Exercise 4.4

Q. 1: Rationalize the denominator of the following.

(i) $\frac{3}{4\sqrt{3}}$

$$= \frac{3}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{3\sqrt{3}}{4(\sqrt{3})^2}$$
$$= \frac{3\sqrt{3}}{4(3)}$$
$$= \frac{\sqrt{3}}{4}$$

(ii) $\frac{14}{\sqrt{98}}$

$$= \frac{14}{\sqrt{98}} \times \frac{\sqrt{98}}{\sqrt{98}}$$
$$= \frac{14\sqrt{98}}{(\sqrt{98})^2}$$
$$= \frac{14\sqrt{49\times2}}{98}$$
$$= \frac{7\sqrt{2}}{7}$$
$$= \sqrt{2}$$

(iii) $\frac{6}{\sqrt{8}\sqrt{27}}$

$$= \frac{6}{\sqrt{8}\sqrt{27}} \times \frac{\sqrt{8}}{\sqrt{8}} \times \frac{\sqrt{27}}{\sqrt{27}}$$
$$= \frac{6\sqrt{8}\sqrt{27}}{(\sqrt{8})^2(\sqrt{27})^2}$$
$$= \frac{6\sqrt{4\times2}\sqrt{9\times3}}{(\sqrt{8})^2(\sqrt{27})^2}$$
$$= \frac{6\times2\sqrt{2}\,3\sqrt{3}}{(\sqrt{8})^2(\sqrt{27})^2}$$
$$= \frac{36\sqrt{2}\sqrt{3}}{(8)(27)}$$
$$= \frac{\sqrt{6}}{6}$$

(iv) $\frac{1}{3+2\sqrt{5}}$

$$= \frac{1}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$
$$= \frac{3-2\sqrt{5}}{(3)^2-(2\sqrt{5})^2}$$
$$= \frac{3-2\sqrt{5}}{9-(4(5))}$$
$$= \frac{3-2\sqrt{5}}{9-20}$$
$$= \frac{3-2\sqrt{5}}{-11}$$

(v) $\frac{15}{\sqrt{31}-4}$

$$= \frac{15}{\sqrt{31}-4} \times \frac{\sqrt{31}+4}{\sqrt{31}+4}$$
$$= \frac{15(\sqrt{31}+4)}{(\sqrt{31})^2-(4)^2}$$
$$= \frac{15(\sqrt{31}+4)}{31-16}$$
$$= \frac{15(\sqrt{31}+4)}{15}$$
$$= \sqrt{31}+4$$

(vi) $\frac{2}{\sqrt{5}-\sqrt{3}}$

$$= \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$
$$= \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$= \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$$
$$= \frac{2(\sqrt{5}+\sqrt{3})}{2}$$
$$= \sqrt{5}+\sqrt{3}$$

(vii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$
$$= \frac{(\sqrt{3})^2+(1)^2-2\sqrt{3}}{(\sqrt{3})^2-(1)^2}$$
$$= \frac{3+1-2\sqrt{3}}{3-1}$$
$$= \frac{4-2\sqrt{3}}{2}$$
$$= \frac{2(2-\sqrt{3})}{2}$$
$$= (2-\sqrt{3})$$

(viii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$
$$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$= \frac{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$$
$$= \frac{5+3+2\sqrt{15}}{5-3}$$
$$= \frac{8+2\sqrt{15}}{2}$$
$$= \frac{2(4+\sqrt{15})}{2}$$
$$= (4+\sqrt{15})$$

