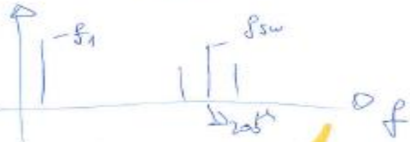
(1) VSI SPWM linear op. mode

RL $R = 10\Omega$ $L = 5mH$ $\hat{V}_{ph} = 250V$

$V_d = 600V$ $f_o = 50Hz$; $\omega_o = 2\pi f_o = 314 \frac{rad}{s}$

$\hat{I}_{ph} = 10A$

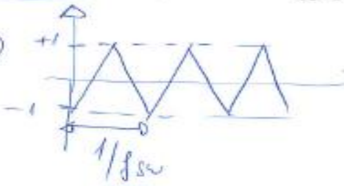
$\cos\theta = 0.7 \Rightarrow \theta = 45.6^\circ$



a) 4

$f_{sw} = 50 \cdot 200\ Hz = 10\,000\ Hz$; $T_{sw} = \frac{1}{f_{sw}} = 100\ \mu s$ 1

carrier sig. $\Rightarrow$



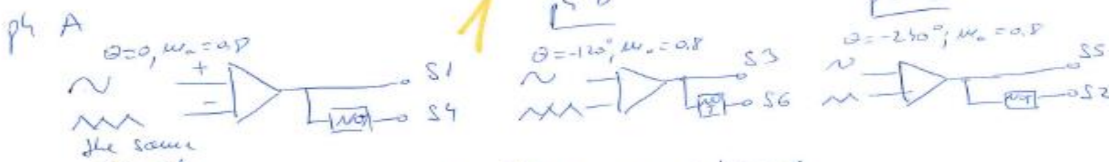
1

mod. sig $\Rightarrow$

1

$m_a = \frac{\hat{V}_{ph}}{\frac{V_{dc}}{2}} = \frac{250}{\frac{600}{2}} = 0.8$

1



no deadtime considered

(b) non ideal switches

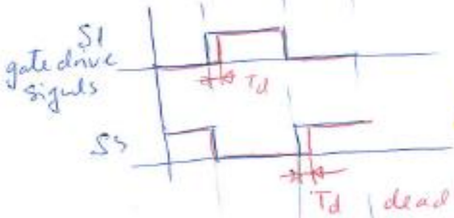
2 $t_{on} = 0.3\mu s$

$t_{off} = 0.65\mu s$

In order to avoid shoot-through of the DC-link, a delay should be inserted in the turn on time of PWM pulses.

This delay is usually called : deadtime.

Ideal PWM for S1 & S4 ~~two example~~



1

$\rightarrow$ T_d should be long enough to make sure S4 is off before S1 ~~is~~ starts turning on.

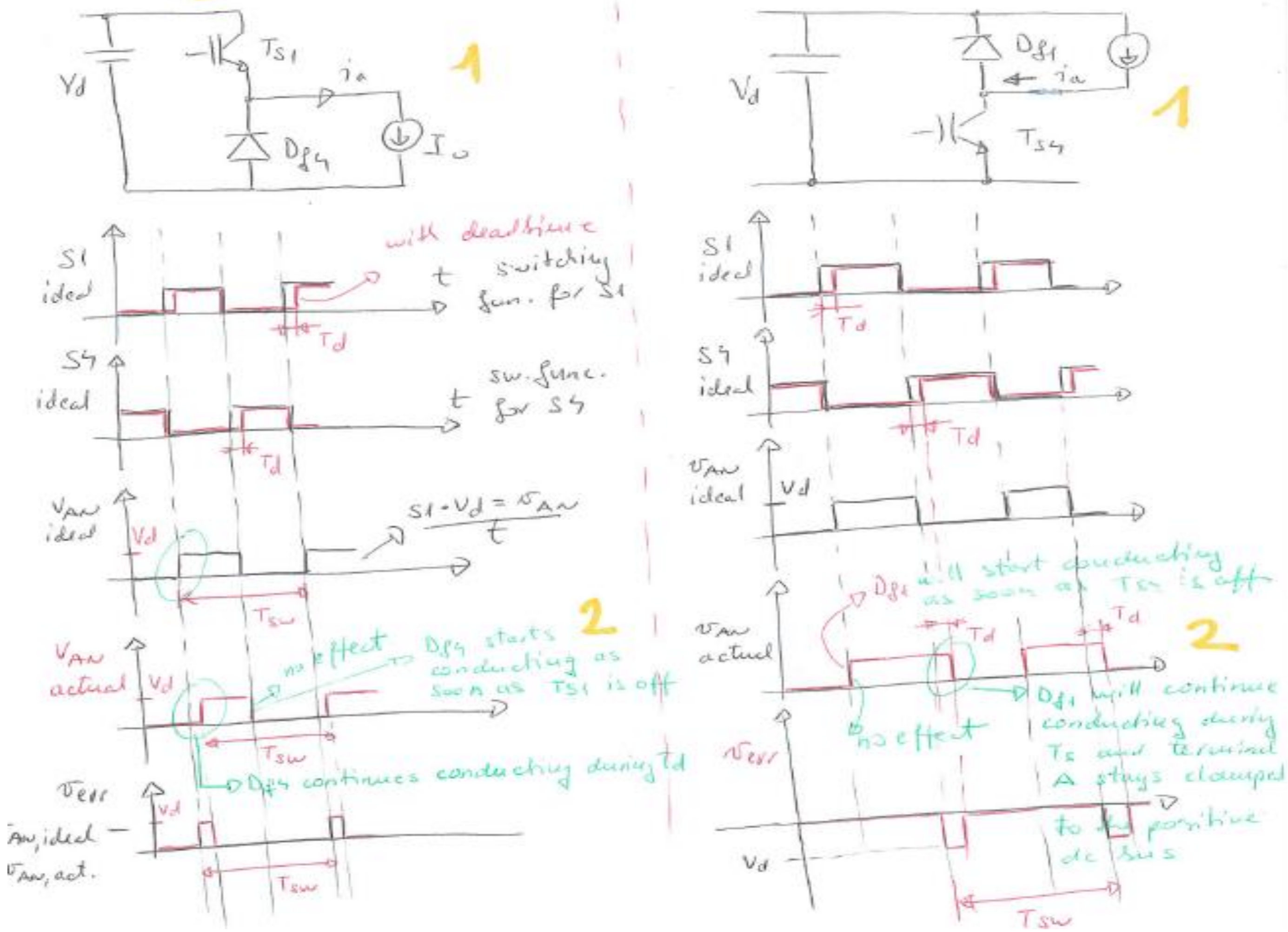
T_d | dead time (only at turn on)

$\rightarrow$ T_d should be long enough to make ~~the~~ sure S1 is completely off before S4 starts turning on 1

