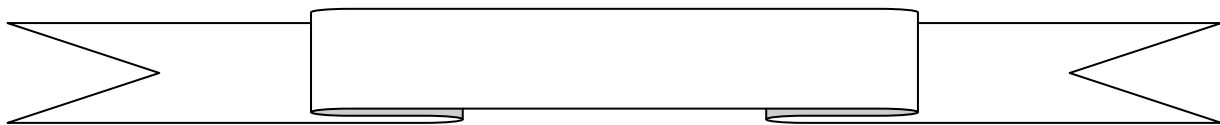




## CHAPTER 9

# SPREAD SPECTRUM



**Spread Spectrum** is an important form of encoding for wireless communications. In spread spectrum (SS), we combine signals from different sources to fit into a larger bandwidth, but our goals are to prevent **eavesdropping and jamming**.

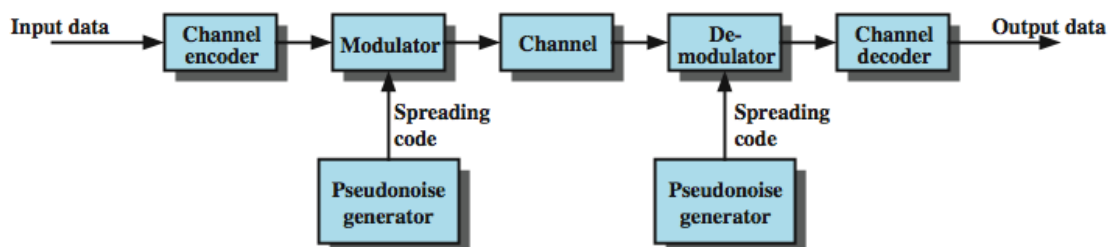


Figure 9.1 General Model of Spread Spectrum System

A signal that occupies a bandwidth of  $B$ , is spread out to occupy a bandwidth of  $B_{ss}$ . All signals are spread to occupy the same bandwidth  $B_{ss}$ . The bandwidth is wider after the signal has been encoded using spread spectrum. ( $B_{ss} \gg B$ ).

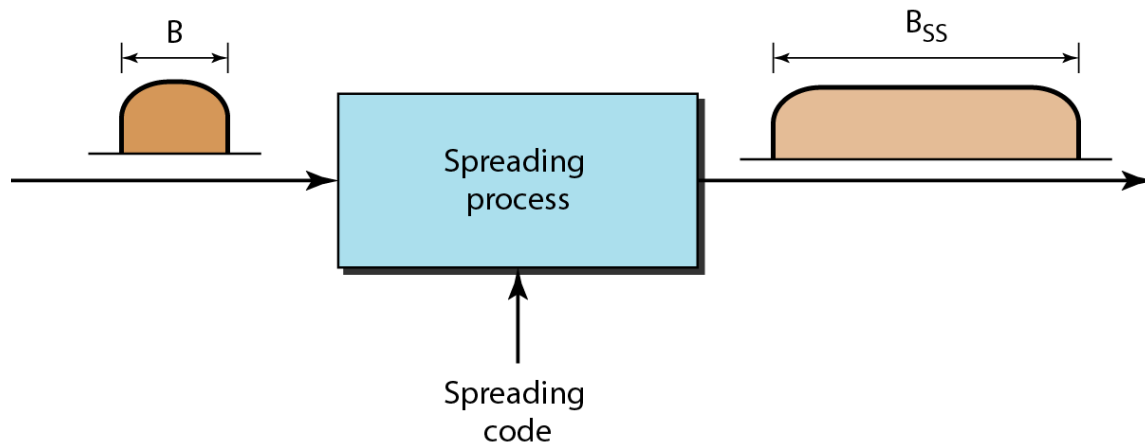


Figure 9.2 Spread Spectrum

**Example:**

9.1 Assume we wish to transmit a 56-kbps data stream using spread spectrum.

- a. Find the channel bandwidth required to achieve a 56-kbps channel capacity when  $\text{SNR} = 0.1, 0.01, \text{ and } 0.001$ .
- b. In an ordinary (not spread spectrum) system, a reasonable goal for bandwidth efficiency might be **1 bps/Hz**. That is, to transmit a data stream of 56 kbps; a bandwidth of 56 kHz is used. In this case, what is the minimum SNR that can be endured for transmission without appreciable errors? Compare to the spread spectrum case.

**Solution:**

a)  $C = B \log_2 (1 + \text{SNR})$ .

For  $\text{SNR} = 0.1$ ,  $B = \boxed{0.41 \text{ MHz}}$

For  $\text{SNR} = 0.01$ ,  $B = \boxed{3.9 \text{ MHz}}$

For  $\text{SNR} = 0.001$ ,  $B = \boxed{38.84 \text{ MHz}}$

Thus, to achieve the desired SNR, the signal must be spread so that 56 KHz is carried in *very large bandwidths*.

- b. For  $1 \text{ bps/Hz} = (C / B)$ , the equation  $C = B \log_2 (1 + \text{SNR})$  becomes

$$\log_2 (1 + \text{SNR}) = 1.$$

$\boxed{\text{SNR} = 1}$ . Thus a far higher SNR is required without spread spectrum.

