Exercise 1.4

Q. 1: Which of the following product of matrices is conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Solution:

(i) The order of first matrix is 2-by-2 and second matrix is 2-by-1
No. of columns of first matrix = No. of Rows of second matrix
So, Conformable for multiplication.

(ii) The order of first matrix is 2-by-2 and second matrix is 2-by-2
No. of columns of first matrix = No. of Rows of second matrix
So, Conformable for multiplication.

(iii) The order of first matrix is 2-by-1 and second matrix is 2-by-2
No. of columns of first matrix ≠ No. of Rows of second matrix
So, not conformable for multiplication.

(iv) The order of first matrix is 3-by-2 and second matrix is 2-by-3
No. of columns of first matrix = No. of Rows of second matrix
So, Conformable for multiplication.

(v) The order of first matrix is 2-by-3 and second matrix is 3-by-2
No. of columns of first matrix = No. of Rows of second matrix
So, Conformable for multiplication.

Q. 2: If A = $\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$, find (i) AB (ii) BA (if possible)

Solution:

AB $= \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 6 + 0 \times 5 \\ -1 \times 6 + 2 \times 5 \end{bmatrix}$

$= \begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix}$

$= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$

BA is not possible because order of B is 2-by-1 and order of A is 2-by-2.
No of columns of B ≠ No of Rows of A

Q. 3: Find the following products.

(i) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ $= [1 \times 4 + 2 \times 0]$

$= [4 + 0]$

$= [4]$

(ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}$ $= [1 \times 5 + 2 \times -4]$

$= [5 - 8]$

$= [-3]$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$ $= [-3 \times 4 + 0 \times 0]$

$= [-12 + 0]$

$= [-12]$

(iv) $\begin{bmatrix}6 & -0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$ $= [6 \times 4 + -0 \times 0]$

$= [24 + 0]$

$= [24]$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix} = \begin{bmatrix}1 \times 4 + 2 \times 0 & 1 \times 5 + 2 \times -4\\-3 \times 4 + 0 \times 0 & -3 \times 5 + 0 \times -4\\6 \times 4 + -1 \times 0 & 6 \times 5 + -1 \times -4\end{bmatrix}$

$= \begin{bmatrix}4 + 0 & 5 - 8\\-12 + 0 & -15 + 0\\24 + 0 & 30 + 4\end{bmatrix}$

$= \begin{bmatrix}4 & -3\\-12 & -15\\24 & 34\end{bmatrix}$

Q. 4: Multiply the following matrices.

(a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix} = \begin{bmatrix}2 \times 2 + 3 \times 3 & 2 \times -1 + 3 \times 0\\1 \times 2 + 1 \times 3 & 1 \times -1 + 1 \times 0\\0 \times 2 + -2 \times 3 & 0 \times -1 + -2 \times 0\end{bmatrix}$

$= \begin{bmatrix}4 + 9 & -2 + 0\\2 + 3 & -1 + 0\\0 - 6 & 0 + 0\end{bmatrix}$

$= \begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$

(b) $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix} = \begin{bmatrix}1 \times 1 + 2 \times 3 + 3 \times -1 & 1 \times 2 + 2 \times 4 + 3 \times 1\\4 \times 1 + 5 \times 3 + 6 \times -1 & 4 \times 2 + 5 \times 4 + 6 \times 1\end{bmatrix}$

$= \begin{bmatrix}1 + 6 - 3 & 2 + 8 + 3\\4 + 15 - 6 & 8 + 20 + 6\end{bmatrix}$

$= \begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}1 \times 1 + 2 \times 4 & 1 \times 2 + 2 \times 5 & 1 \times 3 + 2 \times 6\\3 \times 1 + 4 \times 4 & 3 \times 2 + 4 \times 5 & 3 \times 3 + 4 \times 6\\-1 \times 1 + 1 \times 4 & -1 \times 2 + 1 \times 5 & -1 \times 3 + 1 \times 6\end{bmatrix}$

$= \begin{bmatrix}1 + 8 & 2 + 10 & 3 + 12\\3 + 16 & 6 + 20 & 9 + 24\\-1 + 4 & -2 + 5 & -3 + 6\end{bmatrix}$

$= \begin{bmatrix}9 & 12 & 15\\19 & 26 & 33\\3 & 3 & 3\end{bmatrix}$

(d) $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$ $= \begin{bmatrix} 8\times 2+5\times -4 & 8\times -\frac{5}{2}+5\times 4 \\ 6\times 2+4\times -4 & 6\times -\frac{5}{2}+4\times 4 \end{bmatrix}$

$= \begin{bmatrix} 16-20 & 4\times -5+5\times 4 \\ 12-16 & 3\times -5+4\times 4 \end{bmatrix}$

$= \begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix}$

$= \begin{bmatrix} -4 & 0 \\ -4 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $= \begin{bmatrix} -1\times 0+2\times 0 & -1\times 0+2\times 0 \\ 1\times 0+3\times 0 & 1\times 0+3\times 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Q. 5: For the matrices A = $\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ verify whether

(i) AB = BA

L.H.S = AB

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

$= \begin{bmatrix} -1\times 1+3\times -3 & -1\times 2+3\times -5 \\ 2\times 1+0\times -3 & 2\times 2+0\times -5 \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$

$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$

R.H.S = BA

$= \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1\times -1+2\times 2 & 1\times 3+2\times 0 \\ -3\times -1+-5\times 2 & -3\times 3+-5\times 0 \end{bmatrix}$

