HAMZA BIN ZAHID Mat No: 3405458

Code Algorithem

Solution :-

Given is non Linear System

$$f_1(x,y) = x^2 + y^2 - 9 \quad \text{—— ①}$$
$$f_2(x,y) = x + 2y - 6 \quad \text{—— ②}$$

① Calculating Jacobi Matrix using partial derivatives.

$$\frac{\partial}{\partial x} f_1(x,y) = 2x$$

$$\frac{\partial}{\partial y} f_1(x,y) = 2y$$

$$\frac{\partial}{\partial x} f_2(x,y) = 1$$

$$\frac{\partial}{\partial y} f_2(x,y) = 2$$

$$J = \begin{bmatrix} 2x & 2y \\ 1 & 2 \end{bmatrix}$$

② C++ code will be done in the following flow & .cpp file will be submitted.

③ Initial guess is

$$x_0 = 0.1, \quad y_0 = 0.1$$

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④ Input from user not required in C++ code.

⑤ Run the program for "11" Iterations.

Newton Method Iterations formula:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \end{bmatrix} - J^{-1} * F(x_n)$$ ← Main Eq

Iteration # 01 i.e (initial guess $(x_0 = 0.1, y_0 = 0.1)$)

$$F(x_0, y_0) = \begin{bmatrix} x_0^2 + y_0^2 - 9 \\ x_0 + 2y_0 - 6 \end{bmatrix} = \begin{bmatrix} (0.1)^2 + (0.1)^2 - 9 \\ (0.1) + 2(0.1) - 6 \end{bmatrix} = \begin{bmatrix} -8.98 \\ -5.7 \end{bmatrix}$$

Calculating J^{-1}

$$J^{-1} = \frac{\begin{bmatrix} 2 & -2y_0 \\ -1 & 2x_0 \end{bmatrix}}{4x - 2y}$$

Now guess

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \left\{ \frac{\begin{bmatrix} 2 & -2y_0 \\ -1 & 2x_0 \end{bmatrix}}{4x_0 - 2y_0} \right\} * \begin{bmatrix} -8.98 \\ -5.7 \end{bmatrix}$$

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(2) Point

New guess will be

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} - \left\{ \frac{\begin{bmatrix} 2 & -2(0.1) \\ -1 & 2(0.1) \end{bmatrix}}{4(0.1) - 2(0.1)} \right\} * \begin{bmatrix} -8.98 \\ -5.7 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} - \begin{bmatrix} 2/0.2 & \frac{-0.2}{0.2} \\ -1/0.2 & \frac{0.2}{0.2} \end{bmatrix} * \begin{bmatrix} -8.98 \\ -5.7 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} - \begin{bmatrix} 10 & -1 \\ -5 & 1 \end{bmatrix} * \begin{bmatrix} -8.98 \\ -5.7 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} - \begin{bmatrix} -84.1 \\ 39.2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 84.2 \\ -39.1 \end{bmatrix}$$

Notes

New guess for x_1 & y_1 willbe '84.2 & -39.1'

following the values of x_1, y_1 & updating x & y values each time we will get results repeat this for 11 iterations.

From all 11 iterations we will get the following results (Code in .ipynb file)

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(2) back

Iteration	x	y	Norm
1	84.2	-39.1	92.7871
2	42.7087	-18.3543	46.3887
3	21.9717	-7.98584	23.1847
4	11.6205	-2.81025	11.573
5	6.47935	-0.239673	5.74799
6	3.97605	1.01197	2.79877
7	2.84739	1.57631	1.26189
8	2.86075	1.76963	0.193275
9	2.40146	1.79927	0.0662836
10	2.4	1.8	0.00163537
11	2.4	1.8	9.96706e-07

decrease in Norm value can be seen w.r.t increase in Iterations

Norm Calculation: for (1st Iteration)

$$Norm = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$Norm = \sqrt{(84.2 - 0.1)^2 + (-39.1 - 0.1)^2}$$

$$Norm = 92.7871$$

Note: for each iteration we will calculate Norm using updated values of x & y & then subtracting it from previous value. Norms results are mentioned in above table (C++ Code in .cpp file).

