Exercise 1.1

Q. 1: Write the following quadratic equations in the standard form and point out pure quadratic equations.

(i) $(x + 7)(x-3) = -7$

$x^2 - 3x + 7x - 21 = -7$

$x^2 + 4x - 21 = -7$

$x^2 + 4x - 21 + 7 = 0$

$x^2 + 4x - 14 = 0$

as $b \neq 0$, So the quadratic equation is not pure.

(ii) $\frac{x^2+4}{3} - \frac{x}{7} = 1$

$\frac{7(x^2+4)-3x}{21} = 1$

$\frac{7x^2+28-3x}{21} = 1$

$7x^2 + 28 - 3x = 21$

$7x^2 - 3x + 28 - 21 = 0$

$7x^2 - 3x + 7 = 0$

as $b \neq 0$, So the quadratic equation is not pure.

(iii) $\frac{x}{x+1} - \frac{x+1}{x} = 6$

$\frac{x^2-(x+1)(x+1)}{x(x+1)} = 6$

$\frac{x^2-(x^2+x+x+1)}{x^2+x} = 6$

$\frac{x^2-(x^2+2x+1)}{x^2+x} = 6$

$\frac{x^2-x^2-2x-1}{x^2+x} = 6$

$\frac{-2x-1}{x^2+x} = 6$

$-2x - 1 = 6(x^2 + x)$

$-2x - 1 = 6x^2 + 6x$

$-2x - 1 - 6x^2 - 6x = 0$

$-1 - 6x^2 - 8x = 0$

as $b \neq 0$, So the quadratic equation is not pure.

(iv) $\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0$

$\frac{x(x+4)-(x-2)(x-2)}{x(x-2)} = -4$

$\frac{x^2+4x-(x^2-2x-2x+4)}{x^2-2x} = -4$

$\frac{x^2+4x-(x^2-4x+4)}{x^2-2x} = -4$

$\frac{x^2+4x-x^2+4x-4}{x^2-2x} = -4$

$\frac{8x-4}{x^2-2x} = -4$

$8x - 4 = -4(x^2 - 2x)$

$4(2x-1) = -4(x^2-2x)$

$(2x-1) = \frac{-4(x^2-2x)}{4}$

$2x-1 = -x^2+2x$

$2x-1+x^2-2x = 0$

$x^2-1 = 0$

as b = 0, So the quadratic equation is pure.

(v) $\frac{x+3}{x+4} - \frac{x-5}{x} = 1$

$\frac{x(x+3)-(x-5)(x+4)}{x(x+4)} = 1$

$\frac{x^2+3x-(x^2-5x+4x-20)}{x^2+4x} = 1$

$\frac{x^2+3x-(x^2-x-20)}{x^2+4x} = 1$

$\frac{x^2+3x-x^2+x+20}{x^2+4x} = 1$

$\frac{4x+20}{x^2+4x} = 1$

$4x+20 = 1(x^2+4x)$

$4x+20 = x^2+4x$

$4x+20-x^2-4x = 0$

$-x^2+20 = 0$

$x^2-20 = 0$

as b ≠ 0, So the quadratic equation is not pure.

(vi) $\frac{x+1}{x+2} - \frac{x+2}{x+3} = \frac{25}{12}$

$\frac{(x+1)(x+3)-(x+2)(x+2)}{(x+2)(x+3)} = \frac{25}{12}$

$\frac{(x^2+x+3x+3)-(x^2+2x+2x+4)}{x^2+2x+3x+6} = \frac{25}{12}$

$\frac{(x^2 4x+3)-(x^2+4x+4)}{x^2+5x+6} = \frac{25}{12}$

$\frac{x^2+4x+3-x^2-4x-4}{x^2+5x+6} = \frac{25}{12}$

$\frac{-1}{x^2+5x+6} = \frac{25}{12}$

$-12 = 25(x^2+5x+6)$

$-12 = 25x^2+125x+150$

$-25x^2-125x-12-150 = 0$

$-25x^2-125x-162 = 0$

$25x^2+125x+162 = 0$

as b ≠ 0, So the quadratic equation is not pure.

