Exercise 11.1

Q.1 One angle of a parallelogram in 130°. Find the measures of its remaining angles.

In parallelogram

$m\angle B = 130^\circ$

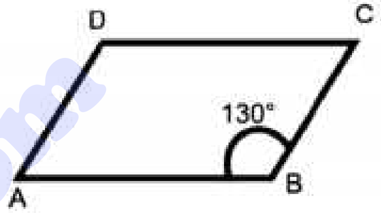
$\angle D = \angle B$ (Opposite angles of a parallelogram)

$m\angle D = m\angle B = 130^\circ$

We know that

$\angle A + \angle B = 180$

$\angle A + 130 = 180$ (sum of int. $\angle$s on same side of a parallelogram is 180°)



$\angle A = 180 - 130$

$\angle A = 50^\circ$

If $\angle D = \angle B$

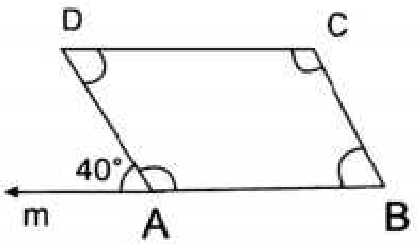
Then

$\angle C = \angle A$

$\angle C = 50^\circ$

Q.2 One exterior angle formed on producing one side of a parallelogram is 40°. Find the measures of its interior angles.

ABCD is a parallelogram. $\overline{BA}$ is produced towards *A*.



$m\angle DAM = 40^\circ$

$m\angle DAB = ?$

$m\angle D = ?$

$m\angle B = ?$

$m\angle C = ?$

$\angle DAM + \angle DAB = 180^\circ$

$40^\circ + \angle DAB = 180^\circ$

$\angle DAB = 180^\circ - 40^\circ$

$\angle DAB = 140^\circ$

$\angle DAB + \angle B = 180^\circ$

$140^\circ + \angle B = 180^\circ$

$\angle B = 180^\circ - 140^\circ$

$\angle B = 40^\circ$

$\angle D = \angle B = 40^\circ$

$\angle D = 40^\circ$

$\angle C = \angle DAB$

$\angle C = 140^\circ$

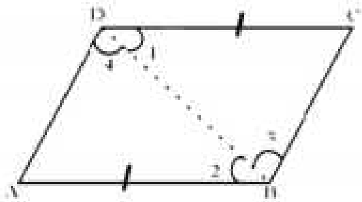
Theorem 11.1.2

Statement: If two opposite sides of quadrilateral are congruent and parallel, it is a parallelogram

Given

In quadrilateral $ABCD$,

$\overline{AB} \cong \overline{DC}$ and $\overline{AB} \parallel \overline{DC}$



To prove

ABCD is a parallelogram

Construction

Join the point B to D and in the figure name the angles as

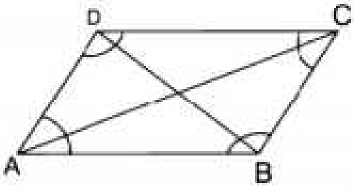
Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\angle 2 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\therefore \Delta ABD \cong \Delta CDB$	SAS postulate
Now $\angle 4 \cong \angle 3$ ....(i)	(Corresponding angles of congruent triangles)
$\therefore \overline{AD} \parallel \overline{BC}$ ....(ii)	from (i)
and $\overline{AD} = \overline{BC}$ ....(iii)	corresponding of sides of congruent triangles
Also $\overline{AB} \parallel \overline{DC}$ ....(iv)	Given
Hence ABCD is a parallelogram	From (ii)-(iv)

Exercise 11.2

Q.1 Prove that a quadrilateral is a parallelogram if its
(a) Opposite angles are congruent
(b) Diagonals bisects each other

