Exercise 5.3

For more educational resources visit
www.taleemcity.com

Q. 1: If $U = \{1, 2, 3, 4, \ldots\ldots, 10\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{1, 4, 7, 10\}$, **Then verify the following questions.**

(i) $A - B = A \cap B'$

$L.H.S = A - B$

$= \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$

$= \{1, 3, 5, 9\}$

$R.H.S = A \cap B'$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 4, 7, 10\})$

$= \{1, 3, 5, 7, 9\} \cap \{1, 2, 3, 5, 6, 8, 9\}$

$= \{1, 3, 5, 9\}$

So, $L.H.S = R.H.S$

(ii) $B - A = B \cap A'$

$L.H.S = B - A$

$= \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$

$= \{4, 10\}$

$R.H.S = B \cap A'$

$= \{1, 4, 7, 10\} \cap (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 3, 5, 7, 9\})$

$= \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\}$

$= \{4, 10\}$

So, $L.H.S = R.H.S$

(iii) $(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)' \quad = U - (A \cup B)$

$= \{1, 2, 3, \ldots\ldots, 10\} - (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\})$

$= \{1, 2, 3, \ldots\ldots, 10\} - \{1, 3, 4, 5, 7, 9, 10\}$

$= \{2, 6, 8\}$

$R.H.S = A' \cap B' \quad = (U - A) \cap (U - B)$

$= (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 4, 7, 10\})$

$= \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 6, 8, 9\}$

$= \{2, 6, 8\}$

So, $L.H.S = R.H.S$

(iv) $(A \cap B)' = A' \cup B'$

$L.H.S = (A \cap B)' \quad = U - (A \cap B)$

$= \{1, 2, 3, \ldots\ldots, 10\} - (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\})$

$= \{1, 2, 3, \ldots\ldots, 10\} - \{1, 7\}$

$= \{2, 3, 4, 5, 6, 8, 9, 10\}$

$R.H.S = A' \cup B' \quad = (U - A) \cup (U - B)$

$= (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 3, 5, 7, 9\}) \cup (\{1, 2, 3, \ldots\ldots, 10\} - \{1, 4, 7, 10\})$

$= \{2,4,6,8,10\} \cup \{2,3,5,6,8,9\}$

$= \{2,3,4,5,6,8,9,10\}$

So, $L.H.S = R.H.S$

(v) $(A-B)' = A' \cup B$

$L.H.S = (A-B)' = U-(A-B)$

$= \{1,2,3,\ldots\ldots,10\} - (\{1,3,5,7,9\} - \{1,4,7,10\})$

$= \{1,2,3,\ldots\ldots,10\} - \{3,5,9\}$

$= \{1,2,4,6,7,8,10\}$

$R.H.S = A' \cup B = (U-A) \cup B$

$= (\{1,2,3,\ldots\ldots,10\} - \{1,3,5,7,9\}) \cup \{1,4,7,10\}$

$= \{2,4,6,8,10\} \cup \{1,4,7,10\}$

$= \{1,2,4,6,7,8,10\}$

So, $L.H.S = R.H.S$

(vi) $(B-A)' = B' \cup A$

$L.H.S = (B-A)' = U-(B-A)$

$= \{1,2,3,\ldots\ldots,10\} - (\{1,4,7,10\} - \{1,3,5,7,9\})$

$= \{1,2,3,\ldots\ldots,10\} - \{4,10\}$

$= \{1,2,3,5,6,7,8,9\}$

$R.H.S = B' \cup A = (U-B) \cup A$

$= (\{1,2,3,\ldots\ldots,10\} - \{1,4,7,10\}) \cup \{1,3,5,7,9\}$

$= \{2,3,5,6,8,9\} \cup \{1,3,5,7,9\}$

$= \{1,2,3,5,6,7,8,9\}$

So, $L.H.S = R.H.S$

Q. 2: **If $U = \{1,2,3,4,\ldots\ldots,10\}$, $A = \{1,3,5,7,9\}$, $B = \{1,4,7,10\}$, $C = \{1,5,8,10\}$**
Then verify the following questions.