Q. 2: Solve the factorization:

(i) $x^2-x-20 = 0$

$x^2-5x+4x-20 = 0$

$x(x-5)+4(x-5) = 0$

$(x-5)(x+4) = 0$

$x-5 = 0$ and $x+4 = 0$

$x = 5$ and $x = -4$

S.S $= \{5, -4\}$

(ii) $3y^2 = y(y-5)$

$3y^2 = y^2 - 5y$

$3y^2 - y^2 + 5y = 0$

$2y^2 + 5y = 0$

$y(2y+5) = 0$

$y = 0$ and $2y + 5 = 0$

$y = 0$ and $2y = -5$

$y = 0$ and $y = \frac{-5}{2}$

S.S $= \left\{0, -\frac{5}{2}\right\}$

(iii) $4 - 32x = 17x^2$

$0 = 17x^2 + 32x - 4$

$17x^2 + 32x - 4 = 0$

$17x^2 + 34x - 2x - 4 = 0$

$17x(x+2) - 2(x+2) = 0$

$(x+2)(17x-2) = 0$

$x + 2 = 0$ and $17x - 2 = 0$

$x = -2$ and $17x = 2$

$x = -2$ and $x = \frac{2}{17}$

S.S $= \left\{-2, \frac{2}{17}\right\}$

(iv) $x^2 - 11x = 152$

$x^2 - 11x - 152 = 0$

$x^2 - 19x + 8x - 152 = 0$

$x(x-19) + 8(x-19) = 0$

$(x-19)(x+8) = 0$

$x - 19 = 0$ and $x + 8 = 0$

$x = 19$ and $x = -8$

S.S $= \{19, -8\}$

(v) $\frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$

$\frac{(x+1)(x+1)+x^2}{x(x+1)} = \frac{25}{12}$

$\frac{(x^2+x+x+1)+x^2}{x^2+x} = \frac{25}{12}$

$\frac{(x^2+2x+1)+x^2}{x^2+x} = \frac{25}{12}$

$\frac{x^2+2x+1+x^2}{x^2+x} = \frac{25}{12}$

$\frac{2x^2+2x+1}{x^2+x} = \frac{25}{12}$

$$12(2x^2+2x+1) = 25(x^2+x)$$
$$24x^2+24x+12 = 25x^2+25x$$
$$0 = 25x^2+25x-24x^2-24x-12$$
$$0 = x^2+x-12$$
$$x^2+x-12 = 0$$
$$x^2+4x-3x-12 = 0$$
$$x(x+4)-3(x+4) = 0$$
$$(x+4)(x-3) = 0$$

$x+4 = 0$ and $x-3 = 0$

$x = -4$ and $x = 3$

S.S $= \{-4, 3\}$

(vi) $\frac{2}{x-9} = \frac{1}{x-3} - \frac{1}{x-4}$

$\frac{2}{x-9} + \frac{1}{x-4} = \frac{1}{x-3}$

$\frac{2(x-4)+x-9}{(x-9)(x-4)} = \frac{1}{x-3}$

$\frac{2x-8+x-9}{x^2-9x-4x+36} = \frac{1}{x-3}$

$\frac{3x-17}{x^2-13x+36} = \frac{1}{x-3}$

$$(x-3)(3x-17) = 1(x^2-13x+36)$$
$$3x^2-9x-17x+51 = x^2-13x+36$$
$$3x^2-26x+51 = x^2-13x+36$$
$$3x^2-26x+51-x^2+13x-36 = 0$$
$$2x^2-13x+15 = 0$$
$$2x^2-10x-3x+15 = 0$$
$$2x(x-5)-3(x-5) = 0$$
$$(x-5)(x-3) = 0$$

$x-5 = 0$ and $x-3 = 0$

$x = 5$ and $x = 3$

S.S $= \{5, 3\}$

Q. 3: Solve the following equations by completing square:

(i) $7x^2+2x-1 = 0$

$7x^2+2x = 1$

Dividing by 7

$\frac{7}{7}x^2+\frac{2}{7}x = \frac{1}{7}$

$x^2+2(x)\frac{1}{7} = \frac{1}{7}$

Adding $\left(\frac{1}{7}\right)^2$ on both sides

$x^2+2(x)\frac{1}{7}+\left(\frac{1}{7}\right)^2 = \frac{1}{7}+\left(\frac{1}{7}\right)^2$

