w^L/_R)^2}} \qquad \angle\left(\frac{v_o}{v_i}\right) = -\tan^{-1}\left(\frac{w^L/_R}{1}\right)$$

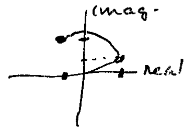


The CR circuit



$$Z_c = \frac{1}{jwC}$$

Impedance of circuit from V_s's point of view is



$$Z = Z_c + R$$

$$I = \frac{v_s}{Z}$$

$$v_o = IR = \frac{v_s}{Z} R = v_s \frac{R}{Z_c + R}$$

$$= v_s \frac{R}{\frac{1}{jwC} + R}$$

$$= v_s \frac{jwCR}{1 + jwCR}$$

CR = time constant of the cct.

$$C = \frac{Q}{V} = \frac{It}{V} = \frac{\text{Amps} \times \text{Time}}{\text{Volts}}.$$

$$R = \frac{\text{Volts}}{\text{Amps}}$$

$$CR = \frac{\cancel{\text{Amps}} \times \text{Time}}{\cancel{\text{Volts}}} \times \frac{\cancel{\text{Volts}}}{\cancel{\text{Amps}}}$$

$$= \text{Time}$$

lets define a frequency domain constant — say w_o or w_c such that w_o (or w_c or ....) $= \frac{1}{CR}$.

$$\therefore \frac{v_o}{v_s} = \frac{j^{w/w_o}}{1 + j^{w/w_o}} = \frac{j^{f/f_o}}{1 + j^{f/f_o}}.$$

$2\log \left|\frac{v_o}{v_s}\right|$ as a function of f.

$$\left|\frac{v_o}{v_s}\right| = \frac{f/f_o}{\sqrt{1+(f/f_o)^2}}$$

$$= \frac{1}{\sqrt{\left(\frac{f_o}{f}\right)^2 + 1}}$$

$$20\log\left|\frac{v_o}{v_s}\right| = 20\log\left[\frac{f/f_o}{\sqrt{1+(f/f_o)^2}}\right]$$

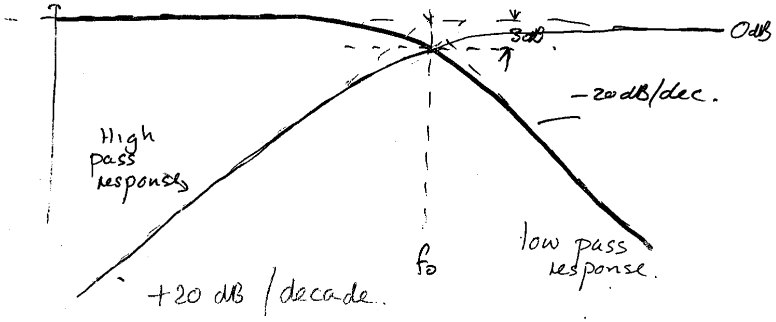
$$= 20\log\frac{f}{f_o} + 20\log\left[\frac{1}{\sqrt{1+(f/f_o)^2}}\right]$$

$f \ll f_o$ $\quad \frac{1}{\sqrt{1+(f/f_o)^2}} \Rightarrow \frac{1}{\sqrt{1}} \Rightarrow 0\,\text{dB}$.
since $\left(\frac{f}{f_o}\right)^2 \ll 1$.

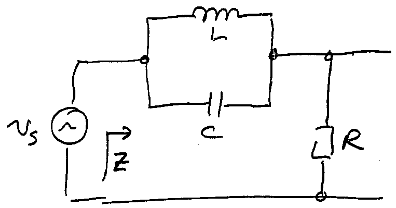
$f = f_o$ $\quad \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} \Rightarrow -3\,\text{dB}$.

$f \gg f_o$ $\quad \frac{1}{\sqrt{f/f_o{}^2}}$ since $\left(\frac{f}{f_o}\right)^2 \gg 1$ $\quad \frac{v_o}{v_s} \Rightarrow \frac{f_o}{f}$

-20 dB/decade
constant slope
on a log-log graph.



