Exercise 2.3

Q. 1: Without solving, find the sum and the product of the roots of the following quadratic equations.

(i) $x^2 - 5x + 3 = 0$

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-5}{1} = 5$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

(ii) $3x^2 + 7x - 11 = 0$

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{7}{3} = \frac{-7}{3}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{-11}{3} = \frac{-11}{3}$

(iii) $px^2 - qx + r = 0$

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-q}{p} = \frac{q}{p}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{r}{p} = \frac{r}{p}$

(iv) $(a+b)x^2 - ax + b = 0$

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-a}{a+b} = \frac{a}{a+b}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{b}{a+b} = \frac{b}{a+b}$

(v) $(l+m)x^2 + (m+n)x + n - l = 0$

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{m+n}{l+m} = -\frac{m+n}{l+m}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{n-l}{l+m} = \frac{n-l}{l+m}$

(vi) $7x^2 - 5mx + 9n = 0$

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-5}{7} = \frac{5m}{7}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{9}{7} = \frac{9n}{7}$

Q. 2: Find the value of k, if

(i) Sum of the roots of the equation $2kx^2 - 3x + 4k = 0$ is twice the product of the roots.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-3}{2k} = \frac{3}{2k}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{4k}{2k} = 2$

According to given condition

$\alpha + \beta = 2\alpha\beta$

$\frac{3}{2k} = 2 \times 2$

$\frac{3}{2k} = 4$

$\frac{3}{8} = k$

$k = \frac{3}{8}$

(ii) Sum of the roots of the equation $x^2 + (3k-7)x + 5k = 0$ is $\frac{3}{2}$ times the product of the roots.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{3k-7}{1} = -3k + 7$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{5k}{1} = 5k$

According to given condition

$$\alpha + \beta = \frac{3}{2}\alpha\beta$$
$$-3k + 7 = \frac{3}{2} \times 5k$$
$$-6k + 14 = 15k$$
$$-6k - 15k = -14$$
$$-21k = -14$$
$$k = \frac{2}{3}$$

Q. 3: Find k, if

(i) sum of the squares of the roots of the equation

$$4kx^2 + 3kx - 8 = 0 \text{ is } 2.$$

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{3k}{4k} = \frac{-3}{4}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{-8}{4k} = \frac{-2}{k}$

According to given condition

$$\alpha^2 + \beta^2 = 2$$

Adding $2\alpha\beta$ on both sides

$$\alpha^2 + \beta^2 + 2\alpha\beta = 2 + 2\alpha\beta$$
$$(\alpha + \beta)^2 = 2 + 2\alpha\beta$$
$$\left(\frac{-3}{4}\right)^2 = 2 + 2 \times \frac{-2}{k}$$
$$\frac{9}{16} = 2 + \frac{-4}{k}$$
$$\frac{9}{16} = \frac{2k-4}{k}$$
$$9k = 32k - 64$$
$$64 = 32k - 9k$$
$$64 = 23k$$
$$\frac{64}{23} = k$$
$$k = \frac{64}{23}$$

(ii) sum of the squares of the roots of the equation

$$x^2 - 2kx + 2k + 1 = 0 \text{ is } 6.$$

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-2k}{1} = 2k$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{2k+1}{1} = 2k + 1$

According to given condition

$$\alpha^2 + \beta^2 = 6$$

Adding $2\alpha\beta$ on both sides

$$\alpha^2 + \beta^2 + 2\alpha\beta = 6 + 2\alpha\beta$$
$$(\alpha + \beta)^2 = 6 + 2\alpha\beta$$
$$(2k)^2 = 6 + 2\,(2k + 1)$$

$$4k^2 = 6 + 4k + 2$$
$$4k^2 = 8 + 4k$$
$$4k^2 - 4k - 8 = 0$$
$$4k^2 - 8k + 4k - 8 = 0$$
$$4k(k-2) + 4(k-2) = 0$$
$$(k-2)(4k+4) = 0$$
$k - 2 = 0$ and $4k + 4 = 0$

