

Exercise 2.4

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Q. 1: If α, β are the roots of the equation $x^2 + px + q = 0$, then evaluate.

(i) $\alpha^2 + \beta^2$

$$\begin{array}{lllll} \text{sum of roots} & = \alpha + \beta & = -\frac{b}{a} & = -\frac{p}{1} & = -p \end{array}$$

$$\begin{array}{lllll} \text{Product of roots} & = \alpha\beta & = \frac{c}{a} & = \frac{q}{1} & = q \end{array}$$

So,

$$\alpha^2 + \beta^2$$

adding and subtracting $2\alpha\beta$

$$= \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-p)^2 - 2(q)$$

$$= p^2 - 2q$$

(ii) $\alpha^3\beta + \alpha\beta^3$

$$\begin{array}{lllll} \text{sum of roots} & = \alpha + \beta & = -\frac{b}{a} & = -\frac{p}{1} & = -p \end{array}$$

$$\begin{array}{lllll} \text{Product of roots} & = \alpha\beta & = \frac{c}{a} & = \frac{q}{1} & = q \end{array}$$

So,

$$\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$$

adding and subtracting $2\alpha\beta$

$$= \alpha\beta(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta)$$

$$= \alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta]$$

$$= q[(-p)^2 - 2(q)]$$

$$= q[p^2 - 2q]$$

$$= qp^2 - 2q^2$$

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$\begin{array}{lllll} \text{sum of roots} & = \alpha + \beta & = -\frac{b}{a} & = -\frac{p}{1} & = -p \end{array}$$

$$\begin{array}{lllll} \text{Product of roots} & = \alpha\beta & = \frac{c}{a} & = \frac{q}{1} & = q \end{array}$$

So,

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

adding and subtracting $2\alpha\beta$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(-p)^2 - 2(q)}{q}$$

$$= \frac{p^2}{q} - \frac{2q}{q}$$

$$= \frac{p^2}{q} - 2$$

Q. 2: If α, β are the roots of the equation $4x^2 - 5x + 6 = 0$, then find the values of

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$

$$\text{sum of roots} = \alpha + \beta = -\frac{b}{a} = -\frac{-5}{4} = \frac{5}{4}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$$

So,

$$\begin{aligned} \frac{1}{\alpha} + \frac{1}{\beta} &= \frac{\alpha + \beta}{\alpha\beta} \\ &= \frac{5/4}{3/2} \\ &= \frac{5}{4} \times \frac{2}{3} \\ &= \frac{5}{6} \end{aligned}$$

(ii) $\alpha^2\beta^2$

$$\text{sum of roots} = \alpha + \beta = -\frac{b}{a} = -\frac{-5}{4} = \frac{5}{4}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$$

So,

$$\begin{aligned} \alpha^2\beta^2 &= (\alpha\beta)^2 \\ &= \left(\frac{3}{2}\right)^2 \\ &= \frac{9}{4} \end{aligned}$$

(iii) $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$

$$\text{sum of roots} = \alpha + \beta = -\frac{b}{a} = -\frac{-5}{4} = \frac{5}{4}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$$

So,

$$\begin{aligned} \frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} &= \frac{\alpha + \beta}{\alpha^2\beta^2} \\ &= \frac{\alpha + \beta}{(\alpha\beta)^2} \\ &= \frac{5/4}{(3/2)^2} \\ &= \frac{5/4}{9/4} \\ &= \frac{5}{4} \times \frac{4}{9} \\ &= \frac{5}{9} \end{aligned}$$

(iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

$$\text{sum of roots} = \alpha + \beta = -\frac{b}{a} = -\frac{-5}{4} = \frac{5}{4}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{6}{4} = \frac{3}{2}$$

So,

$$\begin{aligned}\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} &= \frac{\alpha^3 + \beta^3}{\alpha\beta} \\ &= \frac{(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)}{(\alpha\beta)^2}\end{aligned}$$

adding and subtracting $2\alpha\beta$

$$\begin{aligned}&= \frac{(\alpha + \beta)(\alpha^2 - \alpha\beta - 2\alpha\beta + 2\alpha\beta + \beta^2)}{(\alpha\beta)^2} \\ &= \frac{(\alpha + \beta)(\alpha^2 + 2\alpha\beta + \beta^2 - 3\alpha\beta)}{(\alpha\beta)^2} \\ &= \frac{(\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)}{(\alpha\beta)^2} \\ &= \frac{(5/4)((5/4)^2 - 3(3/2))}{(3/2)^2} \\ &= \frac{(5/4)(25/16 - 9/2)}{9/4} \\ &= \frac{(5/4)(25 - 72/16)}{9/4} \\ &= \frac{(5/4)(-47/16)}{9/4} \\ &= \frac{(-235/64)}{9/4} \\ &= \frac{-235}{64} \times \frac{4}{9} \\ &= \frac{-235}{144}\end{aligned}$$

Q. 3: If α, β are the roots of the equation $lx^2 + mx + n = 0$, then find the values of

(i) $\alpha^3\beta^2 + \alpha^2\beta^3$

$$\begin{array}{llllll}\text{sum of roots} & = \alpha + \beta & = -\frac{b}{a} & = -\frac{m}{l} & = -\frac{m}{l} \\ \text{Product of roots} & = \alpha\beta & = \frac{c}{a} & = \frac{n}{l} & = \frac{n}{l}\end{array}$$

So,

$$\begin{aligned}\alpha^3\beta^2 + \alpha^2\beta^3 &= \alpha^2\beta^2(\alpha + \beta) \\ &= (\alpha\beta)^2(\alpha + \beta) \\ &= \left(\frac{n}{l}\right)^2 \left(-\frac{m}{l}\right) \\ &= \frac{n^2}{l^2} \times \left(-\frac{m}{l}\right) \\ &= -\frac{n^2m}{l^3}\end{aligned}$$

(ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\begin{array}{llllll}\text{sum of roots} & = \alpha + \beta & = -\frac{b}{a} & = -\frac{m}{l} & = -\frac{m}{l} \\ \text{Product of roots} & = \alpha\beta & = \frac{c}{a} & = \frac{n}{l} & = \frac{n}{l}\end{array}$$

So,

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

Adding and subtracting $2\alpha\beta$

$$\begin{aligned}
&= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{(\alpha\beta)^2} \\
&= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \\
&= \frac{\left(-\frac{m}{l}\right)^2 - 2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2} \\
&= \frac{\frac{m^2}{l^2} - \frac{2n}{l}}{\frac{n^2}{l^2}} \\
&= \frac{\frac{m^2 - 2nl}{l^2}}{\frac{n^2}{l^2}} \\
&= \frac{m^2 - 2nl}{l^2} \times \frac{l^2}{n^2} \\
&= \frac{m^2 - 2nl}{n^2}
\end{aligned}$$

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