Another cct from the lab.....



$$Z_1 \| Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z = (Z_L \| Z_C) + R \qquad Z_L = j\omega L, \quad Z_C = \frac{1}{j\omega C}.$$

$$= \frac{j\omega L / j\omega C}{j\omega L + \frac{1}{j\omega C}} + R.$$

$$= \frac{j\omega L}{j\omega L \cdot j\omega C + 1} + R$$

$$= \frac{j\omega L + R(1 + j^2\omega^2 LC)}{1 + j^2\omega^2 LC}$$

but $j^2 = -1$

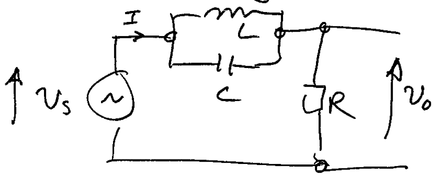
$$= \underline{j\omega L + R - \omega^2 LCR}$$

$$= \frac{j\omega L + R - \omega^2 LCR}{1 - \omega^2 LC}$$

$$= R\left(\frac{j\omega L/R + 1 - \omega^2 LC}{1 - \omega^2 LC}\right)$$

$$|Z| = \frac{R}{1 - \omega^2 LC}\left[(1 - \omega^2 LC)^2 + \frac{\omega^2 L^2}{R^2}\right]^{1/2}$$

To work out gain



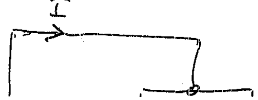
$$I = \frac{v_s}{Z} \quad \therefore \quad v_o = I.R = \frac{v_s R}{Z}$$

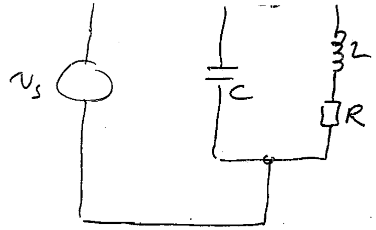
$$\frac{v_o}{v_s} = \frac{R}{Z} = \frac{\cancel{R}}{\cancel{R}\left(\frac{j\omega L/R + 1 - \omega^2 LC}{1 - \omega^2 LC}\right)}$$

$$= \frac{1 - \omega^2 LC}{j\omega \frac{L}{R} + 1 - \omega^2 LC}$$

what happens if L has some resistance?

What is $\frac{V_s}{I}$





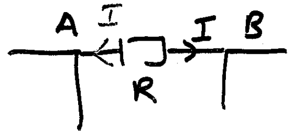
$$Z = Z_C \,//\, (R + Z_L)$$

$$= \frac{1}{j\omega C} \,//\, (R + j\omega L)$$

$$= \frac{\frac{1}{j\omega C}\cdot(R + j\omega L)}{\frac{1}{j\omega C} + R + j\omega L} \times \frac{j\omega C}{j\omega C}$$

$$= \frac{R + j\omega L}{1 + j\omega CR + j^2\omega^2 LC}$$

$$= \frac{R + j\omega L}{1 - \omega^2 LC + j\omega CR}$$



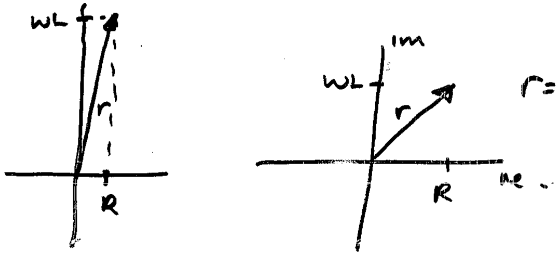
$$I = \frac{V_B - V_A}{R}$$

$$I = \frac{V_A - V_B}{R}$$

$$|Z| = \frac{|R + j\omega L|}{|1 - \omega^2 LC + j\omega CR|}$$

$$= \sqrt{R^2 + \omega^2 L^2}$$

$$= \frac{\sqrt{R^2+\omega^2L^2}}{\sqrt{(1-\omega^2LC)^2+\omega^2C^2R^2}}$$



$$r = \sqrt{R^2+\omega^2L^2}$$

$$\angle Z = \tan^{-1}\frac{\omega L}{R} - \tan^{-1}\frac{\omega CR}{1-\omega^2LC}$$

Power in ac circuits (page 14).

The problem is that V + I are functions of time.

Only makes sense to talk of power in waveform environments where shapes are periodic — ie one cycle is the same as the next.

For power the relationship that never lets you down is

$$P = \frac{1}{T}\int_0^T V(t)\, I(t)\, dt.$$

where T is the periodic time of a periodic waveform.

so if $V(t) = V_p \sin\omega t$
and $I(t) = I_p \sin\omega t$

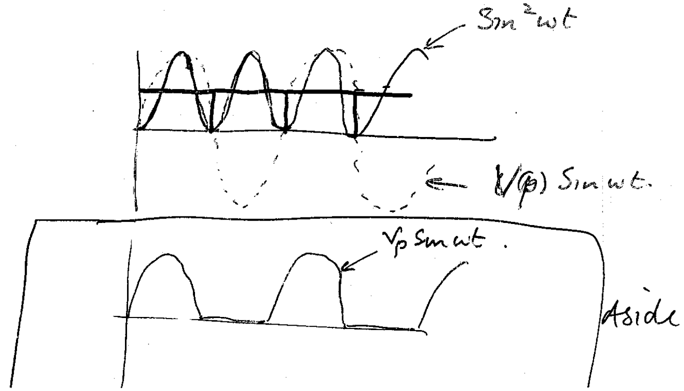
$$P = \frac{1}{T}\int_0^T V_p \sin\omega t \times I_p \sin\omega t\, dt$$

