Department of Arts and Sciences

Program: B.Sc in Computer Science and Engineering

1st Semester of 2nd Year

Course No: Math2101 Course Title: Mathematics-III

Time: 03 (three) hours Full Marks: 70

There are 07 (seven) questions. Answer any 05 (five) questions, taking at least 01(one) from each group.

The figures of the right margin indicate full marks for the respective questions

Group-A

1 (a) Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$. Interpret the result geometrically and deduce that $\left|\alpha + \sqrt{\alpha^2 - \beta^2}\right| + \left|\alpha - \sqrt{\alpha^2 - \beta^2}\right| = |\alpha + \beta| + |\alpha - \beta|$ [07]

(b) Define harmonic function. If $f(z)$ is an analytic function of Z in any domain then prove that

$$\left(\frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2}\right)|f(z)|^n = n^2|f'(z)|^2|f(z)|^{n-2}$$ [07]

2 (a) Define conformal mapping. Find the image of the circle $|z - i| = 1$ under the transformation $w = \frac{1}{z}$. [07]

(b) State Cauchy's integral formula for the derivatives of an analytic function. Find the Taylor's expansion for $f(z) = \cos z$ in power of $z = \frac{\pi}{4}$ [07]

3 (a) State Cauchy's integral theorem. Evaluate the integral $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$ where C is the circle $|z| = 4$ [07]

(b) Obtain the Laurent's expansion for $f(z) = \frac{9z^2 - 4z + 1}{(z-1)(2z-1)(z+2)}$ in the region $1 < |z - 1| < 3$. [07]

Group-B

4 (a) Define Laplace transform of the unit step function. Show that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ using Laplace transform. [7]

(b) Define Laplace transform. Find the Laplace transform of $f(t) = \begin{cases} \cos t, & 0 < t < 2\pi \\ 0, & t > 2\pi \end{cases}$ [7]

(ii) Find the relationship status and its strength between the variables. [03]

6. (a) A soft-drink machine is regulated so that it discharges an average of 200 ml. per cup. If the amount of discharge from the machine is symmetrically distributed with a variance of 225 ml^2,

(i) What fraction of the cups will contain more than 235 ml.? [02]

(ii) What is the percentage that a cup contains from 175 to 225 ml.? [02]

(b) The following table shows the number of constructions (in a year) of some famous construction companies. Represent the data by a suitable diagram. [03]

Company's Names	Frazer Corp. Texas	Kansas USA	Harold Texas	GBR Corp.	Austin Ind.
No. of constructions	11	8	12	9	7

7. BMT, Inc., manufactures performance equipment for cars used in various types of racing. It has gathered the following information on the number of models of engines in different size (in cubic inches) used in the racing market it serves:

Engine size	101 – 150	151 – 200	201 – 250	251 – 300	301 – 350	351 – 400	401 – 450	451 – 500
Number of models	1	7	7	8	17	16	11	7

Find the mean engine size of the models using geometric mean method and also find the variation among the sizes. [03+04]

Table. Values of the standard normal distribution function

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\,du = P(Z \le z)$$

z	0	1	2	3	4	5	6	7	8	9
-3.	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
-2.9	.0019	.0018	.0017	.0017	.0016	.0015	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0020	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.016	.0132	.0129	.0216	.0022	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0102	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0238	.0233
-1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0300	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0570	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
-.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3516	.3121
-.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Values of the standard normal distribution function

