r.1: Capacitance:

7.1)

$C = ?$

$d = 0.1 \times 10^{-3} m$

$S = 1 in \times 1 in$ $\boxed{1 in = 2.54 cm}$

$S = (2.54 \times 10^{-2} \times 2.54 \times 10^{-2}) m^2$

$S = 6.45 \times 10^{-4} m^2$

$\varepsilon_r = 1$

$\varepsilon_r = \varepsilon / \varepsilon_0$

$\varepsilon = \varepsilon_r \varepsilon_0$

$= 1 \cdot \frac{10^{-9}}{36\pi}$

$\varepsilon = 8.8 \times 10^{-12}$

Now,

$C = \varepsilon \frac{S}{d}$

$C = 8.8 \times 10^{-12} \cdot \frac{6.45 \times 10^{-4}}{0.1 \times 10^{-3}}$

$C = 5.67 \times 10^{-11} F$

$\boxed{C = 56.7 pF}$

7.2) $C = ?$

$S = 2mm \times 2 in$

$S = 2 \times 10^{-3} \times 2 \times 2.54 \times 10^{-2}$

$S = 1.016 \times 10^{-4} m^2$

$\varepsilon_r = 5$

$\varepsilon = \varepsilon_r \varepsilon_0$

$\varepsilon = 5 \times \frac{10^{-9}}{36\pi}$

$\varepsilon = 4.4 \times 10^{-11}$

$C = \varepsilon \frac{S}{d}$

$C = 4.4 \times 10^{-11} \times \frac{1.016 \times 10^{-4}}{1 \times 10^{-3}}$

$\boxed{C = 4.4\,pF}$

Q7.3) a)

$S = ?$

$C = 1 \times 10^{-6} F$

$d = 1 \times 10^{-3} m$

$\varepsilon_r = 7500$

$\varepsilon = \varepsilon_r \times \varepsilon_0$

$= 7500 \times \frac{10^{-9}}{36\pi}$

$\varepsilon = 6.6 \times 10^{-8}$

As $S = l \times l = l^2$

So,

$C = \varepsilon \frac{S}{d}$

$C = \varepsilon \frac{l^2}{d}$

$l^2 = 0.015\ m^2$

$l = 0.123\ m$

$\boxed{l = 4.84\ \text{inch}}$

b)

$C = 1F$

$l^2 = \frac{cd}{\varepsilon}$

$l^2 = \frac{1 \times 1 \times 10^{-3}}{6.6 \times 10^{-8}}$

$l^2 = 1.51 \times 10^4$

$\boxed{l = 123\ m}$

Q 7.4)

a) parallel combination

$C_1 = 0.2\ \mu F$

$C_2 = 0.6\ \mu F$

$C_{eq} = C_1 + C_2$

$C_{eq} = 0.8\ \mu F$

$V = 10V$

$q = ?$

$W = ?$

$q = CV$

$$Energy = \frac{1}{2}CV^2 \text{ or } \frac{q^2}{2C}$$

$$w = \frac{q^2}{2C}$$

$$w = \frac{(8\times10^{-6})^2}{2\times0.8\times10^{-6}}$$

$$\boxed{w = 4\times10^{-5}\,J}$$

) Series Combination

$$C_{eq} = \frac{C_1C_2}{C_1+C_2}$$

$$= \frac{0.2\times10^{-6}\times0.6\times10^{-6}}{0.2\times10^{-6}+0.6\times10^{-6}}$$

$$C_{eq} = 1.5\times10^{-7}$$

$$q = C_{eq}V$$

$$q = 1.5\times10^{-7}\times10$$

$$\boxed{q = 1.5\,\mu C}$$

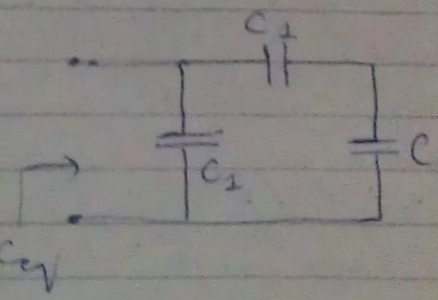
$$w = \frac{q^2}{2C}$$

$$w = \frac{(1.5\times10^{-6})^2}{2\times1.5\times10^{-7}}$$

$$w = 7.5\times10^{-6}\,J$$

Q.5) a)

Ans



$$C_{eq} = C_1 + \frac{CC_1}{C+C_1}$$

$$C_{eq} = \frac{(C+C_1)C_1 + CC_1}{C+C_1}$$

$\therefore C_{eq} = C$

$$C(C+C_1) = CC_1 + C_1^2 + CC_1$$

$$C^2 + CC_1 = C_1^2 + 2CC_1$$

$$C^2 - CC_1 - C_1^2 = 0$$

$$C = \frac{C_1 \pm \sqrt{C_1^2 + 4C_1^2}}{2}$$

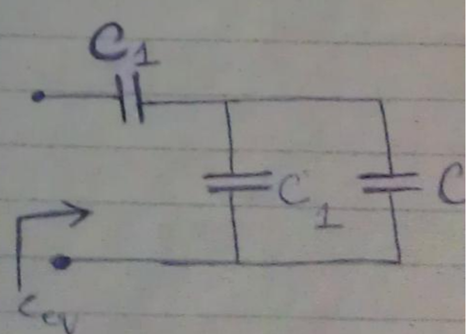
$$C = \frac{C_1 \pm C_1\sqrt{5}}{2}$$

$$C = \frac{C_1(1 \pm \sqrt{5})}{2}$$

ignoring -ve value

$$\boxed{C = \frac{C_1(1+\sqrt{5})}{2}}$$

b)



$$C_{eq} = \frac{(C_1+C)\cdot C_1}{C_1+C+C_1}$$

$\therefore C_{eq} = C$

$$C = \frac{CC_1 + C_1^2}{C+2C_1}$$

$$C(C+2C_1) = CC_1 + C_1^2$$

$$C^2 + 2CC_1 = CC_1 + C_1^2$$

$$C^2 + CC_1 - C_1^2 = 0$$

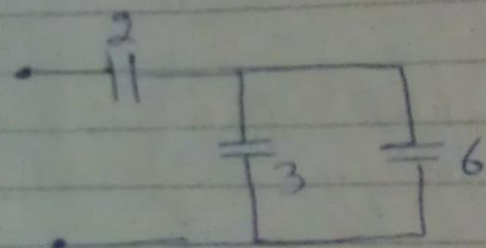
$$C = \frac{-C_1 \pm \sqrt{C_1^2 + 4C_1^2}}{2}$$

$$C = \frac{C_1(-1 \pm \sqrt{5})}{2}$$

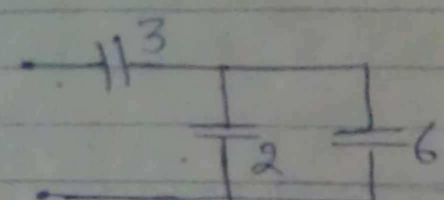
$$\boxed{C = \frac{C_1(\sqrt{5} - 1)}{2}}$$

Q6) 2nF, 3nF, 6nF

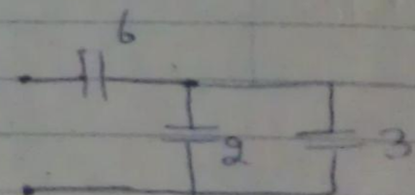
$C_{eq} = 1.6\ nF$



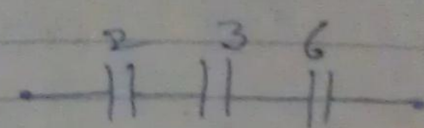
$C_{eq} = 2.1\ nF$



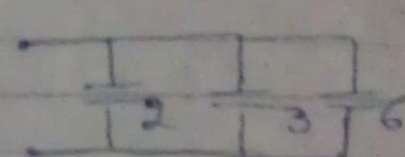
$C_{eq} = 2.7\ nF$



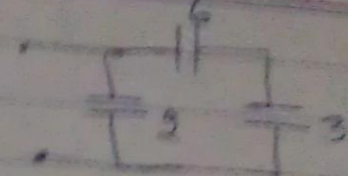
$C_{eq} = 1\ nF$



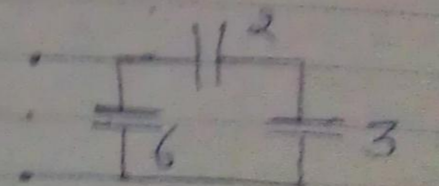
$C_{eq} = 11\ nF$



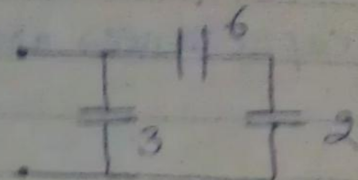
$C_{eq} = 4\,nF$



$C_{eq} = 7.2\,nF$



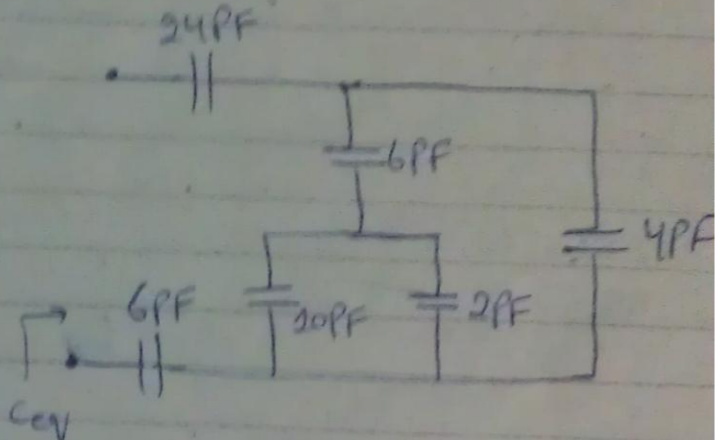
$C_{eq} = 4.5\,nF$



27.7) a)

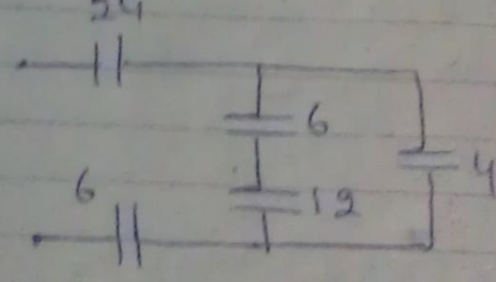
10 || 2

$10 + 2 = 12$



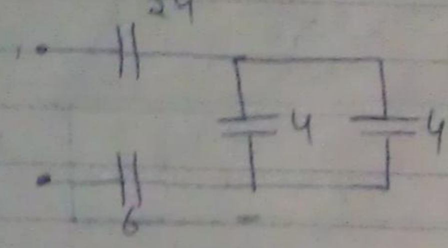
6 Series 12

$= \frac{6 \times 12}{6+12} = 4$

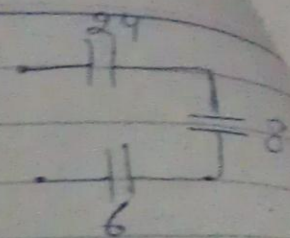


4 || 4

$4 + 4 = 8$



24 Series 8 series 6



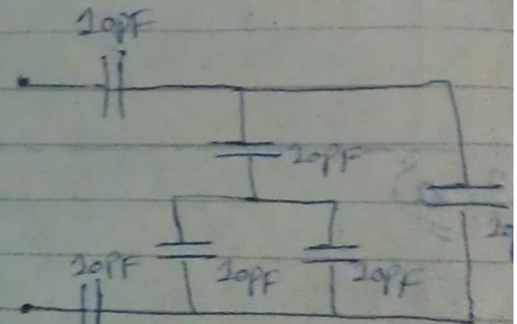
$\frac{24 \times 8}{24+8} = 6$

$\frac{6 \times 6}{6+6} = 3$

So,

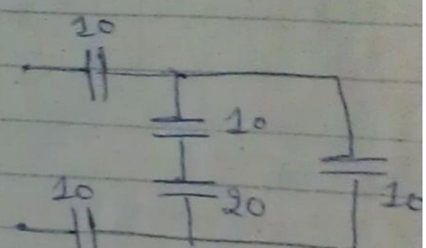
$C_{eq} = 3\,pF$

b) All capacitances are 10pF



$10 \parallel 10 = 20$

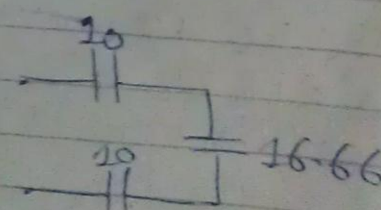
10 Series 20



$\frac{10 \times 20}{10+20} = 6.66$

$6.66 \parallel 10$

$6.66 + 10 = 16.66$

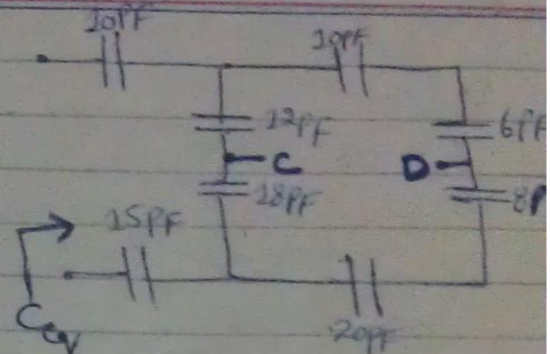


$\frac{10 \times 10}{10+10} = 5$

$\frac{5 \times 16.66}{5+16.66} = 3.84$

$\boxed{C_{eq} = 3.84\,pF}$

a)

