Exercise 7.4

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Things To know:

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

In Problems 1-6, simplify each expressions to a single trigonometric function.

Q. 1: $\frac{sin^2x}{cos^2x}$

$$\frac{sin^2x}{cos^2x} = \left(\frac{sinx}{cosx}\right)^2$$
$$= (tanx)^2$$
$$= tan^2x$$

Q. 2: $\boldsymbol{tanx\ sinx\ sec\ x}$

$$tanx\ sinx\ secx = tanx\ sinx\ \left(\frac{1}{cosx}\right)$$
$$= tanx\ tanx$$
$$= (tanx)^2$$
$$= tan^2x$$

Q. 3: $\frac{tanx}{secx}$

$$\frac{tanx}{secx} = \frac{1}{secx}.tanx$$
$$= cosx\ \left(\frac{sinx}{cosx}\right)$$
$$= sinx$$

Q. 4: $\boldsymbol{1 - cos^2x}$

$$1 - cos^2x = cos^2x + sin^2x - cos^2x$$
$$= sin^2x$$

Q. 5: $\boldsymbol{sec^2x - 1}$

$$sec^2x - 1 = sec^2x - (sec^2x - tan^2x)$$
$$= sec^2x - sec^2x + tan^2x$$
$$= tan^2x$$

Q. 6: $\boldsymbol{sin^2x.cot^2x}$

$$sin^2x.cot^2x = sin^2x.\frac{cos^2x}{sin^2x}$$
$$= cos^2x$$

In problems 7-24, verify the identities.

Q. 7: $\boldsymbol{(1 - sin\theta)(1 + sin\theta) = cos^2x}$

$$L.H.S = (1 - sin\theta)(1 + sin\theta)$$
$$= 1 - sin^2x$$
$$= cos^2x$$
$$= R.H.S$$

Q. 8: $\frac{\sin\theta+\cos\theta}{\cos\theta} = 1 + \tan\theta$

$$L.H.S = \frac{\sin\theta+\cos\theta}{\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$$
$$= \tan\theta + 1$$
$$= R.H.S$$

Q. 9: $(\tan\theta + \cot\theta)\tan\theta = \sec^2\theta$

$$L.H.S = \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\frac{\sin\theta}{\cos\theta}$$
$$= \left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)\frac{\sin\theta}{\cos\theta}$$
$$= \left(\frac{1}{\cos\theta}\right)\frac{1}{\cos\theta}$$
$$= (\sec\theta)\sec\theta$$
$$= \sec^2\theta$$
$$= R.H.S$$

Q. 10: $(\cot\theta + \operatorname{cosec}\theta)(\tan\theta - \sin\theta) = \sec\theta - \cos\theta$

$$L.H.S = \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta} - \sin\theta\right)$$
$$= \left(\frac{\cos\theta+1}{\sin\theta}\right)\left(\frac{\sin\theta-\sin\theta\cos\theta}{\cos\theta}\right)$$
$$= \left(\frac{1+\cos\theta}{\sin\theta}\right)\left(\frac{\sin\theta(1-\cos\theta)}{\cos\theta}\right)$$
$$= (1+\cos\theta)\left(\frac{1-\cos\theta}{\cos\theta}\right)$$
$$= \frac{1-\cos^2\theta}{\cos\theta}$$
$$= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$$
$$= \sec\theta - \cos\theta$$
$$= R.H.S$$

Q. 11: $\frac{\sin\theta+\cos\theta}{\tan^2\theta-1} = \frac{\cos^2\theta}{\sin\theta-\cos\theta}$

$$L.H.S = \frac{\sin\theta+\cos\theta}{\tan^2\theta-1}$$
$$= \frac{\sin\theta+\cos\theta}{\frac{\sin^2\theta}{\cos^2\theta}-1}$$
$$= \frac{\sin\theta+\cos\theta}{\frac{\sin^2\theta-\cos^2\theta}{\cos^2\theta}}$$
$$= (\cos^2\theta)\left(\frac{\sin\theta+\cos\theta}{\sin^2\theta-\cos^2\theta}\right)$$
$$= (\cos^2\theta)\left(\frac{\sin\theta+\cos\theta}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}\right)$$
$$= \frac{\cos^2\theta}{\sin\theta-\cos\theta}$$
$$= R.H.S$$