(a) Given



In quadrilateral ABCD
$m\angle A = m\angle C, m\angle B = m\angle D$

To Prove
$ABCD$ is a parallelogram

Statements	Reasons
$m\angle A = m\angle C \ldots(\text{i})$	Given
$m\angle B = m\angle D \ldots(\text{ii})$	Given
$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$	Angles of quadrilateral
$m\angle A + m\angle B = 180^\circ$	
$m\angle C + m\angle D = 180^\circ$	
$\overline{AD} \parallel \overline{BC}$	
Similarity it can be proved that $\overline{AB} \parallel \overline{DC}$	
Hence ABCD is a parallelogram	

(b) Given
In quadrilateral ABCD, diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other.
i.e. $\overline{OA} = \overline{OC}, \overline{OB} = \overline{OD}$
To prove ABCD is a parallelogram
Proof

Statements	Reasons
In $\Delta ABO \leftrightarrow \Delta CDO$	
$\overline{OA} \cong \overline{OC}$	Given
$\overline{OB} \cong \overline{OD}$	Given
$\angle AOB \cong \angle COD$	Vertical opposite angles
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\Delta ABO \cong \angle CDO$	$S.A.S \cong S.A.S$
Hence, $\overline{AB} \parallel \overline{CD} \ldots(\text{i})$	$\angle 1 \cong \angle 2$
By taking BOC and is ΔAOD it can be prove that	
$\overline{AD} \parallel \overline{BC} \ldots(\text{ii})$	From (i) and (ii)
Hence ABCD is a parallelogram	

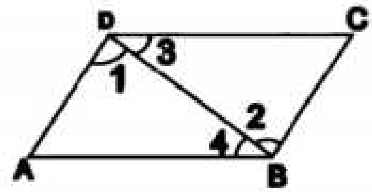
Q.2 Prove that a quadrilateral is a parallelogram if its opposite sides are congruent

Given

In quadrilateral $ABCD$

(i) $\overline{AB} \cong \overline{DC}$

(ii) $\overline{AD} \cong \overline{BC}$



To prove

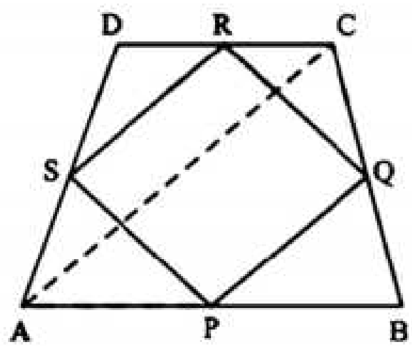
$ABCD$ is a parallelogram i.e. $\overline{AD} \parallel \overline{BC}$

Prove

Statements	Reasons
$\Delta CDB \leftrightarrow \Delta ABD$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	$S.S.S \cong S.S.S$
Thus, $\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 4 \cong \angle 3$	Corresponding angles of congruent triangles
(i) $\overline{AD}$ P $\overline{BC}$	Alternate angles are congruent
$\overline{AB} \parallel \overline{DC}$	Alternate angles are congruent
$\therefore$ ABCD is a parallelogram	

Example

The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.



Given

A quadrilateral $ABCD$, in which P is the mid-point of $\overline{AB}$ Q is the mid-point of $\overline{BC}$ R is the mid-point of $\overline{CD}$ S is the mid-point of $\overline{DA}$

P is joined to Q, Q is joined to R.

R is joined to S and S is joined to P.

To prove

PQRS is a parallelogram.

Construction

Join A to C.

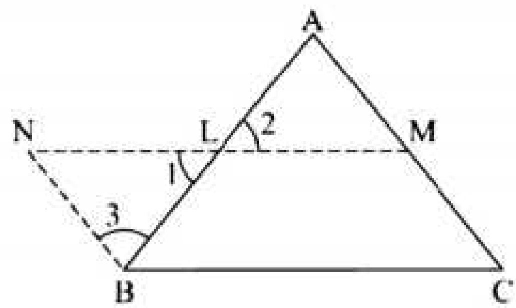
Proof

Statements	Reasons
In ΔDAC,	
$\left.\begin{array}{l}\overline{SR} \parallel \overline{AC} \\ m\overline{SR} = \frac{1}{2} m\overline{AC}\end{array}\right\}$	S is the midpoint of $\overline{DA}$ R is the midpoint of $\overline{CD}$
In ΔBAC,	
$\overline{PQ} \parallel \overline{AC}$	P is the midpoint of $\overline{AB}$