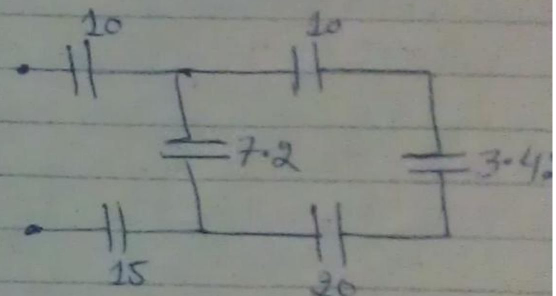


6 : Series 8 =

$\frac{6\times 8}{6+8} = 3.42$

12 Series 18

$\frac{12\times 18}{12+18} = 7.2$



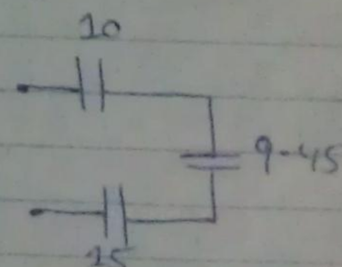
3.42 Series 10 Series 20

$\frac{20\times 10}{20+10} = 6.66$

$\frac{3.42\times 6.66}{3.42+6.66} = 2.25$

$2.25 \parallel 7.2$

$\Rightarrow 2.25 + 7.2 = 9.45$



9.45 Series 10 Series 15

$\frac{10\times 15}{10+15} = 6$

$\frac{6\times 9.45}{6+9.45} = 3.66$

So,

$\boxed{C_{eq} = 3.66\,pF}$

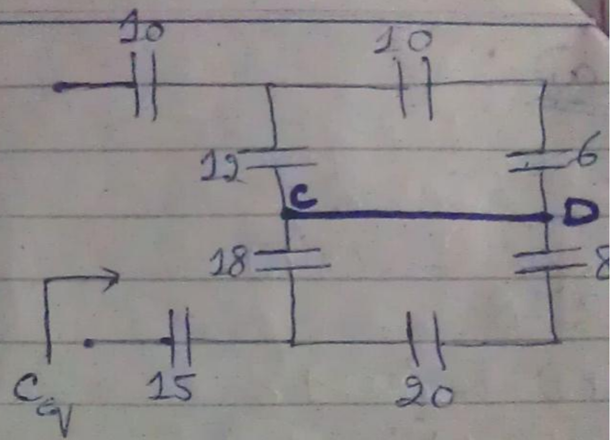
b) C & D are shorted together

6 series 10

$$\frac{6 \times 10}{6+10} = 3.75$$

20 series 8

$$\frac{20 \times 8}{20+8} = 5.71$$

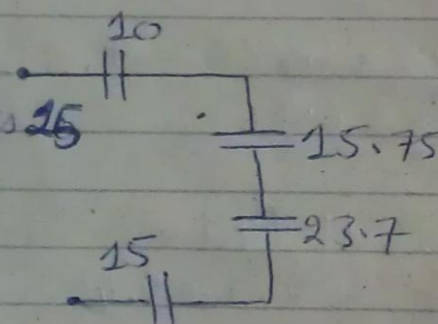


$3.75 \parallel 12$

$3.75 + 12 = 15.75$

$5.71 \parallel 18$

$5.71 + 18 = 23.7$



15.75 series 23.7 series 10 series 15

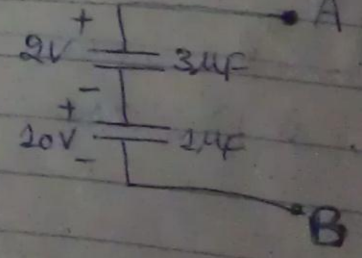
$$\frac{15.75 \times 23.7}{15.75 + 23.7} = 9.46$$

$$\frac{10 \times 15}{10 + 15} = 6$$

$$\frac{6 \times 9.46}{6 + 9.46} = 3.66$$

$$\boxed{C_{eq} = 3.66\,pF}$$

.9) a)



$$W_1 = \frac{1}{2} C_1 V_1^2$$

$$= \frac{1}{2}(3)(2)^2$$

$$W_2 = \frac{1}{2} C_2 V_2^2$$

$$= \frac{1}{2}(1)(10)^2$$

$$W_2 = 50 \mu J$$

$$W = W_1 + W_2$$

$$= 50 + 6$$

$$W = 56 \mu J$$

Now,

$$W' = ?$$

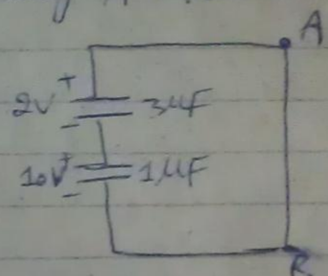
$$C_{eq} = \frac{3 \times 1}{3+1} = 0.75 \mu F$$

$$V_{AB} = V_A + V_B$$

$$V_{AB} = 10 + 2$$

$$V_{AB} = 12V$$

Connecting A & B



$$W' = \frac{1}{2} C V_{AB}^2$$

$$= \frac{1}{2}(0.75)(12)^2$$

$$W' = 54 \mu J$$

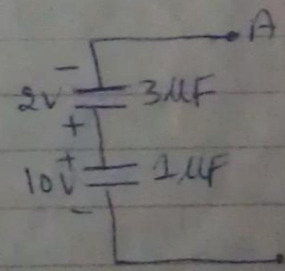
$$\text{Fraction} = \frac{54}{56} \times 100$$

$$\text{Fraction} = 96.42\%$$

b)

$$W = 56 \mu J$$

$$C_{eq} = 0.75 \mu F$$



$V_{AB} = V_B - V_A$
$= 10 - 2$
$V_{AB} = 8V$

$w' = \frac{1}{2} C_{eq} V_{AB}^2$

$w' = \frac{1}{2}(0.75)(8)^2$

$w' = 24\mu J$

Fraction $= \frac{24}{56} \times 100$

Fraction $= 42.85\%$

Q7.10)

$C_1 = 2\mu F$, $V_1 = 5V$
$C_2 = 3\mu F$, $V_2 = 6V$
$C_3 = 6\mu F$, $V_3 = 3V$

$W_1 = \frac{1}{2} C_1 V_1^2$

$= \frac{1}{2}(2)(5)^2$

$W_1 = 25\mu J$

$W_2 = \frac{1}{2} C_2 V_2^2$

$= \frac{1}{2}(3)(6)^2$

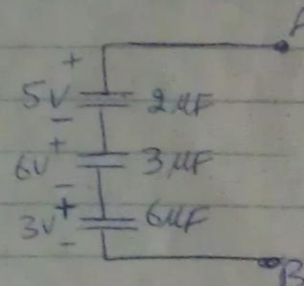
$W_3 = \frac{1}{2}(6)(3)^2$

$W_3 = 27\mu J$

Now,

$C_{eq} = 1\mu F$

For W_{max}



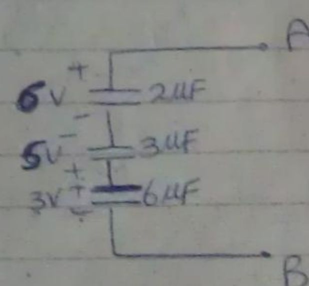
$V_{max} = 5+6+3$
$= 14V$

$W_{max} = \frac{1}{2} C V_{max}^2$

$W_{max} = \frac{1}{2}(1)(14)^2$

$\boxed{W_{max} = 98\mu J}$

For W_{min}



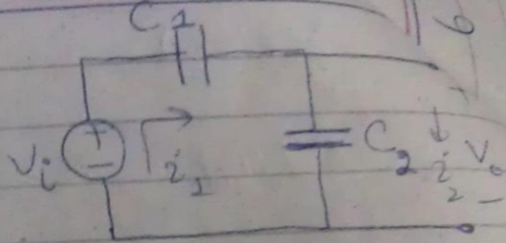
$V_{min} = -5+3+6 = 6-5+3$
$V_{min} = 4V$

$W_{min} = \frac{1}{2} C V_{min}^2$

$= \frac{1}{2}(1)(4)^2$

$\boxed{W_{min} = 8\mu J}$

)a)



$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

$= \frac{C_1 C_2}{C_1 + C_2} \cdot \frac{dv_i}{dt}$

$= C_2 \frac{dv_0}{dt}$

$i_1 = i_2$

$\frac{C_1 \cancel{C_2}}{C_1 + C_2} \frac{dv_i}{dt} = \cancel{C_2} \frac{dv_0}{dt}$

$v_0 = \frac{C_1}{C_1 + C_2} v_i$

OR

By voltage divider rule for capacitor

$V_0 = \frac{C_1}{C_1 + C_2} V_i$

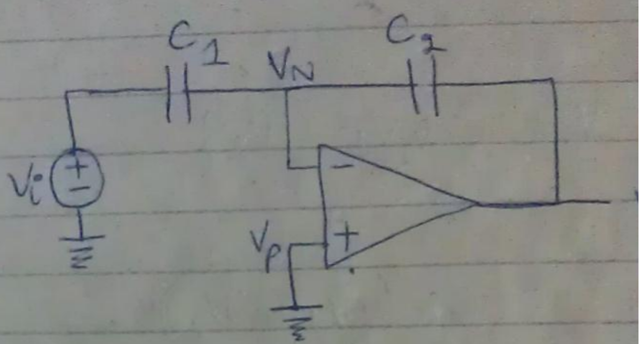
$C_2 = 3nF$
$V_1 = 5V$
$V_0 = 2V$
$C_1 = ?$

$V_0 = \frac{C_1}{C_1 + C_2} v_i$

$2 = \frac{C_1}{C_1 + 3} \times 5$

$$6 + 2c_1 = 5c_1$$
$$6 = 3c_1$$
$$\boxed{c_1 = 2nF}$$

Q7.12) a)



As $V_N = V_p = 0$

$$i_1 = C_1 \frac{dv_i}{dt}$$
$$i_2 = C_2 \frac{dv_o}{dt}$$
$$i_1 + i_2 = 0$$
$$C_1 \frac{dv_i}{dt} + C_2 \frac{dv_o}{dt} = 0$$
$$v_o = -\frac{C_1}{C_2} v_i$$

b)

$C_1 = 40pF$
$C_2 = 10pF$
$V_o = 8V$
$V_i = ?$

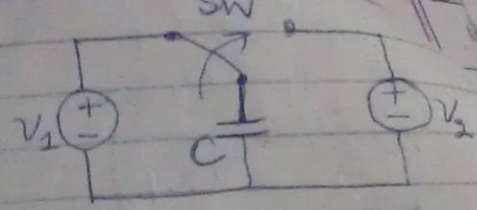
$$V_o = -\frac{C_1}{C_2} v_i$$
$$8 = -\frac{40}{10} v_i$$

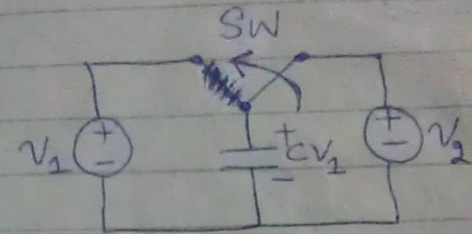
$t_0 = R_{eff} \cdot C$

$\boxed{R_{eff} = \frac{1}{f_0 C}}$

Q7.13) a)

