Exercise 2.8

1. The product of two positive consecutive numbers is 182. Find the numbers.

Let 1st No $= x$

2nd No $= x + 1$

According to the given condition

$$x(x+1) = 182$$
$$x^2 + x = 182$$
$$x^2 + x - 182 = 0$$
$$x^2 + 14x - 13x - 182 = 0$$
$$x(x+14) - 13(x+14) = 0$$
$$(x+14)(x-13) = 0$$

$x + 14 = 0$; $x - 13 = 0$

$x = -14$; $x = 13$

As, the numbers are positive so, $x = 13$

So 1st No $= x = 13$

2nd No $= x + 1 = 14$

2. The sum of the squares of three positive consecutive numbers is 77. Find them.

Let 1st No $= x$

2nd No $= x + 1$

3rd No $= x + 2$

According to the given condition

$$x^2 + (x+1)^2 + (x+2)^2 = 77$$
$$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 77$$
$$3x^2 + 6x + 5 = 77$$
$$3x^2 + 6x - 72 = 0$$
$$3x^2 + 18x - 12x - 72 = 0$$
$$3x(x+6) - 12(x+6) = 0$$
$$(x+6)(3x-12) = 0$$
$$(x+6)(3x-12) = 0$$

$x + 6 = 0$; $3x - 12 = 0$

$x = -6$; $x = 4$

As, the numbers are positive so, $x = 4$

So 1st No $= x = 4$

2nd No $= x + 1 = 5$

3rd No $= x + 2 = 6$

3. The sum of five times a number and the square of the number is 204. Find the number.

Let the No $= x$

According to the given condition

$$5x + x^2 = 204$$

$$x^2 + 5x - 204 = 0$$
$$x^2 + 17x - 12x - 204 = 0$$
$$x(x + 17) - 12(x + 17) = 0$$
$$(x + 17)(x - 12) = 0$$

$x + 17 = 0$; $x - 12 = 0$

$x = -17$; $x = 12$

4. The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Let the No $= x$

According to the given condition

$$(3x - 5)(4x - 1) = 7$$
$$12x^2 - 3x - 20x + 5 = 7$$
$$12x^2 - 23x - 2 = 0$$
$$12x^2 - 24x + x - 2 = 0$$
$$12x(x - 2) + 1(x - 2) = 0$$
$$(x - 2)(12x + 1) = 0$$

$x - 2 = 0$; $12x + 1 = 0$

$x = 2$; $x = -\frac{1}{12}$

So the No $= x = 2$; $x = -\frac{1}{12}$

5. The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Let the No $= x$

According to the given condition

$$x - \frac{1}{x} = \frac{15}{4}$$
$$\frac{x^2 - 1}{x} = \frac{15}{4}$$
$$4x^2 - 4 = 15x$$
$$4x^2 - 15x - 4 = 0$$
$$4x^2 - 16x + x - 4 = 0$$
$$4x(x - 4) + 1(x - 4) = 0$$
$$(x - 4)(4x + 1) = 0$$

$x - 4 = 0$; $4x + 1 = 0$

$x = 4$; $x = -\frac{1}{4}$

So the No $= x = 4$; $x = -\frac{1}{4}$

6. **The sum of the squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number**

Let the digit at 10's place $= x$

And the digit at 1's place $= y$

Then, the Number is given by $= 10x + y$

According to the first condition

$$x^2 + y^2 = 65 \text{ ---------- (i)}$$

According to the second condition

$$10x + y = 9(x + y)$$

$$10x + y = 9x + 9y$$

$$10x + y - 9x - 9y = 0$$

$$x - 8y = 0$$

$$x = 8y \text{ ---------- (ii)}$$

Putting this value in (i)

$$x^2 + y^2 = 65$$

$$(8y)^2 + y^2 = 65$$

$$64y^2 + y^2 = 65$$

$$65y^2 = 65$$

$$y^2 = 1$$

$$y = \pm 1$$

As the number is positive integral so $y = 1$, putting this value in (ii)

