Exercise 3.3

Q. 1: Find a third proportional to

(i) 6, 12

Let x be the third proportional,

$$6 : 12 :: 12 : x$$

Product of Extremes = Product of Means

$$6x = 12 \times 12$$

$$x = \frac{12 \times 12}{6}$$

$$x = 24$$

(ii) $a^3, 3a^2$

Let x be the third proportional,

$$a^3 : 3a^2 :: 3a^2 : x$$

Product of Extremes = Product of Means

$$a^3 \times x = 3a^2 \times 3a^2$$

$$x = \frac{3a^2 \times 3a^2}{a^3}$$

$$x = 9a$$

(iii) $a^2 - b^2, a - b$

Let x be the third proportional,

$$a^2 - b^2 : a - b :: a - b : x$$

Product of Extremes = Product of Means

$$a^2 - b^2 \times x = a - b \times a - b$$

$$x = \frac{a-b \times a-b}{a^2-b^2}$$

$$x = \frac{a-b}{a+b}$$

(iv) $(x - y)^2, x^3 - y^3$

Let c be the third proportional,

$$(x - y)^2 : x^3 - y^3 :: x^3 - y^3 : c$$

Product of Extremes = Product of Means

$$(x - y)^2 \times c = x^3 - y^3 \times x^3 - y^3$$

$$c = \frac{x^3-y^3 \times x^3-y^3}{(x-y)^2}$$

$$c = (x^2 + xy + y^2)(x^2 + xy + y^2)$$

(v) $(x + y)^2, x^2 - xy - 2y^2$

Let c be the third proportional,

$$(x + y)^2 : x^2 - xy - 2y^2 :: x^2 - xy - 2y^2 : c$$

Product of Extremes = Product of Means

$$(x + y)^2 \times c = x^2 - xy - 2y^2 \times x^2 - xy - 2y^2$$

$$c = \frac{(x^2-xy-2y^2)(x^2-xy-2y^2)}{(x+y)^2}$$

$$c = \frac{(x^2-2xy+xy-2y^2)(x^2-2xy+xy-2y^2)}{(x+y)^2}$$
$$= \frac{(x(x-2y)+y(x-2y))(x(x-2y)+y(x-2y))}{(x+y)^2}$$
$$= \frac{((x-2y)(x+y))((x-2y)(x+y))}{(x+y)^2}$$
$$= \frac{(x-2y)(x+y)(x-2y)(x+y)}{(x+y)^2}$$
$$= (x-2y)(x-2y)$$

(vi) $\frac{p^2-q^2}{p^3+q^3}, \frac{p-q}{p^2-pq+q^2}$

Let c be the third proportional,

$$\frac{p^2-q^2}{p^3+q^3} : \frac{p-q}{p^2-pq+q^2} :: \frac{p-q}{p^2-pq+q^2} : c$$

Product of Extremes = Product of Means

$$\frac{p^2-q^2}{p^3+q^3} \times c = \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2}$$
$$c = \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2} \times \frac{p^3+q^3}{p^2-q^2}$$
$$= \frac{p-q}{p^2-pq+q^2} \times \frac{p-q}{p^2-pq+q^2} \times \frac{(p+q)(p^2-pq+q^2)}{(p-q)(p+q)}$$
$$= \frac{p-q}{p^2-pq+q^2}$$

Q. 2: Find a fourth proportional to

(i) 5, 8, 15

Let x be the fourth proportional,

$$5 : 8 :: 15 : x$$

Product of Extremes = Product of Means

$$5x = 8 \times 15$$
$$x = \frac{8 \times 15}{5}$$
$$x = 24$$

(ii) $4x^4, 2x^3, 18x^5$

Let c be the fourth proportional,

$$4x^4 : 2x^3 :: 18x^5 : c$$

Product of Extremes = Product of Means

$$4x^4 \times c = 2x^3 \times 18x^5$$
$$c = \frac{2x^3 \times 18x^5}{4x^4}$$
$$c = \frac{9x^8}{x^4}$$
$$c = 9x^4$$

