Exercise 5.4

Factorize each of the following cubic polynomials by factor theorem.

1. $x^3 - 2x^2 - x + 2$

We have $p(x) = x^3 - 2x^2 - x + 2$

Possible factors of the constant p = 2 are $\pm 1, \pm 2$ and of leading coefficient q = 1 are ± 1.

Thus the expected zeros of $p(x) = 0$ are $\frac{p}{q} = \pm 1, \pm 2$

Using the hit and trial method

for $x = 1$

$P(1)$ = $(1)^3 - 2(1)^2 - (1) + 2$
= $(1) - 2(1) - 1 + 2$
= $1 - 2 - 1 + 2$
= $3 - 3$
= 0

Hence $x - 1$ is a factor of given polynomial.

for $x = -1$

$P(-1)$ = $(-1)^3 - 2(-1)^2 - (-1) + 2$
= $(-1) - 2(1) + 1 + 2$
= $-1 - 2 + 1 + 2$
= $-3 + 3$
= 0

Hence $x + 1$ is a factor of given polynomial.

for $x = 2$

$P(2)$ = $(2)^3 - 2(2)^2 - (2) + 2$
= $(8) - 2(4) - 2 + 2$
= $8 - 8 - 2 + 2$
= $10 - 10$
= 0

Hence $x - 2$ is a factor of given polynomial.

Thus the factorized form of

$p(x) = x^3 - 2x^2 - x + 2$

$p(x) = (x - 1)(x + 1)(x - 2)$

2. $x^3 - x^2 - 22x + 40$

We have $p(x) = x^3 - x^2 - 22x + 40$

Possible factors of the constant p = 40 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$ and of leading coefficient q = 1 are ± 1.

Thus the expected zeros of $p(x) = 0$ are $\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

Using the hit and trial method

for $x = 1$

$P(1)$ = $(1)^3 - (1)^2 - 22(1) + 40$
= $(1) - (1) - 22 + 40$

$= 1 - 1 - 22 + 40$
$= 41 - 23$
$= 18$

Hence $x - 1$ is not a factor of given polynomial.

for $x = -1$

$P(-1) = (-1)^3 - (-1)^2 - 22(-1) + 40$
$= (-1) - (1) + 22 + 40$
$= -1 - 1 + 22 + 40$
$= -2 + 62$
$= 62$

Hence $x + 1$ is not a factor of given polynomial.

for $x = 2$

$P(2) = (2)^3 - (2)^2 - 22(2) + 40$
$= (8) - (4) - 44 + 40$
$= 8 - 4 - 44 + 40$
$= 48 - 48$
$= 0$

Hence $x - 2$ is a factor of given polynomial.

for $x = -2$

$P(-2) = (-2)^3 - (-2)^2 - 22(-2) + 40$
$= (-8) - (4) + 44 + 40$
$= -8 - 4 + 44 + 40$
$= -12 + 84$
$= 72$

Hence $x + 2$ is not a factor of given polynomial.

for $x = 4$

$P(4) = (4)^3 - (4)^2 - 22(4) + 40$
$= (64) - (16) - 88 + 40$
$= 64 - 16 - 88 + 40$
$= 104 - 104$
$= 0$

Hence $x - 4$ is a factor of given polynomial.

for $x = -4$

$P(-4) = (-4)^3 - (-4)^2 - 22(-4) + 40$
$= (-64) - (16) + 88 + 40$
$= -64 - 16 + 88 + 40$
$= -80 + 128$
$= 48$

Hence $x + 4$ is not a factor of given polynomial.

for $x = 5$

$P(5) = (5)^3 - (5)^2 - 22(5) + 40$

= $(125) - (25) - 110 + 40$
= $125 - 25 - 110 + 40$
= $165 - 135$
= 30

Hence $x - 5$ is not a factor of given polynomial.

for $x = -5$

$P(-5)$ = $(-5)^3 - (-5)^2 - 22(-5) + 40$
= $(-125) - (25) + 110 + 40$
= $-125 - 25 + 110 + 40$
= $-150 + 150$
= 0

Hence $x + 5$ is a factor of given polynomial.

Thus the factorized form of

$p(x) = x^3 - x^2 - 22x + 40$

$p(x) = (x - 2)(x - 4)(x + 5)$

3. $x^3 - 6x^2 + 3x + 10$

We have $p(x) = x^3 - 6x^2 + 3x + 10$

Possible factors of the constant p = 10 are $\pm1, \pm2, \pm5$ and of leading coefficient q = 1 are ±1.

