1 Introduction

1.1 Exercise Solutions

Exercise 1–1. Given the pattern in the statement 1 kΩ = 1 kilohm = 1×10^3 ohms, fill in the blanks in the following statements using the standard decimal prefixes.

(a) 10^{-3} is milli or m. 5.0 mW = 5 milliwatts = 5×10^{-3} watts.

(b) d is deci or 10^{-1}. 10.0 dB = 10.0 decibels = 1.0 bel.

(c) p is pico or 10^{-12}. 3.6 ps = 3.6 picoseconds = 3.6×10^{-12} seconds.

(d) micro is 10^{-6} or μ. Since we have less than one microfarad, we can also find expressions in terms of nanofarads with n being 10^{-9}. 0.03 μF or 30 nF = 30 nanofarads = 30.0×10^{-9} farads.

(e) 10^9 is giga or G. 6.6 GHz = 6.6 gigahertz = 6.6×10^9 hertz.

Exercise 1–2. A device dissipates 100 W of power. How much energy is delivered to it in 10 seconds?
Energy is the product of power and time. In this case, we have $w = p\,t = 100\text{ W} \times 10\text{ s} = 1000\text{ J} = 1\text{ kJ}$.

Exercise 1–3. The graph is Figure 1–2(a) shows the charge $q(t)$ flowing past a point in a wire as a function of time.

(a) Find the current $i(t)$ at $t = 1$, 2.5, 3.5, 4.5, and 5.5 ms. Current is the time rate of change of charge, $i = \frac{dq}{dt}$. For each time, compute the slope of the plot and account for the units. At 1 s, we have $i = \frac{-20\,\text{pC}}{2\,\text{ms}} = -10\,\text{nA}$. Similarly for the other times, we have $+40\,\text{nA}$, $0\,\text{nA}$, $-20\,\text{nA}$, and $0\,\text{nA}$.

(b) Sketch the variation of $i(t)$ versus time. The variations in $i(t)$ are shown in Figure 1–2(b) in the textbook and the plot is repeated below in Figure Ex1–3.

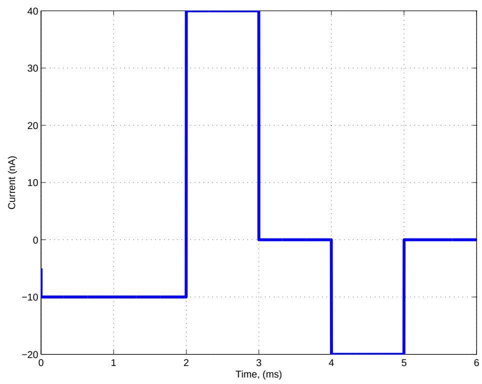


Figure Ex1–3

Exercise 1–4. The working variables of a set of two-terminal electrical devices are observed to be as follows:

	Device 1	Device 2	Device 3	Device 4	Device 5
v	+10 V	?	−15 V	+5 V	?
i	−3 A	−3 A	+10 mA	?	−12 mA
p	?	+40 W	?	+10 mW	−120 mW

Device 1: $p = vi = -30$ W, (delivering power).
Device 2: $v = p/i = -13.3$ V, (absorbing power).

Device 3: $p = vi = -150$ mW, (delivering power).
Device 4: $i = p/v = +2$ mA, (absorbing power).
Device 5: $v = p/i = +10$ V, (delivering power).

1.2 Problem Solutions

Problem 1–1. Express the following quantities to the nearest standard prefix using no more than three digits.

(a). $1,000,000$ Hz $= 1 \times 10^6$ Hz $= 1$ MHz

(b). 102.5×10^9 W $= 103$ GW

(c). 0.333×10^{-7} s $= 33.3 \times 10^{-9}$ s $= 33.3$ ns

(d). 10×10^{-12} F $= 10$ pF

Problem 1–2. Express the following quantities to the nearest standard prefix using no more than three digits.

(a). 0.000222 H $= 222 \times 10^{-6}$ H $= 222\ \mu$H

(b). 20.5×10^5 J $= 2.05 \times 10^6$ J $= 2.05$ MJ

(c). 72.25×10^3 C $= 72.3$ kC

(d). $3{,}264\,\Omega = 3.264 \times 10^3\,\Omega = 3.26\ \mathrm{k}\Omega$

Problem 1–3. An ampere-hour (Ah) meter measures the time-integral of the current in a conductor. During an 8-hour period, a certain meter records 3300 Ah. Find the number of coulombs that flowed through the meter during the recording period.

By definition, 1 ampere = 1 coulomb/second and 1 hour = 3600 seconds. So 3300 Ah = 3300 ampere-hour = 3300 (coulomb/second)(hour)(3600 second/hour) = 11.88 MC.

Problem 1–4. Electric power companies measure energy consumption in kilowatt-hours, denoted kWh. One kilowatt-hour is the amount of energy transferred by 1 kW of power in a period of 1 hour. A power company billing statement reports a user's total energy usage to be 2500 kWh. Find the number of joules used during the billing period.

We have the following relationships: 1 kWh = 1000 watt-hours, 1 watt = 1 joule/second, and 1 hour = 3600 seconds. So 2500 kWh = 2500 kilowatt-hours = 2500000 watt-hours = 2500000 (joules/second)(hours)(3600 second/hour) = 9×10^9 J = 9 GJ.

Problem 1–5. Fill in the blanks in the following statements.

(a). To convert capacitance from picofarads to microfarads, multiply by 10^{-6}. We have 1 pF $= 1 \times 10^{-12}$ F $= (1 \times 10^{-6}) \times 10^{-6}$ F $= 1 \times 10^{-6} \mu$F.

(b). To convert resistance from megohms to kilohms, multiply by 10^3. We have $1\ \mathrm{M}\Omega = 1 \times 10^6\,\Omega = (1 \times 10^3) \times 10^3\,\Omega = 1 \times 10^3\,\mathrm{k}\Omega$.

(c). To convert voltage from millivolts to volts, multiply by 10^{-3}. We have 1 mV $= 1 \times 10^{-3}$ V.

(d). To convert energy from megajoules to joules, multiply by 10^6. We have 1 MJ $= 1 \times 10^6$ J.

Problem 1–6. A wire carries a constant current of 30 mA. How many coulombs flow past a given point in the wire in 5 s?

We know that 1 ampere is equivalent to 1 coulomb/second. Since the current is constant, if we multiply the current by the time, we get the charge flowing past a point over that period of time. We can calculate $q = i \times t = 30$ mA $\times\, 5$ s = (30 millicoulombs/second)(5 seconds) = 150 mC.

Problem 1–7. The net positive charge flowing through a device is $q(t) = 20 + 4t$ mC. Find the current through the device.

The current through the device is the derivative of the charge, $i = dq/dt$.

$$i = \frac{dq}{dt} = \frac{d}{dt}(20 + 4t)\,\text{mC} = 4\,\text{mA}$$

The following MATLAB code calculates the same answer.

```
syms t real
qt = 20 + 4*t;
it = diff(qt,t)
```

Problem 1–8. Figure P1–8 shows a plot of the net positive charge flowing in a wire versus time. Sketch the corresponding current during the same period of time.

Take the derivative of the charge waveform to find the current. The charge waveform is piecewise linear, so calculate the slope of each segment to find the current values. The following table presents the results.

Start Time (s)	Stop Time (s)	Start Charge (C)	End Charge (C)	Current (A)
0	2	10	30	+10
2	3	30	-10	-40
3	5	-10	-20	-5
5	6	-20	30	+50

The following MATLAB code plots the original charge waveform and the corresponding current.

```
syms t
qt =  (10+10*t)*(heaviside(t)-heaviside(t-2))...
    + (110-40*t)*(heaviside(t-2)-heaviside(t-3))...
    + (5-5*t)*(heaviside(t-3)-heaviside(t-5))...
    + (-270+50*t)*(heaviside(t-5)-heaviside(t-6));
tt = 0:0.01:6;
qtt = subs(qt,t,tt);
figure; plot(tt,qtt,'b','LineWidth',3)
xlabel('Time (s)'); ylabel('Charge (C)')
grid on

it = diff(qt,t);
itt = subs(it,t,tt);
figure; plot(tt,itt,'g','LineWidth',3)
xlabel('Time (s)'); ylabel('Current (A)')
grid on; axis([0 6 -60 60])
```

Figure P1–8 displays the resulting plots.

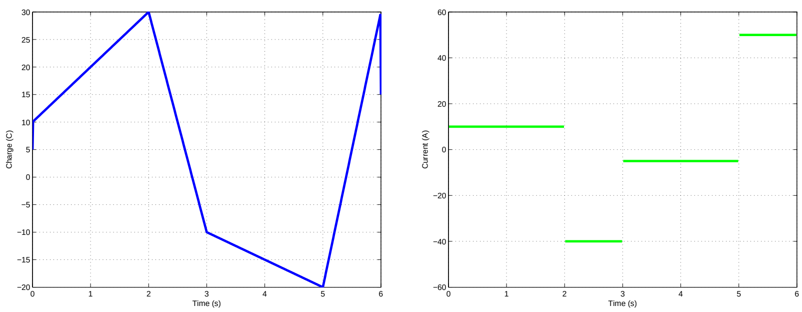


Figure P1–8

Problem 1–9. The net negative charge flowing through a device varies as $q(t) = 3t^2$ C. Find the current through the device at $t = 0$ s, $t = 0.5$ s, and $t = 1$ s.

The current is the derivative of the charge with respect to time, $i = dq/dt$.

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}(3t^2)\,\text{C} = 6t\,\text{A}$$

Evaluate $i(t)$ at 0, 0.5, and 1 s to find the corresponding currents. We have $i(0) = 0$ A, $i(0.5) = 3$ A, and $i(1) = 6$ A. Note that since the negative charge is specified in the problem statement, the current flows in the opposite direction as the charge flow.

The following MATLAB code calculates the same answer.

```
syms t
qt = 3*t^2;
it = diff(qt,t)
tt = [0, 0.5, 1];
itt = subs(it,t,tt)
```

Problem 1–10. A cell-phone charger outputs 9.6 V and is protected by a 50 mA fuse. A 1.5 W cell phone is connected to it to be charged. Will the fuse blow?

If the current to the cell phone exceeds 50 mA, then the fuse will blow. The current is the power divided by voltage, $i = p/v = (1.5\text{ W})/(9.6\text{ V}) = 156.25$ mA. The current is greater than 50 mA, so the fuse will blow.

The following MATLAB code calculates the same answer.

```
p = 1.5;
v = 9.6;
fuse = 50e-3;
i = p/v
FuseBlows = i>fuse
```

Problem 1–11. For $0 \leq t \leq 5$ s, the current through a device is $i(t) = 4t$ A. For $5 < t \leq 10$ s, the current is $i(t) = 40 - 4t$ A, and $i(t) = 0$ A for $t > 10$ s. Sketch $i(t)$ versus time and find the total charge flowing through the device between $t = 0$ s and $t = 10$ s.

The total charge flowing through the device is the integral of the current over time.

$$\begin{aligned} q &= \int_0^{10} i(t)\,dt = \int_0^5 4t\,dt + \int_5^{10} (40-4t)\,dt \\ &= 2t^2\Big|_0^5 + (40t - 2t^2)\Big|_5^{10} = (50-0) + [(400-200) - (200-50)] \\ &= 100\,\text{C} \end{aligned}$$

The following MATLAB code plots the current versus time and calculates the total charge flowing through the device.

```
syms t
it = 4*t*(heaviside(t)-heaviside(t-5))...
    + (40-4*t)*(heaviside(t-5)-heaviside(t-10));
qTotal = int(it,t,0,10)
tt = 0:0.01:10;
itt = subs(it,t,tt);
plot(tt,itt,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Current (A)')
```

Figure P1–11 displays the resulting plot.

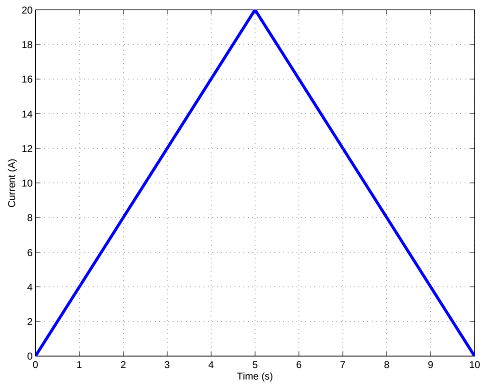


Figure P1–11

Problem 1–12. The charge flowing through a device is $q(t) = 1 - e^{-1000t}\,\mu$C. How long will it take the current to reach 200 μA?

The current is the derivative of the charge with respect to time, $i(t) = \frac{d}{dt}q(t)$. Compute the current and then solve for the time in terms of the current. Substitute in a current of 200 μA to find the corresponding time.

$$i(t) = \frac{d}{dt}q(t) = \frac{d}{dt}(1 - e^{-1000t}) = \frac{e^{-1000t}}{1000}$$

Solve for t in terms of $i(t)$:

$$\begin{aligned}
\frac{e^{-1000t}}{1000} &= i(t) \\
e^{-1000t} &= 1000i(t) \\
-1000t &= \ln\left[1000i(t)\right] \\
t &= -\frac{1}{1000}\ln\left[1000i(t)\right]
\end{aligned}$$

Substitute $i(t) = 200\,\mu$A to get $t = 1.6094$ ms. The following MATLAB code calculates the same answer.

```
syms t
qt = 1e-6*(1-exp(-1000*t));
it = diff(qt,t)
Time200 = solve(it-200e-6,t);
Time200 = vpa(Time200,5)
```

Problem 1–13. The 12-V automobile battery in Figure P1–13 has an output capacity of 100 ampere-hours (Ah) when connected to a head lamp that absorbs 200 watts of power. The car engine is not running and therefore not charging the battery. Assume the battery voltage remains constant.

(a). Find the current supplied by the battery and determine how long can the battery power the headlight.

The current is the power divided by the voltage, $i = p/v = 200\,\text{W}/12\,\text{V} = 16.667$ A. Divide the capacity of the battery by the current to determine how long the battery will power the headlight, $t = 100\,\text{Ah}/16.667\,\text{A} = 6$ hours.

(b). A 100 W device is connected through the utility port. How long can the battery power both the headlight and the device?

The current is the power divided by the voltage, $i = p/v = (200 + 100)\,\mathrm{W}/12\,\mathrm{V} = 25$ A. Note that the power requirement increased by 50%, so the current increased by 50% as well. Divide the capacity of the battery by the current to determine how long the battery will power the headlight, $t = 100\,\mathrm{Ah}/25\,\mathrm{A} = 4$ hours.

The following MATLAB code calculates the same answers.

```
v = 12;
p = 200;
i = p/v
cap = 100;
t = cap/i

p2 = p+100;
i2 = p2/v
t2 = cap/i2
```

Problem 1–14. An incandescent lamp absorbs 100 W when connected to a 120-V source. A energy-efficient compact fluorescent lamp (CFL) producing the same amount of light absorbs 16 W when connected to the same source. How much cheaper is it to operate the CFL versus the incandescent bulb over 1000 hours when electricity costs 7.8 cents/kWh?

Find the energy in kWh used by each type of bulb and then calculate the corresponding costs. For the incandescent lamp, we have $w = (100\,\mathrm{W})(1000\,\mathrm{h}) = 100\,\mathrm{kWh}$. The corresponsding cost is $(100\,\mathrm{kWh})$ $(0.078\,\mathrm{dollars/kWh}) = \$7.80$. For the CFL, we have $w = (16\,\mathrm{W})(1000\,\mathrm{h}) = 16\,\mathrm{kWh}$. The corresponsding cost is $(16\,\mathrm{kWh})$ $(0.078\,\mathrm{dollars/kWh}) = \$1.25$. While operating for 1000 hours, the CFL saves \$6.55. The following MATLAB code calculates the same answer.

```
V = 120;
T = 1000;
P_incand = 100;
rate = 0.078;
kWh_incand = P_incand*T/1000;
cost_incand = rate*kWh_incand

P_fluor = 16;
kWh_fluor = P_fluor*T/1000;
cost_fluor = rate*kWh_fluor

cost_difference = cost_incand - cost_fluor
```

Problem 1–15. The current through a device is zero for $t < 0$ and is $i(t) = 3e^{-2t}$ A for $t \geq 0$. Find the charge $q(t)$ flowing through the device for $t \geq 0$.

The charge flowing through the device is the integral of the current over time.

$$q(t) \;=\; \int_0^t i(\tau)\,d\tau \;=\; \int_0^t 3e^{-2\tau}\,d\tau \;=\; -\frac{3}{2}e^{-2\tau}\Big|_0^t \;=\; -\frac{3}{2}\left(e^{-2t} - 1\right) \;=\; \frac{3}{2}\left(1 - e^{-2t}\right)\ \mathrm{C},\ \ t \geq 0$$

The following MATLAB code calculates the same answer.

```
syms t tau
it = 3*exp(-2*t);
itau = subs(it,t,tau)
qt = simple(int(itau,tau,0,t))
```

Problem 1–16. A string of holiday lights is protected by a 5-A fuse and has 25 bulbs, each of which is rated at 7 W. How many strings can be connected end-to-end across a 120 V circuit without blowing a fuse?

The current is the power divided by the voltage, $i = p/v$. Each string of lights increases the power by 25×7 W = 175 W. Correspondingly, the current increases by $(175\,\text{W})/(120\,\text{V}) = 1.4583$ A. Since each fuse can handle up to 5 A, divide the fuse rating by the current required for each string and round down to get the maximum number of strings. We have (5 A)/(1.4583 A) = 3.4286. The maximum number of strings is three.

Problem 1–17. The i-v relationship for a photocell when illuminated is $i = e^v - 10$ A. For $v = -2$, 2 and 3 V, find the device power and state whether it is absorbing or delivering power.

For each voltage, substitute into the expression for current and solve for the current. Multiply the current and voltage to find power. If the power is positive, the photocell is absorbing power. If the power is negative, the photocell is delivering power. The following table summarizes the results of the calculations.

v (V)	i (A)	p (W)	Absorbing/Delivering
-2	-9.8647	19.7293	Absorbing
2	-2.6109	-5.2219	Delivering
3	10.0855	30.2566	Absorbing

The following MATLAB code calculates the same answer.

```
v = [-2 2 3]
iv = exp(v)-10
p = v.*iv
pAbsorbs = p>0
Results = [v' iv' p' pAbsorbs']
```

Problem 1–18. A new 6 V Alkaline lantern battery delivers 237.5 kJ of energy during its lifetime. How long will the battery last in an application that draws 15 mA continuously. Assume the battery voltage is constant.

The power delivered is the product of the voltage and current, $p = vi = (6\,\text{v})(15\,\text{mA}) = 90\,\text{mW}$. A watt is a joule per second, so the application draws 90 mJ/s. Divide the capacity of the battery by the rate to get the total time, (237.5 kJ)/(90 mJ/s) = 2.6389 Ms = 733.02 h = 30.54 days.

Problem 1–19. The maximum power the device can dissipate is 0.25 W. Determine the maximum current allowed by the device power rating when the voltage is 9 V.

The maximum current will be the maximum power divided by the voltage, $i_{\text{Max}} = p_{\text{Max}}/v = (0.25\text{ W})/(9\text{ V}) = 27.778$ mA.

Problem 1–20. Traffic lights are being converted from incandescent bulbs to LED arrays to save operating and maintenance costs. Typically each incandescent light uses three 100-W bulbs, one for each color R, Y, G. A competing LED array consists of 61 LEDs with each LED requiring 9 V and drawing 20 mA of current. There are three arrays per light - R, Y, G. A small city has 1560 traffic signals. Since one light is always on 24/7, how much can a city save in one year if the city buys their electricity at 7.2 cents per kWh?

To solve this problem, compare two lights and then scale the problem to the number of lights in the city. The incandescent light always has on one 100-W bulb operating 24 hours per day for 365 days, which yields a total of 876 kWh at a cost of $63.072. The LED light always has on one array, using a total power of (61)(9 V)(20 mA) = 10.98 W. Over one year, the LED lights uses 96.185 kWh of energy at a cost of $6.9253. The savings per light per year is $56.147, which, for a total of 1560 lights, translates into a city-wide saving of $87,589 per year. The following MATLAB code calculates the same answer.

```
p_incand = 100/1000;  % Incandescent power in kW
rate = 0.072;         % Dollars per kWh
hours = 24*365;       % Hours per year
cost_incand = p_incand*hours*rate

v_led = 9;            % LED voltage
```

```
i_led = 20e-3;         % LED current
n_led = 61;            % Number of LED lights per array
p_led = n_led*v_led*i_led/1000  % Power in kW
cost_led = p_led*hours*rate

savings_light = cost_incand - cost_led
lights = 1560;
savings_year = lights*savings_light
```

Problem 1–21. Two electrical devices are connected as shown in Figure P1-21. Using the reference marks shown in the figure, find the power transferred and state whether the power is transferred from A to B or B to A when

(a). $v = +11$ V and $i = -1.1$ A

(b). $v = +80$ V and $i = +20$ mA

(c). $v = -120$ V and $i = -12$ mA

(d). $v = -1.5$ V and $i = -600\,\mu$A

The passive sign convention applies to Element B, so if the power is positive the transfer is from A to B, and if the power is negative, the transfer is from B to A. For each case, calculate the power $p = iv$ and determine the direction of the power flow.

The following table summarizes the results of the calculations.

Case	v	i	p	Power Transfer
(a)	+11 V	-1.1 A	-12.1 W	B to A
(b)	+80 V	+20 mA	+1.6 W	A to B
(c)	-120 V	-12 mA	+ 1.44 W	A to B
(d)	-1.5 V	-600 μA	+900 μW	A to B

The following MATLAB code calculates the same answer.

```
v = [11 80 -120 -1.5];
i = [-1.1 20e-3 -12e-3 -600e-6];
p = v.*i
AtoB = p>0
Results = [v' i' p' AtoB']
```

Problem 1–22. Figure P1–22 shows an electric circuit with a voltage and a current variable assigned to each of the six devices. The device voltages and currents are observed to be

	v (V)	i (A)
Device 1	15	-1
Device 2	5	1
Device 3	10	2
Device 4	-10	-1
Device 5	20	-3
Device 6	20	2

Find the power associated with each device and state whether the device is absorbing or delivering power. Use the power balance to check your work.

The power associated with each device is the product of the voltage and currnet, $p = vi$. If the power is positive, the device is absorbing power. If the power is negative, the device is delivering power. The original table is expanded below to include power and the direction of power flow.

	v (V)	i (A)	p (W)	Absorbing/Delivering
Device 1	15	-1	-15	Delivering
Device 2	5	1	5	Absorbing
Device 3	10	2	20	Absorbing
Device 4	-10	-1	10	Absorbing
Device 5	20	-3	-60	Delivering
Device 6	20	2	40	Absorbing

For the power to balance in this system, the sum of the individual device powers should be zero. We have $-15 + 5 + 20 + 10 - 60 + 40 = 0$, so the power balances.

The following MATLAB code calculates the same results.

```
v = [15 5 10 -10 20 20];
i = [-1 1 2 -1 -3 2];
p = v.*i
Absorbing = p>0
Balance = sum(p)
Results = [v' i' p' Absorbing']
```

Problem 1–23. Figure P1–22 shows an electric circuit with a voltage and a current variable assigned to each of the six devices. Use power balance to find v_4 when $v_1 = 20$ V, $i_1 = -2$ A, $p_2 = 20$ W, $p_3 = 10$ W, $i_4 = 1$ A, and $p_5 = p_6 = 2.5$ W. Is device 4 absorbing or delivering power?

First, calculate the power associated with device 1, $p_1 = v_1 i_1 = -40$ W. The power must balance in the circuit, so the sum of all of the device powers is zero. Therefore, we can solve for p_4 and v_4 as follows:

$$
\begin{aligned}
p_4 &= -p_1 - p_2 - p_3 - p_5 - p_6 = 40 - 20 - 10 - 2.5 - 2.5 = 5\,\mathrm{W} \\
v_4 &= p_4/i_4 = 5/1 = 5\,\mathrm{V}
\end{aligned}
$$

The power is positive, so the device is absorbing power.

Problem 1–24. Suppose in Figure P1–22 a ground is connected to the minus ($-$) side of device 6 and another to the junction of devices 2, 3 and 4. Further, assume that the voltage v_4 is 5 V and v_1 is 10 V. What are the voltages v_2, v_3, v_5 and v_6?

Start at the junction of devices 2, 3 and 4, where the voltages is zero because it is connected to ground. Since $v_4 = 5$ V, there is a 5-V drop across device 4 from left to right. Therefore, the voltage at the junction of devices 4, 5 and 6 is -5 V. Devices 5 and 6 are grounded at their negative sides, so the voltage across each is -5 V. Device 3 is grounded on both sides, so its voltage is zero. The negative side of device 1 is grounded and $v_1 = 10$ V, so the voltage at the junction of devices 1 and 2 is 10 V. The negative side of device 2 is grounded, so its voltage is also 10 V. In summary, we $v_2 = 10$ V, $v_3 = 0$ V, $v_5 = -5$ V, and $v_6 = -5$ V.

The following MATLAB code calculates the same results.

```
v4 = 5;
v1 = 10;
v5 = -v4
v6 = v5
v3 = 0
v2 = v1
```

Problem 1–25. For $t \geq 0$ the voltage across and power absorbed by a two-terminal device are $v(t) = 2e^{-t}$ V and $p(t) = 40e^{-2t}$ mW. Find the total charge delivered to the device for $t \geq 0$.

First find the current $i(t) = p(t)/v(t)$ and then integrate the current to find the charge.

$$i(t) = \frac{p(t)}{v(t)} = \frac{40e^{-2t}\,\text{mW}}{2e^{-t}\,\text{V}} = 20e^{-t}\,\text{mA}$$

$$q(t) = \int_0^t i(\tau)\,d\tau = \int_0^t 20e^{-\tau}\,d\tau = -20e^{-\tau}\Big|_0^t = -20\left(e^{-t}-1\right) = 20\left(1-e^{-t}\right)\,\text{mC}$$

To find the total charge delivered to the device for $t \geq 0$, evaluate $q(t)$ in the limit as $t \to \infty$.

$$q_{\text{Total}} = \lim_{t\to\infty} q(t) = \lim_{t\to\infty} 20\left(1-e^{-t}\right) = 20(1-0) = 20\,\text{mC}$$

The following MATLAB code calculates the same results.

```
syms t
vt = 2*exp(-t);
pt = 40e-3*exp(-2*t);
it = simple(pt/vt)
q = simple(int(it,t,0,inf))
```

Problem 1–26. Repeat Problem 1–22 using MATLAB to perform the calculations. Create a vector for the voltage values, `v = [15 5 10 -10 20 20]`, and a vector for the current values, `i = [-1 1 2 -1 -3 2]`. Compute the corresponding vector for the power values, p, using element-by-element multiplication (`.*`) and then use the `sum` command to verify the power balance.

The following MATLAB code provides the solution.

```
device = [1 2 3 4 5 6];
v = [15 5 10 -10 20 20];
i = [-1 1 2 -1 -3 2];
p = v.*i
Absorbing = p>0
Balance = sum(p)
Results = [device' v' i' p' Absorbing']
```

The corresponding MATLAB output is shown below.

```
Balance =
     0.0000e+000
Results =
     1.0000e+000    15.0000e+000    -1.0000e+000   -15.0000e+000     0.0000e+000
     2.0000e+000     5.0000e+000     1.0000e+000     5.0000e+000     1.0000e+000
     3.0000e+000    10.0000e+000     2.0000e+000    20.0000e+000     1.0000e+000
     4.0000e+000   -10.0000e+000    -1.0000e+000    10.0000e+000     1.0000e+000
     5.0000e+000    20.0000e+000    -3.0000e+000   -60.0000e+000     0.0000e+000
     6.0000e+000    20.0000e+000     2.0000e+000    40.0000e+000     1.0000e+000
```

The power balance is zero, as expected, and the other results match those in Problem 1–22.

Problem 1–27. Using the passive sign convention, the voltage across a device is $v(t) = 170\cos(377t)$ V and the current through the device is $i(t) = 2\sin(377t)$ A. Using MATLAB, create a short script (m-file) to assign a value to the time variable, t, and then calculate the voltage, current, and power at that time. Run the script for $t = 5$ ms and $t = 10$ ms and for each result state whether the device is absorbing or delivering power.

The following MATLAB code provides the solution.

```
t = 5e-3
vt = 170*cos(377*t)
```

```
it = 2*sin(377*t)
pt = vt*it
Absorbing = pt>0

t = 10e-3
vt = 170*cos(377*t)
it = 2*sin(377*t)
pt = vt*it
Absorbing = pt>0
```

The corresponding MATLAB output is shown below.

```
t =  5.0000e-003
vt = -52.5401e+000
it = 1.9021e+000
pt = -99.9357e+000
Absorbing = 0

t = 10.0000e-003
vt = -137.5240e+000
it = -1.1757e+000
pt = 161.6889e+000
Absorbing = 1
```

The following table summarizes the results.

t (ms)	$v(t)$ (V)	$i(t)$ (A)	$p(t)$ (W)	Absorbing/Delivering
5	-52.54	1.90	-99.94	Delivering
10	-137.52	-1.18	161.69	Absorbing

Problem 1–28. Power Ratio in dB (A). A stereo amplifier takes the output of a CD player, for example, and increases the power to an audible level. Suppose the output of the CD player is 50 mW and the desired audible output is 100 W per stereo channel, find the power ratio of the amplifier per channel in decibels (dB), where the power ratio in dB is

$$\mathrm{PR_{dB}} = 10\log_{10}\left(\frac{p_2}{p_1}\right)$$

The power values are given, so substitute into the equation for the power ratio and calculate.

$$\mathrm{PR_{dB}} = 10\log_{10}\left(\frac{p_2}{p_1}\right) = 10\log_{10}\left(\frac{100}{0.05}\right) = 10\log_{10}(2000) = (10)(3.301) = 33.01\,\mathrm{dB}$$

Problem 1–29. AC to DC Converter (A). A manufacturer's data sheet for the converter in Figure P1-29 states that the output voltage is $v_{\mathrm{dc}} = 5$ V when the input voltage $v_{\mathrm{ac}} = 120$ V. When the load draws a current $i_{\mathrm{dc}} = 40$ A the input power is $i_{\mathrm{ac}} = 300$ W. Find the efficiency of the converter.

The efficiency of the converter is the percentage of input power that is delivered to the load. The power delivered to the load is the product of the voltage and current, $p_{\mathrm{Load}} = v_{\mathrm{dc}}i_{\mathrm{dc}} = (5\text{ V})(40\text{ A}) = 200$ W. The power input to the converter is 300 W, so the efficiency is (200 W)/(300 W) = 66.67%.

Problem 1–30. Charge-Storage Device (A). A capacitor is a two-terminal device that can store electric charge. In a linear capacitor the amount of charge stored is proportional to the voltage across the device. For a particular device the proportionality is $q(t) = 10^{-7}v(t)$. If $v(t) = 0$ for $t < 0$ and $v(t) = 10(1-e^{-5000t})$ for $t \geq 0$, find the energy stored in the device at $t = 200\,\mu$s.

Take the derivative of the expression for charge to find the expression for current.

$$i(t) = \frac{d}{dt}\left[10^{-7}v(t)\right] = \frac{d}{dt}\left[10^{-6}(1-e^{-5000t})\right] = 10^{-6}\left(5000\,e^{-5000t}\right) = \frac{1}{200}\,e^{-5000t}$$

Multiply the expressions for voltage and current to find the expression for power.

$$p(t) = v(t)\,i(t) = \left[10(1-e^{-5000t})\right]\left[\frac{1}{200}\,e^{-5000t}\right] = \frac{1}{20}\left(e^{-5000t} - e^{-10000t}\right)$$

Integrate the power from $t = 0$ s to $t = 200\,\mu$s to determine the energy stored in the circuit.

$$\begin{aligned} w &= \int_0^{200\,\mu\text{s}} p(t)\,dt = \int_0^{200\,\mu\text{s}} \frac{1}{20}\left(e^{-5000t} - e^{-10000t}\right)dt \\ &= \frac{1}{20}\left(-\frac{e^{-5000t}}{5000} + \frac{e^{-10000t}}{10000}\right)\Bigg|_0^{200\,\mu\text{s}} \\ &= \frac{1}{20}\left(-60.042\times10^{-6} + 100\times10^{-6}\right) = 1.9979\,\mu\text{J} \end{aligned}$$

The following MATLAB code calculates the same answer.

```
syms t
tt = 200e-6;
C = 1e-7;
vt = 10*(1-exp(-5000*t))
qt = C*vt
it = diff(qt,t)
pt = it*vt
wt = double(int(pt,t,0,tt))
```

The corresponding MATLAB output is shown below.

```
vt =
10 - 10/exp(5000*t)
qt =
1/1000000 - 1/(1000000*exp(5000*t))
it =
1/(200*exp(5000*t))
pt =
-(10/exp(5000*t) - 10)/(200*exp(5000*t))
wt =
     1.9979e-006
```

Problem 1–31. Computer Data Sheet (A). A manufacturer's data sheet for a notebook computer lists the power supply requirements as 7.5 A @ 5 V, 2 A @ 15 V, 2.5 A @ -15 V, 2.25 A @ -5 V and 0.5 A @ 12 V. The data sheet also states that the overall power consumption is 115 W. Are these data consistent? Explain.

Compute the power associated with each of the five requirements by multiplying the voltage and current together. The resulting values are 37.5 W, 30 W, -37.5 W, -11.25 W, and 6 W. In this case, it is not reasonable for the computer to supply power back to the power supply, so the negative powers are *not* a correct interpretation of the passive sign convention. We should take the absolute value of the individual powers to determine the total power requirement for the computer. Summing the magnitudes of the powers yields a total of 122.25 W, which is greater than the stated power consumption of 115 W. The stated power consumption is probably a reasonable value, based on the performance of the computer and the fact that, typically, the power supply with not have to deliver the maximum values constantly.

2 Basic Circuit Analysis

2.1 Exercise Solutions

Exercise 2–1. A 6-V lantern battery powers a light bulb that draws 3 mA of current. What is the resistance of the lamp? How much power does the lantern use?

Using Ohm's law, we have $v = iR$ or $R = \frac{v}{i}$, so we can compute the resistance as $R = \frac{6\,\text{V}}{3\,\text{mA}} = 2\,\text{k}\Omega$. The power is $p = vi = (6\,\text{V})(3\,\text{mA}) = 18$ mW.

Exercise 2–2. What is the maximum current that can flow through a $\frac{1}{8}$-W, 6.8-kΩ resistor? What is the maximum voltage that can be across it?

The resistor can dissipate up to 0.125 W of power. We have $p_{\text{MAX}} = i_{\text{MAX}}^2 R$, which we can solved for i_{MAX} and then substitute in values for the power and resistance

$$i_{\text{MAX}} = \sqrt{\frac{p_{\text{MAX}}}{R}} = \sqrt{\frac{0.125}{6800}} = 4.2875\,\text{mA}$$

Similarly, we can use $p_{\text{MAX}} = \frac{v_{\text{MAX}}^2}{R}$ to solve for the maximum voltage as follows:

$$v_{\text{MAX}} = \sqrt{R\,p_{\text{MAX}}} = \sqrt{(6800)(0.125)} = 29.155\,\text{V}$$

Exercise 2–3. A digital clock is a voltage that switches between two values at a constant rate that is used to time digital circuits. A particular clock switches between 0 V and 5 V every 10 μs. Sketch the clock's *i*-*v* characteristics for the times when the clock is at 0 V and at 5 V.

When the clock has a value of 0 V, its voltage is constant and zero for a wide range of currents. In this case, the *i*-*v* characteristic is a vertical line at 0 V. Likewise, when the clock has a value of 5 V, the voltage is constant at 5 V for a wide range of currents. In this case, the *i*-*v* characteristic is a vertical line at 5 V.

Exercise 2–4. Refer to Figure 2–12.

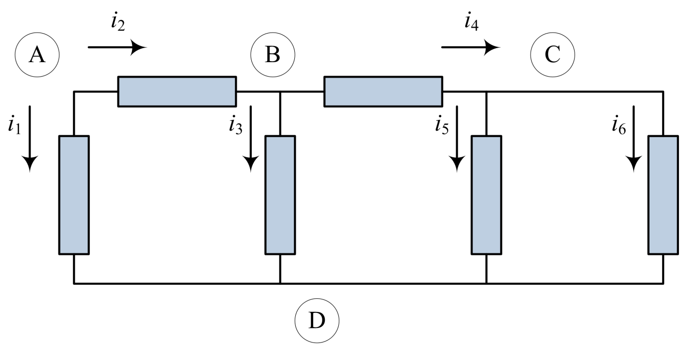


Figure 2–12

(a). Write KCL equations at nodes A, B, C, and D.

KCL states that the sum of the currents entering a node is zero at every instant. As we sum the currents at a node, if the current enters that node, it is positive and if the current leaves the node, it is negative. At node A, both currents i_1 and i_2 are leaving the node, so the equation is $-i_1 - i_2 = 0$. At node B, current i_2 enters the node and currents i_3 and i_4 leave the node, so we have $i_2 - i_3 - i_4 = 0$. At node C, current i_4 enters the node and currents i_5 and i_6 leave the node, so we have $i_4 - i_5 - i_6 = 0$. At node D, currents i_1, i_3, i_5, and i_6 enter the node, so we have $i_1 + i_3 + i_5 + i_6 = 0$.

(b). Given $i_1 = -1$ mA, $i_3 = 0.5$ mA, $i_6 = 0.2$ mA, find i_2, i_4, and i_5.

Applying the KCL equation for node A, we can find $i_2 = -i_1 = 1$ mA. Applying the KCL equation for node B, we have $i_4 = i_2 - i_3 = 1 - 0.5 = 0.5$ mA. Finally, applying the KCL equation for node C, we have $i_5 = i_4 - i_6 = 0.5 - 0.2 = 0.3$ mA.

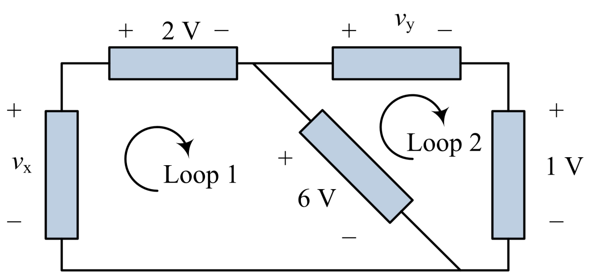


Figure 2–14

Exercise 2–5. Find the voltages v_x and v_y in Figure 2–14.

To find v_x, write the KVL equation around Loop 1 as $-v_x + 2 + 6 = 0$ and solve for $v_x = +8$ V. To find v_y, write the KVL equation around Loop 2 as $v_y + 1 - 6 = 0$ and solve for $v_y = +5$ V.

Exercise 2–6. Find the voltages v_x, v_y, and v_z in Figure 2–15.

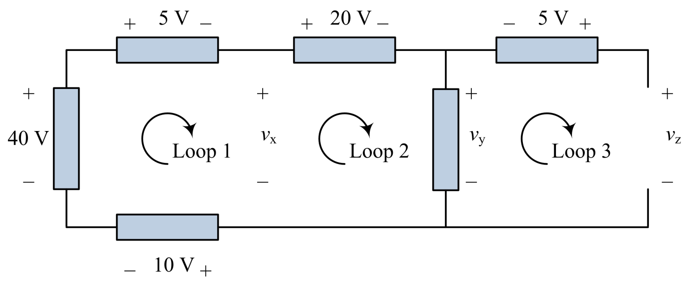


Figure 2–15

In Figure 2–15, some of the unknown voltages do not appear across elements, but we can still write KVL equations. For Loop 1 starting with the lowest element, the KVL equation is $10 - 40 + 5 + v_x = 0$, which can be solved to yield $v_x = 25$ V. For Loop 2, the KVL equation is $-v_x + 20 + v_y = 0$, which can be solved for $v_y = 25 - 20 = 5$ V. Finally, for Loop 3, the KVL equation is $-v_y - 5 + v_z = 0$, which yields $v_z = 5 + 5 = 10$ V.

Exercise 2–7. Identify the elements connected in series or parallel when a short circuit is connected between nodes A and B in each of the circuits in Figure 2–18.

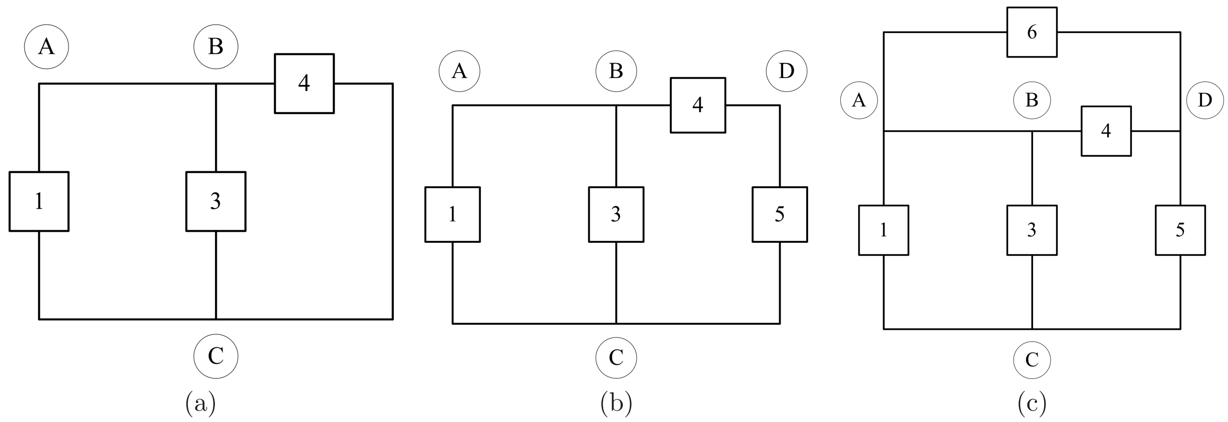


Figure 2–18

In the solution, the short circuit has been applied in each of the circuits and Element 2 has been shorted out of the circuit. For the circuit in Figure 2–18(a), all of the elements share the same two nodes, A and C, so Elements 1, 2, and 3 are in parallel. For the circuit in Figure 2–18(b), Elements 1 and 3 share nodes A and C, so they are in parallel. In addition, Elements 4 and 5 are the only elements connects to node D, so they are in series. For the circuit in Figure 2–18(c), Elements 1 and 3 are in parallel because they share nodes A and C. In addition, Elements 4 and 6 are in parallel, because they share nodes A and D.

Exercise 2–8. Identify the elements in Figure 2–19 that are connected in (a) parallel, (b) series, or (c) neither.

Refer to the figure in the textbook.

(a). Elements 1, 8, and 11 share the upper left node and ground, so they are in parallel. In addition, Elements 3, 4, and 5 share the center node and ground, so they are in parallel.

(b). Elements 9 and 10 are in series, because they share a single node and no other elements with current connect to that node. Likewise, Elements 6 and 7 share a single node with no other elements, so they are also in series.

(c). The remaining element, Element 2, is neither in series nor parallel with any other elements.

Exercise 2–9. A 1-kΩ resistor is added between nodes A and B in Figure 2–20. Find i_x, v_x, i_O, and v_O if $i_S = 1$ mA and $R = 2$ kΩ.

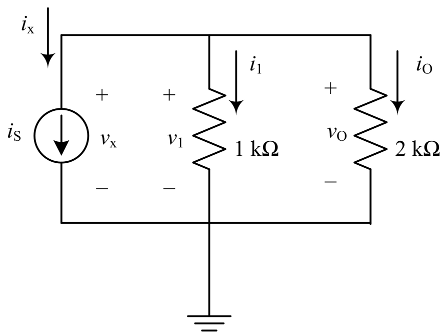


Figure 2–20

The resulting circuit is shown in Figure 2–20. Note that the 1-kΩ resistor has been inserted and the current through it labeled as i_1 and the voltage across it labeled as v_1. Using KCL at the current source, we have $i_x = i_S$. Writing KCL at the top node, we have $-i_x - i_1 - i_O = 0$. Writing KVL around the left loop yields $-v_x + v_1 = 0$. Writing KVL around the right loop yields $-v_1 + v_O = 0$. Alternately, we can see that the three elements all share the top and bottom nodes, so they are all in parallel and have the same voltage, $v_x = v_1 = v_O$. Using these equations and Ohm's law, $v = Ri$, we can solve for the unknown values as follows:

$$
\begin{aligned}
i_{\mathrm{x}} &= i_{\mathrm{S}} = 1\,\mathrm{mA} \\
v_1 &= v_{\mathrm{O}} \\
R_1 i_1 &= R_{\mathrm{O}} i_{\mathrm{O}} \\
1000 i_1 &= 2000 i_{\mathrm{O}} \\
i_1 &= 2 i_{\mathrm{O}} \\
i_{\mathrm{x}} + i_1 + i_{\mathrm{O}} &= 0 \\
i_1 + i_{\mathrm{O}} &= -i_{\mathrm{x}} = -1\,\mathrm{mA} \\
2 i_{\mathrm{O}} + i_{\mathrm{O}} &= -1\,\mathrm{mA} \\
3 i_{\mathrm{O}} &= -1\,\mathrm{mA} \\
i_{\mathrm{O}} &= -333\,\mu\mathrm{A} \\
v_{\mathrm{x}} &= v_{\mathrm{O}} = (2\,\mathrm{k}\Omega)(-333\,\mu\mathrm{A}) = -667\mathrm{mV}
\end{aligned}
$$

Exercise 2–10. The wire connecting R_1 to node B in Figure 2-21 is broken. What would you measure for i_{A}, v_1, i_2, and v_2? Is KVL violated? Where does the source voltage appear across?

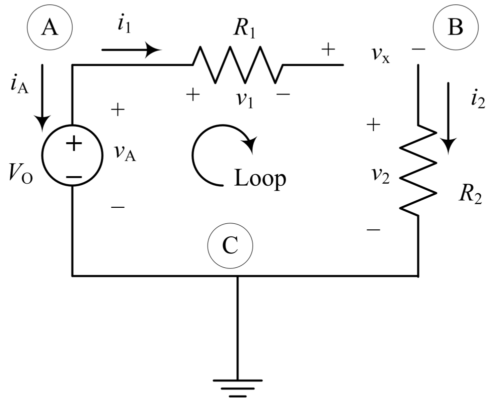


Figure 2–21

Figure 2–21 shows the resulting circuit. If the circuit is broken between R_1 and node B, then no current can flow in the circuit and all currents are zero, $i_{\mathrm{A}} = i_1 = i_2 = 0$. Using Ohm's law, $v = Ri$, for the voltages across the resistors, the current is zero, so the voltages must also be zero and we have $v_1 = v_2 = 0$. Note that a new voltage, v_{x}, has been labeled across the gap where the circuit is broken. We can now write KVL as $-v_{\mathrm{A}} + v_1 + v_{\mathrm{x}} + v_2 = 0$. With $v_1 = v_2 = 0$, we get $v_{\mathrm{x}} = v_{\mathrm{A}} = V_{\mathrm{O}}$. KVL is not violated because the voltage from the source now appears across the gap in the open (broken) circuit.

Exercise 2–11. Repeat the problem of Example 2-10 if the 30-V voltage source is replaced with a 2-mA current source with the arrow pointed up towards node A.

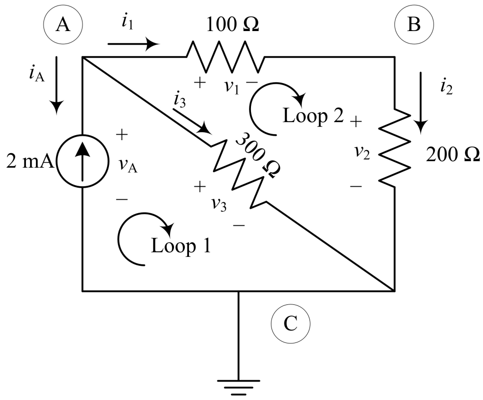


Figure Ex2–11

Figure Ex2–11 shows the resulting circuit. The description of the circuit requires four element equations and four connection equations. The element equations are

$$v_1 = 100i_1$$
$$v_2 = 200i_2$$
$$v_3 = 300i_3$$
$$i_A = -2\,\text{mA}$$

The four connection equations are

$$\text{KCL}: \text{ Node A} \qquad -i_A - i_1 - i_3 = 0$$
$$\text{KCL}: \text{ Node B} \qquad i_1 - i_2 = 0$$
$$\text{KVL}: \text{ Loop 1} \qquad -v_A + v_3 = 0$$
$$\text{KVL}: \text{ Loop 2} \qquad -v_3 + v_1 + v_2 = 0$$

The KCL equation at node B implies $i_1 = i_2$. We can then start with the KVL equation around loop 2 and solve as follows:

$$\begin{aligned} -v_3 + v_1 + v_2 &= 0 \\ v_1 + v_2 &= v_3 \\ 100i_1 + 200i_2 &= 300i_3 \\ 100i_1 + 200i_1 &= 300i_3 \\ 300i_1 &= 300i_3 \\ i_1 &= i_3 \end{aligned}$$

Now using the KCL equation at node A, we have

$$
\begin{aligned}
-i_{\mathrm{A}} - i_1 - i_3 &= 0 \\
i_1 + i_3 &= -i_{\mathrm{A}} = 2\,\mathrm{mA} \\
i_1 + i_1 &= 2\,\mathrm{mA} \\
2i_1 &= 2\,\mathrm{mA} \\
i_1 &= i_3 = i_2 = 1\,\mathrm{mA}
\end{aligned}
$$

Now apply Ohm's law to solve for the voltages

$$
\begin{aligned}
v_1 &= 100i_1 = 100\,\mathrm{mV} \\
v_2 &= 200i_2 = 200\,\mathrm{mV} \\
v_3 &= 300i_3 = 300\,\mathrm{mV}
\end{aligned}
$$

Exercise 2–12. In Figure 2–24, $i_1 = 200$ mA and $i_3 = -100$ mA. Find the voltage v_{x}.

The KCL equation at the center node is $i_1 - i_2 - i_3 = 0$. Solving for i_2, we have $i_2 = i_1 - i_3 = 200 + 100 = 300$ mA. Apply Ohm's law to solve for $v_1 = 100i_1 = (100)(200\,\mathrm{mA}) = 20$ V, and $v_2 = 50i_2 = (50)(300\,\mathrm{mA}) = 15$ V. Write the KVL equation around the left loop as $-v_{\mathrm{x}} + v_1 + v_2 = 0$. Solve for $v_{\mathrm{x}} = v_1 + v_2 = 20 + 15 = 35$ V.

Exercise 2–13. In Figure 2–25(a), the 2-A source is replaced by a 100-V source with the + terminal at the top, and the 3-A source is removed. Find the current and its direction through the voltage source.

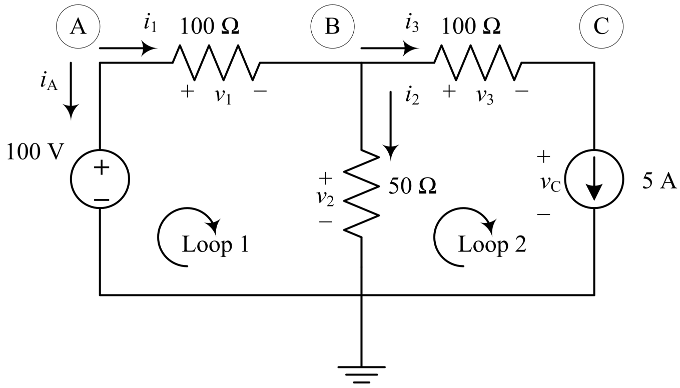


Figure Ex2–13

Figure Ex2–13 shows the resulting circuit. Writing KCL at node C, we have $i_3 - 5 = 0$, which yields $i_3 = 5$ A. Write the KCL equation at node B to get $i_1 - i_2 - i_3 = 0$, which can be solved for $i_1 = i_2 + i_3 = i_2 + 5$. Write the KVL equation around loop 1 to get $-100 + v_1 + v_2 = 0$, which yields the following

$$
\begin{aligned}
v_1 + v_2 &= 100 \\
100i_1 + 50i_2 &= 100 \\
100(i_2 + 5) + 50i_2 &= 100 \\
100i_2 + 500 + 50i_2 &= 100 \\
150i_2 &= -400 \\
i_2 &= -2.667\,\mathrm{A}
\end{aligned}
$$

We can then solve for $i_1 = i_2 + 5 = 2.333$ A and $i_A = -i_1 = -2.333$ A. Since i_A is negative, the current follows in the opposite direction through the voltage source, which is up, and has a magnitude of 2.333 A.

Exercise 2–14. Find the equivalent resistance for the circuit in Figure 2–29.

Redraw the original circuit to an equivalent circuit without the diagonal resistor. Starting from the right side, combine resistors in series or parallel as appropriate to reduce the circuit to a single resistor. The following sequence of circuits shows the progress in reducing the circuit.

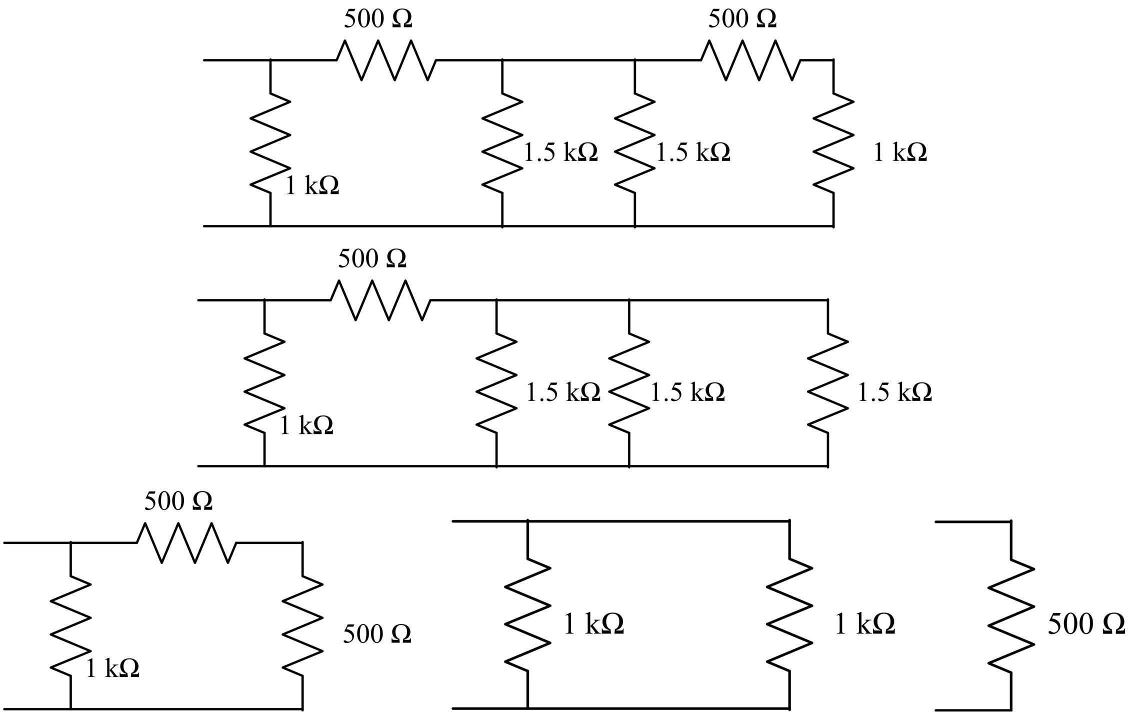


Figure Ex2–14

Starting at the far right, combine the 500-Ω and 1-kΩ resistors in series to get a 1.5-kΩ resistor. Next, combine the three 1.5-kΩ resistors in parallel to get a 500-Ω resistor. Combine the two 500-Ω resistors in series to get a 1-kΩ resistor. Combine the two 1-kΩ resistors in parallel to get the final equivalent resistance of 500-Ω.

Exercise 2–15. Find the equivalent resistance between terminals A–C, B–D, A–D, and B–C in the circuit in Figure 2–30.

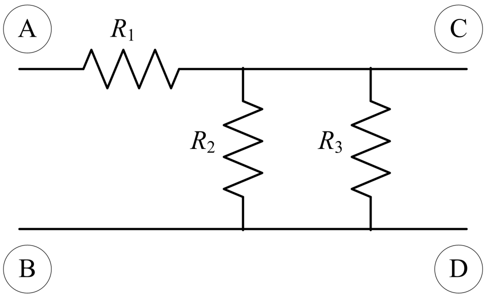


Figure 2–30

If current flows only between terminals A and C, then no current flows through terminals B and D and resistors R_2 and R_3 are not active in the circuit. The equivalent resistance $R_{A-C} = R_1$. If current flows only between terminals B and D, then no current flows through terminals A and C and none of the resistors are active in the circuit. The equivalent resistance $R_{\mathrm{B-D}} = 0$. If current flows between terminals A and D, resistors R_2 and R_3 are in parallel and that combination is in series with R_1. The equivalent resistance $R_{\mathrm{A-D}} = R_1 + R_2 \parallel R_3 = R_1 + \frac{R_2 R_3}{R_2+R_3}$. If current flows between terminals B and C, then no current flows through R_1 and it is not part of the circuit. The equivalent resistance is the parallel combination of R_2 and R_3 or $R_{\mathrm{B-C}} = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2+R_3}$.

Exercise 2–16. Find the equivalent resistance between terminals A–B, A–C, A–D, B–C, B–D, and C–D in the circuit of Figure 2-31. For example: $R_{\mathrm{A-B}} = (80 \parallel 80) + 60 = 100\,\Omega$.

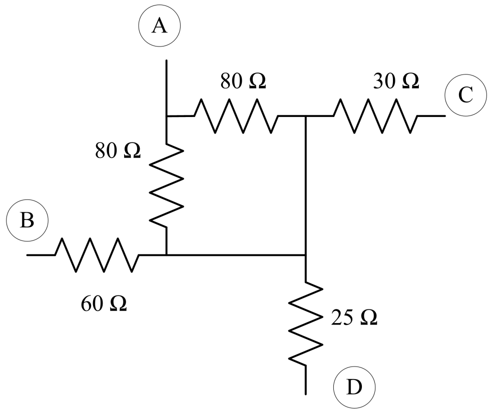


Figure 2–31

If current flows between terminals A and C, then no current flows through the 60-Ω and the 25-Ω resistors and they are not part of the circuit. The two 80-Ω resistors are in parallel and that combination is in series with the 30-Ω resistor, so we have $R_{\mathrm{A-C}} = (80 \parallel 80) + 30 = 70\,\Omega$. If current flows between terminals A and D, then no current flows through the 60-Ω and the 30-Ω resistors and they are not part of the circuit. Again, the two 80-Ω resistors are in parallel and that combination is in series with the 25-Ω resistor, so we have $R_{\mathrm{A-D}} = (80 \parallel 80) + 25 = 65\,\Omega$. If current flows between terminals B and C, then no current flows through the 25-Ω resistor and it is not part of the circuit. In addition, in the remaining circuit, the two 80-Ω resistors are shorted out. The resulting circuit is a series combination of the 60-Ω and 30-Ω resistors, which yields $R_{\mathrm{B-C}} = 60 + 30 = 90\,\Omega$. If current flows between terminals B and D, then no current flows through the 30-Ω resistor and it is not part of the circuit. In addition, in the remaining circuit, the two 80-Ω resistors are again shorted out. The resulting circuit is a series combination of the 60-Ω and 25-Ω resistors, which yields $R_{\mathrm{B-D}} = 60 + 25 = 85\,\Omega$. Finally, with current flowing between terminals C and D, the 60-Ω resistor is not part of the circuit and the two 80-Ω resistors are shorted out. The equivalent resistance is the series combination of the 30-Ω and 25-Ω resistors, which yields $R_{\mathrm{C-D}} = 25 + 30 = 55\,\Omega$.

Exercise 2–17. A practical current source consists of a 2-mA ideal current source in parallel with a 500-Ω resistance. Find the equivalent practical voltage source.

The equivalent practical voltage source will have the same 500-Ω resistance. To transform the current source into a voltage source, we compute $v_{\mathrm{S}} = i_{\mathrm{S}} R = (2\,\mathrm{mA})(500\,\Omega) = 1$ V.

Exercise 2–18. Find the equivalent circuit for each of the following

(a). Three ideal 1.5-V batteries connected in series.

For voltage sources connected in series, the voltages add. Assuming all three sources are oriented in the same direction, the equivalent voltage is $1.5 + 1.5 + 1.5 = 4.5$ V.

(b). A 5-mA current source in series with a 100-kΩ resistor.

A current source in series with a resistor acts as a current source without the resistor, so the equivalent circuits is a single 5-mA current source.

(c). A 40-A ideal current source in parallel with an ideal 10-A current source.

For ideal current sources in parallel, the currents add, so the equivalent circuit is a 50-A current source.

(d). A 100-V source in parallel with two 10-kΩ resistors.

A voltage source in parallel with any resistance acts like a voltage source, so the equivalent circuits is a single 100-V voltage source.

(e). An ideal 15-V source in series with an ideal 10-mA source.

This is not a valid combination of sources and the two cannot be combined in a theoretical perspective.

(f). A 15-V ideal source and a 5-V ideal source connected in parallel.

This is not a valid combination of voltage sources, since a parallel combination of elements must have the same voltage.

Exercise 2–19. Find the voltages v_x, v_y, and v_z in the circuit of Figure 2–39. Show that the sum of all the voltages across each of the individual resistors equals the source voltage.

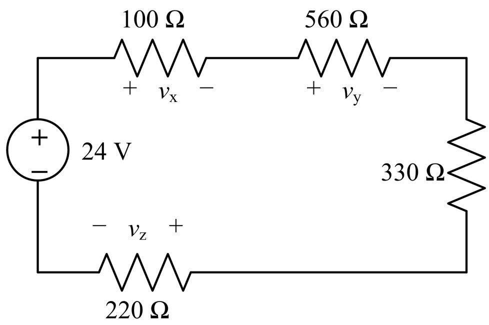


Figure 2–39

For each resistor, use voltage division to find its corresponding voltage.

$$v_x = \left(\frac{100}{100+560+330+220}\right)24 = 1.9835\,V$$

$$v_y = \left(\frac{560}{100+560+330+220}\right)24 = 11.1074\,V$$

$$v_O = \left(\frac{330}{100+560+330+220}\right)24 = 6.5455\,V$$

$$v_z = \left(\frac{220}{100+560+330+220}\right)24 = 4.3636\,V$$

Sum the voltages to get $1.9835 + 11.107 + 6.5455 + 4.3636 = 24$ V, which matches the source voltage.

Exercise 2–20. In Figure 2-40, $R_x = 10\,\text{k}\Omega$. The output voltage $v_O = 20$ V. Find the voltage source that would produce that output. (*Hint:* It is not 10 V.)

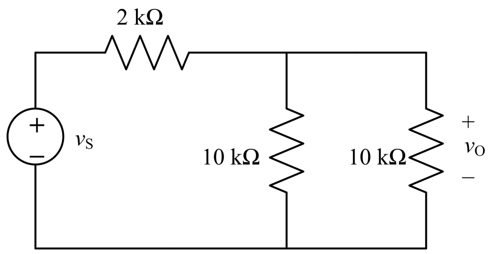


Figure 2–40

Combine the two 10-kΩ resistors in parallel to get a single 5-kΩ resistor in series with the 2-kΩ resistor. The 5-kΩ resistor still has 20 V across it. Use the voltage division equation to solve for the voltage of the source as follows:

$$20 = \left(\frac{5000}{5000+2000}\right) v_s$$

$$v_s = \left(\frac{5000+2000}{5000}\right) 20 = 28\,\text{V}$$

Exercise 2–21. In Figure 2–41, suppose that a resistor R_4 is connected across the output. What value should R_4 be if we want $\frac{1}{2}v_S$ to appear between node A and ground?

Using the concept of voltage division, for one-half of v_S to appear between node A and ground, the resistance between node A and ground will have to match R_1 so that the source voltage divides equally between the two parts of the circuit. The equivalent resistance between node A and ground is the series combination of R_3 and R_4 in parallel with R_2 or $R_{EQ} = R_2 \parallel (R_3 + R_4)$. Setting $R_{EQ} = R_1$ we can solve for R_4 as follows:

$$R_1 = R_2 \parallel (R_3 + R_4) = \frac{R_2(R_3+R_4)}{R_2+R_3+R_4} = \frac{R_2R_3 + R_2R_4}{R_2+R_3+R_4}$$

$$R_1(R_2+R_3+R_4) = R_2R_3 + R_2R_4$$

$$R_1R_4 - R_2R_4 = R_2R_3 - R_1R_2 - R_1R_3$$

$$R_4(R_1 - R_2) = R_2R_3 - R_1R_2 - R_1R_3$$

$$R_4 = \frac{R_2R_3 - R_1R_2 - R_1R_3}{R_1 - R_2} = \frac{R_1R_3 + R_1R_2 - R_3R_2}{R_2 - R_1}$$

Exercise 2–22. Ten volts (v_s) are connected acorss the 10-kΩ potentiometer (R_{TOTAL}) shown in Figure 2–42(c). A load resistor of 10 kΩ is connected across its output. At what resistance should the wiper ($R_{\text{TOTAL}} - R_1$) be set so that 2 V appears at the output, v_O?

To solve this problem, first define $R_2 = R_{\text{TOTAL}} - R_1$, which is the resistance we want to find. For a 10-kΩ potentiometer, $R_1 + R_2 = 10$ kΩ, so $R_1 = 10\,\text{k}\Omega - R_2$. The equivalent resistance of the output is $R_{EQ} = R_2 \parallel 10\,\text{k}\Omega$. Now use the voltage division equation and the specified source and output voltages to solve for R_2 as follows:

$$\begin{aligned}
2 &= \left(\frac{R_{\text{EQ}}}{R_1 + R_{\text{EQ}}}\right)10 = \left[\frac{\dfrac{10^4\,R_2}{10^4+R_2}}{10^4 - R_2 + \left(\dfrac{10^4\,R_2}{10^4+R_2}\right)}\right]10 \\
2\left[10^4 - R_2 + \left(\frac{10^4\,R_2}{10^4+R_2}\right)\right] &= \left(\frac{10^4\,R_2}{10^4+R_2}\right)10 \\
(10^4 - R_2)(10^4+R_2) + 10^4\,R_2 &= (10^4\,R_2)(5) \\
-R_2^2 + 10^8 + 10^4\,R_2 &= (5\times 10^4)R_2 \\
R_2^2 + (4\times 10^4)R_2 - 10^8 &= 0
\end{aligned}$$

Solving for the positive root of the quadratic equation, we get $R_2 = 2.36\,\text{k}\Omega$. The other root is negative, so it is not a valid solution for a resistance.

Exercise 2–23. (a). Find i_{x} and i_{z} in the circuit of Figure 2–46(a).

Use current division to find all of the currents. Note that i_{z} flows through an equivalent resistance of 10 Ω.

$$\begin{aligned}
i_{\text{x}} &= \left(\frac{\frac{1}{20}}{\frac{1}{20}+\frac{1}{20}+\frac{1}{10}}\right)5 = 1.25\,\text{A} \\
i_{\text{y}} &= \left(\frac{\frac{1}{20}}{\frac{1}{20}+\frac{1}{20}+\frac{1}{10}}\right)5 = 1.25\,\text{A} \\
i_{\text{z}} &= \left(\frac{\frac{1}{10}}{\frac{1}{20}+\frac{1}{20}+\frac{1}{10}}\right)5 = 2.5\,\text{A}
\end{aligned}$$

(b). Show that the sum of i_{x}, i_{y}, and i_{z} equals the source current.

Sum the currents found in part (a), $i_{\text{x}} + i_{\text{y}} + i_{\text{z}} = 1.25 + 1.25 + 2.5 = 5$ A.

Exercise 2–24. The circuit in Figure 2-47 shows a delicate device that is modeled by a 90-Ω equivalent resistance. The device requires a current of 1 mA to operate properly. A 1.5-mA fuse is inserted in series with the device to protect it from overheating. The resistance of the fuse is 10 Ω. Without the shunt resistance R_{x}, the source would deliver 5 mA to the device, causing the fuse to blow. Inserting a shunt resistor R_{x} diverts a portion of the available source current around the fuse and device. Select a value of R_{x} so only 1 mA is delivered to the device.

The equivalent resistance of the device and its fuse is 100 Ω. Write the current division equation such that the current through the device is 1 mA and then solve for the shunt resistance R_{x}.

$$\begin{aligned}
1 &= \left(\frac{\frac{1}{100}}{\frac{1}{100}+\frac{1}{R_{\text{x}}}+\frac{1}{100}}\right)10 = \left(\frac{R_{\text{x}}}{R_{\text{x}}+100+R_{\text{x}}}\right)10 = \left(\frac{R_{\text{x}}}{2R_{\text{x}}+100}\right)10 \\
2R_{\text{x}} + 100 &= 10R_{\text{x}} \\
8R_{\text{x}} &= 100 \\
R_{\text{x}} &= 12.5\,\Omega
\end{aligned}$$

Exercise 2–25. Repeat the problem of Example 2–22 if the battery's internal resistance increases to 70 mΩ. Will there be sufficient current available to start the car?

Perform a source transformation with the 12.6-V battery and the 70-mΩ resistor. The resulting current source has a value of 180 A, so it cannot supply 210.1 A to the starter and accessories. Using the second approach described in Example 2–22, the current through the source resistance is 210.1 A and the resistance is 70 mΩ. The voltage drop across the source resistance is $(210.1)(0.070) = 14.707$ V. This voltage is greater than the battery rating, so there will not be sufficient current to start the car.

Exercise 2–26. In Figure 2–51, $R = 15$ kΩ. The voltage source $v_S = 5$ V. Find the power delivered to the circuit by the source.

We can apply the analysis completed in Example 2–24 to solve for the source current

$$i_S = \frac{3}{5}\frac{v_S}{R} = \frac{3}{5}\frac{5}{15\times 10^3} = 200\,\mu\text{A}$$

We can now solve for the source power $p_s = v_S i_S = (5)(200\times 10^{-6}) = 1$ mW.

Exercise 2–27. In Figure 2–53, find the current through the $2R$ resistor.

Using Figure 2–53(b) and current division, we can solve for i_{2R} directly as follows:

$$i_{2R} = \left(\frac{\dfrac{1}{2R}}{\dfrac{1}{R}+\dfrac{1}{2R}+\dfrac{1}{R}}\right)\frac{v_S}{R} = \frac{1}{2+1+2}\frac{v_S}{R} = \frac{v_S}{5R}\text{ A}$$

Exercise 2–28. Find v_x and i_x using circuit reduction on the circuit in Figure 2–54.

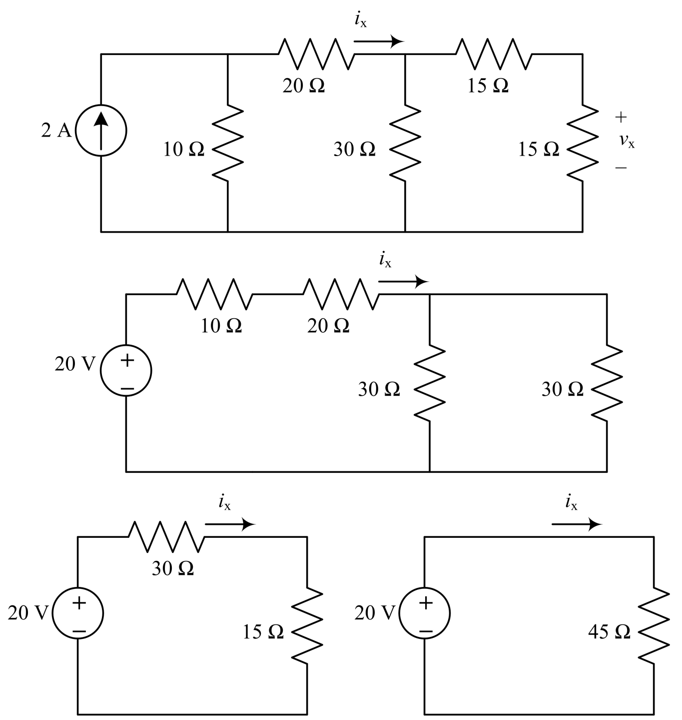


Figure Ex2–28

Figure 2–28 shows the circuit reduction process. In the first step, perform a source transformation and combined the two 15-Ω resistors in series on the right. Next, combined the 10-Ω and 20-Ω resistors in series and the two 30-Ω resistors in parallel. Finally, combine the 30-Ω and 15-Ω resistors in series. Throughout this reduction process we have not disturbed current i_x so we can compute it directly as $i_x = 20/45 = 444$ mA.

Tracing back to the first circuit with the voltage source, we see that i_x enters a circuit where the current divides equally between two 30-Ω resistors. The current through each resistor is half of the original or 222 mA. Therefore, 222 mA flows through each 15-Ω resistor in the original circuit. We can then compute $v_x = (15)(0.222) = 3.33$ V.

Exercise 2–29. Find v_x and v_y using circuit reduction on the circuit in Figure 2–55.

Combine the two voltage sources in series together to get a single 12-V source. To solve for v_x, first note that the voltage source is in parallel with the series combination of resistors on the far right. From the perspective of the left side of the circuit, we can safely ignore the resistors to the right of the voltage source. Perform a source transformation on the 12-V source and the 1-kΩ to its left to get a 12-mA current source in parallel with a 1-kΩ resistor. Now perform current division to find the current through the 1.5-kΩ resistor as follows

$$i_x = \left[\frac{\frac{1}{1.5+2.2}}{\frac{1}{1.5+2.2}+\frac{1}{3.3}+\frac{1}{1}}\right](-12) = -2.0614\,\mathrm{mA}$$

Note the sign convention for v_x introduces the negative sign for the current. Apply Ohm's law to find the voltage $v_x = (1500)(-0.0020614) = -3.092$ V.

To solve for v_y, perform voltage division using the equivalent 12-V source as follows:

$$v_y = \left(\frac{3.3}{1+3.3}\right)12 = 9.2093\,\mathrm{V}$$

Exercise 2–30. Find the voltage across the current source in Figure 2–57.

Combine the resistors on the left in series to get an equivalent resistance of $2.2 + 1.5 + 1 = 4.7\,\mathrm{k\Omega}$. Combine the resistors on the right in series to get an equivalent resistance of $1 + 3.3 = 4.3\,\mathrm{k\Omega}$. Combine the two equivalent resistance in parallel to get a final equivalent resistance of $4.7 \parallel 4.3 = 2.2456\,\mathrm{k\Omega}$. The current flowing through the equivalent resistance yields a voltage of $(2.2456\,\mathrm{k\Omega})(0.1\,\mathrm{mA}) = 224.56$ mV. Given the sign convention in Figure 2–57, the source voltage is negative and $v_S = -224.56$ mV.

2.2 Problem Solutions

Problem 2–1. The current through a 56-kΩ resistor is 2.2 mA. Find the voltage across the resistor.

Using Ohm's law we have $v = Ri = (56 \times 10^3)(2.2 \times 10^{-3}) = 123.2$ V.

Problem 2–2. The voltage across a particular resistor is 6.23 V and the current is 2.75 mA. What is the actual resistance of the resistor? Using the inside back cover, what is the likely standard value of the resistor?

Using Ohm's law we can solve for resistance as $R = v/i = (6.23)/(2.75 \times 10^{-3}) = 2.2655\,\mathrm{k}\Omega$. Using the table of standard values, the resistor is likely marked as a 2.2-kΩ resistor.

Problem 2–3. A 100-kΩ resistor dissipates 100 mW. Find the current through the resistor.

The power dissipated by a resistor is $p = i^2R$. Solving for current, we have $i = \sqrt{p/R} = \sqrt{(10^{-1})/(10^5)} = 1\,\mathrm{mA}$.

Problem 2–4. The conductance of a particular resistor is 0.5 mS. Find the current through the resistor when connected across a 5-V source.

Using the conductance version of Ohm's law, we have $i = Gv = (0.5 \times 10^{-3})(5) = 2.5\,\mathrm{mA}$.

Problem 2–5. In Figure P2–5 the resistor dissipates 25 mW. Find R_x.

The power dissipated by a resistor can be written as $p = v^2/R$. Solving for the resistance, we have $R_x = v^2/p_x = (15^2)/(25 \times 10^{-3}) = 9\,\mathrm{k}\Omega$.

Problem 2–6. In Figure P2–6 find R_x and the power delivered to the resistor.

Using Ohm's law to solve for resistance, we have $R_x = v/i = 100/(10^{-2}) = 10\,\mathrm{k}\Omega$. The power delivered to the resistor is $p = vi = (100)(10^{-2}) = 1$ W.

Problem 2–7. A resistor found in the lab has three orange stripes followed by a gold stripe. An ohmmeter measures its resistance as 34.9 kΩ. Is the resistor properly color coded? (See inside back cover for color code.)

Since there are three colored stripes and a gold stripe, the first two stripes are the significant digits, the third stripe is the multiplier, and the gold stripe is the tolerance. Using the color code table, the significant digits for the first two stripes are 3 and 3. The multiplier associated with orange is 1 k, so we have $33 \times 1000 = 33$ kΩ. The tolerance associated with the gold stripe is ±5%, which gives a range of resistances from 31.35 to 34.65 kΩ. The resistor measured outside of this range, but its measured value is within 10% of 33 kΩ, so it should have a silver tolerance stripe in place of the gold one.

Problem 2–8. The *i*-*v* characteristic of a nonlinear resistor is $v = 82i + 0.18i^3$.

(a). Calculate v and p for $i = \pm 0.5$, ± 1, ± 2, ± 5, and ± 10 A.

Since there are 10 values for the current to examine, this problem is best solved with MATLAB. We will solve the problem for one current and then automate the process. Given $i = +0.5$ A, use the expression to calculate the voltage as $v = 82i + 0.18i^3 = (82)(0.5) + (0.18)(0.5)^3 = 41.023$ V. We can now calculate the power as $p = vi = (41.023)(0.5) = 20.511$ W. To compute the other solutions, we use MATLAB as shown below.

```
ii = [-10, -5, -2, -1, -0.5, 0.5, 1, 2, 5, 10];
v = 82*ii + 0.18*ii.^3;
p = v.*ii;
Results = [ii' v' p']
```

The corresponding MATLAB output is shown below.

```
Results =
   -10.0000e+000    -1.0000e+003    10.0000e+003
    -5.0000e+000  -432.5000e+000     2.1625e+003
    -2.0000e+000  -165.4400e+000   330.8800e+000
    -1.0000e+000   -82.1800e+000    82.1800e+000
  -500.0000e-003   -41.0225e+000    20.5113e+000
```

```
 500.0000e-003    41.0225e+000    20.5113e+000
   1.0000e+000    82.1800e+000    82.1800e+000
   2.0000e+000   165.4400e+000   330.8800e+000
   5.0000e+000   432.5000e+000     2.1625e+003
  10.0000e+000     1.0000e+003    10.0000e+003
```

The results are summarized in the following table.

i (A)	v (V)	p (W)
-10	-1000.00	10000.00
-5	-432.50	2162.50
-2	-165.44	330.88
-1	-82.18	82.18
-0.5	-41.02	20.51
0.5	41.02	20.51
1	82.18	82.18
2	165.44	330.88
5	432.50	2162.50
10	1000.00	10000.00

(b). Find the maximum error in v when the device is treated as a 82-Ω linear resistance on the range $|i| < 0.5$ A.

The term $0.18i^3$ makes the expression for v nonlinear and represents the difference between the actual device and an 82-Ω resistor. The maximum error will occur when the absolute value of the cubic term is maximized. That occurs at the extreme values for i, which are ±0.5 A, in this case. At +0.5 A, the actual voltage is 41.0225 V and the voltage across a 82-Ω resistor is exactly 41 V. The error is 22.5 mV. At −0.5 A, the actual voltage is −41.0225 V and the voltage across a 82-Ω resistor is exactly −41 V, so the error is −22.5 mV. In both cases, the percentage of error is 0.055%, which is exceptionally small.

Problem 2–9. A 100-kΩ resistor has a power rating of 0.125 W. Find the maximum voltage that can be applied to the resistor.

The power dissipated by a resistor can be expressed as $p = v^2/R$. Solving for the voltage, we have $v = \sqrt{pR} = \sqrt{(0.125)(10^5)} = 111.803$ V.

Problem 2–10. A certain type of film resistor is available with resistance values between 10 Ω and 100 MΩ. The maximum ratings for all resistors of this type are 500 V and 0.25 W. Show that the voltage rating is the controlling limit for $R > 1\,\text{M}\Omega$, and that the power rating is the controlling limit when $R < 1\,\text{M}\Omega$.

The power dissipated by a resistor can be expressed as $p = v^2/R$. Solving for the resistance, we have $R = v^2/p$. With both maximum ratings applied, the resistance is $R = (500)^2/(0.25) = 1\,\text{M}\Omega$. Therefore at $R = 1\,\text{M}\Omega$ there are no issues with either type of rating. If the resistance increases above 1 MΩ, then using $R = v^2/p$, the maximum power must be less than 0.25 W. Therefore, the resistor will never dissipate 0.25 W and the voltage rating will be the only active constraint. If the resistance is less than 1 MΩ, then the maximum voltage must be less than 500 V and power rating will be the only active constraint.

Problem 2–11. Figure P2–11 shows the circuit symbol for a class of two-terminal devices called diodes. The i-v relationship for a specific pn junction diode is $i = 2 \times 10^{-16}\left(e^{40v} - 1\right)$ A.

(a). Use this equation to find i and p for $v = 0$, ±0.1, ±0.2, ±0.4, and ±0.8 V. Use these data to plot the i-v characteristic of the element.

For each voltage, use the given equation to compute the current and then use $p = vi$ to compute the associated power. MATLAB is appropriate for these calculations and plotting.

```
v = [-0.8, -0.4, -0.2, -0.1, 0 0.1, 0.2, 0.4, 0.8];
ii = 2e-16*(exp(40*v)-1);
p = v.*ii;
Results = [v' ii' p']
```

The corresponding MATLAB output is shown below followed by a plot of the data in Figure P2–11.

```
Results =
  -800.0000e-003  -200.0000e-018   160.0000e-018
  -400.0000e-003  -200.0000e-018    80.0000e-018
  -200.0000e-003  -199.9329e-018    39.9866e-018
  -100.0000e-003  -196.3369e-018    19.6337e-018
     0.0000e+000     0.0000e+000     0.0000e+000
   100.0000e-003    10.7196e-015     1.0720e-015
   200.0000e-003   595.9916e-015   119.1983e-015
   400.0000e-003     1.7772e-009   710.8888e-012
   800.0000e-003    15.7926e-003    12.6341e-003
```

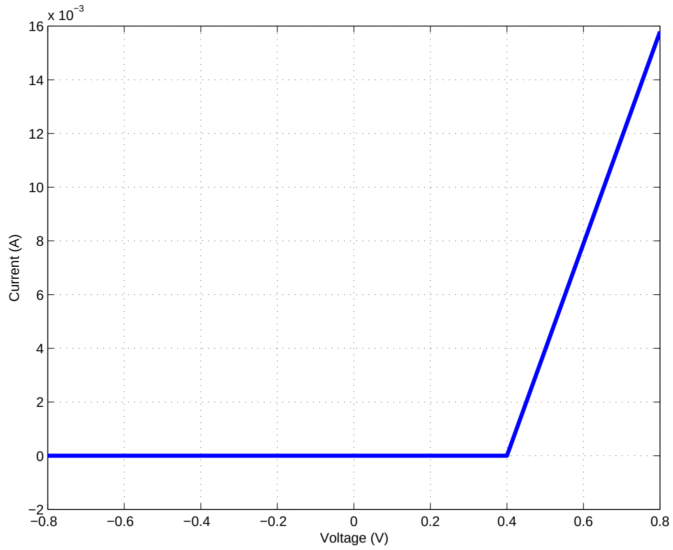


Figure P2–11

(b). Is the diode linear or nonlinear, bilateral or nonbilateral, and active or passive?

The plot in Part (a) shows that the device is nonlinear and nonbilateral. The power for the device is always positive, so it is passive.

(c). Use the diode model to predict i and p for $v = 5$ V. Do you think the model applies to voltages in this range? Explain.

For $v = 5$ V, $i = 1.45 \times 10^{71}$ A and $p = 7.23 \times 10^{71}$ W. The model is not valid because the current and power are too large.

(d). Repeat (c) for $v = -5$ V.

For $v = -5$ V, $i = -2.00 \times 10^{-16}$ A and $p = 1.00 \times 10^{-15}$ W. The model is valid because the current and power are both essentially zero.

Problem 2–12. A thermistor is a temperature-sensing element composed of a semiconductor material which exhibits a large change in resistance proportional to a small change in temperature. A particular thermistor has a resistance of 5 kΩ at 25°C. Its resistance is 340 Ω at 100°C. Assuming a straight-line relationship between these two values, at what temperature will the thermistor's resistance equal 1 kΩ?

Find the rate at which the resistance changes for each degree of temperature.

$$\Delta_\Omega = \frac{5000 - 340}{25 - 100} = \frac{4660}{-75} = -62.13\ \Omega/^\circ\mathrm{C}$$

To go from 5 kΩ to 1 kΩ, the resistance changes by −4 kΩ, which means the temperature change is $-4000/(-62.13) = 64.38°C$. The final temperature is $25 + 64.38 = 89.38°C$.

Problem 2–13. In Figure P2–13 $i_2 = -5$ A and $i_3 = 2$ A. Find i_1 and i_4.
The KCL equations for nodes B and C are

$$
\begin{aligned}
-i_1 - i_2 &= 0 \\
i_2 + i_3 - i_4 &= 0
\end{aligned}
$$

Using the first equation, we can solve for $i_1 = -i_2 = 5$ A. Using the second equation, we can solve for $i_4 = i_2 + i_3 = -5 + 2 = -3$ A.

Problem 2–14. In Figure P2–14 $v_1 = 3$ V and $v_3 = 5$ V. Find v_2, v_4 and v_5.
We can use a KVL equation on the left loop and the two given voltages to solve for v_2. The KVL equation is $-v_1 + v_2 + v_3 = 0$. Solving for $v_2 = v_1 - v_3 = 3 - 5 = -2$ V. In examining the circuit, there is a ground on each side of v_5, so the voltage difference across this element is zero, $v_5 = 0$ V. We can now use KVL around the right loop to solve for v_4. The KVL equation is $-v_3 + v_4 + v_5 = 0$. Solve for $v_4 = v_3 - v_5 = 5 - 0 = 5$ V.

Problem 2–15. For the circuit in Figure P2–15:

(a). Identify the nodes and at least two loops.

The circuit has three nodes and three loops. The nodes are labeled A, B, and C. The first loop contains elements 1 and 2, the second loop contains elements 2, 3, and 4, and the third loop contains elements 1, 3 and 4.

(b). Identify any elements connected in series or in parallel.

Elements 3 and 4 are connected in series. Elements 1 and 2 are connected in parallel.

(c). Write KCL and KVL connection equations for the circuit.

The KCL equations are

$$
\begin{array}{ll}
\text{Node A} & -i_1 - i_2 - i_3 = 0 \\
\text{Node B} & i_3 - i_4 = 0 \\
\text{Node C} & i_1 + i_2 + i_4 = 0
\end{array}
$$

The KVL equations are

$$
\begin{array}{ll}
\text{Loop 1 2} & -v_1 + v_2 = 0 \\
\text{Loop 2 3 4} & -v_2 + v_3 + v_4 = 0 \\
\text{Loop 1 3 4} & -v_1 + v_3 + v_4 = 0
\end{array}
$$

Problem 2–16. In Figure P2–15, $i_2 = -20$ mA and $i_4 = 10$ mA. Find i_1 and i_3.
The KCL equations for the circuit are

$$
\begin{array}{ll}
\text{Node A} & -i_1 - i_2 - i_3 = 0 \\
\text{Node B} & i_3 - i_4 = 0 \\
\text{Node C} & i_1 + i_2 + i_4 = 0
\end{array}
$$

Using the equation for node C, we can solve $i_1 = -i_2 - i_4 = 20 - 10 = 10$ mA. Using the equation for node B, we can solve $i_3 = i_4 = 10$ mA.

Problem 2–17. For the circuit in Figure P2–17:

(a). Identify the nodes and at least three loops in the circuit.

The are four nodes and at least five loops. There are only three independent KVL equations. The nodes are labeled A, B, C, and D. Valid loops include the following sequences of elements: (1, 3, 2), (2, 4, 5), (3, 6, 4), (1, 6, 5), and (2, 3, 6, 5).

(b). Identify any elements connected in series or in parallel.

In this circuit, none of the elements are connected in series and none of them are connected in parallel.

(c). Write KCL and KVL connection equations for the circuit.

The KCL equations are

$$\begin{array}{ll} \text{Node A} & -i_2 - i_3 - i_4 = 0 \\ \text{Node B} & -i_1 + i_3 - i_6 = 0 \\ \text{Node C} & i_1 + i_2 + i_5 = 0 \\ \text{Node D} & i_4 - i_5 + i_6 = 0 \end{array}$$

Three independent KVL equations are

$$\begin{array}{ll} \text{Loop 1 3 2} & -v_1 - v_3 + v_2 = 0 \\ \text{Loop 2 4 5} & -v_2 + v_4 + v_5 = 0 \\ \text{Loop 3 6 4} & v_3 + v_6 - v_4 = 0 \end{array}$$

Problem 2–18. In Figure P2–17 $v_2 = 10$ V, $v_3 = -10$ V, and $v_4 = 3$ V. Find v_1, v_5, and v_6.
The KVL equations are

$$\begin{array}{ll} \text{Loop 1 3 2} & -v_1 - v_3 + v_2 = 0 \\ \text{Loop 2 4 5} & -v_2 + v_4 + v_5 = 0 \\ \text{Loop 3 6 4} & v_3 + v_6 - v_4 = 0 \end{array}$$

Using the first equation, we can solve for $v_1 = v_2 - v_3 = 10 + 10 = 20$ V. Using the second equation, we can solve for $v_5 = v_2 - v_4 = 10 - 3 = 7$ V. Using the third equation, we can solve $v_6 = v_4 - v_3 = 3 + 10 = 13$ V.

Problem 2–19. In many circuits the ground is often the metal case that houses the circuit. Occasionally a failure occurs whereby a wire connected to a particular node touches the case causing that node to become connected to ground. Suppose that in Figure P2–17 Node B accidently touches ground. How would that affect the voltages found in Problem 2–18?

If Node B is connected to ground, then element 6 is connected to ground on both sides, so its voltage is $v_6 = 0$ V. If we define $v_6 = 0$ V, all of the original KVL equations found in Problem 2–17 are still valid. Even though the equations are valid, Problem 2–18 is no longer valid because there is a conflict with the given voltages. Using the KVL equation $v_3 + v_6 - v_4 = 0$ and substituting in $v_6 = 0$, we get $v_3 = v_4$. In Problem 2–18, the given values are $v_3 = -10$ V and $v_4 = 3$ V, which is not possible if Node B is connected to ground.

Problem 2–20. The circuit in Figure P2–20 is organized around the three signal lines A, B, and C.

(a). Identify the nodes and at least three loops in the circuit.

The are four nodes and at least five loops. The nodes are labeled A, B, C, and D. Valid loops include the following sequences of elements: (1, 3, 2), (2, 4, 5), (3, 6, 4), (1, 6, 5), and (2, 3, 6, 5).

(b). Write KCL connection equations for the circuit.

The KCL equations are

$$\begin{array}{ll} \text{Node A} & -i_2 - i_3 - i_4 = 0 \\ \text{Node B} & -i_1 + i_3 - i_6 = 0 \\ \text{Node C} & i_1 + i_2 + i_5 = 0 \\ \text{Node D} & i_4 - i_5 + i_6 = 0 \end{array}$$

(c). If $i_1 = -20$ mA, $i_2 = -12$ mA, and $i_3 = 50$ mA, find i_4, i_5, and i_6.

Using the KCL equation at node A, we can solve for $i_4 = -i_2 - i_3 = 12 - 50 = -38$ mA. Using the KCL equation are node C, we can solve for $i_5 = -i_1 - i_2 = 20 + 12 = 32$ mA. Using the KCL equation at node D, we can solve for $i_6 = i_5 - i_4 = 32 + 38 = 70$ mA.

(d). Show that the circuit in Figure P2–20 is identical to that in Figure P2–17.

The circuits have the same nodes, connections, and current directions, so they must be equivalent.

Problem 2–21. In Figure P2–21 $v_2 = 10$ V, $v_4 = 5$ V, and $v_5 = 15$ V. Find v_1, v_3, and v_6.

The KVL equations for the circuit are

$$\begin{array}{ll} \text{Loop 1 2 3} & -v_1 + v_2 + v_3 = 0 \\ \text{Loop 3 4 5} & -v_3 + v_4 + v_5 = 0 \\ \text{Loop 2 6 4} & -v_2 + v_6 - v_4 = 0 \end{array}$$

Using the second loop equation, we can solve for $v_3 = v_4 + v_5 = 5 + 15 = 20$ V. Using the first loop equation, we can solve for $v_1 = v_2 + v_3 = 10 + 20 = 30$ V. Finally, using the third loop equation, we can solve for $v_6 = v_2 + v_4 = 10 + 5 = 15$ V.

Problem 2–22. In Figure P2–22 $i_1 = 25$ mA, $i_2 = 10$ mA, and $i_3 = -15$ mA. Find i_4 and i_5.

The KCL equations for the circuit are

$$\begin{array}{ll} \text{Node A} & i_1 - i_2 + i_3 - i_4 = 0 \\ \text{Node B} & -i_1 + i_2 - i_5 = 0 \\ \text{Node C} & -i_3 + i_4 + i_5 = 0 \end{array}$$

Using the first node equation, we can solve for $i_4 = i_1 - i_2 + i_3 = 25 - 10 - 15 = 0$ mA. Using the second node equation, we can solve for $i_5 = -i_1 + i_2 = -25 + 10 = -15$ mA.

Problem 2–23.

(a). Use the passive sign convention to assign voltage variables consistent with the currents in Figure P2–22. Write three KVL connection equations using these voltage variables.

Figure P2–23 shows the original Figure P–22 with the voltages labeled following the passive sign convention. The KVL equations for the circuit are

$$\begin{array}{ll} \text{Loop 1 2} & v_1 + v_2 = 0 \\ \text{Loop 2 4 5} & -v_2 + v_4 - v_5 = 0 \\ \text{Loop 3 4} & v_3 + v_4 = 0 \end{array}$$

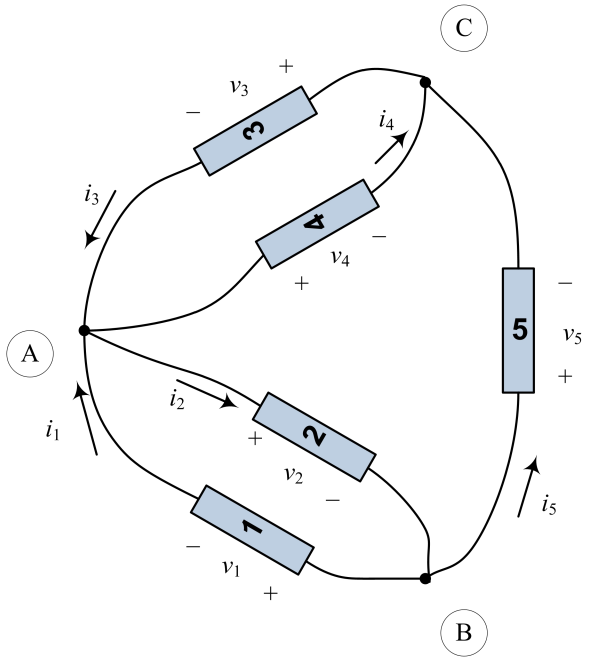


Figure P2–23

(b). If $v_4 = 0$ V, what can be said about the voltages across all the other elements?

If $v_4 = 0$ V, then the third loop equation indicates that $v_3 = 0$ V. Applying these voltages to the other two loop equations, we have $v_1 = -v_2$ and $v_2 = -v_5$.

Problem 2–24. The KCL equations for a three-node circuit are:

$$\begin{array}{ll} \text{Node A} & -i_1 + i_2 - i_4 = 0 \\ \text{Node B} & -i_2 - i_3 + i_5 = 0 \\ \text{Node C} & i_1 + i_3 + i_4 - i_5 = 0 \end{array}$$

Draw the circuit diagram and indicate the reference directions for the element currents.

There are many equivalent diagrams to solve this problem. One possible solution is shown in Figure P2–24.

Problem 2–25. Find v_x and i_x in Figure P2–25.

The current i_x points in the opposite direction as the 500-μA current source, so $i_x = -500\ \mu$A. Using Ohm's law, we have $v_x = Ri_x = (68 \times 10^3)(-500 \times 10^{-6}) = -34$ V.

Problem 2–26. Find v_x and i_x in Figure P2–26.

The circuit has a single current, i_x, which flows clockwise. Label the 22-kΩ resistor as R_1 with voltage drop v_1 following the passive sign convention (positive on left and negative on right). The KVL equation for

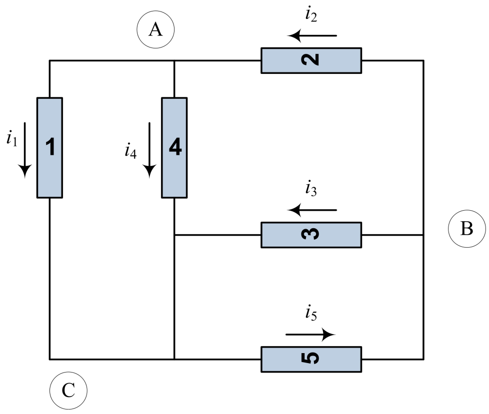


Figure P2–24

the circuit combined with Ohm's law provides the solution.

$$\begin{aligned}
-18 + v_1 + v_\mathrm{x} &= 0 \\
v_1 + v_\mathrm{x} &= 18 \\
R_1 i_\mathrm{x} + R_\mathrm{x} i_\mathrm{x} &= 18 \\
(22 \times 10^3) i_\mathrm{x} + (68 \times 10^3) i_\mathrm{x} &= 18 \\
(90 \times 10^3) i_\mathrm{x} &= 18 \\
i_\mathrm{x} &= 200\,\mu\mathrm{A} \\
v_\mathrm{x} &= (68 \times 10^3) i_\mathrm{x} \quad = \quad 13.6\,\mathrm{V}
\end{aligned}$$

Problem 2–27. Find v_x and i_x in Figure P2–27. Compare the results of your answers with those in Problem 2–26. What effect did adding the 33-kΩ resistor have on the overall circuit? Why isn't i_y zero?

Label the 22-kΩ resistor as R_1 with voltage drop v_1 following the passive sign convention (positive on left and negative on right). Label the 33-kΩ resistor with voltage v_y following the passive sign convention (positive on top and negative on bottom). Label the source current as i_S following the passive sign convention (flowing into the positive terminal of the voltage source). The KVL equations for the circuit are:

$$\begin{aligned}
-18 + v_\mathrm{y} &= 0 \\
-v_\mathrm{y} + v_1 + v_\mathrm{x} &= 0
\end{aligned}$$

The KCL equation for the circuit is

$$-i_\mathrm{S} - i_\mathrm{x} - i_\mathrm{y} = 0$$

Solving the first KVL equation, we have $v_\mathrm{y} = 18$ V. Substituting this result into the second KVL equation, we get $v_1 + v_x = 18$. Current i_x flows through both R_1 and R_x, so we can solve as follows:

$$\begin{aligned} v_1 + v_x &= 18 \\ R_1 i_x + R_x i_x &= 18 \\ (22 \times 10^3)i_x + (68 \times 10^3)i_x &= 18 \\ (90 \times 10^3)i_x &= 18 \\ i_x &= 200\,\mu\text{A} \\ v_x &= (68 \times 10^3)i_x \;=\; 13.6\,\text{V} \end{aligned}$$

These results for v_x and i_x match those in Problem 2–26. We can also find i_y using Ohm's law, $i_y = v_y/R_y = (18)/(33 \times 10^3) = 545.5$ μA. Applying the KCL equation, we get $i_S = -i_x - i_y = -200 - 545.5 = -745.5$ μA. Adding the 33-kΩ resistor increased the amount of current flowing from the source. The current i_y is not zero because there is a voltage across the 33-kΩ resistor.

Problem 2–28. A modeler wants to light his model building using miniature grain-of-wheat light bulbs connected in parallel as shown in Figure P2–28. He uses two 1.5-V "C-cells" to power his lights. He wants to use as many lights as possible, but wants to limit his current drain to 500 μA to preserve the batteries. If each light has a resistance of 36 kΩ, how many lights can he install and still be under his current limit?

The two 1.5-V batteries are connected in series to provide a total of 3 V to the circuit. Since the light bulbs are connected in parallel, the entire 3 V appears across each one. Using Ohm's law, the current through each bulb is $i = v/R = 3/(36 \times 10^3) = 83.3$ μA. The design requires the batteries to provide no more the 500 μA, so we can connect up to $500/83.3 = 6$ bulbs in parallel across the batteries.

Problem 2–29. Find v_x and i_x in Figure P2–29.

In the circuit, 0.5 A flows through the 10-Ω resistor in the center. The voltage drop across this resistor is $v = Ri = (10)(0.5) = 5$ V. The 10-Ω resistor is connected in parallel to the 5-Ω resistor, so they have the same voltage drop. The associated KVL equation verifies this fact. With 5 V across the 5-Ω resistor, the current is $i_x = v/R = 5/5 = 1$ A. KCL at the top node requires that the current entering the node equal the current leaving the node. Since we have $0.5 + 1.0 = 1.5$ A leaving the node, 1.5 A must enter the node through the 4 Ω resistor. The voltage drop across the 4 Ω resistor is $v = Ri = (4)(1.5) = 6$ V. We can now write a KVL equation around the first loop to get $-v_x + 6 + 5 = 0$, which implies $v_x = 11$ V.

Problem 2–30. In Figure P2–30:

(a). Assign a voltage and current variable to every element.

Figure P2–30 shows the voltage and current labels following the passive sign convention.

(b). Use KVL to find the voltage across each resistor.

The KVL equations are

$$\begin{aligned} -v_{S1} + v_1 + v_{S3} &= 0 \\ -v_{S1} + v_2 - v_{S2} &= 0 \\ v_{S2} + v_3 + v_{S3} &= 0 \end{aligned}$$

Solving the first equation, we have $v_1 = v_{S1} - v_{S3} = 10 - 15 = -5$ V. Solving the second equation, we have $v_2 = v_{S1} + v_{S2} = 10 + 5 = 15$ V. Solving the third equation, we have $v_3 = -v_{S2} - v_{S3} = -5 - 15 = -20$ V.

(c). Use Ohm's law to find the current through each resistor.

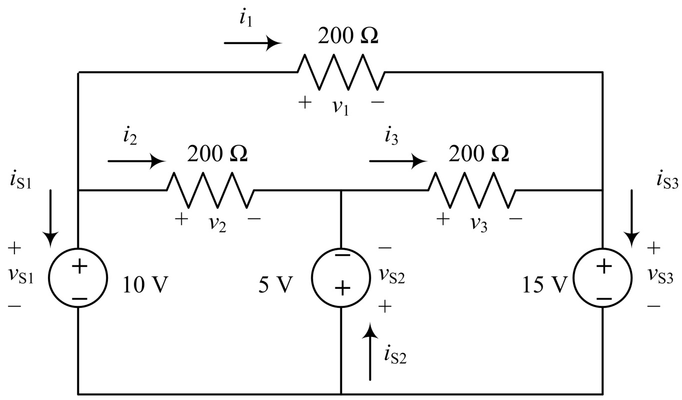


Figure P2–30

Applying $i = v/R$ to each resistor, we have

$$i_1 = \frac{v_1}{R_1} = \frac{-5}{200} = -25\,\text{mA}$$

$$i_2 = \frac{v_2}{R_2} = \frac{15}{200} = 75\,\text{mA}$$

$$i_3 = \frac{v_3}{R_3} = \frac{-20}{200} = -100\,\text{mA}$$

(d). Use KCL to find the current through each voltage source.

The KCL equations are

$$-i_1 - i_2 - i_{S1} = 0$$

$$i_2 - i_3 + i_{S2} = 0$$

$$i_1 + i_3 - i_{S3} = 0$$

Solving the first equation, we have $i_{S1} = -i_1 - i_2 = 25 - 75 = -50$ mA. Solving the second equation, we have $i_{S2} = i_3 - i_2 = -100 - 75 = -175$ mA. Solving the third equation, we have $i_{S3} = i_1 + i_3 = -25 - 100 = -125$ mA.

Problem 2–31. Find the power provided by the source in Figure P2–31.

Figure P2–31 shows the voltage and current labels following the passive sign convention. The KCL

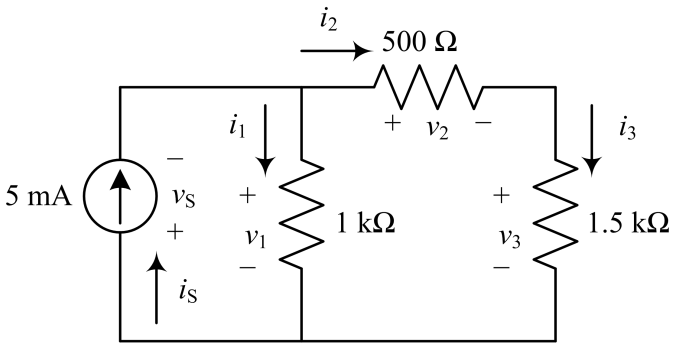


Figure P2–31

equations are

$$\begin{aligned} i_S - i_1 - i_2 &= 0 \\ i_2 - i_3 &= 0 \end{aligned}$$

The KVL equations are

$$\begin{aligned} v_S + v_1 &= 0 \\ -v_1 + v_2 + v_3 &= 0 \end{aligned}$$

The current source requires $i_S = 5$ mA. The first KCL equation implies $i_1 = 5\,\text{mA} - i_2$ and the second implies $i_2 = i_3$. Using Ohm's law and substituting these equations into the second KVL equation, we can solve for the source power as follows:

$$\begin{aligned} v_1 &= v_2 + v_3 \\ R_1 i_1 &= R_2 i_2 + R_3 i_3 \\ R_1(0.005 - i_2) &= R_2 i_2 + R_3 i_2 \\ 1000(0.005 - i_2) &= 500 i_2 + 1500 i_2 \\ 5 - 1000 i_2 &= 2000 i_2 \\ 3000 i_2 &= 5 \\ i_2 &= 1.667\,\text{mA} \\ i_3 &= i_2 = 1.667\,\text{mA} \\ i_1 &= 5 - i_2 = 5 - 1.667 = 3.333\,\text{mA} \\ v_S &= -v_1 = -R_1 i_1 = -(1000)(0.003333) = -3.333\,\text{V} \\ p_S &= v_S\, i_S = (-3.333)(0.005) = -16.667\,\text{mW} \end{aligned}$$

Problem 2–32. Figure P2–32 shows a subcircuit connected to the rest of the circuit at four points.

(a). Use element and connection constraints to find v_x and i_x.

Label the 5-kΩ resistor as R_1 with the current flowing from left to right. Label the 2-kΩ resistor as R_2 with the positive sign at the bottom. Using Ohm's law, we can compute $i_1 = v_1/R_1 = 20/5000 = 4$ mA. The KCL equation at the center node is $4\,\text{mA} + i_1 - i_2 - i_x = 0$. Substituting in the known values, we can solve for i_x as $i_x = 4+i_1-i_2 = 4+4-6 = 2$ mA. Using Ohm's law $v_x = R_x\, i_x = (8000)(0.002) = 16$ V.

(b). Show that the sum of the currents into the rest of the circuit is zero.

The sum of the currents entering the rest of the circuit is $-i_1 + i_2 - 4 + i_x = -4 + 6 - 4 + 2 = 0$ mA.

(c). Find the voltage v_A with respect to the ground in the circuit.

From the ground to v_A there are three voltages. First, there is an increase across the voltage source of 12 V. Next, there is an increase across R_x of 16 V. Finally, there is a decrease across R_2 of $v_2 = R_2 i_2 = (2000)(0.006) = 12$ V. Therefore, $v_A = 12 + 16 - 12 = 16$ V.

Problem 2–33. In Figure P2–33, $i_x = 0.5$ mA. Find the value of R.

Label the left 10-kΩ resistor as R_x, with voltage v_x. Label the unknown resistor as R_1, with current i_1 flowing down. Label the right 10-kΩ resistor as R_2 with the current flowing right to left. Apply the passive sign convention to label the voltages. Use Ohm's law to solve for $v_x = R_x i_x = (10000)(0.0005) = 5$ V. Write the KVL equation for the left side as $-4 - v_x + v_1 = 0$ and solve for v_1 as $v_1 = 4 + v_x = 4 + 5 = 9$ V. Write the KVL equation for the right side as $-v_1 - v_2 + 15 = 0$ and solve for v_2 as $v_2 = 15 - v_1 = 15 - 9 = 6$ V. Use Ohm's law to solve for $i_2 = v_2/R_2 = 6/10000 = 0.6$ mA. Write the KCL equation for the center node as $-i_1 + i_2 - i_x = 0$ and solve for i_1 as $i_1 = i_2 - i_x = 0.6 - 0.5 = 0.1$ mA. Use Ohm's law to find $R_1 = v_1/i_1 = 9/(0.0001) = 90\,\text{k}\Omega$.

Problem 2–34. Figure P2–34 shows a resistor with one terminal connected to ground and the other connected to an arrow. The arrow symbol is used to indicate a connection to one terminal of a voltage source whose other terminal is connected to ground. The label next to the arrow indicates the source voltage at the ungrounded terminal. Find the voltage across, current through, and power dissipated in the resistor.

The voltage across the resistor is the voltage on the left side minus the voltage on the right side. Therefore, $v_x = 0 - (-12) = 0 + 12 = 12$ V. Use Ohm's law to find $i_x = v_x/R_x = 12/(39 \times 10^3) = 307.692\,\mu$A. The power dissipated by the resistor is $p_x = v_x i_x = (12)(307.692 \times 10^{-6}) = 3.6923$ mW.

Problem 2–35. Find the equivalent resistance R_{EQ} in Figure P2–35.

The 10-Ω resistor and the 30-Ω resistor are in parallel. That combination is in series with the 7.5-Ω resistor. We can calculate the equivalent resistance as follows:

$$R_{EQ} = 7.5 + (30 \parallel 10) = 7.5 + \frac{1}{\dfrac{1}{30} + \dfrac{1}{10}} = 7.5 + \frac{(30)(10)}{30 + 10} = 7.5 + 7.5 = 15\,\Omega$$

Problem 2–36. Find the equivalent resistance R_{EQ} in Figure P2–36.

Combine the 33-kΩ and 47-kΩ resistors in series to get an equivalent resistance of $33+47 = 80$ kΩ. The 80-kΩ resistance is in parallel with the 100-kΩ resistor, which yields an equivalent resistance of $100 \parallel 80 = 44.4$ kΩ. That resistance is in series with the 68-kΩ resistor, which yields $R_{EQ} = 68 + 44.4 = 112.4$ kΩ.

Problem 2–37. Find the equivalent resistance R_{EQ} in Figure P2–37.

Working from the right to the left, combine the 10-kΩ resistor in parallel with the 15-kΩ resistor to get an equivalent resistance of 6 kΩ. That resistance is in series with the 33-kΩ resistor, which yields an equivalent resistance of 39 kΩ. Finally, combine the 39-kΩ resistance in parallel with the 56-kΩ resistor to get $R_{EQ} = 22.99$ kΩ.

Problem 2–38. Equivalent resistance is defined at a particular pair of terminals. In the following figure the same circuit is looked at from two different terminal pairs. Find the equivalent resistances R_{EQ1} and R_{EQ2} in Figure P2–38. Note that in calculating R_{EQ2} the 33-kΩ resistor is connected to an open circuit and therefore doesn't affect the calculation.

For R_{EQ1}, ignore the two terminals on the right and collapse the circuit from right to left. The 10-kΩ and the two 22-kΩ resistors are in series; that result is in parallel with the 56-kΩ resistor; and that result is in series with the 33-kΩresistor. We can calculate the equivalent resistance as follows:

$$R_{\text{EQ1}} = 33 + [56 \parallel (10 + 22 + 22)] = 33 + [56 \parallel 54] = 33 + 27.49 = 60.49\,\text{k}\Omega$$

For R_{EQ2}, ignore the two terminals on the left and the 33-kΩ resistor. Collapse the circuit from left to right. The 10, 56, and lower 22-kΩ resistors are in series and that result is in parallel with the right 22-kΩ resistor. We can calculate the equivalent resistance as follows:

$$R_{\text{EQ2}} = [(10 + 56 + 22) \parallel 22] = [88 \parallel 22] = 17.6\,\text{k}\Omega$$

Problem 2–39. Find R_{EQ} in Figure P2–39 when the switch is open. Repeat when the switch is closed.

When the switch is open, the two 100-Ω resistors are in parallel and that result is in series with the two 50-Ω resistors. We can calculate $R_{\text{EQ}} = 50 + (100 \parallel 100) + 50 = 50 + 50 + 50 = 150\ \Omega$. With the switch closed, the wire shorts out the two 100-Ω resistors, so they do not contribute to the equivalent resistance. The results is that the two 50-Ω resistors are in series, so $R_{\text{EQ}} = 50 + 50 = 100\ \Omega$.

Problem 2–40. Find R_{EQ} between nodes A and B for each of the circuits in Figure P2–40. What conclusion can you draw about resistors of the same value connected in parallel?

We can calculate the equivalent resistance for Circuit (a) as follows:

$$R_{\text{EQ}} = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{R}{1+1+1} = \frac{R}{3}$$

For Circuit (b), we have:

$$R_{\text{EQ}} = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R}} = \frac{R}{1+1+1+1+1} = \frac{R}{5}$$

In general, for Circuit (c), we have:

$$R_{\text{EQ}} = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R}} = \frac{R}{1+1+1+\cdots+1} = \frac{R}{n}$$

We can conclude that for identical resistors connected in the parallel, the equivalent resistance is the value of one resistor divided by the number of resistors.

Problem 2–41. Show how the circuit in Figure P2–41 could be connected to achieve a resistance of 100 Ω, 200 Ω, 150 Ω, 50 Ω, 25 Ω, 33.3 Ω, and 133.3 Ω.

For 100 Ω, we need a single 100-Ω resistor, which is a connection between terminals A and D. For 200 Ω, we need two 100-Ω resistors in series, which is a connection between terminals A and B. For 150 Ω, we need a 100-Ω resistor in series with a 50-Ω resistor, which is a connection between terminals A and C. For 50 Ω, we need a single 50-Ω resistor, which is a connection between terminals C and D. We can get 25 Ω by connecting the two 100-Ω resistors in parallel, which yields 50 Ω, and then connecting that result in parallel with the 50-Ω resistor, to get 25 Ω. The required combination is to connect the A, B, and C terminals together on one side and have the D terminal on the other. For 33.3 Ω, connect the 100 and 50-Ω resistors in parallel, which requires B and C to be connected on one side and the D terminal on the other. Finally, to get 133.3 Ω, connect a 100-Ω resistor in series with a parallel combination of a 100 and a 50-Ω resistor. This requires a connection to the A terminal on one side and the B and C terminals connected on the other. The following table summarizes the results.

Resistance (Ω)	Terminal 1	Terminal 2
100	A	D
200	A	B
150	A	C
50	C	D
25	A+B+C	D
33.3	B+C	D
133.3	A	B+C

Problem 2–42. In Figure P2–42 find the equivalent resistance between terminals A-B, A-C, A-D, B-C, B-D, and C-D.

For A-B, ignore the 20-Ω resistor and the 10-Ω resistor connected to terminal D. We then have

$$R_{\text{AB}} = [100 \parallel (60+40)] + 30 = [100 \parallel 100] + 30 = 50 + 30 = 80\,\Omega$$

For A-C, ignore the 30-Ω resistor and the 10-Ω resistor connected to terminal D. We then have

$$R_{\text{AC}} = [60 \parallel (100+40)] + 20 = [60 \parallel 140] + 20 = 42 + 20 = 62\,\Omega$$

For A-D, ignore the 30-Ω resistor and the 20-Ω resistor. We then have

$$R_{\text{AD}} == [60 \parallel (100+40)] + 10 = [60 \parallel 140] + 10 = 42 + 10 = 52\,\Omega$$

For B-C, ignore the A terminal and the 10-Ω resistor. We then have

$$R_{\text{BC}} = 30 + [40 \parallel (100+60)] + 20 = 30 + [40 \parallel 160] + 20 = 30 + 32 + 20 = 82\,\Omega$$

For B-D, ignore the A terminal and the 20-Ω resistor. We then have

$$R_{\text{BD}} = 30 + [40 \parallel (100+60)] + 10 = 30 + [40 \parallel 160] + 10 = 30 + 32 + 10 = 72\,\Omega$$

For C-D, ignore the A terminal and the 30-Ω resistor. In the center of the circuit, the wire shorts out the 60, 100, and 40-Ω resistors, so we then have

$$R_{\text{CD}} = 20 + 0 + 10 = 30\,\Omega$$

Problem 2–43. In Figure P2–43 find the equivalent resistance between terminals A-B, A-C, A-D, B-C, B-D, and C-D.

For R_{AB}, only the 33-kΩ resistor is active, so $R_{\text{AB}} = 33$ kΩ. Similarly for R_{AC}, only the 33-kΩ resistor is active, so $R_{\text{AC}} = 33$ kΩ. For R_{AD}, the two 100-kΩ resistors are in parallel and that result is in series with the 33-kΩ resistor, so $R_{\text{AD}} = 33 + (100 \parallel 100) = 33 + 50 = 83$ kΩ. For R_{BC}, there is a path between the two terminals with no resistors, so $R_{\text{BC}} = 0$ Ω. For R_{BD}, ignore the 33-kΩ resistor, and the two 100-kΩ resistors are in parallel to give $R_{\text{BD}} = 100 \parallel 100 = 50$ kΩ. Similarly for R_{CD}, ignore the 33-kΩ resistor, and the two 100-kΩ resistors are in parallel to give $R_{\text{CD}} = 100 \parallel 100 = 50$ kΩ.

Problem 2–44. Select a value of R_{L} in Figure P2–44 so that $R_{\text{EQ}} = 6$ kΩ. Repeat for $R_{\text{EQ}} = 5$ kΩ.

Create an expression for R_{EQ} in terms of R_{L} and then solve for R_{L}. Use the new expression to find the appropriate values for R_{L} for the given values of R_{EQ}. All resistance are in kilohms.

$$\begin{aligned}
R_{\text{EQ}} &= 10 \parallel (10 + R_{\text{L}}) = \frac{10(10+R_{\text{L}})}{10+10+R_{\text{L}}} = \frac{100+10R_{\text{L}}}{20+R_{\text{L}}} \\
R_{\text{EQ}}(20+R_{\text{L}}) &= 100 + 10R_{\text{L}} \\
20R_{\text{EQ}} + R_{\text{EQ}}R_{\text{L}} &= 100 + 10R_{\text{L}} \\
R_{\text{EQ}}R_{\text{L}} - 10R_{\text{L}} &= 100 - 20R_{\text{EQ}} \\
(R_{\text{EQ}} - 10)R_{\text{L}} &= 100 - 20R_{\text{EQ}} \\
R_{\text{L}} &= \frac{100 - 20R_{\text{EQ}}}{R_{\text{EQ}} - 10}
\end{aligned}$$

For $R_{\text{EQ}} = 6$ kΩ, we have

$$R_{\text{L}} = \frac{100-(20)(6)}{6-10} = \frac{-20}{-4} = 5\,\text{k}\Omega$$

For $R_{\text{EQ}} = 5$ kΩ, we have

$$R_{\text{L}} = \frac{100-(20)(5)}{5-10} = \frac{0}{-5} = 0\,\text{k}\Omega$$

Problem 2–45. Using no more than four 1-kΩ resistors, show how the following equivalent resistors can be constructed: 2 kΩ, 500 Ω, 1.5 kΩ, 333 Ω, 250 Ω, and 400 Ω.

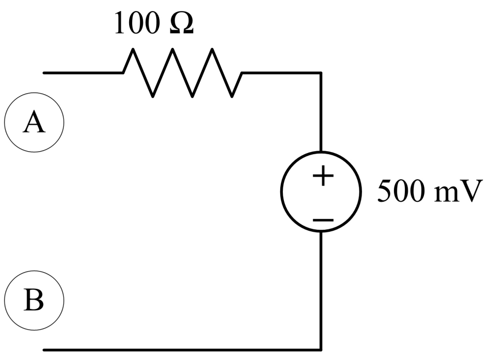
The following table presents the solutions.

R_{EQ} (Ω)	Combination of 1-kΩ Resistors
2000	Two resistors in series: $R + R$
500	Two resistors in parallel: $R \parallel R$
1500	One resistor in series with a parallel combination of two resistors: $R + (R \parallel R)$
333	Three resistors in parallel: $R \parallel R \parallel R$
250	Four resistors in parallel: $R \parallel R \parallel R \parallel R$
400	Two resistors in series in parallel with two resistors in parallel: $(R + R) \parallel R \parallel R$

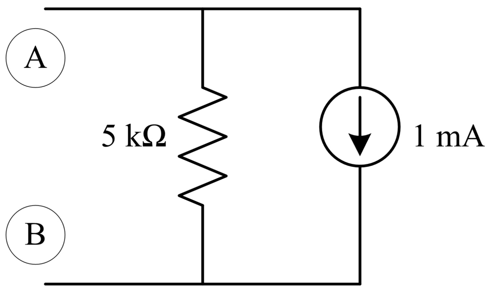
Problem 2–46. Do a source transformation at terminals A and B for each practical source in Figure P2–46.

(a). After the transformation, we will have a voltage source in series with a resistor. The resistance will not change, so $R = 100\ \Omega$. Apply $v_S = i_S R$ to find the voltage source $v_S = (0.005)(100) = 500$ mV. Figure P2–46 (a) shows the results.

(b). After the transformation, we will have a current source in parallel with a resistor. The resistance will not change, so $R = 5$ kΩ. Apply $i_S = v_S / R$ to find the current source $i_S = 5/5000 = 1$ mA. Figure P2–46 (b) shows the results.



(a)



(b)

Figure P2–46

Problem 2–47. Find the equivalent practical voltage source at terminals A and B in Figure P2–47.

A current source in series with a resistor is equivalent to just the current source, so we can remove the 5-Ω resistor without affecting the performance of the circuit between terminals A and B. That leaves a 5-A current source in parallel with a 10-Ω resistor. The current source and parallel resistor can be converted into a voltage source in series with the same resistor. The value for the voltage source follows Ohm's Law, so $v_S = i_S R = (5)(10) = 50$ V. Figure P2–47 shows the resulting circuit.

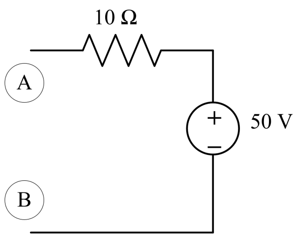


Figure P2–47

Problem 2–48. In Figure P2–48, the i-v characteristic of network N is $v + 50i = 5$ V. Find the equivalent practical current source for the network.

When the circuit is open between nodes A and B, there is no current, $i = 0$ A, and the voltage must be $v = 5$ V in order to satisfy the i-v characteristic. When a short is placed between nodes A and B, the voltage is zero, $v = 0$ V, and the current is $i = 100$ mA in order to satisfy the i-v characteristic. The corresponding practical current source will have a current $i_{\rm S} = 100$ mA and a parallel resistance $R = v_{\rm S}/i_{\rm S} = (5)/(0.1) = 50\ \Omega$.

Problem 2–49. Select the value of $R_{\rm x}$ in Figure P2–49 so that $R_{\rm EQ} = 75$ kΩ.

Combining the resistors from right to left, we can find the following expression for $R_{\rm EQ}$, where all resistances are in kΩ.

$$\begin{aligned}
R_{\rm EQ} &= 15 + 47 + [22 \parallel (R_{\rm x} + 10)] \\
R_{\rm EQ} &= 62 + \left[\frac{22(R_{\rm x}+10)}{22 + R_{\rm x} + 10}\right] = 62 + \left[\frac{22R_{\rm x}+220}{32+R_{\rm x}}\right] \\
(32 + R_{\rm x})R_{\rm EQ} &= (32 + R_{\rm x})62 + 22R_{\rm x} + 220 = 2204 + 84R_{\rm x} \\
32R_{\rm EQ} + R_{\rm EQ}R_{\rm x} &= 2204 + 84R_{\rm x} \\
(84 - R_{\rm EQ})R_{\rm x} &= 32R_{\rm EQ} - 2204 \\
R_{\rm x} &= \frac{32R_{\rm EQ} - 2204}{84 - R_{\rm EQ}}
\end{aligned}$$

For $R_{\rm EQ} = 75$ kΩ, we have

$$R_{\rm x} = \frac{(32)(75) - 2204}{84 - 75} = \frac{196}{9} = 21.78\ {\rm k}\Omega$$

Problem 2–50. Two 10-kΩ potentiometers (a variable resistor whose value between the two ends is 10 kΩ and between one end and the wiper—the third terminal—can range from 0 Ω to 10 kΩ) are connected as shown in Figure P2–50. What is the range of $R_{\rm EQ}$?

At the limits of their settings, the two poteniometers are either in series or parallel. These represent the maximum and minimum equivalent resistances that the combination can take. When the poteniometers are arranged in parallel, the equivalent resistance is $R_{\rm EQ} = 10 \parallel 10 = 5$ kΩ. When the poteniometers are arranged in series, the equivalent resistance is $R_{\rm EQ} = 10 + 10 = 20$ kΩ. The equivalent resistance ranges between 5 and 20 kΩ.

Problem 2–51. Select the value of R in Figure P2–51 so that $R_{\rm AB} = R_{\rm L}$.

Find an expression for $R_{\rm AB}$ in terms of R and $R_{\rm L}$. Set $R_{\rm AB}$ equal to $R_{\rm L}$. Solve for R in terms of $R_{\rm L}$ and choose the positive solution for the resistance.

$$\begin{aligned}
R_{\rm AB} &= R + [4R \parallel (R + R_{\rm L})] = R + \frac{4R(R+R_{\rm L})}{5R+R_{\rm L}} \\
(5R + R_{\rm L})R_{\rm AB} &= (5R + R_{\rm L})R + 4R^2 + 4RR_{\rm L} \\
(5R + R_{\rm L})R_{\rm L} &= 5R^2 + RR_{\rm L} + 4R^2 + 4RR_{\rm L} \\
5RR_{\rm L} + R_{\rm L}^2 &= 9R^2 + 5RR_{\rm L} \\
R^2 &= \frac{R_{\rm L}^2}{9} \\
R &= \frac{R_{\rm L}}{3}
\end{aligned}$$

Problem 2–52. What is the range of R_{EQ} in Figure P2–52?

The poteniometer can range from 0 Ω to 10 kΩ and it is in parallel with a 10-kΩ resistor. That parallel combination is in series with a 5.6-kΩ resistor. When the poteniometer has a value of 0 Ω, it shorts out the 10-kΩ resistor, so only the 5.6-kΩ resistor is active and $R_{\mathrm{EQ}} = 5.6$ kΩ. When the poteniometer has a value of 10 kΩ, the parallel combination is $10 \parallel 10 = 5$ kΩ. That result is in series with the 5.6-kΩ resistor, so we have $R_{\mathrm{EQ}} = 5.6 + 5 = 10.6$ kΩ. R_{EQ} varies between 5.6 and 10.6 kΩ.

Problem 2–53. Find the equivalent resistance between terminals A and B in Figure P2–53.

Place a voltage source, v_{S}, between terminals A and B and redraw the circuit as the equivalent circuit shown in Figure P2–53. The figure is labeled with currents through and voltages across each of the resistors.

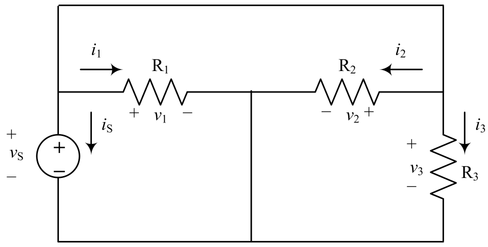


Figure P2–53

Using KVL, we can show that the voltage drop across each resistor is v_{S} and it appears in the direction labeled in the figure. Since the resistors are all equal, the current through each resistor is v_{S}/R. Applying KCL at the node above the voltage source, we have $-i_{\mathrm{S}} - v_{\mathrm{S}}/R - v_{\mathrm{S}}/R - v_{\mathrm{S}}/R = 0$, which implies $i_{\mathrm{S}} = -3v_{\mathrm{S}}/R$. The equivalent resistance is the ratio of v_{S} to the current flowing into the circuit, which $-i_{\mathrm{S}}$. Therefore, we have

$$R_{\mathrm{EQ}} = \frac{v_{\mathrm{S}}}{-i_{\mathrm{S}}} = \frac{v_{\mathrm{S}}}{\frac{3v_{\mathrm{S}}}{R}} = \frac{R}{3}$$

Problem 2–54. Use voltage division in Figure P2–54 to find v_{x}.

Apply the equation for voltage division to get

$$v_{\mathrm{x}} = \left(\frac{4}{2+8+4}\right)(12) = 3.4286\,\mathrm{V}$$

Problem 2–55. Use voltage division in Figure P2–55 to obtain an expression for v_{L} in terms of R, R_{L}, and v_{S}.

The two right resistors are in parallel and the voltage v_{L} appears across that combination. Combine the parallel resistors and then use voltage division to develop the expression for v_{L}.

$$\begin{aligned}
R_{\mathrm{EQ}} &= R \parallel R_{\mathrm{L}} = \frac{RR_{\mathrm{L}}}{R+R_{\mathrm{L}}} \\
v_{\mathrm{L}} &= \left[\frac{R_{\mathrm{EQ}}}{R+R_{\mathrm{EQ}}}\right] v_{\mathrm{S}} = \left[\frac{\frac{RR_{\mathrm{L}}}{R+R_{\mathrm{L}}}}{R+\frac{RR_{\mathrm{L}}}{R+R_{\mathrm{L}}}}\right] v_{\mathrm{S}} = \left[\frac{RR_{\mathrm{L}}}{R^2+RR_{\mathrm{L}}+RR_{\mathrm{L}}}\right] v_{\mathrm{S}} \\
v_{\mathrm{L}} &= \frac{R_{\mathrm{L}}\, v_{\mathrm{S}}}{R+2R_{\mathrm{L}}}
\end{aligned}$$

Problem 2–56. Use current division in Figure P2–56 to find i_{x} and v_{x}.

Combine the 500-Ω and the 1.5-kΩ resistors in series to get an equivalent resistance of 2 kΩ. Now apply current division as follows:

$$\begin{aligned} i_{\mathrm{x}} &= \left(\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}}\right)(3) = \left(\frac{1}{2}\right)(3) = 1.5\,\mathrm{A} \\ v_{\mathrm{x}} &= (1500)(1.5) = 2250\,\mathrm{V} = 2.25\,\mathrm{kV} \end{aligned}$$

Problem 2–57. Use current division in Figure P2–57 to find an expression for v_{L} in terms of R, R_{L}, and i_{S}.

Combine the two right resistors in series to get an equivalent resistance $R_{\mathrm{EQ}} = R + R_{\mathrm{L}}$. Apply the two-path current division rule to solve for the current through R_{L}.

$$i_{\mathrm{L}} = \frac{R}{R+R_{\mathrm{EQ}}}(i_{\mathrm{S}}) = \frac{R}{R+R+R_{\mathrm{L}}}(i_{\mathrm{S}}) = \frac{R}{2R+R_{\mathrm{L}}}(i_{\mathrm{S}})$$

Apply Ohm's law to solve for the voltage v_{L}

$$v_{\mathrm{L}} = R_{\mathrm{L}} i_{\mathrm{L}} = \frac{R R_{\mathrm{L}} i_{\mathrm{S}}}{2R+R_{\mathrm{L}}}$$

Problem 2–58. Find i_{x}, i_{y}, and i_{z} in Figure P2–58.

Combine the 20-Ω and 5-Ω resistors in parallel to get an equivalent resistance of 4 Ω. Combine that result with the 6-Ω resistor in series to get a total equivalent resistance of 10 Ω in the right branch. Apply the two-path current division rule to solve for i_{x} and i_{z}.

$$\begin{aligned} i_{\mathrm{x}} &= \frac{10}{15+10}(200) = 80\,\mathrm{mA} \\ i_{\mathrm{z}} &= \frac{15}{15+10}(200) = 120\,\mathrm{mA} \end{aligned}$$

Apply the two-path current division rule again to solve for i_{y} by dividing i_{z}

$$i_{\mathrm{y}} = \frac{20}{20+5}(120) = 96\,\mathrm{mA}$$

Problem 2–59. Find v_{O} in the circuit of Figure P2–59.

The circuit can be treated as a voltage source in series with three resistors, so voltage division applies. The output voltage, v_{O}, appears across 75% of the 5-kΩ poteniometer or, equivalently, 3.75 kΩ. The 5-kΩ resistor and the remaining 25% of the potentiometer, or 1.25 kΩ, are the other two resistors in the circuit. Compute v_{O} directly as follows:

$$v_{\mathrm{O}} = \frac{3.75}{5+1.25+3.75}(5) = \frac{3.75}{10}(5) = 1.875\,\mathrm{V}$$

Problem 2–60. (A) The 1-kΩ load in Figure P2–60 needs 5 V across it to operate correctly. Where should the wiper on the potentiometer be set (R_{X}) to obtain the desired output voltage?

Figure P2–60 shows an equivalent circuit with the poteniometer split into its two equivalent components. To solve the problem, find an equivalent resistance for the parallel combination of resistors and then apply

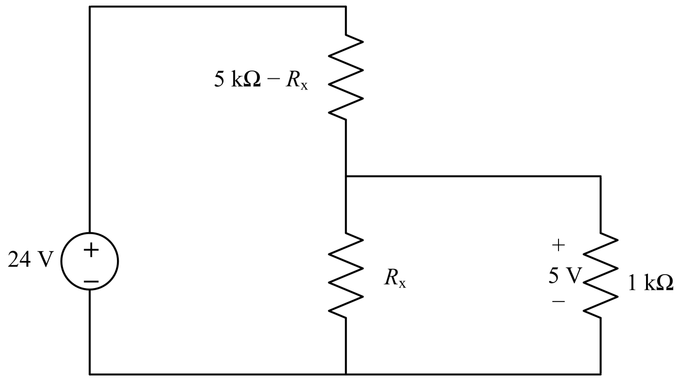


Figure P2–60

voltage division to find an expression for R_x. Solve for R_x and select the positive result.

$$
\begin{aligned}
R_{EQ} &= R_x \parallel 1000 = \frac{1000R_x}{1000+R_x} \\
5\,\text{V} &= \frac{R_{EQ}}{5000 - R_x + R_{EQ}}(24\,\text{V}) \\
5 &= \left[\frac{\dfrac{1000R_x}{1000+R_x}}{5000 - R_x + \dfrac{1000R_x}{1000+R_x}}\right](24) = \frac{24000R_x}{(5000-R_x)(1000+R_x)+1000R_x} \\
1 &= \frac{4800R_x}{5\times 10^6 + 4000R_x - R_x^2 + 1000R_x} \\
-R_x^2 + 5000R_x + 5\times 10^6 &= 4800R_x \\
R_x^2 - 200R_x - 5\times 10^6 &= 0 \\
R_x &= -2138 \text{ or } 2338\,\Omega \\
R_x &= 2.338\,\text{k}\Omega
\end{aligned}
$$

Problem 2–61. Find the range of values of v_O in Figure P2–61.

If we combine the right resistor with the poteniometer in parallel, we can use voltage division to solve for v_O. The poteniometer takes on values from 0 Ω to 1.5 kΩ. When the poteniometer is 0 Ω, the output is shorted out and the voltage is $v_O = 0$ V. When the poteniometer is 1.5 kΩ, the parallel combination is $1500 \parallel 1500 = 750$ Ω. The output voltage is therefore

$$v_O = \frac{750}{1000+750}(50) = 21.4286\,\text{V}$$

Problem 2–62. Use current division in the circuit of Figure P2–62 to find R_X so that the voltage out is 3 V.

If the output voltage is 3 V, then the current flowing through the right branch in the circuit is $i_x = v/R = 3/10 = 300$ mA. Note that R_x is in series with the right 10 Ω resistor. Apply the two-path current division rule to solve for R_x.

$$\begin{aligned} 0.3 &= \frac{10}{10 + R_x + 10}(1) \\ (20 + R_x)(0.3) &= 10 \\ 20 + R_x &= 33.33 \\ R_x &= 13.33\,\Omega \end{aligned}$$

Problem 2–63. (A) Figure P2–63 shows a voltage bridge circuit, that is, two voltage dividers in parallel with a source $v_{S.}$ One resistor R_X is variable. The goal is often to "balance" the bridge by making $v_x = 0$ V. Derive an expression for R_X in terms of the other resistors when the bridge is balanced.

Let the node between resistors R_A and R_B have a voltage v_1 and let the node between resistors R_C and R_X have a voltage v_2. The goal is to make v_1 equal v_2 so that v_x is zero. Use voltage division to derive expressions for v_1 and v_2, set those expressions equal, and solve for R_X.

$$\begin{aligned} v_1 &= \frac{R_B}{R_A + R_B}(v_S) \\ v_2 &= \frac{R_X}{R_C + R_X}(v_S) \\ \frac{R_B v_S}{R_A + R_B} &= \frac{R_X v_S}{R_C + R_X} \\ R_B(R_C + R_X) &= R_X(R_A + R_B) \\ R_B R_C + R_B R_X &= R_A R_X + R_B R_X \\ R_X &= \frac{R_B R_C}{R_A} \end{aligned}$$

Problem 2–64. (A) Ideally, a voltmeter has infinite internal resistance and can be placed across any device to read the voltage without affecting the result. A particular digital multimeter (DMM), a common laboratory tool, is connected across the circuit shown in Figure P2–64. The expected voltage was 10.2 V. However, the DMM reads 7.73 V. The large, but finite, internal resistance of the DMM was "loading" the circuit and causing a wrong measurement to be made. Find the value of the internal resistance R_M of this DMM.

Apply voltage division to find the equivalent resistance of the parallel combination of the 10-MΩ resistor with the DMM.

$$\begin{aligned} 7.73\,\text{V} &= \frac{R_{EQ}}{4.7 + R_{EQ}}(15\,\text{V}) \\ 7.73(4.7 + R_{EQ}) &= 15R_{EQ} \\ 7.27R_{EQ} &= 36.331 \\ R_{EQ} &= 4.99739\,\text{M}\Omega \end{aligned}$$

Now use the expression for a parallel combination of resistors to find the internal resistance of the DMM.

$$\begin{aligned}
R_{\mathrm{EQ}} &= \frac{10R_{\mathrm{M}}}{10+R_{\mathrm{M}}} \\
4.99739 &= \frac{10R_{\mathrm{M}}}{10+R_{\mathrm{M}}} \\
4.99739(10+R_{\mathrm{M}}) &= 10R_{\mathrm{M}} \\
5.00261R_{\mathrm{M}} &= 49.9739 \\
R_{\mathrm{M}} &= 9.98955\,\mathrm{M}\Omega
\end{aligned}$$

Problem 2–65. (D) Select values for R_1, R_2, and R_3 in Figure P2–65 so the voltage divider produces the two output voltages shown.

There are many valid solutions to this problem. One approach is to constrain the resistor values so that the series combination has an equivalent resistance of $R_1 + R_2 + R_3 = 5$ kΩ. Then the current will be $i = v/R_{\mathrm{EQ}} = 5/5000 = 1$ mA. With a current of 1 mA, we must have $R_3 = 1$ kΩ to get a voltage drop of 1 V. The second resistor, R_2, increases the voltage drop by 2.3 V, so we must have $R_2 = 2.3$ kΩ. Finally, the resistors must sum to 5 kΩ, so $R_1 = 1.7$ kΩ.

Problem 2–66. (D) Select a value of R_{x} in Figure P2–66 so that $v_{\mathrm{L}} = 2$ V.

Combine the two right 1-kΩ resistors in parallel to get an equivalent resistance of 500 Ω. Voltage v_{L} appears across the parallel combination, so apply voltage division to solve for R_{x}.

$$\begin{aligned}
v_{\mathrm{L}} \;=\; 2\,\mathrm{V} &= \frac{500}{1000+R_{\mathrm{x}}+500}(12\,\mathrm{V}) \\
2(1500+R_{\mathrm{x}}) &= 6000 \\
2R_{\mathrm{x}} &= 3000 \\
R_{\mathrm{x}} &= 1500\,\Omega \;=\; 1.5\,\mathrm{k}\Omega
\end{aligned}$$

Problem 2–67. (D) Select a value of R_{x} in Figure P2–67 so that $v_{\mathrm{L}} = 2$ V. Repeat for 4 V and 6 V. Caution: R_{x} must be positive.

First, combine R_{x} in parallel with the 50-Ω resistor. Use voltage division with the equivalent resistance to find a general expression for R_x in terms of v_L and then substitute in the desired values for v_L.

$$
\begin{aligned}
R_{EQ} &= \frac{50R_x}{50+R_x} \\
v_L &= \frac{R_{EQ}}{100+R_{EQ}}(12) \\
v_L(100+R_{EQ}) &= 12R_{EQ} \\
100v_L &= (12-v_L)R_{EQ} \\
100v_L &= (12-v_L)\frac{50R_x}{50+R_x} \\
100v_L(50+R_x) &= 50R_x(12-v_L) \\
5000v_L + 100v_LR_x &= 600R_x - 50v_LR_x \\
(600-150v_L)R_x &= 5000v_L \\
R_x &= \frac{5000v_L}{600-150v_L}
\end{aligned}
$$

For $v_L = 2$ V, we get $R_x = 10000/300 = 33.3\ \Omega$. For $v_L = 4$ V, we get $R_x = 20000/0 = \infty\ \Omega$, which is an open circuit. For $v_L = 6$ V, we get $R_x = 30000/(-300) = -100\ \Omega$, which is not possible, so there is no solution for $v_L = 6$ V.

Problem 2–68. Use circuit reduction to find v_x and i_x in Figure P2–68.

Find v_x by combining the 2.2-kΩ and 1-kΩ resistors in series to get $R_{EQ1} = 2.2 + 1 = 3.2$ kΩ and then combine that result in parallel with the 3.3-kΩ resistor to get a total equivalent resistance of $R_{EQ2} = 3.3 \parallel 3.2 = 1.6246$ kΩ. The voltage v_x appears across this equivalent resistance with a current of 300 mA, so we have $v_x = R_{EQ2}i = (1624.6)(0.300) = 487.4$ V. To solve for i_x, perform a source transformation to convert the current source in parallel with a 3.3-kΩ resistor into a voltage source $v_S = i_SR = (0.300)(3300) = 990$ V in series with a 3.3-kΩ resistor. Combine the resulting resistors in series and solve for i_x using Ohm's law, $i_x = v_S/(3300 + 2200 + 1000) = 990/6500 = 152.3$ mA.

Problem 2–69. Use circuit reduction to find v_x, i_x, and p_x in Figure P2–69. Repeat using OrCAD.

Find p_x first by finding an equivalent resistance for all of the resistors combined. To do so, collapse the circuit from right to left to develop the following expression:

$$
\begin{aligned}
R_{EQ} &= 3.3 \parallel \{2.2 + [2 \parallel (1+1)]\} \;=\; 3.3 \parallel \{2.2 + [2 \parallel 2]\} \;=\; 3.3 \parallel \{2.2 + 1\} \;=\; 3.3 \parallel 3.2 \\
R_{EQ} &= 1.6246\,\mathrm{k\Omega}
\end{aligned}
$$

Solve for the power $p_x = i^2R_{EQ} = (0.5)^2(1624.6) = 406.15$ W.

To solve for v_x, perform a source transformation on the left and combine the three resistors on the right to get the circuit shown in Figure P2–69(a). The source transformation yields $v_S = i_SR = (0.500)(3300) = 1650$ V in series with a 3.3 kΩ resistor. Use voltage division to calculate v_x as follows

$$
v_x = \frac{2.2}{3.3+2.2+1}(1650) = 558.46\,\mathrm{V}
$$

To find i_x, leave the resistors on the right intact and perform a source transformation as described above. Combine the 3.3-kΩ and 2.2-kΩ resistors in series to yield the circuit shown in Figure P2–69(b). Perform another source transformation to get a 300-mA current source in parallel with a 5.5-kΩ resistor as shown in Figure P2–69(c). Also in Figure P2–69(c), the two 1-kΩ resistors in series have been combined, since i_x

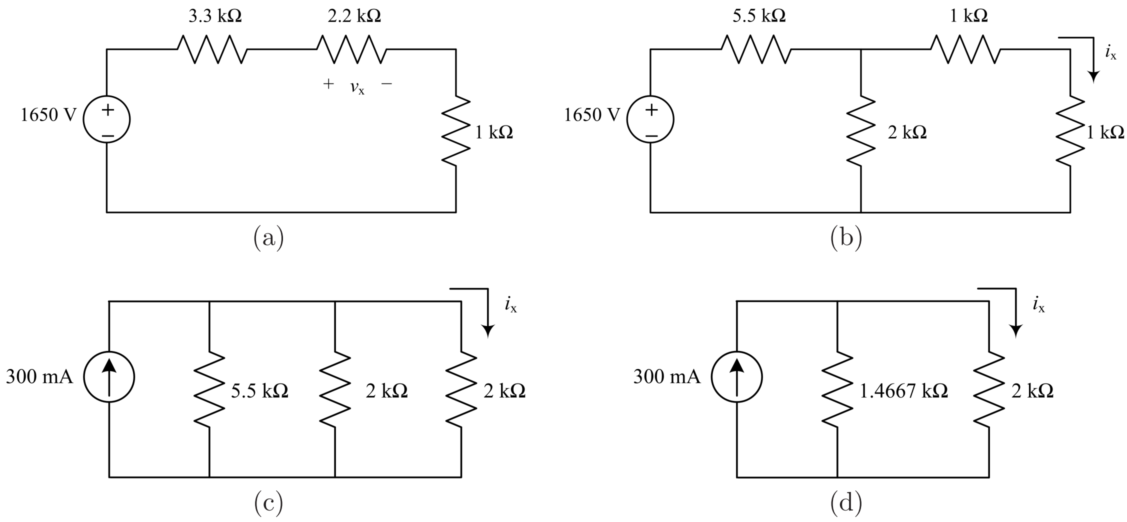
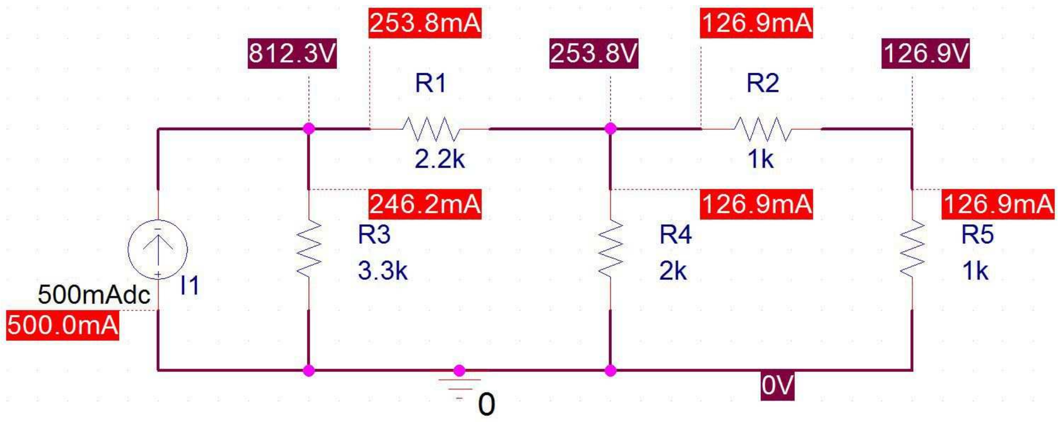


Figure P2–69

flows through both of them. Now combine the 5.5-kΩ and 2-kΩ resistors in parallel to get the circuit in Figure P2–69(d). Apply the two-path current division rule to calculate i_x as follows

$$i_x = \frac{1.4667}{1.4667 + 2}(300\,\text{mA}) = 126.92\,\text{mA}$$

The following OrCAD circuit confirms the solution. In Figure P2–69(e), voltage v_x appears across R_1 and has a value $v_x = 812.3 - 253.8 = 558.5$ V. Current i_x flows through R_5 and has a value $i_x = 126.9$ mA. The power $p_x = (0.5)(812.3) = 406.15$ W.



(e)

Figure P2–69

Problem 2–70. Use circuit reduction to find v_x and i_x in Figure P2–70.

Figure P2–70(a) displays a circuit that is electrically equivalent to the original circuit. Combine the two sets of R and $2R$ resistors in parallel to get the circuit in Figure P2–70(b), which retains v_x. Apply voltage division to solve for v_x as follows

$$v_\text{x} = \frac{\dfrac{2R}{3}}{2R + \dfrac{2R}{3} + \dfrac{2R}{3}}(v_\text{S}) = \frac{2}{6+2+2}(v_\text{S}) = \frac{v_\text{S}}{5}$$

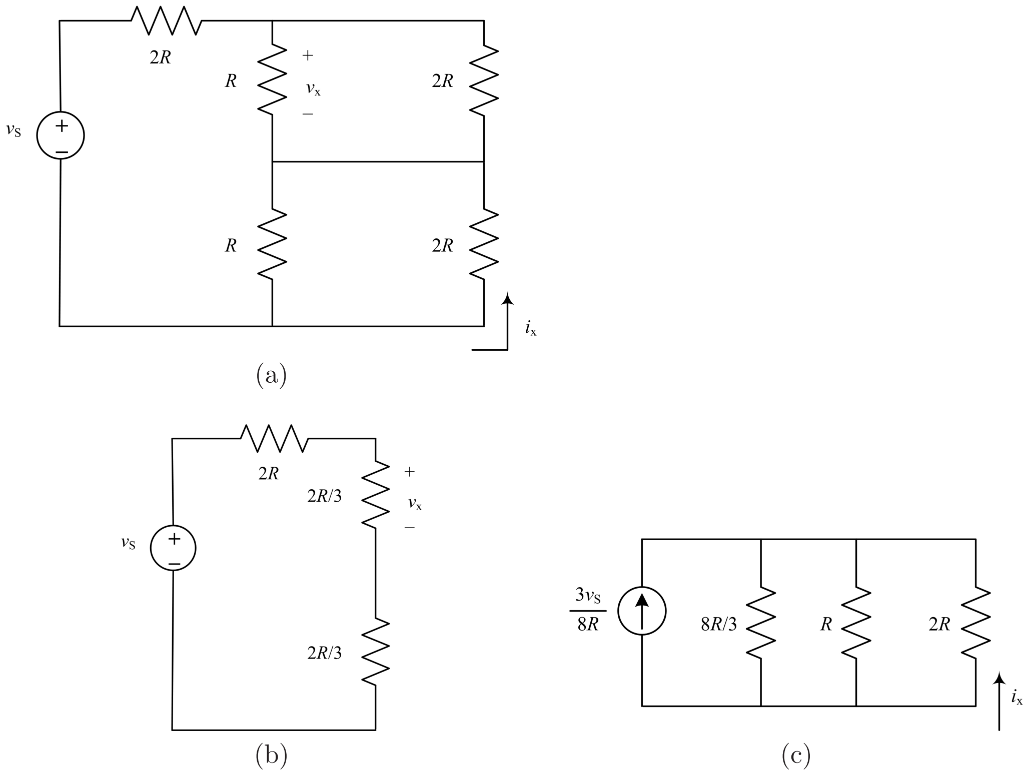


Figure P2–70

To solve for i_x, combine only the top resistors in parallel to get an equivalent resistance of $R_\text{EQ1} = 2R/3$ in series with the top $2R$ resistor and the source. Combine those two resistors in series to get an equivalent resistance of $R_\text{EQ2} = 8R/3$ in series with the source. Perform a source transformation to get the circuit in Figure P2–70(c) with $i_\text{S} = 3v_\text{S}/8R$. Apply current division to solve for i_x, noting that its direction is oppostive that of the source, so it will be negative.

$$i_\text{x} = \frac{\dfrac{1}{2R}}{\dfrac{3}{8R} + \dfrac{1}{R} + \dfrac{1}{2R}}\left(\frac{-3v_\text{S}}{8R}\right) = \frac{4}{3+8+4}\left(\frac{-3v_\text{S}}{8R}\right) = \frac{-v_\text{S}}{10R}$$

Problem 2–71. Use circuit reduction to find v_x, i_x, and p_x in Figure P2–71.