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

$L.H.S = (A \cup B) \cup C$

$= (\{1,3,5,7,9\} \cup \{1,4,7,10\}) \cup \{1,5,8,10\}$

$= \{1,3,4,5,7,9,10\} \cup \{1,5,8,10\}$

$= \{1,3,4,5,7,8,9,10\}$

$R.H.S = A \cup (B \cup C)$

$= \{1,3,5,7,9\} \cup (\{1,4,7,10\} \cup \{1,5,8,10\})$

$= \{1,3,5,7,9\} \cup \{1,4,5,7,8,10\}$

$= \{1,3,4,5,7,8,9,10\}$

So, $L.H.S = R.H.S$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

$L.H.S = (A \cap B) \cap C$

$= (\{1,3,5,7,9\} \cap \{1,4,7,10\}) \cap \{1,5,8,10\}$

$= \{1,7\} \cap \{1,5,8,10\}$

$= \{1\}$

$R.H.S = A \cap (B \cap C)$

$= \{1,3,5,7,9\} \cap (\{1,4,7,10\} \cap \{1,5,8,10\})$

$= \{1,3,5,7,9\} \cap \{1,10\}$

$= \{1\}$

So, $L.H.S = R.H.S$

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$L.H.S = A \cup (B \cap C)$

$= \{1,3,5,7,9\} \cup (\{1,4,7,10\} \cap \{1,5,8,10\})$

$= \{1,3,5,7,9\} \cup \{1,10\}$

$= \{1,3,5,7,9,10\}$

$R.H.S = (A \cup B) \cap (A \cup C)$

$= (\{1,3,5,7,9\} \cup \{1,4,7,10\}) \cap (\{1,3,5,7,9\} \cup \{1,5,8,10\})$

$= \{1,3,4,5,7,9,10\} \cap \{1,3,5,7,8,9,10\}$

$= \{1,3,5,7,9,10\}$

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$L.H.S = A \cap (B \cup C)$

$= \{1,3,5,7,9\} \cap (\{1,4,7,10\} \cup \{1,5,8,10\})$

$= \{1,3,5,7,9\} \cap \{1,4,5,7,8,10\}$

$= \{1,5,7\}$

$R.H.S = (A \cap B) \cup (A \cap C)$

$= (\{1,3,5,7,9\} \cap \{1,4,7,10\}) \cup (\{1,3,5,7,9\} \cap \{1,5,8,10\})$

$= \{1,7\} \cup \{1,5\}$

$= \{1,5,7\}$

Q. 3: If $U = N$, then verify De-Morgans's laws by using $A = \emptyset$, $B = P$.

$\boldsymbol{U} = \{1,2,3,\ldots\ldots\}$

$\boldsymbol{A} = \{\ \}$

$\boldsymbol{B} = \{2,3,5,7,\ldots\ldots\}$

$(A \cap B)' = A' \cup B'$

$L.H.S = (A \cap B)' \quad = U - (A \cap B)$

$= \{1,2,3,\ldots\ldots\} - (\{\ \} \cap \{2,3,5,7,\ldots\ldots\})$

$= \{1,2,3,\ldots\ldots\} - \{\ \}$

$= \{1,2,3,\ldots\ldots\}$

$R.H.S = A' \cup B' \quad = (U - A) \cup (U - B)$

$= (\{1,2,3,\ldots\ldots\} - \{\ \}) \cup (\{1,2,3,\ldots\ldots\} - \{2,3,5,7,\ldots\ldots\})$

$= \{1,2,3,\ldots\ldots\} \cup \{1,4,6,8,9,10,12,\ldots\ldots\}$

$= \{1,2,3,\ldots\ldots\}$

So, $L.H.S = R.H.S$

Now

$(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)' \quad = U - (A \cup B)$

$= \{1,2,3,\ldots\ldots\} - (\{\ \} \cup \{2,3,5,7,\ldots\ldots\})$

$= \{1,2,3,\ldots\ldots\} - \{2,3,5,7,\ldots\ldots\}$

$= \{1, 4, 6, 8, 9, 10, 12, \ldots\ldots\}$

$R.H.S = A' \cap B' \quad = (U - A) \cap (U - B)$

$= (\{1, 2, 3, \ldots\ldots\ldots\} - \{\ \}) \cap (\{1, 2, 3, \ldots\ldots\ldots\} - \{2, 3, 5, 7, \ldots\ldots\ldots\})$