$\left(x+\frac{1}{7}\right)^2 = \frac{1}{7}+\frac{1}{49}$

$$\left(x+\frac{1}{7}\right)^2 = \frac{7+1}{49}$$

$$\left(x+\frac{1}{7}\right)^2 = \frac{8}{49}$$

Taking square on both sides

$$\sqrt{\left(x+\frac{1}{7}\right)^2} = \sqrt{\frac{8}{49}}$$

$$x+\frac{1}{7} = \pm\frac{2\sqrt{2}}{7}$$

$$x = \pm\frac{2\sqrt{2}}{7} - \frac{1}{7}$$

$x = +\frac{2\sqrt{2}}{7} - \frac{1}{7}$ and $x = -\frac{2\sqrt{2}}{7} - \frac{1}{7}$

$x = \frac{2\sqrt{2}-1}{7}$ and $x = \frac{-2\sqrt{2}-1}{7}$

$$S.S = \left\{\frac{2\sqrt{2}-1}{7}, \frac{-2\sqrt{2}-1}{7}\right\}$$

(ii) $ax^2 + 4x - a = 0$

$$ax^2 + 4x = a$$

Dividing by a

$$\frac{a}{a}x^2 + \frac{4}{a}x = \frac{a}{a}$$

$$x^2 + 2(x)\frac{2}{a} = 1$$

Adding $\left(\frac{2}{a}\right)^2$ on both sides

$$x^2 + 2(x)\frac{2}{a} + \left(\frac{2}{a}\right)^2 = 1 + \left(\frac{2}{a}\right)^2$$

$$\left(x+\frac{2}{a}\right)^2 = 1 + \frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2 = \frac{a^2+4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2 = \frac{a^2+4}{a^2}$$

Taking square on both sides

$$\sqrt{\left(x+\frac{2}{a}\right)^2} = \sqrt{\frac{a^2+4}{a^2}}$$

$$x+\frac{2}{a} = \pm\frac{\sqrt{a^2+4}}{a}$$

$$x = \pm\frac{\sqrt{a^2+4}}{a} - \frac{2}{a}$$

$$x = \pm\frac{\sqrt{a^2+4}-2}{a}$$

$$S.S = \left\{\pm\frac{\sqrt{a^2+4}-2}{a}\right\}$$

(iii) $11x^2 - 34x + 3 = 0$

$$11x^2 - 34x = -3$$

Dividing by 11

$$\frac{11}{11}x^2 - \frac{34}{11}x = \frac{-3}{11}$$

$$x^2 + 2(x)\frac{17}{11} = \frac{-3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides

$$x^2 + 2(x)\frac{17}{11} + \left(\frac{17}{11}\right)^2 = \frac{1}{11} + \left(\frac{17}{11}\right)^2$$

$$\left(x + \frac{17}{11}\right)^2 = \frac{1}{11} + \frac{289}{121}$$

$$\left(x + \frac{17}{11}\right)^2 = \frac{11+289}{121}$$

$$\left(x + \frac{17}{11}\right)^2 = \frac{300}{121}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{17}{11}\right)^2} = \sqrt{\frac{300}{121}}$$

$$x + \frac{17}{11} = \pm\frac{10\sqrt{3}}{11}$$

$$x = \pm\frac{10\sqrt{3}}{11} - \frac{17}{11}$$

$x = +\frac{10\sqrt{3}}{11} - \frac{17}{11}$ and $x = -\frac{10\sqrt{3}}{11} - \frac{17}{11}$

$x = \frac{10\sqrt{3}-17}{11}$ and $x = \frac{-10\sqrt{3}-17}{11}$

S.S $= \left\{\frac{10\sqrt{3}-17}{11}, \frac{-10\sqrt{3}-17}{11}\right\}$

(iv) $lx^2 + mx + n = 0$

$$lx^2 + mx = -n$$

Dividing by l

$$\frac{l}{l}x^2 + \frac{m}{l}x = -\frac{n}{l}$$

$$x^2 + 2(x)\frac{m}{2l} = -\frac{n}{l}$$

Adding $\left(\frac{m}{2l}\right)^2$ on both sides

$$x^2 + 2(x)\frac{m}{2l} + \left(\frac{m}{2l}\right)^2 = -\frac{n}{l} + \left(\frac{m}{2l}\right)^2$$

