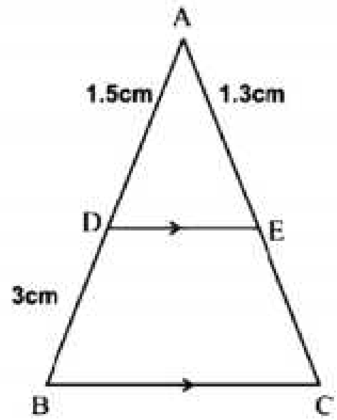
Exercise 14.1

Q.1 In ΔABC

$\overline{DE} \parallel \overline{BC}$



(i) If $\overline{AD}=1.5cm$ $\overline{BD}=3cm$

$\overline{AE}=1.3cm$, then find $\overline{CE}$

$$\frac{\overline{AD}}{\overline{BD}}=\frac{\overline{AE}}{\overline{EC}}$$

By substituting the values of $\overline{AD}, \overline{BD}$ and $\overline{AE}$

So

$$\frac{1.5}{3}=\frac{1.3}{EC}$$

$$\overline{EC}(1.5)=1.3\times 3$$

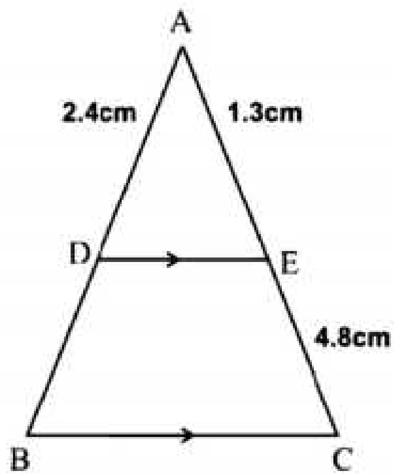
$$\overline{EC}=\frac{1.3\times 3}{1.5}$$

$$\overline{EC}=\frac{3.9}{1.5}$$

$$\overline{EC}=2.6cm$$

(ii) **If AD = 2.4cm** $\overline{AE}=3.2cm$

$\overline{EC}=4.8cm$ find AB



$$\frac{AD}{AB}=\frac{AE}{AC}$$

$$\overline{AC}=\overline{AE}+\overline{EC}$$

$$\overline{AC}=3.2+4.8$$

$$\overline{AC}=8cm$$

$$\therefore \frac{\overline{AD}}{\overline{AB}}=\frac{\overline{AE}}{\overline{AC}}$$

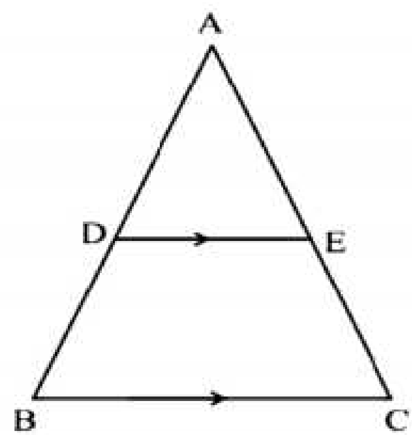
$$\frac{2.4}{AB}=\frac{3.2}{8}$$

$$2.4\times 8=(3.2)\overline{AB}$$

$$\frac{19.2}{3.2}=\overline{AB}$$

$$\overline{AB}=6cm$$

(iii) If $\frac{\overline{AD}}{\overline{BD}}=\frac{3}{5}\ \overline{AC}=4.8cm$ **find** $\overline{AE}$



$$\overline{AC}=\overline{AE}+\overline{EC}$$

$$\overline{AC}=\overline{EC}+\overline{AE}$$

$$\overline{AE}=4.8-\overline{EC}$$

$$\frac{\overline{AD}}{\overline{BD}}=\frac{\overline{AE}}{\overline{EC}}$$

$$\frac{\overline{AD}}{\overline{BD}}=\frac{\overline{AC}-\overline{EC}}{\overline{EC}}$$

