

Date: 10/03/2016

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

First Year, Second Semester

Final Examination, Fall 2016

Course No: CSE 1203

Course Title: Discrete Mathematics

Time: 3 Hours

Full Marks: 70

[There are 7(Seven) questions. The question no.1 is compulsory and answer any four from the rest.]

[Marks allotted are indicated in the right margin within '[]'.]

- Q1 a) Argue that $\exists x \forall y P(x, y) \leftrightarrow \forall x \exists y P(x, y)$ is (or is not) a tautology. [2]
- b) Show that an implication and its contrapositive are equivalent. [2]
- c) Use mathematical induction to prove that one can solve the Tower of Hanoi problem with n disks in $2^n - 1$ moves. [3]
- d) Prove that for any integer x, the integer $x(x + 1)$ is even. [2]
- e) Let K_n be the complete graph with n vertices. For what values of n, K_n admits an Eulerian circuit? [3]
- Q2 a) State and prove De Morgan's laws for sets. [4]
- b) Write what you know about planar graphs. Show that $K_{2,4}$ is a planar graph, where K_5 is not. [3]
- c) Define the generalized pigeonhole principle. [2]
- d) Prove that, "if n is an integer and $n^3 + 7$ is odd then n is even", using –
i. Indirect Proof
ii. Proof by Contradiction [5]

3. a) Given the following statements as premises, all referring to an arbitrary meal: [4]

If he takes coffee, he does not drink milk.
He eats crackers only if he drinks milk.
He does not take soup if he eats crackers.
At noon today, he had coffee.

Draw the conclusion whether he took soup at noon today or not, using rules of inference.

- b) Write short notes on: [4]
i. Spanning tree
ii. Binary search tree
- c) Suppose that the number of bacteria in a colony triples every hour. Set up a recurrence relation for the number of bacteria after n hours have elapsed. [2]

d) Prove using mathematical induction that $1 + 5 + 9 + \dots + (4n+1) = (n+1)(2n+1)$. [4]

4. a) How many eight digit numbers are there that contain a 5 and a 6? Explain. [4]

b) What is the cardinality of each of the following sets? [3]

- i. \mathbb{Z}
- ii. $\mathbb{N} - \mathbb{Z}^+$
- iii. $A - B$



c) Determine whether $(p \wedge (p \vee q)) \leftrightarrow (p \vee (p \wedge q))$ is a tautology using laws of logic. [3]

d) What is the value of each of these expressions? [4]

- i. $- * + - 5 3 2 1 4$
- ii. $5 2 1 - - 3 1 4 + + *$

5. a) Let $A = \{1, 2, 3, 4, 5\}$, determine the truth value of the following: [2]

- i. $(\forall x \in A)(x + 3 = 7)$
- ii. $(\exists x \in A)(x + 3 < 5)$

b) What are the equivalence classes of 1 and 2 for congruence modulo 4? [3]

c) Write a short note on bipartite graphs. Distinguish between Hamiltonian and Eulerian graphs. [4]

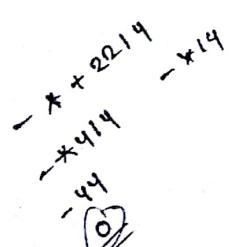
d) Show that a full m -ary tree with i internal vertices has m^i edges. [3]

e) Let $A = \{1, 2, 3\}$, $B = \{p, q\}$ and $C = \{a, b\}$. Let $f: A \rightarrow B$ is $f = \{(1, p), (2, p), (3, a)\}$ and $g: B \rightarrow C$ is given by $\{(p, b), (q, b)\}$. What is the composition of f and g ? [2]

6. a) Let $F(x, y)$ be the statement "x can fool y", where the universe of discourse consists of all people in the world. Use quantifiers to express each of these statements. [4]

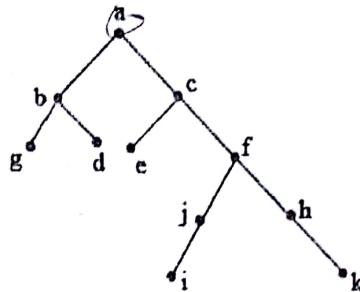
- i. Everybody can fool somebody.
- ii. There is exactly one person whom everybody can fool.
- iii. No one can fool himself or herself.

b) Determine whether $f(x) = 3x + 2$ is a bijection from \mathbb{R} to \mathbb{R} . [2]



c) Show the three types of traversal for the following tree.

[4] ✓



d) Write short notes on:

[4]

- i. Equivalence relation ✓
- ii. Equivalence class

✓ 7. a) Consider the following conditional statement:

[3]

"If the flood destroy my house or the fires destroy my house, then my insurance company will pay me."

Write the converse, inverse and contrapositive of the statement.

b) For what numbers m do we have that $m^2 - 1$ is a prime number?

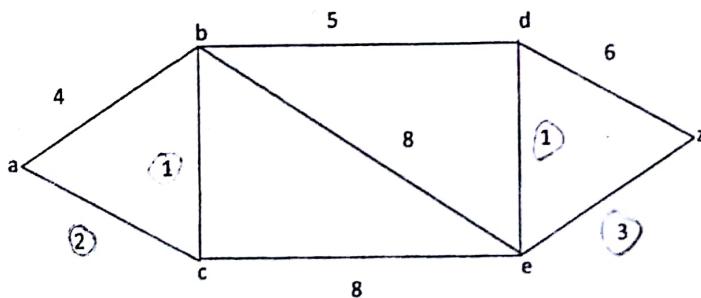
[2]

✓ c) How many bit strings of length six are possible if they start with 1 or end with 0 or have 1s in the two central places?

