5. Induction Motor Drive

Induction motors with squirrel-cage are the workhorse of industry because of their low cost and rugged construction. When supplied directly from the line voltage (50 Hz utility input at essentially a constant voltage), an induction motor operates at a nearly constant speed. However, by means of power electronic converters, and sophisticated control, induction motors can be used as servo drives in computer peripherals, machine tools, and robotics.

5.1 Basic Principles of Induction Motor Operation

In a large majority of applications, induction motor drives incorporate a three-phase, squirrel-cage motor. The stator of an induction motor consists of three phase windings distributed in the stator slots. These three windings are displaced by 120^0 in space, with respect to each other, as shown in Fig. 5.1. The squirrel-cage rotor consists of a stack of insulated laminations. It has electrically conducting bars inserted through it, closed to the periphery in the axial direction, which are electrically shorted at each end of the rotor by end rings, thus producing a cage-like structure. This also illustrates the simple, low cost, and rugged nature of the rotor.

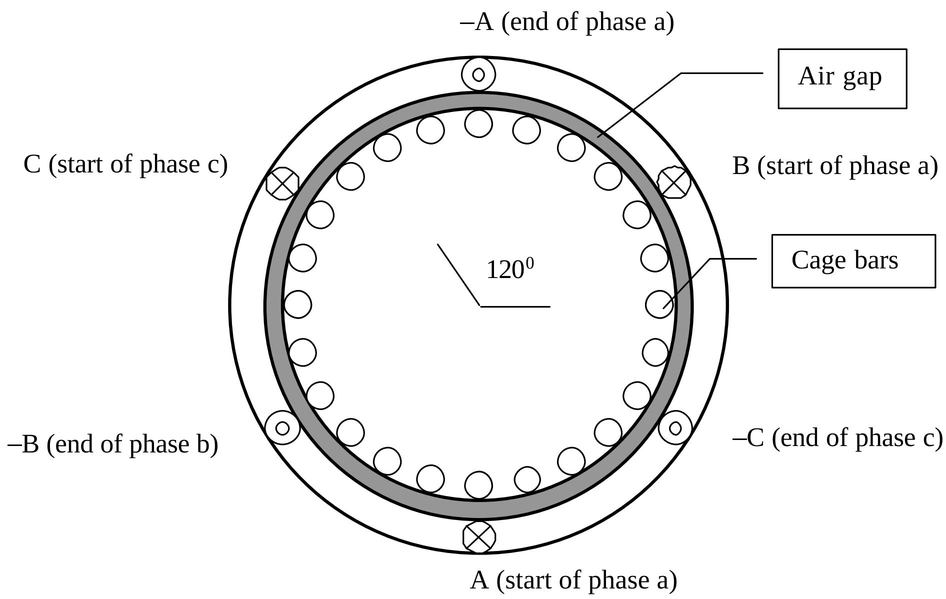

Fig. 5.1 Schematics of an induction motor

As has been shown in section 3.2, when a three-phase winding are supplied by a balanced three-phase sinusoidal voltage source at a frequency of $f = w/2p$, it results in a balanced set of currents, which establishes a flux density distribution B_{ag} in the air gap with the following properties:

(1) it has a constant amplitude

(2) it rotates with a constant speed, also known as the synchronous speed, of ω_s radian per second.

The synchronous speed in a motor with p pole pairs, supplied by frequency f, can be obtained as:

$$w_s = 2pf/p = w/p \tag{5.1}$$

The air gap flux rotates at a synchronous speed relative to the stationary stator windings, and induces winding emfs often called air-gap voltage E_{ag}. This can be illustrated by means of a per-phase equivalent circuit shown in Fig. 5.2, where V_s is the per-phase voltage (equal to line-to-line rms voltage divided by square root of three, and E_{ag} is the air gap voltage. R_s is the resistance of stator winding and L_{ls} is the leakage inductance of the stator winding. The magnetising component I_m of the stator current I_s establishes the air gap flux linkage whose peak value is given by:

$$\Psi_{ag} = \sqrt{2} L_m I_m \tag{5.2}$$

where L_m is the magnetising inductance of the machine, as discussed in section 3.2. Assuming that the air-gap flux linking the stator phase winding to be $y_{ag}(t) = \Psi_{ag} \sin wt$, the induced emf will be:

$$e_{ag} = w\Psi_{ag} \cos wt \tag{5.3}$$

which has an rms value of:

$$E_{ag} = w\Psi_{ag} / \sqrt{2} = 2pf\Psi_{ag} / \sqrt{2} = \sqrt{2}pf\Psi_{ag} = kf\Psi_{ag} \tag{5.4}$$

