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Algorithm A: Ant Colony Optimisation

Algorithm B: Particle Swarm Optimisation

Description of enhancement of Algorithm A:

- Enhancement 1: Hybridisation with Local Search (2-opt) - While ACO excels at global optimisation, it lacks precision in local search. To address this, the algorithm is hybridised with 2-opt. Since running the "Best Improvement" strategy (finding the best possible swap) is computationally expensive, it is applied exclusively to the single best ant of the iteration. To maintain a balance between performance and speed, the "First Improvement" strategy (finding the first better swap) is applied to the remaining elitist ants (e.g the next best 5). This ensures the most promising solutions are aggressively optimised while still refining the elite pool.*

Enhancement 2: Candidate Lists - To mitigate the $O(n^2)$ complexity of calculating transition probabilities for every city at every step, the algorithm utilises candidate lists. A heuristic based on distance precomputes the 20 nearest neighbours for each city. The ants restrict their search to this subset of 20 cities, significantly reducing computation time. The algorithm preserves robustness by falling back to the full city list only if all candidates in the precomputed set have already been visited.

Enhancement 3: Stagnation Recovery - Stagnation occurs when ants converge on a single path, halting exploration. Detecting this via distance comparison is inaccurate, and calculating common edge percentages is too slow. Instead, the algorithm compares the tours of the iteration-best ant and the median ant; if they are highly similar, it is assumed the whole colony has stagnated. When triggered, a weighted function blends current pheromone levels with initial values to inject randomness and restart exploration.

Description of enhancement of Algorithm B:

- Enhancement 1: Normalised Random Keys (NRK) - To avoid the inefficiencies of discretising the TSP (e.g., $O(n^2)$ swap-sequence processing), this enhancement maps the discrete problem into a continuous search space. Cities are mapped to the floating-point values of a particle's position, and the tour is constructed by sorting these values. Crucially, Random Keys suffer from translation invariance, which causes standard PSO vectors to drift to infinity. To solve this, I implemented a Subspace Projection step: after every velocity update, the particle's position is normalised, guaranteeing numerical stability while allowing velocity to naturally adjust a city's "priority" rank.*

Enhancement 2: Dynamic Topology and Time-Varying Inertia - This enhancement dynamically balances exploration and exploitation by synchronising the swarm's physics with its communication graph. Notably, the transition to a continuous search space (via NRK) makes restricted local topologies computationally viable. Continuous $O(n)$ vector arithmetic allows the algorithm to effortlessly process complex neighbourhood interactions without lag. The process initiates with high inertia and a Ring topology (restricted to direct neighbours) to encourage broad, diverse exploration and maintain momentum. As iterations progress, the Linearly Decreasing Inertia Weight (LDIW) removes kinetic energy from the swarm, while the neighbourhood radius is gradually increased. By the final iterations, the structure evolves into a fully connected Star topology, ensuring that the cooling swarm converges strictly on the global optimum.

Enhancement 3: Genetic Particle Crossover - To address PSO's weakness in structural mixing, a genetic crossover operator is integrated into the update loop. Using the Order Crossover (OX1) method, the algorithm probabilistically combines a particle's personal best with its neighbourhood's best. If the resulting offspring yields a superior fitness, the new discrete tour is projected back into the continuous space using an Inverse Rank Mapping, and the particle is re-initialised at this superior location with reset velocity. This works synergistically with the dynamic topology: as the neighbourhood grows, the genetic diversity of the available parents evolves.