Problem 1-1

Express the following quantities to the nearest standard prefix using no more than three digits.

(a) 1,000,000 W (b) 92.5×10^{9} Hz (c) 256.7×10^{-7} s (d) 25×10^{-9} F

Solution:

Use the standard decimal prefixes in Table 1-2 to find the appropriate prefix. MATLAB converts numbers into standard engineering notation.

```
format short eng
a = 1000000
b = 92.5e9
c = 256.7e-7
d = 25e-9
a =
     1.0000e+006
b =
    92.5000e+009
c =
    25.6700e-006
d =
    25.0000e-009
```

Answer:

(a) 1 MW

(b) 92.5 GHz

(c) 25.7 μs

(d) 25 nF

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Problem 1-2

Express the following quantities to the nearest standard prefix using no more than three digits.

(a) 0.000111 A (b) 200×10^5 C (c) 5.02×10^3 J (d) 3,264,000 Ω

Solution:

Use the standard decimal prefixes in Table 1-2 to find the appropriate prefix. MATLAB converts numbers into standard engineering notation.

```
format short eng
a = 0.000111
b = 200e5
c = 5.02e3
d = 3264000
a =
   111.0000e-006
b =
    20.0000e+006
c =
     5.0200e+003
d =
     3.2640e+006
```

Answer:

(a) 111 μA
(b) 20 MC
(c) 5.02 kJ
(d) 3.26 MΩ

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Problem 1-3

An ampere-hour (Ah) meter measures the time-integral of the current in a conductor. During an 8-hour period, a certain meter records 3300 Ah. Find the number of coulombs that flowed through the meter during the recording period.

Solution:

1 ampere = 1 coulomb/second

1 hour = 3600 seconds

3300 Ah = 3300 ampere-hour = 3300 (coulomb/second)(hour)(3600 second/hour) = 11.88 MC

```
format short eng
Ah = 3300;
C = 3300*3600
C =
    11.8800e+006
```

Answer:

11.88 MC

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Problem 1-4

Electric power companies measure energy consumption in kilowatt-hours, denoted kWh. One kilowatt-hour is the amount of energy transferred by 1 kW of power in a period of 1 hour. A power company billing statement reports a user's total energy usage to be 2150 kWh. Find the number of joules used during the billing period.

Solution:

1 kWh = 1000 watt-hours
1 watt = 1 joule/second
1 hour = 3600 seconds

2150 kWh = 2150 kilowatt-hours = 2150000 watt-hours = 2150000 (joules/second)(hours)(3600 second/hour) = 7.74 GJ

```
format short eng
kWh = 2150;
J = kWh*1000*3600
J =
     7.7400e+009
```

Answer:

7.74 GJ

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Problem 1-5

(a) To convert capacitance from picofarads to microfarads, multiply by _____.

(b) To convert resistance from megohms to kilohms, multiply by _____.

(c) To convert voltage from millivolts to volts, multiply by_____.

(d) To convert energy from megajoules to joules, multiply by _____.

Solution:

Assume we have X of the original units. Use identities to determine the multiplier for X in the new units.

(a) X pF = $X \times 10^{-12}$ F = $(X \times 10^{-6}) \times 10^{-6}$ F = $X \times 10^{-6}$ μF
(b) X MΩ = $X \times 10^{6}$ Ω = $(X \times 10^{3}) \times 10^{3}$ Ω = $X \times 10^{3}$ kΩ
(c) X mV = $X \times 10^{-3}$ V
(d) X MJ = $X \times 10^{6}$ J

Answer:

(a) 10^{-6}
(b) 10^{3}
(c) 10^{-3}
(d) 10^{6}

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Problem 1-6

A wire carries a constant current of 300 μA. How many coulombs flow past a given point in the wire in 2 s?

Solution:

```
format short eng
coulomb_per_second = 300e-6;
sec = 2;
coulomb = coulomb_per_second*sec
coulomb =
   600.0000e-006
```

Answer:

600 μC

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Problem 1-7

The net positive charge flowing through a device is $q(t) = 10 + 3t$ mC. Find the current through the device.

Solution:

$$i(t) = \frac{dq(t)}{dt} = 3\,\text{mA}$$

```
syms t real
qt = 10 + 3*t;
it = diff(qt,t)
it =
3
```

Answer:

$i(t) = 3$ mA

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Problem 1-8

Figure P1-8 shows a plot of the net positive charge flowing in a wire versus time. Sketch the corresponding current during the same period of time.

Solution:

Create the original waveform for the charge.

heaviside(t) = u(t) = unit step function.

```
syms t
qt =  (10+10*t)*(heaviside(t)-heaviside(t-2))...
    + (110-40*t)*(heaviside(t-2)-heaviside(t-
3))...
    + (5-5*t)*(heaviside(t-3)-heaviside(t-5))...
    + (-270+50*t)*(heaviside(t-5)-heaviside(t-
6));
tt = 0:0.01:6;
qtt = subs(qt,t,tt);
figure
plot(tt,qtt,'b','LineWidth',3)
xlabel('Time (s)')
ylabel('Charge (C)')
grid on
```

Take the derivative of the charge waveform to find the current.

```
it = diff(qt,t);
itt = subs(it,t,tt);
figure
plot(tt,itt,'g','LineWidth',3)
xlabel('Time (s)')
ylabel('Current (A)')
grid on
axis([0 6 -60 60])
```

Answer:

Shown above

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Problem 1-9

The net positive charge flowing through a device varies as $q(t) = 3t^2$ C. Find the current through the device at $t = 0$ s, $t = 1$ s, and $t = 3$ s.

Solution:

$$i(t) = \frac{dq(t)}{dt} = 6t \text{ A}, \quad i(0) = 0 \text{ A}, \quad i(1) = 6 \text{ A}, \quad i(3) = 18 \text{ A}$$

```
syms t
qt = 3*t^2;
it = diff(qt,t)
tt = [0, 1, 3];
itt = subs(it,t,tt)
it =
6*t
itt =
     0     6    18
```

Answer:

$i(0) = 0$ A; $i(1) = 6$ A; $i(3) = 18$ A

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Problem 1-10
An appliance is protected by a 15 A fuse. What is the maximum number of coulombs that can flow through the device before the breaker trips?

Solution:
The maximum current is 15 A, which is 15 coulombs per second. As long as the rate of coulombs flowing through the device remains below 15 coulombs per second, the total number of coulombs that can flow through it is theoretically infinite.

Answer:
Infinite.

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Problem 1-11

For $0 \le t \le 5$ s, the current through a device is $i(t) = 4t$ A. For $5 < t \le 10$ s, the current is i(t) = 40 – 4*t* A, and i(t) = 0 A for t > 10 s. Sketch *i(t)* versus time and find the total charge flowing through the device between $t = 0$ s and $t = 10$ s.

Solution:

```
syms t
it = 4*t*(heaviside(t)-heaviside(t-5))...
    + (40-4*t)*(heaviside(t-5)-heaviside(t-10));
qTotal = int(it,t,0,10)
tt = 0:0.01:10;
itt = subs(it,t,tt);
plot(tt,itt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
qTotal =
100
```

Answer:

i(*t*) is sketched above.

$$q_{TOTAL} = \int_0^5 4t\,dt + \int_5^{10} (40-4t)\,dt = \frac{1}{2}\times 10\times 20 = 100\,\text{C}$$

q_{TOTAL} = 100 C

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Problem 1-12

The charge flowing through a device is $q(t) = 1 - e^{-500t}$ μC. How long will it take the current to reach 200 μA?

Solution:

$$i(t) = \frac{dq(t)}{dt} = 500\times10^{-6}e^{-500t}\text{ A},\quad 500\times10^{-6}e^{-500t} = 200\times10^{-6},\quad t = \frac{\ln\left(\frac{5}{2}\right)}{500} = 1.8326\,\text{ms}$$

```
syms t
qt = 1e-6*(1-exp(-500*t));
it = diff(qt,t)
Time200 = solve(it-200e-6,t);
Time200 = vpa(Time200,5)
it =
1/2000*exp(-500*t)
Time200 =
.18326e-2
```

Answer:

t = 1.8326 ms

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Problem 1-13

The 12-V automobile battery in Figure P1-13 has an output capacity of 100 ampere-hours (Ah) when connected to a head lamp that absorbs 200 watts of power. Assume the battery voltage is constant.

(a) Find the current supplied by the battery.

(b) How long can the battery power the headlight?

Solution:

```
% Part (a)
v = 12;
p = 200;
i = p/v
% Part (b)
w = 100;
t = w/i
i =
    16.6667e+000
t =
     6.0000e+000
```

Answer:

(a) $i = 16.67$ A

(b) $t = 6$ hours

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Problem 1-14

An incandescent lamp absorbs 100 W when connected to a 120-V source. A fluorescent lamp producing the same amount of light absorbs 12 W when connected to the same source. How much cheaper is it to operate the fluorescent bulb over the incandescent bulb over 1000 hours when electricity costs 7.2 cents/kW-h?

Solution:

```
V = 120;
T = 1000;
P_incand = 100;
kWh_incand = P_incand*T/1000;
cost_incand = 0.072*kWh_incand

P_fluor = 12;
kWh_fluor = P_fluor*T/1000;
cost_fluor = 0.072*kWh_fluor

cost_difference = cost_incand - cost_fluor
cost_incand =
     7.2000e+000
cost_fluor =
   864.0000e-003
cost_difference =
     6.3360e+000
```

Answer:

$6.34

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Problem 1-15

The current through a device is zero for $t < 0$ and is $i(\mathrm{t}) = 3e^{-2t}$ A for $t \geq 0$. Find the charge $q(\mathrm{t})$ flowing through the device for $t \geq 0$.

Solution:

$$q(t) = \int_0^t 3e^{-2\tau} d\tau = \frac{3e^{-2\tau}\Big|_0^t}{-2} = \frac{-3}{2}\left(e^{-2t} - 1\right) = \frac{3}{2}\left(1 - e^{-2t}\right)\mathrm{C}, \quad t \geq 0$$

```
syms t tau
it = 3*exp(-2*t)*heaviside(t);
itau = subs(it,t,tau)
qt = simple(int(itau,tau,-inf,t))
itau =
3*exp(-2*tau)*heaviside(tau)
qt =
-3/2*heaviside(t)*(exp(-2*t)-1)
```

Answer:

$$q(t) = \frac{3}{2}\left[1 - e^{-2t}\right] \text{ C} \quad t \geq 0$$

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Problem 1-16

The current through and voltage across a two-terminal device are $i(t) = 20 \sin(1000t)$ mA and $v = 100 \sin(1000t)$ V. Find the maximum and minimum power delivered to the device.

Solution:

$$p(t) = i(t)v(t) = 2\sin^2(1000t) = 2\frac{1}{2}\left[1-\cos(2000t)\right] = 1-\cos(2000t)\,\text{W}$$

Since $-1 \le \cos(2000t) \le 1$, $p_{MAX} = 2$ W, $p_{MIN} = 0$ W

```
syms t
it = 20e-3*sin(1000*t);
vt = 100*sin(1000*t);
pt = simplify(it*vt)
tt = 0:0.0001:0.005;
ptt = subs(pt,t,tt);
ptMax = max(ptt)
ptMin = min(ptt)
plot(tt,ptt,'b','LineWidth',3)
xlabel('Time (s)')
ylabel('Power (W)')
grid on
pt =
2*sin(1000*t)^2
ptMax =
     1.9997e+000
ptMin =
     0.0000e-003
```

Answer:

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$p_{MAX} = 2$ W
$p_{MIN} = 0$ W

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Problem 1-17

When illuminated the *i*-*v* relationship for a photocell is $i = e^v - 10$ A. For $v = -2$, 2 and 3 V find the device power and state whether it is absorbing or delivering power.

Solution:

```
v = [-2 2 3];
iv = exp(v)-10;
p = v.*iv
pAbsorbs = p>0
p =
    19.7293e+000    -5.2219e+000    30.2566e+000
pAbsorbs =
     1     0     1
```

Answer:

For $v = -2$ V, $p = 19.73$ W, absorbing

For $v = 2$ V, $p = -5.22$ W, delivering

For $v = 3$ V, $p = 30.26$ W, absorbing

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Problem 1-18

A new 1.5 V AA battery delivers 40 kJ of energy during its lifetime. How long will the battery last in an application that draws 10 mA continuously. Assume the battery voltage is constant.

Solution:

```
V = 1.5;
w = 40e3;
Ia = 10e-3;
p = V*Ia;
Tsec = w/p
Tmin = Tsec/60;
Thours = Tsec/3600;
Tdays = Thours/24
Days = floor(Tdays)
Hours = floor((Tsec - Days*3600*24)/3600)
Minutes = floor((Tsec - Days*3600*24 - Hours*3600)/60)
Seconds = Tsec - Days*3600*24 - Hours*3600 - Minutes*60
Tsec =
     2.6667e+006
Tdays =
    30.8642e+000
Days =
    30.0000e+000
Hours =
    20.0000e+000
Minutes =
    44.0000e+000
Seconds =
    26.6667e+000
```

Answer:

30.8642 days or 30 days, 20 hours, 44 minutes, and 26.7 seconds

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Problem 1-19

The maximum power the device can dissipate is 0.5 W. Determine the maximum current allowed by the device power rating when the voltage is 9 V.

Solution:

```
Pmax = 0.5;
V = 9;
Imax = Pmax/V
Imax =
    55.5556e-003
```

Answer:

i_{MAX} = 55.56 mA

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Problem 1-20

A constant current of 2 A charges a battery for 4 hours. During the charging period the battery voltage is $v(t) = 12 - 2e^{-t}$, where t is in hours. Determine the energy stored in the battery.

Solution:

$$p(t) = i(t)v(t) = 24 - 4e^{-t}\ \text{W}, \quad w = \int_0^4 p(t)\,dt = \int_0^4 \left(24 - 4e^{-t}\right)dt = 96 + 4\left(e^{-4} - 1\right) = 92.07\ \text{J}$$

```
syms t
it = 2;
vt = 12-2*exp(-t);
pt = vt*it;
w = double(int(pt,t,0,4))
w =
    92.0733e+000
```

Answer:

$w = 92.07$ J

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Problem 1-21

Two electrical devices are connected as shown in Figure P1-21. Using the reference marks shown in the figure, find the power transferred and state whether the power is transferred from A to B or B to A when

(a) v = + 11 V and i = -1.1 A
(b) v = +80 V and i = +20 mA
(c) v = -120 V and i = -12 mA
(d) v = -1.5 V and i = -600 μA

Solution:

The passive sign convention applies to Element B, so if the power is positive the transfer is from A to B, and if the power is negative, the transfer is from B to A.

```
v = [11 80 -120 -1.5];
i = [-1.1 20e-3 -12e-3 -600e-6];
p = v.*i
AtoB = p>0
p =
   -12.1000e+000     1.6000e+000     1.4400e+000   900.0000e-006
AtoB =
     0     1     1     1
```

Answer:

(a) p = –12.1 W, from B to A
(b) p = 1.6 W, from A to B
(c) p = 1.44 W, from A to B
(d) p = 900 μW, from A to B

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Problem 1-22

Figure P1-22 shows an electric circuit with a voltage and a current variable assigned to each of the six devices. The device voltages and currents are observed to be

	v(V)	i(A)		v(V)	i(A)
Device 1	15	–1	Device 4	–10	–1
Device 2	5	1	Device 5	20	–3
Device 3	10	2	Device 6	20	2

Find the power associated with each device and state whether the device is absorbing or delivering power. Use the power balance to check your work.

Solution:

```
v = [15 5 10 -10 20 20];
ia = [-1 1 2 -1 -3 2];
p = v.*ia
Absorbing = p>0
Balance = sum(p)
p =
   -15.0000e+000     5.0000e+000    20.0000e+000    10.0000e+000   -
60.0000e+000    40.0000e+000
Absorbing =
     0     1     1     1     0     1
Balance =
     0.0000e-003
```

Answer:

Device	p (W)	Absorbing or Delivering
1	–15	Delivering
2	5	Absorbing
3	20	Absorbing
4	10	Absorbing
5	–60	Delivering
6	40	Absorbing

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Power balance is zero, so the values check.

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Problem 1-23

Figure P1-22 shows an electric circuit with a voltage and a current variable assigned to each of the six devices. Use power balance to find v_4 when $v_1 = 20$ V, $i_1 = -2$ A, $p_2 = 20$ W, $p_3 = 10$ W, $i_4 = 1$ A, and $p_5 = p_6 = 2.5$ W. Is device 4 absorbing or delivering power?

Solution:

```
v1 = 20;
i1 = -2;
i4 = 1;
p1 = v1*i1;
p2 = 20;
p3 = 10;
p5 = 2.5;
p6 = 2.5;
p4 = -p1-p2-p3-p5-p6
v4 = p4/i4
Absorbing = p4>0
p4 =
     5.0000e+000
v4 =
     5.0000e+000
Absorbing =
     1
```

Answer:

$v_4 = 5$ V, absorbing power

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Problem 1-24

Suppose in Figure P1-22 a ground is connected to the minus (–) side of element 6 and another to the junction of elements 2, 3 and 4. Further, assume that the voltage v_4 is 5 V and v_1 is 10 V. What are the voltages v_2, v_3, v_5 and v_6?

Solution:

```
v4 = 5;
v1 = 10;
v5 = -v4
v6 = v5
v3 = 0
v2 = v1
v5 =
    -5.0000e+000
v6 =
    -5.0000e+000
v3 =
     0.0000e-003
v2 =
    10.0000e+000
```

Answer:

v_2 = 10 V

v_3 = 0 V

v_5 = –5 V

v_6 = –5 V

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Problem 1-25

For $t \geq 0$ the voltage across and power absorbed by a two-terminal device are $v(t) = 2e^{-t}$ V and $p(t) = 40e^{-2t}$ mW. Find the total charge delivered to the device for $t \geq 0$.

Solution:

$$i(t) = \frac{p(t)}{v(t)} = \frac{40\times10^{-3}e^{-2t}}{2e^{-t}} = 20e^{-t}\text{ mA}, \quad q_{TOTAL} = 20\times10^{-3}\int_0^{\infty} e^{-t}dt = 20\,\text{mC}$$

```
syms t
vt = 2*exp(-t)*heaviside(t);
pt = 40e-3*exp(-2*t)*heaviside(t);
it = simple(pt/vt)
q = simple(int(it,t,0,inf))
it =
1/50/exp(t)
q =
1/50
```

Answer:

$q_{\text{TOTAL}} = 20$ mC

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Problem 1-26

Repeat Problem 1-22 using MATLAB to perform the calculations. Create a vector for the voltage values, `v = [15 5 10 -10 20 20]`, and a vector for the current values, `i = [-1 1 2 -1 -3 2]`. Compute the corresponding vector for the power values, p, using element-by-element multiplication `(.*)` and then use the `sum` command to verify the power balance.

Solution:

```
v = [15 5 10 -10 20 20];
ia = [-1 1 2 -1 -3 2];
p = v.*ia
Absorbing = p>0
Balance = sum(p)
p =
   -15     5    20    10   -60    40
Absorbing =
     0     1     1     1     0     1
Balance =
     0
```

Answer:

Device	p (W)	Absorbing or Delivering
1	–15	Delivering
2	5	Absorbing
3	20	Absorbing
4	10	Absorbing
5	–60	Delivering
6	40	Absorbing

Power balance is zero, so the values check.

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Problem 1-27

Using the passive sign convention, the voltage across a device is $v(t) = 5\cos(10t)$ V and the current through the device is $i(t) = 0.5\sin(10t)$ A. Using MATLAB, create a short script (m-file) to assign a value to the time variable, t, and then calculate the voltage, current, and power at that time. Run the script for $t = 0.2$ s and $t = 0.4$ s and for each result state whether the device is absorbing or delivering power.

Solution:

```
t = 0.2
vt = 5*cos(10*t)
it = 0.5*sin(10*t)
pt = vt*it
Absorbing = pt>0
t = 0.4
vt = 5*cos(10*t)
it = 0.5*sin(10*t)
pt = vt*it
Absorbing = pt>0
t =
    0.2000
vt =
   -2.0807
it =
    0.4546
pt =
   -0.9460
Absorbing =
     0
t =
    0.4000
vt =
   -3.2682
it =
   -0.3784
pt =
    1.2367
Absorbing =
     1
```

Answer:

t	$v(t)$	$i(t)$	$p(t)$	Absorbing or Delivering
0.2 s	–2.08 V	454.6 mA	–946 mW	Delivering
0.4 s	–3.27 V	–378.4 mA	1.237 W	Absorbing

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Problem 1-28

In complete darkness the voltage across and current through a two-terminal light detector are +6.5 V and +7 nA. In full sunlight the voltage and current are +1.1 V and +4.5 mA. Express the light/dark power ratio of the device in decibels (dB), where the power ratio in dB is

$$PR_{dB} = 10 \text{ Log}_{10} (p_2/p_1).$$

Solution:

```
v_dark = 6.5;
i_dark = 7e-9;
p_dark = v_dark*i_dark;
v_sun = 1.1;
i_sun = 4.5e-3;
p_sun = v_sun*i_sun;
PR_dB = 10*log10(p_sun/p_dark)
PR_dB =
   50.3659
```

Answer:

PR_{dB} = 50.3659 dB

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Problem 1-29

A manufacturer's data sheet for the converter in Figure P1-29 states that the output voltage is v_{dc} = 5 V when the input voltage v_{ac} = 120 V. When the load draws a current i_{dc} = 40 A the input power is p_{ac} = 300 W. Find the efficiency of the converter.

Solution:

```
p_ac = 300;
v_dc = 5;
i_dc = 40;
p_dc = v_dc*i_dc;
Efficiency = p_dc/p_ac
Efficiency =
    0.6667
```

Answer:

Efficiency = 66.67%

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Problem 1-30

A capacitor is a two-terminal device that can store electric charge. In a linear capacitor the amount of charge stored is proportional to the voltage across the device. For a particular device the proportionality is $q(t) = 10^{-7}v(t)$. If $v(t) = 0$ for $t < 0$ and $v(t) = 10(1 - e^{-5000t})$ for $t \geq 0$, find the energy stored in the device at $t = 200$ μs.

Solution:

$$q(t) = 10^{-6}\left(1 - e^{-5000t}\right)\text{C}, i(t) = \frac{dq(t)}{dt} = 5e^{-5000t}\text{ mA}, p(t) = i(t)v(t) = 0.05\left(e^{-5000t} - e^{-10000t}\right)\text{W}$$

$$w(200\mu s) = \int_0^{200\times10^{-6}} p(t)\,dt = 0.05\int_0^{200\times10^{-6}} \left(e^{-5000t} - e^{-10000t}\right)dt = 0.05\left[\frac{e^{-1}-1}{-5000} - \frac{e^{-2}-1}{-10000}\right]$$

$$= 1.9979\,\mu\text{J}$$

```
syms t
tt = 200e-6;
C = 1e-7;
vt = 10*(1-exp(-5000*t));
qt = C*vt;
it = diff(qt,t);
pt = it*vt;
wt = double(int(pt,t,0,tt))
wt =
  1.9979e-006
```

Answer:

$w(200\ \mu\text{s}) = 1.9979\ \mu\text{J}$

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Problem 1-31

A manufacturer's data sheet for a desktop computer lists the power supply requirements as 30 A @ 5 V, 5 A @ 15 V, 5 A @ –15 V, 3.25 A @ – 5 V and 0.5 A @ 12 V. The data sheet also states that the overall power consumption is 325 W. Are these data consistent? Explain.

Solution:

```
it = [30 5 5 3.25 0.5];
vt = [5 15 -15 -5 12];
pt = vt.*it
Balance1 = sum(pt)
Balance2 = sum(abs(pt))
pt =
  150.0000   75.0000  -75.0000  -16.2500    6.0000
Balance1 =
  139.7500
Balance2 =
  322.2500
```

Answer:

These data are consistent if the power supply is always supplying power at the stated currents and voltages and the overall power consumption is rounded up to the nearest 5 W. If we calculate the individual powers by multiplying current times voltage for each supply value, we will have positive and negative powers. It is not reasonable that the computer would supply power to a power supply, so the negative power values are not a correct interpretation of the passive sign convention. If we sum the magnitudes of the products of currents and voltages, the total power consumption is 322.25 W, which is very close to the stated value of 325 W.

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Problem 1-32

MATLAB is extremely powerful when linear operations are formatted using vector or matrix notation. The colon operator `(:)` is a special MATLAB operator that allows many calculations to be simplified or expressed in a compact form. For example, we can quickly generate a vector of time values ranging from 0 s to 1 s with increments of 0.1 s using the following command: `t = 0:0.1:1`. We can then substitute this vector into standard MATLAB functions, such as `sin()` or `exp()`, to generate the results of evaluating those functions for every value in the time vector: `v = 5*sin(2*pi*t)`. or `i = 3*exp(-4*t)`. Create the time vector described above and evaluate `v = 5*sin(2*pi*t)` using that time vector. Find the maximum and minimum values for the vector v using the commands `max(v)` and `min(v)`, respectively. Repeat the problem with a time increment that is smaller by a factor of 10 and comment on the results.

Solution:

```
t = 0:0.1:1;
v = 5*sin(2*pi*t);
vMax = max(v)
vMin = min(v)
t2 = 0:0.01:1;
v2 = 5*sin(2*pi*t2);
v2Max = max(v2)
v2Min = min(v2)
vMax =
    4.7553
vMin =
   -4.7553
v2Max =
     5
v2Min =
    -5
```

Answer:

For $\Delta t = 0.1$, $v_{MAX} = 4.7553$ V and $v_{MIN} = -4.7553$ V

For $\Delta t = 0.01$, $v_{MAX} = 5$ V and $v_{MIN} = -5$ V

The smaller time step provides better resolution and allows us to sample the voltage signal at its true maximum and minimum.

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Problem 2-1
The current through a 33-kΩ resistor is 1.2 mA. Find the voltage across the resistor.

Solution:
$v = iR$

```
clear all
R = 33e3;
ii = 1.2e-3;
v = ii*R
v =
    39.6000e+000
```

Answer:
v = 39.6 V

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Problem 2-2
A 6.2-kΩ resistor dissipates 12 mW. Find the current through the resistor.

Solution:
$p = i^2R$

```
clear all
format short eng
R = 6.2e3;
p = 12e-3;
ii = sqrt(p/R)
ii =
     1.3912e-003
```

Answer:
$i = \pm 1.3912$ mA

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Problem 2-3

The conductance of a particular resistor is 1 mS. Find the current through the resistor when connected across a 9 V source.

Solution:

$v = iR$

```
clear all
G = 1e-3;
R = 1/G;
v = 9;
ii = v/R
ii =
     9.0000e-003
```

Answer:

$i = 9$ mA

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Problem 2-4

In Figure P2-4 the resistor dissipates 25 mW. Find R_x.

Solution:

$$p = \frac{v^2}{R}$$

```
clear all
p = 25e-3;
v = 15;
R = v^2/p
R =
     9.0000e+003
```

Answer:

$R = 9\ \text{k}\Omega$

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Problem 2-5

In Figure P2-5 find R_x and the power delivered to the resistor.

Solution:

```
clear all
v = 100;
ii = 10e-3;
R = v/ii
p = v*ii
R =
    10.0000e+003
p =
     1.0000e+000
```

Answer:

R_x = 10 kΩ, p = 1 W

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Problem 2-6

The *i*-*v* characteristic of a nonlinear resistor are $v = 75i + 0.2i^3$.

(a) Calculate *v* and *p* for *i* = ±0.5, ±1, ±2, ±5, and ±10 A.

(b) Find the maximum error in *v* when the device is treated as a 75-Ω linear resistance on the range |i| < 0.5 A.

Solution:

```
clear all
format short eng
ii = [-10, -5, -2, -1, -0.5, 0.5, 1, 2, 5, 10];
v = 75*ii + 0.2*ii.^3;
p = v.*ii;
Results = [ii' v' p']

syms i1
v1 = 75*i1+0.2*i1^3;
v2 = 75*i1;
ii1 = -0.5:0.01:0.5;
vv1 = subs(v1,i1,ii1);
vv2 = subs(v2,i1,ii1);
plot(vv1,ii1,'b','LineWidth',3)
hold on
plot(vv2,ii1,'g','LineWidth',1)
grid on
xlabel('Voltage (V)')
ylabel('Current (A)')
legend('Nonlinear','Linear')
MaxError = max(vv1)-max(vv2)
MaxError2 = subs(v1-v2,i1,0.5)
Results =
   -10.0000e+000  -950.0000e+000     9.5000e+003
    -5.0000e+000  -400.0000e+000     2.0000e+003
    -2.0000e+000  -151.6000e+000   303.2000e+000
    -1.0000e+000   -75.2000e+000    75.2000e+000
  -500.0000e-003   -37.5250e+000    18.7625e+000
   500.0000e-003    37.5250e+000    18.7625e+000
     1.0000e+000    75.2000e+000    75.2000e+000
     2.0000e+000   151.6000e+000   303.2000e+000
     5.0000e+000   400.0000e+000     2.0000e+003
    10.0000e+000   950.0000e+000     9.5000e+003
MaxError =
    25.0000e-003
MaxError2 =
    25.0000e-003
```

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Answer:

(a)

i (A)	*v* (V)	*p* (W)
-10	-950	9500
-5	-400	2000
-2	-151.6	303.2
-1	-75.2	75.2
-0.5	-37.525	18.7625
0.5	37.525	18.7625
1	75.2	75.2
2	151.6	303.2
5	400	2000
10	950	9500

(b) $\text{ERROR}_{\text{MAX}} = 25$ mV

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Problem 2-7

A 10-kΩ resistor has a power rating of ⅛W. Find the maximum voltage that can be applied to the resistor.

Solution:

```
clear all
R = 10e3;
p = 1/8;
v_max = sqrt(p*R)
v_max =
    35.3553e+000
```

Answer:

v_{max} = 35.36 V

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Problem 2-8

A certain type of film resistor is available with resistance values between 10 Ω and 100 MΩ. The maximum ratings for all resistors of this type are 500 V and $^1/_4$ W. Show that the voltage rating is the controlling limit for $R > 1$ MΩ, and that the power rating is the controlling limit when $R < 1$ MΩ.

Solution:

```
clear all
V = 500;
p = 1/4;
R = V^2/p
R =
     1.0000e+006
```

$$p = \frac{v^2}{R}$$

At $R = 1$ MΩ, both p and v can take their maximum values and there are no issues. For $R >$ 1 MΩ, with a maximum voltage, the power must be less than 0.25 W, so the voltage rating on a particular resistor will control the maximum allowable value for the power. For $R < 1$ MΩ, with a maximum voltage, the power will be greater than 0.25 W, so the power rating on a particular resistor will control the maximum allowable value for the voltage.

Answer:

Presented above.

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Problem 2-9

Figure P2-9 shows the circuit symbol for a class of two-terminal devices called diodes. The *i*-*v* relationship for a specific *pn* junction diode is $i = 2\times10^{-16}\left(e^{40v}-1\right)A$

(a) Use this equation to find *i* and *p* for *v* = 0, ±0.1, ±0.2, ±0.4, and ±0.8 V. Use these data to plot the *i*-*v* characteristic of the element.

(b) Is the diode linear or nonlinear, bilateral or nonbilateral, and active or passive?

(c) Use the diode model to predict *i* and *p* for *v* = 5 V. Do you think the model applies to voltages in this range? Explain.

(d) Repeat (c) for *v* = –5 V.

Solution:

```
clear all
v = [-0.8, -0.4, -0.2, -0.1, 0 0.1, 0.2, 0.4, 0.8];
ii = 2e-16*(exp(40*v)-1);
p = v.*ii;
Results = [v' ii' p']
plot(v,ii,'b','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
grid on
v = 5
i5 = 2e-16*(exp(40*v)-1)
v = -5
iNeg5 = 2e-16*(exp(40*v)-1)
Results =
  -800.0000e-003  -200.0000e-018   160.0000e-018
  -400.0000e-003  -200.0000e-018    80.0000e-018
  -200.0000e-003  -199.9329e-018    39.9866e-018
  -100.0000e-003  -196.3369e-018    19.6337e-018
     0.0000e-003     0.0000e-003     0.0000e-003
   100.0000e-003    10.7196e-015     1.0720e-015
   200.0000e-003   595.9916e-015   119.1983e-015
   400.0000e-003     1.7772e-009   710.8888e-012
   800.0000e-003    15.7926e-003    12.6341e-003
v =
     5.0000e+000
i5 =
   144.5195e+069
v =
    -5.0000e+000
iNeg5 =
  -200.0000e-018
```

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Answer:
(a)

v (V)	i (A)	p (W)
-0.8	-2.00E-16	1.60E-16
-0.4	-2.00E-16	8.00E-17
-0.2	-2.00E-16	4.00E-17
-0.1	-1.96E-16	1.96E-17
0	0	0
0.1	1.07E-14	1.07E-15
0.2	5.96E-13	1.19E-13
0.4	1.78E-09	7.11E-10
0.8	1.58E-02	1.26E-02

(b) The plot in Part (a) shows that the device is nonlinear and nonbilateral. The power for the device is always positive, so it is passive.

(c) For $v = 5$ V, $i = 1.45 \times 10^{71}$ A and $p = 7.23 \times 10^{71}$ W. The model is not valid because the current and power are too large.

(d) For $v = -5$ V, $i = -2.00 \times 10^{-16}$ A and $p = 1.00 \times 10^{-15}$ W. The model is valid because the current and power are both essentially zero.

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Problem 2-10

In Figure P2-10 $i_2 = -2$ A and $i_3 = 5$ A. Find i_1 and i_4.

Solution:

Apply KCL at Nodes B and C.

```
clear all
i2 = -2;
i3 = 5;
i1 = -i2
i4 = i2+i3
i1 =
     2.0000e+000
i4 =
     3.0000e+000
```

Answer:

$i_1 = 2$ A and $i_4 = 3$ A.

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Problem 2-11
For the circuit in Figure P2-11:

(a) Identify the nodes and at least two loops.

(b) Identify any elements connected in series or in parallel.

(c) Write KCL and KVL connection equations for the circuit.

Solution:
There are three nodes and three loops.

Answer:
(a) nodes: A, B, C; loops: 1-2; 2-3-4; 1-3-4

(b) series: 3 and 4; parallel: 1 and 2

(c) KCL: node A: $i_1 + i_2 + i_3 = 0$;
node B: $-i_3 + i_4 = 0$;
node C: $-i_1 - i_2 - i_4 = 0$

KVL: loop 1-2: $-v_1 + v_2 = 0$;
loop 2-3-4: $-v_2 + v_3 + v_4 = 0$;
loop 1-3-4: $-v_1 + v_3 + v_4 = 0$

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Problem 2-12

In Figure P2-11, i_2 = –10 mA and i_4 = 20 mA. Find i_1 and i_3.

Solution:

Use the KCL equations developed in the solution to Problem 2-11.

```
clear all
i2 = -10e-3;
i4 = 20e-3;
i3 = i4
i1 = -i2-i3
i3 =
    20.0000e-003
i1 =
   -10.0000e-003
```

Answer:

i_1 = –10 mA and i_3 = 20 mA.

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Problem 2-13
For the circuit in Figure P2-13:

(a) Identify the nodes and at least three loops in the circuit.

(b) Identify any elements connected in series or in parallel.

(c) Write KCL and KVL connection equations for the circuit.

Solution:
There are four nodes and at least five loops. There are only three independent KVL equations.

Answer:
(a) nodes: A, B, C, D;

loops: 1-3-2; 2-4-5; 3-6-4; 1-6-5; 2-3-6-5; 1-6-4-2; 1-3-4-5

(b) series: none; parallel: none

(c) KCL: node A: $i_2 + i_3 + i_4 = 0$;

node B: $i_1 - i_3 + i_6 = 0$;

node C: $-i_1 - i_2 - i_5 = 0$;

node D: $-i_4 + i_5 - i_6 = 0$

KVL: loop 1-3-2: $-v_1 - v_3 + v_2 = 0$;

loop 2-4-5: $-v_2 + v_4 + v_5 = 0$;

loop 3-6-4: $v_3 + v_6 - v_4 = 0$

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Problem 2-14

In Figure P2-13 v_2 = 10 V, v_3 = –10 V, and v_4 = 3 V. Find v_1, v_5, and v_6.

Solution:

Use the KVL equations developed in the solution to Problem 2-13.

```
clear all
v2 = 10;
v3 = -10;
v4 = 3;
v1 = v2 - v3
v5 = v2 - v4
v6 = v4 - v3
v1 =
    20.0000e+000
v5 =
     7.0000e+000
v6 =
    13.0000e+000
```

Answer:

v_1 = 20 V, v_5 = 7 V, and v_6 = 13 V.

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Problem 2-15

The circuit in Figure P2-15 is organized around the three signal lines A, B, and C.

(a) Identify the nodes and at least three loops in the circuit.

(b) Write KCL connection equations for the circuit.

(c) If i_1 = –20 mA, i_2 = –12 mA, and i_3 = 50 mA, find i_4, i_5, and i_6

(d) Show that the circuit in Figure P2-15 is identical to that in Figure P2-13.

Solution:

(a) There are four nodes and at least five loops.

(b) KCL:

node A: $i_2 + i_3 + i_4 = 0$;

node B: $i_1 - i_3 + i_6 = 0$;

node C: $-i_1 - i_2 - i_5 = 0$;

node D: $-i_4 + i_5 - i_6 = 0$

```
clear all
i1 = -20e-3;
i2 = -12e-3;
i3 = 50e-3;
i4 = -i2-i3
i5 = -i1-i2
i6 = i3-i1
i4 =
   -38.0000e-003
i5 =
    32.0000e-003
i6 =
    70.0000e-003
```

Answer:

(a) nodes: A, B, C, D;

loops: 1-3-2; 2-4-5; 3-6-4; 1-6-5; 2-3-6-5; 1-6-4-2; 1-3-4-5

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(b) KCL: node A: $i_2 + i_3 + i_4 = 0$;
node B: $i_1 - i_3 + i_6 = 0$;
node C: $-i_1 - i_2 - i_5 = 0$;
node D: $-i_4 + i_5 - i_6 = 0$

(c) $i_4 = -38$ mA; $i_5 = 32$ mA; $i_6 = 70$ mA

(d) The circuits have the same nodes, connections, and current directions, so they must be equivalent.

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Problem 2-16

In Figure P2-16, v_2 = 10 V, v_3 = 10 V, and v_4 = 10 V. Find v_1 and v_5.

Solution:

Apply KVL to the circuit.

```
clear all
v2 = 10;
v3 = 10;
v4 = 10;
v1 = v2+v3
v5 = v3-v4
v1 =
    20.0000e+000
v5 =
     0.0000e-003
```

Answer:

v_1 = 20 V and v_5 = 0 V.

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Problem 2-17

In Figure P2-17 i_2 = 10 mA, i_3 = –15 mA, and i_4 = 5 mA. Find i_1 and i_5.

Solution:

Apply KCL to the circuit.

```
clear all
i2 = 10e-3;
i3 = -15e-3;
i4 = 5e-3;
i1 = i2-i3+i4
i5 = i2-i1
i1 =
    30.0000e-003
i5 =
   -20.0000e-003
```

Answer:

i_1 = 30 mA and i_5 = −20 mA

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Problem 2-18

(a) Use the passive sign convention to assign voltage variables consistent with the currents in Figure P2-17. Write three KVL connection equations using these voltage variables.

(b) If v_3 = 0 V, what can be said about the voltages across all the other elements?

Solution:

(a) Voltage signs:

Elements 1 and 3: plus on bottom and minus on top

Elements 2 and 4: plus on top and minus on bottom

Element 5: plus on left and minus on right

Write the KVL equations for the loops formed by 1-2, 3-4, and 2-4-5

loop 1-2: $v_1 + v_2 = 0$

loop 3-4: $v_3 + v_4 = 0$

loop 2-4-5: $v_2 - v_4 + v_5 = 0$

(b) If v_3 = 0 V, then v_4 = 0 V. In addition, $v_2 = -v_5$ and $v_1 = v_5$.

Answer:

Presented above.

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Problem 2-19

The KCL equations for a three-node circuit are:

Node A $-i_1 + i_2 - i_4 = 0$

Node B $-i_2 - i_3 + i_5 = 0$

Node C $i_1 + i_3 + i_4 - i_5 = 0$

Draw the circuit diagram and indicate the reference directions for the element currents.

Answer:

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Problem 2-20
Find v_x and i_x in Figure P2-20.

Solution:
Use KCL to find the current and Ohm's Law to find the voltage.

```
clear all
format short eng
is = 2e-3;
ix = -is
vx = 47e3*ix
ix =
    -2.0000e-003
vx =
   -94.0000e+000
```

Answer:
$v_x = -94$ V and $i_x = -2$ mA.

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Problem 2-21
Find v_x and i_x in Figure P2-21.

Solution:
Find the voltage across the 10-Ω resistor using Ohm's Law. The 10-Ω and 5-Ω resistors are in parallel, so they have the same voltage. Find the current through the 5-Ω resistor. The current through the 4-Ω resistor is the sum of the currents through the other two resistors. Find the voltage across the 4-Ω resistor. Then v_x is the sum of the voltages across the 4-Ω and 10-Ω resistors.

```
clear all
i10 = 1/2;
v10 = 10*i10;
v5 = v10;
i5 = v5/5;
ix = i5
i4 = i10+i5;
v4 = 4*i4;
vx = v4+v10
ix =
     1.0000e+000
vx =
    11.0000e+000
```

Answer:
v_x = 11 V and i_x = 1 A

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Problem 2-22
In Figure P 2-22:

(a) Assign a voltage and current variable to every element.

(b) Use KVL to find the voltage across each resistor.

(c) Use Ohm's law to find the current through each resistor.

(d) Use KCL to find the current through each voltage source.

Solution:
(a) For each of the three resistors, the voltage positive sign is on the left and the negative sign is on the right. The current flows from left to right through each element.
Element 1: 50-Ω resistor.
Element 2: left 100-Ω resistor.
Element 3: right 100-Ω resistor.
The left voltage source is v_{S1}, with i_{S1} flowing down.
The center voltage source is v_{S2}, with i_{S2} flowing down.
The right voltage source is v_{S3}, with i_{S3} flowing down.

(b) KVL equations:

$$-v_{S1}+v_1+v_{S3}=0$$

$$-v_{S1}+v_2+v_{S2}=0$$

$$-v_{S2}+v_3+v_{S3}=0$$

```
clear all
format short eng
vs1 = 5;
vs2 = 10;
vs3 = 5;
v1 = vs1-vs3
v2 = vs1-vs2
v3 = vs2-vs3
```

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```
v1 =
     0.0000e-003
v2 =
    -5.0000e+000
v3 =
     5.0000e+000
```

(c) $v = iR$

```
i1 = v1/50
i2 = v2/100
i3 = v3/100
i1 =
     0.0000e-003
i2 =
   -50.0000e-003
i3 =
    50.0000e-003
```

(d) KCL equations

$$i_1 + i_2 + i_{S1} = 0$$
$$-i_2 + i_3 + i_{S2} = 0$$
$$-i_1 - i_3 + i_{S3} = 0$$

```
is1 = -i1-i2
is2 = i2-i3
is3 = i1+i3
is1 =
    50.0000e-003
is2 =
  -100.0000e-003
is3 =
    50.0000e-003
```

Answer:
(a) Presented above.
(b) $v_1 = 0$ V, $v_2 = -5$ V, and $v_3 = 5$ V
(c) $i_1 = 0$ mA, $i_2 = -50$ mA, and $i_3 = 50$ mA
(d) $i_{S1} = 50$ mA, $i_{S2} = -100$ mA, and $i_{S3} = 50$ mA

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Problem 2-23
Find the power dissipated in the 1.5 kΩ resistor in Figure P2-23.

Solution:
Label the elements.
Element 1: 1-kΩ resistor with current flowing down
Element 2: 500-Ω resistor with current flowing to the right
Element 3: 1.5-kΩ resistor with current flowing down

Write KCL, KVL, and Ohm's Law equations:
$i_1 + i_2 - 5\text{ mA} = 0$
$-i_2 + i_3 = 0$
$-v_1 + v_2 + v_3 = 0$
$v_1 = 1000 i_1$
$v_2 = 500 i_2$
$v_3 = 1500 i_3$
Solve the equations for v_3 and i_3 and then compute the power $p_3 = v_3\, i_3$.

```
clear all
format short eng
Eqn1 = 'i1+i2-5e-3';
Eqn2 = '-i2+i3';
Eqn3 = '-v1+v2+v3';
Eqn4 = 'v1-1000*i1';
Eqn5 = 'v2-500*i2';
Eqn6 = 'v3-1500*i3';
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,Eqn5,Eqn6,'v1','v2','v3','i1','i2','i3');
v3 = Soln.v3;
i3 = Soln.i3;
p3 = double(v3*i3)
p3 =
     4.1667e-003
```

Answer:
$P_L = 4.167$ mW

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Problem 2-24

Figure P2-24 shows a subcircuit connected to the rest of the circuit at four points.

(a) Use element and connection constraints to find v_x and i_x.

(b) Show that the sum of the currents into the rest of the circuit is zero.

(c) Find the voltage v_A with respect to the ground in the circuit.

Solution:

(a) Label the elements.

Element 1: 5-kΩ resistor with current flowing from left to right

Element 2: 2-kΩ resistor with positive voltage sign on the bottom

Use Ohm's Law to compute i_1.

Use KCL at the center node to find i_x.

Use Ohm's Law to find v_x.

```
clear all
v1 = 20;
R1 = 5e3;
i1 = v1/R1;
ix = i1+4e-3 - 6e-3
Rx = 8e3;
vx = ix*Rx
ix =
     2.0000e-003
vx =
    16.0000e+000
```

(b) The sum of the currents into the rest of the circuit is $-i_1 + i_2 - 4\text{ mA} + i_x$.

```
i2 = 6e-3;
Current_Out = -i1+i2-4e-3+ix
Current_Out =
     0.0000e-003
```

(c) $v_A = 12 + v_x - v_2$

```
R2 = 2e3;
v2 = i2*R2;
vA = 12+vx-v2
vA =
    16.0000e+000
```

Answer:

(a) $v_x = 16$ V and $i_x = 2$ mA.

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(b) i_{OUT} = 0 mA.
(c) v_A = 16 V.

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Problem 2-25

In Figure P2-25 i_x = –0.5 mA. Find the value of R.

Solution:

Label the circuit elements.

Element 1: Resistor R with current flowing down.

Element 2: 10-kΩ resistor with current flowing from right to left

Compute the voltage v_x using Ohm's Law.

Compute the voltage v_1 using KVL.

Compute the voltage v_2 using KVL.

Compute the current i_2 using Ohm's Law.

Compute the current i_1 using KCL.

Compute the resistance $R_1 = R$ using Ohm's Law.

```
clear all
Rx = 10e3;
R2 = 10e3;
ix = -0.5e-3;
vx = ix*Rx;
v1 = 4-vx;
v2 = 15-v1;
i2 = v2/R2;
i1 = ix+i2;
R1 = v1/i1;
R = R1
R =
    90.0000e+003
```

Answer:

R = 90 kΩ

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Problem 2-26

Figure P2-26 shows a resistor with one terminal connected to ground and the other connected to an arrow. The arrow symbol is used to indicate a connection to one terminal of a voltage source whose other terminal is connected to ground. The label next to the arrow indicates the source voltage at the ungrounded terminal. Find the voltage across, current through, and power dissipated in the resistor.

Solution:

The voltage across the resistor is the voltage on the right side (–15 V) minus the voltage on the left side (0 V), so $v_x = -15 - 0 = -15$ V. Using Ohm's Law, the current is $i_x = v_x / R_x = -150$ μA. The power $p_x = v_x\, i_x = 2.25$ mW.

```
clear all
vx = -15-0
ix = vx/100e3
px = vx*ix
vx =
   -15.0000e+000
ix =
  -150.0000e-006
px =
     2.2500e-003
```

Answer:

$v_x = -15$ V, $i_x = -150$ μA, and $p_x = 2.25$ mW.

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Problem 2-27
Find the equivalent resistance R_{EQ} in Figure P2-27.

Solution:
Combine the resistors working from right to left in the circuit.

```
clear all
R1 = 75;
R2 = 300;
R3 = 100;
R4 = 200;
% Combine R3 and R4 in series
R34 = R3+R4;
% Combine R2 in parallel with the series combination of R3 and R4
R234 = 1/(1/R2 + 1/R34);
% Combine R1 in series with the other combination
R1234 = R1 + R234;
Req = R1234
Req =
   225.0000e+000
```

Answer:
$R_{EQ} = 225\ \Omega$

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Problem 2-28
Find the equivalent resistance R_{EQ} in Figure P2-28.

Solution:
Combine the resistors working from right to left in the circuit.

```
clear all
R1 = 4.7e3;
R2 = 3.3e3;
R3 = 1.5e3;
R4 = 2.2e3;
R34 = 1/(1/R3 + 1/R4);
R234 = R2 + R34;
R1234 = 1/(1/R1 + 1/R234);
Req = R1234
Req =
     2.2157e+003
```

Answer:
R_{EQ} = 2.2157 kΩ

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Problem 2-29

Find R_{EQ} in Figure P2-29 when the switch is open. Repeat when the switch is closed.

Solution:

With the switch open, the 200-Ω and 50-Ω resistors are in parallel. With the switch closed, the 200-Ω and 50-Ω resistors are in parallel with a short circuit, so their equivalent resistance is zero.

```
clear all
R1 = 100;
R2 = 200;
R3 = 50;
R4 = 50;
disp('Switch Open')
Req_open = R1 + 1/(1/R2 + 1/R3) + R4
disp('Switch Closed')
Req_closed = R1 + 0 + R4
Switch Open
Req_open =
   190.0000e+000
Switch Closed
Req_closed =
   150.0000e+000
```

Answer:

Switch open: R_{EQ} = 190 Ω.

Switch closed: R_{EQ} = 150 Ω.

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Problem 2-30

Show how the circuit in Figure P2-30 could be connected to achieve a resistance of 100 Ω, 200 Ω, 150 Ω, 50 Ω, 25 Ω, 33.3 Ω, and 133.3 Ω.

Solution:

The required resistor combinations are described below.

100 Ω: A single 100-Ω resistor.

200 Ω: Two 100-Ω resistors in series.

150 Ω: A 100-Ω resistor in series with a 50-Ω resistor.

50 Ω: A single 50-Ω resistor.

25 Ω: A parallel combination of two 100-Ω resistors and a 50-Ω resistor

33.3 Ω: A parallel combination of a 100-Ω resistor and a 50-Ω resistor

133.3 Ω: A 100-Ω resistor in series with a parallel combination of a 100-Ω resistor and a 50-Ω resistor.

Answer:

The following table summarizes the required connections. A plus sign indicates the nodes are connected together at one of the terminals.

Resistance (Ω)	Terminal 1	Terminal 2
100	A	D
200	A	B
150	A	C
50	C	D
25	A+B+C	D
33.3	B+C	D

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133.3	A	B+C

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Problem 2-31

In Figure P2-31 find the equivalent resistance between terminals A-B, A-C, A-D, B-C, B-D, and C-D.

$R_{C\text{-}D}$ is shown.

Solution:

For each pair of end terminals, combine the appropriate resistors in series and parallel to get the equivalent resistance.

```
clear all
Rab = 1/(1/40 + 1/(40+80)) + 60
Rac = 1/(1/40 + 1/(40+80)) + 30
Rad = 1/(1/40 + 1/(40+80)) + 10
Rbc = 60 + 1/(1/(40+40) + 1/80) + 30
Rbd = 60 + 1/(1/(40+40) + 1/80) + 10
Rcd = 30 + 0 + 10
Rab =
    90.0000e+000
Rac =
    60.0000e+000
Rad =
    40.0000e+000
Rbc =
   130.0000e+000
Rbd =
   110.0000e+000
Rcd =
    40.0000e+000
```

Answer:

R_{AB} = 90 Ω

R_{AC} = 60 Ω

R_{AD} = 40 Ω

R_{BC} = 130 Ω

R_{BD} = 110 Ω

R_{CD} = 40 Ω

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Problem 2-32

Select a value of R_L in Figure P2-32 so that R_{EQ} = 25 kΩ. Repeat for R_{EQ} = 20 kΩ.

Solution:

Find an expression for R_L in terms of R_{EQ} and then solve for R_L for both values of R_{EQ}.

```
clear all
syms RL Req positive
Eqn = 'Req - (10e3 + 1/(1/10e3 + 1/RL) + 10e3)';
Soln = solve(Eqn,'RL')
Req_values = [25e3 20e3];
RL_values = subs(Soln,Req,Req_values)
Soln =
-10000.*(Req-20000.)/(Req-30000.)
RL_values =
    10.0000e+003     0.0000e-003
```

Answer:

For R_{EQ} = 25 kΩ, R_L = 10 kΩ.

For R_{EQ} = 20 kΩ, R_L = 0 kΩ.

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Problem 2-33

Using no more than four 1-kΩ resistors, show how the following equivalent resistors can be constructed: 2 kΩ, 500 Ω, 1.5 kΩ, 333 Ω, 250 Ω, and 400 Ω.

Solution:

R = 2 kΩ: two resistors in series
R = 500 Ω: two resistors in parallel
R = 1.5 kΩ: one resistor in series with a parallel combination of two resistors
R = 333 Ω: three resistors in parallel;
R = 250 Ω: four resistors in parallel;
R = 400 Ω: two resistors in series in parallel with two resistors in parallel

```
clear all
R = 1000;
R2000 = R + R
R500 = 1/(1/R + 1/R)
R1500 = R + 1/(1/R + 1/R)
R333 = 1/(1/R + 1/R + 1/R)
R250 = 1/(1/R + 1/R + 1/R + 1/R)
R400 = 1/(1/(R+R) + 1/R + 1/R)
R2000 =
     2.0000e+003
R500 =
   500.0000e+000
R1500 =
     1.5000e+003
R333 =
   333.3333e+000
R250 =
   250.0000e+000
R400 =
   400.0000e+000
```

Answer:

Presented above.

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Problem 2-34

Find the equivalent practical voltage source at terminals A and B in Figure P2-34

Solution:

A current source in series with a resistor is equivalent to just the current source, so we can remove the 5-Ω resistor. That leaves a 5-A current source in parallel with a 10-Ω resistor. The current source and parallel resistor can be converted into a voltage source in series with the same resistor. The value for the voltage source follows Ohm's Law, so v_S = (5 A)(10 Ω) = 50 V.

Answer:

v_S = 50 V and R_S = 10 Ω.

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Problem 2-35

In Figure P2-35 the *i*-*v* characteristic of network N is $v + 50i = 5$ V. Find the equivalent practical current source for the network.

Solution:

When the circuit is open between nodes A and B, there is no current, $i = 0$ A, and the voltage must be $v = 5$ V. When a short is placed between nodes A and B, the voltage is zero, $v = 0$ V, and the current is $i = 100$ mA. The corresponding practical current source will have a current $i_S = 100$ mA and a parallel resistance $R = (5\text{ V})/(100\text{ mA}) = 50\ \Omega$.

Answer:

$i_S = 100$ mA and $R_S = 50\ \Omega$.

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Problem 2-36

Select the value of R_x in Figure P2-36 so that R_{EQ} = 60 kΩ.

Solution:

Find an expression for R_{EQ} in terms of R_x and then solve for R_x in terms of R_{EQ}.

```
clear all
syms Rx Req positive
Eqn = 'Req - (47e3 + 1/(1/22e3 + 1/(Rx+10e3)))';
Soln = solve(Eqn,'Rx');
Rx_value = subs(Soln,Req,60e3)
Check_Req = 47e3 + 1/(1/22e3 + 1/(Rx_value+10e3))
Rx_value =
    21.7778e+003
Check_Req =
    60.0000e+003
```

Answer:

R_x = 21.78 kΩ.

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Problem 2-37

Two 10-kΩ potentiometers (a variable resistor whose value between the two ends is 10 kΩ and between one end and the wiper – the third terminal – can range from 0 Ω to 10 kΩ) are connected as shown in Figure P2-37. What is the range of R_{EQ}?

Solution:

At the limits of their settings, the two potentiometers are either in series or parallel. These represent the maximum and minimum equivalent resistances that the combination can take.

```
clear all
R = 10e3;
Rmax = R + R
Rmin = 1/(1/R + 1/R)
Rmax =
    20.0000e+003
Rmin =
     5.0000e+003
```

Answer:

5 kΩ ≤ R_{EQ} ≤ 20 kΩ

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Problem 2-38

Select the value of R in Figure P2-38 so that $R_{A\text{-}B} = R_L$.

Solution:

Find an expression for $R_{A\text{-}B}$ in terms of R and R_L. Set $R_{A\text{-}B}$ equal to R_L. Solve for R in terms of R_L and choose the positive solution for the resistance.

```
clear all
syms R Rab RL positive
Eqn = 'RL - (R + 1/(1/4/R + 1/(R+RL)))';
Soln = solve(Eqn,'R')
R = RL/3;
Check_Rab = R + 1/(1/4/R + 1/(R+RL))
Soln =
 -1/3*RL
  1/3*RL
Check_Rab =
RL
```

Answer:

$R = R_L/3$.

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Problem 2-39
What is the range of R_{EQ} in Figure P2-39?

Solution:
The potentiometer varies between 0 and 4 kΩ. Find R_{EQ} with the potentiometer to its maximum and minimum values.

```
clear all
Rp = [0 4e3];
Req = 1e3 + 1./(1./Rp + 1/4e3)
Req =
     1.0000e+003     3.0000e+003
```

Answer:
$1\ \text{k}\Omega \le R_{EQ} \le 3\ \text{k}\Omega$

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Problem 2-40

Find the equivalent resistance between terminals A and B in Figure P2-40

Solution:

Place a voltage source between Nodes A and B and redraw the circuit as the equivalent circuit shown below

Use KVL to show that each resistor experiences a voltage drop equal to the voltage at the source: for R_1, the drop is from left to right, for R_2, the drop is from right to left and for R_3, and the drop is from top to bottom. Therefore, each resistor has current flowing in the direction of the voltage drop equal to v_S/R. Applying KCL at the node above the voltage source, the current flowing out of the voltage source is $i_S = v_S/R + v_S/R + v_S/R = 3\ v_S/R$. The equivalent resistance is the ratio of v_S to i_S, $R_{EQ} = v_S/i_S = R/3$.

Alternatively, rearrangement of the circuit shows that the three resistors are connected in parallel, which yields the same final result.

Answer:

$R_{EQ} = v_S/i_S = R/3$.

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Problem 2-41

Use voltage division in Figure P2-41 to obtain an expression for v_L in terms of R, R_L, and v_S.

Solution:

Find an equivalent resistance for the two resistors in parallel. The voltage v_L appears across this parallel combination, so use voltage division with the equivalent resistance and the resistor in series with the source.

```
clear all
syms vs R RL Req vL
Req = 1/(1/R + 1/RL);
vL = simple(Req*vs/(Req+R))
vL =
vs*RL/(2*RL+R)
```

Answer:

$$v_L = \frac{R_L v_S}{R + 2R_L}$$

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Problem 2-42

Use current division in Figure P2-42 to find v_L if R =10 Ω, R_L = 20 Ω, and i_S = 2.5 mA.

Solution:

Use current division to find the current through the series combination of R and R_L and then use Ohm's Law to compute the voltage v_L.

```
clear all
R = 10;
RL = 20;
is = 2.5e-3;
Req = R + RL;
iL = (1/Req)*is/(1/Req + 1/R);
vL = iL*RL
vL =
    12.5000e-003
```

Answer:

v_L = 12.5 mV.

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Problem 2-43
Find i_x and i_y in Figure P2-43.

Solution:
Combine the 20-Ω and 5-Ω resistors in parallel and then combine that result in series with the 6-Ω resistor. Use current division to find i_x. Use KCL to find the current entering the parallel combination of the 20-Ω and 5-Ω resistors and then use current division again to find the current through the 5-Ω resistor, which is i_y.

```
clear all
Req1 = 1/(1/20 + 1/5);
Req2 = Req1 + 6;
is = 2;
ix = (1/30)*is/(1/30 + 1/Req2)
iReq1 = is - ix;
iy = (1/5)*iReq1/(1/5 + 1/20)
ix =
   500.0000e-003
iy =
     1.2000e+000
```

Answer:
i_x = 500 mA and i_y = 1.2 A.

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Problem 2-44

The 1-kΩ load in Figure P2-44 needs 5 V across it to operate correctly. Where should the wiper on the potentiometer be set (R_X) to obtain the desired output voltage?

Solution:

Redraw the circuit as shown below to clearly identify the potentiometer settings and the resistor values on each side of the wiper.

Find an equivalent resistance for the parallel combination. Use voltage division to find an expression for the voltage across the parallel combination in terms of R_X. Solve for R_X when the voltage across the parallel combination is 5 V. Choose the positive solution.

```
clear all
syms Rx RL Req vL
Eqn1 = 'Req - 1/(1/Rx + 1/1e3)';
Eqn2 = '5 - (Req*24/(Req + 5e3 - Rx))';
Soln = solve(Eqn1,Eqn2,'Req','Rx');
Rx = double(Soln.Rx)
Rx = max(Rx);
Req_Check = 1/(1/Rx + 1/1e3);
V_Check = Req_Check*24/(Req_Check + 5e3 - Rx)
Rx =
     2.3383e+003
    -2.1383e+003
V_Check =
     5.0000e+000
```

Answer:

R_X = 2.3383 kΩ.

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Problem 2-45

Find the range of values of v_O in Figure P2-45.

Solution:

Find an expression for v_O in terms of the value of the potentiometer and then evaluate the expression for the maximum and minimum values of the potentiometer.

```
clear all
Rp = [0,100];
Req = 1./(1./Rp + 1/100);
vs = 24;
vo = Req*vs./(Req + 100)
vo =
     0.0000e-003     8.0000e+000
```

Answer:

$0 \text{ V} \le v_O \le 8 \text{ V}$

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Problem 2-46

Figure P2-46 shows a voltage bridge circuit, that is, two voltage dividers in parallel with a source v_S. One resistor R_X is variable. The goal is often to "balance" the bridge by making $v_X = 0$ V. Derive an expression for R_X in terms of the other resistors for when the bridge is balanced.

Solution:

Let the node between resistors R_A and R_B have a voltage v_1 and let the node between resistors R_C and R_X have a voltage v_2. The goal is to make v_1 equal v_2. Use voltage division to derive expressions for the two voltages. Set the expressions equal and solve for R_X.

$$v_1 = \frac{R_B\ v_S}{R_A + R_B} \text{ and } v_2 = \frac{R_X\ v_S}{R_C + R_X}$$

```
clear all
syms vs Ra Rb Rc Rx
Eqn = 'Rb*vs/(Ra+Rb) - Rx*vs/(Rc+Rx)';
Soln = solve(Eqn,'Rx')
Soln =
Rb*Rc/Ra
```

Answer:

$$R_X = \frac{R_B R_C}{R_A}$$

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Problem 2-47

Ideally, a voltmeter has infinite internal resistance and can be placed across any device to read the voltage without affecting the result. A particular digital multimeter (DMM), a common laboratory tool, is connected across the circuit shown in Figure P2-47. The expected voltage was 5.73 V. However, the DMM reads 3.81 V. The large, but finite, internal resistance of the DMM was "loading" the circuit and causing a wrong measurement to be made. Find the value of the internal resistance of this DMM.

Solution:

The DMM is in parallel with the 6.3-MΩ resistor, so they share the same voltage. Calculate the current through the 6.3-MΩ resistor. The remaining voltage must drop across the 4.7-MΩ resistor, so use that voltage and Ohm's Law to calculate the current through the 4.7-MΩ resistor. Use KCL to find the current through the DMM and then Ohm's Law to find the internal resistance of the DMM.

```
clear all
R1 = 4.7e6;
R2 = 6.3e6;
vs = 10;
v2 = 3.81;
i2 = v2/R2;
v1 = vs - v2;
i1 = v1/R1;
iDMM = i1 - i2;
RDMM = v2/iDMM
RDMM =
     5.3492e+006
```

Answer:

R_{DMM} = 5.3492 MΩ.

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Problem 2-48

Select values for R_1, R_2, and R_3 in Figure P 2-48 so the voltage divider produces the two output voltages shown.

Solution:

If we select the resistors such that the total equivalent resistance seen by the source is 15 kΩ, then 1 mA will flow through each resistor. We can then see that R_1 = 5kΩ, R_2 = 7 kΩ, and R_3 = 3 kΩ will solve the problem.

```
clear all
vs = 15;
R1 = 5e3;
R2 = 7e3;
R3 = 3e3;
v23 = (R2+R3)*vs/(R1+R2+R3)
v3 = R3*vs/(R1+R2+R3)
v23 =
    10.0000e+000
v3 =
     3.0000e+000
```

Answer:

R_1 = 5kΩ, R_2 = 7 kΩ, and R_3 = 3 kΩ. Other solutions are possible.

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Problem 2-49

Select a value of R_x in Figure P2-49 so that v_L = 3 V.

Solution:

The voltage v_L appears across a parallel combination of two 1-kΩ resistors, which have an equivalent resistance of 500 Ω. The current through the equivalent resistor must be 6 mA. That same current must flow through the series combination of the 1-kΩ resistor and R_x and the pair must drop a total of 9 V. The 1-kΩ resistor with 6 mA of current drops 6 V, so R_x must drop 3 V. To carry 6 mA of current and drop 3 V, R_x = 500 Ω.

```
clear all
R = 1e3;
vs = 12;
Req = 1/(1/R+1/R);
vL = 3;
iReq = vL/Req;
is = iReq;
ix = is;
vx = vs-is*R-vL;
Rx = vx/ix
Rx =
   500.0000e+000
```

Answer:

R_x = 500 Ω.

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Problem 2-50

Select a value of R_x in Figure P2-50 so that v_L = 2 V. Repeat for 4 V and 6 V. Caution: R_x must be positive.

Solution:

```
clear all
vL = [2 4 6];
vs = 12;
R1 = 100;
RL = 50;
v1 = vs-vL;
i1 = v1/R1;
iL = vL/RL;
ix = i1-iL;
vx = vL;
Rx = vx./ix;
Rx = Rx'
Rx =
    33.3333e+000
          Inf
  -100.0000e+000
```

Answer:

For v_L = 2 V, select R_x = 33.33 Ω.

For v_L = 4 V, make R_x an open circuit.

For v_L = 6 V, there is no positive value for R_x that will meet this specification.

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Problem 2-51

Use circuit reduction to find v_x and i_x in Figure P2-51.

Solution:

Find the equivalent resistance of the entire circuit. Use Ohm's Law first to compute v_x and then again to compute i_x.

```
clear all
syms R is ix vx Req1 Req2
Req1 = 2*R + R;
Req2 = 1/(1/3/R+1/Req1);
vx = is*Req2
ix = vx/3/R
vx =
3/2*is*R
ix =
1/2*is
```

Answer:

$v_x = 1.5\ R\ i_S$, $i_x = 0.5\ i_S$

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Problem 2-52

Use circuit reduction to find v_x and i_x in Figure P2-52.

Solution:

Find the equivalent resistance of the entire circuit and compute the current through the source. Use the source current and the equivalent resistance seen by v_x to compute v_x. Use the source current and current division to calculate i_x.

```
clear all
syms R vs vx ix Req1 Req2
Req1 = 1/(1/R + 1/R/2);
Req2 = 2*R + 2*Req1;
is = vs/Req2;
vx = Req1*is
ix = (1/R/2)*(-is)/(1/R + 1/R/2)
vx =
1/5*vs
ix =
-1/10*vs/R
```

Answer:

$$v_x = \frac{v_S}{5}$$

$$i_x = \frac{-v_S}{10R}$$

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Problem 2-53

Use circuit reduction to find v_x and the power provided by the source in Figure P2-53.

Solution:

First, find the power provided by the source by combining all of the resistors to determine an overall equivalent resistance. We can then calculate $p = i^2R$. To find v_x, combine the resistors to the right of v_x to determine their equivalent resistance and then use a source transformation to convert the problem into a voltage division problem. Note: You cannot find the power provide by the original source by finding the power provided by an equivalent transformed source.

```
clear all
Req1 = 1/(1/10 + 1/10);
Req2 = 20+Req1;
Req3 = 1/(1/10 + 1/Req2);
Req4 = 20+Req3;
Req = 1/(1/20 + 1/Req4);
is = 3/4;
Rs = 20;
ps = is^2*Req
% Source transformation
vs = is*Rs;
vx = Req3*vs/(Rs + 20 + Req3)
ps =
     6.4773e+000
vx =
     2.2727e+000
```

Answer:

v_x = 2.2727 V and p_S = 6.4773 W.

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Problem 2-54

Use circuit reduction to find v_x and i_x in Figure P2-54.

Solution:

Perform a source transformation and then use current division to find i_x and the current associated with v_x. Use Ohm's Law to find v_x.

```
clear all
vs = 12;
Rs = 6e3;
is = vs/Rs;
ix = (1/12e3)*is/(1/12e3 + 1/6e3 + 1/12e3)
ivx = (1/12e3)*is/(1/12e3 + 1/6e3 + 1/12e3);
vx = ivx*4e3
ix =
   500.0000e-006
vx =
     2.0000e+000
```

Answer:

v_x = 2 V and i_x = 500 μA.

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Problem 2-55

Use source transformation to find i_x in Figure P2-55.

Solution:

Perform a source transformation on the current source and the 30-Ω resistor and then use Ohm's Law to find i_x.

```
clear all
vs1 = 10;
Rs1 = 50;
is2 = 200e-3;
Rs2 = 30;
vs2 = is2*Rs2;
ix = (vs1-vs2)/(Rs1+Rs2)
ix =
    50.0000e-003
```

Answer:

i_x = 50 mA.

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Problem 2-56

Select a value for R_X so that $i_x = 0$ A in Figure P2-56.

Solution:

Perform source transformations on both voltage sources to get the following equivalent circuit:

All three resistors share the same voltage because they are in parallel. If $i_x = 0$ A, then there is no voltage drop across any of the resistors and no current flowing through them. All of the current from the right source must flow through the left source, so we can compute R_X.

```
clear all
vs1 = 24;
vs2 = 12;
Rs2 = 30;
is2 = vs2/Rs2
Rx = vs1/is2
is2 =
   400.0000e-003
Rx =
    60.0000e+000
```

Answer:

$R_X = 60\ \Omega$.

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Problem 2-57

Use source transformations in Figure P2-57 to relate v_O to v_1, v_2, and v_3.

Solution:

Convert each voltage source and resistor pair into a current source in parallel with a resistor to get the following circuit:

We can then add the three current sources, since they are in parallel, and combine the three parallel resistors to get the following circuit:

The output voltage is then the product of the equivalent current source and the equivalent resistance

$$v_O = \frac{v_1 + v_2 + v_3}{R}\frac{R}{3} = \frac{v_1 + v_2 + v_3}{3}$$

```
clear all
syms v1 v2 v3 R i1 i2 i3 Req ieq vo
i1 = v1/R;
i2 = v2/R;
i3 = v3/R;
Req = 1/(1/R + 1/R + 1/R);
ieq = i1 + i2 + i3;
vo = simple(ieq*Req)
vo =
1/3*v1+1/3*v2+1/3*v3
```

Answer:

$$v_O = \frac{v_1 + v_2 + v_3}{3}$$

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Problem 2-58

The current through R_L in Figure P2-58 is 40 mA. Use source transformations to find R_L.

Solution:

Perform a source transformation and then combine the two 100-Ω resistors in parallel to get a 50-Ω equivalent resistor. The resulting current source has a value of 100 mA. Since 40 mA flows through R_L, the remaining 60 mA must flow through the equivalent 50-Ω resistor and it causes a voltage drop of 3 V. The series path with the original 50-Ω resistor and R_L must also drop a total of 3 V, so the total equivalent resistance of the path must be (3 V)/(40 mA) = 75 Ω. Therefore R_L = 25 Ω.

```
clear all
iL = 40e-3;
vs = 10;
Rs = 100;
is = vs/Rs;
Req = 1/(1/100 + 1/100);
iReq = is - iL;
vReq = iReq*Req;
v50L = vReq;
R50L = v50L/iL;
RL = R50L - 50
RL =
    25.0000e+000
```

Answer:

R_L = 25 Ω.

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Problem 2-59

Select R_X so that 25 V are across it in Figure P2-59.

Solution:

Reduce the circuit on both sides of R_X. To the right of R_X, the equivalent resistance is 500 Ω. To the left of R_X, after a source transformations, a parallel resistor combination, and another source transformation, we have a voltage source v_S = 66.7 V in series with a 333-Ω resistor. The resulting equivalent circuit is shown below:

The two known resistors can be combined in series to reduce the circuit further:

If R_X drops 25 V, then the 833-Ω resistor must drop (66.7 – 25) V = 41.7 V. The current through the circuit must be 41.7/833 = 50 mA. We can then calculate R_X = (25 V)/(50 mA) = 500 Ω.

```
clear all
vx = 25;
Req_right = 1/(1/1000 + 1/(800+200));
vs = 100;
Rs = 500;
is = vs/Rs;
Req_left = 1/(1/500 + 1/1000);
vs2 = is*Req_left;
Req = Req_left + Req_right;
vReq = vs2 - vx;
iReq = vReq/Req;
ix = iReq;
Rx = vx/ix
Rx =
   500.0000e+000
```

Answer:

R_X = 500 Ω.

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Problem 2-60

The box in the circuit in Figure P2-60 is a resistor whose value can be anywhere between 8 kΩ and 80 kΩ. Use circuit reduction to find the range of values of v_x.

Solution:

The solution is presented in the commented MATLAB script below.

```
clear all
R = 10e3;
% Source transformation to a current source
vs = 50;
Rs = 10e3;
is = vs/Rs
% Combine parallel resistors
Req = 1/(1/Rs + 1/R)
% Source transformation back to a voltage source
vs2 = is*Req
% Unknown resistor range
Rp = [8e3 80e3];
% Voltage division to calculate vx
vx = R*vs2./(R+Rp+Req)
is =
     5.0000e-003
Req =
     5.0000e+003
vs2 =
    25.0000e+000
vx =
    10.8696e+000     2.6316e+000
```

Answer:

2.6316 V ≤ v_x ≤ 10.8696 V

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Problem 2-61

A circuit is found to have the following element and connection equations:

$v_1 = 24$ V

$v_2 = 8\text{k}\ i_2$

$v_3 = 5\text{k}\ i_3$

$v_4 = 4\text{k}\ i_4$

$v_5 = 16\text{k}\ i_5$

$-v_1 + v_2 + v_3 = 0$

$-v_3 + v_4 + v_5 = 0$

$i_1 + i_2 = 0$

$-i_2 + i_3 + i_4 = 0$

$-i_4 + i_5 = 0$

Use MATLAB to solve for all of the unknown voltages and currents associated with this circuit. Sketch one possible schematic that matches the given equations.

Solution:

Write the equations in matrix form using a vector of unknowns as:

$x = [v_1, v_2, v_3, v_4, v_5, i_1, i_2, i_3, i_4, i_5]$.

```
clear all
A = [1 0 0 0 0 0 0 0 0 0;
    0 1 0 0 0 0 -8000 0 0 0;
    0 0 1 0 0 0 0 -5000 0 0;
    0 0 0 1 0 0 0 0 -4000 0;
    0 0 0 0 1 0 0 0 0 -16000;
    -1 1 1 0 0 0 0 0 0 0;
    0 0 -1 1 1 0 0 0 0 0;
    0 0 0 0 0 1 1 0 0 0;
    0 0 0 0 0 0 -1 1 1 0;
    0 0 0 0 0 0 0 0 -1 1];
B = [24 0 0 0 0 0 0 0 0 0]';
x = A\B
x =
   24.0000
   16.0000
    8.0000
    1.6000
    6.4000
   -0.0020
    0.0020
    0.0016
    0.0004
    0.0004
```

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One possible circuit design that matches the given equations is shown below.

Answer:
[v_1, v_2, v_3, v_4, v_5, i_1, i_2, i_3, i_4, i_5]
= [24 V, 16 V, 8 V, 1.6 V, 6.4 V, –2 mA, 2 mA, 1.6 mA, 0.4 mA, 0.4 mA]
The schematic is shown above.

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Problem 2-62

Consider the circuit of Figure P2-62. Use MATLAB to find all of the voltages and currents in the circuit.

Solution:

Use KVL, KCL, and Ohm's Law to create a set of 14 equations to solve for all of the voltages and currents. Also compute the powers for each element. The unknown vector is:

$x = [v_S, v_1, v_2, v_3, v_4, v_5, v_6, i_S, i_1, i_2, i_3, i_4, i_5, i_6]$

The equations and solution are presented in the following MATLAB code.

```
clear all
format short eng
% Set the matrix to be all zeros and then assign individual values directly
A = zeros(14,14);
A(1,1) = 1;
A(2,2) = 1; A(2,9) = -15e3;
A(3,3) = 1; A(3,10) = -10e3;
A(4,4) = 1; A(4,11) = -33e3;
A(5,5) = 1; A(5,12) = -15e3;
A(6,6) = 1; A(6,13) = -68e3;
A(7,7) = 1; A(7,14) = -22e3;
A(8,8) = 1; A(8,9) = 1;
A(9,9) = 1; A(9,10) = -1; A(9,11) = -1;
A(10,11) = 1; A(10,12) = -1; A(10,13) = -1;
A(11,13) = 1; A(11,14) = -1;
A(12,1) = -1; A(12,2) = 1; A(12,3) = 1;
A(13,3) = -1; A(13,4) = 1; A(13,5) = 1;
A(14,5) = -1; A(14,6) = 1; A(14,7) = 1;
% Display the A matrix
%A
B = zeros(14,1);
B(1) = 120;
% Display the B matrix
%B
% Solve for the output vector
x = A\B;
Results = x';
% Create vectors of the voltages and currents
```

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```
Voltages = x(1:7)
Currents = x(8:14)
% Compute the associated powers
Powers = Voltages.*Currents
PTotal = sum(Powers)
Voltages =
   120.0000e+000
    77.5537e+000
    42.4463e+000
    30.5455e+000
    11.9008e+000
     8.9917e+000
     2.9091e+000
Currents =
    -5.1702e-003
     5.1702e-003
     4.2446e-003
   925.6198e-006
   793.3884e-006
   132.2314e-006
   132.2314e-006
Powers =
  -620.4298e-003
   400.9720e-003
   180.1687e-003
    28.2735e-003
     9.4420e-003
     1.1890e-003
   384.6732e-006
PTotal =
  -101.2103e-018
```

Answer:
The results are summarized in the table below:

Element	Voltage (V)	Current (mA)	Power (mW)
Source	120.0000	-5.1702	-620.4298
1	77.5537	5.1702	400.9720
2	42.4463	4.2446	180.1687
3	30.5455	0.9256	28.2735
4	11.9008	0.7934	9.4420
5	8.9917	0.1322	1.1890
6	2.9091	0.1322	0.3847

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Problem 2-63
Consider the circuit of Figure P2-62 again. Use OrCAD to find all of the voltages and currents in the circuit.

Solution:
The results of the OrCAD simulation are shown below.

Answer:
Presented above.

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Problem 2-64

Use OrCAD to find all of the power dissipated or provided in the circuit of Figure P2-62. Verify that the sum of all power in the circuit is zero.

Solution:

The results of the OrCAD simulation are shown below.

$p_{\text{TOTAL}} = -620.4 + 401.0 + 180.2 + 28.27 + 9.442 + 1.189 + 0.3847 = 0$ mW

Answer:

The schematic is shown above. The sum of the powers is $p = 0$ mW.

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Problem 2-65

Nonlinear Device Characteristics **(A)**

The circuit in Figure P2-65 is a parallel combination of a 50-Ω linear resistor and a varistor whose *i-v* characteristic is $i_V = 2.6 \times 10^{-5} v^3$. For a small voltage, the varistor current is quite small compared to the resistor current. For large voltages, the varistor dominates because its current increases more rapidly with voltage.

(a) Plot the *i-v* characteristic of the parallel combination.

(b) State whether the parallel combination is linear or nonlinear, active or passive, and bilateral or nonbilateral.

(c) Find the range of voltages over which the resistor current is at least 10 times as large as the varistor current.

(d) Find the range of voltages over which the varistor current is at least 10 times as large as the resistor current.

Solution:

The solution is presented in the following commented MATLAB code.

(a)

```
clear all
disp('Part (a)')
% Set the range of voltages to plot
v = -200:1:200;
% Compute the current through the resistor
iR = v/50;
% Compute the current through the varistor
iV = 2.6e-5*v.^3;
% Sum the two path currents to get the total current
iTotal = iR + iV;
% Plot the i-v characteristic
plot(v,iTotal,'b','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
grid on
```

```
Part (a)
```

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(b) The plot created in Part (a) shows that the parallel combination is nonlinear, passive, and bilateral.

(c) Solve the equations for positive voltages and realize that the full range will include negative voltages.

```
clear all
syms v
Eqn = 'v/50 - 10*(2.6e-5*v^3)';
Soln = double(solve(Eqn,'v'))
Soln =
         0
    8.7706
   -8.7706
```

–8.7706 V < v < 8.7706 V

(d) Solve the equations for positive voltages and realize that the full range will include negative voltages.

```
clear all
syms v
Eqn = '2.6e-5*v^3 - v/5';
Soln = double(solve(Eqn,'v'))
Soln =
         0
   87.7058
  -87.7058
```

|v| > 87.7058 V

Answer:

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(a) The plot is shown above.
(b) The parallel combination is nonlinear, passive, and bilateral.
(c) $-8.7706\text{ V} < v < 8.7706\text{ V}$
(d) $|v| > 87.7058\text{ V}$

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Problem 2-66

Center Tapped Voltage Divider **(A)**

Figure P2-66 shows a voltage divider with the center tap connected to ground. Derive equations relating v_A and v_B to v_S, R_1, and R_2.

Solution:

Using the passive sign convention, $i_S = -i_A = i_B$. We can calculate the magnitude of the current by combining the resistors in series and using Ohm's Law.

$$i_A = \frac{v_S}{R_1 + R_2}$$

Now apply Ohm's Law to each resistor to find v_A and v_B.

$$v_A = i_A R_1 = \frac{R_1 v_S}{R_1 + R_2}$$

$$v_B = i_B R_2 = \frac{-R_2 v_S}{R_1 + R_2}$$

Answer:

$$v_A = \frac{R_1 v_S}{R_1 + R_2}$$

$$v_B = \frac{-R_2 v_S}{R_1 + R_2}$$

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Problem 2-67

Active Transducer **(A)**

Figure P2-67 shows an active transducer whose resistance $R(V_T)$ varies with the transducer voltage V_T as $R(V_T) = 0.5\ V_T^2 + 1$. The transducer supplies a current to a 10-Ω load. At what voltage will the load current equal 100 mA?

Solution:

Develop an expression for the current in terms of the transducer voltage and solve for the voltage that will make the current equal 100 mA.

```
clear all
format short eng
syms iL Vtr
Eqn = 'iL - Vtr/(0.5*Vtr^2 + 1 + 10)';
Soln = solve(Eqn,'Vtr');
Vtr_Soln = double(subs(Soln,iL,100e-3))
iL_Check = Vtr_Soln./(0.5*Vtr_Soln.^2+1+10)
Vtr_Soln =
    18.8318e+000
     1.1682e+000
iL_Check =
   100.0000e-003
   100.0000e-003
```

Answer:

There are two answers: $V_T = 1.1682$ V or $V_T = 18.8318$ V.

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Problem 2-68

Programmable Voltage Divider **(A)**

Figure P2-68 shows a programmable voltage divider in which digital inputs b_0 and b_1 control complementary analog switches connecting a multitap voltage divider to the analog output v_O. The switch positions in the figure apply when digital inputs are low. When inputs go high the switch positions reverse. Find the analog output voltage for (b_1,b_0) = (0,0), (0,1), (1,0), and (1,1) when V_{REF} = 12 V.

Solution:

There are four equal resistors in series with a voltage source, so each drops one quarter of the total voltage, or 3 V in this case. As we cycle through the four combinations of the digital inputs, the switches connect the output voltage to be across zero, one, two, or three resistors, in that order. The output voltages are therefore 0 V, 3 V, 6 V, and 9 V.

Answer:

For (b_1,b_0) = (0,0), v_O = 0 V.
For (b_1,b_0) = (0,1), v_O = 3 V.
For (b_1,b_0) = (1,0), v_O = 6 V.
For (b_1,b_0) = (1,1), v_O = 9 V.

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Problem 2-69

Analog Voltmeter Design **(A, D, E)**

Figure P2-69(a) shows a voltmeter circuit consisting of a D'Arsonval meter, two series resistors, and a two-position selector switch. A current of I_{FS} = 400 μA produces full-scale deflection of the D'Arsonval meter, whose internal resistance is R_M = 25 Ω.

(a) **(D)** Select the series resistance R_1 and R_2 so a voltage v_x = 100 V produces full-scale deflection when the switch is in position A, and voltage v_x = 10 V produces full-scale deflection when the switch is in position B.

(b) **(A)** What is the voltage across the 20-kΩ resistor in Figure P2-69(b)? What is the voltage when the voltmeter in part (a) is set to position A and connected across the 20-kΩ resistor? What is the percentage error introduced connecting the voltmeter?

(c) **(E)** A different D'Arsonval meter is available with an internal resistance of 100 Ω and a full-scale deflection current of 100 μA. If the voltmeter in part (a) is redesigned using this D'Arsonval meter, would the error found in part (b) be smaller or larger? Explain.

Solution:

(a) Solve for R_2 first, such that a 10-V input causes 400 μA to flow through the two resistors. Then solve for R_1, such that a 100-V input causes 400 μA to flow through all three resistors.

```
clear all
IFS = 400e-6;
RM = 25;
Req10 = 10/IFS;
R2 = Req10 - RM
Req100 = 100/IFS;
R1 = Req100-Req10
R2 =
    24.9750e+003
R1 =
   225.0000e+003
```

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(b) Using voltage division, the voltage across the 20-kΩ resistor is 20 V when the voltmeter is not connected. When the voltmeter is set in position A and connected in parallel to the 20-kΩ resistor, it is equivalent to placing a 250-kΩ resistor in parallel with the 20-kΩ resistor. We can then find the voltage using voltage division and compute the error.

```
clear all
Req = 1/(1/20e3 + 1/250e3);
vM = Req*50/(Req + 30e3)
ErrorPercent = 100*(20-vM)/20
vM =
    19.0840e+000
ErrorPercent =
     4.5802e+000
```

(c) With a full-scale deflection current of 100 μA for an applied voltage of 100 V, (switch in position A,) the total resistance of the meter must be 1 MΩ. The increased meter resistance will draw less current when it is connected to the 20-kΩ resistor and have a smaller impact on the voltage. The error will decrease. The following calculations verify the results with numerical values.

```
clear all
Req = 1/(1/20e3 + 1/1e6);
vM = Req*50/(Req + 30e3)
ErrorPercent = 100*(20-vM)/20
vM =
    19.7628e+000
ErrorPercent =
     1.1858e+000
```

Answer:
(a) R_1 = 225 kΩ and R_2 = 24.975 kΩ.
(b) v_{ACTUAL} = 20 V, v_{MEAS} = 19.084 V, Error = 4.58%
(c) The error will be smaller as explained and verified above.

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Problem 2-70

MATLAB Function for Parallel Equivalent Resistors (A)

Create a MATLAB function to compute the equivalent resistance of a set of resistors connected in parallel. The function has a single input, which is a vector containing the values of all of the resistors in parallel, and it has a single output, which is the equivalent resistance. Name the function "EQparallel" and test it with at least three different resistor combinations. At least one test should have three or more resistor values.

Solution:

Create a MATLAB script (m-file) named `EQparallel.m` that contains the following code:

```
function Zp = EQparallel(Z)

% Compute the equivalent parallel impedance of a list of impedances

Zinv = 1./Z;
Zp = 1/sum(Zinv);
```

Save the function file and change the MATLAB path to include the location of the function file. Run the following commands to check the function.

```
R1 = EQparallel([1000 1000])
R2 = EQparallel([5e3 20e3])
R3 = EQparallel([4e3 5e3 20e3])
```

The results are:

```
R1 = 500
R2 = 4000
R3 = 2000
```

Answer:

The MATLAB function and examples are presented above. Run the code outside of this Word document.

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Problem 3-1

(a) Formulate node-voltage equations for the circuit in Figure P3-1.

(b) Solve these equations for v_A and v_B.

(c) Use these results to find v_x and i_x.

Solution:

(a)

Node A: $-2+\dfrac{v_A-0}{40}+\dfrac{v_A-v_B}{30}=0$

Node B: $\dfrac{v_B-v_A}{30}+\dfrac{v_B-0}{20}+\dfrac{v_B-0}{80}=0$

(b)

```
clear all
format short eng
syms vA vB
Eqn1 = '-2 + vA/40 + (vA-vB)/30';
Eqn2 = '(vB-vA)/30 + vB/20 + vB/80';
Soln = solve(Eqn1,Eqn2,'vA','vB');
vA = double(Soln.vA)
vB = double(Soln.vB)
vA =
    42.7907e+000
vB =
    14.8837e+000
```

(c)

```
vx = vA-vB
ix = vB/80
vx =
    27.9070e+000
ix =
   186.0465e-003
```

Answer:

(a) Node A: $-2+\dfrac{v_A-0}{40}+\dfrac{v_A-v_B}{30}=0$

Node B: $\dfrac{v_B-v_A}{30}+\dfrac{v_B-0}{20}+\dfrac{v_B-0}{80}=0$

(b) v_A = 42.79 V and v_B = 14.88 V

(c) v_x = 27.91 V and i_x = 186.05 mA

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Problem 3-2

(a) Formulate node-voltage equations for the circuit in Figure P3-2.

(b) Solve these equations for v_A and v_B.

(c) Use these results to find v_x and i_x.

Solution:

The equations are formulated and solved in the following MATLAB code.

```
clear all
syms vA vB
Eqn1 = 'vA/20 + vA/10 + (vA-vB)/8 - 3';
Eqn2 = '3 + (vB-vA)/8 + vB/4 - 1';
Soln = solve(Eqn1,Eqn2,'vA','vB');
vA = double(Soln.vA)
vB = double(Soln.vB)
vx = vB
ix = vA/20
vA =
    10.0000e+000
vB =
    -2.0000e+000
vx =
    -2.0000e+000
ix =
   500.0000e-003
```

Answer:

(a) Node A: $\dfrac{v_A - 0}{20} + \dfrac{v_A - 0}{10} + \dfrac{v_A - v_B}{8} - 3 = 0$

Node B: $3 + \dfrac{v_B - v_A}{8} + \dfrac{v_B - 0}{4} - 1 = 0$

(b) $v_A = 10$ V and $v_B = -2$ V

(c) $v_x = -2$ V and $i_x = 500$ mA

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Problem 3-3

(a) Formulate node-voltage equations for the circuit in Figure P3-3.

(b) Solve these equations for v_A and v_B.

(c) Use these results to find v_x and i_x.

Solution:

The equations are formulated and solved in the following MATLAB code.

```
clear all
syms vA vB
Eqn1 = '20e-3 + vA/4e3 + (vA-vB)/2e3 - 20e-3';
Eqn2 = '20e-3 + (vB-vA)/2e3 + vB/4e3 + vB/2e3';
Soln = solve(Eqn1,Eqn2,'vA','vB');
vA = double(Soln.vA)
vB = double(Soln.vB)
vx = vB/2
ix = (vA-vB)/2e3
vA =
   -14.5455e+000
vB =
   -21.8182e+000
vx =
   -10.9091e+000
ix =
     3.6364e-003
```

Answer:

(a) Node A: $0.02 + \dfrac{v_A - 0}{4000} + \dfrac{v_A - v_B}{2000} - 0.02 = 0$;

Node B: $0.02 + \dfrac{v_B - v_A}{2000} + \dfrac{v_B - 0}{4000} + \dfrac{v_B - 0}{2000} = 0$

(b) $v_A = -14.55$ V; $v_B = -21.82$ V

(c) $v_x = -10.91$ V; $i_x = 3.64$ mA

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Problem 3-4

(a) Formulate node-voltage equations for the circuit in Figure P3-4.

(b) Solve these equations for v_A and v_C.

(c) Use these results to find v_x and i_x.

Solution:

The equations are formulated and solved in the following MATLAB code.

```
clear all
syms vA vB vC
Eqn1 = 'vA/2e3 + (vA-vB)/4e3 - 1e-3';
Eqn2 = 'vB - 5';
Eqn3 = '1e-3 + (vC-vB)/4e3 + vC/4e3';
Soln = solve(Eqn1,Eqn2,Eqn3,'vA','vB','vC');
vA = double(Soln.vA)
vB = double(Soln.vB)
vC = double(Soln.vC)
vx = vA-vC
ix = (vA-vB)/4e3 + (vC-vB)/4e3
vA =
     3.0000e+000
vB =
     5.0000e+000
vC =
   500.0000e-003
vx =
     2.5000e+000
ix =
    -1.6250e-003
```

Answer:

(a) The node-voltage equations are presented above in the MATLAB code.

(b) $v_A = 3$ V and $v_C = 500$ mV

(c) $v_x = 2.5$ V and $i_x = -1.625$ mA

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Problem 3-5

(a) Choose a ground wisely and formulate node-voltage equations for the circuit in Figure P3-5.

(b) Solve for v_x and i_x when $R_1 = R_2 = R_3 = R_4 = 1$ kΩ, $v_S = 20$ V, and $i_S = 1$ mA.

Solution:

(a) Choose the ground to be at the node connected to the negative terminal of the voltage source. Label the other three nodes as A, B, and C from left to right. The node-voltage equations are formulated in the following MATLAB code.

```
clear all
syms is vs R1 R2 R3 R4 vA vB vC
Eqn1 = '-is + (vA-vB)/R4 + (vA-vC)/R3';
Eqn2 = '(vB-vA)/R4 + vB/R1 + (vB-vC)/R2';
Eqn3 = 'vC - vs';
```

(b) Solve the equations and make substitutions for the known parameters.

```
Soln = solve(Eqn1,Eqn2,Eqn3,'vA','vB','vC');
vA = Soln.vA;
vB = Soln.vB;
vC = Soln.vC;
vx = vB-vC;
ix = (vC-vA)/R3;
vA_num = double(subs(vA,{R1,R2,R3,R4,vs,is},{1e3,1e3,1e3,1e3,20,1e-3}));
vB_num = double(subs(vB,{R1,R2,R3,R4,vs,is},{1e3,1e3,1e3,1e3,20,1e-3}));
vC_num = double(subs(vC,{R1,R2,R3,R4,vs,is},{1e3,1e3,1e3,1e3,20,1e-3}));
vx_num = double(subs(vx,{R1,R2,R3,R4,vs,is},{1e3,1e3,1e3,1e3,20,1e-3}))
ix_num = double(subs(ix,{R1,R2,R3,R4,vs,is},{1e3,1e3,1e3,1e3,20,1e-3}))
vx_num =
    -7.8000e+000
ix_num =
     3.4000e-003
```

Answer:

(a) Node A: $-i_S + \dfrac{v_A - v_B}{R_4} + \dfrac{v_A - v_C}{R_3} = 0$;

Node B: $\dfrac{v_B - v_A}{R_4} + \dfrac{v_B - 0}{R_1} + \dfrac{v_B - v_C}{R_2} = 0$;

Node C: $v_C = v_S$

(b) $v_x = -7.80$ V and $i_x = 3.40$ mA

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Problem 3-6

(a) Formulate node-voltage equations for the circuit in Figure P3-6.

(b) Solve for v_x and i_x when $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 10$ kΩ, and $v_S = 24$ V.

Solution:

(a) The node-voltage equations are formulated in the following MATLAB code.

```
clear all
syms vs R1 R2 R3 R4 R5 R6 vA vB
Eqn1 = '(vA-vs)/R1 + (vA-vB)/R2 + (vA-vB)/R3 + vA/R4';
Eqn2 = '(vB-vA)/R2 + (vB-vA)/R3 + vB/R5';
Soln = solve(Eqn1,Eqn2,'vA','vB');
vA = Soln.vA;
vB = Soln.vB;
vx = vA-vB;
ix = (vs-vA)/R1;
vA_num = double(subs(vA,{R1,R2,R3,R4,R5,R6,vs},...
{10e3,10e3,10e3,10e3,10e3,10e3,24}))
vB_num = double(subs(vB,{R1,R2,R3,R4,R5,R6,vs},...
{10e3,10e3,10e3,10e3,10e3,10e3,24}))
vx_num = double(subs(vx,{R1,R2,R3,R4,R5,R6,vs},...
{10e3,10e3,10e3,10e3,10e3,10e3,24}))
ix_num = double(subs(ix,{R1,R2,R3,R4,R5,R6,vs},...
{10e3,10e3,10e3,10e3,10e3,10e3,24}))
vA_num =
     9.0000e+000
vB_num =
     6.0000e+000
vx_num =
     3.0000e+000
ix_num =
     1.5000e-003
```

Answer:

(a) The node-voltage equations are presented above in the MATLAB code.

(b) $v_x = 3$ V and $i_x = 1.5$ mA

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Problem 3-7

(a) Formulate node-voltage equations for the circuit in Figure P3-7.

(b) Use MATLAB to find symbolic expressions for the node voltages in terms of the parameters in the circuit.

(c) Find numeric values for v_A, v_B, and v_C when R_1 = 1.5 kΩ, R_2 = 2.2 kΩ, R_3 = 3.3 kΩ, R_4 = 4.7 kΩ, and $i_{S1} = i_{S2}$ = 2 mA.

(d) Use OrCAD to verify your solution to part (c) is correct.

Solution:

The following MATLAB code presents the solutions to Parts (a), (b), and (c).

```
clear all
syms iS1 iS2 vA vB vC R1 R2 R3 R4 real
NodeA = '-iS1 + (vA-vC)/R4 + (vA-vB)/R1';
NodeB = '(vB-vA)/R1 + (vB-0)/R2 - iS2';
NodeC = 'iS2 + (vC-0)/R3 + (vC-vA)/R4';
vABC = solve(NodeA,NodeB,NodeC,'vA','vB','vC');
vA = simplify(vABC.vA)
vB = simplify(vABC.vB)
vC = simplify(vABC.vC)
vANum = subs(vA,{R1,R2,R3,R4,iS1,iS2},...
    {1500,2200,3300,4700,0.002,0.002})
vBNum = subs(vB,{R1,R2,R3,R4,iS1,iS2},...
    {1500,2200,3300,4700,0.002,0.002})
vCNum = subs(vC,{R1,R2,R3,R4,iS1,iS2},...
    {1500,2200,3300,4700,0.002,0.002})
vA =
(R2*iS1*R4+R4*iS2*R2+R3*R2*iS1+iS1*R4*R1+R1*R3*iS1-R1*iS2*R3)/(R2+R3+R4+R1)
vB =
R2*(iS1*R4+R4*iS2+R1*iS2+R3*iS1)/(R2+R3+R4+R1)
vC =
R3*(R1*iS1+R2*iS1-R1*iS2-R4*iS2)/(R2+R3+R4+R1)
vANum =
     5.9812e+000
vBNum =
     5.3402e+000
vCNum =
    -1.4103e+000
```

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(d) OrCAD simulation:

Answer:

(a) The node-voltage equations are presented above in the MATLAB code.

(b)
```
vA = (R2*iS1*R4+R4*iS2*R2+R3*R2*iS1+iS1*R4*R1+R1*R3*iS1-
R1*iS2*R3)/(R2+R3+R4+R1)
vB = R2*(iS1*R4+R4*iS2+R1*iS2+R3*iS1)/(R2+R3+R4+R1)
vC = R3*(R1*iS1+R2*iS1-R1*iS2-R4*iS2)/(R2+R3+R4+R1)
```

(c) $v_A = 5.9812$ V, $v_B = 5.3402$ V, and $v_C = -1.4103$ V.

(d) The OrCAD results are presented above and they verify the MATLAB results.

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Problem 3-8

(a) Formulate node-voltage equations for the circuit in Figure P3-8.
(b) Solve for v_x and i_x when $R_1 = R_4 = 1$ kΩ, $R_2 = R_3 = 250$ Ω, $R_x = 500$ Ω, and $v_S = 15$ V.
(c) Repeat (b) when R_4 is a variable resistor that varies from 10 Ω to 10,000 Ω. At what value of R_4 is the voltage across $R_X = 0$ V? Hint: Use OrCAD to vary R_4 by trial and error to approach the answer.

Solution:
Ground the negative terminal of the voltage source. Let Nodes A and B be the left and right nodes of the bridge circuit, respectively. Solve Parts (a), (b), and (c) using the following MATLAB code.

```
clear all
% Create the node voltage equations
syms vs R1 R2 R3 R4 Rx vA vB
Eqn1 = '(vA-vs)/R1 + (vA-vB)/Rx + vA/R3';
Eqn2 = '(vB-vs)/R2 + (vB-vA)/Rx + vB/R4';

% Solve the equations
Soln = solve(Eqn1,Eqn2,'vA','vB');
vA = Soln.vA;
vB = Soln.vB;

% Calculate vx and ix
vx = vA-vB;
ix = (vs-vA)/R1 + (vs-vB)/R2;
vA_num = double(subs(vA,{R1,R2,R3,R4,Rx,vs},{1e3,250,250,1e3,500,15}));
vB_num = double(subs(vB,{R1,R2,R3,R4,Rx,vs},{1e3,250,250,1e3,500,15}));
vx_num = double(subs(vx,{R1,R2,R3,R4,Rx,vs},{1e3,250,250,1e3,500,15}))
ix_num = double(subs(ix,{R1,R2,R3,R4,Rx,vs},{1e3,250,250,1e3,500,15}))

% Solve for vx in terms of R4
vx2 = simple(subs(vx,{R1,R2,R3,Rx,vs},{1e3,250,250,500,15}))
R4_vxzero = solve(vx2,R4)

% Allow R4 to vary from 10 to 10000 and solve for vx
R4_var = logspace(1,4,500);
vx_var = double(subs(vx2,R4,R4_var));

% Plot the results
semilogx(R4_var,vx_var,'b','LineWidth',3)
grid on
xlabel('Resistance, (ohms)')
ylabel('v_x (V)')
vx_num =
    -5.0000e+000
ix_num =
    30.0000e-003
vx2 =
-60*(2*R4-125)/(19*R4+3500)
```

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```
R4_vxzero =
125/2
```

(c) OrCAD simulations:

Answer:
(a) The node-voltage equations are presented above in the MATLAB code.
(b) $v_x = -5$ V and $i_x = 30$ mA.
(c) $R_4 = 62.5\ \Omega$.

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Problem 3-9

(a) Formulate mesh-current equations for the circuit in Figure P3-9.

(b) Solve for i_A and i_B.

(c) Use these results to find v_x and i_x.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Formulate the mesh-current equations
syms iA iB
Eqn1 = '-12 + 10e3*iA + 5e3*(iA-iB)';
Eqn2 = '5e3*(iB-iA) + 10e3*iB +4';

% Solve the equations for iA and iB
Soln = solve(Eqn1,Eqn2,'iA','iB');
iA = double(Soln.iA)
iB = double(Soln.iB)

% Calculate vx and ix
vx = 10e3*iA
ix = iA-iB
iA =
   800.0000e-006
iB =
     0.0000e-003
vx =
     8.0000e+000
ix =
   800.0000e-006
```

Answer:

(a) Mesh A: $-12+10\text{k}(i_A)+5\text{k}(i_A-i_B)=0$;

Mesh B: $5\text{k}(i_B-i_A)+10\text{k}(i_B)+4=0$

(b) i_A = 800 μA and i_B = 0 A

(c) v_x = 8 V and i_x = 800 μA

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Problem 3-10
(a) Formulate mesh-current equations for the circuit in Figure P3-10.
(b) Solve for i_A and i_B.
(c) Use these results to find v_x and i_x.

Solution:
The solution is presented in the following MATLAB code.

```
clear all
% Formulate the mesh-current equations
syms iA iB
Eqn1 = '-15 + 4e3*iA + 2e3*iA + 4e3*(iA-iB) + 15';
Eqn2 = '-15 + 4e3*(iB-iA) + 2e3*iB + 4e3*iB';

% Solve the equations for iA and iB
Soln = solve(Eqn1,Eqn2,'iA','iB');
iA = double(Soln.iA)
iB = double(Soln.iB)

% Calculate vx and ix
vx = 4e3*iB
ix = iB-iA
iA =
   714.2857e-006
iB =
     1.7857e-003
vx =
     7.1429e+000
ix =
     1.0714e-003
```

Answer:
(a) The mesh-current equations are presented in the above MATLAB code.
(b) i_A = 714.2857 μA and i_B = 1.7857 mA
(c) v_x = 7.1429 V and i_x = 1.0714 mA

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Problem 3-11

(a) Formulate mesh-current equations for the circuit in Figure P3-11.
(b) Formulate node-voltage equations for the circuit in Figure P3-11.
(c) Which set of equations would be easier to solve? Why?
(d) Using MATLAB, find v_x and i_x in terms of the mesh-current variables.
(e) Using MATLAB, find v_x and i_x in terms of the node-voltage variables.

Solution:

The solutions to Parts (a), (b), (d), and (e) are presented in the following MATLAB code.

```
clear all
syms vS vA vB vC iA iB iC real
syms RA RB RC RD RE positive

MeshA = '-vS+RA*(iA-iC)+RB*(iA-iB)';
MeshB = 'RB*(iB-iA)+RC*(iB-iC)+RD*iB';
MeshC = 'RA*(iC-iA)+RC*(iC-iB)+RE*iC';

NodeA = 'vA-vS';
NodeB = '(vB-vA)/RA + (vB-0)/RB  + (vB-vC)/RC';
NodeC = '(vC-vA)/RE + (vC-vB)/RC + (vC-0)/RD';

iABC = solve(MeshA,MeshB,MeshC,'iA','iB','iC');
iA = simplify(iABC.iA);
iB = simplify(iABC.iB);
iC = simplify(iABC.iC);
vXMesh = simple(RB*(iA-iB))
iXMesh = simple(-iB)

vABC = solve(NodeA,NodeB,NodeC,'vA','vB','vC');
vA = simplify(vABC.vA);
vB = simplify(vABC.vB);
vC = simplify(vABC.vC);
vXNode = simple(vB)
iXNode = simple(-vC/RD)

% Check symbolic values
```

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```
Checkv = simplify(vXNode-vXMesh)
Checki = simplify(iXNode-iXMesh)

vXMesh =
RB*vS*(RD*RA+RD*RC+RD*RE+RC*RE)/(RB*RC*RE+RB*RD*RA+RB*RD*RC+RB*RD*RE+RD*RA*RE+
RC*RA*RE+RD*RA*RC+RB*RA*RE)
iXMesh =
-
vS*(RB*RA+RC*RB+RB*RE+RC*RA)/(RB*RC*RE+RB*RD*RA+RB*RD*RC+RB*RD*RE+RD*RA*RE+RC*
RA*RE+RD*RA*RC+RB*RA*RE)
vXNode =
RB*vS*(RD*RA+RD*RC+RD*RE+RC*RE)/(RB*RC*RE+RB*RD*RA+RB*RD*RC+RB*RD*RE+RD*RA*RE+
RC*RA*RE+RD*RA*RC+RB*RA*RE)
iXNode =
-
vS*(RB*RA+RC*RB+RB*RE+RC*RA)/(RB*RC*RE+RB*RD*RA+RB*RD*RC+RB*RD*RE+RD*RA*RE+RC*
RA*RE+RD*RA*RC+RB*RA*RE)
Checkv =
0
Checki =
0
```

The two check variables above confirm that the mesh-current and node-voltage approaches yield equivalent expressions for v_x and i_x.

Answer:

(a) Mesh A: $-v_S + R_A(i_A - i_C) + R_B(i_A - i_B) = 0$

Mesh B: $R_B(i_B - i_A) + R_C(i_B - i_C) + R_D i_B = 0$

Mesh C: $R_A(i_C - i_A) + R_C(i_C - i_B) + R_E i_C = 0$

(b) Node A: $v_A = v_S$;

Node B: $\dfrac{v_B - v_A}{R_A} + \dfrac{v_B - 0}{R_B} + \dfrac{v_B - v_C}{R_C} = 0$;

Node C: $\dfrac{v_C - v_A}{R_E} + \dfrac{v_C - v_B}{R_C} + \dfrac{v_C - 0}{R_D} = 0$

(c) Node-voltage equations, because there are fewer unknowns with $v_A = v_S$.

(d)
```
vXMesh =
RB*vS*(RD*RA+RD*RC+RD*RE+RC*RE)/(RB*RC*RE+RB*RD*RA+RB*RD*RC+RB*RD*RE+RD*RA*RE+
RC*RA*RE+RD*RA*RC+RB*RA*RE)
iXMesh =
-
vS*(RB*RA+RC*RB+RB*RE+RC*RA)/(RB*RC*RE+RB*RD*RA+RB*RD*RC+RB*RD*RE+RD*RA*RE+RC*
RA*RE+RD*RA*RC+RB*RA*RE)
```

(e)
```
vXNode =
```

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```
RB*vS*(RD*RA+RD*RC+RD*RE+RC*RE)/(RB*RC*RE+RB*RD*RA+RB*RD*RC+RB*RD*RE+RD*RA*RE+
RC*RA*RE+RD*RA*RC+RB*RA*RE)
iXNode =
-
vS*(RB*RA+RC*RB+RB*RE+RC*RA)/(RB*RC*RE+RB*RD*RA+RB*RD*RC+RB*RD*RE+RD*RA*RE+RC*
RA*RE+RD*RA*RC+RB*RA*RE)
```

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Problem 3-12

(a) Formulate mesh-current equations for the circuit in Figure P3-12.

(b) Solve for v_x and i_x when R_1 = 200 Ω, R_2 = 300 Ω, R_3 = 50 Ω, R_4 = 250 Ω, R_5 = 200 Ω, i_S = 100 mA, and v_S = 25 V.

(c) Find the total power dissipated in the circuit.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Create the mesh-current equations
% Note that the variable vis is the voltage across the current source
syms vs is vis R1 R2 R3 R4 R5 iA iB vx ix
Eqn1 = '-vs + R1*iA +R2*iA -vis';
Eqn2 = 'vis + R3*iB + R4*iB + R5*iB';
Eqn3 = 'is - (iB-iA)';

% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,'iA','iB','vis');
iA = simple(Soln.iA);
iB = simple(Soln.iB);
vx = simple(R4*iB);
ix = simple(-iA);

% Calculate numerical values for vx and ix
iA_num = double(subs(iA,{R1,R2,R3,R4,R5,is,vs},{200,300,50,250,200,100e-
3,25}))
iB_num = double(subs(iB,{R1,R2,R3,R4,R5,is,vs},{200,300,50,250,200,100e-
3,25}))
vx_num = double(subs(vx,{R1,R2,R3,R4,R5,is,vs},{200,300,50,250,200,100e-
3,25}))
ix_num = double(subs(ix,{R1,R2,R3,R4,R5,is,vs},{200,300,50,250,200,100e-
3,25}))

% Find the total power in the circuit
R1 = 200;
R2 = 300;
R3 = 50;
R4 = 250;
R5 = 200;
```

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```
is = 100e-3;
vs = 25;
p_Total = (R1+R2)*iA_num^2 + (R3+R4+R5)*iB_num^2 + ix_num*vs -
(R3+R4+R5)*iB_num*is
p_Dissipated = (R1+R2)*iA_num^2 + (R3+R4+R5)*iB_num^2
iA_num =
   -25.0000e-003
iB_num =
    75.0000e-003
vx_num =
    18.7500e+000
ix_num =
    25.0000e-003
p_Total =
     0.0000e-003
p_Dissipated =
     3.1250e+000
```

Answer:
(a) The mesh-current equations are presented above in the MATLAB code.
(b) v_x = 18.75 V and i_x = 25 mA
(c) p = 3.125 W

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Problem 3-13

(a) Formulate mesh-current equations for the circuit in Figure P3-13.

(b) Use MATLAB to find symbolic expressions for v_x and i_x in terms of the parameters in the circuit.

(c) Find numeric values for v_x and i_x when $R_1 = R_2 = 4.7$ kΩ, $R_3 = 2.2$ kΩ, $R_4 = 1$ kΩ, $i_S = 3$ mA, $v_{S1} = 10$ V, and $v_{S2} = 2$ V.

(d) Find the power supplied by v_{S1}.

(e) Use OrCAD to verify your solutions to parts (c) and (d) are correct.

Solution:

The left mesh is Mesh A, the upper right mesh is Mesh B, and the lower right mesh is Mesh C. The solutions to Parts (a), (b), (c), and (d) are presented in the following MATLAB code.

```
clear all
format short eng
syms vS1 vS2 iS iA iB iC real
syms R1 R2 R3 R4 positive

MeshA = '-vS1+R1*(iA-iB)+R2*(iA-iC)';
MeshB = 'iB-iS';
MeshC = 'vS2+R4*iC+R2*(iC-iA)';

iABC = solve(MeshA,MeshB,MeshC,'iA','iB','iC');
iA = simplify(iABC.iA);
iB = simplify(iABC.iB);
iC = simplify(iABC.iC);
vX = simplify(vS1-R3*iB-R4*iC)
iX = iA

% Numerical values
vXNum = subs(vX,{R1,R2,R3,R4,iS,vS1,vS2},...
    {4700,4700,2200,1000,0.003,10,2})
iXNum = double(subs(iX,{R1,R2,R3,R4,iS,vS1,vS2},...
```

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```
    {4700,4700,2200,1000,0.003,10,2}))

pVs1 = iXNum*10
vX =
(vS1*R1*R4+vS1*R1*R2-R3*iS*R1*R4-R3*iS*R1*R2-R3*iS*R2*R4-
R4*R2*R1*iS+R4*R1*vS2+R4*R2*vS2)/(R1*R4+R1*R2+R2*R4)
iX =
(R4*vS1+R4*R1*iS+R2*vS1+R2*R1*iS-R2*vS2)/(R1*R4+R1*R2+R2*R4)
vXNum =
   400.0000e-003
iXNum =
     4.0638e-003
pVs1 =
    40.6383e-003
```

(e) The OrCAD simulation verifies the results above

Answer:

(a) Mesh A: $-v_{S1} + R_1(i_A - i_B) + R_2(i_A - i_C) = 0$;

Mesh B: $i_B = i_S$;

Mesh C: $v_{S2} + R_4 i_C + R_2(i_C - i_A) = 0$;

(b) `vX = (vS1*R1*R4+vS1*R1*R2-R3*iS*R1*R4-R3*iS*R1*R2-R3*iS*R2*R4-R4*R2*R1*iS+R4*R1*vS2+R4*R2*vS2)/(R1*R4+R1*R2+R2*R4)`

`iX = (R4*vS1+R4*R1*iS+R2*vS1+R2*R1*iS-R2*vS2)/(R1*R4+R1*R2+R2*R4)`

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(c) v_x = 400 mV; i_x = 4.0638 mA;

(d) p_{vS1} = 40.638 mW

(e) The OrCAD simulation is shown above.

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Problem 3-14

The circuit in Figure P3-14 seems to require two supermeshes since both current sources appear in two meshes. However, sometimes rearranging the circuit diagram will eliminate the need for a supermesh.

(a) Show that supermeshes can be avoided in Figure P3-14 by rearranging the connection of resistor R_6.

(b) Formulate mesh-current equations for the modified circuit as redrawn in (a).

(c) Solve for v_x when $R_1 = R_2 = R_3 = R_4 = 2$ kΩ, $R_5 = R_6 = 1$ kΩ, $i_{S1} = 40$ mA, and $i_{S2} = 20$ mA.

Solution:

(a) The new, equivalent circuit arrangement is shown below.

(b) and (c) The mesh-current equations and solution are presented in the following MATLAB code. Label the inner three meshes as A, B, and C, from left to right and label the lower mesh as Mesh D.

```
clear all
syms R1 R2 R3 R4 R5 R6 is1 is2 iA iB iC iD
% Formulate the mesh-current equations
Eqn1 = 'iA + is1';
Eqn2 = 'R2*(iB-iA) + R5*iB + R3*(iB-iC)';
Eqn3 = 'iC - is2';
Eqn4 = 'R6*iD + R1*(iD-iA) + R4*(iD-iC)';

% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,'iA','iB','iC','iD');
iC = simple(Soln.iC);
iD = simple(Soln.iD);
vx = simple(R4*(iC-iD));
vx_num = subs(vx,{R1,R2,R3,R4,R5,R6,is1,is2},{2e3,2e3,2e3,2e3,1e3,1e3,40e-
3,20e-3})
vx_num =
56
```

Answer:

(a) The new circuit is presented above.

(b) The mesh-current equations are presented in the MATLAB code.

(c) $v_x = 56$ V.

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Problem 3-15

(a) Formulate mesh-current equations for the circuit in Figure P3-15.
(b) Formulate node-voltage equations for the circuit in Figure P3-15.
(c) Which set of equations would be easier to solve? Why?
(d) Find v_x and i_x using whichever method you prefer.

Solution:

The following MATLAB code contains solutions to Parts (a), (b), and (d).

```
clear all
syms vA vB vC iA iB iC real

MeshA = '-40+2000*(iA-iC)+4000*(iA-iB)';
MeshB = '4000*(iB-iA)+8000*iB-25';
MeshC = 'iC-0.005';

NodeA = 'vA-40';
NodeB = '-0.005+(vB-vA)/2000+vB/4000+(vB-vC)/8000';
NodeC = 'vC+25';

iABC = solve(MeshA,MeshB,MeshC,'iA','iB','iC');
iA = simplify(iABC.iA);
iB = simplify(iABC.iB);
iC = simplify(iABC.iC);
vXMesh = simplify(4000*(iA-iB))
iXMesh = double(-iB)

vABC = solve(NodeA,NodeB,NodeC,'vA','vB','vC');
vA = simplify(vABC.vA);
vB = simplify(vABC.vB);
vC = simplify(vABC.vC);
vXNode = vB
iXNode = double((vC-vB)/8000)

% Check symbolic values
```

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```
Checkv = vXNode-vXMesh
Checki = iXNode-iXMesh
vXMesh =
25.
iXMesh =
    -6.2500e-003
vXNode =
25.
iXNode =
    -6.2500e-003
Checkv =
0.
Checki =
     0.0000e-003
```

Answer:

(a) Mesh A: $-40+2000(i_A - i_C)+4000(i_A - i_B)=0$;

Mesh B: $4000(i_B - i_A)+8000i_B - 25 = 0$;

Mesh C: $i_C = 0.005$;

(b) Node A: $v_A = 40$;

Node B: $-0.005+\dfrac{v_B - v_A}{2000}+\dfrac{v_B - 0}{4000}+\dfrac{v_B - v_C}{8000}=0$;

Node C: $v_C = -25$

(c) Node voltage; there is only one equation to solve.

(d) v_x = 25 V and i_x = −6.25 mA

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Problem 3-16

Use mesh-current, node-voltage or simple engineering intuition to find the input resistance of the circuit in Figure P3-16.

Solution:

The input resistance is 2*R* and this can be determined with the following logic. The bridge connection on the right side of the circuit is balanced, so no current will flow through the horizontal resistor in the middle of the bridge. If no current flows through that resistor, we can remove it from the circuit without changing the equivalent resistance. We then have two parallel branches, each with two resistors in series. Each branch has a resistance of 2*R* and the parallel combination has a resistance of *R*. The parallel combination is in series with the source resistor, so the total resistance is 2*R*.

The following MATLAB code verifies the results using mesh-current analysis.

```
clear all
syms vs R iA iB iC iin Rin
Eqn1 = '-vs + R*iA + R*(iA-iB) + R*(iA-iC)';
Eqn2 = 'R*iB + R*(iB-iC) + R*(iB-iA)';
Eqn3 = 'R*iC + R*(iC-iA) + R*(iC-iB)';
Soln = solve(Eqn1,Eqn2,Eqn3,'iA','iB','iC');
iin = simple(Soln.iA);
Rin = simple(vs/iin)
Rin =
2*R
```

Answer:

$R_{IN} = 2R$.

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Problem 3-17

In Figure P3-17 all of the resistors are 1 kΩ and v_S = 10 V. The voltage at node C is found to be v_C = !2 V when node B is connected to ground. Find the node voltages v_A and v_D, and the mesh currents i_A and i_B.

Solution:

We can solve this problem without node-voltage or mesh-current analysis. If v_B = 0 V and v_C = !2 V, then the current between nodes B and C must be 2 mA through the 1 kΩ resistor. That same current flows from node C to node D. The resistor between nodes C and D also drops 2 V, so we have v_D = !4 V. The source increases the voltage from node D to node A by 10 V, so v_A = 6 V. The current between nodes A and B is therefore 6 mA.

Answer:

v_A = 6 V
v_D = –4 V
i_A = 6 mA
i_B = 2 mA

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Problem 3-18

Use Figure P3-17 and MATLAB to solve the following problems:

(a) Using mesh-current analysis, find a symbolic expression for i_A in terms of the circuit parameters.

(b) Compute the ratio v_S/i_A.

(c) Find a symbolic expression for the equivalent resistance of the circuit by combining resistors in series and parallel. Compare your answer to the results from part (b).

Solution:

The following MATLAB code presents the solutions to parts (a), (b), and (c).

```
clear all
syms vs R1 R2 R3 R4 iA iB Rin1 Rin2

% Formulate the mesh-current equations
Eqn1 = '-vs + R1*iA + R2*(iA-iB)';
Eqn2 = 'R2*(iB-iA) + R3*iB + R4*iB';

% Solve the equations for iA
Soln = solve(Eqn1,Eqn2,'iA','iB');
iA = simple(Soln.iA)

% Compute the input resistance
Rin1 = vs/iA

% Find the equivalent resistance by combining resistors
Rin2 = simplify(R1 + 1/(1/R2 + 1/(R3+R4)))

% Compare the input resistance
Check_Rin = Rin1==Rin2
iA =
vs*(R2+R3+R4)/(R1*R2+R3*R1+R3*R2+R1*R4+R2*R4)
Rin1 =
1/(R2+R3+R4)*(R1*R2+R3*R1+R3*R2+R1*R4+R2*R4)
Rin2 =
1/(R2+R3+R4)*(R1*R2+R3*R1+R3*R2+R1*R4+R2*R4)
Check_Rin =
     1
```

Answer:

(a) $i_A = \dfrac{(R_2 + R_3 + R_4)v_S}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4}$

(b) $\dfrac{v_S}{i_A} = \dfrac{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4}{(R_2 + R_3 + R_4)}$

(c) $R_{IN} = \dfrac{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4}{(R_2 + R_3 + R_4)}$; which matches the answer in Part (b).

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Problem 3-19

Use OrCAD to find the node voltages v_A and v_B and the mesh-currents i_A, i_B and i_C in Figure P3-19.

Solution:

The OrCAD solution is presented below.

Answer:

v_A = 21.54 V

v_B = 18.46 V

i_A = –5.385 mA

i_B = –10 mA

i_C = 3.077 mA

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Problem 3-20

(a) Use MATLAB and mesh-current analysis to solve for the mesh currents i_A, i_B, i_C, and i_D in Figure P3-20.

(b) Solve for the same currents using node-voltage analysis and compare the effort required with each technique.

(c) Use OrCAD to verify your results in the previous two parts.

Solution:

The solutions to Parts (a) and (b) are presented in the following MATLAB code.

```
clear all
syms iA iB iC iD vA vB
% Solve the problem using mesh-current equations
Eqn1 = '4e3*iA + 10e3*(iA-iB) + 10';
Eqn2 = 1e3*iB + 2e3*(iB-iC) + 10e3*(iB-iA)';
Eqn3 = '2e3*(iC-iB) + 8e3*iC + 6e3*(iC-iD)';
Eqn4 = '-5 - 10 + 6e3*(iD-iC)';
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,'iA','iB','iC','iD');
iA = double(Soln.iA);
iB = double(Soln.iB);
iC = double(Soln.iC);
iD = double(Soln.iD);

% Solve the problem using node-voltage analysis
EqnV1 = '(vA-5)/4e3 + (vA-15)/10e3 + (vA-vB)/1e3';
EqnV2 = '(vB-vA)/1e3 + (vB-15)/2e3 + vB/8e3';
SolnV = solve(EqnV1,EqnV2,'vA','vB');
vA = double(SolnV.vA);
vB = double(SolnV.vB);
iA_v = (5-vA)/4e3;
iB_v = (vA-vB)/1e3;
iC_v = vB/8e3;
iD_v = 15/6e3 + iC;
Results = [iA iB iC iD; iA_v iB_v iC_v iD_v]'
Results =
    -1.2565e-003    -1.2565e-003
  -759.1623e-006  -759.1623e-006
```

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```
1.3482e-003     1.3482e-003
3.8482e-003     3.8482e-003
```

(c) The OrCAD solution is presented below and it verifies the MATLAB results.

Answer:
(a) $i_A = -1.2565$ mA, $i_B = -759.1623$ μA, $i_C = 1.3482$ mA, $i_D = 3.8482$ mA
(b) The solution for the currents using node-voltage analysis matches the results in Part (a), but requires only two equations, instead of the four equations used in mesh-current analysis.
(c) The OrCAD results verify the answers through another method.

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Problem 3-21

Find the proportionality constant $K = v_O/i_S$ for the circuit in Figure P3-21.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Define the resistor values
R1 = 68e3;
R2 = 33e3;
R3 = 22e3;

% Use current division to find the output current
syms is io vo K
io = is/(R2+R3)/(1/R1 + 1/(R2+R3));

% Use Ohm's Law to find the output voltage
vo = io*R3;

% Compute the proportionality constant
K = double(vo/is)
K =
    12.1626e+003
```

Answer:

$K = 12163$

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Problem 3-22

Find the proportionality constant $K = i_O/i_S$ for the circuit in Figure P3-22.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms is io R1 R2 R3 R4 K
% Use current division to find io in terms of is
io = is/(R3+R4)/(1/R1 + 1/R2 + 1/(R3+R4));
K = simplify(io/is)
K =
R1*R2/(R1*R2+R3*R1+R3*R2+R1*R4+R2*R4)
```

Answer:

$$K = \frac{R_1 R_2}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_2 R_4}$$

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Problem 3-23

Find the proportionality constant $K = v_O/v_S$ for the circuit in Figure P3-23.

Solution:

Use voltage division

```
clear all
syms vs vo R1 R2 R3 R4 K
vo = -vs*R4/(R2+R4);
K = vo/vs
K =
-R4/(R2+R4)
```

Answer:

$$K = \frac{-R_4}{R_2 + R_4}$$

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Problem 3-24

Use the unit output method to find v_O in Figure P3-24.

Solution:

The steps of the unit output method are shown in the following MATLAB code. Label the resistors R_1, R_2, R_3, R_4, R_5, R_O, from left to right.

```
clear all
% Define the resistor values and the voltage source
R1 = 1e3;
R2 = 5e3;
R3 = 1e3;
R4 = 3e3;
R5 = 1e3;
Ro = 1e3;
vs = 10;

% Create the unit output signal and trace it back to the source
vo = 1;
io = vo/Ro;
i5 = io;
v5 = i5*R5;
v4 = v5 + vo;
i4 = v4/R4;
i3 = i4 + i5;
v3 = i3*R3;
v2 = v3 + v4;
i2 = v2/R2;
i1 = i2 + i3;
v1 = i1*R1;
vs_unit = v1 + v2;
vo_Real = vs/vs_unit
vo_Real =
     1.6484e+000
```

Answer:

v_O = 1.6484 V

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Problem 3-25

Use the unit output method to find i_O in Figure P3-25.

Solution:

The steps of the unit output method are shown in the following MATLAB code. Label the resistors R_1, R_2, R_3, R_4, R_O, from left to right.

```
clear all
% Define the resistor values and the voltage source
R1 = 30;
R2 = 60;
R3 = 15;
R4 = 60;
Ro = 20;
vs = 100;

% Create the unit output signal and trace it back to the source
io = 1;
vo = io*Ro;
v4 = vo;
i4 = v4/R4;
i3 = i4+io;
v3 = i3*R3;
v2 = v3+v4;
i2 = v2/R2;
i1 = i2+i3;
v1 = i1*R1;
vs_unit = v1+v2;
io_real = vs/vs_unit
io_real =
     1.0000e+000
```

Answer:

$i_O = 1$ A

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Problem 3-26

Use the superposition principle to find v_O in Figure P3-26.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Activate the 12-V source and turn off the 6-V source
% Combine the right two resistors in parallel and use voltage division
Req1 = 1/(1/200 + 1/100);
vo1 = Req1*12/(Req1+200);

% Activate the 6-V source and turn off the 12-V source
% Combine the left two resistors in parallel and use voltage division
Req2 = 1/(1/200 + 1/200);
vo2 = Req2*6/(Req2+100);

% Sum the components
vo = vo1+vo2
vo =
     6.0000e+000
```

Answer:

$v_O = 6$ V

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Problem 3-27

Use the superposition principle to find i_O in Figure P3-27. Verify your answer using OrCAD.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Turn on the current source and turn off the voltage source
% Combine the right two resistors in parallel and use current division
Req1 = 1/(1/100+1/100);
io1 = 60e-3/Req1/(1/100 + 1/Req1);

% Turn on the voltage source and turn off the current source
% Perform a source transformation on the voltage source and use current
division
is2 = 12/100;
io2 = is2/100/(1/100+1/100+1/100);

% Sum the components
io = io1+io2
io =
    80.0000e-003
```

The following OrCAD results verify the solution.

Answer:

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i_O = 80 mA

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Problem 3-28

Use the superposition principle to find v_O in Figure P3-28.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Turn on the voltage source and turn off the current source
Req1 = 3.3e3 + 10e3;
Req2 = 1/(1/6.3e3 + 1/Req1);
v1 = Req2*30/(Req2 + 5.6e3);
vo1 = 3.3e3*v1/(3.3e3 + 10e3);

% Turn on the current source and turn off the voltage source
Req3 = 1/(1/5.6e3 + 1/6.3e3);
Req4 = 3.3e3 + Req3;
io2 = 5e-3/Req4/(1/Req4 + 1/10e3);
vo2 = io2*3.3e3;

% Sum the components
vo = vo1+vo2

% Check using node-voltage analysis
syms vA vB
Eqn1 = '(vA-30)/5.6e3 + vA/6.3e3 + (vA-vB)/3.3e3';
Eqn2 = '(vB-vA)/3.3e3 + 5e-3 + vB/10e3';
Soln = solve(Eqn1,Eqn2,'vA','vB');
vA = double(Soln.vA);
vB = double(Soln.vB);
vo_node = vA-vB
vo =
   13.3671
vo_node =
   13.3671
```

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Answer:
v_O = 13.3671 V

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Problem 3-29

Use the superposition principle to find i_O in Figure P3-29. Verify your answer using OrCAD.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Turn on the current source
Req1 = 1/(1/5e3 + 1/15e3 + 1/10e3);
Req2 = 5e3+ Req1;
% Current division twice
i1 = 1e-3/Req2/(1/Req2 + 1/10e3);
io1 = -i1/10e3/(1/5e3 + 1/15e3 + 1/10e3);
% Turn on the 10-V source
vo2 = Req1*10/(Req1+5e3+10e3);
io2 = vo2/10e3;
% Turn on the 20-V source
Req3 = 1/(1/15e3 + 1/15e3 + 1/10e3);
vo3 = Req3*20/(Req3 + 5e3);
io3 = vo3/10e3;

% Sum the components
io = io1+io2+io3
io =
   923.0769e-006
```

The OrCAD results verify the answer above.

Answer:

$i_O = 923.08\ \mu A$

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Problem 3-30

(a) Use the superposition principle to find v_O in terms of v_1, v_2, and R in Figure P3-30.

(b) Use MATLAB and node-voltage analysis to verify your answer symbolically.

Solution:

The following MATLAB code presents the solution.

```
clear all
disp('Part (a)')
syms v1 v2 R vo1 vo2 vo
% Turn on v1
% Perform a source transformation on v1
i1 = v1/2/R;
io1 = i1/2/R/(1/2/R + 1/2/R + 1/2/R);
vo1 = R*io1;

% Turn on v2
% Perform a source transformation on v2
i2 = v2/2/R;
io2 = i2/2/R/(1/2/R + 1/2/R + 1/2/R);
vo2 = R*io2;

% Sum the components
vo = simplify(vo1+vo2)

disp('Part(b)')
% Label the top center node as Node C
syms vC vo_node
Eqn1 = '(vC-v1)/2/R + (vC-v2)/2/R + vC/2/R';
vC = solve(Eqn1,'vC');
% Voltage division
vo_node = simplify(R*vC/(R+R))
Part (a)
vo =
1/6*v1+1/6*v2
Part(b)
vo_node =
1/6*v1+1/6*v2
```

Answer:

(a) $v_O = \dfrac{v_1 + v_2}{6}$

(b) The node-voltage analysis confirms the results from superposition.

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Problem 3-31

(a) Use the superposition principle to find v_O in terms of v_S, i_S, and R in Figure P3-31.

(b) Use MATLAB and node-voltage analysis to verify your answer symbolically.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Solve the problem using superposition
disp('Part (a)')
syms vs is R vo vo1 vo2 vA vB
% Turn on the voltage source
vo1 = R*vs/(R+R+R+R);
% Turn on the current source
io2 = -is/R/2/(1/2/R + 1/2/R);
vo2 = io2*R;
% Sum the components
vo = vo1+vo2

% Solve the problem using node-voltage analysis
% Let the node above the current source be node A
% Let the node above the output resistor be node B
disp('Part (b)')
Eqn1 = '(vA-vs)/2/R + is + (vA-vB)/R';
Eqn2 = '(vB-vA)/R + vB/R';
Soln = solve(Eqn1,Eqn2,'vA','vB');
vo_node = simple(Soln.vB)

Check_vo = vo-vo_node
Part (a)
vo =
1/4*vs-1/2*is*R
Part (b)
vo_node =
1/4*vs-1/2*is*R
Check_vo =
0
```

Answer:

(a) $v_O = \dfrac{v_S}{4} - \dfrac{Ri_s}{2}$

(b) The above MATLAB code confirms the answer.

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Problem 3-32

A linear circuit containing two sources drives a 100-Ω load resistor. Source number 1 delivers 250 mW to the load when source number 2 is off . Source number 2 delivers 4 W to the load when source number 1 is off. Find the power delivered to the load when both sources are on. *Hint*: The answer is not 4.25 W. Why?

Solution:

Power is a nonlinear value. Our approach will be to find the voltage across the resistor associated with each source, add the voltages, and then calculate the total power. The following MATLAB code presents the solution.

```
clear all
R = 100;
p1 = 250e-3;
p2 = 4;
p = [p1 p2];
% Calculate the voltages
v = sqrt(p*R);
v_total = sum(v);
p_total = v_total^2/R
p_total =
    6.2500
```

Answer:

p = 6.25 W

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Problem 3-33

A linear circuit is driven by an independent voltage source v_S = 10 V and an independent current source i_S = 10 mA. The output voltage is v_O = 2 V when the voltage source is on and the current source off . The output is v_O = 1 V when both sources are on . Find the output voltage when v_S = 20 V and i_S = !20 mA.

Solution:

To solve this problem, we need to determine the ratio of the output voltage to each input source. We can then find the output for the new sources. The following MATLAB code presents the solution.

```
clear all
vo1 = 2;
vo = 1;
vo2 = vo-vo1;
vs1 = 10;
is1 = 10e-3;
vs2 = 20;
is2 = -20e-3;
% Determine the voltage gain
Kv = vo1/vs1;
% Determine the ratio of the output voltage to the input current
Kr = vo2/is1;
% Compute the output for the new sources
vo_new = Kv*vs2 + Kr*is2
vo_new =
     6
```

Answer:

v_O = 6 V.

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Problem 3-34

A certain linear circuit has four input voltages and one output voltage v_O. The following table lists the output for different values of the four inputs. Find the input-output relationship for the circuit. Specifically, find an expression for v_O in terms of the four input voltages.

v_{S1}(V)	v_{S2}(V)	v_{S3}(V)	v_{S4}(V)	v_O(V)
2	4	–4	1	20
1	2	2	1.5	–4
1	4	2	2	–1
0	5	3	–1	3

Solution:

Write four linear expressions based on the information above and solve for the gains associated with each source. We will solve for the K_i terms in the following equation:

$$v_O = K_1 v_{S1} + K_2 v_{S2} + K_3 v_{S3} + K_4 v_{S4}$$

The following MATLAB code presents the solution.

```
clear all
% Use matrix notation for formulate and solve the problem
A = [2 4 -4 1;
     1 2 2 1.5;
     1 4 2 2;
     0 5 3 -1];
B = [20; -4; -1; 3];
K = A\B
K =
    1.0000
    2.0000
   -3.0000
   -2.0000
```

Answer:

$$v_O = v_{S1} + 2v_{S2} - 3v_{S3} - 2v_{S4}$$

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Problem 3-35

(a) Find the Thévenin or Norton equivalent circuit seen by R_L in Figure P3-35.

(b) Use the equivalent circuit found in (a) to find i_L if R_L = 47 kΩ.

Solution:

Remove the load. Find the open-circuit voltage and the short-circuit current. Check the look-back resistance. The following MATLAB code presents the solution.

```
clear all
format short eng
disp('Part (a)')
v_oc = 15e3*10/(15e3+10e3);
Req = 10e3 + 1/(1/15e3+1/6.8e3);
i_total = 10/Req;
i_sc = i_total/6.8e3/(1/15e3+1/6.8e3);
% Display the Thevenin and Norton parameters
vT = v_oc
RT = v_oc/i_sc
iN = i_sc
RN = RT

% Check the resistance using the look-back method
RLB = 6.8e3+1/(1/10e3+1/15e3)

disp('Part (b)')
% Use the Thevenin equivalent circuirt to find iL
R = RT + 47e3;
iL = vT/R
Part (a)
vT =
     6.0000e+000
RT =
    12.8000e+003
iN =
   468.7500e-006
RN =
    12.8000e+003
RLB =
    12.8000e+003
Part (b)
iL =
   100.3344e-006
```

Answer:

(a) v_T = 6 V, $R_T = R_N$ = 12.8 kΩ, i_N = 468.75 μA

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(b) $i_L = 100.3344\ \mu A$

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Problem 3-36

(a) Find the Thévenin or Norton equivalent circuit seen by R_L in Figure P3-36.

(b) Use the equivalent circuit found in (a) to find i_L in terms of i_S, R_1, R_2, and R_L.

(c) Check your answer in (b) using current division.

Solution:

The following MATLAB code presents the solution.

```
clear all
disp('Part (a)')
syms is R1 R2 vT RT RN iN iL RL
% Find the open-circuit voltage
vT = is*R1
% Find the short-circuit current
iN = simplify(is/R2/(1/R1+1/R2))
% Find the Thevenin and Norton resistance
RT = simplify(vT/iN)
% Check the resistance using the look-back method
RN = R1+R2

disp('Part (b)')
iL = vT/(RT+RL)

disp('Part (c)')
iL2 = simplify(is/(R2+RL)/(1/R1+1/(R2+RL)))
Check = simplify(iL-iL2)
Part (a)
vT =
is*R1
iN =
is*R1/(R2+R1)
RT =
R2+R1
RN =
R2+R1
Part (b)
iL =
is*R1/(R2+R1+RL)
Part (c)
iL2 =
is*R1/(R2+R1+RL)
Check =
0
```

Answer:

(a) $v_T = i_S R_1$, $R_T = R_N = R_1 + R_2$, $i_N = i_S R_1/(R_1 + R_2)$

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(b) $i_L = i_S R_1/(R_1 + R_2 + R_L)$
(c) The solution using current division agrees with the solution in Part (b).

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Problem 3-37

Find the Thévenin equivalent circuit seen by R_L in Figure P3-37. Find the voltage across the load when R_L = 5 Ω, 10 Ω, and 50 Ω.

Solution:

The following MATLAB code presents the solution.

```
clear all
Req = 1/(1/20+1/20);
vT = 10*15/(10+Req)
RT = 10 + 1/(1/20+1/20+1/10)
RL = [5 10 50];
vL = RL*vT./(RT+RL);
Results = [RL; vL]'
vT =
     7.5000e+000
RT =
    15.0000e+000
Results =
     5.0000e+000     1.8750e+000
    10.0000e+000     3.0000e+000
    50.0000e+000     5.7692e+000
```

Answer:

R_T = 15 Ω; v_T = 7.5 V;

v_L = 1.875 V for R_L = 5 Ω;

v_L = 3 V for R_L = 10 Ω;

v_L = 5.769 V for R_L = 50 Ω

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Problem 3-38

Find the Norton equivalent seen by R_L in Figure P3-38. Find the current through the load when R_L = 4.7 kΩ, 15 kΩ, and 68 kΩ.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Find the short-circuit current and the look-back resistance
iN = 2.5e-3/8.1e3/(1/15e3 + 1/5.6e3 + 1/8.1e3)
RN = 8.1e3+1/(1/15e3 + 1/5.6e3)

% Compute the current through the load using current division
RL = [4.7e3 15e3 68e3];
iL = iN./RL./(1/RN + 1./RL)
iN =
   837.1203e-006
RN =
    12.1777e+003
iL =
   604.0037e-006   375.0938e-006   127.1448e-006
```

Answer:

R_N = 12.1777 kΩ; i_N = 837.1203 μA;

i_L = 604.0037 μA for R_L = 4.7 kΩ;

i_L = 375.0938 μA for R_L = 15 kΩ;

i_L = 127.1448 μA for R_L = 68 kΩ

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Problem 3-39

Find the Thévenin equivalent seen by R_L in Figure P3-39. Find the power delivered to the load when R_L = 50 kΩ and 200 kΩ.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Find the open-circuit voltage and the look-back resistance
vT = 24*30e3/(30e3+60e3)
RT = 40e3 + 1/(1/60e3 + 1/30e3)

% Find the load power
RL = [50e3 200e3];
iL = vT./(RT+RL);
pL = iL.^2.*RL
vT =
     8.0000e+000
RT =
    60.0000e+003
pL =
   264.4628e-006   189.3491e-006
```

Answer:

R_T = 60 kΩ; v_T = 8 V;

p_L = 264 μW for R_L = 50 kΩ;

p_L = 189 μW for R_L = 200 kΩ

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Problem 3-40

(a) Use OrCAD to find the Norton equivalent at terminals A and B in Figure P3-40. Hint: Find the open-circuit voltage and short-circuit current at the requisite terminals.

(b) Use the Norton equivalent circuit found in (a) to determine the power dissipated in R_L when it is equal to 37 kΩ.

(c) Use OrCAD to simulate both the original and the Norton equivalent circuits with R_L=37 kΩ. Verify that the power dissipated by the load is the same in both situations.

Solution:

(a) The circuits for the OrCAD simulations to find the open-circuit voltage and short-circuit current are shown below.

The open-circuit voltage is 8.183 V and the short-circuit current is 221.6 μA. The Norton resistance is therefore 36.9269 kΩ.

(b) Using the Norton equivalent circuit, the following MATLAB code calculates the power dissipated by the load resistor.

```
clear all
iN = 221.6e-6;
RN = 36.9269e3;
RL = 37e3;
iL = iN/RL/(1/RL + 1/RN);
pL = iL^2*RL
pL =
```

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```
453.3378e-006
```

(c) The OrCAD simulations for the original circuit and Norton equivalent circuit are shown below. The power dissipated by the load resistor matches the calculations in Part (b).

Answer:
(a) R_N = 36.9269 kΩ; i_N = 221.6 μA;
(b) p_L = 453.3 μW
(c) The OrCAD simulations verify the Part (b) answer as shown above.

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Problem 3-41

The purpose of this problem is to use Thévenin equivalent circuits to find the voltage v_x in Figure P3-41. Find the Thévenin equivalent circuit seen looking to the left of terminals A and B. Find the Thévenin equivalent circuit seen looking to the right of terminals A and B. Connect these equivalent circuits together and find the voltage v_x.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Left Thevenin equivalent
vT_left = 15*6e3/(3e3+6e3)
RT_left = 1/(1/3e3+1/6e3)

% Right Thevenin equivalent
vT_right = 0
RT_right = 1.2e3+1/(1/1e3+1/4e3)

% Use voltage division to find vx
vx = vT_left*RT_right/(RT_left+RT_right)
vT_left =
    10.0000e+000
RT_left =
     2.0000e+003
vT_right =
     0.0000e-003
RT_right =
     2.0000e+003
vx =
     5.0000e+000
```

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Answer:
$v_x = 5$ V

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Problem 3-42

Assume that Figure P3-42 represents a model of the auxiliary output port of a car. The output current is i = 1 A when v = 0 V. The output voltage is v = 12 V when a 20-Ω resistor is connected between the terminals. Suppose you wanted to charge a 12-V battery by connecting the battery at the port. How much current would the port deliver to the battery?

Solution:

The short circuit current is 1 A, as stated in the problem. When a 20-Ω resistor is connected to the circuit, the equivalent resistance must be 12 Ω to create a 12-V drop. Use MATLAB to solve for the Norton resistance. Then find the Thévenin equivalent circuit and solve for the current into the battery, assuming the battery acts as an ideal source with no internal resistance

```
clear all
iN = 1;
syms RN
Eqn = '1/(1/RN+1/20) - 12';
RN = double(solve(Eqn,'RN'))
vT = iN*RN
RT = RN;
iL = (vT-12)/RT
RN =
    30.0000e+000
vT =
    30.0000e+000
iL =
   600.0000e-003
```

Answer:

i = 600 mA.

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Problem 3-43

The *i*-*v* characteristic of the active circuit in Figure P3-42 is $5v + 500i = 100$. Find the output voltage when a 1-kΩ resistive load is connected.

Solution:

Given the *i*-*v* characteristic of the circuit, if we set the voltage equal to zero, the short-circuit current is 200 mA. If we set the current equal to zero, the open-circuit voltage is 20 V. The Thévenin resistance of the circuit is 100 Ω. The following MATLAB code computes the output voltage.

```
clear all
i_sc = 200e-3;
v_oc = 20;
RT = v_oc/i_sc;
RL = 1e3;
vo = RL*v_oc/(RL+RT)
vo =
    18.1818e+000
```

Answer:

$v_L = 18.18$ V

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Problem 3-44

You successfully completed Circuits I and as an undergraduate work-study, your past professor asked you to help her grade a Circuits I quiz. On the quiz, students were asked to find the power supplied by the source to both the 10 kΩ load (R_L) and to the entire circuit as shown in Figure P3-44. Your professor asks to help her by creating a grading sheet.

(a) Solve the quiz and establish reasonable A, B…etc. cuts for incorrect solutions.

(b) A particular student correctly finds the Thévenin equivalent circuit that the resistive load sees and calculates the power to the load using V_T^2/R_L. He then does a source transformation correctly finding the Norton equivalent of the circuit. He calculates the source power by $V_T \times I_N$. What grade would you give him?

(c) Another student finds $P_L = 5.625$ mW and $P_S = -22.5$ mW, but provides no work to justify her answers. What grade would you give her?

(d) A third student first finds the Norton equivalent, finds the current through the load using a current divider and calculates the power in the load using $I_L^2 R_L$. He figures correctly what the parallel voltage would be across the Norton circuit and the load v_L, and then calculates $P_S = I_N \times v_L$. What grade would you give him?

Solution:

(a) Use node-voltage analysis to solve for the required values as shown in the following MATLAB code. Let the center node be node A and the upper right node be node B. Ground the bottom node.

```
clear all
syms vA vB
Eqn1 = '(vA-15)/10e3 + vA/10e3 + (vA-vB)/10e3';
Eqn2 = '(vB-vA)/10e3 + (vB-15)/10e3 + vB/10e3';
Soln = solve(Eqn1,Eqn2,'vA','vB');
vA = double(Soln.vA);
vB = double(Soln.vB);
vL = 15-vB;
pL = vL^2/10e3
is = (15-vA)/10e3 + (15-vB)/10e3;
ps = 15*is
pL =
     5.6250e-003
ps =
    22.5000e-003
```

(b) The student deserves credit for finding the correct Thévenin and Norton equivalent circuits, but the power calculations are incorrect for both the load and the total power dissipated in the circuit. The grade for this solution should be low, such as a D.

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(c) Give the student full credit and a grade of A. The solution is correct. If the solution had been wrong, then the grade would have been zero or F for not showing any work.

(d) The load power is calculated correctly, but the total power calculation is not valid. Give the student a B grade.

Answer:
(a) p_L = 5.625 mW and p_{TOTAL} = 22.5 mW. The requirements for the grading sheet are instructor specific.
Suggested answers for Parts (b), (c), and (d) are presented above.

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Problem 3-45

The Thévenin equivalent parameters of a practical voltage source are v_T = 30 V and R_T = 300 Ω. Find the smallest load resistance for which the load voltage exceeds 15 V.

Solution:

The following MATLAB code presents the solution.

```
clear all
syms vo RL vT RT
Eqn1 = 'vo - RL*vT/(RL+RT)';
RL = simplify(solve(Eqn1,'RL'))
RL15 = subs(RL,{vT,RT,vo},{30,300,15})
RL =
-vo*RT/(vo-vT)
RL15 =
   300.0000e+000
```

Answer:

R_L > 300 Ω

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Problem 3-46

Use a sequence of source transformations to find the Thévenin equivalent at terminals A and B in Figure P3-46.

Solution:

The sequence of source transformations is illustrated below.

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Answer:
v_T = 10 V and R_T = 10 Ω.

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Problem 3-47

The circuit in Figure P3-47 provides power to a number of loads connected in parallel. The circuit is protected by a ¾ mA fuse with a nominal 100 Ω resistance. Each load is 10 kΩ. What is the maximum number of loads the circuit can drive without blowing the fuse?

Solution:

Find the Thévenin equivalent circuit seen by the load resistors and then calculate the number of load resistors the fuse can support.

```
clear all
% Find the open-circuit voltage and the look-back resistance
vT = 30e3*12/(30e3+10e3)
RT = 100+2.5e3+1/(1/10e3+1/30e3)

% Find the maximum total resistance the fuse can support
i_fuse = 0.75e-3;
R_total = vT/i_fuse
% Find the maximum load resistance
RL_max = R_total-RT

% For a parallel combination of the same resistor value RL_total = RL/N
RL = 10e3;
N = RL/RL_max
% Round the number down
N_max = floor(N)
vT =
     9.0000e+000
RT =
    10.1000e+003
R_total =
    12.0000e+003
RL_max =
     1.9000e+003
N =
     5.2632e+000
N_max =
     5.0000e+000
```

Answer:

$N = 5$

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Problem 3-48
Find the Thévenin equivalent at terminals A and B in Figure P3-48.

Solution:
The following MATLAB code presents the solution.

```
clear all
% Find the open-circuit voltage and the look-back resistance
% Use superposition to find the open-circuit voltage
vT1 = 300*15/(300+300+100);
iT2 = -20e-3/600/(1/600+1/100);
vT2 = iT2*300;
vT = vT1+vT2
RT = 1/(1/300+1/(100+300))
vT =
     5.5714e+000
RT =
   171.4286e+000
```

Answer:
v_T = 5.5714 V and R_T = 171.4286 Ω.

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Problem 3-49

A nonlinear resistor is connected across a two-terminal source whose Thévenin equivalent is v_T = 5 V and R_T = 500 Ω. The i–v characteristic of the resistor is $i = 10^{!4}\,(v + 2\,v^{3.3})$. Use the MATLAB function `plot` to find the operating point for this circuit and determine the voltage across, the current through and the power dissipated in the non-linear resistor.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Create a range of voltages to plot
v = 0:0.01:6;
% Plot the i-v characteristic of the source
i_source = 10e-3-2e-3*v;
% Plot the i-v characteristic of the load
i_load = 1e-4*(v + 2*v.^3.3);
plot(v,i_source,'b','LineWidth',3)
hold on
grid on
plot(v,i_load,'g','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
```

```
% Plot the data again
% Zoom in on the intersection of the two curves
% Create a range of voltages to plot
v = 2.58:0.000001:2.585;
% Plot the i-v characteristic of the source
i_source = 10e-3-2e-3*v;
% Plot the i-v characteristic of the load
i_load = 1e-4*(v + 2*v.^3.3);
plot(v,i_source,'b','LineWidth',3)
```

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```
hold on
grid on
plot(v,i_load,'g','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
axis([2.58 2.585 0.00483 0.00484])
```

Answer:
v_L = 2.582 V; i_L = 4.836 mA; p_L = 12.486 mW

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Problem 3-50

A blue light-emitting diode (LED) is connected across a two-terminal source whose Thévenin equivalent is $v_T = 4$ V and $R_T = 20\ \Omega$. The i–v characteristic of the LED is $i = 10^{!12}\ (e^{10v}-1)$. Figure P3-50 shows the LED's i-v characteristic. Either using MATLAB or a graphical approach determine the voltage across and current through the LED.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Create a range of
voltages to plot
v = -1:0.01:4;
% Plot the i-v characteristic of the source
vT = 4;
RT = 20;
i_source = (vT-v)/RT;
% Plot the i-v characteristic of the LED
i_LED = 1e-12*(exp(10*v)-1);
plot(v,i_source,'b','LineWidth',3)
hold on
grid on
plot(v,i_LED,'g','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
axis([-1 4 -0.05 0.2])


% Zoom in on the intersection
% Create a range of voltages to
plot
v = 2.48:0.00001:2.52;
% Plot the i-v characteristic of
the source
vT = 4;
RT = 20;
i_source = (vT-v)/RT;
% Plot the i-v characteristic of
the LED
i_LED = 1e-12*(exp(10*v)-1);
plot(v,i_source,'b','LineWidth',3)
```

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```
hold on
grid on
plot(v,i_LED,'g','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
%axis([-1 4 -0.05 0.2])
```

Answer:
v_{LED} = 2.5 V and i_{LED} = 75 mA

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Problem 3-51

Find the Norton equivalent seen by R_L in Figure P3-51. Select the value of R_L so that 100 mW is delivered to the load.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Use source transformations to find the Norton equivalent circuit
i1 = 500e-3;
R1 = 20;
v1 = i1*R1;
R2 = R1 + 20;
i2 = v1/R2;
R3 = 1/(1/R2+1/40);
iN = i2
RN = R3

% Find RL so that the load dissipates 100 mW
syms RL iL pL iN RN
Eqn1 = 'pL - iL^2*RL';
Eqn2 = 'iL - iN/RL/(1/RL+1/RN)';
Eqn3 = 'pL - 100e-3';
Soln = solve(Eqn1,Eqn2,Eqn3,'RL','iL','pL');
RL = simplify(Soln.RL);
RL_num = subs(RL,{iN,RN},{i2,R3})
iN =
   250.0000e-003
RN =
    20.0000e+000
RL_num =
     1.9224e+000
   208.0776e+000
```

Answer:

R_N = 20 Ω and i_N = 250 mA.

R_L = 208.1 Ω or R_L = 1.922 Ω

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Problem 3-52

Find the Thévenin equivalent seen by R_L in Figure P3-52.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Find the look-back resistance
RT = 1/(1/1e3 + 1/500 + 1/500)
% Use superposition to find the open-circuit voltage
% Turn on the voltage source
Req = 1/(1/1e3+1/500);
voc1 = 500*10/(500+Req);
% Turn on the curr;nt source
ioc2 = -20e-3/500/(1/Req + 1/500);
voc2 = ioc2*500;
vT = voc1+voc2
RT =
   200.0000e+000
vT =
     2.0000e+000
```

Answer:

R_T = 200 Ω and v_T = 2 V.

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Problem 3-53

Find the Thévenin equivalent seen by R_L in Figure P3-53.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Find the open-circuit voltage and the look-back resistance
vA = 60*60/(60+30);
vB = 20*60/(20+20);
vT = vA-vB
Req1 = 1/(1/30+1/60);
Req2 = 1/(1/20+1/20);
RT = Req1+Req2
vT =
    10.0000e+000
RT =
    30.0000e+000
```

Answer:

R_T = 30 Ω and v_T = 10 V.

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Problem 3-54

The resistance R in Figure P3-54 is adjusted until maximum power is delivered to the load consisting of R and the 15-kΩ resistor in parallel.

(a) Find the required value of R.

(b) How much power is delivered to the load?

Solution:

The following MATLAB code presents the solution.

```
clear all
format short eng
disp('Part (a)')
% Find the Thevenin equivalent for the source circuit
% Use the look-back technique to find the resistance
RT = 2e3+3e3+1e3;
% Open-circuit voltage
vT = 20e-3*3e3;

% Find an expression for R in terms of the total load resistance
syms R RL
Eqn1 = 'RL - 1/(1/R+1/15e3)';
R = solve(Eqn1,'R');
% The total load resistance should equal the Thevenin resistance
% for maximum power transfer
R_num = double(subs(R,RL,RT))

disp('Part (b)')
% Use the Thevenin equivalent circuit to compute the power
% dissipated by the load
pL_max = vT^2/4/RT
Part (a)
R_num =
    10.0000e+003
Part (b)
pL_max =
   150.0000e-003
```

Answer:

(a) $R = 10$ kΩ

(b) $p_L = 150$ mW

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Problem 3-55

When a 5-kΩ resistor is connected across a two-terminal source a current of 15 mA is delivered to the load. When a second 5-kΩ resistor is connected in parallel with the first, a total current of 20 mA is delivered. Find the maximum power available from the source.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Write two linear equations to solve for the Thevenin voltage and resistance
% Note that two 5-kOhm resistors in parallel are equivalent to one 2.5-kOhm
resistor
syms vT RT
Eqn1 = '15e-3 - vT/(5e3+RT)';
Eqn2 = '20e-3 - vT/(2.5e3+RT)';

% Solve the equations for vT and RT
Soln = solve(Eqn1,Eqn2,'vT','RT');
vT = double(Soln.vT);
RT = double(Soln.RT);

% Calculate the maximum power available using the Thevenin equivalent circuit
p_max = vT^2/4/RT
p_max =
     1.1250e+000
```

Answer:

p_{MAX} = 1.125 W for R_L = 5 kΩ

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Problem 3-56

Find the value of *R* in Figure P3-56 so that maximum power is delivered to the 2-kΩ load. Find the value of the maximum power.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Maximum power will be delivered if the Thevenin resistance
% matches the load resistance of 2 kOhms
% Find an expression for the Thevenin resistance in terms of R, solve for R,
and
% evaluate when RT = 2 kOhms
syms RT R
Eqn1 = 'RT - 1/(1/R+1/10e3+1/5e3)';
R = solve(Eqn1,'R');
R = double(subs(R,RT,2e3))

% Find the Thevenin voltage
Req=1/(1/10e3+1/5e3)
vT = 10*Req/(R+Req)

% Calculate the maximum power transfer
p_max = vT^2/4/2e3
R =
     5.0000e+003
Req =
     3.3333e+003
vT =
     4.0000e+000
p_max =
     2.0000e-003
```

Answer:

$R = 5$ kΩ

$p_{MAX} = 2$ mW

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Problem 3-57

Find the value of R_L in Figure P3-57 such that v_L = 6 V. Repeat for v_L = 12 V.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Find the Thevenin equivalent circuit via source transformations
Req1 = 1/(1/1e3+1/3e3);
vT = 10e-3*Req1
RT = Req1+2.75e3

% Solve for RL in terms of a specified vL
syms RL vL
Eqn1 = 'vL - RL*vT/(RL+RT)';
RL = simplify(solve(Eqn1,'RL'));
RL = subs(RL);

% Evaluate RL at the required values for vL
vL_num = [6 12];
RL_num = subs(RL,vL,vL_num)
vT =
     7.5000e+000
RT =
     3.5000e+003
RL_num =
    14.0000e+003    -9.3333e+003
```

Answer:

For v_L = 6 V, R_L = 14 kΩ.

The load voltage v_L = 12 V is impossible, since v_{OC} = 7.5 V.

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Problem 3-58

Find the value of R_L in Figure P3-57 such that maximum power is delivered to the load. Find the value of the maximum power.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Find the Thevenin equivalent circuit via source transformations
Req1 = 1/(1/1e3+1/3e3);
vT = 10e-3*Req1;
RT = Req1+2.75e3;
RL = RT
p_max = vT^2/4/RT
RL =
     3.5000e+003
p_max =
     4.0179e-003
```

Answer:

R_L = 3.5 kΩ and p_{MAX} = 4.0179 mW.

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Problem 3-59

A 100 Ω-load needs 10 mA to operate correctly. Design a practical power source to provide the needed current. The smallest source resistance you can practically design for is 50 Ω, but you can add any other series resistance if you need to.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Find the load voltage
RL = 100;
iL = 10e-3;
vL = iL*RL;

% Use the smallest possible source resistance
Rs = 50

% Calculate the source voltage to provide the required current
vs = iL*(Rs+RL)
Rs =
    50.0000e+000
vs =
     1.5000e+000
```

Answer:

v_S = 1.5 V and R_S = 50 Ω.

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Problem 3-60

A practical source delivers 50 mA to a 300-Ω load. The source delivers 12 V to a 120-Ω load. Find the maximum power available from the source.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Find the Thevenin equivalent source by solving two linear equations
syms vT RT
Eqn1 = '50e-3 - vT/(RT+300)';
Eqn2 = '12 - 120*vT/(120+RT)';
Soln = solve(Eqn1,Eqn2,'vT','RT');
vT = double(Soln.vT);
RT = double(Soln.RT);
% Calculate the maximum power available
p_max = vT^2/4/RT
p_max =
     1.3500e+000
```

Answer:

p_{MAX} = 1.35 W

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Problem 3-61

The source in Figure P3-61 has a 100 mA output current limit. Design an interface circuit so that the load voltage is v_2 = 2 V and the source current is i_1 < 50 mA.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Find the load current
v2 = 2;
R2 = 50;
i2 = v2/R2

% Find the minimum total resistance seen by the source
i1 = 50e-3;
R_total_min = 10/i1
i2 =
    40.0000e-003
R_total_min =
   200.0000e+000

% The required current is less than the maximum current
% Add a series resistor as the interface to create current of 40 mA
Req = 10/i2
R_interface = Req - 50 - 50
Req =
   250.0000e+000
R_interface =
   150.0000e+000
```

The following OrCAD simulation verifies the solution.

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Answer:
$R_{\text{SERIES}} = 150\ \Omega$

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Problem 3-62

Figure P3-62 shows an interface circuit connecting a 15-V source to a diode load. The *i*-*v* characteristic of the diode is $i = 10^{-14}\left(e^{40\,v} - 1\right)$.

(a) Design an interface circuit so that $v = 0.7$ V.

(b) Validate your answer using MATLAB.

Solution:

The following MATLAB code presents the solution.

```
clear all
vs = 15;
% Given the diode voltage, find the diode current
vd = 0.7;
id = 1e-14*(exp(40*vd)-1)

% Find the voltage drop across the interface
v_inter = vs-vd
% If the interface is a series resistor, find the value of the resistance
R_series = v_inter/id
id =
    14.4626e-003
v_inter =
    14.3000e+000
R_series =
   988.7592e+000
```

Answer:

(a) $R_{\text{SERIES}} = 998.7592\ \Omega$.

(b) The MATLAB solution is presented above.

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Problem 3-63

Design the interface circuit in Figure P3-63 so that the voltage delivered to the load is v = 10 V "10%. Use only one or more of the following standard resistors: 130 Ω, 200 Ω, 300 Ω, 430 Ω, 620 Ω, and 910 Ω. These resistors all have a tolerance of "5% which you must account for in your design.

Solution:

The desired voltage delivered to the load is half of the source voltage. If the interface is a series resistor, the load should represent half of the total resistance in the series path. Based on this initial analysis, the interface resistor should be 950 Ω. The closest available standard resistor is 910 Ω. Examine the performance of the system with a 910-Ω resistor in a series interface to see if it meets specifications. The following MATLAB code presents the calculations associated with the solution.

```
clear all
vs = 20;
Rs = 50;
RL = 1e3;
Ri = 910;
Ri_min = 0.95*Ri;
Ri_max = 1.05*Ri;
Ri_all = [Ri_min Ri Ri_max];
% Examine the load voltage for the various interface resistor options
vL = RL*vs./(RL+Rs+Ri_all)
vL =
    10.4466e+000    10.2041e+000     9.9726e+000
```

The allowable range for the load voltage is between 9 V and 11 V, so the 910-Ω resistor satisfies the requirements. The following OrCAD simulations verify the results.

Answer:

R_{SERIES} = 910 Ω. Other solutions are possible using more than one resistor.

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Problem 3-64

Two teams are competing to design the interface circuit in Figure P3-63 so that 25 mW ±10 % is delivered to the 1-kΩ load resistor. Their designs are shown in Figure P3-64. Which solution is better considering the use of standard values, number of parts, and power required by the source? Would your choice be different if the power had to be within ±5 %?

Solution:

Use MATLAB to evaluate the two designs as presented below.

```
clear all
vs = 20;
Rs = 50;
RL = 1e3;
pL = 25e-3;

disp('Design Team A')
iLA = vs/(Rs+2.2e3+680+RL);
pLA = iLA^2*RL
psA = vs*iLA

% Calculate the allowed ranges of load powers for 10 percent and 5 percent
tolerances
pLmin10 = 0.9*pL;
pLmax10 = 1.1*pL;
pLmin5 = 0.95*pL;
pLmax5 = 1.05*pL;
pLRange10 = [pLmin10 pL pLmax10]
pLRange5 = [pLmin5 pL pLmax5]
Check_pLA10 = (pLA>=pLmin10)&&(pLA<=pLmax10)
Check_pLA5 = (pLA>=pLmin5)&&(pLA<=pLmax5)

disp('Design Team B')
Req = 1/(1/33 + 1/33 + 1/1e3);
vLB = Req*vs/(Req+Rs);
pLB = vLB^2/RL
isB = vs/(Rs+Req);
psB = vs*isB
Check_pLB10 = (pLB>=pLmin10)&&(pLB<=pLmax10)
Check_pLB5 = (pLB>=pLmin5)&&(pLB<=pLmax5)
Design Team A
pLA =
    25.8985e-003
psA =
   101.7812e-003
pLRange10 =
```

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```
    22.5000e-003    25.0000e-003    27.5000e-003
pLRange5 =
    23.7500e-003    25.0000e-003    26.2500e-003
Check_pLA10 =
     1
Check_pLA5 =
     1
Design Team B
pLB =
    24.0257e-003
psB =
     6.0394e+000
Check_pLB10 =
     1
Check_pLB5 =
     1
```

Answer:
Team A has the better solution. Both teams meet p_L = 25 mW within 10% and 5%, have standard values, and two parts. Team A requires less source power with p_{SA} = 101.78 mW and p_{SB} = 6.039 W.

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Problem 3-65

The bridge-T attenuation pad shown in Figure P3-65 was found in a drawer. You need an attenuation pad that would match to a 75-Ω source and a 75-Ω load and provide for a –12 dB drop of signal (reduction of four times). Use OrCAD to determine if the device will work.

Solution:

The follow OrCAD simulation shows that the bridge-T attenuation pad meets the requirements.

The voltage at the input of the bridge-T pad is 5 V and the output voltage is 1.25 V, which is a reduction by a factor of 4 or –12.04 dB. If the 75-Ω source experiences a load of 75-Ω, then the source current will be $v_S/150$, which is 10/150 = 66.67 mA, in this case, and agrees with the simulation.

To determine if the load is matched to a 75-Ω equivalent source, perform two more OrCAD simulations to find the open-circuit voltage and short-circuit current from the perspective of the load. To simplify the simulations, replace the load resistor with a large value and then a small value to estimate the open-circuit voltage and short-circuit current

The open-circuit voltage is 2.5 V and the short-circuit current is 33.33 mA. The Thévenin resistance seen by the load is R_T = 2.5 V/33.33 mA = 75 Ω, so the circuit meets all of the specifications.

Answer:

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The device will work as described and verified above.

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Problem 3-66

Select R_L in Figure P3-66 so that the power delivered to the load is 400 mW.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Write an expression for pL in terms of RL.
% Solve for RL in terms of pL and substitute the specified power
syms pL RL
Eqn = 'pL - (10/(50+RL))^2*RL';
RL = simplify(solve(Eqn,'RL'))
RL_num = double(subs(RL,pL,400e-3))

% Check the results
iL_num = 10./(50+RL_num);
pL_num = iL_num.^2.*RL_num
RL =
 -50*(pL-1-(-2*pL+1)^(1/2))/pL
 -50*(pL-1+(-2*pL+1)^(1/2))/pL
RL_num =
   130.9017e+000
    19.0983e+000
pL_num =
   400.0000e-003
   400.0000e-003
```

Answer:

There are two answers: R_L = 19.0983 Ω or R_L = 130.9017 Ω.