z	0	1	2	3	4	5	6	7	8	9
+.0	.5000	.5040	.5080	.5120	.5160	.5190	.5239	.5279	.5319	.5359
+.1	.5398	.5438	.5478	.5517	.5557	.5596	.5363	.5675	.5714	.5753
+.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
+.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
+.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
+.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
+.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
+.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7974	.7823	.7852
+.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
+.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
+1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
+1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
+1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
+1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
+1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
+1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9430	.9441
+1.6	.9452	.9563	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
+1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
+1.8	.9641	.9648	.9656	.9664	.9671	.9678	.9686	.9693	.9700	.9706
+1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9762	.9767
+2.0	.9772	.9778	.9738	.9788	.9793	.9793	.9805	.9808	.9812	.9817
+2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
+2.2	.9861	.9864	.9868	.9871	.9874	.9878	.9881	.9884	.9887	.9890
+2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
+2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
+2.5	.9938	.9940	.9941	.9943	.8845	.9946	.9948	.9949	.9951	.9952
+2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
+2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
+2.8	.9974	.9975	.9967	.9977	.9977	.9978	.9979	.9979	.9980	.9981
+2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.	.9987	.990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Note 1 : If a normal variable X is not "standard", its values must be "standardized" : $Z = (X - \mu) / \sigma$. That is, $P(X \leq x) = \Phi\left(\frac{x - \mu}{\sigma}\right)$.

Note 2 : For $z \geq 4$. $\Phi(x) = 1$ to four decimal places ; for $z \leq -4$. $\Phi(z) = 0$ to four decimal places. Note 3 : The entires opposite z = 3 are for 3.0, 3.1, 3.2, etc.

Ahsanullah University of Science and Technology

Final Examination of Fall Semester 2013

Department of Arts and Sciences

Program: B.Sc in Computer Science and Engineering

1st Semester of 2nd Year

Course No: Math 2101 Course Title: Mathematics-III

Time: 03(three) hours Full Marks: 70

There are 7 (Seven) questions. Answer 5 (Five) questions, taking at least 1(one) from each group.

The figures of the right margin indicate full marks for the respective questions

Group-A

1(a) Determine the set of points in the complex plane which satisfy the inequality $|z-4| \le |z+4|$ and sketch it. [04]

(b) Show that: $|z+\sqrt{2}| \ge |\mathrm{Re}(z)| + |\mathrm{Im}(z)|$, where z is any complex number. [04]

(c) If $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$ then find the analytic function $f(z)$. [06]

2(a) Define conformal mapping. Find the image of the circle $|z-3i| = 3$ under the transformation $w = \dfrac{1}{z}$. [07]

(b) If $f(a) = \displaystyle\int_C \frac{3z^2+7z+1}{z-a}\,dz$, where C is the circle $|z| = 2$ then find $f(4)$, $f'(1)$ and $f''(1)$ [07]

3(a) State Cauchy's residue theorem. Obtain the Laurent's expansion for $f(z) = \dfrac{9z^2-4z+1}{(z-1)(2z-1)(z+2)}$ in the region $1 < |z-i| < 3$. [07]

(b) Using contour integration, evaluate $\displaystyle\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta}\,d\theta$. [07]

Group-B

4(a) Define Laplace transform. Find the Laplace transform of the function $\displaystyle\int_0^t e^{-t}\sin at\,dt$ [07]

(b) State convolution theorem for Laplace transforms. Using convolution theorem find $L^{-1}\left\{\dfrac{s}{(s^2+1)(s^2+4)}\right\}$ [07]

6(a) Prove that $L^{-1}\left\{\frac{1}{s^{\frac{5}{2}}}\right\} = \frac{8}{15\sqrt{\pi}}t^{\frac{3}{2}}$ [06]

(b) Solve the differential equation using Laplace transform $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 8x = e^{2t}$ subject to the conditions $x(0) = 2,\ x'(0) = -2$ [08]

Group-C

6(a) Calculate coefficient of skewness by Karl Pearson's method and the values of β_1 and β_2 from the following data: [14]

Profits(Tk. Lakhs)	10-20	20-30	30-40	40-50	50-60
No.of companies	18	20	30	22	10

7(a) State Bernouli distribution. If X is a binomial variate with parameter n and p then show that $\mu_{r+1} = pq\left[\frac{d\mu_r}{dp} + nr\mu_{r-1}\right]$ [07]

where μ_{r+1}, μ_r and μ_{r-1} are the $(r+1)th$, rth and $(r-1)th$ central moment of the distribution. Also find the first four central moments with the help of this relation.