[4]

✓ d) For the following weighted graph determine the shortest path from z to a and also calculate its length. Draw all the steps to calculate the shortest path.

[5]



Date: 26/09/16

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

First Year, Second Semester

Final Examination, Spring 2016

Course No: CSE1203

Course Title: Discrete Mathematics

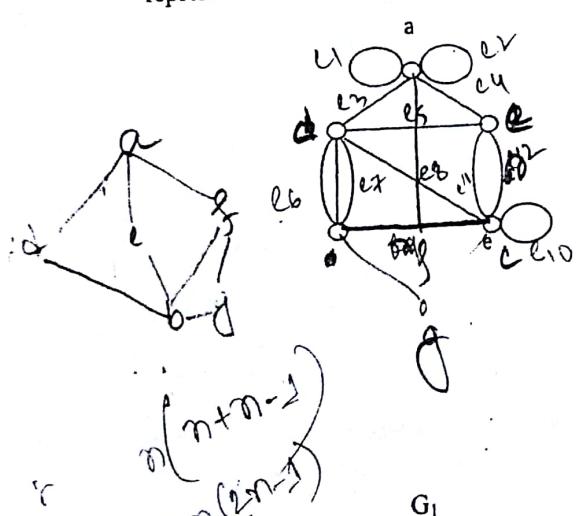
Time: 3 Hours

Full Marks: 70

[There are 7(Seven) questions. The question no.1 is compulsory and answer any four from the rest.]

[Marks allotted are indicated in the right margin within '()' .]

1. a) There are two restaurants next to each other. One has a sign says "Good food is not cheap" and other has a sign that says "Cheap food is not good". Are the signs saying same thing? Explain your answer with propositional logic. [3]
- b) Prove that if n and m are positive even integers, then n^*m is divisible by 4. [2]
- c) There are m functions from a one-element set to the set {1, 2, ..., m}. How many functions are there from a two-element set to {1, 2, ..., m}? Give a recurrence relation for the number T(n) of functions from an n-element set to {1, 2, ..., m}. Solve the recurrence relation. [3]
- d) Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. [4]
- e) Represent using adjacency matrix the graph G_1 given below, and construct graph G_2 represented below using adjacency lists. [2]



G_2
a: d, e, f
b: d, e, f, g
c: e, f, g
d: a, b
e: a, b, c
f: a, b, c
g: b, c

What is a contingency? Show using a truth table whether the following expression is a tautology or not.

$$(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$$

Page 1 of 4



$$1. \frac{(m-1)n+1}{m}$$

$$\text{leaf} = m^n - m^{n-1}$$

- b) Show that a full m -ary tree with n vertices has $I = (n-1)/m$ internal vertices and $L = [(m-1)n+1]/m$ leaves.

- c) Write short notes on:

- i. Power set,
- ii. Set difference,
- iii. Cartesian product.

[3]

- d) Use mathematical induction to show that (n^3-n) is divisible by 3 for all positive integer n . [4]

3. a) Derive r from the given set of facts $\{ t \vee r, s \vee \neg p, t \rightarrow q, \neg s \vee r, \neg q \wedge u \}$ using propositional rules of inference. [4]

- b) Draw a binary tree that represents $((z+5)/3)^x - 5 + (x * 5)$. Now find the prefix and postfix notation of this expression. [4]

- c) Consider the function $f: X \rightarrow Y$ on the sets $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$, where $f(a) = 2, f(b) = 1, f(c) = 2, f(d) = 3$. Determine whether the function is Injective, Surjective and Bijective. If so then why? [3]

- d) Write down the Handshaking theorem and prove it.

$$t \vee r$$

$$s \vee \neg p$$

$$t \rightarrow q$$

$$t s \vee r$$

$$\neg q \wedge u$$

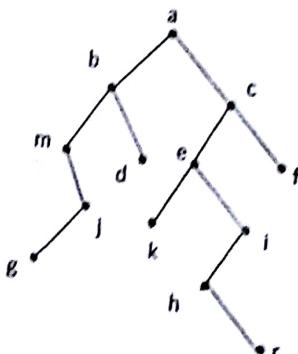
[3]

4. a) Write short notes on:

- i. Quasi order,
- ii. Partitions,
- iii. Inverse functions.

[3]

- b) Show the three types of traversal of the following tree.



$$t \vee r$$

$$s \vee \neg p$$

[4]

$$\begin{aligned} n &= m^i + 1 \\ &= mn + n - 1 \\ &= ml = mn + m + 1 = m \\ &\Rightarrow ml - mn - m = m \end{aligned}$$

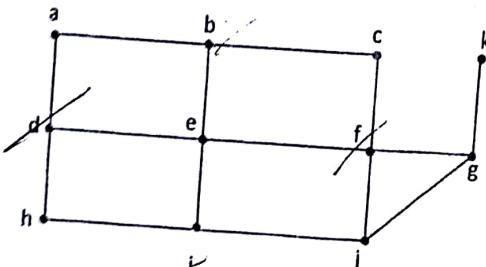
- c) Define the pigeonhole principle. How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen? [3]

- d) Let m be a positive integer greater than 1. Show that the following relation is an equivalence relation on the set of integers.