where $k = \sqrt{2}p \approx 4.44$

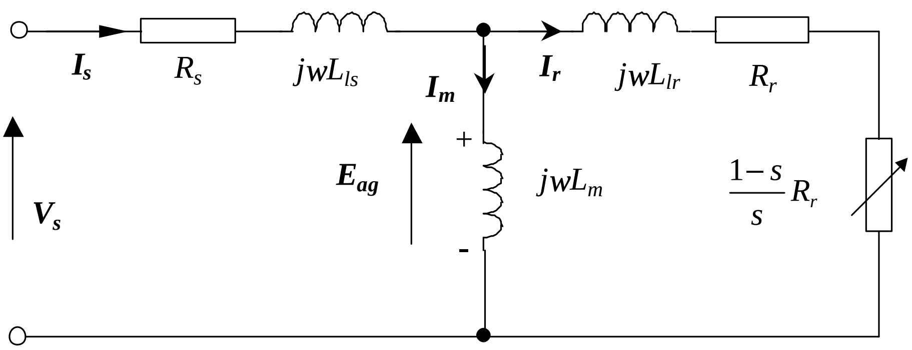

Fig. 5.2 Equivalent circuit of induction motors

Assuming relative motion exists between the rotor and the rotating magnetic field, then a current will be induced in the rotor. This current will be sinusoidally distributed around the surface of the rotor. Torque is therefore produced as a result of interaction between the air gap flux and the rotor current. If the rotor is rotating at the synchronous speed, there will be on relative motion between B_{ag} and the rotor, and hence there will be no induced rotor voltage, currents and torque. At any other speed w_r of the rotor in the same direction of the air gap flux rotation, the rotor is "slipping" with respect to the air gap flux at a relative speed referred to as slip speed w_{sl}, where

$$w_{sl} = w_s - w_r \tag{5.5}$$

The slip speed, normalised by the synchronous speed w_s is simply called the "slip" s:

$$s = w_{sl} / w_s = (w_s - w_r) / w_s \tag{5.6}$$

From Faraday's law, the induced voltages in the rotor circuit are at a slip frequency f_{sl} which is proportional to the slip speed:

$$f_{sl} = (w_{sl} / w_s)f = sf \tag{5.7}$$

The magnitude E_{ra} of this slip-frequency voltage that is induced in any of the rotor conductors can be obtained in a similar manner as the induced voltages in the stator phases. The same air-gap flux F_{ag} links the rotor conductors as the one that links the stator windings. However, the flux-density distribution in the air-gap rotates at a slip speed w_{sl} with respect to the rotor conductors. Therefore, the induced emf E_{ra} in the rotor conductors can be obtained by replacing f in Eqn. (5.4) by the slip frequency f_{sl}. By assuming the squirrel-cage rotor to be represented by a three-phase short-circuited winding with the same equivalent number of turns N_s, per phase as on the stator

$$E_{ra} = k\, f_{sl} \Psi_{ag} \tag{5.8}$$

Since the rotor squirrel-cage winding is short-circuited by the end-rings, these induced voltages at the slip frequency result in rotor currents I_r at the slip frequency f_{sl}

$$E_{ra} = R_r I_r + j2pf_{sl} L_{lr} I_r \tag{5.9}$$

where R_r and L_{lr} are the resistance and the leakage inductance of the per-phase equivalent rotor winding. The slip-frequency rotor currents produce a field that rotates at the slip speed with respect to the rotor and, hence, at the synchronous speed with respect to the stator (since $w_{sl} + w_r = w_s$). The interaction of Y_{ag} and the field produced by the rotor currents results in an electromagnetic torque. These currents also produce losses in the rotor winding resistance:

$$P_r = 3R_r I_r^2 \tag{5.10}$$

Multiplying both sides of Fqn. (5.9) by f/f_{sl} and using Eqns. (5.8) and (5.4)

$$E_{ag} = \frac{f}{f_{sl}} E_{ra} = \frac{f}{f_{sl}} R_r I_r + j2pfL_{lr} I_r \tag{5.11}$$

as shown in Fig. 5.2, where fR_r/f_{sl} is represented as a sum of R_r and $R_r(f - f_{sl})/ f_{sl} = R_r(1\text{-}s)/s$. In Eqn. (5.11), all rotor quantities are referred to N_s (the stator number of turns). By multiplying both sides of the Eqn. (5.11) by I_r the power crossing the air gap, often called the air-gap power P_{ag} can be obtained as:

$$P_{ag} = 3\frac{f}{f_{sl}} R_r I_r^2 = 3\frac{1}{s} R_r I_r^2 \tag{5.12}$$

From Eqns. (5.12) and (5.10), the electromechanical power P_{em} is

$$P_{em} = P_{ag} - P_r = 3R_r I_r^2 \frac{f - f_{sl}}{f_{sl}} = 3R_r I_r^2 \frac{1-s}{s} \tag{5.13}$$

and

$$T_{em} = P_{em} / w_r = 3R_r I_r^2 \frac{1-s}{s} / w_s (1-s) = 3R_r I_r^2 \frac{1}{s}\frac{1}{w_s} = \frac{P_{ag}}{w_s} \tag{5.14}$$

In the equivalent circuit of Fig. 5.2, the loss in the rotor resistance and the per-phase electromechanical power are shown by splitting the resistance $f(R_r/f_{sl})$ in Eqn. (5.11) into R_r and $R_r(1-s)/s$.

The total stator current I_s is the sum of the magnetizing current I_m and the equivalent rotor current I_r (I_r here is the component of the stator current that cancels out the ampere-turns produced by the actual rotor current):

$$I_s = I_m + I_r$$

The phasor diagram for the stator voltages and currents is shown in Fig. 5.3. The magnetizing current I_m which produces Φ_{ag} lags the air-gap voltage by 90^0. I_r which is responsible for producing the electromagnetic torque, lags E_{ag} by the power factor angle q_r of the rotor circuit.

From the phasor diagram, the air-gap power may be obtained as $P_{ag} = 3E_{ag}I_r \cos q_r$, and hence,

$$T_{em} = P_{ag} / w_s = \frac{3w\Psi_{ag} I_r}{\sqrt{2}w_s}\cos q_r = \frac{3p\Psi_{ag}}{\sqrt{2}} I_r \sin d \tag{5.15}$$

where $d = 90^0 - q_r$ is the torque angle between the magnetising current I_m, which produces Ψ_{ag} and I_r. This torque production mechanism is similar to the synchronous machine. The applied per-phase stator voltage V_s is

$$V_s = E_{ag} + (R_s I_s + jwL_{ls} I_s) \tag{5.16}$$

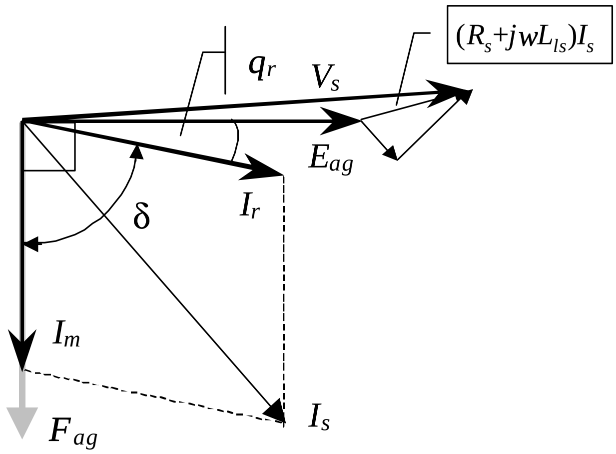

Fig. 5.3 Per phase phasor diagram of Induction motors

In induction motors of normal design, the following condition is true in the rotor circuit at low values of slip frequency, sf, corresponding to normal operation:

$$swL_{sl} << R_r$$

so that $q_r \approx 0$ and $\delta \approx 90^0$. The circuit in Fig. 5.2 is an exact equivalent circuit when a resistance component representing iron losses is included in the magnetising branch. However, it is not very convenient for the performance evaluation. Further approximations are made based on the following factors:

(1) The per-phase stator leakage inductance and stator resistance are small, and therefore even with large stator currents the difference between V_s and E_{ag} is small

(2) The per phase magnetising reactance wL_m is large. Thus I_m is small in comparison with I_s, and are not greatly changed if the element wL_m is moved to the stator terminal as shown in Fig. 5.4