(b) State normal distribution. If X is a possion variate with parameter λ then show that [07]

$\beta_1 = \frac{1}{\lambda}$ and $\beta_2 = 3 + \frac{1}{\lambda}$ where β_1 and β_2 are the measures of skewness and kurtosis.

Program: B.Sc. in Computer Science and Engineering

1st Semester of 2nd Year

Course No: Math 2101 Course Title: Mathematics-III

Time: 03(three) hours Full Marks: 70

Use separate script for each part.

Part-A

There are 5 (Five) questions in Group A and B. Answer 3 (Three) questions, taking at least 1 (one) from each group.

(Marks allotted are indicated in the margin.)

Group-A

1. (a) Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$. Interpret the result geometrically and deduce that $\left|a+\sqrt{a^2-b^2}\right| + \left|a-\sqrt{a^2-b^2}\right| = |a+b| + |a-b|$. 6

(b) Prove that $f(z) = |z|^2$ is continuous everywhere but not differentiable except at the origin. 4

(c) Find the point where the Cauchy's-Riemann equations are satisfied for the function $f(z) = xy^2 + ix^2y$ where does $f'(z)$ exist? Where $f(z)$ is analytic? 4

2. (a) Define conformal mapping. Determine the equation of the curve in the w-plane in to which the straight line $x + y = 1$ is mapped under the transformation $w = \frac{1}{z}$. 7

(b) State Cauchy's integral formula. Show that $\oint_C \frac{e^{tz}}{(z^2+1)^2} dz = \pi i\,(\sin t - t\cos t)$, where C is the circle $|z| = 3$ and $t > 0$. 7

3. (a) State Cauchy's residue theorem. Find the Taylor series expansion of a function of the complex variable $f(z) = \frac{1}{(z-1)(z-3)}$ about $z = 4$. 7

Group-B

4. (a) Define Laplace transform. For Laplace transform of error function show that

$$L\{erf\sqrt{t}\} = \frac{1}{s\sqrt{s+1}}.$$ 7

(b) If $L\{f(t)\} = F(s)$ then show that $L\{f(t-a)\,u(t-a)\} = e^{-as}F(s)$ where $u(t-a)$ is the Heaviside's unit step function. Also find $L\{u(t-a)\}$. 7

5. (a) State convolution theorem for Laplace transform. Evaluate $L^{-1}\left\{\frac{1}{s^2(s^2+4)}\right\}$. 7

(b) Using Laplace transformation solve the differential equation $t\frac{d^2y}{dt^2} + (t-)\frac{dy}{dt} - y = 0$

subject to the conditions $y(0) = 5$ and $y(\infty) = 0$. 7

Part-B

There are 03 (Three) questions. Answer any 02 (Two) questions.
Use graph paper if needed

6. (a) A purchasing agent obtained samples of lamps from two suppliers. He had samples tested in his own laboratory for length of life, with the following results: 8

Length of life (in hours)	Sample A	Sample B
700-900	10	3
900-1100	16	42
1100-1300	26	12
1300-1500	8	3

$L\{erf\sqrt{t}\} = \frac{1}{s\sqrt{s+1}}$. 7

(b) If $L\{f(t)\} = F(s)$ then show that $L\{f(t-a)\,u(t-a)\} = e^{-as}F(s)$ where $u(t-a)$ is the Heaviside's unit step function. Also find $L\{u(t-a)\}$. 7

5. (a) State convolution theorem for Laplace transform. Evaluate $L^{-1}\left\{\frac{1}{s^2(s^2+4)}\right\}$. 7

