Exercise 1.6

Q. 1: Use matrices, if possible, to solve the following systems of linear equations by:

(i) the matrix inversion method (ii) the Cramer's rule

(i) $2x - 2y = 4$; $3x + 2y = 6$

Matrix Inversion Method:

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

So, $AX = B$

And $X = A^{-1}B$ -- (a)

$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = (2 \times 2) - (3 \times -2) = 4 + 6 = 10$

$A^{-1} = \frac{1}{|A|} Adj(A)$

$= \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 2 \times 4 + 2 \times 6 \\ -3 \times 4 + 2 \times 6 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 8 + 12 \\ -12 + 12 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} \frac{20}{10} \\ \frac{0}{10} \end{bmatrix}$

$= \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

So, $x = 2$, $y = 0$

Cramer's Rule:

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = (2 \times 2) - (3 \times -2) = 4 + 6 = 10$

$x = \frac{A_x}{|A|} = \frac{\begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}}{10} = \frac{(4 \times 2)-(6 \times -2)}{10} = \frac{8+12}{10} = \frac{20}{10} = 2$

$y = \frac{A_y}{|A|} = \frac{\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}}{10} = \frac{(2 \times 6)-(3 \times 4)}{10} = \frac{12-12}{10} = \frac{0}{10} = 0$

So, $x = 2$, $y = 0$

(ii) $2x + y = 3$; $6x + 5y = 1$

Matrix Inversion Method:

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

So, $AX = B$

And $X = A^{-1}B$ -- (a)

$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = (2 \times 5) - (6 \times 1) = 10 - 6 = 4$

$A^{-1} = \frac{1}{|A|} Adj(A)$

$= \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 5 \times 3 + -1 \times 1 \\ -6 \times 3 + 2 \times 1 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 15 - 1 \\ -18 + 2 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$

$= \begin{bmatrix} \frac{14}{4} \\ -\frac{16}{4} \end{bmatrix}$

$= \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$

So, $x = \frac{7}{2}$, $y = -4$

Cramer's Rule:

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = (2 \times 5) - (6 \times 1) = 10 - 6 = 4$

$x = \frac{A_x}{|A|} = \frac{\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}}{4} = \frac{(3 \times 5) - (1 \times 1)}{4} = \frac{15 - 1}{4} = \frac{14}{4} = \frac{7}{2}$

$y = \frac{A_y}{|A|} = \frac{\begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}}{4} = \frac{(2 \times 1) - (6 \times 3)}{4} = \frac{2 - 18}{4} = \frac{-16}{4} = -4$

So, $x = \frac{7}{2}$, $y = -4$

(iii) $4x + 2y = 8$; $3x - y = -1$

Matrix Inversion Method:

$$\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}8\\-1\end{bmatrix}$$

Let $A = \begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}$, $X = \begin{bmatrix}x\\y\end{bmatrix}$, $B = \begin{bmatrix}8\\-1\end{bmatrix}$

So, $AX = B$

And $X = A^{-1}B$ -- (a)

$|A| = \begin{vmatrix}4 & 2\\3 & -1\end{vmatrix} = (4 \times -1) - (3 \times 2) = -4 - 6 = -10$

$A^{-1} = \frac{1}{|A|}Adj(A)$

$= \frac{1}{-10}\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

$X = A^{-1}B$

$\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{-10}\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}\begin{bmatrix}8\\-1\end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix}-1 \times 8 + -2 \times -1\\-3 \times 8 + 4 \times -1\end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix}-8 + 2\\-24 - 4\end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix}-6\\-28\end{bmatrix}$

$= \begin{bmatrix}\frac{-6}{-10}\\\frac{-28}{-10}\end{bmatrix}$

$= \begin{bmatrix}\frac{3}{5}\\\frac{14}{5}\end{bmatrix}$

So, $x = \frac{3}{5}$, $y = \frac{14}{5}$

Cramer's Rule:

$$\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}8\\-1\end{bmatrix}$$

Let $A = \begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}$, $X = \begin{bmatrix}x\\y\end{bmatrix}$, $B = \begin{bmatrix}8\\-1\end{bmatrix}$

$|A| = \begin{vmatrix}4 & 2\\3 & -1\end{vmatrix} = (4 \times -1) - (3 \times 2) = -4 - 6 = -10$

