13.11.16

নির্ণয়ক

* Row & Colom সমান হবে।

Square হবে।

3×3

Squar 3^2

matrix

Row & colomn সমান নাও হতে পারে।

* Array of component/element.

* 1 Row থাকলে Row Maxtrix বলে

* 1 টি Colomn থাকলে Colomn Matrix বলে।

* Types of Matrix

* Matrix Operation

1. Addition $\Rightarrow$ Row colom সমান হলে
2. Subtraction $\Rightarrow$ করা যায়।
3. Multiplication $\longrightarrow$ $R_1 \times \boxed{C_1 = R_2}$ C_2
   সমান হবে
4. Inverts ~~Matrix~~

$AB \neq BA$ (Comunicative Law)

Transpose of $A \Rightarrow A^T$ or A'

$$A' = \begin{pmatrix} 3 & 7 & 9 \\ 2 & 5 & 1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$$

*

$$A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \\ 9 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$$

$$AB = \begin{bmatrix} 3+6 & -6+8 \\ 7+15 & -14+20 \\ 9+3 & -18+4 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 2 \\ 22 & 6 \\ 12 & -14 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 9 & 22 & 12 \\ 2 & 6 & -14 \end{bmatrix}$$

$$B^T = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} \quad A^T = \begin{pmatrix} 3 & 7 & 9 \\ 2 & 5 & 1 \end{pmatrix}$$

$$B^T A^T = \begin{bmatrix} 3+6 & 7+15 & 9+3 \\ -6+8 & -14+20 & -18+4 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 22 & 12 \\ 2 & 6 & -14 \end{bmatrix}$$

$\therefore B^T A^T = (AB)^T$

$|A|$ = determinate.

Inverse :- Square Matrix ২৫ ২১০।

$$A^{-1} = \frac{1}{|A|} (\text{adjoint of } A)$$

If $|A| = 0$ then A is called Singular Matrix.
whereas $|A| \neq 0$ then A is non-singular Matrix.

$$A = \begin{pmatrix} 2 & 7 & 1 \\ 0 & 5 & -1 \\ 3 & 2 & -2 \end{pmatrix}_{3\times 3}$$

$$|A| = \begin{vmatrix} 2 & 7 & 1 \\ 0 & 5 & -1 \\ 3 & 2 & -2 \end{vmatrix} = 2(-10+2) + 7(-3+0) + 1(0-15)$$

$$= -16 - 21 - 15 = -52$$

$$= 2(-10+2) - 7(-3-0) + 1(15-0)$$

$$= -16 - 21 \cancel{+} 15$$

"Co-factor $\Rightarrow (-1)^{r+c}$ minor" Co-factor Rule $\longrightarrow (-1)^{r+c}$

$$A = \begin{pmatrix} 2 & 7 & 1 \\ 0 & 5 & -1 \\ 3 & 2 & -2 \end{pmatrix}$$

$= (-2)$ 3rd row 3rd column

$(-1)^{3+3}$

$\Rightarrow (-1)^6$

$= 1$

$\therefore |A| = -52$

Here, the Cofactor of 2 is $+\begin{vmatrix} 5 & -1 \\ 2 & -2 \end{vmatrix}$

$= -8$

the co-factor of 7 is $-\begin{vmatrix} 0 & -1 \\ 3 & -2 \end{vmatrix} = -3$

" " " 1 " $+\begin{vmatrix} 0 & 5 \\ 3 & 2 \end{vmatrix} = -15$

" " " 0 " $-\begin{vmatrix} 7 & 1 \\ 2 & -2 \end{vmatrix} = 16$

" " " 5 " $+\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix}$ $= -4-3$

$= -7$

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the Co-factor of -1 is $-\begin{vmatrix} 2 & 7 \\ 3 & 2 \end{vmatrix} = -(4-21)$

$= +17$

" " of 3 is $+\begin{vmatrix} 7 & 1 \\ 5 & -1 \end{vmatrix} = -7-5$

$= -12$

" " " 2 is $-\begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} = -(-2)$

$= 2$

" " " -2 is $+\begin{vmatrix} 2 & 7 \\ 0 & 5 \end{vmatrix} = 10-0$

$= 10$

The Co-factor matrix of A is

$$\begin{pmatrix} -8 & -3 & -15 \\ 16 & -7 & 17 \\ -12 & 2 & 10 \end{pmatrix}$$

$\therefore$ The Adjoint matrix of A is = Transpose of Co-factor of A matrix.

$$\begin{pmatrix} -8 & 16 & -12 \\ -3 & -7 & 2 \\ -15 & 17 & 10 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{-52} \begin{pmatrix} -8 & 16 & -12 \\ -3 & -7 & 2 \\ -15 & 17 & 10 \end{pmatrix}$$

