Exercise 1.5

Q. 1: Find the determinant of the following matrices.

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ (ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ (iv) $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Solution:

(i) $|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$ = (-1 x 0) – (2 x 1) = 0 – 2 = -2

(ii) $|B| = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix}$ = (1 x -2) – (2 x 3) = –2 – 6 = -8

(iii) $|C| = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$ = (3 x 2) – (3 x 2) = 6 – 6 = 0

(iv) $|D| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$ = (3 x 4) – (1 x 2) = 12 – 2 = 10

Q. 2: Find which of the following matrices are singular or non-singular?

Solution:

(i) $|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$ = (3 x 4) – (2 x 6) = 12 – 12 = 0

As $|A| = 0$. So, The given matrix is a singular matrix.

(ii) $|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$ = (4 x 2) – (3 x 1) = 8 – 3 = 5

As $|B| \neq 0$. So, The given matrix is a non-singular matrix.

(iii) $|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$ = (7 x 5) – (3 x -9) = 35 + 27 = 62

As $|C| \neq 0$. So, The given matrix is a non-singular matrix.

(iv) $|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$ = (5 x 4) – (-2 x -10) = 20 – 20 = 0

As $|D| = 0$. So, The given matrix is a singular matrix.

Q. 3: Find the multiplicative inverse (if it exists) of each.

(i) $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$ = (-1 x 0) – (2 x 3) = 0 – 6 = -6

$A^{-1} = \frac{1}{|A|} Adj(A)$

$= \frac{1}{-6} \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} \frac{0}{-6} & \frac{-3}{-6} \\ \frac{-2}{-6} & \frac{-1}{-6} \end{bmatrix}$

$= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$

(ii) $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$ = (1 x -5) – (-3 x 2) = -5 + 6 = 1

$B^{-1} = \frac{1}{|B|} Adj(B)$

$= \frac{1}{1}\begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

(iii) C $= \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$ = (-2 x -9) – (3 x 6) = 18 – 18 = 0

As the matrix is singular so inverse is not possible.

(iv) D $= \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$

$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix}$ $= (\frac{1}{2} \text{ x } 2) - (1 \text{ x } \frac{3}{4})$

$= 1 - \frac{3}{4}$ $= \frac{4-3}{4}$ $= \frac{1}{4}$

$D^{-1} = \frac{1}{|D|} Adj(D)$

$= \frac{1}{1/4}\begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$

$= \begin{bmatrix} 4 \times 2 & 4 \times -\frac{3}{4} \\ 4 \times -1 & 4 \times \frac{1}{2} \end{bmatrix}$

$= \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$

Q. 4: If A = $\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$, then

(i) A (Adj A) = (Adj A) A = (det A) I

(ii) BB^{-1} = I = $B^{-1}B$

(i) A (Adj A) $= \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \times 6 + 2 \times -4 & 1 \times -2 + 2 \times 1 \\ 4 \times 6 + 6 \times -4 & 4 \times -2 + 6 \times 1 \end{bmatrix}$

$= \begin{bmatrix} 6 - 8 & -2 + 2 \\ 24 - 24 & -8 + 6 \end{bmatrix}$

$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

(Adj A)A $= \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 6 \times 1 + -2 \times 4 & 6 \times 2 + -2 \times 6 \\ -4 \times 1 + 1 \times 4 & -4 \times 2 + 1 \times 6 \end{bmatrix}$

$= \begin{bmatrix} 6 - 8 & 12 - 12 \\ -4 + 4 & -8 + 6 \end{bmatrix}$

$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

(det A)I $= \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= [(1 \times 6) - (4 \times 2)] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= [6 - 8] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= (-2) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

So, A (Adj A) = (Adj A) A = (det A) I

(ii) $|B| = \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}$ $= (3 \times -2) - (2 \times -1) = -6 + 2 = -4$

$B^{-1} = \frac{1}{|B|} Adj(B)$

$= \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{4} & -\frac{1}{4} \\ \frac{2}{4} & -\frac{3}{4} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix}$

$BB^{-1} = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix}$

$= \begin{bmatrix} 3 \times \frac{1}{2} + -1 \times \frac{1}{2} & 3 \times -\frac{1}{4} + -1 \times -\frac{3}{4} \\ 2 \times \frac{1}{2} + -2 \times \frac{1}{2} & 2 \times -\frac{1}{4} + -2 \times -\frac{3}{4} \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{2} - \frac{1}{2} & -\frac{3}{4} + \frac{3}{4} \\ 1 - 1 & -\frac{1}{2} + \frac{3}{2} \end{bmatrix}$

$= \begin{bmatrix} \frac{3-1}{2} & 0 \\ 0 & \frac{-1+3}{2} \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{2} & 0 \\ 0 & \frac{2}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$B^{-1}B = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} \times 3 + -\frac{1}{4} \times 2 & \frac{1}{2} \times -1 + -\frac{1}{4} \times -2 \\ \frac{1}{2} \times 3 + -\frac{3}{4} \times 2 & \frac{1}{2} \times -1 + -\frac{3}{4} \times -2 \end{bmatrix}$

$$= \begin{bmatrix} \frac{3}{2} - \frac{2}{4} & -\frac{1}{2} + \frac{1}{2} \\ \frac{3}{2} - \frac{3}{2} & -\frac{1}{2} + \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6-2}{4} & 0 \\ 0 & \frac{-1+3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{4} & 0 \\ 0 & \frac{2}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

