Lecture-1 12.11.16

Chapter-6 (Roots of ~~No-liner~~ Non-Linear Equation)

Equation

- Algebric
- Polynomial
- Tranceendental

Algebric $\rightarrow$ $x^2 - 5x + 6 = 0$

Polynomial $\rightarrow$ $a_0x^0 + a_1x^1 + a_2x^2 + \cdots + a_nx^n$

Tranceendental $\rightarrow$ log, sinx, cosx / $2\sin x - x = 0$

$e^{x} - x = 0$

$\log x - 2x = 0$

Linear

~~Liner~~ Equation → $y = 3x+5$

2D, 3D Plot graph.

State line তৈরি graph

Non-Linear

~~No-liner~~ equation → $x^3 - 3x^2 + 2x + 5 = 0$

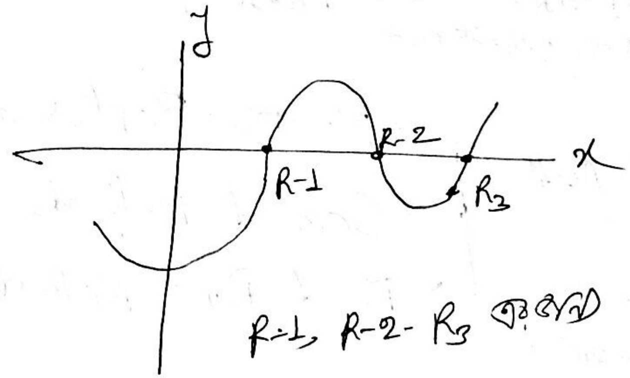
1. Direct Analytical Methods.

$$ax^2 + bx + c = 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Graphical Methods:

$$f(x) = x^3 - 3x^2 + 2x + 5 = 0$$



R-1, R-2- R_3 [illegible] $f(x)$ or $y = 0$

$\therefore$ Root হল R_1, R_2, R_3 point।

Iterative Methods:→

→ Bracketing Methods

→ Open End Methods.

Bracketing Methods → Bisection Methods.
→ False position

(-6,+6) এমন
Limit নিয়ে খুঁজে Root

Open End Methods → Newton Raphson Methods
→ Secant Methods
→ Fixed Point Method

একটা Value হতে
Root খুঁজে বের।

Bisection Methods

By

$$* \; \overset{a_3}{2x^3} \overset{a_2}{-8x^2} + \overset{a_1}{2x} + \overset{a_0}{12x^0} = 0$$

$$|x_{max}| \leq \sqrt{\left(\frac{a_{n-1}}{a_n}\right)^2 - 2\left(\frac{a_{n-2}}{a_n}\right)}$$

$$\leq \sqrt{\left(\frac{-8}{2}\right)^2 - 2\left(\frac{2}{2}\right)}$$

$$\leq \sqrt{14}$$

$\therefore \left(-\sqrt{14}, +\sqrt{14}\right) \rightarrow$ Bracketing Limit পেলাম

$$|X_{max}| = -\frac{a_{n-1}}{a_n}$$

$$= -\frac{-8}{2} = 4$$

4 হলো (X_{max}) এর মান হলো Root গুলো 4 or X_{max} এর বেশি যাবে না। Root গুলো 4 এর নিচে থাকবে।

Bisection Method

$$f(x) = x^2 - 4x - 10 = 0$$

$$|X_{max}| \leq \sqrt{\left(\frac{a_{n-1}}{a_n}\right)^2 - 2\left(\frac{a_{n-2}}{a_n}\right)}$$

$$\leq \left(\frac{-4}{1}\right)^2 - 2\left(\frac{-10}{1}\right)$$

$$\leq \sqrt{36} \leq 6$$

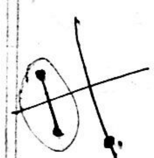
$$\therefore (-6, +6)$$

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$	50	35	22	11	2	-4	-10	-13	-14	-13	-10	-5	2

Step 1: x এর কোন Value এর জন্য $f(x)$ এর মান উল্টে যায় Setuation change হয়।
$f(x)$ এর মান + থেকে (−) হবে or
− থেকে + হবে।

Step 2: সত্যমানের কাছে Setuation হবে।
আগে নিয়ে অনুমান চেক করবো।

Step 3: $f(x_0)f(x_1) < 0$ হলে Root এই Range এর মধ্যে হবে
multiplication or
$f(x_0)f(x_2) < 0$



$f(x)$

$d=$

Error $= 0.005$

সূত্র

$$x_0 = \frac{x_1 + x_2}{2}$$

$$\frac{x_2 - x_1}{x_2} \leq E$$

x_1	x_2	x_0	$f(x_1)$	$f(x_2)$	$f(x_0)$	Decision.
−2	−1	−1.5	2	−4	−1.75	E=0.005
−2	−1.5	−1.75	2	−1.75	0.0625	
−1.75	−1.5	−1.625	0.0625	−1.75	−0.859	
−1.75	−1.625	−1.6875				

