ALGONQUIN COLLEGE

SCHOOL OF ADVANCED TECHNOLOGY

ICT - Applications & Programming
Computer Engineering Technology – Computing Science

Numerical Computing – CST8233

A. Kadri

Lab #5 – Taylor Series & Numerical Differentiation

In this lab, you will write a script to implement the Taylor Series. Also, you will write a script that calculates the first and second orders numerical differentiation.

You will need to show your lab professor to get your grades.

Grades:

2% of your final course mark

Deadline

During the lab period of Week 12 - July 26.

Steps

Step 1. Taylor Series

Taylor series are used to expand a function around a constant value, c. This series is infinite series and is given as follows:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(c)}{n!}(x-c)^n$$

where $f^n(c)$ is the n^{th} derivative of $f(x)$ and c is a constant. In Maclaurin series, the value of this constant is zero.

Step 2. Exercise

A. Find Taylor series for $f(x) = \ln x$. You need to follow the same steps explained in the class except that using the constant c instead of zero. Show that Taylor series is given as follows **when $\boldsymbol{c = 1}$**:

$$\ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(x-c)^n$$

B. Write R program that takes the value of x as an input from the user and then, it computes the value of the series for up to ten terms $n = 1, 2, \ldots, 10$. The output of your code must be a table that shows the value of the function at each term around $c = 1$. Also, the table should show the absolute and relative errors of each result. The output should look like the following:

```
Please enter the value of x:

Term        ln(x)       Absolute error    Relative error
--------------------------------------------------------
1
2
3
4
5
6
7
8
9
10
```

C. Change the number of terms from 10 to 100 and plot the value of the series as a function of the number of terms.

Step 3.

Differentiation is widely used in many applications to find rates of change. It allows us to find the rate of change of a dependent variable y with respect to an independent variable x, which on a graph of y against x is the gradient of the curve. For example, we can find the rate of change of velocity with respect to time to obtain the acceleration.

Numerically, there are three different ways to approximately calculate the first derivative at point x_i:

1) Forward Divided Difference (FDD),
2) Backward Divided Difference (BDD), and
3) Central Divided Difference (CDD).

The formulas are as follows:

$$f^{`}(x_i) \approx \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \text{ (FDD)}$$

$$f^{`}(x_i) \approx \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \text{ (BDD)}$$

$$f^{`}(x_i) \approx \frac{y_{i+1} - y_{i-1}}{x_{i+1} - x_{i-1}} \text{ (CDD)}$$

The second derivative at point x_i can be approximately calculated using the following formula:

$$f^{``}(x_i) \approx \frac{y_{i+1} - 2\, y_i + y_{i-1}}{(x_{i+1} - x_i)^2}$$

Step 4. Exercise

A. Download the file **`rocket.xlsx`**. This data represents the altitude/distance (*m*) of a rocket as a function of time (*sec*.) Examine the data and plot the distance travelled in *(km)* by the rocket as a function of time.

B. Write R program that takes two vectors (`xVec, yVec`) representing a set of data pairs (`x`, `y`) and finds the first derivative using CDD method. The function returns a vector of first derivative at each point of `xVec`. Call this vector **`firstDev`**. Use this function to find the velocity of the rocket at each time.

C. Plot the velocity (*km/sec*) of the rocket as function of time.

D. Write R program that takes two vectors (`xVec, yVec`) representing a set of data pairs (`x`, `y`) and finds the second derivative. The function returns a vector of the second derivative at each point of `xVec`. Call this vector **`secondDev.`** Use this function to find the acceleration of the rocket at each time.

E. Plot the acceleration (*km/sec*^{2}) of the rocket as a function of time.

<u>You need to demo this to your lab professor.</u>