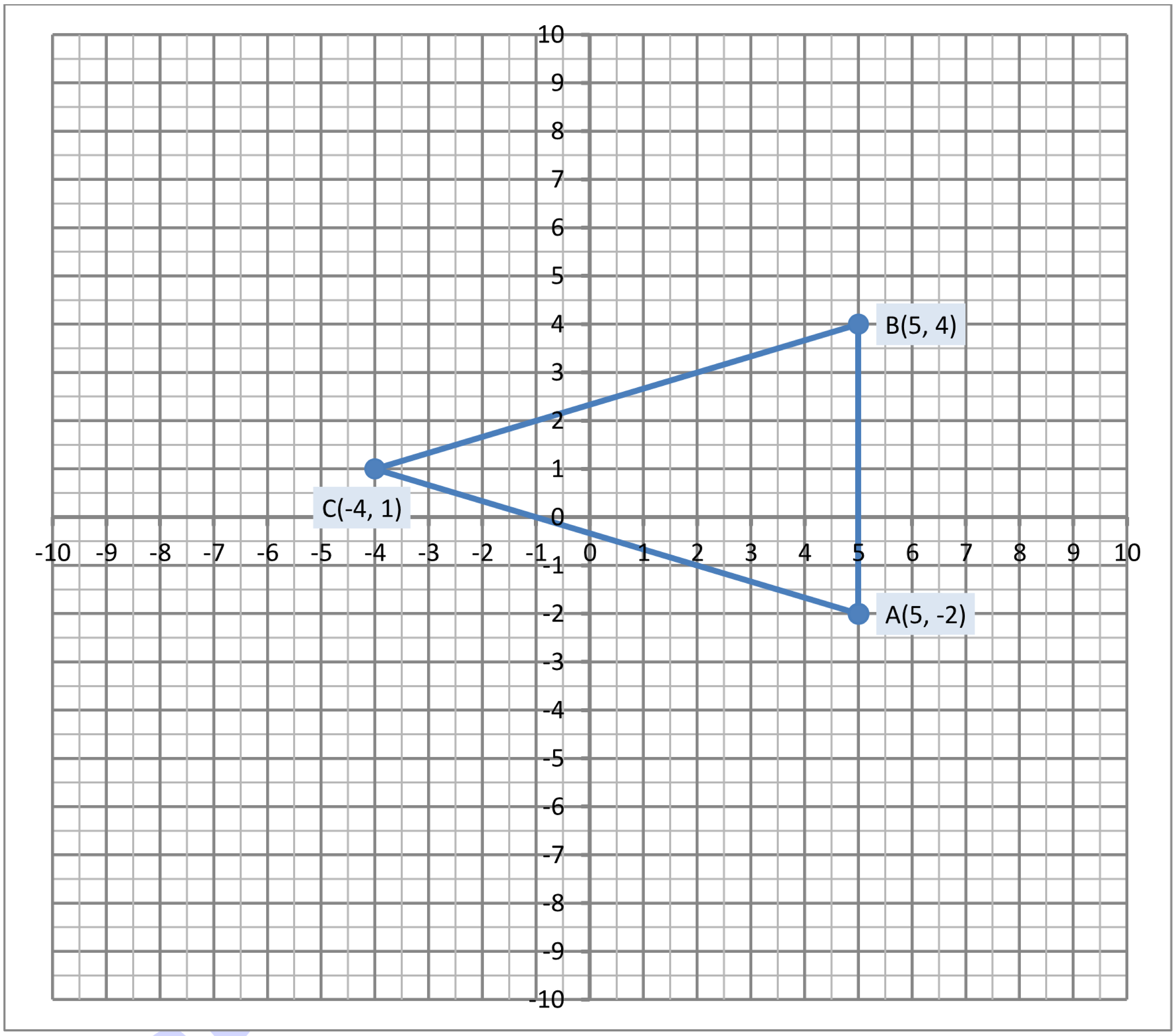
Exercise 9.2

1. Show whether the points with vertices (5, -2), (5, 4) and (-4, 1) are vertices of an equilateral triangle or an isosceles triangle?

x	y
5	-2
5	4
-4	1



By Distance Formula

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5 - 5)^2 + (4 + 2)^2}$

$= \sqrt{(0)^2 + (6)^2}$

$= \sqrt{(6)^2}$

$= 6$

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$= \sqrt{(-4 - 5)^2 + (1 - 4)^2}$

