-:(Applied Physics):-

Semester 01:-

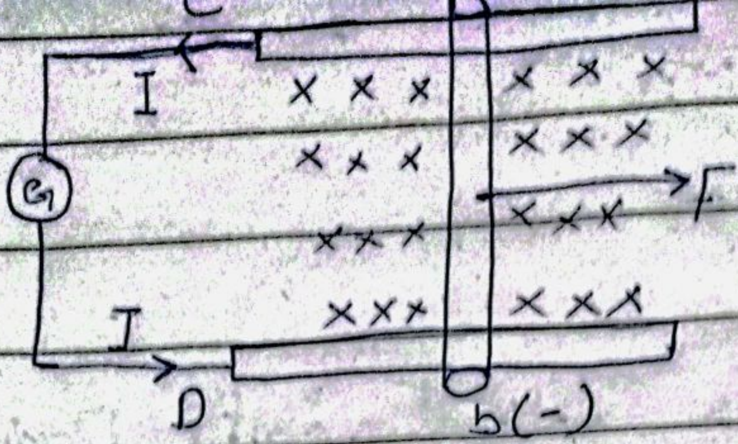
Chapter 10:-

QNo1:-

What is motional emf? Derive it's formula $\varepsilon = -BDV$.
Derive a formula to calculate mechanical Power? to be exerted on a loop of wire of width D and resistance R when it is dragged with velocity V inside and at right angle to the magnetic field?

Ans:-

The emf induced in a conductor when it is moved across a magnetic field is called Motional emf.



Consider a conductor having length D is placed on a conducting rail ~~(with constant velocity. It means)~~ whoes end C and D are connected with a galvanometer.

The galvanometer shows deflection when rod is moved with constant velocity. It means emf is induced in a circuit.

The magnetic force is given as:

$$F_m = q(\vec{V} \times \vec{B})$$

$$F_m = -q(V\hat{i} \times B\hat{K})$$

$$F_m = +qVB\hat{j}$$

Electric field is induced in the rod along +y-axis. The value of electric field is given by:

$$\vec{E} = \frac{\vec{F}_m}{q}$$

$$\vec{E} = \frac{\cancel{q} V B \hat{j}}{\cancel{q}}$$

$$\vec{E} = V B \hat{j}$$

Magnitude is:

$$E = VB$$

The electric field and emf is related as:

$$\mathcal{E} = \int_0^D E \cdot dr = \int E\, dr \cos\theta$$

$$\mathcal{E} = \int_0^D V B\, dr$$

$$\boxed{\mathcal{E} = VBD}$$

(ii) **Induced Current:-**

The amount of current induced in the loop having resistance R due to motional emf is given as:

$$I = \frac{\varepsilon}{R}$$

$$I = \frac{VBD}{R}$$

The direction of induced current is in the loop is anticlockwise according to Lenz's law.

The magnetic force $\vec{F}_L$ acting on the rod is given as:

$$\vec{F}_L = I(\vec{L} \times \vec{B})$$

$$\vec{F}_L = -IDB(\hat{j} \times \hat{k})$$

$$\{B = -B\hat{k} \quad L = D\hat{j}\}$$

$$\vec{F}_L = IDB\hat{i}$$

The magnitude of this force is:

$$F_L = IDB$$

$$F_L = \frac{VBD}{R}(DB)$$

$$F_L = \frac{VB^2D^2}{R} \quad \text{--- (i)}$$

Electrical Power:-

The rate of energy dissipated in the loop having resistance R due to induced current I is called Electrical Power.

~~It is given by:~~

Mechanical Power:-

The rate of doing working is called mechanical Power

It is given by:

$$P_m = F_L V$$

$$P_m = \frac{VB^2D^2}{R}(V)$$

$$P_m = \frac{V^2B^2D^2}{R} \quad \text{--- (ii)}$$

The electrical power is given by:

