

-:(Applied Physics):-

Semester # 01:-

Ch # 09 :-

Q No 1:-

State and explain Biot-Savart law?

Answer:-

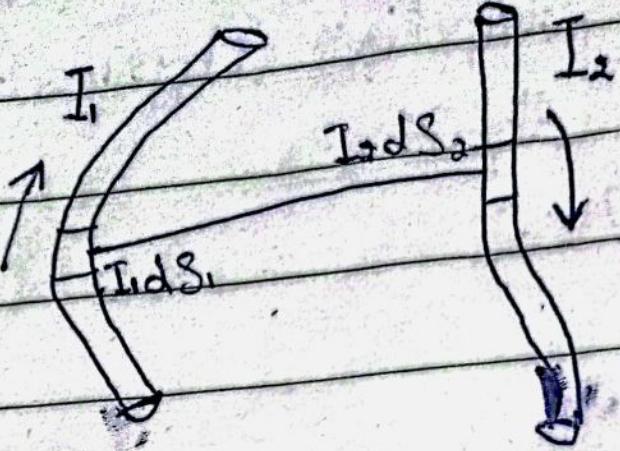
Biot-Savart law:-

The magnitude of magnetic field at a point due to current distribution is directly proportional to amount of current and length of conductor.

And Inversely proportional to the square of distance b/w point and current distribution.

The magnetic field is generated around a conductor when currents flows through it.

Diagram:-



Explanation:-

Now consider two conductors through which current I_1 and I_2 flows in opposite direction.

Divide the first wire into differential element dS_1 and define for each element a length vector $d\vec{S}_1$ that has magnitude dS_1 and direction is the direction of current in dS_1 .

In this way, differential current length element of first wire is $I_1 dS_1$.

Similarly differential current length element of second

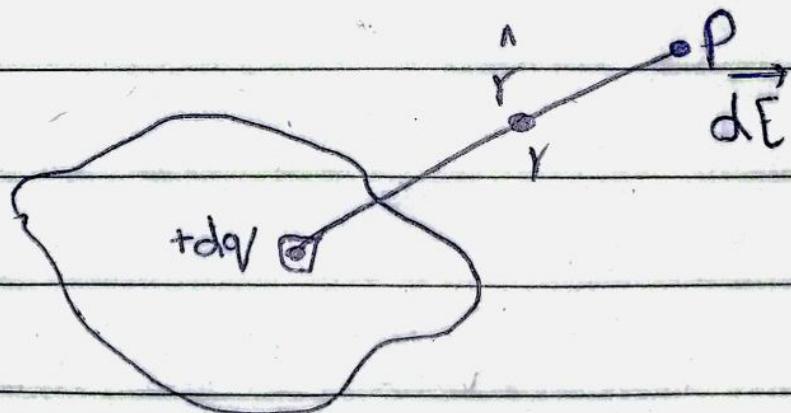
wire is $I_2 \vec{dS}_2$.

The magnetic force $d\vec{F}_{12}$ exerted on current element $I_2 \vec{dS}_2$ which is placed in magnetic field \vec{B}_1 is:

$$d\vec{F}_{12} = I_2 (\vec{dS}_2 \times \vec{B}_1)$$

Similarly The $d\vec{F}_{21}$ placed in \vec{B}_2 is:

$$d\vec{F}_{21} = I_1 (\vec{dS}_1 \times \vec{B}_2)$$



Now Take a point P having distance 'r' from charge dq of some charge distribution.