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Problem 3-67

Design the interface circuit in Figure P3-67 so that R_{IN} = 100 Ω and the current delivered to the 50-Ω load is i = 50 mA. Hint: Use an L-pad.

Solution:

The following MATLAB code presents the solution.

```
clear all
vs = 15;
Rs = 100;
RL = 50;
Rin = 100;
iL = 50e-3;
% If Rin = 100, then we can determine the source current
is = vs/(Rs+Rin)
is =
    75.0000e-003
% The source current is 75 mA and the load current is 50 mA
% Therefore 25 mA is diverted away from the load
% Use an L-pad interface with a resistor, R1, in series with the source
% resistor and another resistor, R2, in parallel with the load
% Compute the load voltage
vL = iL*RL;
% Compute R2
vR2 = vL;
iR2 = is-iL;
R2 = vR2/iR2

% Compute the equivalent resistance for the parallel combination of R2 and RL
Req = 1/(1/R2+1/RL);
% Compute R1
R1 = Rin-Req
R2 =
   100.0000e+000
R1 =
    66.6667e+000
```

Answer:

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One possible design is the third interface circuit in Figure 3-52 with R_1 = 66.7 Ω and R_2 = 100 Ω.

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Problem 3-68

Design the interface circuit in Figure P3-67 so that R_{OUT} = 50 Ω and the voltage delivered to the 50-Ω load is v = 3 V. Hint: Use an L-pad.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Use an L-pad circuit with a resistor R1 in series with
% the source resistor and a resistor R2 in parallel with
% the load resistor
% Develop equations that express the constraints for Rout
% and vL with respect to the two resistors
% Solve the equations

syms R1 R2

% Rout constraint
Eqn1 = '50 - 1/(1/(100+R1)+1/R2)';

% vL constraint
Eqn2 = '3 - 15/(1/R2+1/50)/(100+R1+1/(1/R2+1/50))';

% Solve the equations
Soln = solve(Eqn1,Eqn2,'R1','R2');
R1 = double(Soln.R1)
R2 = double(Soln.R2)
R1 =
    25.0000e+000
R2 =
    83.3333e+000
```

Answer:

One possible design is the third interface circuit in Figure 3-52 with R_1 = 25 Ω and R_2 = 83.33 Ω.

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Problem 3-69

The circuit in Figure P3-69 has a source resistance of 75 Ω and a load resistance of 600 Ω. Design the interface circuit so that the input resistance is R_{IN} = 75 Ω "10% and the output resistance is R_{OUT} =600 Ω "10%.

Solution:

The following MATLAB code presents the solution.

```
clear all
% Rin is less than the load resistance, so it requires a parallel resistor to
% decrease Rin
% Rout is greater than the source resistance, so it requires a series resistor
% to increase Rout
% The fourth design in Figure 3-52, with R1 in parallel with the practical
% source circuit and R2 in series with the load, will meet the requirements.
% Formulate equations for the constraints and solve for R1 and R2

% Rin constraint
Eqn1 = '75 - 1/(1/R1+1/(R2+600))';
% Rout constraint
Eqn2 = '600 - (R2+1/(1/75+1/R1))';

% Solve the equations
Soln = solve(Eqn1,Eqn2,'R1','R2');
R1 = double(Soln.R1)
R2 = double(Soln.R2)
R1 =
    80.1784e+000
   -80.1784e+000
R2 =
   561.2486e+000
  -561.2486e+000

% Keep the positive solutions and check the results
R1 = max(R1);
R2 = max(R2);
Check_Rin = 1/(1/R1+1/(R2+600))
Check_Rout = R2+1/(1/75+1/R1)
Check_Rin =
    75.0000e+000
Check_Rout =
   600.0000e+000
```

Answer:

One possible design is the fourth interface circuit in Figure 3-52 with R_1 = 80.18 Ω and R_2 = 561.25 Ω.

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Problem 3-70

It is claimed that both interface circuits in Figure 3-70 will deliver v = 4 V to the 75-Ω load. Verify this claim. Which interface circuit consumes the least power? Which has an output resistance that best matches the 75-Ω load?

Solution:

For each interface, calculate the load voltage, source power, and output resistance using MATLAB.

```
clear all
vs = 20;
Rs = 150;
RL = 75;

disp('Circuit 1')
R11 = 150;
iL1 = vs/(Rs+R11+RL);
vL1 = RL*iL1
ps1 = iL1*vs
Rout1 = Rs+R11

disp('Circuit 2')
R21 = 100;
R22 = 15;
Req2 = 1/(1/R21+1/(R22+RL));
veq2 = Req2*vs/(Rs+Req2);
vL2 = RL*veq2/(R22+RL)
is2 = vs/(Rs+Req2);
ps2 = is2*vs
Rout2 = 15+1/(1/Rs+1/R21)
Circuit 1
vL1 =
     4.0000e+000
ps1 =
     1.0667e+000
Rout1 =
   300.0000e+000
Circuit 2
vL2 =
     4.0000e+000
ps2 =
     2.0267e+000
Rout2 =
    75.0000e+000
```

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Answer:
The following table summarizes the results

Circuit	v_{LOAD} **(V)**	p_{SOURCE} **(W)**	R_{OUT} **(Ω)**
1	4.00	1.0667	300
2	4.00	2.0267	75

Both circuits provide 4 V to the load. Circuit 1 consumes the least power and Circuit 2 has an output resistance that best matches the 75-Ω load.

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Problem 3-71

Audio Speaker Resistance-Matching Network

A company is producing an interface network that they claim would result in an R_{IN} of 600 Ω ±2 % and R_{OUT} of 16, 8, or 4 Ω ±2 % – depending on whether the connected speakers are 16, 8, or 4 Ω – selectable via a built-in switch. Prove or disprove their claim.

Audio Matching Network

Solution:

The following MATLAB code presents the solution.

```
clear all
% The interface is an L-pad with different values for R1 and R2
% The load resistance value also changes
% Compute Rin and Rout for the three different cases
Rs = 600;
RL = [16 8 4];
R1 = 592+[0 8 12];
R2 = [16 8 4];

% Compute Rin
Rin = R1 + 1./(1./R2+1./RL)

% Compute Rout
Rout = 1./(1./(Rs+R1)+1./R2)
Rin =
   600.0000e+000   604.0000e+000   606.0000e+000
Rout =
    15.7881e+000     7.9470e+000     3.9868e+000
```

Answer:

The claim is true. For R_L = [16, 8, 4] Ω, R_{IN} = [600, 604, 606] Ω, R_{OUT} = [15.79, 7.95, 3.99] Ω. All of the values are within the specified limits.

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Problem 3-72

Attenuation Analysis

In Figure P3-72 a two-port attenuator connects a 600-Ω source to a 600-Ω load . Find the power delivered to the load in terms of v_S. Remove the attenuator and find the power delivered to the load when the source is directly connected to the load. By what fraction does the attenuator reduce the power delivered to the 600-Ω load? Express the fraction in dB.

Solution:

The following MATLAB code presents the solution.

```
clear all
syms vs v1 p1 p2
disp('Attenuator')
Req = 1/(1/450+1/(300+600));
v1 = Req*vs/(600+300+Req);
vL1 = 600*v1/(300+600);
p1 = vL1^2/600

disp('No attenuator')
vL2 = 600*vs/(600+600);
p2 = vL2^2/600

p_ratio = simplify(p1/p2)
p_ratio_dB = 10*log10(double(p_ratio))
Attenuator
p1 =
1/21600*vs^2
No attenuator
p2 =
1/2400*vs^2
p_ratio =
1/9
p_ratio_dB =
    -9.5424e+000
```

Answer:

K = 1/9 or −9.54 dB

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Problem 3-73

Interface Circuit Design

Using no more than three 50-Ω resistors, design the interface circuit in Figure P3-73 so that v # 4 V and i # 100 mA regardless of the value of R_L.

Solution:

We can translate the interface requirements into specifications for Thévenin and Norton equivalent circuits. With v # 4 V, the Thévenin voltage source is less than 4 V. With i # 100 mA, the Norton current source is less than 100 mA. Combining these results, we require the Thévenin resistance to be at least 4 V/100 mA = 40 Ω.

To get an open-circuit voltage of 4 V, we can add two resistors to the interface as shown below.

To increase the Thévenin resistance above 40 Ω, add a resistor in series as shown below.

This interface will meet all of the requirements for any load resistance.

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Answer:
The answer is presented above.

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Problem 3-74

Battery Design

A satellite requires a battery with an open-circuit voltage v_{OC} = 36 V and a Thévenin resistance R_T # 10 Ω. The battery is to be constructed using series and parallel combinations of one of two types of cells. The first type has v_{OC} = 9 V, R_T = 4 Ω, and a weight of 40 grams. The second type has v_{OC} = 4 V, R_T = 0.5 Ω, and a weight of 15 grams. Design a minimum weight battery that meets the open-circuit voltage and Thévenin resistance requirements.

Solution:

Using the first type of cell, we need to connect four cells in series to get 36 V, but the equivalent resistance is 16 Ω, which is above the specification of 10 Ω. To reduce the resistance, connect another set of four cells in series and then connect both sets in parallel to reduce the resistance to 8 Ω while maintaining the voltage level. The total weight is 320 grams.

Using the second type of cell, connect nine cells in series to get 36 V with a resistance of 4.5 Ω. This single set of cells meets both requirements and has a weight of only 135 grams, so it is the better alternative.

Answer:

Use a series combination of nine of the second type of cell.

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Problem 3-75

Design Evaluation

A requirement exists for a circuit to deliver 0 to 5 V to a 100-Ω load from a 20-V source rated at 2.5 W. Two proposed circuits are shown in Figure P3-75. Which one would you choose and why?

Circuit 1

Circuit 2

Solution:

Evaluate the two designs in terms of the voltage provided to the load and the power supplied by the source.

```
clear all
vs = 20;
Rs = 150;
RL = 100;
Ri_max = 100;
Ri = [0 Ri_max];

% Compute vo and ps for Circuit 1
disp('Circuit 1');
Req1 = 1./(1/RL + 1./Ri);
vo1 = Req1*vs./(Rs+Req1)
is1 = vs./(Rs+Req1);
ps1 = is1*vs

% Compute vo and ps for Circuit 2
disp('Circuit 2');
Req2 = 1./(1/RL + 1./Ri);
vo2 = Req2*vs./(Rs+Req2+(Ri_max-Ri))
is2 = vs./(Rs+Req2+(Ri_max-Ri));
ps2 = is2*vs
Circuit 1
Warning: Divide by zero.
vo1 =
     0.0000e-003     5.0000e+000
ps1 =
     2.6667e+000     2.0000e+000
Circuit 2
Warning: Divide by zero.
vo2 =
     0.0000e-003     5.0000e+000
ps2 =
     1.6000e+000     2.0000e+000
```

Answer:

Choose Circuit 2. Both circuits meet the voltage requirements, but the first circuit exceeds the source power rating.

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Problem 3-76

Design Interface Competition.

The output of a transistorized power supply is modeled by the Norton equivalent circuit shown Figure P3-76. Two teams are competing to design the interface circuit so that the 25 mW ±10 % is delivered to the 1-kΩ load resistor. Their designs are shown in Figure P3-76. Which solution is better considering the use of standard values, number of parts, and power required by the source? Would your choice be different if the power had to be within ±5 %?

Solution:

For each interface design, use MATLAB to calculate the power supplied to the load and the power supplied by the source.

```
clear all
is = 400e-3;
Rs = 50;
RL = 1e3;

disp('Team A')
R1 = 3e3;
iL1 = is/(R1+RL)/(1/Rs+1/(R1+RL));
pL1 = iL1^2*RL
vs1 = iL1*(R1+RL);
ps1 = is*vs1

disp('Team B')
R2 = 16;
iL2 = is/RL/(1/Rs+1/R2+1/RL);
pL2 = iL2^2*RL
vs2 = iL2*RL;
ps2 = is*vs2
Team A
pL1 =
    24.3865e-003
ps1 =
     7.9012e+000
Team B
pL2 =
    22.9481e-003
ps2 =
     1.9162e+000
```

Answer:

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Both solutions require one standard part, so they are equivalent in terms of parts. For p_L = 25 mW ±10 %, Team B requires less source power and meets the specs. For p_L = 25 mW ±5 %, only Team A meets the specs.

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Problem 3-77

Analysis of Competing Interface Circuits Using MATLAB.

Figure P3-77 displays two generalized interface circuit designs. In both circuits, resistors R_1 and R_2 connect a Thévenin equivalent circuit to a load resistor. Using MATLAB, develop symbolic expressions for the load current, i_L, and the input resistance, R_{IN}, for each circuit in terms of the given parameters. Using these two expressions, now use the MATLAB command solve to solve for R_1 and R_2 in terms of i_L and R_{IN}. Let v_T = 15 V, R_T = 100 Ω, R_L = 50 Ω, i_L = 50 mA, and R_{IN} = 100 Ω. Can you use both types of interface circuits to find suitable values for R_1 and R_2 to meet these specifications? Compare your interface design(s) with the solution to Problem 3-67.

(a)

(b)

Solution:

The following MATLAB code presents the solution.

```
clear all
syms vT RT RL iL Rin R11 R12 R21 R22
disp('Circuit 1')
% Use a source transformation and current
division to find an
% expression for iL1
Eqn1 = 'iL-
(vT/(RT+R11))/RL/(1/(RT+R11)+1/R12+1/RL)';
% Use equivalent resistance to find Rin1
Eqn2 = 'Rin - (R11 + 1/(1/R12+1/RL))';
% Solve the equations R11 and R12
Soln = solve(Eqn1,Eqn2,'R11','R12');
R11 = simplify(Soln.R11)
R12 = simplify(Soln.R12)
% Substitute numerical values
R11_num = double(subs(R11,{vT,RT,RL,iL,Rin},{15,100,50,50e-3,100}))
R12_num = double(subs(R12,{vT,RT,RL,iL,Rin},{15,100,50,50e-3,100}))
Circuit 1
R11 =
-(Rin*RL*iL+iL*RL*RT-vT*Rin)/vT
R12 =
-iL*RL*(RT+Rin)/(iL*RT+Rin*iL-vT)
R11_num =
    66.6667e+000
R12_num =
   100.0000e+000
clear all;
syms vT RT RL iL Rin R11 R12 R21 R22
disp('Circuit 2')
% Use a source transformation and current division to find an
% expression for iL2
```

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```
Eqn3 = 'iL-(vT/RT)/(R21+RL)/(1/RT+1/R22+1/(R21+RL))';
Eqn4 = 'Rin - 1/(1/R22+1/(R21+RL))';
% Solve the equations R21 and R22
Soln = solve(Eqn3,Eqn4,'R21','R22');
R21 = simplify(Soln.R21)
R22 = simplify(Soln.R22)
% Substitute numerical values
R21_num = double(subs(R21,{vT,RT,RL,iL,Rin},{15,100,50,50e-3,100}))
R22_num = double(subs(R22,{vT,RT,RL,iL,Rin},{15,100,50,50e-3,100}))
Circuit 2
R21 =
-(iL*RT*RL+Rin*RL*iL-Rin*vT)/(Rin+RT)/iL
R22 =
-Rin*vT/(Rin*iL+iL*RT-vT)
R21_num =
   100.0000e+000
R22_num =
   300.0000e+000
```

Answer:
Both circuit designs can meet the specifications.
For Circuit 1, R_1 = 66.67 Ω and R_2 = 100 Ω.
For Circuit 2, R_1 = 100 Ω and R_2 = 300 Ω.

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Problem 3-78

Maximum Power Transfer Using OrCAD

Figure P3-78 shows a circuit with two sources, a fixed load and a resister *R*. Select *R* for maximum power transfer to the load. The result is not an obvious one. Hint: Simulate in OrCAD using the PARAMETER function and sweep R from 0.1 Ω to 100 MΩ.

Solution:

The following OrCAD simulation and results provide a solution to this problem.

Answer:

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For maximum power transfer, select R to be an open circuit.

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Problem 4-1

Find the voltage gain v_O ' v_S and current gain i_O ' i_x in Figure P4-1 for r = 10 kΩ.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms vs ix io vo
r = 10e3;
% Find the current ix
ix = vs/(100+400);
% Find the current io and the voltage vo
io = -r*ix/(500+2e3);
vo = io*2e3;

% Compute the gains
Kv = simplify(vo/vs)
Ki = simplify(io/ix)
Kv =
-16
Ki =
-4
```

Answer:

K_V = −16 and K_I = −4.

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Problem 4-2

Find the voltage gain v_O ' v_1 and the current gain i_O ' i_S in Figure P4-2. For i_S = 2 mA, find the power supplied by the input current source and the power delivered to the 2-kΩ load resistor.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms is v1 i1 io vo
% Find i1 and v1
i1 = is/100/(1/100+1/100);
v1 = i1*100;

% Find io and vo
io = -100*i1/2e3/(1/2e3+1/2e3);
vo = io*2e3;

% Find the gains
Kv = simplify(vo/v1)
Ki = simplify(io/is)

% Find the source and load powers
v1_num = subs(v1,is,2e-3);
is = 2e-3;
ps_num = is*v1_num

vo_num = Kv*v1_num;
io_num = Ki*is;
po_num = vo_num*io_num
Kv =
-1000
Ki =
-25
ps_num =
   200.0000e-006
po_num =
5
```

Answer:

K_V = −1000 and K_I = −25.

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For i_S = 2 mA, p_S = 200 μW and p_L = 5 W.

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Problem 4-3

Find the voltage gain v_O ' v_S and current gain i_O ' i_x in Figure P4-3 for g = 10$^{!3}$ S.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms vs ix vx io vo
g = 1e-3;

% Find ix and vx
ix = vs/(1e3+3e3);
vx = 3e3*ix;

% Find io and vo
io = g*vx/(500+2e3)/(1/10e3+1/(500+2e3));
vo = io*2e3;

% Find the gains
Kv = simplify(vo/vs)
Ki = simplify(io/ix)
Kv =
6/5
Ki =
12/5
```

Answer:

K_V = 1.2 and K_I = 2.4.

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Problem 4-4

(a) Find the voltage gain v_O ' v_S and current gain i_O ' i_x in Figure P4-4.

(b) Validate your answers by simulating the circuit in OrCAD.

Solution:

(a) The solution is presented in the following MATLAB code.

```
clear all
syms vs ix io vo

% Find ix
ix = vs/1.5e3;

% Find io and vo
io = -50*ix;
vo = io*2.2e3;

% Find the gains
Kv = simplify(vo/vs)
Ki = simplify(io/ix)
Kv =
-220/3
Ki =
-50
```

(b) The following OrCAD simulation verifies the answer in Part (a).

Answer:

(a) $K_V = -73.33$ and $K_I = -50$.

(b) The OrCAD simulation verifies the answer in Part (a).

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Problem 4-5

Find the voltage gain v_O ' v_S in Figure P4-5.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms vs vx vo

% Solve for vx in terms of vs
Eqn1 = 'vx-(vs-(-50*vx))';
vx = solve(Eqn1,'vx');

% Find vo
vo = -50*vx;

% Find the gain
Kv = simplify(vo/vs)
Kv_num = double(Kv)
Kv =
50/49
Kv_num =
     1.0204e+000
```

Answer:

K_V = 50/49 = 1.02

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Problem 4-6

Find an expression for the current gain i_O ' i_S in Figure P4-6. *Hint:* Apply KCL at node A.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms vs is ie io B Rs Re Rc vA

% Write a KCL expression at node A
Eqn1 = '(vA-vs)/Rs + vA/Re - B*ie';
% Write additional equations relating the variables in the circuit
Eqn2 = 'ie -vA/Re';
Eqn3 = 'io + B*ie';
Eqn4 = 'is -(vs-vA)/Rs';

% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,'io','is','ie','vA');
io = simplify(Soln.io);
is = simplify(Soln.is);

% Calculate the gain
Ki = io/is
Ki =
B/(-1+B)
```

Answer:

$$K_I = \frac{\beta}{\beta - 1}$$

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Problem 4-7

(a) Find the voltage v_O in Figure P4-7.

(b) Validate your answer by simulating the circuit in OrCAD.

Solution:

(a) The solution is presented in the following MATLAB code.

```
clear all
syms vx vo
% Use node-voltage analysis
Eqn1 = '(vx-10)/2.2e3 + vx/1.5e3 + (vx-vo)/1e3 + 1e-3*vx';
Eqn2 = '-1e-3*vx + (vo-vx)/1e3 + vo/3.3e3';

% Solve the equations for vo
Soln = solve(Eqn1,Eqn2,'vx','vo')
vo = double(Soln.vo)
Soln =
    vo: [1x1 sym]
    vx: [1x1 sym]
vo =
     4.3980e+000
```

(b) The following OrCAD circuit verifies the answer in Part (a).

Answer:

(a) v_O = 4.398 V.

(b) The OrCAD simulation verified the results in Part (a).

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Problem 4-8

(a) Find an expression for the current gain i_O / i_S in Figure P4-8.

(b) Find an expression for the voltage gain v_O / v_S in Figure P4-8.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms vs is vx io vo mu R1 R2 RL

% Write a node-voltage equation
Eqn1 = '(vx-vs)/R1 + (vx-(-mu*vx))/R2';

% Write additional equations relating the circuit parameters
Eqn2 = 'vo + mu*vx';
Eqn3 = 'is - (vs-vx)/R1';
Eqn4 = 'vo - io*RL';

% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,'vx','vo','is','io');
vo = simplify(Soln.vo);
is = simplify(Soln.is);
io = simplify(Soln.io);

% Compute the gains
Ki = simplify(io/is)
Kv = simplify(vo/vs)
Ki =
-mu*R2/RL/(1+mu)
Kv =
-mu*R2/(R2+R1+R1*mu)
```

Answer:

(a) $K_I = \dfrac{-\mu R_2}{(1+\mu)R_L}$

(b) $K_V = \dfrac{-\mu R_2}{(1+\mu)R_1 + R_2}$

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Problem 4-9

Find an expression for the voltage gain v_O ' v_S in Figure P4-9.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms vs vx Rs g Ro vo
% Write a node-voltage equation
Eqn1 = '(vo-vs)/Rs - g*vx + vo/Ro';

% Write additional equations relating the parameters in the circuit
Eqn2 = 'vx - (vs-vo)';

% Solve the equations
Soln = solve(Eqn1,Eqn2,'vx','vo');
vo = simplify(Soln.vo);

% Calculate the gain
Kv = simplify(vo/vs)
Kv =
Ro*(1+g*Rs)/(Ro+g*Rs*Ro+Rs)
```

Answer:

$$K_V = \frac{gR_O R_S + R_O}{gR_O R_S + R_O + R_S}$$

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Problem 4-10

(a) Find an expression for the voltage gain v_O ' v_S in Figure P4-10.

(b) Let R_S = 10 kΩ, R_L = 10 kΩ and μ = 100. Find the voltage gain v_O ' v_S as a function of R_F. What is the voltage gain when R_F is an open circuit, a short circuit, and for R_F = 100 Ω?

(c) Simulate the circuit in OrCAD by varying R_F from 1 Ω to 10 MΩ. Read your output for R_F = 100 Ω. How does your answer compare with part (b)?

Solution:

The solutions to Parts (a) and (b) are presented in the following MATLAB code.

```
clear all
disp('Part (a)')
syms vs vx vo vA Rs Rf mu RL
% Write a node-voltage equation
Eqn1 = '(vA-vs)/Rs + (vA-mu*vx)/Rf';

% Write other equations relating the circuit parameters
Eqn2 = 'vA - (vx + mu*vx)';
Eqn3 = 'vo - mu*vx';

% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,'vA','vx','vo');
vo = simplify(Soln.vo);

% Calculate the gain
Kv = simplify(vo/vs)

disp('Part (b)')
Kv = simplify(subs(Kv,{Rs,RL,mu},{10e3,10e3,100}))
Rf_num = [0 100];
Kv_num = subs(Kv,Rf,Rf_num)
Kv_inf = limit(Kv,Rf,inf)
Part (a)
Kv =
mu/(Rf+Rf*mu+Rs)*Rf
Part (b)
Kv =
100/(101*Rf+10000)*Rf
Kv_num =
     0.0000e-003   497.5124e-003
Kv_inf =
100/101
```

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(c) The OrCAD circuit and simulation results are shown below.

Answer:

(a) $K_V = \dfrac{\mu R_F}{(1+\mu)R_F + R_S}$

(b) $K_V = \dfrac{100R_F}{101R_F + 10000}$. For R_F as an open circuit, a short circuit, and for $R_F = 100\ \Omega$, we have K_V equal to 100/101, 0, and 0.4975, respectively.

(c) The OrCAD results are presented above. With R_F as an open circuit or short circuit, the voltage gain values approach the correct values. For $R_F = 100\ \Omega$, we have $K_V = 0.4975$, which agrees with the calculations in Part (b).

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Problem 4-11

Find the Thévenin equivalent circuit that the load R_L sees in Figure P4-11.

Solution:

Since the circuit has a dependent source, we cannot reliably use the look-back technique to compute the Thévenin resistance. We need to find the open-circuit voltage and the short-circuit current. The solution is presented in the following MATLAB code.

```
clear all
syms vs Rs Rp r is vT RT isc
Eqn1 = 'is - (vs-r*is)/Rs';
Eqn2 = 'vT - r*is';
Eqn3 = 'isc - r*is/Rp';

% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,'is','vT','isc');
vT = simplify(Soln.vT)
isc = simplify(Soln.isc);
RT = simplify(vT/isc)
vT =
r*vs/(Rs+r)
RT =
Rp
```

Answer:

$R_T = R_P$ and $v_T = \dfrac{rV_S}{R_S + r}$.

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Problem 4-12

Find R_{IN} in Figure P4-12.

Solution:

Find the ratio of the input voltage to the input current using MATLAB.

```
clear all
syms is R r iin vin Rin
iin = is;
vin = is*R+r*is;
Rin = simplify(vin/iin)
Rin =
R+r
```

Answer:

$R_{IN} = R + r$.

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Problem 4-13
Find R_{IN} in Figure P4-13.

Solution:
Find the ratio of the input voltage to the input current using MATLAB.

```
clear all
syms is ix vin R Rin B
Eqn1 = 'ix - (is + B*ix)';
ix = solve(Eqn1,'ix');
vin = R*ix;
Rin = vin/is
Rin =
-R/(-1+B)
```

Answer:

$$R_{IN} = \frac{R}{1-\beta}.$$

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Problem 4-14

Find the Thévenin Equivalent circuit seen by the load in Figure P4-14.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms vs Rs Rx ix B Ro vT isc RT
% Find the open-circuit voltage and the short-circuit current
ix = vs/(Rs+Rx);
vT = -B*ix*Ro
isc = -B*ix;
RT = simplify(vT/isc)
vT =
-B*vs/(Rs+Rx)*Ro
RT =
Ro
```

Answer:

$R_T = R_O$ and $v_T = \dfrac{-\beta R_O v_S}{R_S + R_X}$.

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Problem 4-15

Find the Norton Equivalent circuit seen by the load in Figure P4-15.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms is vx g Ro iN vT RN
% Find the short-circuit current
vx = 0;
iN = is

% Find the open-circuit voltage
Eqn1 = 'vx - (is-g*vx)*Ro';
vx = solve(Eqn1,'vx');
vT = simplify(vx);

% Find the Norton resistance
RN = simplify(vT/iN)
iN =
is
RN =
Ro/(1+g*Ro)
```

Answer:

$R_N = \dfrac{R_O}{1+gR_O}$ and $i_N = i_S$.

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Problem 4-16

The circuit parameters in Figure P4-16 are R_B = 50 kΩ, R_C = 4 kΩ, β = 120, V_γ = 0.7 V, and V_{CC} = 15 V. Find i_C and v_{CE} for v_S = 2 V. Repeat for v_S = 5 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
RB = 50e3;
RC = 4e3;
B = 120;
Vg = 0.7;
Vcc = 15;

% Determine the operating mode and find iC and vCE
disp('vs = 2 V')
vs = 2;
if vs<Vg
    disp('Cutoff Mode')
    iC = 0;
    vCE = Vcc;
else
    iB = (vs-Vg)/RB;
    iC = B*iB;
    vCE = Vcc-iC*RC;
    if vCE<0
        disp('Saturation Mode')
        vCE = 0;
        iC = Vcc/RC;
    else
        disp('Active Mode')
    end
end
iC
vCE

% Determine the operating mode and find iC and vCE
disp('vs = 5 V')
vs = 5;
if vs<Vg
    disp('Cutoff Mode')
```

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```
    iC = 0;
    vCE = Vcc;
else
    iB = (vs-Vg)/RB;
    iC = B*iB;
    vCE = Vcc-iC*RC;
    if vCE<0
        disp('Saturation Mode')
        vCE = 0;
        iC = Vcc/RC;
    else
        disp('Active Mode')
    end
end
iC
vCE
vs = 2 V
Active Mode
iC =
     3.1200e-003
vCE =
     2.5200e+000
vs = 5 V
Saturation Mode
iC =
     3.7500e-003
vCE =
     0.0000e-003
```

Answer:
For v_S = 2 V, the transistor is in the active mode and we have i_C = 3.12 mA and v_{CE} = 2.52 V.
For v_S = 5 V, the transistor is in saturation mode and we have i_C = 3.75 mA and v_{CE} = 0 V.

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Problem 4-17

The circuit parameters in Figure P4-16 are R_C = 3 kΩ, β = 100, V_γ = 0.7 V, and V_{CC} = 5 V. Select a value of R_B such that the transistor is in the saturation mode when v_S $ 2 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
RC = 3e3;
B = 100;
Vg = 0.7;
Vcc = 5;
vs = 2;

% In saturation mode vCE = 0 and iC = isc = Vcc/RC
vCE = 0;
iC = Vcc/RC;

% Find iB at the edge of the active mode
iB = iC/B;

% Find RB at the edge of the active mode
RB = (vs-Vg)/iB
RB =
    78.0000e+003
```

If R_B decreases, then i_B increases, which pushes the transistor further into the saturation mode.

Answer:

R_B < 78 kΩ.

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Problem 4-18

The parameters of the transistor in Figure P4-18 are β = 50 and V_γ = 0.7 V. Find i_C and v_{CE} for v_S = 0.8 V. Repeat for v_S = 2 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
RB = 10e3;
B = 50;
Vg = 0.7;

% Find a Thevenin equivalent for the output loop
Vcc = 20e3*15/(20e3+10e3)
RC = 1/(1/20e3+1/10e3)

% Determine the operating mode and find iC and vCE
disp('vs = 0.8 V')
vs = 0.8;
if vs<Vg
    disp('Cutoff Mode')
    iC = 0;
    vCE = Vcc;
else
    iB = (vs-Vg)/RB;
    iC = B*iB;
    vCE = Vcc-iC*RC;
    if vCE<0
        disp('Saturation Mode')
        vCE = 0;
        iC = Vcc/RC;
    else
        disp('Active Mode')
    end
end
iC
vCE

% Determine the operating mode and find iC and vCE
disp('vs = 2 V')
vs = 2;
```

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```
if vs<Vg
    disp('Cutoff Mode')
    iC = 0;
    vCE = Vcc;
else
    iB = (vs-Vg)/RB;
    iC = B*iB;
    vCE = Vcc-iC*RC;
    if vCE<0
        disp('Saturation Mode')
        vCE = 0;
        iC = Vcc/RC;
    else
        disp('Active Mode')
    end
end
iC
vCE
Vcc =
    10.0000e+000
RC =
     6.6667e+003
vs = 0.8 V
Active Mode
iC =
   500.0000e-006
vCE =
     6.6667e+000
vs = 2 V
Saturation Mode
iC =
     1.5000e-003
vCE =
     0.0000e-003
```

Answer:
For v_S = 0.8 V, the transistor is in the active mode and we have i_C = 500 μA and v_{CE} = 6.667 V.
For v_S = 2 V, the transistor is in saturation mode and we have i_C = 1.5 mA and v_{CE} = 0 V.

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Problem 4-19

Two OP AMP circuits are shown in Figure P4-19. Both claim to produce a gain of either ± 10.

(a) Show that the claim is true.

(b) A practical source with a series resistor of 1 kΩ is connected to the input of each circuit. Does the original claim still hold? If it does not, explain why?

Solution:

(a) The solution is presented in the following MATLAB code.

```
clear all
format short eng
disp('Part (a)')
disp('Circuit 1')
% Circuit 1 is a non-inverting amplifier
R1 = 9e3;
R2 = 1e3;
K1 = (R1+R2)/R2

% Circuit 2 is an inverting amplifier
R1 = 1e3;
R2 = 10e3;
K2 = -R2/R1
Part (a)
Circuit 1
K1 =
    10.0000e+000
K2 =
   -10.0000e+000
```

(b) The practical source does not change the gain of Circuit 1, but it does change the gain of Circuit 2.

```
% Circuit 2 is an inverting amplifier
R1 = 1e3+1e3;
R2 = 10e3;
K2 = -R2/R1
K2 =
    -5.0000e+000
```

The gain is reduced for Circuit 2 because the source resistor is in series with the OP AMP's input resistor, which effectively increases the value of R_1 used to compute the gain for the inverting amplifier.

Answer:

(a) $K_1 = 10$ and $K_2 = -10$.

(b) $K_1 = 10$ and $K_2 = -5$. The source resistor loads only Circuit 2.

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Problem 4-20

Suppose the output of the practical source shown in Figure P4-19 needs to be amplified by – 100 and you can use only the two circuits shown. How would you connect the circuits to achieve this? Explain why.

Solution:

Connect the practical source to the input of Circuit 1 and then connect the output of Circuit 1 to the input of Circuit 2. The practical source does not change the gain of Circuit 1, so it still provides a gain of 10. The output of Circuit 1 will not change the gain of Circuit 2, since the output resistance of an OP AMP circuit is very small. The overall gain will be (10)(–10) = –100, as requested.

Answer:

Presented above in the Solution.

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Problem 4-21

(a) Find v_O in terms of v_S in Figure P4-21.

(b) Validate your answer by simulating the circuit in OrCAD.

Solution:

(a) Find the Thévenin equivalent of the input circuit and then analyze the circuit as an inverting OP AMP. The solution is presented in the following MATLAB code.

```
clear all
% Find the Thevenin voltage
syms vs vo
vT = 56e3*vs/(22e3+56e3);
RT = 33e3+1/(1/22e3+1/56e3);

% Find the output voltage in terms of vs
Rf = 330e3;
Kv = -Rf/RT;
vo = Kv*vT
vo_num = vpa(vo,4)
vo =
-840/173*vs
vo_num =
-4.855*vs
```

(b) The following OrCAD simulation verifies the results presented in Part (a).

Answer:

(a) $v_O = -4.86\ v_S$.

(b) The OrCAD simulation verifies the results of Part (a).

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Problem 4-22

What is the range of the gain v_O ' v_S in Figure P4-22?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% The circuit is configured as an inverting amplifier
R1 = 10e3;
% Define the feedback resistor to be its maximum and minimum values
R2 = 100e3+[0 100e3];

% Compute the maximum and minimum gains
K = -R2/R1
K =
   -10.0000e+000   -20.0000e+000
```

Answer:

The gain, K, ranges between –10 and –20.

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Problem 4-23

(a) Find v_O in terms v_S in Figure P4-23.

(b) Find i_O for v_S = 1 V. Repeat for v_S = 3 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
% No current flows into the OP AMP, so the voltage at the positive
% input terminal is vs and the circuit is configured as a non-inverting
% amplifier.  The load resistor does not affect the output voltage.
syms vs vo io
R1 = 150e3;
R2 = 10e3;
RL = 10e3;

% Compute the gain and the output voltage
K = (R1+R2)/R2;
vo = K*vs

disp('Part (b)')
% Compute the output voltage
vs = [1 3];
vo = K*vs;
% Adjust for saturation
vcc = 24;
for n = 1:length(vo)
    if vo(n) > vcc
        vo(n)=vcc;
    elseif vo(n)<-vcc
        vo(n)=-vcc;
    end
end
% Compute the output current
io = vo/RL
Part (a)
vo =
16*vs
Part (b)
io =
     1.6000e-003     2.4000e-003
```

Answer:

(a) $v_O = 16v_S$

(b) For v_S = 1 V, i_O = 1.6 mA.

For v_S = 3 V, i_O = 2.4 mA because of saturation.

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Problem 4-24

(a) Find v_O in terms v_S in Figure P4-24.

(b) Find i_O for v_S = 2 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
% The circuit is connected as a non-inverting amplifier.
% Perform voltage division to find the voltage at the positive input terminal.
syms vs vo io vp
vp = 50e3*vs/(10e3+50e3);

% Find the gain
R1 = 50e3;
R2 = 10e3;
K = (R1+R2)/R2;
vo = K*vp

disp('Part (b)')
io = vo/60e3
io_num = subs(io,vs,2)
Part (a)
vo =
5*vs
Part (b)
io =
1/12000*vs
io_num =
   166.6667e-006
```

Answer:

(a) $v_O = 5\ v_S$.

(b) i_O = 166.67 μA.

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Problem 4-25

What is the gain v_O ' v_S in Figure P4-25 when the switch is in position 1? Repeat for positions 2 and 3.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% The circuit is connected as a non-inverting amplifier
R2 = 10e3;
% Assign values for the feedback resistor for all three switch positions
R1 = [90e3 40e3 0];
% Calculate the gains
K = (R1+R2)/R2
K =
    10.0000e+000     5.0000e+000     1.0000e+000
```

Answer:

For switch positions 1, 2, and 3, the voltage gains are 10, 5, and 1, respectively.

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Problem 4-26

(a) Find v_O in terms of the inputs v_1 and v_2 in Figure P4-26.

(b) If v_1 = 1 V, what is the range of values v_2 can have without saturating the OP AMP?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
syms v1 v2 vo
% The circuit is configured as an inverting summer
R1 = 50e3+50e3;
R2 = 50e3+100e3;
Rf = 300e3;
K1 = -Rf/R1;
K2 = -Rf/R2;
vo = K1*v1+K2*v2
Part (a)
vo =
-3*v1-2*v2

disp('Part (b)')
vcc = 15;
% Set v1=1, and solve for v2 in terms of vo
% Evaluate the expression for vo for the saturation limits
Eqn1 = 'vo+3+2*v2';
v2 = simplify(solve(Eqn1,'v2'));
vo_num = [-vcc vcc];
v2_num = subs(v2,'vo',vo_num)
Part (b)
v2_num =
     6.0000e+000    -9.0000e+000
```

Answer:

(a) $v_O = -3v_1 - 2v_2$.

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(b) $-9\text{ V} < v_2 < 6\text{ V}$.

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Problem 4-27

The input-output relationship for a 3-input inverting summer is

$$v_O = \,!\,[v_1 + 10v_2 + 100v_3]$$

The resistance of the feedback resistor is 100 kΩ. Find the values of the input resistors R_1, R_2, and R_3.

Solution:

The gains on the input paths are −1, −10, and −100, and are equal to −1 times the feedback resistor divided by the input resistor. To achieve these gains, the input resistors must be 100 kΩ, 10 kΩ, and 1 kΩ.

Answer:

R_1 = 100 kΩ, R_2 = 10 kΩ, and R_3 = 1 kΩ.

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Problem 4-28

Find v_O in terms of the inputs v_1 and v_2 in Figure P4-28.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% The circuit is connected as a subtractor
syms v1 v2 vo
R1 = 5e3;
R2 = 25e3;
R3 = 5e3;
R4 = 25e3;

% Compute the gains
K1 = -R2/R1;
K2 = (R1+R2)*(R4)/R1/(R3+R4);

% Find the expression for vo
vo = K1*v1+K2*v2
vo =
-5*v1+5*v2
```

Answer:

$v_O = -5v_1 + 5v_2$.

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Problem 4-29

The switch in Figure P4-29 is open, find v_O in terms of the inputs v_{S1} and v_{S2}. Repeat with the switch closed.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Circuit is connected as a modified subtractor
syms vs1 vs2 v1 v2 vo
disp('Switch Open')
R1 = 30e3;
R2 = 60e3;
R3 = 10e3;
R4 = 20e3;
v1 = vs1+vs2;
v2 = vs2;

% Compute the gains
K1 = -R2/R1;
K2 = (R1+R2)*(R4)/R1/(R3+R4);

% Find the expression for vo
vo = K1*v1+K2*v2

disp('Switch Closed')
R4 = 0;

% Compute the gains
K1 = -R2/R1;
K2 = (R1+R2)*(R4)/R1/(R3+R4);

% Find the expression for vo
vo = K1*v1+K2*v2
Switch Open
vo =
-2*vs1
Switch Closed
vo =
-2*vs2-2*vs1
```

Answer:

With the switch open, $v_O = -2v_{S1} + 0v_{S2}$. With the switch closed, $v_O = -2v_{S1} - 2v_{S2}$.

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Problem 4-30

Find v_O in terms of v_{S1} and v_{S2} in Figure P4-30.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% The circuit is connected as a non-inverting amplifier with a modified input
% Determine the voltage at the positive input terminal in terms of vs1 and vs2
% Use node-voltage analysis
syms vs1 vs2 vo vp R1 R2 R3 R4
Eqn1 = '(vp-vs1)/R1 + (vp-vs2)/R2';
vp = simplify(solve(Eqn1,'vp'))

K = (R3+R4)/R3;

vo = simplify(K*vp)
vp =
(vs1*R2+vs2*R1)/(R2+R1)
vo =
(R3+R4)/R3*(vs1*R2+vs2*R1)/(R2+R1)
```

Answer:

$$v_O = \left[\frac{R_3 + R_4}{R_3}\right]\left[\frac{R_2 v_{S1} + R_1 v_{S2}}{R_1 + R_2}\right]$$

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Problem 4-31

It is claimed that $v_O = v_S$ when the switch is closed in Figure P4-31 and that $v_O = !\,v_S$ when the switch is open. Prove or disprove this claim.

Solution:

With the switch closed, the OP AMP's positive terminal is connected to ground and the circuit is connected as an inverting amplifier with a gain of $-R/R = -1$. The input signal is v_S, so we have $v_O = -v_S$. With the switch open, the input signal appears at the positive terminal, because no current flows into the OP AMP. If $v_P = v_S$, then $v_N = v_S$, and the voltage v_S appears on both sides of the top left resistor. Since there is no voltage drop across the resistor, there is no current through it and there is no current flowing into the OP AMP. Therefore, there is no current flowing through the top right resistor and it does not cause a change in voltage. The output voltage must also match, so $v_O = v_S$.

Answer:

The claim is false. With the switch closed, $v_O = -v_S$, and with the switch open, $v_O = v_S$.

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Problem 4-32

The circuit in Figure P4-32 has a diode in its feedback path and is called a "log-amp" because its output is proportional to the natural log of the input.

(a) Show that $v_O = -V_T \, \ell n\left(1 + \frac{v_S}{R_S I_O}\right)$ if the *i-v* characteristics of the diode are $i_D = I_O\left(e^{\frac{v_D}{V_T}} - 1\right)$.

(b) Using MATLAB plot v_O versus v_S for R_S = 10 kΩ, $I_O = 10^{-14}$ A and V_T = 0.025 V. Plot your results on a semilog plot for 10^{-6} V ≤ v_S ≤ 100 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
syms vo VT vs Rs Io iD vD

% Write a node-voltage equation at the negative input terminal of the OP AMP
% Note that the vp = vn = 0, because the positive terminal is grounded
Eqn1 = '(0-vs)/Rs + Io*(exp(-vo/VT)-1)';
vo = simplify(solve(Eqn1,'vo'))
Part (a)
vo =
-log((vs+Io*Rs)/Io/Rs)*VT

clear all
disp('Part (b)')
Rs = 10e3;
Io = 1e-14;
VT = 0.025;
vs = logspace(-6,2,1000);
vo = -VT*log(1 + vs/Io/Rs);

semilogx(vs,vo,'g','LineWidth',3)
grid on
xlabel('Source Voltage (V)')
ylabel('Output Voltage (V)')
Part (b)
```

Answer:

The answers to Parts (a) and (b) are presented above.

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Problem 4-33

(a) Use node-voltage analysis to find the input-output relationship or K of the circuit in Figure P4-33.

(b) (D) Select values for the resistors so that $K = -3$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
% Write two node-voltage equations
% Note that vp = vn = 0, because the positive input terminal is grounded
syms vs vo vA R R1 R2 R3
Eqn1 = '(0-vs)/R + (0-vA)/R1';
Eqn2 = 'vA/R1 + vA/R2 + (vA-vo)/R3';
% Solve the equations
Soln = solve(Eqn1,Eqn2,'vA','vo');
vo = simplify(Soln.vo);
% Compute the gain
K = simplify(vo/vs)
Part (a)
K =
-(R2*R3+R1*R3+R1*R2)/R2/R

disp('Part (b)')
% Based on the results in Part (a), if all of the resistor values are equal,
% then the gain will be -3
R = 10e3;
R1 = R;
R2 = R;
R3 = R;
K_num = subs(K)
Part (b)
K_num =
    -3.0000e+000
```

Answer:

(a) $K = \dfrac{v_O}{v_S} = \dfrac{-(R_1R_2 + R_1R_3 + R_2R_3)}{RR_2}$.

(b) $R = R_1 = R_2 = R_3 = 10\ \text{k}\Omega$, is one possible solution.

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Problem 4-34

Use node voltage analysis in Figure P4-34 to show that $i_O = -v_S/2R$ regardless of the load. That is, show that the circuit is a voltage-controlled current source.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms vs vo io R vp vn
% Write the node-voltage equations
Eqn1 = 'vn - vp';
Eqn2 = '(vn-vs)/2/R + (vn-vo)/R';
Eqn3 = '(vp-vo)/R + vp/2/R + io';
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,'vp','vo','io');
io = simplify(Soln.io)
io =
-1/2*vs/R
```

Answer:

The solution shows that $i_O = -v_S/2R$, regardless of the load, within the operating limits of the OP AMP circuit.

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Problem 4-35

Find v_O in terms of the inputs v_{S1} and v_{S2} in Figure P4-35.

Solution:

Use superposition and the known OP AMP circuits to determine the input-output relationship. The following MATLAB code contains the necessary steps.

```
clear all
syms vs1 vs2 R1 R2 vo
% Turn on vs1 and ground vs2
% The signal from vs1 passes through two inverting amplifiers
K1 = -R2/R1;
K2 = -R1/R2;
Kvs1 = K1*K2;

% Turn on vs2 and ground vs1
% Analyze each path for vs2 separately
% First path is through both OP AMPs
% One is a non-inverting amplifier and the other is an inverting amplifier
K3 = (R1+R2)/R1;
K4 = K2;
% Second part is through one OP AMP as a non-inverting amplifier
K5 = (R2+R1)/R2;
% Combine the gains for vs2
Kvs2 = K3*K4+K5;

% Create the output expression
vo = Kvs1*vs1+Kvs2*vs2
vo =
vs1
```

Answer:

$v_O = v_{S1} + 0v_{S2}$.

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Problem 4-36

(a) Find v_O in terms of the inputs v_{S1} and v_{S2} in Figure P4-36.

(b) (D) Redesign the circuit using only one OP AMP.

(c) Validate your design using OrCAD.

Solution:

(a) The signal v_{S1} experiences an inverting amplifier followed by an inverting summer. The signal v_{S2} experiences just the inverting summer. The following MATLAB code provides the required steps to solve the problem.

```
clear all
disp('Part (a)')
syms vs1 vs2 vo
% Compute the gains
Kvs1 = (-10e3/1e3)*(-10e3/20e3);
Kvs2 = -10e3/1e3;

% Find the input-output relationship
vo = Kvs1*vs1+Kvs2*vs2
Part (a)
vo =
5*vs1-10*vs2
```

(b) Part (a) shows that v_{S1} experiences a positive gain and v_{S2} experiences a negative gain. To achieve both of these gains with a single OP AMP, we must use a subtractor circuit. Using the subtractor circuit in Figure 4-40 as a model, we set $v_1 = v_{S2}$ and $v_2 = v_{S1}$, to have the correct signs for the gains. To get $K_1 = -10$, choose $R_1 = 1$ kΩ and $R_2 = 10$ kΩ. We can solve for R_3 and R_4 as follows:

$$K_2 = 5 = \left[\frac{R_1 + R_2}{R_1}\right]\left[\frac{R_4}{R_3 + R_4}\right] = \left[\frac{1000 + 10000}{1000}\right]\left[\frac{R_4}{R_3 + R_4}\right] = 11\left[\frac{R_4}{R_3 + R_4}\right]$$

$$\frac{5}{11} = \left[\frac{R_4}{R_3 + R_4}\right]$$

$$R_4 = 5\ \text{k}\Omega$$

$$R_3 = 6\ \text{k}\Omega$$

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(c) Simulate the circuit designed in Part (b) in OrCAD and verify the gain terms. The following two simulations show the results for two inputs sets: (1) v_{S1} = 1 V and v_{S2} = 0 V and (2) v_{S1} = 0 V and v_{S2} = 1 V. Since the circuit is linear, the two results are sufficient to demonstrate it has the correct gain response.

Answer:

(a) $v_O = 5v_{S1} - 10v_{S2}$.

(b) and (c) Convert the circuit to a subtractor OP AMP circuit as shown above and validated in OrCAD.

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Problem 4-37

On an exam, students were asked to design an efficient solution for the following relationship: $v_2 = 3v_1 + 15$. Two of the designs are shown in Figure P4-37. Which, if any, of the designs are correct and what grade would you award each student?

(a)

(b)

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms v1 v2a v2b
disp('Circuit (a)')
% Analyze the gain
experienced by each
input signal
Kv1 = (-10e3/10e3)*(-
20e3/10e3)*(10e3+20e3)/2
0e3;
K5 = (-10e3/10e3)*(-
20e3/10e3)*(10e3+20e3)/20e3;
v2a = Kv1*v1+K5*5

disp('Circuit (b)')
% This is not a standard configuration, so it requires node-voltage analysis
syms vn vp
Eqn1 = '(vp-v1)/10e3 + (vp-5)/10e3';
Eqn2 = vn/10e3 + (vn-v2b)/50e3';
Eqn3 = 'vn-vp';
Soln = solve(Eqn1,Eqn2,Eqn3,'vp','vn','v2b');
v2b = simplify(Soln.v2b)
Circuit (a)
v2a =
3*v1+15
Circuit (b)
v2b =
3.*v1+15.
```

Answer:

Both designs are correct. Design (a) uses standard OP AMP blocks, but requires three OP AMPs and seven resistors. Design (b) is more efficient than Design (a), but does not use a standard approach, so it requires more effort to analyze.

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Problem 4-38

Find the output v_2 in terms of the input v_1 in Figure P4-38.

Solution:

The circuit is a combination of a non-inverting amplifier and an inverting summer with three inputs. The input to the non-inverting amplifier is v_1 and it has a gain of 2, so its output is $2v_1$. The three inputs to the inverting summer are $v_1 + 2$ V, v_1, and the output of the non-inverting amplifier, $2v_1$. The gains associated with the inverting summer are –1, –1, and –2, respectively. The following MATLAB code completes the calculations.

```
clear all
format short eng
syms v1 vo1 v2
% Non-inverting amplifier
K = (10e3+10e3)/10e3;
vo1 = K*v1;

% Inverting summer gains
R1 = 10e3;
R2 = 10e3;
R3 = 5e3;
Rf = 10e3;
K1 = -Rf/R1;
K2 = -Rf/R2;
K3 = -Rf/R3;

% Compute the output, v2
v2 = K1*(v1+2)+K2*v1+K3*vo1
v2 =
-6*v1-2
```

Answer:

$v_2 = -6v_1 - 2$ V.

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Problem 4-39

Find the output v_O in terms of the input v_S in Figure P4-39.

Solution:

The circuit is a combination of an inverting summer followed by a non-inverting amplifier. The two inputs to the inverting summer are v_S and v_O. The input to the non-inverting amplifier is the output of the inverting summer. The following MATLAB code develops the solution

```
clear all
syms vs vo
R1 = 100e3;
R2 = 50e3;
Rf = 100e3;
R3 = 200e3;
R4 = 100e3;

K1 = -Rf/R1;
K2 = -Rf/R2;
K3 = (R3+R4)/R4;

Eqn1 = 'vo - K3*(K1*vo+K2*vs)';
vo = solve(Eqn1,'vo');
vo = subs(vo)
vo =
-3/2*vs
```

Answer:

$v_O = -1.5v_S$

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Problem 4-40

Find the output v_O in terms of the input v_1 in Figure P4-40. Verify your answer using OrCAD.

Solution:

The circuit involves an inverting amplifier, an inverting summer, and a non-inverting amplifier. The following MATLAB code develops the solution.

```
clear all
syms vo v1
K1 = -2;
K2 = -1;
K3 = -1;
K4 = 3;

Eqn1 = 'vo-K4*(K2*K1*v1 + K3*vo)';
vo = solve(Eqn1,'vo');
vo = subs(vo)
vo =
3/2*v1
```

The following OrCAD simulation verifies the result.

Answer:

$v_O = 1.5v_1$.

The OrCAD simulation verifies the result.

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Problem 4-41

Design an OP AMP amplifier with a voltage gain of !10 and an input resistance greater than 10 kΩ using standard 5% resistance values less than 300 kΩ.

Solution:

The following schematic presents one valid solution. The circuit is an inverting amplifier and the input resistance is 15 kΩ. Other solutions are possible.

Answer:

Presented above.

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Problem 4-42
Design an OP AMP amplifier with a voltage gain of 4 using only 10-kΩ resistors and one OP AMP.

Solution:
With a single OP AMP and a positive gain, the circuit must be connected as a non-inverting summer. The following circuit will meet the requirements.

Answer:
Presented above.

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Problem 4-43

Using a single OP AMP, design a circuit with inputs v_1 and v_2 and an output $v_O = 2v_2 - 3v_1$. The input resistance seen by each input should be greater than 5 kΩ.

Solution:

The circuit has two inputs and two distinct gains with different signs. With only one OP AMP, the only design option is a subtractor. The following MATLAB code calculates the required resistor values after selecting two of the resistors values.

```
clear all
format short eng
K1 = -3;
K2 = 2;
% Select R1
R1 = 5e3
% Calculated R2
R2 = -R1*K1
% Select R4
R4 = 5e3;
% Solve for R3
Eqn1 = 'K2 - (R1+R2)*R4/R1/(R3+R4)';
R3 = solve(Eqn1,'R3');
R3 = subs(R3)
R4
R1 =
     5.0000e+003
R2 =
    15.0000e+003
R3 =
     5.0000e+003
R4 =
     5.0000e+003
```

The following circuit meets the specifications:

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Answer:
Presented above.

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Problem 4-44

Design a differential amplifier with inputs v_1 and v_2 and an output $v_O = 10(v_2 \ ! \ v_1)$ using only one OP AMP. All resistances must be between 5 kΩ and 200 kΩ.

Solution:

The circuit has two inputs and two distinct gains with different signs. With only one OP AMP, the only design option is a subtractor. The following MATLAB code calculates the required resistor values after selecting two of the resistors values.

```
clear all
format short eng
K1 = -10;
K2 = 10;
% Select R1
R1 = 10e3
% Calculated R2
R2 = -R1*K1
% Select R4
R4 = 100e3;
% Solve for R3
Eqn1 = 'K2 - (R1+R2)*R4/R1/(R3+R4)';
R3 = solve(Eqn1,'R3');
R3 = subs(R3)
R4
R1 =
    10.0000e+003
R2 =
   100.0000e+003
R3 =
    10.0000e+003
R4 =
   100.0000e+003
```

The following circuit meets the specifications:

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Answer:
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Problem 4-45

Using no more than two OP AMPs, design an OP AMP circuit with inputs v_1, v_2, and v_3 and an output $v_O = -3v_1 + 2v_2 \ ! \ 5v_3$.

Solution:

There are three signals summed together. One signal has a positive gain and the other two have negative gains. To achieve this combination, send signal v_2 through an inverting amplifier with a gain of –1 and then to an inverting summer with a gain of –2 along that input path. Send the other two signals directly to the inverting summer with gains of –3 and –5, respectively. The following circuit meets the specifications.

Answer:

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Problem 4-46

Design the interface circuit in Figure P4-46 so that 10 mW is delivered to the 100-Ω load. Repeat for a 100-kΩ load. Verify your designs using OrCAD.

Solution:

Our approach is to use MATLAB to compute the required output voltages for each resistor and then design OP AMP circuits that meet the specifications.

```
clear all
% Compute the required output voltages for each load
% p = v^2/R
R = [100 100e3];
p = 10e-3;
v = sqrt(p*R)
v =
     1.0000e+000    31.6228e+000
```

We need output voltages of 1 V and 31.6228 V. Since the input voltage is 2 V, we can use a voltage divider and a buffer to produce 1 V at the output. For an output voltage of 31.6628, we can use a non-inverting amplifier with a gain of 15.8114. The required designs are shown below. Other correct designs are possible.

Note: For the second circuit, we have to adjust the maximum and minimum output values for the OP AMP voltages to be ±45 V.

Answer:

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Problem 4-47

Design the interface circuit in Figure P4-47 so that the output is $v_2 = 150v_1 + 1.5$ V.

Solution:

Two signals are combined together, both with positive gains. We need to use an inverting summer to combine the signals and then an inverting amplifier to get the correct gains. The input signal v_1 has a gain of 150 and the input voltage of 1.5 V has a gain of 1. The following design meets the specifications.

Answer:

Presented above.

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Problem 4-48

(a) Design a circuit that can produce $v_O = 1000 v_X + 5\text{ V}$ using two OP AMPs.

(b) Repeat using only one OP AMP.

Solution:

The required circuits are shown below.

(a)

(b)

Answer:

Presented above.

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Problem 4-49

A requirement exists for an OP AMP circuit with the input-output relationship

$$v_O = 5v_{S1} - 2v_{S2}$$

Two proposed designs are shown in Figure P4-49. As the project engineer, you must recommend one of these circuits for production. Which of these circuits would you recommend for production and why? *Hint:* First, verify that the circuits perform the required function.

Solution:

Use MATLAB to calculate the gains for each input signal for each circuit.

```
clear all
syms vs1 vs2 vo
disp('Circuit 1')
% Use superposition to analyze the circuit
K1 = (2e3+3e3)/3e3*(2e3+1e3)/1e3;
K2 = -2e3/1e3;
vo = K1*vs1+K2*vs2

disp('Circuit 2')
K1 = (1e3+5e3)/1e3*(1e3)/(1e3+2e3);
K2 = -5e3/1e3;
vo = K1*vs1+K2*vs2
Circuit 1
vo =
5*vs1-2*vs2
Circuit 2
vo =
2*vs1-5*vs2
```

Answer:

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Circuit 1 meets the specifications. Circuit 2 does not meet the specification, but it would if the input signals were swapped. We recommend Circuit 1 for production.

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Problem 4-50

A requirement exists for an OP AMP circuit to deliver 12 V to a 1-kΩ load using a 4-V source as an input voltage. Two proposed designs are shown in Figure P4-50. Some characteristics of the OP AMP that must be used in the design are

Characteristic	Min	Typical	Max	Units
Open-loop Gain	10^5	2×10^5	-	V/mV
Input Resistance	10^{10}	10^{11}	-	Ω
Output Voltage	–12	-	+15	V
Output Current	-	-	25	mA

Which of these circuits would you recommend for production and why? Hint: First, verify that the circuits perform the required function.

Solution:

The OP AMPs have open-loop gains and input resistances that make the ideal OP AMP assumptions reasonable. The output voltage range includes 12 V, so both approaches are acceptable with respect to the desired output voltage. Circuit 1 uses suspiciously low resistor values, which may increase the required output current beyond the OP AMPs operating range. The following MATLAB code calculates the output current for each design.

```
clear all
disp('Circuit 1')
io1 = 12/(360+180) + 12/1e3
disp('Circuit 2')
io2 = 12/(2e3+1e3) + 12/1e3
Circuit 1
io1 =
    34.2222e-003
Circuit 2
io2 =
    16.0000e-003
```

The output current for Circuit 1 is greater than the maximum value that the OP AMP can produce, so Circuit 1 does not meet specifications. Circuit 2 meets all of the specifications.

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Answer:
Circuit 2.

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Problem 4-51

A particular application requires that an instrumentation interface deliver $v_O = 200v_{TR} - 5\text{ V}\ \pm 2$ % to a DAC. The solution currently in use requires two OP AMPs and is constantly draining the supply batteries. A young engineer designed another tentative solution using just one OP AMP shown in Figure P4-51. As her supervisor, you must determine if it works. Does her design meet the specifications?

Solution:

The circuit is a modified subtractor. To easily analyze the circuit, find the Thévenin equivalent for the input portion of the circuit starting with the 5-V source and then use the standard gain relationships for a subtractor. The following MATLAB code presents the results.

```
clear all
format short eng
% Find the Thevenin equivalent of the constant voltage input
vT = 100*5/(100+20e3)
RT = 3.3e3+1/(1/20e3+1/100)

% Find the subtractor gains
R1 = RT;
R2 = 680e3;
R3 = 3.3e3;
R4 = 680e3;

% Compute the output signal
syms vTR vo
K1 = -R2/R1;
K2 = (R1+R2)*R4/R1/(R3+R4);
v2 = K1*vT + K2*vTR;
v2_num = vpa(v2,5)
vT =
    24.8756e-003
RT =
     3.3995e+003
v2_num =
-4.9759+200.06*vTR
```

Answer:

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The design meets the specifications.

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Problem 4-52

The analog output of a five-bit DAC is 3.90 V when the input code is (1, 1, 0, 0, 1). What is the full-scale output of the DAC? How much does the analog output change when the input LSB changes?

Solution:

Use the formula for the output of a digital-to-analog converter to determine KV_{REF}. We can then determine the full-scale output and the output change when the input LSB changes.

$$v_{\text{O}} = KV_{\text{REF}}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8} + \frac{b_5}{16}\right) = KV_{\text{REF}}\left(1 + \frac{1}{2} + \frac{0}{4} + \frac{0}{8} + \frac{1}{16}\right) = 3.90$$

$$KV_{\text{REF}} = \frac{3.90}{1.5625} = 2.496 \text{ V}$$

$$v_{\text{O, MAX}} = KV_{\text{REF}}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) = (2.496)(1.9375) = 4.836 \text{ V}$$

$$v_{\text{O, MIN}} = KV_{\text{REF}}\left(0 + \frac{0}{2} + \frac{0}{4} + \frac{0}{8} + \frac{1}{16}\right) = (2.496)(0.0625) = 0.156 \text{ V}$$

The following MATLAB code confirms these calculations.

```
clear all
format short eng
vo = 3.9;
KVref = vo/(1+1/2+0/4+0/8+1/16)
vo_max = KVref*(1+1/2+1/4+1/8+1/16)
vo_min = KVref*(1/16)
KVref =
     2.4960e+000
vo_max =
     4.8360e+000
vo_min =
   156.0000e-003
```

Answer:

The full-scale output voltage is $v_{\text{O,MAX}}$ = 4.836 V.

When the LSB changes, the output change is $v_{\text{O,MIN}}$ = 0.156 V.

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Problem 4-53

The full-scale output of a six-bit DAC is 10.5 V. What is the analog output when the input code is (0, 1, 0, 1, 0, 1)? How much does the analog output change when the input LSB changes?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
vo_max = 10.5;
KVref = vo_max/(1 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32)
vo = KVref*(0 + 1/2 + 0/4 + 1/8 + 0/16 + 1/32)
vo_min = KVref*(1/32)
KVref =
     5.3333e+000
vo =
     3.5000e+000
vo_min =
   166.6667e-003
```

Answer:

For input code (0, 1, 0, 1, 0, 1), v_O = 3.5 V.

When the LSB changes, the output change is $v_{O,MIN}$ = 167 mV.

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Problem 4-54

An *R*-2*R* DAC is shown in Figure P4-54. The digital voltages v_1, v_2, etc. can be either 5 V for a logic 1 or 0 V for a logic 0. What is the DAC's output when the logic input is (1, 0, 0 1)?

Solution:

Use node-voltage analysis to find an expression for v_O in terms of the input voltages. The following MATLAB code provides the solution.

```
clear all
syms v1 v2 v3 v4 va vb vc vo R
Eqn1 = '(0-v1)/2/R + (0-va)/R + (0-vo)/2/R';
Eqn2 = '(va-0)/R + (va-v2)/2/R + (va-vb)/R';
Eqn3 = '(vb-va)/R + (vb-v3)/2/R + (vb-vc)/R';
Eqn4 = '(vc-vb)/R + (vc-v4)/2/R + (vc-0)/2/R';
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,'vo','va','vb','vc');
vo = simplify(Soln.vo)
vo_num = subs(vo,{v1,v2,v3,v4},{5,0,0,5})
vo =
-v1-1/4*v3-1/2*v2-1/8*v4
vo_num =
    -5.6250e+000
```

Answer:

For input code (1, 0, 0, 1), $v_O = -5.625$ V.

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Problem 4-55

A Chromel-Constantan thermocouple (curve E) has the characteristics shown in Figure P4-55. Design an interface that will produce a 0- to 5-V output where 0 V refers to 0º C and 5 V refers to 1000º C. The transducer can be modeled as a voltage source in series with a 15 Ω resistor.

Solution:

Using MATLAB, determine the gain and bias required to complete the interface. We can find the gain and bias by solving a pair of linear equations relating the input and output specifications.

```
clear all
syms K vBias
% 0 degrees:  0 V input maps to a 0 V
output
% 1000 degrees:  76 mV input maps to a 5 V output
Eqn1 = '0 - (K*0 + vBias)';
Eqn2 = '5 - (K*76e-3 + vBias)';
Soln = solve(Eqn1,Eqn2,'K','vBias')
K = double(Soln.K)
vBias = double(Soln.vBias)
Soln =
        K: [1x1 sym]
    vBias: [1x1 sym]
K =
    65.7895e+000
vBias =
     0.0000e-003
```

The gain is K = 65.7895 and there is no bias voltage. The following non-inverting amplifier circuit will meet the specifications for the interface.

Answer:

Presented on left.

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Problem 4-56

A small pressure transducer has the characteristics shown in Figure P4-56. Design an interface that will operate between 7- and 32-psi. An input of 7 psi should produce –5 V and 32 psi should produce 5 V. The transducer is modeled as a voltage source in series with a 500 Ω resistor that can vary ± 75 Ω depending on the pressure. The OP AMPs you must use have a maximum closed-loop gain of 2000. Your available bias source is 5 V.

Solution:

Using MATLAB, determine the gain and bias required to complete the interface. We can find the gain and bias by solving a pair of linear equations relating the input and output specifications.

```
clear all
syms K vBias
% 7 psi:  1000 microV input maps to a -5 V output
% 32 psi:  400 microV input maps to a 5 V output
Eqn1 = '-5 - (K*1000e-6 + vBias)';
Eqn2 = '5 - (K*400e-6 + vBias)';
Soln = solve(Eqn1,Eqn2,'K','vBias')
K = double(Soln.K)
vBias = double(Soln.vBias)
Soln =
        K: [1x1 sym]
    vBias: [1x1 sym]
K =
   -16.6667e+003
vBias =
    11.6667e+000
```

The gain is $K = -16667$ and the bias voltage is 11.667 V. The following circuit will meet the specifications for the interface. Note that the bias source is inverted so that the output bias has the correct sign.

Answer:

Presented on left.

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Problem 4-57

A medical grade pressure transducer has been developed for use in invasive blood pressure monitoring. The output voltage of the transducer is $v_{TR} = (0.05P\ !\ 0.75)$ mV, where P is pressure in mmHg. The output resistance of the transducer is 300 Ω. The blood pressure measurement is to be an input to an existing multisensor monitoring system. This system treats a 1-V input as a blood pressure of 20 mmHg and a 10-V input as 200 mmHg. Design an OP AMP circuit to interface the new pressure transducer with the existing monitoring system.

Solution:

Using MATLAB, determine the gain and bias required to complete the interface. We can find the gain and bias by solving a pair of linear equations relating the input and output specifications.

```
clear all
syms K vBias
% Find the transducer voltages related to 20 mmHg and 200 mmHg
vTR1 = (0.05*20-0.75)*1e-3;
vTR2 = (0.05*200-0.75)*1e-3;
% 20 mmHg:  vTR1 input maps to a 1 V output
% 32 mmHg:  vTR2 input maps to a 10 V output
Eqn1 = '1 - (K*vTR1 + vBias)';
Eqn2 = '10 - (K*vTR2 + vBias)';
Soln = solve(Eqn1,Eqn2,'K','vBias');
K = double(subs(Soln.K))
vBias = double(subs(Soln.vBias))
K =
     1.0000e+003
vBias =
   750.0000e-003
```

The gain is $K = 1000$ and the bias voltage is 750 mV. The following circuit will meet the specifications for the interface. The first stage acts as a buffer. The second stage provides gain and combines the transducer signal with the bias. The third stage corrects the sign of the output.

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Answer:
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Problem 4-58

The acid/alkaline balance of a fluid is measured by the pH scale. The scale runs from 0 (extremely acid) to 14 (extremely alkaline,) with pH 7 being neutral. A pH electrode is a sensor that produces a small voltage that is directly proportional to the pH of the fluid in a test chamber. For a certain pH electrode, the proportionality factor is 60 mV/pH. A preamplifier is needed to interface this sensor with a variety of laboratory instruments. The output of the preamp must be 1 V when the sensor is immersed in a test solution with pH = 4 and 1.75 V when it is immersed in a solution with pH = 7. Design an amplifier to meet these requirements.

Solution:

Using MATLAB, determine the gain and bias required to complete the interface. We can find the gain and bias by solving a pair of linear equations relating the input and output specifications.

```
clear all
syms K vBias
% 4 pH:  240 mV input maps to a 1 V output
% 7 pH:  420 mV input maps to a 1.75 V output
Eqn1 = '1 - (K*240e-3 + vBias)';
Eqn2 = '1.75 - (K*420e-3 + vBias)';
Soln = solve(Eqn1,Eqn2,'K','vBias');
K = double(Soln.K)
vBias = double(Soln.vBias)
K =
     4.1667e+000
vBias =
     0.0000e-003
```

The gain is K = 4.1667 and there is no bias voltage. The following circuit will meet the specifications for the interface.

Answer:

Presented above.

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Problem 4-59

A photoresistor varies from 100 Ω in bright sunlight to 200 kΩ in total darkness. Design a suitable circuit using the photoresistor so that total darkness produces 5 V, while bright sunlight produces –5 V, regardless of the load. You have a 5-V source and a ± 15-V source to power any OP AMP you may need.

Solution:

Solve this problem in two stages. First create a circuit that uses the photoresistor to create known voltages for the bright and dark conditions. Then create an interface circuit to translate the known voltages into the required voltages. The following circuit uses a voltage divider to produce known voltages with the photoresistor.

We can use the following MATLAB code to calculate the voltages for the bright and dark conditions.

```
clear all
R_bright = 100;
R_dark = 200e3;
R = 200e3;
vs = 5;
v_bright = vs*R_bright/(R+R_bright)
v_dark = vs*R_dark/(R+R_dark)
v_bright =
     2.4988e-003
v_dark =
     2.5000e+000
```

Now calculate the gain and bias terms.

```
syms K vBias
% Bright:  2.4988 mV input maps to a -5 V output
% Dark:  2.5 V input maps to a 5 V output
Eqn1 = '-5 - (K*2.4988e-3 + vBias)';
Eqn2 = '5 - (K*2.5 + vBias)';
Soln = solve(Eqn1,Eqn2,'K','vBias');
K = double(Soln.K)
vBias = double(Soln.vBias)
K =
     4.0040e+000
vBias =
    -5.0100e+000
```

With minor rounding, the gain is $K = 4$ and the bias voltage is –5 V. The following circuit will meet the specifications for the interface. The first stage acts as a buffer. The second stage provides gain. The third stage combines the transducer signal with the bias and corrects the sign of the gain for the transducer signal.

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Answer:
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Problem 4-60

Your engineering firm needs an instrumentation amplifier that provides the following input-output relationship: $v_O = 10^6 v_{TR} - 3.5\text{ V}$. The transducer is modeled as a voltage source in series with a resistor that varies with the transducer voltage from 40 Ω to 750 Ω. A vendor is offering the amplifier shown in Figure P4-60 and the vendor agrees to make a single change to the amplifier, if needed, for no cost. Would you recommend buying it? Explain your rationale.

Solution:

As shown, the circuit has a gain of $K = (-1000)(-1000) = 10^6$, and a bias of $B = (5\text{ V})(-1\text{ M}/1.42\text{ M}) = -3.52\text{ V}$, which meet the specifications with only minor errors. The circuit appears to be a reasonable design, but when the transducer's output resistance increases above 100 Ω, it will load the first OP AMP stage and decrease the gain of the circuit. To mitigate the effect of the transducer's output resistance, we need to place a buffer at the input of the interface circuit.

Answer:

Purchase the circuit if the vendor will add a buffer at the circuit input without increasing the cost.

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Problem 4-61

The OP AMP in Figure P4-61 operates as a comparator. Find the output voltage when v_S = 2 V. Repeat for v_S = !2 V and v_S = 6 V.

Solution:

The circuit is connected such that the voltage at the negative input terminal is always 5 V and the voltage at the positive input terminal is v_S. In this case, the ideal OP AMP assumptions do not apply and we must use the fundamental OP AMP equation, $v_O = A(v_P - v_N)$, and our knowledge of OP AMP saturation to determine the output voltages. For this circuit, $+v_{CC}$ = 15 V and $-v_{CC}$ = 0 V. We still assume the OP AMP's open-loop gain, A, is very large. When $v_P > v_N$, the OP AMP will try to produce a positive output voltage with a large magnitude and will saturate at its maximum output value, 15 V. When $v_P < v_N$, the OP AMP will try to produce a negative output voltage with a large magnitude and will saturate at its minimum output value, 0 V.

Answer:

The following table summarizes the results described above.

v_S (V)	v_O (V)
2	0
–2	0
6	15

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Problem 4-62

The circuit in Figure P4-62 has V_{CC} = 15 V and V_N = −2 V. Sketch the output voltage v_O on the range 0 # t # 2 s for $v_S(t)$ = 10 sin(2πt) V.

Solution:

The circuit is connected as a comparator. When the input signal is above –2 V, it will produce its maximum output voltage. When the input signal is below –2 V, it will produce its minimum output voltage. The following MATLAB code simulates the response.

```
clear all
t = 0:0.001:2;
vs = 10*sin(2*pi*t);
vo = zeros(size(vs));
v_max = 15;
v_min = 0;
v_compare = -2;
% Compare the signal to the threshold
vo_max = vs>v_compare;
vo_min = vs<=v_compare;
% Construct the output signal
vo = v_max*vo_max+v_min*vo_min;
% Plot the results
plot(t,vs,'b','LineWidth',3)
hold on
grid on
plot(t,vo,'g','LineWidth',3)
xlabel('Time (s)')
ylabel('v_O(t), (V)')
axis([0 2 -15 20])
```

The following OrCAD simulation verifies the MATLAB results.

Answer:

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Problem 4-63

A signal varies as $v_S(t)=10(1-e^{-100t})$ V. Design a comparator circuit that will switch from 0 to 5 V in 2 ms. Use a single OP AMP that has $+V_{CC}$= + 5V and – V_{CC} = 0 V.

Solution:

Determine the voltage of $v_S(t)$, when t = 2 ms, and then design a comparator circuit to switch as specified above.

```
clear all
vs2ms = 10*(1-exp(-100*2e-3))
vs2ms =
     1.8127e+000
```

We want the comparator circuit to produce 0 V when $v_S(t)$ < 1.8127 V, and to produce 5 V when $v_S(t)$ > 1.8127 V. Using Figure P4-61 as a model, the following design provides one possible solution.

The following output plot shows that the circuit meets the specifications.

Answer:

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Problem 4-64

A 5-bit flash ADC in Figure P4-64 uses a reference voltage of 6 V. Find the output code for the analog inputs v_S = 3.5 V, 2.3 V, and 5.3 V. If the reference voltage is changed to 8 V, which of these codes would change?

Solution:

For each OP AMP, if v_S is greater than v_{Ni}, then the output is a binary 1. Otherwise, the output is a binary 0. Using voltage division, with V_{REF} = 6 V, each resistor drops one volt, so v_{N1} = 5 V, v_{N2} = 4 V, v_{N3} = 3 V, v_{N4} = 2 V, and v_{N5} = 1 V. If the reference voltage is increased to 8 V, each resistor drops 1.333 V, so we have v_{N1} = 6.667 V, v_{N2} = 5.333 V, v_{N3} = 4 V, v_{N4} = 2.667 V, and v_{N5} = 1.333 V. The following table summarizes the digital output signals, (b_1, b_2, b_3, b_4, b_5), for each of the analog input signals.

v_S (V)	V_{REF} = 6 V	V_{REF} = 8 V
3.5	(0, 0, 1, 1, 1)	(0, 0, 0, 1, 1)
2.3	(0, 0, 0, 1, 1)	(0, 0, 0, 0, 1)
5.3	(1, 1, 1, 1, 1)	(0, 0, 1, 1, 1)

Answer:

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Problem 4-65

Bipolar Power Supply Voltages

The circuit in Figure P4-65 produces bipolar power supply voltages $V_{POS} > 0$ and $V_{NEG} < 0$ from a floating unipolar voltage source $V_{REF} > 0$. Note that the OP AMP output is grounded and that its $+V_{CC}$ and $!\,V_{CC}$ terminals are connected to V_{POS} and V_{NEG}, respectively.

(a) Show that $V_{POS} = +V_{REF}\,'\,2$ and $V_{NEG} = !\,V_{REF}\,'\,2$ even if the load resistors R_{POS} and R_{NEG} are not equal.

(b) If R_{POS} and R_{NEG} are not equal a current i_G must flow into or out of ground. How does the ungrounded voltage source V_{REF} supply this ground current?

(c) In effect, the OP AMP creates a "virtual ground" at point A ($V_A = 0$) but draws no current in doing so. Why not just connect point A to a "real ground" and do away with the OP AMP?

Solution:

(a) The OP AMP output is connected to ground and the negative input terminal, so $v_N = 0$ V. If the OP AMP is operating in its linear range, then $v_N = v_P = 0$ V, and the node between the two R resistors acts as a ground. The voltage V_{REF} appears across the two R resistors. Under the ideal OP AMP assumptions, no current enters the OP AMP through the positive terminal, so the current through the two R resistors must be $i = V_{REF}/2R$. The voltage drop across each R resistor is therefore $V_{REF}/2$. If the OP AMP causes the center node to have a voltage of 0 V, then the top node must be $V_{POS} = V_{REF}/2$ and the bottom node must be $V_{NEG} = -V_{REF}/2$. The load resistors have no influence on these voltages.

(b) The OP AMP is grounded and it will produce the required currents to satisfy the voltage conditions of the external circuit, as long as the OP AMP is operating in its linear region.

(c) Without the OP AMP, the load resistors R_{POS} and R_{NEG} would interact with the other resistors in the circuit and change the results. The OP AMP serves as a buffer to isolate the load and the reference voltage while still transferring the reference voltage to the output stage in a controlled manner.

Answer:

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Problem 4-66

Thermometer Design Problem

There is a need to design a thermometer that can read from 350º C to 1100º C to monitor the temperature of a rocket's nose cone. The output will feed a 0- to 5-V ADC prior to transmission of the temperature data to the ground. The 350º C will deliver 0 V to the ADC, while 1100º C will deliver 5 V. Select from Figure P4-55 an appropriate thermocouple and design the instrumentation amplifier. Other criteria are: the transducers have a 15-Ω series resistance, the fewer OP AMPs the better, since power is limited, the maximum OP AMP gain is 1000, and the bias voltage, if needed, must be from the available 5-V supply.

Solution:

The temperature range in Figure P4-55 does not extend to 1100° C, but we will assume the thermocouple is linear beyond 1000° C. Thermocouple K is a reasonable choice, since it is linear across the required temperature range. At 350° C, its output voltage is approximately 14 mV. At 1000° C, its output voltage is approximately 42 mV. If we extend the line to 1100° C, the output should be approximately 46 mV, as shown below.

```
clear all
format short eng
% Compute the expected thermocouple voltage at 1100 degrees
% assuming linear operation.
T1 = 350;
T2 = 1000;
T3 = 1100;
V1 = 14e-3;
V2 = 42e-3;
V_per_T = (V2-V1)/(T2-T1);
V3 = V2 + (T3-T2)*V_per_T
V3 =
    46.3077e-003
```

Use MATLAB to calculate the required gain and bias for this interface.

```
syms K vBias
% 350 deg:  14 mV input maps to a 0 V output
% 1100 deg:  46 mV input maps to a 5 V output
Eqn1 = '0 - (K*14e-3 + vBias)';
Eqn2 = '5 - (K*46e-3 + vBias)';
Soln = solve(Eqn1,Eqn2,'K','vBias');
K = double(Soln.K)
vBias = double(Soln.vBias)
K =
   156.2500e+000
vBias =
    -2.1875e+000
```

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The following circuit meets the interface specifications.

The first stage of the circuit is an inverting amplifier with a high input impedance and a gain of –5. The second stage is an inverting summer with gains of –31.25 and –0.4375 to produce the correct final gain and bias for the interface. Other correct designs are possible.

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Problem 4-67

High Bias Design Problem

A particular pressure sensor is designed to operate under constant pressure. The task is to detect a pressure increase and sound an alarm. The sensor produces 1 mV at 100 psi, its usual operating pressure, and increases by 1 μV per psi. The design must sound an alarm if the pressure reaches 150 psi. The transducer is modeled as a voltage source with a series resistor that varies with pressure from 50 Ω at 100 psi to 150 Ω at 150 psi. Two types of OP AMPs are available. Type #1 are single-sided, meaning they have a $+V_{CC}$ of 5 V and a $-V_{CC}$ of 0 V. Type #2 OP AMPs are double-sided with a V_{CC} of ±15 V and have a maximum closed-loop gain of 5000. The alarm can be driven with the + 5 V available from a saturated type #1 OP AMP. Only ± 15-V and + 5-V sources are available. Design the alarm circuit.

Solution:

Connect the transducer model in series with a 5-V source and a 150-Ω resistor to create a voltage divider with the output across the 150-Ω resistor. As the pressure increases, the transducer resistor increases in value and the output voltage decreases. When the output voltage reaches 2.5 V or lower, we want the alarm to sound. Connect the output of the voltage divider to the negative input of a comparator circuit and connect a 2.5-V reference signal to the positive input. Use a Type #1 OP AMP so that it produces the correct signal for the alarm. The following circuit will meet the specifications.

Answer:

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Problem 4-68

Current Switching DAC

The circuit in Figure P4-68 is a 4-bit digital-to-analog converter (DAC). The DAC output is the voltage v_O and the input is the binary code represented by bits b_1, b_2, b_3, and b_4. The input bits are either 0 (low) or 1 (high), and each controls one of the four switches in the figure. When bits are low, their switches are in the left position, directing the $2R$ leg currents to ground. When bits are high, their switches move to the right position, directing the $2R$ leg currents to the OP AMP's inverting input. The $2R$ leg currents do not change when switching from left to right because the inverting input is a virtual ground ($v_N = v_P = 0$). The purpose of this problem is to show that this constant-current switching produces the following input-output relationship.

$$v_O = -\frac{R_F}{2R} V_{REF}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8}\right) \quad \text{(a)}$$

Since the inverting input is a virtual ground, show that the currents in the $2R$ legs are
$i_1 = V_{REF}/2R$, $i_2 = V_{REF}/4R$, $i_3 = V_{REF}/8R$, and $i_4 = V_{REF}/16R$,
regardless of switch positions.

(b) Show that the sum of currents at the inverting input is $b_1 i_1 + b_2 i_2 + b_3 i_3 + b_4 i_4 + i_F = 0$ where bits b_k (k =1, 2, 3, 4) are either 0 or 1.

(c) Use the results in parts (a) and (b) to show that the OP AMP output voltage is

$$v_O = -\frac{R_F}{2R} V_{REF}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8}\right) \quad \text{as stated.}$$

Solution:

(a) The equivalent resistance seen by the reference voltage source is R, so the current flowing out of the source is $i = V_{REF}/R$. The equivalent resistance to the right of node A, is $2R$, so the current splits equally at node A and $i_1 = i/2 = V_{REF}/2R$. The equal current divisions repeat three more times in the same manner to yield $i_2 = V_{REF}/4R$, $i_3 = V_{REF}/8R$, and $i_4 = V_{REF}/16R$.

(b) If a bit is set to 1, the corresponding switch allows the current to flow into the inverting input node. The feedback current i_F always flows into the node, so the corresponding KCL equation is given by $b_1 i_1 + b_2 i_2 + b_3 i_3 + b_4 i_4 + i_F = 0$ as expected.

(c) Given the KCL equation in the solution to Part (b), we can solve for i_F and then v_O as follows.

$$i_F = -b_1 i_1 - b_2 i_2 - b_3 i_3 - b_4 i_4$$

$$v_O = i_F R_F = -R_F(b_1 i_1 + b_2 i_2 + b_3 i_3 + b_4 i_4) = -R_F \frac{V_{REF}}{2R}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8}\right)$$

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Problem 4-69

OP AMP Circuit Analysis and Design

(a) **(A)** Find the input-output relationship of the circuit in Figure P4-69.

(b) **(D)** Design a circuit that realizes the relationship found in (a) using only 10-kΩ resistors and one OP AMP.

Solution:

(a) The circuit is connected as a subtractor in series with an inverting summer. The inputs to the subtractor are v_1 and v_O. The inputs to the summer are the output of the first stage and v_2. The following MATLAB code computes the gains for each stage and determines the overall input output relationship.

```
clear all
syms R v1 v2 vo
K1 = -2*R/R;
K2 = R/(R+2*R)*(R+2*R)/R;
K3 = -R/R;
K4 = -R/(R/2);
Eqn1 = 'va - (K1*v1+K2*vo)';
Eqn2 = 'vo - (K3*va+K4*v2)';
Soln = solve(Eqn1,Eqn2,'va','vo');
vo = simplify(Soln.vo)
vo = subs(vo)
vo =
-(K3*K1*v1+K4*v2)/(-1+K2*K3)
vo =
v1-v2
```

The circuit performs the function of a subtractor.

(b) The following circuit meets the specifications.

Answer:

Presented at right.

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Problem 4-70

Resistance Temperature Transducer

A resistive transducer uses a sensing element whose resistance varies with temperature. For a particular transducer, the resistance varies as $R_{TR} = 0.375T + 100\ \Omega$, where T is temperature in °C. This transducer is to be included in a circuit to measure temperatures in the range from −200° C to 800° C. The circuit must convert the transducer resistance variation over this temperature range into an output voltage in the range from 0 to 5 V. Two proposed circuit designs are shown in Figure P4-70. Which of these circuits would you recommend for production and why? *Hint*: First verify that the circuits perform the required function. Use OrCAD to verify your results.

Solution:

Compute the range of resistance values for the given temperature range.

```
clear all
format short eng
T_range = [-200,800];
RTR_range = 0.375*T_range+100
RTR_range =
    25.0000e+000   400.0000e+000
```

Analyze the performance of each circuit for the given resistor range

```
% Circuit 1
disp('Circuit 1')
% Use voltage division to find vTR
vTR = 5*RTR_range./(RTR_range+100)
% Circuit 1 is configured as a subtractor
vo1 = vTR*(900+600)/900 - 2.5*600/900
Circuit 1
vTR =
     1.0000e+000     4.0000e+000
vo1 =
     0.0000e-003     5.0000e+000
% Circuit 2
disp('Circuit 2')
% Find a Thevenin equivalent for the voltage at the
% negative input terminal
vT = 5*100./(100+RTR_range)
RT = 600 + 1./(1./RTR_range + 1/100)
% Circuit 2 is configured as a subtractor
vo2 = 2.5*(RT+1e3)./RT - vT*1e3./RT
Circuit 2
vT =
```

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```
     4.0000e+000      1.0000e+000
RT =
   620.0000e+000    680.0000e+000
vo2 =
    80.6452e-003      4.7059e+000
```

Circuit 1 meets the specifications, but Circuit 2 does not. Confirm these results using OrCAD.

Circuit 1 simulation Circuit 2 simulation

The output voltages from the two circuits are plotted on the same axes for direct comparison. Although the output voltage from Circuit 2 is similar to the output voltage from Circuit 1, Circuit 2 does not meet the specifications and Circuit 1 does, as predicted with the MATLAB calculations.

Answer:

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Choose Circuit 1 for production because it meets the specifications and Circuit 2 does not.

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Problem 5-1

Sketch the following waveforms:

(a) $v_1(t) = -2u(t) + 2u(t-1)$ V

(b) $v_2(t) = u(t+2) - 2u(t-2)$ V

(c) $v_3(t) = \int_{-\infty}^{t} v_1(x)\,\mathrm{d}x$

(d) $v_4(t) = \dfrac{d\,v_2(t)}{dt}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms x t
% Set the figure LineWidth
LW = 3;
% Create a time vector
% Shift the time vector slightly to see vertical transitions
tt = -3:0.01:3;
tt = tt+2*eps;

% Create the four signals
v1t = -2*heaviside(t) + 2*heaviside(t-1)
v2t = heaviside(t+2) - 2*heaviside(t-2)
v3t = int(subs(v1t,t,x),x,-inf,t)
v4t = diff(v2t,t)

% Substitute in the time vector
v1tt = subs(v1t,t,tt);
v2tt = subs(v2t,t,tt);
v3tt = subs(v3t,t,tt);
v4tt = subs(v4t,t,tt);
v1t =
-2*heaviside(t)+2*heaviside(t-1)
v2t =
heaviside(t+2)-2*heaviside(t-2)
v3t =
-2*heaviside(t)*t+2*heaviside(t-1)*t-2*heaviside(t-1)
v4t =
dirac(t+2)-2*dirac(t-2)

% Plot the results
figure
%subplot(2,2,1)
plot(tt,v1tt,'b','LineWidth',LW)
axis([-3,3,-3,2])
grid on
xlabel('Time, (s)')
ylabel('v_1(t), (V)')
```

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```
title('(a)')
```

```
figure
%subplot(2,2,2)
plot(tt,v2tt,'b','LineWidth',LW)
axis([-3,3,-3,2])
grid on
xlabel('Time, (s)')
ylabel('v_2(t), (V)')
title('(b)')
```

```
figure
%subplot(2,2,3)
plot(tt,v3tt,'b','LineWidth',LW)
axis([-3,3,-3,2])
grid on
xlabel('Time, (s)')
ylabel('v_3(t), (V)')
title('(c)')
```

```
figure
%subplot(2,2,4)
plot(tt,v4tt,'b','LineWidth',LW)
axis([-3,3,-3,2])
grid on
hold on
h1 = stem([-2],[1],'^','fill');
h2 = stem([2],[-2],'v','fill');
set(h1,'LineWidth',2);
set(h2,'LineWidth',2);
xlabel('Time, (s)')
ylabel('v_4(t), (V)')
title('(d)')
```

Answer:
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Problem 5-2

Sketch the following waveforms

(a) $v_1(t) = 1 - u(t)$ V

(b) $v_2(t) = -u(t+3) + 3u(t+1) - 2u(t)$ V

(c) Construct the waveforms in parts (a) and (b) using OrCAD.

Solution:

The solutions for Parts (a) and (b) are presented in the following MATLAB code.

```
clear all
format short eng
syms x t

% Set the figure LineWidth
LW = 3;
% Create a time vector
% Shift the time vector slight to see vertical transitions
tt = -4:0.01:2;
tt = tt+2*eps;

% Create the two signals
v1t = 1-heaviside(t);
v2t = -1*heaviside(t+3) + 3*heaviside(t+1) - 2*heaviside(t);

% Substitute in the time vector
v1tt = subs(v1t,t,tt);
v2tt = subs(v2t,t,tt);

% Plot the results
figure
plot(tt,v1tt,'b','LineWidth',LW)
axis([-4,2,-2,3])
grid on
xlabel('Time, (s)')
ylabel('v_1(t), (V)')
title('(a)')
```

```
% Plot the results
figure
plot(tt,v2tt,'b','LineWidth',LW)
axis([-4,2,-2,3])
grid on
xlabel('Time, (s)')
ylabel('v_2(t), (V)')
title('(b)')
```

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(c) The OrCAD simulations are shown below.

For both signals, $v_1(t)$ and $v_2(t)$, we shifted the signal 4 s forward in time to create the plot and then shifted the time axis back 4 s to generate the correct waveform.

Answer:
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Problem 5-3

Sketch the following waveforms:

(a) $v_1(t) = !2r(t+1) + 2r(t\ !\ 1)$ V

(b) $v_2(t) = r(t+1)\ !\ 2r(t\ !\ 1) + r(t\ !\ 3)$ V

(c) $v_3(t) = \dfrac{d\ v_1(t)}{dt}$

(d) $v_4(t) = \dfrac{d^2\ v_2(t)}{dt^2}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms x t

% Create the time vector
tt = -4:0.02:4;
tt = tt+2*eps;

% Generate the waveforms
v1t = -2*(t+1)*heaviside(t+1) + 2*(t-1)*heaviside(t-1);
v2t = (t+1)*heaviside(t+1) - 2*(t-1)*heaviside(t-1)...
    + (t-3)*heaviside(t-3);
v3t = diff(v1t,t)
v4t = diff(diff(v2t,t),t)

% Substitute in the time vector
v1tt = subs(v1t,t,tt);
v2tt = subs(v2t,t,tt);
v3tt = subs(v3t,t,tt);
v4tt = subs(v4t,t,tt);

% Plot the results
figure
plot(tt,v1tt,'b','LineWidth',3)
axis([-4,4,-5,3])
grid on
xlabel('Time, (s)')
ylabel('v_1(t), (V)')
title('(a)')
v3t =
-2*heaviside(t+1)+(-2*t-
2)*dirac(t+1)+2*heaviside(t-1)+(2*t-
2)*dirac(t-1)
v4t =
2*dirac(t+1)+(t+1)*dirac(1,t+1)-
4*dirac(t-1)-(2*t-2)*dirac(1,t-
1)+2*dirac(t-3)+(t-3)*dirac(1,t-3)
```

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```
figure
plot(tt,v2tt,'b','LineWidth',3)
axis([-4,4,-5,3])
grid on
xlabel('Time, (s)')
ylabel('v_2(t), (V)')
title('(b)')
```

```
figure
plot(tt,v3tt,'b','LineWidth',3)
axis([-4,4,-5,3])
grid on
xlabel('Time, (s)')
ylabel('v_3(t), (V)')
title('(c)')
```

```
figure
plot(tt,v4tt,'b','LineWidth',3)
axis([-4,4,-5,3])
grid on
hold on
h1 = stem([-1],[1],'^','fill');
h2 = stem([1],[-2],'v','fill');
h3 = stem([3],[1],'^','fill');
set(h1,'LineWidth',3);
set(h2,'LineWidth',3);
set(h3,'LineWidth',3);
xlabel('Time, (s)')
ylabel('v_4(t), (V)')
title('(d)')
```

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Answer:
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Problem 5-4

Express each of the following signals as a sum of singularity functions.

(a) $v_1(t) = \begin{cases} 1 & t < 1 \\ -2 & 1 \le t < 2 \\ 0 & 2 \le t \end{cases}$ (b) $v_2(t) = \begin{cases} 0 & t < 0 \\ -4t & 0 \le t < 2 \\ -12 + 2t & 2 \le t < 6 \\ 0 & 6 \le t \end{cases}$

Solution:

(a) Signal $v_1(t)$ has a value of 1 up until t = 1 s, when it decreases by 3. At t = 2, the signal increases by 2. We can construct the waveform with the following step functions:

$$v_1(t) = 1 - 3u(t-1) + 2u(t-2)$$

(b) Signal $v_2(t)$ has a slope of zero up until t = 0, when the slope decreases to –4. At t = 2, the slope changes by 6 to become +2 until t = 6, at which time the slope returns to zero. We can construct this waveform using ramp functions to account for each change in slope.

$$v_2(t) = -4r(t) + 6r(t-2) - 2r(t-6)$$

Answer:

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Problem 5-5

Express each of the waveforms in Figure P 5-5 as a sum of singularity functions.

Solution:

By inspection, we can develop the following waveform expressions:

$$v_1(t) = 2u(t+1) - 4u(t-1) + 2u(t-3) \ \text{ V}$$

$$v_2(t) = 5u(t) - r(t) + r(t-5) + 3\delta(t-10) \ \text{ V}$$

Answer:

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Problem 5-6

Express the waveform in Figure P 5-6 as a sum of step functions.

Solution:

By inspection, we can develop the following waveform expression:

$$v(t) = u(t+1) + u(t) + u(t-1) - u(t-2) - 2u(t-3)$$

The following MATLAB code verifies the results.

```
clear all
syms x t
% Create the time vector
tt = -2:0.02:4;
tt = tt+2*eps;
% Generate the waveforms
vt = heaviside(t+1) + heaviside(t) + heaviside(t-1)...
     - heaviside(t-2) - 2*heaviside(t-3);
% Substitute in the time vector
vtt = subs(vt,t,tt);
% Plot the results
figure
plot(tt,vtt,'b','LineWidth',3)
axis([-2,4,-1,4])
grid on
xlabel('Time, (s)')
ylabel('v(t), (V)')
```

Answer:

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Problem 5-7

Generate on OrCAD a waveform $v(t)$ that starts at t = 1 ms and consists of a pulse train of 1 V pulses with a 1 ms pulse width that repeat every 3 ms.

Solution:

Use a VPULSE source connected across a resistor. Use the following source settings: V1 = 0 V; V2 = 1 V; TD = 1 ms; TR = 0 ms; TF = 0 ms; PW = 1 ms; PER = 3 ms

Answer:

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Problem 5-8

Sketch the following exponential waveforms. Find the amplitude and time constant of each waveform.

(a) $v_1(t) = [5\, e^{!100t}]\, u(t)$ V

(b) $v_2(t) = [2\, e^{!t'2.5}]\, u(t\ !\ 1)$ V

(c) $v_3(t) = [10\, e^{!2(t\ !\ 5)}]\, u(t\ !\ 5)$ V

(d) $v_4(t) = [\,!\,100\, e^{!10000t}]\, u(t)$ V

Solution:

Use MATLAB to plot each waveform. Inspect the waveforms to determine the amplitude and calculate the time constants from the waveform equations.

```
clear all
format short eng
LW = 3;  % Plotting linewidth

syms t v1 v2 v3 v4

% Create the four signals
v1 = 5*exp(-100*t)*heaviside(t);
v2 = 2*exp(-t/2.5)*heaviside(t-1);
v3 = 10*exp(-2*(t-5))*heaviside(t-5);
v4 = -100*exp(-10000*t)*heaviside(t);

% Create appropriate time vectors and
plot the signals
t1 = -0.01+eps:0.0001:0.06;
v1t = subs(v1,t,t1);
plot(t1,v1t,'b','LineWidth',LW);
xlabel('Time (s)')
ylabel('v_1(t) (V)')
grid on
```

```
t2 = -2+eps:0.01:16;
v2t = subs(v2,t,t2);
plot(t2,v2t,'b','LineWidth',LW);
xlabel('Time (s)')
ylabel('v_2(t) (V)')
grid on
```

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```
t3 = 4+3*eps:0.005:8;
v3t = subs(v3,t,t3);
plot(t3,v3t,'b','LineWidth',LW);
xlabel('Time (s)')
ylabel('v_3(t) (V)')
grid on
```

```
t4 = -100e-6+eps:1e-6:600e-6;
v4t = subs(v4,t,t4);
plot(t4,v4t,'b','LineWidth',LW);
xlabel('Time (s)')
ylabel('v_3(t) (V)')
grid on
```

Answer:
The plots are presented above. The following table summarizes the amplitudes and time constants for the signals.

Signal	Amplitude (V)	Time Constant
v_1	5	10 ms
v_2	1.34	2.5 s
v_3	10	500 ms
v_4	-100	100 μs

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Problem 5-9

Write expressions for the derivative ($t > 0$) and integral (from 0 to t) of the exponential waveform $v(t) = [50\, e^{!2000t}]\, u(t)$ V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t v tau
% Create the signal with an expression valid for t > 0
v = 50*exp(-2000*t)

% Calculate the derivative
disp('Derivative')
dv = diff(v,t)

% Calculate the integral
% Use a dummy variable, tau, for integration
disp('Integral')
iv = int(subs(v,t,tau),tau,0,t)
v =
50*exp(-2000*t)
Derivative
dv =
-100000*exp(-2000*t)
Integral
iv =
1/40-1/40*exp(-2000*t)
```

Answer:

$$\frac{dv(t)}{dt} = -100000e^{-2000\,t} \quad \text{V/s}$$

$$\int_0^t v(\tau)\, d\tau = \frac{1}{40}\left(1 - e^{-2000\,t}\right) \quad \text{V} \cdot \text{s}$$

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Problem 5-10

An exponential waveform decays to 50% of its initial (t = 0) amplitude in 10 ms. Find the time constant of the waveform.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms A Tc
Eqn1 = 'A/2 - A*exp(-10e-3/Tc)';
Tc = double(solve(Eqn1,Tc))
% Check the results
Check_v = A*exp(-10e-3/Tc)
Tc =
    14.4270e-003
Check_v =
1/2*A
```

Answer:

T_C = 14.427 ms

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Problem 5-11

The amplitude of an exponential waveform is 6 V at t = 0 and 3.5 V at t = 2 ms. What is its time constant?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms Tc
Eqn1 = '3.5 - 6*exp(-2e-3/Tc)';
Tc = double(solve(Eqn1,Tc))
Tc =
     3.7106e-003
```

Answer:

T_C = 3.7106 ms

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Problem 5-12

Construct an exponential waveform that fits entirely within the un-shaded region in Figure P5-12.

Solution:

Select the initial amplitude to be 0.8, to fit within the available range at $t = 0$. Solve for the time constant such that the waveform has an amplitude of 0.55 at $t = 10$ ms. Plot the waveform to verify it meets the specifications.

```
clear all
syms Tc
A = 0.8
Eqn1 = '0.55 - A*exp(-0.01/Tc)';
Tc = subs(solve(Eqn1,Tc))
Tc_inv = 1/Tc

% Plot the waveform
t = 0:0.0001:0.02;
v = A*exp(-t/Tc);
plot(t,v,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('v(t)  (V)')
axis([0,0.02,0,1.2])
hold on
rectangle('Position',[0,1,0.02,0.2],'FaceColor',[.8,.8,.8],...
'EdgeColor','none')
rectangle('Position',[0.01,0.6,0.01,0.4],'FaceColor',[.8,.8,.8],...
'EdgeColor','none')
rectangle('Position',[0,0,0.01,0.5],'FaceColor',[.8,.8,.8],...
'EdgeColor','none')
```

```
A =
   800.0000e-003
Tc =
    26.6885e-003
Tc_inv =
    37.4693e+000
```

Answer:

$v(t) = 0.8e^{-37.47\,t}$ V.

There are many correct answers.

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Problem 5-13

By direct substitution show that the exponential function $v(t) = V_A e^{-\alpha t}$ satisfies the following first-order differential equation.

$$\frac{dv(t)}{dt} + \alpha v(t) = 0$$

Solution:

$$\frac{dv(t)}{dt} + \alpha v(t) = \frac{d}{dt}\left(V_A e^{-\alpha t}\right) + \alpha V_A e^{-\alpha t} = -\alpha V_A e^{-\alpha t} + \alpha V_A e^{-\alpha t} = 0$$

Answer:

Presented above.

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Problem 5-14

Find the period, frequency, amplitude, time shift, and phase angle of the following sinusoids.

(a) $v_1(t) = 120\cos(120\pi t) + 120\sin(120\pi t)$ V

(b) $v_2(t) = !30\cos(10\text{ k}\pi t) + 40\sin(10\text{ k}\pi t)$ V

Solution:

In both Parts (a) and (b), the two components have the same frequency, so they can be combined into a single sinusoidal expression. The following MATLAB code performs the calculations and presents the requested parameters.

```
clear all
disp('Part (a)')
a = 120;
b = 120;
wo = 120*pi;
fo = wo/2/pi
To = 1/fo
A = sqrt(a^2+b^2)
% Use atan2 to get the correct phase angle
phi = atan2(-b,a);
phi_deg = 180*phi/pi
Ts = -phi*To/2/pi
Part (a)
fo =
    60.0000e+000
To =
    16.6667e-003
A =
   169.7056e+000
phi_deg =
   -45.0000e+000
Ts =
     2.0833e-003

disp('Part (b)')
a = -30;
b = 40;
wo = 10e3*pi;
fo = wo/2/pi
To = 1/fo
A = sqrt(a^2+b^2)
% Use atan2 to get the correct phase angle
phi = atan2(-b,a);
phi_deg = 180*phi/pi
Ts = -phi*To/2/pi
Part (b)
fo =
     5.0000e+003
To =
   200.0000e-006
A =
```

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```
    50.0000e+000
phi_deg =
  -126.8699e+000
Ts =
    70.4833e-006
```

Answer:

Signal	Period	Frequency	Amplitude	Time Shift	Phase Angle
$v_1(t)$	16.67 ms	60 Hz	169.7 V	2.083 ms	–45°
$v_2(t)$	200 μs	5 kHz	50 V	70.48 μs	–126.87°

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Problem 5-15
(a) Plot the waveform of each sinusoid in problem 5-14.
(b) Use OrCAD to produce the waveform in problem 5-14(b).

Solution:
(a) The solution is presented in the following MATLAB code.

```
t1 = 0:0.0001:0.05;
t2 = 0:5e-6:600e-6;

v1t = 120*cos(120*pi*t1)+120*sin(120*pi*t1);
v2t = -30*cos(10e3*pi*t2)+40*sin(10e3*pi*t2);

figure
plot(t1,v1t,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('v_1(t), (V)')


figure
plot(t2,v2t,'b','LineWidth',3)
axis([0,600e-6,-60,60])
grid on
xlabel('Time, (s)')
ylabel('v_2(t), (V)')
```

(b) The OrCAD simulation is presented below. Note that the phase is shifted by 90° to convert from sine to cosine.

Answer:
Presented at right.

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Problem 5-16

Write an expression for the sinusoid in Figure P5-16. What are the phase angle and time shift of the waveform?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% The figure shows the period and amplitude
To = 2e-3;
A = 20
% The time shift is one quarter of a cycle or 2ms/4
Ts = To/4
% Calculate the phase angle in radians and degrees
phi = -2*pi*Ts/To
phi_deg = -360*Ts/To
% Calculate the frequency
fo = 1/To
wo = 2*pi*fo
A =
    20.0000e+000
Ts =
   500.0000e-006
phi =
    -1.5708e+000
phi_deg =
   -90.0000e+000
fo =
   500.0000e+000
wo =
     3.1416e+003
```

Answer:

$v(t) = 20\cos(1000\pi t - \pi/2)\ \text{V} = 20\cos(1000\pi t - 90°)\ \text{V}$

$\phi = -\pi/2$ radians $= -90°$

$T_S = 500\ \mu\text{s}$

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Problem 5-17

Find the Fourier coefficients, cyclic frequency, and radian frequency of the following sinusoids:

(a) $v_1(t) = 50\cos(500\pi t + 36.9^o)$ V

(b) $v_2(t) = 10\cos(500\pi t\ !\ 270^o)$ V

Solution:

```
clear all
disp('Part (a)')
Va = 50;
wo = 500*pi
fo = wo/2/pi
phi = pi*36.9/180;
a = Va*cos(phi)
b = -Va*sin(phi)

disp('Part (b)')
Va = 10;
wo = 500*pi
fo = wo/2/pi
phi = pi*(-270)/180;
a = Va*cos(phi)
b = -Va*sin(phi)
Part (a)
wo =
     1.5708e+003
fo =
   250.0000e+000
a =
    39.9842e+000
b =
   -30.0210e+000
Part (b)
wo =
     1.5708e+003
fo =
   250.0000e+000
a =
    -1.8370e-015
b =
   -10.0000e+000
```

Answer:

(a) $a = 40$ V; $b = -30$ V; $f_0 = 250$ Hz; $\omega_0 = 500\pi$ rad/s;

(b) $a = 0$ V; $b = -10$ V; $f_0 = 250$ Hz; $\omega_0 = 500\pi$ rad/s

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Problem 5-18

Use MATLAB or Excel to display two cycles of the following waveform:

$$v_S = 19.1\sin 1000\pi t + 6.37\sin 3000\pi t + 3.82\sin 5000\pi t + 2.73\sin 7000\pi t + 2.1\sin 9000\pi t \text{ V}$$

What are the period and amplitude of the resulting waveform? What common waveform is this waveform approximating?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Create a time vector
t = 0:0.00001:0.004;

% Create the waveform
vs = 19.1*sin(1000*pi*t) + 6.37*sin(3000*pi*t) ...
    + 3.82*sin(5000*pi*t) + 2.73*sin(7000*pi*t) ...
    + 2.1*sin(9000*pi*t);

% Plot the results
figure
plot(t,vs,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('v_s(t) (V)')
```

Answer:

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The plot is presented above. The period is 2 ms and the amplitude is 15 V. The signal is approximating a square wave.

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Problem 5-19

Find the Fourier coefficients, cyclic frequency, and radian frequency of the following sinusoids:

(a) $v_1(t) = 100 \cos(2\pi 400t + 60^o)$ V

(b) $v_2(t) = 25 \cos(2000\pi t\ !\ 30^o)$ V

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
Va = 100;
wo = 2*pi*400
fo = wo/2/pi
phi = pi*60/180;
a = Va*cos(phi)
b = -Va*sin(phi)

disp('Part (b)')
Va = 25;
wo = 2000*pi
fo = wo/2/pi
phi = pi*(-30)/180;
a = Va*cos(phi)
b = -Va*sin(phi)
Part (a)
wo =
     2.5133e+003
fo =
   400.0000e+000
a =
    50.0000e+000
b =
   -86.6025e+000
Part (b)
wo =
     6.2832e+003
fo =
     1.0000e+003
a =
    21.6506e+000
b =
    12.5000e+000
```

Answer:

(a) $a = 50$ V; $b = -86.6$ V; $f_0 = 400$ Hz; $\omega_0 = 800\pi$ rad/s;

(b) $a = 21.65$ V; $b = 12.5$ V; $f_0 = 1000$ Hz; $\omega_0 = 2000\pi$ rad/s

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Problem 5-20
Find the time shift of each sinusoid in Problem 5-19.

Solution:
The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
wo = 2*pi*400;
fo = wo/2/pi;
To = 1/fo;
phi = pi*60/180;
Ts = -phi*To/2/pi

disp('Part (b)')
wo = 2000*pi;
fo = wo/2/pi;
To = 1/fo;
phi = pi*(-30)/180;
Ts = -phi*To/2/pi
Part (a)
Ts =
  -416.6667e-006
Part (b)
Ts =
    83.3333e-006
```

Answer:
(a) $T_S = -416.67\ \mu s$
(b) $T_S = 83.33\ \mu s$

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Problem 5-21

Consider the following composite waveforms.

(a) Sketch each on paper and then generate each using (b) OrCAD and (c) MATLAB and plot the results.

(a) $v_1(t) = 20\,[1 - e^{-5000t}]u(t)$ V

(b) $v_2(t) = 10\,[e^{-5t} - e^{-10t}]u(t)$ V

Solution:

(a) Hand sketches are not provided in the solutions.

(b) OrCAD simulations.

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(c) MATLAB plots.

```
clear all
% Part (a)
t = 0:0.000001:0.001;
v1t = 20*(1-exp(-
5000*t)).*heaviside(t);
figure
plot(t,v1t,'b','LineWidth',3)
grid on
axis([0 0.001 0 25])
xlabel('Time, (s)')
ylabel('v_1(t), V')
```

```
% Part (b)
t = 0:0.001:1.5;
v2t = 10*(exp(-5*t)-exp(-
10*t)).*heaviside(t);
figure
plot(t,v2t,'b','LineWidth',3)
grid on
axis([0 1.5 0 3])
xlabel('Time, (s)')
ylabel('v_2(t), V')
```

Answer:
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Problem 5-22

Consider the following composite waveforms. (a) Sketch each on paper and then generate each using (b) OrCAD and (c) MATLAB and plot the results.

(a) $v_1(t) = 10 - 10\sin(1000\pi t)$ V

(b) $v_2(t) = 10\,[e^{-1000t} + \cos(2000\pi t)]u(t)$ V

Solution:

(a) Hand sketches are not provided in the solutions.

(b) OrCAD simulations.

Two possible circuits.

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(c) MATLAB plots.

```
clear all
% Part (a)
t = 0:0.000001:0.004;
v1t = 10-10*sin(1000*pi*t);
figure
plot(t,v1t,'b','LineWidth',3)
grid on
%axis([0 0.001 0 25])
xlabel('Time, (s)')
ylabel('v_1(t), V')
```

```
% Part (b)
t = 0:0.000001:0.008;
v2t = 10*(exp(-1000*t) +
cos(2000*pi*t)).*heaviside(t);
figure
```

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```
plot(t,v2t,'b','LineWidth',3)
grid on
%axis([0 0.001 0 25])
xlabel('Time, (s)')
ylabel('v_2(t), V')
```

Answer:
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Problem 5-23

Sketch the damped ramp $v(t) = te^{-50t}u(t)$. Find the maximum value of the waveform and the time at which it occurs.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
t = 0:0.001:0.2;
v = t.*exp(-50*t);
plot(t,v,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

```
% Compute the maximum
syms T
vT = T*exp(-50*T);
dvT = diff(vT,T);
MaxT = double(solve(dvT,'T'))
MaxV = subs(vT,T,MaxT)
MaxT =
    20.0000e-003
MaxV =
     7.3576e-003
```

Answer:

The plot is shown above. The maximum voltage is $v(t)$ = 7.3576 mV at t = 20 ms.

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Problem 5-24

The value of the waveform $v(t) = (V_A - V_B e^{-\alpha t})u(t)$ is 5 V at $t = 0$, 8 V at $t = 5$ ms, and approaches 12 V as $t \to \infty$. (a) Find V_A, V_B, and α then sketch the waveform. (b) Validate your answers by plotting your result in MATLAB.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Create three equations to solve for VA, VB, and alpha
syms VA VB alpha
Eqn1 = '5 - (VA-VB)';
Eqn2 = '8 - (VA-VB*exp(-alpha*5e-3))';
Eqn3 = '12 - (VA-VB*0)';
Soln = solve(Eqn1,Eqn2,Eqn3,'VA','VB','alpha');
VA = double(Soln.VA)
VB = double(Soln.VB)
alpha = double(Soln.alpha)
VA =
    12.0000e+000
VB =
     7.0000e+000
alpha =
   111.9232e+000

% Sketch the waveform
t = 0:40e-6:40e-3;
v = VA-VB*exp(-alpha*t);
plot(t,v,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answer:

$v(t) = (12 - 7\, e^{-111.92t})u(t)$

The plot is presented above.

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Problem 5-25

Write an expression for the composite sinusoidal waveform in Figure P5-25.

Solution:

From Figure P5-25 we can observe that the period is 5 ms and that the waveform is shift by one half of a cycle or 180°. With a period of 5 ms, the frequency is 200 Hz or 400π rads/sec. The sinusoid ranges from 4 to –10, so its amplitude must be half of this range or 7 V. The sinusoid is shifted down and instead of oscillating between 7 and –7 V, the waveform oscillates between 4 and –10 V, so the shift must be –3 V. The following MATLAB code plots the waveform based on the analysis above and confirms that it matches Figure P5-25.

```
clear all
t = 0:0.00001:0.016;
vt = -3+7*cos(400*pi*t-pi);
figure
plot(t,vt,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('v(t), V')
```

Answer:

$v(t) = -3 + 7\cos(400\pi t - \pi)$ V $= -3 + 7\cos(400\pi t - 180°)$ V

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Problem 5-26

A waveform of the form $v(t) = 5 \pm 10\cos(\beta t - 45^{\circ})$ periodically reaches a minimum every 2 ms.

(a) Find the values of V_{max}, V_{min}, and β then sketch the waveform.

(b) Generate the waveform in OrCAD.

Solution:

(a) To reach a minimum every 2 ms, the period must be 2 ms. The solution is presented in the following MATLAB code.

```
clear all
To = 2e-3;
fo = 1/To;
wo = 2*pi*fo;
beta = wo
% Cosine ranges between -1 and 1, so we can easily compute
% the maximum and minimum values for the waveform
v_max = 5+10
v_min = 5-10
beta =
     3.1416e+003
v_max =
    15.0000e+000
v_min =
    -5.0000e+000

% Plot the waveform
t = 0:6e-6:6e-3;
v = 5 - 10*cos(beta*t-45*pi/180);
plot(t,v,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

(b) OrCAD simulation

Answer:

(a) V_{max} = 15 V, V_{min} = –5 V, and $\beta = 1000\pi$.

(b) Presented at right.

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Problem 5-27
Write an expression for the composite exponential waveform in Figure P5-27.

Solution:
The waveform is a decaying exponential combined with a constant signal. The constant signal is 2 V, based on the value the waveform approaches as time increases. The exponential has a magnitude of 2 V at t = 0 s and decays to 50% of that value in 10 ms. We can solve for the time constant with the following MATLAB code:

```
clear all
syms Tc
Eqn1 = 1-2*exp(-10e-3/Tc)';
Tc = double(solve(Eqn1,Tc ))
Tc_inv = 1/Tc
Tc =
    14.4270e-003
Tc_inv =
    69.3147e+000

% Plot the waveform
t = 0:0.00001:0.08;
vt = (2+2*exp(-t/Tc)).*heaviside(t);
figure
plot(t,vt,'b','LineWidth',3)
grid on
axis([0 0.08 0 5])
xlabel('Time, (s)')
ylabel('v(t), V')
```

Answer:
$v(t) = \left(2 + 2e^{-69.3t}\right)u(t)$ V

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Problem 5-28

(a) For the double exponential $v(t) = 10\left(e^{-200t} - e^{-1000t}\right)u(t)$, find the maximum value of the waveform and the time at which it occurs.
(b) Determine the dominant exponential.
(c) Generate the waveform in MATLAB and validate the result.

Solution:
(a) The solution is presented in the following MATLAB code.

```
clear all
format short eng
% Compute the maximum
syms T positive
vT = 10*(exp(-200*T)-exp(-1000*T));
dvT = diff(vT,T);
MaxT = double(solve(dvT,'T'))
MaxV = subs(vT,T,MaxT)
MaxT =
     2.0118e-003
MaxV =
     5.3499e+000
```

The maximum voltage is 5.3499 V and it occurs at t = 2.0118 ms.

(b) The exponential in the signal has two time constants. The time constants are 1/200 = 5 ms and 1/1000 = 1 ms. The dominant exponential is the one with the longer time constant, since it will persist for a longer period of time. Therefore, the dominant exponential is e^{-200t}.

(c) The solution is presented in the following MATLAB code.

```
clear all
t = 0:1e-6:30e-3;
v= 10*(exp(-200*t)-exp(-
1000*t)).*heaviside(t);
plot(t,v,'b','LineWidth',3)
xlabel('Time (s)')
ylabel('Voltage (V)')
grid on
[MaxV,Index] = max(v);
MaxT = t(Index)
MaxV
MaxT =
     2.0120e-003
MaxV =
     5.3499e+000
```

Answer:
(a) The maximum voltage is 5.3499 V and it occurs at t = 2.0118 ms.
(b) The dominant exponential is e^{-200t}.

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(c) The plot is shown above and verifies the results in Part (a).

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Problem 5-29
Write an expression for the damped sine waveform in Figure P5-29.

Solution:
The sinusoid has a period of 10 ms and is a cosine shifted by one quarter of a cycle or 90° = π/2 radians. The frequency is 1/10 ms = 100 Hz = 200π rad/s. The exponential envelope is given as $5e^{-100t}$.
Combining the information, we have the following expression for the waveform.

$$v(t) = 5e^{-100t}\cos(200\pi t - \pi/2)\ \text{V} = 5e^{-100t}\sin(200\pi t)\ \text{V}$$

The following MATLAB code confirms the results.

```
clear all
t = 0:20e-6:20e-3;
v = 5*exp(-100*t).*cos(200*pi*t-pi/2);
e1 = 5*exp(-100*t);
e2 = -5*exp(-100*t);

plot(t,v,'b','LineWidth',3)
hold on
plot(t,e1,'g:','LineWidth',2)
plot(t,e2,'g:','LineWidth',2)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answer:
$$v(t) = 5e^{-100t}\cos(200\pi t - \pi/2)\ \text{V} = 5e^{-100t}\sin(200\pi t)\ \text{V}$$

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Problem 5-30
A circuit response is shown in Figure P5-30. Determine an approximate expression for the waveform.

Solution:
The solution is presented in the following MATLAB code.

```
clear all
% Create a time vector
t = 0:0.00001:0.03;
% Compute the time onstant
% From the plot, measure
the time between the first
%and fifth peak, which represents four complete cycles
dt = 0.025;
% Read the voltages at t=0 and t=dt
v0 = 5;
vdt = -4.2;
% Estimate the final value for the signal
vf = -5;
% Subtract out the final value from the initial voltage
% and the voltage at the fifth peak
v1 = v0 - vf;
v2 = vdt - vf;
% Compute the time constant
Tc = dt/log(v1/v2)
Tc_inv = 1/Tc
% Compute the sinusoidal period
% Number of cycles to average over
n = 4;
T0 = dt/n;
f0 = 1/T0
% There is no phase shift in the
sinusoid
% Create the waveform
v = vf + (v0-vf)*exp(-
t/Tc).*cos(2*pi*f0*t);

% Plot the results
figure
plot(t,v,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
Tc =
     9.8981e-003
Tc_inv =
```

Answer: $v(t) = -5 + 10e^{-101t}\cos(320\pi t)$ V

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```
   101.0291e+000
f0 =
   160.0000e+000
```

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Problem 5-31

Find V_p, V_{pp}, V_{avg}, and V_{rms} for each of the following sinusoids.

(a) $v_1(t) = 100\cos(4000\pi t) + 100\sin(4000\pi t)$ V

(b) $v_2(t) = !30\cos(2000\pi t)\ !\ 40\sin(2000\pi t)$ V

(c) $v_3(t) = 24\cos(4000\pi t\ !\ 45^o)$ V

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
% Convert the signal into a single sinusoid
a = 100;
b = 100;
A = sqrt(a^2+b^2);
Vp = A
Vpp = 2*A
% The sinusoid has no bias term, so the average value is zero
Vavg = 0
% The sinusoid has no bias term, so the RMS value can be computed
% from the amplitude
Vrms = A/sqrt(2)
Part (a)
Vp =
   141.4214e+000
Vpp =
   282.8427e+000
Vavg =
     0.0000e-003
Vrms =
   100.0000e+000

disp('Part (b)')
% Convert the signal into a single sinusoid
a = -30;
b = -40;
A = sqrt(a^2+b^2);
Vp = A
Vpp = 2*A
% The sinusoid has no bias term, so the average value is zero
Vavg = 0
% The sinusoid has no bias term, so the RMS value can be computed
% from the amplitude
Vrms = A/sqrt(2)
Part (b)
Vp =
    50.0000e+000
Vpp =
   100.0000e+000
Vavg =
     0.0000e-003
```

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```
Vrms =
    35.3553e+000

disp('Part (c)')
A = 24;
Vp = A
Vpp = 2*A
% The sinusoid has no bias term, so the average value is zero
Vavg = 0
% The sinusoid has no bias term, so the RMS value can be computed
% from the amplitude
Vrms = A/sqrt(2)
Part (c)
Vp =
    24.0000e+000
Vpp =
    48.0000e+000
Vavg =
     0.0000e-003
Vrms =
    16.9706e+000
```

Answer:

(a) V_p = 141.4 V; V_{pp}, = 282.8 V; V_{avg}, = 0 V; V_{rms} = 100 V;

(b) V_p = 50 V; V_{pp}, = 100 V; V_{avg}, = 0 V; V_{rms} = 35.4 V;

(c) V_p = 24 V; V_{pp}, = 48 V; V_{avg}, = 0 V; V_{rms} = 16.97 V

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Problem 5-32

An exponential waveform given by $v(t) = 10e^{-100t}\ u(t)$ V repeats every five time constants.

(a) Find V_p, V_{pp}, V_{MAX}, and V_{MIN}.

(b) Find V_{avg} and V_{rms}.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
syms t v
v = 10*exp(-100*t)*heaviside(t+eps);
% Note:  The step function is shifted by eps so the value is not NaN
% at t = 0.
Tc = 1/100;
% The maximum occurs at t = 0 and the minimum occurs at 5*Tc
Vmax = double(subs(v,t,0))
Vmin = double(subs(v,t,5*Tc))
Vp = max(abs(Vmax),abs(Vmin))
Vpp = Vmax-Vmin
Part (a)
Vmax =
    10.0000e+000
Vmin =
    67.3795e-003
Vp =
    10.0000e+000
Vpp =
     9.9326e+000

disp('Part (b)')
% The period is five time constants
T = 5*Tc;
% Calculate the average value over one period
Vavg = double(int(v,t,0,T)/T)
% Calculate the RMS value
Vrms = double(sqrt(int(v^2,t,0,T)/T))
Part (b)
Vavg =
     1.9865e+000
Vrms =
     3.1622e+000
```

Answer:

(a) V_p = 10 V; V_{pp}, = 9.9326 V; V_{MAX} = 10 V, and V_{MIN} = 67.3795 mV

(b) V_{avg}, = 1.9865 V; V_{rms} = 3.1622 V

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Problem 5-33

Find V_p, V_{pp}, V_{avg}, and V_{rms} for the periodic waveform in Figure P5-33.

Solution:

By inspection, we can find $V_p = V_{pp} = 15$ V. The following MATLAB code calculates the average and RMS values

```
clear all
To = 4;
Vavg = (0+5+10+15)/To
Vrms = sqrt((0^2+5^2+10^2+15^2)/To)
Vavg =
     7.5000e+000
Vrms =
     9.3541e+000
```

Answer:

$V_p = 15$ V, V_{pp}, = 15 V, V_{avg}, = 7.5 V, and $V_{rms} = 9.3541$ V

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Problem 5-34

Find V_p, V_{pp}, V_{avg}, and V_{rms} for the periodic waveform in Figure P5-34.

Solution:

From the figure, we can observe V_{MAX} = 2 mV and V_{MIN} = –1 mV. With this information, the following MATLAB code computes all of the requested values.

```
clear all
Vmax = 2e-3;
Vmin = -1e-3;
To = 4;
Vp = max(abs(Vmax),abs(Vmin))
Vpp = Vmax-Vmin
Vavg = (2e-3-1e-3)/To
Vrms = sqrt(((2e-3)^2+(-1e-3)^2)/To)
Vp =
     2.0000e-003
Vpp =
     3.0000e-003
Vavg =
   250.0000e-006
Vrms =
     1.1180e-003
```

Answer:

V_p = 2 mV, V_{pp}, = 3 mV, V_{avg}, = 250 μV, and V_{rms} = 1.118 mV

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Problem 5-35

Figure P5-35 displays the response of a circuit to a squarewave signal. The response is a periodic sequence of exponential waveforms. Each exponential has a time constant of 1.6 ms. Use MATLAB to find V_{avg} and V_{rms} for the periodic waveform in Figure P5-35.

4

Solution:

The waveform has a period of 20 ms and each period contains two exponential components. The exponential components have amplitudes of 6 V and approach either V or –2 V. The time constant is 1.6 ms and $1/T_C$ = 625 Hz. We can create an expression for the first cycle of the waveform and then use the formulas to compute the average value and RMS value. The following MATLAB code completes the solution.

```
clear all
% Compute the average value and Vrms
syms x
T = 20e-3;
% Create one period of the signal
vx = (4-6*exp(-625*x))*(heaviside(x)-heaviside(x-0.01))...
    + (-2+6*exp(-625*(x-0.01)))*(heaviside(x-0.01)-heaviside(x-0.02));
vavg = int(vx,x,0,T)/T
vrms = double(sqrt(int(vx^2,x,0,T)/T))
vavg =
1
vrms =
     2.6704e+000
```

Answer:

V_{avg} = 1 V and V_{rms} = 2.6704 V.

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Problem 5-36

Find V_{avg} and V_{rms} of the offset sinewave $v(t) = V_0 + V_A \sin(2\pi t ' T_0)$ in terms of V_0 and V_A.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms v V0 VA Vavg Vrms t To
v = V0 + VA*sin(2*pi*t/To);
Vavg = int(v,t,0,To)/To
Vrms = simplify(sqrt(int(v^2,t,0,To)/To))
Vavg =
V0
Vrms =
1/2*(4*V0^2+2*VA^2)^(1/2)
```

Answer:

$V_{avg} = V_0$

$$V_{rms} = \sqrt{V_0^2 + \frac{V_A^2}{2}}$$

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Problem 5-37

Find V_{avg} and V_{rms} of the fullwave rectified sinewave $v(t) = V_A * \sin(2\pi t ' T_0) *$ in terms of V_A.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Compute the average value and Vrms
syms t real
syms VA To positive
vt = VA*abs(sin(2*pi*t/To));

vavg = int(vt,t,0,To)/To
vrms = sqrt(int(vt^2,t,0,To)/To)
vavg =
2*VA/pi
vrms =
1/2*2^(1/2)*VA
```

Answer:

$V_{avg} = 2V_A/\pi$

$V_{rms} = V_A / \sqrt{2}$

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Problem 5-38

The first cycle ($t > 0$) of a periodic waveform with $T_0 = 30$ ms can be expressed as

$$v(t) = 3u(t) - 6u(t - .01) + 3u(t - .02) \text{ V}$$

Sketch the waveform and find V_{max}, V_{min}, V_p, V_{pp}, and V_{avg}.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
t = 0:30e-6:30e-3;
t = t + 3*eps;
v = 3*heaviside(t)-6*heaviside(t-0.01)+3*heaviside(t-0.02);
plot(t,v,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
axis([0 0.03 -4 4])


% From the plot, we can find Vmax and
Vmin and then compute the other values
Vmax = 3
Vmin = -3
Vp = max(abs(Vmax),abs(Vmin))
Vpp = Vmax-Vmin
Vavg = (3*0.01 + (-3)*0.01 + 0*0.01)/0.03
Vmax =
     3.0000e+000
Vmin =
    -3.0000e+000
Vp =
     3.0000e+000
Vpp =
     6.0000e+000
Vavg =
     0.0000e-003
```

Answer:

$V_{max} = 3$ V, $V_{min} = -3$ V, $V_p = 3$ V, $V_{pp} = 6$ V, and $V_{avg} = 0$ V.

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Problem 5-39

A periodic waveform can be expressed as

$$v(t) = 5 + 5\cos 1000\pi t - 3\sin 2000\pi t + \cos 4000\pi t \text{ V}$$

What is the period of the waveform? What is the average value of the waveform? What is the amplitude of the fundamental (lowest frequency) component? What is the highest frequency in the waveform?

Solution:

Plot the signal to determine the period. Use MATLAB to calculate the average value. The other requested values can be determined by inspection of the waveform.

```
clear all
t = 0:0.00001:0.008;
vt = 5 + 5*cos(1000*pi*t) - 3*sin(2000*pi*t) + cos(4000*pi*t);
plot(t,vt,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('Voltage, (V)')
```

The period is 2 ms.

```
clear all
syms t v
To = 2e-3;
v = 5 + 5*cos(1000*pi*t) -
3*sin(2000*pi*t) +
cos(4000*pi*t);
Vavg = int(v,t,0,To)/To
Vavg =
5
```

Answer:

T_0 = 2 ms, V_{avg} = 5 V, A_1 = 5 V, and f_H = 2000 Hz

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Problem 5-40

Using OrCAD, create a triangular wave that has an amplitude of 5 V and a period of 1 ms. (Hint: the pulse width can be very small but not zero.)

Solution:

Use a voltage pulse with a very small pulse width and set the rise and fall times to cover the period of the signal. The following OrCAD simulation provides the solution.

Answer:

Presented above.

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Problem 5-41

Exponential Signal Descriptors **(A)**

Several of the time descriptors used in digital data communication systems are based on exponential signals. In this problem we explore three of these descriptors.

(a) The *time constant of fall* is defined as the time required for a pulse to fall from 70.7% to 26.0% of its maximum value. Assuming that the pulse decreases as $e^{-\frac{t}{T_C}}$, find the relationship between the time constant of fall and the time constant of the exponential decay.

(b) The *rise time* of a pulse is the time required for a pulse to rise from 10% to 90% of its maximum value. Assuming the pulse increases as $1-e^{-\frac{t}{T_C}}$, find the relationship between rise time and the time constant of the exponential rise.

(c) The *leading-edge pulse time* is defined as the time at which a pulse rises to 50% of its maximum value. Assuming the pulse increases as $1-e^{-\frac{t}{T_C}}$, find the relationship between leading-edge pulse time and the time constant of the exponential rise.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)');
syms t Tc
Eqn1 = '0.707 - exp(-t/Tc)';
Eqn2 = '0.26 - exp(-t/Tc)';
t1 = solve(Eqn1,'t');
t2 = solve(Eqn2,'t');
ta = simplify(t2-t1)
Part (a)
ta =
1.0003490348810450265008399926483*Tc

disp('Part (b)');
Eqn1 = '0.1 - (1-exp(-t/Tc))';
Eqn2 = '0.9 - (1-exp(-t/Tc))';
t1 = solve(Eqn1,'t');
t2 = solve(Eqn2,'t');
tb = simplify(t2-t1)
Part (b)
tb =
2.1972245773362193827904904738451*Tc

disp('Part (c)');
Eqn1 = '0.5 - (1-exp(-t/Tc))';
tc = solve(Eqn1,'t')
Part (c)
tc =
.69314718055994530941723212145818*Tc
```

Answer:

(a) $T_{CF} = 1.000349\ T_C$

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(b) $T_R = 2.197225\ T_C$
(c) $T_{LEP} = 0.693147\ T_C$

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Problem 5-42

Defibrillation Waveforms **(A)**

Ventricular fibrillation is a life threatening loss of synchronous activity in the heart. To restore normal activity, a defibrillator delivers a brief but intense pulse of electrical current through the patient's chest. The pulse waveform is of interest because different waveforms may lead to different outcomes. Figure P5-42 show a waveform known as a *biphasic truncated exponential* used in implantable defibrillators. The waveform is an exponential current whose direction of flow reverses after 4 ms and terminates after 8 ms. Write an expression for this waveform using the basic signals discussed in this chapter.

Solution:

To simplify the solution, allow the rising edge of the waveform to occur at time $t = 0$. Use two points on the exponential curve to determine the time constant and then use step functions to create the waveform. Use MATLAB to perform the calculations and plot the results to verify it matches the figure.

```
clear all
i1 = 35;
i2 = 15;
t1 = 0;
t2 = 4e-3;
Tc = (t2-t1)/log(i1/i2)
Tc_inv = 1/Tc

t = -2e-3:4e-6:10e-3;
t = t+2*eps;
it = 35*exp(-t/Tc).*(heaviside(t)-heaviside(t-4e-3))...
    - 35*exp(-t/Tc).*(heaviside(t-4e-3)-heaviside(t-8e-3));
plot(t,it,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
axis([-2e-3 10e-3 -15 35])
Tc =
     4.7209e-003
Tc_inv =
   211.8245e+000
```

Answer:

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$$i(t) = 35e^{-211.82\,t}\left[u(t) - 2u(t-0.004) + u(t-0.008)\right] \text{ A}$$

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Problem 5-43

Square wave Generator **(D)**

Using an ideal OP AMP (running open-loop, i.e. without feedback) and appropriate resistors, design a way to convert a sinusoid into a square wave that varies from 0 to 5V with a period of 1 ms. The only non-sinusoidal supply available is $V_{CC} = \pm 15$ V.

Solution:

Connect the positive power input for the OP AMP to V_{CC} and the negative power input to ground. Create the comparator circuit shown below to generate the requested square wave.

Answer:

Presented above.

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Problem 5-44

Undesired Oscillations **(A)(E)**

A test is being run in a wind tunnel when a sensor on the trailing edge of a wing produces the response shown in Figure P5-44. When the sensor output reached 1 V, the test was terminated. You are asked to analyze the results. The oscillation could be tolerated if it never reaches 20 V because the on-board computer can mitigate it. However, the response time for the computer to actuate the compensating aileron is 80 ms. Will the compensation occur in time?

Solution:

This problem requires an analysis of a sinusoidal signal multiplied by a growing exponential. If the on-board computer system can immediately detect the signal when it starts, then it will have 80 ms to apply the compensation. Therefore, we need to determine if the signal will reach 20 V in less than 80 ms. Examine the envelope of the signal and determine the time constant to predict the signal amplitude at 80 ms. The following MATLAB code presents the solution.

```
clear all
v1 = 0.03;
v2 = 0.49;
t1 = 0.038;
t2 = 0.057;
Tc = (t2-t1)/log(v2/v1)
Tc_inv = 1/Tc
% Check one of the other points on the graph
% to verify the time constant
Check = v2*exp(.0505/Tc)/exp(t2/Tc);

% Estimate the signal magnitude at t = 80 ms
v3 = v2*exp(.08/Tc)/exp(t2/Tc)
% Find the time at which the signal reaches 20 V
v4 = v2*exp(.08223/Tc)/exp(t2/Tc)
```

```
Tc =
     6.8022e-003
Tc_inv =
   147.0109e+000
v3 =
    14.4097e+000
v4 =
    20.0001e+000
```

Answer:

Based on the estimates used, the system will be able to compensate for the unstable condition if the system can detect the condition within 2 ms of it occurring. Since this is a flight critical system, a more detailed analysis, with access to the data used to create the waveform, would be appropriate to ensure the results are accurate.

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Problem 5-45

Voltmeter Calibration **(A)**

Most dc voltmeters measure the average value of the applied signal. A dc meter that measures the average value can be adapted to indicate the rms value of an ac signal. The input is passed through a rectifier circuit. The rectifier output is the absolute value of the input and is applied to a dc meter whose deflection is proportional to the average value of the rectified signal. The meter scale is calibrated to indicate the rms value of the input signal. A calibration factor is needed to convert the average absolute value into the rms value of the ac signal. What is the required calibration factor for a sinusoid? Would the same calibration factor apply to a square wave?

Solution:

Given a sinusoidal signal with amplitude V_A, the rms value of the signal is $V_{rms} = V_A / \sqrt{2}$. To find the scaling factor requested in the problem statement, find the average value for the absolute value of the sinusoid and compare it to the rms value. The following MATLAB code completes the calculations.

```
clear all
format short eng
syms VA To positive
syms t real
% The integral of the absolute value of a sinusoid over a full
% period is twice the integral of the sinusoid over one half of
% a period
v = VA*sin(2*pi*t/To);
v_abs_avg = 2*int(v,t,0,To/2)/To
% Check the result with the complete integral
v_abs_avg2 = int(abs(v),t,0,To)/To
% Determine the calibration factor
Factor = (VA/sqrt(2))/v_abs_avg
v_abs_avg =
2*VA/pi
v_abs_avg2 =
2*VA/pi
Factor =
1/4*2^(1/2)*pi
```

Answer:

The calibration factor for sinusoidal signal is $K = \sqrt{2}\pi / 4$. The same calibration factor would not apply to a square wave, because the square wave's rms value is different than the sinusoid's rms value.

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Problem 5-46

MATLAB Signal Analyzer **(A)**

Create a MATLAB function to analyze signals represented numerically. The function should have the following two inputs: (1) a vector containing equally spaced samples of the signal of interest and (2) the time step used to sample the signal contained in the vector. The function should display the following descriptors of the signal: V_{MAX}, V_{MIN}, V_p, V_{pp}, V_{avg}, and V_{rms}. The function should also plot the waveform assuming the signal starts at $t = 0$.

Solution:

The following MATLAB function will meet the requirements.

```
function a = analyze(x,dt)
%
% Given samples of a waveform in variable x with time step dt
% between samples, compute the following partial waveform
% descriptors:  % Vmax, Vmin, Vp, Vpp, Vavg, and Vrms.
% The calculations assume the waveform samples cover an integer
% number of periods, if the signal is periodic.
% Plot the signal starting at t=0.

% Find the maximum and minimum values
Vmax = max(x)
Vmin = min(x)

% Find the peak and peak-to-peak values
Vp = max(abs(Vmax),abs(Vmin))
Vpp = Vmax-Vmin

% Compute the average value over the entire signal
n = length(x);
T = (n-1)*dt;
Tf = n*dt;
t = 0:dt:T;
% Use a numerical integral to find the average value
Vavg = sum(dt*x)/Tf

% Use a numerical integral to find the rms value
Vrms = sqrt(dt*sum(x.^2)/Tf)

% Plot the results
figure
plot(t,x,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Signal')
```

Answer:

Presented above.

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Problem 6-1

For t $ 0 the voltage across a 3.3-μF capacitor is $v_C(t) = 5\ e^{!10,000t}$ V. Derive expressions for $i_C(t)$ and $p_C(t)$. Is the capacitor absorbing power, delivering power, or both?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms t vc ic pc
C = 3.3e-6;
vc = 5*exp(-10000*t);
ic = simplify(C*diff(vc,t));
ic_num = vpa(ic,5)
pc = simplify(ic*vc);
pc_num = vpa(pc,5)
ic_num =
-.16500*exp(-10000.*t)
pc_num =
-.82500*exp(-20000.*t)
```

Answer:

$i_C(t) = -165\ e^{-10000t}$ mA, t $ 0

$p_C(t) = -825\ e^{-20000t}$ mW, t $ 0

The capacitor is always delivering power because the power is negative.

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Problem 6-2

The voltage across a 1-nF capacitor is $v_C(t) = 20\cos(2\pi 10^4 t)$ V. Derive expressions for $i_C(t)$ and $p_C(t)$. Is the capacitor absorbing power, delivering power, or both?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms t vc ic pc
C = 1e-9;
vc = 20*cos(2*pi*1e4*t);
ic = simplify(C*diff(vc,t));
ic_num = vpa(ic,5)
pc = simplify(ic*vc);
pc_num = vpa(pc,5)

%tt = 0:1e-6:100e-6;
%pctt = subs(pc,t,tt);
%plot(tt,pctt)
ic_num =
-.12566e-2*sin(62832.*t)
pc_num =
-.25133e-1*sin(62832.*t)*cos(62832.*t)
```

Answer:

$i_C(t) = -1.2566\sin(2\pi 10^4 t)$ mA, t $ 0

$p_C(t) = -25.133\sin(2\pi 10^4 t)\cos(2\pi 10^4 t)$ mW $= -12.566\sin(2\pi\, 20000t)$ mW, t $ 0

The capacitor is both delivering and absorbing power because the power changes sign.

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Problem 6-3

For $t \geq 0$ the current through a 0.25-μF capacitor is $i_C(t) = 10 \sin(2000t)$ mA. Plot $v_C(t)$ versus time when $v_C(0) = -10$ V. *Hint:* In OrCAD use the ISIN source with the frequency in Hz and set the other parameters appropriately. Use the property editor to set the capacitor's initial condition to –10 V. Then plot the output using the transient function. Place a huge, 1000MEG resistor in parallel with the capacitor to avoid "floating nodes". *Caution:* In OrCAD rotating the capacitor so to make it vertical, places the capacitor's number two terminal at the top. When identifying an initial voltage on the capacitor the applied polarity of the voltage is between the number 1 terminal considered positive and the number two terminal considered negative. Hence, the capacitor needs to be "flipped vertically" to place the desired –10 V across the capacitor.

Solution:

The solution is presented in the following MATLAB code and then a second time using OrCAD.

```
clear all
syms t vc ic tau
C = 0.25e-6;
vc0 = -10;
ic = 10e-3*sin(2000*t);
vc = simplify(vc0 +
int(subs(ic,t,tau),tau,0,t)/C);
vc_num = vpa(vc,5)

% Create a numerical time vector to plot
the voltage
tt = 0:5e-6:8e-3;
vctt = subs(vc,t,tt);
plot(tt,vctt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
vc_num =
10.000-20.000*cos(2000.*t)
```

OrCAD simulation:

Answer:

$$v_C(t) = 10 - 20\cos(2000t) \text{ V}$$

The plots are presented above.

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Problem 6-4

The current through a 0.2-μF capacitor is a rectangular pulse with an amplitude of 2 mA and a duration of 4 ms. Find the capacitor voltage at the end of the pulse when the voltage at the beginning is – 2 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vc ic tau
C = 0.2e-6;
vc0 = -2;
ic = 2e-3*(heaviside(t)-heaviside(t-4e-3));
vc = simplify(vc0 + int(ic,t,0,4e-3)/C);
vc_num = vpa(vc,5)
vc_num =
38.000
```

Answer:

$v_C(4\text{ ms}) = 38\text{ V}$

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Problem 6-5

For t $ 0 the current through capacitor is $i_C(t) = 10\ r(t)$ mA. At $t = 0$ the capacitor voltage is 3 V. At $t = 1$ ms the voltage is 8 V. Find the capacitance of the device.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t ic vc C tau
% For t >= 0, we have the following expression for the current
ic = 10e-3*t;
% The initial voltage is given in the problem statement
vc0 = 3;
% Find an expression for the capacitor voltage
vc = vc0+int(subs(ic,t,tau),tau,0,t)/C
% Solve for the voltage expression at t = 1 ms
vc1ms = subs(vc,t,1e-3)
% Solve for C, given the voltage at t = 1 ms is 8 V.
C = double(solve(8-vc1ms,C))
vc =
3+1/200*t^2/C
vc1ms =
3+1/200000000/C
C =
     1.0000e-009
```

Answer:

$C = 1$ nF

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Problem 6-6

The voltage across a 100-nF capacitor is shown in Figure P6-6. Prepare sketches of $i_C(t)$ and $p_C(t)$. Is the capacitor absorbing power, delivering power, or both?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
C = 100e-9;
t1 = 10e-6;
t2 = 30e-6;
t3 = 40e-6;
syms t vc ic pc
vc = 1e6*t*heaviside(t) - 1e6*(t-
t1)*heaviside(t-t1)...
   - 1e6*(t-t2)*heaviside(t-t2) + 1e6*(t-
t3)*heaviside(t-t3);
ic = C*diff(vc,t);
pc = ic*vc;

tt = 0:200e-9:40e-6;
vctt = subs(vc,t,tt);
ictt = subs(ic,t,tt);
pctt = subs(pc,t,tt);

plot(tt,vctt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')


plot(tt,ictt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')


plot(tt,pctt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Power (W)')
```

Answer:

The plots are shown above. The capacitor is both absorbing and delivering power.

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Problem 6-7

The current through a 10-nF capacitor is shown in Figure P6-7. Given that $v_C(0) = !5$ V, at what time will the voltage $v_C(t)$ first reach 5 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t ic vc tau
C = 10e-9;
vc0 = -5;
% Compute the slope of the current
waveform
m = 20e-3/5e-6;
% Construct the current waveform
ic = m*t*heaviside(t) - 2*m*(t-5e-
6)*heaviside(t-5e-6) + m*(t-10e-
6)*heaviside(t-10e-6);
% Calculate the voltage as a function
of time
vc =
simplify(vc0+int(subs(ic,t,tau),tau,0,
t)/C);

% Plot the voltage waveform
tt = eps:100e-9:10e-6;
vctt = subs(vc,t,tt);
plot(tt,vctt,'b','LineWidth',3)
xlabel('Time (s)')
ylabel('Voltage (V)')
grid on
```

Answer:

$t = 10$ μs

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Problem 6-8

The current through a 10-mH inductor is shown in Figure P6-7. Prepare sketches of $v_L(t)$, $p_L(t)$, and $w_L(t)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t iL vL pL wL tau
L = 10e-3;
% Compute the slope of the current waveform
m = 20e-3/5e-6;
% Construct the current waveform
iL = m*t*heaviside(t) - 2*m*(t-5e-6)*heaviside(t-5e-6) + m*(t-10e-
6)*heaviside(t-10e-6);

% Calculate voltage, power, and energy
vL = simplify(L*diff(iL,t));
vL_num = vpa(vL,5);

pL = simplify(iL*vL);
pL_num = vpa(pL,5)

wL = simplify(L/2*iL^2);
wL_num = vpa(wL,5)

% Plot the signals
tt = eps:100e-9:10e-6;
vLtt = subs(vL,t,tt);
plot(tt,vLtt,'b','LineWidth',3)
xlabel('Time (s)')
ylabel('Voltage (V)')
grid on
```

```
vL_num =
40.*heaviside(t)-80.*heaviside(t-.50000e-5)+40.*heaviside(t-.10000e-4)
pL_num =
.16000e6*t*heaviside(t)-.16000e6*heaviside(t-.10000e-4)*t-1.6000*heaviside(t-
.50000e-5)+1.6000*heaviside(t-.10000e-4)
wL_num =
-.80000e-5*heaviside(t-.10000e-
4)+1.6000*heaviside(t-.10000e-4)*t-
80000.*heaviside(t-.10000e-
4)*t^2+.80000e-5*heaviside(t-
.50000e-5)-1.6000*heaviside(t-
.50000e-
5)*t+80000.*heaviside(t)*t^2


pLtt = subs(pL,t,tt);
plot(tt,pLtt,'b','LineWidth',3)
xlabel('Time (s)')
```

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```
ylabel('Power (W)')
grid on
```

```
wLtt = subs(wL,t,tt);
plot(tt,wLtt,'b','LineWidth',3)
xlabel('Time (s)')
ylabel('Energy (J)')
grid on
```

Answer:
The plots of the waveforms are presented above.

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Problem 6-9

For t $ 0 the current through a 100-mH inductor is $i_L(t) = 100\ e^{-10t}$ mA. Find $v_L(t)$, $p_L(t)$ and $w_L(t)$ for t $ 0. Is the inductor absorbing power, delivering power, or both?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t iL vL pL wL tau
L = 100e-3;
% Construct the current waveform
iL = 100e-3*exp(-10*t);

% Calculate voltage, power, and energy
vL = simplify(L*diff(iL,t));
vL_num = vpa(vL,5)

pL = simplify(iL*vL);
pL_num = vpa(pL,5)

wL = simplify(L/2*iL^2);
wL_num = vpa(wL,5)
vL_num =
-.10000*exp(-10.*t)
pL_num =
-.10000e-1*exp(-20.*t)
wL_num =
.50000e-3*exp(-20.*t)
```

Answer:

$v_L(t) = -100\ e^{-10\,t}$ mV

$p_L(t) = -10\ e^{-20\,t}$ mW

$w_L(t) = 500\ e^{-20\,t}$ μJ

The inductor is always delivering power, because its power is always negative.

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Problem 6-10

For t $ 0 the voltage across a 25-mH inductor is $v_L(t) = 20\,e^{!400t}$ V. Plot $i_L(t)$ versus time when $i_L(0) = 3$ A. *Hint:* In OrCAD use the VEXP source, set TD1 to zero and TD2 to 10 s, and set the other parameters appropriately. Use the property editor to set the inductor's initial condition to 3 A. Then plot the output using the transient function. Place a tiny, 1 nΩ resistor in series with the inductor to avoid an "inductor loop".

Solution:

The solution is presented in the following MATLAB code and then a second time using OrCAD.

```
clear all
format short eng
syms t vL iL tau
vL = 20*exp(-400*t);
L = 25e-3;
iL0 = 3;
iL = simplify(iL0 +
int(subs(vL,t,tau),tau,0,t)/L)

tt = 0:150e-6:15e-3;
iLtt = subs(iL,t,tt);
plot(tt,iLtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
iL =
5-2*exp(-400*t)
```

OrCAD simulation:

Answer:

The solutions are presented above.

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Problem 6-11

A voltage $v_L(t) = 5\cos(1000\,n\,t)$ V appears across a 50-mH inductor, where n is a positive integer that controls the frequency of the input signal. The amplitude of the input signal is constant. Assume $i_L(0) = 0$ A. Use MATLAB and symbolic variables to compute an expression for $i_L(t)$. On the same axes, plot $i_L(t)$ versus time for n = 1, 2, 3, 4, and 5, over an appropriate time scale. On another set of axes, plot the amplitude of $i_L(t)$ versus the coefficient n. As n approaches infinity, what happens to the amplitude of the current? What type of circuit element does the inductor behave like as n approaches infinity?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t tau n real

L = 50e-3;
iL0 = 0;

vL = 5*cos(1000*n*t);

% Solve for the current in terms of n and t
iL = simplify(iL0 + int(subs(vL,t,tau),tau,0,t)/L)
iL_num = vpa(iL,5)

% Evaluate the current over a
suitable time span
tt = 0:0.0001:0.01;
iLtt = subs(iL,t,tt);
PlotColor = ['b','g','r','m','c'];

% Plot the result for different
values of n
for N = 1:5
    iLtt2 = subs(iLtt,n,N);

plot(tt,iLtt2,PlotColor(N),'LineWidth
',3)
    grid on
    hold on
end
iL =
1/10/n*sin(1000*n*t)
iL_num =
.10000/n*sin(1000.*n*t)
```

Based on the results above we have $i_L(t) = \dfrac{1}{10n}\sin(1000nt)$ A, so the amplitude is $1/10n$.

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```
% Plot the amplitude versus n
nn = 1:20;
iAmp = 1/10./nn;

figure
plot(nn,iAmp,'b','LineWidth',3)
grid on
xlabel('n')
ylabel('Amplitude')
```

Answer:

$$i_{\mathrm{L}}(t) = \frac{1}{10n}\sin(1000nt)$$

As n approaches infinity, the amplitude of the current approaches zero and the inductor acts like an open circuit.

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Problem 6-12

For $t \geq 0$ the voltage across a 100-mH inductor is $v_L(t) = 1000r(t)$ V. At $t = 5$ ms the inductor current is observed to be zero. Find the value of $i_L(0)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t tau vL iL iL0
L = 100e-3;
vL = 1000*t*heaviside(t);

iL = simplify(iL0 + int(subs(vL,t,tau),tau,0,t)/L)

iL5ms = subs(iL,t,5e-3)
iL0 = solve(iL5ms,iL0)
iL0_num = double(iL0)
iL =
iL0+5000*heaviside(t)*t^2
iL5ms =
iL0+1/8
iL0 =
-1/8
iL0_num =
  -125.0000e-003
```

Answer:

$i_L(0) = -125$ mA.

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Problem 6-13

For t ≥ 0 the current through a 20-mH inductor is $i_L(t) = 200te^{-1000t}$ A. Derive an expression for $v_L(t)$. Is the inductor absorbing power or delivering power or both?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vLt iLt pLt wLt real
L = 20e-3;
iLt = 200*t*exp(-1000*t);
vLt = L*diff(iLt,t)
pLt = simplify(iLt*vLt)

% Plot the power to determine if it is positive or negative
tt = 0:0.000005:0.0025;
pLtt = subs(pLt,t,tt);
plot(tt,pLtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Power (W)')
vLt =
4*exp(-1000*t)-4000*t*exp(-1000*t)
pLt =
-800*t*exp(-2000*t)*(-1+1000*t)
```

Answer:

$v_L(t) = 4\, e^{-1000\, t}\, (1\text{-}1000t)$ V

The inductor is both absorbing and delivering power.

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Problem 6-14

The capacitor in Figure P6-14 carries an initial voltage $v_C(0) = 50$ V. At $t = 0$, the switch is closed, and thereafter the voltage across the capacitor is $v_C(t) = 100 - 50\,e^{-2000t}$ V. Derive expressions for $i_C(t)$ and $p_C(t)$ for $t > 0$. Is the capacitor absorbing power, delivering power, or both?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t ic vc pc
vc = 100-50*exp(-2000*t);
C = 10e-6;
ic = simplify(C*diff(vc,t))
pc = simplify(ic*vc)
% Plot the plot over a suitable time range
tt = 0:30e-6:3e-3;
pctt = subs(pc,t,tt);
plot(tt,pctt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Power (W)')
ic =
exp(-2000*t)
pc =
-50*exp(-2000*t)*(-2+exp(-2000*t))
```

Answer:

$i_C(t) = e^{-2000t}$ A and $p_C(t) = 50[2e^{-2000t} - e^{-4000t}]$ Wfor $t > 0$.

The capacitor is always absorbing power.

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Problem 6-15

A 10-μF capacitor and a 100-mH inductor are connected in parallel with a closed switch. The inductor has 50 mA flowing through it when the switch opens at $t = 0$. (a) Find the initial voltage across the capacitor at $t = 0$. (b) Simulate the circuit using OrCAD. Connect an inductor in parallel with a capacitor and assign the appropriate initial conditions. Plot the voltage across the elements for at least 20 ms. (c) Characterize the response signal.

Solution:

(a) The initial voltage across the capacitor is 0 V, because it is connected in parallel with a closed switch.

(b) OrCAD simulation:

(c) The response signal is sinusoidal. The exact response is $v_C(t) = -5\sin(1000t)$ V for $t \geq 0$.

Answer:

(a) $v_C(0) = 0$ V.

(b) Presented above.

(c) The response is sinusoidal with $v_C(t) = -5\sin(1000t)$ V for $t \geq 0$.

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Problem 6-16

The inductor in Figure P6-16 carries an initial current of $i_L(0) = 0.5$ A. At $t = 0$, the switch opens, and thereafter the current into the rest of the circuit is $i(t) = !0.5\, e^{!2000t}$ A. Derive expressions for $v_L(t)$and $p_L(t)$ for $t > 0$. Is the inductor absorbing or delivering power?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vL iL pL
L = 200e-6;
% With the switch open iL(t) = -i(t)
iL = 0.5*exp(-2000*t);
vL = simplify(L*diff(iL,t))
pL = simplify(iL*vL)
vL =
-1/5*exp(-2000*t)
pL =
-1/10*exp(-4000*t)
```

Answer:

$v_L(t) = -200e^{-2000t}$ mV.

$p_L(t) = -100e^{-4000t}$ mW.

The inductor is delivering power.

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Problem 6-17

The inductor in Figure P6-16 carries an initial current of $i_L(0) = 30$ mA. At $t = 0$, the switch opens, and thereafter the voltage across the inductor is $v_L(t) = !8\, e^{!1000t}$ mV. Derive expressions for $i_L(t)$and $p_L(t)$ for $t > 0$. Is the inductor absorbing or delivering power?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t iL vL pL tau
L = 200e-6;
iL0 = 30e-3;
vL = -8e-3*exp(-1000*t);
iL = simplify(iL0+int(subs(vL,t,tau),tau,0,t)/L)
pL = simplify(iL*vL)

% Plot the power versus time
tt = 0:10e-6:5e-3;
pLtt = subs(pL,t,tt);
plot(tt,pLtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Power (W)')
iL =
-1/100+1/25*exp(-1000*t)
pL =
-1/12500*(-1+4*exp(-1000*t))*exp(-1000*t)
```

Answer:

$i_L(t) = (40\, e^{-1000\, t} - 10)$ mA.

$p_L(t) = 80\, e^{-1000\, t}\, (1 - 4\, e^{-1000\, t})$ μW.

The inductor is both absorbing and delivering power.

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Problem 6-18

A 3.3-μF capacitor is connected in series with a 1-kΩ resistor. The voltage across the capacitor is $v_C(t) = 10\cos(2000t)$ V. What is the voltage across the resistor?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vc ic vR
C = 3.3e-6;
R = 1e3;
vc = 10 *cos(2000*t);
% Find the current through the capacitor
ic = simplify(C*diff(vc,t));
% The current through the resistor must be ic(t) because it is in series
% with the capacitor
vR = R*ic
vR_num = vpa(vR,5)
vR =
-60874255443241523437 5/9223372036854775808*sin(2000*t)
vR_num =
-66.000*sin(2000.*t)
```

Answer:

$v_R(t) = -66\sin(2000t)$ V

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Problem 6-19

A 100-mH inductor is connected in parallel with a 33-kΩ resistor. The current through the inductor is $i_L(t) = 20\ e^{!1000t}$ mA. What is the current through the resistor?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t iL vL iR
L = 100e-3;
R = 33e3;
iL = 20e-3*exp(-1000*t);
vL = simplify(L*diff(iL,t));
% Resistor voltage matches the inductor voltage because
% they are in parallel
iR = simplify(vL/R)
iR_num = vpa(iR,5)
iR =
-1/16500*exp(-1000*t)
iR_num =
-.60606e-4*exp(-1000.*t)
```

Answer:

$i_R(t) = -60.61\ e^{!1000\,t}\ \mu A$

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Problem 6-20

For $t > 0$ the voltage across an energy storage element is $v(t) = 5 \; ! \; 20\, e^{!500t}$ V and the current through the element is $i(t) = 2000t + 16\, e^{!500t}$ mA. What is the element and the element value?

Solution:

If the element is a capacitor, then the current is proportional to the derivative of the voltage. If the element is an inductor, then the voltage is proportional to the derivative of the current. By inspection, the derivative of the voltage will contain only any exponential term and cannot match the current. On the other hand, the derivative of the current will have a constant value and an exponential term, so it can match the voltage. Since the voltage is proportional to the derivative of the current, the element is an inductor. The following MATLAB code calculates the associated inductance.

```
clear all
syms t iL vL
vL = 5-20*exp(-500*t);
iL = 2000e-3*t+16e-3*exp(-500*t);
diL = diff(iL,t)
L = simplify(vL/diL)
diL =
2-8*exp(-500*t)
L =
5/2
```

Answer:

The element is an inductor with $L = 2.5$ H.

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Problem 6-21

The OP AMP integrator in Figure P6-21 has R = 40 kΩ, C = 500 nF, and $v_O(0)$ = 10 V. The input is $v_S(t) = 5\, e^{!500t}\, u(t)$ V. Find $v_O(t)$ for $t > 0$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vs vo tau
R = 40e3;
C = 500e-9;
vo0 = 10;
vs = 5*exp(-500*t);
vo = simplify(vo0 - int(subs(vs,t,tau),tau,0,t)/R/C)
vo_num = vpa(vo,5)
vo =
700976274800962928018/73786976294838203125+36893488147419103232/73786976294838
203125*exp(-500*t)
vo_num =
9.5000+.50000*exp(-500.*t)
```

Answer:

$v_O(t) = 9.5 + 0.5\, e^{-500\, t}$ V for $t > 0$.

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Problem 6-22

An OP AMP integrator with R = 1 MΩ, C = 1 μF, and $v_O(0)$ = 0 V has the input waveform shown in Figure P6-22. Sketch $v_O(t)$ for $t > 0$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vs vo tau
R = 1e6;
C = 1e-6;
vo0 = 0;
vs = -2.5*(heaviside(t)-heaviside(t-
20e-3))...
     +5*(heaviside(t-20e-3) - heaviside(t-30e-3));
vo = simplify(vo0 -
int(subs(vs,t,tau),tau,0,t)/R/C);
% Plot the output voltage
tt = 0:40e-6:40e-3;
vott = subs(vo,t,tt);
plot(tt,vott,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answer:

Presented above.

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Problem 6-23

The OP AMP integrator in Figure P6-21 has R = 50 kΩ, C = 100 μF, and $v_O(0)$ = –2 V. The input is $v_S(t)$ = 10 $u(t)$ V. Use OrCAD to determine how long it takes for the OP AMP to saturate when V_{CC} = ±15 V.

Solution:

The solution is presented in the following MATLAB code and then confirmed with the OrCAD simulation.

```
clear all
syms t positive
syms vs vo vf tau
R = 50e3;
C = 100e-6;
vo0 = -2;
vs = 10*heaviside(t);
vo = simplify(vo0 -
int(subs(vs,t,tau),tau,0,t)/R/C)

% Solve for the time
vf = 15+vo
Tf = solve(vf)

% Plot the output voltage
tt = 0:80e-3:8;
vott = subs(vo,t,tt);
plot(tt,vott,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
vo =
-2-2*t
vf =
13-2*t
Tf =
13/2
```

OrCAD simulation:

Answer:

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$t = 6.5$ s

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Problem 6-24

The OP AMP integrator in Figure P6-21 has R = 20 kΩ, C = 100 nF, and $v_O(0)$ = 0 V. The input is $v_S(t) = 5 \sin(\omega t)\, u(t)$ V. Derive an expression for $v_O(t)$ and find the smallest allowable value of ω for linear operation of the OP AMP. Assume V_{CC} = "15 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms w t positive
syms vs vo vf tau
R = 20e3;
C = 100e-9;
vo0 = 0;
vs = 5*sin(w*t);
vo = simplify(vo0 - int(subs(vs,t,tau),tau,0,t)/R/C)
vo =
2500*(-1+cos(w*t))/w
```

Given the signal for $v_O(t)$, the peak voltage is 5000/ω V. To maintain linear operation for the OP AMP, the peak voltage must remain below 15 V, so ω > 1000/3.