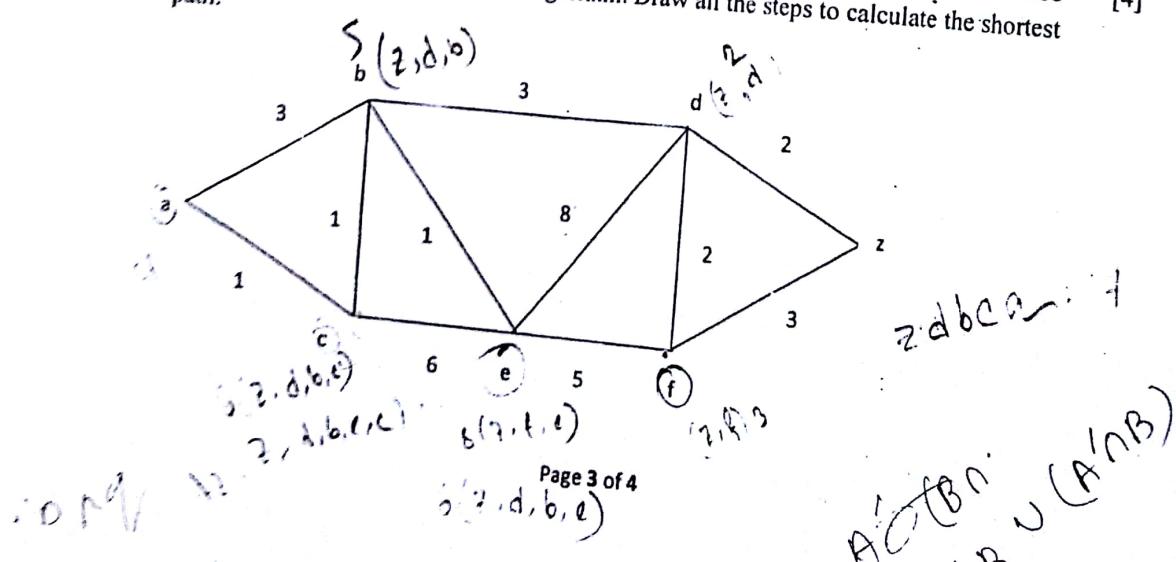
$$R = \{(a, b) \mid a \equiv b \pmod{m}\} \quad [4]$$

$$\begin{aligned} l &= n + \frac{n-1}{m-1} \quad \text{Page 2 of 4} \quad l = m(i-1) + 1 = m \\ &= n + \frac{m}{m-1} \quad \Rightarrow i(m-1) + 1 \\ &= \frac{(n-1)}{m} (m-1) + 1 \quad \frac{(n-1)}{m} (m-1) + 1 \end{aligned}$$

5. a) Translate the following English sentences into logical expressions.
- All your friends are perfect.
 - Someone in your class has exactly one best friend.
- b) Write short note on planar graph. [2]
- c) How many identifiers of length at most 4 are possible under the following conditions? [4]
- i. An identifier starts with an upper case or lower case letter.
 - ii. There are 30 reserved words of length 4, each containing only lowercase letters which cannot be used as identifiers..
 - iii. Digits can also be used.
- d) What is a spanning tree? Use breadth-first search to find a spanning tree for the graph shown below. (Choose the vertex e for the root.) [4]



6. a) What is wrong with the famous supposed "proof" that $1 = 2$? [2]
- b) Devise the "Tower of Hanoi" problem using recurrence relation with initial conditions. [2]
- c) For the following weighted graph determine the shortest path from z to a and also calculate its length using Dijkstra's algorithm. Draw all the steps to calculate the shortest path. [4]



d) Consider the following relations on $\{0, 1, 2, 3\}$:

[4]

- i. $R_1 = \{(0, 0), (1, 1), (2, 2), (3, 3)\}$
- ii. $R_2 = \{(0, 0), (1, 1), (1, 2), (2, 2), (2, 1), (3, 3)\}$
- iii. $R_3 = \{(0, 0), (1, 1), (1, 3), (3, 2), (2, 2), (2, 3), (3, 1), (3, 3)\}$
- iv. $R_4 = \{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

Determine whether the relations are Reflexive, Symmetric, Anti-symmetric or Transitive. Which of these relations are equivalence relations?

7. a) Draw the following graphs:

[3]

- i. C_5
- ii. W_6
- iii. $K_{5,2}$

b) Prove that if $5n+4$ is odd, then n is odd by giving a proof by contradiction.

[3]

c) Prove using the laws of set operations that

[4]

$$((B' \cup A') \cap (A' \cup B)) \cup (A' \cap B) = A'.$$

d) Describe Euler circuit and Hamiltonian circuit with examples.

