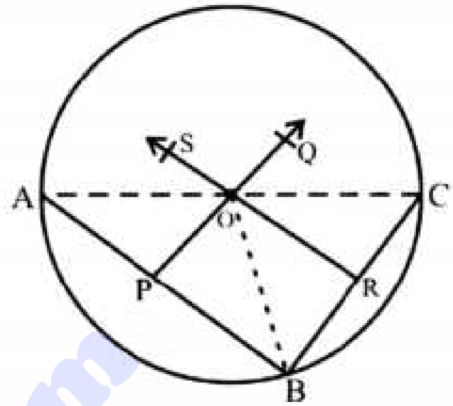
Exercise 12.1

Q.1 Prove that the centre of a circle is on the right bisectors of each of its chords.

Given

A, B, C are the three non-collinear points.

Required: To find the centre of the circle passing through A,B,C



Construction

Join B to C, A take $\overrightarrow{PQ}$ is right bisector of $\overline{AB}$ and $\overline{RS}$ right bisector of BC, they intersect at O.

Join O to A, O to B, O to C.

$\therefore$ O is the centre of circle.

Proof

Statements	Reasons
$\overline{OB} \cong \overline{OC}$ __________ (i)	O is the right bisector of $\overline{BC}$
$\overline{OA} \cong \overline{OB}$ __________ (ii)	O is the right bisector of $\overline{AB}$
$\overline{OA} = \overline{OB} = \overline{OC}$	From (i) and (ii)
Hence is equidistant from the A,B,C	
$\therefore O$ is center of circle which is required	

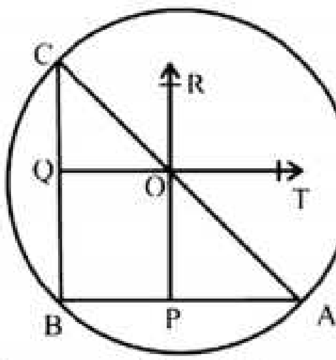
Q.2 Where will the center of a circle passing through three non-collinear points? And Why?

Given

A.B.C are three non collinear points and circle passing through these points.

To prove

Find the center of the circle passing through vertices A, B and C.



Construction

(i) Join B to A and C.

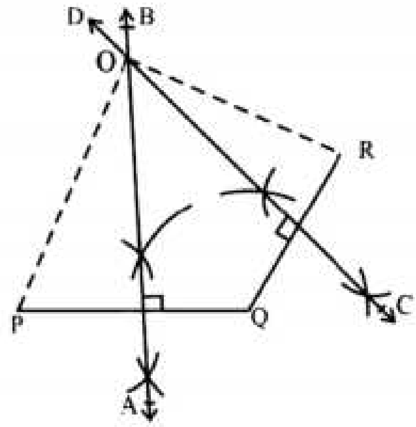
(ii) Take $\overrightarrow{QT}$ right bisector of $\overline{BC}$ and take also $\overrightarrow{PR}$ right bisector of $\overline{AB}$.

$\overrightarrow{PR}$ and $\overrightarrow{QT}$ intersect at point O. joint O to A,B and C. O is the center of the circle.

Proof

Statements	Reasons
$\overline{QO}$ is right bisector $\overline{BC}$	
$\overline{OB} \cong \overline{OC}$ ...(i)	
$\overline{PO}$ is right bisector of $\overline{AB}$	
$\overline{OA} \cong \overline{OB}$ ...(ii)	
So	
$\overline{OA} \cong \overline{OC} \cong \overline{OB}$	From (i) and (ii)
$\therefore$ It is proved that O is the center of the circle.	

Q.3 **Three village P,Q and R are not on the same line. The people of these villages want to make a children park at such a place which is equidistant from these three villages. After fixing the place of children park prove that the park is equidistant from the three villages.**



Given

P,Q,R are three villages not on the same straight line.

To prove

The point equidistant from P,R,Q.

Construction

(i) Joint Q to P and R.

(ii) Take $\overleftrightarrow{AB}$ right bisector of $\overline{PQ}$ and $\overleftrightarrow{CD}$ right bisector of $\overline{QR}$. $\overline{AB}$ and $\overleftrightarrow{CD}$ intersect at O.

(iii) Join 0 to P, Q, R

The place of children part at point O.

