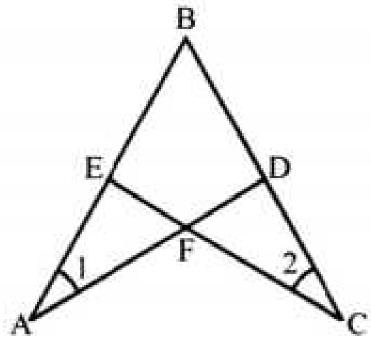
Exercise 10.1

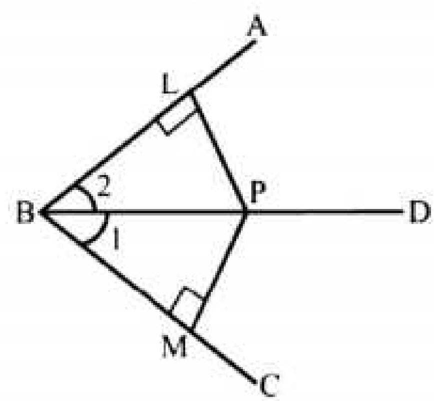
Q.1 **In the given figure**
$\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$
Prove that
$\Delta ABD \cong \Delta CBE$



Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle BAD \cong \angle BCE$	Given $\angle 1 \cong \angle 2$
$\angle ABD \cong \angle CBE$	Common
$\Delta ABD \cong \Delta CBE$	S.A.A $\cong$ S.A.A

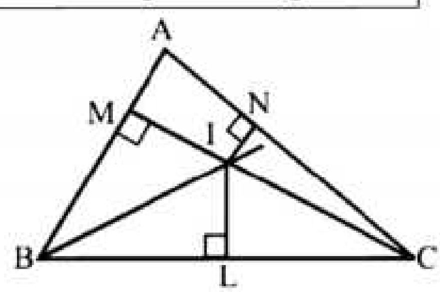
Q.2 **From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.**



Given
$\overline{BD}$ is bisector of $\angle ABC$. P is point on $\overline{BD}$ and $\overline{PL}$ are $\overline{PM}$ are perpendicular to $\overline{AB}$ and $\overline{CB}$ respectively
To prove
$\overline{PL} \cong \overline{PM}$
Proof

Statements	Reasons
In $\Delta BLP \leftrightarrow \Delta BMP$	
$\overline{BP} \cong \overline{BP}$	Common
$\angle BLP \cong \angle BMP$	Each right angle (given)
$\angle LBP \cong \angle MBP$	Given $\overline{BD}$ is bisector of angle B
$\therefore \Delta BLP \cong \Delta BMP$	S.A.A $\cong$ S.A.A
So $\overline{PL} \cong \overline{PM}$	Corresponding sides of congruent triangles

Q.3 **In a triangle ABC, the bisects of $\angle B$ and $\angle C$ meet in point I prove that I is equidistant from the three sides by ΔABC**
Given



In ΔABC, the bisector of $\angle B$ and $\angle C$ meet at I and $\overline{IL}$, $\overline{IM}$, and $\overline{IN}$ are perpendiculars to the three sides of ΔABC.

To prove

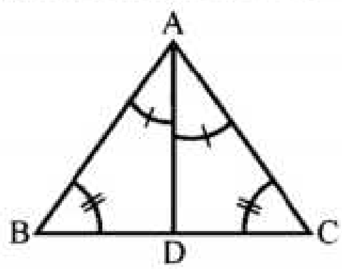
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$

Proof

Statements	Reasons
In $\Delta ILB \leftrightarrow \Delta IMB$	
$\overline{BI} \cong \overline{BI}$	Common
$\angle IBL \cong \angle IBM$	Given BI is bisector of $\angle B$
$\angle ILB \cong \angle IMB$	Given each angle is rights angles
$\Delta ILB \cong \Delta IMB$	$SAA \cong S.A.A$
$\therefore\ \overline{IL} \cong \overline{IM}$ _____ (i)	Corresponding sides of $\cong \Delta$'s
Similarly	
$\Delta IAC \cong \Delta INC$	
So $\overline{IL} \cong \overline{IN}$ _____ (ii)	Corresponding sides of $\cong \Delta$s
from (i) and (ii)	
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$	
$\therefore$ I is equidistant from the three sides of ΔABC.	

