Chapter 4: AC Network Analysis – Instructor Notes

The chapter starts by developing the dynamic equations for energy storage elements in Section 4.1. The analogy between electrical and hydraulic circuits (*Make The Connection: Fluid (hydraulic) Capacitance*, p. 130, *Make The Connection: Fluid (hydraulic) inertance*, p. 139, Table 4.2, p. 139) is introduced early to permit a connection with ideas that may already be familiar to the student from a course in fluid mechanics, such as mechanical, civil, chemical and aerospace engineers are likely to have already encountered.

Next, signal sources are introduced in Section 4.2, with special emphasis on sinusoids. The material in this section can also accompany a laboratory experiment on signal sources. Section 4.3 introduces the formulation (and solution) of circuits described by differential equations, focusing on sinusoidal excitations. The emphasis placed on sinusoidal signals is motivated by the desire to justify the concepts of phasors and impedance, which are introduced next, in Section 4.4. This section covers phasor notation, impedance and admittance. It is followed by Section 4.5, which extends the circuit analysis methods developed in Chapter 3 to AC circuits.

The author has found that presenting the impedance concept early, is an efficient way of using the (invariably too short) semester or quarter. Chapter 4 is specifically designed to permit a straightforward extension of the resistive circuit analysis concepts developed in Chapter 3 to the case of dynamic circuits excited by sinusoids. The ideas of nodal and mesh analysis, and of equivalent circuits, can thus be reinforced at this stage. The treatment of AC circuit analysis methods is reinforced by the usual examples and drill exercises, designed to avoid unnecessarily complicated circuits.

The homework problems in this chapter are mostly mathematical exercises aimed at mastery of the techniques. The 1st Edition of this book includes 67 end-of chapter problems, in addition to 15 fully solved examples.

Learning Objectives for Chapter 4

1. Compute currents, voltages and energy stored in capacitors and inductors.
2. Calculate the average and root-mean-square value of an arbitrary (periodic) signal.
3. Write the differential equation(s) for circuits containing inductors and capacitors.
4. Convert time-domain sinusoidal voltages and currents to phasor notation, and vice-versa, and represent circuits using impedances.

Section 4.1: Energy Storage Elements

Problem 4.1

Solution:

Known quantities:
Inductance value, $L = 0.5$ **H**; the current through the inductor as a function of time.

Find:
The voltage across the inductor, (Eq. 4.9), as a function of time.

Assumptions:
$i_L(t \le 0) = 0$

Analysis:
Using the differential relationship for the inductor, we may obtain the voltage by differentiating the current:

$$v_L(t) = L\frac{di_L(t)}{dt} = 0.5\frac{di_L(t)}{dt} = 0.5 \times \left[-377 \times 2\sin\left(377t + \frac{\pi}{6}\right)\right]$$

$$= 377\sin\left(377t + \frac{\pi}{6} - \pi\right) = 377\sin\left(377t - \frac{5\pi}{6}\right) \quad \mathbf{V}$$

Problem 4.2

Solution:

Known quantities:
Capacitance value $C = 100\ \mu F$; capacitor terminal voltage as a function of time.

Find:
The current through the capacitor as a function of time for each case:
a) $v_c(t) = 40\cos(20t - \pi/2)V$
b) $v_c(t) = 20\sin(100t)V$
c) $v_c(t) = -60\sin(80t + \pi/6)V$
d) $v_c(t) = 30\cos(100t + \pi/4)V$.

Assumptions:
The capacitor is initially discharged: $v_C(t = 0) = 0$

Analysis:
Using the defining differential relationship for the capacitor, (Eq. 4.4), we may obtain the current by differentiating the voltage:

$$i_C(t) = C\frac{dv_C(t)}{dt} = 100 \times 10^{-6}\frac{dv_C(t)}{dt} = 10^{-4}\frac{dv_C(t)}{dt}$$

a)

$$i_C(t) = 10^{-4}\left[-20 \times 40\sin\left(20t - \frac{\pi}{2}\right)\right] = -0.08\sin\left(20t - \frac{\pi}{2}\right)$$

$$= 0.08\sin\left(20t - \frac{\pi}{2} + \pi\right) = 0.08\sin\left(20t + \frac{\pi}{2}\right) \quad \mathbf{A}$$

b)

$$i_C(t) = 10^{-4}[100 \times 20\cos 100t] = 0.2\cos 100t \quad \mathbf{A}$$

c)

$$i_C(t) = 10^{-4}\left[-80\times 60\cos\left(80t+\frac{\pi}{6}\right)\right] = -0.48\cos\left(80t+\frac{\pi}{6}\right)$$

$$= 0.48\cos\left(80t+\frac{\pi}{6}-\pi\right) = 0.48\cos\left(80t-\frac{5\pi}{6}\right) \quad \mathbf{A}$$

d)

$$i_C(t) = 10^{-4}\left[-100\times 30\sin\left(100t+\frac{\pi}{4}\right)\right] = -0.3\sin\left(100t+\frac{\pi}{4}\right)$$

$$= 0.3\sin\left(100t+\frac{\pi}{4}-\pi\right) = 0.3\sin\left(100t-\frac{3\pi}{4}\right) \quad \mathbf{A}$$

Problem 4.3

Solution:

Known quantities:

Inductance value, $L = 250$ **mH**; the current through the inductor, as a function of time.

Find:

The voltage across the inductor as a function of time for each case

a) $i_L(t) = 5\sin 25tA$

b) $i_L(t) = -10\cos 50tA$

c) $i_L(t) = 25\cos(100t+\pi/3)A$

d) $i_L(t) = 20\sin(10t-\pi/12)A$.

Assumptions:

$i_L(t \le 0) = 0$

Analysis:

Using the differential relationship for the inductor, (Eq. 4.9), we may obtain the voltage by differentiating the current:

$$v_L(t) = L\frac{di_L(t)}{dt} = 250\times 10^{-3}\frac{di_L(t)}{dt} = 0.25\frac{di_L(t)}{dt}$$

a)

$$v_L(t) = 0.25[25\times 5\cos 25t] = 31.25\cos 25t \quad \mathbf{V}$$

b)

$$v_L(t) = 0.25[-50\times(-10\sin 50t)] = 125\sin 50t \quad \mathbf{V}$$

c)

$$v_L(t) = 0.25\left[-100\times 25\sin\left(100t+\frac{\pi}{3}\right)\right] = -625\sin\left(100t+\frac{\pi}{3}\right)$$

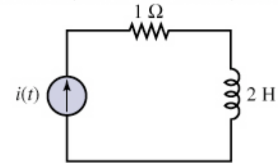
$$= 625\sin\left(100t+\frac{\pi}{3}-\pi\right) = 625\sin\left(100t-\frac{2\pi}{3}\right)\mathbf{V}$$

d)

$$v_L(t) = 0.25\left[10\times 20\cos\left(10t-\frac{\pi}{12}\right)\right] = 50\cos\left(10t-\frac{\pi}{12}\right)\mathbf{V}$$

Problem 4.4

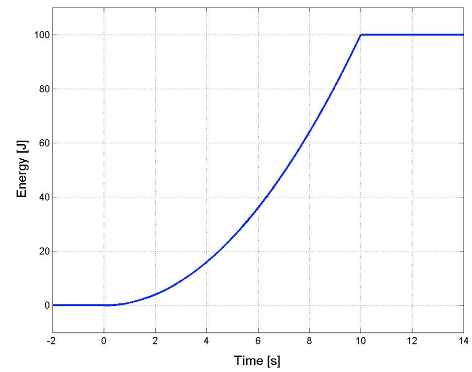
Solution:



Known quantities:
Inductance value; resistance value; the current through the circuit shown in Figure P4.4 as a function of time.

Find:
The energy stored in the inductor as a function of time.

Analysis:
The magnetic energy stored in an inductor may be found from, (Eq. 4.16):



$$w_L(t)=\frac{1}{2}Li(t)^2=\frac{1}{2}(2)i^2(t)=i^2(t)$$

For $-\infty < t < 0$,

$$w_L(t)=0$$

For $0 \le t < 10s$

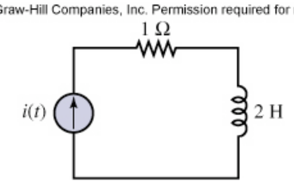
$$w_L(t)=t^2 \text{ J}$$

For $10\ s \le t < +\infty$

$$w_L(t)=100\text{ J}$$

Problem 4.5

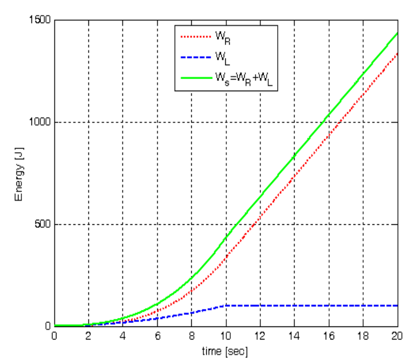
Solution:



Known quantities:
Inductance value; resistance value; the current through the circuit in Figure P4.4 as a function of time.

Find:
The energy delivered by the source as a function of time.

Analysis:



The energy delivered by the source is the sum of energy stored in inductor and the energy dissipated in resistor.

The energy dissipated in resistor is:

$$w_R(t)=\int_0^t p(t)dt = R\int_0^t i^2(t)dt = \int_0^t i^2(t)dt$$

For $-\infty < t < 0$, $w_R(t)=0$. During this time scope,

$$w_S(t)=w_R(t)+w_L(t)=0$$

For $0 \le t < 10s$, $w_R(t)=\int_0^t t^2 dt = \frac{t^3}{3}$. During this time scope,

$$w_S(t)=w_R(t)+w_L(t)=t^2+\frac{t^3}{2}$$. At $t=10\ s$, $w_R(10)=\frac{10^3}{3}=333.3$

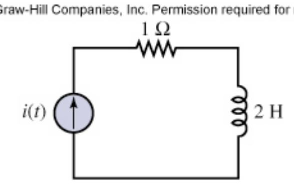
For $10 \le t < +\infty$, $w_R(t)=333.3+\int_{10}^t 10^2\,dt = 333.3+100(t-10)=100t-666.7$ J. During this time scope,

$$w_S(t)=w_R(t)+w_L(t)=-566.7+100t.$$

Problem 4.6

Solution:

Known quantities:



Inductance value; resistance value; the current through the circuit shown in Figure P4.4 as a function of time.

Find:
The energy stored in the inductor and the energy delivered by the source as a function of time.

Analysis:
a) The magnetic energy stored in an inductor may be found from, (Eq. 4.18):

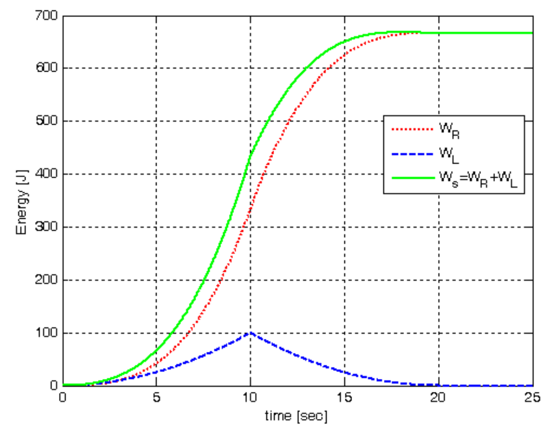
$$w_L(t)=\frac{1}{2}Li(t)^2=\frac{1}{2}(2)i^2(t)=i^2(t)$$

For $-\infty < t < 0$, $w_L(t)=0$

For $0 \le t < 10\,s$, $w_L(t)=t^2$ **J**

For $10 \le t < 20\,s$, $w_L(t)=(20-t)^2=400-40t+t^2$ J

For $20\ s \le t < +\infty$, $w_L(t)=0$ J



b) The energy delivered by the source is the sum of energy stored in inductor and the energy dissipated in resistor.

The energy dissipated in resistor is:

$$w_R(t)=\int_0^t p(t)dt=R\int_0^t i^2(t)dt=\int_0^t i^2(t)dt$$

For $-\infty < t < 0$, $w_R(t)=0$. During this time scope, $w_S(t)=w_R(t)+w_L(t)=0$

For $0 \le t < 10\,s$, $w_R(t)=\int_0^t t^2 dt=\frac{t^3}{3}$. During this time scope, $w_S(t)=w_R(t)+w_L(t)=t^2+\frac{t^3}{2}$. At $t=10\ s$, $w_R(10)=\frac{10^3}{3}=333.3$

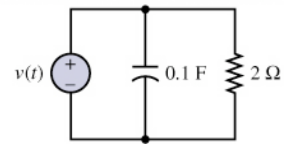
For $10 \le t < 20\,s$, $w_R(t)=333.3+\int_{10}^t (20-t)^2 dt=333.3+\left[400t-40\frac{t^2}{2}+\frac{t^3}{3}\right]_{10}^t=400t-40\frac{t^2}{2}+\frac{t^3}{3}-2000$. During this time scope, $w_S(t)=w_R(t)+w_L(t)=-1600+360t-19t^2+\frac{t^3}{3}$. At $t=20\ s$,

$w_R(20)=-2000+8000-8000+2666.7=666.7$

For $20\ s \le t < +\infty$ $w_R(t)=666.7$. During this time scope, $w_S(t)=w_R(t)+w_L(t)=666.7$ J .

Problem 4.7

Solution:



Known quantities:
Capacitance value; resistance value; the voltage applied to the circuit shown in Figure P4.7 as a function of time.

Find:
The energy stored in the capacitor as a function of time.

Analysis:
The energy stored in a capacitor may be found from:

$$w_C(t) = \frac{1}{2} C v(t)^2 = \frac{1}{2}(0.1) v(t)^2 = 0.05 v(t)^2$$

For $-\infty < t < 0$

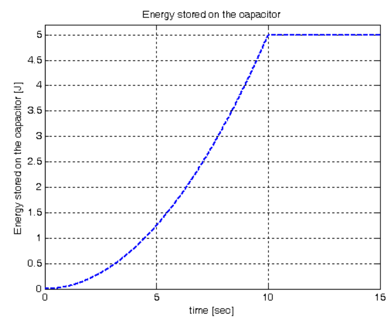
$$w_C(t) = 0$$

For $0 \le t < 10s$

$$w_C(t) = 0.05\,t^2 \text{ J}$$

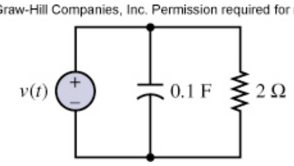
For $10s \le t < +\infty$

$$w_C(t) = 0.05(10)^2 = 5 \text{ J}$$



Problem 4.8

Solution:



Known quantities:
Capacitance value; resistance value; the voltage applied to the circuit shown in Figure P4.7 as a function of time.

Find:
The energy delivered by the source as a function of time.

Analysis:
The energy delivered by the source is the sum of energy stored in capacitor and the energy dissipated in resistor.

The energy dissipated in resistor is:

$$w_R(t) = \int_0^t p(t)dt = \frac{1}{R}\int_0^t v^2(t)dt = 0.5\int_0^t v^2(t)dt$$

For $-\infty < t < 0$, $w_R(t) = 0$. During this time scope,

$$w_S(t) = w_R(t) + w_C(t) = 0$$

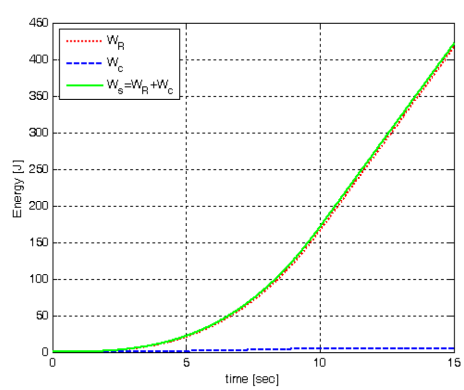
For $0 \le t < 10s$, $w_R(t) = 0.5\int_0^t t^2 dt = 0.1667t^3$. During this time scope, $w_S(t) = w_R(t) + w_L(t) = 0.05t^2 + 0.1667t^3$. At $t = 10\ s$,

$$w_R(10) = 0.1667t^3 = 166.7$$

For $10 \le t < +\infty$,

$$w_R(t) = 166.7 + 0.5\int_{10}^t 10^2 dt = 166.7 + 50(t-10) = 50t - 333.3 \text{ J}.$$

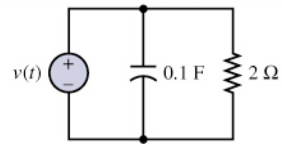
During this time scope, $w_S(t) = w_R(t) + w_L(t) = 50t - 228.3$ J.



