Exercise 1.4

Solve the following equations.

1. $2x + 5 = \sqrt{7x + 16}$

squaring both sides

$$(2x+5)^2 = \left(\sqrt{7x+16}\right)^2$$
$$4x^2 + 20x + 25 = 7x + 16$$
$$4x^2 + 20x + 25 - 7x - 16 = 0$$
$$4x^2 + 13x + 9 = 0$$
$$4x^2 + 9x + 4x + 9 = 0$$
$$x(4x+9) + 1(4x+9) = 0$$
$$(4x+9)(x+1) = 0$$

$4x + 9 = 0$ and $x + 1 = 0$

$4x = -9$ and $x = -1$

$x = -\frac{9}{4}$ and $x = -1$

Checking:

$2\left(\frac{-9}{4}\right) + 5 = \sqrt{7\left(\frac{-9}{4}\right) + 16}$ and $2(-1) + 5 = \sqrt{7(-1) + 16}$

$\frac{-9}{2} + 5 = \sqrt{\frac{-63}{4} + 16}$ and $-2 + 5 = \sqrt{-7 + 16}$

$\frac{-9+10}{2} = \sqrt{\frac{-63+64}{4}}$ and $3 = \sqrt{9}$

$\frac{1}{2} = \sqrt{\frac{1}{4}}$ and $3 = 3$

True and True

So,

S.S $= \left\{-1, -\frac{9}{4}\right\}$

2. $\sqrt{x+3} = 3x - 1$

squaring both sides

$$\left(\sqrt{x+3}\right)^2 = (3x-1)^2$$
$$x + 3 = 9x^2 - 6x + 1$$
$$0 = 9x^2 - 6x + 1 - x - 3$$
$$0 = 9x^2 - 7x - 2$$
$$9x^2 - 7x - 2 = 0$$
$$9x^2 - 9x + 2x - 2 = 0$$
$$9x(x-1) + 2(x-1) = 0$$
$$(x-1)(9x+2) = 0$$

$9x + 2 = 0$ and $x - 1 = 0$

$9x = -2$ and $x = 1$

$x = -\frac{2}{9}$ and $x = 1$

Checking:

$\sqrt{x+3} = 3x-1$

$\sqrt{\left(\frac{-2}{9}\right)+3} = 3\left(\frac{-2}{9}\right)-1$ and $\sqrt{(1)+3} = \sqrt{3(1)-1}$

$\sqrt{\frac{-2}{9}+3} = \frac{-2}{3}-1$ and $\sqrt{4} = 3-1$

$\sqrt{\frac{-2+27}{9}} = \frac{-2-3}{3}$ and $\sqrt{4} = 2$

$\sqrt{\frac{25}{9}} = \frac{-5}{3}$ and $2 = 2$

False and True

So,

S.S $= \{1\}$

3. $4x = \sqrt{13x+14}-3$

$4x+3 = \sqrt{13x+14}$

squaring both sides

$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$

$16x^2+24x+9 = 13x+14$

$16x^2+24x+9-13x-14 = 0$

$16x^2+11x-5 = 0$

$16x^2+16x-5x-5 = 0$

$16x(x+1)-5(x+1) = 0$

$(x+1)(16x-5) = 0$

$16x-5 = 0$ and $x+1 = 0$

$16x = 5$ and $x = -1$

$x = \frac{5}{16}$ and $x = -1$

Checking:

$4x = \sqrt{13x+14}-3$

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right)+14}-3$ and $4(-1) = \sqrt{13(-1)+14}-3$