(b) Using Laplace transformation solve the differential equation $t\frac{d^2y}{dt^2} + (t-)\frac{dy}{dt} - y = 0$

subject to the conditions $y(0) = 5$ and $y(\infty) = 0$. 7

Part-B

There are 03 (Three) questions. Answer any 02 (Two) questions.
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4. (a) Define Laplace transform. For Laplace transform of error function show that

$$L\{\text{erf}\sqrt{t}\} = \frac{1}{s\sqrt{s+1}}.$$ 7

(b) If $L\{f(t)\} = F(s)$ then show that $L\{f(t-a)\,u(t-a)\} = e^{-as}F(s)$ where $u(t-a)$ is the Heaviside's unit step function. Also find $L\{u(t-a)\}$. 7

5. (a) State convolution theorem for Laplace transform. Evaluate $L^{-1}\left\{\frac{1}{s^2(s^2+4)}\right\}$. 7

(b) Using Laplace transformation solve the differential equation $t\frac{d^2y}{dt^2} + (t-)\frac{dy}{dt} - y = 0$

subject to the conditions $y(0) = 5$ and $y(\infty) = 0$. 7

Part-B

There are 03 (Three) questions. Answer any 02 (Two) questions.
Use graph paper if needed

6. (a) A purchasing agent obtained samples of lamps from two suppliers. He had samples tested in his own laboratory for length of life, with the following results: 8

Length of life (in hours)	Sample A	Sample B
700-900	10	3
900-1100	16	42
1100-1300	26	12
1300-1500	8	3

Compare the variability of the two distributions using an appropriate measure. Which company's lamps should be preferred by the buyer and why?

(b) Listed below are the percentage increases in sales for the MG Corporation over the last 6 years. Determine the average percentage increase in sales over the period? 4

9.4%, 13.8%, [illegible] 11.9%, 14.3%, 15.0%

c) Write down the properties of arithmetic mean. 2

7 b) The following data gives the hardness (X) and tensile strength (Y) of 7 copper specimens. Hardness is given in units.

Hardness (X)	140	150	160	170	180	190
Tensile strength (Y)	75	78	77	82	85	86

i) Fit a linear regression equation of Y on X. 6

ii) Estimate the tensile strength for the hardness of 190. 1

iii) Estimate the Coefficient of determination. What does it explain? 3

b) Consider a problem with 100 devices in a box, 15 of which are defective. Three devices are to be selected one after the other. What is the probability of selecting three defective devices one after the other without replacement? 4

8 a) SCCoast, an internet provider in the Southeast, developed the following frequency distribution on the age of internet users. Find the mean, median, and modal age of the users. 10

Age (years)	Frequency
10 up to 20	3
20 up to 30	7
30 up to 40	18
40 up to 50	20
50 up to 60	12

Also show the median value through cumulative frequency polygon.

b) One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that one is white and one is black. 4

Final Examination of Spring Semester 2019

Department of Arts and Science

Program: B.Sc. in Computer Science and Engineering

2nd Year 1st Semester

Course No: Math 2101 | Course Title: Mathematics-III

Time: 3 (three) Hours | Full Marks: 70

Use separate script for each part.

Part-A

There are 5 (Five) questions. Answer 3 (Three) questions,
Taking at least 1 (one) from each group.
(ther figures of the right margin indicate full marks for the respective questions.)

Group-A

1. (a) Determine the set of points in the complex plane which satisfies the inequality $|z+1-i| \le |z-1+i|$ and sketch it. 4

(b) If $w = \varphi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$, determine the function φ. 4

(c) Show that the function $f(z) = u + iv$, where $f(z) = \begin{cases} \frac{x^3(1-i) - y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$

satisfies the Cauchy-Riemann equation at $z = 0$. Is the function analytic at $z = 0$? Justify your answer. 6

2. (a) Define conformal mapping. Find the image of $|z-3i| = 3$ under the mapping $w = \frac{1}{z}$. 7

(b) State Cauchy's integral theorem. Show that $\oint_C \frac{e^{tz}}{z^2+1} dz = 2\pi i \sin t$, where C is the circle $|z| = 1$ and $t \neq 0$. 7