To find p_x, collapse the resistors working from right to left. The two 1-kΩ resistors are in parallel and combine to yield a 500-Ω resistor. That equivalent resistance is in series with the 1.5-kΩ resistor, which combine to yield a 2-kΩ resistor. That equivalent resistance is in parallel with the 3-kΩ resistor, which yields a 1.2-kΩ resistor. That resistor is in series with the 2-kΩ resistor, which yields a total equivalent resistance of 3.2 kΩ. The following expression summarizes these calculations, where all resistances are in kΩ.

$$R_\text{EQ} = 2 + \{3 \parallel [1.5 + (1 \parallel 1)]\} = 2 + \{3 \parallel [1.5 + 0.5]\} = 2 + \{3 \parallel 2\} = 2 + 1.2 = 3.2\,\text{k}\Omega$$

Compute the power as

$$p_x = \frac{v^2}{R_{EQ}} = \frac{(100)^2}{3200} = 3.125\,\mathrm{W}$$

To find i_x, perform a source transformation on the left side and combine the three resistors on the right side as described above. The resulting cicuit is shown in Figure P2–71(a). Apply current division to solve for i_x

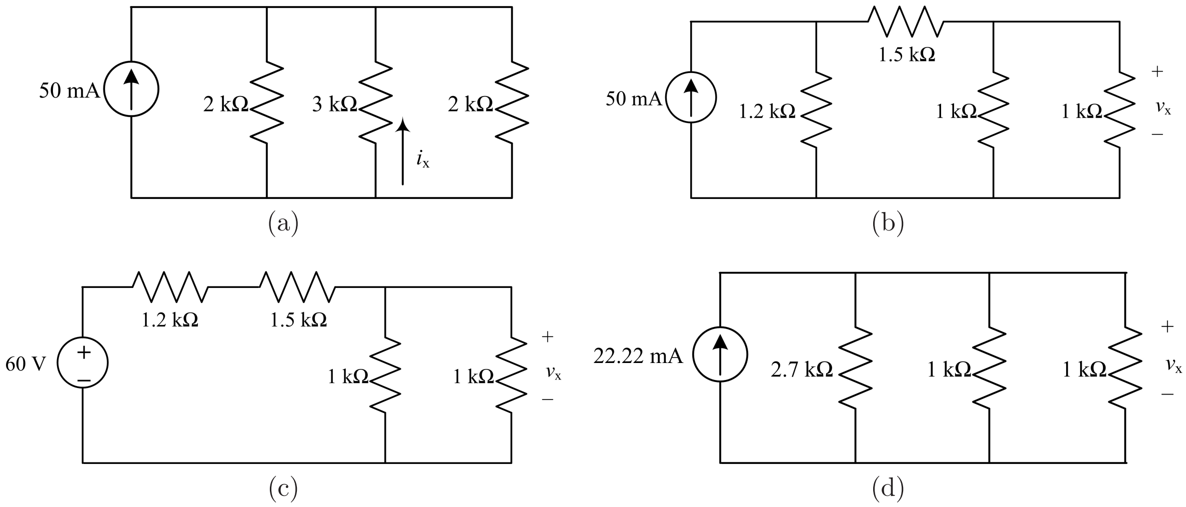


Figure P2–71

as follows, noting the direction of i_x is opposite that of the source

$$i_x = \frac{\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{2}}(-50\,\mathrm{mA}) = \frac{2}{3+2+3}(-50\,\mathrm{mA}) = -12.5\,\mathrm{mA}$$

To find v_x, start with the original circuit and perform a source transformation as above, but leave the three right resistors intact. After the source transformation, combine the 2-kΩ and 3-kΩ resistors in parallel to get the circuit in Figure P2–71(b). Perform another source transformation to get the circuit in Figure P2–71(c). Combine the 1.2-kΩ and 1.5-kΩ resistors in series and perform one final source transformation to get the circuit in Figure P2–71(d). Combine the three resistors in parallel to get a single equivalent resistance of 421.875 Ω, which still has v_x across it. Compute the voltage by multiplying the current by the equivalent resistance to get $v_x = (0.02222)(421.875) = 9.375$ V.

Problem 2–72. Use circuit reduction to find v_x and i_x in Figure P2–72.

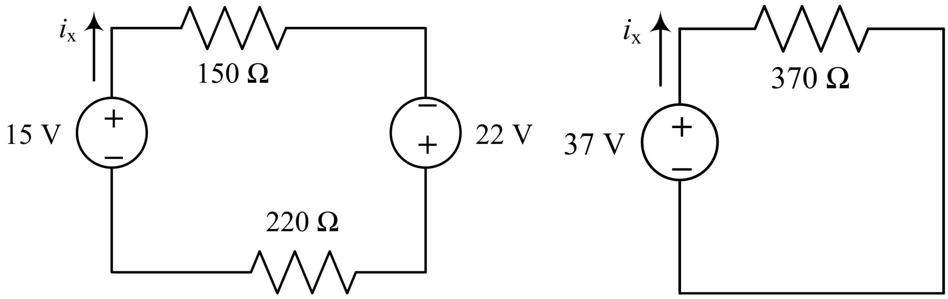
To find i_x, combine the two right resistors in series and perform a source transformation to get an equivalent circuit with a 1.3333-mA current source in parallel with a 18-kΩ and two 12-kΩ resistors. Apply current division to find i_x as follows

$$i_x = \frac{\frac{1}{12}}{\frac{1}{18}+\frac{1}{12}+\frac{1}{12}}(1.3333\,\mathrm{mA}) = 500\,\mu\mathrm{A}$$

In the reduced circuit, there are two 12-kΩ resistors in parallel, so they share the same voltage drop and current. Therefore, 500 μA flows through the 4-kΩ resistor. We can calculate its voltage directly as $v_x = (4000)(500\times10^{-6}) = 2$ V.

Problem 2–73. Use source transformation to find i_x in Figure P2–73.

Perform the source transformation on the current source in parallel with the 220-Ω resistor to get the circuit in Figure P2–73(a). All of the elements in the circuit are in series, so we can combine the two voltage



(a) (b)

Figure P2–73

sources and the two resistors to get the equivalent circuit in Figure P2–73(b). Throughout the process, we have not disrupted i_x, so we can calculate $i_x = 37/370 = 100$ mA.

Problem 2–74. Select a value for R_x so that $i_x = 0$ A in Figure P2–74.

Perform a source transformation on both voltage sources to get the equivalent circuit shown in Figure P2–74. Note the negative voltage source influences the direction of the current in the left transformation. In the

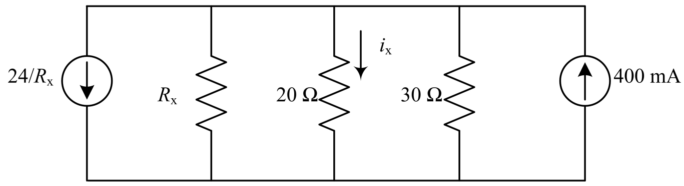


Figure P2–74

equivalent circuit, all three resistors are in parallel and, therefore, share the same voltage. If $i_x = 0$ A, then the voltage drop across each resistor must be zero and no current flows through them. Therefore, all of the current from one source flows through the other source. Solve for R_x as follows

$$\begin{aligned} \frac{24}{R_x} &= 0.4 \\ R_x &= 60\,\Omega \end{aligned}$$

Problem 2–75. Use source transformations in Figure P2–75 to relate v_O to v_1, v_2, and v_3.

Perform a source transformation on each voltage source to get the equivalent circuit shown in Figure P2–75. The current sources are in parallel, so they combine by summing to give an equivalent current source of

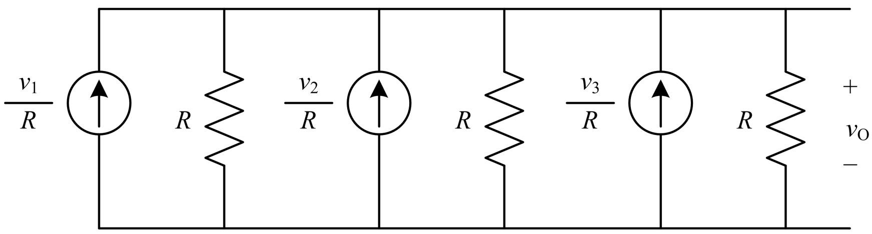


Figure P2–75

$i_S = (v_1 + v_2 + v_3)/R$. The three resistors are in parallel, so they combine to yield an equivalent resistance of $R_{EQ} = R/3$. Apply Ohm's law to compute the output voltage $v_O = i_S R_{EQ} = (v_1 + v_2 + v_3)/3$.

Problem 2–76. The current through R_L in Figure P2–76 is 100 mA. Use source transformations to find R_L. Validate your answer using OrCAD.

Perform a source transformation on the voltage source to get a 1-A current source in parallel with a 100-Ω resistor. Combine the resulting two 100-Ω resistors in parallel to get a 50-Ω resistor. Perform another source transformation to get a 50-V source in series with a 50-Ω resistor, which is also in series with the other 100-Ω resistor and R_L. The 50-V source produces 100 mA through the circuit, so the equivalent resistance is $R_{EQ} = 50/(0.1) = 500$ Ω. The equivalent resistance is also the sum of the three resistors in series $R_{EQ} = 50 + 100 + R_L$, so we can solve for $R_L = 350$ Ω.

The OrCAD circuit in Figure P2–76 confirms the solution.

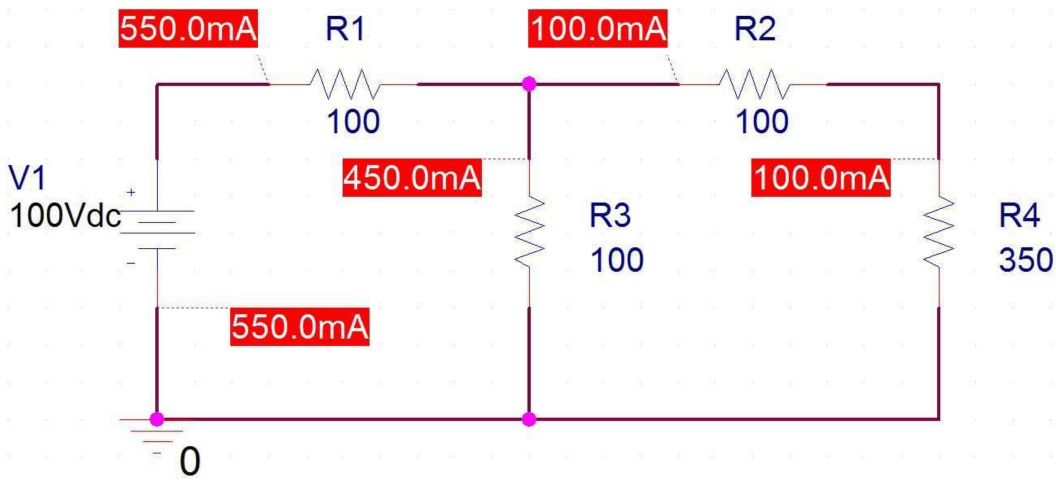


Figure P2–76

Problem 2–77. Select R_x so that 50 V are across it in Figure P2–77.

On the left side of the circuit, perform a source transformation to get a 400-mA current source in parallel with a 500-Ω resistor. Combine the 500-Ω resistor in parallel with the 1-kΩ resistor to get a 333.33-Ω resistor. Perform a second source transformation to get a 133.33-V source in series with the 333.33-Ω resistor. That resistor is in series with R_x. On the right side of the circuit combine the 800-Ω and 200-Ω resistors in series to get a 1-kΩ resistor. That resistor is in parallel with the right 1-kΩ resistor, which combine to yield a 500-Ω resistor. Figure P2–77 displays the resulting circuit. Apply voltage division to solve for R_x

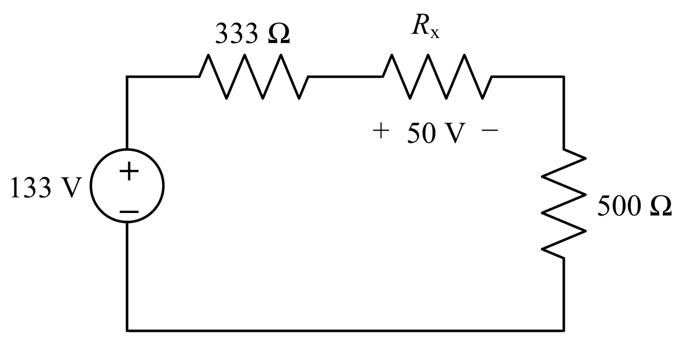


Figure P2–77

$$\begin{aligned}
50\,\text{V} &= \frac{R_\text{x}}{333 + R_\text{x} + 500}(133\,\text{V}) \\
50(R_\text{x} + 833) &= 133R_\text{x} \\
83R_\text{x} &= 41667 \\
R_\text{x} &= 500\,\Omega
\end{aligned}$$

Problem 2–78. The box in the circuit in Figure P2–78 is a resistor whose value can be anywhere between 8 kΩ and 80 kΩ. Use circuit reduction to find the range of values of v_x.

Perform a source transformation to get a 5-mA current source in parallel with a 10-kΩ resistor. In the resulting circuit, combine the two 10-kΩ resistors in parallel to get a 5-kΩ resistor in parallel with the current source. Perform another source transformation to get a 25-V voltage source in series with a 5-kΩ resistor, which are also in series with the variable resistor and the right 10-kΩ resistor. Apply voltage division, once with each extreme value of the variable resistor, to find the range of values for v_x.

$$\begin{aligned}
v_{\text{x,Min}} &= \frac{10}{5 + 80 + 10}(25) = 2.6316\,\text{V} \\
v_{\text{x,Max}} &= \frac{10}{5 + 8 + 10}(25) = 10.8696\,\text{V}
\end{aligned}$$

Problem 2–79. A circuit is found to have the following element and connection equations:

$$\begin{aligned}
v_1 &= 24\,\text{V} & -v_1 + v_2 + v_3 &= 0 \\
v_2 &= 8000\,i_2 & -v_3 + v_4 + v_5 &= 0 \\
v_3 &= 5000\,i_3 & i_1 + i_2 &= 0 \\
v_4 &= 4000\,i_4 & -i_2 + i_3 + i_4 &= 0 \\
v_5 &= 16000\,i_5 & -i_4 + i_5 &= 0
\end{aligned}$$

Use MATLAB to solve for all of the unknown voltages and currents associated with this circuit. Sketch one possible schematic that matches the given equations.

There are many valid approaches to solve this problem using MATLAB. One way is to write the equations in matrix form and solve by inverting the matrix. Choose a vector of variables as

$$\mathbf{x} = [v_1\ v_2\ v_3\ v_4\ v_5\ i_1\ i_2\ i_3\ i_4\ i_5]^\text{T}$$

and write the equations in matrix form as follows.

$$\mathbf{A}\,\mathbf{x} = \mathbf{B}$$

where

$$\mathbf{A} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -8000 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -5000 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -4000 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -16000 \\
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 24 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve for the unknown values by calculating

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{B}$$

The following MATLAB code provides the solution.

```
A = [1 0 0 0 0 0 0 0 0 0;
    0 1 0 0 0 0 -8000 0 0 0;
    0 0 1 0 0 0 0 -5000 0 0;
    0 0 0 1 0 0 0 0 -4000 0;
    0 0 0 0 1 0 0 0 0 -16000;
    -1 1 1 0 0 0 0 0 0 0;
    0 0 -1 1 1 0 0 0 0 0;
    0 0 0 0 0 1 1 0 0 0;
    0 0 0 0 0 0 -1 1 1 0;
    0 0 0 0 0 0 0 0 -1 1];
B = [24 0 0 0 0 0 0 0 0 0]';
x = A\B
```

The corresponding MATLAB output is shown below.

```
x =
   24.0000
   16.0000
    8.0000
    1.6000
    6.4000
   -0.0020
    0.0020
    0.0016
    0.0004
    0.0004
```

Figure P2–79 displays one possible circuit that corresponds to the given equations.

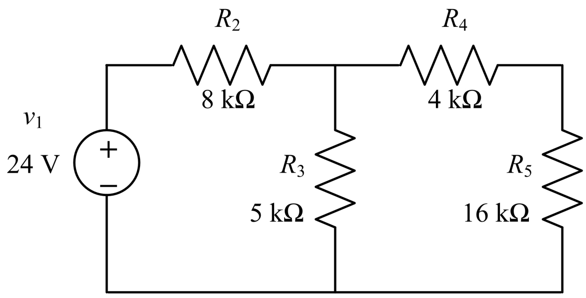


Figure P2–79

In summary, the voltages and currents are as follows

$v_1 = 24\,\text{V}$ $\quad$ $i_1 = -2\,\text{mA}$

$v_2 = 16\,\text{V}$ $\quad$ $i_2 = 2\,\text{mA}$

$v_3 = 8\,\text{V}$ $\quad$ $i_3 = 1.6\,\text{mA}$

$v_4 = 1.6\,\text{V}$ $\quad$ $i_4 = 0.4\,\text{mA}$

$v_5 = 6.4\,\text{V}$ $\quad$ $i_5 = 0.4\,\text{mA}$

Problem 2–80. Consider the circuit of Figure P2–80. Use MATLAB to find all of the voltages and currents in the circuit and find the power provided by the source.

Label the source as v_1 with current i_1 and the resistors from left to right as R_2 to R_7 with corresponding voltages and currents. Write the following element and connection equations by applying Ohm's law, KVL, and KCL:

$$v_1 = 120\,\text{V} \qquad -v_1 + v_2 + v_3 = 0$$

$$v_2 = 150000\,i_2 \qquad -v_3 + v_4 + v_5 = 0$$

$$v_3 = 220000\,i_3 \qquad -v_5 + v_6 + v_7 = 0$$

$$v_4 = 68000\,i_4 \qquad i_1 + i_2 = 0$$

$$v_5 = 56000\,i_5 \qquad -i_2 + i_3 + i_4 = 0$$

$$v_6 = 47000\,i_6 \qquad -i_4 + i_5 + i_6 = 0$$

$$v_7 = 33000\,i_7 \qquad -i_6 + i_7 = 0$$

Using a matrix approach, define a vector of variables as

$$\mathbf{x} = [v_1\ v_2\ v_3\ v_4\ v_5\ v_6\ v_7\ i_1\ i_2\ i_3\ i_4\ i_5\ i_6\ i_7]^{\mathrm{T}}$$

and write the equations in matrix form as follows.

$$\mathbf{A}\,\mathbf{x} = \mathbf{B}$$

The following MATLAB code provides the solution.

```
A = [1 0 0 0 0 0 0 0 0 0 0 0 0 0;
    0 1 0 0 0 0 0 0 -150000 0 0 0 0 0;
    0 0 1 0 0 0 0 0 0 -220000 0 0 0 0;
    0 0 0 1 0 0 0 0 0 0 -68000 0 0 0;
    0 0 0 0 1 0 0 0 0 0 0 -56000 0 0;
    0 0 0 0 0 1 0 0 0 0 0 0 -47000 0;
    0 0 0 0 0 0 1 0 0 0 0 0 0 -33000;
    -1 1 1 0 0 0 0 0 0 0 0 0 0 0;
    0 0 -1 1 1 0 0 0 0 0 0 0 0 0;
    0 0 0 0 -1 1 1 0 0 0 0 0 0 0;
    0 0 0 0 0 0 0 1 1 0 0 0 0 0;
    0 0 0 0 0 0 0 0 -1 1 1 0 0 0;
    0 0 0 0 0 0 0 0 0 0 -1 1 1 0;
    0 0 0 0 0 0 0 0 0 0 0 0 -1 1;
    ];
B = [120 0 0 0 0 0 0 0 0 0 0 0 0 0]';
x = A\B
```

The corresponding MATLAB output is shown below.

```
x =
   120.0000e+000
    82.1192e+000
    37.8808e+000
    25.5188e+000
    12.3620e+000
     7.2627e+000
     5.0993e+000
  -547.4614e-006
   547.4614e-006
   172.1854e-006
```

```
375.2759e-006
220.7506e-006
154.5254e-006
154.5254e-006
```

Problem 2–81. Consider the circuit of Figure P2–80 again. Use OrCAD to find all of the voltages, currents and power delivered or absorbed. Verify that the sum of all power in the circuit is zero.

The OrCAD circuit in Figure P2–81 presents the solution. The power supplied by the voltage source is −65.70 mW and the power dissipated by the resistors is $44.96+6.523+9.577+2.729+1.122+0.788 = 65.70$ mW, so the sum of all power in the circuit is zero.

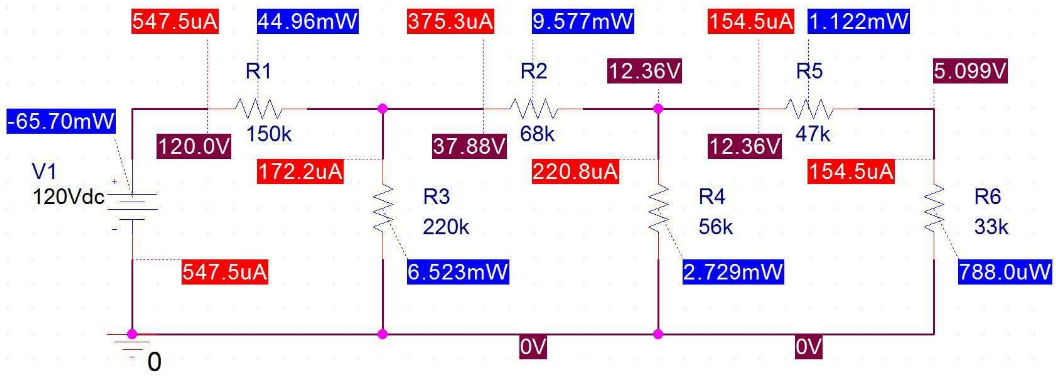


Figure P2–81

Problem 2–82. The circuit of Figure P2–82 is called a "*bridge-T*" circuit. Use OrCAD to find all of the voltages and currents in the circuit.

The OrCAD circuit in Figure P2–82 presents the solution. Note that resistor R_3 has no current following through it.

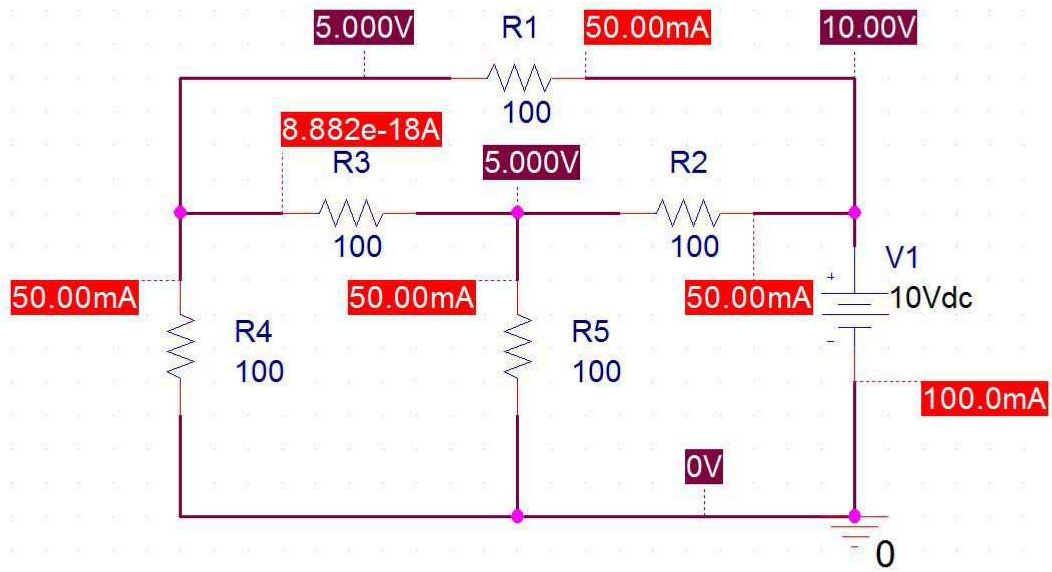


Figure P2–82

Problem 2–83. Nonlinear Device Characteristics **(A)**

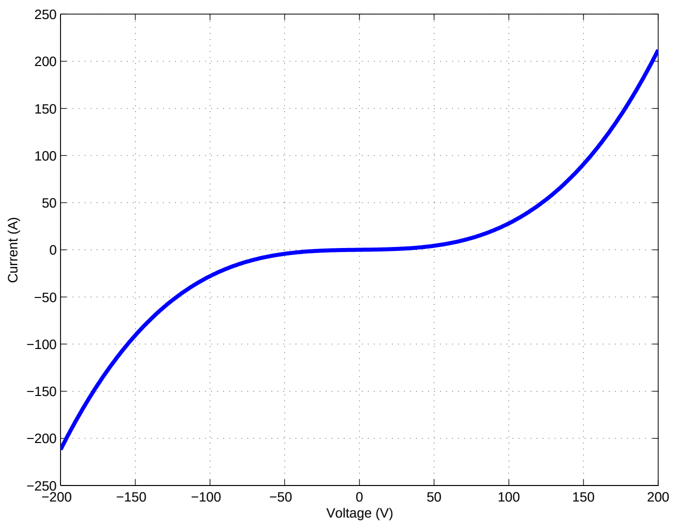
The circuit in Figure P2–83 is a parallel combination of a 50-Ω linear resistor and a varistor whose i-v characteristic is $i_V = 2.6 \times 10^{-5} v^3$. For a small voltage, the varistor current is quite small compared to the resistor current. For large voltages, the varistor dominates because its current increases more rapidly with voltage.

(a). Plot the i-v characteristic of the parallel combination.

For a given voltage v, the current through the 50-Ω resistor is $i_1 = v/50$ and the current through the varistor is $i_V = 2.6 \times 10^{-5} v^3$. The total current is $i = i_1 + i_V$. The following MATLAB code plots the i-v characteristic.

```
% Set the range of voltages to plot
v = -200:1:200;
% Compute the current through the resistor
iR = v/50;
% Compute the current through the varistor
iV = 2.6e-5*v.^3;
% Sum the two path currents to get the total current
iTotal = iR + iV;
% Plot the i-v characteristic
plot(v,iTotal,'b','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
grid on
```

The corresponding MATLAB output is shown below.



(b). State whether the parallel combination is linear or nonlinear, active or passive, and bilateral or nonbilateral.

The parallel combination is nonlinear based on the curved shape of the i-v characteristic. The combination is passive because the power is always positive, which means it is absorbing power. The combination is bilateral because the i-v charateristic has odd symmetry.

(c). Find the range of voltages over which the resistor current is at least 10 times as large as the varistor current.

Solve the following expression for a range of voltages.

$$i_1 \geq 10 i_{\mathrm{V}}$$

$$\frac{v}{50} \geq 10\left(2.6 \times 10^{-5} v^3\right)$$

$$v \geq 0.013 v^3$$

$$v^2 \leq 76.923$$

$$|v| \leq 8.77 \text{ V}$$

(d). Find the range of voltages over which the varistor current is at least 10 times as large as the resistor current.

Solve the following expression for a range of voltages.

$$i_{\mathrm{V}} \geq 10 i_1$$

$$2.6 \times 10^{-5} v^3 \geq 10\left(\frac{v}{50}\right)$$

$$v^2 \geq 7692.3$$

$$|v| \geq 87.7 \text{ V}$$

Problem 2–84. Transistor Biasing **(D)**

The circuit shown in Figure P2–84 is a typical biasing arrangement for a BJT-type transistor. The actual transistor for this problem can be modeled as 0.7-V battery in series with a 200-kΩ resistor. Biasing allows signals that have both a positive and negative variation to be properly amplified by the transistor. Select the two biasing resistors R_{A} and R_{B} so that 3 V ±0.1 V appears across R_{B}.

Label the voltage across the 220-kΩ resistor as v_{T}. Write a KVL equation with resistor R_{B} and the transitor to get

$$-3 + 0.7 + v_{\mathrm{T}} = 0$$

Solve for $v_{\mathrm{T}} = 2.3$ V. The current through the 220-kΩ resistor is $i_{\mathrm{T}} = 2.3/200000 = 11.5\ \mu$A. The voltage across R_{B} is 3 V, which makes the voltage across $R_{\mathrm{A}} = 15 - 3 = 12$ V. As part of the design, choose the current through R_{B} to be approximately equal to the transistor current of 11.5 μA. The required resistance is $R_{\mathrm{B}} = 3/(11.5 \times 10^{-6}) = 260.87$ kΩ. A standard resistor value that is close is 270 kΩ. With $R_{\mathrm{B}} = 270$ kΩ, the current through R_{B} is $i_{\mathrm{B}} = 3/(270000) = 11.11\ \mu$A. Applying KCL, the current through R_{A} is the sum of the currents through R_{B} and the transistor, which yields $i_{\mathrm{A}} = 11.5 + 11.11 = 22.61\ \mu$A. The required resistance is $R_{\mathrm{A}} = 12/(22.61 \times 10^{-6}) = 530.71$ kΩ. Create R_{A} by combining a 330-kΩ and two 100-kΩ resistors in series.

Problem 2–85. Center Tapped Voltage Divider **(A)**

Figure P2–85 shows a voltage divider with the center tap connected to ground. Derive equations relating v_{A} and v_{B} to v_{S}, R_1, and R_2.

Using the passive sign convention and KCL, we have $i_{\mathrm{S}} = -i_{\mathrm{A}} = i_{\mathrm{B}}$. Calculate the magnitude of the current by combining the resistors in series and using Ohm's law.

$$i_{\mathrm{A}} = \frac{v_{\mathrm{S}}}{R_1 + R_2}$$

Apply Ohm's law to each resistor to find v_A and v_B

$$v_A = i_A R_1 = \frac{R_1 v_S}{R_1 + R_2}$$

$$v_B = i_B R_2 = \frac{-R_2 v_S}{R_1 + R_2}$$

Problem 2–86. Active Transducer **(A)**

Figure P2–86 shows an active transducer whose resistance $R(V_T)$ varies with the transducer voltage V_T as $R(V_T) = 0.5V_T^2 + 1$. The transducer supplies a current to a 12-Ω load. At what voltage will the load current equal 100 mA?

The resistors are in series, so $R_{EQ} = R(V_T) + 12$ Ω. Apply Ohm's law to find an expression for i_L in terms of V_T and set $i_L = 100$ mA.

$$i_L = 0.1 = \frac{V_T}{R_{EQ}} = \frac{V_T}{R(V_T) + 12} = \frac{V_T}{0.5V_T^2 + 1 + 12} = \frac{V_T}{0.5V_T^2 + 13}$$

$$(0.1)(0.5V_T^2 + 13) = V_T$$

$$0.05V_T^2 + 1.3 = V_T$$

$$V_T^2 - 20V_T + 26 = 0$$

$$V_T = 1.3977\,\text{V} \quad \text{or} \quad 18.6023\,\text{V}$$

Problem 2–87. Interface Circuit Choice **(E)**

You have a practical voltage source that can be modeled as a 5-V ideal source in series with a 1-kΩ source resistor. You need to use your source to drive a 1-kΩ load that requires exactly 2 V across it. Two solutions are provided to you as shown in Figure P2–87. Validate that both meet the requirement then select the best solution and give the reason for your choice. Consider part count, standard parts, accuracy of meeting the specification, power consumed by the source, etc.

In the circuit with Interface #1, combine the two right resistors in parallel to get an equivalent resistance of 666.7 Ω. Using voltage division, we can confirm that 2 V appears across the 1-kΩ load resistor.

$$v_L = \frac{666.7}{1000 + 666.7}(5) = 2\,\text{V}$$

In the circuit with Interface #2, apply voltage division directly to confirm that 2 V appears across the 1-kΩ load resistor.

$$v_L = \frac{1000}{1000 + 500 + 1000}(5) = 2\,\text{V}$$

Both circuits exactly meet the specification. In addition, both interface designs use a single, non-standard resistor value when considering resistors at the ±10% tolerance level. With Interface #1, the source experiences an equivalent resistance of 1.667 kΩ and provides $p_S = v_S^2/R_{EQ} = 25/1667 = 15$ mW of power. With Interface #2, the source experiences an equivalent resistance of 5 kΩ and provides $p_s = 25/5000 = 5$ mW. If we want to minimize the power provided by the source, Interface #2 is a better choice, since all other factors are equal.

Problem 2–88. Programmable Voltage Divider **(A)**

Figure P2–88 shows a programmable voltage divider in which digital inputs b_0 and b_1 control complementary analog switches connecting a multitap voltage divider to the analog output v_O. The switch positions in the figure apply when digital inputs are low. When inputs go high the switch positions reverse. Find the analog output voltage for $(b_1, b_0) = (0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$ when $v_{REF} = 12$ V.

There are four equal resistors in series with a voltage source, so each drops one quarter of the total voltage, or 3 V in this case. As we cycle through the four combinations of the digital inputs, the switchs connect the output voltage to be across zero, one, two, or three resistors, in that order. The output voltages are therefore 0 V, 3 V, 6 V, and 9 V. The following table summarizes the results.

b_1	b_0	v_O (V)
0	0	0
0	1	3
1	0	6
1	1	9

Problem 2–89. Analog Voltmeter Design **(A, D, E)**

Figure P2–89(a) shows a voltmeter circuit consisting of a D'Arsonval meter, two series resistors, and a two-position selector switch. A current of $i_{FS} = 400\ \mu$A produces full-scale deflection of the D'Arsonval meter, whose internal resistance is $R_M = 25\ \Omega$.

(a). **(D)** Select the series resistances R_1 and R_2 so a voltage $v_x = 100$ V produces full-scale deflection when the switch is in position A, and voltage $v_x = 10$ V produces full-scale deflection when the switch is in position B.

First, solve for R_2 such that a 10-V input at position B causes 400 μA to flow through the two resistors.

$$R_2 + R_M = \frac{v}{i} = \frac{10}{400 \times 10^{-6}} = 25\,\text{k}\Omega$$

Solving for R_2, we get $R_2 = 25000 - 25 = 24.975$ kΩ. Now solve for R_1 such that a 100-V input at position A causes 400 μA to flow through all three resistors.

$$R_1 + R_2 + R_M = \frac{v}{i} = \frac{100}{400 \times 10^{-6}} = 250\,\text{k}\Omega$$

$$R_1 = 225\,\text{k}\Omega$$

(b). **(A)** What is the voltage across the 20-kΩ resistor in Figure P2–89(b)? What is the voltage when the voltmeter in part (a) is set to position A and connected across the 20-kΩ resistor? What is the percentage error introduced connecting the voltmeter?

Using voltage division, the voltage across the 20-kΩ resistor is 20 V when the voltmeter is not connected. When the voltmeter is set in position A and connected in parallel to the 20-kΩ resistor, it is equivalent to placing a 250-kΩ resistor in parallel with the 20-kΩ resistor. The equivalent resistance of the parallel combination is $R_{EQ} = 20 \parallel 250 = 18.5185$ kΩ. Applying voltage division to this case yields the following result:

$$v_M = \frac{18.5185}{30 + 18.5185}(50) = 19.084\,\text{V}$$

The percentage error in this case is 4.58%.

(c). **(E)** A different D'Arsonval meter is available with an internal resistance of 100 Ω and a full-scale deflection current of 100 μA. If the voltmeter in part (a) is redesigned using this D'Arsonval meter, would the error found in part (b) be smaller or larger? Explain.

With a full-scale deflection current of 100 μAfor an applied voltage of 100 V, (switch in position A,) the total resistance of the meter must be 1 MΩ. The increased meter resistance will draw less current when it is connected to the 20-kΩ resistor and have a smaller impact on the voltage. The error will decrease. The new equivalent resistance of the meter in parallel with the 20-kΩ resistor is $R_{EQ} = 19.6078$ kΩ, the measured voltage is 19.7628 V, and the error is 1.19%.

Problem 2–90. MATLAB Function for Parallel Equivalent Resistors **(A)**

Create a MATLAB function to compute the equivalent resistance of a set of resistors connected in parallel. The function has a single input, which is a vector containing the values of all of the resistors in parallel, and it has a single output, which is the equivalent resistance. Name the function "EQparallel" and test it with at least three different resistor combinations. At least one test should have three or more resistor values.

The MATLAB function will compute the reciprocal of each value in the vector of inputs. It will then sum those reciprocals and take the reciprocal of that sum to get the equivalent parallel resistance. The function does not perform any error checking.

The following MATLAB script saved as `EQparallel.m` provides the solution:

```
function Zp = EQparallel(Z)
%
% Compute the equivalent parallel impedance of a list of impedances
%

Zinv = 1./Z;
Zp = 1/sum(Zinv);
```

To test the script, we entered the following three commands:

```
R1 = EQparallel([1000 1000])
R2 = EQparallel([5e3 20e3])
R3 = EQparallel([4e3 5e3 20e3])
```

which yielded the following accurate results:

```
R1 = 500.0000e+000
R2 = 4.0000e+003
R3 = 2.0000e+003
```

Problem 2–91. Finding an Equivalent Resistance using OrCAD **(A)**

Use OrCAD to find the equivalent resistance at terminals A and B of the resistor mesh shown in Figure P2–91. (*Hint:* use a 1-V dc source and measure the current provided by the source.)

The OrCAD circuit in Figure P2–91 presents the solution. The 1-V source causes a current of 9 mA, so the equivalent resistance at terminals A and B is $R_{EQ} = v/i = 1/0.009 = 111.11\ \Omega$.

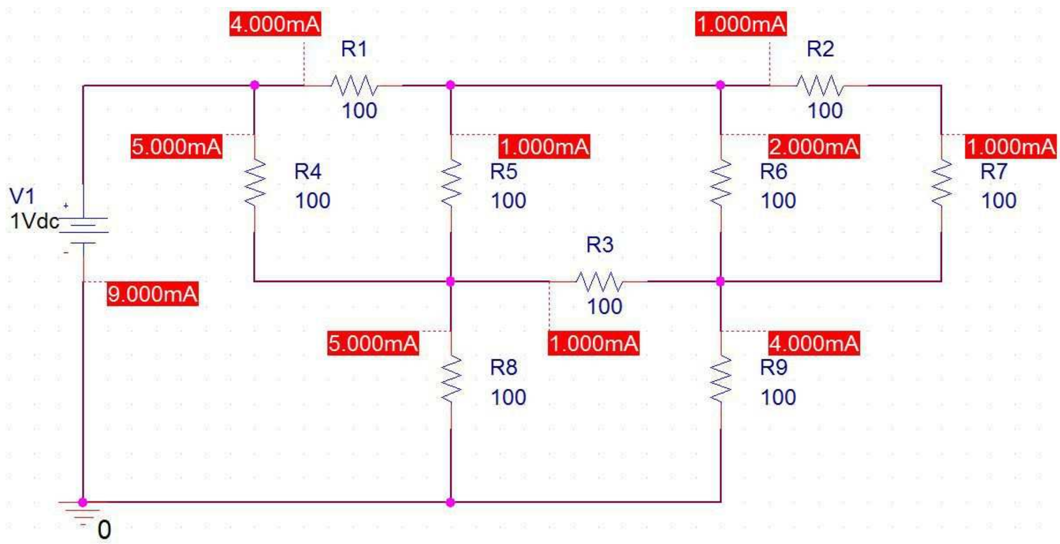


Figure P2–91

3 Circuit Analysis Techniques

3.1 Exercise Solutions

Exercise 3–1. The reference node and node voltages in the bridge circuit of Figure 3–3 are $v_A = 5$ V, $v_B = 10$ V, and $v_C = -3$ V. Find the element voltages.

Apply the node-voltage definitions to find the voltages as follows:

$$v_1 = v_B - 0 = 10 - 0 = 10\,\text{V}$$

$$v_2 = 0 - v_C = 0 - (-3) = 3\,\text{V}$$

$$v_3 = v_B - v_C = 10 - (-3) = 13\,\text{V}$$

$$v_4 = v_A - v_C = 5 - (-3) = 8\,\text{V}$$

$$v_5 = v_A - v_B = 5 - 10 = -5\,\text{V}$$

Exercise 3–2. First find the node voltages in the following circuit and then find v_X and v_Y.

Figure 3–4 defines the reference node as node D with $v_D = 0$ V. Apply the node-voltage definitions to find the node voltages as follows:

$$v_A - v_D = 5\,\text{V} \qquad v_A = 5\,\text{V}$$

$$v_C - v_D = 6\,\text{V} \qquad v_C = 6\,\text{V}$$

$$v_C - v_B = 10\,\text{V} \qquad v_B = -4\,\text{V}$$

Apply the node-voltage definitions again to find the voltages v_X and v_Y.

$$v_X = v_A - v_B = 5 - (-4) = 9\,\text{V}$$

$$v_Y = v_A - v_C = 5 - 6 = -1\,\text{V}$$

Exercise 3–3. For the circuit in Figure 3–6 replace the current source i_{S2} with a resistor R_5.

(a). Using the same node designations and reference node, formulate node-voltage equations for the modified circuit. Place the result in matrix for $\mathbf{Ax} = \mathbf{b}$.

The reference node, node voltages, and element currents are the same as those in Example 3–1. In addition, the KCL equations are the same:

$$\text{Node A}: \; i_0 - i_1 - i_2 = 0$$

$$\text{Node B}: \; i_1 - i_3 + i_5 = 0$$

$$\text{Node C}: \; i_2 - i_4 - i_5 = 0$$

The element equations remain the same except for the current i_5, which now involves a resistor:

$$i_0 = i_{S1} \qquad i_3 = \frac{1}{R_3}v_B$$

$$i_1 = \frac{1}{R_1}(v_A - v_B) \qquad i_4 = \frac{1}{R_4}v_C$$

$$i_2 = \frac{1}{R_2}(v_A - v_C) \qquad i_5 = \frac{1}{R_5}(v_C - v_B)$$

Substitute the element equations into the KCL constraints and arrange the result in standard form to get the following:

$$\text{Node A}: \left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_{\rm A}-\frac{1}{R_1}v_{\rm B}-\frac{1}{R_2}v_{\rm C}=i_{\rm S1}$$

$$\text{Node B}: -\frac{1}{R_1}v_{\rm A}+\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_5}\right)v_{\rm B}-\frac{1}{R_5}v_{\rm C}=0$$

$$\text{Node C}: -\frac{1}{R_2}v_{\rm A}-\frac{1}{R_5}v_{\rm B}+\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)v_{\rm C}=0$$

Write the results in matrix form:

$$\begin{bmatrix} \left(\frac{1}{R_1}+\frac{1}{R_2}\right) & -\frac{1}{R_1} & -\frac{1}{R_2} \\ -\frac{1}{R_1} & \left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_5}\right) & -\frac{1}{R_5} \\ -\frac{1}{R_2} & -\frac{1}{R_5} & \left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right) \end{bmatrix}\begin{bmatrix} v_{\rm A} \\ v_{\rm B} \\ v_{\rm C}\end{bmatrix}=\begin{bmatrix} i_{\rm S1} \\ 0 \\ 0\end{bmatrix}$$

(b). Is the resulting **A** matrix symmetrical?

The matrix is symmetrical because entry (i, j) is the same as entry (j, i) for all off-diagonal terms.

Exercise 3–4. Formulate node-voltage equations for the circuit of Figure 3–7 and place the results in matrix form $\mathbf{Ax} = \mathbf{b}$. Is the resulting matrix $\mathbf{A}$ symmetrical?

The figure is labeled with a reference node, node voltages, and element currents. The KCL constraints at the two nonreference nodes are:

$$\text{Node A}: i_1 - i_2 - i_3 = 0$$

$$\text{Node B}: i_3 - i_4 + i_5 = 0$$

The element equations are as follows:

$$i_1 = 20\,\text{mA} \qquad i_4 = \frac{1}{2000}v_{\rm B}$$

$$i_2 = \frac{1}{1000}v_{\rm A} \qquad i_5 = 50\,\text{mA}$$

$$i_3 = \frac{1}{1500}(v_{\rm A}-v_{\rm B})$$

Substitute the element equations into the KCL constraints and arrange the result in standard form as follows:

$$\text{Node A}: \left(\frac{1}{1000}+\frac{1}{1500}\right)v_{\rm A}-\frac{1}{1500}v_{\rm B}=20\,\text{mA}$$

$$\text{Node B}: -\frac{1}{1500}v_{\rm A}+\left(\frac{1}{1500}+\frac{1}{2000}\right)v_{\rm B}=50\,\text{mA}$$

Write the results in matrix form:

$$\mathbf{Ax}=\begin{bmatrix} 1.6667 & -0.6667 \\ -0.6667 & 1.1667\end{bmatrix}\begin{bmatrix} v_{\rm A} \\ v_{\rm B}\end{bmatrix}=\begin{bmatrix} 20 \\ 50\end{bmatrix}=\mathbf{b}$$

The matrix is symmetrical.

Exercise 3–5. Formulate node-voltage equations for the circuit in Figure 3–10.

The total conductance connected to node A is $1/1000 + 1/2000 = 1.5 \times 10^{-3}$ S and to node B is $1/2000 + 1/500 = 2.5 \times 10^{-3}$ S. The conductance between nodes A and B is 0.5×10^{-3} S. The current source i_{S1} is directed into node A and the current source i_{S2} is directed away from node B. By inspection, the node voltage equations are

$$\text{Node A}: \ (1.5 \times 10^{-3})v_A - (0.5 \times 10^{-3})v_B = i_{S1}$$

$$\text{Node B}: \ -(0.5 \times 10^{-3})v_A + (2.5 \times 10^{-3})v_B = -i_{S2}$$

Exercise 3–6. Solve the node-voltage equations in Exercise 3–5 for v_O in Figure 3–10.

In Figure 3–10, v_O is equal to node voltage v_B. The equations developed in Exercise 3–5 are as follows:

$$\text{Node A}: \ (1.5 \times 10^{-3})v_A - (0.5 \times 10^{-3})v_B = i_{S1}$$

$$\text{Node B}: \ -(0.5 \times 10^{-3})v_A + (2.5 \times 10^{-3})v_B = -i_{S2}$$

Multiply both equations by 1000 to simplify them:

$$\text{Node A}: \ 1.5v_A - 0.5v_B = 1000i_{S1}$$

$$\text{Node B}: \ -0.5v_A + 2.5v_B = -1000i_{S2}$$

Multiply the second equation by three and add the result to the first equation to eliminate v_A as follows:

$$7v_B = 1000i_{S1} - 3000i_{S2}$$

We can then solve for v_B as

$$v_B = v_O = \frac{1000(i_{S1} - 3i_{S2})}{7}$$

The following MATLAB code also provides the solution:

```
syms iS1 iS2 real
A = [1.5e-3 -0.5e-3; -0.5e-3 2.5e-3];
B = [iS1; -iS2];
x = A\B;
vO = x(2)
```

The corresponding MATLAB output is shown below.

```
vO = (1000*iS1)/7 - (3000*iS2)/7
```

Exercise 3–7. Use node-voltage equations to solve for v_1, v_2, and i_3 in Figure 3–12.

Label the lower node as the reference node and the other two nodes as v_A and v_B as shown in Figure Ex3–7. Write the node-voltage equations by inspection, as follows:

$$\text{Node A}: \ \left(\frac{1}{2000} + \frac{1}{3000} + \frac{1}{2000}\right)v_A - \left(\frac{1}{2000}\right)v_B = 0$$

$$\text{Node B}: \ -\left(\frac{1}{2000}\right)v_A + \left(\frac{1}{2000} + \frac{1}{3200}\right)v_B = 20 \times 10^{-3}$$

Multiply both equations by 1000 and simplify to get

$$1.3333v_A - 0.5v_B = 0$$

$$-0.5v_A + 0.8125v_B = 20$$

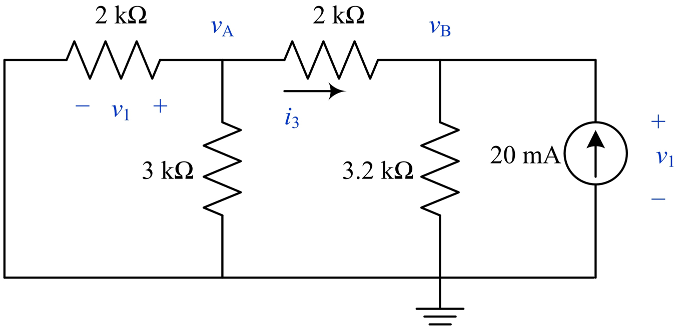


Figure Ex3–7

Multiply the second equation by $1.3333/0.5 = 2.6667$ and add the result to the first equation to eliminate v_A as follows

$$1.6667v_B = 53.3333$$

Solve for v_B to get $v_B = 53.3333/1.6667 = 32$ V. Substitute v_B into the first equation to get

$$1.3333v_A - 16 = 0$$

and solve for v_A to get $v_A = 16/1.3333 = 12$ V. Using the node-voltage definitions, calculate i_3 as follows:

$$i_3 = \frac{v_A - v_B}{2000} = \frac{12 - 32}{2000} = -10\,\text{mA}$$

Finally, $v_1 = v_A = 12$ V and $v_2 = v_B = 32$ V.

Exercise 3–8. Solve Exercise 3–7 using OrCAD.

The required OrCAD circuit with the voltages and currents labeled is shown in Figure Ex3–8.

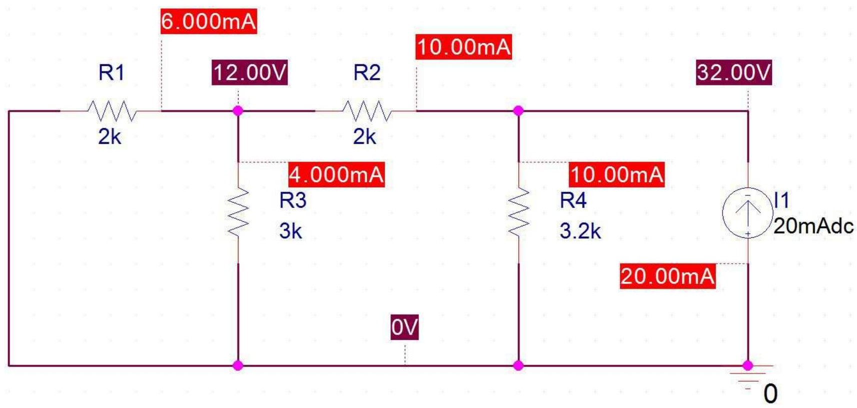


Figure Ex3–8

Exercise 3–9. In Figure 3-14(a), $v_{S1} = 24$ V, $v_{S2} = -12$ V, $R_1 = 3.3$ kΩ, $R_2 = 5.6$ kΩ, and $R_3 = 10$ kΩ. Find v_O using OrCAD.

The required OrCAD circuit with the voltages and currents labeled is shown in Figure Ex3–9. The output voltage is $v_O = 8.82$ V.

Exercise 3–10. For the circuit of Figure 3–15, find i_X.

Using the results of Example 3–5, we have $v_A = 9.5652$ V and $v_B = 10$ V. Solve for i_X directly as follows:

$$i_X = \frac{v_B - v_A}{500} = \frac{10 - 9.5652}{500} = 869.6\,\mu\text{A}$$

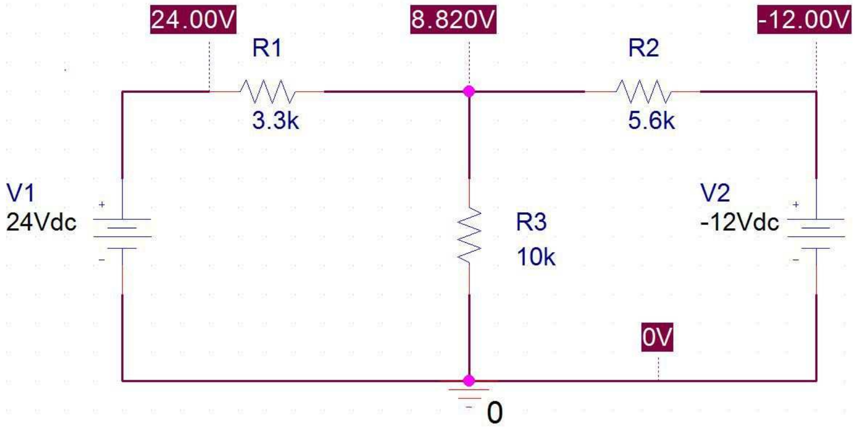


Figure Ex3–9

The following MATLAB code verifies the solution for Example 3–5 and Exercise 3–10:

```
A = [(1/0.5 + 1/1.5 + 1) (-1/1.5);
     (-1/1.5) (1/1.5 + 1/2)];
B = [20; 20];
x = A\B;
vA = x(1)
vD = x(2)
vX = vA-vD
iX = (10-vA)/500
```

The corresponding MATLAB output is shown below.

```
vA =      9.5652e+000
vD =     22.6087e+000
vX =    -13.0435e+000
iX =    869.5652e-006
```

Exercise 3–11. For the circuit in Figure 3–16(a), let $v_{\rm S} = 120$ V and $R = 4$ kΩ.

(a). Use OrCAD to simulate the circuit and find all of the node voltages and the input current.

The required OrCAD circuit with the voltages and currents labeled is shown in Figure Ex3–11.

(b). Verify that the results for the node voltages agree with the symbolic expressions determined in the solution of Example 3–6.

Substitute $v_{\rm S} = 120$ V into the four expressions determined in Example 3–6:

$$v_{\rm B} = \frac{73v_{\rm S}}{263} = 33.31\,{\rm V} \qquad v_{\rm D} = \frac{150v_{\rm S}}{263} = 68.44\,{\rm V}$$

$$v_{\rm C} = \frac{175v_{\rm S}}{263} = 79.85\,{\rm V} \qquad v_{\rm E} = \frac{125v_{\rm S}}{263} = 57.03\,{\rm V}$$

The results match the OrCAD solution.

(c). Use the input current to calculate $R_{\rm IN}$ and compare it with that found in Example 3–6.

Using the input voltage and input current found in the OrCAD simulation, calculate $R_{\rm IN} = v_{\rm IN}/i_{\rm IN} = 120/0.03091 = 3.882$ kΩ. This result agrees with the expression found in Example 3–6, where $R_{\rm IN} = 263R/271 = (263)(4000)/271 = 3.882$ kΩ.

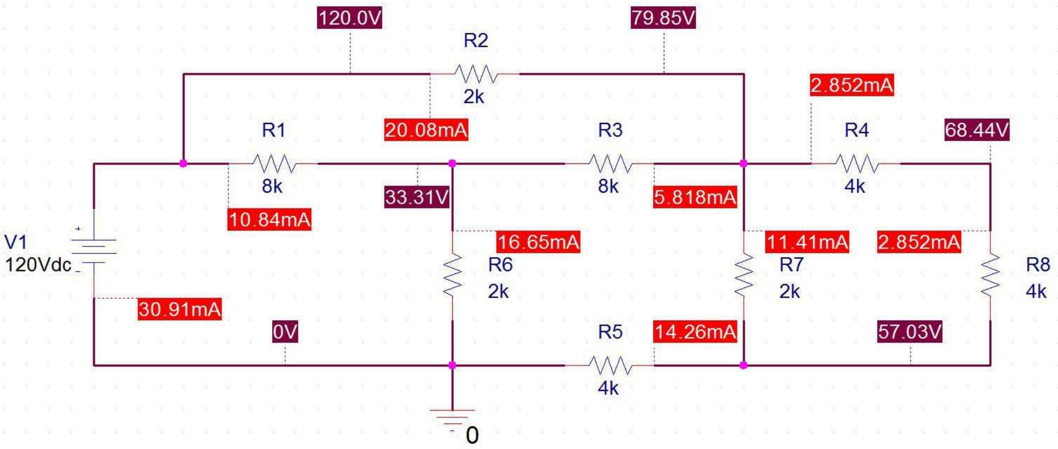


Figure Ex3–11

Exercise 3–12. Find v_O in Figure 3–18 when the element E is

(a). A 10-kΩ resistance.

Define the bottom node to be the reference node and label the other node voltages as shown in Figure Ex3–12(a). The voltage source forces $v_A = 5$ V, so that node voltage is known. Write the other

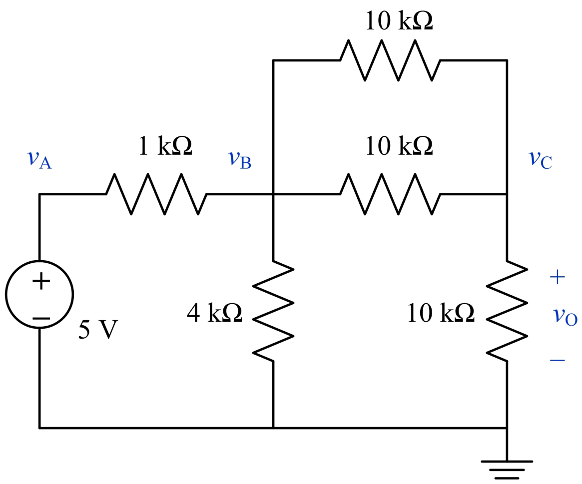


Figure Ex3–12(a)

two node-voltage equations by inspection

$$\text{Node B}: \left(\frac{1}{1000}+\frac{1}{10000}+\frac{1}{10000}+\frac{1}{4000}\right)v_B - \left(\frac{1}{10000}+\frac{1}{10000}\right)v_C = \frac{1}{1000}v_A$$

$$\text{Node C}: -\left(\frac{1}{10000}+\frac{1}{10000}\right)v_B + \left(\frac{1}{10000}+\frac{1}{10000}+\frac{1}{10000}\right)v_C = 0$$

Simplifying the equations, we have

$$14.5v_B - 2v_C = 50$$

$$-2v_B + 3v_C = 0$$

Multiply the second equation by 14.5/2 and add it to the first equation to eliminate v_B and get

$$19.75v_C = 50$$

which yields $v_C = v_O = 2.5316$ V.

(b). A 4-mA independent current source with reference arrow pointing left.

Figure Ex3–12(b) displays the resulting circuit with the reference node and node voltages labeled. As before, the voltage source forces $v_A = 5$ V, so that node voltage is known. Write the other two

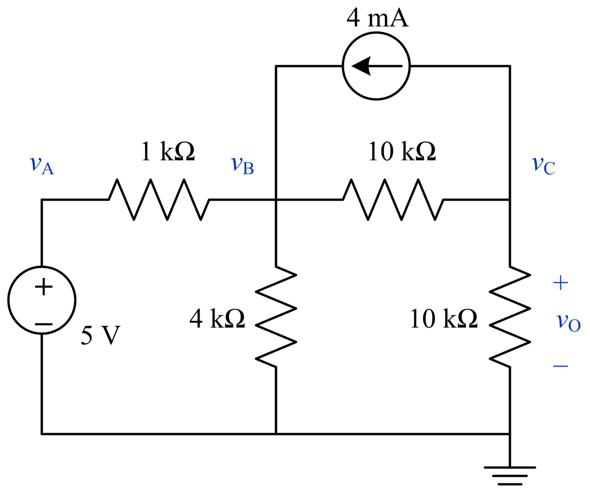


Figure Ex3–12(b)

node-voltage equations by inspection

$$\text{Node B}: \left(\frac{1}{1000}+\frac{1}{10000}+\frac{1}{4000}\right)v_B - \left(\frac{1}{10000}\right)v_C = \frac{1}{1000}v_A + \frac{4}{1000}$$

$$\text{Node C}: -\left(\frac{1}{10000}\right)v_B + \left(\frac{1}{10000}+\frac{1}{10000}\right)v_C = -\frac{4}{1000}$$

Simplifying the equations, we have

$$13.5v_B - v_C = 90$$

$$-v_B + 2v_C = -40$$

Multiply the second equation by 13.5 and add it to the first equation to eliminate v_B and get

$$26v_C = -450$$

which yields $v_C = v_O = -17.3077$ V.

Exercise 3–13. Find v_O in Figure 3–18 when the element E is

(a). An open circuit.

Define the bottom node to be the reference node and label the other node voltages as shown in Figure Ex3–13(a). The voltage source forces $v_A = 5$ V, so that node voltage is known. Write the other two node-voltage equations by inspection

$$\text{Node B}: \left(\frac{1}{1000}+\frac{1}{10000}+\frac{1}{4000}\right)v_B - \left(\frac{1}{10000}\right)v_C = \frac{1}{1000}v_A$$

$$\text{Node C}: -\left(\frac{1}{10000}\right)v_B + \left(\frac{1}{10000}+\frac{1}{10000}\right)v_C = 0$$

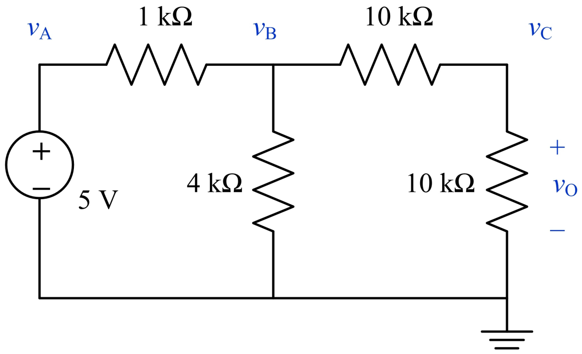


Figure Ex3–13(a)

Simplifying the equations, we have

$$13.5v_B - v_C = 50$$

$$-v_B + 2v_C = 0$$

Multiply the second equation by 13.5 and add it to the first equation to eliminate v_B and get

$$26v_C = 50$$

which yields $v_C = v_O = 1.9231$ V.

(b). A 10-V independent voltage source with the plus reference on the right.

Figure Ex3–13(b) displays the resulting circuit with the reference node and node voltages labeled. As

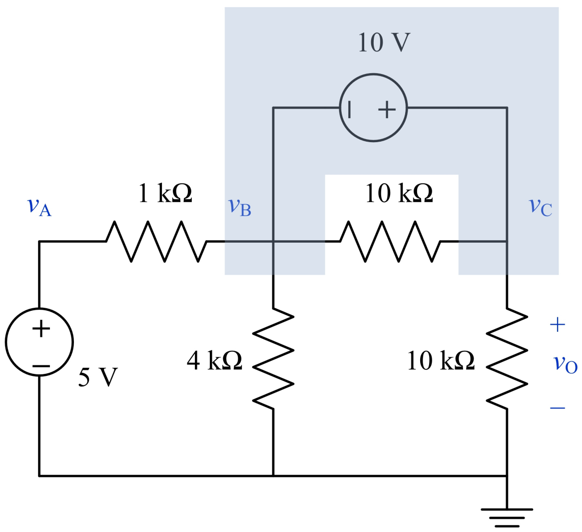


Figure Ex3–13(b)

before, the voltage source forces $v_A = 5$ V, so that node voltage is known. The additional voltage source forces us to use the supernode approach, where the supernode is the shaded section of Figure Ex3–13(b). Write an expression for the currents leaving the supernode as follows:

$$\frac{v_B - v_A}{1000} + \frac{v_B}{4000} + \frac{v_B - v_C}{10000} + \frac{v_C - v_B}{10000} + \frac{v_C}{10000} = 0$$

Simplify this equation to be

$$\left(\frac{1}{1000}+\frac{1}{4000}\right)v_{\mathrm{B}}+\left(\frac{1}{10000}\right)v_{\mathrm{C}}=\frac{v_{\mathrm{A}}}{1000}$$

which is equivalent to

$$12.5v_{\mathrm{B}}+v_{\mathrm{C}}=50$$

The voltage source requires that $v_{\mathrm{C}}-v_{\mathrm{B}}=10$. Substituting we get

$$12.5(v_{\mathrm{C}}-10)+v_{\mathrm{C}}=50$$

$$13.5v_{\mathrm{C}}=175$$

$$v_{\mathrm{C}}=v_{\mathrm{O}}=12.963\,\mathrm{V}$$

Exercise 3–14. In Figure 3–20, the mesh currents are $i_{\mathrm{A}}=10$ A, $i_{\mathrm{B}}=5$ A, and $i_{\mathrm{C}}=-3$ A. Find the element currents i_1 through i_6 and show that KCL is satisfied at nodes A, B, and C.

Write the equation for each element current in terms of the mesh currents and then substitute in the values for the mesh currents as follows:

$$i_1=-i_{\mathrm{A}}=-10\,\mathrm{A}$$

$$i_2=i_{\mathrm{A}}-i_{\mathrm{C}}=10-(-3)=13\,\mathrm{A}$$

$$i_3=i_{\mathrm{A}}-i_{\mathrm{B}}=10-5=5\,\mathrm{A}$$

$$i_4=i_{\mathrm{B}}-i_{\mathrm{C}}=5-(-3)=8\,\mathrm{A}$$

$$i_5=i_{\mathrm{B}}=5\,\mathrm{A}$$

$$i_6=i_{\mathrm{C}}=-3\,\mathrm{A}$$

Verify KCL at nodes A, B, and C as follows:

$$\text{Node A}:\ i_1+i_2+i_6=-10+13-3=0$$

$$\text{Node B}:\ -i_2+i_3+i_4=-13+5+8=0$$

$$\text{Node C}:\ -i_4+i_5-i_6=-8+5-(-3)=0$$

Exercise 3–15. Using Figure 3–18 (see Exercises 3–12 and 3-13), find the current through the 4-kΩ resistor when the element E is

(a). A 10-kΩ resistance.

Figure Ex3–15(a) displays the resulting circuit with the mesh currents labeled as i_{A}, i_{B}, and i_{C}. Write the mesh-current equations by inspection noting that the total resistances in meshes A, B, and C are (1 kΩ + 4 kΩ), (4 kΩ + 10 kΩ + 10 kΩ), and (10 kΩ + 10 kΩ), respectively. The resistance common to meshes A and B is 4 kΩ. The resistance common to meshes B and C is 10 kΩ. There is no resistance common to meshes A and C.

$$\text{Mesh A}:\ (1000+4000)i_{\mathrm{A}}-4000i_{\mathrm{B}}-(0)i_{\mathrm{C}}-5=0$$

$$\text{Mesh B}:\ -4000i_{\mathrm{A}}+(4000+10000+10000)i_{\mathrm{B}}-10000i_{\mathrm{C}}=0$$

$$\text{Mesh C}:\ (0)i_{\mathrm{A}}-10000i_{\mathrm{B}}+(10000+10000)i_{\mathrm{C}}=0$$

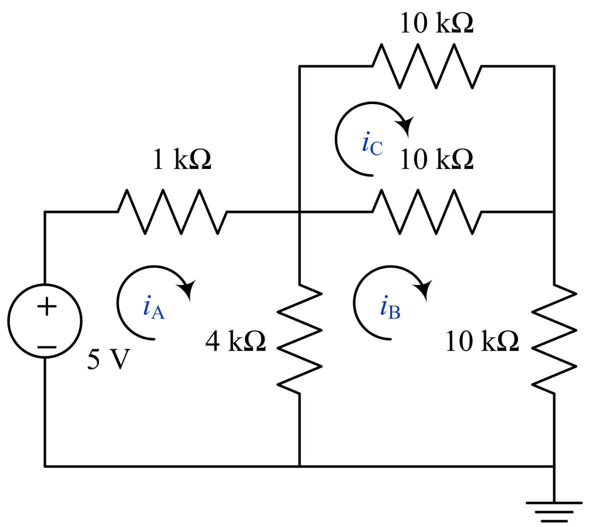


Figure Ex3–15(a)

Simplifying the equations, we get

$$5000i_A - 4000i_B = 5$$

$$-4000i_A + 24000i_B - 10000i_C = 0$$

$$-10000i_B + 20000i_C = 0$$

In matrix form $\mathbf{Ax} = \mathbf{B}$, we have

$$\begin{bmatrix} 5000 & -4000 & 0 \\ -4000 & 24000 & -10000 \\ 0 & -10000 & 20000 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

Apply the following MATLAB code to solve for the mesh currents and the current through the 4-kΩ resistor, which is given by $i_X = i_A - i_B$.

```
A=[5000, -4000, 0;
   -4000, 24000, -10000;
   0, -10000, 20000];
B=[5; 0; 0];
x = A\B;
iA = x(1);
iB = x(2);
iC = x(3);
iX = iA - iB
```

The corresponding MATLAB output is shown below.

```
iX = 949.3671e-006
```

The current flowing down through the 4-kΩ resistor is 949.4 μA.

(b). A 4-mA independent current source with reference arrow pointing left.

Figure Ex3–15(b) displays the resulting circuit with the mesh currents labeled as i_A, i_B, and i_C. By inspection, the independent current source determines $i_C = -4$ mA, because their reference directions

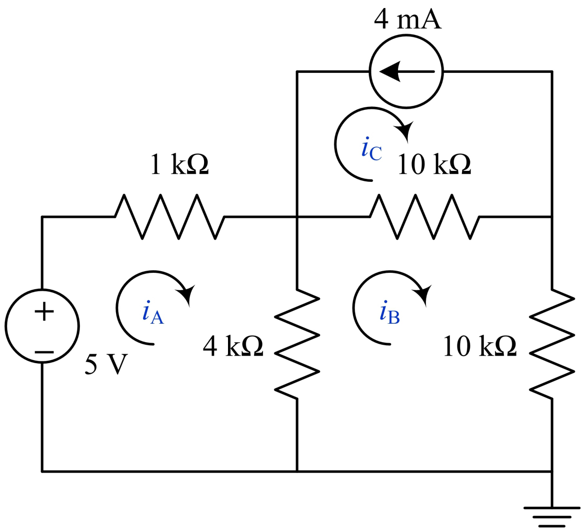


Figure Ex3–15(b)

are opposite. With i_C known, we need just two mesh-current equations to solve for i_A and i_B. Write the equations by inspection.

$$\text{Mesh A}: \ (1000 + 4000)i_A - 4000i_B - (0)i_C - 5 = 0$$

$$\text{Mesh B}: \ -4000i_A + (4000 + 10000 + 10000)i_B - 10000i_C = 0$$

Simplifying the equations, we get

$$5000i_A - 4000i_B = 5$$

$$-4000i_A + 24000i_B = -40$$

To solve for i_A, multiply the first equation by six and add it to the second equation to eliminate i_B as follows:

$$26000i_A = -10$$

Solve for $i_A = -384.615\ \mu$A. Substitute in i_A into the first mesh-current equation to solve for i_B as follows

$$(5000)(-384.615 \times 10^{-6}) - 4000i_B = 5$$

Solve for $i_B = -1.73077$ mA. The current through the 4 kΩ resistor is $i_X = i_A - i_B = (-384.615 \times 10^{-6}) - (-1.73077 \times 10^{-3}) = 1.346$ mA.

Exercise 3–16. In Figure 3–24 replace the 5-V source with a 1-mA dc current source with the arrow pointing up. Use source transformations to reduce the circuit to a single mesh and then solve for i_O.

Figure Ex3–16 displays the resulting circuit and the reduction process using source transformations. In part (b), we eliminate the resistances in series with the left current source and perform a source transformation on the right side. In part (c), we combine the 5-kΩ and 4-kΩ resistors in series and perform another source transformation. In part (d), we combine the two current sources in parallel. In part (e), we perform a final source transformation. We now have a single mesh and can write the mesh-current equation as follows

$$(2000 + 9000)i_A = -17$$

Solving, we get $i_A = -1.5455$ mA. Since i_A and i_O point in opposite directions, $i_O = -i_A = 1.5455$ mA.

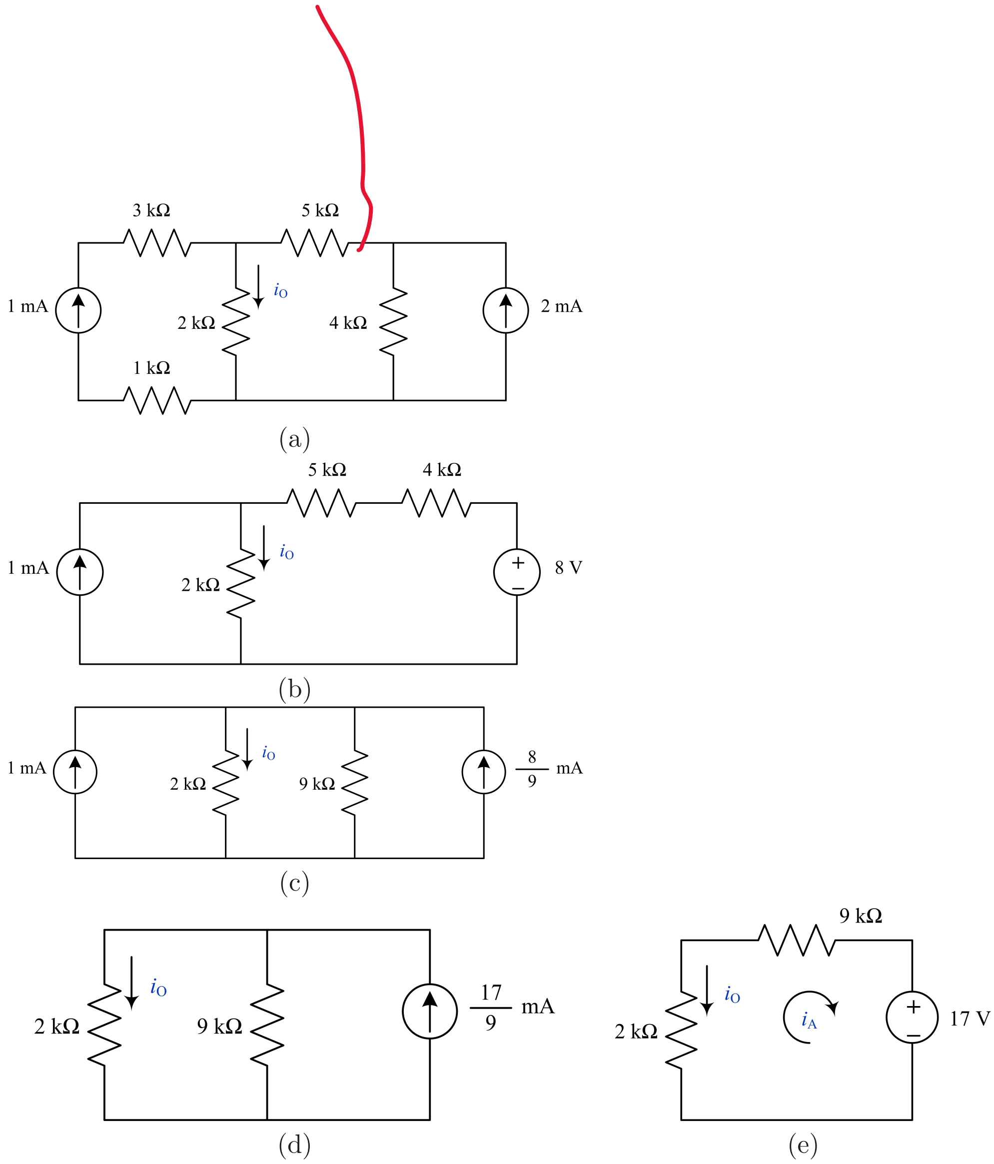


Figure Ex3–16

Exercise 3–17. Use mesh analysis to find the current i_O in Figure 3-26 when the element E is

(a). A 5-V voltage source with the positive reference at the top.

Let the left mesh current be i_A and the right mesh current be i_B. Write the mesh-current equations by inspection.

$$\text{Mesh A}: \quad (5000 + 10000)i_A - 10000i_B = -10$$

$$\text{Mesh B}: \quad -10000i_A + (10000 + 4000)i_B = -5$$

Simplify the equations.

$$15000i_A - 10000i_B = -10$$

$$-10000i_A + 14000i_B = -5$$

Add 1.5 times the second equation to the first equation to eliminate i_A and solve for i_B:

$$i_B = -17.5/11000 = -1.59091\,\text{mA}$$

Substitute into the first equation and solve for i_A:

$$i_A = -25.9091/15000 = -1.72727\,\text{mA}$$

Solve for $i_O = i_A - i_B = -136.364\ \mu\text{A}$.

(b). A 10-kΩ resistor.

Let the left mesh current be i_A and the right mesh current be i_B. Write the mesh-current equations by inspection.

$$\text{Mesh A}: \quad (5000 + 10000)i_A - 10000i_B = -10$$

$$\text{Mesh B}: \quad -10000i_A + (10000 + 4000 + 10000)i_B = 0$$

Simplify the equations.

$$15000i_A - 10000i_B = -10$$

$$-10000i_A + 24000i_B = 0$$

Add 1.5 times the second equation to the first equation to eliminate i_A and solve for i_B:

$$i_B = -10/26000 = -384.615\,\mu\text{A}$$

Substitute into the first equation and solve for i_A:

$$i_A = -13.8462/15000 = -923.077\,\mu\text{A}$$

Solve for $i_O = i_A - i_B = -538.462\ \mu\text{A}$.

Exercise 3–18. Use mesh analysis to find the current i_O in Figure 3-26 when the element E is

(a). A 1-mA current source with the reference arrow directed down.

Let the left mesh current be i_A and the right mesh current be i_B. The current source determines mesh current $i_B = 1$ mA, so we need only one equation to solve for i_A.

$$(5000 + 10000)i_A - 10000i_B = -10$$

$$15000i_A - (10000)(0.001) = -10$$

$$15000i_A = 0$$

$$i_A = 0$$

Solve for $i_O = i_A - i_B = -1$ mA.

(b). Two 20-kΩ resistors in parallel.

Let the left mesh current be $i_{\rm A}$, the center mesh current be $i_{\rm B}$, and the right mesh current be $i_{\rm C}$ as shown in Figure Ex3–18. Write the mesh-current equations by inspection.

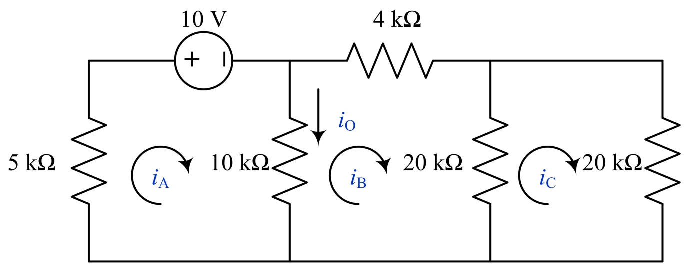


Figure Ex3–18

$$\text{Mesh A}: \ (5000+10000)i_{\rm A} - 10000 i_{\rm B} = -10$$

$$\text{Mesh B}: \ -10000 i_{\rm A} + (10000+4000+20000)i_{\rm B} - 20000 i_{\rm C} = 0$$

$$\text{Mesh C}: \ -20000 i_{\rm B} + (20000+20000)i_{\rm C} = 0$$

Simplify the equations and write them in matrix form.

$$\begin{bmatrix} 15000 & -10000 & 0 \\ -10000 & 34000 & -20000 \\ 0 & -20000 & 40000 \end{bmatrix} \begin{bmatrix} i_{\rm A} \\ i_{\rm B} \\ i_{\rm C} \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 0 \end{bmatrix}$$

Use MATLAB to solve for the mesh currents and then compute $i_{\rm O} = i_{\rm A} - i_{\rm B}$. The following MATLAB code provides the solution.

```
A=[15000, -10000, 0;
   -10000, 34000, -20000;
   0, -20000, 40000];
B=[-10; 0; 0];
x = A\B;
iA = x(1);
iB = x(2);
iC = x(3);
iO = iA - iB
```

The results are

```
iO = -538.4615e-006
```

So we have $i_{\rm O} = -538.462\ \mu$A. This result agrees with that of Exercise 3-17(b), as expected, since the two circuits are equivalent.

Exercise 3–19. Write a set of mesh-current equations for the circuit in Figure 3–27. Do not solve the equations.

Let the left mesh current be $i_{\rm A}$ and the right mesh current be $i_{\rm B}$. Write the mesh-current equations by inspection.

$$\text{Mesh A}: \ (2R+2R)i_{\rm A} - 2R i_{\rm B} = v_1 - v_2$$

$$\text{Mesh B}: \ -2R i_{\rm A} + (2R+R+2R)i_{\rm B} = v_2$$

Simplify the equations.

$$4Ri_{\text{A}} - 2Ri_{\text{B}} = v_1 - v_2$$

$$-2Ri_{\text{A}} + 5Ri_{\text{B}} = v_2$$

Exercise 3–20. Use mesh-current equations to find v_{O} in Figure 3–27.

The following mesh-current equations were developed in Exercise 3–19.

$$4Ri_{\text{A}} - 2Ri_{\text{B}} = v_1 - v_2$$

$$-2Ri_{\text{A}} + 5Ri_{\text{B}} = v_2$$

Multiply the second equation by two and add it to the first equation to eliminate i_{A}

$$8Ri_{\text{B}} = v_1 + v_2$$

Solve for i_{B} and then $v_{\text{O}} = 2Ri_{\text{B}}$.

$$i_{\text{B}} = \frac{v_1 + v_2}{8R}$$

$$v_{\text{O}} = 2Ri_{\text{B}} = \frac{v_1 + v_2}{4}$$

Exercise 3–21. Design a circuit that has $K = i_{\text{O}}/i_{\text{S}} = 0.9$ using 5% tolerance standard value resistors. (See inside back cover.)

There are many solutions using the circuit of Figure 3–29(c). The proportionality constant K is given as

$$K = \frac{R_1}{R_1 + R_2}$$

One possible solution is to select $R_1 = 91$ kΩ and $R_2 = 10$ kΩ, which yields $K = 91/101 = 0.901$.

Exercise 3–22. In Figure 3–30(a), select values of R so that $K = -0.333$.

We have a single constraint on K and four resistor values to select, so there are many possible solutions. With $K < 0$, we require $R_2R_3 < R_1R_4$. We can write K as follows

$$K = \frac{R_2R_3 - R_1R_4}{R_1R_2 + R_1R_4 + R_2R_3 + R_3R_4} = -\frac{1}{3}$$

which implies

$$3R_2R_3 - 3R_1R_4 = -R_1R_2 - R_1R_4 - R_2R_3 - R_3R_4$$

$$R_1R_2 - 2R_1R_4 + 4R_2R_3 + R_3R_4 = 0$$

Since we have only one equation, choose $R_1 = R_2 = R_3 = 1$ kΩ to simplify and solve for R_4.

$$(1)(1) - (2)(1)R_4 + (4)(1)(1) + (1)R_4 = 0$$

$$R_4 = 5\,\text{k}\Omega$$

Exercise 3–23. Find v_{O} in the circuit of Figure 3–31(a) when v_{S} is -5 V, 10 mV, and 3 kV.

Example 3–13 determined the proportionality constant for the circuit was $K = v_{\text{O}}/v_{\text{S}} = 1/3$. Given an input voltage v_{S}, we can calculate the output voltage directly as $v_{\text{O}} = Kv_{\text{S}} = v_{\text{S}}/3$. The corresponding output voltages are -1.667 V, 3.333 mV, and 1 kV.

Exercise 3–24. Use the unit output method to find $K = i_O/i_{IN}$ for the circuit in Figure 3–32. Then use the proportionality constant K to find i_O for the input current shown in the figure.

Assume $i_O = 1$ mA, since the resistors all have units of kilohms. If 1 mA flows through the 1-kΩ and 2-kΩ resistors on the right, then they have voltages of 1 V and 2 V, respectively, with a total voltage drop of 3 V across the series combination. That combination is in parallel with the 3-kΩ resistor, so its voltage drop is $v_1 = 1+2 = 3$ V. The current through the 3-kΩ resistor is therefore, $i_1 = 3/3000 = 1$ mA. Applying KCL, the current flowing through the 500-Ω resistor is $i_2 = i_1 + i_O = 1 + 1 = 2$ mA. The voltage across the 500-Ω resistor is $v_2 = (500)(0.002) = 1$ V. Applying KVL, the voltage drop across the 2-kΩ resistor is $v_3 = v_1+v_2 = 3+1 = 4$ V. The current through the 2-kΩ resistor is $i_3 = 4/2000 = 2$ mA. The corresponding source current $i_S = i_2 + i_3 = 2 + 2 = 4$ mA. The proportionality constant is therefore $K = i_O/i_S = 1/4$. The actual input is 0.6 mA, so the output is $i_O = 0.6/4 = 0.15$ mA.

Exercise 3–25. The circuit of Figure 3–38 contains two R-$2R$ modules. Use superposition to find v_O.

Find v_O by turning off one voltage source at a time and finding the contribution from the voltage source that is active. With $v_{S2} = 0$ V, voltage source v_{S1} is in series with a $2R$, R, and a parallel combination of two $2R$ resistors. The parallel combination is equivalent to a single R resistor, so v_{S1} is in series with an equivalent resistance of $R_{EQ} = 2R + R + (2R \parallel 2R) = 4R$. The output voltage is measured across an equivalent of $2R$ resistors, so $v_{O1} = v_{S1}/2$. With $v_{S1} = 0$ V, voltage source v_{S2} is in series with a $2R$ resistor which is connected in series with a parallel combination of three resistors. Figure Ex3–25 shows an equivalent circuit with the output voltage labeled. Use circuit reduction to find the output voltage. Perform a source

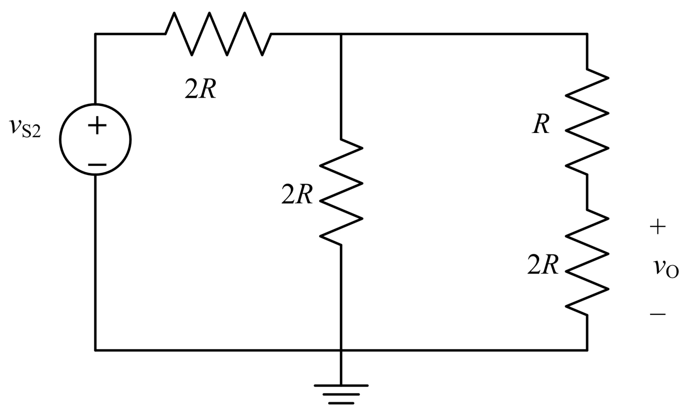


Figure Ex3–25

transformation to get a current source with magnitude $v_{S2}/2R$ in parallel with a $2R$ resistor. Combine the two $2R$ resistors that are now in parallel to get a single R resistor. Apply the two-path current division rule to get the current through the output path as follows

$$i_{O2} = \frac{R}{R+R+2R}\left(\frac{v_{S2}}{2R}\right) = \frac{v_{S2}}{8R}$$

Compute the output voltage

$$v_{O2} = 2Ri_{O2} = \frac{v_{S2}}{4}$$

Combine the two components of the output to get the complete solution

$$v_O = v_{O1} + v_{O2} = \frac{v_{S1}}{2} + \frac{v_{S2}}{4}$$

Exercise 3–26. Repeat Exercise 3–25 with the voltage source v_{S2} replaced by a current source i_{S2} with the current reference arrow directed toward ground.

Voltage source v_{S1} did not change, but the current source replacing v_{S2} has an effect on the circuit connections when it is set to zero. With v_{S1} active and i_{S2} off, voltage source v_{S1} is in series with a series combination of $2R$, R, and $2R$ resistors. The output voltage is measured across the last two resistors, so applying voltage division, the contribution to the output is $v_{O1} = 3v_{S1}/5$. Now consider the contribution with i_{S2} active and v_{S1} off. Figure Ex3–26 shows an equivalent circuit with the output voltage labeled. Note that in Figure Ex3–26 the $2R$ resistor in series with the current source no longer contributes to the

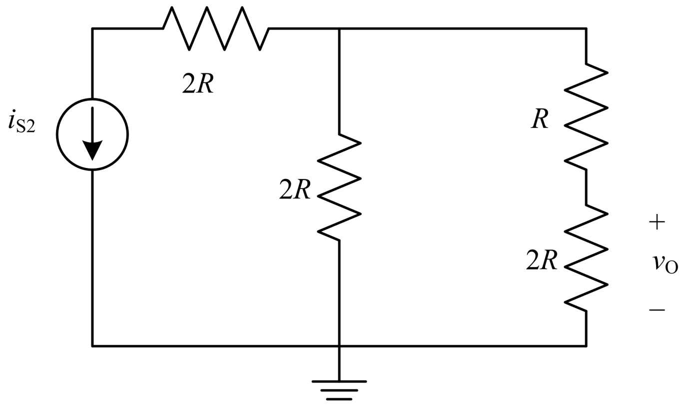


Figure Ex3–26

equivalent resistance. Apply the two-path current division rule directly to find the output current and note the direction of the current source.

$$i_{O2} = -\frac{2R}{2R + R + 2R} i_{S2} = -\frac{2}{5} i_{S2}$$

Compute the output voltage

$$v_{O2} = 2Ri_{O2} = -\frac{4Ri_{S2}}{5}$$

Combine the two components of the output to get the complete solution

$$v_O = v_{O1} + v_{O2} = \frac{3v_{S1}}{5} - \frac{4Ri_{S2}}{5}$$

Exercise 3–27. Use the principle of superposition to find the current i_X in Figure 3–40.

Start with the voltage source active and the current source replaced by an open circuit. Perform a source transformation to get a 1-A current source in parallel with a 12-Ω resistor. The transformed source and 12-Ω resistor are in parallel with a 24-Ω resistor and a $(2 + 10) = 12$-Ω resistor. Apply current division directly to find i_{Xv}.

$$i_{Xv} = \frac{\dfrac{1}{2+10}}{\dfrac{1}{12} + \dfrac{1}{24} + \dfrac{1}{2+10}}(1) = \frac{2}{5}\,\text{A}$$

Now replace the voltage source with a short circuit and solve for i_{Xi} directly using current division.

$$i_{Xi} = \frac{\dfrac{1}{2+10}}{\dfrac{1}{12} + \dfrac{1}{24} + \dfrac{1}{2+10}}(2) = \frac{4}{5}\,\text{A}$$

Combine the two components of the output to get the complete solution

$$i_X = i_{Xv} + i_{Xi} = 0.4 + 0.8 = 1.2\,\text{A}$$

Exercise 3–28. Find the Thévenin equivalent at nodes A and B for the circuit in Figure 3–46.

To find the open-circuit voltage, $v_{OC} = v_T$, combine the resistors to the left of the voltage source into a single equivalent resistor as follows:

$$R_{EQ} = 4.7 \parallel (2.2 + 1.5) = 4.7 \parallel 3.7 = 2.0702\,\text{k}\Omega$$

The 3.3-kΩ resistor has no current flowing through it, so it has no voltage drop. We can then apply voltage division to find the open-circuit voltage

$$v_{\text{OC}} = v_{\text{T}} = \frac{10}{10 + 2.0702}(5) = 4.1424\,\text{V}$$

To find the short-circuit current, $i_{\text{SC}} = i_{\text{N}}$, place a short circuit between nodes A and B. The short-circuit current is then the current through the 3.3-kΩ resistor. Retain the equivalent resistance to the left of the voltage source as $R_{\text{EQ}} = 2.0702$ kΩ. The 3.3-kΩ and 10-kΩ resistors are in parallel and their equivalent resistance is 2.4812 kΩ. Perform voltage division to find the voltage across the 2.4812 kΩ equivalent resistance.

$$v_1 = \frac{2.4812}{2.0702 + 2.4812}(5) = 2.7257\,\text{V}$$

The voltage across the 2.4812 kΩ equivalent resistance is the same as the voltage across the 3.3-kΩ resistor. Calculate the current through the 3.3-kΩ resistor as $i_{\text{SC}} = 2.7257/3300 = 825.98\ \mu$A. Calculate the Thévenin resistance as follows:

$$R_{\text{T}} = \frac{v_{\text{OC}}}{i_{\text{SC}}} = \frac{4.1424}{825.98 \times 10^{-6}} = 5.0152\,\text{k}\Omega$$

In summary, the Thévenin equivalent circuit has $v_{\text{T}} = 4.1424$ V and $R_{\text{T}} = 5.0152$ kΩ.

Exercise 3–29. For the Thévenin circuit of Figure 3–48, select a load R_{L} so that 2.5 V are delivered across it. Choose R_{L} from the 10% values given in the inside back cover. What will be the actual value of voltage delivered to the load?

Apply voltage division to find a candidate value for R_{L}.

$$v_{\text{L}} = 2.5 = \frac{R_{\text{L}}}{9 + R_{\text{L}}}(6)$$

$$22.5 + 2.5R_{\text{L}} = 6R_{\text{L}}$$

$$R_{\text{L}} = 6.4286\,\text{k}\Omega$$

The closest resistor value with 10% tolerance is 6.8 kΩ. With $R_{\text{L}} = 6.8$ kΩ, the actual load voltage is as follows

$$v_{\text{L}} = \frac{6.8}{9 + 6.8}(6) = 2.5823 \pm 10\%\ \text{V}$$

Exercise 3–30. (a). Find the Thévenin and Norton equivalent circuits seen by the load in Figure 3–54.

To find the open-circuit voltage, combine the resistors to the left of the voltage source to get an equivalent resistance $R_{\text{EQ}} = (20 + 20) \parallel 40 = 40 \parallel 40 = 20\ \Omega$. With the interface between nodes A and B an open circuit, the 60-Ω resistor does not contribute to the circuit. Perform voltage division between the 20-Ω equivalent resistance and the 30-Ω resistor, noting the orientation of the voltage source.

$$v_{\text{OC}} = v_{\text{T}} = \frac{30}{20 + 30}(-50) = -30\,\text{V}$$

Use the lookback technique to find the Thévenin resistance R_{T}. Set the voltage source to zero by replacing it with a short circuit and combine the resistors, including the 60-Ω resistor as follows:

$$R_{\text{T}} = 60 + [30 \parallel 40 \parallel (20 + 20)] = 60 + [30 \parallel 40 \parallel 40] = 60 + 12 = 72\,\Omega$$

Compute the Norton current $i_{\text{N}} = v_{\text{T}}/R_{\text{T}} = -30/72 = -416.7$ mA. In summary, we have the following results:

$$v_{\text{T}} = -30\,\text{V}$$

$$i_{\text{N}} = -416.7\,\text{mA}$$

$$R_{\text{T}} = R_{\text{N}} = 72\,\Omega$$

(b). Find the voltage, current, and power delivered to a 50-Ω load resistor.

Connect the 50-Ω load to the Thévenin equivalent circuit. Apply voltage division to determine the load voltage.

$$v_{\mathrm{L}} = \frac{50}{72+50}(-30) = -12.2951\,\mathrm{V}$$

Apply Ohm's law to find the current.

$$i_{\mathrm{L}} = \frac{-30}{72+50} = -245.902\,\mathrm{mA}$$

The power is the product of the voltage and current $p_{\mathrm{L}} = v_{\mathrm{L}}i_{\mathrm{L}} = (-12.2951)(-0.245902) = 3.0234$ W.

Exercise 3–31. Find the current and power delivered to an unknown load in Figure 3–54 when $v = +6$ V.

Model the load as a resistor connected to the Thévenin equivalent circuit and write a KVL equation around the single loop.

$$-(-30) + 72(i) + 6 = 0$$

Solving for the current i, we get $i = -36/72 = -500$ mA. The power supplied to the load is the product of the load voltage and current $p_{\mathrm{L}} = v_{\mathrm{L}}i_{\mathrm{L}} = (6)(-0.5) = -3$ W. The negative power indicates that the load is delivering power to the original circuit.

Exercise 3–32. Suppose for the circuit shown in Figure 3–56(a) that the diode's i-v characteristic can be modeled by the following equation:

$$i = \frac{e^{4v}-1}{10^5}$$

where v is given in volts and i is given in amperes. Note that this i-v characteristic is for a different diode from that shown in Figure 3–56(b). Use manual calculations or the MATLAB function `solve` to find the exact operating point for this circuit.

To solve this problem manually, we can graph the two i-v characteristics to find the intersection (Q-point) as demonstrated in Example 3–21. We could also set the two expressions for current equal and then iteratively search for a valid solution. In this case, MATLAB is a good alternative and the `solve` function is an appropriate choice. To use `solve`, define the i and v variables using the `syms` command and then create two symbolic equations representing the source circuit's and the diode's i-v characteristics. Then apply the `solve` command on the two equations and extract the individual solutions. The following MATLAB code provides the solution:

```
syms i1 v1
Eqn1 = 'i1 = -v1/60 + 2.5/60';
Eqn2 = 'i1 = (exp(4*v1)-1)/100000';
A = solve(Eqn1,Eqn2);
V1 = double(A.v1)
I1 = double(A.i1)
```

The corresponding MATLAB output is shown below.

```
V1 = 1.7746e+000
I1 = 12.0901e-003
```

The solution is $v_{\mathrm{D}} = 1.7746$ V and $i_{\mathrm{D}} = 12.0901$ mA.

MATLAB is also helpful in solving this problem graphically. To do so, create a numerical vector of voltages that ranges between 0 and 3. Use this vector to solve for the corresponding current values associated with both the source circuit and the diode. Plot the two curves and locate the intersection or Q-point. The following MATLAB code demonstrates the approach.

```
vv=0:0.001:3;
ii1 = -vv/60+2.5/60;
```

```
ii2 = (exp(4*vv)-1)/100000;
figure
plot(vv,ii1,'b','LineWidth',2)
hold on
axis([0 3 0 0.05])
plot(vv,ii2,'g','LineWidth',2)
```

The resulting plot is shown in Figure Ex3–32.

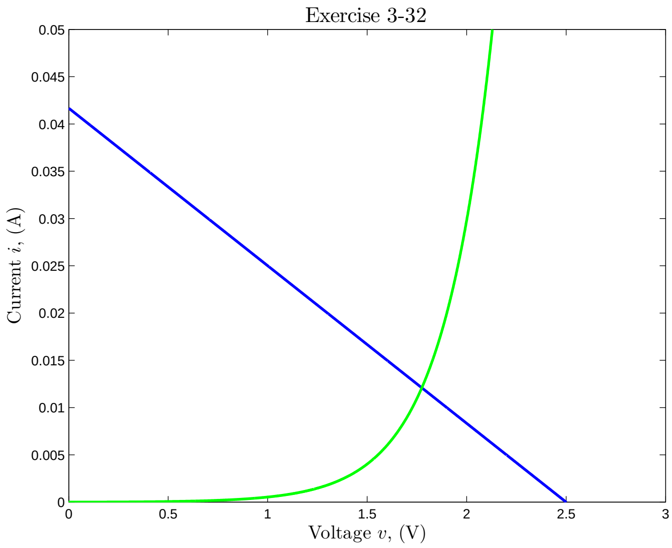


Figure Ex3–32

Exercise 3–33. A source circuit delivers 4 V when a 50-Ω resistor is connected across its output and 5 V when a 75-Ω resistor is connected. Find the maximum voltage, current, and power available from the source.

Use the two results to solve for the Thévenin equivalent voltage and resistance of the source. Apply voltage division for each result to get the following two equations:

$$4 = \frac{50}{R_T + 50}(v_T)$$

$$5 = \frac{75}{R_T + 75}(v_T)$$

Simplify the equations.

$$4R_T + 200 = 50v_T$$

$$5R_T + 375 = 75v_T$$

Solve these equations to get $v_T = 10$ V and $R_T = 75$ Ω. The corresponding Norton current is $i_N = v_T/R_T =$ 133.3 mA. The maximum voltage is the open-circuit voltage $v_{MAX} = v_{OC} = v_T = 10$ V. The maximum current is the short-circuit current $i_{MAX} = i_{SC} = i_N = 133.3$ mA. The maximum power transfer occurs when the load is a 75-Ω resistor and we have

$$p_{MAX} = \left[\frac{v_{OC}}{2}\right]\left[\frac{i_{SC}}{2}\right] = \left[\frac{10}{2}\right]\left[\frac{0.1333}{2}\right] = 333.3\,\text{mW}$$

Exercise 3–34. Select R_L in Figure 3–61 so that $190 \pm 10\%$ mW are delivered to the load. Select a 10% resistor from the inside back cover that will provide the desired voltage.

Solve for the current i flowing in the circuit in terms of $R_{\rm L}$ and then solve for the power delivered to the load in terms of $R_{\rm L}$.

$$i = \frac{10}{100+R_{\rm L}}$$

$$p_{\rm L} = 190\,{\rm mW} = i^2 R_{\rm L} = \left(\frac{10}{100+R_{\rm L}}\right)^2 R_{\rm L} = \frac{100R_{\rm L}}{R_{\rm L}^2+200R_{\rm L}+10^4}$$

$$0.19R_{\rm L}^2 + 38R_{\rm L} + 1900 = 100R_{\rm L}$$

$$0.19R_{\rm L}^2 - 62R_{\rm L} + 1900 = 0$$

Solving the quadratic equation for $R_{\rm L}$ yields $R_{\rm L} = 34.237\ \Omega$ or $R_{\rm L} = 292.078\ \Omega$. Using the table of resistors with 10% tolerance, the closest values are $R_{\rm L} = 33\ \Omega$ and $R_{\rm L} = 270\ \Omega$. With these resistor values, the power delivered to the load is 186.56 mW and 197.22 mW, respectively, both of which are within 10% of the desired 190 mW. Note that the 270-Ω load resistor reduces the circuit current and, therefore, the power delivered by the source ($p_{\rm S} = 270.27$ mW) compared to the 33-Ω resistor ($p_{\rm S} = 751.88$ mW). If we want to minimize the source power, the 270-Ω resistor is a better design.

Exercise 3–35. For the circuit of Figure 3–62, select a load resistor so that 6 A flows through it.

As developed in Example 3–24, the Thévenin equivalent for the source circuit to the left of the load in Figure 3–62 has $v_{\rm T} = 20$ V and $R_{\rm T} = 3.666$ kΩ. If the load is a short circuit, the maximum possible current is $i_{\rm MAX} = v_{\rm T}/R_{\rm T} = 20/3666 = 5.455$ mA. The specification is for 6 A to flow through the load, which is greater than the maximum possible with a resistive circuit. Therefore, there is no load resistor that meets the specification.

Exercise 3–36. For the circuit of Figure 3–62, determine the maximum power available, and if sufficient, select a resistive load that will dissipate 20 mW.

As developed in Example 3–24, the Thévenin equivalent for the source circuit to the left of the load in Figure 3–62 has $v_{\rm T} = 20$ V and $R_{\rm T} = 3.666$ kΩ. The maximum power that can be transferred to the load is given by

$$p_{\rm MAX} = \frac{v_{\rm T}^2}{4R_{\rm T}} = \frac{(20)^2}{(4)(3667)} = 27.27\,{\rm mW}$$

It is possible to transfer 20 mW to the load. Determine the required load resistance.

$$p_{\rm L} = 0.020 = v_{\rm L}^2\frac{1}{R_{\rm L}} = \left[\left(\frac{R_{\rm L}}{3667+R_{\rm L}}\right)(20)\right]^2\frac{1}{R_{\rm L}} = \frac{400R_{\rm L}}{R_{\rm L}^2+7333R_{\rm L}+13.44\times10^6}$$

$$0.02R_{\rm L}^2 - 253.33R_{\rm L} + 268889 = 0$$

Solving the quadratic equation for $R_{\rm L}$ yields $R_{\rm L} = 1.1694$ kΩ or $R_{\rm L} = 11.4973$ kΩ.

Exercise 3–37. Suppose the source circuit in Figure 3–63 is now 12 V in series with a 5-V source resistor. The same LED is used. How does the solution change?

Write KVL around the series loop

$$-12 + (5+R_1)i + 1.5 = 0$$

Set $i = 15$ mA and solve for R_1.

$$R_1 = \frac{10.5}{0.015} - 5 = 695\,\Omega$$

The required resistance is $R_1 = 695\ \Omega$. We can approximate this value by a series combination of a 510-Ω resistor and a 160-Ω resistor to give an equivalent resistance of 670 Ω. Another option would be to use a 510-Ω resistor in series with a parallel combination of two 360-Ω resistors to give an equivalent resistance of 690 Ω. Accounting for the resistors having tolerances of ±5%, both of these options still provide acceptable current levels. For the 670-Ω equivalent resistance, the current ranges from 14.82 mA to 16.37 mA. For the 690-Ω equivalent resistance, the current ranges from 14.39 mA to 15.90 mA.

Exercise 3–38. A Norton source of 300 mA in parallel with a 50-Ω source resistor provides current to a load $R_L = 200\ \Omega$. Your task is to design an interface so that 5 V ±10% are delivered to the load.

(a). Using the series resistor R_S interface shown in Figure 3–59(b), select a 10% resistor from the inside back cover that will provide the desired voltage.

With 5 V across the 200-Ω load resistor, the current through the load is $i_L = 5/200 = 25$ mA. Apply the two-path current division rule to solve for the interface resistor R_S by noting that R_S is in series with the load resistor.

$$i_L = 25 = \left(\frac{50}{50 + R_S + 200}\right)(300) = \frac{15000}{R_S + 250}$$

$$25R_S + 6250 = 15000$$

$$R_S = 350\,\Omega$$

The best match from the 10% resistors is $R_S = 330\ \Omega$. With that choice for a resistor, calculate the actual voltage across the load as follows:

$$i_L = \left(\frac{50}{50 + 330 + 200}\right)(300) = \frac{15000}{580} = 25.8621\,\text{mA}$$

$$v_L = R_L i_L = (200)(0.0258621) = 5.1724\,\text{V}$$

The load voltage is within the specification of 5 V ±10%.

(b). Using the parallel resistor R_P interface shown in Figure 3–59(c), select a 10% resistor from the inside back cover that will provide the desired voltage.

The current required for the load remains 25 mA. With a parallel interface resistor, apply current division to solve for R_P.

$$i_L = 25 = \left(\frac{\frac{1}{200}}{\frac{1}{50} + \frac{1}{R_P} + \frac{1}{200}}\right)(300) = \frac{300R_P}{4R_P + 200 + R_P} = \frac{300R_P}{5R_P + 200}$$

$$125R_P + 5000 = 300R_P$$

$$175R_P = 5000$$

$$R_P = 28.5714\,\Omega$$

The best match from the 10% resistors is $R_P = 27\ \Omega$. With that choice for a resistor, calculate the actual voltage across the load as follows:

$$R_{EQ} = 50 \parallel 27 \parallel 200 = 16.1194\,\Omega$$

$$v_L = 0.300R_{EQ} = (0.300)(16.1194) = 4.8358\,\text{V}$$

The load voltage is within the specification of 5 V ±10%.

(c). Select the solution, series or parallel, that causes the source to provide the desired voltage while delivering the least power. Calculate the power in each case to defend your choice.

Determine the source power for the series interface by computing an equivalent resistance seen by the source.

$$R_{EQ} = 50 \parallel (330 + 200) = 50 \parallel 530 = 45.6897\,\Omega$$

$$p_S = i^2 R_{EQ} = (0.3)^2(45.6897) = 4.11\,\text{W}$$

For the parallel interface, the voltage across the current source is 4.8358 V, so the power it delivers is $p_L = iv = (0.3)(4.8358) = 1.451$ W. The parallel design requires the least power from the source.

Exercise 3–39. Repeat Example 3–27 with the desired $v_2 = 10$ V instead of 2 V.

Apply the same approach used in Example 3–27. Let $R_{EQ} = R_2 \parallel 50$ and $R_{IN} = R_1 + R_{EQ} = 300\ \Omega$. The output port constraint is

$$v_2 = \left(\frac{R_{EQ}}{300 + R_1 + R_{EQ}}\right) 40 = 10\,\text{V}$$

With $R_1 + R_{EQ} = 300\ \Omega$, we get $40R_{EQ} = (10)(600)$, which means $R_{EQ} = 150\ \Omega$. However, $R_{EQ} = R_2 \parallel 50$, and the equivalent resistance of any parallel combination is less than or equal to the smallest resistor value in the combination. Therefore, it is impossible to get $R_{EQ} = 150\ \Omega$ with the selected interface design and we cannot meet both specifications.

Exercise 3–40. Use OrCAD to determine which solution, Figure 3–65 or Figure 3-66, requires less power from the source.

Before solving the problem with OrCAD, we can quickly determine that both designs require the same power from the source. In both cases, the input resistance is 300 Ω. Therefore, the 40-V source sees an equivalent resistance of $R_{EQ} = 300 + 300 = 600\ \Omega$, so its power is

$$p = \frac{v^2}{R_{EQ}} = \frac{40^2}{600} = 2.6667\,\text{W}$$

The following OrCAD simulations in Figure Ex3–40 verify the results.

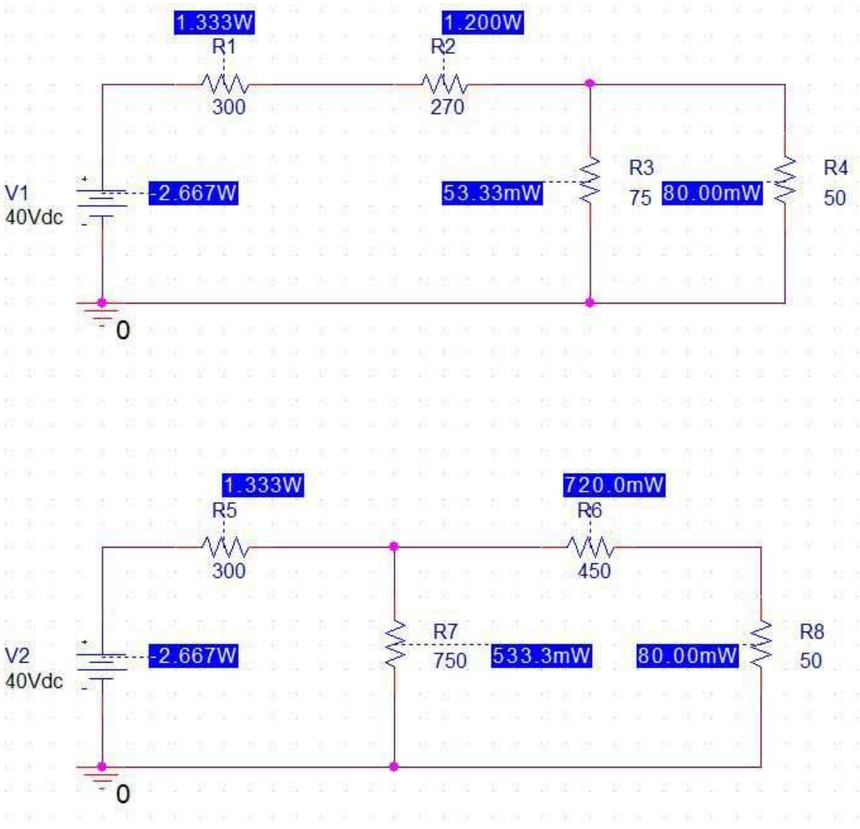


Figure Ex3–40

Exercise 3–41. A common problem is interfacing a TV antenna's 300-Ω line to a 75-Ω cable input on an HDTV set. Repeat Example 3–29 for this particular interface. See Figure 3–68 for a photo of such a device.

Follow the approach developed in Example 3–29. The source should see an equivalent resistance of 300 Ω and the load should see an equivalent resistance of 75 Ω. We have the following equations:

$$R_{\mathrm{AB}} = 300\,\Omega = R_1 + \frac{75R_2}{R_2+75}$$

$$R_{\mathrm{CD}} = 75\,\Omega = \frac{(R_1+300)R_2}{R_1+300+R_2}$$

For this problem, MATLAB is an efficient approach. The following commented MATLAB code provides the solution:

```
% Define the symbolic variables
syms R1 R2 real
% Create the equations for the resistance specifications
Eqn1 = R1 + 75*R2/(R2+75) - 300;
Eqn2 = (R1+300)*R2/(R1+300+R2) - 75;
% Solve the equations
Soln = solve(Eqn1,Eqn2);
% Examine the solutions
RR1 = Soln.R1
RR2 = Soln.R2
% Select the positive results and express as a decimal
RR1 = double(Soln.R1(1))
RR2 = double(Soln.R2(1))
```

The corresponding MATLAB output is shown below.

```
RR1 =
  150*3^(1/2)
 -150*3^(1/2)
RR2 =
  50*3^(1/2)
 -50*3^(1/2)
RR1 = 259.8076e+000
RR2 =  86.6025e+000
```

Therefore, we have $R_1 = 259.81$ Ω and $R_2 = 86.60$ Ω.

Exercise 3–42. Suppose $R_{\mathrm{T}} = 200$ Ω and the loading effect should be less than 1%. What should be the smallest value for R_{L}?

Apply the rule of thumb presented in Example 3–30 to get $R_{\mathrm{L}} \geq 100R_{\mathrm{T}} = (100)(200) = 20$ kΩ. We can be more accurate by applying voltage division. Let $v_{\mathrm{T}} = 100$ V and $v = 99$ V. The appropriate voltage division expression is

$$99 = \frac{R_{\mathrm{L}}}{R_{\mathrm{L}}+200}(100)$$

$$0.99 = \frac{R_{\mathrm{L}}}{R_{\mathrm{L}}+200}$$

$$0.99R_{\mathrm{L}} + 198 = R_{\mathrm{L}}$$

$$0.01R_{\mathrm{L}} = 198$$

$$R_{\mathrm{L}} = 19.8\,\mathrm{k}\Omega$$

Therefore, we need $R_{\mathrm{L}} \geq 19.8$ kΩ to prevent more than 1% loading.

Exercise 3–43. Modify the OrCAD simulation in Example 3–31 to include the 600-Ω load resistor. Create an OrCAD simulation profile to compute the gain between the input voltage and the load voltage. Verify that the results agree with the analysis and discussion presented in Example 3–31.

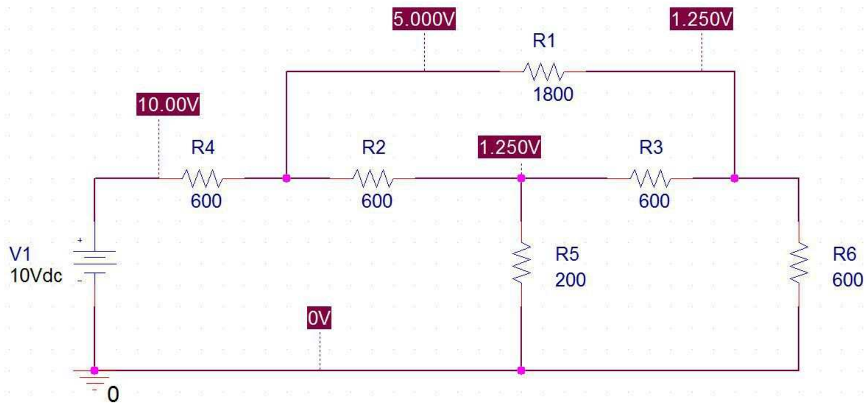


Figure Ex3–43

The OrCAD simulation in Figure Ex3–43 displays the results.
The corresponding OrCAD output is shown below.

```
****     SMALL-SIGNAL CHARACTERISTICS
      V(N00176)/V_V1 =  1.250E-01
      INPUT RESISTANCE AT V_V1 =  1.200E+03
      OUTPUT RESISTANCE AT V(N00176) =  3.000E+02
```

The voltage gain is 0.125 and is consistent with the results found in Example 3–31. The Thévenin source is $v_{\mathrm{T}} = v_{\mathrm{S}}/4$ and the Thévenin resistance is $R_{\mathrm{T}} = 600\ \Omega$. When this source is connected across a 600-Ω load, voltage division indicates that the output voltage is one half of the Thévenin voltage, which is one-eighth of the original source voltage or $v_{\mathrm{S}}/8$. The new output resistance is consistent with two 600-Ω resistors in parallel.

3.2 Problem Solutions

Problem 3–1. Formulate node-voltage equations for the circuit in Figure P3–1. Arrange the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

Write the node-voltage equations by summing the currents leaving each node. Write expressions for each current in terms of the node voltages.

$$-i_{\rm S} + \frac{v_{\rm A} - v_{\rm B}}{R_1} + \frac{v_{\rm A} - v_{\rm C}}{R_5} = 0$$

$$\frac{v_{\rm B} - v_{\rm A}}{R_1} + \frac{v_{\rm B}}{R_2} + \frac{v_{\rm B} - v_{\rm C}}{R_3} = 0$$

$$\frac{v_{\rm C} - v_{\rm A}}{R_5} + \frac{v_{\rm C} - v_{\rm B}}{R_3} + \frac{v_{\rm C}}{R_4} = 0$$

Arrange the results in standard form.

$$\left(\frac{1}{R_1} + \frac{1}{R_5}\right) v_{\rm A} - \frac{1}{R_1} v_{\rm B} - \frac{1}{R_5} v_{\rm C} = i_{\rm S}$$

$$-\frac{1}{R_1} v_{\rm A} + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_{\rm B} - \frac{1}{R_3} v_{\rm C} = 0$$

$$-\frac{1}{R_5} v_{\rm A} - \frac{1}{R_3} v_{\rm B} + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) v_{\rm C} = 0$$

Place the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_5}\right) & -\frac{1}{R_1} & -\frac{1}{R_5} \\ -\frac{1}{R_1} & \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) & -\frac{1}{R_3} \\ -\frac{1}{R_5} & -\frac{1}{R_3} & \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) \end{bmatrix} \begin{bmatrix} v_{\rm A} \\ v_{\rm B} \\ v_{\rm C} \end{bmatrix} = \begin{bmatrix} i_{\rm S} \\ 0 \\ 0 \end{bmatrix}$$

Problem 3–2. (a). Formulate node-voltage equations for the circuit in Figure P3–2. Arrange the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

Write the node-voltage equations by inspection.

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_{\rm A} - \frac{1}{R_2} v_{\rm B} = i_{\rm S}$$

$$-\frac{1}{R_2} v_{\rm A} + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) v_{\rm B} = 0$$

Arrange the results in matrix form.

$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) \end{bmatrix} \begin{bmatrix} v_{\rm A} \\ v_{\rm B} \end{bmatrix} = \begin{bmatrix} i_{\rm S} \\ 0 \end{bmatrix}$$

(b). Solve these equations for $v_{\rm A}$ and $v_{\rm B}$.

Use a matrix approach to solve for the node voltages.

$$\begin{bmatrix} v_\mathrm{A} \\ v_\mathrm{B} \end{bmatrix} = \begin{bmatrix} \left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right) & -\dfrac{1}{R_2} \\ -\dfrac{1}{R_2} & \left(\dfrac{1}{R_2}+\dfrac{1}{R_3}+\dfrac{1}{R_4}\right) \end{bmatrix}^{-1} \begin{bmatrix} i_\mathrm{S} \\ 0 \end{bmatrix}$$

The following MATLAB code performs the calculations:

```
% Define the symbolic variables
syms vA vB iS real
syms R1 R2 R3 R4 positive
% Create the node-voltage equations in matrix form
A = [1/R1+1/R2, -1/R2;...
    -1/R2, 1/R2+1/R3+1/R4];
b = [iS;...
     0];
% Solve for the node voltages
x = A\b
```

The corresponding MATLAB output is shown below.

```
x = (R1*iS*(R2*R3 + R2*R4 + R3*R4))/(R1*R3 + R1*R4 + R2*R3 + R2*R4 + R3*R4)
                   (R1*R3*R4*iS)/(R1*R3 + R1*R4 + R2*R3 + R2*R4 + R3*R4)
```

The node voltages are

$$v_\mathrm{A} = \frac{R_1(R_2R_3 + R_2R_4 + R_3R_4)i_\mathrm{S}}{R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4 + R_3R_4}$$

$$v_\mathrm{B} = \frac{R_1R_3R_4i_\mathrm{S}}{R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4 + R_3R_4}$$

(c). Use these results to find v_x and i_x.

From the circuit, we $v_\mathrm{x} = v_\mathrm{A} - v_\mathrm{B}$ and $i_\mathrm{x} = v_\mathrm{B}/R_4$. Using MATLAB, we can find expressions for these two terms. The following MATLAB code provides the solution:

```
% Solve for vx and ix
vx = simplify(x(1) - x(2))
ix = x(2)/R4
```

The corresponding MATLAB output is shown below.

```
vx = (R1*R2*iS*(R3 + R4))/(R1*R3 + R1*R4 + R2*R3 + R2*R4 + R3*R4)
ix = (R1*R3*iS)/(R1*R3 + R1*R4 + R2*R3 + R2*R4 + R3*R4)
```

The requested values are

$$v_\mathrm{x} = \frac{R_1R_2(R_3 + R_4)i_\mathrm{S}}{R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4 + R_3R_4}$$

$$i_\mathrm{x} = \frac{R_1R_3i_\mathrm{S}}{R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4 + R_3R_4}$$

Problem 3–3. (a). Formulate node-voltage equations for the circuit in Figure P3–3. Arrange the results in matrix form $\mathbf{A}\mathbf{x} = \mathbf{b}$.

Write the node-voltage equations by inspection.

$$\left(\frac{1}{5} + \frac{1}{10} + \frac{1}{8}\right) v_{\mathrm{A}} - \frac{1}{8} v_{\mathrm{B}} = 0$$

$$-\frac{1}{8} v_{\mathrm{B}} + \left(\frac{1}{8} + \frac{1}{4}\right) v_{\mathrm{B}} = 2$$

Arrange the results in matrix form.

$$\begin{bmatrix} \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{8}\right) & -\frac{1}{8} \\ -\frac{1}{8} & \left(\frac{1}{8} + \frac{1}{4}\right) \end{bmatrix} \begin{bmatrix} v_{\mathrm{A}} \\ v_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

(b). Solve these equations for v_{A} and v_{B}.

Multiply both of the original equations by the common denominator of 40 and simplify the results to get the following new, equivalent equations

$$17 v_{\mathrm{A}} - 5 v_{\mathrm{B}} = 0$$

$$-5 v_{\mathrm{A}} + 15 v_{\mathrm{B}} = 80$$

Multiply the first new equation by three and add it to the second equation to eliminate v_{B}.

$$46 v_{\mathrm{A}} = 80$$

Solve for the node voltages.

$$v_{\mathrm{A}} = 1.73913\,\mathrm{V}$$

$$v_{\mathrm{B}} = \frac{17 v_{\mathrm{A}}}{5} = 5.91304\,\mathrm{V}$$

Confirm the results with the following MATLAB approach

```
% Create the node-voltage equations in matrix form
A = [1/5+1/10+1/8, -1/8;...
    -1/8, 1/8+1/4];
b = [0;...
     2];
% Solve for the node voltages
x = A\b;
vA = x(1)
vB = x(2)
```

The corresponding MATLAB output is shown below.

```
vA = 1.7391e+000
vB = 5.9130e+000
```

(c). Use these results to find v_{x} and i_{x}.

By inspection, $v_{\mathrm{x}} = v_{\mathrm{B}}$. Apply Ohm's law to find i_{x}.

$$v_{\mathrm{x}} = v_{\mathrm{B}} = 5.91304\,\mathrm{V}$$

$$i_{\mathrm{x}} = \frac{v_{\mathrm{A}}}{5} = 347.826\,\mathrm{mA}$$

Problem 3–4. (a). Formulate node-voltage equations for the circuit in Figure P3–4.

Write the node-voltage equations by inspection.

$$\left(\frac{1}{20} + \frac{1}{10} + \frac{1}{8}\right) v_{\mathrm{A}} - \frac{1}{8} v_{\mathrm{B}} = 3$$

$$-\frac{1}{8} v_{\mathrm{B}} + \left(\frac{1}{8} + \frac{1}{4}\right) v_{\mathrm{B}} = 1 - 3$$

Arrange the results in matrix form.

$$\begin{bmatrix} \left(\frac{1}{20} + \frac{1}{10} + \frac{1}{8}\right) & -\frac{1}{8} \\ -\frac{1}{8} & \left(\frac{1}{8} + \frac{1}{4}\right) \end{bmatrix} \begin{bmatrix} v_{\mathrm{A}} \\ v_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

(b). Solve these equations for v_{A} and v_{B}.

Multiply both of the original equations by the common denominator of 40 and simplify the results to get the following new, equivalent equations

$$11 v_{\mathrm{A}} - 5 v_{\mathrm{B}} = 120$$

$$-5 v_{\mathrm{A}} + 15 v_{\mathrm{B}} = -80$$

Multiply the first new equation by three and add it to the second equation to eliminate v_{B}.

$$28 v_{\mathrm{A}} = 280$$

Solve for the node voltages.

$$v_{\mathrm{A}} = 10\,\mathrm{V}$$

$$v_{\mathrm{B}} = \frac{(11)(10) - 120}{5} = -2\,\mathrm{V}$$

Confirm the results with the following MATLAB approach

```
% Create the node-voltage equations in matrix form
A = [1/20+1/10+1/8, -1/8;...
    -1/8, 1/8+1/4];
b = [3;...
     -2];
% Solve for the node voltages
x = A\b;
vA = x(1)
vB = x(2)
```

The corresponding MATLAB output is shown below.

```
vA = 10.0000e+000
vB = -2.0000e+000
```

(c). Use these results to find v_{x} and i_{x}.

By inspection, $v_{\mathrm{x}} = v_{\mathrm{B}}$. Apply Ohm's law to find i_{x}.

$$v_{\mathrm{x}} = v_{\mathrm{B}} = -2\,\mathrm{V}$$

$$i_{\mathrm{x}} = \frac{v_{\mathrm{A}}}{20} = 500\,\mathrm{mA}$$

Problem 3–5. (a). Formulate node-voltage equations for the circuit in Figure P3–5. Arrange the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

Combined, the voltage source and reference node (ground) cause the center node to have a voltage of 5 V. Therefore, we have to solve for only two node voltages. Write the node-voltage equations by inspection.

$$\left(\frac{1}{4000} + \frac{1}{4000}\right) v_{\mathrm{A}} = \frac{5}{4000} + 0.01$$

$$\left(\frac{1}{1000} + \frac{1}{4000}\right) v_{\mathrm{B}} = \frac{5}{1000} - 0.01$$

Arrange the results in matrix form.

$$\begin{bmatrix} \left(\frac{1}{4000} + \frac{1}{4000}\right) & 0 \\ 0 & \left(\frac{1}{1000} + \frac{1}{4000}\right) \end{bmatrix} \begin{bmatrix} v_{\mathrm{A}} \\ v_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} 11.25 \times 10^{-3} \\ -5 \times 10^{-3} \end{bmatrix}$$

(b). Solve these equations for v_{A} and v_{B}.

The equations are not coupled, so we can solve each of the original equations independently for the node voltages.

$$(500 \times 10^{-6})v_{\mathrm{A}} = 11.25 \times 10^{-3}$$

$$v_{\mathrm{A}} = 22.5\,\mathrm{V}$$

$$(1.25 \times 10^{-3})v_{\mathrm{B}} = -5 \times 10^{-3}$$

$$v_{\mathrm{B}} = -4\,\mathrm{V}$$

(c). Use these results to find v_{x} and i_{x}.

The voltage v_{x} is the difference between the two node voltages. Apply KCL to solve for i_{x}.

$$v_{\mathrm{x}} = v_{\mathrm{A}} - v_{\mathrm{B}} = 22.5 - (-4) = 26.5\,\mathrm{V}$$

$$i_{\mathrm{x}} = \frac{v_{\mathrm{A}} - 5}{4000} + \frac{v_{\mathrm{B}} - 5}{1000} = \frac{22.5 - 5}{4000} + \frac{-4 - 5}{1000} = -4.625\,\mathrm{mA}$$

Problem 3–6. (a). Choose a ground wisely and formulate node-voltage equations for the circuit in Figure P3–6.

Choose the ground to be at the node connected to the negative terminal of the voltage source. This choice causes the upper right node to have a voltage of $v_{\rm S}$. Label the upper left node as $v_{\rm A}$ and the center node as $v_{\rm B}$. Write the two node-voltage equations by inspection.

$$\left(\frac{1}{R_3}+\frac{1}{R_4}\right)v_{\rm A}-\frac{1}{R_4}v_{\rm B}=i_{\rm S}+\frac{v_{\rm S}}{R_3}$$

$$-\frac{1}{R_4}v_{\rm A}+\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right)v_{\rm B}=\frac{v_{\rm S}}{R_2}$$

(b). Solve for $v_{\rm x}$ and $i_{\rm x}$ when $R_1 = R_2 = R_3 = R_4 = 10$ kΩ, $v_{\rm S} = 25$ V, and $i_{\rm S} = 1$ mA.

First, solve for the node voltages. Since all of the resistor values are equal, multiply both equations by that value to simplify the calculations.

$$2v_{\rm A}-v_{\rm B}=Ri_{\rm S}+v_{\rm S}$$

$$-v_{\rm A}+3v_{\rm B}=v_{\rm S}$$

Add two times the second equation to the first equation to eliminate $v_{\rm A}$ and solve for $v_{\rm B}$.

$$5v_{\rm B}=Ri_{\rm S}+3v_{\rm S}=(10000)(0.001)+(3)(25)=85$$

$$v_{\rm B}=17\,{\rm V}$$

Solve for $v_{\rm A}$.

$$v_{\rm A}=3v_{\rm B}-v_{\rm S}=(3)(17)-25=26\,{\rm V}$$

Solve for $v_{\rm x}$ and $i_{\rm x}$.

$$v_{\rm x}=v_{\rm B}-v_{\rm S}=17-25=-8\,{\rm V}$$

$$i_{\rm x}=\frac{v_{\rm S}-v_{\rm A}}{R_3}=\frac{25-26}{10000}=-100\,\mu{\rm A}$$

Problem 3–7. The following are a set of node-voltage equations; draw the circuit they represent.

$$v_{\rm A}=v_{\rm S}$$

$$\frac{v_{\rm B}-v_{\rm A}}{R_1}+\frac{v_{\rm B}-v_{\rm C}}{R_2}-i_{\rm S}=0$$

$$\frac{v_{\rm C}-v_{\rm A}}{R_3}+\frac{v_{\rm C}-v_{\rm B}}{R_2}+\frac{v_{\rm C}}{R_4}=0$$

$$v_{\rm D}=0$$

The circuit in Figure P3–7 represents one possible solution.

Problem 3–8. (a). Choose a ground wisely and formulate node-voltage equations for the circuit in Figure P3–8.

Choose the ground to be the node connected to the negative terminal of the voltage source. This choice causes the lower node to have a voltage of 15 V. Label the center node as $v_{\rm B}$ and the upper right node

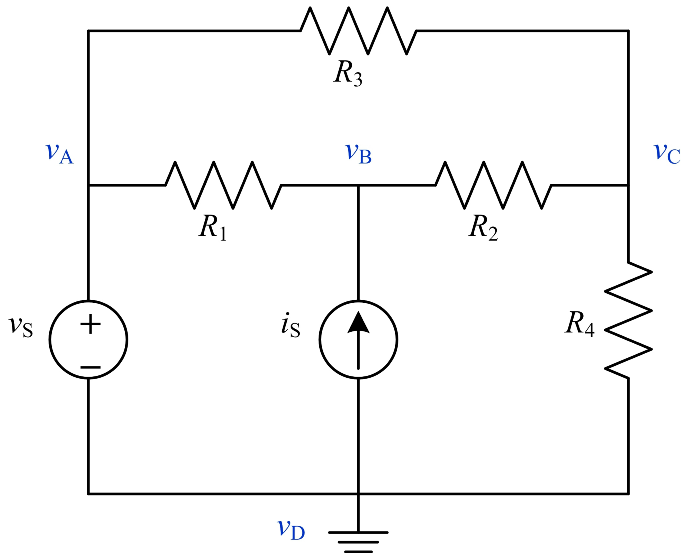


Figure P3–7

as v_C. Write the two node-voltage equations by inspection.

$$\left(\frac{1}{2200}+\frac{1}{1000}+\frac{1}{1000}\right)v_B-\frac{1}{1000}v_C=\frac{15}{1000}-0.01$$

$$-\frac{v_B}{1000}+\left(\frac{1}{1000}+\frac{1}{1000}+\frac{1}{3300}\right)v_C=\frac{15}{3300}+0.01$$

(b). Solve for v_x and i_x.

Use MATLAB to solve the node-voltage equations and then solve for v_x and i_x. The following MATLAB code provides the solution:

```
% Define the symbolic variables
syms vB vC real
% Write the node-voltage equations
Eqn1 = (1/2200 + 1/1000 + 1/1000)*vB - 15/1000 - vC/1000 + 10e-3;
Eqn2 = -vB/1000 + (1/1000 + 1/1000 + 1/3300)*vC - 15/3300 - 10e-3;
% Solve the equations
Soln = solve(Eqn1,Eqn2);
% Examine the solutions
vB1 = double(Soln.vB)
vC1 = double(Soln.vC)
% Solve for the requested voltage and current
vx = vC1 - 0
ix = (0 - vB1)/2200
```

The corresponding MATLAB output is shown below.

```
vB1 = 5.6009e+000
vC1 = 8.7478e+000
vx =  8.7478e+000
ix = -2.5459e-003
```

The results are

$$v_{\mathrm{x}} = 8.7478\,\mathrm{V}$$

$$i_{\mathrm{x}} = -2.5459\,\mathrm{mA}$$

Problem 3–9. (a). Formulate node-voltage equations for the circuit in Figure P3–9.

Write the node-voltage equations by inspection.

$$\left(\frac{1}{R_1}+\frac{1}{R_4}\right)v_{\mathrm{A}} - \frac{1}{R_1}v_{\mathrm{B}} - \frac{1}{R_4}v_{\mathrm{C}} = i_{\mathrm{S1}}$$

$$-\frac{1}{R_1}v_{\mathrm{A}} + \left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_{\mathrm{B}} = i_{\mathrm{S2}}$$

$$-\frac{1}{R_4}v_{\mathrm{A}} + \left(\frac{1}{R_3}+\frac{1}{R_4}\right)v_{\mathrm{C}} = -i_{\mathrm{S2}}$$

(b). Use MATLAB to find symbolic expressions for the node voltages in terms of the parameters in the circuit.

The following MATLAB code provides the solution:

```
% Define the symbolic variables
syms vA vB vC iS1 iS2 real
syms R1 R2 R3 R4 positive
% Write the node-voltage equations
Eqn1 = (1/R1+1/R4)*vA - vB/R1 - vC/R4 - iS1;
Eqn2 = -vA/R1 + (1/R1+1/R2)*vB - iS2;
Eqn3 = -vA/R4 + (1/R3+1/R4)*vC + iS2;
% Solve the equations for the node voltages
Soln = solve(Eqn1,Eqn2,Eqn3,vA,vB,vC);
% Examine the solutions
vA1 = Soln.vA
vB1 = Soln.vB
vC1 = Soln.vC
```

The corresponding MATLAB output is shown below.

```
vA1 =
(R1*R3*iS1 - R1*R3*iS2 + R1*R4*iS1 + R2*R3*iS1 + R2*R4*iS1 + R2*R4*iS2)/(R1 + R2 + R3 + R4)
vB1 =
(R2*(R1*iS2 + R3*iS1 + R4*iS1 + R4*iS2))/(R1 + R2 + R3 + R4)
vC1 =
(R3*(R1*iS1 - R1*iS2 + R2*iS1 - R4*iS2))/(R1 + R2 + R3 + R4)
```

The results are

$$v_{\mathrm{A}} = \frac{(R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4)i_{\mathrm{S1}} + (R_2R_4 - R_1R_3)i_{\mathrm{S2}}}{R_1+R_2+R_3+R_4}$$

$$v_{\mathrm{B}} = \frac{R_2\left[(R_3+R_4)i_{\mathrm{S1}} + (R_1+R_4)i_{\mathrm{S2}}\right]}{R_1+R_2+R_3+R_4}$$

$$v_{\mathrm{C}} = \frac{R_3\left[(R_1+R_2)i_{\mathrm{S1}} - (R_1+R_4)i_{\mathrm{S2}}\right]}{R_1+R_2+R_3+R_4}$$

(c). Find numeric values for v_A, v_B, and v_C when $R_1 = 1.5$ kΩ, $R_2 = 2.2$ kΩ, $R_3 = 3.3$ kΩ, $R_4 = 4.7$ kΩ, and $i_{S1} = i_{S2} = 2$ mA.

Substitute the given values into the symbolic expressions for the node voltages. The following MATLAB code provides the solution:

```
% Substitute in the known values and solve for vA, vB, and vC
R1 = 1500;
R2 = 2200;
R3 = 3300;
R4 = 4700;
iS1 = 2e-3;
iS2 = 2e-3;
vA2 = subs(vA1)
vB2 = subs(vB1)
vC2 = subs(vC1)
```

The corresponding MATLAB output is shown below.

```
vA2 =  5.9812e+000
vB2 =  5.3402e+000
vC2 = -1.4103e+000
```

The results are

$$v_A = 5.9812\,\text{V}$$

$$v_B = 5.3402\,\text{V}$$

$$v_C = -1.4103\,\text{V}$$

(d). Use OrCAD to verify your solution to part (c) is correct.

The OrCAD simulation in Figure P3–9 confirms the results.

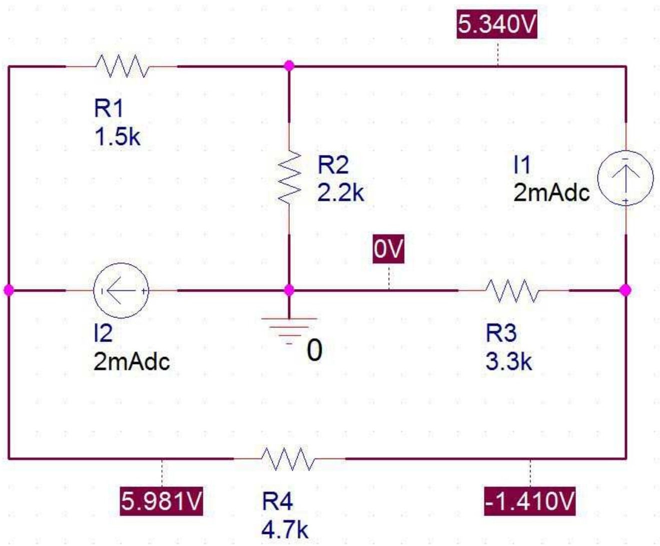


Figure P3–9

Problem 3–10. (a). Formulate node-voltage equations for the circuit in Figure P3–10.

Let the negative terminal of the voltage source be the reference node. The top node then has a voltage of v_S. Let the left corner of the diamond be v_A and the right corner of the diamond be v_B. Write the node-voltage equations by inspection.

$$\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_x}\right)v_A-\frac{1}{R_x}v_B=\frac{v_S}{R_1}$$

$$-\frac{1}{R_x}v_A+\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_x}\right)v_B=\frac{v_S}{R_2}$$

(b). Solve for v_x and i_x when $R_1 = R_4 = 2$ kΩ, $R_2 = R_3 = 500$ Ω, $R_x = 750$ Ω, and $v_S = 15$ V.

Use MATLAB to solve for the node voltages and then compute v_x and i_x.

```
% Define the symbolic variables
syms R1 R2 R3 R4 Rx vA vB vS real
% Write the node-voltage equations
Eqn1 = (1/R1 + 1/R3 + 1/Rx)*vA - vB/Rx - vS/R1;
Eqn2 = -vA/Rx + (1/R2 + 1/R4 + 1/Rx)*vB - vS/R2;
% Solve the equations for vA and vB
Soln = solve(Eqn1,Eqn2,vA,vB);
% Examine the solutions
vA1 = Soln.vA;
vB1 = Soln.vB;
% Substitute in the given values
R1 = 2000;
R2 = 500;
R3 = 500;
R4 = 2000;
Rx = 750;
vS = 15;
vA2 = subs(vA1)
vB2 = subs(vB1)
% Solve for the requested voltage and current
vx = vA2 - vB2
ix = (vS-vA2)/R1 + (vS-vB2)/R2
```

The corresponding MATLAB output is shown below.

```
vA2 =  5.3226e+000
vB2 =  9.6774e+000
vx =  -4.3548e+000
ix =  15.4839e-003
```

The results are

$$v_x = -4.3548\,\text{V}$$

$$i_x = 15.4839\,\text{mA}$$

(c). Repeat (b) when R_4 is a variable resistor that varies from 10 Ω to 10,000 Ω. At what value of R_4 is the voltage across $R_x = 0$ V? (*Hint*: Use OrCAD to vary R_4 by trial and error to approach the answer.)

The approach using OrCAD will yield a solution through an iterative search. We can also use MATLAB to compute the exact value of R_4 that balances the circuit.

```
% Repeat the problem, but allow R4 to vary to make vx = 0
% Define the symbolic variables
syms R1 R2 R3 R4 Rx vA vB vS real
% Write the node-voltage equations
Eqn1 = (1/R1 + 1/R3 + 1/Rx)*vA - vB/Rx - vS/R1;
Eqn2 = -vA/Rx + (1/R2 + 1/R4 + 1/Rx)*vB - vS/R2;
% Solve the equations for vA and vB
Soln = solve(Eqn1,Eqn2,vA,vB);
% Examine the solutions
vA1 = Soln.vA;
vB1 = Soln.vB;
% Create an expression for vx
vx1 = vA1 - vB1;
% Substitute in the known values
R1 = 2000;
R2 = 500;
R3 = 500;
Rx = 750;
vS = 15;
vx2 = subs(vx1);
% Solve for the value of R4 that makes vx = 0
R4_Soln = solve(vx2)
```

The corresponding MATLAB output is shown below.

```
R4_Soln = 125
```

Setting $R_4 = 125\ \Omega$ will balance the circuit and make $v_{\text{x}} = 0$ V.

Problem 3–11. (a). Formulate node-voltage equations for the circuit in Figure P3–11.

Choose the reference node to be the one connected to the negative terminal of the voltage source. This choice makes the node on the left side of the diamond equal to v_{S}. Let the top node be v_{A} and the bottom node be v_{B}. Write the node-voltage equations by inspection.

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{\text{x}}}\right) v_{\text{A}} - \frac{1}{R_{\text{x}}} v_{\text{B}} = \frac{v_{\text{S}}}{R_1}$$

$$-\frac{1}{R_{\text{x}}} v_{\text{A}} + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_{\text{x}}}\right) v_{\text{B}} = \frac{v_{\text{S}}}{R_3}$$

(b). Solve for v_{x} and i_{x} when $R_1 = R_4 = 2\ \text{k}\Omega$, $R_2 = R_3 = 500\ \Omega$, $R_{\text{x}} = 750\ \Omega$, and $v_{\text{S}} = 15$ V.

Use MATLAB to solve for the node voltages and then solve for v_{x} and i_{x}.

```
% Define the symbolic variables
syms vA vB real
% Assign the known values
R1 = 2000;
R2 = 500;
R3 = 500;
R4 = 2000;
Rx = 750;
vS = 15;
% Write the node-voltage equations
Eqn1 = (1/R1 + 1/R2 + 1/Rx)*vA - vB/Rx - vS/R1;
Eqn2 = -vA/Rx + (1/R3 + 1/R4 + 1/Rx)*vB - vS/R3;
% Solve the equations for vA and vB
Soln = solve(Eqn1,Eqn2);
% Examine the solutions
vA1 = Soln.vA
```

```
vB1 = Soln.vB
% Solve for the requested voltage and current
vx = vA1 - vB1
ix = vA1/R2
```

The corresponding MATLAB output is shown below.

```
vA1 = 165/31
vB1 = 300/31
vx = -135/31
ix = 33/3100
```

The results are

$$v_{\mathrm{A}} = \frac{165}{31} = 5.3226\,\mathrm{V}$$

$$v_{\mathrm{B}} = \frac{300}{31} = 9.6774\,\mathrm{V}$$

$$v_{\mathrm{x}} = -\frac{135}{31} = -4.3548\,\mathrm{V}$$

$$i_{\mathrm{x}} = \frac{33}{3100} = 10.6452\,\mathrm{mA}$$

Problem 3–12. (a). Formulate node-voltage equations for the circuit in Figure P3–12. (*Hint*: Use a supernode.)

Let the reference node be the negative terminal of the 15-V voltage source. With that choice, the bottom node has a voltage of 15 V. Let v_{A} be the center node at the positive terminal of the 10-V voltage source and let v_{B} be the upper right node, which is also the negative terminal of the 10-V voltage source. Create a supernode around the 10-V voltage source and write the node-voltage equations.

$$v_{\mathrm{A}} - v_{\mathrm{B}} = 10$$

$$\frac{v_{\mathrm{A}}}{500} + \frac{v_{\mathrm{A}} - 15}{1500} + \frac{v_{\mathrm{B}}}{1000} + \frac{v_{\mathrm{B}} - 15}{2000} = 0$$

(b). Solve for v_{x} and i_{x}.

Use MATLAB to solve for the node voltages and then solve for v_{x} and i_{x}.

```
% Define the symbolic variables
syms vA vB real
% Write the node-voltage equations
Eqn1 = vA-vB-10;
Eqn2 = vA/500 + (vA-15)/1500 + vB/1000 + (vB-15)/2000;
% Solve the equations for vA and vB
Soln = solve(Eqn1,Eqn2);
% Examine the solutions
vA1 = Soln.vA
vB1 = Soln.vB
% Solve for the requested voltage and current
vx = vB1 - 15
ix = vB1/1000
```

The corresponding MATLAB output is shown below.

```
vA1 = 39/5
vB1 = -11/5
vx = -86/5
ix = -11/5000
```

The results are

$$v_\mathrm{A} = \frac{39}{5} = 7.8\,\mathrm{V}$$

$$v_\mathrm{B} = -\frac{11}{5} = -2.2\,\mathrm{V}$$

$$v_\mathrm{x} = -\frac{86}{5} = -17.2\,\mathrm{V}$$

$$i_\mathrm{x} = -\frac{11}{5000} = -2.2\,\mathrm{mA}$$

Problem 3–13. (a). Formulate mesh-current equations for the circuit in Figure P3–13. Arrange the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

Write the mesh-current equations by inspection.

$$(10000 + 5000)i_\mathrm{A} - 5000i_\mathrm{B} = 5$$

$$-5000i_\mathrm{A} + (5000 + 10000)i_\mathrm{B} = -10$$

Arrange the results in matrix form.

$$\begin{bmatrix} (10000+5000) & -5000 \\ & \\ -5000 & (5000+10000) \end{bmatrix} \begin{bmatrix} i_\mathrm{A} \\ i_\mathrm{B} \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

(b). Solve for i_A and i_B.

Use the original mesh-current equations to solve by adding three times the first equation to the second equation to eliminate i_B. Solve for i_A and then substitute back into the first equation to solve for i_B.

$$40000i_\mathrm{A} = 5$$

$$i_\mathrm{A} = 125\,\mu\mathrm{A}$$

$$i_\mathrm{B} = \frac{(15000)(125\times10^{-6}) - 5}{5000} = -625\,\mu\mathrm{A}$$

We can confirm these results with the following MATLAB code:

```
% Create the mesh-current equations in matrix form
A = [15000, -5000;...
    -5000, 15000];
b = [5;...
     -10];
% Solve for the mesh currents
x = A\b;
iA = x(1)
iB = x(2)
```

The corresponding MATLAB output is shown below.

```
iA =  125.0000e-006
iB = -625.0000e-006
```

(c). Use these results to find v_x and i_x.

We can compute the requested values as follows:

$$v_x = 10000 i_A = (10000)(125 \times 10^{-6}) = 1.25\,\text{V}$$

$$i_x = i_A - i_B = 125 - (-625) = 750\,\mu\text{A}$$

Problem 3–14. (a). Formulate mesh-current equations for the circuit in Figure P3–14. Arrange the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

Write the mesh-current equations by inspection.

$$(2200 + 1000)i_A - 1000 i_B - 2200 i_C = -15$$

$$-1000 i_A + (1000 + 1000 + 2000)i_B - 1000 i_C = 0$$

$$-2200 i_A - 1000 i_B + (2200 + 1000 + 1000)i_C = 0$$

Arrange the results in matrix form.

$$\begin{bmatrix} (2200+1000) & -1000 & -2200 \\ -1000 & (1000+1000+2000) & -1000 \\ -2200 & -1000 & (2200+1000+1000) \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} -15 \\ 0 \\ 0 \end{bmatrix}$$

(b). Solve for i_A, i_B, and i_C.

Use MATLAB to solve the equations for the mesh currents.

```
% Create the mesh-current equations in matrix form
A = [3200, -1000, -2200;...
    -1000, 4000, -1000;...
    -2200, -1000, 4200];
b = [-15;...
     0;...
     0];
% Solve for the mesh currents
x = A\b;
iA = x(1)
iB = x(2)
iC = x(3)
```

The corresponding MATLAB output is shown below.

```
iA = -10.4867e-003
iB =  -4.2478e-003
iC =  -6.5044e-003
```

The results are

$$i_A = -10.4867\,\text{mA}$$

$$i_B = -4.2478\,\text{mA}$$

$$i_C = -6.5044\,\text{mA}$$

(c). Use these results to find v_x and i_x.

We can compute the requested values as follows:

$$v_x = 1000\,(i_C - i_B) = -2.2566\,\text{V}$$

$$i_x = i_C = -6.5044\,\text{mA}$$

Problem 3–15. (a). Formulate mesh-current equations for the circuit in Figure P3–15. Arrange the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

Write the mesh-current equations by inspection.

$$(4000 + 1000 + 4000)i_A - 4000i_B = 12 - 12$$

$$-4000i_A + (4000 + 1000 + 4000)i_B = 12$$

Arrange the results in matrix form.

$$\begin{bmatrix} (4000 + 1000 + 4000) & -4000 \\ -4000 & (4000 + 1000 + 4000) \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \end{bmatrix}$$

(b). Solve for i_A and i_B.

Use MATLAB to solve the equations for the mesh currents.

```
% Create the mesh-current equations in matrix form
A = [9000, -4000;...
    -4000, 9000];
b = [0;...
     12];
% Solve for the mesh currents
x = A\b;
iA = x(1)
iB = x(2)
```

The corresponding MATLAB output is shown below.

```
iA = 738.4615e-006
iB = 1.6615e-003
```

The results are

$$i_A = 738.4615\,\mu\text{A}$$

$$i_B = 1.6615\,\text{mA}$$

(c). Use these results to find v_x and i_x.

We can compute the requested values as follows:

$$v_x = 4000 i_B = 6.6462\,\text{V}$$

$$i_x = i_B - i_A = 923.0769\,\mu\text{A}$$

Problem 3–16. (a). Formulate mesh-current equations for the circuit in Figure P3–16. Arrange the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

Write the mesh-current equations by inspection.

$$(R_1 + R_2)\, i_A - R_2 i_B = v_S$$

$$-R_2 i_A + (R_2 + R_3 + R_4)\, i_B = 0$$

Arrange the results in matrix form.

$$\begin{bmatrix} (R_1 + R_2) & -R_2 \\ \\ -R_2 & (R_2 + R_3 + R_4) \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} v_S \\ 0 \end{bmatrix}$$

(b). Solve for i_A and i_B.

