Exercise 9.3

1. Find the mid-point of the line segment joining each of the following pairs of points

(a) $A(9,\ 2), B(7,\ 2)$

If $M(x,\ y)$ is the desired mid-point then,

$x = \frac{9+7}{2} = \frac{16}{2} = 8$ and $y = \frac{2+2}{2} = \frac{4}{2} = 2$

Hence $M(x,\ y) = M(8,\ 2)$

(b) $A(2, -6), B(3, -6)$

If $M(x,\ y)$ is the desired mid-point then,

$x = \frac{2+3}{2} = \frac{5}{2} = 2.5$ and $y = \frac{-6-6}{2} = \frac{-12}{2} = -6$

Hence $M(x,\ y) = M(2.5, -6)$

(c) $A(-8, 1), B(6, 1)$

If $M(x,\ y)$ is the desired mid-point then,

$x = \frac{-8+6}{2} = \frac{-2}{2} = -1$ and $y = \frac{1+1}{2} = \frac{2}{2} = 1$

Hence $M(x,\ y) = M(-1, 1)$

(d) $A(-4, 9), B(-4, -3)$

If $M(x,\ y)$ is the desired mid-point then,

$x = \frac{-4-4}{2} = \frac{-8}{2} = -4$ and $y = \frac{9-3}{2} = \frac{6}{2} = 3$

Hence $M(x,\ y) = M(-4, 3)$

(e) $A(3, -11), B(3, -4)$

If $M(x,\ y)$ is the desired mid-point then,

$x = \frac{3+3}{2} = \frac{6}{2} = 3$ and $y = \frac{-11-4}{2} = \frac{-15}{2} = -7.5$

