or, $L\{y\}\{s^2+25\} = 10\frac{s}{s^2+25} + 2s$

or, $L\{y\} = \frac{10s}{(s^2+25)^2} + \frac{2s}{(s^2+25)}$

Taking Inverse Laplace transform of both sides we get,

$$y = L^{-1}\left\{\frac{10s}{(s^2+25)^2}\right\} + 2L^{-1}\left\{\frac{s}{s^2+25}\right\}$$

$\frac{d}{ds}\frac{1}{(s^2+25)^2}$??

diff. বাদে integration form

$L^{-1}\left\{\frac{d}{ds}\frac{-5}{s^2+25}\right\}$

$= -t\,L^{-1}\left\{\frac{-5}{s^2+25}\right\}$

$$\Rightarrow y = t\,L^{-1}\left\{\int_s^\infty \frac{10s}{(s^2+25)^2}\,ds\right\} + 2\cos 5t \quad \text{----} \ (11)$$

let $s^2+25=x$

$2s.ds = dx$

$$\int \frac{10s}{(s^2+25)^2}\,ds = \int \frac{5dx}{x^2} = 5\frac{(-1)}{x} = -\frac{5}{x} = -\frac{5}{s^2+25}$$

$$\therefore \int_s^\infty \frac{10s}{(s^2+25)^2}\, ds = \left[\frac{-5}{s^2+25}\right]_s^\infty$$

$$= -5\left[0 - \frac{1}{s^2+25}\right]$$

$$= \frac{5}{s^2+25}$$

<u>eqn (ii)</u>

$$y = t L^{-1}\left\{\frac{5}{s^2+25}\right\} + 2\cos 5t$$

$$= t\sin 5t + 2\cos 5t$$

<u>49</u>

Math - III (Sir) 05/08/2017

49, 50, 51, 52, 53, 54

Exercise - 13·6

53

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\sin x \quad \text{---(i)} \quad , \; y(0)=0, \; y'(0)=1.$$

Taking Laplace transform of both sides,

$$L\left\{\frac{d^2y}{dx^2}\right\} + 2L\left\{\frac{dy}{dx}\right\} + 5L\{y\} = L\{e^{-x}\sin x\}$$

or, $s^2L\{y\} - s\,y(0) - y'(0) + 2\{sL\{y\} - y(0)\}$

$$+ 5L\{y\} = \frac{1}{(s+1)^2 + 1^2}$$

or, $L\{y\}(s^2+2s+5) - (s.0) - 1 - (2.0) = \dfrac{1}{s^2+2s+2}$

or, $L\{y\}(s^2+2s+5) = \dfrac{1}{s^2+2s+2} + 1$

or, $L\{y\} = \dfrac{1+s^2+2s+2}{(s^2+2s+2)(s^2+2s+5)}$

Taking inverse Laplace of both sides,

$$y = L^{-1}\left\{\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right\} \quad \text{---} (ii)$$

partial fraction করতে হবে।

Let,

$$\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \frac{As+B}{s^2+2s+2} + \frac{Cs+D}{s^2+2s+5}$$

$$\text{or, } s^2+2s+3 = (As+B)(s^2+2s+5) + (Cs+D)(s^2+2s+5) \quad \text{---} (iii)$$

equating the co-efficient of s^3, s^2, s and constants of both side we get,

$s^3 \longrightarrow \quad 0 = A + C$

$s^2 \longrightarrow \quad 1 = 2A + B + 2C + D$

$s \longrightarrow \quad 2 = 5A + 2B + 2C + 2D$

constant $\longrightarrow \quad 3 = 5B + 2D$

$A = 0$

$C = 0$

$B = \frac{1}{3}$

$D = \frac{2}{3}$

So, from eqⁿ (ii),

$$y = L^{-1}\left\{\frac{1}{3}\frac{1}{s^2+2s+2} + \frac{2}{3}\frac{1}{s^2+2s+5}\right\}$$

$$= \frac{1}{3}L^{-1}\left\{\frac{1}{(s+1)^2+1}\right\} + \frac{2}{3}\cdot\frac{1}{2}L^{-1}\left\{\frac{2}{(s+1)^2+2^2}\right\}$$

$$\text{or, } y = \frac{1}{3}e^{-x}\sin x + \frac{1}{3}e^{-x}\sin 2x$$

$$= \frac{1}{3}e^{-x}(\sin x + \sin 2x)$$

Solution of Simultaneous equation

$\because \frac{d}{dt}$ $\therefore$ x, y are both function of t

57

$$\frac{dx}{dt} + y = 0 \quad \text{---} (i), \quad x(0) = 1$$

$$\frac{dy}{dt} - x = 0 \quad \text{---} (ii), \quad y(0) = 0$$

Taking Laplace transform of both sides of eqn (i) and (ii) we get,

$$L\left\{\frac{dx}{dt} + y\right\} = 0 \qquad L\{0\} = 0$$

$$L\left\{\frac{dy}{dt} - x\right\} = 0$$

or, ~~$s x - x(0)$~~ [where, $L\{x\} = \bar{x}$ and $L\{y\} = \bar{y}$]

$$s\bar{x} - x(0) + \bar{y} = 0$$

or,

$$s\bar{y} - y(0) - \bar{x} = 0$$

or, $s\bar{x} + \bar{y} - 1 = 0$ ---- (iii)

or, $s\bar{y} - \bar{x} = 0$ ---- (iv)