Use MATLAB to solve the equations for the mesh currents.

```
% Define the symbolic variables
syms R1 R2 R3 R4 vS real
% Create the mesh-current equations in matrix form
A = [R1+R2, -R2;...
    -R2, R2+R3+R4];
b = [vS;...
     0];
% Solve for the mesh currents
x = A\b;
iA = x(1)
iB = x(2)
```

The corresponding MATLAB output is shown below.

```
iA = (vS*(R2 + R3 + R4))/(R1*R2 + R1*R3 + R1*R4 + R2*R3 + R2*R4)
iB = (R2*vS)/(R1*R2 + R1*R3 + R1*R4 + R2*R3 + R2*R4)
```

The results are

$$i_A = \frac{(R_2 + R_3 + R_4)\, v_S}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4}$$

$$i_B = \frac{R_2 v_S}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4}$$

(c). Use these results to find v_x and i_x.

We can compute the requested values as follows:

$$v_x = -R_4 i_B = \frac{-R_2R_4v_S}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4}$$

$$i_x = i_B - i_A = \frac{-(R_3 + R_4)\, v_S}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4}$$

Problem 3–17. (a). Formulate mesh-current equations for the circuit in Figure P3–17.

Write the mesh-current equations by inspection.

$$(R_A + R_B)\, i_A - R_B i_B - R_A i_C = v_S$$

$$-R_B i_A + (R_B + R_C + R_D)\, i_B - R_C i_C = 0$$

$$-R_A i_A - R_C i_B + (R_A + R_C + R_E)\, i_C = 0$$

(b). Formulate node-voltage equations for the circuit in Figure P3–17.

Label the node at the negative terminal of the voltage source as the reference node. The node at the positive terminal of the voltage source has a voltage of v_S. Label the center node as v_A and the upper right node as v_B. Write the node-voltage equations by inspection.

$$\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right) v_A - \frac{1}{R_C} v_B = \frac{v_S}{R_A}$$

$$-\frac{1}{R_C} v_A + \left(\frac{1}{R_C} + \frac{1}{R_D} + \frac{1}{R_E}\right) v_B = \frac{v_S}{R_E}$$

(c). Which set of equations would be easier to solve? Why?

The node-voltage equations would be easier to solve, because there are only two unknown voltages, v_A and v_B.

(d). Using MATLAB, find v_x and i_x in terms of the mesh-current variables.

Use MATLAB to set up the mesh-current equations and then solve for v_x and i_x.

```
% Define the symbolic variables
syms iA iB iC vA vB vS real
syms RA RB RC RD RE positive
% Create the mesh-current equations
Eqn1 = (RA+RB)*iA - RB*iB - RA*iC - vS;
Eqn2 = -RB*iA + (RB+RC+RD)*iB - RC*iC;
Eqn3 = -RA*iA - RC*iB + (RA+RC+RE)*iC;
% Solve for the mesh currents
Soln = solve(Eqn1,Eqn2,Eqn3,iA,iB,iC);
iA1 = Soln.iA;
iB1 = Soln.iB;
iC1 = Soln.iC;
% Solve for vx and ix
vx = simplify(RB*(iA1-iB1))
ix = simplify(-iB1)
```

The corresponding MATLAB output is shown below.

```
vx = (RB*vS*(RA*RD + RC*RD + RC*RE + RD*RE))/
(RA*RB*RD + RA*RB*RE + RA*RC*RD + RA*RC*RE + RB*RC*RD + RA*RD*RE + RB*RC*RE + RB*RD*RE)
ix = -(vS*(RA*RB + RA*RC + RB*RC + RB*RE))/
(RA*RB*RD + RA*RB*RE + RA*RC*RD + RA*RC*RE + RB*RC*RD + RA*RD*RE + RB*RC*RE + RB*RD*RE)
```

The results are

$$v_x = \frac{R_B v_S (R_A R_D + R_C R_D + R_C R_E + R_D R_E)}{R_A R_B R_D + R_A R_B R_E + R_A R_C R_D + R_A R_C R_E + R_B R_C R_D + R_A R_D R_E + R_B R_C R_E + R_B R_D R_E}$$

$$i_x = -\frac{v_S (R_A R_B + R_A R_C + R_B R_C + R_B R_E)}{R_A R_B R_D + R_A R_B R_E + R_A R_C R_D + R_A R_C R_E + R_B R_C R_D + R_A R_D R_E + R_B R_C R_E + R_B R_D R_E}$$

(e). Using MATLAB, find v_x and i_x in terms of the node-voltage variables.

Use MATLAB to set up the node-voltage equations and then solve for v_x and i_x.

```
%Create the node-voltage equations
Eqn1 = (1/RA + 1/RB + 1/RC)*vA - vB/RC - vS/RA;
Eqn2 = -vA/RC + (1/RC + 1/RD + 1/RE)*vB - vS/RE;
% Solve for the node voltages
Soln = solve(Eqn1,Eqn2,vA,vB);
vA1 = Soln.vA;
vB1 = Soln.vB;
% Solve for vx and ix
vx = simplify(vA1)
ix = simplify(-vB1/RD)
```

The corresponding MATLAB output is shown below.

```
vx = (RB*vS*(RA*RD + RC*RD + RC*RE + RD*RE))/
(RA*RB*RD + RA*RB*RE + RA*RC*RD + RA*RC*RE + RB*RC*RD + RA*RD*RE + RB*RC*RE + RB*RD*RE)
ix = -(vS*(RA*RB + RA*RC + RB*RC + RB*RE))/
(RA*RB*RD + RA*RB*RE + RA*RC*RD + RA*RC*RE + RB*RC*RD + RA*RD*RE + RB*RC*RE + RB*RD*RE)
```

The answers are the same as part (d).

Problem 3–18. (a). Formulate mesh-current equations for the circuit in Figure P3–18. (*Hint*: Use a supermesh.)

Label the left mesh with current i_A and the right mesh with current i_B. With the current source between the two mesh currents, there is one supermesh around the perimeter of the circuit. The corresponding equations are

$$i_\mathrm{B} - i_\mathrm{A} = i_\mathrm{S}$$

$$(R_1 + R_2)\, i_\mathrm{A} + (R_3 + R_4 + R_5)\, i_\mathrm{B} = v_\mathrm{S}$$

(b). Solve for v_x and i_x when $R_1 = 2$ kΩ, $R_2 = 3$ kΩ, $R_3 = 500$ Ω, $R_4 = 2.5$ kΩ, $R_5 = 2$ kΩ, $i_\mathrm{S} = 10$ mA, and $v_\mathrm{S} = 24$ V.

We have the following expressions for v_x and i_x:

$$v_\mathrm{x} = R_4 i_\mathrm{B}$$

$$i_\mathrm{x} = -i_\mathrm{A}$$

Use MATLAB to solve for the mesh currents with the given values and then solve for v_x and i_x.

```
% Define the symbolic variables
syms iA iB real
% Assign the known values
R1 = 2000;
R2 = 3000;
R3 = 500;
R4 = 2500;
R5 = 2000;
iS = 10e-3;
vS = 24;
% Create the mesh-current equations
Eqn1 = iB - iA - iS;
Eqn2 = (R1+R2)*iA + (R3+R4+R5)*iB - vS;
% Solve for the mesh currents
```

```
Soln = solve(Eqn1,Eqn2);
iA1 = Soln.iA;
iB1 = Soln.iB;
% Solve for vx and ix
vx = simplify(R4*iB1)
ix = simplify(-iA1)
```

The corresponding MATLAB output is shown below.

```
vx = 37/2
ix = 13/5000
```

The results are

$$v_{\text{x}} = \frac{37}{2} = 18.5\,\text{V}$$

$$i_{\text{x}} = \frac{13}{5000} = 2.6\,\text{mA}$$

(c). Find the total power dissipated in the circuit.

The total power dissipated in the circuit can be found by summing the power dissipated in each resistor as $p = i^2R$. Using MATLAB, we have the following:

```
% Find the power dissipated in the circuit
p1 = (R1+R2)*iA1^2 + (R3+R4+R5)*iB1^2
```

The corresponding MATLAB output is shown below.

```
p1 = 769/2500
```

The result is

$$p = \frac{769}{2500} = 307.6\,\text{mW}$$

Problem 3–19. (a). For the circuit of Figure P3–19 solve for i_{A}, i_{B}, and i_{C} using the supermesh principles.

The two current sources connect all three meshes. We can use the current source i_{S1} to establish a relationship between i_{A} and i_{B} and use current source i_{S2} to establish a relationship between i_{A} and i_{C}. Then we need only one more equation to solve for the three mesh currents. We can write the supermesh equation around the perimeter of the circuit, using the appropriate mesh current in each branch. The corresponding system equations are as follows:

$$i_{\text{B}} - i_{\text{A}} = i_{\text{S1}}$$

$$i_{\text{C}} - i_{\text{A}} = i_{\text{S2}}$$

$$R_1 i_{\text{A}} + R_4 i_{\text{B}} + R_2 i_{\text{C}} = 0$$

Note that resistor R_3 does not play a role in the solution because its current is determined completely by the two current sources. Using MATLAB, we can solve for the three mesh currents.

```
% Define the symbolic variables
syms R1 R2 R3 R4 iS1 iS2 iA iB iC real
% Write the mesh-current equations
Eqn1 = iB-iA-iS1;
```

```
Eqn2 = iC-iA-iS2;
Eqn3 = R1*iA + R4*iB + R2*iC;
% Solve the equations for the mesh currents
Soln = solve(Eqn1,Eqn2,Eqn3,iA,iB,iC);
% Examine the solutions
iA1 = Soln.iA
iB1 = Soln.iB
iC1 = Soln.iC
```

The corresponding MATLAB output is shown below.

```
iA1 = -(R2*iS2 + R4*iS1)/(R1 + R2 + R4)
iB1 = (R1*iS1 + R2*iS1 - R2*iS2)/(R1 + R2 + R4)
iC1 = (R1*iS2 - R4*iS1 + R4*iS2)/(R1 + R2 + R4)
```

The results are

$$i_{\mathrm{A}} = -\frac{R_4 i_{\mathrm{S1}} + R_2 i_{\mathrm{S2}}}{R_1 + R_2 + R_4}$$

$$i_{\mathrm{B}} = \frac{(R_1 + R_2) i_{\mathrm{S1}} - R_2 i_{\mathrm{S2}}}{R_1 + R_2 + R_4}$$

$$i_{\mathrm{C}} = \frac{-R_4 i_{\mathrm{S1}} + (R_1 + R_4) i_{\mathrm{S2}}}{R_1 + R_2 + R_4}$$

(b). Use these results to find v_{x}.

From the figure, we have

$$v_{\mathrm{x}} = -R_4 i_{\mathrm{B}}$$

Using the results from part (a), we have

$$v_{\mathrm{x}} = \frac{-R_4[(R_1 + R_2) i_{\mathrm{S1}} - R_2 i_{\mathrm{S2}}]}{R_1 + R_2 + R_4}$$

Problem 3–20. (a). Formulate mesh-current equations for the circuit in Figure P3–20.

The current source defines the mesh in the upper right corner to have a current of i_{S}. Let the left mesh be i_{A} and the lower right mesh be i_{B}. Write the mesh-current equations by inspection.

$$(R_1 + R_2)\, i_{\mathrm{A}} - R_2 i_{\mathrm{B}} = v_{\mathrm{S1}} + R_1 i_{\mathrm{S}}$$

$$-R_2 i_{\mathrm{A}} + (R_2 + R_4)\, i_{\mathrm{B}} = -v_{\mathrm{S2}}$$

(b). Use MATLAB to find symbolic expressions for v_{x} and i_{x} in terms of the parameters in the circuit.

Use MATLAB to solve for the mesh currents and then solve for v_{x} and i_{x}. Note that $i_{\mathrm{x}} = i_{\mathrm{A}}$ and we can use KVL to determine v_{x} as follows:

$$v_{\mathrm{x}} = -R_3 i_{\mathrm{S}} + R_1(i_{\mathrm{A}} - i_{\mathrm{S}}) + v_{\mathrm{S2}}$$

The required MATLAB code is shown below.

```
% Define the symbolic variables
syms vS1 vS2 iS iA iB real
syms R1 R2 R3 R4 positive
% Write the mesh-current equations
Eqn1 = (R1+R2)*iA - R2*iB - vS1 - R1*iS;
Eqn2 = -R2*iA + (R2+R4)*iB + vS2;
```

```
% Solve the equations for the mesh currents
Soln = solve(Eqn1,Eqn2,iA,iB);
% Examine the solution
iA1 = Soln.iA
iB1 = Soln.iB
% Solve for vx and ix
vx = -R3*iS + R1*(iA1-iS) + vS2
ix = iA1
```

The corresponding MATLAB output is shown below.

```
iA1 = (R2*vS1 - R2*vS2 + R4*vS1 + R1*R2*iS + R1*R4*iS)/(R1*R2 + R1*R4 + R2*R4)
iB1 = -(R1*vS2 - R2*vS1 + R2*vS2 - R1*R2*iS)/(R1*R2 + R1*R4 + R2*R4)
vx = vS2 - R3*iS
   - R1*(iS - (R2*vS1 - R2*vS2 + R4*vS1 + R1*R2*iS + R1*R4*iS)/(R1*R2 + R1*R4 + R2*R4))
ix = (R2*vS1 - R2*vS2 + R4*vS1 + R1*R2*iS + R1*R4*iS)/(R1*R2 + R1*R4 + R2*R4)
```

The results are

$$v_{\mathrm{x}} = v_{\mathrm{S2}} - R_3 i_{\mathrm{S}} - R_1 \left[i_{\mathrm{S}} - \frac{R_2 v_{\mathrm{S1}} - R_2 v_{\mathrm{S2}} + R_4 v_{\mathrm{S1}} + R_1 R_2 i_{\mathrm{S}} + R_1 R_4 i_{\mathrm{S}}}{R_1 R_2 + R_1 R_4 + R_2 R_4} \right]$$

$$i_{\mathrm{x}} = \frac{R_2 v_{\mathrm{S1}} - R_2 v_{\mathrm{S2}} + R_4 v_{\mathrm{S1}} + R_1 R_2 i_{\mathrm{S}} + R_1 R_4 i_{\mathrm{S}}}{R_1 R_2 + R_1 R_4 + R_2 R_4}$$

(c). Find numeric values for v_{x} and i_{x} when $R_1 = R_2 = 5.6$ kΩ, $R_3 = 2.2$ kΩ, $R_4 = 1.5$ kΩ, $i_{\mathrm{S}} = 3$ mA, $v_{\mathrm{S1}} = 15$ V, and $v_{\mathrm{S2}} = 5$ V.

Use MATLAB to perform the substitution.

```
% Substitute in numeric values
R1 = 5600;
R2 = 5600;
R3 = 2200;
R4 = 1500;
iS = 3e-3;
vS1 = 15;
vS2 = 5;
vx2 = subs(vx)
ix2 = subs(ix)
```

The corresponding MATLAB output is shown below.

```
vx2 = 4.5977e+000
ix2 = 4.1067e-003
```

The results are

$$v_{\mathrm{x}} = 4.5977\,\mathrm{V}$$

$$i_{\mathrm{x}} = 4.1067\,\mathrm{mA}$$

(d). Find the power supplied by v_{S1}.

The power supplied by voltage source v_{S1} is its voltage times its current.

$$p_{\mathrm{S1}} = v_{\mathrm{S1}} i_{\mathrm{S1}} = v_{\mathrm{S1}} i_{\mathrm{x}} = (15)(4.1067) = 61.6009\,\mathrm{mW}$$

(e). Use OrCAD to verify your solutions to parts (c) and (d) are correct.

The OrCAD simulation in Figure P3–20 confirms the results.

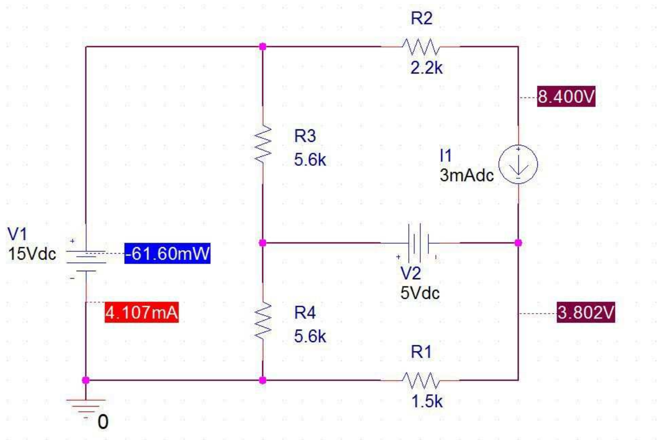


Figure P3–20

Problem 3–21. The circuit in Figure P3–21 seems to require two supermeshes since both current sources appear in two meshes. However, sometimes rearranging the circuit diagram will eliminate the need for a supermesh.

(a). Show that supermeshes can be avoided in Figure P3–21 by rearranging the connection of resistor R_6.

Figure P3–21 shows how to rearrange the circuit to an equivalent circuit that does not require a supermesh to solve for the mesh currents.

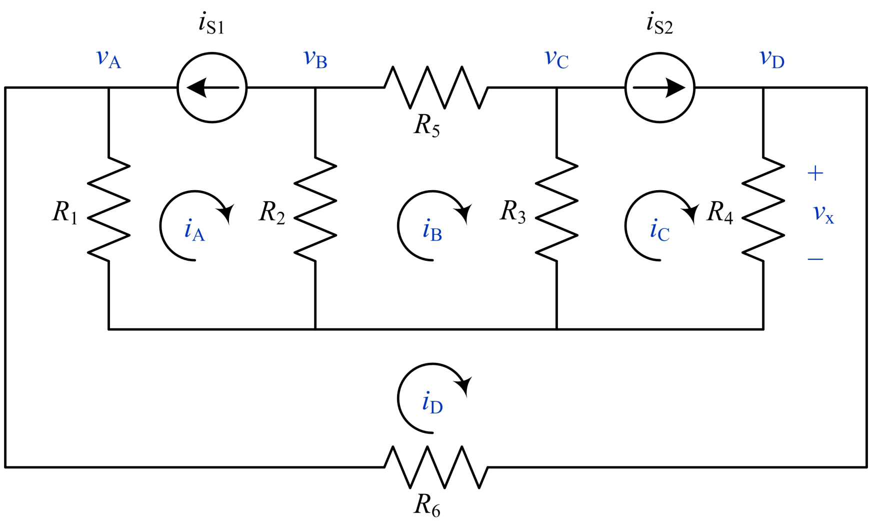


Figure P3–21

(b). Formulate mesh-current equations for the modified circuit as redrawn in part (a).

Write the mesh-current equations by inspection.

$$i_{\mathrm{A}} = -i_{\mathrm{S1}}$$

$$-R_2 i_{\mathrm{A}} + (R_2 + R_3 + R_5)\, i_{\mathrm{B}} - R_3 i_{\mathrm{C}} = 0$$

$$i_{\mathrm{C}} = i_{\mathrm{S2}}$$

$$-R_1 i_{\mathrm{A}} - R_4 i_{\mathrm{C}} + (R_1 + R_4 + R_6)\, i_{\mathrm{D}} = 0$$

(c). Solve for v_{x} when $R_1 = R_2 = R_3 = R_4 = 4\ \mathrm{k\Omega}$, $R_5 = R_6 = 2\ \mathrm{k\Omega}$, $i_{\mathrm{S1}} = 80$ mA, and $i_{\mathrm{S2}} = 40$ mA.

The equation for v_{x} is

$$v_{\mathrm{x}} = R_4\,(i_{\mathrm{C}} - i_{\mathrm{D}})$$

Use MATLAB to compute the mesh currents for the given values of the resistors and current sources and then compute v_{x}.

```
% Define the symbolic variables
syms iA iB iC iD real
% Assign the known values
R1 = 4000;
R2 = 4000;
R3 = 4000;
R4 = 4000;
R5 = 2000;
R6 = 2000;
iS1 = 80e-3;
iS2 = 40e-3;
% Write the mesh-current equations
Eqn1 = iA + iS1;
Eqn2 = -R2*iA + (R2+R3+R5)*iB - R3*iC;
Eqn3 = iC - iS2;
Eqn4 = -R1*iA - R4*iC + (R1+R4+R6)*iD;
% Solve the equations for the mesh currents
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4);
% Examine the solution
iA1 = Soln.iA
iB1 = Soln.iB
iC1 = Soln.iC
iD1 = Soln.iD
% Solve for vx
vx = R4*(iC1-iD1)
```

The corresponding MATLAB output is shown below.

```
iA1 = -2/25
iB1 = -2/125
iC1 =  1/25
iD1 = -2/125
vx = 224
```

The resulting voltage is $v_{\mathrm{x}} = 224$ V.

Problem 3–22. (a). Formulate mesh-current equations for the circuit in Figure P3–22.

Write the mesh-current equations by inspection. Note that the current source defines $i_{\mathrm{C}} = 3$ mA, so only two equations are required to find i_{A} and i_{B}.

$$(2000 + 4000) i_{\mathrm{A}} - 4000 i_{\mathrm{B}} = 20 + (2000)(0.003)$$

$$-4000 i_{\mathrm{A}} + (4000 + 8000) i_{\mathrm{B}} = 15$$

(b). Formulate node-voltage equations for the circuit in Figure P3–22.

Write the node-voltage equations by inspection. Note that the two voltage sources define $v_{\rm A} = 20$ V and $v_{\rm C} = -15$ V, so only one equation is required to find $v_{\rm B}$.

$$\left(\frac{1}{2000} + \frac{1}{4000} + \frac{1}{8000}\right) v_{\rm B} = \frac{20}{2000} - \frac{15}{8000} + 0.003$$

(c). Which set of equations would be easier to solve? Why?

The node-voltage equation will be eaiser to solve, because there is only one equation and one unknown value.

(d). Find $v_{\rm x}$ and $i_{\rm x}$ using whichever method you prefer.

Solve the node-voltage equation.

$$\begin{aligned}
\left(\frac{1}{2000} + \frac{1}{4000} + \frac{1}{8000}\right) v_{\rm B} &= \frac{20}{2000} - \frac{15}{8000} + 0.003 \\
(4+2+1)v_{\rm B} &= (4)(20) - 15 + 24 \\
7v_{\rm B} &= 89 \\
v_{\rm B} &= 12.7143\,\text{V}
\end{aligned}$$

Solve for the requested values.

$$v_{\rm x} = v_{\rm B} = 12.7143\,\text{V}$$

$$i_{\rm x} = -i_{\rm B} = \frac{v_{\rm C} - v_{\rm B}}{8000} = \frac{-15 - 12.7143}{8000} = -3.4643\,\text{mA}$$

Problem 3–23. Use mesh-current, node-voltage or simple engineering intuition to find the input resistance of the circuit in Figure P3–23.

Examining the circuit, the horizontal resistor on the right connects two symmetric paths. The voltage drops along those two paths will be the same, so there will be no voltage difference across the horizontal resistor on the right and no current flowing through it. If there is no current flowing through it, removing it from the circuit does not influence the remaining circuit. We can then combine the remaining resistors to find an equivalent resistance. There are two parallel paths, each with a resistance of $2R$, so their parallel combination is $2R \parallel 2R = R$. That result is in series with the left horizontal resistor, so the total equivalent resistance is $2R$.

Using mesh currents, label the left mesh with $i_{\rm A}$, the upper right mesh with $i_{\rm B}$, and the lower right mesh with $i_{\rm C}$. The following MATLAB code provides a solution.

```
% Define the symbolic variables
syms iA iB iC vS R real
% Write the mesh-current equations
Eqn1 = 3*R*iA - R*iB - R*iC - vS;
Eqn2 = -R*iA + 3*R*iB - R*iC;
Eqn3 = -R*iA - R*iB + 3*R*iC;
% Solve the equations for the mesh currents
Soln = solve(Eqn1,Eqn2,Eqn3,iA,iB,iC);
% Examine the solution
iA1 = Soln.iA
iB1 = Soln.iB
iC1 = Soln.iC
% Solve for the input resistance
RIN1 = vS/iA1
```

The corresponding MATLAB output is shown below.

```
iA1 = vS/(2*R)
iB1 = vS/(4*R)
iC1 = vS/(4*R)
RIN1 = 2*R
```

Using node voltages, label the negative terminal of the voltage source as the reference node. The positive terminal of the voltage source has voltage v_{S}. Label the upper node at v_{A}, the left center node as v_{B}, and the right center node as v_{C}. The following MATLAB code provides a solution.

```
% Define the symbolic variables
syms vA vB vC vS R real
% Write the node-voltage equations
Eqn1 = 3*vA/R - vB/R - vC/R -vS/R;
Eqn2 = -vA/R + 3*vB/R - vC/R;
Eqn3 = -vA/R - vB/R + 3*vC/R;
% Solve the equations for the node voltages
Soln = solve(Eqn1,Eqn2,Eqn3,vA,vB,vC);
% Examine the solution
vA1 = Soln.vA
vB1 = Soln.vB
vC1 = Soln.vC
% Solve for the input resistance
RIN2 = vS/((vS-vA1)/R)
```

The corresponding MATLAB output is shown below.

```
vA1 = vS/2
vB1 = vS/4
vC1 = vS/4
RIN2 = 2*R
```

In all cases, the input resistance is $2R$.

Problem 3–24. In Figure P3–24 all of the resistors are 1 kΩ and $v_{\mathrm{S}} = 10$ V. The voltage at node C is found to be $v_{\mathrm{C}} = -2$ V when node B is connected to ground. Find the node voltages v_{A} and v_{D}, and the mesh currents i_{A} and i_{B}.

We can solve this problem without node-voltage or mesh-current equations. If $v_{\mathrm{B}} = 0$ V and $v_{\mathrm{C}} = -2$ V, then the current from node B to node C can be calculated as follows:

$$i_{\mathrm{B}} = \frac{v_{\mathrm{B}} - v_{\mathrm{C}}}{R_3} = \frac{0 - (-2)}{1000} = 2\,\mathrm{mA}$$

The current i_{B} flows from node C to node D through a 1-kΩ resistor, so the voltage drop is 2 V and we have $v_{\mathrm{D}} = -2 - 2 = -4$ V. The voltage source causes a 10-V voltage increase from node D to node A, so $v_{\mathrm{A}} = -4 + 10 = 6$ V. We can now calculate i_{A} as follows:

$$i_{\mathrm{A}} = \frac{v_{\mathrm{A}} - v_{\mathrm{B}}}{R_1} = \frac{6 - 0}{1000} = 6\,\mathrm{mA}$$

In summary, the results are

$$v_{\mathrm{A}} = 6\,\mathrm{V} \qquad i_{\mathrm{A}} = 6\,\mathrm{mA}$$
$$v_{\mathrm{B}} = 0\,\mathrm{V} \qquad i_{\mathrm{B}} = 2\,\mathrm{mA}$$
$$v_{\mathrm{C}} = -2\,\mathrm{V}$$
$$v_{\mathrm{D}} = -4\,\mathrm{V}$$

Problem 3–25. Use Figure P3–24 and MATLAB to solve the following problems:

(a). Using mesh-current analysis, find a symbolic expression for i_A in terms of the circuit parameters.

By inspection, the mesh-current equations are

$$(R_1 + R_2)\, i_A - R_2 i_B = v_S$$

$$-R_2 i_A + (R_2 + R_3 + R_4)\, i_B = 0$$

The following MATLAB code provides the solution:

```
% Define the symbolic variables
syms iA iB vS R1 R2 R3 R4 real
% Write the mesh-current equations
Eqn1 = (R1+R2)*iA - R2*iB - vS;
Eqn2 = -R2*iA + (R2+R3+R4)*iB;
% Solve the equations for the mesh currents
Soln = solve(Eqn1,Eqn2,iA,iB);
% Examine the solution
iA1 = Soln.iA
```

The corresponding MATLAB output is shown below.

```
iA1 = (vS*(R2 + R3 + R4))/(R1*R2 + R1*R3 + R1*R4 + R2*R3 + R2*R4)
```

The resulting current is

$$i_A = \frac{v_S\,(R_2 + R_3 + R_4)}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_2 R_4}$$

(b). Compute the ratio v_S/i_A.

Using the result from part (a), we have

$$R_{IN} = \frac{v_S}{i_A} = \frac{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_2 R_4}{R_2 + R_3 + R_4}$$

(c). Find a symbolic expression for the equivalent resistance of the circuit by combining resistors in series and parallel. Compare your answer to the results from part (b).

We can calculate the equivalent resistance as follows:

$$R_{EQ} = R_1 + [R_2 \parallel (R_3 + R_4)] = R_1 + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{R_3 + R_4}} = R_1 + \frac{R_2\,(R_3 + R_4)}{R_2 + R_3 + R_4}$$

$$R_{EQ} = \frac{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_2 R_4}{R_2 + R_3 + R_4}$$

The equivalent resistance matches the input resistance calculated in part (b).

Problem 3–26. (a). Formulate mesh-current equations for the circuit in Figure P3–26.

Write the mesh-current equations by inspection.

$$(10000 + 5000 + 10000)\, i_A - 10000 i_B - 5000 i_C = -20$$

$$i_B = -0.01$$

$$-5000 i_A + (5000 + 5000)\, i_C = 20$$

(b). Formulate node-voltage equations for the circuit in Figure P3–26.

To create the node-voltage equations, first perform a source transformation on the 20-V voltage source in series with a 5-kΩ resistor to get a 4-mA current source in parallel with a 5-kΩ resistor connected between v_A and v_B. Write the node-voltage equations by inspection.

$$\left(\frac{1}{10000}+\frac{1}{5000}+\frac{1}{5000}\right)v_A-\left(\frac{1}{5000}+\frac{1}{5000}\right)v_B=0.004$$

$$-\left(\frac{1}{5000}+\frac{1}{5000}\right)v_A+\left(\frac{1}{10000}+\frac{1}{5000}+\frac{1}{5000}\right)v_B=0.01-0.004$$

(c). Which set of equations would be easier to solve? Why?

Both sets of equations require approximately the same amount of effort to solve, since they both have two equations and two unknowns. The mesh-current equations may be slightly easier to solve, since there are fewer fractions, but this difference is minor.

(d). Use OrCAD to find the node voltages v_A and v_B and the mesh-currents i_A, i_B, and i_C in Figure P3–26.

The OrCAD simulation in Figure P3–26 displays the results.

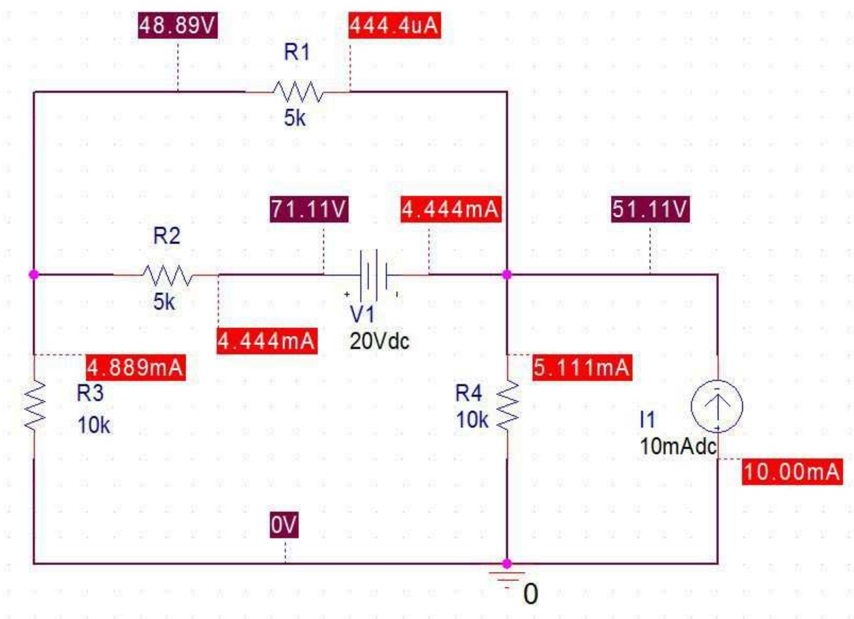


Figure P3–26

The results are

$$v_A = 48.8889\,\text{V} \qquad i_A = -4.8889\,\text{mA}$$

$$v_B = 51.1111\,\text{V} \qquad i_B = -10\,\text{mA}$$

$$i_C = -444.444\,\mu\text{A}$$

Problem 3–27. (a). Formulate mesh-current equations for the circuit in Figure P3–27. Arrange the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

Write the mesh-current equations by inspection.

$$(39000 + 100000)i_{\mathrm{A}} - 100000i_{\mathrm{B}} = -100$$

$$-100000i_{\mathrm{A}} + (100000 + 10000 + 22000)i_{\mathrm{B}} - 22000i_{\mathrm{C}} = 0$$

$$-22000i_{\mathrm{B}} + (68000 + 22000 + 81000)i_{\mathrm{C}} - 68000i_{\mathrm{D}} = 0$$

$$-68000i_{\mathrm{C}} + 68000i_{\mathrm{D}} = 50 + 100$$

Arrange the results in matrix form.

$$\begin{bmatrix} 139000 & -100000 & 0 & 0 \\ -100000 & 132000 & -22000 & 0 \\ 0 & -22000 & 171000 & -68000 \\ 0 & 0 & -68000 & 68000 \end{bmatrix} \begin{bmatrix} i_{\mathrm{A}} \\ i_{\mathrm{B}} \\ i_{\mathrm{C}} \\ i_{\mathrm{D}} \end{bmatrix} = \begin{bmatrix} -100 \\ 0 \\ 0 \\ 150 \end{bmatrix}$$

(b). Use MATLAB and mesh-current analysis to solve for the mesh currents i_{A}, i_{B}, i_{C}, and i_{D} in Figure P3–27.

The following MATLAB code provides the solution:

```
% Write the mesh-current equations in matrix form
A = [139000, -100000, 0, 0;...
    -100000, 132000, -22000, 0;...
    0, -22000, 171000, -68000;...
    0, 0, -68000, 68000];
b = [-100; 0; 0; 150];
% Solve the equations for the mesh currents
x = A\b;
iA = x(1)
iB = x(2)
iC = x(3)
iD = x(4)
```

The corresponding MATLAB output is shown below.

```
iA = -1.2380e-003
iB = -720.8214e-006
iC = 1.3023e-003
iD = 3.5082e-003
```

The results are

$$i_{\mathrm{A}} = -1.2380\,\mathrm{mA}$$

$$i_{\mathrm{B}} = -720.8214\,\mu\mathrm{A}$$

$$i_{\mathrm{C}} = 1.3023\,\mathrm{mA}$$

$$i_{\mathrm{D}} = 3.5082\,\mathrm{mA}$$

(c). Formulate node-voltage equations for the circuit in Figure P3–27. Arrange the results in matrix form $\mathbf{Ax} = \mathbf{b}$.

Label the bottom node as the reference node. The voltage sources then determine that the upper left node has a voltage of 50 V and the middle center node has a voltage of 150 V. Label the upper center node as v_A and the upper right node as v_B. Write the node-voltage equations by inspection.

$$\left(\frac{1}{39000}+\frac{1}{100000}+\frac{1}{10000}\right)v_\mathrm{A}-\frac{1}{10000}v_\mathrm{B}=\frac{50}{39000}+\frac{150}{100000}$$

$$-\frac{1}{10000}v_\mathrm{A}+\left(\frac{1}{10000}+\frac{1}{22000}+\frac{1}{81000}\right)v_\mathrm{B}=\frac{150}{22000}$$

Arrange the results in matrix form.

$$\begin{bmatrix} \left(\frac{1}{39000}+\frac{1}{100000}+\frac{1}{10000}\right) & -\frac{1}{10000} \\ -\frac{1}{10000} & \left(\frac{1}{10000}+\frac{1}{22000}+\frac{1}{81000}\right) \end{bmatrix}\begin{bmatrix} v_\mathrm{A} \\ v_\mathrm{B} \end{bmatrix}=\begin{bmatrix} \frac{50}{39000}+\frac{150}{100000} \\ \frac{150}{22000} \end{bmatrix}$$

(d). Use MATLAB and node-voltage analysis to solve for the mesh currents i_A, i_B, i_C, and i_D in Figure P3–27 and compare the effort required with each technique, i.e., mesh-current versus node-voltage.

The following MATLAB code provides the solution:

```
% Write the node-voltage equations in matrix form
A = [1/39000+1/100000+1/10000, -1/10000;...
    -1/10000, 1/10000+1/22000+1/81000];
b = [50/39000+150/100000; 150/22000];
% Solve the equations for the node voltages
x = A\b;
vA = x(1)
vB = x(2)
% Solve for the mesh currents
iA = (50-vA)/39000
iB = (vA-vB)/10000
iC = vB/81000
iD = 150/68000 + iC
```

The corresponding MATLAB output is shown below.

```
vA =  98.2820e+000
vB = 105.4903e+000
iA = -1.2380e-003
iB = -720.8214e-006
iC = 1.3023e-003
iD = 3.5082e-003
```

The results for the mesh currents match those found in part (b). The approach using node-voltage analysis is slightly simpler, because solving a system of equations with two unknowns is simpler than solving a system of equations with four unknowns. With the node-voltage approach, there are additional calculations required after solving the system of equations, but these are relatively minor compared to solving a system of equations. The MATLAB functions `tic` and `toc` allow us to calculate the time for the two different approaches and the mesh-current approach requires approximately twice the amount of computation time compared to the node-voltage approach.

(e). Use OrCAD to verify your results in the previous two parts.

The OrCAD simulation in Figure P3–27 confirms the results.

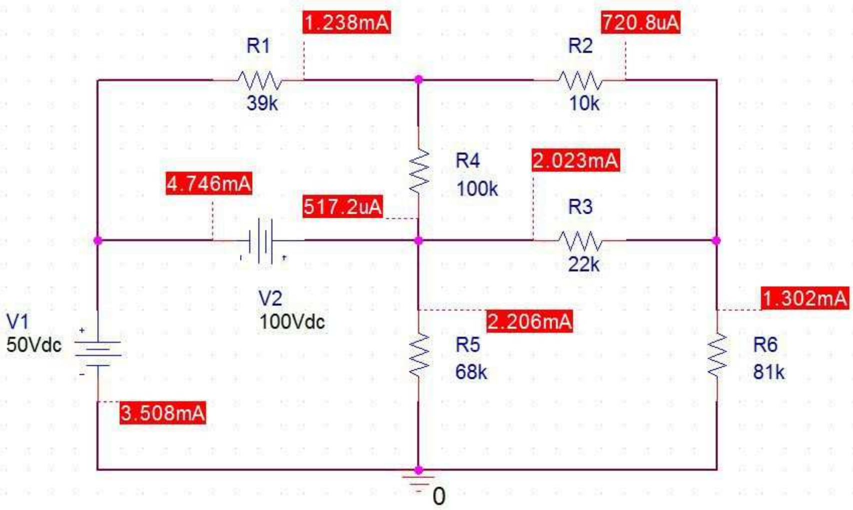


Figure P3–27

Problem 3–28. Find the proportionality constant $K = v_{\mathrm{O}}/v_{\mathrm{S}}$ for the circuit in Figure P3–28.

No current flows through the horizontal 1-kΩ resistor, so it can be ignored in the solution. Use voltage division to find an expression for v_{O} and then compute the gain K.

$$v_{\mathrm{O}} = \frac{1}{2+1}v_{\mathrm{S}} = \frac{v_{\mathrm{S}}}{3}$$

$$K = \frac{v_{\mathrm{O}}}{v_{\mathrm{S}}} = \frac{1}{3}$$

Problem 3–29. Find the proportionality constant $K = i_{\mathrm{O}}/v_{\mathrm{S}}$ for the circuit in Figure P3–29.

Combine the two 1-kΩ resistors in parallel to get an equivalent resistance of 500 Ω. That equivalent resistance is in series with the 2-kΩ resistor, so the total equivalent resistance is 2.5 kΩ. Use the total equivalent resistance to find the current flowing through the voltage source. Perform current division on that current as it splits evenly between the two 1-kΩ paths and use the results to compute the gain K.

$$R_{\mathrm{EQ}} = 2000 + (1000 \parallel 1000) = 2000 + 500 = 2500$$

$$i_{\mathrm{S}} = \frac{v_{\mathrm{S}}}{R_{\mathrm{EQ}}} = \frac{v_{\mathrm{S}}}{2500}$$

$$i_{\mathrm{O}} = \frac{1000}{1000+1000}i_{\mathrm{S}} = \left(\frac{1}{2}\right)\left(\frac{v_{\mathrm{S}}}{2500}\right) = \frac{v_{\mathrm{S}}}{5000}$$

$$K = \frac{i_{\mathrm{O}}}{v_{\mathrm{S}}} = \frac{1}{5000}\,\mathrm{S}$$

Problem 3–30. Find the proportionality constant $K = v_{\mathrm{O}}/i_{\mathrm{S}}$ for the circuit in Figure P3–30.

The 33-kΩ and 22-kΩ resistors are in series, so their equivalent resistance is $R_{\mathrm{EQ}} = 33 + 22 = 55$ kΩ. Use the two-path current division rule to find the current in the output branch. Apply Ohm's law to find

v_O and then solve for the gain K.

$$i_O = \frac{47}{47+55} i_S = \frac{47}{102} i_S$$

$$v_O = i_O R_O = \left(\frac{47}{102} i_S\right)(22000) = \frac{1034000}{102} i_S$$

$$K = \frac{v_O}{i_S} = \frac{1034000}{102} = 10137.3\,\Omega = 10.1373\,\mathrm{k}\Omega$$

Problem 3–31. Find the proportionality constant $K = i_O/i_S$ for the circuit in Figure P3–31.

Resistors R_3 and R_4 are in series. Apply current division to find i_O and then compute the gain K.

$$i_O = \frac{\dfrac{1}{R_3+R_4}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3+R_4}} i_S = \frac{R_1R_2}{R_2(R_3+R_4) + R_1(R_3+R_4) + R_1R_2} i_S$$

$$K = \frac{i_O}{i_S} = \frac{R_1R_2}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4}$$

Problem 3–32. Find the proportionality constant $K = v_O/v_S$ for the circuit in Figure P3–32.

Apply voltage division directly across resistors R_2 and R_4. The polarity of the signs for the output voltage v_O introduce a negative sign in the gain.

$$v_O = -\frac{R_4}{R_2+R_4} v_S$$

$$K = \frac{v_O}{v_S} = -\frac{R_4}{R_2+R_4}$$

Problem 3–33. Use the unit output method to find K and v_O in Figure P3–33.

Let the output voltage be $v_O = 1$ V. Apply Ohm's law to find current flowing through the 22-kΩ resistor.

$$i_O = \frac{1}{22000} = 45.4545\,\mu\mathrm{A}$$

By KCL, the current i_O also flows through the 15-kΩ resistor. Apply Ohm's law to find the voltage drop across the 15-kΩ resistor.

$$v_1 = 15000 i_O = (15000)(45.4545 \times 10^{-6}) = 681.818\,\mathrm{mV}$$

By KVL, the voltage drop across the 47-kΩ resistor is the sum of the voltage drops across the other two resistors.

$$v_2 = v_1 + v_O = 0.681818 + 1 = 1.681818\,\mathrm{V}$$

Apply Ohm's law to find the current through the 47-kΩ resistor.

$$i_2 = \frac{v_2}{47000} = \frac{1.681818}{47000} = 35.7834\,\mu\mathrm{A}$$

Apply KCL to find the total current from the source for a unit output voltage

$$i_S\big|_{\text{for } v_O = 1\,\mathrm{V}} = i_O + i_2 = 45.4545 + 35.7834 = 81.2379\,\mu\mathrm{A}$$

Compute the gain.

$$K = \frac{\text{Output}}{\text{Input}} = \frac{1\,\mathrm{V}}{81.2379\,\mu\mathrm{A}} = 12.3095\,\mathrm{k}\Omega$$

For an input of 10 mA, find the output voltage v_O.

$$v_O = K i_S = (12309.5)(0.01) = 123.095\,\mathrm{V}$$

Problem 3–34. Use the unit output method to find K and v_O in Figure P3–34.

Let the output voltage be $v_O = 1$ V. Apply Ohm's law to find the current through the 2-kΩ resistor.

$$i_O = \frac{1}{2000} = 500\,\mu\text{A}$$

The current i_O flows through the right 1-kΩ resistor. Apply Ohm's law to find its voltage drop.

$$v_1 = 1000 i_O = (1000)(500 \times 10^{-6}) = 500\,\text{mV}$$

Apply KVL to find the voltage across the 3-kΩ resistor as $v_2 = v_1 + v_O = 0.5 + 1 = 1.5$ V. Apply Ohm's law to find the current through the 3-kΩ resistor.

$$i_2 = \frac{v_2}{3000} = \frac{1.5}{3000} = 500\,\mu\text{A}$$

By KCL, the current through the left 1-kΩ resistor is the sum of the two currents in the branches to the right $i_3 = i_2 + i_O = 0.5 + 0.5 = 1\,\text{mA}$. Apply Ohm's law to find the voltage across the left 1-kΩ resistor.

$$v_3 = 1000 i_3 = (1000)(0.001) = 1\,\text{V}$$

Apply KVL to find the voltage across the 2.5-kΩ resistor as $v_4 = v_3 + v_2 = 1 + 1.5 = 2.5$ V. Apply Ohm's law to find the current through the 2.5-kΩ resistor.

$$i_4 = \frac{v_4}{2500} = \frac{2.5}{2500} = 1\,\text{mA}$$

By KCL, the current through the 1.5-kΩ resistor is $i_5 = i_3 + i_4 = 1 + 1 = 2$ mA. Apply Ohm's law to find the voltage across the 1.5-kΩ resistor.

$$v_5 = 1500 i_5 = (1500)(0.002) = 3\,\text{V}$$

Apply KVL to find the voltage across the source.

$$v_S|_{\text{for } v_O = 1\,\text{V}} = v_5 + v_4 = 3 + 2.5 = 5.5\,\text{V}$$

Compute the gain.

$$K = \frac{\text{Output}}{\text{Input}} = \frac{1\,\text{V}}{5.5\,\text{V}} = 0.181818$$

For an input of 10 V, find the output voltage v_O.

$$v_O = K v_S = (0.181818)(10) = 1.81818\,\text{V}$$

Problem 3–35. Use the unit output method to find K and i_O in Figure P3–35.

Let the output current be $i_O = 1$ A. Apply Ohm's law to find the voltage drop across the 20-Ω resistor is 20 V. By KVL, that voltage drop also appears across the 40-Ω resistor, so its current is $i_1 = 20/40 = 0.5$ A. By KCL, the current through the 47-Ω resistor is the sum of the two branch currents to the right $i_2 = i_1 + i_O = 0.5 + 1 = 1.5$ A. By Ohm's law, the voltage across the 47-Ω resistor is $v_2 = (1.5)(47) = 70.5$ V. By KVL, the voltage across the 60-Ω resistor is $v_3 = v_2 + 20 = 90.5$ V. By Ohm's law, the current through the 60-Ω resistor is $i_3 = 90.5/60 = 1.50833$ A. By KCL, the current through the 30-Ω resistor is $i_4 = i_2 + i_3 = 1.5 + 1.50833 = 3.00833$ A. By Ohm's law, the voltage across the 30-Ω resistor is $v_4 = (3.00833)(30) = 90.25$ V. By KVL, the voltage across the source is $v_5 = v_3 + v_4 = 90.5 + 90.25 = 180.75$ V. Compute the gain.

$$K = \frac{\text{Output}}{\text{Input}} = \frac{1\,\text{A}}{180.75\,\text{V}} = 5.5325\,\text{mS}$$

For an input of 100 V, find the output current i_O.

$$i_O = K v_S = (0.0055325)(100) = 553.25\,\text{mA}$$

Problem 3–36. Use the superposition principle to find v_O in Figure P3–36.

Turn on the 12-V source and turn off the 6-V source by replacing it with a short circuit. Combine the two right 200-Ω resistors in parallel to get a 100-Ω resistor. Perform voltage division.

$$v_{O1} = \frac{100}{200+100}(12) = 4\,\mathrm{V}$$

Turn on the 6-V source and turn off the 12-V source by replacing it with a short circuit. Combine the two left 200-Ω resistors in parallel to get a 100-Ω resistor. Perform voltage division.

$$v_{O2} = \frac{100}{100+200}(6) = 2\,\mathrm{V}$$

Sum the component voltages.

$$v_O = v_{O1} + v_{O2} = 4 + 2 = 6\,\mathrm{V}$$

Problem 3–37. Use the superposition principle to find i_O in Figure P3–37. Verify your answer using OrCAD.

Turn on the current source and turn off the voltage source by replacing it with a short circuit. Combine the two right 2-kΩ resistors in parallel to get a 1-kΩ resistor. Perform current division using the two-path current division rule

$$i_{O1} = \frac{1000}{1000+1000}(6) = 3\,\mathrm{mA}$$

Turn on the voltage source and turn off the current source by replacing it with an open circuit. Use node-voltage analysis. Let the top node be the reference node. The lower right node has a voltage of 12 V. The lower left node is the unknown node, v_A. Write the one node-voltage equation and solve for v_A.

$$\left(\frac{1}{1000} + \frac{1}{2000} + \frac{1}{2000}\right) v_A = \frac{12}{2000}$$

$$(2+1+1)\, v_A = 12$$

$$v_A = 3\,\mathrm{V}$$

Use Ohm's law to find the current through the 1-kΩ resistor.

$$i_{O2} = \frac{3}{1000} = 3\,\mathrm{mA}$$

Sum the component currents.

$$i_O = i_{O1} + i_{O2} = 3 + 3 = 6\,\mathrm{mA}$$

The OrCAD simulation in Figure P3–37 displays the results. In the simulation, note how the two center resistors have essentially zero current and 6 mA circulates around the outer loop of the circuit.

Problem 3–38. Use the superposition principle to find v_O in Figure P3–38.

Turn on the voltage source and turn off the current source by replacing it with an open circuit. The 3.3-kΩ and 15-kΩ resistors are in series and that result is in parallel with the 6.8-kΩ resistor. The equivalent resistance is $R_{EQ} = 6.8 \parallel (3.3 + 15) = 6.8 \parallel 18.3 = 4.95777$ kΩ. Perform voltage division to find the voltage across the equivalent resistance.

$$v_1 = \frac{4.95777}{5.6 + 4.95777}(50) = 23.4792\,\mathrm{V}$$

The voltage v_1 appears across the 6.8-kΩ resistor and the series combination of the 3.3-kΩ and 15-kΩ resistors. Apply voltage division a second time to find the voltage across the 3.3-kΩ resistor.

$$v_{O1} = \frac{3.3}{3.3+15}(23.4792) = 4.23396\,\mathrm{V}$$

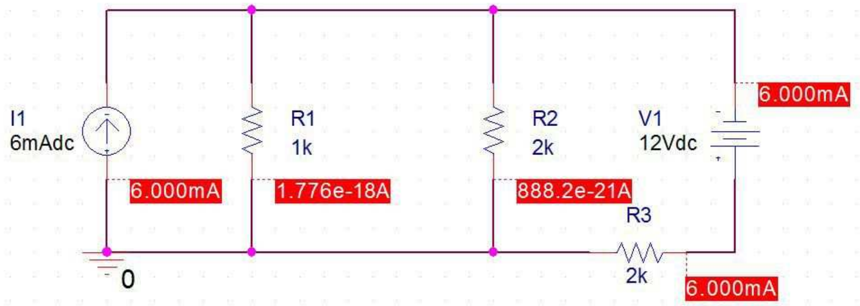


Figure P3–37

Turn on the current source and turn off the voltage source by replacing it with a short circuit. Combine the 5.6-kΩ and 6.8-kΩ resistors in parallel to get $R_{EQ1} = 5.6 \parallel 6.8 = 3.07097$ kΩ. That equivalent resistance is in series with the 3.3-kΩ resistor, which yields $R_{EQ2} = 3.3 + 3.07097 = 6.37097$ kΩ. Perform current division between R_{EQ2} and the 15-kΩ resistor.

$$i_2 = \frac{15}{6.37097 + 15}(10) = 7.01887\,\text{mA}$$

Current i_2 flows through the 3.3-kΩ resistor, so apply Ohm's law to find its voltage drop.

$$v_{O2} = (0.00701887)(3300) = 23.1623\,\text{V}$$

Add the component voltages.

$$v_O = v_{O1} + v_{O2} = 4.23396 + 23.1623 = 27.3962\,\text{V}$$

Problem 3–39. Use the superposition principle to find v_O in Figure P3–39.

Turn on the 10-mA current source and turn off the 20-mA current source by replacing it with an open circuit. The 2-kΩ resistor is then connected in series with the current source and does not influence the results. Perform current division where one path is the 1-kΩ resistor and the other path is the series combination of the 1.5-kΩ resistor and the 2.5-kΩ resistor. Based on the direction of the current, the contribution to the output voltage will be negative.

$$i_{O1} = -\frac{1000}{1000 + 1500 + 2500}(10) = -2\,\text{mA}$$

$$v_{O1} = 2500 i_{O1} = (2500)(-0.002) = -5\,\text{V}$$

Turn on the 20-mA current source and turn off the 10-mA current source by replacing it with an open circuit. Again, the 2-kΩ resistor is in series with the current source, so it does not contribute to the solution. Perform current division where one path is the series combination of the 1-kΩ resistor and the 1.5-kΩ resistor and the other path is the 2.5-kΩ resistor. The contribution to the output voltage will be positive in this case.

$$i_{O2} = \frac{1000 + 1500}{1000 + 1500 + 2500}(20) = 10\,\text{mA}$$

$$v_{O2} = 2500 i_{O2} = 25\,\text{V}$$

Add the component voltages.

$$v_O = v_{O1} + v_{O2} = -5 + 25 = 20\,\text{V}$$

Problem 3–40. Use the superposition principle to find i_O in Figure P3–40. Verify your answer using OrCAD.

Turn on the current source and turn off the two voltage sources by replacing them with short circuits. Let the bottom node be the reference node, the upper left node be v_A and the upper right node be v_B. Apply node-voltage analysis to find the node voltages and then solve for the contribution to the ouput current.

$$\frac{v_A}{5000}+\frac{v_A-v_B}{10000}-0.001=0$$

$$0.001+\frac{v_B-v_A}{10000}+\frac{v_B}{5000}+\frac{v_B}{15000}+\frac{v_B}{10000}=0$$

Solving for the node voltages, we get

$$v_A=2.8205\,\mathrm{V}$$

$$v_B=-1.5385\,\mathrm{V}$$

Find the contribution to the output current.

$$i_{O1}=\frac{v_B}{10000}=-153.85\,\mu\mathrm{A}$$

Turn off the current source by replacing it with an open circuit. Turn on only the 10-V voltage source while leaving the other voltage source off. Use node-voltage analysis and then determine the contribution to the output current. The node-voltage equations are as follows:

$$\frac{v_A-10}{5000}+\frac{v_A-v_B}{10000}=0$$

$$\frac{v_B-v_A}{10000}+\frac{v_B}{5000}+\frac{v_B}{15000}+\frac{v_B}{10000}=0$$

Solving for the node voltages, we get

$$v_A=7.1795\,\mathrm{V}$$

$$v_B=1.5385\,\mathrm{V}$$

Find the contribution to the output current.

$$i_{O2}=\frac{v_B}{10000}=153.85\,\mu\mathrm{A}$$

Turn off the 10-V voltage source and turn on the 20-V voltage source. Leave the current source off. Perform node-voltage analysis, as before.

$$\frac{v_A}{5000}+\frac{v_A-v_B}{10000}=0$$

$$\frac{v_B-v_A}{10000}+\frac{v_B-20}{5000}+\frac{v_B}{15000}+\frac{v_B}{10000}=0$$

Solving for the node voltages, we get

$$v_A=3.0769\,\mathrm{V}$$

$$v_B=9.2308\,\mathrm{V}$$

Find the contribution to the output current.

$$i_{O3}=\frac{v_B}{10000}=923.077\,\mu\mathrm{A}$$

Add the component currents.

$$i_O=i_{O1}+i_{O2}+i_{O3}=-153.85+153.85+923.077=923.077\,\mu\mathrm{A}$$

The OrCAD simulation in Figure P3–40 confirms the results.

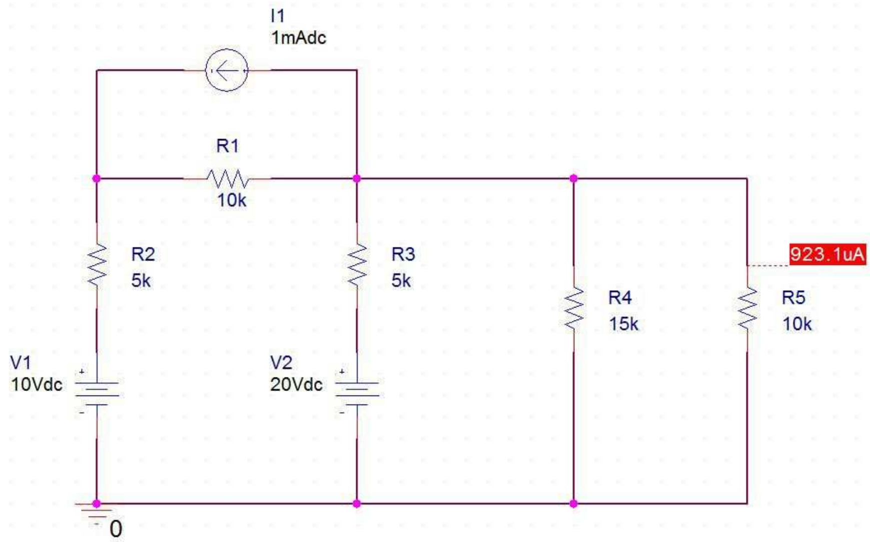


Figure P3–40

Problem 3–41. (a). Use the superposition principle to find v_O in terms of v_1, v_2, and R in Figure P3–41. (This circuit is a 2-bit R-$2R$ network.)

Turn on voltage source v_1 and turn off voltage source v_2 by replacing it with a short circuit. Combine the two R resistors in series and then combine the result in parallel with the center $2R$ resistor to get an equivalent resistance of R. Perform voltage division to find the voltage across the equivalent resistance.

$$v_{11} = \frac{R}{2R+R}v_1 = \frac{v_1}{3}$$

The voltage v_{11} appears across the two R resistors in series and divides equally between them. Compute the contribution to v_O.

$$v_{O1} = \frac{R}{R+R}v_{11} = \frac{v_1}{6}$$

Turn on voltage source v_2 and turn off voltage source v_1 by replacing it with a short circuit. Although the circuit layout is different, the resistors combine in exactly the same way as with voltage source v_1. Combine the two R resistors in series and combine that result in parallel with the left $2R$ resistor. Computer the voltage across the equivalent resistance using voltage division.

$$v_{21} = \frac{R}{2R+R}v_2 = \frac{v_2}{3}$$

The voltage v_{21} appears across the two R resistors in series and divides equally between them. Compute the contribution to v_O.

$$v_{O2} = \frac{R}{R+R}v_{21} = \frac{v_2}{6}$$

Add the component voltages.

$$v_O = v_{O1} + v_{O2} = \frac{v_1}{6} + \frac{v_2}{6} = \frac{v_1+v_2}{6}$$

(b). Use MATLAB and node-voltage analysis to verify your answer symbolically.

Let the node at the negative terminal of the voltage sources be the reference node. Label the top center node as v_{A} and the upper right node as v_{B}. Write the node-voltage equations.

$$\frac{v_{\mathrm{A}}-v_1}{2R}+\frac{v_{\mathrm{A}}-v_2}{2R}+\frac{v_{\mathrm{A}}-v_{\mathrm{B}}}{R}=0$$

$$\frac{v_{\mathrm{B}}-v_{\mathrm{A}}}{R}+\frac{v_{\mathrm{B}}}{R}=0$$

The following MATLAB code provides the solution:

```
% Define the symbolic variables
syms vA vB v1 v2 R real
% Write the node-voltage equations
Eqn1 = (vA-v1)/2/R + (vA-v2)/2/R + (vA-vB)/R;
Eqn2 = (vB-vA)/R + vB/R;
% Solve the equations
Soln = solve(Eqn1,Eqn2,vA,vB);
% Examine the solution
vA1 = Soln.vA;
vB1 = Soln.vB;
% Compute the output voltage
vO = vB1
```

The corresponding MATLAB output is shown below.

```
vO = v1/6 + v2/6
```

The result is the same as that in part (a).

Problem 3–42. (a). Use the superposition principle to find v_{O} in terms of v_{S}, i_{S}, and R in Figure P3–42.

Turn on the voltage source and turn off the current source by replacing it with an open circuit. Perform voltage division to find the contribution to the output voltage.

$$v_{\mathrm{O1}}=\frac{R}{2R+R+R}v_{\mathrm{S}}=\frac{v_{\mathrm{S}}}{4}$$

Turn on the current source and turn off the voltage source by replacing it with a short circuit. Perform two-path current division and then Ohm's law to find the contribution to the output voltage. The sign of the contribution will be negative.

$$i_2=-\frac{2R}{2R+2R}i_{\mathrm{S}}=-\frac{i_{\mathrm{S}}}{2}$$

$$v_{\mathrm{O2}}=Ri_2=-\frac{Ri_{\mathrm{s}}}{2}$$

Add the component voltages.

$$v_{\mathrm{O}}=v_{\mathrm{O1}}+v_{\mathrm{O2}}=\frac{v_{\mathrm{S}}}{4}-\frac{Ri_{\mathrm{S}}}{2}$$

(b). Use MATLAB and node-voltage analysis to verify your answer symbolically.

Let the node at the negative terminal of the voltage source be the reference node. Label the top center node as v_{A} and the upper right node as v_{B}. Write the node-voltage equations.

$$\frac{v_{\mathrm{A}}-v_{\mathrm{S}}}{2R}+i_{\mathrm{S}}+\frac{v_{\mathrm{A}}-v_{\mathrm{B}}}{R}=0$$

$$\frac{v_{\mathrm{B}}-v_{\mathrm{A}}}{R}+\frac{v_{\mathrm{B}}}{R}=0$$

The following MATLAB code provides the solution:

```
% Define the symbolic variables
syms vA vB vS iS R real
% Write the node-voltage equations
Eqn1 = (vA-vS)/2/R + iS + (vA-vB)/R;
Eqn2 = (vB-vA)/R + vB/R;
% Solve the equations
Soln = solve(Eqn1,Eqn2,vA,vB);
% Examine the solution
vA1 = Soln.vA;
vB1 = Soln.vB;
% Compute the output voltage
vO = vB1
```

The corresponding MATLAB output is shown below.

```
vO = vS/4 - (R*iS)/2
```

The result is the same as that in part (a).

Problem 3–43. A linear circuit containing two sources drives a 100-Ω load resistor. Source number 1 delivers 1 W to the load when source number 2 is off. Source number 2 delivers 4 W to the load when source number 1 is off. Find the power delivered to the load when both sources are on. (*Hint*: The answer is not 5 W. Why?)

The power is nonlinear with respect to the sources, so it cannot be added directly. Use $p = v^2/R$ to find the voltage across the load for each source, add the voltages, and then compute the total power.

$$v_{\mathrm{O1}} = \sqrt{p_1 R} = \sqrt{(1)(100)} = 10\,\mathrm{V}$$

$$v_{\mathrm{O2}} = \sqrt{p_2 R} = \sqrt{(4)(100)} = 20\,\mathrm{V}$$

$$v_{\mathrm{O}} = v_{\mathrm{O1}} + v_{\mathrm{O2}} = 10 + 20 = 30\,\mathrm{V}$$

$$p_{\mathrm{L}} = \frac{v_{\mathrm{O}}^2}{R} = \frac{30^2}{100} = 9\,\mathrm{W}$$

Problem 3–44. A linear circuit is driven by an independent voltage source $v_{\mathrm{S}} = 10$ V and an independent current source $i_{\mathrm{S}} = 10$ mA. The output voltage is $v_{\mathrm{O}} = 2$ V when the voltage source is on and the current source off. The output is $v_{\mathrm{O}} = 1$ V when both sources are on. Find the output voltage when $v_{\mathrm{S}} = 20$ V and $i_{\mathrm{S}} = -20$ mA.

Let v_{O1} be the output voltage when the voltage source is on, let v_{O2} be the output voltage when the current source is on, and let v_{O} be the output voltage when both sources are on. We then have the following:

$$v_{\mathrm{O1}} = 2\,\mathrm{V}$$

$$v_{\mathrm{O}} = 1\,\mathrm{V}$$

$$v_{\mathrm{O}} = v_{\mathrm{O1}} + v_{\mathrm{O2}}$$

$$v_{\mathrm{O2}} = 1 - 2 = -1\,\mathrm{V}$$

Compute the gain for each source.

$$K = \frac{\text{Output}}{\text{Input}}$$

$$K_1 = \frac{v_{\text{O}1}}{v_{\text{S}}} = \frac{2}{10} = \frac{1}{5}$$

$$K_2 = \frac{v_{\text{O}2}}{i_{\text{S}}} = \frac{-1}{0.01} = -100\,\Omega$$

Use the gains to compute the output voltage for the new sources.

$$v_{\text{O}} = K_1 v_{\text{S}} + K_2 i_{\text{S}} = \frac{1}{5}(20) + (-100)(-0.02) = 4 + 2 = 6\,\text{V}$$

Problem 3–45. A certain linear circuit has four input voltages and one output voltage v_{O}. The following table lists the output for different values of the four inputs. Find the input-output relationship for the circuit. Specifically, find an expression for v_{O} in terms of the four input voltages.

$v_{\text{S}1}$ (V)	$v_{\text{S}2}$ (V)	$v_{\text{S}3}$ (V)	$v_{\text{S}4}$ (V)	v_{O} (V)
2	4	−4	1	20
1	2	2	1.5	−4
1	4	2	2	−1
0	5	3	−1	3

Since the circuit is linear, their is a gain relating each input to the output. Write four equations relating the inputs to the outputs with the gains. Each equation takes the following form:

$$K_1 v_{\text{S}1} + K_2 v_{\text{S}2} + K_3 v_{\text{S}3} + K_4 v_{\text{S}4} = v_{\text{O}}$$

Substitute in the known values for the source voltages and the output voltage and solve for the four gains. Given the tabular form of the inputs, we can write the equations in matrix form directly.

$$\begin{bmatrix} 2 & 4 & -4 & 1 \\ 1 & 2 & 2 & 1.5 \\ 1 & 4 & 2 & 2 \\ 0 & 5 & 3 & -1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} = \begin{bmatrix} 20 \\ -4 \\ -1 \\ 3 \end{bmatrix}$$

The following MATLAB code provides the solution:

```
% Write the equations in matrix form
A = [2, 4, -4, 1;
    1, 2, 2, 1.5;
    1, 4, 2, 2;
    0, 5, 3, -1];
b = [20; -4; -1; 3];
% Solve the equations for the gains
x = A\b;
K1 = x(1)
K2 = x(2)
K3 = x(3)
K4 = x(4)
```

The corresponding MATLAB output is shown below.

```
K1 =  1.0000e+000
K2 =  2.0000e+000
K3 = -3.0000e+000
K4 = -2.0000e+000
```

The results are

$$K_1 = 1 \qquad K_2 = 2$$

$$K_3 = -3 \qquad K_4 = -2$$

The resulting expression relating the output to the sources is

$$v_O = v_{S1} + 2v_{S2} - 3v_{S3} - 2v_{S4}$$

Problem 3–46. When the current source is turned off in the circuit of Figure P3–46 the voltage source delivers 25 W to the load. How much power does it deliver to the load when both sources are on? Explain your answer.

First, use superposition to analyze this problem. Find the voltage across the load from the voltage source.

$$v_{L1} = \sqrt{p_{L1}R} = \sqrt{(25)(100)} = 50\,\text{V}$$

Turn off the voltage source by replacing it with a short circuit and find the voltage across the load from the current source. Using current division, the current splits equally between the two 100-Ω paths, so 0.5 A flows through the load. The voltage across the load is therefore

$$v_{L2} = i_{L2}R = (0.5)(100) = 50\,\text{V}$$

Add the component voltages.

$$v_L = v_{L1} + v_{L2} = 50 + 50 = 100\,\text{V}$$

Compute the power at the load.

$$p_L = \frac{v_L^2}{R} = \frac{100^2}{100} = 100\,\text{W}$$

From this analysis, it would appear that each source contributes equally to the power dissipated by the load, so we would conclude that the voltage source delivers 50 W to the load when both sources are on. This conclusion is false, because we cannot use superposition to draw conclusion about power contributions. Superposition accurately determines the voltage across the load and the power delivered to it, but it does not allow us to relate those final results directly to the sources. Apply node-voltage analysis to the circuit, with the negative terminal of the voltage source as the reference node. Label the upper right node as v_A. Write the node-voltage equation and solve for v_A.

$$\frac{v_A - 100}{100} - 1 + \frac{v_A}{100} = 0$$

$$v_A - 100 - 100 + v_A = 0$$

$$2v_A = 200$$

$$v_A = 100$$

The voltage on each side of the horizontal 100-Ω resistor is 100 V. Therefore no current flows through that resistor or through the voltage source. When both sources are on, the voltage source provides no power to the circuit and the current source provides the entire 100 W to the load resistor.

Problem 3–47. For the circuit in Figure P3–47 find the Thévenin and Norton equivalent circuits.

Apply voltage division to find the open-circuit voltage. With an open circuit, no current flows through the 5-kΩ resistor, and the open-circuit voltage is the voltage across the 15-kΩ resistor.

$$v_{OC} = v_T = \frac{15}{10+15}(25) = 15\,\text{V}$$

Apply the lookback technique to find the Thévenin resistance. Replace the voltage source with a short circuit. Combine the 10-kΩ and 15-kΩ resistors in parallel and then combine that result in series with the 5-kΩ resistor.

$$R_T = (10 \parallel 15) + 5 = 6 + 5 = 11\,\text{k}\Omega$$

Perform a source transformation to find the Norton current.

$$i_{\rm N} = \frac{v_{\rm T}}{R_{\rm T}} = \frac{15}{11000} = 1.3636\,\text{mA}$$

In summary, the equivalent circuits have the following values:

$$v_{\rm T} = 15\,\text{V} \qquad i_{\rm N} = 1.3636\,\text{mA}$$

$$R_{\rm T} = 11\,\text{k}\Omega \qquad R_{\rm N} = 11\,\text{k}\Omega$$

Problem 3–48. For the circuit in Figure P3–48 find the Thévenin and Norton equivalent circuits.

Apply circuit reduction and a source transformation to find the equivalent circuits. The two 100-Ω resistors are in parallel, so when combined they yield the Norton resistance $R_{\rm N} = 100 \parallel 100 = 50$ Ω. The resulting circuit is a 1-A current source in parallel with a 50-Ω resistor, which is the Norton equivalent circuit. Perform a source transformation to find the Thévenin voltage is $v_{\rm T} = i_{\rm N}R_{\rm N} = (1)(50) = 50$ V. The Thévenin resistance is also 50 Ω. In summary, the equivalent circuits have the following values:

$$v_{\rm T} = 50\,\text{V} \qquad i_{\rm N} = 1\,\text{A}$$

$$R_{\rm T} = 50\,\Omega \qquad R_{\rm N} = 50\,\Omega$$

Problem 3–49. (a). Find the Thévenin or Norton equivalent circuit seen by $R_{\rm L}$ in Figure P3–49.

Remove the load resistor from the circuit. To find the Thévenin or open-circuit voltage, ignore the 5.6 kΩ resistor, since no current flows through it. Apply voltage division to find the voltage across the 15-kΩ resistor.

$$v_{\rm T} = \frac{15}{10+15}(50) = 30\,\text{V}$$

Apply the lookback technique to find the Thévenin resistance. Replace the voltage source with a short circuit. Combine the 10-kΩ and 15-kΩ resistors in parallel and then combine that result in series with the 5.6-kΩ resistor.

$$R_{\rm T} = (10 \parallel 15) + 5.6 = 6 + 5.6 = 11.6\,\text{k}\Omega$$

Perform a source transformation to find the Norton current.

$$i_{\rm N} = \frac{v_{\rm T}}{R_{\rm T}} = \frac{30}{11600} = 2.5862\,\text{mA}$$

The Norton resistance is the same as the Thévenin resistance $R_{\rm N} = R_{\rm T} = 11.6$ kΩ.

(b). Use the equivalent circuit found in part (a) to find $i_{\rm L}$ if $R_{\rm L} = 22$ kΩ.

Use the Thévenin equivalent circuit. The circuit contains a 30-V source connected in series with 11.6-kΩ and 22-kΩ resistors. The single current flows through the load.

$$i_{\rm L} = \frac{30}{11600 + 22000} = 892.857\,\mu\text{A}$$

Problem 3–50. (a). Find the Thévenin or Norton equivalent circuit seen by $R_{\rm L}$ in Figure P3–50.

Remove the load resistor $R_{\rm L}$ from the circuit. Perform a source transformation with the current source $i_{\rm S}$ and R_1 to get a voltage source in series with R_1. The value of the transformed voltage source will be the Thévenin voltage.

$$v_{\rm T} = i_{\rm S}R_1$$

In the transformed circuit, combine the two resistors in series to get the Thévenin resistance.

$$R_{\rm T} = R_1 + R_2$$

Perform another source transformation to find the Norton equivalent circuit.

$$i_{\rm N} = \frac{v_{\rm T}}{R_{\rm T}} = \frac{i_{\rm S}R_1}{R_1 + R_2}$$

The Norton resistance matches the Thévenin resistance $R_{\rm N} = R_{\rm T} = R_1 + R_2$.

(b). Use the equivalent circuit found in part (a) to find i_{L} in terms of i_{S}, R_1, R_2, and R_{L}.

Using the Thévenin equivalent circuit, v_{T} is applied across the series combination of R_{T} and R_{L}. The resulting current i_{L} flows through the load resistor.

$$i_{\text{L}} = \frac{v_{\text{T}}}{R_{\text{T}} + R_{\text{L}}} = \frac{i_{\text{S}} R_1}{R_1 + R_2 + R_L}$$

(c). Check your answer to part (b) using current division.

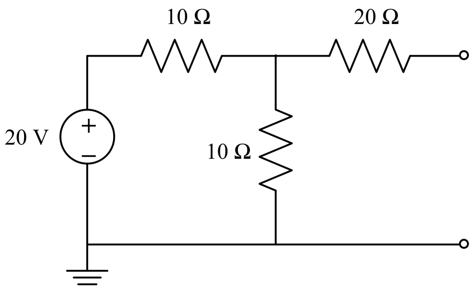
Use the two-path current division rule to check the results. Note that resistors R_2 and R_{L} are in series with an equivalent resistance of $R_{\text{EQ}} = R_2 + R_{\text{L}}$ as the second resistor.

$$i_{\text{L}} = \frac{R_1}{R_1 + R_{\text{EQ}}} i_{\text{S}} = \frac{R_1}{R_1 + R_2 + R_{\text{L}}} i_{\text{S}} = \frac{i_{\text{S}} R_1}{R_1 + R_2 + R_L}$$

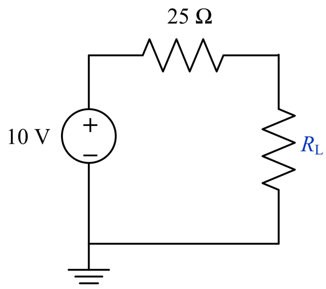
The result is the same as that in part (b).

Problem 3–51. Find the Thévenin equivalent circuit seen by R_{L} in Figure P3–51. Find the voltage across the load when $R_{\text{L}} = 5\ \Omega$, $10\ \Omega$, and $20\ \Omega$.

The two upper left resistors are in parallel and combine to yield a 10-Ω resistor. The two inner resistors are also in parallel and combine to yield another 10-Ω resistor. After these reductions, the resulting circuit is as shown below.



Apply voltage division to show the Thévenin voltage is $v_{\text{T}} = \dfrac{10}{10+10}(20) = 10$ V. The lookback resistance is $R_{\text{T}} = 20 + (10 \parallel 10) = 25\ \Omega$. The resulting Thévenin circuit with the load attached is shown below.



Apply voltage division to find the load voltages:

$$v_{\text{L}} = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{T}}} v_{\text{T}} = \frac{R_{\text{L}}}{R_{\text{L}} + 25}(10)$$

For $R_{\text{L}} = 5,\ 10,\ 20\ \Omega$, the load voltages are $v_{\text{L}} = 1.6667,\ 2.8571,\ 4.4444$ V.

Problem 3–52. Find the Norton equivalent seen by R_{L} in Figure P3–52. Find the current through the load when $R_{\text{L}} = 4.7\ \text{k}\Omega$, $15\ \text{k}\Omega$, and $68\ \text{k}\Omega$.

Combine the two 15-kΩ resistors in parallel to yield a 7.5-kΩ resistor in parallel with the source. Perform a source transformation to yield a 75-V source in series with a 7.5-kΩ resistor. The 7.5-kΩ resistor is now in series with the 8.1-kΩ resistor, so combine them to get a 15.6-kΩ resistor in series with the source. Perform a source transformation to get a 4.808-mA current source in parallel with a 15.6-kΩ resistor as the Norton equivalent circuit.

Apply two-path current division to find the current through the load.

$$i_{\rm L} = \frac{R_{\rm N}}{R_{\rm N}+R_{\rm L}} i_{\rm N} = \frac{15600}{15600+R_{\rm L}}(0.004808)$$

For $R_{\rm L} = 4.7,\ 15,\ 68$ kΩ, the load currents are $i_{\rm L} = 3.69,\ 2.45,\ 0.897$ mA.

Problem 3–53. You need to determine the Thévenin equivalent circuit of a more complex linear circuit. A technician tells you she made two measurements using her DMM. The first was with a 10-kΩ load and the load current was 91 μA. The second was with a 1-kΩ load and the load voltage was 124 mV. Calculate the Thévenin equivalent circuit as shown in Figure P3–53.

Write two equations based on the problem statement using the Thévenin voltage and resistance as the unknowns.

$$91\ \mu{\rm A} = \frac{v_{\rm T}}{R_{\rm T}+10000}$$

$$124\ {\rm mV} = \frac{1000}{1000+R_{\rm T}} v_{\rm T}$$

Solve the two equations for the two unknowns to get

$$v_{\rm T} = 3.0775\ {\rm V}$$

$$R_{\rm T} = 23.8182\ {\rm k}\Omega$$

The following MATLAB code provides the solution:

```
% Define the symbolic variables
syms vT RT real
% Define the known numerical values
RL1 = 10e3;
iL1 = 91.0e-6;
RL2 = 1e3;
vL2 = 124e-3;
% Create the two equations
Eqn1 = vT/(RT+RL1)-iL1;
Eqn2 = RL2*vT/(RT+RL2)-vL2;
% Solve the equations
Soln = solve(Eqn1,Eqn2);
% Examine the solutions
vT1 = double(Soln.vT)
RT1 = double(Soln.RT)
```

Problem 3–54. Find the Thévenin equivalent seen by $R_{\rm L}$ in Figure P3–54. Find the power delivered to the load when $R_{\rm L} = 50$ kΩ and 200 kΩ.

Apply voltage division to calculate the Thévenin voltage

$$v_{\rm T} = \frac{33}{33+47}(24) = 9.9\ {\rm V}$$

Find the lookback resistance as $R_{\rm T} = 91 + (47 \parallel 33) = 91 + 19.388 = 110.388$ kΩ. Use Ohm's law to find the current and then compute the power.

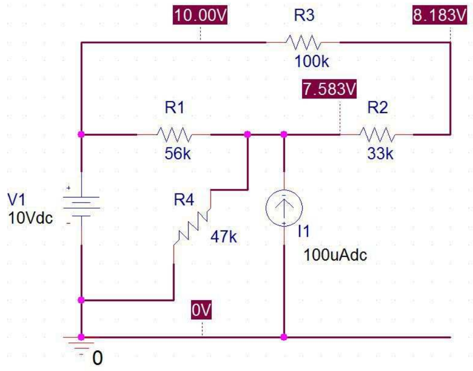
$$i_{\rm L} = \frac{v_{\rm T}}{R_{\rm T}+R_{\rm L}}$$

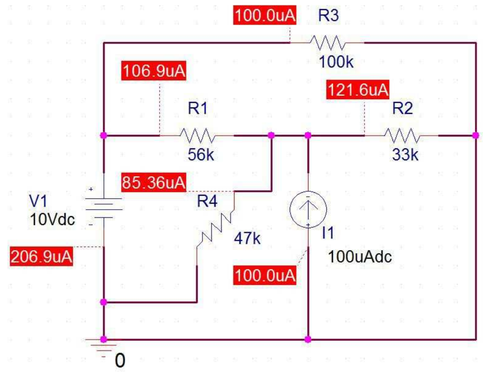
$$p_{\rm L} = i_{\rm L}^2 R_{\rm L}^2$$

For $R_{\rm L} = 50,\ 200$ kΩ, the powers delivered to the load are $p_{\rm L} = 190.5,\ 203.5\ \mu$W.

Problem 3–55. (a). Use OrCAD to find the Norton equivalent at terminals A and B in Figure P3–55. (*Hint:* Find the open-circuit voltage and short-circuit current at the requisite terminals.)

The circuits for the OrCAD simulations to find the open-circuit voltage and short-circuit current are shown below.





The open-circuit voltage is 8.183 V and the short-circuit current is 221.6 μA. The Norton resistance is therefore 36.9269 kΩ.

(b). Use the Norton equivalent circuit found in part (a) to determine the power dissipated in $R_{\rm L}$ when it is equal to 37 kΩ.

Use two-path current division to find the current through the load

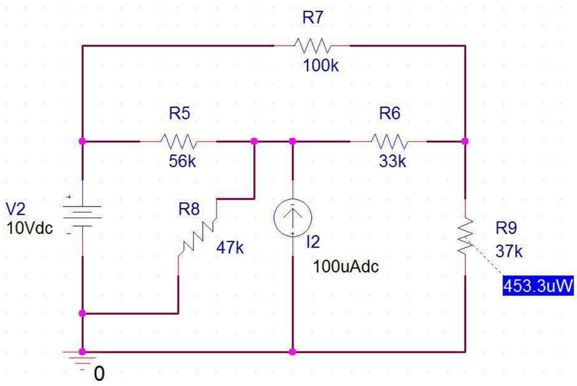
$$i_{\rm L} = \frac{R_{\rm N}}{R_{\rm N}+R_{\rm L}} i_{\rm N} = \frac{36.9269}{36.9269+37}(221.6\ \mu\text{A}) = 110.69\ \mu\text{A}$$

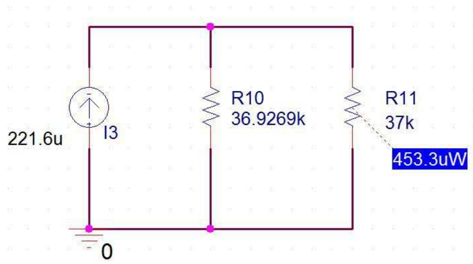
Compute the power dissipated by the load resistor

$$p_{\rm L} = i_{\rm L}^2 R_{\rm L} = (110.69\ \mu\text{A})^2(37\ \text{k}\Omega) = 453.3\ \mu\text{W}$$

(c). Use OrCAD to simulate both the original and the Norton equivalent circuits with $R_{\rm L} = 37$ kΩ. Verify that the power dissipated by the load is the same in both situations.

The following two simulations verify the results.





Problem 3–56. The purpose of this problem is to use Thévenin equivalent circuits to find the voltage $v_{\rm x}$ in Figure P3–56. Find the Thévenin equivalent circuit seen looking to the left of terminals A and B. Find the Thévenin equivalent circuit seen looking to the right of terminals A and B. Connect these equivalent circuits together and find the voltage $v_{\rm x}$.

Looking to the left of terminals A and B, apply voltage division to compute the Thévenin voltage as $v_T = (15)(6)/(6+3) = 10$ V and use the lookback technique to find the Thévenin resistance $R_T = 3 \parallel 6 = 2$ kΩ. Looking to the right of terminals A and B, there is no voltage source, so the Thévenin voltages is zero and the lookback approach yields $R_T = 250 + (1000 \parallel 3000) = 1$ kΩ. Connect the two circuits and apply voltage division to compute v_x

$$v_x = \frac{1}{1+2}(10) = \frac{10}{3} \text{ V}$$

Problem 3–57. The circuit in Figure P3–57 was solved earlier using supermeshes (Problem 3–19.) In this problem solve for the voltage across the load resistor v_L by first finding the Thévenin equivalent circuit seen by the load resistor. Find v_L when $R_L = 2.5$ kΩ.

Replace the load resistor with a short circuit and use superposition to find the short-circuit current. With the 10-mA source active and the 20-mA source replaced by an open circuit, apply two-path current division to get $i_{SC1} = -(1)(0.01)/(1+1.5) = -4$ mA. With the 20-mA source active and the 10-mA source replaced by an open circuit, all 20 mA flows through the short circuit, so $i_{SC2} = 20$ mA. The short-circuit current is $i_{SC} = i_{SC1} + i_{SC2} = 16$ mA. Find the lookback resistance by replacing both current sources with open circuits. The 2-kΩ resistor is not part of a closed path and the other two resistors are in series, so $R_T = 1 + 1.5 = 2.5$ kΩ. Compute the Thévenin voltage $v_T = i_{SC}R_T = (0.016)(2500) = 40$ V. Use voltage division to find v_L

$$v_L = \frac{R_L}{R_T + R_L}(v_T) = \frac{2.5}{2.5+2.5}(40) = 20 \text{ V}$$

Problem 3–58. Assume that Figure P3–58 represents a model of the auxiliary output port of a car. The output current is $i = 1$ A when $v = 0$ V. The output voltage is $v = 12$ V when a 20-Ω resistor is connected between the terminals. Suppose you wanted to charge a 12-V battery by connecting the battery at the port. How much current would the port deliver to the battery?

The short circuit current is 1 A, as stated in the problem. When a 20-Ω resistor is connected to the circuit, the equivalent resistance must be 12 Ω to create a 12-V drop. Therefore, we have $20 \parallel R_N = 12$ Ω. Solve for R_N.

$$12 = 20 \parallel R_N = \frac{20R_N}{20 + R_N}$$

$$240 + 12R_N = 20R_N$$

$$8R_N = 240$$

$$R_N = 30 \ \Omega$$

The Norton equivalent circuit is a 1-A source in parallel with a 30-Ω resistor. The Thévenin equivalent circuit is a 30-V source in series with a 30-Ω resistor. Connect the Thévenin equivalent circuit to the 12-V battery to be charged. The current flowing into the battery is $i_L = (30 \text{ V} - 12 \text{ V})/(30\ \Omega) = (18 \text{ V})/(30\ \Omega) = 600$ mA.

Problem 3–59. The i-v characteristic of the active circuit in Figure P3–58 is $5v + 500i = 100$. Find the output voltage when a 500-Ω resistive load is connected.

The i-v characteristic is equivalent to $v + 100i = 20$. When the two ports are short circuited, the output voltage is zero and the current is $i_{SC} = 20/100 = 200$ mA. When the two ports are open circuited, the current is zero and the voltage is $v_{OC} = 20$ V. The Thévenin resistance is $R_T = 20/0.2 = 100$ Ω. Use the Thévenin equivalent circuit and voltage division to find the output voltage.

$$v_O = \frac{R_L}{R_T + R_L}(v_T) = \frac{500}{100+500}(20) = 16.667 \text{ V}$$

Problem 3–60. You have successfully completed Circuits I, and as part of an undergraduate work-study program your former professor has asked you to help her grade a Circuits I quiz. On the quiz, students were asked to find the power supplied by the source both to the 10-kΩ load (R_L) and to the entire circuit as shown in Figure P3–60. Your professor asks you to help her by creating a grading sheet.

(a). Solve the quiz and establish reasonable A, B, C, D, and F cuts for incorrect solutions.

Use node-voltage analysis to solve for the node voltages. Let the center node be node A and the upper right node be node B. Ground the bottom node. The node-voltage equations are

$$\frac{v_{\rm A}-15}{10000}+\frac{v_{\rm A}}{10000}+\frac{v_{\rm A}-v_{\rm B}}{10000}=0$$

$$\frac{v_{\rm B}-15}{10000}+\frac{v_{\rm B}-v_{\rm A}}{10000}+\frac{v_{\rm B}}{10000}=0$$

Solve for the node voltages as $v_{\rm A}=v_{\rm B}=7.5$ V. Compute the current through the load $i_{\rm L}=(15-7.5)/10000=750\ \mu$A and the power supplied to the load $p_{\rm L}=(750\ \mu{\rm A})^2(10000)=5.625$ mW. Compute the source current $i_{\rm S}=i_{\rm L}+(15-7.5)/10000=1.5$ mA and the power supplied by the source $p_{\rm S}=(0.0015)(15)=22.5$ mW.

(b). A particular student correctly finds the Thévenin equivalent circuit seen by the resistive load and calculates the power to the load using $v_{\rm T}^2/R_{\rm L}$. He then does a source transformation, correctly finding the Norton equivalent of the circuit. He calculates the source power using $v_{\rm T}\times i_{\rm N}$. What grade would you give him?

The student deserves credit for finding the correct Thévenin and Norton equivalent circuits, but the power calculations are incorrect for both the load and the total power dissipated in the circuit. The grade for this solution should be low, such as a D.

(c). Another student finds $p_{\rm L}=5.625$ mW and $p_{\rm S}=22.5$ mW, but provides no work to justify her answers. What grade would you give her?

Give the student full credit and a grade of A. The solution is correct. If the solution had been wrong, then the grade would have been zero or F for not showing any work.

(d). A third student first finds the Norton equivalent, and then finds the current through the load using a current divider and calculates the power in the load using $i_{\rm L}^2R_{\rm L}$. He figures correctly what the parallel voltage would be across the Norton circuit and the load $v_{\rm L}$, and then calculates $p_{\rm S}=i_{\rm N}\times v_{\rm L}$. What grade would you give him?

The load power is calculated correctly, but the total power calculation is not valid. Give the student a B grade.

Problem 3–61. The Thévenin equivalent parameters of a practical voltage source are $v_{\rm T}=30$ V and $R_{\rm T}=300\ \Omega$. Find the smallest load resistance for which the load voltage exceeds 10 V.

Apply voltage division to calculate the solution.

$$v_{\rm L}=\frac{R_{\rm L}}{R_{\rm T}+R_{\rm L}}v_{\rm T}=10$$

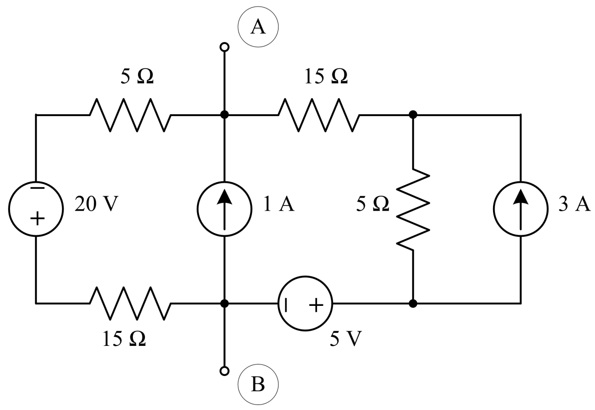
$$10R_{\rm T}+10R_{\rm L}=30R_{\rm L}$$

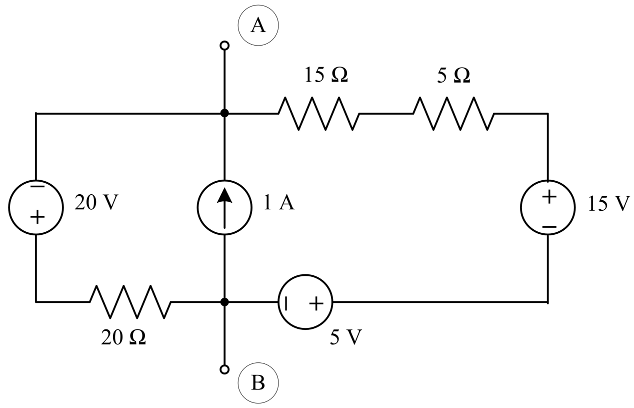
$$20R_{\rm L}=10(300)=3000$$

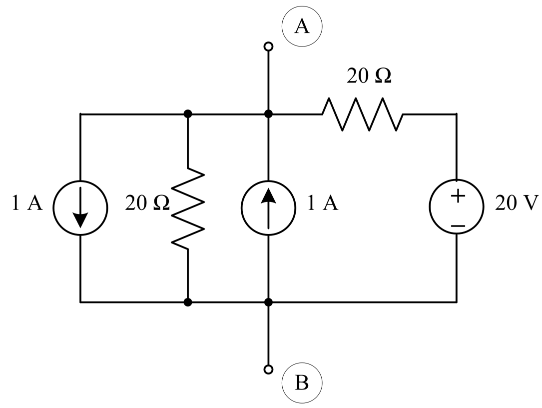
$$R_{\rm L}>150\ \Omega$$

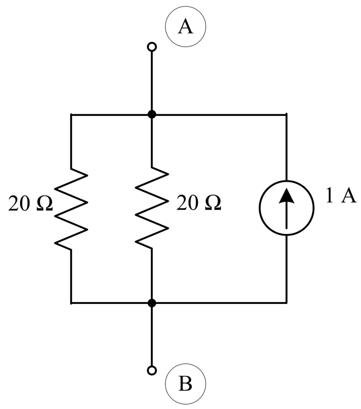
Problem 3–62. Use a sequence of source transformations to find the Thévenin equivalent at terminals A and B in Figure P3–62. Then select a resistor to connect across A and B so that 5 V appears across it.

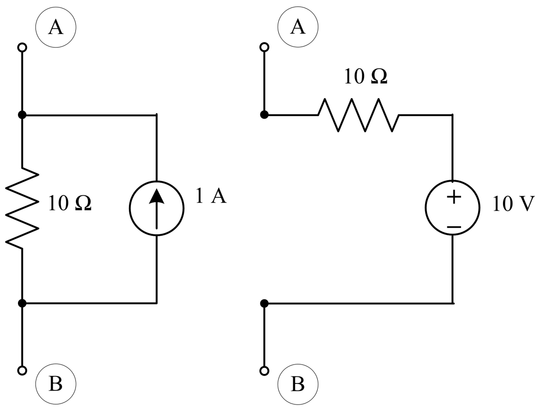
The following sequence of circuit reductions presents the solution.


A
5 Ω
15 Ω
20 V
1 A
5 Ω
3 A
15 Ω
5 V
B


A
15 Ω
5 Ω
20 V
1 A
15 V
20 Ω
5 V
B


A
20 Ω
1 A
20 Ω
1 A
20 V
B


A
20 Ω
20 Ω
1 A
B



To get a 5-V drop across a load resistor, the Thévenin voltage will have to divide equally between the load resistor and the Thévenin resistance, so $R_{\rm L} = R_{\rm T} = 10\ \Omega$.

Problem 3–63. The circuit in Figure P3–63 provides power to a number of loads connected in parallel. The circuit is protected by a 3/4-mA fuse with a nominal 100-Ω resistance. Each load is 10 kΩ. What is the maximum number of loads the circuit can drive without blowing the fuse?

Find the Thévenin equivalent circuit to the left of the load resistors. The open-circuit voltage is $v_{\rm OC} = v_{\rm T} = (8)(30)/(30+10) = 6$ V. The lookback resistance is $R_{\rm T} = 2.5 + 0.1 + (10 \parallel 30) = 2.6 + 7.5 = 10.1$ kΩ. The maximum current from this source is $i_{\rm MAX} = v_{\rm T}/R_{\rm T} = 6/10.1 = 594.06\ \mu$A. The current will never exceed the fuse rating, so any number of load resistors can be connected in parallel with the source circuit.

Problem 3–64. Find the Thévenin equivalent at terminals A and B in Figure P3–64.

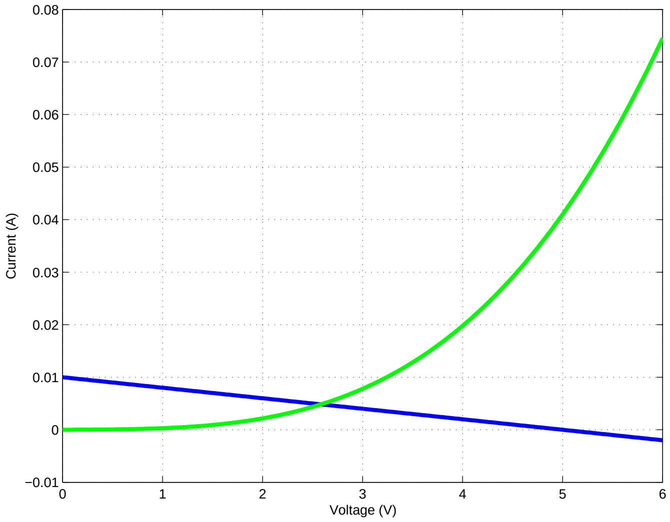
Perform a source transformation on the current source and the 500-Ω resistor to get a 5-V voltage source with the positive terminal at the bottom in series with a 500-Ω resistor. Combine the 500-Ω resistor in series with the 2-kΩ resistor to get a 2.5-kΩ resistor. Combine the two voltage sources in series to get a 10-V source with the positive terminal at the top. Perform a source transformation to get a 4-mA current source in parallel with a 2.5-kΩ resistor. Combine the 2.5-kΩ and 1-kΩ resistors in parallel to get a 714-Ω resistor. Perform another source transformation to get $v_{\rm T} = 2.857$ V and $R_{\rm T} = 714\ \Omega$.

Problem 3–65. A nonlinear resistor is connected across a two-terminal source whose Thévenin equivalent is $v_{\rm T} = 5$ V and $R_{\rm T} = 500\ \Omega$. The i-v characteristic of the resistor is $i = 10^{-4}(v + 2v^{3.3})$. Use the MATLAB function `solve` to find the operating point for this circuit and determine the voltage across, the current through, and the power dissipated in the nonlinear resistor.

The i-v relationship for the source is $v = v_{\rm T} - iR_{\rm T} = 5 - 500i$, which can be rewritten as $i = 0.01 - 0.002v$. Use MATLAB to plot the i-v characteristics for the source and the load and identify the intersection of the two curves. The following MATLAB code plots the curves:

```
% Create a range of voltages to plot
v = 0:0.01:6;
% Plot the i-v characteristic of the source
i_source = 10e-3-2e-3*v;
% Plot the i-v characteristic of the load
i_load = 1e-4*(v + 2*v.^3.3);
figure
plot(v,i_source,'b','LineWidth',3)
hold on
grid on
plot(v,i_load,'g','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
```

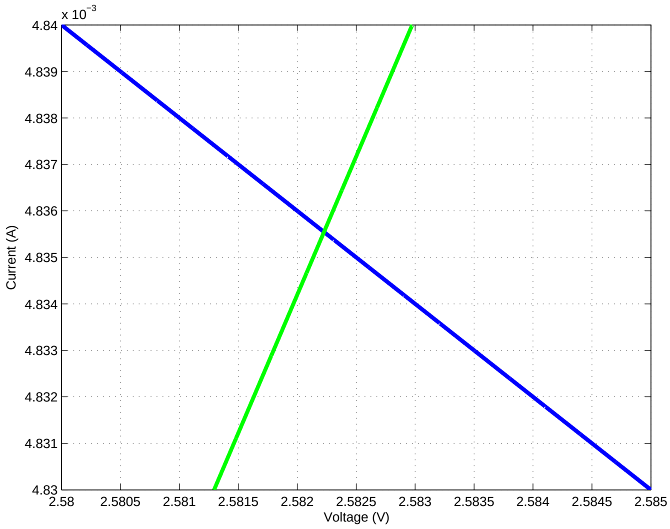
The corresponding plot is shown below:



To find a better solution for the operating point, zoom in on the intersection of the two curves.

```
% Plot the data again
% Zoom in on the intersection of the two curves
% Create a range of voltages to plot
v = 2.58:0.000001:2.585;
% Plot the i-v characteristic of the source
i_source = 10e-3-2e-3*v;
% Plot the i-v characteristic of the load
i_load = 1e-4*(v + 2*v.^3.3);
figure
plot(v,i_source,'b','LineWidth',3)
hold on
grid on
plot(v,i_load,'g','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
axis([2.58 2.585 0.00483 0.00484])
```

The corresponding plot is shown below:



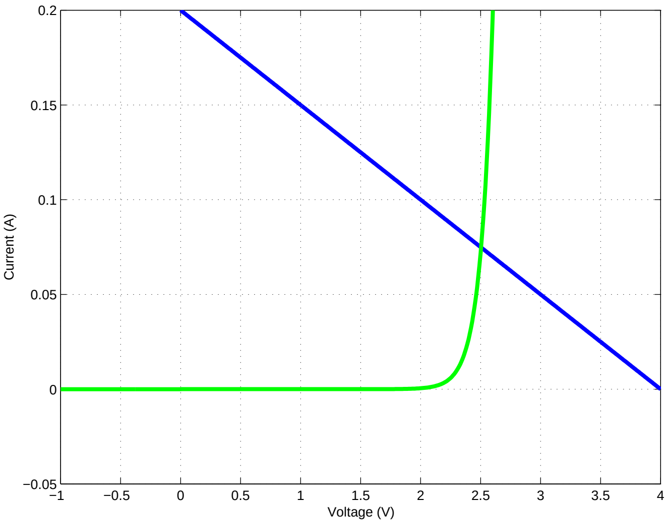
The operating point is $v_{\rm L} = 2.582$ V, $i_{\rm L} = 4.836$ mA, and $p_{\rm L} = iv = 12.486$ mW.

Problem 3–66. A blue light-emitting diode (LED) is connected across a two-terminal source whose Thévenin equivalent is $v_T = 4$ V and $R_T = 20\ \Omega$. The i-v characteristic of the LED is $i = 10^{-12}(e^{10v} - 1)$. Figure P3–66 shows the LED's i-v characteristic. Using either MATLAB or a graphical approach, determine the voltage across and current through the LED.

Use MATLAB to plot the two i-v characteristics and find the intersection of the two curves. The following MATLAB code plots the curves:

```
% Create a range of voltages to plot
v = -1:0.01:4;
% Plot the i-v characteristic of the source
vT = 4;
RT = 20;
i_source = (vT-v)/RT;
% Plot the i-v characteristic of the LED
i_LED = 1e-12*(exp(10*v)-1);
plot(v,i_source,'b','LineWidth',3)
hold on
grid on
plot(v,i_LED,'g','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
axis([-1 4 -0.05 0.2])
```

The corresponding plot is shown below:



Zoom in on the intersection of the two curves to graphically determine the operating point as $v_{\text{LED}} = 2.5038$ V and $i_{\text{LED}} = 74.8$ mA.

Problem 3–67. Find the Norton equivalent seen by R_L in Figure P3–67.

Perform a source transformation on the current source in parallel with the left 20-Ω resistor to get a 10-V voltage source in series with a 20-Ω resistor. Combine the two 20-Ω resistors in series to get a 40-Ω resistor. Perform another source transformation to get a 250-mA current source in parallel with a 40-Ω resistor. Combine the two 40-Ω resistors that are in parallel to get a 20-Ω resistor. The Norton equivalent is $i_N = 250$ mA and $R_N = 20\ \Omega$.

Select the value of R_L so that:

(a). 150 mA is delivered to the load.

Apply current division and solve for R_L

$$i_\mathrm{L} = 0.15 = \frac{20}{20 + R_\mathrm{L}}(0.25)$$

$$3 + 0.15R_\mathrm{L} = 5$$

$$0.15R_\mathrm{L} = 2$$

$$R_\mathrm{L} = 13.33\ \Omega$$

(b). 6 V is delivered to the load.

The maximum load voltage is $v_\mathrm{MAX} = i_\mathrm{N}R_\mathrm{N} = (0.25)(20) = 5$ V, so this result is not possible.

(c). 100 mW is delivered to the load.

Find an expression for the load current and then substitute into the expression for the load power.

$$i_\mathrm{L} = \frac{20}{20 + R_\mathrm{L}}(0.25) = \frac{5}{20 + R_\mathrm{L}}$$

$$p_\mathrm{L} = 0.1 = i_\mathrm{L}^2 R_\mathrm{L} = \left(\frac{25}{R_\mathrm{L}^2 + 40R_\mathrm{L} + 400}\right) R_\mathrm{L}$$

$$1 = \frac{250R_\mathrm{L}}{R_\mathrm{L}^2 + 40R_\mathrm{L} + 400}$$

$$R_\mathrm{L}^2 + 40R_\mathrm{L} + 400 = 250R_\mathrm{L}$$

$$R_\mathrm{L}^2 - 210R_\mathrm{L} + 400 = 0$$

Solving the quadratic equation, we find $R_\mathrm{L} = 208.0776\ \Omega$ or $R_\mathrm{L} = 1.9224\ \Omega$.

Problem 3–68. Find the Thévenin equivalent seen by R_L in Figure P3–68.

With the sources set to zero, the lookback resistance is three 1-kΩ resistors in parallel, so $R_\mathrm{T} = 1000/3 = 333.3\ \Omega$. Use superposition to find the open-circuit voltage. With the voltage source on and the current source replaced by an open-circuit, combine the top two resistors in parallel and apply voltage division to get $v_\mathrm{OC1} = (12)(1000)/(1000{+}500) = 8$ V. With the current source on and the voltage source replaced by a short circuit, combine all three resistors in parallel and apply Ohm's law to get $v_\mathrm{OC2} = (-0.01)(333.3) = -3.333$ V. Note the direction of the current source. The Thévenin voltage is $v_\mathrm{T} = v_\mathrm{OC} = v_\mathrm{OC1} + v_\mathrm{OC2} = 8 - 3.333 = 4.667$ V.

Problem 3–69. Find the Thévenin equivalent seen by R_L in Figure P3–69.

Apply voltage division on each side of the circuit to determine the open-circuit voltage.

$$v_\mathrm{A} = \frac{60}{60 + 30}(60) = 40\ \mathrm{V}$$

$$v_\mathrm{B} = \frac{20}{20 + 20}(60) = 30\ \mathrm{V}$$

$$v_\mathrm{T} = v_\mathrm{OC} = 40 - 30 = 10\ \mathrm{V}$$

Use the lookback technique to find R_T.

$$R_\mathrm{T} = (30 \parallel 60) + (20 \parallel 20) = 20 + 10 = 30\ \Omega$$

Problem 3–70. For the circuit of Figure P3–70 find the value of R_L that will result in:

(a). Maximum voltage. What is that voltage?

Perform a source transformation to get a 60-V voltage source in series with a 3-kΩ resistor and then combine the 3-kΩ and 2-kΩ resistors in series to get the Thévenin equivalent with $v_T = 60$ V and $R_T = 5$ kΩ. For maximum load voltage, let R_L be an open circuit and the voltage will be 60 V.

(b). Maximum current. What is that current?

Using the results from above and performing a source transformation, the Norton equivalent circuit is a 12-mA current source in parallel with a 5-kΩ resistor. The maximum current is 12 mA and occurs with load replaced by a short circuit.

(c). Maximum power. What is that power?

The maximum power occurs with $R_L = R_T = 5$ kΩ and the power delivered to the load is $p_L = v_T^2/(4R_T) = (60)^2/[(4)(5000)] = 180$ mW.

Problem 3–71. For the circuit of Figure P3–71 find the value of R_L that will result in:

(a). Maximum voltage. What is that voltage?

Find the Thévenin equivalent to the left of the load. The lookback resistance is $R_T = 1 + (2.2 \parallel 3.3) = 2.32$ kΩ. Apply voltage division to find the open-circuit voltage as $v_T = (56)(3.3)/(3.3+2.2) = 33.6$ V. For maximum load voltage, let R_L be an open circuit and the voltage will be 33.6 V.

(b). Maximum current. What is that current?

Using the results from above and performing a source transformation, the Norton equivalent circuit is a 14.483-mA current source in parallel with a 2.32-kΩ resistor. The maximum current is 14.483 mA and occurs with load replaced by a short circuit.

(c). Maximum power. What is that power?

The maximum power occurs with $R_L = R_T = 2.32$ kΩ and the power delivered to the load is $p_L = v_T^2/(4R_T) = (33.6)^2/[(4)(2320)] = 121.655$ mW.

Problem 3–72. The resistance R in Figure P3–72 is adjusted until maximum power is delivered to the load consisting of R and the 12-kΩ resistor in parallel.

(a). Find the required value of R.

First, find the lookback resistance to the left of the load as $R_N = 2 + 3 + 1 = 6$ kΩ. For maximum power transfer, the load resistance must match R_N. Two 12-kΩ resistors in parallel will yield a 6-kΩ equivalent resistance, so $R = 12$ kΩ.

(b). How much power is delivered to the load?

Apply current division to find the load current $i_L = (3i_S)/(3 + 1 + 6 + 2) = i_S/4 = 10$ mA. The power delivered to the load is $p_L = i_L^2 R_L = (0.01)^2(6000) = 0.6$ W.

Problem 3–73. When a 5-kΩ resistor is connected across a two-terminal source, a current of 15 mA is delivered to the load. When a second 5-kΩ resistor is connected in parallel with the first, a total current of 20 mA is delivered. Find the maximum power available from the source.

Model the source as a Thévenin equivalent circuit and write two equations describing the conditions in the problem statement. Solve for v_T and R_T.

$$0.015 = \frac{v_T}{R_T + 5000} \qquad 0.02 = \frac{v_T}{R_T + 2500}$$

$$0.015R_T + 75 = v_T \qquad 0.02R_T + 50 = v_T$$

Subtract the left equation from the right equation.

$$0.005R_\mathrm{T} - 25 = 0$$

$$R_\mathrm{T} = \frac{25}{0.005} = 5000\ \Omega$$

Set $R_\mathrm{L} = 5\ \mathrm{k}\Omega$ and solve for $v_\mathrm{T} = 0.02R_\mathrm{T} + 50 = 100 + 50 = 150$ V. The maximum power delivered to the load is $p_\mathrm{L} = v_\mathrm{T}^2/(4R_\mathrm{T}) = (150)^2/[(4)(5000)] = 1.125$ W.

Problem 3–74. Find the value of R in the circuit of Figure P3–74 so that maximum power is delivered to the load. What is the value of the maximum power?

The adjustable resistor is associated with the source and to maximize power transfer, minimize the output resistance of the source. Set $R = 0\ \Omega$ to maximize the power delivered to the load. The equivalent load resistance is $R_\mathrm{EQ} = 5 \parallel 2 = 1.4286\ \mathrm{k}\Omega$. The load voltage is $v_\mathrm{L} = (10)(R_\mathrm{EQ})/(50 + R_\mathrm{EQ}) = 9.66$ V. The power delivered to the load is $p_\mathrm{L} = v_\mathrm{L}^2/R_\mathrm{EQ} = 65.346$ mW.

Problem 3–75. For the circuit of Figure P3–75 find the value of R_L that will result in:

(a). Maximum voltage. What is that voltage?

Find the Thévenin equivalent to the left of the load. Combine the parallel resistors to get an equivalent resistance of 750 Ω and perform a source transformation to get a 7.5-V voltage source in series with a 750-Ω resistor. Combine the resistors in series to get $R_\mathrm{T} = 0.75 + 2.75 = 3.5\ \mathrm{k}\Omega$. For maximum load voltage, let R_L be an open circuit and the voltage will be 7.5 V.

(b). Maximum current. What is that current?

Using the results from above and performing a source transformation, the Norton equivalent circuit is a 2.1429-mA current source in parallel with a 3.5-kΩ resistor. The maximum current is 2.1429 mA and occurs with load replaced by a short circuit.

(c). Maximum power. What is that power?

The maximum power occurs with $R_\mathrm{L} = R_\mathrm{T} = 3.5\ \mathrm{k}\Omega$ and the power delivered to the load is $p_\mathrm{L} = v_\mathrm{T}^2/(4R_\mathrm{T}) = (7.5)^2/[(4)(3500)] = 4.0179$ mW.

Problem 3–76. (D) A 100 Ω-load needs 10 mA to operate correctly. Design a practical power source to provide the needed current. The smallest source resistance you can practically design for is 50 Ω, but you can add any other series resistance if you need to.

There are many correct solutions. If the source has a resistance of 50 Ω, set the source voltage to be 1.5 V to produce a load current of $i_\mathrm{L} = 1.5/(100 + 50) = 10$ mA.

Problem 3–77. A practical source delivers 50 mA to a 300-Ω load. The source delivers 12 V to a 120-Ω load. Find the maximum power available from the source.

With 12 V across a 120-Ω load, the current is $i_\mathrm{L} = 12/120 = 100$ mA. Model the source as a Thévenin equivalent circuit and write two equations describing the conditions in the problem statement. Solve for v_T and R_T.

$$0.05 = \frac{v_\mathrm{T}}{R_\mathrm{T} + 300} \qquad\qquad 0.1 = \frac{v_\mathrm{T}}{R_\mathrm{T} + 120}$$

$$0.05R_\mathrm{T} + 15 = v_\mathrm{T} \qquad\qquad 0.1R_\mathrm{T} + 12 = v_\mathrm{T}$$

Subtract the left equation from the right equation.

$$0.05R_\mathrm{T} - 3 = 0$$

$$R_\mathrm{T} = \frac{3}{0.05} = 60\ \Omega$$

Set $R_\mathrm{L} = 60\ \Omega$ and solve for $v_\mathrm{T} = 0.1R_\mathrm{T} + 12 = 6 + 12 = 18$ V. The maximum power delivered to the load is $p_\mathrm{L} = v_\mathrm{T}^2/(4R_\mathrm{T}) = (18)^2/[(4)(60)] = 1.35$ W.

Problem 3–78. (D) Select R_L and design an interface circuit for the circuit shown in Figure P3–78 so that the load voltage is 2 V.

Perform a source transformation on the voltage source in series with the 10-kΩ resistor to get a 1-mA current source in parallel with a 10-kΩ resistor. Combine the three resistors in parallel to the left of the interface to get a Norton equivalent circuit with $i_N = 1$ mA and $R_N = 2.5$ kΩ. To get a 2-V drop across the load with a 1-mA source, the equivalent resistance must be 2 kΩ. Select a value for R_L so that $R_L \parallel 2.5 = 2$ kΩ.

$$2 = R_L \parallel 2.5 = \frac{2.5R_L}{2.5 + R_L}$$

$$5 + 2R_L = 2.5R_L$$

$$0.5R_L = 5$$

$$R_L = 10\text{ k}\Omega$$

With $R_L = 10$ kΩ, no other resistors are required in the interface—it is a direct connection.

Problem 3–79. (D) The source in Figure P3–79 has a 100-mA output current limit. Design an interface circuit so that the load voltage is $v_2 = 20$ V and the source current is $i_1 < 50$ mA.

With $v_2 = 20$ V across a 500-Ω resistor, the current through the resistor is 40 mA. Make the interface a resistor in series with the other two resistors such that the current is 40 mA. We have $0.04 = 100/R_{EQ}$, so $R_{EQ} = 100/0.04 = 2.5$ kΩ. The existing resistors provide a total of 1 kΩ, so the series interface resistor must be $R_I = 1.5$ kΩ.

Problem 3–80. (D) Figure P3–80 shows an interface circuit connecting a 15-V source to a diode load. The i-v characteristic of the diode is $i = 10^{-14}(e^{40v} - 1)$.

(a). Design an interface circuit so that $v = 0.7$ V.

Find the current through the diode with a voltage drop of 0.7 V.

$$i = 10^{-14}(e^{40v} - 1) = 10^{-14}(e^{28} - 1) = 14.4626\text{ mA}$$

Let the interface be a series resistor. Its current will match that of the diode and its voltage drop will be $15 - 0.7 = 14.3$ V. We have $R_I = 14.3/0.0144626 = 988.76$ Ω.

(b). Validate your answer using MATLAB.

The following MATLAB code provides the solution:

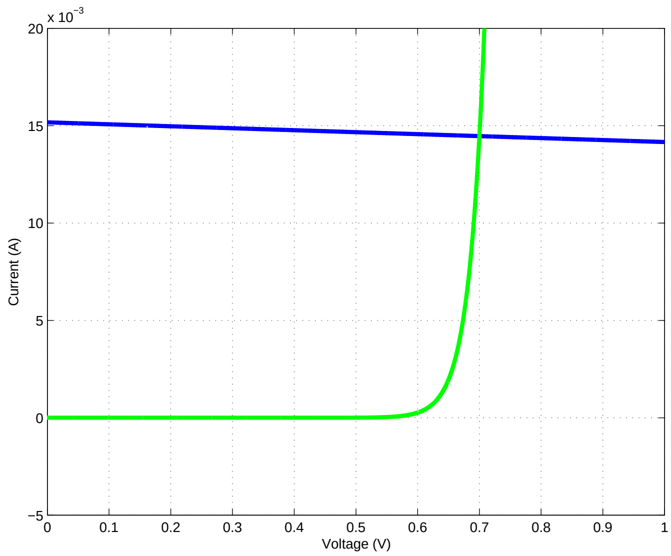
```
% Set the voltage source
vs = 15;
% Given the diode voltage, find the diode current
vd = 0.7;
id = 1e-14*(exp(40*vd)-1)

% Find the voltage drop across the interface
v_inter = vs-vd
% If the interface is a series resistor, find the value of the resistance
R_series = v_inter/id

% Also validate the results graphically
% Create a range of voltages to plot
v = -0.5:0.0001:1;
% Plot the i-v characteristic of the source
vT = 15;
RT = R_series;
i_source = (vT-v)/RT;
% Plot the i-v characteristic of the diode
i_D = 1e-14*(exp(40*v)-1);
```

```
plot(v,i_source,'b','LineWidth',3)
hold on
grid on
plot(v,i_D,'g','LineWidth',3)
xlabel('Voltage (V)')
ylabel('Current (A)')
axis([0 1 -0.005 0.02])
```

The corresponding MATLAB output is shown below and confirms the answer in part (a).

```
id = 14.4626e-003
v_inter = 14.3000e+000
R_series = 988.7592e+000
```

The graphical solution shows that the i-v characteristic for the diode intersects the i-v characteristic of the Thévenin source at 0.7 V.



Problem 3–81. (D) Design the interface circuit in Figure P3–81 so that the voltage delivered to the load is $v = 10$ V $\pm 10\%$. Use one or more of only the following standard resistors: 1.3 kΩ, 2 kΩ, 3 kΩ, 4.3 kΩ, 6.2 kΩ, and 9.1 kΩ. These resistors all have a tolerance of $\pm 5\%$, which you must account for in your design.

The 10-kΩ load has a voltage drop of 10 V, so the desired current through the load is 1 mA. Choose a resistor in series with the other two resistors as the interface design and select the resistance to make the current approximately 1 mA. With a 20-V source, the required equivalent resistance is $R_{\text{EQ}} = 20/0.001 = 20$ kΩ, which means $R_{\text{I}} = 9.5$ kΩ. Using the available resistors, choose $R_{\text{I}} = 1.3+2+6.2 = 9.5$ kΩ. Given the tolerance on these resistors, the possible resistance ranges between 9.025 kΩ and 9.975 kΩ. Apply voltage division to calculate the maximum and minimum output voltages for these extreme values.

$$v_{\text{MAX}} = \frac{10}{10 + 9.025 + 0.5}(20) = 10.2433 \text{ V}$$

$$v_{\text{MIN}} = \frac{10}{10 + 9.975 + 0.5}(20) = 9.7680 \text{ V}$$

The design meets the specifications.

Problem 3–82. (D,E) In this problem you will design two interface circuits that deliver 150 V to the 5-kΩ load.

(a). **(D)** Design a parallel resistor interface to meet the requirements.

The 5-kΩ load has a voltage drop of 150 V, so its current is 30 mA. If the interface circuit is a parallel resistor, then all resistors share the same voltage drop. The current through the other 5-kΩ resistor is also 30 mA and the current through the 10-kΩ resistor is half as much, or 15 mA. The remaining current of $100-30-30-15 = 25$ mA flows through the interface resistor with a voltage drop of 150 V, so the resistance is $R_P = 150/0.025 = 6$ kΩ.

(b). **(D)** Convert the source circuit to its Thévenin equivalent and then design a series interface to meet the requirement.

Combine the parallel resistors and perform a source transformation to get $v_T = 333.3$ V and $R_T = 3.333$ kΩ. The current through the circuit must still be 30 mA, so the equivalent resistance of the series circuit is $333.3/0.03 = 11.111$ kΩ. The series interface resistance is $R_S = 11.111-3.333-5 = 2.778$ kΩ.

(c). **(E)** If minimizing the power that the current source delivers is the primary consideration, which of your two designs best meets the requirement?

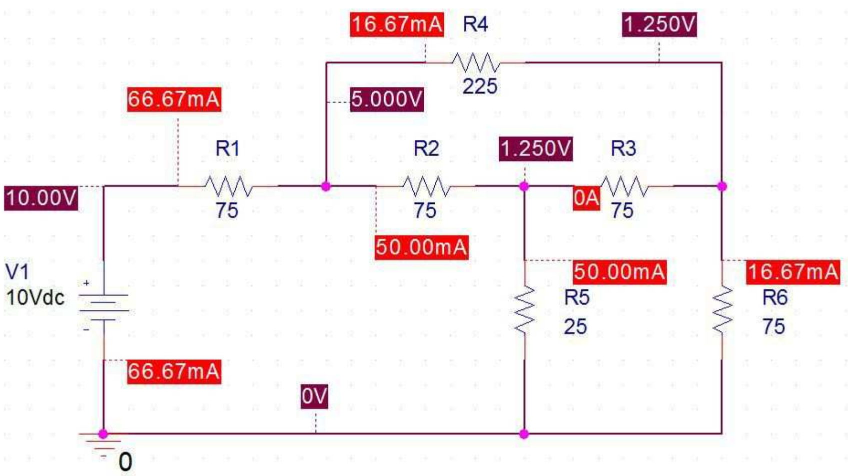
We must analyze the two designs with the original circuit, *not* the Thévenin equivalent. In the parallel design, the source provides $p_S = iv = (0.1)(150) = 15$ W. In the second design, the equivalent resistance seen by the source is $R_{EQ} = 10 \parallel 5 \parallel (2.778 + 5) = 2.333$ kΩ. The source provides $p_S = i^2 R_{EQ} = 23.33$ W. The parallel design minimizes the source power.

Problem 3–83. (E) Two teams are competing to design the interface circuit in Figure P3–83 so that 25 mW ±10% is delivered to the 1-kΩ load resistor. Their designs are shown in Figure P3–83. Which solution is better considering the use of standard values, number of parts, and power required by the source? Would your choice be different if the power had to be within ±5% ?

First, verify that the designs meet the specification. For Team A, combine the series resistors and determine the current through the load as $i_L = 20/(50 + 2200 + 680 + 1000) = 5.089$ mA. The power delivered to the load is $p_L = (5.089 \times 10^{-3})^2(1000) = 25.898$ mW, which meets the specification. For Team B, combine the three parallel resistors to find $R_{EQ} = 33 \parallel 33 \parallel 1000 = 16.23$ Ω. Apply voltage division to find the load voltage $v_L = (20)(16.23)/(50 + 16.23) = 4.9016$ V. The power delivered to the load is $p_L = (4.9016)^2/1000 = 24.026$ mW, which also meets the specification. Both designs meet the specification, have two parts, and all parts use standard values. In the design from Team A, the source delivers $p_S = (20)(0.005089) = 101.8$ mW of power. For Team B, the source sees an equivalent resistance of $R_{EQ} = 66.23$ Ω and delivers $p_S = (20)^2/66.23 = 6.04$ W. Team A has the better design. If the power tolerance was reduced to ±5%, Team A still meets the specification and has the better design.

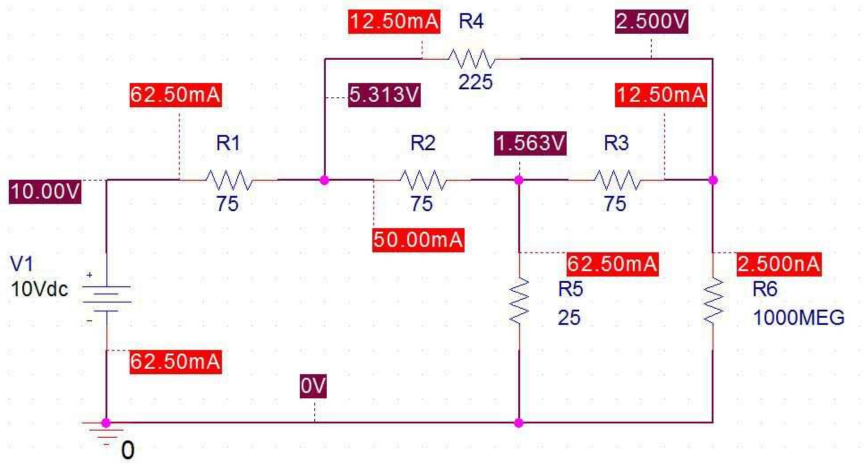
Problem 3–84. (E) The bridge-T attenuation pad shown in Figure P3–84 was found in a drawer. You need an attenuation pad that would match to a 75-Ω source and a 75-Ω load and provide for a −12 dB drop of signal (reduction of four times). Use OrCAD to determine if the device will work.

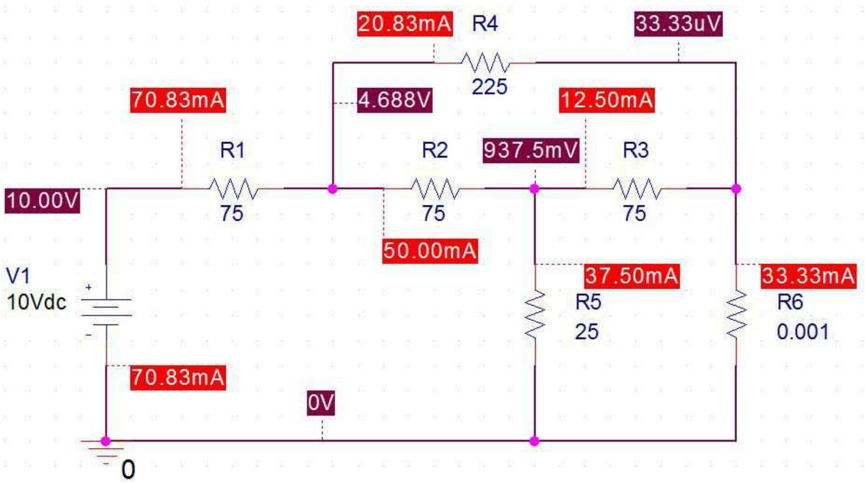
The following OrCAD simulation shows that the bridge-T attenuation pad meets the requirements.



The voltage at the input of the bridge-T pad is 5 V and the output voltage is 1.25 V, which is a reduction by a factor of 4 or 12.04 dB. If the 75-Ω source experiences a load of 75-Ω, then the source current will be $v_S/150 = 10/150 = 66.67$ mA, which agrees with the simulation.

To determine if the load is matched to a 75-Ω equivalent source, perform two more OrCAD simulations to find the open-circuit voltage and short-circuit current from the perspective of the load. To simplify the simulations, replace the load resistor with a large value and then a small value to estimate the open-circuit voltage and short-circuit current.





The open-circuit voltage is 2.5 V and the short-circuit current is 33.33 mA. The Thévenin resistance seen by the load is $R_T = 2.5/0.03333 = 75\ \Omega$, so the circuit meets all of the specifications.

Problem 3–85. (D) Design the interface circuit in Figure P3–85 so that the power delivered to the load is 100 mW.

Find the Thévenin equivalent of the source circuit. Apply voltage division to get $v_T = (10)(50)/(50{+}50) = 5$ V. The lookback resistance is $R_T = 25 + (50 \parallel 50) = 25 + 25 = 50\ \Omega$. Solve for the current through the load when the 50-Ω resistor dissipates 100 mW as $i_L = \sqrt{0.1/50} = 44.72$ mA. Let the interface be a resistor in series with the load. Using the Thévenin equivalent circuit, the total series resistance must be $R_{EQ} = 5/0.04472 = 111.80\ \Omega$. The series interface resistor must be $R_I = 111.8 - 50 - 50 = 11.8\ \Omega$.

Problem 3–86. (D) Design the interface circuit in Figure P3–85 so that the voltage delivered to the load is 1.5 V.

As found in Problem 3-85, the Thévenin equivalent source circuit has $v_T = 5$ V and $R_T = 50\ \Omega$. If the 50-Ω load has a voltage drop of 1.5 V, its current is $i_L = 1.5/50 = 30$ mA. Let the interface be a resistor in series with the load. Using the Thévenin equivalent circuit, the total series resistance must be $R_{\text{EQ}} = 5/0.03 = 166.67\ \Omega$. The series interface resistor must be $R_{\text{I}} = 166.67 - 50 - 50 = 66.67\ \Omega$.

Problem 3–87. (D) Design the interface circuit in Figure P3–87 so that $R_{\text{IN}} = 100\ \Omega$ and the current delivered to the 50-Ω load is $i = 50$ mA. (*Hint:* Use an L-pad.)

Let the L-pad be resistor R_1 in series with the 100-Ω source resistor and R_2 in parallel with the 50-Ω load resistor. With $R_{\text{IN}} = 100\ \Omega$, the source sees an equivalent resistance of 200 Ω and provides a current of $i_{\text{S}} = 15/200 = 75$ mA. With 50 mA delivered to a 50-Ω load, the voltage across the load is 2.5 V. That voltage appears across the load and R_2. Using KVL, the voltage drop across the series combination of the 100-Ω resistor and R_1 is 12.5 V and the current is 75 mA. The total resistance is $100 + R_1 = 12.5/0.075 = 166.67\ \Omega$, so $R_1 = 66.67\ \Omega$. The 75 mA splits between R_2 and the load, with the load carrying 50 mA and R_2 carrying 25 mA. The voltage drop across R_2 is 2.5 V, so $R_2 = 2.5/0.025 = 100\ \Omega$.

Problem 3–88. (D) Design the interface circuit in Figure P3–87 so that $R_{\text{OUT}} = 50\ \Omega$ and the voltage delivered to the 50-Ω load is $v = 2.5$ V. (*Hint:* Use an L-pad.)

Let the L-pad be resistor R_1 in series with the 100-Ω source resistor and R_2 in parallel with the 50-Ω load resistor. With $R_{\text{OUT}} = 50\ \Omega$ and the 2.5 V across the 50-Ω load resistor, the equivalent Thévenin circuit seen by the load is $v_{\text{T}} = 5$ V with $R_{\text{T}} = 50\ \Omega$. With the selected L-pad design, we need 5 V to appear across R_2 and the lookback resistance to be 50 Ω.

$$5 = \frac{R_2}{100 + R_1 + R_2}(15)$$

$$50 = \frac{(100 + R_1)(R_2)}{100 + R_1 + R_2}$$

Simplify the first equation to get

$$100 + R_1 + R_2 = 3R_2$$

$$100 + R_1 = 2R_2$$

Substitute this result into the second equation

$$50 = \frac{(2R_2)(R_2)}{2R_2 + R_2} = \frac{2R_2}{3}$$

We can then solve for $R_2 = 75\ \Omega$ and $R_1 = 50\ \Omega$.

Problem 3–89. (D) The circuit in Figure P3–89 has a source resistance of 75 Ω and a load resistance of 300 Ω. Design the interface circuit so that the input resistance is $R_{\text{IN}} = 75\ \Omega \pm 10\%$ and the output resistance is $R_{\text{OUT}} = 300\ \Omega \pm 10\%$.

Use an L-pad design with R_1 in parallel with the series combination of R_2 and the 300-Ω load resistor. The design constraints yield the following two equations.

$$75 = R_1 \parallel (R_2 + 300) = \frac{R_1(R_2 + 300)}{R_1 + R_2 + 300}$$

$$300 = R_2 + (75 \parallel R_1) = R_2 + \frac{75R_1}{75 + R_1}$$

Use MATLAB to solve these equations.

```
% Define the symbolic variables
syms R1 R2 positive
% Define the known numerical values
RS = 75;
RL = 300;
RIN = 75;
```

```
ROUT = 300;
% Create the equations
Eqn1 = R1*(R2+RL)/(R1+R2+RL) - RIN;
Eqn2 = R2 + RS*R1/(RS+R1) - ROUT;
% Solve the equations
Soln = solve(Eqn1,Eqn2);
% Examine the solutions
R1 = double(Soln.R1)
R2 = double(Soln.R2)
```

The corresponding MATLAB output is shown below.

```
R1 =  86.6025e+000
R2 = 259.8076e+000
```

Select practical values for the resistors and verify that the design still meets the specifications. In this case, choose $R_1 = 82\ \Omega$ and $R_2 = 270\ \Omega$. We then have:

$$R_{\mathrm{IN}} = 82 \parallel (270 + 300) = 71.69\ \Omega$$

$$R_{\mathrm{OUT}} = 270 + (75 \parallel 82) = 309.17\ \Omega$$

The practical design meets the specification within the tolerances.

Problem 3–90. (E) It is claimed that both interface circuits in Figure P3–90 will deliver $v = 4$ V to the 75-Ω load. Verify this claim. Which interface circuit consumes the least power? Which has an output resistance that best matches the 75-Ω load?

With Circuit 1, use voltage division to find the load voltage.

$$v_{\mathrm{L}} = \frac{75}{150 + 150 + 75}(20) = 4\ \mathrm{V}$$

With Circuit 2, find the equivalent resistance of the interface and load combined as $R_{\mathrm{EQ}} = 100 \parallel (15+75) = 47.368\ \Omega$. Apply voltage division to determine the voltage across the interface and then apply it again to find the load voltage.

$$v_{\mathrm{I}} = \frac{47.368}{150 + 47.368}(20) = 4.8\ \mathrm{V}$$

$$v_{\mathrm{L}} = \frac{75}{15 + 75}(4.8) = 4\ \mathrm{V}$$

Both circuits deliver 4 V to the load.

In Circuit 1, the equivalent resistance seen by the source is $R_{\mathrm{EQ}} = 150 + 150 + 75 = 375\ \Omega$, so the power provided by the source is $p_{\mathrm{S}} = (20)^2/375 = 1.067$ W. In Circuit 2, the equivalent resistance seen by the source is $R_{\mathrm{EQ}} = 150 + 47.368 = 197.368\ \Omega$, so the power provided by the source is $p_{\mathrm{S}} = (20)^2/197.368 = 2.027$ W. Circuit 1 consumes the least amount of power.

The output resistance of Circuit 1 is $R_{\mathrm{OUT}} = 150 + 150 = 300\ \Omega$. The output resistance of Circuit 2 is $R_{\mathrm{OUT}} = 15 + (150 \parallel 100) = 15 + 60 = 75\ \Omega$. The output resistance of Circuit 2 best matches that of the load.

Problem 3–91. (E) Audio Speaker Resistance-Matching Network

A company is producing an interface network that they claim would result in an R_{IN} of 600 Ω ±2% and R_{OUT} of 16, 8, or 4 Ω ±2%—depending on whether the connected speakers are 16, 8, or 4 Ω—selectable via a built-in switch. The design is shown in Figure P3–91. Prove or disprove their claim.

The interface is an L-pad design with R_1 in series with the source resistance and R_2 in parallel with the load. The values for R_1 are 592, 600, and 604 Ω, the values for R_2 are 16, 8, and 4 Ω, and the values for R_{L}

are 16, 8, and 4 Ω, respectively. Calculate $R_{\rm IN}$ and $R_{\rm OUT}$ using the following formulas

$$R_{\rm IN} = R_1 + (R_2 \parallel R_{\rm L}) = R_1 + \frac{R_2 R_{\rm L}}{R_2 + R_{\rm L}}$$

$$R_{\rm OUT} = (R_{\rm S} + R_1) \parallel R_2 = \frac{(R_{\rm S} + R_1)R_2}{R_{\rm S} + R_1 + R_2}$$

The claim is true. For $R_{\rm L} = [16,\ 8,\ 4]\ \Omega$, $R_{\rm IN} = [600,\ 604,\ 606]\ \Omega$, $R_{\rm OUT} = [15.79,\ 7.95,\ 3.99]\ \Omega$. All of the values are within the specified limits.

Problem 3–92. (A) Attenuation Analysis

In Figure P3–92 a two-port attenuator connects a 600-Ω source to a 600-Ω load. Find the power delivered to the load in terms of $v_{\rm S}$. Remove the attenuator and find the power delivered to the load when the source is directly connected to the load. By what fraction does the attenuator reduce the power delivered to the 600-Ω load? Express the fraction in dB.

Reduce the circuit from right to left to get a series circuit. We have $R_{\rm EQ} = 450 \parallel (300 + 600) = 300\ \Omega$. Apply voltage division to find the voltage across $R_{\rm EQ}$ as $v_{\rm A} = (v_{\rm s})(300)/(600 + 300 + 300) = v_{\rm S}/4$. Apply voltage division again to get the load voltage $v_{\rm L} = (v_{\rm S}/4)(600)/(300 + 600) = v_{\rm S}/6$. The power delivered to the load is $p_{\rm L} = v_{\rm L}^2/R_{\rm L} = v_{\rm S}^2/21600$. Remove the attenuator and repeat the analysis. Apply voltage division to find the load voltage as $v_{\rm L} = (v_{\rm S})(600)/(600 + 600) = v_{\rm S}/2$. The power delivered to the load is $p_{\rm L} = v_{\rm S}^2/2400$. The power reduction by the attenuator is $2400/21600 = 1/9$, which is equivalent to -9.54 dB.

Problem 3–93. (D) Interface Circuit Design

Using no more than three 50-Ω resistors, design the interface circuit in Figure P3–93 so that $v \leq 5$ V and $i \leq 100$ mA regardless of the value of $R_{\rm L}$.

The specifications indicate that the load must see a source with a Thévenin voltage of no more than 5 V and a Norton current of no more than 100 mA. Use a T-interface with a 50-Ω resistor along each path—one resistor in series with the source resistor, one resistor in series with the load, and the third in parallel with the combination of the 50-Ω resistor and the load. The Thévenin voltage is $v_{\rm T} = (15)(50)/(50 + 50 + 50) = 5$ V. The lookback resistance is $R_{\rm T} = 50 + [50 \parallel (50+50)] = 83.33\ \Omega$. The Norton current is $i_{\rm N} = 5/83.33 = 60$ mA.

Problem 3–94. (D) Battery Design

A satellite requires a battery with an open-circuit voltage $v_{\rm OC} = 36$ V and a Thévenin resistance $R_{\rm T} \leq 10\ \Omega$. The battery is to be constructed using series and parallel combinations of one of two types of cells. The first type has $v_{\rm OC} = 9$ V, $R_{\rm T} = 4\ \Omega$, and a weight of 30 grams. The second type has $v_{\rm OC} = 4$ V, $R_{\rm T} = 2\ \Omega$, and a weight of 18 grams. Design a minimum weight battery that meets the open-circuit voltage and Thévenin resistance requirements.

Using the first type of cell, we need to connect four cells in series to get 36 V, but the equivalent resistance is 16 Ω, which is above the specification of 10 Ω. To reduce the resistance, connect another set of four cells in series and then connect both sets in parallel to reduce the resistance to 8 Ω while maintaining the voltage level. The total weight is 240 grams.

Using the second type of cell, connect nine cells in series to get 36 V with a resistance of 18 Ω, which is above the specified value. Connect another nine cells in series and then connect both sets in parallel to reduce the resistance to 9 Ω with the same voltage level. The total weight is 324 grams.

The first type of cell provides a lighter design.

Problem 3–95. (E) Design Evaluation

A requirement exists for a circuit to deliver 0 to 5 V to a 100-Ω load from a 20-V source rated at 2.5 W. Two proposed circuits are shown in Figure P3–95. Which one would you choose and why?

Examine the extreme cases for each design. In Circuit 1, when the potentiometer is shorted out, the output voltage is $v_{\rm O} = 0$ V, the source current is $i_{\rm S} = 20/150 = 133.3$ mA, and the power provided by the source is $p_{\rm S} = (20)(0.1333) = 2.667$ W, which exceeds its power rating. When the potentiometer has a value of 100 Ω, we have $v_{\rm O} = 5$ V, $i_{\rm S} = 100$ mA, and $p_{\rm S} = 2$ W.

In Circuit 2, when the wire is connected at the bottom, the output voltage is $v_{\rm O} = 0$ V, the source current is $i_{\rm S} = 20/(150 + 100) = 80$ mA, and $p_{\rm S} = 1.6$ W. When the wire is connected at the top, we have $v_{\rm O} = 5$ V, $i_{\rm S} = 100$ mA, and $p_{\rm S} = 2$ W.

In both cases the output voltage will vary smoothly between 0 and 5 V, but Circuit 1 will exceed the source's power rating. Choose Circuit 2.

Problem 3–96. (E) Design Interface Competition

The output of a transistorized power supply is modeled by the Norton equivalent circuit shown Figure P3–96. Two teams are competing to design the interface circuit so that 25 mW ±10% is delivered to the 1-kΩ load resistor. Their designs are shown in Figure P3–96. Which solution is better considering the use of standard values, number of parts, and power required by the source? Would your choice be different if the power had to be within ±5%?

Use current division to find the current supplied to the load in each design and then determine the power delivered to the load.

$$i_{\mathrm{LA}} = \frac{50}{50 + 3000 + 1000}(0.4) = 4.9383 \text{ mA}$$

$$p_{\mathrm{LA}} = i_{\mathrm{LA}}^2 R_{\mathrm{L}} = (0.0049383)^2(1000) = 24.3865 \text{ mW}$$

$$i_{\mathrm{LB}} = \frac{\frac{1}{1000}}{\frac{1}{50} + \frac{1}{16} + \frac{1}{1000}}(0.4) = 4.7904 \text{ mA}$$

$$p_{\mathrm{LB}} = i_{\mathrm{LB}}^2 R_{\mathrm{L}} = (0.0047904)^2(1000) = 22.9481 \text{ mW}$$

Also determine the source powers by finding the equivalent resistance seen by each source. For Team A, the equivalent resistance is $R_{\mathrm{EQ}} = 50 \parallel (3000 + 1000) = 49.38\ \Omega$, so the source power is $p_{\mathrm{S}} = i_{\mathrm{S}}^2 R_{\mathrm{EQ}} = 7.901$ W. For Team B, we have $R_{\mathrm{EQ}} = 50 \parallel 16 \parallel 1000 = 11.98\ \Omega$ and $p_{\mathrm{S}} = 1.916$ W.

Both designs require one standard part, so they are equivalent in terms of parts. Both designs meet the load power specification with 10%, but only Team A meets the specification within 5%. The design from Team B requires less power from the source than the design from Team A. Therefore, with the 10% tolerances, choose Team B, but with the 5% tolerances, choose Team A.

Problem 3–97. (A,E) Analysis of Competing Interface Circuits Using MATLAB

Figure P3–97 displays two generalized interface circuit designs. In both circuits, resistors R_1 and R_2 connect a Thévenin equivalent circuit to a load resistor. Using MATLAB, develop symbolic expressions for the load current, i_{L}, and the input resistance, R_{IN}, for each circuit in terms of the given parameters. Using these two expressions, now use the MATLAB command `solve` to solve for R_1 and R_2 in terms of i_{L} and R_{IN}. Let $v_{\mathrm{T}} = 15$ V, $R_{\mathrm{T}} = 100\ \Omega$, $R_{\mathrm{L}} = 50\ \Omega$, $i_{\mathrm{L}} = 50$ mA, and $R_{\mathrm{IN}} = 100\ \Omega$. Can you use both types of interface circuits to find suitable values for R_1 and R_2 to meet these specifications? Compare your interface design(s) with the solution to Problem 3–87.

In Circuit (a), perform a source transformation to get an equivalent current source of $v_{\mathrm{T}}/(R_{\mathrm{T}} + R_1)$ in parallel with the series combination of $R_{\mathrm{T}} + R_1$. Apply current division to determine i_{L}.

$$i_{\mathrm{La}} = \frac{\frac{1}{R_{\mathrm{L}}}}{\frac{1}{R_{\mathrm{T}}+R_1} + \frac{1}{R_2} + \frac{1}{R_{\mathrm{L}}}}\left(\frac{v_{\mathrm{T}}}{R_{\mathrm{T}} + R_1}\right) = \frac{R_2 v_{\mathrm{T}}}{R_2 R_{\mathrm{L}} + R_{\mathrm{L}} R_{\mathrm{T}} + R_{\mathrm{L}} R_1 + R_2 R_{\mathrm{T}} + R_1 R_2}$$

Determine the input resistance in Circuit (a).

$$R_{\mathrm{INa}} = R_1 + (R_2 \parallel R_{\mathrm{L}}) = R_1 + \frac{R_2 R_{\mathrm{L}}}{R_2 + R_{\mathrm{L}}} = \frac{R_1 R_2 + R_1 R_{\mathrm{L}} + R_2 R_{\mathrm{L}}}{R_2 + R_{\mathrm{L}}}$$

In Circuit (b), perform a source transformation to get an equivalent current source of $v_{\mathrm{T}}/R_{\mathrm{T}}$ in parallel with R_{T}. Apply current division to determine i_{L}.

$$i_{\mathrm{Lb}} = \frac{\frac{1}{R_1+R_{\mathrm{T}}}}{\frac{1}{R_{\mathrm{T}}} + \frac{1}{R_2} + \frac{1}{R_1+R_{\mathrm{L}}}}\left(\frac{v_{\mathrm{T}}}{R_{\mathrm{T}}}\right) = \frac{R_2 v_{\mathrm{T}}}{R_1 R_2 + R_2 R_{\mathrm{L}} + R_1 R_{\mathrm{T}} + R_{\mathrm{T}} R_{\mathrm{L}} + R_2 R_{\mathrm{T}}}$$

Determine the input resistance in Circuit (b).

$$R_{\mathrm{INb}} = R_2 \parallel (R_1 + R_{\mathrm{L}}) = \frac{R_2(R_1 + R_{\mathrm{L}})}{R_2 + R_1 + R_{\mathrm{L}}} = \frac{R_2 R_1 + R_2 R_{\mathrm{L}}}{R_2 + R_1 + R_{\mathrm{L}}}$$

The following MATLAB code provides the solution.

```
syms vT iL
syms RT RL Rin R1 R2 R12
disp('Circuit 1')
% Use a source transformation and current division to find an
% expression for iLa
iLa = (vT/(RT+R1))*(1/RL)/(1/(RT+R1) + 1/R2 + 1/RL);
iLa = simplify(iLa);
% Find the input resistance
Rina = factor(R1 + R2*RL/(R2+RL));

% Solve for R1 and R2 in terms of iL and Rin
Eqn1 = iL*(R2*Rin + RL*Rin - R1*RL - R2*RL) + iL*R2*RT + iL*RL*R1 ...
    + iL*RL*R2 + iL*RL*RT - R2*vT;
R2 = solve(Eqn1,R2)

Eqn2 = R1 - (R2*Rin + RL*Rin - R2*RL)/(R2 + RL);
R1 = solve(Eqn2,R1)

% Substitute numerical values
R1a_num = double(subs(R1,{vT,RT,RL,iL,Rin},{15,100,50,50e-3,100}))
R2a_num = double(subs(R2,{vT,RT,RL,iL,Rin},{15,100,50,50e-3,100}))

disp('Circuit 2')
% Use a source transformation and current division to find an
% expression for iL2
clear R1 R2
syms R1 R2
iLb = (vT/RT)/(R1+RL)/(1/RT+1/R2+1/(R1+RL));
iLb = simplify(iLb);
Rinb = factor(R2*(R1+RL)/(R2+R1+RL));

% Solve for R1 and R2 in terms of iL and Rin
R2 = Rin*(R1+RL)/(R1+RL-Rin);
Eqn1 = iL - (R2*vT)/(R1*R2 + R2*RL + RT*(R1 + R2 + RL));
%Eqn2 = Rin - Rinb
R1 = solve(Eqn1,R1)
R2 = simplify(Rin*(R1+RL)/(R1+RL-Rin))

% Substitute numerical values
R1b_num = double(subs(R1,{vT,RT,RL,iL,Rin},{15,100,50,50e-3,100}))
R2b_num = double(subs(R2,{vT,RT,RL,iL,Rin},{15,100,50,50e-3,100}))
```

The corresponding MATLAB output is shown below.

```
Circuit 1
R2 = -(RL*RT*iL + RL*Rin*iL)/(RT*iL - vT + Rin*iL)
R1 = -(RL*RT*iL - Rin*vT + RL*Rin*iL)/vT
R1a_num =  66.6667e+000
R2a_num = 100.0000e+000

Circuit 2
R1 = -(RL*RT*iL - Rin*vT + RL*Rin*iL)/(RT*iL + Rin*iL)
R2 = -(Rin*vT)/(RT*iL - vT + Rin*iL)
R1b_num = 100.0000e+000
R2b_num = 300.0000e+000
```

The results are

$$R_{1a} = \frac{-i_L R_L (R_T + R_{IN}) + v_T R_{IN}}{v_T}$$

$$R_{2a} = \frac{-i_L R_L (R_T + R_{IN})}{i_L (R_T + R_{IN}) - v_T}$$

$$R_{1b} = \frac{-i_L R_L (R_T + R_{IN}) + v_T R_{IN}}{i_L (R_T + R_{IN})}$$

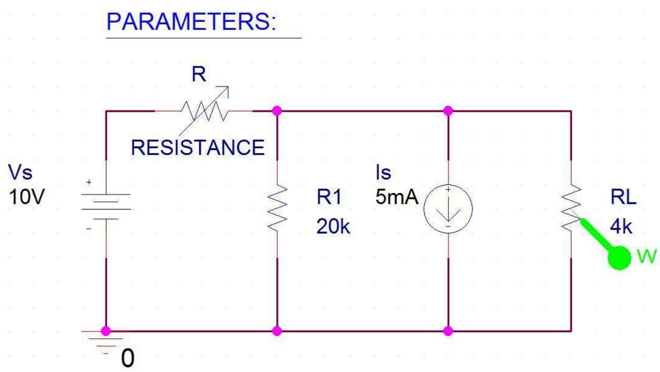
$$R_{2b} = \frac{-v_T R_{IN}}{i_L (R_T + R_{IN}) - v_T}$$

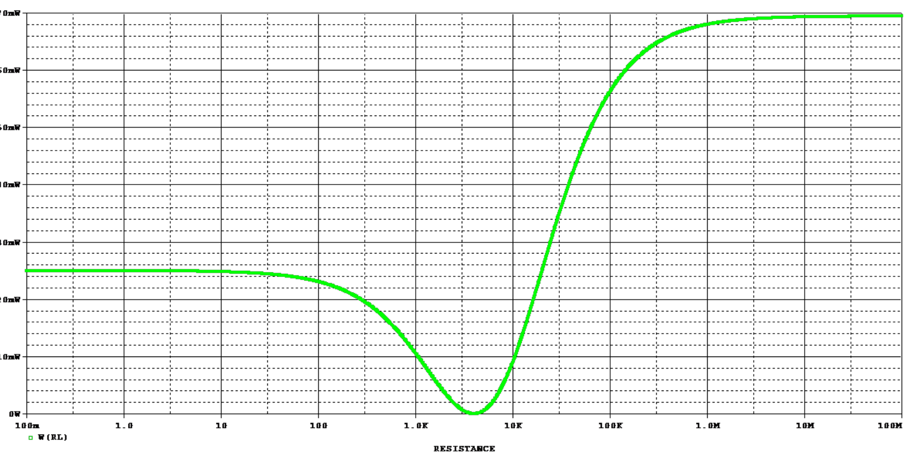
Both designs can find acceptable solutions for the given values and the first design matches the approach used for Problem 3–87. For Circuit (a), choose $R_1 = 66.67\ \Omega$ and $R_2 = 100\ \Omega$. For Circuit (b), choose $R_1 = 100\ \Omega$ and $R_2 = 300\ \Omega$.