$q = C V_1$





$$q = C(v_1 - v_2)$$

$$\frac{q}{t_0} = \frac{C}{t_0}(v_1 - v_2) \qquad \therefore C = \frac{q}{V}$$

$$\frac{q}{t_0} = \frac{q}{V t_0}(v_1 - v_2) \Rightarrow q f_0 = \frac{C}{t_0}(v_1 - v_2)$$

$$\therefore i = \frac{i}{V}(v_1 - v_2)$$

$$\therefore i = \frac{(v_1 - v_2)}{V/i}$$

$$i = \frac{V_1 - V_2}{R_{eff}} \qquad \therefore R_{eff} = \frac{1}{f_0 C}$$

$$\therefore R_{eff} = \frac{t}{C}$$

$$i = \frac{V_1 - V_2}{\cancel{t/C}}$$

$$\cancel{i = V_1 - V_2 / \frac{t}{C}}$$

$$i = V_1 - V_2 \cdot \frac{C}{t_0}$$

$$t_0 = \frac{V_1 - V_2}{i} \cdot C$$

$t_0 = R_{eff} \cdot C$

$$\boxed{R_{eff} = \frac{1}{f_0 C}}$$

a) $R_{eff} = ?$

$C = 10pF$

i) $f_0 = 1KHZ$

$R_{eff} = \frac{1}{f_0 C}$

$R_{eff} = \frac{1}{1\times10^3 \times 10\times10^{-12}}$

$R_{eff} = 1\times10^{+8}\,\Omega$

$$\boxed{R_{eff} = 100M\Omega}$$

ii) $f = 1MHZ$

$R_{eff} = \frac{1}{1\times10^6 \times 10\times10^{-12}}$

$R_{eff} = 1\times10^5\,\Omega$

$$\boxed{R_{eff} = 100K\Omega}$$

Q7.14) $i = ?$

$C = 50nF$

a) $V = 5(1-10^4 t)\,V$

$i = C\frac{dv}{dt}$

$i = 50\times10^{-9} \cdot \frac{d}{dt} 5 - 5\times10^4 t$

$i = 50\times10^{-9}(-5\times10^{4})$

$\boxed{i = -2.5\times10^{-3}A}$

b) $V = 10(1 - e^{-10^4 t})$

$i = 50\times10^{-9}\frac{d}{dt}10 - 10e^{-10^4 t}$

$i = 50\times10^{-9}(-10)\, e^{-10^4 t}\cdot(-10^4)$

$\boxed{i = 5\times10^{-3}e^{-10^4 t}A}$

c) $V = 4\sin10^4 t$

$i = 50\times10^{-9}\frac{d}{dt}4\sin10^4 t$

$i = 50\times10^{-9}\times4\cos10^4 t\cdot10^4$

$\boxed{i = 2\times10^{-3}\cos10^4 tA}$

d) $V = 15V$

$i = 50\times10^{-9}\frac{d}{dt}15$

$\boxed{i = 0A}$

Q7.15) $V = ?$

$V(0) = 5V$

$C = 1\mu F$

$$v = \frac{1}{C}\int i\,dt + v(0)$$

$$v = \frac{1}{1\times10^{-6}}\int 10^{-5}t + 5$$

$$= 1\times10^{6}\times10^{-5}\cdot\frac{t^2}{2} + 5$$

$$= 5t^2 + 5$$

$$\boxed{v = 5(t^2+1)\,V}$$

b) $i = 2e^{-10^5 t}\,A$

$$v = \frac{1}{1\times10^{-6}}\int 2e^{-10^5 t}\,dt + 5$$

$$= 1\times10^{6}\times2\cdot\frac{e^{-10^5 t}}{-10^5} + 5$$

$$= -20e^{-10^5 t} + 5$$

$$\boxed{v = 5(1-4e^{-10^5 t})\,V}$$

c) $i = 0.5\cos 10^5 t\,A$

$$i = \frac{1}{1\times10^{-6}}\int 0.5\cos10^5 t\,dt + 5$$

$$= 1\times10^{6}\times0.5\;\frac{-\sin10^5 t}{10^5} + 5$$

$$= -5\sin10^5 t + 5$$

$$\boxed{i = 5(1-\sin10^5 t)}$$

$0 \rightarrow 2\ \mu s$

$$V = \frac{1}{1\times10^{-6}}\int 5\times10^{-6}\,dt$$

$$\boxed{V = 5t\ V}$$

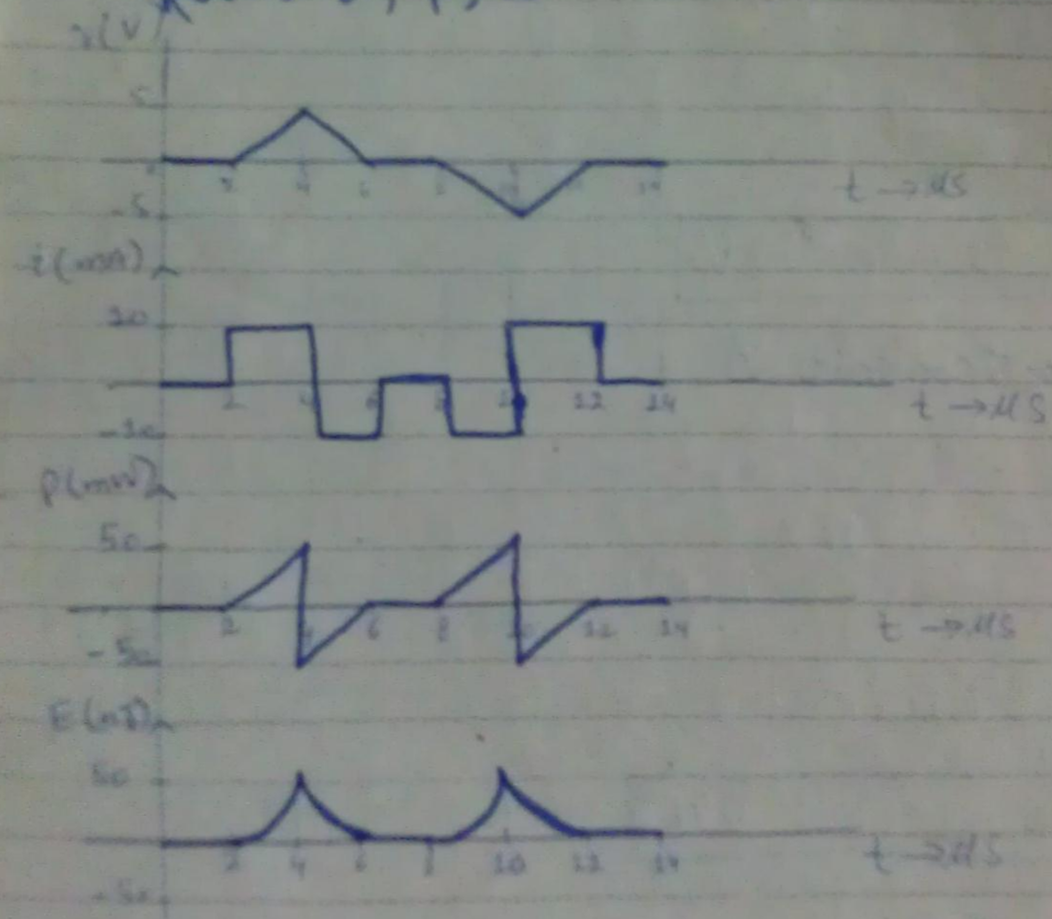
e) $i = 0A$

$$V = \frac{1}{1\times10^{-6}}\int 0\,dt$$

$$\boxed{V = 0V}$$

Q7.16) $C = 4nF$

Sketch i, p, w



0 → 2 μs

$v = 0$

$i = 0$

$P = 0$

2 → 4 μs

$V = \frac{5}{2\times10^{-6}} t - 5$

$V = 2.5\times10^{6} t - 5$

$i = C\frac{dv}{dt}$

$i = 4\times10^{-9}\frac{d}{dt}(2.5\times10^{6}t - 5)$

$i = 10mA$

$P = (2.5\times10^{6}t - 5)\,10\times10^{-3}$

$= 2.5\times10^{4}t - 0.05$

put $t = 4\mu s$

$P = 50mW$

4 → 6 μs

$V = \frac{-5}{2\times10^{-6}} t + 15$

$V = -2.5\times10^{6} t + 15$

$i = 4\times10^{-9}\frac{d}{dt}(-2.5\times10^{6}t) + 15$

$i = -10mA$

$P = (-2.5\times10^{6}t + 15)(-10\times$

$= 2.5\times10^{4}t - 0.15$

put $t = 6\mu s$

$P = 0$

6 → 8 μs

$i = 0$

$P = 0$

8 → 10 μs

-ve slope

$V = -\left(\frac{-0+5}{2\times10^{-6}}t + 20\right)$

$V = -2.5\times10^{6}t + 20$

$i = 4\times10^{-9}\frac{d}{dt}(-2.5\times10^{6}t + 20)$

$i = -10mA$

$P = 25\times10^{6}t - 200$

At $t = 10\mu S$

$P = 50\ mW$

$t = 10 \rightarrow 12\mu S$

$V = 2.5\times10^{6}t - 30$

$i = 4\times10^{-9}\frac{d}{dt}(2.5\times10^{6} - 30)$

$i = 10mA$

$P = 25\times10^{6}t - 300$

At $t = 12\mu S$

$P = 0$

$t = 12 \rightarrow 14\mu S$

$i = 0$

$P = 0$

Energy

$2 \rightarrow 4\mu S$

$W = \frac{1}{2}CV^2$

$= \frac{1}{2}4\times10^{-9}(2.5\times10^{6}t - 5)^2$

$= 2\times10^{-9}(6.25\times10^{12}t^2 + 25 - 2.5\times10^{7}t)$

$= 12500t^2 + 5\times10^{-8} - 0.05t$

For $t = 4\mu S$

$W = 2\times10^{-7} + 5\times10^{-8} - 2\times10^{-7}$

$= 50\times10^{-9}J$

For $4 \rightarrow 6 \mu s$

$$W = \frac{1}{2} 4\times10^{-9}(-2.5\times10^{6}t+15)^2$$

$$= 2\times10^{-9}(6.25\times10^{12}t^2+225-7.5\times10^{7}t)$$
$$= 12500t^2+4.5\times10^{-7}-0.15t$$

For $t = 6 \mu s$

$$W = 4.5\times10^{-7}+4.5\times10^{-7}-9\times10^{-7}$$
$$W = 0$$

For $6 \rightarrow 8 \mu s$

$$W = 0$$

For $8 \rightarrow 10 \mu s$

$$W = \frac{1}{2} 4\times10^{-9}(-2.5\times10^{6}t+20)$$

$$= 2\times10^{-9}(6.25\times10^{12}t^2+400-1\times10^{8}t)$$
$$= 12500t^2+8\times10^{-7}-0.2t$$

At $t = 10 \mu s$

$$W = 1.25\times10^{-6}+8\times10^{-7}-2\times10^{-6}$$
$$W = 50\times10^{-9} J$$

For $10 \rightarrow 12 \mu s$

$$W = \frac{1}{2} 4\times10^{-9}(2.5\times10^{6}t-30)$$

$$= 2\times10^{-9}(6.25\times10^{12}t^2+900-1.5\times10^{8}t)$$
$$= 12500t^2+1.8\times10^{-6}-0.3t$$

At $t = 12 \mu s$

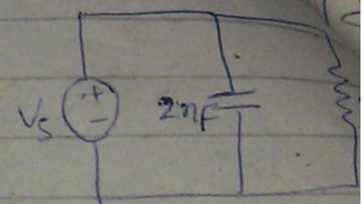
$$W = 1.8\times10^{-6}+1.8\times10^{-6}-3.6\times10^{-6}$$
$$W = 0$$

