☐ Every Composite number has a prime divisor. :

(Alternatively: You can Prove the Next Theorem: (every integer is either prime or Product of primes) - which automatically proves the above theorem. But you will get penalty in marks!!!)

Proof by contradiction:

Suppose, there exists a number which is composite but doesn't have any prime divisor. Let n,

**Informal: Divisors usually appear in pairs ... Suppose: divisors of n = {1, ..., a, ..., b, ..., n} where n= a*b
Now, either a or b are Prime (proved) OR if not (a and b composite), they will gradually break down to smaller numbers (a= p * q, b = r *s where p,q,r,s smaller) until we hit any prime, or go down upto very small primes like 2,3 ... Example: 120 = 10*12 = 2*5*3*4 = 2*5*3*2*2; 256 = 16*16 = 2*8*2*8 = 2*2*2*2*2*2*2*2**

be such smallest number **(this is a prime factorization which is an example of the next theorem)**

Because, n is a composite number, it has a divisor m greater than 1.

Now, m is also a composite number and is not prime according to induction. But, m is smaller than n which contradicts that n is such smallest number. So, no number can be found of such type (Proved).

Any positive integer >1 is either prime or product of primes;

Alternatively (but with marks penalty) You can just State the Fundamental Theorem of Arithmetic and prove => Every integer has a unique prime factorization !!!

$12 = 4 \times 3$
$= 2 \times 2 \times 3$

$$n = P_1 P_2 \ldots P_n = \prod_{k=1}^{n} P_k.$$

Basis: lowest integer greater than 1 is 2. 2 is prime

~~Let~~ Let, this statement is true for

any number between 2 & k+1

all numbers between $2 \le n \le k$.

we show ~~for k+1~~ that this statement is true for $n = k+1$.

if $n = k+1$ is itself prime, then we are done.

So n has a divisor 'a' such that 1n. Since divisors always appear in pairs, Let: n = k+1 = a * b, for some integers a,b

if n is not prime, then n is a composite number. Let $k+1 = a \cdot b$ for some integers a, b. But

$2 \le a < k+1$ and $2 \le b < k+1$.

By induction, a, b are either prime or product of primes.

So, $k+1 = a \cdot b$ is also a product of primes. (Proved).