Problem 3–98. (A) Maximum Power Transfer Using OrCAD

Figure P3–98 shows a circuit with two sources, a fixed load and a resistor R. Select R for maximum power transfer to the load. The result is not an obvious one. (*Hint:* Simulate in OrCAD using the PARAMETER function and sweep R from 0.1 Ω to 100 MΩ.)

The following OrCAD simulation and results provide a solution to this problem.





For maximum power transfer, select R to be an open circuit.

Problem 3–99. (A) Non-inverting Summer

A non-inverting summer interface device is shown in Figure P3–99. Of importance is that the input to the device has infinite resistance—i.e., no current flows into the device. The output voltage for this configuration is two times whatever the input voltage is. Develop a relationship for the voltage v_{L} across R_{L} with respect to the two input voltages and the two input resistances.

Let the input voltage be v_{A} and write a node-voltage equation at the input node. Solve the equation for v_{A}.

$$\frac{v_{\text{A}} - v_{\text{S1}}}{R_1} + \frac{v_{\text{A}} - v_{\text{S2}}}{R_2} = 0$$

$$R_2 v_{\text{A}} - R_2 v_{\text{S1}} + R_1 v_{\text{A}} - R_1 v_{\text{S2}} = 0$$

$$(R_1 + R_2) v_{\text{A}} = R_2 v_{\text{S1}} + R_1 v_{\text{S2}}$$

$$v_{\text{A}} = \frac{R_2 v_{\text{S1}} + R_1 v_{\text{S2}}}{R_1 + R_2}$$

The load voltage is twice the input voltage.

$$v_{\text{L}} = 2\left[\frac{R_2 v_{\text{S1}} + R_1 v_{\text{S2}}}{R_1 + R_2}\right]$$

4 Active Circuits

4.1 Exercise Solutions

Exercise 4–1. Find the output $v_{\rm O}$ in terms of the input $v_{\rm S}$ in the circuit in Figure 4–5.

Apply Ohm's law to compute the source current $i_{\rm x}$.

$$i_{\rm x} = \frac{v_{\rm S}}{R_{\rm S}+R_{\rm P}}$$

Apply voltage division to find the output voltage $v_{\rm O}$ and note the polarity of the dependent source.

$$v_{\rm O} = \left[\frac{R_{\rm L}}{R_{\rm C}+R_{\rm L}}\right](-ri_{\rm x}) = \left[\frac{R_{\rm L}}{R_{\rm C}+R_{\rm L}}\right]\left[\frac{-rv_{\rm S}}{R_{\rm S}+R_{\rm P}}\right] = \left[\frac{-R_{\rm L}r}{(R_{\rm S}+R_{\rm P})(R_{\rm C}+R_{\rm L})}\right]v_{\rm S}$$

Exercise 4–2. With all other resistors set to 1 kΩ and $\mu = 10^5$, select an appropriate design for $R_{\rm F}$ in Figure 4–6 so that the gain $|K|$ can never be larger than 10,000, or smaller than 50.

Use the expression for the gain K and the given resistor values to solve for $R_{\rm F}$ in terms of K and then evaluate the expression at the extreme values in the design.

$$K = \left[\frac{R_{\rm L}}{R_{\rm L}+R_{\rm C}}\right]\left[\frac{-\mu}{1+\dfrac{(1+\mu)R_{\rm S}}{R_{\rm F}}}\right]$$

$$K = \frac{1}{2}\left[\frac{-\mu R_{\rm F}}{R_{\rm F}+1000(1+\mu)}\right]$$

$$2KR_{\rm F} + 2000K(1+\mu) = -\mu R_{\rm F}$$

$$(2K+\mu)R_{\rm F} = -2000K(1+\mu)$$

$$R_{\rm F} = \frac{-2000K(1+\mu)}{2K+\mu} = \frac{-2000K(100001)}{2K+100000}$$

Substitute in $K = -10000$ and $K = -50$ and solve for $R_{\rm F} = 25\times10^6$ and $R_{\rm F} = 100100$. The design for $R_{\rm F}$ should be a 100.1-kΩ resistor in series with a 25-MΩ variable resistor (potentiometer).

Exercise 4–3. For the circuit in Figure 4–7, use the node-voltage equations in Eqs. (4–7) to find the output voltage $v_{\rm O}$ when $R_1 = 1$ kΩ, $R_2 = 3$ kΩ, $R_{\rm B} = 100$ kΩ, $R_{\rm P} = 1.3$ kΩ, $R_{\rm E} = 3.3$ kΩ, and $\beta = 50$.

Substitute the given numerical values into Eqs. (4–7) to get the following expression.

$$(2.11\times10^{-3})v_{\rm C} - (7.69\times10^{-4})v_{\rm D} = (10^{-3})v_{\rm S1} + (3.33\times10^{-4})v_{\rm S2}$$

$$-(3.92\times10^{-2})v_{\rm C} + (3.95\times10^{-2})v_{\rm D} = 0$$

Solve the second equation for $v_{\rm C} = 1.008v_{\rm D}$. Substitute this result into the first equation.

$$v_{\rm O} = v_{\rm D} = 0.736v_{\rm S1} + 0.245v_{\rm S2}$$

Exercise 4–4. (a). Formulate node-voltage equations for the circuit in Figure 4–9.

The node-voltage equations sum the currents leaving each node.

$$-i_{\rm S} + \frac{v_{\rm A}}{1000} + \frac{v_{\rm A}-v_{\rm B}}{2000} = 0$$

$$\frac{v_{\rm B}-v_{\rm A}}{2000} - \frac{v_{\rm x}}{500} + \frac{v_{\rm B}}{500} = 0$$

Write the equations in standard form with the substitution $v_x = v_A$.

$$(1.5 \times 10^{-3})v_A - (0.5 \times 10^{-3})v_B = i_S$$

$$-(2.5 \times 10^{-3})v_A + (2.5 \times 10^{-3})v_B = 0$$

(b). Solve the node-voltage equations for v_O and i_O in terms of i_S.

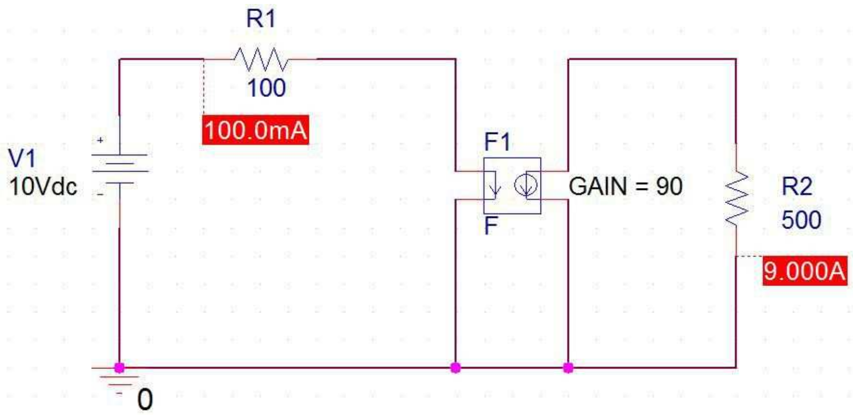
Add five times the first equation to the second equation to eliminate v_B.

$$(5.0 \times 10^{-3})v_A = 5i_S$$

$$v_A = 1000i_S$$

The second node-voltage equation yields $v_B = v_A = 1000i_S$. We have $v_O = v_B = 1000i_S$. Apply Ohm's law to determine the output current $i_O = v_O/500 = 2i_S$.

Exercise 4–5. Find i_O using OrCAD for the circuit of Figure 4–11(a).
The OrCAD simulation is shown below.



The output current is $i_O = -9$ A.

Exercise 4–6. Use node-voltage analysis to find v_O for the circuit in Figure 4–13.
Write the node-voltage equation for the output voltage v_O.

$$\frac{v_O - v_S}{R_x} + \frac{v_O - \mu v_x}{R_2} + \frac{v_O}{R_L} = 0$$

Note that $v_x = v_S - v_O$ and substitute into the node-voltage equation.

$$\frac{v_O - v_S}{R_x} + \frac{v_O}{R_2} - \frac{\mu v_S}{R_2} + \frac{\mu v_O}{R_2} + \frac{v_O}{R_L} = 0$$

$$v_O\left(\frac{1}{R_x} + \frac{1}{R_2} + \frac{\mu}{R_2} + \frac{1}{R_L}\right) = v_S\left(\frac{1}{R_x} + \frac{\mu}{R_2}\right)$$

$$v_O = \frac{\dfrac{1}{R_x} + \dfrac{\mu}{R_2}}{\dfrac{1}{R_x} + \dfrac{1}{R_2} + \dfrac{\mu}{R_2} + \dfrac{1}{R_L}} v_S$$

Exercise 4–7. Write a set of node-voltage equations and use them to find v_O and R_{IN} for the circuit in Figure 4–14. Use the parameter values given in Example 4–6.

Let node A be between R_1 and R_2. The node-voltage equations are

$$\frac{v_{\rm A}-v_{\rm S}}{50}+\frac{v_{\rm A}-v_{\rm O}}{1000}=0$$

$$\frac{v_{\rm O}-v_{\rm A}}{1000}+\frac{v_{\rm O}}{100}+\frac{v_{\rm O}}{5000}-0.1v_{\rm x}=0$$

We also have $v_{\rm x}=v_{\rm A}-v_{\rm O}$. Substitute in for $v_{\rm x}$ and solve for the node voltages.

$$\left(\frac{1}{50}+\frac{1}{1000}\right)v_{\rm A}-\frac{1}{1000}v_{\rm O}=\frac{v_{\rm S}}{50}$$

$$-\left(\frac{1}{1000}+\frac{1}{10}\right)v_{\rm A}+\left(\frac{1}{1000}+\frac{1}{100}+\frac{1}{5000}+\frac{1}{10}\right)v_{\rm O}=0$$

Solving this system of equations, we get $v_{\rm A}=0.99543v_{\rm S}$ and $v_{\rm O}=0.90413v_{\rm S}$. To compute $R_{\rm IN}$, find the input current and divide $v_{\rm S}$ by $i_{\rm IN}$

$$i_{\rm IN}=\frac{v_{\rm S}-v_{\rm A}}{R_1}=\frac{0.00457v_{\rm S}}{50}$$

We have $R_{\rm IN}=v_{\rm S}/i_{\rm IN}=50/0.00457=10.95\ {\rm k}\Omega$.

Exercise 4–8. Use mesh analysis to find the current $i_{\rm O}$ in Figure 4–16 when the element E is a dependent current source $2i_{\rm x}$ with the reference arrow directed down.

Let the left loop have mesh current $i_{\rm A}$ and the right loop have mesh current $i_{\rm B}$. We then have $i_{\rm x}=-i_{\rm A}$ and element E defines $i_{\rm B}=2i_{\rm x}=-2i_{\rm A}$. Write the mesh-current equation for loop A.

$$(5000+10000)i_{\rm A}-10000i_{\rm B}=-10$$

$$(5000+10000+20000)i_{\rm A}=-10$$

$$i_{\rm A}=-285.714\ \mu{\rm A}$$

We then have $i_{\rm B}=-2i_{\rm A}=571.429\ \mu$A and $i_{\rm O}=i_{\rm A}-i_{\rm B}=-857.143\ \mu$A.

Exercise 4–9. Use mesh analysis to find the current $i_{\rm O}$ in Figure 4–16 when the element E is a dependent voltage source $2000i_{\rm x}$ with the plus reference at the top.

Let the left loop have mesh current $i_{\rm A}$ and the right loop have mesh current $i_{\rm B}$. We then have $i_{\rm x}=-i_{\rm A}$ and element E defines a voltage $2000i_{\rm x}=-2000i_{\rm A}$. Write the mesh-current equations.

$$(5000+10000)i_{\rm A}-10000i_{\rm B}=-10$$

$$-10000i_{\rm A}+(10000+4000)i_{\rm B}+2000i_{\rm x}=0$$

Substitute and simplify the equations.

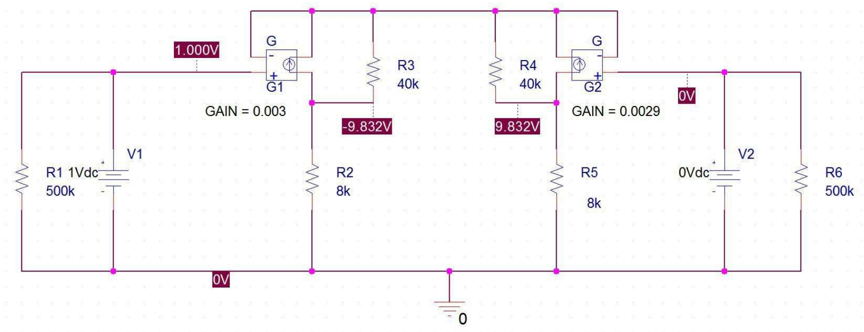
$$15000i_{\rm A}-10000i_{\rm B}=-10$$

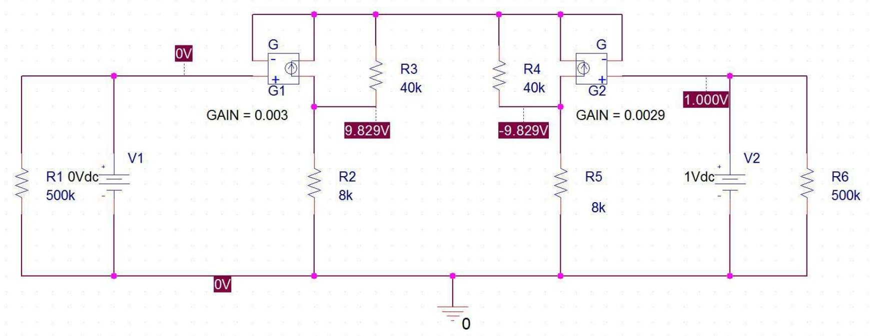
$$-12000i_{\rm A}+14000i_{\rm B}=0$$

Solve for $i_{\rm A}=-1.555$ mA and $i_{\rm B}=-1.333$ mA, which yields $i_{\rm O}=i_{\rm A}-i_{\rm B}=-222.222\ \mu$A.

Exercise 4–10. It is very important in designing difference amplifiers that the two transistors be matched in every way so that the outputs are balanced. Use OrCAD to determine the relationship in the circuit of Figure 4–17(a) if the transconductance of FET G2 is 2.9 mS rather than 3 mS.

The following two OrCAD simulations provide the results.





We have $v_{\rm O} = 9.832v_{\rm S1} - 9.829v_{\rm S2}$, so the gains are different for the two inputs.

Exercise 4–11. For the circuit of Figure 4–18, find the voltage gain $K = v_{\rm O}/v_{\rm IN}$.

The voltage $v_{\rm IN}$ appears across resistor $R_{\rm E}$. The current through $R_{\rm E}$ is $i_{\rm E} = i_{\rm IN} + \beta i_{\rm IN} = (\beta+1)i_{\rm IN}$. So we have $v_{\rm IN} = i_{\rm E}R_{\rm E} = (\beta+1)i_{\rm IN}R_{\rm E}$. The dependent source determines the current through the load resistor and the output voltage as $v_{\rm O} = -\beta i_{\rm IN}R_{\rm L}$. Compute the gain.

$$K = \frac{v_{\rm O}}{v_{\rm IN}} = \frac{-\beta i_{\rm IN}R_{\rm L}}{(\beta+1)i_{\rm IN}R_{\rm E}} = -\frac{\beta R_{\rm L}}{(\beta+1)R_{\rm E}}$$

Exercise 4–12. Find the input resistance and output Thévenin equivalent circuit of the circuit in Figure 4–20.

First note that $v_{\rm F} = v_{\rm S} - \mu v_{\rm F}$, which implies $v_{\rm F} = v_{\rm S}/(1+\mu)$. The input voltage is $v_{\rm S}$ and the input current is $i_{\rm IN}$.

$$i_{\rm IN} = \frac{v_{\rm F}}{R_{\rm F}} = \frac{\dfrac{v_{\rm S}}{1+\mu}}{R_{\rm F}} = \frac{v_{\rm S}}{(1+\mu)R_{\rm F}}$$

Compute the input resistance.

$$R_{\rm IN} = \frac{v_{\rm IN}}{i_{\rm IN}} = \frac{v_{\rm S}}{\frac{v_{\rm S}}{(1+\mu)R_{\rm F}}} = (1+\mu)R_{\rm F}$$

For the Thévenin equivalent, with the load removed, the dependent source determines the open-circuit voltage.

$$v_{\rm OC} = v_{\rm T} = \mu v_{\rm F} = \frac{\mu v_{\rm S}}{1+\mu}$$

The dependent source also controls the short-circuit current.

$$i_{\rm SC} = \frac{\mu v_{\rm F}}{R_{\rm O}} = \frac{\mu v_{\rm S}}{(1+\mu)R_{\rm O}}$$

Compute the Thévenin resistance.

$$R_{\rm T} = \frac{v_{\rm OC}}{i_{\rm SC}} = R_{\rm O}$$

Exercise 4–13. The known parameters in Figure 4–25 are $\beta = 100$, $V_\gamma = 0.7$ V, $R_C = 1$ kΩ, and $V_{CC} = 5$ V. The circuit is to function as a digital inverter that meets two conditions: (1) An input of $v_S = 0$ V must produce an output $v_{CE} = 5$ V. (2) An input of $v_S = 5$ V must produce an output $v_{CE} = 0$ V. Select a value of R_B so that the circuit meets these conditions.

For the first condition, $v_S = 0$ implies $v_S < V_\gamma$ and $i_C < 0$, so the transistor is in cutoff mode. In cutoff mode, we have $i_C = 0$ and $v_{CE} = v_{OC} = V_{CC} = 5$ V, which satisfies the first condition. For the second condition, $v_S = 5$ V and $v_S > V_\gamma$. To get $v_{CE} = 0$ V, we must be in saturation mode with $i_C > i_{SC} = V_{CC}/R_C = 5/1000 = 5$ mA. Use the equation for i_C to solve for R_B.

$$i_C = \beta\left(\frac{v_S - V_\gamma}{R_B}\right) \geq 0.005$$

$$100\left(\frac{5 - 0.7}{R_B}\right) \geq 0.005$$

$$R_B \leq \frac{430}{0.005}$$

$$R_B \leq 86 \text{ k}\Omega$$

Choose $R_B = 56$ kΩ as a reasonable standard value.

Exercise 4–14. Design a noninverting amplifier circuit with a gain of 7 ±10% using standard 10% resistors. (See inside back cover for standard values.)

The gain for a noninverting amplifier is given by $K = (R_1 + R_2)/R_2$ and must fall between 6.3 and 7.7 to be within 10% of 7. The following table presents reasonable solutions.

R_1 (kΩ)	R_2 (kΩ)	K
5.6	1	6.60
6.8	1.2	6.67
8.2	1.5	6.47
10	1.5	7.67
10	1.8	6.56
12	1.8	7.67
12	2.2	6.45
15	2.7	6.56
18	2.7	7.67
18	3.3	6.45
22	3.3	7.67
22	3.9	6.64
27	4.7	6.74
33	5.6	6.89
39	6.8	6.74
47	8.2	6.73

Exercise 4–15. The noninverting amplifier circuit in Figure 4–32(a) is operating with $R_1 = 2R_2$ and $V_{CC} = \pm 12$ V. Over what range of input voltages v_S is the OP AMP in the linear mode?

Find the gain of the amplifier.

$$K = \frac{R_1 + R_2}{R_2} = \frac{2R_2 + R_2}{R_2} = 3$$

Find the corresponding range of input voltages.

$$-12 < v_O < 12$$

$$-12 < Kv_S < 12$$

$$-12 < 3v_S < 12$$

$$-4 < v_S < 4$$

The magnitude of the input voltage should be less than 4 V.

Exercise 4–16. (a). Find v_O in Figure 4–35(a) when $R_1 = R_2 = 1$ kΩ, $v_S = 1$ V, $R_3 = R_4 = 1$ kΩ, and $R_L = 100$ Ω.

Using the results of Example 4–13, the circuit gain is K.

$$K = \left[\frac{R_2}{R_1 + R_2}\right]\left[\frac{R_3 + R_4}{R_4}\right] = \left[\frac{1}{1+1}\right]\left[\frac{1+1}{1}\right] = 1$$

We have $v_O = Kv_S = (1)(1) = 1$ V.

(b). Repeat for when R_3 is a short circuit and the other values are the same.

Using the results of Example 4–13, the circuit gain is K.

$$K = \left[\frac{R_2}{R_1 + R_2}\right]\left[\frac{R_3 + R_4}{R_4}\right] = \left[\frac{1}{1+1}\right]\left[\frac{0+1}{1}\right] = 0.5$$

We have $v_O = Kv_S = (0.5)(1) = 0.5$ V.

Exercise 4–17. A DMM with a known internal resistance of 12.5 MΩ is used to measure voltages across several resistors in the circuit shown in Figure 4–38. What voltage will be measured on the DMM across each resistor?

For each measurement, compute the equivalent resistance of the DMM in parallel with the selected resistor.

$$R_{EQ1} = \frac{R_1 R_M}{R_1 + R_M}$$

Apply voltage division to find the measured voltage.

$$v_1 = \frac{R_{EQ1}}{R_{EQ1} + R_2 + R_3} v_S$$

The following table summarizes the results

Resistor	Expected Voltage (V)	Measured Voltage (V)	Percent Error
50 kΩ	0.237	0.236	0.41%
10 MΩ	47.4	45.5	4.01%
500 kΩ	2.37	2.28	3.68%

Exercise 4–18. The circuits in Figure 4–36 have $v_S = 1.5$ V, $R_S = 2$ kΩ, and $R_L = 1$ kΩ. Compute the maximum power available from the source. Compute the power absorbed by the load resistor in the direct connection in Figure 4–36(b) and in the voltage follower circuit in Figure 4–32(a). Discuss any differences.

The maximum power that the source can deliver to a load occurs when the load resistance matches R_S. The maximum power is then given by $p_{MAX} = v_S^2/(4R_S) = 2.25/8000 = 281.25\ \mu$W. In the direct connection circuit, the current through the load is $i_L = v_S/(R_S + R_L) = 1.5/3000 = 500\ \mu$A. The power delivered to the load is $p_L = i_L^2 R_L = (0.0005)^2(1000) = 250\ \mu$W. In the voltage follower circuit, the voltage across the load is v_S, so the power delivered to the load is $p_L = v_L^2/R_L = 2.25/1000 = 2.25$ mW.

With the direct connection, the power delivered to the load is less than the maximum power available. With the voltage follower circuit, the power delivered to the load is greater than the maximum value specified by the maximum power transfer theorem. However, the maximum power transfer theorem does not apply to the voltage follower circuit since the load power comes from the OP AMP power supply rather than the signal source.

Exercise 4–19. The switch in Figure 4–40 moves from A to B. What is the output voltage v_O when the switch is in position A and in position B?

The gain of the amplifier is $K = -R_F/R_S$. With the switch in position A, the gain is $K = -100/10 = -10$. The input voltage is 1 V, so the output voltage is $v_O = -10$ V. With the switch in position B, the gain is $K = -33$. The gain times the input voltage yields -33 V, but this value is outside the range of V_{CC}. With the switch in position B, the OP AMP is saturated and the output voltage is $v_O = -15$ V.

Exercise 4–20. A 2-mV signal v_S needs to be amplified by a gain of $-450 \pm 10\%$ using standard 10% resistors from the inside back cover. Design an appropriate circuit to amplify the signal.

The gain is negative, so use an inverting amplifier. There are many reasonable designs that fit within the tolerances. One design with $R_1 = 1$ kΩ and $R_2 = 470$ kΩ yields a gain of $K = -470$, which meets the specification.

Exercise 4–21. Find v_O in Figure 4–42 when $v_S = 2$ V. Repeat for $v_S = -4$ V and $v_S = 6$ V.

The inverting OP AMP has a gain of $K = -33/10 = -3.3$. For an input of 2 V, the output is $v_O = (-3.3)(2) = -6.6$ V. With an input of -4 V, the output is $v_O = (-3.3)(-4) = 13.2$ V. With an input of 6 V, the gain times the input voltage is -19.8 V, but this is outside the limits of V_{CC}, so the output is $v_O = -15$ V, because the amplifier is saturated.

Exercise 4–22. In Figure 4–43, $v_1 = 0.6$ V, $v_2 = 0.4$ V, $R_1 = 3.3$ kΩ, $R_2 = 4.7$ kΩ, and $R_F = 15$ kΩ. Find v_O.

Apply the gain formula for the inverting summer OP AMP.

$$v_O = \left(-\frac{R_F}{R_1}\right)v_1 + \left(-\frac{R_F}{R_2}\right)v_2 = \left(-\frac{15}{3.3}\right)(0.6) + \left(-\frac{15}{4.7}\right)(0.4) = -2.7273 - 1.2766 = -4.004 \text{ V}$$

Exercise 4–23. (a). Find v_O in Figure 4–44(a) when $v_1 = 2$ V and $v_2 = -0.5$ V.

Find the expression for the output voltage and substitute in the given values.

$$v_O = \left(-\frac{R_F}{R_1}\right)v_1 + \left(-\frac{R_F}{R_2}\right)v_2 = \left(-\frac{65}{13}\right)(2) + \left(-\frac{65}{5}\right)(-0.5) = -10 + 6.5 = -3.5 \text{ V}$$

(b). If $v_1 = 400$ mV and $V_{CC} = \pm 15$ V, what is the maximum value of v_2 for linear mode operation?

With a maximumized input and linear operation, keep $v_O \geq -V_{CC} = -15$ V.

$$v_O = -5v_1 - 13v_2 \geq -15$$

$$(-5)(0.4) - 13v_2 \geq -15$$

$$-13v_2 \geq -13$$

$$v_2 \leq 1 \text{ V}$$

(c). If $v_1 = 500$ mV and $V_{CC} = \pm 15$ V, what is the minimum value of v_2 for linear mode operation?

With a minimized input and linear operation, keep $v_O \leq +V_{CC} = 15$ V.

$$v_O = -5v_1 - 13v_2 \leq 15$$

$$(-5)(0.5) - 13v_2 \leq 15$$

$$-13v_2 \leq 17.5$$

$$v_2 \geq -1.346 \text{ V}$$

Exercise 4–24. Design a non-inverting summer for four inputs with equal gains of 50.

Following the development in Example 4–17, since the inputs have the same gains, choose the input resistors to be the same with $R = 10\ \text{k}\Omega$, for example. Then choose the gain of the amplifier to be 200, so that the gain for each of the four inputs will be $200/4 = 50$. For a gain of 200, select $R_{\text{A}} = 1\ \text{k}\Omega$ and $R_{\text{B}} = 199\ \text{k}\Omega$.

Exercise 4–25. (a). Find the input-output relationship of the subtractor circuit in Figure 4–47.

Apply the gain relationship for the subtractor OP AMP circuit.

$$v_{\text{O}} = -\left[\frac{R_2}{R_1}\right] v_1 + \left[\frac{R_4}{R_3+R_4}\right]\left[\frac{R_1+R_2}{R_1}\right] v_2 = -\left[\frac{40}{10}\right] v_1 + \left[\frac{15}{10+15}\right]\left[\frac{10+40}{10}\right] v_2 = -4v_1 + 3v_2$$

(b). If $V_{\text{CC}} = \pm 15$ V and $v_1 = 3$ V, what is the allowable range of v_2 for linear operation of the OP AMP?

The output voltage must not saturate the OP AMP.

$$-15 \le v_{\text{O}} \le 15$$

$$-15 \le -4v_1 + 3v_2 \le 15$$

$$-15 \le -4(3) + 3v_2 \le 15$$

$$-15 \le -12 + 3v_2 \le 15$$

$$-3 \le 3v_2 \le 27$$

$$-1\ \text{V} \le v_2 \le 9\ \text{V}$$

Exercise 4–26. Derive an expression for v_{O} in Figure 4–50 in terms of the inputs v_1 and v_2.

The first OP AMP stage is an inverting amplifier with a gain of $K_1 = -40/10 = -4$. The second stage is an inverting summer with gains of $K_{21} = -40/20 = -2$ and $K_{22} = -40/10 = -4$. Cascade the gains together through multiplication to get the overall input-output relationship.

$$v_{\text{O}} = K_1 K_{21} v_1 + K_{22} v_2 = (-4)(-2)v_1 + (-4)v_2 = 8v_1 - 4v_2$$

Exercise 4–27. Using the circuit and analysis shown in Example 4–19, design a circuit using standard 5% resistors from the inside back cover that realizes the expression $v_{\text{O}} = -20v_1 + 10v_2$.

Many solutions are possible. The output voltage for the circuit is

$$v_{\text{O}} = -\left[\frac{R_4}{R_3}\right]\left[\frac{R_1+R_2}{R_1}\right] v_1 + \left[\frac{R_3+R_4}{R_3}\right] v_2 = -K_1 v_1 + K_2 v_2$$

First, select $R_3 = 1\ \text{k}\Omega$ and $R_4 = 9.1\ \text{k}\Omega$ to get $K_2 = 10.1$, which is close to the desired value of 10. Next, choose $R_1 = 9.1\ \text{k}\Omega$ to cancel R_4 in the expression for K_1 and then choose $R_2 = 11\ \text{k}\Omega$ to get $K_1 = 20.1$, which is close to the desired value of 20.

Exercise 4–28. Select values of R_1, R_2, and R_3 in Figure 4–53 so that $v_{\text{O}} = 50(v_2 - v_1)$.

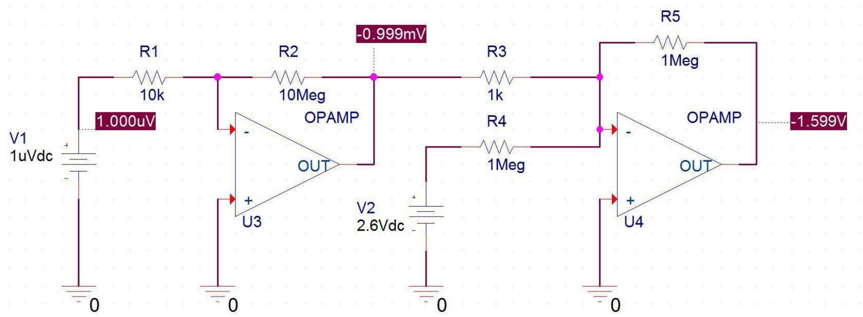
From Example 4–20, the output expression is

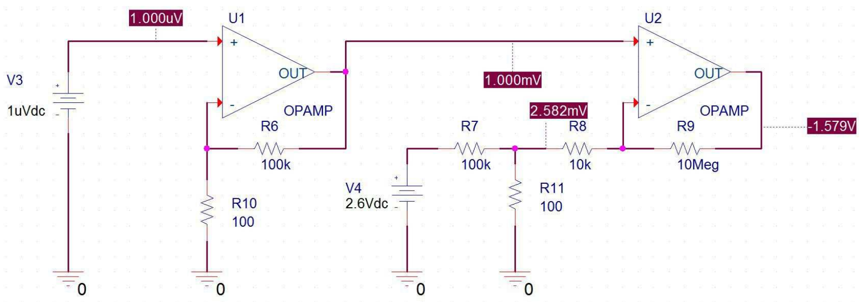
$$v_{\text{O}} = \left[\frac{R_1+R_2+R_3}{R_2}\right](v_2 - v_1)$$

Choose $R_2 = 1\ \text{k}\Omega$ and then $R_1 = R_3 = 24.5\ \text{k}\Omega$ to meet the specifications. There are many possible solutions.

Exercise 4–29. Verify the solutions found in Evaluation Example 4–21 using OrCAD.

The following OrCAD simulations verify the results.





Exercise 4–30. Using the circuits and analyses shown in Design Example 4–22, how much power is being provided by the signal source in each design? What is providing the power to the load?

In Circuit 1, no current flows from the source into the interface circuit. Since no current is flowing, the 5-V source provides no power to the load. In Circuit 2, the resistance seen by the source is 25050 Ω and the power provided by the source is $p_S = v_S^2/R_{IN} = 25/25050 = 998\ \mu$W. In both cases, the load is receiving 200 mW, so the amplifier circuits are providing the power to the load.

Exercise 4–31. A requirement exists for a circuit that implements the block diagram in Figure 4–59(a). The circuit in Figure 4–59(b) is a proposed solution. A breadboard prototype of this circuit failed to pass preliminary testing. Why? Hint: The circuit contains four errors. What are they?

The four errors are as follows: (1) The noninverter produces a gain of 11, not the desired 10. (2) The gain from the noninverter into the inverting summer is −5, not −20. (3) The 5-V source is inverted, resulting in a final value of +5 V instead of the desired −5 V. (4) A common error for new students, the inverting summer has the positive and negative terminals reversed.

Exercise 4–32. The R-$2R$ ladder DAC in Figure 4–62 has $R_F = 40$ kΩ, $R = 10$ kΩ, and $V_{REF} = -3$ V. Find the full-scale output and resolution of the converter.

The full-scale output has all four bits set to one.

$$v_O = -\frac{R_F}{2R}V_{REF}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8}\right) = -\frac{40}{20}(-3)\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) = 11.25 \text{ V}$$

The resolution is the smallest possible voltage change.

$$\text{Res} = \frac{R_F}{2R}\left|V_{REF}\right|\left(\frac{1}{8}\right) = \frac{40}{20}(3)\left(\frac{1}{8}\right) = 0.75 \text{ V}$$

Exercise 4–33. A student chooses to design a 6-bit weighted sum DAC for a project. The requirement is to have errors of 1% or less. What tolerance (accuracy) must the resistors used in the design have to meet the requirement?

Assume we can control the value for R in the design. Whatever value the designer chooses for R, the $32R$ resistor must meet the tolerance specification. Therefore, the tolerance for R is at most $1\%/32 = 0.03125\%$. An R-$2R$ design is probably a better option.

Exercise 4–34. A pressure transducer must be connected to a boiler. The selected transducer is linear between 100 and 1000 psi. Specifically, it has the following characteristics: At 100 psi it produces 10 μV, and at 1000 psi it produces 100 μV. The output needs to be connected to a 0-10 V meter so that 100 psi will give a reading of 0 V and 1000 psi a reading of 10 V. Design a suitable interface using OP AMPs that have a maximum closed-loop gain of 2000.

Compute the gain for the interface.

$$K = \frac{v_{\rm O1} - v_{\rm O2}}{v_{\rm TR1} - v_{\rm TR2}} = \frac{0 - 10}{0.00001 - 0.0001} = 111111$$

Solve for the bias voltage.

$$K(0.00001) + V_{\rm b} = 0$$

$$V_{\rm B} = -1.111 \text{ V}$$

To get a gain of 111111, we must use two OP AMP stages. The first stage is an inverting ammplifier with a gain of $K_1 = -100$. The second stage is an inverting summer. The first stage connects to a gain of $K_{21} = -1111$ and a voltage source of 1.111 V connects to a gain of -1. Figure 4–69 provides one solution.

Exercise 4–35. Use a subtractor to design the interface for the example above.

Use all 1-MΩ resistors in a standard subtractor circuit. Apply the transducer input to the wire leading to the negative terminal of the OP AMP and apply the 10-V bias to the input leading to the positive terminal of the OP AMP. The output is given by the following expression.

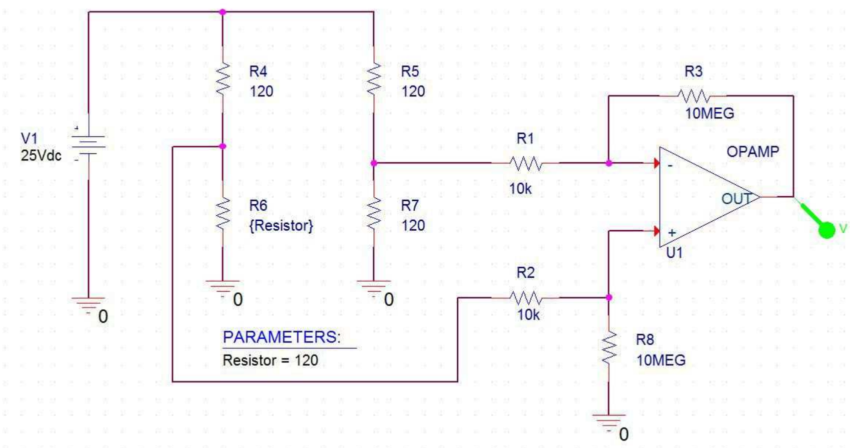
$$v_{\rm O} = -\frac{R_2}{R_1}v_1 + \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)v_2 = -v_1 + v_2 = -v_{\rm TR} + 10 \text{ V}$$

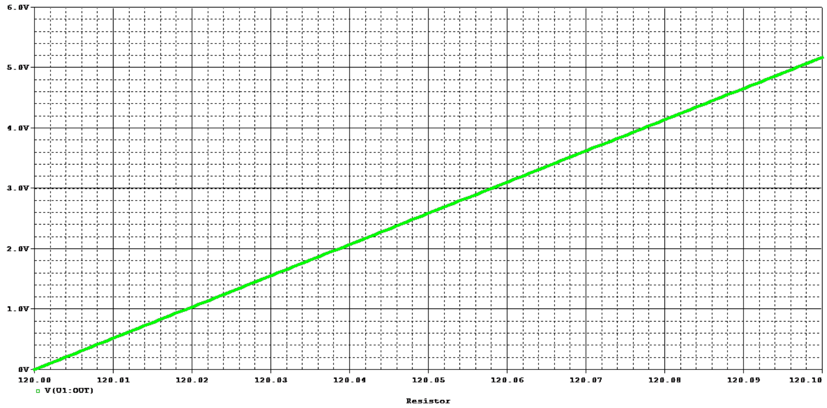
Exercise 4–36. A 2-kΩ potentiometer is connected to the flaps of an unmanned aerial vehicle, or UAV, to detect their position. When the flaps are at their maximum upward extension of $+45^\circ$, the potentiometer is at its maximum resistance of 2 kΩ; when the flaps are flat or 0°, the potentiometer is set at 1 kΩ; and when the flaps are at their minimum downward extension of -45°, the potentiometer is at its minimum resistance of 0 kΩ. Design an interface to a 0-5 V ADC that gives $+45^\circ$ at 5 V and -45° at 0 V.

Connect the potentiometer to a 5-V source. The output voltage is then proportional to the position of the potentiometer. At the maximum upward position, all 5 V appears across the potentiometer. In the flat position, the potentiometer is a voltage divider with two equal resistors, so the output voltage is 2.5 V. In the minimum downward position, the potentiometer has no resistance and no output voltage.

Exercise 4–37. Using the circuit shown in Figure 4–76, simulate the effect in OrCAD of varying the strain gauge $R_{\rm G2}$ from 120 Ω to 120.1 Ω in increments of 0.001 Ω. Plot the resulting output versus percent of stress ($\Delta L/L$).

The following OrCAD simulation and plot provide the results.





Exercise 4–38. Design an instrumentation amplifier using the configuration of Figure 4–78 that has a gain of 10^5. Note that no single stage can have a gain greater than 10^4.

The first stage has a gain of $K_1 = 2R/R_{\rm g} + 1$ and the second stage has a gain of $K_2 = K$. Let $K_1 = 201$ and $K_2 = 500$ to get a total gain of $K_l K_2 = (201)(500) = 100500$, which is close to the desired value of 10^5 and meeting the limitations for gain on each stage. To design K_1, choose $R = 10$ kΩ and $R_{\rm g} = 100$. Then, the KR resistor has a value of 5 MΩ.

Exercise 4–39. Find the comparator output voltage in Figure 4–80 for the following:

(a). $v_1 = 2$ V, $v_2 = 3$ V, $V_{\rm CC} = 5$ V.

We have $v_1 < v_2$, so $v_{\rm O} = -V_{\rm CC} = 0$ V.

(b). $v_1 = 0$ V, $v_2 = -3$ V, $V_{\rm CC} = 10$ V.

We have $v_1 > v_2$, so $v_{\rm O} = +V_{\rm CC} = 10$ V.

(c). $v_1 = -2$ V, $v_2 = -3$ V, $V_{\rm CC} = 3$ V.

We have $v_1 > v_2$, so $v_{\rm O} = +V_{\rm CC} = 3$ V.

Exercise 4–40. The reference voltage in Figure 4–84 is $V_{\rm REF} = 15$ V. What are the output codes corresponding to $v_{\rm S} = 1$, 2, 5, 10, and 14 V?

The voltage ranges for the circuit are 0 to 3, 3 to 6, 6 to 9, 9 to 12, and 12 to 15 V. Using the table of digital output values, the following table summarizes the results.

$v_{\rm S}$ (V)	Output Code
1	(0,0,0,0)
2	(0,0,0,0)
5	(0,0,0,1)
10	(0,1,1,1)
14	(1,1,1,1)

4.2 Problem Solutions

Problem 4–1. Find the voltage gain v_O/v_S and current gain i_O/i_x in Figure P4–1 for $r = 5$ kΩ.

Apply Ohm's law to find $i_x = v_S/(100 + 400) = v_S/500$. The voltage of the dependent source is $-ri_x = -rv_S/500$. Apply voltage division to find the output voltage and then the voltage gain.

$$v_O = \frac{2000}{500 + 2000}(-ri_x) = -\frac{(2000)(5000)}{(2500)(500)}v_S = -8v_S$$

$$\frac{v_O}{v_S} = -8$$

Apply Ohm's law to find the output current $i_O = -ri_x/(500 + 2000) = -5000i_x/2500 = -2i_x$. The current gain is then $i_O/i_x = -2$.

Problem 4–2. Find the voltage gain v_O/v_1 and current gain i_O/i_S in Figure P4–2. For $i_S = 5$ mA, find the power supplied by the input current source and the power delivered to the 2-kΩ load resistor.

On the source side of the circuit, the current divides equally between the two paths with 100-Ω resistors, so $i_1 = i_S/2$ and $v_1 = 100i_S/2 = 50i_S$. The dependent current source is $-100i_1 = -50i_S$. Apply two-path current division to find i_O and then apply Ohm's law to find v_O.

$$i_O = \frac{2000}{2000 + 2000}(-100i_1) = \frac{1}{2}(-50i_S) = -25i_S$$

$$\frac{i_O}{i_S} = -25$$

$$v_O = 2000i_O = (2000)(-25i_S) = -50000i_S$$

$$\frac{v_O}{v_1} = \frac{-50000i_S}{50i_S} = -1000$$

For $i_S = 5$ mA, the input voltage is $v_1 = 50i_S = (50)(0.005) = 250$ mV and the power supplied by the source is $p_S = v_1 i_S = (.25)(0.005) = 1.25$ mW. The load current is $i_O = -25i_S = -125$ mA and the power delivered to the load is $p_L = i_O^2 R_L = (-0.125)^2(2000) = 31.25$ W.

Problem 4–3. Find the voltage gain v_O/v_S and current gain i_O/i_x in Figure P4–3 for $g = 2 \times 10^{-3}$ S. For $v_S = 5$ V, find the power supplied by the input voltage source and the power delivered to the 2-kΩ load resistor.

Apply Ohm's law to find the input current $i_x = v_S/(1000 + 3000) = v_S/4000$. Apply voltage division to find $v_x = 3v_S/(1+3) = 3v_S/4$. Apply two-path current division to find the output current and then find the output voltage and the associated gains.

$$i_O = \frac{10}{10 + 0.5 + 2}(gv_x) = \frac{10}{12.5}(0.002)\frac{3v_S}{4} = \frac{3}{2500}v_S$$

$$v_O = 2000i_O = \frac{6000}{2500}v_S = 2.4v_S$$

$$\frac{v_O}{v_S} = 2.4$$

$$\frac{i_O}{i_x} = \left[\frac{3}{2500}v_S\right]\left[\frac{4000}{v_S}\right] = 4.8$$

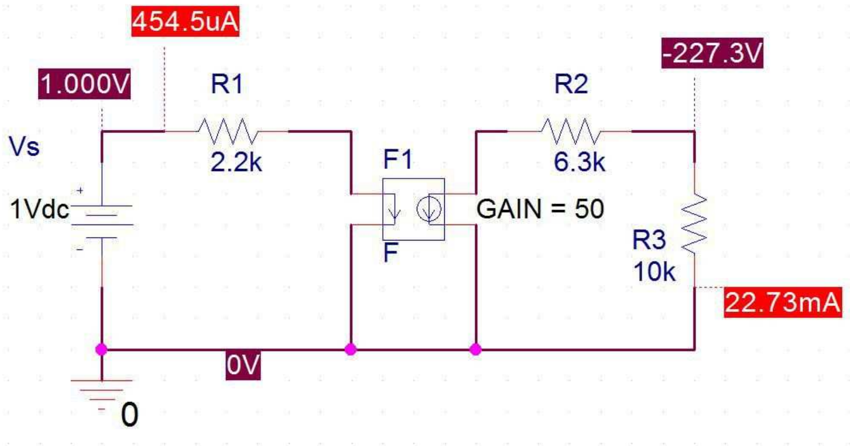
For $v_S = 5$ V, the input current is $i_x = 5/4000 = 1.25$ mA and the power supplied by the source is $p_S = v_S i_x = (5)(0.00125) = 6.25$ mW. The output voltage is $v_O = (2.4)(5) = 12$ V and the output current is $i_O = (3)(5)/2500 = 6$ mA, so the power delivered to the load resistor is $p_L = v_O i_O = (12)(0.006) = 72$ mW.

Problem 4–4. (a). Find the voltage gain v_O/v_S and current gain i_O/i_x in Figure P4–4.

Apply Ohm's law to find the input current $i_x = v_S/2200$. The dependent source current is $50i_x = 50v_s/2200 = v_S/44$. The output current has the opposite direction as the dependent current source, so $i_O = -50i_x = -v_S/44$ and the current gain is $i_O/i_x = -50$. The output voltage is $v_O = 10000i_O = -10000v_S/44 = -227.27v_S$, so the voltage gain is $v_O/v_S = -227.27$.

(b). Validate your answers by simulating the circuit in OrCAD.

The following OrCAD simulation verifies the results.



The voltage gain is $-227.3\,\text{V}/1\,\text{V} = -227.3$ and the current gain is $-22.73\,\text{mA}/454.5\,\mu\text{A} = -50$.

Problem 4–5. Find the voltage gain v_O/v_S in Figure P4–5.

The dependent source is connected in parallel with the output resistor, but has the opposite polarity, so $v_O = -50v_x$. Solve for v_x in terms of v_S.

$$v_x = v_S - (-50v_x) = v_S + 50v_x$$

$$-49v_x = v_S$$

$$v_x = -\frac{v_S}{49}$$

$$v_O = -50v_x = \frac{50}{49}v_S$$

$$\frac{v_O}{v_S} = \frac{50}{49}$$

Problem 4–6. Find an expression for the current gain i_O/i_S in Figure P4–6. (*Hint:* Apply KCL at node A.)

First note that the dependent current source controls the output current $i_O = -\beta i_E$, which implies $i_E = -i_O/\beta$. Apply KCL to sum the currents entering node A.

$$i_S + \beta i_E - i_E = 0$$

$$i_S = (1-\beta)i_E = (1-\beta)\left(\frac{-i_O}{\beta}\right) = \left(\frac{\beta-1}{\beta}\right)i_O$$

$$\frac{i_O}{i_S} = \frac{\beta}{\beta-1}$$

Problem 4–7. (a). Find the voltage v_O in Figure P4–7.

Use node-voltage analysis with nodes v_x and v_O. The corrresponding equations are shown below.

$$\frac{v_x - 5}{1000} + \frac{v_x}{1500} + \frac{v_x - v_O}{1000} + \frac{v_x}{1000} = 0$$

$$-\frac{v_x}{1000} + \frac{v_O - v_x}{1000} + \frac{v_O}{4700} = 0$$

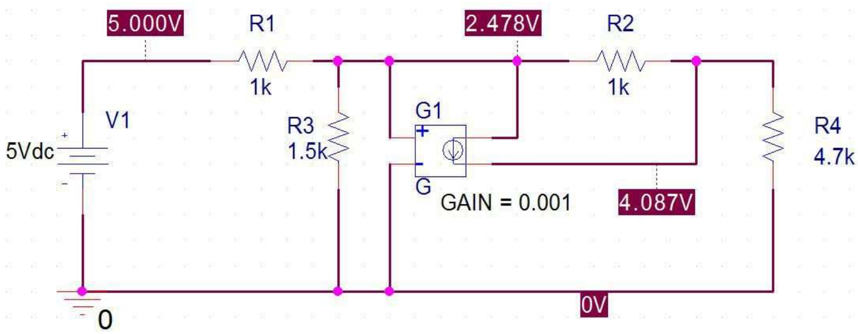
In matrix form, we have the following.

$$\begin{bmatrix} \frac{1}{1000} + \frac{1}{1500} + \frac{1}{1000} + \frac{1}{1000} & -\frac{1}{1000} \\ -\frac{1}{1000} - \frac{1}{1000} & \frac{1}{1000} + \frac{1}{4700} \end{bmatrix} \begin{bmatrix} v_x \\ v_O \end{bmatrix} = \begin{bmatrix} \frac{5}{1000} \\ 0 \end{bmatrix}$$

Solve the equations for $v_x = 2.4783$ V and $v_O = 4.087$ V.

(b). Validate your answer by simulating the circuit in OrCAD.

The following OrCAD simulation verifies the results.



Problem 4–8. (a). Find an expression for the gain i_O/v_S in Figure P4–8 in terms of R_x.

The dependent source directly controls the output current so that $i_O = -1000v_x/2200 = -5v_x/11$. Write a node-voltage equation for voltage v_x and solve for v_x in terms of v_S.

$$\frac{v_x - v_S}{1000} + \frac{v_x - (-1000v_x)}{R_x} = 0$$

$$R_x v_x - R_x v_S + 1000v_x + 1000000v_x = 0$$

$$(R_x + 1001000)v_x = R_x v_S$$

$$v_x = \frac{R_x}{R_x + 1001000} v_S$$

Solve for the output current and the gain.

$$i_O = -\frac{5}{11}v_x = -\frac{5R_x}{11(R_x + 1001000)} v_S$$

$$\frac{i_O}{v_S} = -\frac{5R_x}{11(R_x + 1001000)}$$

(b). Select a value for R_x so that the gain is -0.227.

Apply the results from part (a).

$$-0.227 = -\frac{5R_x}{11(R_x + 1001000)}$$

$$2.497(R_x + 1001000) = 5R_x$$

$$2.503R_x = 2499500$$

$$R_x = 998.6 \text{ k}\Omega$$

A 1-MΩ resistor will work.

Problem 4–9. Find an expression for the voltage gain v_O/v_S in Figure P4–9.

Note that $v_x = v_O - v_S$ and write a node-voltage equation at the output node.

$$\frac{v_O - v_S}{R_S} - gv_x + \frac{v_O}{R_O} = 0$$

$$R_O v_O - R_O v_S - gR_S R_O(v_O - v_S) + R_S v_O = 0$$

$$(R_O - gR_S R_O + R_S)v_O = (R_O - gR_O R_S)v_S$$

$$\frac{v_O}{v_S} = \frac{R_O(1 - gR_S)}{R_O + R_S - gR_O R_S}$$

Problem 4–10. (a). Find an expression for the voltage gain v_O/v_S in Figure P4–10.

Let node A be between resistors R_S and R_F. Note that $v_x = v_A - \mu v_x$, which implies $v_A = (1+\mu)v_x$. Write a node-voltage equation at node A.

$$\frac{v_A - v_S}{R_S} + \frac{v_A - \mu v_x}{R_F} = 0$$

$$\frac{(1+\mu)v_x - v_S}{R_S} + \frac{v_x}{R_F} = 0$$

$$R_F(1+\mu)v_x - R_F v_S + R_S v_x = 0$$

$$[R_F(1+\mu) + R_S]\, v_x = R_F v_S$$

$$v_x = \frac{R_F}{R_F(1+\mu) + R_S} v_S$$

$$v_O = \mu v_x = \frac{\mu R_F}{R_F(1+\mu) + R_S} v_S$$

$$\frac{v_O}{v_S} = \frac{\mu R_F}{R_F(1+\mu) + R_S}$$

(b). Let $R_S = 10$ kΩ, $R_L = 10$ kΩ, and $\mu = 100$. Find the voltage gain v_O/v_S as a function of R_F. What is the voltage gain when R_F is an open circuit, a short circuit, and for $R_F = 100$ Ω.

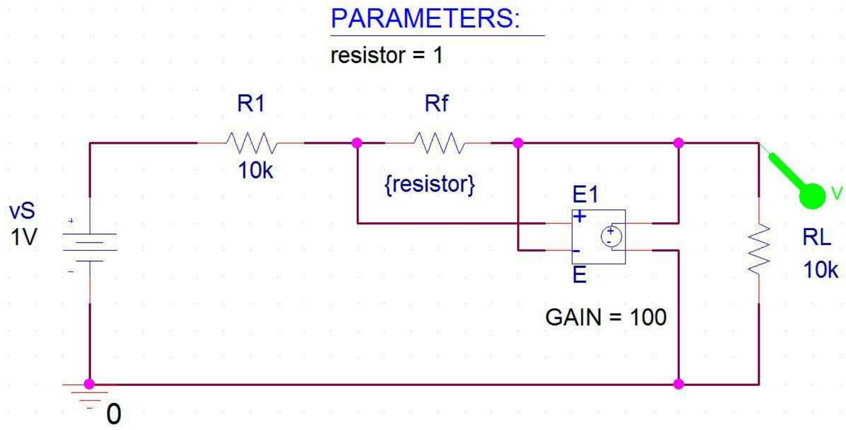
Use the results from part (a).

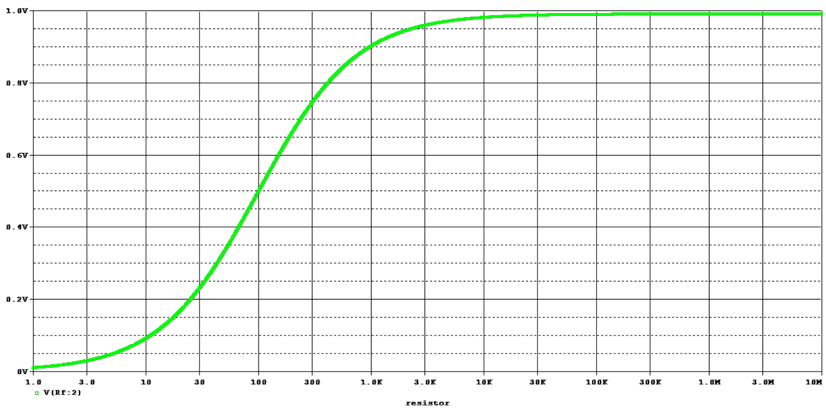
$$\frac{v_O}{v_S} = \frac{\mu R_F}{R_F(1+\mu) + R_S} = \frac{100R_F}{101R_F + 10000}$$

When R_F is an open circuit, $R_F \to \infty$ and the gain is $\mu/(1+\mu) = 100/101$. When R_F is a short circuit, $R_F = 0$ and the gain is zero. When $R_F = 100$ Ω, the gain is 0.4975.

(c). Simulate the circuit in OrCAD by varying R_{F} from 1 Ω to 10 MΩ. Read your output when $R_{\mathrm{F}} = 100$ Ω. How does your answer compare with part (b)?

The following OrCAD simulation presents the results.





The input signal is 1 V. Reading the output plot, the output voltage is approximately 0.5 V with $R_{\mathrm{F}} = 100$ Ω, which agrees with the calculations in part (b) for the gain.

Problem 4–11. Select g in the circuit of Figure P4–11 so that the output voltage is 10 V.

The input side of the circuit is an open circuit, so $v_{\mathrm{x}} = v_{\mathrm{S}} = 1$ mV. The dependent current is $gv_{\mathrm{x}} = 0.001g$. The output voltage is $v_{\mathrm{O}} = i_{\mathrm{O}}R_{\mathrm{O}} = (0.001g)(2200) = 2.2g$. If $v_{\mathrm{O}} = 10$ V, then $g = 10/2.2 = 4.545$ S.

Problem 4–12. Design a dependent source circuit that has a closed-loop voltage gain of −10 using a VCVS with a μ of 100. The source circuit is a voltage source v_{S} in series with a 1-kΩ resistor, and the load is a 3.3-kΩ resistor. (*Hint:* See Figure P4–10.)

Use the circuit in Figure P4–10, but change the polarity of the dependent voltage source. Let the node between R_{S} and R_{F} be node A with voltage v_{A}. We then have $v_{\mathrm{x}} = v_{\mathrm{A}} - (-\mu v_{\mathrm{x}})$, which implies $v_{\mathrm{A}} = (1-\mu)v_{\mathrm{x}}$.

Write the node voltage equation at node A and solve for $R_{\rm F}$ to complete the design.

$$\begin{aligned}
\frac{v_{\rm A}-v_{\rm S}}{R_{\rm S}}+\frac{v_{\rm A}-(-\mu v_{\rm x})}{R_{\rm F}} &= 0 \\
\frac{(1-\mu)v_{\rm x}-v_{\rm S}}{R_{\rm S}}+\frac{v_{\rm x}}{R_{\rm F}} &= 0 \\
R_{\rm F}(1-\mu)v_{\rm x}-R_{\rm F}v_{\rm S}+R_{\rm S}v_{\rm x} &= 0 \\
\left[R_{\rm F}(1-\mu)+R_{\rm S}\right]v_{\rm x} &= R_{\rm F}v_{\rm S} \\
v_{\rm x} &= \frac{R_{\rm F}}{R_{\rm F}(1-\mu)+R_{\rm S}}v_{\rm S} \\
v_{\rm O}=-\mu v_{\rm x} &= -\frac{\mu R_{\rm F}}{R_{\rm F}(1-\mu)+R_{\rm S}}v_{\rm S} \\
\frac{v_{\rm O}}{v_{\rm S}} &= -\frac{\mu R_{\rm F}}{R_{\rm F}(1-\mu)+R_{\rm S}} \\
-10 &= -\frac{100R_{\rm F}}{R_{\rm F}(1-100)+1000} \\
-990R_{\rm F}+10000 &= 100R_{\rm F} \\
1090R_{\rm F} &= 10000 \\
R_{\rm F} &= 9.17\ \Omega
\end{aligned}$$

Problem 4–13. Find the Thévenin equivalent circuit that the load $R_{\rm L}$ sees in Figure P4–13. Repeat the problem with $R_{\rm F}$ replaced by an open circuit.

In the original circuit and with the load removed, the open-circuit voltage is $v_{\rm OC}=v_{\rm T}=-\mu v_{\rm x}$. The short-circuit current $i_{\rm SC}=-\mu v_{\rm x}/R_{\rm P}$. The Thévenin resistance is $R_{\rm T}=v_{\rm OC}/i_{\rm SC}=R_{\rm P}$. Write the node-voltage equation for $v_{\rm x}$.

$$\begin{aligned}
\frac{v_{\rm x}-v_{\rm S}}{R_{\rm S}}+\frac{v_{\rm x}-(-\mu v_{\rm x})}{R_{\rm F}} &= 0 \\
R_{\rm F}v_{\rm x}-R_{\rm F}v_{\rm S}+R_{\rm S}(1+\mu)v_{\rm x} &= 0 \\
\left[R_{\rm F}+R_{\rm S}(1+\mu)\right]v_{\rm x} &= R_{\rm F}v_{\rm S} \\
v_{\rm x} &= \frac{R_{\rm F}}{R_{\rm F}+R_{\rm S}(1+\mu)}v_{\rm S} \\
v_{\rm T}=v_{\rm OC} &= \frac{-\mu R_{\rm F}}{R_{\rm F}+R_{\rm S}(1+\mu)}v_{\rm S}
\end{aligned}$$

Replace $R_{\rm F}$ with an open circuit. The Thévenin resistance is the same with $R_{\rm T}=R_{\rm P}$. The source side of the circuit has an open circuit, so $v_{\rm x}=v_{\rm S}$ and $v_{\rm T}=v_{\rm OC}=-\mu v_{\rm x}=-\mu v_{\rm S}$.

Problem 4–14. Find the Thévenin equivalent circuit that the load $R_{\rm L}$ sees in Figure P4–14.

Find the current $i_{\rm S}$ and then determine the open-circuit voltage and short-circuit current with the load resistor removed.

$$\begin{aligned}
i_{\mathrm{S}} &= \frac{v_{\mathrm{S}} - r i_{\mathrm{S}}}{R_{\mathrm{S}}} \\
R_{\mathrm{S}} i_{\mathrm{S}} &= v_{\mathrm{S}} - r i_{\mathrm{S}} \\
(R_{\mathrm{S}} + r) i_{\mathrm{S}} &= v_{\mathrm{S}} \\
i_{\mathrm{S}} &= \frac{v_{\mathrm{S}}}{R_{\mathrm{S}} + r} \\
v_{\mathrm{T}} = v_{\mathrm{OC}} = r i_{\mathrm{S}} &= \frac{r v_{\mathrm{S}}}{R_{\mathrm{S}} + r} \\
i_{\mathrm{SC}} &= \frac{r i_{\mathrm{S}}}{R_{\mathrm{P}}} = \frac{r v_{\mathrm{S}}}{(R_{\mathrm{S}} + r) R_{\mathrm{P}}} \\
R_{\mathrm{T}} = \frac{v_{\mathrm{OC}}}{i_{\mathrm{SC}}} &= R_{\mathrm{P}}
\end{aligned}$$

Problem 4–15. Find R_{IN} in Figure P4–15.

Find the current i_{S} and then compute $R_{\mathrm{IN}} = v_{\mathrm{S}}/i_{\mathrm{S}}$.

$$\begin{aligned}
i_{\mathrm{S}} &= \frac{v_{\mathrm{S}} - r i_{\mathrm{S}}}{R} \\
R i_{\mathrm{S}} &= v_{\mathrm{S}} - r i_{\mathrm{S}} \\
(R + r) i_{\mathrm{S}} &= v_{\mathrm{S}} \\
i_{\mathrm{S}} &= \frac{v_{\mathrm{S}}}{R + r} \\
R_{\mathrm{IN}} = \frac{v_{\mathrm{S}}}{i_{\mathrm{S}}} &= R + r
\end{aligned}$$

Problem 4–16. Find R_{IN} in Figure P4–16.

The input current is i_{S}. The input voltage is $v_{\mathrm{S}} = i_{\mathrm{x}} R$. Apply KCL to find i_{x} and then find the input resistance.

$$\begin{aligned}
i_{\mathrm{x}} &= i_{\mathrm{S}} + \beta i_{\mathrm{x}} \\
(1 - \beta) i_{\mathrm{x}} &= i_{\mathrm{S}} \\
i_{\mathrm{x}} &= \frac{i_{\mathrm{S}}}{1 - \beta} \\
v_{\mathrm{S}} = i_{\mathrm{x}} R &= \frac{i_{\mathrm{S}} R}{1 - \beta} \\
R_{\mathrm{IN}} = \frac{v_{\mathrm{S}}}{i_{\mathrm{S}}} &= \frac{R}{1 - \beta}
\end{aligned}$$

Problem 4–17. Find the Norton equivalent circuit seen by the load in Figure P4–17.

Apply Ohm's law to find the input current and then determine the parameters for the Norton equivalent circuit with the load removed.

$$i_{\mathrm{x}} = \frac{v_{\mathrm{S}}}{R_{\mathrm{S}} + R_{\mathrm{x}}}$$

$$i_{\mathrm{N}} = i_{\mathrm{SC}} = -\beta i_{\mathrm{x}} = -\frac{\beta v_{\mathrm{S}}}{R_{\mathrm{S}} + R_{\mathrm{x}}}$$

$$v_{\mathrm{OC}} = -\beta i_{\mathrm{x}} R_{\mathrm{O}} = -\frac{\beta v_{\mathrm{S}} R_{\mathrm{O}}}{R_{\mathrm{S}} + R_{\mathrm{x}}}$$

$$R_{\mathrm{N}} = \frac{v_{\mathrm{OC}}}{i_{\mathrm{SC}}} = R_{\mathrm{O}}$$

Problem 4–18. Find the Thévenin equivalent circuit seen by the load in Figure P4–18.

Remove the load from the circuit. The open-circuit voltage is the net current flowing through resistor R_{O} times the resistance R_{O}. The two current sources are in parallel, so they can be combined by adding them together. Note that in this case, we have $v_{\mathrm{x}} = v_{\mathrm{OC}} = v_{\mathrm{T}}$.

$$v_{\mathrm{T}} = v_{\mathrm{OC}} = i_{\mathrm{O}} R_{\mathrm{O}} = (i_{\mathrm{S}} - g v_{\mathrm{x}}) R_{\mathrm{O}}$$

$$v_{\mathrm{T}} = (i_{\mathrm{S}} - g v_{\mathrm{T}}) R_{\mathrm{O}}$$

$$(1 + g R_{\mathrm{O}}) v_{\mathrm{T}} = R_{\mathrm{O}} i_{\mathrm{S}}$$

$$v_{\mathrm{T}} = \frac{R_{\mathrm{O}} i_{\mathrm{S}}}{1 + g R_{\mathrm{O}}}$$

If the load is replaced by a short circuit, the output voltage is zero and $v_{\mathrm{x}} = 0$. The dependent source does not provide any current and the short-circuit current is $i_{\mathrm{SC}} = i_{\mathrm{S}}$. Solve for the Thévenin resistance.

$$R_{\mathrm{T}} = \frac{v_{\mathrm{OC}}}{i_{\mathrm{SC}}} = \frac{R_{\mathrm{O}}}{1 + g R_{\mathrm{O}}}$$

Problem 4–19. The circuit parameters in Figure P4–19 are $R_{\mathrm{B}} = 100$ kΩ, $R_{\mathrm{C}} = 3.3$ kΩ, $\beta = 100$, $V_\gamma = 0.7$ V, and $V_{\mathrm{CC}} = 15$ V. Find i_{C} and v_{CE} for $v_{\mathrm{S}} = 1$ V. Repeat for $v_{\mathrm{S}} = 5$ V.

For $v_{\mathrm{S}} = 1$ V, we have $v_{\mathrm{S}} > V_\gamma$, so the circuit could be active. The short-circuit current is $i_{\mathrm{SC}} = V_{\mathrm{CC}}/R_{\mathrm{C}} = 4.545$ mA. Solve for the collector current.

$$i_{\mathrm{C}} = \beta \left(\frac{v_{\mathrm{S}} - V_\gamma}{R_{\mathrm{B}}} \right) = 100 \left(\frac{1 - 0.7}{100000} \right) = 300\ \mu\mathrm{A}$$

We have $i_{\mathrm{C}} < i_{\mathrm{SC}}$, so the transistor is in the active mode. Calculate v_{CE}.

$$v_{\mathrm{CE}} = V_{\mathrm{CC}} - i_{\mathrm{C}} R_{\mathrm{C}} = 15 - (0.0003)(3300) = 14.01\ \mathrm{V}$$

For $v_{\mathrm{S}} = 5$ V, we have $v_{\mathrm{S}} > V_\gamma$, so the circuit could be active. Solve for the collector current.

$$i_{\mathrm{C}} = 100 \left(\frac{5 - 0.7}{100000} \right) = 4.3\ \mathrm{mA}$$

We have $i_{\mathrm{C}} < i_{\mathrm{SC}}$, so the transistor is in the active mode. Calculate v_{CE}.

$$v_{\mathrm{CE}} = V_{\mathrm{CC}} - i_{\mathrm{C}} R_{\mathrm{C}} = 15 - (0.0043)(3300) = 0.810\ \mathrm{V}$$

Problem 4–20. The circuit parameters in Figure P4–19 are $R_{\mathrm{C}} = 3$ kΩ, $\beta = 120$, $V_\gamma = 0.7$ V, and $V_{\mathrm{CC}} = 5$ V. Select a value of R_{B} such that the transistor is in the saturation mode when $v_{\mathrm{S}} \geq 2$ V.

With $v_S \geq 2$ V, we have $v_S > V_\gamma$, so the transistor is not in cutoff mode. To be in the saturation mode, we need to have $i_C > i_{SC} = V_{CC}/R_C = 5/3000 = 1.667$ mA. Select $i_C = 2$ mA to be in saturation mode. Solve for R_B.

$$i_C = 0.002 = \beta\left(\frac{v_S - V_\gamma}{R_B}\right) = 120\left(\frac{2-0.7}{R_B}\right)$$

$$R_B = 120\left(\frac{1.3}{0.002}\right) = 78\ \text{k}\Omega$$

Problem 4–21. The parameters of the transistor in Figure P4–21 are $\beta = 60$ and $V_\gamma = 0.7$ V. Find i_C and v_{CE} for $v_S = 0.8$ V. Repeat for $v_S = 2$ V.

For both values of v_S, we have $v_S > V_\gamma$, so the transistor is not in cutoff mode. Compute the short-circuit current $i_{SC} = V_{CC}/RRC = 15/10000 = 1.5$ mA. Solve for the collector current with $v_S = 0.8$ V.

$$i_C = \beta\left(\frac{v_S - V_\gamma}{R_B}\right) = 60\left(\frac{0.8-0.7}{10000}\right) = 600\ \mu\text{A}$$

We have $i_C < i_{SC}$, so the transistor is in the active mode. Calculate v_{CE}. First, find the Thévenin equivalent of the right side of the circuit. The open-circuit voltage is $v_{OC} = (15)(20)/(20+10) = 10$ V and the lookback resistance is $R_T = 20 \parallel 10 = 6.667$ kΩ.

$$v_{CE} = V_{CC} - i_C R_C = 10 - (0.0006)(6667) = 6\ \text{V}$$

Solve for the collector current with $v_S = 2$ V.

$$i_C = \beta\left(\frac{v_S - V_\gamma}{R_B}\right) = 60\left(\frac{2-0.7}{10000}\right) = 7.8\ \text{mA}$$

We have $i_C > i_{SC}$, so the transistor is in saturation mode and we have $i_C = i_{SC} = 1.5$ mA and $v_{CE} = 0$ V.

Problem 4–22. An emergency indicator light uses a 10 V, 2-W incandescent lamp. It is to be ON when a digital output is high (5 V). The digital circuit does not have sufficient power to turn on the lamp directly. However, as is common practice, a transistor driver is used as a digital switch. Select R_B in the circuit of Figure 4–22 so to drive the transistor into saturation causing it to act as a short circuit between the lamp and ground when the digital output is high. The Thévenin equivalent for the digital circuit is also shown in the figure.

Calculate the current required to operate the lamp at 10 V and 2 W as $i = p/v = 2/10 = 200$ mA. Therefore, we want to drive the lamp with $i_C = 200$ mA. Assume that $V_\gamma = 0.7$ V, so that the 5-V source is sufficient for $v_S > V_\gamma$ and the transistor is not in cutoff mode. Use the expression for i_C to solve for R_B.

$$i_C = 0.2 = \beta\left(\frac{v_S - V_\gamma}{R_D + R_B}\right) = 50\left(\frac{5-0.7}{R_D + R_B}\right)$$

$$R_D + R_B = \frac{(50)(4.3)}{0.2} = 1075$$

$$R_B = 575\ \Omega$$

A smaller standard value, such as $R_B = 560\ \Omega$ will also work.

Problem 4–23. Find the voltage gain of each OP AMP circuit shown in Figure P4–23.

(a). The circuit is an inverting amplifier, so $K = -R_2/R_1 = -330/33 = -10$.

(b). The circuit is a noninverting amplifier.

$$K = \frac{R_1 + R_2}{R_2} = \frac{330+33}{33} = 11$$

Problem 4–24. Considering simplicity and standard 10% tolerance resistors as major constraints, design OP AMP circuits that produce the following voltage gains ±10%: −100, +200, +1, −0.5, and +0.5.

The following table presents one set of possible designs using standard, single OP AMP circuits.

Desired Gain	OP AMP Design	R_1 (kΩ)	R_2 (kΩ)	Actual Gain
−100	Inverting	1	100	−100
+200	Noninverting	680	3.3	+207.06
+1	Noninverting	0	∞	+1
−0.5	Inverting	56	27	−0.482

To design a circuit with a gain of +0.5, we must use two inverting OP AMP circuits in cascade. The first circuit has a gain of −0.482 with $R_1 = 56$ kΩ and $R_2 = 27$ kΩ and the second circuit has a gain of −1 with $R_1 = R_2 = 10$ kΩ. It is not possible to design a simple noninverting amplifier with a gain less than one.

Problem 4–25. Two OP AMP circuits are shown in Figure P4–25. Both claim to produce a gain of either ±100.

(a). Show that the claim is true.

Circuit 1 is a noninverting amplifier with a gain $K = (R_1 + R_2)/R_2 = (99 + 1)/1 = 100$. Circuit 2 is an inverting amplifier with a gain of $K = -R_2/R_1 = -100/1 = -100$. The claim is true.

(b). A practical source with a series resistor of 1 kΩ is connected to the input of each circuit. Does the original claim still hold? If it does not, explain why?

The claim does not still hold true. The noninverting amplifier maintains its gain of 100. The inverting amplifier now has two resistors in series on the input side and an effective $R_1 = 1 + 1 = 2$ kΩ, so the new gain is −50. To avoid the issue with the inverting OP AMP circuit, add a buffer at the input or scale both resistors in the design to be substantially larger than the source resistance.

Problem 4–26. Suppose the output of the practical source shown in Figure P4–25 needs to be amplified by -10^4 and you can use only the two circuits shown. How would you connect the circuits to achieve this? Explain why.

Connect the source to the input of the noninverting amplifier and connect the output of the noninverting amplifier to the input of the inverting amplifier. The source does not affect the gain of the noninverting amplifier and the noninverting amplifier does not affect the gain of the inverting amplifier, so the overall gain is $K = (100)(-100) = -10^4$.

Problem 4–27.

(a). Find the voltage gain v_O/v_S in Figure P4–27.

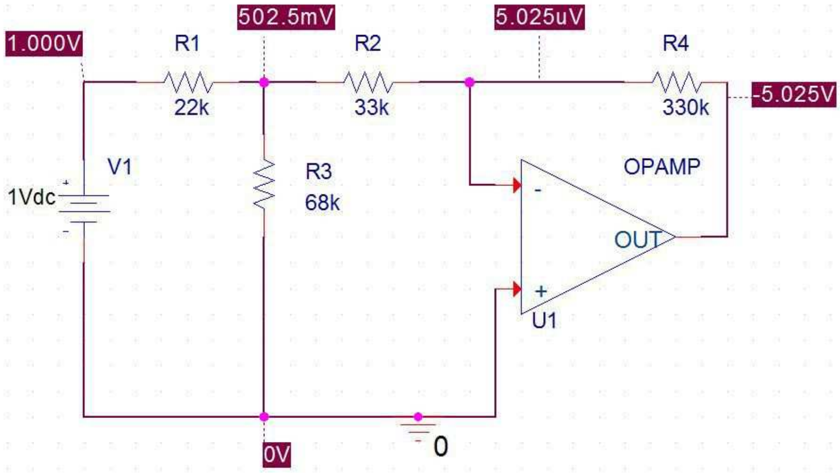
Find the Thévenin equivalent of the source circuit. Apply voltage division to find $v_T = 68v_S/(22{+}68) = 68v_S/90$. The lookback resistance is $R_T = 33 + (22 \parallel 68) = 49.62$ kΩ. The circuit is equivalent to an inverting amplifier with an input of $68v_S/90$ and a gain of $K = -330/49.62$.

$$v_O = \left(\frac{-330}{49.62}\right)\left(\frac{68}{90}\right) v_S = -5.025 v_S$$

$$\frac{v_O}{v_S} = -5.025$$

(b). Validate your answer by simulating the circuit in OrCAD.

The following OrCAD simulation validates the answer.



Problem 4–28. What is the range of the gain v_O/v_S in Figure P4–28?

The circuit is an inverting amplifier design with $R_1 = 2$ kΩ and R_2 ranging from 100 kΩ to 200 kΩ. The gain is $K = -R_2/R_1$, so it ranges from -50 to -100.

Problem 4–29. Design a simple OP AMP circuit that has a variable gain from +10 to +100.

Use a noninverting amplifier with a 1-kΩ resistor connected to ground and a potientiometer for the feedback resistor with a range from 9 to 99 kΩ. One convenient way to design the feedback resistor is to use a 9-kΩ resistor in series with a 90-kΩ potentiometer.

Problem 4–30. For the circuit in Figure P4–30:

(a). Find v_O in terms of v_S.

The circuit is a noninverting amplifier. The 10-kΩ resistor in series with the source has no current flowing through it and does not influence the gain. The input voltage at the positive terminal of the OP AMP is v_S. Likewise, the 10-kΩ resistor at the output does not influence the gain. The gain is $K = (150 + 10)/10 = 16$, so we have $v_O = 16v_S$.

(b). Find i_O for $v_S = 1$ V. Repeat for $v_S = 3$ V.

With $v_S = 1$ V, the output voltage is $v_O = 16v_S = 16$ V and the output current is $i_O = 16/10000 = 1.6$ mA. With $v_S = 3$ V, the output voltage saturates at $v_O = 24$ V and the output current is $i_O = 24/10000 = 2.4$ mA.

Problem 4–31. For the circuit in Figure P4–31:

(a). Find v_O in terms of v_S.

The circuit is a voltage divider connected to a noninverting amplifier. The voltage at the positive input terminal of the OP AMP is $v_P = v_N = 100v_S/(100 + 10) = 10v_S/11$. The gain of the amplifier is $K = (100 + 10)/10 = 11$. The output voltage is $v_O = Kv_P = (11)(10v_S/11) = 10v_S$.

(b). Find i_O for $v_S = 0.5$ V. Repeat for $v_S = 2$ V.

Solve for the output current in terms of v_S.

$$i_O = \frac{v_O - v_N}{100000} = \frac{10v_S - \frac{10}{11}v_S}{100000} = \frac{\frac{100}{11}v_S}{100000} = \frac{v_S}{11000}$$

For $v_S = 0.5$ V, we have $i_O = 0.5/11000 = 45.45$ μA. For $v_S = 2$ V, the amplifier saturates with $v_O = 18$ V and $v_P = v_N = (10)(2)/11 = 20/11$ V. we have $i_O = (18 - 20/11)/100000 = 161.82$ μA.

Problem 4–32. (E) A young designer needed to amplify a 2-V signal by the factors of 1, 5, and 10. Find the problem with the design shown in Figure P4–32. Recommend a fix.

The circuit is a noninverting amplifier with a gain of $K = (R_F + 10)/10$ for three different values of R_F. The gains are 10, 5, and 1 with R_F set to 90, 40, and 0 kΩ, respectively. The problem with the design is that the input signal is 2 V, the maximum gain is 10, and $V_{CC} = \pm 15$ V. This amplifier will produce a maximum output of 15 V and not the 20 V required. Design the circuit with an OP AMP that allows for $V_{CC} = \pm 20$ V or larger.

Problem 4–33. (D) Design two circuits to produce the following output: $v_O = 2v_1 - 3v_2$.

(a). (a) In your first design use a standard subtractor.

With a standard subtractor, apply v_2 to the negative side and v_1 to the positive side. We then have the following output voltage.

$$v_O = -\frac{R_2}{R_1}v_2 + \left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)v_1$$

Select $R_1 = 1$ kΩ and $R_2 = 3$ kΩ to get the correct gain for v_2. Then select $R_3 = R_4 = 1$ kΩ to get the correct gain for v_1.

(b). (b) In your second design both inputs must be into high input resistance amplifiers to avoid loading.

Many designs are possible. The simplest variation on the previous design is to add buffers at the inputs to the standard subtractor design in part (a).

Problem 4–34. For the circuit in Figure P4–34:

(a). Find v_O in terms of the inputs v_1 and v_2.

The circuit is an inverting summer with gains $K_1 = -100/(50+50) = -1$ and $K_2 = -100/(50+100) = -2/3$. The output voltage is $v_O = -v_1 - 2v_2/3$.

(b). If $v_1 = 1$ V, what is the range of values v_2 can have without saturating the OP AMP?

The output voltage is bounded by $V_{CC} = \pm 15$ V.

$$-15 < v_O < 15$$

$$-15 < -v_1 - \frac{2}{3}v_2 < 15$$

$$-15 < -1 - \frac{2}{3}v_2 < 15$$

$$-14 < -\frac{2}{3}v_2 < 16$$

$$21 \text{ V} > v_2 > -24 \text{ V}$$

Problem 4–35. The input-output relationship for a three-input inverting summer is

$$v_O = -\left[v_1 + 10v_2 + 100v_3\right]$$

The resistance of the feedback resistor is 100 kΩ. Find the values of the input resistors R_1, R_2, and R_3.

The gains are $K_x = -R_F/R_x = -100/R_x$ for $x = 1, 2, 3$. Select $R_1 = 100$ kΩ, $R_2 = 10$ kΩ, and $R_3 = 1$ kΩ.

Problem 4–36. Find v_O in terms of the inputs v_1 and v_2 in Figure P4–36.

The circuit is connected as a standard subtractor.

$$v_O = -\frac{R_2}{R_1}v_1 + \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)v_2 = -\frac{33}{10}v_1 + \left(\frac{10 + 33}{10}\right)\left(\frac{33}{10 + 33}\right)v_2 = -3.3v_1 + 3.3v_2$$

Problem 4–37. The switch in Figure P4–37 is open, find v_{O} in terms of the inputs v_{S1} and v_{S2}. Repeat with the switch closed.

With the switch open, the circuit is connected as a modified subtractor. The top input voltage is $v_{\mathrm{S1}}+v_{\mathrm{S2}}$ and has the negative gain. The bottom input voltage is v_{S2} and has the positive gain.

$$\begin{aligned} v_{\mathrm{O}} &= -\frac{R_2}{R_1}(v_{\mathrm{S1}}+v_{\mathrm{S2}}) + \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)v_{\mathrm{S2}} \\ &= -2(v_{\mathrm{S1}}+v_{\mathrm{S2}}) + \left(\frac{90}{30}\right)\left(\frac{60}{90}\right)v_{\mathrm{S2}} \\ &= -2v_{\mathrm{S1}} - 2v_{\mathrm{S2}} + 2v_{\mathrm{S2}} \\ &= -2v_{\mathrm{S1}} \end{aligned}$$

With the switch closed, the bottom 60-kΩ resistor is shorted out and the positive OP AMP terminal is connected directly to ground. The circuit performs as an inverting amplifier with a gain of $K = -60/30 = -2$ and an input voltage of $v_{\mathrm{S1}}+v_{\mathrm{S2}}$, so we have $v_{\mathrm{O}} = -2v_{\mathrm{S1}} - 2v_{\mathrm{S2}}$.

Problem 4–38. Find v_{O} in terms of v_{S1} and v_{S2} in Figure P4–38.

The circuit is configured as a modified noninverting amplifier. Ignoring the portion of the circuit connected to the positive terminal of the OP AMP, the gain of the amplifier is $K = (R_3+R_4)/R_3$. Apply KCL to find the voltage at the positive terminal of the OP AMP and then apply the gain.

$$\begin{aligned} \frac{v_{\mathrm{P}}-v_{\mathrm{S1}}}{R_1} + \frac{v_{\mathrm{P}}-v_{\mathrm{S2}}}{R_2} &= 0 \\ R_2v_{\mathrm{P}} - R_2v_{\mathrm{S1}} + R_1v_{\mathrm{P}} - R_1v_{\mathrm{S2}} &= 0 \\ (R_1+R_2)v_{\mathrm{P}} &= R_2v_{\mathrm{S1}} + R_1v_{\mathrm{S2}} \\ v_{\mathrm{P}} &= \frac{R_2v_{\mathrm{S1}} + R_1v_{\mathrm{S2}}}{R_1+R_2} \\ v_{\mathrm{O}} = Kv_{\mathrm{P}} &= \left(\frac{R_3+R_4}{R_3}\right)\left(\frac{R_2v_{\mathrm{S1}} + R_1v_{\mathrm{S2}}}{R_1+R_2}\right) \end{aligned}$$

Problem 4–39. (E) It is claimed that $v_{\mathrm{O}} = v_{\mathrm{S}}$ when the switch is closed in Figure P4–39 and that $v_{\mathrm{O}} = -v_{\mathrm{S}}$ when the switch is open. Prove or disprove this claim.

With the switch closed, the OP AMP's positive terminal is connected to ground and the circuit is connected as an inverting amplifier with a gain of $K = -R/R = -1$. The input signal is v_{S}, so we have $v_{\mathrm{O}} = -v_{\mathrm{S}}$. With the switch open, the input signal appears at the positive terminal, because no current flows into the OP AMP. If $v_{\mathrm{P}} = v_{\mathrm{S}}$, then $v_{\mathrm{N}} = v_{\mathrm{S}}$, and the voltage v_{S} appears on both sides of the top left resistor. Since there is no voltage drop across the resistor, there is no current through it and there is no current flowing into the OP AMP. Therefore, there is no current flowing through the top right resistor and it does not cause a change in voltage. The output voltage must also match, so $v_{\mathrm{O}} = v_{\mathrm{S}}$. In summary, the claim is false. With the switch closed, $v_{\mathrm{O}} = -v_{\mathrm{S}}$, and with the switch open, $v_{\mathrm{O}} = v_{\mathrm{S}}$.

Problem 4–40. The circuit in Figure P4–40 has a diode in its feedback path and is called a "log-amp" because its output is proportional to the natural log of the input.

(a). Show that $v_{\mathrm{O}} = -V_{\mathrm{T}}\ln\left[1 + v_{\mathrm{S}}/(R_{\mathrm{S}}I_{\mathrm{O}})\right]$ if the *i-v* characteristics of the diode is $i_{\mathrm{D}} = I_{\mathrm{O}}\left(e^{v_{\mathrm{D}}/V_{\mathrm{T}}} - 1\right)$.

The positive terminal of the OP AMP is grounded, so we have $v_{\mathrm{P}} = v_{\mathrm{N}} = 0$ V. The input current is

$i_S = (v_S - 0)/R_S = v_S/R_S$. No current flows into the OP AMP, so $i_D = i_S = v_S/R_S$.

$$i_D = \frac{v_S}{R_S} = I_O\left(e^{v_D/V_T} - 1\right)$$

$$\frac{v_S}{I_O R_S} + 1 = e^{v_D/V_T}$$

$$\ln\left(\frac{v_S}{I_O R_S} + 1\right) = \frac{v_D}{V_T}$$

$$v_D = V_T \ln\left(\frac{v_S}{I_O R_S} + 1\right)$$

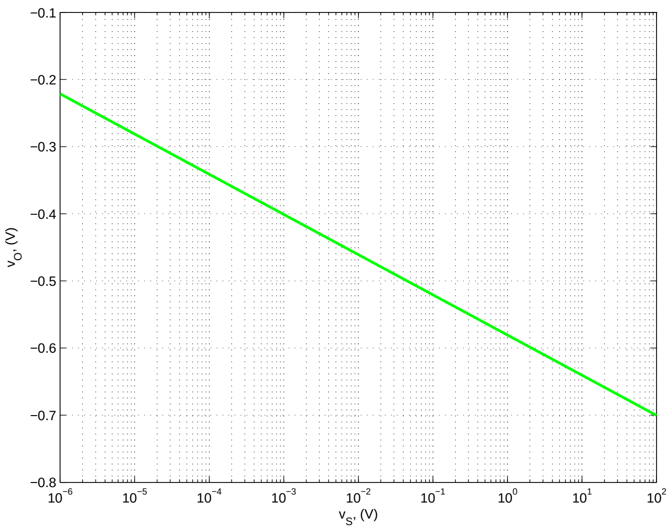
$$v_O = -v_D = -V_T \ln\left(\frac{v_S}{I_O R_S} + 1\right)$$

(b). Using MATLAB, plot v_O versus v_S for $R_S = 10$ kΩ, $I_O = 2 \times 10^{-14}$ A, and $V_T = 0.026$ V. Plot your results on a semilog plot for 10^{-6} V $\leq v_S \leq 100$ V.

The following MATLAB code provides the solution:

```
% Define known parameters
RS = 10e3;
IO = 2e-14;
VT = 0.026;
% Compute the voltages
vS = logspace(-6,2,1000);
vO = -VT*log(1 + vS/RS/IO);
% Plot the results
figure
semilogx(vS,vO,'g','LineWidth',2)
grid on
xlabel('v_S, (V)')
ylabel('v_O, (V)')
```

The corresponding MATLAB output is shown below.



Problem 4–41.

(a). Use node voltage analysis to find the input-output relationship or K of the circuit in Figure P4–41.

Label the node between R_1 and R_3 as node A. Note that $v_P = v_N = 0$ V and write two node-voltage equations.

$$\frac{0 - v_S}{R} + \frac{0 - v_A}{R_1} = 0$$

$$\frac{v_A - 0}{R_1} + \frac{v_A}{R_2} + \frac{v_A - v_O}{R_3} = 0$$

Solve the first equation for $v_A = -R_1 v_S/R$ and substitute into the second equation.

$$R_2R_3v_A + R_1R_3v_A + R_1R_2v_A = R_1R_2v_O$$

$$(R_1R_2 + R_1R_3 + R_2R_3)v_A = R_1R_2v_O$$

$$(R_1R_2 + R_1R_3 + R_2R_3)\left(-\frac{R_1}{R}v_S\right) = R_1R_2v_O$$

$$\frac{v_O}{v_S} = \frac{-(R_1R_2 + R_1R_3 + R_2R_3)}{RR_2}$$

(b). **(D)** Select values for the resistors so that $K = -6$.

Let $R = R_1 = R_2 = 10$ kΩ and solve for R_3.

$$K = \frac{-(R_1R_2 + R_1R_3 + R_2R_3)}{RR_2} = \frac{-(100 + 10R_3 + 10R_3)}{100}$$

$$-6 = \frac{-(100 + 20R_3)}{100}$$

$$600 = 100 + 20R_3$$

$$20R_3 = 500$$

$$R_3 = 25 \text{ k}\Omega$$

Problem 4–42. Use node voltage analysis in Figure P4–42 to show that $i_O = -v_S/(2R)$ regardless of the load. That is, show that the circuit is a voltage-controlled current source.

Write the node-voltage equation at the negative terminal of the OP AMP and simplify.

$$\frac{v_N - v_S}{2R} + \frac{v_N - v_O}{R} = 0$$

$$v_N - v_S + 2v_N - 2v_O = 0$$

$$3v_N = v_S + 2v_O$$

Write the node-voltage equation at the positive terminal of the OP AMP and simplify.

$$\frac{v_P - v_O}{R} + \frac{v_P - 0}{2R} + i_O = 0$$

$$2v_P - 2v_O + v_P + 2Ri_O = 0$$

$$3v_P = 2v_O - 2Ri_O$$

Note that $v_N = v_P$ and set the right sides of the results above equal to each other.

$$v_S + 2v_O = 2v_O - 2Ri_O$$

$$i_O = -\frac{v_S}{2R}$$

Problem 4–43. For the circuit of Figure P4–43:

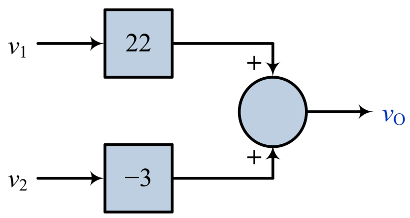
(a). Find the output in terms of v_1 and v_2.

The first stage is an inverting amplifier with a gain of $K_1 = -22/10 = -2.2$. The second stage is an inverting summer with gains of $K_{21} = -100/10 = -10$ and $K_{22} = -100/33 = -3$.

$$v_O = (-2.2)(-10)v_1 + (-3)v_2 = 22v_1 - 3v_2$$

(b). Draw a block diagram for the circuit.

The block diagram is shown below.



Problem 4–44. Find v_O in terms of the inputs v_{S1} and v_{S2} in Figure P4–44.

For the first stage $v_P = v_N = v_{S2}$. Write a node-voltage equation at the negative input terminal and solve for the output voltage for the first stage.

$$\frac{v_{S2} - v_{S1}}{R_1} + \frac{v_{S2} - v_{O1}}{R_2} = 0$$

$$R_2 v_{S2} - R_2 v_{S1} + R_1 v_{S2} - R_1 v_{O1} = 0$$

$$v_{O1} = v_{S2} + \frac{R_2}{R_1}(v_{S2} - v_{S1})$$

For the second stage $v_P = v_N = v_{S2}$. Write a node-voltage equation at the negative input terminal and solve for the output voltage for the second stage.

$$\frac{v_{S2} - v_{O1}}{R_2} + \frac{v_{S2} - v_O}{R_1} = 0$$

$$\frac{-\frac{R_2}{R_1}(v_{S2} - v_{S1})}{R_2} + \frac{v_{S2} - v_O}{R_1} = 0$$

$$\frac{v_{S1} - v_{S2}}{R_1} + \frac{v_{S2} - v_O}{R_1} = 0$$

$$v_{S1} - v_{S2} + v_{S2} - v_O = 0$$

$$v_O = v_{S1}$$

Problem 4–45. For the block diagram of Figure P4–45:

(a). Find an expression for v_O in terms of v_1 and v_2.

Let v_A be the ouput of the summer.

$$v_A = 5v_1 - v_2 + v_O$$
$$v_O = -2v_A$$
$$v_O = -10v_1 + 2v_2 - 2v_O$$
$$3v_O = -10v_1 + 2v_2$$
$$v_O = -\frac{10}{3}v_1 + \frac{2}{3}v_2$$

(b). **(D)** Design a suitable circuit that realizes the block diagram using only one OP AMP.

Use a standard OP AMP subtractor design.

$$v_O = -\frac{R_2}{R_1}v_1 + \left(\frac{R_4}{R_3+R_4}\right)\left(\frac{R_1+R_2}{R_1}\right)v_2$$

To get $K_1 = -10/3$, select $R_1 = 3$ kΩ and $R_2 = 10$ kΩ. To get $K_2 = 2/3$, select $R_3 = 11$ kΩ and $R_4 = 2$ kΩ.

Problem 4–46. For the block diagram of Figure P4–46:

(a). Find an expression for v_O in terms of v_S and the input voltage source.

Let v_A be the output of the first summer and v_B be the output of the second summer.

$$v_A = -2v_S + v_O$$
$$v_B = (-6)(0.5) + 10v_A = -3 + 10(-2v_S + v_O) = -3 - 20v_S + 10v_O$$
$$v_O = -2v_B = -2(-3 - 20v_S + 10v_O) = 6 + 40v_S - 20v_O$$
$$21v_O = 6 + 40v_S$$
$$v_O = \frac{6+40v_S}{21} = \frac{40}{21}v_S + \frac{12}{21}(0.5)$$

(b). **(D)** Design a suitable circuit that realizes the block diagram using only one OP AMP and the 0.5-V source.

Use a standard subtractor circuit. Connect the positive terminal of the v_S source to ground and the negative terminal of the source to the side of the subtractor with negative gain. Connect the 0.5-V source to the side of the subtractor with positive gain.

$$v_O = -\frac{R_2}{R_1}(-v_S) + \left(\frac{R_4}{R_3+R_4}\right)\left(\frac{R_1+R_2}{R_1}\right)(0.5)$$

To get $K_1 = -40/21$, choose $R_1 = 21$ kΩ and $R_2 = 40$ kΩ. To get the correct gain for the 0.5-V source, choose $R_3 = 49$ kΩ and $R_4 = 12$ kΩ.

Problem 4–47. For the circuit in Figure P4–47:

(a). Find v_O in terms of v_S and the input voltage source.

The first stage is an inverting amplifier with a gain of -10. The second stage is an inverting summer with gains of -10 for the input voltage source and -0.5 for the output of the first stage.

$$v_O = (-10)(1) + (-0.5)(-10)v_S = 5v_S - 10$$

(b). Prove that the block diagram provides the same output.

Write the expression for the output voltage and simplify.

$$v_O = 5v_S + (-10)(1) = 5v_S - 10$$

The results is the same as part (a).

(c). **(D)** Redesign the circuit using only one OP AMP.

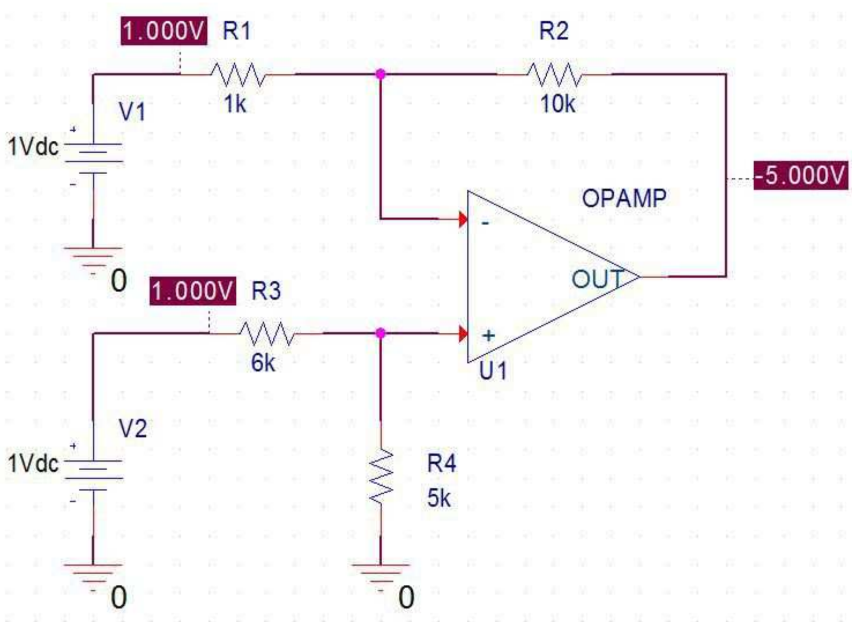
Use a standard subtractor circuit. Connect the 1-V source to the side of the subtractor with negative gain and v_S to the side of the subtractor with positive gain

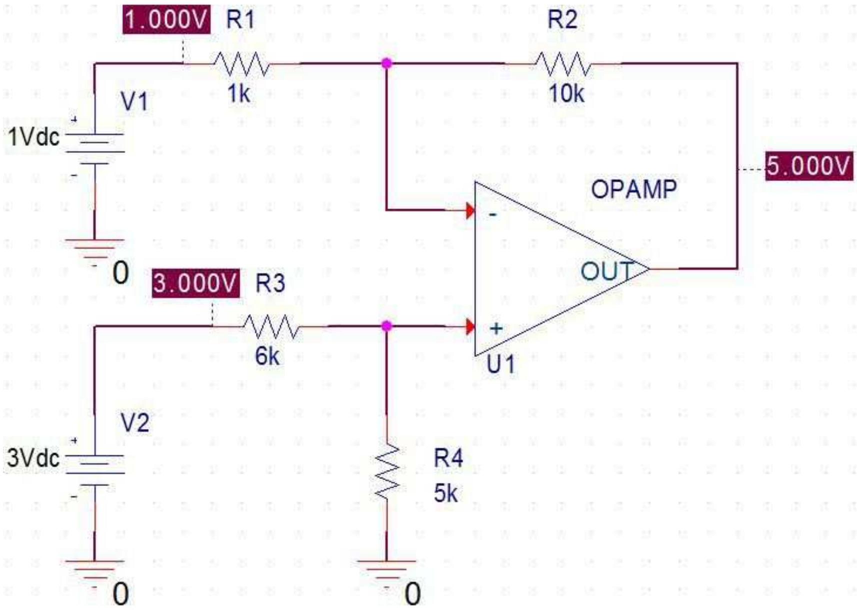
$$v_O = -\frac{R_2}{R_1}(1) + \left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)v_S$$

To get $K_1 = -10$, choose $R_1 = 1\ \text{k}\Omega$ and $R_2 = 10\ \text{k}\Omega$. To get the correct gain for v_S, choose $R_3 = 6\ \text{k}\Omega$ and $R_4 = 5\ \text{k}\Omega$.

(d). Validate your design using OrCAD.

The following two simulations validate the design. Two simulations are required to show how the output scales with respect to v_S. For $v_S = 1$ V, we have $v_O = -5$ V, and for $v_S = 3$ V, we have $v_O = 5$ V.





Problem 4–48. (E) On an exam, students were asked to design an efficient solution for the following relationship: $v_2 = 3v_1 + 15$. Two of the designs are shown in Figure P4–48. Which, if any, of the designs are correct and what grade would you award each student?

Design (a) has three stages: (1) an inverting summer, (2) an inverting amplifier, and (3) a noninverting amplifier. We can compute the input-output relationship as follows.

$$v_2 = \left(\frac{10+20}{20}\right)\left[-\frac{20}{10}\left(-\frac{10}{10}v_1 - \frac{10}{10}(5)\right)\right] = \frac{3}{2}\left[-2\left(-v_1 - 5\right)\right] = 3v_1 + 15$$

The design has the correct relationship.

Design (b) is a noninverting amplifier with a modified input stage and a gain of $K = (50+10)/10 = 6$. Write a node-voltage equation to find $v_P = v_N$ and then compute v_2.

$$\frac{v_P - v_1}{10} + \frac{v_P - 5}{10} = 0$$

$$v_P - v_1 + v_P - 5 = 0$$

$$2v_P = v_1 + 5$$

$$v_P = \frac{v_1 + 5}{2}$$

$$v_2 = 6v_P = 6\left(\frac{v_1+5}{2}\right) = 3v_1 + 15$$

The design also has the correct relationship.

Design (a) uses standard OP AMP blocks, but requires three OP AMPs and seven resistors. Design (b) is more efficient than Design (a), but does not use a standard approach, so it requires more effort to analyze. Since the problem asked students to design an efficient solution for the input-output relationship, give design (a) a grade of B and give design (b) a grade of A.

Problem 4–49. For the circuit of Figure P4–49:

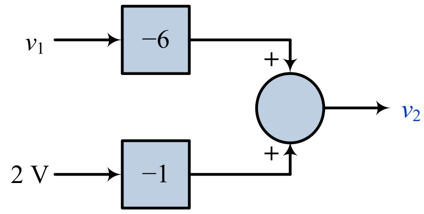
(a). Find the output v_2 in terms of the input v_1 in Figure P4–49.

The left OP AMP is connected as a noninverting amplifier with an input of v_1 and a gain of $K_1 = (10+10)/10 = 2$, so its output is $v_{O1} = 2v_1$. The right OP AMP is connected as an inverting summer with inputs of $v_1 + 2$ V, v_1, and $v_{O1} = 2v_1$, and gains of $K_{21} = -1$, $K_{22} = -1$, and $K_{23} = -2$, respectively.

$$v_2 = (-1)(v_1+2) + (-1)(v_1) + (-2)(2v_1) = -v_1 - 2 - v_1 - 4v_1 = -6v_1 - 2 \text{ V}$$

(b). Draw a representative block diagram for the circuit.

The following block diagram represents the circuit.



Problem 4–50. For the circuit of Figure P4–50:

(a). Use node-voltage analysis to find the output v_O in terms of the input v_S.

Apply node-voltage analysis and note that $v_A = v_S$, $v_B = 0$ V, $v_C = v_E$, and $v_D = v_O$. Write two node-voltage equations, with one at the negative input terminal for each OP AMP.

$$\frac{v_B - v_S}{25} + \frac{v_B - v_C}{100} + \frac{v_B - v_O}{100} = 0$$

$$\frac{v_C - 0}{100} + \frac{v_C - v_O}{300} = 0$$

Substitute $v_B = 0$ V into the first equation to solve for $v_C = -4v_S - v_O$. Solve the second equation for $v_O = 4v_C$. Combine the results to get the following:

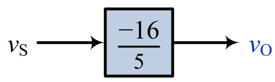
$$v_O = 4v_C = 4(-4v_S - v_O) = -16v_S - 4v_O$$

$$5v_O = -16v_S$$

$$v_O = -\frac{16}{5}v_S$$

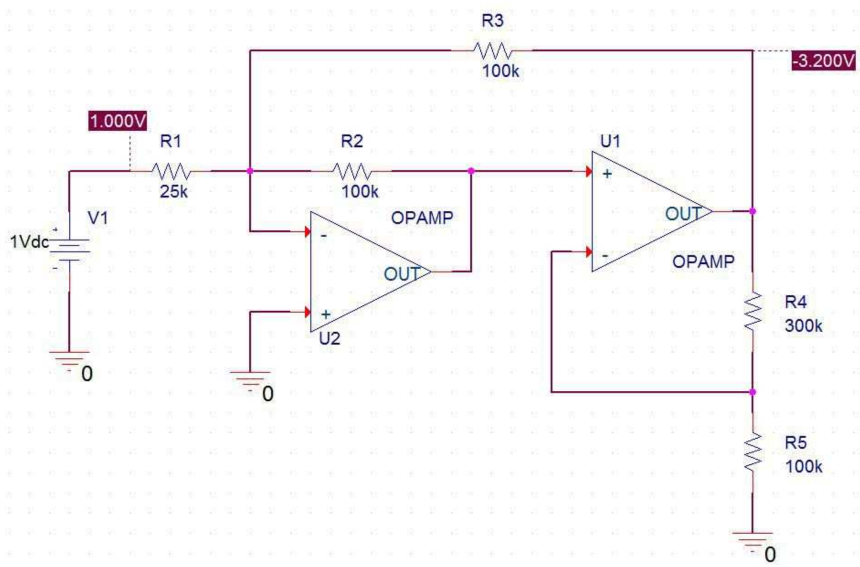
(b). Draw a representative block diagram for the circuit.

The following block diagram represents the circuit.



(c). Verify your answer using OrCAD.

The following OrCAD simulation verifies the results.



Problem 4–51. (E) Faced with having to construct the circuit in Figure P4–51(a), a student offers to build the circuit in Figure P4–51(b) claiming it performs the same task. As the teaching assistant in the course, do you agree with the student's claim?

Compute the gain for circuit (a). The first stage is an inverting amplifier, the second stage is an inverting summer, and the third stage is a noninverting amplifier.

$$v_O = \left(\frac{20+10}{10}\right)\left[\left(-\frac{20}{20}\right)\left(-\frac{20}{10}\right)v_1 - \frac{20}{20}v_O\right]$$

$$v_O = 3\left[2v_1 - v_O\right]$$

$$v_O = 6v_1 - 3v_O$$

$$4v_O = 6v_1$$

$$v_O = 1.5v_1$$

Circuit (b) is a standard noninverting amplifier with a gain of $K = (5+10)/10 = 15/10 = 1.5$, so the circuits do have the same gain. The circuits have different input resistances, with circuit (a) having an input resistance of 10 kΩ and circuit (b) having infinite input resistance. Depending on the application of the design, the input resistance may matter.

Problem 4–52. (D) Design a single OP AMP amplifier with a voltage gain of -100 and an input resistance greater than 5 kΩ using standard 5% resistance values less than 1 MΩ.

Use a standard inverting amplifier design with $R_1 = 5.6$ kΩ and $R_2 = 560$ kΩ.

Problem 4–53. (D) Design an OP AMP amplifier with a voltage gain of 5 using only 10-kΩ resistors and one OP AMP.

Use a standard noninverting amplifier design with R_1 equal to four 10-kΩ resistors connected in series and $R_2 = 10$ kΩ.

Problem 4–54. (D) Using a single OP AMP, design a circuit with inputs v_1 and v_2 and an output $v_O = v_2 - 3v_1$. The input resistance seen by each input should be greater than 5 kΩ.

Use a standard subtractor design with v_1 connected to the side with negative gain and v_2 connected to the side with positive gain.

$$v_O = \left(\frac{R_4}{R_3+R_4}\right)\left(\frac{R_1+R_2}{R_1}\right)(v_2) - \frac{R_2}{R_1}(v_1)$$

Select $R_1 = 10$ kΩ and $R_2 = 30$ kΩ to get the correct gain for v_1. Then select $R_3 = 30$ kΩ and $R_4 = 10$ kΩ to get the correct gain for v_2.

Problem 4–55. (D) Design a difference amplifier with inputs v_1 and v_2 and an output $v_O = 30(v_2 - v_1)$ using only one OP AMP. All resistances must be between 5 kΩ and 200 kΩ.

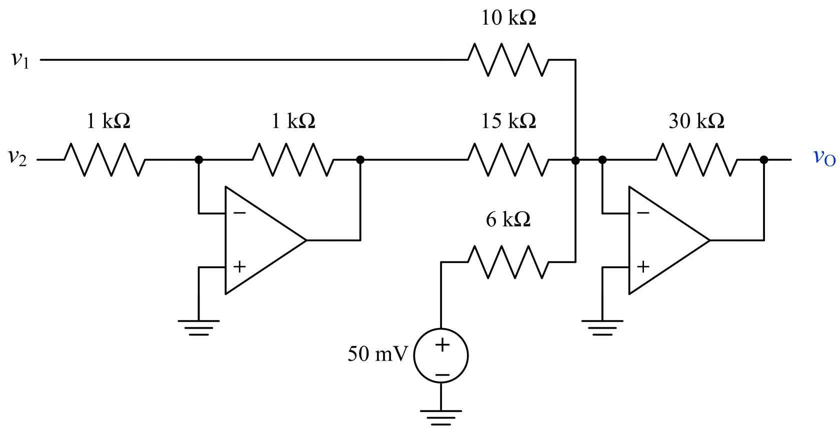
Use a standard subtractor design with v_1 connected to the side with negative gain and v_2 connected to the side with positive gain.

$$v_O = \left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)(v_2) - \frac{R_2}{R_1}(v_1)$$

Select $R_1 = 5$ kΩ and $R_2 = 150$ kΩ to get the correct gain for v_1. Then select $R_3 = 5$ kΩ and $R_4 = 150$ kΩ to get the correct gain for v_2.

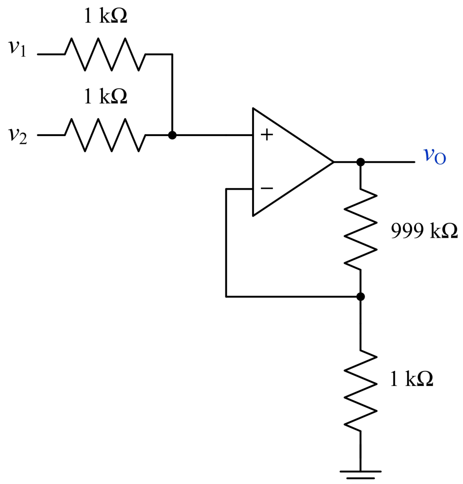
Problem 4–56. (D) Using no more than two OP AMPs, design an OP AMP circuit with inputs v_1, v_2, and 50 mV and an output $v_O = -3v_1 + 2v_2 - 250$ mV.

The following circuit provides one solution. The first stage multiplies v_2 by negative one and the second stage combines the three input signals with the correct gains.



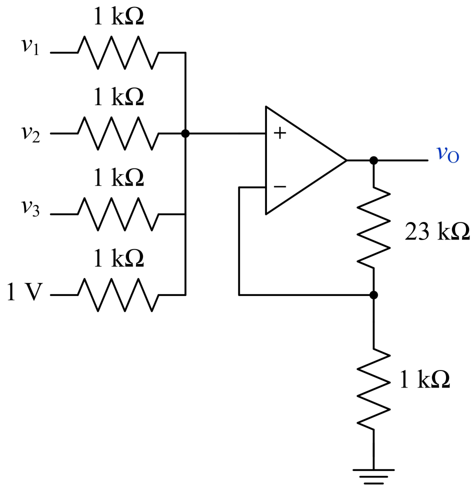
Problem 4–57. (D) Design a two-input noninverting summer that will produce an output $v_O = 500(v_1+v_2)$.

Use the design in Figure 4–45 with equal input resistors and an OP AMP gain of 1000. The following circuit provides one solution.



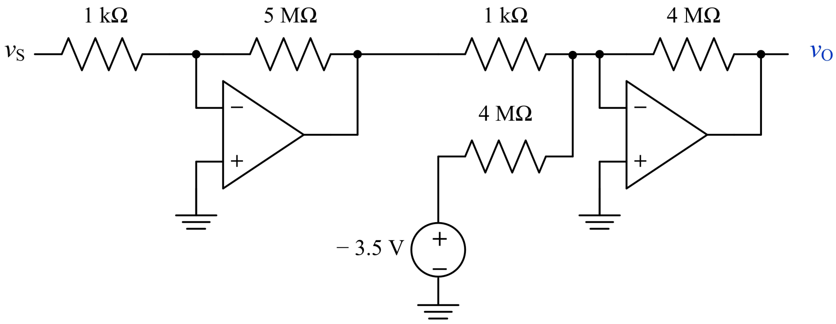
Problem 4–58. (D) Design a four-input noninverting summer that will produce an output $v_O = 6(v_1 + v_2 + v_3 + 1\text{ V})$.

Follow the approach presented in Example 4–17. The four inputs all have the same input resistance and the gain stage has a gain of 24 to compensate for the attenuation at the input stage. The following circuit provides one solution.



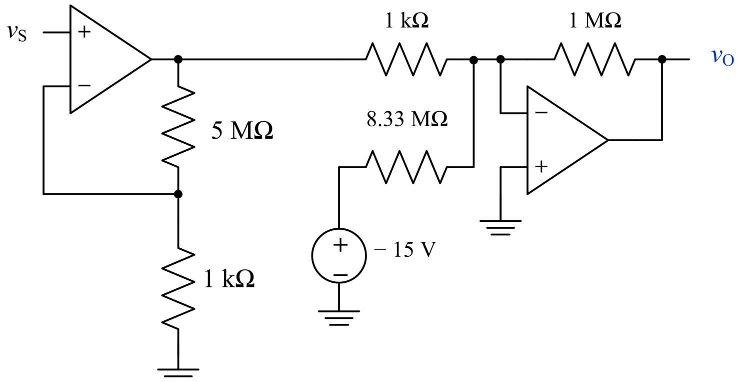
Problem 4–59. (D) Design a cascaded OP AMP circuit that will produce the following output $v_O = 2 \times 10^7 v_S + 3.5$ V. The maximum gain for an OP AMP is 10,000. The input stage must have an input resistance of 1 kΩ or greater.

Use two stages with gains of −5000 and −4000 to get the overall gain of 2×10^7 on v_1. Use an inverting summer to include the 3.5-V voltage. The following circuit provides one solution.



Problem 4–60. (D) Design a cascaded OP AMP circuit that will produce the following output $v_O = -5 \times 10^6 + 1.8$ V. The maximum gain for an OP AMP is 10,000. The input stage must have an input resistance of 1 kΩ or greater. The only voltage source available is the ±15 V used to power the OP AMPs.

Use two stages. The first stage is a noninverting amplifier with a gain of 5000 applied to v_S. The second stage is an inverting summer with a gain of −1000 applied to the output of the first stage and a gain of 0.12 applied to a −15-V source input. The following circuit provides one solution.



Problem 4–61. (D) Using the difference amplifier shown in Figure 4–53 (Example 4–20), design a circuit that will produce the following output $v_O = 500(v_1 - v_2)$.

Using the circuit in Figure 4–53, the output voltage is as follows:

$$v_O = \left[\frac{R_1 + R_2 + R_3}{R_2}\right](v_2 - v_1)$$

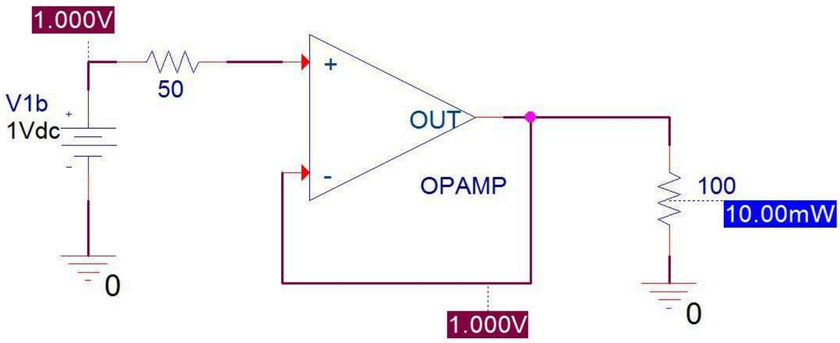
To get the desired output, swap the input sources in Figure 4–53 and use $R_1 = R_3 = 499$ kΩ and $R_2 = 2$ kΩ.

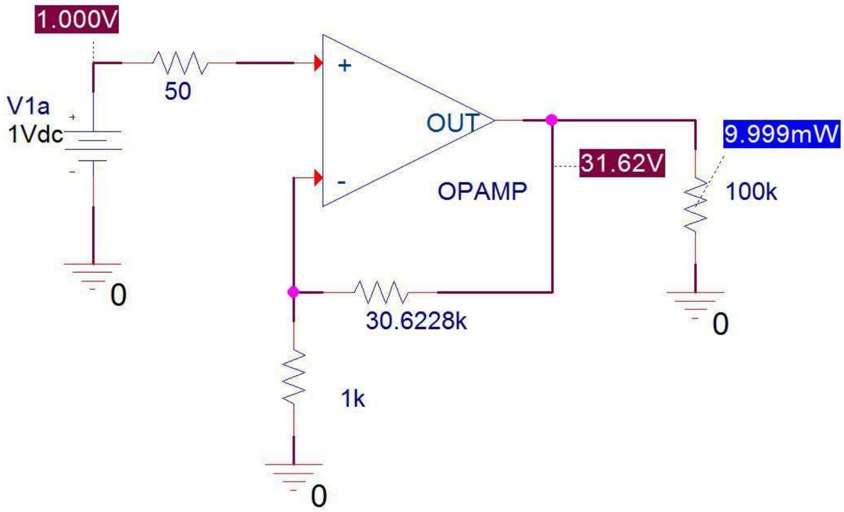
Problem 4–62. (D) Design the interface circuit in Figure P4–62 so that 10 mW is delivered to the 100-Ω load. Repeat for a 100-kΩ load. Verify your designs using OrCAD.

With 10 mW supplied to a 100-Ω load, the voltage across the load is $v = \sqrt{pR} = \sqrt{(0.01)(100)} = 1$ V. The voltage source provides 1 V, so use a buffer to transfer that voltage to the load.

With 10 mW supplied to a 100-kΩ load, the voltage across the load is $v = \sqrt{(0.01)(100000)} = 31.62$ V. Use a noninverting amplifier with a gain of 31.62 to deliver the proper voltage to the load.

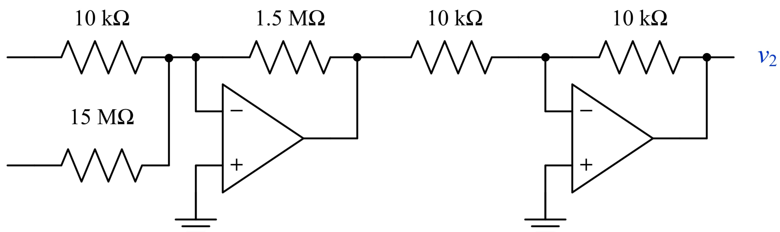
The following two OrCAD simulations verify the results.





Problem 4–63. (D) Design the interface circuit in Figure P4–63 so that the output is $v_2 = 150v_1 + 1.5$ V.

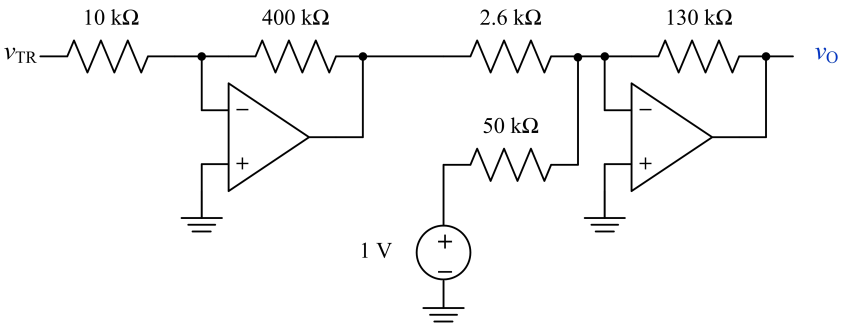
Use a cascade connection of an inverting summer and an inverting amplifier. The inputs to the inverting summer are v_1 and 15 V, with gains of -150 and -0.1, respectively. The output of the summer connects to the inverting amplifier with a gain of -1. The following circuit provides one solution. Note that the input resistance seen by v_1 is large enough to make the 50-Ω source resistance insignificant.



Problem 4–64.

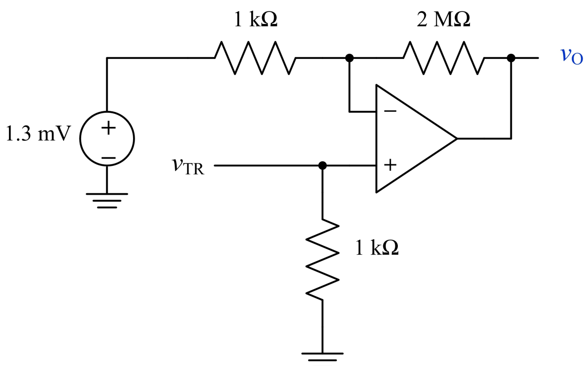
(a). **(D)** Design a circuit that can produce $v_O = 2000v_{TR} - 2.6$ V using two OP AMPs. The input resistance must be greater than 5 kΩ for v_{TR}.

Use two stages in cascade. The first stage is an inverting amplifier with an input of v_{TR} and a gain of -40. The second stage is an inverting summer with one input connected to the output of the first stage and one input connected to a 1-V reference source. The gains for the inverting summer are $K_{21} = -50$ and $K_{22} = -2.6$. The following circuit provides one solution.



(b). Repeat using only one OP AMP.

Use a standard subtractor design. Assume we can create a voltage source with a value of 1.3 mV. The following circuit provides one solution. Note that, mathematically, the voltage v_{TR} has a gain of 2001, but this value is reasonably close to to the desired gain of 2000.



Problem 4–65. (E) A requirement exists for an OP AMP circuit with the input-output relationship $v_O = 5v_{S1} - 2v_{S2}$. Two proposed designs are shown in Figure P4–65. As the project engineer, you must recommend one of these circuits for production. Which of these circuits would you recommend for production and why? (*Hint:* First, verify that the circuits perform the required function.)

Circuit 1 has two stages. The first stage is a noninverting amplifier with input v_{S1} and a gain of $K_1 = (2+3)/3 = 5/3$, so its output is $v_{O1} = 5v_{S1}/3$. The second stage is a subtractor with inputs of v_{O1} and v_{S2} and gains of $K_{21} = (2+1)/1 = 3$ and $K_{22} = -2/1 = -2$. We can calculate v_O as follows:

$$v_O = 3v_{O1} - 2v_{S2} = 3\left(\frac{5}{3}\right)v_{S1} - 2v_{S2} = 5v_{S1} - 2v_{S2}$$

Circuit 1 provides the required input-output relationship.

Circuit 2 is a standard subtractor design.

$$v_O = -\frac{R_2}{R_1}(v_{S2}) + \left(\frac{R_4}{R_3+R_4}\right)\left(\frac{R_1+R_2}{R_1}\right)(v_{S1}) = -\frac{5}{1}v_{S2} + \left(\frac{1}{2+1}\right)\left(\frac{1+5}{1}\right)v_{S1} = -5v_{S2} + 2v_{S1}$$

Circuit 2 does not provide the required input-output relationship. Choose circuit 1 for production because it meets the specifications.

Problem 4–66. (E) A requirement exists for an OP AMP circuit to deliver 12 V to a 1-kΩ load using a 4-V source as an input voltage. Two proposed designs are shown in Figure P4–66. Some characteristics of the OP AMP that must be used in the design are

Characteristic	Min	Typical	Max	Units
Open-loop Gain	10^5	2×10^5	-	V/mV
Input Resistance	10^{10}	10^{11}	-	Ω
Output Voltage	-12	-	$+15$	V
Output Current	-	-	25	mA

Which of these circuits would you recommend for production and why? (*Hint:* First, verify that the circuits perform the required function.)

Circuit 1 has an input of 4 V and a gain $K = (360 + 180)/180 = 3$, so the output is 12 V across the 1-kΩ resistor. Circuit 2 has an input of 4 V and a gain of $K = (2000 + 1000)/1000 = 3$, so the output is 12 V across the 1-kΩ resistor. The key difference between the circuits is the magnitude of the resistors used in the designs. Circuit 1 has relatively small resistor values and the current through the 360-Ω resistor is $i_1 = (12 - 4)/360 = 8/360 = 22.22$ mA and the current through the load resistor is $i_L = 12/1000 = 12$ mA. The OP AMP must supply both of these currents. The total current is 34.22 mA, which exceeds the maximum current output rating for the OP AMP. In contrast, in circuit 2, the current through the 2-kΩ resistor is $i = (12 - 4)/2000 = 4$ mA and the current through the load stays at 12 mA. The total current in circuit 2 is 16 mA, which does not exceed the specification. Recommend circuit 2 for production because it meets the specification within the operational limits of the OP AMP.

Problem 4–67. (E) A particular application requires that an instrumentation interface deliver $v_O = 200v_{TR} - 5$ V ±2% to a DAC. The solution currently in use requires two OP AMPs and is constantly draining the supply batteries. A young engineer designed another tentative solution using just one OP AMP shown in Figure P4–67. As her supervisor, you must determine if it works. Does her design meet the specifications?

The circuit is a modified subtractor. Find the Thévenin equivalent at the negative input terminal. The input voltage is $v_T = (100)(5)/20100 = 24.876$ mV and the lookback resistance is $R_T = 3300 + (20000 \parallel 100) = 3300 + 99.5 = 3.4$ kΩ. Compute the output voltage v_2.

$$v_2 = -\frac{R_2}{R_T}(v_T) + \left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_T + R_2}{R_T}\right)(v_{TR})$$

$$v_2 = -\frac{680}{3.4}(0.024876) + \left(\frac{680}{3.3 + 680}\right)\left(\frac{3.4 + 680}{3.4}\right)v_{TR}$$

$$v_2 = -4.975 + 200.03v_{TR}$$

The design meets the specifications within the allowed tolerances.

Problem 4–68. The analog output of a five-bit DAC is 3.59 V when the input code is (1, 0, 1, 1, 1). What is the full-scale output of the DAC? How much does the analog output change when the input LSB changes?

The input-output relationship for a five-bit DAC is as follows:

$$v_O = KV_{REF}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8} + \frac{b_5}{16}\right)$$

The problem statement yields:

$$3.59 = KV_{REF}\left(1 + \frac{0}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right)$$

$$KV_{REF} = 2.49739 \text{ V}$$

Assume $KV_{REF} = 2.5$ V and compute the full-scale output.

$$v_O = 2.5\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) = 4.84375 \text{ V}$$

When the LSB changes, the output changes by $2.5/16 = 0.15625$ V.

Problem 4–69. The full-scale output of a six-bit DAC is 5.0 V. What is the analog output when the input code is (0, 1, 0, 1, 0, 0)? What is the resolution of this DAC?

The input-output relationship for a six-bit DAC is as follows:

$$v_O = KV_{REF}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8} + \frac{b_5}{16} + \frac{b_6}{32}\right)$$

The problem statement yields:

$$5 = KV_{REF}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}\right)$$

$$KV_{REF} = 2.53968 \text{ V}$$

With an input code (0, 1, 0, 1, 0, 0), the output is

$$v_O = 2.53968\left(0 + \frac{1}{2} + \frac{0}{4} + \frac{1}{8} + \frac{0}{16} + \frac{0}{32}\right) = 1.5873 \text{ V}$$

The resolution is $KV_{REF}/32 = 2.53968/32 = 79.3651$ mV.

Problem 4–70. An R-$2R$ DAC is shown in Figure P4–70. The digital voltages v_1, v_2, etc., can be either 5 V for a logic 1 or 0 V for a logic 0. What is the DAC's output when the logic input is (1, 1, 0 1)?

The input-output relationship for an R-$2R$ DAC is given as

$$v_O = -\frac{R_F}{2R}V_{REF}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8}\right)$$

Compute the output when the logic input is (1, 1, 0 1).

$$v_O = -\frac{2R}{2R}(5)\left(1 + \frac{1}{2} + \frac{0}{4} + \frac{1}{8}\right) = (-5)(1.625) = -8.125 \text{ V}$$

Problem 4–71. A fifth bit is added to the R-$2R$ DAC shown in Figure P4–70. What is the maximum possible magnitude of the output voltage? What is the resolution of the revised DAC?

Use the results developed in Problem 4–70 to compute the requested values.

$$v_O = -5\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8} + \frac{b_5}{16}\right)$$

$$v_{O,MAX} = -5\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) = -9.6875 \text{ V}$$

$$\text{Res} = \frac{5}{16} = 0.3125 \text{ V}$$

Problem 4–72. (D) A Chromel-Constantan thermocouple (curve E) has the characteristics shown in Figure P4–72. Design an interface that will produce a 0- to 5-V output where 0 V refers to 0° C and 5 V refers to 1000° C. The transducer can be modeled as a voltage source in series with a 15-Ω resistor.

A temperature of 0° C produces a voltage of 0 mV, and at the output of the interface it should produce 0 V. A temperature of 1000° C produces a voltage of 75 mV, and at the output of the interface it should produce 5 V. Compute the required gain.

$$K = \frac{v_O(p_1) - v_O(p_2)}{v_{TR}(p_1) - v_{TR}(p_2)} = \frac{0-5}{0-0.075} = 66.67$$

Compute the bias voltage.

$$Kv_{\rm TR} + V_{\rm b} = v_{\rm O}$$

$$(66.67)(0) + V_{\rm b} = 0$$

$$V_{\rm b} = 0 \text{ V}$$

The required gain is $K = 66.67$ and there is no bias voltage. The interface is a standard noninverting amplifier with a gain of 66.67. Select $R_1 = 65.67$ kΩ and $R_2 = 1$ kΩ.

Problem 4–73. (D) A Chromel-Alumel thermocouple (curve K in Figure P4–72) is used to measure the temperature of an electric oven used in the semiconductor industry. Design an interface that will produce a 0- to 5-V output where 0 V refers to 200° C and 5 V refers to 1200° C (assume a straight line out to 1200° C). The transducer can be modeled as a voltage source in series with a 100-Ω resistor.

A temperature of 200° C produces a voltage of 8 mV, and at the output of the interface it should produce 0 V. A temperature of 1200° C produces a voltage of approximately 50 mV, and at the output of the interface it should produce 5 V. Compute the required gain.

$$K = \frac{v_{\rm O}(p_1) - v_{\rm O}(p_2)}{v_{\rm TR}(p_1) - v_{\rm TR}(p_2)} = \frac{0-5}{0.008 - 0.05} = 119$$

Compute the bias voltage.

$$Kv_{\rm TR} + V_{\rm b} = v_{\rm O}$$

$$(119)(0.008) + V_{\rm b} = 0$$

$$V_{\rm b} = -952 \text{ mV}$$

The required gain is $K = 119$ and the bias voltage is $V_{\rm b} = -952$ mV. Use two stages to design the interface. The first stage is an inverting amplifier with input $v_{\rm TR}$, $R_1 = 10$ kΩ, and $R_2 = 1.19$ MΩ to yield an output of $v_{\rm O1} = -119v_{\rm TR}$. The second stage is an inverting summer with two inputs and both gains set to -1. The first input to the summer is the output of the first stage and the second input is a fixed voltage of 952 mV to provide the bais voltage.

Problem 4–74. (D) An analog accelerometer produces a continuous voltage that is proportional to acceleration in gravitational units or g. Figure P4–74 shows the characteristics of the accelerometer in question. The black curve is the actual characteristics; the colored curve is an acceptable linearized model. Design an instrumentation system that will output 0 V for $-1g$ and 5 V for $1.5g$. Note that this accelerometer has an output resistance of 32 kΩ.

An acceleration of $-1g$ produces a voltage of 1.5 V, and at the output of the interface it should produce 0 V. An acceleration of $1.5g$ produces a voltage of 4 V, and at the output of the interface it should produce 5 V. Compute the required gain.

$$K = \frac{v_{\rm O}(p_1) - v_{\rm O}(p_2)}{v_{\rm TR}(p_1) - v_{\rm TR}(p_2)} = \frac{0-5}{1.5-4} = 2$$

Compute the bias voltage.

$$Kv_{\rm TR} + V_{\rm b} = v_{\rm O}$$

$$(2)(1.5) + V_{\rm b} = 0$$

$$V_{\rm b} = -3 \text{ V}$$

The required gain is $K = 2$ and the bias voltage is $V_{\rm b} = -3$ V. Use two stages to design the interface. The first stage is an inverting amplifier with input $v_{\rm TR}$, $R_1 = 0$ kΩ, and $R_2 = 64$ kΩ to yield an output of $v_{O1} = -2v_{TR}$. We set $R_1 = 0$ kΩ because the accelerometer has an output resistance of 32 kΩ, which essentially replaces R_1 in this case. The second stage is an inverting summer with two inputs and both gains set to -1. The first input to the summer is the output of the first stage and the second input is a fixed voltage of 3 V to provide the bais voltage.

Problem 4–75. (D) A small pressure transducer has the characteristics shown in Figure P4–75. Design an interface that will operate between 7- and 32-psi. An input of 7 psi should produce -5 V and 32 psi should produce 5 V. The transducer is modeled as a voltage source in series with a 500-Ω resistor that can vary ± 75 Ω depending on the pressure. The OP AMPs you must use have a maximum closed-loop gain of 2000. Your available bias source is 5 V.

A pressure of 7 psi produces a voltage of 1000 μV, and at the output of the interface it should produce -5 V. A pressure of 32 psi produces a voltage of 400 μV, and at the output of the interface it should produce 5 V. Compute the required gain.

$$K = \frac{v_O(p_1) - v_O(p_2)}{v_{TR}(p_1) - v_{TR}(p_2)} = \frac{-5-5}{0.001 - 0.0004} = -16667$$

Compute the bias voltage.

$$Kv_{TR} + V_b = v_O$$

$$(-16667)(0.001) + V_b = -5$$

$$V_b = 11.667 \text{ V}$$

The required gain is $K = -16667$ and the bias voltage is $V_b = 11.667$ V. Use two stages to design the interface. The first stage is a noninverting amplifier with an input of v_{TR} and a gain of 100 by choosing $R_1 = 99$ kΩ and $R_2 = 1$ kΩ. Note that the noninverting amplifier has a very high input resistance, so the variation in the output resistance of the transducer will not be factor. The second stage is an inverting summer with two inputs. The first input is the output of the first stage and it has a gain of -167 by choosing $R_1 = 3$ kΩ and $R_F = 500$ kΩ. The second input is the negative terminal of the 5-V source, where the positive terminal is connected to ground, and the input has a gain of -2.33 by choosing $R_2 = 214.3$ kΩ with R_F still as 500 kΩ.

Problem 4–76. (D) A medical grade pressure transducer has been developed for use in invasive blood pressure monitoring. The output voltage of the transducer is $v_{TR} = (0.06P - 0.75)$ mV, where P is pressure in mmHg. The output resistance of the transducer is 1 kΩ. The blood pressure measurement is to be an input to an existing multisensor monitoring system. This system treats a 1-V input as a blood pressure of 20 mmHg and a 10-V input as 200 mmHg. Design an OP AMP circuit to interface the new pressure transducer with the existing monitoring system.

A pressure of 20 mmHg produces a voltage of $v_{TR} = [(0.06)(20) - 0.75] = 0.45$ mV, and at the output of the interface it should produce 1 V to match the existing system. A pressure of 200 mmHg produces a voltage of $v_{TR} = [(0.06)(200) - 0.75] = 11.25$ mV, and at the output of the interface it should produce 10 V. Compute the required gain.

$$K = \frac{v_O(p_1) - v_O(p_2)}{v_{TR}(p_1) - v_{TR}(p_2)} = \frac{1-10}{0.00045 - 0.01125} = 833$$

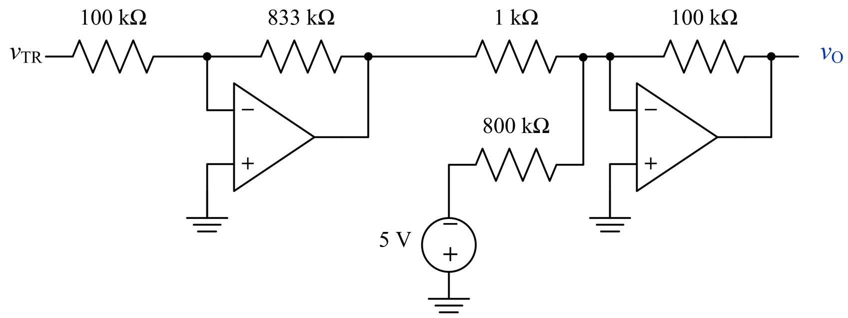
Compute the bias voltage.

$$Kv_{TR} + V_b = v_O$$

$$(833)(0.00045) + V_b = 1$$

$$V_b = 0.625 \text{ V}$$

The required gain is $K = 833$ and the bias voltage is $V_b = 0.625$ V. Use two stages to design the interface. The first stage is an inverting amplifier with an input of v_{TR} and a gain of -8.33. The second stage is an inverting summer with two inputs. The first input is the output of the first stage and has a gain of −100. The second input is from a −5-V source and has a gain of −0.125 to create the bias voltage. One possible design is shown below. Note that the input resistance on the first stage is relatively high to account for the 1-kΩ output resistance of the transducer.



Problem 4–77. (D) The acid/alkaline balance of a fluid is measured by the pH scale. The scale runs from 0 (extremely acid) to 14 (extremely alkaline,) with pH 7 being neutral. A pH electrode is a sensor that produces a small voltage that is directly proportional to the pH of the fluid in a test chamber. For a certain pH electrode, the proportionality factor is 60 mV/pH. A preamplifier is needed to interface this sensor with a variety of laboratory instruments. The output of the preamp must be 1 V when the sensor is immersed in a test solution with pH = 4 and 1.75 V when it is immersed in a solution with pH = 7. Design an amplifier to meet these requirements.

At a pH of 4, the sensor produces (4)(60) = 240 mV, and at the output of the interface it should produce 1 V. At a pH of 7, the sensor produces (7)(60) = 420 mV, and at the output of the interface it should produce 1.75 V. Compute the required gain.

$$K = \frac{v_O(p_1) - v_O(p_2)}{v_{TR}(p_1) - v_{TR}(p_2)} = \frac{1 - 1.75}{0.24 - 0.42} = 4.167$$

Compute the bias voltage.

$$Kv_{TR} + V_b = v_O$$

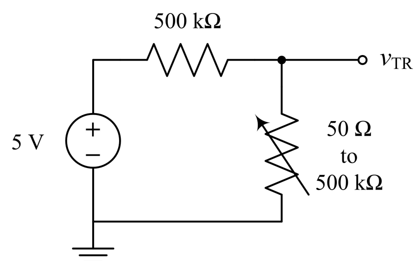
$$(4.167)(0.24) + V_b = 1$$

$$V_b = 0 \text{ V}$$

The required gain is $K = 4.167$ and the bias voltage is $V_b = 0$ V. Since there is no bias voltage, use a noninverting amplifier to create an interface with a gain of 4.167 by selecting $R_1 = 3.167$ kΩ and $R_2 = 1$ kΩ. The input to the interface is v_{TR}.

Problem 4–78. (D) A photoresistor varies from 50 Ω in bright sunlight to 500 kΩ in total darkness. Design a suitable circuit using the photoresistor so that total darkness produces 5 V, while bright sunlight produces −5 V, regardless of the load. You have a 5-V source and a ±15-V source to power any OP AMP you may need.

First, create a simple series circuit with the 5-V source, a fixed 500-kΩ resistor, and the photoresistor. The output of this circuit will be the voltage across the photoresistor, which is the transducer votlage v_{TR}, as shown below.



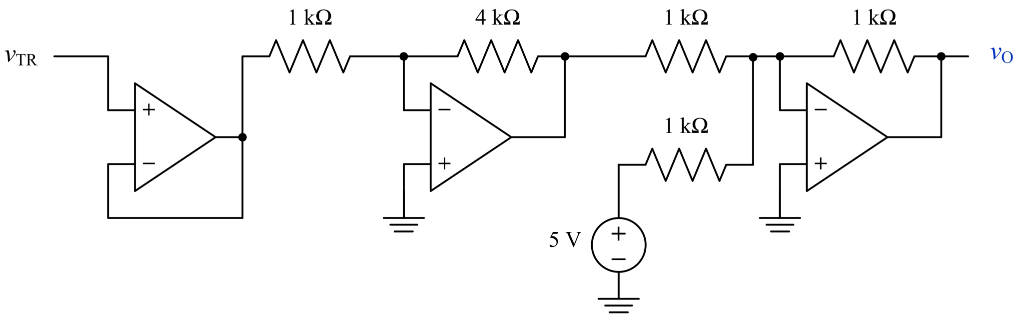
Apply voltage division to determine the transducer output voltage for different lighting conditions. For bright sunlight, $v_{TR} = (5)(50)/(50{+}500000) = 500\ \mu$V. For total darkness, $v_{TR} = (5)(500000)/(500000{+}500000) = 2.5$ V. The interface needs to translate 500 μVinto -5 V and 2.5 V into 5 V. Compute the required gain.

$$K = \frac{v_O(p_1) - v_O(p_2)}{v_{TR}(p_1) - v_{TR}(p_2)} = \frac{-5-5}{0.0005-2.5} = 4$$

Compute the bias voltage.

$$Kv_{TR} + V_b = v_O$$
$$(4)(2.5) + V_b = 5$$
$$V_b = -5\text{ V}$$

The required gain is $K = 4$ and the bias voltage is $V_b = -5$ V. Design an interface with three stages. The first stage is a buffer to isolate the transducer from the rest of the interface. The second stage connects to the buffer and is an inverting amplifier with a gain of -4. The third stage is an inverting summer with two inputs, both with gains of -1. One input is the output of the second stage and the other input is from the 5-V source. One possible design is shown below.

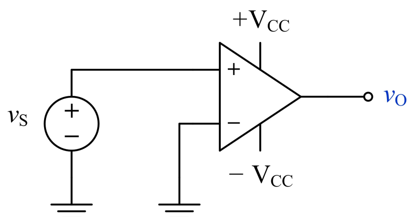


Problem 4–79. (E) Your engineering firm needs an instrumentation amplifier that provides the following input-output relationship: $v_O = 10^6 v_{TR} - 3.5$ V. The transducer is modeled as a voltage source in series with a resistor that varies with the transducer voltage from 40 Ω to 750 Ω. A vendor is offering the amplifier shown in Figure P4–79 and the vendor agrees to make a single change to the amplifier, if needed, for no cost. Would you recommend buying it? Explain your rationale.

As shown, the circuit has an overall gain of $K = (-1000)(-1000) = 10^6$ for v_{TR}, and a bias of $V_b = (5)(-1/1.42) = -3.52$ V, which meet the specifications with only minor errors. The circuit appears to be a reasonable design, but when the transducer's output resistance increases above 100 Ω, it will load the first OP AMP stage and decrease the gain of the circuit. To mitigate the effect of the transducer's output resistance, we need to place a buffer at the input of the interface circuit. Purchase the circuit if the vendor will add a buffer at the circuit input without increasing the cost.

Problem 4–80. (D) Your supervisor drew the Figure of P4–80 on the back of an envelope to show you what he expects as an output to a signal that varies between ±5 V. Design a suitable comparator circuit to achieve his expectation.

The comparator circuit output is +15 V for positive inputs and -15 V for negative inputs. Use an OP AMP with $V_{CC} = \pm 15$ V. Let the input to the positive terminal be v_S and ground the input to the negative terminal. The design is shown below.



Problem 4–81. The OP AMP in Figure P4–81 operates as a comparator. Find the output voltage when $v_S = 2$ V. Repeat for $v_S = -2$ V and $v_S = 6$ V.

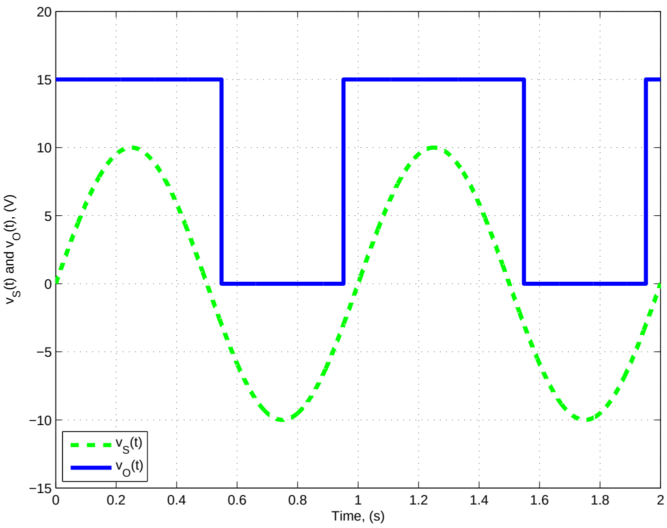
The voltages powering the OP AMP are +5 V and 0 V, so its output will be one of these two values. The input at the positive terminal is v_S and the input to the negative terminal can be found through voltage division as $v_N = (5)(4R)/(4R + R) = 4$ V. In the comparator configuration, if $v_S > 4$ V, the output will be +5 V, and if $v_S < 4$ V, the output will be 0 V. For $v_S = 2$ V, $v_O = 0$ V; for $v_S = -2$ V, $v_O = 0$ V; and for $v_S = 6$ V, $v_O = 5$ V.

Problem 4–82. The circuit in Figure P4–82 has $V_{CC} = 15$ V and $v_N = -3$ V. Sketch the output voltage v_O on the range $0 \leq t \leq 2$ s for $v_S(t) = 10\sin(2\pi t)$ V.

The voltages powering the OP AMP are +15 V and 0 V, so its output will be one of these two values. The input at the positive terminal is $v_S(t)$ and the input to the negative terminal is −3 V. In the comparator configuration, if $v_S > -3$ V, the output will be +15 V, and if $v_S < -3$ V, the output will be 0 V. The following MATLAB code plots $v_S(t)$ and $v_O(t)$.

```
% Create the input signal
t = 0:0.0001:2;
vS = 10*sin(2*pi*t);
% Determine the output signal
VCC = 15;
vN = -3;
vO = VCC*(vS>vN);
% Plot the results
figure
plot(t,vS,'g--','LineWidth',3);
hold on
grid on
axis([0,2,-15,20]);
xlabel('Time, (s)')
ylabel('v_S(t) and v_O(t), (V)')
plot(t,vO,'b','LineWidth',3);
legend('v_S(t)','v_O(t)','Location','SouthWest')
```

The corresponding MATLAB output is shown below.



Problem 4–83. (D) A signal varies as $v_S(t) = 10(1 - e^{-100t})$ V. Design a comparator circuit that will switch from 0 to 5 V at 2 ms. Use a single OP AMP that has $+V_{CC} = +5$ V and $-V_{CC} = 0$ V.

Calculate the voltage for $v_S(t)$ at 2 ms.

$$v_S(0.002) = 10(1 - e^{-100(0.002)}) = 10(1 - 0.818731) = 1.8127 \text{ V}$$

Note that $v_S(t)$ has a value of 0 V at $t = 0$ s and increases to a final value of 10 V. So $v_S(t)$ increases from 0 to 1.8127 V as t increases from 0 to 2 ms. After 2 ms, $v_S(t)$ is larger than 1.8127 V. Design the comparator to switch from 0 V to 5 V when the input voltage rises above 1.8127 V. Use a standard comparator circuit with $v_P = v_S(t)$ and $v_N = 1.8127$ V.

Problem 4–84. A 5-bit flash ADC in Figure P4–84 uses a reference voltage of 6 V. Find the output code for the analog inputs $v_S = 3.5$ V, 2.3 V, and 5.3 V. If the reference voltage is changed to 9 V, which of these codes would change?

For each OP AMP, if v_S is greater than v_{Ni}, then the output is a binary 1. Otherwise, the output is a binary 0. Using voltage division, with $V_{REF} = 6$ V, each resistor drops one volt, so $v_{N1} = 5$ V, $v_{N2} = 4$ V, $v_{N3} = 3$ V, $v_{N4} = 2$ V, and $v_{N5} = 1$ V. If the reference voltage is increased to 9 V, each resistor drops 1.5 V, so we have $v_{N1} = 7.5$ V, $v_{N2} = 6$ V, $v_{N3} = 4.5$ V, $v_{N4} = 3$ V, and $v_{N5} = 1.5$ V. The following table summarizes the digital output signals, $(b_1, b_2, b_3, b_4, b_5)$, for each of the analog input signals.

v_S (V)	$V_{REF} = 6$ V	$V_{REF} = 9$ V	Status
3.5	(0, 0, 1, 1, 1)	(0, 0, 0, 1, 1)	Changed
2.3	(0, 0, 0, 1, 1)	(0, 0, 0, 0, 1)	Changed
5.3	(1, 1, 1, 1, 1)	(0, 0, 1, 1, 1)	Changed

Problem 4–85. (A) Bipolar Power Supply Voltages

The circuit in Figure P4–85 produces bipolar power supply voltages $V_{POS} > 0$ and $V_{NEG} < 0$ from a floating unipolar voltage source $V_{REF} > 0$. Note that the OP AMP output is grounded and that its $+V_{CC}$ and $-V_{CC}$ terminals are connected to V_{POS} and V_{NEG}, respectively.

(a). Show that $V_{POS} = +V_{REF}/2$ and $V_{NEG} = -V_{REF}/2$ even if the load resistors R_{POS} and R_{NEG} are not equal.

The OP AMP output is connected to ground and the negative input terminal, so $v_N = 0$ V. If the OP AMP is operating in its linear range, then $v_N = v_P = 0$ V, and the node between the two R resistors acts as a ground. The voltage V_{REF} appears across the two R resistors. Under the ideal OP AMP assumptions, no current enters the OP AMP through the positive terminal, so the current through the two R resistors must be $i = V_{REF}/2R$. The voltage drop across each R resistor is therefore $V_{REF}/2$. If the OP AMP causes the center node to have a voltage of 0 V, then the top node must be $V_{POS} = V_{REF}/2$ and the bottom node must be $V_{NEG} = -V_{REF}/2$. The load resistors have no influence on these voltages.

(b). If R_{POS} and R_{NEG} are not equal, a current i_G must flow into or out of ground. How does the ungrounded voltage source V_{REF} supply this ground current?

The OP AMP is grounded and it will produce the required currents to satisfy the voltage conditions of the external circuit, as long as the OP AMP is operating in its linear region.

(c). In effect, the OP AMP creates a "virtual ground" at point A ($V_A = 0$) but draws no current in doing so. Why not just connect point A to a "real ground" and do away with the OP AMP?

Without the OP AMP, the load resistors R_{POS} and R_{NEG} would interact with the other resistors in the circuit and change the results. The OP AMP serves as a buffer to isolate the load and the reference voltage while still transferring the reference voltage to the output stage in a controlled manner.

Problem 4–86. (D) Thermometer Design Problem

There is a need to design a thermometer that can read from 250° C to 1000° C to monitor the temperature of a rocket's nose cone. The output will feed a 0- to 5-V ADC prior to transmission of the temperature data to the ground. A reading of 250° C will deliver 0 V to the ADC, while 1000° C will deliver 5 V. Select from Figure P4–72 an appropriate thermocouple with sufficient voltage spread and design the instrumentation amplifier. Other criteria are: the transducers have a 15-Ω series resistance, the fewer OP AMPs the better, since power is limited, the maximum OP AMP gain is 1000, and the bias voltage, if needed, must be from the available 15-V supply.