Answer:

$$v_O(t) = \frac{2500}{\omega}\left[\cos(\omega t) - 1\right] \text{ V}$$

$$\omega > \frac{1000}{3} \text{ rad/s}$$

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Problem 6-25

The OP AMP differentiator in Figure P6-25 with R = 10 kΩ and C = 500 nF has the input $v_S(t) = 6(1 - e^{-50t})u(t)$ V. Find $v_O(t)$ for $t > 0$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t positive
syms vs vo
R = 10e3;
C = 500e-9;
vs = 6*(1-exp(-50*t));
vo = simplify(-R*C*diff(vs,t))
vo_num = vpa(vo,5)
vo =
-3/2*exp(-50*t)
vo_num =
-1.5000*exp(-50.*t)
```

Answer:

$v_O(t) = -1.5e^{-50t}$ for $t > 0$.

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Problem 6-26

The OP AMP differentiator in Figure P6-25 with R = 50 kΩ and C = 20 nF has the input waveform shown in Figure P6-26. Sketch $v_O(t)$ for $t > 0$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t positive
syms vs vo
R = 50e3;
C = 20e-9;
% Define the slopes of the input signal
m1 = 10/2e-3;
m2 = 10/1e-3;
vs = m1*t*heaviside(t) - m1*(t-2e-3)*heaviside(t-2e-3)...
    -m2*(t-2e-3)*heaviside(t-2e-3) + m2*(t-3e-3)*heaviside(t-3e-3);
vo = simplify(-R*C*diff(vs,t));

tt = 0:10e-6:4e-3;
vott = subs(vo,t,tt);
plot(tt,vott,'b','LineWidth',3)
grid on
axis([-1e-3 4e-3 -10 15])
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answer:

Presented at right.

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Problem 6-27

The input to the OP AMP differentiator in Figure P6-25 is $v_S(t) = 10\,[\sin(2\pi\times10^6 t)]\,u(t)$ mV. Select R and C so that the output sinusoid has a maximum of at least ±14 V but does not saturate the OP AMP at ± 15 V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms t R C positive
syms vs vo
vs = 10e-3*sin(2*pi*1e6*t);
vo = simplify(-R*C*diff(vs,t));
vo_num = vpa(vo,5)
vo_num =
-62832.*R*C*cos(.62832e7*t)

% The output amplitude is R*C*2*pi*10e3
% Select a value for C and solve for R for an amplitude of 14.5 V
C_num = 100e-9
vA = R*C_num*2*pi*10e3;
R_num = double(solve(14.5-vA,R))
C_num =
   100.0000e-009
R_num =
     2.3077e+003

% Find the general solution
RC_min = 14/2/pi/10e3
RC_max = 15/2/pi/10e3
RC_min =
   222.8169e-006
RC_max =
   238.7324e-006
```

Answer:

Choose C = 100 nF and R = 2.3077 kΩ. There are other correct answers.

In general, choose R and C such that $222.82 \times 10^{-6} < RC < 238.73 \times 10^{-6}$.

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Problem 6-28

The OP AMP differentiator in Figure P6-25 with R = 10 kΩ and C = 100 pF has the input $v_S(t) = 2[\sin(\omega t)]u(t)$ V. Determine the frequency at which the OP AMP saturates at "15 V. Validate your answer using OrCAD. *Hint:* Use VAC source and an AC sweep.

Solution:

The solution is presented in the following MATLAB code and then verified with an OrCAD simulation.

```
clear all
format short eng
syms t w positive
syms vs vo
R = 10e3;
C = 100e-12;
vs = 2*sin(w*t);
vo = simplify(-R*C*diff(vs,t))
vo_num = vpa(vo,5)
vo =
-1/500000*cos(w*t)*w
vo_num =
-.20000e-5*cos(w*t)*w
```

The magnitude of the output signal is ω/500000 rad/s. Solve for the maximum frequency before saturation.

```
w_max = double(solve(w/500000 - 15,w))
w_max =
     7.5000e+006
```

Confirm the answer with an OrCAD simulation.

Answer:

ω_{MAX} = 7.5 Mrad/s

f_{MAX} = 1.1937 MHz

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Problem 6-29

Find the input-output relationship of the *RC* OP AMP circuit in Figure P6-29.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Write a node-voltage equation at the negative input terminal
% of the OP AMP.
% Use the variable dvs to represent the derivative of vs.
syms vs dvs R C vo ic
Eqn1 = 'ic - C*dvs';
Eqn2 = 'vs/R + ic + (vs-vo)/R';
Soln = solve(Eqn1,Eqn2,'vo','ic');
vo = simplify(Soln.vo)
vo =
2*vs+C*dvs*R
```

Answer:

$$v_O(t) = 2v_S(t) + RC\frac{dv_S(t)}{dt}$$

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Problem 6-30

Show that the RC OP AMP circuit in Figure P6-30 is a noninverting integrator whose input-output relationship is

$$v_O(t) = \frac{1}{RC}\int_0^t v_S(x)dx + v_O(0)$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Let va = vp = vn and write node-voltage equations at the
% input terminals of the OP AMP.
% Use dva, dvs, and dvo to represent the derivatives of the terminal input,
% source, and output signals, respectively

syms va vs vo dva dvs dvo R C
Eqn1 = '(va-vs)/R + C*dva';
Eqn2 = 'va/R + C*(dva-dvo)';
Soln = solve(Eqn1,Eqn2,'dvo','va');
dvo = simplify(Soln.dvo)
dvo =
vs/R/C
```

$$\frac{dv_O(t)}{dt} = \frac{v_S(t)}{RC}$$

$$v_O(t) = \frac{1}{RC}\int_0^t v_S(x)\,dx + v_O(0)$$

Answer:

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Problem 6-31

Design an *RC* OP AMP circuit to implement the block diagram in Figure P6-31.

Solution:

The block diagram has two paths joined by an adder. If we create an inverting differentiator on the top path and an inverting integrator on the bottom path and connect them with an inverting summer, the final signs for each path will be correct. For each path, select $C = 1\ \mu F$ and $R = 50\ k\Omega$. The top path then needs an additional gain of 1000 and the bottom path needs an additional attenuation of 1/1000. The following schematic shows one possible solution.

Answer:

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Problem 6-32
Repeat problem 6-31 except use an *RL* OP AMP circuit.

Solution:
An *RL* integrator has the following format and equation:

$$v_O(t) = v_O(0) - \frac{R}{L}\int_0^t v_S(\tau)\,d\tau$$

An *RL* differentiator has the following format and equation:

$$v_O(t) = -\frac{L}{R}\frac{dv_S(t)}{dt}$$

The complete circuit, using the same approach as in Problem 6-31 is shown below:

Answer:

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Problem 6-33

Design an *RC* OP AMP circuit to implement the input-output relationship

$$v_O(t) = -10\ v_S(t) + 50\int_0^t v_S(x)dx$$

Solution:

The following schematic shows one possible solution.

Answer:

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Problem 6-34

Design an OP AMP circuit to solve the following differential equation

$$v_O(t) = 5\,v_S(t) + \frac{1}{20}\,\frac{dv_S(t)}{dt}$$

Solution:

The following schematic presents one possible solution:

Answer:

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Problem 6-35

Design an RC circuit using only one OP AMP and only one capacitor that implements the input-output relationship

$$v_O(t) = -5\int_0^t v_{S1}(x)\,dx - 10\int_0^t v_{S2}(x)\,dx$$

Solution:

The equation represents an integrating summer and the following schematic shows one possible solution. Use superposition to verify the schematic matches the equation.

Answer:

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Problem 6-36

Find a single equivalent element for each circuit in Figure P6-36.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Circuit C1')
C1 = 10e-6;
C2 = 3.3e-6;
C3 = 1e-6;
C4 = 2.2e-6;
Ceq1 = C1 + C2;
Ceq2 = C3 + C4;
Ceq = 1/(1/Ceq1 + 1/Ceq2)

disp('Circuit C2')
L1 = 150e-6;
L2 = 25e-6;
L3 = 50e-6;
L4 = 100e-6;
L5 = 100e-6;
Leq1 = L2 + L3;
Leq2 = 1/(1/L4 + 1/L5);
Leq3 = 1/(1/Leq1 + 1/Leq2);
Leq = L1 + Leq3
Circuit C1
Ceq =
     2.5794e-006
Circuit C2
Leq =
   180.0000e-006
```

Answer:

Circuit C1, C_{EQ} = 2.5794 μF

Circuit C2, L_{EQ} = 180 μH

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Problem 6-37

A 100-μH inductor is connected in series with a 1-mH inductor and the combination connected in parallel with a 2-mH inductor. Find the equivalent inductance of the connection.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
L1 = 100e-6;
L2 = 1e-3;
L3 = 2e-3;
Leq1 = L1 + L2;
Leq = 1/(1/Leq1 + 1/L3)
Leq =
   709.6774e-006
```

Answer:

L_{EQ} = 709.6774 μH.

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Problem 6-38
Verify Eqs. (6-30) and (6-31).

Solution:
For a series connection of *N* inductors, the total voltage drop is the sum of the individual voltages

$$v(t) = v_1(t) + v_2(t) + \ldots + v_{\mathrm{N}}(t)$$

The current through each inductor in series is the same

$$i_1(t) = i_2(t) = \ldots = i_{\mathrm{N}}(t) = i(t)$$

Express each voltage in terms of the common current and solve for the equivalent inductance

$$v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + \ldots + L_{\mathrm{N}} \frac{di(t)}{dt} = \left(L_1 + L_2 + \ldots + L_{\mathrm{N}}\right)\frac{di(t)}{dt} = L_{\mathrm{EQ}} \frac{di(t)}{dt}$$

For a parallel connection of *N* inductors, the total current is the sum of the individual currents

$$i(t) = i_1(t) + i_2(t) + \ldots + i_{\mathrm{N}}(t)$$

The voltage across each inductor in parallel is the same

$$v_1(t) = v_2(t) = \ldots = v_{\mathrm{N}}(t) = v(t)$$

Express each current in terms of the common voltage and solve for the equivalent inductance

$$i(t) = i_1(0) + \frac{1}{L_1}\int_0^t v(x)dx + i_2(0) + \frac{1}{L_2}\int_0^t v(x)dx + \ldots + i_N(0) + \frac{1}{L_{\mathrm{N}}}\int_0^t v(x)dx$$

$$i(t) = \left[i_1(0) + i_2(0) + \ldots + i_N(0)\right] + \left(\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_{\mathrm{N}}}\right)\int_0^t v(x)dx$$

$$\frac{1}{L_{\mathrm{EQ}}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_{\mathrm{N}}}$$

Answer:
Presented above.

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Problem 6-39

What is the equivalent capacitance and initial voltage of a series connection of a 1-μF capacitor with 10 V stored and a 3.3-μF capacitor with 15 V stored?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
C1 = 1e-6;
C2 = 3.3e-6;
v10 = 10;
v20 = 15;

Ceq = 1/(1/C1 + 1/C2)
veq = v10 + v20
Ceq =
   767.4419e-009
veq =
    25.0000e+000
```

Answer:

C_{EQ} = 767.4419 nF.

$v_C(0)$ = 25 V

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Problem 6-40

For the circuit in Figure P6-40, find an equivalent circuit consisting of one inductor and one capacitor.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
C1 = 10e-6;
C2 = 6e-6;
C3 = 3e-6;
L1 = 0.5;
L2 = 3;
L3 = 3;

Ceq = C1 + 1/(1/C2 + 1/C3)
Leq = L1 + 1/(1/L2 + 1/L3)
Ceq =
    12.0000e-006
Leq =
     2.0000e+000
```

Answer:

The equivalent circuit is a 12-μF capacitor in parallel with a 2-H inductor.

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Problem 6-41

Figure P6-41 is the equivalent circuit of a two-wire feed through capacitor.

(a) What is the capacitance between terminal 1 and ground when terminal 2 is open?

(b) What is the capacitance between terminal 1 and ground when terminal 2 is grounded?

Solution:

(a) The following equivalent circuit clarifies the configuration:

```
clear all
C1 = 25e-9;
C2 = 50e-9;
C3 = 25e-9;
Ceq = C1 + 1/(1/C2 + 1/C3)
Ceq =
    41.6667e-009
```

(b) The following equivalent circuit clarifies the configuration:

```
clear all
C1 = 25e-9;
C2 = 50e-9;
C3 = 25e-9;
Ceq = C1 + C2
Ceq =
    75.0000e-009
```

Answer:

(a) C_{EQ} = 41.67 nF;

(b) C_{EQ} = 75 nF

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Problem 6-42

A capacitor bank is required that can be charged to 5 kV and store at least 250 J of energy. Design a series/parallel combination that meets the voltage and energy requirements using 20-μF capacitors each rated at 2 kV max.

Solution:

To be able to handle 5 kV, we need to combine three capacitors in series, which will have a maximum capacity of 6 kV. The series combination of three 20-μF capacitors will have an equivalent capacitance of 20/3 μF. To be able to store at least 250 J at 5 kV, we need an equivalent capacitance of at least 20 μF, as shown in the following calculations

```
clear all
C = 20e-6;
% Three capacitors in series
Ceq_series = 1/(1/C + 1/C + 1/C)

% Minimum capacitance requirement
% Solve for C in w = 0.5*C*v^2
w = 250;
vC = 5000;
C_min = 2*w/vC^2
Ceq_series =
     6.6667e-006
C_min =
    20.0000e-006
```

If each series combination of three capacitors is equivalent to 20/3 μF, then we will need a parallel combination of three of the series branches to produce the minimum equivalent capacitance. The following schematic shows one possible solution.

Answer:

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Problem 6-43

A switching power supply requires an inductor that can store at least 1 mJ of energy. A list of available inductors is shown below. Select the inductor that best meets the requirement. Consider both meeting the specifications and cost. Explain your choice.

L(μH)	I_{max}(A)	Cost each
10	9.3	$2.75
20	7.2	$3.00
50	5.5	$2.50
100	4.5	$2.75
150	3.5	$2.50
250	2.6	$2.75
500	1.8	$3.00

Solution:

The maximum energy that can be stored in an inductor is

$$w_{\text{MAX}} = \frac{1}{2} L i_{\text{MAX}}^2$$

Use Excel to perform the required calculations

L (microH)	iMAX (A)	Cost	wLMAX (mJ)
10	9.3	$2.75	0.43245
20	7.2	$3.00	0.51840
50	5.5	$2.50	0.75625
100	4.5	$2.75	1.01250
150	3.5	$2.50	0.91875
250	2.6	$2.75	0.84500
500	1.8	$3.00	0.81000

Answer:

Only the 100-μH inductor meets the requirement, so it is the only option available.

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Problem 6-44

The circuits in Figure P6-44 are driven by dc sources. Find the current through the 100-Ω resistor under dc conditions.

C1

C2

Solution:

With dc sources, inductors act like short circuits and capacitors act like open circuits. The following equivalent circuits clarify the resulting connections.

Circuit C1:

```
clear all
R1 = 100;
R2 = 220;
vs = 15;
i100= vs/(R1+R2)
i100 =
    46.8750e-003
```

Circuit C2:

The circuit has no closed paths, so no current flows.

Answer:

Circuit C1: $i(t)$ = 46.875 mA

Circuit C2: $i(t)$ = 0 mA

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Problem 6-45

The circuit in Figure P6-45 is driven by 15-V dc source. Find the energy stored in the capacitor and inductor under dc conditions.

Solution:

Under dc conditions, the capacitor acts like an open circuit and the inductor acts like a short circuit. Since the capacitor and inductor are connected in parallel and the inductor acts like a short circuit, there is no voltage drop across either the capacitor or the inductor. Therefore, the capacitor does not store any energy. The current through the inductor is the same as the current through the resistor and is 15 V/50 Ω = 300 mA. The following MATLAB code computes the energy stored in the inductor.

```
clear all
format short eng
vs = 15;
R = 50;
L = 5e-3;
iL = vs/R
wL = L*iL*iL/2
iL =
   300.0000e-003
wL =
   225.0000e-006
```

Answer:

$w_C = 0$ J

$w_L = 225$ μJ

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Problem 6-46

The OP AMP circuit in Figure P6-46 has a capacitor in its feedback loop. Determine the circuit gain at dc and as the frequency approaches ∞ rad/s.

Solution:

Without the capacitor, the circuit is connected as a non-inverting amplifier. At dc, the capacitor acts like an open circuit, so it is effectively not a part of the circuit. In this case, the gain of the circuit will be $(R + R)/R = 2$. As the frequency approaches ∞ rad/s, the capacitor acts like a short circuit. To verify this fact, let the current through the capacitor be $i_C(t) = \cos(\omega t)$ A. We can then compute the voltage across the capacitor as follows

```
clear all
syms t w ic vc C tau
ic = cos(w*t)
vc0 = 0;
vc = vc0 + int(subs(ic,t,tau),tau,0,t)/C
ic =
cos(w*t)
vc =
1/w*sin(w*t)/C
```

The amplitude of the voltage across the capacitor is proportional to $1/\omega$, so as the frequency increases, the voltage approaches zero for a constant current. This is exactly the characteristic of a short circuit. The combination of a short circuit in parallel with the feedback resistor is essentially a short circuit, so the gain of the circuit will be $(0 + R)/R = 1$.

Answer:

At dc, $K = 2$.

As the frequency approaches ∞ rad/s, $K = 1$.

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Problem 6-47

Piezoelectric Transducer **(A)**

Piezoelectric transducers (sensors) measure dynamic phenomena such as pressure and force. These phenomena cause stresses that "squeeze" a quantity of electric charge from piezoelectric material in the transducer (the term piezo means "squeeze" in Greek). The amount of charge $q(t)$ is directly proportional to the measured variable $x(t)$, that is $q(t) = \alpha x(t)$. Signal amplification is needed because the amount of charge produced is on the order of pC. Figure P6-47 shows an OP AMP charge amplifier that provides the necessary gain. First show that the OP AMP output is $v_O(t) = !Kq(t)$. Then select a value of C so that the charge amplifier gain is $K = 5$ mV ' pC.

Solution:

In the circuit, $v_N = v_P = 0$ V, because the positive input terminal is grounded. We can write the following node-voltage equation at the negative input terminal and solve for the output voltage.

$$-\frac{dq(t)}{dt} + i_C = 0$$

$$-\frac{dq(t)}{dt} + C\frac{d[0 - v_O(t)]}{dt} = 0$$

$$C\frac{dv_O(t)}{dt} = -\frac{dq(t)}{dt}$$

$$v_O(t) = -\frac{1}{C}q(t)$$

The required relationship holds with $K = 1/C$. To get $K = 5$ mV ' pC $= 5 \times 10^9$ V/C, we need to choose $C = 200$ pF.

Answer:

The output relationship is derived above and the required capacitance is $C = 200$ pF.

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Problem 6-48

LC Circuit Response **(A)**

At t = 0 the switch in Figure P6-48 is closed and thereafter the voltage across the capacitor is

$$v_C(t) = (20 + 10000t)\, e^{-8000\,t} \text{ V}$$

Use MATLAB to solve all of the following problems.

(a) Use the capacitor's i-v characteristic to find the current $i(t)$ for t $ 0.

(b) Use the inductor's i-v characteristic and $i(t)$ to find $v_L(t)$ for t $ 0.

(c) Use $v_C(t)$, $v_L(t)$, and KVL to find the voltage $v(t)$ delivered to the rest of the circuit.

(d) The $v(t)$ found in (c) should be proportional to the $i(t)$ found in (a). If so, what is the equivalent resistance looking into the rest of the circuit?

(e) On the same axes, plot $v_C(t)$, $v_L(t)$, and $v(t)$. Use a different color for each waveform. Use the plots to verify KVL for the circuit.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t positive
syms vC iC vL vt it Req
L = 2.5e-3;
C = 25e-6;
vC = (20+10000*t)*exp(-8000*t);

disp('Part (a)')
iC = simplify(C*diff(vC,'t'));
it = -iC

disp('Part (b)')
vL = simplify(L*diff(it,'t'))

disp('Part (c)')
% KVL;  -vC + vL + vt = 0
vt = simplify(vC - vL)

disp('Part (d)')
Req = simplify(vt/it)

disp('Part (e)')
tt = 0:0.000001:0.0006;
```

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```
vCtt = subs(vC,t,tt);
vLtt = subs(vL,t,tt);
vtt = subs(vt,t,tt);

plot(tt,vCtt,'b','LineWidth',3);
hold on
grid on
plot(tt,vLtt,'g','LineWidth',3);
plot(tt,vtt,'r','LineWidth',3);
xlabel('Time (s)')
ylabel('Voltage (V)')
legend('v_C(t)','v_L(t)','v(t)')
Part (a)
it =
5/4*exp(-8000*t)*(3+1600*t)
Part (b)
vL =
-10*exp(-8000*t)*(7+4000*t)
Part (c)
vt =
10*exp(-8000*t)*(9+5000*t)
Part (d)
Req =
8*(9+5000*t)/(3+1600*t)
Part (e)
```

Answer:

(a) $i(t) = \frac{5}{4}[3+1600t]e^{-8000\,t}$

(b) $v_{\mathrm{L}}(t) = -10[7+4000t]e^{-8000\,t}$

(c) $v(t) = 10[9+5000t]e^{-8000\,t}$

(d) $R_{\mathrm{EQ}} = \frac{8(9+5000t)}{3+1600t}$

(e) Presented above.

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Problem 6-49

Supercapacitor **(A)**

Supercapacitors have very large capacitances (typically from 0.1 to 50 F,) very long charge holding times, and small sizes making them useful in non-battery backup power applications. To measure its capacitance, a supercapacitor is charged to an initial voltage V_0 = 5.5 V. At t = 0 the device undergoes a constant current discharge of i_D = 2 mA. At t = 2500 s the voltage remaining on the capacitor is 3 V. Find the device capacitance.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
iD = 2e-3;
% The capacitor current is opposite of the discharge current
iC = -iD;
% Find the time rate of change of the capacitor voltage
dv = 3-5.5;
dt = 2500-0;
dvdt = dv/dt;
% Use the capacitor i-v characteristic to solve for the capacitance
C = iC/dvdt
C =
     2.0000e+000
```

Answer:

C = 2 F.

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Problem 6-50

Analog Computer Solution **(D)**

Design an OP AMP circuit that solves the following second-order differential equation for $v_O(t)$. Solve for the response for $v_O(t)$ using OrCAD. *Caution:* Avoid saturating the OP AMPs by distributing the gain across several OP AMPs.

$$10^{-6}\frac{d^2v_O(t)}{dt^2}+\tfrac{1}{2}\times10^{-3}\frac{dv_O(t)}{dt}+v_O(t)=2\,u(t)$$

Solution:

Reformulate the equation as an integral equation.

$$10^{-6}\frac{d^2v_O(t)}{dt^2}=-\tfrac{1}{2}\times10^{-3}\frac{dv_O(t)}{dt}-v_O(t)+2\,u(t)$$

$$\frac{d^2v_O(t)}{dt^2}=-500\frac{dv_O(t)}{dt}-10^6v_O(t)+2\times10^6\,u(t)$$

$$v_O(t)=500\int\left\{-v_O(t)+2000\left[\int\left(-v_O(t)+2\,u(t)\right)\right]\right\}$$

Re-write the last equation into a form that will be easier to implement with OP AMPs.

$$v_O(t)=(-10)(-50)\int\left\{-v_O(t)+(-40)(-50)\left[\int\left(-v_O(t)-(-2)\,u(t)\right)\right]\right\}$$

Now create a circuit that will perform the integration.

Set the initial voltages on both capacitors to zero. The resulting output voltage is shown in the following plot.

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We can verify these results using MATLAB as follows.

```
clear all
syms vo t
vo = dsolve('1e-6*D2vo+0.5e-3*Dvo+vo=2','vo(0)=0','Dvo(0)=0')
pretty(vo)
% Plot the results
tt = 0:50e-6:25e-3;
vott = subs(vo,t,tt);
plot(tt,vott,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
vo =
-2/15*exp(-250*t)*sin(250*15^(1/2)*t)*15^(1/2)-2*exp(-
250*t)*cos(250*15^(1/2)*t)+2

                                 1/2      1/2                            1/2
  - 2/15 exp(-250 t) sin(250 15    t) 15    - 2 exp(-250 t) cos(250 15    t)

         + 2
```

Answer:
Presented above.

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Problem 6-51

RC OP AMP Circuit Design **(A, E)**

An upgrade to one of your company's robotics products requires a proportional plus integral compensator that implements the input-output relationship

$$v_O(t) = v_S(t) + 50\int_0^t v_S(x)\,dx$$

The input voltage $v_S(t)$ comes from an OP AMP, and the output voltage $v_O(t)$ drives a 10-kΩ resistive load. Two competing designs are shown in Figure P6-51. As the project engineer you are responsible for recommending one of these designs for production. Which design would you recommend and why? Your mentor, a wise senior engineer, suggests that you first check that both designs implement the required signal processing function.

Solution:

In Design #1, the right OP AMP is a subtractor and the left OP AMP is an inverting integrator. For the subtractor, the input leading to the positive input terminal is $v_S(t)$ and it experiences a total gain of (0.5)(2) = 1. The input leading to the negative input terminal of the subtractor is the result of integrating $v_S(t)$ with a gain of −50 and then multiplying by a gain of −1 from the subtractor. Combining the results of the subtractor shows that the circuit implements the required input-output relationship.

In Design #2, the circuit is not a standard configuration, so we will use node-voltage analysis to determine the input-output relationship

$$\frac{v_S(t)}{50\text{ k}} + (400\text{ n})\frac{d[v_S(t) - v_O(t)]}{dt} = 0$$

$$50v_S(t) + \frac{dv_S(t)}{dt} = \frac{dv_O(t)}{dt}$$

$$v_O(t) = v_S(t) + 50\int_0^t v_S(x)\,dx$$

The circuit implements the required input-output relationship.

Since $v_S(t)$ comes from an OP AMP, there will be no loading on the input side of the circuit. Both circuits have an OP AMP at the output, so there will be no loading issues at the output either. Since both designs meet the requirements, choose Design #2, because the circuit is significantly simpler and has fewer parts.

Answer:

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Design #2, because it is simpler and meets the requirements.

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Problem 6-52

Tunable Capacitor **(A)**

In Section 6-1, Figure 6-2 shows an air-tunable capacitor as one of the example capacitor types. This type of device can vary its capacitance similar to how a potentiometer can vary its resistance. Changing the capacitance in a circuit can change the frequency at which it operates. Suppose we are given the circuit in Figure P6-52 with a capacitor connected in parallel with an inductor. There are no other devices in the circuit. The capacitor has an initial voltage $v_C(0) = V_0$ V and the inductor's initial current is $i_L(0) = 0$ A.

The differential equation for the voltage across the capacitor in this circuit is given by

$$\frac{d^2 v_C(t)}{dt^2} + \frac{1}{LC} v_C(t) = 0$$

We will learn more about solving this type of differential equation in the next chapter and beyond. The solution to this differential equation is

$$v_C(t) = V_0 \cos\left(\frac{t}{\sqrt{LC}}\right), \quad t \geq 0$$

Using MATLAB, plot on a semi-log scale (logarithmic on the horizontal and linear on the vertical,) the radian frequency of $v_C(t)$ versus the capacitance of the circuit. Use capacitances scaled logarithmically from 1 nF to 1 μF. Let the inductance be 1 mH. In a separate MATLAB figure, use the `subplot` command to plot $v_C(t)$ versus time for C = 1 nF, 10 nF, 100 nF, and 1 μF. Using the plots you created and your knowledge of how a capacitor stores charge, explain why changing the capacitance of a parallel LC circuit changes the frequency at which is oscillates.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t positive
syms L C V0

% Solve the differential equation
vct = dsolve('D2y + y/L/C =
0','y(0)=V0','Dy(0)=0','t')

% Specify numerical values for the
components
C = logspace(-9,-6,100);
L = 1e-3;

% Evaluate the frequency
w = 1./sqrt(L*C);

% Plot the results
```

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```
semilogx(C,w,'b','LineWidth',3);
grid on
xlabel('Capacitance, (F)')
ylabel('Radian Frequency, (Rad/s)')
vct =
V0*cos(1/L^(1/2)/C^(1/2)*t)


% Plot the output waveform for different capacitor values
tt = 0:0.0000001:0.0006;
C = [1e-9 10e-9 100e-9 1e-6];
for n = 1:length(C)
    vctt = cos(tt/sqrt(L*C(n)));
    subplot(4,1,n)
    plot(tt,vctt)
    xlabel('Time (s)')
    ylabel('Voltage (V)')
    title(['C = ',num2str(C(n)),' F'])
end
```

Answer:
The required plots are presented above.
As the capacitance increases, the capacitor can store more charge for a given voltage level. Since there is more stored charge, it will take longer for the capacitor to discharge and re-charge as its voltage changes sign. Therefore, the period of the voltage oscillations will increase as the capacitance increases.

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Problem 7-1

Find the function $i(t)$ that satisfies the following differential equation and the initial condition:

$$500\frac{di(t)}{dt} + 2500i(t) = 0, \quad i(0) = 10\ mA$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms it t
it = dsolve('500*Dit+2500*it','it(0) = 10e-3','t')
it =
1/100*exp(-5*t)
```

Answer:

$i(t) = 10\,e^{-5t}$ mA, $t \geq 0$.

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Problem 7-2

Find the function $v(t)$ that satisfies the following differential equation and initial condition:

$$10^{-2}\frac{dv(t)}{dt} + v(t) = 0, \quad v(0) = 100\ V$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vt
vt = dsolve('1e-2*Dvt+vt','vt(0)=100','t')
vt =
100*exp(-100*t)
```

Answer:

$v(t) = 100\, e^{-100\,t}$ V , $t \geq 0$.

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Problem 7-3
Find the time constants of the circuits in Figure P7-3.

Solution:
The solution is presented in the following MATLAB code.

```
clear all
disp('Circuit C1')
L1 = 15e-3;
L2 = 10e-3;
R1 = 100;
R2 = 50;
Leq = L1+L2;
Req = R1+R2;
TC = Leq/Req

disp('Circuit C2')
R1 = 100;
L1 = 5e-3;
L2 = 10e-3;
L3 = 10e-3;
Leq = L1 + 1/(1/L2 + 1/L3);
TC = Leq/R1
Circuit C1
TC =
   166.6667e-006
Circuit C2
TC =
   100.0000e-006
```

Answer:
Circuit C1, T_C = 166.67 μs.
Circuit C2, T_C = 100 μs.

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Problem 7-4

Find the time constants of the circuits in Figure P7-4.

C1

C2

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Circuit C1')
Ceq = 0.33e-6;
Req = 2e3+3e3 + 1/(1/6e3 + 1/(1e3+4e3+1e3));
TC = Req*Ceq

disp('Circuit C2')
Req = 1/(1/22e3 + 1/47e3);
Ceq = 10e-9 + 10e-9;
TC = Req*Ceq
Circuit C1
TC =
     2.6400e-003
Circuit C2
TC =
   299.7101e-006
```

Answer:

Circuit C1, T_C = 2.64 ms.

Circuit C2, T_C = 299.71 μs.

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Problem 7-5

The switch in Figure P7-5 is closed at $t = 0$. The initial voltage on the capacitor is $v_C(0) = 20$ V.

(a) Find $v_C(t)$ and $i_O(t)$ for $t \geq 0$.

(b) Use MATLAB to plot the waveforms for $v_C(t)$ and $i_O(t)$.

(c) Simulate the problem using OrCAD and compare the results to the plots in part (b).

Solution:

The solutions to Parts (a) and (b) are presented in the following MATLAB code.

```
clear all
disp('Part (a)')
syms t vc io
Req = 1/(1/33e3 + 1/15e3);
C = 100e-9;
TC = Req*C;
vc0 = 20;
vc = vc0*exp(-t/TC)
vc_num = vpa(vc,5)
io = vc/15e3
io_num = vpa(io,5)
Part (a)
vc =
20*exp(-32000/33*t)
vc_num =
20.*exp(-969.70*t)
io =
1/750*exp(-32000/33*t)
io_num =
.13333e-2*exp(-969.70*t)

disp('Part (b)')
tt = 0:10e-6:6e-3;
vctt = subs(vc,t,tt);
iott = subs(io,t,tt);

plot(tt,vctt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
Part (b)


plot(tt,iott,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

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(c) The follow OrCAD simulation verifies the MATLAB results.

The green curve shows the capacitor voltage and the red curve shows the output current.

Answer:

(a) $v_C(t) = 20\left[e^{-969.7t}\right]u(t)$ V and $i_O(t) = 1.333\left[e^{-969.7t}\right]u(t)$ mA.

(b) Presented above.

(c) Presented above and the results agree with the MATLAB simulations.

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Problem 7-6

In Figure P7-6 the initial current through the inductor is $i_L(0) = 5$ mA.
Find $i_L(t)$ and $v_O(t)$ for $t \geq 0$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t iL vo
Req = 1/(1/10e3 + 1/22e3);
L = 0.4;
TC = L/Req;
iL0 = 5e-3;
iL = iL0*exp(-t/TC)
iL_num = vpa(iL,5)
vo = L*diff(iL,t)
vo_num = vpa(vo,5)
iL =
1/200*exp(-34375/2*t)
iL_num =
.50000e-2*exp(-17188.*t)
vo =
-275/8*exp(-34375/2*t)
vo_num =
-34.375*exp(-17188.*t)
```

Answer:

$i_L(t) = 5\,e^{-17188\,t}$ mA, $t \geq 0$.

$v_O(t) = -34.375\,e^{-17188\,t}$ V, $t \geq 0$.

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Problem 7-7

The switch in Figure P7-7 has been in position A for a long time and is moved to position B at $t = 0$. Find $v_C(t)$ for $t \geq 0$. Repeat the problem for the switch being in position B for a long time and moving to position A at $t = 0$.

Solution:

With the switch moving from position A to position B, we have the following:

Initial value: $v_C(0) = 5$ V

Final value: $v_C(\infty) = 0$ V

Time constant: $T_C = RC = (100\ \text{k}\Omega)(100\ \text{nF}) = 10$ ms

$$v_C(t) = \left[(5-0)e^{-t/10m} + 0\right]u(t) = 5\,e^{-100\,t}\ u(t)\ \text{ V}$$

With the switch moving from position B to position A, we have the following:

Initial value: $v_C(0) = 0$ V

Final value: $v_C(\infty) = 5$ V

Time constant: $T_C = RC = (10\ \text{k}\Omega)(100\ \text{nF}) = 1$ ms

$$v_C(t) = \left[(0-5)e^{-t/1m} + 5\right]u(t) = \left[5 - 5\,e^{-1000\,t}\right]u(t)\ \text{ V}$$

Answer:

When the switch moves from A to B: $v_C(t) = 5\left[e^{-100\,t}\right]u(t)$ V .

When the switch moves from B to A: $v_C(t) = \left[5 - 5e^{-1000\,t}\right]u(t)$ V .

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Problem 7-8

The switch in Figure P7-8 has been open for a long time and is closed at $t = 0$. Find $i_L(t)$ for $t \geq 0$.

Solution:

With a dc source and the switch open, we can find the initial current through the inductor using current division. The following MATLAB code presents the solution

```
clear all
format short eng
syms t iL
iL0 = 50e-3/40/(1/10 + 1/40);
iLf = 0;
L = 20e-3;
R = 40;
Tc = L/R;
iL = (iL0-iLf)*exp(-t/Tc)+iLf
iL =
1/100*exp(-2000*t)
```

Answer:

$i_L(t) = 10\, e^{-2000\, t}\, u(t)$ mA.

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Problem 7-9

The circuit in Figure P7-9 is in the zero state.

(a) Find the voltage $v_O(t)$ for $t \geq 0$ when an input of $i_S(t) = I_A\, u(t)$ is applied. Identify the forced and natural components in the output.

(b) Repeat when the input is $i_S(t) = I_A\, e^{-\alpha t}\, u(t)$.

Solution:

(a) Perform a source transformation to have a voltage source in series with a resistor and the capacitor. The voltage source will be $v_S(t) = RI_A\, u(t)$ and the resistor will be R.

Initial value: $v_C(0) = 0$ V

Final value: $v_C(\infty) = RI_A$ V

Time constant: $T_C = RC$

$$v_C(t) = \left[(0 - RI_A)e^{-t/RC} + RI_A\right]u(t) = RI_A\left[1 - e^{-t/RC}\right]u(t) \text{ V}.$$

The forced response is the constant term and the natural response is the exponential term.

(b) Perform the same source transformation to get $v_S(t) = RI_A\, e^{-\alpha t}\, u(t)$ with resistance R in series with the capacitor.

The natural response takes the same form as before

$$v_N(t) = K\, e^{-\frac{t}{RC}} \quad t \geq 0$$

The forced response is an exponential

$$v_F(t) = K_F e^{-\alpha t}$$

with

$$K_F = \frac{RI_A}{1 - \alpha RC}$$

and

$$K = -\frac{RI_A}{1 - \alpha RC}$$

We therefore have

$$v(t) = v_N(t) + v_F(t) = \frac{-RI_A}{1 - \alpha RC} e^{-t/RC} + \frac{RI_A}{1 - \alpha RC} e^{-\alpha t}$$

where the first exponential is the natural response and the second exponential is the forced response.

We can confirm both answers with the following MATLAB code.

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```
clear all
syms t vc R C Ia alpha
disp('Part (a)')
vc = dsolve('R*C*Dvc+vc=R*Ia','vc(0)=0','t')

disp('Part (b)')
vc = simplify(dsolve('R*C*Dvc+vc=R*Ia*exp(-alpha*t)','vc(0)=0','t'))
Part (a)
vc =
R*Ia-exp(-1/R/C*t)*R*Ia
Part (b)
vc =
-R*Ia*(exp(-t*(alpha*R*C-1)/R/C)-1)*exp(-1/R/C*t)/(alpha*R*C-1)
```

Answer:

(a) $v_O(t) = v_{ON}(t) + v_{OF}(t) = I_A R - I_A R e^{-t/RC}$ V for $t \geq 0$.

(b) $v_O(t) = v_{ON}(t) + v_{OF}(t) = \dfrac{-RI_A}{1-\alpha RC} e^{-t/RC} + \dfrac{RI_A}{1-\alpha RC} e^{-\alpha t}$ V for $t \geq 0$.

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Problem 7-10

The circuit in Figure P7-10 is in the zero state when the input $v_S(t) = V_A u(t)$ is applied. Find $v_O(t)$ for $t \geq 0$. Identify the forced and natural components in the output.

Solution:

Solve for the current through the inductor and then compute the voltage across the resistor. The initial current through the inductor is 0 A. Since we have a dc source, the final current will be V_A/R. The time constant will be L/R. The following MATLAB code completes the solution

```
clear all
syms t va R L iL vo
Tc = L/R;
iL0 = 0;
iLf = va/R;
iL = (iL0-iLf)*exp(-t/Tc)+iLf;
iR = iL;
vo = simplify(R*iR)
vo =
-va*(exp(-t/L*R)-1)
```

Answer:

$v_O(t) = V_A\left[1 - e^{-Rt/L}\right]u(t)$ V.

The forced response is the constant term and the natural response is the exponential term.

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Problem 7-11

The circuit in Figure P7-11 is in the zero state when the input $v_S(t) = 10\ u(t)$ is applied. If C = 0.1 μF and R = 10 kΩ, find $v_O(t)$ for $t \geq 0$. Identify the forced and natural components in the output.

Solution:

Find the voltage across the capacitor as a function of time and then use KVL to find the output voltage across the resistor. The solution is presented in the following MATLAB code.

```
clear all
syms t vs vc vo
vc0 = 0;
vcf = 10;
R = 10e3;
C = 0.1e-6;
Tc = 2*R*C;
vc = simplify((vc0-vcf)*exp(-t/Tc)+vcf)
% The voltage is the same across each resistor, since they
% are in series and have the same resistance.
vo = (10-vc)/2
vc =
-10*exp(-500*t)+10
vo =
5*exp(-500*t)
```

Answer:

$v_O(t) = v_{ON}(t) + v_{OF}(t) = 5\left[e^{-500\,t}\right] + 0$ V for $t \geq 0$.

There is no forced response and the exponential signal is the natural response in the output voltage.

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Problem 7-12

The circuit in Figure P7-12 is in the zero state when the input $v_S(t) = 6\ u(t)$ is applied. If $L = 100$ mH and $R = 100\ \Omega$, find $v_O(t)$ for $t \geq 0$. Identify the forced and natural components in the output. On a single set of axes, use MATLAB to plot the forced response, the natural response, and the complete response.

Solution:

Perform a source transformation on the circuit and then combine the two parallel resistors. The resulting circuit will be a 60-mA current source in parallel with a 50-Ω resistor in parallel with a 100-mH inductor. We can solve for the current through the inductor and then the voltage across it. The following MATLAB code presents the solution.

```
clear all
syms t iL vo vof von
L = 100e-3;
R = 50;
Tc = L/R;
iL0 = 0;
iLf = 60e-3;
iL = simplify((iL0-iLf)*exp(-t/Tc)+iLf)
vo = simplify(L*diff(iL,t))
iL =
-3/50*exp(-500*t)+3/50
vo =
3*exp(-500*t)

% There is no forced response
% The output voltage contains only a
natural response
von = vo;
tt = 0:50e-6:10e-3;
vott = subs(vo,t,tt);
vontt = subs(von,t,tt);
voftt = zeros(size(tt));
plot(tt,vott,'b','LineWidth',4)
hold on
grid on
plot(tt,vontt,'g','LineWidth',2)
plot(tt,voftt,'r','LineWidth',3)
xlabel('Time (s)')
ylabel('Voltage (V)')
legend('Complete','Natural','Forced')
```

Answer:

$v_O(t) = v_{ON}(t) + v_{OF}(t) = 3\left[e^{-500\,t}\right] + 0$ V for $t \geq 0$.

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There is no forced response and the exponential signal is the natural response in the output voltage. The plot is shown above.

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Problem 7-13

The switch in Figure P7-13 has been in position A for a long time and is moved to position B at $t = 0$. Find $v_C(t)$ for $t \geq 0$. Identify the forced and natural components in the response.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vc
% The capacitor is in parallel with one of the resistors
% Use voltage division to find the initial and final voltages
R = 10e3;
C = 0.1e-6;
vs = 12;
vc0 = vs*R/(R+R+R);
vcf = vs*R/(R+R);
% Find the equivalent resistance after the switch moves
Req = 1/(1/R+1/R);
Tc = Req*C;
vc = simplify((vc0-vcf)*exp(-t/Tc)+vcf)
vc =
-2*exp(-2000*t)+6
```

Answer:

$v_C(t) = v_{CN}(t) + v_{CF}(t) = -2\,e^{-2000\,t} + 6$ V for $t \geq 0$.

The exponential term is the natural response and the constant term is the forced response.

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Problem 7-14

Repeat Problem 7-13. However, after the switch is moved to position B at t = 0, the switch is moved back to position A at t = 1 ms. Find $v_C(t)$ for $t \geq 0$.

Solution:

After the switch moves from position A to position B, the response is the same as the solution to Problem 7-13 up until t = 1 ms. At t = 1 ms, find the voltage across the capacitor and use it as the initial voltage to find the response after the switch moves back to position A. The solution is presented in the following MATLAB code.

```
clear all
syms t vc
% The capacitor is in parallel with one of the resistors
% Use voltage division to find the initial and final voltages
R = 10e3;
C = 0.1e-6;
vs = 12;
vc0 = vs*R/(R+R+R);
vcf = vs*R/(R+R);
% Find the equivalent resistance after the switch moves
Req = 1/(1/R+1/R);
Tc = Req*C;
vc = simplify((vc0-vcf)*exp(-t/Tc)+vcf)

% Find the capacitor voltage at t = 1 ms and use it as the new initial voltage
vc1ms = subs(vc,t,1e-3);
vc02 = vc1ms;
vcf2 = vc0;
Req2 = 1/(1/R + 1/(R+R));
Tc2 = Req2*C;
vc2 = simplify((vc02-vcf2)*exp(-(t-1e-3)/Tc2)+vcf2)
vc2_num = vpa(vc2,5)

%tt = 0:3e-6:3e-3;
%vtt = subs(vc*(heaviside(t)-heaviside(t-0.001))+vc2*heaviside(t-0.001),t,tt);
%plot(tt,vtt)
%grid on
vc =
-2*exp(-2000*t)+6
vc2 =
1947051848108003/1125899906842624*exp(-1500*t+3/2)+4
vc2_num =
1.7293*exp(-1500.*t+1.5000)+4.
```

Answer:

$$v_C(t) = \left[6 - 2e^{-2000\,t}\right]\left[u(t) - u(t-0.001)\right] + \left[4 + 1.7293e^{-1500\,(t-0.001)}\right]u(t-0.001) \text{ V.}$$

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Problem 7-15

Find the function that satisfies the following differential equation and the initial condition for an input $v_S(t) = 25\cos(20t)$ V:

$$\frac{dv(t)}{dt} + 10v(t) = v_S(t), \quad v(0) = 0\text{ V}$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t v
v = simplify(dsolve('Dv+10*v=25*cos(20*t)','v(0)=0','t'))
v =
1/2*cos(20*t)+sin(20*t)-1/2*exp(-10*t)
```

Answer:

$v(t) = \left[-0.5e^{-10t} + 0.5\cos(20t) + \sin(20t)\right]u(t)$ V

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Problem 7-16

Repeat Problem 7-15 for $v_S(t) = 25e^{-0.2t}\,u(t)$ V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t v
v = simplify(dsolve('Dv+10*v=25*exp(-0.2*t)','v(0)=0','t'))
v =
125/49*exp(-1/5*t)-125/49*exp(-10*t)
```

Answer:

$$v(t) = \frac{125}{49}\left[e^{-t/5} - e^{-10t}\right]u(t) \text{ V}.$$

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Problem 7-17

The switch in Figure P7-17 has been open long enough for $i_L(0)$ to reach 0 A and is closed at t = 0.

(a) Use OrCAD to find $v_L(t)$ for $t \geq 0$ if the input is $v_S(t)$ = 10 e^{-100t} V.

(b) Use OrCAD to find $v_L(t)$ for $t \geq 0$ if the input is $v_S(t)$ = 10 cos(100t) V.

Solution:

First, solve the problems using MATLAB and then simulate the circuits in OrCAD to verify the results.

(a)
```
clear all
syms t DiLa DiLb iLa iLb vsa vsb
disp('Part (a)')
% Perform a source transformation
vsa = 10*exp(-100*t);
isa = vsa/220;
Req = 1/(1/220 + 1/330);
L = 1;
Eqna = subs('L*DiLa/Req + iLa = isa')
Part (a)
Eqna =
1/132*DiLa+iLa = 1/22*exp(-100*t)

iLa = simplify(dsolve('1/132*DiLa+iLa
= 1/22*exp(-100*t)','iLa(0)=0','t'))
vLa = simplify(L*diff(iLa,t))
iLa =
3/16*(exp(32*t)-1)*exp(-132*t)
vLa =
-75/4*exp(-100*t)+99/4*exp(-132*t)

% Plot the results
tt = 0:100e-6:80e-3;
vLatt = subs(vLa,t,tt);
plot(tt,vLatt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

OrCAD simulation:

Answer:

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(a) $v_L(t) = \frac{1}{4}\left[99e^{-132t} - 75e^{-100t}\right]$ V for $t \geq 0$.

(b)
```
disp('Part (b)')
% Perform a source transformation
vsb = 10*cos(100*t);
isb = vsb/220;
Req = 1/(1/220 + 1/330);
L = 1;
Eqnb = subs('L*DiLb/Req + iLb = isb')
Part (b)
Eqnb =
1/132*DiLb+iLb = 1/22*cos(100*t)

iLb = simplify(dsolve('1/132*DiLb+iLb
= 1/22*cos(100*t)','iLb(0)=0','t'))
vLb = simplify(L*diff(iLb,t))
iLb =
99/3428*cos(100*t)+75/3428*sin(100*t)-
99/3428*exp(-132*t)
vLb =
-
2475/857*sin(100*t)+1875/857*cos(100*t)+3267/857*exp(-132*t)

% Plot the results
tt = 0:200e-6:200e-3;
vLbtt = subs(vLb,t,tt);
plot(tt,vLbtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

OrCAD simulation

The OrCAD simulations match the MATLAB results.

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Answer:

(b) $v_L(t) = \frac{1}{857}\left[3267e^{-132t} + 1875\cos(100t) - 2475\sin(100t)\right]$ V for $t \geq 0$.

The OrCAD simulations for both parts are presented above.

Problem 7-18

The switch in Figure P7-18 has been in position A for a long time and is moved to position B at $t = 0$. Find $i_L(t)$ for $t \geq 0$ and use MATLAB to plot the waveform.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t iL
vs = 5;
R1 = 5e3;
R2 = 50e3;
L = 100e-3;
iL0 = vs/R1;
iLf = vs/R2;
Tc = L/R2;
iL = simplify((iL0-iLf)*exp(-t/Tc)+iLf);
iL_num = vpa(iL,6)
iL_num =
.900000e-3*exp(-500000.*t)+.100000e-3

tt = 0:10e-9:12e-6;
iLtt = subs(iL,t,tt);
plot(tt,iLtt,'b','LineWidth',3)
grid on
xlabel('Time (s)'); ylabel('Current
(A)')
```

Answer:

$i_L(t) = \left[100 + 900e^{-500000t}\right]$ μA for $t \geq 0$.

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Problem 7-19

The switch in Figure P7-19 has been in position A for a long time and is moved to position B at $t = 0$.
(a) Find $v_C(t)$ for $t \geq 0$ and sketch its waveform. You may use OrCAD.
(b) Use OrCAD to solve the same initial problem except that the switch suddenly returns to position A after 10 ms.
Hint: OrCAD switches are located in the EVAL library and are named Sw_tClose and Sw_tOpen, depending on if you want the switch to close or open at a certain time.

Solution:
(a) The solution is presented in the following MATLAB code and then verified with an OrCAD simulation

```
clear all
syms t vc
vs = 10;
R1 = 10e3;
R2 = 90e3;
R3 = 10e3;
C = 1e-6;
vc0 = R2*vs/(R1+R2);
vcf = R3*vs/(R1+R3);
Req = 1/(1/R1 + 1/R3);
Tc = Req*C;
vc = simplify((vc0-vcf)*exp(-t/Tc)+vcf)

tt = 0:50e-6:30e-3;
vctt = subs(vc,t,tt);
plot(tt,vctt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
vc =
4*exp(-200*t)+5
```

OrCAD simulation:

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(b) The following MATLAB code presents an analytical solution and then an OrCAD simulation verifies the results.

```
% The solution is the same as Part (a) for the first 10 ms
vcb0 = subs(vc,t,10e-3);
vcbf = vc0;
Req2 = 1/(1/R1 + 1/R2);
Tc2 = Req2*C;
vcb = vc*(heaviside(t)-heaviside(t-10e-3)) ...
      + ((vcb0-vcbf)*exp(-(t-10e-3)/Tc2)+vcbf)*heaviside(t-10e-3)
vcb_num = vpa(vcb,5)

tt = 0:100e-6:50e-3;
vcbtt = subs(vcb,t,tt);
plot(tt,vcbtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
vcb =
(4*exp(-200*t)+5)*(heaviside(t)-
heaviside(t-1/100))+(-
1947051848108003/562949953421312*exp
(-1000/9*t+10/9)+9)*heaviside(t-
1/100)
vcb_num =
(4.*exp(-200.*t)+5.)*(heaviside(t)-
1.*heaviside(t-.10000e-1))+(-
3.4587*exp(-
111.11*t+1.1111)+9.)*heaviside(t-.10000e-1)
```

OrCAD simulation

Answer:

(a) $v_C(t) = \left[5 + 4e^{-200\,t}\right]u(t)$ V.

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(b) $v_{\mathrm{C}}(t) = \left[5 + 4e^{-200\,t}\right]\left[u(t) - u(t-0.01)\right] + \left[9 - 3.4587e^{-1000\,(t-0.01)/9}\right]\left[u(t-0.01)\right]$ V .

The plots and simulation results are presented above.

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Problem 7-20

Switches 1 and 2 in Figure P7-20 have both been in position A for a long time. Switch 1 is moved to position B at $t = 0$ and Switch 2 is moved to position B at $t = 20$ ms. Find the voltage across the 0.1-μF capacitor for $t \geq 0$ and sketch its waveform.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vc
vs = 10;
R1 = 150e3;
R2 = 50e3;
C = 0.1e-6;
Tc1 = (R1+R2)*C;
Tc2 = R2*C;
vc0 = vs;
vcf = 0;
vc1 = simplify((vc0-vcf)*exp(-t/Tc1)+vcf);
vc20 = subs(vc1,t,20e-3);
vc2 = simplify((vc20-vcf)*exp(-(t-20e-3)/Tc2)+vcf);
vc = vc1*(heaviside(t)-heaviside(t-20e-3)) + vc2*heaviside(t-20e-3);
vc_num = vpa(vc,5)

tt = eps:100e-6:50e-3;
vctt = subs(vc,t,tt);
plot(tt,vctt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
vc_num =
10.*exp(-50.*t)*(heaviside(t)-
1.*heaviside(t-.20000e-
1))+3.6788*exp(-
200.*t+4.)*heaviside(t-.20000e-1)
```

Answer:

$$v_C(t) = \left[10e^{-50t}\right]\left[u(t) - u(t-0.02)\right] + \left[3.6788e^{-200(t-0.02)}\right]\left[u(t-0.02)\right] \text{ V}$$

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Problem 7-21

The switch in Figure P7-21 has been open for a long time and is closed at $t = 0$. The switch is reopened at $t = 4$ ms. Find $v_C(t)$ for $t \geq 0$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vc
vs = 15;
R1 = 1.5e3;
R2 = 15e3;
C = 1e-6;
Req = 1/(1/R1+1/R2);
Tc1 = Req*C;
Tc2 = R2*C;
vc0 = 0;
vcf = vs*R2/(R1+R2);
vc1 = simplify((vc0-vcf)*exp(-t/Tc1)+vcf);
T2 = 4e-3;
vc20 = subs(vc1,t,T2);
vc2f = 0;
vc2 = simplify((vc20-vc2f)*exp(-(t-T2)/Tc2)+vc2f);
vc = vc1*(heaviside(t)-heaviside(t-T2)) + vc2*heaviside(t-T2);
vc_num = vpa(vc,6)
vc_num =
(-13.6364*exp(-733.333*t)+13.6364)*(heaviside(t)-1.*heaviside(t-.400000e-
2))+12.9106*exp(-66.6667*t+.266667)*heaviside(t-.400000e-2)
```

Answer:

$$v_C(t) = 13.6364\left[1 - e^{-733\,t}\right]\left[u(t) - u(t - 0.004)\right] + 12.9106\left[e^{-66.7\,(t-0.004)}\right]\left[u(t - 0.004)\right] \text{ V}$$

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Problem 7-22

Find the sinusoidal steady-state response of $v_C(t)$ in Figure P7-22 when R = 10 kΩ, C = 3.3 μF, and the input voltage is $v_S(t)$ = 100 cos(2000π t) $u(t)$ V. On a single set of axes, use MATLAB to plot the steady-state response and the input voltage waveforms. Describe the differences between the waveforms.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vs vc
R = 10e3;
C = 3.3e-6;
vs = 100*cos(2000*pi*t);
vc = simplify(dsolve('10e3*3.3e-6*Dvc+vc=100*cos(2000*pi*t)','vc(0)=0','t'))
vc =
100*(-exp(-1000/33*t)+cos(2000*pi*t)+66*sin(2000*pi*t)*pi)/(1+4356*pi^2)

% Form the steady-state response
vcss = 100*(cos(2000*pi*t)+66*sin(2000*pi*t)*pi)/(1+4356*pi^2);
tt = 0:2e-6:2e-3;
vstt = subs(vs,t,tt);
vcsstt = subs(vcss,t,tt);
plot(tt,vstt,'b','LineWidth',3)
hold on
plot(tt,vcsstt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
legend('v_s(t)','v_C(t)')
```

It is difficult to compare the signals, because the amplitude of the capacitor voltage is much smaller than the amplitude of the source. Plot 100 times the capacitor voltage along with the source signal.

```
plot(tt,vstt,'b','LineWidth',3)
hold on
plot(tt,100*vcsstt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
legend('v_s(t)','100 v_C(t)')
 % Steady state response
vcss_num = vpa(vcss,5)
vcss_num =
.23260e-
2*cos(6283.2*t)+.48227*sin(6283.2*t)
```

Answer: $v_{CSS}(t) = 2.326\cos(2000\pi\, t) + 482.27\sin(2000\pi\, t)$ V

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The plots are presented above. The capacitor voltage has a much smaller amplitude compared to the source voltage and the phase of the sinusoid is shifted by approximately −90°.

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Problem 7-23

For $t \geq 0$ the zero-input response of the circuit in Figure P7-23 is $v_C(t) = 10e^{-10kt}$ V.

(a) Find C and $i_C(t)$ when $R = 10$ kΩ.

(b) Find the energy stored in the capacitor at $t = 2$ ms.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
disp('Part (a)')
syms t vc ic
Tc = 1/10e3;
R = 10e3;
C = Tc/R
vc = 10*exp(-t/Tc);
ic = C*diff(vc,t)

disp('Part (b)')
vc2ms = subs(vc,t,2e-3);
w = (C*vc2ms^2)/2
Part (a)
C =
    10.0000e-009
ic =
-1/1000*exp(-10000*t)
Part (b)
w =
     2.1242e-024
```

Answer:

(a) $C = 10$ nF and $i_C(t) = -e^{-10k\,t}$ mA for $t \geq 0$.

(b) $w_C(2\text{ ms}) = 2.1242 \times 10^{-24}$ J.

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Problem 7-24

For $t \geq 0$ the zero-input response of the circuit in Figure P7-24 is $i_L(t) = 50e^{-500t}$ mA.

(a) Find L and $v_L(t)$ when $R = 500\ \Omega$.

(b) Find the energy stored in the inductor at $t = 0.5$ ms.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
syms t iL vL
Tc = 1/500;
R = 500;
L = R*Tc
iL = 50e-3*exp(-500*t);
vL = L*diff(iL,t)

disp('Part (b)')
iL05ms = subs(iL,t,0.5e-3);
w = (L*iL05ms^2)/2
Part (a)
L =
     1.0000e+000
vL =
-25*exp(-500*t)
Part (b)
w =
   758.1633e-006
```

Answer:

(a) $L = 1$ H and $v_L(t) = -25e^{-500t}$ V for $t \geq 0$.

(b) $w(0.5\text{ ms}) = 758.1633\ \mu\text{J}$

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Problem 7-25

For $t \geq 0$ the step response of the voltage across the capacitor in Figure P7-25 is $v_C(t) = 5 - 10e^{-1000t}$ V. Find the IV, FV, T_C, R, and $i_C(t)$ when $C = 1$ μF.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t vc ic
C = 1e-6;
vc = 5-10*exp(-1000*t);
IV = limit(vc,t,0)
FV = limit(vc,t,inf)
Tc = 1/1000
R = Tc/C
ic = C*diff(vc,t)
IV =
-5
FV =
5
Tc =
     1.0000e-003
R =
     1.0000e+003
ic =
1/100*exp(-1000*t)
```

Answer:

IV = −5 V

FV = 5 V

$T_C = 1$ ms

$R = 1$ kΩ

$i_C(t) = 10\, e^{-1000\, t}$ mA for $t \geq 0$

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Problem 7-26

Design a first-order *RC* circuit that will produce the following voltage across the capacitor: $v_C(t) = 5 - 10e^{-2000t}$ V.

Solution:

Select a capacitor value for the circuit of C = 100 nF. Analyze the voltage using the following MATLAB code.

```
clear all
syms t vc
C = 100e-9;
vc = 5-10*exp(-2000*t);
IV = limit(vc,t,0)
FV = limit(vc,t,inf)
Tc = 1/2000;
R = Tc/C

% Plot the response to verify the
circuit design
tt = 0:30e-6:3e-3;
vctt = subs(vc,t,tt);
plot(tt,vctt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
IV =
-5
FV =
5
R =
     5.0000e+003
```

The initial voltage is −5 V, the final voltage is 5 V, and for a 100-nF capacitor, the series resistance must be 5 kΩ. The following circuit provides the required response.

Answer:

Presented above.

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Problem 7-27

For $t \geq 0$ the step response of the current through and voltage across the inductor in Figure P7-27 is $i_L(t) = 5 - 10e^{-2000t}$ mA and $v_L(t) = e^{-2000t}$ V

Find IV, FV, T_C, R, and L.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t iL vL
iL = 5e-3-10e-3*exp(-2000*t);
vL = exp(-2000*t);
diL = diff(iL,t);
L = double(vL/diL)
Tc = 1/2000
R = L/Tc
IV = limit(iL,t,0)
FV = limit(iL,t,inf)
L =
    50.0000e-003
Tc =
   500.0000e-006
R =
   100.0000e+000
IV =
-1/200
FV =
1/200
```

Answer:

IV = −5 mA

FV = 5 mA

T_C = 500 μs

R = 100 Ω

L = 50 mH

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Problem 7-28

Design a first-order *RL* circuit that will produce the following current through the inductor: $i_L(t) = 50 + 100e^{-1000t}$ mA.

Solution:

Use MATLAB to determine the key design values and to plot the desired inductor current. Select an inductor value of 1 H. Create the circuit in OrCAD and verify it meets the design requirements.

```
clear all
syms t iL
L = 1;
iL = 50e-3 + 100e-3*exp(-1000*t);
Tc = 1/1000;
R = L/Tc
IV = limit(iL,t,0)
FV = limit(iL,t,inf)
% Plot the response to verify the
circuit design
tt = 0:30e-6:6e-3;
iLtt = subs(iL,t,tt);
plot(tt,iLtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
R =
     1.0000e+003
IV =
3/20
FV =
1/20
```

The initial current is 150 mA, the final current is 50 mA, and for a 1-H inductor, the parallel resistance must be 1kΩ. The following circuit provides the required response.

Answer:

Presented above.

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Problem 7-29

The switch in Figure P7-29 has been in position B for a long time and is moved to position A at t = 0. Design the first-order RC interface circuit such that $v_O(t) = 10 \ ! \ 10e^{!5000t}$ V.

Solution:

Use MATLAB to analyze the response and then design the circuit

```
clear all
syms t vo
vo = 10 - 10*exp(-5000*t);
IV = double(limit(vo,t,0))
FV = double(limit(vo,t,inf))
Tc = 1/5000
IV =
     0.0000e+000
FV =
    10.0000e+000
Tc =
   200.0000e-006
```

With an initial value of 0 V, there is no source within the interface. With a final value of 10 V, the interface circuit must drop 5 V from the 15-V source. If we place a 50-Ω resistor in series with the source, (after the switch moves to position A,) then we will have the correct voltage drop at the output. To complete the interface, we need to place a capacitor in parallel with the 100-Ω resistor and select its value to create the correct time constant. The capacitor will experience an equivalent resistance of 50 Ω in parallel with 100 Ω, which is 100/3 Ω. The time constant is 200 μs, as computed above. The following MATLAB code solves for the required capacitor value.

```
Req = 1/(1/50+1/100);
C = Tc/Req
C =
     6.0000e-006
```

The capacitance is 6 μF. The following interface will meet the specifications.

Answer:

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Problem 7-30

The switch in Figure P7-29 has been in position A for a long time and is moved to position B at $t = 0$. Design the first-order *RC* interface circuit such that $v_O(t) = 10e^{!5000t}$ V.

Solution:

We need to have an initial voltage of 10 V across the output and then a time constant of 1/5000 = 200 µs. To create the initial voltage, place a 50-Ω resistor in series with the source to drop 5 V. The remaining 10 V will drop across the 100-Ω resistor. We will place a capacitor in parallel with the output resistor. The equivalent resistance experienced by the capacitor is the parallel combination of the 50-Ω and 100-Ω resistors, which is 100/3 Ω. To create a time constant of 200 µs, the required capacitance is 6 µF. The following circuit will satisfy the requirements.

Answer:

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Problem 7-31

A timing circuit is required that feeds into an OP AMP's non-inverting terminal (i.e. draws no current.) The circuit's output response $v_O(t)$ must be

$$v_O(t) = 5\left(1 - e^{-1000\,t}\right)u(t)\ \text{V}$$

Figure P7-31 shows two commercial products and the vendors claim each will meet the requirement. Which will you select and why?

Vendor A Unit cost: $7.00

Vendor B Unit cost: $4.50

Solution:

Given the required output voltage signal, the initial value is 0 V, the final value is 5 V, and the time constant is 1/1000 = 1 ms. Examine each of the circuits to see if they also have these values.

The circuit from Vendor A has the output voltage across a capacitor, so we need to find the voltages associated with the capacitor. Assuming the switch has been in the down position for a long time, the initial voltage will be 0 V. The final voltage will be 5 V, because the capacitor will drop all of the voltage from the source. The time constant is (1 kΩ)(1 μF) = 1 ms. The circuit from Vendor A meets the specifications and the cost is $7.00.

The circuit from Vendor B has the output voltage across a resistor in series with an inductor. We will find the values for the inductor current and then translate them into output voltages across the resistor. If the switch has been in the down position for a long time, the initial current is 0 mA and the initial voltage across the resistor is 0 V. The final current through the inductor will be 5 V/1 kΩ = 5 mA and the final voltage across the resistor will be 5 V. The time constant is 1 H/1 kΩ = 1 ms. The circuit from Vendor B meets the specifications and the cost is $4.50.

Answer:

Choose Vendor B because it is less expensive and meets the specification.

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Problem 7-32

A product line needs an *RC* circuit that will meet the following response specifications ± 5%:

IV	FV	T_C	R	C
–10 V	+5V	100 μs	≥1 kΩ	≤0.1 μF

Design a circuit to meet the specifications and validate your results using OrCAD.

Solution:

Use MATLAB to iteratively pick resistor and capacitor values to meet the specifications and then design a switching circuit that meets the requirements.

```
clear all
format short eng
Tc = 100e-6;
% Select a capacitor and calculate the corresponding resistance
C = 0.01e-6
R = Tc/C
C =
    10.0000e-009
R =
    10.0000e+003
```

The following OrCAD circuit meets the specifications. Its resulting output voltage plot is shown below.

Answer:

Presented above.

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Problem 7-33

Find the $v(t)$ that satisfies the following differential equation and initial conditions:

$$\frac{d^2v(t)}{dt^2} + 10\frac{dv(t)}{dt} + 36v(t) = 0, \quad v(0) = 0 \text{ V}, \quad \frac{dv(0)}{dt} = 10 \text{ V/s}$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t v
v = dsolve('D2v+10*Dv+36*v=0','v(0)=0','Dv(0)=10','t')
v_num = vpa(v,5)
v =
10/11*11^(1/2)*exp(-5*t)*sin(11^(1/2)*t)
v_num =
3.0151*exp(-5.*t)*sin(3.3166*t)
```

Answer:

$$v(t) = \frac{10\sqrt{11}}{11} e^{-5t} \sin\left(\sqrt{11}\, t\right) \text{ V} \quad \text{for } t \geq 0.$$

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Problem 7-34

Find the $v(t)$ that satisfies the following differential equation and initial conditions:

$$\frac{d^2v(t)}{dt^2} + 15\frac{dv(t)}{dt} + 50v(t) = 0, \quad v(0) = 5\text{ V}, \quad \frac{dv(0)}{dt} = 10\ \text{ V/s}$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t v
v = dsolve('D2v+15*Dv+50*v=0','v(0)=5','Dv(0)=10','t')
v_num = vpa(v,5)
v =
12*exp(-5*t)-7*exp(-10*t)
v_num =
12.*exp(-5.*t)-7.*exp(-10.*t)
```

Answer:

$v(t) = 12e^{-5t} - 7e^{-10t}$ V for $t \geq 0$.

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Problem 7-35

Find the $v(t)$ that satisfies the following differential equation and initial conditions:

$$\frac{d^2v(t)}{dt^2} + 10\frac{dv(t)}{dt} + 125v(t) = 250u(t), \quad v(0) = 0\text{ V}, \quad \frac{dv(0)}{dt} = 25\ \text{ V/s}$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t v
v = dsolve('D2v+10*Dv+125*v=250','v(0)=0','Dv(0)=25','t')
v_num = vpa(v,5)
v =
3/2*exp(-5*t)*sin(10*t)-2*exp(-5*t)*cos(10*t)+2
v_num =
1.5000*exp(-5.*t)*sin(10.*t)-2.*exp(-5.*t)*cos(10.*t)+2.
```

Answer:

$v(t) = 2 + e^{-5t}\left[1.5\sin(10t) - 2\cos(10t)\right]$ V for $t \geq 0$.

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Problem 7-36

Find the $i(t)$ that satisfies the following differential equation and initial conditions:

$$\frac{d^2 i(t)}{dt^2} + 4\frac{di(t)}{dt} + 8i(t) = 24u(t), \quad i(0) = 0, \quad \frac{di(0)}{dt} = 0$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t it
it = dsolve('D2it+4*Dit+8*it=24','it(0)=0','Dit(0)=0','t')
it_num = vpa(it,5)
it =
-3*exp(-2*t)*sin(2*t)-3*exp(-2*t)*cos(2*t)+3
it_num =
-3.*exp(-2.*t)*sin(2.*t)-3.*exp(-2.*t)*cos(2.*t)+3.
```

Answer:

$i(t) = 3 - 3e^{-2t}\left[\sin(2t) + \cos(2t)\right]$ A for $t \geq 0$.

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Problem 7-37

The switch in Figure P7-37 has been open for a long time and is closed at $t = 0$. The circuit parameters are $L = 1$ H, $C = 0.5$ μF, $R = 2$ kΩ, and $v_C(0) = 10$ V.

(a) Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

(b) Is the circuit overdamped, critically damped, or underdamped?

(c) Use OrCAD to simulate your results.

Solution:

(a) The circuit is a series *RLC* circuit. The initial capacitor voltage is 10 V and the initial value of the derivative of the capacitor voltage is $i_C(t)/C = i_L(t)/C = 0$ A/1 H = 0 A. The following MATLAB code solves the differential equation for the series *RLC* circuit to find $v_C(t)$ and then $i_L(t)$.

```
clear all
format short eng
disp('Part (a)')
syms t vc ic iL
L = 1;
C = 0.5e-6;
R = 2e3;
vc = dsolve('1*0.5e-6*D2vc + 2e3*0.5e-6*Dvc + vc = 0','vc(0) = 10','Dvc(0) =
0','t')
vc_num = vpa(vc,5)
ic = C*diff(vc,t);
% The inductor current flows in the opposite direction of
% the capacitor current
iL = simplify(-ic)
iL_num = vpa(iL,5)
Part (a)
vc =
10*exp(-1000*t)*sin(1000*t)+10*exp(-1000*t)*cos(1000*t)
vc_num =
10.*exp(-1000.*t)*sin(1000.*t)+10.*exp(-1000.*t)*cos(1000.*t)
iL =
295147905179352812 5/29514790517935282 5856*exp(-1000*t)*sin(1000*t)
iL_num =
.10000e-1*exp(-1000.*t)*sin(1000.*t)

tt = 0:12e-6:6e-3;
vctt = subs(vc,t,tt);
iLtt = subs(iL,t,tt);
plot(tt,vctt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

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```
plot(tt,iLtt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

(b) The circuit response is a damped sinusoid, so the circuit is underdamped.

(c) The following OrCAD simulation verifies the results.

Answer:

(a) $v_C(t) = 10e^{-1000t}\left[\sin(1000t) + \cos(1000t)\right]$ V and $i_L(t) = 10e^{-1000t}\left[\sin(1000t)\right]$ mA

(b) The circuit is underdamped.

(c) Presented above.

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Problem 7-38

The switch in Figure P7-38 has been open for a long time and is closed at $t = 0$. The circuit parameters are $L = 0.2$ H, $C = 0.5$ μF, $R = 2$ kΩ, and $v_C(0) = 60$ V.

(a) Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

(b) Is the circuit overdamped, critically damped, or underdamped?

(c) Use OrCAD to simulate your results.

Solution:

(a) The circuit is a parallel *RLC* circuit. The initial inductor current is 0 A and the initial value of the derivative of the inductor current is $v_L(t)/L = v_C(t)/L = 60$ V/0.2 H = 300 A. The following MATLAB code solves the differential equation for the parallel *RLC* circuit to find $i_L(t)$ and then $v_C(t)$.

```
clear all
format short eng
disp('Part (a)')
syms t iL vL vc
L = 0.2;
C = 0.5e-6;
R = 2e3;
iL = dsolve('0.2*0.5e-6*D2iL + 0.2*DiL/2e3 + iL = 0','iL(0) = 0','DiL(0) =
300','t')
iL_num = vpa(iL,5)
vL = L*diff(iL,t);
vc = simplify(vL)
vc_num = vpa(vc,5)
Part (a)
iL =
1/65*39^(1/2)*exp(-500*t)*sin(500*39^(1/2)*t)
iL_num =
.96079e-1*exp(-500.*t)*sin(3122.5*t)
vc =
20/13*exp(-500*t)*(-sin(500*39^(1/2)*t)*39^(1/2)+39*cos(500*39^(1/2)*t))
vc_num =
1.5385*exp(-500.*t)*(-6.2450*sin(3122.5*t)+39.*cos(3122.5*t))
```

```
tt = 0:12e-6:12e-3;
vctt = subs(vc,t,tt);
iLtt = subs(iL,t,tt);
plot(tt,vctt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

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```
plot(tt,iLtt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

(b) The circuit response is a decaying sinusoidal signal, so the circuit is underdamped.

(c) The following OrCAD simulations verify the results.

Answer:

(a) $v_C(t) = 1.5385e^{-500t}\left[39\cos(3122.5t) - 6.2450\sin(3122.5t)\right]$ V and

$i_L(t) = 96.079e^{-500t}\left[\sin(3122.5t)\right]$ mA

(b) The circuit is underdamped.

(c) Presented above.

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Problem 7-39

Use OrCAD to study how the voltage across the circuit in Figure P7-38 changes as the value of the resistor is varied. Use the Parameter function (PARAM) found in the Special Library and do a Transient Analysis for 3 ms. Select Parametric Sweep and then select Global Parameter. Use the same name as for the Parametric function and vary the resistor from 10 Ω to 10 kΩ using the logarithmic choice and 3 points per decade. This will generate a family of curves corresponding to different values of resistance. By clicking on the various curves, determine if the response becomes more or less damped as R varies from 10 Ω to 10 kΩ.

Solution:

The required circuit is shown below along with the resulting parametric plots. As the resistance increases, the response becomes less damped. This result makes sense, because if the resistance was infinite, then it would be equivalent to not having a resistor in the circuit and there would be no damping.

Answer:

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The circuit becomes less damped as the resistance increases. The plots are presented above.

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Problem 7-40

The switch in Figure P7-40 has been open for a long time and is closed at $t = 0$. The circuit parameters are $L = 0.8$ H, $C = 1.25$ μF, $R_1 = 2$ kΩ, $R_2 = 3$ kΩ, and $V_A = 15$ V.

(a) Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

(b) Is the circuit overdamped, critically damped, or underdamped?

(c) Use OrCAD to simulate your results.

Solution:

(a) Since the switch has been open for a long time, the initial capacitor voltage will be 15 V and the initial inductor current will be zero. After the switch closes, the right hand side of the circuit will be a series *RLC* circuit without a forcing function. We can use the following MATLAB code to solve for the capacitor voltage and then the inductor current

```
clear all
disp('Part (a)')
syms t vc ic iL
R1 = 2e3;
R2 = 3e3;
L = 0.8;
C = 1.25e-6;
vc = dsolve('0.8*1.25e-6*D2vc+3e3*1.25e-6*Dvc+vc=0',...
     'vc(0)=15','Dvc(0)=0','t')
vc_num = vpa(vc,5)
ic = C*diff(vc,t);
iL = simplify(ic)
iL_num = vpa(iL,5)
Part (a)
vc =
(15/2+225/322*161^(1/2))*exp(125*(-15+161^(1/2))*t)+(-
225/322*161^(1/2)+15/2)*exp(-125*(15+161^(1/2))*t)
vc_num =
16.367*exp(-288.88*t)-1.3666*exp(-3461.1*t)
iL =
11068046444225731875/23759406366937
902481408*161^(1/2)*(-exp(125*(-
15+161^(1/2))*t)+exp(-
125*(15+161^(1/2))*t))
iL_num =
-.59110e-2*exp(-288.88*t)+.59110e-
2*exp(-3461.1*t)

tt = 0:40e-6:20e-3;
vctt = subs(vc,t,tt);
iLtt = subs(iL,t,tt);
plot(tt,vctt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

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```
plot(tt,iLtt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

(b) The circuit is overdamped because it has two exponentials with different time constants.

(c) The following OrCAD simulation verifies the results.

Answer:

(a) $v_C(t) = 16.367e^{-288.88t} - 1.367e^{-3461.1t}$ V and $i_L(t) = -5.911\left[e^{-288.88t} - e^{-3461.1t}\right]$ mA .

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(b) The circuit is overdamped.

(c) Presented above.

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Problem 7-41

The switch in Figure P7-41 has been open for a long time and is closed at $t = 0$. The circuit parameters are $L = 1.25$ H, $C = 50$ nF, $R_1 = 4$ kΩ, $R_2 = 4$ kΩ, and $V_A = 20$ V.

(a) Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

(b) Is the circuit overdamped, critically damped, or underdamped?

(c) Use OrCAD to simulate your results.

Solution:

(a) Since the switch has been open for a long time, the initial capacitor voltage and inductor current will both be zero. Perform a source transformation and combine the two resistors in parallel to get a parallel *RLC* circuit with a step response. Use MATLAB to compute the inductor current and then the capacitor voltage.

```
clear all
disp('Part (a)')
syms t vc iL vL
R1 = 4e3;
R2 = 4e3;
R = 1/(1/R1 + 1/R2)
L = 1.25;
C = 50e-9;
iL = dsolve('1.25*50e-9*D2iL+1.25*DiL/2e3+iL=20/4e3',...
     'iL(0)=0','DiL(0)=0','t')
iL_num = vpa(iL,5)
vL = L*diff(iL,t);
vc = simplify(vL)
vc_num = vpa(vc,5)
```

```
Part (a)
R =
     2.0000e+003
iL =
-1/150*exp(-2000*t)+1/600*exp(-
8000*t)+1/200
iL_num =
-.66667e-2*exp(-2000.*t)+.16667e-
2*exp(-8000.*t)+.50000e-2
vc =
50/3*exp(-2000*t)-50/3*exp(-8000*t)
vc_num =
16.667*exp(-2000.*t)-16.667*exp(-
8000.*t)
```

```
tt = 0:5e-6:3e-3;
vctt = subs(vc,t,tt);
iLtt = subs(iL,t,tt);
plot(tt,vctt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

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```
plot(tt,iLtt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

(b) The circuit is overdamped because it has two exponential terms with different time constants.

(c) The following OrCAD simulation verifies the results.

Answer:

(a) $v_C(t) = \frac{50}{3}\left[e^{-2000\,t} - e^{-8000\,t}\right]$ V for $t \geq 0$ and $i_L(t) = 5 - \frac{20}{3}e^{-2000\,t} + \frac{5}{3}e^{-8000\,t}$ mA for $t \geq 0$.

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(b) The circuit is overdamped.

(c) Presented above.

Problem 7-42

Repeat Problem 7-41 with L = 560 mH.

Solution:

(a) Since the switch has been open for a long time, the initial capacitor voltage and inductor current will both be zero. Perform a source transformation and combine the two resistors in parallel to get a parallel *RLC* circuit with a step response. Use MATLAB to compute the inductor current and then the capacitor voltage.

```
clear all
disp('Part (a)')
syms t vc iL vL
R1 = 4e3;
R2 = 4e3;
R = 1/(1/R1 + 1/R2);
L = 560e-3;
C = 50e-9;
iL = dsolve('560e-3*50e-9*D2iL+560e-3*DiL/2e3+iL=20/4e3',...
     'iL(0)=0','DiL(0)=0','t')
iL_num = vpa(iL,5)
vL = L*diff(iL,t);
vc = simplify(vL)
vc_num = vpa(vc,5)
Part (a)
R =
     2.0000e+003
iL =
-1/600*exp(-5000*t)*sin(5000/7*21^(1/2)*t)*21^(1/2)-1/200*exp(-
5000*t)*cos(5000/7*21^(1/2)*t)+1/200
iL_num =
-.76378e-2*exp(-5000.*t)*sin(3273.3*t)-.50000e-2*exp(-
5000.*t)*cos(3273.3*t)+.50000e-2
vc =
20/3*exp(-
5000*t)*sin(5000/7*21^(1/2)*t)*21^(1/2)
vc_num =
30.551*exp(-5000.*t)*sin(3273.3*t)

tt = 0:2e-6:1e-3;
vctt = subs(vc,t,tt);
iLtt = subs(iL,t,tt);
plot(tt,vctt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

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```
plot(tt,iLtt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

(b) The circuit is underdamped.

(c) The following OrCAD simulation verifies the results.

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Answer:

(a) $v_C(t) = 30.551e^{-5000t}\sin(3273.3t)$ V for $t \geq 0$ and

$i_L(t) = 5 - e^{-5000t}\left[7.6378\sin(3273.3t) + 5\cos(3273.2t)\right]$ mA for $t \geq 0$.

(b) The circuit is underdamped.

(c) Presented above.

Problem 7-43

The switch in Figure P7-43 has been in position A for a long time. At $t = 0$ it is moved to position B. The circuit parameters are $R_1 = 20$ kΩ, $R_2 = 4$ kΩ, $L = 1.6$ H, $C = 1.25$ μF, and $V_A = 24$ V.

(a) Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

(b) Is the circuit overdamped, critically damped, or underdamped?

(c) Use OrCAD to simulate your results.

Solution:

(a) After the switch moves, perform a source transformation to create a parallel *RLC* circuit. In a parallel *RLC* circuit, we will solve for the inductor current, so we need to know the initial values for the current and its derivative. Before the switch moves, the current through the inductor is 24 V/(20 + 4 kΩ) = 1 mA. The voltage across the inductor before the switch moves is zero, so the initial value of the derivative of the current is zero. We can now use MATLAB to solve the differential equation.

```
clear all
format short eng
disp('Part (a)')
syms t vc iL vL
R = 4e3;
L = 1.6;
C = 1.25e-6;
iA = -24/R;
iL = dsolve('1.6*1.25e-6*D2iL+1.6*DiL/4e3+iL=-24/4e3',...
     'iL(0)=1e-3','DiL(0)=0','t')
iL_num = vpa(iL,5)
vL = L*diff(iL,t);
vc = simplify(vL)
vc_num = vpa(vc,5)
Part (a)
iL =
1/1000*exp(-
100*t)*sin(700*t)+7/1000*exp(-
100*t)*cos(700*t)-3/500
iL_num =
.10000e-2*exp(-
100.*t)*sin(700.*t)+.70000e-
```

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```
2*exp(-100.*t)*cos(700.*t)-.60000e-2
vc =
-8*exp(-100*t)*sin(700*t)
vc_num =
-8.*exp(-100.*t)*sin(700.*t)
```

```
tt = 0:100e-6:50e-3;
vctt = subs(vc,t,tt);
iLtt = subs(iL,t,tt);
plot(tt,vctt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')

plot(tt,iLtt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

(b) The circuit is underdamped because the response is a decaying sinusoid.

(c) The following OrCAD simulation verifies the results.

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Answer:

(a) $v_{\mathrm{C}}(t) = -8e^{-100\,t}\sin(700t)$ V for $t \geq 0$ and

$i_{\mathrm{L}}(t) = e^{-100\,t}\left[7\cos(700t) + \sin(700t)\right] - 6$ mA for $t \geq 0$.

(b) The circuit is underdamped.

(c) Presented above.

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Problem 7-44

The switch in Figure P7-44 has been in position A for a long time and is moved to position B at $t = 0$. The circuit parameters are $R_1 = 500\ \Omega$, $R_2 = 330\ \Omega$, $L = 250$ mH, $C = 3.3\ \mu$F, and $V_A = 5$ V.

(a) Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

(b) Is the circuit overdamped, critically damped, or underdamped?

(c) Use OrCAD to simulate your results.

Solution:

(a) This is a series *RLC* circuit, so we will solve for the capacitor voltage. The initial capacitor voltage is the same as the source voltage, 5 V. The initial capacitor current is zero, so the derivate of the capacitor voltage has an initial value of zero. Use MATLAB to solve for the capacitor voltage and inductor current.

```
clear all
disp('Part (a)')
syms t vc ic iL
R = 330;
L = 250e-3;
C = 3.3e-6;
vc = dsolve('250e-3*3.3e-6*D2vc+330*3.3e-6*Dvc+vc=0',...
     'vc(0)=5','Dvc(0)=0','t')
vc_num = vpa(vc,5)
ic = C*diff(vc,t);
iL = simplify(ic)
iL_num = vpa(iL,5)
Part (a)
vc =
165/64063*2114079^(1/2)*exp(-660*t)*sin(20/33*2114079^(1/2)*t)+5*exp(-
660*t)*cos(20/33*2114079^(1/2)*t)
vc_num =
3.7449*exp(-660.*t)*sin(881.21*t)+5.*exp(-660.*t)*cos(881.21*t)
iL =
-
20291418481080507812 5/12999291421534
605056933888*2114079^(1/2)*exp(-
660*t)*sin(20/33*2114079^(1/2)*t)
iL_num =
-.22697e-1*exp(-
660.*t)*sin(881.21*t)

tt = 0:10e-6:8e-3;
vctt = subs(vc,t,tt);
iLtt = subs(iL,t,tt);
plot(tt,vctt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

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```
plot(tt,iLtt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

(b) The circuit is underdamped.

(c) The following OrCAD simulation verifies the results.

Answer:

(a) $v_C(t) = e^{-660\,t}\left[3.7449\sin(881.25t) + 5\cos(881.25t)\right]$ V for $t \geq 0$ and

$i_L(t) = -22.697e^{-660\,t}\sin(881.25t)$ mA for $t \geq 0$.

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(b) The circuit is underdamped.
(c) Presented above.

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Problem 7-45

The switch in Figure P7-44 has been in position B for a long time and is moved to position A at $t = 0$. The circuit parameters are $R_1 = 500\ \Omega$, $R_2 = 500\ \Omega$, $L = 250$ mH, $C = 1\ \mu$F, and $V_A = 5$ V.

(a) Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

(b) Is the circuit overdamped, critically damped, or underdamped?

(c) Use OrCAD to simulate your results.

Solution:

(a) This is a series *RLC* circuit, so we will solve for the capacitor voltage. The initial capacitor voltage is 0 V and the initial capacitor current is zero, so the derivate of the capacitor voltage has an initial value of zero. Use MATLAB to solve for the capacitor voltage and inductor current.

```
clear all
disp('Part (a)')
syms t vc ic iL
R = 500+500;
L = 250e-3;
C = 1e-6;
vc = dsolve('250e-3*1e-6*D2vc+1e3*1e-6*Dvc+vc=5',...
     'vc(0)=0','Dvc(0)=0','t')
vc_num = vpa(vc,5)
ic = C*diff(vc,t);
iL = simplify(ic)
iL_num = vpa(iL,5)
Part (a)
vc =
-5*exp(-2000*t)-10000*exp(-2000*t)*t+5
vc_num =
-5.*exp(-2000.*t)-10000.*exp(-2000.*t)*t+5.
iL =
20*exp(-2000*t)*t
iL_num =
20.*exp(-2000.*t)*t

tt = 0:12e-6:6e-3;
vctt = subs(vc,t,tt);
iLtt = subs(iL,t,tt);
plot(tt,vctt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

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```
plot(tt,iLtt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

(b) The circuit is critically damped.
(c) The following OrCAD simulation verifies the results.

Answer:

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(a) $v_C(t) = 5 - e^{-2000\,t}\left[5 + 10000t\right]$ V for $t \geq 0$ and $i_C(t) = 20te^{-2000\,t}$ A for $t \geq 0$;

(b) The circuit is critically damped.
(c) Presented above.

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Problem 7-46

The circuit in Figure P7-46 is in the zero state when the step function input is applied. The circuit parameters are L = 0.3 H, C = 1 μF, R = 2.2 kΩ, and V_A = 90 V. Find $v_O(t)$ for $t \geq 0$. *Hint: find the capacitor voltage first*.

Solution:

This is a series *RLC* circuit, so we will find the capacitor voltage first. We can then compute the current in the circuit and use the current to find the voltage across the inductor. The following MATLAB code presents the solution.

```
clear all
syms t vc ic iL vo vR
vA = 90;
R = 2.2e3;
L = 0.3;
C = 1e-6;
vc = dsolve('0.3*1e-6*D2vc+2.2e3*1e-6*Dvc+vc=90',...
     'vc(0)=0','Dvc(0)=0','t');
ic = C*diff(vc,t);
vo = simplify(L*diff(ic,t));
vo_num = vpa(vo,5)
% Compute the resistor voltage and check KVL
vR = R*ic;
CheckKVL = simplify(-vA+vR+vc+vo)
vo_num =
-6.891*exp(-486.86*t)+96.891*exp(-6846.3*t)
CheckKVL =
0
```

Answer:

$v_O(t) = \left[96.891e^{-6846.3t} - 6.891e^{-486.86t}\right]$ V for $t \geq 0$.

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Problem 7-47

The circuit in Figure P7-47 is in the zero state when the step function input is applied. The circuit parameters are R = 15 kΩ, L = 0.5 H, C = 250 nF, and V_A = 70 V. Find $v_O(t)$ for $t \geq 0$. *Hint: find the inductor current first.*

Solution:

If we perform a source transformation, the circuit is a parallel *RLC* circuit. We will find the inductor current and then the inductor voltage, which will match the capacitor voltage in the parallel circuit. The following MATLAB code presents the solution.

```
clear all
syms t iL vo ic iR
R = 15e3;
L = 0.5;
C = 250e-9;
iA = 70/R;
iL = dsolve('0.5*250e-9*D2iL+0.5*DiL/15e3+iL=70/15e3',...
     'iL(0)=0','DiL(0)=0','t');
vo = L*diff(iL,t)
vo_num = vpa(vo,5)
% Computer ic and iR to confirm KCL
ic = C*diff(vo,t);
iR = vo/R;
CheckKCL = vpa(-iA+iL+ic+iR,5)
vo =
140/449*exp(-400/3*t)*sin(400/3*449^(1/2)*t)*449^(1/2)
vo_num =
6.6070*exp(-133.33*t)*sin(2825.3*t)
CheckKCL =
.99661e-20*exp(-133.33*t)*sin(2825.3*t)-.21118e-18*exp(-
133.33*t)*cos(2825.3*t)
```

Answer:

$v_O(t) = 6.6070e^{-133.33t}\sin(2825.3t)$ V for $t \geq 0$.

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Problem 7-48

The circuit in Figure P7-48 is in the zero state when the step function input is applied.

(a) If V_A = 100 V, R = 500 Ω, L=2.5 H, and C = 2.5 μF, derive an expression for the voltage $v_O(t)$ for $t \geq 0$.

(b) Validate your solution by plotting it using MATLAB and comparing it to an OrCAD simulation of the same circuit.

Solution:

(a) The voltage source is in series with the resistor, so we can perform a source transformation to create a parallel *RLC* circuit. We can then solve for the inductor current and the inductor voltage. Once we know the inductor current, we can use KVL to compute the output voltage. The following MATLAB code presents the solution.

```
clear all
syms t iL vL vo
vA = 100;
R = 500;
L = 2.5;
C = 2.5e-6;
iA = vA/R;
iL = dsolve('2.5*2.5e-6*D2iL+2.5*DiL/500+iL=100/500',...
     'iL(0)=0','DiL(0)=0','t');
vL = L*diff(iL,t);
vo = simplify(vA - vL)
vo =
100-80000*exp(-400*t)*t
```

(b) Plot the output voltage using MATLAB and then via an OrCAD simulation.

```
tt = 0:30e-6:20e-3;
vott = subs(vo,t,tt);
plot(tt,vott,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

OrCAD simulation:

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The MATLAB and OrCAD simulation plots agree.

Answer:

(a) $v_O(t) = 100 - 80000te^{-400t}$ V for $t \geq 0$.

(b) Presented above.

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Problem 7-49

Derive expressions for the damping ratio and undamped natural frequency of the circuit in Figure P7-49 in terms of the circuit parameters *R*, *L*, and *C*.

Solution:

If we perform a source transformation, combine the two resistors in parallel, and then perform another source transformation, we have a series *RLC* circuit with resistance *R*/2, inductance *L* and capacitance *C*. The corresponding differential equation is

$$LC\frac{d^2v_C}{dt^2}+\frac{RC}{2}\frac{dv_C}{dt}+v_C=\frac{v_S}{2}$$

which can be re-written as

$$\frac{d^2v_C}{dt^2}+\frac{R}{2L}\frac{dv_C}{dt}+\frac{1}{LC}v_C=\frac{v_S}{2LC}$$

Therefore, we have

$$\omega_0^2=\frac{1}{LC} \qquad \text{and} \qquad 2\zeta\omega_0=\frac{R}{2L}$$

We can then solve for ω_0 and ζ.

Answer:

$$\omega_0=\frac{1}{\sqrt{LC}} \text{ and } \zeta=\frac{R}{4}\sqrt{\frac{C}{L}}\,.$$

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Problem 7-50
Repeat problem 7-49 for the circuit in Figure P7-50.

Solution:
If we perform a source transformation, combine the two resistors in series, and perform another source transformation, we will have a parallel *RLC* circuit with resistance 3*R*, inductance *L* and capacitance *C*. The corresponding differential equation is

$$LC\frac{d^2 i_L}{dt^2}+\frac{L}{3R}\frac{di_L}{dt}+i_L=\frac{i_S}{3}$$

which can be re-written as

$$\frac{d^2 i_L}{dt^2}+\frac{1}{3RC}\frac{di_L}{dt}+\frac{1}{LC}i_L=\frac{i_S}{3LC}$$

Therefore, we have

$$\omega_0^2=\frac{1}{LC} \qquad \text{and} \qquad 2\zeta\omega_0=\frac{1}{3RC}$$

We can then solve for ω_0 and ζ.

Answer:
$\omega_0=\dfrac{1}{\sqrt{LC}}$ and $\zeta=\dfrac{1}{6R}\sqrt{\dfrac{L}{C}}$.

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Problem 7-51

In a series *RLC* circuit the step response across the 4-μF capacitor is

$v_C(t) = 15 \ ! \ e^{!100\,t}\,[15\cos(400t) + 3\sin(400t)]$ V $t \geq 0$

(a) Find *R* and *L*.

(b) Find $i_L(t)$ for $t \geq 0$.

Solution:

(a) Using the following equations and the given step response, we can calculate *R* and *L*.

$$\alpha = \zeta\omega_0$$

$$\beta = \omega_0\sqrt{1-\zeta^2}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$2\zeta\omega_0 = \frac{R}{L}$$

The following MATLAB code presents the solution.

```
clear all
syms a B w0 z R L C
Eqn1 = 'z*w0-100';
Eqn2 = 'w0*sqrt(1-z^2)-400';
Eqn3 = 'w0^2-1/L/C';
Eqn4 = '2*z*w0-R/L';
Eqn5 = 'C-4e-6';
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,Eqn5,'w0','z','R','L','C');
R = double(Soln.R)
L = double(Soln.L)
R =
   294.1176e+000
L =
     1.4706e+000
```

(b) Use MATLAB to compute the corresponding inductor current.

```
syms t vc iL
vc = 15-exp(-100*t)*(15*cos(400*t)+3*sin(400*t));
C = 4e-6;
iL = simplify(C*diff(vc,t));
iL_num = vpa(iL,5)
iL_num =
.12000e-2*exp(-100.*t)*(cos(400.*t)+21.*sin(400.*t))
```

Answer:

(a) $R = 294.1176\ \Omega$ and $L = 1.4706$ H.

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(b) $i_{\mathrm{L}}(t) = 1.2e^{-100t}\left[\cos(400t) + 21\sin(400t)\right]$ mA for $t \geq 0$

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Problem 7-52

In a parallel *RLC* circuit the zero-input response in the 100-mH inductor is

$i_L(t) = 50e^{!4000t} \; ! \; 40e^{!5000t}$ mA $\quad t \geq 0$

(a) Find *R* and *C*.

(b) Find $v_C(t)$ for $t \geq 0$.

Solution:

(a) Using the following equations and the given step response, we can calculate *R* and *C*.

$$\alpha_1 = \zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1}$$

$$\alpha_2 = \zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$2\zeta\omega_0 = \frac{1}{RC}$$

The following MATLAB code presents the solution.

```
clear all
syms a1 a2 w0 z R L C
Eqn1 = 'z*w0+w0*sqrt(z^2-1)-5000';
Eqn2 = 'z*w0-w0*sqrt(z^2-1)-4000';
Eqn3 = 'w0^2-1/L/C';
Eqn4 = '2*z*w0-1/R/C';
Eqn5 = 'L-100e-3';
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,Eqn5,'w0','z','R','L','C');
R = double(Soln.R)
C = double(Soln.C)
w0 = double(Soln.w0);
z = double(Soln.z);
R =
   222.2222e+000
C =
   500.0000e-009
```

(b) Use MATLAB to compute the corresponding capacitor voltage

```
syms t iL vc
iL = 50e-3*exp(-4000*t)-40e-3*exp(-5000*t);
L = 100e-3;
vc = L*diff(iL,t);
vc_num = vpa(vc,5)
vc_num =
-20.*exp(-4000.*t)+20.*exp(-5000.*t)
```

Answer:

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(a) R = 222.2222 Ω and C = 500 nF.

(b) $v_C(t) = 20\left[e^{-5000\,t} - e^{-4000\,t}\right]$ V for $t \geq 0$

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Problem 7-53

In a parallel *RLC* circuit the state variable responses are

$$v_C(t) = e^{!100t}[5\cos(300t) + 15\sin(300t)] \text{ V} \qquad t \geq 0$$

$$i_L(t) = 20 \text{ ! } 25e^{!100t}\cos(300t) \text{ mA} \qquad t \geq 0$$

Find *R*, *L*, and *C*.

Solution:

In a parallel *RLC* circuit, all of the elements share the same voltage. We can take the derivative of the inductor current to compute the voltage divided by the inductance. We can then divide the capacitor voltage by this number to determine the inductance. From there, we use the current response characteristics to determine the resistance and capacitance. The following MATLAB presents the solution.

```
clear all
format short eng
syms t vc iL diL
vc = exp(-100*t)*(5*cos(300*t)+15*sin(300*t));
iL = 20e-3 - 25e-3*exp(-100*t)*cos(300*t);
diL = diff(iL,t);
L1 = double(simplify(vc/diL))
syms w0 z R L C
Eqn1 = 'z*w0-100';
Eqn2 = 'w0*sqrt(1-z^2)-300';
Eqn3 = 'w0^2-1/L/C';
Eqn4 = '2*z*w0-1/R/C';
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,'w0','z','R','C');
R = subs(Soln.R,L,L1)
C = subs(Soln.C,L,L1)
L1 =
     2.0000e+000
R =
     1.0000e+003
C =
     5.0000e-006
```

Answer:

$R = 1\ \text{k}\Omega$, $L = 2$ H, and $C = 5\ \mu\text{F}$.

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Problem 7-54

The zero-input response of a series *RLC* circuit with $R = 80\ \Omega$ is

$$v_C(t) = 2e^{-2000t}\cos(1000t) - 4\,e^{-2000t}\sin(1000t)\ \text{V} \qquad t \geq 0$$

If the initial conditions remain the same, what is the zero-input response when $R = 40\ \Omega$?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t w0 z R L C vc dvc
% Find the initial value and the initial value
% of the derivative of vc(t)
vc = 2*exp(-2000*t)*cos(1000*t)-4*exp(-2000*t)*sin(1000*t);
dvc = diff(vc,t);
vc0 = limit(vc,t,0)
dvc0 = limit(dvc,t,0)

% Analyze the response to find L and C
Eqn1 = 'z*w0-2000';
Eqn2 = 'w0*sqrt(1-z^2)-1000';
Eqn3 = 'w0^2-1/L/C';
Eqn4 = '2*z*w0-R/L';
Eqn5 = 'R-80';
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,Eqn5,'w0','z','R','L','C');
L = double(Soln.L)
C = double(Soln.C)
vc0 =
2
dvc0 =
-8000
L =
    20.0000e-003
C =
    10.0000e-006

% Solve the differential equation for the new value of R
vc2 =dsolve('20e-3*10e-6*D2vc2+40*10e-6*Dvc2+vc2=0','vc2(0)=2','Dvc2(0)=-
8000','t')
vc2 =
-3*exp(-1000*t)*sin(2000*t)+2*exp(-1000*t)*cos(2000*t)
```

Answer:

$v_C(t) = e^{-1000t}\,[2\cos(2000t) - 3\sin(2000t)]$ V, for $t \geq 0$.

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Problem 7-55

In a parallel *RLC* circuit the inductor current is observed to be

$$i_L(t) = 10\, e^{!10\,t} \sin(20t) \quad \text{mA} \qquad t \geq 0$$

Find $v_C(t)$ when $v_C(0) = 10$ V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t iL diL
% Take the derivative of the inductor current to get
% an expression for vL(t)/L = vc(t)/L
iL = 10e-3*exp(-10*t)*sin(20*t);
diL = diff(iL,t);
% Evaluate the derivative of the inductor current at zero
% Divide the given vc(0) by the result to find the inductance
diL0 = subs(diL,t,0);
vc0 = 10;
L = double(vc0/diL0);
% The capacitor voltage equals the inductor voltage in the
% parallel circuit.
vc = simplify(L*diL)
vc_num = vpa(vc,5)
vc =
-5*exp(-10*t)*(sin(20*t)-2*cos(20*t))
vc_num =
-5.*exp(-10.*t)*(sin(20.*t)-2.*cos(20.*t))
```

Answer:

$v_C(t) = -5e^{-10\,t}\left[\sin(20t) - 2\cos(20t)\right]$ V for $t \geq 0$.

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Problem 7-56

Design a parallel *RLC* circuit whose natural response has the form

$$v_L(t) = K_1\, e^{!2000t} + K_2\, t\, e^{!2000t} \text{ V} \qquad t \geq 0$$

Solution:

Based on the form of the natural response, the circuit is critically damped with repeated roots at 2000 rad/s. The corresponding characteristic equation is

$$(s+2000)^2 = s^2 + 4000s + 4000000 = 0$$

In general, the characteristic equation for a parallel *RLC* circuit is

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

By equating like coefficients in the two characteristic equations, we can solve for the parameter values. We have three parameters and two equations, so we will have to select one of the values. The following MATLAB code completes the calculations

```
clear all
% Select a capacitor value
C = 250e-9
% Solve for the inductance
L = 1/C/4e6
% Solve for the resistance
R = 1/C/4000
C =
   250.0000e-009
L =
     1.0000e+000
R =
     1.0000e+003
```

We can now design the required circuit as shown below.

Answer:

Presented above. There are many correct answers.

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Problem 7-57
Design a series *RLC* circuit with $\zeta = 0.5$ and $\omega_0 = 25$ krad/s.
(a) What is the form of the natural response of $v_C(t)$ for your design?
(b) Simulate your circuit in OrCAD.

Solution:
The characteristic equation for a series *RLC* circuit is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

Substituting in the given values we have

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 25000s + (25000)^2 = 0$$

Select a value for the inductance and then solve for the resistance and capacitance.

```
clear all
L = 200e-3
R = 25e3*L
C = 1/L/(25e3)^2
L =
   200.0000e-003
R =
     5.0000e+003
C =
     8.0000e-009
```

The following circuit will meet the specifications

(a) With $\zeta < 1$, the circuit is underdamped. We can calculate the corresponding values for α and β using the following MATLAB code

```
% Calculate alpha and beta
z = 0.5;
w0 = 25e3;
alpha = z*w0
beta = w0*sqrt(1-z^2)
alpha =
    12.5000e+003
beta =
    21.6506e+003
```

The natural response will take the form

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$$v_C(t) = e^{-12500\,t}\left[K_1 \cos(21650.6t) + K_2 \sin(21650.6t)\right] \text{ V}$$

(b) The results of a circuit simulation are shown below

Answer:
The circuit design is presented above.
(a) $v_C(t) = e^{-12500\,t}\left[K_1 \cos(21650.6t) + K_2 \sin(21650.6t)\right]$ V for $t \geq 0$.
(b) Presented above.

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Problem 7-58

Design a series *RLC* circuit with $\zeta = 1$ and $\omega_0 = 10$ krad/s.

(a) What is the form of the natural response of $v_C(t)$ for your design?

(b) Simulate your circuit in OrCAD.

Solution:

The characteristic equation for a series *RLC* circuit is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

Substituting in the given values we have

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 20000s + (10000)^2 = 0$$

Select a value for the inductance and then solve for the resistance and capacitance.

```
clear all
L = 200e-3
R = 20e3*L
C = 1/L/(10e3)^2
L =
   200.0000e-003
R =
     4.0000e+003
C =
    50.0000e-009
```

The following circuit will meet the specifications

(a) The natural response will take the form

$$v_C(t) = e^{-10000\,t}\left[K_1 + K_2\,t\right] \text{ V}$$

(b) The results of a circuit simulation are shown below

Answer:

The circuit design is presented above.

(a)

$v_C(t) = e^{-10000\,t}\left[K_1 + K_2\,t\right]$ V

for $t \geq 0$.

(b) Presented above.

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Problem 7-59

A circuit is needed to produce the following step response

$$v_C(t) = 10 - 13.3\,e^{-200t} + 3.3\,e^{-800t} \text{ V} \qquad t > 0$$

A vendor has proposed using the circuit shown in Figure P7-59 to produce the desired response. The vendor realizes that the proposed circuit does not exactly meet the desired response and is willing to make a single change for no extra charge. What change should the vendor make? *Hint: Use MATLAB to generate the desired response and then simulate the vendor's corrected circuit using OrCAD and verify that the responses match.*

Solution:

The desired step response is overdamped with two distinct, real roots. The proposed circuit is a series *RLC* circuit and the required characteristic equation is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = (s+200)(s+800) = s^2 + 1000s + 160000 = 0$$

The following MATLAB code computes the relevant values from the vendor's circuit.

```
clear all
R = 3.3e3;
L = 6.25;
C = 1e-6;
RL = R/L
w02 = 1/L/C
RL =
   528.0000e+000
w02 =
   160.0000e+003
```

Based on these results, the $1/LC$ term is correct, but we need to adjust the R/L term to get the correct response. If we change only the resistance, then the $1/LC$ term will remain the same and still be correct.

```
% Determine the scaling factor for the resistance
Rscale = 1000/RL
Rnew = R*Rscale
Rscale =
     1.8939e+000
Rnew =
     6.2500e+003
```

If we change the resistor to have a value of 6.25 kΩ, the circuit will have the correct type of response. The following MATLAB code will plot the desired response and the OrCAD simulation verifies the results.

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```
syms t vc
vc = 10-13.3*exp(-200*t)+3.3*exp(-
800*t);
tt = 0:30e-6:30e-3;
vctt = subs(vc,t,tt);
plot(tt,vctt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

The OrCAD circuit and simulation are shown below

Answer:
The vendor should change the resistor to be R = 6.25 kΩ.
The MATLAB and OrCAD simulation results are shown above.

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Problem 7-60

What range of damping ratios is available in the circuit in Figure P7-60.

Solution:

The circuit is a series *RLC* circuit with fixed values for the capacitance and inductance and a variable resistance. The following MATLAB code calculates the available range of damping ratios.

```
clear all
C = 1e-6;
L = 2.5e-3;
% Set the resistance to the maximum and minimum values.
R = 33+[0,500];
% Calculate the range of values for the damping ratio
w0 = sqrt(1/L/C);
z = R/2/L/w0
z =
   330.0000e-003     5.3300e+000
```

Answer:

$0.33 \le \zeta \le 5.33$.

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Problem 7-61

A particular parallel *RLC* circuit has the step response observed on an oscilloscope and shown in Figure P7-61. Four points on the waveform were measured and are shown. Determine the circuit's initial value, final value, the dominant exponential's time constant, and the likely case (A, B, or C) of the circuit response.

Solution:
Observing the step response plot, the initial value is 0 mA and the response is approaching 1 mA for its final value.

The dominant (slowest) exponential is the majority of the signal after 0.2 s, so use the points at 0.2 s and 0.4 s to find the time constant.

```
clear all
% Record the data from the plot
t1 = 0.2004;
t2 = 0.4004;
% Subtract the final value from the current values
i1 = 2.734-1;
i2 = 1.268-1;
% Calculate the time constant
Tc = (t2-t1)/log(i1/i2)
Tc =
   107.1123e-003
```

The results show the time constant is 107.1123 ms. It is possible that the actual time constant is an integer number of milliseconds, so we'll estimate the time constant as 100 ms.

The circuit response is probably overdamped because it appears to contain two exponential components with distinct time constants.

Answer:
$i_L(0) = 0$ mA.
$i_L(\infty) = 1$ mA.
$T_C = 100$ ms.

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Case A or overdamped.

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Problem 7-62

<u>First-order OP AMP Circuit Step Response</u> **(A)**

Find the zero-state response of the OP AMP output voltage in Figure P7-62 when the input is $v_S(t) = V_A\,u(t)$.

Solution:

In the zero-state, the initial voltage across the capacitor is 0 V. Label the node between R_1 and the capacitor as voltage v_1 and note that the node to the right of the capacitor is fixed at 0 V, because of the ideal OP AMP. Write the KCL equation at node 1 as follows

$$\frac{v_1 - v_S}{R_1} + i_C = 0$$

Substitute for the capacitor current and write a first-order differential equation:

$$\frac{v_1 - v_S}{R_1} + C_1\frac{dv_1}{dt} = 0$$

$$R_1C_1\frac{dv_1}{dt} + v_1 = v_S = V_A u(t)$$

Solve the differential equation for v_1.

```
clear all
v1 = dsolve('R1*C1*Dv1+v1=VA','v1(0)=0','t')
v1 =
VA-exp(-1/R1/C1*t)*VA
```

Write the KCL equation at the negative input terminal of the OP AMP and solve for the output voltage.

$$-i_C + \frac{0 - v_O}{R_2} = 0$$

$$v_O = -R_2C_1\frac{dv_1}{dt}$$

Calculate the output voltage based on the results found above:

```
% Find vo(t)
syms R2 C1 t R1
vo = -R2*C1*diff(v1,t)
vo =
-R2/R1*exp(-1/R1/C1*t)*VA
```

Answer:

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$$v_{\mathrm{O}}(t) = \frac{-R_2 V_{\mathrm{A}}}{R_1} e^{-t/R_1C_1} \text{ for } t \geq 0.$$

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Problem 7-63

<u>*RC* Circuit Design</u> **(D)**

Design the first-order *RC* circuit in Figure P7-63 so an input $v_S(t) = 15\ u(t)$ V produces a zero-state response $v_O(t) = 15 - 5e^{-500\,t}$ V.

Solution:

The initial output voltage is 10 V, which indicates we need to divide the source voltage with a voltage divider. The final output voltage is 15 V. Since the source is a scaled step function, the final output voltage will appear across the capacitor. To arrange for both the initial voltage of 10 V and the final voltage, we need to include a resistor in series with the capacitor as the output portion of the circuit. The following circuit will meet the specifications.

In the circuit, the voltage source has the correct amplitude and the capacitor is in the zero state with an initial condition of 0 V. The output voltage is taken across the capacitor and resistor R_2. At time $t = 0$, the capacitor has a voltage drop of 0 V and the remainder of the source voltage must drop across the resistors in the series circuit. Using voltage division, resistor R_2 will drop 10 V, so the initial output voltage is correct. After a long time, the capacitor acts like an open circuit, so it will drop the full 15 V from the source to meet the condition on the final output voltage. We arbitrarily chose $R_1 = 10$ kΩ, which required $R_2 = 20$ kΩ for the voltage divider. We then calculated the capacitance using $RC = 1/T_C = 1/500$ to get C = 66.667 nF.

The following MATLAB code analyzes the voltage across the capacitor and then computes the current in the circuit. Using the results we can calculate the output voltage and compare it to the simulation results for the circuit above.

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```
clear all
format short eng
syms t vc ic vR2 vo
vc0 = 0;
vcf = 15;
R1 = 10e3;
R2 = 2*R1;
Tc = 1/500;
C = 1/500/(R1+R2)
vc = (vc0-vcf)*exp(-t/Tc)+vcf
ic = C*diff(vc,t);
ic_num = vpa(ic,5)
vR2 = R2*ic;
vo = simplify(vc+vR2);
vo_num = vpa(vo,5)
C =
    66.6667e-009
vc =
-15*exp(-500*t)+15
ic_num =
.50000e-3*exp(-500.*t)
vo_num =
-5.0000*exp(-500.*t)+15.

tt = 0:10e-6:10e-3;
vott = subs(vo,t,tt);
plot(tt,vott,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

The follow output plot from the OrCAD simulation matches the MATLAB results.

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Answer:
The required circuit is presented above.

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Problem 7-64

Sample Hold Circuit **(D)**

Figure P7-64 is a simplified diagram of a sample hold circuit. When the switch is in position A, the circuit is in the sample mode and the capacitor voltage must charge to at least 99% of the source voltage V_A in less than 1 μs. When the switch is moved to position B, the circuit is in the hold mode and the capacitor must retain at least 99% of V_A for at least 1 ms. Select a capacitance that meets these constraints.

Solution:

The charging specification sets an upper bound on the capacitor value and the discharging specification sets a lower bound on the capacitor value. The corresponding inequalities for these constraints are

$$1-e^{-1\,\mu\text{s}/R_1C} > 0.99 \quad \text{and} \quad e^{-1\,\text{ms}/R_2C} > 0.99$$

where R_1 = 50 Ω and R_2 = 50 MΩ. Use MATLAB to solve these equations for the maximum and minimum capacitor values and pick a suitable capacitance in the range.

```
clear all
syms Cmin Cmax
Cmax = double(solve('1-exp(-1e-6/50/Cmax)-0.99','Cmax'))
Cmin = double(solve('exp(-1e-3/50e6/Cmin)-0.99','Cmin'))
Cmax =
     4.3429e-009
Cmin =
     1.9900e-009
```

We can choose C = 3 nF to meet the specifications.

Answer:

C = 3 nF.

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Problem 7-65

Super Capacitor **(A)**

Super capacitors have very large capacitance (typically from 0.1 to 50 F), small sizes, and very long charge holding times, making them useful in non-battery backup power applications. The charge holding quality of a super capacitor is measured using the circuit in Figure P7-65. The switch is closed for a long time (say, 24 hours) and the capacitor charged to 5 V. The switch is then opened and the capacitor allowed to self-discharge through any leakage resistance for 24 hours. Suppose that after 24 hours the voltage across a 0.47 F super capacitor is 4.5 V. What is the equivalent leakage resistance in parallel with the capacitor?

Solution:

To solve this problem, we need to find the time constant associated with the capacitor. The solution is presented in the following MATLAB code.

```
clear all
% Set the time points
t1 = 0;
t2 = 24*60*60;
% Set the voltages
i1 = 5;
i2 = 4.5;
% Calculate the time constant
Tc = (t2-t1)/log(i1/i2)
% Calculate the equivalent parallel resistance
C = 0.47;
R = Tc/C
Tc =
   820.0415e+003
R =
     1.7448e+006
```

Answer:

R_L = 1.7448 MΩ.

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Problem 7-66

Cost-Conscious *RLC* Circuit Design **(D)**

You are assigned a task to design a series, passive *RLC* circuit with a characteristic equation of $s^2 + 2000s + 5\times10^6 = 0$. To save money your supervisor wants you to use a previously purchased 100 mH inductor with a 10 Ω parasitic resistance. The *RLC* circuit will be used to interface to a Thèvenin source with a 75 Ω series output resistance. Your circuit must demonstrate the desired response with the source circuit connected.

Solution:

The corresponding series *RLC* characteristic equation is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + 2000s + 5000000 = 0$$

Given a value for the inductance, we can solve for the resistance and capacitance using the characteristic equation.

```
clear all
L = 100e-3
C = 1/L/5e6
R_total = 2000*L
R_design = R_total - 10 -75
L =
   100.0000e-003
C =
     2.0000e-006
R_total =
   200.0000e+000
R_design =
   115.0000e+000
```

Answer:

Use the inductor provided and connect it in series with the source, a 115-Ω resistor, and a 2-μF capacitor in order to meet the specifications.

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Problem 7-67

Combined First- and Second-Order Response (**A**)

The switch in Figure P7-67 has been in position A for a long time and is moved to position B at t = 0 and then to position C when t = 10 ms. For 0 < t < 10 ms the capacitor voltage is a charging exponential $v_C(t) = 10(1 - e^{-100t})$ V. For t > 10 ms the capacitor voltage is a sinusoid $v_C(t) = 6.321 \cos[1000(t - 0.01)]$ V.

(a) Suppose the resistance is reduced to 1 kΩ and the switching sequence repeated.

Will the amplitude of the sinusoid increase, decrease, or stay the same?

Will the frequency of the sinusoid increase, decrease or stay the same?

(b) Suppose the inductance is reduced to 100 mH and the switching sequence repeated.

Will the amplitude of the sinusoid increase, decrease, or stay the same?

Will the frequency of the sinusoid increase, decrease or stay the same?

Solution:

(a) If the resistance is reduced, the time constant to charge the capacitor will be reduced and the capacitor will charge at a faster rate. Since it charges faster, the final voltage on the capacitor will be greater when the switch moves a second time. With a higher initial voltage at the time of the second switch, the amplitude of the sinusoid will increase. The frequency of the sinusoid depends on the values of only the capacitor and the inductor. Since these values have not changed, the frequency of the sinusoid will stay the same.

(b) Changing the inductance will not change the initial voltage on the capacitor when the switch moves a second time, so the amplitude of the sinusoid will stay the same. The frequency of the sinusoid is proportional to $1/\sqrt{LC}$, so if the inductance is reduced, the frequency will increase.

Answer:

Presented above.

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Problem 7-68

Optimum Fusing **(E)**

A sensitive instrument that can be modeled by a series *RLC* circuit, shown in Figure P7-68, is to be protected by a fuse. The voltage across the capacitor is

$$v_C(t) = e^{-10t}(-\cos 103t + .0971\sin 103t)\,u(t) \quad \text{V}$$

The peak current was found to occur at about 14 ms after $t = 0$. Engineer A suggests a 1 mA fuse, Engineer B suggests a 5 mA fuse while Engineer C suggests a 10 mA fuse. The criterion is to select a fuse that is closest to the peak value of current expected without blowing. Which engineer is correct? Provide evidence for your decision.

Solution:

Given the voltage across a capacitor in a series *RLC* circuit, we can calculate the current through the circuit. We will use MATLAB to calculate an expression for the current through the circuit and then plot the waveform to find the maximum current. We can then select the appropriate fuse.

```
clear all
format short eng
syms t vc ic
C = 92.5e-6;
vc = exp(-10*t)*(-1*cos(103*t)+0.0971*sin(103*t));
ic = C*diff(vc,t);
ic_num = vpa(vc,5)
% Plot the current
tt = 0:20e-6:20e-3;
ictt = subs(ic,t,tt);
plot(tt,ictt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
ic_num =
exp(-10.*t)*(-
1.*cos(103.*t)+.97100e-
1*sin(103.*t))
```

The maximum current is above 8 mA, so we must use the 10-mA fuse to protect the circuit.

Answer:

Engineer C is correct. Use the 10-mA fuse.

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Problem 7-69

Lightning Pulser Design **(D)**

The circuit in Figure P7-69 is a simplified diagram of a pulser that delivers simulated lightning transients to the test article at the output interface. Closing the switch must produce a short-circuit current of the form $i_{SC}(t) = I_A e^{!\alpha t}\cos(\beta t)$, with α = 100 krad/s, β = 200 krad/s, and I_A = 2 kA. Select the values of L, C, and V_0.

Solution:

When the switch is closed, the circuit is a parallel *RLC* circuit with an initial voltage on the capacitor and zero initial current through the inductor. The desired circuit response is an underdamped response, so we have the following characteristic equation.

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = (s+\alpha - j\beta)(s+\alpha + j\beta) = s^2 + 2\alpha s + \alpha^2 + \beta^2 = 0$$

Given R = 50 Ω and assuming the test article has negligible resistance, we can calculate the capacitance and then the inductance using the following MATLAB code.

```
clear all
R = 50;
a = 100e3;
B = 200e3;
C = 1/R/2/a
L = 1/C/(a^2+B^2)
C =
   100.0000e-009
L =
   200.0000e-006
```

When the switch closes, there is no current flowing through the inductor, so all of the current must flow through the resistor. The magnitude of the current will be V_0/50 Ω. The following MATLAB code computes the required source voltage.

```
IA = 2000;
V0 = 50*IA
V0 =
   100.0000e+003
```

Answer:

L = 200 μH.

C = 100 nF.

V_0 = 100 kV.

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Problem 7-70

RLC Circuit Design **(E)**

Losses in real inductors can be modeled by the series resistor as shown in Figure P7-70. In this problem, we include the effect of this resistor on the design of the series *RLC* circuit shown in the figure. The design requirements include a source resistance of 50 Ω, an undamped natural frequency of 50 kHz, and a damping ratio less than 0.1.

The characteristics of the available inductors are listed below.

L(mH)	R_L(Ω)	L(mH)	R_L(Ω)
8.2	433	4.7	266
6.8	371	3.9	229
5.6	306		

Which inductor would you use in your design and why?

Solution:

The characteristic equation for a series *RLC* circuit is shown below.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

Given the inductance and undamped natural frequency, we can calculate the required capacitance and the circuit's damping ratio. We will use MATLAB to compute these values for all of the inductor options and then review the results.

```
clear all
format short eng
w0 = 2*pi*50e3;
w02 = w0^2;
L = 1e-3*[8.2, 6.8, 5.6, 4.7, 3.9];
RL = [433, 371, 306, 266, 229];
R = 50+RL;
C = 1/w02./L;
z = R/2/w0./L;
Results = [L' RL' C' z']
Results =
     8.2000e-003   433.0000e+000     1.2356e-009    93.7461e-003
     6.8000e-003   371.0000e+000     1.4900e-009    98.5356e-003
     5.6000e-003   306.0000e+000     1.8093e-009   101.1771e-003
     4.7000e-003   266.0000e+000     2.1558e-009   107.0063e-003
     3.9000e-003   229.0000e+000     2.5980e-009   113.8570e-003
```

The results indicate that two of the inductor options provide damping ratios less than 0.1. All of the capacitor values are reasonable, so we will pick the option that has the lowest damping ratio.

Answer:

Choose the 8.2-mH inductor because it gives the lowest damping ratio.

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Problem 7-71

Competing Circuit Designs **(E)**

Figure P7-71 shows the step responses $v_C(t)$ of two competing series *RLC* circuits from two different vendors. The circuits are designed to switch from 0 to 10 V and to meet a specification for a desired circuit with the following characteristic equation:

$$s^2 + 10\,s + 100 = 0$$

Not all of the details of the actual circuits have been provided because of proprietary reasons, but each vendor has given some information about their respective circuit. Both claim they meet the required specifications.

Vendor A	Vendor B
$R_T = 100\ \Omega$	$R/L = 10$ *rad/s*
$1/LC = 100$ *(rad/s)*^{2}	$\omega_0 = 10$ *rad/s*
$C = 2$ *mF*	

Which vendor would you choose and why?

Solution:

The characteristic equation for a series *RLC* circuit is shown below.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

The required undamped natural frequency is 10 rad/s and the damping ratio is 0.5. Use the information provided to find the undamped natural frequency and damping ratio for each vendor's circuit. The following MATLAB code completes the calculations.

```
clear all
disp('Vendor A')
w02 = 100;
w0 = sqrt(w02)
R = 100;
C = 2e-3;
L = 1/100/C;
z = R/2/w0/L
disp('Vendor B')
w0 = 10
RL = 10;
z = RL/2/w0
Vendor A
w0 =
    10.0000e+000
z =
     1.0000e+000
Vendor B
w0 =
    10.0000e+000
z =
   500.0000e-003
```

Answer:

Choose Vendor B because the circuit meets the specifications.

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Problem 7-72

Solving Differential Equations with MATLAB

MATLAB has a built-in function for solving ordinary differential equations called `dsolve`. We can use this function to quickly explore the solution to a second-order differential equation when the forcing function is a sinusoidal or exponential signal. Suppose we have a series *RLC* circuit in the zero state connected to a voltage source $v_T(t)$. The parameter values are R = 4 kΩ, L = 1 H, and C = 1 μF. The differential equation for the voltage across the capacitor is given by Eq. (7-33). If $v_T(t) = 10\,u(t)$ V, we can use the following MATLAB code to solve for the capacitor voltage and plot the results

```
% Define the symbolic time variable
syms t
% Define the parameter values
R = 4000;
L = 1;
C = 1e-6;
% Solve the differential equation for the series RLC circuit with zero
% initial conditions
vCt = dsolve('L*C*D2v + R*C*Dv + v = 10*heaviside(t)',...
    'v(0)=0','Dv(0)=0','t');
% Create a time vector for plotting and substitute in numerical values
tt = 0:0.0001:0.04;
vCt = subs(vCt);
vCtt = subs(vCt,t,tt);
% Plot the results
figure
plot(tt,vCtt,'b','LineWidth',2)
grid on
xlabel('Time, (s)')
ylabel('v_C(t), (V)')
title('Problem 7-72')
```

Run the given MATLAB code and examine the results. Modify the code to solve the same problem when the input voltage is $v_T(t) = 10\cos(200\,\pi\,t)$ V. Solve the problem a third time for $v_T(t) = 10\,e^{-2000t}$ V. Compare and comment on the responses for the three different types of inputs signals.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Define the symbolic time variable
syms t
% Define the parameter values
R = 4000;
L = 1;
C = 1e-6;
% Solve the differential equation for the series RLC circuit with zero
% initial conditions
vCt = dsolve('L*C*D2v + R*C*Dv + v = 10*heaviside(t)',...
```

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```
    'v(0)=0','Dv(0)=0','t');
% Create a time vector for
plotting and substitute in
numerical values
tt = 0:0.0001:0.04;
vCt = subs(vCt);
vCtt = subs(vCt,t,tt);
% Plot the results
figure
plot(tt,vCtt,'b','LineWidth',2)
grid on
xlabel('Time, (s)')
ylabel('v_C(t), (V)')
title('Problem 7-72 Part 1')
```

```
clear all
% Define the symbolic time
variable
syms t
% Define the parameter values
R = 4000;
L = 1;
C = 1e-6;
% Solve the differential equation for the series RLC circuit with zero
% initial conditions
vCt = dsolve('L*C*D2v + R*C*Dv + v = 10*cos(200*pi*t)',...
    'v(0)=0','Dv(0)=0','t');
% Create a time vector for plotting and substitute in numerical values
tt = 0:0.0001:0.04;
vCt = subs(vCt);
vCtt = subs(vCt,t,tt);
% Plot the results
figure
plot(tt,vCtt,'b','LineWidth',2)
grid on
xlabel('Time, (s)')
ylabel('v_C(t), (V)')
title('Problem 7-72 Part 2')
```

```
clear all
% Define the symbolic time
variable
syms t
% Define the parameter values
R = 4000;
L = 1;
C = 1e-6;
% Solve the differential equation
for the series RLC circuit with zero
% initial conditions
```

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```
vCt = dsolve('L*C*D2v + R*C*Dv + v = 10*exp(-2000*t)',...
    'v(0)=0','Dv(0)=0','t');
% Create a time vector for plotting and substitute in numerical values
tt = 0:0.0001:0.04;
vCt = subs(vCt);
vCtt = subs(vCt,t,tt);
% Plot the results
figure
plot(tt,vCtt,'b','LineWidth',2)
grid on
xlabel('Time, (s)')
ylabel('v_C(t), (V)')
title('Problem 7-72 Part 3')
```

Answer:
The MATLAB code and resulting plots are presented above. By changing the source voltage, we get three distinct types of responses. The first case has an exponential response, the second case has a sinusoidal response, and the third case is a sum of exponentials.

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Problem 8-1

Transform the following sinusoids into phasor form and draw a phasor diagram. Use the additive property of phasors to find $v_1(t) + v_2(t)$.

(a) $v_1(t) = 25 \cos(\omega t + 30\mathsf{E})$ V

(b) $v_2(t) = 100 \cos(\omega t - 150^{\circ})$ V

Solution:

The following MATLAB code creates the phasors, sums them, and plots the phasor diagram

```
clear all
% Create the phasors.
% Convert the degrees to radians for the exponential's argument
v1 = 25*exp(30*j*pi/180)
v2 = 100*exp(-150*j*pi/180)
% Sum the phasors
v12 = v1 + v2
% Find the magnitude and angle of the sum
[Anglev12,Magv12] = cart2pol(real(v12),imag(v12));
Magv12
Anglev12 = 180*Anglev12/pi
v1 =
  21.6506 +12.5000i
v2 =
 -86.6025 -50.0000i
v12 =
 -64.9519 -37.5000i
Magv12 =
    75
Anglev12 =
 -150.0000

% Create the phasor plot
axis([-100 100 -100 100])
line([-100 100],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],[-100 100],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
line([0,real(v12)],[0,imag(v12)],...
    'LineWidth',3,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','m')
line([0,real(v1)],[0,imag(v1)],...
    'LineWidth',2,...
    'Marker','o',...
```

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```
    'MarkerSize',3,...
    'Color','b')
line([0,real(v2)],[0,imag(v2)],...
    'LineWidth',2,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','r')
```

Answers:

(a) $\mathbf{V}_1 = 25\, e^{j30°} = 21.6506 + j\,12.5$

(b) $\mathbf{V}_2 = 100\, e^{-j150°} = -86.6025 - j\,50.0$

$v_1(t) + v_2(t) = 75\cos(\omega t - 150°)$ V

The phasor diagram is shown above.

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Problem 8-2

Transform the following sinusoids into phasor form and draw a phasor diagram. Use the additive property of phasors to find $i_1(t) + i_2(t)$.

(a) $i_1(t) = 4 \sin(\omega t)$ A

(b) $i_2(t) = 3 \cos(\omega t$! 45E) A

Solution:

The following MATLAB code creates the phasors, sums them, and plots the phasor diagram

```
clear all
% Create the phasors.
% Convert the degrees to radians for the exponential's argument
% Note that sin(wt) = cos(wt-pi/2) = cos(wt-90)
i1 = 4*exp(-90*j*pi/180)
i2 = 3*exp(-45*j*pi/180)
% Sum the phasors
i12 = i1 + i2
% Find the magnitude and angle of the sum
[Anglei12,Magi12] = cart2pol(real(i12),imag(i12));
Magi12
Anglei12 = 180*Anglei12/pi
i1 =
   0.0000 - 4.0000i
i2 =
   2.1213 - 2.1213i
i12 =
   2.1213 - 6.1213i
Magi12 =
    6.4785
Anglei12 =
  -70.8864

% Create the phasor plot
axis(8*[-1 1 -1 1])
line([-100 100],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],[-100 100],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
line([0,real(i12)],[0,imag(i12)],...
    'LineWidth',3,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','m')
line([0,real(i1)],[0,imag(i1)],...
    'LineWidth',2,...
```

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```
    'Marker','o',...
    'MarkerSize',3,...
    'Color','b')
line([0,real(i2)],[0,imag(i2)],...
    'LineWidth',2,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','r')
```

Answers:

(a) $\mathbf{I}_1 = 4\,e^{-j90°} = 0 - j\,4$

(b) $\mathbf{I}_2 = 3\,e^{-j45°} = 2.1213 - j\,2.1213$

$i_1(t) + i_2(t) = 6.4785\cos(\omega t - 70.8864°)$ A

The phasor diagram is shown above.

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Problem 8-3

Convert the following phasors into sinusoidal waveforms.

(a) $\mathbf{V_1} = 100e^{-j30^\circ}\text{V}, \omega = 10^3$ rad/s

(b) $\mathbf{V_2} = 120e^{j45^\circ}\text{V}, \omega = 10^3$ rad/s

(c) $\mathbf{I_1} = 240e^{-j26.6^\circ}\text{mA}, \omega = 2\times10^4$ rad/s

(d) $\mathbf{I_2} = 100e^{-j45^\circ}\text{mA}, \omega = 2\times10^4$ rad/s

Solution:

Given a phasor of the form $Me^{-j\phi}$, with radian frequency ω, the corresponding sinusoid is $M\cos(\omega t-\phi)$. The answers below can be written by inspection.

Answers:

(a) $v_1(t) = 100\cos(10^3 t - 30^\circ)$ V

(b) $v_2(t) = 120\cos(10^3 t + 45^\circ)$ V

(c) $i_1(t) = 240\cos(2\times10^4 t - 26.6^\circ)$ mA

(d) $i_2(t) = 100\cos(2\times10^4 t - 45^\circ)$ mA

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Problem 8-4

Use the phasors in Problem 8-3 and the additive property to find the sinusoidal waveforms $v_1(t) + v_2(t)$ and $2i_1(t) - 3i_2(t)$.

Solution:

Note that the frequencies for the signals being combined are compatible, so we can use the phasor approach. The solution is presented in the following MATLAB code.

```
clear all
format short eng
% Create the phasors.
% Convert the degrees to radians for the exponential's argument
v1 = 100*exp(-30*j*pi/180);
v2 = 120*exp(45*j*pi/180);
i1 = 240e-3*exp(-26.6*j*pi/180);
i2 = 100e-3*exp(-45*j*pi/180);
% Combine the phasors
v12 = v1+v2
[Anglev12,Magv12] = cart2pol(real(v12),imag(v12));
Magv12
Anglev12 = 180*Anglev12/pi
i12 = 2*i1 - 3*i2
[Anglei12,Magi12] = cart2pol(real(i12),imag(i12));
Magi12
Anglei12 = 180*Anglei12/pi
v12 =
   171.4554e+000 + 34.8528e+000i
Magv12 =
   174.9619e+000
Anglev12 =
    11.4903e+000
i12 =
   217.0620e-003 -  2.7923e-003i
Magi12 =
   217.0800e-003
Anglei12 =
  -737.0234e-003
```

Answers:

$$v_1(t) + v_2(t) = 174.9619\cos(10^3 t + 11.49°) \text{ V}$$

$$2i_1(t) - 3i_2(t) = 217.08\cos(2\times10^4 t - 0.737°) \text{ mA}$$

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Problem 8-5

The phasor representation of a sinusoid with ω = 500 rad/s is **V** = 15 + *j*10 V. Use the phasor derivative property to find the time derivative of the sinusoid.

Solution:

To take the derivative of a phasor, multiply the phasor by *j*ω. The solution is presented in the following MATLAB code.

```
clear all
V = 15+10j
w = 500
dV = j*w*V
MagdV = abs(dV)
PhasedV = angle(dV)*180/pi
V =
    15.0000e+000 + 10.0000e+000i
w =
   500.0000e+000
dV =
    -5.0000e+003 +  7.5000e+003i
MagdV =
     9.0139e+003
PhasedV =
   123.6901e+000
```

Answer:

$$\frac{dv(t)}{dt} = 9013.9\cos(500t + 123.69°) \text{ V/s}$$

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Problem 8-6
Convert the following phasors into sinusoids:

(a) $\mathbf{V}_1 = 20 + j15 \text{ V}, \omega = 10 \text{ rad/s}$

(b) $\mathbf{V}_2 = 6(8 - j3) \text{ V}, \omega = 20 \text{ krad/s}$

(c) $\mathrm{I}_1 = 12 - j5 + \dfrac{4}{j} \text{ A}, \omega = 3 \text{ krad/s}$

(d) $\mathrm{I}_2 = \dfrac{330 + j810}{2200 - j560} \text{ A}, \omega = 50 \text{ rad/s}$

Solution:
Use MATLAB to find the magnitude and phase of each phasor and then combine the results with the specified frequency to create the sinusoid.

```
clear all
V1 = 20+15j;
MagV1 = abs(V1)
PhaseV1 = angle(V1)*180/pi

V2 = 6*(8-3j);
MagV2 = abs(V2)
PhaseV2 = angle(V2)*180/pi

I1 = 12-5j+4/j;
MagI1 = abs(I1)
PhaseI1 = angle(I1)*180/pi

I2 = (330+810j)/(2200-560j);
MagI2 = abs(I2)
PhaseI2 = angle(I2)*180/pi
MagV1 =
    25.0000e+000
PhaseV1 =
    36.8699e+000
MagV2 =
    51.2640e+000
PhaseV2 =
   -20.5560e+000
MagI1 =
    15.0000e+000
PhaseI1 =
   -36.8699e+000
MagI2 =
   385.2790e-003
PhaseI2 =
    82.1147e+000
```

Answers:

(a) $v_1(t) = 25\cos(10t + 36.8699°)$ V

(b) $v_2(t) = 51.264\cos(20000t - 20.556°)$ V

(c) $i_1(t) = 15\cos(3000t - 36.8699°)$ A

(d) $i_2(t) = 385.279\cos(50t + 82.1147°)$ mA

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Problem 8-7

Use phasors to find the sinusoid $v_2(t)$, where

$v_2(t) = \frac{1}{10}\frac{dv_1(t)}{dt} + 20\,v_1(t)$ and $v_1(t) = 10\cos(250t + 90^o)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
V1 = 10*exp(90*j*pi/180);
w = 250;
dV1 = j*w*V1;
V2 = dV1/10 + 20*V1;
MagV2 = abs(V2)
PhaseV2 = angle(V2)*180/pi
MagV2 =
   320.1562e+000
PhaseV2 =
   141.3402e+000
```

Answer:

$v_2(t) = 320.1562\cos(250t + 141.34^o)$ V.

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Problem 8-8

Given the sinusoids $v_1(t) = 500\ \cos(\omega t - 45°)\text{V}$ and $v_2(t) = 250\ \sin(\omega t)\ \text{V}$ use the additive property of phasors to find $v_3(t)$ such that $v_1 + v_2 + v_3 = 0$.

Solution:

Solving the equation for v_3, we have

$$v_3 = -v_1 - v_2$$

Use MATLAB to complete the calculations

```
clear all
V1 = 500*exp(-45*j*pi/180);
% For v2(t), convert the sine to a cosine
V2 = 250*exp(-90*j*pi/180);
V3 = -V1-V2;
MagV3 = abs(V3)
PhaseV3 = angle(V3)*180/pi
MagV3 =
   699.4832e+000
PhaseV3 =
   120.3612e+000
```

Answer:

$v_3(t) = 699.48\ \cos(\omega t + 120.36°)\text{V}$

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Problem 8-9

Graphically add the following three phasors and determine their sum:

$\mathbf{V}_1 = 9.85 + j\,1.74$ V, $\mathbf{V}_2 = 10\,\angle 130^\circ$ V, $\mathbf{V}_3 = -3.42 - j\,9.40$ V.

Solution:

Use MATLAB to add the phasors algebraically and then graphically.

```
clear all
V1 = 9.85+1.74j;
V2 = 10*exp(130*j*pi/180);
V3 = -3.42-9.4j;
V = V1+V2+V3
MagV = abs(V)
PhaseV = angle(V)*180/pi

% Create the phasor plot
V12 = V1+V2;
axis(10*[-1 1 -1 1])
line([-100 100],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],[-100 100],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
line([0,real(V)],[0,imag(V)],...
    'LineWidth',3,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','g')
line([0,real(V1)],[0,imag(V1)],...
    'LineWidth',2,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','b')
line([real(V1),real(V12)],[imag(V1),imag(V12)],...
    'LineWidth',2,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','b')
line([real(V12),real(V)],[imag(V12),imag(V)],...
    'LineWidth',2,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','b')
```

```
V =
     2.1239e-003 +444.4312e-006i
MagV =
     2.1699e-003
PhaseV =
```

Answer:

$\mathbf{V} = 2.1239 + j\,0.4444$ mV $= 2.17\,e^{j\,11.82^\circ}$ mV.

The phasor plot is shown above.

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```
11.8187e+000
```

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Problem 8-10

Given a sinusoid $v_1(t)$ whose phasor is $\mathbf{V}_1 = !3 + j4$ V, use phasor methods to find a voltage $v_2(t)$ that leads $v_1(t)$ by 90E and has an amplitude of 10 V.

Solution:

When Phasor A leads Phasor B by 90°, Phasor A has a phase that is 90° greater than the phase of Phasor B. We will use the following MATLAB code to complete the solution.

```
clear all
V1 = -3+4j;
PhaseV2 = angle(V1)*180/pi + 90
PhaseV2 =
   216.8699e+000
```

Answer:

$v_2(t) = 10\cos(\omega t + 216.8699°)$ V .

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Problem 8-11

Find the equivalent impedance Z in Figure P8-11 when ω = 2000 rad/s. Express the result in both polar and rectangular form.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
R1 = 10;
R2 = 20;
L = 10e-3;
C = 50e-6;
w = 2000;

ZR1 = R1
ZR2 = R2
ZL = j*w*L
ZC = 1/j/w/C

Zeq = ZR1 + ZL + 1/(1/ZR2+1/ZC)
MagZeq = abs(Zeq)
PhaseZeq = angle(Zeq)*180/pi
ZR1 =
    10.0000e+000
ZR2 =
    20.0000e+000
ZL =
     0.0000e+000 + 20.0000e+000i
ZC =
     0.0000e+000 - 10.0000e+000i
Zeq =
    14.0000e+000 + 12.0000e+000i
MagZeq =
    18.4391e+000
PhaseZeq =
    40.6013e+000
```

Answer:

$Z = 14 + j\,12 = 18.4391\ \angle 40.6013°\ \Omega$.

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Problem 8-12

Find the equivalent impedance Z in Figure P8-12. If ω = 10 krad/s what two elements (R, L, and/or C) could be used to replace the phasor circuit?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
Z = 25 - 25j + 1/(1/(20+50j)+1/(-100j))
MagZ = abs(Z)
PhaseZ = angle(Z)*180/pi
Z =
    93.9655e+000 + 47.4138e+000i
MagZ =
   105.2501e+000
PhaseZ =
    26.7749e+000
```

The equivalent impedance is $Z = 93.97 + j47.41\ \Omega$. This can be represented by a resistor with $R = 93.97\ \Omega$, in series with an inductor with $\omega L = 47.41$.

```
% Compute the inductance
wL = imag(Z);
w = 10e3;
L = wL/w
L =
     4.7414e-003
```

Answers:

$Z = 93.97 + j47.41\ \Omega = 105.25\angle 26.77°\ \Omega$.

The circuit can be replaced by a resistor and inductor in series with $R = 93.97\ \Omega$ and L = 4.7414 mH.

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Problem 8-13

Find the equivalent impedance Z in Figure P8-13 when ω = 1.5 krad/s. What two elements (R, L, and/or C) could be used to replace the phasor circuit?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
w = 1.5e3;
R1 = 50;
ZC = 1/j/w/5e-6;
ZL = j*w*2e-3;
R2 = 100;
Z = 50 + 1/(1/ZC + 1/(ZL+R2))
MagZ = abs(Z)
PhaseZ = angle(Z)*180/pi
Z =
   115.8759e+000 - 47.4751e+000i
MagZ =
   125.2242e+000
PhaseZ =
   -22.2792e+000
```

The circuit can be replaced with a resistor and a capacitor.

```
% Compute the capacitance
wC = -1/imag(Z);
C = wC/w
CheckZC = 1/j/w/C
C =
    14.0424e-006
CheckZC =
     0.0000e+000 - 47.4751e+000i
```

Answers:

$Z = 115.9 - j47.48\ \Omega = 125.2\angle{-22.28°}\ \Omega$.

The circuit can be replaced by a resistor and capacitor in series with R = 115.9 Ω and C = 14.04 μF.

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Problem 8-14

Find the equivalent impedance Z in Figure P8-14. If ω = 100 krad/s what two elements (R, L, and/or C) could be used to replace the phasor circuit?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
Z = 150j + 1/(1/(-300j) + 1/(600+900j))
MagZ = abs(Z)
PhaseZ = angle(Z)*180/pi
Z =
    75.0000e+000 -225.0000e+000i
MagZ =
   237.1708e+000
PhaseZ =
   -71.5651e+000
```

The circuit can be represented by a resistor in series with a capacitor.

```
% Compute the capacitance
w = 100e3;
wC = -1/imag(Z);
C = wC/w
CheckZC = 1/j/w/C
C =
    44.4444e-009
CheckZC =
     0.0000e+000 -225.0000e+000i
```

Answers:

$Z = 75 - j225\ \Omega = 237.17\angle{-71.57°}\ \Omega$.

The circuit can be replaced by a resistor and capacitor in series with R = 75 Ω and C = 44.44 nF.

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Problem 8-15

The circuit in Figure P8-14 is operating in the sinusoidal steady-state with ω = 5 krad/s.

(a) How would the element impedances change if the steady-state frequency were reduced to 500 rad/s?

(b) What is the equivalent impedance Z at this new frequency?

(c) What two elements (R, L, and/or C) could be used to replace the phasor circuit?

Solution:

(a) The impedance for the resistor would not change because it does not depend on frequency. For the inductors, since $Z_L = j\omega L$, if ω is reduced by a factor of 10, then Z_L is also reduced by a factor of 10. The new impedances associated with the inductors would be $j15$ Ω and $j90$ Ω. For the capacitor, $Z_C = 1/(j\omega C)$, so reducing ω by a factor of 10 will increase Z_C by a factor of 10 and the new impedance for the inductor will be $-j3000$ Ω.

(b) The new equivalent impedance is calculated in the following MATLAB code.

```
clear all
Z = 15j + 1/(1/(-3000j) + 1/(600+90j))
MagZ = abs(Z)
PhaseZ = angle(Z)*180/pi
Z =
   611.6831e+000 - 18.3367e+000i
MagZ =
   611.9579e+000
PhaseZ =
    -1.7171e+000
```

(c) The circuit can be represented by a resistor in series with a capacitor.

```
% Compute the capacitance
w = 500;
wC = -1/imag(Z);
C = wC/w
CheckZC = 1/j/w/C
C =
   109.0707e-006
CheckZC =
     0.0000e+000 - 18.3367e+000i
```

Answers:

(a) Z_R does not change; Z_L decreases by a factor of 10; Z_C increases by a factor of 10.

(b) $Z_{EQ} = 611.7 - j18.34\ \Omega = 611.96\angle{-1.717°}\ \Omega$.

(c) The circuit can be replaced by a resistor in series with a capacitor, with $R = 611.7$ Ω and $C = 109.07$ μF.

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Problem 8-16

The circuit in Figure P8-16 is operating in the sinusoidal steady state with ω = 20 krad/s.

(a) Find the equivalent impedance *Z*.

(b) What circuit element can be added in series with the equivalent impedance to place the circuit in resonance?

Solution:

(a) The solution is presented in the following MATLAB code.

```
clear all
w = 20e3;
ZC = 1/j/w/5e-6;
ZL = j*w*2.5e-3;
R = 20;
Z = ZC + 1/(1/ZL + 1/(ZC+R))
MagZ = abs(Z)
PhaseZ = angle(Z)*180/pi
Z =
    25.0000e+000 - 10.0000e+000i
MagZ =
    26.9258e+000
PhaseZ =
   -21.8014e+000
```

(b) To place the circuit in resonance, we need to cancel the imaginary portion of the impedance. We need to add an impedance of $j10\ \Omega$, which would come from an inductor. With $Z_L = j\omega L$, if ω = 20 krad/s, then L = 500 μH.

```
% Compute the inductance
wL = -1*imag(Z);
L = wL/w
L =
   500.0000e-006
```

Answers:

(a) $Z = 25 - j10\ \Omega = 26.93\angle{-21.8°}\ \Omega$.

(b) A 500-μH inductor.

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Problem 8-17

Repeat Problem 8-16 with ω = 5 krad/s.

Solution:

(a) The solution is presented in the following MATLAB code.

```
clear all
w = 5e3;
ZC = 1/j/w/5e-6;
ZL = j*w*2.5e-3;
R = 20;
Z = ZC + 1/(1/ZL + 1/(ZC+R))
MagZ = abs(Z)
PhaseZ = angle(Z)*180/pi
Z =
     2.7027e+000 - 23.7838e+000i
MagZ =
    23.9369e+000
PhaseZ =
   -83.5169e+000
```

(b) To place the circuit in resonance, we need to cancel the imaginary portion of the impedance. We need to add an impedance of $j23.78\ \Omega$, which would come from an inductor. With $Z_L = j\omega L$, if ω = 5 krad/s, then L = 4.757 mH.

```
% Compute the inductance
wL = -1*imag(Z);
L = wL/w
L =
     4.7568e-003
```

Answers:

(a) $Z_{EQ} = 2.7027 - j23.78\ \Omega = 23.94\angle{-83.51°}\ \Omega$.

(b) Add an inductor with L = 4.757 mH.

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Problem 8-18

The equivalent impedance in Figure P8-18 is known to be $Z = 60 + j180\ \Omega$. Find the impedance of the inductor.

Solution:

Find an expression for the equivalent impedance of the circuit, set it equal to $Z = 60 + j180\ \Omega$, and solve for Z_L. The solution is presented in the following MATLAB code.

```
clear all
syms ZL
Eqn1 = 1/(1/600 + 1/(ZL-200j)) - (60+180j);
ZL = solve(Eqn1,ZL)
ZL =
400*i

% Check the results
Z = 1/(1/600 + 1/(double(ZL)-200j))
Z =
    60.0000e+000 +180.0000e+000i
```

Answer:

$Z_L = j400\ \Omega$.

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Problem 8-19

A capacitor C is connected in parallel with a resistor R. Select values of R and C so that the equivalent impedance of the parallel combination is 300 ! j400 Ω at ω = 2 Mrad/s.

Solution:

Find a general expression for the equivalent impedance of a parallel combination of a resistor with a capacitor. Set the expression equal to the desired impedance and solve the two equations (one for the real part and one for the imaginary part) for the resistance and capacitance. The solution is presented in the following MATLAB code.

```
clear all
syms w R C positive
syms ZC Z
ZC = 1/j/w/C;
Z = simplify(1/(1/R + 1/ZC))
Zreal = real(Z);
Zimag = imag(Z);
Eqn1 = Zreal-300;
Eqn2 = Zimag+400;
Eqn3 = w-2e6;
Soln = solve(Eqn1,Eqn2,Eqn3,R,C,w);
R = double(Soln.R)
C = double(Soln.C)
Z =
R/(1+i*w*C*R)
R =
   833.3333e+000
C =
   800.0000e-012
```

Answers:

R = 833.3 Ω and C = 800 pF.

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Problem 8-20

An 800-Ω resistor is connected in parallel with a 1-nF capacitor. The impedance of the parallel combination is 400 – *j*400 Ω. Find the frequency.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms w
R = 800;
C = 1e-9;
ZC = 1/j/w/C;
Eqn1 = 1/(1/R + 1/ZC) - (400-400j);
w = double(solve(Eqn1,w))
CheckZ = 1/(1/R + j*w*C)
w =
     1.2500e+006
CheckZ =
   400.0000e+000 -400.0000e+000i
```

Answer:

The frequency is ω = 1.25 Mrad/s.

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Problem 8-21

A voltage $v(t) = 100 \cos (2000t)$ V is applied across a series connection of a 220-Ω resistor and 110-mH inductor. Find the steady-state current $i(t)$ through the series connection.

Solution:

Find the phasor representation of the voltage and the impedance of the circuit. Compute the corresponding phasor for the current and convert back to a time-domain signal. The following MATLAB code completes the calculations.

```
clear all
syms t
w = 2000;
% Compute the impedance of the series combination
R = 220;
L = 110e-3;
ZL = j*w*L;
Z = R+ZL;
% Find the phasor voltage
Vt = 100*exp(0*j*pi/180);
% Compute the phasor current
It = Vt/Z
MagIt = abs(It)
PhaseIt = angle(It)*180/pi
It =
   227.2727e-003 -227.2727e-003i
MagIt =
   321.4122e-003
PhaseIt =
   -45.0000e+000
```

Answer:

$i(t) = 321.4 \cos (2000t - 45°)$ mA.

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Problem 8-22

The circuit in Figure P8-22 is operating in the sinusoidal steady state with $v_S(t) = V_A \cos(\omega t)$. Derive a general expression for the phasor response $\mathbf{I}_L$ and the voltage $\mathbf{V}_O$.

Solution:

There are several ways to solve this problem. One approach is to combine the impedances of the parallel inductor and resistor and then use voltage division to find the output voltage in phasor form. We can then calculate the inductor current in phasor form. The following MATLAB code completes the calculations

```
clear all
format short eng
syms R L positive
syms w VA t IL Vo Vs ZL Zeq
% Create a phasor for the source voltage
Vs = VA*exp(0*j*pi/180);
% Calculate the inductor impedance and the equivalent
% impedance of the parallel inductor-resistor combination
ZL = j*w*L;
Zeq = 1/(1/ZL + 1/R);
% Use voltage division to find the output voltage phasor
Vo = simplify(Zeq*Vs/(Zeq+R))
% Find the phasor inductor current
IL = simplify(Vo/ZL)
Vo =
-VA*w*L/(-2*w*L+i*R)
IL =
i*VA/(-2*w*L+i*R)
```

Answers:

$$\mathbf{I}_L = \frac{V_A}{R + 2j\omega L}$$

$$\mathbf{V}_O = \frac{j\omega L V_A}{R + 2j\omega L}$$

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Problem 8-23

A current source delivering $i(t) = 240 \cos(500t)$ mA is connected across a parallel combination of a 1-kΩ resistor and a 2-μF capacitor. Find the steady-state current $i_R(t)$ through the resistor and the steady-state current $i_C(t)$ through the capacitor. Draw a phasor diagram showing **I**, $\mathbf{I}_C$, and $\mathbf{I}_R$.

Solution:

This problem can be solved by converting to the phasor domain, using current division, and then converting back to the time domain. The following MATLAB code completes the calculations.

```
clear all
w = 500;
R = 1e3;
C = 2e-6;
% Create the source phasor
Is = 240e-3*exp(0*j*pi/180);
% Create the capacitor impedance
ZC = 1/j/w/C;
% Use current division to find the resistor current
IR = Is/R/(1/R+1/ZC);
MagIR = abs(IR)
PhaseIR = angle(IR)*180/pi
% Use current division to find the capacitor current
IC = Is/ZC/(1/R+1/ZC);
MagIC = abs(IC)
PhaseIC = angle(IC)*180/pi
% Verify KCL holds
CheckKCL = Is-IR-IC
MagIR =
   169.7056e-003
PhaseIR =
   -45.0000e+000
MagIC =
   169.7056e-003
PhaseIC =
    45.0000e+000
CheckKCL =
    13.8778e-018 - 13.8778e-018i

% Create the phasor plot
axis(0.25*[-1 1 -1 1])
line([-100 100],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],[-100 100],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
```

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```
line([0,real(Is)],[0,imag(Is)],...
    'LineWidth',3,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','g')
line([0,real(IR)],[0,imag(IR)],...
    'LineWidth',2,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','b')
line([0,real(IC)],[0,imag(IC)],...
    'LineWidth',2,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','b')
```

Answers:
$i_R(t) = 169.7 \cos(500t - 45°)$ mA.
$i_C(t) = 169.7 \cos(500t + 45°)$ mA.

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Problem 8-24

The circuit in Figure P8-24 is operating in the sinusoidal steady state with $i_S(t) = I_A \cos(\omega t)$. Derive general expressions for the steady-state responses $\mathbf{V}_R$ and $\mathbf{I}_C$.

Solution:

This problem can be solved by using current division in the phasor domain. The following MATLAB code presents the required calculations.

```
clear all
syms w C R positive
syms IA IC IR VR Is ZC
% Create the source current
Is = IA*exp(0*j*pi/180);
% Create the capacitor impedance
ZC = 1/j/w/C;
% Use current division to find the two currents in the circuit
IC = simplify(Is/ZC/(1/ZC + 1/(R+R)))
IR = Is/(R+R)/(1/ZC + 1/(R+R));
% Apply Ohm's Law to find the output voltage
VR = simplify(R*IR)
% Check KCL
CheckKCL = simplify(Is-IC-IR)
IC =
2*i*IA*w*C*R/(2*i*w*C*R+1)
VR =
IA*R/(2*i*w*C*R+1)
CheckKCL =
0
```

Answers:

$$\mathbf{V}_R = \frac{RI_A}{1+2j\omega RC}$$

$$\mathbf{I}_C = \frac{2j\omega RCI_A}{1+2j\omega RC}$$

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Problem 8-25

A voltage $v(t) = 24 \cos(2000t - 45^\circ)$ V is applied across a parallel connection of a 5-kΩ resistor and a 100-nF capacitor. Find the steady-state current $i_C(t)$ through the capacitor and the steady-state current $i_R(t)$ through the resistor. Draw a phasor diagram showing **V**, $\mathbf{I}_C$, and $\mathbf{I}_R$.

Solution:

Since the source voltage is applied across a parallel combination, we can use Ohm's Law to find the required phasor currents and then convert back into the time domain. The solution is presented in the following MATLAB code.

```
clear all
Vs = 24*exp(-45*j*pi/180);
R = 5e3;
C = 100e-9;
w = 2000;
ZC = 1/j/w/C;
IC = Vs/ZC;
MagIC = abs(IC)
PhaseIC = angle(IC)*180/pi
IR = Vs/R;
MagIR = abs(IR)
PhaseIR = angle(IR)*180/pi
MagIC =
     4.8000e-003
PhaseIC =
    45.0000e+000
MagIR =
     4.8000e-003
PhaseIR =
   -45.0000e+000

% Create the phasor plot
axis(25*[-1 1 -1 1])
line([-100 100],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],[-100 100],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
line([0,real(Vs)],[0,imag(Vs)],...
    'LineWidth',3,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','g')
line([0,1000*real(IR)],[0,1000*imag(IR)],...
    'LineWidth',2,...
    'Marker','o',...
```

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```
    'MarkerSize',3,...
    'Color','b')
line([0,1000*real(IC)],[0,1000*imag(IC)],...
    'LineWidth',2,...
    'Marker','o',...
    'MarkerSize',3,...
    'Color','b')
```

Answers:
$i_R(t) = 4.8 \cos(2000t - 45°)$ mA.
$i_C(t) = 4.8 \cos(2000t + 45°)$ mA.
The phasor diagram is shown above. Note that the current phasors are plotted with units of milliamps so that they can be seen on the same scale as the voltage phasor. Also note that the voltage phasor for the source has the same angle as the current phasor for the resistor.

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Problem 8-26

The circuit in Figure P8-26 is operating in the sinusoidal steady state. Find the steady-state response $v_x(t)$.

Solution:

Convert the circuit into the phasor domain. Combine the inductor and the 250-Ω resistor in series and then combine the result in parallel with the capacitor. Use voltage division to find the voltage across the equivalent impedance and then again to find $v_x(t)$. The following MATLAB code presents the remainder of the solution.

```
clear all
w = 1000;
R1 = 500;
R2 = 250;
C = 2e-6;
L = 0.25;
ZC = 1/j/w/C;
ZL = j*w*L;
Zeq1 = ZL+R2;
Zeq2 = 1/(1/ZC+1/Zeq1);
Vs = 20*exp(0*j*pi/180);
V1 = Zeq2*Vs/(R1+Zeq2);
Vx = R2*V1/(R2+ZL);
MagVx = abs(Vx)
PhaseVx = angle(Vx)*180/pi
Zeq2 =
   500.0000e+000 +108.4202e-015i
MagVx =
     7.0711e+000
PhaseVx =
   -45.0000e+000
```

Answer:

$v_x(t) = 7.0711 \cos(1000t - 45°)$ V.

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Problem 8-27

The circuit in Figure P8-27 is operating in the sinusoidal steady state. Find the steady-state response $v_x(t)$.

Solution:

Convert the circuit to the phasor domain and then use current division to find the current in the output branch. Use Ohm's Law to find the output voltage. The following MATLAB code presents the remainder of the solution.

```
clear all
w = 2500;
R = 200;
C = 2e-6;
ZC = 1/j/w/C;
Zeq = R+ZC;
Is = 1*exp(0*j*pi/180);
Ix = Is/Zeq/(1/Zeq + 1/R);
Vx = R*Ix;
MagVx = abs(Vx)
PhaseVx = angle(Vx)*180/pi
MagVx =
    89.4427e+000
PhaseVx =
    26.5651e+000
```

Answer:

$v_x(t) = 89.44 \cos(2500t + 26.565°)$ V.

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Problem 8-28

The circuit in Figure P8-28 is driven by a 10-krad/s source and is operating in the sinusoidal steady state. Use OrCAD to find the steady-state phasor response $\mathbf{V}_x$.

Solution:

First, use MATLAB to compute the two capacitor values and to determine the output phasor. Then use OrCAD to verify the results.

```
clear all
w = 10e3;
Vs = 100*exp(0*j*pi/180);
% Find the capacitor values
wC1 = 1/20;
C1 = wC1/w
wC2 = 1/50;
C2 = wC2/w
C1 =
     5.0000e-006
C2 =
     2.0000e-006

% Find the output phasor voltage by collapsing the circuit from right
% to left using equivalent impedances
Zeq1 = 50-50j;
Zeq2 = 1/(1/100+1/Zeq1);
% Use voltage division twice to find the output phasor voltage
V1 = Zeq2*Vs/(Zeq2-20j);
Vx = -50j*V1/(50-50j)
MagVx = abs(Vx)
PhaseVx = angle(Vx)*180/pi
Zeq1 =
    50.0000e+000 - 50.0000e+000i
Vx =
    50.0000e+000 - 25.0000e+000i
MagVx =
    55.9017e+000
PhaseVx =
   -26.5651e+000
```

Now use OrCAD to simulate the circuit and measure the output phasor voltage.

The relevant portion of the output file is shown below.

```
FREQ        VM(N00196,0)VP(N00196,0)VR(N00196,0)VI(N00196,0)

1.592E+03   5.590E+01  -2.657E+01   5.000E+01  -2.500E+01
```

The OrCAD results match the MATLAB analysis.

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Answer:
$\mathbf{V}_x = 50 - j25$ V = 55.9017 ∠−26.5651° V.

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Problem 8-29

The circuit in Figure P8-29 is operating in the sinusoidal steady state. Use superposition to find the phasor response $\mathbf{I}_x$.

Solution:

The following MATLAB code uses superposition to calculate the phasor components of the output current and then sums the results to get the final answer.

```
clear all
% Create the source phasors
Is = 0.5*exp(0*j*pi/180);
Vs = 50*exp(90*j*pi/180);
% Turn off the voltage source
% Use current division to find Ix1
Ix1 = Is/50/(1/50 + 1/(-50j) + 1/25);
% Turn off the current source
% Find the equivalent impedance to determine Ix2
Zeq = 50 + 1/(1/25 + 1/(-50j));
Ix2 = -1*Vs/Zeq;
Ix = Ix1 + Ix2
MagIx = abs(Ix)
PhaseIx = angle(Ix)*180/pi
Ix =
   250.0000e-003 -750.0000e-003i
MagIx =
   790.5694e-003
PhaseIx =
   -71.5651e+000
```

Answer:

$\mathbf{I}_x = 250 - j750$ mA $= 790.57 \angle{-71.5651°}$ mA.

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Problem 8-30

The circuit in Figure P8-30 is operating in the sinusoidal steady state. Use superposition to find the response $v_x(t)$. *Note*: The sources do not have the same frequency.

Solution:

Since the sources do not have the same frequency, we cannot combine the two components into a single sinusoid. In addition, the different frequencies will create different impedances in the circuit. The following MATLAB code presents the solution.

```
clear all
w1 = 3000;
w2 = 1000;
R = 1000;
C = 1e-6;
ZC1 = 1/j/w1/C;
ZC2 = 1/j/w2/C;
Vs1 = 5*exp(0*j*pi/180);
Vs2 = 10*exp(0*j*pi/180);
% Use voltage division to find the output voltage components
Vx1 = ZC1*Vs1/(ZC1+R);
MagVx1 = abs(Vx1)
PhaseVx1 = angle(Vx1)*180/pi
Vx2 = ZC2*Vs2/(ZC2+R);
MagVx2 = abs(Vx2)
PhaseVx2 = angle(Vx2)*180/pi
MagVx1 =
     1.5811e+000
PhaseVx1 =
   -71.5651e+000
MagVx2 =
     7.0711e+000
PhaseVx2 =
   -45.0000e+000
```

Answer:

$v_x(t) = 1.5811 \cos(3000t - 71.5651°) + 7.0711 \cos(1000t - 45°)$ V.

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Problem 8-31

The circuit in Figure P8-31 is operating in the sinusoidal steady state. Use superposition to find the response $v_x(t)$. *Note*: The sources do not have the same frequency.

Solution:

Since the sources do not have the same frequency, we cannot combine the two components into a single sinusoid. In addition, the different frequencies will create different impedances in the circuit. The following MATLAB code presents the solution.

```
clear all
w1 = 1000;
w2 = 2000;
Is = 200e-3*exp(-30*j*pi/180);
Vs = 10*exp(0*j*pi/180);
C = 2e-6;
ZC1 = 1/j/w1/C;
ZC2 = 1/j/w2/C;
R1 = 50;
R2 = 200;
% Turn on the current source and use current
% division to find Vx1
Ix1 = Is/(ZC1+R2)/(1/(ZC1+R2) + 1/R1);
Vx1 = Ix1*R2;
MagVx1 = abs(Vx1)
PhaseVx1 = angle(Vx1)*180/pi
% Turn on the voltage source and use voltage division
% to find Vx2
Vx2 = -1*R2*Vs/(R1+R2+ZC2);
MagVx2 = abs(Vx2)
PhaseVx2 = angle(Vx2)*180/pi
MagVx1 =
     3.5777e+000
PhaseVx1 =
    33.4349e+000
MagVx2 =
     5.6569e+000
PhaseVx2 =
  -135.0000e+000
```

Answer:

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$v_x(t) = 3.578 \cos(1000t + 33.43°) + 5.657 \cos(2000t - 135°)$ V

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Problem 8-32

The circuit in Figure P8-32 is operating in the sinusoidal steady state.

(a)What impedance, if any, should be connected across $\mathbf{V}_X$ to cancel the reactance in the circuit?

(b) Is the bridge balanced, i.e. is $\mathbf{V}_X = 0$?

Solution:

(a) Compute the equivalent impedance of the circuit using MATLAB.

```
clear all
Z1 = 20-10j;
Z2 = 20+40j;
Zeq = 1/(1/Z1+1/Z2)
Zeq =
    20.0000e+000
```

The equivalent impedance is purely resistive, so nothing needs to be connected across $\mathbf{V}_X$ to cancel the reactance in the circuit.

(b) Use voltage division to find the voltage at the left node and then again to find the voltage at the right node. Subtract the two voltages to get the desired voltage. The following MATLAB code completes the calculations.

```
clear all
Vs = 50*exp(-60*j*pi/180);
V1 = -10j*Vs/(20-10j)
V2 = 20*Vs/(20+40j)
Vx = V1-V2
MagVx = abs(Vx)
PhaseVx = angle(Vx)*180/pi
V1 =
   -12.3205e+000 - 18.6603e+000i
V2 =
   -12.3205e+000 - 18.6603e+000i
Vx =
     0.0000e+000
MagVx =
     0.0000e+000
PhaseVx =
     0.0000e+000
```

The bridge is balanced, because $\mathbf{V}_X = 0$.

Answers:

(a) No impedance is required.

(b) The bridge is balanced.

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Problem 8-33

The circuit in Figure P8-33 is operating in the sinusoidal steady state. Use the unit output method to find the phasor response $\mathbf{V}_x$.

