LDA - ASSIGNMENT-8

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(1) we have equation:

$$y_i = w^T x_i + b + \epsilon_i$$

(ignoring noise ϵ_i)

$$y_i = w^T x_i + b$$

$$\Rightarrow y_i = w_1 x_{i1} + w_2 x_{i2} + \cdots w_n x_{in} + b$$

I can write the above equation in matrix form as below:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_i \\ y_m \end{bmatrix}_{m\times 1} = \begin{bmatrix} x_{11} & \cdots & x_{1n} & 1 \\ x_{21} & \cdots & x_{2n} & 1 \\ x_{i1} & \cdots & x_{in} & 1 \\ x_{m1} & \cdots & x_{mn} & 1 \end{bmatrix}_{m\times(n+1)} \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ b \end{bmatrix}_{(n+1)\times 1}$$

$$Y = AW$$

It is in the form as taught in class so we can use the formulae for Linear Regression derived

$$\Rightarrow \boxed{W_0 = (A^T A)^{-1} A^T Y}$$ Ans

The above formulae is derived for general case

where m = no. of data points

n = no. of features

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}_{m\times 1} \quad A = \begin{bmatrix} \cancel{x_{11}} & x_1 & 1 \\ & x_2 & 1 \\ & \vdots & \\ & x_m & 1 \end{bmatrix}_{m\times(n+1)} = \begin{bmatrix} x_{11} & \cdots & x_{1n} & 1 \\ \vdots & & & \\ x_{m1} & \cdots & x_{mn} & 1 \end{bmatrix}_{m\times(n+1)}$$

$$W_0 = \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ b \end{bmatrix}_{(n+1)\times 1}$$

$$\boxed{W_0 = (A^T A)^{-1} A^T Y}$$

Yes the Solution is unique.

Here what we are doing is Solving for following equation

$AW = \text{Projection}_A(Y)$ || Projection of Y over Range Space of A.

But here as $m >> n+1$, we have assumed

$\text{Rank}(A^T A) = \text{Rank}(A) = n+1$

Hence Null space of $A = \{0\}$

$\Rightarrow \dim(N(A)) = 0$

Hence if $Aw = y$ || y is in Range space of A

then there is only one such w.

(because of injectivity)