PODS Lab 12: Confidence Intervals, Bootstrapping, and Bayes' Theorem

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1. Confidence Intervals (CI)
2. Bootstrapping
3. Bayes' Theorem

Moving From a Point Estimate to an Interval Estimate

- **So far,** we have focused on **point estimates:**
  - **Examples:** Effect size, test statistic, p-value, power, sample mean $\bar{X}$
- **Now,** we shift to **interval estimates.**

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- Suppose we want to estimate the population mean μ.
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- **Option 2:** Construct a **range of plausible values** – an **interval**.

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Definition: Confidence Interval (CI)

An interval estimate that captures plausible values for the true population parameter.

Idea of Confidence Intervals

- **Sampling Error:** Every time we sample from a population, our sample statistic (e.g., $\bar{X}$) can differ due to random variability.
- **Consequence:** Different samples would give different estimates.
- **Big Idea:**
  - If we repeatedly sample from the same population and construct a confidence interval for each sample,
  - Then approximately 95% of those intervals would contain the true population parameter (e.g., the population mean), **assuming no sampling bias**.

Clarifying Points

- **Question:** How can we build a confidence interval for something we don't know (population parameter)?
- **Answer:** The **Central Limit Theorem (CLT)** gives us the structure to do it.
  - **CLT:** If we repeatedly sample from a population and compute the sample mean $\bar{X}$ each time, then the distribution of the sample means will tend toward a **normal distribution**, regardless of the shape of the original population, provided the sample size is **large enough**.

In short: The sampling distribution of a sample statistic (e.g. $\bar{X}$) becomes approximately normal even if the population itself is not.

How to Calculate Confidence Intervals

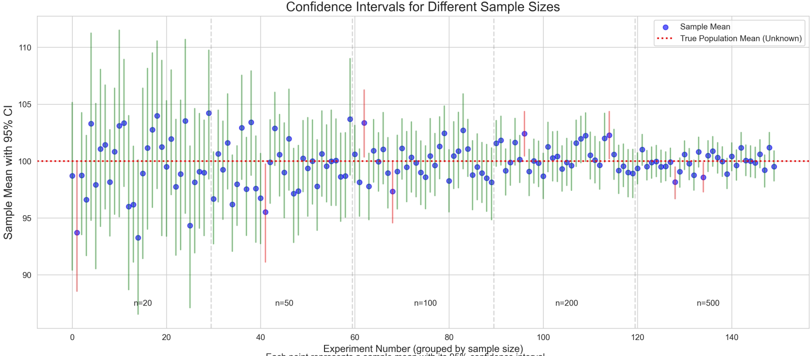
1. Declare a **confidence level:** 99.9%, 99%, 95%, 50%
   - Find the **critical value** z that corresponds to your confidence level
2. Pick a **sample size** n
3. **Measure** the sample statistic (e.g., sample mean $\bar{X}$)
4. Plug into the confidence interval formula

Confidence Interval Formula

$$\text{CI} = \bar{X} \pm (z_{\alpha/2} \times \text{SE}) \qquad \text{,where}$$

- **SE** $= \frac{\sigma}{\sqrt{n}}$ or $\frac{s}{\sqrt{n}}$ if σ is unknown.
- z **(Confidence level):** 99.9% $\rightarrow$ 3.29; 99% $\rightarrow$ 2.58; 95% $\rightarrow$ 1.96; 90% $\rightarrow$ 1.645; 80% $\rightarrow$ 1.28; 50% $\rightarrow$ 0.674 (From z-table)

Confidence Intervals Visualization and interpretation



Each point represents a sample mean with its 95% confidence interval.
Green intervals contain the true population mean, red intervals do not.
Note how larger sample sizes produce narrower confidence intervals.

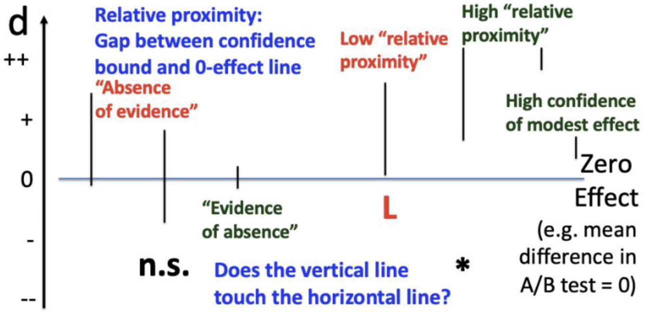
Observe:

1. At the same confidence level (95%) and sample size, **confidence intervals may vary** due to sampling variability.
2. **Larger samples yield narrower intervals** because standard error decreases as n increases.

