Exercise 3.6

Q. 1: If a : b = c : d, (a, b, c, d $\neq$ 0), then show that

(i) $\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

$\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$

Putting the values

$\frac{4(bk)-9b}{4(bk)+9b} = \frac{4(dk)-9d}{4(dk)+9d}$

$\frac{4bk-9b}{4bk+9b} = \frac{4dk-9d}{4dk+9d}$

$\frac{b(4k-9)}{b(4k+9)} = \frac{d(4k-9)}{d(4k+9)}$

$\frac{4k-9}{4k+9} = \frac{4k-9}{4k+9}$

L.H.S = R.H.S

(ii) $\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

$\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$

Putting the values

$\frac{6(bk)-5b}{6(bk)+5b} = \frac{6(dk)-5d}{6(dk)+5d}$

$\frac{6bk-5b}{6bk+5b} = \frac{6dk-5d}{6dk+5d}$

$\frac{b(6k-5)}{b(6k+5)} = \frac{d(6k-5)}{d(6k+5)}$

$\frac{6k-5}{6k+5} = \frac{6k-5}{6k+5}$

L.H.S = R.H.S

(iii) $\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

$\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$

Putting the values

$\frac{bk}{b} = \sqrt{\frac{(bk)^2+(dk)^2}{b^2+d^2}}$

$k \quad = \sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$

$k \quad = \sqrt{\frac{k^2(b^2+d^2)}{b^2+d^2}}$

$k \quad = \sqrt{k^2}$

$k \quad = k$

L.H.S = R.H.S

(iv) $a^6 + c^6 : b^6 + d^6 \quad = a^3c^3 : b^3d^3$

$\frac{a^6+c^6}{b^6+d^6} = \frac{a^3c^3}{b^3d^3}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

Putting the values

$\frac{(bk)^6+(dk)^6}{b^6+d^6} = \frac{(bk)^3(dk)^3}{b^3d^3}$

$\frac{b^6k^6+d^6k^6}{b^6+d^6} = \frac{b^3k^3d^3k^3}{b^3d^3}$

$\frac{k^6(b^6+d^6)}{b^6+d^6} = \frac{b^3d^3k^6}{b^3d^3}$

$k^6 \quad = k^6$

L.H.S = R.H.S

(v) $p(a+b) + qb : p(c+d) + qd \quad = a : c$

$\frac{p(a+b)+qb}{p(c+d)+qd} = \frac{a}{c}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

Putting the values

$\frac{p(bk+b)+qb}{p(dk+d)+qd} = \frac{bk}{dk}$

$\frac{b[p(k+1)+q]}{d[p(k+1)+q]} = \frac{bk}{dk}$

$\frac{b}{d} = \frac{b}{d}$

L.H.S = R.H.S

(vi) $a^2 + b^2 : \frac{a^3}{a+b} = c^2 + d^2 : \frac{c^3}{c+d}$

$\frac{a^2+b^2}{\frac{a^3}{a+b}} = \frac{c^2+d^2}{\frac{c^3}{c+d}}$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

Putting the values

$$\frac{(bk)^2+b^2}{\frac{(bk)^3}{bk+b}} = \frac{(dk)^2+d^2}{\frac{(dk)^3}{dk+d}}$$

$$\frac{b^2k^2+b^2}{\frac{b^3k^3}{bk+b}} = \frac{d^2k^2+d^2}{\frac{d^3k^3}{dk+d}}$$

$$\frac{b^2(k^2+1)}{\frac{b^3k^3}{b(k+1)}} = \frac{d^2(k^2+1)}{\frac{d^3k^3}{d(k+1)}}$$

$$\frac{b^2(k^2+1)}{\frac{b^2k^3}{(k+1)}} = \frac{d^2(k^2+1)}{\frac{d^2k^3}{(k+1)}}$$

$$\frac{(k^2+1)}{\frac{k^3}{(k+1)}} = \frac{(k^2+1)}{\frac{k^3}{(k+1)}}$$

$$\frac{(k^2+1)(k+1)}{k^3} = \frac{(k^2+1)(k+1)}{k^3}$$

L.H.S = R.H.S

(vii) $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

$$\frac{\frac{a}{a-b}}{\frac{a+b}{b}} = \frac{\frac{c}{c-d}}{\frac{c+d}{d}}$$

as $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then $a = bk$ and $c = dk$

Putting the values

$$\frac{\frac{bk}{bk-b}}{\frac{bk+b}{b}} = \frac{\frac{dk}{dk-d}}{\frac{dk+d}{d}}$$

$$\frac{\frac{bk}{b(k-1)}}{\frac{b(k+1)}{b}} = \frac{\frac{dk}{d(k-1)}}{\frac{d(k+1)}{d}}$$

$$\frac{\frac{k}{(k-1)}}{(k+1)} = \frac{\frac{k}{(k-1)}}{(k+1)}$$

$$\frac{k}{(k-1)(k+1)} = \frac{k}{(k-1)(k+1)}$$

L.H.S = R.H.S

Q. 2: If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ (a, b, c, d, e, f $\neq$ 0), then show that

(i) $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

as $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then $a = bk$, $c = dk$ and $e = fk$

Putting the values

$$\frac{bk}{b} = \sqrt{\frac{(bk)^2+(dk)^2+(fk)^2}{b^2+d^2+f^2}}$$

$$k = \sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$$

$k \quad = \sqrt{\frac{k^2(b^2+d^2+f^2)}{b^2+d^2+f^2}}$

$k \quad = \sqrt{k^2}$

$k \quad = k$

L.H.S = R.H.S

(ii) $\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf}\right]^{2/3}$

as $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then $a = bk$, $c = dk$ and $e = fk$

Putting the values

$\frac{bkdk+dkfk+fkbk}{bd+df+fb} = \left[\frac{bkdkfk}{bdf}\right]^{2/3}$

$\frac{k^2(bd+df+fb)}{bd+df+fb} = \left[\frac{k^3bdf}{bdf}\right]^{2/3}$

$k^2 = [k^3]^{2/3}$

$k^2 = k^2$

L.H.S = R.H.S

(iii) $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

as $\frac{a}{b} = \frac{c}{d} = \frac{e}{g}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{g} = k$

Then $a = bk$, $c = dk$ and $e = fk$

Putting the values

$\frac{bkdk}{bd} + \frac{dkfk}{df} + \frac{fkbk}{fb} = \frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$

$\frac{bdk^2}{bd} + \frac{dfk^2}{df} + \frac{fbk^2}{fb} = \frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$

$k^2 + k^2 + k^2 = k^2 + k^2 + k^2$

$3k^2 = 3k^2$

$k^2 = k^2$

L.H.S = R.H.S