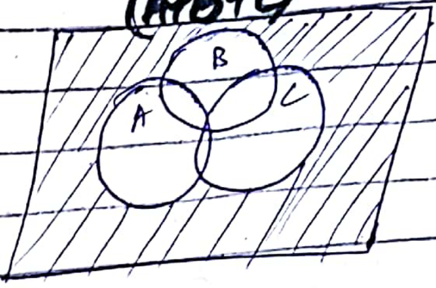
Problem Set 1

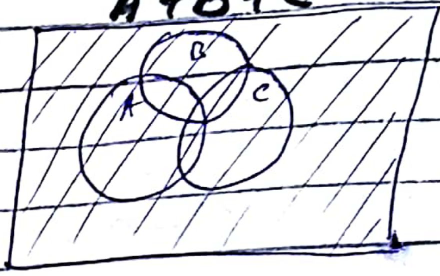
Q.No.1

(a) NOT Valid

$(A+B+C)^c$

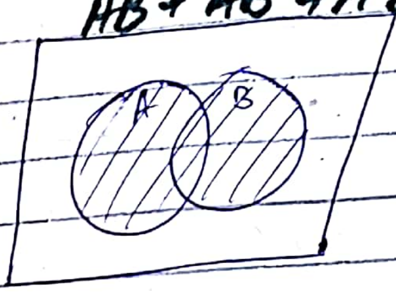


$A^c+B^c+C^c$

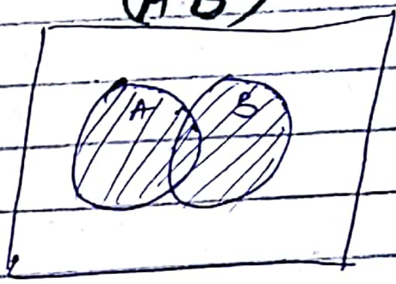


(b) Valid

$AB+AB^c+A^cB$



$(A^cB^c)^c$



Q.No.2

Given
$P(A) = 0.55$
$P(B^c) = 0.35$
$P(A \cup B) = 0.75$

(a) $P(B) = 1 - P(B^c) = 1 - 0.35 = 0.65$

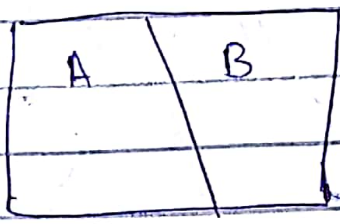
(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

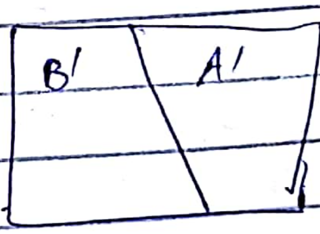
$P(A \cap B) = 0.55 + 0.65 - 0.75$

$= 0.45$

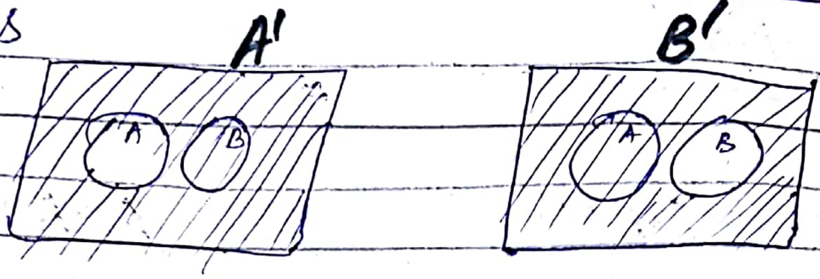
Q.No.3

(a) Yes





(b) Yes



Q. No. 4

(a) P(Atleast one Ace) = 1 − P(No Ace)

$= 1 - \frac{48}{52}\frac{48}{52}$

$= \frac{25}{169}$

(b) P(Two Cards of Same Suit) $= 1 \cdot \frac{13}{52} = \frac{1}{4}$

1st Card be Any Suit

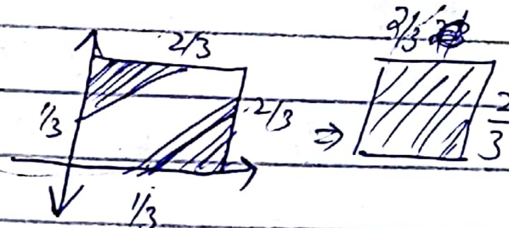
2nd Card from same suit.

(c) P(Neither Ace) $= \frac{48}{52} \cdot \frac{48}{52}$

(d) P(Neither Card Diamond or Club) $= \frac{26}{52}\frac{26}{52}$

Q. No. 5

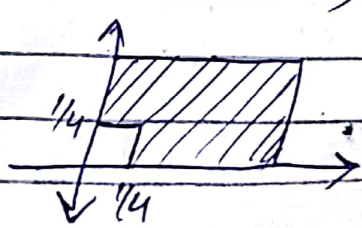
(a) $P(A) = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$



(b) $P(B) = 1 - P(\text{Both Smaller Than } 1/4)$

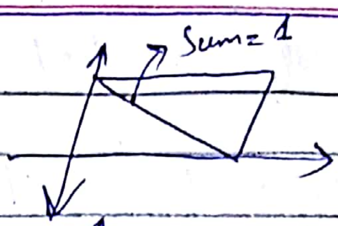
$= 1 - \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$

