Chapter 2 Instructor Notes

Chapter 2 develops the foundations for the first part of the book. Coverage of the entire Chapter would be typical in an introductory course. The first four sections provide the basic definitions and cover Kirchoff's Laws and the passive sign convention; the box *Focus on Methodology: The Passive Sign Convention* (p. 35) and two examples illustrate the latter topic. The sidebars *Make The Connection: Mechanical Analog of Voltage Sources* (p. 20) and *Make The Connection: Hydraulic Analog of Current Sources* (p. 22) present the concept of analogies between electrical and other physical domains; these analogies will continue through the first six chapters.

Sections 2.5and 2.6 introduce the *i-v* characteristic and the resistance element. Tables 2.1 and 2.2 on p. 41 summarize the resistivity of common materials and standard resistor values; Table 2.3 on p. 44 provides the resistance of copper wire for various gauges. The sidebar *Make The Connection: Electric Circuit Analog of Hydraulic Systems – Fluid Resistance* (p. 40) continues the electric-hydraulic system analogy.

Finally, Sections 2.7 and 2.8 introduce some basic but important concepts related to ideal and non-ideal current sources, and measuring instruments.

The Instructor will find that although the material in Chapter 2 is quite basic, it is possible to give an applied flavor to the subject matter by emphasizing a few selected topics in the examples presented in class. In particular, a lecture could be devoted to *resistance devices*, including the resistive displacement transducer of *Focus on Measurements: Resistive throttle position sensor* (pp. 52-54), the resistance strain gauges of *Focus on Measurements: Resistance strain gauges* (pp. 54-55), and *Focus on Measurements: The Wheatstone bridge and force measurements* (pp. 55-56). The instructor wishing to gain a more in-depth understanding of resistance strain gauges will find a detailed analysis in[1].

Early motivation for the application of circuit analysis to problems of practical interest to the non-electrical engineer can be found in the *Focus on Measurements: The Wheatstone bridge and force measurements*. The Wheatstone bridge material can also serve as an introduction to a laboratory experiment on strain gauges and the measurement of force (see, for example[2]). Finally, the material on practical measuring instruments in Section2.8b can also motivate a number of useful examples.

The homework problems include a variety of practical examples, with emphasis on instrumentation. Problem 2.36 illustrates analysis related to fuses; problems 2.44-47 are related to wire gauges; problem 2.52 discusses the thermistor; problems 2.54 and 2.55 discuss moving coil meters; problems 2.52 and 2.53 illustrate calculations related to temperature sensors; an problems 2.56-66 present a variety of problems related to practical measuring devices.

It has been the author's experience that providing the students with an early introduction to practical applications of electrical engineering to their own disciplines can increase the interest level in the course significantly.

Learning Objectives

1. Identify the principal *elements of electrical circuits*: nodes, loops, meshes, branches, and voltage and current sources.
2. Apply *Kirchhoff's Laws* to simple electrical circuits and derive the basic circuit equations.
3. Apply the *passive sign convention* and *compute power* dissipated by circuit elements.
4. Apply the *voltage and current divider laws* to calculate unknown variables in simple series, parallel and series-parallel circuits.
5. Understand the rules for connecting *electrical measuring instruments* to electrical circuits for the measurement of voltage, current, and power.

[1] E. O. Doebelin, Measurement Systems – Application and Design, 4th Edition, McGraw-Hill, New York, 1990.

[2] G. Rizzoni, A Practical Introduction to Electronic Instrumentation, 3rd Edition, Kendall-Hunt, 1998.

Section 2.1: Definitions

Problem 2.1

Solution:

Known quantities:

Initial Coulombic potential energy; initial velocity; final Coulombic potential energy.

Find:

The change in velocity of the electron.

Assumptions:

$$\Delta PE_g << \Delta PE_c$$

Analysis:

Using the first law of thermodynamics, we obtain the final velocity of the electron:

$$Q_{heat} - W = \Delta KE + \Delta PE_c + \Delta PE_g + \ldots$$

Heat is not applicable to a single particle. W=0 since no external forces are applied.

$$\Delta KE = -\Delta PE_c$$

$$\frac{1}{2} m_e (U_f^2 - U_i^2) = -Q_e (V_f - V_i)$$

$$U_f^2 = U_i^2 - \frac{2Q_e}{m_e}(V_f - V_i)$$

$$= \left(93\,M\,\frac{m}{s}\right)^2 - \frac{2(-1.6\times10^{-19}\,C)}{9.11\times10^{-34}\,kg}(6V - 17V)$$

$$= 8.649\times10^{15}\,\frac{m^2}{s^2} - 3.864\times10^{15}\,\frac{m^2}{s^2}$$

$$U_f = 6.917\times10^{7}\,\frac{m}{s}$$

$$\left|U_f - U_i\right| = 93\,M\,\frac{m}{s} - 69.17\,M\,\frac{m}{s} = 23.83\,M\,\frac{m}{s}.$$

Problem 2.2

Solution:

Known quantities:

MKSQ units.

Find:

Equivalent units of volt, ampere and ohm.

Analysis:

$$\text{Voltage} = Volt = \frac{Joule}{Coulomb} \quad V = \frac{J}{C}$$

$$\text{Current} = Ampere = \frac{Coulomb}{second} \quad a = \frac{C}{s}$$

$$\text{Resistance} = Ohm = \frac{Volt}{Ampere} = \frac{Joule \times second}{Coulomb^2} \quad \Omega = \frac{J \cdot s}{C^2}$$

$$\text{Conductance} = Siemen\ or\ Mho = \frac{Ampere}{Volt} = \frac{C^2}{J \cdot s}$$

Problem 2.3

Solution:

Known quantities:
Battery nominal rate.

Find:
Charge potentially derived from the battery, electrons contained in that charge.

Assumptions:
Battery fully charged.

Analysis:

a)

$$100A \times 1hr = \left(100\frac{C}{s}\right)(1hr)\left(3600\frac{s}{hr}\right)$$
$$= 360000\,C$$

b)

charge on electron: $-1.602 \times 10^{-19}\ C$

no. of electrons =

$$\frac{360 \times 10^3}{1.602 \times 10^{-19}} = 224.7 \times 10^{22}$$

Problem 2.4

Solution:

Known quantities:
Diagram of current vs. time; diagram of voltage vs. time.

Find:

The charge transferred to the battery; the energy transferred to the battery.

Analysis:

a) To find the charge delivered to the battery during the charge cycle we examine the charge-current relationship:

$$i = \frac{dq}{dt} \quad or \quad dq = i \cdot dt$$

thus:

$$Q = \int_{t_0}^{t} i(t)dt$$

$$Q = \int_{0}^{5hrs} 50mAdt + \int_{5hrs}^{10hrs} 20mAdt$$

$$= \int_{0}^{18000s} 0.05dt + \int_{18000}^{36000s} 0.02dt$$

$$= 900 + 360$$

$$= 1260C$$

b) To find the energy transferred to the battery, we examine the energy relationship

$$p = \frac{dw}{dt} \quad or \quad dw = p(t)dt$$

$$w = \int_{t_0}^{t} p(t)dt = \int_{o}^{t} v(t)i(t)dt$$

observing that the energy delivered to the battery is the integral of the power over the charge cycle. Thus,

$$w = \int_{0}^{18000} 0.05(1 + \frac{0.75t}{18000})\,dt + \int_{18000}^{36000} 0.02(1 + \frac{0.25t}{18000})\,dt$$

$$= (0.05t + \frac{0.75}{36000}t^2)\Big|_{0}^{18000} + (0.02t + \frac{0.25}{36000}t^2)\Big|_{18000}^{36000}$$

$$\boxed{w = 1732.5\ J}$$

Problem 2.5

Solution:

Known quantities:

Rated voltage of the battery; rated capacity of the battery.

Find:

The rated chemical energy stored in the battery; the total charge that can be supplied at the rated voltage.

Analysis:

a)

$$\Delta V \equiv \frac{\Delta PE_c}{\Delta Q} \quad I = \frac{\Delta Q}{\Delta t}$$

$$\begin{aligned}
&Chemical\ energy = \Delta PE_c \\
&\quad = \Delta V \cdot \Delta Q \\
&\quad = \Delta V \cdot (I \cdot \Delta t) \\
&\quad = 12\ V\ 350\ a\,hr\ 3600\ \frac{s}{hr} \\
&\quad = 15.12\ MJ.
\end{aligned}$$

As the battery discharges, the voltage will decrease below the rated voltage. The remaining chemical energy stored in the battery is less useful or not useful.

b) ΔQ is the total charge passing through the battery and gaining 12 J/C of electrical energy.

$$\begin{aligned}
&\Delta Q = I \cdot \Delta t = 350\ a\,hr = 350\ \frac{C}{s}\ hr \cdot 3600\ \frac{s}{hr} \\
&\quad = 1.26\ MC.
\end{aligned}$$

Problem 2.6

Solution:

Known quantities:

Resistance of external circuit.

Find:

Current supplied by an ideal voltage source; voltage supplied by an ideal current source.

Assumptions:

Ideal voltage and current sources.

Analysis:

a) An ideal voltage source produces a constant voltage at or below its rated current. Current is determined by the power required by the external circuit (modeled as R).

$$I = \frac{V_s}{R} \quad P = V_s \cdot I$$

b) an ideal current source produces a constant current at or below its rated voltage. Voltage is determined by the power demanded by the external circuit (modeled as R).

$$V = I_s \cdot R \quad P = V \cdot I_s$$

A real source will overheat and, perhaps, burn up if its rated power is exceeded.

Sections 2.2, 2.3: KCL, KVL

Problem 2.7

Solution:

Known quantities:

Schematic; value of the resistors and of the voltage source.

Find:

The current in the 15-Ω resistors.

Analysis:

Since the 3 resistors must have equal currents,

$$I_{15\Omega} = \frac{1}{3} \cdot I$$

and,

$$I = \frac{V_S}{R_1 + R_2 \| R_3 + R_4 \| R_5 \| R_6} = \frac{10}{20+8+5} = \frac{10}{33} = 303 \text{ mA}$$

Therefore, $I_{15\Omega} = \frac{10}{99} = 101 \text{ mA}$

Problem 2.8

Solution:

Known quantities:

Schematic; value of the currents $I_0 = -2$ A, $I_1 = -4$ A, $I_S = 8$ A and of the voltage source $V_S = 12$ V.