Problem 4.9

Solution:

Known quantities:
Capacitance value; resistance value; the voltage applied to the circuit shown in Figure P4.7 as a function of time.



Find:
The energy stored in the capacitor and the energy delivered by the source as a function of time.

Analysis:
a) The energy stored in a capacitor may be found from: $w_C(t)=\frac{1}{2}Cv^2(t)=\frac{1}{2}(0.1)v^2(t)=0.05v^2(t)$

For $-\infty < t < 0 \quad w_C(t)=0$

For $0 \le t < 10s \quad w_C(t)=0.05\,t^2$ J

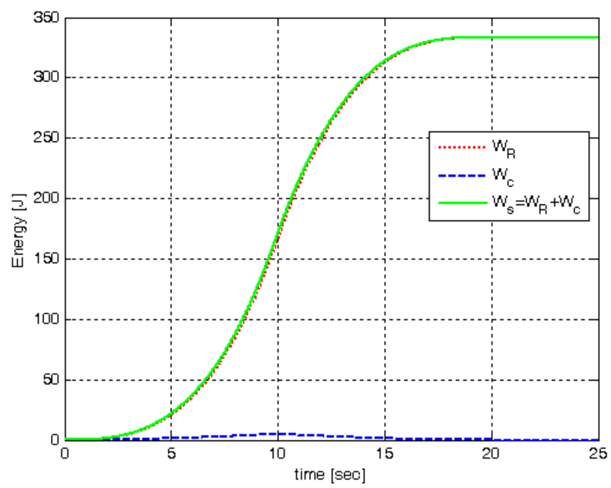
For $10 \le t < 20s \quad w_C(t)=0.05\,(20-t)^2=20-2t+0.05t^2$ J

For $10s \le t < +\infty \quad w_C(t)=0$

b) The energy delivered by the source is the sum of the energy dissipated by the resistance and the energy stored in the capacitor.

The energy dissipated in resistor is:

$w_R(t)=\int_0^t p(t)dt=\frac{1}{R}\int_0^t v^2(t)dt=0.5\int_0^t v^2(t)dt$



For $-\infty < t < 0$, $w_R(t)=0$. During this time scope, $w_S(t)=w_R(t)+w_L(t)=0$

For $0 \le t < 10s$, $w_R(t)=0.5\int_0^t t^2dt=0.1667t^3$.

During this time scope, $w_S(t)=w_R(t)+w_L(t)=0.05t^2+0.1667t^3$. At $t=10\ s$, $w_R(10)=0.1667(10)^3=166.7$

For $10 \le t < 20s$,

$w_R(t)=166.7+0.5\int_{10}^t (20-t)^2dt=200t-10t^2+0.166$

. During this time scope, $w_S(t)=w_R(t)+w_L(t)=-980+198t-9.95t^2+0.1667t^3$. At $t=20\ s$,

$w_R(20)=-1000+4000-4000+1333.6=333.6$

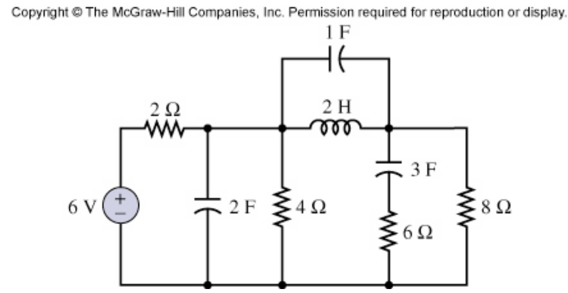
For $20\ s \le t < +\infty \quad w_R(t)=333.6$. During this time scope, $w_S(t)=w_R(t)+w_L(t)=333.6$ J.

Problem 4.10

Solution:

Known quantities:

Capacitance, resistance and inductance values; the voltage $v_S = 6\ V$ applied to the circuit of Fig. P4.10.



Find:

The energy stored in each capacitor and inductor.

Analysis:

Under steady-state conditions, all the currents are constant, no current can flow through the capacitors, and the voltage across any inductor is equal to zero.

$$v_{2F} = v_{4\Omega} \Rightarrow \frac{6 - v_{4\Omega}}{2} = \frac{v_{4\Omega}}{4} + \frac{v_{4\Omega}}{8} \Rightarrow v_{4\Omega} = 3.43\mathbf{V} \qquad \Rightarrow w_{2F} = \frac{1}{2} C_{2F} v_{2F}^2 = \frac{1}{2}(2\mathbf{F})(3.43\mathbf{V})^2 = 11.76\ \mathbf{J}$$

$$v_{1F} = v_{2H} = 0 \qquad \Rightarrow w_{1F} = \frac{1}{2} C_{1F} v_{1F}^2 = \frac{1}{2}(1\ F)(0)^2 = 0$$

$$i_{2H} = \frac{v_{4\Omega}}{8} = 0.43\ \mathbf{A} \quad \Rightarrow w_{2H} = \frac{1}{2} L_{2H} i_{2H}^2 = \frac{1}{2}(2\ \mathbf{H})(0.43\ \mathbf{A})^2 = 0.18\ \ \mathbf{J}$$

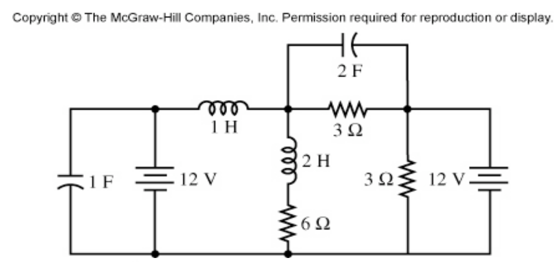
$$v_{3F} = v_{4\Omega} = 3.43\ \mathbf{V} \quad \Rightarrow w_{3F} = \frac{1}{2} C_{3F} v_{3F}^2 = \frac{1}{2}(3\ \mathbf{F})(3.43\ \mathbf{V})^2 = 17.65\ \ \mathbf{J}$$

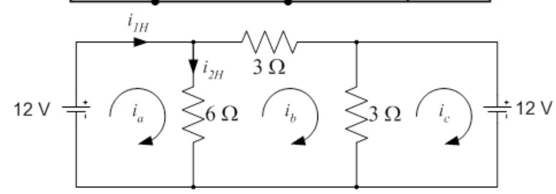
Problem 4.11

Solution:

Known quantities:

Capacitance, resistance and inductance values; the voltage $v_A = 12\ \mathbf{V}$ applied to the circuit shown in Figure P4.11.





Find:

The energy stored in each capacitor and inductor.

Analysis:

Under steady-state conditions, all the currents are constant, no current can flow across the capacitors, and the voltage across any inductor is equal to zero.

The voltage for the 1-F capacitor is equal to the 12-Volt input. Since the voltage is the same on either end of the 3-Ω resistor in parallel with the 2-F capacitor, there is no voltage drop through either component.
Finally, since there is no voltage drop through the 3-Ω resistor in parallel with the 2-F capacitor, there is no current flow through the resistor, and the current through the 1-H inductor is equal to the current through the 2-H inductor. Therefore,

$$v_{1F} = v_A = 12\ \mathbf{V} \Rightarrow w_{1F} = \frac{1}{2} C_{1F} v_{1F}^2 = \frac{1}{2}(1\ \mathbf{F})(12\ \mathbf{V})^2 = 72\ \ \mathbf{J}$$

$$i_{1H} = \frac{12\mathbf{V}}{6\Omega} = 2\ \mathbf{A} \quad \Rightarrow w_{1H} = \frac{1}{2} L_{1H} i_{1H}^2 = \frac{1}{2}(1\ \mathbf{H})(2\ \mathbf{A})^2 = 2\ \ \mathbf{J}$$

$$i_{2H} = i_{1H} = 2\ \mathbf{A} \Rightarrow w_{2H} = \frac{1}{2} L_{2H} i_{2H}^2 = \frac{1}{2}(2\ \mathbf{H})(2\ \mathbf{A})^2 = 4\ \ \mathbf{J}$$

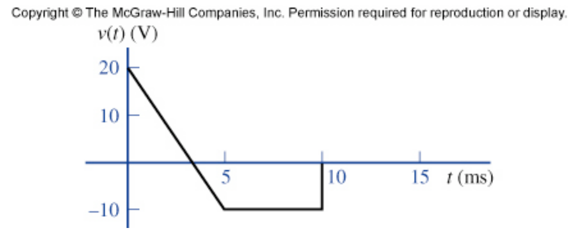
$$v_{2F} = 0\ \mathbf{V} \Rightarrow w_{2F} = \frac{1}{2} C_{2F} v_{2F}^2 = \frac{1}{2}(2\ \mathbf{F})(0\ \mathbf{V})^2 = 0\ \ \mathbf{J}$$

Problem 4.12

Solution:

Known quantities:

Capacitance value $C = 80\ \mu F$; the voltage applied to the capacitor as a function of time as shown in Figure P4.12.



Find:

The current through the capacitor as a function of time.

Analysis:

Since the voltage waveform is piecewise continuous, the derivative must be evaluated over each continuous segment.

For $0 < t < 5$ **ms**

$$v_C(t) = m_{v_C} t + q_{v_C}$$

where:

$$m_{v_C} = \frac{[-10\ \mathbf{V}] - [+20\ \mathbf{V}]}{[5\ \mathbf{ms}] - [0]} = -6\ \frac{\mathbf{V}}{\mathbf{ms}}$$

$$q_{v_C} = +20\ \mathbf{V}$$

$$i_C = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}\left[m_{v_C} t + q_{v_C}\right] = Cm_{v_C} = (80\ \mu F)\left(-6\ \frac{\mathbf{V}}{\mathbf{ms}}\right) = -480\ \mathbf{mA}$$

For $5\ \mathbf{ms} < t < 10\ \mathbf{ms}$

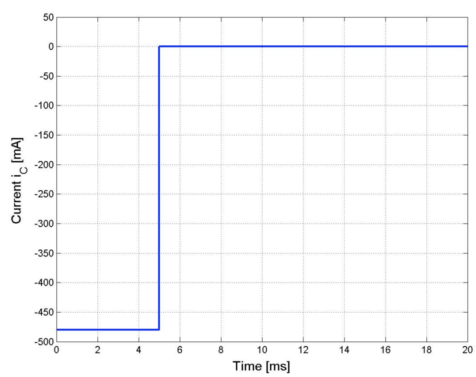
$$v_C(t) = -10\ \mathbf{V}$$

$$i_C = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}[-10\ \mathbf{V}] = 0$$

For $t > 10$ ms

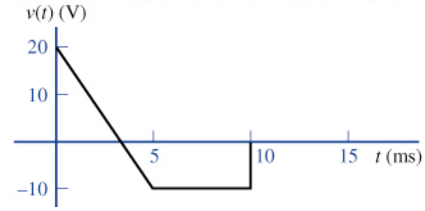
$$v_C(t) = 0$$

$$i_C = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}[0] = 0$$



A capacitor is fabricated from two conducting plates separated by a dielectric constant. Dielectrics are also insulators; therefore, current cannot really flow through a capacitor. Positive charge, however, entering one plate exerts a repulsive force on and forces positive carriers to exit the other plate. Current then *appears* to flow through the capacitor. Such currents are called *electric displacement currents*.

Problem 4.13



Solution:

Known quantities:

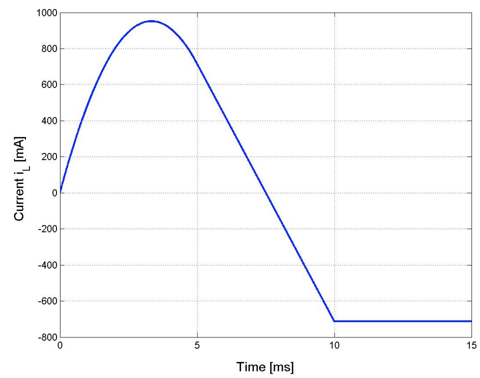
Inductance value, $L = 35$ **mH**; the voltage applied to the inductor as shown in Figure P4.12; the initial condition for the current $i_L(0) = 0$.

Find:

The current across the inductor as a function of time.

Analysis:

Since the voltage waveform is piecewise continuous, integration must be performed over each continuous segment. Where not indicated t is supposed to be expressed in seconds.



For $0 < t \le 5$ **ms**

$$v_L(t) = m_{v_L} t + q_{v_L}$$

where:

$$m_{v_L} = \frac{[-10\ \mathbf{V}] - [+20\ \mathbf{V}]}{[5\ \mathbf{ms}] - [0]} = -6\ \frac{\mathbf{V}}{\mathbf{ms}}$$

$$q_{v_L} = +20\ \mathbf{V}$$

$$i_L = i_L(0) + \frac{1}{L}\int_0^t v_L(\tau)d\tau = 0 + \frac{1}{L}\int_0^t \left(m_{v_L}\tau + q_{v_L}\right)d\tau = \frac{1}{L}\left[\frac{1}{2}m_{v_L}\tau^2 + q_{v_L}\tau\right]_0^t =$$

$$= \frac{1}{2L}\left(m_{v_L}t^2 + q_{v_L}t\right) = \frac{1}{2\cdot 35\ \mathbf{mH}}\left(-6\ \frac{\mathbf{V}}{\mathbf{ms}}\cdot t^2 + 20\ \mathbf{V}\cdot t\right) = \left(-85.71\cdot 10^3 t^2 + 571.4t\right)\mathbf{A}$$

$$i_L(t = 5\ \mathbf{ms} = 0.005\ \mathbf{s}) = \left(-85.71\cdot 10^3\cdot(0.005)^2 + 571.4\cdot 0.005\right)\mathbf{A} = 714.3\ \mathbf{mA}$$

For 5 **ms** $< t \le 10$ **ms**

$$v_L(t) = c_{v_L} = -10\ \mathbf{V}$$

$$i_L = i_L(0.005) + \frac{1}{L}\int_{0.005}^t v_L(\tau)d\tau = i_L(0.005) + \frac{1}{L}\int_{0.005}^t \left(c_{v_L}\right)d\tau = i_L(0.005) + \frac{1}{L}\left[c_{v_L}\tau\right]_{0.005}^t =$$

$$= 714.3\ \mathbf{mA} - \frac{1}{35\ \mathbf{mH}}\cdot(-10\ \mathbf{V})\cdot(t - 0.005\ \mathbf{s}) = (2.143 - 285.7t)\ \mathbf{A}$$

$$i_L(t = 10\ \mathbf{ms} = 0.01\ \mathbf{s}) = \left(2.143 - 285.7(0.01)\right)\mathbf{A} = -713.5\ \mathbf{mA}$$

For $t > 10$ **ms**

$$v_L(t) = c_{v_L} = 0$$

$$i_L = i_L(0.01) + \frac{1}{L}\int_{0.01}^t v_L(\tau)d\tau = i_L(0.01) + \frac{1}{L}\int_{0.01}^t (0)d\tau = i_L(0.01) + \frac{1}{L}[0]_{0.005}^t =$$

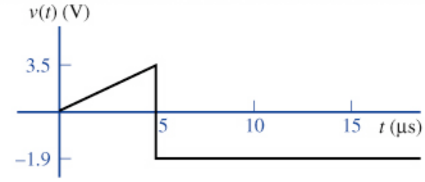
$$= i_L(0.01) = -713.5\ \mathbf{mA}$$

Problem 4.14

Solution:

Known quantities:

Inductance value $L = 0.75$ **mH**; the voltage applied to the inductor as a function of time as shown in Figure P4.14.



Find:

The current through the inductor at the time $t = 15\ \mu s$.

