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The University Of Sheffield.

DEPARTMENT OF ELECTRONIC AND ELECTRICAL ENGINEERING

Spring Semester 2010-2011 (2 hours)

EEE6033 Digital Signal Processing 6

Answer **THREE** questions. **No marks will be awarded for solutions to a fourth question.** Solutions will be considered in the order that they are presented in the answer book. Trial answers will be ignored if they are clearly crossed out. **The numbers given after each section of a question indicate the relative weighting of that section.**

1. **a.** i) In the context of a discrete-time system, explain the concepts of causality, stability, linearity and time invariance. (4 marks)

ii) Determine whether the following two systems are (a) causal, (b) stable, (c) linear time-invariant: (6 marks)

1) $y[n] = \sum_{k=-1}^{10} x[n-k]$ 2) $y[n]=2^{x[n]}$ **(10)**

b. The following is a linear time-invariant (LTI) system (Figure 1) with an input x[n] and an output y[n]. It consists of three sub-systems with impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$, respectively.

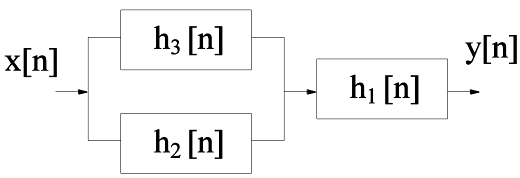

Figure 1

Suppose their impulse responses are given by

$$h_1[n] = h_2[n] = \begin{cases} 1 & n = 0,1 \\ 0 & otherwise \end{cases} \quad \text{and} \quad h_3[n] = \begin{cases} 1 & n = 1,2 \\ 0 & otherwise \end{cases}$$

i) Calculate the impulse response of the whole LTI system. (4 marks)

ii) Without explicitly calculating the magnitude response, state the gain of the system for $\Omega = 0$ and $\Omega=\pi$. (2 marks) **(6)**

c. State the Nyquist sampling theorem and determine the minimum sampling frequency required for sampling the following continuous-time signal x(t)

$$x(t)=\cos(20\pi t)+\cos(100\pi t)$$ **(4)**

2. **a.** For a particular linear discrete-time filtering system, its output y[n] for each time index n is given by the average of its inputs at n and $n-1$.

- **i)** Obtain the linear constant coefficient difference equation describing the behaviour of the filter. (2 marks)
- **ii)** Determine the z-transform H(z) for this system and the associated pole-zero plot. (4 marks)
- **iii)** Is this system a minimum phase system? Explain your answer. (3 marks)

(9)

b. As far as possible, derive the transfer function for an IIR filter which has the z-plane pole-zero plot shown in the following (Figure 2), where there are 2 poles and 2 zeros (3 marks). Sketch the frequency response of the filter (no details needed) — Does it possess a lowpass, highpass, bandpass or bandstop characteristic (3 marks)?

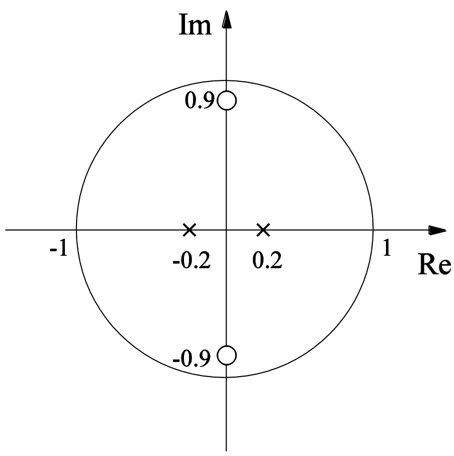

Figure 2

(6)

c. The impulse response h[n] of an LTI discrete-time system is given by

$$h[n]= \delta[n]+3\delta[n-1]-\delta[n-2].$$

Use z-transforms to calculate the output y[n] of the system given the input signal

$$x[n]=\delta[n]+3\delta[n-1]-\delta[n-2]+3\delta[n-3].$$

(5)

3. **a.** Calculate the Discrete Fourier Transform (DFT) of the discrete series x[n]={0.5, 1, 1, 0.5}.

