Ex 2

f est dérivable en $x_0 \overset{?}{\Longrightarrow} f$ est continue en x_0

$$\lim_{x_0} \frac{f(x)-f(x_0)}{x-x_0} = l \overset{?}{\Longrightarrow} \lim_{x_0} f(x) = f(x_0).$$

$$\lim_{x_0} f(x) = \lim_{x_0} \left[f(x) - f(x_0) + f(x_0) \right]$$

$$= \lim_{x_0} \left[f(x) - f(x_0) \right] + f(x_0)$$

~~on~~ montrons que $\lim_{x_0} \left(f(x) - f(x_0) \right) = 0$.

$$\lim_{x_0} f(x) - f(x_0) = \lim_{x_0} \frac{f(x)-f(x_0)}{x-x_0} (x-x_0)$$

$$= f'(x_0) \times \lim_{x_0} (x-x_0) = l \times 0 = 0$$

Ainsi $\lim_{x_0} f(x) = f(x_0)$, donc f est continue en x_0.