QNo7:-

Define and explain electric flux?

Ans:-

Electric Flux:-

The total number of electric field lines passing through unit area is called Electric Flux. It is denoted by 'ϕ'.

In Mathematically:-

The dot product of Electric field intensity and vector Area is Known as Electric Flux.

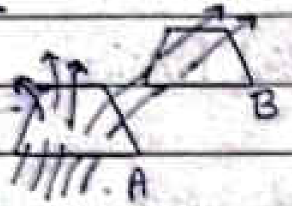
$$d\phi_e = \vec{E} \cdot \vec{A}$$

$$d\phi_e = EA\cos\theta$$

B The net electric flux is:-

$$\phi_e = \int EA\cos\theta$$

Diagram:-



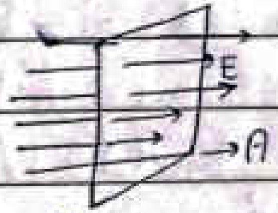
Formula: $\phi = E \cdot A \cos\theta$

Unit: Nm^2C^{-1}

It is a scalar quantity.

Maximum Flux:-

If area is Parallel ~~(perpendicular)~~ to the direction of electric field then flux will be maximum.



$\theta = 0$

Mathematically:-

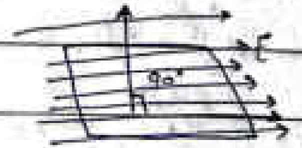
$$\phi = EA\cos\theta$$

$$\phi = EA\cos(0^\circ)$$

$$\boxed{\phi = EA}$$

Minimum Flux:-

If the area is Perpendicular ~~(parallel)~~ to the direction of electric fields lines then flux will be minimum.



Mathematically:-

$$\phi = EA\cos\theta$$

$$\phi = EA\cos(90^\circ)$$

$$\boxed{\phi = 0}$$

-:(Applied Physics):-

Semestor 01:-

QNo8:-

State and prove Gauss's law and write integral and differential form?

Answer:-

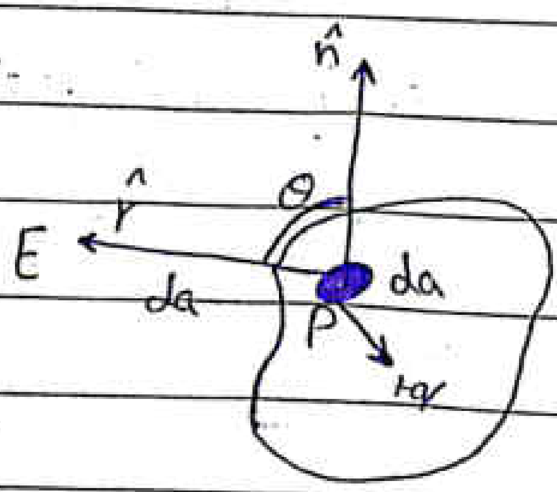
Gauss's Law:-

The Gauss's law state that:
The electric flux passing through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times charge enclosed by that surface.

$$\phi = \frac{q}{\varepsilon}$$

Proof:-

Diagram:-



Consider a +q charge is placed in a closed surface. The electric field lines come out of the surface. Take small element $\vec{da} = da\,\hat{n}$ of the surface at point P.
The electric field at point 'P' due to +q charge having distance 'r' is given by:

$$\vec{E} = \frac{Kq}{r^2}\hat{r}$$

The electric flux is:

$$d\phi \cdot \vec{E}\cdot\vec{da}$$

Integrate:

$$\phi = \int \vec{E}\cdot\vec{da}$$

$$\phi = \int \frac{Kq}{r^2}\hat{r}\cdot da\hat{n}$$

$$\phi = \frac{Kq}{r^2}\int da(\hat{r}\cdot\hat{n})$$

$$\phi = \frac{Kq}{r^2}\int da(\hat{r}\cdot\hat{n})$$

$$\phi = Kq \int \frac{da}{r^2} (\hat{r} \cdot \hat{n})$$

$$\phi = Kq \int d\Omega$$

$$\phi = Kq (4\pi) \longrightarrow \text{steradian}$$

Putting value of K:-

$$\phi = \frac{1}{\cancel{4\pi}\varepsilon_0} q \cancel{(4\pi)}$$

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$ Hence proved.