	Q is the midpoint of $\overline{BC}$
$\overline{SR}$ □ $\overline{PQ}$	Each □ $\overline{AC}$
$m\overline{SR} = m\overline{PQ}$	Each$=\frac{1}{2}\overline{AC}$
Thus $PQRS$ is a parallelogram	$\overline{SR}$ □ $\overline{PQ}, m\overline{SR} = m\overline{PQ}$ (proved)

Theorem 11.1.3

The line segment, joining the midpoint of two sides of triangle, is parallel to the third side and is equal to one half of its length.



Given

In ΔABC, the mid-point of $\overline{AB}$ and $\overline{AC}$ are L and M respectively

To prove

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2} m\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$

Join N to B and in the figure, name the angles $\angle 1$, $\angle 2$ and $\angle 3$ as shown.

Proof

Statements	Reasons
In $\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
$\therefore \Delta BLN \cong \Delta ALM$	S.A.S postulate
$\therefore \angle A \cong \angle 3$...(i)	(Corresponding angles of congruent triangles)
And $\overline{NB} \cong \overline{AM}$...(ii)	(Corresponding sides of congruent triangles)
But $\overline{NB} \parallel \overline{AM}$	from (i), alternative <s
Thus	
$\overline{NB} \parallel \overline{MC}$ ....(iii)	(M is a point of $\overline{AC}$)
$\overline{MC} \cong AM$ ....(iv)	Given
$\overline{NB} \cong \overline{MC}$ ....(v)	from (ii) and (iv)
$BCMN$ is a parallelogram	From (iii) and (v)
$\therefore \overline{BC} \parallel \overline{LM}$ *or* $\overline{BC} \parallel \overline{NL}$	(Opposite sides of a parallelogram BCMN)

	(Opposite sides of a parallelogram)
$\overline{BC} \cong \overline{NM}$ ....(vi)	Construction.
$m\overline{LM} = \frac{1}{2} m\overline{NM}$ ....(vii)	
Hence, $m\overline{LM} = \frac{1}{2} m\overline{BC}$	from (vi) and (vii)

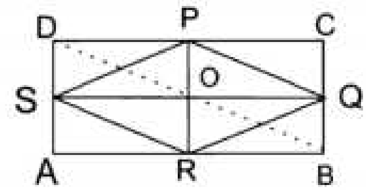
Exercise 11.3

Q.1 Prove that the line segments joining the midpoint of the opposite side of a quadrilateral bisect each other.

Given

$ABCD$ is quadrilaterals point $QRSP$ are the mid point of the sides $\overline{RP}$ and $\overline{SQ}$ are joined they meet at O.

$\overline{OP} \cong \overline{OR}$ $\overline{OQ} \cong \overline{OS}$



Construction

Join P, Q, R and S in order join C to A or A to C

Proof

Statements	Reasons
$SP \parallel AC$ … (i)	In $\Delta ADC, S, P$ are mid point of AD, DC
$m\overline{SP} = \frac{1}{2} m\overline{AC}$…(ii)	
$\overline{AC} \parallel \overline{RQ}$…(iii)	In ΔABC, Q, R are midpoint of $\overline{BC}$, $\overline{AB}$
$m\overline{RQ} = \frac{1}{2}\overline{AC}$…(iv)	
$m\overline{SP} \parallel \overline{RQ}$…(v)	
and $\overline{RQ} = \overline{SP}$…(vi)	From (ii) and (iv)
Now $\overline{RP}$ *and* $\overline{QS}$ diagonals of parallelogram PQRS intersect at O.	
$\therefore$ $\overline{OP} \cong \overline{OR}$ $\overline{OS} \cong \overline{OQ}$	Diagonals of a parallelogram bisects each other.