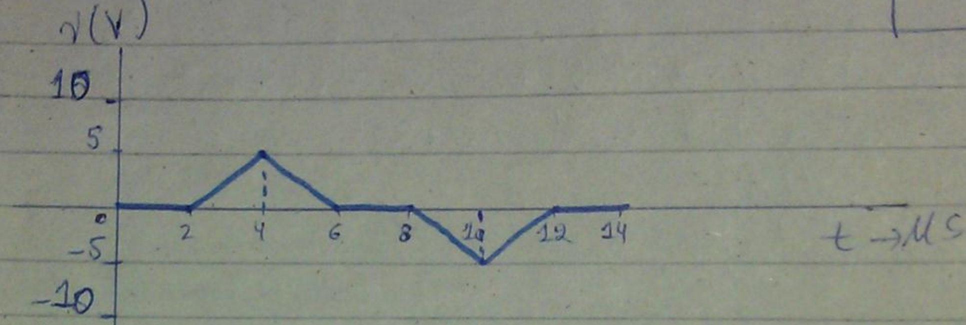
For $12 \rightarrow 14 \mu s$

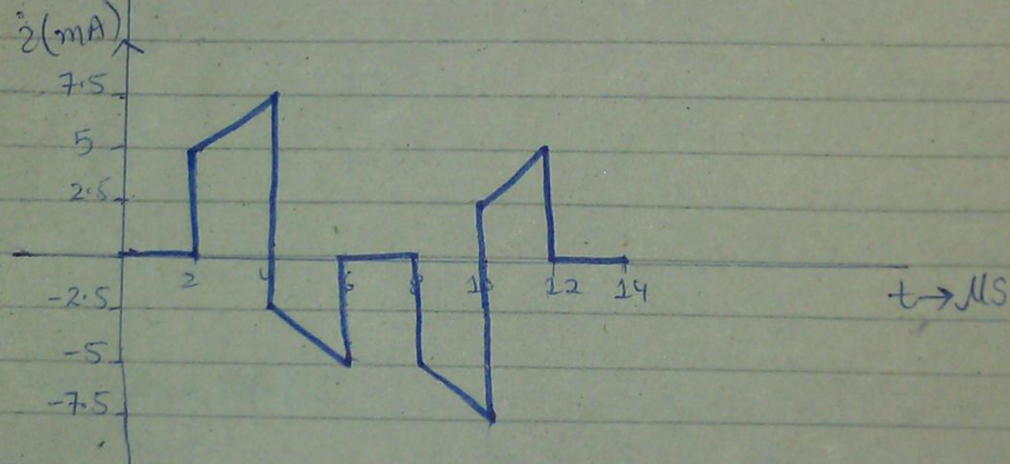
$$W = 0$$

Q17) $C = 2nF$

sketch current







$C = 2nF$

$$i = \frac{V}{R} + C \cdot \frac{dv}{dt}$$

$0 \rightarrow 2\mu s$

$V = 0$

$i = 0$

$2 \rightarrow 4\mu s$

$V = 2.5 \times 10^{6} t - 5$

$$i = \frac{2.5 \times 10^{6} t - 5}{2 \times 10^{3}} + 2 \times 10^{-9} \frac{d}{dt} 2.5 \times 10^{6} t - 5$$

At $t = 2\mu s$	At $t = 4\mu s$
$i = 7.5 mA$	$i = 5 mA$

4 → 6 μs

$V = -2.5\times10^6 t + 15$

$$i = \frac{-2.5\times10^6 t + 15}{2\times10^3} + 2\times10^{-9}\frac{d}{dt}(-2.5\times10^6 t + 15)$$

At $t = 4\mu s$	At $t = 6\mu s$
$i = -2.5 mA$	$i = -5 mA$

6 → 8 μs

$i = 0$

8 → 10 μs

$V = -2.5\times10^6 t + 20$

$$i = \frac{-2.5\times10^6 t + 20}{2\times10^3} + 2\times10^{-9}\frac{d}{dt}(-2.5\times10^6 t + 20$$

At $t = 8\mu s$	At $t = 10\mu s$
$i = -5 mA$	$i = -7.5 mA$

10 → 12 μs

$V = 2.5\times10^6 t - 30$

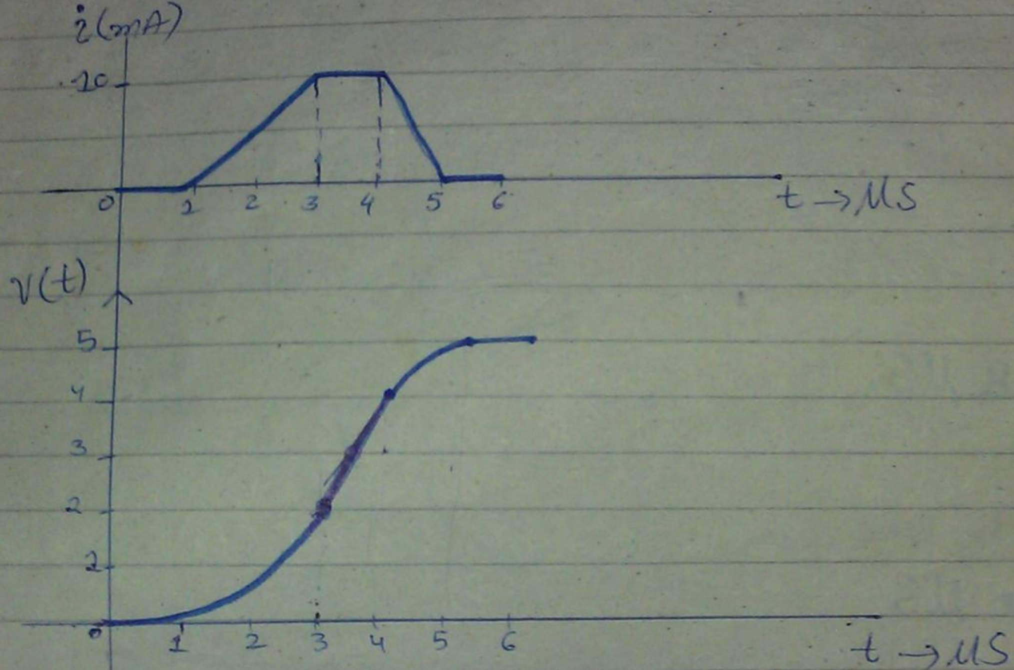
$$i = \frac{2.5\times10^6 t - 30}{2\times10^3} + 2\times10^{-9}\frac{d}{dt}(2.5\times10^6 t - 3$$

At $t = 10\mu s$	At $t = 12\mu s$
$i = 2.5 mA$	$i = 5 mA$

$12 \rightarrow 14\,\mu s$

$i = 0$

Q7.18) $C = 5nF$



$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\xi)\,d\xi - v(0) = \frac{1}{C}\int_{0}^{t} i(\xi)\,d\xi$$

so,

$$v(t) = \frac{1}{C}\int_{0}^{t} i(\xi)\,d\xi + v(0)$$

$1 \longrightarrow 3\,\mu sec$

$$i(t) = \frac{10\times10^{-3}}{2\times10^{-6}}t - 5\times10^{-3}$$

$$i(t) = 5\times10^{3}t - 5\times10^{-3}$$

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(\xi)\,d\xi$$

$$= 0 + \frac{1}{5\times10^{-9}}\int_{1\times10^{-6}}^{3\times10^{-6}} (5\times10^{3}t - 5\times10^{-3})\,dt$$

$$= \frac{1}{5\times10^{-9}}\left|5\times10^{3}\frac{t^2}{2}\right|_{1\times10^{-6}}^{3\times10^{-6}} - \left|5\times10^{-3}t\right|_{1\times10^{-6}}^{3\times10^{-6}}$$

$$= \frac{1}{5\times10^{-9}}\left[5\times10^{3}\left\{\frac{(3\times10^{-6})^2}{2} - \frac{(1\times10^{-6})^2}{2}\right\}\right]$$
$$-5\times10^{-3}\left[3\times10^{-6} - 1\times10^{-6}\right]$$

$$= \frac{1}{5\times10^{-9}}\left[5\times10^{3}(4.5\times10^{-2} - 5\times10^{-3}) - 5\times10^{-3}(2\times10^{-6})\right]$$

$$= \frac{1}{5\times10^{-9}}(200 - 1\times10^{-8})$$

$$= \frac{200}{5\times10^{-9}}$$

$$v(t) = 2\,V$$

For in between shape

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(\xi)\,d\xi$$

$$= 0 + \frac{1}{5\times10^{-9}}\int_{1\times10^{-6}}^{t}(5\times10^{3}\xi - 5\times10^{-3})\,d\xi$$

$$= \frac{1}{5\times10^{-9}}\left[\frac{(5\times10^{3})(\xi)^2}{2} - 5\times10^{-3}\xi\right]_{1\times10^{-6}}^{t}$$

$$= \frac{1}{5\times10^{-9}}\left[5\times10^{3}\frac{t^2}{2} - 5\times10^{-3}t - \frac{5\times10^{3}}{2}(1\times10^{-6})^2\right.$$
$$\left. + (5\times10^{-3})(1\times10^{-6})\right]$$

$v(t) = \frac{1}{5\times10^{-9}}\left[\frac{5\times10^{3}}{2}t^{2} - 5\times10^{-3}t - \frac{5\times10^{-2}}{2} + 5\times10^{-3}\right]$

$v(t) = \frac{1}{2}\times10^{12}t^{2} - 10^{6}t + \frac{1}{2}$ → Eq. of parabola

put $t = 3\mu s$

$v(t) = 2V$

3 → 4 μs

$i(t) = 10mA$

$$v(t) = v(3\times10^{-6}) + \frac{1}{5\times10^{-9}}\int_{3\times10^{-6}}^{t} 10\times10^{-3}\,d\xi$$

$$= 2 + \frac{1}{5\times10^{-9}}\left[10\times10^{-3}\xi\right]_{3\times10^{-6}}^{t}$$

$$= 2 + \frac{1}{5\times10^{-9}}\left[10\times10^{-3}t - 30\times10^{-9}\right]$$

$$= 2 + (2\times10^{6}t - 6)$$

$v(t) = 2\times10^{6}t - 4$

put $t = 4\mu s$

$v(t) = 4V$

Now, initial condition is 4.

4 → 5 μs

$i(t) = -10\times10^{3}t + 50\times10^{-3}$

$$v(t) = 4 + \frac{1}{C}\int_{4\times10^{-6}}^{t} i(\xi)\,d\xi$$

$$v(t) = 4 + \frac{1}{5\times10^{-9}} \int_{4\times10^{-6}}^{t} -10\times10^{3}\xi + 50\times10^{-3}\, d\xi$$

$$= 4 + \frac{1}{5\times10^{-9}} \left[-10\times10^{3}\frac{\xi^2}{2} + 50\times10^{-3}\xi \right]_{4\times10^{-6}}^{t}$$

$$= 4 + \left[-10\times10^{3}\frac{t^2}{2} + 10\times10^{3}\frac{(4\times10^{-6})^2}{2} \right] + \left[50\times10^{-3}t - (50\times10^{-3})(4\times \right.$$

