## Exercise 2.7

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Solve the following simultaneous equations.

1. 
$$x + y = 5$$
 ;  $x^2 - 2y - 14 = 0$ 

$$x + y = 5$$
 ----- (i)

$$x^2 - 2y - 14 = 0$$
 ----- (ii)

From equation (i) we have:

$$y = 5 - x$$
 -----(iii)

Putting in equation (ii)

$$x^2 - 2y - 14 = 0$$

$$x^2 - 2(5 - x) - 14 = 0$$

$$x^2 - 10 + 2x - 14 = 0$$

$$x^2 + 2x - 24 = 0$$

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x+6) - 4(x+6) = 0$$

$$(x+6)(x-4) = 0$$

$$x + 6 = 0$$

$$x = -6$$

$$x-4=0$$

Putting the values of x in equation (iii)

$$y = 5 - x$$

$$= 5 - x$$

$$y = 5 - (-6)$$

$$y = 5 + 6$$

$$y = 11$$

$$y = 5 - x$$

$$v = 5 - (4)$$

$$y = 5 - 4$$

$$v = 1$$

So,

$$S.S = \{(-6, 11), (4, 1)\}$$

2. 
$$3x - 2y = 1$$
 ;  $x^2 + xy - y^2 = 1$ 

$$3x - 2y = 1$$
 ---- (i)

$$x^2 + xy - y^2 = 1$$
 ----- (ii)

From equation (i) we have:

$$-2y = 1 - 3x$$

$$y = \frac{1 - 3x}{-2}$$

$$y = \frac{3x - 1}{2}$$
 (iii)

Putting in equation (ii)

$$x^2 + xy - y^2 = 1$$

$$x^2 + x\left(\frac{3x-1}{2}\right) - \left(\frac{3x-1}{2}\right)^2 = 1$$

$$x^2 + \frac{3x^2 - x}{2} - \frac{9x^2 - 6x + 1}{4} = 1$$

$$\frac{4x^2+2(3x^2-x)-(9x^2-6x+1)}{4} = 1$$

$$\frac{4x^{2}+6x^{2}-2x-9x^{2}+6x-1}{4} = 1$$

$$\frac{x^{2}+4x-1}{4} = 1$$

$$x^{2}+4x-1 = 4$$

$$x^{2}+4x-1-4 = 0$$

$$x^{2}+4x-5 = 0$$

$$x^{2}+4x-5 = 0$$

$$x^{2}+5x-x-5 = 0$$

$$x(x+5)-1(x+5) = 0$$

$$(x+5)(x-1) = 0$$

$$x+5 = 0$$

$$x = -5$$
;  $x-1 = 0$ 

$$x = 1$$

Putting the values of x in equation (iii)

$$y = \frac{3x-1}{2}$$
 ;  $y = \frac{3x-1}{2}$   
 $y = \frac{3(-5)-1}{2}$  ;  $y = \frac{3(1)-1}{2}$   
 $y = \frac{-16}{2}$  ;  $y = \frac{2}{2}$   
 $y = -8$  ;  $y = 1$ 

So,

3.

S.S = 
$$\{(-5, -8), (1, 1)\}$$
  
 $x - y = 7$  ;  $\frac{2}{x} - \frac{5}{y} = 2$ 

$$x - y = 7$$
 ----- (i)  
 $\frac{2}{x} - \frac{5}{y} = 2$   
 $\frac{2y - 5x}{xy} = 2$   
 $2y - 5x = 2xy$   
 $2y - 5x - 2xy = 0$  ----- (ii)

From equation (i) we have:

$$x - y = 7$$
  
 $-y = 7 - x$   
 $y = x - 7$  (iii)

