ICS 2020 Problem Sheet #8

Problem 8.1:

a)

$y=((A\uparrow B)\uparrow C)\uparrow(\neg A\uparrow\neg B)$

b)we know that $X \uparrow Y := \neg(X \land Y)$

$(A\uparrow B)= \neg(A \land B)$

$((A\uparrow B)\uparrow C)=(\neg(A \land B) \uparrow C)=\neg(\neg(A \land B) \land C)$

$(\neg A\uparrow\neg B)=\neg(\neg A \land \neg B)$

then $Y= (\neg(\neg(A \land B) \land C))\uparrow(\neg(\neg A \land \neg B))$

$Y=\neg(\neg(\neg(A \land B) \land C)) \land (\neg(\neg A \land \neg B))$

$Y=\neg(((A \land B) \lor \neg C)\land(A\lor B))$

$Y=((\neg A \lor \neg B) \land C)\lor (\neg A\land \neg B)$

Problem 8.2:

a) Write both functions as a disjunction of product terms:

A	B	Cin	Cout	S
0	0	0	0	0
0	1	0	0	1
1	0	0	0	1
1	1	0	1	0
0	0	1	0	1
0	1	1	1	0
1	0	1	1	0
1	1	1	1	1

a DNF can be obtained by writing down a conjunction of the input values for every row where the result is 1:

$DNF(S)=(\neg A \land B \land \neg Cin) \lor (A \land \neg B \land \neg Cin) \lor (\neg A \land \neg B \land Cin) \lor (A \land B \land Cin)$

$DNF(Cout)=(A \land B \land \neg Cin) \lor (\neg A \land B \land Cin) \lor (A \land \neg B \land Cin) \lor (A \land B \land Cin)$

b)Write both functions as a conjunction of sum terms:

a CNF can be obtained by writing down a disjunction of the negated input values for every row where the result is 0:

$CNF(S)= (\neg A \lor \neg B \lor \neg Cin) \land (A \lor B \lor \neg Cin) \land (\neg A \lor B \lor Cin) \land (A \lor \neg B \lor Cin)$

$CNF(Cout)= (\neg A \lor \neg B \lor \neg Cin) \land (\neg A \lor B \lor \neg Cin) \land (A \lor \neg B \lor \neg Cin) \land (\neg A \lor \neg B \lor Cin)$

c)Write both functions using only not ($\neg$) and not-and ($\uparrow$) operations:

for S:

$S = A \dot{\lor} B \dot{\lor} Cin$

$X \dot{\lor} Y= (X \lor Y) \land \neg(X \land Y)$

$=\neg\neg(X \lor Y) \land (X \uparrow Y)$

$=\neg(\neg X \land \neg Y) \land (X \uparrow Y)$

$=(\neg X \uparrow \neg Y) \land (X \uparrow Y)$

$=\neg\neg((\neg X \uparrow \neg Y) \land (X \uparrow Y))$

$=\neg((\neg X \uparrow \neg Y)\uparrow(X \uparrow Y))$

since this is true for any literals (X and Y random)

then $A \dot{\lor} B=\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B))$

now for $X=A \dot{\lor} B=\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B))$ and $Y =Cin$

$S=(\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B))) \dot{\lor} Cin= X \dot{\lor} Y=\neg((\neg X \uparrow \neg Y)\uparrow(X \uparrow Y))$

we replace:

$S=\neg((((\neg A \uparrow \neg B)\uparrow(A \uparrow B)) \uparrow \neg Cin)\uparrow((\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B))) \uparrow Cin))$

For Cout:

$Cout = (A \land B) \lor (Cin \land (A \dot{\lor} B))$

we know that $A \dot{\lor} B=\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B))$

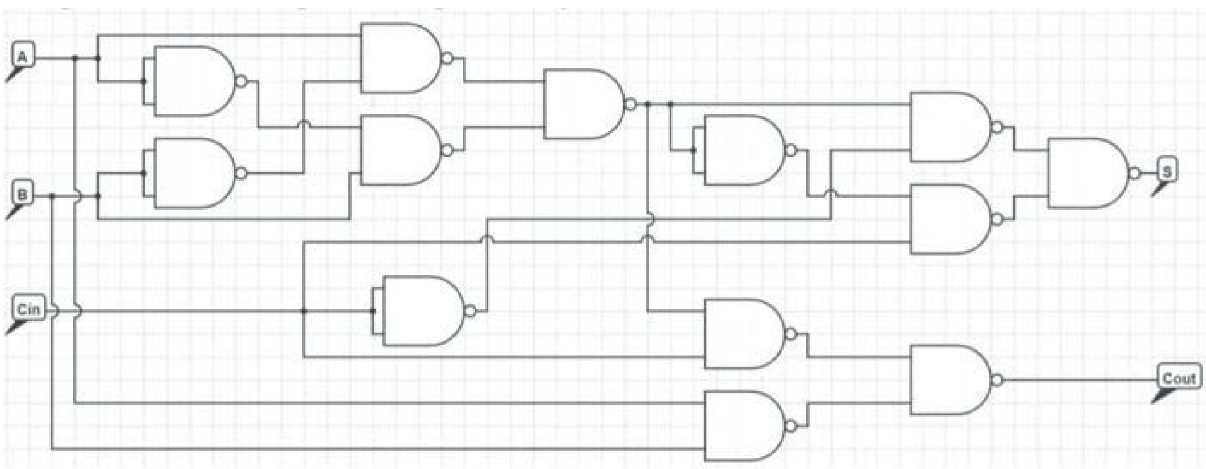
then $Cout = (A \land B) \lor (Cin \land (\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B))))$

$=\neg\neg(A \land B) \lor \neg\neg(Cin \land (\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B))))$

$=\neg(A \uparrow B) \lor \neg(Cin \uparrow (\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B))))$

$=\neg((A \uparrow B) \land (Cin \uparrow (\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B)))))$

$=(A \uparrow B) \uparrow (Cin \uparrow (\neg((\neg A \uparrow \neg B)\uparrow(A \uparrow B))))$



Problem 8.3:

a)

```
fizzbuzz :: Integer -> String
fizzbuzz i
    | i `mod` 3 == 0 && i `mod` 5 == 0 = "FizzBuzz"
    | i `mod` 3 == 0                    = "Fizz"
    | i `mod` 5 == 0                    = "Buzz"
    | otherwise                         = show i
```