Assumptions:

$i_L(t \le 0) = 0$

Analysis:

Since the voltage waveform is a piecewise continuous function of time, integration must be performed over each continuous segment. Where not indicated, *t* is expressed in seconds.

For $0 < t \le 5$ μs

$$v_L(t) = m_{v_L} t + q_{v_L} \text{ where: } m_{v_L} = \frac{[3.5\ \mathbf{V}] - [0\ \mathbf{V}]}{[5\ \mu\mathbf{s}] - [0]} = 0.7\ \frac{\mathbf{V}}{\mu s}$$

$q_{v_L} = 0\ \mathbf{V}$

For $t > 5\ \mu\mathbf{s}$

$$v_L(t) = c_{v_L} = -1.9\ \mathbf{V}$$

Therefore:

$$i_L(t = 15\ ?\mathbf{s}) = \frac{1}{L}\int_{-\infty}^{15\ \mu s} v_L(\tau)d\tau = i_L(0) + \frac{1}{L}\int_0^{5\ \mu s}\left(m_{v_L}\tau\right)d\tau + \frac{1}{L}\int_{5\ \mu s}^{15\ \mu s}\left(c_{v_L}\right)d\tau =$$

$$= i_L(0) + \frac{1}{L}\left[\frac{1}{2}m_{v_L}\tau^2\right]_0^{5\ \mu s} + \frac{1}{L}\left[c_{v_L}\tau\right]_{5\ \mu s}^{15\ \mu s} = 0 + \frac{1}{0.75\ \mathbf{mH}} \cdot \frac{0.7\ \frac{\mathbf{V}}{\mathbf{ms}}}{2} \cdot \left((5\ \mu\mathbf{s})^2 - 0\right) +$$

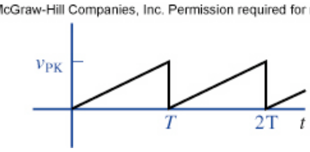
$$+ \frac{1}{0.75\ \mathbf{mH}} \cdot (-1.9\ \mathbf{V}) \cdot (15\ \mu\mathbf{s} - 5\ \mu\mathbf{s}) = -13.67\ \mathbf{mA}$$

Problem 4.15

Solution:

Known quantities:

Capacitance value $C = 680$ **nF**; the periodic voltage applied to the capacitor as shown in Figure P4.15: $v_{Peak} = 20$ V, $T = 40\ \mu\mathbf{s}$.



Find:

The waveform and the plot for the current through the capacitor as a function of time.

Analysis:

Since the voltage waveform is not a continuous function of time, differentiation can be performed only over each continuous segment. In the discontinuity points the derivative of the voltage will assume an infinite value, the sign depending on the sign of the step. Where not indicated, *t* is expressed in seconds.

For each period $0 < t < T$, $T < t < 2T$, … the behavior of the capacitor will be the same; thus, we consider only the first period:

For $0 < t < T$

$$v_C(t) = m_{v_C} t + q_{v_C} \text{ where: } m_{v_C} = \frac{[v_{Peak}] - [0]}{[T] - [0]} = 0.5 \ \frac{\mathbf{V}}{\mathbf{?s}} \quad q_{v_C} = 0 \ \mathbf{V}$$

$$i_C = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}\left[m_{v_C} t\right] = Cm_{v_C} = \left(480 \ \frac{\mathbf{nA-s}}{\mathbf{V}}\right)\left(0.5 \ \frac{\mathbf{V}}{\mu\mathbf{s}}\right) = 340 \ \mathbf{mA}$$

For $t = T, 2T, \ldots$

$$i_C = C\frac{dv_C(t)}{dt} = C[-\infty]$$

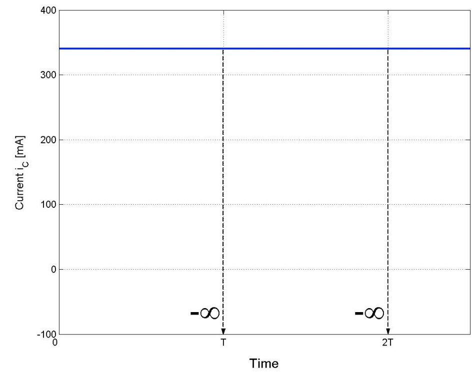


Figure P 4.15 shows the current waveform.

Note: For the voltage across the capacitor to decrease instantaneously to zero at $t = T, 2T, \ldots$, the charge on the plates of the capacitor should be instantaneously discharged. This requires an infinite current which is not physically possible.

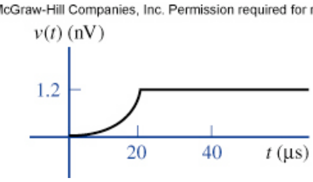
If this were a practical waveform, the slope at $t = T, 2T, \ldots$, would be finite, not infinite. A large negative spike of current over a finite period of time would result instead of the infinite spike over zero time. These large spike of current (or voltage) degrade the performance of many circuits.

Problem 4.16

Solution:

Known quantities:

Inductance value, $L = 16 \ \mu\mathbf{H}$; the voltage applied to the inductor as a function of time as shown in Figure P4.16; the initial condition for the current $i_L(0) = 0$.



Find:

The current through the inductor at $t = 30 \ \mu\mathbf{s}$.

Analysis:

Since the voltage waveform is piecewise continuous, the integration can be performed over each continuous segment. Where not indicated t is supposed to be expressed in seconds.

$$i_L(t = 30 \ \mu\mathbf{s}) = \frac{1}{L}\int_{-\infty}^{30 \ \mu s} v_L(\tau)d\tau = i_L(0) + \frac{1}{L}\int_{0}^{20 \ \mu s} v_L(\tau)d\tau + \frac{1}{L}\int_{20 \ \mu s}^{30 \ \mu s} v_L(\tau)d\tau =$$

$$= i_L(0) + \frac{1}{L}\left[\frac{3}{3}\tau^3 \ \frac{V}{s^2}\right]_0^{20 \ \mu s} + \frac{1}{L}\left[1.2\tau \ \mathbf{nV}\right]_{20 \ \mu s}^{30 \ \mu s} = 0 + \frac{1}{16 \ \mu\mathbf{H}} \cdot 1 \ \frac{\mathbf{V}}{\mathbf{s}^2} \cdot \left((20 \ \mu\mathbf{s})^3 - 0\right) +$$

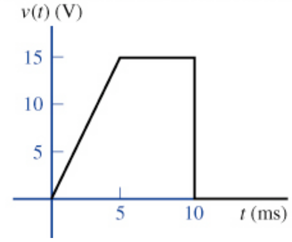
$$+ \frac{1}{16 \ \mu\mathbf{H}} \cdot (1.2 \ \mathbf{nV}) \cdot (30 \ \mu\mathbf{s} - 20 \ \mu\mathbf{s}) = 1.250 \ \mathbf{nA}$$

Problem 4.17

Solution:

Known quantities:

Resistance value $R = 7\ \Omega$; inductance value $L = 7\ \mathbf{mH}$; capacitance value $C = 0.5\ \mu\mathbf{F}$; the voltage across the components as shown in Figure P4.17.



Find:

The current through each component.

Assumptions:

$i_R(t \le 0) = i_L(t \le 0) = i_C(t \le 0) = 0$

Analysis:

Since the voltage waveform is piecewise continuous, integration and differentiation can only be performed over each continuous segment. Where not indicated, t is expressed in seconds.

For $t \le 0$:

$$i_R(t) = i_L(t) = i_C(t) = 0$$

For $0 < t < 5\ \text{ms}$:

$$v(t) = m_v t + q_v$$

where:

$$m_v = \frac{[15\ \mathbf{V}] - [0]}{[5\ \mathbf{ms}] - [0]} = 3\ \frac{\mathbf{V}}{\mathbf{ms}}$$

$$q_{v_C} = 0\ \mathbf{V}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{m_v \cdot t}{R} = \frac{3\ \frac{\mathbf{V}}{\mathbf{ms}} \cdot t}{7\ \Omega} = 428.6 \cdot t\ \mathbf{A}$$

$$i_L(t) = \frac{1}{L}\int_0^t v(\tau)d\tau = \frac{1}{L}\int_0^t m_v \cdot \tau\, d\tau = \frac{1}{L}\left[\frac{1}{2} m_v \cdot \tau^2\right]_0^t = \frac{1}{2L} m_v \cdot t^2 =$$

$$= \frac{1}{2 \cdot 7\ \mathbf{mH}} \cdot 3\ \frac{\mathbf{V}}{\mathbf{ms}} \cdot t^2 = 214.3 \cdot 10^3 \cdot t^2\ \mathbf{A}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}[m_v t] = C m_v = \left(0.5\ \frac{\mu\mathbf{As}}{\mathbf{V}}\right)\left(3\ \frac{\mathbf{V}}{\mathbf{ms}}\right) = 1.5\ \mathbf{mA}$$

For $t = 5\ \mathbf{ms}$:

$$i_R\left(5 \cdot 10^{-3}\right) = 428.6 \cdot t\ \mathbf{A} = 428.6 \cdot 5 \cdot 10^{-3}\ \mathbf{A} = 2.143\ \mathbf{A}$$

$$i_L\left(5 \cdot 10^{-3}\right) = 214.3 \cdot 10^3 \cdot t^2\ \mathbf{A} = 214.3 \cdot 10^3 \cdot \left(5 \cdot 10^{-3}\right)^2\ \mathbf{A} = 5.357\ \mathbf{A}$$

$$i_C\left(5 \cdot 10^{-3}\right) = 1.5\ \mathbf{mA}$$

For $5\ \mathbf{ms} < t < 10\ \mathbf{ms}$:

$$v(t) = c_v = 15\ \mathbf{V}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{c_v}{R} = \frac{15\ \mathbf{V}}{7\ \Omega} = 2.143\ \mathbf{A}$$

$$i_L(t) = i_L\left(5\cdot 10^{-3}\right) + \frac{1}{L}\int_{5\cdot 10^{-3}}^{t} v(\tau)d\tau = i_L\left(5\cdot 10^{-3}\right) + \frac{1}{L}\int_{5\cdot 10^{-3}}^{t} c_v\, d\tau =$$

$$= i_L\left(5\cdot 10^{-3}\right) + \frac{1}{L}\left[c_v\cdot\tau\right]_{5\cdot 10^{-3}}^{t} = i_L\left(5\cdot 10^{-3}\right) + \frac{1}{L}\cdot c_v\cdot\left(t - 5\cdot 10^{-3}\right) =$$

$$= 5.357\ \mathbf{A} + \frac{1}{7\ \mathbf{mH}}\cdot 15\ \mathbf{V}\cdot\left(t - 5\cdot 10^{-3}\ \mathbf{s}\right) = -5.357 + 2.143\cdot 10^{3}\cdot t\ \mathbf{A}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}\left[c_v\right] = 0$$

For $t = 10\ \mathbf{ms}$:

$$i_R(0.01) = 2.143\ \mathbf{A}$$

$$i_L(0.01) = -5.357 + 2.143\cdot 10^{3}\cdot t\ \mathbf{A} = -5.357 + 2.143\cdot 10^{3}\cdot 0.01\ \mathbf{A} = 16.07\ \mathbf{A}$$

$$i_C(0.01) = 0$$

For $t > 10\ \mathbf{ms}$:

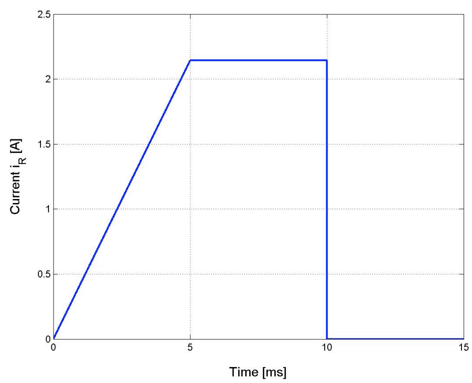
$$v(t) = 0$$

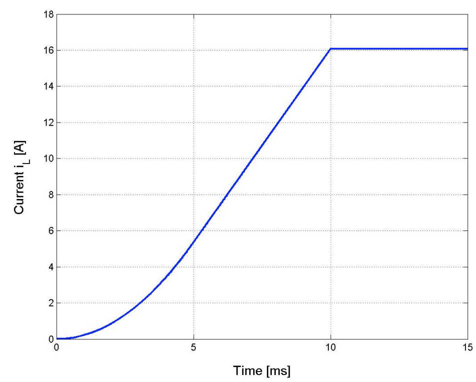
$$i_R(t) = \frac{v(t)}{R} = \frac{0}{R} = 0$$

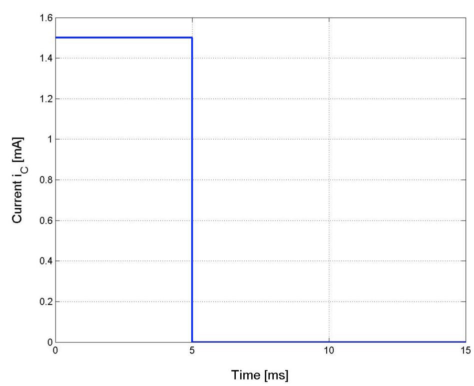
$$i_L(t) = i_L\left(10\cdot 10^{-3}\right) + \frac{1}{L}\int_{10\cdot 10^{-3}}^{t} v(\tau)d\tau = i_L\left(10\cdot 10^{-3}\right) + \frac{1}{L}\int_{10\cdot 10^{-3}}^{t} 0\, d\tau =$$

$$= i_L\left(10\cdot 10^{-3}\right) + \frac{1}{L}\left[0\right]_{10\cdot 10^{-3}}^{t} = i_L\left(10\cdot 10^{-3}\right) = 16.07\ \mathbf{A}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}\left[0\right] = 0$$

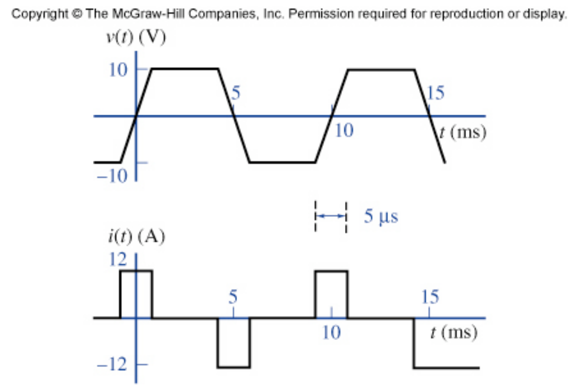






Problem 4.18

Solution:



Known quantities:
The voltage across and the current through an ideal capacitor as shown in Figure P4.18.

Find:
The capacitance of the capacitor.

Analysis:
Considering the period: $-2.5\ \mu\mathbf{s} < t < +2.5\ \mu\mathbf{s}$:

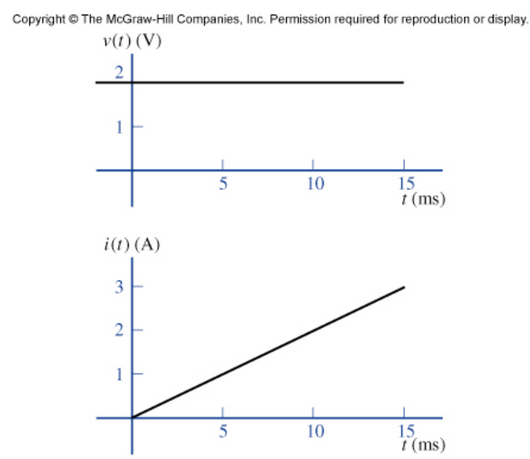
$$i_c = C\frac{dv_c}{dt} = C\frac{\Delta v_c}{\Delta t}$$, since the voltage has a linear waveform.

Substituting:

$$12\ \mathbf{A} = C\frac{[+10\ \mathbf{V}]-[-10\ \mathbf{V}]}{5\ \mu\mathbf{s}} \Rightarrow C = 12\ \mathbf{A}\cdot\frac{5}{20}\frac{\mu\mathbf{s}}{\mathbf{V}} = 3\ \mu\mathbf{F}$$

Problem 4.19

Solution:



Known quantities:
The voltage across and the current through an ideal inductor as shown in Figure P4.19.

