Exercise 13.1

Q.1 Two sides of a triangle measure 10cm and 15 cm which of the following measure is possible for the third side?

(a) $5cm$
(b) $20\ cm$
(c) $25\ cm$
(d) $30\ cm$

Solution

Lengths of two sides are 15 and 10 cm.
So, sum of two lengths of triangle = 10 + 15 = 25 m
10 + 15 > 20
10 + 20 > 15
15 + 20 > 10
$\therefore$ 20 cm is possible for third side
Or
Sum of length of two sides is always greater than the third sides of a triangle.

Given

Q.2 Point O is interior of ΔABC

Show that

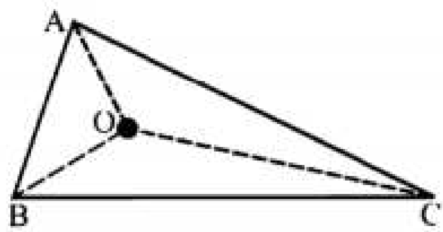
$$m\overline{OA}+m\overline{OB}+m\overline{OC}>\frac{1}{2}\left(m\overline{AB}+m\overline{BC}+m\overline{CA}\right)$$

Given

Point O is interior of ΔABC

To prove:

$$m\overline{OA}+m\overline{OB}+m\overline{OC}<\frac{1}{2}\left(m\overline{AB}+m\overline{BC}+m\overline{AC}\right)$$



Construction

Join O with A, B and C.
So that we get three triangle ΔOAB, ΔOBC and ΔOAC

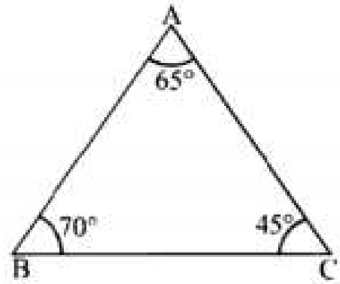
Proof

Statements	Reasons
In ΔOAB $m\overline{OA}+OB > m\overline{AB}$ _____(i)	In any triangle the sum of length of two sides is greater then the third sides.
In ΔOAC $m\overline{OC}+m\overline{OA} > m\overline{AC}$ _____(ii)	As in (i)
In ΔOBC $mOB+\overline{OC} > m\overline{BC}$ _____(iii)	As in (i)
Adding equation i, ii and iii $\overline{OA}+\overline{OC}+\overline{OA}+\overline{OB}+\overline{OB}+\overline{OC}>\overline{AC}+\overline{AB}+\overline{BC}$ $2\overline{OA}+2\overline{OC}+2\overline{OB}>\overline{AB}+\overline{BC}+\overline{CA}$ $2(OA+OC+OB)>\overline{AB}+\overline{BC}+\overline{CA}$	
$\frac{\cancel{2}(OA+OC+OB)}{\cancel{2}}>\frac{\overline{AB}+\overline{BC}+\overline{CA}}{2}$	Dividing both sides by 2

$(OA+OC+OB)>\frac{1}{2}(\overline{AB}+\overline{BC}+\overline{CA})$	

Q.3 **In the ΔABC m∠B = 70° and m∠C = 45° which of the sides of the triangle is longest and which is the shortest.**

Solution



Sum of three angle in a triangle is 180°

$\angle A + \angle B + \angle C = 180$

$\angle A + 70 + 45 = 180$

$\angle A + 115 = 180$

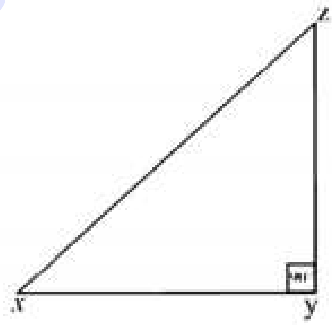
$\angle A = 180 - 115$

$\angle A = 65^\circ$

Sides of the triangle depend upon the angles largest angle has largest opposite side and smallest angle has smallest opposite side here $\angle B$ is largest so, $\overline{AC}$ is largest $\angle C$ is smallest, so $\overline{AB}$ is smallest side.

Q.4 **Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.**

Solution



Sum of three angles in a triangle is equal to 180°. So in a triangle one angle will be equal to 90° and rest of two angles are acute angle (less than 90°)

$\therefore$ m∠y = 90

And m∠x + m∠z = 90

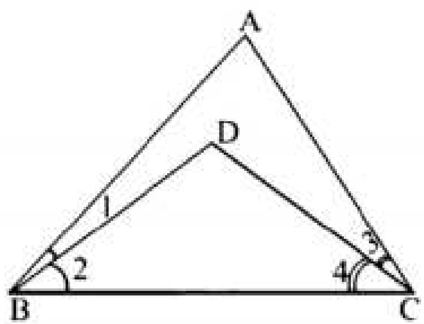
So m∠x and m∠z are acute angle

$\therefore$ Opposite to m∠y = 90° is hypotenuse

It is largest side.