$$= -10^{12}t^2 + 10\times10^{6}t - 20 \rightarrow \text{parabola}$$

At $t = 5\mu sec$

$v(t) = 5\,V$

Q7.19) $C = 0.5\,\mu F$

$v = 5\times \sin 10^{3}t\ V$ $\quad 0 \leq 10^{3}t \leq 2\pi$

$i, P, w = ?$

take $10^{3}t = \theta$

$$\boxed{v_c = 5\sin\theta}$$

$$i_c = C\frac{dv_c}{dt}$$

$$= 0.5\times10^{-6} \times 5\times10^{3}\cos 10^{3}t$$

$$= 2.5\times10^{-3}\cos 10^{3}t$$

$$i_c = 2.5\cos 10^{3}t\ mA$$

$$\boxed{i_c = 2.5\cos\theta\ mA}$$

$$P = v_c i_c$$

$$= 12.5\sin 10^{3}t\cos 10^{3}t\ mW$$

$$= 6.25\sin 2\times10^{3}t\ mW$$

$$\boxed{P = 6.25 \sin 2\theta \text{ mW}}$$

$$w = \frac{1}{2} C V^2$$

$$= \frac{1}{2} \times (0.5 \times 10^{-6}) \times 25 \sin^2 10^3 t$$

$$= \frac{6.25 \times 10^{-6}}{2} \left[1 - \cos 2 \times 10^3 t\right]$$

$$= 3.125 \left[1 - \cos 2 \times 10^3 t\right] \mu J$$

$$\boxed{w = 3.125 (1 - \cos 2\theta) \mu J}$$

At $\theta = 0$

V (volts)	i (mA)	p (mW)	w (μJ)
$V = 0$	$i = 2.5$	$P = 0$	$w = 0$

At $\theta = \pi/2$

$V = 5$	$i = 0$	$P = 0$	$w = 6.25$

At $\theta = \pi$

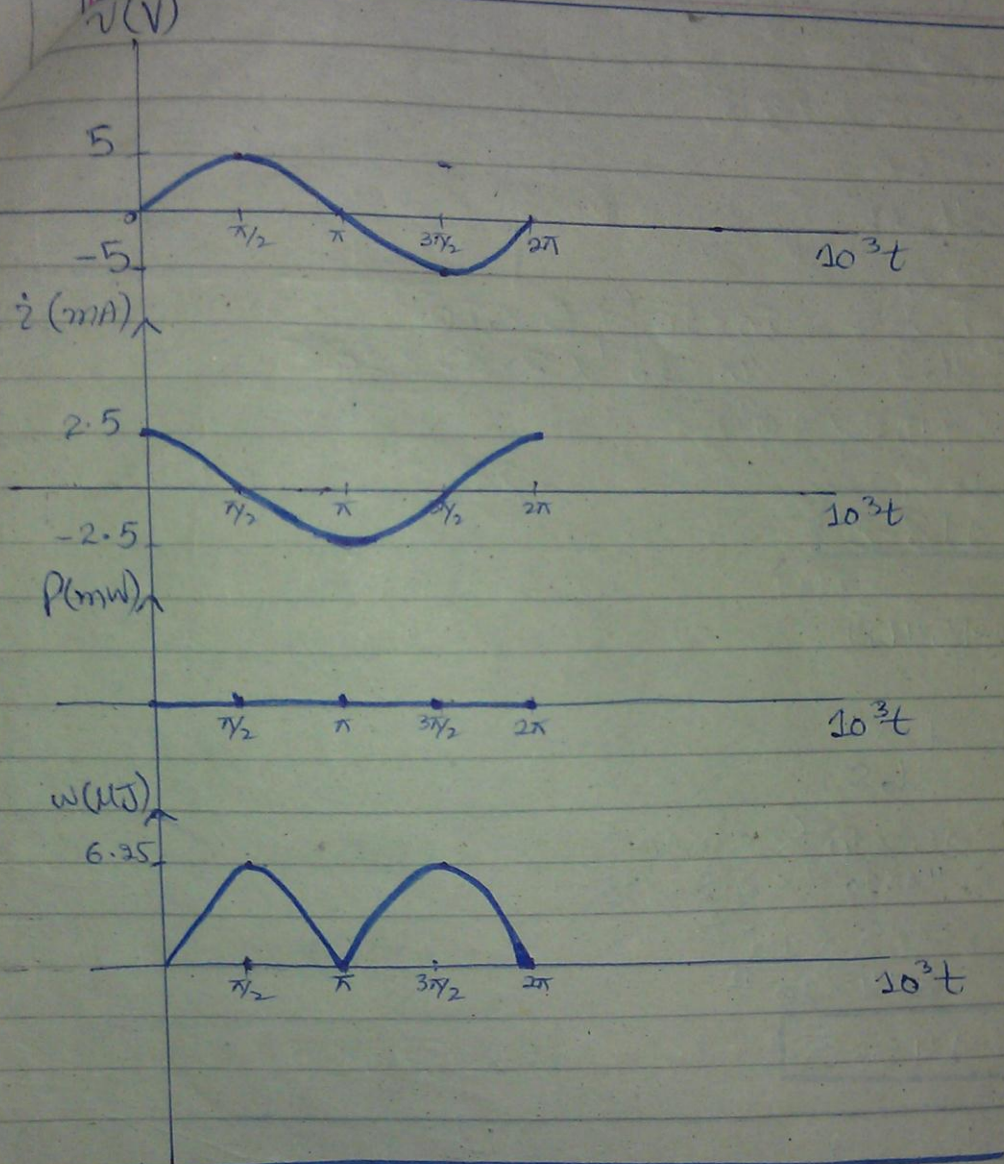
$V = 0$	$i = -2.5$	$P = 0$	$w = 0$

At $\theta = 3\pi/2$

$V = -5$	$i = 0$	$P = 0$	$w = 6.25$

At $\theta = 2\pi$

$v = 0$	$i = 2.5$	$P = 0$	$w = 0$



7.2: Inductance:

Q7.20) a) $N = ?$

$d = 6\,mm = 6 \times 10^{-3}\,m$

$l = 5\,cm = 5 \times 10^{-2}\,m$

$L = 10\,\mu H$

$\mu_0 = 4\pi \times 10^{-7}$

$r = \frac{d}{2} = 3 \times 10^{-3}\,m$

$$S = \pi r^2$$
$$S = 2.82 \times 10^{-5}$$

$$L = \mu_0 N^2 \frac{S}{l}$$

$$N^2 = \frac{Ll}{\mu_0 S} = \frac{10 \times 10^{-6} \times 5 \times 10^{-2}}{4\pi \times 10^{-7} \times 2.82 \times 10^{-5}}$$

$$N^2 = 1.412 \times 10^4$$

$$\boxed{N = 118.8}$$

b) $L = 15\,\mu H$

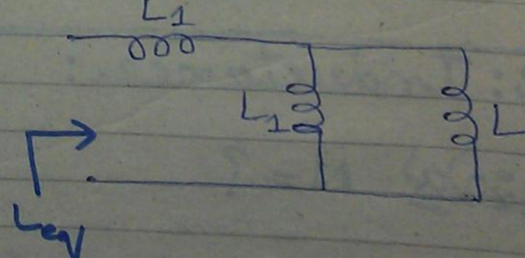
$$N^2 = \frac{Ll}{\mu_0 S}$$

$$N^2 = \frac{15 \times 10^{-6} \times 5 \times 10^{-2}}{4\pi \times 10^{-7} \times 2.82 \times 10^{-5}}$$

$$N^2 = \frac{7.5 \times 10^{-7}}{3.54 \times 10^{-11}}$$

$$\boxed{N = 145.5}$$

Q7.21) a) $L_{eq} = L$



$$L_{eq} = \frac{L_1 L}{L_1 + L} + L_1$$

$$L = \frac{L_1 L + L_1^2 + L L_1}{L_1 + L}$$

$$L L_1 + L^2 = 2 L L_1 + L_1^2$$

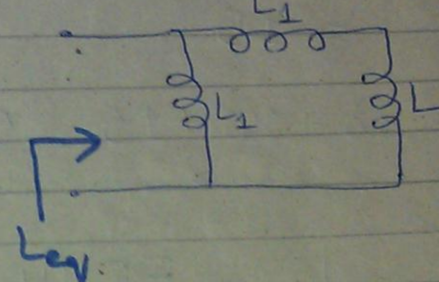
$$L^2 - L L_1 - L_1^2 = 0$$

$$L = \frac{L_1 \pm \sqrt{L_1^2 + 4L_1^2}}{2}$$

$$L = \frac{L_1 \pm L_1\sqrt{5}}{2}$$

$$\boxed{L = \frac{L_1(1+\sqrt{5})}{2}}$$

b)



$$L_{eq} = \frac{(L+L_1)L_1}{L+L_1+L_1}$$

$$L = \frac{LL_1 + L_1^2}{L+2L_1}$$

$$L^2 + 2LL_1 = LL_1 + L_1^2$$

$$L^2 + LL_1 - L_1^2 = 0$$

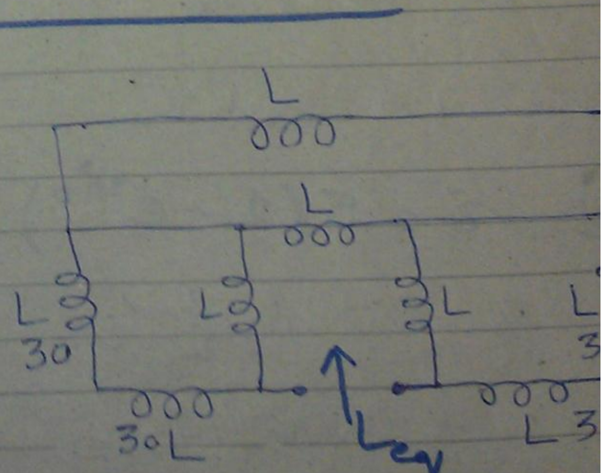
$$L = \frac{-L_1 \pm \sqrt{L_1^2 + 4L_1}}{2}$$

$$L = \frac{-L_1 \pm L_1\sqrt{5}}{2}$$

$$\boxed{L = \frac{L_1(\sqrt{5}-1)}{2}}$$

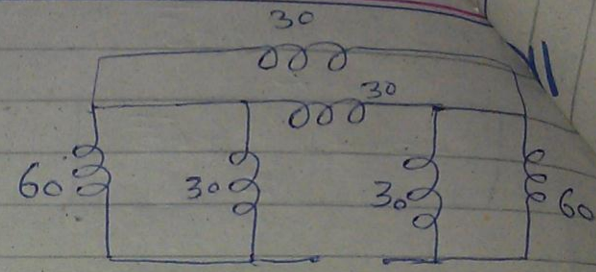
Q7.22) $L_{eq} = ?$
$L = 30\,mH$

30 series 30
= 60



$60 \parallel 30$

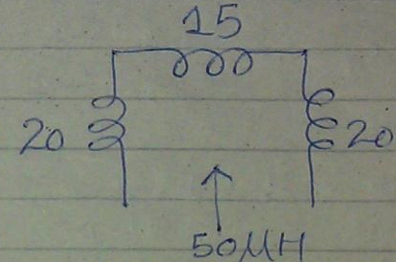
$= \frac{60 \times 30}{60+30} = 20$



$30 \parallel 30$

$= 15$

$20 + 20 + 15 = 55$



So,

$L_{eq} = 55 \mu H$

Q7.23) a) $L = 25 mH$

$V_L = ?$

a) $i = 10(1 - 10^2)t \; A$

$v = L \frac{di}{dt}$

$= 25 \times 10^{-3} \frac{d}{dt}(10 - 10 \times 10^2 t)$

$= 25 \times 10^{-3}(-10 \times 10^2)$

$V = -25 V$

b) $i = 5(1 - e^{-10^2 t}) A$

$v = 25 \times 10^{-3} \frac{d}{dt}(5 - 5e^{-10^2 t})$

$v = 25 \times 10^{-3}(-5)e^{-10^2 t} \cdot (-10^2)$

$v = 12.5 e^{-10^2 t} \; V$

$i = 10mA$

$v = 25\times10^{-3}\frac{d}{dt}\,10\times10^{-3}$

$v = 0\,V$

d) $i = 2\sin10^2t\ A$

$v = 25\times10^{-3}\frac{d}{dt}(2\sin10^2t)$

$v = 25\times10^{-3}\times 2\cos10^2t\times10^2$

$v = 5\cos10^2t\ V$

Q7.24) $L = 1mH$

$i(0) = 10\,mA$

a) $v = 10^{-5}t$

$i = \frac{1}{L}\int v\,dt + i(0)$

$= \frac{1}{1\times10^{-3}}\int10^{-5}t\,dt + 10\times10^{-3}$

$= 1\times10^{3}\times10^{-5}\frac{t^2}{2} + 10\times10^{-3}$

$= 5\times10^{-3}t^2 + 10\times10^{-3}$

$i = 5\times10^{-3}(5\times10^{-3}+t^2)\ A$

b) $v = 2e^{-10^5t}$

$i = \frac{1}{1\times10^{-3}}\int2e^{-10^5t}dt + 10\times10^{-3}$

$= 1\times10^{3}\cdot\frac{2e^{-10^5t}}{-10^5} + 10\times10^{-3}$

$$i = -0.02e^{-10^5 t} + 10\times10^{-3}$$
$$= 20\times10^{-3}e^{-10^5 t} + 10\times10^{-3}$$

$$i = 10\times10^{-3}\left(1 - 10\times10^{-3}e^{-10^5 t}\right) A$$

c) $v = 0.5\cos 10^5 t$

$$i = \frac{1}{1\times10^{-3}}\int 0.5\cos 10^5 t \, dt + 10\times10^{-3}$$
$$= 1\times10^{3}(0.5)\frac{(-\sin 10^5 t)}{10^5} + 10\times10^{-3}$$
$$= -5\times10^{-3}\sin 10^5 t + 10\times10^{-3}$$

$$i = 5\times10^{-3}\left(5\times10^{-3} - \sin 10^5 t\right) A$$

d) $v = 15V$

$$i = \frac{1}{1\times10^{-3}}\int 15\, dt + 10\times10^{-3}$$
$$= 1\times10^{3}\times15t + 10\times10^{-3}$$

