Solution 1:

P(A) = 0.3

P(B) = 0.4

P(A∩B) = 0.2

(a) P(exactly one occurs) = P(A) + P(B) - 2xP(A∩B)
= 0.3

(b) P(atleast one occurs) = P(AUB) = P(A) + P(B) - P(A∩B)
= 0.5

(c) P(none occurs) = 1 - P(AUB) = 0.5

Solution 2:

Let the chosen door be door 1

P(car is behind door 1 initially) = ⅓

A: door 1 does not have a car behind it

B: Monty shows a door with a goat behind it

P(A|B) = $\frac{P(A \cap B)}{P(B)}$

P(B) = 1, because there are 2 goats and the player is choosing 1 door, so one of the two doors left always has a goat behind it and Monty knows what door it is.

P(A∩B) = P(B|A)xP(A)

P(B|A) is also 1

P(A) = ⅔

Therefore, P(A|B) = ⅔

Therefore, contestant should switch door

Solution 3:

A: first 3 draws are red

B: all balls are red

P(B|A) = $\frac{P(A \cap B)}{P(A)}$ = $\frac{P(B)\,P(A|B)}{P(A)}$

6 cases: 1 R, 2 R,3 R, 4 R, 5 R, 6 R

Each has probability 1/6

P(A) = $\frac{1}{6}$ ($\frac{1}{20} + \frac{4}{20} + \frac{10}{20} + \frac{20}{20}$) = 7/24

P(A|B) = 1

P(B) = ⅙

Therefore, P(B|A) = 4/7

Solution 4:

P(X<0.5) = P(X=0.2) + P(X=0.4) = 0.3

P(0.25<X<0.75) = P(0.4) + P(0.5) = 0.4

P(X=0.2|X<0.6) = $\frac{P(X=0.2)}{P(X<0.6)}$ = 0.2

Solution 5:

1) $4c^2 - 9c + 6 = 4$
   $\Rightarrow c = 0.25, 2$
   For c = 2, F(1)<0
   Therefore, c=0.25

2) P(1<X<2) = F(2-) - F(1) = 0
   P(2<=X<3) = F(3-) - F(2-) = 1/12
   P(0<X<=1) = F(1) - F(0) = ¼
   P(1<=X<=2) = F(2) - F(1-) = ⅓
   P(X>=3) = 1 - F(3-) = 0

Solution 6:

a) E(X) = $\int_0^1 x\, dx$ = ½

b) E(X^2) = $\int_0^1 x^2 dx$ = ⅓

V(X) = $E(X^2)$ - $E(X)^2$ = 1/12

c) $E(X^2) + E(Y^2) = 1$

$\Rightarrow E(Y^2) = 2/3$

V(Y) = 5/9

E(Y) = ⅓

d) E(X+Y) = E(X) + E(Y) = ⅚

<u>Answers:</u>

1) (a) 0.3
   (b) 0.5
   (c) 0.5

2) Switch doors

3) 4/7

4) $P(X<0.5) = 0.3$
   $P(0.25<X<0.75) = 0.4$
   $P(X=0.2|X<0.6) = 0.2$

5) (a) $c=0.25$
   (b) $P(1<X<2) = 0$
   $P(2<=X<3) = 1/12$
   $P(0<X<=1) = \frac{1}{4}$
   $P(1<=X<=2) = \frac{1}{3}$
   $P(X>=3) = 0$

6) (a) 0.5
   (b) 1/12
   (c) $\frac{1}{3}$
   (d) $\frac{5}{6}$