$T_d > T_{off}$

$\boxed{T_d \sim 1\mu s}$

c) $i_a > 0$ $\boxed{6}$ $\qquad$ $i_a < 0$



d) $\boxed{4}$

$$v_{A_{Z_{1fund.}}}(t) = m_a \cdot \frac{V_d}{2} \cdot \sin(\omega_o \cdot t) - \frac{4}{\pi} \cdot V_d \cdot \frac{T_d}{T_s} \cdot \sin(\omega_o \cdot t - \theta) \quad 1$$

$$= 0.8 \cdot \frac{600}{2} \cdot \sin(314 \cdot t) - \frac{4}{\pi} \cdot 600 \cdot \frac{1\,\mu s}{100\,\mu s} \cdot \sin\left(314 \cdot t - 45.6 \cdot \frac{\pi}{180}\right) \quad 1$$

$$= 240 \cdot \sin(314 \cdot t) - 7.64 \cdot \sin(314 \cdot t - 0.79587)$$

$$= 240 \cdot \sin(314 \cdot t) - 7.64 \left\{ \sin(314 \cdot t) \cdot \underbrace{\cos(0.79587)}_{=0.7} - \cos(314 \cdot t) \cdot \underbrace{\sin(0.79587)}_{=0.7145} \right\}$$

$$= 240 \cdot \sin(314 \cdot t) - 5.348 \cdot \sin(314 \cdot t) + 5.4588 \cdot \cos(314 \cdot t) \quad 1$$

$$= 234.6520 \cdot \sin(314 \cdot t) + 5.4588 \cdot \cos(314 \cdot t)$$

$$a \cos x + b \sin x = R \cos(x - \alpha)$$

$$R = \sqrt{a^2 + b^2} \quad ; \quad \alpha = a\tan\left(\frac{b}{a}\right)$$

$$= R \cos(314 \cdot t - \alpha)$$

$$R = \sqrt{5.4588^2 + 234.6520^2} = 234.72\,V \quad 1$$

$$\alpha = a\tan\left(\frac{234.6520}{5.4588}\right) = 1.5475$$

(2)

a)

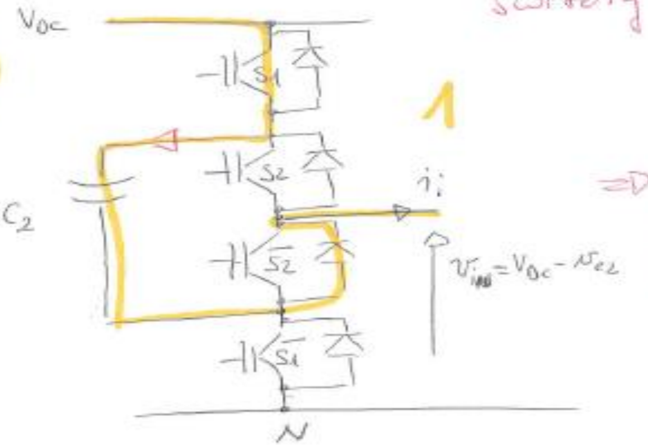
S_{1i}	S_{2i}	$\bar{S}_{2i}$	$\bar{S}_{1i}$	v_{AN}
1	1	0	0	V_{DC}
0	0	1	1	0
1	0	1	0	$V_{DC} - v_{C2i}$
0	1	0	1	v_{C2i}

if $v_{C2i} = \frac{V_{dc}}{2}$ then these are redundant states

$\boxed{2}$

---

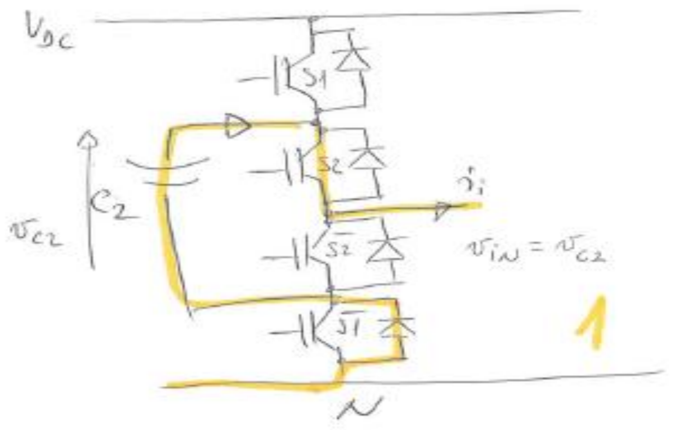
b) 6



switching state 10
$S1 = ON$
$S2 = OFF$
$i_i > 0$

$\Rightarrow$ The entire load current i_i goes into C_2 and will cause charging of $C2$

1 1



switching state 01
$S1 = OFF$
$S2 = ON$
$i_i > 0$

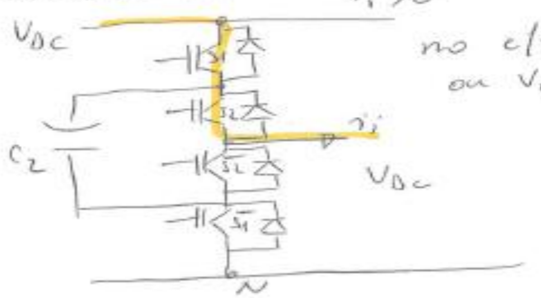
The current goes out of node i and will cause discharging of C_2

1 1

For $i_i < 0$ the effect is opposite.
Therefore if v_{C2} is not balanced v_{in} during sw states 10 & 01 is a function of load current.

1

Sw. state 11 $i_i > 0$



no effect on V_{iN}

V_{DC}

1

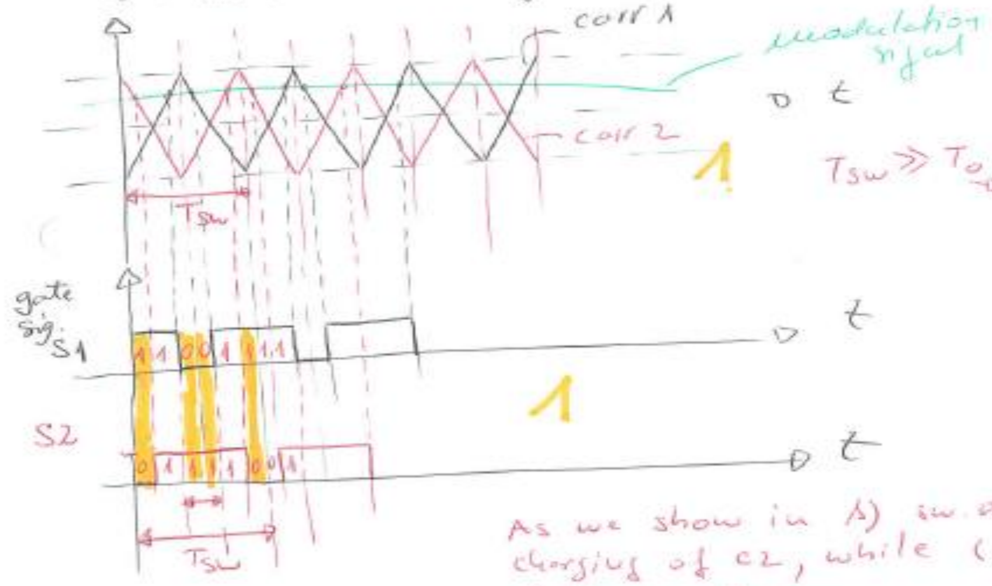
00

$v_{in} = 0$

(2)

c) Phase-shifted carrier PWM
For 3-level Flying capacitor multilevel converter we have common modulation signal for all switches in one leg and two carrier signals (one for modulation of S1 & $\bar{S}_1$ and the another for modulation of S_2 & $\bar{S}_2$) which are phase-shifted by $\delta = \frac{2\pi}{n-1}$ $(n=3)$ $= \underline{\underline{180^\circ}}$

The carrier freq. is much higher than the freq. of the modulation signal.