## Spread Spectrum Approaches are:

1. Frequency-hopping spread spectrum.
2. Direct sequence spread spectrum.

**Frequency-hopping spread spectrum:** is a form of spread spectrum in which the signal is broadcast over a seemingly random series of radio frequencies, hopping from frequency to frequency at fixed intervals.

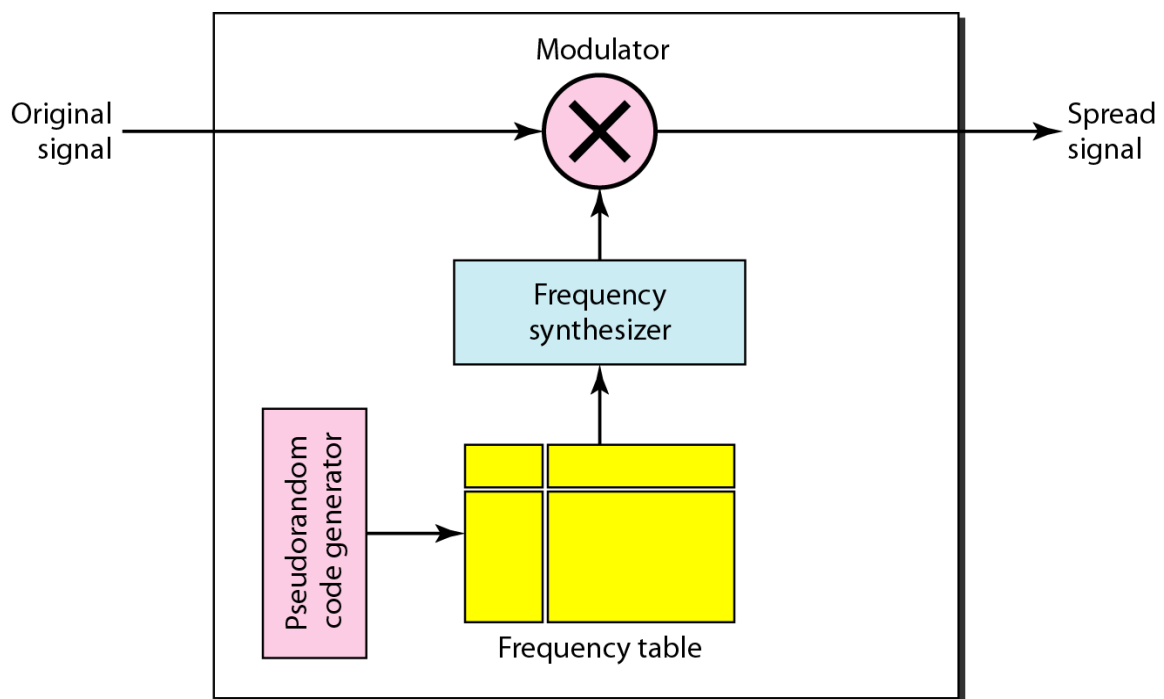


Figure 9.3 Frequency hopping spread spectrum (FHSS)