"The Norm decreases to a very small value indicating convergence to exact solution"

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(3) Front

Now calculating convergence order using the given formula.

$$P_n = \frac{\log(Norm_n / Norm_{n+1})}{\log(Norm_{n+1} / Norm_{n+2})}$$

Using Norm value from the table as we get.

[92.7871, 46.3887, 23.1847, 11.573, 5.74799, 2.79877
1.26189, 0.432275, 0.0662836, 0.00163537, 9.96706×10^{-7}]

Now for

n = 1

$$P_1 = \frac{\log(92.7871/46.3887)}{\log(46.3887/23.1847)}$$

$P_1 = 0.999960$

n = 2

$$P_2 = \frac{\log(46.3887/23.1847)}{\log(23.1847/11.573)}$$

$P_2 = 0.998141$

n = 3

$$P_3 = \frac{\log(23.1847/11.573)}{\log(11.573/5.74799)}$$

$P_3 = 0.992823$

n = 4

$$P_4 = \frac{\log(11.573/5.74799)}{\log(5.74799/2.79877)}$$

$P_4 = 0.971513$

n = 5

$$P_5 = \frac{\log(5.74799/2.79877)}{\log(2.79877/1.26189)}$$

$P_5 = 0.903502$

n = 6

$$P_6 = \frac{\log(2.79877/1.26189)}{\log(1.26189/0.432275)}$$

$P_6 = 0.743551$

n = 7

$$P_7 = \frac{\log(1.26189/0.432275)}{\log(0.432275/0.0662836)}$$

$P_7 = 0.571325$

n = 8

$$P_8 = \frac{\log(0.432275/0.0662836)}{\log(0.0662836/0.00163537)}$$

$P_8 = 0.506505$

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(3) back

n = 9

$$P_9 = \frac{\log(0.0662836/0.00163537)}{\log(0.00163537/9.96706\times10^{57})}$$

$P_9 = 0.500082$

Now Taking Av. of convagence order =>

$$\frac{0.99960 + 0.998141 + 0.992823 + 0.971513 + 0.903502 + 0.743551 + 0.571325 + 0.506505 + 0.500082}{9}$$

Av. Convagence Order => 0.798560222 2

Note To round of this number to nearest 0.5 precision we will consider figure after point i.e, 7 if it is greater than 0.25 then we will round up to nearest 0.5 if less than 0.25 we will round down to nearest 0.5

In this case number after point is 7 80; greater than 0.25 round up to nearest 0.5 number will became 1.

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Iteration vs Norm Graphical Representation using (Matlab):

