PODS Lab 8: Sampling and Hypothesis Testing

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1. **Inference**
2. **Sampling**
3. **Hypothesis Testing Framework**
    - Null (H_0) and Alternative Hypothesis (H_1)
    - Significance Level
    - Statistical Significance

Switching from prediction to inference

- **Before:** Characterizing data (central tendency and dispersion), correlation, regression (prediction)
- **Now:** Inference, sampling, hypothesis testing

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- **Ideally,** we would literally measure every female's height in the U.S. and take the average
- **Problem:** Not possible! Cannot coordinate, too many data points, expensive, time constraint, etc
- **Solution:** Take a subset of the female population (e.g. 1000), measure their heights, and take the average

- We are estimating the height of the entire U.S. female population using a sample

What is Statistical Inference?

- **Inference:** The process of drawing conclusions about a population using a sample.
- **Key Idea:** We rely on a subset of the population (sample) to estimate unknown population characteristics.
- **Example:** Estimating the average height of U.S. females by measuring a subset of the population.

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- **Example:** Estimating the average height of U.S. females by measuring a subset of the population.
- Since sampling is a core idea in inference, let's talk about it

Sampling Terminology

- **Population:** The entire group we want to study (e.g., all U.S. females).
  - **Population Parameter (θ):** A fixed, unknown value describing the population (e.g., true average height of all U.S. females, denoted as μ).

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- **Sample:** A subset of the population used for analysis.
  - **Sample Statistic ($\hat{\theta}$):** A measurable value computed from the sample that estimates the population parameter (e.g., sample mean $\bar{x}$).

	Parameter (θ)	**Statistic ($\hat{\theta}$)**
Mean	μ	$\bar{x}$
Standard Deviation	σ	s

Key Idea: Statistical inference estimates a population parameter θ using a sample statistic $\hat{\theta}$.

Why Does This Work? Law of Large Numbers (LLN)

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- **Intuition:**
  - With a small sample, randomness can cause large fluctuations.
  - As the sample grows, these fluctuations average out, bringing the estimate closer to the true value.
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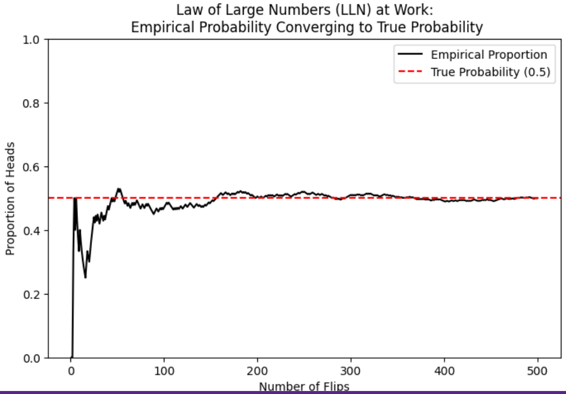
- **Assumptions of LLN:**
  1. **Independence:** Each observation in the sample is independent of the others.
  2. **Random Sampling:** The sample is selected randomly to avoid bias.

Example: Law of Large Numbers in Coin Flipping

- **Key Idea:** As the number of coin flips increases, the **sample statistic** (empirical probability of heads) converges to the **true probability** (0.5).
- **Observation:** The fluctuations are large for small samples but stabilize as the sample size grows.



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- **Example:**
  - Survey people about their sleeping habits at 7 AM on the street.
  - You find that most respondents are early risers.
  - **This leads to a biased conclusion:** You vastly overestimate the proportion of early risers in the population.

Sampling Distribution

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- **Example:**
  - Draw 1,000 random samples of 50 people ($n = 50$) from a population
  - Calculate the mean height of each sample
    - $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_{1000}$
  - The distribution of those sample means would be the **sampling distribution of the mean.**

Central Limit Theorem (CLT) – Link for CLT Simulation (clickable!)

- **Central Limit Theorem (CLT):** Regardless of the population's shape, the sampling distribution of the sample mean becomes approximately normal as the sample size increases.
- **Implication:**
  - Even if the population is skewed, multimodal, or discrete, the sample mean will **follow a normal distribution** for sufficiently large *n*.

