

Exercise 4.1**Q. 1: Identify whether the following algebraic expressions are polynomials (Yes or No).**

- (i) $3x^2 + \frac{1}{x} - 5$ No
- (ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$ No
- (iii) $x^2 - 3x + \sqrt{2}$ Yes
- (iv) $\frac{3x}{2x-1} + 8$ No

Q. 2: State whether each of the following expressions is a rational expression or not.

- (i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$ No
- (ii) $\frac{x^3-2x^2+\sqrt{3}}{2+3x-x^2}$ Yes
- (iii) $\frac{x^2+6x+9}{x^2-9}$ Yes
- (iv) $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$ Yes

Q. 3: Reduce the following rational expressions to the lowest form.

- (i) $\frac{120x^2y^3z^5}{30x^3yz^2} = \frac{4x^2y^3z^5}{x^3yz^2}$
 $= 4x^{2-3}y^{3-1}z^{5-2}$
 $= 4x^{-1}y^2z^3$
 $= \frac{4y^2z^3}{x}$
- (ii) $\frac{8a(x+1)}{2(x^2-1)} = \frac{8a(x+1)}{2(x-1)(x+1)}$
 $= \frac{4a}{x-1}$
- (iii) $\frac{(x+y)^2-4xy}{(x-y)^2} = \frac{x^2+y^2+2xy-4xy}{(x-y)^2}$
 $= \frac{x^2+y^2-2xy}{(x-y)^2}$
 $= \frac{(x-y)^2}{(x-y)^2}$
 $= 1$
- (iv) $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)} = \frac{(x^3-y^3)(x^2-2xy+y^2)}{x^3-y^3}$
 $= (x^2-2xy+y^2)$
 $= (x-y)^2$
- (v) $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)} = \frac{(x+2)(x-1)(x+1)}{(x+1)(x-2)(x+2)}$
 $= \frac{(x-1)}{(x-2)}$
 $= \frac{x-1}{x-2}$
- (vi) $\frac{x^2-4x+4}{2x^2-8} = \frac{(x-2)^2}{2(x^2-4)}$
 $= \frac{(x-2)(x-2)}{2(x-2)(x+2)}$
 $= \frac{x-2}{2(x+2)}$

$$\begin{aligned}
 \text{(vii)} \quad \frac{64x^5 - 64x}{(8x^2 + 8)(2x + 2)} &= \frac{64x(x^4 - 1)}{8(x^2 + 1)2(x + 1)} \\
 &= \frac{4x((x^2)^2 - 1)}{(x^2 + 1)(x + 1)} \\
 &= \frac{4x(x^2 - 1)(x^2 + 1)}{(x^2 + 1)(x + 1)} \\
 &= \frac{4x(x^2 - 1)}{(x + 1)} \\
 &= \frac{4x(x - 1)(x + 1)}{(x + 1)} \\
 &= 4x(x - 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad \frac{9x^2 - (x^2 - 4)^2}{4 + 3x - x^2} &= \frac{(3x)^2 - (x^2 - 4)^2}{4 + 3x - x^2} \\
 &= \frac{[3x - (x^2 - 4)][3x + (x^2 - 4)]}{4 + 3x - x^2} \\
 &= \frac{[3x - x^2 + 4][3x + x^2 - 4]}{4 + 3x - x^2} \\
 &= \frac{[4 + 3x - x^2][3x + x^2 - 4]}{4 + 3x - x^2} \\
 &= x^2 + 3x - 4
 \end{aligned}$$

Q. 4: Evaluate (a) $\frac{x^3y - 2z}{xz}$ for

(i) $x = 3, y = -1, z = -2$

$$\begin{aligned}
 \frac{x^3y - 2z}{xz} &= \frac{(3)^3(-1) - 2(-2)}{(3)(-2)} \\
 &= \frac{(27)(-1) - 2(-2)}{(3)(-2)} \\
 &= \frac{-27 + 4}{-6} \\
 &= \frac{-23}{-6} \\
 &= \frac{23}{6}
 \end{aligned}$$

(ii) $x = -1, y = -9, z = 4$

$$\begin{aligned}
 \frac{x^3y - 2z}{xz} &= \frac{(-1)^3(-9) - 2(4)}{(-1)(4)} \\
 &= \frac{(-1)(-9) - 2(4)}{(-1)(4)} \\
 &= \frac{9 - 8}{-4} \\
 &= \frac{1}{-4}
 \end{aligned}$$

(b) $\frac{x^2y^3 - 5z^4}{xyz}$ for $x = 4, y = -2, z = -1$

$$\begin{aligned}
 \frac{x^2y^3 - 5z^4}{xyz} &= \frac{(4)^2(-2)^3 - 5(-1)^4}{(4)(-2)(-1)} \\
 &= \frac{(16)(-8) - 5(1)}{8} \\
 &= \frac{-128 - 5}{8} \\
 &= \frac{-133}{8} = -16\frac{5}{8}
 \end{aligned}$$