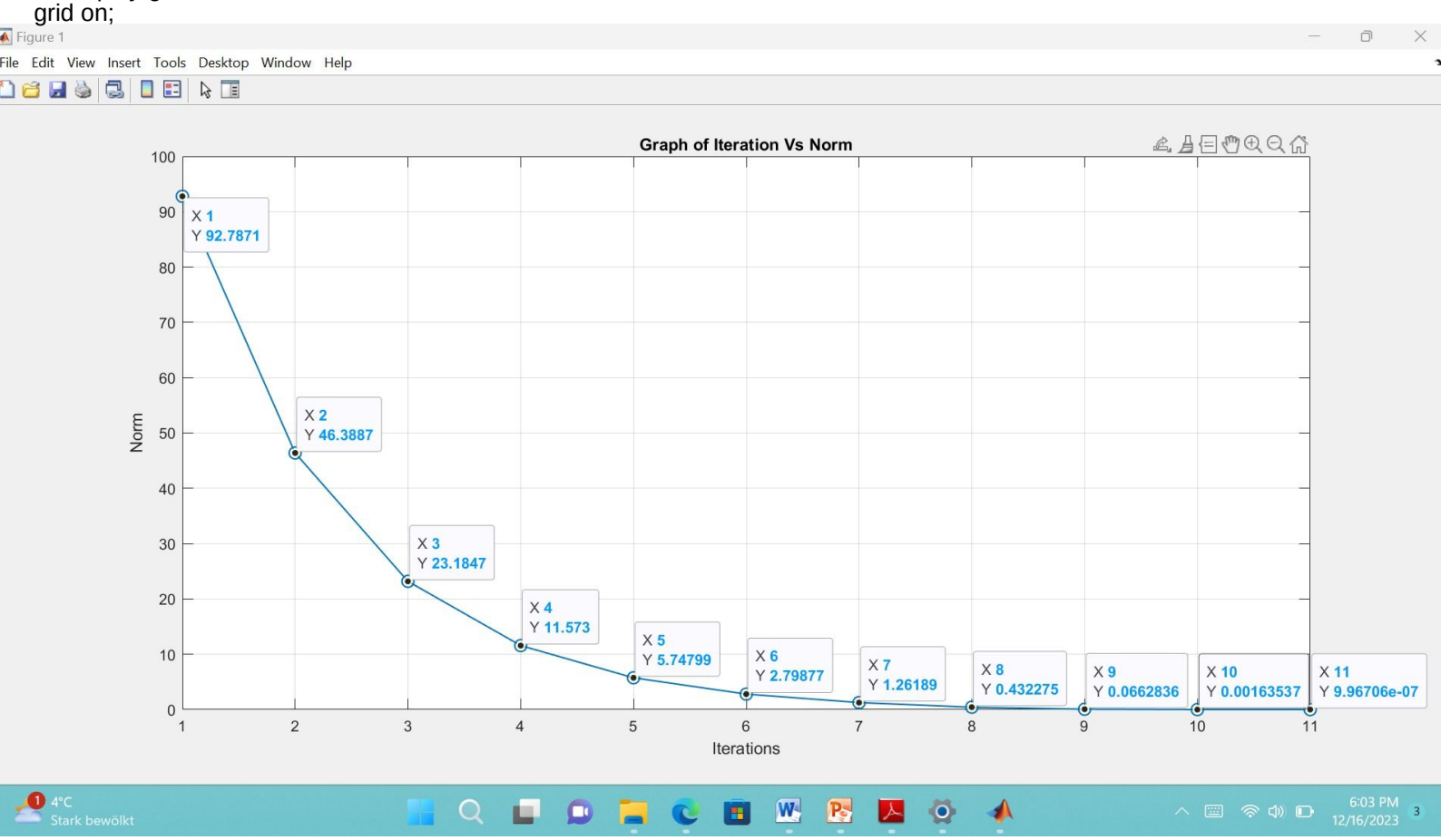
//Norm values are taken from C++ code results whereby matlab used for plotting against each iteration as instructed in question

Matlab Code:

```
% Given points
Iterations = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11];
Norm = [92.7871, 46.3887, 23.1847, 11.573, 5.74799, 2.79877, 1.26189, 0.432275, 0.0662836, 0.00163537, 9.96706*10^-7];

% Plot the given points
plot(Iterations, Norm, 'o-', 'LineWidth', 1, 'MarkerSize', 8);

% Add labels and title
xlabel('Iterations');
ylabel('Norm');
title('Graph of Iteration Vs Norm');
% Display grid
grid on;
```



Additional Comment:

"The Norm values is decreasing to a very small value indicating convergence to exact solution" the above graph depicts the same

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C++ Code Output:

```
Initial Values for x is 0.1
Initial Values for y is 0.1
Iteration x y Norm
   1      84.2      -39.1      92.7871
   2      42.7087      -18.3543      46.3887
   3      21.9717      -7.98584      23.1847
   4      11.6205      -2.81025      11.573
   5      6.47935      -0.239673      5.74799
   6      3.97605      1.01197      2.79877
   7      2.84739      1.57631      1.26189
   8      2.46075      1.76963      0.432275
   9      2.40146      1.79927      0.0662836
   10      2.4      1.8      0.00163537
   11      2.4      1.8      9.96706e-07
Average Convergence Order: 0.79866
Rounded Convergence Order (0.5 precision): 1
```