Q.2 Prove that the line segments joining the midpoint of the opposite sides of a rectangle are the right bisectors of each other.

[Hint: Diagonals of a rectangle are congruent]

Given

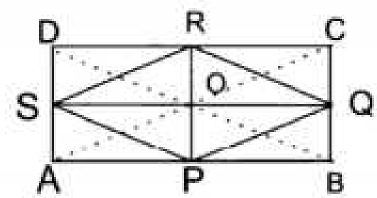
(i) $ABCD$ is a rectangle

(ii) P, Q, R, S are the midpoints of $\overline{AB}$, $\overline{CD}$ and $\overline{DA}$

(iii) $\overline{SQ}$ and $\overline{RP}$ cut each other at point O

$\overline{OS} \cong \overline{OQ}$

$\overline{OP} \cong \overline{OR}$



Construction

Join P to Q and Q to R and R to S and S to P

Join A to C and B to D

Proof

Statements	Reasons
Midpoint of $\overline{BC}$ is Q	Given
Midpoint of $\overline{AB}$ is P	Given
$\therefore \overline{AC} \parallel \overline{PQ}$..................(i)	
$\frac{1}{2}\overline{AC} = \overline{PQ}$.............(ii)	
In ΔADC	
$\overline{AC} \parallel \overline{SR}$.................(iii)	
$\frac{1}{2}\overline{AC} = \overline{SR}$.............(iv)	
$\overline{PQ} = \overline{SR}$	From equation (i) and (ii) each are parallel to
$\overline{SP} = \overline{RQ}$	$\overline{AC}$ each are half of $\overline{DB}$
By joined B to D we can prove	
$\overline{RQ} \parallel \overline{SP}$	
$m\overline{SR} \parallel m\overline{PQ}$	Each of them = $\frac{1}{2}\overline{AC}$
$m\overline{AC} \parallel m\overline{BD}$	
PQRS is a parallelogram all it sides are equal	
$\overline{OP} \cong \overline{OR}$	
$\overline{OS} \cong \overline{OQ}$	
$\Delta OQR \leftrightarrow \Delta OQP$	
$\overline{OR} \cong \overline{OP}$	Proved
$\overline{OQ} \cong \overline{OQ}$	Common
$\overline{RQ} \cong \overline{PQ}$	Adjacent
$\therefore \Delta OQR \cong \Delta OQP$	
$\angle ROQ \cong \angle POQ$...............(vii)	
$\angle ROQ + \angle POQ = 180$.........(viii)	Supplementary angle
$\angle ROQ = \angle POQ = 90°$	From (vii) and (viii)
Thus $\overline{PR} \perp \overline{QS}$	

Q.3 Prove that line segment passing the midpoint of one side and parallel to other side of a triangle also bisects the third side.

Given

In ΔABC, R is the midpoint of $\overline{AB}, \overline{RQ} \parallel \overline{BC}$

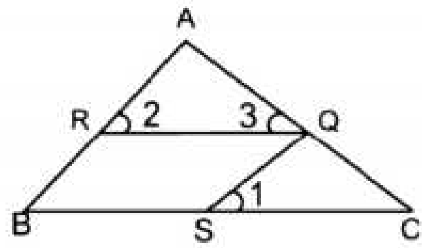
$\overline{RQ} \parallel \overline{BS}$

To prove

$\overline{AQ} = \overline{QC}$

Construction

$\overline{QS} \parallel \overline{AB}$



Proof

Statements	Reasons
$\overline{RQ} \parallel \overline{BS}$	Given
$\overline{QS} \parallel \overline{BR}$	Construction
$RBSQ$ is a Parallelogram	
$\overline{QS} \cong \overline{BR}$...(i)	Opposite side
$\overline{AR} \cong \overline{RB}$...(ii)	Given
$\overline{QS} \cong \overline{AR}$...(iii)	From (i) and (ii)
$\angle 1 \cong \angle B$ and $\angle 1 \cong \angle 2$...(iv)	
$\Delta ARQ \leftrightarrow \Delta QSC$	
$\angle 2 \cong \angle 1$	From (iv)
$\angle 3 \cong \angle C$	
$\overline{AR} \cong \overline{SQ}$	From (iii)
Hence, $\Delta ARQ \cong \Delta QSC$	$A.A.S \cong A.A.S$
$\overline{AQ} \cong \overline{QC}$	Corresponding sides