How to Interpret Confidence Intervals

- **A 95% confidence interval means:**
  - If we repeated sampling many times and constructed a confidence interval each time,
  - About 95% of those intervals would contain the true population parameter.
- **Important:**
  - It does **not** mean there is a 95% probability that the true parameter is inside **this particular** interval.
  - The true parameter is fixed; the interval is random.
- **Tradeoff Between Confidence and Precision:**
  - Higher confidence levels (e.g., 99%) result in wider intervals, as they must include more possible values.
  - Narrower intervals offer greater precision but are less likely to include the true parameter.

How to Interpret Confidence Intervals



Interval	Interpretation
Contains 0	No significant difference between groups
Entirely positive (+, +)	Group 1 $>$ Group 2 (Significant)
Entirely negative ($-$, $-$)	Group 1 $<$ Group 2 (Significant)

Assumptions of Confidence Intervals

1. Random Sampling
2. Independence of Observations
3. Sufficiently Large Sample Size
4. No Sampling Bias
5. **Normality of the Population Distribution**:
    - For **small sample sizes**, many confidence interval methods assume that the population from which the sample is drawn is normally distributed.
    - For **larger sample sizes**, the **Central Limit Theorem (CLT)** ensures that the sampling distribution of the statistic becomes approximately normal, even if the population itself is skewed.

- **Problem:** Not all statistics naturally distribute normally – especially for small samples, ratios, or rare events.
    - **Solution:** Bootstrap!

Bootstrapping

- **Bootstrapping:** A resampling technique to approximate the sampling distribution of a statistic.

1. Obtain one sufficiently large and representative sample (the **original sample**).
2. Sample **with replacement** from the original sample to create many **bootstrap samples**.
   - We treat the original sample as if it were the entire population.
3. Calculate the sample statistic (e.g., mean, median, effect size) for each bootstrap sample.
4. Build a sampling distribution from these bootstrap statistics.

Two Options for Building Confidence Intervals (Bootstrapping)

- After generating bootstrap samples, you have two common ways to construct a confidence interval:

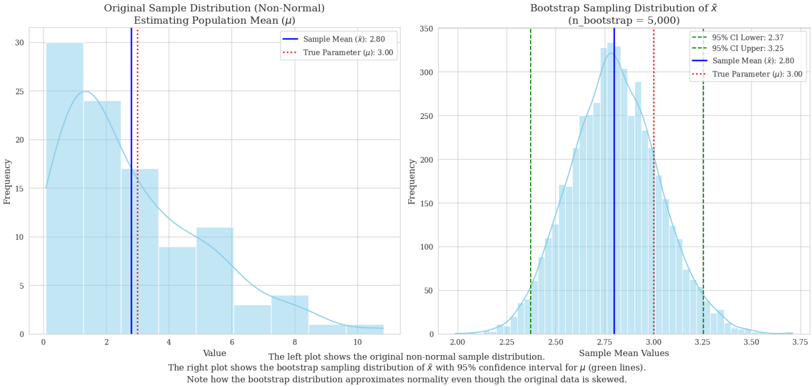
1. **Percentile Method:** (Our Method)

$$\text{CI}_{95\%} = [\text{2.5th percentile}, \ \text{97.5th percentile}]$$

2. **Standard Error Method** (similar to before):

$$\text{CI} = \bar{x}_{\text{original}} \pm (z \times \text{SE}_{\text{bootstrapped}})$$

Bootstrapping and Confidence Interval Visualization



- From the original sample, we drew 5,000 bootstrap samples (sampling with replacement).
- We calculated the sample mean for each bootstrap sample to build the sampling distribution.
- The 95% confidence interval is obtained by slicing the bootstrap distribution at the 2.5th and 97.5th percentiles.