In Figure P4–72, thermocouple K is a reasonable choice, since it is linear across the required temperature range. At 250° C, the output voltage is approximately 10 mV and should be adjusted to provide 0 V to the

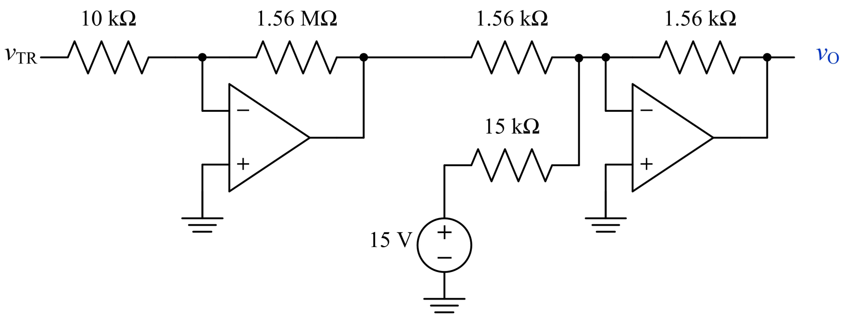
ADC. At 1000° C, the output voltage is approximately 42 mV and should be adjusted to provide 5 V to the ADC. Compute the required gain.

$$K = \frac{v_{\rm O}(p_1) - v_{\rm O}(p_2)}{v_{\rm TR}(p_1) - v_{\rm TR}(p_2)} = \frac{0-5}{0.01-0.042} = 156.25$$

Compute the bias voltage.

$$\begin{aligned} Kv_{\rm TR} + V_{\rm b} &= v_{\rm O} \\ (156.25)(0.01) + V_{\rm b} &= 0 \\ V_{\rm b} &= -1.56 \text{ V} \end{aligned}$$

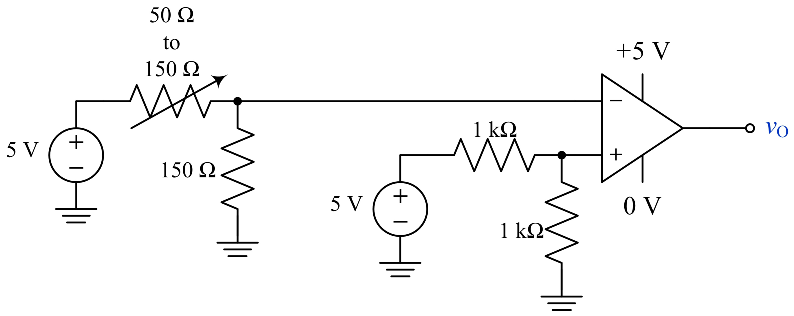
The required gain is $K = 156$ and the bias voltage is $V_{\rm b} = -1.56$ V. To minimize the number of OP AMPs, consider a single subtractor to create the interface. In this case, for the required gain and the bias voltage, a subtractor design cannot meet the specifications. Instead, use a two-stage design. The first stage is an inverting amplifier connected to the transducer output with a gain of -156 and $R_1 = 10$ kΩ and $R_2 = 1.56$ MΩ. The second stage is an inverting summer with two inputs. The first input is connected to the output of the first stage and has a gain of -1. The second input is connected to the 15-V supply and has a gain of $-1.56/15 = -0.104$ to provide the correct bias. Choose $R_{22} = 15$ kΩ and $R_{\rm F} = 1.56$ kΩ to generate the correct bias voltage. The design is shown below.



Problem 4–87. (D) High Bias Design Problem

A particular pressure sensor is designed to operate under constant pressure. The task is to detect a pressure increase and sound an alarm. The sensor produces 1 mV at 100 psi, its usual operating pressure, and increases by 1 μV per psi. The design must sound an alarm if the pressure reaches 150 psi. The transducer is modeled as a voltage source with a series resistor that varies with pressure from 50 Ω at 100 psi to 150 Ω at 150 psi. Two types of OP AMPs are available. Type 1 are single-sided, meaning they have a $+V_{\rm CC}$ of 5 V and a $-V_{\rm CC}$ of 0 V. Type 2 OP AMPs are double-sided with a $V_{\rm CC}$ of ±15 V and have a maximum closed-loop gain of 5000. The alarm can be driven with the +5 V available from a saturated type 1 OP AMP. Only ±15-V and +5-V sources are available. Design the alarm circuit.

Connect the transducer model in series with a 5-V source and a fixed 150-Ω resistor to create a voltage divider with the output across the 150-Ω resistor. As the pressure increases, the transducer resistor increases in value and the output voltage decreases. When the output voltage reaches 2.5 V or lower, we want the alarm to sound. Connect the output of the voltage divider to the negative input of a comparator circuit and connect a 2.5-V reference signal to the positive input. Use a Type 1 OP AMP so that it produces the correct signal for the alarm. The following circuit will meet the specifications.



Problem 4–88. (A) Current Switching DAC

The circuit in Figure P4–88 is a 4-bit digital-to-analog converter (DAC). The DAC output is the voltage v_O and the input is the binary code represented by bits b_1, b_2, b_3, and b_4. The input bits are either 0 (low) or 1 (high), and each controls one of the four switches in the figure. When bits are low, their switches are in the left position, directing the $2R$ leg currents to ground. When bits are high, their switches move to the right position, directing the $2R$ leg currents to the OP AMP's inverting input. The $2R$ leg currents do not change when switching from left to right because the inverting input is a virtual ground ($v_N = v_P = 0$). The purpose of this problem is to show that this constant-current switching produces the following input-output relationship.

$$v_O = -\frac{R_F}{2R}V_{REF}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8}\right)$$

(a). Since the inverting input is a virtual ground, show that the currents in the $2R$ legs are $i_1 = V_{REF}/2R$, $i_2 = V_{REF}/4R$, $i_3 = V_{REF}/8R$, and $i_4 = V_{REF}/16R$, regardless of switch positions.

The equivalent resistance seen by the reference voltage source is R, so the current flowing out of the source is $i = V_{REF}/R$. The equivalent resistance to the right of node A, is $2R$, so the current splits equally at node A and $i_1 = i/2 = V_{REF}/2R$. The equal current divisions repeat three more times in the same manner to yield $i_2 = V_{REF}/4R$, $i_3 = V_{REF}/8R$, and $i_4 = V_{REF}/16R$. The switch positions do not matter, since the lower side of each $2R$ resistor is effectively 0 V in both cases.

(b). Show that the sum of currents at the inverting input is

$$b_1i_1 + b_2i_2 + b_3i_3 + b_4i_4 + i_F = 0$$

where bits b_k $(k = 1, 2, 3, 4)$ are either 0 or 1.

If a bit is set to 1, the corresponding switch allows the current to flow into the inverting input node. The feedback current i_F always flows into the node, so the corresponding KCL equation is given by

$$b_1i_1 + b_2i_2 + b_3i_3 + b_4i_4 + i_F = 0$$

as expected.

(c). Use the results in parts (a) and (b) to show that the OP AMP output voltage is

$$v_O = -\frac{R_F}{2R}V_{REF}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8}\right)$$

as stated.

Given the KCL equation in the solution to part (b), we can solve for i_F and then v_O as follows.

$$i_F = -b_1i_1 - b_2i_2 - b_3i_3 - b_4i_4$$

$$v_O = i_FR_F = -R_F\left(b_1i_1 + b_2i_2 + b_3i_3 + b_4i_4\right)$$

$$v_O = -R_F\left(b_1\frac{V_{REF}}{2R} + b_2\frac{V_{REF}}{4R} + b_3\frac{V_{REF}}{8R} + b_4\frac{V_{REF}}{16R}\right)$$

$$v_O = -R_F\frac{V_{REF}}{2R}\left(b_1 + \frac{b_2}{2} + \frac{b_3}{4} + \frac{b_4}{8}\right)$$

Problem 4–89. (A,D) OP AMP Circuit Analysis and Design

(a). Find the input-output relationship of the circuit in Figure P4–89.

The circuit is connected as a subtractor in series with an inverting summer. The inputs to the subtractor are v_1 and v_O. The inputs to the summer are the output of the first stage and v_2. Compute the output of the first stage and then the output of the second stage.

$$v_{O1} = -\frac{2R}{R}v_1 + \left(\frac{R+2R}{R}\right)\left(\frac{R}{2R+R}\right)v_O = -2v_1 + v_O$$

$$v_O = -\frac{R}{R}v_{O1} - \frac{R}{\frac{R}{2}}v_2$$

$$v_O = -v_{O1} - 2v_2 = 2v_1 - v_O - 2v_2$$

$$2v_O = 2v_1 - 2v_2$$

$$v_O = v_1 - v_2$$

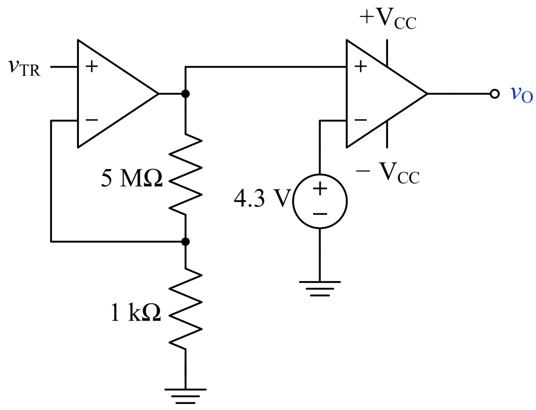
(b). Design a circuit that realizes the relationship found in part (a) using only 10-kΩ resistors and one OP AMP.

Use a standard subtractor design with v_1 connected to the side with positive gain and v_2 connected to the side with negative gain. Set all four resistors to 10 kΩ such that $R_1 = R_2 = R_3 = R_4 = 10$ kΩ to get $v_O = v_1 - v_2$.

Problem 4–90. (D) Instrumentation Amplifier with Alarm

Strain gauges measuring the deflection of a sintered metal column are connected to a Wheatstone bridge. The output of the bridge is balanced when there is no strain producing 0 V output. As the column is deflected, the bridge produces 200 μV per ohm change caused by the strain gauges. The maximum possible defection would result in a 5-Ω change. Design an instrumentation amplifier that can take the voltage created in the bridge and send it to a 0-5 V ADC, where no defection produces 0 V at the ADC and maximum deflection produces 5 V. To avoid loading, send the output of the bridge to a very high input resistance difference amplifier. The columns being tested are brittle and can shatter violently if the conditions cause the strain gauges to change by more than 4.5 Ω. For safety, connect the output of your instrumentation amplifier to a comparator circuit that will trigger an alarm when the strain causes the resistance to change by 4.3 Ω. The alarm needs to be triggered by 15 V.

With no deflection, the strain gauge produces 0 V, which needs to be translated to 0 V for the ADC. At a maximum deflection of 5 Ω the strain gauge produces 1 mV, which needs to be translated to 5 V for the ADC. The gain required for the interface is $(0 - 5)/(0 - 0.001) = 5000$. No bias voltage is required. The interface is a simple noninverting amplifier connected to the output of the strain gauge and with a gain of 5000. Select $R_1 = 5$ MΩ and $R_2 = 1$ kΩ to create the required gain. (Technically, with these resistor values the gain is 5001, but that is sufficiently close to 5000.) Connect the output of the interface to the positive terminal of a comparator circuit. Connect a constant voltage of 4.3 V to the negative terminal of the comparator. The comparator will provie 15 V to the alarm when the output of the interface rises above 4.3 V. The following circuit will meet the specifications



Problem 4–91. (E) Resistance Temperature Transducer

A resistive transducer uses a sensing element whose resistance varies with temperature. For a particular transducer, the resistance varies as $R_{TR} = 0.375T + 100\ \Omega$, where T is temperature in °C. This transducer is to be included in a circuit to measure temperatures in the range from −200° C to 800° C. The circuit must convert the transducer resistance variation over this temperature range into an output voltage in the range from 0 to 5 V. Two proposed circuit designs are shown in Figure P4–91. Which of these circuits would you recommend for production and why? (*Hint:* First verify that the circuits perform the required function.) Use OrCAD to verify your results.

First, find the range of resistor values created by the allowed range of temperatures by evaluating the expression for R_{TR} at the temperature limits.

$$R_{TR}(-200^\circ) = (0.375)(-200) + 100 = 25\ \Omega$$

$$R_{TR}(800^\circ) = (0.375)(800) + 100 = 400\ \Omega$$

Both circuits are configured as subtractors. In each case, determine the Thévenin equivalent of the source input for the possible range of resistor values. For Circuit 1, $v_P = (5R_{TR})/(100 + R_{TR})$, so v_P ranges from 1 to 4 V. Since the voltage source is connected to the positive input terminal, the Thévenin resistance does not influence the results. For Circuit 1 we have

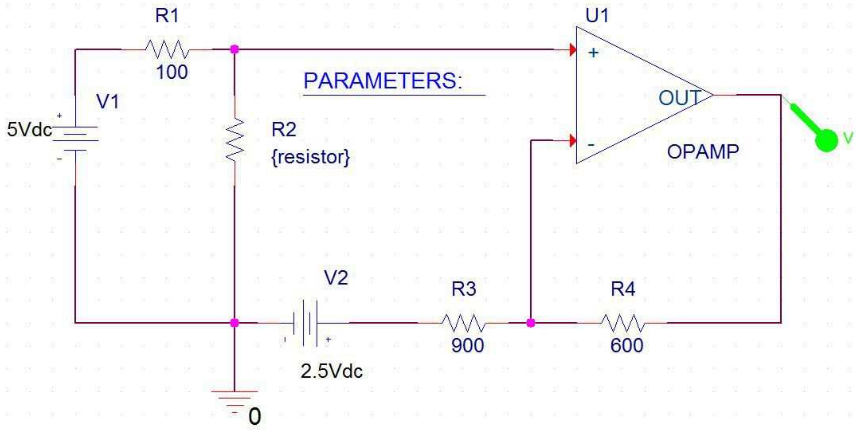
$$v_O = \frac{900 + 600}{900} v_{TR} - \frac{600}{900}(2.5) = \frac{5}{3} v_{TR} - \frac{5}{3}$$

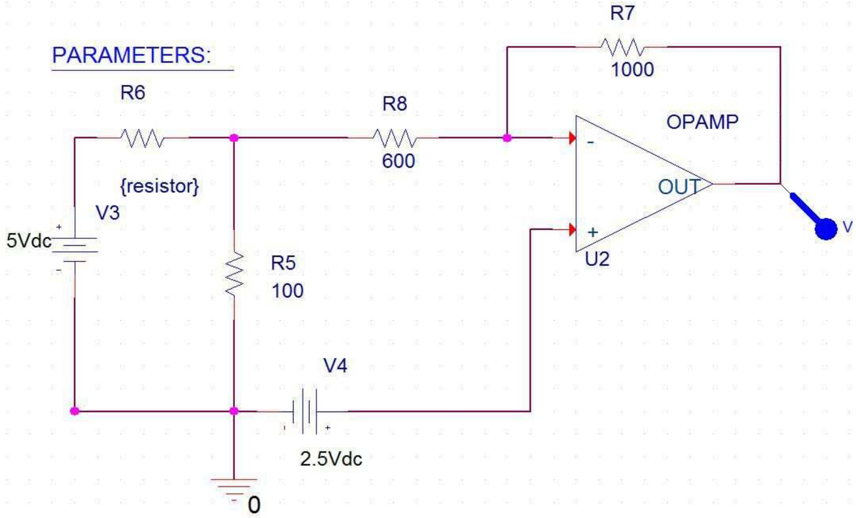
For the possible values of v_{TR}, v_O ranges from 0 to 5 V, which meets the specification.

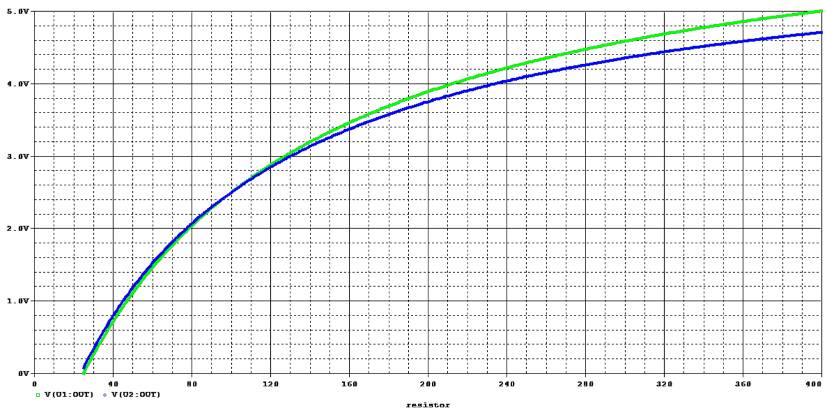
Consider Circuit 2 and find the Thévenin equivalent at the negative input terminal. The voltage is $v_{TR} = (5)(100)/(100 + R_{TR})$ and ranges from 4 to 1 V. The resistance is $R_T = 600 + (R_{TR} \parallel 100)$ and ranges from 620 to 680 Ω. Compute the output voltage for the range of possible temperatures.

$$v_O = -\frac{1000}{R_T}(v_{TR}) + \frac{R_T + 1000}{R_T}(2.5)$$

Over the range of temperatures, v_O ranges from 0.0806 to 4.7059 V, which does not meet the specifications. Choose Circuit 1, since it is the only one that meets the specifications. The following two OrCAD simulations and the associated plot verify these results.


R1
100
U1
PARAMETERS:
V1
5Vdc
R2
{resistor}
OUT
OPAMP
V
V2
R3
R4
900
600
2.5Vdc
0


R7
PARAMETERS:
1000
R6
R8
OPAMP
600
{resistor}
OUT
V3
5Vdc
V
U2
R5
100
V4
2.5Vdc
0


5.0V
4.0V
3.0V
2.0V
1.0V
0V
0
40
80
120
160
200
240
280
320
360
400
V(U1:OUT)
V(U2:OUT)
resistor

5 Signal Waveforms

5.1 Exercise Solutions

Exercise 5–1. Write an expression using unit step functions for the waveform in Figure 5–6.

The signal has a positive transition with a magnitude of 10 at $t = -2$ s, a negative transition with a magnitude of 15 at $t = 2$ s, and a positive transition with a magnitude of 5 at $t = 4$ s. The following expression describes the waveform.

$$v(t) = 10u(t+2) - 15u(t-2) + 5u(t-4) \text{ V}$$

Exercise 5–2. Figure 5–9 purports to be an alternative description of an impulse function as $\varepsilon \to 0$. Prove or disprove the claim.

From the figure, the area of the triangle is $\frac{1}{2}(2\varepsilon)(1/\varepsilon) = 1$. As $\varepsilon \to 0$, the base of the triangle shrinks to zero but the amplitude grows to infinity. The area always remains at one. Hence, this is equivalent to the definition of an impulse and proves the claim.

Exercise 5–3. Write an expression using ramp functions to describe the waveform shown in Figure 5–12.

The waveform has a ramp function that starts at $t = -1$ s and has a slope of $2/(1-(-1)) = +1$. The waveform then changes from a slope of $+1$ to -1 at $t = 1$ s. Finally, at $t = 3$ s, the slope changes from -1 to zero. The following expression describes the waveform.

$$v(t) = r(t+1) - 2r(t-1) + r(t-3) \text{ V}$$

Exercise 5–4. Express the following signals in terms of singularity functions:

(a).

$$v_1(t) = \begin{cases} 0 & t < 2 \\ 4 & 2 < t < 4 \\ -4 & 4 < t \end{cases}$$

The signal transitions from 0 to 4 at $t = 2$ and then transitions from 4 to -4 at $t = 4$. We can create this signal with two step functions.

$$v_1(t) = 4u(t-2) - 8u(t-4)$$

(b).

$$v_2(t) = \begin{cases} 0 & t < 2 \\ 4 & 2 < t < 4 \\ -2t + 12 & 4 < t \end{cases}$$

The signal transitions from 0 to 4 at $t = 2$ and then transitions to have a slope of -2 at $t = 4$. We can create this signal with a step function and a ramp function.

$$v_2(t) = 4u(t-2) - 2r(t-4)$$

(c).

$$v_3(t) = \int_{-\infty}^{t} v_1(x)\, dx$$

The integral of a step function is a ramp function. We can write $v_3(t)$ directly by inspection.

$$v_3(t) = 4r(t-2) - 8r(t-4)$$

(d).

$$v_4(t) = \frac{dv_2(t)}{dt}$$

The derivative of a step function is an impulse function and the derivative of a ramp function is a step function. We can write $v_4(t)$ directly by inspection.

$$v_4(t) = 4\delta(t-2) - 2u(t-4)$$

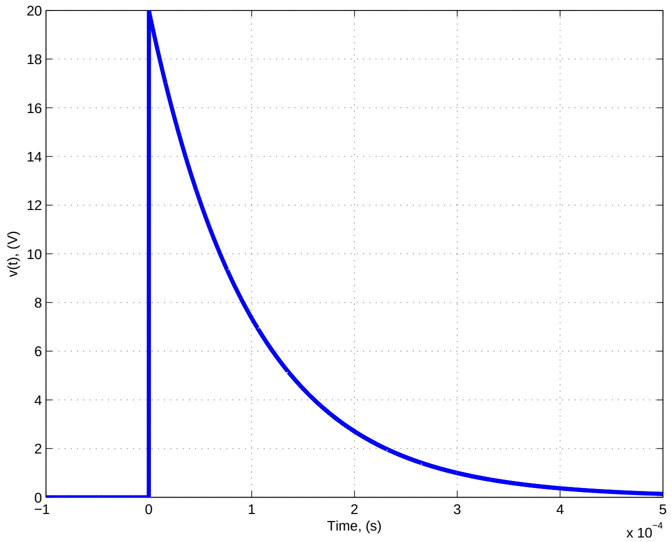
Exercise 5–5. Sketch the waveform described by

$$v(t) = 20e^{-10000t}u(t) \text{ V}$$

The waveform starts at $t = 0$ with a magnitude of 20 V. The waveform then exponentially decays with a time constant of $T_C = 1/10000 = 100$ μs. The following MATLAB code plots the waveform.

```
% Create a time vector
t = -100e-6:100e-9:500e-6;
% Create the signal
vt = 20*exp(-10000*t).*heaviside(t);
% Plot the results
figure
plot(t,vt,'b','LineWidth',3)
grid on
axis([-100e-6,500e-6,0,20]);
xlabel('Time, (s)')
ylabel('v(t), (V)')
```

The corresponding MATLAB output is shown below.



Exercise 5–6. Figure 5–20 contains three exponential waveforms. Match each curve with the appropriate expression.

1. $v_1(t) = 100e^{-t/100\mu}u(t-100\mu)$ V
2. $v_2(t) = 100e^{-t/100\mu}u(t)$ V
3. $v_1(t) = 100e^{-(t-100\mu)/100\mu}u(t-100\mu)$ V

Expression 1 starts at $t = 100\ \mu$s, but the time component in the exponent was not shifted, so the signal has decayed from 100 at $t = 0$ to a lower value at $t = 100\ \mu$s. Expression 1 corresponds to Signal (c). Expression 2 is the only one that starts at $t = 0\ \mu$s, so it corresponds to Signal (a). Expression 3 starts at $t = 100\ \mu$s and the exponent's time component was also shifted, so the magnitude at $t = 100\ \mu$s is 100. Expression 3 corresponds to Signal (b).

Exercise 5–7.

(a). An exponential waveform has $v(0) = 1.2$ V and $v(3) = 0.5$ V. What are V_A and T_C for this waveform?

The amplitude is given as $V_\mathrm{A} = 1.2$ V. Apply the decrement property to find T_C.

$$T_\mathrm{C} = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{3-0}{\ln\left[\dfrac{1.2}{0.5}\right]} = 3.43\ \mathrm{s}$$

(b). An exponential waveform has $v(0) = 5$ V and $v(2) = 1.25$ V. What are the values of $v(t)$ at $t = 1$ s and $t = 4$ s?

The amplitude is given as $V_\mathrm{A} = 5$ V. Apply the decrement property to find T_C.

$$T_\mathrm{C} = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{2-0}{\ln\left[\dfrac{5}{1.25}\right]} = 1.4427\ \mathrm{s}$$

Compute the waveform value at the requested times.

$$v(t) = 5e^{-t/1.4427}u(t)$$

$$v(1) = 5e^{-1/1.4427}u(1) = 2.5\ \mathrm{V}$$

$$v(4) = 5e^{-4/1.4427}u(4) = 0.3125\ \mathrm{V}$$

(c). An exponential waveform has $v(0) = 5$ V and an initial ($t = 0$) slope of -25 V/s. What are V_A and T_C for this waveform?

The amplitude is given as $V_\mathrm{A} = 5$ V. The initial slope of an exponential is given as follows

$$\frac{d}{dt}\left(V_\mathrm{A}e^{-t/T_\mathrm{C}}\right) = -\frac{V_\mathrm{A}}{T_\mathrm{C}}e^{-t/T_\mathrm{C}}$$

So the initial slope is $-V_\mathrm{A}/T_\mathrm{C} = -25$. With $V_\mathrm{A} = 5$, solve for $T_\mathrm{C} = 1/5 = 200$ ms.

(d). An exponential waveform decays to 10% of its initial value in 3 ms. What is T_C for this waveform?

We do not need to know the amplitude to solve this problem. The statement indicates the following relationship, which can be solved for T_C.

$$0.1 = e^{-0.003/T_\mathrm{C}}$$

$$\ln(0.1) = -\frac{0.003}{T_\mathrm{C}}$$

$$T_\mathrm{C} = 1.303\ \mathrm{ms}$$

(e). A waveform has $v(2) = 4$ V, $v(6) = 1$ V, and $v(10) = 0.5$ V. Is it an exponential waveform?

Apply the decrement property twice to determine if it gives the same value for T_C for different combinations of the given points.

$$T_{C1} = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{6-2}{\ln\left[\dfrac{4}{1}\right]} = 2.8854 \text{ s}$$

$$T_{C2} = \frac{10-6}{\ln\left[\dfrac{1}{0.5}\right]} = 5.7708 \text{ s}$$

The waveform is not exponential because it violates the decrement property.

Exercise 5–8. Find the amplitude and time constant for each of the following exponential signals:

(a). $v_1(t) = \left[-15e^{-1000t}\right] u(t)$ V

By inspection, the amplitude is $V_A = -15$ V and the time constant is $T_C = 1/1000 = 1$ ms.

(b). $v_2(t) = \left[+12e^{-t/10}\right] u(t)$ mV

By inspection, the amplitude is $V_A = 12$ mV and the time constant is $T_C = 10$ s.

(c). $i_3(t) = \left[15e^{-500t}\right] u(-t)$ mA

By inspection, the amplitude is $I_A = 15$ mA and the time constant is $T_C = 1/500 = 2$ ms. The fact the the step function has a time argument with a negative sign does not alter the time constant.

(d). $i_4(t) = \left[4e^{-200(t-100)}\right] u(t-100)$ A

By inspection, the amplitude is $I_A = 4$ A and the time constant is $T_C = 1/200 = 5$ ms. In this case, the signal is shifted to start at $t = 100$ s, but the amplitude and time constant are still visible in the original expression.

Exercise 5–9. Derive an expression for the sinusoid displayed in Figure 5–23 when $t = 0$ is placed in the middle of the display.

The display is calibrated to have 0.1 ms per division horizontally and 5 V per division vertically. The waveform has an amplitude of four divisions or 20 V and is symmetric about the horizontal axis, so there is no average value. The first peak appears one half division after the middle of the display, which is a delay of 50 μs. One half cylce of the waveform spans four divisions, so a complete cycle is eight divisions or 800 μs. Given these elements, we can write the expression for the waveform directly.

$$v(t) = V_A \cos\left[\frac{2\pi(t-T_S)}{T_0}\right] = 20\cos\left[\frac{2\pi(t-50\mu)}{800\mu}\right] \text{ V} = 20\cos(7854t - 22.5^\circ) \text{ V}$$

Exercise 5–10. Sketch the waveform described by

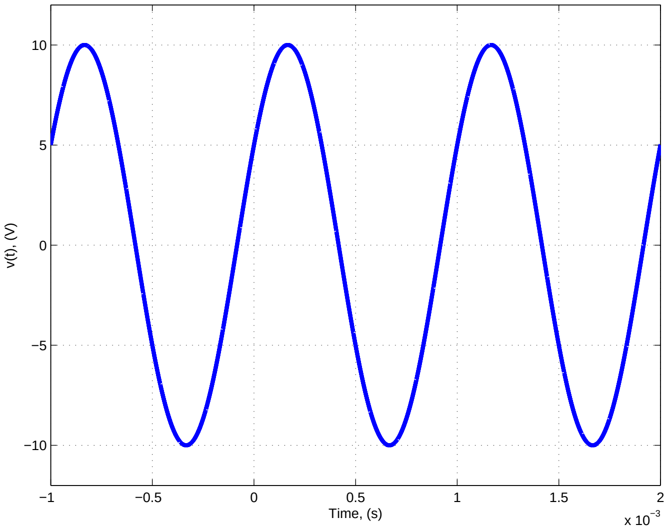
$$v(t) = 10\cos(2000\pi t - 60^\circ) \text{ V}$$

The waveform has an amplitude of 10 V, a frequency of 1 kHz, a period of $T_0 = 1/f_0 = 1/1000 = 1$ ms, a phase angle of -60°, and a time shift of $T_S = (-\phi T_0)/360^\circ = 167$ μs. The following MATLAB code plots the waveform.

```
% Create a time vector
t = -1e-3:1e-7:2e-3;
% Create the signal
vt = 10*cos(2000*pi*t-pi/3);
% Plot the results
figure
plot(t,vt,'b','LineWidth',3)
```

```
grid on
axis([-1e-3,2e-3,-12,12]);
xlabel('Time, (s)')
ylabel('v(t), (V)')
```

The resulting plot is shown below.



Exercise 5–11. Write an equation for the waveform obtained by integrating and differentiating the following signals:

(a). $v_1(t) = 30\cos(10t - 60^\circ)$ V

Differentiate the signal.

$$\frac{dv_1(t)}{dt} = (-10)(30)\sin(10t - 60^\circ) = -300\sin(10t - 60^\circ) = 300\sin(10t + 120^\circ)$$

$$= 300\cos(10t + 30^\circ) \text{ V/s}$$

Integrate the signal.

$$\int v_1(t)\,dt = \int 30\cos(10t - 60^\circ)\,dt = \frac{30}{10}\sin(10t - 60^\circ) = 3\cos(10t - 150^\circ) \text{ V-s}$$

(b). $v_2(t) = 3\cos(4000\pi t) - 4\sin(4000\pi t)$ V

Differentiate the signal.

$$\frac{dv_2(t)}{dt} = (-4000\pi)(3)\sin(4000\pi t) - (4000\pi)(4)\cos(4000\pi t)$$

$$= -12000\pi\sin(4000\pi t) - 16000\pi\cos(4000\pi t)$$

$$= 20000\pi\cos(4000\pi t + 143.13^\circ) \text{ V/s}$$

Integrate the signal.

$$\int v_2(t)\,dt = \int [3\cos(4000\pi t) - 4\sin(4000\pi t)]\;dt = \frac{1}{4000\pi}[3\sin(4000\pi t) + 4\cos(4000\pi t)]$$

$$= \frac{1}{800\pi}\cos(4000\pi t - 36.87^\circ) \text{ V-s}$$

Exercise 5–12. A sinusoid has a period of 5 μs. At $t = 0$, the amplitude is 12 V. The waveform reaches its first positive peak after $t = 0$ at $t = 4$ μs. Find its amplitude, frequency, and phase angle.

Given the information in the problem statement, write the following expression for the waveform and then solve for the amplitude.

$$v(t) = V_{\rm A}\cos\left[\frac{2\pi(t - T_{\rm S})}{T_0}\right] = V_{\rm A}\cos\left[\frac{2\pi(t - 4\mu)}{5\mu}\right] \text{ V}$$

$$v(0) = 12 = V_{\rm A}\cos\left[\frac{2\pi(-4\mu)}{5\mu}\right] = V_{\rm A}(0.309017)$$

$$V_{\rm A} = 38.83 \text{ V}$$

The amplitude is $V_{\rm A} = 38.83$ V, the frequency is $f_0 = 1/T_0 = 1/0.000005 = 200$ kHz, and the phase angle is $\phi = (-360°)(4)/5 = -288° = +72°$.

Exercise 5–13. Describe the following waveform:

$$v(t) = [V_{\rm A}u(t) - V_{\rm A}u(-t)]\,[\delta(t+1) + \delta(t) + \delta(t-1)] \text{ V}$$

Perform the multiplication and use the impulse function property to eliminate terms that are zero.

$$\begin{aligned} v(t) &= V_{\rm A}u(t)\delta(t+1) + V_{\rm A}u(t)\delta(t) + V_{\rm A}u(t)\delta(t-1) - V_{\rm A}u(-t)\delta(t+1) - V_{\rm A}u(-t)\delta(t) - V_{\rm A}u(-t)\delta(t-1) \\ &= 0 + V_{\rm A}\delta(t) + V_{\rm A}\delta(t-1) - V_{\rm A}\delta(t+1) - V_{\rm A}\delta(t) - 0 \\ &= V_{\rm A}\delta(t-1) - V_{\rm A}\delta(t+1) \end{aligned}$$

The resulting waveform contains two impulse functions. One impulse function appears at $t = -1$ s with an amplitude of $-V_{\rm A}$ and the other appears at $t = 1$ s with an amplitude of $V_{\rm A}$.

Exercise 5–14.

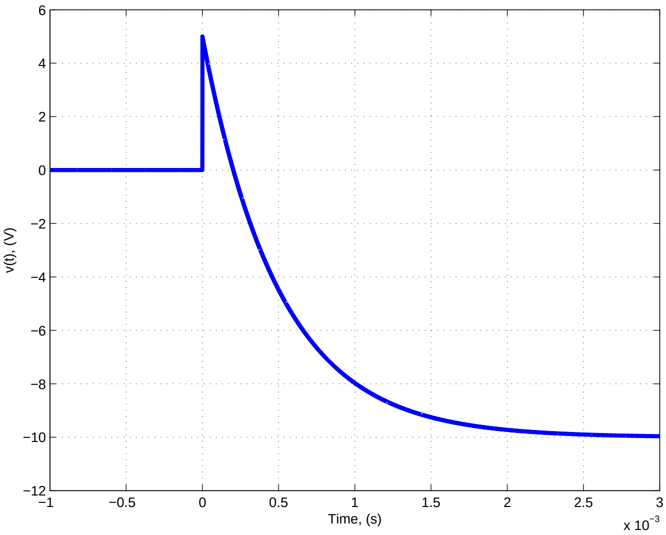
(a). Sketch the waveform described by the following:

$$v(t) = \left[15e^{-2000t} - 10\right] u(t) \text{ V}$$

The waveform has an initial value of 5 V and decays to a final value of -10 V. The following MATLAB code plots the waveform.

```
% Create a time vector
t = -1e-3:1e-7:3e-3;
% Create the signal
vt = (15*exp(-2000*t)-10).*heaviside(t);
% Plot the results
figure
plot(t,vt,'b','LineWidth',3)
grid on
axis([-1e-3,3e-3,-12,6]);
xlabel('Time, (s)')
ylabel('v(t), (V)')
```

The resulting plot is shown below.



(b). What is the value of the voltage at $t = T_C$?

Compute the value at $t = T_C = 1/2000$ s.

$$v(T_C) = \left[15e^{-2000/2000} - 10\right] u(1/2000) = [(15)(0.367879) - 10]\,(1) = -4.4818 \text{ V}$$

Exercise 5–15. Figure 5–28 contains a waveform that is called a double-sided exponential, which is defined as the sum of a normal exponential and a reversed exponential. This waveform is 1 at $t = 0$ and decays exponentially to zero in both directions along the time axis. Write an expression for this waveform.

For $t > 0$, the appropriate expression is $v_P(t) = e^{-\alpha t}u(t)$, where $\alpha > 0$. For $t < 0$, we want to reverse the signal with respect to time, so replace t with $-t$ to get $v_N(t) = e^{-\alpha(-t)}u(-t)$. The complete expression is the sum of the two components.

$$v(t) = v_P(t) + v_N(t) = e^{-\alpha t}u(t) + e^{\alpha t}u(-t) = e^{-\alpha|t|} \text{ V}$$

Exercise 5–16.

(a). Determine the time that the damped ramp reaches its maximum value.

$$v(t) = V_A\left[\left(\frac{t}{T_C}\right)e^{-t/T_C}\right]u(t) \text{ V}$$

To find the maximum value, take the first derivative of the signal and set it equal to zero. We can safely ignore the step function in the calculations.

$$\frac{d\,v(t)}{dt} = 0 = \left(\frac{V_A t}{T_C}\right)\left(-\frac{1}{T_C}e^{-t/T_C}\right) + \left(\frac{V_A}{T_C}\right)\left(e^{-t/T_C}\right)$$

$$0 = -\frac{t}{T_C} + 1$$

$$t = T_C$$

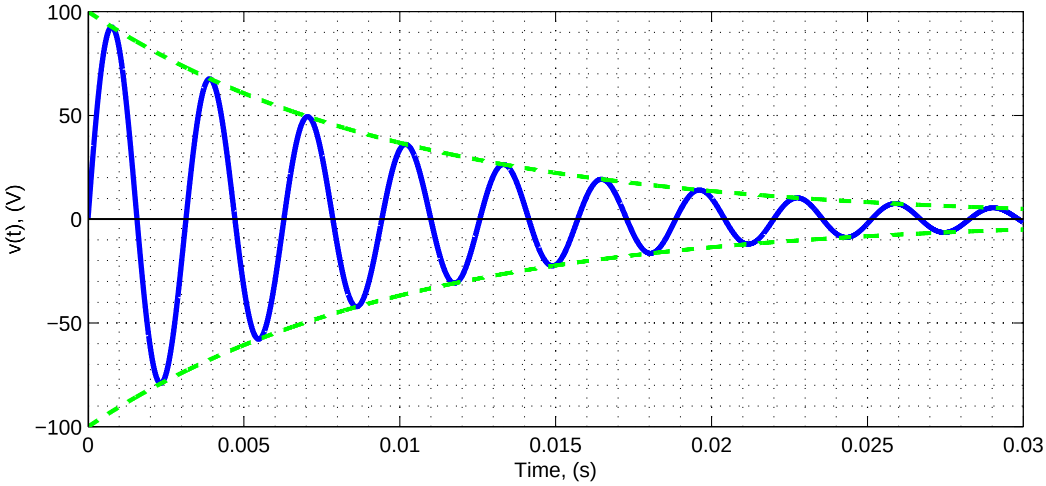
(b). What is the value of $v(t)$ at the maximum?

Evaluate $v(t)$ at $t = T_C$.

$$v(T_C) = V_A\left[\left(\frac{T_C}{T_C}\right)e^{-T_C/T_C}\right]u(T_C) = V_A e^{-1} = 0.368V_A$$

Exercise 5–17. For the damped sinusoid waveform shown in Figure 5–32, determine an approximate expression for $v(t)$.

The waveform is a damped sinusoid with an initial value of zero at $t = 0$ s. Sketch an envelope for the signal as shown in the following figure.



The envelope indicates the amplitude of the signal is $V_A = 100$ V. The time required to complete eight cycles of the sinusoid is 25 ms, so the period is $T_0 = 0.025/8 = 3.125$ ms. The frequency is $f_0 = 1/T_0 = 1/0.003125 = 320$ Hz. The radian frequency is $\omega_0 = (2\pi)(320) = 2011$ rad/s and we will round this value to 2000 rad/s. Apply the decrement property to find the time constant. Use the peaks at $t = 0.6$ ms and $t = 22.6$ ms, with amplitudes of 92 V and 10 V, respectively. Note that these peaks are seven cycles apart, which could be used to develop another estimate for the period of the sinusoid.

$$T_C = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{0.0226 - 0.0006}{\ln\left[\dfrac{92}{10}\right]} = 9.91347 \text{ ms}$$

Note that $1/T_C = 100.87$ Hz and we will round this value to 100 Hz. Combining the results, the expression is as follows:

$$v(t) = 100e^{-100t}\sin(2000t)u(t) \text{ V}$$

Exercise 5–18. A double exponential waveform is given as

$$v(t) = 10\left[e^{-1000t} - e^{-2500t}\right]u(t) \text{ V}$$

(a). What is the value of $v(t)$ at the maximum, and at what time does it occur?

Take the derivative of the signal and set it equal to zero to find the time at which the maximum occurs. We can safely ignore the step function in these calculations.

$$\frac{d\,v(t)}{dt} = 0 = 10\left[-1000e^{-1000t} + 2500e^{-2500t}\right]$$

$$1000e^{-1000t} = 2500e^{-2500t}$$

$$e^{1500t} = 2.5$$

$$1500t = \ln(2.5) = 0.916291$$

$$t = 610.86\ \mu\text{s}$$

Calculate the value at this time.

$$v(0.00061086) = 10\left[e^{-0.61086} - e^{-1.52715}\right]u(0.00061086) = 3.257 \text{ V}$$

(b). What is the time constant of the dominant (longer-lasting) exponential?

The two time constants are $T_{C1} = 1/1000 = 1$ ms and $T_{C2} = 1/2500 = 400$ μs. The dominant exponential is the first one and its time constant is $T_{C1} = 1$ ms.

Exercise 5–19. Find the maximum amplitude and the approximate duration of the following composite waveforms:

(a). $v_1(t) = [25\sin(1000t)]\,[u(t) - u(t-10)]$ V

The step functions allow the sinusoid to be active from $t = 0$ to $t = 10$ s, so the duration is 10 s. The sinusoid will reach a maximum value of 25 V during this time period.

(b). $v_2(t) = [50\cos(1000t)]\left[e^{-200t}\right]u(t)$ V

At $t = 0$, the waveform has a maximum value of 50 V. The duration is approximately five time constants. We have $T_C = 1/200 = 5$ ms, so the duration is approximately 25 ms.

(c). $i_3(t) = \left[3000te^{-1000t}\right]u(t)$ mA

As determined in Exercise 5–16, a damped ramp has its maximum at T_C. In this case, $T_C = 1/1000 = 1$ ms. Evaluate the waveform at $t = 1$ ms.

$$i_3(0.001) = (3000)(0.001)e^{-1000/1000} = 3e^{-1} = 1.104 \text{ mA}$$

The duration of the signal is approximately five time constants or 5 ms.

(d). $i_4(t) = 10e^{-5000|t|}$ A

The signal is a double-sided exponential with a maximum of 10 A at $t = 0$. Since the waveform is double-sided, its duration is 10 time constants. We have $T_C = 1/5000 = 0.2$ ms, so the duration is 2 ms.

Exercise 5–20. Characterize the following waveform defined by

$$v(t) = \sum_{n=1}^{\infty}\left[b_n \sin(2\pi n f_0 t)\right] \text{ V}$$

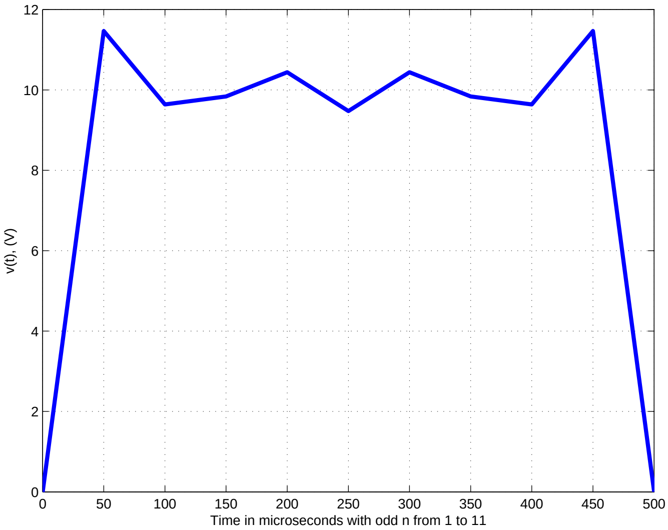
where $b_n = 4V_A/\pi n$, $V_A = 10$ V, $f_0 = 1000$ Hz, and $n = 1, 3, 5, 7, 9, 11, \ldots$, by plotting the first six n terms for a half period of the fundamental frequency f_0. (*Hint:* Vary t from 0 to 500 μs in 50 μs steps. Excel is useful here.) What waveform in Figure 5–2 does this function best resemble?

The following MATLAB code provides the solution:

```
% Define the given values
VA = 10;
f0 = 1000;
T0 = 1/f0;
t0 = 0;
tf = T0/2;
tStep = tf/10;
t = t0:tStep:tf;
vt = zeros(size(t));
nMax = 11;
% Compute the sum
for n = 1:2:nMax
    n
    bn = 4*VA/pi/n;
    vt = vt+bn*sin(2*pi*n*f0*t);
end
% Plot the results
figure
plot(1e6*t,vt,'b','LineWidth',3)
```

```
grid on
axis([0,1e6*tf,0,12])
xlabel(['Time in microseconds with odd n from 1 to ',num2str(nMax)])
ylabel('v(t), (V)')
```

The resulting plot is shown below.



The plot resembles a portion of a square wave.

Exercise 5–21. For the pulse waveform in Figure 5–38, find $V_{\rm MAX}$, $V_{\rm MIN}$, $V_{\rm p}$, $V_{\rm pp}$, and $V_{\rm avg}$.

By inspection, we have $V_{\rm MAX} = V_{\rm A}$ and $V_{\rm MIN} = -V_{\rm A}$. Then determine $V_{\rm p} = V_{\rm A}$ and $V_{\rm pp} = 2V_{\rm A}$. Finally, the waveform appears to have equal areas above and below the horizontal axis, so $V_{\rm avg} = 0$ V.

Exercise 5–22. Find the peak, peak-to-peak, average, and rms values of the periodic waveform in Figure 5–40.

By inspection, we have $V_{\rm MAX} = 2V_{\rm A}$ and $V_{\rm MIN} = -V_{\rm A}$. Then determine $V_{\rm p} = 2V_{\rm A}$ and $V_{\rm pp} = 3V_{\rm A}$. Compute the average value by summing the area under the waveform in one period and dividing by the period.

$$V_{\rm avg} = \frac{1}{T_0}\left[(2V_{\rm A})\frac{T_0}{4} - (V_{\rm A})\frac{T_0}{4}\right] = \frac{V_{\rm A}}{4}$$

Compute the rms value by summing the area of the square of the waveform in one period, dividing by the period, and then taking the square root of the result.

$$V_{\rm rms} = \sqrt{\frac{1}{T_0}\left[(4V_{\rm A}^2)\frac{T_0}{4} + (V_{\rm A}^2)\frac{T_0}{4}\right]} = \frac{\sqrt{5}}{2}V_{\rm A}$$

Exercise 5–23. Classify each of the following signals as periodic or aperiodic and causal or noncausal. Then calculate the average and rms values of the periodic waveforms, and the peak and peak-to-peak values of the other waveforms.

(a). $v_1(t) = 99\cos(3000t) - 132\sin(3000t)$ V

The signal is periodic because it is the sum of two periodic signals with the same frequency. The signal is noncausal because the sinusoids persist for all time. Combine the two terms to get a single sinusoid with an amplitude of $V_{\rm A} = \sqrt{a^2+b^2} = \sqrt{99^2+132^2} = 165$ V. The rms values is $V_{\rm rms} = V_{\rm A}/\sqrt{2} = 116.7$ V.

(b). $v_2(t) = 34\,[\sin(800\pi t)]\,[u(t) - u(t-0.03)]$ V

The signal is aperiodic because it does not repeat for all time. The signal is causal because it has a clear starting time. Since the signal is a sinusoid, by inspection $V_{\rm p} = 34$ V and $V_{\rm pp} = 68$ V.

(c). $i_3(t) = 120\,[u(t+5) - u(t-5)]$ mA

The signal is aperiodic because it does not repeat for all time. The signal is causal because it has a clear starting time. The signal has a constant value of 120 mA between $t = -5$ and $t = 5$, so $V_{\mathrm{p}} = 120$ mA and $V_{\mathrm{pp}} = 120$ mA.

(d). $i_4(t) = 50$ A

The signal is aperiodic because it does not repeat for all time. The signal is noncausal because it persists for all time. The peak value is $V_{\mathrm{p}} = 50$ A, but the peak-to-peak value is $V_{\mathrm{pp}} = 0$ A, because the signal is constant.

5.2 Problem Solutions

Problem 5–1. Sketch the following waveforms:

(a). $v_1(t) = 5u(t) - 5u(t-1)$ V

(b). $v_2(t) = 3u(t+2) - 2u(t-2)$ V

(c). $v_3(t) = \int_{-\infty}^{t} v_1(x)\,dx$

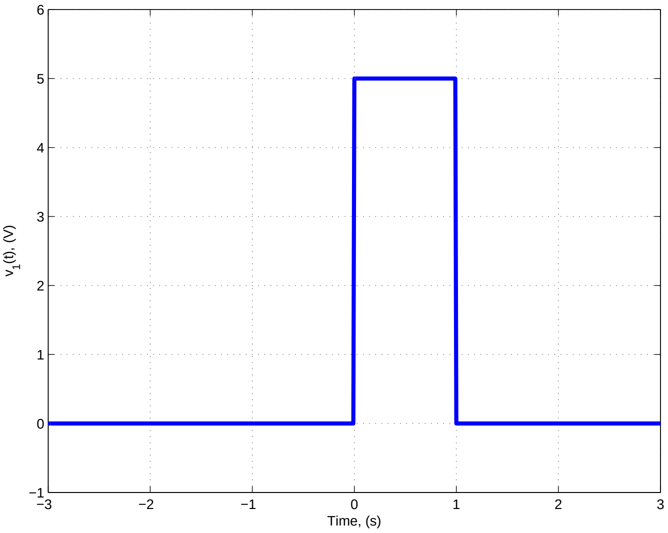
(d). $v_4(t) = dv_2(t)/dt$

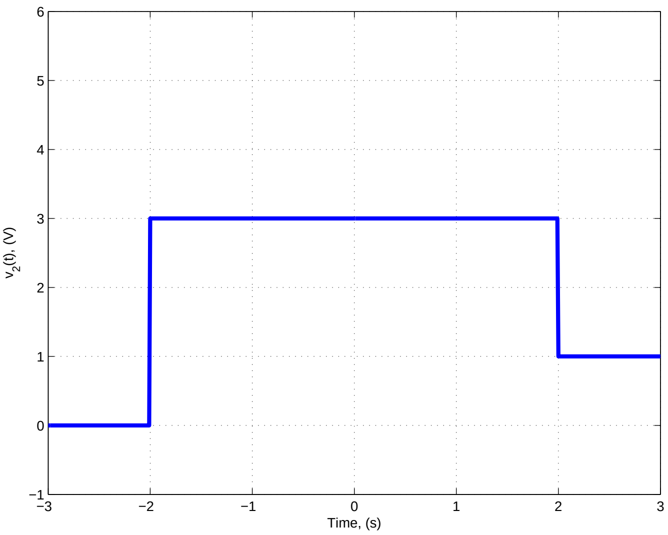
The following MATLAB code plots the waveforms:

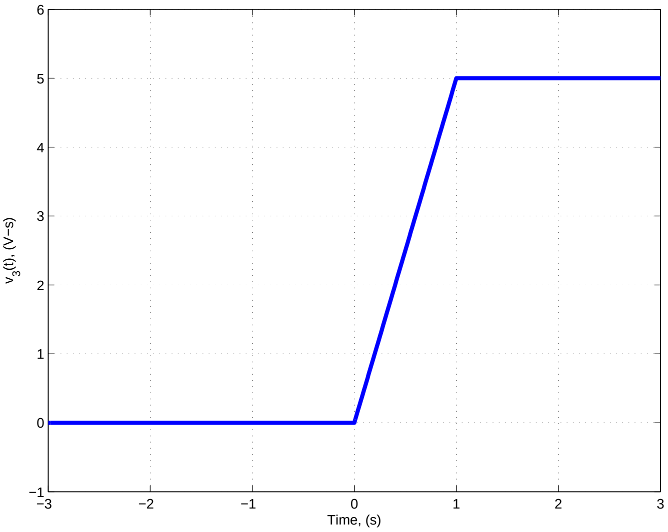
```
syms x t

% Set the figure LineWidth
LW = 3;

% Create a time vector
% Shift the time vector slight to see vertical transitions
tt = -3:0.01:3;
tt = tt+2*eps;

% Create the signals
v1t = 5*heaviside(t) - 5*heaviside(t-1)
v2t = 3*heaviside(t+2) - 2*heaviside(t-2)
v3t = int(subs(v1t,t,x),x,-inf,t)
v4t = diff(v2t,t)

% Substitute in the time vector
v1tt = subs(v1t,t,tt);
v2tt = subs(v2t,t,tt);
v3tt = subs(v3t,t,tt);
v4tt = subs(v4t,t,tt);

% Plot the results
figure
plot(tt,v1tt,'b','LineWidth',LW)
axis([-3,3,-1,6])
grid on
xlabel('Time, (s)')
ylabel('v_1(t), (V)')

figure
plot(tt,v2tt,'b','LineWidth',LW)
axis([-3,3,-1,6])
grid on
xlabel('Time, (s)')
ylabel('v_2(t), (V)')

figure
plot(tt,v3tt,'b','LineWidth',LW)
axis([-3,3,-1,6])
grid on
xlabel('Time, (s)')
ylabel('v_3(t), (V)')

figure
plot(tt,v4tt,'b','LineWidth',LW)
axis([-3,3,-3,4])
grid on
hold on
h1 = stem([-2],[3],'^','fill');
h2 = stem([2],[-2],'v','fill');
set(h1,'LineWidth',2);
set(h2,'LineWidth',2);
```

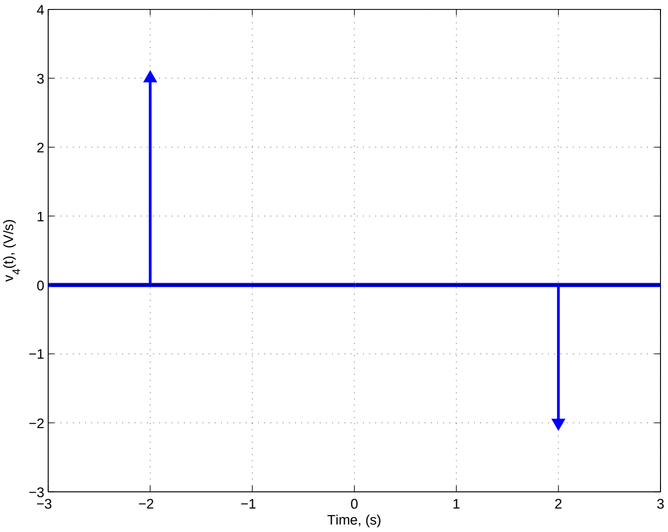
```
xlabel('Time, (s)')
ylabel('v_4(t), (V)')
```

The resulting plots are shown below.









Problem 5–2. Using appropriate step functions, write an expression for each waveform in Figure P5–2.

(a). The function has a negative transition of magnitude 0.5 at $t = -0.5$, a positive transition of magnitude 1 at $t = 0.5$, and a negative transition of magnitude 0.5 at $t = 1.5$.

$$i(t) = -0.5u(t + 0.5) + u(t - 0.5) - 0.5u(t - 1.5)$$

(b). The function has a positive transition of magnitude 60 at $t = -9$, a negative transition of magnitude 120 at $t = -3$, and a positive transition of magnitude 60 at $t = 3$.

$$v(t) = 60u(t + 9) - 120u(t + 3) + 60u(t - 3)$$

Problem 5–3. Sketch the following waveforms

(a). $v_1(t) = 2 - u(t)$ V

The waveform is a constant value of 2 V for $t < 0$ and then decreases to 1 V at $t = 0$.

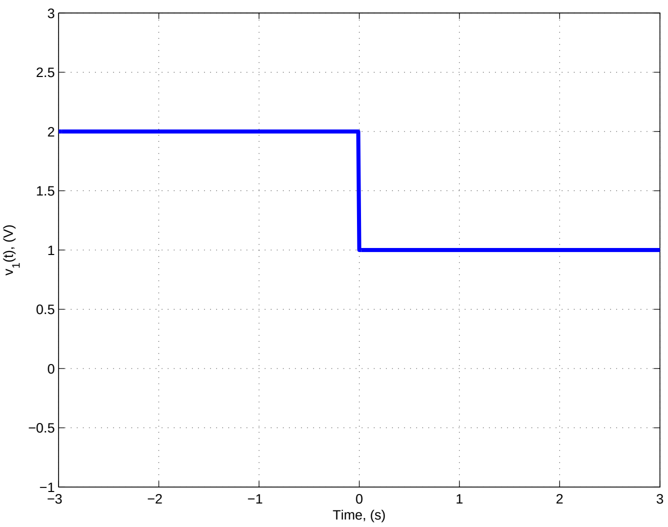
(b). $v_2(t) = -2u(t+200) + 3u(t+100) - u(t)$ V

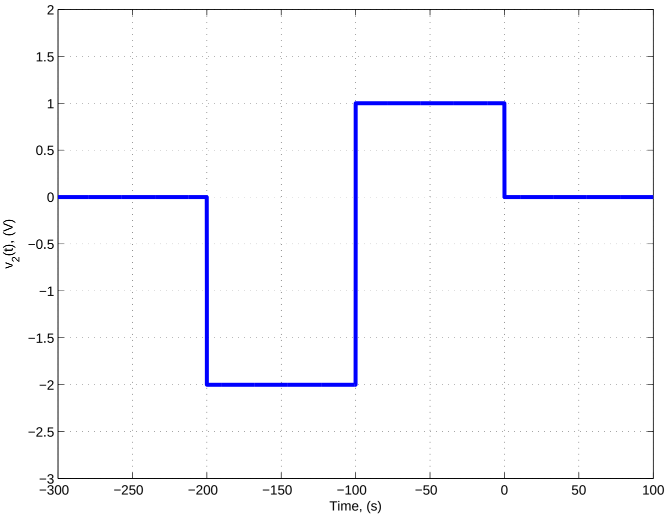
The waveform transitions from 0 V to -2 V at $t = -200$, transitions from -2 V to 1 V at $t = -100$, and then transitions to 0 V at $t = 0$.

The following MATLAB code plots the two waveforms:

```
syms x t

% Set the figure LineWidth
LW = 3;

% Create a time vector
% Shift the time vector slight to see vertical transitions
tt = -300:0.01:100;
tt = tt+2*eps;

% Create the signals
v1t = 2 - heaviside(t)
v2t = -2*heaviside(t+200) + 3*heaviside(t+100) - heaviside(t)

% Substitute in the time vector
v1tt = subs(v1t,t,tt);
v2tt = subs(v2t,t,tt);

% Plot the results
figure
plot(tt,v1tt,'b','LineWidth',LW)
axis([-3,3,-1,3])
grid on
xlabel('Time, (s)')
ylabel('v_1(t), (V)')

figure
plot(tt,v2tt,'b','LineWidth',LW)
axis([-300,100,-3,2])
grid on
xlabel('Time, (s)')
ylabel('v_2(t), (V)')
```

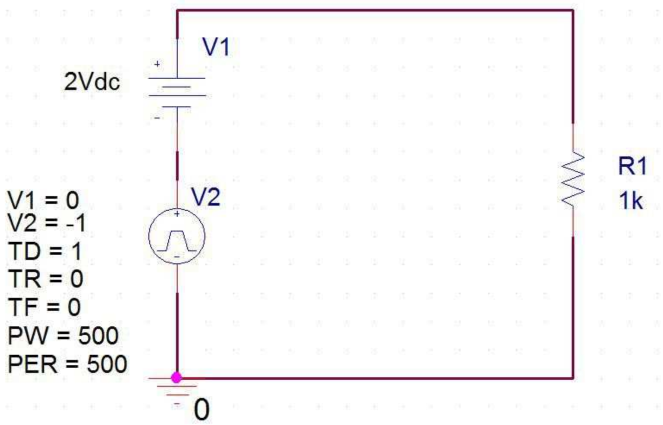
The corresponding plots are shown below.

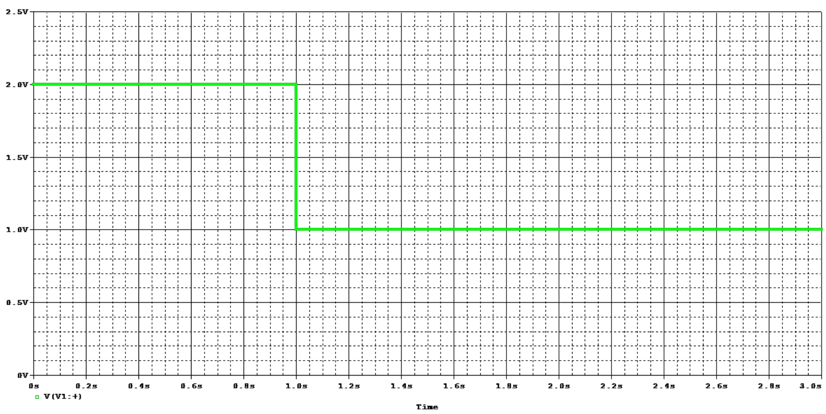


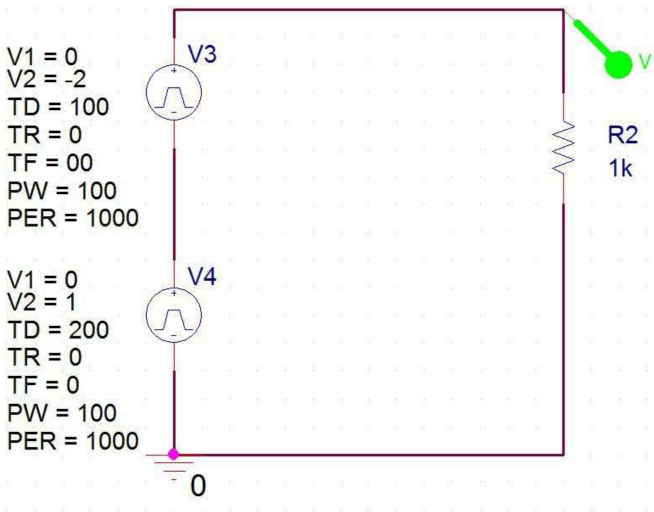


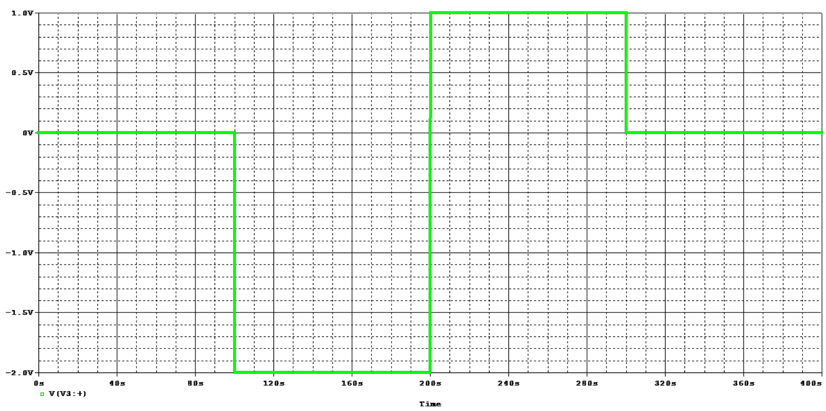
(c). Construct the waveforms in parts (a) and (b) using OrCAD.

OrCAD does not directly allow for simulations with negative time variables. The first simulation is shifted later in time by 1 s and the second simulation is shifted later in time by 300 s to be able to see the transitions. The simulations and the corresponding plots are shown below.









Problem 5–4. Sketch the following waveforms:

(a). $v_1(t) = 4r(t+1) - 4r(t-1)$ V

The waveform is a ramp with a slope of +4 that starts at $t = -1$ and ends at $t = 1$ with a final value of 8 V.

(b). $v_2(t) = 2 + r(t+1) - 2r(t-1) + r(t-3)$ V

The waveform has a constant value of 2 V up until $t = -1$, has a slope of +1 from $t = -1$ to $t = 1$, has a slope of -1 from $t = 1$ to $t = 3$, and has a slope of 0 and a final value of 2 V after $t = 3$.

(c). $v_3(t) = dv_1(t)/dt$

Take the derivative of $v_1(t)$ to get $v_3(t) = 4u(t+1) - 4u(t-1)$ V/s. The waveform has a positive transition of magnitude 4 V/s at $t = -1$ and a negative transition of magnitude 4 V/s at $t = 1$.

(d). $v_4(t) = d^2v_2(t)/dt^2$

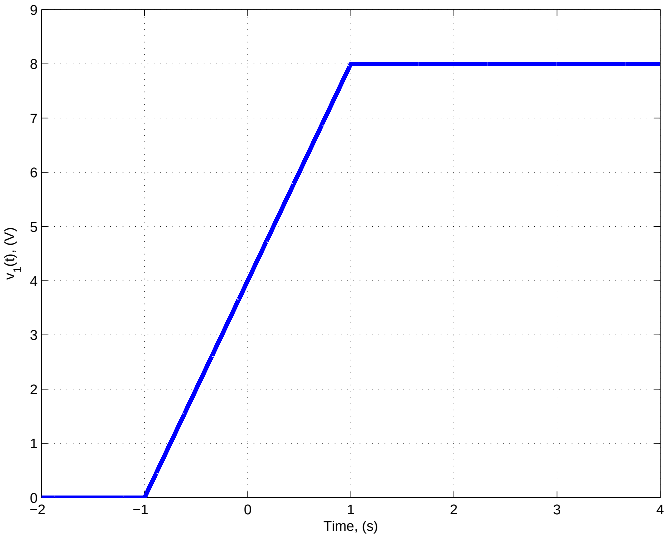
Take the second derivative of $v_2(t)$ to get $v_4(t) = \delta(t+1) - 2\delta(t-1) + \delta(t-3)$ V/s^2. The waveform is a series of three impulse functions.

The following MATLAB code plots the waveforms:

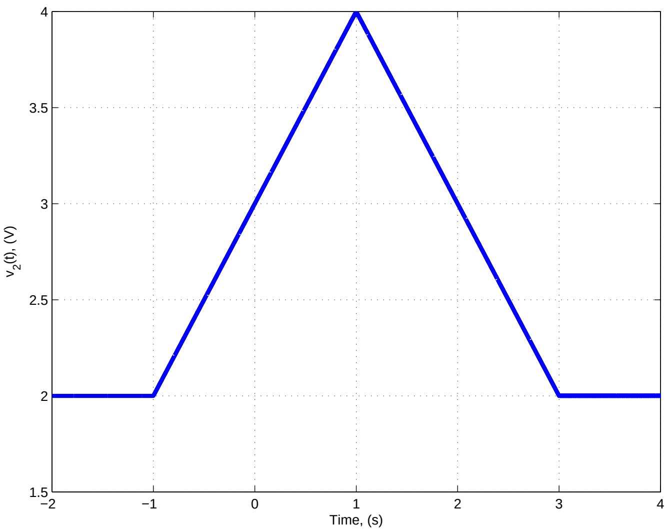
```
% Create the symbolic variables
syms x t
```

```
% Set the figure LineWidth
LW = 3;

% Create a time vector
% Shift the time vector slight to see vertical transitions
tt = -2:0.0001:4;
tt = tt+2*eps;

% Create the signals
v1t = 4*(t+1)*heaviside(t+1) - 4*(t-1)*heaviside(t-1);
v2t = 2 + (t+1)*heaviside(t+1) - 2*(t-1)*heaviside(t-1) ...
        + (t-3)*heaviside(t-3);
v3t = diff(v1t,t);
v4t = diff(diff(v2t,t),t);

% Substitute in the time vector
v1tt = subs(v1t,t,tt);
v2tt = subs(v2t,t,tt);
v3tt = subs(v3t,t,tt);
v4tt = subs(v4t,t,tt);

% Plot the results
figure
plot(tt,v1tt,'b','LineWidth',LW)
%axis([-3,3,-1,6])
grid on
xlabel('Time, (s)')
ylabel('v_1(t), (V)')

figure
plot(tt,v2tt,'b','LineWidth',LW)
%axis([-3,3,-1,6])
grid on
xlabel('Time, (s)')
ylabel('v_2(t), (V)')

figure
plot(tt,v3tt,'b','LineWidth',LW)
%axis([-3,3,-1,6])
grid on
xlabel('Time, (s)')
ylabel('v_3(t), (V/s)')

figure
plot(tt,v4tt,'b','LineWidth',LW)
%axis([-3,3,-3,4])
grid on
hold on
h1 = stem([-1],[1],'^','fill');
h2 = stem([1],[-2],'v','fill');
h3 = stem([3],[1],'^','fill');
set(h1,'LineWidth',2);
set(h2,'LineWidth',2);
set(h3,'LineWidth',2);
xlabel('Time, (s)')
ylabel('v_4(t), (V/s^2)')
```

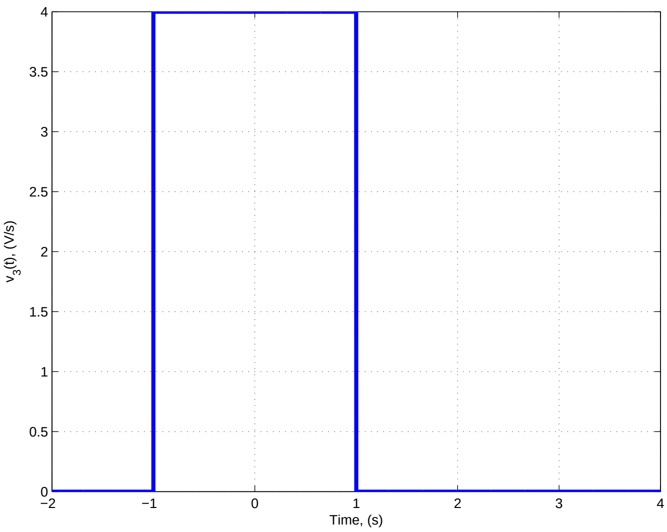
The resulting plots are shown below.



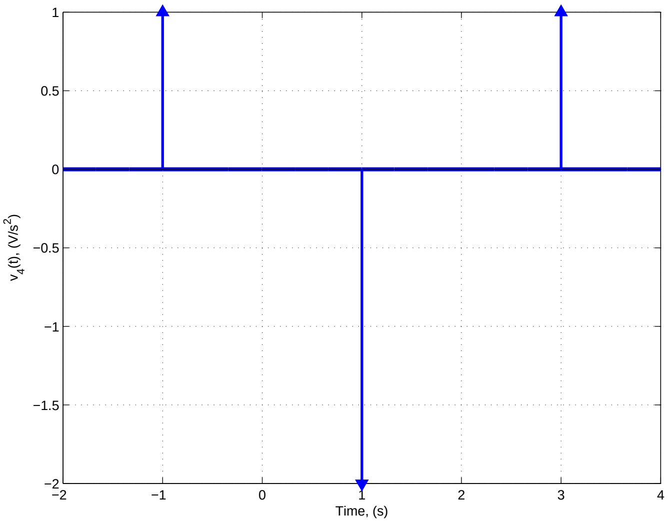
$v_1(t)$, (V)
Time, (s)



$v_2(t)$, (V)
Time, (s)



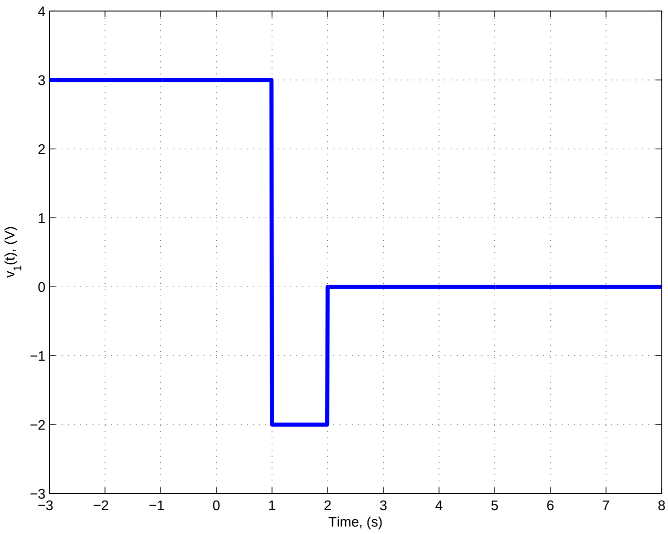
$v_3(t)$, (V/s)
Time, (s)



Problem 5–5. Express each of the following signals as a sum of singularity functions.

(a). $v_1(t) = \begin{cases} 3 & t < 1 \\ -2 & 1 \le t < 2 \\ 0 & 2 \le t \end{cases}$

Sketch the waveform.

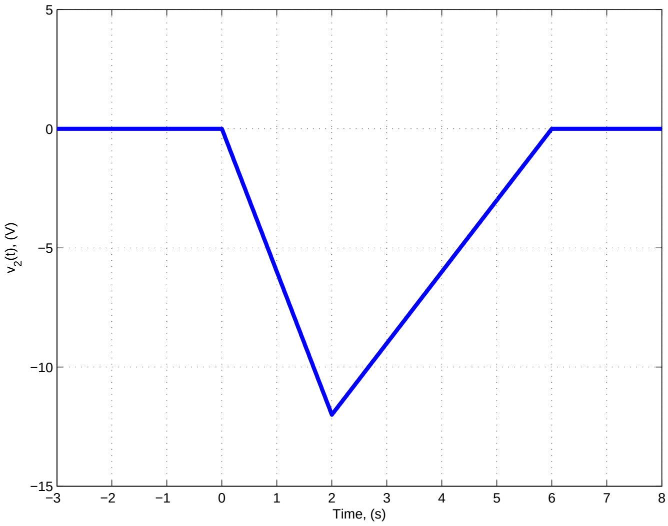


The waveform can be expressed as follows:

$$v_1(t) = 3 - 5u(t-1) + 2u(t-2) = 5u(1-t) - 2u(2-t)$$

(b). $v_2(t) = \begin{cases} 0 & t < 0 \\ -6t & 0 \le t < 2 \\ -18 + 3t & 2 \le t < 6 \\ 0 & 6 \le t \end{cases}$

Sketch the waveform.



The waveform can be expressed as follows:

$$v_2(t) = -6r(t) + 9r(t-1) - 3r(t-6)$$

Problem 5–6. Express the waveform in Figure P5–6 as a sum of step functions.

The waveform can be expressed as follows:

$$v(t) = -6u(t+1) + 18u(t-1) - 24u(t-3) + 12u(t-4.5)$$

Problem 5–7. Express each of the waveforms in Figure P5–7 as a sum of singularity functions.

(a). The waveform can be expressed as follows:

$$v_1(t) = 5r(t+1) - 10r(t) + 10r(t-2) - 5r(t-3)$$

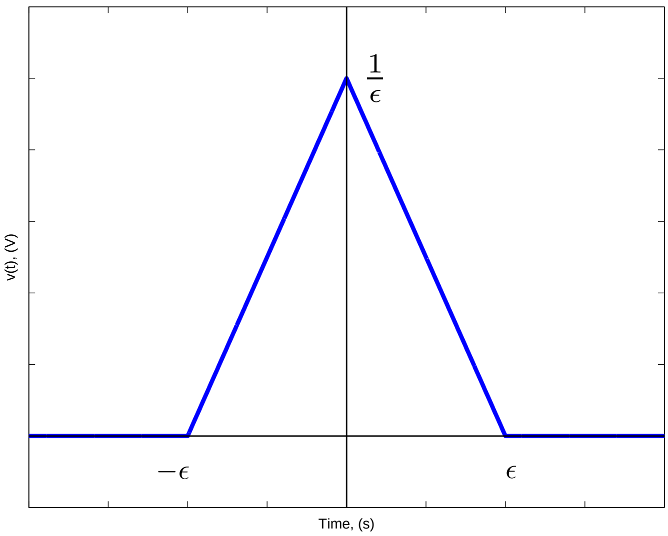
(b). The waveform can be expressed as follows:

$$v_2(t) = 5u(t) - 2.5r(t) + 5r(t-2) - 2.5r(t-4) - 5u(t-4)$$

Problem 5–8. Sketch the waveform described by the following:

$$v(t) = \left[\frac{1}{\varepsilon^2}t + \frac{1}{\varepsilon}\right][u(t+\varepsilon) - u(t)] + \left[-\frac{1}{\varepsilon^2}t + \frac{1}{\varepsilon}\right][u(t) - u(t-\varepsilon)] \text{ V}$$

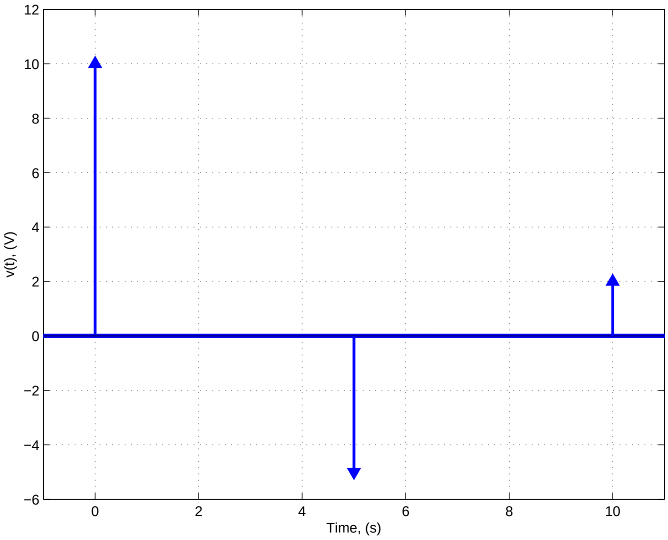
For $-\varepsilon < t < 0$, the waveform is a ramp with slope $1/\varepsilon^2$ that starts with a value of 0 V and rises to a value of $1/\varepsilon$ V. For $0 < t < \varepsilon$, the waveform is a ramp with a slope of $-1/\varepsilon^2$ that starts with a value of $1/\varepsilon$ V and falls to a value of 0 V. The sketch is shown below.



Problem 5–9. Sketch the waveform described by the following:

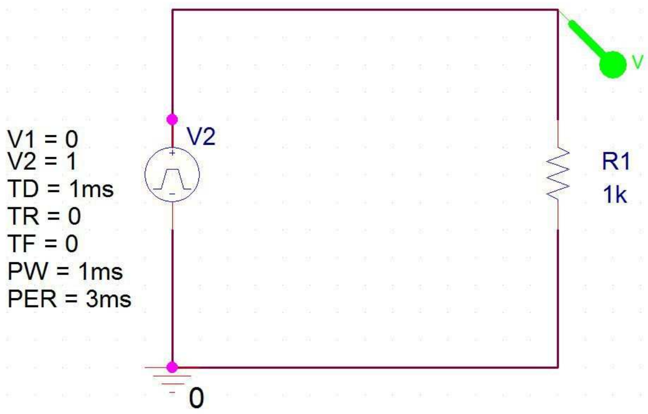
$$v(t) = 10\delta(t) - 5\delta(t-5) + 2\delta(t-10) \text{ V}$$

The sketch is shown below.

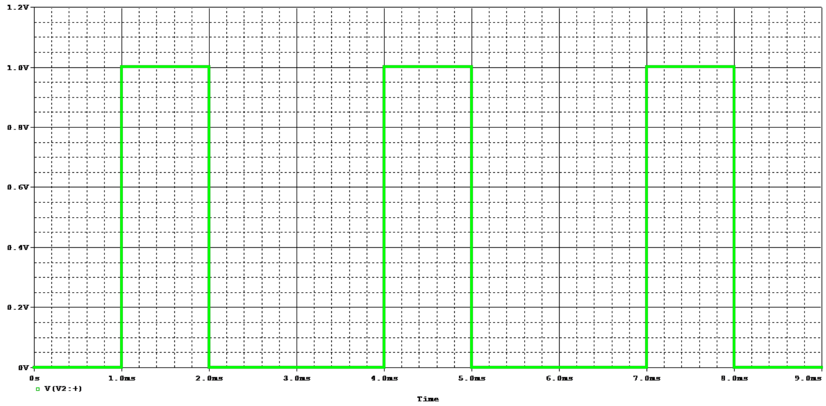


Problem 5–10. Generate on OrCAD a waveform $v(t)$ that starts at $t = 1$ ms and consists of a pulse train of 1 V pulses with a 1 ms pulse width that repeat every 3 ms.

The following OrCAD simulation creates the appropriate plot.



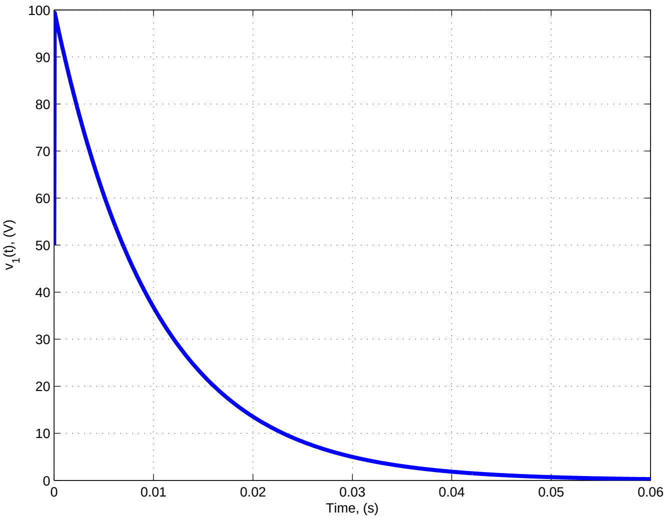
The resulting plot is shown below.



Problem 5–11. Sketch the following exponential waveforms. Find the amplitude and time constant of each waveform.

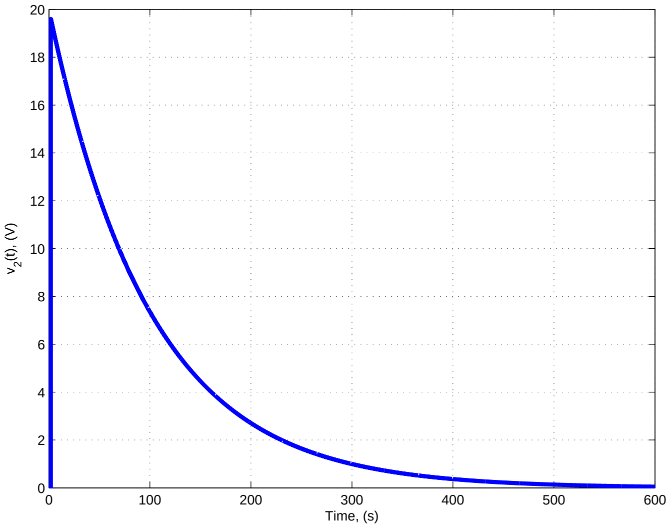
(a). $v_1(t) = \left[100e^{-100t}\right] u(t)$ V

The amplitude is $V_\text{A} = 100$ V and the time constant is $T_\text{C} = 1/100 = 10$ ms. The sketch is shown below.



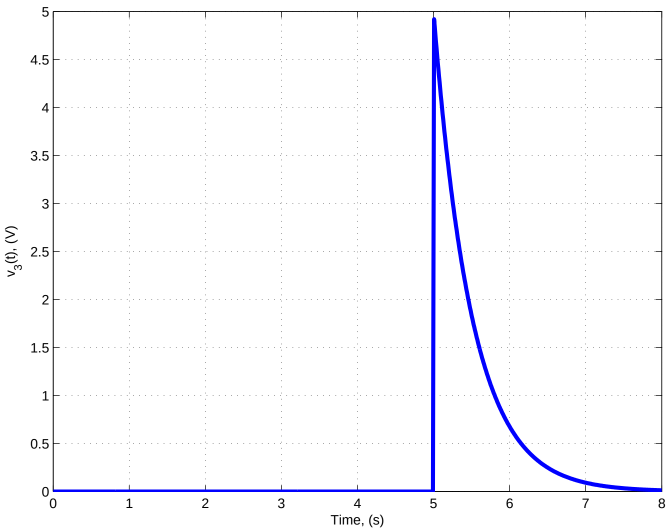
(b). $v_2(t) = \left[20e^{-t/100}\right] u(t-2)$ V

The amplitude is $V_\mathrm{A} = 20e^{-0.02} = 19.6$ V and the time constant is $T_\mathrm{C} = 100$ s. The sketch is shown below.



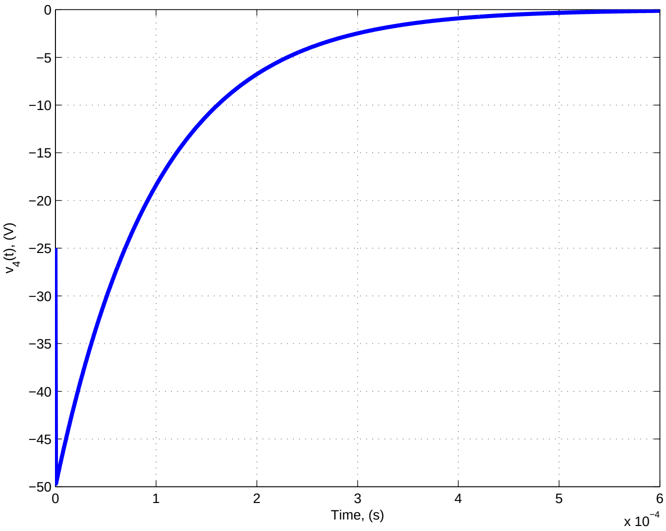
(c). $v_3(t) = \left[5e^{-2(t-5)}\right] u(t-5)$ V

The amplitude is $V_\mathrm{A} = 5$ V and the time constant is $T_\mathrm{C} = 1/2 = 500$ ms. The sketch is shown below.



(d). $v_4(t) = \left[-50e^{-10000t}\right] u(t)$ V

The amplitude is $V_\mathrm{A} = -50$ V and the time constant is $T_\mathrm{C} = 1/10000 = 100$ μs. The sketch is shown below.



Problem 5–12. Write expressions for the derivative ($t > 0$) and integral (from 0 to t) of the exponential waveform $i(t) = \left[500e^{-5000t}\right] u(t)$ μA.

Take the derivative.

$$\frac{di(t)}{dt} = \frac{d}{dt}\left[500e^{-5000t}\right] = (-5000)(500)e^{-5000t} = -2500000e^{-5000t}\ \mu\text{A/s} = -2.5e^{-5000t}\ \text{A/s}$$

Compute the integral.

$$\int_0^t 500e^{-5000\tau}\,d\tau = -\frac{500}{5000}e^{-5000\tau}\Big|_0^t = -0.1\left[e^{-5000t} - 1\right] = 0.1\left[1 - e^{-5000t}\right]\ \mu\text{A-s}$$

Problem 5–13. An exponential waveform decays to 50% of its initial ($t = 0$) amplitude in 25 ms. Find the time constant of the waveform.

The following calcuations yield the time constant.

$$e^{-0.025/T_C} = 0.5$$

$$-\frac{0.025}{T_C} = \ln(0.5)$$

$$T_C = \frac{-0.025}{\ln(0.5)} = \frac{-0.025}{-0.693147} = 36.0674\ \text{ms}$$

Problem 5–14. Write an expression for the waveform in Figure P5–14.

The waveform is an expoential with an amplitude $V_A = 10$, a time constant $T_C = (0.02–0.01)/\ln(10/3.68) = 10$ ms, and a time shift of 10 ms.

$$v(t) = 10e^{-\frac{t-0.01}{0.01}}u(t - 0.01) = 10e^{-100(t-0.01)}u(t - 0.01)$$

Problem 5–15. The amplitude of an exponential waveform is 6 V at $t = 0$ and 3.5 V at $t = 3$ ms. What is its time constant?

Apply the time decrement property.

$$T_C = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{0.003 - 0}{\ln\left[\dfrac{6}{3.5}\right]} = 5.566\ \text{ms}$$

Problem 5–16. Construct an exponential waveform that fits entirely within the nonshaded region in Figure P5–16.

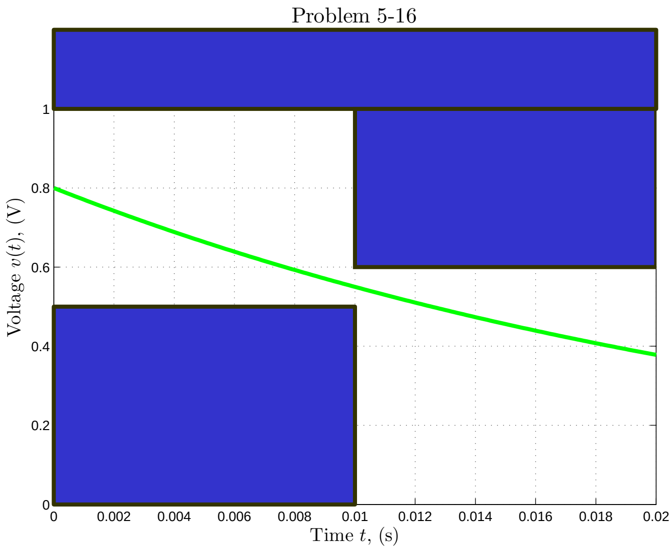
Let the waveform have an initial value of 0.8 V at $t = 0$ s and a desired value of 0.55 V at $t = 0.01$ s. Solve for the time constant using the decrement property.

$$T_\mathrm{C} = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{0.01-0}{\ln\left[\dfrac{0.8}{0.55}\right]} = 26.688 \text{ ms}$$

Calculate $1/T_\mathrm{C} = 37.47$ Hz and write the expression for the waveform as follows:

$$v(t) = 0.8e^{-37.47t}u(t)$$

The following plot of the function confirms the solution.



Problem 5–17. Construct an exponential waveform that fits entirely within the nonshaded region in Figure P5–17.

The function will be a rising exponential with an initial value of zero and a final value between 6 and 10. The time constant is approximately 3 to 4 ms. One way to solve this problem is through trial and error using MATLAB to plot the function. One valid results is

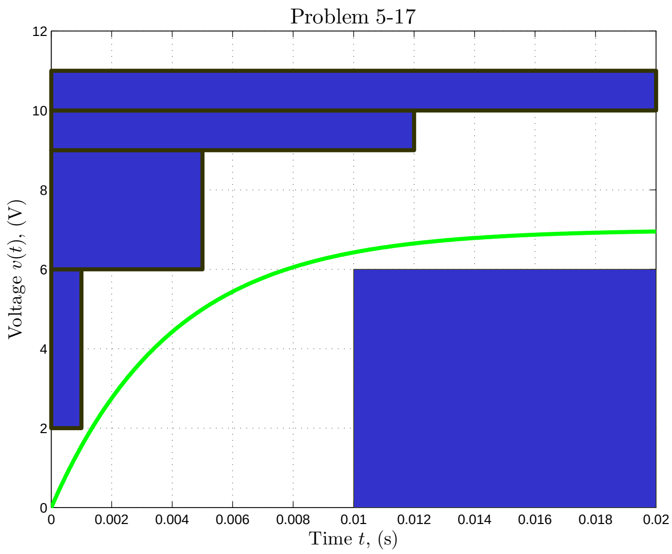
$$v(t) = 7\left[1 - e^{-250t}\right]u(t)$$

The following MATLAB code helps find the solution

```
% Define the exponential parameters
VA = 7;
TC = 4e-3;
t0 = 0;
ts = 50e-6;
tf = 20e-3;
t = t0:ts:tf;
% Create the waveform
vt = VA*(1-exp(-t/TC));
% Plot the results
figure
plot(t,vt,'g','LineWidth',3)
grid on
axis([0,tf,0,12])
xlabel('Time $t$, (s)',...
```

```
    'Interpreter','latex',...
    'FontSize',14)
ylabel('Voltage $v(t)$, (V)',...
    'Interpreter','latex',...
    'FontSize',14)
title('Problem 5-17',...
    'Interpreter','latex',...
    'FontSize',16)
hold on

BlockColor = [.2,.2,.8];
MyEdges = [.2,.2,0];
rectangle('Position',[0.01,0,0.01,6],'FaceColor',BlockColor,...
'EdgeColor',MyEdges)
rectangle('Position',[0,2,0.001,4],'FaceColor',BlockColor,...
'EdgeColor',MyEdges)
rectangle('Position',[0,6,0.005,3],'FaceColor',BlockColor,...
'EdgeColor',MyEdges)
rectangle('Position',[0,9,0.012,1],'FaceColor',BlockColor,...
'EdgeColor',MyEdges)
rectangle('Position',[0,10,0.020,1],'FaceColor',BlockColor,...
'EdgeColor',MyEdges)
```

The corresponding MATLAB plot is shown below.



Problem 5–18. By direct substitution show that the exponential function $v(t) = V_{\mathrm{A}}e^{-\alpha t}$ satisfies the following first-order differential equation.

$$\frac{dv(t)}{dt} + \alpha v(t) = 0$$

Substitute the expression into the equation and solve.

$$\begin{aligned}\frac{dv(t)}{dt} + \alpha v(t) &= 0 \\ \frac{d}{dt}\left(V_{\mathrm{A}}e^{-\alpha t}\right) + \alpha V_{\mathrm{A}}e^{-\alpha t} &= 0 \\ -\alpha V_{\mathrm{A}}e^{-\alpha t} + \alpha V_{\mathrm{A}}e^{-\alpha t} &= 0 \\ 0 &= 0\end{aligned}$$

Problem 5–19. Find the period, frequency, amplitude, time shift, and phase angle of the following sinusoids.

(a). $v_1(t) = 240\cos(120\pi t) + 240\sin(120\pi t)$ V

Compute the magnitude and phase angle to write the sinusoid in standard form and then determine the other values.

$$V_{\text{A}} = \sqrt{a^2+b^2} = \sqrt{240^2+240^2} = 339.41 \text{ V}$$

$$\phi = \tan^{-1}\left(\frac{-b}{a}\right) = \tan^{-1}\left(\frac{-240}{240}\right) = -45^\circ$$

$$v_1(t) = V_{\text{A}}\cos(2\pi f_0 t + \phi) = 339.41\cos(120\pi t - 45^\circ) \text{ V}$$

$$f_0 = \frac{120\pi}{2\pi} = 60 \text{ Hz}$$

$$T_0 = \frac{1}{f_0} = \frac{1}{60} = 16.67 \text{ ms}$$

$$T_{\text{S}} = \frac{-\phi}{360^\circ}T_0 = \frac{45^\circ}{360^\circ}(0.01667) = 2.083 \text{ ms}$$

(b). $v_2(t) = -40\cos(100000\pi t) + 30\sin(100000\pi t)$ V

Compute the magnitude and phase angle to write the sinusoid in standard form and then determine the other values.

$$V_{\text{A}} = \sqrt{a^2+b^2} = \sqrt{(-40)^2+30^2} = 50 \text{ V}$$

$$\phi = \tan^{-1}\left(\frac{-b}{a}\right) = \tan^{-1}\left(\frac{-30}{-40}\right) = -143.1^\circ$$

$$v_2(t) = V_{\text{A}}\cos(2\pi f_0 t + \phi) = 50\cos(100000\pi t - 143.13^\circ) \text{ V}$$

$$f_0 = \frac{100000\pi}{2\pi} = 50 \text{ kHz}$$

$$T_0 = \frac{1}{f_0} = \frac{1}{50000} = 20 \text{ } \mu\text{s}$$

$$T_{\text{S}} = \frac{-\phi}{360^\circ}T_0 = \frac{143.1^\circ}{360^\circ}(0.00002) = 7.952 \text{ } \mu\text{s}$$

Problem 5–20.

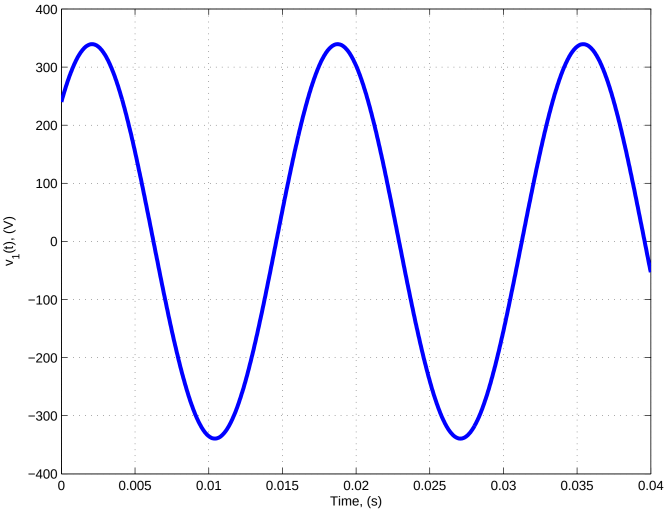
(a). Plot the waveform of each sinusoid in Problem 5–19.

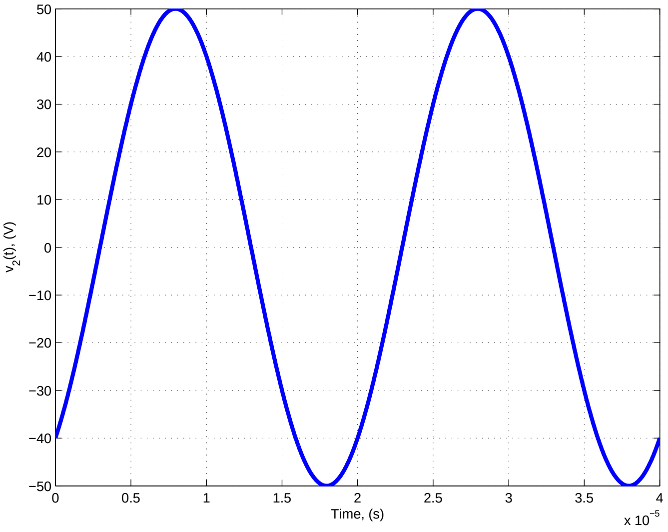
The following MATLAB code plots the waveforms.

```
% Create the signal
t = 0:40e-6:40e-3;
v1t = 240*cos(120*pi*t)+240*sin(120*pi*t);

% Plot the results
figure
plot(t,v1t,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
```

```
ylabel('v_1(t), (V)')

% Create the signal
t = 0:40e-9:40e-6;
v2t = -40*cos(100e3*pi*t)+30*sin(100e3*pi*t);

% Plot the results
figure
plot(t,v2t,'b','LineWidth',3)
grid on
axis([0,40e-6,-50,50]);
xlabel('Time, (s)')
ylabel('v_2(t), (V)')
```

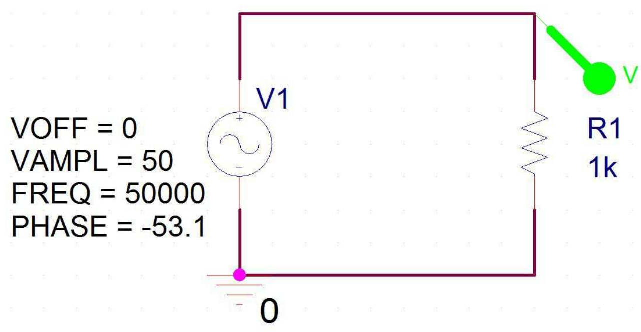
The resulting plots are shown below.



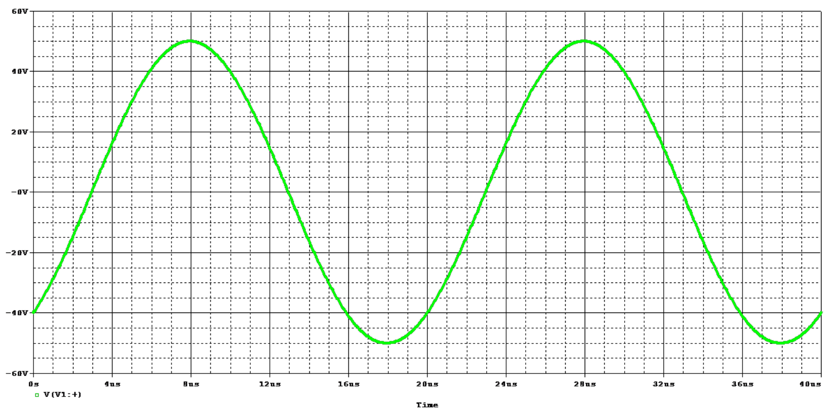


(b). Use OrCAD to produce the waveform in Problem 5–19(b).

The following OrCAD simulation produces the waveform. Note that the phase is set to $-143.1^\circ + 90^\circ = -53.1^\circ$ because OrCAD uses a sine function instead of a cosine function and $\cos(a) = \sin(a + 90^\circ)$.



The resulting plot is shown below.



Problem 5–21. Write an expression for the sinusoid in Figure P5–21. What are the phase angle and time shift of the waveform?

The sinusoid has an amplitude of $V_A = 5$ V and a period of $T_0 = 10$ ms. The frequency is $f_0 = 1/T_0 = 1/0.01 = 100$ Hz. The waveform is shifted one quarter of a cycle, which is a time shift of $T_S = T_0/4 = 2.5$ ms and a phase angle of $\phi = -360°T_S/T_0 = -90°$.

$$v(t) = 5\cos(200\pi t - \pi/2) \text{ V } = 5\cos(200\pi t - 90°) \text{ V} = 5\cos\left[\frac{2\pi(t - 0.0025)}{0.01}\right] \text{ V}$$

Problem 5–22. Write an expression for the sinusoid in Figure P5–22. What are the phase angle and time shift of the waveform?

The sinusoid has an amplitude of $V_A = 3$ V and a period of $T_0 = 2$ μs. The frequency is $f_0 = 1/T_0 = 1/0.000002 = 500$ kHz. The time shift is $T_S = 1.6$ μs and a phase angle of $\phi = -360°T_S/T_0 = -288° = -288° + 360° = 72°$.

$$v(t) = 3\cos\left(1000000\pi t + \frac{2\pi}{5}\right) \text{ V } = 3\cos(1000000\pi t + 72°) \text{ V} = 3\cos\left[\frac{2\pi(t - 0.0000016)}{0.000002}\right] \text{ V}$$

Problem 5–23. Find the Fourier coefficients, cyclic frequency, and radian frequency of the following sinusoids:

(a). $v(t) = 120\cos(120\pi t + 36.9°)$ V

The desired values are calculated below.

$$a = V_\text{A}\cos(\phi) = 120\cos(36.9^\circ) = 95.96 \text{ V}$$
$$b = -V_\text{A}\sin(\phi) = -120\sin(36.9^\circ) = -72.05 \text{ V}$$
$$f_0 = \frac{120\pi}{2\pi} = 60 \text{ Hz}$$
$$\omega_0 = 2\pi f_0 = 376.99 = 120\pi \ \frac{\text{rad}}{\text{s}}$$

(b). $i(t) = 12\cos(120\pi t - 270^\circ)$ A

The desired values are calculated below.

$$a = V_\text{A}\cos(\phi) = 12\cos(-270^\circ) = 0 \text{ A}$$
$$b = -V_\text{A}\sin(\phi) = -12\sin(-270^\circ) = -12 \text{ A}$$
$$f_0 = \frac{120\pi}{2\pi} = 60 \text{ Hz}$$
$$\omega_0 = 2\pi f_0 = 376.99 = 120\pi \ \frac{\text{rad}}{\text{s}}$$

Problem 5–24. Use MATLAB or Excel to display two cycles of the following waveform:

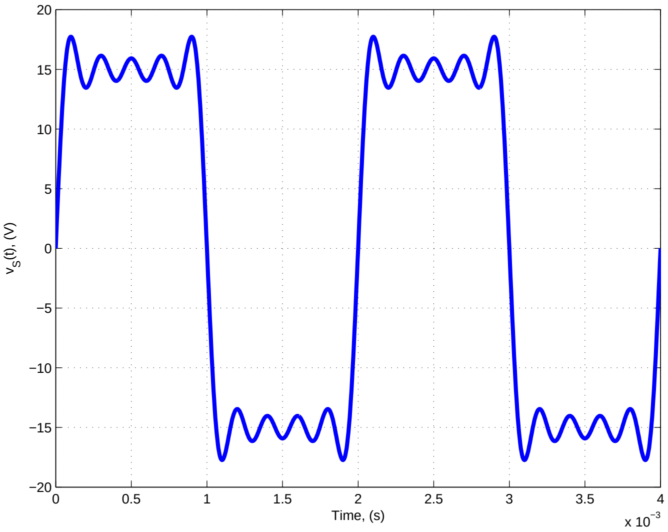
$$v_\text{S} = 19.1\sin(1000\pi t) + 6.37\sin(3000\pi t) + 3.82\sin(5000\pi t) + 2.73\sin(7000\pi t) + 2.1\sin(9000\pi t) \text{ V}$$

What are the period and amplitude of the resulting waveform? What common waveform is this waveform approximating?

The following MATLAB code plots the waveform:

```
% Create the signal
t = 0:4e-6:4e-3;
vS = 19.1*sin(1000*pi*t)+6.37*sin(3000*pi*t)+3.82*sin(5000*pi*t)...
    +2.73*sin(7000*pi*t)+2.1*sin(9000*pi*t);

% Plot the results
figure
plot(t,vS,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('v_S(t), (V)')
```

The corresponding MATLAB output is shown below.



The period of the waveform is 2 ms and the amplitude is 15 V. The waveform has the shape of a periodic square wave.

Problem 5–25. For the following sinusoid:

$$v(t) = 100\cos(2\pi 400t + 30^\circ)\text{ V}$$

(a). Find the Fourier coefficients, cyclic frequency, and radian frequency.

The desired values are calculated below.

$$a = V_\text{A}\cos(\phi) = 100\cos(30^\circ) = 86.60\text{ V}$$

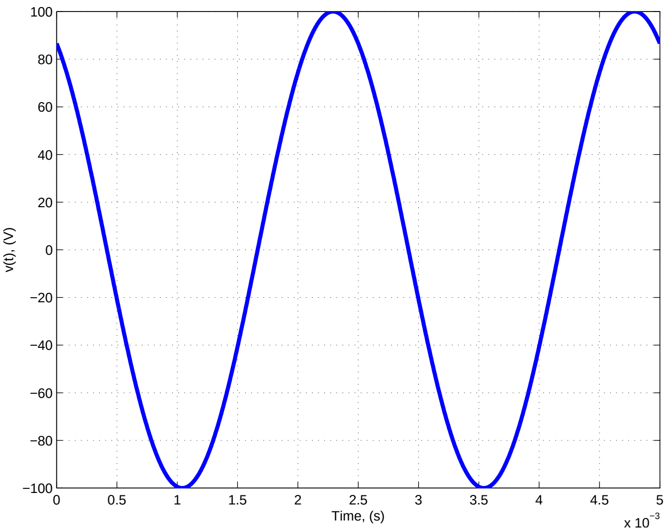
$$b = -V_\text{A}\sin(\phi) = -100\sin(30^\circ) = -50\text{ V}$$

$$f_0 = 400\text{ Hz}$$

$$\omega_0 = 2\pi f_0 = 800\pi = 2513.3\ \frac{\text{rad}}{\text{s}}$$

(b). Plot the waveform.

The plot is shown below.



(c). Use MATLAB to produce the waveform.

The following MATLAB code produced the plot shown in part (b).

```
% Define the parameters
Va = 100;
Vdeg = 30;
wo = 2*pi*400
fo = wo/2/pi
phi = pi*Vdeg/180;
a = Va*cos(phi)
b = -Va*sin(phi)

% Plot the signal
t=0:5e-6:5e-3;
vt = Va*cos(wo*t+phi);
plot(t,vt,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('v(t), (V)')
```

Problem 5–26. Find the time shift of each sinusoid

(a). in Problem 5–19

In part (a), the phase angle is $\phi = -45°$ and the period is $T_0 = 16.67$ ms, so the time shift is $T_S = -\phi T_0/360° = (0.01667)(45/360) = 2.083$ ms. In part (b), the phase angle is $\phi = -143.1°$ and the period is $T_0 = 20$ μs, so the time shift is $T_S = -\phi T_0/360° = (0.000002)(143.1/360) = 7.952$ μs.

(b). in Problem 5–23

In part (a), the phase angle is $\phi = 36.9°$ and the period is $T_0 = 1/f_0 = 1/60 = 16.67$ ms, so the time shift is $T_S = -\phi T_0/360° = (0.01667)(-36.9/360) = -1.708$ ms. In part (b), the phase angle is $\phi = -270°$ and the period is $T_0 = 1/f_0 = 1/60 = 16.67$ ms, so the time shift is $T_S = -\phi T_0/360° = (0.01667)(270/360) = 12.5$ ms.

(c). in Problem 5–25

The phase angle is $\phi = 30°$ and the period is $T_0 = 1/f_0 = 1/400 = 2.5$ ms, so the time shift is $T_S = -\phi T_0/360° = (0.0025)(-30/360) = -208.33$ μs.

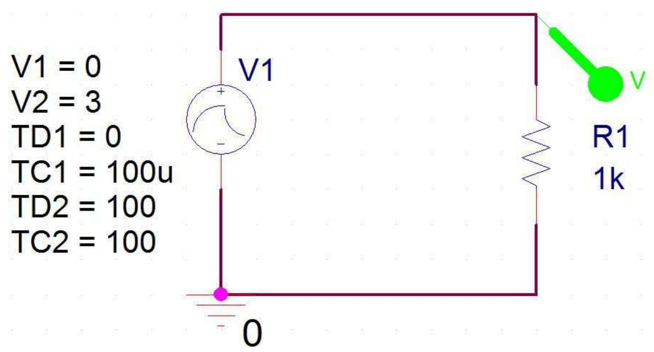
Problem 5–27. Consider the following composite waveforms.

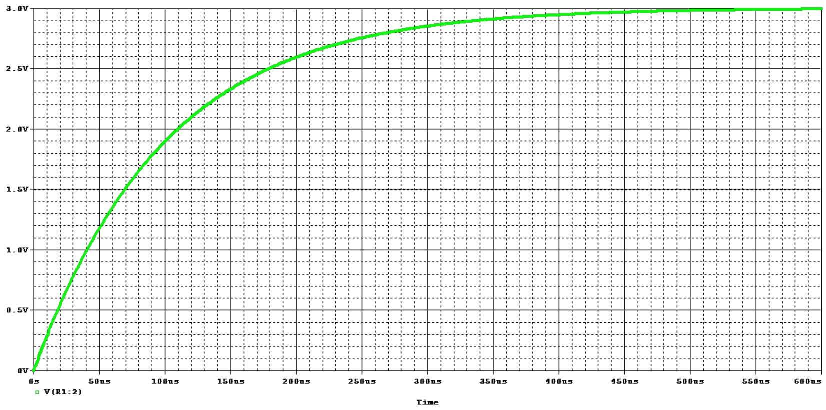
(a). $v_1(t) = 3\left[1 - e^{-10000t}\right]u(t)$ V

(b). $v_2(t) = 15\left[e^{-10t} - e^{-5t}\right]u(t)$ V

Sketch each on paper and then generate each using OrCAD and MATLAB and plot the results.

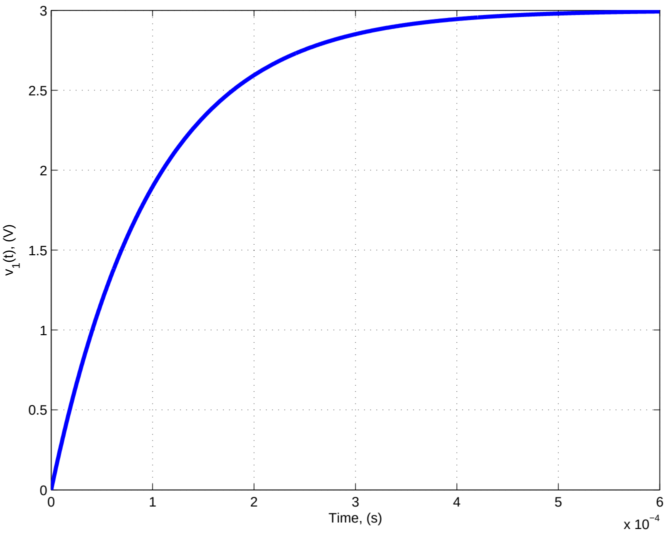
(a). The signal has an initial value of 0 V and a final value of 3 V. The exponential signal has a time constact of 100 μs, so it reaches the final value in approximately 500 μs. The following OrCAD simulation and plot demonstrate one way to create the signal.



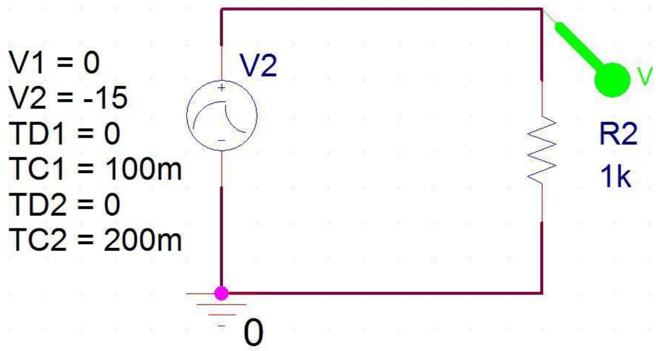


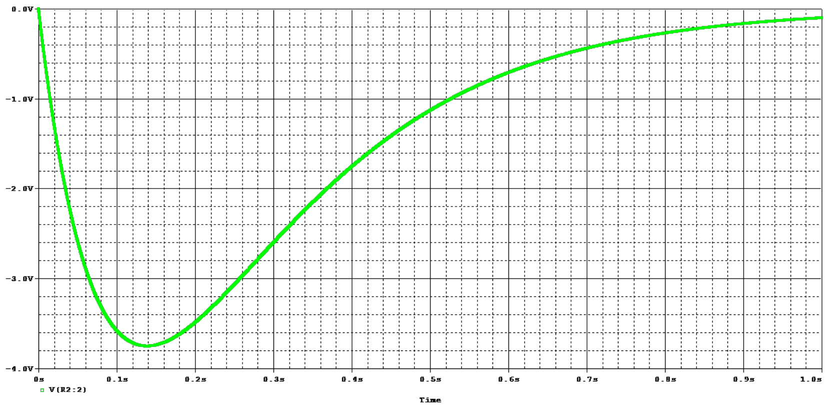
The following MATLAB code and resulting plot show another way to create the signal.

```
t = 0:6e-7:6e-4;
v1t = 3*(1-exp(-10000*t)).*heaviside(t);
figure
plot(t,v1t,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('v_1(t), (V)')
```



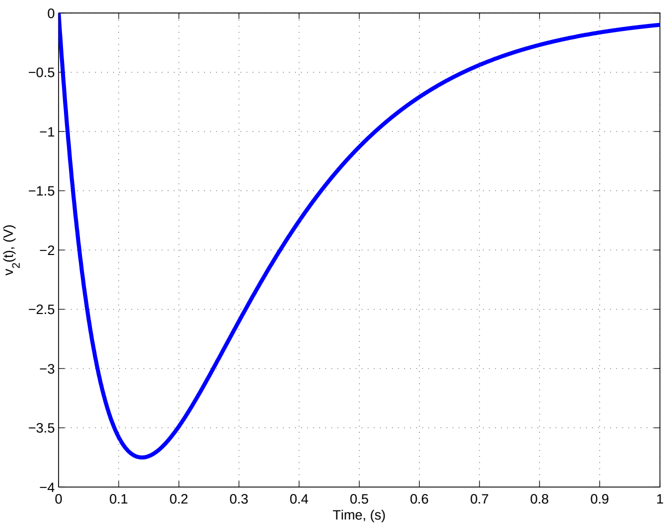
(b). The signal has an initial value of 0 V and a final value of 0 V. The exponent with the positive magnitude has a smaller time constant, so it decays to zero faster than the exponent with the negative time constant. Therefore, the negative magnitude dominates and the waveform is always negative. The waveform starts at 0 V, initially decreases, and then increases to return to zero. Take the derivative of the waveform and set it equal to zero to determine the minimum occurs at $t = 138.63$ ms and has a value of -3.75 V. The signal returns to 0 V after approximately five time constants of the dominant exponential, or at $t = 1$ s. The following OrCAD simulation and plot demonstrate one way to create the signal.





The following MATLAB code and resulting plot show another way to create the signal.

```
t = 0:0.001:1.5;
v2t = 15*(exp(-10*t)-exp(-5*t)).*heaviside(t);
figure
plot(t,v2t,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('v_2(t), (V)')
```



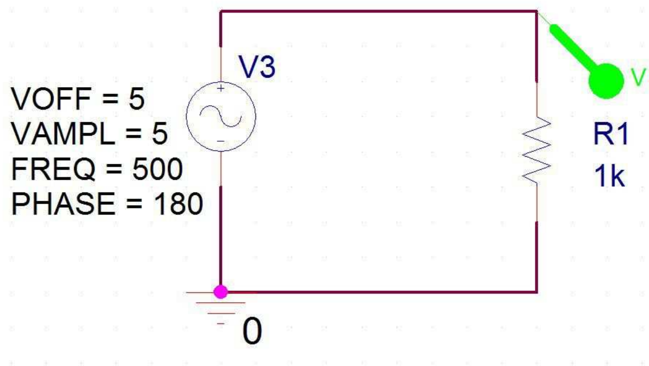
Problem 5–28. Consider the following composite waveforms.

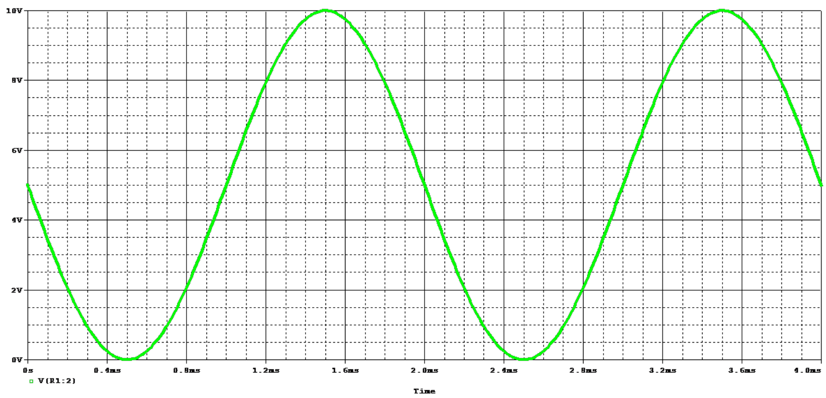
(a). $i_1(t) = 5 - 5\sin(1000\pi t)u(t)$ A

(b). $i_2(t) = 350\left[e^{-1000t} + \cos(2000\pi t)\right]u(t)$ mA

Sketch each on paper and then generate each using OrCAD and MATLAB and plot the results.

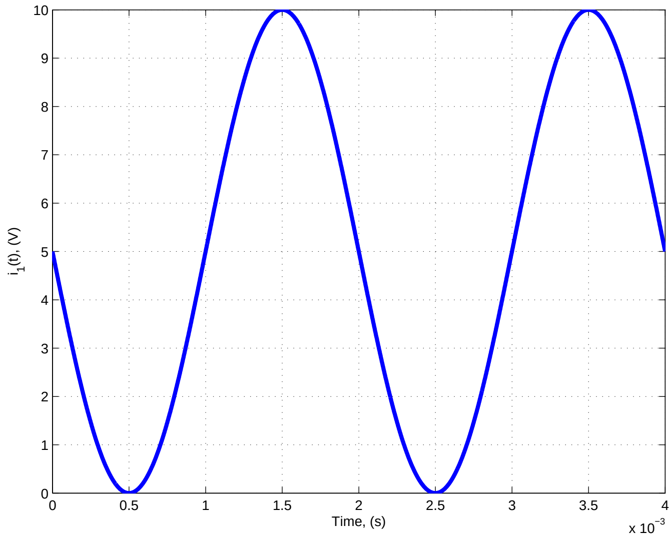
(a). The waveform is a sinusoid with a magnitude of 5 A and a period of $T_0 = 1/500 = 2$ ms, that has been inverted (by the negative sign) and shifted 5 A in the positive direction. The initial value is 5 A and the sinusoid decreases to a value of 0 A at 0.5 ms before is rises to a value of 10 A at 1.5 ms and returns to 5 A at 2 ms. The following OrCAD simulation and plot demonstrate one way to create the signal.



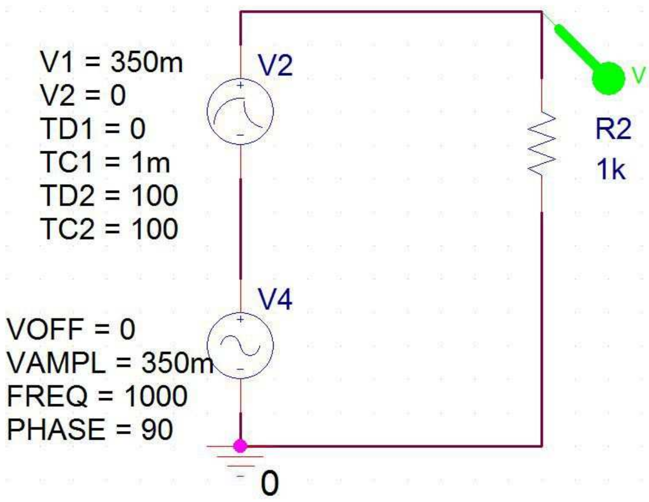


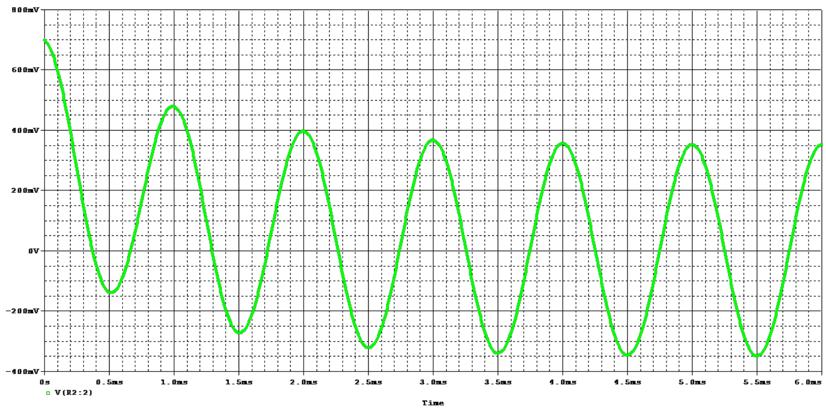
The following MATLAB code and resulting plot show another way to create the signal.

```
t = 0:2e-6:4e-3;
i1t = 5-5*sin(1000*pi*t);
figure
plot(t,i1t,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('i_1(t), (V)')
```



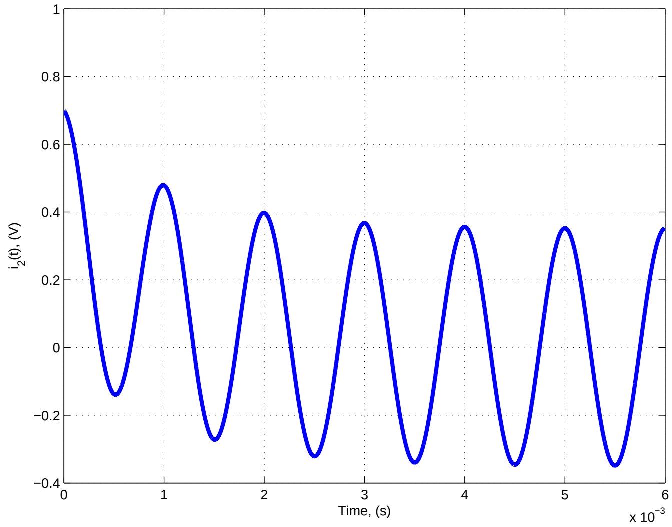
(b). The signal is an exponential plus a sinusoid. The exponent decreases from one to zero in approximately 5 ms, at which time only the sinusoid is visible in the results. The initial value of the overall signal is 700 mA. The sinusoid has an amplitude of 350 mA and a period of 1 ms. The following OrCAD simulation and plot demonstrate one way to create the signal.





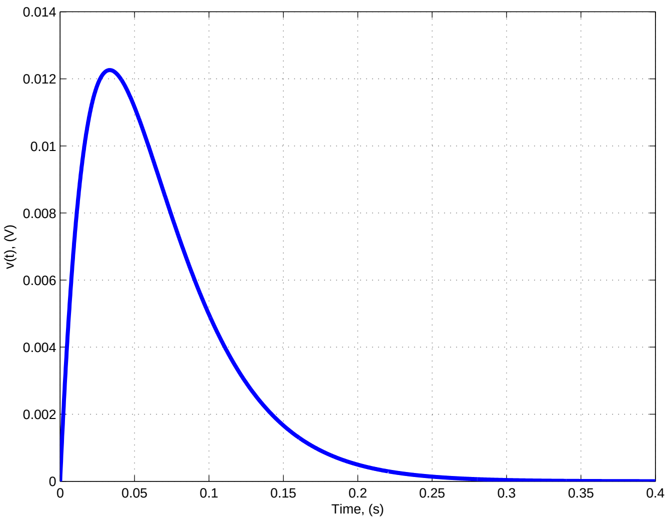
The following MATLAB code and resulting plot show another way to create the signal.

```
t = 0:1e-6:6e-3;
i2t = 350e-3*(exp(-1000*t)+cos(2000*pi*t));
figure
plot(t,i2t,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('i_2(t), (V)')
```



Problem 5–29. Sketch the damped ramp $v(t) = te^{-30t}u(t)$. Find the maximum value of the waveform and the time at which it occurs.

The waveform sketch is shown below.



Take the derivative of the expression and set it equal to zero. Solve for t to determine that the maximum occurs at $t = T_{\mathrm{C}} = 1/30 = 33.33$ ms and has a value of 12.263 mV.

$$\frac{dv(t)}{dt} = 0 = \frac{d}{dt}\left[te^{-30t}\right]$$
$$0 = -30te^{-30t} + (1)e^{-30t}$$
$$30te^{-30t} = e^{-30t}$$
$$30t = 1$$
$$t = \frac{1}{30} = 33.33 \text{ ms}$$
$$v(0.03333) = (0.03333)e^{-30(0.03333)} = 12.263 \text{ mV}$$

Problem 5–30. The value of the waveform $v(t) = (V_{\mathrm{A}} - V_{\mathrm{B}}e^{-\alpha t})u(t)$ is 5 V at $t = 0$, 8 V at $t = 5$ ms, and approaches 12 V as $t \to \infty$.

(a). Find V_{A}, V_{B}, and α, and then sketch the waveform.

As $t \to \infty$, the exponential term decays to zero and the only term remaining in the expression is V_{A}, so $V_{\mathrm{A}} = 12$ V. At $t = 0$, the expression is $V_{\mathrm{A}} - V_{\mathrm{B}} = 5$ V, so $V_{\mathrm{B}} = 7$ V. Use the value at $t = 5$ ms to compute α as follows:

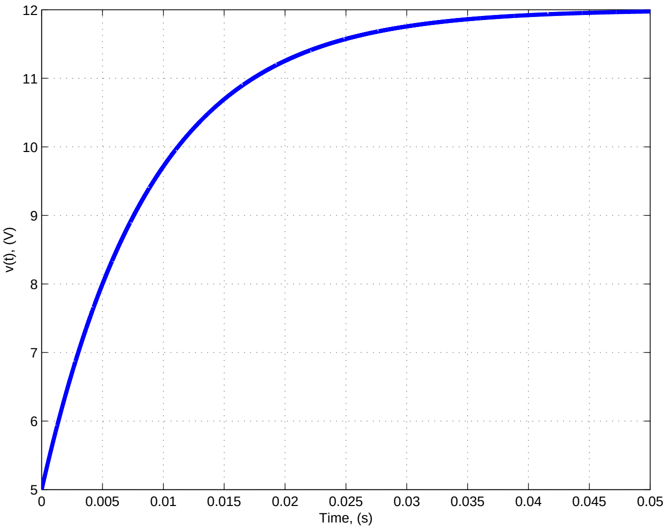
$$8 = \left(12 - 7e^{-\alpha(0.005)}\right)$$
$$e^{-\alpha(0.005)} = \frac{12-8}{7} = 0.571429$$
$$-\alpha(0.005) = -0.559616$$
$$\alpha = 111.923 \text{ Hz}$$

The sketch is shown in part (b).

(b). Validate your answers by plotting your result in MATLAB.

The following MATLAB code and resulting plot validate the results in part (a).

```
t = 0:5e-6:50e-3;
VA = 12;
VB = 7;
a = log(4/7)/(-0.005)
vt = VA - VB*exp(-a*t);
figure
plot(t,vt,'b','LineWidth',3)
grid on
xlabel('Time, (s)')
ylabel('v(t), (V)')
```



Problem 5–31. Write an expression for the composite sinusoidal waveform in Figure P5–31.

The sinusoid has a period of $T_0 = 10$ ms and is shifted by one half of a cycle or $\phi = -180°$. The frequency is $f_0 = 1/T_0 = 1/0.01 = 100$ Hz. The maximum value is 8 V and the minimum value is -20 V, so the average value is $V_{\text{avg}} = (8-20)/2 = -6$ V and the amplitude is $V_{\text{A}} = [8-(-20)]/2 = 14$ V. We can write the following expression for the waveform.

$$v(t) = [-6 + 14\cos(200\pi t - 180°)]\,u(t) \text{ V}$$

Problem 5–32. Write an expression for the composite sinusoidal waveform in Figure P5–32.

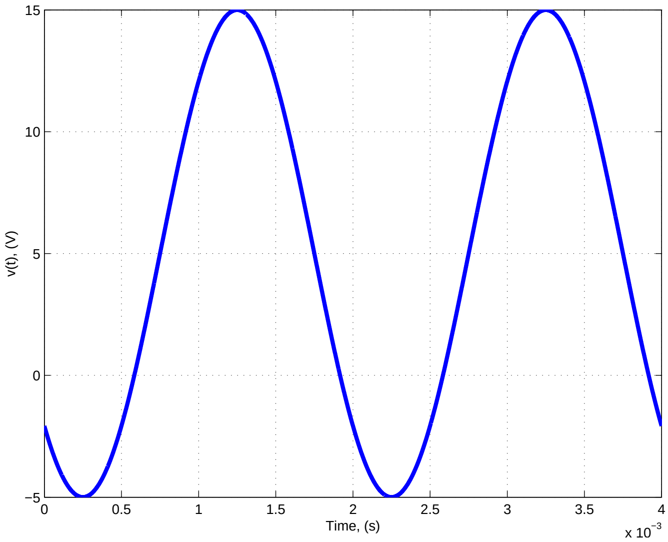
The sinusoid has an amplitude of $V_{\text{A}} = 3$ V and a period of $T_0 = 2$ μs. The frequency is $f_0 = 1/T_0 =$ 500 kHz. The time shift is $T_{\text{S}} = 1.6$ μs, so the phase shift is $\phi = -360° T_{\text{S}}/T_0 = -288° = +72°$. For $t < 1.6$ μs, the signal has an average value of 0 V. For $t \geq 1.6$ μs, the average value is $V_{\text{avg}} = (5-1)/2 = 2$ V. We can write the following expression for the waveform.

$$v(t) = 2u(t - 0.0000016) + 3\cos(1000000\pi t + 72°) \text{ V}$$

Problem 5–33. A waveform of the form $v(t) = 5 - 10\cos(\beta t - 45°)$ periodically reaches a minimum every 2 ms.

(a). Find the values of V_{MAX}, V_{MIN}, and β, and then sketch the waveform.

The maximum value is $V_{\text{MAX}} = 5 + 10 = 15$ V and the minimum value is $V_{\text{MIN}} = 5 - 10 = -5$ V. The period is $T_0 = 2$ ms, so the frequency is $f_0 = 1/T_0 = 1/0.002 = 500$ Hz. The radian frequency is $\omega_0 = \beta = 2\pi f_0 = 1000\pi$ rad/s. The sketch is shown below.

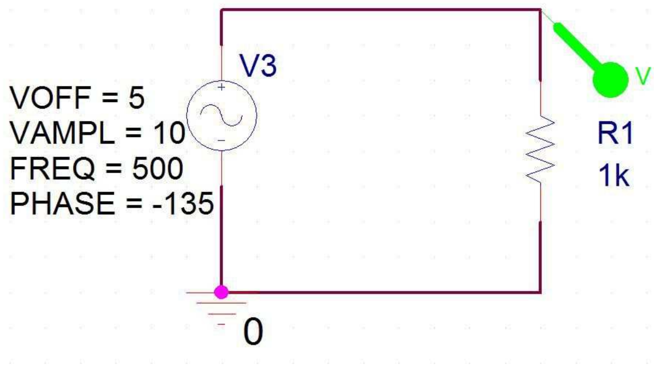


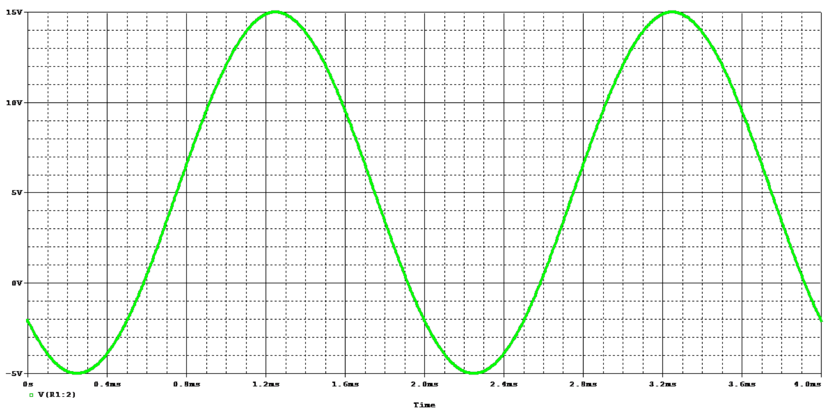
(b). Generate the waveform in OrCAD.

To plot the waveform in OrCAD, note that the following expressions are equivalent.

$$v(t) = 5 - 10\cos(1000\pi t - 45°) = 5 + 10\cos(1000\pi t - 225°) = 5 + 10\sin(1000\pi t - 135°)\text{ V}$$

The following OrCAD simulation will generate the appropriate waveform, as shown below.





Problem 5–34. Write an expression for the composite exponential waveform in Figure P5–34.

The waveform has an initial value of 20 V and a final value of 10 V. The final value is a constant added to the exponential term. Use the decrement property to find the time constant. To use the decrement property in this case, we need to subtract out the average value from the two voltages.

$$T_{\rm C} = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{0.002-0}{\ln\left[\dfrac{20-10}{15-10}\right]} = 2.8854 \text{ ms}$$

Calculate $1/T_{\rm C} = 1/0.0028854 = 346.57$ Hz. We can write the following expression for the waveform.

$$v(t) = \left[10 + 10e^{-346.57t}\right] u(t) \text{ V}$$

Problem 5–35. Write an expression for the composite exponential waveform in Figure P5–35.

Both parts of the exponential waveform approach a final value of 2 V. Find the time constant for each segment and account for the 2-V average value.

$$T_{\rm C1} = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{0.003-0}{\ln\left[\dfrac{4-2}{2.736-2}\right]} = 3.00 \text{ ms}$$

$$T_{\rm C2} = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{0.008-0.005}{\ln\left[\dfrac{0-2}{1.264-2}\right]} = 3.00 \text{ ms}$$

Both exponentials have the same time constant of 3 ms and the same amplitude of 2 V. We can write the following expression for the waveform.

$$v(t) = 2 + 2e^{-t/0.003}\left[u(t) - u(t-0.005)\right] - 2e^{-(t-0.005)/0.003}u(t-0.005) \text{ V}$$

Problem 5–36. For the double exponential $v(t) = 20\left(e^{-100t} - e^{-1000t}\right) u(t)$ V

(a). Find the maximum value of the waveform and the time at which it occurs.

To determine the time at which the maximum value occurs, take the derivative of the expression, set it equal to zero, and solve for t. Evaluate the expression at that time to find the maximum value.

$$\begin{aligned}
\frac{dv(t)}{dt} = 0 &= \frac{d}{dt}\left[20\left(e^{-100t} - e^{-1000t}\right)\right] \\
0 &= 20\left(-100e^{-100t} + 1000e^{-1000t}\right) \\
100e^{-100t} &= 1000e^{-1000t} \\
e^{900t} &= 10 \\
900t &= \ln(10) = 2.30259 \\
t &= 2.55843 \text{ ms} \\
v(0.00255843) &= 20\left(e^{-0.255843} - e^{-2.55843}\right) = 13.937 \text{ V}
\end{aligned}$$

(b). Determine the dominant exponential.

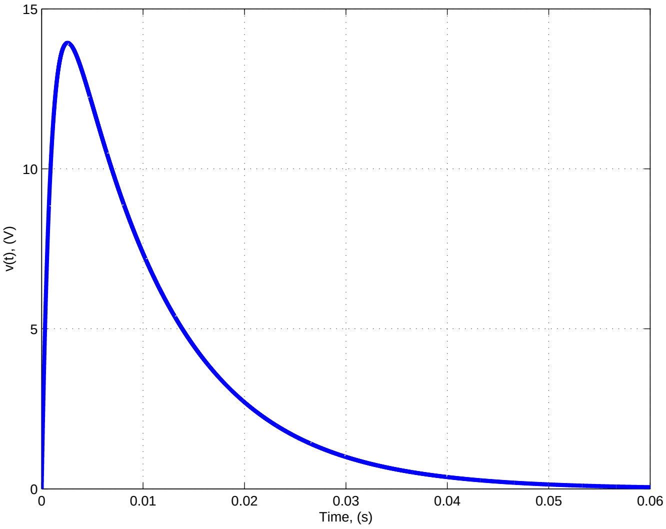
The dominant exponential is the one with the larger time constant. The time constants are $T_{\rm C1} = 1/100 = 10$ ms and $T_{\rm C2} = 1/1000 = 1$ ms, so the first exponential dominates. Since the exponential with the positive magnitude dominates, the waveform is always positive.

(c). Generate the waveform in MATLAB and validate the result.

The following MATLAB code plots the waveform.

```
t = 0:1e-6:60e-3;
vt = 20*(exp(-100*t) - exp(-1000*t));
figure
plot(t,vt,'b','LineWidth',3)
grid on
axis([0,0.06,0,15]);
xlabel('Time, (s)')
ylabel('v(t), (V)')
```

The corresponding MATLAB output is shown below and validates the results in parts (a) and (b).



Problem 5–37. Write an expression for the damped sine waveform in Figure P5–37.

The sinusoid has a period of $T_0 = 10$ ms, which corresponds to a frequency of $f_0 = 1/0.01 = 100$ Hz. The waveform is a cosine shifted by one quarter of a cycle or $\phi = -90°$. The exponential has an amplitude of $V_A = 5$ V. Use the decrement property applied at the peaks of the sinusoid to find the time constant.

$$T_C = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{0.01}{\ln\left[\dfrac{3.9}{1.4}\right]} = 9.76 \text{ ms}$$

Round the time constant to $T_C = 10$ ms and note that $1/T_C = 1/0.01 = 100$ Hz. Combining the results, we can write the following expression for the waveform:

$$v(t) = 5e^{-100t}\cos(200\pi t - 90°) \text{ V} = 5e^{-100t}\sin(200\pi t) \text{ V}$$

Problem 5–38. A circuit response is shown in Figure P5–38. Determine an approximate expression for the waveform.

The signal is a sinusoid with an exponential decay that approaches a final value of -5 V. The waveform starts at a peak, so there is no phase shift and $\phi = 0°$. The initial value is 5 V, so with a final value of -5 V, the amplitude of the signal is 10 V. The first peak has an amplitude of 5 V and a time of $t = 0$ ms. The fourth peak has an amplitude of approximately -3.5 V at a time of approximately 18.5 ms. The fourth peak occurs after three complete cycles, so the time for one cycle is $T_0 = 18.5/3 = 6.17$ ms. The frequency for the sinusoid is $f_0 = 1/T_0 = 162$ Hz. Apply the decrement property and account for the average value to find the time constant for the exponential.

$$T_C = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{0.0185 - 0}{\ln\left[\dfrac{5-(-5)}{-3.5-(-5)}\right]} = 9.75 \text{ ms}$$

Note that $1/T_{\mathrm{C}} = 103$ Hz. We can write the following approximate expression for the waveform:

$$v(t) = V_{\text{avg}} + V_{\text{A}}e^{-t/T_{\text{C}}}\cos(2\pi f_0 t + \phi) = -5 + 10e^{-103t}\cos(324\pi t) \text{ V}$$

Problem 5–39. Find V_{p}, V_{pp}, V_{avg}, and V_{rms} for each of the following sinusoids.

(a). $v_1(t) = 120\cos(377t) + 120\sin(377t)$ V

Find the amplitude of the sinusoid.

$$V_{\text{A}} = \sqrt{a^2 + b^2} = \sqrt{120^2 + 120^2} = 169.7 \text{ V}$$

There is no constant term, so the average value is $V_{\text{avg}} = 0$ V, the peak value is $V_{\text{p}} = 169.7$ V, and the peak-to-peak value is $V_{\text{pp}} = (2)(169.7) = 339.4$ V. For a sinusoid with no average value, the rms value is $V_{\text{rms}} = V_{\text{A}}/\sqrt{2} = 169.7/\sqrt{2} = 120$ V.

(b). $v_2(t) = -40\cos(2000\pi t) - 30\sin(2000\pi t)$ V

Find the amplitude of the sinusoid.

$$V_{\text{A}} = \sqrt{a^2 + b^2} = \sqrt{(-40)^2 + (-30)^2} = 50 \text{ V}$$

There is no constant term, so the average value is $V_{\text{avg}} = 0$ V, the peak value is $V_{\text{p}} = 50$ V, and the peak-to-peak value is $V_{\text{pp}} = (2)(50) = 100$ V. For a sinusoid with no average value, the rms value is $V_{\text{rms}} = V_{\text{A}}/\sqrt{2} = 50/\sqrt{2} = 35.36$ V.

(c). $v_3(t) = 10 + 24\cos(10000\pi t - 45^\circ)$ V

The maximum value is $V_{\text{MAX}} = 10 + 24 = 34$ V and the minimum value is $V_{\text{MIN}} = 10 - 24 = -14$ V. The peak value is $V_{\text{p}} = 34$ V and the peak-to-peak value is $V_{\text{pp}} = 34 - (-14) = 48$ V. The average value is, by inspection, $V_{\text{avg}} = 10$ V. For a sinusoid with an average value, we can compute the rms value as follows.

$$V_{\text{rms}} = \sqrt{V_{\text{avg}}^2 + \frac{V_{\text{A}}^2}{2}} = \sqrt{10^2 + \frac{24^2}{2}} = 19.70 \text{ V}$$

Problem 5–40. An exponential waveform given by $v(t) = 15e^{-1000t}u(t)$ V repeats every five time constants.

(a). Find V_{p}, V_{pp}, V_{MAX}, and V_{MIN}.

At $t = 0$, the signal is at its maximum value of $V_{\text{MAX}} = 15$ V. At $t = 5T_{\text{C}}$, the signal is at its minimum value $V_{\text{MIN}} = 15e^{-5} = 101$ mV. The peak value is $V_{\text{p}} = 15$ V and the peak-to-peak value is $V_{\text{pp}} = 15 - 0.101 = 14.899$ V.

(b). Find V_{avg} and V_{rms}.

Compute the average value over one period of the waveform.

$$V_{\text{avg}} = \frac{1}{T_0}\int_0^{T_0} v(t)\,dt = \frac{1}{5T_{\text{C}}}\int_0^{5T_{\text{C}}} 15e^{-t/T_{\text{C}}}\,dt$$

$$V_{\text{avg}} = \frac{1}{5T_{\text{C}}}(-T_{\text{C}})15e^{-t/T_{\text{C}}}\Big|_0^{5T_{\text{C}}}$$

$$V_{\text{avg}} = -\frac{15}{5}\left(e^{-5} - 1\right)$$

$$V_{\text{avg}} = 2.9798 \text{ V}$$

Compute the rms value over one period of the waveform.

$$V_{\text{rms}} = \sqrt{\frac{1}{T_0}\int_0^{T_0} [v(t)]^2\, dt} = \sqrt{\frac{1}{5T_{\text{C}}}\int_0^{5T_{\text{C}}} 225e^{-2t/T_{\text{C}}}\, dt}$$

$$V_{\text{rms}} = \sqrt{\left(\frac{225}{5T_{\text{C}}}\right)\left(-\frac{T_{\text{C}}}{2}\right)e^{-2t/T_{\text{C}}}\Big|_0^{5T_{\text{C}}}}$$

$$V_{\text{rms}} = \sqrt{-\frac{225}{10}\left(e^{-10}-1\right)}$$

$$V_{\text{rms}} = 4.7433 \text{ V}$$

The following MATLAB code confirms the results.

```
syms t
VA = 15;
TC = 1e-3;
vt = VA*exp(-t/TC)
T0 = 5*TC;
Vavg = int(vt,t,0,T0)/T0
Vavg_num = double(Vavg)
Vrms = sqrt(int(vt^2,t,0,T0)/T0)
Vrms_num = double(Vrms)
```

(c). Find the period T_0 of the waveform.

The time constant for the exponential is $T_{\text{C}} = 1/1000 = 1$ ms. The exponential repeats every five time constants, so the period is $T_0 = 5T_{\text{C}} = 5$ ms.

Problem 5–41. Find V_{p}, V_{pp}, V_{avg}, V_{rms}, and T_0 for the periodic waveform in Figure P5–41.

The maximum value is $V_{\text{MAX}} = 15$ V and the minimum value is $V_{\text{MIN}} = 0$ V. The peak value is $V_{\text{p}} = 15$ V and the peak-to-peak value is $V_{\text{pp}} = 15$ V. Compute the area under one period of the waveform and divide by the period to determine the average value.

$$V_{\text{avg}} = \frac{1}{T_0}\left[(0)\left(\frac{T_0}{4}\right)+(5)\left(\frac{T_0}{4}\right)+(10)\left(\frac{T_0}{4}\right)+(15)\left(\frac{T_0}{4}\right)\right] = \frac{1}{4}\left[5+10+15\right] = 7.5 \text{ V}$$

Compute the area under one period of the square of the waveform, divide by the period and take the square root to determine the rms value.

$$V_{\text{rms}} = \sqrt{\frac{1}{T_0}\left[(0)\left(\frac{T_0}{4}\right)+(25)\left(\frac{T_0}{4}\right)+(100)\left(\frac{T_0}{4}\right)+(225)\left(\frac{T_0}{4}\right)\right]} = \sqrt{\frac{1}{4}\left[25+100+225\right]} = 9.3541 \text{ V}$$

The period of the waveform is $T_0 = 20$ ms.

Problem 5–42. Find V_{p}, V_{pp}, V_{avg}, V_{rms}, and T_0 for the periodic waveform in Figure P5–42.

The maximum value is $V_{\text{MAX}} = 2$ mV and the minimum value is $V_{\text{MIN}} = -1$ mV. The peak value is $V_{\text{p}} = 2$ mV and the peak-to-peak value is $V_{\text{pp}} = 2-(-1) = 3$ mV. Compute the area under one period of the waveform and divide by the period to determine the average value.

$$V_{\text{avg}} = \frac{1}{T_0}\left[(2)\left(\frac{T_0}{4}\right)+(0)\left(\frac{T_0}{4}\right)+(-1)\left(\frac{T_0}{4}\right)+(0)\left(\frac{T_0}{4}\right)\right] = \frac{1}{4}\left[2-1\right] = 250\ \mu\text{V}$$

Compute the area under one period of the square of the waveform, divide by the period and take the square root to determine the rms value.

$$V_{\text{rms}} = \sqrt{\frac{1}{T_0}\left[(4)\left(\frac{T_0}{4}\right)+(0)\left(\frac{T_0}{4}\right)+(1)\left(\frac{T_0}{4}\right)+(0)\left(\frac{T_0}{4}\right)\right]} = \sqrt{\frac{1}{4}\left[4+1\right]} = 1.118 \text{ mV}$$

The period of the waveform is $T_0 = 4$ s.

Problem 5–43. Figure P5–43 is the result of the sum of a fundamental and one of its harmonics. Find $V_{\rm p}$, $V_{\rm pp}$, $V_{\rm avg}$, $V_{\rm rms}$, and T_0 for the waveform.

The maximum value is $V_{\rm MAX} = 120$ V and the minimum value is $V_{\rm MIN} = -115$ V. The peak value is $V_{\rm p} = 120$ V and the peak-to-peak value is $V_{\rm pp} = 120 - (-115) = 235$ V. Since the waveform is the sum of two sinusoids, the average value will be zero, $V_{\rm avg} = 0$ V. The signal has a period of $T_0 = 2$ ms and a frequency of $f_0 = 1/T_0 = 500$ Hz. To compute the rms value we need to know the amplitudes of the two components. Assume both components have no phase shift. At $t = 0$, the amplitude of the sum is 120 V and at $t = 1$ ms, the amplitude is -80 V. One way to achieve this result would be to have the lower frequency component have an amplitude of 100 V and the higher frequency component have an amplitude of 20 V. The higher frequency component completes 10 cycles in the time required for the lower frequency component to complete one cycle. Therefore, the higher frequency component has a frequency $f = 10f_0 = 5000$ Hz. We can estimate the expression for the waveform as follows:

$$v(t) = 100\cos(1000\pi t) + 20\cos(10000\pi t) \text{ V}$$

Compute the rms value for a sum of sinusoids as follows:

$$V_{\rm rms} = \sqrt{\frac{V_{\rm A1}^2}{2} + \frac{V_{\rm A2}^2}{2}} = \sqrt{\frac{100^2}{2} + \frac{20^2}{2}} = 72.11 \text{ V}$$

The following MATLAB code confirms the results.

```
% Plot the waveform
t = 0:1e-8:4e-3;
vt = 100*cos(1000*pi*t)+20*cos(10000*pi*t);
figure
plot(t,vt,'b','LineWidth',2)
grid minor
axis([0,4e-3,-125 125])
xlabel('Time, (s)')
ylabel('v(t), V')

% Compute the rms and average values
syms a1 a2 T0 t
vt = a1*cos(2*pi*t/T0)+a2*cos(20*pi*t/T0);
vrms = sqrt(int(vt^2,t,0,T0)/T0)
pretty(vrms)
vrms_num = subs(vrms,{a1,a2},{100,20})
vavg = int(vt,t,0,T0)/T0
```

Problem 5–44. Figure P5–44 displays the response of a circuit to a square wave signal. The response is a periodic sequence of exponential waveforms. Each exponential has a time constant of 1.6 ms.

(a). Find $V_{\rm p}$, $V_{\rm pp}$, and T_0 for the waveform.

The maximum value is $V_{\rm MAX} = 4$ V and the minimum value is $V_{\rm MIN} = -2$ V. The peak value is $V_{\rm p} = 4$ V and the peak-to-peak value is $V_{\rm pp} = 4 - (-2) = 6$ V. The period of the waveform is 20 ms.

(b). Use MATLAB to find $V_{\rm avg}$ and $V_{\rm rms}$.

Examine the first period of the waveform. For the first half, the waveform starts at an amplitude of -2 V and approaches 4 V over 10 ms. The time constant is $T_{\rm C} = 1.6$ ms, so $1/T_{\rm C} = 1/0.0016 = 625$ Hz. For the second half, the waveform starts at an amplitude of 4 V and approaches -2 V over 10 ms. We can write an approximate expression for the waveform as follows:

$$v(t) = \left[4 - 6e^{-625t}\right][u(t) - u(t-0.01)] + \left[-2 + 6e^{-625(t-0.01)}\right][u(t-0.01) - u(t-0.02)] \text{ V}$$

The following MATLAB code computes the average and rms values.

```
% Compute the average and rms values
syms t
T0 = 20e-3;
TC = 1.6e-3;
% Create one period of the signal
vt = (4-6*exp(-t/TC))*(heaviside(t)-heaviside(t-0.01))...
  + (-2+6*exp(-(t-0.01)/TC))*(heaviside(t-0.01)-heaviside(t-0.02));
vavg = int(vt,t,0,T0)/T0
vrms = sqrt(int(vt^2,t,0,T0)/T0)
vrms_num = double(vrms)
```

The corresponding MATLAB output is shown below.

```
vavg = 1
vrms = (144/(25*exp(25/4)) - 72/(25*exp(25/2)) + 178/25)^(1/2)
vrms_num = 2.6704e+000
```

In summary, we have $V_{\text{avg}} = 1$ V and $V_{\text{rms}} = 2.6704$ V.