$$x = 8y$$

$$x = 8(1)$$

$$x = 8$$

So, the number $= 10x + y$

$$= 10(8) + 1$$

$$= 81$$

7. **The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.**

Let the x co-ordinate $= x$

And the y co-ordinate $= y$

According to the first condition

$$x + y = 9$$

$$y = 9 - x \text{ ---------- (i)}$$

According to the second condition

$$x^2 + y^2 = 45 \text{ ---------- (ii)}$$

Putting the value of y from (i) in (ii)

$$x^2 + y^2 = 45$$

$$x^2 + (9 - x)^2 = 45$$

$$x^2 + 81 - 18x + x^2 = 45$$

$$2x^2 - 18x + 36 = 0$$

$$2x^2 - 12x - 6x + 36 = 0$$
$$2x(x-6) - 6(x-6) = 0$$
$$(x-6)(2x-6) = 0$$

$x - 6 = 0$; $2x - 6 = 0$

$x = 6$; $x = 3$

Put in equation (i)

$y = 9 - x$; $y = 9 - x$

$y = 9 - 6$; $y = 9 - 3$

$y = 3$; $y = 6$

So, the point is $(3, 6)$ or $(6, 3)$

8. Find two integers whose sum is 9 and the difference of their squares is also 9.

Let 1st integer $= x$

And 2nd integer $= y$

According to the first condition

$$x + y = 9$$
$$y = 9 - x \text{ ---------- (i)}$$

According to the second condition

$$x^2 - y^2 = 9 \text{ ---------- (ii)}$$

Putting the value of y from (i) in (ii)

$$x^2 - y^2 = 9$$
$$x^2 - (9-x)^2 = 9$$
$$x^2 - (81 - 18x + x^2) = 9$$
$$x^2 - 81 + 18x - x^2 = 9$$
$$-81 + 18x = 9$$
$$18x = 90$$
$$x = 5$$

Put in equation (i)

$$y = 9 - x$$
$$y = 9 - 5$$
$$y = 4$$

So, the integers are 5, 4

9. Find two integers whose difference is 4 and whose squares differ by 72.

Let 1st integer $= x$

And 2nd integer $= y$

According to the first condition

$$x - y = 4$$
$$-y = 4 - x$$
$$y = x - 4 \text{ ---------- (i)}$$

According to the second condition

$$x^2 - y^2 = 72 \text{ ---------- (ii)}$$

Putting the value of y from (i) in (ii)

$$x^2 - y^2 = 72$$
$$x^2 - (x-4)^2 = 72$$
$$x^2 - (x^2 - 8x + 16) = 72$$
$$x^2 - x^2 + 8x - 16 = 72$$
$$8x = 72 + 16$$
$$8x = 88$$
$$x = 11$$

Put in equation (i)

$$y = x - 4$$
$$y = 11 - 4$$
$$y = 7$$

So, the integers are 11, 7

10. Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375 cm^2.

Let the 1st dimension of rectangle = x

And the 2nd dimension of rectangle = y

According to the first condition

$$Perimeter = 80$$

as we know the $Perimeter = 2(x + y)$ So,

$$2(x + y) = 80$$
$$x + y = 40$$
$$y = 40 - x \text{ ---------- (i)}$$

According to the second condition

$$Area = 375$$

as we know the $Area = x \times y$ So,

$$xy = 375 \text{ ---------- (ii)}$$

Putting the value of y from (i) in (ii)

$$x(40 - x) = 375$$
$$40x - x^2 = 375$$
$$0 = 375 - 40x + x^2$$
$$x^2 - 40x + 375 = 0$$
$$x^2 - 25x - 15x + 375 = 0$$
$$x(x - 25) - 15(x - 25) = 0$$
$$(x - 25)(x - 15) = 0$$

$x - 25 = 0$	;	$x - 15 = 0$
$x = 25$	;	$x = 15$

Put in equation (i)

$y = 40 - x$	;	$y = 40 - x$
$y = 40 - 25$	;	$y = 40 - 15$
$y = 15$	;	$y = 25$

we have 25cm by 15cm or 15cm by 25cm as dimensions.