Find:
The inductance of the inductor.

Analysis:

$$v_L = L\frac{di_L}{dt} = L\frac{\Delta i_L}{\Delta t}$$, since the current has a linear waveform.

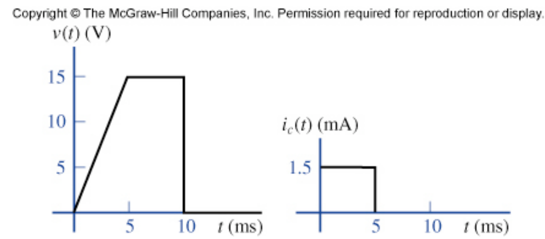
Substituting:

$$2\ \mathbf{V} = L\frac{[2\ \mathbf{A}]-[1\ \mathbf{A}]}{10\ \mathbf{ms}-5\ \mathbf{ms}} \Rightarrow L = 2\ \mathbf{V}\cdot\frac{5}{1}\frac{\mathbf{ms}}{\mathbf{A}} = 10\ \mathbf{mH}$$

Problem 4.20

Solution:

Known quantities:
The voltage across and the current through an ideal capacitor as shown in Figure P4.20.



Find:
The capacitance of the capacitor.

Analysis:
Considering the period: $0 < t < 5$ **ms**:

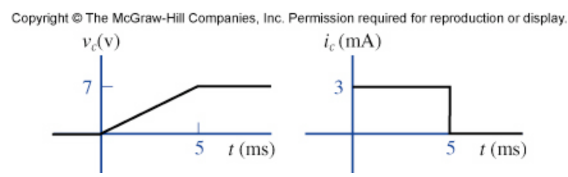
$$i_c = C\frac{dv_c}{dt} = C\frac{\Delta v_c}{\Delta t}$$, since the voltage has a linear waveform. Substituting:

$$1.5\ \mathbf{mA} = C\frac{[15\ \mathbf{V}]-[0]}{5\ \mathbf{ms}} \Rightarrow C = 1.5\ \mathbf{mA}\cdot\frac{5}{15}\frac{\mathbf{ms}}{\mathbf{V}} = 0.5\ \mu\mathbf{F}$$

Problem 4.21

Solution:

Known quantities:
The voltage across and the current through an ideal capacitor as shown in Figure P4.21.



Find:
The capacitance of the capacitor.

Analysis:
Considering the period: $0 < t < 5$ **ms**:

$$i_c = C\frac{dv_c}{dt} = C\frac{\Delta v_c}{\Delta t}$$, since the voltage has a linear waveform.

Substituting:

$$3\ \mathbf{mA} = C\frac{[7\ \mathbf{V}]-[0]}{5\ \mathbf{ms}} \Rightarrow C = 3\ \mathbf{mA}\cdot\frac{5}{7}\frac{\mathbf{ms}}{\mathbf{V}} = 2.14\ \mu\mathbf{F}$$

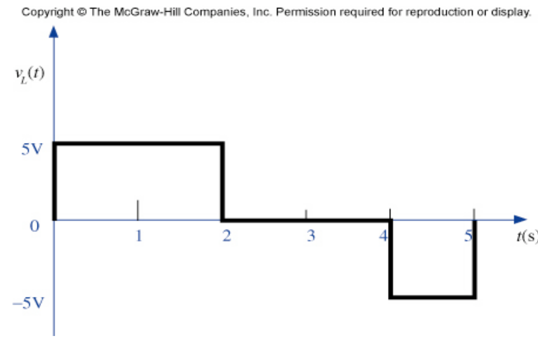
Problem 4.22

Solution:

Known quantities:
The voltage across the inductor as shown in Figure P4.22.

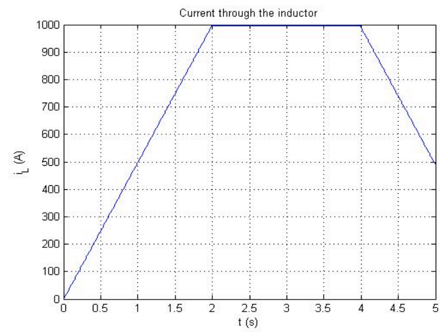
Find:
The current in the inductor.

Analysis:



$$i_L(t) = \frac{1}{L}\int_0^t v_L(t')dt$$

The current in the inductor is shown in the figure:



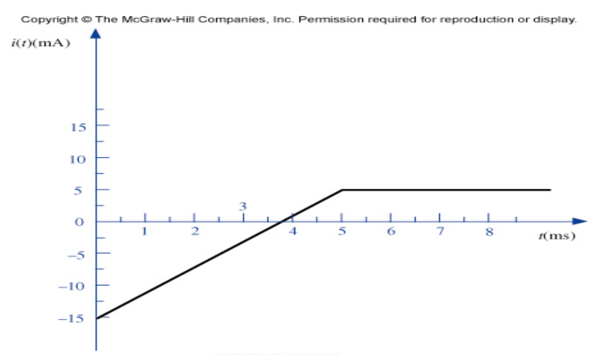
Problem 4.23

Solution:

Known quantities:
The current through the inductor as shown in Figure P4.23.

Find:
The voltage across the inductor.

Analysis:

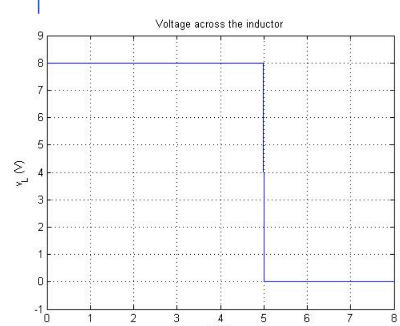


$$i(t) = \begin{cases} 4t - 15\,\text{mA} & 0 < t < 5\text{ms} \\ 5\,\text{mA} & t > 5\text{ms} \end{cases}$$

For a 2 H inductor, since $v_L = L\dfrac{di_L}{dt}$,

$$v_L(t) = \begin{cases} 8\text{ V} & 0 < t < 5\text{ms} \\ 0 & t > 5\text{ms} \end{cases}$$

The voltage waveform is sketched in the figure.



Problem 4.24

Solution:

Known quantities:

The voltage across the inductor as shown in Figure P4.24.

Find:

The current through the inductor and the capacitor.

Analysis:

$$v(t) = \begin{cases} \dfrac{15}{0.004} t \text{ V} & 0 < t < 4\text{ms} \\ 30 - \dfrac{15}{0.004} t \text{ V} & 4\text{ms} < t < 6\text{ms} \end{cases}$$

$$= \begin{cases} 3750t \text{ V} & 0 < t < 4\text{ms} \\ 30 - 3750\, t \text{ V} & 4\text{ms} < t < 6\text{ms} \end{cases}$$

The capacitor current is

$$i_C = C\frac{dv}{dt} = 500 \times 10^{-6} \frac{dv}{dt}$$

$$= \begin{cases} 1.875\, A & 0 < t < 4ms \\ -1.875\, A & 4ms < t < 6ms \end{cases}$$

The inductor current is

$$i_L = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau = i_L(t_0) + 10\int_{t_0}^{t} v(\tau)d\tau$$

Assume $i_L(0) = 0$.

For $0 < t < 4$ms, we have

$$i_L = 0 + 10\int_0^t 3750\tau d\tau = 37500\left.\frac{t^2}{2}\right|_0^t = 18750t^2 \text{ A}$$

For 4ms $< t < 6$ms, we have

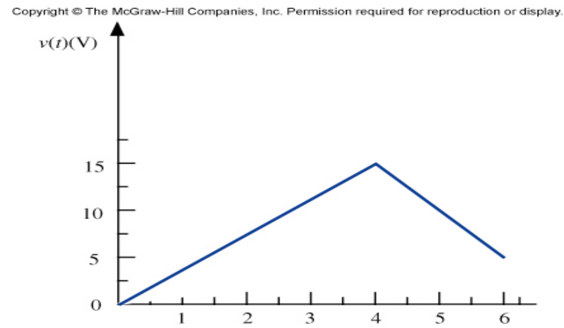
$$i_L = 18750(0.004)^2 + 10\int_{0.004}^{t}(30 - 3750\tau)d\tau$$

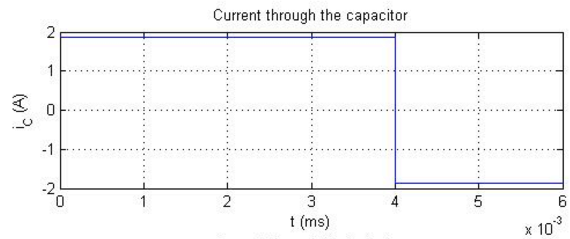
$$= 0.3 + \left[300\tau - 18750\tau^2\right]_{0.004}^{t}$$

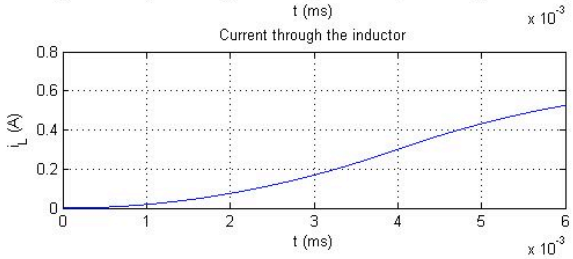
$$= 0.3 + 300t - 18750t^2 - 300(0.004) + 18750(0.004)^2$$

$$= 300t - 18750t^2 - 0.6 \text{ A}$$

The two functions are sketched in the figures:



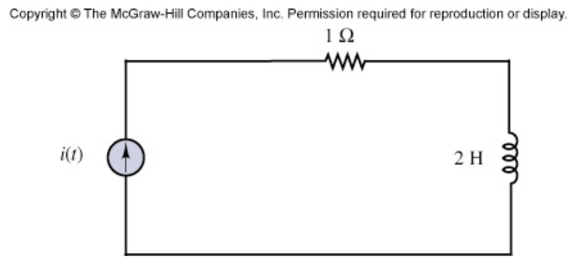




Problem 4.25

Solution:

Known quantities:
Circuit as shown in Figure P4.25 and the value of current source.



Find:
The energy stored in the inductor.

Analysis:
$$w_L(t) = \frac{1}{2}Li^2 = \frac{1}{2}(2)i^2 = i^2$$

For $-\infty < t < 0$,

$$w_L(t) = 0$$

For $0 \le t < 1s$

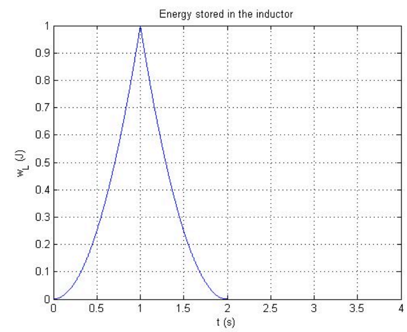
$$w_L(t) = t^2 \text{ J}$$

For $1s \le t < 2s$

$$w_L(t) = [-(t-2)]^2 = t^2 - 4t + 4 \text{ J}$$

For $2s \le t < \infty$

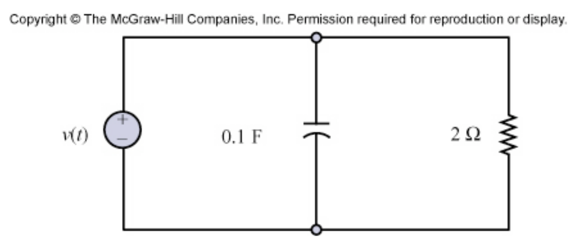
$$w_L(t) = 0$$



Problem 4.26

Solution:

Known quantities:
Circuit as shown in Figure P4.26 and the value of voltage source.



Find:
The energy stored in the capacitor.

Analysis:
$$w_C(t) = \frac{1}{2}Cv^2 = \frac{1}{2}(0.1)v^2 = 0.05v^2$$

For $-\infty < t < 0$

$$w_C(t) = 0$$

For $0 \le t < 1s$

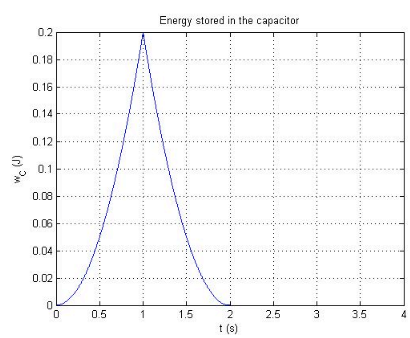
$$w_C(t) = 0.05(2t)^2 = 0.2t^2 \; J$$

For $1s \le t < 2s$

$$w_C(t) = 0.05[-(2t-4)]^2 = 0.2t^2 - 0.8t + 0.8 \, J$$

For $2s \le t < \infty$

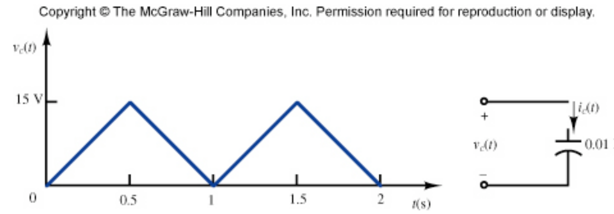
$$w_C(t) = 0$$



Problem 4.27

Solution:

Known quantities:
Circuit as shown in Figure P4.27 and the value of voltage source.



Find:
The current through the capacitor.

Analysis:
In an ideal capacitor,

$$i_c = C\frac{dV_C}{dt}$$

For $0 \le t \le 0.5$ s, the voltage is:

$$V_c = 30t$$

$$\therefore \quad i_c = 0.01 \times 30 = 0.3 \text{ A}$$

For $0.5 \le t \le 1$ s,

$$V_c = -30t + 30$$

$$\therefore \quad i_c = -0.01 \times 30 = -0.3 \text{ A}$$

For $1 \le t \le 1.5$ s,

$$V_c = 30t - 30$$

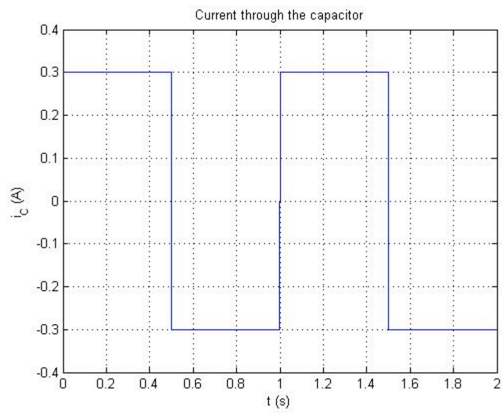
$$\therefore \quad i_c = 0.01 \times 30 = 0.3 \text{ A}$$

For $1.5 \le t \le 2$ s,

$$V_c = -30t + 60$$

$$\therefore \quad i_c = -0.01 \times 30 = -0.3 \text{ A}$$

The waveform of $i_C(t)$ is shown in the right hand side.

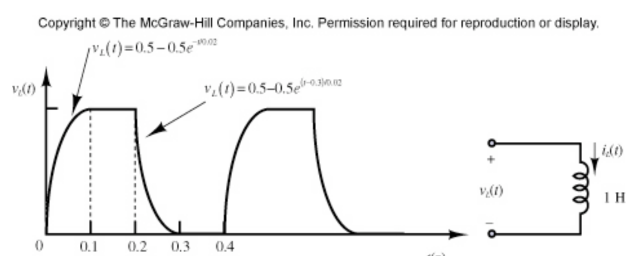


Problem 4.28

Solution:

Known quantities:
Circuit as shown in Figure P4.28 and the value of voltage source.

Find:
The current through the inductor.



