Projet examen finance des marchés

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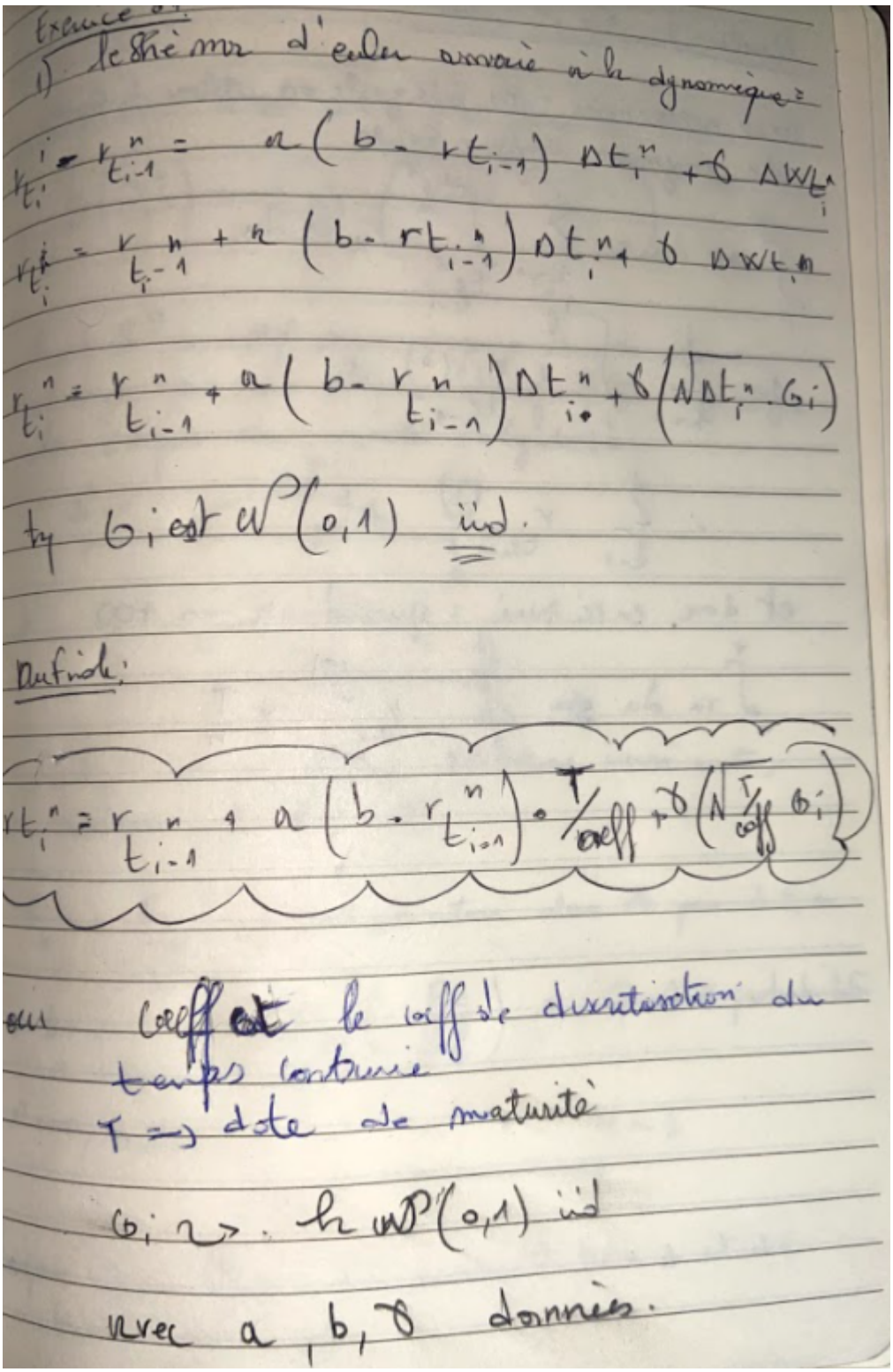
Question 1 :



Question 2 :

In [114…

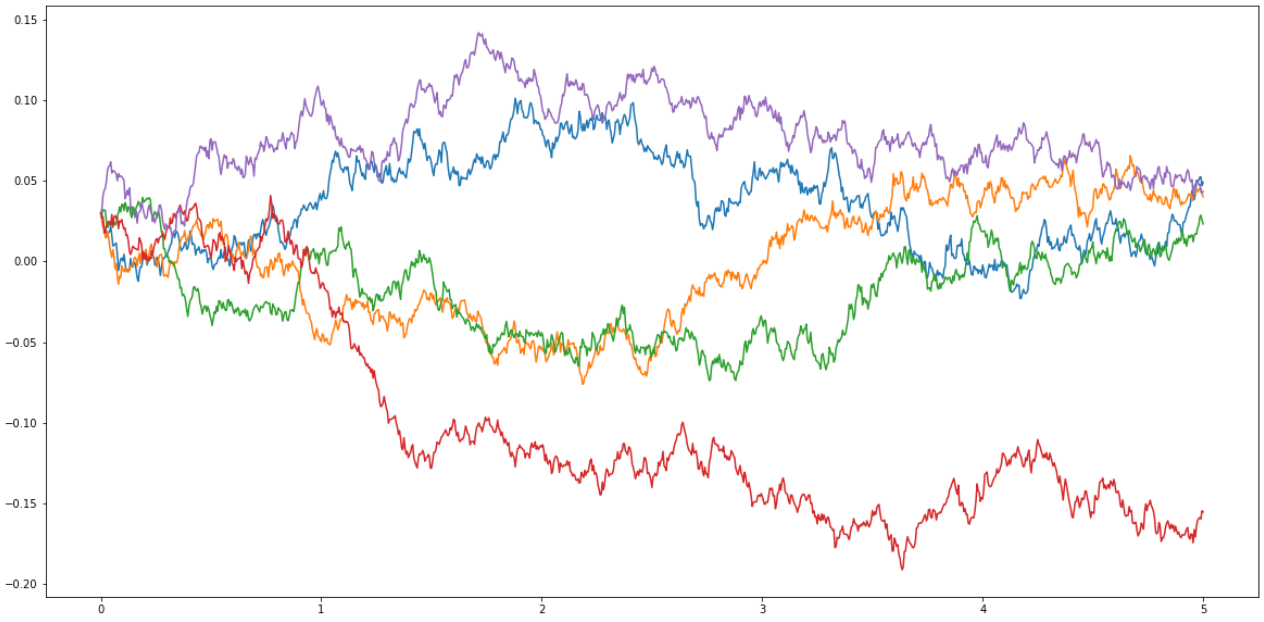
```
import numpy as np
import numpy.random as sim
import matplotlib.pyplot as plt
import matplotlib
matplotlib.rcParams['figure.figsize'] = (20.0, 10.0)
```

