Exercise 2.5

Q. 1: Write the quadratic equations having following roots.

(a) $1, 5$

sum of roots $= S = \alpha + \beta = 1 + 5 = 6$

Product of roots $= P = \alpha\beta = 1 \times 5 = 5$

So,

$$x^2 - Sx + P = 0$$
$$x^2 - 6x + 5 = 0$$

(b) $4, 9$

sum of roots $= S = \alpha + \beta = 4 + 9 = 13$

Product of roots $= P = \alpha\beta = 4 \times 9 = 36$

So,

$$x^2 - Sx + P = 0$$
$$x^2 - 13x + 36 = 0$$

(c) $-2, 3$

sum of roots $= S = \alpha + \beta = -2 + 3 = 1$

Product of roots $= P = \alpha\beta = -2 \times 3 = -6$

So,

$$x^2 - Sx + P = 0$$
$$x^2 - x - 6 = 0$$

(d) $0, -3$

sum of roots $= S = \alpha + \beta = 0 - 3 = -3$

Product of roots $= P = \alpha\beta = 0 \times -3 = 0$

So,

$$x^2 - Sx + P = 0$$
$$x^2 + 3x + 0 = 0$$
$$x^2 + 3x = 0$$

(e) $2, -6$

sum of roots $= S = \alpha + \beta = 2 - 6 = -4$

Product of roots $= P = \alpha\beta = 2 \times -6 = -12$

So,

$$x^2 - Sx + P = 0$$
$$x^2 + 4x - 12 = 0$$

(f) $-1, -7$

sum of roots $= S = \alpha + \beta = -1 - 7 = -8$

Product of roots = P = $\alpha\beta$ = -1×-7 = 7

So,

$$x^2 - Sx + P = 0$$
$$x^2 + 8x + 7 = 0$$

(g) $1 + i, 1 - i$

sum of roots = S = $\alpha + \beta$ = $1 + i + 1 - i$ = 2

Product of roots = P = $\alpha\beta$ = $(1 + i)(1 - i)$ = $1 - i^2$ = 1+1 =2

So,

$$x^2 - Sx + P = 0$$
$$x^2 - 2x + 2 = 0$$

(h) $3 + \sqrt{2}, 3 - \sqrt{2}$

sum of roots = S = $\alpha + \beta$ = $3 + \sqrt{2} + 3 - \sqrt{2}$ = 6

Product of roots = P = $\alpha\beta$ = $(3 + \sqrt{2})(3 - \sqrt{2})$ = $9 - (\sqrt{2})^2$ = 9 - 2 = 7

So,

$$x^2 - Sx + P = 0$$
$$x^2 - 6x + 7 = 0$$

Q. 2: If α, β are the roots of the equation $x^2 - 3x + 6 = 0$, Form equations whose roots are

(a) $2\alpha + 1,\ 2\beta + 1$

For given equation

sum of roots = $\alpha + \beta$ = $-\frac{b}{a}$ = $-\frac{-3}{1}$ = 3

Product of roots = $\alpha\beta$ = $\frac{c}{a}$ = $\frac{6}{1}$ = 6

So the equation for the given roots will be driven as follows,

S = $2\alpha + 1 + 2\beta + 1 = 2\alpha + 2\beta + 2$
$= 2(\alpha + \beta) + 2$
$= 2(3) + 2$
$= 6 + 2$
$= 8$

P = $(2\alpha + 1)(2\beta + 1) = 4\alpha\beta + 2\alpha + 2\beta + 1$
$= 4\alpha\beta + 2(\alpha + \beta) + 1$
$= 4(6) + 2(3) + 1$
$= 24 + 6 + 1$
$= 31$

So,

$$x^2 - Sx + P = 0$$
$$x^2 - 8x + 31 = 0$$

(b) $\alpha^2,\ \beta^2$

For given equation

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-3}{1}$ $= 3$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{6}{1}$ $= 6$

So the equation for the given roots will be driven as follows,

$S = \alpha^2 + \beta^2 = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= (3)^2 - 2(6)$

$= 9 - 12$

$= -3$

$P = \alpha^2\beta^2 = (\alpha\beta)^2$

$= (6)^2$

$= 36$

So,

$x^2 - Sx + P = 0$

$x^2 + 3x + 36 = 0$

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$

For given equation

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-3}{1}$ $= 3$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{6}{1}$ $= 6$

So the equation for the given roots will be driven as follows,

$S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{3}{6}$

$= \frac{1}{2}$

$P = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta}$

$= \frac{1}{6}$

So,

$x^2 - Sx + P = 0$

$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$

multiplying by 6 on b.s.

$6x^2 - 3x + 1 = 0$

(d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

For given equation

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-3}{1}$ $= 3$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{6}{1}$ $= 6$

So the equation for the given roots will be driven as follows,

$$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$
$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$$
$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$
$$= \frac{3^2 - 2(6)}{6}$$
$$= \frac{9-12}{6}$$
$$= \frac{-3}{6}$$
$$= \frac{-1}{2}$$

$$P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$$

So,

$$x^2 - Sx + P = 0$$
$$x^2 - \frac{-1}{2}x + 1 = 0$$
$$x^2 + \frac{1}{2}x + 1 = 0$$

multiplying by 2 on b.s.

$$2x^2 + x + 2 = 0$$

(e) $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

For given equation

sum of roots $= \alpha + \beta$ $= -\frac{b}{a}$ $= -\frac{-3}{1}$ $= 3$

Product of roots $= \alpha\beta$ $= \frac{c}{a}$ $= \frac{6}{1}$ $= 6$

So the equation for the given roots will be driven as follows,

$$S = \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} = (\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta}$$
$$= (3) + \frac{3}{6}$$
$$= 3 + \frac{1}{2}$$
$$= \frac{6+1}{2}$$
$$= \frac{6+1}{2}$$
$$= \frac{7}{2}$$

$$P = (\alpha + \beta).\frac{\alpha + \beta}{\alpha\beta} = (3) \times \frac{3}{6}$$
$$= \frac{3}{2}$$

So,

$x^2 - Sx + P = 0$

$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

multiplying by 2 on b.s.

$2x^2 - 7x + 3 = 0$

Q. 3: If α, β are the roots of the equation $x^2 + px + q = 0$, Form equations whose roots are

(a) $\alpha^2,\ \beta^2$

For given equation

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{p}{1} \quad = -p$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{q}{1} \quad = q$

So the equation for the given roots will be driven as follows,

S = $\alpha^2 + \beta^2 = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= (-p)^2 - 2(q)$

$= p^2 - 2q$

P = $\alpha^2\beta^2 = (\alpha\beta)^2$

$= (q)^2$

$= q^2$

So,

$x^2 - Sx + P = 0$

$x^2 - (p^2 - 2q)x + q^2 = 0$

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

For given equation

sum of roots $= \alpha + \beta \quad = -\frac{b}{a} \quad = -\frac{p}{1} \quad = -p$

Product of roots $= \alpha\beta \quad = \frac{c}{a} \quad = \frac{q}{1} \quad = q$

So the equation for the given roots will be driven as follows,

S = $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{p^2 - 2q}{q}$

$= \frac{p^2 - 2q}{q}$

P = $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

So,

$$x^2 - Sx + P \quad = 0$$

$$x^2 - \frac{p^2 - 2q}{q}x + 1 \qquad = 0$$

multiplying by q on b.s.

$$qx^2 - (p^2 - 2q)x + q = 0$$