$\frac{5}{4} = \sqrt{\frac{65}{16}+14}-3$ and $-4 = \sqrt{-13+14}-3$

$\frac{5}{4} = \sqrt{\frac{65+224}{16}}-3$ and $-4 = \sqrt{1}-3$

$\frac{5}{4} = \sqrt{\frac{289}{16}}-3$ and $-4 = \sqrt{1}-3$

$\frac{5}{4} = \frac{17}{4}-3$ and $-4 = 1-3$

$\frac{5}{4} = \frac{17-12}{4}$ and $-4 = -2$

$\frac{5}{4} = \frac{5}{4}$ and $-4 = -2$

True and False

So,

S.S $= \left\{\frac{5}{16}\right\}$

4. $\sqrt{3x+100} - x = 4$

$\sqrt{3x+100} = x+4$

squaring both sides

$$\left(\sqrt{3x+100}\right)^2 = (x+4)^2$$
$$3x+100 = x^2+8x+16$$
$$0 = x^2+8x+16-3x-100$$
$$0 = x^2+5x-84$$
$$x^2+5x-84 = 0$$
$$x^2+12x-7x-84 = 0$$
$$x(x+12)-7(x+12) = 0$$
$$(x+12)(x-7) = 0$$

$x+12 = 0$ and $x-7 = 0$

$x = -12$ and $x = 7$

Checking:

$\sqrt{3x+100} - x = 4$

$\sqrt{3(-12)+100} - (-12) = 4$ and $\sqrt{3(7)+100} - (7) = 4$

$\sqrt{-36+100} + 12 = 4$ and $\sqrt{21+100} - 7 = 4$

$\sqrt{64} + 12 = 4$ and $\sqrt{121} - 7 = 4$

$8 + 12 = 4$ and $11 - 7 = 4$

$20 = 4$ and $4 = 4$

False and True

So,

S.S $= \{7\}$

5. $\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$

Squaring both sides

$$\left(\sqrt{x+5} + \sqrt{x+21}\right)^2 = \left(\sqrt{x+60}\right)^2$$
$$\left(\sqrt{x+5}\right)^2 + \left(\sqrt{x+21}\right)^2 + 2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right) = x+60$$
$$x+5+x+21+2\sqrt{(x+5)(x+21)} = x+60$$
$$2x+26+2\sqrt{x^2+21x+5x+105} = x+60$$
$$2x+26+2\sqrt{x^2+26x+105} = x+60$$
$$2x+26-x-60 = -2\sqrt{x^2+26x+105}$$
$$x-34 = -2\sqrt{x^2+26x+105}$$

squaring both sides

$$\left(-2\sqrt{x^2+26x+105}\right)^2 = (x-34)^2$$
$$4(x^2+26x+105) = x^2-68x+1156$$

$$4x^2 + 104x + 420 = x^2 - 68x + 1156$$
$$4x^2 + 104x + 420 - x^2 + 68x - 1156 = 0$$
$$3x^2 + 172x - 736 = 0$$
$$3x^2 + 184x - 12x - 736 = 0$$
$$x(3x + 184) - 4(3x + 184) = 0$$
$$(3x + 184)(x - 4) = 0$$

$3x + 184 = 0$ and $x - 4 = 0$

$3x = -184$ and $x = 4$

$x = -\frac{184}{3}$ and $x = 4$

Checking:

$\sqrt{x+5} + \sqrt{x+21} = \sqrt{x+60}$

$\sqrt{\frac{-184}{3}+5} + \sqrt{\frac{-184}{3}+21} = \sqrt{\frac{-184}{3}+60}$ and $\sqrt{4+5} + \sqrt{4+21} = \sqrt{4+60}$

$\sqrt{\frac{-184+15}{3}} + \sqrt{\frac{-184+63}{3}} = \sqrt{\frac{-184+180}{3}}$ and $\sqrt{9} + \sqrt{25} = \sqrt{64}$

$\sqrt{\frac{-169}{3}} + \sqrt{\frac{-121}{3}} = \sqrt{\frac{-4}{3}}$ and $3 + 5 = 8$