Q.5 **In the triangular figure $\overline{AB} > \overline{AC}$. $\overline{BD}$ and $\overline{CD}$ are the bisectors of ∠B and ∠C respectively prove that $\overline{BD} > \overline{DC}$**

Given



In ΔABC

$\overline{AB} > \overline{AC}$

$\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ and $\angle C$

To prove

$\overline{BD} > \overline{CD}$

Construction

Label the angles $\angle 1, \angle 2, \angle 3$ and $\angle 4$

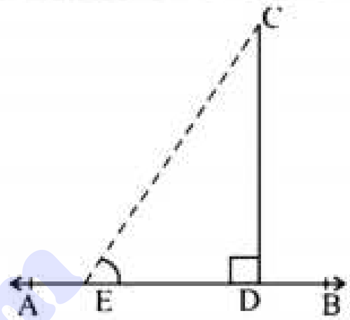
Proof

Statements	Reasons
In ΔABC $\overline{AB} > \overline{AC}$ $\overline{BD}$ is the bisector of $\angle B$ $\frac{1}{2}m\angle ACB > \frac{1}{2}m\angle ABC$	Given

$m\angle ABC$ $m\angle 2 < m\angle 4$ $\overline{CD}$ is the bisector of $\angle C$	Given
InΔBCD $\overline{BD} > \overline{DC}$	Side opposite to greater angle is greater

Theorem 13.1.4

From a point, out side a line, the perpendicular is the shortest distance from the point to the line.



Given:

A line AB and a point C

(Not lying on $\overleftrightarrow{AB}$)and a point D on $\overleftrightarrow{AB}$ such that $\overline{CD} \perp \overleftrightarrow{AB}$

To prove

$m\overline{CD}$ is the shortest distance from the point C to $\overleftrightarrow{AB}$

Construction

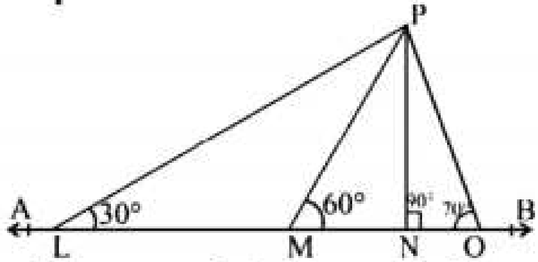
Take a point E on $\overleftrightarrow{AB}$ .Join C and E to form a ΔCDE

Proof

Statements	Reasons
In ΔCDE	
$m\angle CDB > m\angle CED$	(An exterior angle of a triangle is greater than non adjacent interior angle)
But $m\angle CDB = m\angle CDE$	Supplement of right angle
$\therefore$ $m\angle CDE > m\angle CED$	
Or $m\angle CED < m\angle CDE$	
Or $m\overline{CD} < m\overline{CE}$	Side opposite to greater angle is greater.
But E is any point on $\overleftrightarrow{AB}$	
Hence $m\overline{CD}$ is the shortest distance from C to $\overleftrightarrow{AB}$	

Exercise 13.2

Q.1 In the figure P is any point and AB is a line which of the following is the shortest distance between the point P and the line AB.

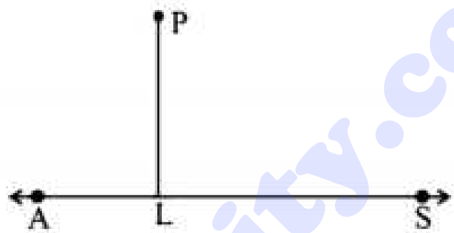


(a) $m\overline{PL}$ (b) $m\overline{PM}$ (c) $m\overline{PN}$ (d) $m\overline{PO}$

As we know that $\overline{PN} \perp \overleftrightarrow{AB}$

So $\overline{PN}$ is the shortest distance

Q.2 In the figure, P is any point lying away from the line $\overline{AB}$. Then $m\overline{PL}$ will be the shortest distance if



(a) $m\angle P \angle A = 80^\circ$ (b) $m\angle P\angle B = 100^\circ$ (c) $m\angle P\angle A = 90^\circ$

Solution:

$m\angle PLA = 90^\circ$

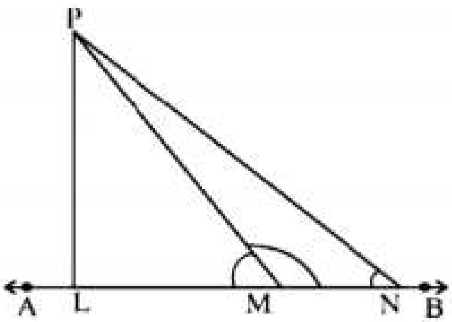
$\overline{PL} \perp \overleftrightarrow{AS}$

PL is the shortest distance

So $\angle PLA$ or PLS equal to 90°

Q.3 In the figure, $\overline{PL}$ is perpendicular to the line AB and $\overline{LN} > m\overline{LM}$. Prove that $m\overline{PN} > m\overline{PM}$.

Given



$\overline{PL} \perp \overleftrightarrow{AB}$

$m\overline{LN} > m\overline{LM}$

To proved:

$m\overline{PN} > m\overline{PM}$

Proof

Statements	Reasons
ΔPLM	
$\angle PLM = 90^o$	
$\therefore \angle PMN > \Delta PLM$	Exterior angle
$\angle PMN > 90^o$	
InΔPLN	
$\angle PLN = 90^o$	
$m\angle PNL < 90^o$	Acute angle
ΔPMN	
$m\angle PMN > m\angle PNL$	
$\therefore \overline{PN} > \overline{PM}$	