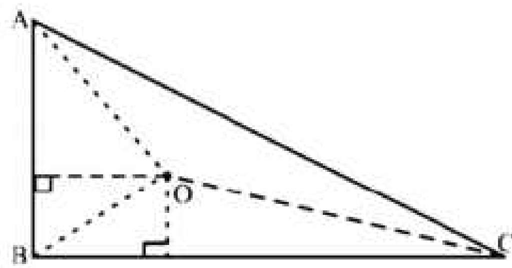
Proof

Statements	Reasons
$\overline{OQ} \cong \overline{OR}$ ________ (i)	O is on the right bisector of $\overline{QR}$
$\overline{OP} \cong \overline{OQ}$ ________ (ii)	O is on the right bisector of $\overline{PQ}$
$\overline{OP} \cong \overline{OQ} \cong \overline{OR}$ ________ (iii)	From (i) and (ii)
$\therefore O$ is on the bisector of $\angle P$	
Hence $\overline{PO}$ is bisector of $\angle P$	

O is equidistant from P,Q and R

Theorem 12.1.3

The right bisectors of the sides of a triangle are concurrent.



Given

ΔABC

To prove

The right bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are concurrent.

Construction

Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which meet each other at the point O. Join O to A, B and C.

Proof

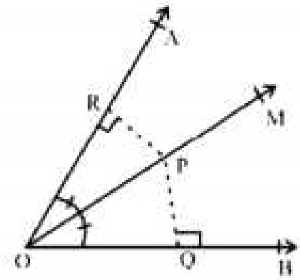
Statements	Reasons
$\overline{OA} \cong \overline{OB}$ $\longrightarrow$ (i)	(Each point on right bisector of a segment is equidistant from its end points)
$\overline{OB} \cong \overline{OC}$ $\longrightarrow$ (ii)	As in (i)
$\overline{OA} \cong \overline{OC}$	from (i) and (ii)
$\therefore$ Point O is on the right bisector of $\overline{CA}$ $\rightarrow$ (iv)	(O is equidistant from A and C)
But point O is on the right bisector of $\overline{AB}$ and of $\overline{BC}$ $\longrightarrow$ (v)	**Construction**
Hence the right bisectors of the three sides of triangle are concurrent at O	{from (iv) and (v)}

Theorem 12.1.4

Any point on the bisector of an angle is equidistant from its arms.

Given

A point P is on $\overrightarrow{OM}$, the bisector of $\angle AOB$



To Prove

$\overline{PQ} \cong \overline{PR}$ i.e P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$

Construction

Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$

Proof

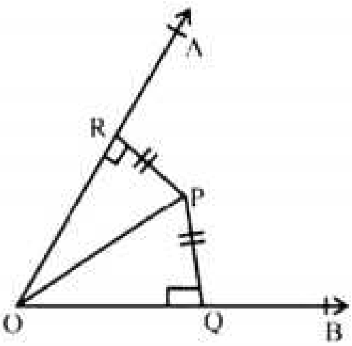
Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PQO \cong \angle PRO$	Construction
$\angle POQ \cong \angle POR$	Given
$\therefore \Delta POQ \cong \Delta POR$	$S.A.A \cong S.A.A$
Hence $\overline{PQ} \cong \overline{PR}$	(Corresponding sides of congruent triangles)

Theorem 12.1.5 (Converse of Theorem 12.1.4)

Any point inside an angle, equidistant from its arms, is on the bisector of it.

Given

Any point P lies inside $\angle AOB$, such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overrightarrow{OB}$ and $\overline{PR} \perp \overrightarrow{OA}$



To prove

Point P is on the bisector of $\angle AOB$

Construction

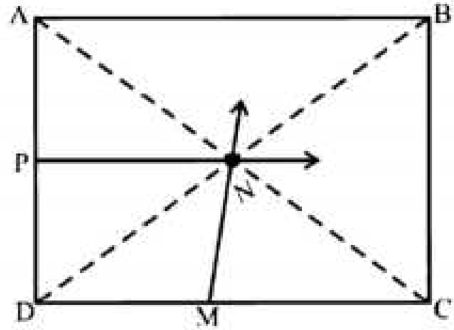
Join P to O

Proof

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$	Given (Right angles)
$\overline{PO} \cong \overline{PO}$	Common
$\overline{PQ} \cong \overline{PR}$	Given
$\therefore \Delta POQ \cong \Delta POR$	$H.S \cong H.S$
Hence $\angle POQ \cong \angle POR$	(Corresponding angles of congruent triangles)
i.e, P is on the bisector of $\angle AO$	

