Exercise 1.3

Solve the following equations.

1. $2x^4 - 11x^2 + 5 = 0$

$2(x^2)^2 - 11(x^2) + 5 = 0$

let $x^2 = y$----------(a)

So,

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(y-5)(2y-1) = 0$

$y - 5 = 0$ and $2y - 1 = 0$

$y = 5$ and $y = \frac{1}{2}$

Put the values y in equ---(a)

$x^2 = 5$ and $x^2 = \frac{1}{2}$

Taking square root of both sides

$\sqrt{x^2} = \pm\sqrt{5}$ and $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\sqrt{5}$ and $x = \pm\frac{1}{\sqrt{2}}$

S.S $= \left\{\pm\sqrt{5}, \pm\frac{1}{\sqrt{2}}\right\}$

2. $2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$

$2(x^2)^2 - 9(x^2) + 4 = 0$

let $x^2 = y$----------(a)

So,

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(y-4)(2y-1) = 0$

$y - 4 = 0$ and $2y - 1 = 0$

$y = 4$ and $y = \frac{1}{2}$

Put the values y in equ---(a)

$x^2 = 4$ and $x^2 = \frac{1}{2}$

Taking square root of both sides

$\sqrt{x^2} = \pm\sqrt{4}$ and $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm 2$ and $x = \pm\frac{1}{\sqrt{2}}$

S.S $= \left\{\pm 2, \pm \frac{1}{\sqrt{2}}\right\}$

3. $5x^{1/2} = 7x^{1/4} - 2$

$5x^{1/2} - 7x^{1/4} + 2 = 0$

$5\left(x^{1/4}\right)^2 - 7\left(x^{1/4}\right) + 2 = 0$

let $x^{1/4} = y$----------(a)

So,

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(5y-2)(y-1) = 0$

$5y - 2 = 0$ and $y - 1 = 0$

$y = \frac{2}{5}$ and $y = 1$

Put the values y in equ---(a)

$x^{1/4} = \frac{2}{5}$ and $x^{1/4} = 1$

Taking power 4 on both sides

$\left(x^{1/4}\right)^4 = \left(\frac{2}{5}\right)^4$ and $\left(x^{1/4}\right)^4 = (1)^4$

$x = \frac{16}{625}$ and $x = 1$

S.S $= \left\{1, \frac{16}{625}\right\}$

4. $x^{2/3} + 54 = 15x^{1/3}$

$x^{2/3} - 15x^{1/3} + 54 = 0$

$\left(x^{1/3}\right)^2 - 15\left(x^{1/3}\right) + 54 = 0$

let $x^{1/3} = y$----------(a)

So,

$y^2 - 15y + 54 = 0$

$y^2 - 6y - 9y + 54 = 0$

$y(y-6) - 9(y-6) = 0$

$(y-6)(y-9) = 0$

$y - 6 = 0$ and $y - 9 = 0$

$y = 6$ and $y = 9$

Put the values y in equ---(a)

$x^{1/3} = 6$ and $x^{1/3} = 9$

Taking power 4 on both sides

$\left(x^{1/3}\right)^3 = (6)^3$ and $\left(x^{1/3}\right)^3 = (9)^3$

$x = 216$ and $x = 729$

S.S $= \{216, 729\}$

5. $3x^{-2} + 5 = 8x^{-1}$

$3x^{-2} - 8x^{-1} + 5 = 0$

$3(x^{-1})^2 - 8(x^{-1}) + 5 = 0$

let $x^{-1} = y$----------(a)

So,

$3y^2 - 8y + 5 = 0$

$3y^2 - 5y - 3y + 5 = 0$

$y(3y - 5) - 1(3y - 5) = 0$

$(3y - 5)(y - 1) = 0$

$3y - 5 = 0$ and $y - 1 = 0$

$y = \frac{5}{3}$ and $y = 1$

Put the values y in equ---(a)

$x^{-1} = \frac{5}{3}$ and $x^{-1} = 1$

Taking power 4 on both sides

$x = \frac{3}{5}$ and $x = 1$

S.S $= \left\{\frac{3}{5}, 1\right\}$

6. $(2x^2 + 1) + \frac{3}{(2x^2+1)} = 4$

let $(2x^2 + 1) = y$----------(a)

So,

$y + \frac{3}{y} = 4$

$\frac{y^2+3}{y} = 4$

$y^2 + 3 = 4y$

$y^2 - 4y + 3 = 0$

$y^2 - 3y - y + 3 = 0$

$y(y - 3) - 1(y - 3) = 0$

$(y - 3)(y - 1) = 0$

$y - 3 = 0$ and $y - 1 = 0$

$y = 3$ and $y = 1$

Put the values y in equ---(a)