(i) Integral form of Gauss's law:-

we Know that, the volume charge density is:

$$\rho = \frac{dq}{dv}$$

$$dq = \rho\, dv \quad \text{— (a)}$$

The net electric flux is:

$$\phi = \int \vec{E} \cdot \vec{da} \quad \text{— (b)}$$

put eq (a) and (b) in:

$$\phi = \frac{q}{\varepsilon_0}$$

Putting values:

$$\boxed{\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int(\rho\, dv)}$$

This is called Integral form of Gauss's law.

(ii) Differential form of Gauss's law:-

we know that:

$$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\rho\left(\int dv\right) \quad \text{--- (i)}$$

Applying divergence theorem:

$$\oint \vec{E}\cdot d\vec{a} = \int(\vec{\nabla}\cdot\vec{E})\,dv \quad \text{--- (ii)}$$

Compairing eq (i) and (ii)

$$\frac{1}{\varepsilon_0}\rho\left(\int dv\right) = \int(\vec{\nabla}\cdot\vec{E})\,dv$$

$$0 = \int(\vec{\Delta}\cdot\vec{E})\,dv - \frac{1}{\varepsilon_0}\rho\left(\int dv\right)$$

$$0 = \int\left((\vec{\Delta}\cdot\vec{E}) - \frac{\rho}{\varepsilon_0}\right)dv$$

$$\boxed{\frac{\rho}{\varepsilon_0} = \vec{\Delta}\cdot\vec{E}}$$ This is called differential form of gauss's law.

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:(Applied Physics):-

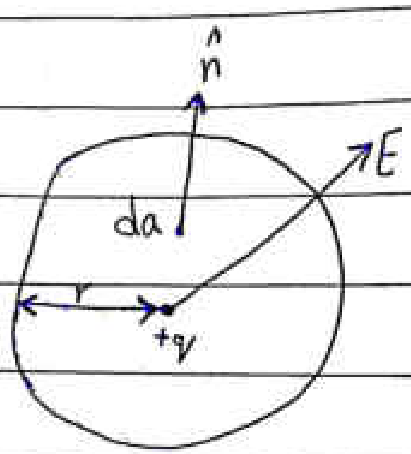
Semester 01:-

Qno9:-

Derive Columb's law from Gauss's law?

Ans:-

Diagram:-



Explanation:-

Consider a +q charge is placed at the center of the sphere having radius r. The sphere is called gussasian sphere.

Take small area element $\vec{da}$ of sphere. The unit vector $\hat{n}$ indicates the direction of 'da'.

The angle between $\hat{n}$ and $\vec{E}$ is zero.

The flux through surface of sphere is:

$$\phi_e = E.A$$

$$d\phi_e = E \cdot da$$

$$d\phi_e = E da \cos(0)^\circ$$

$$d\phi_e = E da$$

Integration:-

$$\phi_e = E\int da$$

$$\phi_e = E(4\pi r^2) \quad \text{—— (i)}$$

According to Gauss's law:

$$\phi_e = \frac{q}{\varepsilon_o} \quad \text{—— (ii)}$$

Compairing (i) and (ii)

$$E(4\pi r^2) = \frac{q}{\varepsilon_o}$$

$$E = \frac{q}{4\pi r^2 \varepsilon_o} = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$$

