Exercise 4.3

Resolve into partial fractions.

1. $\frac{3x-11}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+c}{(x^2+1)}$ .... (i)

multiplying by $(x+3)(x^2+1)$ we get

$3x-11 = A(x^2+1) + (Bx+c)(x+3)$ .... (ii)

$3x-11 = A(x^2+1) + B(x^2+3x) + C(x+3)$

$3x-11 = A(x^2+1) + B(x^2+3x) + C(x+3)$ .... (iii)

put $x = -3$ in (ii)

$3(-3)-11 = A((-3)^2+1) + (B(-3)+c)(-3+3)$

$-9-11 = A(9+1)$

$-20 = 10A$

$A = -2$

Now, comparing coefficients of equation (iii)

x^2; $A + B = 0$

as A = -2

$-2 + B = 0$

$B = 2$

x; $3B + C = 3$

as B = 2

$6 + C = 3$

$C = -3$

put the values in (i) we get

$\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{(x+3)} + \frac{2x-3}{(x^2+1)}$

2. $\frac{3x+7}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+c}{(x^2+1)}$ .... (i)

multiplying by $(x+3)(x^2+1)$ we get

$3x+7 = A(x^2+1) + (Bx+c)(x+3)$ .... (ii)

$3x+7 = A(x^2+1) + B(x^2+3x) + C(x+3)$

$3x+7 = A(x^2+1) + B(x^2+3x) + C(x+3)$ .... (iii)

put $x = -3$ in (ii)

$3(-3)+7 = A((-3)^2+1) + (B(-3)+c)(-3+3)$

$-9+7 = A(9+1)$

$-2 = 10A$

$A = \frac{-1}{5}$

Now, comparing coefficients of equation (iii)

x^2; $A + B = 0$

as A = $\frac{-1}{5}$

$\frac{-1}{5} + B = 0$

$B \quad = \frac{1}{5}$

$x;$ $\quad 3B + C = 3$

as B = $\frac{1}{5}$

$\frac{3}{5} + C \quad = 3$

$C \quad = 3 - \frac{3}{5}$

$C \quad = \frac{15-3}{5}$

$C \quad = \frac{12}{5}$

put the values in (i) we get

$$\frac{3x+7}{(x+3)(x^2+1)} = \frac{\frac{-1}{5}}{(x+3)} + \frac{\frac{1}{5}x+\frac{12}{5}}{(x^2+1)}$$

$$= \frac{\frac{-1}{5}}{(x+3)} + \frac{\frac{x+12}{5}}{(x^2+1)}$$

$$= \frac{-1}{5(x+3)} + \frac{x+12}{5(x^2+1)}$$

3. $\frac{1}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+c}{(x^2+1)}$ .... (i)

multiplying by $(x+1)(x^2+1)$ we get

$1 \quad = A(x^2+1) + (Bx+c)(x+1)$ .... (ii)

$1 \quad = A(x^2+1) + B(x^2+x) + C(x+1)$

$1 \quad = A(x^2+1) + B(x^2+x) + C(x+1)$ .... (iii)

put $x = -1$ in (ii)

$1 \quad = A((-1)^2+1) + (B(-1)+c)(-1+1)$

$-9+7 \quad = A(1+1)$

$-2 \quad = 2A$

$A \quad = -1$

Now, comparing coefficients of equation (iii)

$x^2;$ $\quad A + B \; = 0$

as A = -1

$-1 + B = 0$

$B \quad = 1$

$x;$ $\quad B + C = 0$

as B = 1

$1 + C \; = 0$

$C \quad = -1$

put the values in (i) we get

$$\frac{1}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+c}{(x^2+1)}$$

$$= \frac{-1}{(x+1)} + \frac{x-1}{(x^2+1)}$$

4. $\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+c}{(x^2+1)}$ .... (i)

multiplying by $(x+3)(x^2+1)$ we get

$$9x-7 = A(x^2+1)+(Bx+c)(x+3) \text{ .... (ii)}$$
$$9x-7 = A(x^2+1)+B(x^2+3x)+C(x+3)$$
$$9x-7 = A(x^2+1)+B(x^2+3x)+C(x+3) \text{ .... (iii)}$$

put $x=-3$ in (ii)

$$9(-3)-7 = A((-3)^2+1)+(B(-3)+c)(-3+3)$$
$$-27-7 = A(9+1)$$
$$-34 = 10A$$
$$A = \frac{-17}{5}$$

Now, comparing coefficients of equation (iii)

x^2; $A+B=0$

as A = $\frac{-17}{5}$

$\frac{-17}{5}+B=0$

$B = \frac{17}{5}$

x; $3B+C=9$

as B = $\frac{17}{5}$

$\frac{51}{5}+C=9$

$C = 9-\frac{51}{5}$

$C = \frac{45-51}{5}$

$C = \frac{-6}{5}$

put the values in (i) we get

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{\frac{-17}{5}}{(x+3)}+\frac{\frac{17}{5}x+\frac{-6}{5}}{(x^2+1)}$$
$$= \frac{\frac{-17}{5}}{(x+3)}+\frac{\frac{17x-6}{5}}{(x^2+1)}$$
$$= \frac{-17}{5(x+3)}+\frac{17x-6}{5(x^2+1)}$$

5. $\frac{3x+7}{(x+3)(x^2+4)} = \frac{A}{(x+3)}+\frac{Bx+c}{(x^2+4)}$ .... (i)

multiplying by $(x+3)(x^2+4)$ we get

$$3x+7 = A(x^2+4)+(Bx+c)(x+3) \text{ .... (ii)}$$
$$3x+7 = A(x^2+4)+B(x^2+3x)+C(x+3)$$
$$3x+7 = A(x^2+4)+B(x^2+3x)+C(x+3) \text{ .... (iii)}$$

put $x=-3$ in (ii)

$$3(-3)+7 = A((-3)^2+4)+(B(-3)+c)(-3+3)$$
$$-9+7 = A(9+4)$$
$$-2 = 13A$$
$$A = \frac{-2}{13}$$