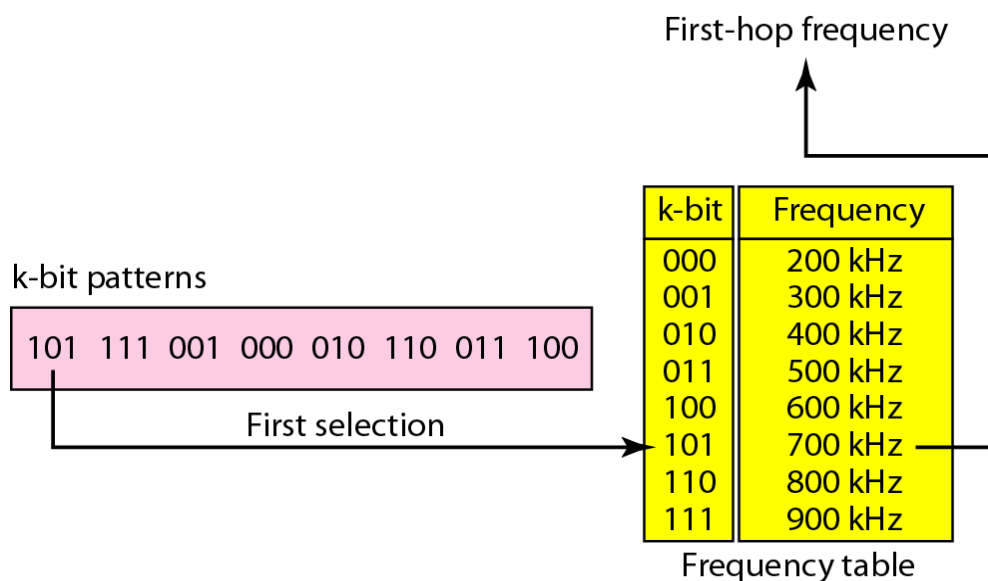


Figure 9.4 Frequency selection in FHSS

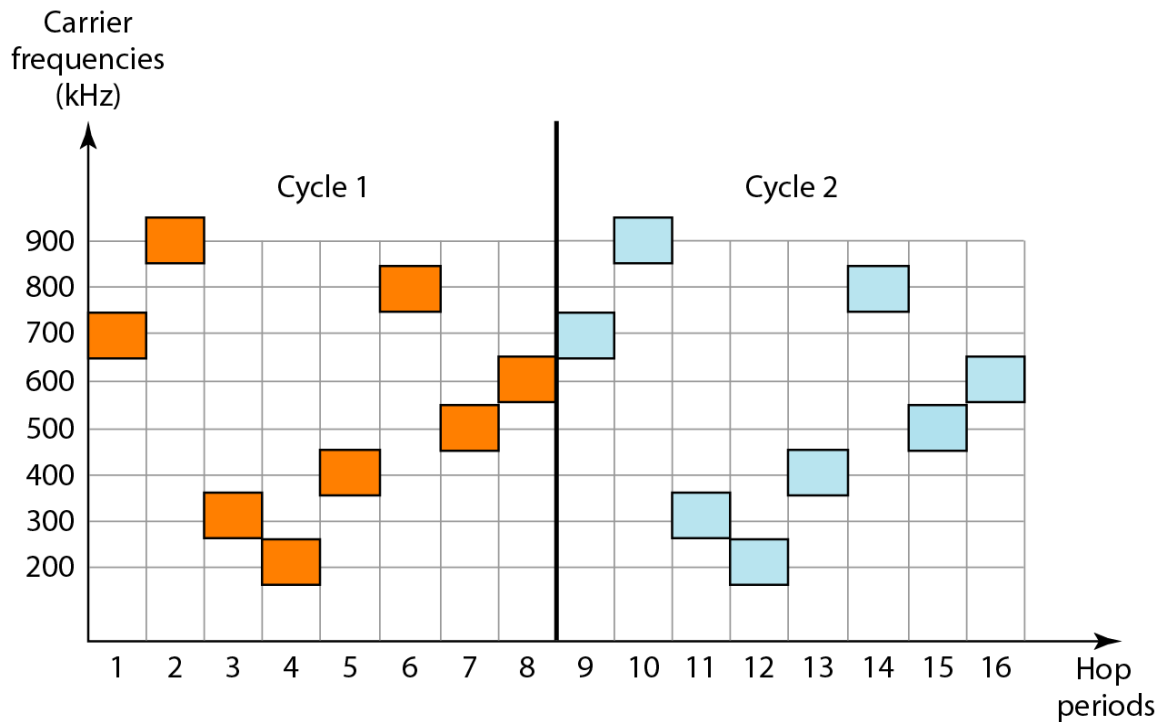


Figure 9.5 FHSS cycles

### Examples:

1. What is the minimum number of bits in a PN sequence if we use FHSS with a channel bandwidth of  $B = 4$  KHz and  $B_{ss} = 100$  KHz?

### Solution:

The number of hops =  $100 \text{ KHz} / 4 \text{ KHz} = 25$ .

So we need  $\log_2 25 = 4.64 \approx \boxed{5 \text{ bits}}$

2. An FHSS system uses a 4-bit PN sequence. If the bit rate of the PN is 64 bits per second, answer the following questions:

- What is the total number of possible hops?
- What is number of finished cycles per time of PN?

### Solution:

a.  $2^4 = \boxed{16 \text{ hops}}$

b.  $(64 \text{ bits/s}) / 4 \text{ bits} = \boxed{16 \text{ cycles/s}}$

3. A pseudorandom number generator uses the following formula to create a random series:

$$N_{i+1} = (5 + 7N_i) \bmod 17-1$$

In which  $N_i$  defines the current random number and  $N_{i+1}$  defines the next random number. The term mod means the value of the remainder when dividing  $(5 + 7N_i)$  by 17. Find the random numbers.

**Solution:**

$$i=0 \longrightarrow N_1 = (5 + 7N_0) \bmod 17-1 = 11, \text{ where } N_0 = 1$$

$$i=1 \longrightarrow N_2 = (5 + 7*11) \bmod 17-1 = 13$$

$$i=2 \longrightarrow N_3 = (5 + 7*13) \bmod 17-1 = 10$$

$$i=3 \longrightarrow N_4 = (5 + 7*10) \bmod 17-1 = 6$$

$$i=4 \longrightarrow N_5 = (5 + 7*6) \bmod 17-1 = 12$$

$$i=5 \longrightarrow N_6 = (5 + 7*12) \bmod 17-1 = 3$$

$$i=6 \longrightarrow N_7 = (5 + 7*3) \bmod 17-1 = 8$$

$$i=7 \longrightarrow N_8 = (5 + 7*8) \bmod 17-1 = 9$$

$$i=8 \longrightarrow N_9 = (5 + 7*9) \bmod 17-1 = -1, \text{ Now we stop because we have a negative value.}$$

**Random numbers** are 11, 13, 10, 6, 12, 3, 8, 9

4. (Q9.2) An FHSS system employs a total bandwidth of  $W_s = 400$  MHz and an individual channel bandwidth of 100 Hz. What is the minimum number of PN bits required for each frequency hop?

