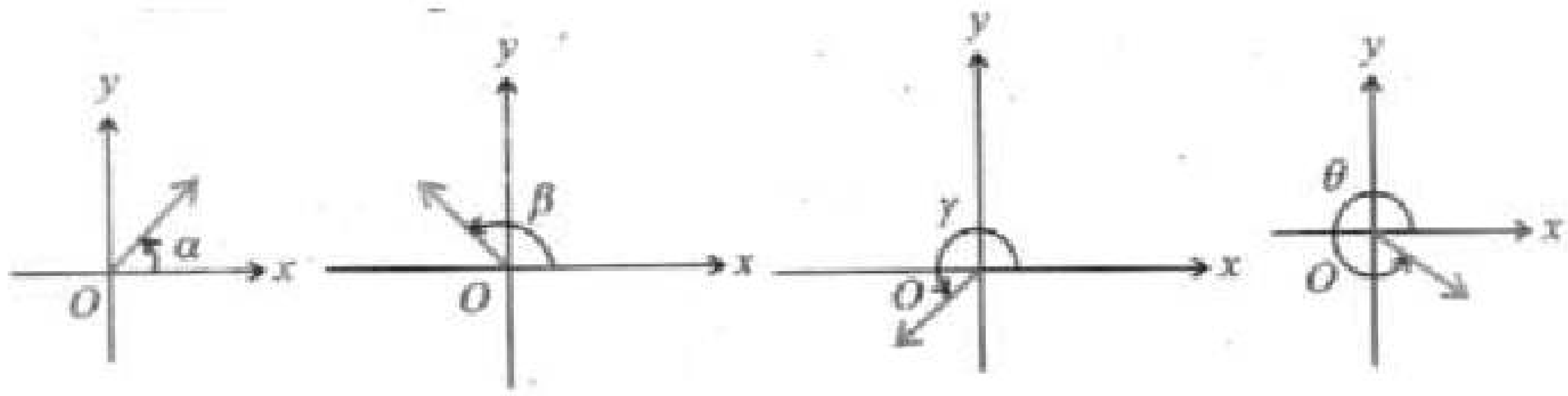
Exercise 7.3

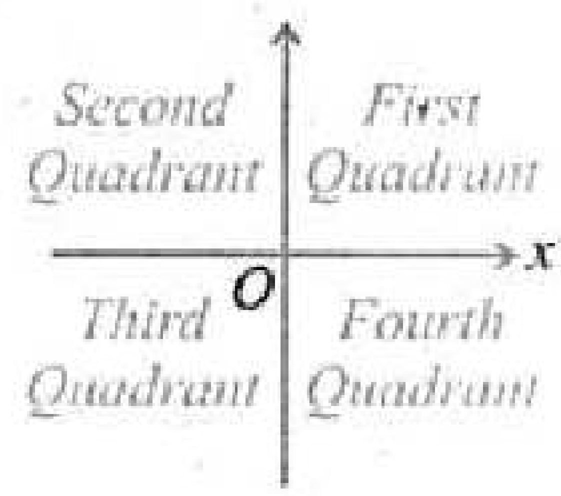
Things To know:

1. Angles in standard position:



2. The Quadrants and Quadrantal Angles:

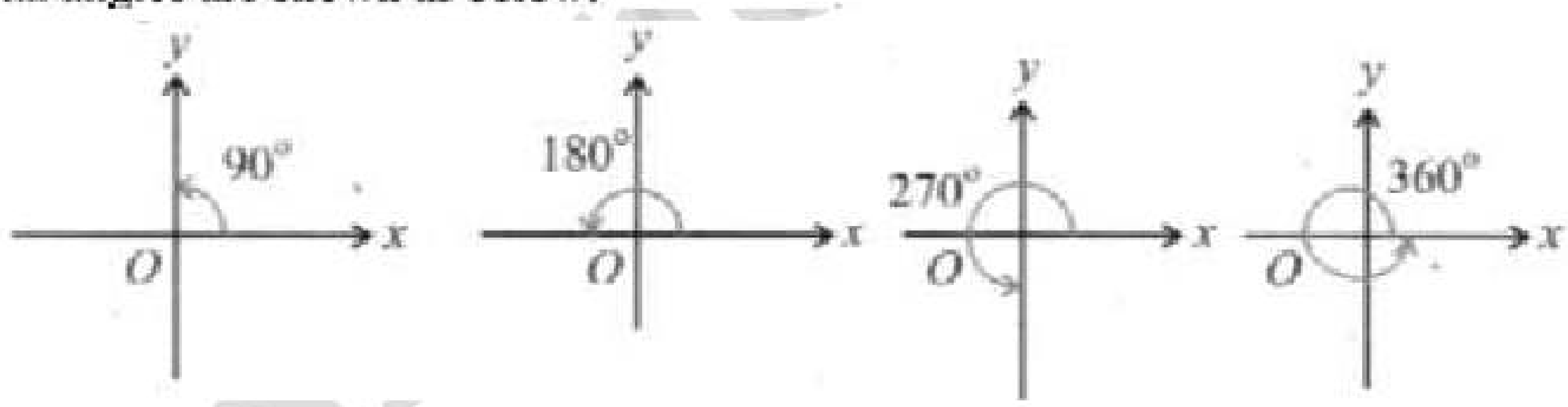
The x-axis and y-axis divides the plane in four regions, called quadrants, when they intersect each other at right angle. The point of intersection is called origin and is denoted by O.