In [115…

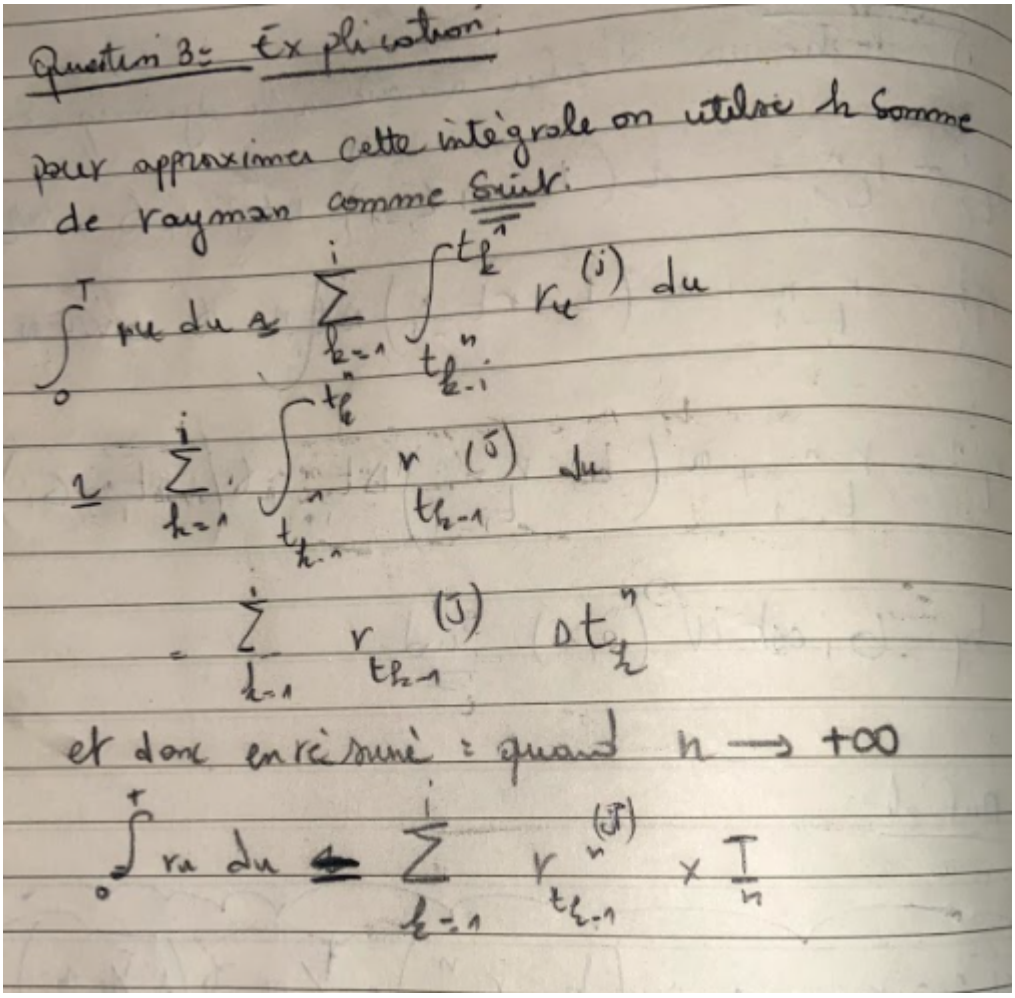
```
n =1000
r0 =3/100
alpha = 0.1
beta =1/100
theta = 5/100
T=5
pas = T/n
N=5
r=np.ones((n+1,N))*r0

for j in range(N):
    for i in range(1,n+1):
        deltaWt = np.sqrt(pas)*sim.randn()
        r[i,j] = r[i-1,j] + alpha*(beta-r[i-1,j])*pas+ theta*deltaWt

dates=np.linspace(0,T,n+1) # n+1 dates
graph=plt.plot(dates,r)
plt.show()
```



Question 3 :



In [116…

```
n =1000
r0 =3/100
alpha = 0.1
beta =1/100
theta = 5/100
T=5
pas = T/n
N=5
r=np.ones((n+1,N))*r0
integ=np.zeros((n+1,N))*r0
for j in range(N):
    for i in range(1,n+1):
        deltaWt = np.sqrt(pas)*sim.randn()
        r[i,j] = r[i-1,j] + alpha*(beta-r[i-1,j])*pas+ theta*deltaWt
        integ[i,j] = pas* np.sum(r[0:i-1,j]) #ici on a calculé l'intégrale pour chaque

print(integ)
```

```
[[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00]
 [ 1.50000000e-04  1.50000000e-04  1.50000000e-04  1.50000000e-04
   1.50000000e-04]
 ...
 [-2.66025954e-01 -1.66335087e-01 -7.92488926e-02  2.42862488e-01
   2.53967123e-01]
 [-2.66571362e-01 -1.66422262e-01 -7.93945190e-02  2.43004285e-01
```

```
  2.53719508e-01]
[-2.67137784e-01 -1.66519203e-01 -7.95460164e-02  2.43114941e-01
  2.53486992e-01]]
```

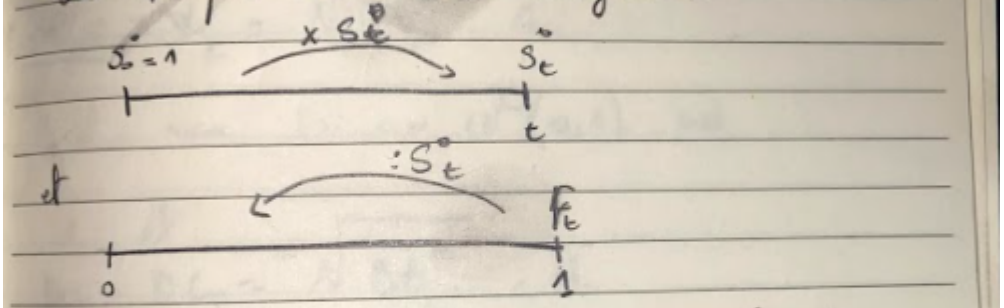
Question 4 :

Question 4:

Explication d'où vient la formule

$$D(0,T) = E_Q\left(e^{-\int_0^T r_u\,du}\right)$$

* Soit S^0 un actif sans risque dans le sens où il n'y a pas de risque de faillite mais il y'a le risque du marché obligataire

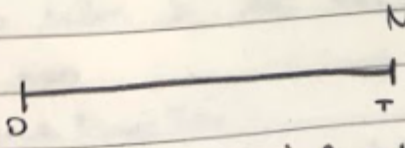


et

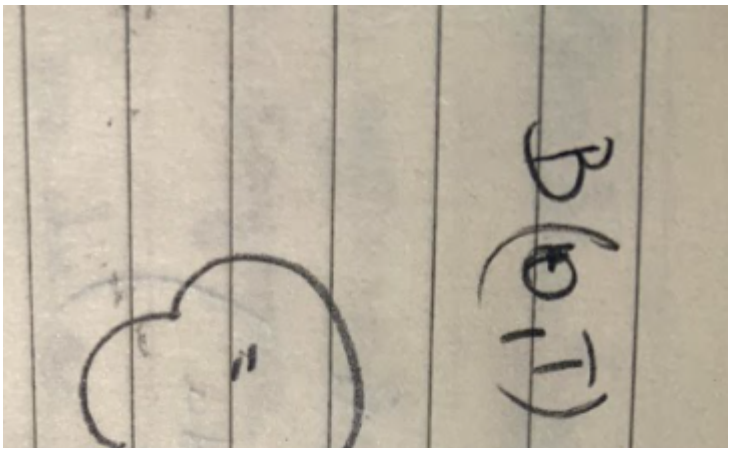
si F_t est un flux aléatoire versé en t, sa valeur actualisée est

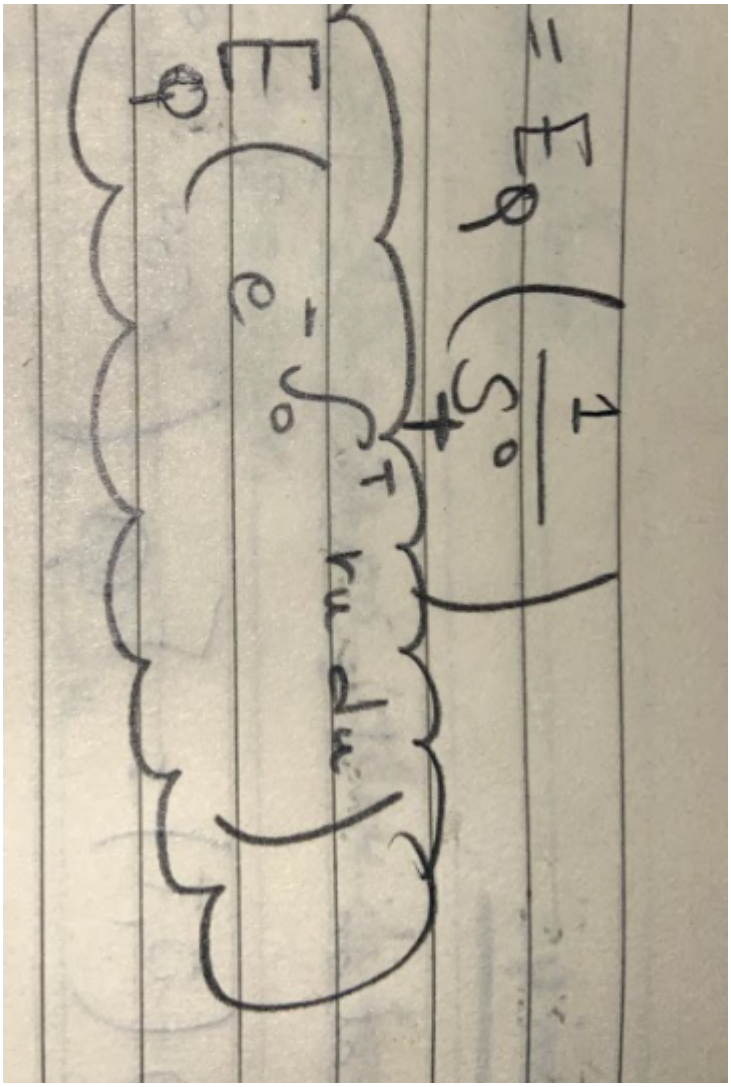
$\tilde{f}_t = \frac{F_t}{S_t^0}$ qui est aléatoire donc le prix de f_t en $t=0$ est: $E_Q\left(\frac{F_t}{S_t^0}\right)$ où Q est la probabilité de risque neutre

Ainsi, pour:



le prix d'un ZC de nominal N = 1 et de maturité T est =





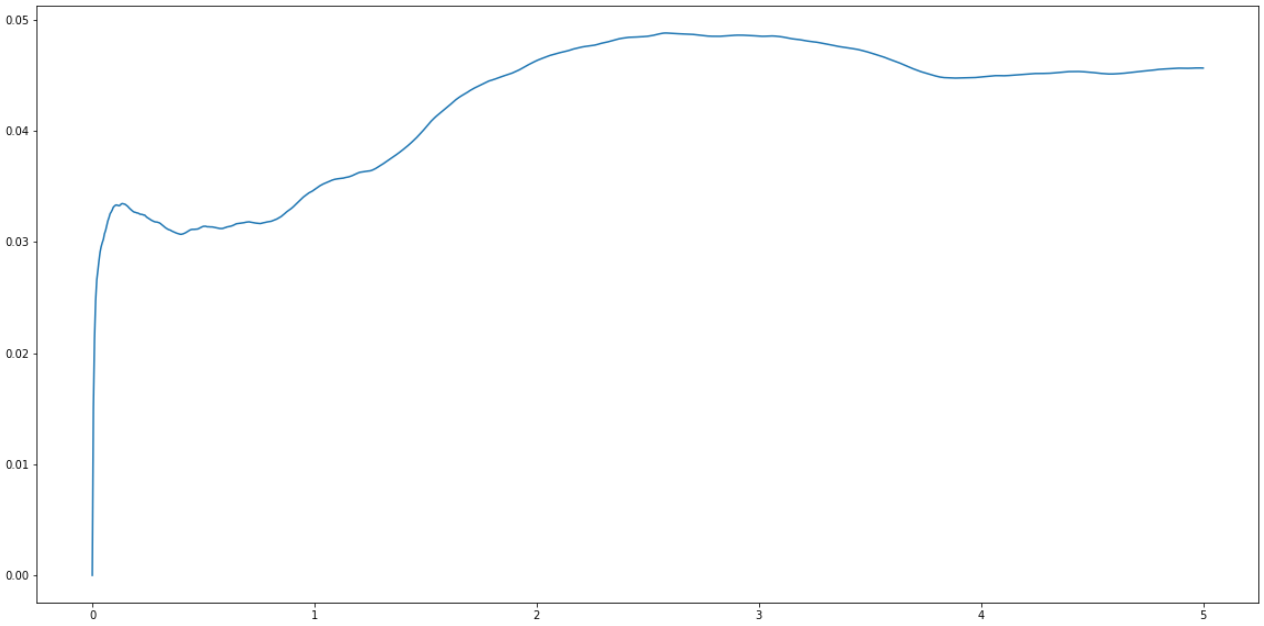
In [117…

```
n =1000
r0 =3/100
alpha = 0.1
beta =1/100
theta = 5/100
T=5
pas = T/n
N=5
r=np.ones((n+1,N))*r0
integ=np.zeros((n+1,N))*r0
Bt=[1] # le vecteur qui stock les Bt(i,n)
R=[]
CoeffActualisation= np.ones((n+1,N)) # Cette variable sert a stocké l'esperence de l'ex
for j in range(N):
    for i in range(1,n+1):
        deltaWt = np.sqrt(pas)*sim.randn()
        r[i,j] = r[i-1,j] + alpha*(beta-r[i-1,j])*pas+ theta*deltaWt
        integ[i,j] = pas* np.sum(r[0:i-1,j]) #ici on a calculé l'intégrale pour chaque
        CoeffActualisation[i,j] = np.exp(-integ[i,j])

 # 2eme etape sommer les Bt(i,n)
for i in range(1,n+1):
    moyenneBt=np.mean(CoeffActualisation[i,:])
    Bt.append(moyenneBt)
    R.append(moyenneBt**(-1/(pas*i))-1)
```

```
R.append(R[-1])
# le vecteur ou on va stocké les valeur du taux Zc R(t)
#on a mis une valeur a cause du problème de n+1 car la longeur de R est n et le rajout

dates=np.linspace(0,T,n+1) # n+1 dates
graph=plt.plot(dates,R)
plt.show()
```