Solution:

We will perform the calculations using MATLAB after we outline the steps. Assume that the output phasor is $1\angle 0°$ V. Use that voltage to calculate the current through the right capacitor. That current also flows through the right resistor. Compute the voltage across that resistor and then use KVL to find the voltage across the left capacitor. Compute the current through the left capacitor and use KCL to find the current through the left resistor. Find the voltage across the left resistor and then use KVL to find the voltage across the source. Divide the actual source voltage by the calculated voltage to determine the actual phasor voltage at the output.

```
clear all
format short eng
Vx = 1*exp(0*j*pi/180);
Ix = Vx/(-100e3j);
VR2 = Ix*200e3;
VC1 = VR2+Vx;
IC1 = VC1/(-20e3j);
IR1 = IC1+Ix;
VR1 = IR1*10e3;
Vs = VR1+VC1;
Vx_actual = 100*exp(0*j*pi/180)/Vs
MagVx_actual = abs(Vx_actual)
PhaseVx_actual = angle(Vx_actual)*180/pi
Vx_actual =
     0.0000e+000 - 38.4615e+000i
MagVx_actual =
    38.4615e+000
PhaseVx_actual =
   -90.0000e+000
```

Answer:

$\mathbf{V}_x = -j38.46$ V $= 38.46\angle -90°$ V.

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Problem 8-34

Use OrCAD to find the Thévenin equivalent of the source circuit to the left of the interface in Figure P8-34. Then use the equivalent circuit to find the steady-state voltage $v(t)$ and current $i(t)$ delivered to the load.

Solution:

Convert the source to a cosine, since $30 \sin(10^6 t)$ V = $30 \cos(10^6 t - 90°)$ V. Compute the answers using MATLAB and then use OrCAD to check the results using the approach described in the problem statement.

```
clear all
Vs = 30*exp(-90*j*pi/180);
w = 1e6;
L = 1e-3;
C = 500e-12;
R1 = 1e3;
R2 = 500;
ZL = j*w*L
ZC = 1/j/w/C
Zeq = 1/(1/ZC + 1/(R1+R2));
% Use voltage division twice to find the load voltage
V1 = Zeq*Vs/(ZL+Zeq);
VL = R2*V1/(R1+R2)
MagVL = abs(VL)
PhaseVL = angle(VL)*180/pi
IL = VL/R2
MagIL = abs(IL)
PhaseIL = angle(IL)*180/pi
ZL =
     0.0000e+000 +  1.0000e+003i
ZC =
     0.0000e+000 -  2.0000e+003i
VL =
    -9.6000e+000 -  7.2000e+000i
MagVL =
    12.0000e+000
PhaseVL =
  -143.1301e+000
IL =
   -19.2000e-003 - 14.4000e-003i
MagIL =
    24.0000e-003
```

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```
PhaseIL =
  -143.1301e+000
```

Now use OrCAD to find the solution. The following two circuits allow us to find the open circuit voltage and the short circuit current associated with the original circuit.

The simulation results are shown below.

```
FREQ        IM(V_PRINT2)IP(V_PRINT2)IR(V_PRINT2)II(V_PRINT2)
1.592E+05   2.683E-02  -1.534E+02  -2.400E-02  -1.200E-02

FREQ        VM(N00172,0)VP(N00172,0)VR(N00172,0)VI(N00172,0)
1.592E+05   6.000E+01  -9.000E+01   3.674E-15  -6.000E+01
```

The Thévenin impedance is the open circuit voltage divided by the short circuit current. Using the results from the OrCAD simulation above, we have:

```
Voc = 0-60j;
Isc = -2.4e-2 - 1.2e-2j;
ZT = Voc/Isc
ZT =
     1.0000e+003 +  2.0000e+003i
```

The Thévenin equivalent source is a 60-V voltage source with a phase of −90° and an impedance of 1 + j2 kΩ. With ω = 1 Mrad/s, we can create the Thévenin impedance with a 1-kΩ resistor and a 2-mH inductor. The above OrCAD circuit will produce the correct response.

The results from the simulation with the Thévenin equivalent circuit are shown below

```
FREQ        IM(V_PRINT3)IP(V_PRINT3)IR(V_PRINT3)II(V_PRINT3)
 1.592E+05   2.400E-02  -1.431E+02  -1.920E-02  -1.440E-02

FREQ        VM(N02912,0)VP(N02912,0)VR(N02912,0)VI(N02912,0)
 1.592E+05   1.200E+01  -1.431E+02  -9.600E+00  -7.200E+00
```

The OrCAD results agree with the MATLAB calculations.

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Answers:
$v(t) = 12 \cos(10^6 t - 143.1°)$ V.
$i(t) = 24 \cos(10^6 t - 143.1°)$ mA.

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Problem 8-35

Find the phasor Thévenin equivalent of the source circuit to the left of the interface in Figure P8-35. Then use the equivalent circuit to find the phasor voltage **V** and current **I** delivered to the load.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
w = 20000;
C = 2e-6;
L = 10e-3;
R1 = 50;
R2 = 100;
R3 = 100;
ZC = 1/j/w/C;
ZL = j*w*L;
Zload = R3 + ZL;
% Use the look-back method to find the Thévenin impedance
ZT = R2 + 1/(1/R1 + 1/ZC)
% Use superposition to find the open-circuit voltage
Vs = 10*exp(-90*j*pi/180);
Is = 5*exp(0*j*pi/180);
Voc1 = Vs*ZC/(ZC+R1);
Voc2 = Is/(1/R1 + 1/ZC);
Voc = Voc1+Voc2
MagVoc = abs(Voc)
PhaseVoc = angle(Voc)*180/pi
% Compute the load voltage and current
Iload = Voc/(ZT+Zload)
MagIload = abs(Iload)
PhaseIload = angle(Iload)*180/pi
Vload = Iload*Zload
MagVload = abs(Vload)
PhaseVload = angle(Vload)*180/pi
ZT =
   110.0000e+000 - 20.0000e+000i
Voc =
    46.0000e+000 -102.0000e+000i
MagVoc =
   111.8928e+000
PhaseVoc =
   -65.7256e+000
Iload =
  -113.7255e-003 -388.2353e-003i
MagIload =
   404.5493e-003
PhaseIload =
  -106.3269e+000
Vload =
    66.2745e+000 - 61.5686e+000i
MagVload =
```

Answers:

The phasor equivalent source is

$\mathbf{V}_T = 111.89\angle{-65.73°}$ V $= 46 - j102$ V with $Z_T = 110 - j20\ \Omega$.

$\mathbf{V} = 66.27 - j61.57 = 90.46\angle{-42.89°}$ V .

$\mathbf{I} = -113.7 - j388.2 = 404.5\angle{-106.33°}$ mA .

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```
    90.4600e+000
PhaseVload =
   -42.8919e+000
```

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Problem 8-36

The circuit in Figure P8-36 is operating in the sinusoidal steady state. When $Z_L = 0$, the phasor current at the interface is $\mathbf{I} = 4.8 - j3.6$ mA. When $Z_L = -j20$ kΩ, the phasor interface current is $\mathbf{I} = 10 + j0$ mA. Find the Thévenin equivalent of the source circuit.

Solution:

When $Z_L = 0$, the load is a short circuit, so the short-circuit current is $\mathbf{I}_{SC} = 4.8 - j3.6$ mA. Model the source as a Norton equivalent circuit to find the Norton impedance as follows. When $Z_L = -j20$ kΩ, we know the load current is $\mathbf{I} = 10 + j0$ mA, so the remainder of the Norton source current must flow through the Norton impedance. We can find the output voltage from the information about the load and then determine the Norton impedance. Finally, we can perform a source transformation to get the Thévenin equivalent source. The following MATLAB code completes the calculations

```
clear all
Isc = 4.8e-3 - 3.6e-3j;
I1 = Isc - 10e-3;
V1 = 10e-3*(-20e3j);
ZT = V1/I1
VT = Isc*ZT
MagVT = abs(VT)
PhaseVT = angle(VT)*180/pi
% Check the conditions in the problem statement
% Both results should be zero
CheckI1 = VT/(ZT) - Isc
CheckI2 = VT/(ZT-20e3j) - 10e-3
ZT =
    18.0000e+003 + 26.0000e+003i
VT =
   180.0000e+000 + 60.0000e+000i
MagVT =
   189.7367e+000
PhaseVT =
    18.4349e+000
CheckI1 =
  -867.3617e-021 +867.3617e-021i
CheckI2 =
     0.0000e+000 +355.2714e-021i
```

Answer:

The phasor equivalent source is $\mathbf{V}_T = 189.74\angle 18.43°$ V $= 180 + j60$ V with $Z_T = 18 + j26$ kΩ.

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Problem 8-37

Design a linear circuit that will deliver an output phasor $\mathbf{V}_O = 100\angle{-45^\circ}$ V when an input phasor $\mathbf{V}_S = 240\angle 0^\circ$ V is applied in Figure P8-37.

Solution:

There are many possible answers to this problem. One approach is to use a pair of impedances connected in series with the source and the output voltage measured across one of the impedances. The circuit would look like the following:

Using voltage division to analyze the circuit, we have the following relationship:

$$\mathbf{V}_O = \frac{Z_2\mathbf{V}_S}{Z_1+Z_2} = \frac{Z_2 240\angle 0^\circ}{Z_1+Z_2} = 100\angle -45^\circ \text{ V}$$

We can satisfy this equation by choosing $Z_2 = 100\angle{-45^\circ}$ kΩ and then choosing Z_1 such that $Z_1 + Z_2 = 240\angle 0^\circ$ kΩ to cancel the source voltage in the expression above. The following MATLAB code completes the calculations

```
clear all
Z2 = 100e3*exp(-45*j*pi/180)
Z1 = 240e3 - Z2
Z2 =
    70.7107e+003 - 70.7107e+003i
Z1 =
   169.2893e+003 + 70.7107e+003i
```

Answer:

$Z_1 = 169.29 + j70.71$ kΩ in series with $\mathbf{V}_S$ and $Z_2 = 70.71 - j70.71$ kΩ in parallel with $\mathbf{V}_O$ as shown in the schematic above.

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Problem 8-38

A load of $Z_L = 100 + j100\ \Omega$ is to be driven by a phasor source $\mathbf{V}_S = 120\ \angle 0^o$ V. The voltage across the load needs to be $\mathbf{V}_L = 100\ \angle 0^o$ V. Design an interface that will meet these conditions. Validate your answer using OrCAD.

Solution:

The following MATLAB code steps through the design process to solve this problem.

```
clear all
Vs = 120*exp(0*j*pi/180);
ZL = 100+100j;
VL = 100*exp(0*j*pi/180);
% Find the load current
IL = VL/ZL;
% Assume the interface is an impedance in series with the
% source and the load
% Find the total equivalent impedance of the circuit by dividing
% the source voltage by the load current
Zeq = Vs/IL;
% The required interface impedance is the difference between the
% total equivalent impedance and the load impedance
ZI = Zeq - ZL
ZI =
    20.0000e+000 + 20.0000e+000i

% The required impedance is a resistor in series with an inductor.
% The parameter values are one-fifth the values for the load impedance.
% To simulate the circuit in OrCAD, choose an operating frequency
% w = 1000 rad/s, so that f = 1000/2/pi = 159.155 Hz.
% Calculate the required impedances
wLL = 100;
w = 1000;
LL = wLL/w
wLI = 20;
LI = wLI/w
LL =
   100.0000e-003
LI =
    20.0000e-003
```

The following OrCAD simulation and results verify the design

FREQ	VM(N03361,0)	VP(N03361,0)	VR(N03361,0)	VI(N03361,0)
1.592E+02	1.000E+02	7.124E-15	1.000E+02	1.243E-14

Answer:

The interface is a series impedance with $Z_I = 20 + j20\ \Omega$.

The OrCAD simulation is shown above.

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Problem 8-39

Design a two-port circuit so that an input voltage $v_S(t) = 100 \cos (10^4 t)$ V delivers a steady-state output current of $i_O(t) = 10 \cos (10^4 t$! 30E) mA to a 100-Ω resistive load. Validate your answer using OrCAD.

Solution:

There are many possible solutions to this problem. If we make the interface an impedance in series with the source and the load, we can solve the problem by finding the equivalent impedance of the circuit and then selecting the devices to achieve that impedance. The following MATLAB code completes the calculations.

```
clear all
Vs = 100;
Io = 10e-3*exp(-30*j*pi/180);
R = 100;
Zeq = Vs/Io;
ZI = Zeq - R
ZI =
     8.5603e+003 +  5.0000e+003i
```

The required series impedance is 8.5603 + j5.0000 kΩ, which requires a resistor and an inductor. We can calculate the inductor's value as follows:

```
% Calculate the inductor value
w = 1e4;
wL = imag(ZI);
L = wL/w
L =
   500.0000e-003
```

The following OrCAD simulation verifies the results.

```
FREQ        IM(V_PRINT1)IP(V_PRINT1)IR(V_PRINT1)II(V_PRINT1)
1.592E+03   1.000E-02  -3.000E+01   8.660E-03  -5.000E-03
```

Answer:

Use a series RL circuit with R = 8560 Ω and L = 500 mH.

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The OrCAD simulation is shown above.

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Problem 8-40

Design a two-port circuit so that an input voltage $v_S(t) = 50 \cos(1000t)$ V delivers a steady-state output voltage of $v_O(t) = 50 \cos(1000t$! 45E) V.

Solution:

There are many possible answers to this problem. One approach is to use a pair of impedances connected in series with the source and the output voltage measured across one of the impedances. The circuit would look like the following:

Using voltage division to analyze the circuit, we have the following relationship:

$$\mathbf{V}_O = \frac{Z_2 \mathbf{V}_S}{Z_1 + Z_2} = \frac{Z_2 50\angle 0°}{Z_1 + Z_2} = 50\angle -45°$$

We can satisfy this equation by choosing $Z_2 = 50\angle -45°$ kΩ and then choosing Z_1 such that $Z_1 + Z_2 = 50\angle 0°$ kΩ to cancel the source voltage in the expression above. The following MATLAB code completes the calculations

```
clear all
Z2 = 50e3*exp(-45*j*pi/180)
Z1 = 50e3 - Z2
Z2 =
    35.3553e+003 - 35.3553e+003i
Z1 =
    14.6447e+003 + 35.3553e+003i
```

Answer:

$Z_1 = 14.64 + j35.36$ kΩ in series with $\mathbf{V}_S$ and $Z_2 = 35.36 - j35.36$ kΩ in parallel with $\mathbf{V}_O$ as shown in the schematic above.

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Problem 8-41

The circuit in Figure P8-41 is operating in the sinusoidal steady state with ω = 4 krad/s. Use node-voltage analysis to find the steady-state response $v_x(t)$.

Solution:

Convert the circuit into the phasor domain and then write a node-voltage equation for the voltage $\mathbf{V}_X$. We can use MATLAB to complete the calculations.

```
clear all
w = 4e3;
Vs1 = 20*exp(0*j*pi/180);
Vs2 = 20*exp(-45*j*pi/180);
L = 0.5;
R = 10e3;
C = 250e-9;
ZL = j*w*L;
ZC = 1/j/w/C;
syms Vx
Eqn1 = (Vx-Vs1)/ZL + Vx/R + (Vx-Vs2)/ZC;
Vx = double(solve(Eqn1,Vx))
MagVx = abs(Vx)
PhaseVx = angle(Vx)*180/pi
Vx =
    13.4049e+000 - 25.6033e+000i
MagVx =
    28.9002e+000
PhaseVx =
   -62.3651e+000
```

Answer:

$v_x(t) = 28.90 \cos (4000t - 62.37°)$ V

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Problem 8-42

Use node-voltage analysis to find the steady-state phasor response $\mathbf{V}_O$ in Figure P8-42.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms V1 Vo
Vs = 120*exp(0*j*pi/180);
Eqn1 = (V1-Vs)/20j + V1/400 + (V1-Vo)/100j;
Eqn2 = (Vo-V1)/100j + Vo/200;
Soln = solve(Eqn1,Eqn2,V1,Vo);
Vo = double(Soln.Vo)
MagVo = abs(Vo)
PhaseVo = angle(Vo)*180/pi
Vo =
    85.2071e+000 - 56.8047e+000i
MagVo =
   102.4062e+000
PhaseVo =
   -33.6901e+000
```

Answer:

$\mathbf{V}_O = 85.2071 - j56.8047$ V $= 102.4062 \angle{-33.6901°}$ V.

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Problem 8-43

The circuit in Figure P8-43 is operating with ω = 10 krad/s. Use OrCAD to find the node voltage phasors $\mathbf{V}_A$ and $\mathbf{V}_B$.

Solution:

We will use MATLAB to find the solution and then also use OrCAD to find the answers using an alternative approach. We will set the operating frequency at ω = 10 krad/s so that we can calculate values for the inductor and capacitors.

```
clear all
syms VA VB
Eqn1 = VA/(-200j) + VA/100 - 2 + (VA-VB)/400;
Eqn2 = (VB-VA)/400 + 2 + VB/200j + VB/(-50j);
Soln = solve(Eqn1,Eqn2,VA,VB);
VA = double(Soln.VA)
MagVA = abs(VA)
PhaseVA = angle(VA)*180/pi
VB = double(Soln.VB)
MagVB = abs(VB)
PhaseVB = angle(VB)*180/pi
VA =
   141.1765e+000 - 35.2941e+000i
MagVA =
   145.5214e+000
PhaseVA =
   -14.0362e+000
VB =
   -23.5294e+000 +105.8824e+000i
MagVB =
   108.4652e+000
PhaseVB =
   102.5288e+000
% Calculate the values for the inductor and capacitors for the given
% operating frequency
w = 10e3;
wL = 200;
L = wL/w
wC1 = 1/200;
C1 = wC1/w
wC2 = 1/50;
C2 = wC2/w
L =
    20.0000e-003
C1 =
   500.0000e-009
C2 =
     2.0000e-006
```

Simulate the circuit in OrCAD. The simulation results are:

```
FREQ        VM(N04612,0)VP(N04612,0)VR(N04612,0)VI(N04612,0)
1.592E+03   1.455E+02  -1.404E+01   1.412E+02  -3.529E+01
FREQ        VM(N04634,0)VP(N04634,0)VR(N04634,0)VI(N04634,0)
1.592E+03   1.085E+02   1.025E+02  -2.353E+01   1.059E+02
```

Answers:

$\mathbf{V}_A = 141.2 - j35.29$ V = 145.5∠−14.04° V.

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$\mathbf{V}_B = -23.53 + j105.9$ V = $108.5\angle 102.5°$ V.
The OrCAD simulation is shown above.

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Problem 8-44

Use mesh-current analysis to find the phasor branch currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit shown in Figure P8-44.

Solution:

Convert the circuit into the phasor domain and then solve for the two mesh currents, $\mathbf{I}_A$ and $\mathbf{I}_B$, circulating in the clockwise direction. We can then use the mesh currents to find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.

```
clear all
syms IA IB
w = 1e4;
R = 1e3;
L = 2e-3;
C = 4e-6;
ZL = j*w*L;
ZC = 1/j/w/C;
% Convert the sources into the phasor domain
Vs1 = 100*exp(-90*j*pi/180);
Vs2 = -100*exp(0*j*pi/180);
Eqn1 = -Vs1 +R*IA + ZL*(IA-IB);
Eqn2 = ZL*(IB-IA) + ZC*IB + Vs2;
Soln = solve(Eqn1,Eqn2,IA,IB);
IA = double(Soln.IA);
IB = double(Soln.IB);
I1 = IA
MagI1 = abs(I1)
PhaseI1 = angle(I1)*180/pi
I2 = IA-IB
MagI2 = abs(I2)
PhaseI2 = angle(I2)*180/pi
I3 = -IB
MagI3 = abs(I3)
PhaseI3 = angle(I3)*180/pi
I1 =
  -405.9406e-003 - 59.4059e-003i
MagI1 =
   410.2643e-003
PhaseI1 =
  -171.6743e+000
I2 =
    -2.0297e+000 - 20.2970e+000i
MagI2 =
    20.3983e+000
PhaseI2 =
   -95.7106e+000
I3 =
    -1.6238e+000 - 20.2376e+000i
MagI3 =
    20.3027e+000
PhaseI3 =
```

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```
   -94.5873e+000


% Check the node voltage using the 3 currents
% All of the following values should be equal
V1 = Vs1-I1*R
V2 = I2*ZL
V3 = Vs2 - ZC*I3
V1 =
   405.9406e+000 - 40.5941e+000i
V2 =
   405.9406e+000 - 40.5941e+000i
V3 =
   405.9406e+000 - 40.5941e+000i
```

Answers:

$\mathbf{I}_1 = -405 - j59.41$ mA $= 410.26\angle -171.67°$ mA.

$\mathbf{I}_2 = -2.03 - j20.30$ A $= 20.4\angle -95.71°$ A.

$\mathbf{I}_3 = -1.62 - j20.24$ A $= 20.3\angle -94.59°$ A.

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Problem 8-45

Use mesh-current analysis to find the phasor currents $\mathbf{I}_A$ and $\mathbf{I}_B$ in Figure P8-45.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms IA IB
Eqn1 = (30+40j)*IA + (20+10j)*(IA-IB) + 120;
Eqn2 = -120 +(20+10j)*(IB-IA) + (30+20j)*IB;
Soln = solve(Eqn1,Eqn2,IA,IB);
IA = double(Soln.IA)
MagIA = abs(IA)
PhaseIA = angle(IA)*180/pi
IB = double(Soln.IB)
MagIB = abs(IB)
PhaseIB = angle(IB)*180/pi
IA =
  -829.7398e-003 +838.6617e-003i
MagIA =
     1.1798e+000
PhaseIA =
   134.6936e+000
IB =
     1.4721e+000 -713.7546e-003i
MagIB =
     1.6360e+000
PhaseIB =
   -25.8664e+000
```

Answers:

$\mathbf{I}_A = -829.7 + j838.7$ mA $= 1.1798 \angle 134.69°$ A.

$\mathbf{I}_B = 1.472 - j0.714$ A $= 1.636 \angle -25.87°$ A.

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Problem 8-46

At what frequency is the magnitude of the output voltage equal to half of the magnitude of the input voltage in the circuit of Figure 8-46?

Solution:

Solve for the input-output relationship with the frequency ω as a variable. Then determine the frequency that makes the output magnitude half of the magnitude of the input voltage. The OP AMP circuit is connected as an inverting amplifier, so we can find the input-output relationship using the impedances at the input and in the feedback path.

```
clear all
syms w positive
syms K
% Find the two impedances
Z1 = 10e3+1/j/w/1.5e-9;
Z2 = 10e3;
K = simplify(-Z2/Z1);
K_num = vpa(K,6)
% We want the magnitude of K to be 0.5
% Since K can be complex and depends on w,
% it may be easier to work with the magnitude
% squared and find the frequency at which it is 0.25
K2 = simplify(K*conj(K));
Eqn1 = K2-0.25;
w = double(solve(Eqn1,w))
K_num =
-.453347e19*w/(.453347e19*w-.302231e24*i)
w =
    38.4900e+003
   -38.4900e+003
```

Choose the positive solution for the frequency.

Answer:

ω = 38.49 krad/s.

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Problem 8-47

Use MATLAB to find the phasor current $\mathbf{I}_O$ in Figure P8-47.

Solution:

Use mesh-current analysis to solve for the mesh currents. The solution is presented in the following MATLAB code.

```
clear all
syms IA IB IC
Vs = 200;
Is = 2;
Eqn1 = -Vs+200*(IA-IB) -50j*(IA-IC);
Eqn2 = (400-300j)*IB + 200j*(IB-IC) + 200*(IB-IA);
Eqn3 = IC + Is;
Soln = solve(Eqn1,Eqn2,Eqn3,IA,IB,IC);
IA = double(Soln.IA);
IC = double(Soln.IC);
IO = IA-IC
MagIO = abs(IO)
PhaseIO = angle(IO)*180/pi
IO =
     3.4462e+000 +430.7692e-003i
MagIO =
     3.4730e+000
PhaseIO =
     7.1250e+000
```

Answer:

$\mathbf{I}_O = 3.4462 + j0.4308$ A $= 3.473 \angle 7.125°$ A.

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Problem 8-48

The circuit in Figure P8-48 is operating with ω = 10 krad/s. Find the phasor outputs $\mathbf{V}_O$ and $\mathbf{I}_O$ in Figure P8-48 when μ = 10 and the phasor input is $\mathbf{I}_S = 1 + j0$ mA.

Solution:

Use node-voltage analysis to solve for the phasor voltage $\mathbf{V}_X$. We can then compute the output voltage, since it is in parallel with the dependent source.

```
clear all
syms Vx
mu = 10;
Is = 1e-3+0j;
% Write a node-voltage equation at the node above
% the current source
Eqn1 = -Is + Vx/2e3 + (Vx-mu*Vx)/20e3j;
Vx = double(solve(Eqn1,Vx));
Vo = mu*Vx
MagVo = abs(Vo)
PhaseVo = angle(Vo)*180/pi
Io = Vo/(-50e3j)
MagIo = abs(Io)
PhaseIo = angle(Io)*180/pi
Vo =
    11.0497e+000 -  9.9448e+000i
MagVo =
    14.8659e+000
PhaseVo =
   -41.9872e+000
Io =
   198.8950e-006 +220.9945e-006i
MagIo =
   297.3177e-006
PhaseIo =
    48.0128e+000
```

Answers:

$\mathbf{V}_O$ = 11.0497 − 9.9448 V = 14.8659 ∠−41.9872 V.

$\mathbf{I}_O$ = 198.895 + 220.9945 μA = 297.3177 ∠ 48.0128° μA.

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Problem 8-49

Find the phasor responses $\mathbf{I}_{IN}$ and $\mathbf{V}_O$ in Figure P8-49 when $\mathbf{V}_S = 1 + j0$ V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms Iin Vo
Vs = 1+0j;
% Write an equation for the input side of the circuit
Eqn1 = Iin - (Vs+Vo/40)/(5+5j);
% Find the equivalent impedance in the output stage
Zeq = 1/(1/50+1/(-50j));
% Write an equation for the output side of the circuit
Eqn2 = 50*Iin*Zeq + Vo;
Soln = solve(Eqn1,Eqn2,Iin,Vo);
Iin = double(Soln.Iin)
MagIin = abs(Iin)
PhaseIin = angle(Iin)*180/pi
Vo = double(Soln.Vo)
MagVo = abs(Vo)
PhaseVo = angle(Vo)*180/pi
Iin =
    18.0967e-003 + 13.1045e-003i
MagIin =
    22.3432e-003
PhaseIin =
    35.9097e+000
Vo =
   -39.0016e+000 +  6.2402e+000i
MagVo =
    39.4976e+000
PhaseVo =
   170.9097e+000
```

Answers:

$\mathbf{I}_{IN} = 18.1 + j13.1$ mA $= 22.34 \angle 35.91°$ mA.

$\mathbf{V}_O = -39.0 + j6.24$ V $= 39.50 \angle 170.91°$ V.

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Problem 8-50

The OP AMP circuit in Figure P8-50 is operating in the sinusoidal steady state with ω = 1 krad/s. Find the ratio of the output phasor $\mathbf{V}_O$ to the input phasor $\mathbf{V}_S$. Repeat for ω = 10 krad/s, ω = 100 krad/s, ω = 1 Mrad/s, and ω = 10 Mrad/s. Use MATLAB to plot the log of the magnitude of $\mathbf{V}_O/\mathbf{V}_S$ versus the log of the frequency ω. Comment on the plot.

Solution:

Solve this problem with ω as a variable and then substitute in the required values. The OP AMP is connected as an inverting amplifier, so we need to calculate the input and feedback impedances to determine the input-output relationship.

```
clear all
syms w positive
R = 10e3;
C = 1e-9;
ZC = 1/j/w/C;
Z1 = R;
Z2 = 1/(1/R+1/ZC);
K = -Z2/Z1;
% Use the variable ww to represent the specific frequencies
% at which to evaluate the input-output ratio
ww = [1e3 10e3 100e3 1e6 10e6];
Kww = subs(K,w,ww);
MagKww = abs(Kww);
PhaseKww = angle(Kww)*180/pi;
% Present the results in a convenient format
Results = [ww.' Kww.' MagKww.' PhaseKww.']
Results =
  Columns 1 through 2
     1.0000e+003                 -999.9000e-003 +  9.9990e-003i
    10.0000e+003                 -990.0990e-003 + 99.0099e-003i
   100.0000e+003                 -500.0000e-003 +500.0000e-003i
     1.0000e+006                   -9.9010e-003 + 99.0099e-003i
    10.0000e+006                  -99.9900e-006 +  9.9990e-003i
  Columns 3 through 4
   999.9500e-003                  179.4271e+000
   995.0372e-003                  174.2894e+000
   707.1068e-003                  135.0000e+000
```

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```
    99.5037e-003                    95.7106e+000
     9.9995e-003                    90.5729e+000
```

```
% Plot the log of the magnitude of the
% voltage ratio versus the log of the frequency
ww = logspace(2,9,100);
Kww = subs(K,w,ww);
MagKww = abs(Kww);
loglog(ww,MagKww,'b','LineWidth',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude Vo / Vs')
```

Answer:
The following table summarizes the results for the requested frequencies.

ω (rad/s)	$\mathbf{V}_O / \mathbf{V}_S$	$\lvert\mathbf{V}_O / \mathbf{V}_S\rvert$	$\angle\mathbf{V}_O / \mathbf{V}_S$
1 k	−999.9 + *j*9.999 m	999.95 m	179.43°
10 k	−990.1 + *j*99.01 m	995.04 m	174.29°
100 k	−500 + *j*500 m	707.11 m	135°
1 M	−9.901 + *j*99.01 m	99.50 m	95.71°
10 M	−99.99 + *j*9.999 m	9.9995 m	90.57°

The plot of the magnitude of the input-output relationship versus frequency is shown above. The plot is approximately constant with a value near one up until a frequency of approximately 100 krad/s. After this frequency, the plot decreases linearly on the log-log scale. The magnitude of the ratio decreases by a factor of 10 every time the frequency increases by a factor of 10.

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Problem 8-51

Find the phasor input $\mathbf{V}_S$ in Figure P8-51 when the phasor output is $\mathbf{V}_O = 200 + j100$ V.

Solution:

The output phasor voltage appears across two elements. Use the voltage and the values of those elements to determine their corresponding currents. Sum the currents to find the current flowing through the other two elements in series. Compute the total voltage across the source.

```
clear all
Vo = 200+100j;
Io1 = Vo/100;
Io2 = Vo/(-200j);
Is = Io1+Io2;
Vs = (10+20j)*Is + Vo
MagVs = abs(Vs)
PhaseVs = angle(Vs)*180/pi
Vs =
   175.0000e+000 +150.0000e+000i
MagVs =
   230.4886e+000
PhaseVs =
    40.6013e+000
```

Answer:

$\mathbf{V}_S = 175 + j150$ V $= 230.4886 \angle 40.60°$ V.

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Problem 8-52

The dependent source circuit in Figure P8-52 is operating in the sinusoidal steady state with ω = 1 krad/s and μ = 100. Find the phasor gain $K = \mathbf{V}_O/\mathbf{V}_S$ and the input impedance Z_{IN} seen by $\mathbf{V}_S$. Validate your answer using OrCAD.

Solution:

Use node-voltage analysis to determine expressions for the voltages in the circuit and then find the input-output ratio. Once we know the voltages, we can also compute the input impedance. The solution using MATLAB is presented below.

```
clear all
syms Vs Vo Vx V1
mu = 100;
Eqn1 = (V1-Vs)/10e3 + (V1-Vx)/10e3 + (V1-mu*Vx)/(-10e3j);
Eqn2 = (Vx-V1)/10e3 + Vx/(-10e3j);
Soln = solve(Eqn1,Eqn2,V1,Vx);
Vx = Soln.Vx;
Vo = mu*Vx;
K = double(Vo/Vs)
MagK = abs(K)
PhaseK = angle(K)*180/pi
% Compute the input impedance
V1 = Soln.V1;
Is = (Vs-V1)/10e3;
Zin = double(simplify(Vs/Is))
K =
     0.0000e+000 +  1.0309e+000i
MagK =
     1.0309e+000
PhaseK =
    90.0000e+000
Zin =
     9.8969e+003 +100.9891e+000i
```

Verify the MATLAB results using OrCAD. First compute the capacitance at ω = 1 krad/s.

```
% Compute the capacitance
w = 1e3;
wC = 1/10e3;
C = wC/w
C =
   100.0000e-009
```

The required circuit is shown below.

The results are shown below.

```
FREQ        VM(N05696,0)VP(N05696,0)VR(N05696,0)VI(N05696,0)
1.592E+02   1.031E+00   9.000E+01   1.227E-11   1.031E+00
```

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```
  FREQ        IM(V_PRINT1)IP(V_PRINT1)IR(V_PRINT1)II(V_PRINT1)
  1.592E+02   1.010E-04  -5.846E-01   1.010E-04  -1.031E-06
```

We can compute the input impedance by taking the ratio of the input voltage to the input current. The input voltage is 1 ∠0° V.

```
% Compute input impedance
Iin2 = 1.01e-4 - 1.031e-6j;
Zin2 = 1/Iin2
Zin2 =
     9.9000e+003 +101.0580e+000i
```

The OrCAD results agree with the MATLAB calculations.

Answers:
$K = \mathbf{V}_O/\mathbf{V}_S = j1.0309 = 1.0309\angle 90°$.
$Z_{IN} = 9.8969 + j0.101$ kΩ.

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Problem 8-53

Find the phasor gain $K = \mathbf{V}_O/\mathbf{V}_S$ and input impedance Z_{IN} of the circuit in Figure P8-53.

Solution:

Use node-voltage analysis to solve the problem.

```
clear all
syms Vs Va Vo
% Create the node-voltage equations
Eqn1 = (Va-Vs)/(-15e3j) + Va/20e3 + (Va-Vo)/(-25e3j);
Eqn2 = (Vo-Va)/(-25e3j) + (Vo-Vs)/10e3;
Soln = solve(Eqn1,Eqn2,Va,Vo);
Vo = Soln.Vo;
K = double(Vo/Vs)
MagK = abs(K)
PhaseK = angle(K)*180/pi
% Find the input current to compute the input impedance
Iin1 = (Vs-Vo)/10e3;
Va = Soln.Va;
Iin2 = (Vs-Va)/(-15e3j);
Iin = Iin1+Iin2;
Zin = double(simplify(Vs/Iin))
K =
   847.2873e-003 - 28.1313e-003i
MagK =
   847.7542e-003
PhaseK =
    -1.9016e+000
Zin =
    21.3235e+003 -  9.7059e+003i
```

Answers:

$K = \mathbf{V}_O/\mathbf{V}_S = 0.8473 - j0.0281 = 0.8478 \angle -1.9016°$.

$Z_{IN} = 21.32 - j9.706$ kΩ.

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Problem 8-54

Find the phasor gain $K = \mathbf{V}_O/\mathbf{V}_S$ and input impedance Z_{IN} of the circuit in Figure P8-54.

Solution:

Use node-voltage analysis to solve the problem.

```
clear all
syms Vs Va Vo
Eqn1 = (Va-Vs)/100 + Va/(-100j) + (Va-Vo)/100j;
Eqn2 = (Vo-Va)/100j + Vo/100;
Soln = solve(Eqn1,Eqn2,Va,Vo);
Vo = Soln.Vo;
K = double(simplify(Vo/Vs))
MagK = abs(K)
PhaseK = angle(K)*180/pi
% Compute the input current to find Zin
Va = Soln.Va;
Iin = (Vs-Va)/100;
Zin = double(simplify(Vs/Iin))
K =
   200.0000e-003 -400.0000e-003i
MagK =
   447.2136e-003
PhaseK =
   -63.4349e+000
Zin =
   200.0000e+000 -100.0000e+000i
```

Answers:

$K = \mathbf{V}_O/\mathbf{V}_S = 0.2 - j0.4 = 0.4472 \angle{-63.43°}$.

$Z_{IN} = 200 - j100\ \Omega$.

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Problem 8-55

Given the circuit in Figure P8-55:

(a) Use node-voltage or mesh-current analysis to develop a set of matrix equations for the circuit.

(b) Use MATLAB to solve the matrix equations and then find the phasor gain $K = \mathbf{V}_O/\mathbf{V}_S$ and input impedance Z_{IN} of the circuit.

(c) Without using the matrix equations, use the MATLAB command `solve` on the original node-voltage or mesh-current equations to solve the equations and then find the phasor gain and input impedance.

Solution:

(a) The node-voltage equations are presented below.

$$\frac{\mathbf{V}_A - \mathbf{V}_S}{15} + \frac{\mathbf{V}_A - \mathbf{V}_B}{60} + \frac{\mathbf{V}_A - \mathbf{V}_O}{j50} = 0$$

$$\frac{\mathbf{V}_B - \mathbf{V}_A}{60} + \frac{\mathbf{V}_B}{-j90} + \frac{\mathbf{V}_B - \mathbf{V}_O}{50} = 0$$

$$\frac{\mathbf{V}_O}{100} + \frac{\mathbf{V}_O - \mathbf{V}_B}{50} + \frac{\mathbf{V}_O - \mathbf{V}_A}{j50} = 0$$

These equations can be re-written as follows:

$$\mathbf{V}_A\left(\frac{1}{15} + \frac{1}{60} + \frac{1}{j50}\right) - \mathbf{V}_B\left(\frac{1}{60}\right) - \mathbf{V}_O\left(\frac{1}{j50}\right) = \frac{\mathbf{V}_S}{15}$$

$$-\mathbf{V}_A\left(\frac{1}{60}\right) + \mathbf{V}_B\left(\frac{1}{60} + \frac{1}{-j90} + \frac{1}{50}\right) - \mathbf{V}_O\left(\frac{1}{50}\right) = 0$$

$$-\mathbf{V}_A\left(\frac{1}{j50}\right) - \mathbf{V}_B\left(\frac{1}{50}\right) + \mathbf{V}_O\left(\frac{1}{100} + \frac{1}{50} + \frac{1}{j50}\right) = 0$$

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These equations can be written in matrix form, as will be shown in the MATLAB code that follows.

(b) Use MATLAB to create the matrix equations and solve for the required values

```
clear all
syms Vs Va Vb Vo
A = [1/15+1/60+1/50j, -1/60, -1/50j; ...
    -1/60, 1/60+1/(-90j)+1/50, -1/50; ...
    -1/50j, -1/50, 1/100+1/50+1/50j];
B = [Vs/15; 0; 0];
Vans = A\B;
Va = Vans(1);
Vb = Vans(2);
Vo = Vans(3);
K = double(simplify(Vo/Vs))
MagK = abs(K)
PhaseK = angle(K)*180/pi
% Compute the input current to find the input impedance
Iin = (Vs-Va)/15;
Zin = double(simplify(Vs/Iin))
K =
   646.1342e-003 -405.7975e-003i
MagK =
   762.9948e-003
PhaseK =
   -32.1304e+000
Zin =
    84.9075e+000 - 20.1201e+000i
```

(c) Use the MATLAB command solve to verify the results

```
clear all
syms Vs Va Vb Vo
Eqn1 = (Va-Vs)/15+(Va-Vb)/60+(Va-Vo)/50j;
Eqn2 = (Vb-Va)/60+Vb/(-90j)+(Vb-Vo)/50;
Eqn3 = Vo/100+(Vo-Vb)/50+(Vo-Va)/50j;
Soln2 = solve(Eqn1,Eqn2,Eqn3,Va,Vb,Vo);
Va2 = Soln2.Va;
Vo2 = Soln2.Vo;

K2 = double(simplify(Vo2/Vs))

Iin2 = simplify((Vs-Va2)/15);
Zin2 = double(simplify(Vs/Iin2))
K2 =
   646.1342e-003 -405.7975e-003i
Zin2 =
    84.9075e+000 - 20.1201e+000i
```

The results agree with those calculated above.

Answers:

(a) The equations are presented above.

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(b) $K = 0.6461 - j0.4058 = 0.76299 \angle -32.13°$.
$Z_{IN} = 84.907 - j20.12\ \Omega$.

(c) The answers using the second MATLAB approach match those found previously.

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Problem 8-56

A load consisting of a 1-kΩ resistor in series with a 12-μF capacitor is connected across a voltage source $v_S(t) = 150 \cos(377t)$ V. Find the phasor voltage, current, and average power delivered to the load.

Solution:

In this case, the load voltage equals the source voltage, because the load is connected in parallel with the source. The solution is presented in the following MATLAB code.

```
clear all
w = 377;
Vs = 150;
R = 1e3;
C = 12e-6;
ZC = 1/j/w/C;
% Find the load impedance
ZL = R+ZC;
% Find the load current
IL = Vs/ZL
MagIL = abs(IL)
PhaseIL = angle(IL)*180/pi
% Compute the average power
PLavg = real(ZL)/2*MagIL^2
IL =
   143.0124e-003 + 31.6119e-003i
MagIL =
   146.4645e-003
PhaseIL =
    12.4644e+000
PLavg =
    10.7259e+000
```

Answers:

$\mathbf{V}_L = 150 + j0$ V $= 150\angle 0°$ V.

$\mathbf{I}_L = 143.01 + j31.61$ mA $= 146.46\angle 12.46°$ mA.

$P = 10.7259$ W.

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Problem 8-57

A load consisting of a 50-Ω resistor in parallel with a 8-μF capacitor is connected across a current source delivering $i_S(t) = 50 \cos(2500t)$ mA. Find the average power delivered to the load.

Solution:

The load receives the entire current from the source, so we know the magnitude of the load current equals the magnitude of the source current. We can combine the parallel impedances to find the equivalent impedance and then use the real part to compute the average power delivered to the load.

```
clear all
IL = 50e-3;
w = 2500;
R = 50;
C = 8e-6;
ZC = 1/j/w/C;
ZL = 1/(1/R+1/ZC);
PLavg = real(ZL)/2*IL^2
PLavg =
    31.2500e-003
```

Answer:

P = 31.25 mW.

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Problem 8-58

Use OrCAD to find the average power delivered to the 50-Ω resistor in Figure P8-58.

Solution:

First, we will find the answer using MATLAB and then we will simulate the circuit in OrCAD.

```
clear all
% Use superposition to find the voltage across the load.
% Then compute the load current and power.
Vs1 = 12;
Vs2 = 6*exp(-90*j*pi/180);
% Turn on the left source
Zeq = 1/(1/40j+1/50);
VL1 = Zeq*Vs1/(Zeq-20j);
% Turn on the right source
Zeq = 1/(1/(-20j) + 1/40j);
VL2 = -50*Vs2/(Zeq+50);
% Combine the results
VL = VL1+VL2;
IL = VL/50;
MagIL = abs(IL);
PLavg = 50/2*MagIL^2
PLavg =
     3.7317e+000
```

Now use OrCAD to simulate the circuit. Select an operating frequency of ω = 10 krad/s so we can find values for the capacitor and inductor.

```
% Calculate capacitor and inductor values
w = 10e3;
wC = 1/20;
C = wC/w
wL = 40;
L = wL/w
C =
     5.0000e-006
L =
     4.0000e-003
```

The OrCAD circuit is show at right. The OrCAD results are shown below.

```
FREQ        IM(V_PRINT1)IP(V_PRINT1)IR(V_PRINT1)II(V_PRINT1)
1.592E+03   3.864E-01   5.270E+01   2.341E-01   3.073E-01
```

Using these results, we can calculate the average power as follows

```
% Use the OrCAD results to calculate average power.
MagIL = 3.864e-1;
R = 50;
PLavg = R/2*MagIL^2
PLavg =
     3.7326e+000
```

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The results are consistent with the MATLAB results.
Answer: P = 3.7317 W.

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Problem 8-59

(a) Find the average power delivered to the load in Figure P8-59.

(b) Find the maximum available average power at the interface shown in the figure.

(c) Specify the load required to extract the maximum average power.

Solution:

(a) We will use node-voltage analysis and MATLAB to find the average power delivered to the load.

```
clear all
w = 1e6;
Vs = 30;
L = 1e-3;
C = 500e-12;
R1 = 1e3;
R2 = 500;
ZL = j*w*L;
ZC = 1/j/w/C;
% Use node-voltage analysis
syms Va
Eqn1 = (Va-Vs)/ZL + Va/ZC + Va/(R1+R2);
Va = double(solve(Eqn1,Va))
IL = Va/(R1+R2);
MagIL = abs(IL);
PLavg = 500/2*MagIL^2
Va =
    21.6000e+000 - 28.8000e+000i
PLavg =
   144.0000e-003
```

(b) To find the maximum available average power, we need to find the Thévenin equivalent for the source.

```
% Find the Thevenin equivalent circuit
VT = ZC*Vs/(ZC+ZL)
MagVT = abs(VT);
ZT = R1 + 1/(1/ZL+1/ZC)
RT = real(ZT)
% Compute the maximum available average power
Pmax = (MagVT^2)/8/RT
```

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```
VT =
    60.0000e+000
ZT =
     1.0000e+003 +  2.0000e+003i
RT =
     1.0000e+003
Pmax =
   450.0000e-003
```

(c) To achieve the maximum power delivered to the load, the load impedance must be the complex conjugate of the Thévenin impedance.

```
% Compute the load impedance to maximize power delivery
ZLmax = conj(ZT)
ZLmax =
     1.0000e+003 -  2.0000e+003i
```

Answers:

(a) $P_L = 144$ mW.

(b) $P_{MAX} = 450$ mW.

(c) $Z_L = 1 - j2$ kΩ.

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Problem 8-60

(a) Find the maximum average power available at the interface in Figure P8-60.

(b) Specify the values of *R* and *C* that will extract the maximum power from the source circuit.

Solution:

(a) If we convert the circuit to the phasor domain, the left side of the interface is a Thévenin equivalent circuit. We can use the Thévenin circuit to find the maximum average power available and the matching load impedance.

```
clear all
w = 8000;
L = 500e-3;
ZL = j*w*L;
Rs = 1e3;
ZT = ZL+Rs;
RT = real(ZT);
VT = 10;
Pmax = (VT^2)/8/RT
Pmax =
    12.5000e-003
```

(b) To extract the maximum power from the source, the load impedance must be the complex conjugate of the Thévenin impedance of the source. We will find the required load impedance and calculate the values of the components required to match the source impedance.

```
% The desired load impedance is the complex conjugate of ZT
Zd = conj(ZT)
syms R C positive
% Find an expression for the load impedance
ZC = 1/j/w/C;
Zload = 1/(1/ZC + 1/R);
% Create equations for the real and imaginary components
% of the load to match the desired values
Eqn1 = real(Zd)-real(Zload);
Eqn2 = imag(Zd)-imag(Zload);
Soln = solve(Eqn1,Eqn2,R,C);
R = double(Soln.R)
C = double(Soln.C)
Zd =
     1.0000e+003 -  4.0000e+003i
R =
    17.0000e+003
C =
    29.4118e-009
```

Answers:

(a) P_{MAX} = 12.5 mW.

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(b) R = 17 kΩ and C = 29.4118 nF.

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Problem 8-61

AC Voltage Measurement **(A)**

An ac voltmeter measurement indicates the amplitude of a sinusoid and not its phase angle. The magnitude and phase can be inferred by making several measurements and using KVL. For example, Figure P8-61 shows a relay coil of unknown resistance and inductance. The following ac voltmeter readings are taken with the circuit operating in the sinusoidal steady state at f = 1 kHz: $|\mathbf{V}_S|$ = 12 V, $|\mathbf{V}_1|$= 5 V, and $|\mathbf{V}_2|$ = 9 V. Find R and L.

Solution:

Given the magnitude of the voltage across the 200-Ω resistor, we can find the magnitude of the current through the series circuit. We can use this current to form two equations that can be solved for the unknown resistance and inductance. One equation involves the magnitude of the voltage across the relay coil and the other involves the magnitude of the voltage across the source. The following MATLAB code completes the calculations.

```
clear all
format short eng
Vs = 12;
V1 = 5;
V2 = 9;
R1 = 200;
w = 2*pi*1e3;
syms R L positive
% Find the magnitude of the current in the circuit
I1 = V1/R1;
% The same current flows through the relay coil.  The impedance of
% the relay coil is R+j*w*L.  The magnitude squared of the impedance
% of the relay coil is R^2 + w^2*L^2
Z = R + j*w*L;
Z2 = Z*conj(Z);
% The magnitude squared of the impedance times the square of the
% magnitude of the current will equal the square of the voltage drop
Eqn1 = Z2*I1^2-V2^2;
% Use the same concept to form another equation with
% the voltage drop across the source
ZT = R1+Z;
ZT2 = ZT*conj(ZT);
Eqn2 = ZT2*I1^2-Vs^2;
% Solve the two equations for R and L
Soln = solve(Eqn1,Eqn2,R,L);
R = double(Soln.R)
L = double(Soln.L)
R =
   152.0000e+000
   152.0000e+000
L =
    51.9382e-003
   -51.9382e-003
```

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Choose the positive results.

Answer:
R = 152 Ω and L = 51.9382

Problem 8-62

OP AMP Band Pass Filter **(D)**

Use the analysis methods discussed in Example 8-23 to find the input-output relationship $\mathbf{V}_O/\mathbf{V}_S$ for the active band pass filter of Figure P8-62. Treat each stage separately and then multiply the input-output relationships from each stage to obtain the overall input-output relationship. Select R and C values so that the low cutoff frequency is 500 rad/s, the upper cutoff frequency is 50 krad/s, and the magnitude of the passband gain is 100.

Solution:

We will use MATLAB to perform the calculations for this problem.

```
clear all
syms R1 R2 R3 R4 C2 C3 w K1 K2 K
% The OP AMPs are connected as inverting amplifiers
% Stage 1
Z1 = R1;
Z2 = 1/(1/R2 + j*w*C2);
K1 = -Z2/Z1
% Stage 2
Z3 = 1/j/w/C3 + R3;
Z4 = R4;
K2 = -Z4/Z3
% Multiply the individual gains to get the overall gain
K = K1*K2
K1 =
-1/(1/R2+i*w*C2)/R1
K2 =
-R4/(-i/w/C3+R3)
K =
1/(1/R2+i*w*C2)/R1*R4/(-i/w/C3+R3)
```

We have the following result:

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$$K = \left[\frac{1}{R_1\left(\frac{1}{R_2} + j\omega C_2\right)}\right]\left[\frac{R_4}{\frac{-j}{\omega C_3} + R_3}\right] = \left[\frac{\frac{R_2}{R_1}\frac{1}{R_2C_2}}{j\omega + \frac{1}{R_2C_2}}\right]\left[\frac{\frac{R_4}{R_3}j\omega}{j\omega + \frac{1}{R_3C_3}}\right]$$

Examine the two expressions in brackets above. In the left expression, at low frequencies, the gain is approximately R_2/R_1 and the gain begins to decrease significantly after the frequency of $\omega = 1/(R_2C_2)$. In the right expression, at low frequencies, the gain is small, but at frequencies above $\omega = 1/(R_3C_3)$, the gain approaches R_4/R_3. The left expression passes the low frequencies with some gain and the right expression passes the high frequencies with some gain. If we select the resistor and capacitor values such that there is a region where both stages allow the signals to pass, we will create the desired bandpass filter. We want the left expression to control the upper frequency of 50 krad/s and we want the right expression to control the lower frequency of 500 rad/s. Allow each stage to have a gain of 10 to achieve an overall gain of 100. Select the capacitor values and then determine the required resistor values to meet the specifications. If the resistor values are not reasonable, then adjust the capacitor values and calculate the new corresponding resistor values.

```
% Select capacitor values
C2 = 100e-12;
C3 = 100e-9;
% Compute the resistor values to meet the cutoff frequencies
R2 = 1/C2/50e3
R3 = 1/C3/500
% Compute the resistor values to meet the gain requirements
R1 = R2/10
R4 = R3*10
R2 =
   200.0000e+003
R3 =
    20.0000e+003
R1 =
    20.0000e+003
R4 =
   200.0000e+003
```

Answers:
$R_1 = R_3 = 20$ kΩ and $R_2 = R_4 = 200$ kΩ.
$C_2 = 100$ pF and $C_3 = 100$ nF.
There are many other valid designs.

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Problem 8-63

Power Transmission Efficiency **(A)**

A power transmission circuit with a source voltage of $\mathbf{V}_S = 440 + j0$ V can be modeled as shown in Figure P8-63. Find the average power produced by the source, lost in the wires, and delivered to the load. What is the transmission efficiency?

Solution:

The solution is presented in the following MATLAB code.

```
clear all
% Find the current in the series circuit
Vs = 440+0j;
Zw = 0.5+4j;
ZL = 50+50j;
Zeq = Zw+ZL+Zw;
Is = Vs/Zeq;
MagIs = abs(Is);
MagIs2 = MagIs^2;
% Find the average power produced by the source
Rs = real(Zeq);
Ps = Rs/2*MagIs2
% Find the average power in the wires
Rw = real(2*Zw);
Pw = Rw/2*MagIs2
% Find the average power in the load
RL = real(ZL);
PL = RL/2*MagIs2
% The efficiency is the ratio of the output power to the input power
n = PL/Ps
Ps =
   827.6278e+000
Pw =
    16.2280e+000
PL =
   811.3998e+000
n =
   980.3922e-003
```

Answers:

$P_S = 827.63$ W.

$P_W = 16.23$ W.

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P_L = 811.40 W.
The transmission efficiency is η = 98.039 %.

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Problem 8-64

60 Hz Filter **(E)**

A 1-kΩ resistor models an important and sensitive laboratory instrument. The desired signal that the instrument measures, varies from 1 Hz to 500 Hz. However, interference from power lines in the laboratory causes the instrument to saturate. A vendor has designed a device that he claims will essentially eliminate a 60-Hz signal from the instrument with only the smallest attenuation to other frequencies. The interface circuit is shown in Figure P8-64. Connect the vendor's circuit to the model for the instrument and analyze its performance to determine if it will do the job.

Solution:

To analyze this circuit, we will determine the magnitude of the voltage across the output (the laboratory instrument) for the range of possible frequencies.

```
clear all
syms w
C = 7.03e-6;
ZC = 1/j/w/C;
L = 1;
ZL = j*w*L;
R1 = 10;
RL = 1e3;
% Use voltage division to find the ratio of the output
% voltage to the input voltage
Zeq = 1/(1/(ZC+ZL) + 1/RL);
K = simplify(Zeq/(Zeq+R1));

% Plot the gain versus frequency
f = logspace(0,3,1000);
ww = 2*pi*f;
Kww = subs(K,w,ww);
MagKww = abs(Kww);
loglog(f,MagKww,'b','LineWidth',3)
grid on
xlabel('Frequency, (Hz)')
ylabel('Gain')
```

Answer:

The plot shows a clear decrease in the magnitude of the output signal when the frequency of the input signal is 60 Hz. This circuit should perform its intended function.

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Problem 8-65

AC Circuit Design **(D)**

Select values of L and C in Figure P8-65 so that the input impedance seen by the voltage source is $50 + j0\ \Omega$ when the frequency is $\omega = 10^6$ rad/s. For these values of L and C, find the output Thévenin impedance seen by the 600-Ω load resistor.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
w = 1e6;
syms L C positive
Zeq = j*w*L + 1/(j*w*C + 1/600);
% Use the real and imaginary part of the equivalent impedance
% to form 2 equations to solve for L and C
Eqn1 = real(Zeq)-50;
Eqn2 = imag(Zeq)-0;
Soln = solve(Eqn1,Eqn2,L,C);
L = double(Soln.L)
C = double(Soln.C)
L =
   165.8312e-006
  -165.8312e-006
C =
     5.5277e-009
    -5.5277e-009

% Calculate the output impedance seen by the load
% Choose the positive results for the capacitance and inductance
L = max(L);
C = max(C);
ZL = j*w*L;
ZC = 1/j/w/C;
R1 = 50;
Zout = 1/(1/(R1+ZL)+1/ZC)
Zout =
   600.0000e+000 -312.2502e-015i
```

Answers:

L = 165.83 μH.

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C = 5.5277 nF.
Z_T = 600 Ω.

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Problem 8-66

AC Circuit Analysis **(E)**

Ten years after graduating with a BSEE, you decide to go to graduate schools for a masters degree. In desperate need of income, you agree to sign on as a grader in the basic circuit analysis course. One of the problems asks the students to find $v(t)$ in Figure P8-66 when the circuit operates in the sinusoidal steady state. One of the students offers the following solution:

$$\begin{aligned} v(t) &= (R + j\omega L)\times i(t) \\ &= (20 + j20)\times 0.5\cos 200t \\ &= 10\cos 200t + j10\cos 200t \\ &= 10\sqrt{2}\cos(200t + 45°) \end{aligned}$$

Is the answer correct? If not what grade would you give the student? If correct, what comments would you give the student about the method of solution?

Solution:

Solve the problem using MATLAB to determine if the student's solution is correct.

```
clear all
It = 0.5;
w = 200;
L = 0.1;
ZL = j*w*L;
R = 20;
Zeq = R+ZL;
Vt = It*Zeq
MagVt = abs(Vt)
PhaseVt = angle(Vt)*180/pi
Vt =
    10.0000e+000 + 10.0000e+000i
MagVt =
    14.1421e+000
PhaseVt =
    45.0000e+000
```

The student has the correct solution, but the approach is not valid. In this case, the answer happens to be correct, because the current flowing in the circuit has zero phase. If the current had non-zero phase, this approach would not always give the correct answer or the required analysis would be much more difficult. The fundamental error in the approach is that the impedances are in the phasor domain, which depends on frequency, and the sinusoid is in the time domain. The two domains are not compatible for analysis and calculations.

Answer:

The answer is correct, but the approach is not valid.

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Problem 8-67

MATLAB Phasor Visualization **(A)**

MATLAB efficiently performs phasor calculations with complex numbers, but it can also be used to better visualize the concept of a rotating phasor. Recall that a sinusoidal signal is the real part of a phasor times $e^{j\omega t}$, where the ω term represents a rotation. The following MATLAB code demonstrates the rotation of a phasor and its projections onto the axes:

```
% Create a figure and draw square axes
figure
axis([-1.5 1.5 -1.5 1.5]);
axis square
% Create reference axes
line([-2 2],[0 0],'Color','k')
line([0 0],[-2 2],'Color','k')
grid on
% Label the plot
title('Rotating Phasor Demonstration')
xlabel('Real')
ylabel('Imaginary')
% Create a rotating unit phasor
% Plot its real and imaginary components
ArcStep = 100;   % Increase to slow the rotations
Rotations = 2;   % Number of rotations
for p = 0:pi/ArcStep:2*Rotations*pi
    x = cos(p);
    y = sin(p);
    L1 = line([0 x],[0 0],'Color','b','LineWidth',3);
    L2 = line([0 0],[0 y],'Color','r','LineWidth',3);
    L3 = line([0 x],[0 y],'Color','m','LineWidth',3);
    drawnow
    set(L1,'Visible','off')
    set(L2,'Visible','off')
    set(L3,'Visible','off')
end
```

As given, the code plots a rotating phasor with a magnitude of one and an angle of zero. Run the code to understand how a rotating phasor creates a sinusoidal signal with its projection onto the real axis. Modify the MATLAB code to plot a rotating sum of phasors. Add the phasor 0.5 ∠90° to the original phasor and plot the two phasors head-to-tail. Also, plot the sum of the two phasors and the projection of the sum onto the real and imaginary axes. Rotate the results as shown in the original code.

Solution:

The solution is presented as the following MATLAB code

```
% Create a figure and draw square axes
figure
axis([-1.5 1.5 -1.5 1.5]);
axis square
```

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```
% Create reference axes
line([-2 2],[0 0],'Color','k')
line([0 0],[-2 2],'Color','k')
grid on

% Label the plot
title('Rotating Phasor Demonstration')
xlabel('Real')
ylabel('Imaginary')

% Create a rotating unit phasor
% Plot its real and imaginary components
ArcStep = 100;   % Increase to slow the rotations
Rotations = 2;   % Number of rotations
M2 = 0.5;
P2 = pi/2;
for p = 0:pi/ArcStep:2*Rotations*pi
    x = cos(p);
    y = sin(p);
    x2 = M2*cos(p + P2);
    y2 = M2*sin(p + P2);
    x3 = x + x2;
    y3 = y + y2;
    L1 = line([0 x3],[0 0],'Color','b','LineWidth',3);
    L2 = line([0 0],[0 y3],'Color','r','LineWidth',3);
    L3 = line([0 x],[0 y],'Color','m','LineWidth',3);
    L4 = line([x x3],[y y3],'Color','m','LineWidth',3);
    L5 = line([0 x3],[0 y3],'Color','g','LineWidth',4);

    drawnow
    set(L1,'Visible','off')
    set(L2,'Visible','off')
    set(L3,'Visible','off')
    set(L4,'Visible','off')
    set(L5,'Visible','off')

end

L3 = line([0 x],[0 y],'Color','m','LineWidth',3);
L4 = line([x x3],[y y3],'Color','m','LineWidth',3);
L5 = line([0 x3],[0 y3],'Color','g','LineWidth',4);
L1 = line([0 x3],[0 0],'Color','b','LineWidth',3);
L2 = line([0 0],[0 y3],'Color','r','LineWidth',3);
```

Answer:
The required MATLAB code is presented above.

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Problem 9-1

Find the Laplace transform of $f(t) = 10[e^{!50t} \ ! \ 2e^{!100t}]\,u(t)$. Locate the poles and zeros of $F(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms t s
f = 10*(exp(-50*t) - 2*exp(-100*t));
F = laplace(f)
F =
-10*s/(s+50)/(s+100)
```

Answer:

$$F(s) = \frac{-10s}{(s+50)(s+100)}$$

The poles are located at $s = -50$ and $s = -100$. The zero is located at $s = 0$.

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Problem 9-2

Find the Laplace transform of $f(t) = A[(3 - \alpha t)e^{-\alpha t}]u(t)$. Locate the poles and zeros of $F(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms A a t s
f = A*((3-a*t)*exp(-a*t));
F = laplace(f)
F =
A*(3*s+2*a)/(s+a)^2
```

Answer:

$$F(s) = \frac{A(3s + 2\alpha)}{(s + \alpha)^2}$$

There are two poles located at $s = -\alpha$. The zero is located at $s = -2\alpha/3$.

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Problem 9-3

Find the Laplace transform of $f(t) = 25[1 - 3\cos(500t)]u(t)$. Locate the poles and zeros of $F(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
f = 25*(1-3*cos(500*t));
F = simplify(laplace(f))
F =
-50*(s^2-125000)/s/(s^2+250000)
```

Answer:

$$F(s) = \frac{-50\,(s^2 - 125000)}{s(s^2 + 250000)}$$

The poles are located at $s = 0$, $j500$, and $-j500$. The zeros are located at $s = 353.55$ and -353.55.

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Problem 9-4

Find the Laplace transform of $f(t) = A[\cos(\beta t) - \sin(\beta t)]u(t)$. Locate the poles and zeros of $F(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms A B t s
f = A*(cos(B*t)-sin(B*t));
F = laplace(f)
F =
A*(s-B)/(s^2+B^2)
```

Answer:

$$F(s) = \frac{A(s-\beta)}{(s^2+\beta^2)}$$

The poles are located at $s = j\beta$ and $-j\beta$. The zero is located at $s = \beta$.

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Problem 9-5

Find the Laplace transform of $f(t) = 2\delta(t) - 100e^{-10t}\cos(10t)\,u(t)$. Locate the poles and zeros of $F(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
f = 2*dirac(t)-100*exp(-10*t)*cos(10*t);
F = laplace(f)
F =
2*(s^2-30*s-300)/(s^2+20*s+200)

% Find the poles and zeros
p = roots([1 20 200])
z = roots([1 -30 -300])
p =
   -10.0000e+000 + 10.0000e+000i
   -10.0000e+000 - 10.0000e+000i
z =
    37.9129e+000
    -7.9129e+000
```

Answer:

$$F(s) = \frac{2\,(s^2 - 30s - 300)}{s^2 + 20s + 200}$$

The poles are located at $s = -10 + j10$ and $-10 - j10$.

The zeros are located at $s = 37.9129$ and -7.9129.

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Problem 9-6

Find the Laplace transform of $f(t) = 30[2 - 5t - 2e^{-5t}]\,u(t)$. Locate the poles and zeros of $F(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
f = 30*(2 - 5*t - 2*exp(-5*t));
F = laplace(f)
F =
150*(s-5)/s^2/(s+5)
```

Answer:

$$F(s) = \frac{150\,(s-5)}{s^2(s+5)}$$

There are two poles at $s = 0$ and another pole at $s = -5$. The zero is located at $s = 5$.

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Problem 9-7

Find the Laplace transforms of the following waveforms and plot their pole-zero diagrams:

(a) $f_1(t) = \left[15e^{-5t} - 20e^{-10t}\right]u(t)$

(b) $f_2(t) = \left[5\cos 10t + 10\cos 20t\right]u(t)$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
f1 = 15*exp(-5*t) - 20*exp(-10*t);
F1 = laplace(f1)
Part (a)
F1 =
-5*(s-10)/(s+5)/(s+10)

% Create the pole-zero diagram
close all
Q = 15;
axis(Q*[-1 1 -1 1])
line(Q*[-1 1],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],Q*[-1 1],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
p = [-5,-10];
z = [10];
MSize = 16;
LW = 2.5;
% Plot poles
for n = 1:length(p)

plot(real(p(n)),imag(p(n)),'x',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end

% Plot zeros
for n = 1:length(z)
    plot(real(z(n)),imag(z(n)),'o',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end
```

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```
disp('Part (b)')
f2 = 5*cos(10*t) + 10*cos(20*t);
F2 = simplify(laplace(f2))
% Create the pole-zero diagram
Part (b)
F2 =
15*s*(s^2+200)/(s^2+100)/(s^2+400)

% Create the pole-zero diagram
close all
Q = 25;
axis(Q*[-1 1 -1 1])
line(Q*[-1 1],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],Q*[-1 1],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
p = roots(conv([1 0 100],[1 0 400]));
z = roots([1 0 200 0]);
MSize = 16;
LW = 2.5;
% Plot poles
for n = 1:length(p)
    plot(real(p(n)),imag(p(n)),'x',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end

% Plot zeros
for n = 1:length(z)
    plot(real(z(n)),imag(z(n)),'o',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end
```

Answers:

(a) $F_1(s) = \dfrac{-5\,(s-10)}{(s+5)(s+10)}$

(b) $F_2(s) = \dfrac{15\,s(s^2+200)}{(s^2+100)(s^2+400)}$

The pole-zero plots are shown above.

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Problem 9-8

Find the Laplace transforms of the following waveforms and plot their pole-zero diagrams.

(a) $f_1(t) = 4\delta(t) + \left[20e^{-20t} + 40e^{-40t}\right]u(t)$

(b) $f_2(t) = \left[15 + 15\cos 500t\right]u(t)$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
f1 = 4*dirac(t) + 20*exp(-20*t) + 40*exp(-40*t);
F1 = simplify(laplace(f1))
Part (a)
F1 =
4*(s^2+75*s+1200)/(s+20)/(s+40)

% Create the pole-zero diagram
close all
Q = 60;
axis(Q*[-1 1 -1 1])
line(Q*[-1 1],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],Q*[-1 1],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
p = [-20,-40];
z = roots([1 75 1200])
MSize = 16;
LW = 2.5;
% Plot poles
for n = 1:length(p)
    plot(real(p(n)),imag(p(n)),'x',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end

% Plot zeros
for n = 1:length(z)
    plot(real(z(n)),imag(z(n)),'o',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end
z =
   -51.8614e+000
```

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```
   -23.1386e+000
```

```
clear all
syms t s
disp('Part (b)')
f2 = 15+15*cos(500*t);
F2 = simplify(laplace(f2))
Part (b)
F2 =
30*(s^2+125000)/s/(s^2+250000)

% Create the pole-zero diagram
close all
Q = 1000;
axis(Q*[-1 1 -1 1])
line(Q*[-1 1],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],Q*[-1 1],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
p = roots([1 0 250000 0]);
z = roots([1 0 125000]);
MSize = 16;
LW = 2.5;
% Plot poles
for n = 1:length(p)
    plot(real(p(n)),imag(p(n)),'x',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end

% Plot zeros
for n = 1:length(z)
    plot(real(z(n)),imag(z(n)),'o',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end
```

Answers:

(a) $F_1(s) = \dfrac{4\,(s^2 + 75s + 1200)}{(s+20)(s+40)}$

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(b) $F_2(s) = \dfrac{30\,(s^2 + 125000)}{s(s^2 + 250000)}$

The pole-zero plots are shown above.

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Problem 9-9

Use MATLAB to find the Laplace transforms of the following waveforms. Locate the poles and zeros of $F(s)$.

(a) $f_1(t) = \delta(t) + \left(625t\, e^{-25t}\right)u(t)$

(b) $f_2(t) = \left[10 + e^{-10t} + 5\cos 10t + 5\sin 10t\right]u(t)$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
f1 = dirac(t) + 625*t*exp(-25*t);
F1 = simplify(laplace(f1))

disp('Part (b)')
f2 = 10 + exp(-10*t) + 5*cos(10*t) + 5*sin(10*t);
F2 = laplace(f2)
Part (a)
F1 =
(s^2+50*s+1250)/(s+25)^2
Part (b)
F2 =
8*(2*s^3+200*s+25*s^2+1250)/s/(s+10)/(s^2+100)

% Find the zeros
z1 = roots([1 50 1250])
z2 = roots([2 25 200 1250])
z1 =
   -25.0000e+000 + 25.0000e+000i
   -25.0000e+000 - 25.0000e+000i
z2 =
    -9.0714e+000
    -1.7143e+000 +  8.1215e+000i
    -1.7143e+000 -  8.1215e+000i
```

Answers:

(a) $F_1(s) = \dfrac{s^2 + 50s + 1250}{(s+25)^2}$

There are two poles located at $s = -25$. The zeros are located at $s = -25 + j25$ and $-25 - j25$.

(b) $F_2(s) = \dfrac{8\,(2s^3 + 25s^2 + 200s + 1250)}{s\,(s+10)(s^2+100)}$

The poles are located at $s = 0$, -10, $j10$, and $s = -j10$. The zeros are located at $s = -9.07$, $-1.71 + j8.12$, and $-1.71 - j8.12$.

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Problem 9-10

Find the Laplace transforms of the following waveforms.

(a) $f_1(t) = 3\delta(t-2)$

(b) $f_2(t) = 10e^{-50(t-1)}u(t-1)$

(c) $f_3(t) = 10e^{-50(t-2)}u(t-2)$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
f1 = 3*dirac(t-2);
F1 = laplace(f1)

disp('Part (b)')
f2 = 10*exp(-50*(t-1))*heaviside(t-1);
F2 = laplace(f2)

disp('Part (c)')
f3 = 10*exp(-50*(t-2))*heaviside(t-2);
F3 = laplace(f3)
Part (a)
F1 =
3*exp(-2*s)
Part (b)
F2 =
10/(s+50)*exp(-s)
Part (c)
F3 =
10/(s+50)*exp(-2*s)
```

Answers:

(a) $F_1(s) = 3\,e^{-2s}$

(b) $F_2(s) = \dfrac{10\,e^{-s}}{s+50}$

(c) $F_3(s) = \dfrac{10\,e^{-2s}}{s+50}$

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Problem 9-11

Use MATLAB to find the Laplace transform of the following waveform

$$f(t) = \left[10 + 2e^{-10t}\right] u(t) + \left[5\cos 100(t - 0.05)\right] u(t - 0.05)$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
f = (10+2*exp(-10*t))*heaviside(t) + 5*cos(100*(t-0.05))*heaviside(t-0.05);
F = laplace(f)
F =
10/s+2/(s+10)+5*exp(-1/20*s)*s/(s^2+10000)
```

Answer:

$$F(s) = \frac{10}{s} + \frac{2}{s+10} + \frac{5s\, e^{-s/20}}{s^2 + 10000}$$

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Problem 9-12

Find the Laplace transform of the following waveforms.

(a) $f_1(t) = \frac{d}{dt}\left(100e^{-5000t}\cos 20000t\right)u(t)$

(b) $f_2(t) = \int_0^t 50e^{-50x}dx + 100\,u(t) + 5\frac{de^{-50t}}{dt}u(t)$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t x
disp('Part (a)')
f1 = diff(100*exp(-5000*t)*cos(20000*t),t);
F1 = laplace(f1)

disp('Part (b)')
f2 = int(50*exp(-50*x),x,0,t) + 100 + 5*diff(exp(-50*t),t);
F2 = laplace(f2)
Part (a)
F1 =
-500000*(s+85000)/(s^2+10000*s+425000000)
Part (b)
F2 =
-50*(3*s-101)/s/(s+50)
```

Answers:

(a) $F_1(s) = \frac{-500000\,(s+85000)}{s^2+10000s+425000000}$

(b) $F_2(s) = \frac{-50\,(3s-101)}{s(s+50)}$

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Problem 9-13

(a) Write an expression for the waveform $f(t)$ in Fig. P9-13 using step and ramp functions.

(b) Use the time-domain translation property to find the Laplace transform of the waveform $f(t)$ found in part (a).

(c) Verify the Laplace transform found in (b) by applying the definition of the Laplace transformation in Eq. (9-2) to the waveform $f(t)$ found in (a).

Solution:

(a) The function is a ramp with a slope of A/T over the interval from $t = 0$ to $t = T$. We can write the following expression for the waveform.

$$f(t) = \frac{A}{T} r(t)[u(t) - u(t-T)]$$

(b) First, we need to re-write $f(t)$ so that the time-domain translation property can be applied.

$$f(t) = \frac{A}{T} r(t)[u(t) - u(t-T)] = \frac{A}{T}[r(t)u(t) - r(t)u(t-T)]$$

$$= \frac{A}{T}[tu(t)u(t) - tu(t)u(t-T)] = \frac{A}{T}[tu(t) - tu(t-T)]$$

$$= \frac{A}{T}[tu(t) - (t-T+T)u(t-T)] = \frac{A}{T}[tu(t) - (t-T)u(t-T) - Tu(t-T)]$$

Now we can take the Laplace transform.

$$F(s) = \frac{A}{T}\left[\frac{1}{s^2} - \frac{e^{-Ts}}{s^2} - \frac{Te^{-Ts}}{s}\right] = \frac{A}{T}\left[\frac{1 - e^{-Ts}(1+Ts)}{s^2}\right]$$

(c) We can use MATLAB to confirm the answer in Part (b).

```
clear all
syms t s A
syms T positive
f = (A/T)*t*heaviside(t)*(heaviside(t)-heaviside(t-T));
F = laplace(f)
F =
(1-(s*T+1)*exp(-s*T))/T/s^2*A
```

Answers:

(a) $f(t) = \frac{A}{T} r(t)[u(t) - u(t-T)]$

(b) $F(s) = \frac{A[1 - e^{-Ts}(1+Ts)]}{Ts^2}$

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(c) The MATLAB calculations above confirm the answer in Part (b).

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Problem 9-14

(a) Write an expression for the waveform $f(t)$ in Fig. P9-14 using step functions.

(b) Use the time-domain translation property to find the Laplace transform of the waveform $f(t)$ found in part (a).

(c) Verify the Laplace transform found in (b) by applying the definition of the Laplace transformation in Eq. (9-2) to the waveform $f(t)$ found in (a).

Solution:

(a) By inspection, we can write the following waveform:

$$f(t) = Au(t) - 2Au(t-T) + Au(t-2T)$$

(b) Applying the time-domain translation property, we get the following Laplace transform:

$$F(s) = \frac{A}{s} - \frac{2Ae^{-Ts}}{s} + \frac{Ae^{-2Ts}}{s}$$

(c) Using MATLAB, we can verify the result in Part (b).

```
clear all
syms t s A
syms T positive
f = A*heaviside(t)-2*A*heaviside(t-T)+A*heaviside(t-2*T);
F = laplace(f)
F =
1/s*(1-2*exp(-s*T)+exp(-2*s*T))*A
```

Answers:

(a) $f(t) = Au(t) - 2Au(t-T) + Au(t-2T)$

(b) $F(s) = \frac{A}{s} - \frac{2Ae^{-Ts}}{s} + \frac{Ae^{-2Ts}}{s}$

(c) The MATLAB calculations above confirm the answer in Part (b).

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Problem 9-15

(a) Find the Laplace transform of the following function. Locate the poles and zeros of $F(s)$.

$$f(t) = \left[500 + 75e^{-200t}t\,\mathbf{sin}\,200t\right]u(t)$$

(b) Validate your result using MATLAB.

Solution:

We will use MATLAB to solve this problem, so Parts (a) and (b) will be combined.

```
clear all
syms t s
f = 500 + 75*exp(-200*t)*t*sin(200*t);
F = simplify(laplace(f))
F =
500*(s^4+800*s^3+320060*s^2+64012000*s+6400000000)/s/(s^2+400*s+80000)^2

% Locate the poles and zeros
p = roots([1 400 80000])
z = roots([1 800 320060 64012000 6.4e9])
p =
  -200.0000e+000 +200.0000e+000i
  -200.0000e+000 -200.0000e+000i
z =
  -201.7622e+000 +204.2927e+000i
  -201.7622e+000 -204.2927e+000i
  -198.2378e+000 +195.7823e+000i
  -198.2378e+000 -195.7823e+000i
```

Answers:

(a) $F(s) = \dfrac{500\left(s^4 + 800s^3 + 320060s^2 + 64012000s + 6.4\times10^9\right)}{s\,(s^2 + 400s + 80000)^2}$

There is a pole at $s = 0$ and two poles each at $s = -200 + j200$ and $-200 - j200$.

The four zeros are located at $s = -201.76 + j204.29$, $-201.76 - j204.29$, $-198.24 + j195.78$, and $-198.24 - j195.78$.

(b) Presented above.

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Problem 9-16

Find the inverse Laplace transforms of the following functions:

(a) $F_1(s) = \dfrac{s+100}{s(s+50)}$

(b) $F_2(s) = \dfrac{s+10}{s(s+50)(s+100)}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = (s+100)/s/(s+50);
f1 = ilaplace(F1)

disp('Part (b)')
F2 = (s+10)/s/(s+50)/(s+100);
f2 = ilaplace(F2)
Part (a)
f1 =
-exp(-50*t)+2
Part (b)
f2 =
-9/500*exp(-100*t)+2/125*exp(-50*t)+1/500
```

Answers:

(a) $f_1(t) = \left[2 - e^{-50t}\right]u(t)$

(b) $f_2(t) = \left[\dfrac{1}{500} + \dfrac{2}{125}e^{-50t} - \dfrac{9}{500}e^{-100t}\right]u(t)$

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Problem 9-17

Find the inverse Laplace transforms of the following functions:

(a) $F_1(s) = \dfrac{50(s+10)}{(s+5)(s+20)}$

(b) $F_2(s) = \dfrac{2s^2}{(s+200)(s+500)}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 50*(s+10)/(s+5)/(s+20);
f1 = ilaplace(F1)

disp('Part (b)')
F2 = 2*s^2/(s+200)/(s+500);
f2 = ilaplace(F2)
Part (a)
f1 =
100/3*exp(-20*t)+50/3*exp(-5*t)
Part (b)
f2 =
2*dirac(t)+800/3*exp(-200*t)-5000/3*exp(-500*t)
```

Answers:

(a) $f_1(t) = \left[\dfrac{50}{3}e^{-5t} + \dfrac{100}{3}e^{-20t}\right]u(t)$

(b) $f_2(t) = 2\delta(t) + \left[\dfrac{800}{3}e^{-200t} - \dfrac{5000}{3}e^{-500t}\right]u(t)$

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Problem 9-18

Find the inverse Laplace transforms of the following functions:

(a) $F_1(s) = 400\dfrac{1}{(s+10)^2 + 20^2}$

(b) $F_2(s) = \dfrac{2(s+10)}{(s+10)^2 + 20^2}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 400/((s+10)^2+20^2);
f1 = ilaplace(F1)

disp('Part (b)')
F2 = 2*(s+10)/((s+10)^2+20^2);
f2 = ilaplace(F2)
Part (a)
f1 =
20*exp(-10*t)*sin(20*t)
Part (b)
f2 =
2*exp(-10*t)*cos(20*t)
```

Answers:

(a) $f_1(t) = 20e^{-10t}\sin(20t)\,u(t)$

(b) $f_2(t) = 2e^{-10t}\cos(20t)\,u(t)$

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Problem 9-19

Find the inverse Laplace transforms of the following functions and sketch their waveforms for β > 0:

(a) $F_1(s) = \dfrac{\beta(s+\beta)}{s(s^2+\beta^2)}$

(b) $F_2(s) = \dfrac{s(s+\beta)}{s^2+\beta^2}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
syms B positive
disp('Part (a)')
F1 = B*(s+B)/s/(s^2+B^2);
f1 = ilaplace(F1)

disp('Part (b)')
F2 = s*(s+B)/(s^2+B^2);
f2 = ilaplace(F2)
Part (a)
f1 =
-cos(B*t)+sin(B*t)+1
Part (b)
f2 =
dirac(t)+B*(cos(B*t)-sin(B*t))
```

Answers:

(a) $f_1(t) = [1 - \cos(\beta t) + \sin(\beta t)]u(t)$

(b) $f_2(t) = \delta(t) + \beta\,[\cos(\beta t) - \sin(\beta t)]u(t)$

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Problem 9-20

Find the inverse Laplace transforms of the following functions:

(a) $F_1(s) = \dfrac{\alpha^2}{s^2(s+\alpha)}$

(b) $F_2(s) = \dfrac{\alpha^2}{s(s+\alpha)^2}$

Solution:

We will assume that α > 0. The solution is presented in the following MATLAB code.

```
clear all
syms t s
syms a positive
disp('Part (a)')
F1 = a^2/s^2/(s+a);
f1 = ilaplace(F1)

disp('Part (b)')
F2 = a^2/s/(s+a)^2;
f2 = ilaplace(F2)
Part (a)
f1 =
a*t+exp(-a*t)-1
Part (b)
f2 =
1-exp(-a*t)*(a*t+1)
```

Answers:

(a) $f_1(t) = \left[\alpha t + e^{-\alpha t} - 1\right]u(t)$

(b) $f_2(t) = \left[1 - (\alpha t + 1)e^{-\alpha t}\right]u(t)$

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Problem 9-21

Find the inverse Laplace transforms of the following functions:

(a) $F_1(s) = \dfrac{600}{(s+10)(s+20)(s+40)}$

(b) $F_2(s) = \dfrac{6(s+10)}{(s+5)(s+20)}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 600/(s+10)/(s+20)/(s+40);
f1 = ilaplace(F1)

disp('Part (b)')
F2 = 6*(s+10)/(s+5)/(s+20);
f2 = ilaplace(F2)
Part (a)
f1 =
-3*exp(-20*t)+exp(-40*t)+2*exp(-10*t)
Part (b)
f2 =
4*exp(-20*t)+2*exp(-5*t)
```

Answers:

(a) $f_1(t) = \left[2e^{-10t} - 3e^{-20t} + e^{-40t}\right]u(t)$

(b) $f_2(t) = \left[2e^{-5t} + 4e^{-20t}\right]u(t)$

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Problem 9-22

Use MATLAB to find the inverse transforms of the following functions:

(a) $F_1(s) = \dfrac{16s}{(s+2)(s^2+12s+13)}$

(b) $F_2(s) = \dfrac{4(s^2+4)}{s(s^2+16)}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 16*s/(s+2)/(s^2+12*s+13);
f1 = simplify(ilaplace(F1))

disp('Part (b)')
F2 = 4*(s^2+4)/s/(s^2+16);
f2 = ilaplace(F2)
Part (a)
f1 =
32/7*exp(-2*t)-32/7*exp(-6*t)*cosh(t*23^(1/2))-16/161*exp(-
6*t)*23^(1/2)*sinh(t*23^(1/2))
Part (b)
f2 =
3*cos(4*t)+1

% The results for f1(t) are not presented in a convenient format
% The following approach gives a better numerical result
% See the MATLAB Help on the residue command to interpret the results
num = [16 0];
den = conv([1 2],[1 12 13]);
[r,p,k]=residue(num,den)
r =
    -2.0474e+000
     4.5714e+000
    -2.5240e+000
p =
   -10.7958e+000
    -2.0000e+000
    -1.2042e+000
k =
     []
```

Answers:

(a) $f_1(t) = \left[-2.524e^{-1.2042\,t} + 4.5714e^{-2\,t} - 2.0474e^{-10.7958\,t}\right]u(t)$

(b) $f_2(t) = [1 + 3\cos(4t)]u(t)$

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Problem 9-23

Find the inverse transforms of the following functions:

(a) $F_1(s) = \dfrac{(s+4000)(s+8000)}{s(s+2000)(s+6000)}$

(b) $F_2(s) = \dfrac{3s^4 + 10s^2 + 4}{s(s^2+1)(s^2+4)}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = (s+4000)*(s+8000)/s/(s+2000)/(s+6000);
f1 = simplify(ilaplace(F1))

disp('Part (b)')
F2 = (3*s^4+10*s^2+4)/s/(s^2+1)/(s^2+4);
f2 = ilaplace(F2)
Part (a)
f1 =
-3/2*exp(-2000*t)-1/6*exp(-6000*t)+8/3
Part (b)
f2 =
cos(2*t)+cos(t)+1
```

Answers:

(a) $f_1(t) = \left[\dfrac{8}{3} - \dfrac{3}{2}e^{-2000\,t} - \dfrac{1}{6}e^{-6000\,t}\right]u(t)$

(b) $f_2(t) = [1 + \cos(t) + \cos(2t)]u(t)$

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Problem 9-24

Find the inverse transforms of the following functions:

(a) $F_1(s) = \dfrac{300(s+20)}{s(s^2+40s+300)}$

(b) $F_2(s) = \dfrac{200s}{(s+40)(s^2+4s+8)}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 300*(s+20)/s/(s^2+40*s+300);
f1 = simplify(ilaplace(F1))

disp('Part (b)')
F2 = 200*s/(s+40)/(s^2+4*s+8);
f2 = ilaplace(F2)
Part (a)
f1 =
-15*exp(-10*t)-5*exp(-30*t)+20
Part (b)
f2 =
-1000/181*exp(-40*t)+100/181*exp(-2*t)*(10*cos(2*t)-9*sin(2*t))
```

Answers:

(a) $f_1(t) = \left[20 - 15e^{-10t} - 5e^{-30t}\right]u(t)$

(b) $f_2(t) = \left[-\dfrac{1000}{181}e^{-40t} + \dfrac{100}{181}e^{-2t}\left(10\cos(2t) - 9\sin(2t)\right)\right]u(t)$

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Problem 9-25

Find the inverse transforms of the following functions:

(a) $F_1(s) = \dfrac{4\left(s^2+16\right)}{s\left(s^2+8s+32\right)}$

(b) $F_2(s) = \dfrac{2\left(s^2+30s+800\right)}{s\left(s^2+50s+400\right)}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 4*(s^2+16)/s/(s^2+8*s+32);
f1 = simplify(ilaplace(F1))

disp('Part (b)')
F2 = 2*(s^2+30*s+800)/s/(s^2+50*s+400);
f2 = ilaplace(F2)
Part (a)
f1 =
2+2*exp(-4*t)*cos(4*t)-6*exp(-4*t)*sin(4*t)
Part (b)
f2 =
2*exp(-40*t)-4*exp(-10*t)+4
```

Answers:

(a) $f_1(t) = \left\{2 + 2e^{-4t}\left[\cos(4t) - 3\sin(4t)\right]\right\}u(t)$

(b) $f_2(t) = \left[4 - 4e^{-10t} + 2e^{-40t}\right]u(t)$

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Problem 9-26

Find the inverse transforms of the following functions and validate your answers using MATLAB:

(a) $F_1(s) = \dfrac{(s+50)^2}{(s+20)^2(s+100)}$

(b) $F_2(s) = \dfrac{(s+20)^2}{(s+50)^2(s+100)}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = (s+50)^2/(s+20)^2/(s+100);
f1 = simplify(ilaplace(F1))

disp('Part (b)')
F2 = (s+20)^2/(s+50)^2/(s+100);
f2 = ilaplace(F2)
Part (a)
f1 =
25/64*exp(-100*t)+39/64*exp(-20*t)+45/4*exp(-20*t)*t
Part (b)
f2 =
64/25*exp(-100*t)+3/25*exp(-50*t)*(150*t-13)
```

Answers:

(a) $f_1(t) = \left[\left(\dfrac{45}{4}t + \dfrac{39}{64}\right)e^{-20t} + \dfrac{25}{64}e^{-100t}\right]u(t)$

(b) $f_2(t) = \left[\dfrac{3}{25}(150t - 13)e^{-50t} + \dfrac{64}{25}e^{-100t}\right]u(t)$

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Problem 9-27

A certain transform has a simple pole at $s = -20$, a simple zero at $s = -\gamma$, and a scale factor of $K = 1$. Select values for γ so the inverse transform is

(a) $f(t) = \delta(t) - 5e^{-20t}$

(b) $f(t) = \delta(t)$

(c) $f(t) = \delta(t) + 5e^{-20t}$

Solution:

The given transform has the general form

$$F(s) = \frac{(s+\gamma)}{(s+20)}$$

To solve this problem, take the Laplace transform of each of the given signals and compare the result to the general form to determine the require value for the γ parameter.

```
clear all
syms t s
fa = dirac(t)-5*exp(-20*t);
Fa = laplace(fa)
fb = dirac(t);
Fb = laplace(fb)
fc = dirac(t)+5*exp(-20*t);
Fc = laplace(fc)
Fa =
(s+15)/(s+20)
Fb =
1
Fc =
(s+25)/(s+20)
```

Answers:

(a) $\gamma = 15$

(b) $\gamma = 20$

(c) $\gamma = 25$

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Problem 9-28

Find the inverse transforms of the following functions:

(a) $F_1(s) = \dfrac{e^{-3s}(s+20)}{(s+10)(s+30)}$

(b) $F_2(s) = \dfrac{se^{-3s}+20}{(s+10)(s+30)}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = exp(-3*s)*(s+20)/(s+10)/(s+30);
f1 = simplify(ilaplace(F1))

disp('Part (b)')
F2 = (s*exp(-3*s)+20)/(s+10)/(s+30);
f2 = ilaplace(F2)
Part (a)
f1 =
heaviside(t-3)*exp(-20*t+60)*cosh(10*t-30)
Part (b)
f2 =
2*exp(-20*t)*sinh(10*t)+1/2*(-exp(-10*t+30)+3*exp(90-30*t))*heaviside(t-3)
```

Answers:

(a) $f_1(t) = \dfrac{1}{2}\left[e^{-10(t-3)} + e^{-30(t-3)}\right]u(t-3)$

(b) $f_2(t) = \left[e^{-10t} - e^{-30t}\right]u(t) - \dfrac{1}{2}\left[e^{-10(t-3)} - 3e^{-30(t-3)}\right]u(t-3)$

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Problem 9-29

Use MATLAB to find the inverse transform and plot the poles and zeros of the following function:

$$F(s) = \frac{100s\left(s^2 + 30s + 400\right)}{(s+25)\left(s^3 + 6s^2 + 16s + 16\right)}$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F = 100*s*(s^2+30*s+400)/(s+25)/(s^3+6*s^2+16*s+16);
f = simplify(ilaplace(F))
f_num = vpa(f,6)

% Find the poles and zeros
p = [roots([1 6 16 16]);-25]
z = [roots([1 30 400]);0]
```

```
Part (a)
f =
687500/12259*exp(-25*t)+422000/533*exp(-2*t)*cos(2*t)+370400/533*exp(-
2*t)*sin(2*t)-17200/23*exp(-2*t)
f_num =
56.0812*exp(-25.*t)+791.745*exp(-2.*t)*cos(2.*t)+694.934*exp(-2.*t)*sin(2.*t)-
747.826*exp(-2.*t)
p =
    -2.0000e+000 +  2.0000e+000i
    -2.0000e+000 -  2.0000e+000i
    -2.0000e+000
   -25.0000e+000
z =
   -15.0000e+000 + 13.2288e+000i
   -15.0000e+000 - 13.2288e+000i
     0.0000e+000
```

```
% Create the pole-zero diagram
close all
Q = 30;
axis(Q*[-1 1 -1 1])
line(Q*[-1 1],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],Q*[-1 1],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
```

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```
MSize = 16;
LW = 2.5;
% Plot poles
for n = 1:length(p)
    plot(real(p(n)),imag(p(n)),'x',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end

% Plot zeros
for n = 1:length(z)
    plot(real(z(n)),imag(z(n)),'o',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end
```

Repeat the pole-zero diagram on a different scale to see the elements close to the origin.

```
% Create the pole-zero diagram
close all
Q = 4;
axis(Q*[-1 1 -1 1])
line(Q*[-1 1],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],Q*[-1 1],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
MSize = 16;
LW = 2.5;
% Plot poles
for n = 1:length(p)
    plot(real(p(n)),imag(p(n)),'x',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end

% Plot zeros
for n = 1:length(z)
    plot(real(z(n)),imag(z(n)),'o',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end
```

Answers:

$$f(t) = \left\{56.081e^{-25t} + e^{-2t}\left[791.74\cos(2t) + 694.93\sin(2t) - 747.83\right]\right\}u(t)\,.$$

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The poles are located at $s = -25, -2, -2 + j2$, and $-2 - j2$.
The zeros are located at $s = 0, -15 + j13.2288$ and $-15 - j13.2288$.
The pole-zero plot is shown above.

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Problem 9-30

Use MATLAB to find the inverse transform and plot the poles and zeros of the following function:

$$F(s) = \frac{50\left(s^3 + 2s^2 + s + 2\right)}{s\left(s^3 + 4s^2 + 4s + 16\right)}$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F = 50*(s^3+2*s^2+s+2)/s/(s^3+4*s^2+4*s+16);
f = ilaplace(F)
f_num = vpa(f,6)

% Find the poles and zeros
p = [roots([1 4 4 16]);0]
z = roots([1 2 1 2])
Part (a)
f =
45*cos(t)^2-15*sin(t)*cos(t)-65/4+85/4*exp(-4*t)
f_num =
45.*cos(t)^2-15.*sin(t)*cos(t)-16.2500+21.2500*exp(-4.*t)
p =
    -4.0000e+000
  -277.5558e-018 +  2.0000e+000i
  -277.5558e-018 -  2.0000e+000i
     0.0000e+000
z =
    -2.0000e+000
  -138.7779e-018 +  1.0000e+000i
  -138.7779e-018 -  1.0000e+000i

% Create the pole-zero diagram
close all
Q = 4;
axis(Q*[-1 1 -1 1])
line(Q*[-1 1],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],Q*[-1 1],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
MSize = 16;
LW = 2.5;
```

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```
% Plot poles
for n = 1:length(p)
    plot(real(p(n)),imag(p(n)),'x',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end

% Plot zeros
for n = 1:length(z)
    plot(real(z(n)),imag(z(n)),'o',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end
```

Answer:

$$f(t) = \left[45\cos^2(t) - 15\sin(t)\cos(t) + \frac{85}{4}e^{-4t} - \frac{65}{4}\right]u(t).$$

The pole-zero plot is shown above.

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Problem 9-31

Use the Laplace transformation to find the $v(t)$ that satisfies the following first-order differential equations:

(a) $500\dfrac{dv(t)}{dt} + 2500v(t) = 0, \quad v(0-) = 100 \ \text{ V}$

(b) $\dfrac{dv(t)}{dt} + 200v(t) = 400u(t), \quad v(0-) = -100 \ \text{ V}$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s Vs vt
disp('Part (a)')
v0 = 100;
% Take the Laplace transform of the differential equation
Eqn1 = 500*(s*Vs - v0)+2500*Vs;
% Solve for V(s)
Vs = solve(Eqn1,Vs)
% Take the inverse Laplace transform to find v(t)
vt = ilaplace(Vs)
Part (a)
Vs =
100/(s+5)
vt =
100*exp(-5*t)

syms t s Vs vt
disp('Part (b)')
v0 = -100;
% Take the Laplace transform of the differential equation
Eqn1 = (s*Vs - v0)+200*Vs-400*laplace(heaviside(t));
% Solve for V(s)
Vs = solve(Eqn1,Vs)
% Take the inverse Laplace transform to find v(t)
vt = ilaplace(Vs)
Part (b)
Vs =
-100*(s-4)/s/(s+200)
vt =
2-102*exp(-200*t)
```

Answers:

(a) $v(t) = \left[100e^{-5t}\right]u(t) \ \text{ V}$

(b) $v(t) = \left[2 - 102e^{-200t}\right]u(t) \ \text{ V}$

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Problem 9-32

Use the Laplace transformation to find the $i(t)$ that satisfies the following first-order differential equation:

$$\frac{di(t)}{dt} + 500\, i(t) = \left[0.100\, e^{-200t}\right] u(t), \quad i(0-) = 0 \text{ A}$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s Is it
i0 = 0;
% Take the Laplace transform of the differential equation
Eqn1 = (s*Is - i0)+500*Is-laplace(0.1*exp(-200*t));
% Solve for I(s)
Is = solve(Eqn1,Is)
% Take the inverse Laplace transform to find i(t)
it = ilaplace(Is)
Is =
1/10/(s+200)/(s+500)
it =
1/1500*exp(-350*t)*sinh(150*t)
```

Answer:

$$i(t) = \frac{1}{3000}\left[e^{-200t} - e^{-500t}\right] u(t)$$

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Problem 9-33

The switch in Fig. P9-33 has been open for a long time and is closed at $t = 0$. The circuit parameters are $R = 400\ \Omega$, $L = 330$ mH, and $V_A = 15$ V.

(a) Find the differential equation for the inductor current $i_L(t)$ and initial condition $i_L(0)$.

(b) Solve for $i_L(t)$ using the Laplace transformation.

Solution:

(a) Since the switch has been open for a long time, the initial current through the inductor is zero. Perform a source transformation on the voltage source. After the switch closes, there is a current source with magnitude V_A/R, in parallel with two resistors, R, and the inductor. The equivalent resistance is $R/2$. The corresponding differential equation for this RL circuit is

$$\frac{2L}{R}\frac{di_L(t)}{dt} + i_L(t) = \frac{V_A}{R}u(t)$$

with initial condition $i_L(0^-) = 0$.

(b) The solution is presented in the following MATLAB code.

```
clear all
syms t s it Is
R = 400;
L = 330e-3;
VA = 15;
i0 = 0;
% Take the Laplace transform of the differential equation
Eqn1 = 2*L/R*(s*Is-i0) + Is -VA/R*laplace(heaviside(t));
% Solve for I(s)
Is = solve(Eqn1,Is)
% Take the inverse Laplace transform to find i(t)
it = ilaplace(Is)
Is =
750/s/(33*s+20000)
it =
3/80-3/80*exp(-20000/33*t)
```

Answers:

(a) $0.00165\dfrac{di_L(t)}{dt} + i_L(t) = 0.0375u(t)$ and $i_L(0^-) = 0$

(b) $i_L(t) = \dfrac{3}{80}\left[1 - e^{-20000\ t/33}\right]u(t)$ A

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Problem 9-34

The switch in Figure P9-33 has been open for a long time and is closed at $t = 0$. The circuit parameters are $R = 200\ \Omega$, $L = 200$ mH, and $V_A = 50\ e^{-1000t}$ V.

(a) Find the differential equation for the inductor current $i_L(t)$ and initial condition $i_L(0)$.

(b) Solve for $i_L(t)$ using the Laplace transformation.

Solution:

(a) Since the switch has been open for a long time, the initial current through the inductor is zero. Perform a source transformation on the voltage source. After the switch closes, there is a current source with magnitude V_A/R, in parallel with two resistors, R, and the inductor. The equivalent resistance is $R/2$. The corresponding differential equation for this RL circuit is

$$\frac{2L}{R}\frac{di_L(t)}{dt} + i_L(t) = \frac{V_A}{R}u(t)$$

with initial condition $i_L(0^-) = 0$.

(b) The solution is presented in the following MATLAB code.

```
clear all
syms t s it Is
R = 200;
L = 200e-3;
VA = 50*exp(-1000*t);
i0 = 0;
% Take the Laplace transform of the differential equation
Eqn1 = 2*L/R*(s*Is-i0) + Is - laplace(VA/R);
% Solve for I(s)
Is = solve(Eqn1,Is)
% Take the inverse Laplace transform to find i(t)
it = ilaplace(Is)
Is =
125/(s+1000)/(s+500)
it =
1/2*exp(-750*t)*sinh(250*t)
```

Answers:

(a) $0.002\dfrac{di_L(t)}{dt} + i_L(t) = 0.25\,e^{-1000\,t}\ u(t)$ and $i_L(0^-) = 0$

(b) $i_L(t) = \dfrac{1}{4}\left[e^{-500\,t} - e^{-1000\,t}\right]u(t)$ A

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Problem 9-35

The switch in Fig. P9-35 has been open for a long time. At $t = 0$ the switch is closed.

(a) Find the differential equation for the capacitor voltage and initial condition $v_C(0)$.

(b) Find $v_O(t)$ using the Laplace transformation for $v_S(t) = 25\,u(t)$ V.

Solution:

(a) We will solve this problem by first solving for the voltage across the capacitor. Since the switch has been open for a long time, the initial voltage across the capacitor is zero. Perform a source transformation, combine all of the resistors, and then perform another source transformation to reduce the circuit to a Thévenin equivalent source in series with the capacitor. The Thévenin voltage will be $v_S(t)/2$ and the Thévenin resistance will be 4 kΩ. We can now write the following differential equation for the capacitor voltage.

$$R_T C \frac{dv_C(t)}{dt} + v_C(t) = v_T(t)$$

$$(4\text{ k})(125\text{ n})\frac{dv_C(t)}{dt} + v_C(t) = \frac{v_S(t)}{2}$$

The initial condition is $v_C(0^-) = 0$.

(b) First find the capacitor voltage and then find the output voltage.

```
clear all
syms t s vCt VCs vst vo
R = 4e3;
C = 125e-9;
vst = 25*heaviside(t);
vc0 = 0;
% Take the Laplace transform of the differential equation
Eqn1 = R*C*(s*VCs-vc0)+VCs-laplace(vst/2);
% Solve for VC(s)
VCs = solve(Eqn1,VCs);
% Take the inverse Laplace transform to find vC(t)
vCt = ilaplace(VCs);
% Use voltage division to find the output voltage
vo = 5e3*vCt/(5e3+3e3)
vo =
125/16-125/16*exp(-2000*t)
```

Answers:

(a) $500\times10^{-6}\dfrac{dv_C(t)}{dt} + v_C(t) = 12.5\,u(t)$ with $v_C(0^-) = 0$

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(b) $v_{\mathrm{O}}(t) = \dfrac{125}{16}\left[1 - e^{-2000\,t}\right]u(t)$ V

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Problem 9-36

Repeat Problem 9-35 for the input waveform $v_S(t)$ = 100 [cos 1000t]$u(t)$ V.

Solution:

(a) Changing the source voltage only affects the forcing function in the differential equation.

$$R_T C \frac{dv_C(t)}{dt} + v_C(t) = v_T(t)$$

$$(4\text{ k})(125\text{ n})\frac{dv_C(t)}{dt} + v_C(t) = \frac{v_S(t)}{2} = 50[\cos(1000t)]u(t)$$

The initial condition is $v_C(0^-) = 0$.

(b) First find the capacitor voltage and then find the output voltage.

```
clear all
syms t s vCt VCs vst vo
R = 4e3;
C = 125e-9;
vst = 100*cos(1000*t)*heaviside(t);
vc0 = 0;
% Take the Laplace transform of the differential equation
Eqn1 = R*C*(s*VCs-vc0)+VCs-laplace(vst/2);
% Solve for VC(s)
VCs = solve(Eqn1,VCs);
% Take the inverse Laplace transform to find vC(t)
vCt = ilaplace(VCs);
% Use voltage division to find the output voltage
vo = 5e3*vCt/(5e3+3e3)
vo =
-25*exp(-2000*t)+25*cos(1000*t)+25/2*sin(1000*t)
```

Answers:

(a) $500\times10^{-6}\dfrac{dv_C(t)}{dt} + v_C(t) = [50\cos(1000t)]u(t)$ with $v_C(0^-) = 0$

(b) $v_O(t) = 25\left[\cos(1000t) + \dfrac{1}{2}\sin(1000t) - e^{-2000\,t}\right]u(t)$ V

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Problem 9-37

Use the Laplace transformation to find the $v(t)$ that satisfies the following second-order differential equation:

$$\frac{d^2 v(t)}{d t^2} + 25\frac{d v(t)}{d t} + 100v(t) = 0\,, \quad v(0\text{-}) = 10 \text{ V and } \frac{dv(0-)}{dt} = 0\,.$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s Vs vt
v0 = 10;
dv0 = 0;
% Take the Laplace transform of the differential equation
Eqn1 = (s^2*Vs - s*v0-dv0)+25*(s*Vs-v0)+100*Vs;
% Solve for V(s)
Vs = solve(Eqn1,Vs)
% Take the inverse Laplace transform to find v(t)
vt = ilaplace(Vs)
Vs =
10*(s+25)/(s^2+25*s+100)
vt =
-10/3*exp(-20*t)+40/3*exp(-5*t)
```

Answer:

$$v(t) = \frac{10}{3}\left[4e^{-5t} - e^{-20t}\right]u(t) \text{ V}$$

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Problem 9-38

Use the Laplace transformation to find the $v(t)$ that satisfies the following second-order differential equation:

$$\frac{d^2 v(t)}{d t^2} + 100\frac{d v(t)}{d t} + 500\, v(t) = 0\,,\ \ v(0^-) = 0 \text{ and } \frac{dv(0^-)}{dt} = 100\,\text{V/s}.$$

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s Vs vt
v0 = 0;
dv0 = 100;
% Take the Laplace transform of the differential equation
Eqn1 = (s^2*Vs-s*v0-dv0)+100*(s*Vs-v0)+500*Vs;
% Solve for V(s)
Vs = solve(Eqn1,Vs)
% Take the inverse Laplace transform to find v(t)
vt = ilaplace(Vs)
vt_num = vpa(vt,6)
Vs =
100/(s^2+100*s+500)
vt =
5^(1/2)*exp(-50*t)*sinh(20*t*5^(1/2))
vt_num =
2.23607*exp(-50.*t)*sinh(44.7214*t)
```

Answer:

$$v(t) = 1.11804\left[e^{-5.2786\,t} - e^{-94.7214\,t}\right]u(t)$$

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Problem 9-39

The switch in Figure P9-39 has been open for a long time and is closed at $t = 0$. The circuit parameters are $R = 500\ \Omega$, $L = 2.5$ H, $C = 2.5\ \mu$F, and $V_A = 20$ V.

(a) Find the circuit differential equation in $i_L(t)$ and the initial conditions $i_L(0)$ and $v_C(0)$.

(b) Use Laplace transforms to solve for $i_L(t)$ for $t \geq 0$.

Solution:

(a) Since the switch has been open for a long time, the capacitor behaves like an open circuit and the inductor behaves likes a short circuit. The initial voltage across the capacitor is 0 V and the initial current through the inductor is $V_A/(2R)$. After the switch closes, the circuit is a parallel *RLC* circuit with the following differential equation:

$$LC\frac{d^2 i_L(t)}{dt^2} + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = 0$$

(b) The solution is presented in the following MATLAB code.

```
clear all
syms s t it Is
R = 500;
L = 2.5;
C = 2.5e-6;
VA = 20;
iL0 = VA/2/R;
diL0 = 0;
% Transform the differential equation into the Laplace domain
k1 = 1/R/C;
k2 = 1/L/C;
%Eqn1 = L*C*(s^2*Is-s*iL0-diL0)+L/R*(s*Is-iL0)+Is
Eqn1 = (s^2*Is-s*iL0-diL0)+k1*(s*Is-iL0)+k2*Is;
% Solve for IL(s)
Is = solve(Eqn1,Is);
Is_num = vpa(simplify(Is),6);
% Take the inverse Laplace transform
it = ilaplace(Is)
it =
1/50*exp(-400*t)*(1+400*t)
```

Answers:

(a) $6.25\times10^{-6}\dfrac{d^2 i_L(t)}{dt^2} + 5\times10^{-3}\dfrac{di_L(t)}{dt} + i_L(t) = 0$, $i_L(0^-) = 20$ mA , and $v_C(0^-) = 0$ V .

(b) $i_L(t) = \dfrac{1}{50}e^{-400t}(1+400t)u(t)$ A

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Problem 9-40
Repeat Problem 9-39 with R = 5 kΩ.

Solution:
(a) Since the switch has been open for a long time, the capacitor behaves like an open circuit and the inductor behaves likes a short circuit. The initial voltage across the capacitor is 0 V and the initial current through the inductor is $V_A/(2R)$. After the switch closes, the circuit is a parallel RLC circuit with the following differential equation:

$$LC\frac{d^2 i_L(t)}{dt^2}+\frac{L}{R}\frac{di_L(t)}{dt}+i_L(t)=0$$

(b) The solution is presented in the following MATLAB code.

```
clear all
syms s t it Is
R = 5000;
L = 2.5;
C = 2.5e-6;
VA = 20;
iL0 = VA/2/R;
diL0 = 0;
% Transform the differential equation into the Laplace domain
k1 = 1/R/C;
k2 = 1/L/C;
%Eqn1 = L*C*(s^2*Is-s*iL0-diL0)+L/R*(s*Is-iL0)+Is
Eqn1 = (s^2*Is-s*iL0-diL0)+k1*(s*Is-iL0)+k2*Is;
% Solve for IL(s)
Is = solve(Eqn1,Is);
Is_num = vpa(simplify(Is),6);
% Take the inverse Laplace transform
it = ilaplace(Is)
it_num = vpa(it,6)
it =
1/16500*exp(-40*t)*(33*cos(120*11^(1/2)*t)+11^(1/2)*sin(120*11^(1/2)*t))
it_num =
.606061e-4*exp(-40.*t)*(33.*cos(397.994*t)+3.31662*sin(397.994*t))
```

Answers:

(a) $6.25\times10^{-6}\dfrac{d^2 i_L(t)}{dt^2}+500\times10^{-6}\dfrac{di_L(t)}{dt}+i_L(t)=0$, $i_L(0^-)=2$ mA, and $v_C(0^-)=0$ V.

(b) $i_L(t)=60.6061\,e^{-40t}\left[33\cos(397.994t)+3.31662\sin(397.994t)\right]u(t)$ μA.

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Problem 9-41

The switch in Figure P9-41 has been closed for a long time and is opened at $t = 0$.

(a) Find the circuit differential equation in $v_C(t)$ and the initial conditions $i_L(0)$ and $v_C(0)$.

(b) The circuit parameters are $L = 5$ H, $C = 2.5$ μF, $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, and $v_S = 20u(t)$ V. Use Laplace transforms and MATLAB to solve for the $v_C(t)$ for $t \geq 0$.

Solution:

(a) With the switch closed, there is no current flowing through the inductor and no voltage drop across the capacitor, so both initial conditions are zero. After the switch moves, the circuit is a series *RLC* circuit with $R = R_1 + R_2$, and a forcing function of $v_S(t)$. The corresponding differential equation is shown below.

$$LC\frac{d^2v_C(t)}{dt^2} + (R_1 + R_2)C\frac{dv_C(t)}{dt} + v_C(t) = v_S(t)$$

$$12.5\times10^{-6}\frac{d^2v_C(t)}{dt^2} + 7.5\times10^{-3}\frac{dv_C(t)}{dt} + v_C(t) = 20u(t)$$

with initial conditions: $i_L(0^-) = 0$ mA and $v_C(0^-) = 0$ V .

(b) The solution is presented in the following MATLAB code.

```
clear all
syms s t vt Vs
R1 = 1e3;
R2 = 2e3;
L = 5;
C = 2.5e-6;
VA = 20*heaviside(t);
vC0 = 0;
dvC0 = 0;
% Transform the differential equation into the Laplace domain
k1 = (R1+R2)/L;
k2 = 1/L/C;
Eqn1 = (s^2*Vs-s*vC0-dvC0)+k1*(s*Vs-vC0)+k2*Vs-k2*laplace(VA);
% Solve for VC(s)
Vs = solve(Eqn1,Vs);
Vs_num = vpa(simplify(Vs),6);
% Take the inverse Laplace transform
vt = ilaplace(Vs)
vt_num = vpa(vt,6);
vt =
-40*exp(-200*t)+20*exp(-400*t)+20
```

Answers:

(a) $12.5\times10^{-6}\frac{d^2v_C(t)}{dt^2} + 7.5\times10^{-3}\frac{dv_C(t)}{dt} + v_C(t) = 20u(t)$, $i_L(0^-) = 0$ mA , and $v_C(0^-) = 0$ V .

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(b) $v_C(t) = \left[20 - 40e^{-200\,t} + 20e^{-400\,t}\right]u(t)$ V

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Problem 9-42
Repeat problem 9-41 with R_2 = 500 Ω.

Solution:
(a) With the switch closed, there is no current flowing through the inductor and no voltage drop across the capacitor, so both initial conditions are zero. After the switch moves, the circuit is a series *RLC* circuit with $R = R_1 + R_2$, and a forcing function of $v_S(t)$. The corresponding differential equation is shown below.

$$LC\frac{d^2v_C(t)}{dt^2}+(R_1+R_2)C\frac{dv_C(t)}{dt}+v_C(t)=v_S(t)$$

$$12.5\times10^{-6}\frac{d^2v_C(t)}{dt^2}+3.75\times10^{-3}\frac{dv_C(t)}{dt}+v_C(t)=20u(t)$$

with initial conditions: $i_L(0^-)=0$ mA and $v_C(0^-)=0$ V.

(b) The solution is presented in the following MATLAB code.

```
clear all
syms s t vt Vs
R1 = 1e3;
R2 = 500;
L = 5;
C = 2.5e-6;
VA = 20*heaviside(t);
vC0 = 0;
dvC0 = 0;
% Transform the differential equation into the Laplace domain
k1 = (R1+R2)/L;
k2 = 1/L/C;
Eqn1 = (s^2*Vs-s*vC0-dvC0)+k1*(s*Vs-vC0)+k2*Vs-k2*laplace(VA);
% Solve for VC(s)
Vs = solve(Eqn1,Vs);
Vs_num = vpa(simplify(Vs),6);
% Take the inverse Laplace transform
vt = ilaplace(Vs)
vt_num = vpa(vt,6)
vt =
20-20/23*exp(-150*t)*(23*cos(50*t*23^(1/2))+3*23^(1/2)*sin(50*t*23^(1/2)))
vt_num =
20.-.869565*exp(-150.*t)*(23.*cos(239.792*t)+14.3875*sin(239.792*t))
```

Answers:

(a) $12.5\times10^{-6}\frac{d^2v_C(t)}{dt^2}+3.75\times10^{-3}\frac{dv_C(t)}{dt}+v_C(t)=20u(t)$, $i_L(0^-)=0$ mA, and $v_C(0^-)=0$ V.

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(b) $v_C(t) = \left\{20 - 0.869565e^{-150t}\left[23\cos(239.792t) + 14.3875\sin(239.792t)\right]\right\}u(t)$ V

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Problem 9-43

The *RLC* circuit in Figure P9-43 is in the zero state when at $t = 0$ an exponential source, $v_S(t) = V_A e^{-\alpha t}$ V, is suddenly connected to it.

(a) Find the circuit integrodifferential equation that describes the behavior of the current in the circuit.

(b) If $R = 100\ \Omega$, $L = 100$ mH, $C = 10\ \mu$F, $V_A = 10$ V, and $\alpha = 500\ \text{s}^{-1}$, use Laplace transforms and MATLAB to solve for $i(t)$ for $t \geq 0$.

Solution:

(a) The circuit is a series *RLC* circuit, so every element has the same current. Write a KVL expression for the circuit and express all of the voltages in terms of the current.

$$v_R(t) + v_L(t) + v_C(t) = v_S(t)$$

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(x)\,dx = v_S(t)$$

$$100i(t) + 0.1\frac{di(t)}{dt} + 100000\int_0^t i(x)\,dx = 10e^{-500\,t}\ u(t)$$

(b) The MATLAB solution is presented below.

```
clear all
syms t s it Is
R = 100;
L = 100e-3;
C = 10e-6;
VA = 10;
a = 500;
vs = VA*exp(-a*t);
i0 = 0;
% Take the Laplace transform of the integrodifferential equation
Eqn1 = R*Is+L*(s*Is-i0)+(1/C)*Is/s-laplace(vs);
% Solve for Is
Is = solve(Eqn1,Is);
% Take the inverse Laplace transform
it = ilaplace(Is)
it =
1/15*exp(-500*t)*(-1+cos(500*3^(1/2)*t)+3^(1/2)*sin(500*3^(1/2)*t))
```

Answers:

(a) $0.1\dfrac{di(t)}{dt} + 100000\displaystyle\int_0^t i(x)\,dx + 100i(t) = 10e^{-500\,t}$, $i_L(0^-) = 0$ mA, and $v_C(0^-) = 0$ V.

(b) $i(t) = \dfrac{1}{15}e^{-500t}\left[-1 + \cos\left(500\sqrt{3}\,t\right) + \sqrt{3}\sin\left(500\sqrt{3}\,t\right)\right]u(t)$ A.

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Problem 9-44

Repeat problem 9-43 when an exponential source, $v_S(t) = 10(1 - e^{-500t})$ V, is suddenly connected to the circuit.

Solution:

(a) The circuit is a series *RLC* circuit, so every element has the same current. Write a KVL expression for the circuit and express all of the voltages in terms of the current.

$$v_R(t) + v_L(t) + v_C(t) = v_S(t)$$

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(x)\,dx = v_S(t)$$

$$100i(t) + 0.1\frac{di(t)}{dt} + 100000\int_0^t i(x)\,dx = 10\left(1 - e^{-500\,t}\right)u(t)$$

(b) The MATLAB solution is presented below.

```
clear all
syms t s it Is
R = 100;
L = 100e-3;
C = 10e-6;
VA = 10;
a = 500;
vs = VA*(1-exp(-a*t));
i0 = 0;
% Take the Laplace transform of the integrodifferential equation
Eqn1 = R*Is+L*(s*Is-i0)+(1/C)*Is/s-laplace(vs);
% Solve for Is
Is = solve(Eqn1,Is);
% Take the inverse Laplace transform
it = ilaplace(Is)
it =
1/15*exp(-500*t)*(1-cos(500*3^(1/2)*t))
```

Answers:

(a) $100i(t) + 0.1\frac{di(t)}{dt} + 100000\int_0^t i(x)\,dx = 10\left(1 - e^{-500\,t}\right)u(t)$, $i_L(0^-) = 0$ mA, and $v_C(0^-) = 0$ V.

(b) $i(t) = \frac{1}{15}e^{-500t}\left[1 - \cos\left(500\sqrt{3}\,t\right)\right]u(t)$ A.

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Problem 9-45

Use the initial and final value properties to find the initial and final values of the waveforms corresponding to the transforms below. If either property is not applicable, explain why.

(a) $F_1(s) = \dfrac{16s}{(s+2)(s^2+12s+13)}$

(b) $F_2(s) = \dfrac{s+10}{s(s+50)(s+100)}$

Solution:

Both transforms are proper rational functions, so the initial value property applies to both. The poles for $sF(s)$ have negative real parts for both transforms, so the final value property also applies. The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 16*s/(s+2)/(s^2+12*s+13);
f10 = limit(s*F1,s,inf);
f1f = limit(s*F1,s,0);
% Check the results by finding f1(t)
f1 = ilaplace(F1);
f120 = limit(f1,t,0);
f12f = limit(f1,t,inf);
Results1 = [f10,f1f;f120,f12f]

disp('Part (b)')
F2 = (s+10)/s/(s+50)/(s+100);
f20 = limit(s*F2,s,inf);
f2f = limit(s*F2,s,0);
% Check the results by finding f2(t)
f2 = ilaplace(F2);
f220 = limit(f2,t,0);
f22f = limit(f2,t,inf);
Results2 = [f20,f2f;f220,f22f]
Part (a)
Results1 =
[ 0, 0]
[ 0, 0]
Part (b)
Results2 =
[     0, 1/500]
[     0, 1/500]
```

Answers:

(a) $f_1(0) = 0$ and $f_1(\infty) = 0$.

(b) $f_2(0) = 0$ and $f_2(\infty) = 0.002$.

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Problem 9-46

Use the initial and final value properties to find the initial and final values of the waveforms corresponding to the transforms below. If either property is not applicable, explain why.

(a) $F_1(s) = 400\dfrac{1}{(s+10)^2 + 20^2}$

(b) $F_2(s) = \dfrac{3s^4 + 10s^2 + 4}{s(s^2+1)(s^2+4)}$

Solution:

Both transforms are proper rational functions, so the initial value property applies to both. The poles for $sF(s)$ have negative real parts for the transform in Part (a), but not for the transform in Part (b), so the final value property only applies to Part (a). The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 400/((s+10)^2+20^2);
f10 = limit(s*F1,s,inf);
f1f = limit(s*F1,s,0);
% Check the results by finding f1(t)
f1 = ilaplace(F1);
f120 = limit(f1,t,0);
f12f = limit(f1,t,inf);
Results1 = [f10,f1f;f120,f12f]

disp('Part (b)')
F2 = (3*s^4+10*s^2+4)/s/(s^2+1)/(s^2+4);
f20 = limit(s*F2,s,inf);
f2f = limit(s*F2,s,0);
% Check the results by finding f2(t)
f2 = ilaplace(F2);
f220 = limit(f2,t,0);
f22f = limit(f2,t,inf);
Results2 = [f20,f2f;f220,f22f]
Part (a)
Results1 =
[ 0, 0]
[ 0, 0]
Part (b)
Results2 =
[       3,       1]
[       3, -1 .. 3]
```

Answers:

(a) $f_1(0) = 0$ and $f_1(\infty) = 0$.

(b) $f_2(0) = 3$ and $f_2(\infty)$ does not exist, because $sF_2(s)$ has poles with non-negative real parts.

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Problem 9-47

Use the initial and final value properties to find the initial and final values of the waveforms corresponding to the following transforms. If either property is not applicable, explain why.

(a) $F_1(s) = \dfrac{50\left(s^2 + 5s + 6\right)}{(s+2)(s+6)(s+12)}$

(b) $F_2(s) = \dfrac{10\left(s^2 + 10s + 40\right)}{s\left(s^2 - 625\right)}$

Solution:

Both transforms are proper rational functions, so the initial value property applies to both. The poles for $sF(s)$ have negative real parts for the transform in Part (a), but not for the transform in Part (b), so the final value property only applies to Part (a). The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 50*(s^2+5*s+6)/(s+2)/(s+6)/(s+12);
f10 = limit(s*F1,s,inf);
f1f = limit(s*F1,s,0);
% Check the results by finding f1(t)
f1 = ilaplace(F1);
f120 = limit(f1,t,0);
f12f = limit(f1,t,inf);
Results1 = [f10,f1f;f120,f12f]

disp('Part (b)')
F2 = 10*(s^2+10*s+40)/s/(s^2-625);
f20 = limit(s*F2,s,inf);
f2f = limit(s*F2,s,0);
% Check the results by finding f2(t)
f2 = ilaplace(F2);
f220 = limit(f2,t,0);
f22f = limit(f2,t,inf);
Results2 = [f20,f2f;f220,f22f]
Part (a)
Results1 =
[ 50,  0]
[ 50,  0]
Part (b)
Results2 =
[     10, -16/25]
[     10,    Inf]
```

Answers:

(a) $f_1(0) = 50$ and $f_1(\infty) = 0$.

(b) $f_2(0) = 10$ and $f_2(\infty)$ does not exist, because $sF_2(s)$ has poles with non-negative real parts.

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Problem 9-48

Use the initial and final value properties to find the initial and final values of the waveforms corresponding to the following transforms. If either property is not applicable, explain why.

(a) $F_1(s) = \dfrac{12s\,(s+6)}{s^2+6s+9}$

(b) $F_2(s) = \dfrac{20\,(s^2+10s-20)}{s\,(s^2+100)}$

Solution:

The transform in Part (a) is not a proper rational function, so the initial value property does not apply to it, but it does apply to Part (b). The poles for $sF(s)$ have negative real parts for the transform in Part (a), but not for the transform in Part (b), so the final value property only applies to Part (a). The solution is presented in the following MATLAB code.

```
clear all
syms t s
disp('Part (a)')
F1 = 12*s*(s+6)/(s^2+6*s+9);
f10 = limit(s*F1,s,inf);
f1f = limit(s*F1,s,0);
% Check the results by finding f1(t)
f1 = ilaplace(F1);
f120 = limit(f1,t,0);
f12f = limit(f1,t,inf);
Results1 = [f10,f1f;f120,f12f]

disp('Part (b)')
F2 = 20*(s^2+10*s-20)/s/(s^2+100);
f20 = limit(s*F2,s,inf);
f2f = limit(s*F2,s,0);
% Check the results by finding f2(t)
f2 = ilaplace(F2);
f220 = limit(f2,t,0);
f22f = limit(f2,t,inf);
Results2 = [f20,f2f;f220,f22f]
Part (a)
Results1 =
[ Inf,   0]
[   0,   0]
Part (b)
Results2 =
[        20,        -4]
[        20, -48 .. 40]
```

Answers:

(a) $f_1(0)$ does not exist because $F(s)$ is an improper rational function and $f_1(\infty) = 0$.

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(b) $f_2(0) = 20$ and $f_2(\infty)$ does not exist, because $sF_2(s)$ has poles with non-negative real parts.

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Problem 9-49

Use the initial and final value properties to find the initial and final values of the waveform corresponding to the following transform. If either property is not applicable, explain why.

$$F(s) = \frac{80\left(s^3 + 2s^2 + s + 2\right)}{s\left(s^3 + 4s^2 + 4s + 16\right)}$$

Solution:

The transform is a proper rational function, so the initial value property applies. We can use MATLAB to find the poles of $sF(s)$ as follows

```
clear all
p = roots([1 4 4 16])
p =
    -4.0000e+000
  -277.5558e-018 +  2.0000e+000i
  -277.5558e-018 -  2.0000e+000i
```

Two of the poles have (nearly) zero real parts, so the final value property does not apply.

```
syms t s
F1 = 80*(s^3+2*s^2+s+2)/s/(s^3+4*s^2+4*s+16);
f10 = limit(s*F1,s,inf);
f1f = limit(s*F1,s,0);
% Check the results by finding f1(t)
f1 = ilaplace(F1);
f120 = limit(f1,t,0);
f12f = limit(f1,t,inf);
Results1 = [f10,f1f;f120,f12f]
Results1 =
[        80,        10]
[        80, -38 .. 58]
```

Answer:

$f(0) = 80$ and $f(\infty)$ does not exist, because $sF(s)$ has poles with non-negative real parts.

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Problem 9-50

The MATLAB function `limit` can be used to take the limit of a symbolic expression. Use MATLAB and the initial and final value properties to find the initial and final values of the waveforms corresponding to the following transforms. If either property is not applicable, explain why. Use MATLAB again to compute the waveforms corresponding to each transform and then take limits in the time domain to verify the answers found using the initial and final value properties.

(a) $F_1(s) = \dfrac{(s+50)^2}{(s+20)^2(s+100)}$

(b) $F_2(s) = \dfrac{(s+20)^2}{(s+50)^2(s+100)}$

Solution:

For both parts, the initial and final value properties apply.

```
clear all
syms t s
disp('Part (a)')
F1 = (s+50)^2/(s+20)^2/(s+100);
f10 = limit(s*F1,s,inf);
f1f = limit(s*F1,s,0);
% Check the results by finding f1(t)
f1 = ilaplace(F1);
f120 = limit(f1,t,0);
f12f = limit(f1,t,inf);
Results1 = [f10,f1f;f120,f12f]

disp('Part (b)')
F2 = (s+20)^2/(s+50)^2/(s+100);
f20 = limit(s*F2,s,inf);
f2f = limit(s*F2,s,0);
% Check the results by finding f2(t)
f2 = ilaplace(F2);
f220 = limit(f2,t,0);
f22f = limit(f2,t,inf);
Results2 = [f20,f2f;f220,f22f]
Part (a)
Results1 =
[ 1, 0]
[ 1, 0]
Part (b)
Results2 =
[ 1, 0]
[ 1, 0]
```

Answers:

(a) $f_1(0) = 1$ and $f_1(\infty) = 0$.

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(b) $f_2(0) = 1$ and $f_2(\infty) = 0$.

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Problem 9-51

<u>The Dominant Pole Approximation</u> **(A)**

When a transform $F(s)$ has widely separated poles, then those closest to the j-axis tend to dominate the response because they have less damping. An approximation to the waveform can be obtained by ignoring the contributions of all except the dominant poles. We can ignore the other poles by simply discarding their terms in the partial fraction expansion of $F(s)$. The purpose of this example is to examine a dominant pole approximation of the transform

$$F(s) = 10^6 \frac{s + 1000}{(s + 4000)\left[(s + 25)^2 + 100^2\right]}$$

(a) Construct a partial-fraction expansion of $F(s)$ and find $f(t)$.
(b) Construct a pole-zero diagram of $F(s)$ and identify the dominant poles.
(c) Construct a dominant pole approximation $g(t)$ by discarding the other poles in the partial fraction expansion in (a).
(d) Plot $f(t)$ and $g(t)$ and comment on the accuracy of the approximation.

Solution:

(a) The partial-fraction expansion is shown below.

$$F(s) = 10^6 \frac{s + 1000}{(s + 4000)\left[(s + 25)^2 + 100^2\right]} = \frac{k_1}{s+4000} + \frac{k_2 s + k_3}{(s+25)^2 + 100^2}$$

Use MATLAB to solve for k_1, k_2, and k_3. First, we have

$$k_1\left[(s+25)^2 + 100^2\right] + (k_2 s + k_3)(s+4000) = 10^6(s+1000)$$
$$k_1 + k_2 = 0$$
$$50k_1 + 4000k_2 + k_3 = 10^6$$
$$10625k_1 + 4000k_3 = 10^9$$

```
clear all
A = [1,1,0; 50,4000,1; 10625,0,4000];
B = [0; 1e6; 1e9];
K = A\B
K =
  -189.7458e+000
   189.7458e+000
   250.5040e+003
```

$$F(s) = \frac{-189.7458}{s+4000} + \frac{189.7458s + 250504}{(s+25)^2 + 100^2}$$

Use MATLAB to complete the inverse Laplace transform.

```
syms t s f F
```

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```
F = 1e6*(s+1000)/(s+4000)/((s+25)^2+100^2);
f = ilaplace(F)
f_num = vpa(f,6)
f =
-4800000/25297*exp(-4000*t)+10000/25297*exp(-
25*t)*(480*cos(100*t)+6217*sin(100*t))
f_num =
-189.746*exp(-4000.*t)+.395304*exp(-
25.*t)*(480.*cos(100.*t)+6217.*sin(100.*t))
```

$$f(t) = \left\{-189.746e^{-4000t} + e^{-25t}\left[189.746\cos(100t) + 2457.6\sin(100t)\right]\right\}u(t)$$

(b) The pole-zero diagram is shown below.

```
% Create the pole-zero diagram
close all
Q = 4000;
axis(Q*[-1 1 -1 1])
line(Q*[-1 1],[0 0],...
    'Color','k',...
    'LineWidth',0.5)
line([0 0],Q*[-1 1],...
    'Color','k',...
    'LineWidth',0.5)
grid on
hold on
xlabel('Real')
ylabel('Imag')
p = [roots([1 50 10625]);-4000]
z = [-1000]
MSize = 16;
LW = 2.5;
% Plot poles
for n = 1:length(p)
    plot(real(p(n)),imag(p(n)),'x',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end

% Plot zeros
for n = 1:length(z)
    plot(real(z(n)),imag(z(n)),'o',...
    'LineWidth',LW,...
    'MarkerSize',MSize)
end
p =
   -25.0000e+000 +100.0000e+000i
   -25.0000e+000 -100.0000e+000i
    -4.0000e+003
z =
    -1.0000e+003
```

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The dominant poles are at $s = -25 + j100$ and $-25 - j100$.

(c) Construct the dominant-pole approximation as follows.

$$G(s) = \frac{189.7458s + 250504}{(s+25)^2 + 100^2}$$

Solve for $g(t)$.

```
G = (189.7458*s+250504)/((s+25)^2+100^2);
g = simplify(ilaplace(G))
g_num = vpa(g,6)
g =
1/3518437208883200*exp(-
25*t)*(667608682949309900*cos(100*t)+8646923775003443853*sin(100*t))
g_num =
.284217e-15*exp(-25.*t)*(.667609e18*cos(100.*t)+.864692e19*sin(100.*t))
```

$$g(t) = \left\{e^{-25t}\left[189.746\cos(100t) + 2457.6\sin(100t)\right]\right\}u(t)$$

(d) Plot the waveforms

```
close all
tt = 0:500e-6:300e-3;
ftt = subs(f,t,tt);
gtt = subs(g,t,tt);
plot(tt,ftt,'g','LineWidth',4)
hold on
grid on
plot(tt,gtt,'b','LineWidth',2)
xlabel('Time (s)')
ylabel('Signals f(t) and g(t)')
```

Examine just the initial part of each waveform in more detail

```
close all
tt = 0:1e-6:2e-3;
ftt = subs(f,t,tt);
gtt = subs(g,t,tt);
plot(tt,ftt,'g','LineWidth',4)
hold on
grid on
plot(tt,gtt,'b','LineWidth',2)
xlabel('Time (s)')
ylabel('Signals f(t) and g(t)')
```

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The two waveforms converge within 1 ms.

Answers:

(a) $F(s) = \dfrac{-189.7458}{s+4000} + \dfrac{189.7458s + 250504}{(s+25)^2 + 100^2}$

$f(t) = \left\{-189.746e^{-4000t} + e^{-25t}\left[189.746\cos(100t) + 2457.6\sin(100t)\right]\right\}u(t)$

(b) The pole-zero diagram is shown above and the dominant poles are at $s = -25 + j100$ and $-25 - j100$.

(c) $g(t) = \left\{e^{-25t}\left[189.746\cos(100t) + 2457.6\sin(100t)\right]\right\}u(t)$

(d) The plots are shown above and the two waveforms are nearly identical after 1 ms. The approximate is very good.

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Problem 9-52

First-Order Circuit Step Response **(A)**

In Chapter 7 we found that the step response of a first-order circuit can be written as

$$f(t) = f(\infty) + \left[f(0) - f(\infty)\right] e^{-t/T_C}$$

where $f(0)$ is the initial value, $f(4)$ is the final value, and T_C is the time constant. Show that the corresponding transform has the form

$$F(s) = K\left[\frac{s+\gamma}{s(s+\alpha)}\right]$$

and relate the time-domain parameters $f(0)$, $f(4)$, and T_C to the s-domain parameters K, γ, and α.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s IV FV Tc f F
f = FV+(IV-FV)*exp(-t/Tc);
F = laplace(f)
F =
(FV+IV*Tc*s)/s/(s*Tc+1)
```

$$F(s) = \frac{f(0)\,T_C s + f(\infty)}{s(T_C s+1)} = \frac{f(0)\,s + \dfrac{f(\infty)}{T_C}}{s\left(s+\dfrac{1}{T_C}\right)} = f(0)\left[\frac{s+\dfrac{f(\infty)}{f(0)\,T_C}}{s\left(s+\dfrac{1}{T_C}\right)}\right] = K\left[\frac{s+\gamma}{s(s+\alpha)}\right]$$

Answers:

$K = f(0)$

$\gamma = \dfrac{f(\infty)}{f(0)\,T_C}$

$\alpha = \dfrac{1}{T_C}$

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Problem 9-53

Inverse Transform for Complex Poles **(A)**

In Section 9-4 we learned that complex poles occur in conjugate pairs and that for simple poles the partial fraction expansion of $F(s)$ will contain two terms of the form

$$F(s) = \ldots \frac{k}{s+\alpha-j\beta} + \frac{k^*}{s+\alpha+j\beta} + \ldots$$

Show that when the complex conjugate residues are written in rectangular form as

$$k = a + jb \textbf{ and } k^* = a - jb$$

the corresponding term in the waveform $f(t)$ is

$$f(t) = \ldots + 2e^{-\alpha t}\left[a\cos(\beta t) - b\sin(\beta t)\right] + \ldots$$

Solution:

$$F(s) = \frac{k}{s+\alpha-j\beta} + \frac{k^*}{s+\alpha+j\beta} = \frac{a+jb}{s+\alpha-j\beta} + \frac{a-jb}{s+\alpha+j\beta}$$

$$F(s) = \frac{(a+jb)(s+\alpha+j\beta)+(a-jb)(s+\alpha-j\beta)}{(s+\alpha)^2+\beta^2}$$

$$F(s) = \frac{2a(s+\alpha)-2b\beta}{(s+\alpha)^2+\beta^2} = 2a\frac{(s+\alpha)}{(s+\alpha)^2+\beta^2} - 2b\frac{\beta}{(s+\alpha)^2+\beta^2}$$

$$f(t) = 2ae^{-\alpha t}\cos(\beta t) - 2be^{-\alpha t}\sin(\beta t)$$

$$f(t) = 2e^{-\alpha t}\left[a\cos(\beta t) - b\sin(\beta t)\right]$$

The solution is also presented in the following MATLAB code.

```
clear all
syms t s a b A B f F
F = (a+j*b)/(s+A-j*B) + (a-j*b)/(s+A+j*B);
f = ilaplace(F)
f =
2*exp(-t*A)*(a*cos(B*t)-b*sin(B*t))
```

Answer:

Presented above.

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Problem 9-54

Solving State Variable Equations **(A)**

With zero input, a series *RLC* circuit can be described by the following coupled first-order equations in the inductor current $i_L(t)$ and capacitor voltage $v_C(t)$.

$$\frac{dv_C(t)}{dt} = \frac{1}{C}\, i_L(t)$$

$$\frac{di_L(t)}{dt} = -\frac{1}{L}\, v_C(t) - \frac{R}{L}\, i_L(t)$$

(a) Transform these equations into the *s* domain and solve for the transforms $I_L(s)$ and $V_C(s)$ in terms of the initial conditions $i_L(0) = I_0$ and $v_C(0) = V_0$.

(b) Find $i_L(t)$ and $v_C(t)$ for $R = 1$ kΩ, $L = 1$ H, $C = 5$ μF, $I_0 = 10$ mA and $V_0 = 5$ V.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
disp('Part (a)')
syms t s I0 V0 Is Vs it vt R L C
% Transform the equations into the s domain
Eqn1 = s*Vs-V0 -Is/C;
Eqn2 = s*Is-I0+Vs/L+R*Is/L;
Soln = solve(Eqn1,Eqn2,Is,Vs);
Is = Soln.Is
Vs = Soln.Vs
Part (a)
Is =
C*(s*I0*L-V0)/(s^2*L*C+1+R*s*C)
Vs =
(s*L*V0*C+I0*L+R*V0*C)/(s^2*L*C+1+R*s*C)

disp('Part (b)')
R = 1e3;
L = 1;
C = 5e-6;
I0 = 10e-3;
V0 = 5;
Is = subs(Is);
it = ilaplace(Is);
it_num = vpa(it,6)
Vs = subs(Vs);
vt = ilaplace(Vs);
vt_num = vpa(vt,6)
Part (b)
it_num =
.573972e-37*exp(-500.000*t)*(-
.779156e36*sinh(223.607*t)+.174225e36*cosh(223.607*t))
vt_num =
.126218e-26*exp(-
500.000*t)*(.396141e28*cosh(223.607*t)+.159444e29*sinh(223.607*t))
```

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```
% Using another approach, we have
Inum = [C*I0*L, -C*V0];
Iden = [L*C, R*C, 1];
Vnum = [L*V0*C, I0*L+R*V0*C];
Vden = [L*C, R*C, 1];

[Ir,Ip,Ik] = residue(subs(Inum),subs(Iden))
[Vr,Vp,Vk] = residue(subs(Vnum),subs(Vden))
Ir =
    27.3607e-003
   -17.3607e-003
Ip =
  -723.6068e+000
  -276.3932e+000
Ik =
     []
Vr =
    -7.5623e+000
    12.5623e+000
Vp =
  -723.6068e+000
  -276.3932e+000
Vk =
     []
```

Answers:

(a) $I_{\mathrm{L}}(s) = \dfrac{C(LI_0 s - V_0)}{LCs^2 + RCs + 1}$

$V_{\mathrm{C}}(s) = \dfrac{LCV_0 s + RCV_0 + LI_0}{LCs^2 + RCs + 1}$

(b) $i_{\mathrm{L}}(t) = \left[-17.36e^{-276.39t} + 27.36e^{-723.61t}\right]u(t)$ mA

$v_{\mathrm{C}}(t) = \left[12.56e^{-276.39t} - 7.56e^{-723.61t}\right]u(t)$ V

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Problem 9-55

Complex Differentiation Property **(A)**

The complex differentiation property of the Laplace transformation states that

$$\textbf{If } \mathscr{L}\{f(t)\} = F(s) \textbf{ then } \mathscr{L}\{tf(t)\} = -\frac{d}{ds}F(s)$$

Use this property to find the Laplace transforms of $f(t) = \{tg(t)\}u(t)$ when $g(t) = e^{-\alpha t}$. Repeat for $g(t) = \sin \beta t$ and $g(t) = \cos \beta t$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms t s g G F F2 a B
g = exp(-a*t);
G = laplace(g)
F = -1*diff(G,s)
F2 = laplace(t*g)
G =
1/(s+a)
F =
1/(s+a)^2
F2 =
1/(s+a)^2

g = sin(B*t);
G = laplace(g)
F = -1*diff(G,s)
F2 = laplace(t*g)
G =
B/(s^2+B^2)
F =
2*B/(s^2+B^2)^2*s
F2 =
2*B/(s^2+B^2)^2*s

g = cos(B*t);
G = laplace(g)
F = -1*diff(G,s)
F2 = laplace(t*g)
G =
s/(s^2+B^2)
F =
-1/(s^2+B^2)+2*s^2/(s^2+B^2)^2
F2 =
(s^2-B^2)/(s^2+B^2)^2
```

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Answer:

For $g(t) = e^{-\alpha t}$, $F(s) = \dfrac{1}{(s+\alpha)^2}$

For $g(t) = \sin \beta t$, $F(s) = \dfrac{2\beta s}{(s^2+\beta^2)^2}$

For $g(t) = \cos \beta t$, $F(s) = \dfrac{s^2-\beta^2}{(s^2+\beta^2)^2}$

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Problem 10-1

Find $Z_{EQ}(s)$ in Figure P10-1. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms s R C ZC Zeq
ZC = 1/s/C;
% Combine the resistor in series with the capacitor
% and then combine the result in parallel with the other
% resistor
Zeq = simplify(1/(1/(2*R+ZC)+1/(2*R)))
Zeq =
2*(2*R*s*C+1)*R/(4*R*s*C+1)
```

Answers:

$$Z_{EQ}(s) = \frac{2R(2RCs+1)}{4RCs+1}$$

The pole is located at $s = -1/4RC$ and the zero is located at $s = -1/2RC$.

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Problem 10-2

Find $Z_{EQ}(s)$ in Figure P10-2. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R C Z1 Z2 Zeq
% Combine the resistors in parallel and the capacitors in parallel
% The results are then in series and can be combined
Z1 = 1/(1/R + 1/(2*R));
Z2 = 1/(1/(1/s/C) + 1/(1/s/(C/2)));
Zeq = simplify(Z1+Z2)
Zeq =
2/3*(R*s*C+1)/s/C
```

Answers:

$$Z_{EQ}(s) = \frac{2\,(RCs+1)}{3\,Cs}$$

The pole is located at $s = 0$ and the zero is located at $s = -1/RC$.

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Problem 10-3

Find $Z_{EQ}(s)$ in Figure P10-3. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros for R = 2 kΩ, L = 0.4 H, and C = 100 nF.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R L C Zeq ZL ZC
% The inductor and capacitor are in series
% The result is in parallel with the resistor
ZL = s*L;
ZC = 1/s/C;
Zeq = simplify(1/(1/(ZL+ZC) + 1/R))
% Substitute numerical values
R = 2e3;
L = 0.4;
C = 100e-9;
Zeq_num = simplify(subs(Zeq))
Zeq =
(s^2*L*C+1)*R/(R*s*C+s^2*L*C+1)
Zeq_num =
2000*(s^2+25000000)/(5000*s+s^2+25000000)

% Find the poles and zeros
p = roots([1 R/L 1/L/C])
z = roots([1 0 1/L/C])
p =
    -2.5000e+003 +  4.3301e+003i
    -2.5000e+003 -  4.3301e+003i
z =
     0.0000e+000 +  5.0000e+003i
     0.0000e+000 -  5.0000e+003i
```

Answers:

$$Z_{EQ}(s) = \frac{R(LCs^2+1)}{LCs^2+RCs+1} = \frac{R(s^2+1/LC)}{s^2+Rs/L+1/LC} = \frac{2000(s^2+25000000)}{s^2+5000s+25000000};$$

The poles are located at $s = -2500 \pm j4330$ rad/s and the zeros are located at $s = \pm j5000$ rad/s.

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Problem 10-4

Find $Z_{EQ}(s)$ in Figure P10-4. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R L ZL Zeq
ZL = s*L;
% The inductor and the R resistor are in parallel and the
% combination is in series with the 2R resistor
Zeq = simplify(2*R + 1/(1/ZL + 1/R))
Zeq =
R*(2*R+3*s*L)/(R+s*L)
```

Answers:

$$Z_{EQ}(s) = \frac{R(3Ls + 2R)}{Ls + R}$$

The pole is located at $s = -R/L$ and the zero is located at $s = -2R/3L$.

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Problem 10-5

Find $Z_{EQ}(s)$ in Figure P10-5. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R L ZL Zeq
ZL = s*2*L;
% The inductor and the resistor are in parallel and the
% combination is in series with the other inductor
Zeq = simplify(ZL + 1/(1/ZL + 1/R))
Zeq =
4*s*L*(R+s*L)/(R+2*s*L)
```

Answers:

$$Z_{EQ}(s) = \frac{4Ls(Ls+R)}{2Ls+R}$$

The pole is locate at $s = -R/2L$ and the zeros are located at $s = 0$ and $s = -R/L$.

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Problem 10-6

Find the poles and zeros of Z_{EQ} in Figure P10-6 with R = 1 kΩ, L = 2 H, and C = 500 nF.

Solution:

The resistor is in parallel with the capacitor and the result is in series with the inductor. The solution is presented in the following MATLAB code.

```
clear all
syms s
R = 1e3;
L = 2;
C = 500e-9;
ZL = s*L;
ZC = 1/s/C;
Zeq = ZL + 1/(1/R + 1/ZC)
Zeq_num = vpa(Zeq,6)
Zeq_num2 = simplify(Zeq_num)
Zeq =
2*s+1/(1/1000+4722366482869645/9444732965739290427392*s)
Zeq_num =
2.*s+1/(.100000e-2+.500000e-6*s)
Zeq_num2 =
2.*(2000.*s+s^2+1000000.)/(2000.+s)

% Calculate the poles and zeros
p = roots([1 2000])
z = roots([1 2000 1000000])
p =
       -2000
z =
  1.0e+003 *
   -1.0000
   -1.0000
```

Answer:

The pole is locate at s = −2000 and the two zeros are both located at s = −1000.

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Problem 10-7

Find $Z_{EQ1}(s)$ and $Z_{EQ2}(s)$ in Figure P10-7. Express each impedance as a rational function and locate its poles and zeros.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R C
R1 = R;
R2 = 2*R;
C1 = C;
C2 = C/2;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
% Computer Zeq1
% The 2R resistor is in series with the C/2 capacitor
% That result is in parallel with the C capacitor
% That result is in series with the R resistor
Zeq1 = simplify(R1 + 1/(1/ZC1 + 1/(R2+ZC2)))
% Compute Zeq2
% The 2R resistor is in series with the C capacitor
% That result is in parallel with the C/2 capacitor
% That result is in series with the R resistor
Zeq2 = simplify(R1 + 1/(1/ZC2 + 1/(R2+ZC1)))
Zeq1 =
(2*R^2*s^2*C^2+5*R*s*C+2)/s/C/(2*R*s*C+3)
Zeq2 =
(2*R^2*s^2*C^2+7*R*s*C+2)/s/C/(2*R*s*C+3)

% We can find the poles by inspection
% Find the zeros for the two impedances
Eqn1 = 2*R^2*s^2*C^2+5*R*s*C+2;
Eqn2 = 2*R^2*s^2*C^2+7*R*s*C+2;
z1 = solve(Eqn1,s)
z2 = solve(Eqn2,s)
z1 =
   -2/R/C
 -1/2/R/C
z2 =
 1/4*(-7+33^(1/2))/R/C
 -1/4*(7+33^(1/2))/R/C
```

Answers:

$$Z_{EQ1}(s) = \frac{2R^2C^2s^2 + 5RCs + 2}{Cs\,(2RCs + 3)}$$

Poles are at $s = 0$ and $s = -3/2RC$.

Zeros are at $s = -2/RC$ and $s = -1/2RC$.

$$Z_{EQ2}(s) = \frac{2R^2C^2s^2 + 7RCs + 2}{Cs\,(2RCs + 3)}$$

Poles are at $s = 0$ and $s = -3/2RC$.

Zeros are at $s = \dfrac{-7 \pm \sqrt{33}}{4RC}$.

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Problem 10-8

Find $Z_{EQ}(s)$ in Figure P10-8. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R C
C1 = C;
C2 = 2*C;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Zeq = simplify(ZC2 + 1/(1/(2*R) + 1/(2*R+ZC1)))
Zeq =
1/2*(8*R*s*C+1+8*R^2*s^2*C^2)/s/C/(4*R*s*C+1)

% Calculate the zeros
Eqn1 = 8*R*s*C+1+8*R^2*s^2*C^2;
Z = solve(Eqn1,s)
Z =
 1/4*(-2+2^(1/2))/R/C
 -1/4*(2+2^(1/2))/R/C
```

Answers:

$$Z_{EQ}(s) = \frac{8R^2C^2s^2 + 8RCs + 1}{2Cs\,(4RCs + 1)}$$

The poles are located at $s = 0$ and $s = -1/4RC$.

The zeros are located at $s = \dfrac{-2 \pm \sqrt{2}}{4RC}$.

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Problem 10-9

Find $Z_{EQ1}(s)$ and $Z_{EQ2}(s)$ in Figure P10-9. Express each impedance as a rational function and locate its poles and zeros.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R L
R1 = R;
R2 = 2*R;
L1 = L;
L2 = 2*L;
ZL1 = s*L1;
ZL2 = s*L2;
% Compute Zeq1
Zeq1 = simplify(ZL1+1/(1/R1+1/(R2+ZL2)))
% Computer Zeq2
Zeq2 = simplify(1/(1/(R1+R2)+1/ZL2))
Zeq1 =
(5*s*L*R+2*s^2*L^2+2*R^2)/(3*R+2*s*L)
Zeq2 =
6*s*L*R/(3*R+2*s*L)

% Compute the zeros for Zeq1
Eqn1 = 5*s*L*R+2*s^2*L^2+2*R^2;
z1 = solve(Eqn1,s)
z1 =
   -2*R/L
 -1/2*R/L
```

Answers:

$$Z_{EQ1}(s) = \frac{2L^2s^2 + 5RLs + 2R^2}{2Ls + 3R}$$

The pole is located at $s = -3R/2L$.

The zeros are located at $s = -2R/L$ and $s = -R/2L$.

$$Z_{EQ2}(s) = \frac{6RLs}{2Ls + 3R}$$

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The pole is located at $s = -3R/2L$.
The zero is located at $s = 0$.

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Problem 10-10

Find the equivalent impedance between terminals 1 and 2 in Figure P10-10. Select values of R and L so that $Z_{EQ}(s)$ has a pole at $s = !2000$ rad/s. Locate the zeros of $Z_{EQ}(s)$ for your choice of R and L.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R L
R1 = R;
R2 = 2*R;
L1 = L;
L2 = 2*L;
ZL1 = s*L1;
ZL2 = s*L2;
Zeq = simplify(ZL2 + 1/(1/R1+1/ZL1) + R2)
Zeq =
(5*s*L*R+2*s^2*L^2+2*R^2)/(R+s*L)
```

The poles is located at $s = -R/L$. Choose $L = 1$ H and $R = 2$ kΩ to place the pole at $s = !2000$ rad/s. Calculate the zeros.

```
% Find the zeros
Eqn1 = 5*s*L*R+2*s^2*L^2+2*R^2;
z = solve(Eqn1,s)
z_num = subs(z,{R,L},{2e3,1})
z =
   -2*R/L
 -1/2*R/L
z_num =
       -4000
       -1000
```

Answers:

$$Z_{EQ}(s) = \frac{2L^2s^2 + 5RLs + 2R^2}{Ls + R}$$

With a pole located at $s = !2000$ rad/s, the zeros are located at $s = -1000$ rad/s and $s = -4000$ rad/s.

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Problem 10-11

The switch in Figure P10-11 has been in position A for a long time and is moved to position B at $t = 0$. Transform the circuit into the s domain and solve for $I_L(s)$ and $i_L(t)$ in symbolic form.

Solution:

Since the switch has been in position A for a long time, the inductor acts like a short circuit and its initial current is $i_L(0) = V_A/R$. Convert the circuit into the s domain after the switch moves. The two resistors do not change. The inductor is replaced by an impedance of Ls in series with a voltage source of magnitude $Li_L(0) = LV_A/R$, with the positive terminal to the right if the source is in the top branch of the circuit. We can find the current in the series circuit by dividing the magnitude of the voltage source by the total impedance in the circuit. The following MATLAB code completes the calculations.

```
clear all
format short eng
syms s t L R VA
% Compute the initial current and corresponding voltage source
iL0 = VA/R;
Vs = L*iL0;
% Compute the equivalent impedance
Zeq = s*L + R + R;
% Find the current in the Laplace domain
Is = simplify(Vs/Zeq)
% Find the current in the time domain
it = ilaplace(Is)
Is =
L*VA/R/(2*R+s*L)
it =
VA/R*exp(-2*R*t/L)
```

Answers:

$$I_L(s) = \frac{V_A}{R}\frac{1}{s + 2R/L}$$

$$i_L(t) = \frac{V_A}{R}\left[e^{-2Rt/L}\right]u(t)$$

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Problem 10-12

The switch in Figure P10-11 has been in position B for a long time and is moved to position A at $t = 0$. Transform the circuit into the s domain and solve for $I_L(s)$ and $i_L(t)$ in symbolic form.

Solution:

Since the switch has been in position B for a long time, the initial current through the inductor is 0 A after the switch moves. If we transform the circuit into the s domain after the switch moves, the resistor does not change, the inductor is replaced by its impedance Ls with no voltage source in series, and the dc voltage source is replaced with V_A/s, since it appears to the rest of the circuit as a scaled step function in the time domain. We can now find the current by dividing the voltage source by the equivalent impedance in the circuit. The following MATLAB code completes the solution.

```
clear all
syms s t L R VA
% Convert the source into the Laplace domain
Vs = VA/s;
% Compute the equivalent impedance
Zeq = s*L + R;
% Find the current in the Laplace domain
Is = simplify(Vs/Zeq)
% Find the current in the time domain
it = ilaplace(Is)
Is =
VA/s/(R+s*L)
it =
VA/R*(1-exp(-R*t/L))
```

Answers:

$$I_L(s) = \frac{V_A}{s\,(Ls+R)} = \frac{V_A}{L}\frac{1}{s\,(s+R/L)}$$

$$i_L(t) = \frac{V_A}{R}\left[1-e^{-Rt/L}\right]u(t)$$

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Problem 10-13

The switch in Figure P10-13 has been in position A for a long time and is moved to position B at $t = 0$. Transform the circuit into the s domain and solve for $I_C(s)$ and $i_C(t)$ in symbolic form.

Solution:

Before the switch moves, the voltage across the capacitor is $V_A/2$, because the capacitor is in parallel with one of the two resistors. The capacitor acts like an open circuit, so we can use voltage division to find its initial voltage. When we convert the circuit into the s domain the resistors do not change, the voltage source is replaced with V_A/s, and the capacitor is replaced with an impedance of $1/Cs$ in series with a voltage source of magnitude $v_C(0)/s$ with its positive terminal at the top. We can use node-voltage analysis to find the voltage across the capacitor and then the current through the capacitor as shown in the following MATLAB code.

```
clear all
syms s t R C VA VCs ICs iCt
% Use voltage division to find the initial capacitor voltage
vC0 = VA/2;
VC0s = vC0/s;
% Write a node-voltage equation for the unknown node voltage VCs
ZC = 1/s/C;
% Solve for the unknown voltage
Eqn1 = (VCs+VA/s)/R + VCs/R + (VCs-VC0s)/ZC;
VCs = solve(Eqn1,VCs)
%vct = ilaplace(VCs)
% Compute the capacitor current
ICs = simplify((VCs-VC0s)/ZC)
% Convert to the time domain
iCt = ilaplace(ICs)
VCs =
1/2*VA*(-2+R*s*C)/s/(R*s*C+2)
ICs =
-2*VA*C/(R*s*C+2)
iCt =
-2*VA/R*exp(-2*t/R/C)
```

Answers:

$$I_C(s) = \frac{-2CV_A}{RCs+2}$$

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$$i_{\mathrm{C}}(t) = \left[\frac{-2V_{\mathrm{A}}}{R} e^{-2t/RC} \right] u(t)$$

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Problem 10-14

The switch in Figure P10-13 has been in position B for a long time and is moved to position A at $t = 0$. Transform the circuit into the s domain and solve for $V_C(s)$ and $v_C(t)$ in symbolic form.

Solution:

Before the switch moves, the voltage across the capacitor is $-V_A/2$, because the capacitor is in parallel with one of the two resistors. The capacitor acts like an open circuit, so we can use voltage division to find its initial voltage. When we convert the circuit into the s domain the resistors do not change, the voltage source is replaced with $-V_A/s$, and the capacitor is replaced with an impedance of $1/Cs$ in series with a voltage source of magnitude $v_C(0)/s = V_A/2s$, with its positive terminal at the bottom. We can use node-voltage analysis to find the voltage across the capacitor as shown in the following MATLAB code.

```
clear all
syms s t R C VA VCs
% Use voltage division to find the initial capacitor voltage
vC0 = VA/2;
VC0s = vC0/s;
% Write a node-voltage equation for the unknown node voltage VCs
ZC = 1/s/C;
% Solve for the unknown voltage
Eqn1 = (VCs-VA/s)/R + VCs/R + (VCs+VC0s)/ZC;
VCs = solve(Eqn1,VCs)
vCt = ilaplace(VCs)
VCs =
-1/2*VA*(-2+R*s*C)/s/(R*s*C+2)
vCt =
1/2*VA*(1-2*exp(-2*t/R/C))
```

Answers:

$$V_C(s) = \frac{V_A\,(2 - RCs)}{2s\,(RCs + 2)}$$

$$v_C(t) = V_A\left[\frac{1}{2} - e^{-2t/RC}\right]u(t)$$

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Problem 10-15

Transform the circuit in Figure P10-15 into the s domain and find: $I_L(s)$, $i_L(t)$, $V_L(s)$, and $v_L(t)$ when $v_1(t) = V_A\,u(t)$ and $i_L(0) = I_A$.

Solution:

When we convert the circuit into the s domain, the resistors do not change, the voltage source is replaced by V_A/s, and the inductor is replaced by an impedance Ls in series with a voltage source of magnitude LI_A, with the positive terminal at the bottom. Use node-voltage analysis to find the inductor voltage and then solve for the inductor current as shown in the following MATLAB code.

```
clear all
syms s t R L VA IA VLs vLt ILs iLt
ZL = s*L;
% Write the node-voltage equation
Eqn1 = (VLs-VA/s)/R + VLs/R + (VLs+L*IA)/ZL;
VLs = solve(Eqn1,VLs)
vLt = ilaplace(VLs)
% Find the inductor current
ILs = simplify((VLs+L*IA)/ZL)
iLt = ilaplace(ILs)
Check_vLt = L*diff(iLt,t)
VLs =
-(-VA+IA*R)/(R+2*s*L)*L
vLt =
1/2*exp(-1/2*R*t/L)*(VA-IA*R)
ILs =
(VA+2*IA*s*L)/(R+2*s*L)/s
iLt =
VA/R+exp(-1/2*R*t/L)*(-VA+IA*R)/R
Check_vLt =
-1/2*exp(-1/2*R*t/L)*(-VA+IA*R)
```

Answers:

$$I_L(s) = \left(\frac{V_A}{R} - I_A\right)\frac{R/2L}{s(s+R/2L)} + \frac{I_A}{s} = \frac{2LI_A s + V_A}{s\,(2Ls+R)}$$

$$i_L(t) = \left[\frac{V_A}{R} + \left(I_A - \frac{V_A}{R}\right)e^{-Rt/2L}\right]u(t)$$

$$V_L(s) = \left(\frac{V_A}{R} - I_A\right)\frac{R/2}{s+R/2L}$$

$$v_L(t) = \left[\frac{R}{2}\left(\frac{V_A}{R} - I_A\right)e^{-Rt/2L}\right]u(t)$$

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Problem 10-16

Transform the circuit in Figure P10-15 into the s domain and find $I_L(s)$, $i_L(t)$, $V_L(s)$, and $v_L(t)$ when $v_1(t) = V_A\, e^{-\alpha t}\, u(t)$ and $i_L(0) = I_A$.

Solution:

Use the same approach presented in Problem 10-15 with the new voltage source.

```
clear all
syms s t R L VA IA a V1s VLs vLt ILs iLt
ZL = s*L;
V1s = laplace(VA*exp(-a*t));
% Write the node-voltage equation
Eqn1 = (VLs-V1s)/R + VLs/R + (VLs+L*IA)/ZL;
VLs = solve(Eqn1,VLs)
vLt = ilaplace(VLs)
% Find the inductor current
ILs = simplify((VLs+L*IA)/ZL)
iLt = ilaplace(ILs)
Check_vLt = simplify(L*diff(iLt,t))
VLs =
-(-VA*s+IA*R*s+IA*R*a)/(s+a)/(R+2*s*L)*L
vLt =
VA*a/(-R+2*a*L)*exp(-a*t)*L+1/2*(-VA+IA*R-2*IA*a*L)*exp(-1/2*R*t/L)/(-
R+2*a*L)*R
ILs =
(VA+2*IA*s*L+2*IA*a*L)/(s+a)/(R+2*s*L)
iLt =
exp(-a*t)/(R-2*a*L)*VA+(VA-IA*R+2*IA*a*L)/(-R+2*a*L)*exp(-1/2*R*t/L)
Check_vLt =
-1/2*(-2*a*exp(-a*t)*VA*L+R*exp(-1/2*R*t/L)*VA-R^2*exp(-
1/2*R*t/L)*IA+2*R*exp(-1/2*R*t/L)*IA*a*L)/(-R+2*a*L)
```

Answers:

$$I_L(s) = \frac{2I_A Ls + V_A + 2I_A \alpha L}{(s+\alpha)(2Ls+R)}$$

$$i_L(t) = \left[\frac{V_A}{R-2\alpha L} e^{-\alpha t} + \frac{V_A - RI_A + 2I_A \alpha L}{2\alpha L - R} e^{-Rt/2L}\right] u(t)$$

$$V_L(s) = \frac{L(V_A s - RI_A s - RI_A \alpha)}{(s+\alpha)(2Ls+R)}$$

$$v_L(t) = \left[\frac{\alpha L V_A}{2\alpha L - R} e^{-\alpha t} + \frac{1}{2}\frac{R(RI_A - V_A - 2I_A \alpha L)}{2\alpha L - R} e^{-Rt/2L}\right] u(t)$$

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Problem 10-17

The switch in Figure P10-17 has been in position A for a long time and is moved to position B at $t = 0$.

(a) Transform the circuit into the s domain and solve for $I_L(s)$ in symbolic form.

(b) Repeat (a) using MATLAB.

(c) Find $i_L(t)$ for $R_1 = R_2 = 500\ \Omega$, $R_3 = 1\ \text{k}\Omega$, $L = 250$ mH, $C = 4\ \mu$F, and $V_A = 15$ V.

Solution:

(a) Since the switch has been in position A for a long time, the initial inductor current is zero and the initial capacitor voltage matches the source voltage. If we transform the circuit into the s domain, the two resistors do not change, the inductor is replaced by its impedance Ls, and the capacitor is replaced by it impedance $1/Cs$ in series with a voltage source of magnitude $v_C(0)/s = V_A/s$, with the positive terminal at the top. We can find $I_L(s)$ by dividing the voltage source by the equivalent impedance in the series circuit (after the switch moves) and accounting for the direction of the current flow.

$$I_L(s) = \frac{-v_C(0)/s}{Z_{EQ}} = \frac{-V_A}{s}\frac{1}{Z_{EQ}} = \frac{-V_A}{s\,(R_2 + R_3 + Ls + 1/Cs)} = \frac{-V_A C}{LCs^2 + (R_2 + R_3)Cs + 1}$$

(b) Now solve the problem using MATLAB.

```
clear all
syms s t R1 R2 R3 L C VA
vC0 = VA/s;
ZL = s*L;
ZC = 1/s/C;
Zeq = R2 + R3 + ZL + ZC
ILs = simplify(-vC0/Zeq)
Zeq =
R2+R3+s*L+1/s/C
ILs =
-VA*C/(R2*s*C+R3*s*C+s^2*L*C+1)
```

(c) We can use MATLAB to perform the substitutions and take the inverse Laplace transform.

```
R1 = 500;
R2 = 500;
R3 = 1e3;
L = 250e-3;
C = 4e-6;
VA = 15;
ILs_num = subs(ILs);
iLt = vpa(ilaplace(ILs_num),6)
iLt =
-.212132e-1*exp(-3000.*t)*sinh(2828.44*t)
```

Answers:

(a) $I_L(s) = \dfrac{-V_A C}{LCs^2 + (R_2 + R_3)Cs + 1}$

(b) The MATLAB solution presented above matches the answer in Part (a).

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(c) $i_{\mathrm{L}}(t) = 10.61\left[e^{-5828.5\,t} - e^{-171.6\,t}\right]u(t)$ mA

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Problem 10-18

The switch in Figure P10-17 has been in position B for a long time and is moved to position A at $t = 0$.

(a) Transform the circuit into the s domain and solve for $V_C(s)$ in symbolic form.

(b) Find $v_C(t)$ for $R_1 = R_2 = 250\ \Omega$, $R_3 = 1\ \text{k}\Omega$, $L = 500$ mH, $C = 1\ \mu\text{F}$, and $V_A = 15$ V.

(c) Repeat (b) using OrCAD.