$$= \frac{1}{T}\int_0^T \frac{V_p^2}{R} \sin^2\omega t\, dt$$

$$= \frac{1}{T}\int_0^T I_p^2 R \sin^2\omega t\, dt.$$

$$= \frac{1}{T}\int_0^T \frac{V_p^2}{R}\left(\frac{1-\cos 2\omega t}{2}\right) dt.$$

$$= \frac{1}{T}\frac{V_p^2}{2R}\int_0^T (1-\cos 2\omega t)\, dt.$$



$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{or} \quad T = \frac{2\pi}{\omega}$$

$$\text{or} \quad \frac{T}{2} = \frac{\pi}{\omega}.$$

$$P = \frac{1}{\pi/\omega} \frac{V_p^2}{2R} \int_0^{\pi/\omega} (1 - \cos 2\omega t)\, dt$$

$$= \frac{V_p^2 \omega}{2R\pi} \left[t - \frac{1}{2\omega} \sin 2\omega t \right]_0^{\pi/\omega}$$

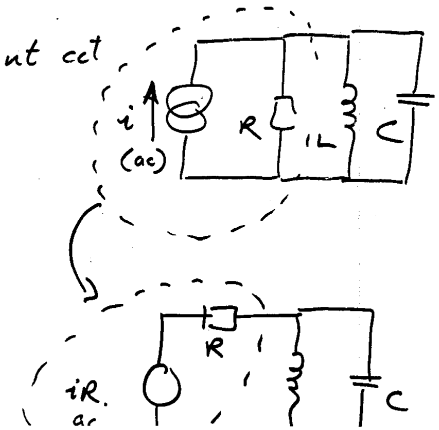
$$= \frac{V_p^2 \omega}{2R\pi} \left[\frac{\pi}{\omega} - 0 - \cancel{\frac{1}{2\omega} \sin \frac{2\omega\pi}{\omega}} + \cancel{\frac{1}{2\omega} \sin 0} \right]$$

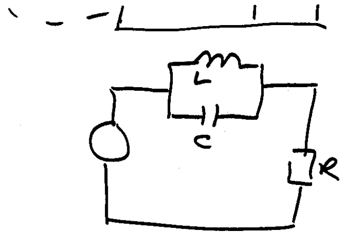
$$= \frac{V_p^2 \cancel{\omega} \cancel{\pi}}{2R \cancel{\pi} \cancel{\omega}} = \frac{1}{R} \cdot \frac{V_p^2}{2}$$

mean squared voltage.

ASIDE

classic // resonant cct





mean squared voltage is often square rooted to give an equivalent voltage with units of volts. This is V_{RMS} → the root-mean-square voltage.

$$\sqrt{\overline{V^2_{(t)}}} = \text{RMS voltage}.$$

Phase difference

$$V_s = V_p \sin \omega t$$

$$I = I_p \sin(\omega t + \phi)$$

$$\frac{V_p}{I_p} = |Z|$$

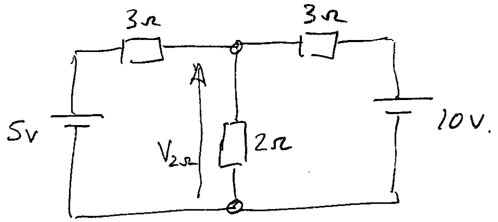
$$P = \frac{1}{T} \int_0^T V_s(t)\, I(t)\, dt = \frac{1}{T} \int_0^T V_p \sin \omega t\, I_p \sin(\omega t + \phi)\, dt$$

gives

$$P = \frac{V_p I_p}{2} \cos\phi = \frac{V_p^2}{2} \cos\phi = \frac{V_{RMS}^2}{2} \cos\phi$$

$$P = \frac{V_p I_p}{2}\cos\phi = \frac{V_p^2}{2|Z|}\cos\phi = \frac{V_{RMS}^2}{|Z|}\cos\phi$$

$$= \frac{I_p^2}{2}|Z|\cos\phi$$



what is the power dissipated in the 2Ω resistor?

