# Problem Set 2

**Q.No.1** S ⇒ Spam , FM ⇒ Free Money , NS ⇒ Not Spam

**Given** $P(S) = 0.8$ , $P(NS) = 0.2$

$P(FM|S) = 0.1$ , $P(FM|NS) = 0.01$

$$P(S|FM) = \frac{P(FM|S) \cdot P(S)}{P(FM|S) \cdot P(S) + P(FM|NS) \cdot P(NS)}$$

$$= \frac{0.1 \times 0.8}{0.1 \times 0.8 + 0.01 \times 0.2}$$

**Q.No.2** F ⇒ Fool , T ⇒ Theif

**Given**

$P(F) = 0.6$ , $P(T) = 0.7$ , $P(F^c \cap T^c) = 0.25$

As $P(F) = P(F \cap T) + P(F \cap T^c)$

$P(T) = P(T \cap F) + P(T \cap F^c)$


~~So P(F~~ And By Total probability Theorem

$P(F \cap T^c) + P(T \cap F^c) + P(F \cap T) + P(F^c \cap T^c) = 1$ — ①

$P(F \cap T) + P(F \cap T^c) = 0.6$ — ②

$P(T \cap F) + P(T \cap F^c) = 0.7$ — ③

Putting ② in ①

$P(T \cap F^c) = 1 - 0.6 - 0.25$

$P(T \cap F^c) = 0.15$

Putting In ③

$P(T \cap F) = 0.7 - 0.15$

$P(T \cap F) = 0.55$

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②

Putting $P(T\cap F)$ in ②

$P(F\cap T^c) = 0.6 - 0.55 = 0.05$

So Now

(a) $P(F\cap T^c) + P(T\cap F^c) = 0.15 + 0.05$

$= 0.2$

(b) $P(T|F^c) = \dfrac{P(T\cap F^c)}{P(F^c)}$

$= \dfrac{0.15}{0.4} = 3.75$

**Q.No.3** FW $\Rightarrow$ Fischer Wins, ~~Spass~~ | P

$g\le 5 \Rightarrow$ games less than 5 |

(a) $P(FW) = p + (1-p-q)p + (1-p-q)^2 p + (1-p-q)^3 p + \cdots$

$= p(1 + (1-p-q) + (1-p-q)^2 + (1-p-q)^3 + \cdots)$

$= p \cdot \dfrac{1}{1-(1-p-q)} = \dfrac{p}{p+q}$

$\left(\because 1 + x + x^2 + x^3 + \cdots = \dfrac{1}{1-x}\right)$

(b) $P(g\le 5) = (p+q) + (1-p-q)(p+q) + (1-p-q)^2(p+q) + \cdots + (1-p-q)^4(p+q)$

$= (p+q)(1 + (1-p-q) + (1-p-q)^2 + (1-p-q)^3 + (1-p-q)^4)$

$FW_1 \Rightarrow$ Fischer Wins first game.

$P(FW_1|g\le 5) = \dfrac{P(FW_1 \cap g\le 5)}{P(g\le 5)}$

$= \dfrac{P(FW_1)}{P(g\le 5)}$ $(\because FW_1 \subset g\le 5)$

$= \dfrac{p}{(p+q)(1+(1-p-q)+(1-p-q)^2+(1-p-q)^3+(1-p-q)^4)}$

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(c) $$P(FW|g\le 5) = \frac{P(FW\cap g\le 5)}{P(g\le 5)}$$

As we know

$$P(FW) = p(1+(1-p-q)+(1-p-q)^2+(1-p-q)^3+(1-p-q)^4+\cdots$$

$$P(g\le 5) = p(1+(1-p-q)+(1-p-q)^2+(1-p-q)^3+(1-p-q)^4)$$
$$+q(1+(1-p-q)+(1-p-q)^2+(1-p-q)^3+(1-p-q)^4)$$

So

$$P(FW\cap g\le 5) \cancel{p(1.P(g\le}$$
$$= p(1+(1-p-q)+(1-p-q)^2+(1-p-q)^3+(1-p-q)^4)$$

$$P(FW|g\le 5) = \frac{p}{p+q}$$

(d) $$P(g\le 5|FW) = \frac{P(g\le 5\cap FW)}{P(FW)}$$

$$= \frac{p(1+(1-p-q)+(1-p-q)^2+\cdots\cdots+(1-p-q)^4)}{p(1+(1-p-q)+(1-p-q)^2+\cdots\cdots)}$$

