Exercise 4.2

Q. 1: (i) If a + b = 10 and a – b = 6, then find the value of ($a^2 + b^2$).

$a + b = 10$

Squaring both sides

$(a+b)^2 = 100$

$a^2 + b^2 + 2ab = 100$ ------------ (i)

Now we have

$a - b = 6$

Squaring both sides

$(a-b)^2 = 36$

$a^2 + b^2 - 2ab = 36$ ------------ (ii)

Adding equation (i) and (ii)

$2a^2 + 2b^2 = 136$

$2(a^2 + b^2) = 136$

$a^2 + b^2 = 68$

(ii) If a + b = 5 and a – b = $\sqrt{17}$, then find the value of ab.

$a + b = 5$

Squaring both sides

$(a+b)^2 = 25$

$a^2 + b^2 + 2ab = 25$ ------------ (i)

Now we have

$a - b = \sqrt{\mathbf{17}}$

Squaring both sides

$(a-b)^2 = 17$

$a^2 + b^2 - 2ab = 17$ ------------ (ii)

Subtracting equation (ii) from (i)

$4ab = 8$

$ab = 2$

Q. 2: $a^2 + b^2 + c^2 = 45$ and $a + b + c = -1$, then find the value of $ab + bc + ca$.

As we know

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(-1)^2 = 45 + 2(ab + bc + ca)$

$1 = 45 + 2(ab + bc + ca)$

$-44 = 2(ab + bc + ca)$

$-22 = ab + bc + ca$

Q. 3: $m + n + p = 10$ and $mn + np + mp = 27$, then find the value of $m^2 + n^2 + p^2$.

As we know

$(m+n+p)^2 = m^2 + n^2 + p^2 + 2(mn + np + mp)$

$(10)^2 = m^2 + n^2 + p^2 + 2(27)$

$100 = m^2 + n^2 + p^2 + 54$

$46 = m^2 + n^2 + p^2$

Q. 4: $\boldsymbol{x^2 + y^2 + z^2 = 78}$ **and** $\boldsymbol{xy + yz + zx = 59}$**, then find the value of** $\boldsymbol{x + y + z}$**.**

As we know

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$(x + y + z)^2 = 78 + 2(59)$

$(x + y + z)^2 = 78 + 118$

$(x + y + z)^2 = 196$

Taking square root on both sides

$x + y + z = \pm 14$

Q. 5: $\boldsymbol{x + y + z = 12}$ **and** $\boldsymbol{x^2 + y^2 + z^2 = 64}$**, then find the value of** $\boldsymbol{xy + yz + zx}$**.**

As we know

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$(12)^2 = 64 + 2(xy + yz + zx)$

$144 = 64 + 2(xy + yz + zx)$

$144 - 64 = 2(xy + yz + zx)$

$80 = 2(xy + yz + zx)$

$40 = xy + yz + zx$

$xy + yz + zx = 40$

Q. 6: **if** $\boldsymbol{x + y = 7}$ **and** $\boldsymbol{xy = 12}$**, then find the value of** $\boldsymbol{x^3 + y^3}$**.**

As we know

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$(7)^3 = x^3 + y^3 + 3(12)(7)$

$343 = x^3 + y^3 + 252$

$343 - 252 = x^3 + y^3$

$91 = x^3 + y^3$

$x^3 + y^3 = 91$

Q. 7: **if** $\boldsymbol{3x + 4y = 11}$ **and** $\boldsymbol{xy = 12}$**, then find the value of** $\boldsymbol{27x^3 + 64y^3}$**.**

As we know

$(3x + 4y)^3 = (3x)^3 + (4y)^3 + 3(3x)(4y)(3x + 4y)$

$(3x + 4y)^3 = 27x^3 + 64y^3 + 36xy(3x + 4y)$

$(11)^3 = 27x^3 + 64y^3 + 36(12)(11)$

$1331 = 27x^3 + 64y^3 + 4752$

$1331 - 4752 = 27x^3 + 64y^3$

$-3421 = 27x^3 + 64y^3$

$27x^3 + 64y^3 = -3421$

Q. 8: **if** $\boldsymbol{x - y = 4}$ **and** $\boldsymbol{xy = 21}$**, then find the value of** $\boldsymbol{x^3 - y^3}$**.**

As we know

$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(4)^3 = x^3 - y^3 - 3(21)(4)$

$64 = x^3 - y^3 - 252$

$316 = x^3 - y^3$

$x^3 - y^3 = 316$

Q. 9: if $5x - 6y = 13$ and $xy = 6$, then find the value of $125x^3 - 216y^3$.

As we know

$$(5x-6y)^3 = (5x)^3 - (6y)^3 - 3(5x)(6y)(5x-6y)$$
$$(5x-6y)^3 = 125x^3 - 216y^3 - 90xy(5x-6y)$$
$$(13)^3 = 125x^3 - 216y^3 - 90(6)(13)$$
$$2197 = 125x^3 - 216y^3 - 7020$$
$$2197 + 7020 = 125x^3 - 216y^3$$
$$9217 = 125x^3 - 216y^3$$
$$125x^3 - 216y^3 = 9217$$