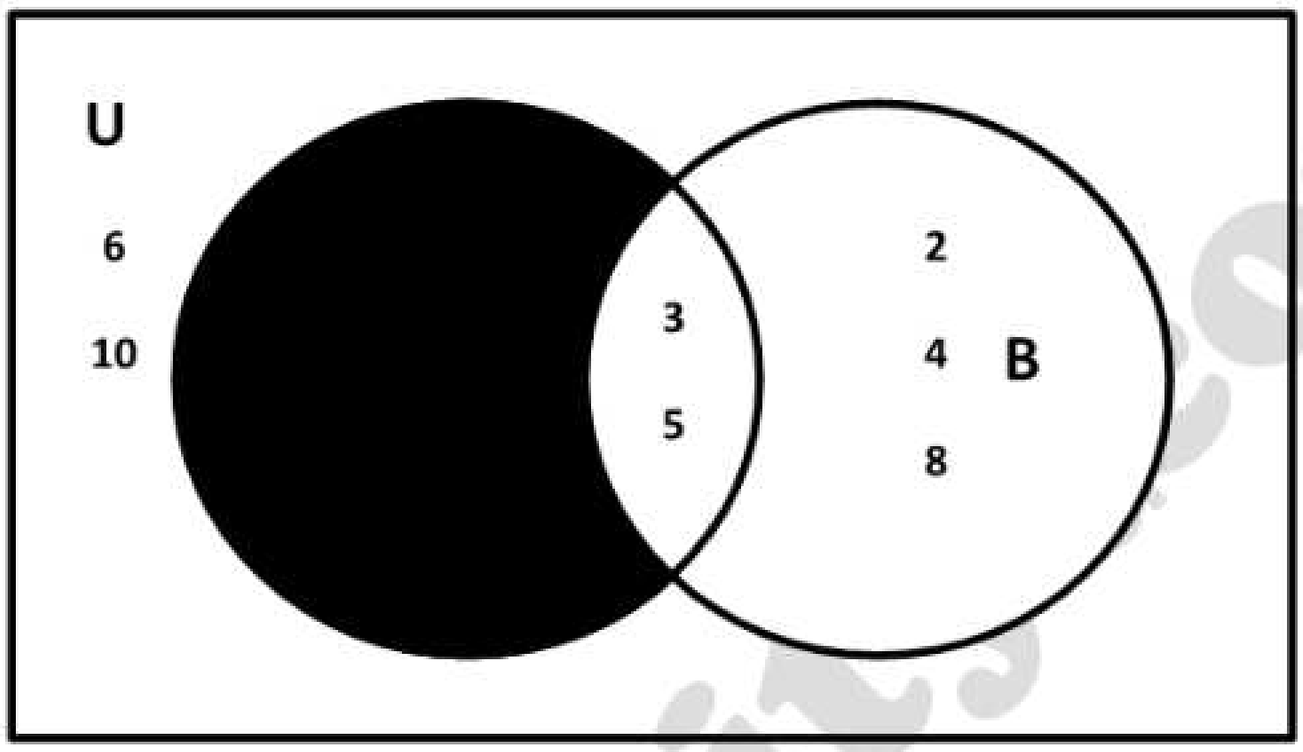
$= \{1, 2, 3, \ldots\ldots\ldots\} \cap \{1, 4, 6, 8, 9, 10, 12, \ldots\ldots\}$

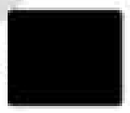
$= \{1, 4, 6, 8, 9, 10, 12, \ldots\ldots\}$