Q. 5: Perform the indicated operation and simplify.

$$\begin{aligned}
 \text{(i)} \quad \frac{15}{2x-3y} - \frac{4}{3y-2x} &= \frac{15}{2x-3y} + \frac{4}{2x-3y} \\
 &= \frac{15+4}{2x-3y} \\
 &= \frac{19}{2x-3y}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{1+2x}{1-2x} - \frac{1-2x}{1+2x} &= \frac{(1+2x)^2 - (1-2x)^2}{(1-2x)(1+2x)} \\
 &= \frac{1+4x+4x^2 - (1-4x+4x^2)}{(1-4x^2)} \\
 &= \frac{1+4x+4x^2 - 1 + 4x - 4x^2}{(1-4x^2)} \\
 &= \frac{8x}{1-4x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{x^2-25}{x^2-36} - \frac{x+5}{x+6} &= \frac{(x+5)(x-5)}{(x-6)(x+6)} - \frac{x+5}{x+6} \\
 &= \frac{x+5}{x+6} \left[\frac{x-5}{x-6} - 1 \right] \\
 &= \frac{x+5}{x+6} \left[\frac{x-5-(x-6)}{x-6} \right] \\
 &= \frac{x+5}{x+6} \left[\frac{x-5-x+6}{x-6} \right] \\
 &= \frac{x+5}{x+6} \left[\frac{1}{x-6} \right] \\
 &= \frac{x+5}{x^2-36}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2-y^2} &= \frac{x(x+y)-y(x-y)}{x^2-y^2} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x^2+xy-xy+y^2}{x^2-y^2} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x^2+y^2}{x^2-y^2} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x^2+y^2-2xy}{x^2-y^2} \\
 &= \frac{(x-y)^2}{(x-y)(x+y)} \\
 &= \frac{x-y}{x+y}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad \frac{x-2}{x^2+6x+9} - \frac{x+2}{2x^2-18} &= \frac{x-2}{(x+3)^2} - \frac{x+2}{2(x^2-9)} \\
 &= \frac{x-2}{(x+3)(x+3)} - \frac{x+2}{2(x+3)(x-3)} \\
 &= \frac{1}{x+3} \left[\frac{x-2}{x+3} - \frac{x+2}{2(x-3)} \right] \\
 &= \frac{1}{x+3} \left[\frac{2(x-2)(x-3) - (x+2)(x+3)}{2(x+3)(x-3)} \right] \\
 &= \frac{1}{x+3} \left[\frac{2(x^2-3x-2x+6) - (x^2+3x+2x+6)}{2(x+3)(x-3)} \right] \\
 &= \frac{1}{x+3} \left[\frac{2(x^2-5x+6) - (x^2+5x+6)}{2(x+3)(x-3)} \right] \\
 &= \frac{1}{x+3} \left[\frac{2x^2-10x+12-x^2-5x-6}{2(x+3)(x-3)} \right] \\
 &= \frac{1}{x+3} \left[\frac{x^2-15x+6}{2(x+3)(x-3)} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{x^2 - 15x + 6}{2(x+3)^2(x-3)} \\
 \text{(vi)} \quad \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1} &= \frac{(x+1)-(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4-1} \\
 &= \frac{x+1-x+1}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1} \\
 &= \frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1} \\
 &= \frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)} - \frac{4}{x^4-1} \\
 &= \frac{2x^2+2-2x^2+2}{x^4-1} - \frac{4}{x^4-1} \\
 &= \frac{4}{x^4-1} - \frac{4}{x^4-1} \\
 &= 0
 \end{aligned}$$

Q. 6: Perform the indicated operation and simplify.

$$\begin{aligned}
 \text{(i)} \quad (x^2 - 49) \cdot \frac{5x+2}{x+7} &= (x+7)(x-7) \cdot \frac{5x+2}{x+7} \\
 &= (x-7)(5x+2)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+6x+9} &= \frac{4(x-3)}{(x+3)(x-3)} \div \frac{18-2x^2}{x^2+6x+9} \\
 &= \frac{4}{(x+3)} \div \frac{-2(x^2-9)}{x^2+2(x)(3)+(3)^2} \\
 &= \frac{4}{(x+3)} \div \frac{-2(x-3)(x+3)}{(x+3)^2} \\
 &= \frac{4}{(x+3)} \div \frac{-2(x-3)}{(x+3)} \\
 &= \frac{4}{(x+3)} \times \frac{(x+3)}{-2(x-3)} \\
 &= \frac{2}{3-x}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4) &= \frac{x^6-y^6}{(x^2-y^2)} \times \frac{1}{(x^4+x^2y^2+y^4)} \\
 &= \frac{x^6-y^6}{(x^2-y^2)((x^2)^2+x^2y^2+(y^2)^2)} \\
 &= \frac{x^6-y^6}{x^6-y^6} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \frac{x^2-1}{x^2+2x+1} \cdot \frac{x+5}{1-x} &= \frac{(x-1)(x+1)}{(x+1)^2} \cdot \frac{x+5}{-(x-1)} \\
 &= \frac{(x-1)(x+1)}{(x+1)(x+1)} \cdot \frac{x+5}{-(x-1)} \\
 &= -\frac{(x+5)}{(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad \frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y} &= \frac{x(x+y)}{y(x+y)} \cdot \frac{x(x+y)}{y(x+y)} \div \frac{x(x-1)}{y(x-2)} \\
 &= \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{y(x-2)}{x(x-1)} \\
 &= \frac{x(x-2)}{y(x-1)}
 \end{aligned}$$