Thus the expected zeros of $p(x) = 0$ are $\frac{p}{q} = \pm1, \pm2, \pm5$

Using the hit and trial method

for $x = 1$

$P(1)$ = $(1)^3 - 6(1)^2 + 3(1) + 10$
= $(1) - 6(1) + 3 + 10$
= $1 - 6 + 3 + 10$
= $14 - 6$
= 8

Hence $x - 1$ is not a factor of given polynomial.

for $x = -1$

$P(-1)$ = $(-1)^3 - 6(-1)^2 + 3(-1) + 10$
= $(-1) - 6(1) - 3 + 10$
= $-1 - 6 - 3 + 10$
= $-10 + 10$
= 0

Hence $x + 1$ is a factor of given polynomial.

for $x = 2$

$P(2)$ = $(2)^3 - 6(2)^2 + 3(2) + 10$
= $(8) - 6(4) + 6 + 10$
= $8 - 24 + 6 + 10$
= $24 - 24$
= 0

Hence $x-2$ is a factor of given polynomial.

for $x=-2$

$P(-2)$ $=(-2)^3-6(-2)^2+3(-2)+10$
$=(-8)-6(4)-6+10$
$=-8-24-6+10$
$=-38+10$
$=-28$

Hence $x+2$ is not a factor of given polynomial.

for $x=5$

$P(5)$ $=(5)^3-6(5)^2+3(5)+10$
$=(125)-6(25)+15+10$
$=125-150+15+10$
$=150-150$
$=0$

Hence $x-5$ is a factor of given polynomial.

Thus the factorized form of
$p(x)=x^3-6x^2+3x+10$
$p(x)=(x+1)(x-2)(x-5)$

4. $x^3+x^2-10x+8$

We have $p(x)=x^3+x^2-10x+8$

Possible factors of the constant p = 10 are $\pm1,\pm2,\pm4$ and of leading coefficient q = 1 are ±1.

Thus the expected zeros of $p(x)=0$ are $\frac{p}{q}=\pm1,\pm2,\pm4$

Using the hit and trial method

for $x=1$

$P(1)$ $=(1)^3+(1)^2-10(1)+8$
$=(1)+(1)-10+8$
$=1+1-10+8$
$=10-10$
$=0$

Hence $x-1$ is a factor of given polynomial.

for $x=-1$

$P(-1)$ $=(-1)^3+(-1)^2-10(-1)+8$
$=(-1)+(1)+10+8$
$=-1+1+10+8$
$=-1+19$
$=18$

Hence $x+1$ is not a factor of given polynomial.

for $x=2$

$P(2)$ $=(2)^3+(2)^2-10(2)+8$
$=(8)+(4)-20+8$

$= 8 + 4 - 20 + 8$
$= 20 - 20$
$= 0$

Hence $x - 2$ is a factor of given polynomial.

for $x = -2$

$P(-2) \quad = (-2)^3 + (-2)^2 - 10(-2) + 8$
$= (-8) + (4) + 20 + 8$
$= -8 + 4 + 20 + 8$
$= -8 + 32$
$= 24$

Hence $x + 2$ is not a factor of given polynomial.

for $x = 4$

$P(4) \quad = (4)^3 + (4)^2 - 10(4) + 8$
$= (64) + (16) - 40 + 8$
$= 64 + 16 - 40 + 8$
$= 88 - 40$
$= 48$

Hence $x - 4$ is not a factor of given polynomial.

for $x = -4$

$P(-4) \quad = (-4)^3 + (-4)^2 - 10(-4) + 8$
$= (-64) + (16) + 40 + 8$
$= -64 + 16 + 40 + 8$
$= -64 + 64$
$= 0$

Hence $x + 4$ is a factor of given polynomial.

Thus the factorized form of

$p(x) = x^3 + x^2 - 10x + 8$

$p(x) = (x - 1)(x - 2)(x + 4)$

5. $x^3 - 2x^2 - 5x + 6$

We have $p(x) = x^3 - 2x^2 - 5x + 6$

Possible factors of the constant p = 6 are $\pm1, \pm2, \pm3$ and of leading coefficient q = 1 are ±1.