Formal Statement:

- If we take a large number of random samples of size *n* from any population with mean μ and standard deviation σ, then the sample means $\bar{x}$ follow:

$$\bar{x} \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Standard Error of the Mean (SEM)

- **Standard Error of the Mean (SEM):** The standard deviation of the **sampling distribution** of the sample mean.

Theoretical (True Population)	**Practical (Estimate w/ Data)**
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$SEM = \frac{s}{\sqrt{n}}$

- **Where:**
  - $\sigma_{\bar{x}}$: Standard deviation of sample mean (i.e., the standard error).
  - σ: Population standard deviation.
    - Practically, we use the sample standard deviation s since σ is unknown.
  - n is the sample size.

- **Note!** Decreases as a function of the square root of the sample size.

Hypothesis Testing Framework

- **What is Hypothesis Testing? (A falsification approach)**
  - A statistical method used to make inferences about a population based on sample data.
  - It helps us determine whether an observed effect is real or due to random chance.

- **Hypothesis Testing Framework:**
  1. **Formulate Null (H_0) and Alternative (H_1) Hypotheses**
     - H_0: Assumption of no effect or no difference.
     - H_1: Assumption of an effect or a difference.
  2. **Choose a Significance Level (α)**
     - Common choices: $\alpha = 0.05, 0.01$.
  3. **Determine Test Statistic**
     - Depends on data type and assumptions.
     - Examples: Z-test, T-test, KS test, Mann-Whitney U test.
  4. **Compute P-value**
     - **Reject H_0:** If $p \leq \alpha$ (statistically significant).
     - **Fail to reject H_0:** If $p > \alpha$ (not enough evidence).

Null (H_0) and Alternative (H_1) Hypothesis

1. **Null Hypothesis (H_0):** The assumption that there is no effect, no difference, or no relationship in the population.
   - Represents the status quo or baseline assumption.
   - We assume H_0 is true unless we have strong evidence against it.
2. **Alternative Hypothesis (H_1):** The hypothesis that there is an effect, a difference, or a relationship in the population.
   - Represents what we want to test for.
   - If there is enough evidence, we reject H_0 in favor of H_1.

P-Value and Interpretation

- **What is the P-value?** Probability of obtaining a test statistic as extreme as (or more extreme than) the observed one, assuming the null hypothesis H_0 is true.

- **P-Value Calculation Algorithm**

  1. Compute the test statistic.
  2. Find the probability of obtaining that test statistic using a corresponding probability distribution.

- **Interpretation:**

  - The probability of observing the data (or something more extreme) given the null hypothesis H_0 is true.

Significance Level (α) and Statistical Significance

- **Significance level (α):**
  - A threshold that is set to determine whether the results of a statistical test are significant or not
  - **Common convention:** $\alpha = 0.05$ (5%) is widely used.
  - **Why is 0.05 standard?**
    - Ronald Fisher (1925) found it convenient as a practical cutoff
- **Statistical Significance:**
  1. **Statistically significant ($p \leq \alpha$)**
     - The observed pattern of data is **unlikely due to chance alone**
     - **Decision:** Reject the null hypothesis (H_0)
  2. **NOT statistically significant ($p > \alpha$)**
     - The observed pattern of data is **plausible under random chance**.
     - **Decision:** Fail to reject the null hypothesis (H_0)

Full Example: Z-test as a Test Statistic (one-tail)

- Algorithm:
  1. Compute Z-score
  2. Look up the Z-score in a standard Z-table (Clickable link!).
  3. Determine the corresponding probability to the Z-score (P-value)

- **Example:** $\mu = 82$, $\sigma = 13$, $\bar{x} = 85$, $n = 200$, $H_0 : \mu = 82$, $H_1 : \mu > 82$

$$Z = \frac{\bar{x} - \mu}{SEM} \approx \frac{85 - 82}{0.92} \approx 3.26$$

- $P(Z \geq 3.26) = 1 - .99944 \approx 0$
- Since $p = 0.00093$ is **less than** $\alpha = 0.05$, we **reject** H_0 (statistically significant result).

Errors

- Our conclusion can be wrong!

		Reality	
		Yes	No
Significant?	Yes	Correct (1-β)	Type I Error False Positive (α)
	No	Type II Error False Negative (β)	Correct (1-α)

Table: Confusion Matrix for errors we might see