Angles between 0° and 90° are in the first quadrant.
Angles between 90° and 180° are in the second quadrant.
Angles between 180° and 270° are in the third quadrant.
Angles between 270° to 360° are in the fourth quadrant.
An angle in standard position is said to lie in a quadrant if its terminal side lies in that quadrant. Angles α, β, γ and θ lie in I, II, III and IV quadrant respectively in figure 7.3.1.

Quadrantal Angles

If the terminal side of an angle in standard position falls on x-axis or y-axis, then it is called a quadrantal angle *i.e.*, 90°, 180°, 270° and 360° are quadrantal angles. The quadrantal angles are shown as below:



3. Trigonometric ratios and their reciprocals with the help of a unit circle:

There are six fundamental trigonometric ratios called sine, cosine, tangent, cotangent, secant and cosecant. To define these functions we use circular approach which involves the unit circle.

Let θ be a real number, which represents the radian measure of an angle in standard position. Let $P(x, y)$ be any point on the unit circle lying on terminal side of θ as shown in the figure.

We define sine of θ, written as $\sin\theta$ and cosine of θ written as $\cos\theta$, as:

$$\sin\theta = \frac{EP}{OP} = \frac{y}{1} \Rightarrow \sin\theta = y$$

and $$\cos\theta = \frac{OE}{OP} = \frac{x}{1} \Rightarrow \cos\theta = x$$

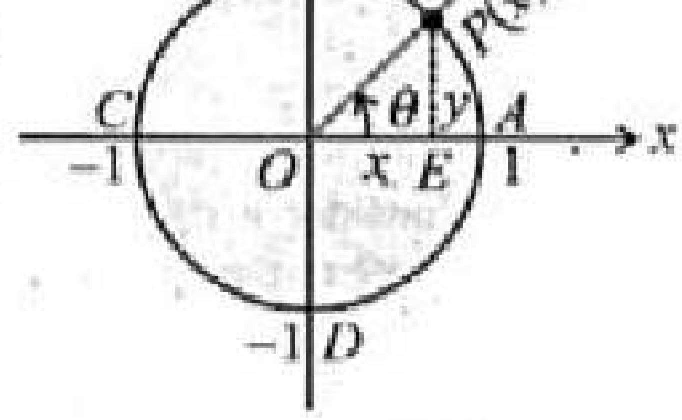


Fig. 7.3.3

$\tan\theta = \frac{EP}{OE} = \frac{y}{x} \Rightarrow \tan\theta = \frac{y}{x} \quad (x \neq 0)$

since $y = \sin\theta$ and $x = \cos\theta \Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{x}{y} \ (y \neq 0) \Rightarrow \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sec\theta = \frac{1}{x} (x \neq 0)$ and $\operatorname{cosec}\theta = \frac{1}{y} \quad (y \neq 0)$

