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AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department : Arts and Sciences
Program : B. Sc. in Computer Science and Engineering
Semester Final Examination, Spring 2017
Year : 1st Semester: 2nd
Course No. : MATH 1219 Course Name: Mathematics II
Time: 3 (Three) hours Full Marks: 70
Instruction: There are 7 (seven) questions. Answer 5 (five) questions, taking any 2(two) from Group- A and 3 (three) from Group- B. Marks allotted are indicated in the right margin.

Group-A

1. (a) Integrate the followings: [9]

(i) $\int \frac{e^x(x+1)}{\sin^2(xe^x)}dx$, (ii) $\int \frac{2x^2-x+3}{x^2-x-6}dx$, (iii) $\int x^3\sqrt{4+9x^2}\,dx$.

(b) Find the area bounded by the loop of the curve $a^2y^2 = x^3(3a-x)$. [5]

2. (a) Evaluate (i) $\int_{-5}^{5} x^{15}(2-15x^2)^{16}\,dx$, (ii) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$. [4]

(b) Derive a reduction formula for $\int \sin^n x\,dx$ and hence calculate $\int \sin^5 x\,dx$. [5]

(c) Compute the volume of the solid generated by revolving the regions bounded by $y = x - x^2$ and $y=0$ about the x-axis. [5]

3. (a) State Walli's formula. Hence determine $\int_0^{\pi/2} \cos^7 x\,dx$. [4]

(b) Prove that $\beta(m,n) = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}}dy$. Hence evaluate $\int_0^{\infty} \frac{x^8}{(1+x)^{15}}dx$. [5]

(c) Calculate the arc length S of the graph $f(x) = \frac{1}{12}x^3 + x^{-1}$ over $[1,3]$. [5]

Group-B

4. (a) Form the differential equation of the lowest order by eliminating arbitrary constants of $y = e^{mx}(a\cos nx + b\sin nx)$, and hence write down the order and degree of the differential equation obtained. [4]

(b) Solve: $(x^2+2xy-y^2)dx + (y^2+2xy-x^2)dy = 0$. [5]

(c) Solve: $(x^2+y^2+x)dx + xydy = 0$. [5]

5. (a) A population grows at the rate of 5% per year. How long does it take for the population to double? Use differential equation for it. [4]

(b) Solve: $(D^2+2D+4)y = e^x \sin 2x$. [5]

(c) Find the solution of the following initial value problem [5]

$(D^3-6D^2+9D)y=0$, subject to $y(0)=0$, $y'(0)=2$, $y''(0)=-6$, where $D=d/dx$.

6. (a) Solve: $(x^2D^2-2xD+2)y=x^3$, where $D=d/dx$. [4]

(b) Solve: $\frac{dx}{dt}+2x-3y=t$, $\frac{dy}{dt}-3x+2y=e^{2t}$. [5]

(c) A circuit has in series an electromotive force given by $E=100\sin 60t\,V$, a resistor of 2Ω, an inductor of $0.1H$ and a capacitor of $\frac{1}{260}$ farads. If the initial current and initial charge on the capacitor are both zero, find the charge on the capacitor at any time $t>0$. [5]

7. (a) Classify and write the name of the following partial differential equations: [4]

(i) $\alpha\frac{\partial^2 u(x,t)}{\partial x^2}-\frac{\partial u(x,t)}{\partial t}=0$; (ii) $\frac{\partial^2 u(x,t)}{\partial x^2}+\frac{\partial^2 u(x,t)}{\partial t^2}=0$. Hence describe the importance of the above partial differential equations in the field of Computer Science and Engineering.

(b) Derive a partial differential equation from the equation $z=e^{(\alpha x+\beta y)}\phi(\alpha x-\beta y)$. [5]

(c) Obtain the general integral of the equation $-2xyp+(x^2+z^2-y^2)q+2yz=0$, [5]

where $p=\partial z/\partial x$ and $q=\partial z/\partial y$.