**Solution:**

$$\# \text{ of hops} = (400 * 10^6) / 100 = 4 * 10^6$$

$$\text{The minimum number of PN bits} = \lceil \log_2 (4 \times 10^6) \rceil = \span style="border: 1px solid black; padding: 2px;">22 \text{ bits}$$

### Slow and Fast FHSS

- Commonly use multiple FSK (MFSK), MFSK uses  $M = 2^L$  different frequencies to encode the digital input  $L$  bits at a time.
- Total MFSK bandwidth  $W_d = 2^L f_d$
- Total FHSS bandwidth  $W_s = 2^k W_d$

- have frequency shifted every  $T_c$  seconds
- duration of signal element is  $T_s$  seconds

**Slow FHSS** = multiple signal elements per hop; has  $T_c \geq T_s$ , ( $T_c = 2T_s = 4T$ ).

**Fast FHSS** = multiple hops per signal element; has  $T_c < T_s$ , ( $T_s = 2T_c = 2T$ ).

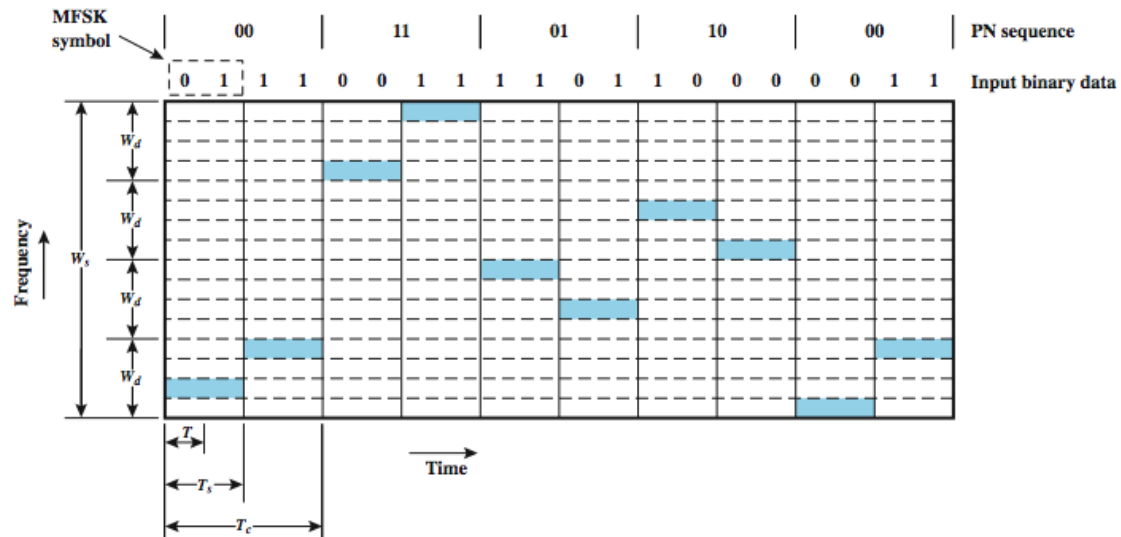


Figure 9.6 Slow MFSK FHSS( $M = 4$ ,  $k = 2$ )

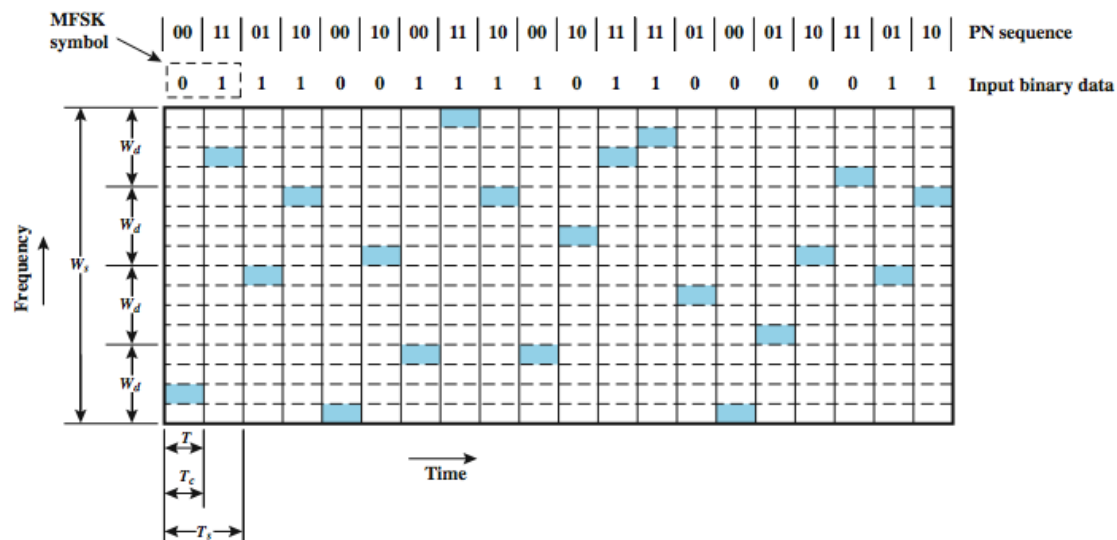


Figure 9.6 Fast MFSK FHSS ( $M = 4$ ,  $k = 2$ )

### Examples:

1. (Q9.4) The following table illustrates the operation of FHSS system for one complete period of the PN sequence.