Some people have used superposition

$$V_{2\Omega} \text{ due to } 5V = 5 \times \frac{2\|3}{3+2\|3} = \frac{5 \times 6/5}{21/5}$$

$$= \frac{6 \times 5}{21} = \frac{10}{7}\ V$$

So $P_{2\Omega}$ due to $5V = \left(\frac{10}{7}\right)^2 / R = \frac{100}{2 \times 49}$ W. $\approx 1W$. ✗

$$V_{2\Omega} \text{ due to } 10V = 10 \times \frac{2\|3}{3+2\|3} = \frac{20}{7}\ V$$ ✗

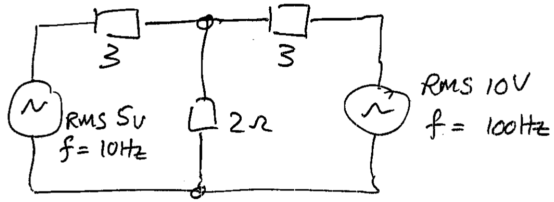
$$P_{2\Omega} \text{ due to } 10V = \frac{400}{2 \times 49} \approx 4W$$

Some people said $P_{2\Omega} = \dfrac{\left(V_{2\Omega}\big|_{5V} + V_{2\Omega}\big|_{10V}\right)^2}{R}$

$$= \left(\frac{30}{7}\right)^2 \times \frac{1}{2}$$

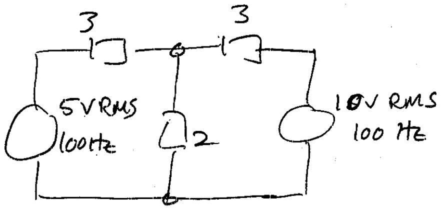
$$= \frac{900}{2 \times 49} \approx 9W. ✓$$

Now consider a different problem



$P_{2\Omega}$ due to 5V 10Hz $\approx 1W$
$P_{2\Omega}$ due to 10V 100Hz $\approx 4W$
$\Big\}$ $P_{TOT} \approx 5W$.

If voltages are added first
$P_{2\Omega} = 9W$.



Same result as the d.c source case.

The mean squared value of

Consider a sum of two a c voltage sources.

$$\overline{(V_1+V_2)^2} = \overline{V_1^2 + 2V_1V_2 + V_2^2}$$

$$= \overline{V_1^2} + \overline{V_2^2} + \overline{2.V_1V_2}$$

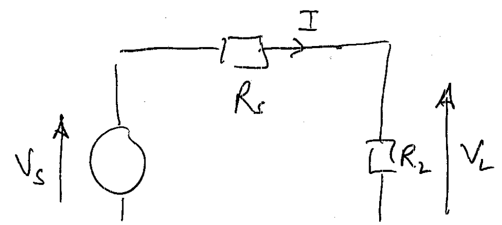
— one dc + one ac voltage

$$\overline{(V_1 + V_p \sin wt)^2}$$

$$= \overline{V_1^2} + \overline{V_p^2 \sin^2 wt} + \overline{2V_1 V_p \sin wt}$$

$$\overline{2V_{p1}V_{p2} \sin w_1 t \sin w_2 t.}$$

Maximum Power Transfer.



want to find the value of R_L that will maximise transfer of power from V_s to R_L.

$$I = \frac{V_s}{R_s + R_L}$$

$$V_L = IR_L = \frac{V_s}{R_s+R_L} \cdot R_L$$

P_L = power transferred to R_L

$$= \frac{V_L^2}{R_L} = \frac{\frac{V_s^2 R_L^2}{(R_s+R_L)^2}}{R_L}$$

$$= \frac{V_s^2 R_L}{(R_s+R_L)^2}$$

now we need to differentiate P_L w.r.t R_L to find max P_L

alternatively look for a minimum in $\frac{1}{P_L}$

$$\frac{d\frac{1}{P_L}}{dR_L} = \frac{d\left(\frac{(R_s+R_L)^2}{V_s^2 R_L}\right)}{dR_L}$$

$$= d\left(\frac{R_s^2 + 2R_sR_L + R_L^2}{V_s^2 R_L}\right)$$

$$\frac{V_s^2 R_L}{dR_L}$$

$$= \frac{1}{V_s^2} \frac{d\left(\frac{R_s^2}{R_L} + 2R_s + R_L\right)}{dR_L}$$

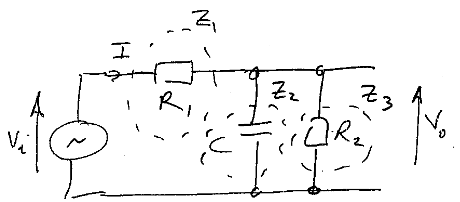
$$0 = \frac{1}{V_s^2}\left[-1R_L^{-2}R_s^2 + 0 + 1\right]$$

$$0 = \frac{1}{V_s^2}\left[-\frac{R_s^2}{R_L^2} + 1\right]$$

$R_s^2 = R_L^2$ for min in $\frac{1}{P_L}$ or max in P_L.