So, $BB^{-1} = I = B^{-1}B$

Q. 5: Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 3\times 7 + 5\times -4 & 3\times -5 + 5\times 3 \\ 4\times 7 + 7\times -4 & 4\times -5 + 7\times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 20 & -15 + 15 \\ 28 - 28 & -20 + 21 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

So, the given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1\times -3 + 2\times 2 & 1\times 2 + 2\times -1 \\ 2\times -3 + 3\times 2 & 2\times 2 + 3\times -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 + 4 & 2 - 2 \\ -6 + 6 & 4 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

So, the given matrices are multiplicative inverse of each other.

Q. 6: If

$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$

Verify that (i) $(AB)^{-1} = B^{-1}A^{-1}$ (ii) $(DA)^{-1} = A^{-1}D^{-1}$

(i) L.H.S $= (AB)^{-1}$

$$AB = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times -4 + 0\times 1 & 4\times -2 + 0\times -1 \\ -1\times -4 + 2\times 1 & -1\times -2 + 2\times -1 \end{bmatrix}$$

$$= \begin{bmatrix} -16 + 0 & -8 + 0 \\ 4 + 2 & 2 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$$

= (-16 x 0) – (6 x -8) = 0+48 = 48

$(AB)^{-1} = \frac{1}{|AB|} Adj(AB)$

$= \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$

$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ -\frac{6}{48} & -\frac{16}{48} \end{bmatrix}$

$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$

R.H.S $= B^{-1}A^{-1}$

$|B| = \begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$

$= (-4 \times -1) - (1 \times -2) = 4 + 2 \quad = 6$

$(B)^{-1} = \frac{1}{|B|} Adj(B)$

$= \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$

$= \begin{bmatrix} -\frac{1}{6} & \frac{2}{6} \\ -\frac{1}{6} & -\frac{4}{6} \end{bmatrix}$

$= \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ -\frac{1}{6} & -\frac{2}{3} \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$

$= (4 \times 2) - (-1 \times 0) = 8 + 0 \quad = 8$

$(A)^{-1} = \frac{1}{|A|} Adj(A)$

$= \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{8} & \frac{0}{8} \\ \frac{1}{8} & \frac{4}{8} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$

$B^{-1}A^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ -\frac{1}{6} & -\frac{2}{3} \end{bmatrix}\begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$

$= \begin{bmatrix} -\frac{1}{6} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{8} & -\frac{1}{6} \times 0 + \frac{1}{3} \times \frac{1}{2} \\ -\frac{1}{6} \times \frac{1}{4} + -\frac{2}{3} \times \frac{1}{8} & -\frac{1}{6} \times 0 + -\frac{2}{3} \times \frac{1}{2} \end{bmatrix}$

$= \begin{bmatrix} -\frac{1}{24} + \frac{1}{24} & 0 + \frac{1}{6} \\ -\frac{1}{24} - \frac{2}{24} & 0 - \frac{2}{6} \end{bmatrix}$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{3}{24} & -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

L.H.S = R.H.S

(ii) L.H.S $= (DA)^{-1}$

DA $= \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 3\times 4+1\times -1 & 3\times 0+1\times 2 \\ -2\times 4+2\times -1 & -2\times 0+2\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12-1 & 0+2 \\ -8-2 & 0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$$

$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix}$

= (11 x 4) – (-10 x 2) = 44 + 20= 64

$(DA)^{-1} = \frac{1}{|DA|}Adj(DA)$

$$= \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{64} & -\frac{2}{64} \\ \frac{10}{64} & \frac{11}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & -\frac{1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

R.H.S $= A^{-1}D^{-1}$

$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$

= (4 x 2) – (-1 x 0) = 8 + 0 = 8

$(A)^{-1} = \frac{1}{|A|}Adj(A)$

$$= \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{8} & \frac{0}{8} \\ \frac{1}{8} & \frac{4}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

$|D| = \begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}$

= (3 x 2) – (-2 x 1) = 6 + 2 = 8

$(D)^{-1} = \frac{1}{|D|}Adj(D)$

$= \frac{1}{8}\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{8} & -\frac{1}{8} \\ \frac{2}{8} & \frac{3}{8} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{4} & -\frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}$

$A^{-1}D^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} \frac{1}{4} & -\frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{4}\times\frac{1}{4}+0\times\frac{1}{4} & \frac{1}{4}\times-\frac{1}{8}+0\times\frac{3}{8} \\ \frac{1}{8}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4} & \frac{1}{8}\times-\frac{1}{8}+\frac{1}{2}\times\frac{3}{8} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{16}+0 & -\frac{1}{32}+0 \\ \frac{1}{32}+\frac{1}{8} & -\frac{1}{64}+\frac{3}{16} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{16} & -\frac{1}{32} \\ \frac{1+4}{32} & \frac{-1+12}{64} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{16} & -\frac{1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$

L.H.S = R.H.S