## Q.No.4

(a) $P(n^{th}\text{ throw olive}) = \left(\frac{1}{2}\right)\left(\frac{5}{6}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2/3$

(b) $P(n^{th}\ \&\ (n+1)^{st}\text{ throw olive}) = \left(\frac{1}{2}\right)\left(\frac{5}{6}\right)^2+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2$

(c) $$P((n+1)^{st}\text{ olive}|n\text{ olives}) = \frac{P(n+1\text{ olives})}{P(n\text{ olives})}$$

$$= \frac{\frac{1}{2}\left[\left(\frac{5}{6}\right)^{n+1}+\left(\frac{1}{2}\right)^{n+1}\right]}{\frac{1}{2}\left[\left(\frac{5}{6}\right)^{n}+\left(\frac{1}{2}\right)^{n}\right]}$$

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**Q.No.5** A ⇒ Dog in forest A | FA ⇒ Found in forest A
B ⇒ " " " B | FB ⇒ Found in forest B

$$P(\text{Dog die} \mid L_n \,\&\, F_n^c) = \frac{n}{n+2}$$

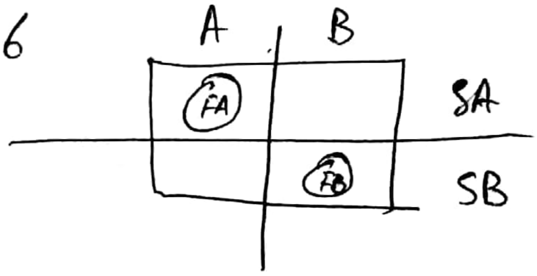
SA ⇒ search in forest A
SB ⇒ Search in forest B

Given

$P(A) = 0.4$ , $P(B) = 0.6$

$P(FA \mid A \cap SA) = 0.25$

$P(FB \mid B \cap SB) = 0.15$




(a) $$P(F \mid SA) = \frac{P(FA \cap A \cap SA)}{P(SA)}$$ ($\because F = FA \cup FB$ but $FB \cap SA = \phi$ And $P(FA \cap SA) = P(FA \cap SA \cap A)$)

$$= \frac{P(FA \cap A \cap SA)}{P(SA) \cdot P(A)} \cdot P(A)$$

$$= \frac{P(FA \cap A \cap SA)}{P(SA \cap A)} \cdot P(A)$$

$$= P(FA \mid A \cap SA) \cdot P(A)$$

$$P(F \mid SA) = 0.25 \times 0.4 = 0.1$$

Similarly

$$P(F \mid SB) = 0.15 \times 0.6 = 0.09$$

(b) $$P(A \mid SA \cap FA_1^c) = \frac{P(A \cap SA \cap FA_1^c)}{P(SA \cap FA_1^c)}$$

Nominator ⇒ $P(A \cap SA \cap FA_1^c) = P(FA_1^c \mid A \cap SA_1) \cdot P(A \cap SA_1)$

$$= 0.75 \times P(A) \cdot P(SA_1) \quad \text{--- (1)}$$

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Denominator

$$P(SA \cap FA_1^c) = P(SA_1 \cap FA_1^c \cap A) + P(SA_1 \cap FA_1^c \cap B)$$

$$P(SA_1 \cap FA_1^c \cap B) = P(FA_1^c \mid B \cap SA_1) \cdot P(B) \cdot P(SA_1)$$
$$= P(B) \cdot P(SA_1) \quad (\because P(FA_1^c \mid B \cap SA_1) = 1)$$

So

$$P(A \mid SA \cap FA_1^c) = \frac{0.75 \times 0.4 \cdot P(SA_1)}{0.75 \times 0.4 \times P(SA_1) + 0.6\, P(SA_1)}$$

$$= \frac{0.3}{0.3 + 0.6} = 0.33$$

**(c)** Due to coin toss $P(SA_1) = P(SB_1) = 1/2$

$$P(SA_1 \mid F_1) = \frac{P(SA_1 \cap F_1)}{P(F_1)} = \frac{P(SA_1 \cap FA_1 \cap A)}{P(FA_1) + P(FB_1)}$$

~~From~~

$$P(SA_1 \cap FA_1 \cap A) = 0.25 \times P(A) \times P(SA_1)$$
$$P(FA_1) = P(FA_1 \cap SA_1 \cap A)$$
$$P(FB_1) = P(FB_1 \cap B \cap SB_1)$$