As we show in b) sw. state (10) causes charging of C2, while (01) causes discharging of C2.

• The inherent characteristic of PSC PWM is that maintains almost equal time intervals of states (10) & (01) during one sw. period and on this way keeps the voltage across C2 constant.

2

d) PSC PWM

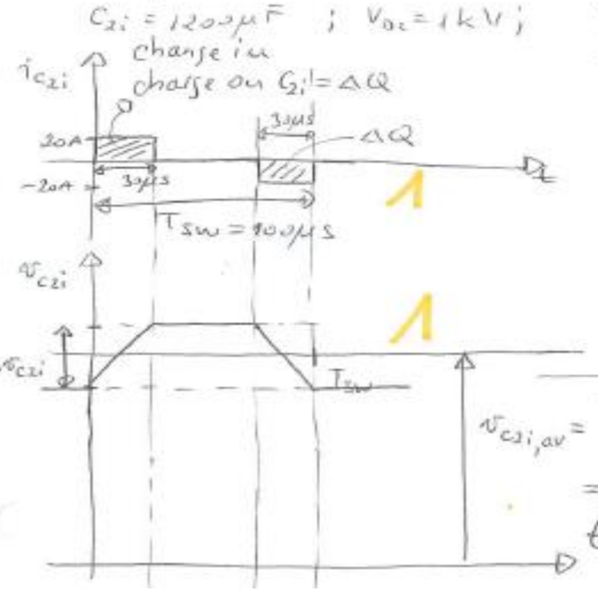
$\Delta v_{C_{2i}} = ?$

$C_{2i} = 1200\mu F$; $V_{DC} = 1kV$;

$i_i = I_o = 20A$

$t_{v_{C_{2i}P}} = 30\mu s = t_{v_{C_{2i}b}}$ in one $T_{sw} = \frac{1}{10kHz} = 100\mu s$

inherent to PSC PWM is $\Delta t_{v_{C_{2i}P}} \simeq \Delta t_{v_{C_{2i}b}}$ in each switching period

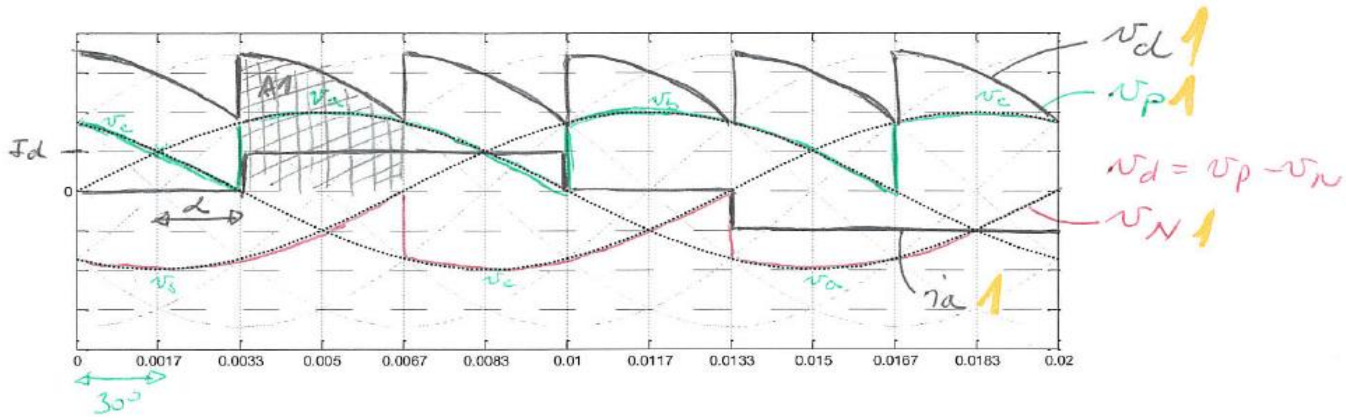


• The change in charge on C_{2i}, will cause the change in the voltage across it:

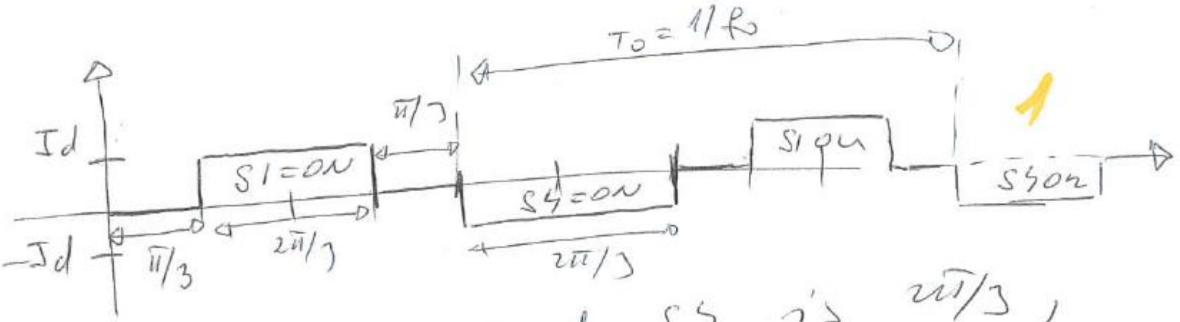
$$\Delta v_{C_{2i}} = \frac{\Delta Q}{C_{2i}} = \frac{20A \cdot 30\mu s}{1200\mu F} = \underline{0.5V} \quad 1$$

$$v_{C_{2i},av} = \frac{V_{dc}}{2} = \frac{1000}{2} = \underline{500V} \quad 1$$

a) $\boxed{4}$



b) 2



conduction period of S4 is $2\pi/3$, and its switching freq. is 50 Hz (line freq.) 1

c) $\boxed{4}$

shown in a)

$$V_d = \frac{\text{area } A1}{\pi/3} = \frac{1}{\pi/3}\int_{\pi/6+\alpha}^{\pi/6+\alpha+\frac{\pi}{3}} v_{as}\cdot d(\omega t) = \int_{\pi/6+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{2}\cdot V_{ee\,rms}\sin\left(\omega t+\frac{\pi}{6}\right)\cdot d(\omega t) =$$

$$= \frac{3}{\pi}\sqrt{2}\cdot V_{ll\,rms}\left\{\cos\left(\frac{\pi}{6}+\alpha+\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{2}+\alpha+\frac{\pi}{6}\right)\right\} =$$

$$= \frac{3}{\pi}\cdot\sqrt{2}\cdot V_{ll,rms}\left\{\cos\left(\alpha+\frac{\pi}{3}\right) - \cos\left(\alpha+\frac{2\pi}{3}\right)\right\} =$$

$$= \frac{3}{\pi}\cdot\sqrt{2}\cdot V_{ll,rms}\left\{\cos\alpha\cdot\cos\frac{\pi}{3} - \cancel{\sin\alpha\cdot\sin\frac{\pi}{3}} - \left(\cos\frac{2\pi}{3}\cdot\cos\alpha - \cancel{\sin\frac{2\pi}{3}\cdot\sin\alpha}\right)\right\}$$

$$= \frac{3}{\pi}\cdot\sqrt{2}\cdot V_{ll,rms}\cdot 2\cdot\frac{1}{2}\cdot\cos\alpha = \frac{3}{\pi}\cdot\sqrt{2}\cdot V_{ll,rms}\cdot\cos\alpha$$