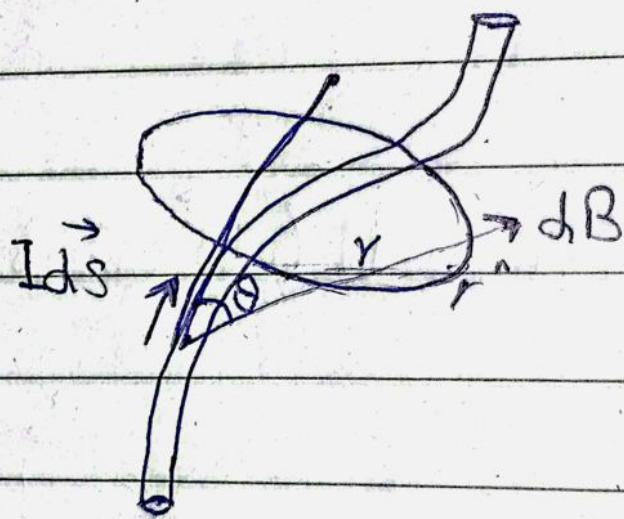
The electric field \vec{dE} is given as:

$$d\vec{E} = K dI \times \frac{\vec{r}}{r^2}$$

Similarly, take a point P having distance r from current element $Id\vec{s}$ of some current distribution.

The magnetic field $d\vec{B}$ is

Given as:



$$d\vec{B} = K Id\vec{s} \times \frac{\vec{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{s}}{r^2}$$

Ques:-

By using Biot-Savart law, determine the magnetic field at a point having distance R away from the long straight current carrying wire?

Ans:-

"The magnetic field is generated when current flows through a straight conductor."

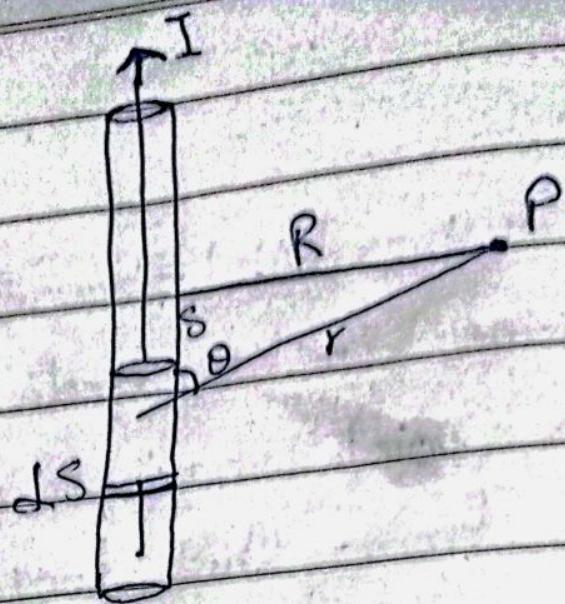
Consider a long straight wire to infinite length through which current I flows.

Take a point P having perpendicular distance R from wire. Φ

By using Biot-Savart law:

$$dB = \frac{\mu_0}{4\pi} \frac{I dS \sin\theta}{r^2}$$

i



By Pythagoras Theorem:

$$r^2 = R^2 + S^2$$

$$\text{and } \sin\theta = \frac{R}{r} = \frac{R}{\sqrt{R^2 + S^2}}$$

Put in i)

$$dB = \frac{\mu_0}{4\pi} \left(\frac{I dS}{R^2 + S^2} \right) \left(\frac{R}{\sqrt{R^2 + S^2}} \right)$$

$$dB = \frac{\mu_0 I R}{4\pi} \frac{dS}{(R^2 + S^2)^{3/2}}$$

The net magnetic field is

$$B = \int_{-\infty}^{+\infty} dB = \int_{-\infty}^{+\infty} 2dB = \int_{-\infty}^{+\infty} 2dB$$

$$B = 2 \int_{-\infty}^{+\infty} dB$$

$$B = 2 \int_{-\infty}^{+\infty} \frac{\mu_0 I R}{4\pi} \frac{dS}{(R^2 + S^2)^{3/2}}$$