Theorem 10.1.2

If two angles of a triangles are congruent then the sides opposite to them are also congruent



Given

In ΔABC, $\angle B \cong \angle C$

To prove

$\overline{AB} \cong \overline{AC}$

Construction

Draw the bisector of $\angle A$, meeting $\overline{BC}$ at point D

Proof

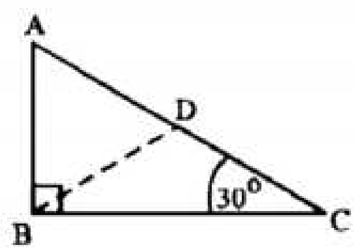
Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\Delta ABD \cong \Delta ACD$	$S.A.A \cong S.A.A$
Hence $\overline{AB} \cong \overline{AC}$	(Corresponding sides of congruent triangles)

Example 1

If one angle of a right triangle is of 30°, the hypotenuse is twice as long as the side opposite to the angle.

Given

In ΔABC, $m\angle B=90^\circ$ and $m\angle C = 30^\circ$



To prove

$m\overline{AC}=2m\overline{AB}$

Constructions

At, B construct $\angle CBD$ of 30°

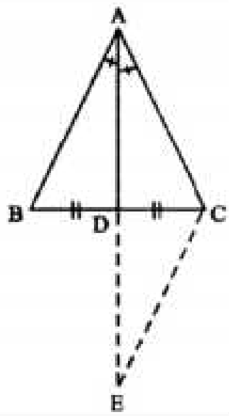
Let $\overline{BD}$ cut $\overline{AC}$ at the point D.

Proof

Statements	Reasons
In $\Delta ABD, m\angle A = 60^\circ$	$m\angle ABC = 90^\circ, m\angle C = 30^\circ$
$m\angle ABD = m\angle ABC$, $mCBD = 60^\circ$	$m\angle ABC = 90^\circ, m\angle CBD = 30^\circ$
$\therefore mADB = 60^\circ$	Sum of measures of $\angle$s of a Δ is 180°
$\therefore \Delta ABD$ is equilateral	Each of its angles is equal to 60°
$\therefore \overline{AB} \cong \overline{BD} \cong \overline{CD}$	Sides of equilateral Δ
In $\Delta BCD, \overline{BD} \cong \overline{CD}$	**$\angle C = \angle CBD$ (each of 30),**
Thus $m\overline{AC} = m\overline{AD} + m\overline{CD}$	
$= m\overline{AB} + m\overline{AB}$	$\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BD} \cong \overline{AB}$
$= 2(m\overline{AB})$	

Example 2

If the bisector of an angle of a triangle bisects the side opposite to it, the triangle is isosceles.



Given

In $\Delta ABC, \overline{AD}$ bisect $\angle A$ and $BD \cong CD$

To prove

$\overline{AB} \cong \overline{AC}$

Construction

Produce $\overline{AD}$ to E, and take $\overline{ED} \cong \overline{AD}$

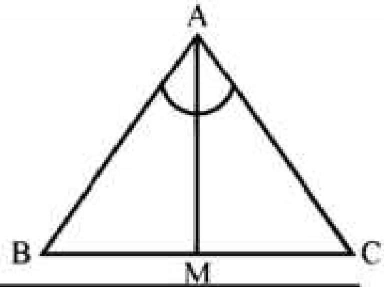
Joint C to E

Proof

Statements	Reasons
In $\Delta ADB \leftrightarrow EDC$	
$\overline{AD} \cong \overline{ED}$	Construction
$\angle ADB \cong \angle EDC$	Vertical angles
$\overline{BD} \cong \overline{CD}$	Given
$\therefore \Delta ADB \cong \Delta EDC$	S.A.S. Postulate
$\therefore \overline{AB} \cong \overline{EC}$ ...(i)	Corresponding sides
and $\angle BAD \cong \angle E$	Corresponding angles
But $\angle BAD \cong \angle CAD$	Given
$\therefore \angle E \cong \angle CAD$	Each $\cong \angle BAD$
In $\Delta ACE, \overline{AC} \cong \overline{EC}$ ...(ii)	$\angle E \cong \angle CAD$ (proved)
Hence $\overline{AB} \cong \overline{AC}$	From (i) and (ii)

Exercise 10.2

Q.1 Prove that any two medians of an equilateral triangle are equal in measure.