$2x^2 + 1 = 3$ and $2x^2 + 1 = 1$

Taking power 4 on both sides

$2x^2 = 3 - 1$ and $2x^2 = 1 - 1$

$2x^2 = 2$ and $2x^2 = 0$

$x^2 = \frac{2}{2}$ and $x^2 = \frac{0}{2}$

$x^2 = 1$ and $x^2 = 0$

$x = \pm 1$ and $x = 0$

S.S $= \{\pm 1, 0\}$

7. $\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4$

let $\frac{x}{x-3} = y$----------(a)

So,

$y + \frac{4}{y} = 4$

$\frac{y^2+4}{y} = 4$

$y^2 + 4 = 4y$

$y^2 - 4y + 4 = 0$

$(y-2)^2 = 0$

Taking root on both sides

$(y-2) = 0$

$y = 2$

Put the value y in equ---(a)

$\frac{x}{x-3} = 2$

Taking power 4 on both sides

$x = 2(x-3)$

$x = 2x - 6$

$x - 2x = -6$

$-x = -6$

$x = \frac{-6}{-1}$

S.S $= \{6\}$

8. $\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}$

let $\frac{4x+1}{4x-1} = y$----------(a)

So,

$y + \frac{1}{y} = \frac{13}{6}$

$\frac{y^2+1}{y} = \frac{13}{6}$

$6(y^2+1) = 13y$

$6y^2 + 6 = 13y$

$6y^2 - 13y + 6 = 0$

$6y^2 - 9y - 4y + 6 = 0$

$3y(2y-3)-2(2y-3) = 0$

$(2y-3)(3y-2) = 0$

$2y-3=0$ and $3y-2=0$

$2y = 3$ and $3y = 2$

$y = \frac{3}{2}$ and $y = \frac{2}{3}$

Put the values y in equ---(a)

$\frac{4x+1}{4x-1} = \frac{3}{2}$ and $\frac{4x+1}{4x-1} = \frac{2}{3}$

$2(4x+1) = 3(4x-1)$ and $3(4x+1) = 2(4x-1)$

$8x+2 = 12x-3$ and $12x+3 = 8x-2$

$8x-12x = -3-2$ and $12x-8x = -2-3$

$-4x = -5$ and $4x = -5$

$x = \frac{-5}{-4}$ and $x = \frac{-5}{4}$

$x = \frac{5}{4}$ and $x = \frac{-5}{4}$

S.S $= \left\{\frac{5}{4}, \frac{-5}{4}\right\}$

9. $\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12}$

let $\frac{x-a}{x+a} = y$----------(a)

So,

$y - \frac{1}{y} = \frac{7}{12}$

$\frac{y^2-1}{y} = \frac{7}{12}$

$12(y^2-1) = 7y$

$12y^2-12 = 7y$

$12y^2-7y-12 = 0$

$12y^2-16y+9y-12 = 0$

$4y(3y-4)-3(3y-4) = 0$

$(3y-4)(4y-3) = 0$

$3y-4=0$ and $4y-3=0$

$3y = 4$ and $4y = 3$

$y = \frac{4}{3}$ and $y = \frac{3}{4}$

Put the values y in equ---(a)

$\frac{x-a}{x+a} = \frac{4}{3}$ and $\frac{x-a}{x+a} = \frac{3}{4}$

$3(x-a) = 4(x+a)$ and $4(x-a) = 3(x+a)$

$3x-3a = 4x+4a$ and $4x-4a = 3x+3a$

$3x-4x = +3a+4a$ and $4x-3x = 3a+4a$

$-x = 7a$ and $x = 7a$

$x = -7a$ and $x = 7a$

S.S $= \{-7a, 7a\}$

10. $x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

dividing by x^2

$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$

$x^2 + \frac{1}{x^2} - 2x + \frac{2}{x} - 2 = 0$

Adding and subtracting 2 on L.H.S

$x^2 + \frac{1}{x^2} - 2 + 2 - 2x + \frac{2}{x} - 2 = 0$

$\left(x - \frac{1}{x}\right)^2 + 2 - 2\left(x - \frac{1}{x}\right) - 2 = 0$

$\left(x - \frac{1}{x}\right)^2 - 2\left(x - \frac{1}{x}\right) = 0$

let $x - \frac{1}{x} = y$----------(a)

So,

$y^2 - 2y = 0$

$y(y - 2) = 0$

$y = 0$ and $y - 2 = 0$

$y = 0$ and $y = 2$

Put the values y in equ---(a)