False and True

So,

S.S $= \{4\}$

6. $\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$

Squaring both sides

$$\left(\sqrt{x+1} + \sqrt{x-2}\right)^2 = \left(\sqrt{x+6}\right)^2$$
$$\left(\sqrt{x+1}\right)^2 + \left(\sqrt{x-2}\right)^2 + 2\left(\sqrt{x+1}\right)\left(\sqrt{x-2}\right) = x + 6$$
$$x + 1 + x - 2 + 2\sqrt{(x+1)(x-2)} = x + 6$$
$$2x - 1 + 2\sqrt{x^2 - x - 2} = x + 6$$
$$2x - 1 - x - 6 = -2\sqrt{x^2 - x - 2}$$
$$x - 7 = -2\sqrt{x^2 - x - 2}$$

squaring both sides

$$\left(-2\sqrt{x^2 - x - 2}\right)^2 = (x - 7)^2$$
$$4(x^2 - x - 2) = x^2 - 14x + 49$$
$$4x^2 - 4x - 8 = x^2 - 14x + 49$$
$$4x^2 - 4x - 8 - x^2 + 14x - 49 = 0$$
$$3x^2 + 10x - 57 = 0$$
$$3x^2 + 19x - 9x - 57 = 0$$
$$x(3x + 19) - 3(3x + 19) = 0$$
$$(3x + 19)(x - 3) = 0$$

$3x + 19 = 0$ and $x - 3 = 0$

$3x = -19$ and $x = 3$

$x = -\frac{19}{3}$ and $x = 3$

Checking:

$\sqrt{x+1} + \sqrt{x-2} = \sqrt{x+6}$

$\sqrt{\frac{-19}{3}+1} + \sqrt{\frac{-19}{3}-2} = \sqrt{\frac{-19}{3}+6}$ and $\sqrt{3+1} + \sqrt{3-2} = \sqrt{3+6}$

$\sqrt{\frac{-19+3}{3}} + \sqrt{\frac{-19-6}{3}} = \sqrt{\frac{-19+18}{3}}$ and $\sqrt{4} + \sqrt{1} = \sqrt{9}$

$\sqrt{\frac{-16}{3}} + \sqrt{\frac{-13}{3}} = \sqrt{\frac{-1}{3}}$ and $2 + 1 = 3$

False and True

So,

S.S $= \{3\}$

7. $\sqrt{11-x} - \sqrt{6-x} = \sqrt{27-x}$

Squaring both sides

$(\sqrt{11-x} - \sqrt{6-x})^2 = (\sqrt{27-x})^2$

$(\sqrt{11-x})^2 + (\sqrt{6-x})^2 - 2(\sqrt{11-x})(\sqrt{6-x}) = 27 - x$

$11 - x + 6 - x - 2\sqrt{(11-x)(6-x)} = 27 - x$

$-2x + 17 - 2\sqrt{66 - 11x - 6x + x^2} = 27 - x$

$-2x + 17 - 2\sqrt{66 - 17x + x^2} = 27 - x$

$-2x + 17 + x - 27 = 2\sqrt{66 - 17x + x^2}$

$-x - 10 = 2\sqrt{66 - 17x + x^2}$

$-(x + 10) = 2\sqrt{66 - 17x + x^2}$

squaring both sides

$(2\sqrt{66 - 17x + x^2})^2 = (-(x + 10))^2$

$4(66 - 17x + x^2) = x^2 + 20x + 100$

$264 - 68x + 4x^2 = x^2 + 20x + 100$

$4x^2 - 68x + 264 - x^2 - 20x - 100 = 0$

$3x^2 - 88x + 164 = 0$

$3x^2 - 82x - 6x + 164 = 0$

$x(3x - 82) - 2(3x - 82) = 0$

$(3x - 82)(x - 2) = 0$

$3x - 82 = 0$ and $x - 2 = 0$

$3x = 82$ and $x = 2$

$x = \frac{82}{3}$ and $x = 2$

Checking:

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$\sqrt{11-\frac{82}{3}}+\sqrt{6-\frac{82}{3}} = \sqrt{27-\frac{82}{3}}$ and $\sqrt{11-2}+\sqrt{6-2} = \sqrt{27-2}$