$$i = 15\times10^{3}t + 10\times10^{-3} A$$

e) $v = 0V$

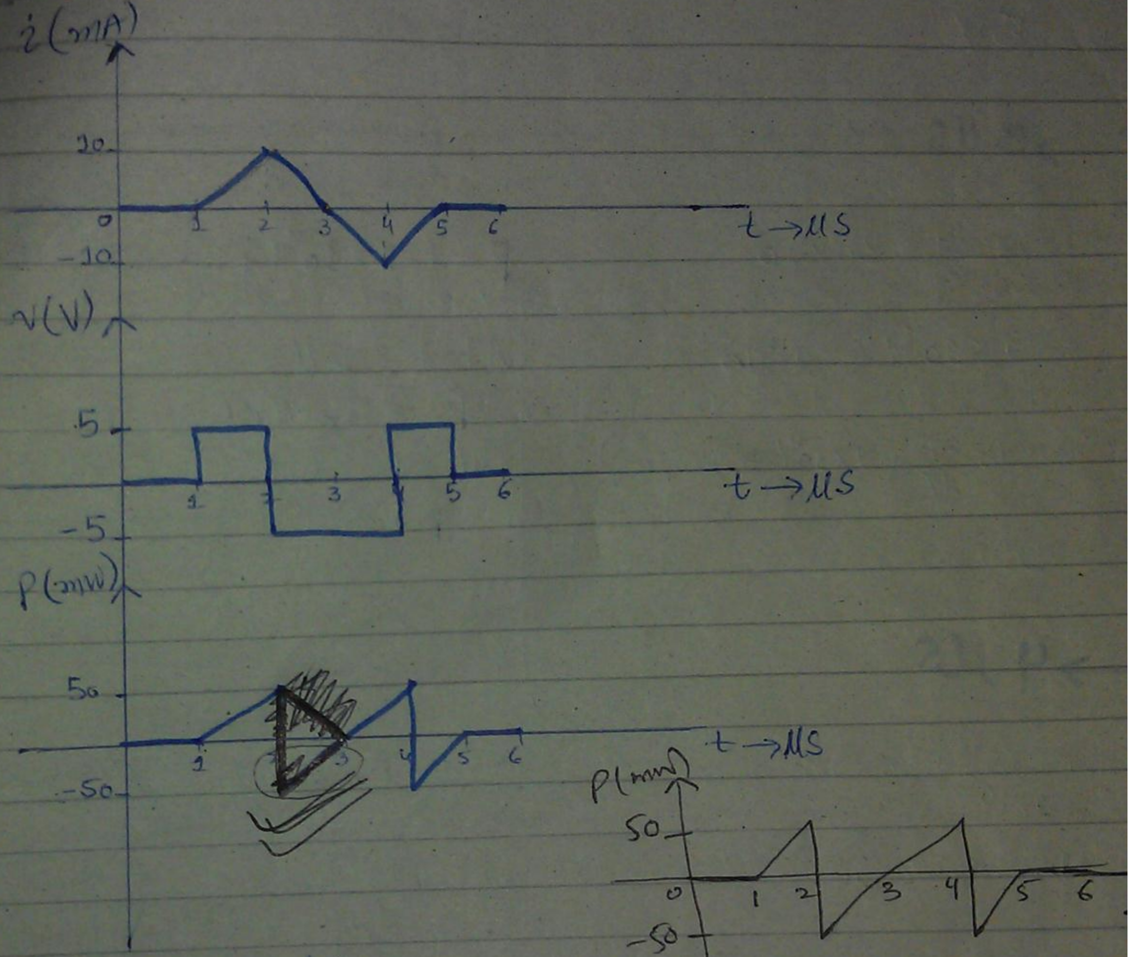
$$i = \frac{1}{1\times10^{-3}}\int 0\, dt + 10\times10^{-3}$$

$$i = 10\times10^{-3} A$$

$$i = 10 mA$$

1.25) $L = 500\,\mu H$

$V, P = ?$



$$v = L\frac{di}{dt}$$

0 → 1 μS

$i = 0$

$V = 0$

$P = 0$

1 → 2 μS

$$i = \frac{10\times10^{-3}}{1\times10^{-6}}t - 10\times10^{-3}$$

$$i = 1\times10^{4}t - 10\times10^{-3}$$

$P = vi$

$P = (1\times10^{4}t - 10\times10^{-3})$

put $t = 2\,\mu S$

$P = 50\,mW$

$v = 500 \times 10^{-6} \frac{d}{dt} 1 \times 10^{4} t - 10 \times 10^{-3}$

$v = 5V$

2 → 3 μS

$i = \frac{-10 \times 10^{-3}}{1 \times 10^{-6}} t + 30 \times 10^{-3}$

$= -1 \times 10^{4} t + 30 \times 10^{-3}$

$v = 500 \times 10^{-6} \frac{d}{dt} (-1 \times 10^{4} t + 30 \times 10^{-3})$

$v = -5V$

$P = (-1 \times 10^{4} t + 30 \times 10^{-3})(-5)$

At $t = 2 \mu S$

$P = -50 mW$

At $t = 3 \mu S$

$P = 0$

3 → 4 μS

$i = \frac{-10 \times 10^{-3}}{1 \times 10^{-6}} + 30 \times 10^{-6}$

$i = -1 \times 10^{4} t + 30 \times 10^{-3}$

$v = 500 \times 10^{-6} \frac{d}{dt} (-1 \times 10^{4} + 30)$

$v = -5V$

$P = (-1 \times 10^{4} t + 30 \times 10^{-3})(-5)$

At $t = 3 \mu S$

$P = 0$

At $t = 4 \mu S$

$P = 50 mW$

4 → 5 μS

$i = \frac{10 \times 10^{-3} t}{1 \times 10^{-6}} - 50 \times 10^{-3}$

$i = 1 \times 10^{4} t - 50 \times 10^{-3}$

$v = 500 \times 10^{-6} \frac{d}{dt} (1 \times 10^{4} t - 50 \times 10^{-3})$

$v = 5V$

$P = (1 \times 10^{4} t - 50 \times 10^{-3})(5)$

At $t = 4 \mu S$

$P = -50 mW$

At $t = 5 \mu S$

$P = 0$

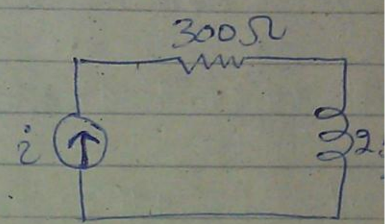
5 → 6 μS

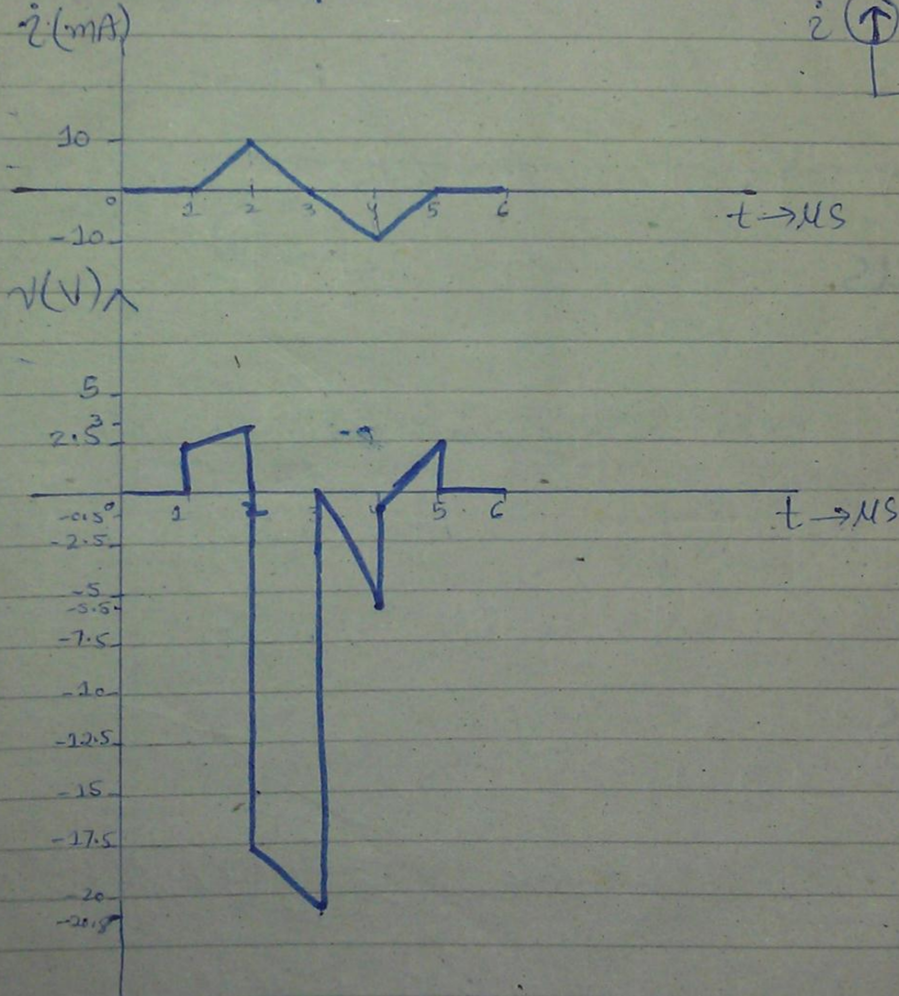
5 → 6 μS

$i = 0$
$v = 0$

$P = 0$

Q7.26 $L = 250\ \mu H$
$R = 300\Omega$
$v = ?$





$$v = iR + L\frac{di}{dt}$$

$0 \to 1\,\mu s$

$i = 0$

$v = 0$

$1 \to 2\,\mu s$

$i = 1 \times 10^4 t - 10 \times 10^{-3}$

$v = (1 \times 10^4 t - 10 \times 10^{-3})(300) + 250 \times 10^{-6} \frac{d}{dt}(1 \times 10^4 t - 10 \times 10^{-3})$

At $t = 1\,\mu s$	At $t = 2\,\mu s$
$v = 2.5\,V$	$v = 3\,V$

$2 \to 3\,\mu s$

$i = -1 \times 10^4 t - 30 \times 10^{-3}$

$v = (-1 \times 10^4 t - 30 \times 10^{-3})(300) + 250 \times 10^{-6} \frac{d}{dt}(-1 \times 10^4 t - 30 \times 10^{-3})$

At $t = 2\,\mu s$	At $t = 3\,\mu s$
$v = -17.5\,V$	$v = -20.5\,V$

$3 \to 4\,\mu s$

$i = -1 \times 10^4 t + 30 \times 10^{-3}$

$v = (-1 \times 10^4 t + 30 \times 10^{-3})(300) + 250 \times 10^{-6} \frac{d}{dt}(-1 \times 10^4 t + 30 \times 10^{-3})$

At $t = 3\,\mu s$	At $t = 4\,\mu s$
$v = 0\,V$	$v = -5.5\,V$

$4 \rightarrow 5\,\mu s$

$i = 1\times10^{4}t - 50\times10^{-3}$

$v = (1\times10^{4}t - 50\times10^{-3})(300) + 250\times10^{-6}\frac{d}{dt}(1\times10^{4}t - 50\times10^{-3})$