So

$$P(SA_1 \mid F_1) = \frac{0.25 \times P(SA_1) \times P(A_1)}{0.25 \times P(SA_1) \times P(A) + 0.15 \times P(SB_1) \cdot P(B_1)}$$

$$= \frac{0.25 \times \frac{1}{2} \times 0.4}{0.25 \times \frac{1}{2} \times 0.4 + 0.15 \times \frac{1}{2} \times 0.6}$$

$$= \frac{0.1}{0.1 + 0.09} = 0.526$$

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$L_1 \Rightarrow$ Dog is alive on Day 1 ...

**(d)** $P(FA_2 \mid SA_1 \cap SA_2) = P(FA_2 \cap FA_1^c \cap L_1 \mid SA_1 \cap SA_2)$

$$= \frac{P(FA_2 \cap FA_1^c \cap L_1 \cap SA_1 \cap SA_2 \cap A)}{P(SA_1 \cap SA_2)}$$

$$= \frac{P(FA_2 \mid FA_1^c \cap L_1 \cap SA_1 \cap SA_2 \cap A) \cdot P(FA_1^c \cap L_1 \cap SA_1 \cap SA_2 \cap A)}{P(SA_1 \cap SA_2)}$$

$$= \frac{0.25 \times P(L_1 \mid FA_1^c \cap SA_1 \cap SA_2 \cap A) . P(FA_1^c \cap SA_1 \cap SA_2 \cap A)}{P(SA_1 \cap SA_2)}$$

$$P(L_1 \mid FA_1^c \cap SA_1 \cap SA_2 \cap A) = 1 - P(L_1^c \mid FA_1^c \cap SA_1 \cap SA_2 \cap A)$$

$$= 1 - \frac{1}{1+2}$$

$$= 2/3$$

$$P(FA_1^c \cap SA_1 \cap SA_2 \cap A) = P(FA_1^c \cap SA_1 \cap A) . P(SA_2)$$

$$= 0.75 \times P(SA_1) P(A) P(SA_2) \quad \left(\text{using eq. } 1 \text{ of part (b)}\right)$$

AND $P(SA_1 \cap SA_2) = P(SA_1) \cdot P(SA_2)$

So

$$P(FA_2 \mid SA_1 \cap SA_2) = \frac{0.25 \times \left(\frac{2}{3}\right) \times 0.75 \times \cancel{P(SA_1)} P(A) \cancel{P(SA_2)}}{\cancel{P(SA_1)} \cdot \cancel{P(SA_2)}}$$

$$= 0.25 \times \frac{2}{3} \times 0.75 \times 0.4$$

$$= 0.05$$

**(e)** $L_1^c \Rightarrow$ Dog dies on first Day evening.

$P(FA_2 \cap L_1^c \mid FA_1^c \cap SA_1 \cap SA_2)$ → Firstly we are calculating The Probability of finding dog dead on day 2 Then we will use compliment to find alive Dog

$$= \frac{P(FA_2 \cap L_1^c \cap FA_1^c \cap SA_1 \cap SA_2)}{P(FA_1^c \cap SA_1 \cap SA_2)}$$

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After Some Manipulation

$$= \frac{P(FA_2 | L_1^c \cap FA_1^c \cap SA_1 \cap SA_2 \cap A) \cdot P(L_1^c | FA_1^c \ldots) \cdot P(FA_1^c | A \cap SA_1) \cdot P(A \cap SA_1 \cap SA_2)}{[P(FA_1^c \cap SA_1 \cap A) + P(FA_1^c \cap SA_1 \cap B)]\ P(SA_2)}$$

$$= \frac{0.25 \times \frac{1}{3} \times 0.75 \times 0.4}{0.75 \times 0.4 + 0.6} =$$