$= \sqrt{(-9)^2 + (-3)^2}$

$= \sqrt{81 + 9}$

$= \sqrt{90}$

$= 3\sqrt{10}$

$|CA| = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2}$

$= \sqrt{(5+4)^2 + (-2-1)^2}$

$= \sqrt{(9)^2 + (-3)^2}$

$= \sqrt{81 + 9}$

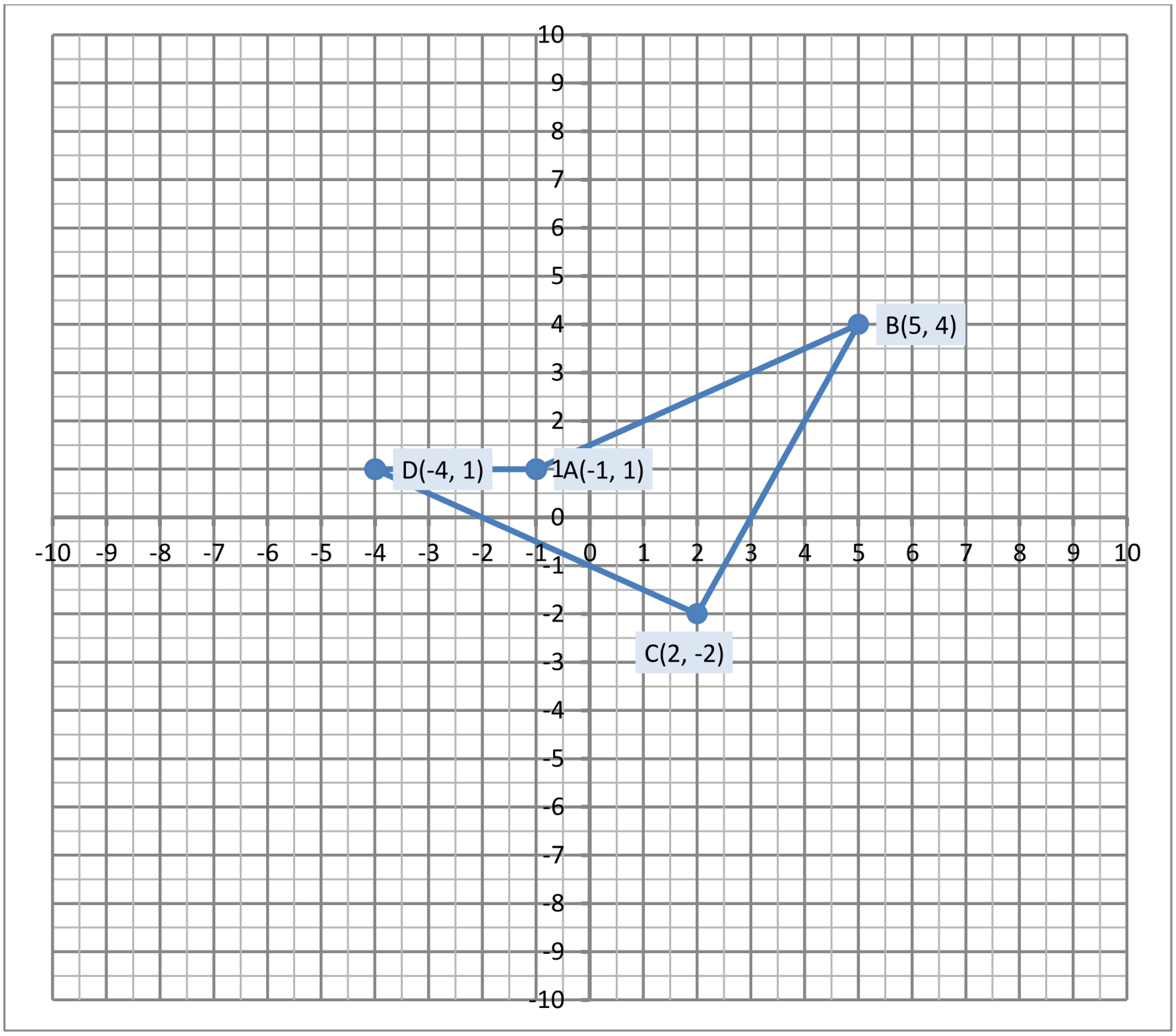
$= \sqrt{90}$

$= 3\sqrt{10}$

As $|BC| = |CA| \neq |AB|$. So, The given points are vertices of an isosceles triangle.