Put $S = R \cot \theta$ then $dS = -R \cosec^2 \theta d\theta$

when $S = 0$

$$0 = R \cot \theta$$

$$\cot \theta = 0$$

$$\cot \theta = 0$$

$$\sin \theta$$

$$\cos \theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\boxed{\theta = 90^\circ}$$

when $S = \infty$

~~$\cot \theta = \infty$~~

~~$\cot \theta = \infty$~~

$$\infty = \cot \theta R$$

$$\frac{\infty}{R} = \cot \theta$$

$$\frac{R}{\infty} = \tan \theta$$

$$\tan \theta = 0$$

$$\frac{\sin \theta}{\cos \theta} = 0$$

$$\sin \theta = 0$$

$$\theta = \sin^{-1}(0)$$

$$\boxed{\theta = 0^\circ}$$

$$B = \frac{\mu_0 I R}{2\pi} \int_{0}^{90^\circ} \frac{-R \cosec^2 \theta d\theta}{(R^2 + R^2 \cot^2 \theta)^{3/2}}$$

$$B = \frac{\mu_0 I R}{2\pi} \int_{0}^{90^\circ} \frac{-R \cosec^2 \theta d\theta}{(R^2)^{3/2} (1 + \cot^2 \theta)^{3/2}}$$

$$B = \frac{\mu_0 I R}{2\pi R^{\frac{3}{2}}} \int_{0}^{90^\circ} \frac{-R \cosec^2 \theta d\theta}{(\cosec^2 \theta)^{3/2}}$$

$$B = \frac{\mu_0 I}{2\pi R} \int_{0^\circ}^{90^\circ} -R \csc^2 \theta \ d\theta$$

$$B = -\frac{\mu_0 I}{2\pi R} \int_{0^\circ}^{90^\circ} \csc^3 \theta \ d\theta$$

$$B = -\frac{\mu_0 I}{2\pi R} \int_{0^\circ}^{90^\circ} \sin \theta \ d\theta$$

$$B = -\frac{\mu_0 I}{2\pi R} [\cos \theta]_{0^\circ}^{90^\circ}$$

$$B = -\frac{\mu_0 I}{2\pi R} (\cos(90^\circ) - \cos(0^\circ))$$

$$B = -\frac{\mu_0 I}{2\pi R} (0 - 1)$$

$$B = \frac{\mu_0 I}{2\pi R}$$

This is the magnitude of magnetic field at a point P having perpendicular distance R from current carrying straight conductor.

-:(Physics):-

Semester 01:-

Ch # 09:-

Qn 04:-

Two long parallel wires carrying currents I_1 and I_2 are separated by a distance d . Find magnitude of magnetic force experienced by each wire.

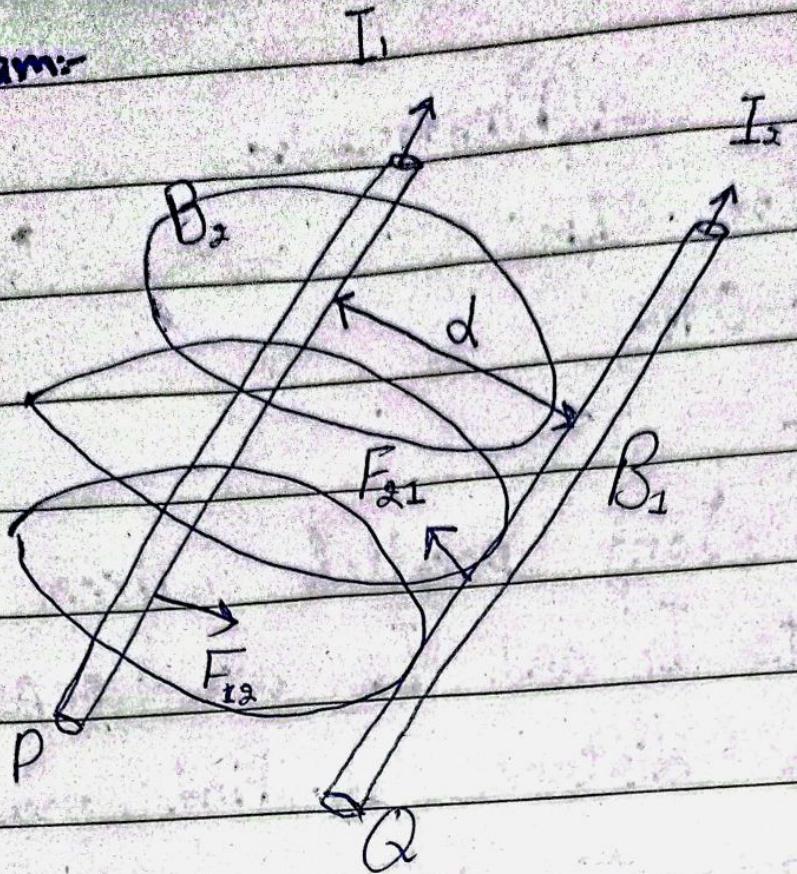
Alternatively, show that parallel currents attract and anti-parallel currents repel each other.

