

∴ (Applied Physics):-

Semester # 01:-

Qno5:-

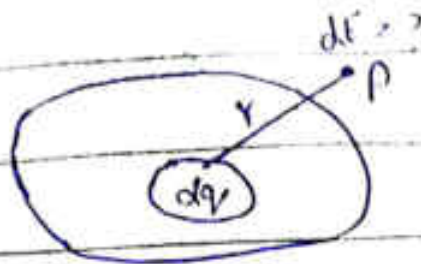
Calculate electric field
due to continuous charge
distribution:-

Answer:-

The electric charge is quantized. The collection of large number of charges is called continuous charge distribution. The continuous charge distribution has three types.

- (i) Linear charge density.
- (ii) Surface charge density.
- (iii) Volume charge density.

Surface Charge Density:-



Consider a surface charge (density) distribution. Take a small area 'da' of this distribution having charge 'dq'.

$$\text{Surface charge density} = \sigma = \frac{dq}{da}$$

$$\sigma = \frac{dq}{da}$$

$$dq = \sigma da \quad \text{--- (i)}$$

we know that:

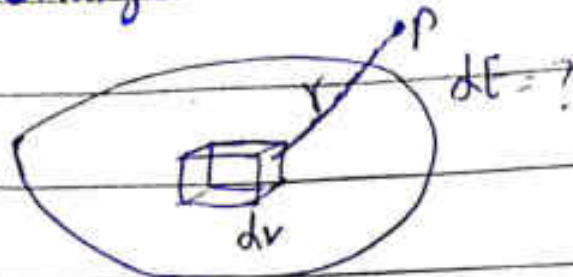
$$E = \frac{Kq}{r^2} \Rightarrow dE = \frac{K dq}{r^2}$$

$$dE = \frac{K \sigma da}{r^2}$$

The net electric field at P is:

$$E = K \int \frac{\sigma}{r^2} da$$

Volume Charge Density:-



Consider a volume charge distribution.

Take a small volume element dv of this distribution having charge dq .

$$\text{Volume charge density} = \rho = \frac{q}{V}$$

$$\rho = \frac{dq}{dv} \Rightarrow dq = \rho dv \quad (1)$$

we know that:

$$E = \frac{Kq}{r^2} \Rightarrow dE = \frac{K dq}{r^2}$$

$$dE = \frac{K \rho dv}{r^2}$$

The net electric field at P is:

$$E = K \int \frac{\rho}{r^2} dv$$

QNo6:-

What is an electric dipole?
Calculate electric field at a point due to electric dipole?

Ans:-

Electric Dipole:-

A pair of positive charge and negative charge of same magnitude having constant distance between them is called

Electric Dipole. It is useful concept in dielectrics and other applications in solid and liquid materials.

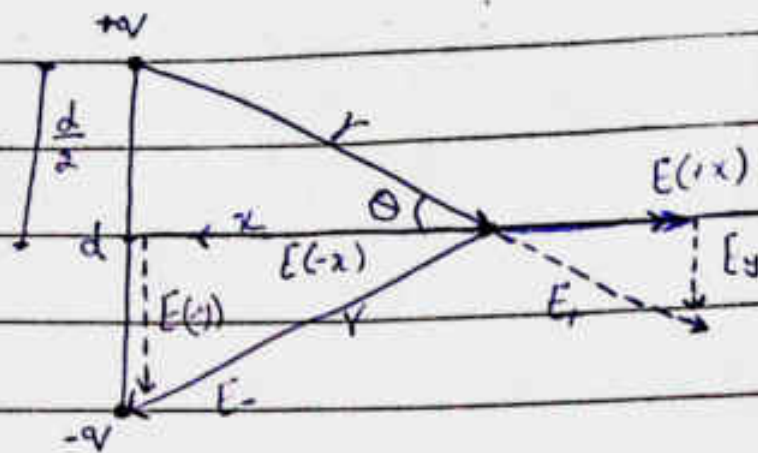
Electric Dipole Moment:-

The product of magnitude of charge and separation between them is called electric dipole moment and it is written as:

$$P = qd$$

Electric field due to electric dipole:-

Diagram:-



Consider a charge $+q$ and $-q$ having distance b/w them. This is called dipole. The dipole moment is given as $P = qd$.