2. Show whether or not the points with vertices (-1, 1), (5, 4), (2, -2), (-4, 1) form a square?

x	y
-1	1
5	4
2	-2
-4	1



By Distance Formula

$|AB| = \sqrt{(5+1)^2 + (4-1)^2}$

$= \sqrt{(6)^2 + (3)^2}$

$= \sqrt{36 + 9}$

$= \sqrt{45}$

$= 3\sqrt{5}$

$|BC| = \sqrt{(2-5)^2 + (-2-4)^2}$

$= \sqrt{(-3)^2 + (-6)^2}$
$= \sqrt{9 + 36}$
$= \sqrt{45}$
$= 3\sqrt{5}$

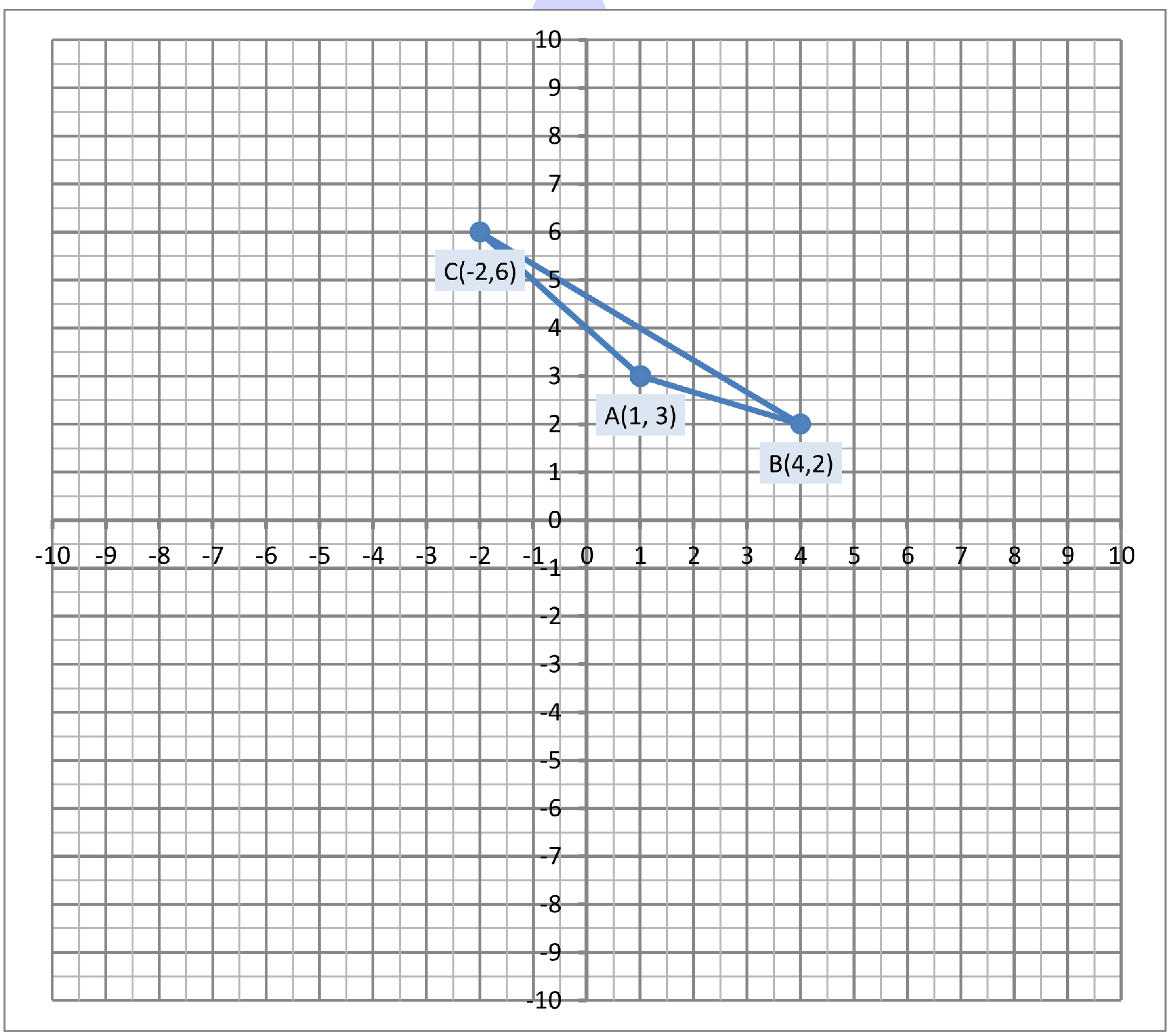
$|DC| = \sqrt{(2+4)^2 + (-2-1)^2}$
$= \sqrt{(6)^2 + (-3)^2}$
$= \sqrt{36 + 9}$
$= \sqrt{45}$
$= 3\sqrt{5}$

$|AD| = \sqrt{(-4+1)^2 + (1-1)^2}$
$= \sqrt{(-3)^2 + (0)^2}$
$= \sqrt{9}$
$= 3$

As $|AB| = |BC| = |DC| \neq |AD|$. So, The given points are not vertices of a square.