Answer:-

(i) Parallel Current Attract:-

The parallel current flowing through conductors attract each other. Consider two parallel conductors P and Q each of length L having separation d.

Diagram:-



The magnitude of magnetic field B_1 where Q is placed is given by Biot-Savart law.

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

The magnetic force experienced by conductor Q having current I_2 is:

$$F_{21} = I_2 L B_1$$

$$F_{21} = I_2 L \frac{\mu_0 I_1}{2\pi d}$$

i

This is magnitude of magnetic force which is acting on conductor Q.

Similarly, The magnetic field B_2 where conductor P is placed is given by Biot-Savart law:

$$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

The magnetic force experienced by conductor P having current I_1 is :

$$F_{12} = I_1 L B_2$$

$$F_{12} = I_1 L \frac{\mu_0 I_2}{2\pi d}$$

(ii)

Comparing eq (i) and eq (ii)

$$F_{12} = F_{21}$$

It shows that F_{12} and F_{21} are equal in magnitude but

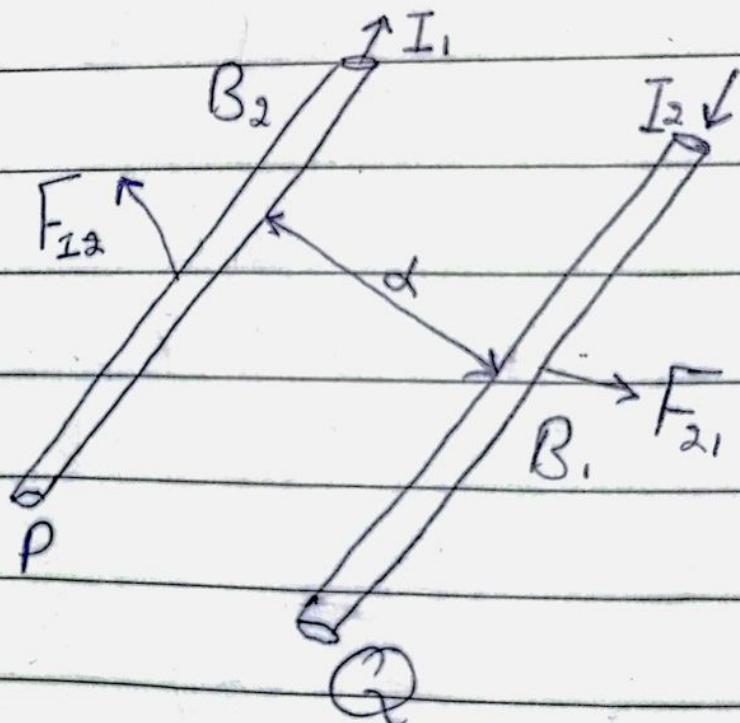
towards each other and their point of application is not same.

(ii) Anti-Parallel current Repels:-

The anti-parallel currents flowing through conductors repel each other.

Consider two parallel conductors P and Q each of length L having separation d. ~~The current~~

Diagram:-



The magnitude of magnetic field B_1 where conductor Q is placed given by Biot-Savart law.