$k = 2$ and $k = -1$

Q. 4: Find p, if

(i) the roots of the equation $x^2 - x + p^2 = 0$ differ by unity.

let 1st root $= \alpha$

then according to given condition

2nd root $= \alpha - 1$

sum of roots $= \alpha + \alpha - 1 = -\frac{b}{a} = -\frac{-1}{1} = 1$

so, $2\alpha - 1 = 1$

$2\alpha = 2$

$\alpha = 1$ ---------------------------- (i)

Product of roots $= \alpha(\alpha - 1) = \frac{c}{a} = \frac{p^2}{1} = p^2$

so, $\alpha^2 - \alpha = p^2$ ---------------------------- (ii)

putting the value from equ. (i) in equ. (ii)

$\alpha^2 - \alpha = p^2$

$(1)^2 - 1 = p^2$

$1 - 1 = p^2$

$0 = p^2$

$p = 0$

(ii) the roots of the equation $x^2 + 3x + p - 2 = 0$ differ by 2.

let 1st root $= \alpha$

then according to given condition

2nd root $= \alpha - 2$

sum of roots $= \alpha + \alpha - 2 = -\frac{b}{a} = -\frac{3}{1} = -3$

so, $2\alpha - 2 = -3$

$2\alpha = -1$

$\alpha = \frac{-1}{2}$ ---------------------------- (i)

Product of roots $= \alpha(\alpha - 2) = \frac{c}{a} = \frac{p-2}{1} = p - 2$

so, $\alpha^2 - 2\alpha = p - 2$ ---------------------------- (ii)

putting the value from equ. (i) in equ. (ii)

$\alpha^2 - 2\alpha = p - 2$

$$\left(\frac{-1}{2}\right)^2 - 2\left(\frac{-1}{2}\right) = p-2$$
$$\frac{1}{4}+1 = p-2$$
$$\frac{1+4}{4} = p-2$$
$$\frac{5}{4} = p-2$$
$$5 = 4p-8$$
$$4p-8 = 5$$
$$4p = 5+8$$
$$4p = 13$$
$$p = \frac{13}{4}$$

Q. 5: Find m, if

(i) the roots of the equation $x^2 - 7x + 3m - 5 = 0$ satisfy the relation $3\alpha + 2\beta = 4$.

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-7}{1} = 7$

so, $\alpha + \beta = 7$

$\beta = 7 - \alpha$ --------------------- (i)

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{3m-5}{1} = 3m-5$

so, $\alpha\beta = 3m-5$ ------------------- (ii)

putting the value of β in $3\alpha + 2\beta = 4$.

$$3\alpha + 2\beta = 4$$
$$3\alpha + 2(7-\alpha) = 4$$
$$3\alpha + 14 - 2\alpha = 4$$
$$\alpha = 4 - 14$$
$$\alpha = -10$$

putting the value of α in equ (i)

$$\beta = 7 - \alpha$$
$$\beta = 7-(-10)$$
$$\beta = 7+10$$
$$\beta = 17$$

putting the values of α, β in equ (ii)

$$\alpha\beta = 3m-5$$
$$(-10)(17) = 3m-5$$
$$-170 = 3m-5$$
$$3m-5 = -170$$
$$3m = -170+5$$
$$3m = -165$$
$$m = -\frac{165}{3}$$
$$m = -55$$

(ii) the roots of the equation $x^2 + 7x + 3m - 5 = 0$ satisfy the relation $3\alpha - 2\beta = 4$.

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{7}{1} = -7$

so, $\alpha + \beta = -7$

$\beta = -7 - \alpha$ ---------------------- (i)

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{3m-5}{1} = 3m - 5$

so, $\alpha\beta = 3m - 5$ ------------------- (ii)

putting the value of β in $3\alpha - 2\beta = 4$.

$$3\alpha - 2\beta = 4$$
$$3\alpha - 2(-7 - \alpha) = 4$$
$$3\alpha + 14 + 2\alpha = 4$$
$$5\alpha = 4 - 14$$
$$\alpha = \frac{-10}{5}$$
$$\alpha = -2$$

putting the value of α in equ (i)

$$\beta = -7 - \alpha$$
$$\beta = -7 - (-2)$$
$$\beta = -7 + 2$$
$$\beta = -5$$

putting the values of α, β in equ (ii)

$$\alpha\beta = 3m - 5$$
$$(-2)(-5) = 3m - 5$$
$$10 = 3m - 5$$
$$10 = 3m - 5$$
$$3m = -15$$
$$m = \frac{-15}{3}$$
$$m = -5$$

(iii) the roots of the equation $3x^2 - 2x + 7m + 2 = 0$ satisfy the relation $7\alpha - 3\beta = 18$.