Given

In ΔABC, $\overline{AB} \cong \overline{AC}$ and M is midpoint of $\overline{BC}$

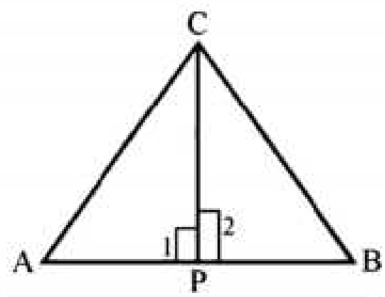
To prove

$\overline{AM}$ bisects $\angle A$ and $\overline{AM}$ is perpendicular to $\overline{BC}$

Proof

Statements	Reasons
In $\Delta ABM \leftrightarrow \Delta ACM$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BM} \cong \overline{CM}$	Given M is midpoint of BC
$\overline{AM} \cong \overline{AM}$	Common
$\therefore \Delta ABM \cong \Delta ACM$	S.S.S $\cong$ S.S.S
So $\angle BAM \cong \angle CAM$	Corresponding angles of congruents triangle
$m\angle AMB + m\angle AMC = 180^o$	
$\therefore$ $m\angle AMB = m\angle AMC$	
i.e $\overline{AM}$ is perpendicular to $\overline{BC}$	

Q.2 Prove that a point which is equidistant from the end points of a line segment, is on the right bisector of line segment



Given

$\overline{AB}$ is line segment. The point C is such that $\overline{CA} \cong \overline{CB}$

To prove

Point C lies on the right bisector of $\overline{AB}$

Construction

(i) Take P as midpoint of $\overline{AB}$ i.e. $\overline{AP} \cong \overline{BP}$

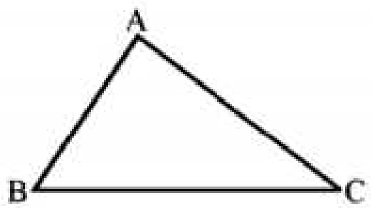
(ii) Joint point C to A, P, B

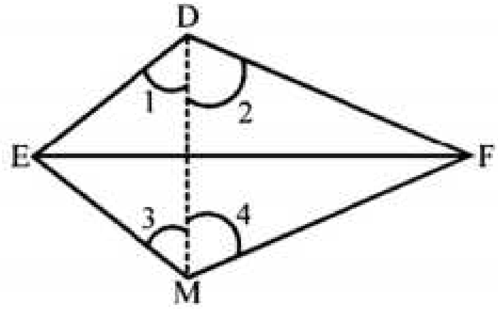
Proof:

Statements	Reasons
In ΔABC	
$\overline{CA} \cong \overline{CB}$	Given
$\angle A \cong \angle B$	Corresponding angles of congruent triangles
Δ CBP $\leftrightarrow \Delta$CAP	
$\overline{CB} \cong \overline{CA}$	
$\Delta CAP \cong \Delta CBP$	S.A.S $\cong$ S.A.S
$\therefore$ $\angle 1 \cong \angle 2$	
$m\angle 1 + m\angle 2 = 180^o$	Adjacent angles on one side of a line
Thus m $\angle 1$ = $m\angle 2 = 90$	
Hence $\overline{CP}$ is right bisector of $\overline{AB}$ and point C lies on $\overline{CB}$	

Theorem 10.1.3

In a correspondence of two triangles if three sides of one triangle are congruent to the corresponding three sides of the other. Then the two triangles are congruent (S.S.S $\cong$ S.S.S)





Given:

In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To prove

$\Delta ABC \cong \Delta DEF$

Construction

Suppose that in ΔDEF the side $\overline{EF}$ is not smaller than any of the remaining two sides. On $\overline{EF}$ construct a ΔMEF in which, $\angle FEM \cong \angle B$ and $\overline{ME} \cong \overline{AB}$. Join D and M. as shown in the above figures we label some of the angles as 1, 2, 3, and 4

Proof:

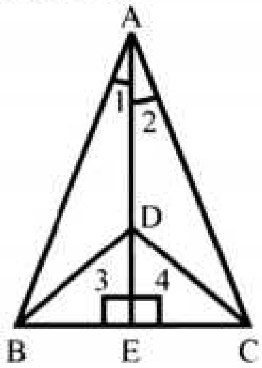
Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta MEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B \cong \angle FEM$	Construction
$\overline{AB} \cong \overline{ME}$	Construction
$\therefore\ \Delta ABC \cong \Delta MEF$	S.A.S Postulate
and $\overline{CA} \cong \overline{FM}$ ____(i)	(Corresponding sides of congruent triangles)
also $\overline{CA} \cong \overline{FD}$ ___(ii)	Given
$\therefore\ \overline{FM} \cong \overline{FD}$	{ From (i) and (ii) }
In ΔFDM	
$\angle 2 \cong \angle 4$_____(iii)	$\overline{FM} \cong \overline{FD}$ (proved)
Similarly $\angle 1 \cong \angle 3$ ______(iv)	
$\therefore\ m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$	{ from (iii) and iv }
$\therefore\ m\angle EDF = m\angle EMF$	
Now in $\Delta DEF \leftrightarrow \Delta MEF$	
$\overline{FD} \cong \overline{FM}$	Proved
and $m\angle EDF \cong \angle EMF$	Proved
$\overline{DE} \cong \overline{ME}$	Each one $\cong \overline{AB}$
$\therefore\ \Delta DEF \cong \Delta MEF$	S.A.S postulates
also $\Delta ABC \cong \Delta MEF$	Proved
Hence $\Delta ABC \cong \Delta DEF$	Each $\Delta \cong \Delta MEF$ (proved)

Example 1

If two isosceles triangles are formed on the same side of their common base, the line through their vertices would be the right bisector of their common base.

Given

ΔABC and ΔDBC formed on the same side of $\overline{BA}$ such that
$\overline{BA} \cong \overline{AC}, \overline{DB} \cong \overline{DC}, \overline{AD}$ meets $\overline{BC}$ at E.

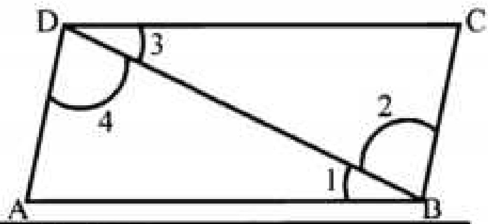


To prove
$\overline{BE} \cong \overline{CE}, \overline{AE} \perp \overline{BC}$
Proof

Statements	Reasons
In $\Delta ADB \leftrightarrow \Delta ADC$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{DB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\therefore \Delta ADB \cong \Delta ADC$	S.S.S ≅ S.S.S
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of $\cong \Delta s$
In $\Delta ABE \leftrightarrow \Delta ACE$	
$\overline{AB} \cong \overline{AC}$	Given
$\angle 1 \cong \angle 2$	Proved
$\Delta ABE \cong \Delta ACE$	S.A.S postulate
$\overline{AE} \cong \overline{AE}$	Common
$\therefore \overline{BE} \cong \overline{CE}$	Corresponding sides of $\cong \Delta s$
$\angle 3 \cong \angle 4$	Corresponding angles of $\cong \Delta s$
$m\angle 3 + m\angle 4 = 180^\circ$	Supplementary angles postulate
$m\angle 3 = m\angle 4 = 90^\circ$	From I and II
Hence $\overline{AE} \perp \overline{BC}$	

Exercise 10.3

Q.1 In the figure, $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$ prove that $\angle A = \angle C, \angle ABC \cong \angle ADC$



Given

In the figure $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$

To prove

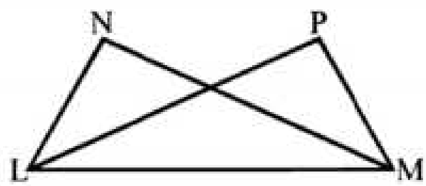
$\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	S.S.S $\cong$ S.S.S
$\therefore$ Hence $\angle A \cong \angle C$	Corresponding angles of congruent triangles
$\angle 1 \cong \angle 3$	Corresponding angles of congruent triangles
$\angle 2 \cong \angle 4$	Corresponding angles of congruent triangles
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	
or $m\angle ABC = m\angle ADC$	
$\angle ABC \cong \angle ADC$	

Q.2 In the figure $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$ prove that $\angle N \cong \angle P, \angle NML \cong \angle PLM$



Given

In the figure

$\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$

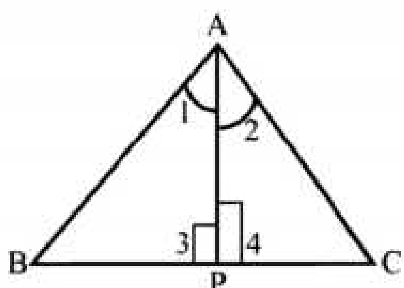
To prove

$\angle N \cong \angle P$ and $\angle NML \cong \angle PLM$

Proof

Statements	Reasons
$\Delta LMN \leftrightarrow \Delta MLP$	
$\overline{LN} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{ML}$	Common
$\Delta LMN \cong \Delta MLP$	S.S.S $\cong$ S.S.S
$\angle N \cong \angle P$	Corresponding angles of congruent triangles
$\angle NML \cong \angle PLM$	Corresponding angles of congruent triangles

