Exercise 6.3

Q.1 Use factorization to find the square root of the following expression.

(i) $4x^2-12xy+9y^2$

Solution: $4x^2-12xy+9y^2$

$4x^2-12xy+9y^2=4x^2-6xy-6xy+9y^2$

$=2x(2x-3y)-3y(2x-3y)$

$=(2x-3y)(2x-3y)$

$4x^2-12y+9y^2=(2x-3y)^2$

Taking square root on both side

$\sqrt{4x^2-12xy+9y^2}=\sqrt{[2x-3y]^2}$

$=\pm(2x-3y)$

(ii) $x^2-1+\frac{1}{4x^2}$

Solution: $x^2-1+\frac{1}{4x^2}$

$=(x)^2-2(x)\left[\frac{1}{2x}\right]+\left[\frac{1}{2x}\right]^2$

$=\left[x-\frac{1}{2x}\right]^2$

Taking square root

$\sqrt{x^2-1+\frac{1}{4x^2}}=\sqrt{\left[x-\frac{1}{2x}\right]^2}$

$\sqrt{x^2-1+\frac{1}{4x^2}}=\pm\left(x-\frac{1}{2x}\right)$

(iii) $\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2$

Solution: $\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2$

$=\left(\frac{1}{4}x\right)^2-2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right)+\left(\frac{1}{6}y\right)^2$

$=\left(\frac{x}{4}-\frac{y}{6}\right)^2$

Taking the square root

$$\sqrt{\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2}=\sqrt{\left(\frac{1}{4}x-\frac{1}{6}y\right)^2}$$
$$=\pm\left(\frac{1}{4}x-\frac{1}{6}y\right)$$
$$=\pm\left(\frac{x}{4}-\frac{y}{6}\right)$$

(iv) $4(a+b)^2-12(a2+b^2)+9(a-b)^2$

Solution: $4(a+b)^2-12(a2+b^2)+9(a-b)^2$

$$=\left[2(a+b)^2\right]-2\left[2(a+b)\right]\left[3(a-b)\right]+\left[3(a-b)\right]^2$$
$$=\left[2(a+b)-3(a-b)\right]^2$$

Taking square root

$$\sqrt{4(a+b)^2-12(a2+b^2)+9(a-b)^2}=\sqrt{\left[2(a+b)-3(a-b)\right]^2}$$
$$=\pm\left[2a+2b-3a+3b\right]$$
$$=\pm(5b-a)$$

(v) $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Solution: $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

$$=\frac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^3)^2+2(3x^2)(4y^2)+(4y^2)^2}$$

=

$$=\frac{\left[2x^3-3y^3\right]^2}{\left[3x^3+4y^2\right]^2}$$

Taking square root

$$=\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}}$$
$$=\pm\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)$$

(vi) $\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right),(x\neq 0)$

Solution: $\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right),(x\neq 0)$

By adding and substituting 4

$$= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right)$$

$$= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right) - 4 + 4$$

$$= x^2 + \frac{1}{x^2} - 2 - 4\left(x - \frac{1}{x}\right) + 4$$

$$= \left(x - \frac{1}{x}\right)^2 - 2\left(x - \frac{1}{x}\right)(2) + (2)^2$$

$$\left[\left(x - \frac{1}{x}\right) - 2\right]^2$$

Taking square root

$$\sqrt{\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right)} = \sqrt{\left[x - \frac{1}{x} - 2\right]^2}$$

$$= \pm\left(x - \frac{1}{x} - 2\right)$$

(vii) $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

Solution: $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 4\left[x^2 + \frac{1}{x^2} + 2\right] + 12$$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 4x^2 - \frac{4}{x^2} - 8 + 12$$

$$= \left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x^2 + \frac{1}{x^2}\right) + 4$$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 2\left[x^2 + \frac{1}{x^2}\right](2) + (2)^2$$

$$= \left[x^2 + \frac{1}{x^2} - 2\right]^2$$

Taking square root

$$= \sqrt{\left[x^2 + \frac{1}{x^2}\right] - 4\left[x + \frac{1}{x}\right]2 + 12}$$