Putting in equation (ii)

$$2y - 5x - 2xy = 0$$

$$2(x - 7) - 5x - 2x(x - 7) = 0$$

$$2x - 14 - 5x - 2x^{2} + 14x = 0$$

$$-2x^{2} + 11x - 14 = 0$$

$$2x^{2} - 11x + 14 = 0$$

$$2x^{2} - 7x - 4x + 14 = 0$$

$$x(2x - 7) - 2(2x - 7) = 0$$

$$(2x-7)(x-2) = 0$$

$$2x - 7 = 0$$
 ;  $x - 2 = 0$   
 $2x = 7$  ;  $x = 2$   
 $x = \frac{7}{2}$  ;  $x = 2$ 

Putting the values of x in equation (iii)

$$y = x - 7$$
 ;  $y = x - 7$   
 $y = \frac{7}{2} - 7$  ;  $y = 2 - 7$   
 $y = \frac{7-14}{2}$  ;  $y = -5$   
 $y = -\frac{7}{2}$  ;  $y = -5$ 

So,

4.

S.S = 
$$\left\{ \left( \frac{7}{2}, -\frac{7}{2} \right), (2, -5) \right\}$$
  
 $x + y = a - b$  ;  $\frac{a}{x} - \frac{b}{x} = 2$ 

$$x + y = a - b - (i)$$

$$\frac{a}{x} - \frac{b}{y} = 2$$

$$\frac{ay - bx}{xy} = 2$$

$$ay - bx = 2xy$$

$$ay - bx - 2xy = 0$$
 ----- (ii)

From equation (i) we have:

$$x + y = a - b$$
  
 $y = a - b - x$   
 $y = a - b - x$  (iii)

Putting in equation (ii)

$$ay - bx - 2xy = 0$$

$$a(a - b - x) - bx - 2x(a - b - x) = 0$$

$$a^{2} - ab - ax - bx - 2ax + 2bx + 2x^{2} = 0$$

$$2x^{2} - 3ax + bx + a^{2} - ab = 0$$

$$2x^{2} - (3a - b)x + a^{2} - ab = 0$$

using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-(3a-b)) \pm \sqrt{(-(3a-b))^2 - 4(2)(a^2 - ab)}}{2(-2)}$$

$$x = \frac{3a-b \pm \sqrt{(3a-b)^2 - 8(a^2 - ab)}}{2(2)}$$

$$x = \frac{3a-b \pm \sqrt{9a^2 + b^2 - 6ab - 8a^2 + 8ab}}{4}$$

$$x = \frac{3a-b\pm\sqrt{a^2+b^2+2ab^2}}{4}$$

$$x = \frac{3a-b\pm\sqrt{(a+b)^2}}{4}$$

$$x = \frac{3a-b\pm(a+b)}{4}$$

$$x = \frac{3a-b+(a+b)}{4}$$
 ;  $x = \frac{3a-b-(a+b)}{4}$   
 $x = \frac{3a-b+a+b}{4}$  ;  $x = \frac{3a-b-a-b}{4}$   
 $x = \frac{4a}{4}$  ;  $x = \frac{2a-2b}{4}$   
 $x = a$  ;  $x = \frac{a-b}{2}$ 

Putting the values of x in equation (iii)

$$y = a - b - x$$
 ;  $y = a - b - x$   
 $y = a - b - a$  ;  $y = a - b - (\frac{a - b}{2})$   
 $y = a - b - a$  ;  $y = \frac{2a - 2b - (a - b)}{2}$   
 $y = -b$  ;  $y = \frac{2a - 2b - a + b}{2}$   
 $y = -b$  ;  $y = \frac{a - b}{2}$ 

So,

5.

$$x^{2} + (y-1)^{2} = 10$$
 ;  $x^{2} + y^{2} + 4x = 1$   
 $x^{2} + y^{2} - 2y + 1 = 10$   
 $x^{2} + y^{2} - 2y = 9$  .....(i)

 $S.S = \left\{ (a, -b), \left( \frac{a-b}{2}, \frac{a-b}{2} \right) \right\}$ 

Subtracting equation (i) from (ii)

$$x^{2} + y^{2} + 4x = 1$$

$$-x^{2} - y^{2} + 2y = -9$$

$$4x + 2y = -8$$

$$2x + y = -4$$

$$y = -4 - 2x - ----- (iii)$$

 $x^2 + y^2 + 4x = 1$  ----- (ii)