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

The magnetic force experienced by conductor Q having current I_2 is:

$$F_{21} = I_2 L B_1$$

$$F_{21} = I_2 L \frac{\mu_0 I_1}{2\pi d}$$

i

This is magnitude of magnetic force which is acting on conductor Q.

Similarly, the magnitude of magnetic field B_2 where the conductor P is placed is given by Biot-Savart law:

$$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

The magnetic force experienced by conductor P carrying current I_1 is:

$$F_{12} = I_1 L B_2$$

$$F_{12} = I_1 L \frac{\mu_0 I_2}{2\pi d}$$

is

Comparing eq (i) and (ii)

$$F_{12} = F_{21}$$

It shows that F_{21} and F_{12} are equal in magnitude but acting away from each other.

-: (Applied Physics) :-

Semester 01:-

Ch # 09:-

Qnos:-

State and explain Ampere law, write its integral and differential form?

Answer:-

Ampere's Law:-

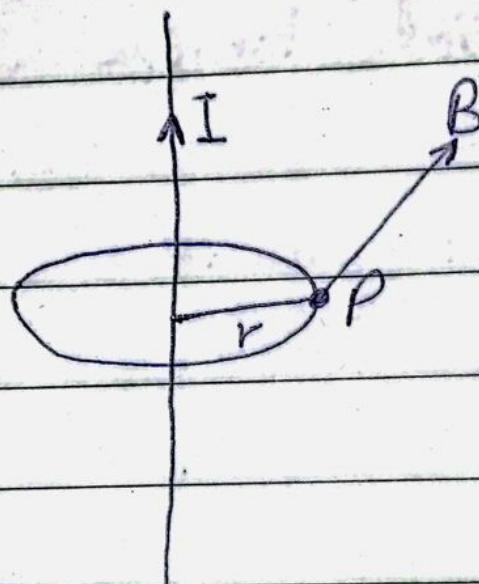
The Ampere's law state that, a magnetic field is generated around a conductor when current flows through it.

The magnitude of generated magnetic field at a point is directly proportional to amount of ~~work~~ current and inversely proportional to the distance between point

and straight current carrying conductor.

The direction of magnetic field is given by the Right hand Rule.

Diagram:-



Consider a straight conductor through which current I flows.

The magnetic field B is generated around it.

The magnitude of magnetic field B depends upon current I and distance r between conductor and point P .

B \propto I

Or

$$B = \frac{\mu_0}{2\pi} \left(\frac{I}{r} \right)$$

where μ_0 is constant
 $\frac{2\pi}{l}$

μ_0 is called permeability of free space. Its value is $4\pi \times 10^{-7}$
where $2\pi r$ is the length of path denoted by l.

$$B = \cancel{\left(\frac{I}{r} \right)} \frac{\mu_0}{l} \frac{l I}{2\pi r}$$

$$BI = \mu_0 I$$

$$BI \cos 0^\circ = \mu_0 I$$

$$\vec{B} \cdot \vec{l} = \mu_0 I$$

Take a small length element dr.

$$\int \vec{B} \cdot d\vec{r} = \mu_0 I$$

$$\boxed{\int B dr \cos 0^\circ = \mu_0 I}$$

Integral Form

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$$

The value of current in terms of current density is:

$$I = \int \vec{J} \cdot d\vec{a}$$

So,

$$\boxed{\oint \vec{B} \cdot d\vec{r} = \mu_0 \int \vec{J} \cdot d\vec{a}} \quad i$$

This is called integral form of Ampere's law

Differential form:-

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \int \vec{J} \cdot d\vec{a}$$

By Stoke's law :

$$\oint \vec{B} \cdot d\vec{r} = \int (\vec{\nabla} \times \vec{B}) \cdot \vec{da}$$

Put in above eq.

$$\int (\vec{\nabla} \times \vec{B}) \cdot \vec{da} = \mu_0 \int \vec{J} \cdot \vec{da}$$

$$\oint (\vec{\nabla} \times \vec{B} - \mu_0 \vec{J}) \cdot d\vec{a} = 0$$

$$\vec{\nabla} \times \vec{B} - \mu_0 \vec{J} = 0$$

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

This is called differential form of Ampere law.