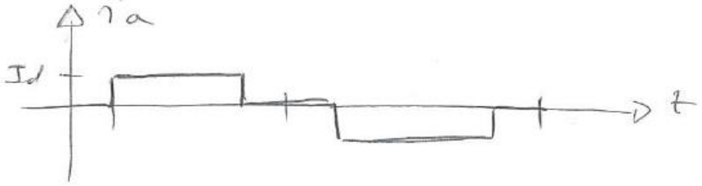
d i)

$$\hat{I}_{a1} = \frac{1}{\pi}\int_{\pi/6}^{\pi/6+\frac{2\pi}{3}} I_d \cdot \sin(\omega t)\,d\omega t + \frac{1}{\pi}\int_{7\pi/6}^{7\pi/6+\frac{2\pi}{3}} (-I_d)\cdot \sin(\omega t)\cdot d\omega t = \quad 1$$

$$= \frac{1}{\pi}\cdot I_d\,(-\cos\omega t)\Big|_{\pi/6}^{5\pi/6} + \frac{1}{\pi}\int_{7\pi/6}^{11\pi/6} (-I_d)\cdot \sin(\omega t)\cdot d(\omega t) = \quad 1$$

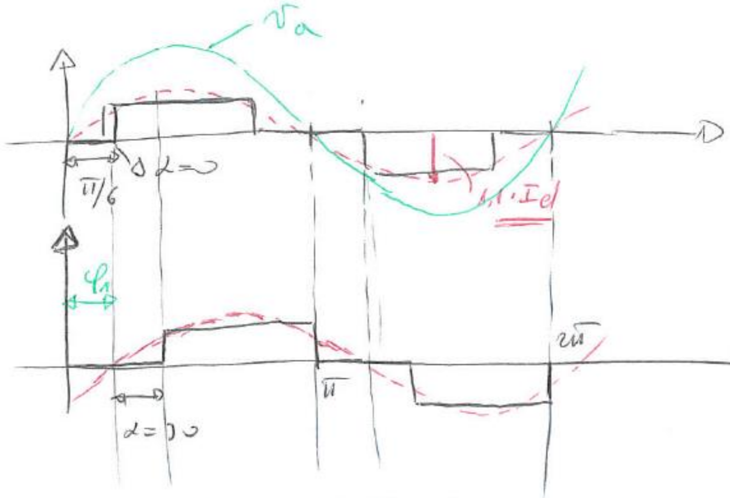
$$= \frac{1}{\pi}\cdot I_d\cdot\left(\cos\frac{\pi}{6} - \cos\frac{5\pi}{6}\right) + \frac{1}{\pi} I_d\cdot\left(\cos\left(\frac{11\pi}{6}\right) - \cos\left(\frac{7\pi}{6}\right)\right) \quad 1$$

$$= \frac{1}{\pi}\cdot I_d\cdot\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) + \frac{1}{\pi}\cdot I_d\cdot\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right) = \frac{2}{\pi}\cdot I_d\cdot\sqrt{3} = \underline{\underline{1.1\,I_d}} \quad 1$$



for $\alpha = 0$ the amplitude is the same only the component is shifted by α

e ii)



$\alpha = 0$

1

$\alpha = 30$

φ_1 – phase shift between v_a & i_{a1}

$= \frac{\pi}{6}$

Reactive power $\quad Q = 3\cdot V_{ph,rms}\cdot I_{ph,rms}\cdot \sin(\varphi_1)$

$$= 3\cdot\frac{V_{ll,rms}}{\sqrt{3}}\cdot I_{ph,rms}\cdot\sin(\varphi_1) \quad 1$$

$$= \sqrt{3}\cdot 2300\cdot 1.1\cdot I_d\cdot\sin\left(\frac{\pi}{6}\right)$$

$$= 325.8\,\text{kVAr} \quad *\frac{1}{\sqrt{2}}$$

$$I_d = \frac{P}{U_{d,av}} = \frac{400\,\text{kW}}{\frac{3}{\pi}\cdot\sqrt{2}\cdot 2300\cdot\cos\left(\frac{\pi}{6}\right)} = \underline{\underline{148.7\,\text{V}}}$$

(6)

a) 4

$$\Delta\Theta = 73.6^\circ - 70^\circ = \underline{\underline{3.6^\circ}} = 2\pi f_o \cdot T_s \cdot \frac{180}{\pi}$$

$$T_s = \frac{1}{f_{sw}} = \frac{1}{10kHz} = 100\mu s$$

$$f_o = \frac{\Delta\Theta}{2\pi \cdot T_s} = \frac{3.6 \cdot \pi/180}{2\pi \cdot 100\mu s}$$

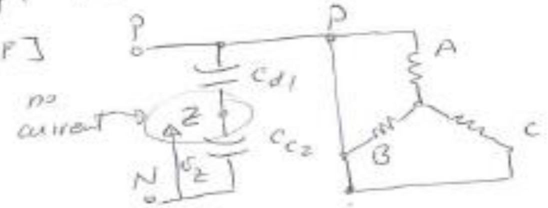
$$f_o = 100 Hz$$ 2

The rotation freq. of the ref. vector coresponds to the fundamental freq. of the inverter. The ref. vec. is synthesised a finit, discrete number of times (N) during one rotation period which coresponds to the fund. period of the output voltge. N in our case is $N = \frac{T_o}{T_s} = \frac{10\ ms}{100\mu s} = \underline{\underline{100 \text{ times}}}$ 2

b) 4

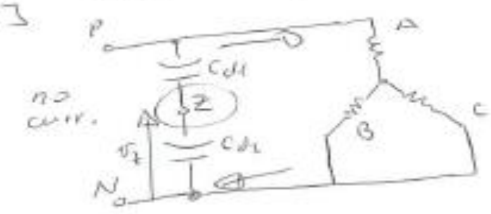
The zero & large vectors do not affect the neutral point Z, as no current flows through the neutral point.

for example: zero vector [PPP]



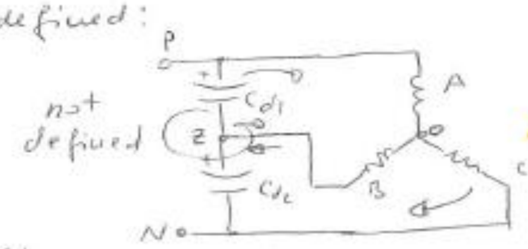
0,5 v_Z not affected

large vector [PNN]



0,5

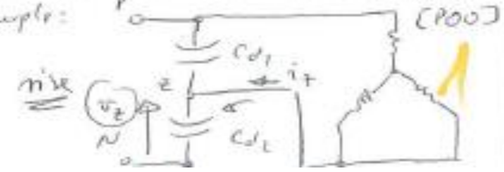
For medium vector it is not defined:



1

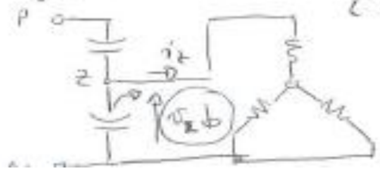
Small vectors have dominant influence on v_Z.

