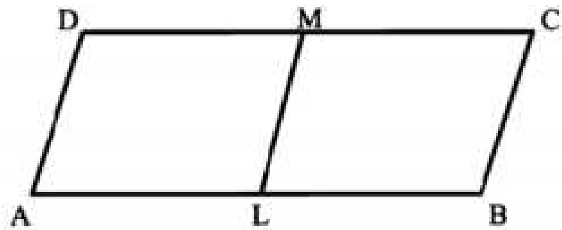
Exercise 16.1

Q.1 Show that the line segment joining the mid point of opposite sides of a purallogram divides it into two equal parallelograms.



Given

ABCD is a parallelogram. L is the midpoint of $\overline{AB}$ and M is the midpoint of $\overline{DC}$

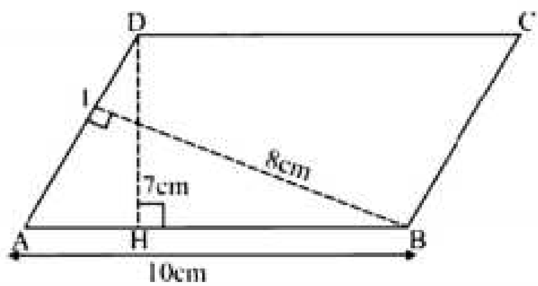
To prove

Area of parallelogram ALMD = area of parallelogram LBCM.

Proof

Statements	Reasons
$\overline{AB} \parallel \overline{DC}$	Opposite sides of parallelogram ABCD.
$\overline{AL} \cong \overline{LB}$ ....(i)	L is midpoint of $\overline{AB}$
The parallelograms ALMD and LBCM are on equal bases and between the same parallel lines $\overline{AB}$ and $\overline{DC}$	From equation (i)
Hence area of parallelogram ALMD= area of parallelogram LBCM.	They have equal areas

Q.2 In a parallelogram ABCD, m $\overline{AB}$ =10cm the altitudes Corresponding to Sides AB and AD are respectively 7cm and 8cm Find $\overline{AD}$



$\overline{AB} = 10$ cm

$\overline{DH} = 7$cm

$\overline{MB} = 8$cm

$\overline{AD} = ?$

Formula

Area of parallelogram = base x altitude

$\overline{AB} \times \overline{DH} = \overline{AD} \times \overline{IB}$

$10 \times 7 \quad = \overline{AD} \times 8$

$\dfrac{\cancel{70}^{35}}{\cancel{8}_{4}} = \overline{AD}$

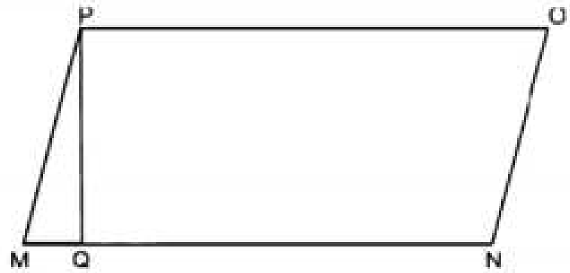
$\dfrac{35}{4} = \overline{AD}$

$\overline{AD} = \dfrac{35}{4}$

Or

$\overline{AD} = 8.75$cm

Q.3 If two parallelograms of equal areas have the same or equal bases, their altitude are equal



In parallelogram opposite side and opponents angles are Congruent.

Given

Parallelogram ABCD and parallelogram MNOP

OD is altitude of parallelogram ABCD

PQ is altitude of parallelogram MNOP

Area of ABCD $\|^{gm}$ $\cong$ Area of MNOP $\|^{gm}$

To prove

$m\overline{OD} \cong m\overline{PQ}$

Proof

Statements	Reasons
Area of parallelogram ABCD= Area of parallelogram MNOP	Given
Area of parallelogram= base × height	Given
$\overline{AB} \times \overline{OD} = \overline{MN} \times \overline{PQ}$	
We know that	
$\overline{AB} = \overline{MN}$	
So	
$\frac{\cancel{AB}}{\cancel{AB}} \times \overline{OD} = \overline{PQ}$	Proved
$\overline{OD} = \overline{PQ}$	

<u>Theorem 16.1.3</u>

Triangle on the same base and of the same (i.e…equal) altitudes are equal in area

