ICS 2020 Problem Sheet #6

Problem 6.1:

Prove that the two elementary boolean functions → (implication) and ¬ (negation) are universal:
we know that
for implication :

x	y	x → y
0	0	1
0	1	1
1	0	0
1	1	1

for negation:

x	¬x
0	1
1	0

x	y	¬x	¬y	(¬x → y)	(x→¬y)	¬(x→¬y)
0	0	1	1	0	1	0
0	1	1	0	1	1	0
1	0	0	1	1	1	0
1	1	0	0	1	0	1

we conclude that (¬x → y) is equivalent to (x ∨ y)
and that ¬(x→¬y) is equivalent to (x ∧ y)

we can find ‘and’ and ‘or’ using the boolean functions implication and negation;
we can therefore find (using implication and negation) other boolean functions .
We conclude that implication and negation are universal.

<u>Problem 6.2:</u>

a) $\phi(A, B) = (\neg A \lor \neg B) \land (\neg A \lor B) \land (A \lor \neg B)$

$\phi(A, B) = (\neg A \lor \neg B) \land (\neg A \lor B) \land (A \lor \neg B)$

$\phi(A, B) = \neg A \lor (\neg B \land B) \land (A \lor \neg B)$ (distributivity)

$\phi(A, B) = \neg A \lor (0) \land (A \lor \neg B)$ (identity)

$\phi(A, B) = \neg A \land (A \lor \neg B)$ (distributivity)

$\phi(A, B) = (\neg A \land A) \lor (\neg A \land \neg B)$

$\phi(A, B) = 0 \lor (\neg A \land \neg B)$ (identity)

$\phi(A, B) = (\neg A \land \neg B)$(de Morgan's laws)

$\phi(A, B) = \neg(A \land B)$

b)

$\phi(A, B, C) = (A \land \neg B) \lor (A \land \neg B \land C)$

$\phi(A, B, C) = (A \land \neg B) \lor (A \land \neg B \land C)$

$\phi(A, B, C) = (A \land \neg B) \lor ((A \land \neg B) \land C)$ (associativity)

$\phi(A, B, C) = (A \land \neg B)$ (absorption laws)

c) $\phi(A, B, C, D) = (A \lor \neg(B \land A)) \land (C \lor (D \lor C))$

$\phi(A, B, C, D) = (A \lor \neg(B \land A)) \land (C \lor (D \lor C))$

$\phi(A, B, C, D) = (A \lor (\neg B \land \neg A)) \land (C \lor (D \lor C))$(de Morgan's laws)

$\phi(A, B, C, D) = ((A \lor \neg B) \land (A \lor \neg A)) \land (C \lor (D \lor C))$(distributivity)

$\phi(A, B, C, D) = ((A \lor \neg B) \land 1) \land (C \lor (D \lor C))$

$\phi(A, B, C, D) = (A \lor \neg B) \land (C \lor (D \lor C))$(identity)

$\phi(A, B, C, D) = (A \lor \neg B) \land (D \lor (C \lor C))$(associativity)

$\phi(A, B, C, D) = (A \lor \neg B) \land (D \lor C)$ (idempotency)

d)

$\phi(A, B, C) = (\neg(A \land B) \lor \neg C) \land (\neg A \lor B \lor \neg C)$ $\phi(A, B, C)$

$= (\neg(A \land B) \lor \neg C) \land (\neg A \lor B \lor \neg C)$ $\phi(A, B, C)$

$= \neg((A \land B) \lor C) \land (\neg A \lor B \lor \neg C)$ (de Morgan's laws)

$\phi(A, B, C) = (\neg((A \land B) \lor C) \land \neg(A \lor \neg B \lor C))$(double negation)

$\phi(A, B, C) = \neg(((A \land B) \lor C) \land (A \lor B \lor C))$(de Morgan's laws)

$\phi(A, B, C) = \neg(((A \land B) \lor C) \land (A \lor B)) \lor ((A \land B) \lor C) \land C))$ (distributivity)

$\phi(A, B, C) = \neg(((A \land B) \lor C) \lor ((A \land B) \lor C))$(idempotency)

$\phi(A, B, C) = \neg((A \land B) \lor C)$ (idempotency)

e)

$\phi(A, B) = (A \lor B) \land (\neg A \lor B) \land (A \lor \neg B) \land (\neg A \lor \neg B)$

$\phi(A, B) = (A \lor B) \land (\neg A \lor B) \land (A \lor \neg B) \land (\neg A \lor \neg B)$ $\phi(A, B)$

$= (A \lor B) \land (\neg A \lor \neg B) \land (\neg A \lor B) \land (A \lor \neg B)$ (commutativity)

$\phi(A, B) = (A \lor B) \land \neg(A \lor B) \land (\neg A \lor B) \land (A \lor \neg B)$(de Morgan's laws)

$\phi(A, B) = (A \lor B) \land \neg(A \lor B) \land (\neg A \lor B) \land \neg(\neg A \lor B)$(double negation)

$\phi(A, B) = 0 \land 0$ $\phi(A, B) = 0$

Problem 6.3:

a)The truth table:

P	Q	R	S	(¬P ∨ Q)	(¬Q ∨ R)	(¬R ∨ S)	(¬S ∨ P)	ϕ
0	0	0	0	1	1	1	1	1
0	0	0	1	1	1	1	0	0
0	0	1	0	1	1	0	1	0
0	0	1	1	1	1	1	0	0
0	1	0	0	1	0	1	1	0
0	1	0	1	1	0	1	0	0
0	1	1	0	1	1	0	1	0
0	1	1	1	1	1	1	0	0
1	0	0	0	0	1	1	1	0
1	0	0	1	0	1	1	1	0
1	0	1	0	0	1	0	1	0
1	0	1	1	0	1	1	1	0
1	1	0	0	1	0	1	1	0
1	1	0	1	1	0	1	1	0
1	1	1	0	1	1	0	1	0
1	1	1	1	1	1	1	1	1

ϕ is satisfied for 2 interpretations.

b)the conditions that satisfy ϕ are :

P	Q	R	S
1	1	1	1

and

P	Q	R	S

0	0	0	0

we can therefore conclude by DNF that :

$\phi = (P \land Q \land R \land S) \lor (\neg P \land \neg Q \land \neg R \land \neg S)$