Putting in equation (ii)

$$x^{2} + y^{2} + 4x = 1$$

$$x^{2} + (-4 - 2x)^{2} + 4x = 1$$

$$x^{2} + (-(4 + 2x))^{2} + 4x = 1$$

$$x^{2} + (2x + 4)^{2} + 4x = 1$$

$$x^{2} + 4x^{2} + 16 + 16x + 4x = 1$$

$$5x^{2} + 20x + 15 = 0$$

$$x^{2} + 4x + 3 = 0$$

$$x^{2} + 3x + x + 3 = 0$$

$$x(x + 3) + 1(x + 3) = 0$$

$$(x + 3)(x + 1) = 0$$

$$x + 3 = 0 ; x + 1 = 0$$

$$x = -3 ; x = -1$$
Putting the values of x in equation (iii)
$$y = -4 - 2x ; y = -4 - 2(-1)$$

$$y = -4 + 6 ; y = -4 + 2$$

$$y = 2 ; y = -2$$
So,
$$S.S = \{(-3, 2), (-1, -2)\}$$

$$(x + 1)^{2} + (y + 1)^{2} = 5 ; (x + 2)^{2} + y^{2} = 5$$

$$x^{2} + 2x + 1 + y^{2} + 2y + 1 = 5$$

$$x^{2} + y^{2} + 2x + 2y + 2 = 5$$

$$x^{2} + y^{2} + 2x + 2y + 3 = 0$$

$$x^{2} + 4x + 4 + y^{2} = 5$$

$$x^{2} + y^{2} + 4x - 1 = 0$$

$$-x^{2} - y^{2} - 2x - 2y + 3 = 0$$

$$2x - 2y + 2 = 0$$

$$2x - 2y = -2$$

$$2(x - y) = -2$$

$$x - y = -1$$

$$-y = x + 1$$
(iii)
Putting in equation (ii)

6.

$$x^{2} + y^{2} + 4x = 1$$

$$x^{2} + (x+1)^{2} + 4x = 1$$

$$x^{2} + x^{2} + 2x + 1 + 4x = 1$$

$$2x^{2} + 6x = 0$$

$$2x(x+3) = 0$$

$$2x = 0$$
 ;  $x + 3 = 0$    
  $x = 0$  ;  $x = -3$ 

Putting the values of x in equation (iii)

$$y = x + 1$$
 ;  $y = x + 1$   
 $y = 0 + 1$  ;  $y = -3 + 1$   
 $y = 1$  ;  $y = -2$ 

So,

$$S.S = \{(0, 1), (-3, -2)\}$$

7. 
$$x^2 + 2y^2 = 22$$

$$5x^2 + y^2 = 29$$

$$x^2 + 2y^2 = 22$$
 ----- (i)

$$5x^2 + y^2 = 29$$
 ----- (ii)

multiplying equation (i) by 5

$$5x^2 + 10y^2 = 110$$
 ----- (iii)

Subtracting equation (ii) from (iii)

$$5x^2 + 10y^2 = 110$$
$$-5x^2 - y^2 = -29$$

-----

$$9y^2 = 81$$

$$y^2 = 9$$

$$y = \pm 3$$

$$y = 3$$
;

$$y = -3$$

Putting the values of x in equation (i)

$$x^2 + 2y^2 = 22$$
 ;

$$x^2 + 2(3)^2 = 22$$

$$x^2 + 2(9) = 22$$

$$x^2 + 18 = 22$$

$$x^2 = 4$$

$$x = \pm 2$$

$$x^2 + 2y^2 = 22$$

$$x^2 + 2(-3)^2 = 22$$

$$x^2 + 2(9) = 22$$

$$x^2 + 18 = 22$$

$$x^2 = 4$$

$$x = \pm 2$$

So,

8.