Take a point P having distance ' x ' which is perpendicular bisector of distance.

The magnitude of electric field at 'P' due to $+q$ charge having distance ' r ' is :

$$E_+ = \frac{kq}{r^2} \quad \text{--- (i)}$$

The magnitude of electric field due to $-q$ charge having

distance 'r' is:

$$E_- = \frac{Kq}{r^2} \quad \text{--- (ii)}$$

Comparing (i) and (ii)

$$E_+ = E_- \quad \text{--- (iii)}$$

To calculate net electric field at 'P' due to dipole, resolve the electric field E_+ and E_- into components.

Rectangular component of E_+ is:

$$E(+x) = E_+ \cos \theta, \quad E(+y) = E_+ \sin \theta$$

Rectangular components of E_- is:

$$E(-x) = E_- \cos \theta, \quad E(-y) = E_- \sin \theta$$

The resultant x-component of electric field is:

$$E_x = (+E(+x)) + (E(-x))$$

$$E_x = +E \cos \theta + (-E \cos \theta)$$

$$E_x = +E \cos \theta - E \cos \theta$$

$$\boxed{E_x = 0} \quad \text{--- (a)}$$

The resultant y-component of electric field is:

$$E_y = (E \sin \theta) + (E \sin \theta)$$

$$E_y = -E \sin \theta + (-E \sin \theta)$$

$$E_y = -E \sin \theta - E \sin \theta$$

$$\boxed{E_y = -2E \sin \theta} \quad (b)$$

Magnitude of resultant electric field E is:

$$E = \sqrt{(E_x)^2 + (E_y)^2}$$

$$E^2 = (E_x)^2 + (E_y)^2$$

$$E^2 = (0)^2 + (-2E \sin \theta)^2$$

$$E^2 = (-2E \sin \theta)^2$$

Taking $\sqrt{\quad}$ on b. sides

$$E = -2E \sin \theta$$

From (i)

$$E = -2 \frac{Kq}{r^2} \frac{d}{2}$$

$$E = -\frac{2K(qd)}{r^3}$$

From dipole moment

$$E = - \frac{KP}{r^3}$$

$$E = -KP r^{-3} \quad \text{--- (iv)}$$

By Pythagorean theorem:-

$$c^2 = a^2 + b^2$$

$$r^2 = \left(\frac{d}{2}\right)^2 + x^2$$

$$r^2 = x^2 + \frac{d^2}{4} \Rightarrow r = \sqrt{\frac{d^2}{4} + x^2}$$

$$r = \left(x^2 + \frac{d^2}{4}\right)^{\frac{1}{2}}$$

Put in eq (iv)

$$E = -KP \left(x^2 + \frac{d^2}{4}\right)^{-3/2}$$

$$E = -KP \left[x^2 \left(1 + \frac{d^2}{4x^2}\right) \right]^{-3/2}$$

$$E = -KP \left[(x^2)^{-3/2} \left(1 + \frac{d^2}{4x^2}\right)^{-3/2} \right]$$

$$E = -KP x^{-3} \left(1 + \frac{d^2}{4x^2}\right)^{-3/2}$$

By Binomial Series:-

$$E_z = KPx^{-3} \left(1 + \frac{3}{2} \frac{d^2}{4x^2} + \dots \right)$$

~~E~~ Neglect the higher order terms, we get:

$$E = \frac{KP}{x^3}$$

This is the electric field at P due to electric dipole.

Where x is distance between point P and bisecting point ~~od~~ of distance b/w +q and -q.

