Exercise 5.5

Q. 1: If $L = \{a, b, c\}$, $M = \{3, 4\}$, then find two binary relations of $L \times M$ and $M \times L$.

$L \times M = \{a, b, c\} \times \{3, 4\}$

$= \{(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$

$M \times L = \{3, 4\} \times \{a, b, c\}$

$= \{(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)\}$

$R_1 = \{(a, 3), (b, 4), (c, 3)\}$

$R_2 = \{(a, 4), (b, 3), (c, 4)\}$

$R_3 = \{(3, a), (4, a)\}$

$R_4 = \{(3, b), (4, b), (3, c), (4, c)\}$

Q. 2: If $Y = \{-2, 1, 2\}$, then make two binary relations $Y \times Y$. Also find their domain and range.

$Y \times Y = \{-2, 1, 2\} \times \{-2, 1, 2\}$

$= \{(-2, -2), (-2, 1), (-2, 2), (1, -2), (1, 1), (1, 2), (2, -2), (2, 1), (2, 2)\}$

$R_1 = \{(-2, -2), (-2, 1), (1, 2), (2, 2)\}$

$Dom\ R_1 = \{-2, 1, 2\} = L$

$Range\ R_1 = \{-2, 1, 2\}$

$R_2 = \{(-2, 1), (1, 1), (-2, 2)\}$

$Dom\ R_2 = \{-2, 1\}$

$Range\ R_2 = \{1, 2\}$

Q. 3: If $L = \{a, b, c\}$ and $L = \{d, e, f, g\}$, then find two binary relations in each:

(i) $L \times L$

$L \times L = \{a, b, c\} \times \{a, b, c\}$

$= \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$

$R_1 = \{(a, a), (a, b)\}$

$R_2 = \{(b, c), (c, c)\}$

(ii) $L \times M$

$L \times M = \{a, b, c\} \times \{d, e, f, g\}$

$= \{(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g)\}$

$R_1 = \{(a, d), (b, g)\}$

$R_2 = \{(a, f), (b, e), (c, f)\}$

(iii) $M \times M$

$M \times M = \{d, e, f, g\} \times \{d, e, f, g\}$

$= \{(d, d), (d, e), (d, f), (d, g), (e, d), (e, e), (e, f), (e, g), (f, d), (f, e), (f, f), (f, g), (g, d), (g, e), (g, f), (g, g)\}$

$R_1 = \{(d, e), (d, f)\}$

$R_2 = \{(e, e), (f, f), (g, g)\}$

Q. 4: If set M has 5 elements, then find the number of binary relations in M.

No. of Elements in M $= m = 5$

No. of binary relations in M $= 2^{m \times m}$

$= 2^{5\times5}$
$= 2^{25}$

Q. 5: If $L = \{x|x \in N \wedge x \leq 5\}$, $M = \{x|x \in P \wedge x \leq 10\}$, then make the following relations from *L* to *M*. Also write the domain and range of each relation.

So, we have from the question

$L = \{1,2,3,4,5\}$

$M = \{2,3,5,7\}$

$L \times M = \{1,2,3,4,5\} \times \{2,3,5,7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),(4,5),(4,7),(5,2),(5,3),(5,5),(5,7)\}$

(i) $R_1 = \{(x,y)|y < x\}$

$= \{(3,2),(4,2),(5,2),(4,3),(5,3)\}$

$Dom\ R_1 = \{3,4,5\}$

$Range\ R_1 = \{2,3\}$

(ii) $R_2 = \{(x,y)|y = x\}$

$= \{(2,2),(3,3),(5,5)\}$

$Dom\ R_2 = \{2,3,5\}$

$Range\ R_2 = \{2,3,5\}$

(iii) $R_3 = \{(x,y)|x + y = 6\}$

$= \{(1,5),(3,3),(4,2)\}$

$Dom\ R_3 = \{1,3,4\}$

$Range\ R_3 = \{2,3,5\}$

(iv) $R_4 = \{(x,y)|y - x = 2\}$

$= \{(1,3),(3,5),(5,7)\}$

$Dom\ R_4 = \{1,3,5\}$

$Range\ R_4 = \{3,7\}$

Q. 6: Indicate relations, into function, one-one function, onto function, and bijective function from the following. Also find their domain and range.

(i) $R_1 = \{(1,1),(2,2),(3,3),(4,4)\}$

$Dom\ R_1 = \{1,2,3,4\}$

$Range\ R_1 = \{1,2,3,4\}$

As, we know A relation becomes a function if

$Dom f = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

So, the given relation is function.

As, All distinct elements of A have distinct images in B so, the given relation is One-One.

Also, every element of set B is an image of at least one element of set A i.e. $Range\ of\ f = B$. So, given relation is also Onto function.

As, the given relation is One-One as well as Onto function so, it is <u>bijective function</u>.