3. Show whether or not the points with coordinates (1, 3), (4, 2) and (-2, 6) are vertices of a right triangle?

x	y
1	3
4	2
-2	6



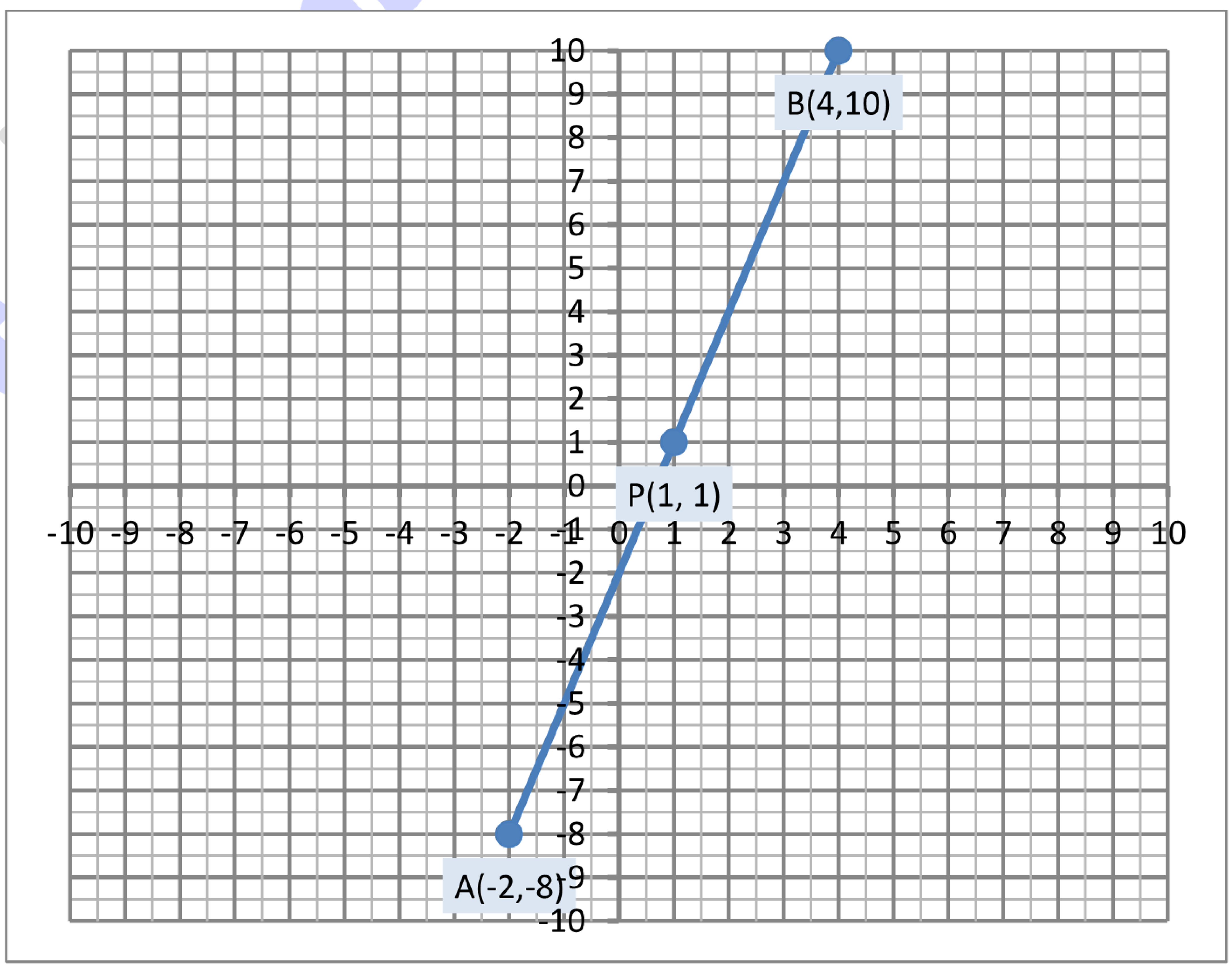
By Distance Formula

$|AB| = \sqrt{(4-1)^2 + (2-3)^2}$
$= \sqrt{(3)^2 + (-1)^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$
$|AB|^2 = 10$
$|BC| = \sqrt{(-2-4)^2 + (6-2)^2}$
$= \sqrt{(-6)^2 + (4)^2}$
$= \sqrt{36+16}$
$= \sqrt{52}$
$|BC|^2 = 52$
$|AC| = \sqrt{(-2-1)^2 + (6-3)^2}$
$= \sqrt{(-3)^2 + (3)^2}$
$= \sqrt{9+9}$
$= \sqrt{18}$
$|AC|^2 = 18$

As $|BC|^2 \neq |AC|^2 + |AB|^2$. So, The given points are not vertices of a right triangle.

4. Use the distance formula to prove whether or not points (1, 1), (-2, -8) and (4, 10) lie on a straight line?

x	y
1	1
-2	-8
4	10



By Distance Formula

$|AP| = \sqrt{(1+2)^2 + (1+8)^2}$
$= \sqrt{(3)^2 + (9)^2}$
$= \sqrt{9+81}$
$= \sqrt{90}$

$|PB| = \sqrt{(4-1)^2 + (10-1)^2}$
$= \sqrt{(3)^2 + (9)^2}$
$= \sqrt{9+81}$
$= \sqrt{90}$

$|AB| = \sqrt{(4+2)^2 + (10+8)^2}$
$= \sqrt{(6)^2 + (18)^2}$
$= \sqrt{36+324}$
$= \sqrt{360}$
$= 2\sqrt{90}$

As $|AP| + |PB| = |AB|$. So, The given points lie on a straight line.