Find:

The unknown currents.

Analysis:

Applying KCL to node (a) and node (b):

$$\begin{cases} I_0 + I_1 + I_2 = 0 \\ I_0 + I_S + I_1 - I_3 = 0 \end{cases} \Rightarrow \begin{cases} I_2 = -(I_0 + I_1) = 6 \text{ A} \\ I_3 = I_0 + I_S + I_1 = 2 \text{ A} \end{cases}$$

Problem 2.9

Solution:

Known quantities:

Schematic of the circuit; value of the currents, of the voltage source $V_S = 24$ V and of the resistance $R_0 = 2\,\Omega$.

Find:

The unknown resistances R_1, R_2, R_3, R_4 and R_5.

Assumption:

In order to solve the problem we need to make further assumptions on the value of the resistors. For example, we may assume that $R_4 = \frac{2}{3}R_1$ and $R_2 = \frac{1}{3}R_1$.

Analysis:

We can express each current in terms of the adjacent node voltages:

$$I_0 = \frac{v_a - v_b}{R_0 + R_4} = \frac{v_a - v_b}{2 + \frac{2}{3}R_1} = -2$$

$$I_1 = \frac{v_a - v_b}{R_1} = -4$$

$$I_2 = \frac{v_a}{R_2} = \frac{v_a}{\frac{1}{3}R_1} = 6$$

$$I_3 = \frac{v_b}{R_3} = 2$$

$$I_S = \frac{V_S - v_b}{R_5} = \frac{12 - v_b}{R_5} = 8$$

Solving the system we obtain:

$v_a = 3\,\text{V}$, $v_b = 9\,\text{V}$, $R_1 = 1.5\,\Omega$, $R_2 = 0.5\,\Omega$, $R_3 = 4.5\,\Omega$, $R_4 = 1\,\Omega$ and $R_5 = 0.375\,\Omega$.

Problem 2.10 (correction Vs=54 V and I2 = 4 A)

Solution:

Known quantities:

Schematic of the circuit; value of the resistors R_1, R_2, R_3, R_4, R_5, of the voltage source $V_S = 54$ V and of the current $I_2 = 4$ A.

Find:

The unknown currents I_0, I_1, I_3, I_S and the resistor R_0.

Analysis:

We can express each current in terms of the adjacent node voltages:

$$I_0 = \frac{v_a - v_b}{R_0 + R_4}$$

$$I_1 = \frac{v_a - v_b}{R_1}$$

$$I_2 = \frac{v_a}{R_2} = 4 \Rightarrow v_a = 4 \cdot 5 = 20 \text{ V}$$

$$I_3 = \frac{v_b}{R_3}$$

$$I_S = \frac{V_S - v_b}{R_5}$$

Applying KCL to node (a) and (b) :

$$\begin{cases} I_0 + I_1 + I_2 = 0 \\ I_0 + I_S + I_1 - I_3 = 0 \end{cases} \Rightarrow \begin{cases} \dfrac{20 - v_b}{R_0 + 1} + \dfrac{20 - v_b}{2} + 4 = 0 \\ \dfrac{20 - v_b}{R_0 + 1} + \dfrac{54 - v_b}{3} + \dfrac{20 - v_b}{2} - \dfrac{v_b}{4} = 0 \end{cases}$$

Solving the system we obtain: $v_b = 24$ V and $R_0 = 1\,\Omega$.

Problem 2.11

Solution:

Known quantities:

Schematic of the circuit; value of the resistors and of the voltage source $V_S = 12$ V.

Find:

The mesh currents i_a, i_b, i_c and the current through each resistors.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b)R_1 = 0 \\ (i_a - i_b)R_1 - i_b R_2 + (i_c - i_b)R_3 = 0 \\ V_S = (i_c - i_b)R_3 \end{cases} \Rightarrow \begin{cases} 2i_a + (i_a - i_b) = 0 \\ (i_a - i_b) - \frac{4}{3} i_b + 6(i_c - i_b) = 0 \\ 6(i_c - i_b) = 12 \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = 2\text{ A} \\ i_b = 6\text{ A} \\ i_c = 8\text{ A} \end{cases} \Rightarrow \begin{cases} I_{R_0} = i_a = 2\text{ A} & \text{(positive in the direction of } i_a) \\ I_{R_1} = i_b - i_a = 4\text{ A} & \text{(positive in the direction of } i_b) \\ I_{R_2} = i_b = 6\text{ A} & \text{(positive in the direction of } i_b) \\ I_{R_3} = i_c - i_b = 2\text{ A} & \text{(positive in the direction of } i_c) \end{cases}$$

Problem 2.12

Solution:

Known quantities:

Schematic of the circuit; value of the resistors and of the voltage source $V_S = 24$ V.

Find:

The mesh currents i_a, i_b, i_c and the voltage across each resistance.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b)R_1 = 0 \\ (i_a - i_b)R_1 - i_b R_2 + (i_c - i_b)R_3 = 0 \\ V_S = (i_c - i_b)R_3 \end{cases} \Rightarrow \begin{cases} 2i_a + 2(i_a - i_b) = 0 \\ 2(i_a - i_b) - 5i_b + 4(i_c - i_b) = 0 \\ 4(i_c - i_b) = 24 \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = 2\text{ A} \\ i_b = 4\text{ A} \\ i_c = 10\text{ A} \end{cases} \Rightarrow \begin{cases} V_{R_0} = R_0 i_a = 4\text{ V} & (\oplus\text{ up}) \\ V_{R_1} = R_1(i_b - i_a) = 4\text{ V} & (\oplus\text{ down}) \\ V_{R_2} = R_2 i_b = 20\text{ V} & (\oplus\text{ up}) \\ V_{R_3} = R_3(i_c - i_b) = 24\text{ V} & (\oplus\text{ up}) \end{cases}$$

Problem 2.13

Solution:

Known quantities:

Schematic of the circuit; value of the resistors and of the current source $I_S = 12$ A.

Find:

The voltage across each resistance.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b)R_1 = 0 \\ (i_a - i_b)R_1 - i_b R_2 + (i_c - i_b)R_3 = 0 \\ i_c = I_S \end{cases} \Rightarrow \begin{cases} i_a + 3(i_a - i_b) = 0 \\ 3(i_a - i_b) - 2i_b - 4i_b + 48 = 0 \\ i_c = 12 \text{ A} \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = \dfrac{16}{3} \text{ A} \\ i_b = \dfrac{64}{9} \text{ A} \\ i_c = 12 \text{ A} \end{cases} \Rightarrow \begin{cases} V_{R_0} = R_0 i_a = 5.33 \text{ V} & (\oplus \text{ up}) \\ V_{R_1} = R_1(i_b - i_a) = 5.33 \text{ V} & (\oplus \text{ down}) \\ V_{R_2} = R_2 i_b = 14.22 \text{ V} & (\oplus \text{ up}) \\ V_{R_3} = R_3(i_c - i_b) = 19.55 \text{ V} & (\oplus \text{ up}) \end{cases}$$

Section 2.4: Passive Sign Convention

Focus on Methodology: Passive Sign Convention

1. Choose an arbitrary direction of current flow.
2. Label polarities of all active elements (current and voltage sources).
3. Assign polarities to all passive elements (resistors and other loads); for passive elements, current always flows into the positive terminal.
4. Compute the power dissipated by each element according to the following rule: If positive current flows into the positive terminal of an element, then the power dissipated is positive (i.e., the element absorbs power); if the current leaves the positive terminal of an element, then the power dissipated is negative (i.e., the element delivers power).

Problem 2.14

Solution:

Known quantities:

Direction and magnitude of the current through the elements; voltage at the terminals.

Find:

Class of the components A and B.

Analysis:

The current enters the negative terminal of element B and leaves the positive terminal: its coulombic potential energy increases.

Element B is a power source. It must be either a voltage source or a current source.

The reverse is true for element A. The current loses energy as it flows through element A.

Element A could be 1. a resistor or 2. a power source through which current is being forced to flows 'backwards'.

Problem 2.15

Solution:

Known quantities:

Current absorbed by the heater; voltage at which the current is supplied; cost of the energy.

Find:

Power consumption, energy dissipated in 24 hr.

Assumptions:

The heater works for 24 hours continuously.

Analysis:

a)

$$P = VI = 110\,V\,(23\,a) = 2.53\,K\frac{J}{a}\frac{a}{s} = 2.53\,KW$$

b)

$$W = Pt = 2.53\ K\frac{J}{s}\ 24\ hr\ 3600\frac{s}{hr} = 218.6\ MJ$$

c)

$$Cost = (Rate)W = 6\frac{cents}{KW\ hr}(2.53\ KW)(24\ hr) = 364.3\ cents = \$3.64$$

Problem 2.16 (Correction: voltage across C = -50 V)

Solution:

Known quantities:

Current through elements A, B and C; voltage across elements A, B and C.

Find:

Which components are absorbing power, which are supplying power; verify the conservation of power.

Analysis:

A absorbs $(35\ V)(15\ A) = 525\ W$

B absorbs $(15\ V)(15\ A) = 225\ W$

C supplies $(50\ V)(15\ A) = 750\ W$

Total power supplied $= P_A = 750\ W$

Total power absorbed $= P_B + P_C = 225\ W + 500\ W = 750\ W$

Total power supplied = Total power absorbed, so conservation of power is satisfied.

Problem 2.17

Solution:

Known quantities:

Voltage of the source; internal resistance of the source; resistance of the load.

Find:

The terminal voltage of the source; the power supplied to the circuit, the efficiency of the circuit.

Assumptions:

We assume that the only loss is due to the internal resistance of the source.

Analysis:

$$KVL: \quad -V_S + I_T R_S + V_T = 0$$

$$OL: \quad V_T = I_T R_L \quad \therefore \quad I_T = \frac{V_T}{R_L}$$

$$-V_S + \frac{V_T}{R_L} R_S + V_T = 0$$

$$V_T = \frac{V_S}{1 + \frac{R_S}{R_L}} = \frac{12\ V}{1 + \frac{5\ K\Omega}{7\ K\Omega}} = 7\ V \quad or \quad VD: V_T = \frac{V_S R_L}{R_S + R_L} = \frac{12\ V\ 7\ K\Omega}{5\ K\Omega + 7\ K\Omega} = 7\ V.$$

$$P_L = \frac{V_R^2}{R_L} = \frac{V_T^2}{R_L} = \frac{(7\ V)^2}{7\ K\frac{V}{a}} = 7\ mW$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{R_L}}{P_{R_S} + P_{R_L}} = \frac{\cancel{I_T^2} R_L}{\cancel{I_T^2} R_S + \cancel{I_T^2} R_L} = \frac{7\ K\Omega}{5\ K\Omega + 7\ K\Omega} = 0.5833 \quad or \quad 58.33\%.$$

Problem 2.18 (missing data: current through E = 1 A)

Solution:

Known quantities:

Current through elements D and E; voltage across elements B, C and E.