(f) $P(L_1 \cap L_2 \cap L_3 | FB_4 \cap SA_1 \cap SA_2 \cap SA_3 \cap SB_4)$

$$= \frac{P(L_1 \cap L_2 \cap L_3 \cap FB_4 \cap SA_1 \cap SA_2 \cap SA_3 \cap SB_4 \cap FA_1^c \cap FA_2^c \cap FA_3^c)}{P(FB_4 \cap SA_1 \cap SA_2 \cap SA_3 \cap SB_4 \cap FA_1^c \cap FA_2^c \cap FA_3^c)}$$

$$= \frac{P(L_3 | L_1 \cap L_2 \cdots) P(L_1 \cap L_2 \cap FB_4 \cap FA_1^c \cdots)}{P(\cdots)}$$

$$= \frac{\left(1 - \frac{3}{5}\right) P(L_2 | \cdots) P(L_1 \cap FB_4 \cap FA_1^c \cdots)}{P(\cdots)}$$

$$= \frac{\left(1 - \frac{3}{5}\right)\left(1 - \frac{2}{4}\right) P(L_1 | FB_4 \cap FA_1^c \cdots) \cdot \cancel{P(FB_4 \cap FA_1^c \cdots)}}{\cancel{P(FB_4 \cap FA_1^c \cdots)}}$$

$$= \left(1 - \frac{3}{5}\right)\left(1 - \frac{2}{4}\right)\left(1 - \frac{1}{3}\right)$$

$$= \frac{2}{5} \times \frac{2}{4} \times \frac{2}{3} =$$

$= 0.133$

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$$P(L_1 \cap L_2 \cap L_3 \mid F_4 \cap SA_1 \cap SA_2 \cap SB_3 \cap SB_4)$$

$$= P(L_3 \mid L_2 \cdots) P(L_2 \mid L_1 \cdots) P(L_1 \mid F_1^c \cdots)$$

$$= \left(1 - \frac{3}{5}\right)\left(1 - \frac{2}{4}\right)\left(1 - \frac{1}{3}\right)$$

$$= 0.133$$

## Q. No. 6

### (a) TRUE

$$P(A|B) = P(A)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

This tells us that A & B are independent.
If A & B are independant then $A^c$ & B are also independent so

$$P(B/A^c) = \frac{P(A^c \cap B)}{P(A^c)} = \frac{\cancel{P(A^c)} \cdot P(B)}{\cancel{P(A^c)}} = P(B)$$

### (b) FALSE

ST $\Rightarrow$ 5 out of 10 tosses are tails
$T_1 \Rightarrow$ First 1 is tail

$$P(T_1 \mid 5T) = \frac{\binom{9}{4}}{\binom{10}{5}} \qquad P(T_{10} \mid 5T) = \frac{\binom{9}{4}}{\binom{10}{5}}$$

$$P(T_1 \cap T_{10} \mid 5T) = \frac{\binom{8}{3}}{\binom{10}{5}} \neq P(T_1 \mid 5T) \cdot P(T_2 \mid 5T)$$

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(c) TRUE

$P(T_1 | 10T) = 1$ , $P(T_{10} | 10T) = 1$

$P(T_1 \cap T_{10} | 10T) = 1 = P(T_1 \cap 10T) \cdot P(T_2 \cap 10T)$

(d) FALSE

Since $A_i$'s are disjoint

L.H.S $P(B|C) = \frac{P(B \cap C)}{P(C)} = \sum_{i=1}^{n} \frac{P(A_i) P(B \cap C | A_i)}{P(C)}$

$= \sum_{i=1}^{n} \frac{P(A_i \cap B \cap C)}{P(C)}$

R.H.S

$\sum_{i=1}^{n} P(A_i|B) P(B|A_i) = \sum_{i=1}^{n} \frac{P(A_i \cap C)}{P(C)} \cdot \frac{P(B \cap A_i)}{P(A_i)}$

$= \sum_{i=1}^{n} \frac{P(A_i \cap B \cap C)}{P(A_i) \cdot P(C)} \neq$ L.H.S

## Q. No. 7

$P(Spam | W_1^c \dots W_{22}^c, W_{23}, W_{24}^c, \dots W_{63}^c, W_{64}, W_{65}, W_{66}^c, \dots W_{100}^c)$

$= \frac{P(W_1^c, \dots | Spam) \cdot P(Spam)}{P(W_1^c, \dots | Spam) \cdot P(Spam) + P(W_1^c, \dots | Not\ Spam) \cdot P(Not\ Spam)}$

$= \frac{(1-p_1)(1-p_2) \dots p_{23} \dots (p_{64})(p_{65}) \dots (1-p_{100}) \cdot p}{(1-p_1) \dots p_{23}(1-p_{24}) \dots (p_{64})(p_{65})(1-p_{66}) \dots (1-p_{100}) \cdot p + ((1-r_1) \dots r_{23}(1-r_{24}) \dots r_{64} r_{65} \dots)(1-p)}$

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