5. Find k, given that the point (2, k) is equidistant from (3, 7) and (9,1).

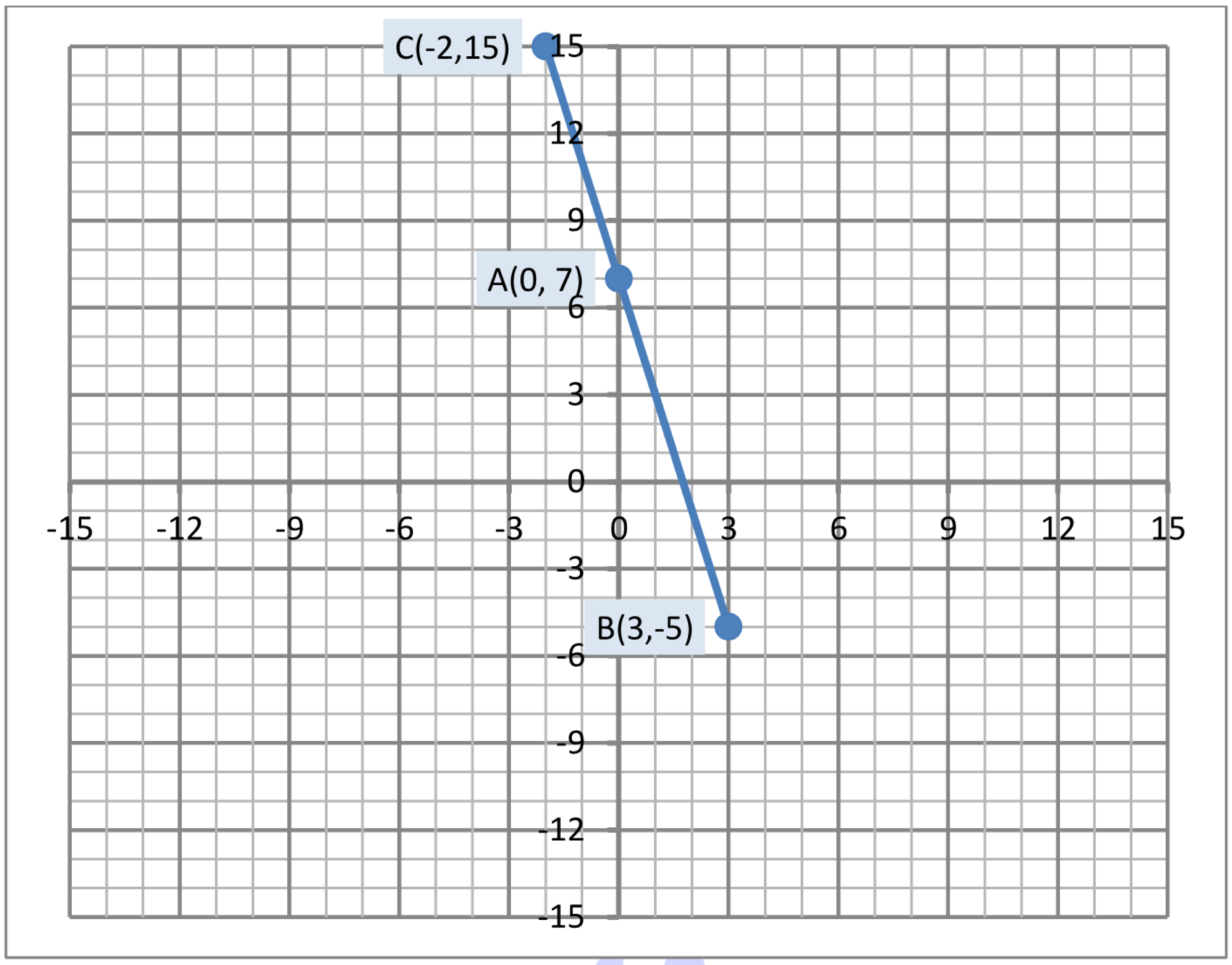
We have P(2, k), A(3, 7) and B(9,1)

By Distance Formula

$|PA| = |PB|$
$\sqrt{(3-2)^2 + (7-k)^2} = \sqrt{(9-2)^2 + (1-k)^2}$
$\sqrt{(1)^2 + (7-k)^2} = \sqrt{(7)^2 + (1-k)^2}$
$\sqrt{1 + (7-k)^2} = \sqrt{49 + (1-k)^2}$
$1 + (7-k)^2 = 49 + (1-k)^2$
$1 + 49 - 14k + k^2 = 49 + 1 - 2k + k^2$
$50 - 14k + k^2 = 50 - 2k + k^2$
$-14k = -2k$
$-14k + 2k = 0$
$-12k = 0$
$k = 0$

6. Use distance formula to verify that the points A(0, 7), B(3, -5), C(-2, 15) are collinear.

x	y
0	7
3	-5
-2	15



By Distance Formula

$|AB| = \sqrt{(3-0)^2 + (-5-7)^2}$
$= \sqrt{(3)^2 + (-12)^2}$
$= \sqrt{9 + 144}$
$= \sqrt{153}$
$= 3\sqrt{17}$