Q. 2: Find the conjugate of $x+\sqrt{y}$

(i) $3+\sqrt{7}$

Conjugate $= 3-\sqrt{7}$

(ii) $4-\sqrt{5}$

Conjugate $= 4+\sqrt{5}$

(iii) $2+\sqrt{3}$

Conjugate $= 2-\sqrt{3}$

(iv) $2+\sqrt{5}$

Conjugate $= 2-\sqrt{5}$

(v) $5+\sqrt{7}$

Conjugate $= 5 - \sqrt{7}$

(vi) $4 - \sqrt{15}$

Conjugate $= 4 + \sqrt{15}$

(vii) $7 - \sqrt{6}$

Conjugate $= 7 + \sqrt{6}$

(viii) $9 + \sqrt{2}$

Conjugate $= 9 - \sqrt{2}$

Q. 3: Simplify by combining similar terms

(i) if $x = 2 - \sqrt{3}$, find $\frac{1}{x}$

As $x = 2 - \sqrt{3}$

then $\frac{1}{x} = \frac{1}{2-\sqrt{3}}$

Multiplying and dividing by $2 + \sqrt{3}$

$$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2+\sqrt{3}}{4-3}$$

$$= 2 + \sqrt{3}$$

(ii) if $x = 4 - \sqrt{17}$, find $\frac{1}{x}$

As $x = 4 - \sqrt{17}$

then $\frac{1}{x} = \frac{1}{4-\sqrt{17}}$

Multiplying and dividing by $4 + \sqrt{17}$

$$= \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$

$$= \frac{4+\sqrt{17}}{(4)^2-\left(\sqrt{17}\right)^2}$$

$$= \frac{4+\sqrt{17}}{16-17}$$

$$= \frac{4+\sqrt{17}}{-1}$$

(iii) if $x = \sqrt{3} + 2$, find $\frac{1}{x}$

As $x = \sqrt{3} + 2$

then $\frac{1}{x} = \frac{1}{\sqrt{3}+2}$

Multiplying and dividing by $\sqrt{3} - 2$

$$= \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$= \frac{\sqrt{3}-2}{\left(\sqrt{3}\right)^2-(2)^2}$$

$$= \frac{\sqrt{3}-2}{3-4}$$

$$= \frac{\sqrt{3}-2}{-1}$$

$$= -\sqrt{3} + 2$$

$$x + \frac{1}{x} = \sqrt{3} + 2 - \sqrt{3} + 2$$

$$= 4$$

Q. 4: Simplify

(i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})+(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{2\sqrt{5}-2\sqrt{6}}{5-3}$$

$$= \frac{2\sqrt{5}-2\sqrt{6}}{2}$$

$$= \frac{2(\sqrt{5}-\sqrt{6})}{2}$$

$$= \sqrt{5} - \sqrt{6}$$

(ii) $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2+\sqrt{5}}$

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} + \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{2-\sqrt{5}}{(2)^2-(\sqrt{5})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} + \frac{2(\sqrt{5}+\sqrt{3})}{5-3} + \frac{2-\sqrt{5}}{4-5}$$

$$= \frac{2-\sqrt{3}}{1} + \frac{2(\sqrt{5}+\sqrt{3})}{2} + \frac{2-\sqrt{5}}{-1}$$

$$= 2 - \sqrt{3} + \sqrt{5} + \sqrt{3} - 2 + \sqrt{5}$$

$$= 2\sqrt{5}$$

(iii) $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$

$$= \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} - \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$= \frac{2\sqrt{5}-2\sqrt{3}}{5-3} + \frac{\sqrt{3}-\sqrt{2}}{3-2} - \frac{3\sqrt{5}-3\sqrt{2}}{5-2}$$

$$= \frac{2\sqrt{5}-2\sqrt{3}}{2} + \frac{\sqrt{3}-\sqrt{2}}{1} - \frac{3\sqrt{5}-3\sqrt{2}}{3}$$

$$= \frac{3(2\sqrt{5}-2\sqrt{3})+6(\sqrt{3}-\sqrt{2})-2(3\sqrt{5}-3\sqrt{2})}{6}$$

$$= \frac{6\sqrt{5}-6\sqrt{3}+6\sqrt{3}-6\sqrt{2}-6\sqrt{5}+6\sqrt{2}}{6}$$

$$= \frac{0}{6}$$

$$= 0$$

Q. 5: (i) If $x = 2 + \sqrt{3}$, find the value of $x - \frac{1}{x}$ and $\left(x - \frac{1}{x}\right)^2$

As $x = 2 + \sqrt{3}$

then $\frac{1}{x} = \frac{1}{2+\sqrt{3}}$

Multiplying and dividing by $2 - \sqrt{3}$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$= \frac{2-\sqrt{3}}{4-3}$