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Bisection Method :→

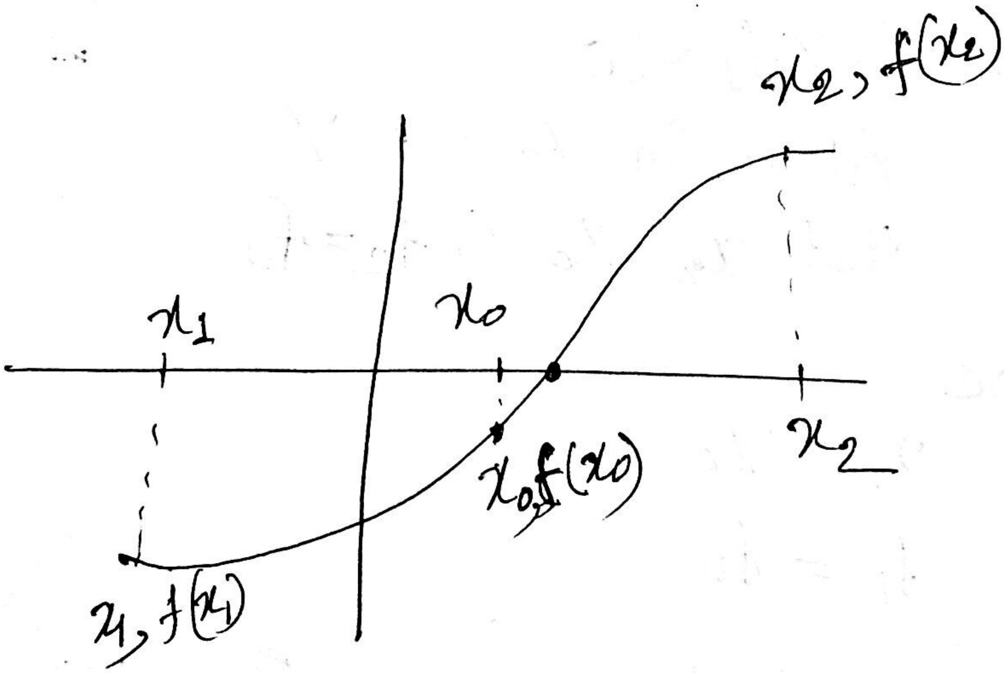
1. $f(x)$

$x_1 < x < x_2$

$f(x_1)$ $\quad$ $f(x_2)$

$\pm$ $\quad$ $\pm$

$f(x_1) * f(x_2) < 0$



1. x_1, x_2 $E = 0.001$ initialize

2. $x_0 = \dfrac{x_1 + x_2}{2}$, f_0

3. $f_1 * f_2 > 0$ হয় তবে কোনো Root নাই।

→ print, x_1 & x_2 এর মাঝে Root নাই।

4. $f(x_0) = 0$; Print "Root $= x_0$"

5. if $f_0 \times f_1 < 0$

/*Root in x_0 & x_1 */

Set $x_2 = x_0$ & $f_2 = f_0$

else

$x_1 = x_0$

$f_1 = f_0$

6. If $\left| \dfrac{x_2 - x_1}{x_2} \right| < E$; root $= \dfrac{x_1 + x_2}{2}$

print root

stop

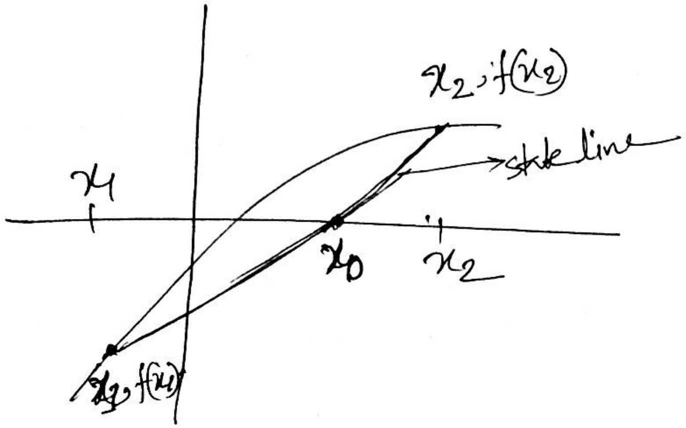
*

else

go to step 2;

$f(x_1) \times ($

*"False Position Method"



$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x) - f(x_1)}{x - x_1}$$

$$\Rightarrow \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{-f(x_1)}{x_0 - x_1}$$

$$\Rightarrow x_0 - x_1 = -\frac{f(x_1) \cdot (x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$\boxed{\therefore x_0 = x_1 - \frac{f(x_1) \cdot (x_2 - x_1)}{f(x_2) - f(x_1)}}$$

বাকি সব Bisection Method এর মত।

if $f(x_0) * f(x_1) < 0$

set $x_2 = x_0$

otherwise

set $x_1 = x_0$

$x^2-2x+x-2=0$

$x(x-2)+1(x-2)=0$

$(x-2)(x-1)=0$

* Math

$f(x) = x^2-x-2$

$1<x<3$

Find Root ?