Solution:

(a) With the switch in position B for a long time, the initial inductor current and capacitor voltage are both zero. If we transform the circuit into the s domain, the voltage source becomes V_A/s, the resistors do not change, the inductor is replaced by its impedance Ls, and the capacitor is replaced by it impedance $1/Cs$. We can then use voltage division to find the capacitor voltage. The following MATLAB code completes the calculations.

```
clear all
syms s t R1 R2 R3 L C VA
Vs = VA/s;
ZL = s*L;
ZC = 1/s/C;
Zeq = R1 + R2 + ZL + ZC;
VCs = simplify(ZC*Vs/Zeq)
VCs =
VA/s/(R1*s*C+R2*s*C+s^2*L*C+1)
```

(b) We can use MATLAB to perform the substitutions and take the inverse Laplace transform.

```
R1 = 250;
R2 = 250;
R3 = 1e3;
L = 500e-3;
C = 1e-6;
VA = 15;
VCs = subs(VCs);
vCt = ilaplace(VCs)
vCt =
15-15/7*exp(-
500*t)*(7*cos(500*7^(1/2)*t)
+7^(1/2)*sin(500*7^(1/2)*t))

% Plot the voltage
tt = 0:20e-6:12e-3;
vCtt = subs(vCt,t,tt);
plot(tt,vCtt,'g','LineWidth'
,3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

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(c) The following OrCAD simulation verifies the results found above.

Answers:

(a) $V_C(s) = \dfrac{V_A}{s\left(LCs^2 + (R_1 + R_2)Cs + 1\right)}$

(b) $v_C(t) = \left\{15 - \dfrac{15}{7}e^{-500t}\left[7\cos\left(500\sqrt{7}t\right) + \sqrt{7}\sin\left(500\sqrt{7}t\right)\right]\right\}u(t)$

(c) The OrCAD simulation is presented above and verifies the results.

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Problem 10-19

The circuit in Figure P10-19 is in the zero state. The *s*-domain relationship between the input $I_1(s)$ and the output $I_R(s)$ is usually given as a ratio called a *network function*. Find $I_R(s)/ I_1(s)$.

Solution:

Convert the circuit into the *s* domain by replacing the capacitor and inductor with their impedances. Use current division to find the resistor current $I_R(s)$ in terms of the source current $I_1(s)$ and then divide to find the network function.

```
clear all
syms s R L C I1s IRs Ts
ZC = 1/s/C;
ZL = s*L;
IRs = I1s/R/(1/R + 1/(ZC+ZL));
Ts = simplify(IRs/I1s)
Ts =
(1+s^2*L*C)/(1+s^2*L*C+R*s*C)
```

Answer:

$$\frac{I_R(s)}{I_1(s)} = \frac{LCs^2 + 1}{LCs^2 + RCs + 1}$$

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Problem 10-20

The circuit in Figure P10-20 is in the zero state. Find the *s*-domain relationship between the input $I_1(s)$ and the output $V_O(s)$.

Solution:

Replace the capacitors with their impedances. Combine all of the impedances to find the equivalent impedance seen by the current source. Compute the output voltage and find the ratio of the output voltage to the input current in the *s* domain.

```
clear all
syms s R C VOs I1s Ts
R1 = 2*R;
R2 = R;
C1 = C;
C2 = C/2;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
% Combine the series paths together
Zeq1 = R1+ZC1;
Zeq2 = R2+ZC2;
% Find the overall equivalent resistance
Zeq = 1/(1/Zeq1+1/Zeq2);
VOs = I1s*Zeq;
Ts = simplify(VOs/I1s)
Ts =
1/3*(2*R*s*C+1)*(R*s*C+2)/s/C/(R*s*C+1)
```

Answer:

$$\frac{V_O(s)}{I_1(s)} = \frac{(2RCs+1)(RCs+2)}{3Cs\,(RCs+1)}$$

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Problem 10-21

The initial conditions in Figure P10-21 are $v_C(0) = 0$ and $i_L(0) = I_0$. Transform the circuit into the s domain and use superposition and voltage division to find the zero-state and zero-input components of $V_C(s)$.

Solution:

Replace the inductor with its impedance in series with a voltage source of magnitude $Li_L(0) = LI_0$, with the positive terminal at the bottom. Replace the capacitor with its impedance. Use MATLAB to complete the calculations.

```
clear all
syms s R L C Vs I0
ZL = s*L;
ZC = 1/s/C;
% Use the voltage source to find the zero-state component.
% Combine the inductor and capacitor in parallel
% and then use voltage division
ZLC = 1/(1/ZL+1/ZC);
VCzs = simplify(Vs*ZLC/(ZLC+R))
% Use the source associated with the initial conditions
% to find the zero-input component
% Combine the resistor and capacitor in parallel and
% then use voltage division
% Note the sign change on the source
ZRC = 1/(1/R+1/ZC);
VCzi = simplify(-L*I0*ZRC/(ZRC+ZL))
VCzs =
L*s*Vs/(s*L+R+R*s^2*L*C)
VCzi =
-R*L*I0/(s*L+R+R*s^2*L*C)
```

Answers:

$$V_{\text{Czs}}(s) = \frac{Ls\,V_{\text{S}}(s)}{RLCs^2 + Ls + R}$$

$$V_{\text{Czi}}(s) = \frac{-RL\,I_0}{RLCs^2 + Ls + R}$$

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Problem 10-22

(a) Transform the circuit in Figure P10-21 into the *s* domain and use superposition and voltage division to find the zero-state and zero-input components of $V_R(s)$. The initial conditions are $v_C(0) = 0$ and $i_L(0) = I_0$.

(b) Solve for $v_R(t)$ using OrCAD if $R = 1.5$ kΩ, $L = 500$ mH, $C = 1$ μF, $v_S(t) = 15\,u(t)$ V, $i_L(0) = 10$ mA, and $v_C(0) = 0$.

Solution:

(a) Replace the inductor with its impedance in series with a voltage source of magnitude $Li_L(0) = LI_0$, with the positive terminal at the bottom. Replace the capacitor with its impedance. Use MATLAB to complete the calculations.

```
clear all
syms s t R L C Vs I0
ZL = s*L;
ZC = 1/s/C;
% Use the voltage source to find the zero-state component.
% Combine the inductor and capacitor in parallel
% and then use voltage division
ZLC = 1/(1/ZL+1/ZC);
VRzs = simplify(Vs*R/(ZLC+R))
% Use the source associated with the initial conditions
% to find the zero-input component
% Combine the resistor and capacitor in parallel and
% then use voltage division
ZRC = 1/(1/R+1/ZC);
VRzi = simplify(L*I0*ZRC/(ZRC+ZL))
VRzs =
R*Vs*(1+s^2*L*C)/(s*L+R+R*s^2*L*C)
VRzi =
R*L*I0/(s*L+R+R*s^2*L*C)
```

(b) First, find the solution using MATLAB for the numerical values.

```
% Combine the components from superposition
VR = VRzs+VRzi;
% Substitute the given values
R = 1.5e3;
L = 500e-3;
C = 1e-6;
Vs = laplace(15*heaviside(t));
I0 = 10e-3;
VR = simplify(subs(VR));
VRzs = subs(VRzs);
VRzi = subs(VRzi);
% Take the inverse Laplace transform
vR = ilaplace(VR)
vRzs = ilaplace(VRzs);
vRzi = ilaplace(VRzi);
% Plot the resistor voltage
```

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```
% Also plot the zero-state and zero-input responses
tt = 0:50e-6:20e-3;
vRtt = subs(vR,t,tt);
vRzstt = subs(vRzs,t,tt);
vRzitt = subs(vRzi,t,tt);
close all
plot(tt,vRtt,'g','LineWidth',3)
grid on
hold on
plot(tt,vRzstt,'b','LineWidth',2.5)
plot(tt,vRzitt,'r','LineWidth',2.5)
legend('Total','Zero-State','Zero-
Input');
xlabel('Time (s)')
ylabel('Voltage (V)')
vR =
15+50085/125437859701224950988 8*exp(-
1000/3*t)*(-
17*cos(1000/3*t*17^(1/2))+17^(1/2)*sin
(1000/3*t*17^(1/2)))
```

The time varying portion of the zero-state response exactly cancels the time-varying portion of the zero-input response. The final result is a dc signal for the resistor voltage. Now simulate the circuit in OrCAD.

Answer:

(a) $V_{\text{Rzs}}(s) = \dfrac{R\left(LCs^2+1\right)V_S(s)}{RLCs^2+Ls+R}$

$V_{\text{Rzi}}(s) = \dfrac{RL\,I_0}{RLCs^2+Ls+R}$

(b) The complete response is a dc voltage with $v_R(t)$ = 15 V.

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Problem 10-23

There is no energy stored in the capacitor in Figure P10-23 at $t = 0$. Transform the circuit into the s domain and use current division to find $v_O(t)$ when the input is $i_S(t) = 2.5\, e^{!200t} u(t)$ mA. Identify the forced and natural poles in $V_O(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t it Is vOt VOs
R1 = 10e3;
R2 = 10e3;
R3 = 5e3;
C = 100e-9;
ZC = 1/s/C;
it = 2.5e-3*exp(-200*t);
Is = laplace(it);
Z1 = R1+ZC;
Z2 = R2+R3;
% Use current division to find IOs
IOs = Is/Z2/(1/Z1+1/Z2);
VOs = R3*IOs;
vOt = simplify(ilaplace(VOs))
vOt =
-15*exp(-400*t)+20*exp(-200*t)
```

Answers:

$v_O(t) = \left[20e^{-200\,t} - 15e^{-400\,t}\right]u(t)$ V

The forced pole is located at $s = -200$ rad/s and the natural pole is located at $s = -400$ rad/s.

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Problem 10-24

Repeat Problem 10-23 when $i_S(t) = 2.5\, e^{!400t} u(t)$ mA.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t it Is vOt VOs
R1 = 10e3;
R2 = 10e3;
R3 = 5e3;
C = 100e-9;
ZC = 1/s/C;
it = 2.5e-3*exp(-400*t);
Is = laplace(it);
Z1 = R1+ZC;
Z2 = R2+R3;
% Use current division to find IOs
IOs = Is/Z2/(1/Z1+1/Z2);
VOs = simplify(R3*IOs)
vOt = simplify(ilaplace(VOs))
VOs =
5*(s+1000)/(s+400)^2
vOt =
5*exp(-400*t)*(1+600*t)
```

Answers:

$v_O(t) = \left[5(1+600t)e^{-400\,t}\right]u(t)$ V

The forced pole and the natural pole are both located at $s = -400$ rad/s.

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Problem 10-25

The circuit in Figure P10-25 is in the zero state. Use a Thévenin equivalent to find the *s*-domain relationship between the input $I_S(s)$ and the interface current $I(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R C Iss Is
ZC = 1/s/C;
% Find the Thevenin equivalent source in the s domain
% The open-circuit voltage is the voltage across the capacitor
VTs = Iss*ZC
% The Thevenin resistance is the series combination of
% the resistor and the capacitor
ZT = ZC+R
% Find the interface current
Is = VTs/(ZT+2*R)
% Find the ratio of I(s)/I_s(s)
Ts = simplify(Is/Iss)
VTs =
Iss/s/C
ZT =
1/s/C+R
Is =
Iss/s/C/(1/s/C+3*R)
Ts =
1/(1+3*R*s*C)
```

Answer:

$$\frac{I(s)}{I_S(s)} = \frac{1}{1+3RCs}$$

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Problem 10-26

Find the Thévenin equivalent circuit that the 2*R* load resistor sees in the circuit of Figure 10-25 when $v_C(0) = V_0$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s R C Iss V0
ZC = 1/s/C;
% Find the Thevenin equivalent source in the s domain
% The open-circuit voltage is the voltage across the capacitor
% Since there is an initial capacitor voltage, we can use
% superposition to find the Thevenin equivalent source
% Using the voltage source
VT1 = Iss*ZC;
% Using the capacitor initial condition
VT2 = V0/s;
% Combine the voltages
VTs = simplify(VT1+VT2)
% The Thevenin resistance is the series combination of
% the resistor and the capacitor
ZT = simplify(ZC+R)
VTs =
(Iss+V0*C)/s/C
ZT =
(1+R*s*C)/s/C
```

Answers:

$$V_T(s) = \frac{I_S(s) + CV_0}{Cs}$$

$$Z_T(s) = \frac{RCs + 1}{Cs}$$

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Problem 10-27

The circuit in Figure P10-27 is in the zero state. Find the Thévenin equivalent to the left of the interface.

Solution:

Use voltage division to find the open-circuit voltage and the look-back method to find the Thévenin impedance.

```
clear all
syms s t VT ZT
Vs = laplace(100*heaviside(t));
R1 = 2e3;
R2 = 2e3;
C = 1e-6;
ZC = 1/s/C;
% Use voltage division
VT = simplify(R2*Vs/(R1+ZC+R2))
% Use the look-back method
ZT = simplify(1/(1/(R1+ZC) + 1/R2))
VT =
50/(s+250)
ZT =
1000*(s+500)/(s+250)
```

Answers:

$$V_{\mathrm{T}}(s) = \frac{50}{s+250} \text{ V-s}$$

$$Z_{\mathrm{T}}(s) = \frac{1000(s+500)}{s+250}\ \Omega$$

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Problem 10-28

There is no initial energy stored in the circuit in Figure P10-28. Transform the circuit into the *s* domain and use superposition to find $V(s)$. Identify the forced and natural poles in $V(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t R L VA IB B
% Transform into the s domain
ZL = s*L;
Vs = laplace(VA*sin(B*t));
Is = laplace(IB*heaviside(t));
% Find the response from the voltage source
Zeq = 1/(1/ZL+1/R);
V1 = Zeq*Vs/(Zeq+R);
% Find the response from the current source
I2 = Is/ZL/(1/R+1/ZL+1/R);
V2 = ZL*I2;
% Combine the components
V = simplify(V1+V2)
V =
(s*VA*B+IB*R*s^2+IB*R*B^2)*L/(R+2*s*L)/(s^2+B^2)
```

Answer:

$$V(s) = \frac{L\left(RI_{\mathrm{B}}s^2 + \beta V_{\mathrm{A}}s + \beta^2 RI_{\mathrm{B}}\right)}{(2Ls+R)\left(s^2+\beta^2\right)}$$

The natural pole is located at $s = -R/2L$. The forced poles are located at $s = \pm j\beta$.

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Problem 10-29

The equivalent impedance between a pair of terminals is

$$Z_{EQ}(s) = 1000\left[\frac{s+2000}{s+1000}\right]$$

A voltage $v(t) = 10u(t)$ is applied across the terminals. Find the resulting current response $i(t)$.

Solution:

Convert the voltage into the s domain, solve for the current, and then convert back into the time domain.

```
clear all
syms s t
vt = 10*heaviside(t);
Vs = laplace(vt);
Zeq = 1000*(s+2000)/(s+1000);
Is = Vs/Zeq;
it = simplify(ilaplace(Is))
it =
1/200+1/200*exp(-2000*t)
```

Answer:

$i(t) = 5\left[1 + e^{-2000\,t}\right]u(t)$ mA

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Problem 10-30

There is no initial energy stored in the circuit in Figure P10-30. Use circuit reduction to find the output network function $V_2(s)/V_1(s)$. Then select values of R and C so that the poles of the network function are approximately −2618 and −382 rad/s.

Solution:

To use circuit reduction in this case, we will perform a series of source transformations until we can use voltage division to find the requested ratio.

```
clear all
syms s V1 V2 R C
ZC = 1/s/C;
% Transform the voltage source into a current source
Ia = V1/ZC;
% Combine the now parallel impedances
Za = 1/(1/ZC+1/R);
% Transform back to a voltage
Vb = Ia*Za;
% Combine the series impedances
Zb = Za+R;
% Use voltage division to find V2
V2 = ZC*Vb/(ZC+Zb);
% Find the ratio
Ts = simplify(V2/V1)
Ts =
R*s*C/(1+3*R*s*C+R^2*s^2*C^2)

% Find the poles
Eqn1 = 1+3*R*s*C+R^2*s^2*C^2;
p = solve(Eqn1,s)
p_num = vpa(p,6)
p =
  1/2*(5^(1/2)-3)/R/C
 -1/2*(3+5^(1/2))/R/C
p_num =
 -.381965/R/C
 -2.61804/R/C
```

To create the requested poles, we need $1/RC = 1000$, so choose $R = 1$ kΩ and $C = 1$ μF.

Answer:

$$\frac{V_2(s)}{V_1(s)} = \frac{RCs}{R^2C^2s^2 + 3RCs + 1}$$

Choose $R = 1$ kΩ and $C = 1$ μF, or any other combination such that $1/RC = 1000$, to place the poles at −2618 and −382 rad/s.

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Problem 10-31

There is no initial energy stored in the circuit in Figure P10-31.

(a) Transform the circuit into the *s* domain and formulate mesh-current equations.

(b) Solve these equations for $I_2(s)$ in symbolic form.

(c) Identify the poles and zeros of $I_2(s)$.

(d) Find $i_2(t)$ for $v_1(t) = 50u(t)$ V, $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $L = 2$ H, and $C = 500$ nF.

Solution:

(a) Replace the inductor and capacitor with their impedances and replace the voltage source with its *s* domain equivalent. We can then write two mesh-current equations. Let the left mesh be Loop A and the right mesh be Loop B.

Loop A: $-V_1(s) + I_A(s)[Ls] + [I_A(s) - I_B(s)][R_1] = 0$

Loop B: $[I_B(s) - I_A(s)][R_1] + I_B(s)\left[R_2 + \dfrac{1}{Cs}\right] = 0$

(b) Use MATLAB to solve the mesh-current equations

```
clear all
syms s t R1 R2 L C V1 IA IB
Eqn1 = -V1+IA*L*s+(IA-IB)*R1;
Eqn2 = (IB-IA)*R1+IB*(R2+1/s/C);
Soln = solve(Eqn1,Eqn2,IA,IB);
IA = simplify(Soln.IA);
IB = simplify(Soln.IB);
I2 = IB
I2 =
V1*R1*s*C/(R1*s^2*C*L+R2*s^2*C*L+R2*s*C*R1+s*L+R1)
```

(c) The positions of the poles and zeros will depend on the type of input signal. If the input signal is a step function, then there are no finite zeros. The poles can be found using MATLAB.

```
% Find the poles
Eqn3 = R1*s^2*C*L+R2*s^2*C*L+R2*s*C*R1+s*L+R1;
p = simplify(solve(Eqn3,s))
p =
 -1/2*(R2*C*R1+L-(R2^2*C^2*R1^2-2*R2*C*R1*L+L^2-4*R1^2*C*L)^(1/2))/C/L/(R1+R2)
 -1/2*(R2*C*R1+L+(R2^2*C^2*R1^2-2*R2*C*R1*L+L^2-4*R1^2*C*L)^(1/2))/C/L/(R1+R2)
```

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(d) Use MATLAB to perform the substitutions and find the inverse Laplace transform.

```
R1 = 1e3;
R2 = 2e3;
L = 2;
C = 500e-9;
v1 = 50*heaviside(t);
V1 = laplace(v1);
I2 = subs(I2);
i2 = ilaplace(I2);
i2_num = vpa(i2,6)
i2_num =
.288676e-1*exp(-500.*t)*sin(288.676*t)
```

Answers:

(a) Loop A: $-V_1(s) + I_A(s)[Ls] + [I_A(s) - I_B(s)][R_1] = 0$;

Loop B: $[I_B(s) - I_A(s)][R_1] + I_B(s)\left[R_2 + \dfrac{1}{Cs}\right] = 0$;

(b) $I_2(s) = I_B(s) = \dfrac{R_1 Cs\, V_1(s)}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1}$

(c) The poles are located at $s = \left(-\dfrac{1}{2}\right)\dfrac{L + R_1R_2C \pm \sqrt{(R_1R_2C)^2 - 2R_1R_2LC + L^2 - 4R_1^2LC}}{LC(R_1 + R_2)}$.

There are no finite zeros if the source is a step function.

(d) $i_2(t) = \left[28.87e^{-500t}\sin(288.68t)\right]u(t)$ mA

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Problem 10-32

There is no initial energy stored in the circuit in Figure P10-31.

(a) Transform the circuit into the *s* domain and formulate node-voltage equations.

(b) Solve these equations for $V_2(s)$ in symbolic form.

(c) Identify the natural and forced poles of $V_2(s)$.

(d) Find $v_2(t)$ for $v_1(t) = 15u(t)$ V, $R_1 = R_2 = 500\ \Omega$, $L = 0.5$ H, and $C = 2\ \mu$F.

Solution:

(a) Replace the inductor and capacitor with their impedances and replace the voltage source with its *s* domain equivalent. We can then write two node-voltage equations. Let Node A be the node to the left of R_2 and left Node B be the node to the right of R_2.

Node A: $$\frac{V_A(s)-V_1(s)}{Ls}+\frac{V_A(s)}{R_1}+\frac{V_A(s)-V_B(s)}{R_2}=0$$

Node B: $$\frac{V_B(s)-V_A(s)}{R_2}+\frac{V_B(s)}{1/Cs}=0$$

(b) Use MATLAB to solve the equations.

```
clear all
syms s t R1 R2 L C V1 VA VB
Eqn1 = (VA-V1)/L/s + VA/R1 + (VA-VB)/R2;
Eqn2 = (VB-VA)/R2 + VB/(1/s/C);
Soln = solve(Eqn1,Eqn2,VA,VB);
VA = simplify(Soln.VA);
VB = simplify(Soln.VB);
V2 = VB
V2 =
R1*V1/(R1*s^2*C*L+R2*s^2*C*L+R2*s*C*R1+s*L+R1)
```

(c) The forced poles will depend on the form of $V_1(s)$. The natural poles are found using MATLAB.

```
% Compute the poles
Eqn1 = R1*s^2*C*L+R2*s^2*C*L+R2*s*C*R1+s*L+R1;
p = simplify(solve(Eqn1,s))
p =
 -1/2*(R2*C*R1+L-(R2^2*C^2*R1^2-2*R2*C*R1*L+L^2-4*R1^2*C*L)^(1/2))/C/L/(R1+R2)
 -1/2*(R2*C*R1+L+(R2^2*C^2*R1^2-2*R2*C*R1*L+L^2-4*R1^2*C*L)^(1/2))/C/L/(R1+R2)
```

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(d) Use MATLAB to perform the substitutions and find the inverse Laplace transform.

```
v1 = 15*heaviside(t);
V1 = laplace(v1);
R1 = 500;
R2 = 500;
L = 0.5;
C = 2e-6;
% Perform the substitution
V2 = subs(V2);
% Take the inverse Laplace transform
v2 = ilaplace(V2);
v2_num = vpa(v2,6)
v2_num =
15.-15.*exp(-500.*t)*(cos(500.*t)+sin(500.*t))
```

Answers:

(a) Node A: $\dfrac{V_A(s)-V_1(s)}{Ls}+\dfrac{V_A(s)}{R_1}+\dfrac{V_A(s)-V_B(s)}{R_2}=0$

Node B: $\dfrac{V_B(s)-V_A(s)}{R_2}+\dfrac{V_B(s)}{1/Cs}=0$

(b) $V_2(s)=\dfrac{R_1V_1(s)}{(R_1+R_2)LCs^2+(R_1R_2C+L)s+R_1}$.

(c) The natural poles are located at $s=\left(-\dfrac{1}{2}\right)\dfrac{L+R_1R_2C\pm\sqrt{(R_1R_2C)^2-2R_1R_2LC+L^2-4R_1^2LC}}{LC(R_1+R_2)}$.

The forced poles will depend on the form of $V_1(s)$.

(d) $v_2(t)=15\left\{1-e^{-500t}\left[\cos(500t)+\sin(500t)\right]\right\}u(t)$ V .

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Problem 10-33

There is no initial energy stored in the circuit in Figure P10-33.

(a) Transform the circuit into the *s* domain and formulate node-voltage equations.

(b) Solve these equations for $V_2(s)$ in symbolic form.

(c) Insert a buffer at point A and solve for $V_2(s)$ in symbolic form. How did inserting the buffer change the denominator, and therefore, the location of the poles?

Solution:

(a) Let the node between the capacitors be Node A and the node to the right of capacitor C_2 be Node 2. We can then write the following node-voltage equations on the *s* domain circuit.

Node A: $$\frac{V_A(s)-V_1(s)}{1/C_1 s}+\frac{V_A(s)}{R_1}+\frac{V_A(s)-V_2(s)}{1/C_2 s}=0$$

Node 2: $$\frac{V_2(s)-V_A(s)}{1/C_2 s}+\frac{V_2(s)}{R_2}=0$$

(b) We can use MATLAB to find the symbolic solution

```
clear all
syms s t VA V1 V2 R1 R2 C1 C2
Eqn1 = (VA-V1)/(1/s/C1) + VA/R1 + (VA-V2)/(1/s/C2);
Eqn2 = (V2-VA)/(1/s/C2) + V2/R2;
Soln = solve(Eqn1,Eqn2,VA,V2);
V2 = simplify(Soln.V2)
V2 =
s^2*C2*R2*C1*R1*V1/(s^2*C1*R1*C2*R2+s*C1*R1+s*C2*R2+1+s*C2*R1)
```

(c) The buffer serves to isolate two voltage dividers in the circuit. We can directly compute $V_2(s)$ by multiplying the two voltage dividers together.

```
% Use voltage dividers to find the new output voltage
VA = R1*V1/(R1+1/s/C1);
V2 = simplify(R2*VA/(R2+1/s/C2))
V2 =
s^2*C2*R2*C1*R1*V1/(s*C1*R1+1)/(s*C2*R2+1)
```

Answers:

(a) Node A: $$\frac{V_A(s)-V_1(s)}{1/C_1 s}+\frac{V_A(s)}{R_1}+\frac{V_A(s)-V_2(s)}{1/C_2 s}=0$$

Node 2: $$\frac{V_2(s)-V_A(s)}{1/C_2 s}+\frac{V_2(s)}{R_2}=0$$

(b) $$V_2(s)=\frac{R_1R_2C_1C_2s^2V_1(s)}{R_1R_2C_1C_2s^2+(R_1C_1+R_2C_2+R_1C_2)s+1}$$

The denominator is missing the term R_1C_2s that was present in the answer to part (b). Since the coefficient for the *s* term is different, the poles will be different.

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(c) $V_2(s) = \dfrac{R_1 R_2 C_1 C_2 s^2 V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2)s + 1}$

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Problem 10-34

There is no initial energy stored in the circuit in Figure P10-33. The Thévenin equivalent circuit to the left of point A when a unit step is applied is

$$V_T(s) = \frac{1}{s+10^3}\ \text{V - s, and } Z_T(s) = \frac{10^6}{s+10^3}\,\Omega$$

Select values for R_2 and C_2 such that the output transform is

$$V_2(s) = \frac{s}{s^2+3000s+10^6}\ \text{V} - \text{s}$$

Solution:

We can use voltage division to find an expression for $V_2(s)$ in terms of R_2 and C_2 for the given Thévenin equivalent circuit. We can then solve for R_2 and C_2 to meet the specification.

```
clear all
syms s R2 C2
VT = 1/(s+1e3);
ZT = 1e6/(s+1e3);
V2 = simplify(R2*VT/(R2 + 1/s/C2 + ZT))
V2 =
s*C2*R2/(s^2*C2*R2+1000*s*C2*R2+s+1000+1000000*s*C2)
```

So we have

$$V_2(s) = \frac{R_2C_2s}{R_2C_2s^2+(1000R_2C_2+1000000C_2+1)s+1000}$$

$$V_2(s) = \frac{s}{s^2+\left(1000+\dfrac{1000000}{R_2}+\dfrac{1}{R_2C_2}\right)s+\dfrac{1000}{R_2C_2}}$$

If we select R_2 = 1 kΩ and C_2 = 1 μF, then $1/R_2C_2$ = 1000 and we can meet the specification.

Answer:

Select R_2 = 1 kΩ and C_2 = 1 μF.

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Problem 10-35

There is no initial energy stored in the bridged-T circuit in Figure P10-35.
(a) Transform the circuit into the *s* domain and formulate mesh-current equations.
(b) Use the mesh-current equations to find the *s*-domain relationship between the input $V_1(s)$ and the output $V_2(s)$.

Solution:
(a) Let the lower mesh be Loop A and the upper mesh be Loop B. We can then write the following mesh-current equations on the *s* domain circuit by inspection.

Loop A: $-V_1(s)+[I_A(s)-I_B(s)]\left(\frac{1}{C_1 s}\right)+I_A(s)R_1=0$

Loop B: $[I_B(s)-I_A(s)]\left(\frac{1}{C_1 s}\right)+I_B(s)\left(R_2+\frac{1}{C_2 s}\right)=0$

(b) We can use MATLAB to solve these equations and then find the relationship between the input $V_1(s)$ and the output $V_2(s)$.

```
clear all
syms s V1 V2 IA IB R1 R2 C1 C2
Eqn1 = -V1+(IA-IB)*(1/s/C1)+IA*R1;
Eqn2 = (IB-IA)*(1/s/C1)+IB*(R2+1/s/C2);
Soln = solve(Eqn1,Eqn2,IA,IB);
IA = simplify(Soln.IA);
IB = simplify(Soln.IB);
% Find an expression for V2(s)
V2 = simplify(IB*(1/s/C2)+IA*R1);
Ts = simplify(V2/V1)
Ts =
(1+s^2*C1*R1*C2*R2+s*C2*R1+s*C1*R1)/(s^2*C1*R1*C2*R2+s*C1*R1+s*C2*R2+1+s*C2*R1
)
```

Answers:

(a) Loop A: $-V_1(s)+[I_A(s)-I_B(s)]\left(\frac{1}{C_1 s}\right)+I_A(s)R_1=0$

Loop B: $[I_B(s)-I_A(s)]\left(\frac{1}{C_1 s}\right)+I_B(s)\left(R_2+\frac{1}{C_2 s}\right)=0$

(b) $V_2(s)=\left[\frac{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2)s+1}{R_1R_2C_1C_2s^2+(R_1C_1+R_2C_2+R_1C_2)s+1}\right]V_1(s)$

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Problem 10-36

There is no initial energy stored in the bridged-T circuit in Figure P10-35.

(a) Transform the circuit into the *s* domain and formulate node-voltage equations.

(b) Use the node-voltage equations to find the *s*-domain relationship between the input $V_1(s)$ and the output $V_2(s)$.

Solution:

(a) Let the node between the capacitors be node A and the node to the right of capacitor C_2 be Node 2. We can now write the node-voltage equations as follows.

Node A: $$\frac{V_A(s)-V_1(s)}{1/C_1 s}+\frac{V_A(s)}{R_1}+\frac{V_A(s)-V_2(s)}{1/C_2 s}=0$$

Node 2: $$\frac{V_2(s)-V_A(s)}{1/C_2 s}+\frac{V_2(s)-V_1(s)}{R_2}=0$$

(b) We can use MATLAB to find the input-output voltage relationship.

```
clear all
syms s t VA V1 V2 R1 R2 C1 C2
Eqn1 = (VA-V1)/(1/s/C1) + VA/R1 + (VA-V2)/(1/s/C2);
Eqn2 = (V2-VA)/(1/s/C2) + (V2-V1)/R2;
Soln = solve(Eqn1,Eqn2,VA,V2);
V2 = simplify(Soln.V2)
Ts = simplify(V2/V1)
V2 =
V1*(1+s^2*C1*R1*C2*R2+s*C2*R1+s*C1*R1)/(s^2*C1*R1*C2*R2+s*C1*R1+s*C2*R2+1+s*C2
*R1)
Ts =
(1+s^2*C1*R1*C2*R2+s*C2*R1+s*C1*R1)/(s^2*C1*R1*C2*R2+s*C1*R1+s*C2*R2+1+s*C2*R1
)
```

Answers:

(a) Node A: $$\frac{V_A(s)-V_1(s)}{1/C_1 s}+\frac{V_A(s)}{R_1}+\frac{V_A(s)-V_2(s)}{1/C_2 s}=0$$

Node 2: $$\frac{V_2(s)-V_A(s)}{1/C_2 s}+\frac{V_2(s)-V_1(s)}{R_2}=0$$

(b) $$\frac{V_2(s)}{V_1(s)}=\frac{R_1R_2C_1C_2s^2+R_1(C_1+C_2)s+1}{R_1R_2C_1C_2s^2+(R_1C_1+R_2C_2+R_1C_2)s+1}$$

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Problem 10-37

There is no initial energy stored in the circuit in Figure P10-37.

(a) Find the zero-state mesh currents $i_A(t)$ and $i_B(t)$ when $v_1(t) = 100\ e^{-200t}\ u(t)$ V.

(b) Validate your answers using OrCAD.

Solution:

(a) The solution is presented in the following MATLAB code.

```
clear all
format short eng
syms s t IA IB iA iB
% Convert the source into the s domain
v1t = 100*exp(-200*t);
V1s = laplace(v1t);
% Write the mesh-current equations
Eqn1 = -V1s + IA*(1e3+1/s/10e-6) + (IA-IB)*(1/s/10e-9);
Eqn2 = (IB-IA)*(1/s/10e-9) + IB*(250+750);
Soln = solve(Eqn1,Eqn2,IA,IB);
IA = Soln.IA;
IB = Soln.IB;
iA = ilaplace(IA);
iB = ilaplace(IB);
iA_num = vpa(iA,6)
iB_num = vpa(iB,6)

iA_num =
.665777e-1*exp(-200.*t)+.166778e-10*exp(-
100050.*t)*(.200400e10*cosh(100000.*t)-.400100e10*sinh(100000.*t))
iB_num =
.667111e-1*exp(-200.*t)+.166778e-7*(-
.400000e7*cosh(100000.*t)+.200200e7*sinh(100000.*t))*exp(-100050.*t)

% Plot the waveforms so we can compare them to the OrCAD results
% First re-write the currents in terms
of exponentials only
iA2 = .665777e-1*exp(-200.*t)-
.166527833000e-1*exp(-50.*t)...
     +.50075094500e-1*exp(-200050.*t);
iB2 = .667111e-1*exp(-200.*t)-.166611e-
1*exp(-50.*t)...
     -.500501e-1*exp(-200050.*t);
tt = 0:20e-6:120e-3;
iAtt = subs(iA2,t,tt);
iBtt = subs(iB2,t,tt);
close all
plot(tt,iAtt,'g','LineWidth',4)
hold on
plot(tt,iBtt,'r','LineWidth',2.5)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

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(b) The following OrCAD simulation validates the results.

Answers:

(a) $i_A(t) = \left[66.578e^{-200\,t} - 16.653e^{-50\,t} + 50.075e^{-200050\,t}\right]u(t)$ mA

$i_B(t) = \left[66.711e^{-200\,t} - 16.661e^{-50\,t} - 50.050e^{-200050\,t}\right]u(t)$ mA

(b) The OrCAD results are presented above.

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Problem 10-38

There is no external input in the circuit in Figure P10-38.

(a) Find the zero-input node voltages $v_A(t)$ and $v_B(t)$, and the voltage across the capacitor $v_C(t)$ when $v_C(0) = 5$ V and $i_L(0) = 0$ A.

(b) Use MATLAB to plot your results in (a).

(c) Use OrCAD to validate your results in (a).

(d) Compare the MATLAB and OrCAD plots. Are they the same?

Solution:

(a) Convert the circuit into the s domain by replacing the inductor by its impedance and the capacitor by its impedance in series with a voltage source of magnitude $v_C(0)/s = 5/s$, with its positive terminal on the left. The node between the new voltage source and the capacitor has a voltage of $[V_B(s)+5/s]$ and we will use the supernode approach to handle this source. We can now write node-voltage equations to solve for the node voltages.

```
clear all
syms s t VA VB VC
C = 10e-6;
L = 0.1;
ZC = 1/s/C;
ZL = s*L;
% Write the node-voltage equations for VA and VB
Eqn1 = VA/100 + (VA-(VB+5/s))/ZC;
Eqn2 = (VB+5/s-VA)/ZC + VB/100 + VB/ZL;
Soln = solve(Eqn1,Eqn2,VA,VB);
VA = simplify(Soln.VA);
VB = simplify(Soln.VB);
VC = VA-VB;
vA = ilaplace(VA)
vB = ilaplace(VB)
vC = ilaplace(VC)
vA =
5/2*exp(-500*t)*(cos(500*t)+sin(500*t))
vB =
5/2*exp(-500*t)*(-cos(500*t)+sin(500*t))
vC =
5*exp(-500*t)*cos(500*t)
```

(b) Plot the voltages using MATLAB.

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```
close all
tt = 0:20e-6:12e-3;
vAtt = subs(vA,t,tt);
vBtt = subs(vB,t,tt);
vCtt = subs(vC,t,tt);
plot(tt,vAtt,'g','LineWidth',3)
hold on
plot(tt,vBtt,'r','LineWidth',3)
plot(tt,vCtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
legend('v_A(t)','v_B(t)','v_C(t)')
```

(c) The OrCAD simulation and results are shown below.

(d) The OrCAD and MATLAB results agree.

Answers:

(a) $v_A(t) = \frac{5}{2}e^{-500t}\left[\cos(500t) + \sin(500t)\right]u(t)$ V.

$v_B(t) = \frac{5}{2}e^{-500t}\left[-\cos(500t) + \sin(500t)\right]u(t)$ V.

$v_A(t) = 5e^{-500t}\left[\cos(500t)\right]u(t)$ V.

(b) The MATLAB plots are shown above.

(c) The OrCAD results are shown above.

(d) The MATLAB and OrCAD results agree.

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Problem 10-39

The circuit in Figure P10-39 is in the zero state. Use node-voltage equations to find the circuit determinant. Select values of R, C, and μ so that the circuit has ω_0 = 5 krad/s and $\zeta = 0.707$.

Solution:

Let the node between the capacitors be Node A. We can now write node-voltage equations at Node A and Node X to find the circuit determinant.

Node A: $$\frac{V_A(s)-V_1(s)}{1/Cs}+\frac{V_A(s)-V_x(s)}{1/Cs}+\frac{V_A(s)-\mu V_x(s)}{R}=0$$

Node X: $$\frac{V_x(s)-V_A(s)}{1/Cs}+\frac{V_x(s)}{R}=0$$

Re-writing the equations, we get

Node A: $$V_A(s)\left[Cs+Cs+\frac{1}{R}\right]-V_x(s)\left[Cs+\frac{\mu}{R}\right]=CsV_1(s)$$

Node X: $$-V_A(s)[Cs]+V_x(s)\left[Cs+\frac{1}{R}\right]=0$$

We can now use MATLAB to find the determinant.

```
clear all
syms s R C u
A = [C*s+C*s+1/R, -(C*s+u/R); -C*s, C*s+1/R];
DetA = det(A)
DetA =
-(-s^2*C^2*R^2+s*C*R*u-3*s*C*R-1)/R^2
```

We can write the determinant as

$$\Delta(s)=\frac{1}{R^2}\left[R^2C^2s^2+RC(3-\mu)s+1\right]=C^2\left[s^2+\frac{(3-\mu)}{RC}s+\frac{1}{R^2C^2}\right]$$

To get ω_0 = 5 krad/s, we need to choose R and C such that $1/RC$ = 5 krad/s. We then can choose μ such that $(3-\mu)/RC = 2\zeta\omega_0$ with $\zeta = 0.707$. Pick C = 100 nF and solve for the other two parameters.

```
C = 100e-9;
w0 = 5e3;
z = 1/sqrt(2);
R = 1/C/w0
u = 3-2*z*w0*R*C
R =
     2.0000e+003
```

Answers:

$$\Delta(s)=\frac{1}{R^2}\left[R^2C^2s^2+RC(3-\mu)s+1\right]=C^2\left[s^2+\frac{(3-\mu)}{RC}s+\frac{1}{R^2C^2}\right]$$

One possible solution is to choose R = 2 kΩ, C = 100 nF, and $\mu = 3-\sqrt{2}$.

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```
u =
     1.5858e+000
```

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Problem 10-40

The circuit in Figure P10-40 is in the zero state. Use mesh-current equations to find the circuit determinant. Select values of R, L, and C so that the circuit has ω_0 = 5 krad/s and $\zeta = 0.707$.

Solution:

We can write the following mesh-current equations.

Loop A: $-V_1(s) + I_A(s)R + [I_A(s) - I_B(s)]\left(Ls + \frac{1}{Cs}\right) = 0$

Loop B: $[I_B(s) - I_A(s)]\left(Ls + \frac{1}{Cs}\right) + I_B(s)(R) = 0$

Re-writing the equations, we get

Loop A: $+I_A(s)\left(R + Ls + \frac{1}{Cs}\right) - I_B(s)\left(Ls + \frac{1}{Cs}\right) = V_1(s)$

Loop B: $-I_A(s)\left(Ls + \frac{1}{Cs}\right) + I_B(s)\left(R + Ls + \frac{1}{Cs}\right) = 0$

We can now use MATLAB to find the determinant.

```
clear all
syms s R L C
ZL = s*L;
ZC = 1/s/C;
A = [R+ZL+ZC, -(ZL+ZC); -(ZL+ZC), R+ZL+ZC];
DetA = det(A)
DetA =
1/s/C*R*(s*C*R+2*s^2*L*C+2)
```

We can write the determinant as

$$\Delta(s) = \frac{R}{Cs}\left[2LCs^2 + RCs + 2\right] = \frac{2RL}{s}\left[s^2 + \frac{R}{2L}s + \frac{1}{LC}\right]$$

To get ω_0 = 5 krad/s and $\zeta = 0.707$, we select a capacitor and solve for the inductor and resistor.

```
w0 = 5e3;
w02 = w0^2;
z = 1/sqrt(2);
C = 100e-9
L = 1/w02/C
R = 4*L*z*w0
C =
   100.0000e-009
L =
   400.0000e-003
```

Answers:

$$\Delta(s) = \frac{R}{Cs}\left[2LCs^2 + RCs + 2\right] = \frac{2RL}{s}\left[s^2 + \frac{R}{2L}s + \frac{1}{LC}\right]$$

One possible solution is to choose R = 5.6569 kΩ, L = 400 mH, and C = 100 nF.

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```
R =
      5.6569e+003
```

Problem 10-41

The circuit in Figure P10-41 is in the zero state. Use node-voltage equations to find the circuit determinant. Select values of R, C_1, and C_2 so that the circuit has ω_0 = 5 krad/s and $\zeta = 0.707$.

Solution:

Let the node between the resistors be Node A and the positive input terminal be Node 2.

Node A: $$\frac{V_A(s)-V_1(s)}{R}+\frac{V_A(s)-V_2(s)}{R}+\frac{V_A(s)-V_2(s)}{1/C_1 s}=0$$

Node 2: $$\frac{V_2(s)-V_A(s)}{R}+\frac{V_2(s)}{1/C_2 s}=0$$

Re-writing the equations, we get

Node A: $$V_A(s)\left[\frac{1}{R}+\frac{1}{R}+C_1 s\right]-V_2(s)\left[\frac{1}{R}+C_1 s\right]=\frac{V_1(s)}{R}$$

Node 2: $$-V_A(s)\left[\frac{1}{R}\right]+V_2(s)\left[\frac{1}{R}+C_2 s\right]=0$$

We can now use MATLAB to find the determinant.

```
clear all
syms s R L C1 C2
A = [1/R+1/R+C1*s, -(1/R+C1*s); -1/R, 1/R+C2*s];
DetA = det(A)
DetA =
(1+2*C2*s*R+C1*s^2*R^2*C2)/R^2
```

The circuit determinant is

$$\Delta(s)=\frac{1}{R^2}\left[R^2C_1C_2s^2+2RC_2s+1\right]=C_1C_2\left[s^2+\frac{2}{RC_1}s+\frac{1}{R^2C_1C_2}\right]$$

If we select C_1 = 100 nF, we can solve for R and C_2 to get ω_0 = 5 krad/s and $\zeta = 0.707$.

```
w0 = 5e3;
w02 = w0^2;
z = 1/sqrt(2);
C1 = 100e-9
Eqn1 = 2/R/C1 - 2*z*w0;
Eqn2 = 1/R^2/C1/C2 - w02;
Soln = solve(Eqn1,Eqn2,R,C2);
R = double(Soln.R)
C2 = double(Soln.C2)
C1 =
```

Answers:

$$\Delta(s)=\frac{1}{R^2}\left[R^2C_1C_2s^2+2RC_2s+1\right]=C_1C_2\left[s^2+\frac{2}{RC_1}s+\frac{1}{R^2C_1C_2}\right]$$

One possible solution is to choose $R = 2000\sqrt{2}\ \Omega$, C_1 = 100 nF, and C_2 = 50 nF.

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```
   100.0000e-009
R =
     2.8284e+003
C2 =
    50.0000e-009
```

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Problem 10-42

Three node voltages are shown in Figure P10-42.

(a) Explain why only one of the node voltages is independent.

(b) Write a node voltage equation in the independent node voltage.

Solution:

(a) Only one node voltage is independent because $V_A(s)$ depends on the source voltage and the voltage $V_X(s)$ and the dependent source link node voltages $V_A(s)$ and $V_B(s)$, as shown below.

$$V_A(s) = V_S(s)$$

$$V_X(s) = V_A(s) - V_B(s) = V_A(s) - \mu V_X(s)$$

$$V_X(s) = \frac{V_A(s)}{1+\mu}$$

$$V_B(s) = \frac{\mu V_A(s)}{1+\mu} = \frac{\mu V_S(s)}{1+\mu}$$

The independent voltage is $V_C(s)$.

(b) The node-voltage equation for $V_C(s)$ is given below.

$$\frac{V_C(s) - V_B(s)}{R_2} + \frac{V_C(s)}{1/Cs} = 0$$

$$\frac{V_C(s) - \dfrac{\mu V_S(s)}{1+\mu}}{R_2} + \frac{V_C(s)}{1/Cs} = 0$$

Answers:

(a) Presented above.

(b) $$\frac{V_C(s) - \dfrac{\mu V_S(s)}{1+\mu}}{R_2} + \frac{V_C(s)}{1/Cs} = 0$$

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Problem 10-43

Three mesh currents are shown in Figure P10-43.

(a) Explain why only two of these mesh currents are independent.

(b) Write *s*-domain mesh-current equations in the two independent mesh currents.

Solution:

(a) Mesh current $I_B(s)$ depends on mesh current $I_A(s)$, as shown below.

$$I_X(s) = I_A(s) - I_B(s) = I_A(s) + \beta I_X(s)$$

$$I_X(s) = \frac{I_A(s)}{1-\beta}$$

$$I_B(s) = \frac{\beta I_A(s)}{\beta - 1}$$

(b) We can write mesh-current equations for the two independent mesh currents:

Mesh A: $-V_S(s) + [I_A(s) - I_C(s)]R_1 + \left[I_A(s) - \frac{\beta}{\beta-1} I_A(s)\right] Ls = 0$

Mesh C: $[I_C(s)]\frac{1}{Cs} + \left[I_C(s) - \frac{\beta}{\beta-1} I_A(s)\right] R_2 + [I_C(s) - I_A(s)]R_1 = 0$

Answers:

(a) Mesh currents $I_A(s)$ and $I_B(s)$ are dependent with $I_B(s) = \frac{\beta}{\beta-1} I_A(s)$.

(b) Mesh A: $-V_S(s) + [I_A(s) - I_C(s)]R_1 + \left[I_A(s) - \frac{\beta}{\beta-1} I_A(s)\right] Ls = 0$

Mesh C: $[I_C(s)]\frac{1}{Cs} + \left[I_C(s) - \frac{\beta}{\beta-1} I_A(s)\right] R_2 + [I_C(s) - I_A(s)]R_1 = 0$

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Problem 10-44

The switch in Figure P10-44 has been in position A for a long time and is moved to position B at $t = 0$.

(a) Write an appropriate set of node-voltage or mesh-current equations in the s domain.

(b) Use MATLAB to solve for $V_C(s)$ and $v_C(t)$. Also using MATLAB, plot $v_C(t)$ and the exponential source on the same axes.

(c) Validate your results for $v_C(t)$ using OrCAD. Include the exponential source in your Probe plot.

Solution:

(a) Since the switch has been in position A for a long time, the inductor acts like a short circuit and the capacitor acts like an open circuit. The initial current through the inductor will be 5 mA and the initial voltage on the capacitor will be 5 V. Convert the circuit into the s domain by replacing the inductor and capacitor with their impedances in series with voltage sources representing the initial conditions. We can then write mesh-current equations to find the mesh currents and then the desired capacitor voltage. Let the left loop be Mesh A and the right loop be Mesh B.

Mesh A: $$-V_S(s) + I_A(s)[1000] + [I_A(s) - I_B(s)][1000 + s] - \frac{1}{200} = 0$$

Mesh B: $$\frac{1}{200} + [I_B(s) - I_A(s)][s + 1000] + I_B(s)\left[2000 + \frac{20\times10^6}{s}\right] + \frac{5}{s} = 0$$

(b) The MATLAB solution is presented below.

```
clear all
syms s t IA IB VC vC
vs = 5*exp(-2000*t);
Vs = laplace(vs);
% Solve the mesh-current equations
Eqn1 = -Vs + IA*1000 + (IA-IB)*(1000+s)
- 1/200;
Eqn2 = 1/200 + (IB-IA)*(s+1000) +
IB*(2000 + 20e6/s) + 5/s;
Soln = solve(Eqn1,Eqn2,IA,IB);
IA = simplify(Soln.IA);
IB = simplify(Soln.IB);
VC = IB/s/50e-9 + 5/s;
VC_num = simplify(vpa(VC,6))
vc = ilaplace(VC);
vc_num = vpa(vc,6)
VC_num =
5.*(-
10000000.+11000.*s+3.*s^2)/(31000.*s^2+90000000.*s+3.*s^3+80000000000.)
vc_num =
-.226259e-15-50.0000*exp(-2000.*t)+.172414*exp(-
4166.67*t)*(319.*cosh(2006.94*t)+276.957*sinh(2006.94*t))

% Find the time-domain coefficients in a more convenient
```

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```
% format
[r,p,k]=residue(5*[3 11e3 -1e7],[3 31e3 9e7 8e10])
r =
     3.6244e+000
    51.3756e+000
   -50.0000e+000
p =
    -6.1736e+003
    -2.1597e+003
    -2.0000e+003
k =
     []
% Plot the capacitor voltage and the source voltage
close all
tt = 0:5e-6:3e-3;
vctt = subs(vc,t,tt);
vstt = subs(vs,t,tt);
plot(tt,vctt,'g','LineWidth',3)
hold on
plot(tt,vstt,'r','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
legend('v_C(t)','v_S(t)')
```

(c) The following OrCAD simulation verifies the MATLAB results.

Answers:

(a) Mesh A: $-V_S(s)+I_A(s)[1000]+[I_A(s)-I_B(s)][1000+s]-\dfrac{1}{200}=0$

Mesh B: $\dfrac{1}{200}+[I_B(s)-I_A(s)][s+1000]+I_B(s)\left[2000+\dfrac{20\times10^6}{s}\right]+\dfrac{5}{s}=0$

(b) $V_C(s)=\dfrac{5(3s^2+11000s-10000000)}{3s^3+31000s^2+90000000s+8\times10^{10}}$

$$v_C(t)=\left[-50e^{-2000t}+51.3756e^{-2159.73t}+3.6244e^{-6173.61t}\right]u(t)\ \text{V}$$

The MATLAB plots are presented above.

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(c) The OrCAD results are presented above and agree with the MATLAB results.

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Problem 10-45

There is no energy stored in the circuit in Figure P10-45 at $t = 0$. Transform the circuit into the s domain and solve for $V_O(s)$ and $v_O(t)$.

Solution:

Use node-voltage analysis with the node to the right of the 10-kΩ resistor being node A. The solution is presented in the following MATLAB code.

```
clear all
syms s t VA VO
vs = 100*heaviside(t);
Vs = laplace(vs);
% Write the node-voltage equations
Eqn1 = (VA-Vs)/10e3 + VA/(1/s/100e-9) + (VA-VO)/(1/s/100e-9);
Eqn2 = (VO-VA)/(1/s/100e-9) + VO/(1/s/100e-9) + VO/1e3;
Soln = solve(Eqn1,Eqn2,VA,VO);
VO = simplify(Soln.VO)
vO = ilaplace(VO)
vO_num = vpa(vO,6)
% Find a more convenient expression for vo(t)
[r,p,k] = residue([100000],[3 22e3 1e7])
VO =
100000/(3*s^2+22000*s+10000000)
vO =
100/91*91^(1/2)*exp(-11000/3*t)*sinh(1000/3*t*91^(1/2))
vO_num =
10.4828*exp(-3666.67*t)*sinh(3179.79*t)
r =
    -5.2414e+000
     5.2414e+000
p =
    -6.8465e+003
  -486.8693e+000
k =
     []
```

Answers:

$$V_O(s) = \frac{100000}{3s^2 + 22000s + 1\times 10^7}$$

$$v_O(t) = 5.2414\left[e^{-486.9t} - e^{-6846.5t}\right]u(t) \text{ V}$$

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Problem 10-46

The switch in Figure P10-46 has been open for a long time and is closed at $t = 0$. Transform the circuit into the s domain and solve for $I_O(s)$ and $i_O(t)$.

Solution:

Before the switch closes, the current through the left inductor is 10 mA and the current through the right inductor is 5 mA. Use mesh-current analysis with the left loop being Mesh A and the right loop being Mesh B. When the switch closes, the 750-Ω resistor is no longer part of the circuit. We can transform the circuit into the s domain and solve for the output current as shown in the following MATLAB code.

```
clear all
syms s t IA IB vs Vs IO iO
vs = 15*heaviside(t);
Vs = laplace(vs);
% Write the mesh-current equations
Eqn1 = -Vs + IA*(500+0.5*s) + (IA-IB)*500 – 0.5*10e-3;
Eqn2 = (IB-IA)*500 + IB*(0.5*s+500) – 0.5*5e-3;
Soln = solve(Eqn1,Eqn2,IA,IB);
IO = simplify(Soln.IB)
iO = ilaplace(IO)
IO =
1/200*(6000000+s^2+4000*s)/s/(3000000+4000*s+s^2)
iO =
1/100-3/400*exp(-1000*t)+1/400*exp(-3000*t)
```

Answers:

$$I_O(s) = \frac{s^2 + 4000s + 6000000}{200s\left(s^2 + 4000s + 3000000\right)}$$

$$i_O(t) = \left[10 - 7.5e^{-1000t} + 2.5e^{-3000t}\right]u(t) \text{ mA}$$

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Problem 10-47

There is no initial energy stored in the circuit in Figure P10-47.

(a) Transform the circuit into the *s* domain and solve for $V_O(s)$ in symbolic form.

(b) If $R_1 = R_2 = 2$ kΩ, select values of *L* and *C* to produce $\zeta = 0.707$ and $\omega_0 = 707$ rad/s. Hint: Try values of *L* first, i.e., 1 H, 2 H, etc.

Solution:

(a) Using node-voltage analysis and MATLAB, we can develop the solution as follows.

```
clear all
syms s Vs VA VO R1 R2 L C;
ZL = s*L;
ZC = 1/s/C;
Eqn1 = (VA-Vs)/R1 + VA/ZL + (VA-VO)/ZC;
Eqn2 = (VO-VA)/ZC + VO/R2;
Soln = solve(Eqn1,Eqn2,VA,VO);
VO = simplify(Soln.VO)
VO =
s^2*C*R2*L*Vs/(s^2*L*C*R2+s*L+R1*s*C*R2+R1+s^2*C*R1*L)
```

(b) We will use MATLAB to perform the substitutions and calculations

```
R1 = 2e3;
R2 = 2e3;
z = 1/sqrt(2);
w0 = 1000/sqrt(2);
w02 = w0^2;
Eqn1 = (R1*R2*C+L)/((R1+R2)*L*C) - 2*z*w0;
Eqn2 = R1/((R1+R2)*L*C) - w02;
Soln = solve(Eqn1,Eqn2,L,C);
C = double(Soln.C)
L = double(Soln.L)
C =
   500.0000e-009
   500.0000e-009
L =
     2.0000e+000
     2.0000e+000
```

Answers:

(a) $$V_O(s) = \frac{R_2 LCs^2 V_S(s)}{(R_1 + R_2)LCs^2 + (R_1 R_2 C + L)s + R_1}$$

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(b) L = 2 H and C = 500 nF.

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Problem 10-48

The input to the integrator shown in Figure P10-48 is $v_1(t) = \cos 10t$ V. The desired output is $v_2(t) = -\sin 10t$ V. Working in the s domain, select values of *R* and *C* to produce the desired output.

Solution:

Write a node-voltage equation at the negative input terminal of the OP AMP to find a relationship in the *s* domain between the input and output voltages. Then use the desired input and output and a selected value for the capacitor to compute the corresponding resistor value. The MATLAB solution is presented below

```
clear all
syms s t R C V1 V2 v1d V1d v2d V2d
v1d = cos(10*t);
V1d = laplace(v1d);
v2d = -1*sin(10*t);
V2d = laplace(v2d);
% Write the node-voltage equation
% The voltage at the negative terminal is zero
Eqn1 = (0-V1)/R + (0-V2)/(1/s/C);
V2 = solve(Eqn1,V2);
% Find the ratio of the output voltage to the input voltage
Ts = simplify(V2/V1)
% Find the desired input-output ratio
Td = simplify(V2d/V1d)
% Select a value for C and substitute
C = 1e-6;
Ts = subs(Ts);
Td = subs(Td);
% Solve for the resistance
R = solve(Ts-Td,R)
Ts =
-1/R/s/C
Td =
-10/s
R =
100000
```

Answer:

One possible solution is with R = 100 kΩ and C = 1 μF.

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Problem 10-49

Show that the circuit in Figure P10-49 has natural poles at $s = -4/RC$ and $s = -2/RC \pm j2/RC$ when $L = R^2C/4$.

Solution:

Use node-voltage analysis to find the relationship between the output and input voltages in the s domain. Then substitute the specified inductance values and find the corresponding poles.

```
clear all
syms s VA VO VS
syms R L C positive
ZL = s*L;
ZC = 1/s/C;
Eqn1 = (VA-VS)/(R+ZL) + VA/ZC + (VA-VO)/ZL;
Eqn2 = (VO-VA)/ZL + VO/R;
Soln = solve(Eqn1,Eqn2,VA,VO);
VO = Soln.VO
Ts = simplify(VO/VS)
VO =
R*VS/(2*s^2*C*L*R+2*s*L+s*C*R^2+s^3*C*L^2+2*R)
Ts =
R/(2*s^2*C*L*R+2*s*L+s*C*R^2+s^3*C*L^2+2*R)
```

The input-output relationship is

$$\frac{V_O(s)}{V_S(s)} = \frac{R}{L^2Cs^3 + 2RLCs^2 + (2L + R^2C)s + 2R} = \frac{R}{(R + Ls)(LCs^2 + RCs + 2)}$$

We can compute the poles as follows:

```
Eqn3 = 2*s^2*C*L*R+2*s*L+s*C*R^2+s^3*C*L^2+2*R;
p = solve(Eqn3,s)
p =
                                  -1/L*R
 -1/2*(C*R-(C^2*R^2-8*C*L)^(1/2))/C/L
 -1/2*(C*R+(C^2*R^2-8*C*L)^(1/2))/C/L
```

Substitute in the specified value for the inductance.

```
p2 = subs(p,{L},{R*R*C/4})
p2 =
                                -4/C/R
 -2*(C*R-(-C^2*R^2)^(1/2))/C^2/R^2
 -2*(C*R+(-C^2*R^2)^(1/2))/C^2/R^2
```

Answer:

The natural poles meet the specifications.

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Problem 10-50

Find the range of the gain μ for which the circuit in Figure P10-50 is stable.

Solution:

The circuit will be stable if the poles remain on the left hand side of the complex plane. Use node-voltage analysis to solve for $V_X(s)$ and determine the pole locations as a function of the gain parameter μ.

```
clear all
syms s VX R C VS u
ZC = 1/s/C;
Eqn1 = (VX-VS)/R + VX/ZC + (VX-u*VX)/R;
VX = solve(Eqn1,VX)
VX =
-VS/(-2-C*R*s+u)
```

We have

$$V_X(s) = \frac{V_S(s)}{RCs + 2 - \mu}$$

The pole is located at $s = (\mu\text{-}2)/RC$, so to maintain negative poles, we require $\mu < 2$.

Answer:

The circuit is stable for $\mu < 2$.

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Problem 10-51

Use symbolic operations in MATLAB to solve for the node voltages in the circuit in Figure P10-51. The circuit is in the zero state.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s VA VB VC VD VS IS R1 R2 C1 C2 L
ZL = s*L;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
% Write the node-voltage equations
Eqn1 = VA-VS;
Eqn2 = (VB-VA)/ZC1 + (VB-VD)/R1;
Eqn3 = (VC-VA)/ZL + VC/ZC2 + (VC-VD)/R2;
Eqn4 = (VD-VB)/R1 + (VD-VC)/R2 - IS;
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,VA,VB,VC,VD);
VA = simplify(Soln.VA)
VB = simplify(Soln.VB)
VC = simplify(Soln.VC)
VD = simplify(Soln.VD)
VA =
VS
VB =
(s^3*L*C1*R1*VS*C2+s^3*C2*L*R2*C1*VS+s^2*L*C1*VS+s^2*C2*L*R2*IS+s*C1*R1*VS+s*C
1*R2*VS+s*L*IS+VS+IS*R2)/(R1*s^3*C1*C2*L+s^2*C2*L+s^3*C1*C2*L*R2+R2*s*C1+s*C1*
R1+1+s^2*L*C1)
VC =
(R1*s^2*L*C1*IS+s*L*IS+s*C1*R2*VS+s*C1*R1*VS+s^2*L*C1*VS+VS)/(R1*s^3*C1*C2*L+s
^2*C2*L+s^3*C1*C2*L*R2+R2*s*C1+s*C1*R1+1+s^2*L*C1)
VD =
(R2*R1*s*C1*IS+R1*s^2*L*C1*IS+s^3*C2*L*R2*C1*VS+s^2*L*C1*VS+s*L*IS+s^2*C2*L*R2
*IS+s^3*C2*L*R2*R1*C1*IS+s*C1*R1*VS+VS+IS*R2+s*C1*R2*VS)/(R1*s^3*C1*C2*L+s^2*C
2*L+s^3*C1*C2*L*R2+R2*s*C1+s*C1*R1+1+s^2*L*C1)
```

Answer:

The MATLAB solution is presented above.

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Problem 10-52

Thévenin's Theorem from Time-domain Data

A black box containing a linear circuit has an on-off switch and a pair of external terminals. When the switch is turned on, the open-circuit voltage between the external terminals is observed to be

$$v_{\mathrm{OC}}(t) = \left(18e^{-10t} - 12e^{-50t}\right)u(t) \ \ \mathrm{V}$$

The short circuit current was observed to be

$$i_{\mathrm{SC}}(t) = \left[\left(138.5e^{-10t} - 174.3e^{-50t} + 32.9e^{-96.6t}\right)\right]u(t) \ \ \mathrm{mA}$$

A 50-Ω load resistance is connected across the terminals and the switch turned on again. What is the voltage delivered to the load?

Solution:

Convert the open-circuit voltage and the short-circuit current into the *s* domain. Take the ratio of the two *s*-domain quantities to find the Thévenin impedance of the circuit. Use the Thévenin impedance and the open-circuit voltage to find the voltage across the load via voltage division. The calculations are completed below.

```
clear all
syms s t voc Voc isc Isc ZT VL vL
voc = 18*exp(-10*t)-12*exp(-50*t);
Voc = laplace(voc);
isc = 138.5e-3*exp(-10*t)-174.3e-3*exp(-50*t)+32.9e-3*exp(-96.6*t);
Isc = laplace(isc);
% Compute the Thevenin impedance
ZT = simplify(Voc/Isc);
% Use voltage division to find the load voltage
VL = simplify(50*Voc/(50+ZT))
vL = simplify(ilaplace(VL));
vL_num = vpa(vL,6);
% Express the results in a more convenient form
% Use VL(s) to find the expression for vL(t)
num = conv(-6*[1 130],[145 -184886 -25851560]);
den = conv(conv([1 10],[1 50]),[5855 1544486 101199560]);
[r,p,k] = residue(num,den);
ResPoles = [r p]
VL =
-6*(s+130)*(145*s^2-184886*s-
25851560)/(s+10)/(s+50)/(5855*s^2+1544486*s+101199560)
ResPoles =
  -169.3055e-003  -142.4726e+000
    68.2271e-003  -121.3166e+000
    -5.0485e+000   -50.0000e+000
     5.0010e+000   -10.0000e+000
```

Answer:

$$v_{\mathrm{L}}(t) = \left[5.001e^{-10t} - 5.0485e^{-50t} + 0.0682271e^{-121.3166t} - 0.1693055e^{-142.4726t}\right]u(t) \ \ \mathrm{V}$$

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Problem 10-53

Design a Load Impedance

In order to match the Thévenin impedance of a source, the load impedance in Figure P10-53 must be $Z_L(s) = \dfrac{s+5}{s+10}$.

(a) What impedance $Z_2(s)$ is required?

(b) How would you realize $Z_2(s)$ using only resistors, inductors, and/or capacitors?

Hint: Write $Z_L(s)$ as a sum of admittances, then solve for $Y_2(s)$.

Solution:

(a) The impedance $Z_2(s)$ is in parallel with a 20-Ω resistor and the combination must match the specified $Z_L(s)$. Find a general expression for the parallel impedance. Compare the result to the specified value for $Z_L(s)$ and solve for $Z_2(s)$.

```
clear all
syms s ZL Z2
ZL = (s+5)/(s+10);
Zp = 1/(1/20 + 1/Z2);
Eqn1 = Zp-ZL;
Z2 = solve(Eqn1,Z2)
Z2 =
20*(s+5)/(19*s+195)
```

$$Z_2(s) = \frac{20(s+5)}{19s+195}$$

(b) Write the admittance associated with $Z_2(s)$ as follows:

$$Y_2(s) = \frac{1}{Z_2(s)} = \frac{19s+195}{20(s+5)} = \frac{\frac{19}{20}(20s+100)+100}{20s+100} = \frac{19}{20} + \frac{100}{20s+100} = \frac{19}{20} + \frac{1}{\frac{s}{5}+1} = \frac{1}{Z_3(s)} + \frac{1}{Z_4(s)}$$

The equivalent admittance of a parallel combination is the sum of the admittances. If we now choose $Z_3(s) = R_3 = 20/19$ and $Z_4(s) = Ls + R_4 = s/5 + 1$, then we will meet the specification. The resulting circuit for $Z_2(s)$ is shown below.

Verify the impedance designed above

```
R3 = 20/19;
L4 = 200e-3;
R4 = 1;
ZL4 = s*L4;
Z2Design = simplify(1/(1/R3+1/(ZL4+R4)))
Z2Design =
20*(s+5)/(19*s+195)
```

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Answers: (a) $Z_2(s) = \dfrac{20(s+5)}{19s+195}$ (b) The design and verification are shown above.

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Problem 10-54

RC Circuit Analysis and Design

The *RC* circuits in Figure P10-54 represent the situation at the input to an oscilloscope. The parallel combination of R_1 and C_1 represent the probe used to connect the oscilloscope to a test point. The parallel combination of R_2 and C_2 represent the input impedance of the oscilloscope.

(a) **(A)** Assuming zero initial conditions, transform the circuit into the *s*-domain and find the relationship between the test point voltage $V_S(s)$ and the voltage $V_O(s)$ at the oscilloscope's input.

(b) **(D)** For R_2 = 10 MΩ and C_2 = 2 pF, determine the values of R_1 and C_1 that make the input voltage a scaled duplicate of the test point voltage.

Solution:

(a) Use voltage division and MATLAB to find the solution.

```
clear all
syms s R1 R2 C1 C2 VO VS
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Z1 = 1/(1/R1 + 1/ZC1);
Z2 = 1/(1/R2 + 1/ZC2);
VO = Z2*VS/(Z1+Z2);
T = simplify(VO/VS)
T =
R2*(1+s*C1*R1)/(R1+R1*C2*s*R2+R2+R2*s*C1*R1)
```

$$\frac{V_O(s)}{V_S(s)} = \frac{R_2(R_1C_1s+1)}{R_1R_2(C_1+C_2)s+R_1+R_2}$$

(b) Substitute in the given values and examine the resulting relationship.

```
T2 = simplify(subs(T,{R2,C2},{10e6,2e-12}));
T2_num = vpa(T2,6)
T2_num =
.193428e33*(1.+s*C1*R1)/(.193428e26*R1+.386856e21*R1*s+.193428e33+.193428e33*s
*C1*R1)
```

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We need to choose R_1 and C_1 such that the denominator cancels the term in parentheses in the numerator. Let's pick the ratio of the numerator to denominator to be $1/r$, where r is a positive integer greater than one. If we write the ratio with R_2 and C_2 substituted in, we have

$$\frac{V_O(s)}{V_S(s)} = \frac{k_1 R_1 C_1 s + k_1}{(k_1 R_1 C_1 + k_2 R_1)s + k_1 + \frac{k_1}{10^7} R_1}$$

where k_1 and k_2 are known coefficients in the MATLAB results. Then we need to solve for R_1 and C_1 such that

$$\frac{1}{r} = \frac{k_1 R_1 C_1}{k_1 R_1 C_1 + k_2 R_1} \quad \text{and} \quad \frac{1}{r} = \frac{k_1}{k_1 + \frac{k_1}{10^7} R_1}$$

Using MATLAB, the solution is calculated below:

```
% Choose an input output ratio
r = 2;
k1 = .193428e33;
k2 = .386856e21;
syms R1 C1
Eqn1 = r*k1*R1*C1-k1*R1*C1-k2*R1;
Eqn2 = r*k1-k1-k1*R1/1e7;
Soln = solve(Eqn1,Eqn2,R1,C1);
R1 = double(Soln.R1)
C1 = double(Soln.C1)
R1 =
    10.0000e+006
C1 =
     2.0000e-012
```

Answers:

(a) $\dfrac{V_O(s)}{V_S(s)} = \dfrac{R_2(R_1 C_1 s + 1)}{R_1 R_2 (C_1 + C_2)s + R_1 + R_2}$

(b) One possible solution is R_1 = 10 MΩ and C_1 = 2 pF.

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Problem 10-55

s-domain OP AMP Circuit Analysis

The OP AMP circuit in Figure P10-55 is in the zero-state. Transform the circuit into the *s*-domain and use the OP AMP circuit analysis techniques developed in Section 4-4 to find the relationship between the input $V_1(s)$ and the output $V_2(s)$.

Solution:

Write node-voltage equations at the node between the resistors (Node A) and at the positive input terminal to the OP AMP. Note that the positive input terminal has the same voltage as the output, so we will label it with voltage $V_2(s)$. The remainder of the solution is presented in the following MATLAB calculations.

```
clear all
syms s V1 V2 VA R1 R2 C1 C2
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Eqn1 = (VA-V1)/R1 + (VA-V2)/ZC2 + (VA-V2)/R2;
Eqn2 = (V2-VA)/R2 + V2/ZC1;
Soln = solve(Eqn1,Eqn2,VA,V2);
V2 = simplify(Soln.V2);
T = simplify(V2/V1)
T =
1/(1+R2*s*C1+R1*C2*s^2*R2*C1+s*C1*R1)
```

Answer:

$$\frac{V_2(s)}{V_1(s)} = \frac{1}{R_1R_2C_1C_2s^2 + C_1(R_1+R_2)s + 1}$$

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Problem 10-56

Pulse Conversion Circuit

The purpose of the test setup in Figure P10-56 is to deliver damped sine pulses to the test load. The excitation comes from a 1-Hz square wave generator. The pulse conversion circuit must deliver damped sine waveforms with $\zeta < 0.5$ and $\omega_0 > 10$ krad/s to 50-Ω and 600-Ω loads. The recommended values for the pulse conversion circuit are L = 10 mH and C = 100 nF. Verify that the test setup meets the specifications. *Hint*: Compute the voltage across the load for an input signal equal to a unit step function, $u(t)$, and then again for a negative unit step function, $-u(t)$. Note that the output of a square wave generator is the sum of a series of step functions.

Solution:

Transform the circuit into the s domain and use voltage division to find the output voltage responses for the range of load resistors. Determine the damping ratio and the undamped natural frequency to verify the circuit meets the specifications

```
clear all
syms s t vt Vs VO
% First examine a positive unit step
vt = heaviside(t);
Vs = laplace(vt);
L = 10e-3;
C = 100e-9;
ZL = s*L;
ZC = 1/s/C;
Rs = 50;
RL = 50;
VO = simplify(RL*Vs/(Rs+ZL+ZC+RL))
VO =
5000/(10000*s+s^2+1000000000)

% Find the damping ratio and undamped natural frequency
w0 = sqrt(1000000000)
z = 10000/2/w0
w0 =
    31.6228e+003
z =
   158.1139e-003
```

The results verify that ζ and ω_0 meet the specifications. Now check for a 600-Ω load resistor.

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```
RL = 600;
VO = simplify(RL*Vs/(Rs+ZL+ZC+RL))
VO =
60000/(65000*s+s^2+1000000000)

% Find the damping ratio and undamped natural frequency
w0 = sqrt(1000000000)
z = 65000/2/w0
w0 =
    31.6228e+003
z =
     1.0277e+000
```

The results show that ω_0 meets the specifications, but ζ does not.

Repeat the tests for a negative unit step function.

```
clear all
syms s t vt Vs VO
% First examine a positive unit step
vt = -heaviside(t);
Vs = laplace(vt);
L = 10e-3;
C = 100e-9;
ZL = s*L;
ZC = 1/s/C;
Rs = 50;
RL = 50;
VO = simplify(RL*Vs/(Rs+ZL+ZC+RL))
VO =
-5000/(10000*s+s^2+1000000000)

RL = 600;
VO = simplify(RL*Vs/(Rs+ZL+ZC+RL))
VO =
-60000/(65000*s+s^2+1000000000)
```

The damping ratio and undamped natural frequency did not change when the input is a negative step function. The circuit still meets the specifications for the 50-Ω load, but not for the 600-Ω load.

Answer:
The circuit meets the specification for the 50-Ω load, but not for the 600-Ω load.

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Problem 11-1

Find the driving point impedance seen by the voltage source in Figure P11-1 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

Solution:

Find the equivalent impedance seen by the source and use voltage division to find the relationship between the input and output voltages. The solution is presented in the following MATLAB code.

```
clear all
syms s R C
ZC = 1/s/C;
R1 = R;
R2 = 2*R;
Zin = simplify(R1 + R2 + ZC)
Tv = simplify((R2+ZC)/(R1+R2+ZC))
Zin =
(3*R*s*C+1)/s/C
Tv =
(2*R*s*C+1)/(3*R*s*C+1)
```

Answers:

$$Z_{\text{IN}}(s) = \frac{3RCs+1}{Cs}$$

$$T_{\text{V}}(s) = \frac{2RCs+1}{3RCs+1}$$

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Problem 11-2

Find the driving point impedance seen by the voltage source in Figure P11-2 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

Solution:

Find the equivalent impedance seen by the source and use voltage division to find the relationship between the input and output voltages. The solution is presented in the following MATLAB code.

```
clear all
syms s R L
ZL = s*L;
R1 = R;
R2 = 2*R;
Z1 = 1/(1/R2 + 1/ZL);
Z2 = R1;
Zin = simplify(Z1+Z2)
Tv = simplify(Z2/(Z1+Z2))
Zin =
R*(3*s*L+2*R)/(s*L+2*R)
Tv =
(s*L+2*R)/(3*s*L+2*R)
```

Answers:

$$Z_{\text{IN}}(s) = \frac{R(2R+3Ls)}{2R+Ls}$$

$$T_{\text{V}}(s) = \frac{2R+Ls}{2R+3Ls}$$

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Problem 11-3

Find the driving point impedance seen by the voltage source in Figure P11-3 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

Solution:

The input impedance is the parallel combination of the capacitor and resistor R_1. We can find the transfer function by treating the OP AMP as an inverting amplifier and finding the input and feedback impedances.

```
clear all
syms s R1 R2 R3 C V1 V2
ZC = 1/s/C;
Zin = simplify(1/(1/R1+1/ZC))
Zf = simplify(1/(1/R2+1/R3));
Tv = simplify(-Zf/Zin)
Zin =
R1/(1+s*C*R1)
Tv =
-R2*R3/(R3+R2)/R1*(1+s*C*R1)
```

Answers:

$$Z_{\text{IN}}(s) = \frac{R_1}{R_1 Cs + 1}$$

$$T_{\text{V}}(s) = \frac{-R_2 R_3 (R_1 Cs + 1)}{R_1 (R_2 + R_3)}$$

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Problem 11-4

Find the driving point impedance seen by the voltage source in Figure P11-4 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

Solution:

For an ideal OP AMP, the input current is zero, so the impedance seen by the voltage source is infinite. The circuit is configured as a non-inverting amplifier.

```
clear all
syms s R C
ZC = 1/s/C;
R1 = R;
R2 = 2*R;
Z1 = R1;
Z2 = 1/(1/ZC + 1/R2);
Tv = simplify((Z1+Z2)/Z2)
Tv =
R*s*C+3/2
```

Answers:

$Z_{IN}(s) = \infty$

$$T_V(s) = \frac{2RCs + 3}{2}$$

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Problem 11-5

Find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ in Figure P11-5.

Solution:

Use node-voltage analysis to find an expression for $V_2(s)$ in terms of $V_1(s)$.

```
clear all
syms s L R C V1 VA V2
ZL = s*L;
ZC = 1/s/C;
R1 = R;
R2 = 2*R;
Eqn1 = (VA-V1)/ZL + VA/R1 + (VA-V2)/R2;
Eqn2= (V2-VA)/R2 + V2/ZC;
Soln = solve(Eqn1,Eqn2,VA,V2);
V2 = simplify(Soln.V2);
Tv = simplify(V2/V1)
Tv =
R/(2*R^2*s*C+R+3*s^2*L*R*C+s*L)
```

Answer:

$$T_V(s) = \frac{R}{3RLCs^2 + (2R^2C + L)s + R}$$

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Problem 11-6

Find the driving point impedance seen by the voltage source in Figure P11-6 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$. Insert a follower at A and repeat.

Solution:

For the circuit as given, use circuit reduction to find the equivalent impedance and then use node-voltage analysis to find the transfer function. With the follower inserted at point A, the input impedance is the series combination of the first resistor with the first inductor. We can find the second transfer function by finding the transfer function on each side of the follower and multiplying the results together.

```
clear all
disp('Original Circuit')
syms s R L V1 VA V2
ZL = s*L;
Zin = simplify(R + 1/(1/ZL + 1/(R+ZL)))
% Node-voltage analysis
Eqn1 = (VA-V1)/R + VA/ZL + (VA-V2)/R;
Eqn2 = (V2-VA)/R + V2/ZL;
Soln = solve(Eqn1,Eqn2,VA,V2);
V2 = simplify(Soln.V2);
Tv = simplify(V2/V1)
Original Circuit
Zin =
(R^2+3*R*s*L+s^2*L^2)/(R+2*s*L)
Tv =
s^2*L^2/(R^2+3*R*s*L+s^2*L^2)

disp('Circuit with follower')
Zin = simplify(R+ZL)
T1 = ZL/(R+ZL);
T2 = ZL/(R+ZL);
Tv = simplify(T1*T2)
Circuit with follower
Zin =
R+s*L
Tv =
s^2*L^2/(R+s*L)^2
```

Answers:

For the original circuit:

$$Z_{IN}(s) = \frac{L^2s^2 + 3RLs + R^2}{2Ls + R} \quad \text{and} \quad T_V(s) = \frac{L^2s^2}{L^2s^2 + 3RLs + R^2}$$

For the circuit with the follower inserted:

$$Z_{IN}(s) = R + Ls \quad \text{and} \quad T_V(s) = \left(\frac{Ls}{R + Ls}\right)^2 = \frac{L^2s^2}{L^2s^2 + 2RLs + R^2}$$

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Problem 11-7

Find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ in Figure P11-7. Select values of R and C so that $T_V(s)$ has a pole at $s = -1$ krad/s.

Solution:

The circuit is connected as a non-inverting amplifier.

```
clear all
syms s R C
ZC = 1/s/C;
Z1 = 1/(1/R + 1/ZC);
Z2 = R;
Tv = simplify((Z1+Z2)/Z2)
Tv =
(2+R*s*C)/(1+R*s*C)
```

The poles is located at $s = -1/RC$, so any resistor-capacitor combination that produces RC = 1/1000 s will meet the specifications. If we select a value for the capacitor, we can calculate the corresponding resistor.

```
C = 100e-9
R = 1/(1000*C)
Check_RC = R*C
C =
   100.0000e-009
R =
    10.0000e+003
Check_RC =
     1.0000e-003
```

Answers:

$$T_V(s) = \frac{RCs + 2}{RCs + 1}$$

Choose any combination that produces RC = 1/1000 s. For example R = 10 kΩ and C = 100 nF.

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Problem 11-8

Find the current transfer function $T_I(s) = I_2(s)/I_1(s)$ in Figure P11-8. Select values of R and L so that $T_I(s)$ has a pole at $s = -500$ rad/s.

Solution:

Use current division to find the transfer function.

```
clear all
syms s R L
Z1 = R;
Z2 = 2*R+s*L;
TI = simplify(1/Z2/(1/Z1+1/Z2))
TI =
R/(s*L+3*R)
```

The pole is located at $s = -3R/L$. Choose a value for the inductor and calculate the corresponding resistor value.

```
p = 500;
L = 1
R = p*L/3
L =
     1.0000e+000
R =
   166.6667e+000
```

Answers:

$$T_I(s) = \frac{R}{Ls + 3R}$$

One possible combination to achieve the desired poles is $R = 166.67\ \Omega$ and $L = 1$ H.

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Problem 11-9

Find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ of the cascade connection in Figure P11-9. Locate the poles and zeros of the transfer function.

Solution:

Both OP AMP circuits are connected as inverting amplifiers, but they have different values for their components. Find the transfer functions associated with each OP AMP and then multiply the results together.

```
clear all
syms s T1 T2 Tv
% First OP AMP
Z1 = 10e3;
ZC = 1/s/100e-9;
Z2 = 1/(1/10e3 + 1/ZC);
T1 = -Z2/Z1;
% Second OP AMP
Z1 = 10e3;
ZC = 1/s/100e-9;
Z2 = 1/(1/100e3 + 1/ZC);
T2 = -Z2/Z1;
% Combine the results
Tv = simplify(T1*T2)
Tv =
1000000/(1000+s)/(100+s)
```

Answers:

$$T_V(s) = \frac{10^6}{(s+100)(s+1000)}$$

The poles are located at $s = -100$ rad/s and -1000 rad/s. There are no finite zeros.

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Problem 11-10

Find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ of the cascade connection in Figure P11-10. Locate the poles and zeros of the transfer function.

Solution:

The first part of the circuit (to the left of the port connection) can be analyzed as a voltage divider, since the OP AMP will not draw any current from it. The OP AMP is connected as a non-inverting amplifier. We can multiply the individual transfer functions together to get the overall transfer function.

```
clear all
syms s T1 T2 Tv
% R-C circuit
ZC = 1/s/100e-9;
T1 = 20e3/(20e3+ZC);
% OP AMP circuit
Z1 = 10e3;
Z2 = 10e3+1/s/200e-9;
T2 = (Z1+Z2)/Z2;
% Combine the results
Tv = simplify(T1*T2)
Tv_num = vpa(Tv,6)
Tv_num2 = simplify(Tv_num)
Tv =
2*s*(590295810358705625*s+147573952589676412928)/(s+500)/(590295810358705625*s
+295147905179352825856)
Tv_num =
2.*s*(.590296e18*s+.147574e21)/(s+500.)/(.590296e18*s+.295148e21)
Tv_num2 =
2.*s*(s+250.)/(s+500.)^2
```

Answers:

$$T_V(s) = \frac{2s(s+250)}{(s+500)^2}$$

There are two poles and both are located at $s = -500$ rad/s. The zeros are located at $s = 0$ rad/s and -250 rad/s.

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Problem 11-11
Find the input impedance Z(s) in Figure P11-10.

Solution:
The ideal OP AMP draws no current from the initial *RC* circuit, so the input impedance is just the series combination of the first capacitor and resistor.

```
clear all
syms s
Zin = simplify(1/s/100e-9 + 20e3)
Zin =
20000*(s+500)/s
```

Answer:

$$Z(s) = \frac{20000\,(s+500)}{s}\ \Omega.$$

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Problem 11-12

Find $v_2(t)$ in Figure P11-12 when $v_1(t) = \delta(t)$. Repeat for $v_1(t) = u(t)$.

Solution:

Convert the circuit to the *s* domain and find the transfer function. Use the transfer function to find the impulse response and step response.

```
clear all
format short eng
syms s t Ts Hs Gs ht gt
R1 = 150;
R2 = 50;
L = 100e-3;
ZL = s*L;
Ts = R2/(R1+ZL+R2);
Hs = Ts;
ht = ilaplace(Hs)
Gs = Hs/s;
gt = ilaplace(Gs)
ht =
500*exp(-2000*t)
gt =
1/4-1/4*exp(-2000*t)
```

Answers:

For $v_1(t) = \delta(t)$, $v_2(t) = 500e^{-2000\,t}\,u(t)$.

For $v_1(t) = u(t)$, $v_2(t) = \frac{1}{4} - \frac{1}{4}e^{-2000\,t}\,u(t)$.

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Problem 11-13

Find $v_2(t)$ in Figure P11-13 when $v_1(t) = \delta(t)$. Repeat for $v_1(t) = u(t)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t Ts Hs Gs ht gt
C = 1e-6;
ZC = 1/s/C;
R1 = 5e3;
R2 = 10e3;
Z1 = 1/(1/R1+1/ZC);
Ts = R2/(Z1+R2);
Hs = Ts;
ht = ilaplace(Hs)
Gs = Hs/s;
gt = ilaplace(Gs)
ht =
dirac(t)-100*exp(-300*t)
gt =
1/3*exp(-300*t)+2/3
```

Answers:

For $v_1(t) = \delta(t)$, $v_2(t) = \delta(t) - 100e^{-300t}\,u(t)$.

For $v_1(t) = u(t)$, $v_2(t) = \frac{1}{3}\left[2 + e^{-300\,t}\right]\mu(t)$.

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Problem 11-14

Find $h(t)$ and $g(t)$ for the circuit in Figure P11-14.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t Ts Hs Gs ht gt
R = 1e3;
L = 2;
C = 2e-6;
ZL = s*L;
ZC = 1/s/C;
Z1 = R+ZL;
Z2 = 1/(1/R+1/ZC);
Ts = Z2/(Z1+Z2);
ht = ilaplace(Ts);
ht_num = vpa(ht,6)
Gs = Ts/s;
gt = ilaplace(Gs);
gt_num = vpa(simplify(vpa(gt,6)),4)
ht_num =
500.000*exp(-500.000*t)*sin(500.000*t)
gt_num =
.5000-.5000*exp(-500.*t)*cos(500.*t)-.5000*exp(-500.*t)*sin(500.*t)
```

Answers:

$$h(t) = 500e^{-500\,t}\sin(500t)\,u(t)$$

$$g(t) = \left\{\frac{1}{2} - \frac{1}{2}e^{-500\,t}\left[\cos(500t) + \sin(500t)\right]\right\}u(t)$$

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Problem 11-15
Find $h(t)$ and $g(t)$ for the circuit in Figure P11-15.

Solution:
The solution is presented in the following MATLAB code.

```
clear all
syms s t Ts Hs Gs ht gt
Z1 = 10e3;
C = 20e-9;
ZC = 1/s/C;
Z2 = 1/(1/100e3 + 1/ZC);
Ts = -Z2/Z1;
ht = ilaplace(Ts);
ht_num = vpa(ht,6)
Gs = Ts/s;
gt = ilaplace(Gs);
gt_num = vpa(gt,6)
ht_num =
-5000.00*exp(-500.000*t)
gt_num =
-10.+10.*exp(-500.000*t)
```

Answers:

$$h(t) = -5000e^{-500\,t}u(t)$$

$$g(t) = 10\left[e^{-500\,t} - 1\right]u(t)$$

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Problem 11-16

Find $v_2(t)$ in Figure P11-16 when $v_1(t) = \delta(t)$. Repeat for $v_1(t) = u(t)$.

Solution:

The circuit is connected as a non-inverting amplifier. The solution is presented in the following MATLAB code.

```
clear all
syms s t Ts Hs Gs ht gt
C = 2e-6;
ZC = 1/s/C;
Z1 = 1/(1/10e3 + 1/ZC);
Z2 = 50e3;
Ts = (Z1+Z2)/Z2;
ht = ilaplace(Ts);
ht_num = vpa(ht,6)
Gs = Ts/s;
gt = ilaplace(Gs);
gt_num = vpa(gt,6)
ht_num =
10.0000*exp(-50.0000*t)+dirac(t)
gt_num =
-.200000*exp(-50.0000*t)+1.20000
```

Answers:

For $v_1(t) = \delta(t)$, $v_2(t) = \delta(t) + 10e^{-50t}\,u(t)$.

For $v_1(t) = u(t)$, $v_2(t) = \left[1.2 - 0.2e^{-50t}\right]u(t)$.

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Problem 11-17

The impulse response of a linear circuit is $h(t) = (30e^{!20t} ! 30e^{!10t})u(t)$. Find the circuit's step response $g(t)$, impulse response transform $H(s)$, step response transform $G(s)$, and the circuit's transfer function $T(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t ht gt Hs Gs Ts
ht = 30*exp(-20*t) - 30*exp(-10*t);
Hs = laplace(ht)
Ts = Hs
Gs = Hs/s
gt = ilaplace(Gs)
Hs =
-300/(s+20)/(s+10)
Ts =
-300/(s+20)/(s+10)
Gs =
-300/(s+20)/(s+10)/s
gt =
-3/2*exp(-20*t)-3/2+3*exp(-10*t)
```

Answers:

$$g(t) = \frac{3}{2}\left[2e^{-10t} - e^{-20t} - 1\right]u(t)$$

$$H(s) = \frac{-300}{(s+10)(s+20)}$$

$$G(s) = \frac{-300}{s(s+10)(s+20)}$$

$$T(s) = \frac{-300}{(s+10)(s+20)}$$

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Problem 11-18

The impulse response of a linear circuit is $h(t) = 2\delta(t)\ !\ 1000e^{!500\,t}u(t)$. Find the circuit's step response $g(t)$, impulse response transform $H(s)$, step response transform $G(s)$, and the circuit's transfer function $T(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t ht gt Hs Gs Ts
ht = 2*dirac(t) - 1000*exp(-500*t);
Hs = laplace(ht)
Ts = Hs
Gs = Hs/s
gt = ilaplace(Gs)
Hs =
2*s/(s+500)
Ts =
2*s/(s+500)
Gs =
2/(s+500)
gt =
2*exp(-500*t)
```

Answers:

$$g(t) = \left[2e^{-500\,t}\right]u(t)$$

$$H(s) = \frac{2s}{s+500}$$

$$G(s) = \frac{2}{s+500}$$

$$T(s) = \frac{2s}{s+500}$$

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Problem 11-19

The step response of a linear circuit is $g(t) = [1 \ ! \ e^{!1000\,t}]u(t)$. Find the circuit's impulse response $h(t)$, impulse response transform $H(s)$, step response transform $G(s)$, and the circuit's transfer function $T(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t ht gt Hs Gs Ts
gt = 1-exp(-1000*t);
Gs = laplace(gt)
Hs = s*Gs
Ts = Hs
ht = ilaplace(Hs)
Gs =
1000/s/(1000+s)
Hs =
1000/(1000+s)
Ts =
1000/(1000+s)
ht =
1000*exp(-1000*t)
ht2 =
1000*exp(-1000*t)
```

Answers:

$$h(t) = 1000e^{-1000\,t}u(t)$$

$$H(s) = \frac{1000}{s+1000}$$

$$G(s) = \frac{1000}{s(s+1000)}$$

$$T(s) = \frac{1000}{s+1000}$$

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Problem 11-20

The step response of a linear circuit is $g(t) = 10(e^{!20\,t}\ !\ e^{!30\,t})u(t)$. Find the circuit's impulse response $h(t)$, impulse response transform $H(s)$, step response transform $G(s)$, and the circuit's transfer function $T(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t ht gt Hs Gs Ts
gt = 10*(exp(-20*t)-exp(-30*t));
Gs = laplace(gt)
Hs = s*Gs
Ts = Hs
ht = ilaplace(Hs)
Gs =
100/(s+20)/(s+30)
Hs =
100*s/(s+20)/(s+30)
Ts =
100*s/(s+20)/(s+30)
ht =
300*exp(-30*t)-200*exp(-20*t)
```

Answers:

$$h(t) = \left[300e^{-30\,t} - 200e^{-20\,t}\right]u(t)$$

$$H(s) = \frac{100s}{(s+20)(s+30)}$$

$$G(s) = \frac{100}{(s+20)(s+30)}$$

$$T(s) = \frac{100s}{(s+20)(s+30)}$$

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Problem 11-21

The impulse response of a linear circuit is $h(t) = 2000[e^{!2000\,t}]u(t)$. Find the output waveform when the input is $x(t) = 5tu(t)$ V.

Solution:

Convert the input into the *s* domain. Convert the impulse response into the transfer function. Multiply the two quantities in the *s* domain and take the inverse Laplace transform to find the output waveform. The solution is presented in the following MATLAB code.

```
clear all
syms s t ht Hs xt Xs yt Ys
ht = 2000*exp(-2000*t);
Hs = laplace(ht);
xt = 5*t*heaviside(t);
Xs = laplace(xt);
Ys = Xs*Hs;
yt = ilaplace(Ys)
yt =
5*t-1/400+1/400*exp(-2000*t)
```

Answer:

$$y(t) = \left[5t + \frac{1}{400}\left(e^{-2000\,t} - 1\right)\right]u(t) \text{ V}$$

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Problem 11-22

The step response of a linear circuit is $g(t) = 0.5[1 - e^{-100\,t}]u(t)$. Find the output waveform when the input is $v_1(t) = [10e^{-200t}]u(t)$. Use MATLAB to find the Laplace transforms of $g(t)$ and $v_1(t)$. Then find $V_2(s)$. Finally, use the inverse Laplace function to find the waveform $v_2(t)$ and plot the results.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t gt Gs Hs v1t V1s v2t V2s
gt = 0.5*(1-exp(-100*t));
Gs = laplace(gt);
v1t = 10*exp(-200*t);
V1s = laplace(v1t);
Hs = s*Gs;
V2s = V1s*Hs
v2t = ilaplace(V2s)
V2s =
500/(s+200)/(s+100)
v2t =
10*exp(-150*t)*sinh(50*t)

% Plot the output waveform
tt = 0:50e-6:50e-3;
v2tt = subs(v2t,t,tt);
plot(tt,v2tt,'b','LineWidth',3)
grid on;
xlabel('Time (s)');
ylabel('Voltage (V)')
```

Answer:

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$$v_2(t) = 5\left[e^{-100\,t} - e^{-200\,t}\right]u(t)$$

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Problem 11-23

The step response of a linear circuit is $g(t) = 10[e^{!100\,t} \sin 200t]u(t)$. Find the circuit's impulse response $h(t)$, impulse response transform $H(s)$, step response transform $G(s)$, and the circuit's transfer function $T(s)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t ht gt Hs Gs Ts
gt = 10*exp(-100*t)*sin(200*t);
Gs = laplace(gt)
Hs = s*Gs
Ts = Hs
ht = ilaplace(Hs)
Gs =
2000/(s^2+200*s+50000)
Hs =
2000*s/(s^2+200*s+50000)
Ts =
2000*s/(s^2+200*s+50000)
ht =
1000*exp(-100*t)*(2*cos(200*t)-sin(200*t))
```

Answers:

$$h(t) = 1000e^{-100\,t}\left[2\cos(200t) - \sin(200t)\right]u(t)$$

$$H(s) = \frac{2000s}{s^2 + 200s + 50000}$$

$$G(s) = \frac{2000}{s^2 + 200s + 50000}$$

$$T(s) = \frac{2000s}{s^2 + 200s + 50000}$$

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Problem 11-24

The impulse response transform of a linear circuit is $H(s) = (s + 2000)/(s + 1000)$. Find the output waveform when the input is $x(t) = 5e^{-1000\,t}\,u(t)$. Use MATLAB to find the Laplace transform of $x(t)$. Then find $Y(s)$. Finally, use the inverse Laplace function to find the waveform $y(t)$ and plot the results.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t Hs xt Xs yt Ys
Hs = (s+2000)/(s+1000);
xt = 5*exp(-1000*t);
Xs = laplace(xt);
Ys = Xs*Hs
yt = ilaplace(Ys)
Ys =
5/(1000+s)^2*(s+2000)
yt =
5*(1+1000*t)*exp(-1000*t)

% Plot the results
tt = 0:5e-6:10e-3;
ytt = subs(yt,t,tt);
plot(tt,ytt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answer:

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$$y(t) = \left[5(1+1000t)e^{-1000\,t}\right]u(t) \ \text{ V}$$

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Problem 11-25

The impulse response of a linear circuit is $h(t) = 20u(t) + \delta(t)$. Find the output waveform $y(t)$ when the input is $x(t) = [e^{!20\,t}]u(t)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t ht Hs xt Hs yt Ys
ht = 20*heaviside(t)+dirac(t);
Hs = laplace(ht);
xt = exp(-20*t);
Xs = laplace(xt);
Ys = Xs*Hs;
yt = ilaplace(Ys)
yt =
1
```

Answer:

$y(t) = u(t)$

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Problem 11-26

The circuit in Figure P11-26 is in the steady state with $v_1(t) = 5 \cos 225.08t$ V. Find $v_{2SS}(t)$. Repeat for $v_1(t) = 5 \cos 10\text{ k}t$ V. And without doing any calculations, repeat for $v_1(t) = 5$ V.

Solution:

Find the transfer function for the circuit. Evaluate the transfer function at $j\omega$, where ω is the frequency of the input signal. Use the magnitude and phase of the result to scale and shift the input sinusoid to find the output sinusoid.

```
clear all
format short eng
syms s w
R = 100;
L = 25e-3;
C = 20e-6;
ZL = s*L;
ZC = 1/s/C;
Ts = (ZL+ZC)/(R+ZL+ZC);
V1 = 5;
w1 = 225.08;
p1 = 0;
Tjw = subs(Ts,s,j*w1);
Magv2 = V1*abs(Tjw)
Phasev2 = p1 + 180*angle(Tjw)/pi
Magv2 =
     4.5392e+000
Phasev2 =
   -24.7903e+000

% Repeat for the other signal
V1 = 5;
w1 = 10e3;
p1 = 0;
Tjw = subs(Ts,s,j*w1);
Magv2 = V1*abs(Tjw)
Phasev2 = p1 + 180*angle(Tjw)/pi
Magv2 =
     4.6292e+000
Phasev2 =
    22.2035e+000
```

For $v_1(t) = 5$ V, the inductor acts like a short circuit and the capacitor acts like an open circuit, so the output signal will be 5 V.

Answers:

For $v_1(t) = 5 \cos 225.08t$ V, $v_{2SS}(t) = 4.5392 \cos(225.08t - 24.7903°)$ V.

For $v_1(t) = 5 \cos 10\text{ k}t$ V, $v_{2SS}(t) = 4.6292 \cos(10\text{ k}t + 22.2035°)$ V.

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For $v_1(t)$ = 5 V, $v_{2SS}(t)$ = 5 V.

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Problem 11-27

The circuit in Figure P11-27 is in the steady state with $v_1(t) = 10 \cos 2000t$ V. Find $v_{2SS}(t)$. Repeat for $v_1(t) = 3 \cos 10\text{ k}t$ V.

Solution:

Find the transfer function for the circuit. Evaluate the transfer function at $j\omega$, where ω is the frequency of the input signal. Use the magnitude and phase of the result to scale and shift the input sinusoid to find the output sinusoid.

```
clear all
format short eng
syms s w
R = 10;
L = 5e-3;
C = 50e-6;
ZL = s*L;
ZC = 1/s/C;
Ts = ZL/(R+ZL+ZC);
V1 = 10;
w1 = 2000;
p1 = 0;
Tjw = subs(Ts,s,j*w1);
Magv2 = V1*abs(Tjw)
Phasev2 = p1 + 180*angle(Tjw)/pi
Magv2 =
    10.0000e+000
Phasev2 =
    90.0000e+000

% Repeat for the second signal
V1 = 3;
w1 = 10e3;
p1 = 0;
Tjw = subs(Ts,s,j*w1);
Magv2 = V1*abs(Tjw)
Phasev2 = p1 + 180*angle(Tjw)/pi
Magv2 =
     3.0593e+000
Phasev2 =
    11.7683e+000
```

Answers:

For $v_1(t) = 10 \cos 2000t$ V, $v_{2SS}(t) = 10 \cos(2000t + 90°)$ V

For $v_1(t) = 3 \cos 10\text{ k}t$ V, $v_{2SS}(t) = 3.0593 \cos(10\text{ k}t + 11.7683°)$ V

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Problem 11-28

The circuit in Figure P11-28 is in the steady state with $i_1(t) = 10 \cos 500t$ mA, $R_1 = 100\ \Omega$, $R_2 = 400\ \Omega$, and $L = 100$ mH. Find $i_{2SS}(t)$. Repeat for $i_1(t) = 10 \cos\ 5000t$ mA.

Solution:

Use current division to find the transfer function for the circuit. Evaluate the transfer function at $j\omega$, where ω is the frequency of the input signal. Use the magnitude and phase of the result to scale and shift the input sinusoid to find the output sinusoid.

```
clear all
syms s
R1 = 100;
R2 = 400;
L = 100e-3;
ZL = s*L;
Ts = -1/(R2+ZL)/(1/R1 + 1/(R2+ZL));
I1 = 10e-3;
w1 = 500;
p1 = 0;
Tjw = subs(Ts,s,j*w1);
Magi2 = I1*abs(Tjw)
Phasei2 = p1 + 180*angle(Tjw)/pi
Ts =
-1000/(5000+s)
Magi2 =
     1.9901e-003
Phasei2 =
   174.2894e+000

% Repeat for the other input signal
I1 = 10e-3;
w1 = 5000;
p1 = 0;
Tjw = subs(Ts,s,j*w1);
Magi2 = I1*abs(Tjw)
Phasei2 = p1 + 180*angle(Tjw)/pi
Magi2 =
     1.4142e-003
Phasei2 =
   135.0000e+000
```

Answers:

For $i_1(t) = 10 \cos 500t$ mA, $i_{2SS}(t) = 1.9901 \cos(500t + 174.2894°)$ mA.

For $i_1(t) = 10 \cos\ 5000t$ mA, $i_{2SS}(t) = 1.4142 \cos(500t + 135°)$ mA.

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Problem 11-29

The OrCAD circuit drawn in Figure P11-29 is in the steady state with $i_1(t) = 5 \cos 1000t$ mA, $R = 1$ kΩ, $L = 2$ H, and $C = 500$ nF. Use OrCAD to find $i_{2SS}(t)$. Copy the figure in OrCAD. Draw your circuit and use IAC for a source. The IPRINT element on the right side of the figure is an ammeter and it comes from the Special Library. It has a polarity as indicated by the small "–" on the element. You should ensure that it will read the correct current direction. Before you can use the IPRINT element, it needs to be set up to display the current magnitude and phase. Double click on the element and in its property editor place a "y" in AC, MAG, and PHASE boxes. Click the Apply button and close the property editor. To obtain the simulation select AC Sweep/Noise. Set the start frequency at 159 Hz (1000 rad/s) and the stop frequency at the same. Place a 1 in Points/Decade. Run the simulation. Under View, select Output File. Scroll down until you see the IPRINT output. Repeat for $i_1(t) = 5 \cos 10\text{ k}t$ mA.

Solution:

Create the above circuit in OrCAD and run the simulations specified in the problem.

For a source frequency of 1000 rad/s or 159 Hz, the results are:

```
FREQ        IM(V_PRINT1)IP(V_PRINT1)
1.592E+02   5.000E-03   1.800E+02
```

The corresponding output signal is $i_{2SS}(t) = 5 \cos (1000t + 180°)$ mA.

For a source frequency of 10 krad/s or 1592 Hz, the results are:

```
FREQ        IM(V_PRINT1)IP(V_PRINT1)
1.592E+03   9.901E-04   1.014E+02
```

The corresponding output signal is $i_{2SS}(t) = 0.990 \cos (10\text{ k}t + 101.4°)$ mA.

We can verify these results with the following MATLAB calculations. Note that following the passive sign convention for the current source as oriented in the OrCAD simulation, the phase of the source is 180°, which means the current flows down through the current source.

```
clear all
syms s
R = 1e3;
```

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```
L = 2;
C = 500e-9;
ZL = s*L;
ZC = 1/s/C;
% Use current division to find the transfer function
Ts = 1/R/(1/R + 1/ZL + 1/ZC);
I1 = 5e-3;
w1 = 1000;
p1 = 180;
Tjw = subs(Ts,s,j*w1);
Magi2 = I1*abs(Tjw)
Phasei2 = p1 + 180*angle(Tjw)/pi
Magi2 =
     5.0000e-003
Phasei2 =
   180.0000e+000

% Repeat for the other input signal
I1 = 5e-3;
w1 = 10000;
p1 = 180;
Tjw = subs(Ts,s,j*w1);
Magi2 = I1*abs(Tjw)
Phasei2 = p1 + 180*angle(Tjw)/pi
Magi2 =
   990.0990e-006
Phasei2 =
   101.4212e+000
```

The MATLAB results verify the OrCAD simulations.

Answers:
For $i_1(t) = 5 \cos 1000t$ mA, $i_{2SS}(t) = 5 \cos(1000t + 180°)$ mA.

For $i_1(t) = 5 \cos 10\ \text{k}t$ mA, $i_{2SS}(t) = 0.990 \cos(10\ \text{k}t + 101.4°)$ mA.

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Problem 11-30

The circuit in Figure P11-30 is in the steady state with $i_1(t) = 10 \cos 5000t$ mA. Find the steady-state voltage $v_{2SS}(t)$. Repeat for $i_1(t) = 5 \cos 2500t$ mA.

Solution:

Use current division to find the output current and then Ohm's law to find the voltage. Combine the results to get the overall transfer function.

```
clear all
syms s
R = 10e3;
C = 10e-9;
ZC = 1/s/C;
T1 = 1/(R+ZC)/(1/R + 1/(R+ZC));
Ts = R*T1;
I1 = 10e-3;
w1 = 5000;
p1 = 0;
Tjw = subs(Ts,s,j*w1);
Magv2 = I1*abs(Tjw)
Phasev2 = p1 + 180*angle(Tjw)/pi
Magv2 =
    35.3553e+000
Phasev2 =
    45.0000e+000

% Repeat for the other signal
I1 = 5e-3;
w1 = 2500;
p1 = 0;
Tjw = subs(Ts,s,j*w1);
Magv2 = I1*abs(Tjw)
Phasev2 = p1 + 180*angle(Tjw)/pi
Magv2 =
    11.1803e+000
Phasev2 =
    63.4349e+000
```

Answers:

For $i_1(t) = 10 \cos 5000t$ mA, $v_{2SS}(t) = 35.3553 \cos(5000t + 45°)$ V.

For $i_1(t) = 5 \cos 2500\, t$ mA, $v_{2SS}(t) = 11.1803 \cos(2500t + 63.4349°)$ V.

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Problem 11-31

The transfer function of a linear circuit is $T(s) = (s + 500)'(s + 50)$. Find the sinusoidal steady-state output for an input $x(t) = 5 \cos 500t$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t xt Xs Ts yt Ys
Ts = (s+500)/(s+50);
mx = 5;
wx = 500;
px = 0;
Tjw = subs(Ts,s,j*wx);
Mag_y = mx*abs(Tjw)
Phase_y = px + 180*angle(Tjw)/pi
Mag_y =
     7.0360e+000
Phase_y =
   -39.2894e+000

% Verify the results using another approach
xt = 5*cos(500*t);
Xs = laplace(xt);
Ys = Xs*Ts;
yt = ilaplace(Ys)
yt =
550/101*cos(500*t)+450/101*sin(500*t)-45/101*exp(-50*t)
```

The exponential portion of the response will decay to zero in steady state. Combine the cosine and sine terms into a single cosine expression

```
Mag_y2 = sqrt((550/101)^2+(450/101)^2)
Phase_y2 = 180*atan2(-450/101,550/101)/pi
Mag_y2 =
     7.0360e+000
Phase_y2 =
   -39.2894e+000
```

The results confirm the previous answer.

Answer:

$y_{SS}(t) = 7.036 \cos(500t - 39.29°)$

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Problem 11-32

The step response of a linear circuit is $g(t) = [5e^{!500\,t}]u(t)$. Find the sinusoidal steady-state output for an input $x(t) = 2\cos 1000t$.

Solution:

Use the step response to find the transfer function and then evaluate the transfer function at the frequency of the input signal.

```
clear all
syms s t
gt = 5*exp(-500*t);
Gs = laplace(gt);
Ts = s*Gs;
mx = 2;
wx = 1000;
px = 0;
Tjw = subs(Ts,s,j*wx);
Mag_y = mx*abs(Tjw)
Phase_y = px + 180*angle(Tjw)/pi
Mag_y =
     8.9443e+000
Phase_y =
    26.5651e+000
```

Answer:

$y_{SS}(t) = 8.9443\cos(500t + 26.5651°)$

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Problem 11-33

The step response of a linear circuit is $g(t) = [2e^{+100\,t}]u(t)$. Find the sinusoidal steady-state output for an input $x(t) = 5 \cos 500t$.

Solution:

If we use the same approach as in the previous problem, we get an answer that appears to be reasonable, as shown in the following MATLAB calculations.

```
clear all
syms s t
gt = 2*exp(100*t);
Gs = laplace(gt);
Ts = s*Gs;
mx = 5;
wx = 500;
px = 0;
Tjw = subs(Ts,s,j*wx);
Mag_y = mx*abs(Tjw)
Phase_y = px + 180*angle(Tjw)/pi
Mag_y =
     9.8058e+000
Phase_y =
   -11.3099e+000
```

This result is not valid, because the transfer function is not stable. Analyze the results in the Laplace domain as shown below.

```
% Examine the results using another approach
xt = 5*cos(500*t);
Xs = laplace(xt);
Ys = Xs*Ts;
yt = ilaplace(Ys)
yt =
125/13*cos(500*t)+25/13*sin(500*t)+5/13*exp(100*t)
```

The output response has an increasing exponential, so there is no steady-state response.

Answer:

The circuit is not stable, so there is no steady-state response.

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Problem 11-34

The impulse response of a linear circuit is $h(t) = [800e^{-1000t}]u(t) - \delta(t)$. Find the sinusoidal steady-state output for an input $x(t) = 3\cos 400t$.

Solution:

Use the impulse response to find the transfer function and then evaluate the transfer function at the frequency of the input signal.

```
clear all
syms s t
ht = 800*exp(-1000*t) - dirac(t);
Ts = laplace(ht);
mx = 3;
wx = 400;
px = 0;
Tjw = subs(Ts,s,j*wx);
Mag_y = mx*abs(Tjw)
Phase_y = px + 180*angle(Tjw)/pi
Mag_y =
     1.2457e+000
Phase_y =
  -138.3665e+000

% Examine the results using another approach
xt = 3*cos(400*t);
Xs = laplace(xt);
Ys = Xs*Ts;
yt = ilaplace(Ys)
yt =
-60/29*exp(-1000*t)-27/29*cos(400*t)+24/29*sin(400*t)

Mag_y2 = sqrt((-27/29)^2+(24/29)^2)
Phase_y2 = 180*atan2(-24/29,-27/29)/pi
Mag_y2 =
     1.2457e+000
Phase_y2 =
  -138.3665e+000
```

Answer:

$y_{SS}(t) = 1.2457\cos(400t - 138.3665°)$

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Problem 11-35

The impulse response of a linear circuit is $h(t) = 400[e^{!100t} ! e^{!400t}]u(t)$. Use MATLAB to find the sinusoidal steady-state output for an input $x(t) = 5 \cos 200t$. Use MATLAB to plot $y(t)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t
ht = 400*(exp(-100*t)-exp(-400*t));
Ts = laplace(ht);
mx = 5;
wx = 200;
px = 0;
Tjw = subs(Ts,s,j*wx);
Mag_y = mx*abs(Tjw)
Phase_y = px + 180*angle(Tjw)/pi
Mag_y =
     6.0000e+000
Phase_y =
   -90.0000e+000

% Examine the results using another approach
xt = 5*cos(200*t);
Xs = laplace(xt);
Ys = Xs*Ts;
yt = ilaplace(Ys)
yt =
6*sin(200*t)-8*exp(-250*t)*sinh(150*t)

% Plot the input and the output
tt = 0:100e-6:100e-3;
xtt = subs(xt,t,tt);
ytt = subs(yt,t,tt);
plot(tt,xtt,'b','LineWidth',3)
hold on
plot(tt,ytt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Output')
legend('x(t)','y(t)')
```

After the initial transient response (approximately 50 ms), the output signal has the correct amplitude and phase shift compared to the input signal.

Answer:

$y_{SS}(t) = 6 \cos(200t - 90°)$

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The MATLAB results are shown above.

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Problem 11-36

The step response of a linear circuit is $g(t) = [e^{-60t} \sin 80t]u(t)$. Find the sinusoidal steady-state response for an input $x(t) = 20 \cos 100t$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t
gt = exp(-60*t)*sin(80*t);
Gs = laplace(gt);
Ts = s*Gs;
mx = 20;
wx = 100;
px = 0;
Tjw = subs(Ts,s,j*wx);
Mag_y = mx*abs(Tjw)
Phase_y = px + 180*angle(Tjw)/pi
Mag_y =
    13.3333e+000
Phase_y =
     0.0000e+000

% Examine the results using another approach
xt = 20*cos(100*t);
Xs = laplace(xt);
Ys = Xs*Ts;
yt = ilaplace(Ys)
yt =
40/3*cos(100*t)+10/3*(-4*cos(80*t)+3*sin(80*t))*exp(-60*t)
```

Answer:

$y_{SS}(t) = 13.3333 \cos(100t)$

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Problem 11-37

The step response of a linear circuit is $g(t) = [1 - 20te^{-10t}]u(t)$. Find the sinusoidal steady-state response for an input $x(t) = 50 \cos 10t$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t
gt = heaviside(t)-20*t*exp(-10*t);
Gs = laplace(gt);
Ts = s*Gs;
mx = 50;
wx = 10;
px = 0;
Tjw = subs(Ts,s,j*wx);
Mag_y = mx*abs(Tjw)
Phase_y = px + 180*angle(Tjw)/pi
Mag_y =
     0.0000e+000
Phase_y =
     0.0000e+000

% Examine the results using another approach
xt = 50*cos(10*t);
Xs = laplace(xt);
Ys = Xs*Ts;
yt = ilaplace(Ys)
yt =
-50*(-1+10*t)*exp(-10*t)
```

Answer:

The sinusoidal steady-state response vanishes for the given input signal.

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Problem 11-38

The impulse response of a linear circuit is $h(t) = [u(t) - u(t\ !\ 2)]$. Use the convolution integral to find the response due to an input $x(t) = u(t\ !\ 2)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s
syms t tau real
syms T positive
% Create the impulse response and the input signal
ht = heaviside(t)-heaviside(t-2);
xt = heaviside(t-2);
% Convert the signals to the convolution integral form
httau = subs(ht,t,T-tau);
xtau = subs(xt,t,tau);
hx = simplify(httau*xtau);
% Perform the convolution integral to generate the output
yt = int(hx,tau,0,T)

% Check the results using Laplace techniques
Hs = laplace(ht);
Xs = laplace(xt);
Ys = Xs*Hs;
yt2 = ilaplace(Ys)
yt =
PIECEWISE([0, T < 2],[T-2, T < 4],[2, 4 <= T])
yt2 =
heaviside(t-2)*(t-2)-heaviside(t-4)*(t-4)
```

Answer:

$$y(t) = (t-2)u(t-2) - (t-4)u(t-4) = r(t-2) - r(t-4)$$

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Problem 11-39

The impulse response of a linear circuit is $h(t) = [u(t) - u(t\ !\ 1)]$. Use the convolution integral to find the response due to an input $x(t) = u(t)\ !\ u(t\ !\ 2)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s
syms t tau real
syms T positive
% Create the impulse response and the input signal
ht = heaviside(t)-heaviside(t-1);
xt = heaviside(t)-heaviside(t-2);
% Convert the signals to the convolution integral form
httau = subs(ht,t,T-tau);
xtau = subs(xt,t,tau);
hx = simplify(httau*xtau);
% Perform the convolution integral to generate the output
yt = int(hx,tau,0,T)

% Check the results using Laplace techniques
Hs = laplace(ht);
Xs = laplace(xt);
Ys = Xs*Hs;
yt2 = ilaplace(Ys)
yt =
PIECEWISE([T, T < 1],[1, T < 2],[3-T, T < 3],[0, 3 <= T])
yt2 =
t-heaviside(t-2)*(t-2)-heaviside(t-1)*(t-1)+heaviside(t-3)*(t-3)
```

The output signal is zero for $t < 0$.

Answer:

$$y(t) = tu(t) - (t-1)u(t-1) - (t-2)u(t-2) + (t-3)u(t-3) = r(t) - r(t-1) - r(t-2) + r(t-3)$$

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Problem 11-40

The impulse response of a linear circuit is $h(t) = 2t[u(t) - u(t\ !\ 1)]$. Use the convolution integral to find the response due to an input $x(t) = u(t\ !\ 1)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s
syms t tau real
syms T positive
% Create the impulse response and the input signal
ht = 2*t*(heaviside(t)-heaviside(t-1));
xt = heaviside(t-1);
% Convert the signals to the convolution integral form
httau = subs(ht,t,T-tau);
xtau = subs(xt,t,tau);
hx = simplify(httau*xtau);
% Perform the convolution integral to generate the output
yt = int(hx,tau,0,T)

% Check the results using Laplace techniques
Hs = laplace(ht);
Xs = laplace(xt);
Ys = Xs*Hs;
yt2 = ilaplace(Ys)
yt =
PIECEWISE([0, T <= 1],[2*T*(T-1)-T^2+1, T <= 2],[2*T-T^2+(T-1)^2, 2 < T])
yt2 =
heaviside(t-1)*(t-1)^2-heaviside(t-2)*t*(t-2)
```

Answer:

$$y(t) = (t-1)^2 u(t-1) - (t^2 - 2t)u(t-2)$$

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Problem 11-41

The impulse response of a linear circuit is $h(t) = e^{!\,t}u(t)$. Use the convolution integral to find the response due to an input $x(t) = u(t)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s
syms t tau real
syms T positive
% Create the impulse response and the input signal
ht = exp(-t)*heaviside(t);
xt = heaviside(t);
% Convert the signals to the convolution integral form
httau = subs(ht,t,T-tau);
xtau = subs(xt,t,tau);
hx = simplify(httau*xtau);
% Perform the convolution integral to generate the output
yt = simplify(int(hx,tau,0,T))

% Check the results using Laplace techniques
Hs = laplace(ht);
Xs = laplace(xt);
Ys = Xs*Hs;
yt2 = ilaplace(Ys)
yt =
(exp(T)-1)*exp(-T)
yt2 =
1-exp(-t)
```

Answer:

$y(t) = \left[1 - e^{-t}\right]u(t)$

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Problem 11-42

The impulse response of a linear circuit is $h(t) = t[u(t) - u(t - 1)]$. Use the convolution integral to find the response due to an input $x(t) = e^{-t}u(t)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s
syms t tau real
syms T positive
% Create the impulse response and the input signal
ht = t*(heaviside(t)-heaviside(t-1));
xt = exp(-t)*heaviside(t);
% Convert the signals to the convolution integral form
httau = subs(ht,t,T-tau);
xtau = subs(xt,t,tau);
hx = simplify(httau*xtau);
% Perform the convolution integral to generate the output
yt = simplify(int(hx,tau,0,T))

% Check the results using Laplace techniques
Hs = laplace(ht);
Xs = laplace(xt);
Ys = Xs*Hs;
yt2 = ilaplace(Ys)
yt =
PIECEWISE([T-1+exp(-T), T <= 1],[exp(-T), 1 < T])
yt2 =
-1+t+exp(-t)-heaviside(t-1)*(-exp(-t+1)-2*exp(-1/2*t+1/2)*sinh(1/2*t-1/2)+t)
```

Answer:

$$y(t) = \left[t - 1 + e^{-t}\right]u(t) - \left[t - 1\right]u(t - 1)$$

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Problem 11-43

The impulse response of a linear circuit is $h(t) = 2e^{!t}u(t)$. Use the convolution integral to find the response due to an input $x(t) = tu(t)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s
syms t tau real
syms T positive
% Create the impulse response and the input signal
ht = 2*exp(-t)*heaviside(t);
xt = t*heaviside(t);
% Convert the signals to the convolution integral form
httau = subs(ht,t,T-tau);
xtau = subs(xt,t,tau);
hx = simplify(httau*xtau);
% Perform the convolution integral to generate the output
yt = simplify(int(hx,tau,0,T))

% Check the results using Laplace techniques
Hs = laplace(ht);
Xs = laplace(xt);
Ys = Xs*Hs;
yt2 = ilaplace(Ys)
yt =
2*(T*exp(T)-exp(T)+1)*exp(-T)
yt2 =
2*t-2+2*exp(-t)
```

Answer:

$$y(t) = 2\left[t - 1 + e^{-t}\right]u(t)$$

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Problem 11-44

Show that $f(t) * \delta(t) = f(t)$. That is, show that convolving any waveform $f(t)$ with an impulse leaves the waveform unchanged.

Solution:

$$f(t) * \delta(t) = \int_0^t \delta(t-\tau) f(\tau) d\tau$$

The expression $\delta(t - \tau)$ is zero everywhere except when $t = \tau$. We can re-write the integral as follows:

$$f(t) * \delta(t) = \int_{t^-}^{t^+} \delta(t-\tau) f(\tau) d\tau = f(t) \int_{t^-}^{t^+} \delta(t-\tau) d\tau = f(t)[1] = f(t)$$

Where above we have used the property that the integral of $\delta(t)$ over all t is one.

Answer:

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Problem 11-45

Show that if $h(t) = u(t)$, then output $y(t)$ for any input $x(t)$ is $y(t) = \int_0^t x(\tau)\,d\tau$.

That is, a circuit whose impulse response is a step function operates as an integrator.

Solution:

The following set of equalities proves the property.

$$y(t) = \int_0^t h(t-\tau)x(\tau)\,d\tau = \int_0^t u(t-\tau)x(\tau)\,d\tau = \int_0^t (1)\,x(\tau)\,d\tau = \int_0^t x(\tau)\,d\tau$$

Where above we have used the fact that $u(t - \tau) = 1$ for $0 \le \tau \le t$, which are the limits of integration for the convolution integral.

Answer:
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Problem 11-46

Use the convolution integral to show that if the input to a linear circuit is $x(t) = u(t)$ then

$$y(t) = g(t) = \int_0^t h(\tau)\,d\tau$$

That is, show that the step response is the integral of the impulse response.

Solution:

The following set of equalities proves the property.

$$y(t) = \int_0^t x(t-\tau)h(\tau)\,d\tau = \int_0^t u(t-\tau)h(\tau)\,d\tau = \int_0^t (1)h(\tau)\,d\tau = \int_0^t h(\tau)\,d\tau = g(t)$$

Where above we have used the fact that $u(t - \tau) = 1$ for $0 \le \tau \le t$, which are the limits of integration for the convolution integral.

Answer:

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Problem 11-47

If the input to a linear circuit is $x(t) = tu(t)$, then the output $y(t)$ is called the ramp response. Use the convolution integral to show that

$$\frac{dy(t)}{dt} = \int_0^t h(\tau)\,d\tau = g(t)$$

That is, show that the derivative of the ramp response is the step response.

Solution:

The following set of equalities proves the property.

$$y(t) = \int_0^t h(t-\tau)x(\tau)\,d\tau = \int_0^t x(t-\tau)h(\tau)\,d\tau = \int_0^t (t-\tau)u(t-\tau)h(\tau)\,d\tau$$

$$y(t) = \int_0^t (t-\tau)\,(1)\,h(\tau)\,d\tau = t\int_0^t h(\tau)\,d\tau - \int_0^t \tau\,h(\tau)\,d\tau$$

$$\frac{dy(t)}{dt} = th(t) + (1)\int_0^t h(\tau)\,d\tau - th(t) = \int_0^t h(\tau)\,d\tau = g(t)$$

Answer:

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Problem 11-48

The impulse response of a linear circuit is $h(t) = u(t)$. Use MATLAB to compute the convolution integral and find the response due to an input $x(t) = t[u(t) - u(t - 1)]$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s
syms t tau real
syms T positive
% Create the impulse response and the input signal
ht = heaviside(t);
xt = t*(heaviside(t)-heaviside(t-1));
% Convert the signals to the convolution integral form
httau = subs(ht,t,T-tau);
xtau = subs(xt,t,tau);
hx = simplify(httau*xtau);
% Perform the convolution integral to generate the output
yt = simplify(int(hx,tau,0,T))

% Check the results using Laplace techniques
Hs = laplace(ht);
Xs = laplace(xt);
Ys = Xs*Hs;
yt2 = ilaplace(Ys)
yt =
PIECEWISE([1/2*T^2, T <= 1],[1/2, 1 < T])
yt2 =
1/2*t^2-1/2*heaviside(t-1)*(-1+t^2)
```

Answer:

$$y(t) = \frac{1}{2}t^2\ u(t) - \frac{1}{2}\left[t^2 - 1\right]u(t-1)$$

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Problem 11-49

The impulse response transform of a linear circuit is $H(s) = 25/(s + 5)$ and $x(t) = tu(t)$. Use s-domain convolution to find the zero-state response $y(t)$.

Solution:

The solution is presented in the following MATLAB code.

```
clear all
syms s t
Hs = 25/(s+5);
xt = t*heaviside(t);
Xs = laplace(xt);
Ys = Xs*Hs;
yt = ilaplace(Ys)
yt =
5*t-1+exp(-5*t)
```

Answer:

$$y(t) = \left[5t - 1 + e^{-5t}\right]u(t)$$

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Problem 11-50

The impulse responses of two linear circuits are $h_1(t) = e^{!2t}u(t)$ and $h_2(t) = 4e^{!4t}u(t)$. What is the impulse response of a cascade connection of these two circuits?

Solution:

Convert the impulse responses into the corresponding transfer functions by taking their Laplace transforms. Multiply the two transfer functions together and take the inverse Laplace transform of the result to find the impulse response of the cascade connection.

```
clear all
syms s t
h1t = exp(-2*t);
h2t = 4*exp(-4*t);
H1s = laplace(h1t);
H2s = laplace(h2t);
Hs = H1s*H2s;
ht = ilaplace(Hs)
ht =
4*exp(-3*t)*sinh(t)
```

Answer:

$$h(t) = 2\left[e^{-2t} - e^{-4t}\right]u(t)$$

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Problem 11-51

Design a circuit to realize the transfer function below using only resistors, capacitors, and OP AMPs. Scale the circuit so that all resistors are greater than 10 kΩ and all capacitors are less the 1 μF.

$$T_V(s) = \pm \frac{2\times 10^5}{(s+100)(s+10{,}000)}$$

Solution:

Write the transfer function as the product of three terms:

$$T_V(s) = \pm\left(\frac{100}{s+100}\right)\left(\frac{2}{10}\right)\left(\frac{10000}{s+10000}\right) = \pm\left(\frac{\frac{100}{s}}{1+\frac{100}{s}}\right)\left(\frac{2}{10}\right)\left(\frac{\frac{10000}{s}}{1+\frac{10000}{s}}\right)$$

The first and last terms represent voltage division stages with an *RC* circuit, where the resistor is 1 Ω and the capacitor has an impedance of 100/*s* or 10000/*s*. The center term represents a gain of 0.2, which can be realized by an inverting amplifier. The two *RC* stages need to be scaled by 10000, to increase the resistor values to meet specifications (10 kΩ). The first capacitor would then have a value of (1/100)/10000 = 1 μF and the second a value of (1/10000)/10000 = 10 nF. Finally, to prevent loading, the first stage and the amplifier must be separated by a buffer.

The following circuit will meet the specifications:

Answer:

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Problem 11-52

Design a circuit to realize the transfer function below using only resistors, capacitors and not more than one OP AMP. Scale the circuit so that all capacitors are exactly 10 nF.

$$T_V(s) = \pm\frac{100\,(s+1000)}{(s+100)(s+10{,}000)}$$

Solution:

Write the transfer function as the product of two terms

$$T_V(s) = \pm\left(\frac{100}{s+100}\right)\left(\frac{s+1000}{s+10000}\right) = \pm\left(\frac{s+1000}{s+10000}\right)\left(\frac{\frac{100}{s}}{1+\frac{100}{s}}\right)$$

Note that we reversed the order of the terms in the second equality. We can now create circuits to achieve each stage in the circuit as follows:

We need to scale the circuit to make all capacitor values 10 nF. Scale the left stage by 100 M and scale the right stage by 1 M. The resulting circuit is shown below.

Use MATLAB to verify the transfer function of the resulting circuit

```
clear all
syms s
C = 10e-9;
ZC = 1/s/C;
R1 = 100e3;
R2 = 10e3;
R3 = 1e6;
Z1 = 1/(1/R1 + 1/ZC);
Z2 = 1/(1/R2 + 1/ZC);
T1 = -Z2/Z1;
T2 = ZC/(R3+ZC);
T = simplify(T1*T2)
T =
-100*(1000+s)/(s+100)/(10000+s)
```

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Answer:
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Problem 11-53

Design a circuit to realize the transfer function below using only resistors, capacitors and not more than one OP AMP. Scale the circuit so that the final design uses only 20-kΩ resistors.

$$T_V(s) = \pm\frac{10{,}000\,s}{(s+1000)(s+5000)}$$

Solution:

Write the transfer function as the product of three terms

$$T_V(s) = \pm\left(\frac{5000}{s+5000}\right)(2)\left(\frac{s}{s+1000}\right) = \pm\left(\frac{\frac{5000}{s}}{1+\frac{5000}{s}}\right)(2)\left(\frac{1}{1+\frac{1000}{s}}\right)$$

We can now design the following circuit:

Scale the first and last stages by 20 k so that the resistors have the correct values. The resulting circuit is shown below.

The following MATLAB calculations verify the circuit provides the correct transfer function.

```
clear all
syms s
R = 20e3;
C1 = 10e-9;
C2 = 50e-9;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
T1 = ZC1/(R+ZC1);
T2 = (R+R)/R;
T3 = R/(R+ZC2);
T = simplify(T1*T2*T3);
T_num = simplify(vpa(T,6))
T_num =
10000.*s/(s+5000.)/(1000.+s)
```

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Answer:
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Problem 11-54

Design a *passive* circuit to realize the transfer function below using only resistors, capacitors, and inductors. Scale the circuit so that all inductors are 50 mH or less.

$$T_V(s) = \frac{s^2}{(s+2000)^2}$$

Solution:

Write the transfer function as follows:

$$T_V(s) = \frac{s^2}{s^2 + 4000s + 4\times10^6} = \frac{s}{s + 4000 + \dfrac{4\times10^6}{s}}$$

We can now create the following circuit to realize the transfer function.

Scale the circuit by a factor of 1/20 to reduce the inductor's value to 50 mH. The resulting circuit is shown below.

The following MATLAB calculations verify the transfer function.

```
clear all
syms s
R = 200;
C = 5e-6;
L = 50e-3;
ZC = 1/s/C;
ZL = s*L;
T = vpa(ZL/(ZL+R+ZC),6);
T_num = simplify(vpa(T,6))
T_num =
s^2/(s^2+4000.*s+4000000.)
```

Answer:

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Problem 11-55

A circuit is needed to realize the transfer function listed below.

$$T_V(s) = \pm\frac{(s+100)(s+500)}{(s+200)(s+1000)}$$

(a) Design the circuit using two OP AMPs.
(b) Design the circuit using only one OP AMP.
(c) Design the circuit using no OP AMPs.

In all cases, scale the circuit so that all parts use practical values.

Solution:

(a) To design the circuit with two OP AMPs, write the transfer function as follows:

$$T_V(s) = \pm\left(\frac{s+100}{s+200}\right)\left(\frac{s+500}{s+1000}\right)$$

The following circuit will yield the desired transfer function.

If we scale the circuit by 10^6 to get more reasonable parameter values, the resulting circuit is as follows:

The following MATLAB calculations verify that the circuit yields the correct transfer function.

```
clear all
syms s
C = 1e-6;
ZC = 1/s/C;
Z1 = 1/(1/ZC+1/10e3);
Z2 = 1/(1/ZC+1/5e3);
Z3 = 1/(1/ZC+1/2e3);
Z4 = 1/(1/ZC+1/1e3);
T1 = -Z2/Z1;
T2 = -Z4/Z3;
T = simplify(T1*T2)
```

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```
T =
(s+100)*(s+500)/(s+200)/(s+1000)
```

(b) To design the circuit with only one OP AMP, write the transfer function as follows:

$$T_V(s) = \pm\left(\frac{s+100}{s+200}\right)\left(\frac{s+500}{s+1000}\right) = \pm\left(\frac{1+\frac{100}{s}}{1+\frac{100}{s}+\frac{100}{s}}\right)\left(\frac{1+\frac{500}{s}}{1+\frac{500}{s}+\frac{500}{s}}\right)$$

The following circuit will yield the desired transfer function.

If we scale the circuit by 10^4 to get more reasonable parameter values, the resulting circuit is as follows:

The following MATLAB calculations verify that the circuit yields the correct transfer function.

```
clear all
syms s
C1 = 1e-6;
C2 = 200e-9;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
T1 = (10e3+ZC1)/(10e3+ZC1+ZC1);
T2 = (10e3+ZC2)/(10e3+ZC2+ZC2);
T = simplify(T1*T2);
T_num = simplify(vpa(T,6))
T_num =
(s+100.)*(s+500.)/(s+200.)/(s+1000.)
```

(c) To design the circuit with no OP AMPs, write the transfer function as follows:

$$T_V(s) = \pm\left(\frac{s^2+600s+50000}{s^2+1200s+200000}\right) = \pm\left(\frac{s+600+\frac{50000}{s}}{s+1200+\frac{200000}{s}}\right) = \pm\left(\frac{s+600+\frac{50000}{s}}{s+600+\frac{50000}{s}+600+\frac{150000}{s}}\right)$$

The following circuit will yield the desired transfer function.

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If we scale the circuit by 10 to get more reasonable parameter values, the resulting circuit is as follows:

The following MATLAB calculations verify that the circuit yields the correct transfer function.

```
clear all
syms s
R = 6e3;
L = 10;
C1 = 666.67e-9;
C2 = 2e-6;
ZL = s*L;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
T = (ZL+R+ZC2)/(ZL+R+ZC2+R+ZC1);
T_num = simplify(vpa(T,6))
T_num =
(s^2+600.*s+50000.)/(s^2+1200.*s+199999.)
```

Answer:
The circuits are presented above.

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Problem 11-56

Design a circuit to realize the transfer function below using only resistors, capacitors, and OP AMPs. Scale the circuit so that all capacitors are exactly 100 nF.

$$T_V(s) = -\frac{400\ (s+100)}{s\,(s+10{,}000)}$$

Solution:

Write the transfer function as follows:

$$T_V(s) = -\left(\frac{400}{s}\right)\left(\frac{s+100}{s+10000}\right) = -\left(\frac{\frac{400}{s}}{1}\right)\left(\frac{s+100}{s+10000}\right)$$

The following circuit will yield the desired transfer function.

If we scale the first stage by 25×10^3 and the last stage by 10^7, then all of the capacitors will be 100 nF. The resulting circuit is shown below.

The following MATLAB calculations verify that the circuit yields the correct transfer function.

```
clear all
syms s
C = 100e-9;
ZC = 1/s/C;
T1 = -ZC/25e3;
T2 = -1e3/1e3;
Z1 = 1/(1/ZC+1/100e3);
Z2 = 1/(1/ZC+1/1e3);
T3 = -Z2/Z1;
T = simplify(T1*T2*T3)
T =
-400*(s+100)/s/(s+10000)
```

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Answer:
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Problem 11-57

A circuit is needed to realize the impulse response transform listed below. Scale the circuit so that all parts use practical values.

$$H(s) = \pm \frac{200s + 10^6}{s^2 + 200s + 10^6}$$

Solution:

The impulse response transform is the same as the transfer function. Write the transfer function as follows:

$$T(s) = \pm\left(\frac{200s + 10^6}{s^2 + 200s + 10^6}\right) = \pm\left(\frac{200 + \frac{10^6}{s}}{s + 200 + \frac{10^6}{s}}\right)$$

We can realize the transfer function with the following circuit:

The circuit does not need any scaling to have practical values. We re-write the circuit with a more common expression for the capacitor.

The following MATLAB calculations verify that the circuit yields the correct transfer function.

```
clear all
syms s
C = 1e-6;
L = 1;
R = 200;
ZC = 1/s/C;
ZL = s*L;
T = simplify((R+ZC)/(R+ZC+ZL))
T =
200*(s+5000)/(200*s+1000000+s^2)
```

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Answer:
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Problem 11-58

It is claimed that both circuits in Figure P11-58 realize the transfer function

$$T_V(\mathbf{s}) = K\left(\frac{\mathbf{s}+\mathbf{2000}}{\mathbf{s}+\mathbf{1000}}\right)$$

(a) Verify that both circuits realize the specified $T_V(s)$.

(b) Which circuit would you choose if the output must drive a 1-kΩ load?

(c) Which circuit would you choose if the input comes from a 50-Ω source?

(d) It is further claimed that connecting the two circuits in cascade produces an overall transfer function of $[T_V(s)]^2$ no matter which circuit is the first stage and which is the second stage.

Do you agree or disagree? Explain.

Solution:

(a) The following MATLAB calculations examine the transfer functions of the circuits. Note that the first circuit is connected as a non-inverting amplifier.

```
clear all
syms s
disp('Circuit 1')
R = 10e3;
C = 100e-9;
ZC = 1/s/C;
Z1 = R;
Z2 = 1/(1/R+1/ZC);
T1 = simplify((Z1+Z2)/Z1)
disp('Circuit 2')
R = 10e3;
C = 50e-9;
ZC = 1/s/C;
T2 = (R+ZC)/(R+ZC+R);
T2_num = vpa(simplify(vpa(T2,6)),6)
Circuit 1
T1 =
(s+2000)/(s+1000)
Circuit 2
T2_num =
.500000*(s+2000.)/(s+1000.)
```

Both circuits realize the transfer function.

(b) To drive a 1-kΩ load, use the first circuit, because its output is the output of an OP AMP and the load will not influence the transfer function. In the second circuit, a 1-kΩ load will alter the transfer function because it will influence the rest of the circuit.

(c) Either circuit will work with a 50-Ω source. The non-inverting amplifier draws no current from the source and the 50-Ω resistor is not significant compared to the 10-kΩ resistor at the input of the second circuit.

(d) The claim is correct because the OP AMP circuit is a non-inverting amplifier. The *RC* circuit (second circuit) can be connected to either the input or the output of the OP AMP without altering the behavior of either circuit.

Answer:

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Problem 11-59

It is claimed that both circuits in Figure P11-59 realize the transfer function

$$T_V(s) = \frac{\pm 1000\,s}{(s+1000)(s+4000)}$$

(a) Verify that both circuits realize the specified $T_V(s)$.

(b) Which circuit would you choose if the output must drive a 1-kΩ load?

(c) Which circuit would you choose if the input comes from a 50-Ω source?

(d) It is further claimed that connecting the two circuits in cascade produces an overall transfer function of $[T_V(s)]^2$ no matter which circuit is the first stage and which is the second stage.

Do you agree or disagree? Explain.

Solution:

(a) The following MATLAB calculations examine the transfer functions of the circuits.

```
clear all
syms s
disp('Circuit 1')
R = 10e3;
C1 = 25e-9;
C2 = 100e-9;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Z1 = R+ZC1;
Z2 = 1/(1/R+1/ZC2);
T1 = simplify(vpa(-Z2/Z1,6))
disp('Circuit 2')
R1 = 20e3;
R2 = 10e3;
C1 = 25e-9;
C2 = 50e-9;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Z1 = R1+ZC1;
Z2 = 1/(1/R2+1/ZC2);
T2 = simplify(vpa(Z2/(Z1+Z2),6))
Circuit 1
T1 =
-1000.*s/(s+1000.)/(s+4000.)
Circuit 2
T2 =
1000.*s/(s^2+5000.*s+4000000.)
```

Both circuits realize the transfer function.

(b) To drive a 1-kΩ load, use the first circuit, because its output is the output of an OP AMP and the load will not influence the transfer function. In the second circuit, a 1-kΩ load will alter the transfer function because it will influence the rest of the circuit.

(c) Either circuit will work with a 50-Ω source. A 50-Ω resistor is not significant compared to the 10-kΩ and 20-kΩ resistors at the inputs of the circuits.

(d) The claim is not correct. If the second circuit is connected after the first circuit, the resulting transfer function will be $[T_V(s)]^2$, because the second circuit will not influence the behavior of the OP AMP circuit. If the second circuit is connected before the first circuit, the circuits will influence each other's behavior and the overall transfer function will not be $[T_V(s)]^2$.

Answer:

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Problem 11-60

Design a circuit that produces the following step response.

$$g(t) = 10\left[1 - e^{-50t} - 50t\,e^{-50t}\right]u(t)$$

Solution:

Convert the step response into the Laplace domain and then find the corresponding transfer function. We can then design an appropriate circuit.

```
clear all
syms s t
gt = 10*(1-exp(-50*t)-50*t*exp(-50*t));
Gs = laplace(gt);
Hs = s*Gs
Hs =
25000/(s+50)^2
```

The resulting transfer function is expanded in a more convenient format below.

$$T(s) = \frac{25000}{(s+50)^2} = \left(\frac{50}{s+50}\right)(10)\left(\frac{50}{s+50}\right) = \left(\frac{\frac{50}{s}}{1+\frac{50}{s}}\right)(10)\left(\frac{\frac{50}{s}}{1+\frac{50}{s}}\right)$$

We can now create a circuit to realize the transfer function.

Scale the first and third stages by 10^5 to get more reasonable values, as shown in the final circuit below.

The following MATLAB calculations verify that the circuit yields the correct transfer function.

```
clear all
syms s
R = 100e3;
C = 200e-9;
ZC = 1/s/C;
T1 = ZC/(R+ZC);
T2 = (1e3+9e3)/1e3;
T3 = ZC/(R+ZC);
```

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```
T = simplify(vpa(T1*T2*T3,6))
T =
25000./(s+50.)^2
```

Answer:
Presented above.

Problem 11-61

A circuit is needed that will take an input of $v_1(t) = 2\,e^{-100t}\,u(t)$ V and produce an output of $v_2(t) = 5\,e^{-1000t}\,u(t)$ V. Design such a circuit using practical parts values. Validate your design by using OrCAD.

Solution:

Convert the input and output into the Laplace domain and divide the results to determine the transfer function. We can then design an appropriate circuit.

```
clear all
syms s t
v1t = 2*exp(-100*t);
v2t = 5*exp(-1000*t);
V1s = laplace(v1t);
V2s = laplace(v2t);
Ts = simplify(V2s/V1s)
Ts =
5/2/(s+1000)*(s+100)
```

The resulting transfer function is expanded in a more convenient format below.

$$T(s) = \frac{2.5(s+100)}{(s+1000)} = (2.5)\left(\frac{s+100}{s+1000}\right) = (2.5)\left(\frac{1+\dfrac{100}{s}}{1+\dfrac{100}{s}+\dfrac{900}{s}}\right)$$

We can now create a circuit to realize the transfer function.

We need to scale the output stage by 10^4 to have more practical values. The resulting circuit is shown below.

The circuit shown above will produce floating node errors in OrCAD because of how the OP

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AMP is connected to the source and one of the capacitors. To correct this simulation problem, attach two high-valued resistors as shown below.

The two additional resistors allow OrCAD to successfully simulate the circuit and do not significantly alter the results. The simulation produces the following output, which meets the specifications.

Answer:
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Problem 11-62

A circuit is needed that will take an input of $v_1(t) = [1 - e^{-10{,}000t}]\, u(t)$ V and produce a constant – 5 V output. Design such a circuit using practical parts values. Validate your design using OrCAD.

Solution:

Convert the input and output voltage signals into the Laplace domain and then divide the results to determine the transfer function. We can then design an appropriate circuit to realize the transfer function.

```
clear all
syms s t
v1t = heaviside(t)-exp(-10000*t);
v2t = -5*heaviside(t);
V1s = laplace(v1t);
V2s = laplace(v2t);
Ts = simplify(V2s/V1s)
Ts =
-1/2000*s-5
```

The resulting transfer function is expanded in a more convenient format below.

$$T(s) = \frac{-s}{2000} - 5 = \frac{-(s+10000)}{2000} = \frac{-\left(1+\dfrac{10000}{s}\right)}{\dfrac{2000}{s}} = (-1)\frac{\left(1+\dfrac{8000}{s}+\dfrac{2000}{s}\right)}{\dfrac{2000}{s}}$$

We can now create a circuit to realize the transfer function.

We need to scale the output stage by 5000 to create reasonable component values.

The circuit shown above will produce floating node errors in OrCAD because of how the OP AMP is connected to one of the capacitors. To correct this simulation problem, attach a high-valued resistor as shown below.

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The simulation results are shown below with plots of the input voltage and the output signal.

Answer:
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Problem 11-63

First-Order Circuit Impulse and Step Responses (A, D)

Each row in the table shown in Figure P11-63 refers to a first-order circuit with an impulse response $h(t)$ and a step response $g(t)$. Fill in the missing entries in the table.

Circuit	$h(t)$	$g(t)$
	$\delta(t)-\left[\alpha e^{-\alpha t}\right]u(t)$	
		$\left(\frac{1+e^{-\alpha t}}{2}\right)u(t)$

Solution:

Analyze each row separately, starting with the first row.

Use MATLAB to find the impulse response and step response.

```
clear all
syms s t R a
C = 1/a/R;
ZC = 1/s/C;
Hs = simplify(ZC/(R+ZC));
ht = ilaplace(Hs)
Gs = simplify(Hs/s);
gt = ilaplace(Gs)
ht =
a*exp(-a*t)
gt =
1-exp(-a*t)
```

The impulse response and step response for the first row are

$$h(t)=\alpha e^{-\alpha t}\,u(t) \quad \text{and} \quad g(t)=\left[1-e^{-\alpha t}\right]u(t)\,.$$

For the second row, we will transform the impulse response into the Laplace domain to find the transfer function and to compute the step response.

```
clear all
syms s t a
```

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```
ht = dirac(t)-a*exp(-a*t);
Hs = laplace(ht)
Gs = Hs/s;
gt = ilaplace(Gs)
Hs =
s/(s+a)
gt =
exp(-a*t)
```

The step response is $g(t) = \left[e^{-\alpha t}\right]u(t)$.

The transfer function is

$$H(s) = \frac{s}{s+\alpha} = \frac{1}{1+\frac{\alpha}{s}} = \frac{R}{R+\frac{R\alpha}{s}}.$$

The following circuit will realize the transfer function with a = α.

For the third row, convert the step response into the Laplace domain and then find the transfer function and the impulse response.

```
clear all
syms s t a
gt = (1+exp(-a*t))/2;
Gs = laplace(gt);
Hs = simplify(s*Gs)
ht = ilaplace(Hs)
Hs =
1/2*(2*s+a)/(s+a)
ht =
dirac(t)-1/2*a*exp(-a*t)
```

The impulse response is $h(t) = \left[\delta(t) - \frac{\alpha}{2}e^{-\alpha t}\right]u(t)$.

The transfer function is

$$H(s) = \frac{1}{2}\frac{2s+\alpha}{s+\alpha} = \frac{s+\frac{\alpha}{2}}{s+\alpha} = \frac{1+\frac{\alpha}{2s}}{1+\frac{\alpha}{2s}+\frac{\alpha}{2s}} = \frac{R+\frac{R\alpha}{2s}}{R+\frac{R\alpha}{2s}+\frac{R\alpha}{2s}}.$$

The following circuit will realize the transfer function with a = α.

Answers:

First row: $h(t) = \alpha e^{-\alpha t}\,u(t)$ and $g(t) = \left[1 - e^{-\alpha t}\right]u(t)$.

Second row: $g(t) = \left[e^{-\alpha t}\right]u(t)$ and the circuit is shown above.

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Third row: $h(t) = \left[\delta(t) - \frac{\alpha}{2} e^{-\alpha t}\right] u(t)$ and the circuit is shown above.

The completed table is shown below.

Circuit	$h(t)$	$g(t)$
	$\alpha e^{-\alpha t}\, u(t)$	$\left[1 - e^{-\alpha t}\right] u(t)$
	$\delta(t) - \left[\alpha e^{-\alpha t}\right] u(t)$	$\left[e^{-\alpha t}\right] u(t)$
	$\left[\delta(t) - \frac{\alpha}{2} e^{-\alpha t}\right] u(t)$	$\left(\frac{1 + e^{-\alpha t}}{2}\right) u(t)$

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Problem 11-64

OP AMP Modules and Loading (A)

Figure P11-64 shows an interconnection of three basic OP AMP modules.

(a) Does this interconnection involve loading?

(b) Find the overall transfer function of the interconnection and locate its poles and zeros.

(c) Find the output $v_2(t)$ when the input is $v_1(t) = \cos 500t$ V. Repeat for $v_1(t) = \cos 10\text{ k}t$ V and again for $v_1(t) = \cos 200\text{ k}t$ V.

(d) Can you think of a use for this circuit?

Solution:

(a) The circuit does not involve loading. All of the inputs are either sources or the outputs of OP AMP stages, which cannot be loaded. All of the outputs are from OP AMP stages.

(b) Since the circuit does not involve loading, we can find the transfer function of each stage and then combine them to get the overall transfer function. The first two stages are inverting amplifiers and the third stage is an inverting summer. The following MATLAB calculations determine the overall transfer function.

```
clear all
syms s
% First stage
C = 1e-9;
ZC = 1/s/C;
Z1 = ZC+10e3;
Z2 = 10e3;
T1 = -Z2/Z1;
% Second stage
C = 100e-9;
ZC = 1/s/C;
Z1 = 10e3;
Z2 = 1/(1/10e3+1/ZC);
T2 = -Z2/Z1;
% Third stage
K = -10e3/10e3;
% The third stage adds the two input signals together after
% scaling by gain K
T = simplify(K*T1+K*T2)
T =
(2000*s+s^2+100000000)/(100000+s)/(1000+s)
```

$$T(s) = \frac{s^2 + 2000s + 10^8}{(s+1000)(s+100000)}$$

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```
% Find the zeros
z = roots([1 2000 1e8])
z =
 -1000.0000e+000 +  9.9499e+003i
 -1000.0000e+000 -  9.9499e+003i
```

The poles are located at $s = -1000$ rad/s and $s = -100000$ rad/s.
The zeros are located at $s = -1000 \pm j9949.9$.

(c) For each input signal, we will find the steady-state output signal using MATLAB.

```
disp('v1(t) = cos(500*t) V')
w = 500;
Magv1 = 1;
Phasev1 = 0;
Tjw = subs(T,s,j*w);
Magv2 = Magv1*abs(Tjw)
Phasev2 = Phasev1+180*angle(Tjw)/pi
v1(t) = cos(500*t) V
Magv2 =
   892.2248e-003
Phasev2 =
   -26.2772e+000

disp('v1(t) = cos(10e3*t) V')
w = 10e3;
Magv1 = 1;
Phasev1 = 0;
Tjw = subs(T,s,j*w);
Magv2 = Magv1*abs(Tjw)
Phasev2 = Phasev1+180*angle(Tjw)/pi
v1(t) = cos(10e3*t) V
Magv2 =
    19.8020e-003
Phasev2 =
     1.0178e-015

disp('v1(t) = cos(200e3*t) V')
w = 200e3;
Magv1 = 1;
Phasev1 = 0;
Tjw = subs(T,s,j*w);
Magv2 = Magv1*abs(Tjw)
Phasev2 = Phasev1+180*angle(Tjw)/pi
v1(t) = cos(200e3*t) V
Magv2 =
   892.2248e-003
Phasev2 =
    26.2772e+000
```

For $v_1(t) = \cos 500t$ V, the steady-state output is $v_2(t) = 0.8922 \cos(500t - 26.28°)$ V.

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For $v_1(t)$ = cos 10 kt V, the steady-state output is $v_2(t)$ = 0.0198 cos(10 kt) V.
For $v_1(t)$ = cos 200 kt V, the steady-state output is $v_2(t)$ = 0.8922 cos(200 kt + 26.28°) V.

(d) This circuit could be used to block signals with frequencies in a certain range and to pass signals with frequencies either higher or lower than that range. It is a form a band stop filter.

Answers:
(a) The circuit does not involve loading. All of the inputs are either sources or the outputs of OP AMP stages, which cannot be loaded. All of the outputs are from OP AMP stages.

(b) $T(s) = \dfrac{s^2 + 2000s + 10^8}{(s+1000)(s+100000)}$

The poles are located at s = −1000 rad/s and s = −100000 rad/s.
The zeros are located at s = −1000 ± j9949.9.

(c) For $v_1(t)$ = cos 500t V, the steady-state output is $v_2(t)$ = 0.8922 cos(500t − 26.28°) V.
For $v_1(t)$ = cos 10 kt V, the steady-state output is $v_2(t)$ = 0.0198 cos(10 kt) V.
For $v_1(t)$ = cos 200 kt V, the steady-state output is $v_2(t)$ = 0.8922 cos(200 kt + 26.28°) V.

(d) This circuit could be used to block signals with frequencies in a certain range and to pass signals with frequencies either higher or lower than that range. It is a form a band stop filter.

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Problem 11-65

OP AMP Modules and Stability (A)

Figure P11-65 shows an interconnection of three basic circuit modules. Does this interconnection involve loading? Find the overall transfer function of the interconnection and locate its poles and zeros. Is the circuit stable?

Solution:

The circuit does not involve loading. In the first stage, the two inputs are from an OP AMP and a source and the output is from an OP AMP. The second stage, the *RC* circuit, has an input from an OP AMP and its output goes to a non-inverting amplifier, which does not cause loading. The last stage is the non-inverting amplifier, so it does not cause or experience loading.

To find the overall transfer function, consider the three stages of the circuit. The first stage is a summing integrator and one of its inputs is the circuit output, which is a form of feedback. The second stage is a first-order *RC* voltage divider circuit. The third stage is a non-inverting amplifier with a gain of $(R+R)/R = 2$. The following MATLAB calculations determine the overall transfer function.

```
clear all
% Let V1 and V2 be the input and output signals in the s domain
% Let T1, T2, and T3 be the outputs of the three stages
syms s V1 V2 T1 T2 T3 C R
% First stage
Z2 = 1/s/C;
Z1 = R;
K = -Z2/Z1;
T1 = K*V1+K*V2;
% Second stage
ZC = 1/s/C;
T2 = T1*ZC/(R+ZC);
% Third stage
K = (R+R)/R;
T3 = simplify(K*T2);
% To find the transfer function, observe T3 = V2 and
% solve for V2.  Then divide by V1 to get the transfer function
Eqn1 = T3-V2;
V2 = solve(Eqn1,V2);
T = simplify(V2/V1)
T =
-2/(2+s^2*C^2*R^2+s*C*R)
```

$$T(s) = \frac{-2}{R^2C^2s^2 + RCs + 2}$$

```
% Find the poles of the circuit
Eqn1 = 2+s^2*C^2*R^2+s*C*R;
p = solve(Eqn1,s)
p =
```

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```
1/2*(-1+i*7^(1/2))/C/R
-1/2*(1+i*7^(1/2))/C/R
```

The circuit does not have any finite zeros and the poles are located at $s = \frac{-1 \pm j\sqrt{7}}{2RC}$.

The circuit is stable because the poles are located in the left half of the complex plane.

Answers:

$$T(s) = \frac{-2}{R^2C^2s^2 + RCs + 2}$$

The circuit does not have any finite zeros and the poles are located at $s = \frac{-1 \pm j\sqrt{7}}{2RC}$.

The circuit is stable because the poles are located in the left half of the complex plane.

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Problem 11-66

Step Response and Fan-Out (A)

The fan-out of a digital device is defined as the maximum number of inputs to similar devices that can be reliably driven by the device output. Figure P11-66 is a simplified diagram of a device output driving *n* identical capacitive inputs. To operate reliably a 5-V step function at the device output must drive the capacitive inputs to 3.7 V in 10 ns or less. Determine the device fan-out for R = 1 kΩ and C = 3 pF.

Solution:

In Figure P11-66, all of the load capacitors are connected in parallel, so they can be modeled as a single capacitor with magnitude nC = (3n) pF, as shown at right.

The response must climb from 0 V to 3.7 V in 10 ns or less and we can use this information to calculate the maximum time constant for the circuit. For a 5-V step input the circuit response will be $(5-5e^{-t/T_C})\,u(t)$ V, where T_C is the time constant. The time constant is $T_C = nRC$. We need to find the maximum allowable time constant and then determine the maximum number of capacitors that the circuit can support. The following MATLAB calculations complete the solution.

```
clear all
syms s TC
% Find the time constant
Eqn1 = 5-5*exp(-10e-9/TC) - 3.7;
TC = double(solve(Eqn1,TC))
% Compute the maximum number of capacitors
R = 1e3;
C = 3e-12;
RC = R*C;
% Round down the results
n = floor(TC/RC)
TC =
  7.4235e-009
n =
     2
```

Answer:

The device fan-out for R = 1 kΩ and C = 3 pF, is n = 2.

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Problem 11-67

Designing to Specifications (D)

A particular circuit needs to be designed that has the following transfer function requirements: Poles at $s = -100$ and $s = -10{,}000$; zeros at $s = 0$ and $s = -1000$; and a gain of 10 as $s \to \infty$. Find the circuit's transfer function and use MATLAB to plot its step response. Then design a circuit that will meet that requirement. Finally, use OrCAD to validate that your circuit has the same step response as found using MATLAB.

Solution:

We can write the transfer function directly from the problem description.

$$T(s) = \frac{10s\,(s+1000)}{(s+100)(s+10000)}$$

Use the transfer function to compute the transform of the step response and then take the inverse Laplace transform to find the step response.

```
clear all
syms s t
Ts =
10*s*(s+1000)/(s+100)/(s+10000);
Gs = Ts/s;
gt = ilaplace(Gs);
tt = 0:2e-6:2e-3;
gtt = subs(gt,t,tt);
plot(tt,gtt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Step Response')
```

To create a circuit that realizes the transfer function, write the transfer function as follows:

$$T(s) = \frac{10s\,(s+1000)}{(s+100)(s+10000)} = \left(\frac{s}{s+100}\right)(10)\left(\frac{s+1000}{s+10000}\right) = \left(\frac{1}{1+\frac{100}{s}}\right)(10)\left(\frac{1+\frac{1000}{s}}{1+\frac{1000}{s}+\frac{9000}{s}}\right)$$

We can realize this transfer function with the circuit on the right:

To have more practical component values, scale the first and last stages by 10^4 to produce the following circuit:

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The circuit shown above will not simulate correctly because of how the capacitor is connected to the OP AMP output. To allow for the simulation without significantly changing the results, add a 100-MΩ resistor in parallel with the 11.11 nF capacitor, as shown below.

The circuit's transfer function is

$$T(s) = \frac{10s\,(s+1000)}{(s+100)(s+10000)}$$

The simulation results are shown below.

Answers:
The circuit design, transfer function, and simulation results are shown above.

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Problem 12-1

Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12-1.

(a) Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

(b) Sketch the straight-line approximations of the gain and phase responses.

(c) Calculate the gain at $\omega = 0.5\omega_C$, ω_C, and $2\omega_C$.

(d) Use OrCAD to plot the Bode magnitude gain response of the circuit. Validate your answers for part (c).

Solution:

The following MATLAB calculations yield the circuit's transfer function.

```
clear all
format short eng
syms s
L = 100e-3;
R1 = 50;
R2 = 100;
ZL = s*L;
T = simplify(R2/(R1+ZL+R2))
T =
1000/(1500+s)
```

$$T_V(s) = \frac{1000}{s+1500}$$

(a) By inspection of the transfer function, we can determine the dc gain is 1000/1500 = 2/3 and that the infinite frequency gain is zero. The cutoff frequency is 1500 rad/s. The circuit is a low-pass filter.

(b) The following MATLAB script creates the straight-line approximations of the gain and phase responses and plots the actual responses.

```
% Create the actual gain responses
close all
ww = logspace(2,5,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
Mag0 = 20*log10(1000/1500);
line([100,1500],[Mag0,Mag0],'LineWidth',
3,'LineStyle',':')
```

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```
line([1500,150000],[Mag0,Mag0-40],'LineWidth',3,'LineStyle',':')
axis([100 1e5 -40 0])
% Create the phase response
close all
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual phase response
semilogx(ww,PhaseTjw,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
% Plot the straight-line approximation
line([100,150],[0,0],'LineWidth',3,'LineSt
yle',':')
line([150,15000],[0,-
90],'LineWidth',3,'LineStyle',':')
line([15000,1e5],[-90,-
90],'LineWidth',3,'LineStyle',':')
```

(c) We can use MATLAB to calculate the gain at $\omega = 0.5\omega_C$, ω_C, and $2\omega_C$.

```
% Calculate the requested gains
wc = 1500;
w2 = wc*[0.5 1 2];
Tw2 = subs(T,s,j*w2);
MagTw2 = abs(Tw2);
Results = [w2' MagTw2']
Results =
   750.0000e+000   596.2848e-003
     1.5000e+003   471.4045e-003
     3.0000e+003   298.1424e-003
```

$$|T_V(0.5\omega_C)| = 0.5963,\ |T_V(\omega_C)| = 0.4714,\ |T_V(2\omega_C)| = 0.2981$$

(d) The OrCAD simulation and resulting Bode magnitude gain response are shown below.

The horizontal axis in the OrCAD simulation has units of hertz, which differs from the MATLAB simulation with units of rad/s. If we adjust the MATLAB simulation

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to have units of hertz, the resulting plot is shown below and agrees with the OrCAD simulation.

```
% Plot the actual gain response with frequency units of hertz
close all
semilogx(ww/2/pi,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (Hz)')
ylabel('Magnitude, (dB)')
```

Answers:

$$T_V(s) = \frac{1000}{s+1500}$$

(a) $|T_V(0)| = 2/3$, $|T_V(\infty)| = 0$, ω_C = 1500 rad/s, low pass

(b) The plots are shown above.

(c) $|T_V(0.5\omega_C)| = 0.5963$, $|T_V(\omega_C)| = 0.4714$, $|T_V(2\omega_C)| = 0.2981$

(d) The OrCAD simulation and results are shown above.

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Problem 12-2

Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12-2.

(a) Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

(b) Sketch the straight-line approximation of the gain response.

(c) Calculate the gain at $\omega = 0.5\omega_C$, ω_C, and $2\omega_C$.

(d) Use OrCAD to plot the Bode magnitude gain response of the circuit. Validate your answers for part (c).

Solution:

The following MATLAB calculations yield the circuit's transfer function.

```
clear all
syms s
L = 20e-3;
R = 600;
ZL = s*L;
Z1 = R;
Z2 = 1/(1/ZL+1/R);
T = simplify(Z2/(Z1+Z2))
T =
1/2*s/(15000+s)
```

The resulting transfer function is

$$T_V(s) = \frac{1}{2}\frac{s}{s+15000}$$

(a) By inspection of the transfer function, we can determine the dc gain is zero and that the infinite frequency gain is one half. The cutoff frequency is 15000 rad/s. The circuit is a high-pass filter.

(b) The following MATLAB script creates the straight-line approximations of the gain and phase responses and plots the actual responses.

```
% Create the actual gain responses
close all
ww = logspace(2,6,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
Mag0 = 20*log10(0.5);
```

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```
line([15000,1e6],[Mag0,Mag0],'LineWidth',3,'LineStyle',':')
line([15,15000],[Mag0-60,Mag0],'LineWidth',3,'LineStyle',':')
axis([100 1e6 -40 0])
% Create the phase response
close all
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual phase response
semilogx(ww,PhaseTjw,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
% Plot the straight-line approximation
line([10,1500],[90,90],'LineWidth',3,'Li
neStyle',':')
line([1500,150000],[90,0],'LineWidth',3,
'LineStyle',':')
line([150000,1e6],[0,0],'LineWidth',3,'L
ineStyle',':')
```

(c) We can use MATLAB to calculate the gain at $\omega = 0.5\omega_C$, ω_C, and $2\omega_C$.

```
% Calculate the requested gains
wc = 15000;
w2 = wc*[0.5 1 2];
Tw2 = subs(T,s,j*w2);
MagTw2 = abs(Tw2);
Results = [w2' MagTw2']
Results =
     7.5000e+003   223.6068e-003
    15.0000e+003   353.5534e-003
    30.0000e+003   447.2136e-003
```

$$|T_V(0.5\omega_C)| = 0.2236,\ |T_V(\omega_C)| = 0.3536,\ |T_V(2\omega_C)| = 0.4472$$

(d) The OrCAD simulation and resulting Bode magnitude gain response are shown below.

The horizontal axis in the OrCAD simulation has units of hertz, which differs from the MATLAB simulation with units of rad/s. If we adjust the MATLAB simulation to have units of hertz,

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the resulting plot is shown below and agrees with the OrCAD simulation.

```
% Plot the actual gain response with frequency units of hertz
close all
semilogx(ww/2/pi,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (Hz)')
ylabel('Magnitude, (dB)')
```

Answers:

$$T_V(s) = \frac{1}{2}\frac{s}{s+15000}$$

(a) $|T_V(0)| = 0$, $|T_V(\infty)| = 0.5$, ω_C = 15000 rad/s, high pass

(b) The plots are shown above.

(c) $|T_V(0.5\omega_C)| = 0.2236$, $|T_V(\omega_C)| = 0.3536$, $|T_V(2\omega_C)| = 0.4472$

(d) The OrCAD simulation and results are shown above.

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Problem 12-3

Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12-3.

(a) Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

(b) Sketch the straight-line approximation of the gain response.

(c) Calculate the gain at $\omega = 0.5\omega_C$, ω_C, and $2\omega_C$.

(d) Use OrCAD to plot the Bode magnitude gain response of the circuit. Validate your answers for part (c).

(e) What element values would you change to increase the passband gain to 10?

Solution:

By inspection, the non-inverting amplifier has a gain of two. The following MATLAB calculations determine the overall transfer function

```
clear all
syms s
C = 250e-9;
R = 20e3;
ZC = 1/s/C;
% Gain from the non-inverting amplifier
K = (30e3+30e3)/30e3;
T = simplify(vpa(K*ZC/(ZC+R),6))
T =
400./(200.+s)
```

The resulting transfer function is

$$T_V(s) = \frac{400}{s+200} = (2)\frac{200}{s+200}$$

(a) By inspection of the transfer function, we can determine the dc gain is two and that the infinite frequency gain is zero. The cutoff frequency is 200 rad/s. The circuit is a low-pass filter.

(b) The following MATLAB script creates the straight-line approximations of the gain and phase responses and plots the actual responses.