P-type small vector makes v_Z rise, for example: [POO]



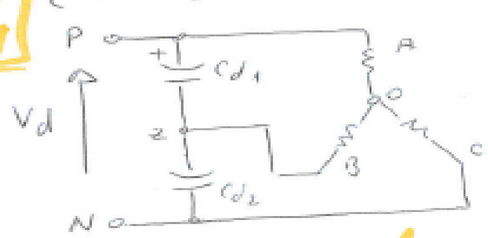
1

N-type small vector makes v_Z decline [ONN]



1

c) [P O N] 4



1

$V_{PN} = V_d$

$V_{Ao} = V_d/2$ 1

$V_{Co} = -V_d/2$

$V_{Co} = 0$

$\vec{V}(t) = \frac{2}{3}\left[v_{Ao}(t)\cdot e^{j0} + v_{Bo}(t)\cdot e^{j2\pi/3} + v_{Co}(t)\cdot e^{j4\pi/3}\right]$

$= \frac{2}{3}\left[\frac{V_d}{2}e^{j0} - \emptyset\cdot e^{j2\pi/3} - \frac{V_d}{2}e^{j4\pi/3}\right]$

$= \frac{2}{3}\left[\frac{V_d}{2} - \frac{V_d}{2}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right]$

$= \frac{2}{3}\left[\frac{V_d}{2} + \frac{V_d}{4} + j\frac{V_d}{4}\cdot\sqrt{3}\right]$

$= \frac{2}{3}\left[\frac{3V_d}{4} + j\frac{V_d}{4}\cdot\sqrt{3}\right] =$

$= \frac{2}{3}\cdot\frac{\sqrt{3}}{\cancel{2}}\left[\frac{\sqrt{3}V_d}{2} + j\frac{V_d}{2}\right]$

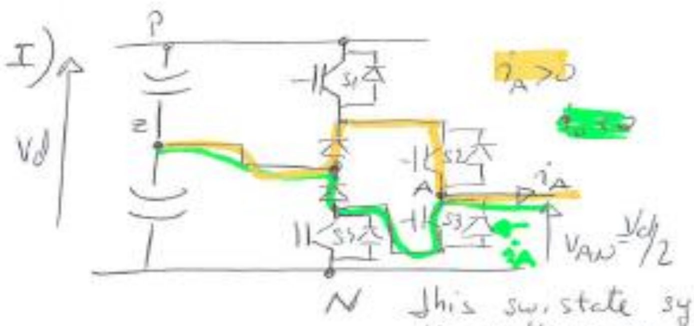
$= \frac{V_d\cdot\sqrt{3}}{3}\cdot e^{j\pi/6}$ 2

d) 4

	S1	S2	S3	S4	v_{AN}	
I	0	1	1	0	$V_d/2$	$i_A > 0$ & $i_A < 0$
II	1	0	0	1	0	$i_A > 0$
III	1	0	0	1	V_d	$i_A < 0$

} forbidden state

I)

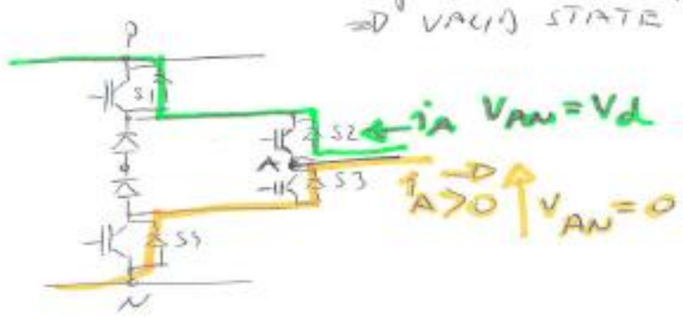


2

this sw. state synthesises the voltage $V_d/2$ independently of the sign of curr. $i_A \gtrless 0$

$\Rightarrow$ VALID STATE

II)



$V_{AN} = f(\text{sign of load current})$

$\Downarrow$ forbidden state

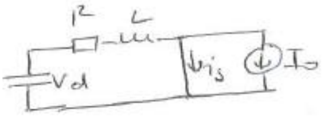
2

③ a) In the res. dc-link voltage inverter, input voltage remains zero during a finite period of time when the inverter switches can change their states at zero voltage.

The operat. cycle can be devided in 3 parts:

1) S=ON – charging of L

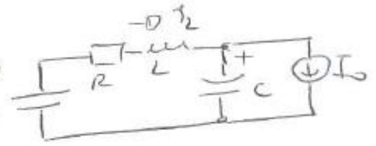
Assumption: capacitor C is discharged in the preceding resonance cycle so S in turned on at ZERO voltage. ⇒