Analysis:

$$v_L = L\frac{di_L}{dt}$$

$$i_L = \frac{1}{L}\int v_L dt$$

For $0 < t < 0.1$ s

$$i_L = \int 0.5 - 0.5e^{-t/0.02}$$

$$= 0.5t + (0.5 \times 0.02)e^{-t/0.02} - 0.01 = 0.5t + 0.01e^{-t/0.02} - 0.01 \quad \text{A}$$

For $0.1 < t < 0.2$ s

$$i_L = i_L(0.1) + \int 0.4866 d\tau = 0.0501 + 0.4966(t - 0.1) = 0.4966t + 0.00044 \text{ A}$$

For $0.2 < t < 0.3$

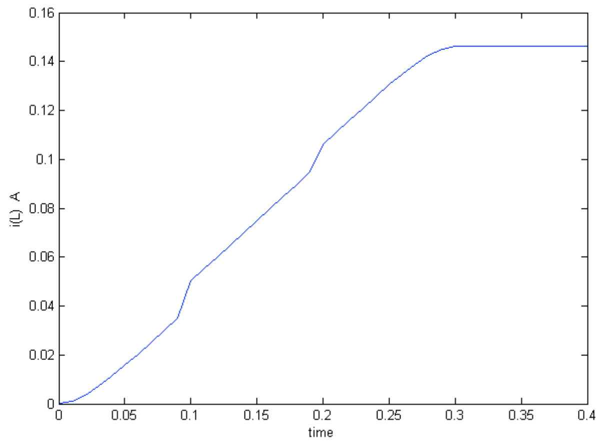
$i_L = i_L(0.2) + \int (0.5 - 0.5e^{(\tau - 0.3)/0.02})d\tau$

$= 0.99364 + 0.5(t - 0.2) - (0.5 \times 0.02)(e^{(t-0.3)/0.02} - e^{-5}) = 0.006283 + 0.5t - 0.01e^{(t-0.3)/0.02}$ A

For 0.3 < t < 0.4

$i_L = i_L(0.3) = 0.146283$ A

The resulting waveform is shown in the right hand side.



Section 4.2: Time-Dependent Signals

Problem 4.29

Solution:

Known quantities:

The signal $x(t) = 2\cos(\omega t) + 2.5$.

Find:

The average and rms value of the signal.

Analysis:

The average value is:

$$\langle v(t) \rangle = \frac{\omega}{2\pi}\left[\int_0^{\frac{2\pi}{\omega}} 2\cos(\omega t)dt + \int_0^{\frac{2\pi}{\omega}} (2.5)dt\right] = \frac{1}{2\pi}\left[-\sin(\omega t)\Big|_0^{\frac{2\pi}{\omega}} + 2.5t\Big|_0^{\frac{2\pi}{\omega}}\right]$$

$$= \frac{1}{2\pi}\left[\sin(0) - \sin(2\pi)\right] + 2.5 = \frac{1}{2\pi}[0 - 0] + 2.5 = 2.5$$

The rms value is:

$$x_{rms} = \sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} (2\cos(\omega t) + 2.5)^2 dt} = \sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \left[4\cdot\left[\cos(\omega t)\right]^2 + 10\cdot\cos(\omega t) + 6.25\right]dt} =$$

$$= \sqrt{\frac{\omega}{2\pi}\cdot\left[4\cdot\int_0^{\frac{2\pi}{\omega}}\left[\cos(\omega t)\right]^2 dt + 10\cdot\int_0^{\frac{2\pi}{\omega}}\sin(\omega t)dt + 6.25\cdot\int_0^{\frac{2\pi}{\omega}} dt\right]} =$$

$$= \sqrt{\frac{\omega}{2\pi}\cdot\left[4\cdot\frac{1}{2}\cdot\frac{2\pi}{\omega} + 10\cdot 0 + 6.25\cdot\frac{2\pi}{\omega}\right]} = \sqrt{8.25} = 2.87$$

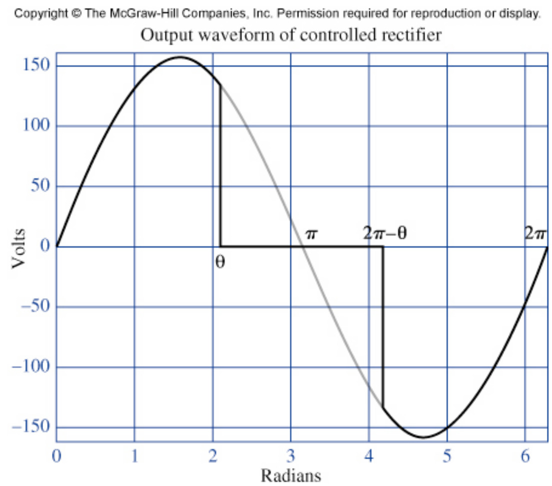
Note: The integral of a sinusoid over an integer number of period is identically zero. This is a useful and important result.

Problem 4.30

Solution:

Known quantities:

The sinusoidal voltage $v(t)$ of 110 V rms shown in Figure P4.30.



Find:

The average and rms voltage.

Analysis:

The rms value of a sinusoidal is equal to 0.707 times the peak value:

$V_{peak} = 110\sqrt{2}$

The average value is:

$$\langle v(t)\rangle = \frac{1}{2\pi}\left[\int_0^{\theta} 110\sqrt{2}\sin(t)dt + \int_{2\pi-\theta}^{2\pi} 110\sqrt{2}\sin(t)dt\right] =$$

$$= \frac{1}{2\pi}\left[-110\sqrt{2}\cos(t)\Big|_0^{\theta} - 110\sqrt{2}\cos(t)\Big|_{2\pi-\theta}^{2\pi}\right] =$$

$$= -\frac{50\sqrt{2}}{\pi}\left[\cos(\theta) - 1 + \cos(2\pi) - \cos(2\pi - \theta)\right] = 0$$

The rms value is:

$$v_{rms} = \left\{\frac{1}{2\pi}\left(\int_0^{\theta}\left(110\sqrt{2}\sin(t)\right)^2 dt + \int_{2\pi-\theta}^{2\pi}\left(110\sqrt{2}\sin(t)\right)^2 dt\right)\right\}^{1/2} =$$

$$= \left\{\frac{12100}{\pi}\left(\frac{1}{2}\left(-\cos(t)\sin(t) + t\right)\Big|_0^{\theta} + \frac{1}{2}\left(-\cos(t)\sin(t) + t\right)\Big|_{2\pi-\theta}^{2\pi}\right)\right\}^{1/2} =$$

$$= \left\{\frac{6050}{\pi}\left(-\cos(\theta)\sin(\theta) + \theta + 2\pi - \left(-\cos(2\pi-\theta)\sin(2\pi-\theta) + 2\pi - \theta\right)\right)\right\}^{1/2} =$$

$$= \left\{\frac{6050}{\pi}(2\theta)\right\}^{1/2} = 110\sqrt{\frac{\theta}{\pi}}$$

Problem 4.31

Solution:

Known quantities:

The sinusoidal voltage $v(t)$ of 110 V rms shown in Figure P4.30.

Find:

The angle θ that correspond to delivering exactly one-half of the total available power in the waveform to a resistive load.

Analysis:

From $v_{rms} = 110\sqrt{\frac{\theta}{\pi}}$, we obtain:

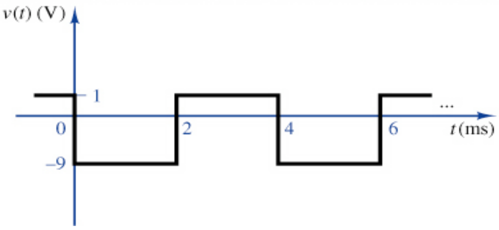
$$v_{rms}^2 = 110^2\frac{\theta}{\pi} = \frac{110^2}{2} \quad\Rightarrow\quad \theta = \frac{\pi}{2}$$

Problem 4.32

Solution:

Known quantities:

The signal $v(t)$ shown in Figure P4.32.



Find:

The ratio between average and rms value of the signal.

Analysis:

The average value is:

$$\langle v \rangle = \frac{1}{0.004}\left[\int_0^{0.002}(-9)dt + \int_{0.002}^{0.004}(1)dt\right] = 250(-0.018 + 0.002) = -4 \text{ V}$$

The rms value is: $v_{rms} = \sqrt{\frac{1}{0.004}\left[\int_0^{0.002}(-9)^2 dt + \int_{0.002}^{0.004}(1)^2 dt\right]} = \sqrt{250\cdot[81\cdot 0.002 + 0.004 - 0.002]} = 6.40$ **V**

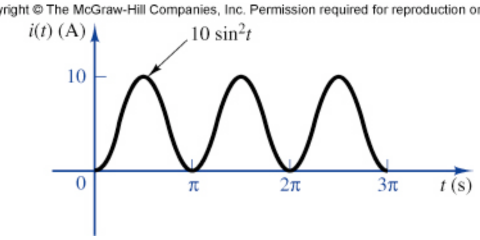
Therefore, $\frac{\langle v \rangle}{v_{rms}} = -\frac{4}{6.40} = -0.625$

Problem 4.33

Solution:

Known quantities:

The signal $i(t)$ shown in Figure P4.33.



Find:

The power dissipated by a 1-Ω resistor.

Analysis:

The rms value is: $i_{rms} = \sqrt{\frac{1}{p}\int_0^p \left(10\cdot\sin^2(t)\right)^2 dt} = \sqrt{\frac{1}{p}\int_0^p 100\cdot\sin^4(t)dt} = \sqrt{\frac{1}{p}\cdot 100\cdot\frac{3p}{8}} = 6.12$ **A**

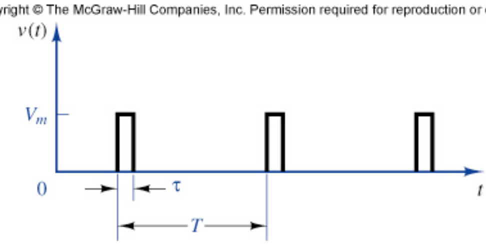
Therefore, the power dissipated by a 1-Ω resistor is: $P_{1\Omega} = Ri_{rms}^2 = (1)(6.12)^2 \text{ W} = 37.5 \text{ W}$

Problem 4.34

Solution:

Known quantities:

The signal $x(t)$ shown in Figure P4.34.



Find:

The average and rms value of the signal.

Analysis:

The average value is: $\langle V \rangle = \frac{1}{T}\int_{t_0}^{t_0+\tau} V_m dt = \frac{t}{T}V_m$ where t_0 is the left-hand side of the pulse.

The rms value is: $V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+\tau} V_m^2 dt} = \sqrt{\frac{t}{T}}V_m$

Therefore,

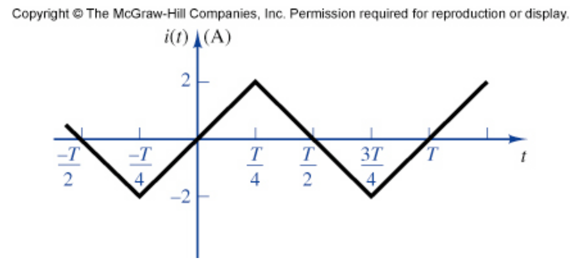
$$\frac{\langle V \rangle}{V_{rms}} = \sqrt{\frac{t}{T}}$$

Problem 4.35

Solution:

Known quantities:

The signal $i(t)$ shown in Figure P4.35.



Find:

The rms value of the signal.

Analysis:

The rms value is:

$$i_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{\frac{T}{4}}\left(\frac{8}{T}t\right)^2 dt + \int_{\frac{T}{4}}^{\frac{3T}{4}}\left(-\frac{8}{T}t+4\right)^2 dt + \int_{\frac{3T}{4}}^{T}\left(\frac{8}{T}t-8\right)^2 dt\right]} =$$

$$= \sqrt{\frac{1}{T}\left[\int_0^{\frac{T}{4}}\left(\frac{64}{T^2}t^2\right)dt + \int_{\frac{T}{4}}^{\frac{3T}{4}}\left(\frac{64}{T^2}t^2 - \frac{64}{T}t+16\right)^2 dt + \int_{\frac{3T}{4}}^{T}\left(\frac{64}{T^2}t^2 - \frac{128}{T}t+64\right)^2 dt\right]} =$$

$$= \sqrt{\frac{1}{T}\left[\frac{1}{3}T + 9T - 18T + 12T - \frac{1}{3}T + 2T - 4T + \frac{64}{3}T - 64T + 64T - 9T + 36T - 48T\right]} = \sqrt{\frac{1}{T}\left[\frac{4}{3}T\right]} = \frac{2}{\sqrt{3}} = 1.15 \text{ } \mathbf{A}$$

Problem 4.36

Solution:

Known quantities:

The signal $v(t)$.

Find:

The rms value of the signal.

Analysis:

The rms value is: $v_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(v(t))^2 d(\omega t)}$

$$v^2{}_{rms} = \frac{1}{2\pi}\int_0^{2\pi}\left(V_{DC}+V_0\cos(\omega t)\right)^2 d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi}\left(V^2{}_{DC}+2V_{DC}V_0\cos(\omega t)+V_0^2\cos^2(\omega t)\right)d(\omega t) =$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left(V_{DC}^2+2V_{DC}V_0\cos(\omega t)+\frac{V_0^2}{2}+\frac{V_0^2}{2}\cos^2(\omega t)\right)d(\omega t) =$$

$$= \frac{1}{2\pi}\left(V_{DC}^2[\omega t]_0^{2\pi} + 0 + \frac{V_0^2}{2}[\omega t]_0^{2\pi} + 0\right) = \frac{1}{2\pi}\left(V_{DC}^2(2\pi-0)+0+\frac{V_0^2}{2}(2\pi-0)+0\right)$$

$$v_{rms} = \sqrt{V_{DC}^2 + \frac{V_0^2}{2}} = \sqrt{(50 \text{ } \mathbf{V})^2 + \frac{1}{2}(70.7 \text{ } \mathbf{V})^2} = 70.7 \text{ } \mathbf{V}$$

Notes:

1. T = period in units of time and ωt = period in angular units, i.e., 2π radians. Considering ωt as a single variable is useful when dealing with sinusoids.
2. The integral of a sinusoid over one or more whole periods is equal to 0.

Problem 4.37

Solution:

Known quantities:
Functions.

Find:
The phasor form.

Analysis:
In phasor form:

a) $V(jw) = 155\angle\text{-}25° \text{ V}$

b) $V(jw) = 5\angle\text{-}130° \text{ V}$

c) $I(jw) = 10\angle 63° + 15\angle\text{-}42° = (4.54 + j8.91) + (11.15 - j\,10.04) = 15.69 - j1.13 = 15.73\angle\text{-}4.12° \text{ A}$

d) $I(jw) = 460\angle\text{-}25° - 220\angle 75° = (416.90 - j194.40) - (56.94\text{-}j212.50) = 359.96 + j18.10 = 360.4\angle 2.88° \text{ A}$

Problem 4.38

Solution:

Known quantities:
Complex number.

Find:
The polar form.

Analysis:

a) $4 + j4 = 4\sqrt{2}\angle 45° = 5.66\angle 45°$

b) $-3 + j\,4 = 5\,\angle 126.9°$

c) $j + 2 - j4 - 3 = -1 - j\,3 = 3.16\angle\text{-}108.4°$

Problem 4.39

Solution:

Known quantities:
Complex number.

Find:
The polar form.

Analysis:

a) $(50 + j10)(4 + j8) = (50.99\angle 11.30°)(8.94\angle 63.43°) = 456.1\angle 74.7°$

$(50 + j10)(4 + j8) = 200 + j400 + j40 + j^2\,80 = 120 + j440 = 456.1\angle 74.7°$

b) $(j2 - 2)(4 + j5)(2 + j7) = (2.82\angle 135°)(6.40\angle 51.34°)(7.28\angle 74.05°) = 131.8\angle 260.4° = 131.8\angle\text{-}99.6°$

$(j2 - 2)(4 + j5)(2 + j7) = -36 - j126 - j4 - j^2\,14 = -22 - j130 = 131.8\angle\text{-}99.6°$

Problem 4.40

Solution:

Known quantities:
Complex number.

Find:
a) Complex conjugate
b) Polar form, by first multiplying numerator and denominator by the complex conjugate.
c) Polar form, by converting into polar coordinates.