Problem 5–45. Find V_{avg} and V_{rms} of the offset sine wave $v(t) = V_0 + V_{\text{A}}\cos(2\pi t/T_0)$ in terms of V_0 and V_{A}.

Over one period, a sinusoid has an average value of zero, so the average value of the offset sine wave is just the offset, $V_{\text{avg}} = V_0$. Compute the rms value as follows, where we will apply the identity $\cos^2(\alpha) = \frac{1}{2}[1+\cos(2\alpha)]$ and make use of the fact that the average value of a sinusoid over an integer number of periods is zero.

$$\begin{aligned}
V_{\text{rms}} &= \sqrt{\frac{1}{T_0}\int_0^{T_0}[v(t)]^2\,dt} \\
&= \sqrt{\frac{1}{T_0}\int_0^{T_0}[V_0 + V_{\text{A}}\cos(2\pi t/T_0)]^2\,dt} \\
&= \sqrt{\frac{1}{T_0}\int_0^{T_0}[V_0^2 + 2V_0V_{\text{A}}\cos(2\pi t/T_0) + V_{\text{A}}^2\cos^2(2\pi t/T_0)]\,dt} \\
&= \sqrt{\frac{1}{T_0}\int_0^{T_0}\left[V_0^2 + 2V_0V_{\text{A}}\cos(2\pi t/T_0) + \frac{V_{\text{A}}^2}{2}(1+\cos(4\pi t/T_0))\right]dt} \\
&= \sqrt{\frac{1}{T_0}\int_0^{T_0}\left[V_0^2 + \frac{V_{\text{A}}^2}{2} + 2V_0V_{\text{A}}\cos(2\pi t/T_0) + \frac{V_{\text{A}}^2}{2}\cos(4\pi t/T_0)\right]dt} \\
&= \sqrt{\frac{1}{T_0}\int_0^{T_0}\left[V_0^2 + \frac{V_{\text{A}}^2}{2}\right]dt} \\
&= \sqrt{V_0^2 + \frac{V_{\text{A}}^2}{2}}
\end{aligned}$$

The following MATLAB code confirms the results.

```
syms v V0 VA Vavg Vrms t To
v = V0 + VA*cos(2*pi*t/To);
Vavg = int(v,t,0,To)/To
Vrms = simplify(sqrt(int(v^2,t,0,To)/To))
```

The corresponding MATLAB output is shown below.

```
Vavg = V0
Vrms = (V0^2 + VA^2/2)^(1/2)
```

Problem 5–46. Find $V_{\rm avg}$ and $V_{\rm rms}$ of the full-wave rectified sine wave $v(t) = V_{\rm A}\left|\sin(2\pi t/T_0)\right|$ in terms of $V_{\rm A}$.

The original sinusoid has a period of T_0, but the rectified sinusoid has a period of $T_0/2$. Compute the average value.

$$\begin{aligned}
V_{\rm avg} &= \frac{1}{T_0/2}\int_0^{T_0/2} V_{\rm A}\sin(2\pi t/T_0)\,dt \\
&= \left(\frac{2V_{\rm A}}{T_0}\right)\left(-\frac{T_0}{2\pi}\right)\cos(2\pi t/T_0)\Big|_0^{T_0/2} \\
&= -\frac{V_{\rm A}}{\pi}\left[\cos\left(\frac{2\pi T_0}{2T_0}\right) - 1\right] \\
&= \frac{V_{\rm A}}{\pi}\left[1-\cos(\pi)\right] \\
&= \frac{2V_{\rm A}}{\pi}
\end{aligned}$$

Compute the rms value.

$$\begin{aligned}
V_{\rm rms} &= \sqrt{\frac{1}{T_0/2}\int_0^{T_0/2}\left[V_{\rm A}\sin(2\pi t/T_0)\right]^2\,dt} \\
&= \sqrt{\frac{2V_{\rm A}^2}{T_0}\int_0^{T_0/2}\left[\sin^2(2\pi t/T_0)\right]\,dt} \\
&= \sqrt{\frac{2V_{\rm A}^2}{T_0}\int_0^{T_0/2}\left[\frac{1}{2}-\frac{1}{2}\cos(4\pi t/T_0)\right]\,dt} \\
&= \sqrt{\frac{2V_{\rm A}^2}{T_0}\int_0^{T_0/2}\left[\frac{1}{2}\right]\,dt} \\
&= \sqrt{\frac{V_{\rm A}^2}{T_0}\int_0^{T_0/2}(1)\,dt} \\
&= \sqrt{\left(\frac{V_{\rm A}^2}{T_0}\right)\left(\frac{T_0}{2}\right)} \\
&= \frac{V_{\rm A}}{\sqrt{2}}
\end{aligned}$$

The following MATLAB code confirms the results.

```
% Compute the average and rms values
syms t real
```

```
syms VA T0 positive
vt = VA*sin(2*pi*t/T0);
vavg = int(vt,t,0,T0/2)/(T0/2)
vrms = sqrt(int(vt^2,t,0,T0/2)/(T0/2))
```

The corresponding MATLAB output is shown below.

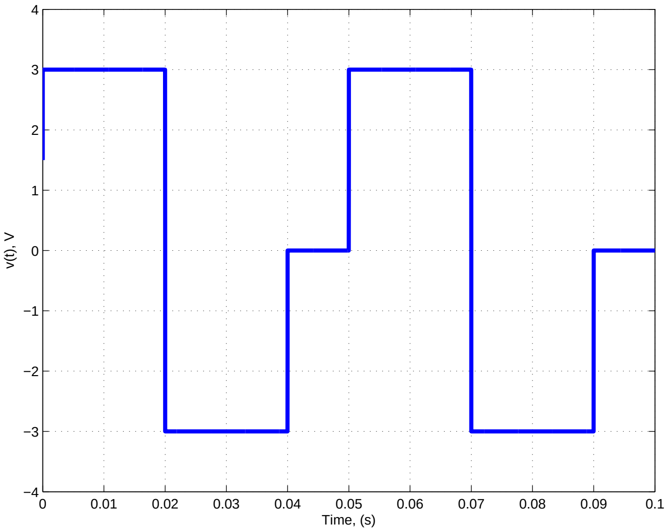
```
vavg = (2*VA)/pi
vrms = (2^(1/2)*VA)/2
```

Problem 5–47. The first cycle ($t > 0$) of a periodic waveform with $T_0 = 50$ ms can be expressed as

$$v(t) = 3u(t) - 6u(t - 0.02) + 3u(t - 0.04)\text{ V}$$

Sketch the waveform and find V_{MAX}, V_{MIN}, V_{p}, V_{pp}, and V_{avg}.

The waveform is sketched below.



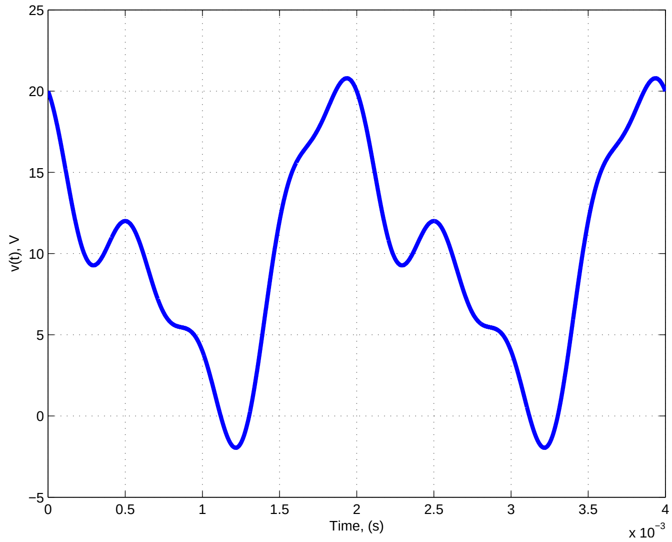
From the plot, we have $V_{\text{MAX}} = 3$ V, $V_{\text{MIN}} = -3$ V, $V_{\text{p}} = 3$ V, and $V_{\text{pp}} = 6$ V. The waveform has equal area above and below the horizontal axis, so the average value is $V_{\text{avg}} = 0$ V.

Problem 5–48. A periodic waveform can be expressed as

$$v(t) = 10 + 8\cos(1000\pi t) - 4\sin(2000\pi t) + 2\cos(4000\pi t)\text{ V}$$

What is the period of the waveform? What is the average value of the waveform? What is the amplitude of the fundamental (lowest frequency) component? What is the highest frequency in the waveform?

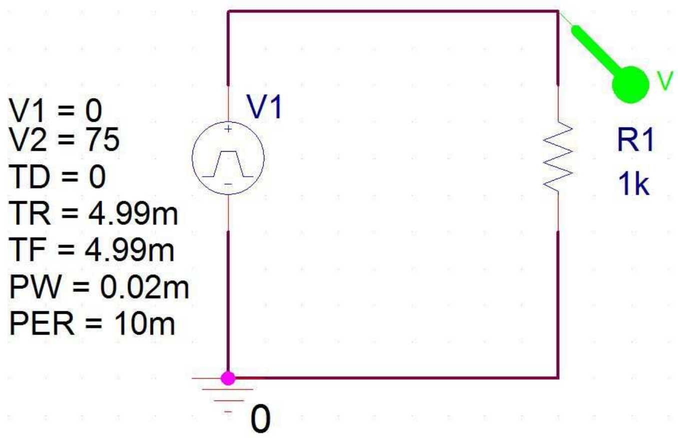
Plot the waveform to simplify the analysis.

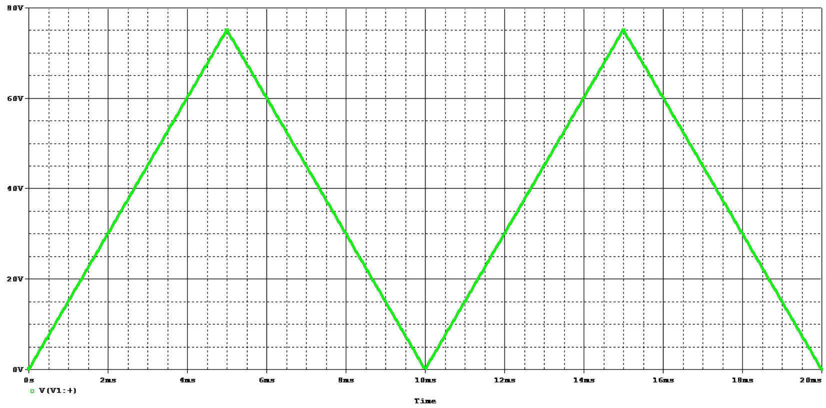


The waveform is periodic with a period equal to the period of the lowest frequency component, $T_0 = 1/500 = 2$ ms. The sinusoidal components all have frequencies that are integer multiples of the lowest frequency $f_0 = 500$ Hz, so the average values of the sinusoidal components are all zero. Therefore, the average value for the waveform is the offset, $V_{\text{avg}} = 10$ V. The amplitude of the fundamental component is $V_1 = 8$ V. The highest frequency in the waveform is $f_4 = 2000$ Hz.

Problem 5–49. Using OrCAD, create a triangular wave that has an amplitude of 75 V and a period of 10 ms. (*Hint:* The pulse width can be very small but not zero. See also Web Appendix C for use of OrCAD.)

In OrCAD, use a pulse that has a long rise time, a very short duration, and a long fall time. The following OrCAD simulation and the resulting plot provide the solution.





Problem 5–50. Consider the waveforms described in the cited problems and determine for each if they are causal or non-causal.

(a). Problem 5–24: The waveform is non-causal because it persists for all time.

(b). Problem 5–28: In part (a), the waveform is non-causal because the average value persists for all time. In part (b), the waveform is causal because of the step function.

(c). Problem 5–39: All of the waveforms are non-causal because they persists for all time.

(d). Problem 5–44: As displayed, the signal is causal because it starts at $t = 0$ s.

(e). Problem 5–48: The waveform is non-causal because it persists for all time.

Problem 5–51. (A) Gated Function

Some radars use a modulated pulse to determine range and target information. A gated modulated pulse is shown in Figure P5–51. Determine an expression for the waveform.

The waveform is a sinusoid multiplied by a pulse. The amplitude of the sinusoid is 30 V. The signal complete 12 cycles in 5 s, so the period is $T_0 = 416.67$ ms and the frequency is $f_0 = 1/T_0 = 2.4$ Hz. A peak occurs at $t = 0$, so there is no phase shift $\phi = 0°$. The sinusoid is active from $t = -2$ s to $t = 3$s. We can write the following expression for the waveform.

$$v(t) = 30\cos(4.8\pi t)\,[u(t+2) - u(t-3)] \text{ V}$$

Problem 5–52. (A) Exponential Signal Descriptors

Several of the time descriptors used in digital data communication systems are based on exponential signals. In this problem we explore three of these descriptors.

(a). The *time constant of fall* is defined as the time required for a pulse to fall from 70.7% to 26.0% of its maximum value. Assuming that the pulse decreases as e^{-t/T_C}, find the relationship between the time constant of fall and the time constant of the exponential decay.

Find the two times in terms of the time constant and subtract.

$$0.707 = e^{-t_1/T_\mathrm{C}}$$

$$\ln(0.707) = -\frac{t_1}{T_\mathrm{C}}$$

$$t_1 = (-T_\mathrm{C})[\ln(0.707)] = 0.346725T_\mathrm{C}$$

$$0.26 = e^{-t_2/T_\mathrm{C}}$$

$$\ln(0.26) = -\frac{t_2}{T_\mathrm{C}}$$

$$t_2 = (-T_\mathrm{C})[\ln(0.26)] = 1.34707T_\mathrm{C}$$

$$T_\mathrm{F} = t_2 - t_1 = 1.34707T_\mathrm{C} - 0.346725T_\mathrm{C} = T_\mathrm{C}$$

(b). The *rise time* of a pulse is the time required for a pulse to rise from 10% to 90% of its maximum value. Assuming the pulse increases as $1 - e^{-t/T_\mathrm{C}}$, find the relationship between rise time and the time constant of the exponential rise.

Find the two times in terms of the time constant and subtract.

$$0.1 = 1 - e^{-t_1/T_\mathrm{C}}$$

$$e^{-t_1/T_\mathrm{C}} = 0.9$$

$$-\frac{t_1}{T_\mathrm{C}} = \ln(0.9)$$

$$t_1 = (-T_\mathrm{C})[\ln(0.9)] = 0.105361T_\mathrm{C}$$

$$0.9 = 1 - e^{-t_2/T_\mathrm{C}}$$

$$e^{-t_2/T_\mathrm{C}} = 0.1$$

$$-\frac{t_2}{T_\mathrm{C}} = \ln(0.1)$$

$$t_2 = (-T_\mathrm{C})[\ln(0.1)] = 2.30259T_\mathrm{C}$$

$$T_\mathrm{R} = t_2 - t_1 = 2.30259T_\mathrm{C} - 0.105361T_\mathrm{C} = 2.19722T_\mathrm{C}$$

(c). The *leading-edge pulse time* is defined as the time at which a pulse rises to 50% of its maximum value. Assuming the pulse increases as $1 - e^{-t/T_\mathrm{C}}$, find the relationship between leading-edge pulse time and the time constant of the exponential rise.

Find the requested time directly

$$0.5 = 1 - e^{-t_1/T_C}$$

$$e^{-t_1/T_C} = 0.5$$

$$-\frac{t_1}{T_C} = \ln(0.5)$$

$$t_1 = (-T_C)[\ln(0.5)] = 0.693147T_C$$

$$T_{LE} = t_1 = 0.693147T_C$$

Problem 5–53. (A) Defibrillation Waveforms

Ventricular fibrillation is a life threatening loss of synchronous activity in the heart. To restore normal activity, a defibrillator delivers a brief but intense pulse of electrical current through the patient's chest. The pulse waveform is of interest because different waveforms may lead to different outcomes. Figure P5–53 shows a waveform known as a *biphasic truncated exponential* used in implantable defibrillators. The waveform is an exponential current whose direction of flow reverses after 4 ms and terminates after 8 ms. Write an expression for this waveform using the basic signals discussed in this chapter.

To simplify the solution, allow the rising edge of the waveform to occur at time $t = 0$. Use two points on the exponential curve to determine the time constant and then use step functions to create the waveform. Apply the decrement property to find the time constant.

$$T_C = \frac{\Delta t}{\ln\left[\dfrac{v(t)}{v(t+\Delta t)}\right]} = \frac{0.004}{\ln\left[\dfrac{35}{15}\right]} = 4.7209 \text{ ms}$$

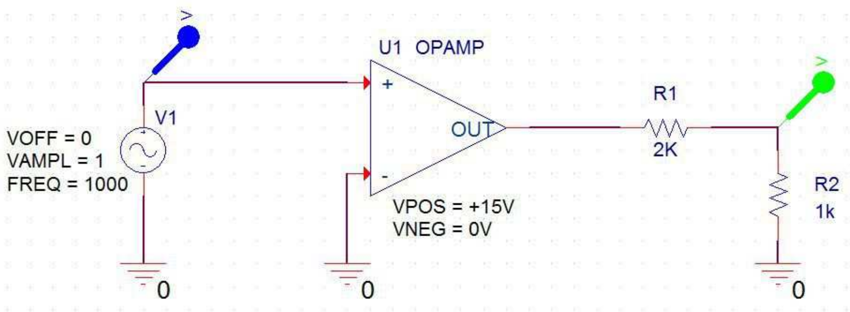
Note that $1/T_C = 211.82$ Hz. The amplitude is $V_A = 35$ A and the sign change occurs at $t = 4$ ms. The exponential is multiplied by a value of one for $0 < t < 0.004$ and is multiplied by a value of negative one for $0.004 < t < 0.008$. Use step functions to create the correct multipliers for the exponential signal. We can write the following expression for the waveform.

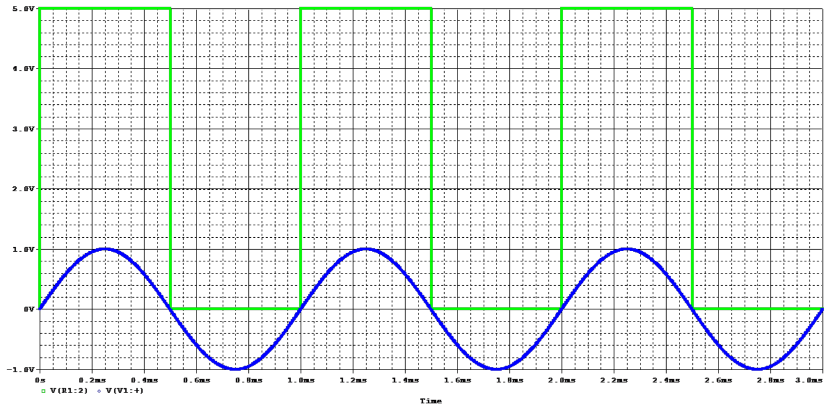
$$i(t) = 35e^{-211.82t}\left[u(t) - 2u(t-0.004) + u(t-0.008)\right] \text{ A}$$

Problem 5–54. (D) Digital Clock Generator

Timing digital circuits is vital to the operation of any digital device. Using an ideal OP AMP (running open-loop, i.e., without feedback) and appropriate resistors, design a way to convert a sinusoid into a square wave that varies from 0 to 5V with a period of 1 μs. The only non-sinusoidal supply available is $V_{CC} = \pm 15$ V.

Assume a sinusoidal voltage source with the correct period is available. Connect the voltage source to a comparator circuit and the output of the comparator to a voltage divider. For the comparator, connect the positive power input for the OP AMP to V_{CC} and the negative power input to ground. The voltage divider reduces the 15-V output down to 5 V. The required circuit and a sample output are shown below.





Problem 5–55. (A,E) Undesired Oscillations

A test is being run in a wind tunnel when a sensor on the trailing edge of a wing produces the response shown in Figure P5–55. When the sensor output reached 1 V, the test was terminated. You are asked to analyze the results. The oscillation could be tolerated if it never reaches 20 V because the on-board computer can mitigate it. However, the response time for the computer to actuate the compensating aileron is 80 ms. Will the compensation occur in time?

This problem requires an analysis of a sinusoidal signal multiplied by a growing exponential. If the on-board computer system can immediately detect the signal when it starts, then it will have 80 ms to apply the compensation. Therefore, we need to determine if the signal will reach 20 V in less than 80 ms. Examine the envelope of the signal and apply the decrement property to determine the time constant. One peak occurs at $t_1 = 0.038$ s with a magnitude of $v_1 = 0.03$ V. Another peak occurs at $t_2 = 0.057$ s with a magnitude of $v_2 = 0.49$ V.

$$T_C = \frac{t_2 - t_1}{\ln\left[\dfrac{v_2}{v_1}\right]} = \frac{0.057 - 0.038}{\ln\left[\dfrac{0.49}{0.03}\right]} = 6.8022 \text{ ms}$$

Note that $1/T_C = 147$ Hz. Estimate the amplitude of the exponential and then the amplitude of the signal at 80 ms.

$$v(t) = V_A e^{t/T_C}$$

$$v(0.057) = 0.49 = V_A e^{147(0.057)}$$

$$V_A = 112.5\ \mu\text{V}$$

$$v(0.08) = (0.0001125)e^{147(0.08)} = 14.4 \text{ V}$$

$$v(0.08223) = (0.0001125)e^{147(0.08223)} = 20.0 \text{ V}$$

The signal will reach 14.4 V at 80 ms and 20 V at 82.23 ms. The system will be able to compensate for the unstable condition if the system can detect the condition within 2 ms of it occurring. Since this is a flight critical system, a more detailed analysis, with access to the data used to create the waveform, would be appropriate to ensure the results are accurate.

Problem 5–56. (V) Voltmeter Calibration

Most dc voltmeters measure the average value of the applied signal. A dc meter that measures the average value can be adapted to indicate the rms value of an ac signal. The input is passed through a rectifier circuit. The rectifier output is the absolute value of the input and is applied to a dc meter whose deflection is proportional to the average value of the rectified signal. The meter scale is calibrated to indicate the rms value of the input signal. A calibration factor is needed to convert the average absolute value into the rms value of the ac signal. What is the required calibration factor for a sinusoid? Would the same calibration factor apply to a square wave?

In Problem 5–46, we found the average and rms values for a rectified sinusoid as follows:

$$V_{\rm avg} = \frac{2V_{\rm A}}{\pi}$$

$$V_{\rm rms} = \frac{V_{\rm A}}{\sqrt{2}}$$

We need to find K such that $KV_{\rm avg} = V_{\rm rms}$.

$$K = \frac{V_{\rm rms}}{V_{\rm avg}} = \frac{\dfrac{V_{\rm A}}{\sqrt{2}}}{\dfrac{2V_{\rm A}}{\pi}} = \frac{\pi}{2\sqrt{2}} = \frac{\sqrt{2}\pi}{4}$$

The calibration factor is $K = \sqrt{2}\pi/4$. The same calibration factor would not apply to a square wave, because the square wave's rms value is different than the sinusoid's rms value.

Problem 5–57. (A) MATLAB Signal Analyzer

Create a MATLAB function to analyze signals represented numerically. The function should have the following two inputs: (1) a vector containing equally spaced samples of the signal of interest and (2) the time step used to sample the signal contained in the vector. The function should display the following descriptors of the signal: $V_{\rm MAX}$, $V_{\rm MIN}$, $V_{\rm p}$, $V_{\rm pp}$, $V_{\rm avg}$, and $V_{\rm rms}$. The function should also plot the waveform assuming the signal starts at $t = 0$.

The following MATLAB code provides the solution:

```
function a = analyze(x,dt)
%
% Given samples of a waveform in variable x with time step dt
% between samples, compute the following partial waveform
% descriptors:  Vmax, Vmin, Vp, Vpp, Vavg, and Vrms.
% The calculations assume the waveform samples cover an integer
% number of periods, if the signal is periodic.
% Plot the signal starting at t=0.

% Find the maximum and minimum values
Vmax = max(x)
Vmin = min(x)

% Find the peak and peak-to-peak values
Vp = max(abs(Vmax),abs(Vmin))
Vpp = Vmax-Vmin

% Compute the average value over the entire signal
n = length(x);
T = (n-1)*dt;
Tf = n*dt;
t = 0:dt:T;
% Use a numerical integral to find the average value
Vavg = sum(dt*x)/Tf

% Use a numerical integral to find the rms value
Vrms = sqrt(dt*sum(x.^2)/Tf)

% Plot the results
figure
plot(t,x,'b','LineWidth',3)
grid on
xlabel('Time (s)')
ylabel('Signal')
```

6 Capacitance and Inductance

6.1 Exercise Solutions

Exercise 6–1. A 1-μF capacitor has no voltage across it at $t = 0$. A current flowing through the capacitor is given as $i_C = 2u(t) - 3u(t-2) + u(t-4)\mu$A. Find the voltage across the capacitor at $t = 4$ s.

Apply the capacitor i-v relationship to find the voltage.

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i_C(x)\,dx$$

$$v_C(4) = 0 + \frac{1}{10^{-6}}\int_0^4 i_C(x)\,dx$$

$$v_C(4) = 10^6\left[\int_0^2 (2\times 10^{-6})\,dx - \int_2^4 (1\times 10^{-6})\,dx\right]$$

$$v_C(4) = [(2)(2-0) - (1)(4-2)]$$

$$v_C(4) = 2 \text{ V}$$

Exercise 6–2.

(a). The voltage across a 10-μF capacitor is $25[\sin(2000t)]u(t)$ V. Derive an expression for the current through the capacitor.

Apply the capacitor i-v relationship to find the voltage.

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

$$i_C(t) = 10\times 10^{-6}\frac{d}{dt}[25\sin(2000t)] = (10\times 10^{-6})(2000)(25)\cos(2000t)$$

$$i_C(t) = 500\cos(2000t) \text{ mA}, \quad t > 0$$

(b). At $t = 0$ the voltage across a 100-pF capacitor is -5 V. The current through the capacitor is $10[u(t) - u(t-10^{-4})]$ μA. What is the voltage across the capacitor for $t > 0$?

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i_C(x)\,dx$$

$$v_C(t) = -5 + \frac{1}{100\times 10^{-12}}\int_0^t (10\times 10^{-6})[u(t) - u(t-10^{-4})]\,dx$$

$$v_C(t) = -5 + 10^5 t \text{ V}, \quad \text{for } 0 < t < 10^{-4}$$

$$v_C(t) = -5 + (10^5)(10^{-4}) = -5 + 10 = 5 \text{ V}, \quad \text{for } t \geq 10^{-4}$$

Exercise 6–3. For $t \geq 0$ the voltage across a 200-pF capacitor is $5e^{-4000t}$ V.

(a). What is the charge on the capacitor at $t = 0$ and $t = +\infty$?

Find the voltage at the given times and then compute the charge.

$$q(t) = Cv_C(t)$$

$$q(0) = Cv_C(0) = (200 \times 10^{-12})(5)(e^0) = 1 \text{ nC}$$

$$q(\infty) = Cv_C(0) = (200 \times 10^{-12})(5)(e^{-\infty}) = 0 \text{ C}$$

(b). Derive an expression for the current through the capacitor for $t \geq 0$.

The current through a capacitor is the capacitance times the derivative of the voltage.

$$i_C(t) = C\frac{dv_C(t)}{dt} = (200 \times 10^{-12})\frac{d}{dt}\left(5e^{-4000t}\right) = (200 \times 10^{-12})(5)(-4000)e^{-4000t} = -4e^{-4000t} \ \mu\text{A}$$

(c). For $t > 0$ is the device absorbing or delivering power?

Compute the power and examine the sign.

$$p_C(t) = i_C(t)v_C(t) = \left(-4e^{-4000t} \ \mu\text{A}\right)\left(5e^{-4000t} \text{ V}\right) = -20e^{-8000t} \ \mu\text{W}$$

The power is always negative, so the capacitor is always delivering power.

Exercise 6–4. Find the power and energy for the capacitors in Exercise 6–2.

(a). Compute the power.

$$p_C(t) = i_C(t)v_C(t) = [0.5\cos(2000t)]\,[25\sin(2000t)u(t)]$$

$$= 12.5\cos(2000t)\sin(2000t)\,u(t) \text{ W} = 6.25\sin(4000t)\,u(t) \text{ W}$$

Compute the energy.

$$w_C(t) = \frac{1}{2}Cv_C^2(t) = \frac{1}{2}(10^{-5})\,[25\sin(2000t)u(t)]^2 = 3.125\sin^2(2000t)\,u(t) \text{ mJ}$$

(b). Compute the power for the two time intervals.

$$p_C(t) = i_C(t)v_C(t) = (10^{-5})(-5 + 10^5 t) = -50 \times 10^{-6} + t \text{ W} \ \text{ for } \ 0 < t < 0.1 \text{ ms}$$

$$p_C(t) = (0)(5) = 0 \text{ W} \ \text{ for } \ t \geq 0.1 \text{ ms}$$

Compute the energy for the two time intervals.

$$w_C(t) = \frac{1}{2}Cv_C^2(t) = \frac{1}{2}(10^{-10})(25 - 10^6 t + 10^{10}t^2)$$

$$= 1.25 - (5 \times 10^4)t + (5 \times 10^8)t^2 \text{ nJ} \ \text{ for } \ 0 < t < 0.1 \text{ ms}$$

$$w_C(t) = \frac{1}{2}(10^{-10})(5)^2 = 1.25 \text{ nJ} \ \text{ for } \ t \geq 0.1 \text{ ms}$$

Exercise 6–5. Find the power and energy for the capacitor in Exercise 6–3.

Compute the power.

$$p_C(t) = i_C(t)v_C(t) = \left[-4e^{-4000t} \ \mu\text{A}\right]\left[5e^{-4000t} \text{ V}\right] = -20e^{-8000t} \ \mu\text{W}$$

Compute the energy.

$$w_C(t) = \frac{1}{2}Cv_C^2(t) = \frac{1}{2}(200 \times 10^{-12})\left[5e^{-4000t}\right]^2 = 2.5e^{-8000t} \text{ nJ}$$

Exercise 6–6. For $t > 0$, the voltage across a 4-mH inductor is $v_{\text{L}}(t) = 20e^{-2000t}$ V. The initial current is $i_{\text{L}}(0) = 0$.

(a). What is the current through the inductor for $t > 0$?

Compute the current.

$$i_{\text{L}}(t) = i_{\text{L}}(0) + \frac{1}{L}\int_0^t v_{\text{L}}(x)\,dx = 0 + \frac{1}{0.004}\int_0^t 20e^{-2000x}\,dx$$
$$= -\frac{(250)(20)}{2000}(e^{-2000t} - 1) = 2.5(1 - e^{-2000t}) \text{ A}$$

(b). What is the power for $t > 0$?

Compute the power.

$$p_{\text{L}}(t) = i_{\text{L}}(t)v_{\text{L}}(t) = \left[2.5(1 - e^{-2000t})\right]\left[20e^{-2000t}\right] = 50(e^{-2000t} - e^{-4000t}) \text{ W}$$

(c). What is the energy for $t > 0$?

Compute the energy.

$$w_{\text{L}}(t) = \frac{1}{2}Li_{\text{L}}^2(t) = \frac{1}{2}(0.004)\left[2.5(1 - e^{-2000t})\right]^2 = 12.5\left(1 - 2e^{-2000t} + e^{-4000t}\right) \text{ mJ}$$

Exercise 6–7. For $t < 0$, the current through a 100-mH inductor is zero. For $t \geq 0$, the current is $i_{\text{L}}(t) = 20e^{-2000t} - 20e^{-4000t}$ mA.

(a). Derive an expression for the voltage across the inductor for $t > 0$.

Compute the voltage.

$$v_{\text{L}}(t) = L\frac{di_{\text{L}}(t)}{dt} = (0.1)(0.02)(-2000e^{-2000t} + 4000e^{-4000t}) = -4e^{-2000t} + 8e^{-4000t} \text{ V}$$

(b). Find the time $t > 0$ at which the inductor voltage passes through zero.

Compute the time.

$$v_{\text{L}}(t) = 0 = -4e^{-2000t} + 8e^{-4000t}$$
$$4e^{-2000t} = 8e^{-4000t}$$
$$e^{2000t} = 2$$
$$2000t = \ln(2)$$
$$t = \frac{\ln(2)}{2000} = 346.6\ \mu\text{s}$$

(c). Derive an expression for the inductor power for $t > 0$.

Compute the power.

$$\begin{aligned} p_{\mathrm{L}}(t) = i_{\mathrm{L}}(t)v_{\mathrm{L}}(t) &= (0.02)\left(e^{-2000t} - e^{-4000t}\right)\left(-4e^{-2000t} + 8e^{-4000t}\right) \\ &= -0.08e^{-4000t} + 0.16e^{-6000t} + 0.08e^{-6000t} - 0.16e^{-8000t} \\ &= -80e^{-4000t} + 240e^{-6000t} - 160e^{-8000t} \text{ mW} \end{aligned}$$

(d). Find the time interval over which the inductor absorbs power and the interval over which it delivers power.

The current is always positive. The voltage is positive on $0 < t < 346.6$ μs and negative for $t > 346.6$ μs. Therefore, the power is positive on $0 < t < 346.6$ μs and the inductor is absorbing power, while the power is negative for $t > 346.6$ μs and the inductor is delivering power.

Exercise 6–8. A 50-mH inductor has an initial current of $i_{\mathrm{L}}(0) = 0$ A. The following voltage is applied across the inductor starting at $t = 0$:

$$v_{\mathrm{L}}(t) = -\frac{5}{\pi}\sin(400\pi t) - \frac{5}{2\pi}\sin(800\pi t) - \frac{5}{3\pi}\sin(1200\pi t) \text{ V}$$

For $t \geq 0$, use MATLAB to determine the inductor current, power, and energy. Plot those three signals and the inductor voltage for $0 \leq t \leq 10$ ms.

The following MATLAB code provides the solution:

```
% Create symbolic results
syms t x real
L = 50e-3;
iL0 = 0;
vL = -5*sin(400*pi*t)/pi - 5*sin(800*pi*t)/2/pi - 5*sin(1200*pi*t)/3/pi
vLx = subs(vL,t,x);
iL = iL0 + int(vLx,x,0,t)/L
pL = simplify(iL*vL)
wL = simplify(L*iL^2/2)
```

The corresponding MATLAB output is shown below.

```
vL = - (5*sin(400*pi*t))/pi - (5*sin(800*pi*t))/(2*pi) - (5*sin(1200*pi*t))/(3*pi)
iL = -(36*sin(200*pi*t)^2 + 9*sin(400*pi*t)^2 + 4*sin(600*pi*t)^2)/(72*pi^2)
pL = (5*(6*sin(400*pi*t) + 3*sin(800*pi*t) + 2*sin(1200*pi*t))*
     (36*sin(200*pi*t)^2 + 9*sin(400*pi*t)^2 + 4*sin(600*pi*t)^2))/(432*pi^3)
wL = (36*sin(200*pi*t)^2 + 9*sin(400*pi*t)^2 + 4*sin(600*pi*t)^2)^2/(207360*pi^4)
```

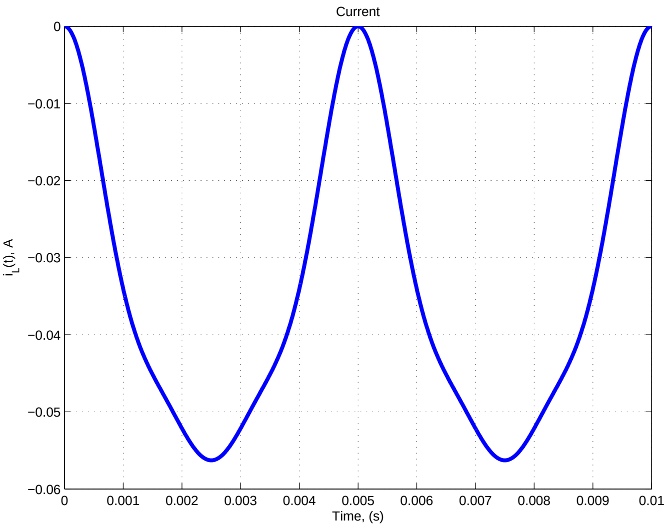
The results are as follows:

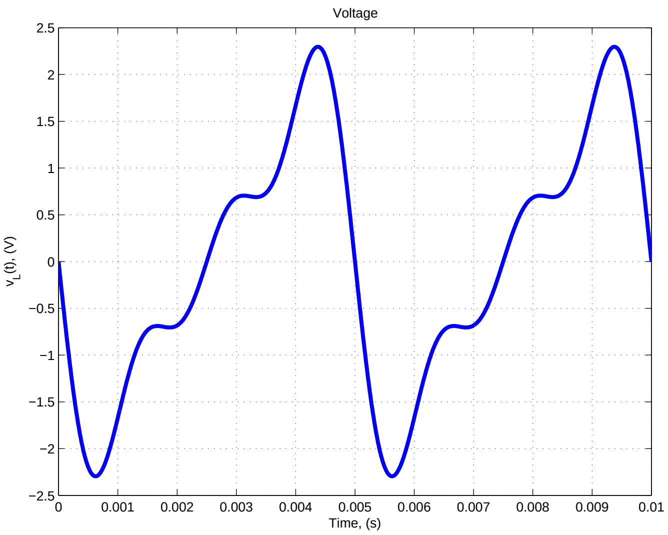
$$i_{\mathrm{L}}(t) = -\frac{1}{72\pi^2}\left[36\sin^2(200\pi t) + 9\sin^2(400\pi t) + 4\sin^2(600\pi t)\right]$$

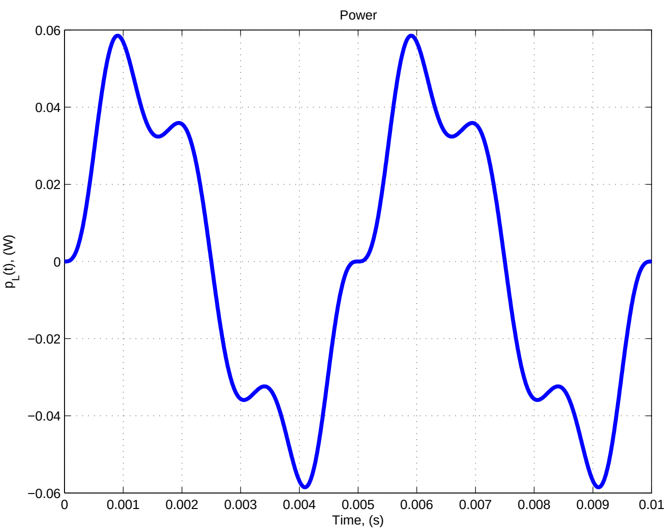
$$\begin{aligned} p_{\mathrm{L}}(t) = \frac{1}{432\pi^3}\Big\{ & 5\left[6\sin(400\pi t) + 3\sin(800\pi t) + 2\sin(1200\pi t)\right] \\ & \left[36\sin^2(200\pi t) + 9\sin^2(400\pi t) + 4\sin^2(600\pi t)\right]\Big\} \end{aligned}$$

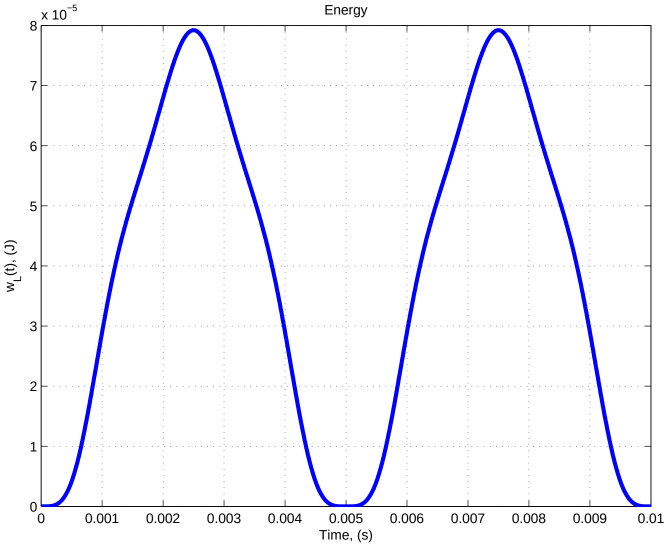
$$w_{\mathrm{L}}(t) = \frac{1}{207360\pi^4}\left[36\sin^2(200\pi t) + 9\sin^2(400\pi t) + 4\sin^2(600\pi t)\right]^2$$

The corresponding plots are shown below.


Current
$i_L(t)$, A
Time, (s)


Voltage
$v_L(t)$, (V)
Time, (s)


Power
$p_L(t)$, (W)
Time, (s)



Exercise 6–9. The input to the circuit in Figure 6–18 is $v_{\mathrm{S}}(t) = 10e^{-5t}u(t)$ V.

(a). For $v_{\mathrm{C}}(0) = 0$, derive an expression for the output voltage, assuming the OP AMP is in its linear range.

The circuit is an inverting integrator.

$$
\begin{aligned}
v_{\mathrm{O}}(t) &= v_{\mathrm{O}}(0) - \frac{1}{RC}\int_0^t v_{\mathrm{S}}(x)\,dx \\
&= 0 - \frac{1}{(10^6)(10^{-6})}\int_0^t 10e^{-5x}\,dx \\
&= \frac{-10}{-5}e^{-5x}\bigg|_0^t = 2\left(e^{-5t} - 1\right)u(t) \text{ V}
\end{aligned}
$$

(b). Does the OP AMP saturate with the given input?

The output voltage for the OP AMP has an initial value of 0 V and a final value of -2 V. The OP AMP will not saturate for $V_{\mathrm{CC}} \geq 2$ V.

Exercise 6–10. The input to the circuit in Figure 6–19(a) is $v_{\mathrm{S}}(t) = V_{\mathrm{A}}\cos(2000t)$. The OP AMP saturates when $v_{\mathrm{O}}(t) = \pm 15$ V.

(a). Derive an expression for the output, assuming that the OP AMP is in the linear mode.

The circuit is an inverting differentiator.

$$
\begin{aligned}
v_{\mathrm{O}}(t) &= -RC\frac{dv_{\mathrm{S}}(t)}{dt} \\
&= -(10^4)(10^{-7})\frac{d}{dt}\left[V_{\mathrm{A}}\cos(2000t)\right] \\
&= (-10^{-3})(-2000)V_{\mathrm{A}}\sin(2000t) = 2V_{\mathrm{A}}\sin(2000t) \text{ V}
\end{aligned}
$$

(b). What is the maximum value of VA for linear operation?

For linear operation, we require $|2V_{\mathrm{A}}| \leq V_{\mathrm{CC}}$, so $|V_{\mathrm{A}}| \leq 7.5$ V.

Exercise 6–11. Find the input-output relationship of the circuit in Figure 6–21.

The circuit contains two stages. The first stage is a standard subtractor circuit with all resistors equal, so both inputs see unity gain and the output of the first stages is $v_{O1}(t) = v_{S2}(t) - v_{S1}(t)$. The second stage is an inverting integrator operating on the output of the first stage. Combining the results, we have the following expression for the output.

$$v_O(t) = v_O(0) + \frac{1}{RC}\int_0^t [v_{S1}(x) - v_{S2}(x)]\, dx$$

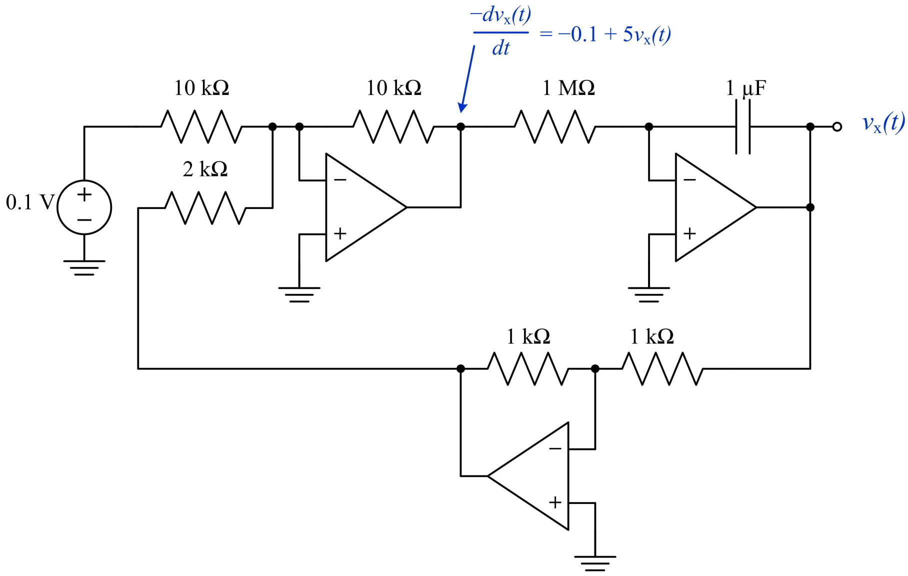
Exercise 6–12. Design a circuit to solve the following differential equation:

$$10\frac{dv_x(t)}{dt} + 50v_x(t) = 1 \text{ V}$$

Solve the given expression for the derivative.

$$\frac{dv_x(t)}{dt} = \frac{1}{10} - 5v_x(t)$$

Let $v_x(t)$ be the output of an inverting integrator with a gain of one. The input to that integrator is therefore the negative of the derivative $-dv_x(t)/dt$. Design a summing circuit that yields the negative of the derivative $-0.1 + 5v_x(t)$ and connect its output to the inverting integrator. One possible solution is shown belown.



Exercise 6–13.

(a). A 150-μH inductor is in parallel with two other, identical inductors. That combination is in series with a 100-μH inductor. What is the equivalent inductance of the connection?

Combine the inductors in parallel and add the result to series inductor.

$$L_{EQ} = 100 + \frac{1}{\frac{1}{150} + \frac{1}{150} + \frac{1}{150}} = 100 + \frac{150}{3} = 150\ \mu\text{H}$$

(b). A capacitor bank consists of five hundred 400-VDC, 10-mF capacitors in parallel. How much energy can the bank store when the capacitors are fully charged?

Find the equivalent capacitance and then the energy stored.

$$C_{EQ} = (500)(0.01) = 5 \text{ F}$$

$$w_{C,MAX} = \frac{1}{2}Cv_{MAX}^2 = \frac{1}{2}(5)(400)^2 = 400 \text{ kJ}$$

Exercise 6–14. Find the equivalent capacitance and the initial stored voltage for the circuit in Figure 6–29.

Combine the 0.1 μF and 0.022 μF capacitors in series, combine that result in parallel with the 0.15 μF capacitor, and combine that result in series with the 0.033 μF capacitor.

$$C_{\mathrm{EQ}} = \frac{1}{\left[\dfrac{1}{\left(\dfrac{1}{\dfrac{1}{0.1}+\dfrac{1}{0.022}}\right)+0.15}\right]+\dfrac{1}{0.033}} = 0.027583\ \mu\mathrm{F}$$

The initial stored voltage is the sum of the voltage drops through either path through the circuit, $v_{\mathrm{C}}(0) = 20 + 15 = 35$ V.

Exercise 6–15. The current through a series connection of two 1-μF capacitors is a rectangular pulse with an amplitude of 2 mA and a duration of 10 ms. At $t = 0$ the voltage across the first capacitor is +10 V and across the second is zero.

(a). What is the voltage across the series combination at $t = 10$ ms?

Combine the capacitors in series to find their equivalent capacitance.

$$C_{\mathrm{EQ}} = \frac{1}{\frac{1}{1}+\frac{1}{1}} = 0.5\ \mu\mathrm{F}$$

The initial voltage across the series combination is $v_{\mathrm{C}}(0) = 10 + 0 = 10$ V. Compute the expression for the voltage across the capacitors.

$$v_{\mathrm{C}}(t) = v_{\mathrm{C}}(0) + \frac{1}{C}\int_0^t i_{\mathrm{C}}(x)\,dx$$

$$v_{\mathrm{C}}(0.01) = 10 + \frac{1}{0.5\times 10^{-6}}\int_0^{0.01}(0.002)\,dx$$

$$= 10 + \frac{1}{0.5\times 10^{-6}}(0.002)(0.01) = 10 + 40 = 50\ \mathrm{V}$$

(b). What is the maximum instantaneous power delivered to the series combination?

The power is the product of the current and voltage. Since the current is constant, the maximum power occurs when the voltage is at its maximum value at $t = 10$ ms, $p_{\mathrm{C,MAX}} = (i_{\mathrm{C}})(v_{\mathrm{C,MAX}}) = (0.002)(50) = 100$ mW.

(c). What is the energy stored on the first capacitor at $t = 0$ and $t = 10$ ms?

Compute the energy stored at the two times. Note that the voltage increase across the series combination is split equally between the two capacitors, since they have the same value for capacitance. Therefore, the voltage across the first capacitor at $t = 10$ ms is $v_{\mathrm{C1}}(0.01) = 10 + 20 = 30$ V.

$$w_{\mathrm{C}}(t) = \frac{1}{2}Cv_{\mathrm{C}}^2(t)$$

$$w_{\mathrm{C}}(0) = \frac{1}{2}(1\times 10^{-6})(10^2) = 50\ \mu\mathrm{J}$$

$$w_{\mathrm{C}}(0.01) = \frac{1}{2}(1\times 10^{-6})(30^2) = 450\ \mu\mathrm{J}$$

Exercise 6–16. Find the OP AMP output voltage in Figure 6–31.

The input voltage is a dc source, so the two capacitors are equivalent to open circuits. No current flows from the source into the OP AMP and the resulting circuit is configured as a standard noninverting amplifier.

$$v_{\mathrm{O}}(t) = \frac{R_1 + R_2}{R_1} v_{\mathrm{dc}}$$

6.2 Problem Solutions

Problem 6–1. For $t > 0$, the voltage across a 3.3-μF capacitor is $v_C(t) = 5u(t)$ V. Derive expressions for $i_C(t)$ and $p_C(t)$. Is the capacitor absorbing power, delivering power, or both?

The voltage is constant for $t > 0$. The current is proportional to the derivative of the voltage, so the current is zero, $i_C(t) = 0$, since the voltage is not changing. The power is the product of the current and voltage, so it is also zero, $p_C(t) = 0$. The capacitor is neither absorbing nor delivering power.

Problem 6–2. For $t > 0$, the voltage across a 0.022-μF capacitor is $v_C(t) = 10e^{-1000t}u(t)$ V. Derive expressions for $i_C(t)$ and $p_C(t)$. Is the capacitor absorbing power, delivering power, or both?

Compute the current and the power.

$$i_C(t) = C\frac{dv_C(t)}{dt} = (0.022\,\mu)\frac{d}{dt}\left(10e^{-1000t}\right) = (0.022\,\mu)(10)(-1000)e^{-1000t} = -220e^{-1000t}u(t)\ \mu\text{A}$$

$$p_C(t) = i_C(t)v_C(t) = \left(-220\,\mu\,e^{-1000t}\right)\left(10e^{-1000t}\right)u(t) = -2.2e^{-2000t}u(t)\ \text{mW}$$

The power is always negative, so the capacitor is delivering power.

Problem 6–3. The voltage across a 1000-pF capacitor is $v_C(t) = 50\cos(2\pi 10^4 t)$ V. Derive expressions for $i_C(t)$ and $p_C(t)$. Is the capacitor absorbing power, delivering power, or both?

Compute the current and the power.

$$i_C(t) = C\frac{dv_C(t)}{dt} = (1000\,\text{p})\frac{d}{dt}\left[50\cos(2\pi 10^4 t)\right] = (1000\,\text{p})(50)(-2\pi 10^4)\sin(2\pi 10^4 t) = -\pi\sin(2\pi 10^4 t)\ \text{mA}$$

$$p_C(t) = i_C(t)v_C(t) = \left[-\pi\sin(2\pi 10^4 t)\right]\left[50\cos(2\pi 10^4 t)\right]\ \text{mW} = -25\pi\sin(4\pi 10^4 t)\ \text{mW}$$

The power is both positive and negative, so the capacitor is both absorbing and delivering power.

Problem 6–4. The current through a 0.2-μF capacitor is a rectangular pulse with an amplitude of 3 mA and a duration of 5 ms. Find the capacitor voltage at the end of the pulse when the capacitor voltage at the beginning of the pulse is -1 V.

Compute the voltage.

$$\begin{aligned}
v_C(t) &= v_C(0) + \frac{1}{C}\int_0^t i_C(x)\,dx \\
&= -1 + \frac{1}{0.2\,\mu}\int_0^{0.005}(0.003)\,dx \\
&= -1 + (5\times 10^6)(0.003)(0.005-0) \\
&= -1 + 75 = 74\ \text{V}
\end{aligned}$$

Problem 6–5. For $t \geq 0$, the current through capacitor is $i_C(t) = 10r(t)$ mA. At $t = 0$ the capacitor voltage is 3 V. At $t = 1$ ms, the voltage is 8 V. Find the capacitance of the device.

Find an expression for the capacitor voltage and then solve for the capacitance.

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i_C(x)\,dx$$

$$= 3 + \frac{1}{C}\int_0^t 0.01x\,dx = 3 + \frac{1}{C}(0.005)(t^2 - 0) = 3 + \frac{0.005t^2}{C}$$

$$v_C(0.001) = 8 = 3 + \frac{0.005(0.001)^2}{C}$$

$$5 = \frac{0.000000005}{C}$$

$$C = 1000\text{ pF}$$

Problem 6–6. The voltage across a 0.1-μF capacitor is shown in Figure P6–6. Prepare sketches of $i_C(t)$, $p_C(t)$, and $w_C(t)$. Is the capacitor absorbing power, delivering power, or both?

Compute the current for the three segments of the voltage and write the expressions for the power and energy.

$$i_C(t) = C\frac{dv_C(t)}{dt} = (0.1\,\mu)\frac{50}{10\,\text{m}} = 500\,\mu\text{A}, \quad \text{for} \;\; 0 < t < 10\text{ ms}$$

$$i_C(t) = 0 \;\; \text{for} \;\; 10 < t < 30\text{ ms}$$

$$i_C(t) = (0.1\,\mu)\frac{-50}{10\,\text{m}} = -500\,\mu\text{A}, \quad \text{for} \;\; 30 < t < 40\text{ ms}$$

$$i_C(t) = 0 \;\; \text{for} \;\; 40 < t\text{ ms}$$

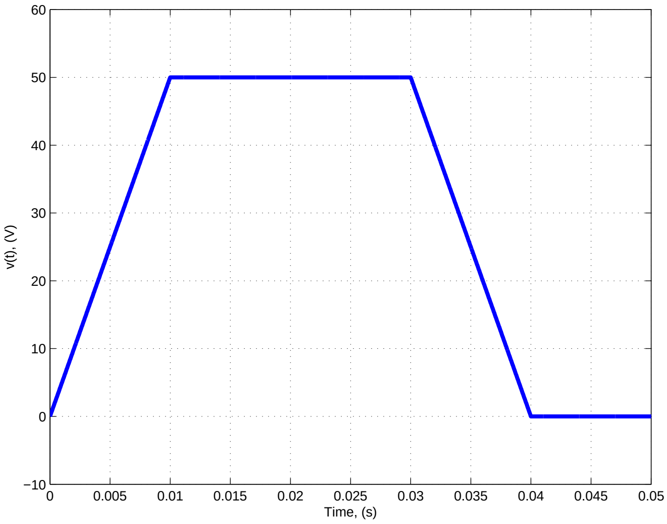
$$p_C(t) = i_C(t)v_C(t)$$

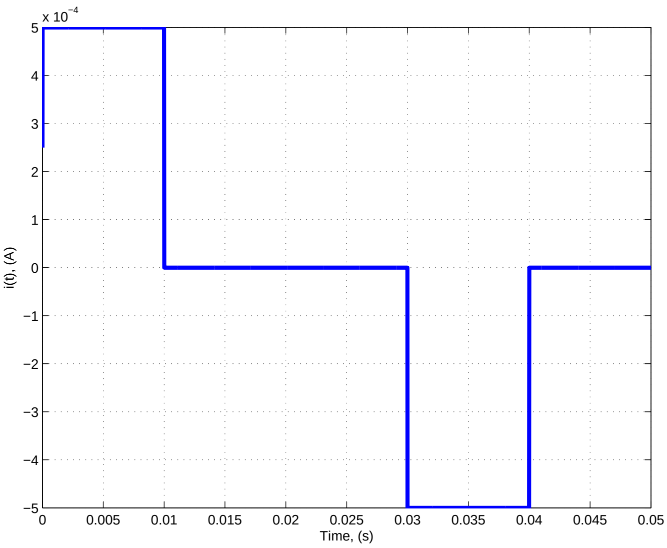
$$w_C(t) = \frac{1}{2}Cv_C^2(t)$$

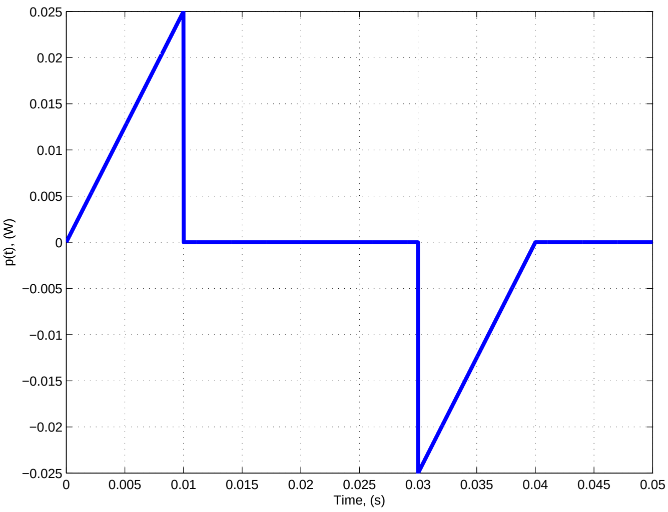
Use MATLAB to calculate the power and energy using the code shown below.

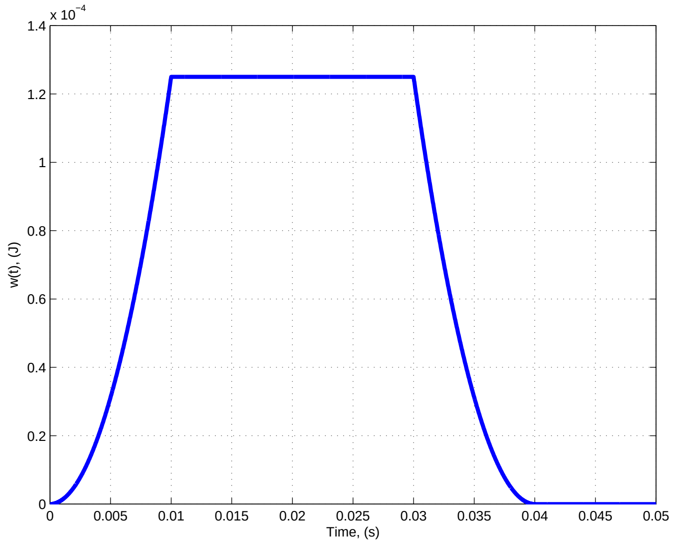
```
% Create the time vector
t = 0:10e-6:50e-3;
% Compute the slope of the waveform
m = 50/10e-3;
% Set the important times
t1 = 10e-3;
t2 = 30e-3;
t3 = 40e-3;
% Create the waveform
vt = m*t.*heaviside(t)-m*(t-t1).*heaviside(t-t1)...
    - m*(t-t2).*heaviside(t-t2) + m*(t-t3).*heaviside(t-t3);
%Compute the current, power, and energy
C = 0.1e-6;
ic1 = C*m;
ic2 = 0;
ic3 = -C*m;
it = ic1*heaviside(t)-ic1*heaviside(t-t1)+ic3*heaviside(t-t2)...
    - ic3*heaviside(t-t3);
pt = it.*vt;
wt = vt.^2*C/2;
```

The corresponding plots are shown below.


v(t), (V)
Time, (s)


x 10^−4
i(t), (A)
Time, (s)


p(t), (W)
Time, (s)



The capacitor both absorbs and delivers power.

Problem 6–7. The voltage across a 0.01-μF capacitor is shown in Figure P6–7. Prepare sketches of $i_C(t)$, $p_C(t)$, and $w_C(t)$. Is the capacitor absorbing power, delivering power, or both?

Compute the current for the three segments of the voltage and write the expressions for the power and energy.

$$i_C(t) = C\frac{dv_C(t)}{dt} = (0.01\,\mu)\frac{20}{50\,\mu} = 4 \text{ mA} \;\; \text{for} \;\; 0 < t < 50\;\mu\text{s}$$

$$i_C(t) = 0 \;\; \text{for} \;\; 50 < t < 75\;\mu\text{s}$$

$$i_C(t) = (0.01\,\mu)\frac{-20}{25\,\mu} = -8 \text{ mA}, \;\; \text{for} \;\; 75 < t < 100\;\mu\text{s}$$

$$i_C(t) = 0 \;\; \text{for} \;\; 100 < t\;\mu\text{s}$$

$$p_C(t) = i_C(t)v_C(t)$$

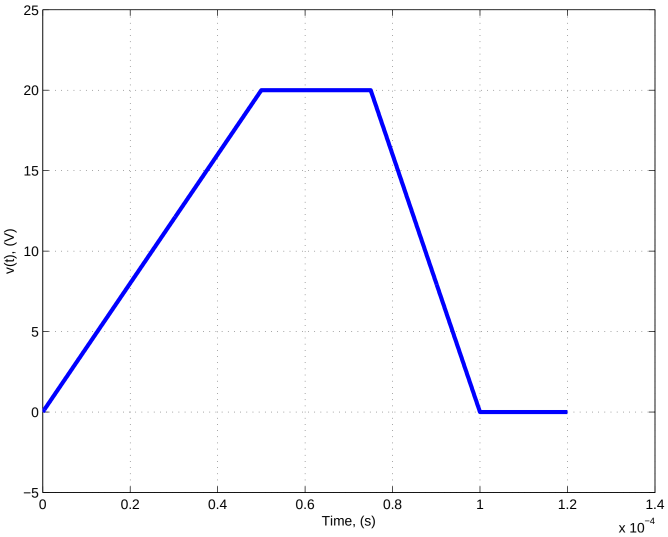
$$w_C(t) = \frac{1}{2}Cv_C^2(t)$$

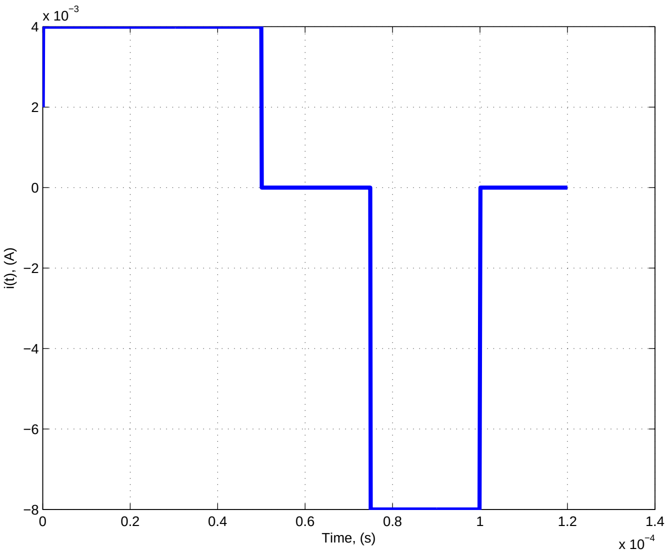
Use MATLAB to calculate the power and energy using the code shown below.

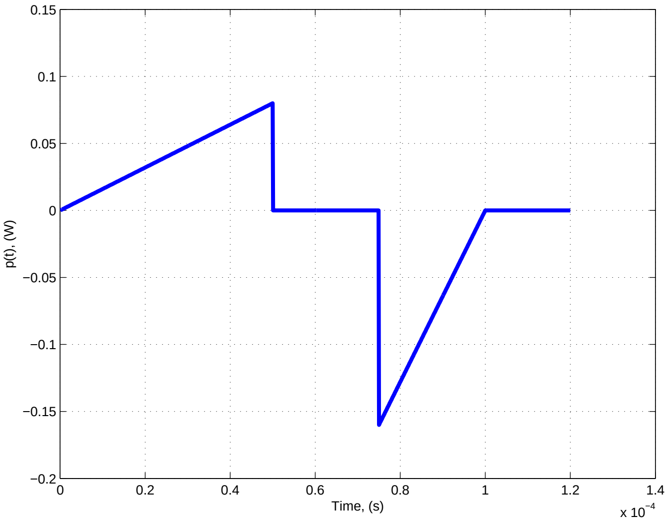
```
% Create the time vector
t = 0:1e-7:120e-6;
% Compute the slope of the waveform
m = 20/50e-6;
% Set the important times
t1 = 50e-6;
t2 = 75e-6;
t3 = 100e-6;
% Create the waveform
vt = m*t.*heaviside(t)-m*(t-t1).*heaviside(t-t1)...
    - 2*m*(t-t2).*heaviside(t-t2) + 2*m*(t-t3).*heaviside(t-t3);
%Compute the current, power, and energy
C = 0.01e-6;
ic1 = C*20/50e-6;
ic2 = 0;
ic3 = C*20/25e-6;
it = ic1*heaviside(t)-ic1*heaviside(t-t1)-ic3*heaviside(t-t2)...
```

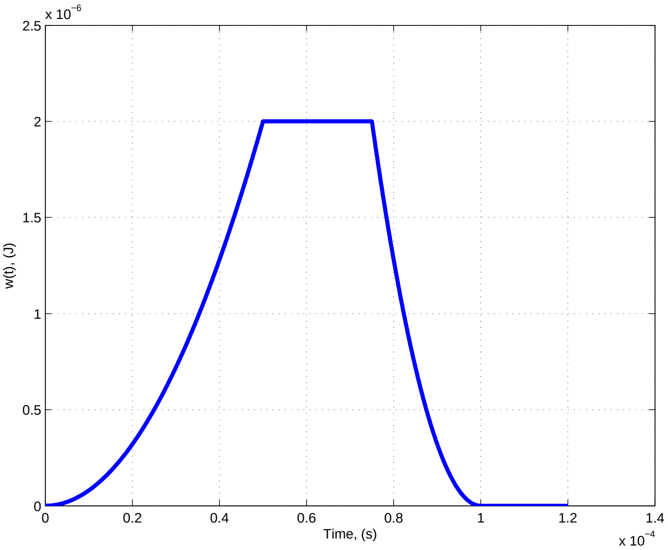
```
    + ic3*heaviside(t-t3);
pt = it.*vt;
wt = vt.^2*C/2;
```

The corresponding plots are shown below.









The capacitor both absorbs and delivers power.

Problem 6–8. The current through a 1500-pF capacitor is shown in Figure P6–8. Given that $v_{\rm C}(0) = -5$ V, at what time will the voltage $v_{\rm C}(t)$ first reach 50 V?

Compute the voltage after 10 μs. Note that the slope of the current waveform is $\pm 0.02/5\,\mu = \pm 4000$.

$$\begin{aligned}
v_{\rm C}(t) &= v_{\rm C}(0) + \frac{1}{C}\int_0^t i_{\rm C}(x)\,dx \\
&= -5 + \frac{1}{1500\,{\rm p}}\left[\int_0^{5\,\mu} 4000x\,dx + \int_{5\,\mu}^{10\,\mu}(0.04 - 4000x)\,dx\right] \\
&= -5 + \frac{1}{1500\,{\rm p}}\left(\frac{1}{2}\right)(0.02)\,(10\,\mu) \\
&= -5 + 66.67 = 61.67\ {\rm V}
\end{aligned}$$

The voltage increases from -5 V to 61.67 V over 10 μs. Based on the symmetric shape of the current waveform, the voltage increases by 33.33 V over $0 < t < 5$ μs, and increases by 33.33 V over $5 < t < 10$ μs.

We can write the following expression for the voltage as a function of time for $t > 5\ \mu$s.

$$v_{\rm C}(t) = 50 = -5 + 33.33 + \frac{1}{1500\,\rm p}\int_{5\,\mu}^{t}(0.04 - 4000x)\,dx$$

$$50 = 28.33 + \frac{1}{1500\,\rm p}(0.04x - 2000x^2)\Big|_{5\,\mu}^{t}$$

$$50 = 28.33 + \frac{1}{1500\,\rm p}\left[0.04t - 2000t^2 - (0.04)(5\,\mu) + (2000)(5\,\mu)^2\right]$$

$$50 = 28.3333 - (1.3333 \times 10^{12})t^2 + (26.6667 \times 10^6)t - 133.3333 + 33.3333$$

$$0 = (1.3333 \times 10^{12})t^2 - (26.6667 \times 10^6)t + 121.6667$$

$$t = 7.042\ \mu\text{s} \ \text{ or } \ 12.958\ \mu\text{s}$$

Select the answer that is less than 10 μs, so $t = 7.042\ \mu$s.

Problem 6–9. For $t \geq 0$ the current through a 0.22-μF capacitor is $i_{\rm C}(t) = 6\sin(1000\pi t)$ mA. Plot $v_{\rm C}(t)$ versus time when $v_{\rm C}(0) = -10$ V. [*Hint:* In OrCAD use the ISIN source—the frequency is in Hz—and set the other parameters appropriately. Use the Property Editor to set the capacitor's initial condition to -10 V. Then plot the output using the transient function. Place a huge, 1000MEG resistor in parallel with the capacitor to avoid "floating nodes." *Caution:* In OrCAD rotating the capacitor to make it vertical places the capacitor's number 2 terminal at the top. In the identification of an initial voltage on the capacitor, the applied polarity of the voltage is between the number 1 terminal (considered positive) and the number 2 terminal (considered negative). Hence, the capacitor needs to be "flipped vertically" to place the desired -10 V across the capacitor.]

First, solve the problem analytically.

$$v_{\rm C}(t) = v_{\rm C}(0) + \frac{1}{C}\int_0^t i_{\rm C}(x)\,dx$$

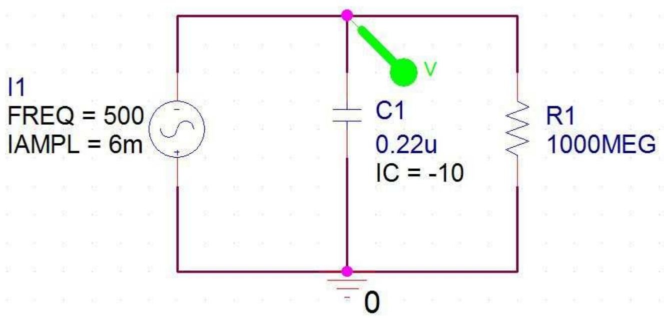
$$= -10 + \frac{1}{0.22\,\mu}\int_0^t 0.006\sin(1000\pi x)\,dx$$

$$= -10 + \left(\frac{0.006}{0.22\,\mu}\right)\left(\frac{-1}{1000\pi}\right)[\cos(1000\pi x)]\Big|_0^t$$

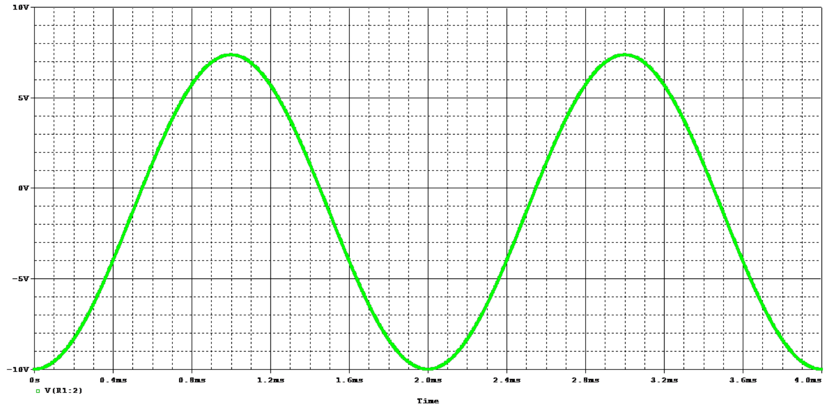
$$= -10 - \frac{300}{11\pi}[\cos(1000\pi t) - 1]$$

$$= -10 + \frac{300}{11\pi}[1 - \cos(1000\pi t)]$$

The following OrCAD simulation also provides the solution.



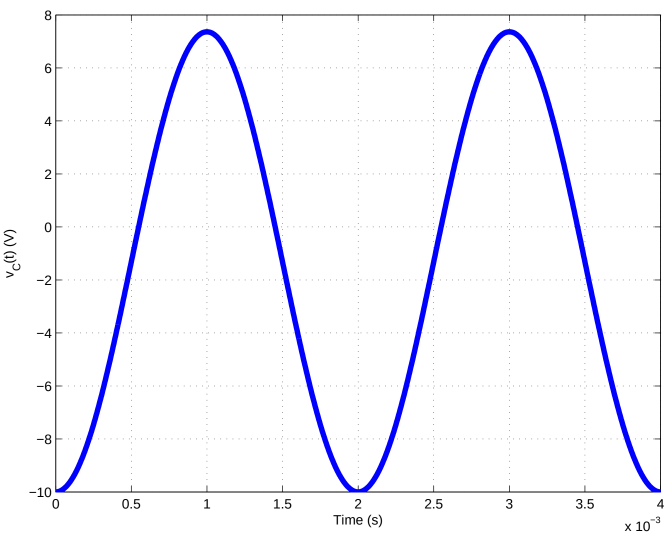
The resulting OrCAD output is shown below.



The problem can also be solved with the following MATLAB code.

```
syms t x real
C = 0.22e-6;
vC0 = -10;
iCt = 6e-3*sin(1000*pi*t);
iCx = subs(iCt,t,x);
vCt = vC0 + int(iCx,x,0,t)/C
t0 = 0;
tf = 4e-3;
ts = tf/1000;
tt = t0:ts:tf;
vCtt = subs(vCt,t,tt);

figure
plot(tt,vCtt,'b','LineWidth',4)
grid on
hold on
xlabel('Time (s)')
ylabel('v_C(t) (V)')
```

The corresponding MATLAB plot is shown below.



Problem 6–10. The current through a 10-mH inductor is shown in Figure P6–10. Prepare sketches of $v_{\mathrm{L}}(t)$, $p_{\mathrm{L}}(t)$, and $w_{\mathrm{L}}(t)$.

Compute the voltages for each segment of the current and determine the expressions for the power and energy.

$$v_{\mathrm{L}}(t) = L\frac{di_{\mathrm{L}}(t)}{dt}$$

$$v_{\mathrm{L}}(t) = (0.01)\frac{0.02}{5\,\mu} = 40 \text{ V} \text{ for } 0 < t < 5\,\mu\text{s}$$

$$v_{\mathrm{L}}(t) = (0.01)\frac{-0.01}{5\,\mu} = -20 \text{ V} \text{ for } 5 < t < 10\,\mu\text{s}$$

$$v_{\mathrm{L}}(t) = 0 \text{ for } 10 < t\,\mu\text{s}$$

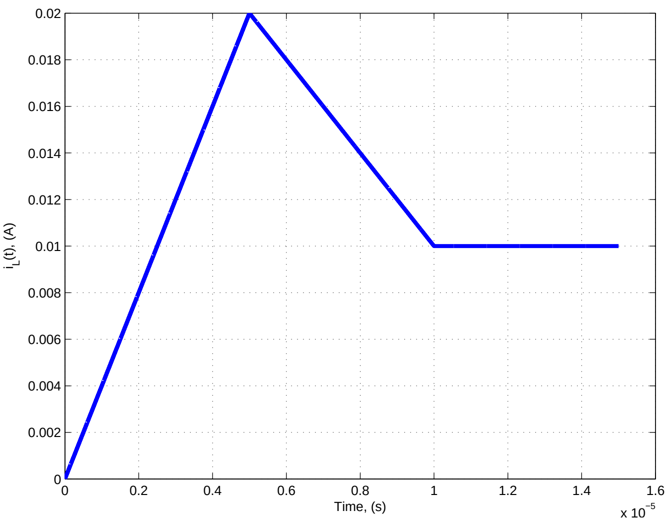
$$p_{\mathrm{L}}(t) = i_{\mathrm{L}}(t)v_{\mathrm{L}}(t)$$

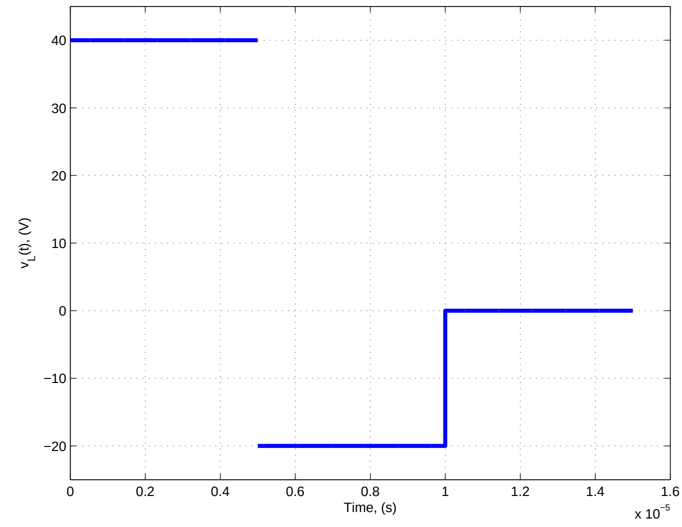
$$w_{\mathrm{L}}(t) = \frac{1}{2}Li_{\mathrm{L}}^2(t)$$

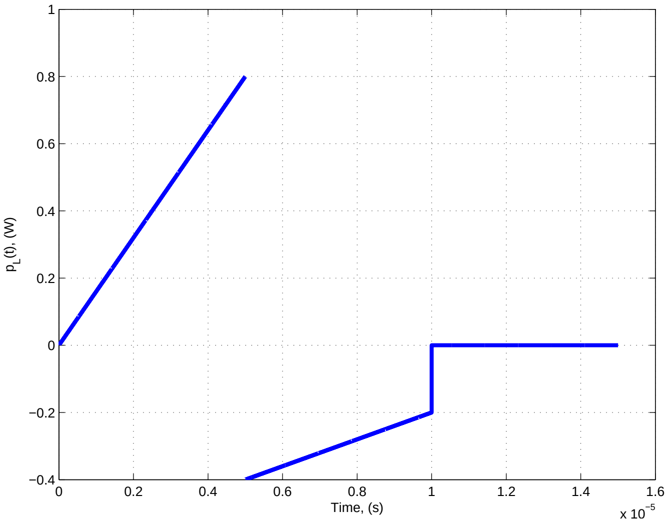
The following MATLAB code computes the required values.

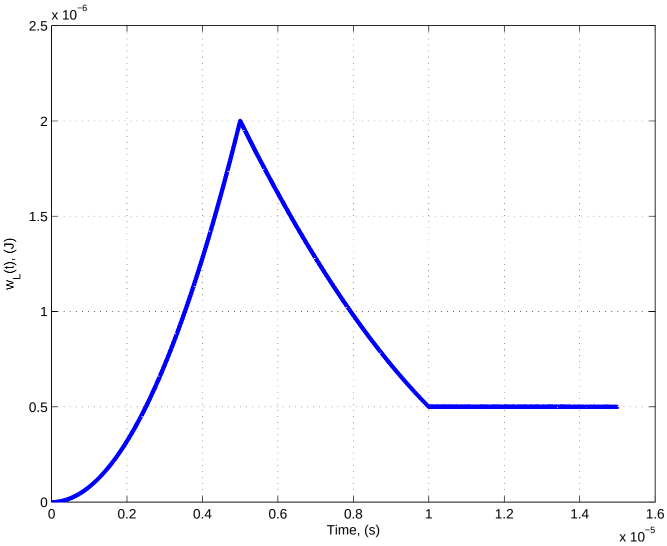
```
syms t x real
L = 10e-3;
t0 = 0;
t1 = 5e-6;
t2 = 10e-6;
t3 = 15e-6;
ts = 1e-9;
tt = t0:ts:t3;
iLt = 4000*t*heaviside(t) - 6000*(t-t1)*heaviside(t-t1) ...
    + 2000*(t-t2)*heaviside(t-t2);
vLt = L*diff(iLt,t);
iLtt = subs(iLt,t,tt);
vLtt = subs(vLt,t,tt);
pLtt = iLtt.*vLtt;
wLtt = L*iLtt.^2/2;
```

The corresponding MATLAB plots are shown below.









Problem 6–11. For $t \geq 0$ the current through a 560-μH inductor is $i_L(t) = 1e^{-10000t}$ A. Find $v_L(t)$, $p_L(t)$, and $w_L(t)$ for $t \geq 0$. Is the inductor absorbing power, delivering power, or both?

Compute the voltage, power, and energy associated with the inductor.

$$v_L(t) = L\frac{di_L(t)}{dt} = (560\,\mu)\frac{d}{dt}\left(e^{-10000t}\right) = (560\,\mu)(-10000)e^{-10000t} = -5.6e^{-10000t}\text{ V}$$

$$p_L(t) = i_L(t)v_L(t) = \left(e^{-10000t}\right)\left(-5.6e^{-10000t}\right) = -5.6e^{-20000t}\text{ W}$$

$$w_L(t) = \frac{1}{2}Li_L^2(t) = (0.5)(560\,\mu)(e^{-10000t})^2 = 280e^{-20000t}\,\mu\text{J}$$

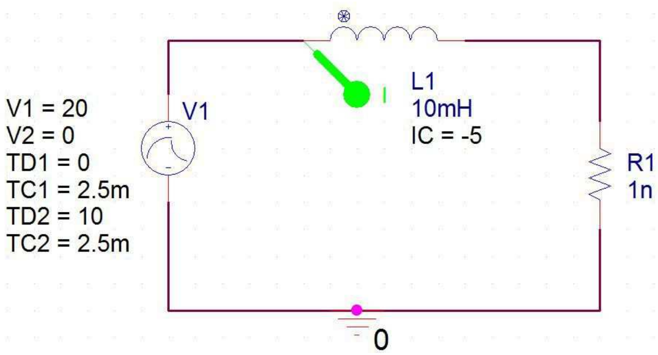
The power is always negative, so the inductor is always delivering power.

Problem 6–12. For $t > 0$ the voltage across a 10-mH inductor is $v_L(t) = 20e^{-400t}$ V. Plot $i_L(t)$ versus time when $i_L(0) = -5$ A. [*Hint:* In OrCAD use the VEXP source, set TD1 to zero and TD2 to 10 s, and set the other parameters appropriately. Use the Property Editor to set the inductor's initial condition to -5 A. Then plot the output using the transient function. Place a tiny, 1-nΩ resistor in series with the inductor to avoid an "inductor loop."]

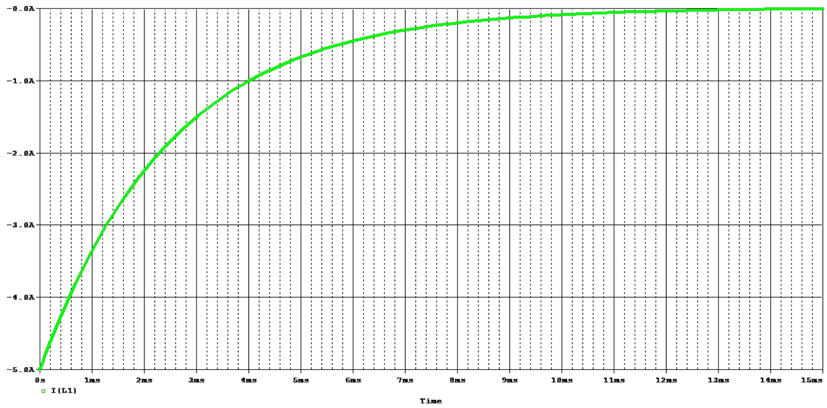
First, solve the problem analytically.

$$\begin{aligned} i_L(t) &= i_L(0) + \frac{1}{L}\int_0^t v_L(x)\,dx \\ &= -5 + \frac{1}{0.01}\int_0^t 20e^{-400x}\,dx \\ &= -5 + \left[\frac{(100)(20)}{-400}\right]e^{-400x}\Big|_0^t \\ &= -5 - 5\left(e^{-400t} - 1\right) = -5e^{-400t}\text{ A} \end{aligned}$$

The following OrCAD simulation also provides the solution.



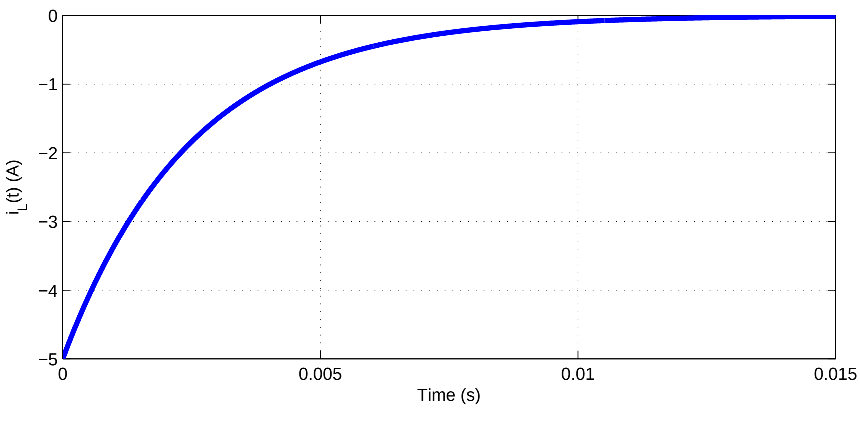
The resulting OrCAD output is shown below.



The problem can also be solved with the following MATLAB code.

```
syms t x real
L = 10e-3;
iL0 = -5;
vLt = 20*exp(-400*t);
vLx = subs(vLt,t,x);
iLt = iL0 + int(vLx,x,0,t)/L
t0 = 0;
tf = 15e-3;
ts = tf/1000;
tt = t0:ts:tf;
iLtt = subs(iLt,t,tt);

figure
plot(tt,iLtt,'b','LineWidth',3)
grid on
hold on
xlabel('Time (s)')
ylabel('i_L(t) (A)')
```

The corresponding MATLAB plot is shown below.



Problem 6–13. A voltage $v_L(t) = 5\cos(1000nt)$ V appears across a 50-mH inductor, where n is a positive integer that controls the frequency of the input signal. The amplitude of the input signal is constant. Assume $i_L(0) = 0$ A. Use MATLAB and symbolic variables to compute an expression for $i_L(t)$. On the same axes, plot $i_L(t)$ versus time for $n = 1, 2, 3, 4$, and 5, over an appropriate time scale. On another set of axes, plot the amplitude of $i_L(t)$ versus the coefficient n. As n approaches infinity, what happens to the amplitude of the current? What type of circuit element does the inductor behave like as n approaches infinity?

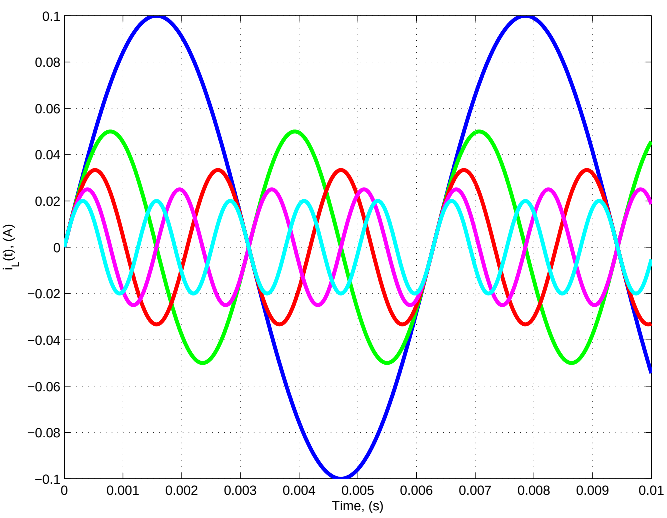
The following MATLAB code provides the solution:

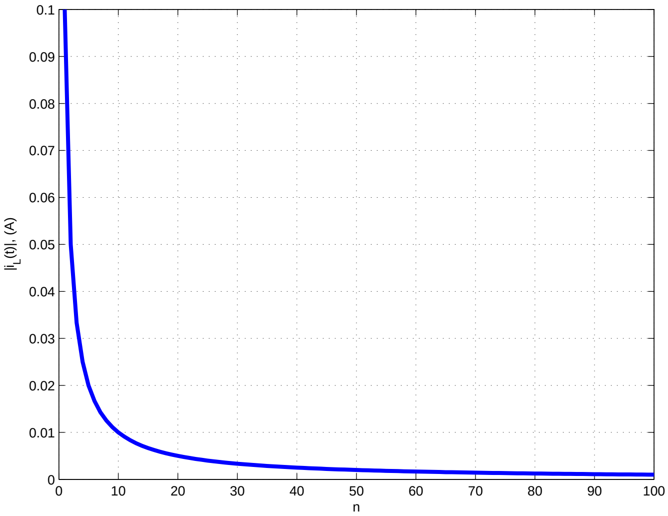
```
syms t tau n real
L = 50e-3;
iL0 = 0;
vL = 5*cos(1000*n*t);

% Solve for the current in terms of n and t
%iL = simplify(iL0 + int(subs(vL,t,tau),tau,0,t)/L)
iL = iL0 + int(subs(vL,t,tau),tau,0,t)/L

% Evaluate the current over a suitable time span
tt = 0:0.00001:0.01;
iLtt = subs(iL,t,tt);
PlotColor = ['b','g','r','m','c'];

% Plot the result for different values of n
for N = 1:5
    iLtt2 = subs(iLtt,n,N);
    plot(tt,iLtt2,PlotColor(N),'LineWidth',3)
    grid on
    hold on
end
xlabel('Time, (s)')
ylabel('i_L(t), (A)')

% Plot current amplitude versus n
N = 1:100;
iN = 1/10./N;
figure
plot(N,iN,'b','LineWidth',3)
grid on
xlabel('n')
ylabel('|i_L(t)|, (A)')
```

The corresponding MATLAB output is shown below.

```
iL = sin(1000*n*t)/(10*n)
```

The corresponding MATLAB plots are shown below.





The inductor current is $i_{\rm L}(t) = (1/10n)\sin(1000nt)$. As n approaches infinity, the amplitude of the current approaches zero. Therefore, the inductor acts like an open circuit for high-frequency signals.

Problem 6–14. For $t \geq 0$ the voltage across a 50-mH inductor is $v_{\rm L}(t) = 1000r(t)$ V. At $t = 5$ ms the inductor current is observed to be zero. Find the value of $i_{\rm L}(0)$.

Evaluate the expression for the current at 5 ms and solve for $i_{\rm L}(0)$.

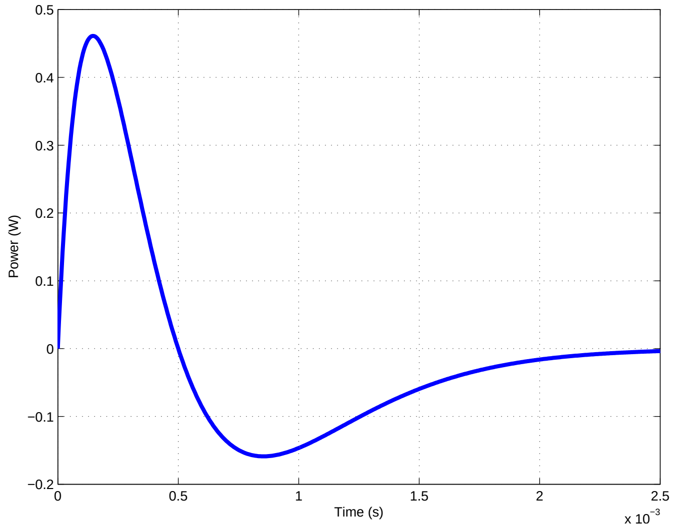
$$\begin{aligned}
i_{\rm L}(0.005) &= i_{\rm L}(0) + \frac{1}{L}\int_0^{0.005} v_{\rm L}(t)\,dt \\
0 &= i_{\rm L}(0) + \frac{1}{0.05}\int_0^{0.005} 1000t\,dt \\
0 &= i_{\rm L}(0) + (20)(1000)(0.5)t^2\Big|_0^{0.005} \\
0 &= i_{\rm L}(0) + 10000\left[(0.005)^2 - 0\right] \\
i_{\rm L}(0) &= -250\text{ mA}
\end{aligned}$$

Problem 6–15. For $t \geq 0$ the current through a 200-mH inductor is $i_{\rm L}(t) = 200te^{-2000t}$ A. Derive an expression for $v_{\rm L}(t)$. Is the inductor absorbing power or delivering power or both?

Compute the voltage and the power.

$$\begin{aligned}
v_{\rm L}(t) &= L\frac{di_{\rm L}(t)}{dt} = (0.2)\frac{d}{dt}\left(200te^{-2000t}\right) \\
v_{\rm L}(t) &= (0.2)(200)\left(-2000te^{-2000t} + e^{-2000t}\right) = 40e^{-2000t}\left(1 - 2000t\right)\text{ V} \\
p_{\rm L}(t) &= i_{\rm L}(t)v_{\rm L}(t) = \left[200te^{-2000t}\right]\left[40e^{-2000t}(1 - 2000t)\right] = 8000te^{-4000t}(1 - 2000t)\text{ W}
\end{aligned}$$

At $t = 0$, the power is positive and for $t > 500$ μs, the power is negative, so the inductor both absorbs and delivers power. The following plot of the power confirms this result.



Problem 6–16. The capacitor in Figure P6–16 carries an initial voltage $v_C(0) = 25$ V. At $t = 0$, the switch is closed, and thereafter the voltage across the capacitor is $v_C(t) = 100 - 75e^{-2000t}$ V. Derive expressions for $i_C(t)$ and $p_C(t)$ for $t > 0$. Is the capacitor absorbing power, delivering power, or both?

Compute the current and power.

$$i_C(t) = C\frac{dv_C(t)}{dt} = (10\,\mu)\frac{d}{dt}\left(100 - 75e^{-2000t}\right)$$

$$i_C(t) = (10\,\mu)(-75)(-2000)e^{-2000t} = 1.5e^{-2000t}\text{ A}$$

$$p_C(t) = i_C(t)v_C(t) = \left(1.5e^{-2000t}\right)\left(100 - 75e^{-2000t}\right)$$

$$p_C(t) = 150e^{-2000t} - 112.5e^{-4000t}\text{ W}$$

The expression for the power is always positive, so the capacitor is always absorbing power.

Problem 6–17. A 50-μF capacitor and a 100-mH inductor are connected in parallel with a closed switch as shown in Figure P6–17. The inductor has 25 mA flowing through it at $t = 0^-$. The switch opens at $t = 0$.

(a). Find the initial voltage across the capacitor at $t = 0$.

The voltage across the capacitor cannot change instantaneously. Since the capactor is connected in parallel with a closed switch at $t = 0^-$, the capacitor voltage is zero at $t = 0$, so we have $v_C(0) = 0$ V.

(b). Write an equation for the voltage across the elements for $t > 0$. Do not solve it.

With the switch open, we have the following relationships from KVL, KCL, and the i-v relationships for capacitors and inductors.

$$v_C(t) = v_L(t)$$

$$i_C(t) = -i_L(t)$$

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

Use substitution to develop the following equation.

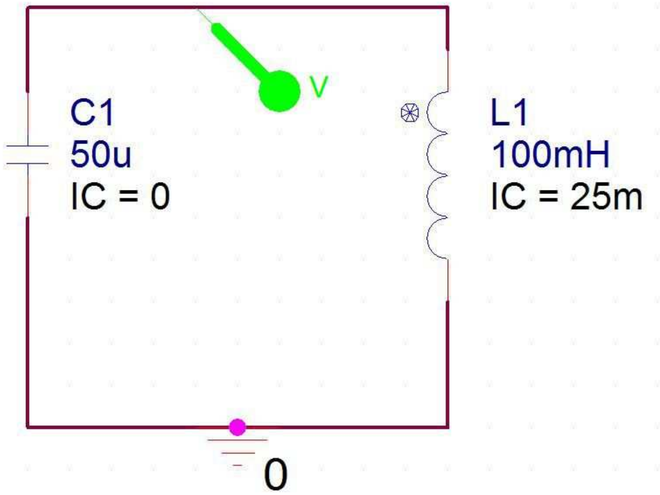
$$v_L(t) = L\frac{di_L(t)}{dt} = -L\frac{di_C(t)}{dt} = -LC\frac{d^2v_C(t)}{dt^2}$$

$$v_L(t) = -LC\frac{d^2v_L(t)}{dt^2}$$

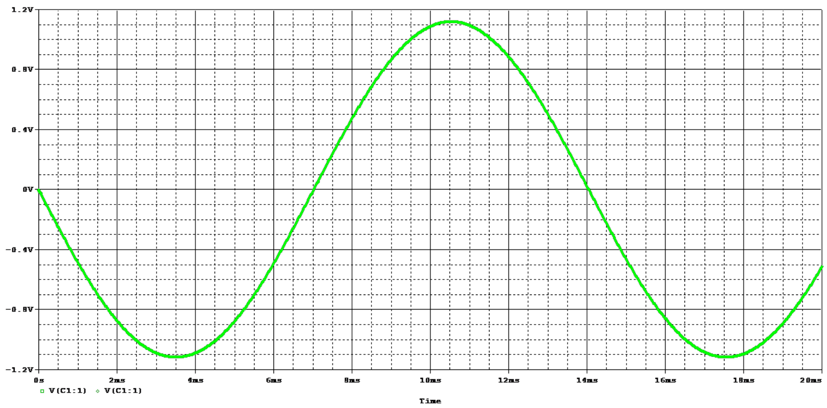
The initial conditions for the differential equation are $v_C(0) = 0$ V and $i_L(0) = 25$ mA.

(c). Simulate the circuit using OrCAD. Connect an inductor in parallel with a capacitor and assign the appropriate initial conditions and run a transient analysis. Plot the voltage across the elements for at least 20 ms.

The following OrCAD simulation provides the solution.



The corresponding output plot is shown below.



(d). Characterize the response signal.

The response signal is a sinusoidal waveform with an amplitude of 1.12 V and a frequency of 71.2 Hz.

Problem 6–18. The inductor in Figure P6–18 carries an initial current of $i_L(0) = 0.5$ A. At $t = 0$, the switch opens, and thereafter the current into the rest of the circuit is $i(t) = -0.5e^{-2000t}$ A. Derive expressions for $v_L(t)$ and $p_L(t)$ for $t > 0$. Is the inductor absorbing or delivering power?

With the switch open, by KCL we have $i_{\rm L}(t) = -i(t)$. Compute the voltage and power.

$$v_{\rm L}(t) = L\frac{di_{\rm L}(t)}{dt} = (100\,\mu)\frac{d}{dt}\left(0.5e^{-2000t}\right)$$

$$v_{\rm L}(t) = (100\,\mu)(0.5)(-2000)e^{-2000t} = -100e^{-2000t} \text{ mV}$$

$$p_{\rm L}(t) = i_{\rm L}(t)v_{\rm L}(t) = \left(0.5e^{-2000t}\right)\left(-0.1e^{-2000t}\right)$$

$$p_{\rm L}(t) = -50e^{-4000t} \text{ mW}$$

The power is always negative, so the inductor is delivering power to the rest of the circuit.

Problem 6–19. The inductor in Figure P6–18 carries an initial current of $i_{\rm L}(0) = 30$ mA. At $t = 0$, the switch opens, and thereafter the voltage across the inductor is $v_{\rm L}(t) = -8e^{-1000t}$ mV. Derive expressions for $i_{\rm L}(t)$ and $p_{\rm L}(t)$ for $t > 0$. Is the inductor absorbing or delivering power?

Compute the inductor current and power.

$$i_{\rm L}(t) = i_{\rm L}(0) + \frac{1}{L}\int_0^t v_{\rm L}(x)\,dx$$

$$i_{\rm L}(t) = 0.03 + \frac{1}{100\,\mu}\int_0^t \left(-0.008e^{-1000x}\right)\,dx$$

$$i_{\rm L}(t) = 0.03 + \frac{-0.008}{(100\,\mu)(-1000)}\left(e^{-1000t} - 1\right)$$

$$i_{\rm L}(t) = 0.03 + 0.08\left(e^{-1000t} - 1\right)$$

$$i_{\rm L}(t) = 80e^{-1000t} - 50 \text{ mA}$$

$$p_{\rm L}(t) = i_{\rm L}(t)v_{\rm L}(t) = \left(0.08e^{-1000t} - 0.05\right)\left(-0.008e^{-1000t}\right)$$

$$p_{\rm L}(t) = 400e^{-1000t} - 640e^{-2000t}\ \mu\text{W}$$

The power changes sign from negative to positive, so the inductor is both delivering and absorbing power to the rest of the circuit.

Problem 6–20. A 0.056-μF capacitor is connected in series with a 1-kΩ resistor. The voltage across the capacitor is $v_{\rm C}(t) = 10\cos(5000t)$ V. What is the voltage across the resistor?

Find the current through capacitor, which equals the current through the resistor. Apply Ohm's law to find the voltage across the resistor.

$$i_{\rm C}(t) = C\frac{dv_{\rm C}(t)}{dt} = (0.056\,\mu)\frac{d}{dt}\left[10\cos(5000t)\right]$$

$$i_{\rm C}(t) = (0.056\,\mu)(10)(-5000)\sin(5000t) = -2.8\sin(5000t) \text{ mA}$$

$$v_{\rm R}(t) = i_{\rm R}(t)R = i_{\rm C}(t)R = \left[-0.0028\sin(5000t)\right]\left[1000\right] = -2.8\sin(5000t)\text{ V}$$

Problem 6–21. A 50-mH inductor is connected in parallel with a 33-kΩ resistor. The current through the inductor is $i_{\rm L}(t) = 20e^{-1000t}$ mA. What is the current through the resistor?

Since the inductor and resistor are connected in parallel, they share the same voltage. Find the voltage across the inductor, which matches the voltage across the resistor, and then apply Ohm's law to find the current through the resistor.

$$v_{\rm L}(t) = L\frac{di_{\rm L}(t)}{dt} = (0.05)(0.02)(-1000)e^{-1000t} = -e^{-1000t} \text{ V}$$

$$i_{\rm R}(t) = \frac{v_{\rm R}(t)}{R} = \frac{-e^{-1000t}}{33000} = -30.3030e^{-1000t} \ \mu\text{A}$$

Problem 6–22. For $t > 0$ the voltage across an energy storage element is $v(t) = 5e^{-100t}$ V and the current through the element is $i(t) = 10-5e^{-100t}$ A. What is the element, the element value, and its initial condition?

Since the current is an exponential plus a constant value and the voltage does not have a constant value in its expression, the voltage is proportional to the derivative of the current. Therefore, the element is an inductor. Solve for L and the initial condition.

$$v_{\rm L}(t) = L\frac{di_{\rm L}(t)}{dt}$$

$$5e^{-100t} = L\frac{d}{dt}\left(10 - 5e^{-100t}\right) = L(-5)(-100)e^{-100t} = 500Le^{-100t}$$

$$L = \frac{5}{500} = 10 \text{ mH}$$

$$i_{\rm L}(0) = 10 - 5 = 5 \text{ A}$$

Problem 6–23. For $t > 0$ the voltage across an energy storage element is $v(t) = 5 - 20e^{-500t}$ V and the current through the element is $i(t) = 2000t + 16e^{-500t}$ mA. What is the element, the element value, and its initial condition?

Given the expressions for the voltage and current, the voltage must be proportional to the derivative of the current. The key to determining this fact is that one term in the expression for current is proportional to t and the corresponding term in the expression for voltage is a constant. Solve for L and the initial condition.

$$v_{\rm L}(t) = L\frac{di_{\rm L}(t)}{dt}$$

$$5 - 20e^{-500t} = L\frac{d}{dt}\left(2t + 0.016e^{-500t}\right) = L\left[2 + (0.016)(-500)e^{-500t}\right] = L\left[2 - 8e^{-500t}\right]$$

$$L = \frac{5 - 20e^{-500t}}{2 - 8e^{-500t}} = 2.5 \text{ H}$$

$$i_{\rm L}(0) = 0 + 16 = 16 \text{ mA}$$

Problem 6–24. The OP AMP integrator in Figure P6–24 has $R = 33$ kΩ, $C = 0.056\ \mu$F, and $v_{\rm O}(0) = 10$ V. The input is $v_{\rm S}(t) = 5e^{-500t}u(t)$ V. Find $v_{\rm O}(t)$ for $t > 0$.

Apply the equation for an OP AMP integrator to calculate the output voltage.

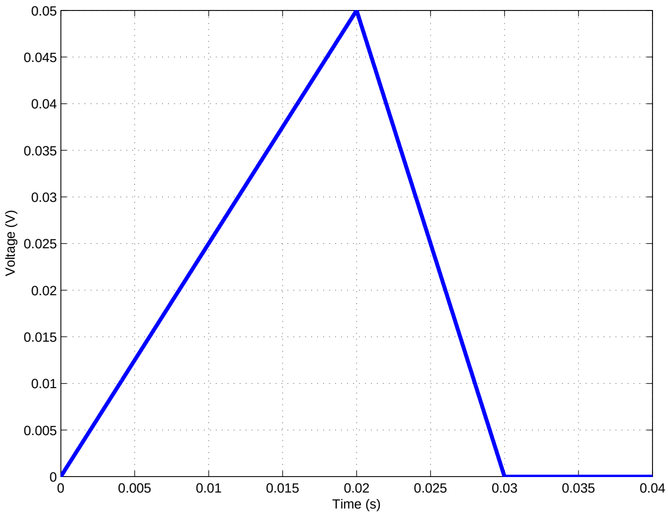
$$\begin{aligned}
v_{\rm O}(t) &= v_{\rm O}(0) - \frac{1}{RC}\int_0^t v_{\rm S}(x)\,dx \\
&= 10 - \frac{1}{(33\,\text{k})(0.056\,\mu)}\int_0^t 5e^{-500x}\,dx \\
&= 10 - \frac{5}{(33\,\text{k})(0.056\,\mu)(-500)}\left(e^{-500t} - 1\right) \\
&= 10 + 5.41126\left(e^{-500t} - 1\right) = 4.58874 + 5.41126e^{-500t} \text{ V}, \quad t > 0
\end{aligned}$$

Problem 6–25. An OP AMP integrator with $R = 1$ MΩ, $C = 1$ μF, and $v_O(0) = 0$ V has the input waveform shown in Figure P6–25. Sketch $v_O(t)$ for $t > 0$.

With zero initial voltage at the output, we have the following expression for the output voltage:

$$\begin{aligned}
v_O(t) &= -\frac{1}{RC}\int_0^t v_S(x)\,dx \\
&= -\frac{1}{(1\,\mathrm{M})(1\,\mu)}\int_0^t [-2.5u(t) + 7.5u(t-0.02) - 5u(t-0.3)]\,dx \\
&= \int_0^t [2.5u(t) - 7.5u(t-0.02) + 5u(t-0.3)]\,dx \\
&= 2.5r(t) - 7.5r(t-0.02) + 5r(t-0.03)\text{ V} \quad t > 0
\end{aligned}$$

Plot the results.



Problem 6–26. An OP AMP circuit from Figure 6–17 is in the box shown in Figure 6–26. The input and outputs are given. What is the function of the circuit in the box if:

(a). $v_S(t) = \cos(500t)$ mV and $v_O(t) = 5\sin(500t)$ V?

The derivative of $\cos(t)$ is $-\sin(t)$ and a differentiator circuit has a negative gain. With an input of $0.001\cos(500t)$ V, the output of the differentiator circuit is $(-RC)(0.001)(-500)\sin(500t) = (0.5)RC\sin(500t)$ V. To match the given output, we require $RC = 10$ in the differentiator circuit.

(b). $v_S(t) = \cos(500t)$ mV and $v_O(t) = -20\sin(500t)$ μV?

The integral of $\cos(t)$ is $\sin(t)$ and an integrator circuit has a negative gain. With an input of $0.001\cos(500t)$ V, the output of the integrator circuit is

$$(-1/RC)(0.001)(1/500)\sin(500t) = -1/500000RC\sin(500t)\text{ V}$$

To match the given output, we require $RC = 1/10 = 0.1$ in the integrator circuit.

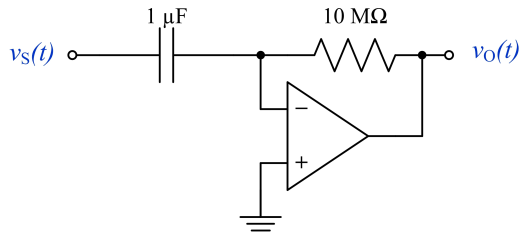
(c). $v_S(t) = \cos(500t)$ mV and $v_O(t) = -2\cos(500t)$ V?

The output has the oppositve sign of the input and a gain of 2000. The circuit is an inverting amplifier with a gain $K = -2000$.

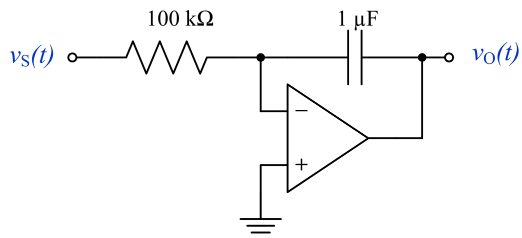
Problem 6–27. Design appropriate OP AMP circuits that will realize each of the functions in Problem 6–26.

The corresponding designs are shown below. There are many correct answers.

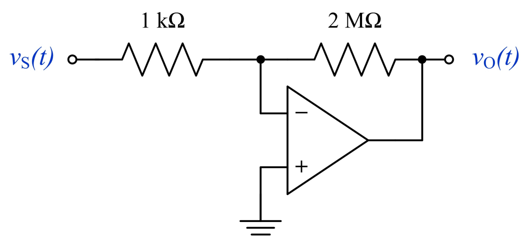
(a).



(b).



(c).

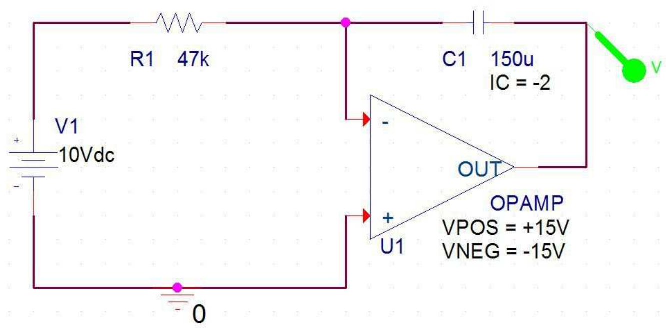


Problem 6–28. The OP AMP integrator in Figure P6–24 has $R = 47$ kΩ, $C = 150$ μF, and $v_O(0) = -2$ V. The input is $v_S(t) = 10u(t)$ V. Use OrCAD to determine how long it takes for the OP AMP to saturate when $V_{CC} = \pm 15$ V.

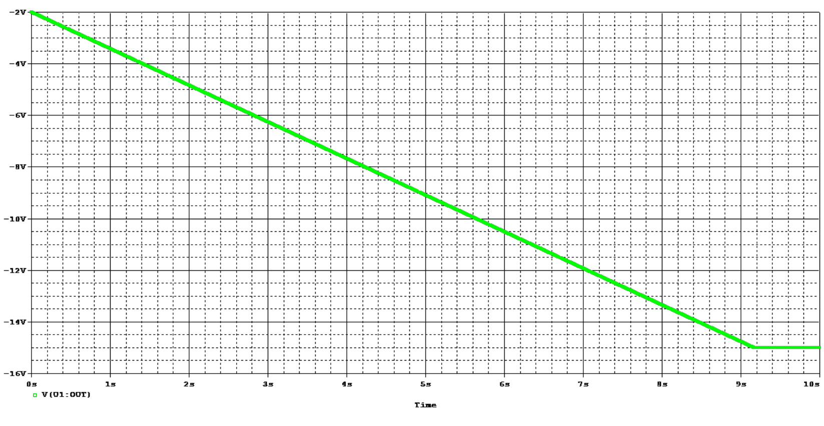
Apply the equation for an OP AMP integrator to calculate the output voltage and then determine the time at which the output voltage reaches ±15 V.

$$
\begin{aligned}
v_{\mathrm{O}}(t) &= v_{\mathrm{O}}(0) - \frac{1}{RC}\int_0^t v_{\mathrm{S}}(x)\,dx \\
&= -2 - \frac{1}{(47\,\mathrm{k})(150\,\mu)}\int_0^t 10\,dx \\
&= -2 - \frac{10}{(47\,\mathrm{k})(150\,\mu)}(t-0) \\
&= -2 - \frac{10}{7.05}t \\
-15 &= -2 - \frac{10}{7.05}t \\
t &= \frac{(13)(7.05)}{10} = 9.165 \text{ s}
\end{aligned}
$$

The following OrCAD simulation verifies the results.



The corresponding OrCAD output is shown below.



Problem 6–29. The OP AMP integrator in Figure P6–24 has $R = 22$ kΩ, $C = 0.001$ μF, and $v_{\mathrm{O}}(0) = 0$ V. The input is $v_{\mathrm{S}}(t) = 2\sin(\omega t)u(t)$ V. Derive an expression for $v_{\mathrm{O}}(t)$ and find the smallest allowable value of ω for linear operation of the OP AMP. Assume $V_{\mathrm{CC}} = \pm 15$ V.

Apply the equation for an OP AMP integrator to calculate the output voltage and then determine the smallest allowable value of ω for linear operation.

$$\begin{aligned}
v_{\mathrm{O}}(t) &= v_{\mathrm{O}}(0) - \frac{1}{RC}\int_0^t v_{\mathrm{S}}(x)\,dx \\
&= 0 - \frac{1}{(22\,\mathrm{k})(0.001\,\mu)}\int_0^t 2\sin(\omega x)\,dx \\
&= \left(\frac{-2}{22\,\mu}\right)\left(\frac{-1}{\omega}\right)\cos(\omega x)\Big|_0^t \\
&= \frac{1}{(11\,\mu)\omega}\left[\cos(\omega t) - 1\right] \\
-15 &= \frac{1}{(11\,\mu)\omega}\left[\cos(\omega t) - 1\right] = \frac{1}{(11\,\mu)\omega}\left[-1-1\right] \\
-15 &= \frac{-2}{(11\,\mu)\omega} \\
\omega &= \frac{2}{(11\,\mu)(15)} = 12.1212\ \mathrm{krad/s}
\end{aligned}$$

We require $\omega \geq 12.1212$ krad/s. In the solution, we used the extreme value of $\cos(\omega t) = -1$ to find the extreme value for ω such that the output voltage never exceeds $V_{\mathrm{CC}} = \pm 15$ V.

Problem 6–30. The OP AMP differentiator in Figure P6–30 with $R = 15$ kΩ and $C = 0.56$ μF has the input $v_{\mathrm{S}}(t) = 6(1 - e^{-50t})u(t)$ V. Find $v_{\mathrm{O}}(t)$ for $t > 0$.

Apply the equation for an OP AMP differentiator.

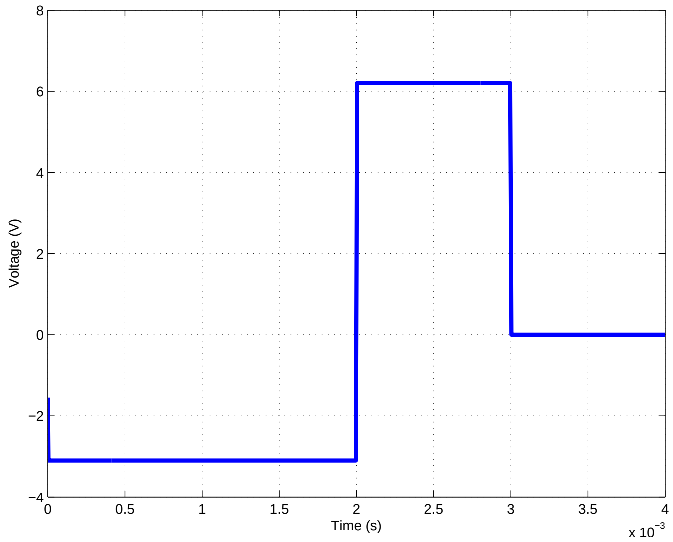
$$\begin{aligned}
v_{\mathrm{O}}(t) &= -RC\frac{dv_{\mathrm{S}}(t)}{dt} \\
&= -(15\,\mathrm{k})(0.56\,\mu)\frac{d}{dt}\left[6(1 - e^{-50t})\right] \\
&= -(15\,\mathrm{k})(0.56\,\mu)(-6)(-50)e^{-50t} = -2.52e^{-50t}\ \mathrm{V}, \quad t > 0
\end{aligned}$$

Problem 6–31. The OP AMP differentiator in Figure P6–30 with $R = 47$ kΩ and $C = 0.022$ μF has the input waveform shown in Figure P6–31. Sketch $v_{\mathrm{O}}(t)$ for $t > 0$.

Apply the equation for an OP AMP differentiator.

$$\begin{aligned}
v_{\mathrm{O}}(t) &= -RC\frac{dv_{\mathrm{S}}(t)}{dt} \\
&= -(47\,\mathrm{k})(0.022\,\mu)\frac{dv_{\mathrm{S}}(t)}{dt} = -(1.034\,\mathrm{m})\frac{dv_{\mathrm{S}}(t)}{dt}
\end{aligned}$$

The derivative of the input is the slope of the line. For $0 < t < 2$ ms, the slope is $6/0.002 = 3000$ V/s, and for $2 < t < 3$ ms, the slope is $-6/0.001 = -6000$ V/s. Multiplying by the $-RC$ factor, for $0 < t < 2$ ms, $v_{\mathrm{O}}(t) = -3.102$ V, and for $2 < t < 3$ ms, $v_{\mathrm{O}}(t) = 6.204$ V. The output is zero otherwise. The sketch is shown below.



Problem 6–32. The OP AMP differentiator shown in Figure P6–30 has $R = 100$ kΩ and $C = 0.1$ μF and an output $v_{\rm O}(t) = 2[\sin(100t)]u(t)$ V. What is its input $v_{\rm S}(t)$?

Apply the equation for an OP AMP differentiator and solve for the input.

$$v_{\rm O}(t) = -RC\frac{dv_{\rm S}(t)}{dt}$$

$$2\sin(100t) = -(100\,{\rm k})(0.1\,\mu)\frac{dv_{\rm S}(t)}{dt} = -(10\,{\rm m})\frac{dv_{\rm S}(t)}{dt}$$

$$\frac{dv_{\rm S}(t)}{dt} = -200\sin(100t)$$

$$v_{\rm S}(t) = 2\cos(100t)u(t) + K \text{ V}$$

The input could have a constant bias, which is denoted by K.

Problem 6–33. The input to the OP AMP differentiator in Figure P6–30 is $v_{\rm S}(t) = 10[\sin(2\pi\times 10^6 t)]u(t)$ mV. Select R and C so that the output sinusoid has a maximum of at least ± 14 V but does not saturate the OP AMP at ± 15 V.

Apply the equation for an OP AMP differentiator and solve for the values of R and C.

$$v_{\rm O}(t) = -RC\frac{dv_{\rm S}(t)}{dt}$$

$$v_{\rm O}(t) = -RC\frac{d}{dt}[0.01\sin(2\pi\times 10^6 t)] = -RC(0.01)(2\pi\times 10^6)\cos(2\pi\times 10^6 t)$$

$$v_{\rm O}(t) = -RC(2\pi\times 10^4)\cos(2\pi\times 10^6 t)$$

The amplitude of the sinusoid must be between 14 and 15, so solve the following inequality.

$$14 < RC(2\pi 10^4) < 15$$

$$\frac{14}{2\pi 10^4} < RC < \frac{15}{2\pi 10^4}$$

$$222.82\,\mu < RC < 238.73\,\mu$$

There are many possible solutions, with one being $R = 15$ kΩ and $C = 0.015$ μF.

Problem 6–34. The OP AMP differentiator in Figure P6–30 with $R = 5$ kΩ and $C = 220$ pF has the input $v_S(t) = 2[\sin(\omega t)]u(t)$ V. Determine the frequency at which the OP AMP saturates at ±15 V. Validate your answer using OrCAD. [*Hint:* Use a VAC source and an AC sweep.]

Apply the equation for an OP AMP differentiator.

$$v_O(t) = -RC\frac{dv_S(t)}{dt}$$

$$v_O(t) = -(5\,\text{k})(220\,\text{p})\frac{d}{dt}[2\sin(\omega t)]$$

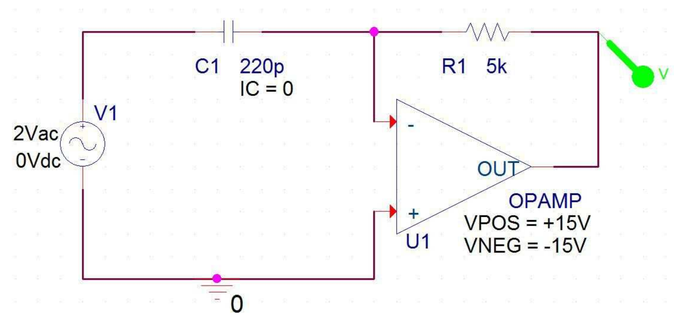
$$v_O(t) = -(5\,\text{k})(220\,\text{p})(2)(\omega)\cos(\omega t) = (-2.2\,\mu)(\omega)\cos(\omega t)$$

$$15 = (2.2\,\mu)(\omega)$$

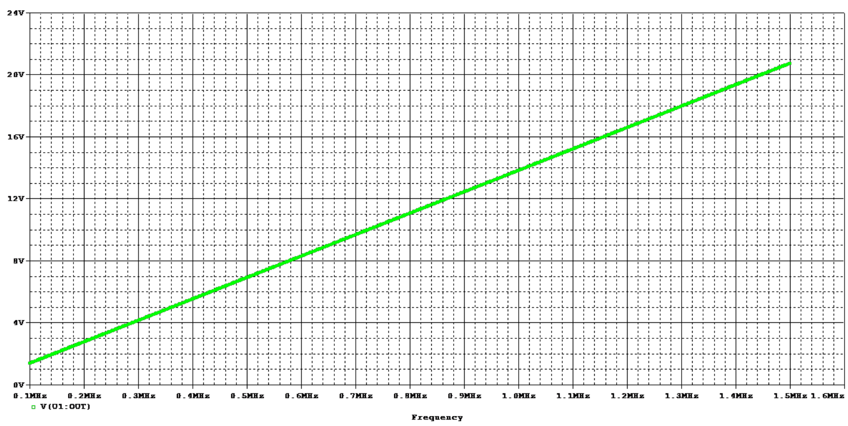
$$\omega = 6.81818\ \text{Mrad/s}$$

$$f = 1.08515\,\text{MHz}$$

The following OrCAD simulation verifies the results.



The corresponding output is shown below.



The output voltages exceeds 15 V at a frequency of 1.08 MHz.

Problem 6–35. Find the input-output relationship of the *RC* OP AMP circuit in Figure P6–35.

Define the current through the capacitor, $i_C(t)$, as flowing from left to right. Let the voltage at the node between the capacitor and the left resistor be $v_A(t)$. The voltage at the node between the two resistors is 0 V because the positive input terminal to the OP AMP is grounded. Write node-voltage equations at those two nodes and solve for the relationship between $v_{\mathrm{O}}(t)$ and $v_{\mathrm{S}}(t)$.

$$i_{\mathrm{C}}(t) = C\frac{d}{dt}v_{\mathrm{C}}(t) = C\frac{d}{dt}\left[v_{\mathrm{s}}(t) - v_{\mathrm{A}}(t)\right]$$

$$-i_{\mathrm{C}}(t) + \frac{v_{\mathrm{A}}(t) - 0}{R} = 0$$

$$C\frac{d}{dt}\left[v_{\mathrm{A}}(t) - v_{\mathrm{s}}(t)\right] + \frac{v_{\mathrm{A}}(t)}{R} = 0$$

$$\frac{0 - v_{\mathrm{A}}(t)}{R} + \frac{0 - v_{\mathrm{O}}(t)}{R} = 0$$

$$v_{\mathrm{A}}(t) = -v_{\mathrm{O}}(t)$$

$$C\frac{d}{dt}\left[-v_{\mathrm{O}}(t) - v_{\mathrm{s}}(t)\right] + \frac{-v_{\mathrm{O}}(t)}{R} = 0$$

$$C\frac{d}{dt}\left[v_{\mathrm{O}}(t) + v_{\mathrm{s}}(t)\right] + \frac{v_{\mathrm{O}}(t)}{R} = 0$$

$$\frac{d}{dt}\left[v_{\mathrm{O}}(t) + v_{\mathrm{s}}(t)\right] + \frac{v_{\mathrm{O}}(t)}{RC} = 0$$

The differential equation expresses the relationship between $v_{\mathrm{O}}(t)$ and $v_{\mathrm{S}}(t)$.

Problem 6–36. Find the input-output relationship of the RC OP AMP circuit in Figure P6–36.

The voltage at both input terminals of the OP AMP is $v_{\mathrm{S}}(t)$. Write the node-voltage equation at the negative terminal of the OP AMP.

$$i_{\mathrm{C}}(t) + \frac{v_{\mathrm{S}}(t)}{R} + \frac{v_{\mathrm{S}}(t) - v_{\mathrm{O}}(t)}{R} = 0$$

$$C\frac{dv_{\mathrm{S}}(t)}{dt} + \frac{v_{\mathrm{S}}(t)}{R} + \frac{v_{\mathrm{S}}(t) - v_{\mathrm{O}}(t)}{R} = 0$$

$$RC\frac{dv_{\mathrm{S}}(t)}{dt} + 2v_{\mathrm{S}}(t) = v_{\mathrm{O}}(t)$$

Problem 6–37. Show that the RC OP AMP circuit in Figure P6–37 is a noninverting integrator whose input-output relationship is

$$v_{\mathrm{O}}(t) = \frac{1}{RC}\int_0^t v_{\mathrm{S}}(x)\,dx + v_{\mathrm{O}}(0)$$

Let $v_{\mathrm{A}}(t)$ be the voltage at the input terminals of the OP AMP. Write node-voltage equations at the two input terminals.

$$\frac{v_{\mathrm{A}}(t)}{R} + C\frac{d[v_{\mathrm{A}}(t) - v_{\mathrm{O}}(t)]}{dt} = 0$$

$$\frac{v_{\mathrm{A}}(t) - v_{\mathrm{S}}(t)}{R} + C\frac{dv_{\mathrm{A}}(t)}{dt} = 0$$

Rewrite the equations as follows and substitute.

$$\frac{v_{\mathrm{A}}(t)}{R} + C\frac{dv_{\mathrm{A}}(t)}{dt} - C\frac{dv_{\mathrm{O}}(t)}{dt} = 0$$

$$\frac{v_{\mathrm{A}}(t)}{R} + C\frac{dv_{\mathrm{A}}(t)}{dt} = \frac{v_{\mathrm{S}}(t)}{R}$$

$$\frac{v_{\mathrm{S}}(t)}{R} - C\frac{dv_{\mathrm{O}}(t)}{dt} = 0$$

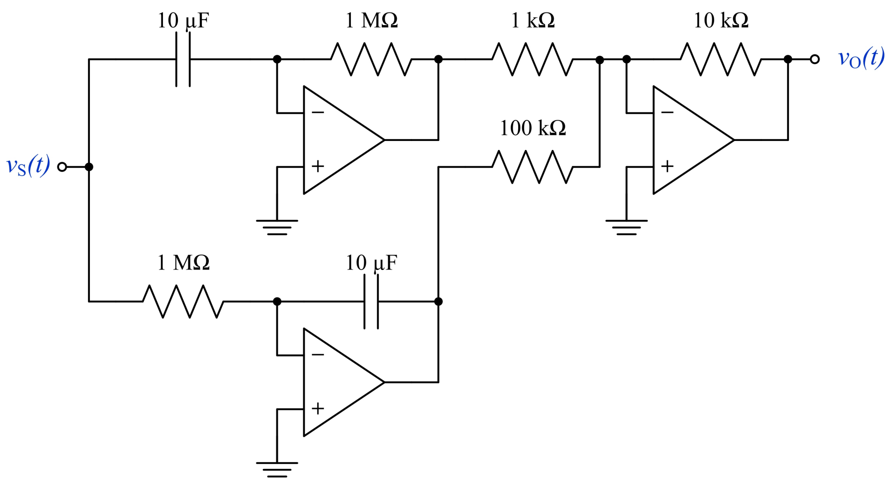
$$\frac{v_{\mathrm{S}}(t)}{R} = C\frac{dv_{\mathrm{O}}(t)}{dt}$$

$$v_{\mathrm{S}}(t) = RC\frac{dv_{\mathrm{O}}(t)}{dt}$$

$$v_{\mathrm{O}}(t) = v_{\mathrm{O}}(0) + \frac{1}{RC}\int_0^t v_{\mathrm{S}}(x)\,dx$$

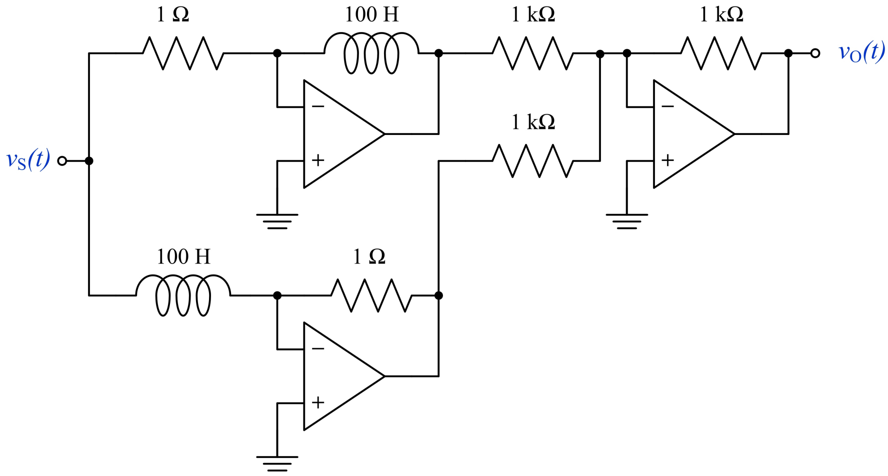
Problem 6–38. Design an *RC* OP AMP circuit to implement the block diagram in Figure P6–38.

The top branch is a differentiator with a gain of 100 and the bottom branch is an integrator with a gain of 1/100. The branches are combined by a summer. If we use an inverting summer to connect a differentiator with a gain of −10 and an integrator with a gain of −1/10, and then repeat those two gains as part of the summer, we will achieve the same result. The following design implements the block diagram.



Problem 6–39. Repeat Problem 6–38 except use an *RL* OP AMP circuit.

To create an *RL* differentiator, start with an *RC* integrator and replace the capacitor with an inductor. The gain of the differentiator will be $-L/R$. To create an *RL* integrator, start with an *RC* differentiator and replace the capacitor with an inductor. The gain of the integrator is $-R/L$. The following circuit implements the block diagram.



Note that the resistor and inductor values are not practical for this design.

Problem 6–40. In this problem you will design an oscillator. The equation for your oscillator is

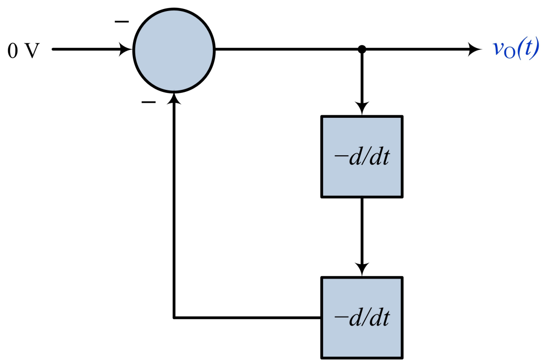
$$\frac{d^2v_O(t)}{dt^2} + v_O(t) = 0 \text{ V}$$

(a). Draw a block diagram to solve your equation ($v_O(t)$ should be your output) using differentiators.

The appropriate equation is

$$v_O(t) = -\frac{d^2v_O(t)}{dt^2}$$

The corresponding block diagram is shown below.

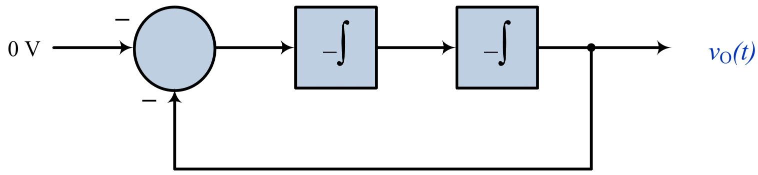


(b). Draw a block diagram to solve your equation ($v_O(t)$ should be your output) using integrators.

The appropriate equation is

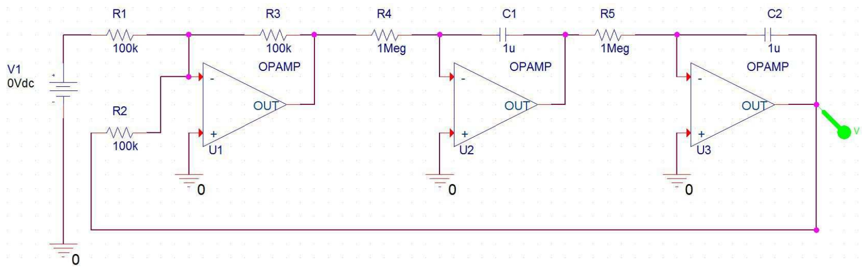
$$v_O(t) = \int\int [0 - v_O(t)]$$

The corresponding block diagram is shown below.



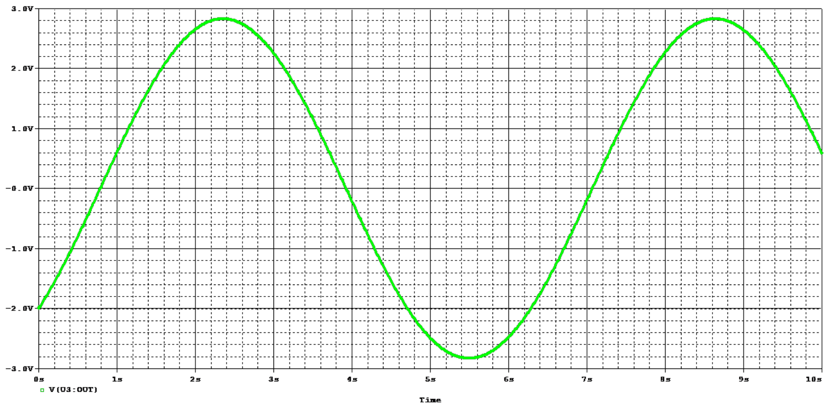
(c). Design a circuit using OP AMP integrators to realize your block diagram.

The following circuit implements the integrator design.



(d). Using OrCAD, simulate your circuit and show that your output is an oscillator. Give both capacitors in the circuit the same non-zero initial conditions. What is its oscillating frequency? What could you do to alter the oscillating frequency?

The results of the simulation are shown below.

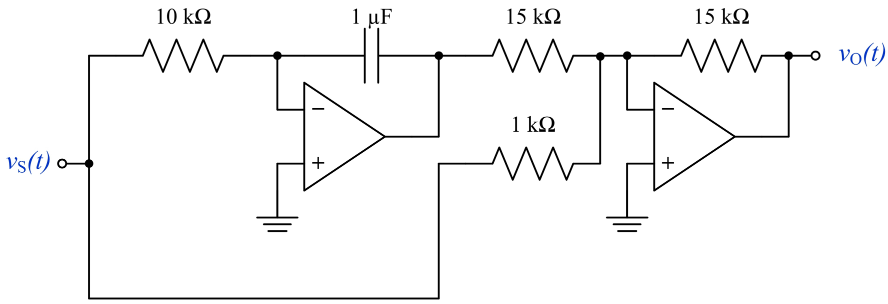


The oscillating frequency is 1 rad/s and we can adjust the frequency by changing the resistor or capacitor values.

Problem 6–41. Design an *RC* OP AMP circuit to implement the input-output relationship

$$v_O(t) = -15v_S(t) + 100\int_0^t v_S(x)\,dx$$

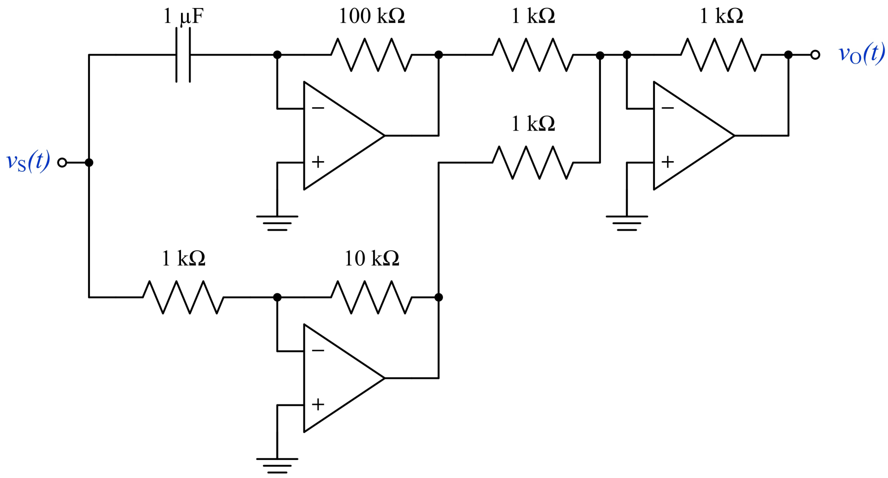
The output is a sum of a scaled version of the input and a scaled version of the integral of the input. The following block diagram provides a solution.



Problem 6–42. Design an OP AMP circuit to solve the following differential equation

$$v_O(t) = 10v_S(t) + \frac{1}{10}\frac{dv_S(t)}{dt}$$

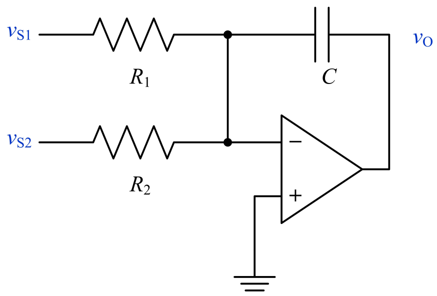
The output is a sum of a scaled version of the input and a scaled version of the derivative of the input. The following block diagram provides a solution.



Problem 6–43. Design an *RC* circuit using only one OP AMP and only one capacitor that implements the input-output relationship

$$v_O(t) = -15\int_0^t v_{S1}(x)\,dx - 20\int_0^t v_{S2}(x)\,dx$$

The circuit is a summing integrator. Using superposition and node-voltage analysis, we can show that the following circuit provides a solution when $R_1 = 66.7$ kΩ, $R_2 = 50$ kΩ, and $C = 1$ μF.



Problem 6–44. Find a single equivalent element for each circuit in Figure P6–44.

For circuit C1, the top two capacitors are in parallel, so they sum together, and the bottom two capacitors are in parallel, so they also sum together. The two combinations are in series, so they sum reciprocally. The equivalent capacitance is

$$C_{EQ} = \frac{1}{\dfrac{1}{(0.5+3.3)} + \dfrac{1}{(1+2.2)}} = 1.73714\ \mu\text{F}$$

For circuit C2, add the two 25-μH inductors in series. Combine that result in parallel with the two 100-μH inductors. Add that results to the 75-μH inductor in series. The equivalent inductance is

$$L_{EQ} = 75 + [(25+25) \parallel (100 \parallel 100)] = 75 + [50 \parallel 50] = 75 + 25 = 100\ \mu\text{H}$$

Problem 6–45. (E) A 50-μH inductor is connected in series with a 2-mH inductor and the combination is connected in parallel with a 1-mH inductor. All are ±5%. Find the equivalent inductance of the connection. Which inductor played no effective role in this combination and could have been ignored?

Add the 50-μH and 2-mH inductors in series and combine the result in parallel with the 1-mH inductor. The equivalent inductance is

$$L_{EQ} = (2 + 0.05) \parallel 1 = 0.672131\ \text{mH} = 672.131\ \mu\text{H}$$

The 50-μH inductor is insignificant and could be removed. Without it, the equivalent inductance is 666.667 μH, which is a difference less than the tolerance of the larger inductors.

Problem 6–46. Use the look-back method to find the equivalent capacitance of the circuit shown in Figure 6–46.

Replace the voltage source with a short circuit. The two left capacitors are in parallel and have an equivalent capacitance of $2C$. Similarly, the two right capacitors are in parallel and have an equivalent capacitance of $2C$. The two combinations are in series, so the resulting capacitance is $C_{\rm EQ} = C$.

Problem 6–47. Verify Equations (6–30) and (6–31).

For a series connection of N inductors, the total voltage drop is the sum of the individual voltages

$$v(t) = v_1(t) + v_2(t) + \cdots + v_{\rm N}(t)$$

The current through each inductor in series is the same

$$i_1(t) = i_2(t) = \cdots = i_{\rm N}(t) = i(t)$$

Express each voltage in terms of the common current and solve for the equivalent inductance

$$v(t) = L_1\frac{di(t)}{dt} + L_2\frac{di(t)}{dt} + \cdots + L_{\rm N}\frac{di(t)}{dt} = (L_1 + L_2 + \cdots + L_{\rm N})\frac{di(t)}{dt} = L_{\rm EQ}\frac{di(t)}{dt}$$

For a parallel connection of N inductors, the total current is the sum of the individual currents

$$i(t) = i_1(t) + i_2(t) + \cdots + i_{\rm N}(t)$$

The voltage across each inductor in parallel is the same

$$v_1(t) = v_2(t) = \cdots = v_{\rm N}(t) = v(t)$$

Express each current in terms of the common voltage and solve for the equivalent inductance

$$i(t) = i_1(0) + \frac{1}{L_1}\int_0^t v(x)\,dx + i_2(0) + \frac{1}{L_2}\int_0^t v(x)\,dx + \cdots + i_{\rm N}(0) + \frac{1}{L_{\rm N}}\int_0^t v(x)\,dx$$

$$i(t) = [i_1(0) + i_2(0) + \cdots + i_{\rm N}(0)] + \left[\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_{\rm N}}\right]\int_0^t v(x)\,dx$$

$$\frac{1}{L_{\rm EQ}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_{\rm N}}$$

Problem 6–48. What is the equivalent capacitance and initial voltage of a series connection of a 10-μF capacitor with 100 V stored and a 33-μF capacitor with 200 V stored?

Combine the capacitors in series as follows:

$$C_{\rm EQ} = \frac{1}{\dfrac{1}{10} + \dfrac{1}{33}} = 7.67442\ \mu\text{F}$$

In series, the stored voltages add, so the initial voltage is $v_{\rm C}(0) = 100 + 200 = 300$ V.

Problem 6–49. What is the equivalent capacitance and initial voltage for the capacitor bank shown in Figure P6–49?

Combine the two 0.01-μF capacitors in parallel to get an equivalent of a 0.02-μF capacitor with an initial voltage of 10 V. Combine that result in series with the 0.022-μF capacitor to get an equivalent of a 0.010476-μF capacitor with an initial voltage of 15 V. Combine that result in parallel with the 0.056-μF capacitor to get an equivalent of a 0.066476-μF capacitor with an initial voltage of 15 V. Combine that result in series with the upper 0.01-μF capacitor to get an equivalent of a 0.0086924-μF capacitor with an initial voltage of 35 V.

Problem 6–50. What is the equivalent inductance and initial current for the inductors shown in Figure P6–50?

Combine the two left inductors in series by adding their values to get an equivalent inductance of $25+15 = 40\ \mu\text{H}$ with an initial current of 50 μA. Combine that result in parallel with the other two inductors by taking the reciprocal of the sum of the reciprocals of the inductances and adding the initial currents to get an equivalent inductance of 18.1818 μH and an initial current of 180 μA.

Problem 6–51. For the circuit in Figure P6–51, find an equivalent circuit consisting of one inductor and one capacitor.

Combine the 6-μF and 4-μF capacitors in series to get an equivalent capacitance of 2.4 μF. Combine that result in parallel with the 10-μF capacitor to get an equivalent capacitance of 12.4 μF. Combine the two 3-H inductors in parallel to get an equivalent inductance of 1.5 H. Combine that result in series with the 1.5-H inductor to get an equivalent inductance of 3 H. The final result is a 12.4-μF capacitor in parallel with a 3-H inductor.

Problem 6–52. Figure P6–52 is the equivalent circuit of a two-wire feed-through capacitor.

(a). What is the capacitance between terminal 1 and ground when terminal 2 is open?

In this configuration, the 50-pF capacitor is in series with the lower 25-pF capacitor and that result is in parallel with the upper 25-pF capacitor. The equivalent capacitance is

$$C_{\text{EQ}} = 25 + \frac{1}{\dfrac{1}{25} + \dfrac{1}{50}} = 25 + 16.667 = 41.667 \text{ pF}$$

(b). What is the capacitance between terminal 1 and ground when terminal 2 is grounded?

In this configuration, the lower 25-pF capacitor is grounded on both sides and does not contribute to the equivalent capacitance. The result is that the 25-pF and 50-pF capacitors are in parallel, so the equivalent capacitance is their sum, 75 pF.

Problem 6–53. (D) A capacitor bank is required that can be charged to 5 kV and store at least 250 J of energy. Design a series/parallel combination that meets the voltage and energy requirements using 22-μF capacitors each rated at 2 kV max.

To achieve a charging voltage of 5 kV, three capacitors must be connected in series, which will yield a maximum voltage of 6 kV. The equivalent capacitance of three 22-μF capacitors in series is 7.333 μF. To store 250 J at 5 kV, we require a capacitance of $C = 2w/v_{\text{C}}^2 = (2)(250)/5000^2 = 20\ \mu\text{F}$. Combine three of the three-capacitor series connections in parallel to get an overall capacitance of 22 μF to meet the specifications.

Problem 6–54. (E) A switching power supply requires an inductor that can store at least 1 mJ of energy. A list of available inductors is shown below. Select the inductor that best meets the requirement. Consider both meeting the specifications and cost. Explain your choice.

L (μH)	I_{MAX} (A)	Cost Each
10	9.2	\$ 4.75
20	7.0	\$ 5.00
50	5.5	\$ 4.50
100	4.3	\$ 4.75
150	3.8	\$ 4.50
250	2.5	\$ 4.75
500	2.1	\$ 5.00

The energy stored in an inductor is given by $w_{\text{L}} = (1/2)Li_{\text{L}}^2$. Use MATLAB to examine each option and select the inductor that meets the specification with the lowest cost. The corresponding MATLAB code is shown below.

```
L = 1e-6*[10 20 50 100 150 250 500];
Imax = [9.2 7.0 5.5 4.3 3.8 2.5 2.1];
wmax = L.*Imax.^2/2;
Results = [L' Imax' wmax']
```

The corresponding MATLAB output is shown below.

```
Results =
    10.0000e-006     9.2000e+000   423.2000e-006
    20.0000e-006     7.0000e+000   490.0000e-006
    50.0000e-006     5.5000e+000   756.2500e-006
   100.0000e-006     4.3000e+000   924.5000e-006
   150.0000e-006     3.8000e+000     1.0830e-003
   250.0000e-006     2.5000e+000   781.2500e-006
   500.0000e-006     2.1000e+000     1.1025e-003
```

Two inductors can store at least 1 mJ of energy. The 150-μH inductor costs less and is the best choice.

Problem 6–55. The circuits in Figure P6–55 are driven by dc sources. Find the current through the 330-Ω resistor under dc conditions.

Under dc conditions, inductors appear to be short circuits and capacitors appear to be open circuits. In circuit C1, the equivalent circuit is the 15-V source in series with the two resistors. The current through the 330-Ω resistor is $i = 15/(330 + 220) = 27.27$ mA. In circuit C2, the circuit is open and no current flows through the 330-Ω resistor.

Problem 6–56. The circuit in Figure P6–56 is driven by 15-V dc source. Find the energy stored in the capacitor and inductor under dc conditions.

Under dc conditions, inductors appear to be short circuits and capacitors appear to be open circuits. If the inductor appears as a short circuit, then there is no voltage across it. The inductor is in parallel with the capacitor, so there is no voltage across the capacitor and the capacitor stores no energy. The current through the inductor is $i_{\rm L} = 15/50 = 300$ mA, so the energy stored in the inductor is $w_{\rm L} = (1/2)Li_{\rm L}^2 = (1/2)(0.005)(0.3)^2 = 225\ \mu$J.

Problem 6–57. The OP AMP circuit in Figure P6–57 has a capacitor in its feedback loop. Determine the circuit gain at dc and as the frequency approaches ∞ rad/s.

At dc, the capacitor appears as an open circuit, so the resulting OP AMP circuit is an inverting amplifier with a gain of $K = -R_2/R_1$. As the input signal frequency approaches ∞ rad/s, the capacitor appears as a short circuit, which effectively shorts out R_2 and the gain approaches $K = -0/R_1 = 0$. To understand how a capacitor behaves as a short circuit at high frequencies, consider the following justification. A sinusoidal input causes a sinusoidal current to flow through R_1. A sinusoidal current through R_1 will cause a sinusoidal current through the capacitor and R_2. Assume the current through the capacitor is proportional to $\cos(\omega t)$ and find the voltage across the capacitor.

$$i_{\rm C}(t) = \cos(\omega t)$$

$$v_{\rm C}(t) = \frac{1}{C}\int_0^t \cos(\omega x)\,dx = \frac{1}{\omega C}\sin(\omega t)$$

For a current with a constant amplitude, as the frequency ω increases, the voltage drop across the capacitor decreases. As $\omega \to \infty$, we have $v_{\rm C} \to 0$, so the output voltage approaches the voltage at the negative input terminal of the OP AMP, which is zero in this case. Therefore the gain approaches zero as $\omega \to \infty$.

Problem 6–58. The OP AMP circuit in Figure P6–58 has a capacitor in its feedback loop. Determine the circuit gain at dc and as the frequency approaches ∞ rad/s.

At dc, the capacitor appears as an open circuit, so the resulting OP AMP circuit is a noninverting amplifier with a gain of $K = (R + R)/R = 2$. As discussed in Problem 6–58, as the input signal frequency approaches ∞ rad/s, the capacitor appears as a short circuit and the output voltage will match the voltage at the negative input terminal to the OP AMP. In this case the voltage at the negative input terminal is $v_{\rm S}(t)$, so the output equals the input and the gain is $K = 1$.

Problem 6–59. (A) Piezoelectric Transducer

Piezoelectric transducers (sensors) measure dynamic phenomena such as pressure and force. These phenomena cause stresses that “squeeze” a quantity of electric charge from piezoelectric material in the transducer (the term piezo means “squeeze” in Greek). The amount of charge $q(t)$ is directly proportional to the measured variable $x(t)$, that is $q(t) = \alpha x(t)$. Signal amplification is needed because the amount of charge produced is on the order of pC. Figure P6–59 shows an OP AMP charge amplifier that provides the necessary gain. First show that the OP AMP output is $v_{\rm O}(t) = -Kq(t)$. Then select a value of C so that the charge amplifier gain is $K = 6$ mV/pC.