$$E = \frac{Kq}{r^2} \quad \text{—— (iii)}$$

we Know that $\Rightarrow$ $E = \frac{F}{q_o}$

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So,

$$\frac{F}{q_\circ} = \frac{kq}{r^2}$$

$$\boxed{F = \frac{kqq_\circ}{r^2}}$$

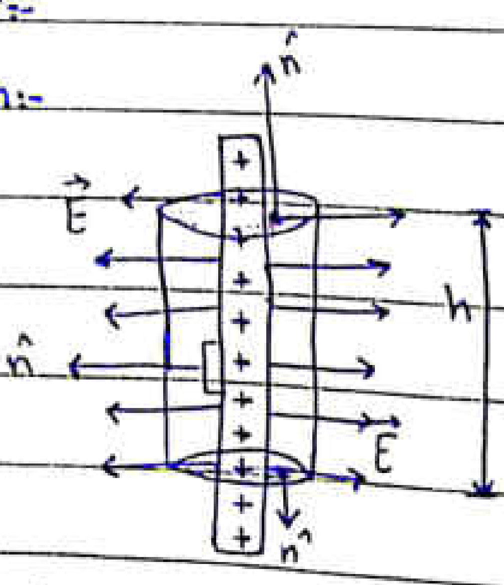
This is statement of Columb's law.

QNo10:-

Calculate electric field due to infinite line of charges by using Gauss's law?

Answer:-

Diagram:-



Explanation:-

Consider a straight positively charged wire having infinite length

Now take a cylinder called gussasian surface having radius 'r' and height 'h'

Take a small length element 'dz' of the line having charge 'dq'.

The linear charge density is:

$$\lambda = \frac{dq}{dz}$$

$$dq = \lambda dz$$

Integration :-

$$q = \lambda \int dz$$

$$q = \lambda h \quad \text{———} \quad (i)$$

The flux through surface is:

$$d\phi = \vec{E} \cdot \vec{da} = E da \cos(0^\circ)$$

$$d\phi = E da$$

Integration:-

$$\phi = E \int da = E(L \times W)$$

$$\phi = E(2\pi r h) \quad \text{———} \quad (ii)$$

by Gauss's law :-

$$\phi = \frac{q}{\varepsilon_0}$$

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From eq (i)

$$\phi = \frac{\lambda h}{\varepsilon_o} \quad \text{(iii)}$$

Compairing eq (ii) and (iii)

$$E(2\pi r h) = \frac{\lambda h}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi r \varepsilon_o} = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{r}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_o r}}$$

In vector form:-

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_o r}\,\hat{r}$$

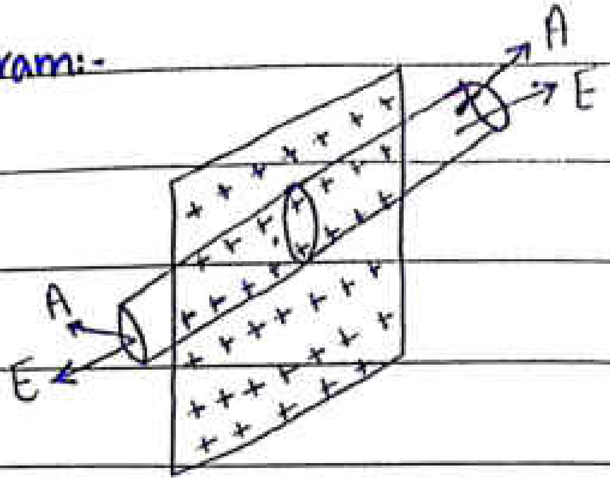
-: Long :-

Q.No 11.-

Calculate the electric field intensity due to infinite sheet of charges?

Answer:-

Diagram:-



Explanation:-

Consider a positively charged sheet having infinite length. We have to calculate electric field $\vec{E}$ at points near the sheet by Gauss's law:

Imagine a cèlender/cylinder called guassion surface having cross-sectional Area 'A'.

Take small area element 'da' of sheet having charge 'dq'.

