Limites-Équivalents

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Limites usuels : $\alpha,\ \beta > 0,\ \ a \in \mathbb{R}^*$

$\dfrac{\sin(ax)}{x} \xrightarrow[x\longrightarrow 0]{} a$; $\dfrac{\cos x - 1}{x} \xrightarrow[x\longrightarrow 0]{} 1$; $\dfrac{1-\cos^2 x}{x} \xrightarrow[x\longrightarrow 0]{} \dfrac{1}{2}$; $\dfrac{\tan ax}{x} \xrightarrow[x\longrightarrow 0]{} a$

$\ln x \xrightarrow[x\longrightarrow 0^+]{} -\infty$; $\ln x \xrightarrow[x\longrightarrow +\infty]{} +\infty$; $x^\alpha |\ln x|^\beta \xrightarrow[x\longrightarrow 0^+]{} 0$; $\dfrac{\ln(1+x)}{x} \xrightarrow[x\longrightarrow 0]{} 1$; $\dfrac{(\ln x)^\alpha}{x^\beta} \xrightarrow[x\longrightarrow +\infty]{} 0$

$e^x \xrightarrow[x\longrightarrow -\infty]{} 0$; $e^x \xrightarrow[x\longrightarrow +\infty]{} +\infty$; $xe^x \xrightarrow[x\longrightarrow -\infty]{} 0$; $\dfrac{e^{\alpha x}}{x^\beta} \xrightarrow[x\longrightarrow +\infty]{} +\infty$; $\dfrac{e^x - 1}{x} \xrightarrow[x\longrightarrow 0]{} 1$

Équivalents usuels : $\alpha \in \mathbb{R}^*$

$$f \underset{x_0}{\sim} g \Longleftrightarrow \lim_{x_0} \frac{f(x)}{g(x)} = 1, \quad g(x) \neq 0 \text{ au } V(x_0)$$

$\sin(x) \underset{0}{\sim} x$; $\tan x \underset{0}{\sim} x$; $1-\cos x \underset{0}{\sim} \dfrac{1}{2}x^2$; $(1+x)^\alpha - 1 \underset{0}{\sim} \alpha x$; $\ln(1+x) \underset{0}{\sim} x$; $e^x - 1 \underset{0}{\sim} x$

Si $u(x) \xrightarrow[x\longrightarrow x_0]{} 0$, alors

$\sin(u(x)) \underset{x_0}{\sim} u(x)$; $\tan(u(x)) \underset{x_0}{\sim} u(x)$; $1-\cos(u(x)) \underset{x_0}{\sim} \dfrac{1}{2}u^2(x)$; $(1+u(x))^\alpha - 1 \underset{x_0}{\sim} \alpha u(x)$; $\ln(1+u(x)) \underset{x_0}{\sim} u(x)$; $e^{u(x)} - 1 \underset{x_0}{\sim} u(x)$

Opérations

Si $f_1 \underset{x_0}{\sim} g_1$ et $f_2 \underset{x_0}{\sim} g_2$, alors $f_1 \times f_2 \underset{x_0}{\sim} g_1 \times g_2$ et $\dfrac{f_1}{f_2} \underset{x_0}{\sim} \dfrac{g_1}{g_2}$

Soit $P_n(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$, $a_n \neq 0$. $P_n(x) \underset{\infty}{\sim} a_n x^n$.

Soit $Q(x) = \dfrac{a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n}{b_0 + b_1 x + b_2 x^2 + \ldots + b_m x^m}$, $a_n \neq 0$ et $b_m \neq 0$. $Q(x) \underset{\infty}{\sim} \dfrac{a_n x^n}{b_m x^m}$.