2) S=off ; diode D is off

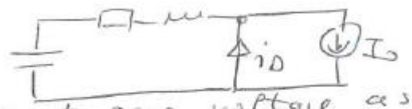
⇒ resonance

S opens & energy stored in L is discharged through the capacitor C. ⇒



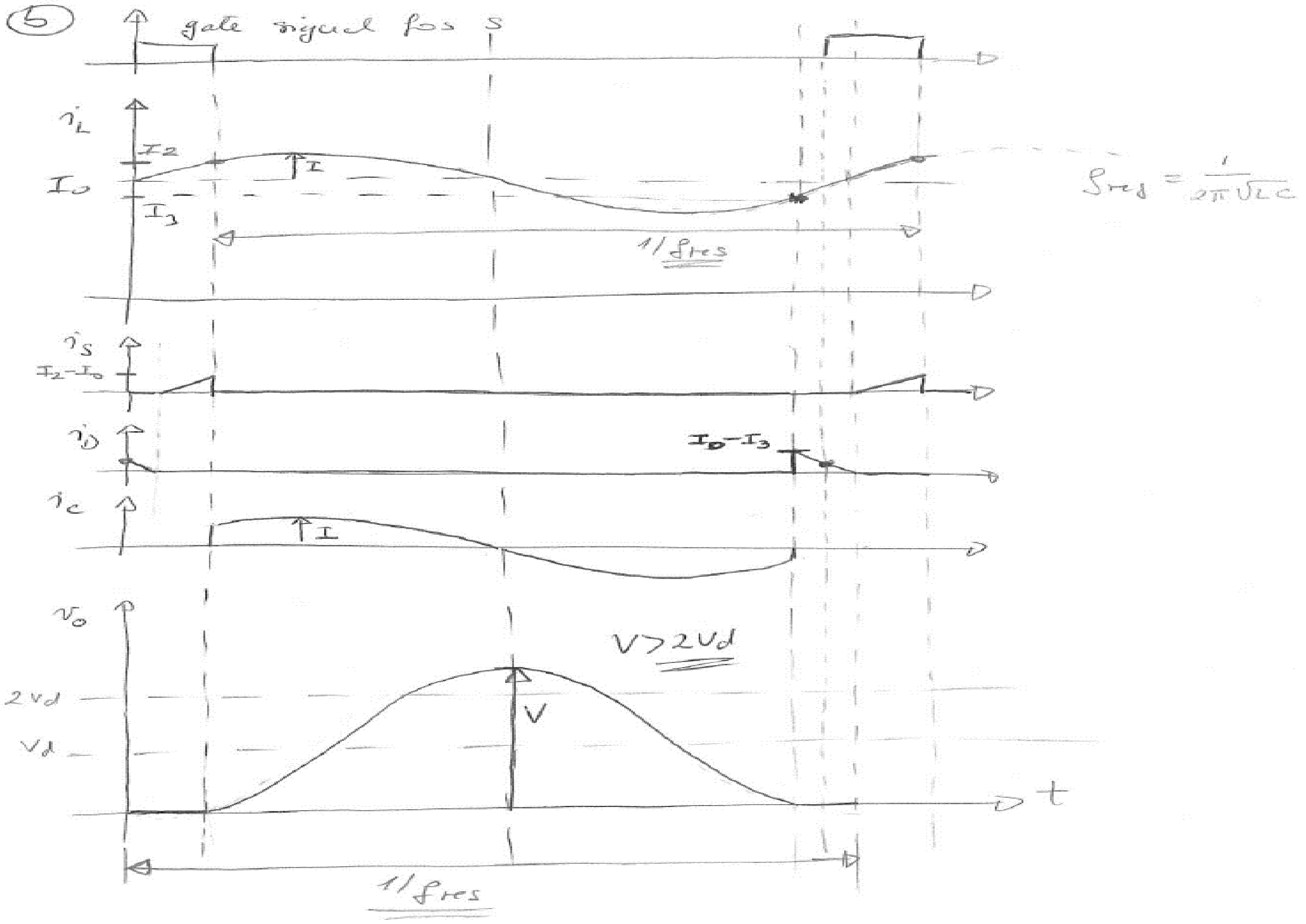
3) Diode D in ON

At the instant when voltage V_c drops to zero, the freewheeling diode D becomes forward biased & it shorts the output of the circuit. ⇒

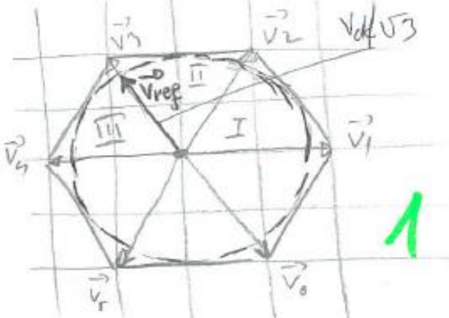


The output remains at zero voltage as long as the diode is conducting. Therefore, switch S has to be turned on before the diode current reaches zero so that the switch is turned on at zero voltage.

⑤



c) $\boxed{4}$



1

$|\vec{V_{ref}}| = \frac{V_{dc}}{\sqrt{3}}$ (max achievable)

$m_a = \frac{|\vec{V_{ref}}|}{\frac{V_{dc}}{\sqrt{3}}} = \underline{\underline{1}}$ 1

$\theta = 120°$ ⟹ Sector $\underline{II}$; $\vec{V_{ref}}$ can be synthesised by $\vec{V_2}$, $\vec{V_3}$ and $\vec{V_0}$.

Eq. (3.1) & (3.2) are valid for Sector I

For Sector $\underline{II}$ we have:

$$T_{V_2} = \underbrace{\sqrt{3} \cdot \frac{V_{ref}}{V_{dc}}}_{} \cdot T_s \cdot \sin\left(\frac{\pi}{3} - \theta'\right)$$

$$T_{V_3} = \underbrace{\sqrt{3} \cdot \frac{V_{ref}}{V_{dc}}}_{m_a} \cdot T_s \cdot \sin(\theta') \qquad 0 < \theta' < \frac{\pi}{3}$$ 1

$$\theta' = 120° - \frac{\pi}{3} = \underline{\underline{\frac{\pi}{3}}}$$

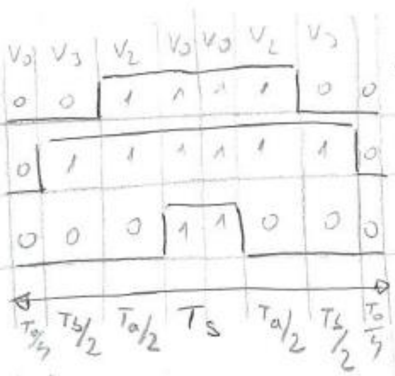
$$T_a = T_{V_2} = m_{a,max} \cdot T_s \cdot \sin\left(\frac{\pi}{3} - \frac{\pi}{3}\right) = \emptyset$$

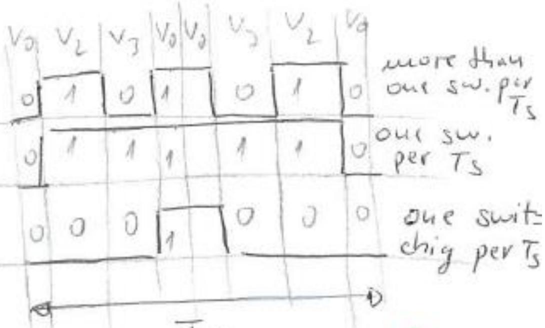
$$T_b = T_{V_3} = m_{a,max} \cdot T_s \cdot \sin\left(\frac{\pi}{3}\right) = 1 \cdot 200\mu s \cdot \frac{\sqrt{3}}{2} = 173\mu s$$

$$T_0 = T_s - T_{V_2} - T_{V_3} = 200\mu s - 173\mu s = \underline{\underline{27\mu s}}$$ 1

d) Sector $\underline{II}$

7-segment sw. sequence

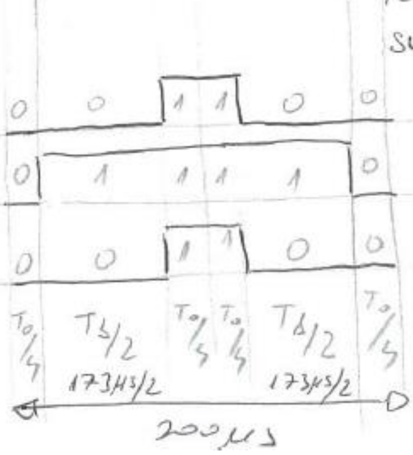




$\boxed{4}$ 1

better option to minimise switching and losses 1

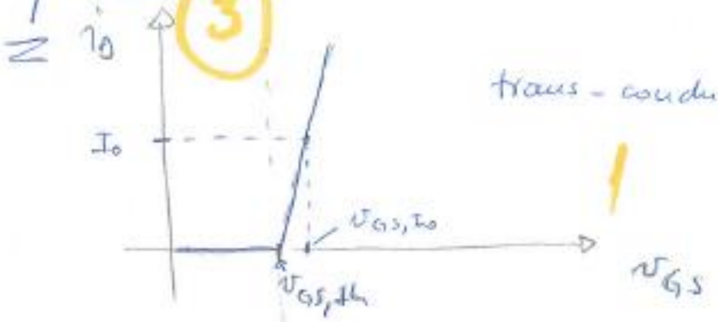
1



1

Question 4

4 a.i 3



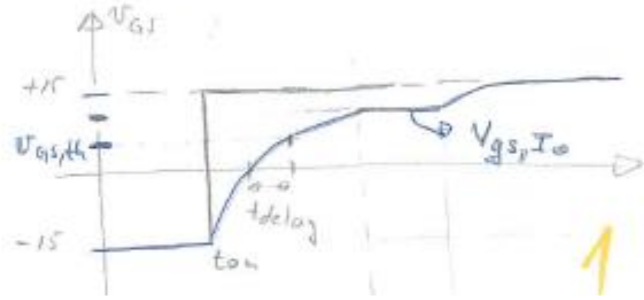
trans-conductance $g_{fs} = \frac{\Delta i_D}{\Delta v_{GS}} = \frac{I_o}{v_{GS,I_o} - v_{GS,th}}$ 1

