Exercise 15

Q.1 Verify that the Δs having the following measures of sides are right-angled to verify weather the Δs are right angled or not we use Pythagoras Theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Perpendicular})^2$

(i) a = 5cm
b = 12cm
c = 13cm
$a^2 = 25cm^2$
$b^2 = 144cm^2$
$c = 169cm^2$
Larger Size is Hypotenuse So
169 = 25 + 144
169 = 169
L.H.S = R.H.S
So it is right angled triangle

(ii) a = 1.5cm
b = 2cm
c = 2.5cm
$a^2 = 2.25cm^2$
$b^2 = 4cm^2$
$c^2 = 6.25$
6.25 = 2.25 + 4
6.25 = 6.25
L.H.S = R.H.S
So it is right-angled triangle

(iii) a = 9cm
b = 12cm
c = 15cm
$a^2 = 81cm^2$
$b^2 = 144cm^2$
$c = 225cm^2$
$225cm^2 = 8.1cm + 144cm$
$225cm^2 = 225cm^2$
L.H.S = R.H.S
So it is right angled triangle

(iv) a = 16cm
b = 30cm
c = 34cm
$a^2 = 256cm^2$
$b^2 = 900cm$
$c^2 = 1156cm^2$
1156 = 256 + 900
1156 = 1156
L.H.S = R.H.S
It is right angled triangle

Q.2 Verify that a^2+b^2, a^2-b^2 and 2ab are the measures of the sides of a right angled Triangle where a and b are any two real numbers (a >b)

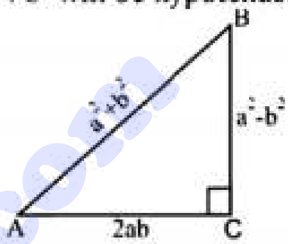
Let $a = z$ and $b = 1$

$a^2+b^2 = (2)^2+(1)^2 = 4+1 = 5$

$a^2-b^2 = (2)^2-(1)^2 = 4-1 = 3$

$2ab = 2(2)(1) = 4$

Since a^2+b^2 is the largest side so a^2+b^2 will be hypotenuse



So

$(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2$

$(a^2+b^2)^2 = (2ab)^2 + (a^2-b^2)^2$

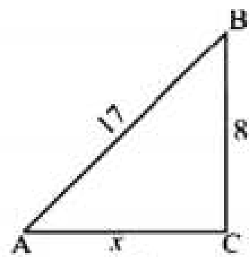
$a^4+b^4+2a^2b^2 = 4a^2b^2+a^4+b^4-2a^2b^2$

$a^4+b^4+2a^2b^2 = a^4+b^4+2a^2b^2$

L.H.S = R.H.S

It is proved that it is a right angled triangle

Q.3 The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of right angled triangle by Pythagoras theorem



$(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2$

$(17)^2 = (x)^2 + (8)^2$

$289 = x^2 + 64$

$289 - 64 = x^2$

$x^2 = 225$

Taking square root both side

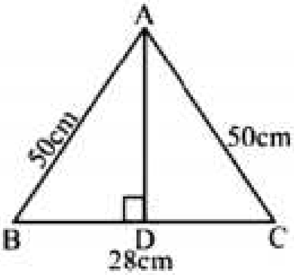
$\sqrt{x^2} = \sqrt{225}$

$x = 15$

Q.4 In an isosceles Δ the base $\overline{BC} = 28\text{cm}$ and

$\overline{AB} = \overline{AC} = 50\text{cm}$

If $\overline{AD} \perp \overline{BC}$ then find



(i) Length of $\overline{AD}$

Solution:

$\overline{AD} \perp \overline{BC}$

So $\overline{BD} = \overline{CD}$

$\frac{1}{2}\overline{BC} = \frac{1}{2}(28)$

$\frac{1}{2}\overline{BC} = 14$

So

$\overline{BD} = \overline{CD} = 14$

$(\overline{AB})^2 = (\overline{BD})^2 + (\overline{AD})^2$

$2500 = (14)^2 + (\overline{AD})^2$

$2500 = 196 + (\overline{AD})^2$

\ $2500 - 196 = (\overline{AD})^2$

$(\overline{AD})^2 = 2304$

Taking square root on both side

$\sqrt{(\overline{AD})^2} = \sqrt{2304}$

$\overline{AD} = 48\text{cm}$

(ii) Area of Δ ABC

Area of Δ ABC $= \frac{1}{2}$ (base) (height)

$= \frac{1}{2}(28)(48)$

$= (14)(48)$

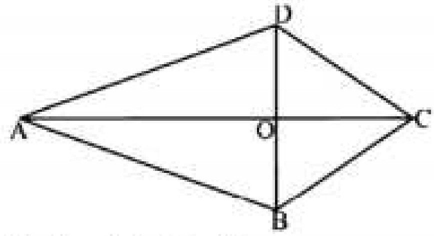
$= 672\ \text{cm}^2$

Q.5 In a quadrilateral ABCD the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other.