At $t = 4\,\mu s$	At $t = 5\,\mu s$
$v = -0.5\,V$	$v = 2.5\,V$

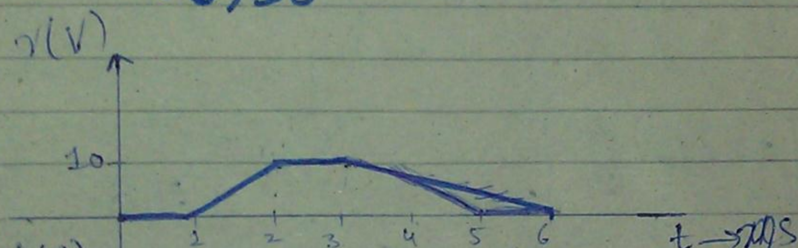
$5 \rightarrow 6\,\mu s$

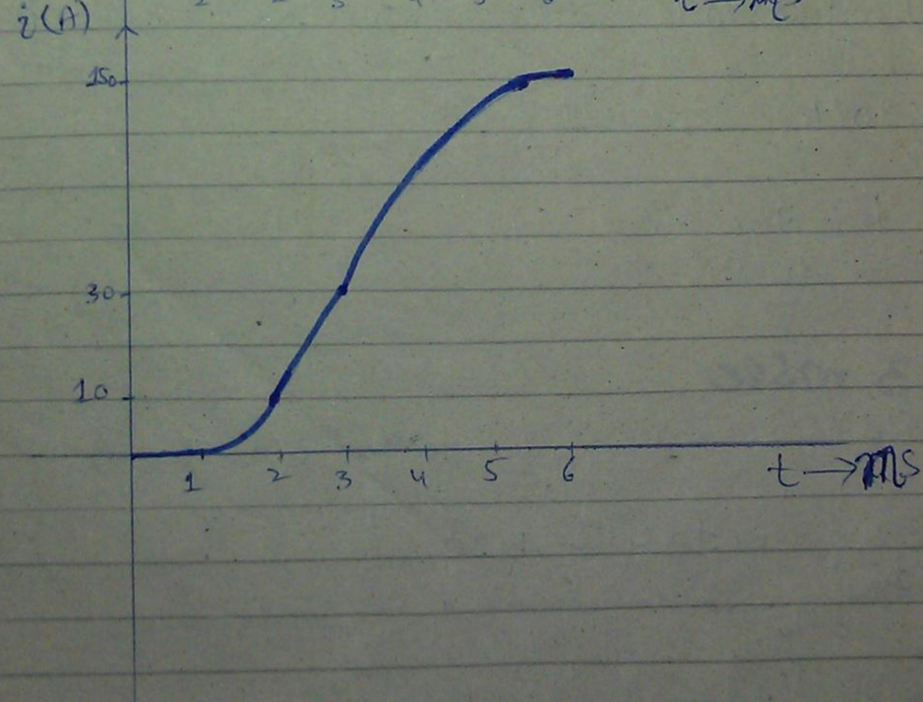
$i = 0$

$v = 0$

Q7.27) $L = 2mH$

$i(0) = 0$





$$i(t) = \frac{1}{L}\int_0^t v(\xi)\,d\xi + i(0)$$

$0 \to 1$ msec

$v = 0$

$i = 0$

$1 \to 2$ msec

$$v = \frac{10}{1\times10^{-3}}t - 10$$

$$v = 10\times10^{3}t - 10$$

$$i = \frac{1}{2\times10^{-3}}\int_{1\times10^{-3}}^{t} 10\times10^{3}\xi - 10\,d\xi + 0$$

$$= 2\times10^{3}\left[10\times10^{3}\frac{\xi^2}{2}\right]_{1\times10^{-3}}^{t} - \left|10\xi\right|_{1\times10^{-3}}^{t}$$

$$= 2\times10^{3}\left\{\left[10\times10^{3}\frac{t^2}{2} - 5\times10^{-3}\right] - \left[10t - 0.01\right]\right\}$$

$$= 1\times10^{7}t^2 - 2\times10^{4}t + 10$$

put $t = 2$ msec

$i = 10\,A$

$2 \to 3$ msec

$v = 10\,V$

$$i = \frac{1}{2\times10^{-3}}\int_{2\times10^{-3}}^{t} 10\,d\xi + 10$$

$$= 2\times10^{3}\left|10\xi\right|_{2\times10^{-3}}^{t} + 10$$

$= 2\times10^{3}[10t - 0.02] + 10$

$i = 2\times10^{4}t - 30$

put $t = 3$ msec

$i = 30A$

3 → 5 msec

$V = \frac{-10}{2\times10^{-3}}t + 50$

$V = -5\times10^{3}t + 50$

$i = \frac{1}{2\times10^{-3}}\int_{3\times10^{-3}}^{t} -5\times10^{3}\xi + 50\, d\xi + 30$

$= 2\times10^{3}\left|-5\times10^{3}\frac{\xi^{2}}{2}\right|_{3\times10^{-3}}^{t} + \left|50\xi\right|_{3\times10^{-3}}^{t} + 30$

$= 2\times10^{3}\left\{\left[-5\times10^{3}\frac{t^{2}}{2} + 22.5\times10^{-3}\right] + \left[50t - 0.15\right]\right\} + 30$

$i = -5\times10^{6}t^{2} + 1\times10^{5}t - 225$

put $t = 5$ msec

$i = 150A$

5 → 6 msec

$V = 0$

$i = 0$

Q7.28) $L = 1.5\,mH$
$i = 2 \times \sin 10^6 t \; mA$

Take $10^6 t = \theta$ $\quad 0 \le \theta \le 2\pi$

$\boxed{i = 2\sin\theta \; mA}$

$v = ?$

$V = L\frac{di}{dt}$

$= 1.5\times10^{-3}\frac{d}{dt}(2\times\sin 10^6 t)\,m$

$= 2\times1.5\times10^{-3}(\cos 10^6 t)\times10^6\,m$

$\boxed{V = 3\cos\theta \; V}$

$P = vi$
$= (3\cos\theta)(2\sin\theta)$
$= 3.2\sin\theta\cos\theta$

$\boxed{P = 3\sin 2\theta \; mW}$

$w = \frac{1}{2}LI^2$

$w = \frac{1}{2}(1.5\times10^{-3})(2\sin\theta\,m)^2$

$\boxed{w = 3(1-\cos 2\theta)\,nJ}$

Put $\theta = 0$

$v\,(V)$	$i\,(mA)$	$P\,(mW)$	$w\,(nJ)$
$V = 3$	$i = 0$	$P = 0$	$w = 0$

$\theta = \pi/2$

$V = 0$	$i = 2$	$P = 0$	$w = 6$

$\theta = \pi$

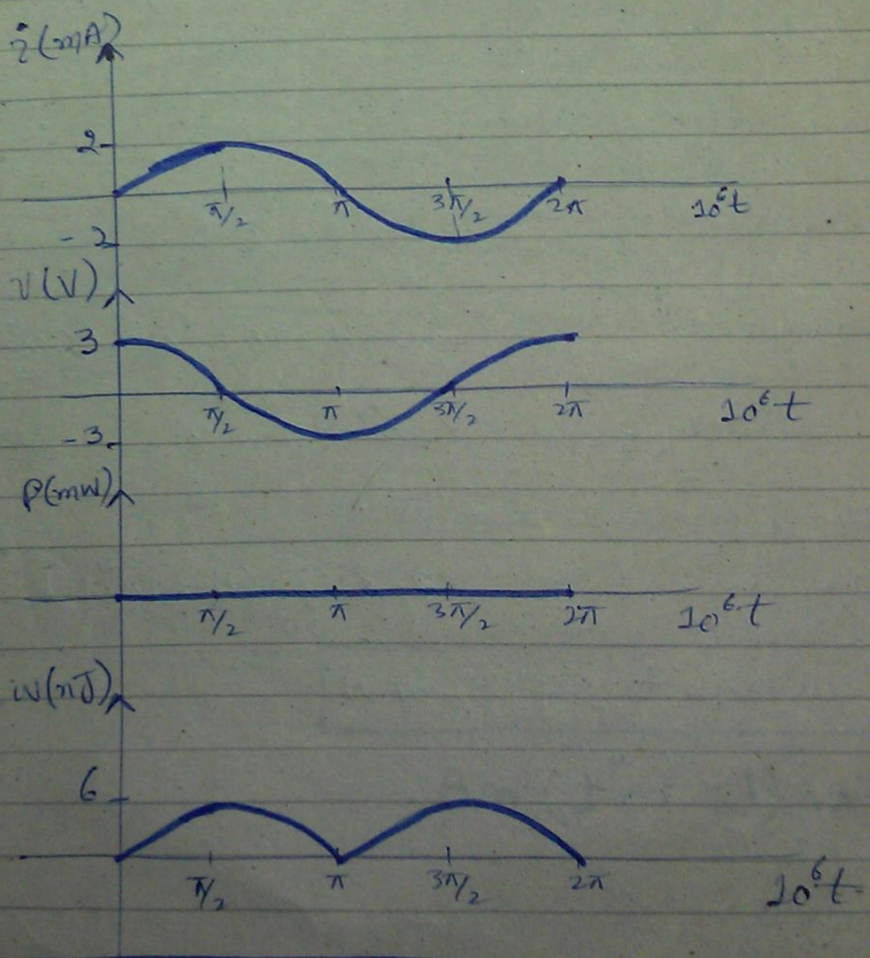
$V = -3$	$i = 0$	$P = 0$	$w = 0$

$\theta = 3\pi/2$

$V = 0$	$i = -2$	$P = 0$	$w = 6$

$\theta = 2\pi$

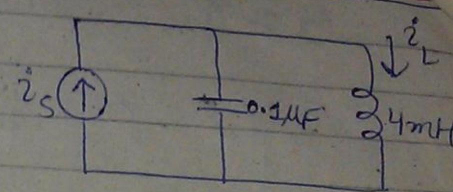
$V = 3$	$i = 0$	$P = 0$	$w = 0$



Q7.29)

a) $i_L = 20\sin(4\times10^4 t)\,mA$

$i_s = ?$



$i_s = i_c + i_L$

$V_L = L\dfrac{di_L}{dt}$

$= 4\times10^{-3}\dfrac{d}{dt}\,20\sin(4\times10^4 t)\,mA$

$= 4\times10^{-3}\times20\cos(4\times10^4 t).\,4\times10^4$

$V_L = 3.2\cos(4\times10^4 t)V$

$V_L = V_C = V_S$

$i_c = C\dfrac{dV_c}{dt}$

$= 0.1\times10^{-6}.\dfrac{d}{dt}(3.2\cos4\times10^4 t)$

$= 0.1\times10^{-6}\times3.2(-\sin4\times10^4 t)\times4\times10^4$

$i_c = 12.8(-\sin4\times10^4 t)\,mA$

$i_s = i_L + i_c$

$i_s = 20\sin(4\times10^4 t) - 12.8\sin(4\times10^4 t)$

$\boxed{i_s = \sin4\times10^4 t\times7.2\ mA}$

b) $i_L = 20\sin(6\times10^4 t)\,mA$

$V_L = L\dfrac{di_L}{dt}$

$$V_L = 4\times10^{-3}\frac{d}{dt}\,20\sin(6\times10^4 t)$$

$$= 4\times10^{-3}\times20\cos(6\times10^4 t)\times6\times10^4$$

$$= 4800\cos(6\times10^4 t)\ mV$$

$$V_L = 4.8\cos(6\times10^4 t)\ V$$

$$i_c = C\frac{dV_c}{dt}$$

$$= 0.1\times10^{-6}\frac{d}{dt}(4.8\cos(6\times10^4 t))\ V$$

$$= 0.1\times10^{-6}\times4.8(-\sin 6\times10^4 t)\times6\times10^4$$

$$i_c = -2.8\sin(6\times10^4 t)\ mA$$

$$i_s = 20\sin(6\times10^4 t) - 2.8\sin(6\times10^4 t)$$

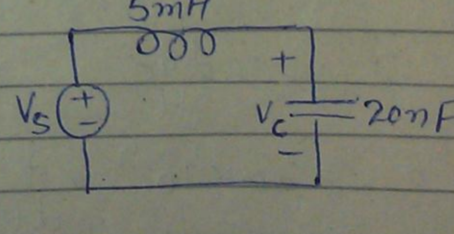
$$\boxed{i_s = 17.2\sin 6\times10^4 t\ mA}$$

Q7.30)

$V_s = ?$

a)

$V_c = 10\sin 10^4 t\ V$



$$i_c = C\frac{dV_c}{dt}$$

$$= 20\times10^{-9}\frac{d}{dt}\,10\sin10^4 t$$

$$i_c = 20\times10^{-9}\times10\cos(10^4 t)\times10^4$$

$$i_c = 2\cos(10^4 t)\ mA$$

As, $i_C = i_L = i_S$

$v_L = L \frac{di_L}{dt}$

$= 5\times10^{-3} \frac{d}{dt} (2\cos 10^4 t) \times 10^{-3}$

$V_L = 5\times10^{-3} \times 2\times10^{-3} (-\sin 10^4 t)\times 10^4$

$V_L = -0.1 \sin(10^4 t)\ V$

$v_S = V_C + V_L$

$v_S = 10\sin 10^4 t - 0.1\sin 10^4 t$

$\boxed{v_S = \sin 10^4 t\,(9.9)\ V}$

b)

$V_C = 10\sin 10^5 t\ V$

$i_C = C \frac{dV_C}{dt}$

$= 20\times10^{-9} \times \frac{d}{dt}\, 10\sin 10^5 t$

$i_C = 20\times10^{-9} \times 10\cos(10^5 t)\times 10^5$

$i_C = 0.02\cos 10^5 t\ A$

b)

