

Exercise 5.2

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Q. 1: If $X = \{1, 3, 5, 7, \dots, 19\}$, $Y = \{0, 2, 4, 6, \dots, 20\}$ and $Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$,

Then find the following:

- (i) $X \cup (Y \cup Z) = \{1, 3, 5, 7, \dots, 19\} \cup (\{0, 2, 4, 6, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
 $= \{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$
 $= \{0, 1, 2, 3, 4, \dots, 20, 23\}$
- (ii) $(X \cup Y) \cup Z = (\{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, \dots, 20\}) \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
 $= \{0, 1, 3, 4, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
 $= \{0, 1, 2, 3, \dots, 20, 23\}$
- (iii) $X \cap (Y \cap Z) = \{1, 3, 5, 7, \dots, 19\} \cap (\{0, 2, 4, 6, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
 $= \{1, 3, 5, 7, \dots, 19\} \cap \{2\}$
 $= \{ \}$
- (iv) $(X \cap Y) \cap Z = (\{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, \dots, 20\}) \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
 $= \{ \} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
 $= \{ \}$
- (v) $X \cup (Y \cap Z) = \{1, 3, 5, 7, \dots, 19\} \cup (\{0, 2, 4, 6, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
 $= \{1, 3, 5, 7, \dots, 19\} \cup \{2\}$
 $= \{1, 2, 3, 5, 7, \dots, 19\}$
- (vi) $(X \cup Y) \cap (X \cup Z) =$
 $(\{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, \dots, 20\}) \cap (\{1, 3, 5, 7, \dots, 19\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
 $= \{0, 1, 2, 3, \dots, 20\} \cap \{1, 2, 3, 5, 7, \dots, 19, 23\}$
 $= \{1, 2, 3, 5, 7, \dots, 19\}$
- (vii) $X \cap (Y \cup Z) = \{1, 3, 5, 7, \dots, 19\} \cap (\{0, 2, 4, 6, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
 $= \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$
 $= \{3, 5, 7, 11, 13, 17, 19\}$
- (viii) $(X \cap Y) \cup (X \cap Z) =$
 $(\{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, \dots, 20\}) \cup (\{1, 3, 5, 7, \dots, 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$
 $= \{ \} \cup \{3, 5, 7, 11, 13, 17, 19\}$
 $= \{3, 5, 7, 11, 13, 17, 19\}$

Q. 2: If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 4, 8\}$

Prove the following identities

- (i) $A \cap B = B \cap A$
 $L.H.S = A \cap B$
 $= \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$
 $= \{2, 4, 6\}$
 $R.H.S = B \cap A$
 $= \{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\}$
 $= \{2, 4, 6\}$
- (ii) $A \cup B = B \cup A$

$$\begin{aligned}
 L.H.S &= A \cup B \\
 &= \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\} \\
 &= \{1, 2, 3, 4, 5, 6, 8\}
 \end{aligned}$$

$$\begin{aligned}
 R.H.S &= B \cup A \\
 &= \{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5, 6\} \\
 &= \{1, 2, 3, 4, 5, 6, 8\}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \\
 L.H.S &= A \cap (B \cup C) \\
 &= \{1, 2, 3, 4, 5, 6\} \cap (\{2, 4, 6, 8\} \cup \{1, 4, 8\}) \\
 &= \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\} \\
 &= \{1, 2, 4, 6\}
 \end{aligned}$$

$$\begin{aligned}
 R.H.S &= (A \cap B) \cup (A \cap C) \\
 &= (\{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}) \cup (\{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\}) \\
 &= \{2, 4, 6\} \cup \{1, 4\} \\
 &= \{1, 2, 4, 6\}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\
 L.H.S &= A \cup (B \cap C) \\
 &= \{1, 2, 3, 4, 5, 6\} \cup (\{2, 4, 6, 8\} \cap \{1, 4, 8\}) \\
 &= \{1, 2, 3, 4, 5, 6\} \cup \{4, 8\} \\
 &= \{1, 2, 3, 4, 5, 6, 8\}
 \end{aligned}$$

$$\begin{aligned}
 R.H.S &= (A \cup B) \cap (A \cup C) \\
 &= (\{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}) \cap (\{1, 2, 3, 4, 5, 6\} \cup \{1, 4, 8\}) \\
 &= \{1, 2, 3, 4, 5, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\} \\
 &= \{1, 2, 3, 4, 5, 6, 8\}
 \end{aligned}$$

Q. 3: If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7\}$, then verify the De-Morgan's Laws

$$\text{i.e., } (A \cap B)' = A' \cup B' \quad \text{and} \quad (A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

$$\begin{aligned}
 L.H.S &= (A \cap B)' = U - (A \cap B) \\
 &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}) \\
 &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3, 5, 7\} \\
 &= \{1, 2, 4, 6, 8, 9, 10\}
 \end{aligned}$$

$$\begin{aligned}
 R.H.S &= A' \cup B' = (U - A) \cup (U - B) \\
 &= (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\}) \cup (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 3, 5, 7\}) \\
 &= \{2, 4, 6, 8, 10\} \cup \{1, 4, 6, 8, 9, 10\} \\
 &= \{1, 2, 4, 6, 8, 9, 10\}
 \end{aligned}$$

$$\text{So, } L.H.S = R.H.S$$

Now

$$(A \cup B)' = A' \cap B'$$

$$\begin{aligned}
 L.H.S &= (A \cup B)' = U - (A \cup B) \\
 &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\})
 \end{aligned}$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 5, 7, 9\}$$

$$= \{4, 6, 8, 10\}$$

$$R.H.S = A' \cap B' = (U - A) \cap (U - B)$$

$$= (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 3, 5, 7\})$$

$$= \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$$

$$= \{4, 6, 8, 10\}$$

So, $L.H.S = R.H.S$

Q. 4: If $U = \{1, 2, 3, \dots, 20\}$, $X = \{1, 3, 7, 9, 15, 18, 20\}$, $Y = \{1, 3, 5, \dots, 17\}$,
then show that

(i) $X - Y = X \cap Y'$

$$L.H.S = X - Y$$

$$= \{1, 3, 7, 9, 15, 18, 20\} - \{1, 3, 5, \dots, 17\}$$

$$= \{18, 20\}$$

$$R.H.S = X \cap Y'$$

$$= \{1, 3, 7, 9, 15, 18, 20\} \cap (\{1, 2, 3, \dots, 20\} - \{1, 3, 5, \dots, 17\})$$

$$= \{1, 3, 7, 9, 15, 18, 20\} \cap \{2, 4, 6, \dots, 18, 19, 20\}$$

$$= \{18, 20\}$$

So, $L.H.S = R.H.S$

(ii) $Y - X = Y \cap X'$

$$L.H.S = Y - X$$

$$= \{1, 3, 5, \dots, 17\} - \{1, 3, 7, 9, 15, 18, 20\}$$

$$= \{5, 11, 13, 17\}$$

$$R.H.S = Y \cap X'$$

$$= \{1, 3, 5, \dots, 17\} \cap (\{1, 2, 3, \dots, 20\} - \{1, 3, 7, 9, 15, 18, 20\})$$

$$= \{1, 3, 5, \dots, 17\} \cap \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19\}$$

$$= \{5, 11, 13, 17\}$$

So, $L.H.S = R.H.S$