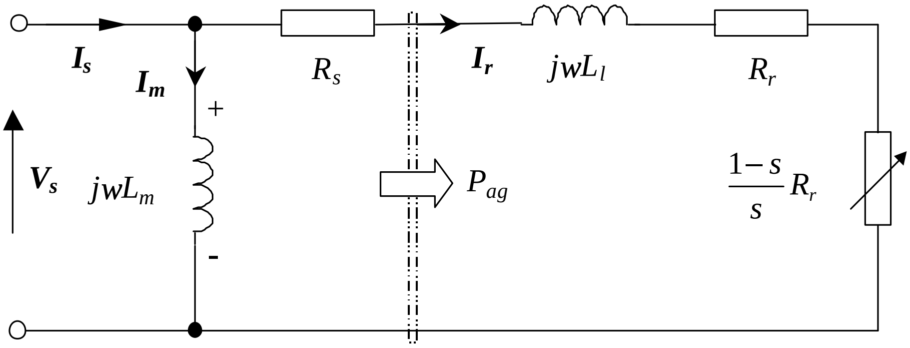

Fig. 5.4 Simplified equivalent circuit of induction motors

where $L_l = L_{ls} + L_{lr}$ is the total leakage inductance of the machine. Since variation of L_m with changes in V_s/ω is so small, it may be shown as constant circuit parameters.

5.2 Torque-speed characteristic and speed control of induction motors

5.2.1 Torque-speed characteristic

From Fig. 5.4, the rotor current may be obtained as:

$$I_r^2 = \frac{V_s^2}{(R_s + R_r/s)^2 + (wL_l)^2} = \frac{s^2 V_s^2}{(sR_s + R_r)^2 + (swL_l)^2} \tag{5.17}$$

The electromagnetic torque becomes

$$T_{em} = 3R_r I_r^2 \frac{1}{s}\frac{1}{w_s} = \frac{3pR_r V_s^2}{w} \frac{s}{(sR_s + R_r)^2 + (swL_l)^2} \tag{5.18}$$

With constant voltage V_s and frequency w, the torque produced is a function of the form:

$$\frac{s}{(sR_s + R_r)^2 + (swL_l)^2}$$

Fig. 5.5 shows the typical torque-speed characteristic for $V_s = 240$ (V), $p = 2$, $w = 2\pi 50 = 314$, $R_s = 0.32\Omega$, $R_r = 0.34\Omega$, and $wL_l = 1.95\Omega$.

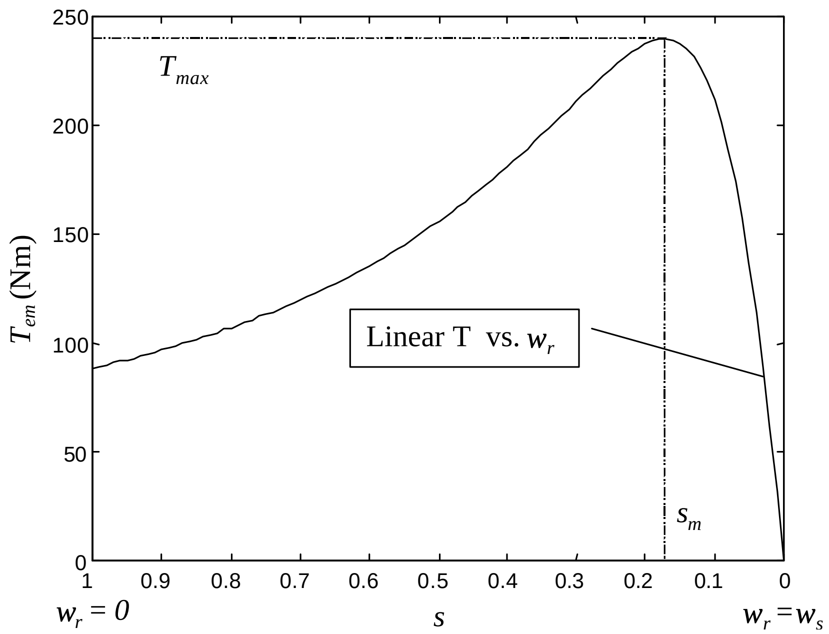

Fig. 5.5 Torque speed characteristic of induction machine

Several observations can be made:

(1) For small values of s, sR_s and $swL_l << R_r$ and can be neglected, thus torque is linear with respect to slip s or the rotor speed w_r:

$$T_{em} = \frac{3pV_s^2}{wR_r} s = \frac{3pV_s^2}{wR_r}\left(1 - \frac{w_r}{w_s}\right) \tag{5.19}$$

(2) There exists a maximum torque which can be determined by setting $dT_{em}/ds = 0$:

$$\begin{aligned} s_m &= \frac{R_r}{\sqrt{R_s^2 + (wL_l)^2}} \\ T_{\max} &= \frac{3pV_s^2}{2w\left(R_s + \sqrt{R_s^2 + (wL_l)^2}\right)} \end{aligned} \tag{5.20}$$

This maximum torque is also known as pull-out torque.