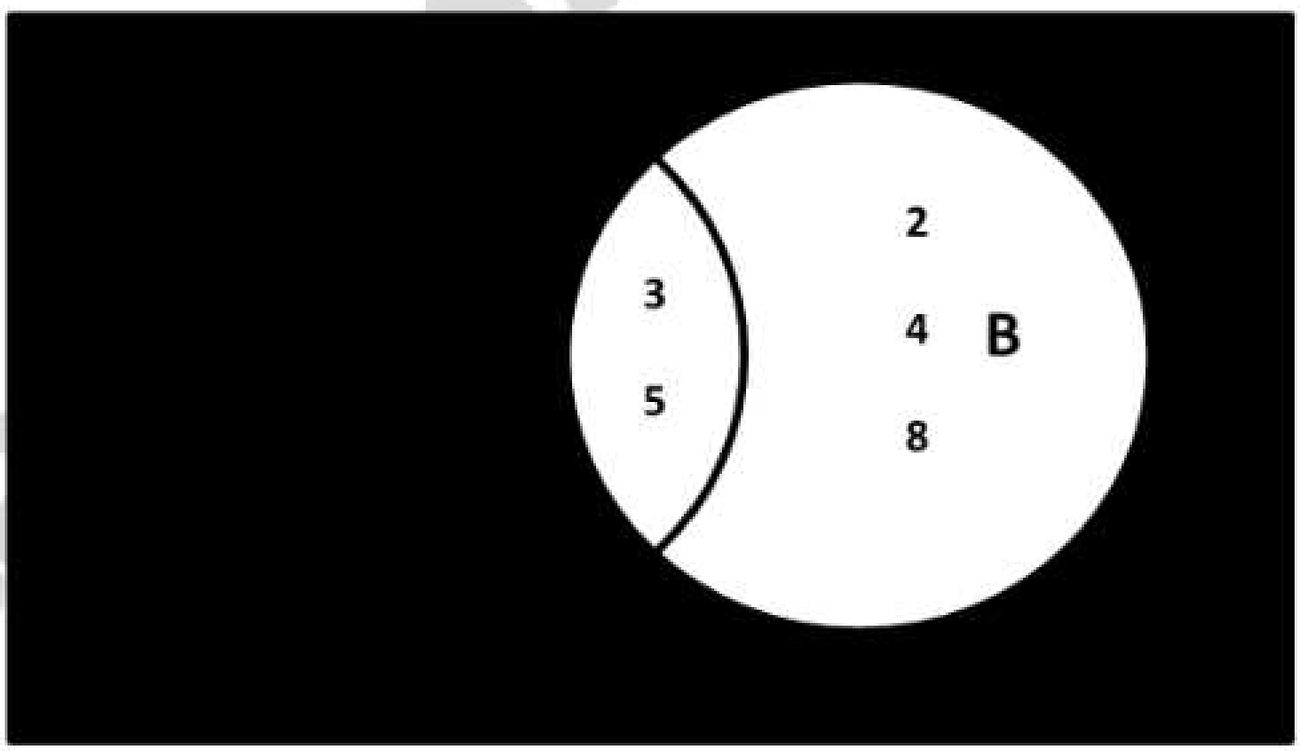
So, $L.H.S = R.H.S$

Q. 4: If $U = \{1, 2, 3, 4, \ldots\ldots, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 4, 5, 8\}$, then prove the following questions by Venn diagram:

(i) $A - B = A \cap B'$



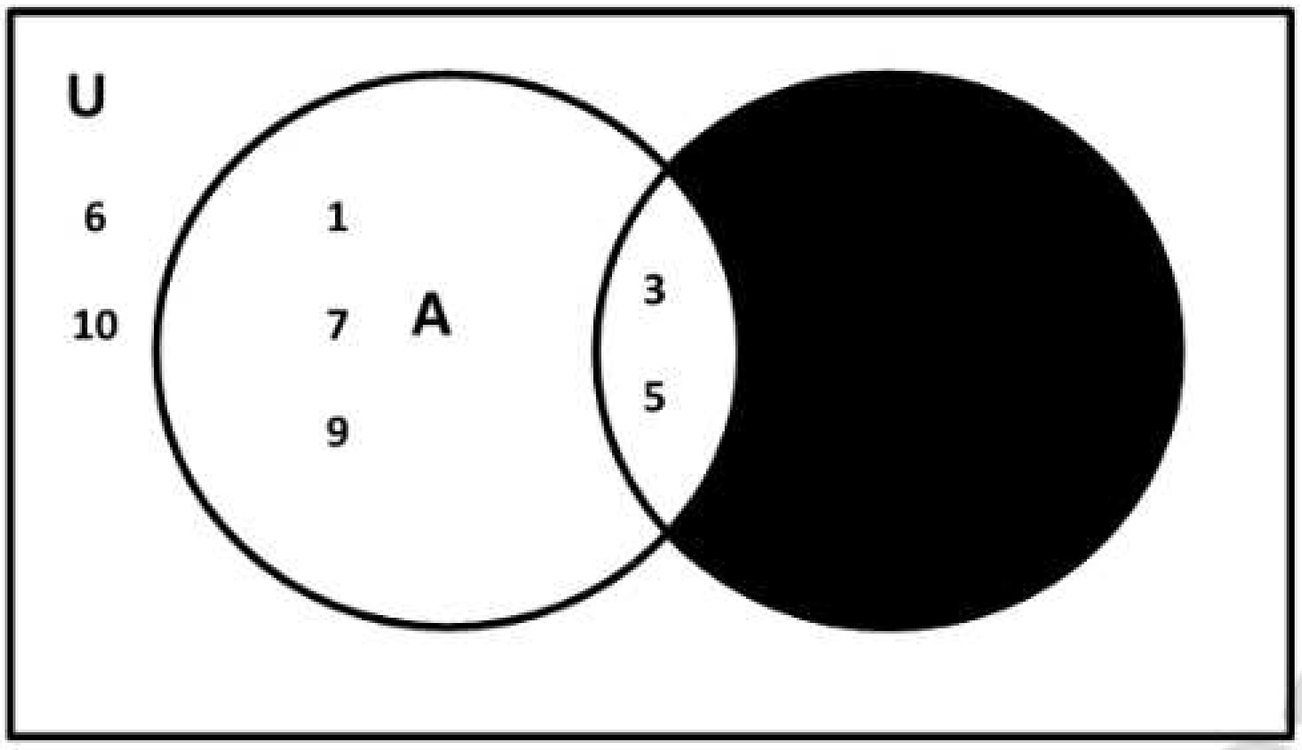
$A - B =$ 



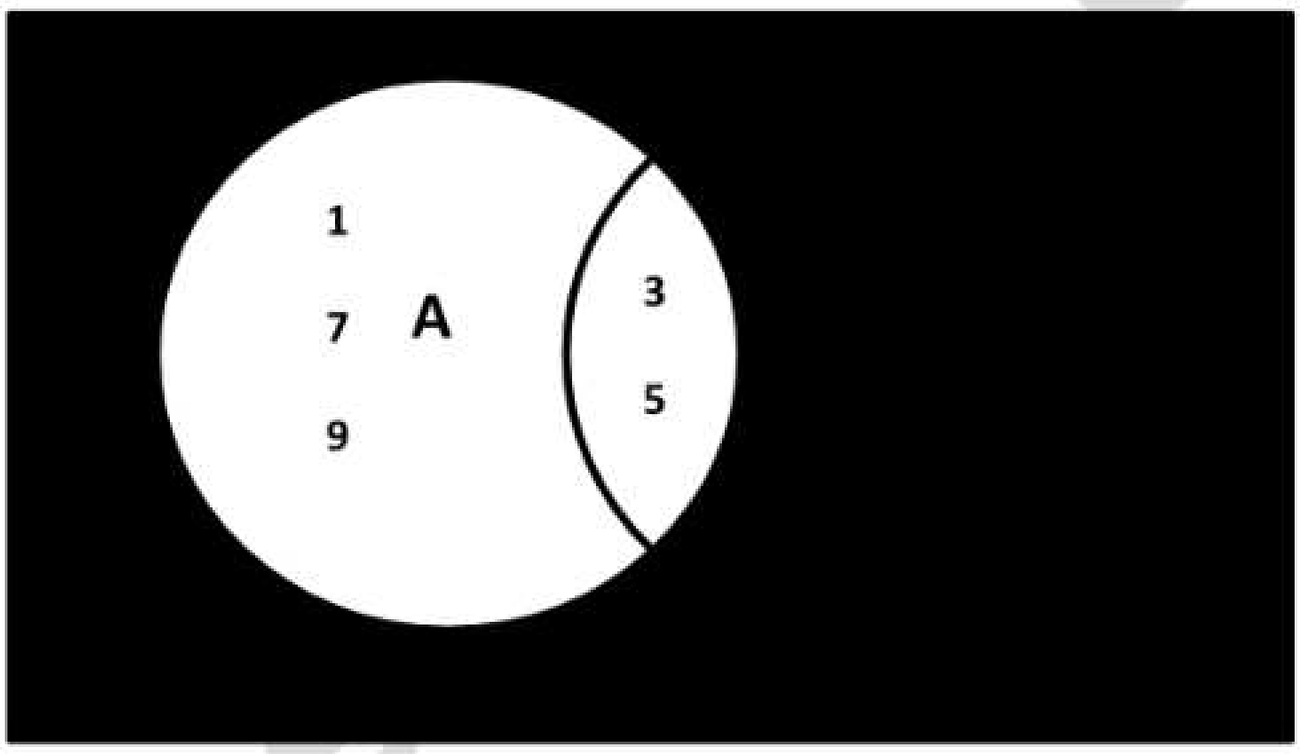
$U - B =$ 

$A \cap B' =$ 

(ii) $B - A = B \cap A'$



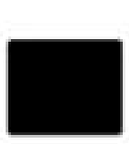
$B - A =$ ■