$x = \frac{A_x}{|A|} = \frac{\begin{vmatrix}8 & 2\\-1 & -1\end{vmatrix}}{-10} = \frac{(8 \times -1)-(2 \times -1)}{-10} = \frac{-8+2}{-10} = \frac{-6}{-10} = \frac{3}{5}$

$y = \frac{A_y}{|A|} = \frac{\begin{vmatrix}4 & 8\\3 & -1\end{vmatrix}}{-10} = \frac{(4 \times -1)-(3 \times 8)}{-10} = \frac{-4-24}{-10} = \frac{-28}{-10} = \frac{14}{5}$

So, $x = \frac{3}{5}$, $y = \frac{14}{5}$

(iv) 3x - 2y = -6 ; 5x - 2y = -10

Matrix Inversion Method:

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$

So, AX = B

And $X = A^{-1}B$ -- (a)

$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$ = (3 x -2) – (5 x -2) = -6 + 10 = 4

$A^{-1} = \frac{1}{|A|} Adj(A)$

$= \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -2 \times -6 + 2 \times -10 \\ -5 \times -6 + 3 \times -10 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 12 - 20 \\ 30 - 30 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -8 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-8}{4} \\ \frac{0}{4} \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

So, $x = -2$, $y = 0$

Cramer's Rule:

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$ = (3 x -2) – (5 x -2) = -6 + 10 = 4

$$x = \frac{A_x}{|A|} = \frac{\begin{vmatrix} -6 & -2 \\ -10 & -2 \end{vmatrix}}{4} = \frac{(-6 \times -2)-(-10 \times -2)}{4} = \frac{12-20}{4} = \frac{-8}{4} = -2$$

$$y = \frac{A_y}{|A|} = \frac{\begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}}{4} = \frac{(3 \times -10)-(5 \times -6)}{4} = \frac{-30+30}{4} = \frac{0}{4} = 0$$

So, x = -2, y = 0

(v) 3x - 2y = 4 ; -6x + 4y = 7

Matrix Inversion Method:

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Let A = $\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 4 \\ 7 \end{bmatrix}$

So, AX = B

And X = A^{-1}B -- (a)

$|A|$ = $\begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix}$ = (3 x 4) – (-6 x -2) = 12 - 12 = 0

So, Solution is not possible

Cramer's Rule:

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Let A = $\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 4 \\ 7 \end{bmatrix}$

$|A|$ = $\begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix}$ = (3 x 4) – (-6 x -2) = 12 - 12 = 0

So, Solution is not possible

(vi) 4x + y = 9 ; -3x - y = -5

Matrix Inversion Method:

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Let A = $\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

So, AX = B

And X = A^{-1}B -- (a)

$|A|$ = $\begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$ = (4 x -1) – (-3 x 1) = -4 + 1 = -1

A^{-1} = $\frac{1}{|A|}Adj(A)$

= $\frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

X = A^{-1}B

$\begin{bmatrix} x \\ y \end{bmatrix}$ = $\frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

= $\frac{1}{-1}\begin{bmatrix} -1 \times 9 + -1 \times -5 \\ 3 \times 9 + 4 \times -5 \end{bmatrix}$

= $\frac{1}{-1}\begin{bmatrix} -9 + 5 \\ 27 - 20 \end{bmatrix}$

= $\frac{1}{-1}\begin{bmatrix} -4 \\ 7 \end{bmatrix}$

= $\begin{bmatrix} \frac{-4}{-1} \\ \frac{7}{-1} \end{bmatrix}$

$= \begin{bmatrix} 4 \\ -7 \end{bmatrix}$

So, x = 4, y = -7

Cramer's Rule:

$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

Let A = $\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$|A|$ = $\begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$ = (4 x -1) – (-3 x 1) = -4 + 1 = -1

x $= \frac{A_x}{|A|}$ $= \frac{\begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix}}{-1}$ $= \frac{(9\times -1)-(-5\times 1)}{-1}$ $= \frac{-9+5}{-1}$ $= \frac{-4}{-1}$ = 4

y $= \frac{A_y}{|A|}$ $= \frac{\begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix}}{-1}$ $= \frac{(4\times -5)-(-3\times 9)}{-1}$ $= \frac{-20+27}{-1}$ $= \frac{7}{-1}$ $= -7$