The positive input terminal to the OP AMP is grounded, so the voltage at the negative input terminal is also zero. Write a node-voltage equation at the negative input terminal and solve for the output voltage.

$$-\frac{dq(t)}{dt} + i_{\rm C} = 0$$

$$-\frac{dq(t)}{dt} + C\frac{d[0 - v_{\rm O}(t)]}{dt} = 0$$

$$C\frac{dv_{\rm O}(t)}{dt} = -\frac{dq(t)}{dt}$$

$$v_{\rm O}(t) = -\frac{1}{C}q(t)$$

The required relationship holds with $K = 1/C$. To get $K = (6\,\text{mV})/\text{pC} = 6 \times 10^9$ V/C, we need to choose $C = 1/(6 \times 10^9) = 166.67$ pF.

Problem 6–60. (A) LC Circuit Response

At $t = 0$ the switch in Figure P6–60 is closed and thereafter the voltage across the capacitor is

$$v_{\rm C}(t) = (20 + 10000t)e^{-8000t} \text{ V}$$

Use MATLAB to solve all of the following problems.

(a). Use the capacitor’s i-v characteristic to find the current $i(t)$ for $t \geq 0$.

Apply the equation for the current through a capacitor.

$$i_{\rm C}(t) = C\frac{dv_{\rm C}(t)}{dt}$$

$$i_{\rm C}(t) = (25\,\mu)\frac{d}{dt}\left[(20 + 10000t)e^{-8000t}\right]$$

$$i_{\rm C}(t) = (25\,\mu)\left[(20 + 10000t)(-8000)e^{-8000t} + 10000e^{-8000t}\right]$$

$$i_{\rm C}(t) = (25\,\mu)\left[-160000 - 80000000t + 10000\right]e^{-8000t} = (-2000t - 3.75)\,e^{-8000t}$$

$$i(t) = -i_{\rm C}(t) = (2000t + 3.75)\,e^{-8000t} \text{ A}$$

The following MATLAB code confirms the solution.

```
syms t positive
syms vC iC vL vt it Req
L = 2.5e-3;
C = 25e-6;
vC = (20+10000*t)*exp(-8000*t);
iC = simplify(C*diff(vC,'t'));
it = -iC
```

The corresponding MATLAB output is shown below.

```
it = (5*(1600*t + 3))/(4*exp(8000*t))
```

(b). Use the inductor's i-v characteristic and $i(t)$ to find $v_{\rm L}(t)$ for $t \geq 0$.

Compute the voltage across the inductor.

$$v_{\rm L}(t) = L\frac{di_{\rm L}(t)}{dt}$$

$$v_{\rm L}(t) = (0.0025)\frac{d}{dt}(2000t + 3.75)e^{-8000t} = (0.0025)\left[(2000t + 3.75)(-8000)e^{-8000t} + 2000e^{-8000t}\right]$$

$$v_{\rm L}(t) = -(40000t + 70)e^{-8000t}\ \text{V}$$

The following MATLAB code confirms the solution.

```
vL = simplify(L*diff(it,'t'))
```

The corresponding MATLAB output is shown below.

```
vL = -(10*(4000*t + 7))/exp(8000*t)
```

(c). Use $v_{\rm C}(t)$, $v_{\rm L}(t)$, and KVL to find the voltage $v(t)$ delivered to the rest of the circuit.

Apply KVL.

$$-v_{\rm C}(t) + v_{\rm L}(t) + v(t) = 0$$

$$v(t) = v_{\rm C}(t) - v_{\rm L}(t) = (20 + 10000t)e^{-8000t} + (40000t + 70)e^{-8000t}$$

$$v(t) = (50000t + 90)e^{-8000t}\ \text{V}$$

The following MATLAB code confirms the solution.

```
vt = simplify(vC - vL)
```

The corresponding MATLAB output is shown below.

```
vt = (10*(5000*t + 9))/exp(8000*t)
```

(d). The $v(t)$ found in part (c) should be proportional to the $i(t)$ found in part (a). If so, what is the equivalent resistance looking into the rest of the circuit?

Find the equivalent resistance.

$$R_{\rm EQ} = \frac{v(t)}{i(t)} = \frac{(50000t + 90)e^{-8000t}}{(2000t + 3.75)e^{-8000t}} = \frac{40000t + 72}{1600t + 3}\ \Omega$$

The following MATLAB code confirms the solution.

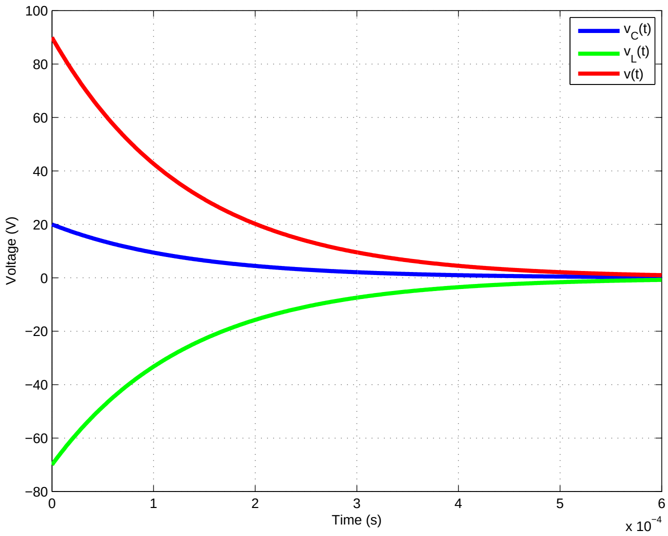
```
Req = simplify(vt/it)
```

The corresponding MATLAB output is shown below.

```
Req = 25 - 3/(1600*t + 3)
```

(e). On the same axes, plot $v_C(t)$, $v_L(t)$, and $v(t)$. Use a different color for each waveform. Use the plots to verify KVL for the circuit.

The appropriate plot is shown below and confirms KVL.



Problem 6–61. (A) Supercapacitor

Supercapacitors have very large capacitances (typically from 0.1 to 50 F,) very long charge holding times, and small sizes, making them useful in non-battery backup power applications. To measure its capacitance, a supercapacitor is charged to an initial voltage $V_0 = 5.5$ V. At $t = 0$ the device undergoes a constant current discharge of $i_D = 2$ mA. At $t = 2500$ s the voltage remaining on the capacitor is 3 V. Find the device capacitance.

Use the equation for the voltage across a capacitor to solve for the capacitance.

$$v_C(t) = v_O(0) + \frac{1}{C}\int_0^t i_C(x)\,dx$$

$$v_C(2500) = 3 = 5.5 + \frac{1}{C}\int_0^{2500}(-0.002)\,dx = 5.5 - \frac{1}{C}(0.002)(2500)$$

$$\frac{5}{C} = 2.5$$

$$C = 2\text{ F}$$

Problem 6–62. (D) Analog Computer Solution

Design an OP AMP circuit that solves the following second-order differential equation for $v_O(t)$. Solve for the response for $v_O(t)$ using OrCAD. *Caution:* Avoid saturating the OP AMPs by distributing the gain across several OP AMPs.

$$10^{-6}\frac{d^2v_O(t)}{dt^2} + \frac{1}{2}\times 10^{-3}\frac{dv_O(t)}{dt} + v_O(t) = 1.5u(t)$$

Solve the given equation for the highest derivative and reformulate the equation as an integral equation.

$$10^{-6}\frac{d^2v_O(t)}{dt^2} = -\frac{1}{2}\times 10^{-3}\frac{dv_O(t)}{dt} - v_O(t) + 1.5u(t)$$

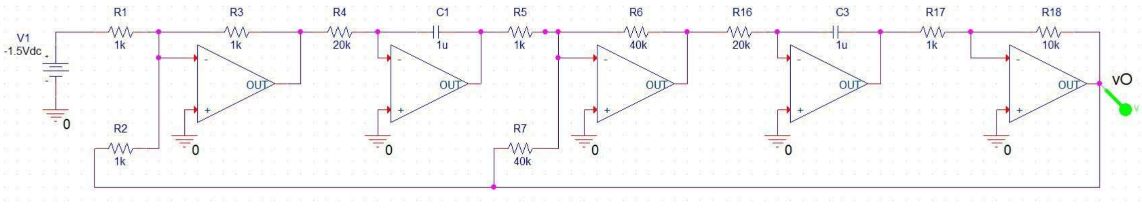
$$\frac{d^2v_O(t)}{dt^2} = -500\frac{dv_O(t)}{dt} - 10^6 v_O(t) + 1.5\times 10^6 u(t)$$

$$v_O(t) = 500\int\left\{-v_O(t) + 2000\left[\int(-v_O(t)+1.5u(t))\right]\right\}$$

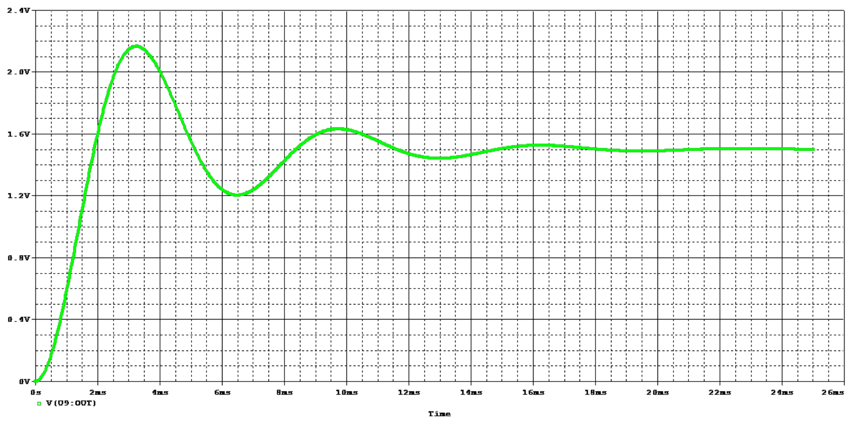
Rewrite the last expression in a form that will be easier to implement with standard OP AMP circuits.

$$v_O(t) = (-10)(-50)\int\left\{-v_O(t) + (-40)(-50)\left[\int(-v_O(t) - (-1.5)u(t))\right]\right\}$$

The following circuit will solve the differential equation.



Set the initial voltages on both capacitors to zero. The resulting output voltage is shown in the following plot.



Problem 6–63. (A,E) *RC* OP AMP Circuit Design

An upgrade to one of your company's robotics products requires a proportional plus integral compensator that implements the input-output relationship

$$v_O(t) = v_S(t) + 50\int_0^t v_S(x)\,dx$$

The input voltage $v_S(t)$ comes from an OP AMP, and the output voltage $v_O(t)$ drives a 10-kΩ resistive load. Two competing designs are shown in Figure P6–63. As the project engineer, you are responsible for recommending one of these designs for production. Which design would you recommend and why? (Your mentor, a wise senior engineer, suggests that you first check that both designs implement the required signal processing function.)

The circuit in Design 1 is a subtractor with two inputs. The upper input uses voltage division to divide the input by two and goes to the positive terminal of the subtractor. The lower input integrates the input and feeds the negative side of the subtractor. Combining these components, we get the following input-output relationship.

$$\begin{aligned} v_{\mathrm{O}}(t) &= \left(\frac{1}{2}\right)(2)v_{\mathrm{S}}(t) - \left[\frac{-1}{(1\,\mu)(20\,\mathrm{k})}\right]\int_0^t v_{\mathrm{S}}(x)\,dx \\ &= v_{\mathrm{S}}(t) + 50\int_0^t v_{\mathrm{S}}(x)\,dx \end{aligned}$$

The first design meets the specification. Apply node-voltage analysis to the circuit in Design 2.

$$\frac{v_{\mathrm{S}}(t)}{R} + C\frac{d(v_{\mathrm{S}}(t) - v_{\mathrm{O}}(t))}{dt} = 0$$

$$\frac{dv_{\mathrm{O}}(t)}{dt} = \frac{dv_{\mathrm{S}}(t)}{dt} + \frac{1}{RC}v_{\mathrm{S}}(t)$$

$$v_{\mathrm{O}}(t) = v_{\mathrm{S}}(t) + \frac{1}{RC}\int_0^t v_{\mathrm{S}}(x)\,dx$$

$$v_{\mathrm{O}}(t) = v_{\mathrm{S}}(t) + \frac{1}{(50\,\mathrm{k})(0.4\,\mu)}\int_0^t v_{\mathrm{S}}(x)\,dx$$

$$v_{\mathrm{O}}(t) = v_{\mathrm{S}}(t) + 50\int_0^t v_{\mathrm{S}}(x)\,dx$$

The second design also meets the specification. Since both designs meet the specification, choose Design 2, since it is simpler than Design 1.

Problem 6–64. (A) Tunable Capacitor

In Section 6–1, Figure 6–2(c) shows an air-tunable capacitor as one example of the capacitor types. This type of device can vary its capacitance similarly to how a potentiometer can vary its resistance. Changing the capacitance in a circuit can change the frequency at which it operates. Suppose we are given the circuit in Figure P6–64 with a capacitor connected in parallel with an inductor. There are no other devices in the circuit. The capacitor has an initial voltage $v_{\mathrm{C}}(0) = V_0$ V and the inductor's initial current is $i_{\mathrm{L}}(0) = 0$ A. The differential equation for the voltage across the capacitor in this circuit is given by

$$\frac{d^2 v_{\mathrm{C}}(t)}{dt^2} + \frac{1}{LC}v_{\mathrm{C}}(t) = 0$$

We will learn more about solving this type of differential equation in the next chapter and beyond. The solution to this differential equation is

$$v_{\mathrm{C}}(t) = V_0\cos\left(\frac{t}{\sqrt{LC}}\right), \quad t \geq 0$$

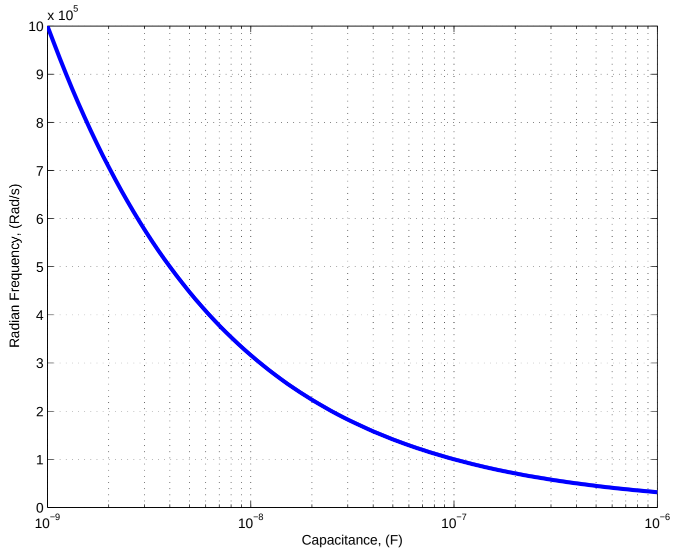
Using MATLAB, plot on a semi-log scale (logarithmic on the horizontal and linear on the vertical,) the radian frequency of $v_{\mathrm{C}}(t)$ versus the capacitance of the circuit. Use capacitances scaled logarithmically from 0.001 μF to 1 μF. In a separate MATLAB figure, use the subplot command to plot $v_{\mathrm{C}}(t)$ versus time for $C = 0.001$ μF, 0.01 μF, 0.1 μF, and 1μF. Using the plots you created and your knowledge of how a capacitor stores charge, explain why changing the capacitance of a parallel LC circuit changes the frequency at which is oscillates.

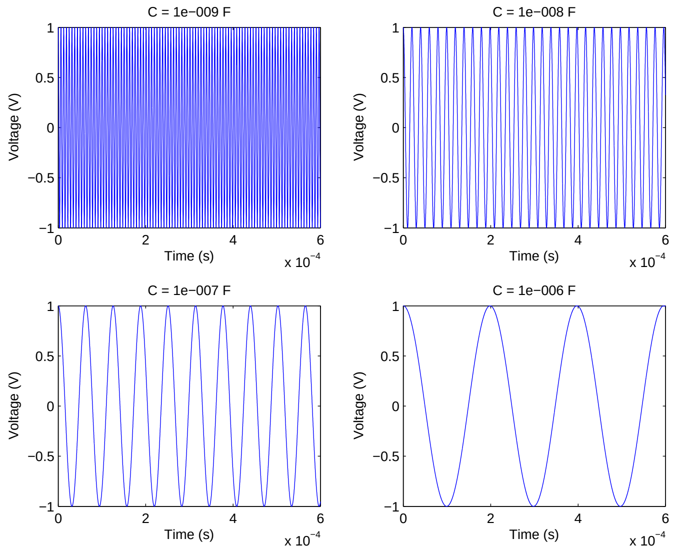
The following MATLAB code provides the solution:

```
syms t L C V0 real

% Solve the differential equation
vct = dsolve('D2y + y/L/C = 0','y(0)=V0','Dy(0)=0','t')

% Specify numerical values for the components
C = logspace(-9,-6,100);
L = 1e-3;

% Evaluate the frequency
w = 1./sqrt(L*C);

% Plot the results
figure
semilogx(C,w,'b','LineWidth',3);
hold on
grid on
xlabel('Capacitance, (F)')
ylabel('Radian Frequency, (Rad/s)')

% Plot the output waveform for different capacitor values
figure
tt = 0:0.0000001:0.0006;
C = [1e-9 10e-9 100e-9 1e-6];
for n = 1:length(C)
    vctt = cos(tt/sqrt(L*C(n)));
    subplot(2,2,n)
    plot(tt,vctt)
    xlabel('Time (s)')
    ylabel('Voltage (V)')
    title(['C = ',num2str(C(n)),' F'])
end
```

The corresponding MATLAB output and plots are shown below.

```
vct = (V0*exp((t*(-C*L)^(1/2))/(C*L)))/2 + V0/(2*exp((t*(-C*L)^(1/2))/(C*L)))
```





The MATLAB result for the capacitor voltage is equivalent to the result presented in the problem statement. As the capacitance increases, the capacitor can store more charge for a given voltage level. Since there is more stored charge, it will take longer for the capacitor to discharge and re-charge as its voltage changes sign. Therefore, the period of the voltage oscillations will increase as the capacitance increases.

Problem 6–65. (A) Equivalent Capacitance Bridge

Find the equivalent capacitance of the capacitance bridge shown in Figure 6–65. (*Hint:* Use Node Analysis.)

Apply the input $v_{\rm S}(t)$ at the top node and ground the bottom node. Label the left node with $v_{\rm A}(t)$ and the right node with $v_{\rm B}(t)$. The circuit is completely symmetric, so voltages $v_{\rm A}(t)$ and $v_{\rm B}(t)$ must match and there is no voltage across the center capacitor. We can remove the center capacitor without changing the performance of the circuit. With the center capacitor removed, we can combine each side of the circuit with series capacitors to get $C/2$ on each side and then combine the two parallel sides to get an overall equivalent capacitance of $C_{\rm EQ} = C$. A formal analysis with node-voltage techniques yields the same results:

$$C\frac{d[v_{\rm A}(t) - v_{\rm S}(t)]}{dt} + C\frac{d[v_{\rm A}(t) - v_{\rm B}(t)]}{dt} + C\frac{dv_{\rm A}(t)}{dt} = 0$$

$$C\frac{d[v_{\rm B}(t) - v_{\rm S}(t)]}{dt} + C\frac{d[v_{\rm B}(t) - v_{\rm A}(t)]}{dt} + C\frac{dv_{\rm B}(t)}{dt} = 0$$

Cancel the capacitance since it is common to all terms. Rewrite the equations and solve for $v_{\rm A}(t)$ and $v_{\rm B}(t)$

in terms of $v_\mathrm{S}(t)$.

$$\frac{d}{dt}\left[v_\mathrm{A}(t) - v_\mathrm{S}(t) + v_\mathrm{A}(t) - v_\mathrm{B}(t) + v_\mathrm{A}(t)\right] = 0$$

$$\frac{d}{dt}\left[v_\mathrm{B}(t) - v_\mathrm{S}(t) + v_\mathrm{B}(t) - v_\mathrm{A}(t) + v_\mathrm{B}(t)\right] = 0$$

$$3v_\mathrm{A}(t) - v_\mathrm{B}(t) = v_\mathrm{S}(t)$$

$$-v_\mathrm{A}(t) + 3v_\mathrm{B}(t) = v_\mathrm{S}(t)$$

$$v_\mathrm{A}(t) = \frac{v_\mathrm{S}(t)}{2}$$

$$v_\mathrm{B}(t) = \frac{v_\mathrm{S}(t)}{2}$$

As presented above, $v_\mathrm{A}(t) = v_\mathrm{B}(t)$, so the analysis above holds and $C_\mathrm{EQ} = C$.

7 First- and Second-Order Circuits

7.1 Exercise Solutions

Exercise 7–1. Find the time constant T_C for circuit C3 in Figure 7–4.

For an RL parallel circuit, the time constant is the equivalent inductance divided by the equivalent resistance, $T_C = L_{EQ}/R_{EQ}$. In Figure 7–4, the resistors are in series and the inductors are in parallel, which gives the following results.

$$R_{EQ} = R_1 + R_2$$

$$L_{EQ} = \frac{L_1 L_2}{L_1 + L_2}$$

$$T_{EQ} = \frac{L_{EQ}}{R_{EQ}} = \frac{L_1 L_2}{(L_1 + L_2)(R_1 + R_2)} \text{ s}$$

Exercise 7–2. The switch in Figure 7–6 closes at $t = 0$. For $t \geq 0$ the current through the resistor is $i_R(t) = e^{-100t}$ mA.

(a). What is the capacitor voltage at $t = 0$?

At $t = 0$, the current through the resistor is $i_R(0) = 1$ mA, so the voltage across the resistor is $v_R(0) = (1\,\text{mA})(10\,\text{k}\Omega) = 10$ V. With the switch closed, the resistor and capacitor are in parallel, so the capacitor voltages is $v(0) = 10$ V.

(b). Write an equation for $v(t)$ for $t \geq 0$.

The capacitor voltage equals the resistor voltage.

$$v(t) = v_R(t) = i_R(t)R = (e^{-100t}\,\text{mA})(10\,\text{k}\Omega) = 10e^{-100t}\,\text{V}$$

(c). Write an equation for the power absorbed by the resistor for $t \geq 0$.

The power absorbed by the resistor is the product of the current and the votlage.

$$p_R(t) = i_R(t)v_R(t) = (e^{-100t}\,\text{mA})(10e^{-100t}\,\text{V}) = 10e^{-200t}\,\text{mW}$$

(d). How much energy does the resistor dissipate for $t \geq 0$?

Integrate the expression for the resistor's power to determine the energy dissipated.

$$w_R = \int_0^\infty 0.01e^{-200t}\,dt = \frac{0.01}{-200}e^{-200t}\bigg|_0^\infty = (-50\,\mu)(0-1) = 50\,\mu\text{J}$$

(e). How much energy is stored in the capacitor at $t = 0$?

Compute the energy in the capacitor.

$$w_C(0) = \frac{1}{2}Cv^2(0) = (0.5)(1\,\mu\text{F})(10\,\text{V})^2 = 50\,\mu\text{J}$$

Exercise 7–3. Find the current through R_2 in Example 7–3.

Apply two-path current division to the current flowing through the inductor.

$$i_{R_2}(t) = \frac{R_1}{R_1 + R_2}i(t) = \frac{2}{2+6}(0.1e^{-37500t}) = 0.025e^{-37500t}\text{ A} \quad t > 0$$

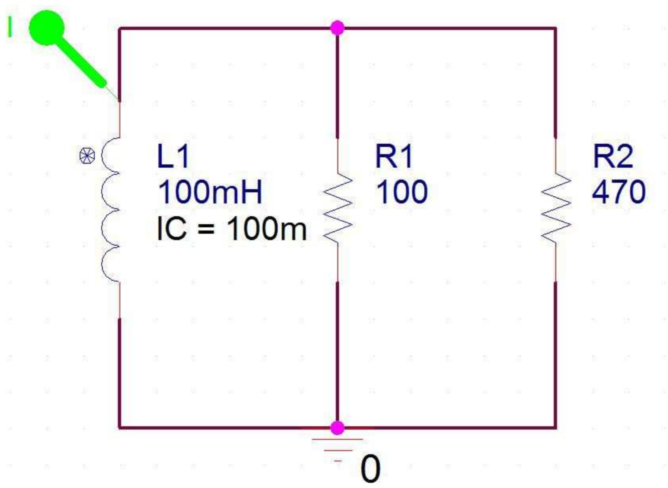
Exercise 7–4. The switch in the RL circuit of Figure 7–9 moves from position A to position B at $t = 0$. If the current flowing through the inductor at $t = 0$ is 1 mA, how long after the switch moves to position B does it take for the voltage across the resistor to reach -5 V?

After the switch moves, we have an RL parallel circuit with a time constant $T_{\rm C} = L/R = 0.1/10000 = 10\ \mu$s. The current through the inductor also flows through the resistor. Compute the requested time as follows.

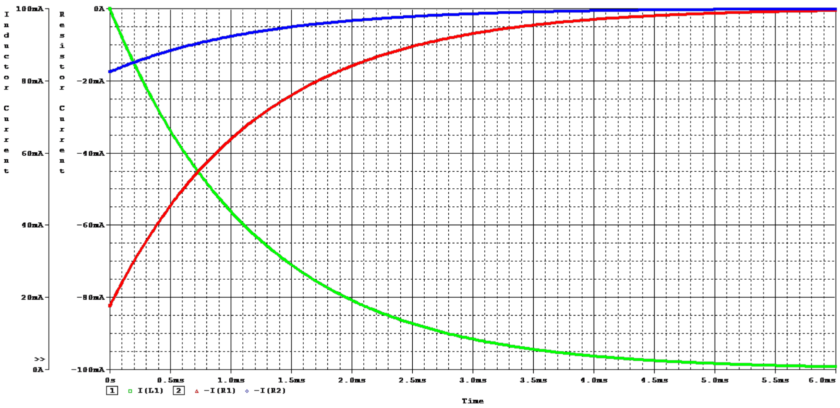
$$\begin{aligned}
T_{\rm C} &= 10\ \mu\text{s} \\
\frac{1}{T_{\rm C}} &= 100000 \\
i_{\rm L}(0) &= 1\ \text{mA} \\
i_{\rm L}(t) &= i_{\rm L}(0)e^{-t/T_{\rm C}} = e^{-100000t}\ \text{mA} \\
v_{\rm R}(t) &= -i_{\rm L}(t)R = -10e^{-100000t}\ \text{V} \\
-5 &= -10e^{-100000t} \\
\frac{1}{2} &= e^{-100000t} \\
t &= 6.93147\ \mu\text{s}
\end{aligned}$$

Exercise 7–5. A 100-mH inductor and two resistors are all connected in parallel. One resistor is 100 Ω and the second is 470 Ω. At time $t = 0$, the inductor has 100 mA flowing through it. Use OrCAD to find a plot of the current through the inductor for $t \geq 0$. Then on a separate y-axis, plot the current through each resistor.

The following OrCAD simulation provides the solution.



The corresponding plot is shown below.



Exercise 7–6. Use the results from Example 7–6 and find the current through R_1 in Figure 7–15 for $t \geq 0$.

Apply KVL to solve for the voltage across R_1 and then apply Ohm's law to solve for the current through R_1.

$$R_1 = 30 \text{ k}\Omega$$

$$v(t) = 25 - 10e^{-1600t} \text{ V}$$

$$V_{\text{A}} = v_{\text{R}_1}(t) + v(t)$$

$$v_{\text{R}_1}(t) = V_{\text{A}} - v(t) = 100 - 25 + 10e^{-1600t} = 75 + 10e^{-1600t} \text{ V}$$

$$i_{\text{R}_1}(t) = \frac{v_{\text{R}_1}(t)}{R_1} = 2.5 + 0.333e^{-1600t} \text{ mA} \quad t \geq 0$$

Exercise 7–7. Use the results from Example 7–7 and find the voltage across the current source in Figure 7–16(a) for $t \geq 0$.

The voltage across the current source is the same as the voltage across resistor R_1.

$$i(t) = \left[I_0 - \frac{R_1 I_{\text{A}}}{R_1 + R_2}\right] e^{-(R_1+R_2)t/L} + \frac{R_1 I_{\text{A}}}{R_1 + R_2} \quad t \geq 0$$

$$i_{\text{R}_1}(t) = I_{\text{A}} - i(t) = I_{\text{A}} - \frac{R_1 I_{\text{A}}}{R_1 + R_2} + \left[\frac{R_1 I_{\text{A}}}{R_1 + R_2} - I_0\right] e^{-(R_1+R_2)t/L}$$

$$v_{\text{S}}(t) = v_{\text{R}_1}(t) = i_{\text{R}_1}(t) R_1$$

$$v_{\text{S}}(t) = R_1 I_{\text{A}} \left[1 - \frac{R_1}{R_1 + R_2}\right] + \left[\frac{R_1 I_{\text{A}}}{R_1 + R_2} - I_0\right] R_1 e^{-(R_1+R_2)t/L} \text{ V} \quad t \geq 0$$

Exercise 7–8. Given the first-order circuit step response

$$v_{\text{C}}(t) = 20 - 20e^{-1000t} \text{ V} \quad t \geq 0$$

(a). What is the amplitude of the step input?

The amplitude of the step is the constant value in the expression for $v_{\text{C}}(t)$, which is 20 V.

(b). What is the circuit time constant?

From the exponential, the time constant is $T_{\text{C}} = 1/1000 = 1$ ms.

(c). What is the initial value of the state variable?

The initial value is $v_{\text{C}}(0) = 20 - 20 = 0$ V.

(d). What is the circuit differential equation?

Given the element identified above, we can write the differential equation as follows.

$$T_{\mathrm{C}}\frac{dv_{\mathrm{C}}(t)}{dt} + v_{\mathrm{C}}(t) = V_{\mathrm{A}}u(t)$$

$$\frac{1}{1000}\frac{dv_{\mathrm{C}}(t)}{dt} + v_{\mathrm{C}}(t) = 20u(t)$$

Exercise 7–9. Find the solutions of the following first-order differential equations:

(a). $10^{-4}\dfrac{dv_{\mathrm{C}}(t)}{dt} + v_{\mathrm{C}}(t) = -5u(t)$, $v_{\mathrm{C}}(0) = 5$ V.

The solution has the following form

$$v_{\mathrm{C}}(t) = (V_0 - V_{\mathrm{A}})\, e^{-t/T_{\mathrm{C}}} + V_{\mathrm{A}}$$

$$T_{\mathrm{C}} = 10^{-4}$$

$$\frac{1}{T_{\mathrm{C}}} = 10000$$

$$V_0 = v_{\mathrm{C}}(0) = 5 \text{ V}$$

$$V_{\mathrm{A}} = -5 \text{ V}$$

$$v_{\mathrm{C}}(t) = (5 - (-5))\, e^{-10000t} - 5$$

$$v_{\mathrm{C}}(t) = -5 + 10e^{-10000t} \text{ V} \quad t \geq 0$$

(b). $5 \times 10^{-2}\dfrac{di_{\mathrm{L}}(t)}{dt} + 2000i_{\mathrm{L}}(t) = 10u(t)$, $i_{\mathrm{L}}(0) = -5$ mA.

Rewrite the original expression to put it in a standaard form.

$$25 \times 10^{-6}\frac{di_{\mathrm{L}}(t)}{dt} + i_{\mathrm{L}}(t) = 0.005u(t)$$

The initial current does not change. The solution has the following form

$$i_{\mathrm{L}}(t) = (I_0 - I_{\mathrm{A}})\, e^{-t/T_{\mathrm{C}}} + I_{\mathrm{A}}$$

$$T_{\mathrm{C}} = 25 \times 10^{-6}$$

$$\frac{1}{T_{\mathrm{C}}} = 40000$$

$$I_0 = i_{\mathrm{L}}(0) = -5 \text{ mA}$$

$$I_{\mathrm{A}} = 5 \text{ mA}$$

$$i_{\mathrm{L}}(t) = (-5 - 5)\, e^{-40000t} + 5$$

$$i_{\mathrm{L}}(t) = 5 - 10e^{-40000t} \text{ mA} \quad t \geq 0$$

Exercise 7–10. The switch in Figure 7–18 closes at $t = 0$. Find the zero-state response of the capacitor voltage for $t \geq 0$.

Find the Thévenin equivalent voltage across the capacitor and the time constant. Treat the capacitor as an open circuit to find the Thévenin voltage. Combine the right 10-kΩ and 5-kΩ resistors in series and then combine that result, 15 kΩ, in parallel with the center 10-kΩ resistor. The result is a 6-kΩ resistor in series with the left 10-kΩ resistor. Perform voltage division to get 3.75 V across the 6-kΩ equivalent resistance. Perform voltage division a second time to get 2.5 V across the right 10-kΩ resistor, which is the Thévenin voltage seen by the capacitor. Use the lookback technique to find the equivalent resistance as follows

$$R_{\mathrm{EQ}} = 10 \parallel [5 + (10 \parallel 10)] = 5 \text{ k}\Omega$$

The time constant is $T_{\mathrm{C}} = R_{\mathrm{EQ}}C = (5000)(1\,\mu) = 5$ ms and $1/T_{\mathrm{C}} = 1/0.005 = 200$ Hz. Write the zero-state response as follows

$$v_{\mathrm{C}}(t) = V_{\mathrm{A}}(1 - e^{-t/T_{\mathrm{C}}}) = 2.5(1 - e^{-200t}) \text{ V}$$

Exercise 7–11. The switch in Figure 7–19 opens at $t = 0$. Find the zero-state response of the inductor current for $t \geq 0$.

With the switch closed, all of the current flows through the switch's short circuit. After the switch opens, treat the inductor as a short circuit to find the current. The current splits equally between the two paths, because the resistors are equal. The current seen by the inductor is 5 mA. The equivalent resistance seen by the inductor is the series equivalent of the two 25-Ω resistors, which is 50 Ω. The time constant is $T_{\mathrm{C}} = L/R = 0.2/50 = 4$ ms and $1/T_{\mathrm{C}} = 1/0.004 = 250$. Write the zero-state response as follows

$$i_{\mathrm{L}}(t) = I_{\mathrm{A}}(1 - e^{-t/T_{\mathrm{C}}}) = 5(1 - e^{-250t}) \text{ mA}$$

Exercise 7–12. The switch in Figure 7–22(a) has been closed for a long time. The switch opens at $t = 0$. Find the inductor current for $t \geq 0$.

With the switch closed, resistor R_2 is not contributing to the circuit and the inductor acts as a short circuit. The initial current through the inductor is $i_{\mathrm{L}}(0) = V_{\mathrm{A}}/R_1$. After the switch opens, resistor R_2 contributes to the circuit and the final current flowing through the inductor will be $i_{\mathrm{L}}(\infty) = V_{\mathrm{A}}/(R_1 + R_2)$. The time constant is $T_{\mathrm{C}} = L/R_{\mathrm{EQ}} = L/(R_1 + R_2)$. Write the expression for the inductor current directly

$$i_{\mathrm{L}}(t) = [i_{\mathrm{L}}(0) - i_{\mathrm{L}}(\infty)]\, e^{-t/T_{\mathrm{C}}} + i_{\mathrm{L}}(\infty)$$

$$i_{\mathrm{L}}(t) = \left[\frac{V_{\mathrm{A}}}{R_1} - \frac{V_{\mathrm{A}}}{R_1 + R_2}\right] e^{-(R_1+R_2)t/L} + \frac{V_{\mathrm{A}}}{R_1 + R_2} \text{ A} \quad t \geq 0$$

Exercise 7–13. The switch in the circuit of Figure 7–23 has been closed for a long time. It opens at $t = 0$. Find the voltage $v_{\mathrm{C}}(t)$ and the current $i_{\mathrm{R}}(t)$ for $t \geq 0$.

Find the capacitor voltage first, and then solve for the resistor current. With the switch closed, the capacitor is connected across one resistor in a voltage divider circuit. The resistors are equal, so the initial capacitor voltage is half of the source voltage, $v_{\mathrm{C}}(0) = 25$ V. After the switch opens, the capacitor is still connected across a voltage divider, but the voltage has changed to $v_{\mathrm{C}}(\infty) = (50)(20 + 10)/(20 + 10 + 20) = 30$ V. With the switch open the equivalent resistance is $R_{\mathrm{EQ}} = 20 \parallel (10 + 20) = 12$ kΩ and the time constant is $T_{\mathrm{C}} = R_{\mathrm{EQ}}C = (12000)(0.1\,\mu) = 1.2$ ms. We also have $1/T_{\mathrm{C}} = 1/0.0012 = 833.3$ Hz. Write the expression for the capacitor voltage directly and then compute the resistor current.

$$v_{\mathrm{C}}(t) = [v_{\mathrm{C}}(0) - v_{\mathrm{C}}(\infty)]\, e^{-t/T_{\mathrm{C}}} + v_{\mathrm{C}}(\infty)$$

$$v_{\mathrm{C}}(t) = [25 - 30]\, e^{-833.3t} + 30 = 30 - 5e^{-833.3t} \text{ V} \quad t \geq 0$$

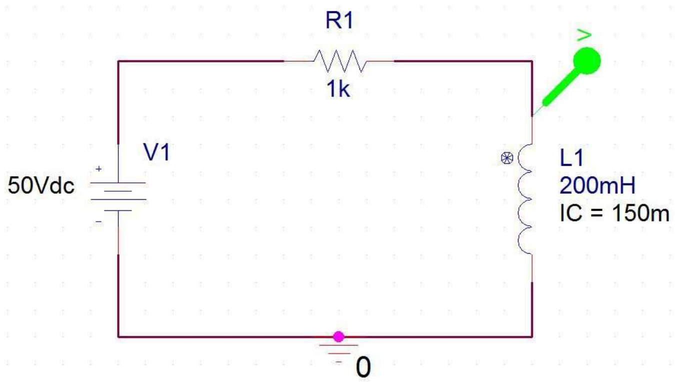
$$i_{\mathrm{R}}(t) = \frac{v_{\mathrm{C}}(t)}{(10 + 20)\,\text{k}\Omega} = 1 - 0.1667e^{-833.3t} \text{ mA} \quad t \geq 0$$

Exercise 7–14. Design a first-order RL circuit that will produce the following current through the inductor: $i_{\mathrm{L}}(t) = 5 - 5e^{-500t}$ mA for $t > 0$. Use standard values for the components.

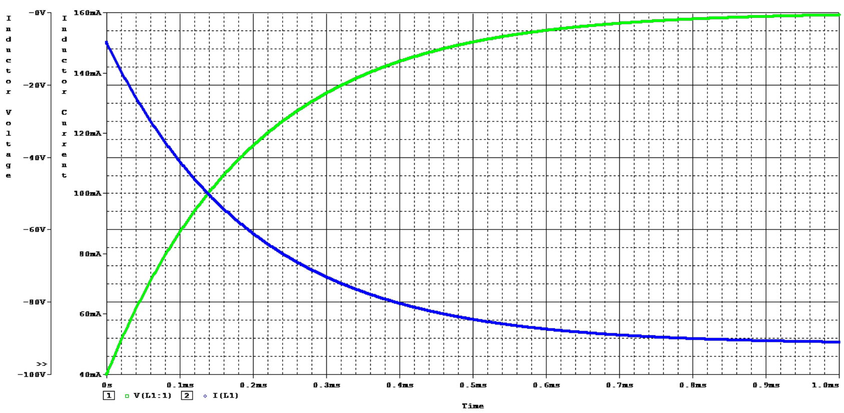
The initial current through the inductor is $i_L(0) = 5 - 5 = 0$ mA. The final current through the inductor is $i_L(\infty) = 5 - 0 = 5$ mA. The time constant is $T_C = L/R = 1/500 = 2$ ms. Choose $L = 2$ mH. We then need $R = L/T_C = 0.002/0.002 = 1\ \Omega$ for the equivalent resistance. The final current is 5 mA, so connect a $5u(t)$-mA current source in parallel with a 2-mH inductor and a 1-Ω resistor to create the circuit. The inductor has no initial current before the source is connected, $i_L(0) = 0$.

Exercise 7–15. Use OrCAD to simulate a transient response for the circuit in Figure 7–25 and find $i_L(t)$ and $v_L(t)$. (*Hint:* Be certain to add the initial condition to the inductor and to orient it correctly.)

The following OrCAD simulation provides the results.



The corresponding plot is shown below.



Exercise 7–16. In each circuit shown in Figure 7–27 the switch has been in position A for a long time and is moved to position B at $t = 0$. Find the circuit state variable for each circuit for $t \geq 0$.

(a). Apply KVL to find the initial voltage $v_C(0) = V_A$, since no current is flowing before the switch moves. After the switch moves, the final voltage will be $v_C(\infty) = 0$ V. The time constant is $T_C = (R_1 + R_2)C$.

$$v_C(t) = [v_C(0) - v_C(\infty)]\, e^{-t/T_C} + v_C(\infty)$$

$$v_C(t) = V_A e^{-t/(R_1+R_2)C} \text{ V}$$

(b). The initial current is $i_L(0) = V_A/R_2$. The final current is $i_L(\infty) = 0$ A. The time constant is $T_C = L/(R_1 + R_2)$.

$$i_L(t) = [i_L(0) - i_L(\infty)]\, e^{-t/T_C} + i_L(\infty)$$

$$i_L(t) = \frac{V_A}{R_2} e^{-(R_1+R_2)t/L} \text{ A}$$

Exercise 7–17. In the circuit in Figure 7–28 the switch has been in position A for a long time and is moved to position B at $t = 0$. For $t \geq 0$ find the output voltage $v_O(t)$.

Find the inductor current first and then the output voltage. Before the switch moves, the inductor acts as a short circuit and its initial current is $i_L(0) = (10\,\text{V})/(50\,\Omega) = 0.2$ A. After the switch moves, the final current is $i_L(\infty) = (-10\,\text{V})/(50\,\Omega) = -0.2$ A. After the switch moves, the equivalent resistance seen by the inductor is $R_{EQ} = 50 \parallel (50 + 25) = 30\ \Omega$. The time constant is $T_C = L/R_{EQ} = 0.15/30 = 5$ ms and $1/T_C = 200$ Hz. Write the expression for the inductor current, solve for the inductor voltage, and then apply voltage division to determine the output voltage.

$$i_L(t) = [i_L(0) - i_L(\infty)]\, e^{-t/T_C} + i_L(\infty)$$

$$i_L(t) = [0.2 - (-0.2)]\, e^{-200t} - 0.2 = 0.4e^{-200t} - 0.2 \text{ A}$$

$$v_L(t) = L\frac{di_L(t)}{dt} = (0.15)(0.4)(-200)e^{-200t} = -12e^{-200t} \text{ V}$$

$$v_O(t) = \left(\frac{25}{50 + 25}\right) v_L(t) = -4e^{-200t} \text{ V}$$

Exercise 7–18. The capacitor in the circuit of Figure 7–29 is in the zero state. Find the voltage across and the current through the capacitor for $t \geq 0$.

Find the Thévenin equivalent source seen by the capacitor. The lookback resistance is $R_{EQ} = 300 \parallel 200 = 120$ kΩ. Apply voltage division to find the voltage $v_T(t) = (200v_S(t))/(200 + 300) = 0.4v_S(t) = 40e^{-1000t}u(t)$ V. The time constant for the circuit is $T_C = R_{EQ}C = (120\,\text{k}\Omega)(0.001\,\mu\text{F}) = 120\ \mu$s and $1/T_C = 8333$ Hz. The natural and forced responses have the following forms:

$$v_N(t) = Ke^{-8333t}$$

$$v_F(t) = K_F e^{1000t}$$

The differential equation for the force response is

$$(120\,\mu)\frac{dv_F(t)}{dt} + v_F(t) = 40e^{-1000t}$$

Substitute in for $v_F(t)$ and solve for K_F. Then solve for $v_C(t)$.

$$(120\,\mu)\frac{dv_F(t)}{dt} + v_F(t) = 40e^{-1000t}$$

$$(120\,\mu)(K_F)(-1000)e^{-1000t} + K_F e^{-1000t} = 40e^{-1000t}$$

$$K_F = \frac{40}{1 - (1000)(120\,\mu)} = 45.455$$

$$v_C(t) = v_N(t) + v_F(t) = Ke^{-8333t} + 45.455e^{-1000t}$$

$$v_C(0) = 0 = K + 45.455$$

$$K = -45.455$$

$$v_C(t) = 45.455\left(e^{-1000t} - e^{-8333t}\right) \text{ V} \quad t \geq 0$$

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

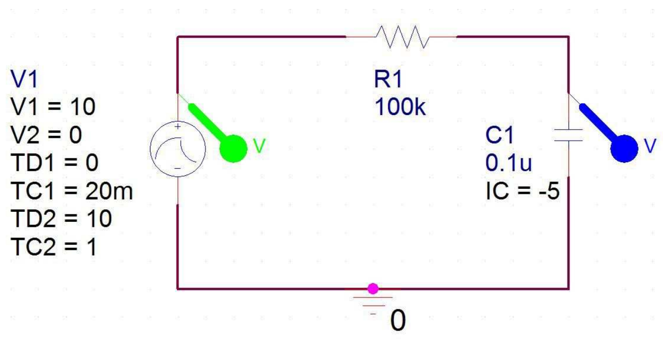
$$i_C(t) = (0.001\,\mu)(45.455)\left[-1000e^{-1000t} - (-8333)e^{-8333t}\right]$$

$$i_C(t) = -45.455e^{-1000t} + 378.79e^{-8333t}\ \mu\text{A} \quad t \geq 0$$

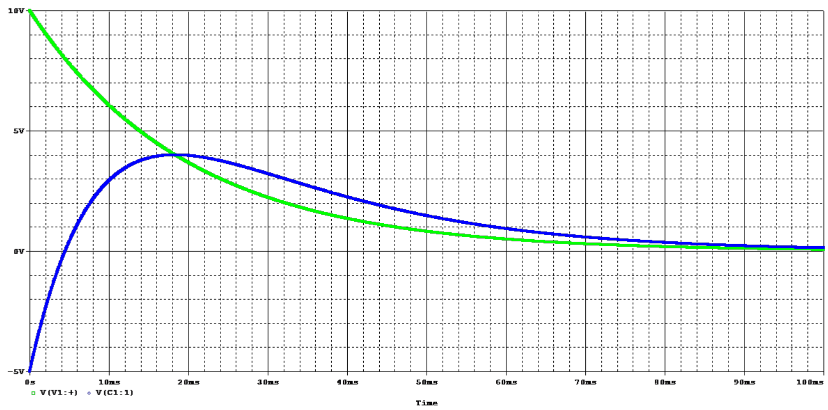
Exercise 7–19. The circuit in Figure 7–2(a) has $R_T = 100$ kΩ, and $C = 0.1\mu$F, and it is driven by $v_T(t) = 10e^{-50t}$ V. The capacitor has an initial voltage of -5 V.

(a). Use OrCAD to find the transient response of the capacitor voltage.

The following OrCAD simulation provides the results.



The corresponding plot is shown below.



(b). Determine which is the dominant exponential.

The forcing function has the dominant exponential with a 20-ms time constant. The circuit's time constant is only $T_C = (100\,\text{k}\Omega)(0.1\mu\text{F}) = 10$ ms and it quickly decays away, leaving only that of the forcing function.

Exercise 7–20. Find the sinusoidal steady-state response of the output voltage $v_O(t)$ in Figure 7–32 when the input current is $i_S(t) = [I_A \cos(\omega t)]\, u(t)$ A.

Write the corresponding differential equation and then develop the solution.

$$\frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = I_A \cos(\omega t)u(t)$$

$$i_L(t) = a\cos(\omega t) + b\sin(\omega t)$$

$$\frac{di_L(t)}{dt} = -a\omega\sin(\omega t) + b\omega\cos(\omega t)$$

$$\frac{L}{R}\left[-a\omega\sin(\omega t) + b\omega\cos(\omega t)\right] + a\cos(\omega t) + b\sin(\omega t) = I_A\cos(\omega t)$$

$$\left(\frac{Lb\omega}{R} + a - I_A\right)\cos(\omega t) + \left(b - \frac{a\omega L}{R}\right)\sin(\omega t) = 0$$

$$a + \frac{L\omega}{R} b = I_{\mathrm{A}}$$

$$-\frac{\omega L}{R} a + b = 0$$

$$a = \frac{I_{\mathrm{A}} R^2}{L^2\omega^2 + R^2}$$

$$b = \frac{I_{\mathrm{A}} LR\omega}{L^2\omega^2 + R^2}$$

$$i_{\mathrm{L}}(t) = \frac{I_{\mathrm{A}} R}{L^2\omega^2 + R^2}\left[R\cos(\omega t) + L\omega\sin(\omega t)\right]$$

$$v_{\mathrm{L}}(t) = L\frac{di_{\mathrm{L}}(t)}{dt} = \frac{I_{\mathrm{A}} RL\omega}{L^2\omega^2 + R^2}\left[-R\sin(\omega t) + L\omega\cos(\omega t)\right]$$

$$\theta = \tan^{-1}\left(\frac{-b}{a}\right) = \tan^{-1}\left(\frac{R}{L\omega}\right)$$

$$V_{\mathrm{A}} = \sqrt{a^2 + b^2} = \sqrt{\frac{I_{\mathrm{A}}^2 R^2 L^2\omega^2(R^2 + L^2\omega^2)}{(R^2 + L^2\omega^2)^2}} = \frac{I_{\mathrm{A}} L\omega}{\sqrt{1 + \left(\frac{L\omega}{R}\right)^2}}$$

$$v_{\mathrm{O}}(t) = V_{\mathrm{A}}\cos(\omega t + \theta)\ \mathrm{V} \quad t \geq 0$$

Exercise 7–21. Find the forced component solution of the differential equation

$$10^{-3}\frac{dv(t)}{dt} + v(t) = 10\cos(\omega t)\ \mathrm{V}$$

for the following frequencies:

(a). $\omega = 500$ rad/s

The solution to the differential equation has the form

$$v(t) = a\cos(\omega t) + b\sin(\omega t)$$

Solve for a and b in terms of ω and substitute in the different values for ω.

$$\frac{dv(t)}{dt} = -a\omega\sin(\omega t) + b\omega\cos(\omega t)$$

$$\frac{1}{1000}\left[-a\omega\sin(\omega t) + b\omega\cos(\omega t)\right] + a\cos(\omega t) + b\sin(\omega t) = 10\cos(\omega t)$$

$$a + \frac{\omega}{1000} b = 10$$

$$-\frac{\omega}{1000} a + b = 0$$

$$a = \frac{10^7}{\omega^2 + 10^6}$$

$$b = \frac{10^4\omega}{\omega^2 + 10^6}$$

For $\omega = 500$ rad/s, $a = 8$ and $b = 4$, which yields

$$v_\text{F}(t) = 8\cos(500t) + 4\sin(500t) \text{ V} \quad t \geq 0$$

(b). $\omega = 1000$ rad/s

Using the results from part (a), for $\omega = 1000$ rad/s, $a = 5$ and $b = 5$, which yields

$$v_\text{F}(t) = 5\cos(1000t) + 5\sin(1000t) \text{ V} \quad t \geq 0$$

(c). $\omega = 2000$ rad/s

Using the results from part (a), for $\omega = 2000$ rad/s, $a = 2$ and $b = 4$, which yields

$$v_\text{F}(t) = 2\cos(2000t) + 4\sin(2000t) \text{ V} \quad t \geq 0$$

Exercise 7–22. The circuit in Figure 7–33 is operating in the sinusoidal steady state with

$$v_\text{O}(t) = 10\cos(100t - 45^\circ) \text{ V}$$

Find the source voltage $v_\text{S}(t)$.

Find the current through the circuit and then find the voltage across the inductor. Sum the inductor and resistor voltages to find the source voltage.

$$v_\text{O}(t) = 10\cos(100t - 45^\circ) = 7.07\cos(100t) + 7.07\sin(100t) \text{ V}$$

$$i(t) = \frac{v_\text{O}(t)}{100} = 0.0707\cos(100t) + 0.0707\sin(100t) \text{ A}$$

$$v_\text{L}(t) = L\frac{di_\text{L}(t)}{dt} = (1)[-7.07\sin(100t) + 7.07\cos(100t)] \text{ V}$$

$$v_\text{S}(t) = v_\text{L}(t) + v_\text{O}(t) = 14.14\cos(100t) = 10\sqrt{2}\cos(100t) \text{ V}$$

Exercise 7–23. For a series RLC circuit:

(a). Find the roots of the characteristic equation when $R_\text{T} = 2$ kΩ, $L = 100$ mH, and $C = 0.4$ μF.

The roots of the characteristic equation for a series RLC circuit are

$$s_1,\ s_2 = \frac{-R_\text{T}C \pm \sqrt{(R_\text{T}C)^2 - 4LC}}{2LC}$$

Substitute in the given values to find $s_1 = -1340$, $s_2 = -18660$.

(b). For $L = 100$ mH, select the values of R_T and C so the roots of the characteristic equation are s_1, $s_2 = -1000 \pm j2000$.

We can write the characteristic equation as follows

$$(s - s_1)(s - s_2) = s^2 - (s_1 + s_2)s + s_1 s_2 = 0$$

$$\frac{1}{s_1 s_2}s^2 - \frac{s_1 + s_2}{s_1 s_2}s + 1 = 0$$

$$(s + 1000 - j2000)(s + 1000 + j2000) = s^2 + 2000s + 5000000 = 0$$

$$\frac{1}{5000000}s^2 + \frac{2000}{5000000}s + 1 = 0$$

Compare the characteristic equation to the standard form.

$$LCs^2 + R_{\mathrm{T}}Cs + 1 = 0$$

Solve for R_{T} and C.

$$LC = \frac{1}{5000000}$$

$$C = \frac{1}{5000000L} = \frac{1}{(5000000)(0.1)} = 2\ \mu\mathrm{F}$$

$$R_{\mathrm{T}}C = \frac{2000}{5000000}$$

$$R_{\mathrm{T}} = \frac{2000}{5000000C} = \frac{2000}{(5000000)(2\,\mu)} = 200\ \Omega$$

(c). Select the values of R_{T}, L, and C so $s_1 = s_2 = -10^4$.

Write the characteristic equation, put it in standard form, select a value for C, and solve for the other two values.

$$(s - s_1)(s - s_2) = s^2 - (s_1 + s_2)s + s_1s_2 = s^2 + 20000s + 10^8 = 0$$

$$\frac{1}{10^8}s^2 + \frac{20000}{10^8}s + 1 = 0$$

$$LCs^2 + R_{\mathrm{T}}Cs + 1 = 0$$

$$C = 1\ \mu\mathrm{F}$$

$$L = \frac{1}{10^8C} = \frac{1}{(10^8)(10^{-6})} = 10\ \mathrm{mH}$$

$$R_{\mathrm{T}} = \frac{20000}{10^8C} = \frac{20000}{(10^8)(10^{-6})} = 200\ \Omega$$

There are other correct answers.

Exercise 7–24. The circuit in Figure 7–35 has $C = 0.02\ \mu$F and $L = 100$ mH. Select a value for R that will produce the critically damped case.

For the critically damped case, there are two real, equal roots and we have the following relationships:

$$R^2C^2 = 4LC$$

$$R^2 = \frac{4L}{C}$$

$$R = 2\sqrt{\frac{L}{C}} = 2\sqrt{\frac{0.1}{0.02\,\mu}} = 4.472\ \mathrm{k}\Omega$$

Exercise 7–25. In a series RLC circuit, $R = 250\ \Omega$, $L = 10$ mH, $C = 1\ \mu$F, $V_0 = 0$ V, and $I_0 = 30$ mA. Find the capacitor voltage and inductor current for $t \geq 0$.

The characteristic equation is

$$LCs^2 + RCs + 1 = 0$$

The roots of the equation are

$$s_1,\ s_2 = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC}$$

Solving for the given values, we have $s_1 = -5000$ and $s_2 = -20000$. The expression for the capacitor voltage has the following form

$$v_C(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} = K_1 e^{-5000t} + K_2 e^{-20000t}$$

Use the initial conditions to solve for K_1 and K_2.

$$v_C(0) = 0 = K_1 + K_2$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\left(-5000K_1 e^{-5000t} - 20000K_2 e^{-20000t}\right)$$

$$i_C(0) = 0.03 = -5000CK_1 - 20000CK_2$$

$$K_1 = 2$$

$$K_2 = -2$$

$$v_C(t) = 2e^{-5000t} - 2e^{-20000t} \text{ V}$$

$$i_C(t) = -10e^{-5000t} + 40e^{-20000t} \text{ mA}$$

Exercise 7–26. In a series RLC circuit the zero-input responses are

$$v_C(t) = 2000te^{-500t} \text{ V}$$

$$i_L(t) = 3.2e^{-500t} - 1600te^{-500t} \text{ mA}$$

(a). Find the circuit characteristic equation.

Based on the form of the zero-input responses, the circuit has repeated real roots at $s = -500$. The characteristic equation is

$$(s + 500)^2 = s^2 + 1000s + 250000 = 0$$

(b). Find the initial values of the state variables.

Evaluate the expressions at $t = 0$.

$$v_C(0) = (2000)(0)(1) = 0 \text{ V}$$

$$i_L(0) = (3.2)(1) - (1600)(0)(1) = 3.2 \text{ mA}$$

(c). Find R, L, and C.

Use the expressions for voltage and current to find C. Write the characteristic equation in terms of R,

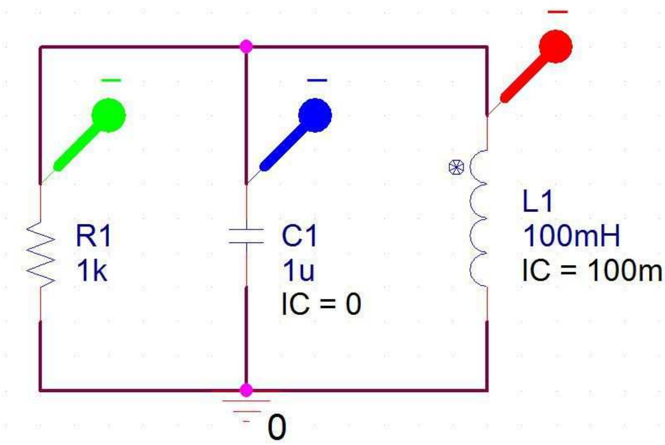
L, and C and then solve for R and L.

$$i_{\mathrm{L}}(t) = i_{\mathrm{C}}(t) = C\frac{dv_{\mathrm{C}}(t)}{dt}$$
$$0.0032e^{-500t} - 1.6te^{-500t} = C\left[(2000t)(-500)e^{-500t} + 2000e^{-500t}\right]$$
$$0.0032 = 2000C$$
$$C = 1.6\ \mu\mathrm{F}$$
$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$
$$\frac{1}{LC} = 250000$$
$$L = \frac{1}{250000C} = 2.5\ \mathrm{H}$$
$$\frac{R}{L} = 1000$$
$$R = 1000L = 2.5\ \mathrm{k}\Omega$$

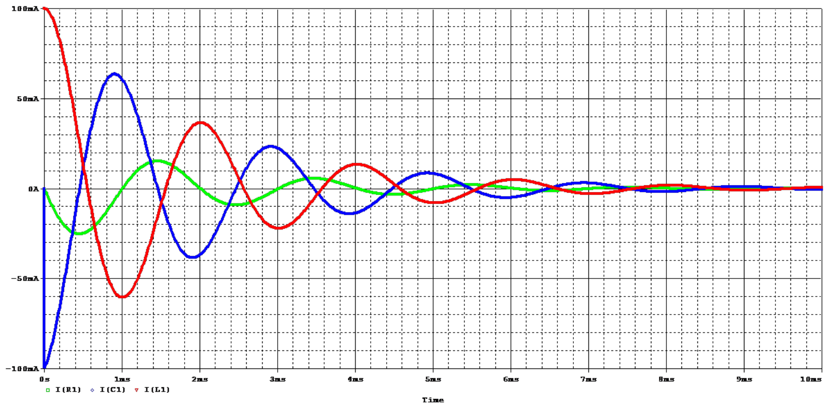
Exercise 7–27. A parallel RLC circuit has $R = 1$ kΩ, $C = 1$ μF, and $L = 100$ mH. The initial conditions are $I_0 = 100$ mA and $V_0 = 0$ V.

(a). Use OrCAD to plot the zero-input response of the inductor, resistor, and capacitor currents on one y-axis.

The following OrCAD simulation provides the results.



The corresponding plot is shown below.



(b). From your current plots, show that Kirchhoffs current law (KCL) holds for every instant.

At any point in time, the three currents always sum to zero, thereby validating Kirchhoffs current law. See the plot of the sum of all three currents in part (a).

Exercise 7–28. The zero-input responses of a parallel RLC circuit are observed to be

$$i_{\rm L}(t) = 10te^{-2000t} \text{ A}$$

$$v_{\rm C}(t) = 10e^{-2000t} - 20000te^{-2000t} \text{ V}$$

(a). What is the circuit characteristic equation?

The circuit has repeated real poles at $s = -2000$, so the characteristic equation is

$$(s + 2000)^2 = s^2 + 4000s + 4000000 = 0$$

(b). What are the initial values of the state variables?

Evaluate the expressions for the state variables at $t = 0$.

$$i_{\rm L}(0) = (10)(0)(1) = 0 \text{ A}$$

$$v_{\rm C}(0) = (10)(1) - (20000)(0)(1) = 10 \text{ V}$$

(c). What are the values of R, L, and C?

Use the expressions for current and voltage to find L. Write the characteristic equation in terms of R,

L, and C and then solve for R and C.

$$v_{\mathrm{L}}(t) = v_{\mathrm{C}}(t) = L\frac{di_{\mathrm{L}}(t)}{dt}$$

$$10e^{-2000t} - 20000te^{-2000t} = L\left[(10t)(-2000)e^{-2000t} + 10e^{-2000t}\right]$$

$$10 = 10L$$

$$L = 1 \text{ H}$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$\frac{1}{LC} = 4000000$$

$$C = \frac{1}{4000000L} = 0.25\ \mu\text{F}$$

$$\frac{1}{RC} = 4000$$

$$R = \frac{1}{4000C} = 1\text{ k}\Omega$$

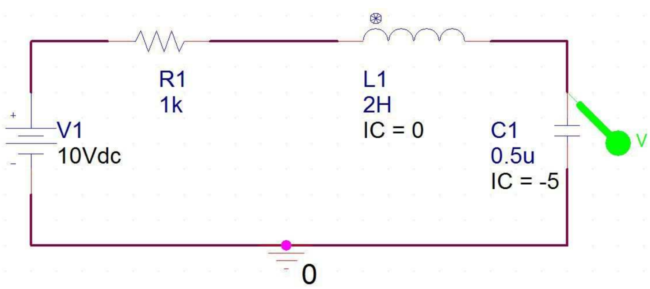
(d). Write an expression for the current through the resistor.

The voltage across the capacitor also appears across the resistor.

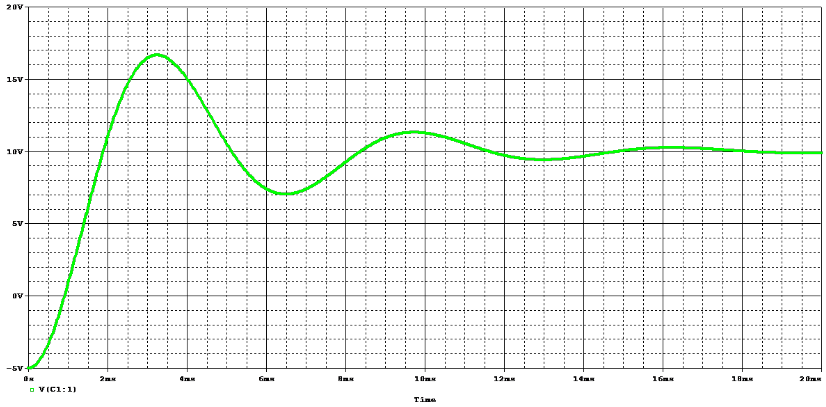
$$i_{\mathrm{R}}(t) = \frac{v_{\mathrm{C}}(t)}{R} = \frac{10e^{-2000t} - 20000te^{-2000t}}{1000} = 10e^{-2000t} - 20000te^{-2000t}\text{ mA} \quad t \geq 0$$

Exercise 7–29. The series RLC circuit of Figure 7–40(a) is excited by a 10-V step and has the initial conditions $i_{\mathrm{L}}(0) = 0$ A and $v_{\mathrm{C}}(0) = -5$ V. Use OrCAD to plot the response of the capacitor voltage for $t \geq 0$.

The following OrCAD simulation provides the results.

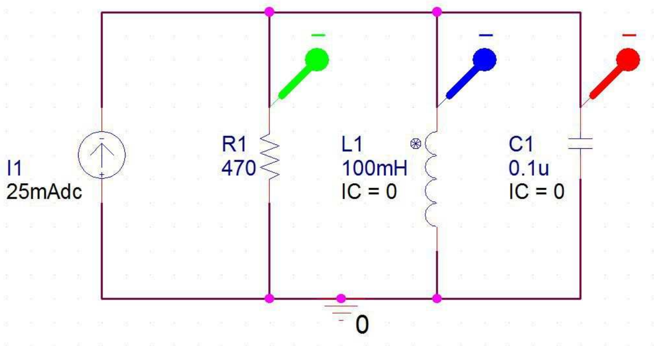


The corresponding plot is shown below.

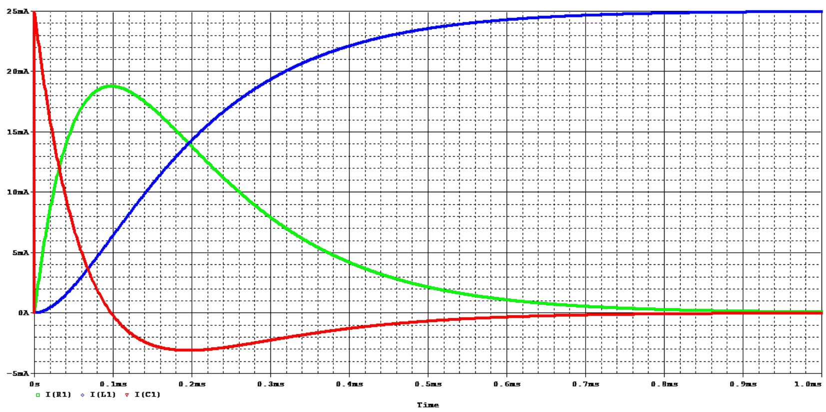


Exercise 7–30. Use OrCAD to plot the currents through the three elements in circuit of Figure 7–42. Show that sum of all three element currents equals the source current for all time.

The following OrCAD simulation provides the results.



The corresponding plot is shown below.



The plot also includes a trace of the sum of the three element currents, which is always equal to the source current.

Exercise 7–31. Find the zero-state response of $v_O(t)$ in Figure 7–44 for $v_S(t) = 60u(t)$ V.

Perform a source transformation on the voltage source and resistor to get an equivalent current source of $i_S(t) = 60/200 = 0.3$ A and $R_N = 200\ \Omega$. The circuit is now a parallel RLC circuit. The corresponding differential equation is

$$LC\frac{d^2 i_{\mathrm{L}}(t)}{dt} + \frac{L}{R_{\mathrm{N}}}\frac{di_{\mathrm{L}}(t)}{dt} + i_{\mathrm{L}}(t) = i_{\mathrm{S}}(t)$$

$$6.25 \times 10^{-8}\frac{d^2 i_{\mathrm{L}}(t)}{dt} + 6.25 \times 10^{-4}\frac{di_{\mathrm{L}}(t)}{dt} + i_{\mathrm{L}}(t) = 0.3$$

The forced response is $i_{\mathrm{LF}}(t) = 0.3$ A. Write the homogeneous equation in standard form.

$$\frac{d^2 i_{\mathrm{L}}(t)}{dt} + 10^4\frac{di_{\mathrm{L}}(t)}{dt} + 16 \times 10^6 i_{\mathrm{L}}(t) = 0$$

The corresponding characteristic equation is

$$s^2 + 10^4 s + 16 \times 10^6 = 0$$

The roots of the characteristic equation are $s_1 = -2000$ and $s_2 = -8000$. The natural response has the form $i_{\mathrm{LN}}(t) = K_1 e^{-2000t} + K_2 e^{-8000t}$. The complete response is

$$i_{\mathrm{L}}(t) = i_{\mathrm{LF}}(t) + i_{\mathrm{LN}}(t) = 0.3 + K_1 e^{-2000t} + K_2 e^{-8000t}$$

Apply the initial conditions to solve for K_1 and K_2.

$$i_{\mathrm{L}}(0) = 0 = 0.3 + K_1 + K_2$$

$$\frac{di_{\mathrm{L}}(0)}{dt} = 0 = \frac{1}{L}v_{\mathrm{C}}(0) = -2000K_1 - 8000K_2$$

$$K_1 = -0.4$$

$$K_2 = 0.1$$

$$i_{\mathrm{L}}(t) = 0.3 - 0.4e^{-2000t} + 0.1e^{-8000t} \text{ A}$$

$$v_{\mathrm{O}}(t) = L\frac{di_{\mathrm{L}}(t)}{dt} = 0.125(800e^{-2000t} - 800e^{-8000t})$$

$$v_{\mathrm{O}}(t) = 100\left(e^{-2000t} - e^{-8000t}\right) \text{ V}$$

Exercise 7–32.

(a). Select a value for R that will cause the RLC circuit of Figure 7–44 to produce a critically damped response. All parameters except R remain the same.

The characteristic equation is

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

For a critically damped response, $\zeta = 1$. Solve for R.

$$\omega_0^2 = \frac{1}{LC} = 16 \times 10^6$$

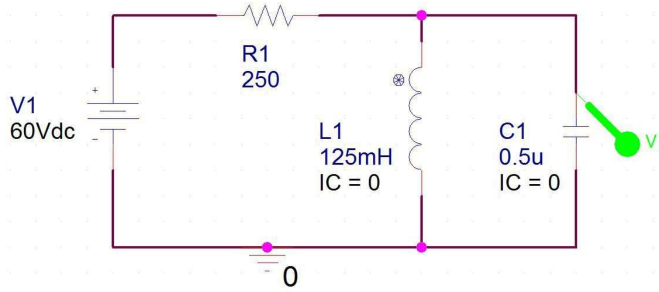
$$\omega_0 = 4000$$

$$\frac{1}{RC} = 2\zeta\omega_0 = 2\omega_0$$

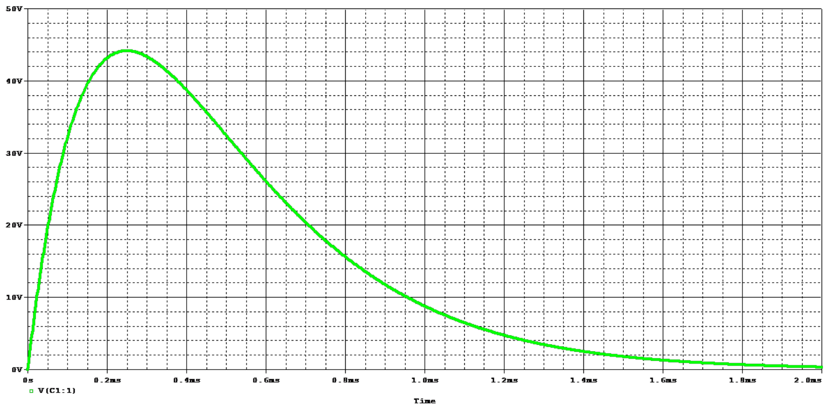
$$R = \frac{1}{2\omega_0 C} = \frac{1}{(2)(4000)(0.5\,\mu)} = 250\ \Omega$$

(b). Use OrCAD to determine the maximum value of $v_O(t)$ and the time at which it reaches that value.

The following OrCAD simulation provides the results.



The corresponding plot is shown below.



The maximum output voltage is 44.15 V and it occurs at $t = 250\ \mu$s.

(c). What is the value of the maximum power delivered by the source?

The maximum current of 240 mA occurs at $t = 0$ and again as $t \to \infty$. Hence, the maximum power delivered by the source is 14.4 W.

Exercise 7–33. The step response of a series RLC circuit is observed to be

$$v_C(t) = 15 - 15e^{-1000t}\cos(1000t) \text{ V} \quad t \geq 0$$

$$i_L(t) = 45e^{-1000t}\cos(1000t) + 45e^{-1000t}\sin(1000t) \text{ mA} \quad t \geq 0$$

(a). What is the circuit characteristic equation?

From the form of the response, we have $\alpha = 1000$ and $\beta = 1000$. The characteristic equation is

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + 2\alpha s + (\alpha^2 + \beta^2) = s^2 + 2000s + 2000000 = 0$$

(b). What are the initial values of the state variables?

Evaluate the expressions at $t = 0$.

$$v_C(0) = 15 - (15)(1)(1) = 0 \text{ V}$$

$$i_L(0) = (45)(1)(1) + (45)(1)(0) = 45 \text{ mA}$$

(c). What is the amplitude of the step input?

The amplitude of the step input is the constant term in the expression for $v_{\rm C}(t)$, so $V_{\rm A} = 15$ V.

(d). What are the values of R, L, and C?

Use the equations for voltage and current to solve for C. Then use the characteristic equation to solve for R and L.

$$i_{\rm L}(t) = i_{\rm C}(t) = C\frac{dv_{\rm C}(t)}{dt}$$

$$\begin{aligned}
0.045e^{-1000t}\cos(1000t) + 0.045e^{-1000t}\sin(1000t) &= C\Big[(-15)(-1000)e^{-1000t}\sin(1000t) \\
&\quad + (-15)(-1000)e^{-1000t}\cos(1000t)\Big] \\
15000C &= 0.045 \\
C &= 3\ \mu\text{F} \\
\frac{1}{LC} &= 2000000 \\
L &= \frac{1}{2000000C} = 166.7\ \text{mH} \\
\frac{R}{L} &= 2000 \\
R &= 2000L = 333.3\ \Omega
\end{aligned}$$

(e). What is the voltage across the resistor?

All elements share the same current.

$$v_{\rm R}(t) = i_{\rm L}(t)R = 15e^{-1000t}\cos(1000t) + 15e^{-1000t}\sin(1000t)\ \text{V}$$

Exercise 7–34.

(a). What range of source resistance will produce an underdamped natural response in a series RLC circuit with $L = 200$ mH and $C = 0.032\ \mu$F?

For an underdamped natural response, $\zeta < 1$. The characteristic equation is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

Solve for ω_0 and then the valid range for R.

$$\begin{aligned}
\omega_0^2 &= \frac{1}{LC} = 156250000 \\
\omega_0 &= 12500 \\
\frac{R}{L} &= 2\zeta\omega_0 \\
R &= 2\zeta\omega_0 L < 2\omega_0 L \\
R &< (2)(12500)(0.2) = 5\ \text{k}\Omega
\end{aligned}$$

(b). Compare your answer with the parallel circuit solution in Example 7–25.

In a parallel circuit, as R_{N} increases, it has less influence on the circuit response—an open circuit would ideally let the circuit oscillate forever. For a series circuit, the dual is true: as R_{T} decreases, it has less influence on the energy oscillating between the inductor and the capacitor, and a short circuit would ideally let the circuit oscillate forever.

Exercise 7–35. Design a series RLC circuit with $\zeta = 1.5$ and $\omega_0 = 50$ krad/s. You must use a 0.1-μF capacitor.

Use the characteristic equation to determine the values for R and L.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

$$\frac{R}{L} = 2\zeta\omega_0 = (2)(1.5)(50000) = 150000$$

$$\frac{1}{LC} = \omega_0^2 = (50000)^2 = 2.5 \times 10^9$$

$$L = \frac{1}{C\omega_0^2} = \frac{1}{(0.1\,\mu)(2.5 \times 10^9)} = 4 \text{ mH}$$

$$R = 150000L = 600\ \Omega$$

7.2 Problem Solutions

Problem 7–1. Find the function $i(t)$ that satisfies the following differential equation and the initial condition:

$$500\frac{di(t)}{dt} + 25000i(t) = 0, \quad i(0) = 15 \text{ mA}$$

Write the differential equation in standard form.

$$0.02\frac{di(t)}{dt} + i(t) = 0$$

The solution has the following form.

$$i(t) = Ke^{-t/0.02} = Ke^{-50t}$$

The initial condition determines K.

$$i(0) = 0.015 = (K)(1) = K$$

The solution is $i(t) = 15e^{-50t}$ mA and can be verified by direct substitution into the original equation.

Problem 7–2. Find the function $v(t)$ that satisfies the following differential equation and initial condition:

$$10^{-3}\frac{dv(t)}{dt} + v(t) = 0, \quad v(0) = 50 \text{ V}$$

The equation is written in standard form and has a solution with the following form.

$$v(t) = Ke^{-t/0.001} = Ke^{-1000t}$$

The initial condition determins K.

$$v(0) = 50 = (K)(1) = K$$

The solution is $v(t) = 50e^{-1000t}$ V and can be verified by direct substitution into the original equation.

Problem 7–3. Find the time constants of the circuits in Figure P7–3.

For circuit C1, combine the inductors in series to get an equivalent inductance of $L_{\text{EQ}} = 150 + 100 = 250$ mH and combine the resistors in series to get an equivalent resistance of $R_{\text{EQ}} = 100 + 150 = 250\ \Omega$. The time constant is $T_{\text{C1}} = L_{\text{EQ}}/R_{\text{EQ}} = 0.25/250 = 1$ ms.

For circuit C2, combine the two resistors in parallel to get $R_{\text{EQ}} = 100 \parallel 100 = 50\ \Omega$. Combine the parallel inductors and add the result to the series inductor to get $L_{\text{EQ}} = 5 + (10 \parallel 10) = 5 + 5 = 10$ mH. The time constant is $T_{\text{C2}} = L_{\text{EQ}}/R_{\text{EQ}} = 0.01/50 = 200\ \mu$s.

Problem 7–4. Find the time constants of the circuits in Figure P7–4.

For circuit C1, find the equivalent resistance by combining resistors in series and parallel from right to left as follows:

$$\begin{aligned}R_{\text{EQ}} &= 3 + [6 \parallel (1 + 4 + 1)] + 3\\ &= 3 + [6 \parallel 6] + 3\\ &= 3 + 3 + 3\\ &= 9 \text{ k}\Omega\end{aligned}$$

The equivalent capacitance is $C_{\text{EQ}} = 0.33\ \mu$F. The time constant is $T_{\text{C1}} = R_{\text{EQ}}C_{\text{EQ}} = (9000)(0.33\,\mu) = 2.97$ ms.

For circuit C2, combine the resistors in parallel to $R_{\text{EQ}} = 33 \parallel 47 = 19.3875$ kΩ. Combine the capacitors in parallel to get $C_{\text{EQ}} = 0.01 + 0.01 = 0.02\ \mu$F. The time constant is $T_{\text{C2}} = R_{\text{EQ}}C_{\text{EQ}} = (19387.5)(0.02\,\mu) = 387.75\ \mu$s.

Problem 7–5. Each of the two circuits in Figure P7–5 have a switch that affects their time constants. For circuit C1 find the time constant when the switch is in position A and repeat for position B. For circuit C2 find the time constant when the switch is closed and repeat when it is open.

For circuit C1 and with the switch in position A, resistors R_2 and R_3 are in parallel and the two capacitors are in parallel. Resistor R_1 does not contribute to the time constant. We have the following values for the time constant:

$$R_{\rm EQ} = R_2 \parallel R_3 = \frac{R_2R_3}{R_2+R_3}$$

$$C_{\rm EQ} = C_1 \parallel C_2 = C_1 + C_2$$

$$T_{\rm C1A} = R_{\rm EQ}C_{\rm EQ} = \frac{R_2R_3(C_1+C_2)}{R_2+R_3}$$

For circuit C1 and with the switch in position B, the result is similar to that for the switch being in position A, but replace R_2 with R_1 as follows:

$$R_{\rm EQ} = R_1 \parallel R_3 = \frac{R_1R_3}{R_1+R_3}$$

$$C_{\rm EQ} = C_1 \parallel C_2 = C_1 + C_2$$

$$T_{\rm C1B} = R_{\rm EQ}C_{\rm EQ} = \frac{R_1R_3(C_1+C_2)}{R_1+R_3}$$

For circuit C2 and with the switch closed, resistors R_1, R_4, and R_3 are in parallel with the capacitor. We have the following values for the time constant:

$$R_{\rm EQ} = R_1 \parallel R_4 \parallel R_3 = \frac{R_1R_3R_4}{R_1R_3+R_1R_4+R_3R_4}$$

$$C_{\rm EQ} = C$$

$$T_{\rm C2,Closed} = R_{\rm EQ}C_{\rm EQ} = \frac{CR_1R_3R_4}{R_1R_3+R_1R_4+R_3R_4}$$

For circuit C2 and with the switch open, the result is similar to that for the switch being closed, but replace R_4 with the series combination of $R_2 + R_4$ as follows:

$$R_{\rm EQ} = R_1 \parallel (R_2+R_4) \parallel R_3 = \frac{R_1R_3(R_2+R_4)}{R_1R_2+R_1R_3+R_1R_4+R_2R_3+R_3R_4}$$

$$C_{\rm EQ} = C$$

$$T_{\rm C2,Open} = R_{\rm EQ}C_{\rm EQ} = \frac{CR_1R_3(R_2+R_4)}{R_1R_2+R_1R_3+R_1R_4+R_2R_3+R_3R_4}$$

Problem 7–6. The switch in Figure P7–6 is closed at $t = 0$. The initial voltage on the capacitor is $v_{\rm C}(0) = 250$ V.

(a). Find $v_{\rm C}(t)$ and $i_{\rm O}(t)$ for $t \geq 0$.

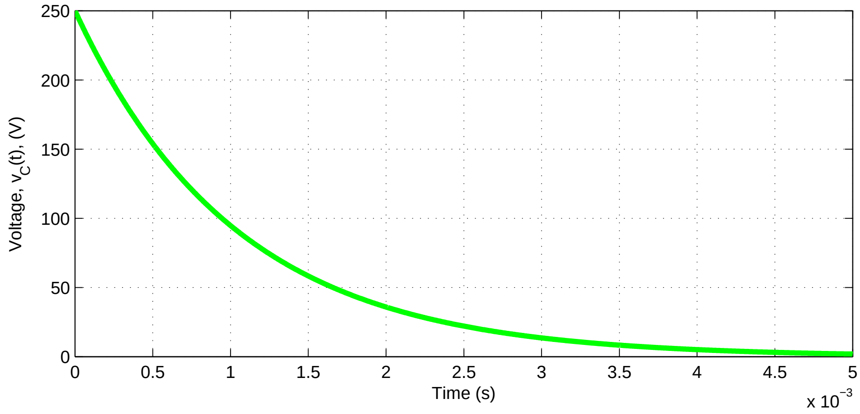
The equivalent resistance is $R_{\rm EQ} = 33 \parallel 15 = 10.3125$ kΩ. The time constant is $T_{\rm C} = R_{\rm EQ}C = 1.03125$ ms and $1/T_{\rm C} = 969.7$ Hz. The capacitor voltage will have an exponential decay with an initial value of 250 V and a time constant of 1.03125 ms. The capacitor voltage appears across the 15-kΩ resistor, which can be used to find $i_{\rm O}(t)$.

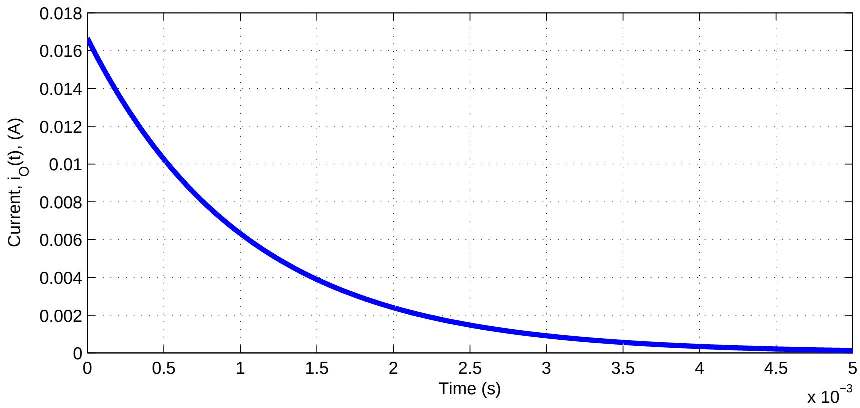
$$v_{\rm C}(t) = V_0e^{-t/T_{\rm C}} = 250e^{-969.7t} \text{ V}$$

$$i_{\rm O}(t) = \frac{v_{\rm C}(t)}{15000} = 16.67e^{-969.7t} \text{ mA}$$

(b). Use MATLAB to plot the waveforms for $v_C(t)$ and $i_O(t)$.

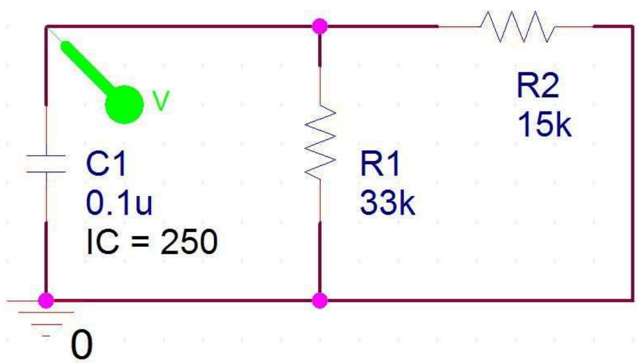
The corresponding MATLAB plots are shown below.



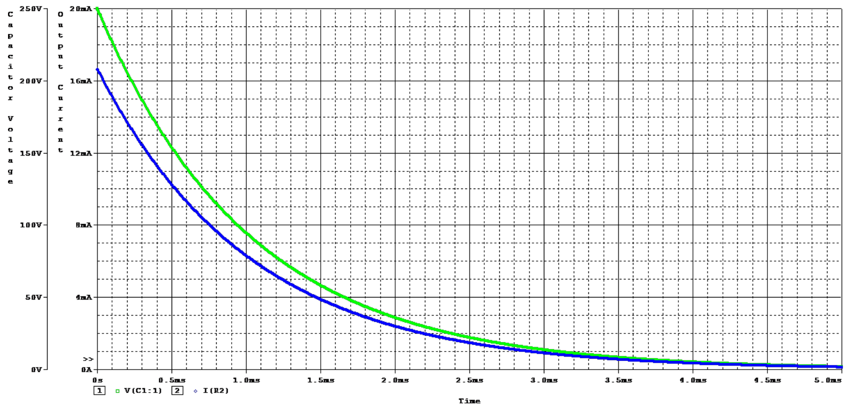


(c). Simulate the problem using OrCAD and compare the results to the plots in part (b).

The following OrCAD simulation provides the results.



The corresponding plot is shown below.



The results agree with the plots in part (b).

Problem 7–7. In Figure P7–7 the initial current through the inductor is $i_L(0) = 3$ mA. Find $i_L(t)$ and $v_O(t)$ for $t \geq 0$.

The equivalent resistance in the circuit is $R_{EQ} = 33 \parallel 47 = 19.3875$ kΩ. The time constant is $T_C = L/R_{EQ} = 0.5/19387.5 = 25.79$ μs and $1/T_C = 38775$ Hz. Use the initial condition and time constant to solve for the current through the inductor and then use Ohm's law and the equivalent resistance to solve for the output voltage. Note the polarity of the output voltage.

$$i_L(t) = I_0 e^{-t/T_C} = 3e^{-38775t} \text{ mA}$$

$$v_O(t) = -i_L(t)R_{EQ} = -58.1625e^{-38775t} \text{ V}$$

Problem 7–8. The switch in Figure P7–8 has been in position A for a long time and is moved to position B at $t = 0$. Find $i_L(t)$ for $t \geq 0$.

With the switch in position A, the current through the inductor is −10 mA, which is the initial current after the switch moves. After the switch moves to position B, the equivalent resistance is $R_{EQ} = 100 + 50 = 150$ Ω. The time constant is $T_C = L/R_{EQ} = 0.015/150 = 100$ μs and $1/T_C = 10000$ Hz. The current through the inductor is

$$i_L(t) = I_0 e^{-t/T_C} = -10e^{-10000t} \text{ mA}$$

Problem 7–9. The circuit was in the zero state when the source in Figure P7–9 is applied. Find $v_C(t)$ for $t \geq 0$.

With the circuit initially in the zero state, we have $v_C(0) = 0$ V. After a long time, the capacitor appears as an open circuit and its voltage matches the voltage across the right 100-kΩ resistor. Apply voltage division to get $v_C(\infty) = (100)(25)/(100 + 100) = 12.5$ V. The equivalent resistance seen by the capacitor is $R_{EQ} = 50 + (100 \parallel 100) = 50 + 50 = 100$ kΩ. The time constant is $T_C = R_{EQ}C = (100\,\text{k})(100\,\text{p}) = 10$ μs and $1/T_C = 100000$ Hz. The voltage across the capacitor is

$$v_C(t) = [v_C(0) - v_C(\infty)]\, e^{-t/T_C} + v_C(\infty) = [0 - 12.5]\, e^{-100000t} + 12.5 = 12.5\left(1 - e^{-100000t}\right) \text{ V}$$

Problem 7–10. The switch in Figure P7–10 has been in position A for a long time and is moved to position B at $t = 0$. Find $v_C(t)$ for $t \geq 0$.

The initial voltage across the capacitor is $v_C(0) = 5$ V. After the switch moves, the final voltage across the capacitor will be $v_C(\infty) = 0$ V. The time constant is $T_C = RC = (50\,\text{k})(0.05\,\mu) = 2.5$ ms and $1/T_C = 400$ Hz. The voltage across the capacitor is

$$v_C(t) = [v_C(0) - v_C(\infty)]\, e^{-t/T_C} + v_C(\infty) = 5e^{-400t} \text{ V}$$

Problem 7–11. The switch in Figure P7–11 has been open for a long time and is closed at $t = 0$. Find $i_L(t)$ for $t \geq 0$.

Apply two-path current division to find the initial current through the inductor.

$$i_{\mathrm{L}}(0) = \frac{15}{15+47}(10\,\mathrm{m}) = 2.4194 \text{ mA}$$

After the switch closes, the time constant is $T_{\mathrm{C}} = L/R = 0.01/47 = 212.766$ μs and $1/T_{\mathrm{C}} = 4700$ Hz. The final inductor current is $i_{\mathrm{L}}(\infty) = 0$ mA. The current through the inductor is

$$i_{\mathrm{L}}(t) = [i_{\mathrm{L}}(0) - i_{\mathrm{L}}(\infty)]\, e^{-t/T_{\mathrm{C}}} + i_{\mathrm{L}}(\infty) = 2.4194e^{-4700t} \text{ mA}$$

Problem 7–12. The circuit in Figure P7–12 is in the zero state. Find the voltage $v_{\mathrm{O}}(t)$ for $t \geq 0$ when an input of $i_{\mathrm{S}}(t) = I_{\mathrm{A}}u(t)$ is applied. Identify the forced and natural components in the output.

Perform a source transformation to have a voltage source in series with a resistor and the capacitor. The voltage source will be $v_{\mathrm{S}}(t) = RI_{\mathrm{A}}u(t)$ and the resistor will be R. The initial voltage is $v_{\mathrm{C}}(0) = 0$ V and the final voltage will be $v_{\mathrm{C}}(\infty) = v_{\mathrm{S}}(t) = RI_{\mathrm{A}}$ V. The time constant is $T_{\mathrm{C}} = RC$. The capacitor voltage is the output voltage.

$$v_{\mathrm{O}}(t) = v_{\mathrm{C}}(t) = [v_{\mathrm{C}}(0) - v_{\mathrm{C}}(\infty)]\, e^{-t/T_{\mathrm{C}}} + v_{\mathrm{C}}(\infty) = [0 - RI_{\mathrm{A}}]\, e^{-t/RC} + RI_{\mathrm{A}} = RI_{\mathrm{A}}\left(1 - e^{-t/RC}\right) \text{ V}$$

The forced component is the constant term, $v_{\mathrm{OF}}(t) = RI_{\mathrm{A}}$ V, and the natural component is the exponential term, $v_{\mathrm{ON}}(t) = -RI_{\mathrm{A}}e^{-t/RC}$ V.

Problem 7–13. The circuit in Figure P7–13 is in the zero state when the input $v_{\mathrm{S}}(t) = V_{\mathrm{A}}u(t)$ is applied. Find $v_{\mathrm{O}}(t)$ for $t > 0$. Identify the forced and natural components in the output.

Find the inductor current and then determine the output voltage. The initial inductor current is $i_{\mathrm{L}}(0) = 0$ A and the final inductor current is $i_{\mathrm{L}}(\infty) = v_{\mathrm{S}}(t)/R = V_{\mathrm{A}}/R$. The time constant is $T_{\mathrm{C}} = L/R$.

$$i_{\mathrm{L}}(t) = [i_{\mathrm{L}}(0) - i_{\mathrm{L}}(\infty)]\, e^{-t/T_{\mathrm{C}}} + i_{\mathrm{L}}(\infty) = \left[0 - \frac{V_{\mathrm{A}}}{R}\right] e^{-Rt/L} + \frac{V_{\mathrm{A}}}{R}$$

$$i_{\mathrm{L}}(t) = \frac{V_{\mathrm{A}}}{R}\left(1 - e^{-Rt/L}\right)$$

$$v_{\mathrm{O}}(t) = i_{\mathrm{L}}(t)R = V_{\mathrm{A}}\left(1 - e^{-Rt/L}\right)$$

The forced component in the output is the constant voltage $v_{\mathrm{OF}}(t) = V_{\mathrm{A}}$ and the natural component is the exponential term, $v_{\mathrm{ON}}(t) = -V_{\mathrm{A}}e^{-Rt/L}$.

Problem 7–14. The circuit in Figure P7–14 is in the zero state when the input $v_{\mathrm{S}}(t) = 100u(t)$ is applied. If $C = 0.01$ μF and $R = 100$ kΩ, find $v_{\mathrm{O}}(t)$ for $t \geq 0$. Identify the forced and natural components in the output.

Find the capacitor voltage and then the output voltage. The initial capacitor voltage is $v_{\mathrm{C}}(0) = 0$ V and the final voltage is $v_{\mathrm{C}}(\infty) = v_{\mathrm{S}}(t) = 100$ V. The equivalent resistance is $R_{\mathrm{EQ}} = R + R = 2R$. The time constant is $T_{\mathrm{C}} = R_{\mathrm{EQ}}C = 2RC = (2)(100\,\mathrm{k})(0.01\,\mu) = 2$ ms and $1/T_{\mathrm{C}} = 500$ Hz. Determine the capacitor voltage, capacitor current, and output voltage.

$$v_{\mathrm{C}}(t) = [v_{\mathrm{C}}(0) - v_{\mathrm{C}}(\infty)]\, e^{-t/T_{\mathrm{C}}} + v_{\mathrm{C}}(\infty) = [0 - 100]\, e^{-500t} + 100 = 100\left(1 - e^{-500t}\right) \text{ V}$$

$$i_{\mathrm{C}}(t) = C\frac{dv_{\mathrm{C}}(t)}{dt} = (0.01\,\mu)(100)(-500)(-e^{-500t}) = 500e^{-500t}\ \mu\text{A}$$

$$v_{\mathrm{O}}(t) = i_{\mathrm{C}}(t)R = (100\,\mathrm{k})(500\,\mu)e^{-500t} = 50e^{-500t} \text{ V}$$

There is no forced response in the output and the natural response is the entire output signal.

Problem 7–15. The circuit in Figure P7–15 is in the zero state when the input $v_{\mathrm{S}}(t) = 20u(t)$ is applied. If $L = 100$ mH and $R = 1$ kΩ, find $v_{\mathrm{O}}(t)$ for $t \geq 0$. Identify the forced and natural components in the output.

On a single set of axes, use MATLAB to plot the forced response, the natural response, and the complete response.

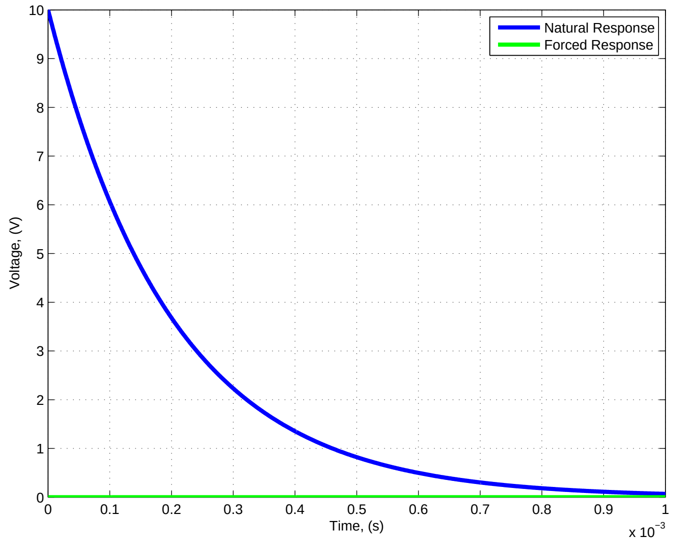
Find the inductor current and then solve for the output voltage. The initial inductor current is $i_{\rm L}(0) = 0$ A and the final inductor current is $i_{\rm L}(\infty) = v_{\rm S}(t)/R = 20/1000 = 20$ mA. The equivalent resistance seen by the inductor is the parallel combination of the two resistors, $R_{\rm EQ} = R \parallel R = R/2 = 500\ \Omega$. The time constant is $T_{\rm C} = L/R_{\rm EQ} = 0.1/500 = 200\ \mu$s and $1/T_{\rm C} = 5000$ Hz. We can now solve for the requested values.

$$i_{\rm L}(t) = [i_{\rm L}(0) - i_{\rm L}(\infty)]\, e^{-t/T_{\rm C}} + i_{\rm L}(\infty) = [0 - 20]\, e^{-5000t} + 20 \text{ mA}$$

$$i_{\rm L}(t) = 20\left(1 - e^{-5000t}\right) \text{ mA}$$

$$v_{\rm O}(t) = v_{\rm L}(t) = L\frac{di_{\rm L}(t)}{dt} = (0.1)(0.02)(-5000)(-e^{-5000t}) = 10e^{-5000t} \text{ V}$$

The output voltage is a natural response and there is no forced response. The following MATLAB plot displays the results.



Problem 7–16. The switch in Figure P7–16 has been in position A for a long time and is moved to position B at $t = 0$. Find $v_{\rm C}(t)$ for $t > 0$. Identify the forced and natural components in the response.

Before the switch moves, the capacitor is in parallel with one of three resistors and after the switch moves the capacitor is in parallel with one of two resistors. Apply voltage division to get $v_{\rm C}(0) = (10)(24)/(10 + 10 + 10) = 8$ V and apply it again to get $v_{\rm C}(\infty) = (10)(24)/(10 + 10) = 12$ V. The equivalent resistance seen by the capacitor is $R_{\rm EQ} = 10 \parallel 10 = 5$ kΩ. The time constant is $T_{\rm C} = R_{\rm EQ}C = (5000)(0.02\,\mu) = 100\ \mu$s and $1/T_{\rm C} = 10000$ Hz. Write the expression for the capacitor voltage.

$$v_{\rm C}(t) = [v_{\rm C}(0) - v_{\rm C}(\infty)]\, e^{-t/T_{\rm C}} + v_{\rm C}(\infty) = [8 - 12]\, e^{-10000t} + 12 = 12 - 4e^{-10000t} \text{ V}$$

The forced component is the constant term, $v_{\rm CF}(t) = 12$ V, and the natural component is the exponential term, $v_{\rm CN}(t) = -4e^{-10000t}$ V.

Problem 7–17. Repeat Problem 7–16. However, after the switch is moved to position B at $t = 0$, the switch is moved back to position A at $t = 100\ \mu$s. Find $v_{\rm C}(t)$ for $t \geq 0$.

The results from Problem 7–16 are valid for $0 \leq t \leq 100\ \mu$s. Find the capacitor voltage at $t = 100\ \mu$s and use it as a new initial voltage for the circuit.

$$v_{\rm C}(100\,\mu) = 12 - 4e^{-10000(100\,\mu)} = 12 - 4e^{-1} = 10.5285 \text{ V}$$

Determine the new time constant after the switch returns to position A. The equivalent resistance is $R_{\rm EQ} = 10 \parallel (10 + 10) = 10 \parallel 20 = 6.667$ kΩ. The time constant is $T_{\rm C} = R_{\rm EQ}C = (6667)(0.02\,\mu) = 133.33\ \mu$s and

$1/T_\mathrm{C} = 7500$ Hz. With the switch in position A, the final voltage for the capacitor is $v_\mathrm{C}(\infty) = 8$ V, as developed in Problem 7–16. We can now combine these results with the solution in Problem 7–16 to write the expression for the capacitor voltage.

$$v_\mathrm{C}(t) = \left[12 - 4e^{-10000t}\right][u(t) - u(t-100\,\mu)] + \left[(10.5285 - 8)e^{-7500(t-100\,\mu)} + 8\right]u(t-100\,\mu)$$

$$v_\mathrm{C}(t) = \left[12 - 4e^{-10000t}\right][u(t) - u(t-100\,\mu)] + \left[8 + 2.5285e^{-7500(t-100\,\mu)}\right]u(t-100\,\mu) \text{ V}$$

Problem 7–18. Find the function that satisfies the following differential equation and the initial condition for an input $v_\mathrm{S}(t) = 50\cos(200t)$ V:

$$\frac{dv(t)}{dt} + 50v(t) = v_\mathrm{S}(t), \quad v(0) = 0 \text{ V}$$

Based on the differential equation, the natural response has the form $v_\mathrm{N}(t) = Ke^{-50t}$. Based on the input, the forced response has the form $v_\mathrm{F}(t) = a\cos(200t) + b\sin(200t)$. Solve for the forced response and then the complete response.

$$\frac{dv_\mathrm{F}(t)}{dt} + 50v_\mathrm{F}(t) = v_\mathrm{S}(t)$$

$$\frac{d}{dt}\left[a\cos(200t) + b\sin(200t)\right] + 50\left[a\cos(200t) + b\sin(200t)\right] = 50\cos(200t)$$

$$-200a\sin(200t) + 200b\cos(200t) + 50a\cos(200t) + 50b\sin(200t) = 50\cos(200t)$$

$$50a + 200b = 50$$

$$-200a + 50b = 0$$

$$a = \frac{1}{17}$$

$$b = \frac{4}{17}$$

$$v_\mathrm{F}(t) = \frac{1}{17}\cos(200t) + \frac{4}{17}\sin(200t)$$

$$v(t) = v_\mathrm{N}(t) + v_\mathrm{F}(t) = Ke^{-50t} + \frac{1}{17}\cos(200t) + \frac{4}{17}\sin(200t)$$

$$v(0) = 0 = K + \frac{1}{17} + 0$$

$$K = -\frac{1}{17}$$

$$v(t) = -\frac{1}{17}e^{-50t} + \frac{1}{17}\cos(200t) + \frac{4}{17}\sin(200t)$$

The following MATLAB code efficiently provides the solution.

```
syms t v
v = simplify(dsolve('Dv+50*v=50*cos(200*t)','v(0)=0','t'))
```

The corresponding MATLAB output is shown below and agrees with the answer above.

```
v = cos(200*t)/17 - 1/(17*exp(50*t)) + (4*sin(200*t))/17
```

Problem 7–19. Repeat Problem 7–18 for $v_S(t) = 100e^{-0.02t}u(t)$ V.

Based on the differential equation, the natural response has the form $v_N(t) = K_1e^{-50t}$. Based on the input, the forced response has the form $v_F(t) = K_2e^{-0.02t}$. Solve for the forced response and then the complete response.

$$\frac{dv_F(t)}{dt} + 50v_F(t) = v_S(t)$$

$$\frac{d}{dt}\left[K_2e^{-0.02t}\right] + 50K_2e^{-0.02t} = 100e^{-0.02t}$$

$$-0.02K_2e^{-0.02t} + 50K_2e^{-0.02t} = 100e^{-0.02t}$$

$$49.98K_2 = 100$$

$$K_2 = \frac{5000}{2499}$$

$$v_F(t) = \frac{5000}{2499}e^{-0.02t}$$

$$v(t) = v_N(t) + v_F(t) = K_1e^{-50t} + \frac{5000}{2499}e^{-0.02t}$$

$$v(0) = 0 = K_1 + \frac{5000}{2499}$$

$$K_1 = -\frac{5000}{2499}$$

$$v(t) = -\frac{5000}{2499}e^{-50t} + \frac{5000}{2499}e^{-0.02t}$$

The following MATLAB code efficiently provides the solution.

```
syms t v
v = simplify(dsolve('Dv+50*v=100*exp(-0.02*t)','v(0)=0','t'))
```

The corresponding MATLAB output is shown below and agrees with the answer above.

```
v = (5000*(exp((2499*t)/50) - 1))/(2499*exp(50*t))
```

Problem 7–20. The switch in Figure P7–20 has been open long enough for $i_L(0)$ to reach 0 A and is closed at $t = 0$.

(a). If $v_S(t) = 10u(t)$ V, find $v_L(t)$ for $t \geq 0$.

Use a source transformation to find the equivalent circuit seen by the inductor. Replace the voltage source in series with the 330-Ω resistor with a current source $i_S(t) = v_S(t)/330$ in parallel with a 330-Ω resistor. Combine the two resistors in parallel to get $R_{EQ} = 330 \parallel 470 = 193.875\ \Omega$. The time constant is $T_C = L/R_{EQ} = 1/194$ s and $1/T_C = 194$ Hz. The initial current is $i_L(0) = 0$ A. The input is a step function, so solve for the inductor current directly. The final inductor current is

$i_L(\infty) = i_S(t) = 10/330$ A. Write the expression for the complete response of the inductor current and then solve for the inductor voltage

$$i_L(t) = [i_L(0) - i_L(\infty)]\, e^{-t/T_C} + i_L(\infty) = \left[0 - \frac{10}{330}\right] e^{-194t} + \frac{10}{330}$$

$$i_L(t) = \frac{10}{330}\left(1 - e^{-194t}\right) \text{ A}$$

$$v_L(t) = L\frac{di_L(t)}{dt} = (1)\left(\frac{10}{330}\right)(-194)(-e^{-194t}) = 5.8788e^{-194t} \text{ V}$$

(b). If $v_S(t) = 10\cos(100t)$ V, find $v_L(t)$ for $t \geq 0$.

The natural response will have the form of the expression found in part (a), $i_{LN}(t) = Ke^{-194t}$. The forced response will have the form $i_{LF}(t) = a\cos(100t) + b\sin(100t)$. Write the differential equation for the circuit, solve for the forced response and then the complete response.

$$\frac{L}{R}\frac{di_{LF}(t)}{dt} + i_{LF}(t) = \frac{v_S(t)}{330}$$

$$\frac{1}{194}\frac{d}{dt}\left[a\cos(100t) + b\sin(100t)\right] + a\cos(100t) + b\sin(100t) = \frac{10\cos(100t)}{330}$$

$$-100a\sin(100t) + 100b\cos(100t) + 194a\cos(100t) + 194b\sin(100t) = \frac{1940\cos(100t)}{330}$$

$$-33000a\sin(100t) + 33000b\cos(100t) + 64020a\cos(100t) + 64020b\sin(100t) = 1940\cos(100t)$$

$$64020a + 33000b = 1940$$

$$-33000a + 64020b = 0$$

$$a = 0.02394$$

$$b = 0.01234$$

$$i_{LF}(t) = 0.02394\cos(100t) + 0.01234\sin(100t) \text{ A}$$

$$i_L(t) = i_{LN}(t) + i_{LF}(t) = Ke^{-194t} + 0.02394\cos(100t) + 0.01234\sin(100t)$$

$$i_L(0) = 0 = K + (0.02394)(1) + (0.01234)(0)$$

$$K = -0.02394$$

$$i_L(t) = -23.94e^{-194t} + 23.94\cos(100t) + 12.34\sin(100t) \text{ mA}$$

$$v_L(t) = L\frac{di_L(t)}{dt} = 4.6447e^{-194t} - 2.394\sin(100t) + 1.234\cos(100t) \text{ V}$$

(c). If $v_S(t) = 10e^{-100t}$ V, find $v_L(t)$ for $t \geq 0$.

The natural response will have the form of the expression found in part (a), $i_{LN}(t) = K_1e^{-194t}$. The forced response will have the form $i_{LF}(t) = K_2e^{-100t}$. Write the differential equation for the circuit, solve for the forced response and then the complete response.

$$\frac{L}{R}\frac{di_{\rm LF}(t)}{dt} + i_{\rm LF}(t) = \frac{v_{\rm S}(t)}{330}$$

$$\frac{1}{194}\frac{d}{dt}\left[K_2e^{-100t}\right] + K_2e^{-100t} = \frac{10e^{-100t}}{330}$$

$$-33000K_2e^{-100t} + 64020K_2e^{-100t} = 1940e^{-100t}$$

$$31020K_2 = 1940$$

$$K_2 = 0.06254$$

$$i_{\rm LF}(t) = 0.06254e^{-100t} \text{ A}$$

$$i_{\rm L}(t) = i_{\rm LN}(t) + i_{\rm LF}(t) = K_1e^{-194t} + 0.06254e^{-100t}$$

$$i_{\rm L}(0) = 0 = K_1 + 0.06254$$

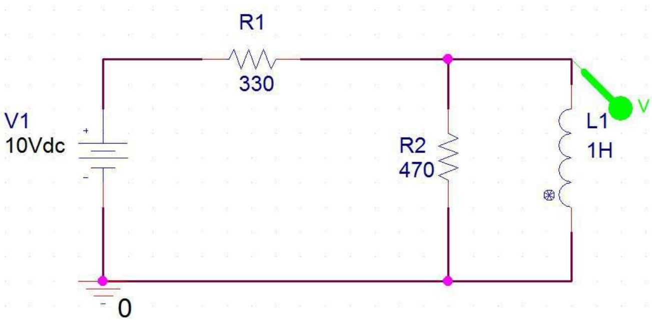
$$K_1 = -0.06245$$

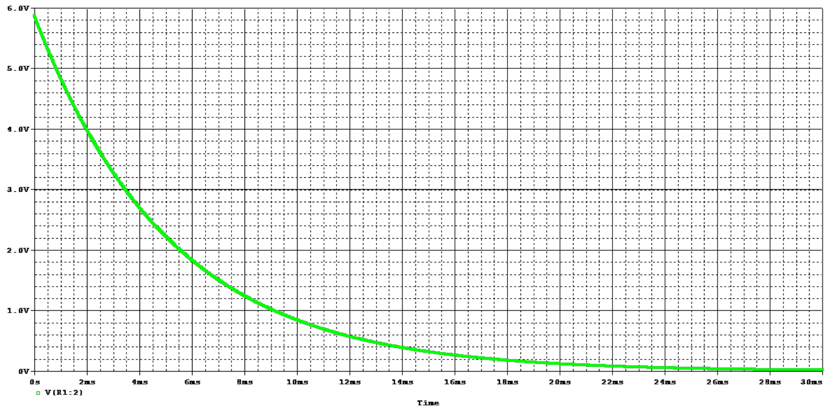
$$i_{\rm L}(t) = -0.06254e^{-194t} + 0.06254e^{-100t}$$

$$v_{\rm L}(t) = L\frac{di_{\rm L}(t)}{dt} = 12.133e^{-194t} - 6.254e^{-100t} \text{ V}$$

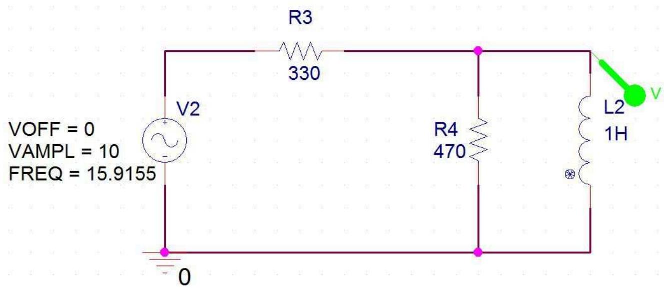
Problem 7–21. Repeat Problem 7–20 using OrCAD.

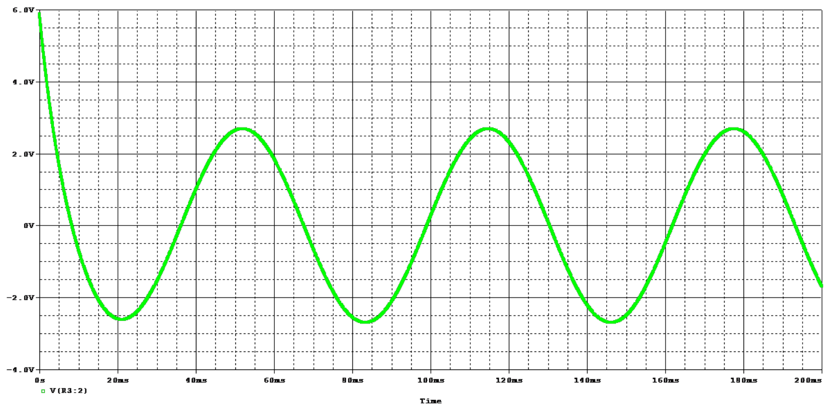
The following OrCAD simulations and plots present the solutions. In all of the simulations, the initial inductor current is zero.

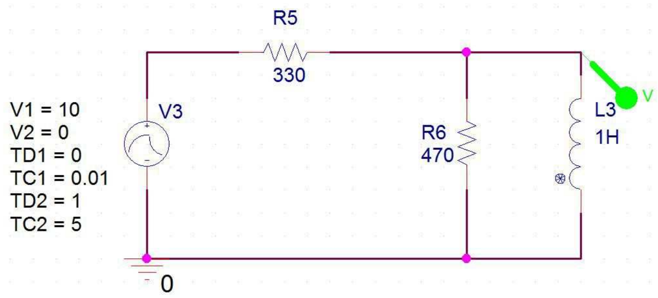


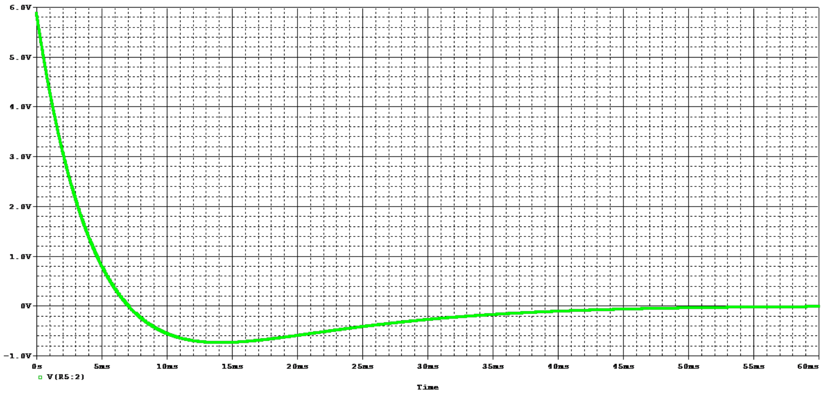


The phase angle of the source is 90° to account for the fact that OrCAD uses a sine function instead of a cosine function for the sinusoidal voltage source.







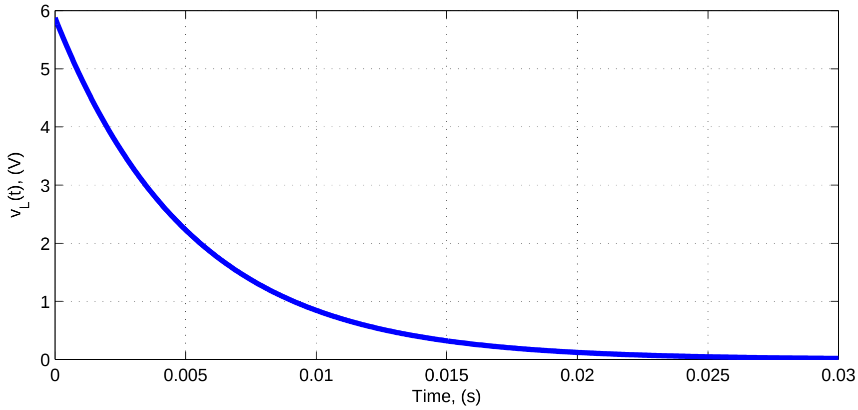


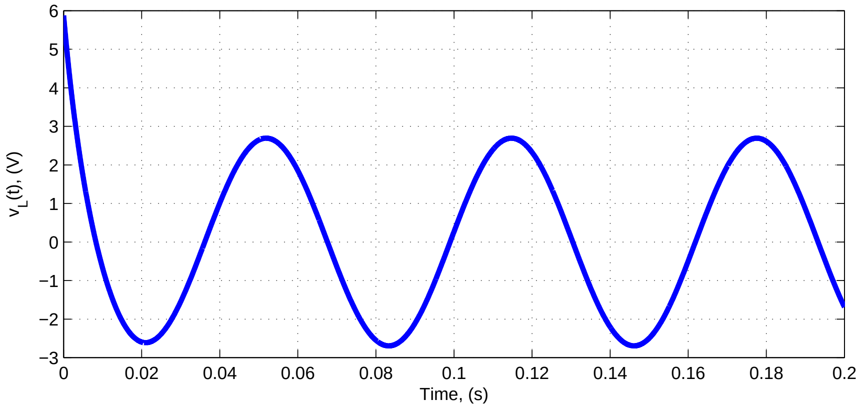
Problem 7–22. Repeat Problem 7–20 using MATLAB to plot the waveforms.

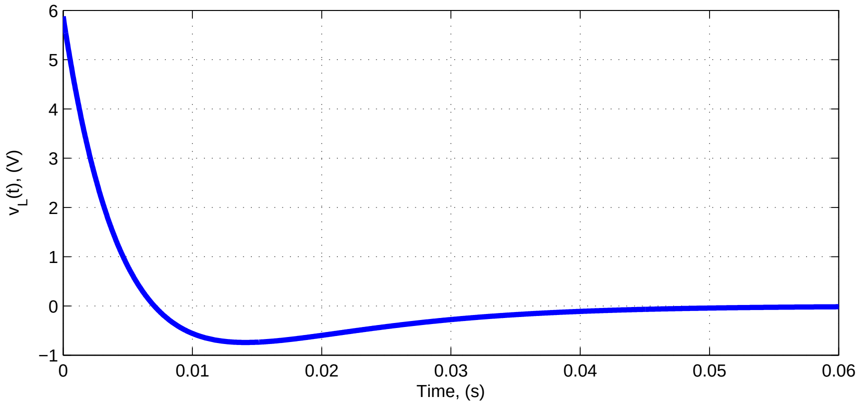
The following MATLAB code provides the solution:

```
syms t iL
L = 1;
disp('Part (a)')
iL = dsolve('1/194*DiL + iL = 10/330','iL(0)=0','t')
vLa = L*diff(iL,t)
tt = 0:50e-6:30e-3;
vLatt = subs(vLa,t,tt);
figure('Position',[500,500,900,400])
plot(tt,vLatt,'b','LineWidth',3)
set(gca,'PlotBoxAspectRatio',[9 4 1])
grid on
xlabel('Time, (s)')
ylabel('v_L(t), (V)')

disp('Part (b)')
iL = dsolve('1/194*DiL + iL = 10/330*cos(100*t)','iL(0)=0','t')
vLb = L*diff(iL,t)
tt = 0:50e-6:200e-3;
vLbtt = subs(vLb,t,tt);
figure('Position',[500,500,900,400])
plot(tt,vLbtt,'b','LineWidth',3)
set(gca,'PlotBoxAspectRatio',[9 4 1])
grid on
xlabel('Time, (s)')
ylabel('v_L(t), (V)')

disp('Part (c)')
iL = dsolve('1/194*DiL + iL = 10/330*exp(-100*t)','iL(0)=0','t')
vLc = L*diff(iL,t)
tt = 0:50e-6:60e-3;
vLctt = subs(vLc,t,tt);
figure('Position',[500,500,900,400])
plot(tt,vLctt,'b','LineWidth',3)
set(gca,'PlotBoxAspectRatio',[9 4 1])
grid on
xlabel('Time, (s)')
ylabel('v_L(t), (V)')
```

The corresponding MATLAB plots are shown below.







Problem 7–23. The switch in Figure P7–23 has been in position A for a long time and is moved to position B at $t = 0$. Find $i_{\rm L}(t)$ for $t \geq 0$.

The initial current through the inductor is $i_{\rm L}(0) = 15/10000 = 1.5$ mA and the final current through the inductor is $i_{\rm L}(\infty) = 15/100000 = 0.15$ mA. After the switch moves, the time constant is $T_{\rm C} = L/R = 0.1/100000 = 1$ μs and $1/T_{\rm C} = 10^6$ Hz. The expression for the inductor current is

$$i_{\rm L}(t) = [i_{\rm L}(0) - i_{\rm L}(\infty)]\, e^{-t/T_{\rm C}} + i_{\rm L}(\infty) = [1.5 - 0.15]\, e^{-10^6 t} + 0.15$$

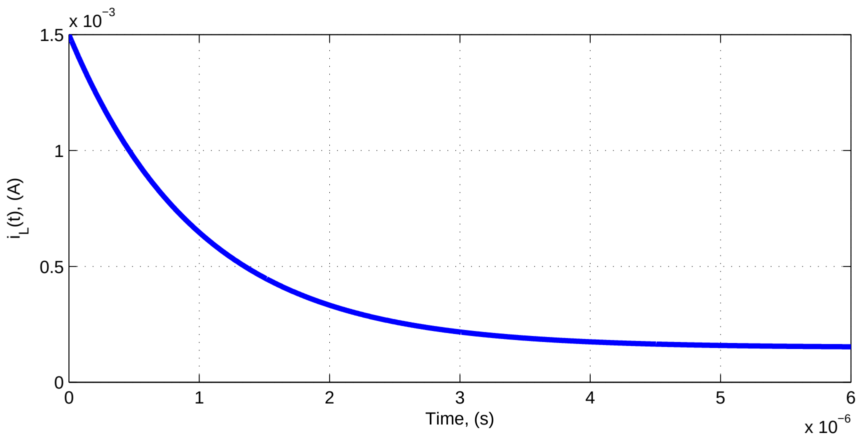
$$i_{\rm L}(t) = 1.35e^{-10^6 t} + 0.15 \text{ mA}$$

Problem 7–24. Repeat Problem 7–23 using MATLAB to plot the waveform.

The following MATLAB code provides the solution:

```
syms t iL
vs = 15;
R1 = 10e3;
R2 = 100e3;
L = 100e-3;
iL0 = vs/R1;
iLf = vs/R2;
Tc = L/R2;
iL = simplify((iL0-iLf)*exp(-t/Tc)+iLf);
iL_num = vpa(iL,6)

tt = 0:5e-9:6e-6;
iLtt = subs(iL,t,tt);
figure('Position',[500,500,900,400])
plot(tt,iLtt,'b','LineWidth',3)
set(gca,'PlotBoxAspectRatio',[9 4 1])
grid on
xlabel('Time, (s)')
ylabel('i_L(t), (A)')
```

The corresponding MATLAB plot is shown below.



Problem 7–25. The switch in Figure P7–25 has been in position A for a long time and is moved to position B at $t = 0$. The switch suddenly returns to position A after 10 ms. Find $v_C(t)$ for $t > 0$ and sketch its waveform.

Treat the problem as two separate step responses. Apply voltage division to find the initial voltage as $v_C(t) = 80$ V and the final voltage as $v_C(\infty) = 50$ V. With the switch in position B, the equivalent resistance is $R_{EQ} = 20 \parallel 20 = 10$ kΩ. The time constant is $T_C = R_{EQ}C = (10000)(1\,\mu) = 10$ ms and $1/T_C = 100$ Hz. Find the response with the switch in position B

$$v_C(t) = [v_C(0) - v_C(\infty)]\, e^{-t/T_C} + v_C(\infty) = [80 - 50]\, e^{-100t} + 50 = 30e^{-100t} + 50 \text{ V}$$

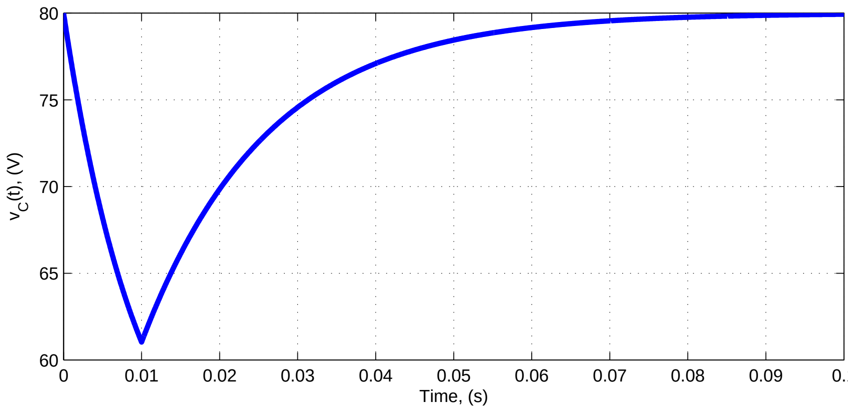
This response is only valid for $0 \le t < 10$ ms. Evaluate the expression at $t = 10$ ms to find the capacitor voltage $v_C(0.01) = 61.036$ V, which is the initial voltage after the switch returns to position A. The new final voltage will be $v_C(\infty) = 80$ V, the time constant is $T_C = (16000)(1\,\mu) = 16$ ms, and $1/T_C = 62.5$ Hz. The response after the switch moves back to position A has the form:

$$v_C(t) = [61.036 - 80]\, e^{-62.5t} + 80 = 80 - 18.964e^{-62.5t} \text{ V}$$

Shift the second response to the correct time and combine it with the first response to get the complete response:

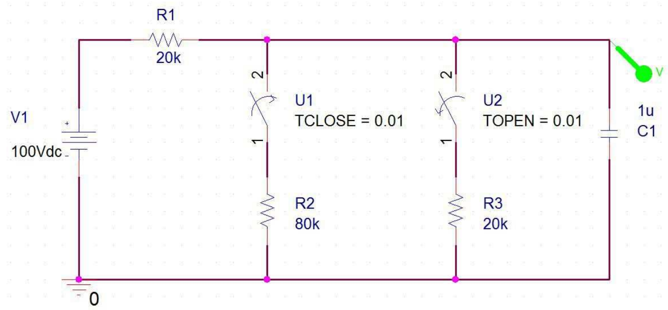
$$v_C(t) = \left[50 + 30e^{-100t}\right][u(t) - u(t-0.01)] + \left[80 - 18.964e^{-62.5(t-0.01)}\right] u(t-0.01)$$

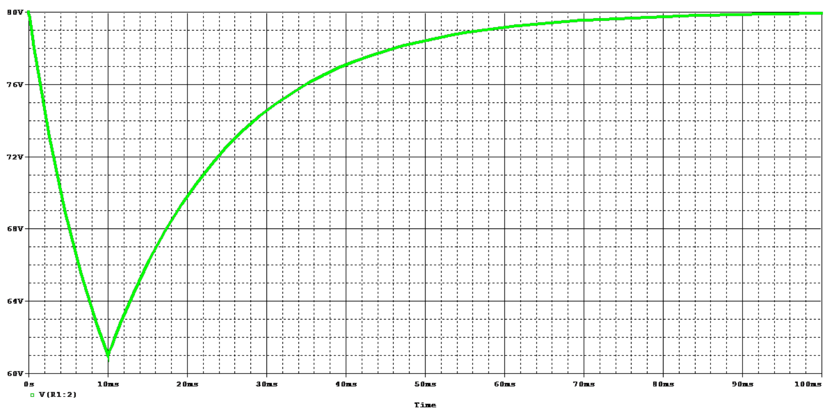
The corresponding MATLAB plot is shown below.



Problem 7–26. Use OrCAD to solve Problem 7–25. (*Hint:* OrCAD switches are located in the EVAL library and are named `Sw_tClose` and `Sw_tOpen`, depending on if you want the switch to close or open at a certain time.)

The following OrCAD simulation and plot present the solutions. The initial voltage on the capacitor is 80 V.





Problem 7–27. Switches 1 and 2 in Figure P7–27 have both been in position A for a long time. Switch 1 is moved to position B at $t = 0$ and Switch 2 is moved to position B at $t = 20$ ms. Find the voltage across the 0.1-μF capacitor for $t \geq 0$ and sketch its waveform.

With both switches in position A, the initial voltage across the capacitor is $v_C(0) = 100$ V. Once switch 1 moves to position B, the final voltage across the capaitor will be $v_C(\infty) = 0$ V and that remains true after switch 2 moves. With only switch 1 in position B, the equivalent resistance is $R_{EQ} = 150 + 50 = 200$ kΩ. The time constant is $T_{C1} = R_{EQ}C = (200\,\text{k})(0.1\,\mu) = 20$ ms and $1/T_{C1} = 50$ Hz. After switch 2 moves to position B, the equivalent resistance decreases to 50 kΩ, the time constant becomes $T_{C2} = 5$ ms, and

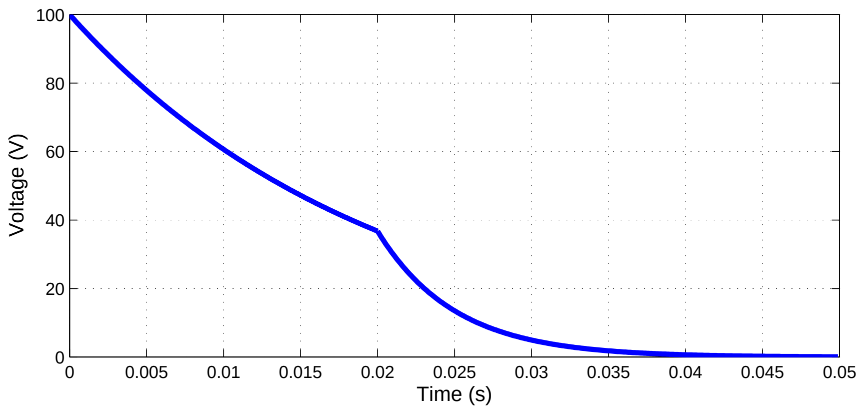
$1/T_{C2} = 200$ Hz. Write the expression for the capacitor voltage with only switch 1 in position B, find the voltage at $t = 20$ ms, and then construct the complete expression for the capacitor voltage.

$$v_{C1}(t) = [100 - 0]e^{-50t} + 0 = 100e^{-50t} \text{ V}$$

$$v_{C1}(0.02) = (100)(e^{-1}) = 36.788 \text{ V}$$

$$v_C(t) = 100e^{-50t}[u(t) - u(t-0.02)] + 36.788e^{-200(t-0.02)}u(t-0.02) \text{ V}$$

The corresponding MATLAB plot is shown below.



Problem 7–28. The switch in Figure P7–28 has been open for a long time and is closed at $t = 0$. The switch is reopened at $t = 3$ ms. Find $v_C(t)$ for $t \geq 0$.

With the switch open, the initial voltage across the capacitor is $v_C(0) = 0$ V. Apply voltage division to find the final voltage across the capacitor with the switch closed, $v_C(\infty) = (15)(24)/(1.5 + 15) = 21.818$ V. The equivalent resistance seen by the capacitor with the switch closed is $R_{EQ} = 1.5 \parallel 15 = 1.3636$ kΩ. The time constant is $T_{C1} = R_{EQ}C = (1363.6)(1\,\mu) = 1.3636$ ms and $1/T_{C1} = 733$ Hz. When the switch reopens, the new final voltage will be $v_C(\infty) = 0$ V, the equivalent resistance will be $R_{EQ} = 15$ kΩ, the time constant will be $T_{C2} = 15$ ms, and $1/T_{C2} = 66.67$ Hz. Write the expression for the capacitor voltage with the switch closed, find the capacitor voltage when the switch reopens, and then construct the complete expression for the capacitor voltage.

$$v_{C1}(t) = [0 - 21.818]e^{-733t} + 21.818 = 21.818(1 - e^{-733t}) \text{ V}$$

$$v_{C1}(0.003) = 21.818(1 - e^{-2.2}) = 19.4 \text{ V}$$

$$v_C(t) = 21.818(1 - e^{-733t})[u(t) - u(t-0.003)] + 19.4e^{-66.67(t-0.003)}u(t-0.003) \text{ V}$$

Problem 7–29. Find the sinusoidal steady-state response of $v_C(t)$ in Figure P7–29 when $R = 100$ kΩ, $C = 0.01\ \mu$F, and the input voltage is $v_S(t) = 10\cos(100t)u(t)$ V. Repeat for an input voltage of $v_S(t) = 10\cos(1000t)u(t)$ V and one more time for an input voltage of $v_S(t) = 10\cos(10000t)u(t)$ V. Describe how changing the frequency affects the output's amplitude and phase. You may choose to use MATLAB and plot the steady-state responses of each input on a single set of axes.

The forced response has the following differential equation:

$$RC\frac{dv_{CF}(t)}{dt} + v_{CF}(t) = V_A\cos(\omega t)$$

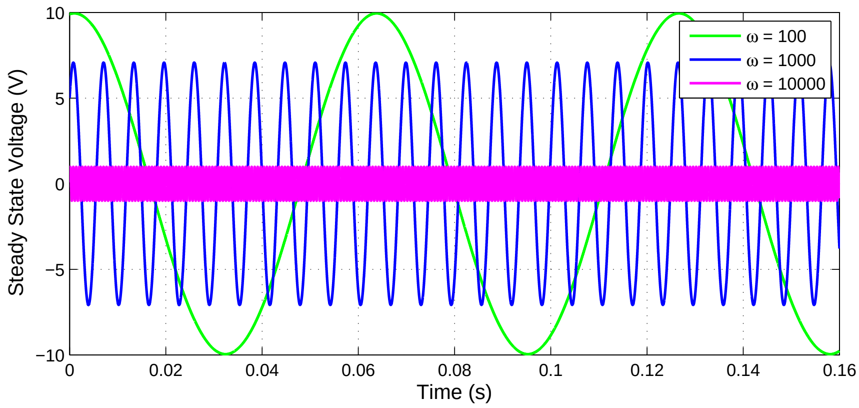
and, as developed in the text, the corresponding solution is:

$$v_{CF}(t) = \frac{V_A}{1 + (\omega RC)^2}[\cos(\omega t) + \omega RC\sin(\omega t)] = \frac{V_A}{\sqrt{1 + (\omega RC)^2}}\cos(\omega t + \theta)$$

where $\theta = \tan^{-1}(-\omega RC)$. For each sinusoidal input, $V_A = 10$ V and the frequency ω varies. Compute the magnitude and phase for each sinusoidal steady-state response. For $v_S(t) = 10\cos(\omega t)u(t)$ V, with $\omega = 100$ rad/s, $v_C(t) = 9.9504\cos(100t - 5.71°)$ V; with $\omega = 1000$ rad/s, $v_C(t) = 7.0711\cos(1000t - 45°)$ V; and with $\omega = 10000$ rad/s, $v_C(t) = 0.995\cos(10000t - 84.29°)$ V. As the frequency increases, the amplitude decreases and the phase shift approaches $-90°$. The following table summarizes the results.

ω (rad/s)	Amplitude (V)	Phase (Degrees)
100	9.9504	-5.71
1000	7.0711	-45.00
10000	0.995	-84.29

The following plot displays the steady-state responses for the various input frequencies.



Problem 7–30. On the circuit of Figure P7–29 the input is $v_S(t) = 10e^{-1000t}u(t)$ V. Find the output $v_C(t)$ when $R = 100$ kΩ, $C = 0.01$ μF, and $v_C(0) = 0$.

The differential equation for this circuit is

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 10e^{-1000t}u(t)$$

The time constant for the natural response is $T_C = RC = (100\,\text{k})(0.01\,\mu) = 1$ ms and $1/T_C = 1000$ Hz. The natural response will have the form $v_{CN}(t) = K_1e^{-1000t}$. The natural response has the same form as the input, so the forced response will have the form $v_{CF}(t) = K_2e^{-1000t} + K_3te^{-1000t}$. Substitute the forced response into the differential equation to solve for K_2 and K_3.

$$\frac{1}{1000}\left(-1000K_2e^{-1000t} - 1000K_3te^{-1000t} + K_3e^{-1000t}\right) + K_2e^{-1000t} + K_3te^{-1000t} = 10e^{-1000t}$$

$$-K_2 - K_3t + \frac{K_3}{1000} + K_2 + K_3t = 10$$

$$K_3 = 10000$$

The K_2 terms cancel, so set $K_2 = 0$ without any loss in generality. The forced response is $v_{CF}(t) = 10000te^{-1000t}$. Apply the initial conditions to find the complete response.

$$v_C(t) = v_{CN}(t) + v_{CF}(t) = K_1e^{-1000t} + 10000te^{-1000t}$$

$$v_C(0) = 0 = (K_1)(1) + (10000)(0)(1) = K_1$$

$$v_C(t) = 10000te^{-1000t} \text{ V}$$

The following MATLAB code confirms the solution.

```
syms t real
vCt = dsolve('1e-3*Dx + x = 10*exp(-1000*t)','x(0)=0','t')
```

The corresponding MATLAB output is shown below.

```
vCt = (10000*t)/exp(1000*t)
```

Problem 7–31. For $t \geq 0$ the zero-input response of the circuit in Figure P7–31 is $v_C(t) = 20e^{-10000t}$ V.

(a). Find C and $i_C(t)$ when $R = 10$ kΩ.

The time constant is $T_C = 1/10000 = 100$ μs. Compute the capacitance $C = T_C/R = (100\,\mu)/(10000) = 0.01$ μF. Compute the current.

$$i_C(t) = C\frac{dv_C(t)}{dt} = (0.01\,\mu)(20)(-10000)e^{-10000t} = -2e^{-10000t} \text{ mA}$$

(b). Find the energy stored in the capacitor at $t = 2$ ms.

Find the voltage at $t = 2$ ms and then the energy stored.

$$v_C(0.002) = 20e^{-20} = 41.223 \text{ nV}$$

$$w_C(0.002) = \frac{1}{2}Cv_C^2(0.002) = (0.5)(0.01\,\mu)(41.223\,\text{n})^2 = 8.4967 \times 10^{-24} \text{ J}$$

Problem 7–32. For $t > 0$ the zero-input response of the circuit in Figure P7–32 is $i_L(t) = 150e^{-500t}$ mA.

(a). Find L and $v_L(t)$ when $R = 500$ Ω.

The time constant is $T_C = 1/500 = 2$ ms. Compute the inductance $L = T_C R = (0.002)(500) = 1$ H. Compute the voltage.

$$v_L(t) = L\frac{di_L(t)}{dt} = (1)(0.15)(-500)e^{-500t} = -75e^{-500t} \text{ V}$$

(b). Find the energy stored in the inductor at $t = 0.5$ ms.

Find the current at $t = 0.5$ ms and then the energy stored.

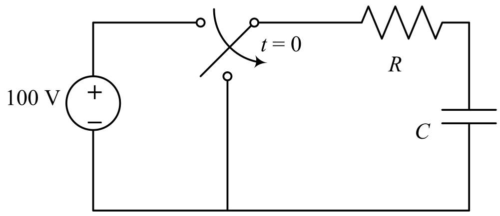
$$i_L(0.0005) = 0.15e^{-0.25} = 0.11682 \text{ A}$$

$$w_C(0.0005) = \frac{1}{2}Li_L^2(0.0005) = (0.5)(1)(0.11682) = 6.8235 \text{ mJ}$$

Problem 7–33. Design a series RC circuit using a dc voltage source that delivers the following voltage across the capacitor for $t > 0$.

$$v_C(t) = 100e^{-2000t} \text{ V} \quad t \geq 0$$

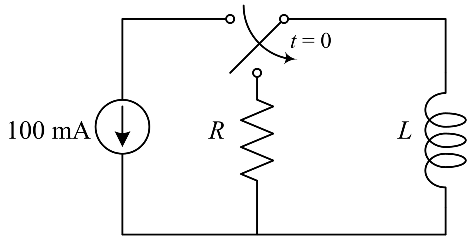
The initial capacitor voltage is $v_C(0) = 100$ V and the final voltage is $v_C(\infty) = 0$ V, so the capacitor must be initially charged and then allowed to discharge. The time constant is $T_C = 1/2000 = 500$ μs. Choose $C = 1$ μF and solve for $R = T_C/C = 500$ Ω. The following circuit provides one possible solution.



Problem 7–34. Design a parallel RL circuit using a dc current source that delivers the following voltage across the resistor for $t > 0$.

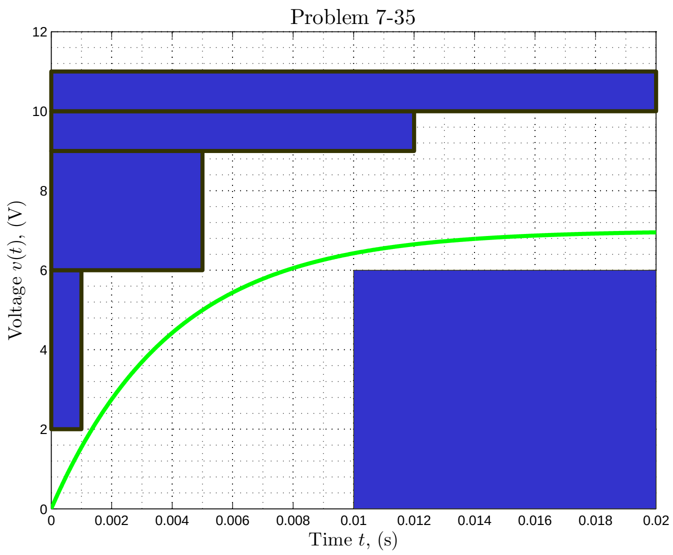
$$v_{\mathrm{R}}(t) = 100e^{-2000t} \text{ V} \quad t \geq 0$$

The initial resistor voltage is $v_{\mathrm{R}}(0) = 100$ V and the final voltage is $v_{\mathrm{R}}(\infty) = 0$ V, so the inductor must have an initial current that dissipates through the resistor. The time constant is $T_{\mathrm{C}} = 1/2000 = 500$ μs. Choose $R = 1$ kΩ and solve for $L = T_{\mathrm{C}}R = 500$ mH. With $R = 1$ kΩ, the initial current must be $i_{\mathrm{L}}(0) = 100$ mA. The following circuit provides one possible solution.

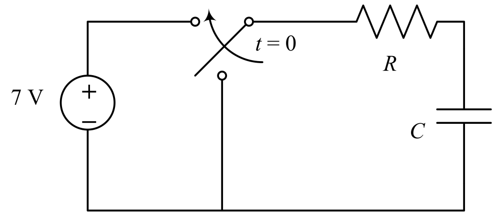


Problem 7–35. Design a series RC circuit using a dc voltage source that delivers a voltage across the capacitor for $t > 0$ that fits entirely within the non-shaded region of Figure P7–35.

The circuit charges a capacitor to a final voltage between 6 and 10 V with a time constant of approximately 4 ms. Use MATLAB to plot exponential signals with different final values and time constants to find one that fits within the specified region. One solution is shown below.

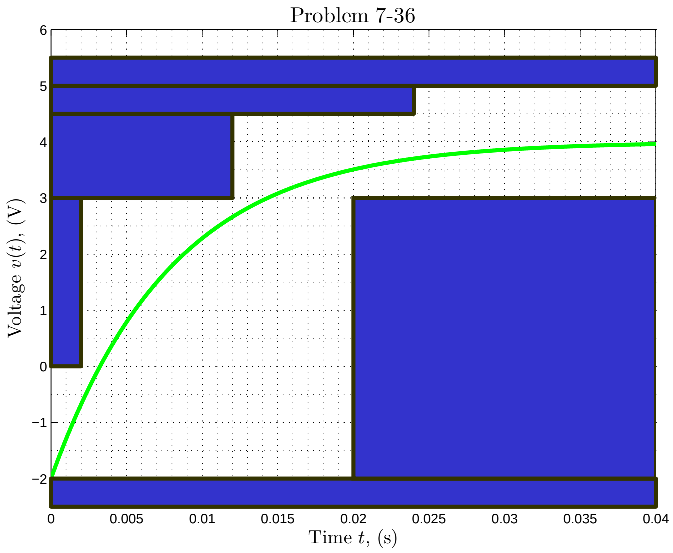


The plot shown used a final value of 7 V and a time constant of 4 ms. Choose $C = 1$ μF and solve for $R = T_{\mathrm{C}}/C = 4$ kΩ. The following circuit provides one possible solution.

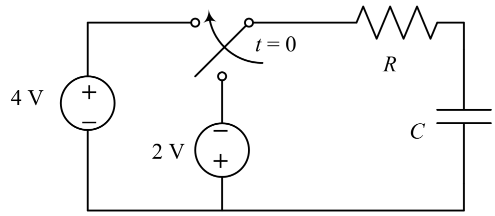


Problem 7–36. Design a series RC circuit using dc voltage sources that delivers a voltage across the capacitor for $t > 0$ that fits entirely within the non-shaded region of Figure P7–36.

The initial voltage is -2 V and the final voltage is between 3 and 5 V. The time constant is approximately 8 ms. Use MATLAB to examine a solution with a final voltage of 4 V and a time constant of 8 ms. The corresponding plot is shown below and meets the requirements.



Create a series RC circuit with two voltage sources that will meet the specifications. Choose $C = 1\ \mu$F and solve for $R = T_C/C = 8$ kΩ. The following circuit provides one possible solution.



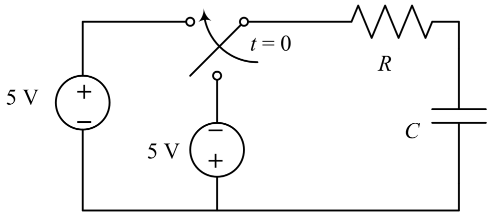
Problem 7–37. For $t \geq 0$ the step response of the voltage across the capacitor in Figure P7–37 is $v_C(t) = 5 - 10e^{-1000t}$ V. Find the IV, FV, T_C, R, and $i_C(t)$ when $C = 1.5\ \mu$F.

The initial value is $v_C(0) = 5 - (10)(1) = -5$ V and the final value is $v_C(\infty) = 5 - (10)(0) = 5$ V. The time constant is $T_C = 1/1000 = 1$ ms and the resistance is $R = T_C/C = 666.67\ \Omega$. The capacitor current is

$$i_C(t) = C\frac{dv_C(t)}{dt} = (1.5\,\mu)(-10)(-1000)(e^{-1000t} = 15e^{-1000t} \text{ mA}$$

Problem 7–38. Design a first-order RC circuit using standard parts (see inside rear cover) that will produce the following voltage across the capacitor: $v_C(t) = 5 - 10e^{-10000t}$ V.

The initial capacitor voltage is $v_C(0) = -5$ V and the final voltage is $v_C(\infty) = 5$ V. The time constant is $T_C = 1/10000 = 100\ \mu$s. Choose $C = 1\ \mu$F and solve for $R = T_C/C = 100\ \Omega$. With those element values, the following circuit will provide the required response.



Problem 7–39. For $t \geq 0$ the step responses of the current through and voltage across the inductor in Figure P7–39 are $i_L(t) = 5 - 10e^{-2000t}$ mA and $v_L(t) = e^{-2000t}$ V. Find IV, FV, T_C, R, and L.

The initial inductor current is $i_L(0) = -5$ mA and the final current is $i_L(\infty) = 5$ mA. The time constant is $T_C = 1/2000 = 500\ \mu$s. Use the inductor i-v relationship to find the inductance and then use the time constant, $T_C = L/R$, to find the value for the resistance.

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$e^{-2000t} = L\frac{d}{dt}(0.005 - 0.01e^{-2000t}) = (L)(-0.01)(-2000)e^{-2000t} = 20Le^{-2000t}$$

$$L = \frac{1}{20} = 50 \text{ mH}$$

$$R = \frac{L}{T_C} = \frac{0.05}{500\,\mu} = 100\ \Omega$$

Problem 7–40. Design a first-order RL circuit that will produce the following current through the inductor: $i_L(t) = 50 + 100e^{-20000t}$ mA for $t \geq 0$.

The initial inductor current is $i_L(0) = 150$ mA and the final current $i_L(\infty) = 50$ mA. The time constant is $T_C = 1/20000 = 50\ \mu$s. Choose $R = 1$ kΩ and solve for $L = T_C R = (50\,\mu)(1000) = 50$ mH. A simple design would be a 1-kΩ resistor in parallel with a 50-mH inductor and both in parallel with two current sources. One current source is 50 mA and the other is 100 mA. Both current sources have been connected to the RL circuit for a long time and the 100-mA source is disconnected at time $t = 0$.

Problem 7–41. The switch in Figure P7–41 has been in position B for a long time and is moved to position A at $t = 0$. Design the first-order RC interface circuit such that $v_O(t) = 15 - 15e^{-5000t}$ V.

The initial output voltage is $v_O(0) = 0$ V and the final voltage is $v_O(\infty) = 15$ V. The time constant is $T_C = 1/5000 = 200\ \mu$s. To get a final voltage of 15 V, put a 100-Ω resistor in series with the source. By voltage division, the final voltage will be $v_O(\infty) = (300)(20)/(100 + 300) = 15$ V. To get the correct exponential response, place a capacitor in parallel with the 300-Ω resistor. After the switch moves, the capacitor sees an equivalent resistance of $R_{EQ} = 100 \parallel 300 = 75\ \Omega$. Choose the capacitance to create the correct time constant, $C = T_C/R = (200\,\mu)/75 = 2.67\ \mu$F.

Problem 7–42. The switch in Figure P7–41 has been in position A for a long time and is moved to position B at $t = 0$. Design the first-order RC interface circuit such that $v_O(t) = 15e^{-5000t}$ V.

The initial output voltage is $v_O(0) = 15$ V and the final voltage is $v_O(\infty) = 0$ V. The time constant is $T_C = 1/5000 = 200\ \mu$s. To get the initial voltage of 15 V, put a 100-Ω resistor in series with the source. By voltage division, the initial voltage will be $v_O(\infty) = (300)(20)/(100 + 300) = 15$ V. Place a capacitor in parallel with the 300-Ω resistor. Once the switch moves, the source will be disconnected and the output voltage will naturally decay to zero. Choose the capacitor to get the correct time constant. The equivalent resistance is $R_{EQ} = 100 \parallel 300 = 75\ \Omega$, so we have $C = T_C/R = (200\,\mu)/75 = 2.67\ \mu$F. The design is the same as that for Problem 7–41.

Problem 7–43. A timing circuit is required that feeds into an OP AMP's non-inverting terminal (i.e., draws no current.) The circuit's output response $v_O(t)$ must be

$$v_O(t) = 5\left(1 - e^{-1000t}\right)u(t) \text{ V}$$

Figure P7–43 shows two commercial products and the vendors claim each will meet the requirement. Which will you select and why?

Given the required output voltage signal, the initial value is $v_O(0) = 0$ V, the final value is $v_O(\infty) = 5$ V, and the time constant is $T_C = 1/1000 = 1$ ms. Examine each of the circuits to see if they also have these values.

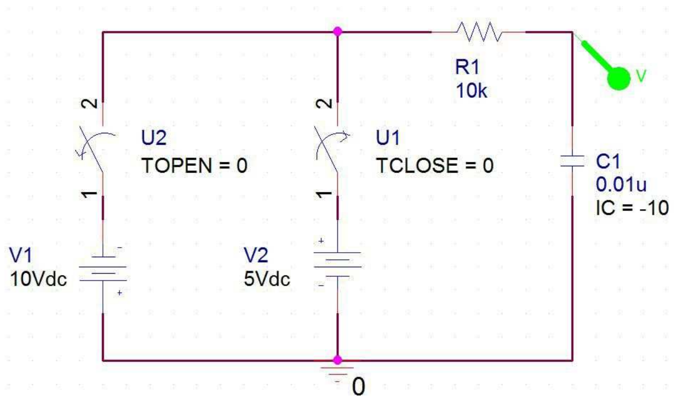
The circuit from Vendor A has the output voltage across a capacitor, so we need to find the voltages associated with the capacitor. Assuming the switch has been in the down position for a long time, the initial voltage will be 0 V. The final voltage will be 5 V, because the capacitor will drop all of the voltage from the source. The time constant is $T_C = (1\,\text{k})(1\,\mu) = 1$ ms. The circuit from Vendor A meets the specifications and the cost is \$7.00.