```
% Create the actual gain responses
close all
ww = logspace(0,5,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
```

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```
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line
approximation
Mag0 = 20*log10(2);
line([1,200],[Mag0,Mag0],'LineWidth
',3,'LineStyle',':')
line([200,200000],[Mag0,Mag0-
60],'LineWidth',3,'LineStyle',':')
axis([1 1e5 -40 10])


% Create the phase response
close all
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual phase response
semilogx(ww,PhaseTjw,'g','LineWidth
',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
% Plot the straight-line
approximation
line([1,20],[0,0],'LineWidth',3,'Li
neStyle',':')
line([20,2000],[0,-
90],'LineWidth',3,'LineStyle',':')
line([2000,2e5],[-90,-
90],'LineWidth',3,'LineStyle',':')
```

(c) We can use MATLAB to calculate the gain at $\omega = 0.5\omega_C$, ω_C, and $2\omega_C$.

```
% Calculate the requested gains
wc = 200;
w2 = wc*[0.5 1 2];
Tw2 = subs(T,s,j*w2);
MagTw2 = abs(Tw2);
Results = [w2' MagTw2']
Results =
   100.0000e+000     1.7889e+000
   200.0000e+000     1.4142e+000
   400.0000e+000   894.4272e-003
```

$|T_V(0.5\omega_C)| = 1.7889$; $|T_V(\omega_C)| = 1.4142$; $|T_V(2\omega_C)| = 0.8944$

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(d) The OrCAD simulation and resulting Bode magnitude gain response are shown below.

The horizontal axis in the OrCAD simulation has units of hertz, which differs from the MATLAB simulation with units of rad/s. If we adjust the MATLAB simulation to have units of hertz, the resulting plot is shown below and agrees with the OrCAD simulation.

```
% Plot the actual gain response with
frequency units of hertz
close all
semilogx(ww/2/pi,MagTjwdB,'g','LineW
idth',3)
grid on
hold on
xlabel('Frequency, (Hz)')
ylabel('Magnitude, (dB)')
```

(e) To increase the pass band gain, we need to increase the gain in the non-inverting amplifier. If we replace the horizontal 30-kΩ resistor with a 270-kΩ resistor, the pass band gain will be 10.

Answers:

$$T_V(s) = \frac{400}{s+200} = (2)\frac{200}{s+200}$$

(a) $|T_V(0)| = 2$; $|T_V(\infty)| = 0$; ω_C = 200 rad/s; low pass;

(b) The plots are shown above.

(c) $|T_V(0.5\omega_C)| = 1.7889$; $|T_V(\omega_C)| = 1.4142$; $|T_V(2\omega_C)| = 0.8944$

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(d) The OrCAD simulation and results are shown above.

(e) Increase the horizontal 30-kΩ resistor to 270-kΩ

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Problem 12-4

Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12-4.

(a) Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

(b) Sketch the straight-line approximation of the gain response.

(c) Calculate the gain at $\omega = 0.5\omega_C$, ω_C, and $2\omega_C$.

(d) Use OrCAD to plot the Bode magnitude gain response of the circuit. Validate your answers for part (c).

(e) What element values would you change to double the passband gain without changing the cutoff frequency?

Solution:

The following MATLAB calculations determine the overall transfer function

```
clear all
syms s
C = 50e-9;
R1 = 5e3;
R2 = 10e3;
ZC = 1/s/C;
Z1 = R1;
Z2 = 1/(1/R2+1/ZC);
T = simplify(vpa(-Z2/Z1,6))
T =
-4000./(2000.+s)
```

The resulting transfer function is

$$T_V(s) = \frac{-4000}{s+2000} = (-2)\frac{2000}{s+2000}$$

(a) By inspection of the transfer function, we can determine the magnitude of the dc gain is two and that the infinite frequency gain is zero. The cutoff frequency is 2000 rad/s. The circuit is a low-pass filter.

(b) The following MATLAB script creates the straight-line approximations of the gain and phase responses and plots the actual responses.

```
% Create the actual gain responses
close all
ww = logspace(1,6,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
```

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```
% Plot the straight-line approximation
Mag0 = 20*log10(2);
line([10,2000],[Mag0,Mag0],'LineWidth',3,'LineStyle',':')
line([2000,2000000],[Mag0,Mag0-60],'LineWidth',3,'LineStyle',':')
axis([10 1e6 -40 10])
```

```
% Create the phase response
close all
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual phase response
semilogx(ww,PhaseTjw,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
% Plot the straight-line approximation
line([10,200],[180,180],'LineWidth',3,'
LineStyle',':')
line([200,20000],[180,90],'LineWidth',3
,'LineStyle',':')
line([20000,2e6],[90,90],'LineWidth',3,
'LineStyle',':')
```

(c) We can use MATLAB to calculate the gain at $\omega = 0.5\omega_C$, ω_C, and $2\omega_C$.

```
% Calculate the requested gains
wc = 2000;
w2 = wc*[0.5 1 2];
Tw2 = subs(T,s,j*w2);
MagTw2 = abs(Tw2);
Results = [w2' MagTw2']
Results =
     1.0000e+003     1.7889e+000
     2.0000e+003     1.4142e+000
     4.0000e+003   894.4272e-003
```

$|T_V(0.5\omega_C)| = 1.7889$; $|T_V(\omega_C)| = 1.4142$; $|T_V(2\omega_C)| = 0.8944$

(d) The OrCAD simulation is shown above and resulting Bode magnitude gain response is shown below.

The horizontal axis in the OrCAD simulation has units of hertz, which differs from the MATLAB simulation with units of rad/s. If we adjust the MATLAB simulation to have units of hertz, the

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resulting plot is shown below and agrees with the OrCAD simulation.

```
% Plot the actual gain response with frequency units of hertz
close all
semilogx(ww/2/pi,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (Hz)')
ylabel('Magnitude, (dB)')
```

(e) To double the passband gain, we need to increase the gain at low frequencies when the capacitor is acting as an open circuit. To make this change without influencing the cutoff frequency, we need to adjust the 5-kΩ resistor. Replace the 5-kΩ resistor with a 2.5-kΩ resistor.

Answers:

$$T_V(s) = \frac{-4000}{s+2000} = (-2)\frac{2000}{s+2000}$$

(a) $|T_V(0)| = 2$; $|T_V(\infty)| = 0$; ω_C = 2000 rad/s; low pass;

(b) The plots are shown above.

(c) $|T_V(0.5\omega_C)| = 1.7889$; $|T_V(\omega_C)| = 1.4142$; $|T_V(2\omega_C)| = 0.8944$

(d) The OrCAD simulation and results are shown above.

(e) Replace the 5-kΩ resistor with a 2.5-kΩ resistor.

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Problem 12-5

Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12-5.

(a) Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

(b) Use OrCAD to plot the Bode magnitude gain response of the circuit.

(c) What element value would you change to increase the cutoff frequency by one decade?

Solution:

The following MATLAB calculations yield the circuit's transfer function.

```
clear all
syms s
% Gain of non-inverting amplifier
K = (60e3+15e3)/15e3;
C = 0.1e-6;
R = 10e3;
ZC = 1/s/C;
T = simplify(K*R/(R+ZC+R))
T =
5/2*s/(s+500)
```

The resulting transfer function is

$$T_V(s) = \frac{5}{2}\frac{s}{s+500}$$

(a) By inspection of the transfer function, we can determine the dc gain is zero and that the infinite frequency gain is 2.5. The cutoff frequency is 500 rad/s. The circuit is a high-pass filter.

(b) The OrCAD simulation above and resulting Bode magnitude gain response at left are shown.

(c) The cutoff frequency is proportional to $1/C$, so if we want to increase the cutoff frequency by a factor of 10, we need to decrease the value of the capacitor by a factor of 10. Therefore, we need to decrease the 100-nF capacitance to 10-nF.

Answers:

(a) $|T_V(0)| = 0$, $|T_V(\infty)| = 2.5$, ω_C = 500 rad/s, high pass

(b) The OrCAD simulation and results are shown above.

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(c) Decrease the 100-nF capacitance to 10-nF.

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Problem 12-6

Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12-6. What type of gain response does the circuit have? What is the passband gain? Select values of R and C so that the cutoff frequency is 10 kHz.

Solution:

The input signal passes through an RC voltage divider and then into a non-inverting amplifier. To find the overall transfer function, multiply the result of the voltage divider by the gain of the amplifier as shown below.

```
clear all
syms s C R
ZC = 1/s/C;
T1 = R/(ZC+R);
T2 = (R+R)/R;
T = simplify(T1*T2)
T =
2*R*s*C/(1+R*s*C)
```

The transfer function is

$$T_V(s) = \frac{2RCs}{RCs+1} = \frac{2s}{s+\dfrac{1}{RC}}$$

The transfer function represents a high-pass filter with a passband gain of two and a cutoff frequency of $1/RC$. To realize a cutoff frequency of 10 kHz = 20π krad/s, we can choose C = 10 nF and then calculate the corresponding resistor value as follows.

```
fc = 10e3;
wc = 2*pi*fc;
C = 10e-9;
R = 1/wc/C
Check_fc = 1/R/C/2/pi  R =
     1.5915e+003
Check_fc =
    10.0000e+003
```

Answers:

$$T_V(s) = \frac{2RCs}{RCs+1} = \frac{2s}{s+\dfrac{1}{RC}}$$

The transfer function represents a high-pass filter with a passband gain of two.
For f_C = 10 kHz, choose C = 10 nF and R = 1.5915 kΩ. Other solutions are possible.

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Problem 12-7

A first-order high-pass circuit has a passband gain of 20 dB and a cutoff frequency of 500 rad/s. Find the gain (in dB) at ω = 0, 200, 500, and 1000 rad/s.

Solution:

We can use the description to find the transfer function of the circuit and then evaluate it at the other frequencies.

```
clear all
syms s
% Convert the dB gain to a multiplier
KdB = 20;
K = 10^(KdB/20);
wc = 500;
% Form the first-order high-pass filter transfer function
T = K*s/(s+wc)
T =
10*s/(s+500)
```

$$T(s) = \frac{10s}{s+500}$$

Now evaluate the transfer function at the specified frequencies.

```
% Create a vector of frequencies
w = [0 200 500 1000];
Tjw = subs(T,s,j*w);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
Results = [w' MagTjwdB']
Results =
     0.0000e+000         -Inf
   200.0000e+000    11.3966e+000
   500.0000e+000    16.9897e+000
     1.0000e+003    19.0309e+000
```

Answers:

$|T_V(0)| = -\infty$ dB, $|T_V(200)| = 11.3966$ dB, $|T_V(500)| = 16.9897$ dB, $|T_V(1000)| = 19.0309$ dB.

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Problem 12-8

A first-order low-pass circuit has a passband gain of 5 dB and a cutoff frequency of 2 krad/s. Find the gain (in dB) at ω = 0, 200 rad/s, 20 krad/s, and 200 krad/s.

Solution:

We can use the description to find the transfer function of the circuit and then evaluate it at the other frequencies.

```
clear all
syms s
% Convert the dB gain to a multiplier
KdB = 5;
K = 10^(KdB/20);
wc = 2e3;
% Form the first-order low-pass filter transfer function
T = vpa(K*wc/(s+wc),6)
T =
3556.56/(2000.+s)
```

$$T(s) = \frac{3556.56}{s + 2000}$$

Now evaluate the transfer function at the specified frequencies.

```
% Create a vector of frequencies
w = [0 200 20e3 200e3];
Tjw = subs(T,s,j*w);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
Results = [w' MagTjwdB']
Results =
     0.0000e+000     5.0000e+000
   200.0000e+000     4.9568e+000
    20.0000e+003   -15.0432e+000
   200.0000e+003   -35.0004e+000
```

Answers:

$|T_V(0)| = 5$ dB, $|T_V(200)| = 4.9568$ dB, $|T_V(20000)| = -15.0432$ dB, $|T_V(200000)| = -35.0004$ dB.

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Problem 12-9

The transfer function of a first-order circuit is

$$T(s) = \frac{10{,}000}{s+1000}$$

(a) Identify the type of gain response. Find the cutoff frequency and the passband gain.
(b) Use MATLAB to plot the magnitude of the Bode gain response.
(c) Design a circuit to realize the transfer function.
(d) Use OrCAD to validate your circuit design by comparing its frequency response to the MATLAB output.

Solution:

(a) We can rewrite the transfer function as follows:

$$T(s) = \frac{10000}{s+1000} = (10)\frac{1000}{s+1000}$$

The transfer function represents a low-pass filter with a cutoff frequency of 1000 rad/s and a passband gain of 10.

(b) The MATLAB plot of the magnitude of the Bode gain response is shown below

```
clear all
syms s
T = 10000/(s+1000);
ww = logspace(1,5,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
semilogx(ww,MagTjwdB,'g','Line
Width',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
```

(c) The circuit is a combination of a low-pass filter and a gain stage with a gain of 10. The following circuit uses a passive *RC* circuit for the low-pass filter followed by a non-inverting amplifier for the gain stage. We chose C = 100 nF and then calculated R such that $1/RC$ = 1000 rad/s.

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(d) The results of the OrCAD simulation are shown below.

The OrCAD results verify the MATLAB calculations if we adjust for the 2π scaling factor between the plots.

Answers:

(a) It is a low-pass filter with $\omega_C = 1000$ rad/s and $K = 10$.

(b), (c), and (d) Presented above.

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Problem 12-10

The transfer function of a first-order circuit is

$$T(s) = \frac{s}{s+3000}$$

(a) Identify the type of gain response. Find the cutoff frequency and the passband gain.
(b) Use MATLAB to plot the magnitude of the Bode gain response.
(c) Design a circuit to realize the transfer function.
(d) Use OrCAD to validate your circuit design by comparing its frequency response to the MATLAB output.

Solution:

(a) By inspection, the transfer function represents a high-pass filter with a cutoff frequency of 3000 rad/s and a passband gain of one.

(b) The MATLAB plot of the magnitude of the Bode gain response is shown at right.

```
clear all
syms s
T = s/(s+3000);
ww = logspace(1,5,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
semilogx(ww,MagTjwdB,'g','LineWidth'
,3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
```

(c) A high-pass filter with a gain of one can be realized by a simple *RC* circuit. We chose C = 10 nF and then calculated R such that $1/RC$ = 3000 rad/s. The resulting circuit is shown below.

(d) The results of the OrCAD simulation are shown at right.

The OrCAD results verify the MATLAB calculations if we adjust for the 2π scaling factor between the plots.

Answers:

(a) It is a high-pass filter with ω_C = 3000 rad/s and K = 1.

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(b), (c), and (d) Presented above.

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Problem 12-11

Design a passive first-order low-pass filter with a dc gain of 0 dB and a cutoff frequency of 5 kHz. Use OrCAD to validate your response.

Solution:

A dc gain of 0 dB represents an absolute gain of one. Therefore we can create the low-pass filter as a simple combination of a resistor in series with a capacitor. The cutoff frequency will be 1/*RC*, so we can select a capacitor value and then calculate the appropriate resistor value as follows:

```
clear all
fc = 5e3;
wc = 2*pi*fc;
% Select a capacitor value
C = 10e-9;
% Calculate the corresponding resistor value
R = 1/C/wc
Check_fc = 1/R/C/2/pi
R =
     3.1831e+003
Check_fc =
     5.0000e+003
```

The following circuit will meet the specifications.

The simulation results are shown below.

Answers:

The circuit design and simulation results are shown above.

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Problem 12-12

Design a first-order high-pass filter with a high-frequency gain of 50 and a cutoff frequency of 500 Hz. Use OrCAD to validate your response.

Solution:

Since the circuit has gain in the passband, we need to pair an *RC* high-pass filter with an amplifier. The *RC* circuit will have a cutoff frequency of 1/*RC*, which must equal 1000π rad/s. If we choose a capacitor value, we can calculate the corresponding resistor value.

```
clear all
fc = 500;
wc = 2*pi*fc;
% Select a capacitor value
C = 100e-9;
% Calculate the corresponding resistor value
R = 1/C/wc
Check_fc = 1/R/C/2/pi
R =
     3.1831e+003
Check_fc =
   500.0000e+000
```

The following circuit will meet the specifications.

The simulation results are shown below.

Answers:

The circuit design and simulation results are shown at left.

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Problem 12-13

Design a first-order low-pass filter with a dc gain of 2 and a gain of −3 dB at 1000 rad/sec. Use OrCAD to validate your response.

Solution:

First, we need to determine the cutoff frequency for the circuit. We know the dc gain is 2, so the transfer function for a first-order low-pass filter has the following form $T(s) = \dfrac{2\omega_C}{s + \omega_C}$.

If we evaluate the transfer function at $s = j1000$, the gain is −3 dB = 0.7071. We can use MATLAB to calculate the cutoff frequency

```
clear all
syms s
syms wc positive
T = 2*wc/(s+wc);
w1 = 1000;
Tw1 = subs(T,s,j*w1);
% Find the magnitude squared of the transfer
% function to eliminate the imaginary part
Tw2 = Tw1*conj(Tw1);
Eqn1 = Tw2-1/2;
WC = double(solve(Eqn1,wc))
% Check the results
wcc = abs(WC(1));
Ts = vpa(subs(T,wc,wcc),6)
MagTsw1 = abs(subs(Ts,s,j*w1))
WC =
   377.9645e+000
  -377.9645e+000
Ts =
755.929/(s+377.964)
MagTsw1 =
   707.1069e-003
```

The transfer function is

$$T(s) = \frac{755.929}{s + 377.964}$$

We can design the circuit as an *RC* low-pass filter in series with a non-inverting amplifier as shown below.

```
wc = wcc;
% Select a capacitor value
C = 100e-9;
% Calculate the corresponding resistor value
R = 1/C/wc
Check_wc = 1/R/C
R =
    26.4575e+003
Check_wc =
```

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```
377.9645e+000
```

The corresponding circuit is shown below:

The simulation results are shown below:

Answers:
The circuit design and simulation results are shown above.

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Problem 12-14

The circuit in Figure P12-14 produces a bandpass response for a suitable choice of element values. Identify the elements that control the two cutoff frequencies. Select the element values so that the passband gain is 30 and the cutoff frequencies are 100 rad/s and 2500 rad/s. Use practical element values with R $ 10 kΩ and C # 1 μF. Use OrCAD to validate your design.

Solution:

The circuit has three stages: (1) a low-pass filter, (2) a gain stage, and (3) a high-pass filter. Components R_1 and C_1 control the cutoff frequency of the low-pass filter, which is responsible for the higher cutoff frequency or 2500 rad/s. Components R_2 and C_2 control the cutoff frequency of the high-pass filter, which is responsible for the lower cutoff frequency or 100 rad/s. Components R_3 and R_4 control the gain through a non-inverting amplifier.

Select both capacitors to be 10 nF and calculate the required resistor values to realize the desired cutoff frequencies.

```
clear all
wc1 = 100;
wc2 = 2500;
C = 10e-9;
R1 = 1/C/wc2
R2 = 1/C/wc1
R1 =
     40.0000e+003
R2 =
      1.0000e+006
```

The circuit at right above will meet the specifications:
The simulation results are shown below.

Answers:
The circuit design and simulation results are shown at right and above.

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Problem 12-15

Repeat Problem 12-14 for the circuit in Figure P12-15. Use OrCAD to validate your design.

Solution:

The circuit has two stages: (1) a low-pass filter with gain and (2) a high-pass filter with gain. Components R_2 and C_1 control the cutoff frequency of the low-pass filter, which is responsible for the higher cutoff frequency or 2500 rad/s. Components R_3 and C_2 control the cutoff frequency of the high-pass filter, which is responsible for the lower cutoff frequency or 100 rad/s. Components R_1 and R_4 influence the gain of each stage.

Select both capacitors to be 10 nF and calculate the required resistor values to realize the desired cutoff frequencies.

```
clear all
wc1 = 100;
wc2 = 2500;
C = 10e-9;
R2 = 1/C/wc2
R3 = 1/C/wc1
R2 =
    40.0000e+003
R3 =
     1.0000e+006
```

With the resistor values identified above, choose R_1 = 10 kΩ and R_4 = 7.5 MΩ to achieve gains of 4 and 7.5 in the two stages, respectively. The overall gain will then be 30.

The circuit above will meet the specifications:

The simulation results are shown at left.

Answers:
The circuit design and simulation results are shown at left and above.

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Problem 12-16

Suppose the circuits you designed in problems 12-14 and 12-15 had to feed a 500 Ω recorder. Which circuit would you select and why?

Solution:

If the circuit in Problem 12-14 is connected to a 500-Ω load, the load will appear to be in parallel with the 1-MΩ output resistor, which will drastically change the cutoff frequency of the high-pass filter. The circuit will no longer perform as specified. With the circuit designed in Problem 12-15, the output is the output of an OP AMP, so the load resistance will have no effect on the performance of the circuit. The circuit in Problem 12-15 is the only option that maintains the desired filter specifications.

Answer:

Use the circuit in Problem 12-15 because it will not be affected by the load.

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Problem 12-17

The circuit in Figure P12-17 produces a bandstop response for a suitable choice of element values.

(a) Find the circuit's transfer function.

(b) Identify the elements that control the two cutoff frequencies. Select the element values so that the cutoff frequencies are 100 krad/s and 1000 krad/s. Use practical element values with $R \geq 10$ kΩ, $L \leq 100$ mH, and $C \leq 1$ μF.

(c) Use MATLAB to plot the Bode magnitude plot for the values you selected.

(d) What is the passband gain for your design?

(e) Simulate your circuit using OrCAD and compare the results to the MATLAB output.

Solution:

(a) The circuit is the sum of the outputs of an *RL* high-pass filter and an *RC* low-pass filter. The following MATLAB calculations determine the transfer function.

```
clear all
syms s R1 R2 R L C
ZL = s*L;
ZC = 1/s/C;
T1 = ZL/(R1+ZL);
T2 = ZC/(ZC+R2);
K = -R/R;
T = simplify(K*T1+K*T2)
T =
-(2*s*L+s^2*L*R2*C+R1)/(R1+s*L)/(1+R2*s*C)
```

$$T_V(s) = \frac{-(R_2LCs^2 + 2Ls + R_1)}{(Ls + R_1)(R_2Cs + 1)}$$

(b) The transfer function indicates that the cutoff frequencies are R_1/L and $1/R_2C$. From the circuit configuration, we can determine that the inductor is part of the high-pass filter and that the capacitor is part of the low-pass filter, so we can match each cutoff frequency to the

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appropriate filter. Choose values for the inductor and capacitor and then calculate the corresponding resistor values to meet the specifications.

```
% Cutoff frequencies
wc1 = 100e3;
wc2 = 1000e3;
L = 10e-3
C = 1e-9
R1 = L*wc2
R2 = 1/C/wc1
L =
    10.0000e-003
C =
     1.0000e-009
R1 =
    10.0000e+003
R2 =
    10.0000e+003
```

(c) Substitute the selected values into the transfer function and plot the magnitude response.

```
% Substitute
T = subs(T);
% Plot the results
close all
ww = logspace(3,8,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
semilogx(ww,MagTjwdB,'g','LineWidth
',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
```

(d) The passband gain is 1 or 0 dB.

(e) The corresponding OrCAD circuit is shown below.

The simulation results are shown below.

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The results agree with the MATLAB simulation if we correct for the 2π scaling difference between the frequency axes.

Answers:

(a) $T_V(s) = \dfrac{-(R_2LCs^2 + 2Ls + R_1)}{(Ls + R_1)(R_2Cs + 1)}$;

(b) $\omega_{C1} = 1/R_2C$, $\omega_{C2} = R_1/L$; one possible solution: R_1 = 10 kΩ, R_2 = 10 kΩ, R = 10 kΩ, L = 10 mH, C = 1 nF

(c) The MATLAB plot is shown above.

(d) The passband gain is one.

(e) The OrCAD simulation and results are shown above.

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Problem 12-18

The following problem involves a cascade connection of two first-order passive circuits separated by a non-inverting OP AMP. The source has a 50-Ω series resistor and the output of the filter feeds a 10-kΩ load. Design a bandpass circuit with the following specifications all ± 5%: f_{CL} = 20 Hz, f_{CH} = 20 kHz, B = 19980 Hz. Passband gain 20 dB. Use OrCAD to validate your results.

Solution:

Make the low-pass filter the first stage of the circuit. It will be responsible for the upper cutoff frequency of 20 kHz = 40π krad/s and we will choose the resistance value such that the source resistance is not a significant factor. The non-inverting amplifier will provide the gain of 20 dB, which is equivalent to an absolute gain of 10. The high-pass filter will be the third stage and it will use the 10-kΩ load as part of its design to achieve the lower cutoff frequency of 20 Hz = 40π rad/s. Compute the component values and then construct and simulate the circuit.

```
clear all
disp('Low pass filter')
wc = 2*pi*20e3;
C = 10e-12
R = 1/C/wc
disp('High pass filter')
wc = 2*pi*20;
R = 10e3
C = 1/R/wc
Low pass filter
C =
    10.0000e-012
R =
   795.7747e+003
High pass filter
R =
    10.0000e+003
C =
   795.7747e-009
```

Round the calculated values and verify that the resulting filters still meet the specifications.

```
disp('Low pass filter')
R1 = 800e3
C1 = 10e-12
fchigh = 1/R1/C1/2/pi
disp('High pass filter')
R2 = 10e3
C2 = 800e-9
fclow = 1/R2/C2/2/pi
BW = fchigh-fclow
Low pass filter
R1 =
   800.0000e+003
C1 =
    10.0000e-012
fchigh =
```

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```
    19.8944e+003
High pass filter
R2 =
    10.0000e+003
C2 =
   800.0000e-009
fclow =
    19.8944e+000
BW =
    19.8745e+003
```

We must meet all specifications within 5%, so establish the acceptable ranges for the frequencies and bandwidth.

```
range = [0.95 1.05];
fclow = 20*range
fchigh = 20e3*range
BW = 19980*range
fclow =
    19.0000e+000    21.0000e+000
fchigh =
    19.0000e+003    21.0000e+003
BW =
    18.9810e+003    20.9790e+003
```

All of the calculated values are within specifications.

The resulting circuit is shown at right.

The simulation results are shown below. The simulation verifies that the circuit meets all of the design specifications.

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Answers:
The circuit design and simulation results are presented above.

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Problem 12-19

The following problem involves a cascade connection of two first-order active OP AMP circuits. The source has a 50-Ω series resistor and the output of the filter feeds a 10-kΩ load. Design a bandpass circuit with the following specifications all ± 5%: f_{CL} = 20 Hz, f_{CH} = 20 kHz. Passband gain 20 dB, B = 19980 Hz. Use OrCAD to validate your results.

Solution:

Make the low-pass filter the first stage of the circuit. It will be responsible for the upper cutoff frequency of 20 kHz = 40π krad/s. We will make this stage of the circuit have a gain of one, such that the resistors are equal and we will choose the resistance value such that the source resistance is not a significant factor. The high-pass filter will be the second stage and it will use a 10-kΩ resistor to achieve the lower cutoff frequency of 20 Hz = 40π rad/s. The high-pass filter stage will also realize the 20-dB gain, which is equivalent to an absolute gain of 10. Compute the component values and then construct and simulate the circuit.

```
clear all
disp('Low pass filter')
wc = 2*pi*20e3;
C = 10e-12
R = 1/C/wc
disp('High pass filter')
wc = 2*pi*20;
R = 10e3
C = 1/R/wc
Low pass filter
C =
    10.0000e-012
R =
   795.7747e+003
High pass filter
R =
    10.0000e+003
C =
   795.7747e-009
```

Round the calculated values and verify that the resulting filters still meet the specifications.

```
disp('Low pass filter')
R1 = 800e3
C1 = 10e-12
fchigh = 1/R1/C1/2/pi
disp('High pass filter')
R2 = 10e3
C2 = 800e-9
fclow = 1/R2/C2/2/pi
BW = fchigh-fclow
Low pass filter
R1 =
   800.0000e+003
C1 =
    10.0000e-012
fchigh =
```

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```
    19.8944e+003
High pass filter
R2 =
    10.0000e+003
C2 =
   800.0000e-009
fclow =
    19.8944e+000
BW =
    19.8745e+003
```

We must meet all specifications within 5%, so establish the acceptable ranges for the frequencies and bandwidth.

```
range = [0.95 1.05];
fclow = 20*range
fchigh = 20e3*range
BW = 19980*range
fclow =
    19.0000e+000    21.0000e+000
fchigh =
    19.0000e+003    21.0000e+003
BW =
    18.9810e+003    20.9790e+003
```

All of the calculated values are within specifications.

The resulting circuit is shown at right.

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The simulation results are shown below. The simulation verifies that the circuit meets all of the design specifications.
Answers: The circuit design and simulation results are presented above.
Problem 12-20

The following problem involves a cascade connection of two first-order passive circuits separated by a non-inverting OP AMP. The source has a 50-Ω series resistor and the output of the filter feeds a 10-kΩ load. Design a bandpass circuit with the following specifications all ± 5%: f_{CL} = 2 kHz, f_{CH} = 10 kHz, B = 8 kHz. Passband gain 20 dB. Use OrCAD to validate your results. Even though problems 12-18 and 12-20 share similar constructs one cannot achieve the desired specifications in 12-20. What is the reason for the problem?

Solution:
Follow the same approach as in the solution to Problem 12-18.

Make the low-pass filter the first stage of the circuit. It will be responsible for the upper cutoff frequency of 10 kHz = 20π krad/s and we will choose the resistance value such that the source resistance is not a significant factor. The non-inverting amplifier will provide the gain of 20 dB, which is equivalent to an absolute gain of 10. The high-pass filter will be the third stage and it will use the 10-kΩ load as part of its design to achieve the lower cutoff frequency of 2 kHz = 4π krad/s. Compute the component values and then construct and simulate the circuit.

```
clear all
disp('Low pass filter')
wc = 2*pi*10e3;
C = 10e-12
R = 1/C/wc
disp('High pass filter')
```

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```
wc = 2*pi*2e3;
R = 10e3
C = 1/R/wc
Low pass filter
C =
    10.0000e-012
R =
     1.5915e+006
High pass filter
R =
    10.0000e+003
C =
     7.9577e-009
```

Round the calculated values and verify that the resulting filters still meet the specifications.

```
disp('Low pass filter')
R1 = 1.6e6
C1 = 10e-12
fchigh = 1/R1/C1/2/pi
disp('High pass filter')
R2 = 10e3
C2 = 8e-9
fclow = 1/R2/C2/2/pi
BW = fchigh-fclow
Low pass filter
R1 =
     1.6000e+006
C1 =
    10.0000e-012
fchigh =
     9.9472e+003
High pass filter
R2 =
    10.0000e+003
C2 =
     8.0000e-009
fclow =
     1.9894e+003
BW =
     7.9577e+003
```

We must meet all specifications within 5%, so establish the acceptable ranges for the frequencies and bandwidth.

```
range = [0.95 1.05];
fclow = 2e3*range
fchigh = 10e3*range
BW = 8e3*range
fclow =
     1.9000e+003     2.1000e+003
fchigh =
     9.5000e+003    10.5000e+003
BW =
```

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```
7.6000e+003     8.4000e+003
```

All of the calculated frequency values are within specifications.

The resulting circuit is shown at right.

The simulation results are shown below.

The simulation shows that the circuit does not meet all of the specifications. The passband gain is not within 5% of 20 dB. If we increase the gain in the non-inverting amplifier stage to achieve

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a passband gain of 20 dB, then the resulting magnitude gain response is as follows:

With this magnitude response, the cutoff frequencies are not correct and the bandwidth is too large. We cannot achieve the specifications because the cutoff frequencies are too close to each other for a pair of first-order filters. In Problem 12-18, the bandwidth was large enough to allow the high-pass filter to approach its maximum gain before the low-pass filter began attenuating the signal. That is not the case with this circuit, so we cannot meet the specifications with the proposed circuit type.

Answer:
The circuit design and simulation results are presented above. The circuit cannot meet the specifications.

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Problem 12-21

A series *RLC* bandpass circuit with $R = 30\ \Omega$ is designed to have a bandwidth of 5 Mrad/s and a center frequency of 50 Mrad/s. Find *L*, *C*, *Q*, and the two cutoff frequencies. Could you design this circuit using a cascade connection of two first-order filters separated by a follower? Why or why not?

Solution:

The following MATLAB calculations determine the requested values.

```
clear all
format short eng
R = 30;
B = 5e6;
w0 = 50e6;
w02 = w0^2;
L = R/B
C = 1/L/w02
Q = w0/B
wC1 = -R/2/L + sqrt((R/2/L)^2+1/L/C)
wC2 =  R/2/L + sqrt((R/2/L)^2+1/L/C)
BCheck = wC2 - wC1
L =
     6.0000e-006
C =
    66.6667e-012
Q =
    10.0000e+000
wC1 =
    47.5625e+006
wC2 =
    52.5625e+006
BCheck =
     5.0000e+006
```

The specifications require a quality factor of $Q = 10$, which indicates we cannot construct this filter using a cascade connection of two first-order filters separated by a follower. The high quality factor means the magnitude response is narrow band, and not suitable for a cascade design.

Answers:

$L = 6\ \mu\text{H}$, $C = 66.67$ pF, $Q = 10$, $\omega_{C1} = 47.5625$ Mrad/s, and $\omega_{C2} = 52.5625$ Mrad/s.

The filter cannot be designed with a cascade connection of first-order filters because the quality factor is too high.

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Problem 12-22

A parallel *RLC* bandpass circuit with C = 10 nF and Q = 10 has a center frequency of 500 krad/s. Find R, L, and the two cutoff frequencies. Could you design this circuit using a cascade connection of two first-order filters separated by a follower? Why or why not?

Solution:

The following MATLAB calculations determine the requested values.

```
clear all
C = 10e-9;
Q = 10;
w0 = 500e3;
w02 = w0^2;
B = w0/Q;
R = 1/B/C
L = 1/C/w02
wc1 = -1/2/R/C + sqrt((1/2/R/C)^2+1/L/C)
wc2 = 1/2/R/C + sqrt((1/2/R/C)^2+1/L/C)
Check_BW = wc2-wc1
Check_w0 = sqrt(wc1*wc2)
R =
     2.0000e+003
L =
   400.0000e-006
wc1 =
   475.6246e+003
wc2 =
   525.6246e+003
Check_BW =
    50.0000e+003
Check_w0 =
   500.0000e+003
```

The specifications require a quality factor of Q = 10, which indicates we cannot construct this filter using a cascade connection of two first-order filters separated by a follower. The high quality factor means the magnitude response is narrow band, and not suitable for a cascade design.

Answers:

R = 2 kΩ, L = 400 μH, ω_{C1} = 475.625 krad/s, and ω_{C2} = 525.625 krad/s.

The filter cannot be designed with a cascade connection of first-order filters because the quality factor is too high.

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Problem 12-23

A 20-mH inductor with an internal series resistance of 20 Ω is connected in series with a capacitor and a voltage source with a Thévenin resistance of 60 Ω.

(a) What value of C is needed to produce ω_0 = 6 krad/s?

(b) Find the bandwidth and quality factor of the circuit.

Solution:

(a) This is a series *RLC* circuit, so we know the center frequency is related to the inductance and capacitance as follows:

$$\omega_0^2 = \frac{1}{LC}$$

The follow MATLAB calculations determine the capacitance

```
clear all
format short eng
w0 = 6e3;
w02 = w0^2;
L = 20e-3;
C = 1/L/w02
C =
     1.3889e-006
```

(b) The total resistance in the circuit is the sum of the source resistance and the internal resistance of the inductor. The following MATLAB calculations yield the bandwidth and quality factor.

```
R = 60+20;
B = R/L
Q = w0/B
B =
     4.0000e+003
Q =
     1.5000e+000
```

Answers:

(a) C = 1.3889 μF;

(b) B = 4 krad/s and Q = 1.5.

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Problem 12-24

In a series *RLC* circuit, which element would you adjust (and by how much) to

(a) Double the bandwidth without changing the center frequency?

(b) Double the center frequency without changing the bandwidth?

(c) Repeat parts (a) and (b) for a parallel *RLC* circuit.

Solution:

(a) In a series *RLC* circuit, the bandwidth is $B = R/L$, and the center frequency is $\omega_0^2 = 1/LC$. To double the bandwidth without changing the center frequency, we need to double the value of the resistance.

(b) To double the center frequency without changing the bandwidth, we need to reduce the capacitance by a factor of four.

(c) In a parallel *RLC* circuit, the bandwidth is $B = 1/RC$, and the center frequency is $\omega_0^2 = 1/LC$. To double the bandwidth, divide R by two. To double the center frequency, divide L by four.

Answers:

(a) Double R.

(b) Divide C by four.

(c) Divide R by two. Divide L by four.

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Problem 12-25

A parallel *RLC* circuit with $R = 2\ \text{k}\Omega$ has a center frequency of 10 MHz and a bandwidth of 50 kHz. Find the values of *L* and *C*. Could you design this circuit using a cascade connection of two first-order filters separated by a follower? Why or why not?

Solution:

The following MATLAB calculations find the requested values.

```
clear all
R = 2e3;
w0 = 2*pi*10e6;
w02 = w0^2;
B = 2*pi*50e3;
Q = w0/B
% Use the bandwidth to compute the capacitance
C = 1/R/B
% Use the center frequency squared to find the inductance
L = 1/C/w02
Q =
   200.0000e+000
C =
     1.5915e-009
L =
   159.1549e-009
```

The specifications require a quality factor of $Q = 200$, which indicates we cannot construct this filter using a cascade connection of two first-order filters separated by a follower. The high quality factor means the magnitude response is narrow band, and not suitable for a cascade design.

Answers:

$L = 159.15$ nH and $C = 1.5915$ nF.

The filter cannot be designed with a cascade connection of first-order filters because the quality factor is too high.

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Problem 12-26

A series *RLC* bandpass filter is required to have resonance at f_0 = 50 kHz. The circuit is driven by a sinusoidal source with a Thévenin resistance of 60 Ω. The following standard capacitors are available in the stock room: 1 μF, 680 nF, 470 nF, 330 nF, 200 nF, and 120 nF. The inductor will be custom-designed to match the capacitor used. Select the available capacitor that minimizes the filter bandwidth.

Solution:

In a series *RLC* circuit, the bandwidth is $B = R/L$, and the center frequency is $\omega_0^2 = 1/LC$. To minimize the bandwidth, we need to maximize the value of the inductor. The center frequency is fixed, so to maximize L, we need to minimize C. Choose C = 120 nF.

Use MATLAB to verify the results by calculating the bandwidth for all of the possible capacitor values.

```
clear all
R = 60;
w0 = 2*pi*50e3;
w02 = w0^2;
C = [1e-6 680e-9 470e-9 330e-9 200e-9 120e-9];
L = 1/w02./C;
B = R./L;
disp('C    L    B')
Results = [C' L' B']
C    L    B
Results =
     1.0000e-006    10.1321e-006     5.9218e+006
   680.0000e-009    14.9002e-006     4.0268e+006
   470.0000e-009    21.5577e-006     2.7832e+006
   330.0000e-009    30.7034e-006     1.9542e+006
   200.0000e-009    50.6606e-006     1.1844e+006
   120.0000e-009    84.4343e-006   710.6115e+003
```

Answer:

C = 120 nF.

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Problem 12-27

A series *RLC* bandstop circuit is to be used as a notch filter to eliminate a bothersome 60-Hz hum in an audio channel. The signal source has a Thévenin resistance of 500 Ω. Select values of *L* and *C* so the upper cutoff frequency of the stopband is below 100 Hz. Use OrCAD to verify your design. Could you design this circuit using a cascade connection of two first-order filters separated by a follower? Why or why not?

Solution:

Set the upper cutoff frequency to be 90 Hz and the lower cutoff frequency to be such that the center frequency is 60 Hz. Recall that the center frequency is the geometric mean of the two cutoff frequencies. Find the bandwidth. Then, given the resistance is 500 Ω, use the bandwidth to find the inductance. Finally, use the center frequency to find the capacitance. The MATLAB calculations provide the numerical results.

```
clear all
R = 500;
w0 = 2*pi*60;
w02 = w0^2;
% Select the upper cutoff frequency
wc2 = 2*pi*90;
% Calculate the lower cutoff frequency
wc1 = w02/wc2
% Find the bandwidth and quality factor
B = wc2-wc1
Q = w0/B
% Find the inductance and capacitance
L = R/B
C = 1/L/w02
wc1 =
   251.3274e+000
B =
   314.1593e+000
Q =
     1.2000e+000
L =
     1.5915e+000
C =
     4.4210e-006
```

The following OrCAD simulation verifies the results.

The output voltage is shown below. The center frequency is at 60 Hz and the upper cutoff frequency is at 90 Hz.

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This response cannot be realized using a cascade connection of two first-order filters separated by a follower, because a bandstop filter requires two first-order filters combined by a summer.

Answer:
L = 1.5915 H and C = 4.4210 μF.
The simulation results are presented above.
This response cannot be realized using a cascade connection of two first-order filters separated by a follower, because a bandstop filter requires two first-order filters combined by a summer.

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Problem 12-28

Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ for the bandpass circuit in Figure P12-28. Use MATLAB to visualize the Bode characteristics if $R = 50\ \Omega$, $L = 50\ \mu$H, and $C = 20$ nF. Design another circuit to meet those characteristics. Verify your design using OrCAD.

Solution:

Use MATLAB to calculate the transfer function.

```
clear all
syms s R L C
ZC = 1/s/C;
ZL = s*L;
Z1 = R;
Z2 = 1/(1/ZL+1/ZC);
Ts = simplify(Z2/(Z1+Z2))
Ts =
s*L/(R+R*s^2*C*L+s*L)
```

$$T_V(s) = \frac{Ls}{RLCs^2 + Ls + R}$$

Use MATLAB to plot the magnitude response.

```
close all
R = 50;
L = 50e-6;
C= 20e-9;
T = subs(Ts);
ww = logspace(4,8,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
semilogx(ww,MagTjwdB,'g','LineWi
dth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
```

We can rewrite the transfer function as follows:

$$T_V(s) = \frac{Ls}{RLCs^2 + Ls + R} = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

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Examining the denominator of the transfer function, the center frequency squared is 1/*LC*, and the bandwidth is 1/*RC*. Substituting the numerical values, we have:

```
w02 = 1/L/C;
w0 = sqrt(w02)
B = 1/R/C
w0 =
     1.0000e+006
B =
     1.0000e+006
```

We can design a series *RLC* circuit to meet these specifications. Select a value for the resistance and then calculate the inductance and capacitance.

```
R2 = 1e3
L2 = R2/B
C2 = 1/w02/L2
R2 =
     1.0000e+003
L2 =
     1.0000e-003
C2 =
     1.0000e-009
```

The following OrCAD circuit and simulation verify the results.

Answer:

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$$T_{\mathrm{V}}(s) = \frac{Ls}{RLCs^2 + Ls + R}$$

The MATLAB results, circuit design, and OrCAD simulation are presented above.

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Problem 12-29

Show that the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12-29 has a bandstop filter characteristic. Derive expressions relating the notch frequency and the cutoff frequencies to R, L, and C. Then select values of R, L, and C so that the bandwidth and the center frequency are both 1 krad/s. Validate you design using OrCAD.

Solution:

Use MATLAB to determine the transfer function.

```
clear all
syms s
syms R L C positive
ZL = s*L;
ZC =1/s/C;
Z1 = 1/(1/ZL+1/ZC);
Z2 = R;
Ts = simplify(Z2/(Z1+Z2))
Ts =
R*(1+s^2*C*L)/(s*L+R+R*s^2*C*L)
```

$$T_V(s) = \frac{R(LCs^2+1)}{RLCs^2+Ls+R}$$

The transfer function has the form of a bandstop filter.

The notch frequency occurs at the zero of the transfer, which is at $s^2 = -1/LC$. When we substitute $s = j\omega$, this yields

$$\omega_0^2 = \frac{1}{LC}$$

At both low and high frequencies, the gain of the transfer function is one. To find the cutoff frequencies, we need to find when the transfer function has a gain of 0.7071. Substitute $s = j\omega$, and use MATLAB to find the cutoff frequencies.

```
syms wc real
Tjw = subs(Ts,s,j*wc)
Eqn2 = Tjw*conj(Tjw)-1/2;
Soln2 = simplify(solve(Eqn2,wc))
Tjw =
R*(1-wc^2*C*L)/(i*wc*L+R-R*wc^2*C*L)
Soln2 =
  1/2*(-L+L^(1/2)*(L+4*R^2*C)^(1/2))/R/C/L
  -1/2*(L+L^(1/2)*(L+4*R^2*C)^(1/2))/R/C/L
   1/2*(L+L^(1/2)*(L+4*R^2*C)^(1/2))/R/C/L
 -1/2*(-L+L^(1/2)*(L+4*R^2*C)^(1/2))/R/C/L
```

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Choose the positive solutions.

$$\omega_{C1} = \frac{-L+\sqrt{L}\sqrt{L+4R^2C}}{2RLC} = \frac{-1+\sqrt{1+\frac{4R^2C}{L}}}{2RC} = -\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}$$

$$\omega_{C2} = \frac{L+\sqrt{L}\sqrt{L+4R^2C}}{2RLC} = \frac{1+\sqrt{1+\frac{4R^2C}{L}}}{2RC} = \frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}$$

The bandwidth is the difference between the cutoff frequencies, $B = 1/RC$. Choose a value for the resistor and calculate the other values.

```
B = 1e3;
w0 = 1e3;
w02 = w0^2;
R = 1e3
C = 1/R/B
L = 1/C/w02
R =
      1.0000e+003
C =
      1.0000e-006
L =
      1.0000e+000
```

The OrCAD circuit above and its simulation results are shown below.

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Answers:

$$T_{\mathrm{V}}(s) = \frac{R(LCs^2+1)}{RLCs^2+Ls+R}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_{\mathrm{C1}} = \frac{-L+\sqrt{L}\sqrt{L+4R^2C}}{2RLC} = \frac{-1+\sqrt{1+\frac{4R^2C}{L}}}{2RC} = -\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}$$

$$\omega_{\mathrm{C2}} = \frac{L+\sqrt{L}\sqrt{L+4R^2C}}{2RLC} = \frac{1+\sqrt{1+\frac{4R^2C}{L}}}{2RC} = \frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}$$

R = 1 kΩ
L = 1 H
C = 1 μF

The circuit design and simulation results are presented above.

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Problem 12-30

Figure P12-30 shows an *RLC* filter with an input current and an output voltage. The purpose of this problem is to determine the filter type using informal circuit analysis. Use the element impedances and basic analysis tools to find the magnitude of the output voltage $*V_2(j\omega)*$ at ω = 0, ω = 4, and $\omega = \omega_0 = 1 ' \sqrt{LC}$. What is the filter type?

Solution:

At ω = 0, the capacitor acts like an open circuit and the inductor acts like a short circuit. The current flows through the resistor and the inductor and the output voltage is zero.

At ω = 4, the capacitor acts like a short circuit and the inductor acts like an open circuit. The current flows through the capacitor only and the output voltage is again zero.

At $\omega = \omega_0 = 1 ' \sqrt{LC}$, we can find the impedance of the capacitor and the inductor and then use current division to find the output current. Once we know the output current, we can find the output voltage. The following MATLAB calculations provide the solution.

```
clear all
syms s
syms w0 R L C positive
ZC = 1/s/C;
ZL = s*L;
w0 = 1/sqrt(L*C);
% Current division
T1 = 1/(R+ZL)/(1/(R+ZL) + 1/ZC);
% Output voltage
T = T1*ZL;
Tjw0 = simplify(subs(T,s,j*w0))
Tjw0 =
L/C/R
```

At $\omega = \omega_0 = 1 ' \sqrt{LC}$, $*V_2(j\omega)* = *I_1(j\omega)*$ (L/RC). There is a signal at the center frequency, so this circuit is a bandpass filter.

Answer:

At ω = 0 and ω = 4, the output signal is zero.

At $\omega = \omega_0 = 1 ' \sqrt{LC}$, $*V_2(j\omega)* = *I_1(j\omega)*$ (L/RC).

The circuit is a bandpass filter.

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Problem 12-31

Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ for the circuit in Figure P12-31.

(a) Construct the straight-line Bode plot of the gain and phase of the transfer function. Use the straight-line plots to estimate the amplitude and phase of the steady-state output for $v_1(t) = 10 \sin 30t$ V.

(b) Calculate the actual output amplitude and phase for this input and compare the two results.

(c) Use MATLAB to generate a Bode plot of your transfer function. Compare your answers from the straight-line plot and your hand calculations, with that produced by MATLAB.

Solution:

The following MATLAB calculations determine the transfer function.

```
clear all
syms s
C = 1e-6;
R1 = 100e3;
R2 = 10e3;
ZC = 1/s/C;
Z1 = 1/(1/ZC+1/R1);
Z2 = R2;
T = simplify(Z2/(Z1+Z2))
T =
(10+s)/(110+s)
```

$$T_V(s) = \frac{s+10}{s+110}$$

(a) and (c) Plot the actual and straight-line approximations on the same axes.

```
% Create the actual gain responses
close all
ww = logspace(-1,4,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
Mag0 = 20*log10(10/110);
MagInf = 20*log10(1/1);
line([.1,10],[Mag0,Mag0],'LineWidth',3,
'LineStyle',':')
line([10,110],[Mag0,MagInf],'LineWidth',3,'LineStyle',':')
line([110,1e4],[MagInf,MagInf],'LineWidth',3,'LineStyle',':')
```

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```
% Create the phase response
close all
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual phase response
semilogx(ww,PhaseTjw,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
% Plot the straight-line approximation
% The maximum phase is 45 deg/decade
times the log of
% the ratio of the start and end
frequencies
PhaseMax = 45*log10(11/1);
line([.1,1],[0,0],'LineWidth',3,'LineSty
le',':')
line([1,11],[0,PhaseMax],'LineWidth',3,'
LineStyle',':')
line([11,100],[PhaseMax,PhaseMax],'LineWidth',3,'LineStyle',':')
line([100,1100],[PhaseMax,0],'LineWidth',3,'LineStyle',':')
line([1100,1e4],[0,0],'LineWidth',3,'LineStyle',':')
```

Estimate the signal amplitude and phase at ω = 30 rad/s using the straight-line approximations.

```
% Magnitude estimate
% Starting dB is Mag0 calculated above
% Change is 20 dB/decade over log10(30/10) decades
Mag30dB = Mag0+20*log10(30/10);
Mag30 = 10^(Mag30dB/20);
% Scale by the magnitude of the input
MagOut = 10*Mag30

% Phase estimate
% 30 rad/s occurs at the flat portion of the approximation
% Note the input phase is -90 degrees because of the sine
PhaseIn = -90;
PhaseOut = PhaseMax + PhaseIn
MagOut =
     2.7273e+000
PhaseOut =
   -43.1373e+000
```

(b) Find the actual magnitude and phase using the transfer function.

```
Tj30 = subs(T,s,j*30);
MagTj30 = abs(Tj30);
MagOut = 10*MagTj30
PhaseOut = PhaseIn + 180*angle(Tj30)/pi
MagOut =
     2.7735e+000
```

Answers:

$$T_V(s) = \frac{s+10}{s+110}$$

(a) A_{SL} = 2.7273 V and ϕ_{SL} = −43.14°

(b) A_{ACTUAL} = 2.7735 V and ϕ_{ACTUAL} = −33.69°

(c) The MATLAB plots are shown above. In the plots, the magnitude approximation and actual values are in close agreement, which is reflected in the similarity of the actual and approximate output amplitudes. The phase approximation is not very close to the actual phase response at 30 rad/s, as shown in the plot. Therefore, the approximated and actual output phases differ by approximately 10°, which is consistent with the

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```
PhaseOut =
   -33.6901e+000
```

Problem 12-32
Repeat Prob. 12-31 using the circuit in Figure P12-32.

Solution:
The circuit is a non-inverting amplifier. The following MATLAB calculations determine the transfer function.

```
clear all
syms s
C = 1e-6;
R1 = 10e3;
R2 = 100e3;
ZC = 1/s/C;
Z1 = R1;
Z2 = 1/(1/ZC+1/R2);
T = simplify((Z1+Z2)/Z1)
T =
(110+s)/(10+s)
```

$$T_{\mathrm{V}}(s) = \frac{s+110}{s+10}$$

(a) and (c) Plot the actual and straight-line approximations on the same axes.

```
% Create the actual gain responses
close all
ww = logspace(-1,4,500);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth'
,3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line
approximation
Mag0 = 20*log10(110/10);
MagInf = 20*log10(1/1);
line([.1,10],[Mag0,Mag0],'LineWidth'
,3,'LineStyle',':')
line([10,110],[Mag0,MagInf],'LineWid
```

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```
th',3,'LineStyle',':')
line([110,1e4],[MagInf,MagInf],'LineWidth',3,'LineStyle',':')


% Create the phase response
close all
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual phase response
semilogx(ww,PhaseTjw,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
% Plot the straight-line approximation
% The maximum phase is 45 deg/decade
times the log of
% the ratio of the start and end
frequencies
PhaseMin = -45*log10(11/1);
line([.1,1],[0,0],'LineWidth',3,'LineSt
yle',':')
line([1,11],[0,PhaseMin],'LineWidth',3,
'LineStyle',':')
line([11,100],[PhaseMin,PhaseMin],'Line
Width',3,'LineStyle',':')
line([100,1100],[PhaseMin,0],'LineWidth
',3,'LineStyle',':')
line([1100,1e4],[0,0],'LineWidth',3,'LineStyle',':')
```

Estimate the signal amplitude and phase at ω = 30 rad/s using the straight-line approximations.

```
% Magnitude estimate
% Starting dB is Mag0 calculated above
% Change is -20 dB/decade over log10(30/10) decades
Mag30dB = Mag0-20*log10(30/10);
Mag30 = 10^(Mag30dB/20);
% Scale by the magnitude of the input
MagOut = 10*Mag30

% Phase estimate
% 30 rad/s occurs at the flat portion of the approximation
% Note the input phase is -90 degrees because of the sinusoid
PhaseIn = -90;
PhaseOut = PhaseMin + PhaseIn
MagOut =
    36.6667e+000
PhaseOut =
  -136.8627e+000
```

(b) Find the actual magnitude and phase using the transfer function.

```
Tj30 = subs(T,s,j*30);
```

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Answers:

$$T_V(s) = \frac{s+110}{s+10}$$

(a) A_{SL} = 36.6667 V and ϕ_{SL} = −136.86°

(b) A_{ACTUAL} = 36.0555 V and ϕ_{ACTUAL} = −146.31°

(c) The MATLAB plots are shown above. In the plots, the magnitude approximation and actual values are in close agreement, which is reflected in the similarity of the actual and approximate output amplitudes. The phase approximation is not very close to the actual phase response at 30 rad/s, as shown in the plot. Therefore, the approximated and actual output phases differ by approximately 10°, which is consistent with the plots.

```
MagTj30 = abs(Tj30);
MagOut = 10*MagTj30
PhaseOut = PhaseIn + 180*angle(Tj30)/pi
MagOut =
    36.0555e+000
PhaseOut =
  -146.3099e+000
```

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Problem 12-33

For the following transfer function

$$T(s) = \frac{(s+100)}{(s+10)(s+1000)}$$

(a) Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency and passband gain.

(b) Use MATLAB to plot the Bode magnitude of the transfer function.

(c) Design a circuit using practical values for the components that has the same transfer function.

(d) Use OrCAD to compare the frequency response of your designed circuit with the MATLAB Bode plot.

Solution:

(a) and (b) Plot the actual and straight-line approximation to the magnitude together.

```
clear all
% Create the actual gain responses
close all
syms s
T = (s+100)/(s+10)/(s+1000);
ww = logspace(-1,5,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
Mag0 = 20*log10(100/10/1000);
```

```
line([.1,10],[Mag0,Mag0],'LineWidth',3,'LineStyle',':')
line([10,100],[Mag0,Mag0-20],'LineWidth',3,'LineStyle',':')
line([100,1000],[Mag0-20,Mag0-20],'LineWidth',3,'LineStyle',':')
line([1000,100000],[Mag0-20,Mag0-60],'LineWidth',3,'LineStyle',':')
```

This is a low-pass filter with a cutoff frequency of 10 rad/sec and a passband gain of approximately −40 dB.

(c) Write the transfer function as follows:

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$$T(s)=\frac{s+100}{(s+10)(s+1000)}=\left(\frac{1}{s+10}\right)\left(\frac{s+100}{s+1000}\right)=\left(\frac{\frac{1}{s}}{1+\frac{1}{s}+\frac{9}{s}}\right)\left(\frac{s+100}{s+1000}\right)$$

Using the techniques from Chapter 11, we can create the following circuit.

Note that the 1000-MΩ resistor at the output is added to the circuit to allow OrCAD to complete the simulation.

(d) The OrCAD simulation results are shown below and agree with the MATLAB results.

Answers:

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(a) This is a low-pass filter with a cutoff frequency of 10 rad/sec and a passband gain of approximately −40 dB.
(b) The MATLAB plot is shown above.
(c) The circuit is shown above.
(d) The simulation results are shown above and match the MATLAB results.

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Problem 12-34

For the following transfer function

$$T(s) = \frac{10^6 s^2}{(s+1000)^2}$$

(a) Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency and passband gain.

(b) Use MATLAB to plot the Bode magnitude of the transfer function.

(c) Design a circuit using practical values for the components that has the same transfer function.

(d) Use OrCAD to compare the frequency response of your designed circuit with the MATLAB Bode plot.

Solution:

(a) and (b) Plot the actual and straight-line approximation to the magnitude together.

```
clear all
% Create the actual gain responses
close all
syms s
T = 1e6*s^2/(s+1000)/(s+1000);
ww = logspace(1,5,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
MagInf = 20*log10(1e6);
line([10,1000],[MagInf-
80,MagInf],'LineWidth',3,'LineStyle',':')
line([1000,100000],[MagInf,MagInf],'LineWidth',3,'LineStyle',':')
```

This is a high-pass filter with a corner frequency of 1 krad/s in the straight-line approximation. The cutoff frequency is slightly higher because there are two poles acting at 1 krad/s. The cutoff frequency is approximately 1.56 krad/s and the passband gain is approximately 120 dB.

(c) Write the transfer function as follows:

$$T(s) = \left[\frac{1000s}{s+1000}\right]^2 = \left[\frac{1000}{1+\frac{1000}{s}}\right]^2$$

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The circuit can be formed using two identical stages. Using the techniques from Chapter 11, we can create the following circuit.

(d) The OrCAD simulation results are shown below and agree with the MATLAB results.

Answers:
(a) This is a high-pass filter. The cutoff frequency is approximately 1.56 krad/s and the passband gain is approximately 120 dB.
(b) The MATLAB plot is shown above.
(c) The circuit is shown above.
(d) The simulation results are shown above and match the MATLAB results.

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Problem 12-35

For the following transfer function

$$T(s) = \frac{250\,s\,(s+300)}{(s+10)(s+1000)}$$

(a) Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency and passband gain.

(b) Use MATLAB to plot the Bode magnitude of the transfer function.

(c) Design a circuit using practical values for the components that has the same transfer function.

(d) Use OrCAD to compare the frequency response of your designed circuit with the MATLAB Bode plot.

Solution:

(a) and (b) Plot the actual and straight-line approximation to the magnitude together.

```
clear all
% Create the actual gain responses
close all
syms s
T = 250*s*(s+300)/(s+10)/(s+1000);
ww = logspace(-1,5,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
% Find the important straight-line
gains
MagInf = 20*log10(250)
MagMid = MagInf + 20*log10(300/1000);
line([.1,10],[MagMid-40,MagMid],'LineWidth',3,'LineStyle',':')
line([10,300],[MagMid,MagMid],'LineWidth',3,'LineStyle',':')
line([300,1000],[MagMid,MagInf],'LineWidth',3,'LineStyle',':')
line([1000,100000],[MagInf,MagInf],'LineWidth',3,'LineStyle',':')
MagInf =
   47.9588
```

This is a high-pass filter with a cutoff frequency of 1 krad/sec and a passband gain of approximately 48 dB.

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(c) Write the transfer function as follows: $T(s) = \left(\frac{250s}{s+10}\right)\left(\frac{s+300}{s+1000}\right) = \left(\frac{250}{1+\frac{10}{s}}\right)\left(\frac{s+300}{s+1000}\right)$

Using the techniques from Chapter 11, we can create the following circuit.

(d) The OrCAD simulation results are shown below and agree with the MATLAB results.

Answers:
(a) This is a high-pass filter with a cutoff frequency of 1 krad/sec and a passband gain of approximately 48 dB.
(b) The MATLAB plot is shown above.
(c) The circuit is shown above.
(d) The simulation results are shown above and match the MATLAB results.

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Problem 12-36

For the following transfer function $T_V(s) = V_2(s)/V_1(s)$

$$T_V(s) = \frac{10(s+5)(s+20)}{(s+1)(s+100)}$$

(a) Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency(ies) and passband gain.

(b) Use MATLAB to plot the Bode magnitude and phase of the transfer function.

(c) What is the output $v_2(t)$, if $v_1(t) = 10 \cos(1000t + 45°)$ V?

Solution:

(a) and (b) Plot the actual and straight-line approximation to the magnitude together.

```
clear all
% Create the actual gain responses
close all
syms s
T = 10*(s+5)*(s+20)/(s+1)/(s+100);
ww = logspace(-2,4,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
% Find the important straight-line
gains
Mag0 = 20*log10(10*5*20/1/100);
MagInf = 20*log10(10);
MagMid = Mag0 - 20*log10(5/1);
line([.01,1],[Mag0,Mag0],'LineWidth',3,'LineStyle',':')
line([1,5],[Mag0,MagMid],'LineWidth',3,'LineStyle',':')
line([5,20],[MagMid,MagMid],'LineWidth',3,'LineStyle',':')
line([20,100],[MagMid,MagInf],'LineWidth',3,'LineStyle',':')
line([100,10000],[MagInf,MagInf],'LineWidth',3,'LineStyle',':')
```

This is a bandstop filter with cutoff frequencies at 1 rad/s and 100 rad/s. The passband gain is 20 dB in both passbands.

(b) Plot the actual phase response.

```
% Create the phase response
close all
PhaseTjw = 180*angle(Tjw)/pi;
semilogx(ww,PhaseTjw,'g','LineWidth',3)
grid on
```

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```
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
```

(c) Find the magnitude and phase of the transfer function at 1 krad/s to determine the magnitude and phase of the output for the given input signal.

```
Tj1000 = subs(T,s,j*1000);
MagTj1000 = abs(Tj1000);
PhaseTj1000 = 180*angle(Tj1000)/pi;
MagV2 = 10*MagTj1000
PhaseV2 = 45+PhaseTj1000
MagV2 =
   99.5248
PhaseV2 =
   49.3356
```

For $v_1(t) = 10 \cos (1000t + 45°)$ V, the output signal is $v_2(t) = 99.52 \cos (1000t + 49.34°)$ V.

Answers:

(a) This is a bandstop filter with cutoff frequencies at 1 rad/s and 100 rad/s. The passband gain is 20 dB.

(b) The plots are shown above.

(c) For $v_1(t) = 10 \cos (1000t + 45°)$ V, the output signal is $v_2(t) = 99.52 \cos (1000t + 49.34°)$ V.

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Problem 12-37

For the following transfer function $T_V(s) = V_2(s)/V_1(s) = \dfrac{10^6(s+10)^2}{(s+100)^4}$

(a) Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency(ies) and passband gain.
(b) Use MATLAB to plot the Bode magnitude and phase of the transfer function.
(c) What is the output $v_2(t)$, if $v_1(t) = 10 \cos(50t - 60°)$ V?

Solution:
(a) and (b) Plot the actual and straight-line approximation to the magnitude together.

```
clear all
% Create the actual gain responses
close all
syms s
T = 1e6*(s+10)^2/(s+100)^4;
ww = logspace(-1,4,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
% Find the important straight-line
gains
Mag0 = 20*log10(1e6*10^2/100^4);
line([.1,10],[Mag0,Mag0],'LineWidth',3,'LineStyle',':')
line([10,100],[Mag0,Mag0+40],'LineWidth',3,'LineStyle',':')
line([100,10000],[Mag0+40,Mag0+40-80],'LineWidth',3,'LineStyle',':')
```

This is a low-pass filter with a peak. The cutoff frequency is approximately 1 krad/s and the passband gain is 0 dB.

(b) Plot the actual phase response.

```
% Create the phase response
close all
PhaseTjw = 180*angle(Tjw)/pi;
semilogx(ww,PhaseTjw,'g','LineWidth',
3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
```

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(c) Find the magnitude and phase of the transfer function at 50 rad/s to determine the magnitude and phase of the output for the given input signal.

```
w1 = 50;
Tjw1 = subs(T,s,j*w1);
MagTjw1 = abs(Tjw1);
PhaseTjw1 = 180*angle(Tjw1)/pi;
MagV2 = 10*MagTjw1
PhaseV2 = -60+PhaseTjw1
MagV2 =
  166.4000
PhaseV2 =
   -8.8801
```

For $v_1(t) = 10 \cos (50t - 60°)$ V, the output signal is $v_2(t) = 166.4 \cos (50t - 8.88°)$ V.

Answers:
(a) This is a low-pass filter with a peak. The cutoff frequency is approximately 1 krad/s and the passband gain is 0 dB.
(b) The plots are shown above.
(c) For $v_1(t) = 10 \cos (50t - 60°)$ V, the output signal is $v_2(t) = 166.4 \cos (50t - 8.88°)$ V.

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Problem 12-38

For the following transfer function $T_V(s) = V_2(s)/V_1(s)$

$$T_V(s) = \frac{4s}{0.04\,s^2 + 0.2\,s + 1}$$

(a) Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function?

(b) Use the straight-line plot to estimate the maximum gain and the frequency at which it occurs.

(c) Use MATLAB to plot the Bode magnitude and phase of the transfer function.

Solution:

(a) and (c) We can rewrite the transfer function as follows:

$$T_V(s) = \frac{4s}{0.04\,s^2 + 0.2\,s + 1} = \frac{100s}{s^2 + 5s + 25} = \frac{100s}{s^2 + 5s + 5^2}$$

The transfer function has a zero at 0 rad/s and complex poles at 5 rad/s. The straight-line approximation enters with a slope of +20 dB/decade up until ω = 5 rad/s, where the complex poles contribute to the response. After ω = 5 rad/s, the slope is −20 dB/decade. There are no other changes to the Bode magnitude plot. The damping ratio is 0.5, so we do not expect a significant resonant peak. The gain at the peak is calculated in the following MATLAB script. Plot the actual and straight-line approximation to the magnitude together.

```
clear all
% Create the actual gain responses
close all
syms s
T = 4*s/(0.04*s^2+0.2*s+1);
ww = logspace(-2,2,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
% Find the magnitude at the peak
MagTj5 = abs(subs(T,s,j*5));
MagTj5dB = 20*log10(MagTj5)
% Find the important straight-line
gains
MagMid = 20*log10(MagTj5);
MagMin = MagMid+20*log10(0.01/5);
MagMax = MagMid+20*log10(5/100);
line([.01,5],[MagMin,MagMid],'LineWidth',3,'LineStyle',':')
line([5,100],[MagMid,MagMax],'LineWidth',3,'LineStyle',':')
```

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```
MagTj5dB =
   26.0206
```

This is a bandpass filter.

(b) The maximum gain is approximately 26 dB and it occurs at 5 rad/s.

(c) The magnitude response is plotted above. Plot the actual phase response.

```
% Create the phase response
close all
PhaseTjw = 180*angle(Tjw)/pi;
semilogx(ww,PhaseTjw,'g','LineWidth',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
```

Answers:
(a) The plots are shown above. The transfer function represents a bandpass filter.
(b) The maximum gain is approximately 26 dB and it occurs at 5 rad/s.
(c) The phase response is shown above.

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Problem 12-39

Consider the gain plot in Figure P12-39.

(a) Find the transfer function corresponding to the straight-line gain plot.

(b) Use MATLAB to plot the Bode magnitude of the transfer function.

(c) Compare the straight-line gain and the actual gain at ω = 10 and 400 rad/s.

Solution:

(a) Since the gain plot is horizontal for low frequencies, there are no poles or zeros at 0 rad/s. Based on the changes in the plot, a single pole occurs at 20 rad/s and a single zero at 200 rad/s. The transfer function takes the following form:

$$T(s) = K\frac{s+200}{s+20}$$

The high-frequency gain is 0 dB, which is an absolute gain of one, so $K = 1$.

(b) Plot the Bode magnitude of the transfer function.

```
clear all
% Create the actual gain responses
close all
syms s
T = (s+200)/(s+20);
ww = logspace(0,4,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
```

(c) The straight-line gain at ω = 10 rad/s is 20 dB and the straight-line gain at ω = 400 rad/s is 0 dB. Compute the actual gains at ω = 10 rad/s and ω = 400 rad/s.

```
ww = [10 400];
Tjww = subs(T,s,j*ww);
MagTjww = abs(Tjww)
MagTjwwdB = 20*log10(MagTjww)
MagTjww =
    8.9554    1.1166
MagTjwwdB =
   19.0417    0.9583
```

Answers:

(a) $T(s) = \dfrac{s+200}{s+20}$

(b) The MATLAB results are shown above.

(c) For ω = 10 rad/s, the straight-line gain is 20 dB and the actual gain is 19.0417 dB. For ω = 400 rad/s, the straight-line gain is 0 dB and the actual gain is 0.9583 dB. The straight-line gain is a reasonably good approximation at these two frequencies.

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Problem 12-40
Consider the gain plot in Figure P12-40.
(a) Find the transfer function corresponding to the straight-line gain plot.
(b) Use MATLAB to plot the Bode magnitude of the transfer function.
(c) Compare the straight-line gain and the actual gain at ω = 100 and 500 rad/s.
(d) Design a circuit that will realize the transfer function found in (a).
(e) Use OrCAD to verify your circuit design.

Solution:
(a) There are no zeros or poles at 0 rad/s. There is a zero located at 5 rad/s. The two poles are located at 50 rad/s and 250 rad/s. The transfer function has the following form:

$$T(s) = K\frac{s+5}{(s+50)(s+250)}$$

The gain at low frequencies is −10 dB, which is an absolute gain of 0.3162. If we substitute $s = j\omega$ for a small value of ω, the approximate gain of the transfer function is $(5K)/(50\times250) = K/2500 = 0.3162$. Solving for K, we have $K = 790.57$.

(b) Plot the Bode magnitude of the transfer function.

```
clear all
% Create the actual gain responses
close all
syms s
K = 2500*10^(-10/20)
T = K*(s+5)/(s+50)/(s+250);
ww = logspace(-1,4,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
semilogx(ww,MagTjwdB,'g','LineWidth
',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
K =
  790.5694
```

(c) The straight-line gain at ω = 100 rad/s is 10 dB and the straight-line gain at ω = 500 rad/s is calculated below. Also compute the actual gains at ω = 100 rad/s and ω = 500 rad/s.

```
% Straight-line gain at w = 500
MagSL500dB = 10+20*log10(250/500)
% Actual gains at w = 100 and w = 500
ww = [100 500];
Tjww = subs(T,s,j*ww);
```

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```
MagTjww = abs(Tjww)
MagTjwwdB = 20*log10(MagTjww)
MagSL500dB =
    3.9794
MagTjww =
    2.6294    1.4073
MagTjwwdB =
    8.3972    2.9675
```

(d) Rewrite the transfer function as follows:

$$T(s) = (790.57)\frac{s+5}{(s+50)(s+250)} = \left[\frac{250}{s+250}\right]\left[3.16\right]\left[\frac{s+5}{s+50}\right] = \left[\frac{\frac{250}{s}}{1+\frac{250}{s}}\right]\left[3.16\right]\left[\frac{s+5}{s+50}\right]$$

Using the techniques from Chapter 11, we can design the following circuit:

(e) The OrCAD simulation results are shown below. The plot closely matches the MATLAB results, if we account for the 2π scaling difference between the frequency axes.

Answers:
(a)

$$T(s) = (790.57)\frac{s+5}{(s+50)(s+250)}$$

(b) The MATLAB results are shown above.

(c) For ω = 100 rad/s, the straight-line gain is 10 dB and the actual gain is 8.3972 dB. For ω = 500 rad/s, the straight-line gain is 3.9794 dB and the actual gain is 2.9675 dB. The straight-line gain is a reasonably good approximation at these two frequencies.

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(d) The circuit design is presented above.

(e) The OrCAD simulation results are presented above.

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Problem 12-41
Consider the gain plot in Figure P12-41.
(a) Find the transfer function corresponding to the straight-line gain plot.
(b) Use MATLAB to plot the Bode magnitude of the transfer function.
(c) Design a circuit that will realize the transfer function found in (a).
(d) Use OrCAD to verify your circuit design.

Solution:
(a) The gain plot has an initial slope of +1, so it must have a zero at 0 rad/s. The transfer function has poles located at 100 rad/s and 50 krad/s. The general form of the transfer function is shown below.

$$T(s) = \frac{Ks}{(s+100)(s+50000)}$$

To find the gain, K, we can evaluate the transfer function at 1000 rad/s, which is in the center of the flat portion of the response where the gain is 5. We can approximate the gain as follows:

$$|T(s)| = 5 = \frac{K(1000)}{(1000)(50000)}$$

Solving for the gain, we have K = 250000, so the transfer function is

$$T(s) = \frac{250000s}{(s+100)(s+50000)}$$

(b) Plot the Bode magnitude of the transfer function.

```
clear all
% Create the actual gain responses
close all
syms s
T = 250000*s/(s+100)/(s+50000);
ww = logspace(0,6,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
loglog(ww,MagTjw,'g','LineWidth',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude')
```

(c) Rewrite the transfer function as follows:

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$$T(s)=\frac{250000s}{(s+100)(s+50000)}=\left(\frac{s}{s+100}\right)(5)\left(\frac{50000}{s+50000}\right)=\left(\frac{1}{1+\frac{100}{s}}\right)(5)\left(\frac{\frac{50000}{s}}{1+\frac{50000}{s}}\right)$$

We can design the following circuit to realize the transfer function.

(d) The OrCAD simulation results are shown below and verify the MATLAB results.

Answers:

(a) $T(s)=\dfrac{250000s}{(s+100)(s+50000)}$

(b) The MATLAB plot is presented above.

(c) The circuit design is presented above.

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(d) The OrCAD simulation results are presented above.

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Problem 12-42

Consider the gain plot in Figure P12-42.

(a) Find the transfer function corresponding to the straight-line gain plot.

(b) Use MATLAB to plot the Bode magnitude of the transfer function.

(c) Design a circuit that will realize the transfer function found in (a).

(d) Use OrCAD to verify your circuit design.

Solution:

(a) The transfer function has no poles or zeros at 0 rad/s. There is a pole at 10 rad/s and another pole at 5000 rad/s. The two zeros are located at 100 rad/s and 500 rad/s. The gain at 0 rad/s is 1. The transfer function has the following form.

$$T(s)=\frac{K(s+100)(s+500)}{(s+10)(s+5000)}=\frac{(s+100)(s+500)}{(s+10)(s+5000)}$$

If we evaluate the transfer function at 0 rad/s and set the gain equal to 1, the gain constant $K = 1$, as shown above.

(b) Plot the Bode magnitude of the transfer function.

```
clear all
% Create the actual gain responses
close all
syms s
T = (s+100)*(s+500)/(s+10)/(s+5000);
ww = logspace(-1,5,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
loglog(ww,MagTjw,'g','LineWidth',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude')
```

(c) Rewrite the transfer function as follows:

$$T(s)=\frac{(s+100)(s+500)}{(s+10)(s+5000)}=\left(\frac{s+100}{s+10}\right)\left(\frac{s+500}{s+5000}\right)$$

We can design the following circuit to realize the transfer function.

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(d) The OrCAD simulation results are shown below and verify the MATLAB results.

Answers:

(a) $T(s) = \dfrac{(s+100)(s+500)}{(s+10)(s+5000)}$

(b) The MATLAB plot is presented above.

(c) The circuit design is presented above.

(d) The OrCAD simulation results are presented above.

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Problem 12-43

Consider the gain plot in Figure P12-43.

(a) Find the transfer function corresponding to the straight-line gain plot.

(b) Use MATLAB to plot the Bode magnitude of the transfer function.

(c) Design a circuit that will realize the transfer function found in (a).

(d) Use OrCAD to verify your circuit design.

Solution:

(a) The transfer function has two zeros at 0 rad/s, a pole at 10 rad/s, and another pole at 250 rad/s. The gain at high frequencies is 10. The transfer function has the following form:

$$T(s) = \frac{Ks^2}{(s+10)(s+250)}$$

If we evaluate the transfer function at 1000 rad/s and set the gain equal to 10, the gain constant K can be calculated as follows:

$$|T(s)| = \frac{K(1000)^2}{(1000)(1000)} = 10$$

The gain constant K = 10.

(b) Plot the Bode magnitude of the transfer function.

```
clear all
% Create the actual gain
responses
close all
syms s
T = 10*s^2/(s+10)/(s+250);
ww = logspace(-1,4,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
loglog(ww,MagTjw,'g','LineWidth',
3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude')
```

(c) Rewrite the transfer function as follows:

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$$T(s)=\frac{10s^2}{(s+10)(s+250)}=\left(\frac{s}{s+10}\right)(10)\left(\frac{s}{s+250}\right)=\left(\frac{1}{1+\frac{10}{s}}\right)(10)\left(\frac{1}{1+\frac{250}{s}}\right)$$

We can design the following circuit to realize the transfer function.

(d) The OrCAD simulation results are shown below and verify the MATLAB results.

Answers:

(a) $T(s)=\dfrac{10s^2}{(s+10)(s+250)}$

(b) The MATLAB plot is presented above.

(c) The circuit design is presented above.

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(d) The OrCAD simulation results are presented above.

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Problem 12-44

For the following transfer function $T_V(s) = V_2(s)/V_1(s)$

$$T_V(s) = \frac{10(s+20)}{s+1}$$

(a) Construct the straight-line Bode plot of the phase.

(b) Use the straight-line phase diagram to estimate the phase at ω = 1, 10, and 100 rad/s.

(c) Use MATLAB to plot the Bode gain and phase and compare the phase plot to the straight-line estimate.

Solution:

(a) and (c) Plot the straight-line Bode plot or the phase and the actual phase plot on the same axes.

```
clear all
close all
syms s
T = 10*(s+20)/(s+1);
ww = logspace(-3,4,1000);
Tjw = subs(T,s,j*ww);
% Create the phase response
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual phase response
semilogx(ww,PhaseTjw,'g','LineWid
th',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
% Plot the straight-line
approximation
% Find the phase at the middle
frequencies
PhaseMid = -45*log10(2/.1);
line([1e-3,0.1],[0,0],'LineWidth',3,'LineStyle',':')
line([0.1,2],[0,PhaseMid],'LineWidth',3,'LineStyle',':')
line([2,10],[PhaseMid,PhaseMid],'LineWidth',3,'LineStyle',':')
line([10,200],[PhaseMid,0],'LineWidth',3,'LineStyle',':')
line([200,10000],[0,0],'LineWidth',3,'LineStyle',':')
```

The two plots are in close agreement except near the critical frequencies.

(b) Using the straight-line approximation, we can estimate the phase at ω = 1, 10, and 100 rad/s as follows. At ω = 1 rad/s, the approximation is one decade after the plots starts to decrease at −45°/decade, so the value is −45°. At ω = 10 rad/s, the plot is at the right edge of the horizontal portion, so the value is −45°[*log*(2/.1)] = −58.55°. At ω = 100 rad/s, the plot has increased from its lowest value at +45°/decade for one decade, so the value is −13.55°.

(c) We plotted the phase above, so we need to plot the magnitude response as shown below.

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```
close all
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude')
```

Answers:
(a) The plot is shown above.
(b) At ω = 1 rad/s, the phase is −45°. At ω = 10 rad/s, the phase is −58.55°. At ω = 100 rad/s, the phase is −13.55°.
(c) The MATLAB plot is shown above.

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Problem 12-45

For the following transfer function $T_V(s) = V_2(s)/V_1(s)$

$$T_V(s) = \frac{10(s+5)}{(s+1)(s+100)}$$

(a) Construct the straight-line Bode plot of the phase.

(b) Use the straight-line phase diagram to estimate the phase at ω = 1, 10, and 100 rad/s.

(c) Use MATLAB to plot the Bode gain and phase and compare the phase plot to the straight-line estimate.

Solution:

(a) and (c) Plot the straight-line Bode plot or the phase and the actual phase plot on the same axes.

```
clear all
close all
syms s
T = 10*(s+5)/(s+1)/(s+100);
ww = logspace(-3,4,1000);
Tjw = subs(T,s,j*ww);
% Create the phase response
PhaseTjw = 180*angle(Tjw)/pi;
% Plot the actual phase response
semilogx(ww,PhaseTjw,'g','LineWidt
h',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Phase, (degrees)')
% Plot the straight-line
approximation
% Find the phase at the middle
frequencies
PhaseMid = -45*log10(.5/.1);
PhaseInf = PhaseMid-45*log10(1000/50);
line([1e-3,0.1],[0,0],'LineWidth',3,'LineStyle',':')
line([0.1,0.5],[0,PhaseMid],'LineWidth',3,'LineStyle',':')
line([0.5,50],[PhaseMid,PhaseMid],'LineWidth',3,'LineStyle',':')
line([50,1000],[PhaseMid,PhaseInf],'LineWidth',3,'LineStyle',':')
line([1000,10000],[-90,-90],'LineWidth',3,'LineStyle',':')
```

The two plots are similar, but there are significant differences for frequencies between 1 and 100 rad/s.

(b) Using the straight-line approximation, we can estimate the phase at ω = 1, 10, and 100 rad/s as follows. At ω = 1 rad/s, the approximation is in the center horizontal portion, so the phase is −45°[*log*(.5/.1)] = −31.45°. At ω = 10 rad/s, the plot is also at the center horizontal portion, so the phase is −31.45°. At ω = 100 rad/s, the plot has decreased from the center section by −45°[*log*(100/50)] = −13.55°, to give a value of −45°.

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(c) We plotted the phase above, so we need to plot the magnitude response as shown below.

```
close all
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude')
```

Answers:
(a) The plot is shown above.
(b) At ω = 1 rad/s, the phase is −31.45°. At ω = 10 rad/s, the phase is −31.45°. At ω = 100 rad/s, the phase is −45°.
(c) The MATLAB plot is shown above.

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Problem 12-46

The step response of a linear circuit is

$$g(t) = \left[12 - 10\,e^{-10t} - 2\,e^{-100t}\right]u(t)$$

(a) Is the circuit a low-pass, high-pass, bandpass, or bandstop filter?

(b) Construct the straight-line Bode gain plot and estimate the cutoff frequency and passband gain.

(c) Use MATLAB to plot the Bode magnitude and step responses.

(d) Design a circuit to achieve the transfer function.

(e) Use OrCAD to verify the step and frequency responses.

Solution:

(a) Find the transfer function.

```
clear all
syms s t
gt = 12*heaviside(t)-10*exp(-10*t)-2*exp(-100*t);
Gs = laplace(gt);
T = simplify(s*Gs)
T =
300*(s+40)/(s+10)/(s+100)
```

$$T(s) = \frac{300(s+40)}{(s+10)(s+100)}$$

Based on the transfer function, the filter is a low-pass filter.

(b) and (c) Plot the Bode magnitude response approximation and actual values.

```
% Create the actual gain response
close all
ww = logspace(-1,4,1000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth'
,3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line
approximation
% Find the important straight-line gains
MagMin = 20*log10(300*40/10/100);
MagMid = MagMin - 20*log10(40/10);
MagInf = MagMid - 20*log10(10000/100);
line([.1,10],[MagMin,MagMin],'LineWidth',3,'LineStyle',':')
line([10,40],[MagMin,MagMid],'LineWidth',3,'LineStyle',':')
```

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```
line([40,100],[MagMid,MagMid],'LineWidth',3,'LineStyle',':')
line([100,1e4],[MagMid,MagInf],'LineWidth',3,'LineStyle',':')
```

The cutoff frequency is approximately 10 rad/s and the passband gain is 12 = 21.58 dB.

(c) The actual Bode magnitude response is shown above. Plot the step response.

```
% Plot the step response
close all
tt = 0:600e-6:600e-3;
gtt = subs(gt,t,tt);
plot(tt,gtt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Step Response')
```

(d) Rewrite the transfer function as follows:

$$T(s)=\frac{300(s+40)}{(s+10)(s+100)}=\left(\frac{s+40}{s+10}\right)(3)\left(\frac{100}{s+100}\right)=\left(\frac{s+40}{s+10}\right)(3)\left(\frac{\frac{100}{s}}{1+\frac{100}{s}}\right)$$

The following circuit will realize the transfer function:

(e) The following OrCAD simulation results validate the circuit design.

Frequency response:

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Step response:

Answers:
(a) Low-pass filter.
(b) The plot is shown above. The cutoff frequency is approximately 10 rad/s and the passband gain is 12 = 21.58 dB.
(c) The MATLAB plots are presented above.
(d) The circuit is presented above.
(e) The OrCAD results are presented above and they verify the step and frequency responses.

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Problem 12-47

The step response of a linear circuit is

$$g(t) = \left[3 - 2\,e^{-20t} + 2\,e^{-500t}\right]u(t)$$

(a) Is the circuit a low-pass, high-pass, bandpass, or bandstop filter?

(b) Construct the straight-line Bode gain plot and estimate the cutoff frequency and passband gain.

(c) Use MATLAB to plot the Bode magnitude and step responses.

(d) Design a circuit to achieve the transfer function.

(e) Use OrCAD to verify the step and frequency responses.

Solution:

(a) Find the transfer function.

```
clear all
syms s t
gt = 3*heaviside(t)-2*exp(-20*t)+2*exp(-500*t);
Gs = laplace(gt);
T = simplify(s*Gs)
T =
3*(s^2+200*s+10000)/(s+20)/(s+500)
```

$$T(s) = \frac{3(s^2 + 200s + 10000)}{(s+20)(s+500)} = \frac{3(s+100)^2}{(s+20)(s+500)}$$

Based on the transfer function, the filter is a bandstop filter.

(b) and (c) Plot the Bode magnitude response approximation and actual values.

```
% Create the actual gain response
close all
ww = logspace(-1,4,2000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line approximation
% Find the important straight-line
gains
MagMin = 20*log10(3*10000/20/500);
MagMid = MagMin - 20*log10(100/20);
MagInf = MagMin;
line([.1,20],[MagMin,MagMin],'LineWidth',3,'LineStyle',':')
line([20,100],[MagMin,MagMid],'LineWidth',3,'LineStyle',':')
line([100,500],[MagMid,MagInf],'LineWidth',3,'LineStyle',':')
line([500,1e4],[MagInf,MagInf],'LineWidth',3,'LineStyle',':')
```

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The cutoff frequencies are approximately 20 rad/s and 500 rad/s. The passband gain is 3 = 9.54 dB.

(c) The actual Bode magnitude response is shown above. Plot the step response.

```
% Plot the step response
close all
tt = 0:300e-6:300e-3;
gtt = subs(gt,t,tt);
plot(tt,gtt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Step Response')
```

(d) Rewrite the transfer function as follows:

$$T(s) = \frac{3(s+100)^2}{(s+20)(s+500)} = (3)\left(\frac{s+100}{s+20}\right)\left(\frac{s+100}{s+500}\right)$$

The following circuit will realize the transfer function:

(e) The following OrCAD simulation results validate the circuit design.

Frequency response:

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Step response:

Answers:

(a) Bandstop filter.

(b) The plot is shown above. The cutoff frequencies are approximately 20 rad/s and 500 rad/s. The passband gain is 3 = 9.54 dB.

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(c) The MATLAB plots are presented above.
(d) The circuit is presented above.
(e) The OrCAD results are presented above and they verify the step and frequency responses.

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Problem 12-48

The transfer function of a linear circuit is

$$H(s) = \frac{7800s}{(s+50)(s+2000)}$$

(a) Is the circuit a low-pass, high-pass, bandpass, or bandstop filter?

(b) Construct the straight-line Bode gain plot and estimate the cutoff frequency and passband gain.

(c) Use MATLAB to plot the Bode magnitude and step responses.

(d) Design a circuit to achieve the transfer function.

(e) Use OrCAD to verify the step and frequency responses.

Solution:

(a) The circuit is a bandpass filter.

(b) and (c) Plot the Bode magnitude response approximation and actual values.

```
clear all
syms s t
T = 7800*s/(s+50)/(s+2000);
% Create the actual gain
response
close all
ww = logspace(0,5,2000);
Tjw = subs(T,s,j*ww);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWi
dth',3)
grid on
hold on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
% Plot the straight-line
approximation
% Find the important straight-line gains
MagMid = 20*log10(7800*1000/1000/2000);
MagMin = MagMid - 20*log10(50/1);
MagInf = MagMin;
line([1,50],[MagMin,MagMid],'LineWidth',3,'LineStyle',':')
line([50,2000],[MagMid,MagMid],'LineWidth',3,'LineStyle',':')
line([2000,100000],[MagMid,MagInf],'LineWidth',3,'LineStyle',':')
```

The cutoff frequencies are approximately 50 rad/s and 2000 rad/s. The passband gain is 3.9 = 11.82 dB.

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(c) The actual Bode magnitude response is shown above. Plot the step response.

```
% Plot the step response
Gs = T/s;
gt = ilaplace(Gs);
close all
tt = 0:120e-6:120e-3;
gtt = subs(gt,t,tt);
plot(tt,gtt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Step Response')
```

(d) Rewrite the transfer function as follows:

$$T(s)=\frac{7800s}{(s+50)(s+2000)}=\left(\frac{s}{s+50}\right)(3.9)\left(\frac{2000}{s+2000}\right)=\left(\frac{1}{1+\frac{50}{s}}\right)(3.9)\left(\frac{\frac{2000}{s}}{1+\frac{2000}{s}}\right)$$

The following circuit will realize the transfer function:

(e) The following OrCAD simulation results validate the circuit design.

Frequency response:

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Step response:

Answers:
(a) Bandpass filter.
(b) The plot is shown above. The cutoff frequencies are approximately 50 rad/s and 2000 rad/s. The passband gain is 3.9 = 11.82 dB.
(c) The MATLAB plots are presented above.
(d) The circuit is presented above.
(e) The OrCAD results are presented above and they verify the step and frequency responses.

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Problem 12-49

The straight-line gain response of a linear circuit is shown in Figure P12-39. What are the initial and final values of the circuit step response? What is the approximate duration of the transient response?

Solution:

First, find the transfer function corresponding to the straight-line gain response. The circuit has a pole located at 20 rad/s and a zero at 200 rad/s. The high-frequency gain is 0 dB, so we can write the following transfer function.

$$T(s) = \frac{s+200}{s+20}$$

The step response is calculated below.

```
clear all
syms s t
Ts = (s+200)/(s+20);
Gs = Ts/s;
gt = ilaplace(Gs)
gt =
-9*exp(-20*t)+10
```

We can find the initial and final values by taking the appropriate limits

```
IV = limit(gt,t,0)
FV = limit(gt,t,inf)
IV =
1
FV =
10
```

The time constant for the step response is T_C = 1/20 s. The duration of the transient response is approximately $5T_C$ = 5/20 s = 250 ms.

Answer:

$$g(t) = \left[10 - 9e^{-20t}\right]u(t)$$

$g(0) = 1$

$g(\infty) = 10$

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The duration of the transient response is approximately 250 ms.

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Problem 12-50

The straight-line gain response of a linear circuit is shown in Figure P12-41. What are the initial and final values of the circuit step response? What is the approximate duration of the transient response?

Solution:

First, find the transfer function corresponding to the straight-line gain response. The circuit has a zero located at 0 rad/s and poles located at 100 rad/s and 50 krad/s. The mid-frequency gain is 5, so we can write the following transfer function.

$$T(s) = \frac{250000s}{(s+100)(s+50000)}$$

The step response is calculated below.

```
clear all
syms s t
Ts = 250000*s/(s+100)/(s+50000);
Gs = Ts/s;
gt = ilaplace(Gs)
gt =
5000/499*exp(-25050*t)*sinh(24950*t)
```

$$g(t) = \frac{2500}{499}\left[e^{-100\,t} - e^{-50000\,t}\right]u(t)$$

We can find the initial and final values by taking the appropriate limits

```
IV = limit(gt,t,0)
FV = limit(gt,t,inf)
IV =
0
FV =
0
```

The time constant for the step response is the dominant (slowest) exponential, so T_C = 1/100 s. The duration of the transient response is approximately $5T_C$ = 5/100 s = 50 ms. Plot the step response to verify its duration

```
close all
tt = 0:60e-6:60e-3;
gt=2500/499*(exp(-100*t)-exp(-50000*t));
gtt = subs(gt,t,tt);
plot(tt,gtt,'b','LineWidth',3)
grid on
```

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```
xlabel('Time (s)')
ylabel('Step Response')
```

Answers:

$$g(t) = \frac{2500}{499}\left[e^{-100\,t} - e^{-50000\,t}\right]u(t)$$

$g(0) = 0$

$g(\infty) = 0$

The duration of the transient response is approximately 50 ms.

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Problem 12-51

There is a need for a passive notch filter at 10 krad/s. The narrower the notch the better, but there should be minimal ringing of the signals passing through. The transforms of three filters were submitted for consideration. Which would you recommend and why?

$$T_1(s) = \frac{s^2 + 100s + 10^8}{s^2 + 10^4 s + 10^8}$$

$$T_2(s) = \frac{s^2 + 500s + 10^8}{s^2 + 2500s + 10^8}$$

$$T_3(s) = \frac{s^2 + 10^8}{s^2 + 5000s + 10^8}$$

Solution:

Plot the magnitude response and the step response for each signal.

```
clear all
close all
syms s t
% Form the transfer functions
T1 =
(s^2+100*s+1e8)/(s^2+1e4*s+1e8);
T2 =
(s^2+500*s+1e8)/(s^2+2500*s+1e8);
T3 = (s^2+1e8)/(s^2+5000*s+1e8);
% Find the step responses
g1 = ilaplace(T1/s);
g2 = ilaplace(T2/s);
g3 = ilaplace(T3/s);
% Compute the magnitude response
and plot them
ww = logspace(3,5,2000);
MagT1jwdB =
20*log10(abs(subs(T1,s,j*ww)));
MagT2jwdB =
20*log10(abs(subs(T2,s,j*ww)));
MagT3jwdB =
20*log10(abs(subs(T3,s,j*ww)));
% Plot the actual gain response
semilogx(ww,MagT1jwdB,'g','LineWidth',2)
grid on
hold on
semilogx(ww,MagT2jwdB,'r','LineWidth',2)
semilogx(ww,MagT3jwdB,'b','LineWidth',2)
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
legend('T1','T2','T3')

% Plot the step responses
close all
tt = 0:2e-6:2e-3;
```

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```
g1tt = subs(g1,t,tt);
g2tt = subs(g2,t,tt);
g3tt = subs(g3,t,tt);
plot(tt,g1tt,'g','LineWidth',2)
grid on
hold on
plot(tt,g2tt,'r','LineWidth',2)
plot(tt,g3tt,'b','LineWidth',2)
xlabel('Time (s)')
ylabel('Step Response')
legend('T1','T2','T3')
```

Answer:
$T_2(s)$ has the narrowest notch and the lowest amplitude ringing

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Problem 12-52

Step Response of an *RLC* bandpass circuit **(A) (D)**

The step response of a series *RLC* bandpass circuit is

$$g(t) = \left[\frac{4}{5} e^{-200t} \sin(500t)\right] u(t)$$

(a) Find the passband center frequency and the two cutoff frequencies.

(b) Design a circuit that would possess the above step response.

Solution:

(a) Find the corresponding transfer function and plot its magnitude response.

```
clear all
format short eng
close all
syms s t
% Form the transfer functions
gt = 4/5*exp(-200*t)*sin(500*t);
Gs = laplace(gt);
Ts = simplify(s*Gs)
ww = logspace(1,4,1000);
MagTjwdB =
20*log10(abs(subs(Ts,s,j*ww)));
% Plot the actual gain response
semilogx(ww,MagTjwdB,'g','LineWi
dth',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
Ts =
400*s/(s^2+400*s+290000)
```

The transfer function is

$$T(s) = \frac{400s}{s^2 + 400s + 290000}$$

The center frequency is $\omega_0 = \sqrt{290000}$ rad/s = 538.52 rad/s and the bandwidth is B = 400 rad/s. We can use this information to solve for the cutoff frequencies as follows.

```
% Solve for the cutoff frequencies
syms wc1 wc2
w02 =290000;
B = 400;
Eqn1 = wc1*wc2-w02;
Eqn2 = wc2-wc1-400;
Soln = solve(Eqn1,Eqn2,wc1,wc2);
```

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```
wc1 = double(Soln.wc1)
wc2 = double(Soln.wc2)
wc1 =
   374.4563e+000
  -774.4563e+000
wc2 =
   774.4563e+000
  -374.4563e+000
```

Choose the positive solutions, so we have ω_{C1} = 374.46 rad/s and ω_{C2} = 774.46 rad/s. The magnitude response plot agrees with this numerical solution.

(b) Choose a value for the resistor and then solve for the corresponding inductor and capacitor values for the series *RLC* circuit design.

```
R = 1e3
L = R/B
C = 1/L/w02
R =
     1.0000e+003
L =
     2.5000e+000
C =
     1.3793e-006
```

Create the circuit and verify the magnitude response.

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The simulation results are shown below and agree with the MATLAB results if we scale the frequency axis by 2π.

Verify the step response of the circuit.

```
close all
tt = 0:30e-6:30e-3;
gtt = subs(gt,t,tt);
plot(tt,gtt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Step Response')
```

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Answers:

(a) $\omega_0 = \sqrt{290000}$ rad/s = 538.52 rad/s, ω_{C1} = 374.46 rad/s, and ω_{C2} = 774.46 rad/s.

(b) The circuit is presented above and the design was verified through simulation.

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Problem 12-53

A Tunable Tank Circuit **(A)**

The *RLC* circuit in Figure P12-53 (often called a tank circuit) has R = 10 kΩ, C = 125 pF, and an adjustable (tunable) L ranging from 64 to 640 μH.

(a) Show that the circuit is a bandpass filter.

(b) Find the frequency range (in Hz) over which the center frequency can be tuned.

(c) Find the bandwidth (in Hz) at the end points of this range.

Solution:

(a) Find the transfer function for the circuit and show that it has the characteristic structure of a bandpass filter.

```
clear all
syms s R L C
ZL = s*L;
ZC = 1/s/C;
Z1 = R;
Z2 = 1/(1/ZC+1/ZL);
Ts = simplify(Z2/(Z1+Z2))
Ts =
s*L/(R*s^2*C*L+R+s*L)
```

$$T(s) = \frac{Ls}{RLCs^2 + Ls + R}$$

The transfer function has the characteristic structure of a bandpass filter.

(b) If we write the transfer function as follows:

$$T(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

The center frequency is $\omega_0 = 1/\sqrt{LC}$. Given the capacitance and a range for the inductance, we can compute the range of center frequencies.

```
C = 125e-12;
L = [64e-6 640e-6];
w0 = 1./sqrt(L*C)
f0 = w0/2/pi
w0 =
    11.1803e+006     3.5355e+006
```

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```
f0 =
     1.7794e+006   562.6977e+003
```

The center frequency ranges between 3.5355 Mrad/s and 11.1803 Mrad/s. In hertz, the center frequency ranges between 562.6977 kHz and 1.7794 MHz.

(c) The bandwidth is 1/*RC*, so it does not depend on the inductance

```
R = 10e3;
B = 1/R/C
B2 = B/2/pi
B =
   800.0000e+003
B2 =
   127.3240e+003
```

The bandwidth is 800 krad/s or 127.324 kHz at both end points of the range of center frequencies.

Answers:
(a) The transfer function has the characteristic structure of a bandpass filter.
(b) The center frequency ranges between 562.6977 kHz and 1.7794 MHz.
(c) The bandwidth is 127.324 kHz at both end points of the range of center frequencies.

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Problem 12-54

Filter Design Specification**(D)**

Construct a transfer function whose gain response lies entirely within the non-shaded region in Figure P12-54.

Solution:

The transfer function must have the response of a high-pass filter. If we set the passband gain to 20 dB, which is an amplitude of 10, then the response will be within the non-shaded region at the upper frequency limit. The response can enter with a slope of +20 dB per decade, so we can have a zero at $\omega = 0$ rad/s. Between $\omega = 10^3$ rad/s and $\omega = 10^4$ rad/s, the response must rise by at least 34 dB. Since the rise is at a rate of more than 20 dB per decade, we will need two zeros active in this frequency range to get the desired response. Place the other zero at $\omega = 10^3$ rad/s. We will then need two poles at $\omega = 10^4$ rad/s to realize the passband of the high-pass filter.

The initial filter design has the following transfer function:

$$T(s) = \frac{10s(s+1000)}{(s+10000)^2}$$

Plot the magnitude response of this transfer function to verify it meets the specifications.

```
clear all
syms s
Ts = 10*s*(s+1000)/(s+10000)^2;
ww = logspace(2,5,1000);
MagTjwdB =
20*log10(abs(subs(Ts,s,j*ww)));
semilogx(ww,MagTjwdB,'g','LineWidt
h',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
line([100 1000],[-40 -
20],'LineWidth',3,'LineStyle',':')
line([1000 1000],[-20
20],'LineWidth',3,'LineStyle',':')
line([1000 100000],[20
20],'LineWidth',3,'LineStyle',':')
line([10000 10000],[-40
14],'LineWidth',3,'LineStyle',':')
line([10000 100000],[14
14],'LineWidth',3,'LineStyle',':')
```

The initial design does not meet the specification, so we need to adjust the transfer function to meet the specifications. Replace the zero at $\omega = 1000$ rad/s with a zero at $\omega = 0$ rad/s and decrease the frequency of one of the repeated poles until the response meets specifications.

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```
Ts = 10*s^2/(s+9900)/(s+10000);
ww = logspace(2,5,1000);
MagTjwdB =
20*log10(abs(subs(Ts,s,j*ww)));
semilogx(ww,MagTjwdB,'g','LineWidth
',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
axis([100 1e5 -40 25])
line([100 1000],[-40 -
20],'LineWidth',3,'LineStyle',':')
line([1000 1000],[-20
20],'LineWidth',3,'LineStyle',':')
line([1000 100000],[20
20],'LineWidth',3,'LineStyle',':')
line([10000 10000],[-40
14],'LineWidth',3,'LineStyle',':')
line([10000 100000],[14
14],'LineWidth',3,'LineStyle',':')
```

The resulting transfer function is

$$T(s) = \frac{10s^2}{(s+9900)(s+10000)}$$

Check the response at the critical frequencies.

```
w2 = [1e2 1e3 1e4 1e5];
MagTjw2dB = 20*log10(abs(subs(Ts,s,j*w2)));
Results = [w2' MagTjw2dB']
Results =
   100.0000e+000   -59.9136e+000
     1.0000e+003   -20.0000e+000
    10.0000e+003    14.0228e+000
   100.0000e+003    19.9144e+000
```

The design meets all of the specifications.

Answer:

$$T(s) = \frac{10s^2}{(s+9900)(s+10000)}$$

Many other solutions are possible.

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Problem 12-55

Chip *RC* Networks **(E)**

Integrated circuit (chip) *RC* networks are used at parallel data ports to suppress radio frequency noise. In a certain application, RF noise at 3.2 MHz is interfering with a 4-bit parallel data signal operating at 1.1 MHz. A chip *RC* network is to be used to reduce the RF noise on the parallel bus by at least 7 dB without reducing the data signals by more than 2 dB. A vendor offers a family of chip *RC* networks connected as shown in Figure P12-55. The available circuit parameters are shown in Table P12-55. Select the part number that best meets the noise suppression requirements.

Solution:

Each signal passes through a passive *RC* low-pass filter with the following transfer function

```
clear all
syms s R C
ZC = 1/s/C;
Ts = simplify(ZC/(R+ZC))
Ts =
1/(R*s*C+1)
```

For each *RC* combination, find the magnitude of the response at 3.2 MHz and 1.1 MHz to determine which parts satisfy the specifications.

```
% Define the part values
format short
syms w
CC = 1e-12*[220 470 1000 220 470 1000 220 470 1000 220 470 1000];
RR = [47 47 47 100 100 100 220 220 220 470 470 470];
Parts = [14 15 16 24 25 26 34 35 36 44 45 46];
% Create the transfer functions
TT = 1./(RR.*CC*s+1);
% Compute the gains at the critical frequencies
MagTT11dB = 20*log10(abs(subs(TT,s,j*2*pi*1.1e6)));
MagTT32dB = 20*log10(abs(subs(TT,s,j*2*pi*3.2e6)));
% Identify the designs that meet the specifications
Specs = (MagTT11dB>=-2)&(MagTT32dB<-7);
% Display the results
Results = [RR' 1e9*CC' MagTT11dB' MagTT32dB' Specs' Parts']
Results =
   47.0000    0.2200   -0.0221   -0.1838         0   14.0000
   47.0000    0.4700   -0.1001   -0.7819         0   15.0000
   47.0000    1.0000   -0.4357   -2.7715         0   16.0000
  100.0000    0.2200   -0.0993   -0.7761         0   24.0000
  100.0000    0.4700   -0.4357   -2.7715         0   25.0000
  100.0000    1.0000   -1.6958   -7.0265    1.0000   26.0000
  220.0000    0.2200   -0.4607   -2.8937         0   34.0000
  220.0000    0.4700   -1.7919   -7.2609    1.0000   35.0000
  220.0000    1.0000   -5.2009  -13.1315         0   36.0000
  470.0000    0.2200   -1.7919   -7.2609    1.0000   44.0000
  470.0000    0.4700   -5.2257  -13.1653         0   45.0000
  470.0000    1.0000  -10.6266  -19.5569         0   46.0000
```

Answer:

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Parts ZAEN26, ZAEN35, and ZAEN44 meet the specifications, so any of them can be used for this application.

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Problem 12-56

Design Evaluation **(E)**

Your company issued a request for proposals listing the following design requirements and evaluation criteria.

Design Requirements: Design a low-pass filter with a passband gain of 9 ± 10% and a cutoff frequency of 90 ± 10% krad/s. A sensor drives the filter input with a 1-kΩ source resistance and an open-circuit voltage range of ± 1.6 V.

Evaluation Criteria: Filter performance, parts count, use of standard parts, and cost.

The two vendors have responded with the designs shown in Figure P12-56.

As a junior engineer, the project manager asks you evaluate the designs and recommend a vendor. Which vendor would you recommend and why?

First-Order Filter Company

Unit Cost: $29.99

Simply Filters, Ltd

Unit Cost: $20.99

Solution:

Find the transfer function for each circuit and analyze the frequency response. Note that in the second circuit, we must account for the source resistance since it is in series with the 3.3-kΩ resistor. The source resistance is not an issue with the first circuit, because it is connected to the positive input terminal of an OP AMP.

```
clear all
syms s
% Circuit 1
K = (8.2e3+1e3)/1e3;
R = 3.3e3;
C = 3.3e-9;
ZC = 1/s/C;
T1 = vpa(simplify(vpa(K*ZC/(R+ZC),8)),8)
% Circuit 2
K = (68e3+8.2e3)/8.2e3;
R = 1e3+3.3e3;
C = 3.3e-9;
ZC = 1/s/C;
T2 = vpa(simplify(vpa(K*ZC/(R+ZC),8)),8)
T1 =
9292929.3/(11.*s+1010101.)
T2 =
```

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```
28159645./(43.*s+3030303.)
```

Rewriting the results, we have:

$$T_1(s) = \frac{9.2(91827)}{s+91827} \text{ and } T_2(s) = \frac{9.29(70472)}{s+70472}$$

Both circuits meet the gain specification, but the second circuit does not meet the cutoff frequency specification because it failed to account for the input resistance.

Check the first circuit for saturation of the OP AMP. The gain is 9.2 and the maximum signal amplitude is 1.6 V, so the maximum OP AMP output is (9.2)(1.6 V) = 14.72 V. The OP AMP will not saturate, so the first circuit is still a good choice.

Answer:
Select the circuit from the First-Order Filter Company because it is the only one that meets specifications.

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Problem 12-57

Design Evaluation **(E)**

In a research laboratory, you need a bandpass filter to meet the following requirements:

Design Requirements: Passband gain: 10 ± 5%, B = 10 krad/s ± 5%, ω_0 = 5 krad/s ± 2%, ω_{CL} = 2 krad/s ± 10%.

Evaluation Criteria: Filter performance, parts count, use of standard parts, and cost.

The two vendors have responded with the designs shown in Figure P12-57.

As a research assistant, your supervisor asks you to evaluate the two designs and recommend a vendor. Which vendor would you recommend and why?

Vendor #1

Vendor #2

Solution:

Find the transfer function for each design and plot the magnitude response

```
clear all
format short eng
syms s
% Circuit 1
R = 1e3;
L = 100e-3;
C = 400e-9;
ZL = s*L;
ZC = 1/s/C;
K = (90e3+10e3)/10e3;
T1 = simplify(K*R/(R+ZC+ZL));
T1_num = simplify(vpa(T1,6))
% Circuit 2
R1 = 1e3;
R2 = 1e3;
C1 = 485e-9;
C2 = 82.4e-9;
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
K = (10.7e3+1e3)/1e3;
T2 =
simplify(K*(R1/(R1+ZC1))*(ZC2/(R2+ZC2)
));
T2_num = simplify(vpa(T2,6))
T1_num =
94447300000.*s/(9444730000.*s+23611800
000000.+944473.*s^2)
T2_num =
136363793103448.27586206896551724*s/(1
43147.*s+295148000.)/(6709.*s+81420000
.)
```

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```
% Plot the magnitude response for each circuit
close all
ww = logspace(2,5,1000);
MagT1dB = 20*log10(abs(subs(T1,s,j*ww)));
MagT2dB = 20*log10(abs(subs(T2,s,j*ww)));
semilogx(ww,MagT1dB,'g','LineWidth',2.5)
hold on
grid on
semilogx(ww,MagT2dB,'r','LineWidth',2.5)
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
```

Plot the critical region of the magnitude response

```
close all
semilogx(ww,MagT1dB,'g','LineWidth',2.
5)
hold on
grid on
semilogx(ww,MagT2dB,'r','LineWidth',2.
5)
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
axis([1000 20000 15 20])
```

The bandwidth of the second circuit does not meet the specifications. The following MATLAB calculations using results from the transfer functions confirm this analysis.

```
% Circuit 1
B1 = 9444730000/944473
% Circuit 2
B2 = (143147*81420000+295148000*6709)/(143147*6709)
B1 =
    10.0000e+003
B2 =
    14.1978e+003
```

The first circuit also meets the specification for the lower cutoff frequency, so it is an appropriate choice.

Answer:
Select the circuit from Vendor #1, because it is the only one that meets all of the specifications.

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Problem 13-1

The equation for the first cycle ($0 \le t \le T_0$) of a periodic waveform is

$$v(t) = V_A e^{-5t/T_0} \text{ V}$$

(a) Sketch the first two cycles of the waveform.

(b) Derive expressions for the Fourier coefficients a_n and b_n.

Solution:

(a) Use MATLAB to plot the waveform with both V_A and T_0 normalized to values of one.

```
clear all
format short eng
VA = 1;
T0 = 1;
syms t
vt = VA*exp(-5*t/T0)*(heaviside(t)-
heaviside(t-T0))...
     + VA*exp(-5*(t-T0)/T0)*(heaviside(t-
T0)-heaviside(t-2*T0));
tt = 0:T0/1000:2*T0;
vtt = subs(vt,t,tt+eps);
close all
plot(tt,vtt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

(b) Compute the Fourier coefficients

```
clear all
syms t VA real
syms T0 n positive
vt = VA*exp(-5*t/T0);
a0 = int(vt,t,0,T0)/T0;
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0)/T0;
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0)/T0;
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
a02 =
-1/5*VA*(-1+exp(-5))
an2 =
(10*VA-10*VA*exp(-5)*cos(2*pi*n)+4*VA*exp(-
5)*pi*n*sin(2*pi*n))/(25+4*pi^2*n^2)
bn2 =
(4*VA*pi*n-4*VA*exp(-5)*pi*n*cos(2*pi*n)-10*VA*exp(-
5)*sin(2*pi*n))/(25+4*pi^2*n^2)
```

Since n is an integer, it is always true that $\cos(2\pi n) = 1$ and $\sin(2\pi n) = 0$. We can use this information to simplify the expressions generated above.

Answer:

(a) The sketch is shown above.

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(b) $a_0 = \frac{V_A}{5}\left[1-e^{-5}\right]$, $a_n = \frac{10\,V_A\left[1-e^{-5}\right]}{25+4\pi^2 n^2}$, and $b_n = \frac{4\,V_A\,\pi n\left[1-e^{-5}\right]}{25+4\pi^2 n^2}$ for all n.

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Problem 13-2

The equation for the first cycle ($0 \le t \le T_0$) of a periodic pulse train is

$$v(t) = V_A[3u(t) - 2u(t - T_0/2)] \text{ V}$$

(a) Sketch the first two cycles of the waveform.

(b) Derive expressions for the Fourier coefficients a_n and b_n.

Solution:

(a) Use MATLAB to plot the waveform with both V_A and T_0 normalized to values of one.

```
clear all
format short eng
VA = 1;
T0 = 1;
syms t
vt = VA*(3*heaviside(t)-
2*heaviside(t-T0/2))...
     + VA*(2*heaviside(t-T0)-
2*heaviside(t-3*T0/2));
tt = 0:T0/1000:2*T0;
vtt = subs(vt,t,tt+eps);
close all
plot(tt,vtt,'g','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
axis([0 2*T0 0 4*VA])
```

(b) Compute the Fourier coefficients

```
clear all
syms t VA real
syms T0 n positive
vt = VA*(3*heaviside(t)-2*heaviside(t-T0/2));
a0 = int(vt,t,0,T0)/T0;
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0)/T0;
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0)/T0;
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
a02 =
2*VA
an2 =
2*VA*sin(pi*n)*(cos(pi*n)+1)/pi/n
bn2 =
-2*VA*(cos(pi*n)^2-2+cos(pi*n))/pi/n
```

Since n is an integer, it is always true that $\sin(\pi n) = 0$ and $[\cos(\pi n)]^2 = 1$. We can use this information to simplify the expressions generated above.

Answer:

(a) The sketch is shown above.

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(b) $a_0 = 2V_{\mathrm{A}}$, $a_n = 0$, and $b_n = \dfrac{2\,V_{\mathrm{A}}\,[1-\cos(n\pi)]}{n\pi}$ for all n.

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Problem 13-3

Derive expressions for the Fourier coefficients of the periodic waveform in Figure P13-3.

Solution:
One period of the function is a linear expression:

$$v(t) = V_A - \frac{2V_A t}{T_0}$$

Now evaluate the integrals to compute the Fourier coefficients.

```
clear all
syms t VA real
syms T0 n positive
vt = VA*(1-2*t/T0);
a0 = int(vt,t,0,T0)/T0;
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0)/T0;
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0)/T0;
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
a02 =
0
an2 =
(VA-VA*pi*n*sin(2*pi*n)-VA*cos(2*pi*n))/pi^2/n^2
bn2 =
2*VA*cos(pi*n)*(cos(pi*n)*pi*n-sin(pi*n))/pi^2/n^2
```

Since n is an integer, it is always true that $\sin(2\pi n) = \sin(\pi n) = 0$, $\cos(2\pi n) = 1$, and $[\cos(\pi n)]^2 = 1$. We can use this information to simplify the expressions generated above.

Answer:

$a_0 = 0$, $a_n = 0$, and $b_n = \dfrac{2V_A}{n\pi}$ for all n.

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Problem 13-4

Derive expressions for the Fourier coefficients of the periodic waveform in Figure P13-4.

Solution:

One period of the non-zero portion of the function is a linear expression:

$$v(t) = \frac{2V_A t}{T_0}$$

Now evaluate the integrals to compute the Fourier coefficients.

```
clear all
syms t VA real
syms T0 n positive
vt = 2*VA*t/T0*(heaviside(t)-heaviside(t-T0/2));
a0 = int(vt,t,0,T0)/T0;
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0)/T0;
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0)/T0;
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
a02 =
1/4*VA
an2 =
VA*(-1+cos(pi*n)+pi*n*sin(pi*n))/pi^2/n^2
bn2 =
VA*(sin(pi*n)-pi*n*cos(pi*n))/pi^2/n^2
```

Since n is an integer, it is always true that $\sin(\pi n) = 0$. We can use this information to simplify the expressions generated above.

Answer:

$a_0 = \frac{V_A}{4}$, $a_n = \frac{V_A[\cos(n\pi)-1]}{n^2\pi^2}$, and $b_n = \frac{-V_A\cos(n\pi)}{n\pi}$ for all n.

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Problem 13-5

Use the results in Figure 13-4 to calculate the Fourier coefficients of the square wave in Figure P13-5. Write an expression for the first four nonzero terms in the Fourier series.

Solution:

The signal in Figure P13-5 is a square wave with A = 5 V and T_0 = 4 ms. Using the results in Figure 13-4, we have the following coefficients.

$a_0 = 0$

$a_n = 0$ for all n

$b_n = \dfrac{4A}{n\pi} = \dfrac{20}{n\pi}$ for n odd

$b_n = 0$ for n even

Compute the first four nonzero coefficients.

```
clear all
format short eng
syms t
A = 5;
T0 = 4e-3;
n = [1 3 5 7];
bn = 4*A/pi./n
sn = sin(2*pi*n*t/T0)
vn = vpa(sum(bn.*sn),6)
bn =
     6.3662e+000     2.1221e+000     1.2732e+000   909.4568e-003
sn =
[  sin(500*t*pi), sin(1500*t*pi), sin(2500*t*pi), sin(3500*t*pi)]
vn =
6.36620*sin(1570.80*t)+2.12207*sin(4712.38*t)+1.27324*sin(7853.98*t)+.909457*s
in(10995.6*t)
```

Answers:

$a_0 = 0$, $a_n = 0$, and $b_n = \dfrac{10\left[1-\cos(\pi n)\right]}{\pi n}$ for all n.

$v(t) \approx 6.3662\sin(2\pi\,250\,t) + 2.1221\sin(2\pi\,750\,t) + 1.2732\sin(2\pi\,1250\,t) + 0.9095\sin(2\pi\,1750\,t)$ V

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Problem 13-6

Use the results in Figure 13-4 to calculate the Fourier coefficients of the shifted triangular wave in Figure P13-6. Write an expression for the first four nonzero terms in the Fourier series.

Solution:

For a non-shifted triangular wave, only the a_n coefficients are nonzero. The triangular wave is shifted by $-T_0/4$ in Figure P13-6. We can use the following equivalent expressions to find the coefficients for a shifted version of the triangular wave.

$$v(t) = f(t - T_0/4) = \sum_{\text{n odd}} a_n \cos[2\pi n(t - T_0/4)/T_0]$$

$$= \sum_{\text{n odd}} a_n \cos[2\pi nt/T_0 - \pi n/2]$$

$$= \sum_{\text{n odd}} a_n [\cos(2\pi nt/T_0)\cos(\pi n/2) + \sin(2\pi nt/T_0)\sin(\pi n/2)]$$

$$= \sum_{\text{n odd}} a_n [\sin(2\pi nt/T_0)\sin(\pi n/2)]$$

```
clear all
syms t
A = 40;
T0 = 2e-3;
n = [1 3 5 7];
bn = 8*A/pi^2./(n.^2).*sin(pi*n/2)
sn = sin(2*pi*n*t/T0)
vn = vpa(sum(bn.*sn),6)
bn =
    32.4228e+000    -3.6025e+000     1.2969e+000  -661.6894e-003
sn =
[ sin(1000*t*pi), sin(3000*t*pi), sin(5000*t*pi), sin(7000*t*pi)]
vn =
32.4228*sin(3141.59*t)-3.60253*sin(9424.77*t)+1.29691*sin(15708.0*t)-
.661689*sin(21991.1*t)
```

Plot the results to verify the waveform has the correct form.

```
close all
tt = 0:4e-6:4e-3;
vtt = subs(vn,t,tt);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answer:

$a_0 = 0$, $a_n = 0$, and $b_n = \dfrac{320\sin(\pi n/2)}{(\pi n)^2}$ for all n.

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Note that $\sin(n\pi/2) = 0$ for n even.

$$v(t) \approx 32.4228\sin(2\pi\, 500\, t) - 3.6025\sin(2\pi\, 1500\, t) + 1.2969\sin(2\pi\, 2500\, t) - 0.6617\sin(2\pi\, 3500\, t) \text{ V}$$

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Problem 13-7

Use the results in Figure 13-4 to calculate the Fourier coefficients of the full-wave rectified sine wave in Figure P13-7. Use MATLAB to verify your results. Write an expression for the first four nonzero terms in the Fourier series.

Solution:

The signal has an amplitude of 170 V and the period of the original sinusoid is 1/60 s. Using Figure 13-4, the corresponding Fourier coefficients are:

$a_0 = 2A/\pi = 340/\pi$

$a_n = \dfrac{4A/\pi}{1-n^2} = \dfrac{680/\pi}{1-n^2}$ for n even

$a_n = 0$ for n odd

$b_n = 0$ for all n

Use MATLAB to verify the results.

```
clear all
format short eng
syms t VA real
syms T0 n positive
% The period of the original sinusoid is T0, but
% the period of the rectified signal is T0/2.
% Adjust the integrals accordingly
vt = VA*sin(2*pi*t/T0);
a0 = int(vt,t,0,T0/2)/(T0/2);
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0/2)/(T0/2);
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0/2)/(T0/2);
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
a02 =
2*VA/pi
an2 =
-2*VA*(1+cos(pi*n))/pi/(-1+n^2)
bn2 =
-2*VA/pi/(-1+n^2)*sin(pi*n)
```

These results are consistent with the ones obtained above if we note that $\sin(n\pi) = 0$ for all n, so that all b_n terms are zero, and that $1+\cos(n\pi) = 0$ for n odd and $1+\cos(n\pi) = 2$ for n even.

Find the first four nonzero terms.

```
A = 170;
T0 = 1/60;
a0 = subs(a0,VA,A)
nn = [2 4 6 8];
```

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```
an = subs(an2,VA,A);
ann = subs(an,n,nn)
cn = cos(2*pi*nn*t/T0)
vn = vpa(a0+sum(ann.*cn),6)
a0 =
   108.2254e+000
ann =
   -72.1502e+000   -14.4300e+000    -6.1843e+000    -3.4357e+000
cn =
[ cos(240*t*pi), cos(480*t*pi), cos(720*t*pi), cos(960*t*pi)]
vn =
108.225-72.1502*cos(753.982*t)-14.4300*cos(1507.96*t)-6.18431*cos(2261.94*t)-
3.43573*cos(3015.93*t)
```

Plot the results to verify the waveform has the correct form.

```
close all
tt = 0:20e-6:20e-3;
vtt = subs(vn,t,tt);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answer:

$a_0 = \dfrac{340}{\pi}$, $a_n = \dfrac{340\left[\cos(\pi n)+1\right]}{\pi(1-n^2)}$, and $b_n = 0$ for all n.

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$$v(t) \approx 108.225 - 72.1502\cos(2\pi\, 120\, t) - 14.43\cos(2\pi\, 240\, t)$$
$$- 6.1843\cos(2\pi\, 360\, t) - 3.4357\cos(2\pi\, 480\, t) \text{ V}$$

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Problem 13-8

A half-wave rectified sine wave has an amplitude of 120 V and a fundamental frequency of 50 Hz. Use the results in Figure 13-4 to write an expression for the first four nonzero terms in the Fourier series. Use MATLAB to plot the amplitude spectrum of the signal.

Solution:

Using the results in Figure 13-4, we have the following coefficients:

$a_0 = A/\pi = 120/\pi$

$a_n = \dfrac{2A/\pi}{1-n^2} = \dfrac{240/\pi}{1-n^2}$ for n even

$a_n = 0$ for n odd

$b_1 = A/2$ for $n = 1$

$b_n = 0$ for $n \neq 1$

```
clear all
syms t
A = 120;
T0 = 1/50;
a0 = A/pi;
b1 = A/2;
n = [2 4 6 8];
an = 2*A/pi./(1-n.^2)
cn = cos(2*pi*n*t/T0)
vn = vpa(a0+b1*sin(2*pi*t/T0)+sum(an.*cn),6)
an =
   -25.4648e+000    -5.0930e+000    -2.1827e+000    -1.2126e+000
cn =
[ cos(200*t*pi), cos(400*t*pi), cos(600*t*pi), cos(800*t*pi)]
vn =
38.1972+60.*sin(314.159*t)-25.4648*cos(628.318*t)-5.09296*cos(1256.64*t)-
2.18270*cos(1884.95*t)-1.21261*cos(2513.27*t)
```

Plot the results to verify the waveform has the correct form.

```
close all
tt = 0:40e-6:40e-3;
vtt = subs(vn,t,tt);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Plot the amplitude spectrum.

```
close all
% Create vectors for the
frequencies and amplitudes
```

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```
ff = [0 50 100 200 300 400];
aa = abs([a0 b1 an]);
% Plot the spectrum
stem(ff,aa,'b','LineWidth',3,'MarkerSize',10,'MarkerFaceColor','b')
xlabel('Frequency, (Hz)')
ylabel('Amplitude')
```

Answers:

$$v(t) \approx 38.1972 + 60\sin(2\pi\, 50\, t) - 25.4648\cos(2\pi\, 100\, t) - 5.093\cos(2\pi\, 200\, t)$$
$$- 2.1827\cos(2\pi\, 300\, t) - 1.2126\cos(2\pi\, 400\, t) \ \text{ V}$$

The amplitude spectrum is plotted above.

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Problem 13-9

The waveform $f(t)$ is a 2-kHz triangular wave with a *peak-to-peak* amplitude of 8 V. Use the results in Figure 13-4 to write an expression for the first four nonzero terms in the Fourier series of $g(t) = 4 + f(t)$ and plot its amplitude spectrum. Use MATLAB to plot two periods of $g(t)$ and an estimate for $g(t)$ using the first four nonzero terms in the Fourier series. Comment on the differences between the two waveforms.

Solution:

The average value for a triangular wave is zero, so the average value of $g(t)$ will be 4 V, because it is a triangular wave shifted by 4 V. We can calculate the other coefficients as follows:

```
clear all
syms t
A = 8/2;
T0 = 1/2e3;
a0 = 4;
n = [1 3 5 7];
an = 8*A./(n*pi).^2
cn = cos(2*pi*n*t/T0)
vn = vpa(a0+sum(an.*cn),6)
an =
     3.2423e+000   360.2531e-003   129.6911e-003    66.1689e-003
cn =
[  cos(4000*t*pi), cos(12000*t*pi), cos(20000*t*pi), cos(28000*t*pi)]
vn =
4.+3.24228*cos(12566.4*t)+.360253*cos(37699.1*t)+.129691*cos(62831.8*t)+.66168
9e-1*cos(87964.5*t)
```

Plot the amplitude spectrum.

```
close all
% Create vectors for the frequencies and amplitudes
ff = [0 n/T0];
aa = abs([a0 an]);
% Plot the spectrum
stem(ff,aa,'b','LineWidth',3,'MarkerSize'
,10,'MarkerFaceColor','b')
xlabel('Frequency, (Hz)')
ylabel('Amplitude')
```

Plot the results to verify the waveform has the correct form.

```
close all
tt = 0:1e-6:1e-3;
vtt = subs(vn,t,tt);
% Construct the actual signal
m = 4*A/T0;
```

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```
gt = 8-m*t*heaviside(t)+2*m*(t-T0/2)*heaviside(t-T0/2)...
     -2*m*(t-T0)*heaviside(t-T0)+2*m*(t-3*T0/2)*heaviside(t-3*T0/2);
gtt = subs(gt,t,tt);
plot(tt,vtt,'b','LineWidth',4)
grid on
hold on
plot(tt,gtt,'g','LineWidth',2)
xlabel('Time (s)')
ylabel('Voltage (V)')
legend('v(t)','g(t)')
```

Answers:

$$g(t) \approx 4 + 3.2423\cos(2\pi\, 2000\, t) + 0.3603\cos(2\pi\, 6000\, t)$$
$$+ 0.1297\cos(2\pi\, 10000\, t) + 0.0662\cos(2\pi\, 14000\, t) \ \text{ V}$$

The amplitude spectrum is presented above.

The plot of the original function and its Fourier series estimate are extremely similar. The estimate does not have transitions that are as sharp as the original signal, but it is a very accurate estimate in the linear portions of the waveform.

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Problem 13-10

A sawtooth wave has *peak-to-peak* amplitude of 16 V and a fundamental frequency of 4 kHz. Use the results in Figure 13-4 to write an expression for the first four nonzero terms in the Fourier series and plot the amplitude spectrum of the signal. Use MATLAB to plot two periods of the original signal and an estimate for the signal using the first four nonzero terms in the Fourier series. Comment on the differences between the two waveforms.

Solution:

We can calculate the coefficients as follows:

```
clear all
syms t
A = 16;
T0 = 1/4e3;
a0 = A/2;
n = [1 2 3 4];
bn = -A./n/pi
sn = sin(2*pi*n*t/T0)
vn = vpa(a0+sum(bn.*sn),6)
bn =
    -5.0930e+000    -2.5465e+000    -1.6977e+000    -1.2732e+000
sn =
[  sin(8000*t*pi), sin(16000*t*pi), sin(24000*t*pi), sin(32000*t*pi)]
vn =
8.-5.09296*sin(25132.7*t)-2.54648*sin(50265.4*t)-1.69765*sin(75398.2*t)-
1.27324*sin(100531.*t)
```

Plot the amplitude spectrum.

```
close all
% Create vectors for the frequencies and amplitudes
ff = [0 n/T0];
aa = abs([a0 bn]);
% Plot the spectrum
stem(ff,aa,'b','LineWidth',3,'MarkerSize',10,'MarkerFaceColor','b')
xlabel('Frequency, (Hz)')
ylabel('Amplitude')
```

Plot the results to verify the waveform has the correct form.

```
close all
tt = 0:500e-9:500e-6;
vtt = subs(vn,t,tt);
% Construct the actual signal
m = A/T0;
gt = m*t*heaviside(t)-A*heaviside(t-T0);
gtt = subs(gt,t,tt+eps);
plot(tt,vtt,'b','LineWidth',4)
```

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```
grid on
hold on
plot(tt,gtt,'g','LineWidth',2)
xlabel('Time (s)')
ylabel('Voltage (V)')
legend('v(t)','g(t)')
```

Answers:

$$f(t) \approx 8 - 5.0930\sin(2\pi\, 4000\, t) - 2.5465\sin(2\pi\, 8000\, t)$$
$$-1.6977\sin(2\pi\, 12000\, t) - 1.2732\sin(2\pi\, 16000\, t) \ \text{ V}$$

The amplitude spectrum is presented above.

The plot of the original function and its Fourier series estimate are similar. The estimate does not have transitions that are as sharp as the original signal and the estimated linear portions have some fluctuations, but the general shape is correct.

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Problem 13-11

The equation for the first cycle ($0 \le t \le T_0$) of a periodic pulse train is

$$v(t) = V_A[u(3t - T_0) - u(3t - 2T_0)] \text{ V}$$

(a) Sketch the first two cycles of the waveform and identify a related signal in Figure 13-4.

(b) Use the Fourier series of the related signal to find the Fourier coefficients of $v(t)$.

Solution:

(a) Use MATLAB to sketch the signal with the amplitude and period normalized to values of one.

```
clear all
close all
syms t
VA = 1;
T0 = 1;
vt = VA*(heaviside(3*t-T0)-
heaviside(3*t-2*T0))...
     +VA*(heaviside(3*(t-T0)-T0)-
heaviside(3*(t-T0)-2*T0));
tt = 0:T0/1e3:2*T0;
vtt = subs(vt,t,tt+eps);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

The signal is similar to a rectangular pulse from Figure 13-4. Since the portion of the signal at 0 V is centered at $t = 0$, we can create $v(t)$ by subtracting a rectangular pulse from a constant value of with amplitude V_A.

(b) Using the results in Figure 13-4, $v(t) = 1 - f(t)$, where $f(t)$ is a rectangular pulse with $A = V_A$, $T_0 = T_0$, and $T = 2T_0/3$. The Fourier coefficients are calculated as follows:

$$a_0 = V_A - \frac{V_A T}{T_0} = V_A - \frac{2V_A}{3} = \frac{V_A}{3}$$

$$a_n = -\frac{2V_A}{n\pi}\sin\left(\frac{n\pi T}{T_0}\right) = -\frac{2V_A}{n\pi}\sin\left(\frac{2n\pi}{3}\right) \text{ for all } n$$

$b_n = 0$ for all n

Answers:

(a) The sketch is presented above. $v(t)$ is equivalent to V_A minus a rectangular pulse with $A = V_A$, $T_0 = T_0$, and $T = 2T_0/3$.

(b) $a_0 = \dfrac{V_A}{3}$, $a_n = \dfrac{-2V_A \sin(2\pi n/3)}{\pi n}$, and $b_n = 0$ for all n.

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Problem 13-12

The equation for a periodic waveform is

$$v(t) = V_A[\sin(4\pi t/T_0) + |\sin(4\pi t/T_0)|]$$

(a) Sketch the first two cycles of the waveform and identify a related signal in Figure 13-4.
(b) Use the Fourier series of the related signal to find the Fourier coefficients of $v(t)$.
(c) Use MATLAB to sketch an estimate for the signal using the Fourier coefficients for $n \leq 6$.

Solution:

(a) Use MATLAB to sketch the signal with the amplitude and period normalized to values of one.

```
clear all
close all
syms t
VA = 1;
T0 = 1;
vt =
VA*(sin(4*pi*t/T0)+abs(sin(4*pi*t/T0)
));
tt = 0:T0/1e3:T0;
vtt = subs(vt,t,tt+eps);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

The related signal in Figure 13-4 is a half-wave rectified sine wave.

(b) Using the results in Figure 13-4, $v(t) = f(t)$, where $f(t)$ is a half-wave rectified sine wave with $A = 2V_A$ and $T_0 = T_0/2$. The Fourier coefficients are calculated as follows:

$$a_0 = \frac{2V_A}{\pi}$$

$$a_n = \frac{4V_A/\pi}{1-n^2} \text{ for } n \text{ even}$$

$$a_n = 0 \text{ for } n \text{ odd}$$

$b_1 = V_A$ for $n = 1$

$b_n = 0$ for $n \neq 1$

(c) Plot an estimate for the signal.

```
close all
syms t
a0 = 2*VA/pi;
b1 = VA;
n = [2 4 6];
```

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```
an = 4*VA/pi./(1-n.^2);
cn = cos(2*pi*n*t/T0);
vn = vpa(a0+b1*sin(2*pi*t/T0)+sum(an.*cn),6);
tt = 0:T0/1e3:2*T0;
vtt = subs(vn,t,tt);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answers:

(a) The sketch is presented above. The signal is a form of a half-wave rectified sine wave.

(b) $a_0 = \dfrac{2V_A}{\pi}$

$a_n = \dfrac{4V_A/\pi}{1-n^2}$ for n even

$a_n = 0$ for n odd

$b_1 = V_A$ for $n = 1$

$b_n = 0$ for $n \neq 1$

(c) The MATLAB plot is presented above.

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Problem 13-13

The first four terms in the Fourier series of a periodic waveform are

$$v(t) = 20\left[\sin(10\pi t) - \frac{1}{9}\sin(30\pi t) + \frac{1}{25}\sin(50\pi t) - \frac{1}{49}\sin(70\pi t)\right] \text{ V}$$

(a) Find the period and fundamental frequency in rad/s and Hz. Identify the harmonics present in the first four terms.

(b) Identify the symmetry features of the waveform.

(c) Write the first four terms in the Fourier series for the waveform $v(t - T_0/4)$.

Solution:

(a) The frequencies present in the terms are 5 Hz, 15 Hz, 25 Hz, and 35 Hz, which are equivalent to 10π rad/s, 30π rad/s, 50π rad/s, and 70π rad/s. The fundamental frequency is the lowest frequency, or 5 Hz = 10π rad/s. The period is 200 ms. The third, fifth, and seventh harmonics appear in the first four terms.

(b) To identify the symmetry features, plot an estimate for the waveform.

```
clear all
close all
syms t
vn = 20*(sin(10*pi*t)-
sin(30*pi*t)/9+sin(50*pi*t)/25-
sin(70*pi*t)/49);
tt = 0:1e-3:0.5;
vtt = subs(vn,t,tt);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

The waveform is odd and has half-wave symmetry.

(c) The harmonics in the waveform have components with sinusoids $\sin(2\pi n5t)$ where n is odd. Determine how this type of sinusoid changes with a shift of $-T_0/4$.

$$\begin{aligned}
\sin[2\pi n5(t - T_0/4)] &= \sin[2\pi n5t - 2\pi n5T_0/4] = \sin[2\pi n5t]\cos[-\pi n5T_0/2] - \cos[2\pi n5t]\sin[\pi n5T_0/2] \\
&= \sin[2\pi n5t]\cos[\pi n5T_0/2] - \cos[2\pi n5t]\sin[\pi n5T_0/2] \\
&= \sin[2\pi n5t]\cos[\pi n/2] - \cos[2\pi n5t]\sin[\pi n/2] \\
&= -\cos[2\pi n5t]\sin[\pi n/2]
\end{aligned}$$

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We know that $\sin(n\pi/2) = 1$ for $n = 1, 5, 9, \ldots$ and $\sin(n\pi/2) = -1$ for $n = 3, 7, 11, \ldots$. Applying this information to the original expression for $v(t)$, we can write the following expression for $v(t - T_0/4)$.

$$v(t) = -20\left[\cos(10\pi t) + \frac{1}{9}\cos(30\pi t) + \frac{1}{25}\cos(50\pi t) + \frac{1}{49}\cos(70\pi t)\right] \text{ V}$$

Plot this signal to confirm the results.

```
close all
syms t
vn = -20*(cos(10*pi*t)+cos(30*pi*t)/9+cos(50*pi*t)/25+cos(70*pi*t)/49);
tt = 0:1e-3:0.5;
vtt = subs(vn,t,tt);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answers:

(a) The fundamental frequency is the lowest frequency, or 5 Hz = 10π rad/s. The period is 200 ms. The third, fifth, and seventh harmonics appear in the first four terms.

(b) The waveform is odd and has half-wave symmetry.

(c) $$v(t) = -20\left[\cos(10\pi t) + \frac{1}{9}\cos(30\pi t) + \frac{1}{25}\cos(50\pi t) + \frac{1}{49}\cos(70\pi t)\right] \text{ V}$$

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Problem 13-14

The first five terms in the Fourier series of a periodic waveform are

$$v(t) = 30 + 15\left[\frac{\pi}{4}\sin(500t) - \frac{1}{3}\cos(1000t) - \frac{1}{15}\cos(1500t) - \frac{1}{35}\cos(2500t)\right] \text{ V}$$

(a) Find the period and fundamental frequency in rad/s and Hz. Identify the harmonics present in the first five terms.
(b) Use MATLAB to plot two periods of $v(t)$.
(c) Identify the symmetry features of the waveform.
(d) Write the first five terms of the Fourier series for the waveform $v(-t)$.

Solution:

(a) The fundamental frequency is 500 rad/s, which is equivalent to 250/π Hz. The period is π/250 s, which is approximately 12.57 ms. The second, third, and fifth harmonics appear in the first five terms.

(b) Plot two periods of the waveform.

```
clear all
close all
syms t
vn = 30+15*(pi/4*sin(500*t)-
cos(1000*t)/3-cos(1500*t)/15-
cos(2500*t)/35);
tt = 0:30e-6:30e-3;
vtt = subs(vn,t,tt);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

(c) The waveform has no symmetry features.

(d) The following equalities allow us to find $v(-t)$:

$$v(t) = 30 + 15\left[\frac{\pi}{4}\sin(500t) - \frac{1}{3}\cos(1000t) - \frac{1}{15}\cos(1500t) - \frac{1}{35}\cos(2500t)\right]$$

$$v(-t) = 30 + 15\left[\frac{\pi}{4}\sin(-500t) - \frac{1}{3}\cos(-1000t) - \frac{1}{15}\cos(-1500t) - \frac{1}{35}\cos(-2500t)\right]$$

$$v(-t) = 30 + 15\left[-\frac{\pi}{4}\sin(500t) - \frac{1}{3}\cos(1000t) - \frac{1}{15}\cos(1500t) - \frac{1}{35}\cos(2500t)\right]$$

Plot this signal to confirm the results.

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```
close all
syms t
vn = 30+15*(-pi/4*sin(500*t)-cos(1000*t)/3-cos(1500*t)/15-cos(2500*t)/35);
tt = 0:30e-6:30e-3;
vtt = subs(vn,t,tt);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

Answers:

(a) The fundamental frequency is 500 rad/s, which is equivalent to 250/π Hz. The period is π/250 s, which is approximately 12.57 ms. The second, third, and fifth harmonics appear in the first five terms.

(b) The MATLAB plot is presented above.

(c) The waveform has no symmetry features.

(d) $v(-t) = 30 + 15\left[-\frac{\pi}{4}\sin(500t) - \frac{1}{3}\cos(1000t) - \frac{1}{15}\cos(1500t) - \frac{1}{35}\cos(2500t)\right]$

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Problem 13-15

The equation for a full-wave rectified cosine is $v(t) = V_A\,|\cos(2\pi t/T_0)|$ V.

(a) Sketch $v(t)$ for $-T_0 \le t \le T_0$.

(b) Compute the Fourier coefficients for $v(t)$.

(c) Use the Fourier coefficients to plot an estimate for $v(t)$.

Solution:

(a) Plot the signal with the amplitude and period normalized to values of one.

```
clear all
close all
syms t
VA = 1;
T0 = 1;
vt = VA*abs(cos(2*pi*t/T0));
tt = -T0:T0/1e3:T0;
vtt = subs(vt,t,tt+eps);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
```

(b) The following calculations determine the Fourier coefficients.

```
clear all
format short eng
syms t VA real
syms T0 n positive
% The period of the original sinusoid is T0, but
% the period of the rectified signal is T0/2.
% Adjust the integrals accordingly
T02 = T0/2;
% Define the function on the interval -T02/2 to T02/2
vt = VA*cos(2*pi*t/T0);
a0 = int(vt,t,-T02/2,T02/2)/T02;
an = 2*int(vt*cos(2*pi*n*t/T02),t,-T02/2,T02/2)/T02;
bn = 2*int(vt*sin(2*pi*n*t/T02),t,-T02/2,T02/2)/T02;
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
a02 =
2*VA/pi
an2 =
-4*VA/pi/(-1+4*n^2)*cos(pi*n)
bn2 =
0
```

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$$a_0 = \frac{2V_A}{\pi}$$

$$a_n = \frac{4V_A \cos(n\pi)}{\pi(4n^2 - 1)} \text{ for all } n$$

$$b_n = 0 \text{ for all } n$$

(c) Plot an estimate for the signal.

```
close all
VA = 1;
T0 = 1;
T02 = T0/2;
a0 = subs(a02);
an = subs(an2);
nn = [1 2 3 4 5];
an = subs(an,n,nn);
cn = cos(2*pi*nn*t/T02);
vn = vpa(a0+sum(an.*cn),6);
tt = -T0:T0/1e3:T0;
vtt = subs(vn,t,tt);
plot(tt,vtt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Voltage (V)')
axis([-T0 T0 0 1.2])
```

Answers:

(a) The plot is presented above.

(b) $a_0 = \frac{2V_A}{\pi}$

$$a_n = \frac{4V_A \cos(n\pi)}{\pi(4n^2 - 1)} \text{ for all } n$$

$$b_n = 0 \text{ for all } n$$

Note: These coefficients are based on an analysis of the signal having a period that is $T_0/2$ when the Fourier series is expanded.

(c) The plot is presented above.

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Problem 13-16

The periodic pulse train in Figure P13-16 is applied to the *RL* circuit shown in the figure.

(a) Use the results in Figure 13-4 to find the Fourier coefficients of the input for $V_A = 4$ V, $T_0 = 2\pi$ ms, and $T = T_0/4$.

(b) Find the first four nonzero terms in the Fourier series of $v_O(t)$ for $R = 200\ \Omega$, and $L = 100$ mH.

Solution:

(a) Using the results in Figure 13-4, we have the following coefficients:

$$a_0 = \frac{V_A T}{T_0} = \frac{V_A T_0}{4T_0} = \frac{V_A}{4} = 1\text{ V}$$

$$a_n = \frac{2V_A}{n\pi}\sin\left(\frac{n\pi T}{T_0}\right) = \frac{8}{n\pi}\sin\left(\frac{n\pi}{4}\right)\text{V for all } n$$

$$b_n = 0 \text{ for all } n$$

(b) Find the transfer function for the *RL* circuit and evaluate it at the frequencies for the sinusoidal components of the input Fourier series.

```
clear all
syms s
syms w positive
VA = 4;
T0 = pi*2e-3;
% Compute the transfer function
R = 200;
L = 100e-3;
ZL = s*L;
Ts = simplify(R/(ZL+R));
% Find the input frequencies
n = [1 2 3];
w = 2*pi*n/T0
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
Tjw2 = Tjw.*conj(Tjw);
MagTjw = sqrt(Tjw2)
% The angle of the transfer function has the following form
```

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```
AngleTjw = -180*atan2(w,2000)/pi
% Find the output coefficients
a0 = 1;
an = 8*sin(n*pi/4)./n/pi
anOut = an.*MagTjw
% Since we only have an Fourier coefficients, all of the input
% signals are cosines and the output phase is the phase of the
% transfer function plus 180 degrees when the input coefficient
% has a negative sign
PhaseIn = 180*(an<0)
PhaseOut = PhaseIn+AngleTjw
w =
     1.0000e+003     2.0000e+003     3.0000e+003
MagTjw =
   894.4272e-003   707.1068e-003   554.7002e-003
AngleTjw =
   -26.5651e+000   -45.0000e+000   -56.3099e+000
an =
     1.8006e+000     1.2732e+000   600.2109e-003
anOut =
     1.6105e+000   900.3163e-003   332.9371e-003
PhaseIn =
     0.0000e+000     0.0000e+000     0.0000e+000
PhaseOut =
   -26.5651e+000   -45.0000e+000   -56.3099e+000
```

$$v_{\mathrm{O}}(t) = 1 + 1.6105\cos(1000t - 26.57°) + 0.9003\cos(2000t - 45°) + 0.3329\cos(1000t - 56.31°)$$

Answers:

(a) $a_0 = 1\,\mathrm{V}$

$a_{\mathrm{n}} = \dfrac{8}{n\pi}\sin\left(\dfrac{n\pi}{4}\right)\mathrm{V}$ for all n

$b_{\mathrm{n}} = 0$ for all n

(b) $v_{\mathrm{O}}(t) = 1 + 1.6105\cos(1000t - 26.57°) + 0.9003\cos(2000t - 45°) + 0.3329\cos(1000t - 56.31°)$

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Problem 13-17

The periodic triangular wave in Figure P13-17 is applied to the *RC* circuit shown in the figure. The Fourier coefficients of the input are:

$$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{8V_A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)$$

If V_A = 20 V and T_0 = 10π ms, find the first four nonzero terms in the Fourier series of $v_O(t)$ for R = 50 kΩ , and C = 50 nF.

Solution:

Find the first four nonzero input terms and their frequencies.

```
clear all
VA = 20;
T0 = pi*10e-3;
n = 1:8;
bn = 8*VA*sin(n*pi/2)./(n*pi).^2;
w = 2*pi*n/T0;
Results = [n' bn' w']
Results =
     1.0000e+000    16.2114e+000   200.0000e+000
     2.0000e+000   496.3307e-018   400.0000e+000
     3.0000e+000    -1.8013e+000   600.0000e+000
     4.0000e+000  -248.1653e-018   800.0000e+000
     5.0000e+000   648.4556e-003     1.0000e+003
     6.0000e+000   165.4436e-018     1.2000e+003
     7.0000e+000  -330.8447e-003     1.4000e+003
     8.0000e+000  -124.0827e-018     1.6000e+003
```

Only the odd values of n have nonzero components, so n = 1, 3, 5, and 7, and the corresponding frequencies are 200, 600, 1000, and 1400 rad/s.

Find the transfer function and evaluate it at the frequencies corresponding to non-zero components.

```
syms s
% Compute the transfer function
R = 50e3;
C = 50e-9;
ZC = 1/s/C;
```

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```
Ts = simplify(ZC/(R+ZC));
% Find the input frequencies
n = 1:2:7;
w = 2*pi*n/T0
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw)
PhaseTjw = 180*angle(Tjw)/pi
% Find the output coefficients
bn = 8*VA*sin(n*pi/2)./(n*pi).^2
bnOut = abs(bn).*MagTjw
% Since we only have bn Fourier coefficients, all of the input
% signals are sines.  The input phase is -90 degrees if the input
% coefficient is positive and the input phase is +90 degrees if
% the input coefficient is negative
PhaseIn = 180*(bn<0)-90
PhaseOut = PhaseIn+PhaseTjw
w =
   200.0000e+000   600.0000e+000     1.0000e+003     1.4000e+003
MagTjw =
   894.4272e-003   554.7002e-003   371.3907e-003   274.7211e-003
PhaseTjw =
   -26.5651e+000   -56.3099e+000   -68.1986e+000   -74.0546e+000
bn =
    16.2114e+000    -1.8013e+000   648.4556e-003  -330.8447e-003
bnOut =
    14.4999e+000   999.1623e-003   240.8304e-003    90.8900e-003
PhaseIn =
   -90.0000e+000    90.0000e+000   -90.0000e+000    90.0000e+000
PhaseOut =
  -116.5651e+000    33.6901e+000  -158.1986e+000    15.9454e+000
```

Answer:

$$v_{\mathrm{O}}(t) = 14.4999\cos(200t - 116.5651°) + 0.9992\cos(600t + 33.6901°)$$
$$+ 0.2408\cos(1000t - 158.1986°) + 0.0909\cos(1400t + 15.9454°) \ \ \mathrm{V}$$

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Problem 13-18

The periodic sawtooth wave in Figure P13-18 drives the OP AMP circuit shown in the figure.

(a) Use the results in Figure 13-4 to find the Fourier coefficients of the input for V_A = 10 V and $T_0 = 8\pi$ ms.

(b) Find the first four nonzero terms in the Fourier series of $v_O(t)$ for R_1 = 20 kΩ , R_2 = 40 kΩ , and C = 100 nF.

Solution:

(a) The input waveform is a sawtooth wave. The Fourier coefficients are as follows:

$$a_0 = \frac{V_A}{2} = 5\text{ V}$$

$$a_n = 0\text{ V} \quad \text{for all } n$$

$$b_n = -\frac{V_A}{n\pi} = -\frac{10}{n\pi}\text{ V} \quad \text{for all } n$$

Find the first four nonzero input terms and their frequencies. Note that the circuit is a high-pass filter, so the dc component will not pass through.

```
clear all
VA = 10;
T0 = pi*8e-3;
n = 1:4;
bn = -VA./n/pi;
w = 2*pi*n/T0;
Results = [n' bn' w']
Results =
     1.0000e+000    -3.1831e+000   250.0000e+000
     2.0000e+000    -1.5915e+000   500.0000e+000
     3.0000e+000    -1.0610e+000   750.0000e+000
     4.0000e+000  -795.7747e-003     1.0000e+003
```

The nonzero values of n = 1, 2, 3, and 4, and the corresponding frequencies are 250, 500, 750, and 1000 rad/s.

Find the transfer function and evaluate it at the frequencies corresponding to non-zero components.

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```
syms s
% Compute the transfer function
R1 = 20e3;
R2 = 40e3;
C = 100e-9;
ZC = 1/s/C;
Ts = simplify(-R2/(R1+ZC));
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw)
PhaseTjw = 180*angle(Tjw)/pi
% Find the output coefficients
bn
bnOut = abs(bn).*MagTjw
% Since we only have bn Fourier coefficients, all of the input
% signals are sines.  The input phase is -90 degrees if the input
% coefficient is positive and the input phase is +90 degrees if
% the input coefficient is negative
PhaseIn = 180*(bn<0)-90
PhaseOut = PhaseIn+PhaseTjw
MagTjw =
   894.4272e-003     1.4142e+000     1.6641e+000     1.7889e+000
PhaseTjw =
  -116.5651e+000  -135.0000e+000  -146.3099e+000  -153.4349e+000
bn =
    -3.1831e+000    -1.5915e+000    -1.0610e+000  -795.7747e-003
bnOut =
     2.8471e+000     2.2508e+000     1.7657e+000     1.4235e+000
PhaseIn =
    90.0000e+000    90.0000e+000    90.0000e+000    90.0000e+000
PhaseOut =
   -26.5651e+000   -45.0000e+000   -56.3099e+000   -63.4349e+000
```

Answers:

(a)

$$a_0 = \frac{V_A}{2} = 5\text{ V}$$

$$a_n = 0\text{ V} \quad \text{for all } n$$

$$b_n = -\frac{V_A}{n\pi} = -\frac{10}{n\pi}\text{ V} \quad \text{for all } n$$

(b)

$$v_O(t) = 2.8471\cos(250t - 26.5651°) + 2.2508\cos(500t - 45°)$$
$$+1.7657\cos(750t - 56.3099°) + 1.4235\cos(1000t - 63.4349°)\ \text{ V}$$

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Problem 13-19

The periodic sawtooth wave in Figure P13-18 above drives the OP AMP circuit shown in the figure.

(a) Use the results in Figure 13-4 to find the Fourier coefficients of the input for V_A = 8 V and T_0 = 20π ms

(b) Find the first four nonzero terms in the Fourier series of $i(t)$ for R_1 = 100 kΩ, R_2 = 20 kΩ , and C = 400 nF.

Solution:

(a) The input waveform is a sawtooth wave. The Fourier coefficients are as follows:

$$a_0 = \frac{V_A}{2} = 4 \text{ V}$$

$$a_n = 0 \text{ V} \quad \text{for all } n$$

$$b_n = -\frac{V_A}{n\pi} = -\frac{8}{n\pi} \text{ V} \quad \text{for all } n$$

(b) Find the first four nonzero input terms and their frequencies. Note that the circuit is a high-pass filter, so the dc component will not pass through the circuit to the output voltage.

```
clear all
VA = 8;
T0 = pi*20e-3;
n = 1:4;
bn = -VA./n/pi;
w = 2*pi*n/T0;
Results = [n' bn' w']
Results =
     1.0000e+000    -2.5465e+000   100.0000e+000
     2.0000e+000    -1.2732e+000   200.0000e+000
     3.0000e+000  -848.8264e-003   300.0000e+000
     4.0000e+000  -636.6198e-003   400.0000e+000
```

The nonzero values of n = 1, 2, 3, and 4, and the corresponding frequencies are 100, 200, 300, and 400 rad/s.

Find the transfer function and evaluate it at the frequencies corresponding to non-zero components.

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```
syms s
% Compute the transfer function
R1 = 100e3;
R2 = 20e3;
C = 400e-9;
ZC = 1/s/C;
Zin = R1+ZC;
Ts = simplify(1/Zin);
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw)
PhaseTjw = 180*angle(Tjw)/pi
% Find the output coefficients
bn
bnOut = abs(bn).*MagTjw
% Since we only have bn Fourier coefficients, all of the input
% signals are sines.  The input phase is -90 degrees if the input
% coefficient is positive and the input phase is +90 degrees if
% the input coefficient is negative
PhaseIn = 180*(bn<0)-90
PhaseOut = PhaseIn+PhaseTjw
MagTjw =
     9.7014e-006     9.9228e-006     9.9655e-006     9.9805e-006
PhaseTjw =
    14.0362e+000     7.1250e+000     4.7636e+000     3.5763e+000
bn =
    -2.5465e+000    -1.2732e+000  -848.8264e-003  -636.6198e-003
bnOut =
    24.7045e-006    12.6341e-006     8.4589e-006     6.3538e-006
PhaseIn =
    90.0000e+000    90.0000e+000    90.0000e+000    90.0000e+000
PhaseOut =
   104.0362e+000    97.1250e+000    94.7636e+000    93.5763e+000
```

Answers:

(a)

$$a_0 = \frac{V_A}{2} = 4\text{ V}$$

$$a_n = 0\text{ V} \quad \text{for all } n$$

$$b_n = -\frac{V_A}{n\pi} = -\frac{8}{n\pi}\text{ V} \quad \text{for all } n$$

(b)

$$i(t) = 24.7\cos(100t + 104.04°) + 12.63\cos(200t + 97.13°)$$
$$+ 8.46\cos(300t + 94.76°) + 6.35\cos(400t + 93.58°)\ \ \mu\text{A}$$

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Problem 13-20

The periodic triangular wave in Figure P13-20 is applied to the *RLC* circuit shown in the figure.
(a) Use the results in Figure 13-4 to find the Fourier coefficients of the input for V_A = 10 V and T_0 = 400π μs.
(b) Find the amplitude of the first five nonzero terms in the Fourier series for $i(t)$ when R = 1 Ω, L = 8 mH, and C = 200 nF. What term in the Fourier series tends to dominate the response? Explain.

Solution:
(a) Using Figure 13-4, we have the following coefficients:

$$a_0 = 0 \text{ V}$$
$$a_n = \frac{8V_A}{(n\pi)^2} = \frac{80}{(n\pi)^2} \text{ V for } n \text{ odd}$$
$$a_n = 0 \text{ V for } n \text{ even}$$
$$b_n = 0 \text{ V for all } n$$

(b) Find the first five nonzero input terms and their frequencies.

```
clear all
VA = 10;
T0 = pi*400e-6;
n = [1 3 5 7 9];
an = 8*VA./(n*pi).^2;
w = 2*pi*n/T0;
Results = [n' an' w']
Results =
     1.0000e+000     8.1057e+000     5.0000e+003
     3.0000e+000   900.6327e-003    15.0000e+003
     5.0000e+000   324.2278e-003    25.0000e+003
     7.0000e+000   165.4223e-003    35.0000e+003
     9.0000e+000   100.0703e-003    45.0000e+003
```

The nonzero values of n = 1, 3, 5, 7, and 9, and the corresponding frequencies are 5, 15, 25, 35, and 45 krad/s.

Find the transfer function and evaluate it at the frequencies corresponding to non-zero components.

```
syms s
% Compute the transfer function
R = 1;
L = 8e-3;
```

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```
C = 200e-9;
ZL = s*L;
ZC = 1/s/C;
Zin = R+ZL+ZC;
Ts = simplify(1/Zin);
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw)
% Find the output coefficients
an
anOut = abs(an).*MagTjw
Results = [w' anOut']
MagTjw =
     1.0417e-003     4.6874e-003     1.0000e+000     7.2915e-003     4.0178e-
003
an =
     8.1057e+000   900.6327e-003   324.2278e-003   165.4223e-003   100.0703e-
003
anOut =
     8.4434e-003     4.2217e-003   324.2278e-003     1.2062e-003   402.0649e-
006
Results =
     5.0000e+003     8.4434e-003
    15.0000e+003     4.2217e-003
    25.0000e+003   324.2278e-003
    35.0000e+003     1.2062e-003
    45.0000e+003   402.0649e-006
```

The current amplitudes are shown above and the response at an input frequency of 25 krad/s is the largest, because the *RLC* circuit is resonant at that frequency and the transfer function between the input voltage and current is a bandpass filter.

Answers:

(a)

$a_0 = 0\text{ V}$

$a_n = \dfrac{8V_A}{(n\pi)^2} = \dfrac{80}{(n\pi)^2}\text{ V}$ for n odd

$a_n = 0\text{ V}$ for n even

$b_n = 0\text{ V}$ for all n

(b) At ω = 5 krad/s, $|i(t)|$ = 8.44 mA.

At ω = 15 krad/s, $|i(t)|$ = 4. 22 mA.

At ω = 25 krad/s, $|i(t)|$ = 324.23 mA.

At ω = 35 krad/s, $|i(t)|$ = 1.21 mA.

At ω = 45 krad/s, $|i(t)|$ = 0.402 mA.

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The Fourier series term appearing at 25 krad/s dominants the expression because the circuit is resonant at that frequency.

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Problem 13-21

A sawtooth wave with V_A = 12 V and T_0 = 20π ms drives a circuit with a transfer function $T(s)$ = $s/(s + 200)$. Find the amplitude of the first five nonzero terms in the Fourier series of the steady-state output. Construct plots of the amplitude spectra for the input and output waveforms and comment on any differences.

Solution:

Using the results in Figure 13-4, we have the following coefficients:

$$a_0 = 6\text{ V}$$
$$a_n = 0\text{ V} \text{ for all } n$$
$$b_n = -\frac{12}{n\pi}\text{ V} \text{ for all } n$$

The transfer function is a high-pass filter, so the dc component will not appear in the output. The first five terms will be for n = 1, 2, 3, 4, and 5. Find the associated input frequencies and then determine the magnitude and phase of the transfer function at those frequencies.

```
clear all
VA = 12;
T0 = pi*20e-3;
n = 1:5;
bn = -VA./n/pi;
w = 2*pi*n/T0;
Results = [n' bn' w']
Results =
     1.0000e+000    -3.8197e+000   100.0000e+000
     2.0000e+000    -1.9099e+000   200.0000e+000
     3.0000e+000    -1.2732e+000   300.0000e+000
     4.0000e+000  -954.9297e-003   400.0000e+000
     5.0000e+000  -763.9437e-003   500.0000e+000
```

Each input component is a sine function with a negative amplitude, so its phase will be +90 degrees. Find the transfer function and evaluate it at the frequencies corresponding to non-zero components.

```
syms s
% Compute the transfer function
Ts = s/(s+200);
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw)
PhaseTjw = 180*angle(Tjw)/pi
% Find the output coefficients
bn
bnOut = abs(bn).*MagTjw
PhaseIn = 180*(bn<0)-90
PhaseOut = PhaseIn+PhaseTjw
```

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```
MagTjw =
   447.2136e-003   707.1068e-003   832.0503e-003   894.4272e-003   928.4767e-
003
PhaseTjw =
    63.4349e+000    45.0000e+000    33.6901e+000    26.5651e+000
21.8014e+000
bn =
    -3.8197e+000    -1.9099e+000    -1.2732e+000  -954.9297e-003  -763.9437e-
003
bnOut =
     1.7082e+000     1.3505e+000     1.0594e+000   854.1151e-003   709.3039e-
003
PhaseIn =
    90.0000e+000    90.0000e+000    90.0000e+000    90.0000e+000
90.0000e+000
PhaseOut =
   153.4349e+000   135.0000e+000   123.6901e+000   116.5651e+000
111.8014e+000
```

Plot the amplitude spectra of the input and output.

```
close all
% Plot the spectrum
stem([0 w],[VA/2 abs(bn)],...

'b','LineWidth',3,'MarkerSize',10,
'MarkerFaceColor','b')
hold on
grid on
stem(w,bnOut,'g','LineWidth',3,'Ma
rkerSize',10,'MarkerFaceColor','g'
)
xlabel('Frequency, (rad/s)')
ylabel('Amplitude')
legend('Input','Output')
```

The output signal does not have the dc component and the lower frequency components are attenuated more than the higher frequency components.

Answers:

$$v_O(t) = 1.7082\cos(100t + 153.43°) + 1.3505\cos(200t + 135°)$$
$$+1.0594\cos(300t + 123.69°) + 0.8541\cos(400t + 116.57°) + 0.7093\cos(500t + 111.80°) \text{ V}$$

The amplitude spectra are shown above.

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Problem 13-22
Repeat Problem 13-21 for $T(s) = 200/(s + 200)$.

Solution:
Using the results in Figure 13-4, we have the following coefficients:

$$a_0 = 6\text{ V}$$
$$a_n = 0\text{ V} \text{ for all } n$$
$$b_n = -\frac{12}{n\pi}\text{ V} \text{ for all } n$$

The transfer function is a low-pass filter, so the dc component will appear in the output. The gain at dc is one. The first five terms will include the dc term and n = 1, 2, 3, and 4. Find the associated input frequencies and then determine the magnitude and phase of the transfer function at those frequencies.

```
clear all
VA = 12;
T0 = pi*20e-3;
n = 1:4;
bn = -VA./n/pi;
w = 2*pi*n/T0;
Results = [n' bn' w']
Results =
     1.0000e+000    -3.8197e+000   100.0000e+000
     2.0000e+000    -1.9099e+000   200.0000e+000
     3.0000e+000    -1.2732e+000   300.0000e+000
     4.0000e+000  -954.9297e-003   400.0000e+000
```

Each input component is a sine function with a negative amplitude, so its phase will be +90 degrees. Find the transfer function and evaluate it at the frequencies corresponding to non-zero components.

```
syms s
% Compute the transfer function
Ts = 200/(s+200);
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw)
PhaseTjw = 180*angle(Tjw)/pi
% Find the output coefficients
bn
bnOut = abs(bn).*MagTjw
PhaseIn = 180*(bn<0)-90
PhaseOut = PhaseIn+PhaseTjw
MagTjw =
   894.4272e-003   707.1068e-003   554.7002e-003   447.2136e-003
PhaseTjw =
   -26.5651e+000   -45.0000e+000   -56.3099e+000   -63.4349e+000
```

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```
bn =
    -3.8197e+000    -1.9099e+000    -1.2732e+000  -954.9297e-003
bnOut =
     3.4165e+000     1.3505e+000   706.2662e-003   427.0575e-003
PhaseIn =
    90.0000e+000    90.0000e+000    90.0000e+000    90.0000e+000
PhaseOut =
    63.4349e+000    45.0000e+000    33.6901e+000    26.5651e+000
```

Plot the amplitude spectra of the input and output.

```
close all
% Plot the spectrum
stem([0 w],[VA/2 abs(bn)],...
     'b','LineWidth',3,'MarkerSize',10,'MarkerFaceColor','b')
hold on
grid on
stem([0 w],[VA/2 bnOut],...

'g','LineWidth',3,'MarkerSize',10
,'MarkerFaceColor','g')
xlabel('Frequency, (rad/s)')
ylabel('Amplitude')
legend('Input','Output')
```

The output signal's dc component matches the dc component of the input. The higher frequency components of the output are attenuated more than the lower frequency components.

Answers:

$$v_O(t) = 6 + 3.4165\cos(100t + 63.43°) + 1.3505\cos(200t + 45°)$$
$$+ 0.7063\cos(300t + 33.69°) + 0.4271\cos(400t + 26.57°) \text{ V}$$

The amplitude spectra are shown above.

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Problem 13-23

The voltage across a 1-nF capacitor is a triangular wave with V_A = 15 V and f_0 = 1 kHz. Construct plots of the amplitude spectra of the capacitor voltage and current. Discuss any differences in spectral content.

Solution:

The coefficients of the input signal are for a_n, with n odd. All other coefficients are zero.

$$a_n = \frac{120}{(n\pi)^2}\ \text{V} \quad \text{for } n \text{ odd.}$$

The transfer function from the capacitor voltage to the capacitor current is the reciprocal of the impedance, which is sC.