3. (a) State Cauchy's residue theorem. Expand $f(z) = \frac{z^2}{(z-1)(z-2)}$ in a Laurent Series for the region $1 < |z| < 2$. 7

(b) Using contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$. 7

4. (a) Define Laplace transform. For Laplace transform of Bessel functions, prove that $L\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}$. 7

(b) State second translation property for inverse Laplace transforms. By expanding $e^{-k\sqrt{s}}$

Where k is a positive constant then show that $L^{-1}\left\{e^{-k\sqrt{s}}\right\} = \frac{k}{2t^{3/2}\sqrt{\pi}} e^{\frac{k^2}{4t}}$. 7

5. (a) Using Laplace transformation, Prove that $\int_0^{00} \cos x^2\, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$. 7

(b) Using Laplace transformation solve the differential equation $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = t + sint$ Subject to the conditions x(0) = 1 and -2. Also verify the result. 7

Part-B (Statistics)

There are 3 (Three) questions. Answer any 2 (Two) questions.

1. a) Define with example- i) Classical probability 5
ii) Empirical Probability
iii) Subjective Probability

b) The following frequency distribution gives the lifetime of 200 incandescent lamps:

Life in hours	No of lamps
500-600	2
600-700	5
700-800	12
800-900	25
900-1000	58
1000-1100	41
1100-1200	43
1200-1300	7
1300-1400	6
1400-1500	1

i) Determine the mean. Median and modal lifetimes of the lamps 6

ii) Show the median value graphically. 3

2. a) What is the coefficient of variation? When do we use it? 2

b) In a certain box, there are 15 defective devices out of 100 devices. Three devices are to be selected one after the other. Find the probability of selecting three devices without replacement 3

c) Suppose that samples of polythene bags from two manufacturers A and B are tested by a prospective buyer for bursting pressure, with the following results: 9

Bursting Pressure (lbs)	Number of bags	
	A	B
5-10	2	9
10-15	9	11
15-20	29	18
20-25	54	32
25-30	11	27
30-35	5	13

Which set of bags has the highest average bursting pressure? Which has more uniform bursting pressure? If the prices are same, which manufacturer's bags would be preferred by the buyer? Why?

3. a) A personal manager of an electronic manufacturing company devises a manual dexterity test for job applicants to predict their production rating in assembly department. In order to do this he selects a random sample of 10 applicants. They are given the test and later assigned a production rating. Results are as follows:

Worker	A	B	C	D	E	F	G	H	I	J
Test score	53	36	88	84	86	64	45	48	39	69
Production rating	45	43	89	79	84	66	49	48	43	76

i) Fit and interpret a regression equation of production rating on test score. 6

ii) Evaluate the coefficient of determination. Comment about the explanatory power of the regression equations. 3

iii) Also calculate the standard error when production rating is dependent on test score. 2

b) In a factory, there are 100 skilled, 250 semi-skilled and 150 unskilled workers. It has been observed that on an average a particular fabric is woven by a skilled worker in 3 hours, by a semi-skilled worker in 4 hours and by unskilled worker in 5 hours. Find the weighted average of time needed to complete a particular fabric by the workers. 3

Date: 27 / 09 / 2016

Ahsanullah University of Science and Technology

Final Examination of Spring Semester 2016

Department of Arts and Sciences

Program: B.Sc in Computer Science and Engineering

1st Semester of 2nd Year

Course No: Math 2101 Course Title: Mathematics-III

Time: 03(three) hours Full Marks: 70

There are 2(two) parts. Use separate scripts for each part.