Prove that

$(\overline{AB})^2 + (\overline{CD})^2 = (\overline{AD})^2 + (\overline{BC})^2$

ΔAOB



$(\overline{AB})^2 = (\overline{OB})^2 + (\overline{OA})^2 \longrightarrow$ (i)

ΔBOC

$(\overline{BC})^2 = (\overline{OB})^2 + (\overline{OC})^2 \longrightarrow$ (ii)

ΔCOD

$(\overline{CD})^2 = (\overline{OD})^2 + (\overline{OC})^2 \longrightarrow$ (iii)

ΔDOA

$(\overline{AD})^2 = (\overline{OA})^2 + (\overline{OD})^2 \longrightarrow$ (iv)

By adding (i) and (iii)

$(\overline{AB})^2 + (\overline{CD})^2 = (\overline{OB})^2 + (\overline{OA})^2 + (\overline{OD})^2 + (\overline{OC})^2 \rightarrow$ (v)

By adding (ii) and (iv)

$(\overline{AD})^2 + (\overline{BC})^2 = (\overline{OB})^2 + (\overline{OC})^2 + (\overline{OA})^2 + (\overline{OD})^2 \rightarrow$ (vi)

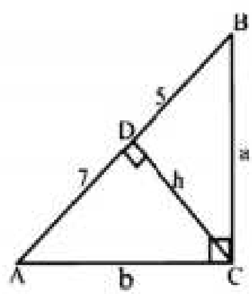
By comparing v and vi

$(\overline{AB})^2 + (\overline{CD})^2 = (\overline{AD})^2 + (\overline{BC})^2$

Hence proved

Q.6 the ΔABC as shown in the figure m$\angle$ACB = 90° and $\overline{CD} \perp \overline{AB}$ find the length a, h and b if $m\overline{BD} = 5$ units and $m\overline{AD} = 7$ units

(i)



ΔACB

$(7+5)^2 = (b)^2 + (a)^2$

$a^2 + b^2 = (12)^2$

$a^2 + b^2 = 144$ ______ (i)

ΔADC

$(b)^2 = (7)^2 + (h)^2$

$b^2 - h^2 = 49$ ________ (ii)

ΔCDB

$a^2 = (5)^2 + (h)^2$

$a^2 - h^2 = 25$ _______ (iii)

Subtracting ii from iii

$$\begin{array}{r} a^2 - \cancel{h^2} = 25 \\ \pm b^2 \text{ m } \cancel{h^2} = \pm 49 \\ \hline a^2 - b^2 = -24 \end{array}$$

$a^2 - b^2 = -24$ __________ (iv)

Adding equation I and IV

$$\begin{array}{r} a^2 + \cancel{b^2} = 144 \\ a^2 - \cancel{b^2} = -24 \\ \hline 2a^2 = 120 \end{array}$$

$2a^2 = 120$

$a^2 = \dfrac{\cancel{120}^{60}}{\cancel{2}}$ $\quad 2\times 2\times 15$

$a^2 = 60$ $\quad 4\times 15$

Prime factor

2	60
2	30
	15

$a^2 = 4 \times 15$

Taking square root both side

$\sqrt{a^2} = \sqrt{4\times 15}$

$a = 2\sqrt{15}$

Putting the value of a in equation

(i)

$\left(2\sqrt{15}\right)^2 + b^2 = 144$

Prime factor

$4\times 15 + b^2 = 144$

$60 + b^2 = 144$

$b^2 = 144 - 60$

$b^2 = 84$

$b^2 = 4\times 21$

$2\times 2\times 21$

4×21

2	84
2	42
21	21
	1

Taking square root both side

$b^2 = \sqrt{4\times 21}$

$b = 2\sqrt{21}$

Putting the value of b in equation

(ii)

$\left(2\sqrt{21}\right)^2 - h^2 = 49$

$4\times 21 - 49 = h^2$

$h^2 = 84 - 49$

$h^2 = 35$

Taking square root both side

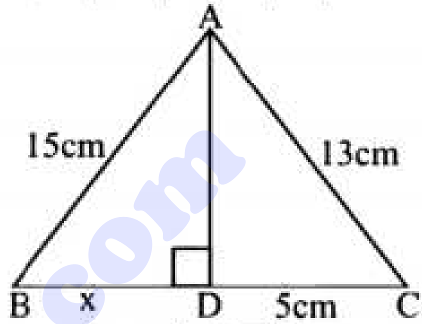
$\sqrt{h^2} = \sqrt{35}$

$h = \sqrt{35}$

(ii) Find the value of x in the shown figure

From ΔADC

$\left(\overline{AC}\right)^2 = \left(\overline{DC}\right)^2 + \left(\overline{AD}\right)^2$