$= \frac{2-\sqrt{3}}{1}$

$= 2-\sqrt{3}$

$x-\frac{1}{x} = 2+\sqrt{3}-(2-\sqrt{3})$

$= 2+\sqrt{3}-2+\sqrt{3}$

$x-\frac{1}{x} = 2\sqrt{3}$

Squaring both sides

$\left(x-\frac{1}{x}\right)^2 = (2\sqrt{3})^2$

$\left(x-\frac{1}{x}\right)^2 = 4(3)$

$\left(x-\frac{1}{x}\right)^2 = 12$

(ii) If $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, find the value of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$ and $x^3+\frac{1}{x^3}$

As $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

Multiplying and dividing by $\sqrt{5}-\sqrt{2}$

$= \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{(\sqrt{5}-\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}$

$= \frac{(\sqrt{5})^2+(\sqrt{2})^2-2\sqrt{10}}{5-2}$

$= \frac{7-2\sqrt{10}}{3}$

then $\frac{1}{x} = \frac{3}{7-2\sqrt{10}}$

Multiplying and dividing by $7+2\sqrt{10}$

$= \frac{3}{7-2\sqrt{10}} \times \frac{7+2\sqrt{10}}{7+2\sqrt{10}}$

$= \frac{3(7+2\sqrt{10})}{(7)^2-(2\sqrt{10})^2}$

$= \frac{3(7+2\sqrt{10})}{49-40}$

$= \frac{3(7+2\sqrt{10})}{9}$

$= \frac{7+2\sqrt{10}}{3}$

$x+\frac{1}{x} = \frac{7-2\sqrt{10}}{3} + \frac{7+2\sqrt{10}}{3}$

$= \frac{7-2\sqrt{10}+7+2\sqrt{10}}{3}$

$x+\frac{1}{x} = \frac{14}{3}$ -------------------- (i)

Squaring both sides

$$\left(x+\frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2$$
$$x^2+\frac{1}{x^2}+2 = \frac{196}{9}$$
$$x^2+\frac{1}{x^2} = \frac{196}{9}-2$$
$$x^2+\frac{1}{x^2} = \frac{196-18}{9}$$
$$x^2+\frac{1}{x^2} = \frac{178}{9}$$

Taking cube on both sides of equation (i)

$$\left(x+\frac{1}{x}\right)^3 = \left(\frac{14}{3}\right)^3$$
$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right) = \frac{2744}{27}$$
$$x^3+\frac{1}{x^3}+3\left(\frac{14}{3}\right) = \frac{2744}{27}$$
$$x^3+\frac{1}{x^3}+14 = \frac{2744}{27}$$
$$x^3+\frac{1}{x^3} = \frac{2744}{27}-14$$
$$x^3+\frac{1}{x^3} = \frac{2744-378}{27}$$
$$x^3+\frac{1}{x^3} = \frac{2366}{27}$$

Q. 6: Determine the rational numbers a and b if $\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$.

Rationalizing L.H.S

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)+(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$
$$= \frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{(\sqrt{3})^2-(1)^2}$$
$$= \frac{(\sqrt{3})^2+1-2\sqrt{3}+(\sqrt{3})^2+1+2\sqrt{3}}{3-1}$$
$$= \frac{3+1-2\sqrt{3}+3+1+2\sqrt{3}}{2}$$
$$= \frac{8}{2}$$
$$= 4$$

as, R.H.S $= a+b\sqrt{3}$

So, $a+b\sqrt{3} = 4$

equating them we get a = 4 and b = 0.