Exercise 5.3

Q. 1: Use the remainder theorem to find the remainder when

(i) $3x^3 - 10x^2 + 13x - 6$ is divided by $(x-2)$

for $x - 2 = 0$, we have $x = 2$

$P(2) = 3(2)^3 - 10(2)^2 + 13(2) - 6$
$= 3(8) - 10(4) + 26 - 6$
$= 24 - 40 + 26 - 6$
$= 50 - 46$
$= 4$

So, Remainder = 4

(ii) $4x^3 - 4x + 3$ is divided by $(2x-1)$

for $2x - 1 = 0$, we have $x = \frac{1}{2}$

$P\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right) + 3$
$= 4\left(\frac{1}{8}\right) - 2 + 3$
$= \frac{1}{2} + 1$
$= \frac{1+2}{2}$
$= \frac{3}{2}$

So, Remainder = $\frac{3}{2}$

(iii) $6x^4 + 2x^3 - x + 2$ is divided by $(x+2)$

for $x + 2 = 0$, we have $x = -2$

$P(-2) = 6(-2)^4 + 2(-2)^3 - (-2) + 2$
$= 6(16) + 2(-8) + 2 + 2$
$= 96 - 16 + 4$
$= 100 - 16$
$= 84$

So, Remainder = 84

(iv) $(2x-1)^3 + 6(3+4x)^2 - 10$ is divided by $(2x+1)$

for $2x + 1 = 0$, we have $x = -\frac{1}{2}$

$P\left(-\frac{1}{2}\right) = \left(2\left(-\frac{1}{2}\right) - 1\right)^3 + 6\left(3 + 4\left(-\frac{1}{2}\right)\right)^2 - 10$
$= (-1-1)^3 + 6(3-2)^2 - 10$
$= (-2)^3 + 6(1)^2 - 10$
$= -8 + 6 - 10$
$= -12$

So, Remainder = -12

(v) $x^3 - 3x^2 + 4x - 14$ is divided by $(x+2)$

for $x + 2 = 0$, we have $x = -2$

$P(-2) = (-2)^3 - 3(-2)^2 + 4(-2) - 14$

$= (-8) - 3(4) - 8 - 14$
$= -8 - 12 - 8 - 14$
$= -20 - 22$
$= -42$

So, Remainder = -42

Q. 2:

(i) If $(x+2)$ is a factor of $3x^2 - 4kx - 4k^2$, then find the value(s) of k.

for $x + 2 = 0$, we have $x = -2$

$P(-2) = 3(-2)^2 - 4k(-2) - 4k^2$
$= 3(4) + 8k - 4k^2$
$= 12 + 8k - 4k^2$

as $(x+2)$ is a factor of given polynomials. So, Remainder = 0

$12 + 8k - 4k^2 = 0$
$-4(k^2 - 2k - 3) = 0$
$k^2 - 2k - 3 = 0$
$k^2 - 3k + k - 3 = 0$
$k(k-3) + 1(k-3) = 0$
$(k-3)(k+1) = 0$

So, $k = 3$ and $k = -1$

(ii) If $(x-1)$ is a factor of $x^3 - kx^2 + 11x - 6$, then find the value(s) of k.

for $x - 1 = 0$, we have $x = 1$

$P(1) = (1)^3 - k(1)^2 + 11(1) - 6$
$= 1 - k + 11 - 6$
$= 6 - k$

as $(x-1)$ is a factor of given polynomials. So, Remainder = 0

$6 - k = 0$
$k = 6$

So, $k = 6$

Q. 3: Without actual long division determine whether

(i) $(x-2)$ and $(x-3)$ are factors of $p(x) = x^3 - 12x^2 + 44x - 48$.

for $x - 2 = 0$, we have $x = 2$

$P(2) = (2)^3 - 12(2)^2 + 44(2) - 48$
$= 8 - 12(4) + 88 - 48$
$= 8 - 48 + 88 - 48$
$= 96 - 96$
$= 0$

So $(x-2)$ is a factor of given polynomial.

for $x - 3 = 0$, we have $x = 3$

$P(3) = (3)^3 - 12(3)^2 + 44(3) - 48$
$= 27 - 12(9) + 132 - 48$
$= 27 - 108 + 132 - 48$

$= 159 - 156$
$= 3$

So $(x-3)$ is not factor of given polynomial.