Now, comparing coefficients of equation (iii)

$x^2;\quad A + B = 0$

as A = $\frac{-2}{13}$

$\frac{-2}{13} + B = 0$

$B = \frac{2}{13}$

$x;\quad 3B + C = 3$

as B = $\frac{2}{13}$

$\frac{6}{13} + C = 3$

$C = 3 - \frac{6}{13}$

$C = \frac{39-6}{13}$

$C = \frac{33}{13}$

put the values in (i) we get

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{\frac{-2}{13}}{(x+3)} + \frac{\frac{2}{13}x+\frac{33}{13}}{(x^2+4)}$$

$$= \frac{\frac{-2}{13}}{(x+3)} + \frac{\frac{2x+33}{13}}{(x^2+1)}$$

$$= \frac{-2}{13(x+3)} + \frac{2x+33}{13(x^2+1)}$$

6. $\frac{x^2}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{Bx+c}{(x^2+4)}$ .... (i)

multiplying by $(x+2)(x^2+4)$ we get

$x^2 = A(x^2+4) + (Bx+c)(x+2)$ .... (ii)

$x^2 = A(x^2+4) + B(x^2+2x) + C(x+2)$

$x^2 = A(x^2+4) + B(x^2+2x) + C(x+2)$ .... (iii)

put $x = -2$ in (ii)

$(-2)^2 = A((-2)^2+4) + (B(-2)+c)(-2+3)$

$4 = A(4+4)$

$4 = 8A$

$A = \frac{1}{2}$

Now, comparing coefficients of equation (iii)

$x^2;\quad A + B = 1$

as A = $\frac{1}{2}$

$\frac{1}{2} + B = 1$

$B = 1 - \frac{1}{2}$

$B = \frac{1}{2}$

$x;\quad 2B + C = 0$

as B = $\frac{1}{2}$

$1 + C = 0$

$C = -1$

put the values in (i) we get

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{\frac{1}{2}}{(x+2)} + \frac{\frac{1}{2}x-1}{(x^2+4)}$$

$$= \frac{\frac{1}{2}}{(x+3)} + \frac{\frac{x-2}{2}}{(x^2+1)}$$

$$= \frac{1}{2(x+3)} + \frac{x-2}{2(x^2+1)}$$

7. $\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)} + \frac{Bx+c}{(x^2-x+1)}$ .... (i)

multiplying by $(x+1)(x^2-x+1)$ we get

$1 = A(x^2-x+1) + (Bx+c)(x+1)$ .... (ii)

$1 = A(x^2-x+1) + B(x^2+x) + C(x+1)$

$1 = A(x^2-x+1) + B(x^2+x) + C(x+1)$ .... (iii)

put $x = -1$ in (ii)

$(-1)^2 = A((-1)^2 - (-1) + 1) + (B(-1) + c)(-1+1)$

$1 = A(1+1+1)$

$1 = 3A$

$A = \frac{1}{3}$

Now, comparing coefficients of equation (iii)

$x^2;$ $A + B = 0$

as A = $\frac{1}{3}$

$\frac{1}{3} + B = 0$

$B = 0 - \frac{1}{3}$

$B = -\frac{1}{3}$

$Const; A + C = 1$

as A = $\frac{1}{3}$

$\frac{1}{3} + C = 1$

$C = 1 - \frac{1}{3}$

$C = \frac{3-1}{3}$

$C = \frac{2}{3}$

put the values in (i) we get

$$\frac{1}{x^3+1} = \frac{\frac{1}{3}}{(x+1)} + \frac{-\frac{1}{3}x+\frac{2}{3}}{(x^2-x+1)}$$

$$= \frac{\frac{1}{3}}{(x+1)} + \frac{\frac{-x+2}{3}}{(x^2-x+1)}$$

$$= \frac{1}{3(x+1)} + \frac{-x+2}{3(x^2-x+1)}$$

8. $\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)} + \frac{Bx+c}{(x^2-x+1)}$ .... (i)

multiplying by $(x+1)(x^2-x+1)$ we get

$x^2+1 = A(x^2-x+1) + (Bx+c)(x+1)$ .... (ii)

$x^2+1 = A(x^2-x+1) + B(x^2+x) + C(x+1)$

$x^2+1 = A(x^2-x+1) + B(x^2+x) + C(x+1)$ .... (iii)

put $x = -1$ in (ii)

$(-1)^2+1 = A((-1)^2-(-1)+1) + (B(-1)+c)(-1+1)$

$2 = A(1+1+1)$

$2 = 3A$

$A = \frac{2}{3}$

Now, comparing coefficients of equation (iii)

$x^2;\quad A+B = 1$

as A = $\frac{2}{3}$

$\frac{2}{3} + B = 1$

$B = 1 - \frac{2}{3}$

$B = 1 - \frac{2}{3}$

$B = \frac{3-2}{3}$

$B = \frac{1}{3}$

$const;\ A + C = 1$

as A = $\frac{2}{3}$

$\frac{2}{3} + C = 1$

$C = 1 - \frac{2}{3}$

$C = \frac{3-2}{3}$

$C = \frac{1}{3}$

put the values in (i) we get

$$\frac{x^2+1}{x^3+1} = \frac{\frac{2}{3}}{(x+1)} + \frac{\frac{1}{3}x+\frac{1}{3}}{(x^2-x+1)}$$

$$= \frac{\frac{2}{3}}{(x+1)} + \frac{\frac{x+1}{3}}{(x^2-x+1)}$$

$$= \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}$$