Find:

Which components are absorbing power, which are supplying power; verify the conservation of power.

Analysis:

By KCL, the current through element B is 5 A, to the right. By KVL,

$v_D = v_E = 10\ V$ (positive at the top)

$v_A + 5 - 10 - 10 = 0$

Therefore the voltage across element A is

$v_A = 15\ V$ (positive on top)

A supplies $(15\ V)(5\ A) = 75\ W$

B supplies $(5\ V)(5\ A) = 25\ W$

C absorbs $(10\ V)(5\ A) = 50\ W$

D absorbs $(10\ V)(4\ A) = 40\ W$

E absorbs $(10\ V)(1\ A) = 10\ W$

Total power supplied $= P_A + P_B = 75\ W + 25\ W = 100\ W$

Total power absorbed $= P_C + P_D + P_E = 50\ W + 40\ W + 10\ W = 100\ W$

Total power supplied = Total power absorbed, so conservation of power is satisfied.

Problem 2.19

Solution:

Known quantities:

Voltage of the battery; power absorbed by each headlight.

Find:

Resistance of each headlight; total resistance seen by the battery.

Analysis:

Headlight no. 1:

$$P = v \times i = 100\ W = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{100} = \frac{576}{100} = 5.76\Omega$$

Headlight no. 2:

$$P = v \times i = 75\ W = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{75} = \frac{576}{75} = 7.68\Omega$$

The total resistance is given by the parallel combination:

$$\frac{1}{R_{TOTAL}} = \frac{1}{5.76\Omega} + \frac{1}{7.68\Omega} \quad \text{or } R_{TOTAL} = 3.29\ \Omega$$

Problem 2.20

Solution:

Known quantities:

Voltage of the battery; power absorbed by each headlight; power absorbed by each taillight.

Find:

Equivalent resistance seen by the battery.

Analysis:

The resistance corresponding to a 75-W headlight is:

$$R_{75W} = \frac{v^2}{75} = \frac{576}{75} = 7.68\Omega$$

For each 15-W tail light we compute the resistance:

$$R_{15W} = \frac{v^2}{15} = \frac{576}{15} = 38.4\Omega$$

Therefore, the total resistance is computed as:

$$\frac{1}{R_{TOTAL}} = \frac{1}{7.68\Omega} + \frac{1}{7.68\Omega} + \frac{1}{38.4\Omega} + \frac{1}{38.4\Omega} \text{ or } R_{TOTAL} = 3.2\,\Omega$$

Problem 2.21

Solution:

Known quantities:

Voltage of the source; value of one of the resistors.

Find:

The power absorbed by variable resistor R (ranging from 0 to 20 Ω).

Analysis:

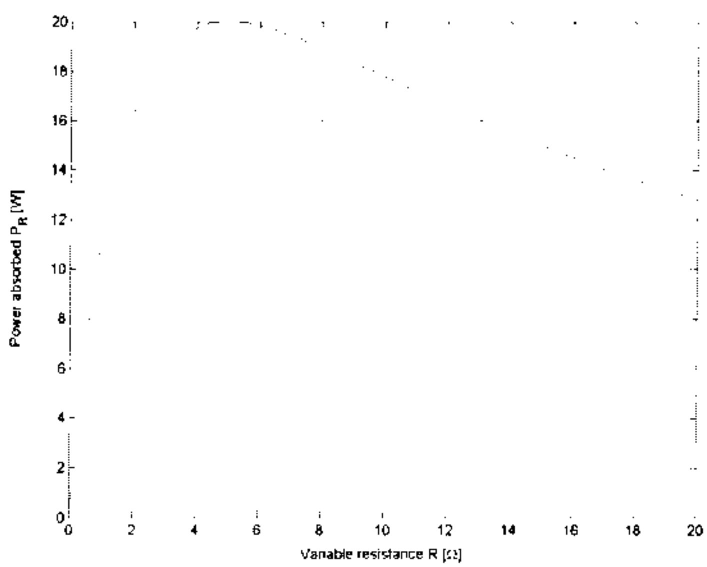
The current flowing clockwise in the series circuit is:

$$i = \frac{20}{5+R}$$

The voltage across the variable resistor R, positive on the left, is:

$$v_R = Ri = \frac{20R}{R+5}$$

Therefore, $P_R = v_R i = \left(\frac{20}{5+R}\right)^2 R$



Problem 2.22

Solution:

Known quantities:

Voltage of the source; internal resistance of the source.

Find:

The power supplied by the source as a function of current; the power supplied by an ideal source as a function of current.

Assumptions:

There are no other losses except that on Rs.

Analysis:

Rs = equivalent resistance for internal losses

$$P_{loss} = I_T^2 R_S$$

Ps = power supplied by the source $= V_S I_S = V_S I_T$.

V_T = voltage at the battery terminals:

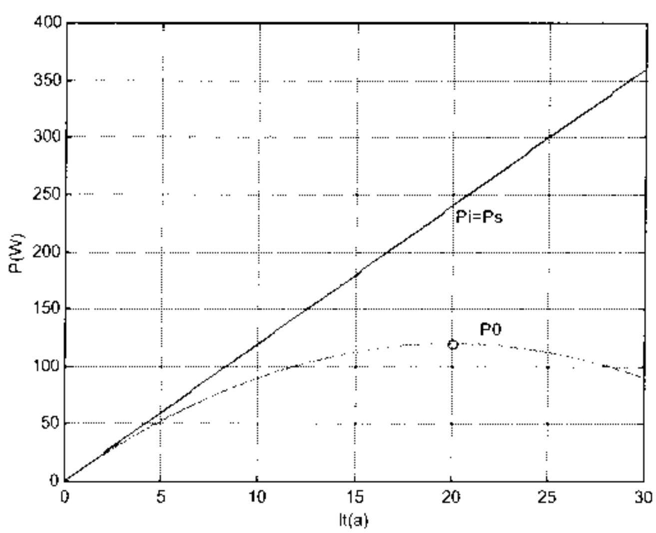
$$VD: \quad V_T = V_S - R_S I_T$$

P_0 = power supplied to the circuit (R_L in this case) $= I_T V_T$.

Conservation of energy:

$$P_S = P_{loss} + P_0 .$$

$I_T(a)$	$P_S(W)$	$P_{loss}(W)$	$P_0(W)$
0	0	0	0
2	30	1.875	28.13
5	60	7.5	52.5
10	120	30	90
20	240	120	120
30	360	270	90



Note that the power supplied to the circuit is maximum when $I_T = 20a$.

$$R_L = \frac{P_0}{I_T^2} = \frac{120\ Va}{(20\ a)^2} = 30\ m\frac{V}{a} = 30\ m\Omega$$

$$R_S = \frac{P_{loss}}{I_T^2} = \frac{120\ Va}{(20\ a)^2} = 30\ m\Omega$$

$$R_L = R_S$$

Problem 2.23

Solution:

Known quantities:

Power delivered by the source; voltage v to v_1 ratio; circuit diagram.

Find:

The resistance R, the current i and the two voltages v and v_1

Analysis:

$$P = v \cdot i = 40\ mW \quad \text{(eq. 1)}$$

$$\mathrm{v}_1 = 10000 \cdot i = \frac{\mathrm{v}}{8} \quad \text{(eq. 2)}$$

From eq.1 and eq.2, we obtain:

i = 0.7 mA and v = 56 V.

Applying KVL for the loop:

$$-v + 8000i + 10000i + Ri + 12000i = 0 \text{ or, } 0.0007R = 35$$

Therefore,

R = 50kΩ and $\mathrm{v}_1 = 7V$.

Problem 2.24

Solution:

Known quantities:

Rated power; rated optical power; operating life; rated operating voltage; open-circuit resistance of the filament.

Find:

The resistance of the filament in operation; the efficiency of the bulb.

Analysis:

a)

$$P = VI \quad \therefore I = \frac{P_R}{V_R} = \frac{60\ Va}{115\ V} = 521.7\ ma$$

$$\text{OL: } R = \frac{V}{I} = \frac{V_R}{I} = \frac{115\,V}{521.7\,ma} = 220.4\,\Omega$$

b)
Efficiency is defined as the ratio of the useful power dissipated by or supplied by the load to the total power supplied by the source. In this case, the useful power supplied by the load is the optical power. From any handbook containing equivalent units: 680 lumens=1 W

$$P_{o,out} = \text{Optical Power Out} = 820\,lum\,\frac{W}{680\,lum} = 1.206\,W$$

$$\eta = \text{efficiency} = \frac{P_{o,out}}{P_R} = \frac{1.206\,W}{60\,W} = 0.02009 = 2.009\,\%.$$

Problem 2.25

Solution:

Known quantities:

Rated power; rated voltage.

Find:

The power dissipated by a series of three light bulbs connected to the nominal voltage.

Analysis:

When connected in series, the voltage of the source will divide equally across the three bulbs. The across each bulb will be 1/3 what it was when the bulbs were connected individually across the source. Power dissipated in a resistance is a function of the voltage squared, so the power dissipated in each bulb when connected in series will be 1/9 what it was when the bulbs were connected individually, or 11.11 W:

$$\text{OL: } P = IV_B = I^2 R_B = \frac{V_B^2}{R_B}$$

$$V_B = V_S = 110\,V$$

$$R_B = \frac{V_B^2}{P} = \frac{(110\,V)^2}{100\,Va} = 121\,\Omega$$

Connected in series and assuming the resistance of each bulb remains the same as when connected individually:

$$\text{KVL: } -V_S + V_{B1} + V_{B2} + V_{B3} = 0$$

$$\text{OL: } -V_S + IR_{B3} + IR_{B2} + IR_{B1} = 0$$

$$I = \frac{V_S}{R_{B1} + R_{B2} + R_{B3}} = \frac{110\,V}{121 + 121 + 121\,\frac{V}{a}} = 303ma$$

$$P_{B1} = I^2 R_{B1} = (303\,ma)^2 \left(121\,\frac{V}{a}\right) = 11.11\,W = \frac{1}{9}\,100\,W.$$

Problem 2.26

Solution:

Known quantities:

Rated power of the two light bulbs; rated voltage.