Transfer curve (linearised)

$I_o = g_{fs}\,(v_{GS,I_o} - v_{GS,th})$

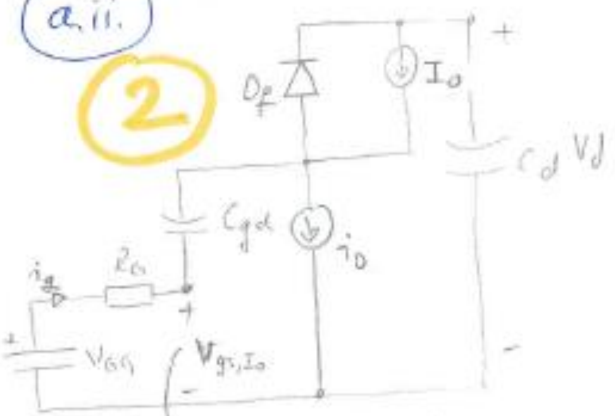
$\frac{I_o}{g_{fs}} + v_{GS,th} = v_{GS,I_o}$

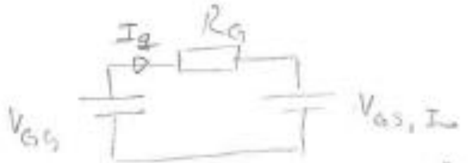
$\frac{300}{60} + 6 = \underline{\underline{11V}}$ 1