Thus the expected zeros of $p(x) = 0$ are $\frac{p}{q} = \pm1, \pm2, \pm3$

Using the hit and trial method

for $x = 1$

$P(1) \quad = (1)^3 - 2(1)^2 - 5(1) + 6$
$= (1) - 2(1) - 5 + 6$
$= 1 - 2 - 5 + 6$
$= 7 - 7$
$= 0$

Hence $x - 1$ is a factor of given polynomial.

for $x = -1$

$P(-1)$ $= (-1)^3 - 2(-1)^2 - 5(-1) + 6$

$= (-1) - 2(1) + 5 + 6$

$= -1 - 2 + 5 + 6$

$= -3 + 11$

$= 8$

Hence $x + 1$ is not a factor of given polynomial.

for $x = 2$

$P(2)$ $= (2)^3 - 2(2)^2 - 5(2) + 6$

$= (8) - 2(4) - 10 + 6$

$= 8 - 8 - 10 + 6$

$= 14 - 18$

$= -4$

Hence $x - 2$ is not a factor of given polynomial.

for $x = -2$

$P(-2)$ $= (-2)^3 - 2(-2)^2 - 5(-2) + 6$

$= (-8) - 2(4) + 10 + 6$

$= -8 - 8 + 10 + 6$

$= -16 + 16$

$= 0$

Hence $x + 2$ is a factor of given polynomial.

for $x = 3$

$P(3)$ $= (3)^3 - 2(3)^2 - 5(3) + 6$

$= (27) - 2(9) - 15 + 6$

$= 27 - 18 - 15 + 6$

$= 33 - 33$

$= 0$

Hence $x - 3$ is a factor of given polynomial.

Thus the factorized form of

$p(x) = x^3 - 2x^2 - 5x + 6$

$p(x) = (x - 1)(x + 2)(x - 3)$

6. $x^3 + 5x^2 - 2x - 24$

We have $p(x) = x^3 + 5x^2 - 2x - 24$

Possible factors of the constant p = 24 are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12$ and of leading coefficient q = 1 are ±1.

Thus the expected zeros of $p(x) = 0$ are $\frac{p}{q} = \pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12$

Using the hit and trial method

for $x = 1$

$P(1)$ $= (1)^3 + 5(1)^2 - 2(1) - 24$

$= (1) + 5(1) - 2 - 24$

$= 1 + 5 - 2 - 24$
$= 6 - 26$
$= -20$

Hence $x - 1$ is not a factor of given polynomial.

for $x = -1$

$P(-1) = (-1)^3 + 5(-1)^2 - 2(-1) - 24$
$= (-1) + 5(1) + 2 - 24$
$= -1 + 5 + 2 - 24$
$= -25 + 7$
$= -18$

Hence $x + 1$ is not a factor of given polynomial.

for $x = 2$

$P(2) = (2)^3 + 5(2)^2 - 2(2) - 24$
$= (8) + 5(4) - 4 - 24$
$= 8 + 20 - 4 - 24$
$= 28 - 28$
$= 0$

Hence $x - 2$ is a factor of given polynomial.

for $x = -2$

$P(-2) = (-2)^3 + 5(-2)^2 - 2(-2) - 24$
$= (-8) + 5(4) + 4 - 24$
$= -8 + 20 + 4 - 24$
$= -32 + 24$
$= -8$

Hence $x + 2$ is not a factor of given polynomial.

for $x = 3$

$P(3) = (3)^3 + 5(3)^2 - 2(3) - 24$
$= (27) + 5(9) - 6 - 24$
$= 27 + 45 - 6 - 24$
$= 72 - 30$
$= 42$

Hence $x - 3$ is not a factor of given polynomial.

for $x = -3$

$P(-3) = (-3)^3 + 5(-3)^2 - 2(-3) - 24$
$= (-27) + 5(9) + 6 - 24$
$= -27 + 45 + 6 - 24$
$= -51 + 51$
$= 0$

Hence $x + 3$ is a factor of given polynomial.

for $x = 4$

$P(4) = (4)^3 + 5(4)^2 - 2(4) - 24$

$= (64) + 5(16) - 8 - 24$
$= 64 + 80 - 8 - 24$
$= 144 - 32$
$= 112$

Hence $x - 4$ is not a factor of given polynomial.

for $x = -4$

$P(-4) \quad = (-4)^3 + 5(-4)^2 - 2(-4) - 24$
$= (-64) + 5(16) + 8 - 24$
$= -64 + 80 + 8 - 24$
$= -88 + 88$
$= 0$

Hence $x + 4$ is a factor of given polynomial.