Iteration 1:- $x_1 = 1$ & $x_2 = 3$

$\therefore f(x_1) = -2$ | From $f(x)$ equation

$f(x_2) = 4$

$\therefore x_0 = 1.667 \rightarrow$ | From False Position method |

$\therefore f(x_0) = -0.889$

$\therefore$ For ~~Error~~ $\frac{x_2-x_1}{x_2} < E$

Iteration 2:

$x_1 = 1.6667$ | Because $f(x_0) \times f(x_1) < 0$ False So, $x_1 = x_0$

$x_2 = 3$

$x_0 = 1.909$

$f(x_1) = -0.8889$

$f(x_2) = 4$

$f(x_0) = -0.264$

Iteration 3:

$x_1 = 1.909$

$x_2 = 3$

$x_0 = 1.986$

6.8

Algorithm ওয়া 6.4

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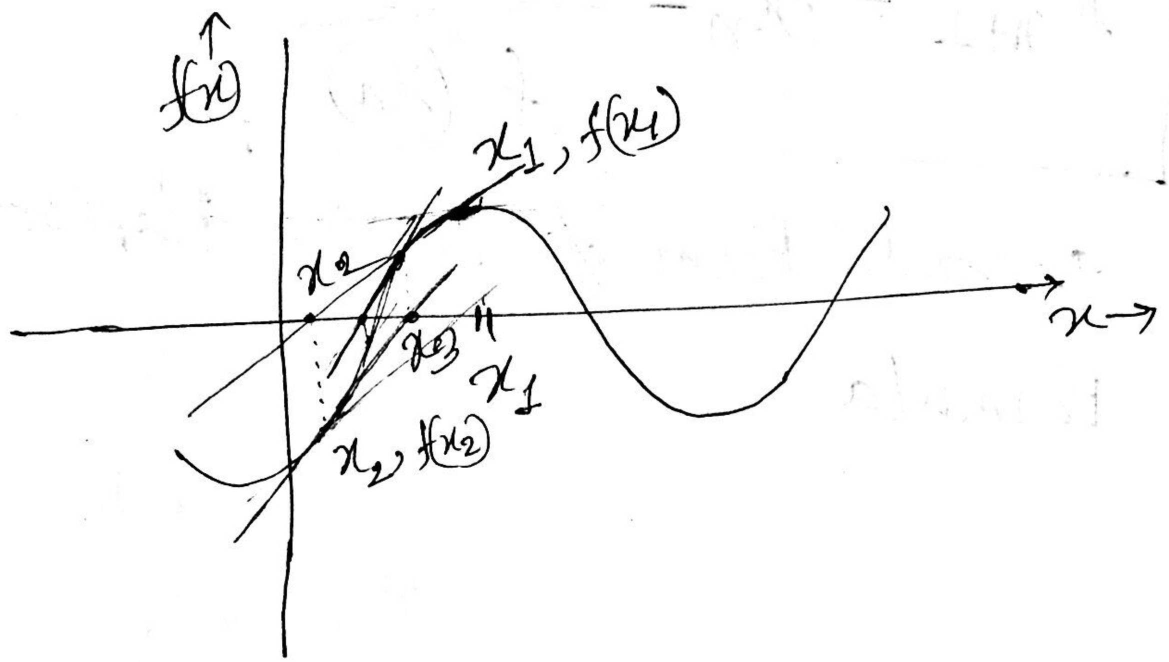
1. Newton–Raphson Method



1. Let Root $= x_1$

2. The slope of the tangent, $\tan\alpha = \dfrac{f(x_1)}{x_1 - x_2}$

$f(x) = x^2 - 3x + 2$

$f'(x) = 2x - 3$

$\Rightarrow f'(x_1) = \dfrac{f(x_1)}{x_1 - x_2}$

$\Rightarrow x_1 - x_2 = \dfrac{f(x_1)}{f'(x_1)}$

$\Rightarrow x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

general Form of Newton-Raphson Formula

$x^2 - 3x + 2$

$x^2 - 2x - x + 2$

$\Rightarrow x(x-2) - 1(x-2)$

$\Rightarrow x = 2, 1$

Example → 6.7

$$f(x) = x^2 - 3x + 2$$

$$x_1 = 0$$

$$f'(x) = 2x - 3$$

$$f(x_1) = 2$$

$$f'(x_1) = 3$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.6667$$

$$\left|\frac{x_2 - x_1}{x_2}\right| \leq E = 0.001$$

Iteration 1:-

$x_1 = 0.6667$

$f(x_1) =$

$f'(x_1) =$

$x_2 = 0.9333$

Iteraton 2:

$x_3 = 0.9959$

4

Iteration 3:

$x_4 = 0.99999$

Iteration 4:

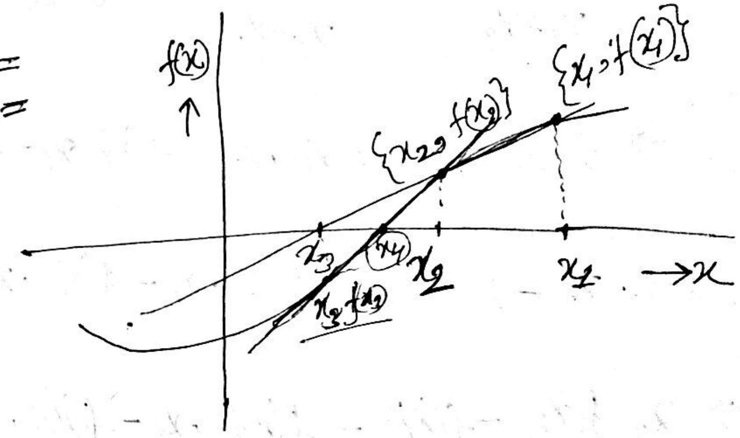
$x_5 = 0.9999$

Iteration 5

$x_6 = 1.000$

* Second Method

$x_1 =$
$x_2 =$



* Let, x_1 & x_2 | Add to this point.

$$\frac{f(x_1)}{x_1 - x_3} = x_1 \text{ ∴ slop} = \tan\alpha$$

$$\frac{f(x_2)}{x_2 - x_3} = x_2 \text{ ∴ slop} = \tan\alpha$$

$$\therefore \frac{f(x_1)}{x_1 - x_3} = \frac{f(x_2)}{x_2 - x_3}$$ [ঢাল ও Same]

$$\Rightarrow f(x_1)\, x_2 - f(x_1)\cdot x_3 = f(x_2)\, x_1 - f(x_2)\cdot x_3$$

$$\Rightarrow f(x_2)\, x_3 - f(x_1)\, x_3 = f(x_2)\cdot x_1 - f(x_1)\cdot x_2$$

$$\Rightarrow x_3 \left(f(x_2) - f(x_1) \right) = f(x_2)\cdot x_1 - f(x_1)\cdot x_2$$

$$\Rightarrow x_3 = \frac{f(x_2)\cdot x_1 - f(x_1)\cdot x_2}{f(x_2) - f(x_1)}$$

$$\boxed{\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$ → ~~newton~~ Raphson

(+ −) করে

$$\Rightarrow x_3 = \frac{f(x_2)\cdot x_1 - f(x_1)\, x_2 + f(x_2)\cdot x_2 - f(x_2)\cdot x_2}{f(x_2) - f(x_1)}$$

~~$f(x_2)\cdot x_2$~~

$$\Rightarrow x_3 = \frac{x_2\, f(x_2) - f(x_1) - f(x_2)\cdot(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$\Rightarrow x_2 = x_2 - \frac{f(x_2)\,(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$\boxed{\therefore x_{n+1} = x_n - \frac{f(x_n)\,(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}}$$