[4]

edge OR vertex repeat 2/0 1/-

$$\begin{aligned} & \cancel{A \cap B \cap (A \cup B)} \cup \cancel{A' \cap B' \cap (A' \cup B')} \cup A' \cap B \\ \Rightarrow & \cancel{A \cap B \cap A' \cap B'} \cup \cancel{A' \cap B \cap A'' \cap B''} \cup A' \cap B \\ \Rightarrow & \cancel{A' \cap B' \cap A'' \cap B''} \cup \cancel{A'' \cap B'' \cap (A' \cup B')} \cup A' \cap B \\ \Rightarrow & \cancel{A' \cap B' \cap A'' \cap B''} \Rightarrow A' \cap B' \cap A \cap (A' \cap B) \\ \Rightarrow & A' \cap B' \cap A \cap (A' \cap B) \\ \Rightarrow & A' \cap B' \\ \Rightarrow & A' \\ \Rightarrow & A \end{aligned}$$

*(a+b) * (a-b)*

a x a x

a/b P' V q'

Ahsanullah University of Science and Technology**Department of Computer Science and Engineering****First Year, Second Semester****Final Examination, Fall 2015****Course No: CSE 1203****Course Title: Discrete Mathematics****Time: 3 Hours****Full Marks: 70****[There are 7(Seven) questions. The question no.1 is compulsory. Answer any four from the rest.]****[Marks allotted are indicated in the right margin within '()' .]**

1. a) A set S is defined recursively by [2]

Basis: $0 \in S$

Recursive step: if $a \in S$ then $a + 3 \in S$ and $a + 5 \in S$.

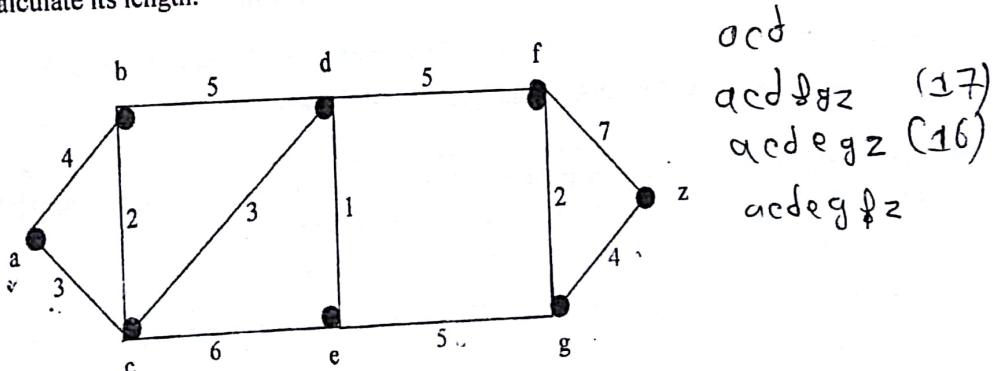
Determine the set $S \cap \{a \in \mathbb{Z} \mid 0 < a < 12\}$.

- b) Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent using laws of [2]
logical equivalences.

- c) Use mathematical induction to show that $2^n > n^2$ where $n > 4$. [4]

- d) A perfect number is a positive integer n such that the sum of the factors of n is equal to
 $2^n - 1$ (1 and n are considered factors of n). So 6 is a perfect number as the sum of the
factors of 6 is $1+2+3+6=12=2^3-1$. Now prove that no prime number is a perfect number. [2]

- e) For the following weighted graph identify the shortest path between a and z, and also [4]
calculate its length.



2. a) Suppose that on an island there are three types of people, knights, knaves and normals. Knights always speak the truth, knaves always lie and normals sometimes lie and sometimes tell the truth. Detectives questioned three inhabitants of the island - X, Y, and Z as part of the investigation of a crime. The detectives knew that one of the three committed the crime, but not which one. They also knew that the criminal was a knight, but the other two were not knights. Additionally the detectives recorded these statements: [4]

X: "I am innocent."
 Y: "What X says is true."
 Z: "Y is not a normal."

After analyzing their information, the detectives identified the guilty party. Who was it?
 Justify your answer.

- b) Translate the following English sentences into logical expressions using predicates and quantifiers. [3]

- i. Every guest gets exactly one gift.
- ii. There is a person in your school who is not happy.

- c) Consider the following relations on a set of integers {1, 2, 3, 4}. [4]

$$\begin{aligned} R1 &= \{(a, b) \mid a \geq b\} \\ R2 &= \{(a, b) \mid a = b - 1\} \\ R3 &= \{(a, b) \mid a < b\} \\ R4 &= \{(a, b) \mid a + b \leq 4\}. \end{aligned}$$

Determine whether the relations are reflexive, irreflexive, Symmetric, Anti-symmetric and/or transitive.

- d) Write down the Handshaking Theorem and prove it. [3]

- ✓ 3. a) A group of ten people begin a chain letter, with each person sending the letter to five other people. Each of these people sends the letter to five additional people. [4]

- i. Find a recurrence relation for the number of letters sent at the nth stage of this chain letter, if no person ever receives more than one letter.
- ii. What are the initial conditions for the recurrence relation?
- iii. How many letters are sent at the nth stage of the chain letter?

- b) Let f and g be the functions from the set of integers to the set of integers defined by $f(y) = 5y+9$ and $g(y) = 2y+5$. What is the composition of f and g? What is the composition of g and f? [3]

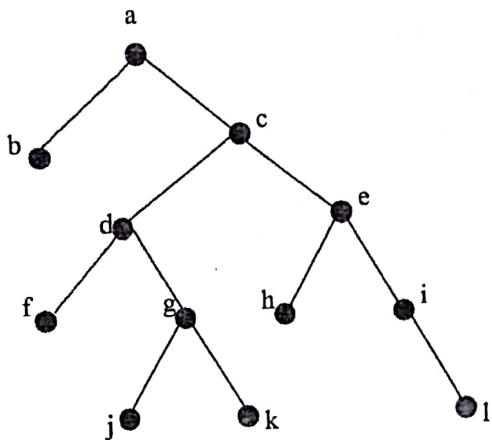
- c) Write what you know about Planar Graphs? Show that, $k_{2,5}$ is a Planar Graph, where k_5 is not. [3]

