Exercise 4.1

Resolve into partial fractions.

1. $\frac{7x-9}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$ ..... (i)

multiplying by $(x+1)(x-3)$ we get

$$7x-9 = A(x-3)+B(x+1) \text{ ..... (ii)}$$

put $x=-1$ in (ii)

$$7(-1)-9 = A(-1-3)+B(-1+1)$$
$$-7-9 = A(-4)$$
$$-16 = -4A$$
$$A = \frac{-16}{-4}$$
$$A = 4$$

put $x=3$ in (ii)

$$7(3)-9 = A(3-3)+B(3+1)$$
$$21-9 = B(4)$$
$$12 = 4B$$
$$B = \frac{12}{4}$$
$$B = 3$$

put the values in (i) we get

$$\frac{7x-9}{(x+1)(x-3)} = \frac{4}{(x+1)} + \frac{3}{(x-3)}$$

2. $\frac{x-11}{(x-4)(x+3)} = \frac{A}{(x-4)} + \frac{B}{(x+3)}$ ..... (i)

multiplying by $(x-4)(x+3)$ we get

$$x-11 = A(x+3)+B(x-4) \text{ ..... (ii)}$$

put $x=4$ in (ii)

$$4-11 = A(4+3)+B(4-4)$$
$$-7 = A(7)$$
$$-7 = 7A$$
$$A = \frac{-7}{7}$$
$$A = -1$$

put $x=-3$ in (ii)

$$-3-11 = A(-3+3)+B(-3-4)$$
$$-14 = B(-7)$$
$$-14 = -7B$$
$$B = \frac{-14}{-7}$$
$$B = 2$$

put the values in (i) we get

$$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{(x-4)} + \frac{2}{(x+3)}$$

3. $\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$ ..... (i)

multiplying by $(x-1)(x+1)$ we get

$$3x-1 = A(x+1) + B(x-1) \text{ ..... (ii)}$$

put $x = 1$ in (ii)

$$3(1)-1 = A(1+1) + B(1-1)$$
$$3-1 = A(2)$$
$$2 = 2A$$
$$A = \frac{2}{2}$$
$$A = 1$$

put $x = -1$ in (ii)

$$3(-1)-1 = A(-1+1) + B(-1-1)$$
$$-3-1 = B(-2)$$
$$-4 = -2B$$
$$B = \frac{-4}{-2}$$
$$B = 2$$

put the values in (i) we get

$$\frac{3x-1}{x^2-1} = \frac{1}{(x-1)} + \frac{2}{(x+1)}$$

4. $\frac{x-5}{x^2+2x-3} = \frac{x-5}{x^2+3x-x-3} = = \frac{x-5}{x(x+3)-1(x+3)}$

$$\frac{x-5}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)} \text{ ..... (i)}$$

multiplying by $(x+3)(x-1)$ we get

$$x-5 = A(x-1) + B(x+3) \text{ ..... (ii)}$$

put $x = -3$ in (ii)

$$-3-5 = A(-3-1) + B(-3+3)$$
$$-8 = A(-3-1)$$
$$-8 = -4A$$
$$A = \frac{-8}{-4}$$
$$A = 2$$

put $x = 1$ in (ii)

$$1-5 = A(1-1) + B(1+3)$$
$$-4 = B(4)$$
$$-4 = 4B$$
$$B = \frac{-4}{4}$$
$$B = -1$$

put the values in (i) we get

$$\frac{x-5}{(x+3)(x-1)} = \frac{2}{(x+3)} + \frac{-1}{(x-1)}$$

5. $\frac{3x+3}{(x-1)(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x+2)}$ ..... (i)

multiplying by $(x-1)(x+2)$ we get

$3x+3 = A(x+2) + B(x-1)$ ..... (ii)

put $x = 1$ in (ii)

$3(1)+3 = A(1+2) + B(1-1)$

$3+3 = A(3)$

$6 = 3A$

$A = \frac{6}{3}$

$A = 2$

put $x = -2$ in (ii)

