Date of Examination: 17/09/2017

AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
Department of Arts and Sciences
Program: B. Sc. in Computer Science and Engineering
Semester Final Examination, Spring 2017
Year: 1st Semester: 1st

Course No: MATH 1115 **Course Name: Mathematics I**
Time: 3 (three) hours **Full Marks: 70**

There are 07(seven) questions. Answer any 05 (five) questions, taking 03 (three) from Group-A and 02 (two) from Group-B. Marks allotted are indicated in the right margin.

Group-A

1. a. Define continuity and differentiability of a function $f(x)$ at $x=a$. Show that $f(x)=|x|$ is continuous but not differentiable at $x=0$. 7

   b. State the Leibnitz's theorem. By following the theorem prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$, where $y=e^{a\sin^{-1}x}$. 7

2. a. State the Rolle's and First Mean Value (FMV) theorems. Verify these for $f(x)=x^2-3x+2$ in [1, 2]. Show that FMV theorem can be expressed in the following alternative form, $f(a+h)=f(a)+hf'(a+\theta h)$, where $a\leq x\leq a+h,\ \ 0<\theta<1$. 7

   b. Define tangent and normal of a curve with figure. Prove that the segment (between the coordinate's axes) of a tangent of astroid $x^{2/3}+y^{2/3}=a^{2/3}$ is a constant length. 7

3. a. (i) State the Euler's theorem for homogeneous function. By applying the theorem prove that $x\frac{\partial F}{\partial x}+y\frac{\partial F}{\partial y}=\frac{1}{2}\tan F$, where $F(x,y)=\sin^{-1}\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$. 3.5

   (ii) State the L'Hospital's rule. By applying the rule prove that $\lim_{x\to 1} x^{1/(1-x)}=1/e$. 3.5

   b. Define the maximum and minimum of a function. Find the dimension of the largest rectangle which can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 7

4. a. Define the curvature of a curve with figure and show that the radius of curvature of the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ of the curve $x^3+y^3=3axy$ is $\frac{-3a}{8\sqrt{2}}$. 7

   b. Define asymptote of a curve with example. Find all the asymptotes of the curve $4(x^4+y^4)-17x^2y^2-4x(4y^2-x^2)+2(x^2-2)=0$. 7

Group-B

5. a. Show that the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two parallel lines if $a:h=h:b=g:f$. Also show that the distance between them is $2\sqrt{\dfrac{(g^2-ac)}{a(a+b)}}$. 7

   b. Reduce the general equation of second degree $4x^2-4xy+y^2-8x-6y+5=0$ to the standard form. Find its any four properties. 7

6. a. Define director circle. Find the co-ordinates of the limiting point of the co-axial system determined by circles $x^2+y^2-2x+8y+11=0$ and $x^2+y^2+4x+2y+5=0$. 7

   b. Define direction cosines and direction ratios of a line. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\dfrac{4}{3}$. 7

7. a. Define a plane and write the conditions of perpendicularity and parallelism of two planes. Find the equation of the plane passing through the points (1, -2, 2), (-3, 1, -2) and perpendicular to the plane $2x+y-z+6=0$. 7

   b. Write the general equation of a straight line in three dimensions. Find the equation of line perpendicular to both the lines $\dfrac{x-1}{1}=\dfrac{y-1}{2}=\dfrac{z+2}{3}$, $\dfrac{x+2}{2}=\dfrac{y-5}{-1}=\dfrac{z+3}{2}$ and passing through their intersection. 7

$r+1 = 2r_1-2$

$r = 2r_1-3$ $\quad 3r_1-9$

$\Rightarrow 5r_1 = 10$

$r_1 = 2$