(ii) $(x-2)$, $(x+3)$ and $(x-4)$ are factors of $p(x) = x^3 + 2x^2 - 5x - 6$.

for $x - 2 = 0$, we have $x = 2$

$P(2) = (2)^3 + 2(2)^2 - 5(2) - 6$
$= 8 + 2(4) - 10 - 6$
$= 8 + 8 - 10 - 6$
$= 16 - 16$
$= 0$

So $(x-2)$ is a factor of given polynomial.

for $x + 3 = 0$, we have $x = -3$

$P(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 6$
$= -27 + 2(9) + 15 - 6$
$= -27 + 18 + 15 - 6$
$= -33 + 33$
$= 0$

So $(x+3)$ is a factor of given polynomial.

for $x - 4 = 0$, we have $x = 4$

$P(4) = (4)^3 + 2(4)^2 - 5(4) - 6$
$= 64 + 2(16) - 20 - 6$
$= 64 + 32 - 20 - 6$
$= 96 - 26$
$= 70$

So $(x-4)$ is not a factor of given polynomial.

Q. 4: For what value of m is the polynomial $P(x) = 4x^3 - 7x^2 + 6x - 3m$ is exactly divisible by x+2?

for $x + 2 = 0$, we have $x = -2$

$P(-2) = 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m$
$= 4(-8) - 7(4) + 6(-2) - 3m$
$= -32 - 28 - 12 - 3m$
$= -72 - 3m$

as $(x+2)$ is a factor of given polynomials. So, Remainder = 0

$-72 - 3m = 0$
$-3m = 72$

So, $m = -24$

Q.5: Determine the value of k if $p(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x-3)$.

for $x - 3 = 0$, we have $x = 3$

$P(3) = k(3)^3 + 4(3)^2 + 3(3) - 4$
$= k(27) + 4(9) + 3(3) - 4$
$= 27k + 36 + 9 - 4$

$= 27k + 41$

$q(3) = (3)^3 - 4(3) + k$
$= (27) - 4(3) + k$
$= 27 - 12 + k$
$= 15 + k$

as the remainder is same so

$p(3) = q(3)$
$27k + 41 = 15 + k$
$27k - k = 15 - 41$
$26k = -26$

So, $k = -1$

Q.6: The remainder after dividing the polynomial $p(x) = x^3 + ax^2 + 7$ by $(x + 1)$ is 2b. Calculate the value of a and b if this expression leaves remainder of $(b + 5)$ on being divided by $(x - 2)$.

for $x + 1 = 0$, we have $x = -1$

$P(-1) = (-1)^3 + a(-1)^2 + 7$
$= -1 + a(1) + 7$
$= a + 6$

as remainder in this case is 2b so

$2b = a + 6$
$a - 2b = -6$ ------------------ (i)

for $x - 2 = 0$, we have $x = 2$

$P(2) = (2)^3 + a(2)^2 + 7$
$= 8 + a(4) + 7$
$= 4a + 15$

as remainder in this case is b + 5 so

$b + 5 = 4a + 15$
$4a - b = -10$ --------------- (ii)

multiplying equation (ii) by 2

$8a - 2b = -20$ --------------- (iii)

subtracting equation (i) from (iii)

$8a - 2b = -20$
$-a + 2b = +6$

we have

$7a = -14$
$a = -2$

put in equation (i)

$-2b = -4$
$b = 2$

So, $a = -2$ and $b = 2$

Q. 7: The polynomial $x^3 + lx^2 + mx + 24$ has a factor $(x + 4)$ and it leaves remainder of 36 when divided by $(x - 2)$. Find the values of l and m.

for $x+4=0$, we have $x=-4$

$P(-4)$ $= (-4)^3 + l(-4)^2 + m(-4) + 24$

$= -64 + l(16) - 4m + 24$

$= -40 + 16l - 4m$

as $x+4$ is the factor of given polynomial so R = 0.