Q. 10: if $x + \frac{1}{x} = 13$, then find the value of $x^3 + \frac{1}{x^3}$.

As we know

$$x + \frac{1}{x} = 13$$

Taking cube on both sides

$$\left(x + \frac{1}{x}\right)^3 = (13)^3$$
$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 2197$$
$$x^3 + \frac{1}{x^3} + 3(13) = 2197$$
$$x^3 + \frac{1}{x^3} + 39 = 2197$$
$$x^3 + \frac{1}{x^3} = 2158$$

Q. 11: if $x - \frac{1}{x} = 7$, then find the value of $x^3 - \frac{1}{x^3}$.

As we know

$$x - \frac{1}{x} = 7$$

Taking cube on both sides

$$\left(x - \frac{1}{x}\right)^3 = (7)^3$$
$$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 343$$
$$x^3 - \frac{1}{x^3} - 3(7) = 343$$
$$x^3 - \frac{1}{x^3} - 21 = 343$$
$$x^3 - \frac{1}{x^3} = 364$$

Q. 12: if $3x + \frac{1}{3x} = 5$, then find the value of $27x^3 + \frac{1}{27x^3}$.

As we know

$$3x + \frac{1}{3x} = 5$$

Taking cube on both sides

$$\left(3x + \frac{1}{3x}\right)^3 = (5)^3$$

$$(3x)^3 + \frac{1}{(3x)^3} + 3\left(3x + \frac{1}{3x}\right) = 125$$

$$27x^3 + \frac{1}{27x^3} + 3(5) = 125$$

$$27x^3 + \frac{1}{27x^3} + 15 = 125$$

$$27x^3 + \frac{1}{27x^3} = 110$$

Q. 13: if $5x - \frac{1}{5x} = 6$, then find the value of $125x^3 - \frac{1}{125x^3}$.

As we know

$$5x - \frac{1}{5x} = 6$$

Taking cube on both sides

$$\left(5x - \frac{1}{5x}\right)^3 = (6)^3$$

$$(5x)^3 - \frac{1}{(5x)^3} - 3\left(5x - \frac{1}{5x}\right) = 216$$

$$125x^3 - \frac{1}{125x^3} - 3(6) = 216$$

$$125x^3 - \frac{1}{125x^3} - 18 = 216$$

$$125x^3 - \frac{1}{125x^3} = 234$$

Q. 14: Factorize (i) $x^3 - y^3 - x + y$ (ii) $8x^3 - \frac{1}{27x^3}$

(i) $x^3 - y^3 - x + y$

$$= [(x)^3 - (y)^3] - (x - y)$$

$$= [(x - y)(x^2 + xy + y^2)] - [x - y]$$

$$= (x - y)[(x^2 + xy + y^2) - 1]$$

$$= (x - y)[x^2 + xy + y^2 - 1]$$

(ii) $8x^3 - \frac{1}{27y^3}$

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$

$$= \left(2x - \frac{1}{3y}\right)\left((2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right)$$

$$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$$

Q. 15: Find the products using formulas.

(i) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$$= (x^2 + y^2)((x^2)^2 - x^2y^2 + (y^2)^2)$$

$$= ((x^2)^3 + (y^2)^3)$$

$$= (x^6 + y^6)$$

(ii) $(x^3 + y^3)(x^6 - x^3y^3 + y^6)$

$$= (x^3 + y^3)((x^3)^2 - x^3y^3 + (y^3)^2)$$

$$= ((x^3)^3 + (y^3)^3)$$

$$= (x^9 + y^9)$$

(iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$

$$= [(x - y)(x^2 + xy + y^2)][(x + y)(x^2 - xy + y^2)][(x^2 + y^2)(x^4 - x^2y^2 + y^4)]$$

$$= (x^3 - y^3)(x^3 + y^3)[(x^2 + y^2)((x^2)^2 - x^2y^2 + (y^2)^2)]$$

$$= [(x^3)^2 - (y^3)^2][(x^2)^3 + (y^2)^3]$$

$$= [x^6 - y^6][x^6 + y^6]$$

$$= (x^6)^2 - (y^6)^2$$

$= x^{12} - y^{12}$

(iv) $(2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)(4x^4 - 2x^2 + 1)$

$= [(2x^2 - 1)(4x^4 + 2x^2 + 1)][(2x^2 + 1)(4x^4 - 2x^2 + 1)]$

$= [(2x^2 - 1)((2x^2)^2 + (2x^2)(1) + (1)^2)][(2x^2 + 1)((2x^2)^2 - (2x^2)(1) + (1)^2)]$

$= [(2x^2)^3 - (1)^3][(2x^2)^3 + (1)^3]$

$= [8x^6 - 1][8x^6 + 1]$

$= (8x^6)^2 - (1)^2$

$= 64x^{12} - 1$