The surface charge density is:

$$\sigma = \frac{dq}{da} \Rightarrow dq = \sigma da$$

Integration:-

$$q = \sigma \int da$$

$$q = \sigma A \quad \text{(i)}$$

The flux through surface is:

$$d\phi_e = Eda\cos(0^\circ) + Eda\cos(0^\circ)$$

$$d\phi_e = Eda + Eda$$

$$d\phi_e = 2Eda$$

Integration:-

$$\phi_e = 2E\int da$$

$$\phi_e = 2EA \quad \text{——— (ii)}$$

By Gauss's law:

$$\phi_e = \frac{q}{\varepsilon_o}$$

from (i)

$$\phi_e = \frac{\sigma A}{\varepsilon_o} \quad \text{——— (iii)}$$

Compairing (ii) and (iii)

$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_o}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_o}}$$

In vector form

$$\vec{E} = \frac{\sigma}{2\varepsilon_o} \times \hat{n}$$

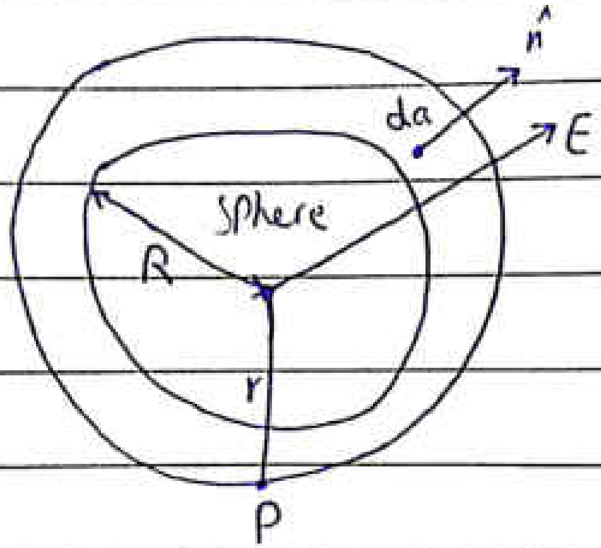
-: Long:-

QNo12:-

Calculate the electric field at a point due to spherical volume charge distribution?

Answer:-

Diagram:-



Explanation:-

Consider a sphere of rudius R having uniform positive charge. Take a small volume element 'dV' of the sphere having charge 'dq'.

The volume charge density is:

$$\rho = \frac{dq}{dv}$$

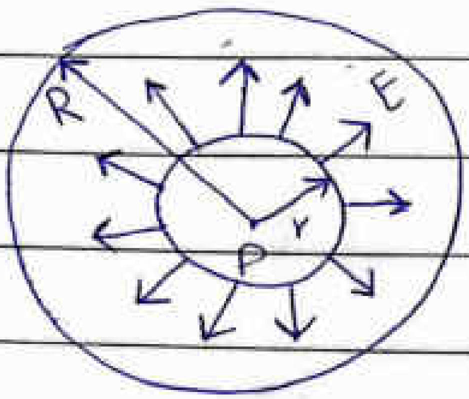
$$dq = \rho dv$$

-:(Applied Physics):-

Semester 01:-

13:- **Electric field inside spherical charge distribution:-**

Diagram:-



Explanation:-

Consider a uniformly charged sphere having radius R. Take a small volume element dv of the sphere having charge dq. The volume charge density is defined as:

$$\rho = \frac{dq}{dv}$$

$$dq = \rho\, dv$$

Integration:-

$$q = \int \rho \, dv$$

$$q = \frac{4}{3}\pi R^3 \rho \quad \text{———} \quad (i)$$

Now take a point P inside charged sphere having distance 'r' from central charged sphere.

$$q' = \frac{4}{3}\pi r^3 \rho \quad \text{———} \quad (ii)$$

divided eq(ii) by eq(i)

$$\frac{q'}{q} = \frac{r^3}{R^3}$$

$$q' = \frac{q r^3}{R^3} \quad \text{———} \quad (iii)$$

The electric flux is :

$$\phi = \int E \, da \cos\theta \quad \{\theta = 0^\circ\}$$

$$\phi = E \int da$$

$$\phi = E(4\pi r^2) \quad \text{———} \quad (iv)$$

By Gauss's law:

$$\phi = \frac{q'}{\varepsilon_0}$$

Putting values

$$E(4\pi r^2) = \frac{q r^3}{R^3 \varepsilon_0}$$

$$E = \frac{q r^3}{4\pi\varepsilon_0 r^2 R^3}$$

$$E = \frac{q r}{4\pi\varepsilon_0 R^3}$$

Electric field at surface of sphere:

$$E = \frac{q r}{4\pi\varepsilon_0 R^3}$$

Put $r = R$, to calculate electric field at the surface of sphere.