Let $$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

$$\boxed{\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

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Ex 6.7

$f(x) = x^2 - 3x + 2$

$x_1 = 0$

$x_2 = x_1 -$

Ex 6.9 :-

$x^2 - 4x - 10 = 0$

$x_1 = 4$

Iteration 1: $x_2 = 2$

$f(x_1) = -10$

$f(x_2) = -14$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = 9$$

iteration 2:-

$x_1 = 2$

$x_2 = 9$

$f(x_1) =$

$f(x_2) =$

$x_3 =$

$x^2-2x+x+2$

Fixed Point Method :-

$\frac{1 \pm \sqrt{1-8}}{2}$

$f(x) = 0$

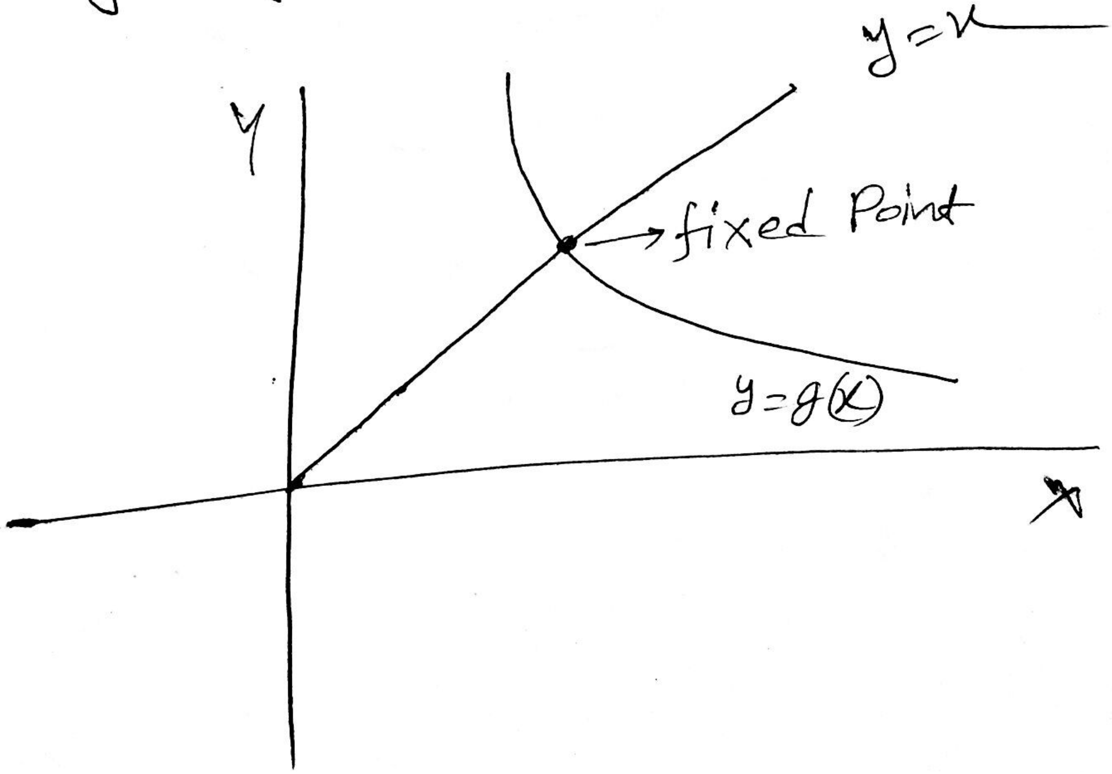
$\Rightarrow x^2 - x + 2 = 0$

$\Rightarrow x = x^2 + 2$

$\Rightarrow \underline{x = g(x)} \rightarrow$ Fixed Point equation.

$\therefore y = x$

$\therefore y = gx$



$$\cancel{x^2 - 2x + x + 2 = 0}$$

$$\cancel{x(x-2) + 1(x+2) = 0}$$

Let $x_0 = ?$

$\therefore x_1 = g(x_0)$

$x_2 = g(x_1)$

$\vdots$

$\boxed{\therefore x_{i+1} = g(x_i)}$ → Fixed Point Formula.

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$x = -2, 1$

Ex: 6.11

$x^2 + x - 2 = 0$

$x = 2 - x^2 = g(x)$

$\therefore x_0 = 0$

$x_1 = 2$

$\therefore x_1 - x_0 = 2$

$x_2 = -2$

$x_2 - x_1 = -4$

$\therefore x_3 = -2$

$x_3 - x_2 = 0$

$\therefore \boxed{\text{Root} \Rightarrow x_3 = -2}$

$x^2 - 5 = 0$

$x = \frac{5}{x} = g$

$x_0 = 1$

$x_1 = 5$

$x_2 = 1$

$x_3 = 5$

$x_4 = 1$

$x_5 = 1$

Oscillatory divergence

If: $x = x^2 + x - 5 = g(x)$

$\therefore x_0 = 0$

$x_1 = -5$

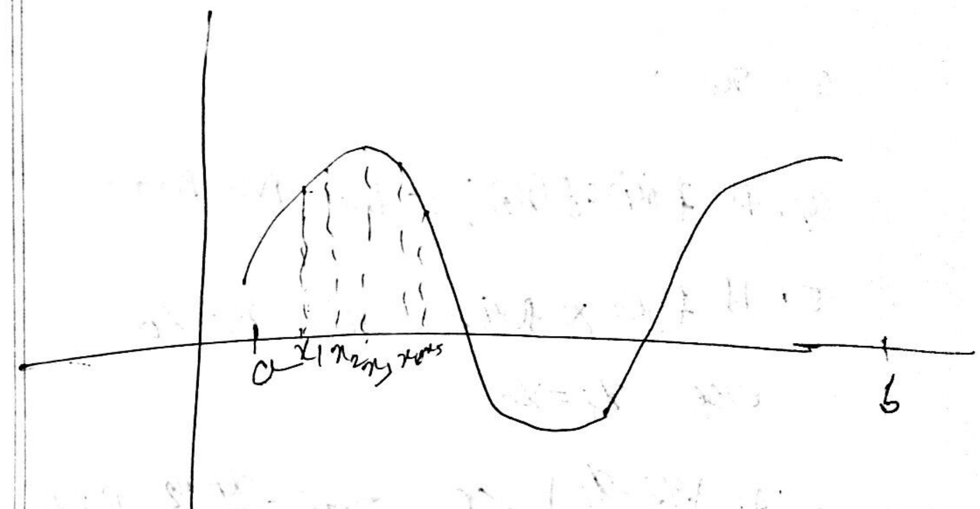
$x_2 = 15$

$x_3 = 235$

$x_4 = 55453$

monotone divergence

* Determining all possible roots:-



Let:

upper limit = b

lower limit = a

set a, b, f

$x_1 = a$

$x_2 = \Delta x + x_1$

$$\boxed{\Delta x = \frac{b-a}{1000} \qquad \Delta x = 0.0001}$$

খুব ছোট নিব

1. set $a, b, \varepsilon, \Delta x$

2. $x_1 = a, \quad x_2 = x_1 + \Delta x$

3. $x_0,$

4. If $f(x_1) \times f(x_2) > 0$ then No Root

5. if $f(x_0) \times f(x_1) < 0$ then $x_2 = x_0$

else, $x_1 = x_0$

6. If $\left| \frac{x_2 - x_1}{x_2} \right| < \varepsilon$ root $= \frac{x_1 + x_2}{2}$, Print

{

if $(x_2 < b)$ then $a = x_2$ go to step 1

else stop

}

else go to step 2

7. Stop

$(((a_3)b+a_2)b+a_1)b+a_0$

6.5

* Evaluation of Polynomials :→

$$f(x) = x^3 - 4x^2 + \cancel{x} + 6$$

$$f(x) = \sum_{i=0}^{n} a_i x^i$$

$$= a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x^1 + a_0 x^0$$

$$= ((\cdots(\underbrace{\underbrace{a_n}_{P_n} x + a_{n-1})}_{P_{n-1}} x + \cdots + a_2)x + a_1)x + a_0$$