5.2.2 Speed control

The torque speed characteristic of Eqns. (5.18) and (5.19) shows that the speed of induction machines is dependent on the supply frequency. Therefore, the speed of an induction machine can simply be controlled by variation of supply frequency. However, how the magnitude of supply voltage V_s changes accordingly needs careful examinations:

CONSTANT VOLTAGE OPERATION

The supply voltage is kept constant while the supply frequency is varied. Fig. 5.6 shows the torque speed characteristics under this operation. It is apparent from Eqns. (5.18)~(5.20) that the electromagnetic torque decreases as the frequency increases whilst the output power is kept constant. Since $V_s \approx E_{ag} = 4.44f\ \Psi_{ag}$, the air gap flux linkage decreases as the frequency increases, which explain why the torque is reduced. This operation is useful when flux weakening is necessary in order to extend the operation speed above its rated value. However, the constant voltage operation will not be appropriate if the torque capability of the motor is required to equal the rated torque at any supply frequency.

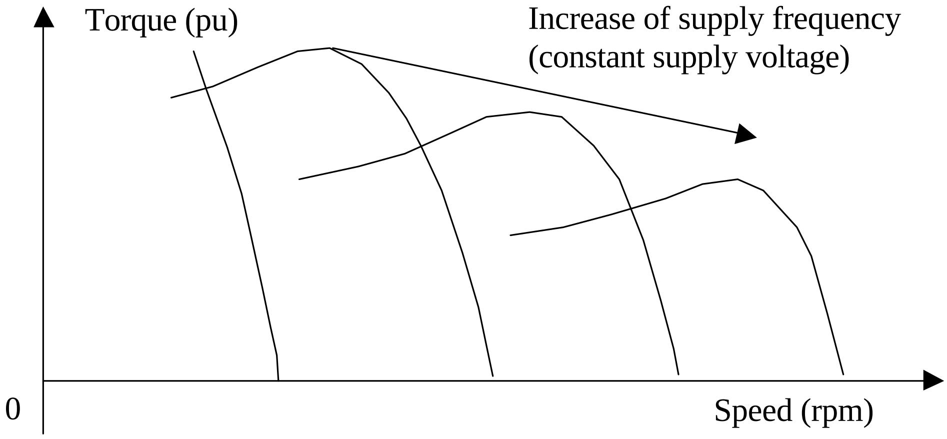

Fig. 5.6 Torque speed characteristics under variable frequency, constant voltage operation

CONSTANT (V_s/f) OPERATION

In order to maintain approximately constant torque capability for all frequencies, the ratio of supply voltage to frequency, (V_s/f) may be kept constant. The torque speed characteristics with this control strategy is shown in Fig. 5.7

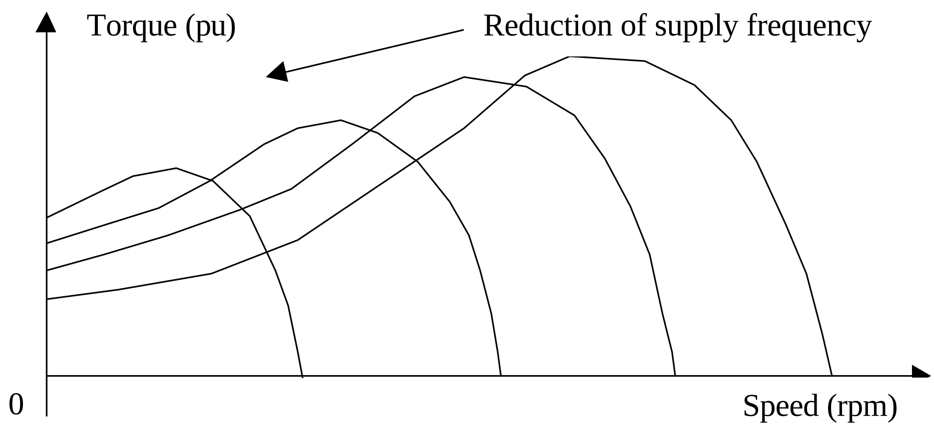

Fig. 5.7 Torque speed characteristics under variable frequency, constant (V_s/f) operation

As can be seen from Eqn. (5.20), when supply frequency is high, $wL_l >> R_s$. Hence T_{max} may be approximated as:

$$T_{\max} = \frac{3pV_s^2}{2w\left(R_s + \sqrt{R_s^2 + (wL_l)^2}\right)} \approx \frac{3pV_s^2}{2w(wL_l)} = \frac{3p}{2L_l}\left(\frac{V_s}{w}\right)^2 \propto \left(\frac{V_s}{f}\right)^2 \tag{5.21-a}$$

