# Comprehensive simulation and analysis of four bar mechanism

## 1 Overview

Four bar Analyzer is developed as a Flash application using ActionScript 3.0 as the programming language. For its development, the following features were considered.

a) It should have preloaded mechanism, thus saving students' efforts and time required to build a model for analysis. Thus, making the tool attractive to students.
b) It should be easy to use.
c) It should be able to animate and plot graphs or vector diagrams of analyses results.
d) Since, it has to be easily distributed to the end users, it has to be less dependent on other applications.
e) It should contain all the concepts associated with a mechanism like branching, configurations, ability to trace coupler curves, perform FKin and IDyn.
f) The size of the application should be as low as possible so that it is easily downloadable even with slow internet connections.

### 1.1 Workflow

Four bar Analyzer requires the length of the links of four bar mechanism as the input to perform position, velocity, acceleration and inverse dynamics. The illustration of its workflow is given in Fig. (1).

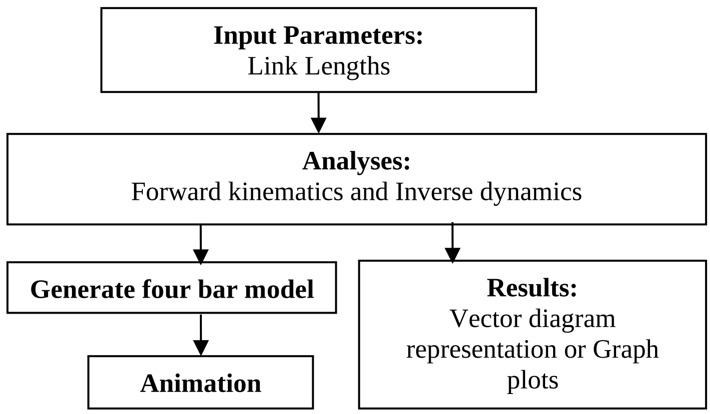


Figure 1 Workflow of Four bar Analyzer

### 1.2 Analyses

The forward kinematics of the four bar mechanisms is performed based on the input parameters entered by the user. The position analyses is performed based on the formulations of geometric approach, whereas for velocity and acceleration analyses, numerical differentiation is performed. Inverse dynamics of four bar mechanism is performed based on the principle of virtual work. The torque required by the driving joint is calculated when an external force is applied to one of the links of four bar linkage. The location and angle of application of the external force can be changed and the corresponding results are displayed through graph plots. The formulations involving these calculations are covered in detail in Section 3.

## 1.3 Generation of four bar model and animation

The shape and orientation of the links are generated based on length of the input links and position analysis results. The limits of the animation are set by calculating the maximum and minimum angle of the driving joint. The mechanism is made to run between the calculated initial and final position in a loop. The user interface of the Four bar Analyzer is shown in Fig. (2).

