

**Exercise 5.2****Q. 1: Factorize**

$$\begin{aligned}
 \text{(i)} \quad x^4 + \frac{1}{x^4} - 3 &= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2 - 1 \\
 &= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2(x^2)\left(\frac{1}{x^2}\right) - 1 \\
 &= \left(x^2 - \frac{1}{x^2}\right)^2 - (1)^2 \\
 &= \left(x^2 - \frac{1}{x^2} + 1\right)\left(x^2 - \frac{1}{x^2} - 1\right) \\
 \text{(ii)} \quad 3x^4 + 12y^4 &= 3(x^4 + 4y^4) \\
 &= 3[(x^2)^2 + (2y^2)^2 + 2(x^2)(2y^2) - 2(x^2)(2y^2)] \\
 &= 3[(x^2 + 2y^2)^2 - (2xy)^2] \\
 &= 3(x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy) \\
 &= 3(x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2) \\
 \text{(iii)} \quad a^4 + 3a^2b^2 + 4b^4 &= a^4 + 3a^2b^2 + a^2b^2 - a^2b^2 + 4b^4 \\
 &= a^4 + 3a^2b^2 + a^2b^2 + 4b^4 - a^2b^2 \\
 &= a^4 + 4a^2b^2 + 4b^4 - a^2b^2 \\
 &= (a^2)^2 + 2(a^2)(2b^2) + (2b^2)^2 - a^2b^2 \\
 &= (a^2 + 2b^2)^2 - (ab)^2 \\
 &= (a^2 + 2b^2 - ab)(a^2 + 2b^2 + ab) \\
 \text{(iv)} \quad 4x^4 + 81 &= (2x^2)^2 + (9)^2 + 2(2x^2)(9) - 2(2x^2)(9) \\
 &= (2x^2)^2 + (9)^2 + 2(2x^2)(9) - 36x^2 \\
 &= (2x^2 + 9)^2 - (6x)^2 \\
 &= (2x^2 + 9 - 6x)(2x^2 + 9 + 6x) \\
 &= (2x^2 - 6x + 9)(2x^2 + 6x + 9) \\
 \text{(v)} \quad x^4 + x^2 + 25 &= (x^2)^2 + 2(x^2)(5) + (5)^2 - 2(x^2)(5) + x^2 \\
 &= (x^2)^2 + 2(x^2)(5) + (5)^2 - 10x^2 + x^2 \\
 &= (x^2)^2 + 2(x^2)(5) + (5)^2 - 9x^2 \\
 &= (x^2 + 5)^2 - (3x)^2 \\
 &= (x^2 + 5 + 3x)(x^2 + 5 - 3x) \\
 &= (x^2 + 3x + 5)(x^2 - 3x + 5) \\
 \text{(vi)} \quad x^4 + 4x^2 + 16 &= (x^2)^2 + 2(x^2)(4) + (4)^2 - 2(x^2)(4) + 4x^2 \\
 &= (x^2)^2 + 2(x^2)(4) + (4)^2 - 8x^2 + 4x^2 \\
 &= (x^2)^2 + 2(x^2)(4) + (4)^2 - 4x^2 \\
 &= (x^2 + 4)^2 - (2x)^2 \\
 &= (x^2 + 4 - 2x)(x^2 + 4 + 2x) \\
 &= (x^2 - 2x + 4)(x^2 + 2x + 4)
 \end{aligned}$$

**Q. 2:**

$$\begin{aligned}
 \text{(i)} \quad x^2 + 14x + 48 &= x^2 + 6x + 8x + 48 \\
 &= x(x + 6) + 8(x + 6) \\
 &= (x + 6)(x + 8)
 \end{aligned}$$



$$\begin{aligned}
 \text{(ii)} \quad x^2 - 21x + 108 &= x^2 - 12x - 9x + 108 \\
 &= x(x - 12) - 9(x - 12) \\
 &= (x - 12)(x - 9) \\
 \text{(iii)} \quad x^2 - 11x - 42 &= x^2 + 3x - 14x - 42 \\
 &= x(x + 3) - 14(x + 3) \\
 &= (x + 3)(x - 14) \\
 \text{(iv)} \quad x^2 + x - 132 &= x^2 + 12x - 11x - 132 \\
 &= x(x + 12) - 11(x + 12) \\
 &= (x + 12)(x - 11)
 \end{aligned}$$