$= \frac{1}{\cos\theta} \qquad\qquad = \frac{1}{\sin\theta}$

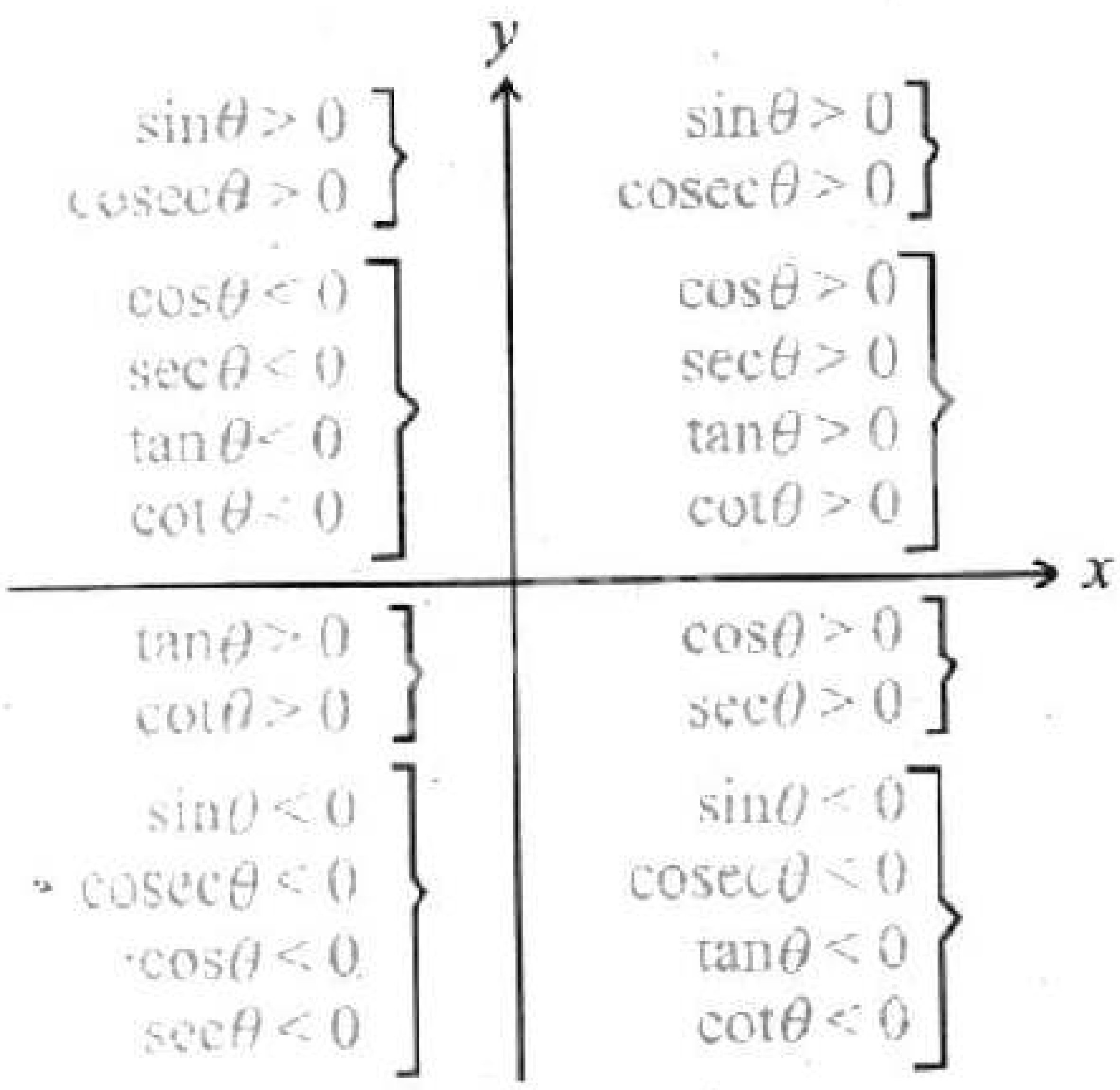
Reciprocal Identities

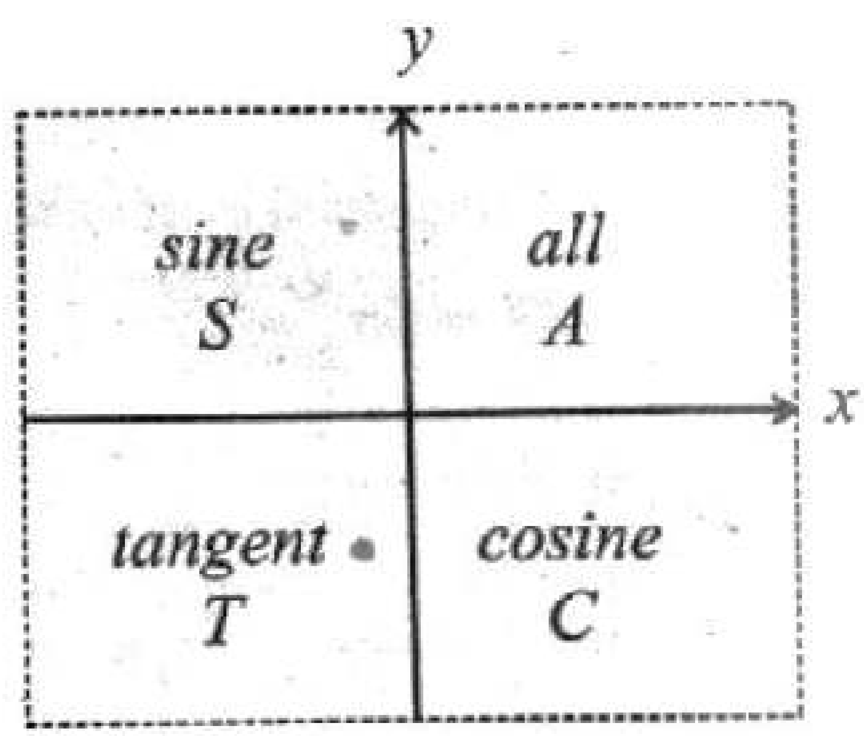
$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$

Signs of trigonometric ratios in different Quadrants:





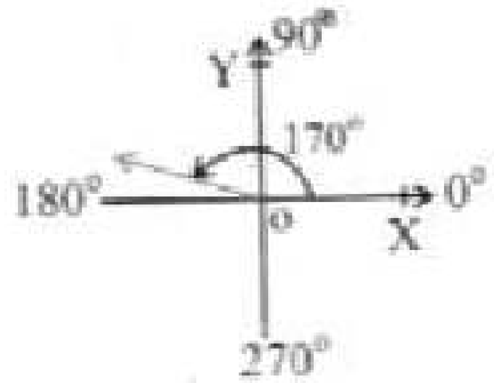
Read 'ASTC' as
'Add Sugar To Coffee'

Allied Angles:

$$sin(-\Theta) = -sin\Theta \qquad cos(-\Theta) = cos\Theta$$
$$sin(90 - \Theta) = cos\Theta \qquad cos(90 - \Theta) = sin\Theta$$
$$sin(90 + \Theta) = cos\Theta \qquad sin(90 + \Theta) = -sin\Theta$$
$$sin(180 - \Theta) = sin\Theta \qquad cos(180 - \Theta) = -cos\Theta$$
$$sin(180 + \Theta) = -sin\Theta \qquad cos(180 + \Theta) = -cos\Theta$$

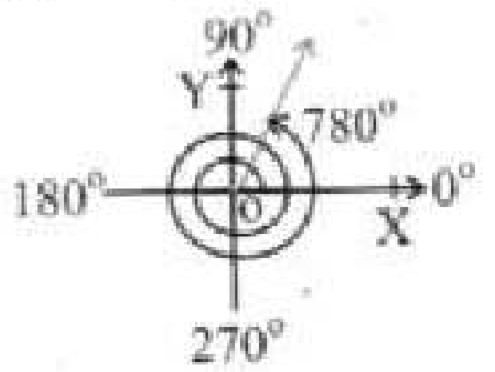
Q. 1: Locate each of the following angles in standard position using protractor or fair free hand guess. Also find a positive and negative angle coterminal with each given angle.