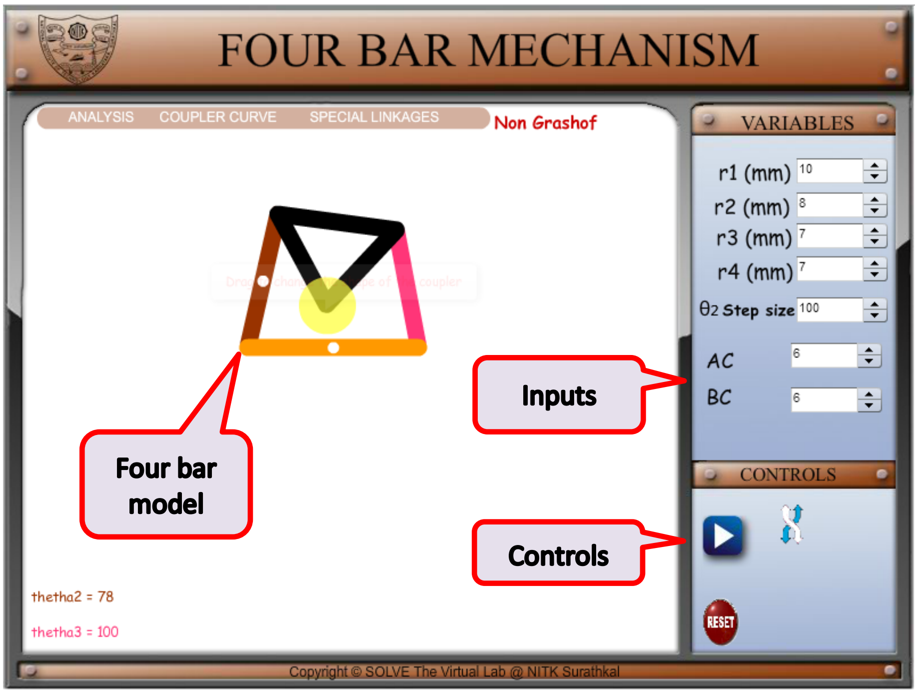


Figure 2 : User interface of Four bar Analyzer

To make the learning more fun and realistic, a lot of the functions can be performed through the mouse. Panning of the four bar model, length of the coupler links, crank rotation, location and the angle of the external force in inverse dynamics etc. can all be changed through mouse inputs. The coupler curves can be traced for any valid link lengths with different colours for the sake of visual comparison, as shown in Fig. (3). Moreover, the coupler curves can be drawn during animation to see the path it follows.

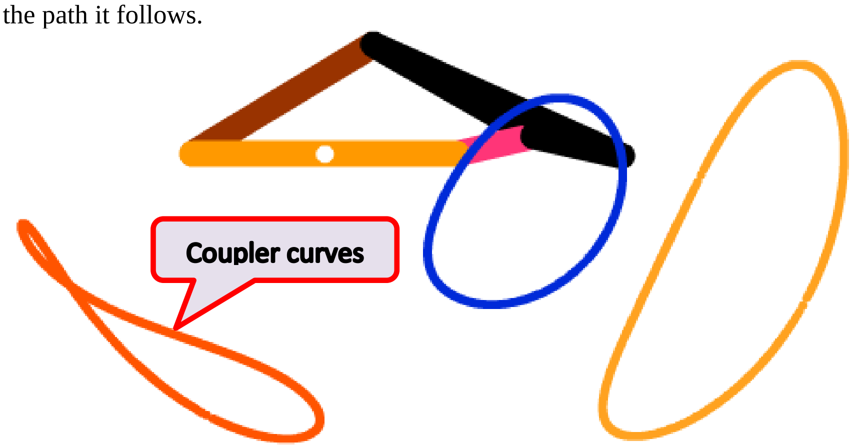


Figure 3 Traces of Coupler curves

## 1.4 Conditions and Configurations

Every four bar linkage follows one of the following two conditions depending on the rotating behavior of the shortest link.

a) Grashof Condition: The shortest link of the mechanism makes a complete rotation with respect to its neighboring link.
b) Non-Grashof condition: None of the links in the mechanism makes a full rotation with respect to its neighboring link.

Unlike most of the applications presented in Table 1, Four bar Analyzer can perform analysis of the above mentioned conditions under a single program. This helps the students to understand how the analyses results and the coupler curve changes when the mechanism switches to a different condition upon changing the link length.

In addition, Grashof four bar mechanism has multiple solutions for any given input crank angle. Four bar Analyzer determines both the solutions and allows a student to swap between the two solutions, which are referred as configurations or branches. Fig. (4). shows the two possible solutions for a particular input angle. This helps the student in better understanding of different solutions of a four bar mechanism.

a) Configuration 1    b) Configuration 2

Figure 4: Two Solution of four bar mechanism

## 1.5 Special mechanisms

Straight line motion generation using simplest linkage mechanisms is one of important requirement in industrial applications. By setting particular link lengths of a four bar mechanism, approximate straight line motion can be generated. Some of these special mechanisms are available in Four bar Analyzer, where students can generate straight line motion by selecting the inbuilt templates and observe the animation. These mechanisms are shown in Fig. (5).