Time	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Input data	0	1	1	1	1	1	1	0	0	0	1	0	0	1	1	1	1	0	1	0
Frequency	$f_1$		$f_3$		$f_{23}$		$f_{22}$		$f_8$		$f_{10}$		$f_1$		$f_3$		$f_2$		$f_2$	
PN sequence	001				110				011				001				001			

- What is the period of the PN sequence, in terms of bits in the sequence?
- The system makes use of a form of FSK. What form of FSK is it?
- What is the number of bits per signal element (symbol)?
- What is the number of FSK frequencies?
- What is the length of a PN sequence per hop?
- Is this a slow or fast FH system?
- What is the total number of possible carrier frequencies?
- Show the variation of the base, or demodulated, frequency with time.

**Solution:**

- Period of the PN sequence is 15
- MFSK
- $L = 2$
- $M = 2^L = 2^2 = 4$
- $k = 3$
- Slow FHSS
- $2^k = 2^3 = 8$
- We have 4 FSK Frequencies ( $f_0 - f_3$ )  
for  $f_{23} \longrightarrow 23 \bmod 4 = 3 \longrightarrow$  So  $f_{23} = f_3$

<b>Time</b>	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
<b>Input data</b>	0	1	1	1	1	1	1	0	0	0	1	0	0	1	1	1	1	0	1	0
<b>Frequency</b>	$f_1$		$f_3$		$f_3$		$f_2$		$f_0$		$f_2$		$f_1$		$f_3$		$f_2$		$f_2$	

- 2. (Q9.5)** The following table illustrates the operation of FHSS system using the same PN sequence as previous questions

<b>Time</b>	0	1	2	3	4	5	6	7	8	9	10	11
<b>Input data</b>	0	1	1	1	1	1	1	0	0	0	1	0
<b>Frequency</b>	$f_1$	$f_{21}$	$f_{11}$	$f_3$	$f_3$	$f_3$	$f_{22}$	$f_{10}$	$f_0$	$f_0$	$f_2$	$f_{22}$
<b>PN sequence</b>	001	110	011	001	001	001	110	011	001	001	001	110

<b>Time</b>	12	13	14	15	16	17	18	19
<b>Input data</b>	0	1	1	1	1	0	1	0
<b>Frequency</b>	$f_9$	$f_1$	$f_3$	$f_3$	$f_{22}$	$f_{10}$	$f_2$	$f_2$
<b>PN sequence</b>	011	001	001	001	110	011	001	001

- What is the period of the PN sequence?
- The system makes use of a form of FSK. What form of FSK is it?
- What is the number of bits per signal element?

- What is the number of FSK frequencies?
- What is the length of a PN sequence per hop?
- Is this a slow or fast FH system?
- What is the total number of possible carrier frequencies?
- Show the variation of the base, or demodulated, frequency with time.

**Solution:**

- Period of the PN sequence is 15
- MFSK
- $L = 2$
- $M = 2^L = 2^2 = 4$
- $k = 3$
- FAST FHSS
- $2^k = 2^3 = 8$
- We have 4 FSK Frequencies ( $f_0 - f_3$ )  
for  $f_{22} \longrightarrow 22 \bmod 4 = 2 \longrightarrow$  So  $f_{22} = f_2$

Time	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Input data	0	1	1	1	1	1	1	0	0	0	1	0	0	1	1	1	1	0	1	0
Frequency	$f_1$		$f_3$		$f_3$		$f_2$		$f_0$		$f_2$		$f_1$		$f_3$		$f_2$		$f_2$	

**Direct sequence spread spectrum:** is a form of spread spectrum in which each bit in the original signal is represented by multiple bits in the transmitted signal, using a spreading code.

For an N-bit spreading code, the bit rate after spreading (usually called the chip rate) is N times the original bit rate.

