Exercise 4.3

Q. 1: Express each of the following surf in the simplest form.

(i) $\sqrt{180}$

$= \sqrt{9 \times 4 \times 5}$

$= \sqrt{(3)^2 \times (2)^2 \times 5}$

$= \sqrt{(3)^2} \times \sqrt{(2)^2} \times \sqrt{5}$

$= 3 \times 2 \times \sqrt{5}$

$= 6\sqrt{5}$

(ii) $3\sqrt{162}$

$= 3\sqrt{2 \times 81}$

$= 3\sqrt{2 \times (9)^2}$

$= 3 \times \sqrt{(9)^2} \times \sqrt{2}$

$= 3 \times 9 \times \sqrt{2}$

$= 27\sqrt{2}$

(iii) $\frac{3}{4}\sqrt[3]{128}$

$= \frac{3}{4}\sqrt[3]{2 \times 64}$

$= \frac{3}{4}\sqrt[3]{2 \times (4)^3}$

$= \frac{3}{4}\sqrt[3]{2} \times \sqrt[3]{(4)^3}$

$= \frac{3}{4}\sqrt[3]{2} \times 4$

$= 3\sqrt[3]{2}$

(iv) $\sqrt[5]{96x^6y^7z^8}$

$= \sqrt[5]{3.32.x^5.x^1.y^5.y^2.z^5.z^3}$

$= \sqrt[5]{3.(2)^5.x^5.x^1.y^5.y^2.z^5.z^3}$

$= \sqrt[5]{(2)^5.x^5.y^5.z^5.3.x^1.y^2.z^3}$

$= \sqrt[5]{(2)^5}.\sqrt[5]{x^5}.\sqrt[5]{y^5}.\sqrt[5]{z^5}.\sqrt[5]{3xy^2z^3}$

$= 2.x.y.z.\sqrt[5]{3xy^2z^3}$

$= 2xyz\sqrt[5]{3xy^2z^3}$

Q. 2: Simplify

(i) $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

$= \frac{\sqrt{2\times3\times3}}{\sqrt{3}\sqrt{2}}$

$= \frac{\sqrt{2}\sqrt{3}\sqrt{3}}{\sqrt{3}\sqrt{2}}$

$= \sqrt{3}$

(ii) $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

$= \frac{\sqrt{7\times3}\sqrt{3\times3}}{\sqrt{7\times3\times3}}$

$= \frac{\sqrt{7}\sqrt{3}\sqrt{3}\sqrt{3}}{\sqrt{7}\sqrt{3}\sqrt{3}}$

$= \sqrt{3}$

(iii) $\sqrt[5]{243x^5y^{10}z^{15}}$

$= \sqrt[5]{(3)^5.x^5.y^{10}.z^{15}}$

$= \sqrt[5]{(3)^5}.\sqrt[5]{x^5}.\sqrt[5]{y^{10}}.\sqrt[5]{z^{15}}$

$= 3.x.y^2.z^3$

$= 3xy^2z^3$

(iv) $\frac{4}{5}\sqrt[3]{125}$ $= \frac{4}{5}\sqrt[3]{125}$

$= \frac{4}{5}\sqrt[3]{(5)^3}$

$= \frac{4}{5} \times 5$

$= 4$

(v) $\sqrt{21} \times \sqrt{7} \times \sqrt{3}$ $= \sqrt{3 \times 7} \times \sqrt{7} \times \sqrt{3}$

$= \sqrt{3} \times \sqrt{7} \times \sqrt{7} \times \sqrt{3}$

$= \sqrt{3} \times \sqrt{3} \times \sqrt{7} \times \sqrt{7}$

$= \left(\sqrt{3}\right)^2 \times \left(\sqrt{7}\right)^2$

$= 3 \times 7$

$= 21$

Q. 3: Simplify by combining similar terms

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$ $= \sqrt{3 \times 3 \times 5} - 3\sqrt{2 \times 2 \times 5} + 4\sqrt{5}$

$= \sqrt{(3)^2 \times 5} - 3\sqrt{(2)^2 \times 5} + 4\sqrt{5}$

$= \sqrt{(3)^2}.\sqrt{5} - 3\sqrt{(2)^2}.\sqrt{5} + 4\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$

$= (3 - 6 + 4)\sqrt{5}$

$= \sqrt{5}$

(ii) $4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300}$ $= 4\sqrt{2 \times 2 \times 3} + 5\sqrt{3 \times 3 \times 3} - 3\sqrt{3 \times 5 \times 5} + \sqrt{2 \times 2 \times 3 \times 5 \times 5}$

$= 4\sqrt{(2)^2 \times 3} + 5\sqrt{(3)^2 \times 3} - 3\sqrt{(5)^2 \times 3} + \sqrt{(2)^2 \times 3 \times (5)^2}$

$= 4\sqrt{(2)^2}.\sqrt{3} + 5\sqrt{(3)^2}.\sqrt{3} - 3\sqrt{(5)^2}.\sqrt{3} + \sqrt{(2)^2}.\sqrt{3}.\sqrt{(5)^2}$

$= 4 \times 2\sqrt{3} + 5 \times 3\sqrt{3} - 3 \times 5\sqrt{3} + 2 \times 5.\sqrt{3}$

$= 8\sqrt{3} + 15\sqrt{3} - 15\sqrt{3} + 10\sqrt{3}$

$= (8 + 10)\sqrt{3}$

$= 18\sqrt{3}$

(iii) $\sqrt{3}\left(2\sqrt{3} + 3\sqrt{3}\right)$ $= \sqrt{3}.\sqrt{3}(2 + 3)$

$= \left(\sqrt{3}\right)^2(5)$

$= 3(5)$

$= 15$

(iv) $2\left(6\sqrt{5} - 3\sqrt{5}\right)$ $= 2\sqrt{5}(6 - 3)$

$= 2\sqrt{5}(3)$

$= 6\sqrt{5}$

Q. 4: Simplify

(i) $\left(3 + \sqrt{3}\right)\left(3 - \sqrt{3}\right)$ $= \left((3)^2 - \left(\sqrt{3}\right)^2\right)$

$= (9 - 3)$

$= 6$

(ii) $\left(\sqrt{5} + \sqrt{3}\right)^2$ $= \left(\left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2 + 2\sqrt{5}\,\sqrt{3}\right)$

$= (5 + 3 + 2\sqrt{15})$

$= (8 + 2\sqrt{15})$

(iii) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$ $= \left((\sqrt{5})^2 - (\sqrt{3})^2\right)$

$= (5 - 3)$

$= 2$

(iv) $\left(\sqrt{2} + \frac{1}{\sqrt{3}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{3}}\right)$ $= \left((\sqrt{2})^2 - \left(\frac{1}{\sqrt{3}}\right)^2\right)$

$= \left(2 - \frac{1}{3}\right)$

$= \frac{6-1}{3}$

$= \frac{5}{3}$

(v) $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})(x + y)(x^2 + y^2)$ $= \left((\sqrt{x})^2 - (\sqrt{y})^2\right)(x + y)(x^2 + y^2)$

$= (x - y)(x + y)(x^2 + y^2)$

$= (x^2 - y^2)(x^2 + y^2)$

$= ((x^2)^2 - (y^2)^2)$

$= (x^4 - y^4)$