Algo-6.1

$$\therefore P_n = a_n$$

$$P_{n-1} = P_n x + a_{n-1}$$

$$P_{n-2} = P_{n-1} x + a_{n-2}$$

$$P_{n-3} = P_{n-2} x + a_{n-3}$$

$$\vdots$$

$$P_j = P_{j+1} x + a_j \rightarrow \text{General Form.}$$

$$\vdots$$

$$P_1 = P_2 x + a_1$$

$$P_0 = P_1 x + a_0$$

$\therefore f(x) = P_0$ [কারণ সবার নিচে থেকে সমস্ত বের করে আসতেছি।

Ex → 6.3

$f(x) = x^3 - 4x^2 + x + 6x^0$ Here, $x = 2$

$$\boxed{\begin{aligned} a_3 &= 1 \\ a_2 &= -4 \\ a_1 &= 1 \\ a_0 &= 6 \\ x &= 2 \end{aligned}} \rightarrow \text{Input.}$$

$\therefore P_3 = a_3 = 1$

$P_2 = P_3 x + a_2 = 1 \times 2 + (-4) = -2$

$P_1 = P_2 x + a_1 = -2 \times 2 + 1 = -3$

$P_0 = P_1 x + a_0 = (-3) \times 2 + 6 = 0$

$\therefore f(x) = P_0 = 0$.

6.13

Roots of Polynomials

The properties of n^{th} degree polynomials.

1. There are n roots (real or complex)
2. A root may be repeated (multiple roots)
3. Complex roots occur in conjugate pairs.
4. If n is odd and all the co-efficients are real then there is at least one real root.

~~5.~~

5. The Polynomial can be expressed as $P(x) = (x - x_r)\, q(x)$ where x_r is a root of $P(x)$ and $q(x)$ is the quotient Polynomial of order $(n-1)$

Descartes Rule:-

1. no. of Positive real roots $\leq$ no. of sign changes.

2. no of negative real roots $\leq$ no of sign changes (If x is replaced by $-x$)

Example:-

$$+x^{3} - 7x^{2} + 15x - 9 = 0$$

→ তিনটি Root আছে।

sign change হয়েছে ৩ বার। So, There is 3 or less then 3 Roots।

x কে replaced by $-x$

$$-x^3 - 7x^2 - 15x - 9 = 0$$

now কোনো sign chang হয় নাই সবগুলোই same। So There is no negative roots!

23/1/16

* Synthetic Division & Deflation

Defination From Book.

$$\therefore P(x) = (x - x_{r})\, q(x)$$

→ quation to Poligonia

$$P(x) = x^3 - 15x^2 + 6x + 0$$

Let $x = 3$ root of $P(x)$.

$$\therefore P(x) = (x-3)\, q(x)$$

P(x) কে এর কোনো ভাবে 1° Reduce করলাম

$q(x) = x^2 - 10x + 9$ → Like equation

again

$= x - 10$ Like equation

Again $P(x) = (x - x_r)\, q(x)$

$$\boxed{\therefore q(x) = \frac{P(x)}{(x - x_r)}}$$ Called Synthetic Division

Ex→6.15

$P(x) = x^3 - 7x^2 + 15x - 9 = 0$ has a root at $x = 3$

Find $q(x) = ?$

$\therefore P(x) = (x - x_r)\, q(x)$

$= (x-3)\, q(x)$

$\therefore a_3 = 1$ | $a_1 = 15$

$a_2 = -7$ | $a_0 = -9$

$\therefore b_3 = 0$

$b_2 = a_3 + 3 \times b_3 = 1 + 3 \times (0) = 1$

$b_1 = a_2 + 3 \times b_2 = -7 + 3(1) = -4$

$b_0 = a_1 + 3 \times b_1 = 15 + 3(-4) = 3$

$\therefore q(x) = 1x^2 - 4x + 3$

P-178

Multiple Roots by Newton's Method :→

1. n, coefficients of Polynomials ← input.
2. x_0 = guess, E = guess
3. do while $(n>1)$
3. $x_r = x_0 - \frac{f(x)}{f'(x)}$
4. Root $(n) = x_r$
5. New Polynomial of degree $(n-1)$ [Synthetic Division]
6. set $X_0 = X_r$