$= 15/16$

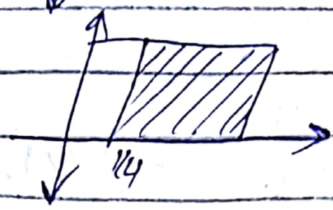


(c) $P(C) = 0$

Line has No Area



(d) $P(D) = 1 \times \frac{3}{4} = \frac{3}{4}$



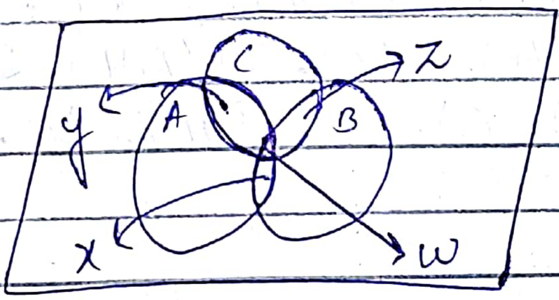
Q.No.6

$P(A \cap B^c) = P(A \text{ but not } B)$
$= P(A) - P(A \cap B)$

$P(A^c \cap B) = P(B \text{ but } A)$
$= P(B) - P(A \cap B)$

$P(A \cap B^c) + P(A^c \cap B) = P(A) + P(B) - 2P(A \cap B)$

Q.No.7



$P(A \cup B \cup C) = P(D_1) + x + y + z - w$ — (1)

Where $x = P(A \cap B)$
$y = P(B \cap C)$
$z = P(A \cap C)$
$w = P(A \cap B \cap C)$

$$P(A\cup(B\cup C)) = P(A) + P(B\cup C) - P(A\cap(B\cup C)) \quad -②$$

$$\Rightarrow P(A\cap(B\cup C)) = P(A\cap B) + P(A\cap C) - P(A\cap B\cap C)$$

↳ Putting in ②

$$P(A\cup B\cup C) = P(A) + P(B\cup C) - P(A\cap B) - P(A\cap C) + P(A\cap B\cap C) \quad -③$$

$$\Rightarrow P(B\cup C) = P(B) + P(C) - P(B\cap C)$$

↳ Putting in ③

(You Don't Need Above Part if you know this)

$$\hookrightarrow P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + 2P(A\cap B\cap C)$$

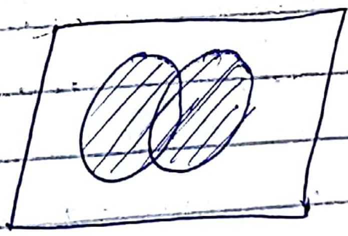
↳ Putting in ①

$$P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + 2P(A\cap B\cap C) = P(D) + P(A\cap B) + P(A\cap C) + P(B\cap C) - P(A\cap B\cap C)$$

$$P(D_1) = P(A) + P(B) + P(C) - 2P(A\cap B) - 2P(A\cap C) - 2P(B\cap C) + 3P(A\cap B\cap C).$$

Q. No. 8

(a)



By Venn-diagram

$$A\cup B = A\cup(B\cap A^c)$$

$$P(A\cup B) = P(A) + P(B\cap A^c)$$

We know that if $(B\cap A^c)\subset B \Rightarrow P(B\cap A^c)\le P(B)$

so

$$P(A\cup B) \le P(A) + P(B)$$

or $P(A) + P(B) \ge P(A\cup B)$

As $A \subset A \cup B$ and $B \subset A \cup B$

so $P(A \cup B) \geq \max\{P(A), P(B)\}$

Finally,

$\Rightarrow P(A) + P(B) \geq P(A \cup B) \geq \max\{P(A), P(B)\}$

As we know

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

As $A \cup B \subset S$ (S means Sample set)

$P(A \cap B) \geq P(A) + P(B) - P(S)$

$P(A \cap B) \geq P(A) + P(B) - 1$ $(\because P(S) = 1)$

As $A \cap B \subset A$ and $A \cap B \subset B$

so $\min\{P(A), P(B)\} \geq P(A \cap B)$

Finally,

$\min\{P(A), P(B)\} \geq P(A \cap B) \geq P(A) + P(B) - 1$

(b) First inequality becomes equality when A and B are mutually exclusive and ~~for~~ second when A and B are collectively exhaustive.

Q.No.9

For $N=4 \Rightarrow P(A)$ ($\because$ 4 Parking places 2 cars & 3 spaces apart is impossible.)

For $N>4$

$$P(A) = \frac{\binom{N}{1}\binom{2}{1} - 8}{\binom{N}{2}} = \frac{2N-8}{N(N-1)}$$

$\binom{N}{1} \Rightarrow$ Mary has N choices to park

$\binom{2}{1} \Rightarrow$ Tom has two choices, 3 spaces apart either on left or right.

$-8 \Rightarrow$ overcounting on both sides first 4 edge places

Q.No.10

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

(a) $P(HHH) = \frac{1}{8}$

(b) $P(HTH) = \frac{1}{8}$

(c) $P(HHT, HTH, THH) = 3/8$

(d) $P(HHT, HTH, THH, HHH) = 4/8$

(e) $P(HHT, HTH, THH) = 3/7$ ($\because$ HHH is excluded from S)

(f) Less than two tails means two or more than two heads so 100% chances of more heads than tails $\Rightarrow P(\text{''}) = 1$