$16l - 4m$ $= 40$

dividing by 4

$4l - m = 10$ ------------------ (i)

for $x-2=0$, we have $x=2$

$P(2)$ $= (2)^3 + l(2)^2 + m(2) + 24$

$= 8 + l(4) + 2m + 24$

$= 32 + 4l + 2m$

as remainder in this case is 36 so

$4l + 2m + 32 = 36$

$4l + 2m$ $= 4$ --------------------- (ii)

subtracting equation (i) from (ii)

$4l + 2m$ $= 4$

$-4l + m$ $= -10$

we have

$3m$ $= -6$

m $= -2$

put in equation (ii)

$4l + 2(-2)$ $= 4$

$4l - 4 = 4$

$4l$ $= 8$

l $= \frac{8}{4}$

l $= 2$

So, $l = 2$ and $m = -2$

Q. 8: The expression $lx^3 + mx^2 - 4$ leaves remainder of -3 and 12 when divided by $(x-1)$ and $(x+2)$ respectively. Calculate the values of l and m.

for $x-1=0$, we have $x=1$

$P(1)$ $= l(1)^3 + m(1)^2 - 4$

$= l + m - 4$

as remainder in this case is -3 so

$l + m - 4$ $= -3$

$l + m$ $= 1$ ------------------ (i)

for $x+2=0$, we have $x=-2$

$P(-2)$ $= l(-2)^3 + m(-2)^2 - 4$

$= l(-8) + 4m - 4$

$= -8l + 4m - 4$

as remainder in this case is 12 so

$-8l + 4m - 4 = 12$

$-8l + 4m \quad = 16$

dividing by 4

$-2l + m \quad = 4$ --------------------- (ii)

subtracting equation (i) from (ii)

$-2l + m \quad = 4$

$-l - m \quad = -1$

we have

$-3l \quad = 3$

$l \quad = -1$

put in equation (i)

$-1 + m \quad = 1$

$m \quad = 2$

So, $l = -1$ and $m = 2$

Q. 9: The expression $ax^3 - 9x^2 + bx + 3a$ is exactly divisible by $x^2 - 5x + 6$.Find the values of a and b.

$x^2 - 5x + 6 \quad = x^2 - 3x - 2x + 6$

$= x(x-3) - 2(x-3)$

$= (x-3)(x-2)$

as the polynomial is divisible by $x^2 - 5x + 6$ so $(x-3)(x-2)$ are also its factors.

for $x - 3 = 0$, we have $x = 3$

$P(3) \quad = a(3)^3 - 9(3)^2 + b(3) + 3a$

$= a(27) - 9(9) + b(3) + 3a$

$= 27a - 81 + 3b + 3a$

$= 30a - 81 + 3b$

as the polynomial is divisible by $x - 3$

$30a - 81 + 3b = 0$

$30a + 3b \quad = 81$

dividing by 3

$10a + b \quad = 27$ ------------------ (i)

for $x - 2 = 0$, we have $x = 2$

$P(2) \quad = a(2)^3 - 9(2)^2 + b(2) + 3a$

$= a(8) - 9(4) + b(2) + 3a$

$= 8a - 36 + 2b + 3a$

$= 11a - 36 + 2b$

as the polynomial is divisible by $x - 2$

$11a - 36 + 2b = 0$

$11a + 2b \quad = 36$

$11a + 2b \quad = 36$ --------------------- (ii)

Multiplying equation (i) by 2

$20a + 2b \quad = 54$ --------------------- (iii) subtracting equation (ii) from (iii)

$20a + 2b \quad = 54$

$-11a - 2b \quad = -36$

we have

$9a \quad = 18$

$a \quad = 2$

put in equation (i)

$10(2) + b \quad = 27$

$20 + b = 27$

$b \quad = 7$

So, $a = 2$ and $b = 7$