Thus the factorized form of

$p(x) = x^3 + 5x^2 - 2x - 24$

$p(x) = (x - 2)(x + 3)(x + 4)$

7. $3x^3 - x^2 - 12x + 4$

We have $p(x) = 3x^3 - x^2 - 12x + 4$

Possible factors of the constant p = 4 are $\pm 1, \pm 2$ and of leading coefficient q = 3 are $\pm 1, \pm 3$.

Thus the expected zeros of $p(x) = 0$ are $\frac{p}{q} = \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

Using the hit and trial method

for $x = 1$

$P(1) \quad = 3(1)^3 - (1)^2 - 12(1) + 4$
$= 3(1) - (1) - 12 + 4$
$= 3 - 1 - 12 + 4$
$= 7 - 13$
$= -6$

Hence $x - 1$ is not a factor of given polynomial.

for $x = -1$

$P(-1) \quad = 3(-1)^3 - (-1)^2 - 12(-1) + 4$
$= 3(-1) - (1) + 12 + 4$
$= -3 - 1 + 12 + 4$
$= -4 + 16$
$= 12$

Hence $x + 1$ is not a factor of given polynomial.

for $x = 2$

$P(2) \quad = 3(2)^3 - (2)^2 - 12(2) + 4$
$= 3(8) - (4) - 24 + 4$
$= 24 - 4 - 24 + 4$
$= 28 - 28$
$= 0$

Hence $x-2$ is a factor of given polynomial.

for $x=-2$

$P(-2)$ $= 3(-2)^3-(-2)^2-12(-2)+4$

$= 3(-8)-(4)+24+4$

$= -24-4+24+4$

$= -28+28$

$= 0$

Hence $x+2$ is a factor of given polynomial.

for $x=\frac{1}{3}$

$P\left(\frac{1}{3}\right)$ $= 3\left(\frac{1}{3}\right)^3-\left(\frac{1}{3}\right)^2-12\left(\frac{1}{3}\right)+4$

$= 3\left(\frac{1}{27}\right)-\left(\frac{1}{9}\right)-4+4$

$= \frac{1}{9}-\frac{1}{9}$

$= 0$

Hence $3x-1$ is a factor of given polynomial.

Thus the factorized form of

$p(x)=3x^3-x^2-12x+4$

$p(x)=(x-2)(x+2)(3x-1)$

8. $2x^3+x^2-2x-1$

We have $p(x)=2x^3+x^2-2x-1$

Possible factors of the constant p = 1 are ± 1 and of leading coefficient q = 2 are $\pm 1, \pm 2$.

Thus the expected zeros of $p(x)=0$ are $\frac{p}{q}=\pm 1, \pm\frac{1}{2}$

Using the hit and trial method

for $x=1$

$P(1)$ $= 2(1)^3+(1)^2-2(1)-1$

$= 2(1)+(1)-2-1$

$= 2+1-2-1$

$= 3-3$

$= 0$

Hence $x-1$ is a factor of given polynomial.

for $x=-1$

$P(-1)$ $= 2(-1)^3+(-1)^2-2(-1)-1$

$= 2(-1)+(1)+2-1$

$= -2+1+2-1$

$= -3+3$

$= 0$

Hence $x+1$ is a factor of given polynomial.

for $x=\frac{1}{2}$

$P\left(\frac{1}{2}\right)$ $= 2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)-1$

$$= 2\left(\frac{1}{8}\right) + \left(\frac{1}{4}\right) - 1 - 1$$
$$= \frac{1}{4} + \frac{1}{4} - 2$$
$$= \frac{1+1-8}{4}$$
$$= \frac{-6}{4}$$

Hence $2x - 1$ is not a factor of given polynomial.

for $x = -\frac{1}{2}$

$$P\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) - 1$$
$$= 2\left(-\frac{1}{8}\right) + \left(\frac{1}{4}\right) + 1 - 1$$
$$= -\frac{1}{4} + \frac{1}{4}$$
$$= 0$$

Hence $2x + 1$ is a factor of given polynomial.

Thus the factorized form of

$p(x) = 2x^3 + x^2 - 2x - 1$

$p(x) = (x - 1)(x + 1)(2x + 1)$