Therefore, at high speed, the pull-out torque is constant provided that (V_s/f) is be kept constant. When frequency (or speed) is low, however, R_s becomes dominant, i.e., $R_s >> wL_l$ and the pull-out torque may be approximated by:

$$T_{\max} = \frac{3pV_s^2}{2w\left(R_s + \sqrt{R_s^2 + (wL_l)^2}\right)} \approx \frac{3pV_s^2}{4wR_s} = \frac{3pw}{4R_s}\left(\frac{V_s}{w}\right)^2 \propto \left(\frac{V_s}{f}\right)^2 f \qquad (5.21\text{-b})$$

Hence, the torque capability decreases as the frequency is reduced. In summary, constant (V_s/f) operation achieves constant torque capability at high speed, but reduced torque at low speed.

CONSTANT AIRGAP FLUX (E_{ag}/f) OPERATION

The decrease in torque capability at low speed under constant (V_s/f) is due to the voltage drop across the stator resistance. At low synchronous speed, the supply voltage V_s is proportionally low. Thus the voltage drop R_sI_s may significantly influence the airgap flux since

$$\Psi_{\mathrm{ag}} \propto E_{ag}/f \approx (V_s/f - R_sI_s/f) = \text{constant} - R_sI_s/f.$$

Therefore the airgap flux decreases as the frequency is reduced. To maintain a constant torque capability, the ratio (E_{ag}/f) should essentially be kept constant. This will ensure the airgap flux linkage will be constant for any supply frequency. Under this condition,

$$I_r^2 = \frac{E_{ag}^2}{\left(R_r/s\right)^2 + (wL_l)^2} = \frac{s^2E_{ag}^2}{R_r^{\ 2} + (swL_l)^2} \qquad (5.22)$$

The electromagnetic torque is now given by

$$\begin{aligned} T_{em} &= 3R_rI_r^2\frac{1}{s}\frac{1}{w_s} = \frac{3pR_r}{w}\frac{sE_{ag}^2}{R_r^{\ 2} + (swL_l)^2} = 3p\left(\frac{E_{ag}}{w}\right)^2\frac{swR_r}{R_r^{\ 2} + (swL_l)^2} \\ &= 3p\Psi_{ag}^2R_r\frac{sw}{R_r^{\ 2} + (swL_l)^2} \end{aligned} \qquad (5.23)$$

The maximum torque is

$$T_{\max} = \frac{3p}{2L_l}\Psi_{ag}^2 = \frac{3p}{2L_l}\left(\frac{E_{ag}}{w}\right)^2 \qquad (5.24)$$

It follows that the pull-out torque is independent of the supply frequency when (E_{ag}/f) is constant. The corresponding torque speed characteristics is shown in Fig. 5.8. This mode of speed control is usually referred to as scalar control, and as can be seen, under this control strategy, the pull-out torque is constant for all supply frequency and the starting torque at low speed is significantly increased. However, the torque speed characteristics are non-linear over the entire speed range.

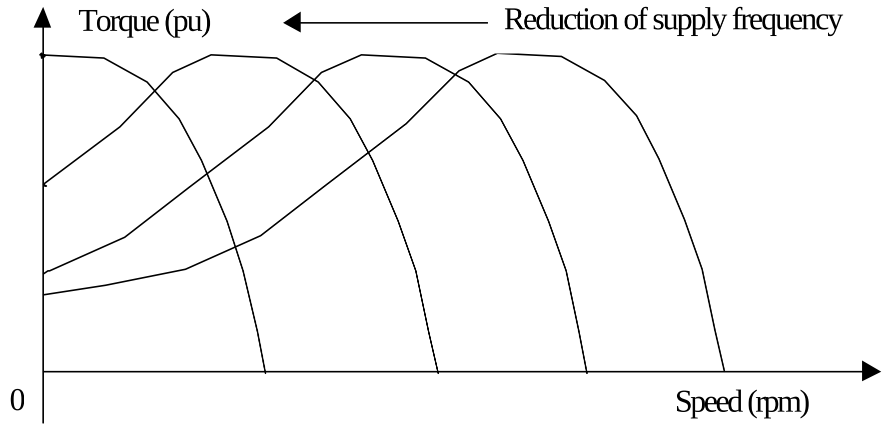

Fig. 5.8 Torque speed characteristics under variable frequency, constant airgap flux (E_{ag}/f) operation

5.2.3 Implementation of scalar speed control

The motor is supplied "open-loop" from a 3 phase variable voltage and frequency source, such as shown in Fig. 5.9. To compensate for the voltage drop across the stator resistance, a pre-set voltage-frequency relationship can be stored in memory, and used to generate the output voltage for a given speed (frequency) demand. This pre-programmed voltage-frequency relationship cannot account for the variation of stator current but for some applications may be adequate. The resulting reference voltage signal (amplitude and angular frequency) is modulated by a PWM scheme such as SPWM or SVPWM discussed in Chapter 3 to form the switching signals for the voltage source inverter.