$3(-2)+3 = A(-2+2) + B(-2-1)$

$-6+3 = B(-3)$

$-3 = -3B$

$B = \frac{-3}{-3}$

$B = 1$

put the values in (i) we get

$\frac{3x+3}{(x-1)(x+2)} = \frac{2}{(x-1)} + \frac{1}{(x+2)}$

6. $\frac{7x-25}{(x-4)(x-3)} = \frac{A}{(x-4)} + \frac{B}{(x-3)}$ ..... (i)

multiplying by $(x-4)(x-3)$ we get

$7x-25 = A(x-3) + B(x-4)$ ..... (ii)

put $x = 4$ in (ii)

$7(4)-25 = A(4-3) + B(4-4)$

$28-25 = A(1)$

$3 = A$

$A = 3$

put $x = 3$ in (ii)

$7(3)-25 = A(3-3) + B(3-4)$

$21-25 = B(-1)$

$-4 = -1B$

$B = \frac{-4}{-1}$

$B = 4$

put the values in (i) we get

$\frac{7x-25}{(x-4)(x-3)} = \frac{3}{(x-4)} + \frac{4}{(x-3)}$

7. $\frac{x^2+2x+1}{(x-2)(x+3)} = \frac{x^2+2x+1}{x^2+3x-2x-6} = \frac{x^2+2x+1}{x^2+x-6} = 1 + \frac{x+7}{x^2+x-6}$ ..... (i)

$\frac{x+7}{(x-2)(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x+3)}$ ..... (ii)

multiplying by $(x-2)(x+3)$ we get

$x+7 = A(x+3) + B(x-2)$ ..... (iii)

put $x = 2$ in (iii)

$$\begin{aligned} 2+7 &= A(2+3)+B(2-2) \\ 9 &= A(5) \\ 9 &= 5A \\ A &= \frac{9}{5} \end{aligned}$$

put $x = -3$ in (iii)

$$\begin{aligned} -3+7 &= A(-3+3)+B(-3-2) \\ 4 &= B(-5) \\ 4 &= -5B \\ B &= \frac{4}{-5} \end{aligned}$$

put the values in (ii) we get

$$\frac{x+7}{(x-2)(x+3)} = \frac{9}{5(x-2)} - \frac{9}{5(x+3)}$$

put the value in (i) we get

$$\frac{x^2+2x+1}{(x-2)(x+3)} = 1 + \frac{9}{5(x-2)} - \frac{9}{5(x+3)}$$

8. $$\frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3+\frac{8x-4}{3x^2-2x-1} \quad \text{..... (i)}$$

$$\frac{8x-4}{3x^2-2x-1} = \frac{8x-4}{(x-1)(3x+1)} = \frac{A}{(x-1)} + \frac{B}{(3x+1)} \quad \text{..... (ii)}$$

multiplying by $(x-1)(3x+1)$ we get

$$8x-4 = A(3x+1)+B(x-1) \quad \text{..... (iii)}$$

put $x = 1$ in (iii)

$$\begin{aligned} 8(1)-4 &= A(3(1)+1)+B(1-1) \\ 4 &= A(3+1) \\ 4 &= 4A \\ A &= 1 \end{aligned}$$

put $x = -\frac{1}{3}$ in (iii)

$$\begin{aligned} 8\left(-\tfrac{1}{3}\right)-4 &= A\left(3\left(-\tfrac{1}{3}\right)+1\right)+B\left(-\tfrac{1}{3}-1\right) \\ -\tfrac{8}{3}-4 &= B\left(-\tfrac{1}{3}-1\right) \\ \frac{-8-12}{3} &= B\left(\frac{-1-3}{3}\right) \\ \frac{-20}{3} &= \frac{-4}{3}B \\ B &= \frac{-20}{3}\times\frac{3}{-4} \\ B &= 5 \end{aligned}$$

put the values in (ii) we get

$$\frac{8x-4}{3x^2-2x-1} = \frac{1}{(x-1)} + \frac{5}{(3x+1)}$$

put the value in (i) we get

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3+\frac{1}{(x-1)}+\frac{5}{(3x+1)}$$