- d) Prove that, "if n is an integer and $3n + 2$ is even then n is even", using –
 a) Indirect Proof
 b) Proof by Contradiction [4]

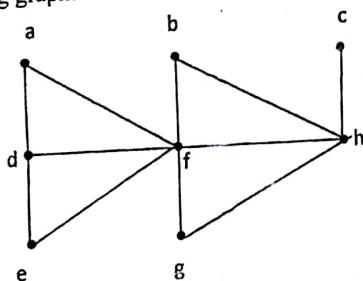
- ✓ 4. a) What is Implication of propositions? State the Converse, Contra-positive and Inverse of the following implication: "If it snows tonight, then I will stay at home". [3]



- b) Verify the distributive properties of set theory using a membership table. [4]
- c) Write short notes on: [3]
- Congruence Modulo,
 - Partial order,
 - Equivalence classes.
- d) Demonstrate the three types of traversals on the following tree. [4]



5. a) Define the "Tower of Hanoi" problem using a recurrence relation with initial conditions. [4]
- b) Illustrate the Pigeonhole Principle including its generalized form. [3]
- c) What is a spanning tree? Construct two spanning trees using DFS and BFS on the following graph. Consider vertex 'f' as the root of the tree. [6]



- d) If $A \subset B$, then what is $|A \cap B|$? [1]

6. a) Prove using laws of set operations that $(A \cap B) \cup (A \cap B') = A$. [4]

- b) Draw a binary tree that represents $((x - 3) + ((x/4) + (x - y)^5))$. Now find the prefix and postfix forms of the expression.
- c) There are 21 different letters in the Italian alphabet, of which 5 are vowels. How many words of 10 letters you can form using the Italian alphabet? How many words of length 8 you can form which contain exactly one vowel?
- d) What is a binary search tree?

7. a) Show that $\neg p \rightarrow \neg q$ and its contrapositive are equivalent. [4]
- b) Is it possible to solve the Konigsberg-bridge problem? Justify your answer. [4]
- c) How many edges do the following graphs have: C_n , W_n , K_n , $K_{n,m}$? Explain your answer. [4]
- d) Show that a full m-ary tree with
- n vertices has $(n-1)/m$ internal vertices and $[(m-1)n + 1]/m$ leaves.
 - k leaves has $(mk-1)/(m-1)$ vertices and $(k-1)/(m-1)$ internal vertices.

Date: 14/10/14

Ahsanullah University of Science and Technology

Department of Computer Science and Engineering

First Year, Second Semester

Course No: CSE 1203

Time: 3 Hours

Final Examination, Spring 2015

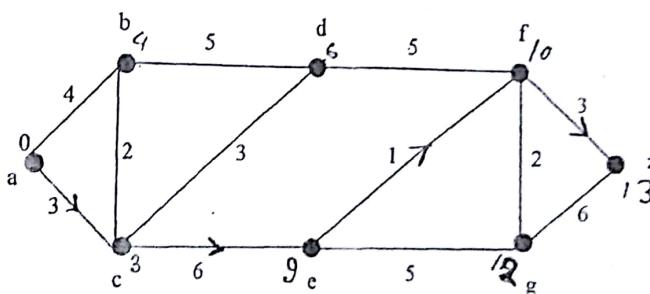
Course Title: Discrete Mathematics

Full Marks: 70

[There are 7(Seven) sets. The set no.1 is compulsory and answer any four sets from the rest six sets]

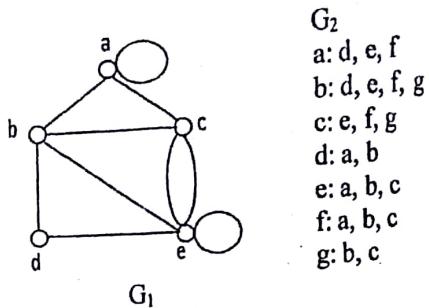
[Marks allotted are indicated in the right margin within '()']

- 1.** a) What is a contradiction? Show by using a truth table whether the expression $(\neg p \vee q) \leftrightarrow (p \wedge \neg q)$ is a contradiction or not. [2]
- b) Express the following statement using predicates and quantifiers-
"Everyone has exactly one best friend" [1]
- c) Consider the function $f: A \rightarrow B$ on the sets $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$, where $f(a) = 3$, $f(b) = 4$, $f(c) = 1$. Determine whether the function is Injective, Surjective and Bijective. If so then why? [3]
- d) Use mathematical induction to show that-
$$1^2 + 2^2 + 3^2 + \dots + n^2 = n^2(n+1)^2/4$$
 [4]
- e) For the following weighted graph identify the shortest path between a and z. Also calculate the length of the path. [4]



- 2.** a) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent using laws of logical equivalence. [3]
- b) Find the number of strings of length 8 of letters of the English alphabet, with repeated letters allowed and without repeated letters,
i. that begin with A and end with Z.
ii. that begin with Z or end with Z. [4]
- c) Prove using the laws of set operations that
$$(A \cap A) \cup (B \cap (B' \cup A))' = U$$
 [4]

- d) Represent using adjacency matrix the graph G_1 given below, and construct graph G_2 represented below using adjacency lists.