$v_L = L\frac{di_L}{dt}$

$v_L = 5\times10^{-3} \frac{d}{dt}(0.02\cos 10^5 t)$

$v_L = 10(-\sin 10^5 t)\,V$

$v_S = v_C + v_L$

$= \cancel{10\sin 10^5 t - 10\sin 10^5 t}$

$\boxed{v_S = 0\,V}$

7.3: Natural Response of RC & RL circuits:

Q7.31

$s = ?$

$$5\times10^{-3}\frac{d}{dt}y(t) + 2y(t) = 10^{-3}\frac{d}{dt}x(t) - x(t) \quad (1)$$

$$\tau\frac{dy(t)}{dt} + y(t) = x(t)$$

$\div$ (1) by 2

$$\frac{5\times10^{-3}}{2}\frac{d}{dt}y(t) + y(t) = \frac{10^{-3}}{2}\frac{dx(t)}{dt} - \frac{1}{2}x(t)$$

$$\tau = \frac{5\times10^{-3}}{2} = 2.5\times10^{-3}\,s$$

$$s = -\frac{1}{\tau} = \frac{-1}{2.5\times10^{-3}}$$

$\boxed{s = -400\ Np/sec}$

Q.7.32)

$V(t) = V(0)e^{-t/\tau}$

$V(1\mu S) = 12.28V$

$V(7\mu S) = 3.7V$

a) $V(0) = ?$

$\tau = ?$

$12.28 = V(0)e^{-\frac{1\times10^{-6}}{\tau}}$ —— ①

$3.7 = V(0)e^{-\frac{7\times10^{-6}}{\tau}}$ —— ②

① ÷ ②

$\frac{12.28}{3.7} = \frac{V(0)\,e^{-\frac{1\times10^{-6}}{\tau}}}{V(0)\,e^{-\frac{7\times10^{-6}}{\tau}}}$

$3.3 = e^{6\times10^{-6}/\tau}$

$\ln 3.3 = \ln e^{6\times10^{-6}/\tau}$

$1.193 = \frac{6\times10^{-6}}{\tau}$

$\boxed{\tau = 5.02\times10^{-6}\ S}$

Put in ①

$12.28 = V(0)\,e^{-1\times10^{-6}/5.02\times10^{-6}}$

$V(0) = 12.28e^{0.2}$

$\boxed{V(0) = 15V}$

$v(10\mu S) = ?$

$v(10\mu S) = v(0)e^{-t/\tau}$

$= 15e^{-10\times10^{-6}/5\times10^{-6}}$

$= 15e^{-2} = 15/e^{2}$

$$\boxed{v(10\mu S) = 2V}$$

Q7.33

$i(t) = i(0)e^{-t/\tau}$

$t = ?\ (6 \rightarrow 4mA)$

$8 \rightarrow 2\,mA$

$t_{\varepsilon} = 5.545\,msec$

$t_{\varepsilon} = -\tau \ln \varepsilon$

$\cancel{t_{\varepsilon} = -5.45 \ln(2-8)}$

$\cancel{= -5.45}$

$5.45 = -\tau \ln(2-8)$

$5.45 = -\tau \ln \frac{2}{8}$

$\tau = 3.95$

$6 \rightarrow 4mA$

$t = -3.95 \ln(4-6)$

$t = -3.95 \ln \frac{4}{6}$

$$\boxed{t = 1.601\ msec}$$

$9 \rightarrow 1\,mA$

$\varepsilon = 1/9$

$t = -3.95 \ln (1/9)$

$\boxed{t = 8.67\, msec}$

Q7.34) Solution = ?

$$2\frac{dy(t)}{dt} + y(t) = 5e^{-t}$$

$$y(0) = 10$$

$\div$ by 2

$$\frac{d}{dt}y(t) + \frac{1}{2}y(t) = \frac{5}{2}e^{-t} \quad (1)$$

$$P = 1/2$$

For Integrating factor

$$e^{\int P dt} = e^{\int 1/2\, dt} = e^{t/2}$$

$(1) \Rightarrow e^{t/2}$

$$e^{t/2}\frac{dy(t)}{dt} + \frac{1}{2}e^{t/2}y(t) = \frac{5}{2}e^{t/2}\cdot e^{-t}$$

product rule

$$\frac{d}{dt}\left(e^{t/2}\cdot y(t)\right) = \frac{5}{2}e^{-t/2}$$

Integrating both sides

$$y(t)e^{t/2} = \frac{5}{2}\,\frac{e^{-t/2}}{-1/2} + c$$

$y(t)e^{t/2} = -5e^{-t/2} + c$ —— ②

using initial condition

$y(0) = 10$

put $t = 0$

② $\Rightarrow$ $y(0)e^{0} = -5e^{0} + c$

$10 = -5 + c$

$c = 15$

put in ②

$y(t)\,e^{t/2} = -5e^{t/2} + 15$

$y(t) = -5\dfrac{e^{t/2}}{e^{t/2}} + \dfrac{15}{e^{t/2}}$

$= -5e^{-t} + 15e^{-t/2}$

$= 5(-e^{-t} + 3e^{-t/2})$

$$\boxed{y(t) = 5(3e^{-t/2} - e^{-t})}$$

Q7.35)

$2\dfrac{dy(t)}{dt} + 3y(t) = x(t)$ —— ①

$x(t) = 2te^{-t}$, $y(0) = 5$

put $x(t)$ in ①

$2\dfrac{dy(t)}{dt} + 3y(t) = 2te^{-t}$

$\div$ by 2

$$\frac{d}{dt}y(t) + \frac{3}{2}y(t) = te^{-t} \quad \text{—— } (2)$$

$$P = 3/2$$

$$e^{\int P dt} = e^{\int 3/2\, dt} = e^{3/2 t}$$

$\times$ (2) by $e^{3/2 t}$

$$e^{3/2 t}\frac{d}{dt}y(t) + \frac{3}{2}y(t)e^{3/2 t} = te^{3/2 t}.e^{-t}$$

$$\frac{d}{dt}\left(y(t).e^{3/2 t}\right) = t.e^{t/2}$$

Integrating both sides

$$y(t)e^{3/2 t} = t.\frac{e^{t/2}}{1/2} - \int \frac{e^{t/2}}{1/2}dt$$

$$= 2te^{t/2} - 2\int e^{t/2}dt$$

$$y(t)e^{3/2 t} = 2te^{t/2} - 4e^{t/2} + C \quad \text{—— } (3)$$

put $y(0) = 5$ & $t = 0$

$$5e^0 = -4e^0 + C$$

$$C = 9$$

put in (3)

$$y(t)e^{3/2 t} = 2te^{t/2} - 4e^{t/2} + 9$$

$$y(t) = 2t\frac{e^{t/2}}{e^{3/2 t}} - \frac{4e^{t/2}}{e^{3/2 t}} + \frac{9}{e^{3/2 t}}$$

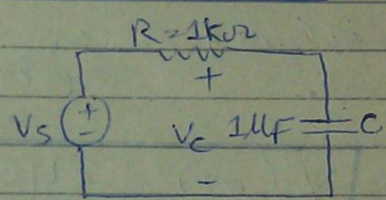
$$\boxed{y(t) = 2e^{-t}(t-2) + 9e^{-3/2 t}}$$

Q 7.36)

$w(0) = 450\,\mu J$

$v(2ms) = 0$

$V_S = v(\infty) = ?$



$$v(t) = [y(0) - y(\infty)]e^{-t/\tau} + y(\infty)$$

$$\tau = R \times C = 1\times10^{-6} \times 1\times10^{3}$$

$$\tau = 1\times10^{-3}$$

$$w = \frac{1}{2}CV_c^2(0)$$

$$450\times10^{-6} = \frac{1}{2}(1\times10^{-6})V^2(0)$$

$$V(0) = 30V$$

$$v(t) = [30 - y(\infty)]e^{-t/1\times10^{-3}} + y(\infty)$$

$$v(2mS) = 0$$

$$0 = [30 - y(\infty)]e^{-2\times10^{-3}/1\times10^{-3}} + y(\infty)$$

$$0 = \frac{30}{e^2} - \frac{y(\infty)}{e^2} + y(\infty)$$

$$y(\infty) = V(\infty)$$

$$V(\infty) + \frac{V(\infty)}{e^2} = \frac{30}{e^2}$$

$$V(\infty)\left[1 + \frac{1}{e^2}\right] = \frac{30}{e^2}$$

$$V(\infty) = \frac{30}{e^2} \times \frac{e^2}{-e^2+1}$$

$$\boxed{V(\infty) = -4.7V}$$

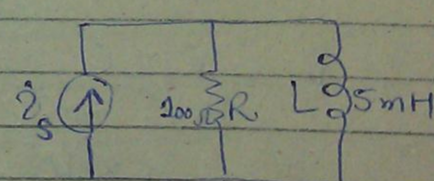
Q7.37)

$i(0) = ?$

$w = ?$

$i(\infty) = I = 25mA$

$i(40\mu S) = 0$



$$\tau = \frac{L}{R} = 5\times10^{-5}$$

$$i(t) = [i(0) - i(\infty)]e^{-t/\tau} + i(\infty)$$

$$i(40\mu S) = 0$$

$$0 = [i(0) - 25\times10^{-3}]e^{-40\times10^{-6}/5\times10^{-5}} + 25\times10^{-3}$$

$$0 = 0.449i(0) - 0.0112 + 25\times10^{-3}$$

$$0.449i(0) = -0.0137$$

$$i(0) = -0.0306A$$

$\boxed{i(0) = -30.6\, mA}$

$w = \frac{1}{2} L I^2$

$= \frac{1}{2}(5\times10^{-3})(-30.6\times10^{-3})^2$

$\boxed{w = 2.34\, \mu J}$

Q.7.38) a)

$v = 5(1 + e^{-t/\tau})\, V$

$v = 5e^{-t/\tau} + 5$

Transient $= [y(0) - y(\infty)] e^{-t/\tau}$

$= 5e^{-t/\tau}$

Steady state $= y(\infty) = 5$

$y(\infty) = 5$

$y(0) - y(\infty) = 5$

$y(0) - 5 = 5$

$y(0) = 10$

$y_{natural} = y(0) e^{-t/\tau}$

$= 10 e^{-t/\tau}$

$$y_{forced} = y(\infty)(1 - e^{-t/\tau})$$
$$= 5(1 - e^{-t/\tau})$$
$$y_{forced} = 5 - 5e^{-t/\tau}$$

b)

$$V = 5(2 - e^{-t/\tau})V$$
$$V = -5e^{-t/\tau} + 10$$
$$V_{transient} = -5e^{-t/\tau}$$
$$V_{steady\,state} = 10$$
$$y(\infty) = 10$$
$$y(0) - y(\infty) = -5$$
$$y(0) - 10 = -5$$
$$y(0) = 5$$
$$y_{natural} = y(0)e^{-t/\tau}$$
$$y_{natural} = 5e^{-t/\tau}$$
$$y_{forced} = y(\infty)[1 - e^{-t/\tau}]$$
$$= 10[1 - e^{-t/\tau}]$$
$$y_{forced} = 10 - 10e^{-t/\tau}$$

$v = 5(3e^{-t/\tau} + 2)\ V$

$v = 15e^{-t/\tau} + 10$

$y(\infty) = 10$

$y(0) - y(\infty) = 15$

$y(0) - 10 = 15$

$y(0) = 25$

$y_{transient} = 15e^{-t/\tau}$

$y_{s.s} = 10$

$y_{natural} = 25e^{-t/\tau}$

$y_{forced} = 10(1 - e^{-t/\tau})$

Q7.39) a) $i = 5(2 - 3e^{-t/\tau})\ A$

$i = -15e^{-t/\tau} + 10$

$i_{transient} = -15e^{-t/\tau}$

$i_{s.s} = 10$

$y(\infty) = 10$

$y(0) - y(\infty) = -15$

$y(0) = -5$

$y_{natural} = y(0)\, e^{-t/\tau}$

$i_{natural} = -5 e^{-t/\tau}$

$i_{forced} = y(\infty)\left(1 - e^{-t/\tau}\right)$

$= 10\left(1 - e^{-t/\tau}\right)$

b) $i = 5e^{-t/\tau}$

$i_{transient} = 5 e^{-t/\tau}$

$i_{s.s} = 0$

$y(\infty) = 0$

$y(0) - y(\infty) = 5$

$y(0) = 5$

$i_{natural} = 5e^{-t/\tau}$

$i_{forced} = i(\infty)\left[1 - e^{-t/\tau}\right]$

$i_{forced} = 0$