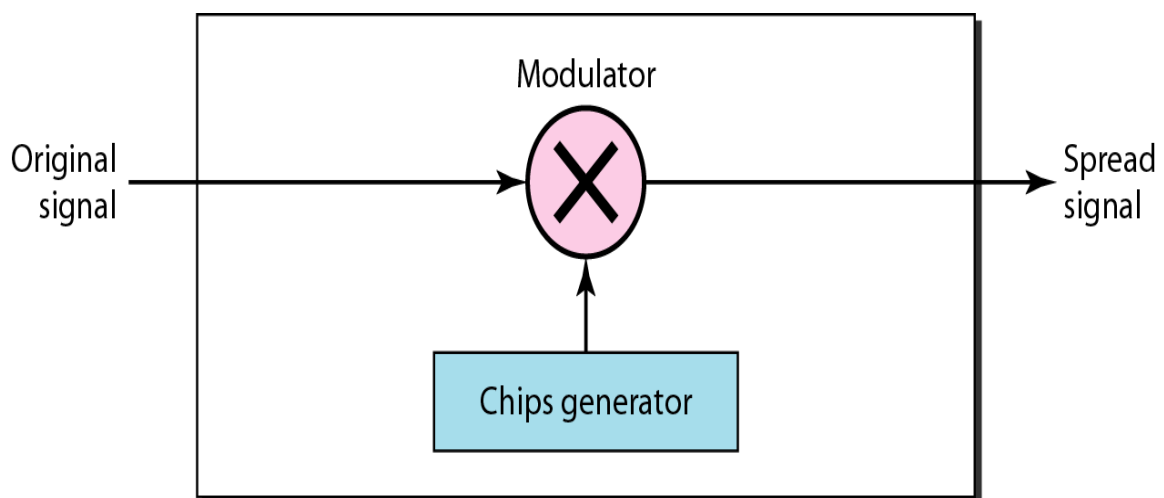


Figure 9.7 Direct Sequence Spread Spectrum



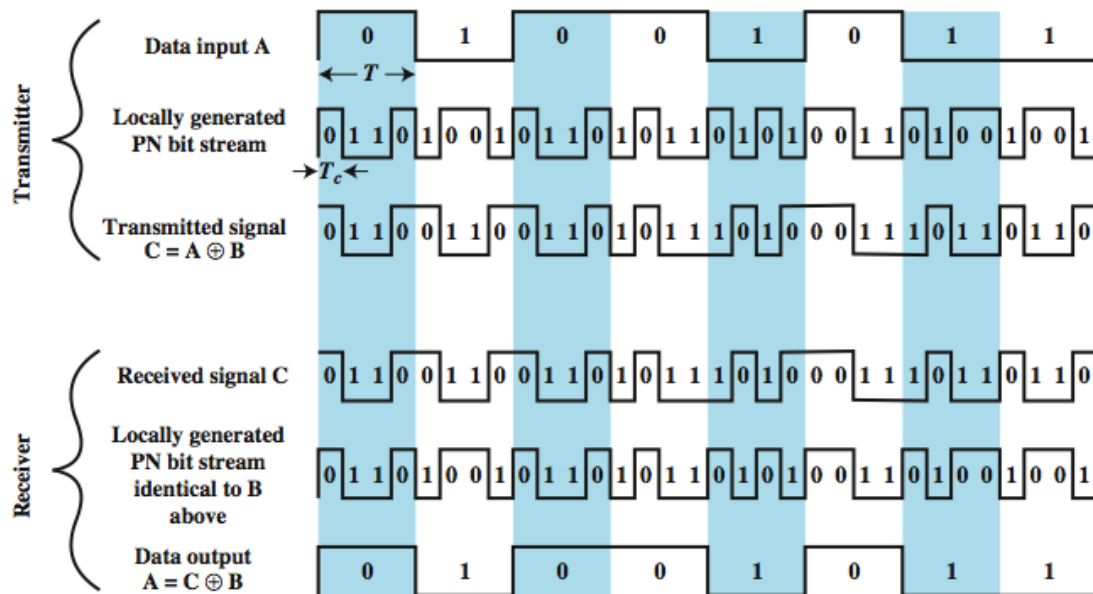


Figure 9.8 Direct Sequence Spread Spectrum Example

In Figure 9.9, the spreading code is 11 bits chip (Barker chip) with the pattern 10110111000 (in this case). If the original signal rate is  $N$ , the rate of the spread signal is  $11N$ . This means that the required bandwidth for the spread signal is 11 times larger than the bandwidth of the original signal.

(Spread signal = Original signal \* Spreading code)

