$L_n = 1 + \frac{n(n+1)}{2}$

Date: 12.09.17

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

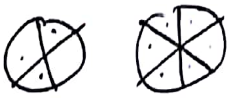
3rd Year, 1st Semester, Final Examination, Spring-2017

Course No: **CSE 3101** Course Title: **Mathematical Analysis for Computer Science**

Time: 3 hours Full Marks: 70

[There are 7 (seven) questions carrying 14 marks each. **Answer any 3 (three) questions from Section A and any 2 (two) questions from Section B**]

[*Marks allotted are indicated in the right margin*]



$J(1) = 1$

SECTION – A

1.a) For the *Lines in the Plane* problem, explain how adding the n-th line creates n new regions. Also, describe a special case when it would create fewer than n regions. Then, find the maximum number of regions that can be obtained from n number of intersecting 'M' shapes. [5]

b) For the Josephus problem starting with n people, prove that $J(n) = 2l + 1$ by using mathematical induction. Also, show that $J(2^m) = 1$. [5]

c) Write the recurrence for Double Tower of Hanoi (DTOH) problem. *Find the closed* form expression for the recurrence. Also, find the minimum number of moves *necessary* to solve the DTOH problem starting with 18 disks. *[4]*

$(2^n - 1)$

2.a) By using the Repertoire method, find the closed form expression for R_n from the following recurrence. [5]

$R_0 = 5$

$R_n = R_{n-1} + 3 + 2n$

b) The average number of comparison steps C_n made by the quicksort algorithm to sort n items satisfies the following recurrence. [5]

$C_0 = 0$

$$C_n = (n+1) + \frac{2}{n}\sum_{k=0}^{n-1} C_k$$

$T_n = \frac{1}{s_n a_1 b_1}$

Find the summation factor for the recurrence and prove that $C_n = 2(n+1)H_n - 2n$.

c) Derive the formula for Perturbation technique. Then, apply the perturbation method to find a closed form expression for the following sum: $S_n = \sum_{0 \le k \le n} k^2$ [4]

3.a) If n is an m-bit integer number, then prove that $m = 1 + \lfloor \lg(n) \rfloor$. Also, prove or disprove that $(x \bmod ny) \bmod y = x \bmod y$, where n is an integer and $y \neq 0$. [5]

b) Prove the Symmetry identity, Addition formula and Absorption identity for binomial coefficients. [5]

c) Prove that, $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$. [4]

4.a) Write down an efficient algorithm to find all the prime integers up to a given integer n. Also, calculate the value of $\varepsilon_3(243!)$. [3]

b) Write and prove the fundamental theorem of arithmetic. [3]

c) What do you understand by relative primality and Stern-Brocot tree? Give an inductive proof of the following property of Stern-Brocot tree — if m/n and m'/n' are consecutive fractions at any stage of the construction, then $m'n - mn' = 1$. [4]

d) Write an algorithm to generate the *L-R* sequence that locates a given fractional value in the Stern-Brocot tree. Also, demonstrate every step of the algorithm by locating the fraction 13/8 in the Stern-Brocot tree. [4]

$\frac{30}{100} \times \frac{40}{100} + \frac{70}{100} \times \frac{20}{100}$

SECTION – B

5.a) In a hospital, 40% of those with high blood pressure have had strokes and 20% of those without high blood pressure have had strokes. If 30% of the patients have high blood pressure, what percent of the patients have had stokes? [3]

b) At a party, each of five men throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. [3]

i) What is the probability that ALL of the five men select their own hats?
ii) What is the probability that EXACTLY four men select their own hats?

c) Define Poisson random variable. Suppose, six fair coins are flipped. If the outcomes are assumed independent, then what is the probability that at least three tails are obtained? [4]

$\binom{n}{i} p^i (1-p)^{n-i}$

d) Suppose that 10 percent of men and 5 percent of women are born color-blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. [4]

	SC	S	G
C	0.5		0.2
S	0.4		0.3
G	0.1		0.5

6.a) For the Gambler's ruin problem, prove that $P_i = \frac{1-(q/p)^i}{1-(q/p)} P_1$, where the symbols have their usual meanings. [5]

b) Write the Chapman-Kolmogorov equation. On any given day, Romi is either cheerful (C), so-so (S) or glum (G). He will be cheerful tomorrow if he is C, S or G today with respective probabilities 0.5, 0.4, 0.1. He will be glum tomorrow if he is C, S or G today with probabilities 0.2, 0.3, 0.5, respectively. Given that Romi is Glum on Monday, what is the probability that he will NOT be Glum on Friday? [5]

c) An airplane can make a successful flight to its destination if at least 50% of its engines remain operative. If the probability of failure of an airplane engine is 0.25 during each flight, then what is the probability that a three engine air plane can make a successful flight to the destination? [4]

7.a) What do you understand by an $M/G/k$ queue? For a single server exponential queuing system, it is given that $P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right)$. *Find the value of W and W_q for this queuing system.* [5]

b) *For the shoeshine shop model, it is given that* $\lambda = 1$, $\mu_1 = 1$ and $\mu_2 = 2$. Calculate the *following quantities for this system.* [5]

i) The probability that the shoeshine shop is empty.

ii) The probability that the customer at chair 1 is blocked.

iii) The mean number of customers in the system.

c) What do you understand by the expectation of a random variable? Find $E[X]$, where X is a geometric random variable with parameter p. [4]