$$\frac{3}{5}=\frac{4.8-EC}{EC}$$

$$3\left(\overline{EC}\right) = 5\left(4.8 - \overline{EC}\right)$$

$$3\left(\overline{EC}\right) = 24 - 5\left(\overline{EC}\right)$$

$$3\left(\overline{EC}\right) + 5\left(\overline{EC}\right) = 24$$

$$8\left(\overline{EC}\right) = 24$$

$$\left(\overline{EC}\right) = \frac{\cancel{24}^{3}}{\cancel{8}}$$

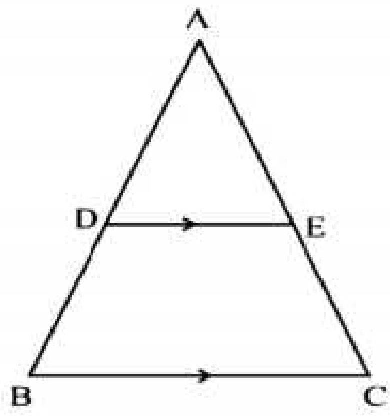
$$\overline{EC}= 3\text{cm}$$

$$\overline{AE}=\overline{AC}-\overline{EC}$$

$$=4.8-3$$

$$=1.8cm$$

(iv) If $\overline{AD}=2.4\text{cm}\,\overline{AE}=3.2\text{cm}\,\overline{DE}=2\text{cm}\,\overline{BC}=5\text{cm}$. **Find** $\overline{AB}, \overline{DB}, \overline{AC}, \overline{CE}$.



$$\frac{\overline{AD}}{\overline{AB}}=\frac{\overline{AE}}{\overline{AC}}=\frac{\overline{DE}}{\overline{BC}}$$

$$\frac{2.4}{AB}=\frac{3.2}{AC}=\frac{2}{5}$$

$$\frac{2.4}{AB}=\frac{2}{5}$$

$$(2.4)5 = 2(AB)$$

$$\frac{12.0}{2}=AB$$

$$\overline{AB}=6\text{cm}$$

$$\frac{3.2}{AC}=\frac{2}{5}$$

$$16.0 = 2(AC)$$

$$\frac{\cancel{16}^{8}}{\cancel{2}}=AC$$

$$\overline{AC}=8\text{cm}$$

$$\overline{DB}=\overline{AB}-\overline{AD}$$

$$\overline{DB}=6-2.4$$

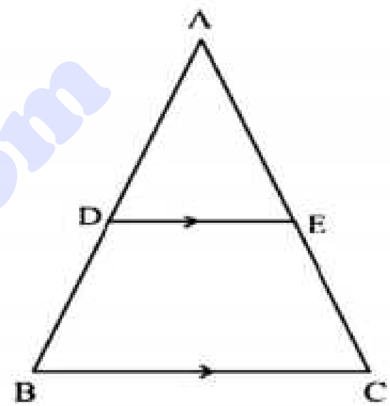
$$\overline{BD}=3.6\text{cm}$$

$$\frac{\overline{AD}}{\overline{AB}}=\frac{\overline{AE}}{\overline{AC}}$$

$$\frac{2.4}{6}=\frac{\overline{AE}}{8}$$
$$\overline{AE}=\frac{2.4}{6}\times 8$$
$$\overline{AE}=\frac{19.2}{6}$$
$$\overline{AE}=3.2cm$$
$$\overline{CE}=\overline{AC}-\overline{AE}$$
$$\overline{CE}=8-3.2$$
$$\overline{CE}=4.8cm$$