$$S.S = \{(\pm 2, \pm 3)\}$$

$$4x^2 - 5y^2 = 6$$

$$3x^2 + y^2 = 14$$

$$4x^2 - 5y^2 = 6$$
 ----- (i)

$$3x^2 + y^2 = 14$$
 ----- (ii)

multiplying equation (ii) by 5

$$15x^2 + 5y^2 = 70$$
 ----- (iii)

Adding equation (i) and (iii)

$$4x^2 - 5y^2 = 6$$

$$15x^2 + 5y^2 = 70$$

-----

$$x^2 = 4$$

$$x = \pm 2$$

$$x = 2 ; x = -2$$

Putting the values of x in equation (i)

$$4x^{2} - 5y^{2} = 6$$
 ;  $4x^{2} - 5y^{2} = 6$   
 $4(2)^{2} - 5y^{2} = 6$  ;  $4(-2)^{2} - 5y^{2} = 6$   
 $4(4) - 5y^{2} = 6$  ;  $4(4) - 5y^{2} = 6$   
 $16 - 5y^{2} = 6$  ;  $16 - 5y^{2} = 6$   
 $-5y^{2} = -10$  ;  $-5y^{2} = -10$   
 $y^{2} = 2$  ;  $y^{2} = 2$   
 $y = \pm \sqrt{2}$  ;  $y = \pm \sqrt{2}$ 

So,

9.

S.S = 
$$\{(\pm 2, \pm \sqrt{2})\}$$
  
 $7x^2 - 3y^2 = 4$ ;  $2x^2 + 5y^2 = 7$ 

$$7x^2 - 3y^2 = 4$$
 ----- (i)

$$2x^2 + 5y^2 = 7$$
 ----- (ii)

multiplying equation (i) by 2

$$14x^2 - 6y^2 = 8$$
 ----- (iii)

multiplying equation (ii) by 7

$$14x^2 + 35y^2 = 49$$
 ----- (iv)

Subtracting equation (iii) from (iv)

$$14x^2 + 35y^2 = 49$$
$$-14x^2 + 6y^2 = -8$$

$$41y^2 = 41$$

$$y^2 = 1$$

$$y = \pm 1$$

$$y = 1$$
 ;  $y = -1$ 

Putting the values of x in equation (i)

$$7x^{2} - 3y^{2} = 4$$
 ;  $7x^{2} - 3y^{2} = 4$   
 $7x^{2} - 3(1)^{2} = 4$  ;  $7x^{2} - 3(-1)^{2} = 4$   
 $7x^{2} - 3 = 4$  ;  $7x^{2} - 3 = 4$   
 $7x^{2} = 4 + 3$  ;  $7x^{2} = 4 + 3$   
 $7x^{2} = 7$  ;  $7x^{2} = 7$   
 $x^{2} = 1$  ;  $x^{2} = 1$ 

$$x = \pm 1 \qquad ; \qquad x = \pm 1$$

So,

$$S.S = \{(\pm 1, \pm 1)\}$$

10. 
$$x^2 + 2y^2 = 3$$
 ;  $x^2 + 4xy - 5y^2 = 0$ 

$$x^2 + 2y^2 = 3$$
 ----- (i)

$$x^2 + 4xy - 5y^2 = 0$$
 ----- (ii)

from equation (ii)

$$x^2 + 4xy - 5y^2 = 0$$

$$x^2 + 4yx - 5y^2 = 0$$

so, a=1, b = 4y, c = 
$$-5y^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4y \pm \sqrt{(4y)^2 - 4(1)(-5y^2)}}{2(1)}$$

$$= \frac{-4y \pm \sqrt{16y^2 + 20y^2}}{2}$$

$$= \frac{-4y \pm \sqrt{36y^2}}{2}$$

$$= \frac{-4y \pm 6y}{2}$$

$$x = \frac{-4y + 6y}{2}$$

$$x = \frac{-4}{}$$

$$x = \frac{2y}{2}$$

$$x = \frac{-10}{2}$$

$$x = y$$
---(iii)