Q. 12: $\frac{\cos^2\theta}{\sin\theta} + \sin\theta = \operatorname{cosec}\theta$

$$L.H.S = \frac{\cos^2\theta}{\sin\theta} + \sin\theta$$

$$= \frac{cos^2\theta + sin^2\theta}{sin\theta}$$
$$= \frac{1}{sin\theta}$$
$$= cosec\theta$$
$$= R.H.S$$

Q. 13: $\boldsymbol{sec\theta - cos\theta = tan\theta sin\theta}$

$L.H.S = sec\theta - cos\theta$
$$= \frac{1}{cos\theta} - cos\theta$$
$$= \frac{1 - cos^2\theta}{cos\theta}$$
$$= \frac{sin^2\theta}{cos\theta}$$
$$= \frac{sin\theta}{cos\theta}.sin\theta$$
$$= tan\theta.sin\theta$$
$$= R.H.S$$

Q. 14: $\boldsymbol{\frac{sin^2\theta}{cos\theta} + cos\theta = sec\theta}$

$L.H.S = \frac{sin^2\theta}{cos\theta} + cos\theta$
$$= \frac{sin^2\theta + cos^2\theta}{cos\theta}$$
$$= \frac{1}{cos\theta}$$
$$= sec\theta$$
$$= R.H.S$$

Q. 15: $\boldsymbol{tan\theta + cot\theta = sec\theta cosec\theta}$

$L.H.S = tan\theta + cot\theta$
$$= \frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}$$
$$= \frac{sin^2\theta + cos^2\theta}{cos\theta sin\theta}$$
$$= \frac{1}{cos\theta sin\theta}$$
$$= sec\theta.cosec\theta$$
$$= R.H.S$$

Q. 16: $\boldsymbol{(tan\theta + cot\theta)(cos\theta + sin\theta) = sec\theta + cosec\theta}$

$L.H.S = (tan\theta + cot\theta)(cos\theta + sin\theta)$
$$= \left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)(cos\theta + sin\theta)$$
$$= \left(\frac{sin^2\theta + cos^2\theta}{cos\theta sin\theta}\right)(cos\theta + sin\theta)$$
$$= \left(\frac{1}{cos\theta sin\theta}\right)(cos\theta + sin\theta)$$
$$= \frac{cos\theta}{cos\theta sin\theta} + \frac{sin\theta}{cos\theta sin\theta}$$
$$= \frac{1}{sin\theta} + \frac{1}{cos\theta}$$
$$= cosec\theta + sec\theta$$
$$= R.H.S$$

Q. 17: $\boldsymbol{sin\theta(tan\theta + cot\theta) = sec\theta}$

$$L.H.S = (tan\theta + cot\theta)sin\theta$$
$$= \left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right) sin\theta$$
$$= \left(\frac{sin^2\theta + cos^2\theta}{cos\theta sin\theta}\right) sin\theta$$
$$= \left(\frac{1}{cos\theta sin\theta}\right) sin\theta$$
$$= \frac{1}{cos\theta}$$
$$= sec\theta$$
$$= R.H.S$$

Q. 18: $\boldsymbol{\frac{1+cos\theta}{sin\theta} + \frac{sin\theta}{1+cos\theta} = 2cosec\theta}$

$$L.H.S = \frac{1+cos\theta}{sin\theta} + \frac{sin\theta}{1+cos\theta}$$
$$= \frac{(1+cos\theta)^2 + sin^2\theta}{sin\theta(1+cos\theta)}$$
$$= \frac{1+cos^2\theta+2cos\theta+sin^2\theta}{sin\theta(1+cos\theta)}$$
$$= \frac{1+2cos\theta+cos^2\theta+sin^2\theta}{sin\theta(1+cos\theta)}$$
$$= \frac{1+2cos\theta+1}{sin\theta(1+cos\theta)}$$
$$= \frac{2+2cos\theta}{sin\theta(1+cos\theta)}$$
$$= \frac{2(1+cos\theta)}{sin\theta(1+cos\theta)}$$
$$= \frac{2}{sin\theta}$$
$$= 2cosec\theta$$
$$= R.H.S$$

