Exercise 7.2

1. Identify the following statements as True or False.

(i) $|x| = 0$ has only one solution. T

(ii) All absolute value equations have two solutions. F

(iii) The equation $|x| = 2$ is equivalent to $x = 2$ or $x = -2$. T

(iv) The equation $|x - 4| = -4$ has no solution. T

(v) The equation $|2x - 3| = 5$ is equivalent to $2x - 3 = 5$ or $2x + 3 = 5$. F

Q. 2: Solve for x

(i) $|3x - 5| = 4$

$+(3x - 5) = 4$ and $-(3x - 5) = 4$

$3x - 5 = 4$ and $3x - 5 = -4$

$3x = 9$ and $3x = 1$

$x = 3$ and $x = \frac{1}{3}$

Check:

$|3(3) - 5| = 4$ and $\left|3\left(\frac{1}{3}\right) - 5\right| = 4$

$|9 - 5| = 4$ and $|1 - 5| = 4$

$|4| = 4$ and $|-4| = 4$

$4 = 4$ and $4 = 4$

So, S.S $= \left\{3, \frac{1}{3}\right\}$

(ii) $\frac{1}{2}|3x + 2| - 4 = 11$

$\frac{1}{2}|3x + 2| = 15$

$|3x + 2| = 30$

$+(3x + 2) = 30$ and $-(3x + 2) = 30$

$3x + 2 = 30$ and $3x + 2 = -30$

$3x = 28$ and $3x = -32$

$x = \frac{28}{3}$ and $x = -\frac{32}{3}$

Check:

$\frac{1}{2}\left|3\left(\frac{28}{3}\right) + 2\right| - 4 = 11$ and $\frac{1}{2}\left|3\left(-\frac{32}{3}\right) + 2\right| - 4 = 11$

$\frac{1}{2}|28 + 2| - 4 = 11$ and $\frac{1}{2}|-32 + 2| - 4 = 11$

$\frac{1}{2}|30| - 4 = 11$ and $\frac{1}{2}|-30| - 4 = 11$

$\frac{1}{2}(30) - 4 = 11$ and $\frac{1}{2}(30) - 4 = 11$

$15 - 4 = 11$ and $15 - 4 = 11$

$11 = 11$ and $11 = 11$

So, S.S $= \left\{\frac{28}{3}, -\frac{32}{3}\right\}$

(iii) $|2x + 5| = 11$

$+(2x + 5) = 11$ and $-(2x + 5) = 11$

$2x + 5 = 11$ and $2x + 5 = -11$

$2x = 6$ and $2x = -16$

$x = 3$ and $x = -8$

Check:

$|2(3) + 5| = 11$ and $|2(-8) + 5| = 11$

$|6 + 5| = 11$ and $|-16 + 5| = 11$

$|11| = 11$ and $|-11| = 11$

$11 = 11$ and $11 = 11$

So, S.S = $\{3, -8\}$

(iv) $|3 + 2x| = |6x - 7|$

$3 + 2x = +(6x - 7)$ and $3 + 2x = -(6x - 7)$

$3 + 2x = 6x - 7$ and $3 + 2x = -6x + 7$

$2x - 6x = -7 - 3$ and $2x + 6x = 7 - 3$

$-4x = -10$ and $8x = 4$

$x = \frac{5}{2}$ and $x = \frac{1}{2}$

Check:

$\left|3 + 2\left(\frac{5}{2}\right)\right| = \left|6\left(\frac{5}{2}\right) - 7\right|$ and $\left|3 + 2\left(\frac{1}{2}\right)\right| = \left|6\left(\frac{1}{2}\right) - 7\right|$

$|3 + (5)| = |3(5) - 7|$ and $|3 + 1| = |3 - 7|$

$|3 + 5| = |15 - 7|$ and $|4| = |-4|$

$8 = 8$ and $4 = 4$

So, S.S = $\left\{\frac{5}{2}, \frac{1}{2}\right\}$

(v) $|x + 2| - 3 = 5 - |x + 2|$

$|x + 2| + |x + 2| = 5 + 3$

$2|x + 2| = 8$

$|x + 2| = 4$

$+(x + 2) = 4$ and $-(x + 2) = 4$

$x + 2 = 4$ and $x + 2 = -4$

$x = 4 - 2$ and $x = -4 - 2$

$x = 2$ and $x = -6$

Check:

$|(2) + 2| - 3 = 5 - |(2) + 2|$ and $|(-6) + 2| - 3 = 5 - |(-6) + 2|$

$|2 + 2| - 3 = 5 - |2 + 2|$ and $|-6 + 2| - 3 = 5 - |-6 + 2|$

$|4| - 3 = 5 - |4|$ and $|-4| - 3 = 5 - |-4|$

$4 - 3 = 5 - 4$ and $4 - 3 = 5 - 4$

So, S.S = $\{2, -6\}$

(vi) $\frac{1}{2}|x + 3| + 21 = 9$

$\frac{1}{2}|x + 3| = 9 - 21$

$\frac{1}{2}|x + 3| = -12$

$|x + 3| = -24$

$+(x + 3) = -24$ and $-(x + 3) = -24$

$x+3 = -24$ and $-x-3 = -24$

$x = -24-3$ and $-x = -24+3$

$x = -27$ and $x = 21$

Check:

$\frac{1}{2}|(-27)+3|+21 = 9$ and $\frac{1}{2}|(21)+3|+21 = 9$

$\frac{1}{2}|-27+3|+21 = 9$ and $\frac{1}{2}|21+3|+21 = 9$

$\frac{1}{2}|-24|+21 = 9$ and $\frac{1}{2}|24|+21 = 9$

$12+21 \neq 9$ and $12+21 \neq 9$

So, S.S = { }

(vii) $\left|\frac{3-5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$\left|\frac{3-5x}{4}\right| = \frac{2}{3}+\frac{1}{3}$

$\left|\frac{3-5x}{4}\right| = 1$

$+\left(\frac{3-5x}{4}\right) = 1$ and $-\left(\frac{3-5x}{4}\right) = 1$

$\frac{3-5x}{4} = 1$ and $\frac{3-5x}{4} = -1$

$3-5x = 4$ and $3-5x = -4$

$-5x = 4-3$ and $-5x = -4-3$

$-5x = 1$ and $-5x = -7$

$x = -\frac{1}{5}$ and $x = \frac{7}{5}$

Check:

$\left|\frac{3-5\left(-\frac{1}{5}\right)}{4}\right| - \frac{1}{3} = \frac{2}{3}$ and $\left|\frac{3-5\left(\frac{7}{5}\right)}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$\left|\frac{3+1}{4}\right| - \frac{1}{3} = \frac{2}{3}$ and $\left|\frac{3-7}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$\left|\frac{4}{4}\right| - \frac{1}{3} = \frac{2}{3}$ and $\left|\frac{-4}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$1-\frac{1}{3} = \frac{2}{3}$ and $1-\frac{1}{3} = \frac{2}{3}$

$\frac{3-1}{3} = \frac{2}{3}$ and $\frac{3-1}{3} = \frac{2}{3}$

$\frac{2}{3} = \frac{2}{3}$ and $\frac{2}{3} = \frac{2}{3}$

So, S.S = $\left\{-\frac{1}{5}, \frac{7}{5}\right\}$

(viii) $\left|\frac{x+5}{2-x}\right| = 6$

$+\left(\frac{x+5}{2-x}\right) = 6$ and $-\left(\frac{x+5}{2-x}\right) = 6$

$\frac{x+5}{2-x} = 6$ and $\frac{x+5}{2-x} = -6$

$x+5 = 6(2-x)$ and $x+5 = -6(2-x)$

$x+5 = 12-6x$ and $x+5 = -12+6x$

$x+6x = 12-5$ and $x-6x = -12-5$

$7x = 7$ and $-5x = -17$

$x = 1$ and $x = \frac{17}{5}$

Check:

$\left|\frac{1+5}{2-1}\right| = 6$ and $\left|\frac{\left(\frac{17}{5}\right)+5}{2-\left(\frac{17}{5}\right)}\right| = 6$

$\left|\frac{1+5}{2-1}\right| = 6$ and $\left|\frac{\frac{17}{5}+5}{2-\frac{17}{5}}\right| = 6$

$\left|\frac{6}{1}\right| = 6$ and $\left|\frac{\frac{17+25}{5}}{\frac{10-17}{5}}\right| = 6$

$|6| = 6$ and $\left|\frac{\frac{42}{5}}{\frac{-7}{5}}\right| = 6$

$|6| = 6$ and $\left|\frac{42}{-7}\right| = 6$

$6 = 6$ and $|-6| = 6$

So, S.S = $\left\{1, \frac{17}{5}\right\}$