** linear equation $\leq \frac{1}{p}$

x এর power

*
$$2x - 3y + z = 5$$
$$x + 2y + z = 0$$
$$3x + 5y - 7z = 10$$

$$\underbrace{\begin{pmatrix} 2 & -3 & 1 \\ 1 & 2 & 1 \\ 3 & 5 & -7 \end{pmatrix}}_{A \ (3\times3)} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X \ (3\times1)} = \underbrace{\begin{pmatrix} 5 \\ 0 \\ 10 \end{pmatrix}}_{C \ (3\times1)}$$

A → ** Co-efficient matrix

→ x, y, z এর Co-efficient নিয়ে গঠিত Matrix কে বলে Co-efficient matrix.

$$\therefore A_{3\times3} X_{(3\times1)} = C_{(3\times1)}$$

NB: Matrix এর operation Constant হয় না।

$$A \quad X = C$$

$$\Rightarrow A^{-1}(AX) = A^{-1}C$$

$$\Rightarrow (A^{-1} \times A) * X = A^{-1}C$$

$$\because A^{-1} * A = I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{\therefore \; IX = A^{-1}C}$$

Now,

$$IX = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= X$$

Now,

$$IX = A^{-1}C$$

$$\therefore X = A^{-1}_{3\times 3} C_{(3\times 1)}$$

$$= K_{(3\times 1)}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}$$

* Crammer's Rules:-

$$x = \frac{D_x}{D}$$

$$y = \frac{D_y}{D}$$

$$z = \frac{D_z}{D}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{152}\begin{pmatrix} \frac{D_x}{D} \\ \frac{D_y}{D} \\ \frac{D_z}{D} \end{pmatrix}$$

If $A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & 2 & 1 \\ 3 & 5 & -7 \end{pmatrix}$

$2x - 3y + z = 5$

$x + 2y + y = 0$

$3x + 5y - 7z = 10$

$AX = C$

$$\begin{vmatrix} 2 & 5 & 1 \\ 1 & 0 & 1 \\ 3 & 10 & \end{vmatrix}$$

21.11.16

* RANK of a Matrix :-

Upper and lower triangular Matrix

$$\begin{pmatrix} 5 & 7 & 9 \\ 2 & 1 & 0 \\ 7 & 3 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 7 & 9 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \text{Upper triangular Matrix}$$

Diagonal এর নিচে [illegible] সংখ্যাগুলি Zero হয়ে থাকে

$$\begin{pmatrix} 5 & 0 & 0 \\ 2 & 1 & 0 \\ 7 & 3 & 2 \end{pmatrix} = \text{Lower triangular Matrix}$$

= Diagonal এর উপরে যদি zero হয় তাহলে।

Symmetry and Non (skew) symmetry Matrix:-

$$\begin{pmatrix} a & f & g \\ f & b & h \\ g & h & c \end{pmatrix} = \text{Symmetry Matrix}$$

= যদি কোন Diagonal এর সাপেক্ষে যদি এর অন্যান্য উপাদান গুলো সমান হয় তবে Symmetry Matrix বলে।

$$\begin{pmatrix} a & f & g \\ -f & b & h \\ -g & -h & c \end{pmatrix} =$$

যদি Diagonal এর সাপেক্ষে যদি যে কোনো উপাদান উল্টো হয় তাহলে Non/skew Symmetry Matrix বলে।

Orthogonal Matrix:-

If A is square and A is satisfies

$$A^TA = AA^T = I \text{ (Identy matris)}$$

then A is orthogonal.

$$\therefore A^T = A^{-1}$$

কোন Matrix এর Transpose and inverse সমান হলে তাকে Orthogonal matrix বলে।

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = zero / Null

$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ = non zero / [illegible] zero না হলে non zero.

Rank of matrix :-

By performing elementary transformation, any non-zero matrix A can be reduced to one of the following four forms, called the Normal or canonical form of A :

(i) I_r

(ii) $[I_r, O]$

(iii) $\begin{bmatrix} I_r \\ O \end{bmatrix}$

(iv) $\begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$

The number r is called the rank of A and we write $\underset{\uparrow \text{row}}{\rho}(A) = r$.

The form $\begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$ is called first canonical form of A.