$x - \frac{1}{x} = 0$ and $x - \frac{1}{x} = 2$

$\frac{x^2-1}{x} = 0$ and $\frac{x^2-1}{x} = 2$

$x^2 - 1 = 0$ and $x^2 - 1 = 2x$

$(x - 1)(x + 1) = 0$ and $x^2 - 2x - 1 = 0$

$x - 1 = 0, x + 1 = 0$ and $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = 1 \ , x = -1$ and $x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-1)}}{2(1)}$

$x = 1 \ , x = -1$ and $x = \frac{2 \pm \sqrt{4+4}}{2(1)}$

$x = 1 \ , x = -1$ and $x = \frac{2 \pm \sqrt{8}}{2}$

$x = 1 \ , x = -1$ and $x = \frac{2 \pm \sqrt{4 \times 2}}{2}$

$x = 1 \ , x = -1$ and $x = \frac{2 \pm 2\sqrt{2}}{2}$

$x = 1 \ , x = -1$ and $x = \frac{2(1 \pm \sqrt{2})}{2}$

S.S $= \{1 \pm \sqrt{2}, \pm 1\}$

11. $2x^4 + x^3 - 6x^2 + x + 2 = 0$

dividing by x^2

$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$

$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$

Adding and subtracting 4 on L.H.S

$2x^2 + \frac{2}{x^2} + 4 - 4 + x + \frac{1}{x} - 6 = 0$

$2\left(x^2 + \frac{1}{x^2} + 2\right) + \left(x + \frac{1}{x}\right) - 10 = 0$

$2\left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 10 = 0$

let $x + \frac{1}{x} = y$----------(a)

So,

$2y^2 + y - 10 = 0$

$2y^2 + 5y - 4y - 10 = 0$

$y(2y + 5) - 2(2y - 5) = 13$

$2y + 5 = 0$	and	$y - 2 = 0$
$2y + 5 = 0$	and	$y = 2$
$2y = -5$	and	$y = 2$
$y = \frac{-5}{2}$	and	$y = 2$

Put the values y in equ---(a)

$x + \frac{1}{x} = \frac{-5}{2}$	and	$x + \frac{1}{x} = 2$
$\frac{x^2+1}{x} = \frac{-5}{2}$	and	$\frac{x^2+1}{x} = 2$
$2x^2 + 2 = -5x$	and	$x^2 + 1 = 2x$
$2x^2 + 5x + 2 = 0$	and	$x^2 - 2x + 1 = 0$
$2x^2 + 4x + x + 2 = 0$	and	$x^2 - 2x + 1 = 0$
$2x(x + 2) + 1(x + 2) = 0$	and	$x^2 - 2x + 1 = 0$
$(x + 2)(2x + 1) = 0$	and	$(x - 1) = 0$
$x + 2 = 0, 2x + 1 = 0$	and	$(x + 1)^2 = 0$
$x = -2, x = \frac{-1}{2}$	and	$x = 1$

S.S $= \left\{-2, \frac{-1}{2}, 1\right\}$

12. $4.2^{2x+1} - 9.2^x + 1 = 0$

$4.2^{2x}.2^1 - 9.2^x + 1 = 0$

$8.2^{2x} - 9.2^x + 1 = 0$

$8(2^x)^2 - 9(2^x) + 1 = 0$

let $2^x = y$----------(a)

So,

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$y - 1 = 0$	and	$8y - 1 = 0$
$y - 1 = 0$	and	$8y = 1$
$y = 1$	and	$y = \frac{1}{8}$

Put the values y in equ---(a)

$2^x = 1$	and	$2^x = \frac{1}{8}$
$2^x = 2^0$	and	$2^x = 2^{-3}$
$x = 0$	and	$x = -3$

S.S $= \{0, -3\}$

13. $3^{2x+2} = 12.3^x - 3$

$3^{2x}.3^2 - 12.3^x + 3 = 0$

$9.3^{2x} - 12.3^x + 3 = 0$

$9(3^x)^2 - 12(3^x) + 3 = 0$

let $3^x = y$----------(a)

So,

$9y^2 - 12y + 3 = 0$

$9y^2 - 9y - 3y + 3 = 0$

$9y(y-1) - 3(y-1) = 0$

$(y-1)(9y-3) = 0$

$y - 1 = 0$	and	$9y - 3 = 0$
$y - 1 = 0$	and	$9y = 3$
$y = 1$	and	$y = \frac{3}{9}$
$y = 1$	and	$y = \frac{1}{3}$

Put the values y in equ---(a)

