CHAPTER-1
RELATIONS AND FUNCTIONS

EXERCISE - 1.3

1. Let f:{1, 3, 4}→ {1, 2, 5} and g: {1, 2, 5}→ {1, 3} be given by $f = \{(1,2),(3,5),(4,1)\} and g = \{(1,3),(2,3),(5,1)\}$.Write down gof.

2. Let f, g, and h be functions from $\mathbf{R}$to$\mathbf{R}$.Show that

$$(f+g)oh = foh + goh$$

$$(f.g)oh = (foh).(goh)$$

3. Find gof and fog,if

   (i)$f(x) = |x|$ and $g(x) = |5x-2|$

   (ii)(x)=8x^3 and $g(x) = x^{\frac{1}{3}}$

4. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of f?

5. State with reason whether the following functions have an inverse.

   (i) $f : \{1,2,3,4\} \to \{10\}$ with

   $f = \{(1,10),(2,10),(3,10),(4,10)\}$

   (ii) $g : \{5,6,7,8\} \to \{1,2,3,4\}$ with

   $g = \{(5,4),(6,3),(7,4),(8,2)\}$

   (iii) $h : \{2,3,4,5\} \to \{7,9,11,13\}$ with

   $h = \{(2,7),(3,9),(4,1),(5,13)\}$

6. Show that$f : [-1,1] \to \mathbf{R}$,given by$f(x) = \frac{x}{(x+2)}$,is one-one.Find the inverse of the function $f : [-1,1] \to$ Range f.
(Hint: For $y \in f$, solve $y = \frac{x}{x+2}$ for some x in $[-1,1]$, i.e.,$x = \frac{2y}{1-y}$).

7. Consider f: $\mathbf{R}\to\mathbf{R}$ given by f(x) = 4x+3.Show that f is invertible.Find the inverse of f.

8. Consider$f : \mathbf{R}_+ \to [4,\infty)$ given by $f(x) = x^2 + 4$.Show that f is invertible with the inverse of $f^{-1}(y)$=$\sqrt{y-4}, where$ $\mathbf{R}_+$ is the set of all non-negative real numbers.

9. Consider$f$: $\mathbf{R}_+ \rightarrow [-5, \infty)$given by $f(x) = 9x^2 + 6x - 5$.Show that f is invertible
with:f$^{-1}(y) = (\frac{(\sqrt{(y+6)}-1}{3})$

10. Let $f : X \rightarrow Y$ be an invertible function.Show that f has a unique inverse. (Hint: Suppose g_1 and g_2 are two inverses of f.Then for all $y \in Y$, $fog_1(y) = 1_Y(y) = fog_2(y)$.) Use one-one ness of f).

11. Consider $f : \{1, 2, 3\} \rightarrow \{a, b, c\}$ given by f(1)=a, f(2)=b and f(3)=c. Find f^{-1} and show that $(f^{-1})^{-1} = f$.

12. Let$f : X \rightarrow Y$be an invertible function.Show that the inverse of f^{-1} is f, i.e.,$(f^{-1})^{-1} = f$.

13. If $f : \mathbf{R} \rightarrow \mathbf{R}$ is given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $fof(x)$ is

(A)$x^{\frac{1}{3}}$ (B) x^3 (C)x   (D)$(3 - x^3)$

14. Let f: $\mathbf{R}$ - $\left\{-\frac{4}{3}\right\}$ be a function defined as: $f(x) = \frac{4x}{3x+4}$. The inverse of

f is the map $g : Range f \rightarrow \mathbf{R} - \left\{-\frac{4}{3}\right\}$ given by

(A) $g(y) = \frac{3y}{3-4y}$ (B) $g(y) = \frac{4y}{4-3y}$

(C) $g(y) = \frac{4y}{3-4y}$ (D) $g(y) = \frac{3y}{4-3y}$