Exercise 12.2

Q.1 In a quadrilateral ABCD $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}, \overline{CD}$ meet each other at point N. Prove that $\overline{BN}$ is a bisector of $\angle ABC$



Given

In the quadrilateral ABCD

$\overline{AB} \cong \overline{BC}$

$\overline{NM}$ is right bisector of $\overline{CD}$

$\overline{PN}$ is right bisector of $\overline{AD}$

They meet at N

To prove

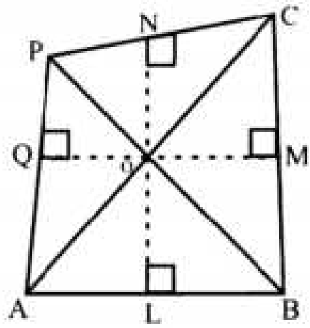
$\overline{BN}$ is the bisector of angle ABC

Construction join N to A,B,C,D

Proof

Statements	Reasons
$\overline{ND} \cong \overline{NA}$ ____________ (i)	N is an right bisector of $\overline{AD}$
$\overline{ND} \cong \overline{NC}$ ____________ (ii)	N is on right bisector of $\overline{DC}$
$\overline{NA} = \overline{NC}$ ____________ (iii)	from (i) and (ii)
$\Delta BNC \leftrightarrow \Delta ANB$	
$\overline{NC} = \overline{NA}$	Already proved (from iii)
$\overline{AB} \cong \overline{CB}$	Given
$\overline{BN} \cong \overline{BN}$	Common
$\therefore \Delta BNA \cong \Delta BNC$	$S.S.S \cong S.S.S$
Hence $\angle ABN \cong \angle NBC$	Corresponding angles of congruent triangles
Hence $\overline{BN}$ is the bisector of $\angle ABC$	

Q.2 The bisectors of $\angle A, \angle B$ and $\angle C$ of a quadrilateral ABCP meet each other at point O. Prove that the bisector of $\angle P$ will also pass through the point O.



Given

ABCP is quadrilateral. $\overline{AO}, \overline{BO}, \overline{CO}$ are bisectors of $\angle A, \angle B$ and $\angle C$ meet at point O.

To prove

$\overline{PO}$ is bisector of $\angle P$

Construction:

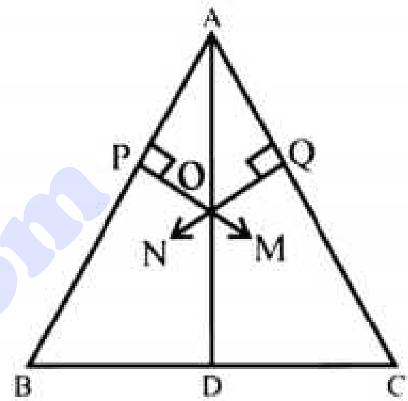
Join P to O.

Draw $\overline{OQ} \perp \overline{AP}$, $\overline{ON} \perp \overline{PC}$ and $\overline{OL} \perp \overline{AB}$, $\overline{OM} \perp \overline{BC}$

Proof:

Statements	Reasons
$\overline{OM} \cong \overline{ON}$ __________ (i)	O is on the bisector of $\angle C$
$\overline{OL} \cong \overline{OM}$ __________ (ii)	O is on the bisector of $\angle B$
$\overline{OL} \cong \overline{OQ}$ __________ (iii)	O is on the bisector of $\angle A$
$\overline{OQ} \cong \overline{ON}$	From i, ii, iii
Point O lines on the bisector of $\angle P$	
$\therefore \overline{OP}$ is the bisector of angle P	

Q.3 Prove that the right bisector of congruent sides of an isosceles triangle and its altitude are concurrent.



Given

ΔABC

$\overline{AB} \cong \overline{AC}$ due to isosceles triangle $\overline{PM}$ is right bisector of $\overline{AB}$

$\overline{QN}$ is right bisector of $\overline{AC}$

$\overleftrightarrow{PM}$ and $\overrightarrow{QN}$ intersect each other at point O

Required

The altitude of ΔABC lies at point O

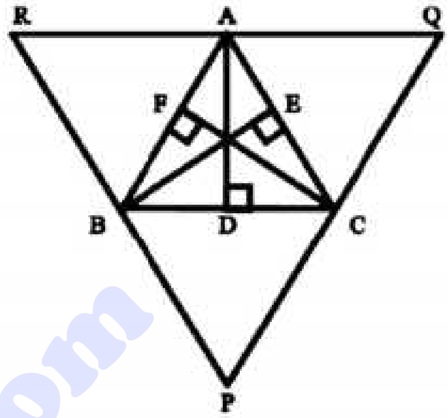
Join A to O and extend it to cut $\overline{BC}$ at D.