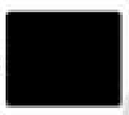


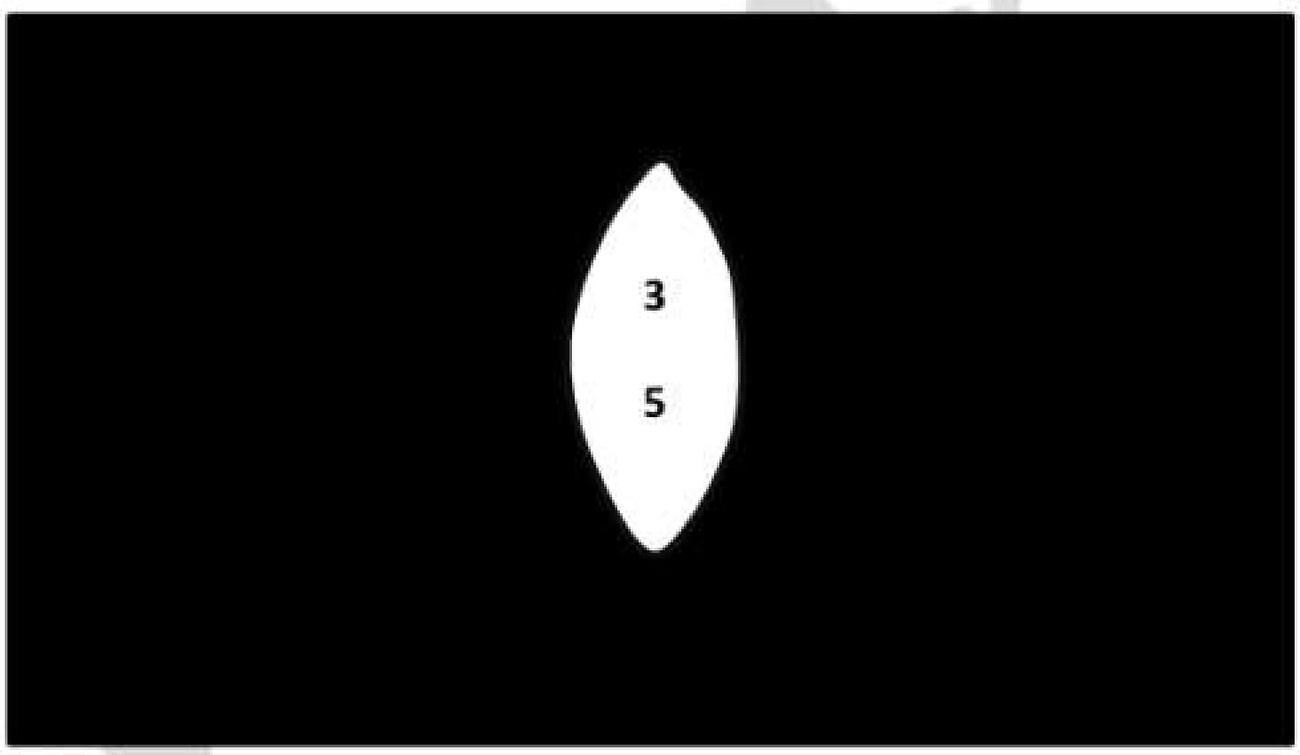
$U - A =$ ■

$B \cap A' =$ ■

(iii) $(A \cup B)' = A' \cap B'$

$A \cup B =$ 

$(A \cup B)' =$ 



$A' =$ 

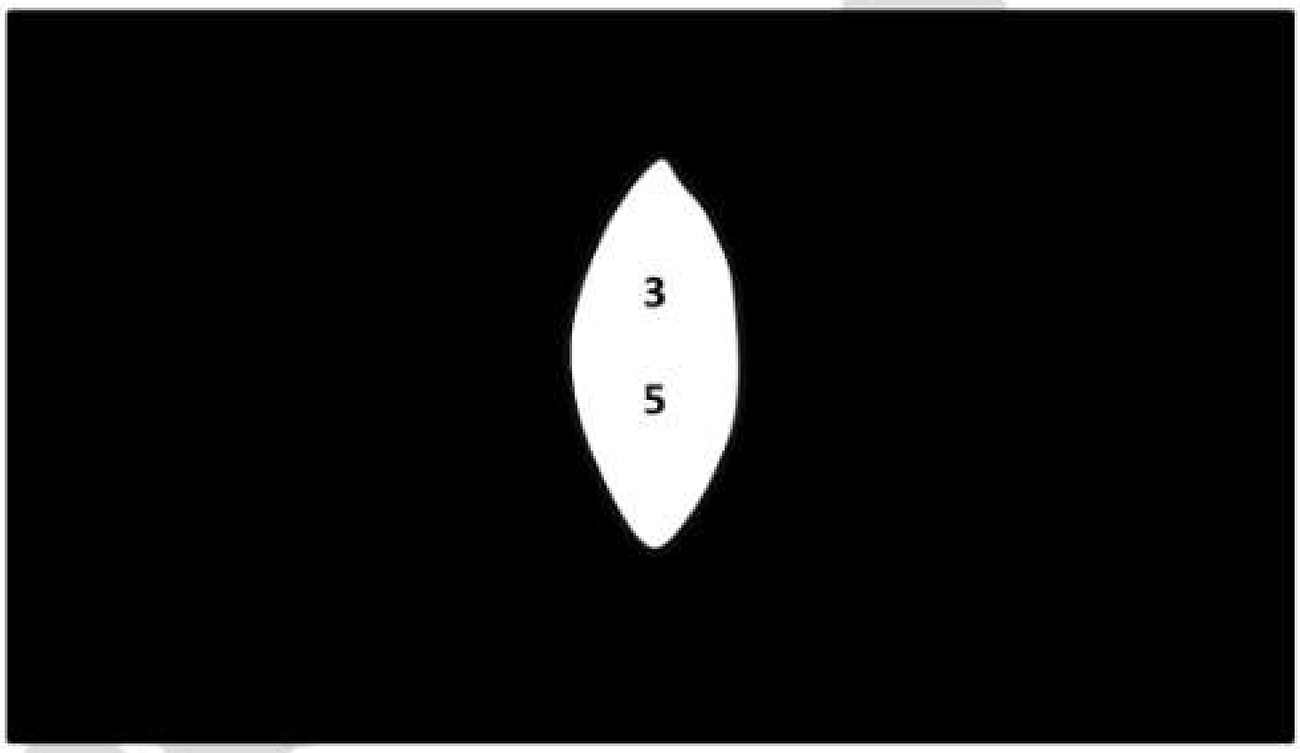
