ALGONQUIN COLLEGE

SCHOOL OF ADVANCED TECHNOLOGY

ICT - Applications & Programming
Computer Engineering Technology – Computing Science

Numerical Computing – CST8233

Term: Summer 2023

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Assignment #3 – Numerical Ordinary Differentiation Equation (ODE)

In this assignment, you will use menus to prompt users with different options. You will also solve ODE numerically using two methods: Euler's and Runge-Kutta 4th Order Methods.

Objectives

- Create a menu system to prompt users with different options,
- Use Euler's method to solve a given ODE,
- Use Runge-Kutta 4th order to solve the same ODE, and
- Compare the numerical solutions to the analytical one.

Grades:

7% of your final course mark

Deadline

August 11th, 2023, 11:59 PM

Important Note

Create only one script file for this assignment. To be able to test the code, I should be able to execute the "`ODEsolver)`" function, which for this assignment serves as the equivalent of "`main()`" in other languages.

You may create other functions if you wish, but we will only use that one function to launch your code for marking.

Tasks

For the thin, glass-walled mercury thermometer system shown in Figure 1, assume that the temperature of the bath changes based on certain chemical process occurring between two substances reacting with each other inside the bath. It is found that the equation that describes this process is given as follows:

$$\frac{d\theta(t)}{dt} + 2\theta(t) = \cos 4t$$

It can be found that the actual solution of the response of the thermometer, $\theta(t)$, is given by the following equation:

$$\theta(t) = 0.1\cos 4t + 0.2\sin 4t + 2.9\, e^{-2t}$$

The ODE given above can be solved using many numerical methods, such as Euler's and Runge-Kutta 2nd Order Methods.

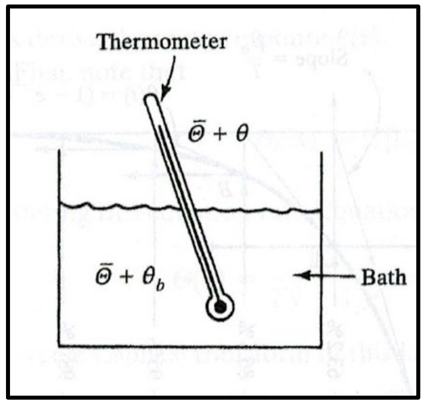

Figure 1 Thin, glass-walled mercury thermometer system

1. Write an R function that computes the solution $\theta(t)$ using Euler's Method. For this step, use the following information: $h = 0.8, 0.2, 0.05$, $\theta_0 = 3$ ℃, $0 \leq t \leq 2$ second. Find the discrete values of $\theta(t)$ at each h step value.

2. Write an R function that uses Runge-Kutta 4th method to solve the same ODE using the following information: $h = 0.8, 0.2, 0.05$, $\theta_0 = 3$ ℃, $0 \leq t \leq 2$ second. Find the discrete values of $\theta(t)$ at each h step value.

3. Calculate the relative error of the resultant solution at each time for each h step. Your output of your code should show a table that shows the exact temperature, the estimated temperature, and the relative error. The user will choose one method and one step size.

Example Output

The output of the code should look like below. The results of test case when using Euler's and Runge-Kutta for $h = 0.2$ are shown in the table below.

```
>> Choose the method for solving the ODE:
1. Euler's Method
2. Runge-Kutta 4th Order Method

>> 1
```

```
>> Choose step size "h" (0.8, 0.2, 0.05)
>> 0.2
```

Time(second)	Exact Temp(C)	Estimated Temp(C)	Percentage Error(%)
0.2	2.157	2.000	7.28
0.4	1.500	1.339	10.71
0.6	0.935	0.798	14.66
0.8	0.474	0.331	30.13
1.0	0.176	-0.001	100.54
1.2	0.073	-0.131	280.86
1.4	0.128	-0.061	148.01
1.6	0.241	0.118	50.86
1.8	0.299	0.270	9.76
2.0	0.236	0.283	19.89

```
>> Choose the method for solving the ODE:
1. Euler's Method
2. Runge-Kutta 4th Order Method

>> 2
>> Choose step size "h" (0.8, 0.2, 0.05)
>> 0.2
```

Time(second)	Exact Temp(C)	Estimated Temp(C)	Percentage Error(%)
0.2	2.157	2.157	0.01
0.4	1.500	1.500	0.02
0.6	0.935	0.935	0.03
0.8	0.474	0.474	0.06
1.0	0.176	0.176	0.15
1.2	0.073	0.073	0.34
1.4	0.128	0.128	0.15
1.6	0.241	0.241	0.05
1.8	0.299	0.299	0.02
2.0	0.236	0.236	0.00

```
>> Choose the method for solving the ODE:
1. Euler's Method
2. Runge-Kutta 4th Order Method
```