$$P_e = I^2 R$$

$$P_e = \left(\frac{VBD}{R}\right)^2 R$$

$$P_e = \frac{V^2 B^2 D^2}{R} \quad \text{---------} \quad (iii)$$

The above eq (ii) and eq (iii) gives

$$\boxed{P_m = P_e}$$ Mechanical

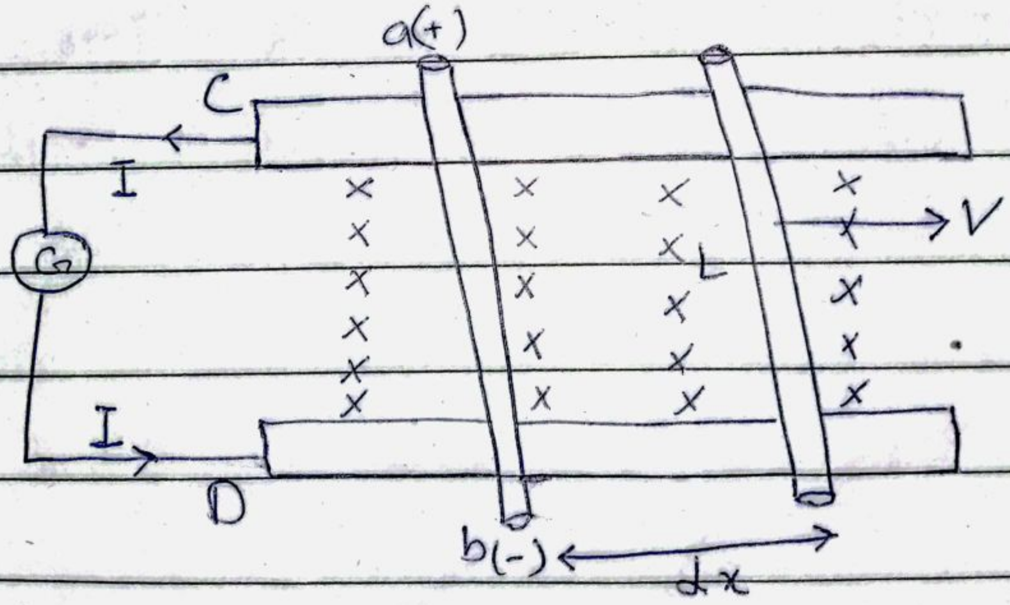
It means ~~electrical~~ Power is completely converted into electrical power.

Qno2:-

State and Explain Faraday's law of electromagnetic induction for a coil of N turns by giving the significance of negative sign. Write its integral and Differential form?

Ans:-

The Faraday's law of electromagnetic induction states that whenever magnetic flux changes in a coil or loop, emf is induced in it. The flux change continue. The magnitude of induced emf is directly proportional to the rate of change in magnetic flux.



Consider a conductor having length L, is placed on conducting rail. The ends of conducting rail are connected with galvanometer.

The galvanometer shows no deflection when rod is at rest. It means emf is not induced in rod.

The emf is induced in rod when it is pulled towards right constant velocity V is given as:

$$\mathcal{E} = -VBL \quad \text{——— (i)}$$

The velocity V of rod which travels distance dx in time dt is:

$$V = \frac{dx}{dt} \quad (\text{Put in (i)})$$

$$\mathcal{E} = -\left(\frac{dx}{dt}\right) BL$$

$$\{ L\,dx = da \}$$

$$\mathcal{E} = -\frac{B\,da}{dt}$$

$$\boxed{\mathcal{E} = -\frac{d\phi_m}{dt}}$$

For N turns:

$$\varepsilon = -N \frac{d\phi_m}{dt} \quad \text{--- (ii)}$$

The negative sign means direction of induced emf is such as it opposes the cause producting it.

$$d\phi_m = \vec{B} \cdot \vec{da}$$

Integrate:

$$\phi_m = \int \vec{B} \cdot \vec{da} \quad \text{(Put in (ii))}$$

$$\varepsilon = -N \frac{d}{dt} \left(\int \vec{B} \cdot \vec{da} \right) \quad \text{--- (iii)}$$

Integral form of Faraday's law:-

Mathematical form of Faraday's law is:

$$\varepsilon = -N \frac{d}{dt} \left(\int \vec{B} \cdot \vec{da} \right)$$

we Know that

$$\varepsilon = \int \vec{E} \cdot \vec{dr}$$

So,

$$\int \vec{E} \cdot \vec{dr} = -N \frac{d}{dt} \left(\int \vec{B} \cdot \vec{da} \right)$$

For N=1

$$\boxed{\int \vec{E}\cdot\vec{dr} = -\frac{d}{dt}\left(\int \vec{B}\cdot\vec{da}\right)}$$

This is called integral form of Faraday's law.

Differential form :-

Since

$$\int \vec{E}\cdot\vec{dr} = -\frac{d}{dt}\left(\int \vec{B}\cdot\vec{da}\right)$$

By stoke's law.

$$\int \vec{E}\cdot\vec{dr} = \int (\vec{\nabla}\times\vec{E})\cdot\vec{da}$$

So,

$$\int (\vec{\nabla}\times\vec{E})\cdot\vec{da} = -\frac{d}{dt}\left(\int \vec{B}\cdot\vec{da}\right)$$

$$\int \left(\vec{\nabla}\times\vec{E} + \frac{d\vec{B}}{dt}\right)\cdot\vec{da} = 0$$

$$\vec{\nabla}\times\vec{E} + \frac{d\vec{B}}{dt} = 0$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}}$$

This is called Differential form of Faraday's law.

QNO 3:-

State and explain Lenz's law and show that it is in accordance with law of conservation of energy?

