

Exercise 2.4**Q. 1: Use laws of exponents to simplify:**

$$(i) \frac{(243)^{-2/3} (32)^{-1/5}}{\sqrt{(196)^{-1}}} = \frac{(3 \times 3 \times 3 \times 3 \times 3)^{-2/3} (2 \times 2 \times 2 \times 2 \times 2)^{-1/5}}{\sqrt{(2 \times 2 \times 7 \times 7)^{-1}}}$$

$$= \frac{(3^5)^{-2/3} (2^5)^{-1/5}}{\sqrt{(2^2 \times 7^2)^{-1}}}$$

$$= \frac{(3)^{-10/3} (2^5)^{-1/5}}{(2^2)^{-1/2} (7^2)^{-1/2}}$$

$$= \frac{(3^{10})^{-1/3} (2)^{-1}}{(2)^{-1} (7)^{-1}}$$

$$= \frac{(3^9 \cdot 3)^{-1/3}}{(7)^{-1}}$$

$$= \frac{(3^9)^{-1/3} (3)^{-1/3}}{(7)^{-1}}$$

$$= \frac{(3)^{-3} (3)^{-1/3}}{(7)^{-1}}$$

$$= \frac{(7)^1}{(3)^3 (3)^{1/3}}$$

$$= \frac{7}{27 \sqrt[3]{3}}$$

$$(ii) = (2x^5y^{-4})(-8x^{-3}y^2)$$

$$= (2 \times -8 \cdot x^5 \cdot x^{-3} \times y^{-4} \cdot y^2)$$

$$= -16x^{5-3} \times y^{-4+2}$$

$$= -16x^2 \times y^{-2}$$

$$= \frac{-16x^2}{y^2}$$

$$(iii) = \left(\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0} \right)^{-3}$$

$$= (x^{-2-4}y^{-1+3}z^{-4-0})^{-3}$$

$$= (x^{-6}y^2z^{-4})^{-3}$$

$$= x^{-6 \times -3} y^{2 \times -3} z^{-4 \times -3}$$

$$= x^{18} y^{-6} z^{12}$$

$$= \frac{x^{18} z^{12}}{y^6}$$

$$(iv) = \frac{(81)^n \cdot 3^5 - (3)^{4n-1} (243)}{(9^{2n})(3^3)}$$

$$= \frac{(3 \times 3 \times 3 \times 3)^n \cdot 3^5 - (3)^{4n} (3)^{-1} (3 \times 3 \times 3 \times 3 \times 3)}{((3 \times 3)^{2n})(3^3)}$$

$$= \frac{(3^4)^n \cdot 3^5 - (3)^{4n} (3)^{-1} (3^5)}{((3^2)^{2n})(3^3)}$$

$$= \frac{3^{4n} \cdot 3^5 - 3^{4n} \cdot 3^{-1+5}}{3^{4n} \cdot 3^3}$$

$$= \frac{3^{4n} \cdot 3^5 - 3^{4n} \cdot 3^4}{3^{4n} \cdot 3^3}$$

$$= \frac{3^{4n} \cdot 3^4 (3-1)}{3^{4n} \cdot 3^3}$$

$$= 3(3-1) = 6$$

Q. 2: Show that

$$\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$$

$$\begin{aligned} \text{L.H.S} &= \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} \\ &= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \\ &= (x)^{a^2-b^2} \times (x)^{b^2-c^2} \times (x)^{c^2-a^2} \\ &= (x)^{a^2-b^2+b^2-c^2+c^2-a^2} \\ &= (x)^0 \\ &= 1 = \text{R.H.S} \end{aligned}$$

Q. 3: Simplify

$$\begin{aligned} \text{(i)} \quad \frac{2^{1/3} \times (27)^{1/3} \times (60)^{1/2}}{(180)^{1/2} \times (4)^{-1/3} \times (9)^{1/4}} &= \frac{2^{1/3} \times (3 \times 3 \times 3)^{1/3} \times (3 \times 2 \times 2 \times 5)^{1/2}}{(3 \times 3 \times 2 \times 2 \times 5)^{1/2} \times (2 \times 2)^{-1/3} \times (3 \times 3)^{1/4}} \\ &= \frac{2^{1/3} \times (3^3)^{1/3} \times (3 \times 2^2 \times 5)^{1/2}}{(3^2 \times 2^2 \times 5)^{1/2} \times (2^2)^{-1/3} \times (3^2)^{1/4}} \\ &= \frac{2^{1/3} \times 3 \times (3)^{1/2} \times (2^2)^{1/2} \times (5)^{1/2}}{(3^2)^{1/2} \times (2^2)^{1/2} \times (5)^{1/2} \times (2)^{-2/3} \times (3)^{1/2}} \\ &= \frac{2^{1/3} \times 3 \times 2 \times (5)^{1/2}}{3 \times 2 \times (5)^{1/2} \times (2)^{-2/3}} \\ &= \frac{2^{1/3}}{(2)^{-2/3}} \\ &= 2^{1/3+2/3} \\ &= 2^{1+2/3} \\ &= 2^{3/3} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \sqrt{\frac{(216)^{2/3} \times (25)^{1/2}}{(.04)^{-1/2}}} &= \sqrt{\frac{(2 \times 2 \times 2 \times 3 \times 3 \times 3)^{2/3} \times (5 \times 5)^{1/2}}{\left(\frac{4}{100}\right)^{-1/2}}} \\ &= \sqrt{\frac{(2 \times 2 \times 2 \times 3 \times 3 \times 3)^{2/3} \times (5 \times 5)^{1/2}}{\left(\frac{2 \times 2}{2 \times 2 \times 5 \times 5}\right)^{-1/2}}} \\ &= \sqrt{\frac{(2^3 \times 3^3)^{2/3} \times (5^2)^{1/2}}{\left(\frac{1}{5^2}\right)^{-1/2}}} \\ &= \sqrt{(2^3 \times 3^3)^{2/3} \times (5^2)^{1/2} \left(\frac{1}{5^2}\right)^{1/2}} \\ &= \sqrt{(2^3)^{2/3} \times (3^3)^{2/3} \times (5^2)^{1/2} \times \frac{1}{5^{2 \times 1/2}}} \\ &= \sqrt{2^2 \times 3^2 \times 5 \times \frac{1}{5}} \\ &= \sqrt{2^2} \times \sqrt{3^2} \end{aligned}$$

$$= 2 \times 3$$
$$= 6$$

$$(iii) \quad 5^{2^3} \div (5^2)^3 = \frac{5^8}{5^6}$$
$$= 5^{8-6}$$
$$= 5^2$$
$$= 25$$

$$(iv) \quad (x^3)^2 \div x^{3^2} = \frac{x^6}{x^9}$$
$$= \frac{1}{x^{9-6}}$$
$$= \frac{1}{x^3}$$

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