Q. 19: $\boldsymbol{\frac{1}{1-cos\theta} + \frac{1}{1+cos\theta} = 2cosec^2\theta}$

$$L.H.S = \frac{1}{1-cos\theta} + \frac{1}{1+cos\theta}$$
$$= \frac{1+cos\theta+1-cos\theta}{(1-cos\theta)(1+cos\theta)}$$
$$= \frac{2}{1-cos^2\theta}$$
$$= \frac{2}{sin^2\theta}$$
$$= 2cosec^2\theta$$
$$= R.H.S$$

Q. 20: $\boldsymbol{\frac{1+sin\theta}{1-sin\theta} - \frac{1-sin\theta}{1+sin\theta} = 4tan\theta sec\theta}$

$$L.H.S = \frac{1+sin\theta}{1-sin\theta} - \frac{1-sin\theta}{1+sin\theta}$$
$$= \frac{(1+sin\theta)^2-(1-sin\theta)^2}{(1-sin\theta)(1+sin\theta)}$$
$$= \frac{1+sin^2\theta+2sin\theta-1-sin^2\theta+2sin\theta}{1-sin^2\theta}$$
$$= \frac{4sin\theta}{1-sin^2\theta}$$

$$= \frac{4sin\theta}{cos^2\theta}$$
$$= \frac{4sin\theta}{cos\theta(cos\theta)}$$
$$= 4tan\theta sec\theta$$
$$= R.H.S$$

Q. 21: $\boldsymbol{sin^3\theta = sin\theta - sin\theta cos^2\theta}$

$R.H.S = sin\theta - sin\theta cos^2\theta$
$= sin\theta(1 - cos^2\theta)$
$= sin\theta(sin^2\theta)$
$= sin^3\theta$
$= L.H.S$

Q. 22: $\boldsymbol{cos^4\theta - sin^4\theta = cos^2\theta - sin^2\theta}$

$L.H.S = cos^4\theta - sin^4\theta$
$= (cos^2\theta)^2 - (sin^2\theta)^2$
$= (cos^2\theta - sin^2\theta)(cos^2\theta + sin^2\theta)$
$= (cos^2\theta - sin^2\theta)(1)$
$= R.H.S$

Q. 23: $\boldsymbol{\sqrt{\frac{1+cos\theta}{1-cos\theta}} = \frac{sin\theta}{1-cos\theta}}$

$L.H.S = \sqrt{\frac{1+cos\theta}{1-cos\theta}}$
$= \sqrt{\frac{(1+cos\theta)(1-cos\theta)}{(1-cos\theta)(1-cos\theta)}}$
$= \sqrt{\frac{(1-cos^2\theta)}{(1-cos\theta)^2}}$
$= \frac{\sqrt{sin^2\theta}}{\sqrt{(1-cos\theta)^2}}$
$= \frac{sin\theta}{1-cos\theta}$
$= R.H.S$

Q. 23: $\boldsymbol{\sqrt{\frac{sec\theta+1}{sec\theta-1}} = \frac{sec\theta+1}{tan\theta}}$

$L.H.S = \sqrt{\frac{sec\theta+1}{sec\theta-1}}$
$= \sqrt{\frac{(sec\theta+1)(sec\theta+1)}{(sec\theta-1)(sec\theta-1)}}$
$= \sqrt{\frac{(sec\theta+1)^2}{(sec^2\theta-1)}}$
$= \frac{\sqrt{(sec\theta+1)^2}}{\sqrt{tan^2\theta}}$
$= \frac{sec\theta+1}{tan\theta}$
$= R.H.S$