<u>**Theorem: 11.1.4**</u>

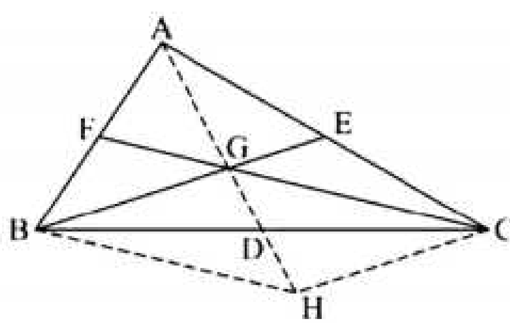
Statement: The median of triangle are concurrent and their point of concurrency is the point of trisection of each median.

Given ΔABC

To prove

The medians of the ΔABC are concurrent and the point of concurrency is the point of trisection of each median

Construction



Draw two medians $\overline{BE}$ and $\overline{CF}$ of the ΔABC which intersect each other at point G. Join A to G and produce it to the point H such that $AG \cong \overline{GH}$ Join H to the points B and C $\overline{AH}$ Intersects $\overline{BC}$ at the point D.

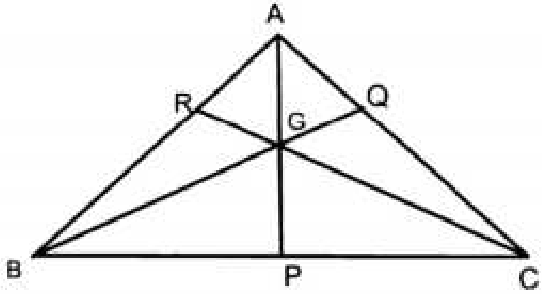
Proof

Statements	Reasons
In Δ ACH,	
$\overline{GE} \parallel \overline{HC}$	G and E are mid-points of sides $\overline{AH}$ and $\overline{AC}$ respectively
Or $\overline{BE} \parallel \overline{HC}$ ....(i)	G is point of $\overline{BE}$ diagonals $\overline{BC}$
Similarly $\overline{CF} \parallel \overline{HB}$...(ii)	
$\therefore$ BHCG is a parallelogram	From (i) and (ii)
And	
$m\overline{GD} = \frac{1}{2} m\overline{GH}$...(iii)	Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram $BHCG$ intersect each other at point D.
$\overline{BD} = \overline{CD}$	
$\overline{AD}$ is a median of ΔABC medians	G is the interesting point of $\overline{BE}$, $\overline{CF}$ and $\overline{AD}$ pass through it.
$\overline{AD}, \overline{BE}$ and $\overline{CF}$ pass through the point G	
Now $\overline{GH} \cong \overline{AG}$...(iv)	Construction
$m\overline{GD} = \frac{1}{2} m\overline{AG}$	From (iii) and (iv)
and G is the point of trisection of $\overline{AD}$...(v)	
similarly it can be proved that G is also the point of trisection of $\overline{CF}$ and $\overline{BE}$	

Exercise 11.4

Q.1 The distance of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2 cm. 1.4 cm and 1.6 cm. Find the length of its medians.