$$x = -5y$$
 ----- (iv)

putting the values of y in equation (i)

$$y^2 + 2y^2 = 3$$

$$25y^2 + 2y^2 = 3$$

$$3y^2 = 3$$

$$27y^2 = 3$$

$$y^2 = 1$$

$$y^2 = \frac{1}{2}$$

$$y = \pm 1$$

$$y = \pm \frac{1}{9}$$

Putting the values of y in equation (iii) and (iv)

$$x = y$$

$$x = -5y$$

$$x = \pm 1$$

$$x = -5y$$

$$x = -5 \times \pm \frac{1}{9}$$

$$x = \pm 1$$

$$x = \mp \frac{5}{9}$$

So,

S.S = 
$$\left\{ (1, 1), (-1, -1), \left( \frac{5}{9}, -\frac{1}{9} \right), \left( -\frac{5}{9}, \frac{1}{9} \right) \right\}$$

11. 
$$3x^2 - y^2 = 26$$

$$3x^2 - 5xy - 12y^2 = 0$$

$$3x^2 - y^2 = 26$$
 ----- (i)

$$3x^2 - 5xy - 12y^2 = 0$$
 ----- (ii)

from equation (ii)

$$3x^2 - 5xy - 12y^2 = 0$$

$$3x^2 - 5yx - 12y^2 = 0$$

so, a=3, b = -5y, c = 
$$-12y^2$$

$$x = \frac{5y+13y}{6}$$
 ;  $x = \frac{5y-13y}{6}$   
 $x = \frac{18y}{6}$  ;  $x = \frac{-8y}{6}$   
 $x = 3y---(iii)$  ;  $x = -\frac{4y}{6}$ -------(iv)

putting the values of x in equation (i)

$$3x^2 - y^2 = 26$$
 ;

$$3x^2 - y^2 = 26$$

$$3(3y)^2 - y^2 = 26$$

$$3\left(-\frac{4y}{3}\right)^2 - y^2 = 26$$

$$27y^2 - y^2 = 26$$

$$3\left(\frac{16y^2}{9}\right) - y^2 = 26$$

$$26y^2 = 26$$

$$\frac{16y^2}{3} - y^2 = 26$$

$$26y^2 = 26$$

$$\frac{16y^2 - 3y^2}{3} = 26$$

$$y^2 = 1$$

$$\frac{13y^2}{3} = 26$$

$$y^2 = 1$$

$$y^2 = 26 \times \frac{3}{13}$$

$$y^2 = 1$$

$$y^2 = 6$$

$$y = \pm 1$$

$$y = \pm \sqrt{6}$$

Putting the values of y in equation (iii) and (iv)

$$x = 3y$$

$$x = -\frac{4y}{3}$$

$$x = 3(\pm 1)$$

$$x = -\frac{4}{3} \times \pm \sqrt{6}$$

$$x = \pm 3$$

$$x = \mp \frac{3}{4\sqrt{6}}$$

So,

S.S = 
$$\left\{ (3, 1), (-3, -1), \left( -\frac{4\sqrt{6}}{3}, \sqrt{6} \right), \left( \frac{4\sqrt{6}}{3}, -\sqrt{6} \right) \right\}$$

12. 
$$x^2 + xy = 5$$

$$y^2 + xy = 3$$

$$x^2 + xy = 5$$
 ----- (i)

$$y^2 + xy = 3$$
 ----- (ii)