G_2

a:	d, e, f
b:	d, e, f, g
c:	e, f, g
d:	a, b
e:	a, b, c
f:	a, b, c
g:	b, c

3. a) Let $F(x, y)$ be the statement "x can fool y", where the universe of discourse consists of all people in the world. Use quantifiers to express each of these statements. [3]
- Everybody can fool somebody.
 - There is exactly one person whom everybody can fool.
 - No one can fool himself or herself.
- b) Draw the graph of the function $f(x) = [2x]$. [3]
- c) Represent the expression $((y - 7) \uparrow 5) * (x - (5+y)) + 9$ using a binary tree. Write this Expression in prefix and postfix notation. [4]
- d) Let m be a positive integer greater than 1. Show that the following relation is an equivalence relation on the set of integers. [4]

$$R = \{(a, b) \mid a \equiv b \pmod{m}\}$$

4. a) Draw the following graphs:
 i. C_5 ii. W_6 iii. K_5 [3]
- b) Devise the "Tower of Hanoi" problem using recurrence relation with initial conditions. Show an iterative approach to solve this recurrence relation. [6]
- c) Determine whether the relations on $\{0, 1, 2, 3\}$ are equivalence relations?
 i. $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$
 ii. $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$ [2]
- d) Derive p applying rules of inference over the set of propositions given below.
 $\{ t \rightarrow q, r \rightarrow t, \neg q \wedge s, \text{ and } r \vee p \}$ [3]

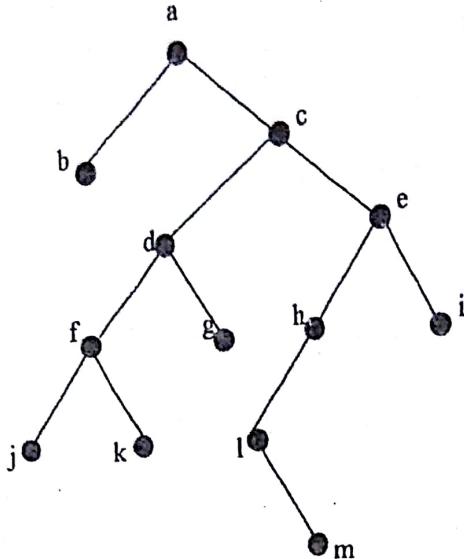
5. a) Suppose that a popular style of running shoe is available for both men and women. The woman's shoe comes in sizes 6, 7, 8 and the man's shoe comes in sizes 8, 9, 10, 11, and 12. The man's shoe comes in white and black, while the woman's shoe comes in white, red, and black. Determine the number of different shoes that a store has to stock to have at least one pair of this type of running shoe for all available sizes and colors for both men and women using counting rules. [3]

- b) Show that a full m-ary tree with
- n vertices has $(n-1)/m$ internal vertices and $[(m-1)n + 1]/m$ leaves. [4]
 - k leaves has $(mk-1)/(m-1)$ vertices and $(k-1)/(m-1)$ internal vertices.
- c) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x+3$ and $g(x) = 3x+2$. What is the composition of f and g ? What is the composition of g and f ? [3]
- d) Use mathematical induction to prove that $2^n < n!$ for every positive integer n with $n \geq 4$. [4]

6. a) Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if
- a is taller than b .
 - a and b were born on the same day.
- b) Verify the associative property of set theory using a membership table. [4]
- c) Write a short note on bipartite graphs. Distinguish between Hamiltonian and Eulerian graphs. [2]
- d) Show that $\neg p \rightarrow q$ and its contrapositive are equivalent. [2]
- e) Write short notes on:
- Power Set
 - Cartesian product

7. a) Prove that, "if n is an integer and $n^3 + 5$ is odd then n is even", using – [5]
- Indirect Proof
 - Proof by Contradiction
- b) Write short notes on: [2]
- Partitions
 - Partial Order
- c) A bowl contains ten red balls and ten blue balls. A man selects balls out at random without looking at them. [2]
- How many balls must be selected to be sure that he has at least three balls of the same color?
 - How many balls must be selected to be sure that he has at least three red balls?

d) Show the three types of traversals of the following tree.



e) Write down the Handshaking Theorem and prove it.

[2]

$$\neg P \rightarrow (q \rightarrow r)$$

$$p \vee q$$

$$q \vee p$$

$$\neg P \rightarrow (\neg q \wedge r)$$

$$\neg P \rightarrow (\neg q \wedge \neg r)$$

$$\neg q \wedge (\neg P \wedge r)$$

$$\neg q \rightarrow \neg P$$

Date: 21-4-15

Ahsanullah University of Science & Technology
Department of Computer Science and Engineering
Year: 1st, Semester: 2nd, Final Examination (Fall 2014)

Course No: CSE 1203
Full Marks: 70

Course Title: Discrete Mathematics
Time: 3 Hours

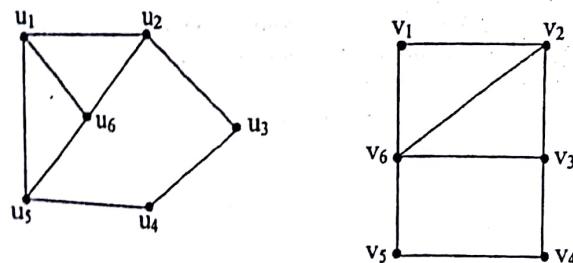
[There are Seven (7) Questions. Answer any Five (5).]
[Marks allotted are indicated in the right margin.]