∴ (Applied Physics) :-

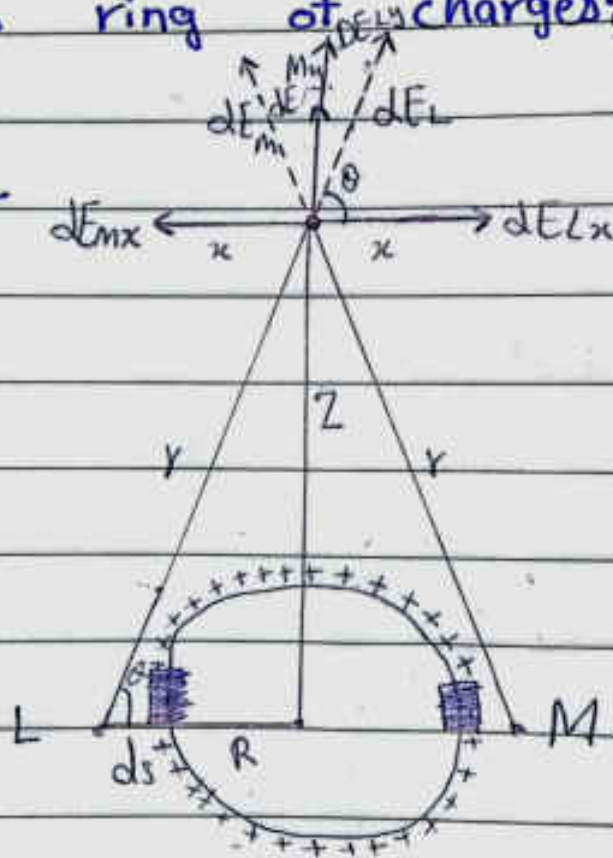
Assignment 01:-

Q No 1:-

Explain the electric field due to ring of charges?

Ans:-

Diagram:-



Explanation:-

Consider a positively charged ring having radius ' R ' on which positive charge ' q ' is distributed uniformly. This is called Linear charge distribution. Take a small

length element ds of ring having charge dq .

The linear charge density is defined as:

$$\lambda = \frac{dq}{ds}$$

$$dq = \lambda ds \quad \text{--- (i)}$$

It is given:

$$dE_r = \frac{Kdq}{r^2} \quad \text{--- (a)}$$

$$dE_m = \frac{Kdq}{r^2} \quad \text{--- (b)}$$

Comparing (a) and (b)

$$dE_r = dE_m$$

To find magnitude of electric field

$$dE = \sqrt{dE_r^2 + dE_y^2} \quad \text{--- (ii)}$$

Component of dE_r :-

$$dE_x = dE_r \cos \theta$$

$$dE_y = dE_r \sin \theta$$

Components of dE_m :-

$$dE_{mx} = dE_m \cos \theta$$

$$dE_{my} = dE_m \sin \theta$$

Resultant x-component:-

$$dE_x = dE_l \cos \theta + (-dE_m \cos \theta)$$

$$dE_x = \cancel{dE_l \cos \theta} - dE_m \cos \theta$$

$$\boxed{dE_x = 0}$$

Resultant Y-component:-

$$dE_y = dE_l \sin \theta + dE_m \sin \theta$$

$$\boxed{dE_y = 2dE \sin \theta}$$

Now eq. (ii) becomes

$$dE = \sqrt{(0)^2 + (2dE \sin \theta)^2}$$

$$dE = 2dE \sin \theta$$

$$dE = \frac{2Kdq \sin \theta}{r^2}$$

$$\left\{ \because \sin \theta = \frac{P}{H} = \frac{z}{r} \right\}$$

$$dE = \frac{2Kdq z}{r^3}$$

from eq (i)

$$dE = \frac{2K \lambda ds z}{r^3} \quad \text{--- (iii)}$$

The value of r is:

$$c^2 = a^2 + b^2$$

$$r^2 = z^2 + R^2$$

$$r = \sqrt{z^2 + R^2}$$

$$r = (z^2 + R^2)^{\frac{1}{2}}$$

Put r in (iii)

$$dE = \frac{2K \lambda ds z}{(z^2 + R^2)^{\frac{3}{2}}}$$

Integrating

$$\int dE = \int \frac{2K \lambda ds z}{(z^2 + R^2)^{\frac{3}{2}}}$$

$$E = \frac{2zK \int \lambda ds}{(z^2 + R^2)^{\frac{3}{2}}}$$

$$E = \frac{2z\lambda K \pi R}{(z^2 + R^2)^{\frac{3}{2}}} \quad \text{--- (iv)}$$

We know that

$$\lambda = \frac{dq}{ds} \Rightarrow dq = \lambda ds \Rightarrow q = \lambda \int ds$$

$$q = \lambda(2\pi R)$$

The equation (iv) is

$$E = \frac{2K\lambda(2\pi R)}{(z^2 + R^2)^{3/2}}$$

$$E = \frac{2Kq}{(z^2 + R^2)^{3/2}}$$

This is called electric field at point P due to ring of charges. When the point P lies at large distance $z \gg R$ The Term R^2 can be neglected, as compare to z^2 .

$$E = \frac{2Kq}{(z^2 + 0)^{3/2}}$$

$$E = \frac{2Kq}{(z^2)^{3/2}} \Rightarrow E = \frac{2Kq}{z^3}$$

$$E = \frac{Kq}{z^2}$$

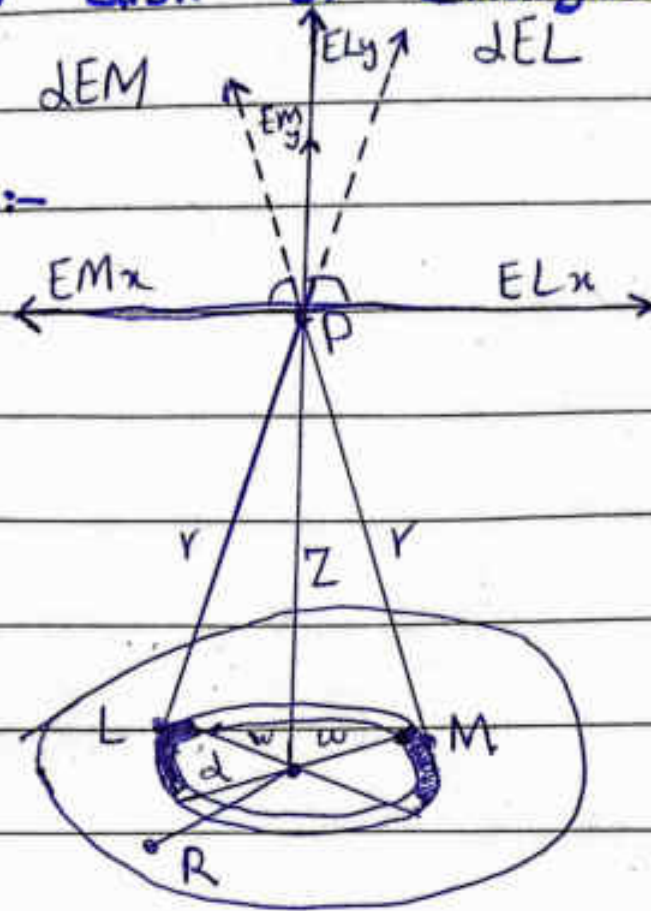


Q No 2:-

Explain the electric field due to disk of charges?

Answer:- dEM dEL

Diagram:-



Explanation:-

Consider a positively charged disk of radius R having surface charge density σ . Divide the disk into small rings. Now consider such a small ring having radius ' w ' and ~~area~~ ' dw '.