$\sqrt{\frac{33-82}{3}}+\sqrt{\frac{18-82}{3}} = \sqrt{\frac{81-82}{3}}$ and $\sqrt{9}+\sqrt{4} = \sqrt{25}$

$\sqrt{\frac{-49}{3}}+\sqrt{\frac{-64}{3}} = \sqrt{\frac{-1}{3}}$ and $3+2 = 5$

False and True

So,

S.S $= \{2\}$

8. $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Squaring both sides

$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2 = \left(\sqrt{a}\right)^2$

$\left(\sqrt{4a+x}\right)^2+\left(\sqrt{a-x}\right)^2-2\left(\sqrt{4a+x}\right)\left(\sqrt{a-x}\right)=a$

$4a+x+a-x-2\sqrt{(4a+x)(a-x)} = a$

$5a-2\sqrt{4a^2-4ax+ax-x^2} = a$

$5a-2\sqrt{4a^2-3ax-x^2} = a$

$5a-a = 2\sqrt{4a^2-3ax-x^2}$

$4a = 2\sqrt{4a^2-3ax-x^2}$

squaring both sides

$\left(2\sqrt{4a^2-3ax-x^2}\right)^2 = (4a)^2$

$4(4a^2-3ax-x^2) = 16a^2$

$4a^2-3ax-x^2 = \frac{16a^2}{4}$

$4a^2-3ax-x^2 = 4a^2$

$-3ax-x^2 = 4a^2-4a^2$

$-3ax-x^2 = 0$

$-x(3a+x) = 0$

$x = 0$ and $3a+x = 0$

$x = 0$ and $x = -3a$

Checking:

$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

$\sqrt{4a+0}-\sqrt{a-0} = \sqrt{a}$ and $\sqrt{4a-3a}-\sqrt{a-(-3a)} = \sqrt{a}$

$\sqrt{4a}-\sqrt{a} = \sqrt{a}$ and $\sqrt{4a-3a}-\sqrt{a+3a} = \sqrt{a}$

$2\sqrt{a}-\sqrt{a} = \sqrt{a}$ and $\sqrt{a}-\sqrt{4a} = \sqrt{a}$

$2\sqrt{a}-\sqrt{a} = \sqrt{a}$ and $\sqrt{a}-2\sqrt{a} = \sqrt{a}$

$\sqrt{a}$ $= \sqrt{a}$ and $-\sqrt{a}$ $= \sqrt{a}$

True and False

So,

S.S $= \{0\}$

9. $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

let $x^2 + x = y$ ---------- (a)

So

$$\sqrt{y+1} - \sqrt{y-1} = 1$$

Squaring both sides

$$\left(\sqrt{y+1} - \sqrt{y-1}\right)^2 = (1)^2$$

$$\left(\sqrt{y+1}\right)^2 + \left(\sqrt{y-1}\right)^2 - 2\left(\sqrt{y+1}\right)\left(\sqrt{y-1}\right) = 1$$

$$y + 1 + y - 1 - 2\sqrt{(y+1)(y-1)} = 1$$

$$2y - 2\sqrt{y^2-1} = 1$$

$$2y - 2\sqrt{y^2-1} = 1$$

$$2y - 1 = 2\sqrt{y^2-1}$$

$$2y - 1 = 2\sqrt{y^2-1}$$

squaring both sides

$$\left(2\sqrt{y^2-1}\right)^2 = (2y-1)^2$$

$$4(y^2-1) = 4y^2 - 4y + 1$$

$$4y^2 - 4 = 4y^2 - 4y + 1$$

$$4y^2 - 4y^2 + 4y = 1 + 4$$

$$4y = 5$$

$$y = \frac{5}{4}$$

put value of y in equation (a)

$$x^2 + x = \frac{5}{4}$$

$$4x^2 + 4x = 5$$

$$4x^2 + 4x - 5 = 0$$

a = 4, b = 4, c = -5

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(4) \pm \sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16+80}}{8}$$

$$x = \frac{-4 \pm \sqrt{96}}{8}$$

$$x = \frac{-4 \pm \sqrt{16 \times 6}}{8}$$

$$x = \frac{-4 \pm 4\sqrt{6}}{8}$$

$$x = \frac{4(-1\pm\sqrt{6})}{8}$$

S.S $= \left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

10. $\sqrt{x^2+3x+8} + \sqrt{x^2+3x+2} = 3$

let $x^2 + 3x = y$ ---------- (a)