$3^x = 1$	and	$3^x = \frac{1}{3}$
$3^x = 3^0$	and	$3^x = 3^{-1}$
$x = 0$	and	$x = -1$

S.S $= \{0, -1\}$

14. $2^x + 64.2^{-x} - 20 = 0$

$2^x + \frac{64}{2^x} - 20 = 0$

let $2^x = y$----------(a)

So,

$y + \frac{64}{y} - 20 = 0$

$\frac{y^2-64-20y}{y} = 0$

$y^2 + 64 - 20y = y \times 0$

$y^2 - 20y + 64 = 0$

$y^2 - 16y - 4y + 64 = 0$

$y(y - 16) - 4(y - 16) = 0$

$(y - 16)(y - 4) = 0$

$y - 16 = 0$	and	$y - 4 = 0$
$y = 16$	and	$y = 4$

Put the values y in equ---(a)

$2^x = 16$	and	$2^x = 4$
$2^x = 2^4$	and	$2^x = 2^2$
$x = 4$	and	$x = 2$

S.S $= \{4,2\}$

15. $(x + 1)(x + 3)(x - 5)(x - 7) = 192$

$(x + 1)(x - 5)(x + 3)(x - 7) = 192$

$(x^2 - 5x + x - 5)(x^2 - 7x + 3x - 21) = 192$

$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192$

let $x^2 - 4x = y$----------(a)

So,

$(y - 5)(y - 21) = 192$

$y^2 - 21y - 5y + 105 = 192$

$y^2 - 26y + 105 - 192 = 0$

$y^2 - 26y - 87 = 0$

$y^2 - 29y + 3y - 87 = 0$

$y(y - 29) + 3(y - 29) = 0$

$(y - 29)(y + 3) = 0$

$y - 29 = 0$	and	$y + 3 = 0$
$y = 29$	and	$y = -3$

Put the values y in equ---(a)

$x^2 - 4x = 29$	and	$x^2 - 4x = -3$
$x^2 - 4x - 29 = 0$	and	$x^2 - 4x + 3 = 0$
$x^2 - 4x - 29 = 0$	and	$x^2 - 4x + 3 = 0$

$a = 1, b = -4, c = -29$ and $x^2 - 3x - x + 3 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ and $x(x-3) - 1(x-3) = 0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2-4(1)(-29)}}{2(-1)}$ and $(x-3)(x-1) = 0$

$x = \frac{4 \pm \sqrt{16+116}}{-2}$ and $x - 3 = 0$, $x - 1 = 0$

$x = \frac{4 \pm \sqrt{132}}{-2}$ and $x = 3$, $x = 1$

$x = \frac{4 \pm 2\sqrt{33}}{-2}$ and $x = 3$, $x = 1$

$x = \frac{2(2 \pm \sqrt{33})}{-2}$ and $x = 3$, $x = 1$

$x = -(2 \pm \sqrt{33})$ and $x = 3$, $x = 1$

S.S $= \{2 \pm \sqrt{33}, 3, 1\}$

16. $(x-1)(x-2)(x-8)(x+5) + 360 = 0$

$(x-1)(x-2)(x-8)(x+5) = -360$

$(x-1)(x-2)(x-8)(x+5) = -360$

$(x^2 - 2x - x + 2)(x^2 + 5x - 8x - 40) = -360$

$(x^2 - 3x + 2)(x^2 - 3x - 40) = -360$

let $x^2 - 3x = y$----------(a)

So,

$(y+2)(y-40) = -360$

$y^2 - 40y + 2y - 80 = -360$

$y^2 - 38y - 80 + 360 = 0$

$y^2 - 38y + 280 = 0$

$y^2 - 28y - 10y + 280 = 0$

$y(y-28) - 10(y-28) = 0$

$(y-28)(y-10) = 0$

$y - 28 = 0$ and $y - 10 = 0$

$y = 28$ and $y = 10$

Put the values y in equ---(a)

$x^2 - 3x = 28$ and $x^2 - 3x = 10$

$x^2 - 3x - 28 = 0$ and $x^2 - 3x - 10 = 0$

$x^2 + 4x - 7x - 28 = 0$ and $x^2 - 5x + 2x - 10 = 0$

$x(x+4) - 7(x+4) = 0$ and $x(x-5) + 2(x-5) = 0$

$(x+4)(x-7) = 0$ and $(x-5)(x+2) = 0$

$x + 4 = 0$, $x - 7 = 0$ and $x - 5 = 0$, $x + 2 = 0$

$x = -4$, $x = 7$ and $x = 5$, $x = -2$

S.S $= \{-4, 7, 5, -2\}$