$$E = \frac{q \cancel{R}}{4\pi\varepsilon_0 R^3} = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}$$

$$\boxed{E = \frac{Kq}{R^3}}$$

-:(Applied Physics):-

Semester # 01:-

Qno24:-

Calculate electric field intensity due to spherical charged shell of constant surface charge density?

Answer:-

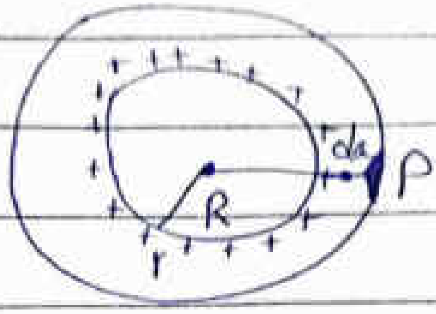
SHELL Theorem:-

There are two shell theorems established by using Gauss's law for uniform spherical shell of charges having constant surface charge density.

1st SHELL Theorem:-

"A uniform spherical shell of charges behaves for external points in such a way that all its charges was concentrated at its center."

Proof:-



The surface charge density is:

$$\sigma = \frac{dq}{da}$$

$$dq = \sigma\, da$$

Integral:

$$q = \sigma \int da$$

$$q = \sigma (4\pi R^3) \quad \text{(i)}$$

Now consider a point P outside shell having distance r from the center of spherical charged shell.

The electric flux is:

$$\phi = \int E\, da$$

$$\phi = E(4\pi r^2) \quad \text{(ii)}$$

By Gauss's law:

$$\phi = \frac{q}{\varepsilon_0}$$

Putting values from (i) and (ii)

$$\phi = \frac{q}{\varepsilon_o}$$

$$E(4\pi r^2) = \frac{\sigma(4\pi R^3)}{\varepsilon_o}$$

$$E = \frac{\sigma(\cancel{4\pi} R^3)}{\cancel{4\pi} r^2 \varepsilon_o}$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_o}$$

$$\boxed{E = \frac{Kq}{r^2}}$$

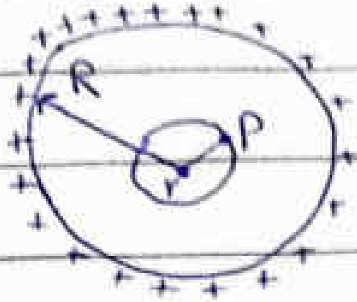
This is electric field at point P due to point charge q.
First shell theorem is proved.

2nd SHELL Theorem:-

"A uniform charged spherical shell exerts no electrostatics force on charged particle placed inside the shell."

Proof:-

Diagram:-



Consider a positively charged spherical shell having radius R and uniform surface density σ. The net charge on shell is q.

Take a point 'P' inside the shell where we want to evaluate electric field.

Electric flux is:

$$d\phi = E\,da$$

Integrate:

$$\phi = E\int da$$

$$\phi = E(4\pi r^2) \quad \text{(i)}$$

By Gauss's law:

$$\phi = \frac{q}{\varepsilon_0}$$

Put value

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$$\Rightarrow E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Inside the guassian surface, charge is zero ($q=0$)

$$E = \frac{0}{4\pi\varepsilon_0 r^2} = 0$$

$$\boxed{E = 0}$$

The electric force on charged placed inside guassian shell.

$$F = qE$$

$$F = q(0)$$

$$\boxed{F = 0}$$

Hence, the 2nd shell theorem is proved.