$= \begin{bmatrix} -1+4 & 3+0 \\ 3+-10 & -9+0 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 \\ -7 & -9 \end{bmatrix}$

L.H.S ≠ R.H.S

(ii) A(BC) = (AB)C

L.H.S = A(BC)

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1\times 2+2\times 1 & 1\times 1+2\times 3 \\ -3\times 2+-5\times 1 & -3\times 1+-5\times 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2+2 & 1+6 \\ -6-5 & -3-15 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$

$= \begin{bmatrix} -1\times 4+3\times -11 & -1\times 7+3\times -18 \\ 2\times 4+0\times -11 & 2\times 7+0\times -18 \end{bmatrix}$

$= \begin{bmatrix} -4-33 & -7-54 \\ 8+0 & 14+0 \end{bmatrix}$

$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$

R.H.S $= (AB)C$

$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right)\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1\times1+3\times-3 & -1\times2+3\times-5 \\ 2\times1+0\times-3 & 2\times2+0\times-5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -10\times2+-17\times1 & -10\times1+-17\times3 \\ 2\times2+4\times1 & 2\times1+4\times3 \end{bmatrix}$

$= \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$

$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$

L.H.S = R.H.S

(iii) $A(B + C) = AB + AC$

L.H.S $= A(B + C)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}+\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$

$= \begin{bmatrix} -1\times3+3\times-2 & -1\times3+3\times-2 \\ 2\times3+0\times-2 & 2\times3+0\times-2 \end{bmatrix}$

$= \begin{bmatrix} -3-6 & -3-6 \\ 6+0 & 6+0 \end{bmatrix}$

$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$

R.H.S $= AB + AC$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}+\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1\times1+3\times-3 & -1\times2+3\times-5 \\ 2\times1+0\times-3 & 2\times2+0\times-5 \end{bmatrix}+\begin{bmatrix} -1\times2+3\times1 & -1\times1+3\times3 \\ 2\times2+0\times1 & 2\times1+0\times3 \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}+\begin{bmatrix} -2+3 & -1+9 \\ 4+0 & 2+0 \end{bmatrix}$

$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}+\begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} -10+1 & -17+8 \\ 2+4 & 4+2 \end{bmatrix}$

$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$

L.H.S = R.H.S

(iv) $A(B - C) = AB - AC$

L.H.S $= A(B - C)$

$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix} - \begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$

$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix}$

$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$

$= \begin{bmatrix}-1\times -1+3\times -4 & -1\times 1+3\times -8\\ 2\times -1+0\times -4 & 2\times 1+0\times -8\end{bmatrix}$

$= \begin{bmatrix}1-12 & -1-24\\ -2+0 & 2+0\end{bmatrix}$

$= \begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$

R.H.S = AB - AC

$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix} - \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$

$= \begin{bmatrix}-1\times 1+3\times -3 & -1\times 2+3\times -5\\ 2\times 1+0\times -3 & 2\times 2+0\times -5\end{bmatrix} - \begin{bmatrix}-1\times 2+3\times 1 & -1\times 1+3\times 3\\ 2\times 2+0\times 1 & 2\times 1+0\times 3\end{bmatrix}$

$= \begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix} - \begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$

$= \begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix} - \begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$

$= \begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$

$= \begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$

L.H.S = R.H.S

Q. 6: For the matrices

$\mathbf{A} = \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}, \mathbf{B} = \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}, \mathbf{C} = \begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Verify that (i) $(AB)^t = B^tA^t$ (ii) $(BC)^t = C^tB^t$

(i) L.H.S $= (AB)^t$

$= \left(\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\right)^t$

$= \begin{bmatrix}-1\times 1+3\times -3 & -1\times 2+3\times -5\\ 2\times 1+0\times -3 & 2\times 2+0\times -5\end{bmatrix}^t$

$= \begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}^t$

$= \begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}^t$

$= \begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$

R.H.S $= B^tA^t$

$= \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}^t \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}^t$

$= \begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$

$= \begin{bmatrix}1\times -1+-3\times 3 & 1\times 2+-3\times 0\\ 2\times -1+-5\times 3 & 2\times 2+-5\times 0\end{bmatrix}$

$= \begin{bmatrix}-1-9 & 2+0\\ -2-15 & 4+0\end{bmatrix}$

$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$

L.H.S = R.H.S

(ii) L.H.S $= (BC)^t$

$= \left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}\right)^t$

$= \begin{bmatrix} 1\times -2 + 2\times 3 & 1\times 6 + 2\times -9 \\ -3\times -2 + -5\times 3 & -3\times 6 + -5\times -9 \end{bmatrix}^t$

$= \begin{bmatrix} -2+6 & 6-18 \\ 6-15 & -18+45 \end{bmatrix}^t$

$= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}^t$

$= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$

R.H.S $= C^tB^t$

$= \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}^t \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t$

$= \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$

$= \begin{bmatrix} -2\times 1 + 3\times 2 & -2\times -3 + 3\times -5 \\ 6\times 1 + -9\times 2 & 6\times -3 + -9\times -5 \end{bmatrix}$

$= \begin{bmatrix} -2+6 & 6-15 \\ 6-18 & -18+45 \end{bmatrix}$

$= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$

L.H.S = R.H.S