Adding equation (i) and (ii)

$$x^2 + 2xy + y^2 = 8$$

$$(x+y)^2 = 8$$

taking square root on both sides

$$x + y = \pm \sqrt{8}$$
  
 $y = \pm 2\sqrt{2} - x$  -----(iii)

putting the values of y in equation (i)

$$x^2 + xy = 5$$

$$x^2 + x(\pm 2\sqrt{2} - x) = 5$$

$$x^2 \pm 2\sqrt{2}x - x^2 = 5$$

$$\pm 2\sqrt{2}x = 5$$

$$x = \pm \frac{5}{2\sqrt{2}}$$

Putting the values of x in equation (iii)

$$y = \pm 2\sqrt{2} - x$$

$$y = \pm 2\sqrt{2} - \left(\pm \frac{5}{2\sqrt{2}}\right)$$

$$y = \pm 2\sqrt{2} \mp \frac{5}{2\sqrt{2}}$$

$$y = +2\sqrt{2} - \frac{5}{2\sqrt{2}}$$

$$y = -2\sqrt{2}$$

$$y = \frac{\left(2\sqrt{2}\right)^2 - 5}{2\sqrt{2}}$$

$$y = \frac{-(2\sqrt{2})^2 + 5}{2\sqrt{2}}$$

$$y = \frac{8-5}{2\sqrt{2}}$$

$$y = \frac{1}{x}$$

$$y = \frac{3}{2\sqrt{3}}$$

$$y = \frac{-3}{2\sqrt{2}}$$

So,

S.S = 
$$\left\{ \left( \frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}} \right), \left( -\frac{5}{2\sqrt{2}}, -\frac{3}{2\sqrt{2}} \right) \right\}$$
  
13.  $x^2 - 2xy = 7$  ;  $xy + 3y^2 = 2$ 

$$x^2 - 2xy = 7$$
 ----- (i)

$$xy + 3y^2 = 2$$
 ----- (ii)

multiplying equ (i) by 2 and (ii) by 7 we get

$$2x^2 - 4xy = 14$$
 ----- (iii)

$$7xy + 21y^2 = 14$$
 ----- (iv)

subtracting (iv) from (iii)

$$2x^2 - 11xy - 21y^2 = 0$$

$$2x^2 - 14xy + 3xy - 21y^2 = 0$$

$$2x(x-7y) + 3y(x-7y) = 0$$

$$(x-7y)(2x+3y)=0$$

$$x - 7y = 0 ; 2x + 3y = 0$$

$$x = 7y ; 2x = -3y$$

$$x = 7y$$
---(v) ;  $x = -\frac{3y}{2}$ ------(vi)

put in equ (ii)

$$xy + 3y^2 = 2$$
 ;  $xy + 3y^2 = 2$ 

$$(7y)y + 3y^2 = 2$$
 ;  $\left(-\frac{3y}{2}\right)y + 3y^2 = 2$ 

$$7y^2 + 3y^2 = 2$$
 ;  $-\frac{3y^2}{2} + 3y^2 = 2$ 

$$10y^2 = 2 ; \frac{-3y^2 + 6y^2}{2} = 2$$

$$y^2 = \frac{2}{10}$$
 ;  $3y^2 = 4$ 

$$y = \pm \frac{1}{\sqrt{5}}$$
 ;  $y^2 = \frac{4}{3}$ 

$$y = \pm \frac{1}{\sqrt{5}}$$
 ;  $y = \pm \frac{2}{\sqrt{3}}$ 

put the values in equation (v) and (vi) respectively.

$$x = 7y ; x = -\frac{3y}{2}$$

$$x = 7y \qquad ; \qquad x = -\frac{1}{2}$$

$$x = 7\left(\pm\frac{1}{\sqrt{5}}\right) \qquad ; \qquad x = -\frac{3}{2} \times \pm\frac{2}{\sqrt{3}}$$

$$x = \pm \frac{7}{\sqrt{5}}$$
 ;  $x = \mp \frac{3}{\sqrt{3}}$ 

$$x = \pm \frac{7}{\sqrt{5}} \qquad ; \qquad x = \mp \sqrt{3}$$

So,

S.S = 
$$\left\{ \left( \frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right), \left( -\frac{7}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right), \left( \sqrt{3}, -\frac{2}{\sqrt{3}} \right), \left( -\sqrt{3}, \frac{2}{\sqrt{3}} \right) \right\}$$