Proof

Statements	Reasons
$m\overline{AB} \cong m\overline{AC}$	Given
$\frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{AC}$	Dividing both side by 2
$\overline{AQ} \cong \overline{AP}$	
In $\Delta AQO \leftrightarrow \Delta APO$	
$\angle APO \cong \angle AQO$	Each 90^o (Given)
$\overline{AQ} \cong \overline{AP}$	Already Proved
$\overline{AO} \cong \overline{AO}$	Common
$\Delta APO \cong \Delta AQO$	$H.S \cong H.S$
$\angle PAO \cong \angle QAO$ (i)	Corresponding angles of congruent triangles
$\Delta BAD \leftrightarrow \Delta CAD$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\angle BAD \cong \angle CAD$	Proved from (i)
$\Delta BAD \cong \Delta CAD$	$S.A.S \cong S.A.S$

$\angle ODB \cong \angle ODC$ $m\angle ODM + m\angle ODC = 180^o$ $\therefore \overline{AD} \perp \overline{BC}$ Point 0 lies on altitude $\overline{AD}$	Each angle is 90^o (Given) Supplementary angle

Q.4 Prove that the altitudes of a triangle are concurrent.



Given

In ΔABC

$\overline{AD}, \overline{BE}, \overline{CF}$ are its altitiudes

i.e $\overline{AD} \perp \overline{BC}, \overline{BE} \perp \overline{AC}, \overline{CF} \perp \overline{AB}$

Required $\overline{AD}, \overline{BE}$ and $\overline{CF}$ are concurrent

Construction:

Passing through A, B, C take

$\overline{RQ} \| \overline{BC}, \overline{RP} \| \overline{AC}$ and $\overline{QP} \| \overline{AB}$ respectively forming a ΔPQR

Proof

Statements	Reasons
$\overline{BC} \| \overline{AQ}$	Construction
$\overline{AB} \| \overline{QC}$	Construction
$\therefore ABCQ$ is a P^{gm}	
Hence $\overline{AQ}$ %$\overline{BC}$	
Similarly $\overline{AB}$ %$\overline{QC}$	
Hence point A is midpoint RQ	
And $\overline{AD} \perp \overline{BC}$	Given
$\overline{BC} \| \overline{RQ}$	Opposite sides of parallelogram ABCQ
$\overline{AD} \| \overline{RQ}$	
Thus $\overline{AD} \perp$ is right bisector of $\overline{RQ}$	
similarly $\overline{BE}$ is a right bisector of $\overline{RP}$ and	
$\overline{CF}$ is right bisector of PQ	
$\therefore \perp^s$ $\overline{AD}, \overline{BE}, \overline{CF}$ are right bisector of sides of ΔPQR	
$\therefore \overline{AD}, \overline{BE}$ and $\overline{CF}$ are Concurrent	

Theorem12.1.6

The bisectors of the angles of a triangle are concurrent

Given

ΔABC

To Prove

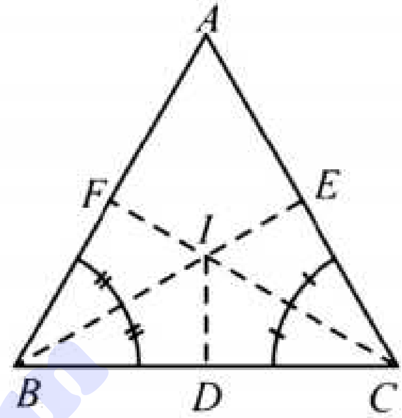
The bisector of $\angle A$, $\angle B$, and $\angle C$ are concurrent

Construction:

Draw the bisectors of $\angle B$ and $\angle C$ which intersect at point I. From I, draw

$\overline{IF} \perp \overline{AB}$, $\overline{ID} \perp \overline{BC}$ and $\overline{IE} \perp \overline{CA}$

