ICS 2020 Problem Sheet #3

Problem 3.1:

a)

prove or disprove that :$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

first let's take an $(x,y) \in (A \cap B) \times (C \cap D)$

$(x,y) \in (A \cap B) \times (C \cap D) \Leftrightarrow x \in (A \cap B)$ and $y \in (C \cap D)$

$\Leftrightarrow$ ($x \in A$ and $x \in B$) and ($y \in C$ and $y \in D$)

$\Leftrightarrow$ ($x \in A$ and $y \in C$) and ($x \in B$ and $y \in D$)

$\Leftrightarrow$ ($(x,y) \in (A \times C)$ and $(x,y) \in (B \times D)$

then we conclude that :

$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

b)

prove or disprove that :$(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

first let's take an $x \in (A \cup B) \times (C \cup D)$

1: $(x,y) \in (A \cup B) \times (C \cup D) \Leftrightarrow x \in (A \cup B)$ and $y \in (C \cup D)$

$\Leftrightarrow$ ($x \in A$ or $x \in B$) and ($y \in C$ or $y \in D$)

2: $(x,y) \in (A \times C) \cup (B \times D) \Leftrightarrow (x,y) \in (A \times C)$ or $(x,y) \in (B \times D)$

$\Leftrightarrow$ ($x \in A$ and $y \in C$) or ($x \in B$ and $y \in D$)

$\Leftrightarrow$ ($x \in A$ or $x \in B$) and ($x \in A$ or $y \in D$)

and ($y \in C$ or $x \in B$) and ($y \in C$ or $y \in D$)

$\Leftrightarrow$ ($x \in A$ or $x \in B$) and ($y \in C$ or $y \in D$) and

($x \in A$ or $y \in D$) and ($y \in C$ or $x \in B$)

we know from 1 that :

($x \in A$ or $x \in B$) and ($y \in C$ or $y \in D$) $\Leftrightarrow (x,y) \in (A \cup B) \times (C \cup D)$

we can then replace in 2 :

$(x,y) \in (A \times C) \cup (B \times D) \Leftrightarrow ((x,y) \in ((A \cup B) \times (C \cup D))$ and ($x \in A$ or $y \in D$) and ($y \in C$ or $x \in B$)

since for any sets A,B,C,D:

$(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D) \Leftrightarrow (A \times B) \cup (C \times D) \neq (A \cup C) \times (B \cup D)$

then we conclude that:

$(A \cup B) \times (C \cup D) \neq (A \times C) \cup (B \times D)$

Problem 3.2:

a)

$R = \{(a, b) | a, b \in Z \land |a - b| \leq 3\}$

determine whether R is reflexive, symmetric, or transitive.
1) let $a \in Z$
let's check if (a,a) checks $|a - b| \leq 3$
$|a - b| \leq 3$ with (a,b)=(a,a) $\Leftrightarrow |a - a| \leq 3$
$\Leftrightarrow 0 \leq 3$
we conclude that R is reflexive.

2) let $(a,b) \in Z$
let $(a,b) \in R$
check if $(b,a) \in R$
$(a,b) \in R \Rightarrow a,b \in Z$ and $|a - b| \leq 3$
$\Rightarrow a,b \in Z$ and $|b - a| \leq 3$
$\Rightarrow (b,a) \in R$
we conclude that R is symmetric.

3) let $a,b,c \in Z$
let $((a, b) \in R \land (b, c) \in R)$
$(a,b) \in R \Rightarrow a,b \in Z$ and $|a - b| \leq 3$

$(b,c) \in R \Rightarrow b,c \in Z$ and $|b - c| \leq 3$

$(a,c) \in R \Rightarrow a,c \in Z$ and $|a - c| \leq 3$

for a=3 b=0 and c=-3
$(3,0) \in R$
$(0,-3) \in R$
let's see if $(a,c) \in R$
$3,-3 \in Z$ but $|3 + 3| > 3$
then We conclude that R is not transitive.

b)
$R = \{(a, b) | a, b \in Z \land (a \bmod 10) = (b \bmod 10)\}$
determine whether R is reflexive, symmetric, or transitive.

1)let $a \in Z$
let's check if $(a,a) \in R$
$(a,a) \in R \Leftrightarrow a,a \in Z \land (a \bmod 10) = (a \bmod 10)$
which is true
we conclude that R is reflexive

2)let $(a,b) \in Z$
let $(a,b) \in R$
check if $(b,a) \in R$
$(a,b) \in R \Rightarrow a,b \in Z$ and $(a \bmod 10) = (b \bmod 10)$

=> a,b∈ Z and (b mod 10) = (a mod 10)
=>(b,a)∈R
we conclude that R is symmetric.

3) let a,b,c ∈ Z
let ((a, b) ∈ R ∧ (b, c) ∈ R)
1: (a,b) ∈R => a,b∈ Z and (a mod 10) = (b mod 10)

2:(b,c) ∈R => b,c∈ Z and (b mod 10) = (c mod 10)

from 1 and 2 we get that (a mod 10) = (b mod 10) = (c mod 10) and that a,c ∈ Z
then (a,c) ∈R => a,c∈ Z
we conclude that R is transitive.

<u>Problem 3.3:</u>

Proof by induction over s that: cnt x (con s t) = (cnt x s) + (cnt x t)
first for s=[ ]
con s t =t
then cnt x (con s t)=cnt x t
since cnt x s=0 because s=[ ]
then we conclude that (cnt x s) + (cnt x t)= cnt x t
then for s=[ ] : cnt x (con s t) = (cnt x s) + (cnt x t)
let's assume that: cnt x (con s t) = (cnt x s) + (cnt x t)
Prove that cnt x (con ss t) = (cnt x ss) + (cnt x t)
since s=[ ] : cnt x (con s t) = (cnt x s) + (cnt x t)
and since we also have: cnt x (con s t) = (cnt x s) + (cnt x t)
then cnt x (con ss t) = (cnt x ss) + (cnt x t)
we conclude that cnt x (con s t) = (cnt x s) + (cnt x t)