Exercise 2.6

Q. 1: Use synthetic division to find the quotient and the remainder, when

(i) $(x^2 + 7x - 1) \div (x + 1)$

From divisor, $x + a$, here $a = -1$

	1	7	-1
-1	↓	-1	-6
	1	6	-7

So the remainder is -7.

(ii) $(4x^3 - 5x + 15) \div (x + 3)$

From divisor, $x + a$, here $a = -3$

	4	0	-5	15
-3	↓	-12	36	-93
	4	-12	31	-78

So the remainder is -78.

(iii) $(x^3 + x^2 - 3x + 2) \div (x - 2)$

From divisor, $x - a$, here $a = 2$

	1	1	-3	2
2	↓	2	6	6
	1	3	3	8

So the remainder is 8.

Q. 2: Find the value of h using synthetic division, if

(i) 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$

	2	-3h	0	9
3	↓	6	18-9h	54-27h
	2	6-3h	18-9h	63-27h

According to given condition

$$63 - 27h = 0$$
$$-27h = -63$$
$$h = \frac{-63}{-27}$$
$$h = \frac{7}{3}$$

(ii) 1 is the zero of the polynomial $x^3 - 2hx^2 + 11$

	1	-2h	0	11
1	↓	1	1-2h	1-2h
	1	1-2h	1-2h	12-2h

According to given condition

$$12 - 2h = 0$$
$$-2h = -12$$
$$h = \frac{-12}{-2}$$
$$h = 6$$

(iii) -1 is the zero of the polynomial $2x^3 + 5hx - 23$

	2	0	5h	-23
-1	↓	-2	2	-5h-2
	2	-2	5h+2	-5h-25

According to given condition

$$-5h - 25 = 0$$
$$-5h = 25$$
$$h = \frac{25}{-5}$$
$$h = -5$$

Q. 3: Use synthetic division to find the values of l and m, if

(i) $(x + 3)$ and $(x - 2)$ are the factors of the polynomial $x^3 + 4x^2 + 2lx + m$

As $x + 3$ is factor of given polynomial

So,

From divisor, $x + a$, here $a = -3$

	1	4	$2l$	m
-3	↓	-3	-3	$-6l + 9$
	1	1	$2l - 3$	$m - 6l + 9$

So,

$$m - 6l + 9 = 0$$
$$m - 6l = -9 \text{ ---------- (i)}$$

As $x - 2$ is factor of given polynomial

From divisor, $x - a$, here $a = 2$

	1	4	$2l$	m
2	↓	2	12	$4l + 24$
	1	6	$2l + 12$	$m + 4l + 24$

So,

$$m + 4l + 24 = 0$$

$$m + 4l = -24 \text{ ---------- (ii)}$$

Subtracting equation (i) from equation (ii)

$$m + 4l = -24$$

$$-m + 6l = +9$$

$$10l = -15$$

$$l = -\frac{3}{2}$$

Putting the value in equation (i)

$$m - 6l = -9$$

$$m - 6\left(-\frac{3}{2}\right) = -9$$

$$m + 9 = -9$$

$$m = -9 - 9$$

$$m = -18$$

(ii) $(x - 1)$ and $(x + 1)$ are the factors of the polynomial $x^3 - 3lx^2 + 2mx + 6$

As $x - 1$ is factor of given polynomial

So,

From divisor, $x - a$, here $a = 1$

	1	$-3l$	$2m$	6
1	↓	1	$-3l + 1$	$2m - 3l + 1$
	1	$-3l + 1$	$2m - 3l + 1$	$2m - 3l + 7$

So,

$$2m - 3l + 7 = 0$$

$$2m - 3l = -7 \text{ ---------- (i)}$$

As $x + 1$ is factor of given polynomial

From divisor, $x + a$, here $a = -1$

	1	$-3l$	$2m$	6
-1	↓	-1	$3l + 1$	$-2m - 3l - 1$
	1	$-3l - 1$	$2m + 3l + 1$	$-2m - 3l + 5$

So,

$$-2m - 3l + 5 = 0$$

$$-2m - 3l = -5 \text{ ---------- (ii)}$$

Adding equation (i) and equation (ii)

$$2m - 3l = -7$$

$$-2m - 3l = -5$$

$$-6l = -12$$

$$l = 2$$

Putting the value in equation (i)

$$2m - 3l = -7$$
$$2m - 3(2) = -7$$
$$2m - 6 = -7$$
$$2m = -7 + 6$$
$$m = -\frac{1}{2}$$

Q. 4: Solve by using synthetic division, if

(i) 2 is the root of the equation $x^3 - 28x + 48 = 0$

as 2 is the root of given equation

	1	0	-28	48
2	↓	2	4	−48
	1	2	−24	0

So,

$$x^2 + 2x - 24 = 0$$
$$x^2 + 6x - 4x - 24 = 0$$
$$x(x+6) - 4(x+6) = 0$$
$$(x+6)(x-4) = 0$$

$x + 6 = 0$ and $x - 4 = 0$

$x = -6$ and $x = 4$

So the roots are 2, -6, 4

(ii) 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$

as 3 is the root of given equation

	2	-3	-11	6
3	↓	6	9	−6
	2	3	−2	0

So,

$$2x^2 + 3x - 2 = 0$$
$$2x^2 + 4x - x - 2 = 0$$
$$2x(x+2) - 1(x+2) = 0$$
$$(x+2)(2x-1) = 0$$

$x + 2 = 0$ and $2x - 1 = 0$

$x = -2$ and $2x = 1$

$x = -2$ and $x = \frac{1}{2}$

So the roots are 3, -2, $\frac{1}{2}$

(iii) -1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$

as -1 is the root of given equation

	4	-1	-11	-6
-1	↓	-4	5	+6
	4	-5	-6	0

So,

$$4x^2 - 5x - 6 = 0$$
$$4x^2 - 8x + 3x - 6 = 0$$
$$4x(x-2) + 3(x-2) = 0$$
$$(x-2)(4x+3) = 0$$

$x - 2 = 0$ and $4x + 3 = 0$

$x = 2$ and $4x = -3$

$x = 2$ and $x = \frac{-3}{4}$

So the roots are -1, 2, $\frac{-3}{4}$

Q. 5: Solve by using synthetic division, if

(i) 1 and 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

	1	0	-10	0	9
1	↓	1	1	-9	-9
	1	1	-9	-9	0
3	↓	3	12	9	
	1	4	3	0	

So,

$$x^2 + 4x + 3 = 0$$
$$x^2 + 3x + x + 3 = 0$$
$$x(x+3) + 1(x+3) = 0$$
$$(x+3)(x+1) = 0$$

$x + 3 = 0$ and $x + 1 = 0$

$x = -3$ and $x = -1$

So the roots are 1, 3, -3, -1

(ii) 3 and -4 are the roots of the equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

	1	2	-13	-14	24
3	↓	3	15	6	-24
	1	5	2	-8	0
-4	↓	-4	-4	8	
	1	1	-2	0	

So,

$$x^2 + x - 2 = 0$$
$$x^2 + 2x - x - 2 = 0$$

$$x(x+2)-1(x+2) = 0$$
$$(x+2)(x-1) = 0$$

$x+2 = 0$ and $x-1 = 0$

$x = -2$ and $x = 1$

So the roots are 3, -4, -2, 1