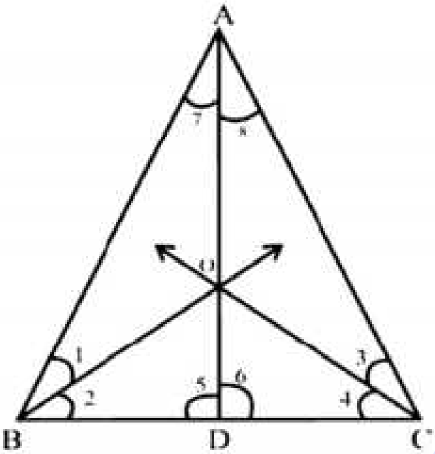
Proof



Statements	Reasons
$\overline{ID} \cong \overline{IF}$	(Any point on bisector of an angle is equidistance from its arms.
Similarly $ID \cong IE$	
$\therefore\ \overline{IE} \cong \overline{IF}$	Each $\cong$ ID
So the point I is on the bisector of $\angle A$ … (i)	
Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$ … (ii)	Construction
Thus the bisector of $\angle A$, $\angle B$ and $\angle C$ are concurrent at I	{From (i) and (ii)}

Exercise 12.3

Q.1 Prove that the bisectors of the angles of base of an isosceles triangle intersect each other on its altitude.



Given

ΔABC

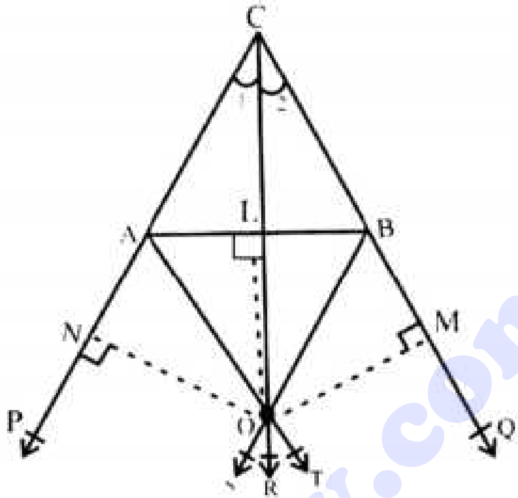
$\overline{AB} = \overline{AC}$ Due to isosceles triangle

Bisect $\angle B$ and $\angle C$ to intersect at point O Join A to D and extend to BC at D $\overline{AD}$ is the altitude of ΔABC $\overline{AD} \perp \overline{BC}$

Proof

Statements	Reasons
In ΔABC	
$\overline{AB} \cong \overline{AC}$	Given
$\angle B \cong \angle C$	Due to isosceles triangle opposite angle are congruent
$\frac{1}{2} m\angle B = \frac{1}{2} m\angle C$	Dividing both side by 2
$\angle 1 \cong \angle 3$	
$\Delta ABO \leftrightarrow \Delta ACO$	
$\overline{AO} = \overline{AO}$	
$\overline{AB} = \overline{AC}$	
$\overline{BO} \cong \overline{CO}$	Given
$\Delta ABO \cong \Delta ACO$	Due to isosceles triangle
$\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	
$\angle 7 \cong \angle 8$	
$\overline{AB} \cong \overline{AC}$	
$\Delta ABD \cong \Delta ACD$	
$\angle 5 + \angle 6 = 180$	
$\angle 5 = \angle 6 = 90^o$	
So $\overline{AD} \perp \overline{BC}$	Supplementary angles
$\overline{AD}$ Passes from point O	

Q.2 **Prove that the bisectors of two exterior and third interior angle of a triangle are concurrent**



Given

ΔABC

Exterior angles are $\angle ABQ$ and $\angle BAP$ $\overrightarrow{AT}$ and $\overrightarrow{BS}$ intersect each other at point O therefore join O to C

Draw the angle bisecter of C

$\angle 1 \cong \angle 2$

Construction

$\overline{OM} \perp \overrightarrow{CQ}, \overline{OL} \perp \overline{AB}, \overline{ON} \perp \overrightarrow{CP}$

Proof

Statements	Reasons
$\overline{ON} \cong \overline{OM}$ ....(i)	
$\overline{OL} \cong \overline{OM}$ ....(ii)	
$\overline{ON} \cong \overline{OL}$	
Hence Angle Bisector of C i,e $\angle 1 \cong \angle 2$	Comparing equation (i) and (ii)