If $\overline{AD}=4x-3$ $\overline{AE}=8x-7$

$\overline{BD}=3x-1$ and $CE=5x-3$ Find the value of x



$$\frac{\overline{AD}}{\overline{BD}}=\frac{\overline{AE}}{\overline{EC}}$$

By putting the value of $\overline{AD}, \overline{AE}, \overline{BD}$ and $\overline{CE}$

$$\frac{4x-3}{3x-1}=\frac{8x-7}{5x-3}$$

By cross multiplying

$(4x-3)(5x-3)=(8x-7)(3x-1)$

$20x^2-12x-15x+9=24x^2-8x-21x+7$

$20x^2-27x+9=24x^2-29x+7$

$0=24x^2-20x^2-29x+27x+7-9$

$4x^2-2x-2=0$

$2(2x^2-x-1)=0$

$2x^2-2x+1x-1=\frac{0}{2}$

$2x(x-1)+1(x-1)=0$

$(x-1)(2x+1)=0$

$x-1=0$ $\quad$ $2x+1=0$

$x=1$ $\quad$ $2x=-1$

$x=-\frac{1}{2}$

Distance is not taken in negative it is always in positive so the value of x = 1.

Q.2 **In ΔABC is an isosceles triangle $\angle A$ is vertex angle and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that**

$m\overline{AD}:m\overline{DB}=m\overline{AE}:m\overline{EC}$

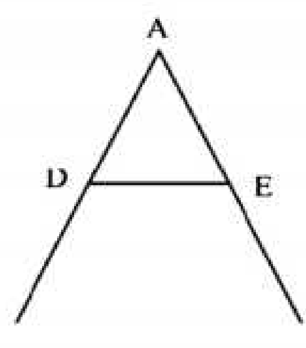
Prove that ΔADE is also an isosceles triangle.

Given:

ΔABC is an isosceles triangle, $\angle A$ is vertex and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$.

$$\frac{m\overline{AD}}{m\overline{BD}}=\frac{m\overline{AE}}{m\overline{EC}}$$

To Prove



$m\overline{AD} = m\overline{AE}$

Proof

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

Or $\dfrac{\overline{BD}}{\overline{AD}} = \dfrac{\overline{EC}}{\overline{AE}}$

Or $\dfrac{\overline{AD}+\overline{BD}}{\overline{AD}} = \dfrac{\overline{AE}+\overline{EC}}{\overline{EC}}$

As we know

$\overline{AB} = \overline{AD} + \overline{BD}$

$\overline{AC} = \overline{AE} + \overline{EC}$

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

From this

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

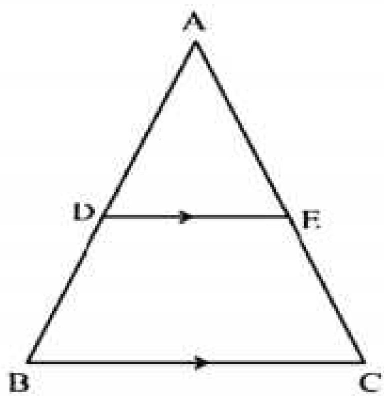
$\overline{AD} = \overline{AE}$

$\overline{AB} = \overline{AC}$ **(Given)**

Q.3 In an equilateral triangle ABC shown in the figure $m\overline{AE} : m\overline{AC} = m\overline{AD} : m\overline{AB}$ find all the three angles of ΔADE and name it also.

Given

ΔABC is equilateral triangle



To prove

To find the angles of ΔADE

Solution:

$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

All angles are equal as it is an equilateral triangle which are equal to 60° each

$\angle A = \angle B = \angle C$

$m\overline{BC} \parallel m\overline{DE}$

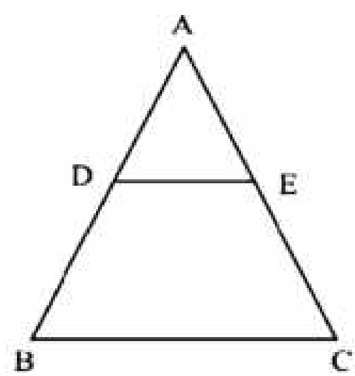
$\angle ADE = \angle ABC = 60^\circ$

$\angle AED = \angle ACB = 60^\circ$

$\angle A = 60^\circ$

ΔADE is an equilateral triangle

Q.4 Prove that line segment drawn through the midpoint of one side of a triangle and parallel to another side bisect the third side