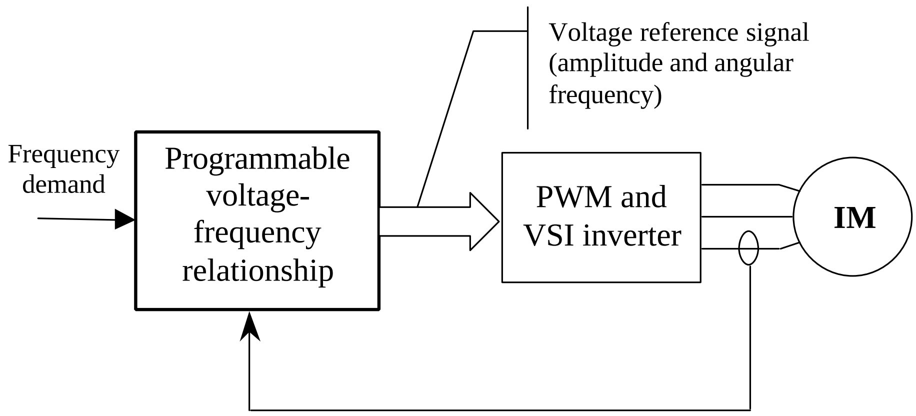

Fig. 5.9 Block diagram of open-loop, scalar control of induction machine

To implement a more precise R_sI_s compensation, some forms of current measurements are necessary, and the level of voltage compensation can be derived from the simplified phasor diagram shown in Fig. 5.10:

$$V_s = \sqrt{(E_{ag} + I_s R_s \cos\varphi)^2 + (I_s R_s \sin\varphi)^2} \qquad (5.25)$$

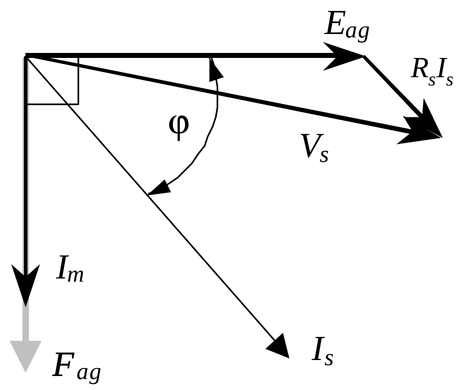

Fig. 5.10 simplified per phase phasor diagram for resistance voltage compensation

where E_{ag} is determined from a linear relationship with the frequency f. A typical V-f curve under this compensation scheme is given in Fig. 5.11.

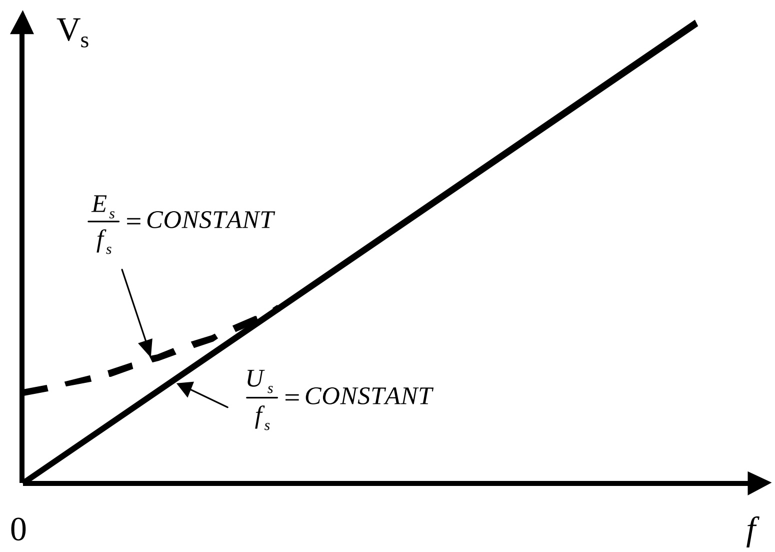

Fig. 5.11 V-f relationship for constant airgap flux operation

5.3 Field Oriented (vector) Control

Although with constant (E_{ag}/f) the torque capability of the motor is improved, the torque – speed characteristics are non-linear, and response is slow all due to the rotor leakage inductance, as can be seen from Eqn. (5.23).