(i) 170°



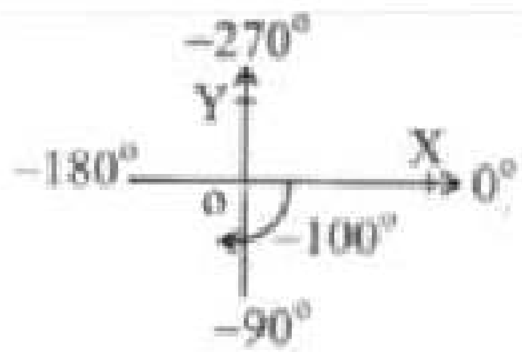
Positive coterminal angle $360^\circ + 170^\circ = 530^\circ$
negative coterminal angle -190°

(ii) 780°



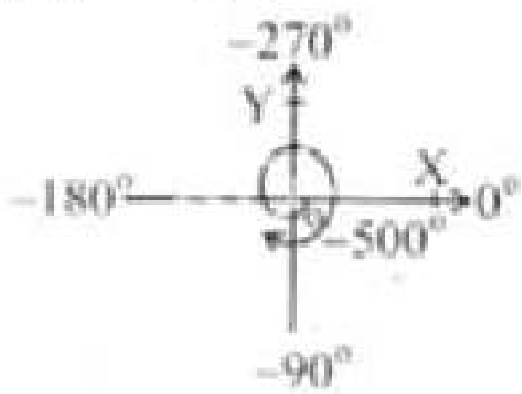
Positive coterminal angle 60°
negative coterminal angle is -300°

(iii) -100°



Positive coterminal angle is 260°
negative coterminal angle $-360^\circ - 100^\circ = -460^\circ$

(iv) -500°



Positive coterminal angle 220°
negative coterminal angle -140°

Q. 2: Identify the closest quadrantal angles between which the following angles lies.

(i) 156°

156° lies between 90° and 180°.

(ii) 318°

318° lies between 270° and 360°.

(iii) 572°

572° lies between 540° and 630°.

(iv) -330°

-330° lies between -270° and -360°. i.e. quadrantal angles are 0° and 90°.

Q. 3: Write the closest quadrantal angles between which the angle lies. Write your answer in radian measure.

(i) $\frac{\pi}{3}$

$\frac{\pi}{3}$ lies between 0 and $\frac{\pi}{2}$

(ii) $\frac{3\pi}{4}$

$\frac{3\pi}{4}$ lies between $\frac{\pi}{2}$ and π

(iii) $\frac{-\pi}{4}$

$\frac{-\pi}{4}$ lies between 0 and $-\frac{\pi}{2}$

(iv) $\frac{-3\pi}{4}$

$\frac{-3\pi}{4}$ lies between $-\frac{\pi}{2}$ and $-\pi$

Q. 4: In which quadrant θ lie when

(i) $sin\theta > 0, tan\theta < 0$

Quadrant II

(ii) $cos\theta < 0, sin\theta < 0$

Quadrant III

(iii) $sec\theta > 0, sin\theta < 0$

Quadrant IV

(iv) $cos\theta < 0, tan\theta < 0$

Quadrant II

(v) $cosec\theta > 0, cos\theta > 0$

Quadrant I

(vi) $sin\theta < 0, sec\theta < 0$

Quadrant III

Q. 5: Fill in the blanks.

(i) cos(-150°) = + cos150°

(ii) sin(-310°) = - sin310°

(iii) tan(-210°) = -tan210°

(iv) cot(-45°) = - cot45°

(v) sec(-60°) = + sec60°

(vi) cosec(-137°) = - cosec137°

Q. 6: The given point P lies on the terminal side of θ. Find quadrant of θ and all six trigonometric ratios.

(i) (-2, 3)

We have $x = -2$ and $y = 3$, so θ lies in Quadrant II.

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

Thus

$sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}}$; $cosec\theta = \frac{\sqrt{13}}{3}$

$cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{13}}$; $sec\theta = -\frac{\sqrt{13}}{2}$

$tan\theta = \frac{y}{x} = \frac{-3}{2}$; $cot\theta = -\frac{2}{3}$

(ii) (-3, -4)

We have $x = -3$ and $y = -4$, so θ lies in Quadrant III.

$r \quad = \sqrt{x^2 + y^2}$

$\quad = \sqrt{(-3)^2 + (-4)^2}$

$\quad = \sqrt{9 + 16}$

$\quad = \sqrt{25}$

$\quad = 5$

Thus

$sin\theta = \frac{y}{r} = \frac{-4}{5}$; $cosec\theta = \frac{-5}{4}$

$cos\theta = \frac{x}{r} = \frac{-3}{5}$; $sec\theta = -\frac{5}{3}$

$tan\theta = \frac{y}{x} = \frac{4}{3}$; $cot\theta = \frac{3}{4}$

(iii) $(\sqrt{2}, 1)$

We have $x = \sqrt{2}$ and $y = 1$, so θ lies in Quadrant II.