**Q. 3:**

$$\begin{aligned}
 \text{(i)} \quad 4x^2 + 12x + 5 &= 4x^2 + 10x + 2x + 5 \\
 &= 2x(2x + 5) + 1(2x + 5) \\
 &= (2x + 5)(2x + 1) \\
 \text{(ii)} \quad 30x^2 + 7x - 15 &= 30x^2 + 25x - 18x - 15 \\
 &= 5x(6x + 5) - 3(6x + 5) \\
 &= (6x + 5)(5x - 3) \\
 \text{(iii)} \quad 24x^2 - 65x + 21 &= 24x^2 - 56x - 9x + 21 \\
 &= 8x(3x - 7) - 3(3x - 7) \\
 &= (3x - 7)(8x - 3) \\
 \text{(iv)} \quad 5x^2 - 16x - 21 &= 5x^2 - 21x + 5x - 21 \\
 &= x(5x - 21) + 1(5x - 21) \\
 &= (5x - 21)(x + 1) \\
 \text{(v)} \quad 4x^2 - 17xy + 4y^2 &= 4x^2 - 16xy - xy + 4y^2 \\
 &= 4x(x - 4y) - y(x - 4y) \\
 &= (x - 4y)(4x - y) \\
 \text{(vi)} \quad 3x^2 - 38xy - 13y^2 &= 3x^2 - 39xy + xy - 13y^2 \\
 &= 3x(x - 13y) + y(x - 13y) \\
 &= (x - 13y)(3x + y) \\
 \text{(vii)} \quad 5x^2 + 33xy - 14y^2 &= 5x^2 + 35xy - 2xy - 14y^2 \\
 &= 5x(x + 7y) - 2y(x + 7y) \\
 &= (x + 7y)(5x - 2y) \\
 \text{(viii)} \quad \left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4 &= \left(5x - \frac{1}{x}\right)^2 + 2\left(5x - \frac{1}{x}\right) + 2\left(5x - \frac{1}{x}\right) + 4 \\
 &= \left(5x - \frac{1}{x}\right)\left(5x - \frac{1}{x} + 2\right) + 2\left(5x - \frac{1}{x} + 2\right) \\
 &= \left(5x - \frac{1}{x} + 2\right)\left(5x - \frac{1}{x} + 2\right)
 \end{aligned}$$

**Q. 4:**

$$\begin{aligned}
 \text{(i)} \quad (x^2 + 5x + 4)(x^2 + 5x + 6) - 3 \\
 \text{let } x^2 + 5x &= y \\
 (y + 4)(y + 6) - 3 &= y^2 + 4y + 6y + 24 - 3 \\
 &= y^2 + 10y + 21
 \end{aligned}$$



$$\begin{aligned}
 &= y^2 + 7y + 3y + 21 \\
 &= y(y + 7) + 3(y + 7) \\
 &= (y + 7)(y + 3)
 \end{aligned}$$

as  $x^2 + 5x = y$ , So,

$$(x^2 + 5x + 4)(x^2 + 5x + 6) - 3 = (x^2 + 5x + 7)(x^2 + 5x + 3)$$

$$(ii) \quad (x^2 - 4x)(x^2 - 4x - 1) - 20$$

let  $x^2 - 4x = y$

$$\begin{aligned}
 y(y - 1) - 20 &= y^2 - y - 20 \\
 &= y^2 - 5y + 4y - 20 \\
 &= y(y - 5) + 4(y - 5) \\
 &= (y - 5)(y + 4)
 \end{aligned}$$

as  $x^2 - 4x = y$ , So,

$$\begin{aligned}
 (x^2 - 4x)(x^2 - 4x - 1) - 20 &= (x^2 - 4x - 5)(x^2 - 4x + 4) \\
 &= (x^2 - 5x + x - 5)(x^2 - 4x + 4) \\
 &= (x(x - 5) + 1(x - 5))(x^2 - 4x + 4) \\
 &= (x - 5)(x + 1)(x - 2)^2
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad (x + 2)(x + 3)(x + 4)(x + 5) - 15 &= (x + 2)(x + 5)(x + 3)(x + 4) - 15 \\
 &= (x^2 + 2x + 5x + 10)(x^2 + 3x + 4x + 12) - 15 \\
 &= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15
 \end{aligned}$$

let  $x^2 + 7x = y$

$$\begin{aligned}
 (y + 10)(y + 12) - 15 &= y^2 + 12y + 10y + 120 - 15 \\
 &= y^2 + 22y + 105 \\
 &= y^2 + 15y + 7y + 105 \\
 &= y(y + 15) + 7(y + 15) \\
 &= (y + 15)(y + 7)
 \end{aligned}$$

as  $x^2 + 7x = y$ , So,

$$(x + 2)(x + 3)(x + 4)(x + 5) - 15 = (x^2 + 7x + 15)(x^2 + 7x + 7)$$

$$\begin{aligned}
 (iv) \quad (x + 4)(x - 5)(x + 6)(x - 7) - 504 &= (x + 4)(x - 5)(x + 6)(x - 7) - 504 \\
 &= (x^2 - 5x + 4x - 20)(x^2 - 7x + 6x - 42) - 504 \\
 &= (x^2 - x - 20)(x^2 - x - 42) - 504
 \end{aligned}$$