So, x = 4, y = -7

(vii) 2x - 2y = 4 ; -5x - 2y = -10

Matrix Inversion Method:

$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$

Let A = $\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 4 \\ -10 \end{bmatrix}$

So, AX = B

And X = A^{-1}B -- (a)

$|A|$ = $\begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$ = (2 x -2) – (-5 x -2) = -4 - 10 = -14

A^{-1} $= \frac{1}{|A|}Adj(A)$

$= \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

X = A^{-1}B

$\begin{bmatrix} x \\ y \end{bmatrix}$ $= \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} -2\times 4 + 2\times -10 \\ 5\times 4 + 2\times -10 \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} -8-20 \\ 20-20 \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{-14} \end{bmatrix}$

$= \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

So, x = 2, y = 0

Cramer's Rule:

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

Let A = $\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 4 \\ -10 \end{bmatrix}$

$|A|$ = $\begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$ = (2 x -2) – (-5 x -2) = -4 - 10 = -14

x = $\frac{A_x}{|A|}$ = $\frac{\begin{vmatrix} 4 & -2 \\ -10 & -2 \end{vmatrix}}{-14}$ = $\frac{(4\times -2)-(-10\times -2)}{-14}$ = $\frac{-8-20}{-14}$ = $\frac{-28}{-14}$ = 2

y = $\frac{A_y}{|A|}$ = $\frac{\begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix}}{-14}$ = $\frac{(2\times -10)-(-5\times 4)}{-14}$ = $\frac{-20+20}{-14}$ = $\frac{0}{-14}$ = 0

So, x = 2, y = 0

(viii) 3x - 4y = 4 ; x + 2y = 8

Matrix Inversion Method:

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Let A = $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 4 \\ 8 \end{bmatrix}$

So, AX = B

And X = A^{-1}B -- (a)

$|A|$ = $\begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$ = (3 x 2) – (1 x -4) = 6 + 4 = 10

A^{-1} = $\frac{1}{|A|} Adj(A)$

= $\frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

X = A^{-1}B

$\begin{bmatrix} x \\ y \end{bmatrix}$ = $\frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \end{bmatrix}$

= $\frac{1}{10}\begin{bmatrix} 2\times 4 + 4\times 8 \\ -1\times 4 + 3\times 8 \end{bmatrix}$

= $\frac{1}{10}\begin{bmatrix} 8+32 \\ -4+24 \end{bmatrix}$

= $\frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix}$

= $\begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$

= $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

So, x = 4, y = 2

Cramer's Rule:

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$ = (3 x 2) – (1 x -4) = 6 + 4 = 10

$x = \frac{A_x}{|A|} = \frac{\begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}}{10} = \frac{(4\times 2)-(8\times -4)}{10} = \frac{8+32}{10} = \frac{40}{10} = 4$

$y = \frac{A_y}{|A|} = \frac{\begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}}{10} = \frac{(3\times 8)-(1\times 4)}{10} = \frac{24-4}{10} = \frac{20}{10} = 2$

So, x = 4, y = 2

Solve the following word problems by using

(i) Matrix inversion method (ii) Crammer's rule

Q. 2: The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150 cm. Find the dimensions of the rectangle.

Let the width of rectangle = x

And the length of rectangle = y

According to 1st condition

y = 4x

-4x + y = 0 ---------------------------- (i)

According to 2nd condition the formula for perimeter of rectangle is P = 2(length + width)

150 = 2(x +y)

2x + 2y = 150 -------------------------- (ii)

Matrix Inversion Method:

$$\begin{bmatrix} -4 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

Let $A = \begin{bmatrix} -4 & 1 \\ 2 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$

So, AX = B

And $X = A^{-1}B$ -- (a)

$|A| = \begin{vmatrix} -4 & 1 \\ 2 & 2 \end{vmatrix}$ = (-4 x 2) – (2 x 1) = -8 - 2 = -10

$A^{-1} = \frac{1}{|A|} Adj(A)$

$= \frac{1}{-10}\begin{bmatrix} 2 & -1 \\ -2 & -4 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} 2 & -1 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} 0 \\ 150 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} 2\times 0 + -1\times 150 \\ -2\times 0 + -4\times 150 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} 0 - 150 \\ 0 - 600 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix}-150\\-600\end{bmatrix}$