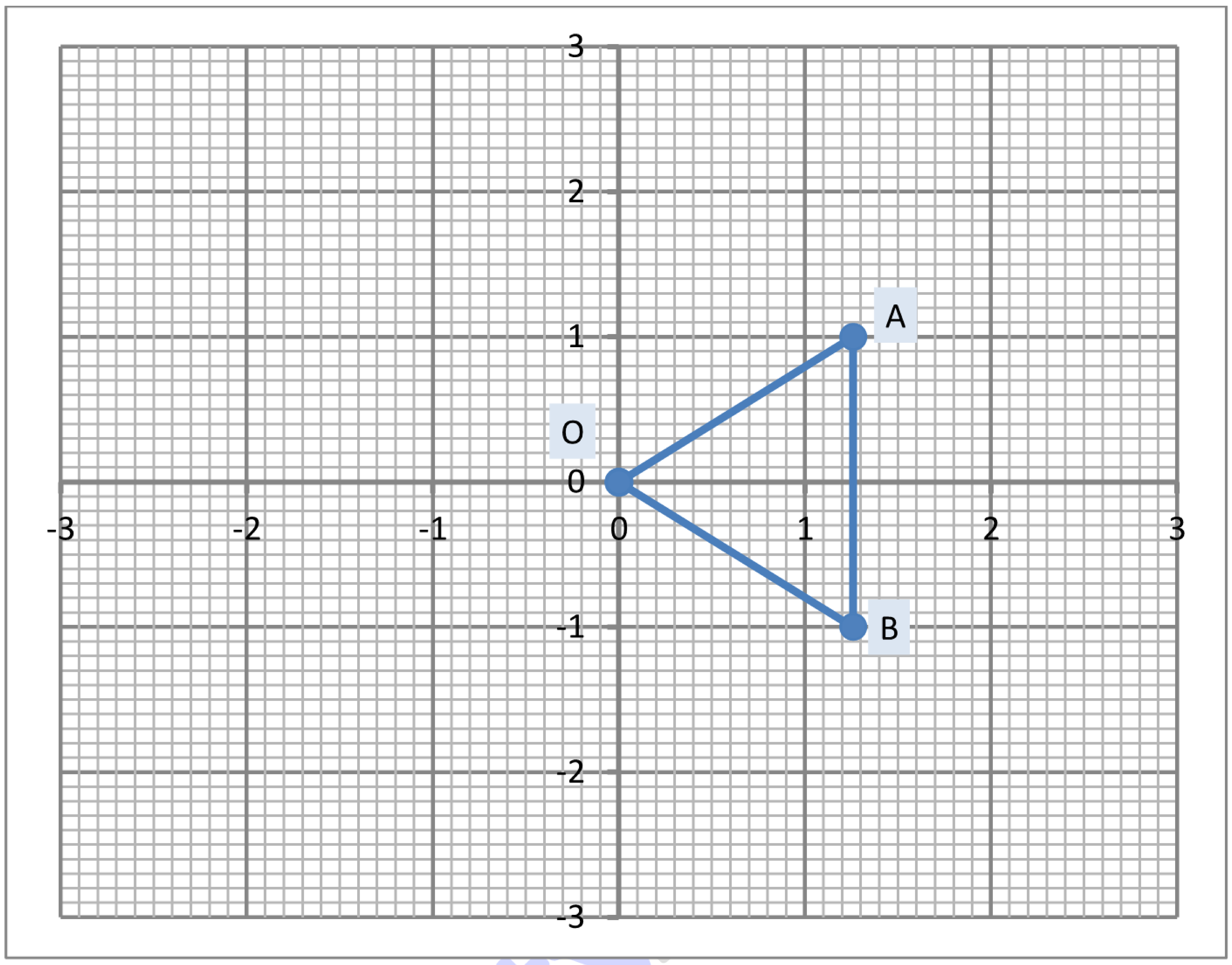
$|AC| = \sqrt{(-2-0)^2 + (15-7)^2}$
$= \sqrt{(-2)^2 + (8)^2}$
$= \sqrt{4 + 64}$
$= \sqrt{68}$
$= 2\sqrt{17}$

$|BC| = \sqrt{(-2-3)^2 + (15+5)^2}$
$= \sqrt{(-5)^2 + (20)^2}$
$= \sqrt{25 + 400}$
$= \sqrt{425}$
$= 5\sqrt{17}$

As $|AB| + |AC| = |BC|$. So, The given points lie on a straight line.

7. Verify whether or not the points O(0, 0), A($\sqrt{3}$, 1), B ($\sqrt{3}$, -1) are the vertices of an equilateral triangle.

x	y
0	0
$\sqrt{3}$	1
$\sqrt{3}$	-1



By Distance Formula

$$|OA| = \sqrt{(\sqrt{3}-0)^2 + (1-0)^2}$$
$$= \sqrt{(\sqrt{3})^2 + (1)^2}$$
$$= \sqrt{3+1}$$
$$= \sqrt{4}$$
$$= 2$$