or $R_s = R_L$

Start with an R-C circuit



what impedance does V_i see (ie what is $\frac{V_i}{I}$?)

what is $\frac{V_o}{V_i}$?

$$Z = \frac{V_i}{I} = Z_1 + Z_2 /\!/ Z_3$$

$$= R_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} = R_1 + \frac{\frac{1}{j\omega C} \cdot R_2}{\frac{1}{j\omega C} + R_2}$$

$$= R_1 + \frac{R_2}{1 + j\omega C R_2}$$

$$= \frac{R_1 + j\omega C R_2 R_1 + R_2}{1 + j\omega C R_2} = \frac{(R_1 + R_2) + j\omega C R_1 R_2}{1 + j\omega C R_2}$$

$$= \frac{(R_1 + R_2)\left(1 + j\omega C \frac{R_1 R_2}{R_1 + R_2}\right)}{1 + j\omega C R_2}$$

what about $\frac{V_o}{V_i}$?

$$I = \frac{V_i}{Z}$$

$$V_o = I \, . \, Z_2 /\!/ Z_3 = I \frac{R_2}{1 + j\omega C R_2}$$

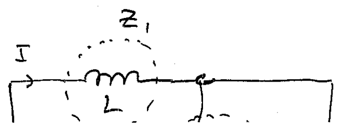
$$\frac{v}{v_i} = \frac{Z_2 /\!/ Z_3}{Z} = \frac{\frac{R_2}{\cancel{1 + j\omega C R_2}}}{(R_1 + R_2)\left(\frac{1 + j\omega C \frac{R_1 R_2}{R_1 + R_2}}{\cancel{1 + j\omega C R_2}}\right)}$$

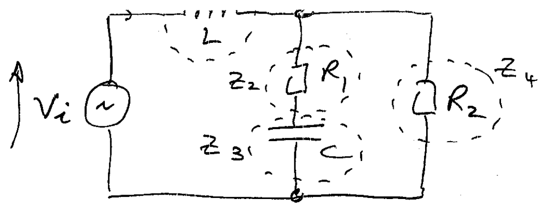
$$= \frac{R_2}{(R_1 + R_2)} \cdot \frac{1}{1 + j\omega C \frac{R_1 R_2}{R_1 + R_2}}$$

$$= k \, . \, \frac{1}{1 + j\, f/f_0}$$

where $k = \frac{R_2}{R_1 + R_2}$

and $f_0 = \frac{1}{2\pi C (R_1 /\!/ R_2)}$





what does Z look like as far as V_i is concerned.

$$Z_T = \frac{V_i}{I} = Z_1 + \left[Z_4 // (Z_2 + Z_3) \right]$$

let $j\omega = s$.

$$Z_1 = j\omega L \Rightarrow sL \qquad Z_2 = R_1 \qquad Z_3 = \frac{1}{j\omega C} \Rightarrow \frac{1}{sC}$$

$$Z_4 = R_2.$$

$$\therefore Z_T = sL + \frac{Z_4(Z_2+Z_3)}{Z_4+Z_2+Z_3}$$

$$= sL + \frac{R_2\left(R_1 + \frac{1}{sC}\right)}{R_2 + R_1 + \frac{1}{sC}}$$ multiply top & bottom by sc.

$$= sL + \frac{R_2(sCR_1+1)}{sC(R_2+R_1)+1}$$

$$= \frac{s^2LC(R_2+R_1) + sL + R_2sCR_1 + R_2}{1+sC(R_2+R_1)}$$

$$= \frac{R_2 + s(L+CR_1R_2) + s^2LC(R_2+R_1)}{1+sC(R_2+R_1)}$$

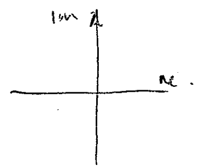
putting $s = j\omega$

$$= \frac{R_2 - \omega^2LC(R_2+R_1) + j\omega(L+CR_1R_2)}{1+j\omega C(R_1+R_2)}$$