$r \quad = \sqrt{x^2 + y^2}$

$\quad = \sqrt{\left(\sqrt{2}\right)^2 + (1)^2}$

$\quad = \sqrt{2 + 1}$

$\quad = \sqrt{3}$

Thus

$sin\theta = \frac{y}{r} = \frac{1}{\sqrt{3}}$; $cosec\theta = \sqrt{3}$

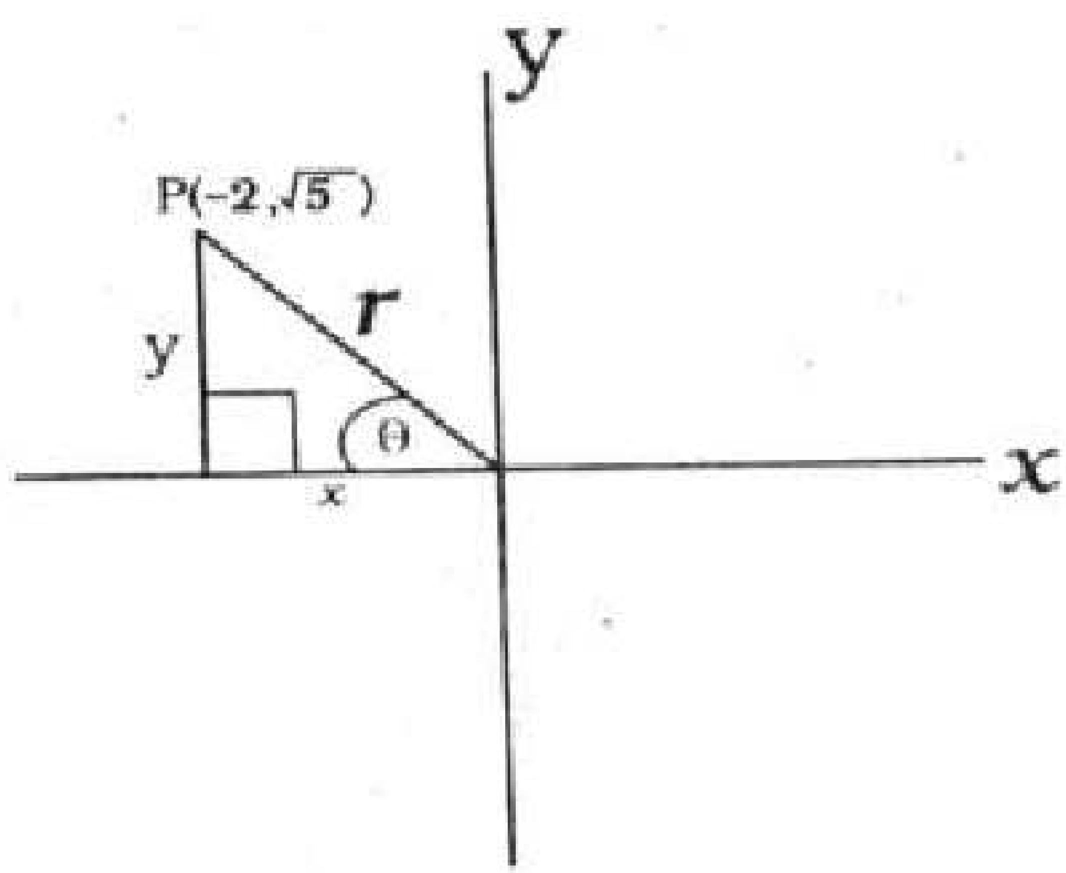
$cos\theta = \frac{x}{r} = \frac{\sqrt{2}}{\sqrt{3}}$; $sec\theta = \frac{\sqrt{3}}{\sqrt{2}}$

$tan\theta = \frac{y}{x} = \frac{1}{\sqrt{2}}$; $cot\theta = \sqrt{2}$

Q. 7: If $cos\theta = \frac{-2}{3}$ and terminal arm of the angle θ is in quadrant II, find the values of remaining trigonometric functions.

In any right triangle XYZ,

$cos\theta = \frac{-2}{3} = \frac{x}{r}$ then, $x = -2$ and $r = 3$



Also,

$sec\theta = \frac{1}{cos\theta} = \frac{-3}{2}$

As we know

$r^2 = x^2 + y^2$

$(3)^2 = (-2)^2 + y^2$

$9 = 4 + y^2$

$5 = y^2$

$y = \pm\sqrt{5}$ so, $y = \sqrt{5}$

$sin\theta = \frac{y}{r} = \frac{\sqrt{5}}{3}$; $cosec\theta = \frac{3}{\sqrt{5}}$

$tan\theta = \frac{y}{x} = \frac{-\sqrt{5}}{2}$; $cot\theta = \frac{-2}{\sqrt{5}}$