Ques:-

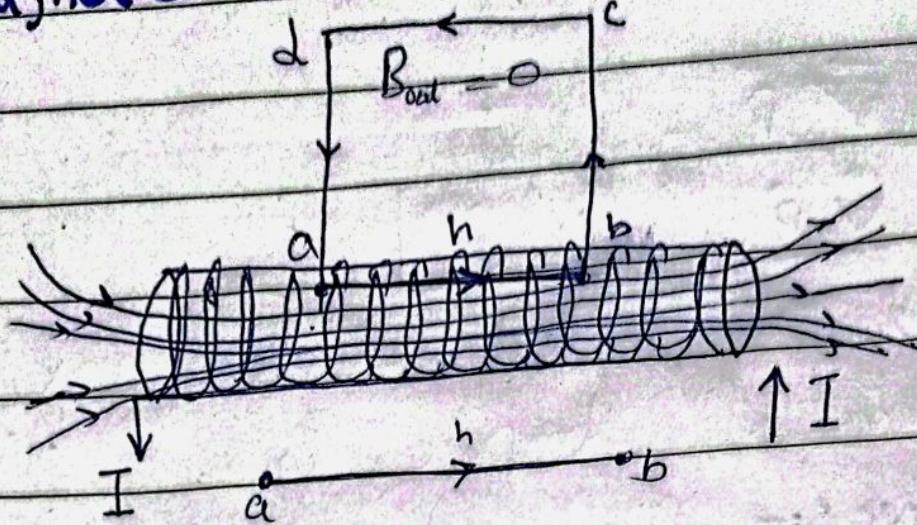
Apply Ampere's law to calculate magnetic field within and outside a current carrying solenoid?

Ans:-

A long tightly wound cylindrical conducting wire is called solenoid.

The magnetic field is generated when current flows through windings of solenoid.

(j) Magnetic field inside solenoid:-



Consider a solenoid having N turns and length l . The current I flows through each turn of solenoid.

$$n = \frac{N}{l}$$

The magnetic field outside the solenoid is zero.

Take a loop in the shape of rectangle to calculate magnetic field inside the solenoid.

By Ampere law:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$$

$$\int_a^b \vec{B} \cdot d\vec{r} + \int_b^c \vec{B} \cdot d\vec{r} + \int_c^d \vec{B} \cdot d\vec{r} + \int_d^a \vec{B} \cdot d\vec{r} = \mu_0 I$$

$$\int_a^b B dr \cos 0^\circ + \int_b^c B dr \cos 90^\circ + \int_c^d B dr \cos 180^\circ + \int_d^a B dr \cos 90^\circ = \mu_0 I$$

$$\int_a^b B dr (1) + 0 + 0 + 0 = \mu_0 I$$

$$\int_a^b B dr = \mu_0 I$$

The length of loop from a to b

is h

so

$$h B = \mu_0 I \quad \text{(i)}$$

Unit length has turn = n

h length has turn = nh

One turn has current = I.

nh turn has current = nhI.

$$I = nhI.$$

(ii)

Put in (i)

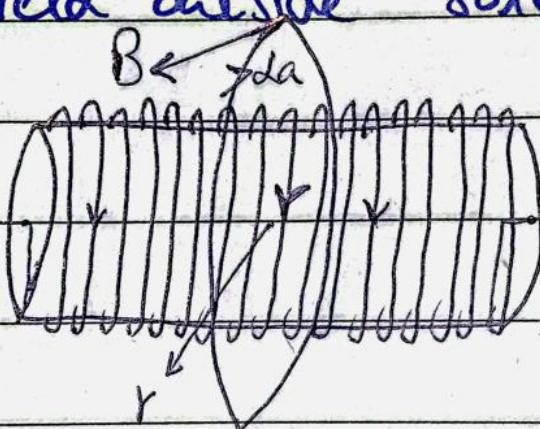
$$B_K = \mu_0 n K I$$

$$B = \mu_0 n I$$

This is the magnitude of magnetic field inside the solenoid.