Part-A

There are **5 (Five)** questions. Answer **3 (Three)** questions,
taking at least **1(One)** from each group.
(The figures of the right margin indicate full marks for the respective questions)

Group-A

1 (a) Prove that, if sum and product of two complex numbers are both real then the two numbers must either be real or conjugate. 4

(b) If z is any complex number then show that $\frac{d}{dz}(\bar{z})$ does not exist anywhere. 3

(c) Define analytic function. If $f(z)$ is an analytic function of z in any domain then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^p = p^2|f'(z)|^2|f(z)|^{p-2}$. 7

2 (a) Show that $u(x,y)=\frac{1}{2}\log(x^2+y^2)$ is harmonic and also find the harmonic conjugate if $f(z)=u+iv$ is analytic. 4

(b) Find the image of the circle $|z-3i|=3$ under the transformation $w=\frac{1}{z}$. 4

(c) Show that $\oint_c \frac{e^{3z}}{z-\pi i}dz = \begin{cases} -2\pi i, & \text{if C is the circle } |z-1|=4 \\ 0, & \text{if C is the ellipse } |z-2|+|z+2|=6. \end{cases}$ 6

(d) State Taylor's theorem for analytic function. Expand $f(z)=\frac{z^2}{(z-1)(z-2)}$ in a Laurent series for the region $1<|z|<2$ and $0<|z|<1$. 7

3 (a) Evaluate $\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}d\theta$ by using the method of contour integration. 7

7 (a) Pull length tests on a sample of 6 soldered leads for a semiconductor device yield the following results in pounds force required to rupture the bond: 8

19.8, 12.7, 13.2, 16.9, 10.6, 18.8

Another sample of 8 leads was tested after encapsulation to determine whether the pull strength has been increased by encapsulation of the device with the following results:

24.9, 22.8, 23.6, 22.1, 20.4, 21.6, 21.8, 22.5

Compare the variability in the two samples using an appropriate measure.

(b) Ecommerce.com, a large internet retailer, is studying the lead time (elapsed time between when an order is placed and when it is filled) for a sample of recent orders. The lead times are reported in days. 6

Lead Time (days)	Frequency
0 up to 5	6
5 up to 10	7
10 up to 15	12
15 up to 20	8
20 up to 25	7

Determine the mean, median and mode of the lead time.

8 (a) The following data relate the number of units of a goods that were ordered as a function of the price of the good at six different locations:

Number ordered	88	112	123	136	158	172
Price	50	40	35	30	20	15

i) Fit a regression line of 'number ordered' on 'price'. How many units do you think would be ordered if the price were 25? 6

ii) Determine the explanatory power of the regression equation. 4

(b) The percentage increase in sales for the last 4 years at Combs Cosmetics were: 4

4.91%, 5.75%, 8.12%, and 21.60%

Determine the average percentage increase in sales.

Date of Examination: 20/09/2017

AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department : Arts and Sciences

Program : B. Sc. in Computer Science and Engineering

Semester Final Examination, Spring 2017

Year : 2nd Semester : 1ST

Course No. : Math- 2101 Course Name: Mathematics III

Time: 3 hours Full Marks: 70

Instruction : Use separate answer script for Part A(Group 1 and Group 2) and Part B. Marks are indicated in the right margin.

Part A

Group- 1:There are 3(Three) questions. Answer any 2 (two)

1. a. Find the image of the infinite strip $\frac{1}{6} < y < \frac{1}{2}$ under the mapping function $w = \frac{1}{z}$. 6

   b. Expand $f(z) = \frac{1}{(z+3)(z+1)}$ in Laurent's series if $1 < |z| < 3$. 4

   c. Use De Moivre's theorem to solve the equation $x^4 + x^3 + x^2 + x + 1 = 0$. 4

2. a. Show that the real and imaginary parts of the functiuon $w = \ln z$ satisfy the Cauchy-Riemann equations when z is not zero. 6

   b. Show that the function $u(x, y) = e^{-x}(x \sin y - y \cos y)$ is a harmonic function. Find the conjugate harmonic function $v(x, y)$ and express $f(z) = u + iv$ in terms of z. 8