In [118…

```
print("les prix du B0,T sont : ")
print(Bt)
```

```
les prix du B0,T sont :
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```
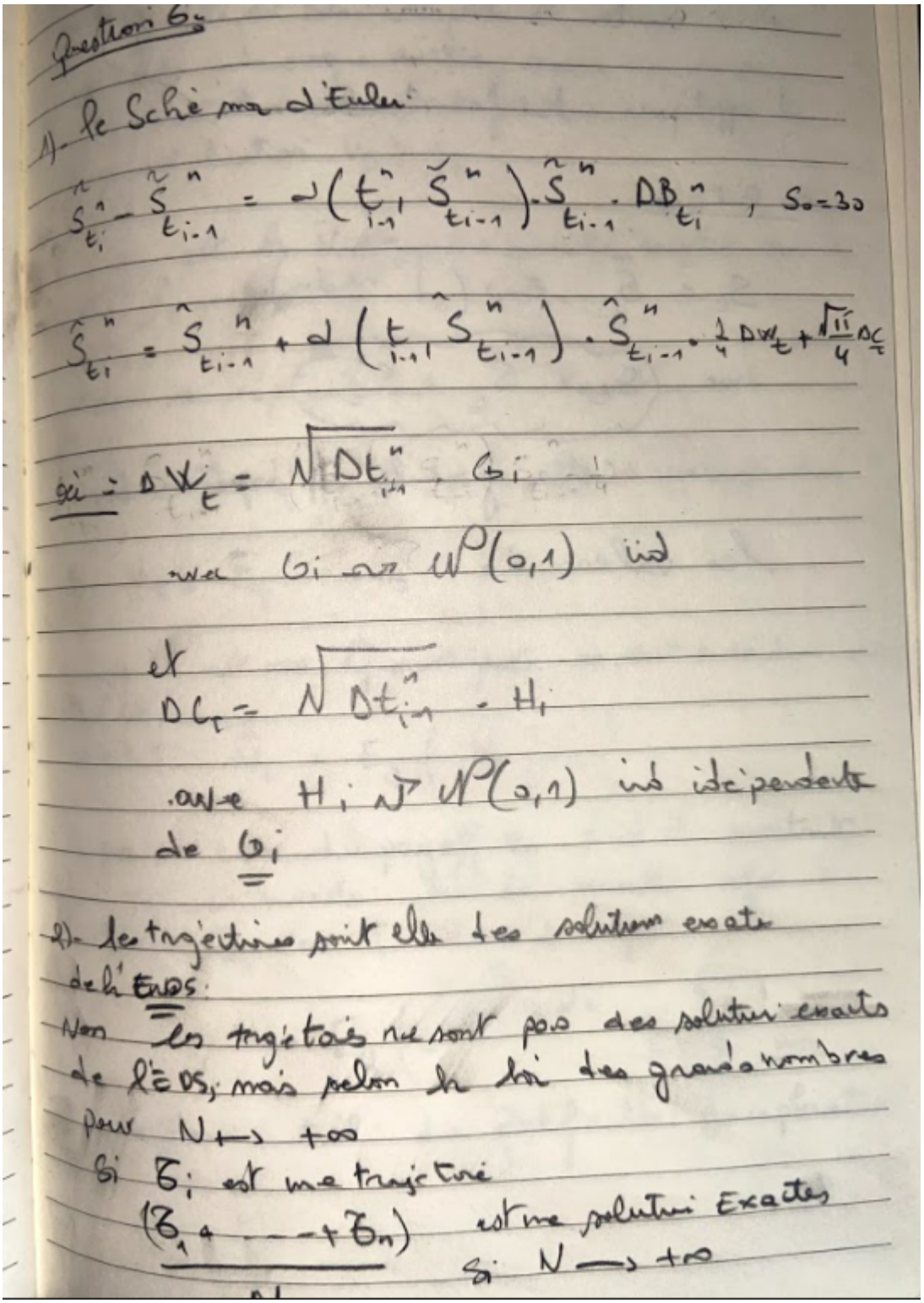

Question 5 :

Les marchés obligataire et l'actif S ne sont pas corrélés car la volatilitée de l'actif S dans l'EDS qui est : $B_t = \frac{1}{4}W_t + \frac{\sqrt{15}}{4}C_t$ où Ct est un mouvement brownien standard independant. On sait que la mouvement brownien du marché et le Wt qui est utilisé dans l'EDS du taux instantané rt. or la volatilité de l'actif S et constitué aussi du movement Ct qui est indépendant du mouvement du marché wt.

donc l'actif S et le marché ne sont pas corrélés.

Question 6 :

Donnez le schéma d'euler :



Question 6 :

1) le Schéma d'Euler :

$$\tilde{S}^n_{t_i} - \tilde{S}^n_{t_{i-1}} = \alpha\left(t_{i-1}, \tilde{S}^n_{t_{i-1}}\right)\cdot \tilde{S}^n_{t_{i-1}} \cdot \Delta B^n_{t_i}, \quad S_0 = 30$$

$$\hat{S}^n_{t_i} = \hat{S}^n_{t_{i-1}} + \alpha\left(t_{i-1}, \hat{S}^n_{t_{i-1}}\right)\cdot \hat{S}^n_{t_{i-1}} \cdot \frac{1}{4}\Delta W_t + \frac{\sqrt{11}}{4}\Delta C_t$$

où : $\Delta W_t = \sqrt{\Delta t^n_{i+1}} \cdot G_i$

avec $G_i \sim \mathcal{N}(0,1)$ iid

et

$\Delta C_t = \sqrt{\Delta t^n_{i+1}} \cdot H_i$

avec $H_i \sim \mathcal{N}(0,1)$ iid indépendante de G_i

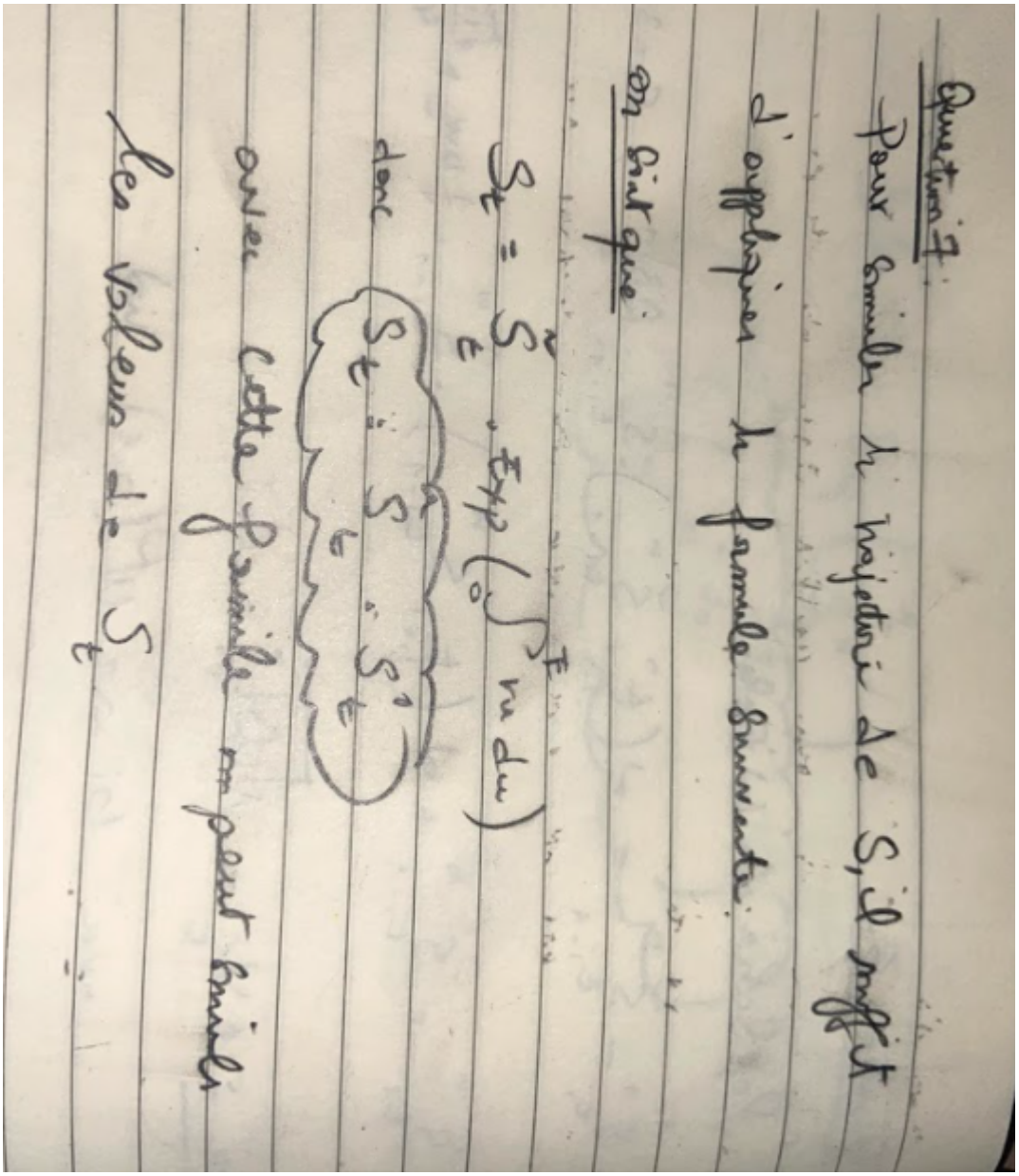
2) les trajectoires sont elles des solutions exacte de l'EDS :

Non les trajectoires ne sont pas des solutions exacts de l'EDS, mais selon la loi des grands nombres pour $N \to +\infty$

Si G_i est une trajectoire

$\frac{(G_1 + \dots + G_n)}{n}$ est une solution Exacte Si $N \to +\infty$

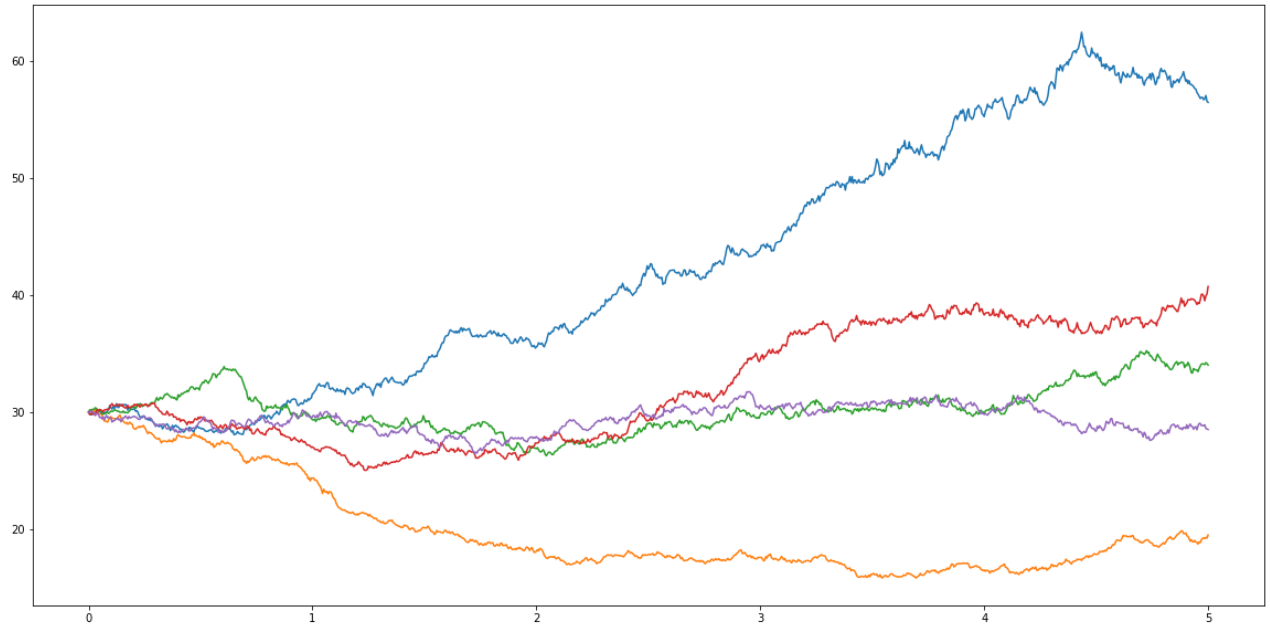
Question 7 :



In [119…