1. a) Write a brief essay on quantified predicates and their negations. [4]
b) Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology using truth table and laws of logical equivalences. [3+3]
c) Write short notes on 'Counter example method' and 'Modus ponens'. [2+2]
2. a) Translate the following English sentences into logical expressions. [2+2]
i. Every guest gets at least one gift.
ii. There is a person in your school who is not happy.
b) Define any two rules of inference for quantified predicates. Show the following argument is valid using the rules of inference.
There is a student such that if he knows programming, then he knows Java.
All students know programming. Therefore, there is a student that knows either Java or C++.
c) Prove that if $3n+2$ is odd, then n is odd by giving a proof by contradiction. [4]
3. a) Use set builder notation and logical equivalences to show that $A \cap (A \cup B) = A$. [3]
b) Prove using the laws of set operations that $(A \cap A) \cup (B \cap (B' \cup A))' = U$. [4]
c) What is wrong with the famous supposed "proof" that $1 = 2$? [2]
d) Prove using mathematical induction that $1+5+9+\dots+(4n+1) = (n+1)(2n+1)$. [4]
- a) Let R and S be relations on a set A represented by the matrices [6]
 $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$
b) Find the matrix that represent $M_R \odot M_S$. Determine whether the relations represented by the matrices are reflexive, symmetric and/or transitive. [2]
What are the equivalence classes of 1 and 2 for congruence modulo 4?

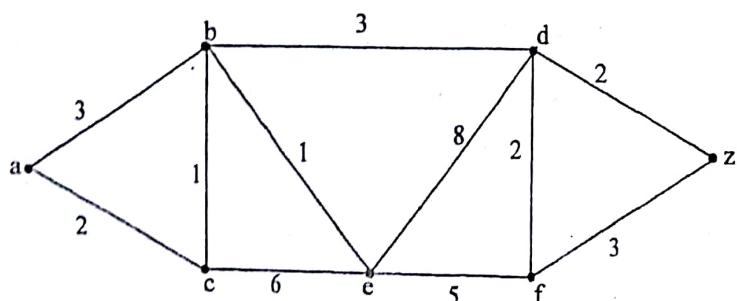
- c) Consider the function $f: X \rightarrow Y$ on the sets $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$, where $f(a) = 2$, $f(b) = 1$, $f(c) = 2$, $f(d) = 3$. Determine whether the function is Injective, Surjective and Bijective. If so then why? [4]
- d) Draw the graph of the function $f(x) = \lceil x \rceil$. [2]

5. a) How many positive integers between 100 and 999 are divisible by 3 or 5 or 7? [4]
- b) Define the inclusion-exclusion principle with example. [3]
- c) Devise the "Tower of Hanoi" problem using recurrence relation with initial conditions. [4]
- d) Define the generalized pigeonhole principle. What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$? [6]

6. a) Draw a graph which consists of 7 vertices and which chromatic number is 2. [2]
- b) Write a short note on planar graphs. Distinguish between a strongly connected and a weakly connected graphs. [1+1]
- c) What is graph isomorphism? What is Adjacency matrix and Incidence matrix? Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists. [1+2+2]



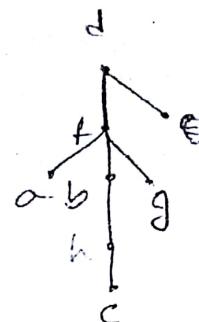
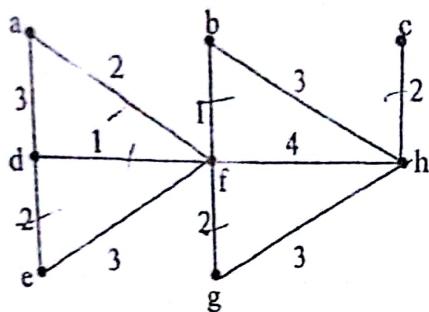
- d) For the following weighted graph determine the shortest path between a and z; also calculate its length. Draw all the steps to calculate the shortest path. [5]



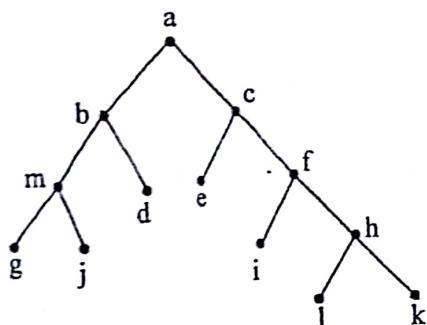
$$H_n = 2 \cdot H_{n-1} + 1$$



7. a) Use Prim's algorithm to find a minimum spanning tree in the weighted graph [6]



- b) Show the three types of traversal of the following tree. [6]



d, f	-	-
f, b	-	-
d, e	-	2
f, a	-	2
f, g	-	2
b, h	-	3
l, c	-	2

- c) Show that a full m -ary tree with n vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n + 1]/m$ leaves. [6]

a - b and c

a b c

Date: 28.8.14

Ahsanullah University of Science & Technology
Department of Computer Science and Engineering
Year: 1st, Semester: 2nd, Final Examination (Spring 2014)

Course No: CSE 1203
 Full Marks: 70

Course Title: Discrete Mathematics
 Time: 3 Hours

[There are Seven (7) Questions. Answer any Five (5).]
 [Marks allotted are indicated in the right margin.]