Let ΔABC with the point of concurrency of medians at G



$\overline{AG}=1.2cm, \overline{BG}=1.4cm \text{ and } \overline{CG}=1.6cm$

$\overline{AP}=\frac{3}{2}\overline{AG}=\frac{3}{2}\times 1.2=1.8cm$

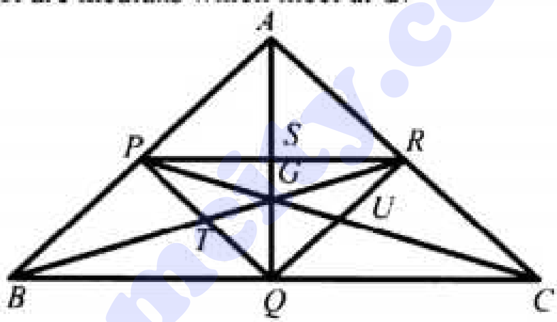
$\overline{BQ}=\frac{3}{2}\overline{BG}=\frac{3}{2}\times 1.4=2.1cm$

$\overline{CR}=\frac{3}{2}\overline{CG}=\frac{3}{2}\times 1.6=2.4cm$

Q.2 Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the midpoint of its sides to the same.

Given

In ΔABC, AQ, CP, BR are medians which meet at G.



To prove

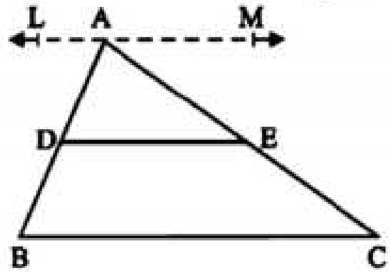
G is the point of concurrency of the medians of ΔABC and ΔPQR

Proof

Statements	Reasons
$\overline{PR} \parallel \overline{BC}$	P, R are midpoint of $\overline{AB}, \overline{AC}$
$\overline{BQ} \parallel \overline{PR}$	
Similarly $\overline{QR} \parallel \overline{BP}$	
$\therefore PBQR$ is a parallelogram it diagonals $\overline{BR}$ and $\overline{PQ}$ bisector each other at T	
Similarly U is the midpoint of QR and S is midpoint of $\overline{PR}$	
$\therefore \overline{PU}, \overline{QS}, \overline{RT}$ are medians of ΔPQR	
(i) $\overline{AQ}$ and $\overline{SQ}$ pass through G	
(ii) $\overline{BR}$ and $\overline{TR}$ pass through G	
(iii) $\overline{UP}$ and $\overline{CP}$ pass through G	
Hence G is point of concurrency of medians of ΔPQR and ΔABC	

Example

A line, through the mid-point of one side, parallel to another side of a triangle, bisects the third side.



Given

In ΔABC, D is the mid-point of $\overline{AB}$.

$\overline{DE}$ □ $\overline{BC}$ which cuts $\overline{AC}$ at E.

To prove

$\overline{AE} \cong \overline{BC}$

Construction

Through A, draw $\overline{LM}$ □ $\overline{BC}$.

Proof

Statements	Reasons
Intercepts cut by $\overline{LM}, \overline{DE}, \overline{BC}$ on $\overline{AC}$ are congruent. i.e., $\overline{AE} \simeq \overline{EC}$.	Intercepts cut by parallels $\overline{LM}, \overline{DE}$, $\overline{BC}$ on $\overline{AB}$ are congruent (given)

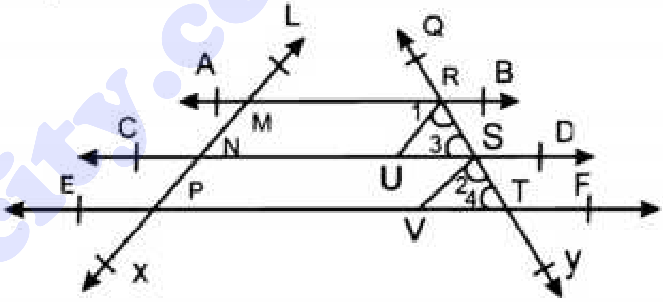
Theorem 11.1.5

Statement: In three or more parallel lines make congruent segments on a traversal they also intercept congruent segments on any other line that cuts them.

Given

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at point R, S and T respectively.