End of do

7. Root $(1) = -\frac{a_0}{a_1}$
8. Stop.

Quiz-1

Ex→6.15

$P(x) = x^3 - 7x^2 + 15x - 9 = 0$ has a root at $x = 3$

Find $q(x) = ?$

$\therefore P(x) = (x - x_r)\, q(x)$
$= (x-3)\, q(x)$

$\therefore a_3 = 1$ | $a_1 = 15$
$a_2 = -7$ | $a_0 = -9$

$\therefore b_3 = 0$

$b_2 = a_3 + 3 \times b_3 = 1 + 3 \times (0) = 1$

$b_1 = a_2 + 3 \times b_2 = -7 + 3(1) = -4$

$b_0 = a_1 + 3 \times b_1 = 15 + 3(-4) = 3$

$\therefore q(x) = 1 \cdot x^2 - 4x + 3$

P-178

Multiple Roots by Newton's Method :→

1. n, coefficients of Polynomials ← input.
2. x_0 = guess, E = guess
3. do while $(n > 1)$

3. $x_r = x_0 - \frac{f(x)}{f'(x)}$

4. Root $(n) = x_r$

5. New Polynomial of degree $(n-1)$
[Synthetic Division]

6. set $x_0 = x_r$

End of do

7. Root $(1) = -\frac{a_0}{a_1}$

8. Stop.

Quiz-1

* Bisection & false position

$10 \rightarrow f(x1) : 0.0040$

$f(x2) : -0.0166$

$\varepsilon \rightarrow 0.0006$

$0.0040 = x^2 - 4x - 10$

$x1 = -1.7422$

$x2 = -1.7412$

$$\text{Error} = \left|\frac{x_2 - x_1}{x_2}\right| = \left|\frac{-1.7412 + 1.7422}{-1.7412}\right|$$

$$= \left|\frac{-0.0010}{-1.7412}\right|$$

$$= 5.7431\,e^{-4}$$

$$= 0.00057431$$

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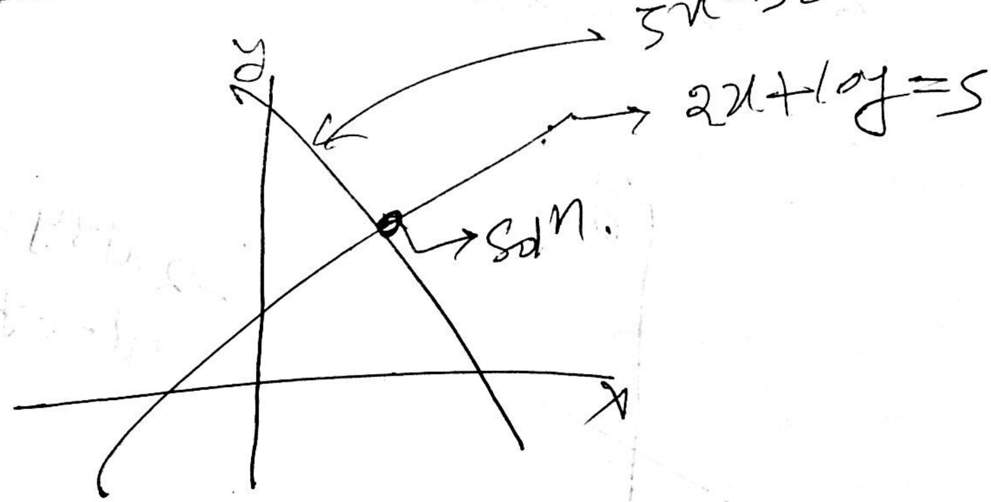
Chapter-7