$$= \sqrt{\left[x^2 \frac{1}{x^2} - 2\right]^2}$$

$$= \pm\left(x^2 + \frac{1}{x^2} - 2\right)$$

(viii) $(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)$

Solution: $(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)$

$=[x^2+2x+x+2][x^2+3x+x+3][x^2+3x+2x+6]$

$=[x(x+2)+1(x+2)][x(x+3)+1(x+3)][x(x+3)+2(x+3)]$

$=(x+2)(x+1)(x+3)(x+1)(x+3)(x+2)$

$=(x+2)^2(x+1)^2(x+3)^2$

Taking square root

$=\sqrt{(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)}$

$=\sqrt{(x+2)^2(x+1)^2(x+3)^2}$

$=\pm(x+1)(x+2)(x+3)$ **Ans**

(ix) $(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)$

Solution: $(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)$

$=(x^2+7x+1x+7)(2x^2-3x+2x-3)(2x^2+14x-3x-21)$

$=[(x(x+7)+1(x+7)][x(2x-x)+1(2x-3)][(2x(x+7)-3(x+7)]$

$=(x+7)(x+1)(2x-3)(x+1)(x+7)(2x-3)$

$=(x+7)^2(x+1)^2(2x-3)^2$

Taking square root

$=\sqrt{(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)}$

$=\sqrt{(x+7)^2(x+1)^2(2x-3)^2}$

$=\pm(x+1)(x+7)(2x-3)$ **Ans**

Q.2 **Use division method to find the square root of the following expression.**

(i) $4x^2+12xy+9y^2+16x+24y+16$

Solution: $4x^2+12xy+9y^2+16x+24y+16$

$$\begin{array}{r|l}
 & 2x+3y+4 \\
\hline
2x & \cancel{4x^2}+12xy+9y^2+16x+24y+16 \\
 & \pm\cancel{4x^2} \\
\hline
4x+3y & 12\cancel{xy}+\cancel{9y^2}+16x+24y+16 \\
 & \pm 12\cancel{xy}\pm\cancel{9y^2} \\
\hline
4x+6y+4 & \cancel{16x}\ \cancel{24y}+\cancel{16} \\
 & \cancel{16x}\pm\cancel{24y}\pm\cancel{16} \\
\hline
 & 0
\end{array}$$

Square root $=\pm(2x+3y+4)$

(ii) $x^4 - 10x^3 + 37x^2 - 60x + 36$

Solution: $x^4 - 10x^3 + 37x^2 - 60x + 36$

$$\begin{array}{r|l}
 & \qquad\qquad\qquad\qquad\quad x^2 - 5x + 6 \\
x^2 & \cancel{x^4} - 10x^3 + 37x^2 - 60x + 36 \\
 & \underline{\pm \cancel{x^4}} \\
2x^2 - 5x & \cancel{10x^3} + 37x^2 - 60x + 36 \\
 & \underline{\pm \cancel{10x^3} \pm 25x^2} \\
2x^2 - 10x + 6 & \cancel{12x^2} + \cancel{60x} + \cancel{36} \\
 & \underline{\pm \cancel{12x^2} \pm \cancel{60x} \pm \cancel{36}} \\
 & \times
\end{array}$$

Square root $= \pm\left(x^2 - 5x + 6\right)$

(iii) $9x^4 - 6x^3 + 7x^2 - 2x + 1$

Solution: $9x^4 - 6x^3 + 7x^2 - 2x + 1$

$$\begin{array}{r|l}
 & \qquad\qquad\qquad\qquad\quad 3x^2 - x + 1 \\
3x^2 & \cancel{9x^4} - 6x^3 + 7x^2 - 2x + 1 \\
 & \underline{\pm \cancel{9x^4}} \\
6x^2 - x & \cancel{6x^3} + \cancel{7x^2} - 2x + 1 \\
 & \underline{\pm \cancel{6x^3} \pm \cancel{7x^2}} \\
6x^2 - 2x + 1 & \cancel{6x^2} - \cancel{2x} + \cancel{1} \\
 & \underline{\pm \cancel{6x^2} \pm \cancel{2x} \pm \cancel{1}} \\
 & \times
\end{array}$$

Square root $\pm\left(= 3x^2 - x + 1\right)$

(iv) $4 + 25x^2 + 7x^2 - 2x + 1$

Solution: $4 + 25x^2 - 12x - 24x^3 + 16x^4$

$$4x^2-3x+2$$

$$4x^2\ \big)\ \cancel{16x^4}-24x^3+25x^2-12x+4$$

$$\underline{\pm\cancel{16x^4}}$$

$$8x^2-3x\ \big)\ -\cancel{24x^3}+25x^2-12x+4$$

$$\underline{\pm\cancel{24x^3}\pm 9x^2}$$

$$8x^2-6x+2\ \big)\ \cancel{16x^2}-\cancel{12x}+\cancel{4}$$

$$\underline{\pm\cancel{16x^2}\pm\cancel{12x}\pm\cancel{4}}$$

$$\times$$

Square root $=\pm\left(4x^2-3x+2\right)$

(v) $\dfrac{x^2}{y^2}-\dfrac{10x}{y}+27-\dfrac{10y}{x}+\dfrac{y^2}{x^2},(x\neq 0,y\neq 0)$

Solution: $\dfrac{x^2}{y^2}-\dfrac{10x}{y}+27-\dfrac{10y}{x}+\dfrac{y^2}{x^2},(x\neq 0,y\neq 0)$

$$\frac{x}{y}-5+\frac{y}{x}$$

$$\frac{x}{y}\ \Big)\ \cancel{\frac{x^2}{y^2}}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},$$

$$\underline{\pm\cancel{\frac{x^2}{y^2}}}$$

$$\frac{2x}{y}-5\ \Big)\ -\cancel{\frac{10x}{y}}+27-\frac{10y}{x}+\frac{y^2}{x^2}$$

$$\underline{\pm\cancel{\frac{x^2}{y^2}}\pm 25}$$

$$\frac{2x}{y}-10+\frac{y}{x}\ \Big)\ +\cancel{2}-\cancel{\frac{10y}{c}}+\cancel{\frac{y^2}{x^2}}$$

$$\underline{\pm\cancel{2}\mp\cancel{\frac{10y}{x}}\pm\cancel{\frac{y^2}{x^2}}}$$

$$\times$$

Square root $=\pm\left(\dfrac{x}{y}-5+\dfrac{y}{x}\right)$

Q.3 Find the value of k for which the following expressions will become a perfect square.