The direction of magnetic field is given by right hand rule.

(ii) Magnetic field outside solenoid:-



Consider a solenoid having n turns per unit length. The current I_o flows through each turn of solenoid.

Imagine an Amperian loop having radius r . The magnetic field lines will be tangential.

By Ampere's law:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$$

$$\oint B_t dr \cos 0^\circ = \mu_0 I.$$

$$B_t \oint dr = \mu_0 I.$$

$$B_t (2\pi r) = \mu_0 I.$$

$$B_t = \frac{\mu_0 I}{2\pi r}. \quad \text{(i)}$$

This is magnetic field outside the solenoid.

We know that

$$B = \mu_0 n I. \quad \text{(ii)}$$

Divide eq (i) by eq (ii)

$$\frac{B_t}{B} = \left(\frac{\mu_0 I}{2\pi r} \right) \left(\frac{1}{\mu_0 n I} \right)$$

$$\frac{B_t}{B} = \frac{1}{2\pi r n}$$

$$B_t = \frac{B}{2\pi r n}$$

It shows B_t smaller than B because $2\pi r n$ is very large factor. That is why the

magnetic field outside the solenoid is weaker than inside.

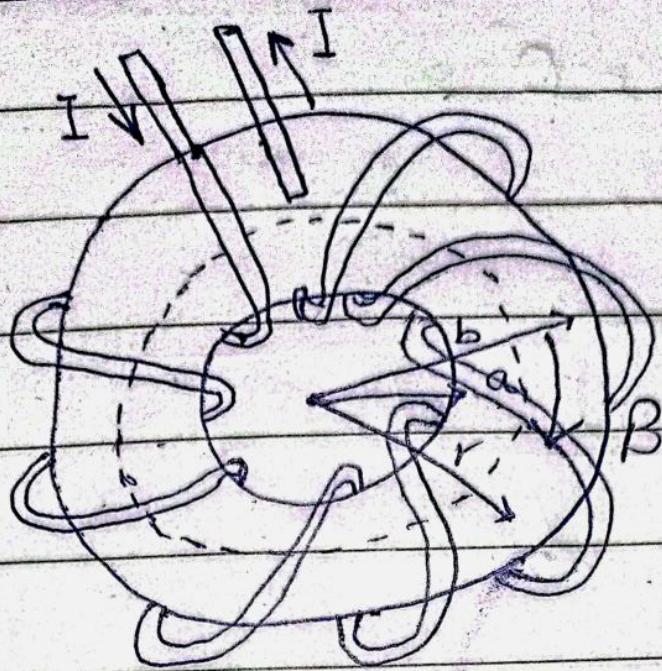
Q No 7:-

Apply Ampere's law to calculate magnetic field at the interior points of a current carrying toroid?

Ans:-

The solenoid bend into circle shape is called Toroid. A strong magnetic field is generated when current flows through windings of solenoid. These magnetic field lines are along the axis of the solenoid.

Magnetic field due to toroid:-



Consider a toroid having N turns and each turn carries current I_0 . The total current is:

$$I = NI_0$$

To calculate magnetic field, B at a point P inside Toroid, imagine an Amperian loop of radius r .

The magnetic field B and small length element dr are along tangent direction so make zero angle.

By Ampere's law:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$$

$$\oint B dr \cos 90^\circ = \mu_0 N I$$

$$B \oint dr = \mu_0 N I$$

$$B(2\pi r) = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

This is magnitude of magnetic field inside toroid. It depends upon turn of toroid and radius r .
The direction of magnetic field within toroid is given by Right hand Rule.