Given

$\overline{AD} = \overline{BD}$

$\overline{DE} \parallel \overline{BC}$

To Prove

$\overline{AE} = \overline{EC}$

In ΔABC

$\overline{DE} \parallel \overline{BC}$

In theorem it is already discussed that

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

As we know $\overline{AD} = \overline{BD}$ or $\overline{BD} = \overline{AD}$

$$\frac{\cancel{\overline{AD}}}{\cancel{\overline{AD}}} = \frac{\overline{AE}}{\overline{EC}}$$

$$1 = \frac{\overline{AE}}{\overline{EC}}$$

$\overline{EC} = \overline{AE}$

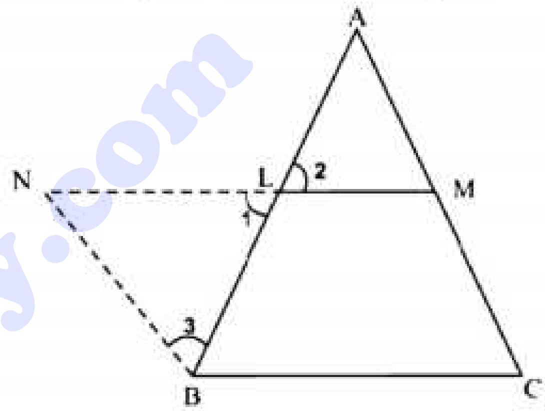
Q.5 Prove that the line segment joining the midpoint of any two sides of a triangle is parallel to the third side

Given

ΔABC the midpoint of $\overline{AB}$ and $\overline{AC}$ are L and M respectively

To Prove

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2}\overline{BC}$



Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$

Join N to B and in the figure name the angles $\angle 1$, $\angle 2$, and $\angle 3$

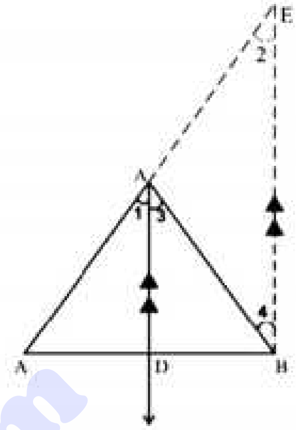
Proof

Statements	Reasons
$\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong AL$	Given
$\angle 2 = \angle 1$ or $\angle 1 = \angle 2$	Vertical angles
$\overline{NL} = \overline{ML}$	Construction
$\therefore\ \Delta BLN \cong \Delta ALM$	Corresponding angle of congruent triangles Given
$\therefore \angle A = \angle 3$	
And $\overline{NB} \cong \overline{AM}$	
$\overline{NB} \parallel \overline{AM}$	
$\overline{ML} = \overline{AM}$	Given
$\overline{NB} \cong \overline{ML}$	
$\overline{BC}\,\overline{MN}$ is parallelogram	
$\therefore\ \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	
$\overline{BC} \cong \overline{NM}$	(Opposite side of parallelogram BCMN)
$mLM = \frac{1}{2} m\overline{NM}$	(Opposite side of parallelogram)

Hence $m\overline{LM} = \frac{1}{2} m\overline{BC}$	

Theorem 14.1.3

The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle.



Given

In ΔABC internal angle bisector of $\angle A$ meets $\overline{CB}$ at the points D.