```
clear all
VA = 15;
f0 = 1e3;
C = 1e-9;
T0 = 1/f0;
n = 1:2:9;
an = 8*VA./(n*pi).^2;
w = 2*pi*n/T0;

syms s
% Compute the transfer function
Ts = s*C;
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
% Find the output coefficients
an;
anOut = abs(an).*MagTjw;
Results = [w' MagTjw' an' anOut']
Results =
     6.2832e+003     6.2832e-006    12.1585e+000    76.3944e-006
    18.8496e+003    18.8496e-006     1.3509e+000    25.4648e-006
    31.4159e+003    31.4159e-006   486.3417e-003    15.2789e-006
    43.9823e+003    43.9823e-006   248.1335e-003    10.9135e-006
    56.5487e+003    56.5487e-006   150.1055e-003     8.4883e-006

close all
% Plot the input spectrum
stem(w,abs(an),...
     'b','LineWidth',3,'MarkerSize',10,'MarkerFaceColor','b')
grid on
xlabel('Frequency, (rad/s)')
ylabel('Amplitude')
title('Input Components')
```

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```
close all
% Plot the output spectrum
stem(w,anOut,...
     'g','LineWidth',3,'MarkerSize',10,'MarkerFaceColor','g')
grid on
xlabel('Frequency, (rad/s)')
ylabel('Amplitude')
title('Output Components')
```

Answers:
The spectra are presented above.

The capacitor acts like a type of high-pass filter between its voltage and current. The higher-frequency components of the current attenuate at a slower rates than those of the voltage signal.

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Problem 13-24

An ideal time delay is a signal processor whose output is $v_O(t) = v_{IN}(t - T_D)$. Write an expression for $v_O(t)$ for $T_D = 0.5$ ms and

$$v_{IN}(t) = 10 + 10\cos(2\pi 500t) + 2.5\cos(2\pi 1000t) + 0.625\cos(2\pi 4000t) \text{ V}$$

Discuss the spectral changes caused by the time delay.

Solution:

The following equalities allow us to determine an expression for the output signal.

$$v_{IN}(t) = 10 + 10\cos(2\pi 500t) + 2.5\cos(2\pi 1000t) + 0.625\cos(2\pi 4000t) \text{ V}$$

$$v_O(t) = v_{IN}(t - T_D) = 10 + 10\cos[2\pi 500(t - T_D)] + 2.5\cos[2\pi 1000(t - T_D)] + 0.625\cos[2\pi 4000(t - T_D)] \text{ V}$$

$$v_O(t) = 10 + 10\cos[2\pi 500t - 2\pi 500T_D] + 2.5\cos[2\pi 1000t - 2\pi 1000T_D] + 0.625\cos[2\pi 4000t - 2\pi 4000T_D] \text{ V}$$

$$v_O(t) = 10 + 10\cos[2\pi 500t - \pi/2] + 2.5\cos[2\pi 1000t - \pi] + 0.625\cos[2\pi 4000t - 4\pi] \text{ V}$$

$$v_O(t) = 10 + 10\sin[2\pi 500t] - 2.5\cos[2\pi 1000t] + 0.625\cos[2\pi 4000t] \text{ V}$$

The amplitude spectrum does not change. The phase spectrum changes, with the amount of phase shift being proportional to the frequency of the component. The linear phase shift allows for the constant time delay.

Answer:

$$v_O(t) = 10 + 10\sin[2\pi 500t] - 2.5\cos[2\pi 1000t] + 0.625\cos[2\pi 4000t] \text{ V}$$

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Problem 13-25

A sawtooth wave with V_A = 8 V and T_0 = 20π ms drives a circuit whose transfer function is

$$T(s) = \frac{100s}{(s+50)^2 + 400^2}$$

(a) Find the amplitude of the first four nonzero terms in the Fourier series of the steady-state output. What term in the Fourier series tends to dominate the response? Explain.

(b) Repeat Part (a) for a new transfer function $R(s) = T(s)/s$. What is the key difference between $T(s)$ and $R(s)$?

Solution:

(a) The transfer function is a bandpass filter, so the dc component of the input will not appear in the output. The first four terms will be for n = 1, 2, 3, and 4. Find the associated input frequencies and then determine the magnitude and phase of the transfer function at those frequencies.

```
clear all
VA = 8;
T0 = pi*20e-3;
n = 1:6;
bn = -VA./n/pi;
w = 2*pi*n/T0;
```

Find the transfer function and evaluate it at the frequencies corresponding to non-zero components.

```
syms s
% Compute the transfer function
Ts = 100*s/((s+50)^2+400^2);
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
% Find the output coefficients
bnOut = abs(bn).*MagTjw;
Results = [n' w' bn' bnOut']
Results =
     1.0000e+000   100.0000e+000    -2.5465e+000   166.6244e-003
     2.0000e+000   200.0000e+000    -1.2732e+000   205.1595e-003
     3.0000e+000   300.0000e+000  -848.8264e-003   324.5503e-003
     4.0000e+000   400.0000e+000  -636.6198e-003   635.3800e-003
     5.0000e+000   500.0000e+000  -509.2958e-003   252.6815e-003
     6.0000e+000   600.0000e+000  -424.4132e-003   123.3683e-003
```

The component at 400 rad/s dominants the output expression. The transfer function is a bandpass filter with a peak response at 400 rad/s.

(b) The transfer function is a low-pass filter, so the dc component of the input will appear in the output. The first four terms will include n = 1, 2, and 3. Find the associated input frequencies and then determine the magnitude and phase of the transfer function at those frequencies.

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```
clear all
VA = 8;
T0 = pi*20e-3;
n = 1:6;
a0 = VA/2;
bn = -VA./n/pi;
abn = [a0 bn];
w = 2*pi*n/T0;
w2 = [0 w];
```

Find the transfer function and evaluate it at the frequencies corresponding to non-zero components.

```
syms s
% Compute the transfer function
Ts = 100*s/((s+50)^2+400^2);
Rs = Ts/s;
% Evaluate the transfer function at the input frequencies
Tjw = subs(Rs,s,j*w2);
MagTjw = abs(Tjw);
% Find the output coefficients
abnOut = abs(abn).*MagTjw;
n2 = [0 n];
Results = [n2' w2' abn' abnOut']
Results =
     0.0000e+000     0.0000e+000     4.0000e+000     2.4615e-003
     1.0000e+000   100.0000e+000    -2.5465e+000     1.6662e-003
     2.0000e+000   200.0000e+000    -1.2732e+000     1.0258e-003
     3.0000e+000   300.0000e+000  -848.8264e-003     1.0818e-003
     4.0000e+000   400.0000e+000  -636.6198e-003     1.5885e-003
     5.0000e+000   500.0000e+000  -509.2958e-003   505.3630e-006
     6.0000e+000   600.0000e+000  -424.4132e-003   205.6138e-006
```

Answers:

(a) For ω = 100 rad/s, the output amplitude is 166 mV.
For ω = 200 rad/s, the output amplitude is 205 mV.
For ω = 300 rad/s, the output amplitude is 325 mV.
For ω = 400 rad/s, the output amplitude is 635 mV.
The component at 400 rad/s dominates the output expression. The transfer function is a bandpass filter with a peak response at 400 rad/s.

(b) For ω = 0 rad/s, the output amplitude is 2.46 mV.
For ω = 100 rad/s, the output amplitude is 1.67 mV.
For ω = 200 rad/s, the output amplitude is 1.03 mV.
For ω = 300 rad/s, the output amplitude is 1.08 mV.
For ω = 400 rad/s, the output amplitude is 1.59 mV.

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The component at 0 rad/s dominates the output expression. The transfer function is a low-pass filter, so the lowest frequency component should have the dominant response. The key difference between $T(s)$ and $R(s)$ is that $T(s)$ is a bandpass filter and $R(s)$ is a low-pass filter.

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Problem 13-26

The current through a 500-Ω resistor is

$$i(t) = 30 + 15\cos(120\pi t - 30^\circ) - 10\cos(360\pi t + 45^\circ) \text{ mA}$$

Find the rms value of the current and the average power delivered to the resistor.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
format short eng
A0 = 30e-3;
An = 1e-3*[15 -10];
Irms = sqrt(A0^2+sum(An.^2)/2)
R = 500;
Pavg = R*Irms^2
Irms =
    32.5960e-003
Pavg =
   531.2500e-003
```

Answers:

I_{rms} = 32.596 mA.

P = 531.25 mW.

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Problem 13-27

The voltage across a 50-Ω resistor is

$$v(t) = 20 + 15\sin(300\pi t) - 5\sin(900\pi t) + 3\sin(1500\pi t) \text{ V}$$

(a) Find expressions for the current through the resistor and the power dissipated by the resistor.
(b) Find the average of the power expression by integrating over one period of the waveform and dividing by the period.
(c) Find the rms value of the voltage by applying Eq (13-19) to the expression for $v(t)$.
(d) Find the rms value of the voltage by applying Eq (13-24) to the Fourier coefficient of $v(t)$ and compare the result to the answer in Part (c).
(e) Compute the average power dissipated by the resistor using $P = V_{\text{rms}}^2 / R$ and compare the result to the answer in Part (b).

Solution:

(a) The solution is presented in the following MATLAB calculations.

```
clear all
syms t
R = 50;
vt = 20 + 15*sin(300*pi*t) - 5*sin(900*pi*t) + 3*sin(1500*pi*t);
it = vt/50
pt = simplify(vt*it)
it =
2/5+3/10*sin(300*t*pi)-1/10*sin(900*t*pi)+3/50*sin(1500*t*pi)
pt =
1/50*(20+15*sin(300*t*pi)-5*sin(900*t*pi)+3*sin(1500*t*pi))^2
```

(b) The fundamental frequency is 150 Hz, so the period is 1/150 s.

```
% Integrate over one period to find the average power
f0 = 150;
T0 = 1/f0;
Pavg = int(pt,t,0,T0)/T0
Pavg =
1059/100
```

(c) Find the rms value of the voltage.

```
Vrms = double(sqrt(int(vt^2,t,0,T0)/T0))
Vrms =
    23.0109e+000
```

(d) Find the rms value of the voltage using the Fourier coefficients.

```
A0 = 20;
An = [15 -5 3];
Vrms = sqrt(A0^2+sum(An.^2)/2)
Vrms =
    23.0109e+000
```

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(e) Compute the average power dissipated in the resistor.

```
Pavg = Vrms^2/R
Pavg =
    10.5900e+000
```

Answers:

(a) $i(t) = 400 + 300\sin(300\pi t) - 100\sin(900\pi t) + 60\sin(1500\pi t)$ mA

$$p(t) = \frac{1}{50}\left[20 + 15\sin(300\pi t) - 5\sin(900\pi t) + 3\sin(1500\pi t)\right]^2 \text{ W}$$

(b) P = 10.59 W

(c) V_{rms} = 23.01 V

(d) V_{rms} = 23.01 V. The answer is the same as Part (c).

(e) P = 10.59 W. The answer is the same as Part (b).

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Problem 13-28

Use MATLAB to find the rms value of a half-wave rectified sine wave. Find the fraction of the total average power carried by the dc component plus the first three nonzero ac components in the Fourier series.

Solution:

Let the half-wave rectified sine wave have a period of T_0 and an amplitude of A. Then we have the following signal

$$f(t) = A\sin(2\pi t/T_0) \text{ for } 0 < t < T_0/2$$
$$f(t) = 0 \text{ for } T_0/2 < t < T_0$$

The following MATLAB calculations determine the rms value.

```
clear all
syms t
syms A T0 R positive
ft = A*sin(2*pi*t/T0);
Vrms = sqrt(int(ft^2,t,0,T0/2)/T0)
P = Vrms^2/R;
Vrms =
1/2*A
```

Find the power carried in the dc component plus the first three nonzero ac components.

```
a0 = A/pi;
b1 = A/2;
n = [2 4];
an = 2*A/pi./(1-n.^2);
A0 = a0;
An = [b1 an];
Vrms2 = sqrt(A0^2+sum(An.^2)/2);
P2 = Vrms2^2/R;
P_ratio = double(P2/P)
P_ratio =
   998.9505e-003
```

Answer:

$V_{rms} = A/2$.

The dc component and the first three nonzero ac components carry 99.895% percent of the average power.

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Problem 13-29

Find the rms value of the periodic waveform in Figure P13-29 and the average power the waveform delivers to a resistor. Find the dc component of the waveform and the average power carried by the dc component. What fraction of the total average power is carried by the dc component? What fraction is carried by the ac components?

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t
syms VA T0 R positive
% Create one period of the signal
vt = 2*VA/T0*t*(heaviside(t)-heaviside(t-T0/2));
% Find the rms value
Vrms = sqrt(int(vt^2,t,0,T0)/T0)
% Compute the average power
P = Vrms^2/R
% The dc component of the waveform is its average value
Vdc = int(vt,t,0,T0)/T0
% Average power of dc component
Pdc = Vdc^2/R
% Fraction of power carried by dc component
Pdc_ratio = double(Pdc/P)
% Fraction of power carried by ac components is the remainder
Pac_ratio = 1-Pdc_ratio
Vrms =
1/6*6^(1/2)*VA
P =
1/6*VA^2/R
Vdc =
1/4*VA
Pdc =
1/16*VA^2/R
Pdc_ratio =
   375.0000e-003
Pac_ratio =
   625.0000e-003
```

Answers:

$$V_{\text{rms}} = \frac{V_{\text{A}}}{\sqrt{6}} \text{ and } P = \frac{V_{\text{A}}^{\ 2}}{6R}$$

$$V_{\text{dc}} = \frac{V_{\text{A}}}{4} \text{ and } P_{\text{dc}} = \frac{V_{\text{A}}^{\ 2}}{16R}$$

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The dc components carry 37.5% of the total average power and the ac components carry 62.5% of the total average power.

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Problem 13-30

Repeat Problem 13-29 for the periodic waveform in Figure P13-30.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t
syms VA T0 R positive
% Create one period of the signal
vt = (VA-VA/T0*t)*(heaviside(t)-heaviside(t-T0));
% Find the rms value
Vrms = sqrt(int(vt^2,t,0,T0)/T0)
% Compute the average power
P = Vrms^2/R
% The dc component of the waveform is its average value
Vdc = int(vt,t,0,T0)/T0
% Average power of dc component
Pdc = Vdc^2/R
% Fraction of power carried by dc component
Pdc_ratio = double(Pdc/P)
% Fraction of power carried by ac components is the remainder
Pac_ratio = 1-Pdc_ratio
Vrms =
1/3*3^(1/2)*VA
P =
1/3*VA^2/R
Vdc =
1/2*VA
Pdc =
1/4*VA^2/R
Pdc_ratio =
   750.0000e-003
Pac_ratio =
   250.0000e-003
```

Answers:

$$V_{\text{rms}} = \frac{V_{\text{A}}}{\sqrt{3}} \text{ and } P = \frac{V_{\text{A}}^{\ 2}}{3R}$$

$$V_{\text{dc}} = \frac{V_{\text{A}}}{2} \text{ and } P_{\text{dc}} = \frac{V_{\text{A}}^{\ 2}}{4R}$$

The dc components carry 75% of the total average power and the ac components carry 25% of the total average power.

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Problem 13-31

A first-order low-pass filter has a cutoff frequency of 800 rad/s and a passband gain of 20 dB. The input to the filter is $x(t) = 5 \cos 400t + 2 \cos 1600t$ V. Find the rms value of the steady-state output.

Solution:

The transfer function for the filter is given by

$$T(s) = \frac{10(800)}{s+800} = \frac{8000}{s+800}$$

Find the magnitude of the transfer function at the input frequencies and then compute the magnitudes of the output components. Next, find the rms value using the Fourier components. The solution is presented in the following MATLAB calculations.

```
clear all
syms s
Ts = 8000/(s+800);
w = [400 1600];
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
MagX = [5 2];
MagY = MagX.*MagTjw;
Yrms = sqrt(sum(MagY.^2)/2)
Yrms =
    32.2490e+000
```

Answer:

$Y_{rms} = 32.249$ V

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Problem 13-32
Repeat Problem 13-31 for a first-order high-pass filter with the same cutoff frequency and passband gain.

Solution:
The transfer function for the filter is given by

$$T(s) = \frac{10s}{s+800}$$

Find the magnitude of the transfer function at the input frequencies and then compute the magnitudes of the output components. Next, find the rms value using the Fourier components. The solution is presented in the following MATLAB calculations.

```
clear all
syms s
Ts = 10*s/(s+800);
w = [400 1600];
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
MagX = [5 2];
MagY = MagX.*MagTjw;
Yrms = sqrt(sum(MagY.^2)/2)
Yrms =
    20.2485e+000
```

Answer:
Y_{rms} = 20.2485 V

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Problem 13-33

Estimate the rms value of the periodic voltage

$$v(t) = V_A\left[2 - \cos(\omega_0 t) + \frac{1}{3}\cos(3\omega_0 t) - \frac{1}{5}\cos(5\omega_0 t) + \frac{1}{7}\cos(7\omega_0 t) - \ldots\right] \text{ V}$$

Solution:

The Fourier coefficients for the ac components have the form 1/*n*, for *n* odd. We can use a partial sum to estimate the rms value of the voltage.

```
clear all
syms VA positive
% Create the dc component
A0 = 2*VA;
% Create the ac components
N = 10000;
n = 1:2:N;
An = VA./n;
% Find the rms value
Vrms = sqrt(A0^2+sum(An.^2)/2);
Vrms_num = vpa(Vrms,6)
Vrms_num =
2.14868*VA
```

Answer:

$V_{rms} = 2.1487\ V_A$

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Problem 13-34

The input to the circuit in Figure P13-34 is the voltage

$$v_S(t) = 16\cos(2\pi 3000t) + 4\cos(2\pi 6000t) \text{ V}$$

Calculate the average power delivered to the 100-Ω resistor.

Solution:

Determine the transfer function from the input voltage source to the output voltage. Calculated the magnitude of the output components and then determine the average power delivered to the load.

```
clear all
syms s
R1 = 50;
R2 = 100;
C = 750e-9;
ZC = 1/s/C;
Z2 = 1/(1/ZC+1/R2);
Ts = Z2/(R1+Z2);
w = 2*pi*[3000 6000];
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
MagVs = [16 4];
MagVo = MagVs.*MagTjw;
Vorms = sqrt(sum(MagVo.^2)/2);
P = Vorms^2/R2
P =
   484.3438e-003
```

Answer:

P = 484.34 mW.

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Problem 13-35

Find an expression for the average power delivered to a resistor R by a triangular wave voltage with amplitude V_A and period T_0. How many components of the Fourier series are required to account for 98% of the average power carried by the waveform?

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t
syms VA T0 R positive
% Create one period of the waveform
vt = (VA-4*VA*t/T0)*(heaviside(t)-heaviside(t-T0/2))...
    + (-3*VA+4*VA*t/T0)*(heaviside(t-T0/2)-heaviside(t-T0));
% Find the rms voltage
vrms = simplify(sqrt(int(vt^2,t,0,T0)/T0))
% Find the average power delivered to a resistor
ptrms = vrms^2/R
vrms =
1/3*3^(1/2)*VA
ptrms =
1/3*VA^2/R
```

Find the power in the Fourier series approximation for different values of n.

```
% Triangular waveform
n = 1:2:9;
an = 8./(n*pi).^2;
% Find the square of the rms voltage for each n
Vrms2 = VA^2*cumsum(an.^2)/2;
P = Vrms2/R;
Pratio = double(P/ptrms);
Results = [n' Pratio']
Results =
     1.0000e+000   985.5343e-003
     3.0000e+000   997.7014e-003
     5.0000e+000   999.2782e-003
     7.0000e+000   999.6887e-003
     9.0000e+000   999.8389e-003
```

Answer:

$$P = \frac{V_A^{\ 2}}{3R}$$

We only need one component of the Fourier series to account for 98% of the average power carried by the waveform.

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Problem 13-36

Use the defining integral to find the Fourier transform of $f(t) = A[u(t) - u(t-2)]$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w A
ft = A*(heaviside(t)-heaviside(t-2));
Fw = int(ft*exp(-j*w*t),t,-inf,inf)
% Check the results
Fw2 = fourier(ft,t,w)
Fw =
A*(-i/w+i/w*exp(-2*i*w))
Fw2 =
i*A*(-1+exp(-2*i*w))/w
```

Answer:

$$F(\omega) = \frac{A}{j\omega}\left[1 - e^{-2j\omega}\right]$$

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Problem 13-37

Use the defining integral to find the Fourier transform of $f(t) = At[u(t) - u(t-2)]$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w A
ft = A*t*(heaviside(t)-heaviside(t-2));
Fw = int(ft*exp(-j*w*t),t,-inf,inf)
% Check the results
Fw2 = fourier(ft,t,w)
Fw =
A*(-1/w^2+(1+2*i*w)*exp(-2*i*w)/w^2)
Fw2 =
A*(2*i*exp(-2*i*w)*w+exp(-2*i*w)-1)/w^2
```

Answer:

$$F(\omega) = \frac{A}{\omega^2}\left[-1 + (1 + 2j\omega)e^{-2j\omega}\right]$$

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Problem 13-38

Use MATLAB and the defining integral to find the Fourier transform of the following waveform:

$$f(t) = A\sin(\pi t/4)[u(t) - u(t-4)]$$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w A
ft = A*sin(pi*t/4)*(heaviside(t)-heaviside(t-4));
Fw = int(ft*exp(-j*w*t),t,-inf,inf)
Fw = simplify(Fw)
% Check the results
Fw2 = fourier(ft,t,w)
Fw =
A*(1/4*pi/(-w^2+1/16*pi^2)+1/4*pi/(-w^2+1/16*pi^2)*exp(-4*i*w))
Fw =
-4*A*(1+exp(-4*i*w))*pi/(16*w^2-pi^2)
Fw2 =
4*A*pi/(-4*w+pi)/(4*w+pi)*(1+exp(-4*i*w))
```

Answer:

$$F(\omega) = \frac{4\pi A\left(1 + e^{-4j\omega}\right)}{\pi^2 - 16\omega^2}$$

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Problem 13-39

Use the inversion integral to find the inverse transform of the following function:

$$F(\omega) = 5\pi[u(\omega+2) - u(\omega-2)]$$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
Fw = 5*pi*(heaviside(w+2)-heaviside(w-2));
ft = int(Fw*exp(j*w*t),w,-inf,inf)/2/pi
ft = simplify(ft)
% Check the results
ft2 = ifourier(Fw,w,t)
ft =
5/2*i/t*exp(-2*i*t)-5/2*i/t*exp(2*i*t)
ft =
5/t*sin(2*t)
ft2 =
5/t*sin(2*t)
```

Answer:

$$f(t) = \frac{5\sin(2t)}{t}$$

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Problem 13-40

Use MATLAB and the inversion integral to find the inverse transform of the following function:

$$F(\omega) = j\pi 8\left[u(\omega+2) - 2u(\omega) + u(\omega-2)\right]$$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
Fw = j*pi*8*(heaviside(w+2)-2*heaviside(w)+heaviside(w-2));
ft = int(Fw*exp(j*w*t),w,-inf,inf)/2/pi
ft = simplify(ft)
% Check the results
ft2 = ifourier(Fw,w,t)
ft =
4*i*(i/t*exp(-2*i*t)-2*i/t+i/t*exp(2*i*t))
ft =
-8*(cos(2*t)-1)/t
ft2 =
16/t*sin(t)^2
```

Answer:

$$f(t) = \frac{8\left[1-\cos(2t)\right]}{t} = \frac{16\sin^2(t)}{t}$$

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Problem 13-41

Use MATLAB and the inversion integral to find the inverse transform of the following function:

$$F(\omega) = \sin(\pi\,\omega/4)\left[u(\omega) - u(\omega - 4)\right]$$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
Fw = sin(pi*w/4)*(heaviside(w)-heaviside(w-4));
ft = int(Fw*exp(j*w*t),w,-inf,inf)/2/pi
ft = simplify(ft)
% Check the results
ft2 = ifourier(Fw,w,t)
ft =
(1/8*pi/(-t^2+1/16*pi^2)+1/8*pi/(-t^2+1/16*pi^2)*exp(4*i*t))/pi
ft =
2*(1+exp(4*i*t))/(-16*t^2+pi^2)
ft2 =
2*(1+exp(4*i*t))/(-4*t+pi)/(4*t+pi)
```

Answer:

$$f(t) = \frac{2(1+e^{j4t})}{\pi^2 - 16t^2}$$

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Problem 13-42

Find the inverse transforms of the following functions:

(a) $F_1(\omega) = \dfrac{400}{(j\omega + 20)(j\omega + 40)}$

(b) $F_2(\omega) = \dfrac{j\omega}{(j\omega + 20)(j\omega + 40)}$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
disp('Part (a)')
Fw = 400/(j*w+20)/(j*w+40);
ft = int(Fw*exp(j*w*t),w,-inf,inf)/2/pi
% Check the results
ft2 = ifourier(Fw,w,t)
disp('Part (b)')
Fw = j*w/(j*w+20)/(j*w+40);
ft = int(Fw*exp(j*w*t),w,-inf,inf)/2/pi
% Check the results
ft2 = ifourier(Fw,w,t)
Part (a)
ft =
10*signum(t)*(signum(t)*exp(20*t)-signum(t)+exp(20*t)-1)*exp(-40*t)
ft2 =
20*heaviside(t)*(exp(-20*t)-exp(-40*t))
Part (b)
ft =
-1/2*signum(t)*(signum(t)*exp(20*t)-2*signum(t)+exp(20*t)-2)*exp(-40*t)
ft2 =
heaviside(t)*(-exp(-20*t)+2*exp(-40*t))
```

Answers:

(a) $f(t) = 20\left(e^{-20t} - e^{-40t}\right)u(t)$

(b) $f(t) = \left(-e^{-20t} + 2e^{-40t}\right)u(t)$

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Problem 13-43

Use MATLAB to find the inverse transforms of the following functions:

(a) $F_1(\omega) = \dfrac{800}{j\omega(j\omega+10)(j\omega+20)}$

(b) $F_2(\omega) = \dfrac{-\omega^2}{(j\omega+10)(j\omega+20)}$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
disp('Part (a)')
Fw = 800/j/w/(j*w+10)/(j*w+20);
ft = ifourier(Fw,w,t)
disp('Part (b)')
Fw = -(w^2)/(j*w+10)/(j*w+20);
ft = ifourier(Fw,w,t)
Part (a)
ft =
16*heaviside(t)*sinh(5*t)^2*exp(-10*t)-2
Part (b)
ft =
dirac(t)+(10*exp(-10*t)-40*exp(-20*t))*heaviside(t)
```

Part (a) can also be solved efficiently using partial fraction expansion.

$$F_1(\omega) = \frac{800}{j\omega(j\omega+10)(j\omega+20)} = \frac{4}{j\omega} + \frac{-8}{j\omega+10} + \frac{4}{j\omega+20}$$

Then apply the basic Fourier transform pairs to find the result.

Part (b) can also be solved efficiently using partial fraction expansion.

$$F_2(\omega) = \frac{-\omega^2}{(j\omega+10)(j\omega+20)} = 1 + \frac{-30j\omega-200}{(j\omega+10)(j\omega+20)} = 1 + \frac{10}{j\omega+10} + \frac{-40}{j\omega+20}$$

Answers:

(a) $f_1(t) = 2\,\mathrm{sgn}(t) + 4\left(e^{-20t} - 2e^{-10t}\right)u(t)$

(b) $f_2(t) = \delta(t) + \left(10e^{-10t} - 40e^{-20t}\right)u(t)$

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Problem 13-44

Find the Fourier transforms of the following waveforms:

(a) $f_1(t) = 2u(-t) - 2$

(b) $f_2(t) = -2\,\mathrm{sgn}(t) - 2u(-t)$

(c) $f_3(t) = -\mathrm{sgn}(t) - 1$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
disp('Part (a)')
ft = 2*heaviside(-t)-2;
Fw = fourier(ft,t,w)
disp('Part (b)')
ft = -2*(2*heaviside(t)-1)-2*heaviside(-t);
Fw = fourier(ft,t,w)
disp('Part (c)')
ft = -1*(2*heaviside(t)-1)-1;
Fw = fourier(ft,t,w)
Part (a)
Fw =
-2*pi*dirac(w)+2*i/w
Part (b)
Fw =
-2*pi*dirac(w)+2*i/w
Part (c)
Fw =
-2*pi*dirac(w)+2*i/w
```

The three waveforms are identical, so they all have the same Fourier transform.

Answers:

(a) $F_1(\omega) = -2\left[\dfrac{1}{j\omega} + \pi\delta(\omega)\right]$

(b) $F_2(\omega) = -2\left[\dfrac{1}{j\omega} + \pi\delta(\omega)\right]$

(c) $F_3(\omega) = -2\left[\dfrac{1}{j\omega} + \pi\delta(\omega)\right]$

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Problem 13-45

Find the Fourier transforms of the following waveforms:

(a) $f_1(t) = 2\,e^{-2t}\,u(t) + 2\,\mathrm{sgn}(t)$

(b) $f_2(t) = 2\,e^{-2t}\,u(t) + 2u(t)$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
disp('Part (a)')
ft = 2*exp(-2*t)*heaviside(t)+2*(2*heaviside(t)-1);
Fw = fourier(ft,t,w)
disp('Part (b)')
ft = 2*exp(-2*t)*heaviside(t)+2*heaviside(t);
Fw = fourier(ft,t,w)
Part (a)
Fw =
2/(w*i+2)/w*(3*w-4*i)
Part (b)
Fw =
2*pi*dirac(w)+2*(2*w-2*i)/(w*i+2)/w
```

Answers:

(a) $F_1(\omega) = \dfrac{6\,j\omega + 8}{j\omega(\,j\omega + 2)}$;

(b) $F_2(\omega) = 2\pi\,\delta(\omega) + \dfrac{4(\,j\omega + 1)}{j\omega(\,j\omega + 2)}$

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Problem 13-46

Use MATLAB to find the Fourier transforms of the following waveforms:

(a) $f_1(t) = 3\sin(t) - \cos(t)$

(b) $f_2(t) = (2/t)\sin(t) - \cos(t)$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
disp('Part (a)')
ft = 3*sin(t)-cos(t);
Fw = fourier(ft,t,w)
disp('Part (b)')
ft = (2/t)*sin(t)-cos(t);
Fw = fourier(ft,t,w)
Part (a)
Fw =
pi*(-dirac(w-1)-3*i*dirac(w-1)-dirac(w+1)+3*i*dirac(w+1))
Part (b)
Fw =
pi*(-dirac(w-1)-dirac(w+1)+2*heaviside(w+1)-2*heaviside(w-1))
```

Answers:

(a) $F_1(\omega) = -\pi\left[(1+3j)\delta(\omega-1) + (1-3j)\delta(\omega+1)\right]$

(b) $F_2(\omega) = -\pi\left[\delta(\omega-1) + \delta(\omega+1) - 2u(\omega+1) + 2u(\omega-1)\right]$

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Problem 13-47

Find the Fourier transforms of the following waveforms:

(a) $f_1(t) = 3\cos[2\pi(t-5)]$

(b) $f_2(t) = 2e^{j2t}\,\mathrm{sgn}(t)$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
disp('Part (a)')
ft = 3*cos(2*pi*(t-5));
Fw = fourier(ft,t,w)
disp('Part (b)')
ft = 2*exp(j*2*t)*(2*heaviside(t)-1);
Fw = fourier(ft,t,w)
Part (a)
Fw =
3*pi*(dirac(w-2*pi)+dirac(w+2*pi))
Part (b)
Fw =
4*i/(-w+2)
```

Answers:

(a) $F_1(\omega) = 3\pi\left[\delta(\omega-2\pi)+\delta(\omega+2\pi)\right]$

(b) $F_2(\omega) = \dfrac{4}{j(\omega-2)}$

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Problem 13-48

Find the inverse transforms of the following functions:

(a) $F_1(\omega) = 4\pi\delta(\omega) + 4\pi\delta(\omega - 2) + 4\pi\delta(\omega + 2)$

(b) $F_2(\omega) = 4\pi\delta(\omega) - j2/\omega$

(c) $F_3(\omega) = 2\pi\delta(\omega) - j2/\omega$

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
disp('Part (a)')
Fw = 4*pi*(dirac(w)+dirac(w-2)+dirac(w+2));
ft = ifourier(Fw,w,t)
disp('Part (b)')
Fw = 4*pi*dirac(w)-j*2/w;
ft = ifourier(Fw,w,t)
disp('Part (c)')
Fw = 2*pi*dirac(w)-j*2/w;
ft = ifourier(Fw,w,t)
Part (a)
ft =
4*cos(2*t)+2
Part (b)
ft =
2*heaviside(t)+1
Part (c)
ft =
2*heaviside(t)
```

Answers:

(a) $f_1(t) = 2 + 4\cos(2t)$

(b) $f_2(t) = 1 + 2u(t)$

(c) $f_3(t) = 2u(t)$

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Problem 13-49

Use the duality property to find the inverse transforms of the following functions:

(a) $F_1(\omega) = 3\cos(10\omega)$

(b) $F_2(\omega) = 6u(\omega) - 3$

(c) $F_3(\omega) = 6e^{-|3\omega|}$

Solution:

The duality transform property is, given $f(t) \Leftrightarrow F(\omega)$, then $F(t) \Leftrightarrow 2\pi f(-\omega)$.

For each transform, set $2\pi f(-\omega)$ equal to the transformed function and then solve for $f(\omega)$. Substitute t for ω to find $f(t)$. Take the transform of $f(t)$ to get $F(\omega)$. Substitute t for ω to find $F(t)$.

(a)

$$2\pi f(-\omega) = 3\cos(10\omega)$$

$$f(\omega) = \frac{3}{2\pi}\cos(-10\omega) = \frac{3}{2\pi}\cos(10\omega)$$

$$f(t) = \frac{3}{2\pi}\cos(10t)$$

$$F(\omega) = \frac{3}{2}\left[\delta(\omega - 10) + \delta(\omega + 10)\right]$$

$$F(t) = \frac{3}{2}\left[\delta(t - 10) + \delta(t + 10)\right]$$

(b)

$$2\pi f(-\omega) = 6u(\omega) - 3$$

$$f(\omega) = \frac{3}{\pi}u(-\omega) - \frac{3}{2\pi}$$

$$f(t) = \frac{3}{\pi}u(-t) - \frac{3}{2\pi}$$

$$F(\omega) = \frac{-3}{j\omega\pi} + 3\delta(\omega) - 3\delta(\omega) = \frac{-3}{j\omega\pi}$$

$$F(t) = \frac{-3}{jt\pi} = \frac{3j}{t\pi}$$

(c)

$$2\pi f(-\omega) = 6e^{-|3\omega|}$$

$$f(\omega) = \frac{3}{\pi}e^{-|3\omega|}$$

$$f(t) = \frac{3}{\pi}e^{-|3t|}$$

$$F(\omega) = \frac{18}{\pi(\omega^2 + 9)}$$

$$F(t) = \frac{18}{\pi(t^2 + 9)}$$

Answers:

(a) $f_1(t) = 1.5\left[\delta(t - 10) + \delta(t + 10)\right]$

(b) $f_2(t) = \dfrac{3j}{\pi t}$

(c) $f_3(t) = \dfrac{18}{\pi\,(9 + t^2)}$

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Problem 13-50

Use the time-shifting property to find the inverse transforms of the following functions:

(a) $F_1(\omega) = [6\pi\delta(\omega) - j3/\omega]e^{-j5\omega}$

(b) $F_2(\omega) = 7e^{-j4\omega}/(j\omega + 3)$

(c) $F_3(\omega) = 5\cos(3\omega)/j\omega$

Solution:

The time-shifting property is, given $f(t) \Leftrightarrow F(\omega)$, then $f(t-T) \Leftrightarrow e^{-j\omega T}F(\omega)$.

(a)

$$F_1(\omega) = [6\pi\delta(\omega) - j3/\omega]e^{-j5\omega} = \left[6\pi\delta(\omega) + \frac{3}{j\omega}\right]e^{-j5\omega} = 3\left[\pi\delta(\omega) + \pi\delta(\omega) + \frac{1}{j\omega}\right]e^{-j5\omega}$$

$$f_1(t) = \frac{3}{2} + 3u(t-5)$$

(b) By inspection we have

$$f_2(t) = 7e^{-3(t-4)}u(t-4)$$

(c)

$$F_3(\omega) = 5\cos(3\omega)/j\omega = \frac{5}{2j\omega}\left[e^{j3\omega} + e^{-j3\omega}\right] = \frac{5}{4}\frac{2}{j\omega}\left[e^{j3\omega} + e^{-j3\omega}\right]$$

$$f_3(t) = \frac{5}{4}[\text{sgn}(t+3) + \text{sgn}(t-3)]$$

Check the results

```
clear all
syms t w
disp('Part (a)')
Fw = (6*pi*dirac(w)-j*3/w)*exp(-j*5*w);
ft = ifourier(Fw,w,t)
disp('Part (b)')
Fw = 7*exp(-j*4*w)/(j*w+3);
ft = ifourier(Fw,w,t)
disp('Part (c)')
Fw = 5*cos(3*w)/j/w;
ft = ifourier(Fw,w,t)
```

```
Part (a)
ft =
3*heaviside(t-5)+3/2
Part (b)
ft =
7*exp(-3*t+12)*heaviside(t-4)
Part (c)
ft =
```

Answers:

(a) $f_1(t) = \frac{3}{2} + 3u(t-5)$

(b) $f_2(t) = 7e^{-3(t-4)}u(t-4)$

(c) $f_3(t) = \frac{5}{4}[\text{sgn}(t+3) + \text{sgn}(t-3)]$

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```
5/2*heaviside(3+t)-5/2*heaviside(-t+3)
```

Problem 13-51

Given that the Fourier transform of $f(t)$ is

$$F(\omega)=\frac{400}{(j\omega+10)(j\omega+20)}$$

use the integration property to find the waveform

$$g(t)=\int_{-\infty}^{t} f(x)\,dx$$

Solution:

Applying the integration property, we have

$$G(\omega)=\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)=\frac{400}{j\omega(j\omega+10)(j\omega+20)}+2\pi\delta(\omega)$$

$$=\frac{2}{j\omega}+\frac{-4}{j\omega+10}+\frac{2}{j\omega+20}+2\pi\delta(\omega)$$

Taking the inverse transform, we have

$$g(t)=\mathrm{sgn}(t)-4e^{-10t}u(t)+2e^{-20t}u(t)+1=2u(t)-4e^{-10t}u(t)+2e^{-20t}u(t)$$

Check the results

```
clear all
syms t w x
Fw = 400/(j*w+10)/(j*w+20);
fx = ifourier(Fw,w,x);
gt = int(fx,x,-inf,t)
gt =
-4*heaviside(t)*exp(-10*t)+2*heaviside(t)+2*heaviside(t)*exp(-20*t)
```

Answer:

$$g(t)=2\left[1-2e^{-10t}+e^{-20t}\right]u(t)$$

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Problem 13-52

Use the reversal property to show that

$$\mathcal{F}\left\{Ae^{-\alpha|t|}\,\mathbf{sgn}(t)\right\} = \frac{-2\,Aj\omega}{\omega^2+\alpha^2}$$

Solution:

The reversal property is, given $f(t) \;\Leftrightarrow\; F(\omega)$, then $f(-t) \;\Leftrightarrow\; F(-\omega)$.

We can rewrite the original signal as follows:

$$f(t) = Ae^{-\alpha|t|}\,\mathrm{sgn}(t) = Ae^{-\alpha t}u(t) - Ae^{\alpha t}u(-t)$$

The Fourier transform is

$$F(\omega) = \frac{A}{\alpha + j\omega} - \frac{A}{\alpha - j\omega} = \frac{A(\alpha - j\omega) - A(\alpha + j\omega)}{\alpha^2+\omega^2} = \frac{-2Aj\omega}{\alpha^2+\omega^2}$$

Answer:

The transform is demonstrated above.

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Problem 13-53

Use the frequency shifting property to prove the modulation property

$$\mathfrak{F}\{f(t)\sin(\beta t)\} = \frac{F(\omega-\beta)}{2j} - \frac{F(\omega+\beta)}{2j}$$

Solution:

The frequency shifting property is, given $f(t) \Leftrightarrow F(\omega)$, then $e^{j\beta t} f(t) \Leftrightarrow F(\omega-\beta)$.

We can rewrite the time-domain signal as follows:

$$f(t)\sin(\beta t) = \frac{1}{2j} f(t)\left[e^{j\beta t} - e^{-j\beta t}\right]$$

We can then take the transform, as follows:

$$f(t)\sin(\beta t) \Leftrightarrow \frac{1}{2j}\left[F(\omega-\beta) - F(\omega+\beta)\right]$$

Answer:

The transform is demonstrated above.

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Problem 13-54

Use the frequency shifting property to show that

$$\mathfrak{F}\{\cos(\beta t)u(t)\} = \frac{j\omega}{\beta^2 - \omega^2} + \frac{\pi}{2}[\delta(\omega - \beta) + \delta(\omega + \beta)]$$

Solution:

The frequency shifting property is, given $f(t) \Leftrightarrow F(\omega)$, then $e^{j\beta t} f(t) \Leftrightarrow F(\omega - \beta)$.

We can rewrite the time-domain signal as follows:

$$\cos(\beta t)u(t) = \frac{1}{2}\left[e^{j\beta t} + e^{-j\beta t}\right]u(t)$$

We can then take the transform, as follows:

$$\begin{aligned}
\cos(\beta t)u(t) &\Leftrightarrow \frac{1}{2}\left[\frac{1}{j(\omega - \beta)} + \pi\delta(\omega - \beta) + \frac{1}{j(\omega + \beta)} + \pi\delta(\omega + \beta)\right] \\
&= \frac{1}{2}\left[\frac{j(\omega + \beta) + j(\omega - \beta)}{-(\omega^2 - \beta^2)} + \pi\delta(\omega - \beta) + \pi\delta(\omega + \beta)\right] \\
&= \frac{1}{2}\left[\frac{2j\omega}{\beta^2 - \omega^2} + \pi\delta(\omega - \beta) + \pi\delta(\omega + \beta)\right] \\
&= \frac{j\omega}{\beta^2 - \omega^2} + \frac{\pi}{2}[\delta(\omega - \beta) + \delta(\omega + \beta)]
\end{aligned}$$

Answer:

The transform is demonstrated above.

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Problem 13-55

The input in Figure P13-55 is $v_1(t) = 5\text{sgn}(t)$ V. Use Fourier transforms to find $v_2(t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w
% Find the transform of the input
v1t = 5*(2*heaviside(t)-1);
V1w = fourier(v1t,t,w);
% Find the transfer function
R = 1e3;
L = 1;
C = 1e-6;
ZL = j*w*L;
ZC = 1/j./w/C;
Tjw = R/(R+ZL+ZC);
% Compute the output signal transform and waveform.
V2w = Tjw*V1w;
v2t = ifourier(V2w,w,t)
v2t =
20/3*3^(1/2)*heaviside(t)*sin(500*t*3^(1/2))*exp(-500*t)
```

Answer:

$$v_2(t) = \frac{20}{\sqrt{3}} e^{-500\,t} \sin\left(500\sqrt{3}\,t\right) u(t)$$

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Problem 13-56

The input in Figure P13-56 is $v_1(t) = 10e^{4t}\ u(-t)$ V. Use Fourier transforms to find $v_2(t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w
% Find the transform of the input
v1t = 10*exp(4*t)*heaviside(-t);
V1w = fourier(v1t,t,w);
% Find the transfer function
R1 = 10e3;
R2 = 20e3;
C = 1e-6;
ZC = 1/j./w/C;
Z2 = 1/(1/ZC+1/R2);
Tjw = Z2/(R1+Z2);
% Compute the output signal transform and waveform.
V2w = Tjw*V1w;
v2t = ifourier(V2w,w,t)
v2t =
500/77*exp(-150*t)*heaviside(t)+500/77*exp(4*t)*heaviside(-t)
```

Answer:

$$v_2(t) = \frac{500}{77}\left[e^{-150t}u(t) + e^{4t}u(-t)\right]$$

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Problem 13-57

The input in Figure P13-56 is $v_1(t) = 10\,\mathrm{sgn}(t)$ V. Use Fourier transforms to find $v_2(t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w
% Find the transform of the input
v1t = 10*(2*heaviside(t)-1);
V1w = fourier(v1t,t,w);
% Find the transfer function
R1 = 10e3;
R2 = 20e3;
C = 1e-6;
ZC = 1/j/w/C;
Z2 = 1/(1/ZC+1/R2);
Tjw = Z2/(R1+Z2);
% Compute the output signal transform and waveform.
V2w = Tjw*V1w;
v2t = ifourier(V2w,w,t)
v2t =
-20/3+20/3*(-2*exp(-150*t)+2)*heaviside(t)
```

Answer:

$$v_2(t) = -\frac{20}{3} + \frac{40}{3}\left[1 - e^{-150t}\right]u(t) = \frac{20}{3}\mathrm{sgn}(t) - \frac{40}{3}e^{-150\,t}\ u(t)$$

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Problem 13-58

The input in Figure P13-58 is $v_1(t) = 8u(-t)$ V. Use Fourier transforms to find $v_2(t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w
% Find the transform of the input
v1t = 8*heaviside(-t);
V1w = fourier(v1t,t,w);
% Find the transfer function
R1 = 10e3;
R2 = 20e3;
C = 1e-6;
ZC = 1/j/w/C;
Z1 = R1+ZC;
Z2 = R2;
Tjw = -Z2/Z1;
% Compute the output signal transform and waveform.
V2w = Tjw*V1w;
v2t = ifourier(V2w,w,t)
v2t =
16*exp(-100*t)*heaviside(t)
```

Answer:

$v_2(t) = 16e^{-100t}\ u(t)$

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Problem 13-59

The input in Figure P13-58 is $v_1(t) = 10e^{-4|t|}$ V. Use MATLAB and Fourier transform techniques to find $v_2(t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w
% Find the transform of the input
v1t = 10*exp(-4*abs(t));
V1w = fourier(v1t,t,w);
% Find the transfer function
R1 = 10e3;
R2 = 20e3;
C = 1e-6;
ZC = 1/j/w/C;
Z1 = R1+ZC;
Z2 = R2;
Tjw = -Z2/Z1;
% Compute the output signal transform and waveform.
V2w = Tjw*V1w;
v2t = ifourier(V2w,w,t)
v2t =
5/78*(13*exp(-4*t)-25*exp(-100*t))*heaviside(t)-10/13*exp(4*t)*heaviside(-t)
```

Answer:

$$v_2(t) = \frac{5}{78}\left[13e^{-4t} - 25e^{-100t}\right]u(t) - \frac{10}{13}e^{4t}\,u(-t)$$

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Problem 13-60

The impulse response of a linear system is $h(t) = e^{-2t}u(t)$. Find the output for an input $x(t) = u(-t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w
% Find the Fourier transform of the impulse response
ht = exp(-2*t)*heaviside(t);
Hw = fourier(ht,t,w);
% Find the Fourier transform of the input signal
xt = heaviside(-t);
Xw = fourier(xt,t,w);
% Find the Fourier transform of the output signal
Yw = Hw*Xw;
% Find the output waveform
yt = ifourier(Yw,w,t)
yt =
1/2*exp(-2*t)*heaviside(t)+1/2*heaviside(-t)
```

Answer:

$$y(t) = \frac{1}{2}e^{-2t}u(t) + \frac{1}{2}u(-t)$$

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Problem 13-61

The impulse response of a linear system is $h(t) = e^{-2|t|}$. Find the output for an input $x(t) = u(t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w
% Find the Fourier transform of the impulse response
ht = exp(-2*abs(t));
Hw = fourier(ht,t,w);
% Find the Fourier transform of the input signal
xt = heaviside(t);
Xw = fourier(xt,t,w);
% Find the Fourier transform of the output signal
Yw = Hw*Xw;
% Find the output waveform
yt = ifourier(Yw,w,t)
yt =
-2*heaviside(t)*sinh(t)^2+1/2*exp(2*t)
```

Answer:

$$y(t) = \left[1 - \frac{1}{2}e^{-2t}\right]u(t) + \frac{1}{2}e^{2t}\left[1 - u(t)\right]$$

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Problem 13-62

The impulse response of a linear system is $h(t) = \delta(t) - 4e^{-2t}u(t)$. Use MATLAB and Fourier transform techniques to find the output for an input $x(t) = \text{sgn}(-t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w
% Find the Fourier transform of the impulse response
ht = dirac(t)-4*exp(-2*t)*heaviside(t);
Hw = fourier(ht,t,w);
% Find the Fourier transform of the input signal
xt = 2*heaviside(-t)-1;
Xw = fourier(xt,t,w);
% Find the Fourier transform of the output signal
Yw = Hw*Xw;
% Find the output waveform
yt = ifourier(Yw,w,t)
yt =
-1+(2-4*exp(-2*t))*heaviside(t)
```

Answer:

$$y(t) = -1 + \left[-4e^{-2t} + 2\right]u(t)$$

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Problem 13-63

The impulse response of a linear system is $h(t) = A[\delta(t) - \alpha e^{-\alpha t}u(t)]$, with $\alpha > 0$. Let $A = 5$ and $\alpha = 2$ and use MATLAB to plot $|H(\omega)|$. On the same axes, plot $|H(\omega)|$ for $A = 5$ and $\alpha = 4$. Describe the system frequency response and the influence of the parameter α.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
close all
syms t w
syms A a positive
ht = A*(dirac(t)-a*exp(-a*t)*heaviside(t));
Hw = fourier(ht,t,w);
% Solve for with the given values for A
and alpha
H1w = subs(Hw,{A,a},{5,2})
H2w = subs(Hw,{A,a},{5,4})
% Compute the responses for values of w
ww = -10:0.01:10;
H1ww = subs(H1w,w,ww);
H2ww = subs(H2w,w,ww);
MagH1ww = abs(H1ww);
MagH2ww = abs(H2ww);
% Plot the results
plot(ww,MagH1ww,'b','LineWidth',3)
hold on
grid on
plot(ww,MagH2ww,'g','LineWidth',3)
xlabel('Frequency')
ylabel('Amplitude')
legend('|H_1(\omega)|','|H_2(\omega)|','Location','SouthEast')
H1w =
5*i*w/(w*i+2)
H2w =
5*i*w/(w*i+4)
```

Answer:

The plot is shown above. The system frequency response is that of a high-pass filter. The parameter α influences the cutoff frequency of the filter. As α increases, the cutoff frequency increases.

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Problem 13-64

The impulse response of a linear system is $h(t) = A[\delta(t) - \sin(\beta t)/\pi t]$. Let A = 5 and β = 2 and use MATLAB to plot $|H(\omega)|$. On the same axes, plot $|H(\omega)|$ for A = 5 and β = 4. Describe the system frequency response and the influence of the parameter β.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
close all
syms t w
syms A B positive
ht = A*(dirac(t)-sin(B*t)/pi/t);
Hw = fourier(ht,t,w);
% Solve for with the given values for A and alpha
H1w = subs(Hw,{A,B},{5,2})
H2w = subs(Hw,{A,B},{5,4})
% Compute the responses for values of w
ww = -10:0.01:10;
H1ww = subs(H1w,w,ww);
H2ww = subs(H2w,w,ww);
MagH1ww = abs(H1ww);
MagH2ww = abs(H2ww);
% Plot the results
plot(ww,MagH1ww,'b','LineWidth',4)
hold on
grid on
plot(ww,MagH2ww,'g','LineWidth',2.5
)
xlabel('Frequency')
ylabel('Amplitude')
axis([-10 10 -1 6])
legend('|H_1(\omega)|','|H_2(\omega
)|','Location','SouthEast')
H1w =
5*heaviside(-w-2)+5*heaviside(w-2)
H2w =
5*heaviside(-w-4)+5*heaviside(w-4)
```

Answer:

The plot is shown above. The system frequency response is that of an ideal high-pass filter. The parameter β influences the cutoff frequency of the filter. As β increases, the cutoff frequency increases.

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Problem 13-65

The impulse response of a linear system is $h(t) = -Ae^{-\alpha t}u(t) + Ae^{-\alpha(-t)}u(-t)$, with α > 0. Let A = 5 and α = 2 and use MATLAB to plot |H(ω)|. On the same axes, plot |H(ω)| for A = 5 and α = 4. Describe the system frequency response and the influence of the parameter α.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
close all
syms t w
syms A a positive
ht = -A*exp(-a*t)*heaviside(t)+A*exp(a*t)*heaviside(-t);
Hw = fourier(ht,t,w);
% Solve for with the given values for A and alpha
H1w = subs(Hw,{A,a},{5,2})
H2w = subs(Hw,{A,a},{5,4})
% Compute the responses for values of
w
ww = -10:0.01:10;
H1ww = subs(H1w,w,ww);
H2ww = subs(H2w,w,ww);
MagH1ww = abs(H1ww);
MagH2ww = abs(H2ww);
% Plot the results
plot(ww,MagH1ww,'b','LineWidth',3)
hold on
grid on
plot(ww,MagH2ww,'g','LineWidth',3)
xlabel('Frequency')
ylabel('Amplitude')
legend('|H_1(\omega)|','|H_2(\omega)|'
,'Location','SouthEast')
H1w =
10*i*w/(w^2+4)
H2w =
10*i*w/(w^2+16)
```

Answer:

The plot is shown above. The system frequency response is that of a bandpass filter. The parameter α influences the center frequency of the pass band. As α increases, the center frequency increases. In addition, as the parameter increases, the magnitude of the pass band gain decreases.

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Problem 13-66

The frequency response of a linear system is shown in Figure P13-66. Find the system impulse response $h(t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w B
Hw = heaviside(w+2*B)+heaviside(w+B)-heaviside(w-B)-heaviside(w-2*B);
ht = ifourier(Hw,w,t)
ht =
2/t*cos(1/2*B*t)*sin(3/2*B*t)/pi
```

Answer:

$$h(t) = \frac{2}{\pi t}\cos(\beta t/2)\sin(3\beta t/2) = \frac{1}{\pi t}\left[\sin(\beta t) + \sin(2\beta t)\right]$$

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Problem 13-67

The frequency response of a linear system is shown in Figure P13-67. Find the system impulse response $h(t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w A B
Hw = A*(w+B)/B*heaviside(w+B)-2*A*w/B*heaviside(w)+A*(w-B)/B*heaviside(w-B);
ht = ifourier(Hw,w,t)
ht =
2*A/B/t^2*sin(1/2*B*t)^2/pi
```

Answer:

$$h(t) = \frac{2A}{\pi\beta t^2}\sin^2\left(\frac{\beta t}{2}\right)$$

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Problem 13-68

Find the 1-Ω energy carried by the signal $F(\omega) = 8/(\omega^2 + 4)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
Fw = 8/(w^2+4);
% Compute the integral in the frequency domain
W = int((abs(Fw))^2,w,-inf,inf)/2/pi
% Check the results with the time-domain integral
ft = ifourier(Fw,w,t);
Wt = int(ft^2,t,-inf,inf)
W =
2
Wt =
2
```

Answer:

$W_{1\Omega}$ = 2 J.

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Problem 13-69

Use MATLAB to compute the 1-Ω energy carried by the signal $f(t) = Ae^{\alpha t}u(-t)$, with α > 0.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
syms A a positive
ft = A*exp(a*t)*heaviside(-t);
% Compute the integral in the time domain
Wt = int(ft^2,t,-inf,inf)
% Compute the integral in the frequency domain
Fw = fourier(ft,t,w);
W = int((abs(Fw))^2,w,-inf,inf)/2/pi
Wt =
1/2*A^2/a
W =
1/2*A^2/a
```

Answer:

$$W_{1\Omega} = \frac{A^2}{2\alpha}$$

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Problem 13-70

Find the 1-Ω energy carried by the signal

$$F(\omega) = \frac{j\omega A}{\omega^2 + \alpha^2}$$

Then find the percentage of the 1-Ω energy carried in the frequency band $|\omega| \le \alpha$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t A a
Fw = j*w*A/(w^2+a^2);
% Compute the integral in the frequency domain
W = int((abs(Fw))^2,w,-inf,inf)/2/pi
% Check the results with the time-domain integral
ft = ifourier(Fw,w,t);
Wt = int(ft^2,t,-inf,inf)
% Find the energy in the requested frequency range
Wa = int((abs(Fw))^2,w,0,a)/pi
Wa_percent = double(Wa/W)
W =
1/4*A^2/a
Wt =
1/4*A^2/a
Wa =
1/8*A^2/a*(pi-2)/pi
Wa_percent =
   181.6901e-003
```

Answers:

$$W_{1\Omega} = \frac{A^2}{4\alpha}$$

The percentage of the 1-Ω energy carried in the frequency band $|\omega| \le \alpha$ is 18.17%.

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Problem 13-71

The transfer function of an ideal high-pass filter is $H(\omega) = 1$ for $|\omega| \geq 3$ krad/s. The filter input signal is $x(t) = 10\, e^{-3000t}\, u(t)$. Find the 1-Ω energy carried by the input signal and the percentage of the input energy that appears in the output.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
syms A a positive
ft = 10*exp(-3000*t)*heaviside(t);
% Compute the integral in the time domain
Wt = int(ft^2,t,-inf,inf)
% Compute the integral in the frequency domain
Fw = fourier(ft,t,w);
W = int((abs(Fw))^2,w,-inf,inf)/2/pi
% Calculate the energy in the output signal by
% integrating across the pass band
Wout = int((abs(Fw))^2,w,3000,inf)/pi
% Find the percentage of energy in the output
Wout_percent = double(Wout/W)
Wt =
1/60
W =
1/60
Wout =
1/120
Wout_percent =
   500.0000e-003
```

Answer:

For the input signal, $W_{1\Omega} = 1/60$ J. The output signal carries 50% of the 1-Ω energy carried by the input signal.

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Problem 13-72

The impulse response of a filter is $h(t) = 20\, e^{-40t}\, u(t)$. Find the 1-Ω energy in the output signal when the input is $x(t) = 5\, e^{-10t}\, u(t)$.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms t w
ht = 20*exp(-40*t)*heaviside(t);
xt = 5*exp(-10*t)*heaviside(t);
% Compute the output signal
Hw = fourier(ht,t,w);
Xw = fourier(xt,t,w);
Yw = Hw*Xw;
% Find the energy in the output signal
W = int((abs(Yw))^2,w,-inf,inf)/2/pi
W =
1/4
```

Answer:

$W_{1\Omega} = 1/4$ J.

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Problem 13-73

The current in a 200-Ω resistor is $i(t) = -4\,u(t+2) + 8\,u(t) - 4\,u(t-2)$. Find the total energy delivered to the resistor.

Solution:

Use Ohm's Law to find the voltage across the resistor. Then find the expression for the power dissipated by the resistor. Integrate the power over all time to find the energy delivered to the resistor. The solution is presented in the following MATLAB calculations.

```
clear all
format short eng
syms t
R = 200;
it = -4*heaviside(t+2)+8*heaviside(t)-4*heaviside(t-2);
vt = it*R;
pt = vt*it;
w_total = double(int(pt,t,-inf,inf))
w_total =
    12.8000e+003
```

Answer:

W_{TOTAL} = 12.8 kJ.

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Problem 13-74

The transfer function of an ideal bandpass filter is $H(\omega) = 1$ for $400 \le \omega \le 600$ rad/s. Use MATLAB to find the 1-Ω energy carried by the output signal when the input is $x(t) = 10\, e^{-500t}\, u(t)$. What percentage of the input signal energy appears in the output?

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
syms w t
syms A a positive
ft = 10*exp(-500*t)*heaviside(t);
% Find the energy of the input signal
% Compute the integral in the time domain
Wt = int(ft^2,t,-inf,inf)
% Compute the integral in the frequency domain
Fw = fourier(ft,t,w);
W = int((abs(Fw))^2,w,-inf,inf)/2/pi
% Calculate the energy in the output signal by
% integrating across the pass band
Wout = double(int((abs(Fw))^2,w,400,600)/pi)
% Find the percentage of energy in the output
Wout_percent = double(Wout/W)
Wt =
1/10
W =
1/10
Wout =
    12.8162e-003
Wout_percent =
   128.1625e-003
```

Answer:

For the output signal, $W_{1\Omega} = 12.82$ mJ. The output signal carries 12.82% of the 1-Ω energy carried by the input signal.

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Problem 13-75

Fourier Series from a Bode Plot **(A)**

The transfer function of a linear circuit has the straight-line gain and phase Bode plots in Figure P13-75. The first four terms in the Fourier series of a periodic input $v_1(t)$ to the circuit are

$$v_1(t) = 21\cos(10t) + 7\cos(30t) + 4.2\cos(50t) + 3\cos(70t) \text{ V}$$

Estimate the amplitudes and phase angles of the first four terms in the Fourier series of the steady-state output $v_2(t)$.

Solution:

Use the Bode plot to find the transfer function for the circuit. Substitute $s = j\omega$ to find the Fourier transform of the impulse response, $H(\omega)$. Use $H(\omega)$ to determine the amplitude and phase changes at the input frequencies. We can then construct the first four terms of the steady-state output.

The transfer function is a low-pass filter with a dc gain of 0 dB = 1. The slope of the transition region is −40 dB/decade, which indicates there is a pair of poles at ω = 20 rad/s and a pair of zeros at ω = 40 rad/s. The transfer function is:

$$T(s) = \frac{1}{4}\frac{(s+40)^2}{(s+20)^2}$$

Complete the other calculations in MATLAB.

```
clear all
syms s w t
Ts = 1/4*(s+40)^2/(s+20)^2;
Tjw = subs(Ts,s,j*w);
ww = [10 30 50 70];
Magv1 = [21 7 4.2 3];
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
PhaseTjww = 180*angle(Tjww)/pi;
% Compute the output magnitudes
Magv2 = Magv1.*MagTjww;
% The input phases are all zero, so the output phases are the
% phases of the transfer function
Phasev2 = PhaseTjww;
Results = [ww' Magv2' Phasev2']
Results =
    10.0000e+000    17.8500e+000   -25.0576e+000
    30.0000e+000     3.3654e+000   -38.8801e+000
    50.0000e+000     1.4845e+000   -33.7168e+000
    70.0000e+000   919.8113e-003   -27.5990e+000
```

Answer:

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$$v_2(t) = 17.85\cos(10t - 25.1°) + 3.37\cos(30t - 38.9°) + 1.48\cos(50t - 33.7°) + 0.92\cos(70t - 27.6°) \text{ V}$$

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Problem 13-76

Spectrum of a Periodic Impulse Train **(A)**

A periodic impulse train can be written as

$$x(t) = T_0 \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

Find the Fourier coefficients of $x(t)$. Plot the amplitude spectrum and comment on the frequencies contained in the impulse train.

Solution:

Use the Fourier integrals to compute the Fourier coefficients.

```
clear all
syms t n T0
ft = T0*dirac(t);
% Integrate from -T0/2 to +T0/2
a0 = int(ft,t,-T0/2,T0/2)/T0
an = 2*int(ft*cos(2*pi*n*t/T0),t,-T0/2,T0/2)/T0
bn = 2*int(ft*sin(2*pi*n*t/T0),t,-T0/2,T0/2)/T0
a0 =
1
an =
2
bn =
0
```

```
% Plot the amplitude spectrum
for T0 normalized to one
T0 = 1;
n = 1:7;
w = 2*pi*n/T0;
an = 2*ones(size(w));
% Include the dc component
w = [0 w];
an = [1 an];
close all
% Plot the spectrum
stem(w,an,...

'b','LineWidth',3,'MarkerSize',1
0,'MarkerFaceColor','b')
grid on
xlabel('Frequency, (rad/s)')
ylabel('Amplitude')
```

Answer:

The Fourier coefficients are $a_0 = 1$, $a_n = 2$, and $b_n = 0$, for all n. The impulse train contains equal amounts of energy at every nonzero frequency.

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Problem 13-77

Power Supply Filter Design **(D)**

The input to a power supply filter is a full-wave rectified sine wave with f_0 = 60 Hz. The filter is a first-order low pass with unity dc gain. Select the cutoff frequency of the filter so that all of the ac components in the filter output are all less than 1% of the dc component.

Solution:

Using the results in Figure 13-4, a full-wave rectified sine wave has a dc component and ac components for even values of n. The gain of a first-order low-pass filter decreases as frequency increases. Therefore, if we design the filter such that the first ac component is less than 1% of the dc component, then all of the ac components will meet the specification. With unity dc gain, the magnitude of the dc component at the output will be $2A/\pi$. The first ac component appears at $n = 2$ or $\omega = 2\pi n/T_0 = 240\pi$, with an amplitude of $4A/(3\pi)$. We need the low-pass filter to have a gain at $\omega = 240\pi$ such that $|T(j\omega)|\times 4A/(3\pi) < 2A/(100\pi)$. Therefore, $|T(j\omega)| < 3/200$. For a first-order low-pass filter we have:

$$T(j\omega) = \frac{\omega_C}{j\omega + \omega_C}$$

Use MATLAB to complete the calculations.

```
clear all
syms wc positive
n = 2;
f0 = 60;
T0 = 1/f0;
w = 2*pi*n/T0;
Tjw = wc/(j*w+wc);
K = 3/200;
K2 = K^2;
% Work with the magnitude squared of the gain
Tjw2 = Tjw*conj(Tjw);
% Solve for the maximum cutoff frequency
Eqn1 = Tjw2-K2;
wc = solve(Eqn1,wc)
% Choose the positive solution
wc_num = max(double(wc))
wc =
  720/39991*39991^(1/2)*pi
 -720/39991*39991^(1/2)*pi
wc_num =
    11.3110e+000
```

Answer:

Use a cutoff frequency such that ω_C < 11.311 rad/s.

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Problem 13-78

Spectrum Analyzer Calibration **(E)**

A certain spectrum analyzer measures the average power delivered to a calibrated resistor by the individual harmonics of periodic waveforms. The calibration of the analyzer has been checked by applying a 1-MHz square wave and the following results reported

f(MHz)	1	3	5	7	9	11
P(dBm)	12.1	2.56	−1.88	−4.80	−6.98	−8.73

The reported power in dBm is $P = 10\ \mathrm{Log}(P_n)$, where P_n is the average power delivered by the nth harmonic in mW. Is the spectrum analyzer correctly calibrated?

Solution:

Using the results of Figure 13-4, the harmonics of a square wave are proportional to $1/n$, and appear only for odd values of n. Assume that the reported results at an input frequency of 1 MHz are correct and determine the expected power levels for the harmonics. If the voltage decreases by $1/n$ for the harmonics, then the power decreases by $1/(n^2)$. The following analysis predicts how the power levels should change:

$$P = 10\log(P_n) = 10\log\left(\frac{P_1}{n^2}\right) = 10\log(P_1) - 10\log(n^2)$$

The power levels should decrease by $10\log(n^2)$ for each harmonic. The following MATLAB calculations predict the output power levels.

```
clear all
n = 1:2:11;
P1 = 12.1;
Pn2 = 10*log10(n.^2);
P_out = P1-Pn2'
P_out =
    12.1000e+000
     2.5576e+000
    -1.8794e+000
    -4.8020e+000
    -6.9849e+000
    -8.7279e+000
```

Answer:

The expected output power levels agree closely with the measured values. The spectrum analyzer is calibrated correctly.

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Problem 13-79

Fourier Series and Fourier Transforms. **(A)**

Given a rectangular pulse as shown in Figure 13-4, with amplitude A, width T, and period T_0, we can compute and plot the coefficients in the corresponding Fourier series. If we allow T_0 to increase to infinity, the waveform is a single pulse and the Fourier series approaches a scaled version of the Fourier transform. To see this graphically, use MATLAB to create the following series of plots. Let $A = 5$, $T = 1$, and $T_0 = 2$. Compute the Fourier series coefficients for $n = 0, 1, 2, \ldots 10\ T_0$. Create a stem plot of $(a_n \times T_0)$ on the vertical axis versus the (n / T_0) on the horizontal axis. Increment T_0 by one and repeat the stem plot. Create plots up until $T_0 = 20$ and comment on the behavior of the results. Now compute the Fourier transform of $f(t) = A[u(t+T/2) - u(t-T/2)]$. Evaluate $F(\omega)$ for $\omega = 0$ to 20π rad/s. On the same axes as your final stem plot, plot $2F(\omega)$ versus $\omega/2\pi$. Comment on the results.

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
A = 5;
T = 1;
T0 = 2;

% Use a for loop to plot the
amplitude spectra
for T0 = 2:20
    n = 1:10*T0;
    n0 = [0 n];
    a0 = A*T/T0;
    an =
2*A*sin(n*pi*T/T0)./n/pi;
    aAll = [a0 an];
    stem(n0/T0,aAll*T0)
    grid on
    if T0==2
        v = axis;
    end
    axis([-1 v(2)+1 v(3) 2*v(4)])
    pause(0.5)
end

% Calculate and plot the Fourier
transform
syms t w

ft = A*(heaviside(t+T/2)-heaviside(t-T/2));
Fw = fourier(ft,t,w);
ww = 0:0.01:10;
Fww = subs(Fw,w,2*pi*ww);
hold on
plot(ww,2*Fww,'g','LineWidth',2)
xlabel('Frequency')
ylabel('Amplitude')
```

Answer:

The resulting plot is shown above. Run the MATLAB code to visualize the progression of plots. The Fourier transform is limiting case of the Fourier series.

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Problem 13-80

Impulse Generator **(A)**

Theoretically, an impulse has an amplitude spectrum that is constant at all frequencies. In practice, a constant spectrum across an infinite bandwidth cannot be achieved, nor is it really necessary. What is required is an amplitude spectrum that is "essentially" constant over the frequency range of interest. Under this concept, an impulse generator is a signal source that produces a pulse waveform whose amplitude spectrum does not vary more than a prescribed amount over a specified frequency range.

Consider a rectangular pulse:

$$f(t) = A[u(t+T/2) - u(t-T/2)]$$

Find $F(\omega)$ and sketch its amplitude spectrum $|F(\omega)|$. Select the pulse duration T such that the amplitude spectrum does not change by more than 10% over a frequency range from 1 MHz to 10 MHz.

Solution:

Use MATLAB to calculate $F(\omega)$. For the initial plot, select a normalized value of one for the amplitude. Iteratively experiment with values of T until we get a reasonable plot in the frequency range from 1 MHz to 10 MHz.

```
clear all
close all
format short eng
syms A T positive
syms w t
ft = A*(heaviside(t+T/2)-heaviside(t-T/2));
Fw = fourier(ft,t,w)
% Substitute values of one for A and T to plot the amplitude spectrum
T1 = 1e-7;
T2 = 1e-8;
Fw1 = subs(Fw,{A,T},{1,T1});
Fw2 = subs(Fw,{A,T},{1,T2});
ff = logspace(1,7,1000);
ww = 2*pi*ff;
Fww1 = subs(Fw1,w,ww);
Fww2 = subs(Fw2,w,ww);
MagFww1 = abs(Fww1);
MagFww2 = abs(Fww2);
plot(ff,MagFww1,'b','LineWidth',3)
hold on
grid on
plot(ff,MagFww2,'g','LineWidth',3)
legend(['T = ',num2str(T1)],['T =
',num2str(T2)])
xlabel('Frequency, (Hz)')
ylabel('Amplitude')
Fw =
2*A/w*sin(1/2*T*w)
```

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The Fourier transform of the rectangular pulse is

$$F(\omega) = \frac{2A}{\omega}\sin\left(\frac{T\omega}{2}\right)$$

Based on the plots above, it appears that a pulse duration between 10 ns and 100 ns will meet the specification on maximum allowed change in the amplitude spectrum. We can find the approximate pulse duration by iteratively evaluating the following MATLAB calculations.

```
w1 = 2*pi*1e6;
w2 = 2*pi*10e6;
T0 = 2.516e-8
Mag1 = abs(subs(Fw,{w,T},{w1,T0}));
Mag2 = abs(subs(Fw,{w,T},{w2,T0}));
MagRatio = double(Mag2/Mag1)
T0 =
    25.1600e-009
MagRatio =
   900.0132e-003
```

Plot the amplitude spectrum for the value of the pulse duration that meets the specifications.

```
close all
Fw0 = subs(Fw,{A,T},{1,T0});
ff = logspace(1,8,1000);
ww = 2*pi*ff;
Fww0 = subs(Fw0,w,ww);
MagFww0 = abs(Fww0);
% Find the magnitude at the two
critical frequencies
Mag1 = abs(subs(Fw0,w,w1))
Mag2 = abs(subs(Fw0,w,w2))
MagRatio = Mag2/Mag1
plot(ff,MagFww0,'b','LineWidth',3)
grid on
xlabel('Frequency, (Hz)')
ylabel('Amplitude')
Mag1 =
    25.1338e-009
Mag2 =
    22.6208e-009
MagRatio =
   900.0132e-003
```

Answers:

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$$F(\omega) = \frac{2A}{\omega}\sin\left(\frac{T\omega}{2}\right)$$

The amplitude spectra are shown above.
Choose the pulse duration T less than 25.16 ns to meet the specifications.

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Problem 14-1

Interchanging the positions of the resistors and capacitors converts the low-pass filter in Figure 14-3(a) into the high-pass filter in Figure 14-9(a). This *CR-RC* interchange involves replacing R_k by $1'C_k s$ and $C_k s$ by $1'R_k$. Show that this interchange converts the low-pass transfer function in Eq. (14-3) into the high-pass function in Eq. (14-9).

Solution:

Start with Equation (14-3) and make the required substitutions. Simplify the results and verify that it matches Equation (14-9).

Figure 14-3(a)

$$T(s)=\frac{\mu}{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2-\mu R_1C_1)s+1}$$

Figure 14-9(a)

$$T(s)=\frac{\mu}{\frac{1}{C_1s}\frac{1}{C_2s}\frac{1}{R_1}\frac{1}{R_2}+\frac{1}{C_1s}\frac{1}{R_1}+\frac{1}{C_1s}\frac{1}{R_2}+\frac{1}{C_2s}\frac{1}{R_2}-\mu\frac{1}{C_1s}\frac{1}{R_1}+1}$$

$$T(s)=\frac{\mu R_1R_2C_1C_2s^2}{1+(R_2C_2+R_1C_2+R_1C_1-\mu R_2C_2)s+R_1R_2C_1C_2s^2}$$

The result matches Equation (14-9). We can also use MATLAB to efficiently perform the substitution as follows.

```
clear all
format short eng
syms u R1 R2 C1 C2 r1 r2 c1 c2 s
TLP = u/(R1*R2*C1*C2*s^2+(R1*C1+R1*C2+R2*C2-u*R1*C1)*s+1);
THP = simplify(subs(TLP,{R1,R2,C1,C2},{1/c1/s,1/c2/s,1/r1/s,1/r2/s}))
THP =
-u*c1*c2*r1*s^2*r2/(-1-s*c2*r2-s*c2*r1-s*c1*r1+s*u*c2*r2-c1*c2*r1*s^2*r2)
```

Answer:

The conversion is shown above.

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Problem 14-2

Show that the circuit in Figure 14-14 has the bandpass transfer function in Eq. (14-15).

Solution:

Label the node between R_1 and C_2 as node A. Write two node-voltage equations to solve for $V_2(s)$ in terms of $V_1(s)$. Find the transfer function to verify the results.

```
clear all
syms s R1 R2 C1 C2 VA V1 V2
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Eqn1 = (VA-V1)/R1 + VA/ZC2 + (VA-V2)/ZC1;
Eqn2 = -VA/ZC2 - V2/R2;
Soln = solve(Eqn1,Eqn2,VA,V2);
V2 = Soln.V2;
Ts = simplify(V2/V1)
Ts =
-1/(1+C2*s*R1+C1*s*R1+R1*R2*C1*C2*s^2)*C2*s*R2
```

The transfer function matches Equation (14-15).

Answer:

$$T(s) = \frac{-R_2 C_2 s}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2)s + 1}$$

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Problem 14-3

Show that the circuit in Figure 14-17 has the bandstop transfer function in Eq. (14-20).

Solution:

Solve the following node-voltage equations for V_2 in terms of V_1:

Node 1: $\dfrac{V_A - V_1}{R_1} + \dfrac{V_A - V_2}{1/C_1 s} + \dfrac{V_A - V_N}{1/C_2 s} = 0$;

Node 2: $\dfrac{V_N - V_A}{1/C_2 s} + \dfrac{V_N - V_2}{R_2} = 0$;

Node 3: $V_N = V_P = \dfrac{R_B V_1}{R_A + R_B}$

```
clear all
syms s R1 R2 C1 C2 RA RB V1 V2 VN VA K
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Eqn1 = (VA-V1)/R1 + (VA-V2)/ZC1 + (VA-VN)/ZC2;
Eqn2 = (VN-VA)/ZC2 + (VN-V2)/R2;
%Eqn3 = (VN-V1)/RA + VN/RB;
% Let K = RB/(RA+RB)
Eqn3 = VN-K*V1;
Soln = solve(Eqn1,Eqn2,Eqn3,VA,VN,V2);
V2 = Soln.V2;
Ts = simplify(V2/V1)
Ts =
(-
s*R2*C2+C1*s^2*R1*R2*C2*K+s*R2*C2*K+K+C2*s*R1*K+C1*s*R1*K)/(1+C2*s*R1+C1*s*R1+
R1*R2*C1*C2*s^2)
```

Factor out K,

$$K = \frac{R_B}{R_A + R_B}$$

to derive the final expression, which matches Equation (14-20).

Answer:

$$T(s) = \frac{R_B}{R_A + R_B}\left[\frac{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 - R_2 C_2 R_A / R_B)s + 1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2)s + 1}\right]$$

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Problem 14-4

Show that the active filter in Figure P14-4 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + R_2 C_2 s + 1}$$

Using $C_1 = C_2 = C$, develop a method of selecting values for C, R_1, and R_2. Then select values so that the filter has a cutoff frequency of 10 krad/s and a ζ of 0.5. Use MATLAB to plot the filter's Bode diagram. Determine the type of filter it is and its roll-off.

Solution:

Let V_A be the voltage at the positive input terminal of the first OP AMP. Use the ideal OP AMP assumptions to write node voltage equations to solve for the transfer function as follows:

```
clear all
syms s R1 R2 C1 C2 V1 V2 VA
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Eqn1 = (VA-V1)/R1 + (VA-V2)/ZC1;
Eqn2 = (V2-VA)/R2 + V2/ZC2;
Soln = solve(Eqn1,Eqn2,VA,V2);
V2 = Soln.V2;
Ts = simplify(V2/V1)
Ts =
1/(1+s*R2*C2+R1*R2*C1*C2*s^2)
```

The transfer function has the correct form.

For $C_1 = C_2 = C$, we have:

$$T(s) = \frac{1}{R_1 R_2 C^2 s^2 + R_2 C s + 1} = \frac{\dfrac{1}{R_1 R_2 C^2}}{s^2 + \dfrac{1}{R_1 C} s + \dfrac{1}{R_1 R_2 C^2}}$$

To solve for the component values, select a value for C and then solve for R_1 and R_2 using the following two equations, in order:

$$\frac{1}{R_1 C} = 2\zeta\omega_0 \quad \text{and} \quad \frac{1}{R_1 R_2 C^2} = \omega_0^2$$

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Use MATLAB to solve for the component values to meet the specifications

```
% Given specifications
w0 = 10e3;
w02 = w0^2;
z = 0.5;
% Select a value for C
C = 10e-9
% Solve the equations for the resistor values
Eqn1 = 1/R1/C-2*z*w0;
Eqn2 = 1/R1/R2/C^2-w02;
Soln = solve(Eqn1,Eqn2,R1,R2);
R1 = double(Soln.R1)
R2 = double(Soln.R2)
C =
    10.0000e-009
R1 =
    10.0000e+003
R2 =
    10.0000e+003
```

Plot the amplitude response.

```
% Substitute values
syms w
C1 = C;
C2 = C;
Ts = subs(Ts);
Tjw = simplify(subs(Ts,s,j*w));
ww = logspace(2,6,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWid
th',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude')
```

Answers:
The derivation of the transfer function is completed above using MATLAB. To solve for the component values, select a value for C and then solve for R_1 and R_2 using the following two equations, in order:

$$\frac{1}{R_1 C} = 2\zeta\omega_0 \quad \text{and} \quad \frac{1}{R_1 R_2 C^2} = \omega_0^2$$

To meet the specifications, we can use C = 10 nF and $R_1 = R_2$ = 10 kΩ.

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The Bode plot is shown above. The filter is a second-order low-pass filter with a roll-off of −40 dB/decade.

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Problem 14-5

The circuit in Figure 14-3(a) has a low-pass transfer function given in Eq. (14-3) and repeated below

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{\mu}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2 + R_2C_2 - \mu R_1C_1)s + 1}$$

In Section 14-2, we developed *equal element* and *unity gain* design methods for this circuit. This problem explores an *equal time constant* design method. Using $R_1C_1 = R_2C_2$ and $\mu = 2$, develop a method of selecting values for C_1, C_2, R_1, and R_2. Then select values so that the filter has a cutoff frequency of 20 krad/s and a ζ of 0.1. Use MATLAB to plot the filter's Bode diagram. Determine the location in rad/s and magnitude in dB of the peak in the frequency response.

Figure 14-3(a)

Solution:

Let $T = R_1C_1 = R_2C_2$ and $\mu = 2$. Substitute into the transfer function.

$$T(s) = \frac{2}{T^2s^2 + (T + R_1C_2 + T - 2T)s + 1} = \frac{\frac{2}{T^2}}{s^2 + \frac{R_1C_2}{T^2}s + \frac{1}{T^2}}$$ We now have the following equations:

$$\frac{1}{T^2} = \frac{1}{(R_1C_1)^2} = \omega_0^2 \quad \text{or} \quad \frac{1}{R_1C_1} = \omega_0$$

and $$\frac{R_1C_2}{T^2} = R_1C_2\omega_0^2 = 2\zeta\omega_0 \quad \text{or} \quad R_1C_2\omega_0 = 2\zeta$$

Select C_1 and then use the first equation to find R_1. Use the second equation to find C_2. Use the equality for the time constant to find R_2.

Use MATLAB to find component values that meet the specifications.

```
clear all
w0 = 20e3;
w02 = w0^2;
z = 0.1;
% Select a capacitor value for C1
C1= 25e-9
R1 = 1/C1/w0
C2 = 2*z/R1/w0
R2 = R1*C1/C2
C1 =
    25.0000e-009
R1 =
     2.0000e+003
```

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```
C2 =
     5.0000e-009
R2 =
    10.0000e+003
```

R_1 = 2 kΩ, R_2 = 10 kΩ, C_1 = 25 nF, and C_2 = 5 nF

Plot the magnitude response

```
syms w s
u = 2;
Ts =
2/(R1*R2*C1*C2*s^2+(R1*C1+R1*C2+R2*C2
-2*R1*C1)*s+1);
Tjw = simplify(subs(Ts,s,j*w));
ww = logspace(2,6,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth'
,3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude')
```

Numerically search for the frequency of the maximum response.

```
ww = 19.7e3:0.001:19.9e3;
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
[MaxT,nn]=max(MagTjww);
MaxTdB = 20*log10(MaxT)
Maxw = ww(nn)
MaxTdB =
    20.0436e+000
Maxw =
    19.7990e+003
```

Answers:
The important equations are

$$\frac{1}{R_1C_1} = \omega_0 \quad \text{and} \quad R_1C_2\omega_0 = 2\zeta \quad \text{and} \quad R_1C_1 = R_2C_2$$

Select C_1 and then use the first equation to find R_1. Use the second equation to find C_2. Use the third equation to find R_2.

The following component values will meet the specifications: R_1 = 2 kΩ, R_2 = 10 kΩ, C_1 = 25 nF, and C_2 = 5 nF.

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The Bode plot is shown above.

The peak frequency response occurs at ω = 19.8 krad/s and has a magnitude of 20.0436 dB.

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Problem 14-6

The active filter in Figure P14-6 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{-R_2/R_1}{R_2R_3C_1C_2s^2 + [R_3C_2 + R_2C_2(1+R_3/R_1)]s + 1}$$

Using $R_1 = R_2 = R_3 = R$, develop a method of selecting values for C_1, C_2, and R. Then select values so that the filter has a cutoff frequency of 100 krad/s and a ζ of 0.05. Use MATLAB to plot the filter's Bode diagram. Build and simulate your circuit in OrCAD and compare the results with MATLAB's. Determine the location in rad/s and magnitude in dB of the peak in the frequency response.

Solution:

Substitute $R_1 = R_2 = R_3 = R$ into the transfer function and simplify as follows.

$$T(s) = \frac{-1}{R^2C_1C_2s^2 + 3RC_2s + 1} = \frac{\dfrac{-1}{R^2C_1C_2}}{s^2 + \dfrac{3}{RC_1}s + \dfrac{1}{R^2C_1C_2}}$$

To solve for the component values, select a value for C_1 and then solve for R and C_2, in order, using the following two equations:

$$\frac{3}{RC_1} = 2\zeta\omega_0 \quad \text{and} \quad \frac{1}{R^2C_1C_2} = \omega_0^2$$

Use MATLAB to compute component values to meet the specifications.

```
clear all
close all
w0 = 100e3;
w02 = w0^2;
z = 0.05;
C1 = 10e-9
R = 3/2/z/w0/C1
C2 = 1/R^2/C1/w02
C1 =
    10.0000e-009
R =
    30.0000e+003
C2 =
    11.1111e-012
```

Component values of R = 30 kΩ, C_1 = 10 nF, and C_2 = 11.11 pF will meet the specifications. Plot the Bode diagram for these component values.

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```
syms w s
R1 = R;
R2 = R;
R3 = R;
Ts = -
R2/R1/(R2*R3*C1*C2*s^2+(R3*C2+R2*C2*
(1+R3/R1))*s+1);
Tjw = simplify(subs(Ts,s,j*w));
ww = logspace(3,7,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth
',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The OrCAD circuit and simulation results are shown below.

The OrCAD results match the MATLAB results if we account for the change in the units for frequency.

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Numerically search for the frequency of the maximum response.

```
ww = 99.5e3:0.001:100e3;
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
[MaxT,nn]=max(MagTjww);
MaxTdB = 20*log10(MaxT)
Maxw = ww(nn)
MaxTdB =
    20.0109e+000
Maxw =
    99.7497e+003
```

Answers:
To solve for the component values, select a value for C_1 and then solve for R and C_2, in order, using the following two equations:

$$\frac{1}{R_1 C} = 2\zeta\omega_0 \quad \text{and} \quad \frac{1}{R^2 C_1 C_2} = \omega_0^2$$

Component values of R = 30 kΩ, C_1 = 10 nF, and C_2 = 11.11 pF will meet the specifications.
The MATLAB Bode amplitude response is shown above.
The OrCAD circuit and simulation are shown above.
The peak frequency response occurs at ω = 99.7497 krad/s and has a magnitude of 20.01 dB.

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Problem 14-7

Show that the active filter in Figure P14-7 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_1 R_2 C_1 C_2 s^2}{R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + 1}$$

Using $C_1 = C_2 = C$, develop a method of selecting values for C, R_1, and R_2. Then select values so that the filter has a cutoff frequency of 10 krad/s and a ζ of 0.5. Use MATLAB to plot the filter's Bode diagram. Determine the type of filter it is and its roll-off. How does the circuit and this transfer function compare with that of Problem 14-4?

Solution:

Use node-voltage analysis to solve for the transfer function. Label the positive input terminal of the first OP AMP as node A.

```
clear all
syms s R1 R2 C1 C2 V1 V2 VA
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Eqn1 = (VA-V1)/ZC1+(VA-V2)/R1;
Eqn2 = (V2-VA)/ZC2+V2/R2;
Soln = solve(Eqn1,Eqn2,VA,V2);
V2 = Soln.V2;
Ts = simplify(V2/V1)
Ts =
s^2*R2*C2*C1*R1/(1+R1*R2*C1*C2*s^2+C1*s*R1)
```

The transfer function matches the one presented above.

With $C_1 = C_2 = C$, we can write the transfer function as follows:

$$T(s) = \frac{R_1 R_2 C^2 s^2}{R_1 R_2 C^2 s^2 + R_1 C s + 1} = \frac{s^2}{s^2 + \frac{1}{R_2 C} s + \frac{1}{R_1 R_2 C^2}}$$

To solve for the component values, select a value for C and then solve for R_2 and R_1 using the following two equations, in order:

$$\frac{1}{R_2 C} = 2\zeta\omega_0 \quad \text{and} \quad \frac{1}{R_1 R_2 C^2} = \omega_0^2$$

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```
% Compute component values to meet the specifications
w0 = 10e3;
w02 = w0^2;
z = 0.5;
C = 100e-9
R2 = 1/2/z/w0/C
R1 = 1/C^2/R2/w02
C =
   100.0000e-009
R2 =
     1.0000e+003
R1 =
     1.0000e+003
```

The following component values meet the specifications: R_1 = 1 kΩ, R_2 = 1 kΩ, and C = 100 nF.

Use MATLAB to plot the amplitude spectrum.

```
% Substitute values
syms w
C1 = C;
C2 = C;
Ts = subs(Ts);
Tjw = simplify(subs(Ts,s,j*w));
ww = logspace(2,6,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWi
dth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The circuit is a high-pass filter with a rolloff of 40 dB/decade.

Answers:
The transfer function matches the one presented above.
To solve for the component values, select a value for C and then solve for R_2 and R_1 using the following two equations, in order:

$$\frac{1}{R_2 C} = 2\zeta\omega_0 \quad \text{and} \quad \frac{1}{R_1 R_2 C^2} = \omega_0^2$$

The following component values meet the specifications: R_1 = 1 kΩ, R_2 = 1 kΩ, and C = 100 nF.
The MATLAB Bode plot is presented above.
The circuit is a high-pass filter with a rolloff of 40 dB/decade.

The circuit in Figure P14-7 is the same as the circuit in Figure P14-4, except that the resistors and capacitors are swapped. The swap causes the low-pass filter in Figure P14-4 to become a

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high-pass filter in Figure P14-7. The transfer functions are similar in their general structure, but the high-pass filter has s^2 in the numerator to achieve the correct gain response.

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Problem 14-8

The circuit in Figure 14-9(a) has a high-pass transfer function given in Eq. (14-9) and repeated below

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{\mu R_1 R_2 C_1 C_2 \; s^2}{R_1 R_2 C_1 C_2 s^2 + \left(R_2 C_2 + R_1 C_1 + R_1 C_2 - \mu R_2 C_2\right) s + 1}$$

In Section 14-2, we developed *equal element* and *unity gain* design methods for this circuit. This problem explores an *equal time constant* design method. Using $R_1C_1 = R_2C_2$ and $\mu = 2$, develop a method of selecting values for C_1, C_2, R_1, and R_2. Then select values so that the filter has a cutoff frequency of 50 krad/s and a ζ of 0.2. Use MATLAB to plot the filter's Bode diagram.

Figure 14-9(a)

Solution:

Let $T = R_1C_1 = R_2C_2$ and $\mu = 2$. Substitute into the transfer function.

$$T(s) = \frac{2T^2 s^2}{T^2 s^2 + (T + T + R_1C_2 - 2T)s + 1} = \frac{2s^2}{s^2 + \frac{R_1C_2}{T^2}s + \frac{1}{T^2}} \text{ W}$$

e now have the following equations:

$$\frac{1}{T^2} = \frac{1}{(R_1C_1)^2} = \omega_0^2 \quad \text{or} \quad \frac{1}{R_1C_1} = \omega_0$$

and

$$\frac{R_1C_2}{T^2} = R_1C_2\omega_0^2 = 2\zeta\omega_0 \quad \text{or} \quad R_1C_2\omega_0 = 2\zeta$$

Select C_1 and then use the first equation to find R_1. Use the second equation to find C_2. Use the equality for the time constant to find R_2.

Use MATLAB to find component values that meet the specifications.

```
clear all
w0 = 50e3;
w02 = w0^2;
z = 0.2;
% Select a capacitor value for C1
C1= 5e-9
R1 = 1/C1/w0
C2 = 2*z/R1/w0
R2 = R1*C1/C2
C1 =
     5.0000e-009
R1 =
```

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```
     4.0000e+003
C2 =
     2.0000e-009
R2 =
    10.0000e+003
```

The following component values will meet the specifications: R_1 = 4 kΩ, R_2 = 10 kΩ, C_1 = 5 nF, and C_2 = 2 nF.

Plot the amplitude response

```
syms w s
u = 2;
Ts =
2*R1*R2*C1*C2*s^2/(R1*R2*C1*C2*s^
2+(R2*C2+R1*C1+R1*C2-
2*R2*C2)*s+1);
Tjw = simplify(subs(Ts,s,j*w));
ww = logspace(3,7,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWi
dth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

Answers:
The important equations are

$$\frac{1}{R_1C_1} = \omega_0 \quad \text{and} \quad R_1C_2\omega_0 = 2\zeta \quad \text{and} \quad R_1C_1 = R_2C_2$$

Select C_1 and then use the first equation to find R_1. Use the second equation to find C_2. Use the third equation to find R_2.

The following component values will meet the specifications: R_1 = 2 kΩ, R_2 = 10 kΩ, C_1 = 25 nF, and C_2 = 5 nF.

The Bode plot is shown above.

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Problem 14-9

Show that the active filter in Figure P14-9 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{-R_1C_1s}{R_1R_2C_1C_2s^2+(R_1C_2 + R_2C_2)s+1}$$

Using $R_1 = R_2 = R$, develop a method of selecting values for C_1, C_2, and R. Then select values so that the filter has a ω_0 of 10 krad/s and a ζ of 0.05. Use MATLAB to plot the filter's Bode diagram. What type of filter is this?

Solution:

Label the node between C_1 and R_2 as node A. Use node-voltage analysis to find the transfer function.

```
clear all
syms s R1 R2 C1 C2 V1 V2 VA
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Eqn1 = (VA-V1)/ZC1+(VA-V2)/R1+VA/R2;
Eqn2 = -VA/R2 - V2/ZC2;
Soln = solve(Eqn1,Eqn2,VA,V2);
V2 = Soln.V2;
Ts = simplify(V2/V1)
Ts =
-C1*s*R1/(R1*R2*C1*C2*s^2+s*R2*C2+1+C2*s*R1)
```

The transfer function matches the one presented above.

With $R_1 = R_2 = R$, we can write the transfer function as follows:

$$T(s) = \frac{-RC_1s}{R^2C_1C_2s^2+2RC_2s+1} = \frac{-\frac{1}{RC_2}s}{s^2+\frac{2}{RC_1}s+\frac{1}{R^2C_1C_2}}$$

We now have the following equations:

$$\frac{2}{RC_1} = 2\zeta\omega_0 \quad \text{and} \quad \frac{1}{R^2C_1C_2} = \omega_0^2$$

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Select a value for C_1 and use the first equation to solve for R. Then use the second equation to solve for C_2.

Solve for component values to meet the specifications.

```
w0 = 10e3;
w02 = w0^2;
z = 0.05;
C1 = 2e-6
R = 2/2/z/w0/C1
C2 = 1/R^2/C1/w02
C1 =
     2.0000e-006
R =
     1.0000e+003
C2 =
     5.0000e-009

syms w s
R1 = R;
R2 = R;
Ts = subs(Ts);
Tjw = simplify(subs(Ts,s,j*w));
ww = logspace(2,6,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineW
idth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter is a bandpass filter.

Answers:
The transfer function matches the one presented above.

Use the following equations to determine the component values:

$$\frac{2}{RC_1} = 2\zeta\omega_0 \quad \text{and} \quad \frac{1}{R^2C_1C_2} = \omega_0^2$$

Select a value for C_1 and use the first equation to solve for R. Then use the second equation to solve for C_2.

The component values R = 1 kΩ, C_1 = 2 μF, and C_2 = 5 nF will meet the specifications.

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The MATLAB Bode plot is shown above.

The filter is a bandpass filter.

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Problem 14-10

The active filter in Figure P14-10 has a transfer function of the form

$$T(s)=\frac{V_2(s)}{V_1(s)}=\frac{-R_3C_2s}{R_1R_3C_1C_2s^2+(R_1C_1+R_1C_2)s+1+R_1/R_2}$$

Using $C_1 = C_2 = C$ and $R_1 = R_2 = R$, develop a method of selecting values for C and R. Then select values so that the filter has a ω_0 of 5 krad/s and a ζ of 0.3. Use MATLAB to plot the filter's Bode diagram. What type of filter is this?

Solution:

With $C_1 = C_2 = C$ and $R_1 = R_2 = R$, we can rewrite the transfer function as follows:

$$T(s)=\frac{-R_3Cs}{RR_3C^2s^2+2RCs+2}=\frac{-\frac{1}{RC}s}{s^2+\frac{2}{R_3C}s+\frac{2}{RR_3C^2}}$$

We now have the following equations:

$$\frac{2}{R_3C}=2\zeta\omega_0 \quad \text{and} \quad \frac{2}{RR_3C^2}=\omega_0^2$$

Select a value for C and use the first equation to solve for R_3. Then use the second equation to solve for R.

Solve for the component values to meet the specifications

```
clear all
w0 = 5e3;
w02 = w0^2;
z = 0.3;
C = 20e-9
R3 = 2/2/z/w0/C
R = 2/R3/C^2/w02
C =
    20.0000e-009
R3 =
    33.3333e+003
R =
     6.0000e+003
```

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The following component values will meet the specifications: R = 6 kΩ, R_3 = 33.33 kΩ, and C = 20 nF. Plot the Bode amplitude response.

```
syms w s
R1 = R;
R2 = R;
C1 = C;
C2 = C
Ts = -
R3*C2*s/(R1*R3*C1*C2*s^2+(R1*C1+R1*
C2)*s+1+R1/R2);
Tjw = simplify(subs(Ts,s,j*w));
ww = logspace(2,6,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidt
h',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
C2 =
    20.0000e-009
```

The filter is a bandpass filter.

Answers:
Use the following equations to find the component values:

$$\frac{2}{R_3C} = 2\zeta\omega_0 \quad \text{and} \quad \frac{2}{RR_3C^2} = \omega_0^2$$

Select a value for C and use the first equation to solve for R_3. Then use the second equation to solve for R.

The following component values will meet the specifications: R = 6 kΩ, R_3 = 33.33 kΩ, and C = 20 nF.

The MATLAB Bode plot is shown above and the filter is a bandpass filter.

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Problem 14-11

The task is to design a second-order low-pass Filter using the three approaches shown in HW Problems 14-4, 14-5, and 14-6. The filter specs are a cutoff frequency of 10 krad/s and a ζ of 0.5. Using OrCAD, build your three designs and select the best one based on how well each meets the specs, the number of parts, and the gain.

Solution:

For each design approach, use the results from each problem to find the component values and then construct and simulate the corresponding circuit.

First, using the solution to Problem 14-4, we have

```
clear all
% Given specifications
w0 = 10e3;
w02 = w0^2;
z = 0.5;
% Select a value for C
C = 10e-9
R1 = 1/C/2/z/w0
R2 = 1/R1/C^2/w02
C =
    10.0000e-009
R1 =
    10.0000e+003
R2 =
    10.0000e+003
```

The corresponding circuit is shown above.

For Problem 14-5, we have:

```
% Select a capacitor value for C1
C1= 10e-9
R1 = 1/C1/w0
C2 = 2*z/R1/w0
R2 = R1*C1/C2
C1 =
    10.0000e-009
R1 =
    10.0000e+003
C2 =
    10.0000e-009
R2 =
    10.0000e+003
```

The corresponding circuit is shown at right above.

For Problem 14-6, we have:

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```
C1 = 10e-9
R = 3/2/z/w0/C1
C2 = 1/R^2/C1/w02
C1 =
    10.0000e-009
R =
    30.0000e+003
C2 =
     1.1111e-009
```

The corresponding circuit is shown at right.

The simulation results for all three circuits are shown below. The plots from Problems 14-4 (green) and 14-6 (blue) are exactly aligned and the plot from Problem 14-5 (red) has double the gain of the other two.

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Answers:

The following table summarizes the results:

Item	Prob 14-4	Prob 14-5	Prob 14-6
Meets specs	Yes	Yes	Yes
Gain	1	2	1
OP AMPs	2	1	1
Resistors	2	4	3
Capacitors	2	2	2

If a larger gain is more important, then choose the circuit in Problem 14-5. If a lower part count is more important, then choose the circuit in Problem 14-6.

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Problem 14-12

Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	$*T(j\omega_0)*$	Constraints
14-12	Low pass	50000	*	5	dc gain of 10

Solution:

For a second-order low-pass filter, the gain at ω_0 is $|K|/2\zeta$. With $K = 10$ and the gain at ω_0 equal to 5, then $\zeta = 1$. We can then construct the transfer function.

$$T(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{(10)(2.5\times10^9)}{s^2 + 10^5 s + 2.5\times10^9}$$

```
clear all
close all
syms w s
K = 10;
w0 = 50000;
w02 = w0^2;
z = 1;
Ts = K*w02/(s^2+2*z*w0*s+w02);
Tjw = subs(Ts,s,j*w);
ww = logspace(3,6,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
loglog(ww,MagTjww,'b','LineWidth',
3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude')
MagTjdc = abs(subs(Tjw,w,0))
MagTjw0 = abs(subs(Tjw,w,w0))
MagTjdc =
    10.0000e+000
MagTjw0 =
     5.0000e+000
```

Answers:

$$T(s) = \frac{(10)(2.5\times10^9)}{s^2 + 10^5 s + 2.5\times10^9}$$

The Bode diagram is shown above and it meets the specifications.

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Problem 14-13

Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	$*T(j\omega_0)*$	Constraints
14-13	High pass	500	0.5	*	infinite frequency gain of 10

Solution:

For a second-order high-pass filter, the gain at ω_0 is $|K|/2\zeta$. With $K = 10$ and $\zeta = 0.5$, we expect the gain at ω_0 to equal 10. We can construct the transfer function as follows:

$$T(s) = \frac{Ks^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{10s^2}{s^2 + 500s + 250000}$$

```
clear all
close all
syms w s
K = 10;
w0 = 500;
w02 = w0^2;
z = 0.5;
Ts = K*s^2/(s^2+2*z*w0*s+w02);
Tjw = subs(Ts,s,j*w);
ww = logspace(1,5,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
loglog(ww,MagTjww,'b','LineWidth'
,3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude')
MagTjInf = abs(subs(Tjw,w,1e20))
MagTjw0 = abs(subs(Tjw,w,w0))
MagTjInf =
    10.0000e+000
MagTjw0 =
    10.0000e+000
```

Answers:

$$T(s) = \frac{10s^2}{s^2 + 500s + 250000}$$

The Bode diagram is shown above and it meets the specifications.

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Problem 14-14

Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	$*T(j\omega_0)*$	Constraints
14-14	Bandpass	10,000	*	5	B = 200 rad/s

Solution:

With ω_0 = 10 krad/s and B = 200 rad/s, we can calculate the damping ratio as $\zeta = B/2\omega_0 = 0.01$. For a second-order bandpass filter, the gain at ω_0 is $|K|/2\zeta$. With $\zeta = 0.01$ and the gain at ω_0 equal to 5, we can find $K = 0.1$. We can construct the transfer function as follows:

$$T(s) = \frac{K\omega_0 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{1000s}{s^2 + 200s + 10^8}$$

```
clear all
close all
syms w s
K = 0.1;
w0 = 10000;
w02 = w0^2;
z = 0.01;
Ts = K*w0*s/(s^2+2*z*w0*s+w02);
Tjw = subs(Ts,s,j*w);
ww = logspace(3,5,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
loglog(ww,MagTjww,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude')
MagTjw0 = abs(subs(Tjw,w,w0))
MagTjw0 =
     5.0000e+000
```

Plot the response so that we can verify the bandwidth.

```
ww = 9800:10200;
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
plot(ww,MagTjww,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude')
line([min(ww),max(ww)],[5/sqrt(2),5/
sqrt(2)],'Color','g','LineWidth',2)
```

Answers:

$$T(s) = \frac{1000s}{s^2 + 200s + 10^8}$$

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The Bode diagram is shown above and it meets the specifications.

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Problem 14-15

Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	$*T(j\omega_0)*$	Constraints
14-15	Bandpass	200,000	0.25	10	dc gain of zero

Solution:

The gain at ω_0 is $|K|/2\zeta$, so we can calculate $K = 5$. We can construct the transfer function as follows:

$$T(s) = \frac{K\omega_0 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{10^6 s}{s^2 + 10^5 s + 4\times 10^{10}}$$

```
clear all
close all
syms w s
K = 5;
w0 = 200000;
w02 = w0^2;
z = 0.25;
Ts = K*w0*s/(s^2+2*z*w0*s+w02);
Tjw = subs(Ts,s,j*w);
ww = logspace(4,6,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
loglog(ww,MagTjww,'b','LineWidth'
,3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude')
axis([min(ww) max(ww) 0.1 10])
MagTjdc = abs(subs(Tjw,w,0))
MagTjw0 = abs(subs(Tjw,w,w0))
MagTjdc =
     0.0000e+000
MagTjw0 =
    10.0000e+000
```

Answers:

$$T(s) = \frac{10^6\ s}{s^2 + 10^5\ s + 4\times 10^{10}}$$

The Bode diagram is shown above and it meets the specifications.

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Problem 14-16

Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	$*T(j\omega_0)*$	Constraints
14-16	Bandstop	1000	0.01	0	dc gain of 10

Solution:

For a bandstop filter, the dc gain is K. We can construct the transfer function as follows:

$$T(s) = \frac{K(s^2+\omega_0^2)}{s^2+2\zeta\omega_0 s+\omega_0^2} = \frac{10(s^2+10^6)}{s^2+20s+10^6}$$

```
clear all
close all
syms w s
K = 10;
w0 = 1000;
w02 = w0^2;
z = 0.01;
Ts =
K*(s^2+w02)/(s^2+2*z*w0*s+w02)
Tjw = subs(Ts,s,j*w);
ww = logspace(2,4,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
loglog(ww,MagTjww,'b','LineWidth',
3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude')
axis([min(ww) max(ww) 1 20])
MagTjdc = abs(subs(Tjw,w,0))
MagTjw0 = abs(subs(Tjw,w,w0))
Ts =
(10*s^2+10000000)/(s^2+20*s+1000000)
MagTjdc =
    10.0000e+000
MagTjw0 =
     0.0000e+000
```

Answers:

$$T(s) = \frac{10(s^2+10^6)}{s^2+20s+10^6}$$

The Bode diagram is shown above and it meets the specifications.

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Problem 14-17

Design second-order active filters that meet the following requirements. Simulate your designs in OrCAD to validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	Constraints
14-17 **(D)**	Low pass	20,000	0.5	Use 10-kΩ resistors

Solution:

Since all of the resistors are 10-kΩ, use the equal element design approach for a low-pass filter.

```
clear all
w0 = 20000;
z = 0.5;
R = 10e3
C = 1/w0/R
u = 3-2*z;
RB = R;
RA = (u-1)*RB
R =
    10.0000e+003
C =
     5.0000e-009
RA =
    10.0000e+003
```

The circuit is shown at right.

The OrCAD simulation results are shown below.

Answer:

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Problem 14-18

Design second-order active filters that meet the following requirements. Simulate your designs in OrCAD to validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	Constraints
14-18 **(D)**	Low pass	2500	0.8	dc gain of 20 dB

Solution:

With a dc gain of 20 dB, which is a multiplicative factor of 10, we will use the unity gain approach along with an amplifier with a gain of 10.

```
clear all
w0 = 2500;
z = 0.8;
C1 = 10e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
C1 =
    10.0000e-009
C2 =
     6.4000e-009
R =
    50.0000e+003
```

The circuit is shown at right.

The OrCAD simulation results are shown below.

Answer:

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Problem 14-19

Design second-order active filters that meet the following requirements. Simulate your designs in OrCAD to validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	Constraints
14-19 **(D)**	Low pass	100,000	0.25	Use 200-nF capacitors

Solution:

With equal valued capacitors, we must use the equal element design approach.

```
clear all
w0 = 100000;
z = 0.25;
C = 200e-9
R = 1/w0/C
u = 3-2*z;
RB = 10e3
RA = (u-1)*RB
C =
   200.0000e-009
R =
    50.0000e+000
RB =
    10.0000e+003
RA =
    15.0000e+003
```

The circuit is shown at right.

The OrCAD simulation results are shown below.

Answer:

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Problem 14-20

Design second-order active filters that meet the following requirements. Simulate your designs in OrCAD to validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	Constraints
14-20 **(D)**	High pass	2500	0.75	Use 200-nF capacitors

Solution:

Since the capacitors are equal, we will use the equal element design approach.

```
clear all
w0 = 2500;
z = 0.75;
C = 200e-9
R = 1/w0/C
u = 3-2*z;
RB = 10e3
RA = (u-1)*RB
C =
   200.0000e-009
R =
     2.0000e+003
RB =
    10.0000e+003
RA =
     5.0000e+003
```

The circuit is shown at right.

The OrCAD simulation results are shown below.

Answer:

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Problem 14-21

Design second-order active filters that meet the following requirements. Simulate your designs in OrCAD to validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	Constraints
14-21 **(D)**	High pass	25,000	0.25	High-frequency gain of 40 dB

Solution:

Since we have a gain specification, we will use the unity gain design approach and include an amplifier to provide the gain of 40 dB, which is a factor of 100.

```
clear all
w0 = 25000;
z = 0.25;
R2 = 100e3
R1 = R2*z^2
C = 1/w0/sqrt(R1*R2)
R2 =
   100.0000e+003
R1 =
     6.2500e+003
C =
     1.6000e-009
```

The circuit is shown below.

The OrCAD simulation results are shown below.

Answer:

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Problem 14-22

Design second-order active filters that meet the following requirements. Simulate your designs in OrCAD to validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	Constraints
14-22 **(D)**	Bandpass	5000	*	Center-frequency gain of 20 dB

Solution:

The center-frequency gain is 10. If we use the equal-capacitor method, we need to choose ζ such that $1/2\zeta^2 = 10$, which means $\zeta = 0.2236$. Now use the equal capacitor method to design the circuit.

```
clear all
w0 = 5000;
Kw0 = 10;
z = sqrt(1/Kw0/2)
R2 = 100e3
R1 = R2*z^2
C = 1/w0/sqrt(R1*R2)
z =
   223.6068e-003
R2 =
   100.0000e+003
R1 =
     5.0000e+003
C =
     8.9443e-009
```

The circuit is shown at right above.

The OrCAD simulation results are shown below.

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Answer:
The design and simulation results are presented above.
Problem 14-23
Design second-order active filters that meet the following requirements. Simulate your designs in OrCAD to validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	Constraints
14-23 **(D)**	Bandpass	3974	*	Bandwidth of 125.5 krad/s.

Solution:
With a bandwidth of 125.5 krad/s = $2\zeta\omega_0$, we can calculate ζ = 15.7901. Design the circuit using the equal-capacitor method.

```
clear all
w0 = 3974;
B = 125.5e3;
z = B/2/w0
R2 = 1e3
R1 = R2*z^2
C = 1/w0/sqrt(R1*R2)
z =
    15.7901e+000
R2 =
     1.0000e+003
R1 =
   249.3284e+003
C =
    15.9363e-009
```

The circuit is shown above. The second stage is included to return the overall gain to one.

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Problem 14-24

Design second-order active filters that meet the following requirements. Simulate your designs in OrCAD to validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	Constraints
14-24 **(D)**	Bandstop	377	0.01	1 kΩ < resistors < 10 MΩ

Solution:

Use the equal-capacitor method with R_2 = 10 MΩ.

```
clear all
w0 = 377;
w02 = w0^2;
z = 0.01;
R2 = 10e6
R1 = R2*z^2
C = 1/w0/sqrt(R1*R2)
RA = 2*R1
RB = R2
R2 =
     10.0000e+006
R1 =
      1.0000e+003
C =
     26.5252e-009
RA =
      2.0000e+003
RB =
     10.0000e+006
```

The circuit is shown at right.

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Problem 14-25

Design second-order active filters that meet the following requirements. Simulate your designs in OrCAD to validate the requirements.

Problem	Type	ω_0(rad/s)	ζ	Constraints
14-25 **(D)**	Bandstop	5000	0.2	Use 200-nF capacitors

Solution:

Use the equal-capacitor method with C = 200 nF.

```
clear all
w0 = 5000;
w02 = w0^2;
z = 0.2;
C = 200e-9
R2 = 1/z/C/w0
R1 = R2*z^2
RA = 2*R1
RB = R2
C =
   200.0000e-009
R2 =
     5.0000e+003
R1 =
   200.0000e+000
RA =
   400.0000e+000
RB =
     5.0000e+003
```

The circuit is shown below.

The OrCAD simulation results are shown below.

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Problem 14-26

Construct the lowest-order transfer functions that meet the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Problem	Pole Type	ω_C(rad/s)	T_{MAX}	ω_{MIN}(rad/s)	T_{MIN}
14-26	First-order Cascade	2000	40 dB	20,000	! 30 dB

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
% Set the filter specifications
TmaxdB = 40;
TmindB = -30;
wC = 2000;
wmin = 20000;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);

% Determine the filter order
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Set the initial filter order to one
n = 1;
% While the filter order is too small to satisfy the specifications,
% increment the filter order
while (Tratio > (1 + wratio^2*(2^(1/n) - 1))^(n/2))&&(n<200)
    n = n + 1;
end
% Display the filter order, cutoff frequency, and gain
FirstOrderCascadeOrder = n
FirstOrderAlpha = wC/sqrt(2^(1/n) - 1)
FirstOrderK = Tmax^(1/n)
FirstOrderCascadeOrder =
     7.0000e+000
FirstOrderAlpha =
     6.1991e+003
FirstOrderK =
     1.9307e+000
```

The filter is a seventh-order filter. The transfer function is

$$T(s) = \left[\frac{(1.9307)(6199.1)}{s + 6199.1}\right]^7$$

Find the magnitude of the gain at $\omega = \omega_C$ and ω_{MIN}.

```
syms s w
```

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```
n = FirstOrderCascadeOrder;
a = FirstOrderAlpha;
K = FirstOrderK;
Ts = (K*a/(s+a))^n;
Tjw = subs(Ts,s,j*w);
MagTjwcdB = 20*log10(abs(subs(Tjw,w,wC)))
MagTjwmindB = 20*log10(abs(subs(Tjw,w,wmin)))
MagTjwcdB =
    36.9897e+000
MagTjwmindB =
   -34.0072e+000
```

Plot the transfer function to verify it meets the specifications.

```
close all
ww = logspace(2,5,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter meets the specifications.

Answers:

$$T(s) = \left[\frac{(1.9307)(6199.1)}{s + 6199.1}\right]^7$$

$$|T(j\omega_C)| = 36.9897 \text{ dB}$$

$$|T(j\omega_{MIN})| = -34.0072 \text{ dB}$$

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Problem 14-27

Construct the lowest-order transfer functions that meet the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Problem	Pole Type	ω_C(rad/s)	T_{MAX}	ω_{MIN}(rad/s)	T_{MIN}
14-27	Butterworth	10,000	0 dB	20,000	! 30 dB

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
% Set the filter specifications
TmaxdB = 0;
TmindB = -30;
wC = 10000;
wmin = 20000;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Compute the Butterworth order
ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
ButterworthOrder =
     5.0000e+000
```

The filter requires a fifth-order Butterworth filter. The corresponding transfer function is given below.

```
syms s w
wc = wC;
K = Tmax;
q5 = (s+1)*(s^2+0.618*s+1)*(s^2+1.618*s+1);
q = subs(q5,s,s/wc);
Ts = simplify(K/q)
Ts =
1000000000000000000000/(s+10000)/(s^2+6180*s+100000000)/(s^2+16180*s+100000000)
```

$$T(s) = \frac{10^{20}}{(s+10000)(s^2+6180s+10^8)(s^2+16180s+10^8)}$$

Find the magnitude of the gain at $\omega = \omega_C$ and ω_{MIN}.

```
Tjw = subs(Ts,s,j*w);
MagTjwcdB = 20*log10(abs(subs(Tjw,w,wC)))
MagTjwmindB = 20*log10(abs(subs(Tjw,w,wmin)))
MagTjwcdB =
    -3.0096e+000
MagTjwmindB =
   -30.1071e+000
```

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Plot the transfer function to verify it meets the specifications.

```
close all
ww = logspace(2,5,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter meets the specifications.

Answers:

$$T(s) = \frac{10^{20}}{(s+10000)(s^2+6180s+10^8)(s^2+16180s+10^8)}$$

$$|T(j\omega_C)| = -3.0096 \text{ dB}$$

$$|T(j\omega_{MIN})| = -30.1071 \text{ dB}$$

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Problem 14-28

Construct the lowest-order transfer functions that meet the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Problem	Pole Type	ω_C(rad/s)	T_{MAX}	ω_{MIN}(rad/s)	T_{MIN}
14-28	Chebychev	25,000	20 dB	100,000	!30 dB

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
% Set the filter specifications
TmaxdB = 20;
TmindB = -30;
wC = 25000;
wc = wC;
wmin = 100000;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Compute the Chebychev order
ChebychevOrder = ceil(acosh(sqrt(Tratio^2-1))/acosh(wratio))
ChebychevOrder =
     4.0000e+000
```

The filter requires a fourth-order Chebychev filter. The corresponding transfer function is given below.

```
syms s w
wc = wC;
K = Tmax;
q4c = ((s/0.9502)^2+0.1789*(s/0.9502)+1)*((s/0.4425)^2+0.9276*(s/0.4425)+1);
q = subs(q4c,s,s/wc);
Ts = K/sqrt(2)/q;
Ts_num = vpa(Ts,6)
Ts_num2= vpa(factor(Ts_num),6)
Ts_num =
7.07105/(.177211e-8*s^2+.753105e-5*s+1.)/(.817134e-8*s^2+.838508e-4*s+1.)
Ts_num2 =
.488315e18/(s^2+10261.6*s+.122379e9)/(s^2+4249.76*s+.564299e9)
```

$$T(s) = \frac{0.488315\times10^{18}}{(s^2+10262s+0.122379\times10^9)(s^2+4250s+0.564299\times10^9)}$$

Find the magnitude of the gain at $\omega = \omega_C$ and ω_{MIN}.

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```
Tjw = subs(Ts,s,j*w);
MagTjwcdB = 20*log10(abs(subs(Tjw,w,wC)))
MagTjwmindB = 20*log10(abs(subs(Tjw,w,wmin)))
MagTjwcdB =
    16.9909e+000
MagTjwmindB =
   -45.6699e+000
```

Plot the transfer function to verify it meets the specifications.

```
close all
ww = logspace(3,5,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter meets the specifications.

Answers:

$$T(s) = \frac{0.488315\times10^{18}}{(s^2 + 10262s + 0.122379\times10^9)(s^2 + 4250s + 0.564299\times10^9)}$$

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$$\left|T(j\omega_{\mathrm{C}})\right| = 16.99 \text{ dB}$$

$$\left|T(j\omega_{\mathrm{MIN}})\right| = -45.67 \text{ dB}$$

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Problem 14-29

Construct the lowest-order transfer functions that meet the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Problem	Pole Type	ω_C(rad/s)	T_{MAX}	ω_{MIN}(rad/s)	T_{MIN}
14-29	Butterworth	200,000	10 dB	400,000	! 10 dB

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
% Set the filter specifications
TmaxdB = 10;
TmindB = -10;
wC = 200000;
wmin = 400000;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Compute the Butterworth order
ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
ButterworthOrder =
     4.0000e+000
```

The filter requires a fourth-order Butterworth filter. The corresponding transfer function is given below.

```
syms s w
wc = wC;
K = Tmax;
q4 = (s^2+0.7654*s+1)*(s^2+1.848*s+1);
q = subs(q4,s,s/wc);
Ts = simplify(K/q)
Ts =
160000000000000000000*10^(1/2)/(s^2+153080*s+40000000000)/(s^2+369600*s+40000
000000)
```

$$T(s) = \frac{\left(1.6\times10^{21}\right)\sqrt{10}}{(s^2+153080s+4\times10^{10})(s^2+369600s+4\times10^{10})}$$

Find the magnitude of the gain at $\omega = \omega_C$ and ω_{MIN}.

```
Tjw = subs(Ts,s,j*w);
MagTjwcdB = 20*log10(abs(subs(Tjw,w,wC)))
MagTjwmindB = 20*log10(abs(subs(Tjw,w,wmin)))
MagTjwcdB =
```

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```
     6.9882e+000
MagTjwmindB =
   -14.1001e+000
```

Plot the transfer function to verify it meets the specifications.

```
close all
ww = logspace(4,6,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter meets the specifications.

Answers:

$$T(s) = \frac{\left(1.6\times10^{21}\right)\sqrt{10}}{(s^2+153080s+4\times10^{10})(s^2+369600s+4\times10^{10})}$$

$$\left|T(j\omega_C)\right| = 6.9882 \text{ dB}$$

$$\left|T(j\omega_{MIN})\right| = -14.1001 \text{ dB}$$

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Problem 14-30

Construct the lowest-order transfer functions that meet the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Problem	Pole Type	ω_C(rad/s)	T_{MAX}	ω_{MIN}(rad/s)	T_{MIN}
14-30	Chebychev	2000	10 dB	3500	! 20 dB

Solution:

The solution is presented in the following MATLAB calculations.

```
clear all
% Set the filter specifications
TmaxdB = 10;
TmindB = -20;
wC = 2000;
wc = wC;
wmin = 3500;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Compute the Chebychev order
ChebychevOrder = ceil(acosh(sqrt(Tratio^2-1))/acosh(wratio))
ChebychevOrder =
     4.0000e+000
```

The filter requires a fourth-order Chebychev filter. The corresponding transfer function is given below.

```
syms s w
wc = wC;
K = Tmax;
q4c = ((s/0.9502)^2+0.1789*(s/0.9502)+1)*((s/0.4425)^2+0.9276*(s/0.4425)+1);
q = subs(q4c,s,s/wc);
Ts = K/sqrt(2)/q;
Ts_num = vpa(Ts,6)
Ts_num2= vpa(factor(Ts_num),6)
Ts_num =
2.23607/(.276892e-6*s^2+.941381e-4*s+1.)/(.127677e-5*s^2+.104814e-2*s+1.)
Ts_num2 =
.632503e13/(s^2+820.931*s+783226.)/(s^2+339.981*s+.361152e7)
```

$$T(s) = \frac{0.632503\times 10^{13}}{(s^2+821s+783226)(s^2+340s+0.361152\times 10^7)}$$

Find the magnitude of the gain at $\omega = \omega_C$ and ω_{MIN}.

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```
Tjw = subs(Ts,s,j*w);
MagTjwcdB = 20*log10(abs(subs(Tjw,w,wC)))
MagTjwmindB = 20*log10(abs(subs(Tjw,w,wmin)))
MagTjwcdB =
     6.9909e+000
MagTjwmindB =
   -24.2424e+000
```

Plot the transfer function to verify it meets the specifications.

```
close all
ww = logspace(2,4,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter meets the specifications.

Answers:

$$T(s) = \frac{0.632503\times10^{13}}{(s^2+821s+783226)(s^2+340s+0.361152\times10^7)}$$

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$$\left|T(j\omega_{\mathrm{C}})\right| = 6.99 \text{ dB}$$
$$\left|T(j\omega_{\mathrm{MIN}})\right| = -24.24 \text{ dB}$$

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Problem 14-31

Design an active low-pass filter to meet the specification in Problem 14-26. Use OrCAD to verify that your design meets the specifications.

Solution:

The filter requires seven identical stages, with each stage having a gain of 1.9307 and a cutoff frequency of 6199.1 rad/s. Use the following first-order, active low-pass filter as the template for one of the stages.

Choose a capacitor value and then calculate the resistor values as follows:

```
clear all
K = 1.9307;
wc = 6199.1;
C2 = 10e-9
R2 = 1/C2/wc
R1 = R2/K
C2 =
    10.0000e-009
R2 =
    16.1314e+003
R1 =
     8.3552e+003
```

Cascade seven of the stages together and simulate the circuit. The complete circuit is shown below.

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The amplitude response is shown below.

Answer:
The design is described above and the circuit meets the specifications.

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Problem 14-32
Design an active low-pass filter to meet the specification in Problem 14-27. Use OrCAD to verify that your design meets the specifications.

Solution:
The transfer function is shown below.

$$T(s) = \frac{10^{20}}{(s+10000)(s^2+6180s+10^8)(s^2+16180s+10^8)}$$

Rewrite the transfer function as follows to identify the three stages of the circuit.

$$T(s) = \left[\frac{10000}{s+10000}\right]\left[\frac{10^8}{s^2+6180s+10^8}\right]\left[\frac{10^8}{s^2+16180s+10^8}\right]$$

The first stage is a first-order low-pass filter with a gain of one and a cutoff frequency of 10 krad/s.

```
clear all
disp('First stage')
w0 = 10e3;
C = 10e-9
R = 1/C/w0
First stage
C =
    10.0000e-009
R =
    10.0000e+003
```

The second and third stages both have ω_0 = 10 krad/s and damping ratios as calculated below.

```
w0 = 10e3;
z1 = 6180/2/w0
z2 = 16180/2/w0
z1 =
   309.0000e-003
z2 =
   809.0000e-003
```

Since each stage has unity gain, use the unity-gain method for designing the second-order stages. Determine the component values as calculated below.

```
disp('Second stage')
z = z1;
C1 = 10e-9
C2 = C1*z^2
```

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```
R = 1/w0/sqrt(C1*C2)
disp('Third stage')
z = z2;
C1 = 10e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
Second stage
C1 =
    10.0000e-009
C2 =
   954.8100e-012
R =
    32.3625e+003
Third stage
C1 =
    10.0000e-009
C2 =
     6.5448e-009
R =
    12.3609e+003
```

Construct the circuit and verify it meets the specification.

The amplitude response is shown below.

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Answer:
The design is described above and the circuit meets the specifications.

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Problem 14-33
Design an active low-pass filter to meet the specification in Problem 14-28. Use OrCAD to verify that your design meets the specifications.

Solution:
The transfer function for the circuit is shown below.

$$T(s) = \frac{0.488315\times10^{18}}{(s^2 + 10262s + 0.122379\times10^9)(s^2 + 4250s + 0.564299\times10^9)}$$

We can rewrite the transfer function as follows:

$$T(s) = \left[\frac{.564299\times10^9}{s^2 + 4250s + 0.564299\times10^9}\right][7.071]\left[\frac{0.122379\times10^9}{s^2 + 10262s + 0.122379\times10^9}\right]$$

For each second-order stage, calculate ω_0, ζ, and the component values for the circuit. Use the unity gain design approach for the second-order stages and then add an amplifier with a gain of 7.071.

```
disp('First stage')
w02 = 0.564229e9;
w0 = sqrt(w02);
z = 4250/2/w0;
C1 = 100e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
disp('Second stage')
w02 = 0.122379e9;
w0 = sqrt(w02);
z = 10262/2/w0;
C1 = 100e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
First stage
C1 =
   100.0000e-009
C2 =
   800.3178e-012
R =
     4.7059e+003
Second stage
C1 =
   100.0000e-009
C2 =
    21.5128e-009
R =
     1.9489e+003
```

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Construct the circuit and verify it meets the specification.

The amplitude response is shown below.

Answer:
The design is described above and the circuit meets the specifications.

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Problem 14-34
Design an active low-pass filter to meet the specification in Problem 14-29. Use OrCAD to verify that your design meets the specifications.

Solution:
The transfer function for the circuit is shown below.

$$T(s) = \frac{\left(1.6\times10^{21}\right)\sqrt{10}}{(s^2+153080s+4\times10^{10})(s^2+369600s+4\times10^{10})}$$

We can rewrite the transfer function as follows:

$$T(s) = \left[\frac{4\times10^{10}}{s^2+153080s+4\times10^{10}}\right]\left[\frac{4\times10^{10}}{s^2+369600s+4\times10^{10}}\right]\left[\sqrt{10}\right]$$

The second-order stages both have ω_0 = 200 krad/s and damping ratios as calculated below.

```
clear all
w0 = 200e3;
z1 = 153080/2/w0
z2 = 369600/2/w0
z1 =
   382.7000e-003
z2 =
   924.0000e-003
```

Use the unity-gain method for designing the second-order stages and then add a gain stage at the end with a gain of 3.1623. Determine the component values as calculated below.

```
disp('First stage')
z = z1;
C1 = 1e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
disp('Second stage')
z = z2;
C1 = 1e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
First stage
C1 =
     1.0000e-009
C2 =
   146.4593e-012
R =
    13.0651e+003
```

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```
Second stage
C1 =
     1.0000e-009
C2 =
   853.7760e-012
R =
     5.4113e+003
```

Construct the circuit and verify it meets the specification.

The amplitude response is shown below.

Answer:
The design is described above and the circuit meets the specifications.

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Problem 14-35
Design an active low-pass filter to meet the specification in Problem 14-30. Use OrCAD to verify that your design meets the specifications.

Solution:
The transfer function for the circuit is shown below.

$$T(s) = \frac{0.632503\times 10^{13}}{(s^2 + 821s + 783226)(s^2 + 340s + 0.361152\times 10^7)}$$

We can rewrite the transfer function as follows:

$$T(s) = \left[\frac{783226}{s^2 + 821s + 783226}\right][2.236]\left[\frac{3611520}{s^2 + 340s + 3611520}\right]$$

For each second-order stage, calculate ω_0, ζ, and the component values for the circuit. Use the unity gain design approach for the second-order stages and then add an amplifier with a gain of 2.236.

```
disp('First stage')
w02 = 783226;
w0 = sqrt(w02);
z = 821/2/w0;
C1 = 100e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
disp('Second stage')
w02 = 3611520;
w0 = sqrt(w02);
z = 340/2/w0;
C1 = 100e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
First stage
C1 =
   100.0000e-009
C2 =
    21.5149e-009
R =
    24.3605e+003
Second stage
C1 =
   100.0000e-009
C2 =
   800.2171e-012
R =
    58.8235e+003
```

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Construct the circuit and verify it meets the specification.

The amplitude response is shown below.

Answer:
The design is described above and the circuit meets the specifications.

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Problem 14-36

A low-pass filter is needed to suppress the harmonics in a periodic waveform with f_0 = 1 kHz. The filter must have unity passband gain, less than !25 dB gain at the 3rd harmonic, and less than !40 dB gain at the 5th harmonic. Since power is at a premium, choose a filter approach that minimizes the number of OP AMPs. Design a filter that meets these requirements. Verify your design using OrCAD.

Solution:

Set the cutoff frequency at 1 kHz and then determine the minimum filter order to meet each of the other specifications. Then choose the minimum filter order that meets both of the requirements.

```
clear all
% Set the filter specifications for the third harmonic
disp('Third Harmonic')
TmaxdB = 0;
TmindB = -25;
wC = 2*pi*1000;
wmin = 2*pi*3000;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Compute the Butterworth order
ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
% Compute the Chebychev order
ChebychevOrder = ceil(acosh(sqrt(Tratio^2-1))/acosh(wratio))
% Set the filter specifications for the third harmonic
disp('Fifth Harmonic')
TmaxdB = 0;
TmindB = -40;
wC = 2*pi*1000;
wmin = 2*pi*5000;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Compute the Butterworth order
ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
% Compute the Chebychev order
ChebychevOrder = ceil(acosh(sqrt(Tratio^2-1))/acosh(wratio))
Third Harmonic
ButterworthOrder =
     3.0000e+000
ChebychevOrder =
     3.0000e+000
Fifth Harmonic
ButterworthOrder =
     3.0000e+000
```

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```
ChebychevOrder =
     3.0000e+000
```

Based on the results, we need a third-order filter and either a Butterworth or a Chebychev design will work. We will design a third-order Butterworth filter using the unity gain approach.

```
% Design the transfer function
K = Tmax;
wc = wC;
syms s w
q3 = (s+1)*(s^2+s+1);
q = subs(q3,s,s/wc);
Ts = simplify(K/q)
Ts =
8000000000/(s+2000*pi)*pi^3/(s^2+2000*s*pi+4000000*pi^2)
```

The transfer function is:

$$T(s) = \left[\frac{2000\pi}{s+2000\pi}\right]\left[\frac{(2000\pi)^2}{s^2+2000\pi s+(2000\pi)^2}\right]$$

The first stage is a first-order low-pass filter and the second stage is a second-order low-pass filter. Determine the component values.

```
disp('First Stage')
w0 = 2000*pi;
C = 100e-9
R = 1/C/w0
disp('Second Stage')
z = 2000*pi/2/w0;
C1 = 100e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
First Stage
C =
   100.0000e-009
R =
     1.5915e+003
Second Stage
C1 =
   100.0000e-009
C2 =
    25.0000e-009
R =
     3.1831e+003
```

Construct the circuit and verify it meets the specification.

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The amplitude response is shown below.

Answer:
The design is described above and the circuit meets the specifications.

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Problem 14-37

Design a low-pass filter with 6 dB passband gain, a cutoff frequency of 3.2 kHz, and a stopband gain of less than −20 dB at 9.6 kHz. The filter must not have an overshoot of more than 12%. Verify your design using OrCAD.

Solution:

Since we want an overshoot of less than 12%, we will use a Butterworth design. Determine the required filter order to meet the specifications and then design the component values.

```
clear all
TmaxdB = 6;
TmindB = -20;
wC = 2*pi*3200;
wmin = 2*pi*9600;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Compute the Butterworth order
ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
ButterworthOrder =
     3.0000e+000
```

We will need a third-order filter.

```
% Design the transfer function
K = Tmax;
wc = wC;
syms s w
q3 = (s+1)*(s^2+s+1);
q = subs(q3,s,s/wc);
Ts = simplify(K/q)
Ts =
140404103409378734375/268435456/(s+6400*pi)*pi^3/(s^2+6400*s*pi+40960000*pi^2)
```

The transfer function is:

$$T(s) = \left[\frac{(2)6400\pi}{s+6400\pi}\right]\left[\frac{(6400\pi)^2}{s^2+6400\pi s+(6400\pi)^2}\right]$$

The first stage is a first-order low-pass filter with a gain of two and the second stage is a second-order low-pass filter. Determine the component values.

```
disp('First Stage')
w0 = 6400*pi;
C = 10e-9
R = 1/C/w0
disp('Second Stage')
z = 6400*pi/2/w0;
C1 = 10e-9
```

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```
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
First Stage
C =
    10.0000e-009
R =
     4.9736e+003
Second Stage
C1 =
    10.0000e-009
C2 =
     2.5000e-009
R =
     9.9472e+003
```

Construct the circuit above and verify it meets the specification.

The amplitude response is shown below.

Also examine the step response to verify the overshoot requirement.

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Answer:
The design is described above and the circuit meets the specifications.

Problem 14-38
Design a low-pass filter with 0 dB passband gain, a cutoff frequency of 1 kHz, and a stopband gain of less than −20 dB at 4 kHz. The filter must not have an overshoot. Verify your design using OrCAD.

Solution:
Since the filter must not have an overshoot, we must use a first-order cascade design. Calculate the filter order and then design the filter components.

```
clear all
% Set the filter specifications
TmaxdB = 0;
TmindB = -20;
wC = 2*pi*1000;
wmin = 2*pi*4000;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);

% Determine the filter order
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Set the initial filter order to one
n = 1;
% While the filter order is too small to satisfy the specifications,
% increment the filter order
while (Tratio > (1 + wratio^2*(2^(1/n) - 1))^(n/2))&&(n<200)
    n = n + 1;
end
% Display the filter order, cutoff frequency, and gain
```

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```
FirstOrderCascadeOrder = n
FirstOrderAlpha = wC/sqrt(2^(1/n) - 1)
FirstOrderK = Tmax^(1/n)
FirstOrderCascadeOrder =
     3.0000e+000
FirstOrderAlpha =
    12.3242e+003
FirstOrderK =
     1.0000e+000

K = FirstOrderK;
wc = FirstOrderAlpha;
C2 = 10e-9
R2 = 1/C2/wc
R1 = R2/K
C2 =
    10.0000e-009
R2 =
     8.1141e+003
R1 =
     8.1141e+003
```

Construct the circuit and verify it meets the specification.

The amplitude response is shown below.

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Also examine the step response to verify the overshoot requirement.

Answer:
The design is described above and the circuit meets the specifications.

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Problem 14-39

Design a low-pass filter with 0 dB passband gain, a cutoff frequency of 1 kHz, and a stopband gain of less than −20 dB at 2 kHz. Overshoot is not a problem, but a low filter order, least number of parts, and a maximum of two OP AMPs is desired. Verify your design using OrCAD.

Solution:

Calculate the filter order using both Butterworth and Chebychev approaches to determine which has the lowest order.

```
clear all
TmaxdB = 0;
TmindB = -20;
wC = 2*pi*1000;
wmin = 2*pi*2000;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Compute the Butterworth order
ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
% Compute the Chebychev order
ChebychevOrder = ceil(acosh(sqrt(Tratio^2-1))/acosh(wratio))
ButterworthOrder =
     4.0000e+000
ChebychevOrder =
     3.0000e+000
```

The Chebychev approach has a lower order and will require only two OP AMPs, so it will yield a valid design. Determine the transfer function.

```
syms s w
wc = wC;
K = Tmax;
q3c = (s/0.298+1)*((s/0.9159)^2+0.3254*(s/0.9159)+1);
q = subs(q3c,s,s/wc);
Ts = K/q;
Ts_num = vpa(Ts,6)
Ts_num2= vpa(factor(Ts_num),6)
Ts_num =
1/(.534076e-3*s+1.)/(.301956e-7*s^2+.565443e-4*s+1.)
Ts_num2 =
.620088e11/(s+1872.39)/(s^2+1872.60*s+.331174e8)
```

$$T(s)=\frac{0.620088\times10^{11}}{(s+1872)(s^2+1873s+0.331174\times10^8)}=\left[\frac{1872}{s+1872}\right]\left[\frac{33117400}{s^2+1873s+33117400}\right]$$

Find the component values.

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```
disp('First Stage')
w0 = 1872;
C = 10e-9
R = 1/C/w0
disp('Second Stage')
w02 = 33117400;
w0 = sqrt(w02);
z = 1873/2/w0;
C1 = 10e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
First Stage
C =
    10.0000e-009
R =
    53.4188e+003
Second Stage
C1 =
    10.0000e-009
C2 =
   264.8252e-012
R =
   106.7806e+003
```

Construct the above circuit and verify it meets the specification.

The amplitude response is shown below.

Answer:

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The design is described above and the circuit meets the specifications.

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Problem 14-40

A pesky signal at 200 kHz is interfering with a desired signal at 25 kHz. A careful analysis suggests that reducing the interfering signal by 30 dB will eliminate the problem, provided the desired signal is not reduced by more than 3 dB. Design an active *RC* filter that meets these requirements. Verify your design using OrCAD.

Solution:

Use a low-pass Butterworth filter design with a cutoff frequency of 25 kHz. Determine the required filter order and the component values.

```
clear all
TmaxdB = 0;
TmindB = -30;
wC = 2*pi*25000;
wmin = 2*pi*200000;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wmin/wC;
% Compute the Butterworth order
ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
ButterworthOrder =
     2.0000e+000
```

We will need a second-order filter.

```
% Design the transfer function
K = Tmax;
wc = wC;
syms s w
q2 = s^2+sqrt(2)*s+1;
q = subs(q2,s,s/wc);
Ts = simplify(K/q)
Ts =
2500000000*pi^2/(s^2+50000*2^(1/2)*s*pi+2500000000*pi^2)
```

The transfer function is:

$$T(s) = \left[\frac{(50000\pi)^2}{s^2 + 50000\pi\sqrt{2}s + (50000\pi)^2} \right]$$

Determine the component values.

```
w0 = 50000*pi;
z = 50000*pi*sqrt(2)/2/w0;
C1 = 1e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
```

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```
C1 =
     1.0000e-009
C2 =
   500.0000e-012
R =
     9.0032e+003
```

Construct the circuit and verify it meets the specification.

The amplitude response is shown below.

Answer:
The design is described above and the circuit meets the specifications.

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Problem 14-41

Construct the lowest order, high-pass transfer functions that meet the following filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Problem	Pole Type	ω_C(rad/s)	T_{MAX}	ω_{MIN}(rad/s)	T_{MIN}
14-41	First-order cascade	2000	40 dB	200	0 dB

Solution:

Determine the filter order and then construct the transfer function.

```
% Set the filter specifications
TmaxdB = 40;
TmindB = 0;
wC = 2000;
wmin = 200;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);

% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wC/wmin;
% Set the initial filter order to one
n = 1;
% While the filter order is too small to satisfy the specifications,
% increment the filter order
while Tratio > (1 + wratio^2*(2^(1/n) - 1))^(n/2)
    n = n + 1;
end
% Display the filter order
FirstOrderCascadeOrder = n
FirstOrderAlpha = wC*sqrt(2^(1/n) - 1)
FirstOrderK = Tmax^(1/n)
FirstOrderCascadeOrder =
     3.0000e+000
FirstOrderAlpha =
     1.0196e+003
FirstOrderK =
     4.6416e+000
```

The filter is a third-order filter. The transfer function is

$$T(s) = \left[\frac{4.6416s}{s+1020}\right]^3$$

Find the magnitude of the gain at $\omega = \omega_C$ and ω_{MIN}.

```
syms s w
n = FirstOrderCascadeOrder;
a = FirstOrderAlpha;
```

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```
K = FirstOrderK;
Ts = (K*s/(s+a))^n;
Tjw = subs(Ts,s,j*w);
MagTjwcdB = 20*log10(abs(subs(Tjw,w,wC)))
MagTjwmindB = 20*log10(abs(subs(Tjw,w,wmin)))
MagTjwcdB =
    36.9897e+000
MagTjwmindB =
    -2.9371e+000
```

Plot the transfer function to verify it meets the specifications.

```
close all
ww = logspace(2,4,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter meets the specifications.

Answers:

$$T(s) = \frac{100s^3}{(s+1020)^3}$$

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$$\left|T(j\omega_{\mathrm{C}})\right| = 36.9879 \text{ dB}$$
$$\left|T(j\omega_{\mathrm{MIN}})\right| = -2.94 \text{ dB}$$

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Problem 14-42

Construct the lowest order, high-pass transfer functions that meet the following filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Problem	Pole Type	ω_C(rad/s)	T_{MAX}	ω_{MIN}(rad/s)	T_{MIN}
14-42	Butterworth	2000	20 dB	200	! 30 dB

Solution:

Determine the filter order and then construct the transfer function.

```
% Set the filter specifications
TmaxdB = 20;
TmindB = -30;
wC = 2000;
wmin = 200;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);

% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wC/wmin;
% Compute the Butterworth order
ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
ButterworthOrder =
     3.0000e+000
```

We need a third-order filter.

```
K = Tmax;
syms s w
wc = wC;
% Butterworth filter design
q3 = (s+1)*(s^2+s+1);
q = subs(q3,s,wc/s);
Ts = simplify(K/q)
Ts =
10*s^3/(2000+s)/(4000000+2000*s+s^2)
```

The transfer function is:

$$T(s) = \left[\frac{10s}{s+2000}\right]\left[\frac{s^2}{s^2+2000s+(2000)^2}\right]$$

Find the magnitude of the gain at $\omega = \omega_C$ and ω_{MIN}.

```
Tjw = subs(Ts,s,j*w);
MagTjwcdB = 20*log10(abs(subs(Tjw,w,wC)))
MagTjwmindB = 20*log10(abs(subs(Tjw,w,wmin)))
```

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```
MagTjwcdB =
    16.9897e+000
MagTjwmindB =
   -40.0000e+000
```

Plot the transfer function to verify it meets the specifications.

```
close all
ww = logspace(2,4,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter meets the specifications.

Answers:

$$T(s) = \left[\frac{10s}{s+2000}\right]\left[\frac{s^2}{s^2+2000s+(2000)^2}\right]$$

$$|T(j\omega_C)| = 16.9879 \text{ dB}$$

$$|T(j\omega_{MIN})| = -40 \text{ dB}$$

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Problem 14-43

Construct the lowest order, high-pass transfer functions that meet the following filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Problem	Pole Type	ω_C(rad/s)	T_{MAX}	ω_{MIN}(rad/s)	T_{MIN}
14-43	Chebychev	2000	0 dB	500	! 25 dB

Solution:

Determine the filter order and then construct the transfer function.

```
clear all
% Set the filter specifications
TmaxdB = 0;
TmindB = -25;
wC = 2000;
wmin = 500;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);

% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wC/wmin;
% Compute the Chebychev order
ChebychevOrder = ceil(acosh(sqrt(Tratio^2-1))/acosh(wratio))
ChebychevOrder =
     2.0000e+000
```

We will need a second-order Chebychev filter. Create the transfer function.

```
K = Tmax;
wc = wC;
syms s w
q2c = ((s/0.8409)^2+0.7654*(s/0.8409)+1);
q = subs(q2c,s,wc/s);
Ts = simplify(K/sqrt(2)/q);
Ts_num = simple(vpa(Ts,6))
Ts_num2= simple(vpa(factor(Ts_num),6))
Ts_num =
104604.*s^2/(836827000000.+269301000.*s+147933.*s^2)
Ts_num2 =
104604.*s^2/(147933.*s^2+.269301e9*s+.836827e12)
```

The transfer function is

$$T(s) = \frac{0.7071\, s^2}{s^2 + 1820s + 5656800}$$

Find the magnitude of the gain at $\omega = \omega_C$ and ω_{MIN}.

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```
Tjw = subs(Ts,s,j*w);
MagTjwcdB = 20*log10(abs(subs(Tjw,w,wC)))
MagTjwmindB = 20*log10(abs(subs(Tjw,w,wmin)))
MagTjwcdB =
    -3.0105e+000
MagTjwmindB =
   -29.8317e+000
```

Plot the transfer function to verify it meets the specifications.

```
close all
ww = logspace(2,4,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter meets the specifications.

Answers:

$$T(s) = \frac{0.7071\,s^2}{s^2 + 1820s + 5656800}$$

$$|T(j\omega_C)| = -3.0105 \text{ dB}$$

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$$\left|T(j\omega_{\text{MIN}})\right| = -29.8317 \text{ dB}$$

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Problem 14-44

Construct the lowest order, high-pass transfer functions that meet the following filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Problem	Pole Type	ω_C(rad/s)	T_{MAX}	ω_{MIN}(rad/s)	T_{MIN}
14-44	You Decide	2000	10 dB	500	! 10 dB

Solution:

Determine the filter order required for both Butterworth and Chebychev design approaches.

```
% Set the filter specifications
TmaxdB = 10;
TmindB = -10;
wC = 2000;
wmin = 500;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);

% Compute ratios of filter specification values
Tratio = Tmax/Tmin;
wratio = wC/wmin;
% Compute the Butterworth order
ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
% Compute the Chebychev order
ChebychevOrder = ceil(acosh(sqrt(Tratio^2-1))/acosh(wratio))
ButterworthOrder =
     2.0000e+000
ChebychevOrder =
     2.0000e+000
```

We need a second-order filter with either approach. Choose a Butterworth design

```
K = Tmax;
syms s w
wc = wC;
% Butterworth filter design
q2 = s^2+sqrt(2)*s+1;
q = subs(q2,s,wc/s);
Ts = simplify(K/q);
Ts_num = vpa(Ts,7)
Ts_num =
3.162279*s^2/(4000000.+2828.428*s+s^2)
```

The transfer function is:

$$T(s) = \frac{3.1623\, s^2}{s^2 + 2828s + (2000)^2}$$

Find the magnitude of the gain at $\omega = \omega_C$ and ω_{MIN}.

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```
Tjw = subs(Ts,s,j*w);
MagTjwcdB = 20*log10(abs(subs(Tjw,w,wC)))
MagTjwmindB = 20*log10(abs(subs(Tjw,w,wmin)))
MagTjwcdB =
     6.9897e+000
MagTjwmindB =
   -14.0993e+000
```

Plot the transfer function to verify it meets the specifications.

```
close all
ww = logspace(2,4,1000);
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagTjwwdB = 20*log10(MagTjww);
semilogx(ww,MagTjwwdB,'b','LineWidth',3)
grid on
xlabel('Frequency (rad/s)')
ylabel('Amplitude (dB)')
```

The filter meets the specifications.

Answers:

$$T(s) = \frac{3.1623\, s^2}{s^2 + 2828s + (2000)^2}$$

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$$\left|T(j\omega_{\mathrm{C}})\right| = 6.9879 \text{ dB}$$
$$\left|T(j\omega_{\mathrm{MIN}})\right| = -14.10 \text{ dB}$$

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Problem 14-45

Design an active high-pass filter to meet the specification in Problem 14-41. Use OrCAD to verify that your design meets the specifications.

Solution:

The transfer function is

$$T(s) = \left[\frac{4.6416s}{s+1020}\right]^3$$

Compute the component values for one of the stages

```
clear all
w0 = 1020;
K = 4.6416;
C1 = 100e-9
R1 = 1/w0/C1
R2 = K*R1
C1 =
   100.0000e-009
R1 =
     9.8039e+003
R2 =
    45.5059e+003
```

Construct the circuit and verify it meets the specification.

The amplitude response is shown below.

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Answer:
The design is described above and the circuit meets the specifications.

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Problem 14-46

Design an active high-pass filter to meet the specification in Problem 14-42. Use OrCAD to verify that your design meets the specifications.

Solution:

The transfer function is:

$$T(s) = \left[\frac{10s}{s+2000}\right]\left[\frac{s^2}{s^2+2000s+(2000)^2}\right]$$

Design the component values.

```
clear all
w0 = 2000;
disp('First-order stage')
K = 10;
C1 = 100e-9
R1 = 1/w0/C1
R2 = K*R1
disp('Second-order stage')
% Unity gain approach
z = 2000/2/w0;
R2 = 10e3
R1 = R2*z^2
C = 1/w0/sqrt(R1*R2)
First-order stage
C1 =
   100.0000e-009
R1 =
     5.0000e+003
R2 =
    50.0000e+003
Second-order
stage
R2 =
    10.0000e+003
R1 =
     2.5000e+003
C =
   100.0000e-009
```

Construct the circuit below and verify it meets the specification.

The amplitude response is shown at left.

Answer:

The design is described above and the circuit meets the specifications.

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Problem 14-47

Design an active high-pass filter to meet the specification in Problem 14-43. Use OrCAD to verify that your design meets the specifications.

Solution:

The transfer function is

$$T(s) = \frac{0.7071\,s^2}{s^2 + 1820s + 5656800}$$

Design the component values using a second-order filter with unity gain followed by an amplifier.

```
clear all
w02 = 5656800;
w0 = sqrt(w02);
% Unity gain approach
z = 1820/2/w0;
R2 = 10e3
R1 = R2*z^2
C = 1/w0/sqrt(R1*R2)
R2 =
    10.0000e+003
R1 =
     1.4639e+003
C =
   109.8901e-009
```

Construct the circuit above and verify it meets the specification.

The amplitude response is shown below.

Answer:

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The design is described above and the circuit meets the specifications.

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Problem 14-48

Design an active high-pass filter to meet the specification in Problem 14-44. Use OrCAD to verify that your design meets the specifications.

Solution:

The transfer function is:

$$T(s) = \frac{3.1623\, s^2}{s^2 + 2828s + (2000)^2}$$

Design the component values.

```
clear all
w0 = 2000;
% Unity gain approach
z = 2828/2/w0;
R2 = 10e3
R1 = R2*z^2
C = 1/w0/sqrt(R1*R2)
R2 =
    10.0000e+003
R1 =
     4.9985e+003
C =
    70.7214e-009
```

Construct the circuit and verify it meets the specification.

The amplitude response is shown below.

Answer:

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The design is described above and the circuit meets the specifications.

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Problem 14-49

A certain instrumentation system for a new hybrid car needs a band-pass filter to limit its output bandwidth prior to digitization. The filter must meet the following specifications:

$T_{MAX} = +20 \pm 1\ dB$ $\omega_{CH} = 5.5\ krad/s\ (875.4\ Hz) \pm 10\%$ $\omega_{CL} = 5\ krad/s\ (795.8\ Hz) \pm 10\%$

$T_{MIN} \leq -20\ dB$ $\omega_{CHMIN} = 55\ krad/s\ (8.754\ kHz)$ $\omega_{CLMIN} = 500\ rad/s\ (79.6\ Hz)$

Two vendors have submitted proposed solutions as OrCAD graphics shown in Figure P14-49. As the requirements engineer on the project, you need to select the best one. You must check that each circuit meets or fails every specification. You should also consider: (1) parts count, (2) ease of maintainability and implementation (fewer parts, number of similar parts, adjustments of potentiometers, standard values), (3) frequency domain response, and (4) cost. You should address each item for each design at least qualitatively. Attach whatever software outputs you think will support your decision.

Solution:

Simulate both circuits in OrCAD to determine how well they meet the specifications.

Vendor #1:

Vendor #2:

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The amplitude responses for both circuits are plotted on the same axes, with Vendor #1 in green and Vendor #2 in red.

Use the OrCAD cursor to help determine the performance of each circuit. The results are summarized in the table below.

Answers:

Item	Specification	Vendor #1	Vendor #2
$\lvert T_{MAX}\rvert$	20 dB	20.29 dB	20.90 dB
ω_{CL}	795.8 Hz	793.9 Hz	723.7 Hz
ω_{CH}	875.4 Hz	877.3 Hz	964.1 Hz
$\lvert T(\omega_{CLMIN})\rvert$	−20 dB	−20.04 dB	−42.4 dB
$\lvert T(\omega_{CHMIN})\rvert$	−20 dB	−20.04 dB	−42.4 dB
Frequency Response Summary		Meets Specs, Peaked pass band	Meets Specs, Flat pass band
Parts Count	Minimum	7	10
Similar Parts	Maximum	1 pair	3 pairs
Potentiometers	Minimum	0	2
Standard Values	Maximum	3	2
Cost	Minimum	$75	$60

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Strictly speaking, the response for Vendor #2 does not meet the specification for ω_{CH} by 1.16 Hz. The response is extremely close to meeting the specification and we would have to analyze actual circuits to determine their performance. From an overall perspective, the circuit from Vendor #1 clearly meets all of the specifications, requires fewer parts, has more similar parts, no potentiometers, and more standard parts. Even though the circuit from Vendor #1 is more expensive, it is probably the best option.

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Problem 14-50

You are working at an aircraft manufacturing plant on an altitude sensor that eventually will be used to retrofit dozens of similar sensors on an upgrade to a current airframe. You have a requirement to find a quality notch filter to eliminate an undesirable interference at 400 Hz. You find a vendor and ask for a suitable product specification and a bid.

The specifications you require are:

1. *Must block signals at* 400 ± 2 *% Hz*
2. *Pass-band gain should be* 0 *dB* ± 1 *dB.*
3. *Bandwidth should be* 20 *Hz* ± 5 *% Hz*

The vendor's proposal, shown as an OrCAD drawing in Figure P14-50, claims to meet all of the design requirements. Determine if it passes all of the stated requirements. Then explain if you would buy the filter and why or why not?

Solution:

Simulate the circuit in OrCAD and analyze its performance against the specifications.

The amplitude response is shown below.

Answer:

The filter has a notch located at 397.9 Hz with a gain of −48.7 dB. The passband gain is 0 dB. The cutoff frequencies occur at 388.0 Hz and 408.0 Hz, so the bandwidth is 20 Hz. From the simulation, the filter meets all of the specifications. It appears that the filter is an acceptable choice, based on the simulation. One concern with the filter is the 625-Ω resistor at the input. Depending on the nature of the sensor that the filter will interface with, there may be loading

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issues at the input. We would either have to isolate the filter with a buffer or examine the sensor in more detail to make sure it would not impact the performance of the filter.

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Problem 14-51

Design Evaluation

A need exists for a third-order Butterworth low-pass filter with a cutoff frequency of 2 krad and a dc gain of 0 dB. The design department has proposed the circuit in Figure P14-51. As a junior engineer in the manufacturing department, you have been asked to verify the design and suggest modifications that would simplify production.

Solution:

Verify the design by simulating the circuit in OrCAD.

The amplitude response is shown below.

The circuit meets the specifications. Since the output stage is a unity-gain low-pass filter, it can be simplified by replacing it with an *RC* circuit with the correct cutoff frequency. The design is shown below.

The new amplitude response is shown below.

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The amplitude response matches that of the original circuit, but the new circuit has fewer parts and a simpler design.

Answer:
The proposed circuit meets the specifications, but a simpler design shown above can also meet the requirements.

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Problem 14-52

Modifying an Existing Circuit

One of your company's products includes the passive *RLC* filter and OP AMP buffer circuit in Figure P14-52. The supplier of the inductor is no longer in business and a suitable replacement is not available, even on *E-Bay*. You have been asked to design a suitable inductorless replacement. To minimize production changes, your design must use the existing OP AMP as is and either the 1-kΩ resistor or the 100-nF capacitor or both, if possible.

Solution:

Use MATLAB to determine the transfer function for the circuit.

```
clear all
syms s
R = 1e3;
L = 0.1;
C = 100e-9;
ZL = s*L;
ZC = 1/s/C;
Ts = simplify(ZC/(R+ZL+ZC))
Ts =
100000000/(10000*s+s^2+100000000)
```

The transfer function is

$$T(s) = \frac{10^8}{s^2 + 10000s + 10^8}$$

The filter is a second-order, unity-gain, low-pass filter with ω_0 = 10 krad/s and ζ = 0.5. Design a replacement low-pass filter using just resistors and capacitors using the unity-gain approach.

```
% Set the specified values
w0 = 10e3;
z = 0.5;
% Select a value for C1 and calculate the other values
C1 = 100e-9
C2 = C1*z^2
R = 1/w0/sqrt(C1*C2)
C1 =
   100.0000e-009
C2 =
    25.0000e-009
R =
     2.0000e+003
```

The resulting circuit design is shown above:

Answer:

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An alternate design is shown above. It uses the OP AMP, the capacitor, and four of the 1-kΩ resistors to make the two 2-kΩ resistors in the new filter.

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Problem 14-53

What's a High-Pass Filter

Ten years after earning a BSEE, you return for a master's degree and sign on as the laboratory instructor for the basic circuit analysis course. One experiment asks the students to build the active filter in Figure P14-53 and measure its gain response over the range from 150 Hz to 15 kHz. The lab instructions say the circuit is a high-pass filter with ω_0 = 10 krad/s and an infinite frequency gain of 0 dB. Everything goes well until a student, intrigued by the concept of infinite frequency, inputs 1 MHz and measures a gain of only 0.7. The student then inputs 2 MHz and measures a gain of 0.45. The high-pass filter appears to be a bandpass filter! Motivated by an insatiable thirst for understanding, the student asks you for an explanation. You first check the student's circuit and find it to be correct. You next replace the OP AMP and get almost the same results. Desperate for an explanation (your credibility is on the line here), you read the course textbook (it is Thomas, Rosa, and Toussaint), and find the answer in Chapter 12 (Example 12-4). What do you tell the student?

Answer:

Real OP AMPs have a finite gain-bandwidth product, typically on the order of 1 MHz. The gain-bandwidth limitation does not affect the high-pass filter gain for test frequencies in the range from 150 Hz to 15 kHz. At 1 MHz, the OP AMP begins to run out of bandwidth and its closed-loop gain decreases. The effect is even more noticeable at 2 MHz. In other words, the OP AMP eventually runs out of bandwidth, so active high-pass filters do not really work at infinite frequency. To design an active high-pass filter, you must choose an OP AMP with sufficient bandwidth to cover the frequency range of interest (150 Hz to 15 kHz in this lab), which must be a finite range.

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Problem 14-54

Bandpass to Bandstop Transformation

The three-terminal circuit in Figure P14-54(a) has a bandpass transfer function of the form

$$T(s) = \frac{V_O(s)}{V_S(s)} = \frac{2\zeta(s/\omega_0)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

Show that the circuit in Figure P14-54(b) has a bandstop transfer function of the form

$$T(s) = \frac{V_O(s)}{V_S(s)} = \frac{(s/\omega_0)^2 + 1}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

That is, show that interchanging the input and ground terminals changes a unity-gain bandpass circuit into a unity-gain bandstop circuit.

(a)

(b)

Solution:

The original input to the circuit is $V_1(s) = V_S(s) - 0$, and the original output is $V_2(s) = V_O(s) - 0$. With the interchange of input source and the ground, the new input signal is $V_1(s) = 0 - V_S(s)$, and the new output is $V_2(s) = V_O(s) - V_S(s)$. Apply the original transfer function to the new input signal to generate a new output signal $V_2(s)$. Add $V_S(s)$ to the new output to determine $V_O(s)$. Divide the result by $V_S(s)$ to determine the new transfer function. The solution is presented in the following MATLAB calculations.

```
clear all
syms s w0 z Vo Vs V1 V2
% Create the original transfer function
Ts1 = 2*z*s/w0/((s/w0)^2+2*z*(s/w0)+1);
% Create the new input signal
V1 = -Vs;
% Calculate the new output signal, V2(s)
V2 = Ts1*V1;
% Calculate the new output signal, Vo(s)
Vo = V2+Vs;
% Determine the new transfer function
Ts2 = simplify(Vo/Vs)
Ts2 =
(s^2+w0^2)/(s^2+2*z*s*w0+w0^2)
```

The resulting transfer function is

$$T(s) = \frac{s^2 + \omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

which is equivalent to the desired transfer function.

Answer:

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The analysis and results are presented above.

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Problem 14-55

Third-Order Butterworth Circuit

Show that the circuit in Figure P14-55 produces a third-order Butterworth low-pass filter with a cutoff frequency of $\omega_C = 1/RC$ and a passband gain of $K = 4$.

Solution:

Use node-voltage analysis to find the transfer function of the circuit. Label the input as V_1 and the output as V_2. Label the nodes to the right of the horizontal resistors as nodes A, B, and C, and note that node C has equations at both input terminals of the OP AMP.

```
clear all
syms s R C V1 V2 VA VB VC
ZC = 1/s/C;
Z2C = 1/s/(2*C);
ZC2 = 1/s/(C/2);
% Write the node-voltage equations
Eqn1 = (VA-V1)/R + VA/Z2C + (VA-VB)/R;
Eqn2 = (VB-VA)/R + (VB-V2)/ZC2 + (VB-VC)/R;
Eqn3 = (VC-VB)/R + VC/ZC;
Eqn4 = VC/R + (VC-V2)/(3*R);
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,VA,VB,VC,V2);
V2 = Soln.V2;
% Create the transfer function
Ts = simplify(V2/V1)
Ts2 = factor(Ts)
Ts =
4/(1+2*s*C*R+2*s^2*C^2*R^2+s^3*C^3*R^3)
Ts2 =
4/(1+s*C*R)/(s^2*C^2*R^2+s*C*R+1)
```

The transfer function is

$$T(s) = \frac{4}{R^3C^3s^3 + 2R^2C^2s^2 + 2RCs + 1} = \frac{4}{(RCs+1)(R^2C^2s^2 + RCs + 1)}$$

Answer:

Compare the transfer function to the polynomials in Table 14-1. The transfer function matches the form of a third-order, low-pass Butterworth filter with a cutoff frequency of $1/RC$ and a passband gain of 4, as expected.

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Problem 14-56

Notch Filter Comparison

To eliminate an interfering signal at 10 krad/s on a new product design, your consulting firm needs to purchase a notch filter with the following specifications:

Center frequency = 10 *krad/s* ± 0.5%

Bandwidth = 200 *rad/s* ± 2%

Depth of notch (attenuation at ω_0*)* = 50 *dB min.*

Two vendors have submitted proposed solutions as OrCAD graphics shown in Figure P14-56. As the owner, you want to select the best one. You must check that each circuit meets or fails every specification. You should also consider: (1) parts count, (2) ease of maintainability and implementation (fewer parts, number of similar parts, adjustments of potentiometers, standard values, and power usage), (3) frequency domain response, and (4) cost. You should address each item for each design at least qualitatively. Attach whatever software outputs you think will support your decision.

Solution:

Simulate the circuits and analyze their amplitude responses.

Simplicity Plus:

Filters-R-Us:

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Amplitude responses with Simplicity Plus in green and Filters-R-Us in red.

The two responses align with each other, so they share the same characteristics with respect to the specifications. The following table summarizes the performance of each filter.

Item	Specification	Simplicity Plus	Filters-R-Us
ω_0	10 krad/s ± 0.5%	10 krad/s	10 krad/s
B	200 rad/s ± 2%	200.4 rad/s	200.4 rad/s
Notch Depth	50 dB	90.4 dB	90.4 dB
Response Summary		Meets Specs	Meets Specs
Parts Count	Minimum	3	7
Similar Parts	Maximum	0 pairs	2 pairs
Potentiometers	Minimum	0	0
Standard Values	Maximum	All	All
Power Usage (OP AMPs)	Minimum	0	1
Cost	Minimum	$15.00	$12.50

Answer:

The circuits have the same frequency responses and both meet the specifications. If using an inductor is not a problem, then choose the design from Simplicity Plus, since it performs better than the other design with respect to most of the criteria. If using an inductor presents a problem, then choose the design from Filters-R-Us, since it meets the specifications.

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Problem 15-1

In Figure P15-1 L_1 = 100 mH, L_2 = 400 mH, M = 180 mH, and $v_S(t)$ = 200 sin 400t V.

(a) Write the i-v relationships for the coupled inductors using the reference marks in the figure.

(b) Find the voltage $v_2(t)$ when the output terminals are open circuited (i_2 = 0).

Solution:

(a) The coupling is additive, so we have the following equations:

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = 0.1 \frac{di_1(t)}{dt} + 0.18 \frac{di_2(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = 0.18 \frac{di_1(t)}{dt} + 0.4 \frac{di_2(t)}{dt}$$

(b) Use MATLAB to solve for voltage $v_2(t)$.

```
clear all
syms t
L1 = 100e-3;
L2 = 400e-3;
M = 180e-3;
v1 = 200*sin(400*t);
di1 = v1/L1;
v2 = M*di1
v2 =
360*sin(400*t)
```

Answers:

(a)

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = 0.1 \frac{di_1(t)}{dt} + 0.18 \frac{di_2(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = 0.18 \frac{di_1(t)}{dt} + 0.4 \frac{di_2(t)}{dt}$$

(b) $v_2(t) = 360 \sin(400t)$ V

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Problem 15-2

In Figure P15-1 L_1 = 100 mH, L_2 = 400 mH, M = 190 mH, $v_S(t)$ = 100 sin 1000t V and $i_2(t)$ = 2 cos 1000t A.

(a) Write the i-v relationships for the coupled inductors using the reference marks in the figure.

(b) Find the output voltage $v_2(t)$.

Solution:

(a) The coupling is additive, so we have the following equations:

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = 0.1 \frac{di_1(t)}{dt} + 0.19 \frac{di_2(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = 0.19 \frac{di_1(t)}{dt} + 0.4 \frac{di_2(t)}{dt}$$

(b) Use MATLAB to solve for voltage $v_2(t)$.