$= \begin{bmatrix}\frac{-150}{-10}\\\frac{-600}{-10}\end{bmatrix}$

$= \begin{bmatrix}15\\60\end{bmatrix}$

So, width = 15 cm, length = 60 cm

Cramer's Rule:

$\begin{bmatrix}-4 & 1\\2 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\150\end{bmatrix}$

Let A = $\begin{bmatrix}-4 & 1\\2 & 2\end{bmatrix}$, X = $\begin{bmatrix}x\\y\end{bmatrix}$, B = $\begin{bmatrix}0\\150\end{bmatrix}$

$|A|$ = $\begin{vmatrix}-4 & 1\\2 & 2\end{vmatrix}$ = (-4 x 2) – (2 x 1) = -8 - 2 = -10

x $= \frac{A_x}{|A|}$ $= \frac{\begin{vmatrix}0 & 1\\150 & 2\end{vmatrix}}{-10}$ $= \frac{(0\times 2)-(150\times 1)}{-10}$ $= \frac{0-150}{-10}$ $= \frac{-150}{-10}$ = 15

y $= \frac{A_y}{|A|}$ $= \frac{\begin{vmatrix}-4 & 0\\2 & 150\end{vmatrix}}{-10}$ $= \frac{(-4\times 150)-(2\times 0)}{-10}$ $= \frac{-600-0}{-10}$ $= \frac{-600}{-10}$ = 60

So, width = 15 cm, length = 60 cm

Q. 3: Two sides of a rectangle differ by 3.5cm. Find the dimensions of the rectangle if its perimeter is 67cm.

Let the width of rectangle = x

And the length of rectangle = y

According to 1st condition

y = x + 3.5

-x + y = 3.5 ----------------------------- (i)

According to 2nd condition the formula for perimeter of rectangle is P = 2(length + width)

67 = 2(x +y)

2x + 2y = 67 ------------------------------ (ii)

Matrix Inversion Method:

$\begin{bmatrix}-1 & 1\\2 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3.5\\67\end{bmatrix}$

Let A = $\begin{bmatrix}-1 & 1\\2 & 2\end{bmatrix}$, X = $\begin{bmatrix}x\\y\end{bmatrix}$, B = $\begin{bmatrix}3.5\\67\end{bmatrix}$

So, AX = B

And $X = A^{-1}B$ -- (a)

$|A|$ = $\begin{vmatrix}-1 & 1\\2 & 2\end{vmatrix}$ = (-1 x 2) – (2 x 1) = -2 - 2 = -4

A^{-1} $= \frac{1}{|A|}Adj(A)$

$= \frac{1}{-4}\begin{bmatrix}-1 & 1\\2 & 2\end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} 2 & -1 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} 2 \times 3.5 + -1 \times 67 \\ -2 \times 3.5 + -1 \times 67 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} 7 - 67 \\ -7 - 67 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -60 \\ -74 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-60}{-4} \\ \frac{-74}{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 18.5 \end{bmatrix}$$

So, width = 15 cm, length = 18.5 cm

<u>Cramer's Rule</u>:

$$\begin{bmatrix} -1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

Let $A = \begin{bmatrix} -1 & 1 \\ 2 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix}$ = (-1 x 2) – (2 x 1) = -2 - 2 = -4

$$x = \frac{A_x}{|A|} = \frac{\begin{vmatrix} 3.5 & 1 \\ 67 & 2 \end{vmatrix}}{-4} = \frac{(3.5 \times 2)-(67 \times 1)}{-4} = \frac{7-67}{-4} = \frac{-60}{-4} = 15$$

$$y = \frac{A_y}{|A|} = \frac{\begin{vmatrix} -1 & 3.5 \\ 2 & 67 \end{vmatrix}}{-4} = \frac{(-1 \times 67)-(2 \times 3.5)}{-4} = \frac{-67-7}{-4} = \frac{-74}{-4} = 18.5$$

So, width = 15 cm, length = 18.5 cm

Q. 4: The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.

Let 1^{st} angle of triangle = x

and 2^{nd} angle of triangle = y

then 3^{rd} angle of triangle = x (because triangle is isosceles)