$$\left(x + \frac{m}{2l}\right)^2 = -\frac{n}{l} + \frac{m^2}{4l^2}$$

$$\left(x + \frac{m}{2l}\right)^2 = \frac{-4ln+m^2}{4l^2}$$

$$\left(x + \frac{m}{2l}\right)^2 = \frac{m^2-4ln}{4l^2}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{m}{2l}\right)^2} = \sqrt{\frac{m^2-4ln}{4l^2}}$$

$$x + \frac{m}{2l} = \pm\frac{\sqrt{m^2-4ln}}{2l}$$

$$x = \pm\frac{\sqrt{m^2-4ln}}{2l} - \frac{m}{2l}$$

S.S $= \left\{\frac{-m\pm\sqrt{m^2-4ln}}{2l}\right\}$

(v) $3x^2 + 7x = 0$

$3x^2 + 7x = 0$

Dividing by l

$\frac{3}{3}x^2 + \frac{7}{3}x = \frac{0}{3}$

$x^2 + 2(x)\frac{7}{6} = 0$

Adding $\left(\frac{7}{6}\right)^2$ on both sides

$x^2 + 2(x)\frac{7}{6} + \left(\frac{7}{6}\right)^2 = \left(\frac{7}{6}\right)^2$

$\left(x + \frac{7}{6}\right)^2 = \frac{49}{36}$

$\left(x + \frac{7}{6}\right)^2 = \frac{49}{36}$

$\left(x + \frac{7}{6}\right)^2 = \frac{49}{36}$

Taking square on both sides

$\sqrt{\left(x + \frac{7}{6}\right)^2} = \sqrt{\frac{49}{36}}$

$x + \frac{7}{6} = \pm\frac{7}{6}$

$x = +\frac{7}{6} - \frac{7}{6}$ and $x = -\frac{7}{6} - \frac{7}{6}$

$x = 0$ and $x = \frac{-7-7}{6}$

$x = 0$ and $x = \frac{-14}{6}$

S.S $= \left\{0, \frac{-7}{3}\right\}$

(vi) $x^2 - 2x - 195 = 0$

$x^2 - 2x = 195$

$x^2 - 2x = 195$

$x^2 - 2(x)(1) = 195$

Adding $(1)^2$ on both sides

$x^2 + 2(x)(1) + (1)^2 = 195 + (1)^2$

$(x + 1)^2 = 195 + 1$

$(x + 1)^2 = 196$

$(x + 1)^2 = 196$

Taking square on both sides

$\sqrt{(x + 1)^2} = \sqrt{196}$

$x + 1 = \pm 14$

$x = +14 - 1$ and $x = -14 - 1$

$x = 13$ and $x = -15$

$x = 13$ and $x = -15$

S.S $= \{13, -15\}$

(vii) $-x^2 + \frac{15}{2} = \frac{7}{2}x$

$$\frac{15}{2} = x^2 + \frac{7}{2}x$$

$$x^2 + \frac{7}{2}x = \frac{15}{2}$$

$$x^2 + 2(x)\frac{7}{4} = \frac{15}{2}$$

Adding $\left(\frac{7}{4}\right)^2$ on both sides

$$x^2 + 2(x)\frac{7}{4} + \left(\frac{7}{4}\right)^2 = \frac{15}{2} + \left(\frac{7}{4}\right)^2$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{15}{2} + \frac{49}{16}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{120+49}{16}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{169}{16}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{7}{4}\right)^2} = \sqrt{\frac{169}{16}}$$

$$x + \frac{7}{4} = \pm\frac{13}{4}$$

$x = +\frac{13}{4} - \frac{7}{4}$ and $x = -\frac{13}{4} - \frac{7}{4}$

$x = 0$ and $x = \frac{-7-7}{6}$

$x = 0$ and $x = \frac{-14}{6}$

S.S $= \left\{0, \frac{-7}{3}\right\}$

(viii) $x^2 + 17x + \frac{33}{4} = 0$

$$x^2 + 17x = -\frac{33}{4}$$

$$x^2 + 2(x)\frac{17}{2} = \frac{-33}{4}$$

Adding $\left(\frac{17}{2}\right)^2$ on both sides

$$x^2 + 2(x)\frac{17}{2} + \left(\frac{17}{2}\right)^2 = \frac{-33}{4} + \left(\frac{17}{2}\right)^2$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33}{4} + \frac{289}{4}$$