Direct Sol^n of liner Equations:→

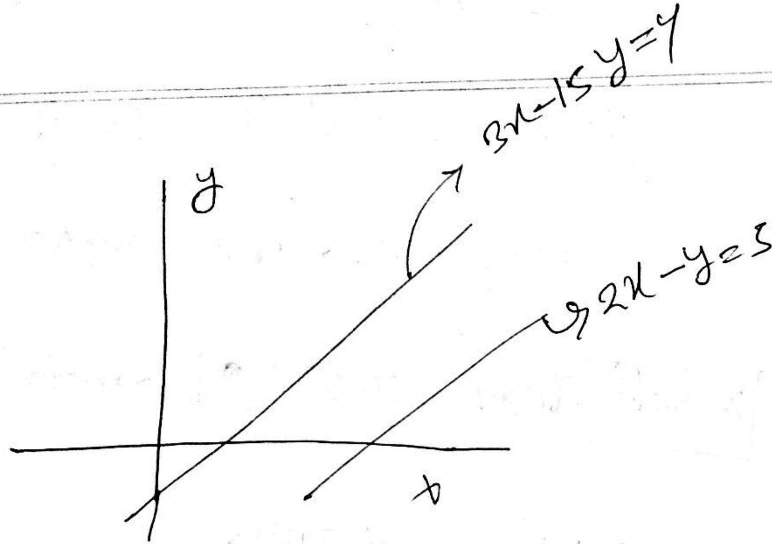
$ax+by=c$ → Liner equation

$\boxed{\text{* Self Study → 7.2, 7.3}}$ → math টা Important

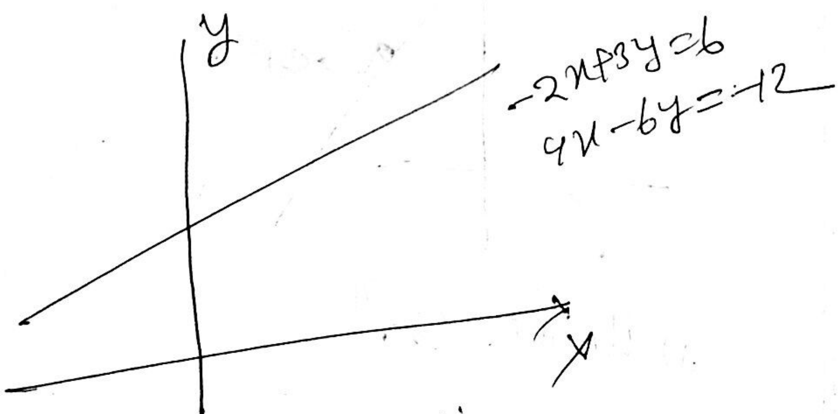
* Existance of Sol^n:→



① Unique Sol^n

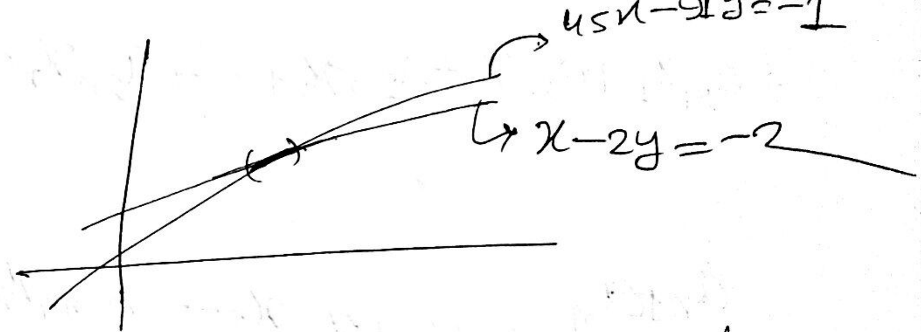


② No Solution



③ Infinite solution.

④ ill conditioned



$$a_{11}x_1 + a_{12}x_2 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$\therefore \boxed{\frac{a_{11}}{a_{12}} \approx \frac{a_{21}}{a_{22}} = \text{tangent}}$$

$$\Rightarrow \boxed{a_{21}a_{12} \approx a_{11}a_{22}}$$

$$\Rightarrow \boxed{a_{11}a_{22} - a_{21}a_{12} \cong 0}$$

* $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$

$\Rightarrow a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$

$\vdots$

$\cancel{a_{n1}x_n}$

$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n = b_n$

Co-officient of matrix

$$Ax = b$$

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

Vector matrix of Unknown Variable

$$x = \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{vmatrix} \qquad b = \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{vmatrix}$$

$\therefore Ax = b$

$A^{-1}Ax = A^{-1}b$

$\Rightarrow x = A^{-1}b$

Direct Sol^n of Liner Equation

- Elimination Approach
  - → ① Gouss Elimination Method
- Iterative approach

Gouse Elimination Method

7.1

$$3x+2y+z=10 \quad \text{—— (i)}$$
$$2x+3y+2z=14 \quad \text{—— (ii)}$$
$$x+2y+3z=14 \quad \text{—— (iii)}$$

(i) First ~~last~~ equation এর কোনো element Remove হবে না

(ii) ② নং এর first এর টা Remove হবে ও 0 হবে

(iii) ③ নং এর first 2টা " " 0 হবে।

⋮

(n) n নং এর first (n−1)টা " " 0 হবে।

* $3x+2y+z=10$

$\Rightarrow \left(\frac{2}{3}\right)$ দিয়ে Multiplie করলাম (2) নং এর সাথে বানানোর জন্য।

$\therefore 2x+\frac{4}{3}y+\frac{2}{3}z=\frac{20}{3}$ — (i)

$\therefore 2x+3y+2z=14$ — (ii)

$\frac{5}{3}y+\frac{4}{3}z=\frac{22}{3}$

(ii) − (i) $\times \frac{1}{3}$

* $x+2y+3z=14$

$\therefore x+\frac{2}{3}y+\frac{1}{3}z=\frac{10}{3}$

$\frac{4}{3}y+\frac{8}{3}z=\frac{32}{3}$

$\Rightarrow y+2z=8$ — (v)

Main

$3x+2y+z=10$ — (i)

$0+5y+4z=22$ — (iv) [(ii) − (i) $\times \frac{2}{3}$]

$y+2z=8$ — (v)

(v) − (iv) $\times \frac{1}{5}$

* $y+2z=8$

$y+\frac{4}{5}z=\frac{22}{5}$

$\frac{6}{5}z=\frac{18}{5}$

$\Rightarrow 6z=18$

$z=3$

Put z in (v) →

$y+6=8$

$y=2$

put z, y in (i)

$3x+4+3=10$

$3x=3$

$x=1$

$\therefore \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$