Automata, Computability, and Complexity

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<u>SHEET #4:</u>

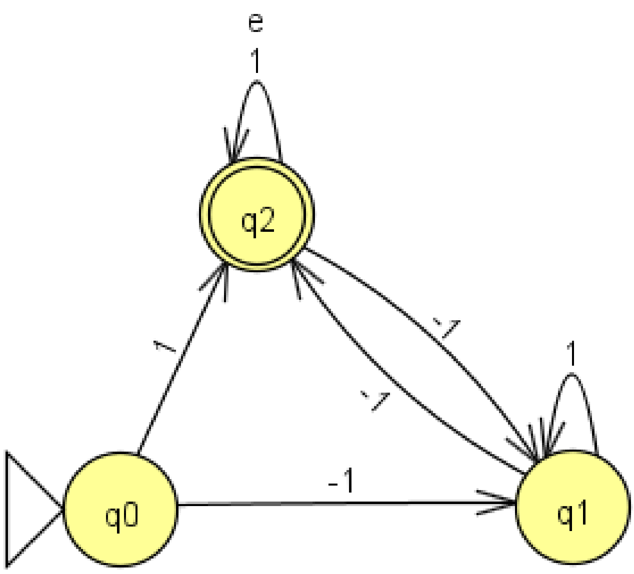
<u>Exercise 1 :</u>

a)

$L1 = \{w \in \Sigma * : w = w1 \ldots wn \text{ and } \boldsymbol{\Pi}\ n\ i=1\ wi = 1\}$

Show that L1 is a regular

$N1 = (\{q_0,q_1,q_2\} , \{1,-1\} , \delta , q_0 , \{q_2\})$



Since L1 is accepted by N1 it is indeed a regular language.

b)

$L2 = \{w \in \Sigma * : w = w1 \ldots wn \text{ and } \boldsymbol{\Sigma}\ n\ i=1\ wi = 0\}$

Show that L2 is not regular.

We assume that L2 is regular. In this case, the Pumping Lemma guarantees the existence of a pumping length p such that if s is any string in L2 of length at least p, then s may be divided into three pieces, s = xyz, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in L2$,
2. $|y| > 0$, and 3. $|xy| \leq p$.

Then we need to find a counterexample:
let s $\in$ L2 such that:
s =-1 -1 1 1 1 -1 = (-1)(-1 1 1 1)(-1) = xyz
following the pumping lemma s = xy^iz for each i ≥ 0 $\in$ L2
for i = 2:
s = (-1)(-1 1 1 1)(-1 1 1 1)(-1) $\notin$ L2
Then L2 is not a regular language.

Exercise 2 :

a)
L3 = {w $\in \Sigma*$: N(w, 0) = 2N(w, 1)} = {w $\in \Sigma*$: w has twice more 0s than 1s}

Show that L3 is not regular.
We assume that L3 is regular. In this case, the Pumping Lemma guarantees the existence of a pumping length p such that if s is any string in L3 of length at least p, then s may be divided into three pieces, s = xyz, satisfying the following conditions:
1. for each i ≥ 0, xyiz $\in$ L3,
2. |y| > 0, and 3. |xy| ≤ p.

Then we need to find a counterexample:
let s $\in$ L3 such that:
s = 1 0 0 = (1)(0)(0) = xyz
following the pumping lemma s = xyiz for each i ≥ 0 $\in$ L3
for i = 2:
s = (1)(0)(0)(0) $\notin$ L3
Then L3 is not a regular language.

b)
L4 = {w $\in \Sigma*$: N(w, 0) = 0, N(w, 1) = p, p prime }

Show that L4 is not regular.

We assume that L4 is regular. In this case, the Pumping Lemma guarantees the existence of a pumping length p such that if s is any string in L4 of length at least p, then s may be divided into three pieces, s = xyz, satisfying the following conditions:
1. for each $i \geq 0$, $xy^iz \in L4$,
2. $|y| > 0$, and 3. $|xy| \leq p$.

Then we need to find a counterexample:
let $s \in L4$ such that:
s = 1 1 1 = (1)(1)(1) = xyz
following the pumping lemma $s = xy^iz$ for each $i \geq 0 \in L3$
for i = 2:
$s = (1)(1)(1)(1) \notin L4$
Then L4 is not a regular language.

Exercise 3:

a) Describe the language of the context-free grammar
G1 = ({A, B}, {0, 1, 2}, R, A) with rules:
$A \rightarrow 00A1 \mid B$
$B \rightarrow \varepsilon \mid 22B$

Starting with A following the rules
$A \rightarrow 00A1 \rightarrow 00B1 \rightarrow 001$
$A \rightarrow 00A1 \rightarrow 0000A11 \rightarrow 0000B11 \rightarrow 000011$
$A \rightarrow 00A1 \rightarrow 00B1 \rightarrow 0022B1 \rightarrow 002222B1 \rightarrow 0022221$
We conclude that the language of the context-free grammar G1 is:
$L1 = 0^{2k} (22)^* 1^k$ with $k \in N$

b) Describe the language of the context-free grammar
G2 = ({X, Y, Z}, {m, n}, R, X) with rules:
$X \rightarrow XZ \mid Y\ m$
$Y \rightarrow Y\ Y \mid m \mid n$
$Z \rightarrow n \mid mm$

Starting with X following the rules

$X \to XZ \to X(Z)^* \to Y\,m(n)^* \to (Y)^*\,m(n)^* \to (m)^*\,m(n)^*$
$X \to XZ \to X(Z)^* \to Y\,m(n)^* \to (Y)^*\,m(n)^* \to (n)^*\,m(n)^*$
s1 = {w | w | w is a string that start by (either
$k \geq 0$ ms or $k \geq 0$ ns) followed by m then $k \geq 0$ ns}

$X \to XZ \to X(Z)^* \to Y\,m(mm)^* \to (Y)^*\,m(mm)^* \to (m)^*\,m(mm)^*$
$X \to XZ \to X(Z)^* \to Y\,m(mm)^* \to (Y)^*\,m(mm)^* \to (n)^*\,m(mm)^*$
s2 = {w | w is a string that start by (either $k \geq 0$ ms or $k \geq 0$ ns) followed by $n = 2j+1$ ms}

We conclude that the language of the context-free grammar G1 is:
$L2 = s1 \cup s2$

<u>Exercise 4:</u>

a)The finite automata can be converted into the CFG:
G = ({X, Y, Z}, {0, 1}, R, X) with rules:
$X \to 0Y \mid 1Z$
$Y \to 0X \mid 1Y \mid e$
$Z \to 11Z \mid e$

b)Convert the context-free grammar
G3 = ({X, Y, Z}, {m, n}, R, X) with rules
$X \to XZ \mid Y\,m$
$Y \to Y\,Y \mid nn$
$Z \to \varepsilon \mid mm$
into Chomsky normal form.

step1:
$X' \rightarrow X$

step 2 :
we have $Z \rightarrow \varepsilon$ and $X \rightarrow XZ$ but since $Z \rightarrow \varepsilon$ we will just remove $Z \rightarrow \varepsilon$

step 3 :
we have $X \rightarrow XZ$ and $X \rightarrow Ym$ then $X \rightarrow YmZ$, so $X \rightarrow Ymmm$
and $Y \rightarrow Y\ Y$ and $Y \rightarrow nn$ then $Y \rightarrow nnnn$
We conclude $X \rightarrow nnnnmmm$