$$\left(x + \frac{17}{2}\right)^2 = \frac{-33+289}{4}$$

$$\left(x + \frac{7}{2}\right)^2 = \frac{256}{4}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{7}{2}\right)^2} = \sqrt{\frac{256}{4}}$$

$$x + \frac{7}{2} = \pm\frac{16}{2}$$

$x = +\frac{16}{2} - \frac{7}{2}$ and $x = -\frac{16}{2} - \frac{7}{2}$

$x = \frac{16-7}{2}$ and $x = \frac{-16-7}{2}$

$x = \frac{9}{2}$ and $x = \frac{-23}{2}$

S.S $= \left\{\frac{9}{2}, \frac{-23}{2}\right\}$

(ix) $4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$

$4 = \frac{3x^2+5}{3x+1} + \frac{8}{3x+1}$

$4 = \frac{3x^2+5+8}{3x+1}$

$4(3x+1) = 3x^2 + 13$

$12x + 4 = 3x^2 + 13$

$0 = 3x^2 + 13 - 12x - 4$

$0 = 3x^2 - 12x - 9$

$3x^2 - 12x - 9 = 0$

$3x^2 - 12x = 9$

Dividing both sides by 3

$\frac{3}{3}x^2 - \frac{12}{3}x = \frac{9}{3}$

$x^2 - 4x = 3$

$x^2 - 2(x)(2) = 3$

Adding $(2)^2$ on both sides

$x^2 + 2(x)(2) + (2)^2 = 3 + (2)^2$

$(x+2)^2 = 3 + 4$

$(x+2)^2 = 7$

$(x+2)^2 = 7$

Taking square on both sides

$\sqrt{(x+2)^2} = \sqrt{7}$

$x + 2 = \pm\sqrt{7}$

$x = -2 \pm \sqrt{7}$

S.S $= \{-2 \pm \sqrt{7}\}$

(x) $7(x+2a)^2 + 3a^2 = 5a(7x+23a)$

$7(x^2 + 2ax + 4a^2) + 3a^2 = 35ax + 115a^2$

$7x^2 + 14ax + 28a^2 + 3a^2 = 35ax + 115a^2$

$7x^2 + 14ax + 31a^2 = 35ax + 115a^2$

$7x^2 + 14ax + 31a^2 - 35ax - 115a^2 = 0$

$7x^2 - 21ax - 84a^2 = 0$

Dividing both sides by 7

$\frac{7}{7}x^2 - \frac{21}{7}ax - \frac{84}{7}a^2 = 0$

$x^2 - 3ax - 12a^2 = 0$

$x^2 - 3ax = 12a^2$

$x^2 - 2(x)\left(\frac{3a}{2}\right) = 12a^2$

Adding $\left(\frac{3a}{2}\right)^2$ on both sides

$$x^2 + 2(x)\frac{3a}{2} + \left(\frac{3a}{2}\right)^2 = 12a^2 + \left(\frac{3a}{2}\right)^2$$

$$\left(x + \frac{3a}{2}\right)^2 = 12a^2 + \frac{9a^2}{4}$$

$$\left(x + \frac{3a}{2}\right)^2 = \frac{48a^2+9a^2}{4}$$

$$\left(x + \frac{3a}{2}\right)^2 = \frac{57a^2}{4}$$

Taking square on both sides

$$\sqrt{\left(x + \frac{3a}{2}\right)^2} = \sqrt{\frac{57a^2}{4}}$$

$$x + \frac{3a}{2} = \pm\frac{\sqrt{57}a}{2}$$

$$x = -\frac{3a}{2} \pm \frac{\sqrt{57}a}{2}$$

$$x = \frac{-3a\pm\sqrt{57}a}{2}$$

$$\text{S.S} = \left\{\frac{-3a\pm\sqrt{57}a}{2}\right\}$$