let  $x^2 - x = y$

$$\begin{aligned}
 (y - 20)(y - 42) - 504 &= y^2 - 42y - 20y + 840 - 504 \\
 &= y^2 - 62y + 336 \\
 &= y^2 - 56y - 6y + 336 \\
 &= y(y - 56) - 6(y - 56) \\
 &= (y - 56)(y - 6)
 \end{aligned}$$

as  $x^2 - x = y$ , So,

$$\begin{aligned}
 (x + 4)(x - 5)(x + 6)(x - 7) - 504 &= (x^2 - x - 56)(x^2 - x - 6) \\
 &= (x^2 - 8x + 7x - 56)(x^2 - 3x + 2x - 6) \\
 &= (x(x - 8) + 7(x - 8))(x(x - 3) + 2(x - 3))
 \end{aligned}$$



$$\begin{aligned}
 &= (x-8)(x+7)(x-3)(x+2) \\
 \text{(v)} \quad (x+1)(x+2)(x+3)(x+6) - 3x^2 &= (x+1)(x+6)(x+2)(x+3) - 3x^2 \\
 &= (x^2 + 6x + x + 6)(x^2 + 3x + 2x + 6) - 3x^2 \\
 &= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2 \\
 &= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2 \\
 &= x^2 \left[ \frac{(x^2 + 6 + 7x)(x^2 + 6 + 5x)}{x^2} - \frac{3x^2}{x^2} \right] \\
 &= x^2 \left[ \left(x + \frac{6}{x} + 7\right) \left(x + \frac{6}{x} + 5\right) - 3 \right]
 \end{aligned}$$

$$\text{let } x + \frac{6}{x} = y$$

$$\begin{aligned}
 x^2[(y+7)(y+5) - 3] &= x^2[y^2 + 5y + 7y + 35 - 3] \\
 &= x^2[y^2 + 12y + 32] \\
 &= x^2[y^2 + 8y + 4y + 32] \\
 &= x^2[y(y+8) + 4(y+8)] \\
 &= x^2[(y+8)(y+4)]
 \end{aligned}$$

$$\text{as } x + \frac{6}{x} = y, \text{ So,}$$

$$(x+1)(x+2)(x+3)(x+6) - 3x^2 = x^2 \left(x + \frac{6}{x} + 8\right) \left(x + \frac{6}{x} + 4\right)$$

**Q. 5:**

$$\begin{aligned}
 \text{(i)} \quad x^3 + 48x - 12x^2 - 64 &= x^3 - 12x^2 + 48x - 64 \\
 &= (x)^3 - 3(x)^2(4) + 3(x)(4)^2 - (4)^3 \\
 &= (x-4)^3 \\
 \text{(ii)} \quad 8x^3 + 60x^2 + 150x + 125 &= 8x^3 + 60x^2 + 150x + 125 \\
 &= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3 \\
 &= (2x+5)^3 \\
 \text{(iii)} \quad x^3 - 18x^2 + 108x - 216 &= x^3 - 18x^2 + 108x - 216 \\
 &= (x)^3 - 3(x)^2(6) + 3(x)(6)^2 - (6)^3 \\
 &= (x-6)^3 \\
 \text{(iv)} \quad 8x^3 - 125y^3 - 60x^2y + 150xy^2 &= 8x^3 - 60x^2y + 150xy^2 - 125y^3 \\
 &= (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3 \\
 &= (2x-5y)^3
 \end{aligned}$$

**Q. 6:**

$$\begin{aligned}
 \text{(i)} \quad 27 + 8x^3 &= (3)^3 + (2x)^3 \\
 &= (3+2x)((3)^2 - (3)(2x) + (2x)^2) \\
 &= (3+2x)(9 - 6x + 4x^2) \\
 \text{(ii)} \quad 125x^3 - 216y^3 &= (5x)^3 - (6y)^3 \\
 &= (5x-6y)((5x)^2 + (5x)(6y) + (6y)^2) \\
 &= (5x-6y)(25x^2 + 30xy + 36y^2) \\
 \text{(iii)} \quad 64x^3 + 27y^3 &= (4x)^3 + (3y)^3 \\
 &= (4x+3y)((4x)^2 - (4x)(3y) + (3y)^2) \\
 &= (4x+3y)(16x^2 - 12xy + 9y^2) \\
 \text{(iv)} \quad 8x^3 + 125y^3 &= (2x)^3 + (5y)^3
 \end{aligned}$$



$$\begin{aligned} &= (2x + 5y)((2x)^2 - (2x)(5y) + (5y)^2) \\ &= (2x + 5y)(4x^2 - 10xy + 25y^2) \end{aligned}$$

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