$B' =$ 

$A' \cap B' =$ 

(iv) $(A \cap B)' = A' \cup B'$

$A \cap B =$

$(A \cap B)' =$

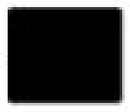


$A' =$

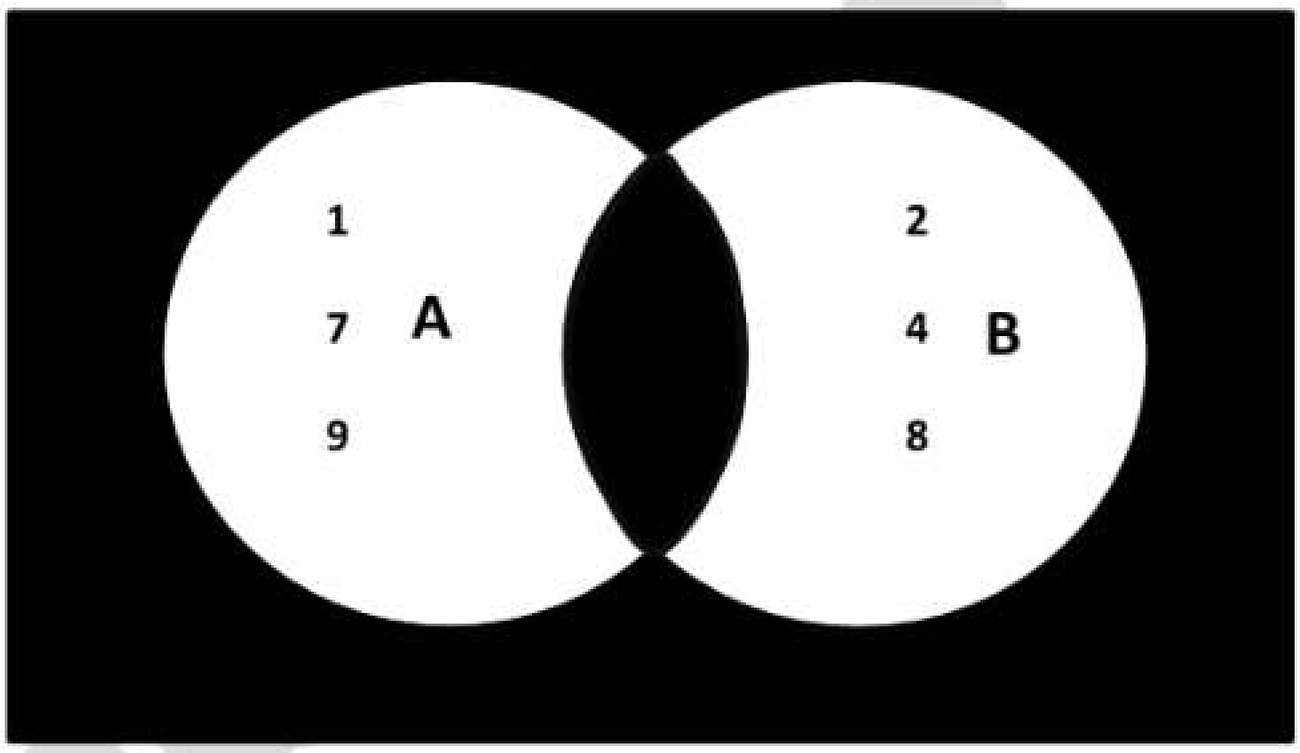
$B' =$

$A' \cup B' =$