1

a.ii. 2

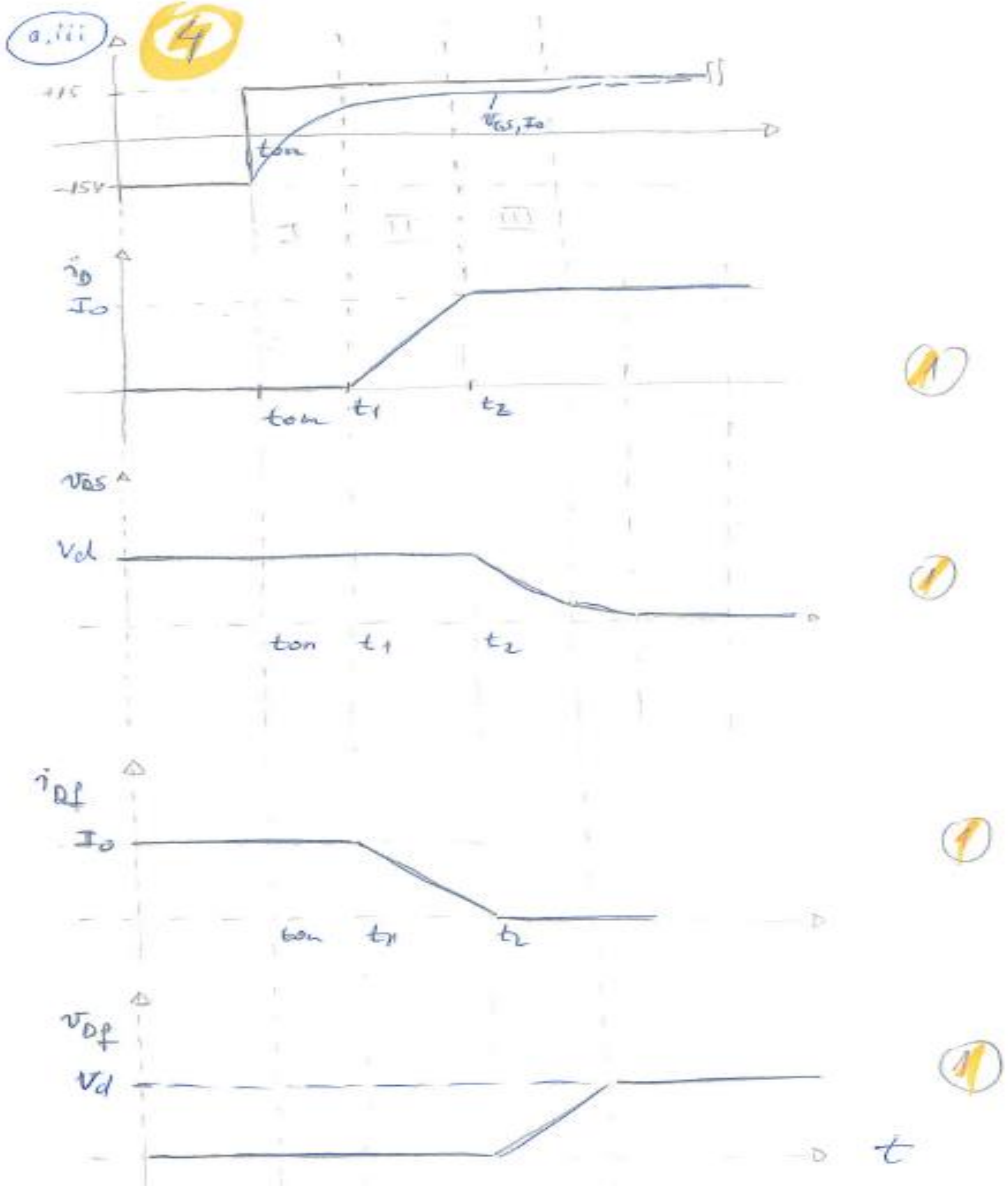


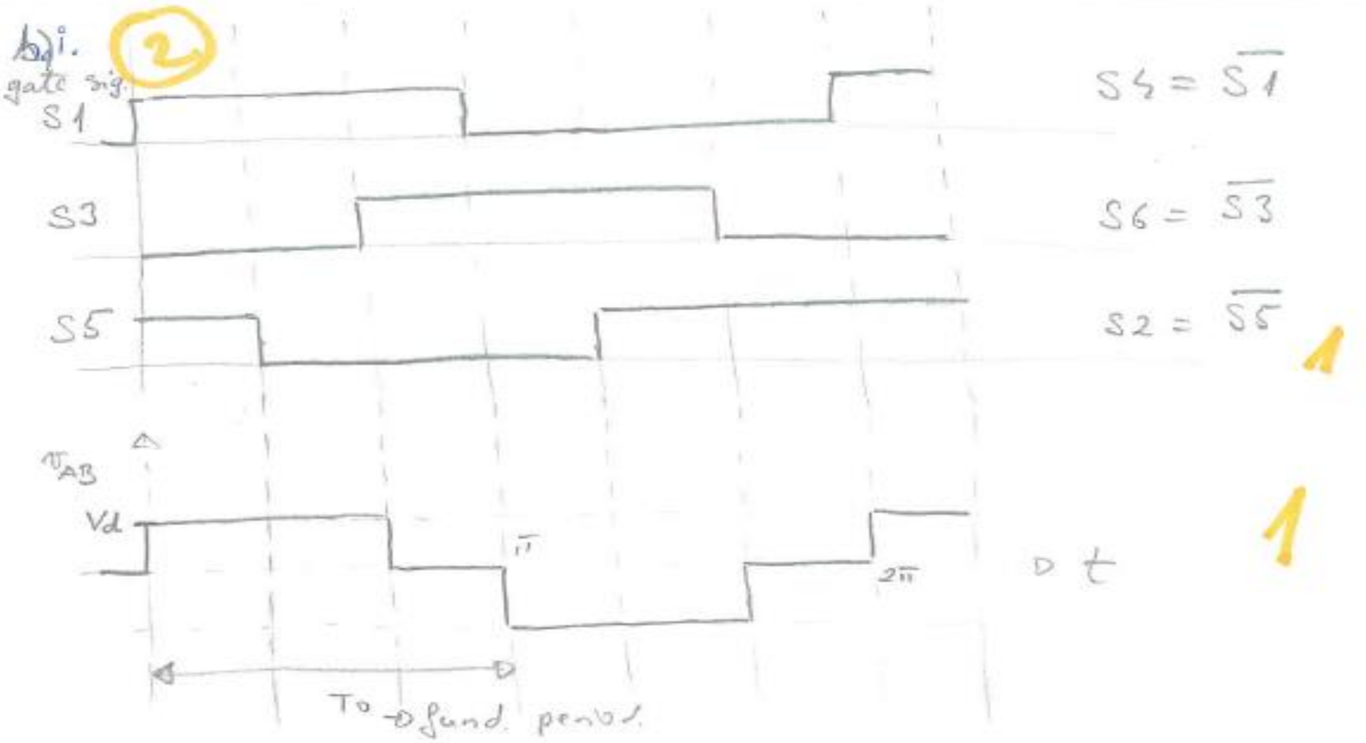


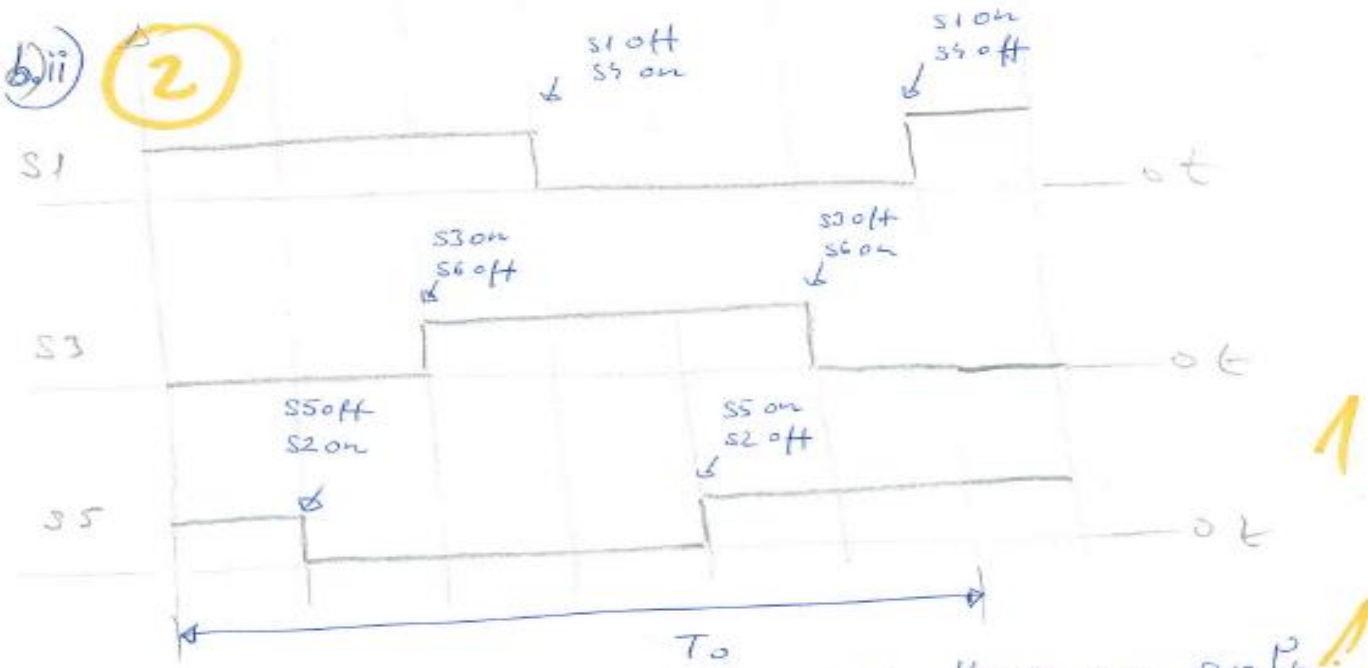
from

1 $I_g = \frac{V_{GG} - V_{GS,I_o}}{R_G} = \frac{15-11}{2} = \underline{\underline{2A}}$

C_{gs} → fully charged and no current flows through C_{gs} = voltage across C_{gs} = const. = V_{GS,I_o} 1


a.iii
4
+15
-15V
v_{GS}, I_o
ton
i_D
I_o
ton
t_1
t_2
v_{DS}
V_d
ton
t_1
t_2
i_{Df}
I_o
ton
t_1
t_2
v_{Df}
V_d
t
1
1
1
1

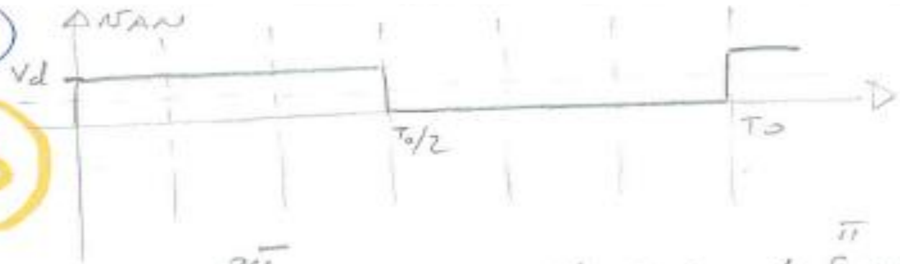




During one fundamental period there are only six switching commutations as above shown. 1

5, iii)

3



T_0 – fun. period

$T_0 = \frac{1}{50} = 20\,ms$

$$A_1 = \frac{1}{\pi}\int_0^{2\pi} u(\omega t)\cdot\sin(\omega t)\cdot d\omega t = \frac{1}{\pi}\int_0^{\pi} V_d\cdot\sin(\omega t)\cdot d\omega t =$$

$$= \frac{V_d}{\pi}(-\cos\omega t)\Big|_0^{\pi} = \frac{V_d}{\pi}(\cos 0 - \cos\pi) = \frac{2V_d}{\pi}$$ 1

rms line-to-line voltage:

$$V_{AB,rms} = \sqrt{3}\cdot\frac{2V_d}{\pi}\cdot\frac{1}{\sqrt{2}}$$ 1

$$460 = \frac{\sqrt{3}\cdot 2\cdot V_d}{\sqrt{2}\cdot\pi}$$

$$V_d \cong 589{,}97V$$ 1

$$\underline{V_d \simeq 600V}$$