Hence $M(x,\ y) = M(3, -7.5)$

(f) $A(0, 0), B(0, -5)$

If $M(x,\ y)$ is the desired mid-point then,

$x = \frac{0+0}{2} = \frac{0}{2} = 0$ and $y = \frac{0-5}{2} = -2.5$

Hence $M(x,\ y) = M(0, -2.5)$

2. The end point P of a line segment PQ is (-3, 6) and its mid-point is (5, 8). Find the coordinates of the end point Q.

Since $M(5, 8)$ is the mid-point of $P(-3, 6)$ and $Q(x, y)$

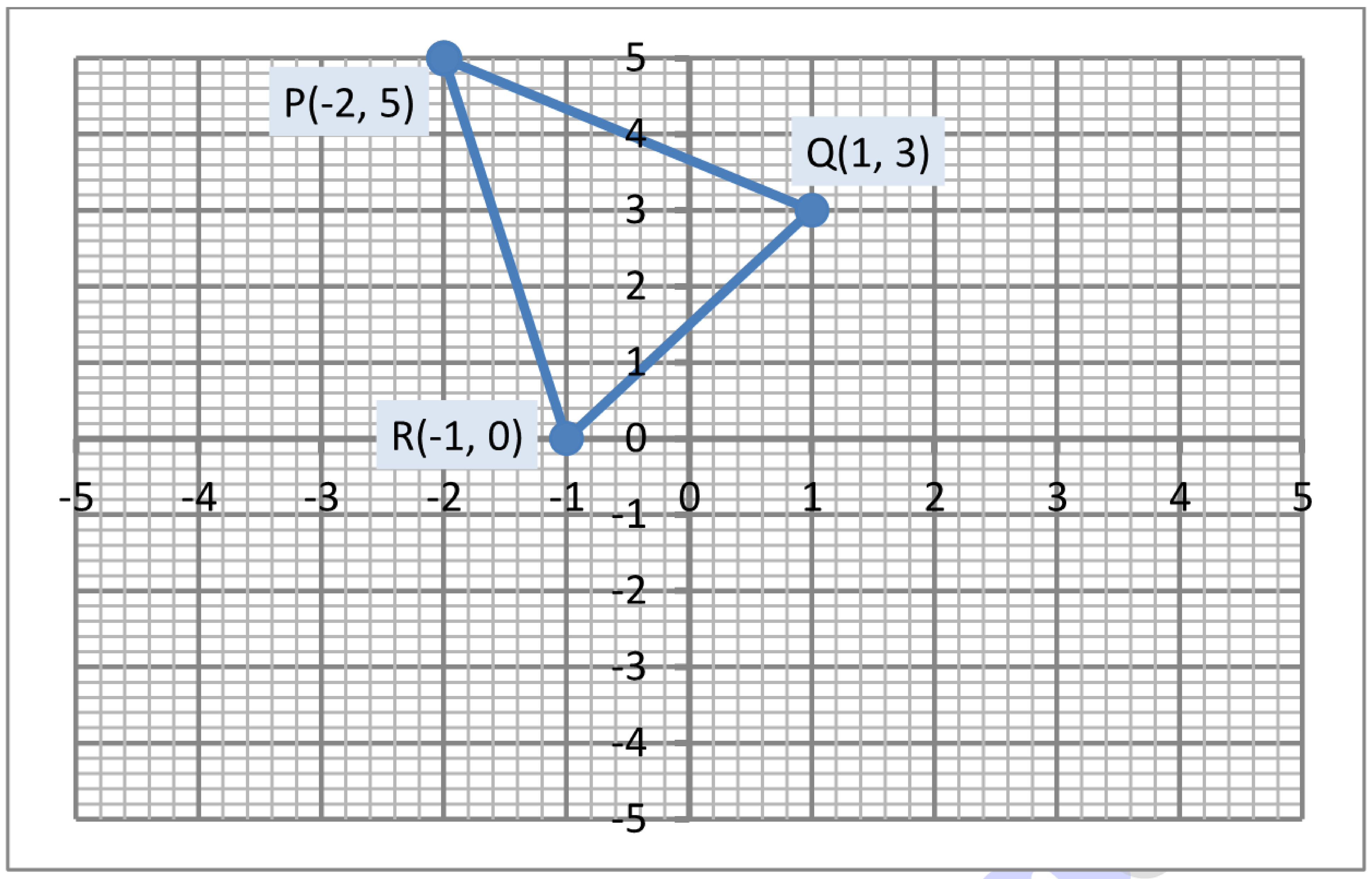
$5 = \frac{-3+x}{2}$ and $8 = \frac{6+y}{2}$

$10 = -3 + x$ and $16 = 6 + y$

$13 = x$ and $10 = y$

3. Prove that mid-point of the hypotenuse of right triangle is equidistant from its three vertices P(-2, 5), Q(1, 3) and R(-1, 0).

x	y
-2	5
1	3
-1	0



By Distance Formula

$|PQ| = \sqrt{(1+2)^2+(3-5)^2} = \sqrt{(3)^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$

$|QR| = \sqrt{(-1-1)^2+(0-3)^2} = \sqrt{(-2)^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$

$|PR| = \sqrt{(-1+2)^2+(0-5)^2} = \sqrt{(1)^2+(-5)^2} = \sqrt{1+25} = \sqrt{26}$

As $|PR|^2 = |PQ|^2 + |QR|^2$. So, $|PR|$ is the hypotenuse of triangle.

If $M(x,\ y)$ is the desired mid-point then

$x = \frac{-2-1}{2} = \frac{-3}{2}$ and $y = \frac{5+0}{2} = \frac{5}{2}$

Hence $M(x,\ y) = M\left(\frac{-3}{2}, \frac{5}{2}\right)$

$|PM| = \sqrt{\left(\frac{-3}{2}+2\right)^2+\left(\frac{5}{2}-5\right)^2} = \sqrt{\left(\frac{-3+4}{2}\right)^2+\left(\frac{5-10}{2}\right)^2}$

$= \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{-5}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{25}{4}} = \sqrt{\frac{26}{4}}$

$|QM| = \sqrt{\left(\frac{-3}{2}-1\right)^2+\left(\frac{5}{2}-3\right)^2} = \sqrt{\left(\frac{-3-2}{2}\right)^2+\left(\frac{5-6}{2}\right)^2}$

$= \sqrt{\left(\frac{-5}{2}\right)^2+\left(\frac{-1}{2}\right)^2} = \sqrt{\frac{25}{4}+\frac{1}{4}} = \sqrt{\frac{26}{4}}$

$|RM| = \sqrt{\left(\frac{-3}{2}+1\right)^2+\left(\frac{5}{2}-0\right)^2} = \sqrt{\left(\frac{-3+2}{2}\right)^2+\left(\frac{5-0}{2}\right)^2}$

$= \sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{5}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{25}{4}} = \sqrt{\frac{26}{4}}$

As, $|PM| = |QM| = |RM|$. So, M is equidistant from P, Q and R.