Analysis:
a)
$$A = 4 + j4, \; A^* = 4 - j4$$
$$B = 2 - j8, \; B^* = 2 + j8$$
$$C = -5 + j2, \; C^* = -5 - j2$$

b)
$$\frac{1+j7}{4+j4} = \frac{(1+j7)(4-j4)}{(4+j4)(4-j4)} = \frac{4 - j4 + j28 - j^2\,28}{16+16} = \frac{32 + j24}{32} = 1 + j\,0.75 = 1.25\angle 36.87°$$
$$\frac{j4}{2-j8} = \frac{j4(2+j8)}{(2-j8)(2+j8)} = \frac{-32 + j8}{4+64} = -\frac{32}{68} + j\frac{8}{68} = 0.485\angle 165.96°$$
$$\frac{1}{-5+j2} = \frac{1(-5-j2)}{(-5+j2)(-5-j2)} = \frac{-5-j2}{25+4} = -\frac{5}{29} - j\frac{2}{29} = 0.1857\angle\text{-}158.2°$$

c) Repeat b) converting to polar form first:
$$\frac{1+j7}{4+j4} = \frac{7.071\angle 81.87°}{4\sqrt{2}\angle 45°} = 1.25\angle 36.87°$$
$$\frac{j4}{2-j8} = \frac{4\angle 90°}{8.246\angle 75.96°} = 0.485\angle 165.96°$$
$$\frac{1}{-5+j2} = \frac{1\angle 0°}{5.385\angle 158.2°} = 0.1857\angle\text{-}158.2°$$

Problem 4.41

Solution:

Known quantities:
Complex number.

Find:
Real-imaginary form

Analysis:
$$j^j = e^{-\pi/2} = 0.2079$$
$$e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 + j0 = -1$$

Problem 4.42

Solution:

Known quantities:

Sinusoidal functions.

Find:

Sum of them, by using trigonometric identity and phasors

Analysis:

(a) Using the trigonometric identity $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$, we expand the voltages:

$$10\cos(\omega t+30°)=10\cos 30°\cos\omega t-10\sin 30°\sin\omega t=10\left(\frac{\sqrt{3}}{2}\right)\cos\omega t-10\left(\frac{1}{2}\right)\sin\omega t=8.66\cos\omega t-5\sin\omega t$$

$$20\cos(\omega t+60°)=20\cos 60°\cos\omega t-20\sin 60°\sin\omega t=20\left(\frac{1}{2}\right)\cos\omega t-20\left(\frac{\sqrt{3}}{2}\right)\sin\omega t=10\cos\omega t-17.32\sin\omega t$$

Hence,

$v(t)=(8.66+10)\cos\omega t-(5+17.32)\sin\omega t=18.66\cos\omega t-22.32\sin\omega t$

Now, $v(t)$ is of the form

$v(t)=A\cos\phi\cos\omega t-A\sin\phi\sin\omega t=A\cos(\omega t+\phi)$

and since $\dfrac{A\sin\varphi}{A\cos\varphi}=\tan\varphi$, we have $\varphi=\tan^{-1}\left(\dfrac{22.32}{18.66}\right)=50.1°$ and $A=\dfrac{18.66}{\cos\varphi}=29.09 \quad \therefore v(t)=29.09\cos(\omega t+50.1°)$

(b) Using phasors,

$V_1(\omega)=10\angle 30°=10\dfrac{\sqrt{3}}{2}+j10\dfrac{1}{2}=8.66+j5 \qquad V_2(\omega)=20\angle 60°=20\dfrac{1}{2}+j20\dfrac{\sqrt{3}}{2}=10+j17.32$

$V(\omega)=(8.66+10)+j(5+17.32)=18.66+j22.32=29.09\angle 50.1°$

$\therefore v(t)=29.09\cos(\omega t+50.1°)$

Section 4.4: Phasor Solution of Circuits with Sinusoidal Excitation

Focus on Methodology: Phasors

1. Any sinusoidal signal may be mathematically represented in one of two ways: a **time domain form**: $v(t) = A\cos(\omega t + \theta)$, and a frequency domain form: $\mathbf{V}(j\omega) = Ae^{j\theta} = A\angle\theta$. Note the $j\omega$ in the notation $\mathbf{V}(j\omega)$, indicating the $e^{j\omega t}$ dependence of the phasor. In the remainder of this chapter, bold uppercase quantities indicate phasor voltages and currents.
2. A phasor is a complex number, expressed in polar form, consisting of a *magnitude* equal to the peak amplitude of the sinusoidal signal and a *phase angle* equalto the phase shift of the sinusoidal signal *referenced to a cosine signal*.
3. When using phasor notation, it is important to note the specific frequency ω of the sinusoidal input.

Problem 4.43

Solution:

Known quantities:

The current through and the voltage across a component.

Find:

a) Whether the component is a resistor, capacitor, inductor
b) The value of the component in ohms, farads, or henrys.

Analysis:

a) The current and the voltage can be expressed in phasor form:

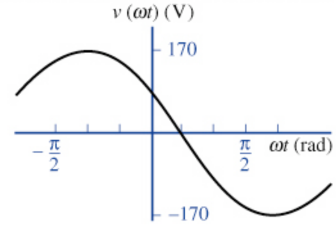
$$\mathbf{I} = 17\angle -15^o \ \mathbf{mA}, \ \mathbf{V} = 3.5\angle 75^o \ \mathbf{V}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{3.5\angle 75^o \ \mathbf{V}}{17\angle -15^o \ \mathbf{mA}} = 205.9\angle 90^o \ \Omega = 0 + j \cdot 205.9 \ \Omega$$

The impedance has a positive imaginary or reactive component and a positive angle of 90 degree indicating that this is an inductor (see Fig. 4.39).

b) $Z_L = j \cdot X_L = j \cdot \omega L = j \cdot 205.9 \ \Omega \ \Rightarrow \ L = \dfrac{205.9 \ \Omega}{628.3 \ \dfrac{\mathbf{rad}}{\mathbf{s}}} = 327.7 \ \mathbf{m}\dfrac{\mathbf{Vs}}{\mathbf{A}} = 327.7 \ \mathbf{mH}$

Problem 4.44



Solution:

Known quantities:
The waveform of a signal shown in Figure P4.44.

Find:
The sinusoidal description of the signal.

Analysis:
From the graph of Figure P4.44:

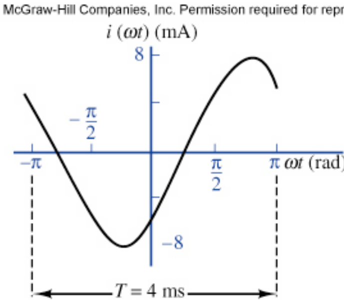
$$\phi = +\frac{\pi}{3}\frac{180^o}{\pi} = 60^o,\ V_0 = 170\ V,\ \omega = 2\pi f = \frac{2\pi}{T}$$

$$v_r(t) = V_0\cos(\omega t + \phi) = 170\cos\left(\omega t + 60^o\right)\mathbf{V}$$

Phasor form:

$$\mathbf{V} = V_0\angle\phi = 170\angle 60^o\ \mathbf{V} = 170\ \mathbf{V}\cdot e^{j\,60^o}$$

Problem 4.45



Solution:

Known quantities:
The waveform of a signal shown in Figure P4.45.

Find:
The sinusoidal description of the signal.

Analysis:
From graph:

$$\phi = -\frac{3\pi}{4}\frac{180^o}{\pi} = -135^o,\ I_0 = 8\ \mathbf{mA},\ \omega = 2\pi f = \frac{2\pi}{T} = 1571\ \frac{\mathbf{rad}}{\mathbf{s}}$$

$$i(t) = I_0\cos(\omega t + \phi) = 8\cos\left(1571\ \frac{\mathbf{rad}}{\mathbf{s}}\cdot t - 135^o\right)\ \mathbf{mA}$$

Phasor form:

$$\mathbf{V} = V_0\angle\phi = 8\angle(-135)^o\ \mathbf{V} = 8\,\mathbf{V}\cdot e^{-j\,135^o}$$

Problem 4.46

Solution:

Known quantities:

The current through $i(t) = I_0 \cos\left(\omega t + 45^o\right)$ $I_0 = 3$ **mA**, $\omega = 6.283\ \frac{\textbf{rad}}{\textbf{s}}$, and the voltage across $v(t) = V_0 \cos(\omega t)$, $V_0 = 700$ **mV**, $\omega = 6.283\ \frac{\textbf{rad}}{\textbf{s}}$ an electrical component.

Find:

a) Whether the component is inductive or capacitive.
b) The waveform of the instantaneous power $p(t)$ as a function of ωt over the range $0 < \omega t < 2\pi$.
c) The average power dissipated as heat in the component.
d) The same as b. and c. with the phase of the current equal to zero.

Analysis:

a) Phasor notation:

$\mathbf{I} = 3\angle 45^o$ **mA**, $\mathbf{V} = 700\angle 0^o$ **mV**

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{700\angle 0^o\ \textbf{mV}}{3\angle 90^o\ \textbf{mA}} = 233.3\angle -45^o\ \Omega = 165.0 - j165.0\Omega$$

The component is inductive because it is lagging.

b)

$$p(t) = v(t)i(t) = V_0 I_0 \cos\left(\omega t + 45^o\right)\cos(\omega t) = \frac{1}{2}V_0 I_0\left(\cos\left(2\omega t + 45^o\right) + \cos\left(45^o\right)\right) =$$

$$= \frac{1}{2}(700\ \textbf{mV})(3\ \textbf{mA})\left(\cos\left(2\omega t + 45^o\right) + 0.707\right) = \left(1050\cos\left(2\omega t + 45^o\right) + 742.4\right)\ \mu\textbf{W}$$

c)

$$P = \frac{1}{\omega T}\int_0^{\omega T} p(t)d\omega t = \frac{1}{\omega T}\int_0^{\omega T}\left(1050\,\mu\textbf{W}\cos\left(2\omega t + 45^o\right) + 742.4\right)d\omega t =$$

$$= \frac{1}{\omega T}(1050\ \mu\textbf{W})\frac{1}{2}\left[\sin\left(2\omega t + 45^o\right)\right]_0^{2\pi} + \frac{1}{\omega T}742.4(\omega t)\Big|_0^{2\pi} =$$

$$= \frac{1}{\omega T}(1050\ \mu\textbf{W})\frac{1}{2}\left[\sin\left(765^o\right) - \sin(45°)\right] + 742.4 =$$

$$= \frac{1}{2\pi}(1050\ \mu\textbf{W})\frac{1}{2}[0 - 0] + 742.4 = 742.4\ \mu\textbf{W}$$

d)

$$p(t) = v(t)i(t) = \frac{1}{2}V_0 I_0 \cos(\omega t)\cos(\omega t) = \frac{1}{2}V_0 I_0\left(\cos(2\omega t) + \cos\left(0^o\right)\right) =$$

$$= \frac{1}{2}(700\ \textbf{mV})(3\ \textbf{mA})\left(\cos(2\omega t) + 1\right) = 1050\left(\cos(2\omega t) + 1\right)\ \mu\textbf{W}$$

$$P = \frac{1}{\omega T}\int_0^{\omega t} p(t)d\omega t = \frac{1}{\omega T}\int_0^{\omega t}\left(1050\ \mu\textbf{W}\left(\cos(2\omega t) + 1\right)\right)d\omega t =$$

$$= \frac{1}{\omega T}(1050\ \mu\textbf{W})\left(\frac{1}{2}\left[\sin(2\omega t)\right]_0^{2\pi} + [\omega t]_0^{2\pi}\right) =$$

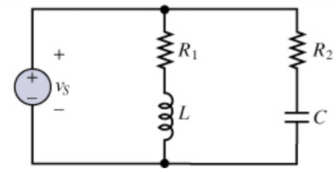
$$= \frac{1}{2\pi}(1050\ \mu\textbf{W})\left(\frac{1}{2}(0 - 0) + (2\pi - 0)\right) = 1050\ \mu\textbf{W}$$

Problem 4.47

Solution:

Known quantities:

The values of the impedance, $R_1 = 2.3\ \mathbf{k\Omega}$, $R_2 = 1.1\ \mathbf{k\Omega}$, $L = 190\ \mathbf{mH}$, $C = 55\ \mathbf{nF}$ and the voltage applied to the circuit shown in Figure P4.47, $v_s(t) = 7\cos\left(3000t + 30^o\right)\mathbf{V}$.



Find:

The equivalent impedance of the circuit.

Analysis:

$$X_L = \omega L = \left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(190\ \mathbf{mH}) = 0.57\ \mathbf{k\Omega} \Rightarrow Z_L = +j\cdot X_L = +j\cdot 0.57\ \mathbf{k\Omega}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(55\ \mathbf{nF})} = 6.061\ \mathbf{k\Omega} \Rightarrow Z_C = -j\cdot X_C = -j\cdot 6.061\ \mathbf{k\Omega}$$

$$Z_{eq1} = Z_{R1} + Z_L = R_1 + jX_L = 2.3 + j\cdot 0.57\ \mathbf{k\Omega} = 2.37\angle 13.92^o\ \mathbf{k\Omega}$$

$$Z_{eq2} = Z_{R1} + Z_C = R_1 - jX_C = 1.1 - j\cdot 6.061\ \mathbf{k\Omega} = 6.16\angle -79.71^o\ \mathbf{k\Omega}$$

$$Z_{eq} = \frac{Z_{eq1}\cdot Z_{eq2}}{Z_{eq1} + Z_{eq2}} = \frac{\left(2.37\angle 13.92^o\ \mathbf{k\Omega}\right)\cdot\left(6.16\angle -79.71^o\ \mathbf{k\Omega}\right)}{\left(2.3 + j\cdot 0.57\ \mathbf{k\Omega}\right) + \left(1.1 - j\cdot 6.061\ \mathbf{k\Omega}\right)} =$$

$$= \frac{14.60\angle -65.79^o\ \mathbf{k\Omega^2}}{3.4 - j\cdot 5.491\ \mathbf{k\Omega}} = \frac{14.60\angle -65.79^o\ \mathbf{k\Omega^2}}{6.458\angle -58.23^o\ \mathbf{k\Omega}} = 2.261\angle -7.56^o\ \mathbf{k\Omega}$$

Problem 4.48

Solution:

Known quantities:

The values of the impedance, $R_1 = 3.3\ \mathbf{k\Omega}$, $R_2 = 22\ \mathbf{k\Omega}$, $L = 1.90\ \mathbf{H}$, $C = 6.8\ \mathbf{nF}$ and the voltage applied to the circuit shown in Figure P4.47, $v_s(t) = 636\cos\left(3000t + 15^o\right)\mathbf{V}$.

Find:

The equivalent impedance of the circuit.

Analysis:

$$X_L = \omega L = \left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(1.90\ \mathbf{H}) = 5.7\ \mathbf{k\Omega} \Rightarrow Z_L = +j\cdot X_L = +j\cdot 5.7\ \mathbf{k\Omega}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(6.8\ \mathbf{nF})} = 49.02\ \mathbf{k\Omega} \Rightarrow Z_C = -j\cdot X_C = -j\cdot 49.02\ \mathbf{k\Omega}$$

$$Z_{eq1} = Z_{R1} + Z_L = R_1 + jX_L = 3.3 + j\cdot 5.7\ \mathbf{k\Omega} = 6.59\angle 59.93^o\ \mathbf{k\Omega}$$

$$Z_{eq2} = Z_{R1} + Z_C = R_1 - jX_C = 22 - j\cdot 49.02\ \mathbf{k\Omega} = 53.73\angle -65.83^o\ \mathbf{k\Omega}$$

$$Z_{eq} = \frac{Z_{eq1} \cdot Z_{eq2}}{Z_{eq1} + Z_{eq2}} = \frac{\left(6.59\angle 59.93^o \ \mathbf{k\Omega}\right) \cdot \left(53.73\angle -65.83^o \ \mathbf{k\Omega}\right)}{\left(3.3 + j \cdot 5.7 \ \mathbf{k\Omega}\right) + \left(22 - j \cdot 49.02 \ \mathbf{k\Omega}\right)} =$$

$$= \frac{354.08\angle -5.9^o \ \mathbf{k\Omega^2}}{25.3 - j \cdot 43.32 \ \mathbf{k\Omega}} = \frac{354.08\angle -5.9^o \ \mathbf{k\Omega^2}}{50.17\angle -59.71^o \ \mathbf{k\Omega}} = 7.05\angle 53.81^o \ \mathbf{k\Omega}$$

Problem 4.49

Solution:

Known quantities:

The current in the circuit,

$i_s(t) = I_0 \cos\left(\omega t + 30^o\right)$ $I_0 = 13$ **mA**, $\omega = 1000 \ \frac{\mathbf{rad}}{\mathbf{s}}$, and the value of the capacitance present in the circuit shown in Figure P4.49 $C = 0.5 \ \mu\mathbf{F}$.