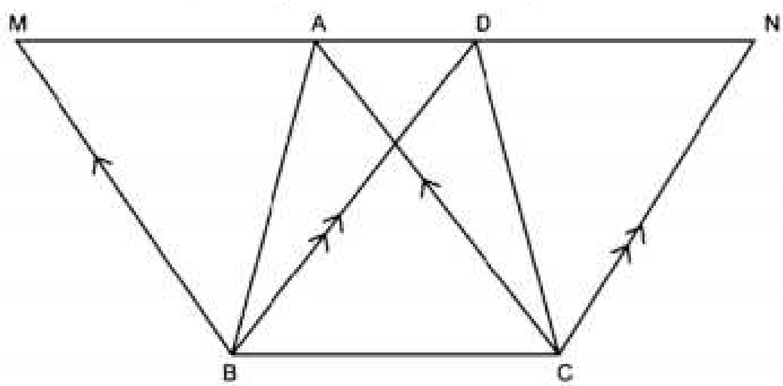
Given

Δ's ABC , DBC on the Same base $\overline{BC}$ and having equal altitudes

To prove

Area of (ΔABC) = area of (ΔDBC)

Construction:

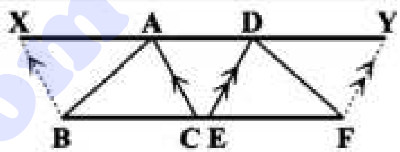


Draw $\overline{BM}$ P to $\overline{CA}$, $\overline{CN}$ P to $\overline{BD}$ meeting $\overline{AD}$ produced in M.N.

Proof

Statements	Reasons
ΔABC and ΔDBC are between the same $\parallel^s$	Their altitudes are equal
Hence MADN is parallel to $\overline{BC}$	
$\therefore$ Area $\parallel^{gm}$ (BCAM)= Area $\parallel^{gm}$ (BCND)	These $\parallel^{gm}$ are on the same base
But $\Delta ABC = \frac{1}{2} \parallel^{gm}$ (BCAM)------(ii)	$\overline{BC}$ and between the same $\parallel^s$
And $\Delta DBC = \frac{1}{2} \parallel^{gm}$ (BCND)-----(iii)	Each diagonal of a $\parallel^{gm}$
Hence area (ΔABC) = Area(ΔDBC)	Bisects it into two congruent triangles From (i) (ii) and (iii)

Theorem 16.1.4



Triangles on equal bases and of equal altitudes are equal in area.

Given

Δs ABC, DEF on equal bases $\overline{BC}$, $\overline{EF}$ and having altitudes equal

To prove

Area (ΔABC) = Area (ΔDEF)

Construction:

Place the Δs ABC and DEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are in the same straight line BCEF and their vertices on the same side of it .Draw $\overline{BX} \parallel \overline{CA}$ and $\overline{FY} \parallel \overline{ED}$ meeting $\overline{AD}$ produced in X, Y respectively

Proof

Statements	Reasons
ΔABC , ΔDEF are between the same parallels	Their altitudes are equal (given)
$\therefore$ XADY is $\parallel^{gm}$ to BCEF	

$\therefore$ area $\parallel^{gm}$ (BCAX) = A area $\parallel^{gm}$ (EFYD)----(i)	These $\parallel^{gm}$ are on equal bases and between the same parallels
But $\Delta ABC = \frac{1}{2}\parallel^{gm}$ (BCAX)----(ii)	Diagonal of a $\parallel^{gm}$ bisect it
And area of $\Delta DEF = \frac{1}{2}$ area of $\parallel^{gm}$ (EFYD)__ (iii)	
$\therefore$ area (ΔABC) = area (ΔDEF)	From (i),(ii)and(iii)