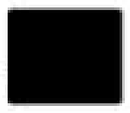
(v) $(A - B)' = A' \cup B$

$A - B =$ 

$(A - B)' =$ 



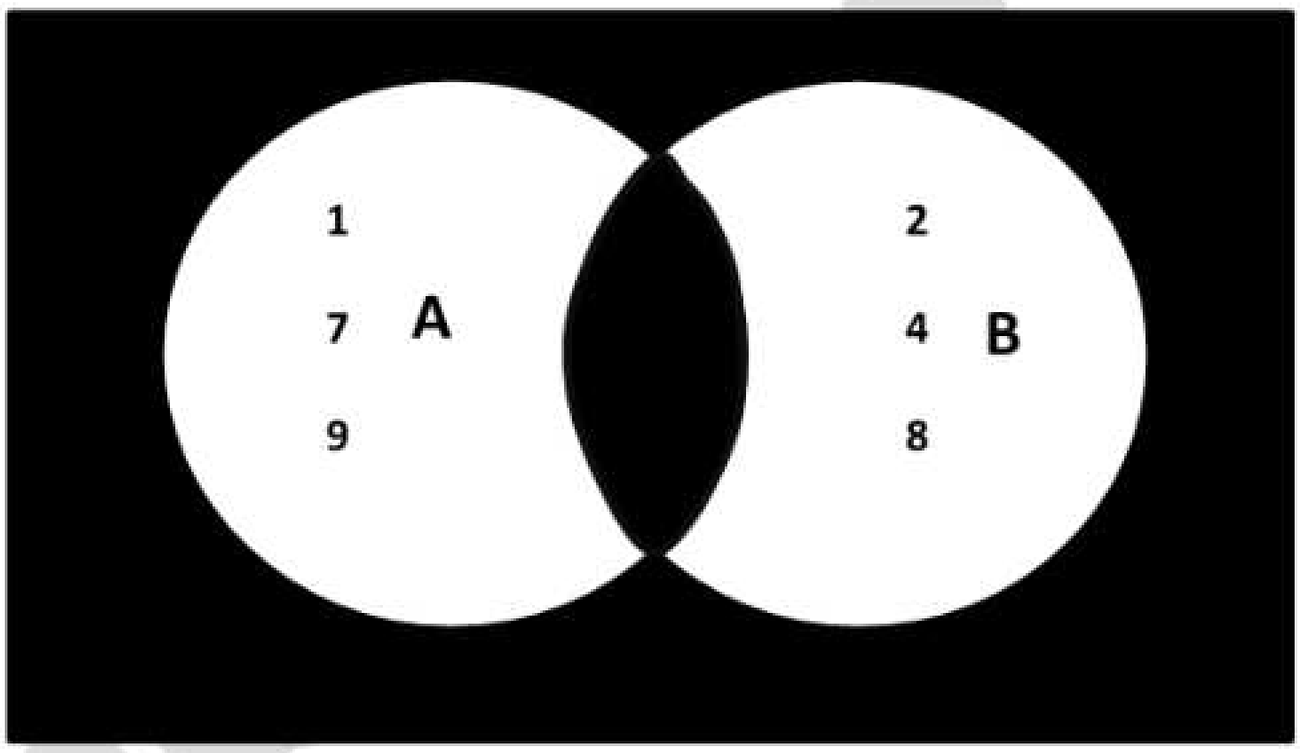
$A' =$ 

$A' \cup B =$ 

(vi) $(B - A)' = B' \cup A$

$B - A =$

$(B - A)' =$



$B' =$

$B' \cup A =$