Exercise 3.7

Q. 1: The surface area A of a cube varies directly as the square of the length l of an edge and A = 27 square units when l = 3 units.

Find (i) A when l = 4 units (ii) l when A = 12 sq. units.

$A \propto l^2$

$A = kl^2$ ----------- (i)

When A = 27 square units, l = 3 units

$(27) = k(3)^2$

$(27) = k(9)$

$k = 3$

So equation (i) becomes

$A = 3l^2$

(i) When l = 4 units

$A = 3(4)^2$

$A = 3(16)$

$A = 48$ sq. units

(ii) When A = 12 sq. units

$12 = 3l^2$

$4 = l^2$

$l = 2$

Q. 2: The surface area S of a sphere varies directly as the square of radius r, and S = 16π when r = 2. Find r when $S = 36\pi$.

$S \propto r^2$

$S = kr^2$ ----------- (i)

S = 16π when r = 2

$(16\pi) = k(2)^2$

$(16\pi) = k(4)$

$k = 4\pi$

So equation (i) becomes

$S = 4\pi r^2$

When S = 36π units

$36\pi = 4\pi r^2$

$r^2 = 9$

$r = 3$

Q. 3: In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S and $F = 32lb$ when $S = 1.6\ in$. Find (i) S when $F = 50\ lb$ (ii) F when $S = 0.8in$.

$F \propto S$

$F = kS$ ----------- (i)

F = 32lb when S = 1.6

$(32) = k(1.6)$

$k = \frac{32}{1.6}$

$k = 20$

So equation (i) becomes

$F = 20S$

(i) When F = 50lb

$50lb = 20S$

$S = 2.5$ in

(ii) When S = 0.8in

$F = 20(0.8)$

$F = 16\ lb$

Q. 4: The intensity I of light from a given source varies inversely as the square of the distance d from it. If the intensity is 20 candlepower at a distance of 12ft. from the source, find the intensity at a point 8ft. from the source.

$I \propto \frac{1}{d^2}$

$I = \frac{k}{d^2}$ ----------- (i)

I = 20 when d = 12ft

$(20) = \frac{k}{(12)^2}$

$20 = \frac{k}{144}$

$k = 2880$

So equation (i) becomes

$I = \frac{2880}{d^2}$

When d = 8ft

$I = \frac{2880}{8^2}$

$I = \frac{2880}{64}$

$I = 45$cp

Q. 5: The pressure P in a body of fluid varies directly as the depth d. If the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 lb/sq. in, how deep must the fluid be to exert a pressure of 9 lb/sq. in?

$P \propto d$

$P = kd$ ----------- (i)

d = 5ft when P = 2.25 lb/sq. in

$2.25 = k(5)$

$k = \frac{2.25}{5}$

$k = 0.45$

So equation (i) becomes

$P = 0.45d$

When P = 9 lb/sq. in

$9 = 0.45d$

$d = \frac{9}{0.45}$

$d = 20$

Q. 6: Labour costs c varies jointly as the number of workers n and the average number of days d, if the cost of 800 workers for 13 days is Rs. 286000, then find the labour cost of 600 workers for 18 days.

$c \propto nd$

$c = knd$ ----------- (i)

c = Rs. 286000, n = 800, d = 13

$286000 = k(800)(13)$

$k = \frac{286000}{10400}$

$k = 27.5$

So equation (i) becomes

$c = 27.5nd$

When n = 600, d = 18

$c = 27.5(600)(18)$

$c = 297000$

Q. 7: The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length l. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4 inch pillar must be to support a load of 28 tons?

$c \propto \frac{d^4}{l^2}$

$c = \frac{kd^4}{l^2}$ ----------- (i)

c = 63 tons, d = 6inch, l = 30feet

$63 = \frac{k(6)^4}{(30)^2}$

$k = \frac{63 \times 900}{1296}$

$k = 43.75$

So equation (i) becomes

$c = \frac{43.75d^4}{l^2}$

When d = 4inch, c = 28 tons

$28 = \frac{43.75(4)^4}{l^2}$

$l^2 = \frac{43.75(4)^4}{28}$

$l^2 = 400$

$l = 20$ feet

Q. 8: The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inversely as the power p of the motor. If 25 sec. are required for a 4-hp motor to lift 500 lb through 40 ft, what power is required to lift 800 lb, through 120 ft in 40 sec.?

$T \propto \frac{wd}{p}$

$T = \frac{kwd}{p}$ ----------- (i)

T=25 sec., p = 4-hp, w = 500lb, d = 40ft

$$25 = \frac{k(500)(40)}{(4)}$$
$$k = \frac{25 \times 4}{500 \times 40}$$
$$k = 0.005$$

So equation (i) becomes

$$T = \frac{0.005wd}{p}$$

When w = 800lb, d = 120ft, T = 40 sec.

$$40 = \frac{0.005 \times 800 \times 120}{p}$$
$$p = \frac{0.005 \times 800 \times 120}{40}$$
$$p = 12 \text{ hp}$$

Q. 9: The kinetic energy (K.E.) of a body varies jointly as the mass "m" of the body and the square of its velocity "v". If the kinetic energy is 4320 ft/lb when the mass is 45 lb and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 lb automobile travelling 44 ft/sec.

$$K.E \propto mv^2$$
$$K.E = kmv^2 \text{ ----------- (i)}$$

K.E=4320 ft/lb, m = 45lb, v = 24 ft/sec.

$$4320 = k(45)(24)^2$$
$$k = \frac{4320}{45 \times 576}$$
$$k = \frac{4320}{25920}$$
$$k = \frac{1}{6}$$

So equation (i) becomes

$$K.E = \frac{mv^2}{6}$$

When m = 3000lb, v = 44 ft/sec.

$$K.E = \frac{(3000)(44)^2}{6}$$
$$K.E = 500 \times 1936$$
$$K.E = 968000 \text{ ft/lb}$$