1. a) Define 'exclusive disjunction of two propositions' and 'contrapositive of a conditional statement'. [2+1]

- b) What is a contingency? Show using a truth table whether the expression $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology or not. [1+3]

- c) Prove using laws of logical equivalences that [4]

$$(\neg q \wedge (\neg p \wedge q)) \wedge (p \vee p) \equiv F.$$

- d) Write down the domination and distributive laws of propositional logic. [3]

2. a) Translate the following English sentences into logical expressions. [2+2]

- i. Every student in this class has taken at least one computer science course.

- ii. All your friends are perfect.

- b) Write short notes on 'counter example method' and 'direct proof method'. [2+2]

- c) Prove that $\sqrt{2}$ is irrational using the method of proof by contradiction. [3]

- d) Differentiate between theorem, lemma and corollary. [3]

3. a) Use set builder notation and logical equivalences to show that [3]

$$\overline{A \cup B} = \overline{A} \cap \overline{B}.$$

- b) Prove using the laws of set operations that [4]

$$((A \cap B) \cup (B \cap C \cap A)) \cup (A \cap B') = A.$$

- c) Illustrate various aspects of set difference operation. [3]

- d) Prove using mathematical induction that $2 + 6 + 10 + \dots + (4n-2) = 2n^2$. [4]

4. a) Consider the following relations on $\{0, 1, 2, 3\}$: [6+1]

- i. $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$

- ii. $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$

- iii. $\{(0, 0), (1, 1), (1, 2), (2, 2), (2, 1), (3, 3)\}$

- iv. $\{(0, 0), (1, 1), (1, 3), (3, 2), (2, 2), (2, 3), (3, 1), (3, 3)\}$

- v. $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

Determine whether the relations are Reflexive, Symmetric, Anti-symmetric or Transitive. Which of these relations are equivalence relations?

- b) Let m be a positive integer greater than 1. Show that the following relation [4]
is an equivalence relation on the set of integers.

$$R = \{(a, b) \mid a \equiv b \pmod{m}\}.$$

- c) Differentiate between injective and surjective functions. [3]

- a) How many bit strings of length four are possible if they start with 1 or end [3] 5.
with 0 or have 1s in the two central places?

- b) How many identifiers of length at most 3 are possible under the following [4] 5.
conditions?

i. An identifier starts with an upper case or lower case letter.

ii. There are 30 reserved words of length 3, each containing only lowercase letters.

iii. Digits can also be used.

- c) Find the solution, up to a_9 , of the following recurrence relation. [4] 5.

$$a_n = 2a_{n-1} + a_{n-2}^2, \text{ for } n \in \mathbb{N}, n \geq 2, a_0 = 0, a_1 = -1.$$

- d) Define the pigeonhole principle. How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen? [1+2] 5.

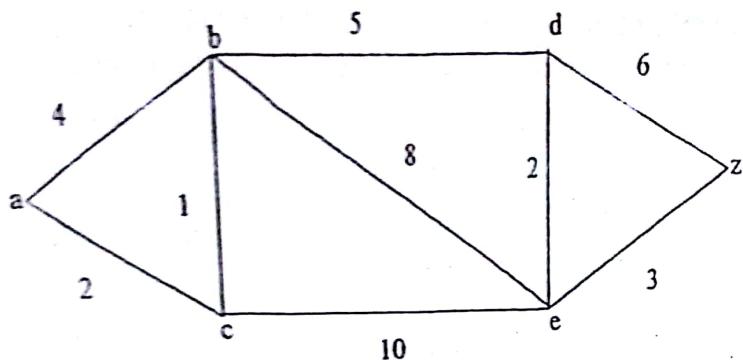
- ⑥ a) Write down the Handshaking theorem and prove it. [3] 5.

- b) Write a short note on bipartite graphs. Distinguish between Hamiltonian and Eulerian graphs. [1+2] 5.

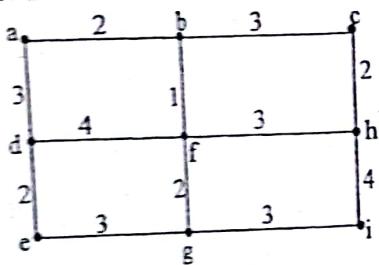
- c) The table below is the adjacency matrix of a directed graph. Find the in-degrees and out-degrees of each of the vertices, and also the number of loops of the graph using the entries in the table. [3+1] 5.

	1	2	3	4	5	6
1	0	1	0	1	0	1
2	0	0	1	1	1	0
3	0	0	1	0	1	0
4	0	0	1	0	1	0
5	1	0	0	0	1	1
6	0	1	0	0	1	0

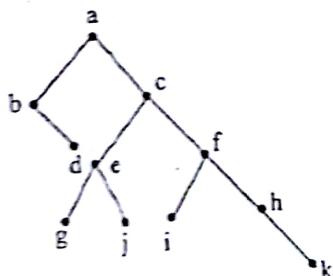
- d) For the following weighted graph determine the shortest path between a and z; also calculate its length. Draw all the steps to calculate the shortest path. [4]



- ⑦ a) Use Kruskal's algorithm to find a minimum spanning tree in the weighted graph shown below. [4]



- b) Show the three types of traversal of the following tree. [6]



- c) Use breadth-first search to find a spanning tree for the graph shown below. [4]
(Choose the vertex e for the root.)