Q.3 Prove that median bisecting the base of an isosceles triangle bisects the vertex angle and it is perpendicular to the base



Given

ΔABC

(i) $\overline{AB} \cong \overline{AC}$

(ii) Point P is mid point of $\overline{BC}$ i.e: $\overline{BP} = \overline{CP}$

P is joined to A, i.e. $\overline{AP}$ is median

To prove

$\angle 1 \cong \angle 2$

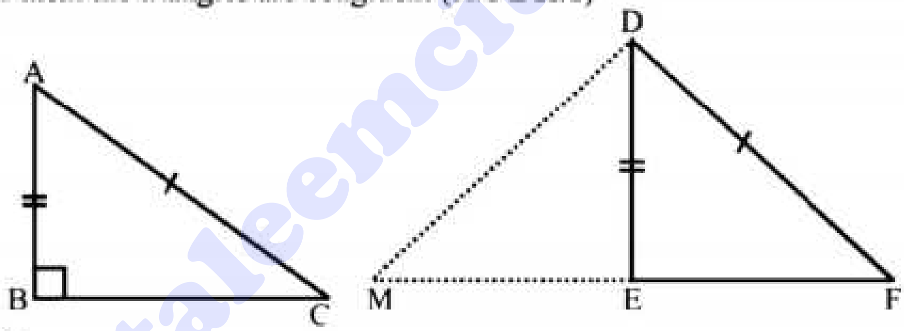
$\overline{AP} \perp \overline{BC}$

Proof

Statements	Reasons
$\Delta ABP \leftrightarrow \Delta ACP$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BP} \cong \overline{CP}$	Given
$\overline{AP} \cong \overline{AP}$	Common
$\Delta ABP \cong \Delta ACP$	S.S.S ≅ S.S.S
$\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 3 \cong \angle 4$ _____(i)	
$m\angle 3 + m\angle 4 = 180^o$ ____(ii)	Corresponding angles of congruent triangles
Thus $m\angle 3 = m\angle 4 = 90$	
Q $\overline{AP} \perp \overline{BC}$	From equation (i) and (ii)

Theorem 10.1.4

If in the corresponding of the two right angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other them the triangles are congruent (H.S ≅ H.S)



Given

$\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$ (right angles)

$\overline{CA} \cong \overline{FD}, \overline{AB} \cong \overline{DE}$

To Prove

$\Delta ABC \cong \Delta DEF$

Construction

Prove $\overline{FE}$ to a point M such that $\overline{EM} \cong \overline{BC}$ and join the point D and M

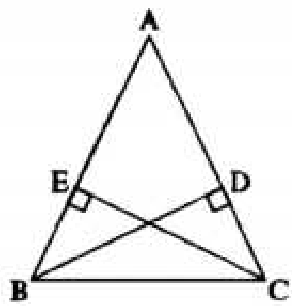
Proof

Statements	Reasons
$m\angle DEF + \angle DEM = 180^o$ ____(i)	Supplementary angles
Now $m\angle DEF = 90^o$ ________(ii)	Given
$\therefore m\angle DEM = 90^o$	{ from (i) and (ii) }
In $\Delta ABC \leftrightarrow \Delta DEM$	
$\overline{BC} \cong \overline{EM}$	Construction
$\angle ABC \cong \angle DEM$	(Each angle equal to 90^o)

$\overline{AB} \cong \overline{DE}$	Given
$\Delta ABC \cong \Delta DEM$	SAS postulate
ad $\angle C = \angle M$	Corresponding angles of congruent triangles
$\overline{CA} \cong \overline{MD}$	Corresponding sides of congruent triangles
But $\overline{CA} \cong \overline{FD}$	Given
$\overline{MD} \cong \overline{FD}$	Each is congruent to $\overline{CA}$
In DMF	
$\angle F \cong \angle M$	$\overline{MD} \cong \overline{FD}$ (proved)
But $\angle C \cong \angle M$	(Proved)
$\angle C \cong \angle F$	Each is congruent to $\angle M$
	Given
$\angle ABC \cong \angle DEF$	Given
$\overline{AB} \cong \overline{DE}$	(Proved)
Hence $\Delta ABC \cong \Delta DEF$	(S.A.A $\cong$ S.A.A)

Example

If perpendiculars from two vertices of a triangle to the opposite sides are congruent, then the triangle is isosceles.