(i) $4x^4-12x^3+37x^2-42x+k$

Solution: $4x^4-12x^3+37x^2-42x+k$

$$2x^2-3x+7$$

$$2x^2\,\big)\,\cancel{4x^4}-12x^3+37x^2-42x+k$$

$$\pm\cancel{4x^4}$$

$$4x^2-3x\,\big)\,-\cancel{12x^3}+37x^2-42x+k$$

$$\pm\cancel{12x^3}\pm 9x^2$$

$$4x^2-6x+7\,\big)\,\cancel{28x^2}-\cancel{42x}+k$$

$$\pm\cancel{28x^2}\pm\cancel{42x}\pm 49$$

$$k-49$$

In the case of perfect square remainder is always is equal to zero so

$k-49=0$

$k=49$

(ii) $x^4-4x^3+10x^2-kx+9$

Solution: $x^4-4x^3+10x^2-kx+9$

$$x^2-2x+3$$

$$=x^2\,\big)\,\cancel{x^4}-4x^3+10x^2-kx+9$$

$$\pm\cancel{x^4}$$

$$2x^2-2x\,\big)\,-\cancel{4x^3}+10x^2-kx+9$$

$$\pm\cancel{4x^3}\pm 4x^2$$

$$2x^2-4x+3\,\big)\,6x^2-kx+9$$

$$-6x^2\mp 12x\pm 9$$

$-kx+12x=0$

In the case of square root remainder is always equal to zero

$-x(k-12)=0$

$k-12=\dfrac{0}{-x}$

$k-12=0$

$k=12$

Q.4 Find the value of l and m for which the following expression will be perfect square

(i) $x^4+4x^3+16x^2+ln+m$

Solution: $x^4+4x^3+16x^2+ln+m$

$$\begin{array}{r|l} & x^2+2x+6 \\ \hline x^2 & \cancel{x^4}+4x^3+16x^2+lx+m \\ & \pm\cancel{x^4} \\ \hline 2x^2+2x & \cancel{4x^3}+16x^2+lx+m \\ & \pm\cancel{4x^3}\pm4x^2 \\ \hline 2x^2+4x+6 & \cancel{12x^2}+lx+m \\ & \pm\cancel{12x^2}\pm24x-36 \\ \hline \end{array}$$

In the case of square root remainder is always zero

$(lx-24x)$, $\quad m-36=0$

$x(l-24)=0$, $\quad m=36$ **Ans**

$l-24=\frac{0}{x}$

$l-24=0$

$l=24$ **Ans**

(ii) $49x^4-70x^3+109x^2+lx-m$

Solution: $49x^4-70x^3+109x^2+lx-m$

$$\begin{array}{r|l} & 7x^2-5x+6 \\ \hline 7x^2 & \cancel{49x^4}-70x^3+109x^2+lx-m \\ & \pm\cancel{49x^4} \\ \hline 14x^2-5x & -\cancel{70x^3}+109x^2+lx-m \\ & \mp\cancel{70x^3}\pm25x^2 \\ \hline 14x^2-10x+6 & \cancel{84x^2}+lx-m \\ & \pm\cancel{84x^2}\mp60x\pm36 \\ \hline & lx+60x-m-36 \\ & (l+60)x-m-36 \end{array}$$

In the case of square root remainder is always equal to zero

$-m-36=0$

$-m=36$

$l+60=0 \qquad m=-36$

$l=-60$ **Ans**

Q.5 **To make the expression** $9x^4 - 12x^3 + 22x^2 - 13x + 12$ **a perfect square**

Solution: $9x^4 - 12x^3 + 22x^2 - 13x + 12$

$$3x^2 - 2x + 3$$

$$= 3x^2 \overline{)\, \cancel{9x^4} - 12x^3 - + 22x^2 - 13x + 12}$$

$$\underline{\pm \cancel{9x^4}}$$

$$6x^2 - 2x \overline{)\, -\cancel{12x^3} + 22x^2 - 13x + 12}$$

$$\underline{\pm \cancel{12x^3} \pm 4x^2}$$

$$6x^2 - 4x + 3 \overline{)\, \cancel{18x^2} - 13x + 12}$$

$$\underline{\pm \cancel{18x^2} \mp 12x \pm 9}$$

$$-x + 3$$

(i) $+x - 3$ **is to be added**

(ii) $-x + 3$ **is to be subtract from it**

(iii) $-x + 3 = 0$

$x = 3$