Find:

The power dissipated by the series of the two light bulbs.

Assumptions:

The resistance of each bulb doesn't vary when connected in series.

Analysis:

When connected in series, the voltage of the source will divide equally across the three bulbs. The across each bulb will be 1/3 what it was when the bulbs were connected individually across the source. Power dissipated in a resistance is a function of the voltage squared, so the power dissipated in each bulb when connected in series will be 1/9 what it was when the bulbs were connected individually, or 11.11 W:

$$\text{OL: } P = IV_B = I^2 R_B = \frac{V_B^2}{R_B}$$

$$V_B = V_S = 110\ V$$

$$R_{60} = \frac{V_B^2}{P_{60}} = \frac{(110\ V)^2}{60\ Va} = 201.7\ \Omega$$

$$R_{100} = \frac{V_B^2}{P_{100}} = \frac{(110\ V)^2}{100\ Va} = 121\ \Omega$$

When connected in series and assuming the resistance of each bulb remains the same as when connected individually:

$$\text{KVL: } -V_S + V_{B60} + V_{B100} = 0$$

$$\text{OL: } -V_S + IR_{B60} + IR_{B100} = 0$$

$$I = \frac{V_S}{R_{B60} + R_{B100}} = \frac{110\ V}{201.7 + 121\ \frac{V}{a}} = 340.9\ ma$$

$$P_{B60} = I^2 R_{B60} = (340.9\ ma)^2 \left(201.7\ \frac{V}{a}\right) = 23.44\ W$$

$$P_{B100} = I^2 R_{B100} = (340.9\ ma)^2 \left(121\ \frac{V}{a}\right) = 14.06\ W$$

Notes: 1.It's strange but it's true that a 60 W bulb connected in series with a 100 W bulb will dissipate more power than the 100 W bulb. 2. If the power dissipated by the filament in a bulb decreases, the temperature at which the filament operates and therefore its resistance will decrease. This made the assumption about the resistance necessary.

Problem 2.27

Solution:

Known quantities:

Schematic of the circuits.

Find:

The resistor values, including the power rating, necessary to achieve the indicated voltages.

Assumptions:

Resistors are available in $\frac{1}{8}$ - $\frac{1}{4}$ - $\frac{1}{2}$ -, and 1-W ratings.

Analysis:

(a)

$$v_{out} = \left(\frac{R_2}{R_2 + R_1}\right) \cdot V = \frac{\mathrm{R}_2}{\mathrm{R}_2 + 10000} \cdot (30) = 10$$

$$\mathrm{R}_2(30 - 10) = 10 \cdot 10 \cdot 10^3$$

$$\mathrm{R}_2 = 5\,k\Omega$$

$$P_2 = I^2 R = \left(\frac{30}{15000}\right)^2 \cdot (5000) = 20\,mW$$

$$P_{R_2} = \frac{1}{8} W$$

$$P_1 = I^2 R_1 = 40\,mW$$

$$P_{R_1} = \frac{1}{8} W.$$

(b)

$$v_{out} = \left(\frac{R_2}{R_2 + R_1}\right) \cdot V = 12 \cdot \left(\frac{140}{140 + R_1}\right) = 8.5$$

$$R_1 = 57\Omega$$

$$I = \frac{V}{R_1 + R_2} = \frac{12\,V}{57\,\Omega + 140\,\Omega} = 61\,ma \Rightarrow P_1 = I^2 R_1 = 212.1\,mW$$

$$P_2 = I^2 R_2 = 520.9\,mW$$

$$P_{R_1} = \frac{1}{4} W$$

$$P_{R_2} = 1\ W$$

Problem 2.28

Solution:

Known quantities:
Schematic of the circuits.

Find:
The resistor values, including the power rating, necessary to achieve the indicated voltages.

Assumptions:
Resistors are available in $\frac{1}{8}$ - , $\frac{1}{4}$ -, $\frac{1}{2}$ - and 1-W ratings.

Analysis:

$$v_{out} = \frac{R_2}{R_0 + R_1 + R_2} V = 110 \cdot \left(\frac{4300}{1600 + R_1 + 4300} \right) = 64.3$$

$$R_1 = 1.45 k\Omega$$

$$I = \frac{V}{R_0 + R_1 + R_2} = \frac{110\,V}{1600\,\Omega + 1450\,\Omega + 4300\,\Omega} = 15\,ma$$

$$P_0 = I^2 R_0 = 360\,mW \Rightarrow P_{R_0} = \frac{1}{2} W$$

$$P_1 = I^2 R_1 = 326.25\,mW \Rightarrow P_{R_1} = \frac{1}{2} W$$

$$P_2 = I^2 R_2 = 967.5\,mW \Rightarrow P_{R2} = 1\ W$$

Problem 2.29

Solution:

Known quantities:
Schematic of the circuit.

Find:
The equivalent resistance seen by the source, the current i, the power delivered by the source, the voltages v_1 and v_2, the minimum power rating required for R_1

Analysis:

a) The equivalent resistance seen by the source is

$$R_{eq} = R_0 + R_1 + R_2 = 8 + 10 + 2 = 20\Omega$$

b) Applying KVL:

$$V - R_{eq} i = 0 \text{, therefore } i = \frac{V}{R_{eq}} = \frac{24\text{V}}{20\Omega} = 1.2\text{A}$$

c)

$$P_{source} = Vi = 24V \cdot 1.2A = 28.8\ W$$

d) Applying Ohm's law:

$$v_1 = R_1 i = 10\Omega \cdot 1.2A = 12\ V \text{, and } v_2 = R_2 i = 2\Omega \cdot 1.2A = 2.4\ V$$

e)

$P_1 = R_1 i^2 = 10\Omega \cdot (1.2A)^2 = 14.4\ W$, therefore the minimum power rating for R_1 is 16 W.

Problem 2.30

Solution:

Known quantities:

Schematic of the circuit, value of the resistors.

Find:

The currents i_1 and i_2; the power delivered by the 3-A current source and the 12-V voltage source; the total power dissipated by the circuit.

Analysis:

KCL at node 1 requires that:

$$\frac{v_1}{R_2} + \frac{v_1 - 12\text{ V}}{R_3} - 3\text{ A} = 0$$

Solving for v_1 we have

$$v_1 = 3\frac{(4 + R_3)R_2}{R_2 + R_3} = 18\text{ V}$$

Therefore,

$$i_1 = -\frac{v_1}{R_2} = -\frac{18}{10} = -1.8\text{ A}$$

$$i_2 = \frac{12 - v_1}{R_3} = -\frac{6}{5} = -1.2\text{ A}$$

and the power delivered by the 3-A source is:

$P_{3\text{-A}} = (v_{3\text{-A}})(3)$

Thus, we can compute the voltage across the 3-A source as

$v_{3\text{-A}} = 3R_1 + v_1 = 3 \cdot 25 + 28 = 103\text{ V}$

Thus,

$P_{3\text{-A}} = (103)(2) = 206$ W.

Similarly, the power supplied by the 12-V source is:

$P_{12\text{-V}} = (12)(I_{12\text{-V}})$

We have $I_{12\text{-V}} = \dfrac{12}{R_4} + i_2 = 514.3$ mA, thus:

$P_{12\text{-V}} = (12)(I_{12\text{-V}}) = 6.17$ W

Since the power dissipated equals the total power supplied:

$P_{diss} = P_{3\text{-}A} + P_{12\text{-}V} = 206 + 6.17 = 212.17$ W

Problem 2.31

Solution:

Known quantities:

Schematic of the circuit.

Find:

The power delivered by the dependent source.

Analysis:

$$i = \frac{24V}{(7+5)\Omega} = \frac{24}{12}\text{ A} = 2\text{ A}$$

$$i_{source} = 0.5i^2 = 0.5\cdot(4) = 2\text{ A}$$

The voltage across the dependent source (+ ref. taken at the top) can be found by KVL:

$$-v_D + (2A)(10\Omega) + 24V = 0 \Rightarrow v_D = 44\text{ V}$$

Therefore, the power delivered by the dependent source is

$$P_D = v_D i_{source} = 44\cdot 2 = 88\text{ W}.$$

Problem 2.32

Solution:

Known quantities:

Schematic of the circuit.

Find:

The load current; the power dissipated by the load; variations in the load current and in the power dissipated by the load due to the parallel connection with a second battery.

Analysis:

a)

$$I_L = \frac{V_1}{R_1 + R_L} = \frac{12}{0.15 + 2.55} = \frac{12}{2.7} = 4.44\text{ A}$$

$$P_{Load} = I_L^2 R_L = 50.4\text{ W}.$$

b) with another source in the circuit we must find the new power dissipated by the load. To do so, we write KVL twice using mesh currents to obtain 2 equations in 2 unknowns:

$$\begin{cases} I_2R_2 - V_1 + V_2 - I_1R_1 = 0 \\ (I_1 + I_2)R_L + I_2R_2 = V_2 \end{cases} \Rightarrow \begin{cases} 0.28\cdot I_2 - 0.15\cdot I_1 = 0 \\ 2.55\cdot(I_1 + I_2) + 0.28\cdot I_2 = 12 \end{cases}$$

Solving the above equations gives us:

$$I_1 = 2.95 \text{ A}, \; I_2 = 1.58 \text{ A} \Rightarrow I_L = I_1 + I_2 = 4.53 \text{ A}$$
$$P_{Load} = I_L^2 R_L = 52.33 \text{ W}$$

This is an increase of 1%.

Problem 2.33

Solution:

Known quantities:

Open-circuit voltage at the terminals of the power source; voltage drop with a 10-W load attached.

Find:

The voltage and the internal resistance of the source; the voltage at its terminals with a 15-Ω load resistor attached; the current that can be derived from the source under short-circuit conditions.