3. a. Evaluate the following integral using Cauchy integral formula $\int_c \frac{z-1}{(z+1)^2(z-2)} dz$ where c is the circle $|z - 1| = 3$. 7

   b. Evaluate $\int_0^{2\pi} \frac{1}{2+\cos\theta} d\theta$ by contour integration in the complex plane. 7

Group 2: There are 2(Two) questions. Answer any 1 (One)

4. a. Define Laplace Transform. Find the Laplace transform of $\frac{1-\cos at}{t}$. 5

   b. Find the Laplace transform of the Periodic function $f(t) = t^2$, $0 < t < 2$ and $f(t + 2) = f(t)$. 5

Evaluate the following integral using Laplace transform 4

$$\int_0^\infty t\, e^t \sin 4t \; dt$$

5. a. Find the inverse Laplace transform of the following functions 7

i) $\ln\left(1+\frac{1}{s^2}\right)$ ii) $\frac{1}{s(s^2+a^2)}$ (Using Convolution theorem)

b. Solve the following initial value problem using the Laplace transform 7

$2y'' + 5y' + 2y = e^{-2t},\ y(0) = 1,\ y'(0) = 1.$

Part -B

There are 03 (three) questions. Answer any 02 (two) questions.

6. a. Suppose you are interested in using past expenditure on research and development (R&D) by a firm to predict current expenditure on research and development by a firm to predict current expenditure on R &D. You got the following data by taking a random sample of firms, where X is the amount on R & D (in lakh Taka) 5 years ago and Y is the amount spent on R & D (in lakh Taka) in the current year: 8

X	30	50	20	80	10	20	20	40
Y	50	80	30	110	20	20	40	50

Find the regression equation of Y on X. Also determine the explanatory power of the regression equation.

b. The average particulate concentration, in micrograms per cubic meter, was measured in petrochemical complex at 36 randomly chosen times, with the following concentrations resulting: 6

5,18,15,7, 23,220,130,85, 103, 25, 80, 7, 24, 6, 13, 65, 37, 25,24, 65, 82, 95, 77, 15, 70, 110, 44, 28, 33, 81, 29, 14, 45, 92, 17,53

Group the data using a proper method.

7. a. The range of two type of mortar shell is being investigated. The observed ranges (in meter) of such shells are as follows: 10

	Ranges	400-500	500-600	600-700	700-800	800-900
Number of shells for two types	Type 1	2	5	12	10	6
	Type 2	1	9	11	9	5

i)Determine the variability in the ranges of mortar shell using an appropriate measure, ii) which type do you prefer better on the basis of variability?

b. The annual rate of growth of sales of Microsoft Corporation is 4

Year	2012	2013	2014	2015	2016
Sales (percent increase)	20.2%	22.5%	23.9%	28.0%	25%

Calculate the average increase in sales over the period.

8. a. A system consists of four components as illustrated in the following figure. The system works if the component X and Y works and either of the components Z or W work. The reliability (probability of working) of each component is also shown in the figure . Find the probability that i) the entire system works ii) The system Z does not work given that the entire system works. Assume that all four components work independently 5

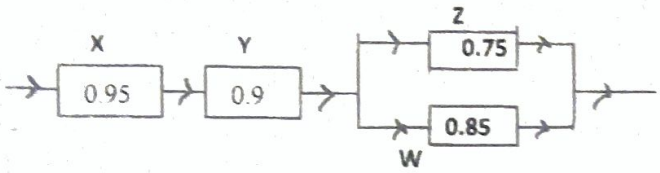

Figure:1

b. Consider a problem with 100 devices in a box, 15 of which are defective. Three devices are to be selected one after the other. What is the probability of selecting three defective devices one after the other without replacement? 4

c. The following are the percentages of ash content in 12 sample of coal found in close proximity 5

9.2, 14.1, 9.8, 12.4, 16.0, 12.6, 22.7, 18.9,21.0, 14.5, 20.4, 16.9

Find the arithmetic mean and median amount of percentage of ash content.