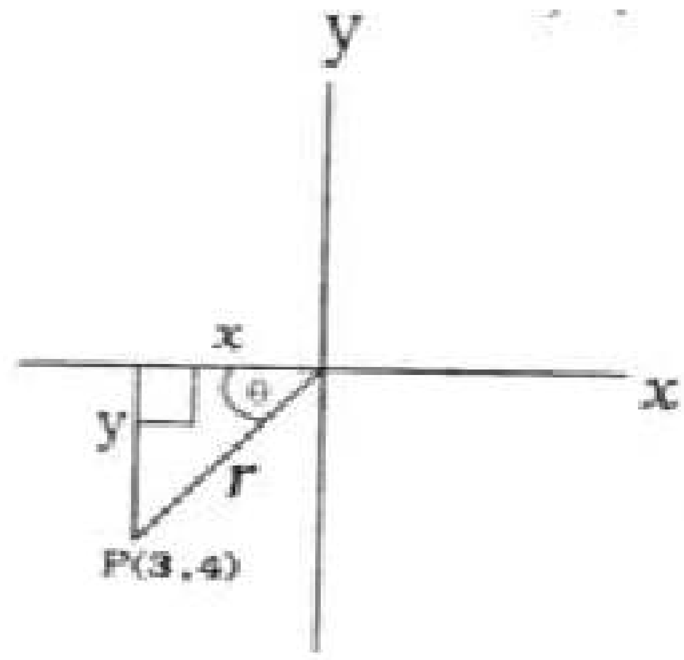
Q. 8: If $tan\theta = \frac{4}{3}$ and $sin\theta < 0$, find the values of other trigonometric functions at θ.

In any right triangle XYZ,

$tan\theta \quad = \frac{4}{3} = \frac{y}{x}$ then, $y = 4$ and $x = 3$

Also,

$cot\theta \quad = \frac{1}{tan\theta} = \frac{3}{4}$



As we know

$r^2 \quad = x^2 + y^2$

$r^2 \quad = (3)^2 + (4)^2$

$r^2 \quad = 9 + 16$

$r^2 \quad = 25$

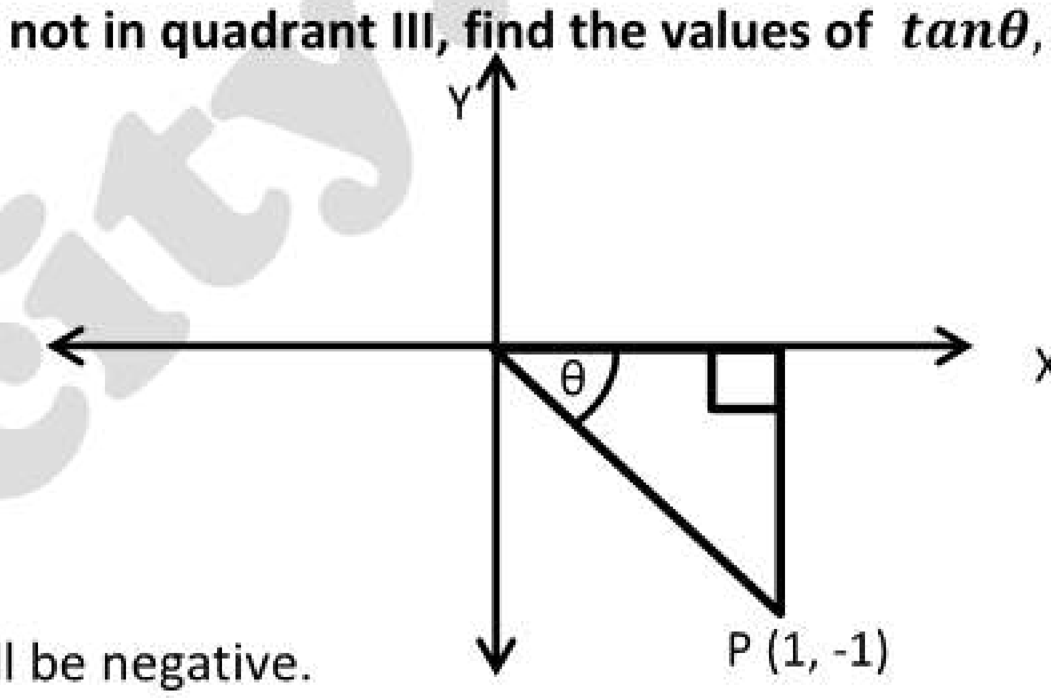
$r \quad = \pm 5$ so, $r \quad = 5$

As, $sin\theta < 0$ and $tan\theta > 0$. So, the terminal arm of angle lies in Quadrant III.

$sin\theta \quad = -\frac{y}{r} = -\frac{4}{5}$; $cosec\theta = \frac{-5}{4}$

$cos\theta \quad = -\frac{x}{r} = \frac{-3}{5}$; $sec\theta \quad = \frac{-5}{3}$

Q. 9: If $sin\theta = \frac{-1}{\sqrt{2}}$ and terminal side of the angle is not in quadrant III, find the values of $tan\theta, sec\theta$ and $cosec\theta$.



In any right triangle XYZ,

$sin\theta \quad = \frac{-1}{\sqrt{2}} = \frac{y}{r}$ then, $y = -1$ and $r = \sqrt{2}$

Also,

$cosec\theta = \frac{1}{sin\theta} = -\sqrt{2}$

As $sin\theta$ is negative in Quadrant III and IV,

therefore in this case θ is in Quadrant IV and $cos\theta$ will be negative.