```
clear all
syms t
L1 = 100e-3;
L2 = 400e-3;
M = 190e-3;
v1 = 100*sin(1000*t);
i2 = 2*cos(1000*t);
di2 = diff(i2,t);
di1 = (v1-M*di2)/L1;
v2 = M*di1+L2*di2
v2 =
112*sin(1000*t)
```

Answers:

(a)

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = 0.1 \frac{di_1(t)}{dt} + 0.19 \frac{di_2(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = 0.19 \frac{di_1(t)}{dt} + 0.4 \frac{di_2(t)}{dt}$$

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(b) $v_2(t) = 112\sin(1000t)$ V

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Problem 15-3

In Figure P15-1 L_1 = 10 mH, L_2 = 5 mH, M = 7 mH, $i_1(t)$ = 2 sin 1000t A, and $i_2(t)$ = −sin 1000t A.

(a) Write the *i-v* relationships for the coupled inductors using the reference marks given.

(b) Find the source voltage $v_S(t)$ and the output voltage $v_2(t)$.

Solution:

(a) The coupling is additive, so we have the following equations:

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = 0.01 \frac{di_1(t)}{dt} + 0.007 \frac{di_2(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = 0.007 \frac{di_1(t)}{dt} + 0.005 \frac{di_2(t)}{dt}$$

(b) Use MATLAB to solve for the voltages $v_S(t)$ and $v_2(t)$.

```
clear all
syms t
L1 = 10e-3;
L2 = 5e-3;
M = 7e-3;
i1 = 2*sin(1000*t);
i2 = -1*sin(1000*t);
di1 = diff(i1,t);
di2 = diff(i2,t);
v1 = L1*di1+M*di2;
vs = v1
v2 = M*di1+L2*di2
vs =
13*cos(1000*t)
v2 =
9*cos(1000*t)
```

Answers:

(a)

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = 0.01 \frac{di_1(t)}{dt} + 0.007 \frac{di_2(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = 0.007 \frac{di_1(t)}{dt} + 0.005 \frac{di_2(t)}{dt}$$

(b) $v_S(t) = 13\cos(1000t)$ V

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$v_2(t) = 9\cos(1000t)$ V

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Problem 15-4

In Figure P15-4 $L_1 = L_2 = 30$ mH, $M = 30$ mH and $i_S(t) = 5 \sin 100t$ A. Find $v_1(t)$ and $v_2(t)$ when the output terminals are open circuited ($i_2 = 0$).

Solution:

The coupling is negative

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt} = 0.03 \frac{di_1(t)}{dt} - 0.03 \frac{di_2(t)}{dt}$$

$$v_2(t) = -M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = -0.03 \frac{di_1(t)}{dt} + 0.03 \frac{di_2(t)}{dt}$$

Use MATLAB to solve for the voltages $v_1(t)$ and $v_2(t)$.

```
clear all
syms t
L1 = 30e-3;
L2 = 30e-3;
M = -30e-3;
i1 = 5*sin(100*t);
di1 = diff(i1,t);
v1 = L1*di1
v2 = M*di1
v1 =
15*cos(100*t)
v2 =
-15*cos(100*t)
```

Answers:

$v_1(t) = 15\cos(100t)$ V

$v_2(t) = -15\cos(100t)$ V

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Problem 15-5

In Figure P15-4 L_1 = 25 mH, L_2 = 30 mH, M = 25 mH and $i_2(t)$ = 0.5 sin 1000t A when the output terminals are short circuited. Find $v_1(t)$ and $i_1(t)$.

Solution:

The coupling is negative

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} = 0.025\frac{di_1(t)}{dt} - 0.025\frac{di_2(t)}{dt}$$

$$v_2(t) = -M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = -0.025\frac{di_1(t)}{dt} + 0.030\frac{di_2(t)}{dt}$$

The output is short circuited, so $v_2(t)$ = 0 V. Solve for $i_1(t)$.

$$\frac{di_1(t)}{dt} = 1.2\frac{di_2(t)}{dt} = (1.2)[500\cos(1000t)] = 600\cos(1000t)$$

$$i_1(t) = 0.6\sin(1000t) \text{ A}$$

Solve for $v_1(t)$.

$$v_1(t) = 0.025\frac{di_1(t)}{dt} - 0.025\frac{di_2(t)}{dt} = (0.025)[600\cos(1000t)] - (0.025)[500\cos(1000t)]$$

$$v_1(t) = 2.5\cos(1000t) \text{ V}$$

Using MATLAB:

```
clear all
syms t
L1 = 25e-3;
L2 = 30e-3;
M = 25e-3;
i2 = 0.5*sin(1000*t);
di2 = diff(i2,t);
di1 = L2*di2/M;
v1 = L1*di1-M*di2
i1 = simple(int(di1))
Check_v1 = L1*di1-M*di2
v1 =
5/2*cos(1000*t)
i1 =
3/5*sin(1000*t)
Check_v1 =
5/2*cos(1000*t)
```

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Answer:
$v_1(t) = 2.5\cos(1000t)$ V and $i_1(t) = 0.6\sin(1000t)$ A .

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Problem 15-6

A pair of coupled inductors have L_1 = 3.6 H, L_2 = 2.4 H, and k = 0.98. When the output terminals are open circuited (i_2 = 0) the output voltage is observed to be $v_2(t)$ = 30 sin 1000t V. Find the input voltage $v_1(t)$ for additive coupling.

Solution:

Use MATLAB to compute the solution.

```
clear all
syms t
L1 = 3.6;
L2 = 2.4;
k = 0.98;
M = k*sqrt(L1*L2);
v2 = 30*sin(1000*t);
di1 = v2/M;
v1 = L1*di1;
v1_num = vpa(v1,6)
v1_num =
37.4922*sin(1000.*t)
```

Answer:

$v_1(t) = 37.4922\sin(1000t)$ V

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Problem 15-7

In Figure P15-7 $L_1 = L_2 = 6$ mH, $M = 4$ mH, and $i_1(t) = 5\cos(1000t)$ A. Find the input voltage $v_x(t)$.

Solution:

The coupling is additive. The currents are equal, so $i_1(t) = i_2(t)$. Using KVL, the input voltage is the sum of the inductor voltages, so $v_x(t) = v_1(t) + v_2(t)$. Use MATLAB to compute the results.

```
clear all
syms t
L1 = 6e-3;
L2 = 6e-3;
M = 4e-3;
i1 = 5*cos(1000*t);
i2 = i1;
di1 = diff(i1,t);
di2 = diff(i2,t);
v1 = L1*di1+M*di2;
v2 = M*di1+L2*di2;
vx = v1+v2
vx =
-100*sin(1000*t)
```

Answer:

$v_x(t) = -100\sin(1000t)$ V

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Problem 15-8

In Figure P15-8 show that $L_{EQ} = L_1(1 - k^2)$, where k is the coupling coefficient.

Solution:

The output is a short circuit, so we have $v_2(t) = 0$. The coupling is additive. Use MATLAB to show the results.

```
clear all
syms L1 L2 M k v1 di1 di2
% Write an equation for M in terms of the coupling coefficient
Eqn1 = M - k*sqrt(L1*L2);
% Write the equations for the coupled inductors
Eqn2 = M*di1 + L2*di2;
Eqn3 = v1 - (L1*di1 + M*di2);
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,v1,M,di2);
v1 = simplify(Soln.v1)
v1 =
L1*di1-k^2*L1*di1
```

We can rewrite the solution as follows:

$$v_1(t) = L_1\frac{di_1}{dt} - k^2 L_1\frac{di_1}{dt} = L_1(1-k^2)\frac{di_1}{dt} = L_{EQ}\frac{di_1}{dt}$$

Answer:

Based on the expression for $v_1(t)$, we have $L_{EQ} = L_1(1 - k^2)$.

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Problem 15-9

In Figure P15-9 L_1 = 800 mH, L_2 = 400 mH and M = 500 mH. The load is a 600-mH inductor and $i_2(t)$ = 0.5 sin 1000t A. Find the source voltage $v_S(t)$.

Solution:

The coupling is additive. Use MATLAB to compute the answer.

```
clear all
syms t
L1 = 800e-3;
L2 = 400e-3;
M = 500e-3;
i2 = 0.5*sin(1000*t);
di2 = diff(i2,t);
L_load = 600e-3;
% Use the output current to find the voltage across the load
v2 = -L_load*di2;
% Find the derivative of the input current
di1 = (v2-L2*di2)/M;
% Compute the input voltage
v1 = L1*di1+M*di2;
vs = v1
vs =
-550*cos(1000*t)
```

Answer:

$v_S(t) = -550\cos(1000t)$ V

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Problem 15-10

In Figure P15-9 L_1 = 800 mH, L_2 = 400 mH and M = 500 mH. The load is a 1-μF capacitor and $v_2(t)$ = 500 cos 1000t V. Find the source voltage $v_S(t)$.

Solution:

The coupling is additive. Use MATLAB to compute the answer.

```
clear all
syms t
L1 = 800e-3;
L2 = 400e-3;
M = 500e-3;
v2 = 500*cos(1000*t);
C_load = 1e-6;
% Use the output voltage to find the current through the load
i2 = -C_load*diff(v2,t);
di2 = diff(i2,t);
% Find the derivative of the input current
di1 = (v2-L2*di2)/M;
% Compute the input voltage
v1 = L1*di1+M*di2;
vs = v1
vs =
730*cos(1000*t)
```

Answer:

$v_S(t) = 730\cos(1000t)$ V

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Problem 15-11

The input voltage to a perfectly coupled transformer with $n = 4$ is $v_1(t) = 120 \cos 377t$ V. An 800-Ω resistive load is connected across the secondary. Find the secondary voltage and the current delivered to the resistive load. Assume additive coupling.

Solution:

Use MATLAB to compute the solution.

```
clear all
syms t
n = 4;
RL = 800;
v1 = 120*cos(377*t);
v2 = n*v1
iL = v2/RL
v2 =
480*cos(377*t)
iL =
3/5*cos(377*t)
```

Answers:

$v_2(t) = 480\cos(377t)$ V

$i_L(t) = 600\cos(377t)$ mA

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Problem 15-12

In Figure P15-12 R_S = 40 Ω, R_L = 1280 Ω, the turns ratio is n = 8, and the source voltage is $v_S(t)$ = 240 cos 377t V. Find expressions for $i_1(t)$ and $i_2(t)$.

Solution:

Find the equivalent resistance seen by the source and then compute the input current. Use the transformer relationships to compute the output current. Use MATLAB to complete the calculations.

```
clear all
syms t
RS = 40;
RL = 1280;
n = 8;
% Find the equivalent load resistance seen by the source
Req = RL/n^2;
% Find the total resistance seen by the source
Rtotal = RS + Req;
vs = 240*cos(377*t);
% Compute the input current
i1 = vs/Rtotal
% Compute the output current
i2 = -i1/n
i1 =
4*cos(377*t)
i2 =
-1/2*cos(377*t)
```

Answers:

$i_1(t) = 4\cos(377t)$ A

$i_2(t) = -0.5\cos(377t)$ A

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Problem 15-13

In Figure P15-12 R_S = 50 Ω, R_L = 2 Ω, the turns ratio is n = 1⁄5, and the source voltage is $v_S(t)$ = 440 cos 400t V. Find expressions for $v_1(t)$ and $v_2(t)$.

Solution:

Find the equivalent load resistance seen by the source. Use voltage division to find the input voltage. Use the transformer relationships to find the output voltage. Use MATLAB to complete the calculations.

```
clear all
syms t
RS = 50;
RL = 2;
n = 1/5;
% Find the equivalent load resistance seen by the source
Req = RL/n^2;
% Use voltage division to find v1
vs = 440*cos(400*t);
v1 = vs*Req/(RS+Req)
% Find the output voltage
v2 = n*v1
v1 =
220*cos(400*t)
v2 =
44*cos(400*t)
```

Answers:

$v_1(t) = 220\cos(400t)$ V

$v_2(t) = 44\cos(400t)$ V

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Problem 15-14

The turns ratio of the ideal transformer in Figure P15-14 is $n = 5$. The source and load impedances are $Z_S = 40 + j45\ \Omega$ and $Z_L = 500 - j350\ \Omega$. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{V}_O$ when $\mathbf{V}_S = 200\angle 0°$ V.

Solution:

Find the equivalent impedance seen by the source. Compute the input and output currents. Compute the output voltage. Use MATLAB to complete the calculations.

```
clear all
format short eng
syms t
Vs = 200;
n = 5;
Zs = 40+45j;
ZL = 500-350j;
Zeq = Zs + ZL/n^2;
I1 = Vs/Zeq
I2 = -I1/n
V2 = -I2*ZL;
Vo = V2
I1 =
     2.6310e+000 -  1.3594e+000i
I2 =
  -526.2004e-003 +271.8702e-003i
Vo =
   167.9456e+000 -320.1052e+000i
```

Answers:

$\mathbf{I}_1 = 2.6310 - j1.3594$ A.

$\mathbf{I}_2 = -526.2004 + j271.8702$ mA.

$\mathbf{V}_O = 167.9456 - j320.1052$ V.

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Problem 15-15

The turns ratio of the ideal transformer in Figure P15-14 is $n = 1/4$. The source and load impedances are $Z_S = 50 + j0\ \Omega$ and $Z_L = 5 + j2\ \Omega$ Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{V}_O$ when $\mathbf{V}_S = 440\angle 0°$.

Solution:

Find the equivalent impedance seen by the source. Compute the input and output currents. Compute the output voltage. Use MATLAB to complete the calculations.

```
clear all
format short eng
syms t
Vs = 440;
n = 1/4;
Zs = 50+0j;
ZL = 5+2j;
Zeq = Zs + ZL/n^2;
I1 = Vs/Zeq
I2 = -I1/n
V2 = -I2*ZL;
Vo = V2
I1 =
     3.1913e+000 -785.5389e-003i
I2 =
   -12.7650e+000 +  3.1422e+000i
Vo =
    70.1094e+000 +  9.8192e+000i
```

Answers:

$\mathbf{I}_1 = 3.1913 - j0.7855$ A.

$\mathbf{I}_2 = -12.765 + j3.1422$ A.

$\mathbf{V}_O = 70.1094 + j9.8192$ V.

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Problem 15-16

In Figure P15-16 the turns ratio is $n = 4$, $X = 45\ \Omega$, and $R_L = 720\ \Omega$. Find $\mathbf{I}_{IN}$ and $\mathbf{V}_O$ when $\mathbf{V}_S = 100\angle 0°$ V.

Solution:

Find the equivalent impedance seen by the source voltage and then compute the input phasor current. Then compute the output voltage.

```
clear all
n = 4;
X = 45;
Z1 = j*X;
RL = 720;
Req = RL/n^2;
Vs = 100;
Zeq = 1/(1/Z1 + 1/Req);
Iin = Vs/Zeq
Vo = n*Vs
Iin =
     2.2222e+000 -  2.2222e+000i
Vo =
   400.0000e+000
```

Answers:

$\mathbf{I}_{IN} = 2.22 - j2.22$ A.

$\mathbf{V}_O = 400$ V.

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Problem 15-17

In Figure P15-16 the turns ratio is $n = 1/2$, $X = 10\ \Omega$, and $R_L = 5\ \Omega$. Find $\mathbf{I}_{IN}$ and $\mathbf{V}_O$ when $\mathbf{V}_S = 200\angle 0°$ V.

Solution:

Find the equivalent impedance seen by the source voltage and then compute the input phasor current. Then compute the output voltage.

```
clear all
n = 1/2;
X = 10;
Z1 = j*X;
RL = 5;
Req = RL/n^2;
Vs = 200;
Zeq = 1/(1/Z1 + 1/Req);
Iin = Vs/Zeq
Vo = n*Vs
Iin =
  10.0000 -20.0000i
Vo =
   100
```

Answers:

$\mathbf{I}_{IN} = 10 - j20$ A.

$\mathbf{V}_O = 100$ V.

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Problem 15-18

A 440-V source with a source resistance of 25 Ω is connected to the primary of an ideal transformer with $n = 2$. Find the average power delivered to a 300-Ω load connected across the secondary.

Solution:

Find the equivalent resistance seen by the source and calculate the primary current. Determine the secondary current and then find the average power delivered to the load.

```
clear all
vs = 440;
n = 2;
Rs = 25;
RL = 300;
Req = RL/n^2;
Rtotal = Rs + Req;
i1 = vs/Rtotal;
i2 = -i1/n;
p2avg = i2^2*RL/2
p2avg =
  726.0000
```

Answer:

$P_{AVG} = 726$ W.

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Problem 15-19

In Figure P15-19 the voltage source output is v_S = 15 sin 377t V. Find the source input current $i_S(t)$.

Solution:

The primary voltage is the same as the source voltage. Calculate the secondary voltage and then the current through the 12-Ω load. Find the current through the 6-Ω resistor and then find the secondary current. Find the primary current and use KCL to find the source current.

```
clear all
syms t
n = 4;
vs = 15*sin(377*t);
v1 = vs;
% Find the secondary voltage
v2 = n*v1;
% Find the load current
iL = v2/12;
% Find the current through the 6-ohm resistor
i6 = (v1-v2)/6;
% Find the secondary current
i2 = i6-iL;
% Find the primary current
i1 = -n*i2;
% Find the source current
is = i1+i6
is =
85/2*sin(377*t)
```

Answer:

$i_S(t) = 42.5\sin(377t)$ A

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Problem 15-20

Show that the equivalent resistance in Figure P15-20 is $R_{EQ} = \left(\frac{N_1 + N_2}{N_2}\right)^2 R_L$

Solution:
The equivalent resistance is the input voltage divided by the input current. If the voltage across the primary is $v_1(t)$, then the voltage across the secondary is $(N_2/N_1)v_1(t)$. Let the primary current be $i_1(t)$. The current in the secondary is $-N_1 i_1(t)/N_2$ and the current through the load resistor is $v_2(t)/R = (N_2/RN_1)v_1(t)$. Use KCL to solve for $i_1(t)$ in terms of $v_1(t)$ and then compute the equivalent resistance. Use MATLAB to complete the calculations.

```
clear all
syms v1 v2 i1 i2 N1 N2 RL
% Find the secondary voltage in terms of the primary voltage
v2 = N2*v1/N1;
% Find the input voltage
vs = v1+v2;
% Find the secondary current in terms of the primary current
i2 = -N1*i1/N2;
% Use Ohm's Law to find the load current
iL = v2/RL;
% Use KCL to solve for the primary current in terms
% of the primary voltage
Eqn1 = i1-i2-iL;
i1 = solve(Eqn1,i1);
% Calculate the equivalent resistance
Req = simplify(vs/i1)
Req =
(N2+N1)^2/N2^2*RL
```

Answer:
The calculations above verify that the equivalent resistance satisfies the following relationship.

$$R_{EQ} = \left(\frac{N_1 + N_2}{N_2}\right)^2 R_L$$

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Problem 15-21

In Figure P15-21 the impedances are $Z_1 = 20 - j45\ \Omega$, $Z_2 = 40 + j30\ \Omega$, and $Z_3 = 360 + j270\ \Omega$. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.

Solution:

Find the equivalent resistance seen by the source and then use the transformer relationships to find the currents.

```
clear all
n1 = 2;
n2 = 3;
Z1 = 20-45j;
Z2 = 40+30j;
Z3 = 360+270j;
Zt2 = Z3/n2^2;
Zt1 = (Z2+Zt2)/n1^2;
Zeq = Z1+Zt1;
Vs = 200;
I1 = Vs/Zeq
I2 = I1/n1
I3 = I2/n2
I1 =
   3.2000 + 2.4000i
I2 =
   1.6000 + 1.2000i
I3 =
   0.5333 + 0.4000i
```

Answers:

$\mathbf{I}_1 = 3.2 + j2.4$ A.

$\mathbf{I}_2 = 1.6 + j1.2$ A.

$\mathbf{I}_3 = 0.5333 + j0.4$ A.

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Problem 15-22

In Figure P15-21 the impedances are $Z_1 = 35 + j20\ \Omega$, $Z_2 = 70 + j8\ \Omega$, and $Z_3 = 270 - j72\ \Omega$. Find the average power delivered to Z_3.

Solution:

Find the equivalent resistance seen by the source and then use the transformer relationships to find the currents. Use the current $\mathbf{I}_3$ to find the average power delivered to Z_3.

```
clear all
n1 = 2;
n2 = 3;
Z1 = 35+20j;
Z2 = 70+8j;
Z3 = 270-72j;
Zt2 = Z3/n2^2;
Zt1 = (Z2+Zt2)/n1^2;
Zeq = Z1+Zt1;
Vs = 200;
I1 = Vs/Zeq;
I2 = I1/n1;
I3 = I2/n2;
Pavg = abs(I3)^2*real(Z3)/2
Pavg =
   37.5000
```

Answer:

$P_{AVG} = 37.5$ W.

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Problem 15-23

An ideal transformer has a turns ratio of $n = 10$. The secondary winding is connected to a load $Z_L = 300 + j100\ \Omega$. The primary is connected to a voltage source with a peak amplitude of 300 V and an internal impedance of $Z_S = j2\ \Omega$. Find the average power delivered to the load.

Solution:

Find the equivalent impedance seen by the source. Find the primary current. Find the secondary current. Calculate the average power delivered to the load.

```
clear all
n = 10;
Zs = 2j;
ZL = 300+100j;
vs = 300;
Zeq = Zs+ZL/n^2;
I1 = vs/Zeq;
I2 = -I1/n;
Pavg = abs(I2)^2*real(ZL)/2
Pavg =
  7.5000e+003
```

Answer:

$P_{AVG} = 7.5$ kW.

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Problem 15-24

The primary winding of an ideal transformer with N_1 = 100 and N_2 = 250 is connected to a 480-V source. A 100-Ω load is connected across the secondary windings. Find amplitudes of the primary and secondary currents.

Solution:

Use MATLAB to calculate the currents.

```
clear all
N1 = 100;
N2 = 250;
n = N2/N1;
Vs = 480;
RL = 100;
Req = RL/n^2;
I1 = Vs/Req
I2 = -I1/n
I1 =
    30
I2 =
   -12
```

Answers:

$|\mathbf{I}_1|$ = 30 A.

$|\mathbf{I}_2|$ = 12 A.

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Problem 15-25

A transformer that can be treated as ideal has 480 turns in the primary winding and 240 turns in the secondary winding. The primary is connected to a 60-Hz source with peak amplitude of 440 V. The secondary winding delivers 5 kW to a resistive load. Find the primary and secondary currents and the impedance seen by the source.

Solution:

If the transformer is ideal, then it does not dissipate any power. Find the secondary voltage and then compute the secondary current based on the power dissipated by the resistive load. Then calculate the primary current and the equivalent impedance seen by the source.

```
clear all
N1 = 480;
N2 = 240;
n = N2/N1;
PL = 5e3;
v1 = 440;
v2 = n*v1;
RL = v2^2/2/PL;
i2 = -v2/RL
i1 = -n*i2
Req = RL/n^2
i2 =
  -45.4545
i1 =
   22.7273
Req =
   19.3600
```

Answers:

$|\mathbf{I}_1| = 22.7273$ A.

$|\mathbf{I}_2| = 45.4545$ A.

$Z_{IN} = 19.36\ \Omega$.

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Problem 15-26

The input voltage to the transformer in Figure P15-26 is a sinusoid $v_1(t) = 100 \cos 4000t$ V. With the circuit operating in the sinusoidal steady state, use mesh-current analysis to find the phasor output voltage $\mathbf{V}_2$ and the input impedance Z_{IN}. Use MATLAB to perform all of the calculations.

Solution:

Let the left mesh be mesh A and the right mesh be mesh B. The solution is presented in the following MATLAB calculations.

```
clear all
format short eng
syms IA IB I1 I2 V1 V2
% Define the given values
Vs = 100;
w = 4000;
Rs = 50;
L1 = 30e-3;
L2 = 120e-3;
M = 50e-3;
RL = 600;
% Write the mesh-current equations
Eqn1 = -Vs+IA*Rs+V1;
Eqn2 = -V2+IB*RL;
% Write the equations for the coupled inductors
Eqn3 = V1-(j*w*L1*IA + j*w*M*(-IB));
Eqn4 = V2-(j*w*M*IA + j*w*L2*(-IB));
% Solve for the voltages and currents
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,V1,V2,IA,IB);
V2 = double(Soln.V2)
% Compute the input impedance
V1 = Soln.V1;
I1 = Soln.IA;
Zin = double(Vs/I1)
V2 =
   122.9487e+000 + 15.8809e+000i
Zin =
    90.6504e+000 + 87.4797e+000i
```

Answers:

$\mathbf{V}_2 = 122.9487 + j15.8809$ V.

$Z_{IN} = 90.6504 + j87.4797\ \Omega$.

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Problem 15-27

Repeat Problem 15-26 with $v_1(t)$ = 100 cos 2000t V.

Solution:

Let the left mesh be mesh A and the right mesh be mesh B. The solution is presented in the following MATLAB calculations.

```
clear all
format short eng
syms IA IB I1 I2 V1 V2
% Define the given values
Vs = 100;
w = 2000;
Rs = 50;
L1 = 30e-3;
L2 = 120e-3;
M = 50e-3;
RL = 600;
% Write the mesh-current equations
Eqn1 = -Vs+IA*Rs+V1;
Eqn2 = -V2+IB*RL;
% Write the equations for the coupled inductors
Eqn3 = V1-(j*w*L1*IA + j*w*M*(-IB));
Eqn4 = V2-(j*w*M*IA + j*w*L2*(-IB));
% Solve for the voltages and currents
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,V1,V2,IA,IB);
V2 = double(Soln.V2)
% Compute the input impedance
V1 = Soln.V1;
I1 = Soln.IA;
Zin = double(Vs/I1)
V2 =
    97.3183e+000 + 51.9031e+000i
Zin =
    64.3678e+000 + 54.2529e+000i
```

Answers:

$\mathbf{V}_2$ = 97.3183 + j51.9031 V.

Z_{IN} = 64.3678 + j54.2529 Ω.

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Problem 15-28

A transformer operating in the sinusoidal steady state with ω = 500 rad/s has self inductances L_1 = 400 mH, L_2 = 200 mH, and k = 0.92. The load connected to the secondary winding is a 100-Ω resistor. Find the transformer input impedance. Assume additive coupling.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2
w = 500;
L1 = 400e-3;
L2 = 200e-3;
k = 0.92;
M = k*sqrt(L1*L2);
RL = 100;
% Write the coupled inductor equations
Eqn1 = V1 - (j*w*L1*I1 + j*w*M*I2);
Eqn2 = V2 - (j*w*M*I1 + j*w*L2*I2);
% Write Ohm's Law for the output current
Eqn3 = V2 + RL*I2;
% Solve the equations for V1 and I1
Soln = solve(Eqn1,Eqn2,Eqn3,V1,V2,I2);
V1 = Soln.V1;
% Compute the input impedance
Zin = double(simplify(V1/I1))
Zin =
    84.6400e+000 +115.3600e+000i
```

Answer:

Z_{IN} = 84.64 + j115.36 Ω.

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Problem 15-29

A transformer operating in the sinusoidal steady state with ω = 377 rad/s has self inductances L_1 = 200 mH, L_2 = 400 mH, and k = 0.95. The load connected across the secondary winding is Z_L = 75 + j150. Find the transformer input impedance. Assume additive coupling.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2
w = 377;
L1 = 200e-3;
L2 = 400e-3;
k = 0.95;
M = k*sqrt(L1*L2);
ZL = 75+150j;
% Write the coupled inductor equations
Eqn1 = V1 - (j*w*L1*I1 + j*w*M*I2);
Eqn2 = V2 - (j*w*M*I1 + j*w*L2*I2);
% Write Ohm's Law for the output current
Eqn3 = V2 + ZL*I2;
% Solve the equations for V1 and I1
Soln = solve(Eqn1,Eqn2,Eqn3,V1,V2,I2);
V1 = Soln.V1;
% Compute the input impedance
Zin = double(simplify(V1/I1))
Zin =
     8.0082e+000 + 43.2820e+000i
```

Answer:

Z_{IN} = 8.0082 + j43.282 Ω.

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Problem 15-30

The circuit in Figure P15-30 is in the sinusoidal steady state with $\mathbf{V}_S$ = 500 V and $Z_L = 10 + j5\ \Omega$. Find $\mathbf{V}_O$, the impedance seen by the source, and the average power delivered to Z_L. Use MATLAB to perform all of the calculations.

Solution:

Note that the inductors have negative coupling.

```
clear all
syms V1 V2 I1 I2
% Define the parameters
Vs = 500;
ZL = 10+5j;
Zs = 50+10j;
L1 = 100j;
L2 = 50j;
M = 70j;
% Write the mesh-current equations
Eqn1 = -Vs+Zs*I1+V1;
Eqn2 = -V2-ZL*I2;
% Write the equations for the coupled inductors
Eqn3 = V1-(L1*I1 - M*I2);
Eqn4 = V2-(-M*I1 + L2*I2);
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,V1,V2,I1,I2);
V0 = double(Soln.V2)
I1 = Soln.I1;
I2 = Soln.I2;
% Find the input impedance
Zin = double(simplify(Vs/I1))
% Find the power delivered to the load
Pavg = double(abs(I2)^2*real(ZL)/2)
V0 =
   -95.8511e+000 - 29.2719e+000i
Zin =
    65.6800e+000 + 23.7600e+000i
Pavg =
   401.7711e+000
```

Answers:

$\mathbf{V}_O = -95.8511 - j29.2719$ V.

$Z_{IN} = 65.68 + j23.76\ \Omega$.

$P_{AVG} = 401.7711$ W.

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Problem 15-31

Repeat Problem 15-30 with $Z_L = 10 - j10\ \Omega$.

Solution:

Note that the inductors have negative coupling.

```
clear all
syms V1 V2 I1 I2
% Define the parameters
Vs = 500;
ZL = 10-10j;
Zs = 50+10j;
L1 = 100j;
L2 = 50j;
M = 70j;
% Write the mesh-current equations
Eqn1 = -Vs+Zs*I1+V1;
Eqn2 = -V2-ZL*I2;
% Write the equations for the coupled inductors
Eqn3 = V1-(L1*I1 - M*I2);
Eqn4 = V2-(-M*I1 + L2*I2);
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,V1,V2,I1,I2);
V0 = double(Soln.V2)
I1 = Soln.I1;
I2 = Soln.I2;
% Find the input impedance
Zin = double(simplify(Vs/I1))
% Find the power delivered to the load
Pavg = double(abs(I2)^2*real(ZL)/2)
V0 =
  -135.2498e+000 + 69.2743e+000i
Zin =
    78.8235e+000 -  5.2941e+000i
Pavg =
   577.2856e+000
```

Answers:

$\mathbf{V}_O = -135.2498 + j69.2743$ V.

$Z_{IN} = 78.8235 - j5.2941\ \Omega$.

$P_{AVG} = 577.2856$ W.

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Problem 15-32

The circuit in Figure P15-32 is in the sinusoidal steady state with $v_S(t) = 200 \cos 2500t$ V. Find the input current $i(t)$, the impedance seen by the voltage source, and the average power supplied by the voltage source.

Solution:

Note that the circuit has negative coupling.

```
clear all
syms V1 V2 I1 I2
w = 2500;
Vs = 200;
Zs = 2e3;
C = 125e-9;
ZC = 1/j/w/C;
% Find the equivalent load impedance
ZL = 1/(1/2e3 + 1/ZC);
L1 = 1/2; L2 = 1/2; M = 1/4;
Z1 = j*w*L1;
Z2 = j*w*L2;
ZM = j*w*M;
% Write the mesh-current equations
Eqn1 = -Vs + Zs*I1 + V1;
Eqn2 = -V2-ZL*I2;
% Write the equations for the coupled inductors
Eqn3 = V1-(j*w*L1*I1 - j*w*M*I2);
Eqn4 = V2-(-j*w*M*I1 + j*w*L2*I2);
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,V1,V2,I1,I2);
% Find the input current
I1 = double(Soln.I1);
MagI1 = abs(I1)
PhaseI1 = 180*angle(I1)/pi
% Find the input impedance
Zin = double(Vs/I1)
% Find the power supplied by the source
Pavg = double(abs(I1)^2*real(Zin)/2)
MagI1 =
    78.4405e-003
PhaseI1 =
   -27.7561e+000
Zin =
     2.2563e+003 +  1.1874e+003i
Pavg =
     6.9415e+000
```

Answers:

$i(t) = 78.44 \cos(2500t - 27.76°)$ mA.

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$Z_{IN} = 2.2563 + j1.1874$ kΩ.
$P_{AVG} = 6.9415$ W.

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Problem 15-33

The circuit in Figure P15-33 is in the sinusoidal steady state with $\mathbf{V}_S$ = 100 V and R_L = 45 Ω. Find $\mathbf{V}_O$, the impedance seen by the source, and the average power delivered to R_L. Use MATLAB to perform all of the calculations.

Solution:

The following MATLAB calculations complete the solution.

```
clear all
syms V1 V2 I1 I2
Vs = 100;
RL = 45;
Zs = 10j;
L1 = 40j;
L2 = 100j;
M = 60j;
ZC = -30j;
% Write the mesh-current equations
Eqn1 = -Vs + Zs*I1 + V1 + ZC*(I1+I2);
Eqn2 = ZC*(-I2-I1) - V2 + RL*(-I2);
% Write the equations for the coupled inductors
Eqn3 = V1-(L1*I1 + M*I2);
Eqn4 = V2-(M*I1 + L2*I2);
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,V1,V2,I1,I2);
V2 = double(Soln.V2);
I1 = Soln.I1;
I2 = Soln.I2;
% Find the output voltage
V0 = double(V2 + ZC*(I2+I1))
V02 = double(-I2*RL);
% Find the input impedance
Zin = double(Vs/I1)
% Find the power delivered to the load
Pavg = double(abs(I2)^2*real(RL)/2)
V0 =
   114.6226e+000 - 63.6792e+000i
Zin =
     5.8484e+000 + 10.9025e+000i
Pavg =
   191.0377e+000
```

Answers:

$\mathbf{V}_O$ = 114.6226 − j63.6792 V.

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Z_{IN} = 5.8484 + j10.9025 Ω.
P_{AVG} = 191.0377 W.
Problem 15-34
Find $\mathbf{V}_1$ and $\mathbf{V}_2$ in Figure P15-34.

Solution:
Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 IA IB
Vs = 120;
R = 5;
n = 5;
ZL = -5j;
% Write the mesh-current equations
Eqn1 = -Vs + V1 + R*(IA-IB);
Eqn2 = R*(IB-IA) - V2 + ZL*IB;
% Write equations for the coupled inductors
Eqn3 = V2 - n*V1;
Eqn4 = IB - IA/n;
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,V1,V2,IA,IB);
V1 = double(Soln.V1)
V2 = double(Soln.V2)
V1 =
   -29.4163e+000 -  9.3385e+000i
V2 =
  -147.0817e+000 - 46.6926e+000i
```

Answers:
$\mathbf{V}_1$ = −29.4163 − j9.3385 V.
$\mathbf{V}_2$ = −147.0817 − j46.6926 V.

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Problem 15-35

The ideal transformer in Figure P15-35 is connected as an auto transformer with $N_1 = N_2 = 500$ turns, $R_L = 60\ \Omega$, and $\mathbf{V}_S = 120\angle 0°$ V. Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_L$, $\mathbf{I}_S$, and the average power delivered to R_L.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms I1 I2 V1 V2
RL = 60;
Vs = 120;
N1 = 500;
N2 = 500;
n = N2/N1;
% Write the equations for the circuit
Eqn1 = V2 - Vs;
Eqn2 = V2 - n*V1;
Eqn3 = I2 + I1/n;
Eqn4 = V1 + V2 + I1*RL;
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,V1,V2,I1,I2);
I1 = double(Soln.I1)
I2 = double(Soln.I2)
Is = I2 - I1
IL = -I1
Pavg = abs(I1)^2*RL/2
I1 =
    -4.0000e+000
I2 =
     4.0000e+000
Is =
     8.0000e+000
IL =
     4.0000e+000
Pavg =
   480.0000e+000
```

Answers:

$\mathbf{I}_1 = -4$ A.

$\mathbf{I}_2 = 4$ A.

$\mathbf{I}_L = 4$ A.

$\mathbf{I}_S = 8$ A.

$P_{AVG} = 480$ W.

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Problem 15-36

Use an equivalent T-circuit to find Z_{EQ} in Figure P15-36.

Solution:

The calculations for the equivalent impedances are shown below.

```
clear all
L1 = 8j;
L2 = 2j;
M = 4j;
% The inductors have positive coupling
LA = L1 - M
LB = L2 - M
LC = M
LA =
     0.0000e+000 +  4.0000e+000i
LB =
     0.0000e+000 -  2.0000e+000i
LC =
     0.0000e+000 +  4.0000e+000i
```

The equivalent T-circuit is shown below.

Find the equivalent impedance.

```
Zeq = 2 + LA + 1/(1/(LC+6j) + 1/(LB+1+5))
Zeq =
     8.0000e+000 +  6.0000e+000i
```

Answer:

$Z_{EQ} = 8 + j6\ \Omega$.

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Problem 15-37

A transformer operating in the sinusoidal steady state with ω = 4 krad/s has self inductances L_1 = 10 mH, L_2 = 40 mH, and k = 0.9. The load connected to the secondary winding is $Z_L = 150 + j75$ Ω. Use an equivalent T-circuit to find the transformer input impedance. Assume additive coupling.

Solution:

Use MATLAB to perform the calculations.

```
clear all
% Define the given parameters
w = 4e3;
L1 = 10e-3;
L2 = 40e-3;
k = 0.9;
% Solve for the mutual inductance
M = k*sqrt(L1*L2);
% Create the impedances
Z1 = j*w*L1;
Z2 = j*w*L2;
ZM = j*w*M;
ZL = 150+75j;
% Find the inductances for the equivalent T-circuit
ZA = Z1-ZM;
ZB = Z2-ZM;
ZC = ZM;
% Find the equivalent impedance
Zeq = ZA + 1/(1/ZC + 1/(ZB+ZL))
Zeq =
    10.0045e+000 + 24.3263e+000i
```

Answer:

$Z_{EQ} = 10.0045 + j24.3263$ Ω.

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Problem 15-38

A transformer operating in the sinusoidal steady state with ω = 4 krad/s has self inductances L_1 = 8 mH, L_2 = 12 mH, and M = 9 mH. The load connected to the secondary winding is $Z_L = 150 - j75\ \Omega$. Use an equivalent T-circuit to find the transformer input impedance. Assume additive coupling.

Solution:

Use MATLAB to perform the calculations.

```
clear all
% Define the given parameters
w = 4e3;
L1 = 8e-3;
L2 = 12e-3;
M = 9e-3;
% Create the impedances
Z1 = j*w*L1;
Z2 = j*w*L2;
ZM = j*w*M;
ZL = 150-75j;
% Find the inductances for the equivalent T-circuit
ZA = Z1-ZM;
ZB = Z2-ZM;
ZC = ZM;
% Find the equivalent impedance
Zeq = ZA + 1/(1/ZC + 1/(ZB+ZL))
Zeq =
     8.3688e+000 + 33.5064e+000i
```

Answer:

$Z_{EQ} = 8.3688 + j33.5064\ \Omega$.

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Problem 15-39

A transformer operating in the sinusoidal steady state has inductances L_1 = 5 mH, L_2 = 20 mH, and M = 10 mH. The load connected across the secondary winding is $Z_L = 100 + j100\ \Omega$. The voltage source connected to the primary winding has a frequency of ω = 10 krad/s and a peak amplitude of 200 V. Sketch the phasor domain circuit diagram using an equivalent T-circuit to represent the transformer. Use MATLAB to analyze the resulting circuit to find the voltage and average power delivered to Z_L. Assume additive coupling.

Solution:

The calculations for the equivalent impedances are shown below.

```
clear all
Vs = 200;
w = 10e3;
L1 = 5e-3;
L2 = 20e-3;
M = 10e-3;
% Create the impedances
Z1 = j*w*L1;
Z2 = j*w*L2;
ZM = j*w*M;
ZL = 100+100j;
% Find the inductances for the equivalent T-circuit
ZA = Z1-ZM
ZB = Z2-ZM
ZC = ZM
ZA =
     0.0000e+000 - 50.0000e+000i
ZB =
     0.0000e+000 +100.0000e+000i
ZC =
     0.0000e+000 +100.0000e+000i
```

The equivalent circuit is shown at right.

Use MATLAB to find the output voltage and the average power dissipated by the load resistor. Use node-voltage analysis

```
syms VA VB
Eqn1 = (VA-Vs)/ZA + VA/ZC + (VA-VB)/ZB;
Eqn2 = (VB-VA)/ZB + VB/ZL;
Soln = solve(Eqn1,Eqn2,VA,VB);
Vo = double(Soln.VB)
IL = Vo/ZL;
Pavg = abs(IL)^2*real(ZL)/2
Vo =
   400.0000e+000
Pavg =
   400.0000e+000
```

Answers:

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$\mathbf{V}_L = 400$ V.
$P_{AVG} = 400$ W.

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Problem 15-40

The self and mutual inductances of a transformer can be calculated from measurements of the steady-state ac voltages and currents with the secondary winding open circuited and short circuited. Suppose the measurements are $|\mathbf{V}_1| = 120$ V, $|\mathbf{I}_1| = 120$ mA, and $|\mathbf{V}_2| = 240$ V when the secondary is open and $|\mathbf{I}_1| = 10$ A and $|\mathbf{I}_2| = 2.2$ A when the secondary is shorted. All measurements were made at $f = 400$ Hz. Find L_1, L_2, and M.

Solution:

Use MATLAB to perform the calculations.

```
clear all
f = 400;
w = 2*pi*f;
% Consider the results with the secondary open
MagV1 = 120;
MagV2 = 240;
MagI1 = 120e-3;
MagI2 = 0;
% Write the equations for the coupled inductors
% Note that the j has been dropped because we are using magnitudes
syms L1 L2 M
Eqn1 = MagV1 - (w*L1*MagI1 + w*M*MagI2);
Eqn2 = MagV2 - (w*M*MagI1 + w*L2*MagI2);
Soln = solve(Eqn1,Eqn2,L1,M);
L1 = double(Soln.L1)
M = double(Soln.M)
% Consider the results with the secondary shorted
MagI1 = 10;
MagI2 = 2.2;
% Write the equation for the secondary magnitudes
Eqn3 = w*M*MagI1 - w*L2*MagI2;
L2 = double(solve(Eqn3,L2))
L1 =
   397.8874e-003
M =
   795.7747e-003
L2 =
     3.6172e+000
```

Answers:

$L_1 = 397.8874$ mH.

$L_2 = 3.6172$ H.

$M = 795.7747$ mH.

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Problem 15-41

Current Transformer **(A)**

A current transformer is an instrumentation device designed to provide an output voltage proportional to an ac current. Figure P15-41 shows a common design with many turns wrapped around a metallic ring surrounding the wire carrying the ac current being measured. In effect, the current transformer has a single turn primary winding and a secondary with several hundred turns N. Derive an expression relating the voltage $v_O(t)$ to the current $i_1(t)$ assuming the transformer is ideal.

Solution:

Use MATLAB to perform the calculations

```
clear all
syms N i1 i2 R vo
% Find the secondary current in terms of the primary current
i2 = -i1/N
% Compute the output voltage
vo = -i2*R
i2 =
-i1/N
vo =
i1/N*R
```

Answer:

$$v_O(t) = \frac{Ri_1(t)}{N}$$

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Problem 15-42

Transformer Thévenin Equivalent **(A)**

Show that the parameters of the Thèvenin equivalent at the output of the transformer in Figure P15-42 are

$$Z_T = jX_2(1 - k^2) \text{ and } \mathbf{V}_T = (X_M/X_1)\mathbf{V}_S$$

where k is the coupling coefficient.

Solution:

For the open-circuit voltage, we have the following coupled inductor equations when we ignore the secondary current that is equal to zero:

$\mathbf{V}_1 = jX_1\mathbf{I}_1$ and $\mathbf{V}_2 = jX_M\mathbf{I}_1$.

Solving for $\mathbf{V}_2 = \mathbf{V}_T$ in terms of $\mathbf{V}_1 = \mathbf{V}_S$, we have:

$\mathbf{V}_2 = (X_M/X_1)\mathbf{V}_1$.

To find the look-back impedance, set the source equal to zero and find the ratio of the secondary voltage to the secondary current. With the source equal to zero, the equation for the primary is

$0 = jX_1\mathbf{I}_1 + jX_M\mathbf{I}_2$

Therefore, we have $\mathbf{I}_1 = (-X_M/X_1)\mathbf{I}_2$. Substitute this result into the equation for the secondary:

$\mathbf{V}_2 = -jX_M(X_M/X_1)\mathbf{I}_2 + jX_2\mathbf{I}_2$

$$Z_T = \frac{\mathbf{V}_2}{\mathbf{I}_2} = -j\frac{X_M^2}{X_1} + jX_2 = j\frac{X_1X_2 - X_M^2}{X_1} = j\frac{X_1X_2 - kX_1X_2}{X_1} = jX_2(1-k^2)$$

Answer:

The result is demonstrated above.

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Problem 15-43

Perfectly Coupled Transformer **(A)**

Figure P15-43 is an equivalent circuit of a perfectly coupled transformer. This model is the basis for the transformer equivalent circuits used in the analysis of power systems. The inductance L_m is called the magnetizing inductance. The current through this inductance represents the current needed to establish the magnetic field in the transformer at no load ($i_2 = 0$). Show that the *i-v* equations for this circuit are:

$$v_1(t) = L_m \frac{di_1(t)}{dt} + nL_m \frac{di_2(t)}{dt}$$

$$v_2(t) = nL_m \frac{di_1(t)}{dt} + n^2 L_m \frac{di_2(t)}{dt}$$

Then show that $n = v_2(t)/v_1(t)$ and $k = 1$.

Solution:

The primary voltage is the inductance L_m times the derivate of the current through the inductor. The current through the inductor is $i_1(t) + ni_2(t)$. Therefore, we have:

$$v_1(t) = L_m \frac{d}{dt}\left[i_1(t) + ni_2(t)\right] = L_m \frac{di_1(t)}{dt} + nL_m \frac{di_2(t)}{dt}$$

which is the first equation above.

The coupled inductors require $v_2(t) = nv_1(t)$, so we can write

$$v_2(t) = nL_m \frac{di_1(t)}{dt} + n^2 L_m \frac{di_2(t)}{dt}$$

which is the second equation above.

Given the approach we used to construct $v_2(t)$, we have

$$\frac{v_2(t)}{v_1(t)} = \frac{nL_m \dfrac{di_1(t)}{dt} + n^2 L_m \dfrac{di_2(t)}{dt}}{L_m \dfrac{di_1(t)}{dt} + nL_m \dfrac{di_2(t)}{dt}} = n$$

Solve for *k*.

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{nL_m}{\sqrt{L_m n^2 L_m}} = 1$$

Answer:

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The results are demonstrated above.

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Problem 16-1

The voltage across and current through the load in Figure P16-1 are

$$v(t) = 150\cos(\omega t - 45°)\text{ V},\quad i(t) = 3\cos(\omega t + 20°)\text{ A}$$

Find the average power, reactive power, and instantaneous power delivered to the load.

Solution:

Use MATLAB to perform the calculations.

```
clear all
format short eng
syms t w
vt = 150*cos(w*t-pi*45/180);
it = 3*cos(w*t+pi*20/180);
% Find the amplitudes and relative phases of the voltage and current
VA = 150;
IA = 3;
vPhase = -45;
iPhase = 20;
theta = pi*(vPhase-iPhase)/180;
% Find the power components
P = VA*IA*cos(theta)/2
Q = VA*IA*sin(theta)/2
pt = vpa(P*(1+cos(2*w*t))-Q*sin(2*w*t),6)
P =
    95.0891e+000
Q =
  -203.9193e+000
pt =
95.0891+95.0891*cos(2.*w*t)+203.919*sin(2.*w*t)
```

Answers:

$P = 95.0891$ W.

$Q = -203.9193$ VAR.

$p(t) = 95.0891[1+\cos(2\omega t)] + 203.9193\sin(2\omega t)$ W.

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Problem 16-2

The voltage across and current through the load in Figure P16-1 are

$$v(t) = 135\cos(\omega t)\text{ V},\quad i(t) = 2\sin(\omega t + 30°)\text{ A}$$

Find the average power, reactive power, and instantaneous power delivered to the load.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms t w
% Find the amplitudes and relative phases of the voltage and current
VA = 135;
IA = 2;
vPhase = 0;
iPhase = 30-90;
theta = pi*(vPhase-iPhase)/180;
% Find the power components
P = VA*IA*cos(theta)/2
Q = VA*IA*sin(theta)/2
pt = vpa(P*(1+cos(2*w*t))-Q*sin(2*w*t),6)
P =
    67.5000e+000
Q =
   116.9134e+000
pt =
67.5000+67.5000*cos(2.*w*t)-116.913*sin(2.*w*t)
```

Answers:

$P = 67.5$ W.

$Q = 116.9134$ VAR.

$p(t) = 67.5[1+\cos(2\omega t)] - 116.9134\sin(2\omega t)$ W .

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Problem 16-3

The following voltage and current phasors apply to the circuit in Figure P16-3. Calculate the average power and reactive power delivered to the impedance *Z*. Find the power factor and state whether the power factor is lagging or leading.

(a) **V** = 250∠0° V (rms), **I** = 0.25∠−15° A (rms)

(b) **V** = 120∠135° V (rms), **I** = 12.5∠165° A (rms)

Solution:

Use MATLAB to perform the calculations.

```
clear all
disp('Part (a)')
Vrms = 250;
vPhase = 0;
Irms = 0.25;
iPhase = -15;
theta = pi*(vPhase-iPhase)/180;
% Average power
P = Vrms*Irms*cos(theta)
% Reactive power
Q = Vrms*Irms*sin(theta)
% Power factor
pf = cos(theta)
if theta == 0
    disp('The voltage and current are in phase.')
elseif theta > 0
    disp('Lagging power factor')
else
    disp('Leading power factor')
end

disp('Part (b)')
Vrms = 120;
vPhase = 135;
Irms = 12.5;
iPhase = 165;
theta = pi*(vPhase-iPhase)/180;
% Average power
P = Vrms*Irms*cos(theta)
% Reactive power
Q = Vrms*Irms*sin(theta)
% Power factor
pf = cos(theta)
if theta == 0
    disp('The voltage and current are in phase.')
elseif theta > 0
    disp('Lagging power factor')
else
    disp('Leading power factor')
end
Part (a)
P =
```

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```
    60.3704e+000
Q =
    16.1762e+000
pf =
   965.9258e-003
Lagging power factor
Part (b)
P =
     1.2990e+003
Q =
  -750.0000e+000
pf =
   866.0254e-003
Leading power factor
```

Answers:
(a) P = 60.3704 W; Q = 16.1762 VAR; pf = 0.9659; lagging.
(b) P = 1.299 kW; Q = −750 VAR; pf = 0.866; leading.

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Problem 16-4

Repeat Problem 16-3 using the following voltage and current phasors.

(a) **V** = 160∠45° V (rms), **I** = 8∠−25° A (rms)

(b) **V** = 220∠30° V (rms), **I** = 0.5∠60° A (rms)

Solution:

Use MATLAB to perform the calculations.

```
clear all
disp('Part (a)')
Vrms = 160;
vPhase = 45;
Irms = 8;
iPhase = -25;
theta = pi*(vPhase-iPhase)/180;
% Average power
P = Vrms*Irms*cos(theta)
% Reactive power
Q = Vrms*Irms*sin(theta)
% Power factor
pf = cos(theta)
if theta == 0
    disp('The voltage and current are in phase.')
elseif theta > 0
    disp('Lagging power factor')
else
    disp('Leading power factor')
end

disp('Part (b)')
Vrms = 220;
vPhase = 30;
Irms = 0.5;
iPhase = 60;
theta = pi*(vPhase-iPhase)/180;
% Average power
P = Vrms*Irms*cos(theta)
% Reactive power
Q = Vrms*Irms*sin(theta)
% Power factor
pf = cos(theta)
if theta == 0
    disp('The voltage and current are in phase.')
elseif theta > 0
    disp('Lagging power factor')
else
    disp('Leading power factor')
end
```

```
Part (a)
P =
   437.7858e+000
```

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```
Q =
     1.2028e+003
pf =
   342.0201e-003
Lagging power factor
Part (b)
P =
    95.2628e+000
Q =
   -55.0000e+000
pf =
   866.0254e-003
Leading power factor
```

Answers:
(a) P = 437.8 W; Q = 1203 VAR; pf = 0.3420; lagging.
(b) P = 95.2628 W; Q = −55 VAR; pf = 0.866; leading.

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Problem 16-5

The following conditions apply to the circuit in Figure P16-3. Find the complex power delivered to the impedance Z.

(a) $\mathbf{V} = 150\angle 45°$ V (rms), $Z = 50\angle 15°\ \Omega$

(b) $Z = 30 - j40\Omega$, $\mathbf{I} = 10\angle -25°$ A (rms)

Solution:

Use MATLAB to perform the calculations.

```
clear all
disp('Part (a)')
Vrms = 150;
vPhase = 45*pi/180;
Zmag = 50;
zPhase = 15*pi/180;
% Determine the current
Irms = Vrms/Zmag;
iPhase = vPhase-zPhase;
theta = vPhase-iPhase;
% Determine the complex power
S = Vrms*Irms*exp(theta*j)

disp('Part (b)')
Z = 30-40j;
Zmag = abs(Z);
zPhase = angle(Z);
Irms = 10;
iPhase = -25*pi/180;
% Determine the current
Vrms = Irms*Zmag;
vPhase = iPhase+zPhase;
theta = vPhase-iPhase;
% Determine the complex power
S = Vrms*Irms*exp(theta*j)
Part (a)
S =
   434.6666e+000 +116.4686e+000i
Part (b)
S =
     3.0000e+003 -  4.0000e+003i
```

Answers:

(a) $S = 434.6666 + j116.4686$ VA.

(b) $S = 3 - j4$ kVA.

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Problem 16-6

The following conditions apply to the circuit in Figure P16-3. Find the impedance Z.

(a) $S = 1000 + j250$ kVA, $|\mathbf{I}| = 100$ A.

(b) $|S| = 15$ kVA, $P = 12$ kW, $Q < 0$, $|\mathbf{I}| = 20$ A.

Solution:

Use MATLAB to perform the calculations.

```
clear all
disp('Part (a)')
S = 1000e3+250e3j;
Irms = 100;
Z = S/Irms^2

disp('Part (b)')
MagS = 15e3;
P = 12e3;
Irms = 20;
R = P/Irms^2;
Q = -1*sqrt(MagS^2-P^2);
X = Q/Irms^2;
Z = R+j*X
Part (a)
Z =
   100.0000e+000 + 25.0000e+000i
Part (b)
Z =
    30.0000e+000 - 22.5000e+000i
```

Answers:

(a) $Z = 100 + j25\ \Omega$.

(b) $Z = 30 - j22.5\ \Omega$.

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Problem 16-7

A load draws an average power of 30 kW at a lagging power factor of 0.6 from a 2.4-kV (rms) source. Find I_{rms}, and the load impedance.

Solution:

Use MATLAB to perform the calculations.

```
clear all
pf = 0.6;
theta = acos(pf);
P = 30e3;
Vrms = 2.4e3;
% Compute the rms current
Irms = P/Vrms/pf
% Determine the complex power
Q = Vrms*Irms*sin(theta);
S = P + j*Q;
% Determine the impedance
Z = S/Irms^2
Irms =
    20.8333e+000
Z =
    69.1200e+000 + 92.1600e+000i
```

Answers:

I_{rms} = 20.8333 A.

Z_L = 69.12 + j92.16 Ω.

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Problem 16-8

A load draws an apparent power of 10 kVA at a leading power factor of 0.9 from a 880-V(rms) source. Find I_{rms}, and the load impedance.

Solution:

Use MATLAB to perform the calculations.

```
clear all
SMag = 10e3;
pf = 0.9;
theta = acos(pf);
% We have a leading power factor, so ensure theta < 0
if theta > 0
    theta = -1*theta;
end
Vrms = 880;
% Determine the rms current
Irms = SMag/Vrms
% Determine the complex power
P = Vrms*Irms*pf;
Q = Vrms*Irms*sin(theta);
S = P + j*Q;
% Determine the impedance
Z = S/Irms^2
Irms =
    11.3636e+000
Z =
    69.6960e+000 - 33.7553e+000i
```

Answers:

I_{rms} = 11.3636 A.

$Z_L = 69.696 - j33.7553\ \Omega$.

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Problem 16-9
A load with a leading power factor of 0.9 draws 15 A (rms) and 5 kW from a 60-Hz source. Find the load impedance.

Solution:
Use MATLAB to perform the calculations.

```
clear all
pf = 0.9;
theta = acos(pf);
% We have a leading power factor, so ensure theta < 0
if theta > 0
    theta = -1*theta;
end
Irms = 15;
P = 5e3;
% Find the rms voltage
Vrms = P/Irms/pf;
% Find the complex power
Q = Vrms*Irms*sin(theta);
S = P+j*Q;
% Find the impedance
Z = S/Irms^2
Z =
    22.2222e+000 - 10.7627e+000i
```

Answer:
$Z_L = 22.2222 - j10.7627\ \Omega$.

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Problem 16-10
A load is rated at 440 V (rms), 7.5 A (rms), and 3.3 kW when connected to a 50-Hz source. Find the load impedance.

Solution:
Use MATLAB to perform the calculations.

```
clear all
Vrms = 440;
Irms = 7.5;
P = 3.3e3;
pf = P/Irms/Vrms
theta = acos(pf);
Q = Vrms*Irms*sin(theta);
S = P+j*Q;
Z = S/Irms^2
pf =
     1.0000e+000
Z =
    58.6667e+000
```

Answer:
$Z_L = 58.6667\ \Omega$.

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Problem 16-11

A load made up of a 100-Ω resistor in parallel with a 150-mH inductor is connected across a 240-V(rms), 60-Hz voltage source. Find the complex power delivered to the load and the load power factor. State whether the power factor is lagging or leading.

Solution:

Use MATLAB to perform the calculations

```
clear all
f = 60;
w = 2*pi*f;
Vrms = 240;
% Determine the load impedance
R = 100;
L = 150e-3;
ZL= j*w*L;
Zeq = 1/(1/R+1/ZL);
% Find the complex power
Irms = Vrms/abs(Zeq);
theta = angle(Zeq);
P = Vrms*Irms*cos(theta);
Q = Vrms*Irms*sin(theta);
S = P+j*Q
pf = cos(theta)
if theta == 0
    disp('The voltage and current are in phase.')
elseif theta > 0
    disp('Lagging power factor')
else
    disp('Leading power factor')
end
S =
   576.0000e+000 +  1.0186e+003i
pf =
   492.2349e-003
Lagging power factor
```

Answer:

$S = 576 + j1018.6$ VA

pf = 0.4922 and it is a lagging power factor.

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Problem 16-12

A load made up of a 50-Ω resistor in series with a 100-mH inductor is connected across a 110-V(rms), 60-Hz voltage source. Find the complex power delivered to the load and the load power factor. State whether the power factor is lagging or leading.

Solution:

Use MATLAB to perform the calculations

```
clear all
f = 60;
w = 2*pi*f;
Vrms = 110;
% Determine the load impedance
R = 50;
L = 100e-3;
ZL= j*w*L;
Zeq = R+ZL;
% Find the complex power
Irms = Vrms/abs(Zeq);
theta = angle(Zeq);
P = Vrms*Irms*cos(theta);
Q = Vrms*Irms*sin(theta);
S = P+j*Q
pf = cos(theta)
if theta == 0
    disp('The voltage and current are in phase.')
elseif theta > 0
    disp('Lagging power factor')
else
    disp('Leading power factor')
end
S =
   154.2886e+000 +116.3309e+000i
pf =
   798.4712e-003
Lagging power factor
```

Answers:

S = 154.2886 + j116.3309 VA

pf = 0.7985 and it is a lagging power factor.

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Problem 16-13

A load made up of a 100-kΩ resistor in series with 20-nF capacitor is connected across a 2400-V (rms), 60-Hz voltage source. Find the complex power delivered to the load and the load power factor. State whether the power factor is lagging or leading.

Solution:

Use MATLAB to perform the calculations

```
clear all
f = 60;
w = 2*pi*f;
Vrms = 2400;
% Determine the load impedance
R = 100e3;
C = 20e-9;
ZC = 1/j/w/C;
Zeq = R+ZC;
% Find the complex power
Irms = Vrms/abs(Zeq);
theta = angle(Zeq);
P = Vrms*Irms*cos(theta);
Q = Vrms*Irms*sin(theta);
S = P+j*Q
pf = cos(theta)
if theta == 0
    disp('The voltage and current are in phase.')
elseif theta > 0
    disp('Lagging power factor')
else
    disp('Leading power factor')
end
S =
    20.8768e+000 - 27.6887e+000i
pf =
   602.0331e-003
Leading power factor
```

Answers:

$S = 20.8768 - j27.6887$ VA

pf = 0.6020 and it is a leading power factor.

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Problem 16-14

In Figure P16-14 the load Z_L is a 60-Ω resistor in series with a capacitor whose reactance is −30 Ω. The source voltage is 440 V (rms). Find the complex power produced by the source and the complex power delivered to the load.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZL = 60 - 30j;
Vs = 440;
Z1 = 20+25j;
Z2 = -20j;
% Find the parallel impedance with the load
Z3 = 1/(1/Z2+1/ZL);
% Use voltage division to find the load voltage
VL = Vs*Z3/(Z1+Z3);
% Compute the complex power at the load
SL = abs(VL)^2/conj(ZL)
% Find the impedance seen by the source
Zin = Z1+Z3;
Ss = abs(Vs)^2/conj(Zin)
SL =
     1.1876e+003 -593.7891e+000i
Ss =
     7.2244e+003 +  2.4989e+003i
```

Answers:

$S_S = 7224 + j2499$ VA.

$S_L = 1188 - j593.8$ VA.

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Problem 16-15

Repeat Problem 16-14 when Z_L is 50-Ω resistor in parallel with an impedance of 40 − j50 Ω.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZL = 1/(1/50 + 1/(40-50j));
Vs = 440;
Z1 = 20+25j;
Z2 = -20j;
% Find the parallel impedance with the load
Z3 = 1/(1/Z2+1/ZL);
% Use voltage division to find the load voltage
VL = Vs*Z3/(Z1+Z3);
% Compute the complex power at the load
SL = abs(VL)^2/conj(ZL)
% Find the impedance seen by the source
Zin = Z1+Z3;
Ss = abs(Vs)^2/conj(Zin)
SL =
     1.4573e+003 -597.2636e+000i
Ss =
     6.1136e+003 +  2.7743e+003i
```

Answers:

S_S = 6114 + j2774 VA.

S_L = 1457 − j597.3 VA.

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Problem 16-16

In Figure P16-16 the load Z_L is a 50-Ω resistor and the source voltage is 480 V(rms). Find the complex power produced by the source.

Solution:

Use MATLAB to perform the calculations.

```
clear all
Vs = 480;
ZL = 50;
Zin = 100j + 1/(1/-25j + 1/(ZL+100j));
Ss = abs(Vs)^2/conj(Zin)
Ss =
   184.3200e+000 +  3.3178e+003i
```

Answer:

$S_S = 184 + j3318$ VA.

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Problem 16-17

Repeat Problem 16-16 when the load Z_L is a 50-Ω resistor in series with a capacitor whose reactance is −100 Ω.

Solution:

Use MATLAB to perform the calculations.

```
clear all
Vs = 480;
ZL = 50-100j;
Zin = 100j + 1/(1/-25j + 1/(ZL+100j));
Ss = abs(Vs)^2/conj(Zin)
Ss =
   354.4615e+000 +  2.8357e+003i
```

Answer:

$S_S = 354 + j2836$ VA.

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Problem 16-18

In Figure P16-18 the three load impedances are: $Z_1 = 25 + j6\ \Omega$, $Z_2 = 16 + j4\ \Omega$, and $Z_3 = 100 + j20\ \Omega$. Use MATLAB to solve the following problems:

(a) Find the phasor currents in lines A, B, and N.

(b) Find the complex power produced by each source.

Solution:

Use MATLAB to perform the calculations.

```
clear all
Z1 = 25+6j;
Z2 = 16+4j;
Z3 = 100+20j;
Vs = 110;
disp('Part (a)')
% Find the current through each load
I1 = Vs/Z1;
I2 = Vs/Z2;
I3 = 2*Vs/Z3;
IA = I1+I3
IB = -I2-I3
IN = I2-I1
disp('Part (b)')
S1 = Vs*conj(IA)
S2 = Vs*conj(-IB)
Part (a)
IA =
     6.2757e+000 -  1.4216e+000i
IB =
    -8.5860e+000 +  2.0407e+000i
IN =
     2.3102e+000 -619.1599e-003i
Part (b)
S1 =
   690.3322e+000 +156.3720e+000i
S2 =
   944.4570e+000 +224.4796e+000i
```

Answers:

(a) $\mathbf{I}_A = 6.2757 - j1.4216$ A.

$\mathbf{I}_B = -8.586 + j2.0407$ A.

$\mathbf{I}_N = 2.3102 - j0.6192$ A.

(b) $S_1 = 690.33 + j156.37$ VA.

$S_2 = 944.46 + j224.48$ VA.

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Problem 16-19

Repeat Problem 16-18 with $Z_1 = 20 + j12\ \Omega$, $Z_2 = 20 + j6\ \Omega$, and $Z_3 = 50 + j0\ \Omega$.

Solution:

Use MATLAB to perform the calculations.

```
clear all
Z1 = 20+12j;
Z2 = 20+6j;
Z3 = 50+0j;
Vs = 110;
disp('Part (a)')
% Find the current through each load
I1 = Vs/Z1;
I2 = Vs/Z2;
I3 = 2*Vs/Z3;
IA = I1+I3
IB = -I2-I3
IN = I2-I1
disp('Part (b)')
S1 = Vs*conj(IA)
S2 = Vs*conj(-IB)
Part (a)
IA =
     8.4441e+000 -  2.4265e+000i
IB =
    -9.4459e+000 +  1.5138e+000i
IN =
     1.0018e+000 +912.7091e-003i
Part (b)
S1 =
   928.8529e+000 +266.9118e+000i
S2 =
     1.0390e+003 +166.5138e+000i
```

Answers:

(a) $\mathbf{I}_A = 8.4441 - j2.4265$ A.

$\mathbf{I}_B = -9.4459 + j1.5138$ A.

$\mathbf{I}_N = 1.0018 + j0.9127$ A.

(b) $S_1 = 928.85 + j266.91$ VA.

$S_2 = 1039 + j166.51$ VA.

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Problem 16-20
A load is rated at 440 V(rms), 30 A(rms), and 10 kW. Find the average power delivered to the load when the load voltage is 450 V(rms).

Solution:
Use MATLAB to perform the calculations

```
clear all
Vrated = 440;
Irated = 30;
Prated = 10e3;
% Find the rated power factor
pf = Prated/Vrated/Irated;
% Find the load impedance
ZL = Vrated/Irated*(pf + j*sqrt(1-pf^2));
% Apply a voltage of 450 V(rms)
Vnew = 450;
SL = abs(Vnew)^2/conj(ZL);
Pavg = real(SL)
Pavg =
    10.4597e+003
```

Answer:
P_L = 10.4597 kW.

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Problem 16-21

The average power delivered to the load Z_L in Figure P16-21 is 40 kW at a lagging power factor of 0.8. The load voltage is 2.4 kV(rms) and the line has an impedance of $Z_W = 1 + j8\ \Omega$/wire. Find the apparent power supplied by the source and magnitude of the source voltage.

Solution:

Use MATLAB to perform the calculations.

```
clear all
PL = 40e3;
pf = 0.8;
VL = 2.4e3;
ZW = 1+8j;
% Find the complex power at the load
SL = PL/pf*(pf+j*sqrt(1-pf^2));
% Find the magnitude of the current
MagIL = abs(SL/VL);
% Find the complex power in the wires
SW = 2*MagIL^2*ZW;
% Find the total complex power
Ss = SL+SW;
% Find the apparent source power
MagSs = abs(Ss)
% Find the magnitude of the source voltage
MagVs = abs(Ss)/MagIL
MagSs =
    55.0917e+003
MagVs =
     2.6444e+003
```

Answers:

The apparent power supplied by the source is 55.0917 kVA.

The magnitude of the source voltage is 2.6444 kV.

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Problem 16-22

Repeat Problem 16-21 with the power factor increased to 0.95.

Solution:

Use MATLAB to perform the calculations.

```
clear all
PL = 40e3;
pf = 0.95;
VL = 2.4e3;
ZW = 1+8j;
% Find the complex power at the load
SL = PL/pf*(pf+j*sqrt(1-pf^2));
% Find the magnitude of the current
MagIL = abs(SL/VL);
% Find the complex power in the wires
SW = 2*MagIL^2*ZW;
% Find the total complex power
Ss = SL+SW;
% Find the apparent source power
MagSs = abs(Ss)
% Find the magnitude of the source voltage
MagVs = abs(Ss)/MagIL
MagSs =
    44.4547e+003
MagVs =
     2.5339e+003
```

Answers:

The apparent power supplied by the source is 44.4547 kVA.

The magnitude of the source voltage is 2.5339 kV.

Increasing the power factor decreases the required source power by nearly 20%.

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Problem 16-23

The complex power delivered to the load Z_L in Figure P16-21 is 20 + *j*15 kVA. The load voltage is 2 kV(rms) and the line has an impedance of Z_W = 2 + j12 Ω per wire. Find the magnitude of the source voltage and the complex power produced by the source.

Solution:

Use MATLAB to perform the calculations.

```
clear all
SL = 20e3+15e3j;
VL = 2e3;
ZW = 2+12j;
% Find the magnitude of the current
MagIL = abs(SL/VL);
% Find the complex power in the wires
SW = 2*MagIL^2*ZW;
% Find the total complex power
Ss = SL+SW
% Find the magnitude of the source voltage
MagVs = abs(Ss)/MagIL
Ss =
    20.6250e+003 + 18.7500e+003i
MagVs =
     2.2299e+003
```

Answers:

The magnitude of the source voltage is 2.2299 kV.

The complex power produced by the source is 20.625 + *j*18.75 kVA.

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Problem 16-24

The complex power delivered to the load Z_L in Figure P16-21 is 20 + j30 kVA. The source produces an average power of 21 kW and the line has an impedance of Z_W = 0.6 + j5 Ω per wire. Find the reactive power produced by the source.

Solution:

Use MATLAB to perform the calculations

```
clear all
SL = 20e3+30e3j;
Ps = 21e3;
ZW = 0.6+5j;
% Find the average power in the line
PW = Ps - real(SL);
% Find the magnitude of the line current
MagIL = sqrt(PW/2/real(ZW));
% Find the total complex power
Ss = SL + 2*MagIL^2*ZW;
% Find the reactive power of the source
Qs = imag(Ss)
Qs =
    38.3333e+003
```

Answer:

The reactive power produced by the source is 38.333 kVAR.

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Problem 16-25

In Figure P16-25 the voltage across the two loads is $|\mathbf{V}_L| = 4.8$ kV (rms). The load Z_1 draws an average power of 12 kW and a lagging power factor of 0.75 lagging. The load Z_2 draws an apparent power of 15 kVA and a lagging power factor of 0.8 lagging. The line has an impedance of $Z_W = 10 + j54\ \Omega$ per wire. Find the apparent power produced by the source and the rms value of the source voltage.

Solution:

Use MATLAB to perform the calculations.

```
clear all
MagVL = 4.8e3;
pf1 = 0.75;
PZ1 = 12e3;
pf2 = 0.8;
PZ2 = 15e3*pf2;
ZW = 10+54j;
% Find the complex power for each load
S1 = PZ1/pf1*(pf1+j*sqrt(1-pf1^2));
S2 = PZ2/pf2*(pf2+j*sqrt(1-pf2^2));
% Find the total complex power at the load
SL = S1+S2;
% Find the magnitude of the current
MagIL = abs(SL)/MagVL;
% Find the complex power in the line
SW = 2*MagIL^2*ZW;
% Find the total complex power
Ss = SL+SW;
% Find the apparent source power
MagSs = abs(Ss)
% Find the rms value of the source voltage
MagVs = MagSs/MagIL
MagSs =
    34.5912e+003
MagVs =
     5.3603e+003
```

Answers:

The apparent power produced by the source is 34.5912 kVA.

The rms value of the source voltage is 5.3603 kV.

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Problem 16-26

The two loads in Figure P16-25 draw apparent powers of $|S_1|$ = 16 kVA at a lagging power factor of 0.8 and $|S_2|$ = 25 kVA at unity power factor. The voltage across the loads is 3.8 kV and the line has an impedance of $Z_W = 5 + j26\ \Omega$ per wire. Find the apparent power produced by the source and the rms value of the source voltage.

Solution:

Use MATLAB to perform the calculations.

```
clear all
MagS1 = 16e3;
pf1 = 0.8;
MagS2 = 25e3;
pf2 = 1;
VL = 3.8e3;
ZW = 5+26j;
% Find the complex power in each load
S1 = MagS1*(pf1+j*sqrt(1-pf1^2));
S2 = MagS2*(pf2+j*sqrt(1-pf2^2));
% Find the load current
MagIL = abs(S1+S2)/abs(VL);
% Find the power in the line
SW = 2*MagIL^2*ZW;
% Find the total complex power
Ss = S1+S2+SW;
% Find the apparent source power
MagSs = abs(Ss)
% Find the rms value of the source voltage
MagVs = MagSs/MagIL
MagSs =
    41.6762e+003
MagVs =
     4.0608e+003
```

Answers:

The apparent power produced by the source is 41.6762 kVA.

The rms value of the source voltage is 4.0608 kV.

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Problem 16-27

A 60-Hz voltage source feeds a two-wire line with $Z_W = 0.6 + j3.4\ \Omega$ per wire. The load at the receiving end of the line draws an apparent power of 5 kVA at a leading power factor 0.8. The voltage across the load is 500 V(rms). Find the apparent power produced by the source and the rms value of the source voltage.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW = 0.6+3.4j;
MagSL = 5e3;
pf = 0.8;
MagVL = 500;
% Find the complex power at the load
SL = MagSL*(pf-j*sqrt(1-pf^2));
% Find the magnitude of the current
MagIL = MagSL/MagVL;
% Find the total complex power
Ss = SL + 2*MagIL^2*ZW;
% Find the apparent source power
MagSs = abs(Ss)
% Find the rms value of the source voltage
MagVs = MagSs/MagIL
MagSs =
     4.7283e+003
MagVs =
   472.8298e+000
```

Answers:

The apparent power produced by the source is 4.7283 kVA.

The rms value of the source voltage is 472.83 V.

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Problem 16-28

In Figure P16-28 the load voltage is $|\mathbf{V}_L| = 5$ kV (rms) at 60 Hz and the load Z_L draws an average power of 10 kW at a lagging power factor of 0.725. Find the value of the parallel capacitance needed to raise the power factor of the combination to unity.

Solution:

Use MATLAB to perform the calculations.

```
clear all
MagVL = 5e3;
PL = 10e3;
pf = 0.725;
% Find the complex power of the load
SL = PL/pf*(pf+j*sqrt(1-pf^2));
% We need the capacitor to cancel the complex power in the load
QC = -imag(SL);
% Solve for the capacitance
C = -QC/2/pi/60/MagVL^2
C =
     1.0080e-006
```

Answer:

$C = 1.008\ \mu$F.

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Problem 16-29

In Figure P16-28 the load voltage is $|\mathbf{V}_L| =$ 440 V(rms) at 60 Hz. The load Z_L draws an apparent power of 880 VA at a lagging power factor of 0.7. Find the value of the capacitance required to raise the power factor of the parallel combination to 0.95. Repeat for a corrected power factor of unity.

Solution:

Use MATLAB to perform the calculations.

```
clear all
MagVL = 440;
MagSL = 880;
pf = 0.7;
% Find the average and reactive power at the load
PL = MagSL*pf;
QL = MagSL*sqrt(1-pf^2);
% Set the new power factor
disp('Correct the power factor to 0.95.')
pfc = 0.95;
% Find the new apparent power
MagSp = PL/pfc;
% Find the new reactive power
Qp = MagSp*sqrt(1-pfc^2);
% Find the required change in reactive power
QC = Qp-QL;
% Solve for the capacitance
C = -QC/2/pi/60/MagVL^2
disp('Correct the power factor to 1.')
pfc = 1;
QC = -QL;
% Solve for the capacitance
C = -QC/2/pi/60/MagVL^2
Correct the power factor to 0.95.
C =
     5.8365e-006
Correct the power factor to 1.
C =
     8.6106e-006
```

Answers:

For a power factor of 0.95, the capacitance is 5.8365 μF.

For a power factor of 1, the capacitance is 8.6106 μF.

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Problem 16-30

In Figure P16-30, $|\mathbf{V}_L|$ = 4.4 kV(rms) at 60 Hz and the load Z_L draws an apparent power of 330 kVA at a lagging power factor of 0.7. Use MATLAB to find the apparent power supplied by the source and the source power factor when C = 25 μF.

Solution:

Use MATLAB to perform the calculations.

```
clear all
MagVL = 4.4e3;
MagSL = 330e3;
pf = 0.7;
C = 25e-6;
ZW = 0.05+2j;
w = 2*pi*60;
QC = -w*C*MagVL^2;
% Find the complex power absorbed by the load and capacitor
SLC = MagSL*(pf+j*sqrt(1-pf^2))+j*QC;
% Find the current
MagIL = abs(SLC)/MagVL;
% Find the total complex power
Ss = SLC + 2*MagIL^2*ZW;
% Find the apparent source power
MagSs = abs(Ss)
% Find the source power factor
pfs = real(Ss)/MagSs
MagSs =
   240.1998e+003
pfs =
   962.9078e-003
```

Answers:

The apparent power supplied by the source is 240.2 kVA.

The source power factor is 0.9629.

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Problem 16-31

In a balanced three-phase circuit the phase voltage magnitude is V_P = 277 V(rms). For a positive phase sequence:

(a) Find all of the line and phase voltage phasors using $\mathbf{V}_{AN}$ as the phase reference.

(b) Sketch a phasor diagram of the line and phase voltages.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VP = 277;
% Positive phase sequence
VAN = VP*exp(0*j*pi/180);
VBN = VAN*exp(-j*2*pi/3);
VCN = VAN*exp(-j*4*pi/3);
VAB = sqrt(3)*VAN*exp(j*pi/6);
VBC = VAB*exp(-j*2*pi/3);
VCA = VAB*exp(-j*4*pi/3);
% Find the magnitude and phase of each voltage
MagVAN = abs(VAN)
MagVBN = abs(VBN)
MagVCN = abs(VCN)
PhaseVAN = 180*angle(VAN)/pi
PhaseVBN = 180*angle(VBN)/pi
PhaseVCN = 180*angle(VCN)/pi
MagVAB = abs(VAB)
MagVBC = abs(VBC)
MagVCA = abs(VCA)
PhaseVAB = 180*angle(VAB)/pi
PhaseVBC = 180*angle(VBC)/pi
PhaseVCA = 180*angle(VCA)/pi
MagVAN =
   277.0000e+000
MagVBN =
   277.0000e+000
MagVCN =
   277.0000e+000
PhaseVAN =
     0.0000e+000
PhaseVBN =
  -120.0000e+000
PhaseVCN =
   120.0000e+000
MagVAB =
   479.7781e+000
MagVBC =
   479.7781e+000
MagVCA =
   479.7781e+000
PhaseVAB =
    30.0000e+000
PhaseVBC =
```

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```
   -90.0000e+000
PhaseVCA =
   150.0000e+000
```

```
% Create the phase diagrams
close all
line([0,real(VAN)],[0,imag(VAN)],'LineWidth
',3,'Color','g')
line([0,real(VBN)],[0,imag(VBN)],'LineWidth
',3,'Color','b')
line([0,real(VCN)],[0,imag(VCN)],'LineWidth
',3,'Color','r')
grid on
axis(300*[-1 1 -1 1])
xlabel('Real')
ylabel('Imaginary')
title('Phase Voltages')
legend('V_{AN}','V_{BN}','V_{CN}')
```

```
close all
line([0,real(VAB)],[0,imag(VAB)],'LineWidth',3,'Color','g')
line([0,real(VBC)],[0,imag(VBC)],'LineWidth',3,'Color','b')
line([0,real(VCA)],[0,imag(VCA)],'LineWidth',3,'Color','r')
grid on
axis(500*[-1 1 -1 1])
xlabel('Real')
ylabel('Imaginary')
title('Line Voltages')
legend('V_{AB}','V_{BC}','V_{CA}')
```

Answers:

(a)

Signal	Amplitude (V)	Angle (degrees)
$\mathbf{V}_{AN}$	277	0
$\mathbf{V}_{BN}$	277	−120
$\mathbf{V}_{CN}$	277	120
$\mathbf{V}_{AB}$	480	30
$\mathbf{V}_{BC}$	480	−90
$\mathbf{V}_{CA}$	480	150

(b) The phasor diagrams are shown above.

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Problem 16-32

In a balanced three-phase circuit the line voltage magnitude is V_L = 2.4 kV(rms). For a positive phase sequence:

(a) Find all of the line and phase voltage phasors using $\mathbf{V}_{AB}$ as the phase reference.

(b) Sketch a phasor diagram of the line and phase voltages.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VL = 2.4e3;
% Positive phase sequence
VAB = VL*exp(0*j*pi/180);
VBC = VAB*exp(-j*2*pi/3);
VCA = VAB*exp(-j*4*pi/3);
VAN = VAB/sqrt(3)*exp(-j*pi/6);
VBN = VAN*exp(-j*2*pi/3);
VCN = VAN*exp(-j*4*pi/3);
% Find the magnitude and phase of each voltage
MagVAN = abs(VAN)
MagVBN = abs(VBN)
MagVCN = abs(VCN)
PhaseVAN = 180*angle(VAN)/pi
PhaseVBN = 180*angle(VBN)/pi
PhaseVCN = 180*angle(VCN)/pi
MagVAB = abs(VAB)
MagVBC = abs(VBC)
MagVCA = abs(VCA)
PhaseVAB = 180*angle(VAB)/pi
PhaseVBC = 180*angle(VBC)/pi
PhaseVCA = 180*angle(VCA)/pi
MagVAN =
     1.3856e+003
MagVBN =
     1.3856e+003
MagVCN =
     1.3856e+003
PhaseVAN =
   -30.0000e+000
PhaseVBN =
  -150.0000e+000
PhaseVCN =
    90.0000e+000
MagVAB =
     2.4000e+003
MagVBC =
     2.4000e+003
MagVCA =
     2.4000e+003
PhaseVAB =
     0.0000e+000
PhaseVBC =
```

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```
  -120.0000e+000
PhaseVCA =
   120.0000e+000

% Create the phase diagrams
close all
line([0,real(VAN)],[0,imag(VAN)],'LineWidth',3,'Color','g')
line([0,real(VBN)],[0,imag(VBN)],'LineWidth',3,'Color','b')
line([0,real(VCN)],[0,imag(VCN)],'LineWidth',3,'Color','r')
grid on
axis(1500*[-1 1 -1 1])
xlabel('Real')
ylabel('Imaginary')
title('Phase Voltages')
legend('V_{AN}','V_{BN}','V_{CN}')


close all
line([0,real(VAB)],[0,imag(VAB)],'LineWid
th',3,'Color','g')
line([0,real(VBC)],[0,imag(VBC)],'LineWid
th',3,'Color','b')
line([0,real(VCA)],[0,imag(VCA)],'LineWid
th',3,'Color','r')
grid on
axis(2400*[-1 1 -1 1])
xlabel('Real')
ylabel('Imaginary')
title('Line Voltages')
legend('V_{AB}','V_{BC}','V_{CA}')
```

Answers:

(a)

Signal	Amplitude (V)	Angle (degrees)
$\mathbf{V}_{AN}$	1386	−30
$\mathbf{V}_{BN}$	1386	−150
$\mathbf{V}_{CN}$	1386	90
$\mathbf{V}_{AB}$	2400	0
$\mathbf{V}_{BC}$	2400	−120
$\mathbf{V}_{CA}$	2400	120

(b) The phasor diagrams are shown above.

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Problem 16-33

In a balanced three-phase circuit $\mathbf{V}_{BC} = j208$ V(rms). Find all the line and phase voltage phasors in polar form for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VBC = 208j;
% Positive phase sequence
VAB = VBC*exp(j*2*pi/3);
VCA = VAB*exp(-j*4*pi/3);
VAN = VAB/sqrt(3)*exp(-j*pi/6);
VBN = VAN*exp(-j*2*pi/3);
VCN = VAN*exp(-j*4*pi/3);
% Find the magnitude and phase of each voltage
MagVAN = abs(VAN);
MagVBN = abs(VBN);
MagVCN = abs(VCN);
PhaseVAN = 180*angle(VAN)/pi;
PhaseVBN = 180*angle(VBN)/pi;
PhaseVCN = 180*angle(VCN)/pi;
MagVAB = abs(VAB);
MagVBC = abs(VBC);
MagVCA = abs(VCA);
PhaseVAB = 180*angle(VAB)/pi;
PhaseVBC = 180*angle(VBC)/pi;
PhaseVCA = 180*angle(VCA)/pi;
Results = [MagVAN MagVBN MagVCN MagVAB MagVBC MagVCA;...
           PhaseVAN PhaseVBN PhaseVCN PhaseVAB PhaseVBC PhaseVCA]'
Results =
   120.0889e+000   180.0000e+000
   120.0889e+000    60.0000e+000
   120.0889e+000   -60.0000e+000
   208.0000e+000  -150.0000e+000
   208.0000e+000    90.0000e+000
   208.0000e+000   -30.0000e+000
```

Answer:

Signal	Amplitude (V)	Angle (degrees)
$\mathbf{V}_{AN}$	120	180
$\mathbf{V}_{BN}$	120	60
$\mathbf{V}_{CN}$	120	−60
$\mathbf{V}_{AB}$	208	−150
$\mathbf{V}_{BC}$	208	90
$\mathbf{V}_{CA}$	208	−30

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Problem 16-34

In a balanced three-phase circuit $\mathbf{V}_{AN} = 300 + j400$ V(rms). Find all the line and phase voltage phasors in polar form for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VAN = 300+400j;
% Positive phase sequence
VBN = VAN*exp(-j*2*pi/3);
VCN = VAN*exp(-j*4*pi/3);
VAB = sqrt(3)*VAN*exp(j*pi/6);
VBC = VAB*exp(-j*2*pi/3);
VCA = VAB*exp(-j*4*pi/3);
% Find the magnitude and phase of each voltage
MagVAN = abs(VAN);
MagVBN = abs(VBN);
MagVCN = abs(VCN);
PhaseVAN = 180*angle(VAN)/pi;
PhaseVBN = 180*angle(VBN)/pi;
PhaseVCN = 180*angle(VCN)/pi;
MagVAB = abs(VAB);
MagVBC = abs(VBC);
MagVCA = abs(VCA);
PhaseVAB = 180*angle(VAB)/pi;
PhaseVBC = 180*angle(VBC)/pi;
PhaseVCA = 180*angle(VCA)/pi;
Results = [MagVAN MagVBN MagVCN MagVAB MagVBC MagVCA;...
           PhaseVAN PhaseVBN PhaseVCN PhaseVAB PhaseVBC PhaseVCA]'
Results =
   500.0000e+000    53.1301e+000
   500.0000e+000   -66.8699e+000
   500.0000e+000   173.1301e+000
   866.0254e+000    83.1301e+000
   866.0254e+000   -36.8699e+000
   866.0254e+000  -156.8699e+000
```

Answer:

Signal	Amplitude (V)	Angle (degrees)
$\mathbf{V}_{AN}$	500	53.13
$\mathbf{V}_{BN}$	500	−66.87
$\mathbf{V}_{CN}$	500	173.13
$\mathbf{V}_{AB}$	866	83.13
$\mathbf{V}_{BC}$	866	−36.87
$\mathbf{V}_{CA}$	866	−156.87

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Problem 16-35

A balanced Δ-connected three-phase source has $\mathbf{V}_{AB} = 208\angle 25°$ V(rms) and a positive phase sequence. Find the three source voltages of the equivalent Y-connected source.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VAB = 208*exp(25*j*pi/180);
VAN = VAB/sqrt(3)*exp(-j*pi/6);
VBN = VAN*exp(-j*2*pi/3);
VCN = VAN*exp(-j*4*pi/3);
% Find the three sources voltages in an equivalent wye source
MagVAN = abs(VAN);
MagVBN = abs(VBN);
MagVCN = abs(VCN);
PhaseVAN = 180*angle(VAN)/pi;
PhaseVBN = 180*angle(VBN)/pi;
PhaseVCN = 180*angle(VCN)/pi;
Results = [MagVAN MagVBN MagVCN;...
           PhaseVAN PhaseVBN PhaseVCN]'
Results =
   120.0889e+000    -5.0000e+000
   120.0889e+000  -125.0000e+000
   120.0889e+000   115.0000e+000
```

Answer:

Signal	Amplitude (V)	Angle (degrees)
$\mathbf{V}_{AN}$	120	−5
$\mathbf{V}_{BN}$	120	−125
$\mathbf{V}_{CN}$	120	115

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Problem 16-36

In a balanced three-phase circuit $\mathbf{V}_{AN} = V_P\angle 0°$. Show that $\mathbf{V}_{AB} = \sqrt{3}\, V_P\angle -30°$ when the phase sequence is negative. That is, show that $\mathbf{V}_{AB}$ lags $\mathbf{V}_{AN}$ by 30° when the phase sequence is ACB.

Solution:

Use MATLAB to perform the calculations

```
clear all
syms VP
VAN = VP*(1+0j);
VBN = VAN*exp(-j*4*pi/3);
VAB = simplify(VAN-VBN)
% Create the expected voltage for VAB
VAB2 = simplify(sqrt(3)*VP*exp(-30*j*pi/180))
% Check that the two are equal
CheckVAB = simplify(VAB-VAB2)
VAB =
3/2*VP-1/2*i*VP*3^(1/2)
VAB2 =
-1/2*VP*3^(1/2)*(-3^(1/2)+i)
CheckVAB =
0
```

Answer:

The MATLAB calculations above verify the results. In a negative phase sequence, $\mathbf{V}_{AB}$ lags $\mathbf{V}_{AN}$ by 30°.

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Problem 16-37

A balanced wye load with $Z_Y = 10 - j5\ \Omega$ per phase is connected in parallel with a balanced delta load with $Z_\Delta = 60 + j15\ \Omega$ per phase. Find the phase impedance of an equivalent delta load.

Solution:

The equivalent delta load is $3Z_Y$ in parallel with Z_Δ. Use MATLAB to perform the calculations.

```
clear all
ZY = 10-5j;
ZD = 60+15j;
ZDeq = 1/(1/(3*ZY) + 1/ZD)
ZDeq =
    22.5000e+000 -  5.0000e+000i
```

Answer:

$Z_{\Delta EQ} = 22.5 - j5\ \Omega$ per phase.

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Problem 16-38

A balanced wye load with $Z_Y = 8 + j6\ \Omega$ per phase is connected in parallel with a balanced delta load with $Z_\Delta = 12 - j18\ \Omega$ per phase. Find the phase impedance of an equivalent wye load.

Solution:

The equivalent wye load is one third of the delta equivalent load of the parallel combination. Use MATLAB to perform the calculations.

```
clear all
ZY = 8+6j;
ZD = 12-18j;
ZDeq = 1/(1/(3*ZY) + 1/ZD);
ZYeq = ZDeq/3
ZYeq =
     5.6667e+000 -  2.0000e+000i
```

Answer:

$Z_{YEQ} = 5.667 - j2\ \Omega$ per phase.

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Problem 16-39

A balanced wye load with $Z_{Y1} = 8 + j5\ \Omega$ per phase is connected in parallel with a second wye load with $Z_{Y2} = 24 + j15\ \Omega$ per phase. Find the phase impedance of the equivalent wye load.

Solution:

The impedance of the equivalent wye load is one third of the parallel combination of the equivalent delta loads. Use MATLAB to perform the calculations.

```
clear all
ZY1 = 8+5j;
ZY2 = 24+15j;
ZDeq = 1/(1/(3*ZY1) + 1/(3*ZY2));
ZYeq = ZDeq/3
ZYeq =
     6.0000e+000 +  3.7500e+000i
```

Answer:

$Z_{YEQ} = 6 + j3.75\ \Omega$ per phase.

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Problem 16-40

In a balanced Δ-Δ circuit the source produces $V_{AB} = 440\angle 35°$ V(rms) and a positive phase sequence. The phase impedance of the load is $Z_\Delta = 12 + j15\ \Omega$ per phase. Find the three source voltages and phase impedance in the equivalent Y-Y circuit.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VAB = 440*exp(35*j*pi/180);
ZD = 12+15j;
VAN = VAB/sqrt(3)*exp(-j*pi/6);
VBN = VAN*exp(-j*2*pi/3);
VCN = VAN*exp(-j*4*pi/3);
MagVAN = abs(VAN);
MagVBN = abs(VBN);
MagVCN = abs(VCN);
PhaseVAN = 180*angle(VAN)/pi;
PhaseVBN = 180*angle(VBN)/pi;
PhaseVCN = 180*angle(VCN)/pi;
Results = [MagVAN MagVBN MagVCN;...
           PhaseVAN PhaseVBN PhaseVCN]'
ZY = ZD/3
Results =
   254.0341e+000     5.0000e+000
   254.0341e+000  -115.0000e+000
   254.0341e+000   125.0000e+000
ZY =
     4.0000e+000 +  5.0000e+000i
```

Answers:

Signal	Amplitude (V)	Angle (degrees)
$\mathbf{V}_{AN}$	254	5
$\mathbf{V}_{BN}$	254	−115
$\mathbf{V}_{CN}$	254	125

$Z_Y = 4 + j5\ \Omega$ per phase.

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Problem 16-41

In a balanced Y-Y circuit the line voltage and phase impedance are V_L = 480 V (rms) and Z_Y = 20 + j10 Ω per phase. Using $\angle \mathbf{V}_{AN} = 0°$ as the phase reference: find the line current and line voltage phasors in polar form for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VL = 480;
ZY = 20+10j;
VAN = VL/sqrt(3)*exp(j*0);
IA = VAN/ZY;
IB = IA*exp(-j*2*pi/3);
IC = IB*exp(-j*2*pi/3);
VAB = sqrt(3)*VAN*exp(j*pi/6);
VBC = VAB*exp(-j*2*pi/3);
VCA = VAB*exp(-j*4*pi/3);
% Find the magnitude and phase
MagIA = abs(IA);
MagIB = abs(IB);
MagIC = abs(IC);
PhaseIA = 180*angle(IA)/pi;
PhaseIB = 180*angle(IB)/pi;
PhaseIC = 180*angle(IC)/pi;
MagVAB = abs(VAB);
MagVBC = abs(VBC);
MagVCA = abs(VCA);
PhaseVAB = 180*angle(VAB)/pi;
PhaseVBC = 180*angle(VBC)/pi;
PhaseVCA = 180*angle(VCA)/pi;
Results = [MagIA MagIB MagIC MagVAB MagVBC MagVCA;...
           PhaseIA PhaseIB PhaseIC PhaseVAB PhaseVBC PhaseVCA]'
Results =
    12.3935e+000   -26.5651e+000
    12.3935e+000  -146.5651e+000
    12.3935e+000    93.4349e+000
   480.0000e+000    30.0000e+000
   480.0000e+000   -90.0000e+000
   480.0000e+000   150.0000e+000
```

Answer:

Signal	Amplitude	Angle (degrees)
$\mathbf{I}_A$	12.39 A	−26.57
$\mathbf{I}_B$	12.39 A	−146.57
$\mathbf{I}_C$	12.39 A	93.43
$\mathbf{V}_{AB}$	480 V	30
$\mathbf{V}_{BC}$	480 V	−90
$\mathbf{V}_{CA}$	480 V	150

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Problem 16-42

In a balanced Y-Y circuit the line voltage is V_L = 680 V (rms). The line and load impedances are $Z_W = 3 + j15\ \Omega$ per phase and $Z_Y = 50 + j40\ \Omega$ per phase. Using $\angle \mathbf{I}_A = 0°$ as the phase reference, find $\mathbf{I}_A$ and $\mathbf{V}_{AB}$ in polar form for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VL = 680;
ZW = 3+15j;
ZY = 50+40j;
VP = VL/sqrt(3);
IA = abs(VP)/abs(ZY)*exp(j*0);
MagIA = abs(IA)
PhaseIA = 180*angle(IA)/pi
VAN = IA*ZY;
VAB = sqrt(3)*VAN*exp(j*pi/6);
MagVAB = abs(VAB)
PhaseVAB = 180*angle(VAB)/pi
MagIA =
     6.1314e+000
PhaseIA =
     0.0000e+000
MagVAB =
   680.0000e+000
PhaseVAB =
    68.6598e+000
```

Answer:

Signal	Amplitude	Angle (degrees)
$\mathbf{I}_A$	6.13 A	0
$\mathbf{V}_{AB}$	680 V	68.66

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Problem 16-43

In a balanced Y-Δ circuit the line voltage and phase impedance are V_L = 880 V (rms) and Z_Δ = 16 + j12 Ω per phase. Using $\angle \mathbf{V}_{AB} = 0°$ as the phase reference, find the line current and phase current phasors in polar form for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VL = 880;
ZD = 16+12j;
VAN = VL/sqrt(3)*exp(-j*pi/6);
ZY = ZD/3;
IA = VAN/ZY;
IB = IA*exp(-j*2*pi/3);
IC = IA*exp(-j*4*pi/3);
IAB = IA/sqrt(3)*exp(j*pi/6);
IBC = IAB*exp(-j*2*pi/3);
ICA = IAB*exp(-j*4*pi/3);
% Find the magnitude and phase
MagIA = abs(IA);
MagIB = abs(IB);
MagIC = abs(IC);
PhaseIA = 180*angle(IA)/pi;
PhaseIB = 180*angle(IB)/pi;
PhaseIC = 180*angle(IC)/pi;
MagIAB = abs(IAB);
MagIBC = abs(IBC);
MagICA = abs(ICA);
PhaseIAB = 180*angle(IAB)/pi;
PhaseIBC = 180*angle(IBC)/pi;
PhaseICA = 180*angle(ICA)/pi;
Results = [MagIA MagIB MagIC MagIAB MagIBC MagICA;...
           PhaseIA PhaseIB PhaseIC PhaseIAB PhaseIBC PhaseICA]'
Results =
    76.2102e+000   -66.8699e+000
    76.2102e+000   173.1301e+000
    76.2102e+000    53.1301e+000
    44.0000e+000   -36.8699e+000
    44.0000e+000  -156.8699e+000
    44.0000e+000    83.1301e+000
```

Answer:

Signal	Amplitude (A)	Angle (degrees)
$\mathbf{I}_A$	76.2	−66.9
$\mathbf{I}_B$	76.2	173.1
$\mathbf{I}_C$	76.2	53.1
$\mathbf{I}_{AB}$	44	−36.9
$\mathbf{I}_{BC}$	44	−156.9

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$\mathbf{I}_{CA}$	44	83.1

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Problem 16-44

In a balanced Y-Δ circuit the line and load impedances are $Z_W = 1.5 + j6.5\ \Omega$ per phase and $Z_\Delta = 15 + j8\ \Omega$ per phase. The line voltage at the source is $V_L = 250$ V (rms). Using $\angle \mathbf{V}_{AB} = 0°$ as the phase reference, find the line current $\mathbf{I}_A$ and phase current $\mathbf{I}_{AB}$ for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW = 1.5+6.5j;
ZD = 15+8j;
VL = 250;
VAB = VL*exp(j*0);
VAN = VAB/sqrt(3)*exp(-j*pi/6)
ZY = ZD/3;
IA = VAN/(ZW+ZY);
IAB = IA/sqrt(3)*exp(j*pi/6);
% Find the magnitude and phase
MagIA = abs(IA)
PhaseIA = 180*angle(IA)/pi
MagIAB = abs(IAB)
PhaseIAB = 180*angle(IAB)/pi
VAN =
   125.0000e+000 - 72.1688e+000i
MagIA =
    12.8445e+000
PhaseIA =
   -84.6599e+000
MagIAB =
     7.4158e+000
PhaseIAB =
   -54.6599e+000
```

Answer:

Signal	Amplitude (A)	Angle (degrees)
$\mathbf{I}_A$	12.84	−84.66
$\mathbf{I}_{AB}$	7.42	−54.66

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Problem 16-45

In a balanced Y-Δ circuit the line voltage and phase impedance are V_L = 2.4 kV (rms) and Z_Δ = 250∠30°. Using ∠$\mathbf{V}_{AN}$ = 0° as the phase reference, find the phase current $\mathbf{I}_{AB}$ and line current $\mathbf{I}_A$ for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VL = 2.4e3;
ZD = 250*exp(30*j*pi/180);
VAB = VL*exp(j*pi/6);
IAB = VAB/ZD;
IA = sqrt(3)*IAB*exp(-j*pi/6);
% Find the magnitude and phase
MagIA = abs(IA)
PhaseIA = 180*angle(IA)/pi
MagIAB = abs(IAB)
PhaseIAB = 180*angle(IAB)/pi
MagIA =
    16.6277e+000
PhaseIA =
   -30.0000e+000
MagIAB =
     9.6000e+000
PhaseIAB =
     0.0000e+000
```

Answer:

Signal	Amplitude (A)	Angle (degrees)
$\mathbf{I}_A$	16.63	−30
$\mathbf{I}_{AB}$	9.6	0

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Problem 16-46

A balanced three-phase source with $\mathbf{V}_{AB} = 440\angle 60°$ supplies a balanced Δ-connected load with a phase impedance of $Z_\Delta = 15\angle -30°$. Find the phase current $\mathbf{I}_{AB}$ and line current $\mathbf{I}_A$ for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VAB = 440*exp(60*j*pi/180);
ZD = 15*exp(-30*j*pi/180);
IAB = VAB/ZD;
IA = sqrt(3)*IAB*exp(-j*pi/6);
% Find the magnitude and phase
MagIA = abs(IA)
PhaseIA = 180*angle(IA)/pi
MagIAB = abs(IAB)
PhaseIAB = 180*angle(IAB)/pi
MagIA =
    50.8068e+000
PhaseIA =
    60.0000e+000
MagIAB =
    29.3333e+000
PhaseIAB =
    90.0000e+000
```

Answer:

Signal	Amplitude (A)	Angle (degrees)
$\mathbf{I}_A$	50.81	60
$\mathbf{I}_{AB}$	29.33	90

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Problem 16-47

In a balanced wye load the line current and load impedance are I_L = 6 A(rms) and $Z_Y = 25 + j15$ Ω per phase. Using $\angle \mathbf{V}_{AN} = 0°$ as the phase reference, find the line current $\mathbf{I}_A$ and line voltage $\mathbf{V}_{AB}$ for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
IL = 6;
ZY = 25+15j;
VAN = IL*abs(ZY);
VAB = sqrt(3)*VAN*exp(j*pi/6);
IA = VAN/ZY;
% Find the magnitude and phase
MagIA = abs(IA)
PhaseIA = 180*angle(IA)/pi
MagVAB = abs(VAB)
PhaseVAB = 180*angle(VAB)/pi
MagIA =
     6.0000e+000
PhaseIA =
   -30.9638e+000
MagVAB =
   302.9851e+000
PhaseVAB =
    30.0000e+000
```

Answer:

Signal	Amplitude	Angle (degrees)
$\mathbf{I}_A$	6 A	−30.96
$\mathbf{V}_{AB}$	303 V	30

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Problem 16-48

In a balanced delta load the phase current and load impedance are I_P = 12 A(rms) and Z_Δ = 200∠60°. Using ∠$\mathbf{V}_{AN}$ = 0° as the phase reference, find the line current $\mathbf{I}_A$ and line voltage $\mathbf{V}_{AB}$ for a positive phase sequence.

Solution:

Use MATLAB to perform the calculations.

```
clear all
IP = 12;
ZD = 200*exp(60*j*pi/180);
VL = IP*abs(ZD);
VAB = VL*exp(j*pi/6);
IAB = VAB/ZD;
IA = sqrt(3)*IAB*exp(-j*pi/6);
% Find the magnitude and phase
MagIA = abs(IA)
PhaseIA = 180*angle(IA)/pi
MagVAB = abs(VAB)
PhaseVAB = 180*angle(VAB)/pi
MagIA =
    20.7846e+000
PhaseIA =
   -60.0000e+000
MagVAB =
     2.4000e+003
PhaseVAB =
    30.0000e+000
```

Answer:

Signal	Amplitude	Angle (degrees)
$\mathbf{I}_A$	20.78 A	−60
$\mathbf{V}_{AB}$	2400 V	30

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Problem 16-49

An average power of 5 kW is delivered to a balanced wye load with $Z_Y = 35 + j40\ \Omega$ per phase. Find I_L, V_L, and the reactive power delivered to the load.

Solution:

Use MATLAB to perform the calculations.

```
clear all
PL = 5e3;
ZY = 35+40j;
% Solve for the current
IL = sqrt(PL/3/real(ZY))
% Find the complex power to the load
SL = 3*IL^2*ZY;
% Find the line voltage
VL = abs(SL)/sqrt(3)/IL
% Find the reactive power delivered to the load
QL = imag(SL)
IL =
     6.9007e+000
VL =
   635.2727e+000
QL =
     5.7143e+003
```

Answers:

I_L = 6.9007 A.

V_L = 635.3 V.

Q_L = 5.7143 kVAR.

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Problem 16-50

An apparent power of 12 kVA is delivered to balanced delta load with $Z_\Delta = 120 + j35\ \Omega$ per phase. Find I_L, V_L, and the complex power delivered to the load.

Solution:

Use MATLAB to perform the calculations.

```
clear all
MagSL = 12e3;
ZD = 120+35j;
ZYeq = ZD/3;
IL = sqrt(MagSL/3/abs(ZYeq))
SL = 3*IL^2*ZYeq
VL = MagSL/sqrt(3)/IL
IL =
     9.7980e+000
SL =
    11.5200e+003 +  3.3600e+003i
VL =
   707.1068e+000
```

Answers:

$I_L = 9.798$ A.

$V_L = 707.11$ V.

$S_L = 11.52 + j3.36$ kVA.

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Problem 16-51

In a balanced three-phase circuit the line and load impedances are $Z_W = 0.5 + j2.8\ \Omega$ per phase and $Z_Y = 65 - j20\ \Omega$ per phase. The line voltage at the source is 440 V(rms). Find I_L and the complex power delivered to the load.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW = 0.5+2.8j;
ZY = 65-20j;
VL = 440;
VP = VL/sqrt(3);
IL = VP/abs(ZW+ZY)
SL = 3*IL^2*ZY
IL =
     3.7512e+000
SL =
     2.7439e+003 -844.2922e+000i
```

Answers:

$I_L = 3.7512$ A.

$S_L = 2744 - j844$ VA.

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Problem 16-52

In a balanced three-phase circuit the line and load impedances are $Z_W = 0.3 + j1.8\ \Omega$ per phase and $Z_\Delta = 200 + j100\ \Omega$ per phase. The line voltage at the delta load is 2.4 kV(rms). Find I_L and the complex power delivered to the load.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW = 0.3+1.8j;
ZD = 200+100j;
VL = 2.4e3;
IP = VL/abs(ZD);
IL = sqrt(3)*IP
SL = 3*IP^2*ZD
IL =
    18.5903e+000
SL =
    69.1200e+003 + 34.5600e+003i
```

Answers:

$I_L = 18.5903$ A.

$S_L = 69.12 + j34.56$ kVA.

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Problem 16-53

In a balanced three-phase circuit the line and load impedances are $Z_W = 15 + j85\ \Omega$ per phase and $Z_Y = 180 + j120\ \Omega$ per phase. The line voltage at the wye load is 4.16 kV(rms). Find I_L and the complex power produced by the source.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW = 15+85j;
ZY = 180+120j;
VL = 4.16e3;
VP = VL/sqrt(3);
IL = VP/abs(ZY)
Ss = 3*IL^2*(ZW+ZY)
IL =
    11.1022e+000
Ss =
    72.1067e+003 + 75.8044e+003i
```

Answers:

$I_L = 11.1022$ A.

$S_S = 72.11 + j75.80$ kVA.

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Problem 16-54

In a balanced three-phase circuit the line and load impedances are $Z_W = 2 + j12\ \Omega$ per phase and $Z_Y = 16 + j10\ \Omega$ per phase. The line current is $I_L = 15$ A(rms). Find the complex power produced by the source and the line voltage magnitude at the source.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW = 2+12j;
ZY = 16+10j;
IL = 15;
Ss = 3*IL^2*(ZW+ZY)
VL = abs(Ss)/sqrt(3)/IL
Ss =
    12.1500e+003 + 14.8500e+003i
VL =
   738.5120e+000
```

Answers:

$S_S = 12.15 + j14.85$ kVA.

$V_L = 738.51$ V.

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Problem 16-55

A balanced three-phase source supplies a balanced load through a three-wire line with $Z_W = 2 + j12\ \Omega$ per phase. The average power delivered to the load is 15 kW at a lagging power factor of 0.85. The line voltage at the load is 480 V(rms). Find the complex power produced by the source and the line voltage magnitude at the source.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW = 2+12j;
PL = 15e3;
pf = 0.85;
VL = 480;
SL = PL/pf*(pf+j*sqrt(1-pf^2));
IL = abs(SL)/sqrt(3)/VL;
Ss = SL + 3*IL^2*ZW
VLs = abs(Ss)/sqrt(3)/IL
Ss =
    17.7033e+003 + 25.5159e+003i
VLs =
   844.7196e+000
```

Answers:

$S_S = 17.70 + j25.52$ kVA.

$V_L = 844.72$ V.

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Problem 16-56

A balanced three-phase source supplies a balanced load through a three-wire line with $Z_W = 5 + j30\ \Omega$ per phase. The apparent power delivered to the load is 25 kVA at a lagging power factor of 0.95. The line current is $I_L = 12$ A(rms). Find the complex power produced by the source and the line voltage magnitude at the source.

Solution:

Use MATLAB to perform the calculations.

```
clear all
format short eng
ZW = 5+30j;
MagSL = 25e3;
pf = 0.95;
IL = 12;
% Find the complex power at the load
SL = MagSL*(pf+j*sqrt(1-pf^2));
% Find the complex power at the source
Ss = SL + 3*IL^2*ZW
% Find the line voltage magnitude at the source
MagVLs = abs(Ss)/sqrt(3)/IL
Ss =
    25.9100e+003 + 20.7662e+003i
MagVLs =
     1.5976e+003
```

Answers:

$S_S = 25.91 + j20.77$ kVA.

$|V_{LS}| = 1.5976$ kV.

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Problem 16-57

In a balanced three-phase circuit the source produces 25 kW at a lagging power factor of 0.75. The line voltage at the source is 4.16 kV(rms) and the 3-wire line connecting to the load has an impedance of $Z_W = 2 + j12\ \Omega$ per phase. Find the complex power delivered to the load and the line voltage magnitude at the load.

Solution:

Use MATLAB to perform the calculations

```
clear all
Ps = 25e3;
pf = 0.75;
VL = 4.16e3;
ZW = 2+12j;
% Find the complex power at the source
Ss = Ps/pf*(pf+j*sqrt(1-pf^2));
% Find the current
IL = abs(Ss)/sqrt(3)/VL;
% Find the complex power at the load
SL = Ss-3*IL^2*ZW
% Find the voltage magnitude at the load
VL = abs(SL)/sqrt(3)/IL
SL =
    24.8716e+003 + 21.2775e+003i
VL =
     4.0848e+003
```

Answers:

$S_L = 24.87 + j21.28$ kVA.

$|V_L| = 4.0848$ kV.

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Problem 16-58

A balanced Y-connected load with $Z_Y = 8 + j6\ \Omega$ is connected in parallel with a balanced Δ-connected load with $Z_\Delta = 12\angle{-40°}\ \Omega$. The line current into the parallel loads is $I_L = 73.5$ A (rms). Find the line voltage and the total complex power delivered to the parallel loads.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZY = 8+6j;
ZD = 12*exp(-40*j*pi/180);
IL = 73.5;
% Find the equivalent Y-load
ZYeq = 1/(3/ZD+1/ZY);
% Find the complex power delivered to the loads
SL = 3*abs(IL)^2*ZYeq
% Find the line voltage
VL = abs(SL)/sqrt(3)/IL
SL =
    52.4733e+003 - 19.4611e+003i
VL =
   439.6179e+000
```

Answers:

$V_L = 439.62$ V.

$S_L = 52.47 - j19.46$ kVA.

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Problem 16-59

The average power delivered to a balanced wye load is 20 kW at a power factor of 0.8. The line voltage at the load is V_L = 480 V(rms). Find the phase impedance of the load.

Solution:

Use MATLAB to perform the calculations.

```
clear all
PL = 20e3;
pf = 0.8;
VL = 480;
% Find the complex power
S = PL/pf*(pf+j*sqrt(1-pf^2));
% Find the current
IL = abs(S)/sqrt(3)/VL;
% Find the load impedance
ZY = S/3/IL^2
ZY =
     7.3728e+000 +  5.5296e+000i
```

Answer:

$Z_Y = 7.3728 + j5.5296\ \Omega$ per phase.

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Problem 16-60

The apparent power delivered to a balanced delta load is 30 kVA at a lagging power factor of 0.72. The line voltage at the load is V_L = 2.4 kV (rms). Find the phase impedance of the load.

Solution:

Use MATLAB to perform the calculations.

```
clear all
PL = 30e3;
pf = 0.72;
VL = 2.4e3;
% Find the complex power
S = PL*(pf+j*sqrt(1-pf^2));
% Find the currents
IL = abs(S)/sqrt(3)/VL;
IP = IL/sqrt(3);
% Find the phase impedance
ZD = S/3/IP^2
ZD =
   414.7200e+000 +399.7291e+000i
```

Answer:

Z_Δ = 414.72 + j399.73 Ω per phase.

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Problem 16-61

In Figure P16-61 the load at bus 2 draws an apparent power of $|S_2|$ = 3 MVA at a lagging power factor of 0.75. The line voltage at bus 2 is V_{L2} = 68 kV (rms) and the line has an impedance of $Z_W = 15 + j100\ \Omega$ per phase. Find complex power produced by the source and the line voltage at bus 1.

Solution:

Use MATLAB to perform the calculations.

```
clear all
MagS2 = 3e6;
pf = 0.75;
VL2 = 68e3;
ZW = 15+100j;
% Find the complex power
S2 = MagS2*(pf+j*sqrt(1-pf^2));
% Find the current
IL = abs(S2)/sqrt(3)/VL2;
% Find the complex power
S1 = S2 + 3*abs(IL)^2*ZW
% Find the line voltage
VL1 = abs(S1)/sqrt(3)/IL
S1 =
     2.2792e+006 +  2.1790e+006i
VL1 =
    71.4721e+003
```

Answers:

S_1 = 2.2792 + j2.179 MVA.

V_{L1} = 71.4721 kV.

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Problem 16-62

In Figure P16-61 the load at bus 2 draws an average power of P_2 = 900 kW at a lagging power factor of 0.8. The line current and line impedance are I_L = 14 A(rms) and Z_W = 10 + j75 Ω per phase. Find the source power factor and the line voltages at bus 1 and bus 2.

Solution:

Use MATLAB to perform the calculations.

```
clear all
P2 = 900e3;
pf = 0.8;
IL = 14;
ZW = 10+75j;
% Find the complex powers
S2 = P2/pf*(pf+j*sqrt(1-pf^2));
S1 = S2 + 3*abs(IL)^2*ZW;
pf1 = real(S1)/abs(S1)
% Find the line voltages
VL1 = abs(S1)/sqrt(3)/abs(IL)
VL2 = abs(S2)/sqrt(3)/abs(IL)
pf1 =
   783.2268e-003
VL1 =
    47.6974e+003
VL2 =
    46.3942e+003
```

Answers:

pf_1 = 0.783.

V_{L1} = 47.6974 kV.

V_{L2} = 46.3942 kV.

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Problem 16-63

In Figure P16-61 the load at bus 2 draws an average power of P_2 = 600 kW at a lagging power factor of 0.8. The source at bus 1 produces an average power of P_1 = 625 kW and the line has an impedance of $Z_W = 10 + j75\ \Omega$ per phase. Find the reactive power produced by the source and line voltages at bus 1 and bus 2.

Solution:

Use MATLAB to perform the calculations.

```
clear all
P2 = 600e3;
pf2 = 0.8;
P1 = 625e3;
ZW = 10+75j;
% Find the complex power S2
S2 = P2/pf2*(pf2+j*sqrt(1-pf2^2));
% Find the line average power and resistance
PW = P1-P2;
RW = real(ZW);
% Find the line current
IL = sqrt(PW/3/RW);
% Find the complex power S1 and reactance Q1
S1 = S2 + 3*abs(IL)^2*ZW;
Q1 = imag(S1)
% Find the line voltages
VL1 = abs(S1)/sqrt(3)/abs(IL)
VL2 = abs(S2)/sqrt(3)/abs(IL)
Q1 =
   637.5000e+003
VL1 =
    17.8553e+003
VL2 =
    15.0000e+003
```

Answers:

Q_1 = 637.5 kVAR.

V_{L1} = 17.8553 kV.

V_{L2} = 15 kV.

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Problem 16-64

In Figure P16-64, the three buses are interconnected by transmission lines with wire impedances of $Z_{W1} = 100 + j600\ \Omega$ per phase and $Z_{W2} = 120 + j800\ \Omega$ per phase. The source at bus 2 produces an apparent power of $|S_2| = 350$ kVA at a lagging power factor of 0.9. The load at bus 3 draws an apparent power of $|S_3| = 600$ kVA at a lagging power factor of 0.8. The line voltage at bus 3 is $V_{L3} = 138$ kV (rms). Use MATLAB to find the apparent power produced by the source at bus 1, the source power factor, and the line voltages at bus 1 and bus 2.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW1 = 100+600j;
ZW2 = 120+800j;
MagS2 = 350e3;
pf2 = 0.9;
MagS3 = 600e3;
pf3 = 0.8;
VL3 = 138e3;
% Find the complex powers are bus 2 and bus 3
S2 = MagS2*(pf2+j*sqrt(1-pf2^2));
S3 = MagS3*(pf3+j*sqrt(1-pf3^2));
% Find the input to line 2 at bus 2
IL2 = MagS3/sqrt(3)/VL3;
SW2 = 3*abs(IL2)^2*ZW2;
VL2 = abs(S3+SW2)/sqrt(3)/IL2
% Find the output of line 1 at bus 2
IL1 = abs(S3+SW2-S2)/sqrt(3)/VL2;
SW1 = 3*abs(IL1)^2*ZW1;
% Find the input to line 1 at bus 1
S1 = S3+SW2-S2+SW1;
MagS1 = abs(S1)
pf1 = real(S1)/MagS1
VL1 = abs(S1)/sqrt(3)/abs(IL1)
% Check the power balance
Check_Balance = S1+S2-SW1-S3-SW2
VL2 =
   140.5260e+003
MagS1 =
   280.5310e+003
pf1 =
   597.6557e-003
VL1 =
   141.5964e+003
Check_Balance =
   -24.5564e-012 - 69.1216e-012i
```

Answers:

$|S_1| = 280.531$ kVA, $\text{pf}_1 = 0.5977$, $V_{L1} = 141.6$ kV, and $V_{L2} = 140.5$ kV.

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Problem 16-65

In Figure P16-64, the three buses are interconnected by transmission lines with wire impedances of $Z_{W1} = 120 + j800\ \Omega$ per phase and $Z_{W2} = 200 + j1200\ \Omega$ per phase. The source at bus 2 produces an apparent power of $|S_2| = 400$ kVA at a lagging power factor of 0.9. The load at bus 3 draws an apparent power of $|S_3| = 650$ kVA at a leading power factor of 0.95. The line current in line 2 is $I_{L2} = 3$ A (rms). Use MATLAB to find the apparent power produced by the source at bus 1, the source power factor, and the line voltages at bus 1, bus 2, and bus 3.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW1 = 120+800j;
ZW2 = 200+1200j;
MagS2 = 400e3;
pf2 = 0.9;
MagS3 = 650e3;
pf3 = 0.95;
IL2 = 3;
% Find the complex powers are bus 2 and bus 3
S2 = MagS2*(pf2+j*sqrt(1-pf2^2));
S3 = MagS3*(pf3-j*sqrt(1-pf3^2));
VL3 = MagS3/sqrt(3)/IL2
SW2 = 3*abs(IL2)^2*ZW2;
VL2 = abs(S3+SW2)/sqrt(3)/IL2
IL1 = abs(S3+SW2-S2)/sqrt(3)/VL2;
SW1 = 3*abs(IL1)^2*ZW1;
S1 = S3+SW2-S2+SW1;
MagS1 = abs(S1)
pf1 = real(S1)/MagS1
VL1 = MagS1/sqrt(3)/abs(IL1)
VL3 =
   125.0926e+003
VL2 =
   124.2900e+003
MagS1 =
   426.8854e+003
pf1 =
   619.2788e-003
VL1 =
   122.3405e+003
```

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Answers:
$|S_1|$ = 426.885 kVA, pf_1 = 0.6193, V_{L1} = 122.3 kV, V_{L2} = 124.3 kV, and V_{L3} = 125.1 kV.

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Problem 16-66

In Figure P16-66, the three buses are interconnected by transmission lines with wire impedances of $Z_{W1} = 100 + j850\ \Omega$ per phase and $Z_{W2} = 50 + j250\ \Omega$ per phase. The load at bus 1 draws an apparent power of $|S_1| = 400$ kVA at a lagging power factor of 0.85. The line voltage at bus 1 is $V_{L1} = 138$ kV (rms). The load at bus 3 draws an apparent power of $|S_3| = 400$ kVA at a lagging power factor of 0.95. The line current in line 2 is $I_{L2} = 1.66$ A(rms). Find the apparent power produced by the source at bus 2, the source power factor, and the line voltages at bus 2 and bus 3.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW1 = 100+850j;
ZW2 = 50+250j;
MagS1 = 400e3;
pf1 = 0.85;
VL1 = 138e3;
MagS3 = 400e3;
pf3 = 0.95;
IL2 = 1.66;
S1 = MagS1*(pf1+j*sqrt(1-pf1^2));
S3 = MagS3*(pf3+j*sqrt(1-pf3^2));
IL1= MagS1/sqrt(3)/VL1;
SW1 = 3*abs(IL1)^2*ZW1;
VL2 = abs(S1+SW1)/sqrt(3)/IL1
VL3 = abs(S3)/sqrt(3)/IL2
SW2 = 3*IL2^2*ZW2;
S2 = S1+SW1+S3+SW2;
MagS2 = abs(S2)
pf2 = real(S2)/MagS2
VL2 =
   139.5578e+003
VL3 =
   139.1205e+003
MagS2 =
   799.4424e+003
pf2 =
   902.1957e-003
```

Answers:

$|S_2| = 799.44$ kVA, $\text{pf}_2 = 0.9022$, $V_{L2} = 139.6$ kV, and $V_{L3} = 139.1$ kV.

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Problem 16-67

In Figure P16-66, the three buses are interconnected by transmission lines with wire impedances of $Z_{W1} = 100 + j800\ \Omega$ per phase and $Z_{W2} = 10 + j50\ \Omega$ per phase. The load at bus 1 draws an apparent power of $|S_1| = 250$ kVA at a lagging power factor of 0.95. The line voltage at bus 1 is $V_{L1} = 115$ kV (rms). The load at bus 3 draws an apparent power of $|S_3| = 500$ kVA at a lagging power factor of 0.75. The line current in line 2 is $I_{L2} = 2.5$ A (rms). Use MATLAB to find the apparent power produced by the source at bus 2, the source power factor, and the line voltages at bus 2 and bus 3.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW1 = 100+800j;
ZW2 = 10+50j;
MagS1 = 250e3;
pf1 = 0.95;
VL1 = 115e3;
MagS3 = 500e3;
pf3 = 0.75;
IL2 = 2.5;
S1 = MagS1*(pf1+j*sqrt(1-pf1^2));
S3 = MagS3*(pf3+j*sqrt(1-pf3^2));
IL1= MagS1/sqrt(3)/VL1;
SW1 = 3*abs(IL1)^2*ZW1;
VL2 = abs(S1+SW1)/sqrt(3)/IL1
VL3 = abs(S3)/sqrt(3)/IL2
SW2 = 3*IL2^2*ZW2;
S2 = S1+SW1+S3+SW2;
MagS2 = abs(S2)
pf2 = real(S2)/MagS2
VL2 =
   115.7604e+003
VL3 =
   115.4701e+003
MagS2 =
   739.5588e+003
pf2 =
   829.0890e-003
```

Answers:

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$|S_2|$ = 739.56 kVA, pf_2 = 0.82909, V_{L2} = 115.8 kV, and V_{L3} = 115.5 kV.

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Problem 16-68

In Figure P16-68, the source at bus 1 supplies two load buses through transmission lines with impedances of $Z_{W1} = 30 + j165\ \Omega$ per phase and $Z_{W2} = 40 + j240\ \Omega$ per phase. The bus 2 load draws an apparent power of $|S_2| = 800$ kVA at a lagging power factor of 0.95. The bus 3 load draws an apparent power of $|S_3| = 600$ kVA at a lagging power factor of 0.8. The line voltage at bus 3 is $V_{L3} = 46$ kV(rms). Use MATLAB to find the complex power produced by the source at bus 1, the source power factor, and the line voltages at bus 1 and bus 2.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW1 = 30+165j;
ZW2 = 40+240j;
MagS2 = 800e3;
pf2 = 0.95;
MagS3 = 600e3;
pf3 = 0.8;
VL3 = 46e3;
S2 = MagS2*(pf2+j*sqrt(1-pf2^2));
S3 = MagS3*(pf3+j*sqrt(1-pf3^2));
IL2 = abs(S3)/sqrt(3)/VL3;
SW2 = 3*abs(IL2)^2*ZW2;
VL2 = abs(S3+SW2)/sqrt(3)/IL2
IL1 = abs(S3+SW2+S2)/sqrt(3)/VL2;
SW1 = 3*abs(IL1)^2*ZW1;
S1 = S3+SW2+S2+SW1
pf1 = real(S1)/abs(S1)
VL1 = abs(S1)/sqrt(3)/abs(IL1)
VL2 =
    48.3453e+003
S1 =
     1.2722e+006 +790.2579e+003i
pf1 =
   849.4542e-003
VL1 =
    51.4839e+003
```

Answers:

$S_1 = 1272 + j790.3$ kVA, $\text{pf}_1 = 0.8495$, $V_{L1} = 51.48$ kV, and $V_{L2} = 48.35$ kV.

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Problem 16-69

In Figure P16-68, the source at bus 1 supplies two load buses through transmission lines with impedances of $Z_{W1} = 4 + j20\ \Omega$ per phase and $Z_{W2} = 3 + j15\ \Omega$ per phase. The bus 2 load draws a complex power of $S_2 = 35 + j10$ MVA. The bus 3 load draws a complex power of $S_3 = 40 + j15$ MVA. The line voltages at bus 3 is $V_{L3} = 161$ kV(rms). Use MATLAB to find the apparent power produced by the source at bus 1, the source power factor, and the line voltages at bus 1 and bus 2.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW1 = 4+20j;
ZW2 = 3+15j;
S2 = 35e6+10e6j;
S3 = 40e6+15e6j;
VL3 = 161e3;
pf2 = real(S2)/abs(S2);
pf3 = real(S3)/abs(S3);
IL2 = abs(S3)/sqrt(3)/VL3;
SW2 = 3*abs(IL2)^2*ZW2;
VL2 = abs(S3+SW2)/sqrt(3)/IL2
IL1 = abs(S3+SW2+S2)/sqrt(3)/VL2;
SW1 = 3*abs(IL1)^2*ZW1;
S1 = S3+SW2+S2+SW1;
MagS1 = abs(S1)
pf1 = real(S1)/abs(S1)
VL1 = MagS1/sqrt(3)/abs(IL1)
VL2 =
   163.1793e+003
MagS1 =
    82.1605e+006
pf1 =
   927.0020e-003
VL1 =
   168.4351e+003
```

Answers:

$|S_1| = 82.16$ MVA, $\text{pf}_1 = 0.927$, $V_{L1} = 168.4$ kV, and $V_{L2} = 163.2$ kV.

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Problem 16-70

In Figure P16-68, the source at bus 1 supplies two load buses through transmission lines with impedances of $Z_{W1} = 6 + j33\ \Omega$ per phase and $Z_{W2} = 3 + j15\ \Omega$ per phase. The load at bus 2 draws a complex power of $S_2 = 3.5 + j1.0$ MVA. The load at bus 3 draws a complex power of $S_3 = 4 + j1.5$ MVA. The line current in line 2 is $I_{L2} = 20$ A(rms). Use MATLAB to find the apparent power produced by the source at bus 1, the source power factor, and the line voltages at bus 1, bus 2 and bus 3.

Solution:

Use MATLAB to perform the calculations.

```
clear all
ZW1 = 6+33j;
ZW2 = 3+15j;
S2 = 3.5e6+1e6j;
S3 = 4e6+1.5e6j;
IL2 = 20;
pf2 = real(S2)/abs(S2);
pf3 = real(S3)/abs(S3);
VL3 = abs(S3)/sqrt(3)/IL2
SW2 = 3*abs(IL2)^2*ZW2;
VL2 = abs(S3+SW2)/sqrt(3)/IL2
IL1 = abs(S3+SW2+S2)/sqrt(3)/VL2;
SW1 = 3*abs(IL1)^2*ZW1;
S1 = S3+SW2+S2+SW1;
MagS1 = abs(S1)
pf1 = real(S1)/abs(S1)
VL1 = MagS1/sqrt(3)/abs(IL1)
VL3 =
   123.3221e+003
VL2 =
   123.6026e+003
MagS1 =
     7.9821e+006
pf1 =
   943.1357e-003
VL1 =
   124.6534e+003
```

Answers:

$|S_1| = 7.9821$ MVA, $\text{pf}_1 = 0.943$, $V_{L1} = 124.7$ kV, $V_{L2} = 123.6$ kV, and $V_{L3} = 123.3$ kV.

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Problem 16-71

Complex Power and Phase Sequence (A)

In a balanced three-phase load $\mathbf{V}_{AB} = 480\angle 30°$ V (rms) and $\mathbf{I}_A = 25\angle -30°$ A (rms). Find the complex power delivered to the load for a positive phase sequence. Repeat for a negative phase sequence. Hint: the result in Problem 16-36 will be useful.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VAB = 480*exp(30*j*pi/180);
IA = 25*exp(-30*j*pi/180);
disp('Positive phase sequence')
% For a positive phase sequence, VAN lags VAB by 30 degrees
VAN = VAB/sqrt(3)*exp(-j*pi/6);
% Impedance of the equivalent Y-load
ZY = VAN/IA;
% Complex power
SL = 3*abs(IA)^2*ZY
disp('Negative phase sequence')
% For a negative phase sequence, VAN leads VAB by 30 degrees
VAN = VAB/sqrt(3)*exp(j*pi/6);
% Impedance of the equivalent Y-load
ZY = VAN/IA;
% Complex power
SL = 3*abs(IA)^2*ZY
Positive phase sequence
SL =
    18.0000e+003 + 10.3923e+003i
Negative phase sequence
SL =
     5.5381e-012 + 20.7846e+003i
```

Answers:

For a positive phase sequence, $S_L = 18 + j10.39$ kVA.

For a negative phase sequence, $S_L = 0 + j20.78$ kVA.

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Problem 16-72

Three-Phase Power Factor Corrections **(A,D)**

The line voltage in Figure P16-72 is V_L = 480 V(rms). The balanced three-phase load operates 8 hours/day and draws an apparent power of 100 kVA at a lagging power factor of 0.8. The electric power supplier charges 7¢/kW-hr when the load power factor is greater than 0.95 and 10¢/kW-hr when the power factor is less than 0.95. The following three-phase capacitor banks are commercially available. The cost of installing a capacitor bank is $20 per kVAR.

480 V 3-Phase Capacitor Equipment

kVAR	Part Number	Unit Price	kVAR	Part Number	Unit Price
10	1N0240A05	$550	40	1N0240A17	$950
20	1N0240A09	$700	50	1N0240A19	$1100
30	1N0240A11	$850	60	1N0240A23	$1500

If one or more capacitor banks are purchased to increase the power factor above 0.95, estimate the time it will take for the accumulated savings in operating costs to equal the cost of purchasing and installing the new equipment.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VL = 480;
MagSL = 100e3;
pf = 0.8;
% Find the complex power at the load
SL = MagSL*(pf+j*sqrt(1-pf^2));
PL = real(SL)
QL = imag(SL)
% Increase the power factor to 0.95
pf = 0.95;
SLc = PL/pf*exp(j*acos(pf));
QC = imag(SLc)-QL
PL =
    80.0000e+003
QL =
    60.0000e+003
QC =
   -33.7053e+003
```

To increase the power factor to 0.95, we need a capacitive reactance of 33.71 kVAR. Use part 1N0240A17, with 40 kVAR at a cost of $950.

```
% Calculate the cost to install
CostInstall = 950+20*40

% Calculate the break-even time
% Savings per day
SavingsPerDay = PL/1000*8*(0.1-0.07)
DaysToSave = CostInstall/SavingsPerDay
CostInstall =
     1.7500e+003
SavingsPerDay =
    19.2000e+000
```

Answer:

It takes slightly more than 91 days to recover the cost for the equipment.

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```
DaysToSave =
    91.1458e+000
```

Problem 16-73

Three-Phase Transformer **(A)**

Figure P16-73 shows three identical ideal transformers connected to form a three-phase transformer. The windings on the primary (source) side are connected in delta and the windings on the secondary (load) side are connected in wye. On the secondary side of the transformer V_L = 600 V(rms) and I_L = 20 A(rms). The turns ratio of each transformer is n = 0.1. Find the line voltage and line current on the primary side.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VL2 = 600;
IL2 = 20;
n = 0.1;
% The voltage across each secondary winding is the phase voltage
VP2 = VL2/sqrt(3);
% The voltage across each primary winding is the line voltage
VL1 = VP2/n
% The current through each primary winding is the phase current
IP1 = n*IL2;
% The current on the primary side is
IL1 = sqrt(3)*IP1
VL1 =
     3.4641e+003
IL1 =
     3.4641e+000
```

Answers:

V_{L1} = 3.4641 kV (rms).

I_{L1} = 3.4641 A (rms).

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Problem 16-74

Phase Converter Efficiency **(A)**

Three-phase motors are often used in equipment because they are more efficient and reliable than single-phase motors. Such equipment may be installed in locations where only single-phase power is available and the cost of installing three-phase service is prohibitive. The rotary phase converter in Figure P16-74 is one way of providing three-phase power from a single-phase source. Simply stated, the converter is a rotating transformer that shifts the phase of a portion of the single-phase input to produce a balanced set of three-phase voltages. In a certain application a converter supplies three-phase power to a 30-hp motor (1 hp = 746 W) that is 85% efficiency at full load. At full load the single-phase inputs are 220 V (rms) and 130 A (rms) at a power factor of 0.95 lagging. Find the efficiency of the converter.

Solution:

Use MATLAB to perform the calculations.

```
clear all
% Find the electrical input power for the 3-phase motor
P3 = 30*746/0.85
% Single phase input
V1 = 220;
I1 = 130;
pf = 0.95;
% Single-phase complex input power
S1 = V1*I1*(pf+j*sqrt(1-pf^2))
% Efficiency
eta = P3/real(S1)
P3 =
    26.3294e+003
S1 =
    27.1700e+003 +  8.9303e+003i
eta =
   969.0619e-003
```

Answer:

The efficiency of the converter is 96.9%.

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Problem 16-75

Three-phase Voltages in the Time Domain (A)

In a balanced 60-Hz three-phase system $\mathbf{V}_{AB} = 147\angle{-23.1°}$ V(rms). For a positive phase sequence, derive a time-domain expression for the phase voltage $v_{AN}(t)$.

Solution:

Use MATLAB to perform the calculations.

```
clear all
VAB = 147*exp(-23.1*j*pi/180);
VAN = VAB/sqrt(3)*exp(-j*pi/6);
MagVAN = abs(VAN)
PhaseVAN = 180*angle(VAN)/pi
syms t
f = 60;
vANt = sqrt(2)*MagVAN*cos(2*pi*f*t-53.1);
vANt_num = vpa(vANt,3)
MagVAN =
    84.8705e+000
PhaseVAN =
   -53.1000e+000
vANt_num =
120.*cos(377.*t-53.1)
```

Answer:

$v_{AN}(t) = 120\cos(377t - 53.1°)$ V.

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Problem 17-1
Find the *z*-parameters of the two-port network in Figure P17-1.

Solution:
Use MATLAB to perform the calculations.

```
clear all
format short eng
z11 = 100 + 1/(1/600 + 1/(400+200));
z21 = 200*(1/(400+200)/(1/600 + 1/(400+200)));
z12 = 600*(1/(600+400)/(1/(600+400) + 1/200));
z22 = 1/(1/(600+400) + 1/200);
z = [z11, z12; z21, z22]
z =
   400.0000e+000   100.0000e+000
   100.0000e+000   166.6667e+000
```

Answers:
z_{11} = 400 Ω
z_{12} = 100 Ω
z_{21} = 100 Ω
z_{22} = 166.67 Ω

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Problem 17-2
Find the *y*-parameters of the two-port network in Figure P17-1.

Solution:
Use MATLAB to perform the calculations.

```
clear all
y11 = 1/(100 + 1/(1/600 + 1/400));
y21 = (-600/1000)/(100 + 1/(1/600 + 1/400));
y12 = y21;
y22 = 1/200 + 1/(400 + 1/(1/100 + 1/600));
y = [y11, y12; y21, y22]
y =
     2.9412e-003    -1.7647e-003
    -1.7647e-003     7.0588e-003
```

Answers:
y_{11} = 2.9412 mS
y_{12} = −1.7647 mS
y_{21} = −1.7647 mS
y_{22} = 7.0588 mS

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Problem 17-3
Find the *z*-parameters of the two-port network in Figure P17-3.

Solution:
Use MATLAB to perform the calculations.

```
clear all
z11 = 1/(1/50j + 1/(100-100j));
z21 = 100*50j/(50j+100-100j);
z12 = 50j*100/(100+50j-100j);
z22 = 1/(1/(50j-100j) + 1/100);
z = [z11, z12; z21, z22]
z =
    20.0000e+000 + 60.0000e+000i   -20.0000e+000 + 40.0000e+000i
   -20.0000e+000 + 40.0000e+000i    20.0000e+000 - 40.0000e+000i
```

Answers:
$z_{11} = 20 + j60\ \Omega$
$z_{12} = -20 + j40\ \Omega$
$z_{21} = -20 + j40\ \Omega$
$z_{22} = 20 - j40\ \Omega$

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Problem 17-4
Find the *y*-parameters of the two-port network in Figure P17-3.

Solution:
Use MATLAB to perform the calculations.

```
clear all
y11 = 1/50j + 1/(-100j);
y21 = (-50j/(50j-100j))/(-100j*50j/(50j-100j));
y12 = y21;
y22 = 1/(-100j) + 1/100;
y = [y11, y12; y21, y22]
y =
     0.0000e+000 - 10.0000e-003i     0.0000e+000 - 10.0000e-003i
     0.0000e+000 - 10.0000e-003i    10.0000e-003 + 10.0000e-003i
```

Answers:
$y_{11} = -j10$ mS
$y_{12} = -j10$ mS
$y_{21} = -j10$ mS
$y_{22} = 10 + j10$ mS

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Problem 17-5
Find the *z*-parameters of the two-port network in Figure P17-5.

Solution:
Use MATLAB to perform the calculations.

```
clear all
syms B V1 V2 I1 I2 R1 R2 R3

V1 = R1*I1 + R2*(1+B)*I1;
z11 = simplify(V1/I1);

V2 = -R3*B*I1;
z21 = simplify(V2/I1);

V1 = 0;
z12 = simplify(V1/I2);

z22 = R3;

z = [z11, z12; z21, z22]
z =
[ R1+R2+R2*B,          0]
[      -R3*B,         R3]
```

Answers:
$z_{11} = R_1 + (1 + \beta)R_2$
$z_{12} = 0$
$z_{21} = -\beta R_3$
$z_{22} = R_3$

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Problem 17-6
Find the *y*-parameters of the two-port network in Figure P17-5.

Solution:
Use MATLAB to perform the calculations.

```
clear all
syms B V1 V2 I1 I2 R1 R2 R3

V1 = R1*I1 + R2*(1+B)*I1;
y11 = simplify(I1/V1);

I2 = B*I1;
V1 = R1*I1 + R2*(1+B)*I1;
y21 = simplify(I2/V1);

I1 = 0;
y12 = simplify(I1/V2);

y22 = 1/R3;

y = [y11, y12; y21, y22]
y =
[ 1/(R1+R2+R2*B),              0]
[ B/(R1+R2+R2*B),           1/R3]
```

Answers:

$$y_{11} = \frac{1}{R_1 + (1+\beta)R_2}$$

$$y_{12} = 0$$

$$y_{21} = \frac{\beta}{R_1 + (1+\beta)R_2}$$

$$y_{22} = \frac{1}{R_3}$$

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Problem 17-7

The z-parameters of a two-port circuit are z_{11} = 1 kΩ and $z_{12} = z_{21} = z_{22}$ = 500 Ω. Find the port currents I_1 and I_2 when a 12-V voltage source is connected at the input port and a 250-Ω resistor is connected at the output port.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2

z11 = 1e3;
z12 = 500;
z21 = 500;
z22 = 500;

Rout = 250;
Vin = 12;

Eqn1 = Vin-z11*I1-z12*I2;
Eqn2 = -Rout*I2-z21*I1-z22*I2;
Soln = solve(Eqn1,Eqn2,I1,I2);
I1 = double(Soln.I1)
I2 = double(Soln.I2)
I1 =
    18.0000e-003
I2 =
   -12.0000e-003
```

Answers:

I_1 = 18 mA

I_2 = −12 mA

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Problem 17-8

The z-parameters of a two-port circuit are z_{11} = 60 kΩ, z_{12} = 0, z_{21} = −250 kΩ, and z_{22} = 5 kΩ. Find the open-circuit (I_2 = 0) voltage gain $T_V = V_2/V_1$.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2

z11 = 60e3;
z12 = 0;
z21 = -250e3;
z22 = 5e3;

I2 = 0;

Eqn1 = V1-z11*I1-z12*I2;
Eqn2 = V2-z21*I1-z22*I2;
Soln = solve(Eqn1,Eqn2,V1,V2);
V1 = simplify(Soln.V1);
V2 = simplify(Soln.V2);
Tv = double(V2/V1)
Tv =
    -4.1667e+000
```

Answer:

T_V = −4.1667

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Problem 17-9

The y-parameters of a two-port circuit are y_{11} = 4 mS, $y_{12} = z_{21}$ = −2 mS, and y_{22} = 2 mS. A 12-V voltage source is connected at the input port, and a 1500-Ω resistor is connected across the output port. Find the port variable responses V_2, I_1, and I_2.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2

y11 = 4e-3;
y12 = -2e-3;
y21 = -2e-3;
y22 = 2e-3;

V1 = 12;
Rout = 1500;

Eqn1 = I1-y11*V1-y12*V2;
Eqn2 = I2 -y21*V1-y22*V2;
Eqn3 = I2+V2/Rout;
Soln = solve(Eqn1,Eqn2,Eqn3,V2,I1,I2);
V2 = double(Soln.V2)
I1 = double(Soln.I1)
I2 = double(Soln.I2)
V2 =
     9.0000e+000
I1 =
    30.0000e-003
I2 =
    -6.0000e-003
```

Answers:

V_2 = 9 V

I_1 = 30 mA

I_2 = −6 mA

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Problem 17-10

The y-parameters of a two-port circuit are $y_{11} = 5 + j20$ mS, $y_{12} = z_{21} = -j20$ mS, and $y_{22} = 0$. Find the input admittance $Y_{IN} = I_1/V_1$ when a 50-Ω resistor is connected at the output port.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2

y11 = 5e-3+20e-3j;
y12 = -20e-3j;
y21 = -20e-3j;
y22 = 0;

Rout = 50;

Eqn1 = I1-y11*V1-y12*V2;
Eqn2 = I2 -y21*V1-y22*V2;
Eqn3 = V2+Rout*I2;
Soln = solve(Eqn1,Eqn2,Eqn3,V1,I1,I2);
V1 = simplify(Soln.V1);
I1 = simplify(Soln.I1);
Yin = double(simplify(I1/V1))
Yin =
    25.0000e-003 + 20.0000e-003i
```

Answer:

$Y_{IN} = 25 + j20$ mS

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Problem 17-11

The output of the two-port network in Figure P17-11 is connected to a load impedance Z_L. Show that the voltage gain $T_V = V_2/V_1$ is

$$T_V = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta_z}$$

where $\Delta_z = z_{11}z_{22} - z_{12}z_{21}$.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 ZL z11 z12 z21 z22

Eqn1 = V1-z11*I1-z12*I2;
Eqn2 = V2-z21*I1-z22*I2;
Eqn3 = V2+ZL*I2;

Soln = solve(Eqn1,Eqn2,Eqn3,V1,V2,I2);
V1 = simplify(Soln.V1);
V2 = simplify(Soln.V2);
Tv = simplify(V2/V1)
Tv =
ZL*z21/(z11*ZL+z11*z22-z12*z21)
```

Answer:

$$T_V = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta_z} \quad \text{where } \Delta_z = z_{11}z_{22} - z_{12}z_{21}.$$

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Problem 17-12

The output of the two-port network in Figure P17-11 is connected to a load impedance Z_L. Show that the current gain $T_I = I_2/I_1$ is

$$T_I = \frac{y_{21}}{y_{11} + \Delta_y Z_L}$$

where $\Delta_y = y_{11}y_{22} - y_{12}y_{21}$.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 ZL y11 y12 y21 y22

Eqn1 = I1-y11*V1-y12*V2;
Eqn2 = I2-y21*V1-y22*V2;
Eqn3 = V2+ZL*I2;

Soln = solve(Eqn1,Eqn2,Eqn3,I1,I2,V2);
I1 = simplify(Soln.I1);
I2 = simplify(Soln.I2);
TI = simplify(I2/I1)
TI =
-y21/(y12*ZL*y21-y11-y11*y22*ZL)
```

Answer:

$$T_I = \frac{y_{21}}{y_{11} + \Delta_y Z_L} \quad \text{where } \Delta_y = y_{11}y_{22} - y_{12}y_{21}.$$

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Problem 17-13
Find the *h*-parameters and the *t*-parameters of the two-port network in Figure P17-13(a).

Solution:
Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 Y

% Set V2 = 0 to solve for h11 and h21
V2 = 0;
V1 = V2;
h11 = simplify(V1/I1);
I2 = -I1;
h21 = simplify(I2/I1);
% Set I1 = 0 to solve for h12 and h22
clear V1 V2
syms V1 V2
I1 = 0;
V1 = V2;
h12 = simplify(V1/V2);
I2 = V2*Y;
h22 = simplify(I2/V2);
h = [h11, h12; h21, h22]

clear all
syms V1 V2 I1 I2 Y
% Set -I2 = 0 to solve for A and C
V2 = V1;
A = simplify(1/(V2/V1));
I1 = V2*Y;
C = simplify(1/(V2/I1));
% Set V2 = 0 to solve for B and D
V2 = 0;
V1 = V2;
B = simplify(V1/(-I2));
I2 = -I1;
D = simplify(1/(-I2/I1));
t = [A, B; C, D]
h =
[  0,  1]
[ -1,  Y]
t =
[ 1, 0]
[ Y, 1]
```

Answer:
$h_{11} = 0$, $h_{12} = 1$, $h_{21} = -1$, $h_{22} = Y$
$A = 1$, $B = 0$, $C = Y$, $D = 1$

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Problem 17-14
Find the *h*-parameters and the *t*-parameters of the two-port network in Figure P17-13(b).

Solution:
Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 Z

% Set V2 = 0 to solve for h11 and h21
V2 = 0;
V1 = Z*I1;
h11 = simplify(V1/I1);
I2 = -I1;
h21 = simplify(I2/I1);
% Set I1 = 0 to solve for h12 and h22
clear V1 V2
syms V1 V2
I1 = 0;
V1 = V2;
h12 = simplify(V1/V2);
I2 = I1;
h22 = simplify(I2/V2);
h = [h11, h12; h21, h22]

clear all
syms V1 V2 I1 I2 Z
% Set -I2 = 0 to solve for A and C
V2 = V1;
A = simplify(1/(V2/V1));
I2 = 0;
I1 = I2;
C = simplify(1/(V2/I1));
clear I1 I2
syms I1 I2
% Set V2 = 0 to solve for B and D
I2 = -I1;
V1 = Z*I1;
B = simplify(V1/(-I2));
D = simplify(1/(-I2/I1));
t = [A, B; C, D]
h =
[  Z,  1]
[ -1,  0]
t =
[ 1, Z]
[ 0, 1]
```

Answer:
$h_{11} = Z$, $h_{12} = 1$, $h_{21} = -1$, $h_{22} = 0$
$A = 1$, $B = Z$, $C = 0$, $D = 1$

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Problem 17-15
Find the *h*-parameters of the two-port network in Figure P17-15.

Solution:
Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 B R1 R2 R3

% Set V2 = 0 to solve for h11 and h21
V1 = (R1+R2)*I1;
h11 = simplify(V1/I1);
I2 = B*I1;
h21 = simplify(I2/I1);
% Set I1 = 0 to solve for h12 and h22
I1 = 0;
V1 = (R1+R2)*I1;
h12 = simplify(V1/V2);
V2 = I2*R3;
h22 = simplify(I2/V2);
h = [h11, h12; h21, h22]
h =
[ R1+R2,     0]
[     B,  1/R3]
```

Answer:
$h_{11} = R_1 + R_2$, $h_{12} = 0$, $h_{21} = \beta$, $h_{22} = 1/R_3$

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Problem 17-16
Find the *t*-parameters of the two-port network in Figure P17-15.

Solution:
Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 B R1 R2 R3

% Set -I2 = 0 to solve for A and C
V1 = (R1+R2)*I1;
V2 = -B*I1*R3;
A = simplify(1/(V2/V1));
C = simplify(1/(V2/I1));
% Set V2 = 0 to solve for B and D
I2 = B*I1;
V1 = (R1+R2)*I1;
B = simplify(1/(-I2/V1));
D = simplify(1/(-I2/I1));
t = [A, B; C, D]
t =
[ -1/B/R3*(R1+R2),    -1/B*(R1+R2)]
[         -1/B/R3,            -1/B]
```

Answers:

$$A = \frac{-(R_1 + R_2)}{\beta R_3}$$

$$B = \frac{-(R_1 + R_2)}{\beta}$$

$$C = \frac{-1}{\beta R_3}$$

$$D = \frac{-1}{\beta}$$

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Problem 17-17
Find the *h*-parameters of the two-port network in Figure P17-17.

Solution:
Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 R1 R2

% Set V2 = 0 to solve for h11 and h21
V1 = R1*I1;
h11 = simplify(V1/I1);
I2 = -(V1-0)/R2;
h21 = simplify(I2/I1);
% Set I1 = 0 to solve for h12 and h22
I1 = 0;
V1 = R1*I1;
h12 = simplify(V1/V2);
V2 = I2*R2;
h22 = simplify(I2/V2);
h = [h11, h12; h21, h22]
h =
[     R1,      0]
[ -R1/R2,   1/R2]
```

Answer:
$h_{11} = R_1$, $h_{12} = 0$, $h_{21} = -R_1/R_2$, $h_{22} = 1/R_2$

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Problem 17-18
Find the *t*-parameters of the two-port network in Figure P17-17.

Solution:
Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 R1 R2

% Set -I2 = 0 to solve for A and C
V1 = V2;
A = simplify(1/(V2/V1));
V2 = R1*I1;
C = simplify(1/(V2/I1));
% Set V2 = 0 to solve for B and D
I2 = (0-V1)/R2;
B = simplify(1/(-I2/V1));
I1 = V1/R1;
D = simplify(1/(-I2/I1));
t = [A, B; C, D]
t =
[       1,      R2]
[    1/R1, 1/R1*R2]
```

Answer:
$A = 1$, $B = R_2$, $C = 1/R_1$, $D = R_2/R_1$

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Problem 17-19

The h-parameters of a two-port network are h_{11} = 500 Ω, h_{12} = 1, h_{21} = −1, and h_{22} = 2 mS. Find the Thévenin equivalent circuit at the output port when a 12-V voltage source is connected at the input port.

Solution:

Find an expression for voltage V_2 in terms of current I_2 to identify the Thévenin parameters as follows:

$$V_2 = V_T + R_T I_2$$

where V_T is the Thévenin voltage and R_T is the Thévenin resistance.

Use MATLAB to perform the calculations

```
clear all
syms V1 V2 I1 I2

h11 = 500;
h12 = 1;
h21 = -1;
h22 = 2e-3;

Eqn1 = V1-h11*I1-h12*V2;
Eqn2 = I2-h21*I1-h22*V2;
Eqn3 = V1-12;
Soln = solve(Eqn1,Eqn2,Eqn3,V1,V2,I1);
V2 = simplify(Soln.V2)
V2 =
6+250*I2
```

Answers:

V_T = 6 V

R_T = 250 Ω

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Problem 17-20

The h-parameters of a two-port network are h_{11} = 6 kΩ, h_{12} = 0, h_{21} = 50, and h_{22} = 0.2 mS. Find the current gain I_2/I_1 when a 20-kΩ resistor is connected at the output port.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2

h11 = 6e3;
h12 = 0;
h21 = 50;
h22 = 0.2e-3;
Rout = 20e3;

Eqn1 = V1-h11*I1-h12*V2;
Eqn2 = I2-h21*I1-h22*V2;
Eqn3 = V2+Rout*I2;
Soln = solve(Eqn1,Eqn2,Eqn3,V2,I1,I2);
I1 = simplify(Soln.I1);
I2 = simplify(Soln.I2);
TI = simplify(I2/I1)
TI =
10
```

Answer:

T_I = 10

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Problem 17-21

The *t*-parameters of a two-port network are $A = 2$, $B = 400\ \Omega$, $C = 2.5$ mS, and $D = 1$.

(a) Find the input resistance when the output port is open-circuited.

(b) Find the input resistance when the output port is short-circuited.

(c) Find the input resistance when the output port is connected to a 400-Ω resistor.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2

A = 2; B = 400; C = 2.5e-3; D = 1;

%disp('Part (a)')
I2 = 0;
Eqn1 = V1-A*V2+B*I2;
Eqn2 = I1-C*V2+D*I2;
Soln = solve(Eqn1,Eqn2,V1,I1);
V1 = simplify(Soln.V1);
I1 = simplify(Soln.I1);
Rina = simplify(V1/I1)

%disp('Part (b)')
clear V1 V2 I1 I2; syms V1 V2 I1 I2
V2 = 0;
Eqn1 = V1-A*V2+B*I2;
Eqn2 = I1-C*V2+D*I2;
Soln = solve(Eqn1,Eqn2,V1,I1);
V1 = simplify(Soln.V1);
I1 = simplify(Soln.I1);
Rinb = simplify(V1/I1)

%disp('Part (c)')
clear V1 V2 I1 I2; syms V1 V2 I1 I2
Eqn1 = V1-A*V2+B*I2;
Eqn2 = I1-C*V2+D*I2;
Eqn3 = V2+400*I2;
Soln = solve(Eqn1,Eqn2,Eqn3,V1,V2,I1);
V1 = simplify(Soln.V1);
I1 = simplify(Soln.I1);
Rinc = simplify(V1/I1)
Rina =
800
Rinb =
400
Rinc =
600
```

Answers:

(a) $R_{IN} = 800\ \Omega$

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(b) $R_{IN} = 400\ \Omega$
(c) $R_{IN} = 600\ \Omega$

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Problem 17-22

The *t*-parameters of a two-port network are $A = 0$, $B = -j50\ \Omega$, $C = -j20$ mS, and $D = 1 - j0.25$. Find the input admittance I_1/V_1 when a 50-Ω resistor is connected at the output port.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2

A = 0;
B = -50j;
C = -20e-3j;
D = 1-0.25j;
Rout = 50;

Eqn1 = V1-A*V2+B*I2;
Eqn2 = I1-C*V2+D*I2;
Eqn3 = V2+Rout*I2;
Soln = solve(Eqn1,Eqn2,Eqn3,V1,I1,V2);
V1 = simplify(Soln.V1);
I1 = simplify(Soln.I1);
Yin = double(simplify(I1/V1))
Yin =
    25.0000e-003 + 20.0000e-003i
```

Answer:

$Y_{IN} = 25 + j20$ mS

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Problem 17-23

The output of the two-port network in Figure P17-11 is connected to a load impedance Z_L. Show that the current gain $T_I = I_2/I_1$ is

$$T_I = \frac{h_{21}}{1 + h_{22} Z_L}$$

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 h11 h12 h21 h22 ZL

Eqn1 = V1-h11*I1-h12*V2;
Eqn2 = I2-h21*I1-h22*V2;
Eqn3 = V2+ZL*I2;
Soln = solve(Eqn1,Eqn2,Eqn3,V1,I1,I2);
I1 = simplify(Soln.I1);
I2 = simplify(Soln.I2);
TI  = simplify(I2/I1)
TI =
1/(1+h22*ZL)*h21
```

Answer:

$$T_I = \frac{h_{21}}{1 + h_{22} Z_L}$$

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Problem 17-24

The output of the two-port network in Figure P17-11 is connected to a load impedance Z_L. Show that the input impedance $Z_{IN} = V_1/I_1$ is

$$Z_{IN} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$$

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 h11 h12 h21 h22 ZL

Eqn1 = V1-h11*I1-h12*V2;
Eqn2 = I2-h21*I1-h22*V2;
Eqn3 = V2+ZL*I2;
Soln = solve(Eqn1,Eqn2,Eqn3,V1,I1,I2);
V1 = simplify(Soln.V1);
I1 = simplify(Soln.I1);
Zin  = simple(V1/I1)
Zin =
(h11+h11*h22*ZL-h12*h21*ZL)/(1+h22*ZL)
```

Answer:

$$Z_{IN} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$$

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Problem 17-25

Starting with the two-port *i*-*v* relationships in terms of *h*-parameters, show that $z_{11} = \Delta_h/h_{22}$, $z_{12} = h_{12}/h_{22}$, $z_{21} = -h_{21}/h_{22}$, and $z_{22} = 1/h_{22}$.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 h11 h12 h21 h22

Eqn1 = V1-h11*I1-h12*V2;
Eqn2 = I2-h21*I1-h22*V2;

% Solve for V2 in terms of I1 and I2 to identify z21 and z22
V2 = simple(solve(Eqn2,V2));

% Solve for V1 in terms of I1 and I2 to identify z11 and z12
V1 = simple(solve(subs(Eqn1),V1))
V2
V1 =
h11*I1-(-h12*I2+h12*h21*I1)/h22
V2 =
(I2-h21*I1)/h22
```

Answers:

$z_{11} = \Delta_h/h_{22}$

$z_{12} = h_{12}/h_{22}$

$z_{21} = -h_{21}/h_{22}$

$z_{22} = 1/h_{22}$

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Problem 17-26

Starting with the two-port *i*-*v* relationships in terms of *t*-parameters, show that $z_{11} = A/C$, $z_{12} = \Delta_t/C$, $z_{21} = 1/C$, and $z_{22} = D/C$.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 A B C D

Eqn1 = V1-A*V2+B*I2;
Eqn2 = I1-C*V2+D*I2;

% Solve for V2 in terms of I1 and I2 to identify z21 and z22
V2 = simple(solve(Eqn2,V2));

% Solve for V1 in terms of I1 and I2 to identify z11 and z12
V1 = simple(solve(subs(Eqn1),V1))
V2
V1 =
(A*I1+A*D*I2-B*I2*C)/C
V2 =
(I1+D*I2)/C
```

Answers:

$z_{11} = A/C$

$z_{12} = \Delta_t/C$

$z_{21} = 1/C$

$z_{22} = D/C$

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Problem 17-27

The *i-v* relationships of a two-port are $V_1 = 2000\ I_1 - 20\ V_2$ and $I_2 = 50\ I_1 + 10^{-2}\ V_2$. Find the *y*-parameters of the two-port. Is the two-port reciprocal?

Solution:

The *i-v* relationships are given in terms of the *h*-parameters. Use the conversions in Table W-2 to determine the *y*-parameters. Use MATLAB to perform the calculations.

```
clear all
h11 = 2000;
h12 = -20;
h21 = 50;
h22 = 0.01;

y11 = 1/h11;
y12 = -h12/h11;
y21 = h21/h11;
y22 = (h11*h22-h12*h21)/h11;

y = [y11, y12; y21, y22]

Reciprocal = y12==y21
y =
   500.0000e-006    10.0000e-003
    25.0000e-003   510.0000e-003
Reciprocal =
     0
```

Answers:

$y_{11} = 500\ \mu S$

$y_{12} = 10$ mS

$y_{21} = 25$ mS

$y_{22} = 510$ mS

The two-port is not reciprocal.

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Problem 17-28

The *i*-*v* relationships of a two-port are $V_1 = 5000\ I_1 + 20\ I_2$ and $V_2 = 500\ I_1 + 3000\ I_2$. Find the *t*-parameters of the two-port. Is the two-port reciprocal?

Solution:

The *i*-*v* relationships are given in terms of the *z*-parameters. Use the conversions in Table W-2 to determine the *t*-parameters. Use MATLAB to perform the calculations.

```
clear all
z11 = 5000;
z12 = 20;
z21 = 500;
z22 = 3000;

A = z11/z21;
B = (z11*z22-z12*z21)/z21;
C = 1/z21;
D = z22/z21;

t = [A, B; C, D]

Reciprocal = A*D-B*C==1
t =
    10.0000e+000    29.9800e+003
     2.0000e-003     6.0000e+000
Reciprocal =
     0
```

Answers:

$A = 10$

$B = 29.98\ \text{k}\Omega$

$C = 2\ \text{mS}$

$D = 6$

The two-port is not reciprocal.

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Problem 17-29

The *h*-parameters of a two-port amplifier are h_{11} = 10 kΩ, h_{12} = 0, h_{21} = −10, and h_{22} = 1 mS. Find the *h*-parameters of a cascade connection of two such amplifiers.

Solution:

To find the parameters for a cascade connection, express the two-port amplifier in terms of the *t*-parameters, combine the two amplifiers, and then convert back to the *h*-parameters. Use MATLAB to perform the calculations.

```
clear all
h11 = 10e3;
h12 = 0;
h21 = -10;
h22 = 1e-3;
hdet = h11*h22-h12*h21;

% Find the t-parameters
A = -hdet/h21;
B = -h11/h21;
C = -h22/h21;
D = -1/h21;
t = [A, B; C, D];

% Combine the two amplifiers in cascade
tc = t*t;
Ac = tc(1,1);
Bc = tc(1,2);
Cc = tc(2,1);
Dc = tc(2,2);
tcdet = Ac*Dc-Bc*Cc;

% Convert back to the h-parameters
hc11 = Bc/Dc;
hc12 = tcdet/Dc;
hc21 = -1/Dc;
hc22 = Cc/Dc;
hc = [hc11, hc12; hc21, hc22]
hc =
    10.0000e+003   126.1617e-018
    -9.0909e+000     1.0000e-003
```

Answers:

h_{11} = 10 kΩ

h_{12} = 0

h_{21} = −9.0909

h_{22} = 1 mS

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Problem 17-30

The *h*-parameters of a two-port amplifier are h_{11} = 10 kΩ, h_{12} = 0, h_{21} = −10, and h_{22} = 1 mS. Find the *h*-parameters of a parallel connection of two such amplifiers. Assume the connection does not change the parameters of either amplifier.

Solution:

To find the parameters for a parallel connection, express the two-port amplifier in terms of the *y*-parameters, combine the two amplifiers, and then convert back to the *h*-parameters. Use MATLAB to perform the calculations.

```
clear all
h11 = 10e3;
h12 = 0;
h21 = -10;
h22 = 1e-3;
hdet = h11*h22-h12*h21;

y11 = 1/h11;
y12 = -h12/h11;
y21 = h21/h11;
y22 = hdet/h11;
y = [y11, y12; y21, y22];

yp = y+y;

yp11 = yp(1,1);
yp12 = yp(1,2);
yp21 = yp(2,1);
yp22 = yp(2,2);
ypdet = yp11*yp22-yp12*yp21;

hp11 = 1/yp11;
hp12 = -yp12/yp11;
hp21 = yp21/yp11;
hp22 = ypdet/yp11;

hp = [hp11, hp12; hp21, hp22]
hp =
     5.0000e+003     0.0000e+000
   -10.0000e+000     2.0000e-003
```

Answers:

h_{11} = 5 kΩ

h_{12} = 0

h_{21} = −10

h_{22} = 2 mS

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Problem 17-31

A two-port network is said to be **unilateral** if excitation applied at the output port produces a zero response at the input port. Show that a two-port is unilateral if $AD - BC = 0$.

Solution:

From Table 17-2, we have the following relationships:

$$z_{12} = \frac{\Delta_t}{C} = z_{21}\Delta_t, \quad y_{12} = \frac{-\Delta_t}{B} = \Delta_t y_{21}, \quad h_{12} = \frac{\Delta_t}{D} = -\Delta_t h_{21}$$

If an excitation at the input produces a nonzero output, then z_{21}, y_{21}, and h_{21} are not zero. When $\Delta_t = 0$, the reverse transfer functions z_{12}, y_{12}, and h_{12} are all zero. Therefore, an excitation applied at the output port produces zero response at the input port.

Answer:

Presented above.

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Problem 17-32

A load impedance Z_L is connected at the output of a two-port network. Show that the input impedance $Z_{IN} = V_1/I_1 = Z_L$ when $A = D$ and $B = C(Z_L)^2$.

Solution:

Use MATLAB to perform the calculations.

```
clear all
syms V1 V2 I1 I2 A B C D ZL

Eqn1 = V1-A*V2+B*I2;
Eqn2 = I1-C*V2+D*I2;
Eqn3 = V2+I2*ZL;

Eqn1a = subs(Eqn1,{A,B},{D,C*ZL^2});

Soln = solve(Eqn1a,Eqn2,Eqn3,V1,I1,V2);
V1 = simplify(Soln.V1)
I1 = simplify(Soln.I1)
Zin = simplify(V1/I1)
V1 =
-D*ZL*I2-C*ZL^2*I2
I1 =
-C*ZL*I2-D*I2
Zin =
ZL
```

Answer:

$Z_{IN} = Z_L$

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