~~Solving eq^n (III) and (IV),~~

$$\cancel{\frac{\bar{x}}{0+1} = \frac{\bar{y}}{-s+0} = \frac{1}{-s-s}}$$

$$\cancel{\bar{x} = -\frac{1}{2s}} \qquad \cancel{\bar{y} = \frac{1}{2}}$$

from eq^n (III) and (IV),

$$s.s\bar{y} + \bar{y} - 1 = 0$$

or, $\bar{y}\left(s^2+1\right) = 1$

or, $\bar{y} = \dfrac{1}{s^2+1}$ ---- (V)

$\therefore \bar{x} = \dfrac{s}{s^2+1}$ ---- (VI)

Taking Inverse Laplace transform of eq^n (V) and (VI) we get,

$$L^{-1}\left\{\bar{y}\right\} = L^{-1}\left\{\frac{1}{s^2+1}\right\}$$

or, $y = \sin t$

and, $L^{-1}\left\{\bar{x}\right\} = L^{-1}\left\{\dfrac{s}{s^2+1}\right\}$

or, $x = \cos t$

58

$$(D+1)y_1 + (D-1)y_2 = e^{-t} \quad \text{.... (i)}$$

$$(D+2)y_1 + (D+1)y_2 = e^{t} \quad \text{.... (ii)}$$

$$\left[\begin{array}{l} y_1(0) = 1 \\ y_2(0) = 0 \end{array}\right]$$

$D = \frac{d}{dt}$

$$(D+1)^2 y_1 + (D-1)(D+1)y_2 = (D+1)e^{-t} \quad \text{.... (iii)}$$

$$(D+2)(D-1) + (D+1)(D-1)y_2 = (D-1)e^{t} \quad \text{.... (iv)}$$

Subtracting (iv) from (iii),

$$(D+1)^2 y_1 - (D+2)(D-1)y_1 = (D+1)e^{-t} - (D-1)e^{t}$$

or, $(D^2+2D+1)y_1 - (D^2+D-2)y_1 = -e^{-t}+e^{-t} - e^{t}+e^{t}$

or, $(D+3)y_1 = 0$

or, $\frac{dy_1}{dt} + 3y_1 = 0$

Taking Laplace transform of both sides,

$sL\{y_1\} - y_1(0) + 3L\{y_1\} = 0$

or, $L\{y_1\}(s+3) - 1 = 0$

or, $L\{y_1\}(s+3) = 1$

or, $L\{y_1\} = \frac{1}{s+3}$

$\therefore y_1 = L^{-1}\left\{\frac{1}{s+3}\right\}$

$= e^{-3t}$

① $\Rightarrow$

$(D+1)e^{-3t} + (D-1)y_2 = e^{-t}$

or, $-3e^{-3t} + e^{-3t} + (D-1)y_2 = e^{-t}$

or, $\frac{dy_2}{dt} - y_2 = e^{t} + 2e^{-3t}$

Taking Laplace transform of both sides,

$sL\{y_2\} - y_2(0) - L\{y_2\} = \frac{1}{s+1} + \frac{2}{s+3}$

or, $L\{y_2\}(s-1) - 0 = \frac{1}{s+1} + \frac{2}{s+3}$

or, $L\{y_2\} = \frac{1}{(s+1)(s-1)} + \frac{2}{(s+3)(s-1)}$

or,

or, $L\{y_2\} = \dfrac{1}{s^2-1} + \dfrac{2}{(s-1)(1+3)} + \dfrac{2}{(3-1)(s+3)}$

$$= \frac{1}{s^2-1} + \frac{1}{2(s-1)} - \frac{1}{2(s+3)}$$

Taking Inverse Laplace of both sides,

$$\cancel{\therefore\ y_2 = L\left\{\frac{1}{s^2-1} + \frac{1}{2(s-1)} + \frac{1}{s+3}\right\}}$$

$$\cancel{= te^t + \frac{1}{2}e^t + e^{-3t}}$$

$$\therefore\ y_2 = L\left\{\frac{1}{s^2-1} + \frac{1}{2(s-1)} - \frac{1}{2(s+3)}\right\}$$

$$= \sinh t + \frac{1}{2}e^t - \frac{1}{2}e^{-3t}$$

06/08/17

Ques Pattern

Set A

2 group — Complex
— Laplace

Def^n থাকতে পারে।
সূত্রের proof থাকবে না।