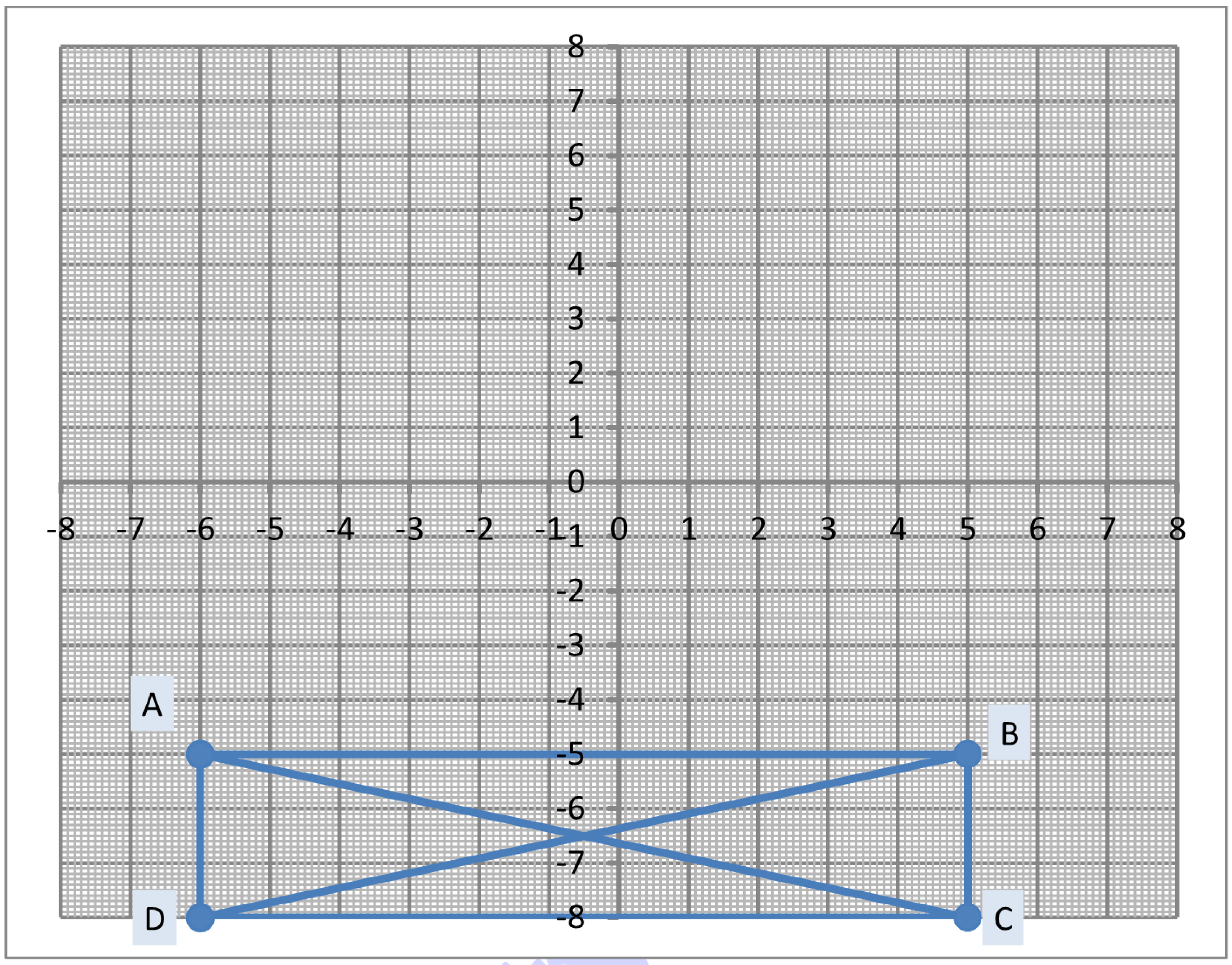
$$|OB| = \sqrt{(\sqrt{3}-0)^2 + (-1-0)^2}$$
$$= \sqrt{(\sqrt{3})^2 + (-1)^2}$$
$$= \sqrt{3+1}$$
$$= \sqrt{4}$$
$$= 2$$

$$|AB| = \sqrt{(\sqrt{3}-\sqrt{3})^2 + (-1-1)^2}$$
$$= \sqrt{(0)^2 + (-2)^2}$$
$$= \sqrt{4}$$
$$= 2$$

As $|OA| = |OB| = |AB|$. So, The given points are vertices of an equilateral triangle.

8. Show that the points A(-6, -5), B(5, -5), C(5, -8) and D(-6, -8) are vertices of a rectangle. Find the lengths of its diagonals. Are they equal?

x	y
-6	-5
5	-5
5	-8
-6	-8



By Distance Formula

$|AB| = \sqrt{(5+6)^2 + (-5+5)^2}$
$= \sqrt{(11)^2 + (0)^2}$
$= \sqrt{(11)^2}$
$= 11$

$|BC| = \sqrt{(5-5)^2 + (-8+5)^2}$
$= \sqrt{(0)^2 + (-3)^2}$
$= \sqrt{(3)^2}$
$= 3$

$|CD| = \sqrt{(-6-5)^2 + (-8+8)^2}$
$= \sqrt{(-11)^2 + (0)^2}$
$= \sqrt{(11)^2}$
$= 11$

$|AD| = \sqrt{(-6+6)^2 + (-8+5)^2}$
$= \sqrt{(0)^2 + (-3)^2}$
$= \sqrt{(3)^2}$
$= 3$

Finding diagonals distances

$|AC| = \sqrt{(5+6)^2 + (-8+5)^2}$

$= \sqrt{(11)^2 + (-3)^2}$

$= \sqrt{121 + 9}$

$= \sqrt{130}$

$|BD| = \sqrt{(-6-5)^2 + (-8+5)^2}$

$= \sqrt{(-11)^2 + (-3)^2}$

$= \sqrt{121 + 9}$

$= \sqrt{130}$

As we can see $|AB| = |CD|$ and $|BC| = |AD|$.

Now checking the second condition for rectangle which is applying Pythagoras Theorem