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-2}{3} = \frac{2}{3}$

so, $\alpha + \beta = \frac{2}{3}$

$\beta = \frac{2}{3} - \alpha$ ---------------------- (i)

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{7m+2}{3} = \frac{7m+2}{3}$

so, $\alpha\beta = \frac{7m+2}{3}$ ------------------- (ii)

putting the value of β in $7\alpha - 3\beta = 18$.

$$7\alpha - 3\beta = 18$$
$$7\alpha - 3\left(\frac{2}{3} - \alpha\right) = 18$$
$$7\alpha - 2 + 3\alpha = 18$$

$$7\alpha + 3\alpha = 18 + 2$$
$$10\alpha = 20$$
$$\alpha = 2$$

putting the value of α in equ (i)

$$\beta = \frac{2}{3} - \alpha$$
$$\beta = \frac{2}{3} - 2$$
$$\beta = \frac{2-6}{3}$$
$$\beta = \frac{-4}{3}$$

putting the values of α, β in equ (ii)

$$\alpha\beta = \frac{7m+2}{3}$$
$$(2)\left(\frac{-4}{3}\right) = \frac{7m+2}{3}$$
$$\frac{-8}{3} = \frac{7m+2}{3}$$
$$-8 = 7m + 2$$
$$7m + 2 = -8$$
$$7m = -8 - 2$$
$$7m = -10$$
$$m = \frac{-10}{7}$$

Q. 6: Find m, if sum and product of the roots of the following equations is equal to a given number $\boldsymbol{\lambda}$.

(i) $(2m + 3)x^2 + (7m - 5)x + (3m - 10) = 0$

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{7m-5}{2m+3} = -\frac{7m-5}{2m+3}$

according to given condition, $\alpha + \beta = \boldsymbol{\lambda}$

$$\boldsymbol{\lambda} = -\frac{7m-5}{2m+3} \text{------------------------- (i)}$$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{3m-10}{2m+3} = \frac{3m-10}{2m+3}$

according to given condition, $\alpha\beta = \boldsymbol{\lambda}$

so, $\boldsymbol{\lambda} = \frac{3m-10}{2m+3}$ ------------------- (ii)

comparing equ (i) and (ii)

$$-\frac{7m-5}{2m+3} = \frac{3m-10}{2m+3}$$
$$-7m + 5 = 3m - 10$$
$$-7m - 3m = -10 - 5$$
$$-10m = -15$$
$$m = \frac{-15}{-10}$$
$$m = \frac{3}{2}$$

(ii) $4x^2 - (3 + 5m)x + (9m - 17) = 0$

If α, β are the roots of the given equation.

sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-(3+5m)}{4} = \frac{3+5m}{4}$

according to given condition, $\alpha + \beta = \boldsymbol{\lambda}$

$$\boldsymbol{\lambda} = \frac{3+5m}{4} \text{ ---------------------- (i)}$$

Product of roots $= \alpha\beta = \frac{c}{a} = -\frac{9m-17}{4} = -\frac{9m-17}{4}$

according to given condition, $\alpha\beta = \boldsymbol{\lambda}$

so, $\boldsymbol{\lambda} = \frac{9m-17}{4}$ ------------------- (ii)

comparing equ (i) and (ii)

$$\frac{3+5m}{4} = -\frac{9m-17}{4}$$

$$3 + 5m = -9m + 17$$

$$5m + 9m = 17 - 3$$

$$14m = 14$$

$$m = \frac{14}{14}$$

$$m = 1$$