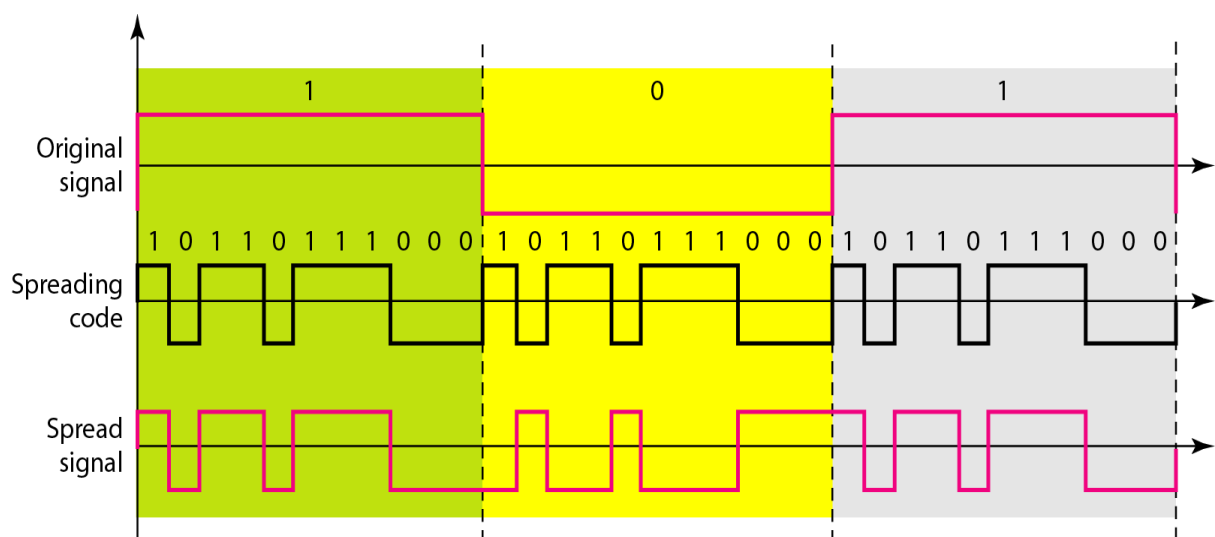


Figure 9.9 DSSS example

### Example:

We have a digital medium with a data rate of 10 Mbps. How many 64-kbps voice channels can be carried by this medium if we use **DSSS** with the **Barker** sequence?

### Solution:

The Barker chip is 11 bits, which means that it increases the bit rate 11 times.

A voice channel of 64 kbps needs  $\longrightarrow 11 \times 64 \text{ kbps} = 704 \text{ kbps}$ .

This means that the bandpass channel can carry:

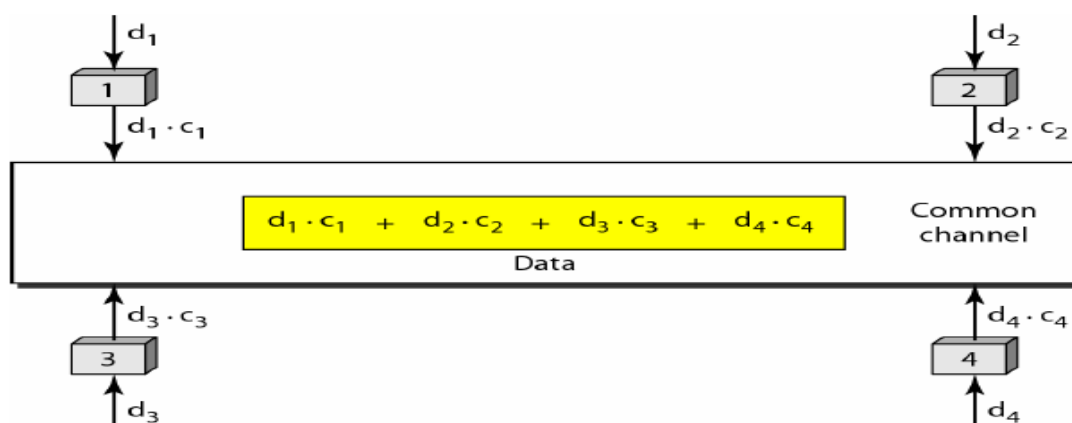
$$(10 \text{ Mbps}) / (704 \text{ kbps}) \approx \boxed{14 \text{ channels.}}$$

### Code Division Multiple Access (CDMA)

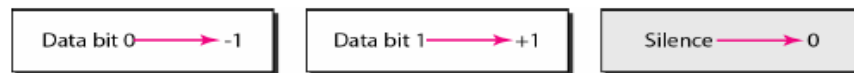
- a multiplexing technique used with spread spectrum
- given a data signal rate  $D$
- break each bit into  $k$  chips according to a fixed chipping code specific to each user
- resulting new channel has chip data rate  $kD$  chips per second
- can have multiple channels superimposed

Code division multiple access exploits the nature of spread spectrum transmission to enable multiple users to independently use the same bandwidth with very little interference.

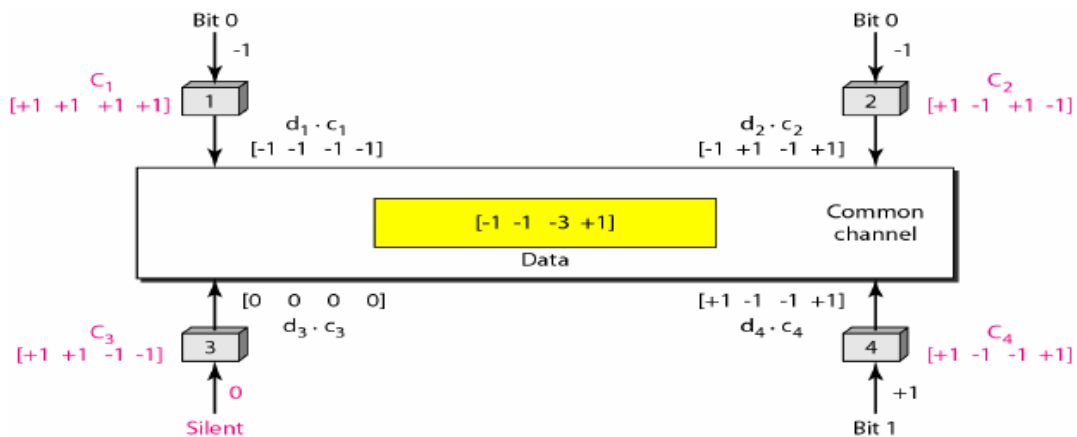
CDMA allows multiple users to transmit over the same wireless channel using spread spectrum. Each user uses a different spreading code. The receiver picks out one signal by matching the spreading code.