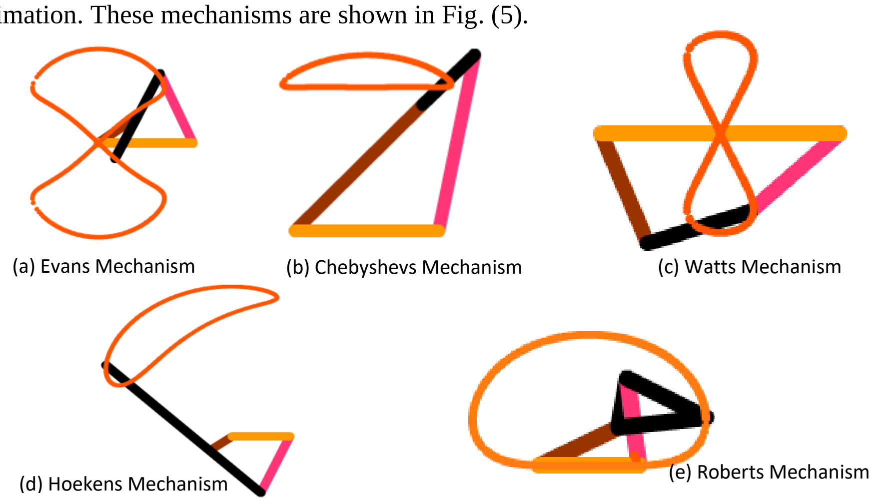


Figure 5: Special four bar mechanisms

# 2 Kinematic Analysis and Inverse dynamics

Kinematic analysis is the study of the position, velocity and acceleration of the links and joints, whereas inverse dynamics of a mechanism deals with the forces and torques acting on it. In any motion analysis tool, the position analysis is done at the beginning to perform the mechanism's animation, followed by velocity and acceleration analyses. Inverse dynamics is performed at the end which is typically shown as graph plots. Four bar Analyzer also follows similar approach and the associated formulations are explained in the following sub-section.

## 2.1 Position Analysis

To perform the position analysis of a mechanism, the position (angle) of the input (driving) joint and the links' lengths are required as input. Four bar Analyzer determines the position of all the ends of the links based on geometric approach. Consequently, the joint angles of the unknown joints are found out. The formulations of position analysis of four bar mechanism are explained next.

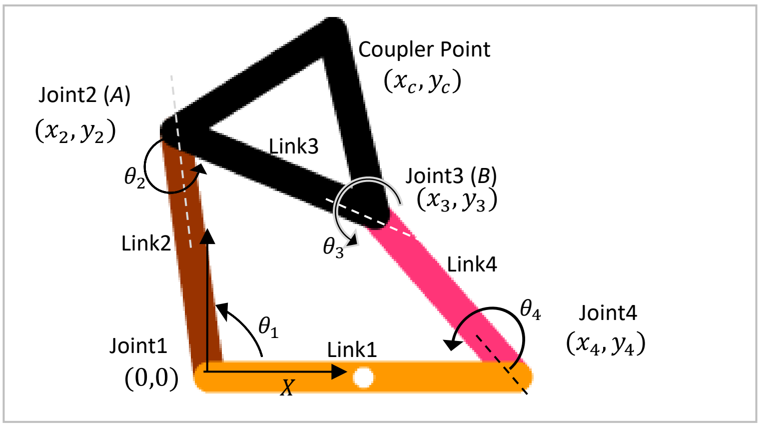


Figure 6: Nomenclature of four-bar mechanism

The nomenclature used in Four bar Analyzer is shown in Fig. (6). Link1 is fixed to ground and the input/driving joint is considered as Joint1. The length of Link1 is considered as $L_1$, length of Link2 as $L_2$ and so on. . For the simplicity of calculations the coordinates of Joint1 is assumed to be (0,0).

The coordinates of Joint2 ($A$) are found from

$$x_2 = L_2\cos\theta_1 \tag{1}$$

$$y_2 = L_2 \sin\theta_1 \tag{2}$$

The coordinates of Joint3 ($B$) are then determined using the equations of circles about Joint2 ($x_2$,$y_2$) and Joint4 ($x_4$, $y_4$)

$${L_3}^2 = (x_3 - x_2)^2 + (y_3 - y_2)^2 \tag{3}$$

$${L_4}^2 = (x_3 - L_1)^2 + {y_3}^2 \tag{4}$$

Eq. (3) and Eq. (4) provide a pair of simultaneous equations in $x_3$ and$y_3$, i.e., the coordinates of point $B$. Subtracting Eq. (4) from Eq. (3) gives an expression for $x_3$ as

$$x_3 = \frac{{L_2}^2 - {L_3}^2 + {L_4}^2 - {L_1}^2}{2(x_2 - L_1)} - \frac{2y_2y_3}{2(x_2 - L_1)} \tag{5}$$