Now,

$r^2 \quad = x^2 + y^2$

$\left(\sqrt{2}\right)^2 = x^2 + (-1)^2$

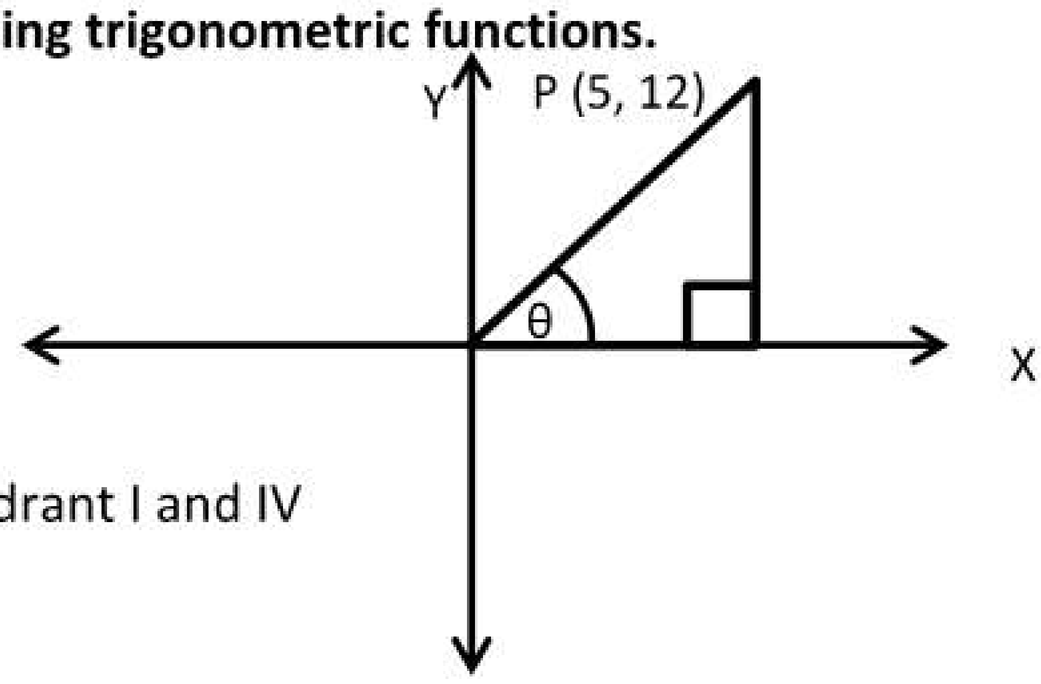
$2 \quad = x^2 + 1$

$1 \quad = x^2$

$x \quad = \pm 1$ so, $x \quad = 1$

$sec\theta \quad = \frac{\sqrt{2}}{1} = \sqrt{2}$; $tan\theta \quad = \frac{y}{x} = \frac{-1}{1} = -1$

Q. 10: If $cosec\theta = \frac{13}{12}$ and $sec\theta > 0$, find the remaining trigonometric functions.



In any right triangle XYZ,

$cosec\theta = \frac{13}{12} = \frac{r}{y}$ then, $y = 12$ and $r = 13$

Also,

$sin\theta = \frac{1}{cosec\theta} = \frac{12}{13}$

As $sin\theta > 0$ in Quadrant I and II and $sec\theta > 0$ in Quadrant I and IV

therefore in this case θ is in Quadrant I.

Now,

$r^2 = x^2 + y^2$

$(13)^2 = x^2 + (12)^2$

$169 = x^2 + 144$

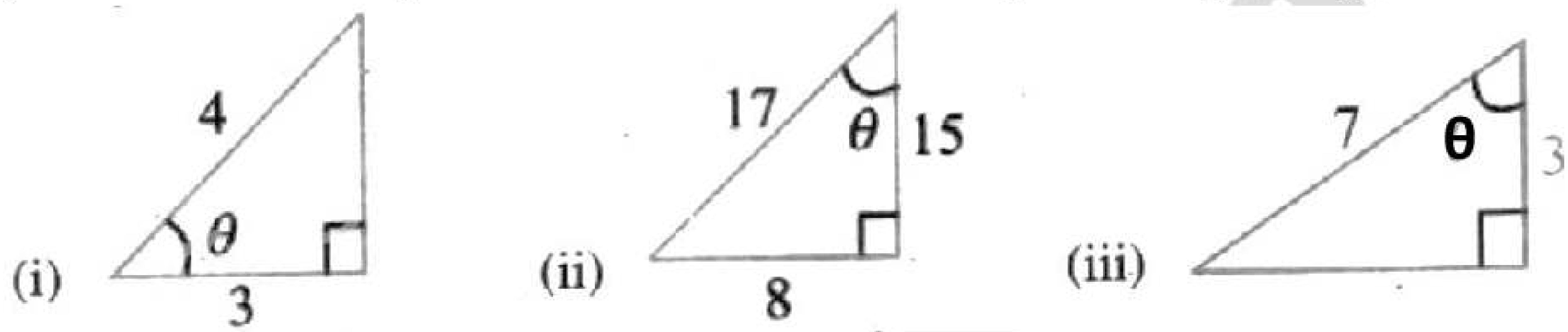
$25 = x^2$

$x = \pm 5$ so, $x = 5$

$\cos\theta = \frac{x}{r} = \frac{5}{13}$; $\sec\theta = \frac{13}{5}$

$\tan\theta = \frac{y}{x} = \frac{12}{5}$; $\cot\theta = \frac{5}{12}$

Q. 11: Find the values of trigonometric functions at the indicated angle θ in the right triangle.



(i) From figure we have

$x = 3$ and $r = 4$

Now,

$r^2 = x^2 + y^2$

$(4)^2 = (3)^2 + y^2$

$16 = 9 + y^2$

$7 = y^2$

$y = \pm\sqrt{7}$ so, $y = \sqrt{7}$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{7}}{4}$; $\operatorname{cosec}\theta = \frac{4}{\sqrt{7}}$