```
def sig(t,x):
    return 0.2* (1+ (np.sqrt(t)/(2*np.sqrt(T))) + (x/ (1+x**2)))


n =1000
r0 =3/100
alpha = 0.1
beta =1/100
theta = 5/100
T=5
pas = T/n
N=5
r=np.ones((n+1,N))*r0
integ=np.zeros((n+1,N))*r0

S0=30
TildeS=S0*np.ones((n+1,N))
S=S0*np.ones((n+1,N))

for j in range(N):
```

```
    for i in range(1,n+1):
        deltaWt = np.sqrt(pas)*sim.randn()
        r[i,j] = r[i-1,j] + alpha*(beta-r[i-1,j])*pas+ theta*deltaWt
        integ[i,j] = pas* np.sum(r[0:i-1,j]) #ici on a calculé l'intégrale pour chaque

        deltaCt=np.sqrt(pas)*sim.randn() # ici on simule le 2e mouvement brownien
        # on utilise l'ancien mouvment brownien Wt
        TildeS[i,j]=TildeS[i-1,j]+sig(pas*(i-1),TildeS[i-1,j])*TildeS[i-1,j]*0.25*delta
        #ici on simule l'actif S selon la formule déduite du tildeS
        S[i,j]=TildeS[i,j]*np.exp(integ[i,j])