$|AC|^2 = |AB|^2 + |BC|^2$ or $|BD|^2 = |CD|^2 + |BC|^2$

in both cases we have

$130 = (11)^2 + (3)^2$

$130 = 121 + 9$

$130 = 121 + 9$

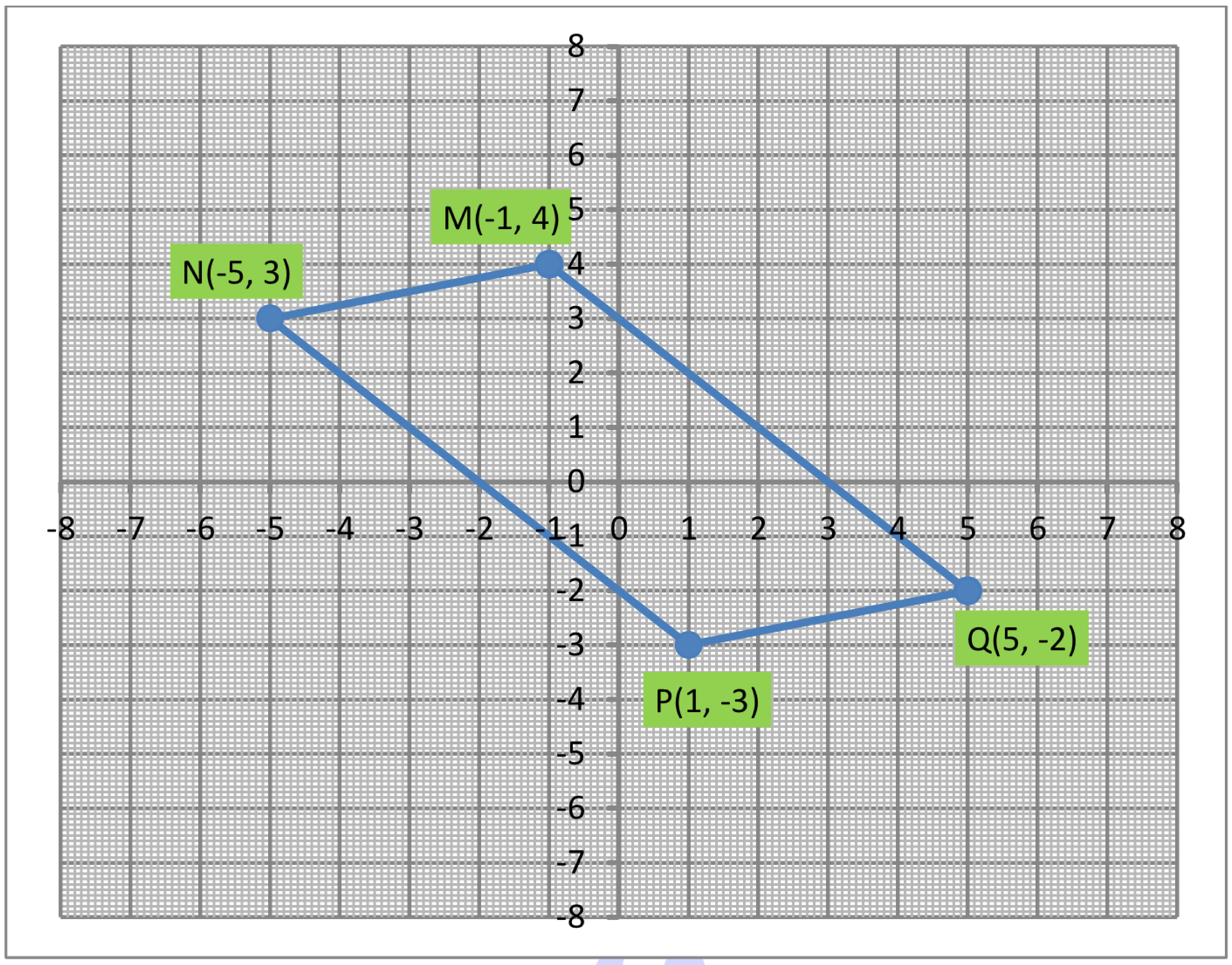
$130 = 130$

So, the vertices make a rectangle.

Also the Length of diagonals is equal i.e. $|AC| = |BD|$

9. Show that the points M(-1, 4), N(-5, 3), P(1, -3) and Q(5, -2) are the vertices of a parallelogram.

x	y
-1	4
−5	3
1	-3
5	-2



By Distance Formula

$|MN| = \sqrt{(-5+1)^2+(3-4)^2}$

$= \sqrt{(-4)^2+(-1)^2}$

$= \sqrt{16+1}$

$= \sqrt{17}$

$|NP| = \sqrt{(1+5)^2+(-3-3)^2}$

$= \sqrt{(6)^2+(-6)^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$

$= 2\sqrt{18}$

$|PQ| = \sqrt{(5-1)^2+(-2+3)^2}$

$= \sqrt{(4)^2+(1)^2}$

$= \sqrt{16+1}$

$= \sqrt{17}$

$|QM| = \sqrt{(-1-5)^2+(4+2)^2}$

$= \sqrt{(-6)^2+(6)^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$

$= 2\sqrt{18}$

Finding diagonals distances

$|MP| = \sqrt{(1+1)^2 + (-3-4)^2}$
$= \sqrt{(2)^2 + (-7)^2}$
$= \sqrt{4+49}$
$= \sqrt{53}$

$|NQ| = \sqrt{(5+5)^2 + (-2-3)^2}$
$= \sqrt{(10)^2 + (-5)^2}$
$= \sqrt{100+25}$
$= \sqrt{125}$

As we can see $|MN| = |PQ|$ and $|NP| = |QM|$.

Now checking the second condition for rectangle which is applying Pythagoras Theorem

$|MP|^2 = |MN|^2 + |NP|^2$

in both cases we have

$53 = 17 + 72$

$53 \neq 89$

So, the vertices make a parallelogram.

10. Find the length of the diameter of the circle having centre at C(-3, 6) and passing through P(1, 3).

According to given condition

Radius of the circle $= |CP|$
$= \sqrt{(1+3)^2 + (3-6)^2}$
$= \sqrt{(4)^2 + (-3)^2}$
$= \sqrt{16+9}$
$= \sqrt{25}$
$= 5$

As we know

Diameter of Circle $= 2(radius\ of\ circle)$
$= 2(5)$
$= 10$