Find:

a) The phasor notation for the source current.
b) The impedance of the capacitor.
c) The voltage across the capacitor, showing all the passages and using phasor notation only.

Analysis:

a) Phasor notation:

$$\mathbf{I}_s = I_0 \ \angle\phi = 13\angle 30^o \ \mathbf{mA}$$

b) $$Z_C = -jX_C - j\frac{1}{\omega C} = -j\frac{1}{\left(1000 \ \frac{\mathbf{rad}}{\mathbf{s}}\right)\left(0.5 \ \Omega\mathbf{F}\right)} = 0 - j2 \ \mathbf{k\Omega} = 2\angle -90^o \ \mathbf{k\Omega}$$

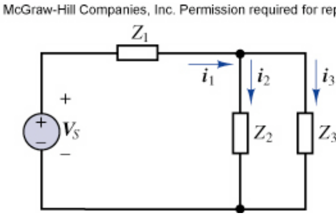
c)

$$\mathbf{V}_C = \mathbf{I}_s \cdot Z_C = \left(13\angle 30^o \ \mathbf{mA}\right) \cdot \left(2\angle -90^o \ \mathbf{k\Omega}\right) = 26\angle -60^o \ \mathbf{V}$$

$$v_C(t) = 26\cos\left(1000t - 60^o\right)\mathbf{V}$$

Note that conversion from phasor notation to time notation or vice versa can be done at any time.

Problem 4.50



Solution:

Known quantities:

The values of two currents in the circuit shown in Figure P4.50: $i_1(t) = 141.4\cos\left(\omega t + 135^o\right)\mathbf{mA}$,

$i_2(t) = 50\cos\left(\omega t + 53.13^o\right)\mathbf{mA}$, $\omega = 377 \ \frac{\mathbf{rad}}{\mathbf{s}}$.

Find:

The current $i_3(t)$.

Analysis:

A solution using trigonometric identities is possible but inefficient, cumbersome, and takes a lot of time. Phasors are better! Note that one current is described with a sine and the other with a cosine function. When using phasors, all currents and voltages must be described with either sine functions or cosine functions. Which does not matter, but it is a good idea to adopt one and use it consistently. Therefore, first converts to cosines.

KCL: $$-i_1(t) + i_2(t) + i_3(t) = 0 \ \Rightarrow \ +i_3(t) = i_1(t) - i_2(t)$$

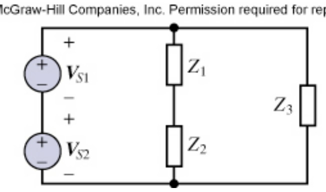
$$i_3(t) = 141.4\cos\left(\omega t + 135^o\right)\mathbf{mA} - 50\sin\left(\omega t - 53.13^o\right)\mathbf{mA} =$$
$$= 141.4\cos\left(\omega t + 135^o\right)\mathbf{mA} - 50\cos\left(\omega t - 53.13^o - 90^o\right)\mathbf{mA}$$

$$\mathbf{I}_3 = 141.4\ \mathbf{mA}\ \angle 135^o - 50\ \mathbf{mA}\ \angle -143.13^o =$$
$$= (-99.98 + j\cdot 99.98)\ \mathbf{mA} - (-40.00 - j\cdot 30.00)\ \mathbf{mA} =$$
$$= (-59.98 + j\cdot 129.98)\ \mathbf{mA} = 143.2\ \mathbf{mA}\ \angle 114.8^o$$

$$i_3(t) = 143.2\cos\left(\omega t + 114.8^o\right)\mathbf{mA}$$

If sine functions were used, the result in phasor notation would differ in phase by 90 degrees.

Problem 4.51



Solution:

Known quantities:

The values of the impedance, $Z_1 = 5.9\angle 7^o\ \mathbf{k\Omega}$, $Z_2 = 2.3\angle 0^o\ \Omega$, $Z_3 = 17\angle 11^o\ \Omega$ and the voltages applied to the circuit shown in Figure P4.51, $v_{s1}(t) = v_{s2}(t) = 170\cos(377t)\ \mathbf{V}$.

Find:

The current through Z_3.

Analysis:

$$\mathbf{V}_{s1} = \mathbf{V}_{s2} = 170\angle 0^o\ \mathbf{V} = (170 + j0)\ \mathbf{V}$$

KVL: $-\mathbf{V}_{s1} - \mathbf{V}_{s2} + \mathbf{I}_3 Z_3 = 0$

$$\mathbf{I}_3 = \frac{\mathbf{V}_{s1} + \mathbf{V}_{s2}}{Z_3} = \frac{170\angle 0^o\ \mathbf{V} + 170\angle 0^o\ \mathbf{V}}{17\angle 11^o\ \Omega} = \frac{340\angle 0^o\ \mathbf{V}}{17\angle 11^o\ \Omega} = 20\angle -11^o\ \mathbf{A}$$

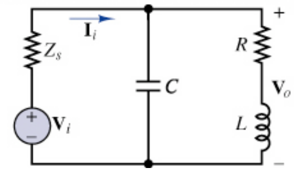
$$i_3(t) = 20\cos\left(377\ \frac{\mathbf{rad}}{\mathbf{s}}\cdot t - 11^o\right)\ \mathbf{A}$$

Note also:

KVL: $-\mathbf{V}_{s1} + \mathbf{I}_1 Z_1 = 0 \Rightarrow \mathbf{I}_1 = \dfrac{\mathbf{V}_{s1}}{Z_1}$, $-\mathbf{V}_{s2} - \mathbf{I}_2 Z_2 = 0 \Rightarrow \mathbf{I}_2 = -\dfrac{\mathbf{V}_{s2}}{Z_2}$

Problem 4.52

Solution:



Known quantities:

The values of the impedance in the circuit shown in Figure P4.52, $Z_S = (13000 + j\omega 3)\ \Omega$, $R = 120\ \Omega$, $L = 19\ \mathbf{mH}$, $C = 220\ \mathbf{pF}$.

Find:

The frequency such that the current $\mathbf{I}_i$ and the voltage $\mathbf{V}_0$ are in phase.

Analysis:

Z_S is not a factor in this solution. Only R, L, and C will determine if the voltage across this combination is in phase with the current through it. If the voltage and current are in phase, then, the equivalent impedance must have an "imaginary" or reactive part which is zero!

$$Z_{eq} = \frac{\mathbf{V}_0}{\mathbf{I}_i} = \left|Z_{eq}\right|\angle 0^o = R_{eq} + jX_{eq},\ X_{eq}(\omega) = 0$$

$$Z_{eq} = \frac{(Z_R + Z_L)\cdot Z_C}{Z_R + Z_L + Z_C} = \frac{(R + jX_L)\cdot(-jX_C)}{R + jX_L - jX_C} = \frac{X_L X_C - jRX_C}{R + j(X_L - X_C)}\frac{R - j(X_L - X_C)}{R - j(X_L - X_C)} =$$

$$= \frac{\left[X_L X_C R - RX_C(X_L - X_C)\right] - j\left[R^2 X_C + X_L X_C(X_L - X_C)\right]}{R^2 + (X_L - X_C)^2}$$

At the resonant frequency the reactive component of this impedance must equal zero:

$$X_{eq}(\omega) = \frac{R^2 X_C + X_L X_C(X_L - X_C)}{R^2 + (X_L - X_C)^2} = 0 \Rightarrow R^2 + X_L(X_L - X_C) = 0$$

$$R^2 + \omega L\left(\omega L - \frac{1}{\omega C}\right) = 0 \Rightarrow \omega^2 L^2 = \frac{L}{C} - R^2$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(19\mathbf{mH})(220\mathbf{pF})} - \frac{(120\Omega)^2}{(19\mathbf{mH})^2}} = \sqrt{239.24\mathbf{G}\frac{\mathbf{rad}}{\mathbf{s}} - 39.89\mathbf{M}\frac{\mathbf{rad}}{\mathbf{s}}} = 489.1\mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}$$

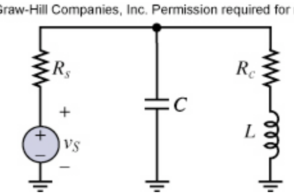
Notes:

1. To separate the equivalent impedance into real (resistive) and "imaginary" (reactive) components, the denominator had to be "rationalized". This was done by multiplying numerator and denominator by the complex conjugate of the denominator, and multiplying term by term. Remember that $j^2 = -1$, etc.
2. The term with *R* had a negligible effect on the resonant frequency in this case. If *R* is sufficiently large, however, it will significantly affect the answer.

Problem 4.53

Solution:

Known quantities:



The values of the impedance, $R_s = 50\,\Omega$, $R_c = 40\,\Omega$, $L = 20\ \mu\mathbf{H}$, $C = 1.25\ \mathbf{nF}$, and the voltage applied to the circuit shown in Figure P4.53,
$v_s(t) = V_0 \cos\left(\omega t + 0^o\right)$ $V_0 = 10\ \mathbf{V}$, $\omega = 6\ \mathbf{M}\dfrac{\mathbf{rad}}{\mathbf{s}}$.

Find:

The current supplied by the source.

Analysis:

Assume clockwise currents:

$$X_L = \omega L = \left(6\ \mathbf{M}\frac{\mathbf{rad}}{\mathbf{s}}\right)\left(20\ \mu\mathbf{H}\right) = 1203\Omega \ \Rightarrow\ Z_L = 0 + j120\ \Omega = 120\angle 90^o\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(6\ \mathbf{M}\frac{\mathbf{rad}}{\mathbf{s}}\right)\left(1.25\ \mathbf{nF}\right)} = 133.3\ \Omega \ \Rightarrow\ Z_C = 0 - j133.3\ \Omega = 133.3\angle -90^o\ \Omega$$

$$Z_{R_c} = 40 - j\,\Omega = 40\angle 0^o\ \Omega,\ Z_{R_s} = 50 - j\ \Omega = 50\angle 0^o\ \Omega$$

Equivalent impedances:

$$Z_{eq1} = Z_{R_c} + Z_L = 40 + j120\ \Omega = 126.5\angle 71.56^o\ \Omega$$

$$Z_{eq} = Z_{R_s} + \frac{Z_C \cdot Z_{eq1}}{Z_C + Z_{eq1}} = 50 + j0\ \Omega + \frac{\left(133.3\angle -90^o\ \Omega\right)\cdot\left(126.5\angle 71.56^o\ \Omega\right)}{133.3\angle -90^o\ \Omega + 126.5\angle 71.56^o\ \Omega} =$$

$$= 50 + j0\ \Omega + \frac{16.87\angle -18.44^o\ \mathbf{k\Omega^2}}{42.161\angle -18.44^o\ \Omega} = 50\angle 0^o\ \Omega + 400\angle 0^o\ \Omega = 450\angle 0^o\ \Omega$$

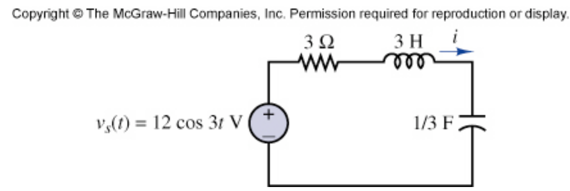
OL: $$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_{eq}} = \frac{10\angle 0^o\ \mathbf{V}}{450\angle 0^o\ \Omega} = 22.22\angle 0^o\ \mathbf{mA} \ \Rightarrow\ i_s(t) = 22.22\cos\left(\omega t + 0^o\right)\mathbf{mA}$$

Note:
The equivalent impedance of the parallel combination is purely resistive; therefore, the frequency given is the resonant frequency of this network.

Problem 4.54

Solution:

Known quantities:
The values of the impedance and the voltage applied to the circuit shown in Figure P4.54.



Find:
The current in the circuit.

Analysis:
Assume clockwise currents:

$$\omega = 3\ \frac{\textbf{rad}}{\textbf{s}},\ V_S = 12\angle 0^o\ \textbf{V}$$

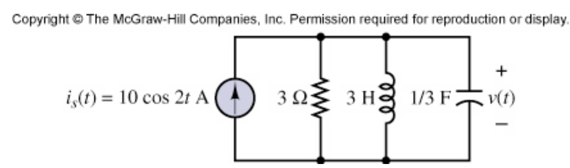
$$Z_C = \frac{1}{j\omega C} = -j\ \Omega,\ Z_L = j\omega L = j9\ \Omega\ \Rightarrow\ Z_{total} = 3 + j9 - j = 3 + j8\quad \Omega$$

$$I = \frac{12}{3 + j8} = 0.4932 - j1.3151\ \textbf{A} = 1.4045\angle -69.44^o\ \textbf{A},\ i(t) = 1.4\cos\left(\omega t - 69.4^o\right)\ \textbf{A}$$

Problem 4.55

Solution:

Known quantities:
The values of the impedance and the current source shown in Figure P4.55.



Find:
The voltage.

Analysis:
Assume clockwise currents:

$$\omega = 2\ \frac{\textbf{rad}}{\textbf{s}},\ \mathbf{I}_S = 10\angle 0^o\ \textbf{A},\ Z_L = j\omega L = j6\ \Omega,\ Z_C = \frac{1}{j\omega C} = -j1.5\Omega$$

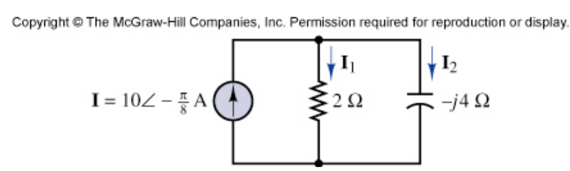
$$Z_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}} = \frac{1}{\frac{1}{3} - j\frac{1}{6} + j\frac{2}{3}} = \frac{1}{0.33 + j0.5} = 0.9231 - j1.3846\quad \Omega$$

$$V = I_S Z_{eq} = 10\ \text{A} \cdot (0.9231 - j1.3846)\ \Omega = 9.231 - j13.846\ a * 10\ \text{V} = 16.641\angle -56.31^o\ \text{V}$$

Problem 4.56

Solution:

Known quantities:
The values of the impedance and the current source for the circuit shown in Figure P4.56.



Find:
The current I_1.

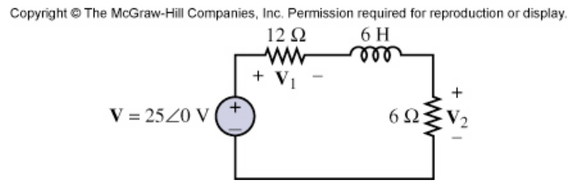
Analysis:
Specifying the positive directions of the currents as in figure P4.45:

$$Z_{eq} = \frac{1}{\frac{1}{2} + \left(\frac{1}{-j4}\right)} = 1.79\angle 26.56^o\quad \Omega$$

$V_S = I_S Z_{eq} = \left(10\angle -22.5^o\right)\mathbf{A} \cdot \left(1.79\angle 26.56^o\right)\Omega = 17.9\angle 4.06^o \quad \mathbf{V}$

$\mathbf{I}_1 = \frac{\mathbf{V}_S}{R} = 8.95\angle 4.06^o \ \mathbf{A}$

Problem 4.57



Solution:

Known quantities:
The values of the impedance and the voltage source for circuit shown in Figure P4.57.

Find:
The voltage $\mathbf{V}_2$.