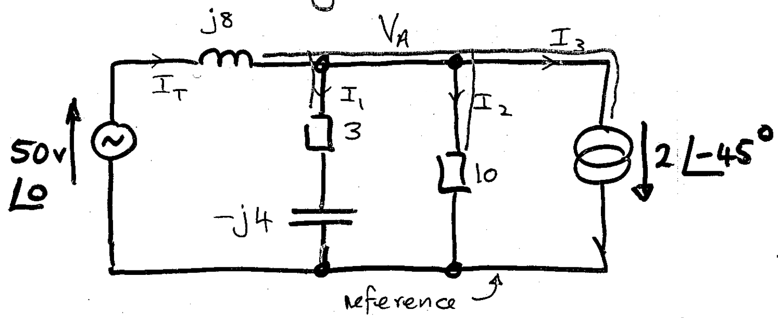
$$|Z| = \frac{|V|}{|I|} = \frac{V_p}{I_p} = \frac{V_{rms}}{I_{rms}}.$$

$$|Z| = \frac{\left[\left[R_2 - \omega^2LC(R_2+R_1)\right]^2 + \omega^2(L+CR_1R_2)^2\right]^{1/2}}{\left[1+\omega^2C^2(R_1+R_2)^2\right]^{1/2}}.$$

$$\phi(z) = \tan^{-1}\frac{\omega(L+CR_1R_2)}{R_2-\omega^2LC(R_2+R_1)} - \tan^{-1}\frac{\omega C(R_1+R_2)}{1}$$



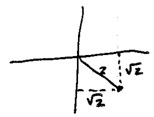
green = V_A node.



What is V_A (with respect to the reference).

Sum currents at V_A node

$$I_T = I_1 + I_2 + I_3$$



$$\frac{50 - V_A}{j8} = \frac{V_A}{3 + (-j4)} + \frac{V_A}{10} + 2\angle -45$$

$$\frac{50 - V_A}{j8} = \frac{V_A}{3 - j4} + \frac{V_A}{10} + \sqrt{2} - j\sqrt{2}$$

$$\frac{-50j + jV_A}{8} = \frac{V_A(3+j4)}{\underbrace{3^2+4^2}_{=25}} + \frac{V_A}{10} + \sqrt{2} - j\sqrt{2}$$

$$-50 \times 25 j + 25 j V_A = 24 V_A + 32 j V_A + 20 V_A + 200\sqrt{2} - 200 j\sqrt{2}$$

$$-1250 j + 200\sqrt{2}\, j - 200\sqrt{2} = V_A\left[24 + 20 + j(32 - 25)\right]$$

$$-967 j - 283 = V_A\left[44 + j(7)\right]$$

$$V_A = \frac{-967 j - 283}{44 + j7} = \frac{1008\angle 73.7 - 180}{44.03\angle 9.04^\circ}$$

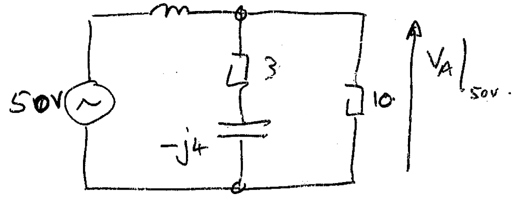
$$= \frac{1008\angle -106.3}{44\angle 9^\circ}$$

$$V_A = \frac{1008}{44}\angle -106.3 - 9$$

$$= 22.9\angle -115.3^\circ$$

Lets try using superposition.

j8 partial cct for 50V source.



V_A due to 50 V

$$= 50\,\frac{10 /\!/ (3 - j4)}{j8 + 10 /\!/ (3 - j4)}$$

$$= 50\,\frac{\frac{10(3-j4)}{10+3-j4}}{j8 + \frac{10(3-j4)}{10+3-j4}}$$

$$= 50\,\frac{10(3-j4)}{j8(10+3-j4) + 10(3-j4)}$$

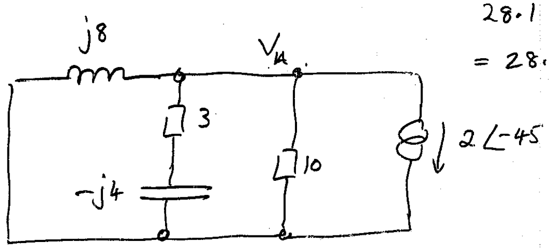
$$= \frac{500(3-j4)}{104j + 32 + 30 - 40j} = \frac{500(3-j4)}{64j + 62}$$

$$= 500 \cdot \frac{5\angle -53}{89\angle 46}$$

$$28.1\angle -53 - 46$$

$$= 28.1\angle -99$$

partial circuit for $2\angle -45$



Impedance seen by current source looking into the circuit is

$10 \parallel j8 \parallel (3-j4).$

$$\therefore \frac{1}{Z_T} = \frac{1}{10} + \frac{1}{3-j4} + \frac{1}{j8}$$

$$= \frac{1}{10} + \frac{3+j4}{25} - \frac{j}{8}$$

$$= \frac{20 + 24 + j32 - j25}{200.} = \frac{44+7j}{200.}$$

$$\therefore Z_T = \frac{200}{44+7j}$$

$$V_A = -\left[2\angle{-45}\right] \times Z_T = \frac{-\sqrt{2} + j\sqrt{2}}{44+7j}.\ 200$$