So

$$\sqrt{y+8} + \sqrt{y+2} = 3$$

Squaring both sides

$$\left(\sqrt{y+8} + \sqrt{y+2}\right)^2 = (3)^2$$

$$\left(\sqrt{y+8}\right)^2 + \left(\sqrt{y+2}\right)^2 + 2\left(\sqrt{y+8}\right)\left(\sqrt{y+2}\right) = 9$$

$$y + 8 + y + 2 + 2\sqrt{(y+8)(y+2)} = 9$$

$$2y + 10 + 2\sqrt{y^2+2y+8y+16} = 9$$

$$2y + 10 + 2\sqrt{y^2+10y+16} = 9$$

$$2y + 10 - 9 = -2\sqrt{y^2+10y+16}$$

$$2y + 1 = -2\sqrt{y^2+10y+16}$$

squaring both sides

$$\left(-2\sqrt{y^2+10y+16}\right)^2 = (2y+1)^2$$

$$4(y^2+10y+16) = 4y^2+4y+1$$

$$4y^2+40y+64 = 4y^2+4y+1$$

$$4y^2+40y+64-4y^2-4y-1 = 0$$

$$36y + 63 = 0$$

$$36y = -63$$

$$y = \frac{-63}{36}$$

$$y = \frac{-7}{4}$$

put value of y in equation (a)

$$x^2 + 3x = -\frac{7}{4}$$

$$4x^2 + 12x = -7$$

$$4x^2 + 12x + 7 = 0$$

a = 4, b = 12, c = 7

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(12)\pm\sqrt{(12)^2-4(4)(7)}}{2(4)}$$

$$x = \frac{-12\pm\sqrt{144-112}}{8}$$

$$x = \frac{-12\pm\sqrt{32}}{8}$$

$$x = \frac{-12\pm\sqrt{16\times2}}{8}$$

$$x = \frac{-12\pm4\sqrt{2}}{8}$$

$$x = \frac{4(-3\pm\sqrt{2})}{8}$$

S.S $= \left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

11. $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

let $x^2+3x=y$ ---------- (a)

So

$$\sqrt{y+9}+\sqrt{y+4}=5$$

Squaring both sides

$$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=(5)^2$$

$$\left(\sqrt{y+9}\right)^2+\left(\sqrt{y+4}\right)^2+2\left(\sqrt{y+9}\right)\left(\sqrt{y+4}\right)=25$$

$$y+9+y+4+2\sqrt{(y+9)(y+4)}=25$$

$$2y+13+2\sqrt{y^2+4y+9y+36}=25$$

$$2y+13+2\sqrt{y^2+13y+36}=25$$

$$2y+13-25=-2\sqrt{y^2+13y+36}$$

$$2y-12=-2\sqrt{y^2+13y+36}$$

$$2(y-6)=-2\sqrt{y^2+13y+36}$$

squaring both sides

$$\left(-\sqrt{y^2+13y+36}\right)^2=(y-6)^2$$

$$y^2+13y+36=y^2-12y+36$$

$$y^2+13y+36=y^2-12y+36$$

$$y^2+13y+36-y^2+12y-36=0$$

$$25y=0$$

put value of y in equation (a)

$$x^2+3x=0$$

$$x(x+3)=0$$

$x = 0$ and $x+3=0$

$x = -3$

S.S $= \{0,-3\}$