Analysis:
Specifying the positive directions as in figure P4.57:
$Z_L = j\omega L = j12 \ \Omega$

$$V_2 = \frac{R_{6\Omega}}{R_{12\Omega} + Z_L + R_{6\Omega}} V = \frac{6 \ \Omega}{\left(12 + j12 + 6\right) \ \Omega} 25\angle 0^o \ \mathbf{V} = \frac{150\angle 0^o \ \Omega}{18 + j12 \ \Omega} \mathbf{V} = 6.93\angle -33.7^o \quad \mathbf{V}$$

Problem 4.58

Solution:

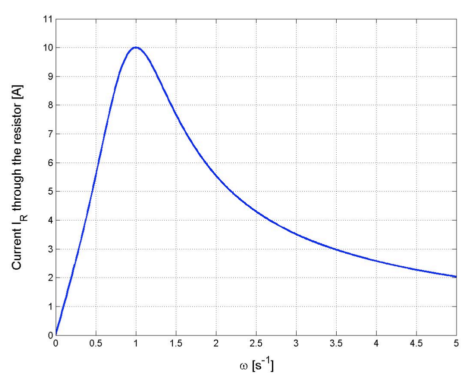
Known quantities:
The values of the impedance and the current source of circuit shown in Figure P4.58.

Find:
The value of ω for which the current through the resistor is maximum.

Analysis:
Assume clockwise currents:

$\mathbf{I}_S = 10\angle 0^o \ \mathbf{A}, \ Z_L = j\omega L = j3\omega \ \Omega, \ Z_C = \frac{1}{j\omega C} = -j\frac{3}{\omega}\Omega$

$$I_R = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}} I_S = \frac{\frac{1}{3}}{\frac{1}{3} - j\frac{1}{3\omega} + j\frac{\omega}{3}} 10\angle 0^o = \frac{10\omega}{\omega + j\left(\omega^2 - 1\right)}$$



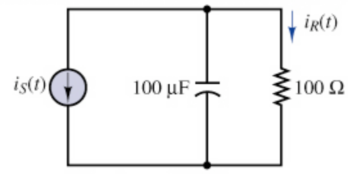
The maximum of I_R is obtained for $\omega = 1.$, therefore
$i_R(t) = 10\cos(t)$.

Problem 4.59

Solution:

Known quantities:

The values of the impedance and the current source for circuit shown in Figure P4.59.



$i_S(t) = 1 \cos(200\pi t)$

Find:

The current through the resistor.

Analysis:

Specifying the positive directions as in figure P4.59:
By current division:

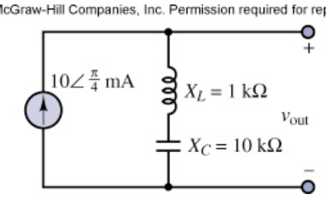
$$\mathbf{I}_R = -\frac{\frac{1}{R}}{\frac{1}{R}+\frac{1}{Z_C}} \cdot \mathbf{I}_s = -\frac{1}{1+\frac{R}{j\omega C}} \cdot \mathbf{I}_s = -\frac{1}{1+j\omega RC} \cdot \mathbf{I}_s = -\frac{1-j\omega RC}{1+(\omega RC)^2} \cdot \mathbf{I}_s =$$

$$= -(0.0247 - j0.1552)\ \mathbf{A} \cdot 1\angle 0^o\ \mathbf{A} = 157 \cdot 10^{-3} \angle 99.04^o\ \mathbf{A}$$

$$i_R(t) = 157 \cos\left(200\pi t + 99.04^o\right) \mathbf{mA}$$

Problem 4.60

Solution:



Known quantities:

The values of the reactance $X_L = 1\ \mathbf{k\Omega}$, $X_C = 10\ \mathbf{k\Omega}$, and the current source $\mathbf{I} = 10\angle 45^o\ \mathbf{mA}$ for circuit shown in Figure P4.60.

Find:

The voltage v_{out}.

Analysis:

Specifying the positive directions of the currents as in Figure P4.60:

$$\mathbf{V}_{out} = Z_{eq}\mathbf{I} = (Z_L + Z_C)\mathbf{I} = (0 + jX_L + 0 - jX_C)\mathbf{I} = (j1\ \mathbf{k\Omega} - j10\ \mathbf{k\Omega}) \cdot 10\angle 45^o\ \mathbf{mA} =$$

$$= (-j9\ \mathbf{k\Omega}) \cdot 10\angle 45^o\ \mathbf{mA} = 9\angle -90^o\ \mathbf{k\Omega} \cdot 10\angle 45^o\ \mathbf{mA} = 90\angle -45^o\ \mathbf{V}$$

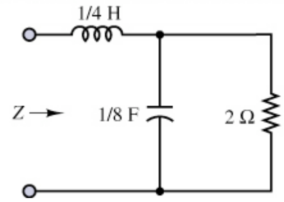
$$v_{out} = 90 \cos(\omega t - 45°)\ \mathbf{V}$$

Problem 4.61

Solution:

Known quantities:

The circuit shown in Figure P4.61, the values of the resistance, $R = 2\ \Omega$, capacitance, $C = 1/8\ \mathbf{F}$, inductance, $L = 1/4\ \mathbf{H}$, and the frequency $\omega = 4\ \dfrac{\mathbf{rad}}{\mathbf{s}}$.



Find:

The impedance Z.

Analysis:

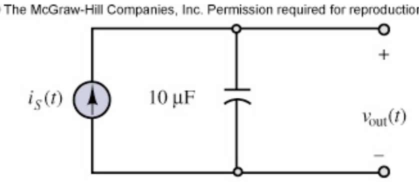
$$Z_L = j\omega L = j4\frac{1}{4}\ \Omega = j\ \Omega,\ Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C} = -j\frac{1}{4\cdot(1/8)} = -j2\ \Omega$$

$$Z = Z_L + Z_C \| R = Z_L + \frac{1}{\dfrac{1}{Z_C}+\dfrac{1}{R}} = j + \frac{1}{\dfrac{1}{-j2}+\dfrac{1}{2}} = j + \frac{j2}{-1+j} = j + \frac{(j2)}{(-1+j)}\frac{(-1-j)}{(-1-j)}$$

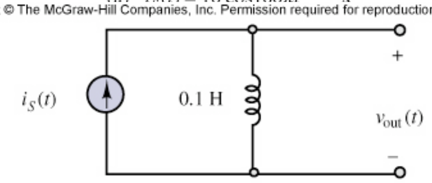
$$= j + \frac{j2(-1-j)}{1+1} = j - j + 1 = 1\ \Omega$$

Problem 4.62

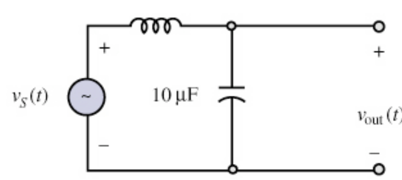
Solution:



(a) $i_S(t) = 10\cos 100\pi t$ A



(b) $i_S(t) = 20\sin 10t$ A



(c) $v_S(t) = 50\sin 100t$ V

Known quantities:

The circuit shown in Figure P4.62, the values of current source and voltage source and components in the circuit.

Find:

The sinusoidal steady-state outputs.

Analysis:

For circuit a):

$$V_{out} = 10\angle 0° \times \left(\frac{1}{\pi 10^{-3}}\angle\text{-}90°\right)$$

$$= \frac{10000}{\pi}\angle\text{-}90° = 3183\angle\text{-}90°$$

$$v_{out}(t) = 3183\cos(100\pi t - 90°)\quad \text{V}$$

For circuit b):

$$\text{Vout} = 20\angle 0° \times 1\angle 90° = 20\angle 90°$$

$$v_{out}(t) = 20\sin(10t + 90°) = 20\cos 10t\ \text{V}$$

For circuit c):

$$V_{out} = \frac{-j1000}{j0.1 - j1000} \times 50\angle 0° \approx 50\angle 0°$$

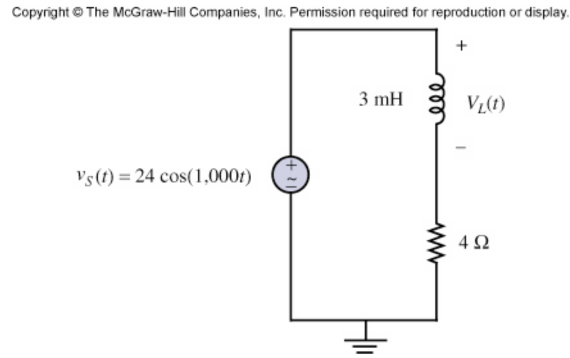
$$v_{out}(t) = 50.0\sin 100t\quad \text{V}$$

Problem 4.63

Solution:

Known quantities:

The circuit shown in Figure P4.63, the values of voltage source and components in the circuit.



Find:

The voltage across the inductor.

Analysis:

We have

$$j\varpi L = j(1000)(0.003) = j3$$

By voltage division:

$$V_L = \frac{j3}{4 + j3}(240)$$

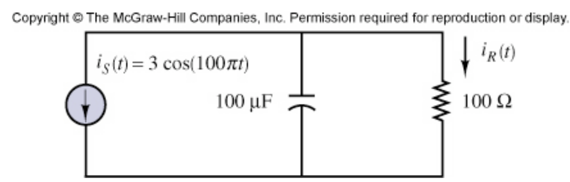
$$= 8.64 + j11.52 = 14.4\angle 53.13°$$

Therefore

$$v_L(t) = 14.4\ \cos(1000t + 53.13°)\ \text{V}$$

Problem 4.64

Solution:



Known quantities:

The circuit shown in Figure P4.64, the values of current source and components in the circuit.

Find:

The current through the capacitor.

Analysis:

We have $C = 100\ \mu\text{F}$, $R = 100\,\Omega$

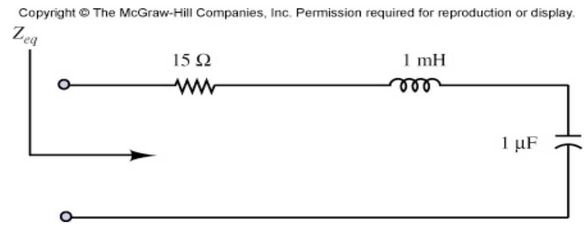
By current division:

$$I_R = -\frac{100}{100 + \dfrac{-j10^6}{100(100\pi)}} \times 1\angle 0°$$

$$= \text{-0.9080 - 0.2890j} = 0.953\angle -162.3°$$

$$i_R(t) = 2.859\cos(100\pi t - 162.3°)\,\text{mA}$$

Problem 4.65



Solution:

Known quantities:

The circuit shown in Figure P4.65, the values of resistance, inductor and capacitor.

Find:

The frequency causes the equivalent impedance to appear to be purely resistive

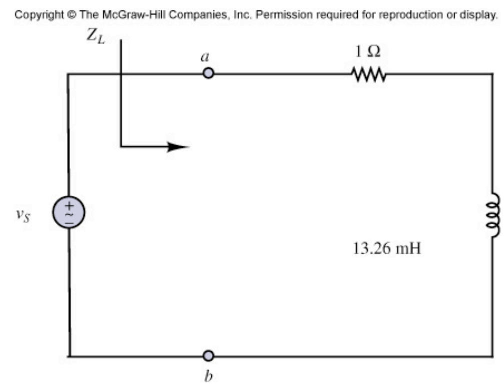
Analysis:

The series impedance of the circuit is $Z_{eq} = R + j\omega + \frac{1}{j\omega} = 15 + j\left(0.001\omega - \frac{1}{10^{-6}\omega}\right)$

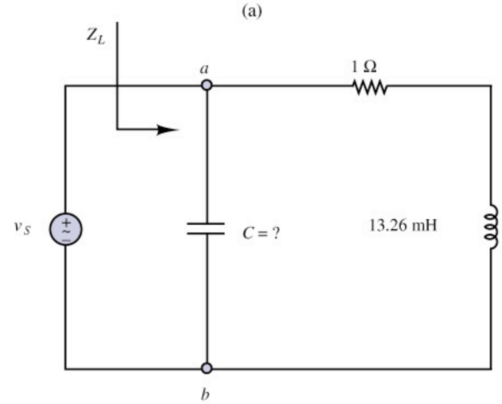
This impedance is purely resistive if its imaginary part (the reactance) is zero.

Therefore, we solve for the frequency, ω, at which $0.001\omega = \frac{1}{10^{-6}\omega}$. So $\omega = 31{,}622.77$ rad/s

Problem 4.66



(a)



(b)

Solution:

Known quantities:

The circuit shown in Figure P4.66, the values of resistance and inductor.

Find:

a) The equivalent impedance seen by the source, if the frequency is 377 rad/s

b) The value of capacitor to make the equivalent impedance purely resistitive

c) The actual impedance with the capacitor included

Analysis:

a) $Z_L = R + jX_L = 1 + j\left(377 \times 13.26 \times 10^{-3}\right) = 1 + j5$

b) $Z_L = Z_L || Z_C$, $Z_C = \frac{1}{j\omega C} = \frac{1}{j377C}$

such that

$$Z_L = \frac{\frac{1}{j377C}(1+j5)}{\frac{1}{j377C} + (1+j5)} = \frac{(1+j5)}{1 + j377C(1+j5)} = \frac{1+j5}{1\text{-}1885C + j377C}$$

The impedance angle is: $\angle Z_L' = \arctan\left(\frac{5}{1}\right) - \arctan\left(\frac{377C}{1-1885C}\right)$

In order to have a strictly real impedance we set $\angle Z_L' = 0$, or:

$\left(\frac{5}{1}\right) = \left(\frac{377\text{C}}{1\text{-}1885\text{C}}\right)$, which leads to: $5 = (377 + 9425)C$ or $C = \frac{5}{9802} = 510.1\ \mu\text{F}$

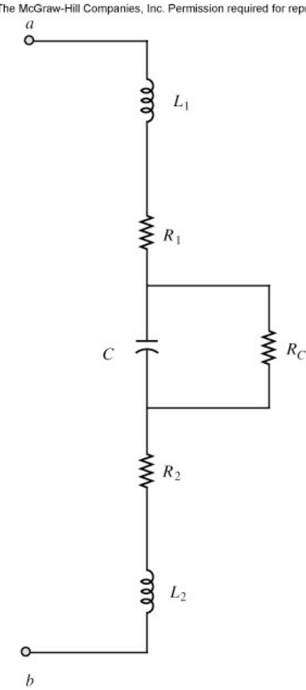
c) $Z_L = \frac{\frac{1}{j0.1923}(1+j5)}{\frac{1}{j0.1923} + (1+j5)} = 26$

Problem 4.67

Solution:

Known quantities:

The circuit shown in Figure P4.67, the values of resistance and capacitor.



Find:

a) Equivalent impedance

b) The range of frequency for which Z_{ab} is capacitive

Analysis:

a) Let Z_1 denote the impedance of the parallel R-C combination:

$$Z_1 = R_C || \frac{1}{jwC} = \frac{R_C \frac{1}{jwC}}{R_C + \frac{1}{jwC}} = \frac{R_C}{jwR_C + 1}$$

Let $R_1 = R_2 = R/2$ and $L_1 = L_2 = L/2$. The total equivalent resistance, Z_{ab}, is given by:

$$Z_{ab} = R_1 + jwL_1 + Z + R_2 + jwL_2$$

$$= R + jwL + \frac{R_C}{jwR_C + 1}$$

$$= \left[R + R_C - w^2 LR_C\right] + j\left[w\left(R\,R_C + L\right)\right]$$

$$= \mathrm{Re}\left(Z_{ab}\right) + j\,\mathrm{Im}\left(Z_{ab}\right)$$

b)

If we set $\mathrm{Im}(Z_{ab}) = 10\mathrm{Re}(Z_{ab})$ and solve for ω, we will find the minimum frequency at which the impedance of the physical capacitor may be considered pure imaginary. Using the result of part a),

$10\left[R + R_C - w^2 LR_C\right] = \left[w\left(R\,R_C + L\right)\right]$ Thus, we need to solve the quadratic equation

$$10LR_C w^2 + \left(RR_C + L\right)w - 10\left(R + R_C\right) = 0$$

or

$$20w^2 + 20w - 1\times 10^8 = 0$$

which has roots $w = -2{,}236.6$ and $w = 2{,}236.6$. Selecting the positive root for physical reason, we conclude that physical capacitor will act very nearly like an ideal capacitor for frequencies above 2.2356×10^3 rad/s (355.6 Hz).