(ii) $R_2 = \{(1,2),(2,1),(3,4),(3,5)\}$

$Dom\ R_2 = \{1, 2, 3\}$
$Range\ R_2 = \{1, 2, 4, 5\}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

As, we can clearly see the 3 is repeated in 3rd and 4th ordered pair so the given relation is not a function, its only a relation.

(iii) $R_3 = \{(b, a), (c, a), (d, a)\}$
$Dom\ R_3 = \{b, c, d\}$
$Range\ R_3 = \{a\}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

So, the given relation is a function.
As, it doesn't fulfill any condition of One-One, Onto or into function so the relation is only a function.

(iv) $R_4 = \{(1, 1), (2, 3), (3, 4), (4, 3), (5, 4)\}$
$Dom\ R_4 = \{1, 2, 3, 4, 5\}$
$Range\ R_4 = \{1, 3, 4, \}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

So, the given relation is a function.
As,
It doesn't fulfill condition of One-One.
Every element of set B is an image of at least one element of set A. So, the given relation is an onto function.

(v) $R_5 = \{(a, b), (b, a), (c, d), (d, e)\}$
$Dom\ R_5 = \{a, b, c, d\}$
$Range\ R_5 = \{a, b, d, e\}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

So, the given relation is a function.
As,
As, All distinct elements of A have distinct images in B so, the given relation is One-One.
It doesn't fulfill condition of Onto function.
So, the given relation is a One-One function.

(vi) $R_6 = \{(1, 2), (2, 3), (1, 3), (3, 4)\}$

$Dom\ R_6$ = $\{1, 2, 3\}$
$Range\ R_6$ = $\{2, 3, 4\}$
As, we know A relation becomes a function if

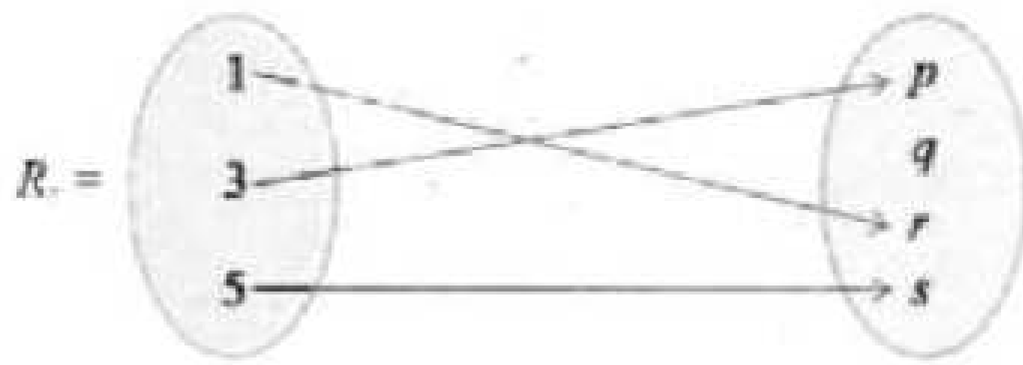
$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

As, we can clearly see the 1 is repeated in 1st and 3rd ordered pair so the given relation is not a function, it's only a relation.

(vii)



R_7 = $\{(1, r), (3, p), (5, s)\}$

$Dom\ R_7$ = $\{1, 3, 5\}$
$Range\ R_7$ = $\{p, r, s\}$
As, we know A relation becomes a function if

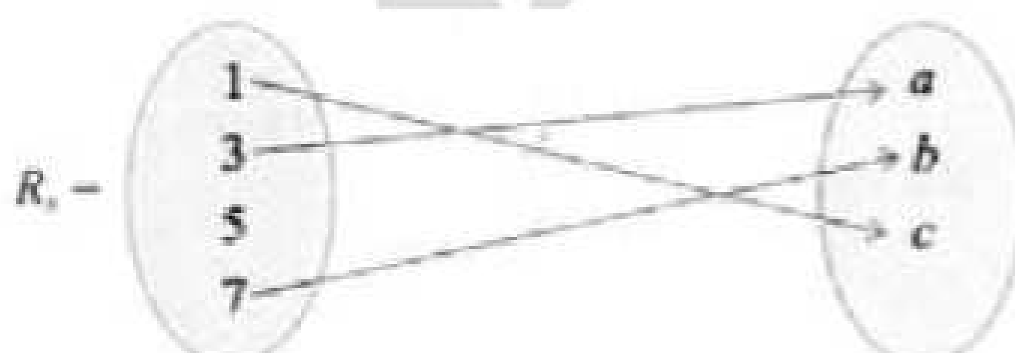
$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

As, All distinct elements of A have distinct images in B so, the given relation is One-One.
It doesn't fulfill condition of Onto function.
So, the given relation is a One-One function.

(viii)



R_7 = $\{(1, c), (3, a), (7, b)\}$

$Dom\ R_8$ = $\{1, 3, 7\}$
$Range\ R_8$ = $\{a, b, c\}$
As, we know A relation becomes a function if

$Domf = A$

and

Every $x \in A$ appears in one and only one ordered pair in f.

But $Domf \neq A$, So the given relation is not a function. it's only a relation.