Substitute the known component of Eq. (5) as $S$, i.e.

$$S = \frac{{L_2}^2 - {L_3}^2 + {L_4}^2 - {L_1}^2}{2(x_2 - L_1)} \tag{6}$$

The expression of $x_3$ is then reduces to

$$x_3 = S - \frac{2y_2y_3}{2(x_2 - L_1)} \tag{7}$$

Substituting (7) into (4) results in a quadratic equation in $y_3$ yielding its solution as

$$y_3 = \frac{-Q \pm \sqrt{Q^2 - 4PR}}{2P} \tag{8}$$

where *P*, *Q* and *R* are given by,

$$P = \frac{{y_2}^2}{(x_2 - L_1)^2} + 1 \tag{9}$$

$$Q = \frac{2y_2(L_1 - S)}{(x_2 - L_1)} \tag{10}$$

$$R = (L_1 - S)^2 - {L_4}^2 \tag{11}$$

It should be noted that the expression for $y_3$ has two possible solutions, which correspond to the two configurations of a four-bar mechanism as explained in Section 2.4. Depending on the configuration chosen, the value of $y_3$ is determined using Eq. (8) and then the value of $x_3$ is calculated using Eq. (7). Knowing the coordinates of Joint2 and Joint3, i.e., points *A* and *B*, the joint angles of Joint2 ($\theta_2$), Joint3 ($\theta_3$) and Joint4 ($\theta_4$) are calculated using simple trigonometry. To perform animation for a range of input motion, the above set of calculations is repeated for different values of $\theta_1$ and the corresponding values of the coordinates of the position of joints are determined.

## 2.2 Velocity and acceleration Analysis

For the velocity analysis, numerical method was used in Four bar Analyzer, as opposed to an analytical method. With numerical method, the velocity of any point on the links and the velocity of a joint can be easily obtained. If the positions of any link or joint are $P_1$ and $P_2$ at time $t_1$ and $t_2$, then the velocity at time $t_2$, i.e., $V_2$, is determined as

$$V_2 = \frac{P_2 - P_1}{t_2 - t_1} \tag{12}$$

Since, the time interval ($t_2$-$t_1$) is kept constant throughout the simulation, the Eq. (12) becomes

$$V_2 = \frac{P_2 - P_1}{t} \tag{13}$$

where,

$$t = t_2 - t_1 \tag{14}$$

The time interval *t* is chosen to be small enough to give accurate results for all configurations.

The approach to determine the angular acceleration of joints or acceleration of any point on links is similar to the calculation methodology discussed above. If $V_1$ and $V_2$ are the velocities at time $t_1$ and $t_2$, then the acceleration at time $t_2$, i.e., $A_2$, is given by

$$A_2 = \frac{V_2 - V_1}{t} \tag{15}$$

## 2.3 Inverse Dynamics

Inverse dynamics is about the calculation of forces and torques involved in a mechanism based on the kinematics of the links and joints. Four bar Analyzer performs Inverse dynamics of a four bar mechanism when an external force is acting on Link4. The location and angle of the application of the external force F are user inputs and can be easily controlled using mouse as shown in Fig. (7).

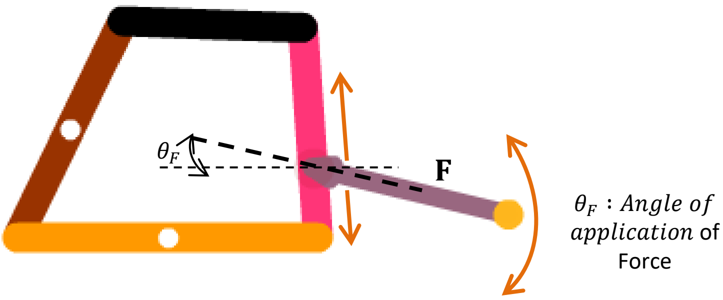


Figure 7 External Force on Four-bar linkage

Inverse dynamics is performed based on the concept of virtual work [7]. The governing equation of virtual work is given below in Eq. (16):

$$\sum_{k=2}^{n} \boldsymbol{F}_k \cdot \boldsymbol{V}_k + \sum_{k=2}^{n} \boldsymbol{T}_k \cdot \boldsymbol{\omega}_k = \sum_{k=2}^{n} m_k\, a_k \cdot v_k + \sum_{k=2}^{n} I_k\, \alpha_k \cdot \omega_k \tag{16}$$

Where,
**F** is (external) Force vector
**V** is the velocity vector of CG of any link
**T** is (external) Torque/moment vector
ω is the angular velocity of any joint
m is the mass of the link
**a** is the acceleration of the CG of any link
**I** is the moment of inertia of any link
α is the angular acceleration of any joint.
n=4 (since the number of links in four bar mechanism is 4)
k represents each of the four links or joints of four-bar linkage. It starts with 2 as Link1 is the ground link.