$$\approx \left[\frac{2\angle 135}{44\angle 9^\circ}\right] 200$$

$$\left[\frac{1}{22}\angle 135-9\right] 200$$

$$= 9.1\angle 126^\circ$$

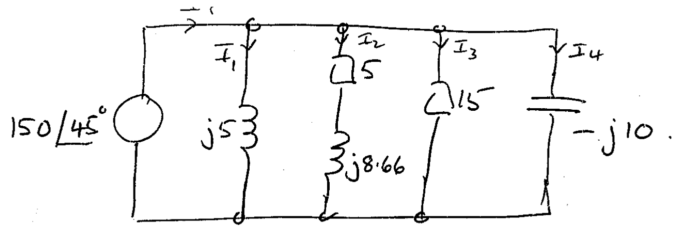
$$V_{A\,TOT} = 28.1\angle{-99^\circ} + 9.0\angle 126^\circ$$

$$-4.41 - 27.7j \qquad -5.28 + j7.27$$

$$= -9.7 - 20.4j$$

$$\equiv 22.6\angle{-115.4}$$

Another circuit example.



Find total current I_T and cct impedance from source point of view.

method 1

$$\frac{1}{Z} = \frac{1}{j5} + \frac{1}{5+j8.66} + \frac{1}{15} + \frac{1}{-j10}.$$

$$= \frac{-j}{5} + \frac{5-j8.66}{100} + \frac{1}{15} + \frac{j}{10}.$$

$$= \frac{-30j + 1.5(5-j8.66) + 10 + 15j}{150}$$

$$= \frac{-27.99j + 17.5}{150.}$$

$$\therefore Z = \frac{150}{17.5 - 27.99j} = \frac{150(17.5 + 27.99j)}{1090.}$$

$$= 0.138(17.5 + 27.99j)$$

$$\equiv 0.138 \times 33\angle 58^\circ$$

$$= 4.55\angle 58^\circ.$$

$$Z = \frac{V}{I_T} \qquad I_T = I_1 + I_2 + I_3 + I_4$$

$$I = 150\angle 45 \qquad I_{\cdot} = \frac{150\angle 45}{}$$

$$I_1 = \frac{150\angle 45}{j5} \qquad I_2 = \frac{150\angle 45}{5+j8.66}$$

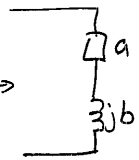
$$I_3 = \frac{150\angle 45}{15} \qquad I_4 = \frac{150\angle 45}{-j10.}$$

move the problem back by 45°

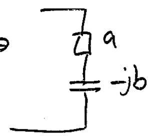
$$I_1 = \frac{150}{j5} \qquad I_2 = \frac{150}{5+j8.66} \qquad I_3 = \frac{15}{15}$$

$$I_4 = \frac{150}{-j10.}$$

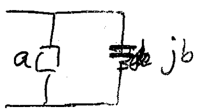
$Z = a + jb.$ $\quad Z \longrightarrow$



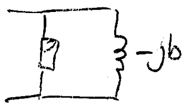
if $Z = a - jb$ $\quad Z \rightarrow$



$Y = a + jb$ $\quad Y \rightarrow$



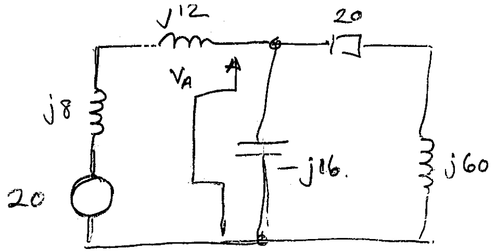
$Y = a - jb.$ $\quad Y =$



$$Z_L = j\omega L$$

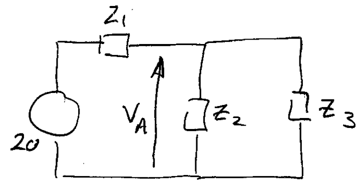
$$Y_L = \frac{1}{j\omega L} = -\frac{j}{\omega L}.$$

j12 20



find V_A.