RANK = The number of non-zero rows in upper triangular matrix.

$$\begin{pmatrix} 5 & 7 & 9 & 1 \\ -3 & 2 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{pmatrix}$$

তারপর Diagonal হয়ে গেল।

তাহলে বাকিগুলো Zero করলেই হবে।

$$\begin{vmatrix} 2 & 3 & \\ & 6 & 7 \\ 8 & 9 & 10 \end{vmatrix}$$

$$(-1)^{1+1} \cdot 2 \begin{vmatrix} 6 & 7 \\ 9 & 10 \end{vmatrix}$$

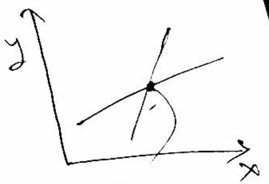
co-factor $= (-1)^{r+c} \cdot$ minor

* System of linear equation:-

→ Solution আছে

Consistance equation

$$2x + 5y = 9$$
$$3x + 2y = 7$$

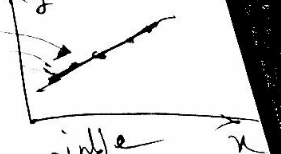


$$\begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$$

$$A * \cancel{X} = C$$

$|A| \neq 0$: Unique Solution = একটি x and y এর unique solution থাকবে।

* $2x + 5y = 9$
  $4x + 10y = 18$



Many or Infinite solution

$$2x + 5y = 9$$

$$y = \frac{9 - 2x}{5}$$ → free variable

NB: অসংখ্য Point এ Intersect করে তাই Infinite Solution

$x = 0,\ y = 9/5$ | $x = 1\ \ y = \frac{7}{5}$ | $x = 2,\ y = 1$

$ax + by = k$

$c_1 - c_2 =$ given

দুইটি Parallel equation

$*\ 2x + 5y = 9$

$4x + 10y = 7$

$|A| = 0$; But no soultion

So, It's called Inconsistency.

Homogeneous System of Linear equation

$$2x + 5y = 0$$
$$3x + 2y = 0$$

$$\begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$AX = O$$

zero হবে তখন

23.11.16

A' or A^T = Transpose

A^{-1} = Inverse Matrix

$2x + 3y = 0$

<u>Arthogonal → $AA^T = A^TA = I$</u>

then $A^T = A' = A^{-1}$

$$\begin{pmatrix} 5 & 7 & 1 \\ \cancel{8} & 9 & 1 \\ \cancel{2} & 1 & 7 \end{pmatrix}$$

→ $C_1' = 7C_1 - 3C_3$

$R_2' = 2$

$C_2' = C_2 * 7 - C_3 \times 1$

$R_2' = R_2 \times 5 - R_1 \times 2$

H.W Is unit and Identity matrix same?

আইগেন

* Characteristic roots or eigen values :→

Characteristic matrix :– Any square matrix A, $A-\lambda I$ matrix is called the characteristic matrix, where λ is scalar and I is unit matrix

Let, $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

$$\therefore A-\lambda I = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix} - \lambda I \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{pmatrix}$$

= characteristic matrix

• The determinant ~~$(A-\lambda I)$~~ $|A-\lambda I|$ when expanded will give a polynomial which we call as characteristic polynomial of matrix A.

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)(6-5\lambda+\lambda^2-2) - 2(2-\lambda-[illegible]) + 1(2-3+\lambda)$$

$$= \lambda^3 - 7\lambda^2 + 11\lambda - 5$$

The equation $|A-\lambda I| = 0$ is called the characteristic equation.

বা: $\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

* The roots of ~~charactoristic~~ characteristic equation are called characteristic roots of matrix A. / or ~~Ideal value~~ eigen vale

eg: $\lambda^3 - 7\lambda^2 + 11\lambda - 50$

$\Rightarrow (\lambda-1)(\lambda-1)(\lambda-5) = 0$

$\therefore \lambda = 1, 1, 5.$

30.11.16

V.V.I

* **Eigenvalues :-**

Properties of eigen values :-

1. Any square matrix A and its transpose A' have the same eigenvalues

2. The sum of the eigen values of matrix is equal to the trace of the matrix.

* Note :- trace = Matrix এর Diagonal element গুলোর sumition.

3. The product of the eigenvalue of a matrix A is equal to the determinate of A

4. If $\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_n$ are the eigenvalues of A, then the eigenvalues of
(i) kA are $k\lambda_1, k\lambda_2, k\lambda_3, \cdots, k\lambda_n$
(ii) A^m are $\lambda_1^m, \lambda_2^m, \lambda_3^m, \cdots, \lambda_n^m$
(iii) A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \cdots, \frac{1}{\lambda_n}$

* System of Solution
* 217, Page, System of Liner equal
47, 48, 49

~~The Inverse matri~~
* System of Liner equation.
* eigen value For [illegible]
* Rank. → elementary Transmiformant

CAYLEY-HAMILTON Theorem:

Statement :→ Every square matrix satisfies its own characteristic equation.

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

Step-1 :→

$$\therefore A - \lambda I = \begin{pmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{pmatrix}$$

charecteris matrix

characteristic polynomial.