dates=np.linspace(0,T,n+1) # n+1 dates
graph=plt.plot(dates,S)
plt.show()
```



Question 8

Question 6

Si le marché est complet, il existe une infinité de probabilité de risque neutre, on n'a pas le prix de réplication mais on peut obtenir un prix de sous réplication $V_t \geq g_T$

où g_T est la V.A dite marché complet qui est $\mathcal{F}_t$ mesurable

et donc sa réplication minimale V_0 est

$$V_0 = \left\{ \sup : E_Q\left(\tilde{g}_T\right) \quad \hat{g}_T \Rightarrow g_T \text{ actualisé} \right.$$

$$\text{et } V_t = \xi_T$$

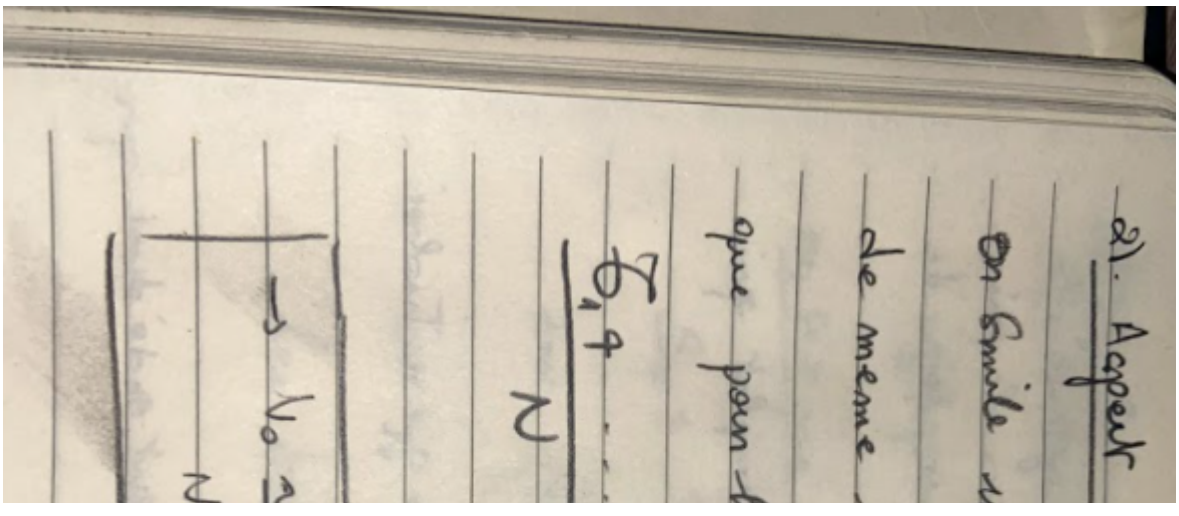
comme $\hat{S}$ est une Q martingale on peut en déduire que V l'est aussi.

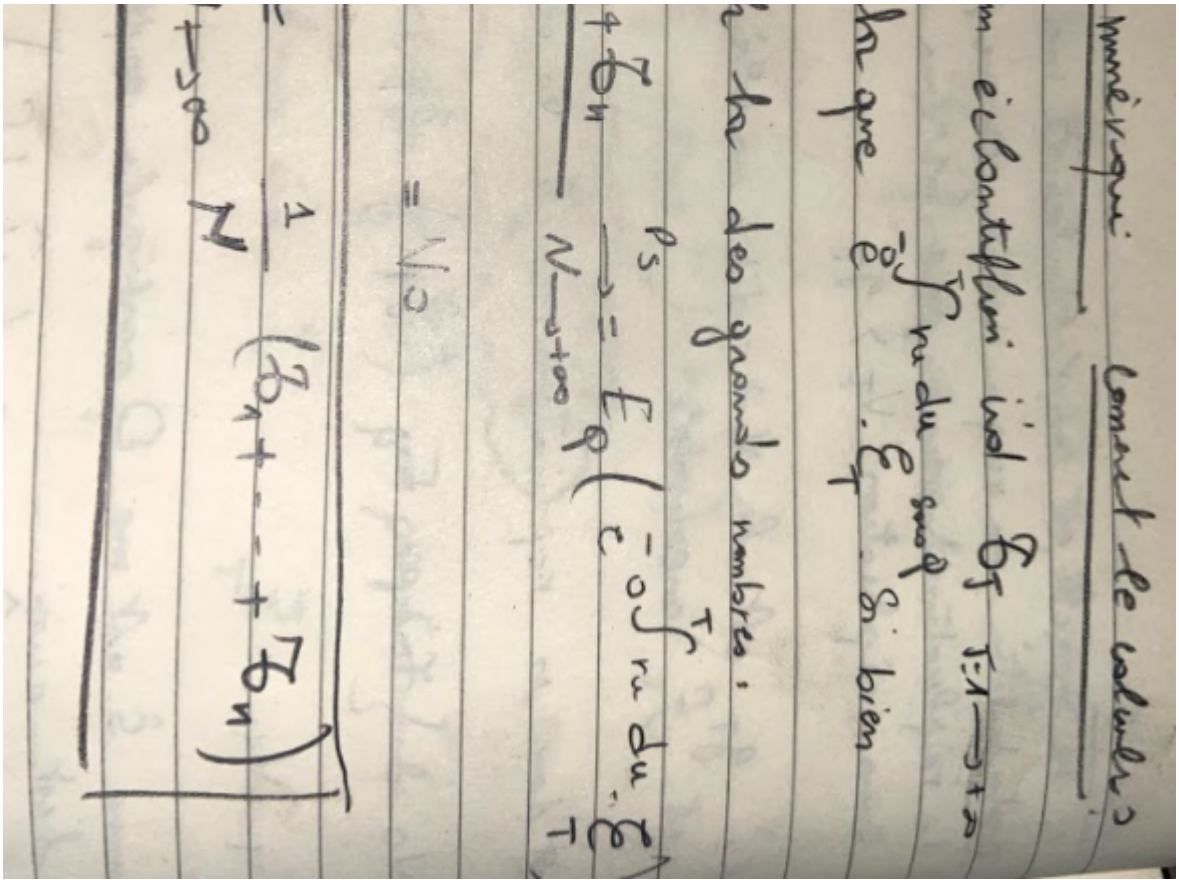
$$\forall t \leq T \quad \hat{V}_t = E_Q\left(\hat{V}_T / \mathcal{F}_{t_1}\right)$$

le prix en $t=0$ du payoff ξ_T c'est le montant que je dois investir pour le marché afin d'obtenir ξ_T en T donc il s'agit

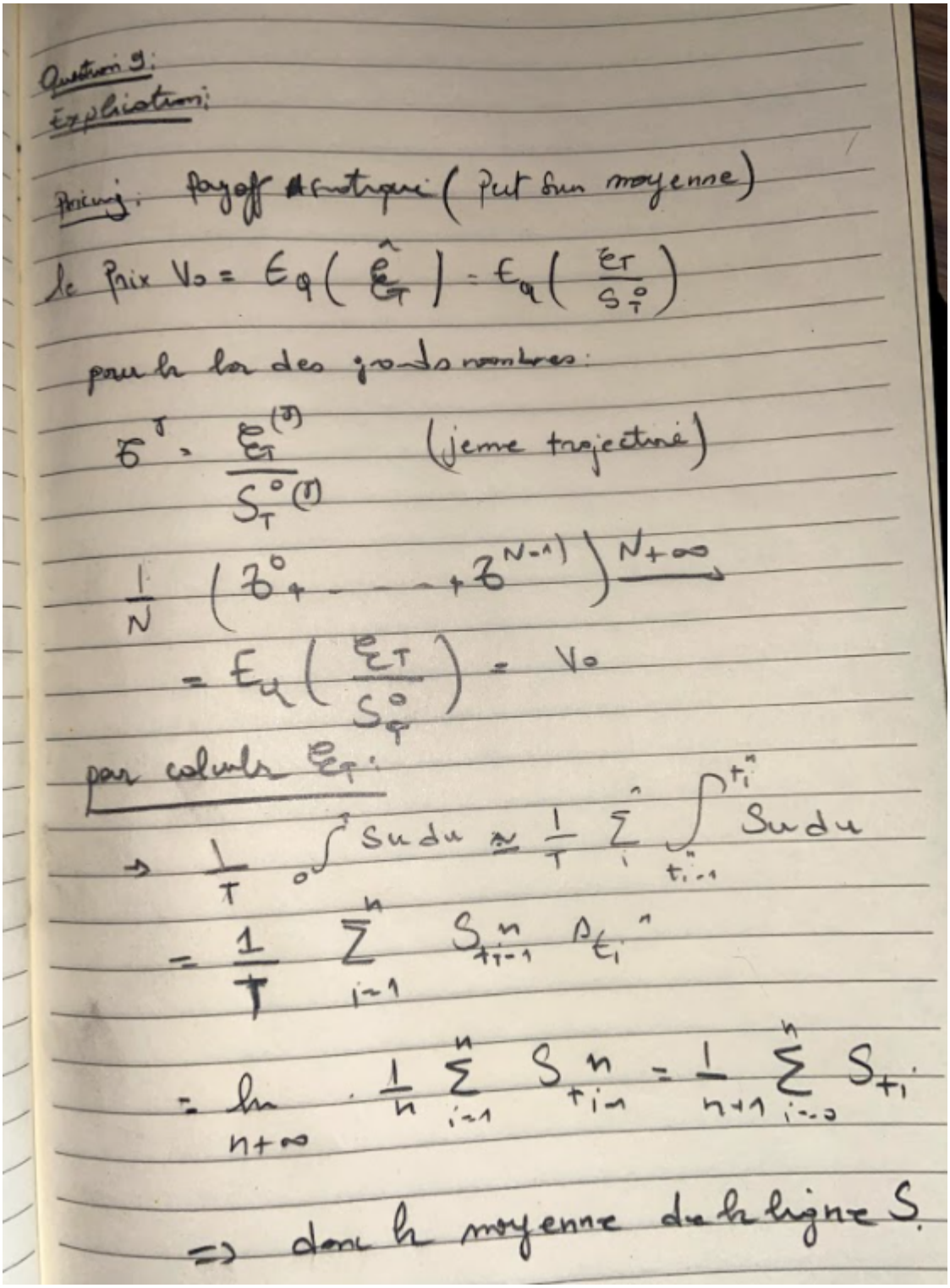
$$\Rightarrow V_0 = E_Q\left(e^{-\int_0^T r_u du} \cdot \xi_T\right)$$

Valeur initial du portfeuille de réplication





Question 9



In [120…