## Data representation



## Encoding and decoding



Now imagine station 3, which we said is silent, is listening to station 2. Station 3 multiplies the total data on the channel by the code for station 2, which is

$[+1 -1 +1 -1]$ , to get:

$$[-1 -1 -3 +1] \cdot [+1 -1 +1 -1] = -4/4 = -1 \dots \text{bit 0}$$

### Example:

Consider a CDMA system in which users A and B have codes  $(-1 \ 1 \ -1 \ -1 \ -1 \ -1)$  and  $(-1 \ -1 \ 1 \ 1 \ -1 \ -1)$  respectively.

(a) Show the output at the receiver if A transmits a data 1 and B does not transmit;

### Solution:

$$d_a \cdot c_a = +1 * [-1 \ 1 \ -1 \ -1 \ -1 \ -1] = [-1 \ 1 \ -1 \ -1 \ -1 \ -1] = \text{Data}$$

$$\text{Output at receiver} = c_a \cdot \text{Data} = [-1 \ 1 \ -1 \ -1 \ -1 \ -1] \cdot [-1 \ 1 \ -1 \ -1 \ -1 \ -1] = \boxed{8}$$

(b) Show the output at the receiver if A transmits a data 0 and B does not transmit;

### Solution:

$$d_a \cdot c_a = -1 * [-1 \ 1 \ -1 \ -1 \ -1 \ -1] = [1 \ -1 \ 1 \ 1 \ 1 \ 1] = \text{Data}$$

$$\text{Output at receiver} = c_a \cdot \text{Data} = [-1 \ 1 \ -1 \ -1 \ -1 \ -1] \cdot [1 \ -1 \ 1 \ 1 \ 1 \ 1] = \boxed{-8}$$

(c) Show the output at the receiver if A transmits a data bit 1 and B transmits a data bit 1. Assume the received power from both A and B is the same.

**Solution:**

A output (data = 1)	-1	1	-1	1	-1	1	-1	1	
B output (data = 1)	-1	-1	1	1	-1	-1	1	1	
Received	-2	0	0	2	-2	0	0	2	
Receiver codeword	-1	1	-1	1	-1	1	-1	1	
Multiplication	2	0	0	2	2	0	0	2	=8

(d) Show the output at the receiver if A transmits a data bit 0 and B transmits a data bit 1. Assume the received power from both A and B is the same.

**Solution:**

A output (data = 0)	1	-1	1	-1	1	-1	1	-1	
B output (data = 1)	-1	-1	1	1	-1	-1	1	1	
Received	0	-2	2	0	0	-2	2	0	
Receiver codeword	-1	1	-1	1	-1	1	-1	1	
Multiplication	0	-2	-2	0	0	-2	-2	0	=-8

(e) Show the output at the receiver if A transmits a data bit 1 and B transmits a data bit 0. Assume the received power from both A and B is the same.

**Solution:**

A output (data = 1)	-1	1	-1	1	-1	1	-1	1	
B output (data = 0)	1	1	-1	-1	1	1	-1	-1	
Received	0	2	-2	0	0	2	-2	0	
Receiver codeword	-1	1	-1	1	-1	1	-1	1	
Multiplication	0	2	2	0	0	2	2	0	=8

(f) Show the output at the receiver if A transmits a data bit 0 and B transmits a data bit 0. Assume the received power from both A and B is the same.

**Solution:**

A output (data = 0)	1	-1	1	-1	1	-1	1	-1	
B output (data = 0)	1	1	-1	-1	1	1	-1	-1	
Received	2	0	0	-2	2	0	0	-2	
Receiver codeword	-1	1	-1	1	-1	1	-1	1	
Multiplication	-2	0	0	-2	-2	0	0	-2	=-8

(g) Show the output at the receiver if A transmits a data bit 1 and B transmits a data bit 1. Assume the received power from B is twice the received power from A. This can be represented by showing the received signal component from A as consisting of elements of magnitude 1(+1, -1) and the received signal component from B as consisting of elements of magnitude 2(+2, -2).

**Solution:**

A output (data = 1)	-1	1	-1	1	-1	1	-1	1	
B output (data = 1)	-2	-2	2	2	-2	-2	2	2	
Received	-3	-1	1	3	-3	-1	1	3	
Receiver codeword	-1	1	-1	1	-1	1	-1	1	
Multiplication	3	-1	-1	3	3	-1	-1	3	=8

(h) Show the output at the receiver if A transmits a data bit 0 and B transmits a data bit 1. Assume the received power from B is twice the received power from A.

**Solution:**

A output (data = 0)	1	-1	1	-1	1	-1	1	-1	
B output (data = 1)	-2	-2	2	2	-2	-2	2	2	
Received	-1	-2	2	1	-1	-3	3	-1	
Receiver codeword	-1	1	-1	1	-1	1	-1	1	
Multiplication	1	-2	-2	1	1	-3	-3	-1	=-8