(iii) $15a^5b^6, 10a^2b^5, 21a^3b^3$

Let c be the fourth proportional,

$$15a^5b^6 : 10a^2b^5 :: 21a^3b^3 : c$$

Product of Extremes = Product of Means

$$15a^5b^6 \times c = 10a^2b^5 \times 21a^3b^3$$

$$c = \frac{10a^2b^5 \times 21a^3b^3}{15a^5b^6}$$

$$c = \frac{2a^2b^5 \times 7a^3b^3}{a^5b^6}$$

$$c = 14a^{2+3-5}b^{5+3-6}$$

$$c = 14b^2$$

(iv) $x^2 - 11x + 24, x - 3, 5x^4 - 40x^3$

Let c be the fourth proportional,

$$x^2 - 11x + 24 : x - 3 :: 5x^4 - 40x^3 : c$$

Product of Extremes = Product of Means

$$x^2 - 11x + 24 \times c = x - 3 \times 5x^4 - 40x^3$$

$$c = \frac{x-3 \times 5x^3(x-8)}{x^2-8x-3x+24}$$

$$c = \frac{x-3 \times 5x^3(x-8)}{x(x-8)-3(x-8)}$$

$$c = \frac{x-3 \times 5x^3(x-8)}{(x-8)(x-3)}$$

$$c = 5x^3$$

(v) $p^3 + q^3, p^2 - q^2, p^2 - pq + q^2$

Let c be the fourth proportional,

$$p^3 + q^3 : p^2 - q^2 :: p^2 - pq + q^2 : c$$

Product of Extremes = Product of Means

$$p^3 + q^3 \times c = p^2 - q^2 \times p^2 - pq + q^2$$

$$c = \frac{(p^2-q^2)(p^2-pq+q^2)}{p^3+q^3}$$

$$c = \frac{(p-q)(p+q)(p^2-pq+q^2)}{p^3+q^3}$$

$$c = \frac{(p-q)(p^3+q^3)}{p^3+q^3}$$

$$c = p - q$$

(vi) $(p^2 - q^2)(p^2 + pq + q^2), p^3 + q^3, p^3 - q^3$

Let c be the fourth proportional,

$$(p^2 - q^2)(p^2 + pq + q^2) : p^3 + q^3 :: p^3 - q^3 : c$$

Product of Extremes = Product of Means

$$(p^2 - q^2)(p^2 + pq + q^2) \times c = p^3 + q^3 \times p^3 - q^3$$

$$c = \frac{(p^3+q^3)(p^3-q^3)}{(p^2-q^2)(p^2+pq+q^2)}$$

$$c = \frac{(p+q)(p^2-pq+q^2)(p-q)(p^2+pq+q^2)}{(p+q)(p-q)(p^2+pq+q^2)}$$

$$c = p^2 - pq + q^2$$

Q. 3: Find a mean proportional between

(i) 20, 45

Let x be the mean proportional,

$$20 : x :: x : 45$$

Product of means = Product of extremes

$$x^2 = 20 \times 45$$

$$x^2 = 900$$
$$x = \pm 30$$

(ii) $20x^3y^5, 5x^7y$

Let c be the mean proportional,

$$20x^3y^5 : c :: c : 5x^7y$$

Product of means = Product of extremes

$$c^2 = 20x^3y^5 \times 5x^7y$$
$$c^2 = 100x^{10}y^6$$
$$c = \pm 10x^5y^3$$

(iii) $15p^4qr^3, 135q^5r^7$

Let c be the mean proportional,

$$15p^4qr^3 : c :: c : 135q^5r^7$$

Product of means = Product of extremes

$$c^2 = 15p^4qr^3 \times 135q^5r^7$$
$$c^2 = 2025p^4q^6r^{10}$$
$$c = \pm 45p^2q^3r^5$$

(iv) $x^2 - y^2, \frac{x-y}{x+y}$

Let c be the mean proportional,

$$x^2 - y^2 : c :: c : \frac{x-y}{x+y}$$

Product of Extremes = Product of Means

$$c^2 = x^2 - y^2 \times \frac{x-y}{x+y}$$
$$c^2 = \frac{(x-y)(x+y)(x-y)}{(x+y)}$$
$$c^2 = (x-y)^2$$
$$c = \pm(x-y)$$

Q. 4: Find the values of the letter involved in the following continued proportions.

(i) 5, p, 45

$$5 : p :: p : 45$$

Product of means = Product of extremes

$$p^2 = 5 \times 45$$
$$p^2 = 225$$
$$p = \pm 15$$

(ii) $8, x, 18$

$$8 : x :: x : 18$$

Product of means = Product of extremes

$$x^2 = 8 \times 18$$
$$x^2 = 144$$
$$x = \pm 12$$

(iii) $12, 3p-6, 27$

Let c be the mean proportional,

$12 : 3p-6 :: 3p-6 : 27$

Product of means = Product of extremes

$(3p-6)^2 = 12 \times 27$

$(3p-6)^2 = 324$

$3p-6 = \pm 18$

$3p-6 = 18$; $3p-6 = -18$

$3p = 24$; $3p = -12$

$p = 8$; $p = -4$

(iv) $7, m-3, 28$

Let c be the mean proportional,

$7 : m-3 :: m-3 : 28$

Product of means = Product of extremes

$(m-3)^2 = 7 \times 28$

$(m-3)^2 = 196$

$m-3 = \pm 14$

$m-3 = 14$; $m-3 = -14$

$m = 17$; $m = -11$