Analysis:

(a)

$$\frac{(49V)^2}{R_L} = 10W \Rightarrow R_L = 2.4k\Omega$$

$v_s = 50.8V$, the open-circuit voltage

$$\frac{R_L}{R_S + R_L} v_S = 49 \Rightarrow \frac{2400}{R_S + 2400} \cdot 50.8 = 49$$
$$R_S = \frac{(2400)(50.8)}{49} - 2400 = 88.2\Omega$$

(b)

$$v = \frac{R_L}{R_S + R_L} v_S = \frac{15}{88.2 + 15} 50.8 = 7.4V$$

(c)

$$i_{CC}(R_L = 0) = \frac{v_S}{R_S} = \frac{50.8}{88.2} = 576 \text{ mA}$$

Problem 2.34

Solution:

Known quantities:

Voltage of the heater, maximum and minimum power dissipation; number of coils, schematics of the configurations.

Find:

The resistance of each coil; the power dissipation of each of the other two possible arrangements.

Analysis:

(a) For the parallel connection, $P = 2000$ W. Therefore,

$$2000 = \frac{(220)^2}{R_1} + \frac{(220)^2}{R_2}$$
$$= (220)^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or,

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{5}{121}.$$

For the series connection, $P = 300$ W. Therefore,

$$300 = \frac{(220)^2}{R_1 + R_2}$$

or,

$$\frac{1}{R_1 + R_2} = \frac{3}{484}.$$

Solving, we find that $R_1 = 131.6\Omega$ and $R_2 = 29.7\Omega$.

(b) the power dissipated by R_1 alone is:

$$P_{R_1} = \frac{(220)^2}{R_1} = 368W$$

and the power dissipated by R_2 alone is

$$P_{R_2} = \frac{(220)^2}{R_2} = 1631W.$$

Sections 2.5, 2.6: Resistance and Ohm's Law

Problem 2.35

Solution:

Known quantities:

Radius of the cylindrical substrate; length of the substrate; conductivity of the carbon.

Find:

The thickness of the carbon film required for a resistance R of 33 KΩ.

Assumptions:

We assume the thickness of the film to be much smaller than the radius, and neglect the end surface of the cylinder.

Analysis:

$$R = \frac{d}{\sigma \cdot A} \cong \frac{d}{\sigma \cdot 2\pi a \cdot \Delta t}$$

$$\Delta t = \frac{d}{R \cdot 2\pi a \cdot \sigma} = \frac{9 \cdot 10^{-3}\ m}{33 \cdot 10^{3}\ \Omega \cdot 2.9 \cdot 10^{6}\ \frac{S}{m} \cdot 2\pi \cdot 1 \cdot 10^{-3}\ m}$$

Problem 2.36

Solution:

Known quantities:

The constants A and k; the open-circuit resistance.

Find:

The rated current at which the fuse blows, showing that this happens at:

$$I = \frac{1}{\sqrt{AkR_0}}.$$

Assumptions:

Here the resistance of the fuse is given by:

$$R = R_0\left[1 + A(T - T_0)\right]$$

where T_0, room temperature, is assumed to be 25°C.

We assume that:

$$T - T_0 = kP$$

where P is the power dissipated by the resistor (fuse).

Analysis:

$$R = R_0(1 + A \cdot \Delta T) = R_0(1 + AkP) = R_0(1 + AkI^2R)$$

$$R - R_0AkI^2R = R_0$$

$$R = \frac{R_0}{1 - R_0 A k I^2} \quad \to \infty \quad \text{when} \quad I - R_0 A k I^2 \to 0$$

$$I = \frac{1}{\sqrt{AkR_0}} = (0.7 \frac{m}{°C} 0.35 \frac{°C}{Va} 0.11 \frac{V}{a})^{-\frac{1}{2}} = 6.09 \text{ a.}$$

Problem 2.37

Solution:

Known quantities:

Schematic; tolerances in the values of the resistances.

Find:

The worst-case output voltages.

Analysis:

a) 10% worst case: low voltage

$R_2 = 9000\ \Omega$, $R_1 = 11000\ \Omega$

$$v_{OUT,MIN} = \frac{9000}{9000 + 11000} 5 = 2.25V$$

10% worst case: high voltage

$R_2 = 11000\ \Omega$, $R_1 = 9000\ \Omega$

$$v_{OUT,MAX} = \frac{11000}{9000 + 11000} 5 = 2.75V$$

b) 5% worst case: low voltage

$R_2 = 9500\ \Omega$, $R_1 = 10500\ \Omega$

$$v_{OUT,MIN} = \frac{9500}{9500 + 10500} 5 = 2.375V$$

5% worst case: high voltage

$R_2 = 10500\ \Omega$, $R_1 = 9500\ \Omega$

$$v_{OUT,MAX} = \frac{10500}{9500 + 10500} 5 = 2.625V$$

Problem 2.38

Solution:

Known quantities:

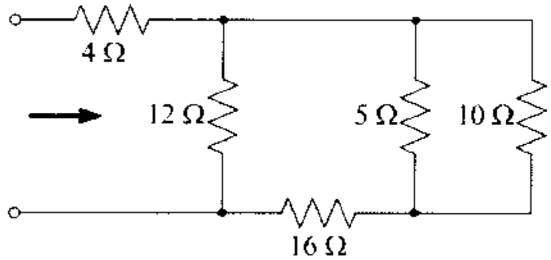
Schematic of the circuit.

Find:

The equivalent resistance of the circuit.

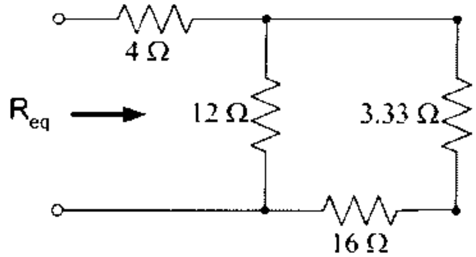
Analysis:

Starting from the right side, we combine the two resistors in series:

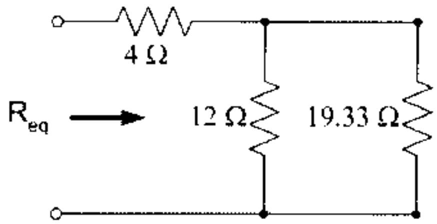


Then, we can combine the two parallel resistors, namely the 5 Ω resistor and 10 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{5} + \frac{1}{10} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = \frac{10}{3}\ \Omega$$

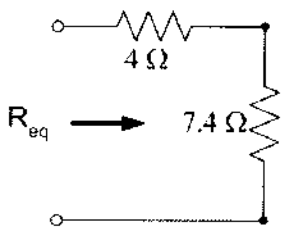


Then, we can combine the two resistors in series, namely the 3.33 Ω and the 16 Ω resistor:



Then, we can combine the two parallel resistors, namely the 12 Ω resistor and 19.33 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{12} + \frac{1}{19.33} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 7.4\ \Omega$$



Therefore, R_{eq} = 4 + 7.4 = 11.4 Ω.

Problem 2.39

Solution:

Known quantities:

Schematic of the circuit.

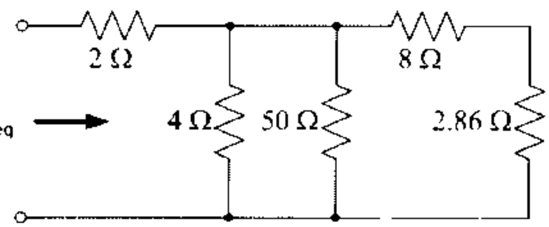
Find:

The equivalent resistance of the circuit seen by the source; the current i through the resistance R_2.

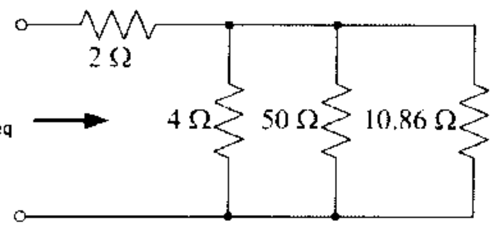
Analysis:

Starting from the right side, we can combine the three parallel resistors, namely the 10 Ω resistor, the 12 Ω resistor and the 6 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{10} + \frac{1}{12} + \frac{1}{6} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = \frac{20}{7}\,\Omega$$

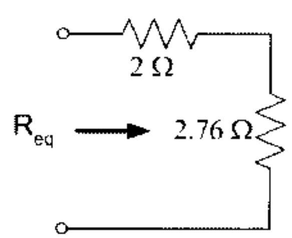


Then, we can combine the two resistors in series, namely the 8 Ω and the 2.86 Ω resistor:



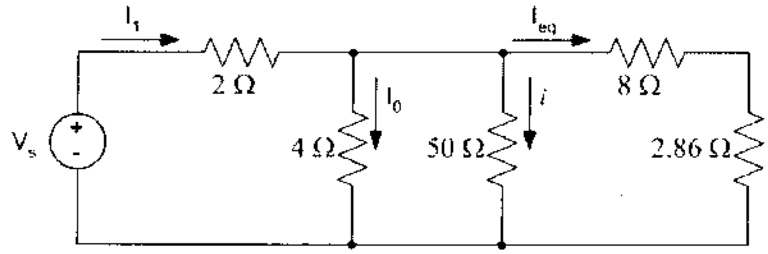
Then, we can combine the three parallel resistors, namely the 4 Ω resistor, the 50 Ω resistor and the 10.86 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{4} + \frac{1}{50} + \frac{1}{10.86} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 2.76\,\Omega$$



Therefore, $R_{eq} = 2 + 2.76 = 4.76\ \Omega$.

Looking at the following equivalent circuit:



We can apply KVL and KCL to the above circuit:

$$\begin{cases} V_S - 2I_1 - 4I_0 = 0 \\ I_1 = I_0 + i + I_{eq} \\ 4I_0 = 50i = 2.86 I_{eq} \end{cases} \Rightarrow \begin{cases} I_0 = 12.5i \\ I_1 = 6 - 25i \\ I_{eq} = 17.48i \\ i = I_1 - I_0 - I_{eq} \end{cases} \Rightarrow i = 107 \text{ mA}$$

Problem 2.40

Solution:

Known quantities:

Schematic of the circuit. Voltage source, value of the resistors R_i (i=1..6) and power absorbed by the 20-Ω resistor.

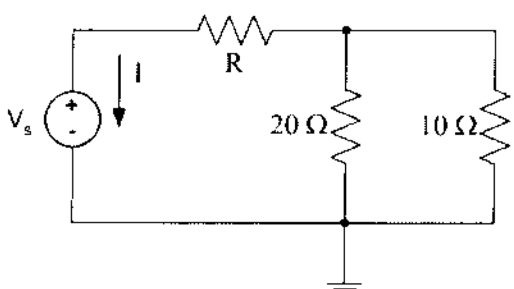
Find:

The resistance R.