$\cos\theta = \frac{x}{r} = \frac{3}{4}$; $\sec\theta = \frac{4}{3}$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{7}}{3}$; $\cot\theta = \frac{3}{\sqrt{7}}$

(ii) From figure we have

$x = 15$, $y = 8$ and $r = 17$

Now,

$\sin\theta = \frac{y}{r} = \frac{8}{17}$; $\operatorname{cosec}\theta = \frac{17}{8}$

$\cos\theta = \frac{x}{r} = \frac{15}{17}$; $\sec\theta = \frac{17}{15}$

$\tan\theta = \frac{y}{x} = \frac{8}{15}$; $\cot\theta = \frac{15}{8}$

(iii) From figure we have

$x = 3$ and $r = 7$

Now,

$r^2 = x^2 + y^2$

$(7)^2 \quad = (3)^2 + y^2$

$49 \quad = 9 + y^2$

$40 \quad = y^2$

$y \quad = \pm 2\sqrt{10}$ so, $y = 2\sqrt{10}$

$sin\theta \quad = \frac{y}{r} = \frac{2\sqrt{10}}{7}$; $cosec\theta = \frac{7}{2\sqrt{10}}$

$cos\theta \quad = \frac{x}{r} = \frac{3}{7}$; $sec\theta \quad = \frac{7}{3}$

$tan\theta \quad = \frac{y}{x} = \frac{2\sqrt{10}}{3}$; $cot\theta \quad = \frac{3}{2\sqrt{10}}$

Q. 12: Find the values of the trigonometric functions. Do not use trigonometric tables or calculator.

(i) $tan30^o$

We know that $2k\pi + \theta = \theta$

$tan30^o \quad = tan\left(2(0)\pi + \frac{\pi}{6}\right) \quad = tan\frac{\pi}{6} \quad = \frac{1}{\sqrt{3}}$

(ii) $tan330^o$

We know that $2k\pi + \theta = \theta$

$tan330^o \quad = tan\left(2(1)\pi - \frac{\pi}{6}\right) \quad = -tan\frac{\pi}{6} \quad = -\frac{1}{\sqrt{3}}$

(iii) $sec330^o$

We know that $2k\pi + \theta = \theta$

$sec330^o \quad = sec\left(2(1)\pi - \frac{\pi}{6}\right) \quad = sec\frac{\pi}{6} \quad = \frac{2}{\sqrt{3}}$

(iv) $cot\frac{\pi}{4}$

We know that $2k\pi + \theta = \theta$

$cot\frac{\pi}{4} \quad = cot\left(2(0)\pi + \frac{\pi}{4}\right) \quad = cot\frac{\pi}{4} \quad = \frac{1}{1} \quad = 1$

(v) $cos\frac{2\pi}{3}$

$cos\frac{3\pi-\pi}{3} \quad = cos\left(\pi - \frac{\pi}{3}\right) \quad = -cos\frac{\pi}{3} \quad = -\frac{1}{2}$

(vi) $cosec\frac{2\pi}{3}$

$cosec\frac{3\pi-\pi}{3} \quad = cosec\left(\pi - \frac{\pi}{3}\right) \quad = cosec\frac{\pi}{3} \quad = \frac{2}{\sqrt{3}}$

(vii) $cos(-450^o)$

We know that $2k\pi + \theta = \theta$

$cot(-360^o - 90^o) = cos\left(2(-1)\pi - \frac{\pi}{2}\right) \quad = cos\frac{\pi}{2} \quad = 0$

(viii) $tan(-9\pi)$

We know that $2k\pi + \theta = \theta$

$tan(-8\pi - \pi) = tan(2(-4)\pi - \pi) \quad = -tan\pi \quad = 0$

(ix) $cos\left(\frac{-5\pi}{6}\right)$

We know that $2k\pi + \theta = \theta$

$cos\left(-\pi + \frac{\pi}{6}\right) = cos\left(-\pi + \frac{\pi}{6}\right) \quad = -cos\frac{\pi}{6} \quad = -\frac{\sqrt{3}}{2}$

(x) $sin\frac{7\pi}{6}$

We know that $2k\pi+\theta=\theta$

$sin\left(\pi+\frac{\pi}{6}\right)$ $= sin\left(\pi+\frac{\pi}{6}\right)$ $= -sin\frac{\pi}{6}$ $= -\frac{1}{2}$

(xi) $cot\frac{7\pi}{6}$

We know that $2k\pi+\theta=\theta$

$cot\left(\pi+\frac{\pi}{6}\right)$ $= cot\left(\pi+\frac{\pi}{6}\right)$ $= cot\frac{\pi}{6}$ $= \frac{1}{\sqrt{3}}$

(x) $cos225^o$

We know that $2k\pi+\theta=\theta$

$cos\frac{5\pi}{4}$ $= cos\left(\pi+\frac{\pi}{4}\right)$ $= -cos\frac{\pi}{4}$ $= -\frac{1}{\sqrt{2}}$