Exercise 16.2

Q.1

Show that

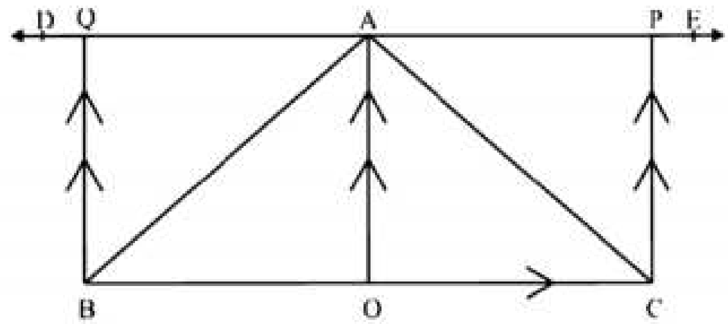
Given

ΔABC, O is the mid point of $\overline{BC}$

$\overline{OB} \cong \overline{OC}$

To prove

Area ΔABO = area ΔACO



Construction

Draw $\overline{DE}$ P $\overline{BC}$

$\overline{CP}$ P $\overline{OA}$

$\overline{BQ}$ P $\overline{OA}$

Proof

Statements	Reasons
$\overline{BQ}$ P $\overline{OA}$	Construction
$\overline{OB}$ P $\overline{AQ}$	Construction
$\|^{gm}$ BOAQ	Base of same
$\|^{gm}$ COAP	Parallel line of $\overset{sur}{DE}$
$\overline{OB} \cong \overline{OC}$	O is the mid point of $\overline{BC}$
Area of $\|^{gm}$ BOAQ= Area of $\|^{gm}$ COAP ... (i)	
Area of $\Delta ABO = \frac{1}{2}$ Area of $\|^{gm}$ BOAQ	
Area of $\Delta ACO = \frac{1}{2}$ Area of $\|^{gm}$ COAP	
Area of ΔABO = Area of ΔACO	Dividing equation (i) both side by (ii)

So median of a triangle divides it into two triangles of equal area.

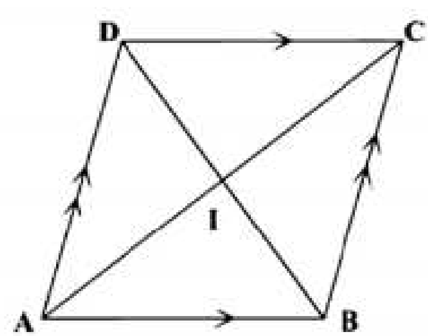
Q.2 **Prove that a parallelogram is divided by its diagonals into four triangles of equal area.**

Given:

In parallelogram ABCD, $\overline{AC}$ and $\overline{BD}$ are its diagonals, which meet at I

To prove:

Triangles ABI, BCI CDI and ADI have equal areas.



Proof:

Triangles ABC and ABD have the same base $\overline{AB}$ and are between the same parallel lines $\overline{AB}$ and $\overline{DC}$ $\therefore$ they have equal areas.

Or area of ΔABC =area of Δ ABD

Or area of Δ ABI + area of Δ BCI= area of Δ ABI+ area of Δ ADI

$\Rightarrow$ Area of Δ BCI = area of Δ ADI ... (i)

Similarly area of Δ ABC = area of Δ BCD

$\Rightarrow$ Area of Δ ABI +area of Δ BCI = area of Δ BCI + area of Δ CDI

$\Rightarrow$ Area of Δ ABI = area of Δ CDI… (ii)

As diagonals of a parallelogram bisect each other I is the midpoint of $\overline{AC}$ so $\overline{BI}$ is a median of Δ ABC

$\therefore$ Area of Δ ABI = area of Δ BCI… (iii)

$\Delta CDI \cong \Delta AOI$

$\overline{BI} \cong \overline{DI}$

Area of Δ ABI = area of Δ BCI = area of Δ CDI= area of Δ ADI

Q.3 Divide a triangle into six equal triangular parts

Given

ΔABC

To prove

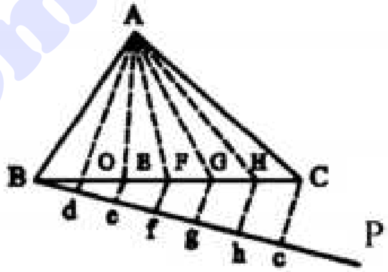
To divide ΔABC into six equal part triangular parts

Construction

Take $\overrightarrow{BP}$ any ray making an acute angle with $\overline{BC}$ draw six arcs of the same radius on $\overrightarrow{BP}$ i.e $m\overline{Bd} = mde = mef = mfg = mgh = mhc$

Join c to C and parallel line segments as

$\overline{cC} \parallel \overline{hH} \parallel \overline{gG} \parallel \overline{fF} \parallel \overline{eE} \parallel \overline{do}$



Join A to O,E,F,G,H

Proof

Base $\overline{BC}$ of ΔABC has been divided to six equal parts.

We get six triangles having equal base and same altitude

Their area is equal

Hence ΔBOA=ΔOEA=ΔEFA=ΔFGA=ΔGHA=ΔHCA