Given

In $\Delta ABC, \overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$

Such that $\overline{BD} \cong \overline{CE}$

To prove

$\overline{AB} \cong \overline{AC}$

Proof

Statements	Reasons
In $\Delta BCD \leftrightarrow \Delta CBC$	
$\angle BDC \cong \angle BEC$	$\overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$ given $\Rightarrow$ each angle $= 90^\circ$
$\overline{BC} \cong \overline{BC}$	Common hypotenuse
$\overline{BD} \cong \overline{CE}$	Given
$\Delta BCD \cong \Delta CBE$	H.S $\cong$ H.S
$\angle BCA \cong \angle CBE$	Corresponding angles Δ s
Thus $\angle BCA \cong \angle CBA$	
Hence $\overline{AB} \cong \overline{AC}$	In $\Delta ABC, \angle BCA \cong \angle CBA$

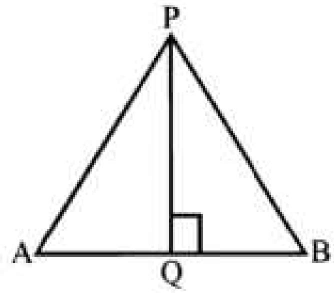
Exercise 10.4

Q.1 **In ΔPAB of figure $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$ prove that $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$**

Given:

In ΔPAB

$\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$



To prove

$\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$

Proof

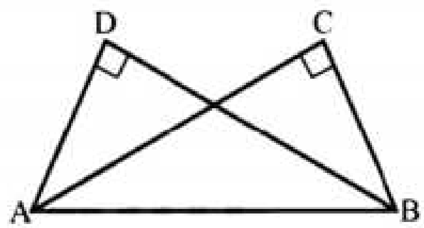
Statements	Reasons
In $\Delta APQ \leftrightarrow \Delta BPQ$	
$\overline{PA} \cong \overline{PB}$	Given
$\angle AQP \cong \angle BQP$	Given $\overline{PQ} \perp \overline{AB}$
$\overline{PQ} \cong \overline{PQ}$	Common
$\therefore \Delta APQ \cong \Delta BPQ$	H.S $\cong$ H.S
So $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of congruent triangles
and $\angle APQ \cong \angle BPQ$	Corresponding angles of congruent triangles

Q.2 **In the figure $m\angle C \cong m\angle D = 90^o$ and $\overline{BC} \cong \overline{AD}$ prove that $\overline{AC} \cong \overline{BD}$ and $\angle BAC \cong \angle ABD$**

Given

In the figure given $m\angle C = m\angle D = 90^o$

$\overline{BC} \cong \overline{AD}$



To Prove

$\overline{AC} \cong \overline{BD}$

$\angle BAC \cong \angle ABD$

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta BAC$	
$\overline{AD} \cong \overline{BC}$	Given
$\angle D \cong \angle C$	Each 90^o
$\overline{AB} \cong \overline{BA}$	Common
Thus $\Delta ABD \cong \Delta BAC$	H-S $\cong$ H-S
$\therefore \overline{AC} \cong \overline{BD}$	Corresponding sides of congruent triangles
$\therefore \angle BAC \cong \angle ABD$	Corresponding angles of congruent triangles

Q.3 **In the figure, $m\angle B = m\angle D = 90^o$ and $\overline{AD} \cong \overline{BC}$ prove that ABCD is a rectangle**

Given

In the figure

$m\angle B = m\angle D$ 90° and $\overline{AD} \cong \overline{BC}$

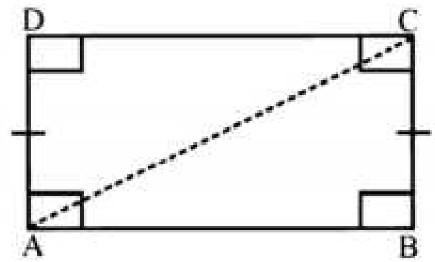
To prove

$ABCD$ is a rectangle

Construction

Join A to C

Proof



Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta CDA$	
$\angle B \cong \angle D$	Given each angle = 90°
$\overline{AC} \cong \overline{CA}$	Common
$\overline{BC} \cong \overline{DA}$	Given
$\therefore \Delta ABC \cong \Delta CDA$	H-S ≅ H-S
$\overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles
and $\angle ACB \cong \angle CAD$	Corresponding angles of congruent triangles
Hence ABCD is a rectangle	