Analysis:

Starting from the right side, we can replace resistors R_i (i=2..6) with a single equivalent resistors:

$$R_{eq} = R_2 + \left(R_3 + \left(R_4 \parallel R_5\right)\right) \parallel R_6 = 10\,\Omega$$



The same voltage appears across both R_1 and R_{eq} and, therefore, these element are in parallel. Applying the voltage divider rule:

$$V_{R_1} = \frac{R_1 \parallel R_{eq}}{R + R_1 \parallel R_{eq}} V_S = \frac{1000}{3R + 20}$$

The power absorbed by the 20-Ω resistor is:

$$P_{20\Omega} = \frac{(V_{R_1})^2}{R_1} = \frac{1}{20}\left(\frac{1000}{3R+20}\right)^2 = \frac{50000}{(3R+20)^2} = 20 \Rightarrow R = 10\,\Omega$$

Problem 2.41

Solution:

Known quantities:

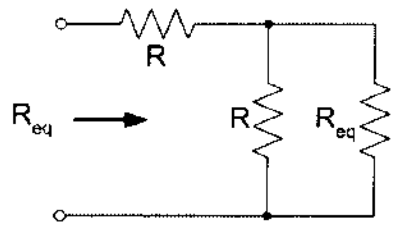
Schematic of the circuit.

Find:

The equivalent resistance R_{eq} of the infinite network of resistors.

Analysis:

We can see the infinite network of resistors as the equivalent to the circuit in the picture:



Therefore,

$$R_{eq} = R + (R \,||\, R_{eq}) = R + \frac{RR_{eq}}{R + R_{eq}} \Rightarrow R_{eq} = \frac{1}{2}(1+\sqrt{5})R$$

Problem 2.42

Solution:

Known quantities:

Schematic of the circuit. The value of the resistors and the source voltage.

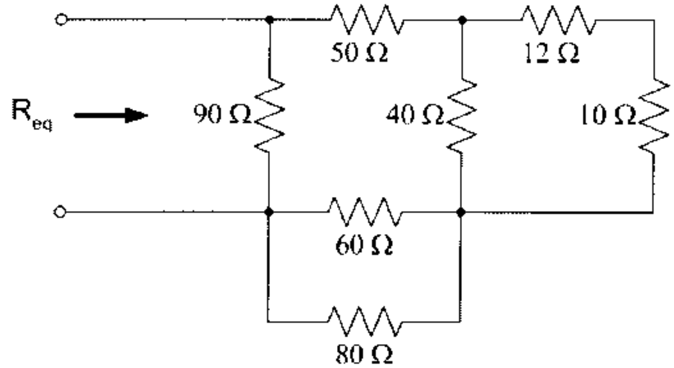
Find:

The equivalent resistance of the circuit seen by the source; the current i through and the power absorbed by the resistance 90-Ω resistance.

Analysis:

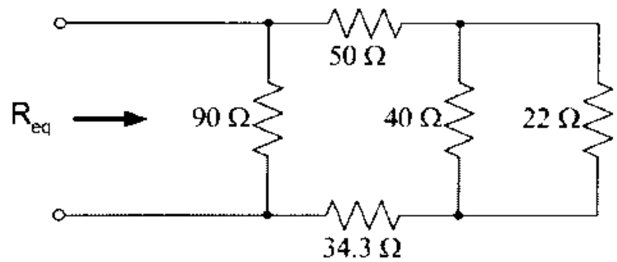
Starting from the right side, we can combine the two parallel resistors, namely the 20 Ω resistor and the 30 Ω resistor:

$$\frac{1}{R_{parallel}} = \frac{1}{20} + \frac{1}{30}\left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 12\,\Omega$$



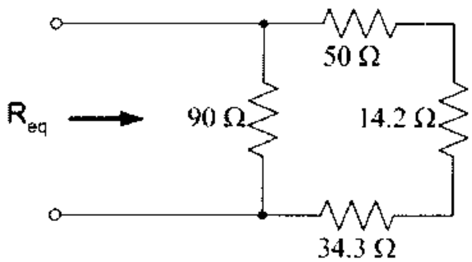
Then we can combine the two parallel resistors in the bottom, namely the 60 Ω resistor and the 80 Ω, and the two resistor in series:

$$\frac{1}{R_{parallel}} = \frac{1}{60} + \frac{1}{80} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 34.3\ \Omega$$



Then we can combine the two parallel resistors on the right, namely the 40 Ω resistor and the 22 Ω:

$$\frac{1}{R_{parallel}} = \frac{1}{40} + \frac{1}{22} \left(\Omega^{-1}\right) \Rightarrow R_{parallel} = 14.2\ \Omega$$



Therefore, $\frac{1}{R_{eq}} = \frac{1}{90} + \frac{1}{(50 + 14.2 + 34.3)} \left(\Omega^{-1}\right) \Rightarrow R_{eq} = 47\ \Omega$.

The current through and the power absorbed by the 90-Ω resistor are:

$$I_{90\Omega} = \frac{V_S}{R_1} = \frac{110}{90} = 1.22\ \text{A}$$

$$P_{90\Omega} = \frac{(V_S)^2}{R_1} = \frac{110^2}{90} = 134.4\ \text{W}$$

Problem 2.43

Solution:

Known quantities:

Schematic of the circuit.

Find:

The equivalent resistance at terminals a,b in the case that terminals c,d are a) open b) shorted; the same for terminals c,d with respect to terminals a,b.

Analysis:

With terminals c-d open, R_{eq}= $(360+540)\|(180+540)\Omega = 400\Omega$,

with terminals c-d shorted, R_{eq}= $(360\|180)+(540\|540)\Omega = 390\Omega$,

with terminals a-b open, R_{eq}= $(540+540)\|(360+180)\Omega = 360\Omega$,

with terminals a-b shorted, R_{eq}= $(360\|540)+(180\|540)\Omega = 351\Omega$.

Problem 2.44

Solution:

Known quantities:

Layout of the site; characteristics of the cables; rated voltage of the generator; range of voltages and currents absorbed by the engine at full load.

Find:

The minimum AWG gauge conductors which must be used in a rubber insulated cable.

Analysis:

The cable must meet two requirements:

1. The conductor current rating must be greater than the rated current of the motor at full load. This requires AWG #14.
2. The voltage drop due to the cable resistance must not reduce the motor voltage below its minimum rated voltage at full load.

$$KVL: \quad -V_G + V_{RC1} + V_{M-Min} + V_{RC2} = 0$$

$$-V_G + I_{M-FL} R_{C1} + V_{M-Min} + I_{M-FL} R_{C2} = 0$$

$$R_{C1} + R_{C2} = \frac{V_G - V_{M-Min}}{I_{M-FL}} =$$

$$= \frac{110\ V - 105\ V}{7.103\ A} = 703.9\ m\Omega$$

$$R_{Max} = \frac{R_{C1}}{d} = \frac{R_{C2}}{d} = \frac{\frac{1}{2}[703.9\ m\Omega]}{150\ m} = 2.346\ m\frac{\Omega}{m}$$

Therefore, AWG #8 or larger wire must be used.

Problem 2.45

Solution:

Known quantities:

Layout of the site; characteristics of the cables; rated voltage of the generator; total electrical load in the building.

Find:

The minimum AWG gauge conductors which must be used in a rubber insulated cable.

Analysis:

The cable must meet two requirements:

3. The conductor current rating must be greater than the rated current of the motor at full load. This requires AWG #4.
4. The voltage drop due to the cable resistance must not reduce the motor voltage below its minimum rated voltage at full load.

$$KVL: \quad -V_S + V_{RC1} + V_{L-Min} + V_{RC2} = 0$$

$$-V_S + I_{L-FL}R_{C1} + V_{L-Min} + I_{L-FL}R_{C2} = 0$$

$$R_{C1} + R_{C2} = \frac{V_S - V_{L-Min}}{I_{L-FL}} =$$

$$= \frac{450\ V - 446\ V}{51.57\ A} = 77.6\ m\Omega$$

$$R_{Max} = \frac{R_{C1}}{d} = \frac{R_{C2}}{d} = \frac{\frac{1}{2}[77.6\ m\Omega]}{85\ m} = 0.4565\ m\frac{\Omega}{m}$$

Therefore, AWG #0 or larger wire must be used.

Problem 2.46

Solution:

Known quantities:

Layout of the site; characteristics of the cables; rated voltage of the generator; electrical characteristics of the engine.

Find:

The maximum length of a rubber insulated cable with AWG #14 which can be used to connect the motor and the generator.

Analysis:

The voltage drop due to the cable resistance must not reduce the motor voltage below its minimum rated voltage at full load.

$$KVL: \quad -V_G + V_{RC1} + V_{M-Min} + V_{RC2} = 0$$

$$-V_G + I_{M-FL}R_{C1} + V_{M-Min} + I_{M-FL}R_{C2} = 0$$

$$R_{C1} + R_{C2} = \frac{V_G - V_{M-Min}}{I_{M-FL}} =$$

$$= \frac{110\ V - 105\ V}{7.103\ A} = 703.9\ m\Omega$$

$$d_{Max} = \frac{R_{C1}}{R_{rated}} = \frac{R_{C2}}{R_{rated}} = \frac{\frac{1}{2}[703.9\ m\Omega]}{8.285\ m\frac{\Omega}{m}} = 42.48\ m$$

Problem 2.47

Solution:

Known quantities:

Layout of the site; characteristics of the cables; rated voltage of the generator; total electrical load in the building.

Find:

The maximum length of a rubber insulated cable with AWG #4 which can be used to connect the source to the load.

Analysis:

The voltage drop due to the cable resistance must not reduce the motor voltage below its minimum rated voltage at full load.

$$KVL: \quad -V_S + V_{RC1} + V_{L-Min} + V_{RC2} = 0$$

$$-V_S + I_{L-FL}R_{C1} + V_{L-Min} + I_{L-FL}R_{C2} = 0$$

$$R_{C1} + R_{C2} = \frac{V_S - V_{L-Min}}{I_{L-FL}} =$$

$$= \frac{450\ V - 446\ V}{51.57\ A} = 77.6\ m\Omega$$

$$d_{Max} = \frac{R_{C1}}{R_{rated}} = \frac{R_{C2}}{R_{rated}} = \frac{\frac{1}{2}[77.6\ m\Omega]}{0.8153\ m\frac{\Omega}{m}} = 47.59\ m$$

Problem 2.48

Solution:

Known quantities:

Schematic of the circuit; value of the resistors.