According to 1^{st} condition

y = 2x - 16

-2x + y = -16 ----------------------------- (i)

As we know

sum of all angles of a triangle = 180°

So,

x + y + x = 180

2x + y = 180 ------------------------------ (ii)

<u>Matrix Inversion Method</u>:

$\begin{bmatrix} -2 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ 180 \end{bmatrix}$

Let A = $\begin{bmatrix} -2 & 1 \\ 2 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} -16 \\ 180 \end{bmatrix}$

So, AX = B

And X = A^{-1}B -- (a)

$|A|$ = $\begin{vmatrix} -2 & 1 \\ 2 & 1 \end{vmatrix}$ = (-2 x 1) – (2 x 1) = -2 - 2 = -4

A^{-1} = $\frac{1}{|A|}Adj(A)$

= $\frac{1}{-4}\begin{bmatrix} 1 & -1 \\ -2 & -2 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

X = A^{-1}B

$\begin{bmatrix} x \\ y \end{bmatrix}$ = $\frac{1}{-4}\begin{bmatrix} 1 & -1 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} -16 \\ 180 \end{bmatrix}$

= $\frac{1}{-4}\begin{bmatrix} 1 \times -16 + -1 \times 180 \\ -2 \times -16 + -2 \times 180 \end{bmatrix}$

= $\frac{1}{-4}\begin{bmatrix} -16 - 180 \\ 32 - 360 \end{bmatrix}$

= $\frac{1}{-4}\begin{bmatrix} -196 \\ -328 \end{bmatrix}$

= $\begin{bmatrix} \frac{-196}{-4} \\ \frac{-328}{-4} \end{bmatrix}$

= $\begin{bmatrix} 49 \\ 82 \end{bmatrix}$

So, 1st angle = 49°, 2nd angle = 82°, 3rd angle = 49°

<u>Cramer's Rule</u>:

$\begin{bmatrix} -2 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ 180 \end{bmatrix}$

Let A = $\begin{bmatrix} -2 & 1 \\ 2 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} -16 \\ 180 \end{bmatrix}$

$|A|$ = $\begin{vmatrix} -2 & 1 \\ 2 & 1 \end{vmatrix}$ = (-2 x 1) – (2 x 1) = -2 - 2 = -4

x = $\frac{A_x}{|A|}$ = $\frac{\begin{vmatrix} -16 & 1 \\ 180 & 1 \end{vmatrix}}{-4}$ = $\frac{(-16 \times 1)-(180 \times 1)}{-4}$ = $\frac{-16-180}{-4}$ = $\frac{-196}{-4}$ = 49

y = $\frac{A_y}{|A|}$ = $\frac{\begin{vmatrix} -2 & -16 \\ 2 & 180 \end{vmatrix}}{-4}$ = $\frac{(-2 \times 180)-(2 \times -16)}{-4}$ = $\frac{-360+32}{-4}$ = $\frac{-328}{-4}$ = 82

So, 1st angle = 49°, 2nd angle = 82°, 3rd angle = 49°

Q. 5: One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of right triangle.

Let 1st angle of triangle = x

and 2nd angle of triangle = y

then 3^{rd} angle of triangle = 90^o (because triangle is right triangle)

According to 1^{st} condition

$$y = 2x + 12$$

$$-2x + y = 12 \text{ ----------------------------- (i)}$$

As we know

sum of all angles of a triangle = 180^o

So,

$$x + y + 90 = 180$$

$$x + y = 90 \text{ ----------------------------- (ii)}$$

<u>Matrix Inversion Method</u>:

$$\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

Let A = $\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 12 \\ 90 \end{bmatrix}$

So, AX = B

And $X = A^{-1}B$ -- (a)

$|A| = \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix}$ = (-2 x 1) – (1 x 1) = -2 - 1 = -3

$A^{-1} = \frac{1}{|A|}Adj(A)$

$= \frac{1}{-3}\begin{bmatrix} 1 & -1 \\ -1 & -2 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} 1 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 12 \\ 90 \end{bmatrix}$

$= \frac{1}{-3}\begin{bmatrix} 1 \times 12 + -1 \times 90 \\ -1 \times 12 + -2 \times 90 \end{bmatrix}$