With reference to the equivalent circuit which is redrawn in a slightly different form in Fig. 5.12, if the voltage across the rotor leakage inductance can be compensated, or in other words, the ratio of E_r/f is kept constant, where E_r is the induced voltage across R_r/s, then the rotor current is

$$I_r^2 = \frac{s^2 E_r^2}{R_r^2} \tag{5.26}$$

From Eqn. (5.14), one has

$$T_{em} = 3R_r I_r^2 \frac{1}{s}\frac{1}{w_s} = 3pR_r \frac{1}{w\,s}\frac{s^2 E_r^2}{R_r^2} = \frac{3p^2}{R_r}\left(\frac{E_r}{w}\right)^2 (w_s - w_r) \tag{5.27}$$

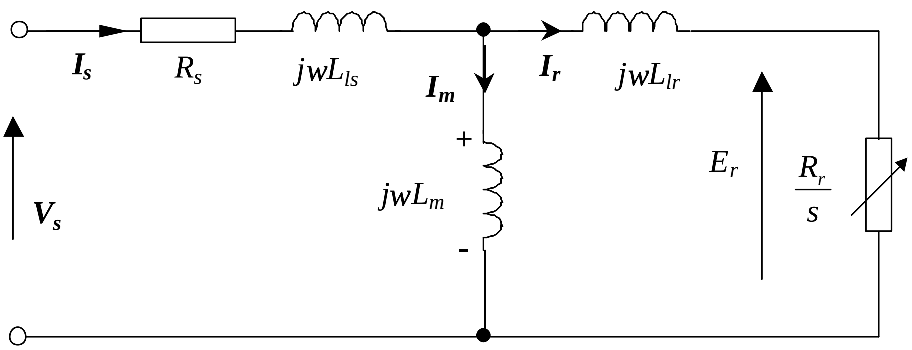

Fig. 5.12 Equivalent circuit of induction motors showing the concept of vector control scheme

It is clear that with a constant E_r/f, the torque speed characteristics now becomes linear, similar to that of DC machines and Brushless AC PM machines under field oriented control as shown in Fig. 5.13, and the influence of rotor leakage inductance is eliminated. Further more, it appears that, theoretically, there is no limit on the maximum torque (in practice, it will be limited by saturation in the machine). This mode of operation indeed requires the rotor flux, Ψ_r being maintained at a constant level for all frequency, which can be realised by so called field-oriented control or vector control. With this control strategy, the induction motor has a characteristic comparable to DC and other machine types. Thus field oriented controlled induction motor drive are increasingly being used in many servo applications.

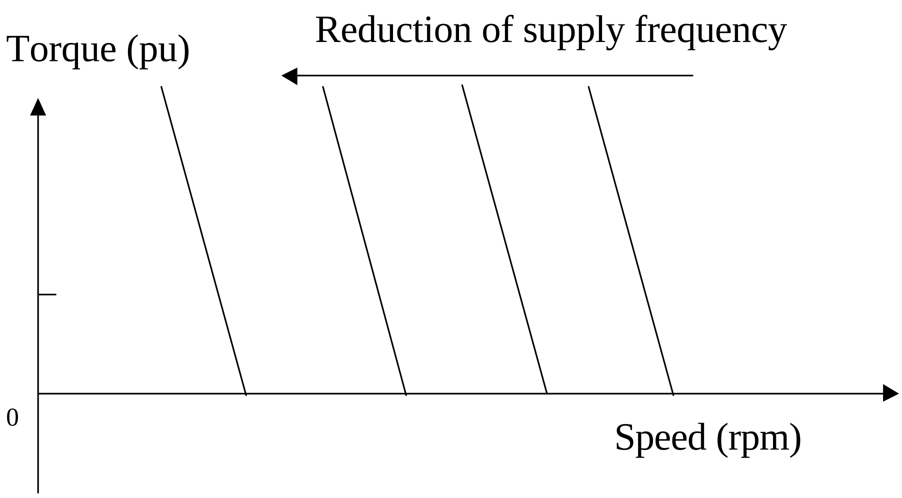

Fig. 5.13 Torque speed characteristics under field oriented control (constant rotor flux (E_r/f) operation)

The implementation of this control strategy entails advanced machine modelling and mathematical development, which are beyond the scope of this course.