The first two terms (in the LHS) in Eq. (16) represents all the external forces and external torques applied to the system respectively. The two terms in the RHS represents the inertia forces and inertia torque present in the system.

Since no external forces are considered on Link2 and Link3 and no external torques on Joint2, Joint3 and Joint4, Eq. (16) can be expanded in the vector form as below:

$$\boldsymbol{F}_4 \cdot V_4 + \boldsymbol{T}_{12} \cdot \omega_2 = \left(m_2 a_{G_2} \cdot v_{G_2} + m_3 a_{G_3} \cdot v_{G_3} + m_4 a_{G_4} \cdot v_4\right) + \left(\boldsymbol{I}_2 \alpha_2 \cdot \omega_2 + \boldsymbol{I}_3 a_3 \cdot \omega_3 + \boldsymbol{I}_4 \alpha_4 \cdot \omega_4\right) \tag{17}$$

Angular velocities and angular accelerations of joints and linear velocity and linear acceleration of CGs of links are already known from the results of kinematic analysis (Section 3.1 and 3.2). Four bar Analyzer solves Eq. (17) by expanding it into scalar equations to determine the value of $T_{12}$, the driving torque. Further, these set of equations are repeatedly solved for every value of $\theta_1$ and the corresponding graph is plotted as shown in Fig. (8).

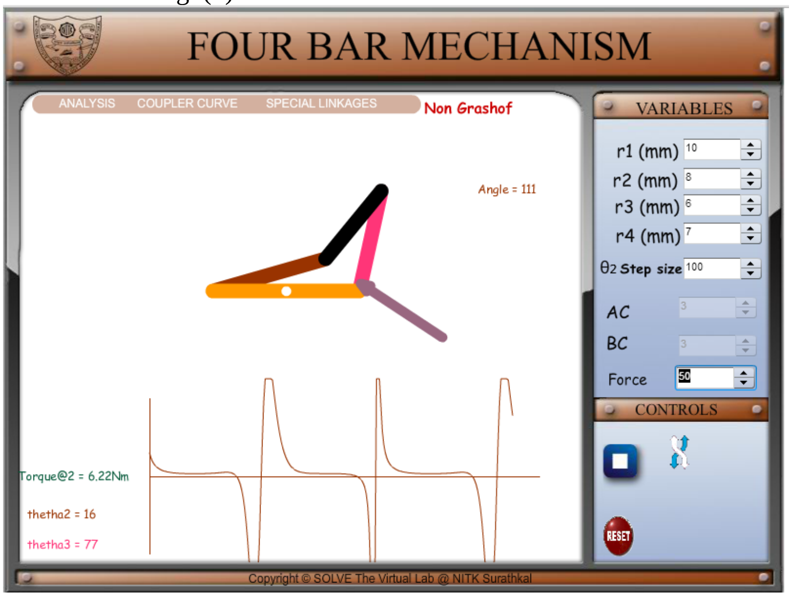


Figure 8 Inverse dynamics in Four bar Analyzer

## 3 Future Scope

Four bar Analyzer has several good features which needs further fine tuning with addition of new features. Some of them which are planned for implementation are given below:

a) Up gradation to HTML5 from Flash environment.
b) Inclusion of inversions and its analyses
c) Optimization
d) Detailed dynamic force analysis
e) Interfacing with MATLAB and other software

## 4 Conclusions

A new online mechanism learning tool is presented in this paper. It performs kinematic analysis of four bar mechanism by taking the length of links as input. The application size is just 132KB and yet it comes loaded with many features like configurations, special mechanisms, FKin and IDyn which can be accessed with user friendly interface. Four bar Analyzer can be downloaded and used for free from http://solve.nitk.ac.in/

## References

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