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C++ Code with detailed comments:
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```
#include <iostream>
#include <cmath>
using namespace std;

int main()

{
    // Initial guess as given in problem
    double x = 0.1;
    double y = 0.1;
    cout << "Initial Values for x is " <<   x <<  endl;
    cout << "Initial Values for y is " <<   y <<  endl;

    // Defining maxIterations as given is 11 in question
    const int maxIterations = 11;
    cout << "Iteration x y Norm" << endl;

    // Loop for iterations till maximum 11
    for (int i = 0; i < maxIterations; ++i)
    {
        // Equations and their derivatives
        double f1 = x * x + y * y - 9; //defining funcion "F1(x)""
        double f2 = x + 2 * y - 6; //defining funcion "F2(x)""
        double df1dx = 2 * x; //defining partial derivative of funcion "F1(x)"" wrt "x"
        double df1dy = 2 * y; //defining partial derivative of funcion "F1(x)"" wrt "y"
        double df2dx = 1; //defining partial derivative of funcion "F2(x)" wrt "x"
        double df2dy = 2; //defining partial derivative of funcion "F2(x)" wrt "y"

        //This part of the loop is where the Newton method is applied.
        //It calculates the Jacobian matrix determinant, checks for division by zero, and then updates
        //the variables (x and y) based on the Newton method's formula.
        //The loop iterates until convergence is achieved or the maximum number of iterations is reached
        // Check for division by zero error "safety check" to check for infinity situation not created in code

        double det = df1dx * df2dy - df1dy * df2dx;// defining and finding Jacobian matrix determinant

        // Check for division by zero error "safety check" to check for infinity situation not created in code
        if (fabs(det) < 1e-10)
        {
            cerr << "Error: Division by zero." << endl;
            return 1;
        }

        // Calculate the change in variables
        double deltaX = (f1 * df2dy - f2 * df1dy) / det;
        double deltaY = (f2 * df1dx - f1 * df2dx) / det;

        // Update variables
        x -= deltaX;
        y -= deltaY;

        // Calculate the Euclidean norm
        double norm = sqrt(deltaX * deltaX + deltaY * deltaY);

        // Print current solution and norm
        cout << "   " << i + 1 << "     " << x << "     " << y << "     " << norm << endl;

        // Check for convergence (convergence vs Iteration graph & code made in Matlab)
        if (norm < 1e-8)
        {
            cout << "Converged to a solution." << endl;
            break;
        }
    }
    //Generated norm value arrays to futhur calculate the covergance order
    double Norm[] = {92.7871, 46.3887, 23.1847, 11.573, 5.74799, 2.79877, 1.26189, 0.432275, 0.0662836, 0.00163537, 9.96706e-7};

    double convergenceOrders[10];
    //Convergance order loop just like in notes I have calculated P1,p2,p3 till P9 convergence convergenceOrders
    for (int i = 0; i < 9; ++i)
    {
        convergenceOrders[i] = log(Norm[i] / Norm[i + 1]) / log(Norm[i + 1] / Norm[i + 2]);
    }

    // Calculate the average convergence order
    double averageConvergenceOrder = 0.0;
    for (int i = 0; i < 9; ++i)
    {
        averageConvergenceOrder += convergenceOrders[i];
    }
    averageConvergenceOrder /= 9;

    // Round to the nearest 0.5 precision
    double roundedConvergenceOrder = round(averageConvergenceOrder * 2) / 2;

    cout << "Average Convergence Order: " << averageConvergenceOrder << endl;
    cout << "Rounded Convergence Order (0.5 precision): " << roundedConvergenceOrder << endl;

    return 0;

}
```