$(13)^2 = (5)^2 + \left(\overline{AD}\right)^2$

$169 = 25 + \left(\overline{AD}\right)^2$

$169 - 25 = \left(\overline{AD}\right)^2$

$\left(\overline{AD}\right)^2 = 144$

Taking square root both side

$\sqrt{\left(\overline{AD}\right)^2} = \sqrt{(144)}$

$\overline{AD} = 12$

From Δ ADB

$\left(\overline{AB}\right)^2 = (BD)^2 + \left(\overline{AD}\right)^2$

$(15)^2 = x^2 + (12)^2$

$225 = x^2 + 144$

$225 - 144 = x^2$

$x^2 = 81$

Taking square on both side

$\sqrt{x^2} = \sqrt{81}$

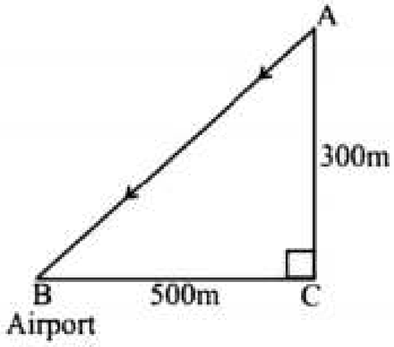
$x = 9$

Q.7 A plane is at a height of 300m and is 500m away from the airport as shown in the figure How much distance will it travel to land at the airport?

ΔABC is right angle triangle

$\left(\overline{AB}\right)^2 = \left(\overline{BC}\right)^2 + \left(\overline{AC}\right)^2$

$\left(\overline{AB}\right)^2 = (500)^2 + (300)^2$



$$\left(\overline{AB}\right)^2 = 250000 + 90000$$

$$\left(\overline{AB}\right)^2 = 340000$$

$$\left(\overline{AB}\right)^2 = 10000 \times 34$$

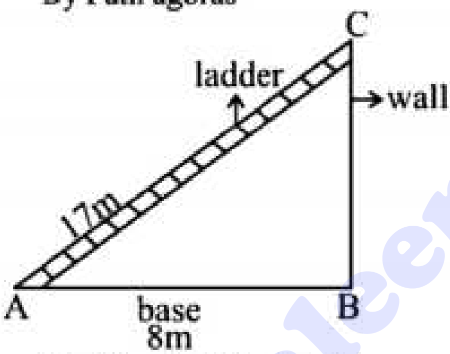
Taking square root on both side

$$\sqrt{\left(\overline{AB}\right)^2} = \sqrt{10000 \times 34}$$

$$AB = 100\sqrt{34}m$$

Q.8 A ladder 17m long rests against a vertical wall. The foot of the ladder is 8m away from the base of thewall. How high up the wall will the ladder reach?

By Path agoras



$$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$$

$$(17)^2 = (8)^2 + \left(\overline{BC}\right)^2$$

$$289 = 64 + \left(\overline{BC}\right)^2$$

$$284 - 64 = \left(\overline{BC}\right)^2$$

$$\left(\overline{BC}\right)^2 = 225$$

Taking square root on both side

$$\sqrt{\left(\overline{BC}\right)^2} = \sqrt{225}$$

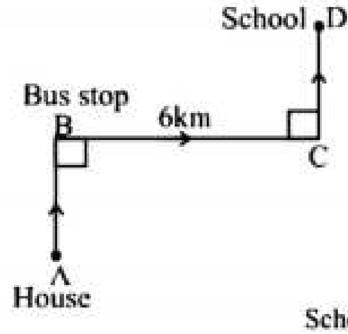
$$\overline{BC} = 15\text{m}$$

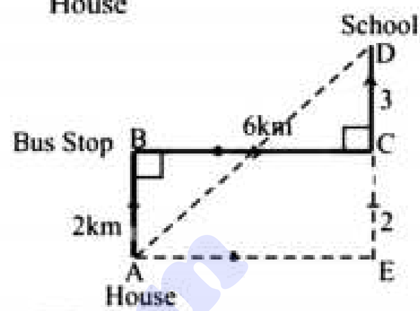
The height of wall $= \overline{BC} = 15m$

Q.9 A student travels to his school by the route as shown in the figure. Find $m\overline{AD}$, the direct distance from his house to school.

Solution:

As we know that in rectangular opposite sides are equal so





$$\overline{AB} = \overline{CE} = 2km$$

$$\overline{BC} = \overline{AE} = 6km$$

$$\overline{DE} = \overline{DC} + \overline{CE}$$

$\therefore$ We get triangle

Δ ADF which is right angled triangle

$$\left(\overline{AD}\right)^2 = \left(\overline{AE}\right)^2 + \left(\overline{ED}\right)^2$$

$$\left(\overline{AD}\right)^2 = (6)^2 + (3+2)^2$$

$$\left(\overline{AD}\right)^2 = 36 + (5)^2$$

$$\left(\overline{AD}\right)^2 = 36 + 25$$

$$\left(\overline{AD}\right)^2 = 61$$

Taking square root on both side

$$\sqrt{\left(\overline{AD}\right)^2} = \sqrt{61}$$

$$\overline{AD} = \sqrt{61}km$$