(5)

b. Consider a sequence $x_1[n]$ whose length is L (nonzero for n=0, 1, …, L-1) and a sequence $x_2[n]$ whose length is P (nonzero for n=0, 1, …, P-1). A linear convolution of these two sequences will generate a third sequence $x_3[n]$. Describe the process involved in calculating this linear convolution using DFT.

(5)

c. A lowpass digital filter is to be designed and the first order lowpass filter given in the following equation is used as a prototype, where ω_b is the filter cutoff frequency.

$$H(s) = \frac{\omega_b}{s + \omega_b}$$

i. Design the digital filter using the Impulse Invariance method if ω_b=5rad/sec and the filter is implemented at a sampling frequency of 8Hz. (4 marks)

ii. Given the same sampling frequency of 8 Hz, design the digital filter using the Bilinear Transform method and make sure that the resultant digital filter has a normalised cutoff frequency corresponding to 5rad/sec. (6 marks)

(10)

4. **a.** What is the output y[n] of an LTI system with impulse response h[n], n= $-\infty$, …, 0, …, $+\infty$, given the sinusoidal input $x[n]=A\sin(\Omega_0 n+\varphi)$ (5 marks)? How does the output change if h[n] is real-valued (3 marks)? **(8)**

b. A digital FIR highpass filter is to be designed for filtering out low frequency noise. It should pass the signal with a frequency higher than 6×10^4 rad/sec (passband cutoff frequency), and attenuate any component below the frequency 6×10^2 rad/sec (stopband cutoff frequency) by more than 40dB. The sampling frequency for implementing this filter is f_s=30kHz.

i) Sketch the magnitude characteristic of the highpass FIR filter in the normalised (digital) frequency domain. (3 marks)

ii) Translate the desired highpass filter characteristic to an appropriate lowpass characteristic that is suitable for design using the 'window' method. Sketch the translated lowpass filter magnitude characteristic with the normalised stopband and passband frequencies. (3 marks)

iii) Design an FIR lowpass filter to meet the lowpass filter characteristic derived in part (ii), using the 'window' method with the aid of the information given below. Obtain the first 4 impulse response coefficients of the resulting lowpass filter. (4 marks)

iv) From the results of part iii), derive the design result for the desired highpass FIR filter and provide the first 4 impulse response coefficients. (2 marks)

Rectangular $w[n]=\begin{cases}1, & 0\le n\le N-1\\ 0, & elsewhere\end{cases}$

Bartlett $w[n]=\begin{cases}2n/(N-1), & 0\le n\le (N-1)/2\\ 2-2n/(N-1), & (N-1)/2\le n\le N-1\\ 0, & elsewhere\end{cases}$

Hanning $w[n]=\begin{cases}\{1-\cos[2\pi n/(N-1)]\}/2, & 0\le n\le N-1\\ 0, & elsewhere\end{cases}$

Hamming $w[n]=\begin{cases}0.54-0.46\cos[2\pi n/(N-1)], & 0\le n\le N-1\\ 0, & elsewhere\end{cases}$

Blackman $w[n]=\begin{cases}0.42-0.5\cos[2\pi n/(N-1)]+0.08\cos[4\pi n/(N-1)], & 0\le n\le N-1\\ 0, & elsewhere\end{cases}$

	Main-lobe width	Peak of first-lobe	Stopband attentuation factor (for lowpass filter)
Rectangular	$\approx 4\pi/N$	-13dB	-21dB
Bartlett	$\approx 8\pi/N$	-27dB	-25dB
Hanning	$\approx 8\pi/N$	-32dB	-44dB
Hamming	$\approx 8\pi/N$	-43dB	-53dB
Blackman	$\approx 12\pi/N$	-58dB	-74dB

(12)

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