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix}$$

$$= \lambda^3 - 7\lambda^2 + 11\lambda - 5$$

$\therefore \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$ characteristic equation

→ এর মান গুলো কে বলে eigen value.

From CAYLEY - HAMILTON Theorem অনুযায়ী,

$$A^3 - 7A^2 + 11A - 5I = 0 \quad \text{(1)}$$

A^{-1} নির্ণয় করার নিয়ম।

→ (1) $\times A^{-1} \Rightarrow$

$$A^2 - 7A + 11I - 5A^{-1} = 0$$

$$\therefore A^{-1} = \frac{1}{5}(A^2 - 7A + 11I)$$

* Characteristic vectors or Eigen vectors :→

Let A be $n \times n$ square matrix and Y and X are two non-zero column vectors such that

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$\therefore Y = AX \Rightarrow$ A transform vector X to vector Y. Two vectors X and Y have the same direction. Here we have to determine those ~~ve~~ vectors X whose images Y are given by $Y = \lambda X$.

* Corresponding to each charectistic root λ we have ~~a zero~~ corresponding non-zero vector X which satisfies the equation $|A-\lambda I|X = 0$. The non-zero vector X is called Eigenvectors.

$$\underbrace{|A-\lambda I|}_{\text{eigen value}} \underbrace{X}_{\text{eigen vector}} = 0$$

Q; Find the eigenvalues and eigenve[ctors] of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}_{3\times 3}$

$$A - \lambda I = \begin{pmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{pmatrix}$$

$$\therefore |A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix}$$

$$= (1-\lambda)\{(2-\lambda)(3-\lambda)-2\} - 1(2-4+2\lambda)$$

$$= (1-\lambda)\{(6-5\lambda-\lambda^2)-2\} - (-2+2\lambda)$$

$$= (1-\lambda)(4-5\lambda-\lambda^2) - (-2+2\lambda)$$

$$= 4-5\lambda-\lambda^2-4\lambda+5\lambda^2+\lambda^3+2\cancel{+}2\lambda$$

$$= \lambda^3+4\lambda^2-11\lambda+6$$

So, when $\lambda = 1$

$\therefore$ Eigen vector

$$X_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

Similarly,
এখানে $\lambda_1 = 2$ ও $\lambda = 3$ এর জন্য Eigen vector
বের করতে হবে।

$\boxed{\text{H.W: Properties of eigen vectors}}$.

Say, $X_2 = \begin{pmatrix} 5 \\ 7 \\ -1 \end{pmatrix}$

$X_3 = \begin{pmatrix} 0 \\ 1 \\ 7 \end{pmatrix}$

অনুরূপভাবে $\lambda = 2$ ও
$\lambda = 3$ এর
জন্য এই মানগুলো আসবে

$\therefore$ Model matrix, $P = (x_1, x_2, x_3)$

$$= \begin{pmatrix} 5 & 5 & 0 \\ -5 & 7 & 1 \\ 0 & -1 & 7 \end{pmatrix}$$

$\therefore$ Now

$$P^{-1}AP = \begin{pmatrix} \end{pmatrix} \begin{pmatrix} \end{pmatrix} \begin{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

= "diagonal matrix"

And এই diagonal matrix এর diagonal এর
মান গুলোই হবে diagonal matrix.

$\therefore |A - \lambda I| = 0$

$\Rightarrow \lambda^3 + 4\lambda^2 - 11\lambda + 6 = 0$

$\Rightarrow \lambda^3 - \lambda^2 + 5\lambda^2 - 5\lambda - 6\lambda + 6 = 0$

$\Rightarrow (\lambda^2 - 5\lambda - 6)(\lambda - 1) = 0$

$\Rightarrow (\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$

$\therefore \lambda = 1, 2, 3$ "Eigenvalues"

Now, $(A - \lambda I)\lambda = 0$

$$\begin{pmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ --- (I)}$$

If, $\lambda = 1$,

$$\begin{pmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$\Rightarrow -x_3 = 0$ --- (i)

$x_1 + x_2 + x_3 = 0$ --- (ii)

$2x_1 + 2x_2 + 2x_3 = 0$ --- (iii)

Homogeneous linear equation

যেহেতু (ii) ও (iii) একই equation

In (ii) put, $x_3 = 0$

$\therefore x_1 + x_2 = 0$

$\therefore x_1 = -x_2$

Let, $x_1 = 1$ → Infinite solution করে নিলাম

$\therefore x_2 = -1$

* Matrix Polynomial :-

$$f(x) = x^3 + 3\cancel{x} + 1$$

$$f(A) = A^3 + 3A + I$$

$$A^3 - 7A^2 + 11A - 5I$$