The surface charge density is:

$$\sigma = \frac{dq}{da}$$

$$\boxed{dq = \sigma da}$$

integrate:

$$q = \sigma \int da$$

$$q = \sigma (dxdy) \quad \text{--- (i)}$$

we have to calculate electric field at point P having distance z from the plane of disk.

$$dE_L = \frac{Kdq}{r^2}$$

$$dE_M = \frac{Kdq}{r^2}$$

Comparing:-

$$dE_L = dE_M$$

To calculate electric field dE at P resolve the electric field into components.

along x-axis:-

$$dE_x = dE_{Lx} + (-E_{Mx})$$

$$dE_x = E_{Lx} - E_{Mx}$$

$$\boxed{dE_x = 0}$$

along y-axis

$$dE_y = E_{Ly} + E_{My}$$

$$dE_y = 2E_{Ly}$$

$$\left\{ \because E_{Ly} = E_{Lm} \right\}$$

$$dE_y = 2E_{Ly} \sin \theta$$

The magnitude of electric field is:

$$dE = \sqrt{dE_x^2 + dE_y^2}$$

$$dE = \sqrt{(0)^2 + (2E_{Ly} \sin \theta)^2}$$

$$dE = 2E_{Ly} \sin \theta$$

$$dE = \frac{2KQda}{r^2} \sin \theta$$

$$\left\{ \begin{array}{l} \sin \theta = \frac{P}{H} = \frac{Z}{r} \end{array} \right\}$$

from eq (i)

$$dE = \frac{2KQda}{r^2} \left(\frac{Z}{r} \right)$$

$$dE = \frac{2ZKQda}{r^3}$$

For 'r' By pythagorean Theorem:-

$$c^2 = a^2 + b^2$$

$$r^2 = Z^2 + R^2$$

$$r = (z^2 + R^2)^{\frac{1}{2}}$$

Put in σ

$$dE = \frac{2zK\sigma da}{r^3}$$

$$dE = \frac{2zK\sigma da}{(z^2 + R^2)^{\frac{3}{2}}}$$

Integrate

$$\int dE = \int_0^R \frac{2zK\sigma (d\alpha dw)}{(z^2 + R^2)^{\frac{3}{2}}}$$

$$\int dE = 2K\sigma z \int_0^R \frac{2wdw}{(z^2 + R^2)^{\frac{3}{2}}} \int_0^\pi d\alpha$$

$$\int dE = K\sigma z \int_0^R (z^2 + R^2)^{-\frac{3}{2}} 2wdw (\pi)$$

$$\int dE = K\sigma z \left| (z^2 + R^2)^{-\frac{1}{2}} \right|_0^R$$

$$\int dE = \frac{\pi K\sigma z}{4\pi\epsilon_0} \left| -2 (z^2 + R^2)^{-\frac{1}{2}} \right|_0^R$$

$$\int dE = \frac{-2\pi\sigma z}{4\pi\epsilon_0} \left(\frac{1}{(z^2 + R^2)^{\frac{1}{2}}} - \frac{1}{(z^2 + 0)^{\frac{1}{2}}} \right)$$

$$\int dE_z = \frac{\sigma z}{2\epsilon_0} \left(\frac{1}{z} - \frac{1}{(z^2 + R^2)^{\frac{1}{2}}} \right)$$

$$\int dE_z = \frac{\sigma}{2\epsilon_0} \left(\frac{z}{z} - \frac{z}{(z^2 + R^2)^{\frac{1}{2}}} \right)$$

$$\int dE_z = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{(z^2 + R^2)^{\frac{1}{2}}} \right)$$

if $R \rightarrow \infty$

$$E_z = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\infty} \right)$$

$$E_z = \frac{\sigma}{2\epsilon_0} (1)$$

$$\boxed{E_z = \frac{\sigma}{2\epsilon_0}}$$

if $z \ll R$

then $z \rightarrow 0$

$$\boxed{E_z = \frac{\sigma}{2\epsilon_0}}$$