Find:

The equivalent resistance between A and B.

Analysis:

Shorting nodes C and D creates a single node to which all four resistors are connected.

$$R_{eq1} = R_1 \| R_3 = \frac{R_1 R_3}{R_1 + R_3} = \frac{[2.2\ K\Omega][4.7\ K\Omega]}{2.2 + 4.7\ K\Omega} = 1.499\ K\Omega$$

$$R_{eq2} = R_2 \| R_4 = \frac{R_2 R_4}{R_2 + R_4} = \frac{[18\ K\Omega][3.3\ K\Omega]}{18 + 3.3\ K\Omega} = 2.789\ K\Omega$$

$$R_{eq} = R_{eq1} + R_{eq2} = 1.499 + 2.789\ K\Omega = 4.288\ K\Omega$$

Problem 2.49

Solution:

Known quantities:

Schematic of the circuit; value of the resistors, voltage of the source.

Find:

The voltage between nodes A and B.

Analysis:

The same current flows through R1 and R3. Therefore, they are connected in series. Similarly, R2 and R4 are connected in series.

SPECIFY THE ASSUMED POLARITY OF THE VOLTAGE BETWEEN NODES A AND B. THIS WILL HAVE TO BE A WILD GUESS AT THIS POINT.

Specify the polarities of the voltage across R3 and R4 which will be determined using voltage division. The actual polarities are not difficult to determine. Do so.

$$VD:\quad V_{R3} = \frac{V_S R_3}{R_1 + R_3} = \frac{[12\ V][6.8\ K\Omega]}{11 + 6.8\ K\Omega} = 4.584\ V$$

$$VD:\quad V_{R4} = \frac{V_S R_4}{R_2 + R_4} = \frac{[12\ V][220\ K\Omega]}{220 + 220\ K\Omega} = 6\ V$$

$$KVL:\quad -V_{R3} + V_{AB} + V_{R4} = 0 \therefore V_{AB} = V_{R3} - V_{R4} = -1.416\ V$$

The voltage is negative indicating that the polarity of V_{AB} is opposite of that specified.

A solution is not complete unless the assumed positive direction of a current or assumed positive polarity of a voltage IS SPECIFIED ON THE CIRCUIT.

Problem 2.50

Solution:

Known quantities:

Schematic of the circuit; value of the resistors, voltage of the source.

Find:

The voltage between nodes A and B.

Analysis:

The same current flows through R1 and R3. Therefore, they are connected in series. Similarly, R2 and R4 are connected in series.
SPECIFY THE ASSUMED POLARITY OF THE VOLTAGE BETWEEN NODES A AND B. THIS WILL HAVE TO BE A WILD GUESS AT THIS POINT.
Specify the polarities of the voltage across R3 and R4 which will be determined using voltage division. The actual polarities are not difficult to determine. Do so.

$$VD: \quad V_{R3} = \frac{V_S R_3}{R_1 + R_3} = \frac{[5\,V][4.7\,K\Omega]}{2.2K\Omega + 4.7\,K\Omega} = 3.406\,V$$

$$VD: \quad V_{R4} = \frac{V_S R_4}{R_2 + R_4} = \frac{[5\,V][3.3\,K\Omega]}{18K\Omega + 3.3\,K\Omega} = 0.917\,V$$

$$KVL: \quad -V_{R3} + V_{AB} + V_{R4} = 0 \Rightarrow V_{AB} = V_{R3} - V_{R4} = 2.489\,V$$

A solution is not complete unless the assumed positive direction of a current or assumed positive polarity of a voltage IS SPECIFIED ON THE CIRCUIT.

Problem 2.51

Solution:

Known quantities:

Schematic of the circuit; value of the resistors, voltage of the source.

Find:

The voltage across the resistance R_3.

Analysis:

The same voltage appears across both R_2 and R_3 and, therefore, these element are in parallel. Applying the voltage divider rule:

$$V_{R_3} = \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} V_S = \frac{2.3k\Omega}{1.7m\Omega + 2.3k\Omega} 12\text{ V} = 11.999991\text{ V} \quad (\oplus\text{ down})$$

Note that since $R_1 << R_2 \| R_3$, then $V_{R_3} \cong V_S$.

Sections 2.7, 2.8: Practical Sources and Measuring Devices

Problem 2.52 (correction: beta, equation and range of T)

Solution:

Known quantities:

Parameters $R_0 = 300\ \Omega$ (resistance at temperature $T_0 = 298$ K), and $\beta = -0.01\,\mathrm{K}^{-1}$, value of the second resistor.

Find:

The diagram $R_{th}(T)$ versus T in the range $350 \le T \le 750$[°K]; the equivalent resistance of the parallel connection with the 250-Ω resistor; the diagram $R_{eq}(T)$ versus T in the range $350 \le T \le 750$[°K].

Assumptions:

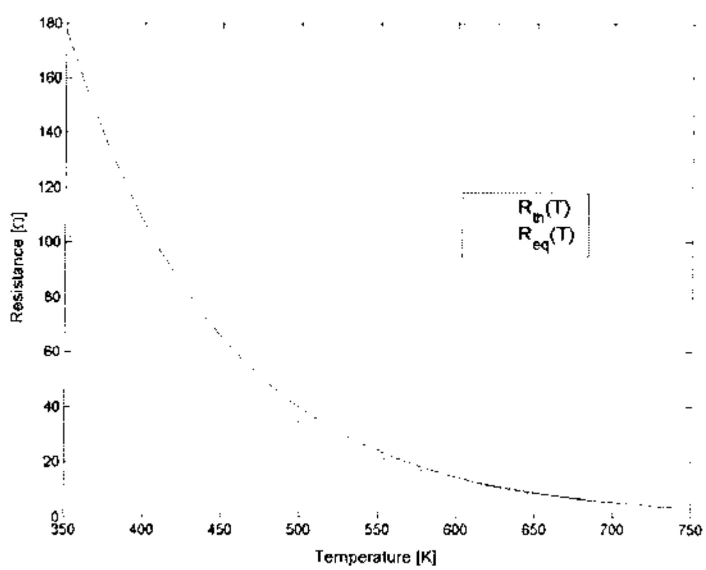
$$R_{th}(T) = R_0 e^{-\beta(T-T_0)}.$$

Analysis:

a) $R_{th}(T) = 300e^{-0.01(T-298)}$

b) $R_{eq}(T) = R_{th}(T) \| 250\Omega = \dfrac{1500\,e^{-0.01(T-298)}}{5+6\,e^{-0.01(T-298)}}$

The two plots are shown below.



In the above plot, the solid line is for the thermistor alone; the dashed line is for the thermistor-resistor combination.

Problem 2.53

Solution:

Known quantities:

The value of the resistance R_m, the total length of the resistor x_T and voltage source v_S.

Find:

The expression for $v_{out}(x)$. The diagram v_{out}/v_S versus x/x_T; the distance x when $v_{out}=5$ V.

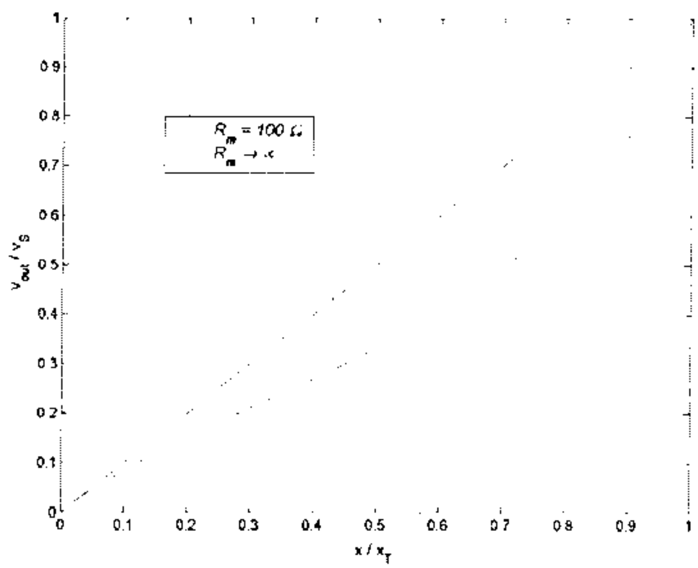
Assumptions:

$$\frac{v_{out}}{v_S}=\frac{1}{1/(x/x_T)+(R_P/R_m)(1-x/x_T)}$$

$$R_P(x)=200e^x$$

Analysis:

a) $$v_{out}(x)=\frac{10}{1/(x/0.02)+(200e^x/100)(1-x/0.02)}=500\frac{x}{1-10^2e^x x+5\cdot10^3e^x x^2}$$



In the above plot, the solid line is for $R_m=100\,\Omega$; the dashed line is for $R_m\to\infty$.

b) $x(v_{out}=5\text{ V})=14.18\text{ cm}$

c) Now $R_m\to\infty$.

$$v_{out}(x)=\frac{10}{1/(x/0.02)+(200e^x/\infty)(1-x/0.02)}\to 500x$$

$$x(v_{out}=5\text{ V})=10\text{ cm}$$

Problem 2.54

Solution:

Known quantities:

Meter resistance of the coil; meter current for full scale deflection; max measurable pressure.

Find:

The circuit required to indicate the pressure measured by a sensor; the value of each component of the circuit; the linear range, and the maximum pressure that can accurately be measured.

Analysis:

a) A series resistor to drop excess voltage is required.
b) At full scale, meter:

$$I_{\hat{m}FS} = 10\mu a$$

$$r_{\hat{m}} = 200\Omega$$

$$0.L.: \quad V_{\hat{m}FS} = I_{\hat{m}FS} r_{\hat{m}} = 2\ mV.$$

at full scale, sensor (from characteristics):

$$P_{FS} = 100 \quad kPa$$

$$V_{TFS} = 9.5 \quad mV$$

$$KVL: \quad -V_{TFS} + V_{RFS} + V_{\hat{m}FS} = 0$$

$$V_{RFS} = V_{TFS} - V_{\hat{m}FS} = 9.5\ mV - 2\ mV = 7.5\ mV$$

$$I_{RFS} = I_{TFS} = I_{\hat{m}FS} = 10\mu a$$

Ohm law: $R = \dfrac{V_{RFS}}{I_{RFS}} = \dfrac{7.5\ mV}{10\ \mu a} = 750\ \Omega$.

c) from sensor characteristic: 30 kPa –110 kPa.