$\equiv$



$$Z_1 = j20.\left[j12 + j8\right]$$

$$Z_2 = -j16$$

$$Z_3 = 20 + j60$$

$$V_A = 20 \times \frac{Z_2 \parallel Z_3}{Z_1 + Z_2 \parallel Z_3}$$

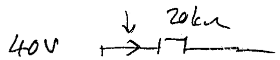
$$= \frac{20 \times \dfrac{-j16(20+j60)}{-j16+20+j60}}{j20 + \dfrac{-j16(20+j60)}{-j16+20+j60}}$$

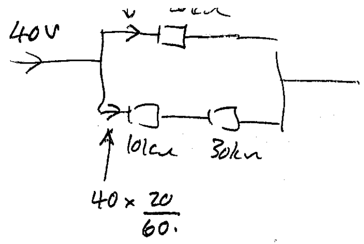
$$= \frac{20 \times \left[-j320 - j^2 960\right]}{j20(-j16+20+j60) - j320 - j^2 960}.$$

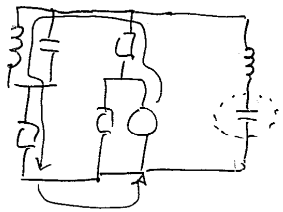
$$= \frac{20 \left[-j320 - j^2 960\right]}{-j^2 320 + 400j + 1200j^2 - 320j - j^2 960}$$

$$j^2 = -1$$

$$40 \times \frac{40}{60}$$





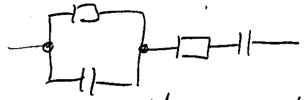


Resonance

Resonance is a property of circuits or systems that contain two or more independent reactances.

→ eg an L and a C

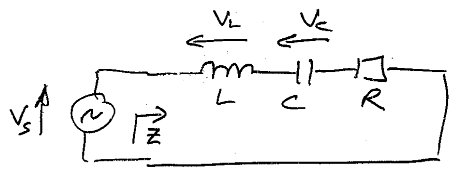
or two Cs separated by resistors in such a way that knowledge of voltage across one C gives no information about voltage across the other or knowledge of current through one gives no info. about current through other.



This circuit can be resonant.

Definition of a resonant condition in a reactive cct is when Z is purely real.

Classic resonant circuits



$$Z = j\omega L + \frac{1}{j\omega C} + R.$$

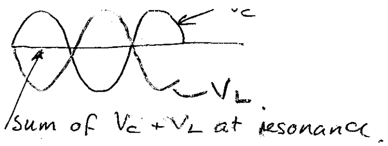
$$= j\left(\omega L - \frac{1}{\omega C}\right) + R.$$

cct resonant when j terms = 0

ie when $\left(\omega L - \frac{1}{\omega C}\right) = 0$

or $\omega^2 = \frac{1}{LC}$ or $\omega = \frac{1}{\sqrt{LC}}$

$V_c + V_L$ at resonance



$$Q = \frac{|V_L|}{|V_s|} = \frac{|V_c|}{|V_s|} \text{ at resonance.}$$

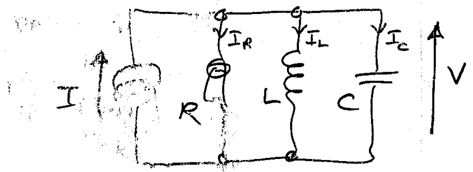
$$\frac{|V_c|}{|V_s|} = \frac{|V_c|}{|V_R|} = \frac{I/\omega C}{IR} = \frac{1}{\omega CR}$$

in terms of L $\quad \frac{\omega L}{R}$

put $\omega = \frac{1}{\sqrt{LC}}$

$$Q = \frac{1}{\frac{1}{\sqrt{LC}} CR} = \frac{\sqrt{LC}}{CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Classical parallel resonant cct



$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{j\omega L + R + j\omega C . j\omega L . R}{R j\omega L}$$

$$Z = \frac{R j\omega L}{R + j\omega L - \omega^2 LCR}$$

$$= \frac{j\omega L}{1 + j\omega \frac{L}{R} - \omega^2 LC} \times \frac{j}{j}$$

$$= \frac{-\omega L}{j(1-\omega^2 LC) - \omega\frac{L}{R}}$$

$$= \frac{\omega L}{\omega\frac{L}{R} - j(1-\omega^2 LC)}$$

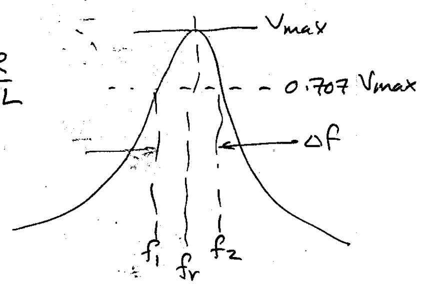
to make Z purely real, equate coefficient of j to zero.

$$1 - \omega^2 LC = 0$$

$$\omega = \frac{1}{\sqrt{LC}}$$

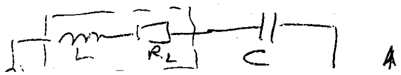
$$Q = \omega CR \text{ or } \frac{R}{\omega L}$$

$$\text{or } R\sqrt{\frac{C}{L}}$$



$$Q = \frac{f_r}{\Delta f}.$$

Non ideal behaviour



Non ideal behaviour