```
def pricingEu(k):
    def sig(t,x):
        return 0.2* (1+ (np.sqrt(t)/(2*np.sqrt(T))) + (x/ (1+x**2)))

    def call_function(x,k):
        return max(x-k,0)

    n =1000
    r0 =3/100
    alpha = 0.1
    beta =1/100
```

```
    theta = 5/100
    T=5
    pas = T/n
    N=100
    r=np.ones((n+1,N))*r0
    integ=np.zeros((n+1,N))*r0

    ListpayoffActualise=[] #cette liste contient toutes les possibilité des payoff poss

    S0=30
    TildeS=S0*np.ones((n+1,N))
    S=S0*np.ones((n+1,N))

    for j in range(N):
        for i in range(1,n+1):
            deltaWt = np.sqrt(pas)*sim.randn()
            r[i,j] = r[i-1,j] + alpha*(beta-r[i-1,j])*pas+ theta*deltaWt
            integ[i,j] = pas* np.sum(r[0:i-1,j]) #ici on a calculé l'intégrale pour cha

            deltaCt=np.sqrt(pas)*sim.randn() # ici on simule le 2e mouvement brownien
            # on utilise l'ancien mouvment brownien Wt
            TildeS[i,j]=TildeS[i-1,j]+sig(pas*(i-1),TildeS[i-1,j])*TildeS[i-1,j]*0.25*d
            #ici on simule l'actif S selon la formule déduite du tildeS
            S[i,j]=TildeS[i,j]*np.exp(integ[i,j])

        payoffActualisee =np.exp(-r*T)* call_function(S[n,j],k)
        ListpayoffActualise.append(payoffActualisee)

    V0Approx=np.mean(ListpayoffActualise)
    print("pour K = ",k)
    print("La valeur apoproximative du prix V0 de l'option Européenne: ", V0Approx)
    print("------------------")
```

In [121…

```
def pricingAsiatique(k):
    def sig(t,x):
        return 0.2* (1+ (np.sqrt(t)/(2*np.sqrt(T))) + (x/ (1+x**2)))

    def put_as(x,k):
        return max(k-x,0)

    n =1000
    r0 =3/100
    alpha = 0.1
    beta =1/100
    theta = 5/100
    T=5
    pas = T/n
    N=10
    r=np.ones((n+1,N))*r0
    integ=np.zeros((n+1,N))*r0

    PayoffAct=[]
    MoyS=[]

    S0=30
    TildeS=S0*np.ones((n+1,N))
    S=S0*np.ones((n+1,N))

    for j in range(N):
```

```
        for i in range(1,n+1):
            deltaWt = np.sqrt(pas)*sim.randn()
            r[i,j] = r[i-1,j] + alpha*(beta-r[i-1,j])*pas+ theta*deltaWt
            integ[i,j] = pas* np.sum(r[0:i-1,j]) #ici on a calculé l'intégrale pour cha
            deltaCt=np.sqrt(pas)*sim.randn() # ici on simule le 2e mouvement brownien
            # on utilise l'ancien mouvment brownien Wt
            TildeS[i,j]=TildeS[i-1,j]+sig(pas*(i-1),TildeS[i-1,j])*TildeS[i-1,j]*0.25*d
            #ici on simule l'actif S selon la formule déduite du tildeS
            S[i,j]=TildeS[i,j]*np.exp(integ[i,j])

        MoyS.append(np.mean(S[:,j]))
        aux=put_as(MoyS[-1],k)
        aux1=aux*np.exp(-integ[n,j])
        PayoffAct.append(aux1)

    V0Approx=np.mean(PayoffAct)
    print("pour K = ",k)
    print("La valeur apoproximative du prix V0 de l'option  Asiatique : ", V0Approx)
    print("------------------")
```

Etude du prix : pour l'option Europenne

In [122…

```
pricingEu(5)
pricingEu(19)
pricingEu(29)
pricingEu(31)
pricingEu(35)
pricingEu(60)
pricingEu(150)
```

```
pour K =  5
La valeur apoproximative du prix V0 de l'option Européenne:  26.36580410691194
------------------
pour K =  19
La valeur apoproximative du prix V0 de l'option Européenne:  14.747737232857975
------------------
pour K =  29
La valeur apoproximative du prix V0 de l'option Européenne:  7.774640420305349
------------------
pour K =  31
La valeur apoproximative du prix V0 de l'option Européenne:  8.140216742637497
------------------
pour K =  35
La valeur apoproximative du prix V0 de l'option Européenne:  7.5713407679087394
------------------
pour K =  60
La valeur apoproximative du prix V0 de l'option Européenne:  1.3703515046106738
------------------
pour K =  150
La valeur apoproximative du prix V0 de l'option Européenne:  0.0
------------------
```

Etude du prix : pour l'option asiatique

In [123…

```
pricingAsiatique(5)
pricingAsiatique(19)
pricingAsiatique(29)
pricingAsiatique(31)
```

```
pricingAsiatique(35)
pricingAsiatique(60)
pricingAsiatique(150)
```

```
pour K =  5
La valeur apoproximative du prix V0 de l'option  Asiatique :  0.0
------------------
pour K =  19
La valeur apoproximative du prix V0 de l'option  Asiatique :  0.0
------------------
pour K =  29
La valeur apoproximative du prix V0 de l'option  Asiatique :  0.7378871368446392
------------------
pour K =  31
La valeur apoproximative du prix V0 de l'option  Asiatique :  0.6208763500875791
------------------
pour K =  35
La valeur apoproximative du prix V0 de l'option  Asiatique :  4.0608439556842955
------------------
pour K =  60
La valeur apoproximative du prix V0 de l'option  Asiatique :  21.73099877725498
------------------
pour K =  150
La valeur apoproximative du prix V0 de l'option  Asiatique :  111.5483005383735
------------------
```