To prove

$m\overline{BD} : m\overline{DC} - m\overline{AB} : m\overline{AC}$

Construction

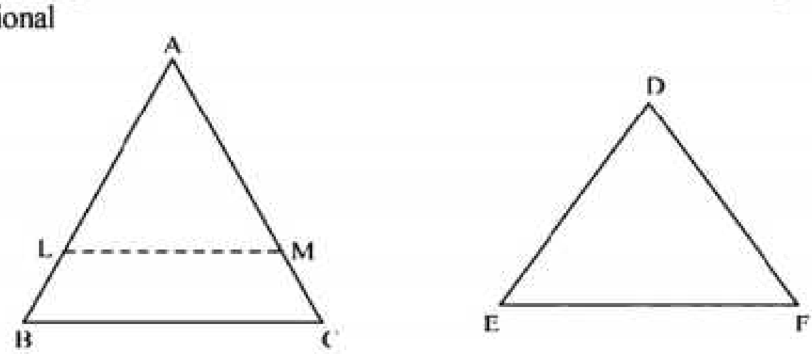
Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ Produced at E

Proof

Statements	Reasons
Q $\overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersect them	Construction
$m\angle 1 = m\angle 2$ ....(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and $\overline{AB}$ intersects them	
$\therefore\ m\angle 3 = m\angle 4$ ....(ii)	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore\ m\angle 2 = m\angle 4$	From (i) and (ii)
And $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	In a Δ, the sides opposite to congruent angles are also congruent
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	$m\overline{EA} = m\overline{AB}$ (proved)
Thus $m\overline{BD} : m\overline{DC} = m\overline{AB} : \overline{AC}$	

Theorem 14.1.4

If two triangles are similar, then the measures of their corresponding sides are proportional



Given

$\Delta ABC \sim \Delta DEF$
i.e $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

Construction

(I) Suppose that $mAB > m\overline{DE}$

(II) $m\overline{AB} \leq m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$

Join L and M by the line segment LM

Proof

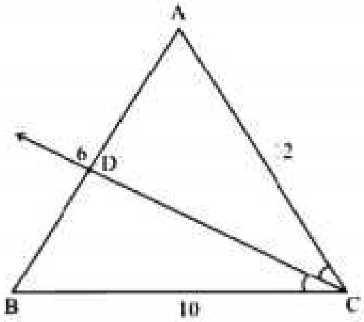
Statements	Reasons
In $\Delta ALM \leftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S Postulate
And $\angle L \cong \angle E$, $\angle M \cong \angle F$	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore$ $\angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AM}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
Or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$ ....(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (Construction)
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that	
$\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$ ....(ii)	
Thus $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}} = \frac{m\overline{EF}}{m\overline{BC}}$	By (i) and (ii)
Or $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$	By taking reciprocals
If $m\overline{AB} = m\overline{DE}$	
Then in $\Delta ABC \leftrightarrow \Delta DEF$	
(II) If $m\overline{AB} < m\overline{DE}$, it can similarly be proved by taking intercepts on the sides of ΔDEF	
$\angle A \cong \angle D$	
$\angle B \cong \angle E$	

And $\overline{AB} \cong \overline{DE}$	
So $\Delta ABC \cong \Delta DEF$	A.S.A $\cong$ A.S.A
Thus $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}} = 1$	$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
Hence the result is true for all the cases.	

Exercise 14.2

Q.1 **In ΔABC as shown in the figure $\overleftrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D.$m\overline{BD}$ is equal to**

(a) 5 **(b)** 16 **(c)** 10 **(d)** 18



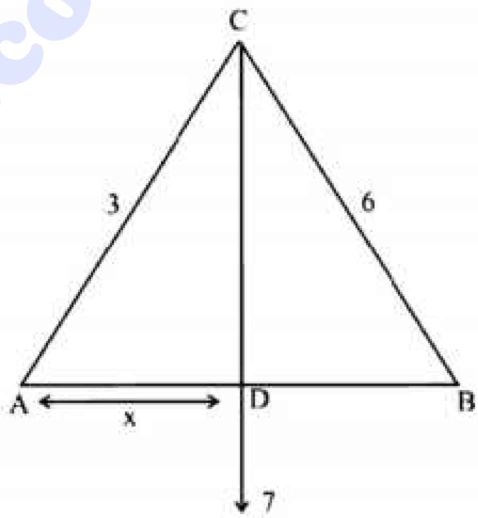
$$\frac{m\overline{BD}}{m\overline{DA}}=\frac{m\overline{BC}}{m\overline{CA}}$$