4. If O(0, 0), A(3, 0) and B(3, 5) are three points in the plane, find M_1 and M_2 as mid-points of the line segments AB and OB respectively. Find $|M_1M_2|$.

If $M_1(x,\ y)$ is the mid-point of AB

$x = \frac{3+3}{2} = \frac{6}{2} = 3$ and $y = \frac{0+5}{2} = \frac{5}{2}$

Hence $M_1(x,\ y) = M_1\left(3, \frac{5}{2}\right)$

If $M_2(x,\ y)$ is the mid-point of OB

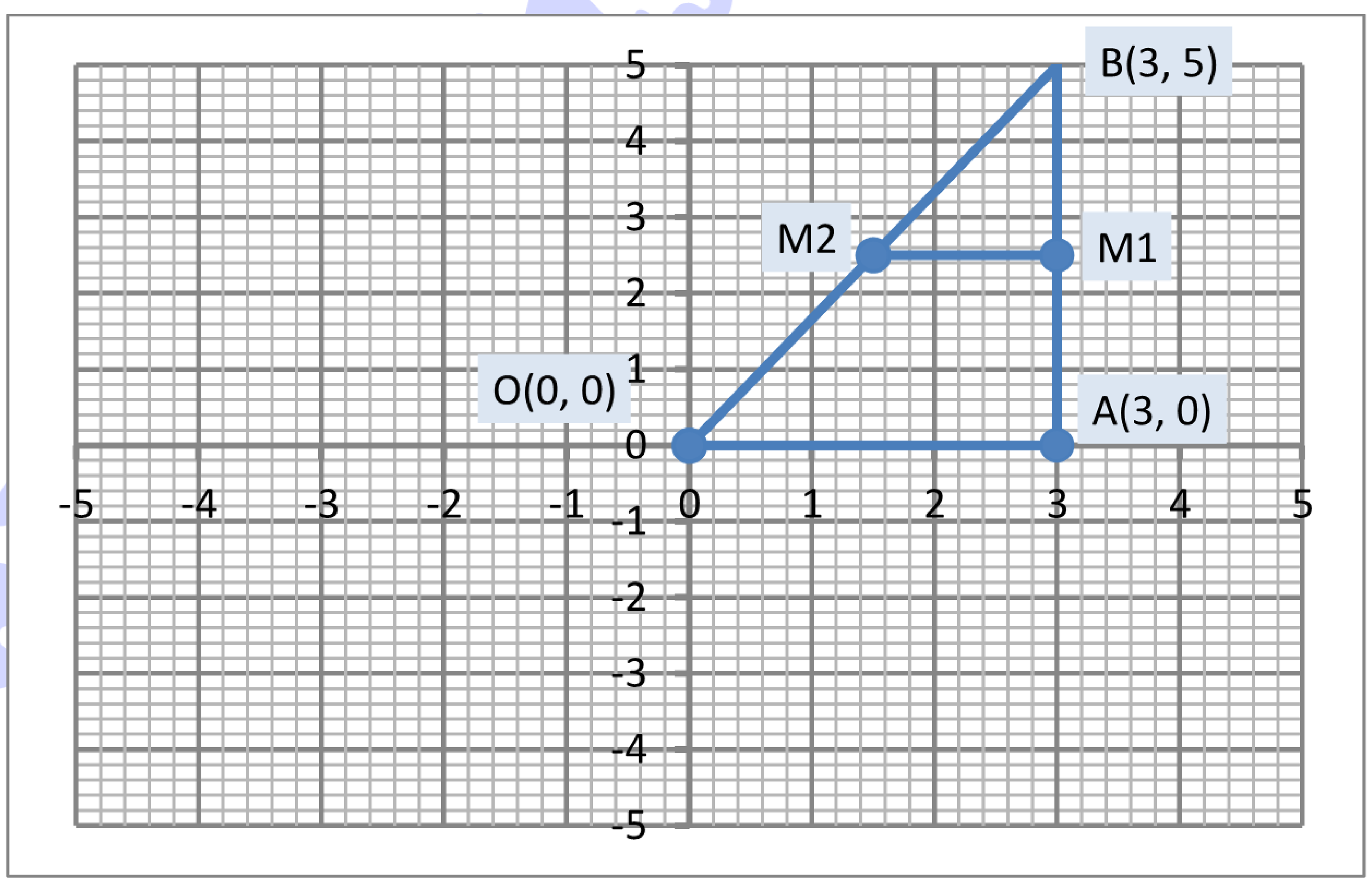
$x = \frac{0+3}{2} = \frac{3}{2}$ and $y = \frac{0+5}{2} = \frac{5}{2}$