$= \frac{1}{-3}\begin{bmatrix} 12 - 90 \\ -12 - 180 \end{bmatrix}$

$= \frac{1}{-3}\begin{bmatrix} -78 \\ -192 \end{bmatrix}$

$= \begin{bmatrix} \frac{-78}{-3} \\ \frac{-192}{-3} \end{bmatrix}$

$= \begin{bmatrix} 26 \\ 64 \end{bmatrix}$

So, 1^{st} angle = 26^o, 2^{nd} angle = 64^o

<u>Cramer's Rule</u>:

$$\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

Let A = $\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 12 \\ 90 \end{bmatrix}$

$|A| = \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix}$ = (-2 x 1) – (1 x 1) = -2 - 1 = -3 x $= \frac{A_x}{|A|}$ $= \frac{\begin{vmatrix} 12 & 1 \\ 90 & 1 \end{vmatrix}}{-3}$ $= \frac{(12\times1)-(90\times1)}{-3}$ $= \frac{12-90}{-3}$ $= \frac{-78}{-3}$ = 26

y $= \frac{A_y}{|A|}$ $= \frac{\begin{vmatrix} -2 & 12 \\ 1 & 90 \end{vmatrix}}{-3}$ $= \frac{(-2\times90)-(1\times12)}{-3}$ $= \frac{-180-12}{-3}$ $= \frac{-192}{-3}$ = 64

So, 1st angle = 26°, 2nd angle = 64°

Q. 6: Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Let the speed of 1st car = x

and the speed of 2nd car = y

According to 1st condition

x - y = 6

x - y = 6 ---------------------------- (i)

According to 2nd condition

Distance covered by 1st car + Distance covered by 2nd car = total distance covered

$4\frac{1}{2}x + 4\frac{1}{2}y = 600-123$

So,

4.5x + 4.5y = 477

4.5x + 4.5y = 477 ----------------------------- (ii)

Matrix Inversion Method:

$$\begin{bmatrix} 1 & -1 \\ 4.5 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

Let A = $\begin{bmatrix} 1 & -1 \\ 4.5 & 4.5 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 6 \\ 477 \end{bmatrix}$

So, AX = B

And X = $A^{-1}B$ -- (a)

$|A|$ = $\begin{vmatrix} 1 & -1 \\ 4.5 & 4.5 \end{vmatrix}$ = (1 x 4.5) – (4.5 x -1) = 4.5 + 4.5 = 9

A^{-1} $= \frac{1}{|A|} Adj(A)$

$= \frac{1}{9}\begin{bmatrix} 4.5 & 1 \\ -4.5 & 1 \end{bmatrix}$

Putting the value of A^{-1} in equ – (a)

X = $A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix}$ $= \frac{1}{9}\begin{bmatrix} 4.5 & 1 \\ -4.5 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 477 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 4.5\times6+1\times477 \\ -4.5\times6+1\times477 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 27+477 \\ -27+477 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 504 \\ 450 \end{bmatrix}$

$$= \begin{bmatrix} \frac{504}{9} \\ \frac{450}{9} \end{bmatrix}$$

$$= \begin{bmatrix} 56 \\ 50 \end{bmatrix}$$

So, the speed of 1st car = 56 km/hr, the speed of 2nd car = 50 km/hr

Cramer's Rule:

$$\begin{bmatrix} 1 & -1 \\ 4.5 & 4.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & -1 \\ 4.5 & 4.5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 477 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -1 \\ 4.5 & 4.5 \end{vmatrix}$ = (1 x 4.5) – (4.5 x -1) = 4.5 + 4.5 = 9

$x = \frac{A_x}{|A|} = \frac{\begin{vmatrix} 6 & -1 \\ 477 & 4.5 \end{vmatrix}}{9} = \frac{(6 \times 4.5)-(477 \times -1)}{9} = \frac{27+477}{9} = \frac{504}{9} = 56$

$y = \frac{A_y}{|A|} = \frac{\begin{vmatrix} 1 & 6 \\ 4.5 & 477 \end{vmatrix}}{9} = \frac{(1 \times 477)-(4.5 \times 6)}{9} = \frac{477-27}{9} = \frac{450}{9} = 50$

So, the speed of 1st car = 56 km/hr, the speed of 2nd car = 50 km/hr