$$\frac{\overline{BD}}{6}=\frac{10}{12}$$

$$\overline{BD}=\frac{\cancel{10}^{5}\times\cancel{6}^{2}}{\cancel{12}_{4_{2}}}\text{ or }\overline{BD}=\frac{10\times 6}{12}=\frac{\cancel{60}^{5}}{\cancel{12}}$$

$$\overline{BD}=5$$

Q.2 **In ΔABC shown in the figure $\overleftrightarrow{CD}$ bisects $\angle C$. If $m\overline{AC}=3$, $\overline{CB}=6$ and $m\overline{AB}=7$ then find $m\overline{AD}$ and $\overline{DB}$**



$$\overline{AB}=\overline{AD}+\overline{BD}$$

$$\overline{AD}=\overline{AB}-\overline{BD}$$

$$\overline{AD}=7-x$$

$$\frac{m\overline{AD}}{m\overline{BD}}=\frac{m\overline{AC}}{m\overline{CB}}$$

$$\frac{x}{7-x}=\frac{\cancel{3}^{1}}{\cancel{6}_{2}}$$

$$\frac{x}{7-x}=\frac{1}{2}$$

$2x = 7-x$

$2x + x = 7$

$3x = 7$

$$x=\frac{7}{3}\quad\text{or}\quad\overline{AD}=\frac{7}{3}$$

$$\overline{AB}=\overline{AD}+\overline{BD}$$

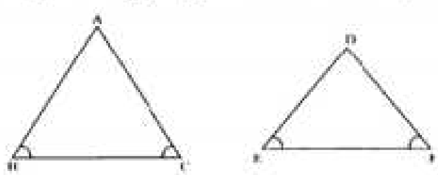
$$7=\frac{7}{3}+\overline{BD}$$

$$7-\frac{7}{3}=\overline{BD}$$

$$\frac{21-7}{3}=\overline{BD}$$

$$\overline{BD}=\frac{14}{3}$$

Q.3 **Show that in any corresponding of two triangles if two angles of one triangle are congruent to the corresponding angles of the other, then the triangle are similar**



Given

ΔABC and Δ DEF

$\angle B \cong \angle E$

$\angle C \cong \angle F$

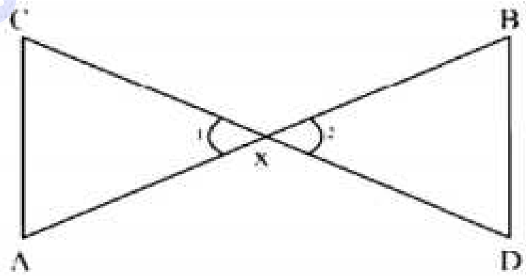
To Prove

$\Delta ABC \cong \Delta DEF$

Proof

Statements	Reasons
$\angle A + \angle B + \angle C = 180^o$ $\angle D + \angle E + \angle F = 180$ $\angle A \cong \angle D$ $\angle B = \angle E$ $\angle C = \angle F$ Hence Δ ABC $\cong \Delta$DEF	Sume of three angles of a triangle = 180^o

Q.4 If line segment $\overline{AB}$ and $\overline{CD}$ are intersecting at point X and $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$ then show that ΔAXC and ΔBXD are similar



Given

Line segment $\overline{AB}$ and $\overline{CD}$ intersect at X

$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$

To Prove

ΔCXA and ΔDXB are similar

Proof

Statements	Reasons
$\frac{\overline{AX}}{\overline{XB}} = \frac{\overline{CX}}{\overline{XD}}$	Given
$\angle 1 \cong \angle 2$	
$\overline{AC} \parallel \overline{BD}$	Vertical angles
$\angle A = m\angle B$	
$m\angle C = m\angle D$	Alternate angles
Hence proved the triangle are similar	