Hence $M_2(x,\ y) = M_2\left(\frac{3}{2}, \frac{5}{2}\right)$

Now we find $|M_1M_2|$

$$|M_1M_2| = \sqrt{\left(\frac{3}{2} - 3\right)^2 + \left(\frac{5}{2} - \frac{5}{2}\right)^2}$$
$$= \sqrt{\left(\frac{3-6}{2}\right)^2 + (0)^2}$$
$$= \sqrt{\left(\frac{-3}{2}\right)^2}$$
$$= \sqrt{\left(\frac{3}{2}\right)^2}$$
$$= \frac{3}{2}$$

x	y
0	0
3	0
3	5



5. Show that the diagonals of the parallelogram having vertices A(1, 2), B(4, 2), C(-1, -3) and D(-4, -3) bisect each other.

If $M_1(x,\ y)$ is the mid-point of DB

$x = \frac{x_1+x_2}{2} = \frac{4-4}{2} = \frac{0}{2} = 0$ and $y = \frac{y_1+y_2}{2} = \frac{2-3}{2} = -\frac{1}{2}$

Hence $M_1(x,\ y) = M_1\left(0, -\frac{1}{2}\right)$

If $M_2(x,\ y)$ is the mid-point of AC

$x = \frac{x_1+x_2}{2} = \frac{1-1}{2} = \frac{0}{2} = 0$ and $y = \frac{y_1+y_2}{2} = \frac{2-3}{2} = -\frac{1}{2}$

Hence $M_2(x,\ y) = M_2\left(0, -\frac{1}{2}\right)$

As M_1 and M_2 coincide hence the diagonals AC and BD bisect each other.

6. The vertices of a triangle are P(4, 6), Q (-2, -4) and R(-8, 2). Show that the length of the line segment joining the mid-points of the line segments PR, QR is $\frac{1}{2}PQ$.

If $M_1(x,\ y)$ is the mid-point of PR

$x=\frac{x_1+x_2}{2}=\frac{4-8}{2}=\frac{-4}{2}=-2$ and $y=\frac{y_1+y_2}{2}=\frac{6+2}{2}=\frac{8}{2}=4$

Hence $M_1(x,\ y)=M_1(-2,4)$

If $M_2(x,\ y)$ is the mid-point of QR

$x=\frac{x_1+x_2}{2}=\frac{-2-8}{2}=\frac{-10}{2}=-5$ and $y=\frac{y_1+y_2}{2}=\frac{-4+2}{2}=-\frac{2}{2}=-1$

Hence $M_2(x,\ y)=M_2(-5,-1)$

Length of line segment joining M_1 and M_2.

$|M_1M_2|$ $=\sqrt{(-5+2)^2+(-1-4)^2}$

$=\sqrt{(-3)^2+(-5)^2}$

$=\sqrt{9+25}$

$=\sqrt{34}$

Length of PQ

PQ $=\sqrt{(-2-4)^2+(-4-6)^2}$

$=\sqrt{(-6)^2+(-10)^2}$

$=\sqrt{36+100}$

$=\sqrt{136}$

$=\sqrt{4(34)}$

$=2\sqrt{34}$

dividing by 2

$\frac{1}{2}PQ$ $=\frac{2\sqrt{34}}{2}$

$=\sqrt{34}$

$=|M_1M_2|$

Hence proved.