Problem 2.55

Solution:

Known quantities:

Meter resistance of the coil; meter current for full scale deflection; max measurable pressure.

Find:

The circuit required to meet the specification; the value of each component of the circuit; the linear range, and the maximum pressure that can accurately be measured.

Analysis:

a) A series resistor to drop excess voltage is required.
b) At full scale, meter:

$$I_{\hat{m}FS} = 50\mu a$$

$$r_{\hat{m}} = 1.8k\Omega$$

$$0.L.: \quad V_{\hat{m}FS} = I_{\hat{m}FS} r_{\hat{m}} = 90\ mV.$$

at full scale, sensor (from characteristics):

$$V_{TFS} = 9.5 \quad V$$
$$KVL: \quad -V_{TFS} + V_{RFS} + V_{\hat{m}FS} = 0$$
$$V_{RFS} = V_{TFS} - V_{\hat{m}FS} = 9.5\ V - 90\ mV = 9.41\ V$$
$$I_{RFS} = I_{TFS} = I_{\hat{m}FS} = 50 \mu a$$

Ohm law: $R = \frac{V_{RFS}}{I_{RFS}} = \frac{9.41\ V}{10\ \mu a} = 188.2\ k\Omega$.

c) from sensor characteristic: 20 kPa -110 kPa.

Problem 2.56

Solution:

Known quantities:

Meter resistance of the coil; meter voltage for full scale deflection; max measurable temperature.

Find:

The circuit required to meet the specifications of the new sensor; the value of each component of the circuit; the linear range, and the maximum temperature that can accurately be measured.

Analysis:

a) A parallel resistor is required to shunt (bypass) the excess current.
b) At full scale, meter:

$$V_{\hat{m}FS} = 250\ mV$$
$$r_{\hat{m}} = 2.5\ k\Omega$$
$$O.L.: \quad I_{\hat{m}FS} = \frac{V_{\hat{m}FS}}{r_{\hat{m}}} = 100\ \mu a.$$

at full scale, sensor (from characteristics):

$$T_{FS} = 400 \quad ^\circ C$$
$$I_{TFS} = 8.5 \quad ma$$
$$KCL: \quad -I_{TFS} + I_{RFS} + I_{\hat{m}FS} = 0$$
$$I_{RFS} = I_{TFS} - I_{\hat{m}FS} = 8.5\ ma - 100\ \mu a = 8.4\ ma$$
$$V_{RFS} = V_{TFS} = V_{\hat{m}FS} = 250\ mV$$

Ohm law: $R = \frac{V_{RFS}}{I_{RFS}} = \frac{250\ mV}{8.4\ ma} = 29.76\ \Omega$.

c) from sensor characteristic: 220 °C 410 °C.

Problem 2.57

Solution:

Known quantities:

Meter resistance of the coil; meter voltage at full scale; max measurable temperature.

Find:

The circuit required to meet the specifications of the new sensor; the value of each component of the circuit; the linear range, and the maximum temperature that can be accurately be measured.

Analysis:

a) A parallel resistor is required to shunt (bypass) the excess current.

b) At full scale, meter:

$$V_{\hat{m}FS} = 250\ mV$$

$$r_{\hat{m}} = 2.5\ k\Omega$$

$$0.L.:\quad I_{\hat{m}FS} = \frac{V_{\hat{m}FS}}{r_{\hat{m}}} = 100\ \mu a.$$

at full scale, sensor (from characteristics):

$$T_{FS} = 400 \quad {}^{\circ}C$$

$$I_{TFS} = 8.5 \quad ma$$

$$KCL:\quad -I_{TFS} + I_{RFS} + I_{\hat{m}FS} = 0$$

$$I_{RFS} = I_{TFS} - I_{\hat{m}FS} = 8.5\ ma - 100\ \mu a = 8.4\ ma$$

$$V_{RFS} = V_{TFS} = V_{\hat{m}FS} = 250\ mV$$

Ohm law: $R = \frac{V_{RFS}}{I_{RFS}} = \frac{250\ mV}{8.4\ ma} = 29.76\ \Omega$.

c) from sensor characteristic: 220 °C –410 °C.

Problem 2.58

Solution:

Known quantities:

Schematic of the circuit; voltage at terminals with switch open and closed for fresh battery; same voltages for the same battery after 1 year.

Find:

The internal resistance of the battery in the two cases.

Analysis:

a)

$$V_{out} = \left(\frac{10}{10 + r_B}\right) V_{oc}$$

$$r_B = 10\left(\frac{V_{oc}}{V_{out}} - 1\right) = 10\left(\frac{2.28}{2.27} - 1\right)$$

$$= 0.044\Omega$$

b)

$$r_B = 10\left(\frac{V_{oc}}{V_{out}} - 1\right) = 10\left(\frac{2.2}{0.31} - 1\right)$$

$$= 60.97\Omega$$

Problem 2.59

Solution:

Known quantities:

Current for full-scale deflection; desired full scale values.

Find:

Value of the resistors required for the given full scale ranges.

Analysis:

We desire R_1, R_2, R_3 such that $I_a = 30\ \mu A$ for $I = 10$ mA, 100 mA, and 1 A, respectively. We use conductances to simplify the arithmetic:

$$G_a = \frac{1}{R_a} = \frac{1}{1000}\ S$$

$$G_{1,2,3} = \frac{1}{R_{1,2,3}}$$

By the current divider rule:

$$I_a = \frac{G_a}{G_a + G_x} I$$

or:

$$G_x = G_a\left(\frac{I}{I_a}\right) - G_a \text{ or } \frac{1}{G_x} = \frac{1}{G_a}\left(\frac{I_a}{I - I_a}\right)$$

$$R_x = R_a\left(\frac{I_a}{I - I_a}\right).$$

We can construct the following table:

x	I	R_x (Approx.)
1	10^{-2} A	3 Ω
2	10^{-1} A	0.3 Ω
3	10^{0} A	0.03 Ω

Problem 2.60 (correction: $I_{meter} = 150\ \mu A$)

Solution:

Known quantities:

Schematic of the circuit; for point b): value of R_p and current displayed on the ammeter.

Find:

The current i; the internal resistance of the meter.

Assumptions:

$$r_a << 50\ k\Omega.$$

Analysis:

a) Assuming that $r_a << 50\ k\Omega$

$$i \approx \frac{V_s}{R_s} = \frac{12}{50000} = 240\ \mu A$$

b) With the same assumption as in part a)

$$i_{meter} = 200 \cdot (10)^{-6} = \frac{R_p}{r_a + R_p} i$$

or:

$$150 \cdot (10)^{-6} = \frac{15}{r_a + 15} 240 \cdot 10^{-6}.$$

Therefore, $r_a = 9\ \Omega$.

Problem 2.61

Solution:

Known quantities:

Voltage read at the meter; schematic of the circuit.

Find:

The internal resistance of the voltmeter.

Analysis:

Using the voltage divider rule:

$$V = 11.81 = \frac{r_m}{r_m + R_s}(12)$$

Therefore, $r_m = 1.55\ M\Omega$.

Problem 2.62

Solution:

Known quantities:

Value of the voltage source; ratios between R_s and r_m.

Find:

The meter reads in the various cases.

Analysis:

By voltage division:

$$V = \frac{r_m}{r_m + R_s}(24)$$

R_S	V
$0.2\, r_m$	20 V
$0.4\, r_m$	17.14 V
$0.6\, r_m$	15 V
$1.2\, r_m$	10.91 V
$4\, r_m$	4.8 V
$6\, r_m$	3.43 V
$10\, r_m$	2.18 V

For a voltmeter, we always desire $r_m >> R_s$.

Problem 2.63

Solution:

Known quantities:
Schematic of the circuit, values of the components.

Find:
The voltage across R_4 with and without the voltmeter.

Assumptions:
The voltmeter behavior is modeled as that of an ideal voltmeter in parallel with a 120- kΩ resistor.

Analysis:
We develop first an expression for V_{R_4} in terms of R_4. Next, using current division:

$$\begin{cases} I_{R_1} = I_S\left(\dfrac{R_S}{R_S + R_1 + R_2 \,||\, (R_3 + R_4)}\right) \\ I_{R_4} = I_{R_1}\left(\dfrac{R_2}{R_2 + R_3 + R_4}\right) \end{cases}$$

Therefore,

$$I_{R_4} = I_S\left(\frac{R_S}{R_S + R_1 + R_2 \,||\, (R_3 + R_4)}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + R_4}\right)$$

$$
\begin{aligned}
V_{R_4} &= I_{R_4} R_4 \\
&= I_S\left(\frac{R_S R_4}{R_S + R_1 + R_2 \| (R_3 + R_4)}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + R_4}\right) \\
&= \frac{66000 \cdot R_4}{R_4 + 2.1352 \cdot 10^6}
\end{aligned}
$$

Without the voltmeter:

a) $V_{R_4} = 3.08$ V

b) $V_{R_4} = 30.47$ V

c) $V_{R_4} - 269.91$ V

d) $V_{R_4} = 1260.7$ V.

Now we must find the voltage drop across R_4 with a 120-kΩ resistor across R_4. This is the voltage that the voltmeter will read.

$$
\begin{aligned}
I_{R_4} &= I_S\left(\frac{R_S}{R_S + R_1 + R_2 \| (R_3 + (R_4 \| 120k\Omega))}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + (R_4 \| 120k\Omega)}\right) \\
V_{R_4} &= I_{R_4} R_4 \\
&= I_S I_S\left(\frac{R_S R_4}{R_S + R_1 + R_2 \| (R_3 + (R_4 \| 120k\Omega)}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + (R_4 \| 120k\Omega)}\right) \\
&= 82.5\frac{(120000 + R_4)\cdot R_4}{7319 R_4 + 320.28 \cdot 10^6}
\end{aligned}
$$

With the voltmeter:

a) $V_{R_4} - 3.08$ V

b) $V_{R_4} = 30.47$ V

c) $V_{R_4} = 272.57$ V

d) $V_{R_4} = 1724.99$ V.

Problem 2.64

Solution:

Known quantities:

Schematic of the circuit, value of the components.

Find:

The current through R_5 both with and without the ammeter, for various values of the resistor R_5.