Prove

$\overline{RS} \cong \overline{ST}$

Construction

From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U, from S draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V. as shown in the figure let the angles be labeled as $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

Proof

Statements	Reasons
$MNUR$ is parallelogram	$\overline{RU} \parallel \overline{LX}$ (Construction) $\overline{AB} \parallel \overline{CO}$ (given)
$\therefore \overline{MN} \cong \overline{RU}$ (i)	(Opposite side of parallelogram).
Similarly.	
$\overline{NP} \cong \overline{SV}$ (ii)	
But $\overline{MN} \cong \overline{NP}$ (iii)	Given
$\therefore \overline{RU} \cong \overline{SV}$	{from (i) (ii) and (iii)} each is $\parallel$ $\overline{LX}$ (construction)
Also $\overline{RU}$ □ $\overline{SV}$	
$\therefore \angle 1 \cong \angle 2$	Corresponding angles
and $\angle 3 \simeq \angle 4$	Corresponding angles
In $\Delta RUS \leftrightarrow \Delta SVT$	

$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\therefore \Delta RUS \cong \Delta SVT$	$S.A.A \cong S.A.A$
Hence $\overline{RS} \cong \overline{ST}$	(Corresponding sides of congruent triangles)

Exercise 11.5

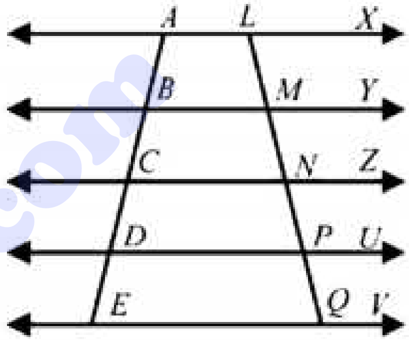
Q.1 **In the given figure**

$\overline{AX} \parallel \overline{BY} \parallel \overline{CZ} \parallel \overline{DU} \parallel \overline{EV}$ and $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$

If $\overline{MN} = 1cm$ then find the length of $\overline{LN}$ and $\overline{LQ}$

$\therefore \overline{PQ} \cong \overline{NP} \cong \overline{MN} \cong \overline{LM}$

$MN = 1cm$



Given

$\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

Therefore, $\overline{LN} = \overline{LM} + \overline{MN}$

$\overline{LM} = \overline{MN}$

so, $\overline{LN} = \overline{MN} + \overline{MN}$

$\overline{LN} = 1 + 1$

$\overline{LN} = 2cm$

$\overline{LM} = \overline{NP} = \overline{PQ} = \overline{MN} = 1cm$

So, $\overline{LM} = 1cm, \overline{NP} = 1cm, \overline{PQ} = 1cm$

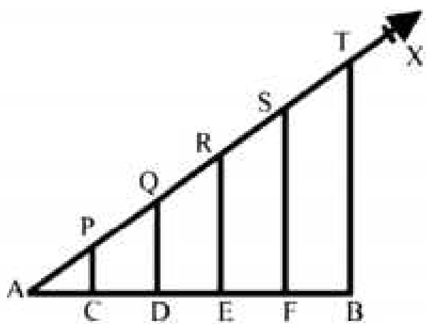
$LQ = \overline{LM} + \overline{MN} + \overline{NP} + \overline{PQ}$

$LQ = 1 + 1 + 1 + 1$

$LQ = 4cm$

Q.2 **Take a line segment of length $5.5cm$ and divide it into five congruent parts [Hint: draw an acute angle $\angle B$ AX. On $\overline{AX}$ take $\overline{AP} \cong \overline{PQ} \cong \overline{RS} \cong \overline{ST}$ join T to B draw lines parallel to $\overline{TB}$ from the point P, Q R and S.**

Proof



Construction:

(i) Take a line segment $AB = 5.5cm$

(ii) Draw any acute angle $\angle BAX$

(iii) Draw arcs on $\overline{AX}$ which are $\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$

(iv) Join T to B

(v) Draw lines $\overline{SF}, \overline{RE}, \overline{QD}, \& \overline{PC}$ Parallel to $\overline{TB}$.

Result line segment $\overline{AB}$ is divided into congruent line segments $\overline{AC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EF} \cong \overline{FB}$.