Ans:-

Lenz's law:-

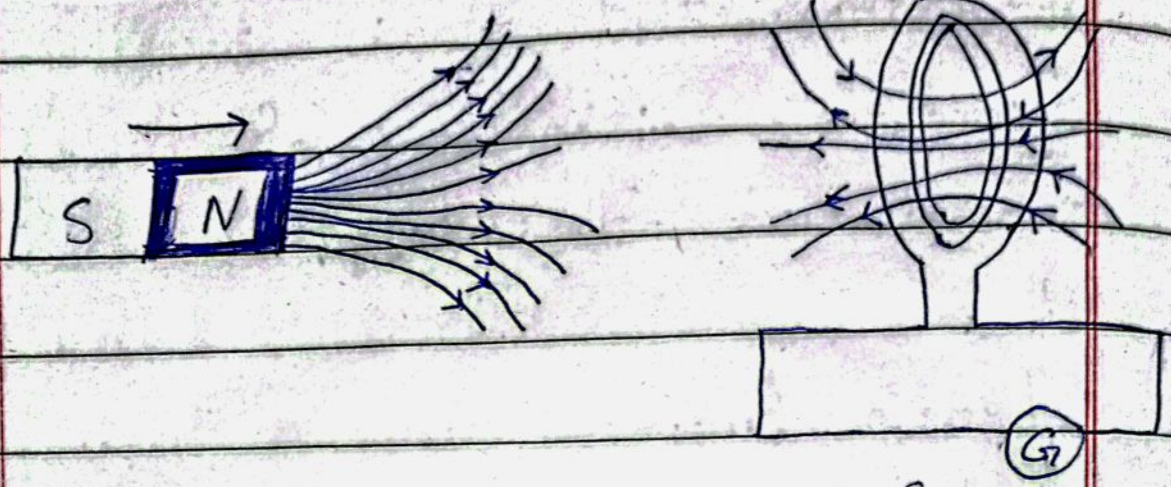
It is states that the direction of induced current in a loop is such as it opposes the cause producting it.

The induced ~~effect~~ magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant.

Explanation of (-)sign in Faraday's law:-

According to Faraday's law emf induced in a coil due to change in magnetic flux is written as:

$$\varepsilon = -N\frac{d\phi}{dt}$$



The negative sign in the formula is very important. It is about the direction of induced emf and explained by Lenz's as:

Explanation:-

Consider a coil connected with galvanometer and a bar magnet. The galvanometer show deflection when bar magnet is moved. It shows no deflection when bar magnet is at rest.

The current induced in loop must flow in clockwise

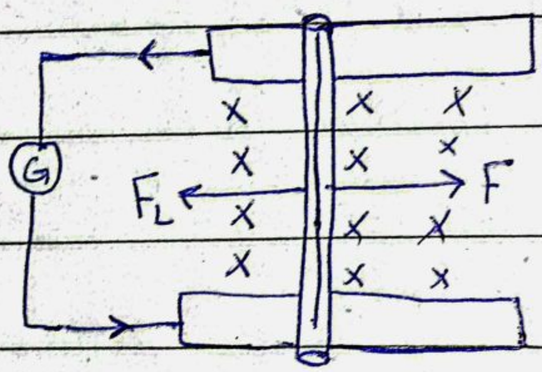
direction or anticlockwise.
The lenz's law decide the direction of current.
The direction of induced current must be anticlockwise when north pole of bar magnet is moved towards the coil.
Similarly, the direction of induced current is clockwise when north pole of bar magnet is ~~moved~~ pulled out from coil.

Conclusion:-

The "Push" of the magnet is the "cause" that produces the induced current.
Similarly "Pull" of the magnet is the "cause" that produces the induced current. Because the magnetic field is generated due to induced current opposes the motion of bar magnet.

Lenz's law and law of conservation of Energy:-

The lenz's law is in accordance to law of conservation of energy because mechanical energy is converted into electrical energy when a conductor is moved across the magnetic field but total energy is constant.



Consider a conductor having length D is placed on a conducting rail when ends are connected with galvanometer. The magnetic field $\vec{B} = -B\hat{k}$ is directed into plane of paper.

The amount of current induced in the loop having resistance R is :

$$I = \frac{\varepsilon}{R}$$

$$I = \frac{VBD}{R}$$

The direction of induced current is anticlockwise according to lenz's law.

The magnetic force $\vec{F}_L$ is :

$$\vec{F}_L = I(\vec{L}\times\vec{B})$$

$$\{ L = D\hat{j} \; ; \; B = -B\hat{k}$$

$$\vec{F}_L = \cancel{I}(-IDB(\hat{j}\times\hat{k})$$

$$\vec{F}_L = IDB\hat{i}$$

$$F_L = IDB = \frac{VBD}{R}(DB)$$

$$F_L = \frac{VB^2D^2}{R}$$

Machenical Power:-

$$P_m = \vec{F}_L V$$

$$P_m = \frac{VB^2D^2}{R}(V)$$

$$P_m = \frac{V^2B^2D^2}{R} \quad \text{——— (i)}$$

Electrical Power:-

$$P_e = I^2 R$$

$$P_e = \left(\frac{VBD}{R}\right)^2 R$$

$$P_e = \frac{V^2B^2D^2\cancel{R}}{R^{\cancel{2}}}$$

$$P_e = \frac{V^2B^2D^2}{R} \quad \text{——— (ii)}$$

$$\boxed{P_m = P_e}$$

It means machenical power is completely converted into electrical power.

Hence Lenz's law is in accordance ~~to~~ with law of conservation of energy.