The circuit from Vendor B has the output voltage across a resistor in series with an inductor. We will find the values for the inductor current and then translate them into output voltages across the resistor. If the switch has been in the down position for a long time, the initial current is 0 mA and the initial voltage across the resistor is 0 V. The final current through the inductor will be $i_L(\infty) = 5/1000 = 5$ mA and the final voltage across the resistor will be 5 V. The time constant is $T_C = 1/1000 = 1$ ms. The circuit from Vendor B meets the specifications and the cost is \$4.50.

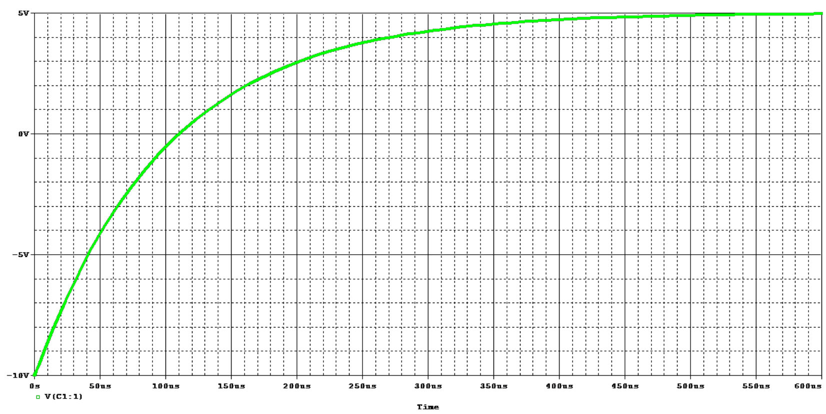
Choose Vendor B because it is less expensive and meets the specification.

Problem 7–44. A product line needs an RC circuit that will meet the following response specifications $\pm 5\%$: IV $= -10$ V, FV $= +5$ V, $T_C = 100$ μs, $R \geq 1$ kΩ, and $C \leq 0.1$ μF. Design a circuit to meet the specifications and validate your results using OrCAD.

Design an RC circuit connected to two voltage sources. One voltage source is -10 V and the other is $+5$ V. The -10-V source has been connected to the RC circuit for a long time and is disconnected at $t = 0$. The $+5$-V source has been disconnected and is connected at $t = 0$. Choose $R = 10$ kΩ and solve for the capacitance $C = T_C/R = (100\,\mu)/(10000) = 0.01$ μF. The following design meets the specifications:



The corresponding simulation is shown below.



Problem 7–45. Find the $v(t)$ that satisfies the following differential equation and initial conditions:

$$\frac{d^2v(t)}{dt^2} + 5\frac{dv(t)}{dt} + 36v(t) = 0, \;\; v(0) = 0 \text{ V}, \;\; \frac{dv(0)}{dt} = 20 \text{ V/s}$$

The corresponding characteristic equation for this differential equation is

$$s^2 + 5s + 36 = 0$$

The roots of the characteristic equation are $s = -2.5 \pm j5.4544$, so the solution has the following form:

$$v(t) = K_1 e^{-2.5t} \cos(5.4544t) + K_2 e^{-2.5t} \sin(5.4544t)$$

Apply the initial conditions to solve for the constants K_1 and K_2

$$v(0) = 0 = (K_1)(1)(1) + (K_2)(1)(0) = K_1$$

$$\frac{dv(0)}{dt} = 20 = \frac{d}{dt}\left[K_2 e^{-2.5t} \sin(5.4544t)\right]$$

$$20 = K_2\left[5.4544e^{-2.5t}\cos(5.4544t) - 2.5e^{-2.5t}\sin(5.4544t)\right]\Big|_{t=0} = K_2\left[5.4544 - 0\right]$$

$$K_2 = \frac{20}{5.4544} = 3.667$$

The final solution is

$$v(t) = 3.667e^{-2.5t} \sin(5.4544t) \text{ V}$$

Problem 7–46. Find the $v(t)$ that satisfies the following differential equation and initial conditions:

$$\frac{d^2v(t)}{dt^2} + 10\frac{dv(t)}{dt} + 100v(t) = 0, \;\; v(0) = 5 \text{ V}, \;\; \frac{dv(0)}{dt} = 0 \text{ V/s}$$

The corresponding characteristic equation for this differential equation is

$$s^2 + 10s + 100 = 0$$

The roots of the characteristic equation are $s = -5 \pm j8.6603$, so the solution has the following form:

$$v(t) = K_1 e^{-5t} \cos(8.6603t) + K_2 e^{-5t} \sin(8.6603t)$$

Apply the initial conditions to solve for the constants K_1 and K_2

$$v(0) = 5 = (K_1)(1)(1) + (K_2)(1)(0) = K_1$$

$$\frac{dv(0)}{dt} = 0 = \frac{d}{dt}\left[5e^{-5t}\cos(8.6603t) + K_2 e^{-5t}\sin(8.6603t)\right]$$

$$0 = \Big[(5)(-8.6603)e^{-5t}\sin(8.6603t) + (5)(-5)e^{-5t}\cos(8.6603t)$$

$$+ K_2(8.6603)e^{-5t}\cos(8.6603t) + K_2(-5)e^{-5t}\sin(8.6603t)\Big]\Big|_{t=0}$$

$$0 = -25 + 8.6603K_2$$

$$K_2 = \frac{25}{8.6603} = 2.887$$

The final solution is

$$v(t) = e^{-5t}\left[5\cos(8.6603t) + 2.887\sin(8.6603t)\right] \text{ V}$$

Problem 7–47. Find the $v(t)$ that satisfies the following differential equation and initial conditions:

$$\frac{d^2v(t)}{dt^2} + 10\frac{dv(t)}{dt} + 125v(t) = 250u(t), \quad v(0) = 10 \text{ V}, \quad \frac{dv(0)}{dt} = 25 \text{ V/s}$$

The input is a step function, so the forced response is a constant. Substitute $v_\text{F}(t) = K$ into the differential equation to get

$$0 + (10)(0) + 125K = 250$$

The forced response is $v_\text{F}(t) = 2$. The corresponding characteristic equation for this differential equation is

$$s^2 + 10s + 125 = 0$$

The roots of the characteristic equation are $s = -5 \pm j10$, so the natural response has the following form:

$$v_\text{N}(t) = K_1e^{-5t}\cos(10t) + K_2e^{-5t}\sin(10t)$$

The complete response has the form

$$v(t) = v_\text{N}(t) + v_\text{F}(t) = 2 + K_1e^{-5t}\cos(10t) + K_2e^{-5t}\sin(10t)$$

Apply the initial conditions to solve for the constants K_1 and K_2

$$v(0) = 10 = 2 + (K_1)(1)(1) + (K_2)(1)(0) = 2 + K_1$$

$$K_1 = 8$$

$$\frac{dv(0)}{dt} = 25 = \frac{d}{dt}\left[2 + 8e^{-5t}\cos(10t) + K_2e^{-5t}\sin(10t)\right]$$

$$25 = \Big[(8)(-10)e^{-5t}\sin(10t) + (8)(-5)e^{-5t}\cos(10t)$$

$$+ K_2(10)e^{-5t}\cos(10t) + K_2(-5)e^{-5t}\sin(10t)\Big]\Big|_{t=0}$$

$$25 = -40 + 10K_2$$

$$K_2 = \frac{65}{10} = 6.5$$

The final solution is

$$v(t) = 2 + e^{-5t}\left[8\cos(10t) + 6.5\sin(10t)\right] \text{ V}$$

Problem 7–48. Find the $i(t)$ that satisfies the following differential equation and initial conditions:

$$\frac{d^2i(t)}{dt^2} + 8\frac{di(t)}{dt} + 16i(t) = 48u(t), \quad i(0) = 0 \text{ A}, \quad \frac{di(0)}{dt} = 0 \text{ A/s}$$

The input is a step function, so the forced response is a constant. Substitute $i_\text{F}(t) = K$ into the differential equation to get

$$0 + (8)(0) + 16K = 48$$

The forced response is $i_\text{F}(t) = 3$. The corresponding characteristic equation for this differential equation is

$$s^2 + 8s + 16 = 0$$

The roots of the characteristic equation are repeated with $s_1 = s_2 = -4$, so the natural response has the following form:

$$i_\text{N}(t) = K_1e^{-4t} + K_2te^{-4t}$$

The complete response has the form

$$i(t) = i_{\mathrm{N}}(t) + i_{\mathrm{F}}(t) = 3 + K_1 e^{-4t} + K_2 t e^{-4t}$$

Apply the initial conditions to solve for the constants K_1 and K_2

$$i(0) = 0 = 3 + K_1 + 0$$

$$K_1 = -3$$

$$\frac{di(0)}{dt} = 0 = \frac{d}{dt}\left(3 - 3e^{-4t} + K_2 t e^{-4t}\right)$$

$$0 = \left[(-3)(-4)e^{-4t} + K_2 t(-4)e^{-4t} + K_2 e^{-4t}\right]\Big|_{t=0} = 12 + K_2$$

$$K_2 = -12$$

The final solution is

$$i(t) = 3 - 3e^{-4t} - 12te^{-4t} \text{ A}$$

Problem 7–49. The switch in Figure P7–49 has been open for a long time and is closed at $t = 0$. The circuit parameters are $L = 1$ H, $C = 0.5$ μF, $R = 1$ kΩ, and $v_{\mathrm{C}}(0) = 10$ V.

(a). Find $v_{\mathrm{C}}(t)$ and $i_{\mathrm{L}}(t)$ for $t \geq 0$.

The circuit is an RLC series circuit with no input signal. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2 v_{\mathrm{C}}(t)}{dt^2} + RC\frac{dv_{\mathrm{C}}(t)}{dt} + v_{\mathrm{C}}(t) = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 1000s + 2000000 = 0$$

The roots of the characteristic equation are $s = -500 \pm j1323$ and the response has the form

$$v_{\mathrm{C}}(t) = K_1 e^{-500t}\cos(1323t) + K_2 e^{-500t}\sin(1323t)$$

Apply the initial conditions to solve for the constants K_1 and K_2.

$$v_{\mathrm{C}}(0) = 10 = (K_1)(1)(1) + (K_2)(1)(0) = K_1$$

$$i_{\mathrm{L}}(0) = i_{\mathrm{C}}(0) = C\frac{dv_{\mathrm{C}}(0)}{dt} = 0 = C\frac{d}{dt}\left[10e^{-500t}\cos(1323t) + K_2 e^{-500t}\sin(1323t)\right]\Big|_{t=0}$$

$$0 = (10)(-500) + 1323K_2$$

$$K_2 = 3.78$$

The final expression for the voltage is

$$v_{\mathrm{C}}(t) = e^{-500t}\left[10\cos(1323t) + 3.78\sin(1323t)\right] \text{ V}$$

Compute the current

$$i_L(t) = i_C(t) = C\frac{dv_C(t)}{dt} = (0.5\,\mu)\frac{d}{dt}\left\{e^{-500t}\left[10\cos(1323t) + 3.78\sin(1323t)\right]\right\}$$

$$= (0.5\,\mu)\Big\{e^{-500t}\left[-13230\sin(1323t) + 5000\cos(1323t)\right]$$

$$-\,500e^{-500t}\left[10\cos(1323t) + 3.78\sin(1323t)\right]\Big\}$$

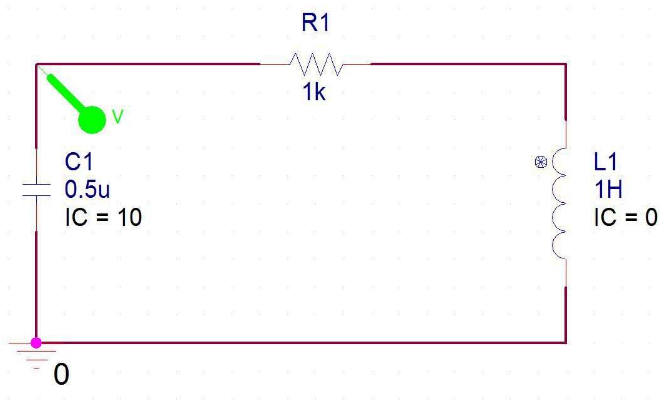
$$= -7.56e^{-500t}\sin(1323t)\text{ mA}$$

(b). Is the circuit overdamped, critically damped, or underdamped?

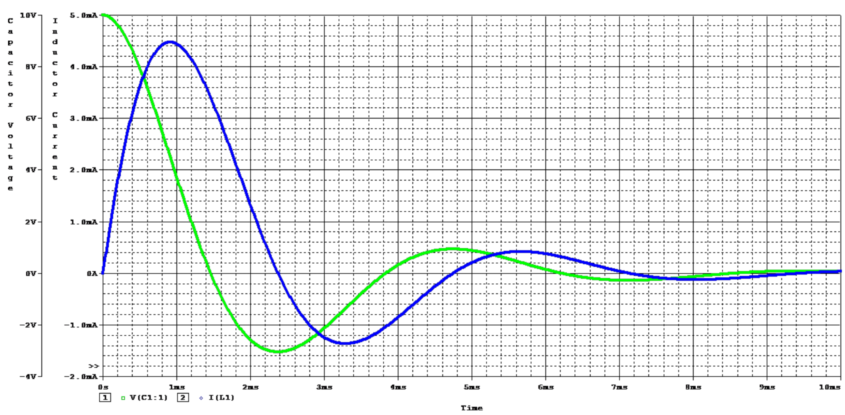
Based on the form of the natural response, the circuit is underdamped.

(c). Use OrCAD to simulate your results.

The following OrCAD simulation provides the results.



The corresponding output plot is shown below.



Problem 7–50. The switch in Figure P7–50 has been open for a long time and is closed at $t = 0$. The circuit parameters are $L = 1$ H, $C = 1\ \mu$F, $R = 500\ \Omega$, and $v_C(0) = 60$ V.

(a). Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

The circuit is an RLC parallel circuit with no input signal. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2 i_{\mathrm{L}}(t)}{dt^2} + \frac{L}{R}\frac{di_{\mathrm{L}}(t)}{dt} + i_{\mathrm{L}}(t) = 0$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s^2 + 2000s + 1000000 = 0$$

The roots of the characteristic equation are repeated with $s_1 = s_2 = -1000$, so the response has the following form:

$$i_{\mathrm{L}}(t) = K_1 e^{-1000t} + K_2 t e^{-1000t}$$

Apply the initial conditions to solve for the constants K_1 and K_2. Since the switch has been open for a long time, the initial current through the inductor is zero.

$$i_{\mathrm{L}}(0) = 0 = (K_1)(1) + (K_2)(0)(1) = K_1$$

$$v_{\mathrm{C}}(0) = 60 = L\frac{di_{\mathrm{L}}(0)}{dt} = (1)\frac{d}{dt}\left[K_2 t e^{-1000t}\right]\Big|_{t=0}$$

$$60 = \left[-1000K_2 t e^{-1000t} + K_2 e^{-1000t}\right]\Big|_{t=0} = K_2$$

The final expression for the inductor current is

$$i_{\mathrm{L}}(t) = 60te^{-1000t} \text{ A}$$

Compute the voltage.

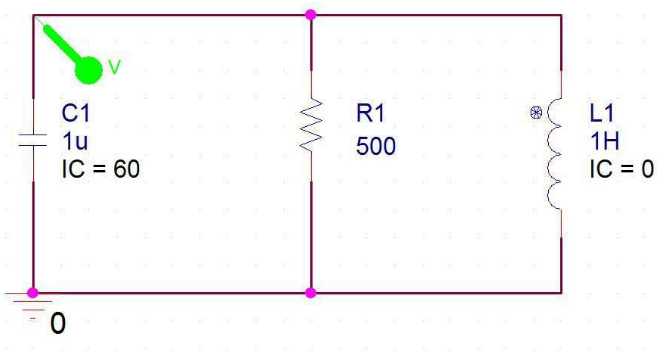
$$v_{\mathrm{C}}(t) = v_{\mathrm{L}}(t) = L\frac{di_{\mathrm{L}}(t)}{dt} = (1)\left[-60000te^{-1000t} + 60e^{-1000t}\right] = e^{-1000t}\left[60 - 60000t\right] \text{ V}$$

(b). Is the circuit overdamped, critically damped, or underdamped?

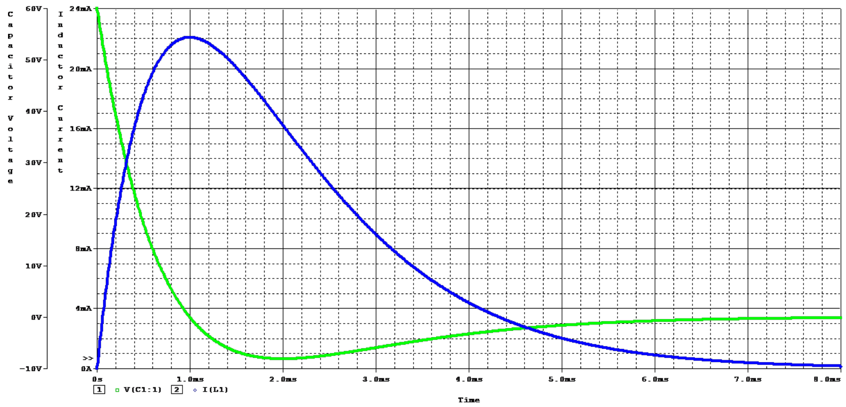
The response has repeated real roots, so it is critically damped.

(c). Use OrCAD to simulate your results.

The following OrCAD simulation provides the results.

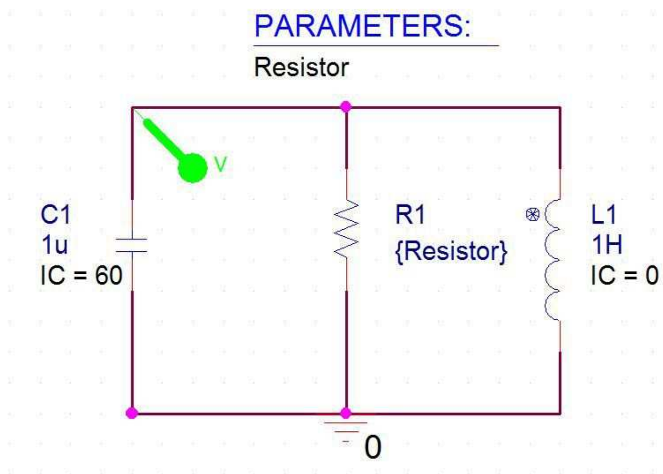


The corresponding output plot is shown below.

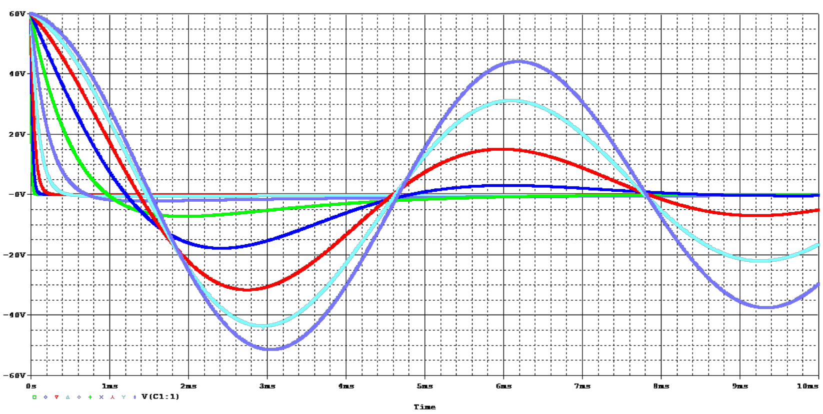


Problem 7–51. Use OrCAD to study how the voltage across the circuit in Figure P7–50 changes as the value of the resistor is varied. Use the Parameter function (PARAM) found in the Special Library and do a Transient Analysis for 10 ms. Select Parametric Sweep and then select Global Parameter. Use the same name as for the Parametric function and vary the resistor from 10 Ω to 10 kΩ using the logarithmic choice and 3 points per decade. This will generate a family of curves corresponding to different values of resistance. By clicking on the various curves, determine if the response becomes more or less damped as R varies from 10 Ω to 10 kΩ.

The following OrCAD simulation provides the results.



The corresponding output plot is shown below.



As the resistance increases, the response become less damped. In the limit with an infinitely large resistor, the response would have no damping and the energy would oscillate between the capacitor and inductor.

Problem 7–52. The switch in Figure P7–52 has been open for a long time and is closed at $t = 0$. The circuit parameters are $L = 0.8$ H, $C = 1.25$ μF, $R_1 = 2$ kΩ, $R_2 = 1$ kΩ, and $V_A = 12$ V.

(a). Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

Before the switch closes, the voltage across the capacitor is $v_C(0) = V_A = 12$ V and the current through the inductor is $i_L(0) = 0$ A. After the switch closes, the circuit is an RLC series circuit with no input signal. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2v_C(t)}{dt^2} + R_2C\frac{dv_C(t)}{dt} + v_C(t) = 0$$

$$s^2 + \frac{R_2}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 1250s + 1000000 = 0$$

The roots of the characteristic equation are $s = -625 \pm j780.6$ and the response has the form

$$v_C(t) = K_1e^{-625t}\cos(780.6t) + K_2e^{-625t}\sin(780.6t)$$

Apply the initial conditions to solve for the constants K_1 and K_2.

$$v_C(0) = 12 = (K_1)(1)(1) + (K_2)(1)(0) = K_1$$

$$i_L(0) = 0 = C\frac{dv_C(0)}{dt} = C\frac{d}{dt}\left[12e^{-625t}\cos(780.6t) + K_2e^{-625t}\sin(780.6t)\right]\Big|_{t=0}$$

$$0 = (-625)(12) + (K_2)(780.6)$$

$$K_2 = 9.608$$

The final expression for the capacitor voltage is

$$v_C(t) = e^{-625t}\left[12\cos(780.6t) + 9.608\sin(780.6t)\right] \text{ V}$$

Compute the inductor current.

$$i_L(t) = i_C(t) = C\frac{dv_C(t)}{dt} = (1.25\,\mu)\frac{d}{dt}\left\{e^{-625t}\left[12\cos(780.6t) + 9.608\sin(780.6t)\right]\right\}$$

$$= (1.25\,\mu)\Big\{e^{-625t}\left[-9367\sin(780.6t) + 7500\cos(780.6)\right]$$

$$- 625e^{-625t}\left[12\cos(780.6t) + 9.608\sin(780.6t)\right]\Big\}$$

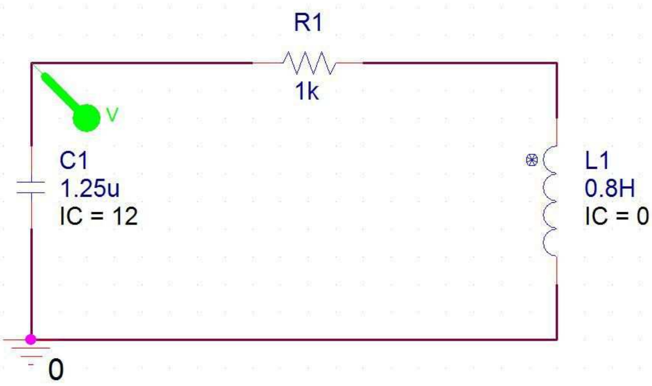
$$= -19.215e^{-625t}\sin(780.6t) \text{ mA}$$

(b). Is the circuit overdamped, critically damped, or underdamped?

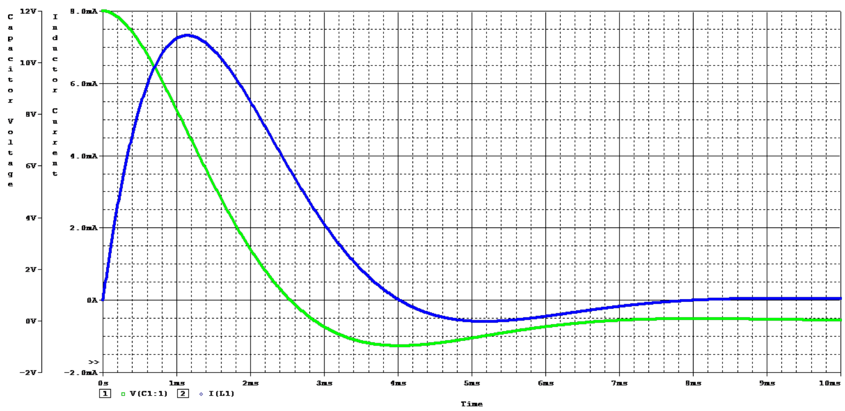
Based on the form of the response, the circuit is underdamped.

(c). Use OrCAD to simulate your results.

The following OrCAD simulation provides the results.



The corresponding output plot is shown below.



Problem 7–53. The switch in Figure P7–53 has been open for a long time and is closed at $t = 0$. The circuit parameters are $L = 1.25$ H, $C = 0.05\ \mu$F, $R_1 = 20$ kΩ, $R_2 = 20$ kΩ, and $V_A = 20$ V.

(a). Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

Before the switch closes, the initial conditions are both zero, with $v_C(0) = 0$ V and $i_L(0) = 0$ A. After the switch closes, the equivalent circuit, after a source transformation and combining the parallel resistors, is a current source $I_A = V_A/R_1 = 1$ mA and a parallel RLC circuit with $R = 20 \parallel 20 = 10$ kΩ. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2 i_L(t)}{dt^2} + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = 0.001 \text{ A}$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s^2 + 2000s + 16000000 = 0$$

The input signal is a constant, so the forced response is also a constant and we have

$$i_{LF}(t) = 0.001 \text{ A}$$

The roots of the characteristic equation are $s = -1000 \pm j3873$, so the natural response has the following form:

$$i_{LN}(t) = K_1 e^{-1000t}\cos(3873t) + K_2 e^{-1000t}\sin(3873t)$$

The complete response has the form

$$i_L(t) = i_{LF}(t) + i_{LN}(t) = 0.001 + K_1 e^{-1000t}\cos(3873t) + K_2 e^{-1000t}\sin(3873t)$$

Apply the initial conditions to find K_1 and K_2.

$$i_L(0) = 0 = 0.001 + (K_1)(1)(1) + (K_2)(1)(0)$$

$$K_1 = -0.001$$

$$v_C(0) = 0 = L\frac{di_L(0)}{dt} = 1.25\frac{d}{dt}\left\{0.001 - 0.001e^{-1000t}\cos(3873t) + K_2e^{-1000t}\sin(3873t)\right\}\Big|_{t=0}$$

$$0 = (-1000)(-0.001) + 3873K_2$$

$$K_2 = -0.0002582$$

The final expression for the inductor current is

$$i_L(t) = 1 - e^{-1000t}\left[\cos(3873t) + 0.2582\sin(3873t)\right] \text{ mA}$$

Compute the capacitor voltage.

$$v_C(t) = v_L(t) = L\frac{di_L(t)}{dt}$$

$$= (1.25)\frac{d}{dt}\left\{0.001 - 0.001e^{-1000t}\left[\cos(3873t) + 0.2582\sin(3873t)\right]\right\}$$

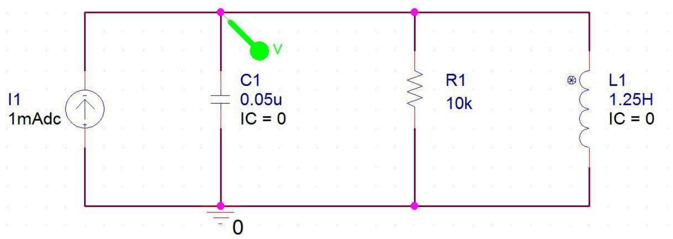
$$= 5.164e^{-1000t}\sin(3873t) \text{ V}$$

(b). Is the circuit overdamped, critically damped, or underdamped?

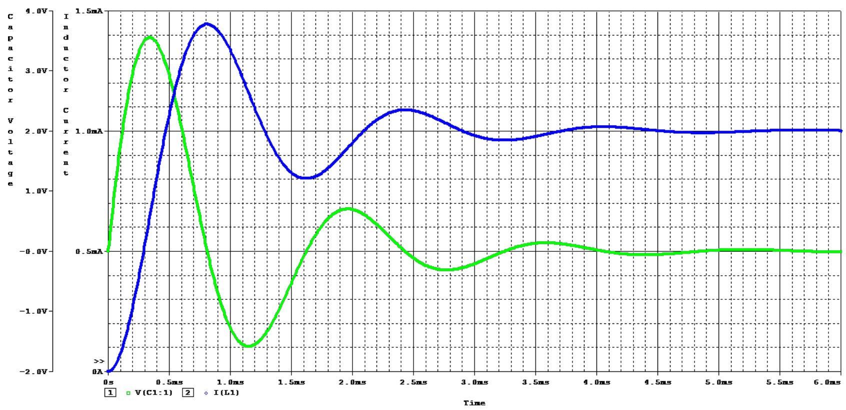
Based on the form of the response, the circuit is underdamped.

(c). Use OrCAD to simulate your results.

The following OrCAD simulation provides the results.



The corresponding output plot is shown below.



Problem 7–54. Repeat Problem 7–53 with $R_1 = 4$ kΩ, $R_2 = 4$ kΩ.

(a). Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

Before the switch closes, the initial conditions are both zero, with $v_C(0) = 0$ V and $i_L(0) = 0$ A. After the switch closes, the equivalent circuit, after a source transformation and combining the parallel resistors, is a current source $I_A = V_A/R_1 = 5$ mA and a parallel RLC circuit with $R = 4 \parallel 4 = 2$ kΩ. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2 i_L(t)}{dt^2} + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = 0.005 \text{ A}$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s^2 + 10000s + 16000000 = 0$$

The input signal is a constant, so the forced response is also a constant and we have

$$i_{LF}(t) = 0.005 \text{ A}$$

The roots of the characteristic equation are $s_1 = -2000$ and $s_2 = -8000$, so the natural response has the following form:

$$i_{LN}(t) = K_1 e^{-2000t} + K_2 e^{-8000t}$$

The complete response has the form

$$i_L(t) = i_{LF}(t) + i_{LN}(t) = 0.005 + K_1 e^{-2000t} + K_2 e^{-8000t}$$

Apply the initial conditions to find K_1 and K_2.

$$i_L(0) = 0 = 0.005 + K_1 + K_2$$

$$v_C(0) = 0 = L\frac{di_L(0)}{dt} = 1.25\frac{d}{dt}\left[0.005 + K_1 e^{-2000t} + K_2 e^{-8000t}\right]\Big|_{t=0}$$

$$0 = -2000K_1 - 8000K_2$$

$$K_1 = -0.006667$$

$$K_2 = 0.001667$$

The final expression for the inductor current is

$$i_L(t) = 5 - 6.667e^{-2000t} + 1.667e^{-8000t} \text{ mA}$$

Compute the capacitor voltage.

$$v_C(t) = v_L(t) = L\frac{di_L(t)}{dt}$$

$$= (1.25)\frac{d}{dt}\left[5 - 6.667e^{-2000t} + 1.667e^{-8000t}\right]$$

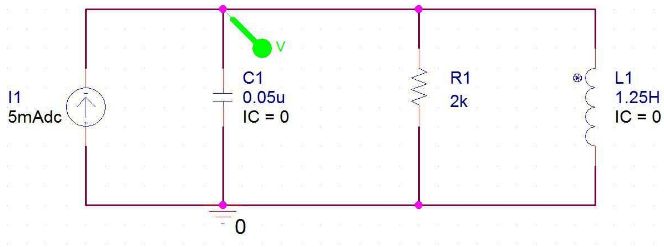
$$= 16.667e^{-2000t} - 16.667e^{-8000t} \text{ V}$$

(b). Is the circuit overdamped, critically damped, or underdamped?

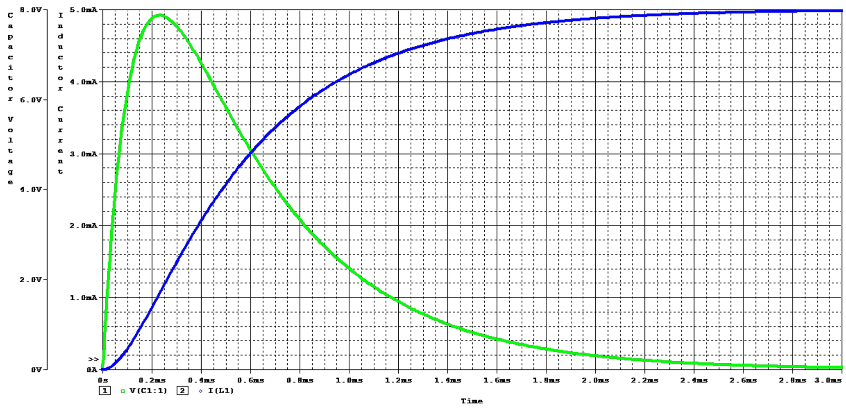
Based on the form of the response, the circuit is overdamped.

(c). Use OrCAD to simulate your results.

The following OrCAD simulation provides the results.



The corresponding output plot is shown below.



Problem 7–55. The switch in Figure P7–55 has been in position A for a long time. At $t = 0$ it is moved to position B. The circuit parameters are $R_1 = 20$ kΩ, $R_2 = 4$ kΩ, $L = 1.6$ H, $C = 1.25$ μF, and $V_A = 24$ V.

(a). Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

Before the switch moves, the current flowing through the inductor is $i_L(0) = V_A/(R_1 + R_2) = 24/24000 = 1$ mA and the voltage across the capacitor is $v_C(0) = 0$ V. After the switch moves, perform a source transformation to get a parallel RLC circuit with current source with magnitude $V_A/R_2 = 24/4000 = 6$ mA, with the reference arrow pointing down. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2 i_L(t)}{dt^2} + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = -0.006 \text{ A}$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s^2 + 200s + 500000 = 0$$

The input signal is a constant, so the forced response is also a constant and we have

$$i_{LF}(t) = -0.006 \text{ A}$$

The roots of the characteristic equation are $s = -100 \pm j700$, so the natural response has the following form:

$$i_{LN}(t) = K_1 e^{-100t}\cos(700t) + K_2 e^{-100t}\sin(700t)$$

The complete response has the form

$$i_L(t) = i_{LF}(t) + i_{LN}(t) = -0.006 + K_1 e^{-100t}\cos(700t) + K_2 e^{-100t}\sin(700t)$$

Apply the initial conditions to find K_1 and K_2.

$$i_{\mathrm{L}}(0) = 0.001 = -0.006 + (K_1)(1)(1) + (K_2)(1)(0)$$

$$K_1 = 0.007$$

$$v_{\mathrm{C}}(0) = 0 = L\frac{di_{\mathrm{L}}(0)}{dt} = 1.6\frac{d}{dt}\left\{-0.006 + 0.007e^{-100t}\cos(700t) + K_2e^{-100t}\sin(700t)\right\}\Big|_{t=0}$$

$$0 = (-100)(0.007) + 700K_2$$

$$K_2 = 0.001$$

The final expression for the inductor current is

$$i_{\mathrm{L}}(t) = -6 + e^{-100t}\left[7\cos(700t) + \sin(700t)\right] \text{ mA}$$

Compute the capacitor voltage.

$$v_{\mathrm{C}}(t) = v_{\mathrm{L}}(t) = L\frac{di_{\mathrm{L}}(t)}{dt}$$

$$= (1.6)\frac{d}{dt}\left\{-0.006 + e^{-100t}\left[0.007\cos(700t) + 0.001\sin(700t)\right]\right\}$$

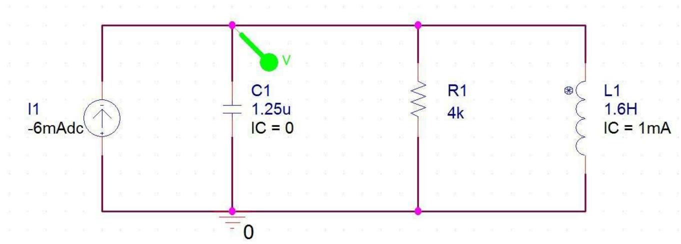
$$= -8e^{-100t}\sin(700t) \text{ V}$$

(b). Is the circuit overdamped, critically damped, or underdamped?

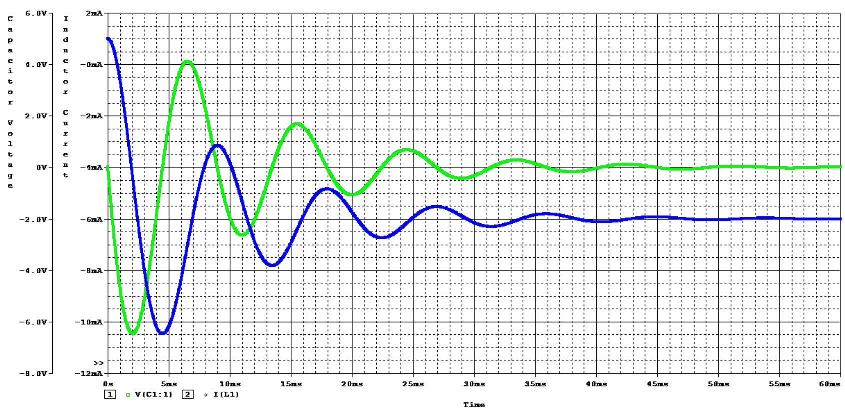
Based on the form of the response, the circuit is underdamped.

(c). Use OrCAD to simulate your results.

The following OrCAD simulation provides the results.



The corresponding output plot is shown below.



Problem 7–56. You have a need for an interface circuit that will connect your source to a load with a very high input resistance as shown in Figure P7–56(a). Your interface must have a response that fits within the boundaries shown in Figure P7–56(b). A vendor offers a suitable circuit shown in Figure P7–56(a) and says that they are willing to change one component without additional cost. Would you purchase the circuit and, if so, what change, if any, would you require? Use MATLAB or OrCAD to verify your result.

The circuits is a series RLC circuit with a dc source. Before the switch moves, the initial capacitor voltages is $v_{\rm C}(0) = 0$ V and the inductor current is $i_{\rm L}(0) = 0$ A. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2v_{\rm C}(t)}{dt^2} + RC\frac{dv_{\rm C}(t)}{dt} + v_{\rm C}(t) = 5$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 1400s + 4000000 = 0$$

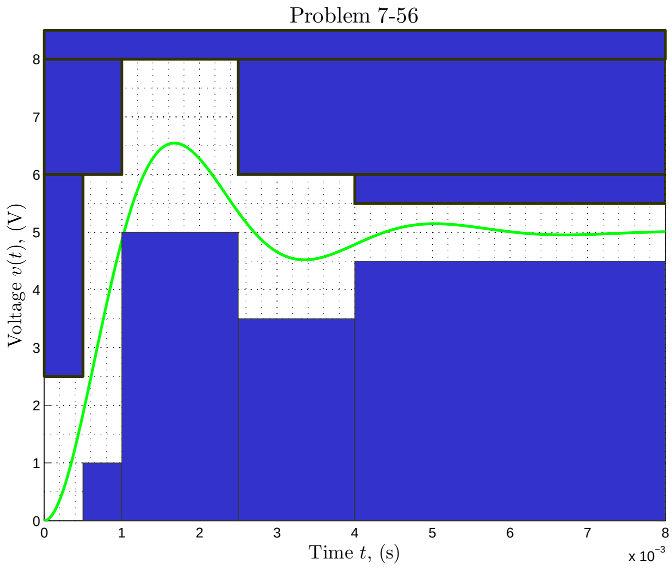
The roots of the characteristic equation are $s = -700 \pm j1873.5$ and the response has the form

$$v_{\rm O}(t) = v_{\rm C}(t) = 5 + K_1e^{-700}\cos(1873.5t) + K_2e^{-700t}\sin(1873.5t)$$

Apply the initial conditions to solve for the constants K_1 and K_2. The final expression for the output voltage is

$$v_{\rm O}(t) = 5 - e^{-700t}\left[5\cos(1873.5t) + 1.8682\sin(1873.5t)\right]\ \text{V}$$

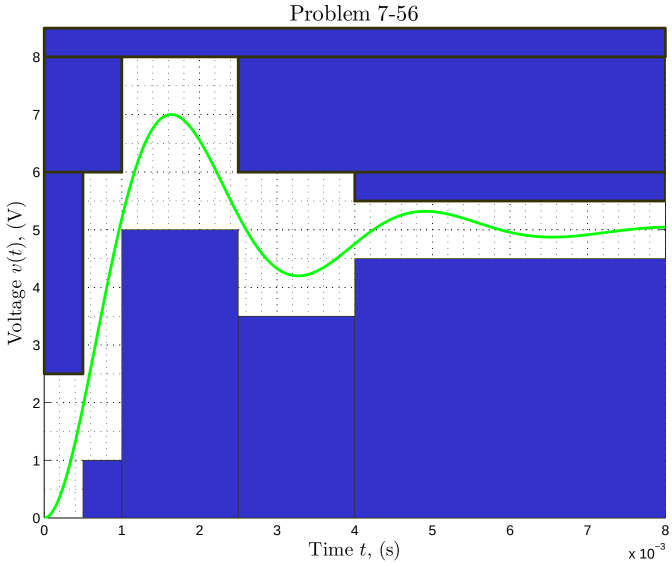
Using MATLAB to simulate this response, the output voltage does not stay within the specified boundaries, as shown below.



To correct the response, reduce the damping by reducing the resistance in the circuit. Reducing the damping will increase the magnitude of the first peak. Replace the 250-Ω resistor with a 180-Ω resistor and repeat the analysis. The corresponding response is

$$v_{\rm O}(t) = 5 - e^{-560t}\left[5\cos(1920t) + 1.4583\sin(1920t)\right]\ \text{V}$$

The following MATLAB plot confirms that the results meet the specifications.



Problem 7–57. The switch in Figure P7–57 has been in position A for a long time and is moved to position B at $t = 0$. The circuit parameters are $R_1 = 500\ \Omega$, $R_2 = 220\ \Omega$, $L = 250$ mH, $C = 3.3\ \mu$F, and $V_A = 15$ V.

(a). Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

Before the switch moves, the voltage across the capacitor is $v_C(0) = V_A = 15$ V and the current through the inductor is $i_L(0) = 0$ A. After the switch moves, the circuit is an RLC series circuit with no input signal. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2v_C(t)}{dt^2} + R_2C\frac{dv_C(t)}{dt} + v_C(t) = 0$$

$$s^2 + \frac{R_2}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 880s + 1212121 = 0$$

The roots of the characteristic equation are $s = -440 \pm j1009.2$ and the response has the form

$$v_C(t) = K_1e^{-440t}\cos(1009.2t) + K_2e^{-440t}\sin(1009.2t)$$

Apply the initial conditions to solve for the constants K_1 and K_2.

$$v_C(0) = 15 = (K_1)(1)(1) + (K_2)(1)(0) = K_1$$

$$i_L(0) = 0 = C\frac{dv_C(0)}{dt} = C\frac{d}{dt}\left[15e^{-440t}\cos(1009.2t) + K_2e^{-440t}\sin(1009.2t)\right]\Big|_{t=0}$$

$$0 = (-440)(15) + (K_2)(1009.2)$$

$$K_2 = 6.5397$$

The final expression for the capacitor voltage is

$$v_C(t) = e^{-440t}\left[15\cos(1009.2t) + 6.5397\sin(1009.2t)\right]\ \text{V}$$

Compute the inductor current.

$$i_L(t) = i_C(t) = C\frac{dv_C(t)}{dt} = (3.3\,\mu)\frac{d}{dt}\left\{e^{-440t}\left[15\cos(1009.2t) + 6.5397\sin(1009.2t)\right]\right\}$$

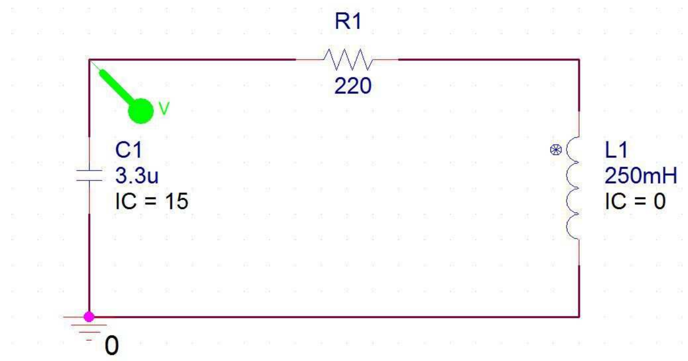
$$= -59.45e^{-440t}\sin(1009.2t)\ \text{mA}$$

(b). Is the circuit overdamped, critically damped, or underdamped?

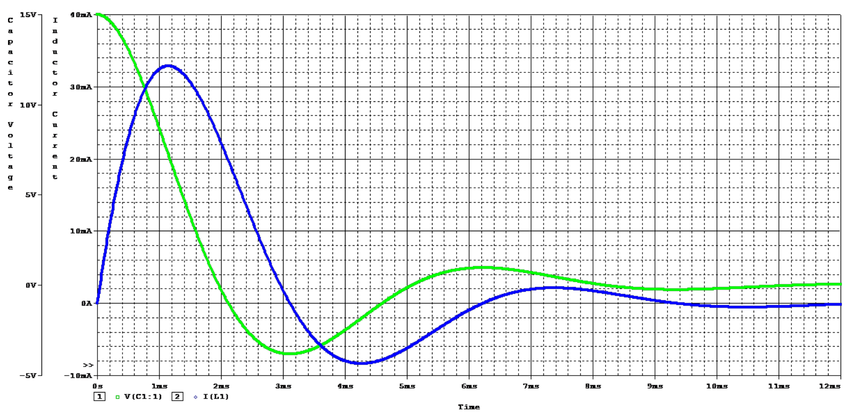
Based on the form of the response, the circuit is underdamped.

(c). Use OrCAD to simulate your results.

The following OrCAD simulation provides the results.



The corresponding output plot is shown below.



Problem 7–58. The switch in Figure P7–57 has been in position B for a long time and is moved to position A at $t = 0$. The circuit parameters are $R_1 = 500\ \Omega$, $R_2 = 500\ \Omega$, $L = 250$ mH, $C = 1\ \mu$F, and $V_A = 5$ V.

(a). Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

Before the switch moves, the voltage across the capacitor is $v_C(0) = 0$ V and the current through the inductor is $i_L(0) = 0$ A. After the switch moves, the circuit is an *RLC* series circuit with an input signal. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2v_C(t)}{dt^2} + (R_1 + R_2)C\frac{dv_C(t)}{dt} + v_C(t) = 5$$

$$s^2 + \frac{R_1 + R_2}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 4000s + 4000000 = 0$$

The roots of the characteristic equation are repeated with $s_1 = s_2 = -2000$ and the response has the form

$$v_C(t) = 5 + K_1e^{-2000t} + K_2te^{-2000t}$$

Apply the initial conditions to solve for the constants K_1 and K_2.

$$v_C(0) = 0 = 5 + (K_1)(1) + (K_2)(0)(1) = 5 + K_1$$

$$K_1 = -5$$

$$i_L(0) = 0 = C\frac{dv_C(0)}{dt} = C\frac{d}{dt}\left[5 - 5e^{-2000t} + K_2te^{-2000t}\right]\Big|_{t=0}$$

$$0 = (-5)(-2000) + K_2$$

$$K_2 = -10000$$

The final expression for the capacitor voltage is

$$v_C(t) = 5 - 5e^{-2000t} - 10000te^{-2000t} \text{ V}$$

Compute the inductor current.

$$i_L(t) = i_C(t) = C\frac{dv_C(t)}{dt} = (1\,\mu)\frac{d}{dt}\left[5 - 5e^{-2000t} - 10000te^{-2000t}\right]$$

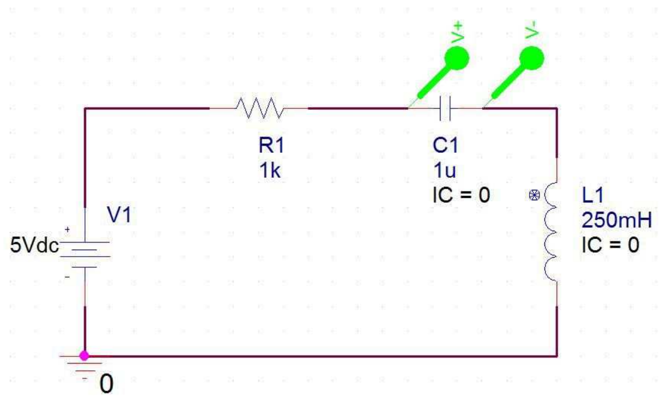
$$= 20te^{-2000t} \text{ A}$$

(b). Is the circuit overdamped, critically damped, or underdamped?

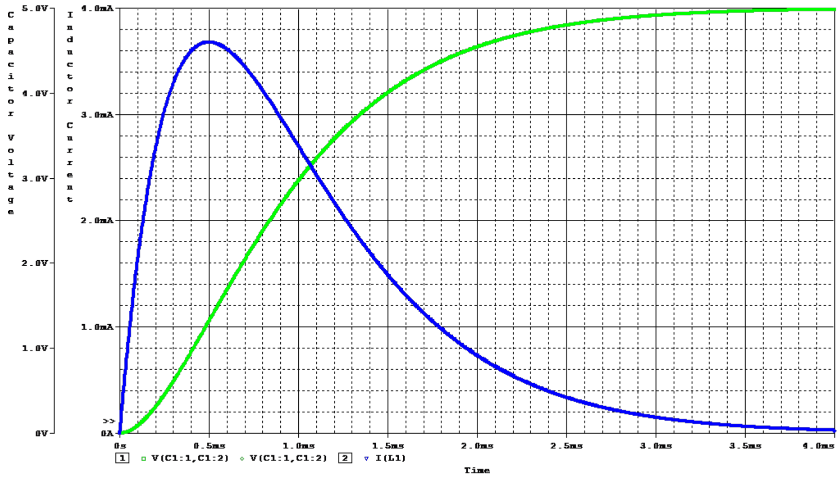
Based on the form of the response, the circuit is critically damped.

(c). Use OrCAD to simulate your results.

The following OrCAD simulation provides the results.



The corresponding output plot is shown below.



Problem 7–59. The circuit in Figure P7–59 is in the zero state when the step function input is applied. The circuit parameters are $L = 250$ mH, $C = 1\ \mu$F, $R = 3.3$kΩ, and $V_{\rm A} = 10$ V. Find $v_{\rm O}(t)$ for $t \geq 0$. (*Hint*: Find the capacitor voltage first.)

The circuit is a series RLC circuit with an input signal. The corresponding differential equation and characteristic equation are

$$LC\frac{d^2v_{\rm C}(t)}{dt^2} + RC\frac{dv_{\rm C}(t)}{dt} + v_{\rm C}(t) = 10$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 13200s + 4000000 = 0$$

The roots of the characteristic equation are $s_1 = -12890$ and $s_2 = -310.33$ and the response has the form

$$v_{\rm C}(t) = 10 + K_1e^{-12890t} + K_2e^{-310.33t}$$

Apply the initial conditions to solve for the constants K_1 and K_2.

$$v_{\rm C}(0) = 0 = 10 + (K_1)(1) + (K_2)(1)$$

$$i_{\rm L}(0) = 0 = C\frac{dv_{\rm C}(0)}{dt} = C\frac{d}{dt}\left[10 + K_1e^{-12890t} + K_2e^{-310.33t}\right]\Big|_{t=0}$$

$$0 = -12890K_1 - 310.33K_2$$

$$K_1 = 0.2467$$

$$K_2 = -10.2467$$

The final expression for the capacitor voltage is

$$v_{\rm C}(t) = 10 + 0.2467e^{-12890t} - 10.2467e^{-310.33t} \text{ V}$$

Compute the inductor current.

$$i_{\rm L}(t) = i_{\rm C}(t) = C\frac{dv_{\rm C}(t)}{dt} = (1\,\mu)\frac{d}{dt}\left[10 + 0.2467e^{-12890t} - 10.2467e^{-310.33t}\right]$$

$$= -3.18e^{-12890t} + 3.18e^{-310.33t} \text{ mA}$$

Compute the output voltage.

$$v_O(t) = v_L(t) = L\frac{di_L(t)}{dt} = (0.25)\frac{d}{dt}\left[-0.00318e^{-12890t} + 0.00318e^{-310.33t}\right]$$
$$= 10.247e^{-12890t} - 0.247e^{-310.33t} \text{ V}$$

Problem 7–60. The circuit in Figure P7–60 is in the zero state when the step function input is applied. If the input source is $V_A = 70$ V and $L = 0.5$ H, select values of R and C so that the circuit's output $v_O(t)$ for $t \geq 0$ is critically damped. Use MATLAB or OrCAD to show your result for $v_O(t)$. (*Hint*: Find the inductor current first.)

Perform a source transformation on the circuit to create a parallel RLC circuit with a current source of magnitude $V_A/R = 70/R$ A. The characteristic equation is

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

For a critically damped system, we have $\zeta = 1$ and the roots of the charactereistic equation are repeated at $s = -\omega_0 = -1/\sqrt{LC}$. Pick $C = 2$ μF and solve for ω_0 and R.

$$\omega_0^2 = \frac{1}{LC} = (1000)^2$$
$$\omega_0 = 1000 \text{ rad/s}$$
$$2\omega_0 = 2000 = \frac{1}{RC}$$
$$R = \frac{1}{2000C} = 250\ \Omega$$

The current source will be $70/250 = 280$ mA and the inductor current will have the following form:

$$i_L(t) = 0.28 + K_1e^{-1000t} + K_2te^{-1000t} \text{ A}$$

Apply the initial conditions to solve for K_1 and K_2.

$$i_L(0) = 0 = 0.28 + K_1$$
$$K_1 = -0.28$$
$$v_C(0) = 0 = v_L(0) = L\frac{di_L(0)}{dt} = L\frac{d}{dt}\left[0.28 - 0.28e^{-1000t} + K_2te^{-1000t}\right]\Big|_{t=0}$$
$$0 = 280 + K_2$$
$$K_2 = -280$$

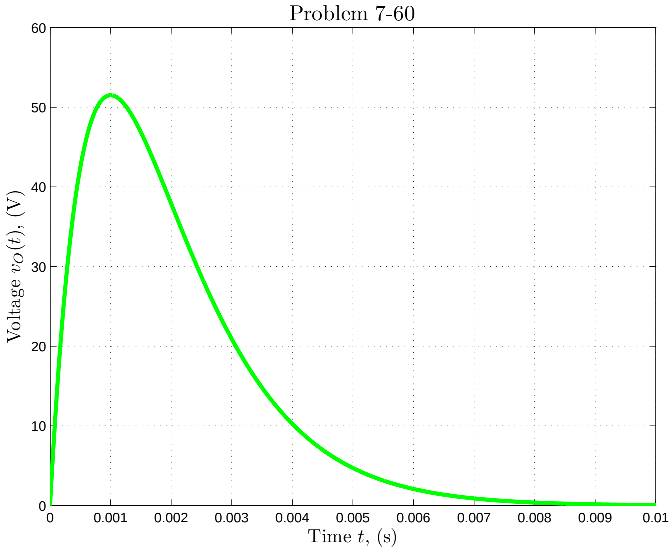
The inductor current has the following form:

$$i_L(t) = 0.28 - e^{-1000t}\left(0.28 + 280t\right) \text{ A}$$

The output voltage equals the inductor voltage.

$$v_O(t) = v_L(t) = L\frac{di_L(t)}{dt} = (0.5)\frac{d}{dt}\left[0.28 - e^{-1000t}\left(0.28 + 280t\right)\right]$$
$$= 140000te^{-1000t} \text{ V}$$

The following MATLAB plot displays the output voltage.



Problem 7–61. The circuit in Figure P7–61 is in the zero state when the step function input is applied.

(a). If $V_A = 24$ V, $R = 1.5$ kΩ, $L = 250$ mH, and $C = 0.25\ \mu$F, derive an expression for the voltage $v_O(t)$ for $t \geq 0$.

Perform a source transformation with the voltage source and resistor to get a current source in parallel with the resistor and the other two elements. The magnitude of the current source is $V_A/R = 24/1500 = 16$ mA. The parallel RLC circuit has the following differential equation and characteristic equation:

$$LC\frac{d^2 i_L(t)}{dt^2} + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = 0.016 \text{ A}$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s^2 + 2667s + 16000000 = 0$$

The roots of the characteristic equation are $s = -1333 \pm j3771$ and the solution to the differential equation has the following form

$$i_L(t) = 0.016 + K_1 e^{-1333t}\cos(3771t) + K_2 e^{-1333t}\sin(3771t)$$

Apply the initial conditions to solve for K_1 and K_2.

$$i_L(0) = 0 = 0.016 + (K_1)(1)(1) + (K_2)(1)(0)$$

$$K_1 = -0.016$$

$$v_C(0) = 0 = v_L(0) = L\frac{di_L(0)}{dt} = L\frac{d}{dt}\left[0.016 - 0.016e^{-1333t}\cos(3771t) + K_2 e^{-1333t}\sin(3771t)\right]\Big|_{t=0}$$

$$0 = (-0.016)(-1333) + (3771)(K_2)$$

$$K_2 = -0.005657$$

The final expression for the inductor current is

$$i_L(t) = 16 - 16e^{-1333t}\cos(3771t) - 5.657e^{-1333t}\sin(3771t) \text{ mA}$$

Compute the inductor voltage.

$$v_{\mathrm{L}}(t) = L\frac{di_{\mathrm{L}}(t)}{dt} = (0.25)\frac{d}{dt}\left[0.016 - 0.016e^{-1333t}\cos(3771t) - 0.005657e^{-1333t}\sin(3771t)\right]$$

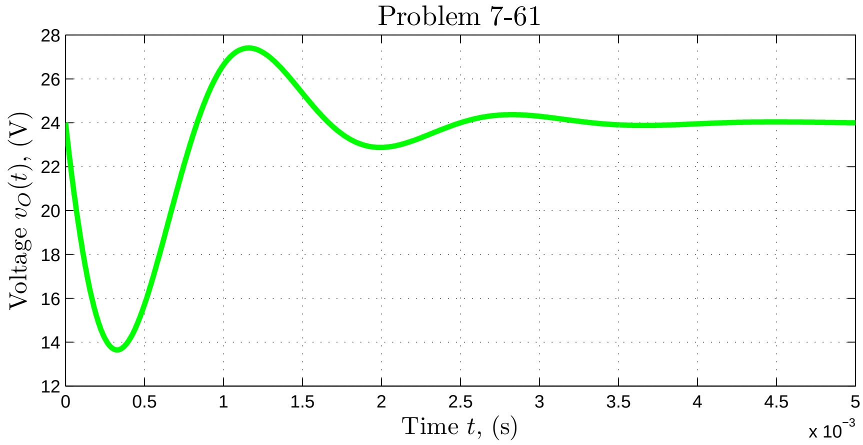
$$v_{\mathrm{L}}(t) = 16.971e^{-1333t}\sin(3771t)\text{ V}$$

Use KVL to compute the output voltage in the original circuit.

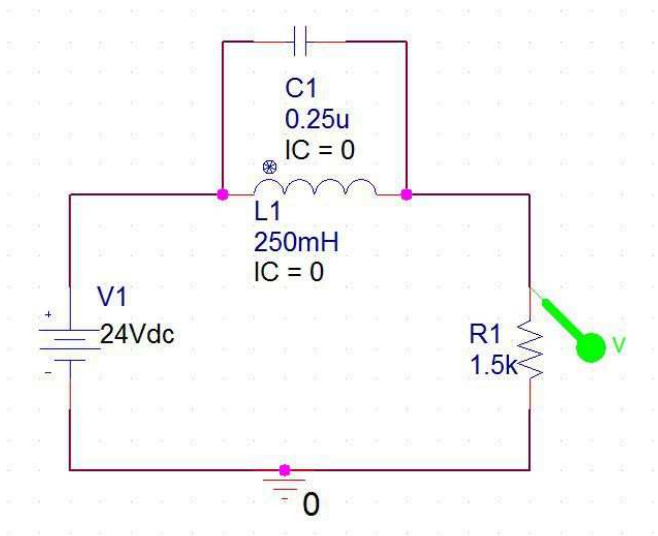
$$v_{\mathrm{O}}(t) = V_{\mathrm{A}} - v_{\mathrm{L}}(t) = 24 - 16.971e^{-1333t}\sin(3771t)\text{ V}$$

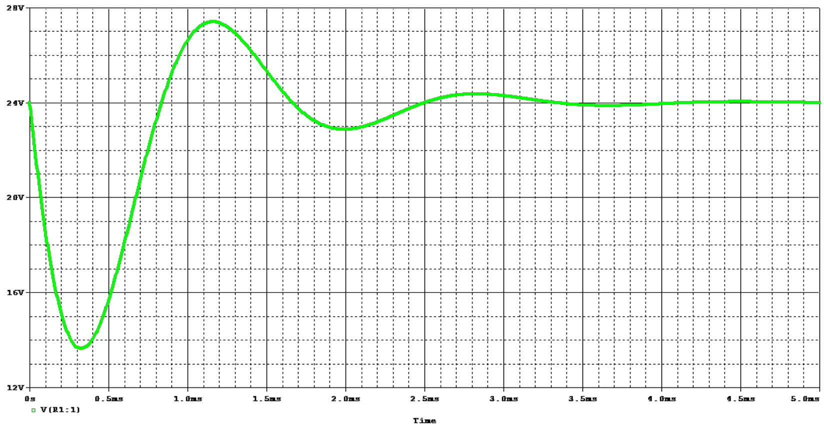
(b). Validate your solution by plotting it using MATLAB and comparing it to an OrCAD simulation of the same circuit.

The MATLAB plot of the output voltage is shown below.



The corresponding OrCAD simulation and the resulting output plot are shown below.





The MATLAB and OrCAD results agree.

Problem 7–62. Derive expressions for the damping ratio and undamped natural frequency of the circuit in Figure P7–62 in terms of the circuit parameters R, L, and C. Which parameter(s) affect the damping ratio? Can you change the damping ratio without affecting the undamped natural frequency?

Perform a source transformation on the original circuit to get a current source with magnitude $v_S(t)/R$ in parallel with the two resistors. Combine the resistors in parallel to get an equivalent resistance of $R/2$. Perform a second source transformation to get a voltage source of magnitude $v_S(t)/2$ is series with a $R/2$ resistor. We now have a series RLC circuit where the resistor is $R/2$. The corresponding chararacteristic equation is

$$s^2 + \frac{R}{2L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\zeta\omega_0 = \frac{R}{2L}$$

$$\zeta = \frac{R}{4\omega_0 L} = \frac{R}{4\frac{1}{\sqrt{LC}}L} = \frac{R\sqrt{LC}}{4L}$$

$$\zeta = \frac{R}{4}\sqrt{\frac{C}{L}}$$

All parameters affect the damping ratio. We can change the damping ratio without affecting the undamped natural frequency by changing the resistance.

Problem 7–63. Derive expressions for the damping ratio and undamped natural frequency of the circuit in Figure P7–63 in terms of the circuit parameters R, L, and C. Which parameter(s) affect the damping ratio? Can you change the damping ratio without affecting the undamped natural frequency?

Perform a source transformation to get a voltage source with magnitude $i_S(t)R$ in series with both resistors. Combine the two resistors in series to get an equivalent resistance of $3R$. Perform a second source transformation to get a current source of magnitude $i_S(t)/3$ in parallel with a resistor $3R$ and the other two elements. The circuit is a parallel RLC circuit with the following characteristic equation:

$$s^2 + \frac{1}{3RC}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\zeta\omega_0 = \frac{1}{3RC}$$

$$\zeta = \frac{1}{6\omega_0 RC} = \frac{\sqrt{LC}}{6RC}$$

$$\zeta = \frac{1}{6R}\sqrt{\frac{L}{C}}$$

All parameters affect the damping ratio. We can change the damping ratio without affecting the undamped natural frequency by changing the resistance.

Problem 7–64. In a series RLC circuit the step response across the 1-μF capacitor is

$$v_C(t) = 15 - e^{-200t}\left[15\cos(1000t) + 3\sin(1000t)\right] \text{ V} \quad t \geq 0$$

(a). Find R and L.

From the voltage response, we have the following:

$$\alpha = 200$$

$$\beta = 1000$$

$$\omega_0^2 = \alpha^2 + \beta^2 = 1040000 = \frac{1}{LC}$$

$$L = \frac{1}{1040000C} = 961.538 \text{ mH}$$

$$\omega_0 = 1019.8 \text{ rad/s}$$

$$2\zeta\omega_0 = 2\alpha = 400 = \frac{R}{L}$$

$$R = 400L = 384.615\ \Omega$$

(b). Find $i_L(t)$ for $t \geq 0$.

Compute the current.

$$\begin{aligned} i_L(t) = i_C(t) &= C\frac{dv_C(t)}{dt} \\ &= (1\,\mu)\frac{d}{dt}\left\{15 - e^{-200t}\left[15\cos(1000t) + 3\sin(1000t)\right]\right\} \\ &= 15.6e^{-200t}\sin 1000t \text{ mA} \end{aligned}$$

Problem 7–65. In a parallel RLC circuit the zero-input response in the 220-mH inductor is

$$i_L(t) = 50e^{-6000t} - 40e^{-3000t} \text{ mA} \quad t \geq 0$$

(a). Find R and C.

For a parallel RLC circuit, we have the following

$$(s+6000)(s+3000) = s^2 + 9000s + 18000000 = s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$\frac{1}{LC} = 18000000$$

$$C = \frac{1}{18000000L} = 0.2525\ \mu\text{F}$$

$$\frac{1}{RC} = 9000$$

$$R = \frac{1}{9000C} = 440\ \Omega$$

(b). Find $v_{\text{C}}(t)$ for $t \geq 0$.

Compute the voltage.

$$v_{\text{C}}(t) = v_{\text{L}}(t) = L\frac{di_{\text{L}}(t)}{dt} = (0.22)\frac{d}{dt}\left[0.05e^{-6000t} - 0.04e^{-3000t}\right]$$

$$= 26.4e^{-3000t} - 66e^{-6000t}\ \text{V}$$

Problem 7–66. In a parallel RLC circuit the state variable responses are

$$v_{\text{C}}(t) = e^{-100t}\left[5\cos(300t) + 15\sin(300t)\right]\ \text{V} \quad t \geq 0$$

$$i_{\text{L}}(t) = 20 - 25e^{-100t}\cos(300t)\ \text{mA} \quad t \geq 0$$

Find R, L, and C.

For a parallel RLC circuit, we have the following relationships:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + 2\alpha s + (\alpha^2 + \beta^2) = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

$$\alpha = 100$$

$$\beta = 300$$

$$v_{\text{C}}(t) = v_{\text{L}}(t) = L\frac{di_{\text{L}}(t)}{dt}$$

$$v_{\text{C}}(t) = L\frac{d}{dt}\left[0.02 - 0.025e^{-100t}\cos(300t)\right]$$

$$v_{\text{C}}(t) = L\left[7.5e^{-100t}\sin(300t) + 2.5e^{-100t}\cos(300t)\right]$$

$$L = 2\ \text{H}$$

$$C = \frac{1}{L\omega_0^2} = 5\ \mu\text{F}$$

$$R = \frac{1}{2\alpha C} = 1\ \text{k}\Omega$$

Problem 7–67. The zero-input response of a series RLC circuit with $R = 80\ \Omega$ is

$$v_{\mathrm{C}}(t) = 2e^{-2000t}\cos(1000t) - 4e^{-2000t}\sin(1000t)\text{ V} \quad t \geq 0$$

If the initial conditions remain the same, what is the zero-input response when $R = 40\ \Omega$?

Find the values for L and C.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\alpha s + (\alpha^2 + \beta^2) = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

$$\alpha = 2000$$

$$\beta = 1000$$

$$\frac{R}{L} = 4000$$

$$L = \frac{R}{4000} = 20\text{ mH}$$

$$C = \frac{1}{\omega_0^2 L} = 10\ \mu\text{F}$$

The initial capacitor voltage is $v_{\mathrm{C}}(0) = 2$ V and the initial inductor current is

$$i_{\mathrm{L}}(0) = i_{\mathrm{C}}(0) = C\frac{dv_{\mathrm{C}}(0)}{dt} = (10\,\mu)\frac{d}{dt}\left[2e^{-2000t}\cos(1000t) - 4e^{-2000t}\sin(1000t)\right]\Big|_{t=0}$$

$$= (10\,\mu)\left[-8000e^{-2000t}\cos(1000t) + 6000e^{-2000t}\sin(1000t)\right]\Big|_{t=0}$$

$$= e^{-2000t}\left[-80\cos(1000t) + 60\sin(1000t)\right]\Big|_{t=0}\ \text{mA}$$

$$= -80\text{ mA}$$

With $R = 40\ \Omega$, the new characteristic equation is

$$s^2 + 2000s + 5000000 = 0$$

The roots of the characteristic equation are $s = -1000 \pm j2000$ and the solution has the form

$$v_{\mathrm{C}}(t) = K_1 e^{-1000t}\cos(2000t) + K_2 e^{-1000t}\sin(2000t)\text{ V} \quad t \geq 0$$

Apply the initial conditions to solve for K_1 and K_2.

$$v_{\mathrm{C}}(0) = 2 = K_1$$

$$i_{\mathrm{L}}(0) = -0.08 = i_{\mathrm{C}}(0) = C\frac{dv_{\mathrm{C}}(0)}{dt} = (10\,\mu)\frac{d}{dt}\left[2e^{-1000t}\cos(2000t) + K_2 e^{-1000t}\sin(2000t)\right]$$

$$-8000 = -2000 + 2000K_2$$

$$K_2 = -3$$

The new zero-input response is

$$v_{\mathrm{C}}(t) = e^{-1000t}\left[2\cos(2000t) - 3\sin(2000t)\right]\text{ V} \quad t \geq 0$$

Problem 7–68. In a parallel RLC circuit the inductor current is observed to be

$$i_{\mathrm{L}}(t) = 20e^{-20t}\sin(20t) \text{ mA} \quad t \geq 0$$

Find $v_{\mathrm{C}}(t)$ when $v_{\mathrm{C}}(0) = 0.5$ V.

For this parallel RLC circuit, we have the following:

$$v_{\mathrm{C}}(0) = 0.5 = v_{\mathrm{L}}(0) = L\frac{di_{\mathrm{L}}(0)}{dt} = L\frac{d}{dt}\left[0.02e^{-20t}\sin(20t)\right]\Big|_{t=0}$$

$$0.5 = L\left[0.4e^{-20t}\cos(20t) - 0.4e^{-20t}\sin(20t)\right]\Big|_{t=0} = 0.4L$$

$$L = 1.25 \text{ H}$$

$$v_{\mathrm{C}}(t) = v_{\mathrm{L}}(t) = L\frac{di_{\mathrm{L}}(t)}{dt} = 1.25\left[0.4e^{-20t}\cos(20t) - 0.4e^{-20t}\sin(20t)\right]$$

$$v_{\mathrm{C}}(t) = 0.5e^{-20t}\left[\cos(20t) - \sin(20t)\right] \text{ V}$$

Problem 7–69. (D) Design a parallel RLC circuit whose natural response has the form

$$v_{\mathrm{L}}(t) = K_1 e^{-10000t} + K_2 t e^{-10000t} \text{ V} \quad t \geq 0$$

The characteristic equation has repeated roots at $s_1 = s_2 = 10000$, so for a parallel RLC circuit, we have

$$(s+\alpha)^2 = s^2 + 2\alpha s + \alpha^2 = s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

Pick a value for the capacitor and then solve for the inductance and resistance.

$$C = 0.1 \ \mu\text{F}$$

$$\frac{1}{LC} = \alpha^2 = (10000)^2$$

$$L = \frac{1}{10^8 C} = 100 \text{ mH}$$

$$\frac{1}{RC} = 2\alpha = 20000$$

$$R = \frac{1}{20000C} = 500 \ \Omega$$

Problem 7–70. (D) Design a series RLC circuit with $\zeta = 0.1$ and $\omega_0 = 50$ krad/s.

(a). What is the form of the natural response of $v_{\mathrm{C}}(t)$ for your design?

The characteristic equation for the circuit is

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + 10000s + 2.5\times 10^9 = 0$$

For a series RLC circuit, we have

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Select $C = 0.001$ μF and solve for the inductance and resistance.

$$L = \frac{1}{(2.5\times 10^9)C} = 400 \text{ mH}$$

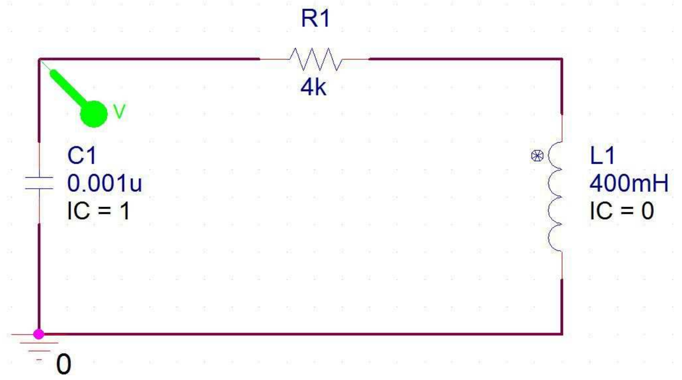
$$R = 10000L = 4 \text{ k}\Omega$$

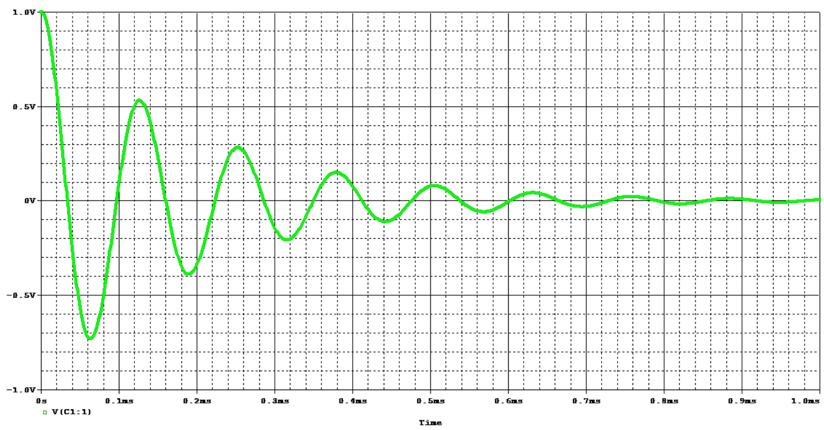
The roots of the characteristic equation are $s = -5000 \pm j49749$, so the natural response has the form

$$v_{\rm C}(t) = e^{-5000t}\left[K_1 \cos(49749t) + K_2 \sin(49749t)\right] \text{ V}$$

(b). Simulate your circuit in OrCAD.

The following OrCAD simulation and the resulting plot are shown below





Problem 7–71. (D) Design a series RLC circuit with $\zeta = 1$ and $\omega_0 = 100$ krad/s.

(a). What is the form of the natural response of $v_{\rm C}(t)$ for your design?

The characteristic equation for the circuit is

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + 200000s + 10^{10} = 0$$

For a series RLC circuit, we have

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Select $C = 0.001\ \mu$F and solve for the inductance and resistance.

$$L = \frac{1}{10^{10}C} = 100 \text{ mH}$$

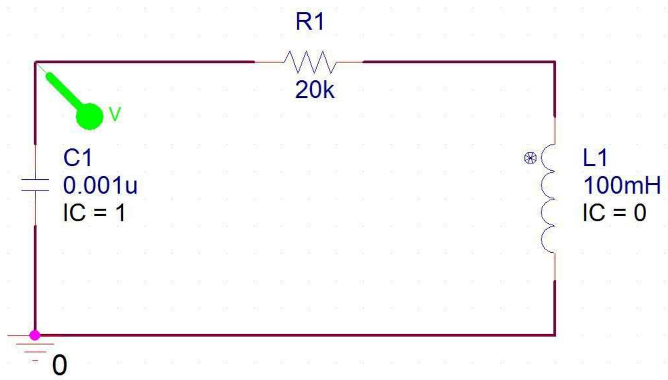
$$R = 200000L = 20 \text{ k}\Omega$$

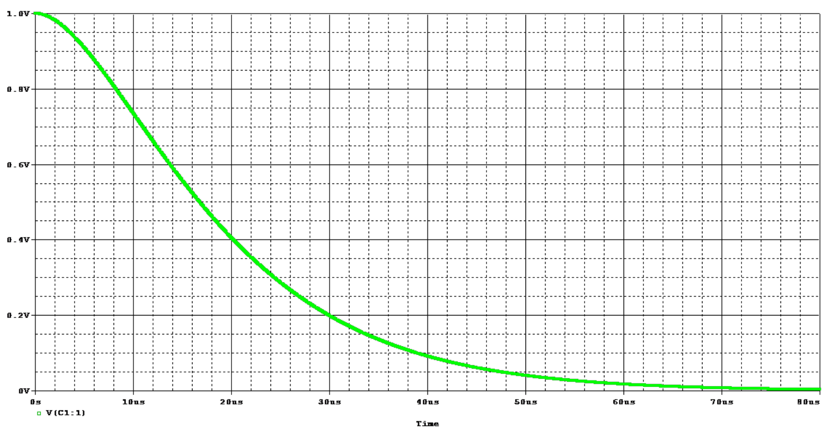
The roots of the characteristic equation are repeated with $s_1 = s_2 = -100000$, so the natural response has the form

$$v_{\rm C}(t) = K_1 e^{-100000t} + K_2 t e^{-100000t} \text{ V}$$

(b). Simulate your circuit in OrCAD.

The following OrCAD simulation and the resulting plot are shown below





Problem 7–72. Design a series *RLC* circuit whose output voltage resides entirely within the non-shaded region of Figure P7–72. Validate your design using MATLAB or OrCAD.

The response approaches a final value of 5 V in approximately 3 ms. Use a 5-V source as the input to the *RLC* circuit that is originally in the zero state and measure the output voltage across the capacitor. Use an overdamped system, so that the response has the form

$$v_{\mathrm{C}}(t) = 5 + K_1 e^{-\alpha_1 t} + K_2 e^{-\alpha_2 t} \text{ V}$$

To satisfy the zero-state initial conditions, we must have

$$K_1 + K_2 = -5$$

$$-\alpha_1 K_1 - \alpha_2 K_2 = 0$$

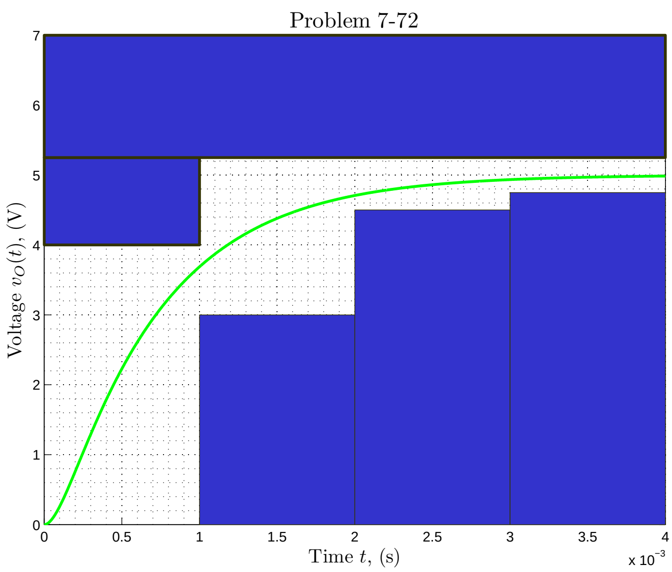
Pick $\alpha_2 = 10000$ so that portion of the response decays very quickly and does not interfere with the other part of the response. Since the dominant response decays in approximately 3 ms, the time constant is approximately 600 μs and α_1 is approximately 1666. For simplicity, try $\alpha_1 = 1000$, 1500, and 2000, to find a match within the specifications. Use MATLAB to compute the constants K_1 and K_2, as well as the corresponding values for R, L, and C, based on the relationship

$$(s + \alpha_1)(s + \alpha_2) = s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

The following MATLAB code provides the solution:

```
a1 = 1500;
a2 = 10000;
w02 = a1*a2;
w0 = sqrt(w02)
z = (a1+a2)/2/w0
C = 0.1e-6
L = 1/w02/C
R = 2*z*w0*L
% Solve for the constants
A = [1, 1; -a1, -a2];
b = [-5; 0];
K = A\b;
K1 = K(1)
K2 = K(2)
syms t real
vCt = 5 + K1*exp(-a1*t) + K2*exp(-a2*t)
tf = 4e-3;
tt = 0:2e-6:tf;
vCtt = subs(vCt,t,tt);
figure
plot(tt,vCtt,'g','LineWidth',2)
axis([0,tf,0,7])
xlabel('Time $t$, (s)',...
    'Interpreter','latex',...
    'FontSize',14)
ylabel('Voltage $v_O(t)$, (V)',...
    'Interpreter','latex',...
    'FontSize',14)
title('Problem 7-72',...
    'Interpreter','latex',...
    'FontSize',16)
grid minor
hold on
```

A value of $\alpha_1 = 1500$ provides an acceptable solution, as shown in the following plot.



The corresponding parameter values are $R = 7.667$ kΩ, $L = 667$ mH, and $C = 0.1$ μF.

Problem 7–73. A circuit is needed to produce the following step response

$$v_C(t) = 10 - 13.3e^{-200t} + 3.3e^{-800t} \text{ V} \quad t \geq 0$$

A vendor has proposed using the circuit shown in Figure P7–73 to produce the desired response. The vendor realizes that the proposed circuit does not exactly meet the desired response and is willing to make a single change for no extra charge. What change should the vendor make? (*Hint:* Use MATLAB to generate the desired response and then simulate the vendor's corrected circuit using OrCAD and verify that the responses match.)

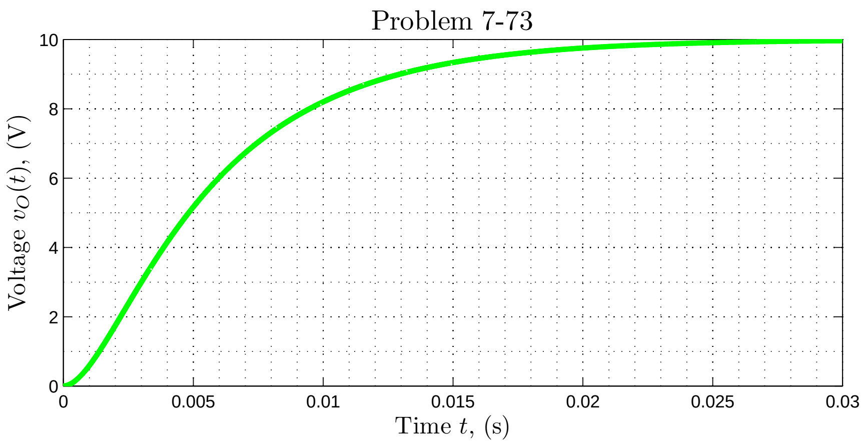
For the vendor's series RLC circuit, we have the following characteristic equation:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 528s + 160000 = 0$$

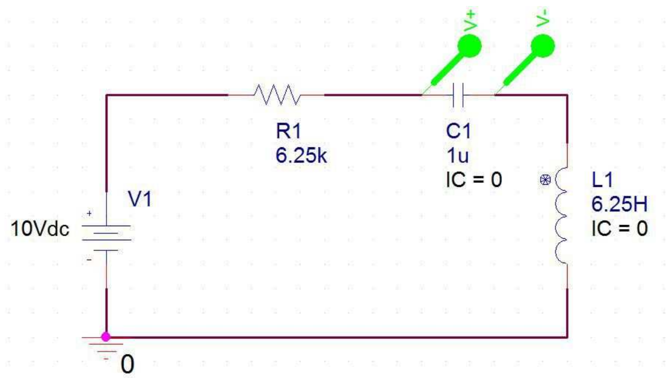
The desired characteristic equation is

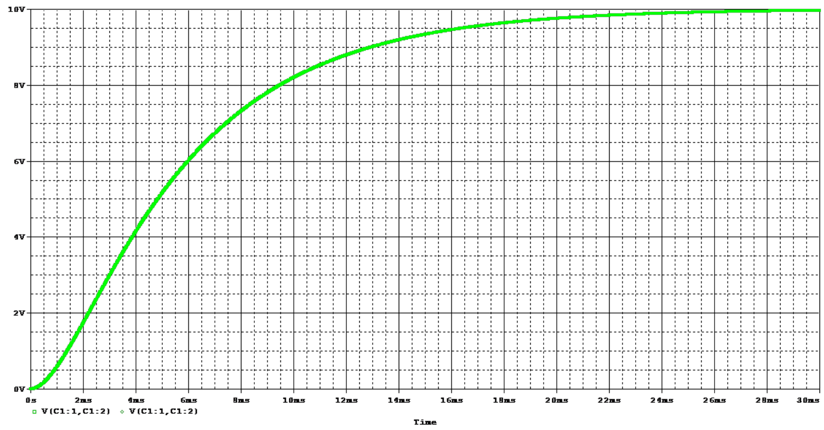
$$(s + 200)(s + 800) = s^2 + 1000s + 160000 = 0$$

We need to change the resistor to $R = 1000L = 6.25$ kΩ to get the proper characteristic equation. Using MATLAB, the desired response is



The OrCAD simulation and resulting plot are shown below and verify that the output matches the response created with MATLAB.





Problem 7–74. What range of damping ratios is available in the circuit in Figure P7–74?

The circuit is a series RLC circuit with a potentiometer. The equivalent resistance ranges from 22 Ω to 222 Ω. The characteristic equation is

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Since L and C are fixed, solve for $\omega_0 = 200$ krad/s. Then compute the damping ratios as $\zeta = R/(2\omega_0 L)$ to get a range from 0.022 to 0.222.

Problem 7–75. A variable capacitor is used in the circuit of P7–75 to vary the damping ratio. What range of damping ratios is available in the circuit?

The corresponding characteristic equation is

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

For the range of capacitor values, solve the following equations:

$$\omega_0^2 = \frac{1}{LC}$$

$$\zeta = \frac{R}{2\omega_0 L}$$

The damping ratio varies from 0.0007906 to 0.025.

Problem 7–76. A particular parallel RLC circuit has the step response observed on an oscilloscope and shown in Figure P7–76. Four points on the waveform were measured and are shown. Determine the circuit's initial value, final value, the dominant exponential's time constant, and the likely case (A, B, or C) of the circuit response.

For a parallel RLC circuit, we have the following characteristic equation:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

The initial inductor current is 0 mA and the final inductor current is 1 mA. The response reaches its final value in approximately 500 ms, so the dominant exponential's time constant is approximately 100 ms. Based on the form of the response, there is a second exponential in the equation for the current, so the response is overdamped and has the form

$$i_{\mathrm{L}}(t) = 0.001 + K_1 e^{-\alpha_1 t} + K_2 e^{-\alpha_2 t} \text{ A}$$

Given the points identified in Figure P7–76, we can write four equations in four unknows and solve for the constants as follows:

$$K_1 = 0.015$$

$$K_2 = -0.016$$

$$\alpha_1 = 10$$

$$\alpha_2 = 20$$

Problem 7–77. (A) First-order OP AMP Circuit Step Response

Find the zero-state response of the OP AMP output voltage in Figure P7–77 when the input is

$$v_S(t) = V_A u(t) \text{ V}$$

Let $v_1(t)$ be the node voltage between R_1 and C_1 and let $i_C(t)$ be the current flowing through the capacitor to the right. Assume the initial capacitor voltage is zero and write a node-voltage equation at the node between R_1 and C_1.

$$v_C(0) = 0 \text{ V}$$

$$\frac{v_1(t) - v_S(t)}{R_1} + i_C(t) = 0$$

$$\frac{v_1(t) - v_S(t)}{R_1} + C_1\frac{dv_1(t)}{dt} = 0$$

$$R_1C_1\frac{dv_1(t)}{dt} + v_1(t) = v_S(t) = V_A u(t)$$

The circuit has a first-order step response with $v_1(0) = 0$ V, $v_1(\infty) = V_A$, and $T_C = R_1C_1$. We have the following relationships:

$$v_1(t) = V_A - V_A e^{-t/R_1C_1}$$

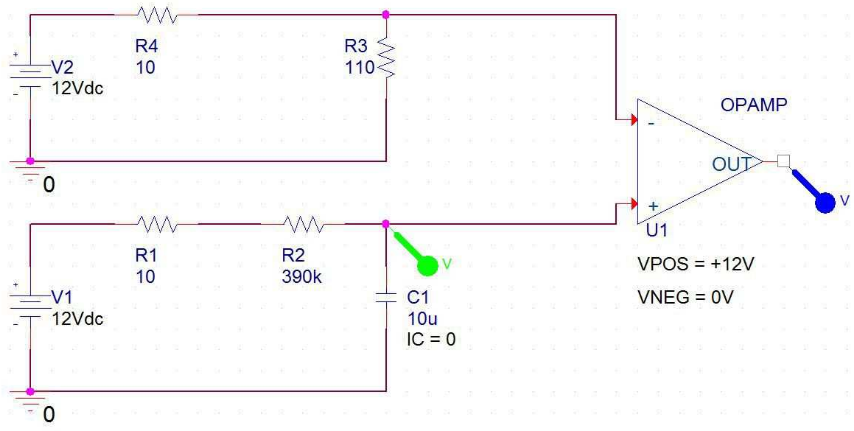
$$v_O(t) = -R_2 i_C(t) = -R_2C_1\frac{dv_1(t)}{dt}$$

$$v_O(t) = (-R_2C_1)(-V_A)\left(-\frac{1}{R_1C_1}\right)e^{-t/R_1C_1} = -\frac{R_2}{R_1}V_A e^{-t/R_1C_1} \text{ V} \quad t \geq 0$$

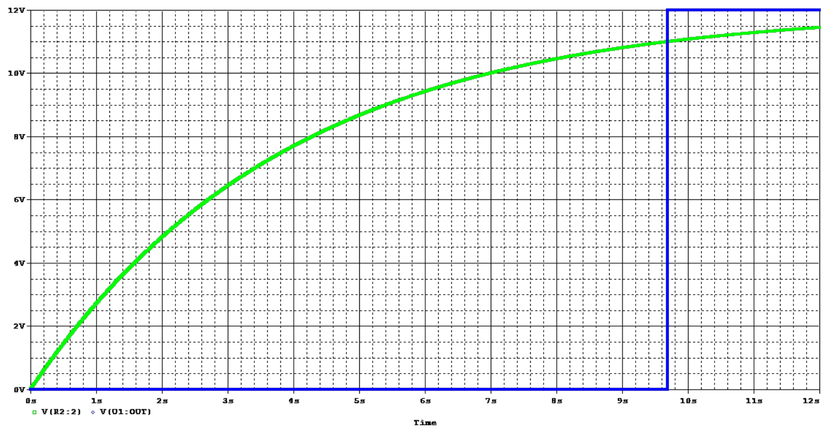
Problem 7–78. (D) Intermittent Timing Circuit for Windshield Wipers

A car maker needs an RC timing circuit to trigger the windshield wiper relay. The circuit should be driver selectable to trigger at 1, 2, 5, and 10 s $\pm 5\%$. The source circuit is the car voltage of 12 V with a series resistance of 10 Ω. You must use standard parts (see inside rear cover). You have a single-sided OP AMP available with a V_{CC} of 0 V and +12 V that you can use to trigger the windshield wiper relay. The relay triggers with an input of +12 V ± 1 V. You should validate your design using OrCAD.

There are many correct solutions for this problem. One approach uses an OP AMP comparator circuit with the negative input connected to a fixed voltage less than 12 V and the positive input connected to an RC circuit that is charging up to 12 V. Design the RC time constants to reach the fixed voltage at the negative input at the appropriate time for the wiper interval. Power the OP AMP with +12 V and 0 V. With R_2 set to 39 kΩ, 82 kΩ, 200 kΩ (100 kΩ + 100 kΩ), and 390 kΩ (as shown), the following circuit will meet the specifications.



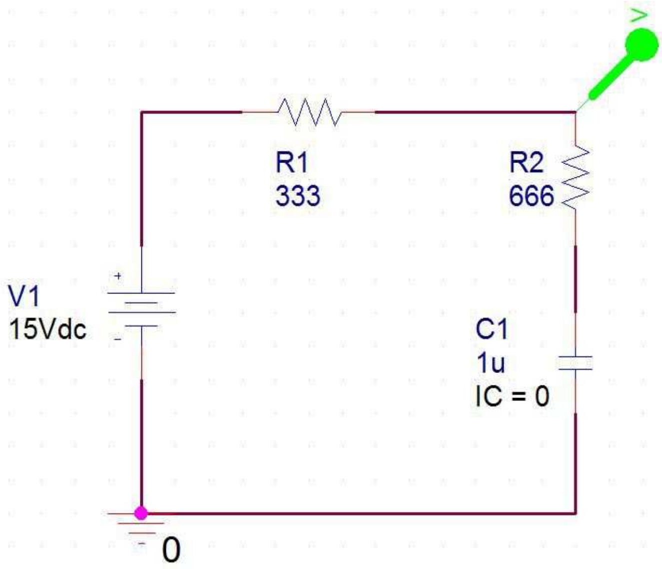
The input at the negative terminal is 11 V and the time constants are set so that the positive terminal reaches 11 V at approximately 2.5 time constants. The corresponding OrCAD output is shown below.

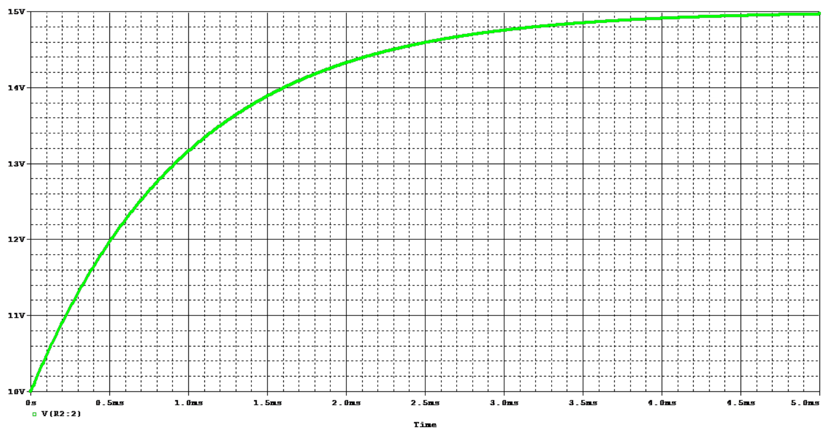


Problem 7–79. (D) RC Circuit Design

Design the first-order RC circuit in Figure P7–79 so an input $v_S(t) = 15u(t)$ V produces a zero-state response $v_O(t) = 15 - 5e^{-1000t}$ V. Validate your design using MATLAB or OrCAD.

The initial voltage is $v_O(0) = 10$ V and the final voltage is $v_O(\infty) = 15$ V. The time constant is $T_C = 1/1000 = 1$ ms. Choose $C = 1\ \mu$F and $R_{EQ} = 1$ kΩ to get the correct time constant. The capacitor has an initial voltage of zero and the resistance is divided into two parts, with $R_1 = 333\ \Omega$ and $R_2 = 666\ \Omega$. Connect the circuit to a 15 V source and measure the output across the series combination of the 666-Ω resistor and the capacitor. The following circuit provides the correct output, as shown in the corresponding OrCAD simulation.





There are many correct solutions.

Problem 7–80. (D) Sample/Hold Circuit

Figure P7–80 is a simplified diagram of a sample/hold circuit. When the switch is in position A, the circuit is in the sample mode and the capacitor voltage must charge to at least 99% of the source voltage V_A in less than 1 μs. When the switch is moved to position B, the circuit is in the hold mode and the capacitor must retain at least 99% of V_A for at least 1 ms. Select a capacitance that meets these constraints.

In the sample mode, we have the following requirement:

$$0.99V_{\mathrm{A}} < V_{\mathrm{A}}\left(1 - e^{-1\,\mu/T_{\mathrm{C}}}\right)$$

$$0.99 < 1 - e^{-1\,\mu/T_{\mathrm{C}}}$$

$$e^{-1\,\mu/T_{\mathrm{C}}} < 0.01$$

$$\frac{-1\,\mu}{T_{\mathrm{C}}} < \ln(0.01)$$

$$T_{\mathrm{C}} < \frac{-1\,\mu}{\ln(0.01)} = 217.147 \text{ ns}$$

$$RC < 217.147 \text{ ns}$$

$$C < 0.0043429\ \mu\text{F}$$

In the hold mode, we have the following requirement:

$$0.99V_{\mathrm{A}} < V_{\mathrm{A}}e^{-1\,\mathrm{m}/T_{\mathrm{C}}}$$

$$0.99 < e^{-1\,\mathrm{m}/R_2C}$$

$$\ln(0.99) < \frac{-1\,\mathrm{m}}{R_2C}$$

$$C > \frac{-1\,\mathrm{m}}{R_2\ln(0.99)} = 0.00199\ \mu\text{F}$$

Choose $C = 0.003\ \mu$F to meet both requirements.

Problem 7–81. (A) Super Capacitor

Super capacitors have very large capacitance (typically from 0.1 to 50 F), small sizes, and very long charge holding times, making them useful in non-battery backup power applications. The charge holding quality of a super capacitor is measured using the circuit in Figure P7–81. The switch is closed for a long time (say, 24 hours) and the capacitor charged to 5 V. The switch is then opened and the capacitor allowed to self-discharge through any leakage resistance for 24 hours. Suppose that after 24 hours the voltage across a 0.47 F super capacitor is 4.5 V. What is the equivalent leakage resistance in parallel with the capacitor?

We have the following relationships

$$5e^{-t/RC} = 4.5$$

$$e^{-t/RC} = 0.9$$

$$\frac{-t}{RC} = \ln(0.9)$$

$$R = \frac{-t}{C\ln(0.9)} = \frac{-(24)(60)(60)}{(0.47)\ln(0.9)}$$

$$R = 1.74477\ \mathrm{M}\Omega$$

Problem 7–82. (D) Cost-Conscious RLC Circuit Design

You are assigned a task to design a series, passive RLC circuit with a characteristic equation of $s^2 + 2000s + 5 \times 10^6 = 0$. To save money, your supervisor wants you to use a previously purchased 150-mH inductor with a 10-Ω parasitic resistance. The RLC circuit will be used to interface to a Thévenin source with a 75-Ω series output resistance. Your circuit must demonstrate the desired response with the source circuit connected.

For a series RLC circuit, the characteristic equation is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2000s + 5 \times 10^6 = 0$$

We have the following relationships:

$$\frac{R}{L} = 2000$$

$$R = 2000L = (2000)(.15) = 300\ \Omega$$

$$\frac{1}{LC} = 5000000$$

$$C = \frac{1}{5000000L} = 1.333\ \mu\text{F}$$

With 75 Ω at the source and 10 Ω at the inductor, we need to add a resistor $R = 215\ \Omega$ to get a total equivalent resistance of 300 Ω. Choose $C = 1.333\ \mu$F to complete the design.

Problem 7–83. (A) Combined First- and Second-Order Response

The switch in Figure P7–83 has been in position A for a long time and is moved to position B at $t = 0$ and then to position C when $t = 10$ ms. For $0 < t < 10$ ms the capacitor voltage is a charging exponential $v_C(t) = 10(1-e^{-100t})$ V. For $t > 10$ ms the capacitor voltage is a sinusoid $v_C(t) = 6.321\cos[1000(t-0.01)]$ V.

(a). Suppose the resistance is reduced to 1 kΩ and the switching sequence repeated. Will the amplitude of the sinusoid increase, decrease, or stay the same? Will the frequency of the sinusoid increase, decrease or stay the same?

If the resistance is reduced, the time constant to charge the capacitor will be reduced and the capacitor will charge at a faster rate. Since it charges faster, the final voltage on the capacitor will be greater when the switch moves a second time. With a higher initial voltage at the time of the second switch, the amplitude of the sinusoid will increase. The frequency of the sinusoid depends on the values of only the capacitor and the inductor. Since these values have not changed, the frequency of the sinusoid will stay the same.

(b). Suppose the inductance is reduced to 100 mH and the switching sequence repeated. Will the amplitude of the sinusoid increase, decrease, or stay the same? Will the frequency of the sinusoid increase, decrease or stay the same?

Changing the inductance will not change the initial voltage on the capacitor when the switch moves a second time, so the amplitude of the sinusoid will stay the same. The frequency of the sinusoid is proportional to $1/\sqrt{LC}$, so if the inductance is reduced, the frequency will increase.

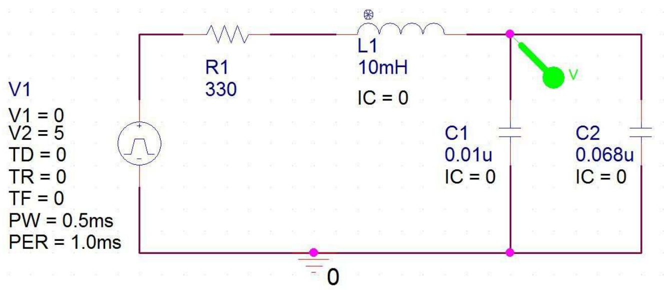
Problem 7–84. (A,D) Undesired Ringing

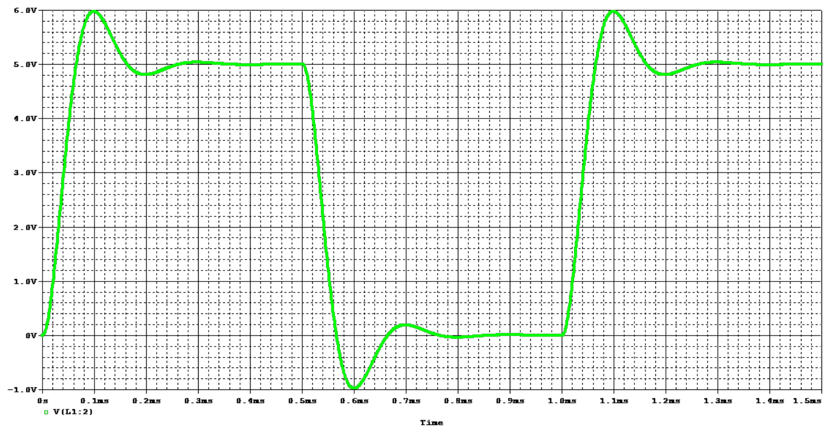
A digital clock has become corrupted by a ringing (undesired oscillations) as shown in Figure P7–84(a). The unwanted oscillations can cause false triggers and must be reduced. The clock can be modeled as an RLC series circuit as shown in Figure P7–84(b) with the voltage taken at node A. The parasitic capacitance is estimated at 0.01 μF and the Thévenin resistance at 330 Ω. From the graph determine the inductance L. Design an interface circuit that significantly reduces the ringing without significantly reducing the rise time (the time it takes the pulse to go from low to high or vice-versa.) The transition must occur in less than 80 μs and the "overshoot" (deviation from 0 or 5 V) must be less than ±1 V. Use standard value components. Use OrCAD to validate your design.

Examine the peaks of the first oscillation. The first peak occurs as $t_1 = 0.033$ ms and the fourth peak occurs at $t_2 = 0.222$ ms, approximately. The time between the peaks is 0.189 ms over three cycles, so the period is approximately $T_0 = 0.063$ ms. The frequencies are $f_0 = 1/T_0 = 15.87$ kHz and $\omega_0 = 2\pi f_0 = 99.7$ krad/s, which is approximately 100 krad/s. Solve for the inductance using $\omega_0^2 = 1/LC$, so $L = 1/\omega_0^2 C = 10$ mH. The characteristic equation for a series RLC circuit is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

Increasing the capacitance in the series RLC circuit will decrease the amplitude of the overshoot and slow the response. Using a single capacitor in parallel with the given capacitor as the interface will allow us to increase the equivalent capacitance of the series RLC circuit without affecting the other two components. By experimenting with different standard values, we can find a balance between the two competing requirements of minimal overshoot and fast rise time. An interface capacitance of 0.068 μF is sufficient. The design is shown in the following OrCAD simulation and the corresponding output plot is shown below.





Problem 7–85. (E) Optimum Fusing

A sensitive instrument that can be modeled by the series RLC circuit shown in Figure P7–85 is to be protected by a fuse. The voltage across the capacitor is

$$v_C(t) = e^{-10t}\left[-\cos(103t) + 0.0971\sin(103t)\right]u(t) \text{ V}$$

The peak current was found to occur at about 13 ms after $t = 0$. Engineer A suggests a 1-mA fuse, Engineer B suggests a 5-mA fuse, while Engineer C suggests a 10-mA fuse. The criterion is to select a fuse that is closest to the peak value of current expected without blowing. Which engineer is correct? Provide evidence for your decision.

Compute the current through the capacitor, which will equal the current through the fuse, and find the maximum value.

$$
\begin{aligned}
i_{\mathrm{C}}(t) &= C\frac{dv_{\mathrm{C}}(t)}{dt} \\
&= (92.5\,\mu)\frac{d}{dt}\left\{e^{-10t}\left[-\cos(103t)+0.0971\sin(103t)\right]\right\} \\
&= (92.5\,\mu)\left\{e^{-10t}\left[103\sin(103t)+10\cos(103t)\right]-10e^{-10t}\left[-\cos(103t)+0.0971\sin(103t)\right]\right\} \\
&= e^{-10t}\left[1.85\cos(103t)+9.4377\sin(103t)\right]\ \text{mA} \\
i_{\mathrm{C}}(0.013) &= 8.4388\ \text{mA}
\end{aligned}
$$

The maximum current is above 5 mA. Use the 10-mA fuse, since it is the only one that will allow the circuit to operate.

Problem 7–86. (D) Lightning Pulser Design

The circuit in Figure P7–86 is a simplified diagram of a pulser that delivers simulated lightning transients to the test article at the output interface. Closing the switch must produce a short-circuit current of the form $i_{\mathrm{SC}}(t) = I_{\mathrm{A}}e^{-\alpha t}\cos(\beta t)$, with $\alpha = 100$ krad/s, $\beta = 200$ krad/s, and $I_{\mathrm{A}} = 2$ kA. Select the values of L, C, and V_0.

For a parallel RLC circuit, we have the following characteristic equation and relationships:

$$
s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + 2\alpha s + (\alpha^2+\beta^2) = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0
$$

$$
\begin{aligned}
\omega_0^2 &= \alpha^2+\beta^2 = 50\times 10^9 \\
\omega_0 &= 223607\ \text{rad/s} \\
\frac{1}{RC} &= 2\alpha = 200000 \\
C &= \frac{1}{200000R} = 0.1\ \mu\text{F} \\
\frac{1}{LC} &= \omega_0^2 = 50\times 10^9 \\
L &= \frac{1}{\omega_0^2 C} = 200\ \mu\text{H} \\
I_{\mathrm{A}} &= \frac{V_{\mathrm{A}}}{R} = \frac{V_{\mathrm{A}}}{50} \\
V_0 &= V_{\mathrm{A}} = 50I_{\mathrm{A}} = (50)(2000) = 100\ \text{kV}
\end{aligned}
$$

Problem 7–87. (E) *RLC* Circuit Design

Losses in real inductors can be modeled by a series resistor as shown in Figure P7–87. In this problem, we include the effect of this resistor on the design of the series RLC circuit shown in the figure. The design requirements include a source resistance of 50 Ω, an undamped natural frequency of 50 kHz, and a damping ratio less than 0.1. The characteristics of the available inductors are listed below.

L (mH)	R_{L} (Ω)	L (mH)	R_{L} (Ω)
10.0	651	4.7	240
7.5	471	3.9	190
6.8	356	3.3	161

Which inductor would you use in your design and why?

For a series RLC circuit, we have the following characteristic equation and relationships:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

$$\frac{R}{L} = 2\zeta\omega_0$$

$$\zeta = \frac{R}{2\omega_0 L}$$

$$\frac{1}{LC} = \omega_0^2$$

$$C = \frac{1}{\omega_0^2 L}$$

$$f_0 = 50000$$

$$\omega_0 = 2\pi f_0 = 100000\pi$$

$$\zeta < 0.1$$

Use MATLAB to evaluate all of the inductor options. The MATLAB code is shown below:

```
f0 = 50e3;
w0 = 2*pi*f0;
R = 50;
RL = [651, 471, 356, 240, 190, 161];
RT = R+RL;
L = [10, 7.5, 6.8, 4.7, 3.9, 3.3]*1e-3;
% Compute the damping ratio
z = RT./L/2/w0;
% Compute the capacitance
C = 1/w0/w0./L;
Results = [L' RL' C' z']
```

The corresponding MATLAB output is shown below.

```
Results =
    10.0000e-003   651.0000e+000     1.0132e-009   111.5676e-003
     7.5000e-003   471.0000e+000     1.3509e-009   110.5596e-003
     6.8000e-003   356.0000e+000     1.4900e-009    95.0249e-003
     4.7000e-003   240.0000e+000     2.1558e-009    98.2020e-003
     3.9000e-003   190.0000e+000     2.5980e-009    97.9415e-003
     3.3000e-003   161.0000e+000     3.0703e-009   101.7627e-003
```

Three options meet the specification for the damping ratio. Choose the inductor with the smallest damping ratio, which is $L = 6.8$ mH with a damping ratio of $\zeta = 0.095$.

Problem 7–88. (E) Competing Circuit Designs

Figure P7–88 shows the step responses $v_C(t)$ of two competing series RLC circuits from two different vendors. The circuits are designed to switch from 0 to 10 V and to meet a specification for a desired circuit with the following characteristic equation:

$$s^2 + 10s + 100 = 0$$

Not all of the details of the actual circuits have been provided because of proprietary reasons, but each vendor has given some information about their respective circuit. Both claim they meet the required specifications.

Vendor A	Vendor B
$R_{\mathrm{T}} = 100\ \Omega$	$R/L = 10$ rad/s
$1/LC = 100\ (\mathrm{rad/s})^2$	$\omega_0 = 10$ rad/s
$C = 2$ mF	

Which vendor would you choose and why?

For a series RLC circuit, we have the following characteristic equation and relationships:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

$$\omega_0 = 10$$

$$2\zeta\omega_0 = 10$$

$$\zeta = 0.5$$

$$L = \frac{1}{\omega_0^2 C}$$

For Vendor A, we have $L = 1/(\omega_0^2 C) = 1/(100)(0.002) = 5$ H and $R/L = 100/5 = 20$, which does not meet the specification set by the characteristic equation. For Vendor B, we have $\omega_0^2 = 100$ and $R/L = 10$, which does meet the specifications. Choose Vendor B.

Problem 7–89. Solving Differential Equations with MATLAB

MATLAB has a built-in function for solving ordinary differential equations called `dsolve`. We can use this function to quickly explore the solution to a second-order differential equation when the forcing function is a sinusoidal or exponential signal. Suppose we have a series RLC circuit in the zero state connected to a voltage source $v_{\mathrm{T}}(t)$. The parameter values are $R = 4$ kΩ, $L = 1$ H, and $C = 1$ μF. The differential equation for the voltage across the capacitor is given by Eq. (7–33). If $v_{\mathrm{T}}(t) = 10u(t)$ V, we can use the following MATLAB code to solve for the capacitor voltage and plot the results

```
% Define the symbolic time variable
syms t real
% Define the parameter values
R = 4000;
L = 1;
C = 1e-6;
% Solve the differential equation for the series RLC circuit with zero
% initial conditions
vCt = dsolve('L*C*D2v + R*C*Dv + v = 10','v(0)=0','Dv(0)=0','t');
% Create a time vector for plotting and substitute in numerical values
tt = 0:0.0001:0.04;
vCt = subs(vCt);
vCtt = subs(vCt,t,tt);
% Plot the results
figure
plot(tt,vCtt,'b','LineWidth',2)
grid on
xlabel('Time, (s)')
ylabel('v_C(t), (V)')
title('Problem 7-89')
```

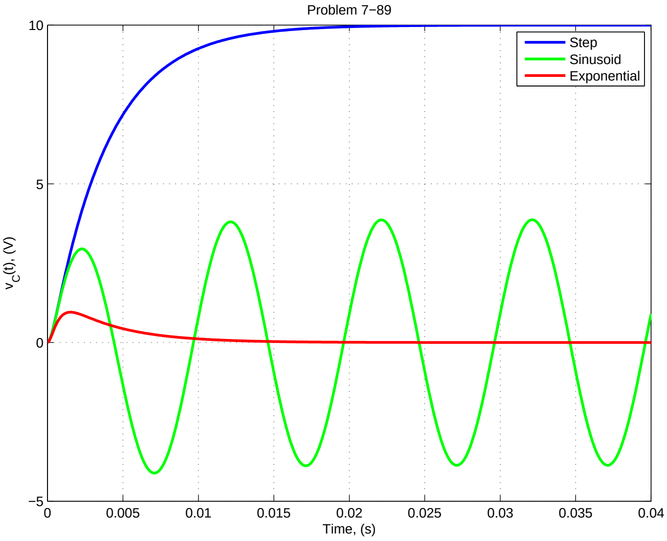
Run the given MATLAB code and examine the results. Modify the code to solve the same problem when the input voltage is $v_{\mathrm{T}}(t) = 10\cos(200\pi t)$ V. Solve the problem a third time for $v_{\mathrm{T}}(t) = 10e^{-2000t}$ V. Compare and comment on the responses for the three different types of inputs signals.

The code modifications for the sinusoidal and exponential inputs are shown below.

```
vCt = dsolve('L*C*D2v + R*C*Dv + v = 10*cos(200*pi*t)',...
    'v(0)=0','Dv(0)=0','t');
```

```
vCt = dsolve('L*C*D2v + R*C*Dv + v = 10*exp(-2000*t)',...
    'v(0)=0','Dv(0)=0','t');
```

Plot the outputs from the three different simulations on a single set of axes, as shown below.



The step response transitions from zero to the step input with an overdamped response. The sinusoidal output has rapid exponential growth before reaching a steady-state response. The response to the exponential input is a sum of exponentials that eventually approaches zero.

8 Sinusoidal Steady-State Response

8.1 Exercise Solutions

Exercise 8–1. Convert the following sinusoids to phasors in polar and rectangular form:

(a). $v(t) = 20\cos(150t - 60°)$ V

By inspection, in polar form we have $\mathbf{V} = 20\angle{-60°}$ V. Compute the rectangular form.

$$\mathbf{V} = 20\cos(-60°) + j20\sin(-60°) = 10 - j17.32 \text{ V}$$

(b). $v(t) = 10\cos(1000t + 180°)$ V

By inspection, in polar form we have $\mathbf{V} = 10\angle 180°$ V. Compute the rectangular form.

$$\mathbf{V} = 10\cos(180°) + j10\sin(180°) = -10 + j0 \text{ V}$$

(c). $i(t) = -4\cos(3t) + 3\cos(3t - 90°)$ A

In polar form we have $\mathbf{I} = -4\angle 0° + 3\angle{-90°}$, which we will simplify after we find the rectangular form.

$$\mathbf{I} = -4\cos(0°) - j4\sin(0°) + 3\cos(-90°) + j3\sin(-90°)$$

$$\mathbf{I} = -4 - j0 + 0 - j3 = -4 - j3 \text{ A}$$

$$\mathbf{I} = \sqrt{(-4)^2 + (-3)^2}\angle\tan^{-1}(-3/-4) = 5\angle{-143°} \text{ A}$$

Exercise 8–2. Convert the following phasors to sinusoids:

(a). $\mathbf{V} = 169\angle{-45°}$ V at $f = 60$ Hz

By inspection, we have $v(t) = 169\cos(120\pi t - 45°)$ V.

(b). $\mathbf{V} = 10\angle 90° + 66 - j10$ V at $\omega = 10$ krad/s

In rectangular form, we have $\mathbf{V} = j10 + 66 - j10 = 66$ V, so the corresponding sinusoid is $v(t) = 66\cos(10000t)$ V.

(c). $\mathbf{I} = 15 + j5 + 10\angle 180°$ mA at $\omega = 1000$ rad/s

In rectangular form, we have $\mathbf{I} = 15 + j5 - 10 = 5 + j5$. In polar form, we have $5\sqrt{2}\angle 45°$, so the corresponding sinusoid is $i(t) = 5\sqrt{2}\cos(1000t + 45°)$ mA.

Exercise 8–3. (a). Construct the phasors for the following signals:

$$i_1(t) = 100\cos(2000t) \text{ mA}$$

$$i_2(t) = 50\cos(2000t - 60°) \text{ mA}$$

By inspection, the phasors are

$$\mathbf{I}_1 = 100\angle 0° = 100 + j0 \text{ mA}$$

$$\mathbf{I}_2 = 50\angle{-60°} = 25 - j43.3 \text{ mA}$$

(b). Use the additive property to find $i(t) = i_1(t) + i_2(t)$ and check the results using MATLAB.
Compute $i(t)$.

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 100 + j0 + 25 - j43.3 = 125 - j43.3 = 132.3\angle{-19.1^\circ} \text{ mA}$$

$$i(t) = 132.3\cos(2000t - 19.1^\circ) \text{ mA}$$

The following MATLAB code verifies the solution:

```
I1 = 100*exp(j*0)
I2 = 50*exp(-j*pi/3)
I = I1+I2
IMag = abs(I)
IPhase = 180*angle(I)/pi
```

The corresponding MATLAB output is shown below.

```
I1 =   100.0000e+000
I2 =    25.0000e+000 - 43.3013e+000i
I =    125.0000e+000 - 43.3013e+000i
IMag = 132.2876e+000
IPhase = -19.1066e+000
```

Exercise 8–4. Show that the phasors $\mathbf{I}_\text{A}$, $\mathbf{I}_\text{B}$, and $\mathbf{I}_\text{C}$ would still sum to zero if they were all rotated 90° counterclockwise.

The new phasors and their sum are shown below

$$\mathbf{I}_\text{A} = 5\angle 140^\circ = -3.83 + j3.21$$

$$\mathbf{I}_\text{B} = 5\angle 260^\circ = -0.87 - j4.92$$

$$\mathbf{I}_\text{C} = 5\angle 20^\circ = 4.70 + j1.71$$

$$\mathbf{I}_\text{A} + \mathbf{I}_\text{B} + \mathbf{I}_\text{C} = (-3.83 - 0.87 + 4.70) + j(3.21 - 4.92 + 1.71) = 0 + j0 \text{ A}$$

Exercise 8–5. Find the phasor corresponding to the time derivative of the waveform:

$$v(t) = 100\cos(1000t) \text{ V}$$

The phasor and its derivative are shown below.

$$\mathbf{V} = 100\angle 0^\circ \text{ V}$$

$$j\omega\mathbf{V} = (j1000)100\angle 0^\circ = (1000\angle 90^\circ)(100\angle 0^\circ) = 10^5\angle 90^\circ \text{ V/s}$$

Exercise 8–6. Find the phasor corresponding to the waveform $v(t) = V_\text{A}\cos(\omega t) + 2V_\text{A}\sin(\omega t)$.

The solution is presented below.

$$v(t) = V_\text{A}\cos(\omega t) + 2V_\text{A}\sin(\omega t)$$

$$v(t) = V_\text{A}\cos(\omega t) + 2V_\text{A}\cos(\omega t - 90^\circ)$$

$$\mathbf{V} = \mathbf{V}_\text{A}\angle 0^\circ + 2\mathbf{V}_\text{A}\angle{-90^\circ} = \mathbf{V}_\text{A} - j2\mathbf{V}_\text{A}$$

$$\mathbf{V} = \sqrt{5}\mathbf{V}_\text{A}\angle{-63.4}$$

Exercise 8–7. A series circuit is composed of a 1-kΩ resistor, a 1-μF capacitor, and a 100-mH inductor.

(a). At what frequency will the magnitude of the impedance of the inductor equal that of the resistor?

We have the following relationships:

$$Z_{\rm R} = R$$

$$Z_{\rm L} = j\omega L$$

$$|Z_{\rm L}| = \omega L$$

$$Z_{\rm C} = \frac{1}{j\omega C}$$

$$|Z_{\rm C}| = \frac{1}{\omega C}$$

$$\omega L = R$$

$$\omega = \frac{R}{L} = \frac{1000}{0.1} = 10 \text{ krad/s}$$

(b). At what frequency will the magnitude of the impedance of the capacitor equal that of the resistor?

We have the following relationship:

$$\frac{1}{\omega C} = R$$

$$\omega = \frac{1}{RC} = \frac{1}{(1000)(1\,\mu)} = 1 \text{ krad/s}$$

(c). At what frequency will the magnitude of the impedance of the inductor equal the magnitude of the impedance of the capacitor? What is this frequency called?

We have the following relationship:

$$\omega L = \frac{1}{\omega C}$$

$$\omega = \sqrt{\frac{1}{LC}} = 3.162 \text{ krad/s}$$

This frequency is known as the resonant frequency.

Exercise 8–8. The circuit in Figure 8–12 is operating in the sinusoidal steady state with $v(t) = 50\cos(500t)$ V and $i(t) = 4\cos(500t - 60^\circ)$ A. Find the impedance of the elements in the box.

Compute the impedance.

$$\mathbf{V} = Z\mathbf{I}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}}$$

$$\mathbf{V} = 50\angle 0^\circ$$

$$\mathbf{I} = 4\angle{-60^\circ}$$

$$Z = \frac{50\angle 0^\circ}{4\angle{-60^\circ}} = 12.5\angle 60^\circ = 6.25 + j10.83\ \Omega$$

Exercise 8–9. A series connection consists of a 12-mH inductor and a 20-pF capacitor. The current flowing through the circuit is $i_{\mathrm{L}}(t) = 20\cos(10^6 t)$ mA.

(a). Find the impedance of each element.

We have the following impedances:

$$Z_{\mathrm{L}} = j\omega L = j(10^6)(0.012) = j12\ \mathrm{k\Omega}$$

$$Z_{\mathrm{C}} = \frac{1}{j\omega C} = \frac{1}{j(10^6)(20\,\mathrm{p})} = -j50\ \mathrm{k\Omega}$$

(b). Find the phasor voltage across each element.

We have the following voltages:

$$\mathbf{V} = Z\mathbf{I}$$

$$\mathbf{I} = 0.02\angle 0^\circ = 0.02 + j0$$

$$\mathbf{V}_{\mathrm{L}} = Z_{\mathrm{L}}\mathbf{I} = (j12000)(0.02) = j240 = 240\angle 90^\circ\ \mathrm{V}$$

$$\mathbf{V}_{\mathrm{C}} = Z_{\mathrm{C}}\mathbf{I} = (-j50000)(0.02) = -j1000 = 1000\angle{-90^\circ}\ \mathrm{V}$$

(c). Find an equation for the waveform of the voltage across each element.

Convert each phasor to a waveform.

$$v_{\mathrm{L}}(t) = 240\cos(10^6 t + 90^\circ)\ \mathrm{V}$$

$$v_{\mathrm{C}}(t) = 1\cos(10^6 t - 90^\circ)\ \mathrm{kV}$$

(d). Does the current in the inductor lead or lage the voltage across it?

In the inductor, the current's angle is 90° less than that of the voltage, so the current lags the voltage in the inductor by 90°.

(e). Does the current in the capacitor lead or lag the voltage across it?

In the capacitor, the current's angle is 90° greater than that of the voltage, so the current leads the voltage in the capacitor by 90°.

Exercise 8–10. The circuit in Figure 8–15(a) is operating in the sinusoidal steady state with $v_{\mathrm{S}}(t) = 100\cos(2000t - 45^\circ)$ V.

(a). Transform the circuit into the phasor domain.

In the phasor domain, we have the following values:

$$\mathbf{V}_{\mathrm{S}} = 100\angle{-45^\circ}\ \mathrm{V}$$

$$Z_{\mathrm{R}} = 50\ \Omega$$

$$Z_{\mathrm{C}} = \frac{1}{j\omega C} = \frac{1}{j(2000)(10\,\mu)} = -j50\ \Omega$$

$$Z_{\mathrm{L}} = j\omega L = j(2000)(0.025) = j50\ \Omega$$

(b). Solve for the phasor current **I**.

Sum the impedances in series and solve for the current.

$$\mathbf{I} = \frac{\mathbf{V}_\mathrm{S}}{Z_\mathrm{EQ}} = \frac{\mathbf{V}_\mathrm{S}}{Z_\mathrm{R} + Z_\mathrm{C} + Z_\mathrm{L}}$$

$$\mathbf{I} = \frac{100\angle{-45^\circ}}{50 - j50 + j50} = \frac{100\angle{-45^\circ}}{50} = 2\angle{-45^\circ}$$

(c). Solve for the phasor voltage across each element.

We have the following voltages:

$$\mathbf{V} = Z\mathbf{I}$$

$$\mathbf{V}_\mathrm{R} = Z_\mathrm{R}\mathbf{I} = (50)(2\angle{-45^\circ}) = 100\angle{-45^\circ}\ \mathrm{V}$$

$$\mathbf{V}_\mathrm{L} = Z_\mathrm{L}\mathbf{I} = (j50)(2\angle{-45^\circ}) = 100\angle 45^\circ\ \mathrm{V}$$

$$\mathbf{V}_\mathrm{C} = Z_\mathrm{C}\mathbf{I} = (-j50)(2\angle{-45^\circ}) = 100\angle{-135^\circ}\ \mathrm{V}$$

(d). Find the waveforms corresponding to the phasors found in (b) and (c).

By inspection, we have the following waveforms:

$$i(t) = 2\cos(2000t - 45^\circ)\ \mathrm{A}$$

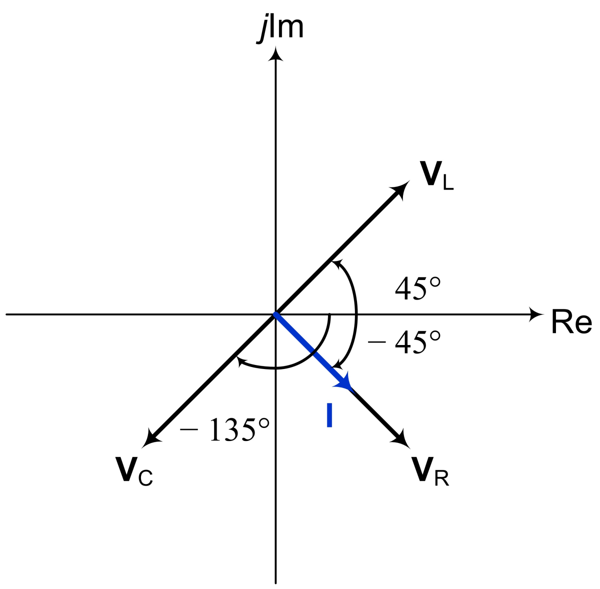
$$v_\mathrm{R}(t) = 100\cos(2000t - 45^\circ)\ \mathrm{V}$$

$$v_\mathrm{L}(t) = 100\cos(2000t + 45^\circ)\ \mathrm{V}$$

$$v_\mathrm{C}(t) = 100\cos(2000t - 135^\circ)\ \mathrm{V}$$

(e). Draw a phasor diagram of all three voltages and the current.

The phasor diagram is shown below.



Exercise 8–11. Design the voltage divider in Figure 8–16(a) so that an input $v_S(t) = 50\cos(2000t)$ V produces an output $v_O(t) = 25\cos(2000t - 30°)$ V.

There are many correct solutions. Assume $Z_2 = 1000 - j1000\ \Omega$. We have the following relationships:

$$\mathbf{V}_O = \frac{Z_2}{Z_1 + Z_2}\mathbf{V}_S$$

$$25\angle{-30°} = \frac{Z_2}{Z_1 + Z_2}50\angle 0°$$

$$(21.65 - j12.5)(Z_1 + Z_2) = 50Z_2$$

$$Z_1 = \frac{50 - (21.65 - j12.5)}{21.65 - j12.5}Z_2 = \frac{50 - (21.65 - j12.5)}{21.65 - j12.5}(1000 - j1000)$$

$$Z_1 = 1732 + j268 = 1753\angle 8.8°\ \Omega$$

$$R_1 = 1.732\ \text{k}\Omega$$

$$R_2 = 1\ \text{k}\Omega$$

$$L = \frac{268}{2000} = 134\ \text{mH}$$

$$C = \frac{1}{(1000)(2000)} = 0.5\ \mu\text{F}$$

Exercise 8–12. The circuit shown in Figure 8–17(a) is operating in the sinusoidal steady state at a frequency of 100 krad/s. It requires a load of $Z_L = 1500\angle{-57.5°}\ \Omega$ to operate properly. Design the load using standard parts to within ±5% of the desired values.

We have the following relationships:

$$\omega = 100000\ \text{rad/s}$$

$$Z_L = 1500\angle{-57.5°} = 805.9 - j1265\ \Omega$$

One way to design the load is with a resistor in series with a capacitor. The resistance should be 805.9 Ω, and the closest standard value is 820 Ω. The capacitor should be $C = 1/(1265\omega) = 7905$ pF, and the closest standard value is 8200 pF. The resulting impedance is $Z_L = 820 - j1220\ \Omega$, which is within ±5% of the desired value.

Exercise 8–13. Consider the Maxwell bridge shown in Figure 8–18. Suppose we know that the unknown impedance is an unknown capacitor C_X in parallel with an unknown resistance R_X. Let Z_1 be a resistance R_1 in series with an inductance L_1. Let Z_2 be a resistance R_2 and Z_3 be a resistance R_3. Find the relationships that will allow the bridge to be balanced.

Using the results from Example 8–8, we have the following relationships:

$$Z_{\text{X}} = R_{\text{X}} + jX_{\text{X}} = \frac{Z_2 Z_3}{Z_1} = \frac{R_2 R_3}{R_1 + j\omega L_1} = \frac{R_2 R_3 / R_1}{1 + j\omega L_1 / R_1}$$

$$Z_{\text{X}} = R_{\text{X}} \parallel \frac{1}{j\omega C_{\text{X}}} = \frac{\dfrac{R_{\text{X}}}{j\omega C_{\text{X}}}}{R_{\text{X}} + \dfrac{1}{j\omega C_{\text{X}}}} = \frac{R_{\text{X}}}{j\omega R_{\text{X}} C_{\text{X}} + 1}$$

$$\frac{R_2 R_3 / R_1}{1 + j\omega L_1 / R_1} = \frac{R_{\text{X}}}{j\omega R_{\text{X}} C_{\text{X}} + 1}$$

$$R_{\text{X}} = \frac{R_2 R_3}{R_1}$$

$$R_{\text{X}} C_{\text{X}} = \frac{L_1}{R_1}$$

$$C_{\text{X}} = \frac{L_1}{R_2 R_3}$$

Exercise 8–14. The circuit in Figure 8–21(a) is operating in the sinusoidal steady state with $i_{\text{S}}(t) = 100\cos(1000t - 45^\circ)$ mA.

(a). Transform the circuit into the phasor domain.

In the phasor domain, we have the following values:

$$\mathbf{I}_{\text{S}} = 0.1\angle{-45^\circ} \text{ A}$$

$$Z_{\text{C}} = \frac{1}{j\omega C} = \frac{1}{j(1000)(1\,\mu)} = -j1000\ \Omega$$

$$Z_{\text{R}} = 500\ \Omega$$

$$Z_{\text{L}} = j\omega L = j(1000)(0.5) = j500\ \Omega$$

(b). Solve for the phasor voltage V.

Find the equivalent impedance and then the phasor voltage.

$$Z_{\text{EQ}} = Z_{\text{C}} \parallel (Z_{\text{R}} + Z_{\text{L}}) = \frac{(-j1000)(500 + j500)}{-j1000 + 500 + j500} = 1000\ \Omega$$

$$\mathbf{V} = Z\mathbf{I}_{\text{S}} = (1000)(0.1\angle{-45^\circ}) = 100\angle{-45^\circ} \text{ V}$$

(c). Solve for the phasor current through each element.

We have the following currents:

$$\mathbf{I} = \frac{\mathbf{V}}{Z}$$

$$\mathbf{I}_{\text{C}} = \frac{100\angle{-45^\circ}}{1000\angle{-90^\circ}} = 100\angle 45^\circ \text{ mA}$$

$$\mathbf{I}_{\text{R}} = \mathbf{I}_{\text{L}} = \frac{100\angle{-45^\circ}}{500\sqrt{2}\angle 45^\circ} = 141.4\angle{-90^\circ} \text{ mA}$$

(d). Find the waveforms corresponding to the phasors found in (b) and (c).

By inspection, we have the following waveforms:

$$v(t) = 100\cos(1000t - 45^\circ)\ \text{V}$$
$$i_{\text{C}}(t) = 100\cos(1000t + 45^\circ)\ \text{mA}$$
$$i_{\text{R}}(t) = i_{\text{L}}(t) = 141.4\cos(1000t - 90^\circ)\ \text{mA}$$

Exercise 8–15. Using the values in Example 8–10, find the voltage $v_{\text{L}}(t)$ across the inductor in the circuit shown in Figure 8–22.

We have the following relationships:

$$\mathbf{V}_{\text{L}} = Z_{\text{L}}\mathbf{I}_{\text{L}}$$
$$\mathbf{I}_{\text{L}} = \mathbf{I} = 0.2\angle 53.1^\circ\ \text{A}$$
$$Z_{\text{L}} = j500 = 500\angle 90^\circ\ \Omega$$
$$\mathbf{V}_{\text{L}} = (500\angle 90^\circ)(0.2\angle 53.1^\circ) = 100\angle 143.1^\circ\ \text{V}$$
$$v_{\text{L}}(t) = 100\cos(2000t + 143.1^\circ)\ \text{V}$$

Exercise 8–16. The circuit in Figure 8–25 is operating in the sinusoidal steady state. If $R = 1$ kΩ, $L = 200$ mH, and $C = 1$ μF:

(a). Find the value of ω that will cause the circuit to be in resonance.

Using the results of Example 8–11, we have the following relationship:

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{1}{(RC)^2}} = \sqrt{\frac{1}{(0.2)(1\,\mu)} - \frac{1}{[(1000)(1\,\mu)]^2}} = 2\ \text{krad/s}$$

(b). What will the value of Z_{EQ} be under those conditions?

Again, using the results of Example 8–11, we have the following relationship:

$$Z_{\text{EQ}} = j\omega L + Z_{\text{RC}} = j\omega L + \frac{R}{1 + j\omega RC} = 200 + j0\ \Omega$$

As expected, at a resonant frequency, the impedance does not have an imaginary component.

Exercise 8–17. The circuit in Figure 8–25 is operating in the sinusoidal steady state at $\omega = 1$ krad/s. If $R = 1$ kΩ, $L = 200$ mH, and $C = 1$ μF:

(a). Find the value of Z_{EQ} classically under those conditions.

We have the following relationships:

$$Z_{\rm EQ} = Z_{\rm L} + (Z_{\rm C} \parallel Z_{\rm R})$$

$$Z_{\rm L} = j\omega L = j(1000)(0.2) = j200$$

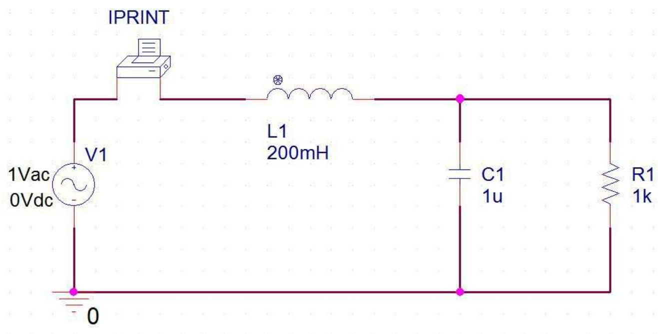
$$Z_{\rm R} = 1000$$

$$Z_{\rm C} = \frac{1}{j\omega C} = \frac{1}{j(1000)(1\,\mu)} = -j1000$$

$$Z_{\rm EQ} = j200 + \frac{(1000)(-j1000)}{1000 - j1000} = 500 - j300 = 583.1\angle{-31^\circ}\ \Omega$$

(b). Repeat the problem using OrCAD. (*Hint*: Drive the circuit with a 1 VAC source at 1 krad/s and use IPRINT to measure the input current. The equivalent impedance then will be $Z_{\rm EQ} = 1\angle 0^\circ/\mathbf{I}\ \Omega$.)

The OrCAD simulation is shown below.



The resulting output is shown below.

```
FREQ        IM(V_PRINT1)IP(V_PRINT1)IR(V_PRINT1)II(V_PRINT1)
 1.592E+02   1.715E-03   3.096E+01   1.471E-03   8.824E-04
```

The input phasor current is $\mathbf{I} = 1.715\angle 30.96^\circ = 1.471 + j0.8824$ mA. Calculate the input impedance.

$$Z_{\rm EQ} = \frac{1}{0.001715\angle 30.96^\circ} = 500 - j300\ \Omega$$

Exercise 8–18. The circuit in Figure 8–27 is operating at $\omega = 10$ krad/s.

(a). Find the equivalent impedance Z.

Compute the impedance.

$$Z = (500 - j125) \parallel (100 + j400)$$

$$Z = \frac{(500 - j125)(100 + j400)}{500 - j125 + 100 + j400}$$

$$Z = 256.1 + j195.1 = 321.96\angle 37.3^\circ\ \Omega$$

(b). What element should be connected in series with Z to make the total reactance zero?

To make the total reactance zero, we need to connect $Z_{\rm C} = -j195.1\ \Omega$. The capacitance is $C = 1/(195.1\omega) = 1/[(195.1)(10000)] = 0.513\ \mu$F.

Exercise 8–19. In Figure 8–28, $v_{\text{S}}(t) = 12.5\cos(1000t)$ V and $i_{\text{S}}(t) = 0.2\cos(1000t - 36.9°)$ A. What is the impedance seen by the voltage source and what element is in the box?

Compute the impedance.

$$Z = \frac{\mathbf{V}_{\text{S}}}{\mathbf{I}_{\text{S}}}$$

$$Z = \frac{12.5\angle 0°}{0.2\angle{-36.9°}}$$

$$Z = 62.5\angle 36.9° = 50 + j37.5\ \Omega$$

The component is an inductor with $L = 37.5/\omega = 37.5/1000 = 37.5$ mH.

Exercise 8–20. For the circuit of Figure 8–29(a), design a low-pass filter using standard parts so that the cutoff frequency is 1000 rad/s.

There are many correct solutions. One approach is to use a series RC circuit with the output taken across the capacitor. Select the components such that the cutoff frequency is $\omega_{\text{C}} = 1/RC = 1000$. Use $R = 1$ kΩ and $C = 1\ \mu$F, as one approach with standard parts.

Exercise 8–21. For the circuit of Figure 8–29(a), replace the capacitor with an inductor.

(a). Find the ratio $|\mathbf{V}_{\text{L}}/\mathbf{V}_{\text{S}}|$.

Use voltage division to find the requested ratio.

$$\mathbf{V}_{\text{L}} = \frac{Z_{\text{L}}}{Z_{\text{R}} + Z_{\text{L}}}\mathbf{V}_{\text{S}}$$

$$\mathbf{V}_{\text{L}} = \frac{j\omega L}{R + j\omega L}\mathbf{V}_{\text{S}}$$

$$\frac{\mathbf{V}_{\text{L}}}{\mathbf{V}_{\text{S}}} = \frac{j\omega}{\frac{R}{L} + j\omega}$$

$$\left|\frac{\mathbf{V}_{\text{L}}}{\mathbf{V}_{\text{S}}}\right| = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}$$

(b). Comment on its behavior as the frequency changes from 0 to ∞.

For small values of ω, the output voltage is small. As ω increases, the output voltage increases and eventually approaches the value of the input voltage. This is a high-pass filter because it blocks low-frequency signals and passes high-frequency signals. The cutoff frequency is $\omega_{\text{C}} = R/L$ rad/s.

Exercise 8–22. Use the unit output method to find the output current $\mathbf{I}_{\text{O}}$ in the circuit of Figure 8–31.

The following calculations provide the solution. First, define the output current to be 1 A.

$$\mathbf{I}_{\text{O}} = 1\text{ A}$$

Compute the voltage across the right horizontal resistor.

$$\mathbf{V}_{\text{C}} = (\mathbf{I}_{\text{O}})(j50) = j50\text{ V}$$

Compute the current through the right vertical resistor.

$$\mathbf{I}_{\text{C}} = \frac{\mathbf{V}_{\text{C}}}{50} = j1\text{ A}$$

Compute the current through the left horizontal resistor.

$$\mathbf{I}_{\mathrm{B}} = \mathbf{I}_{\mathrm{O}} + \mathbf{I}_{\mathrm{C}} = 1 + j1 \text{ A}$$

Compute the voltage across the left vertical resistor

$$\mathbf{V}_{\mathrm{B}} = 100\mathbf{I}_{\mathrm{B}} + \mathbf{V}_{\mathrm{C}} = 100 + j150 \text{ V}$$

Compute the current through the left vertical resistor

$$\mathbf{I}_{\mathrm{A}} = \frac{\mathbf{V}_{\mathrm{B}}}{50\angle 40^\circ} = 3.4605 + j1.0126 \text{ A}$$

Compute the current through the source

$$\mathbf{I}_{\mathrm{S}} = \mathbf{I}_{\mathrm{A}} + \mathbf{I}_{\mathrm{B}} = 4.4605 + j2.0126 \text{ A}$$

Find the unit-output gain

$$K = \frac{1}{\mathbf{I}_{\mathrm{S}}} = \frac{1}{4.4605 + j2.0126} = 0.18627 - j0.084046 = 0.20435\angle -24.3^\circ$$

Compute the actual output current

$$\mathbf{I}_{\mathrm{O}} = K\mathbf{I}_{\mathrm{S}} = (0.20435\angle -24.3^\circ)(50\angle 15^\circ) = 10.22\angle -9.28^\circ \text{ mA}$$

Exercise 8–23. The two sources in Figure 8–34 have the same frequency. Use superposition to find the phasor current $\mathbf{I}_{\mathrm{X}}$.

Replace the voltage source with a short circuit and find the contribution from the current source. Find the equivalent impedance parallel to the 100-Ω resistor and apply two-path current division.

$$Z_{\mathrm{EQ1}} = j75 + (10 \parallel -j100) = 9.901 + j74.01 \ \Omega$$

$$\mathbf{I}_{\mathrm{X1}} = \frac{100}{100 + 9.901 + j74.01}(0.1\angle -90^\circ) = -42.157 - j62.601 \text{ mA}$$

Replace the current source with an open circuit and find the contribution from the voltage source. Find the equivalent impedance in series with the 10-Ω resistor and apply voltage division.

$$Z_{\mathrm{EQ2}} = (100 + j75) \parallel -j100 = 94.118 - j76.471 \ \Omega$$

$$\mathbf{V}_{\mathrm{X2}} = -\frac{94.118 - j76.471}{10 + 94.118 - j76.471}(20\angle 45^\circ) = -13.908 - 12.612 \text{ V}$$

$$\mathbf{I}_{\mathrm{X2}} = \frac{-13.908 - 12.612}{100 + j75} = -149.547 - j13.958 \text{ mA}$$

Sum the component currents.

$$\mathbf{I}_{\mathrm{X}} = \mathbf{I}_{\mathrm{X1}} + \mathbf{I}_{\mathrm{X2}} = -42.157 - j62.601 - 149.547 - j13.958 = -191.704 - j76.559 = 206.426\angle -158.23^\circ \text{ mA}$$

Exercise 8–24. Use superposition to find the output voltage $v_{\mathrm{O}}(t)$ in the circuit of Figure 8–36 if $i_{\mathrm{S}}(t) = 100\cos(10000t)$ mA and $v_{\mathrm{S}}(t) = 20\cos(20000t - 45^\circ)$ V.

The source frequencies are different, so the impedances will differ and we will not be able to combine the sinusoidal expressions. Replace the voltage source with a short circuit, find the equivalent impedance, and the output voltage.

$$Z_{\mathrm{C1}} = \frac{1}{j(10000)(1\,\mu)} = -j100 \ \Omega$$

$$Z_{\mathrm{EQ1}} = 100 \parallel -j100 = 50 - j50 \ \Omega$$

$$\mathbf{V}_{\mathrm{O1}} = Z_{\mathrm{EQ1}}\mathbf{I}_{\mathrm{S}} = (50 - j50)(0.1) = 5 - j5 = 7.07\angle -45^\circ \text{ V}$$

Replace the current source with an open circuit and apply voltage division.

$$Z_{\mathrm{C2}} = \frac{1}{j(20000)(1\,\mu)} = -j50\ \Omega$$

$$\mathbf{V}_{\mathrm{O2}} = \frac{-j50}{100 - j50}(20\angle{-45^\circ}) = -2.828 - j8.485 = 8.944\angle{-108.43^\circ}\ \mathrm{V}$$

Write the final expression.

$$v_{\mathrm{O}}(t) = 7.07\cos(10000t - 45^\circ) + 8.944\cos(20000t - 108.43^\circ)\ \mathrm{V}$$

Exercise 8–25. Analyze the sinusoidal steady-state behavior of the circuit shown in Figure 8–37 in more detail. To do so, find the magnitude of the ratio of the output voltage to the input voltage for the range of frequencies from 1 Hz to 1 kHz. For simplicity, assume that the input signal always has a magnitude of 1 V. Examine at least 500 data points in the frequency range and space them logarithmically with the MATLAB command `logspace`. Plot the results in terms of magnitude versus frequency (in hertz) on log-log axes. Use the plot to verify the results in Example 8–17.

Compute the equivalent impedance to the right of the inductor on the left.

$$Z_{\mathrm{EQ}} = \frac{1}{j\omega C} \parallel (100 + j\omega) = \frac{\left(\dfrac{1}{j\omega C}\right)(100 + j\omega)}{\dfrac{1}{j\omega C} + j\omega + 100} = \frac{100 + j\omega}{1 - \omega^2 C + j100\omega C}$$

Apply voltage division twice to determine the output voltage gain.

$$\mathbf{V}_{\mathrm{A}} = \frac{Z_{\mathrm{EQ}}}{Z_{\mathrm{EQ}} + 1 + j\omega}\mathbf{V}_{\mathrm{S}}$$

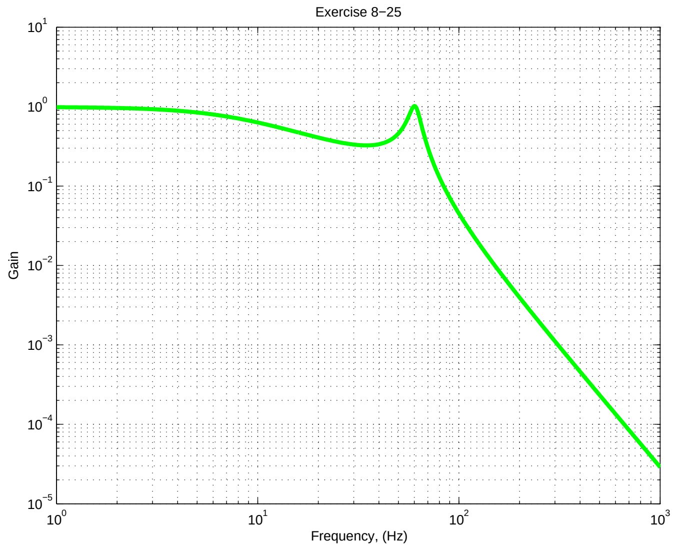
$$\mathbf{V}_{\mathrm{L}} = \frac{100}{100 + j\omega}\mathbf{V}_{\mathrm{A}} = \frac{100 Z_{\mathrm{EQ}}}{(100 + j\omega)(Z_{\mathrm{EQ}} + 1 + j\omega)}\mathbf{V}_{\mathrm{S}}$$

$$K = \frac{\mathbf{V}_{\mathrm{L}}}{\mathbf{V}_{\mathrm{S}}} = \frac{100 Z_{\mathrm{EQ}}}{(100 + j\omega)(Z_{\mathrm{EQ}} + 1 + j\omega)}$$

The following MATLAB code provides the solution.

```
% Define the fixed parameters
Rs = 1;
L1 = 1;
L2 = 1;
C = 14e-6;
RL = 100;
% Create a list of frequencies to evaluate
f = logspace(0,3,1000);
% Create the radian frequencies
w = 2*pi*f;
% Compute the gain for each frequency
Zeq = (100+j*w)./(1+j*100*C-C*w.^2);
K = abs(100*Zeq./((100+j*w).*(Zeq+1+j*w)));
% Plot the results
figure
loglog(f,K,'g','LineWidth',3)
grid on
xlabel('Frequency, (Hz)')
ylabel('Gain')
title('Exercise 8-25')
```

The corresponding MATLAB output is shown below.



Exercise 8–26. The Norton circuit in Figure 8–41(b) has a current source $\mathbf{I}_{\mathrm{N}} = 300 - j400$ mA and a Norton impedance Z_{N} of $100 + j100\ \Omega$. Find the equivalent Thévenin circuit.

We have the following relationships:

$$Z_{\mathrm{T}} = Z_{\mathrm{N}} = 100 + j100 = 141.4\angle 45^\circ\ \Omega$$

$$\mathbf{V}_{\mathrm{T}} = Z_{\mathrm{N}}\mathbf{I}_{\mathrm{N}} = (100 + j100)(0.3 - j0.4) = 70 - j10 = 70.71\angle{-8.13^\circ}\ \mathrm{V}$$

Exercise 8–27. Convert the Thévenin circuit found in Example 8–19 into its Norton equivalent. Then repeat the design task in that example.

Compute the Norton equivalent circuit.

$$Z_{\mathrm{N}} = Z_{\mathrm{T}} = 500 + j500 = 707.1\angle 45^\circ\ \Omega$$

$$\mathbf{I}_{\mathrm{N}} = \frac{\mathbf{V}_{\mathrm{T}}}{Z_{\mathrm{T}}} = \frac{35.35\angle{-45^\circ}}{707.1\angle 45^\circ} = 0 - j50 = 50\angle{-90^\circ}\ \mathrm{mA}$$

For the load design, we have the following relationships:

$$\mathbf{V}_{\mathrm{L}} = 10\angle{-90^\circ}\ \mathrm{V}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{\mathrm{L}}}{Z_{\mathrm{N}}} = \frac{10\angle{-90^\circ}}{707.1\angle 45^\circ} = -0.01 - j0.01\ \mathrm{A}$$

$$\mathbf{I}_{\mathrm{L}} = \mathbf{I}_{\mathrm{N}} - \mathbf{I}_1 = 0 - j0.05 - (-0.01 - j0.01) = 0.01 - j0.04\ \mathrm{A}$$

$$Z_{\mathrm{L}} = \frac{\mathbf{V}_{\mathrm{L}}}{\mathbf{I}_{\mathrm{L}}} = \frac{10\angle{-90^\circ}}{0.01 - j0.04} = 235.3 - j58.8\ \Omega$$

As expected, the load matches the design from Example 8–19.

Exercise 8–28. Repeat Example 8–20 using OrCAD if both sources are operating at a frequency of $\omega =$ 20 krad/s. (*Hint*: Find L_1, L_2, and C from the data in Example 8–20 first.)

Compute the component values.

$$\omega L_1 = 10$$

$$L_1 = \frac{10}{5000} = 2 \text{ mH}$$

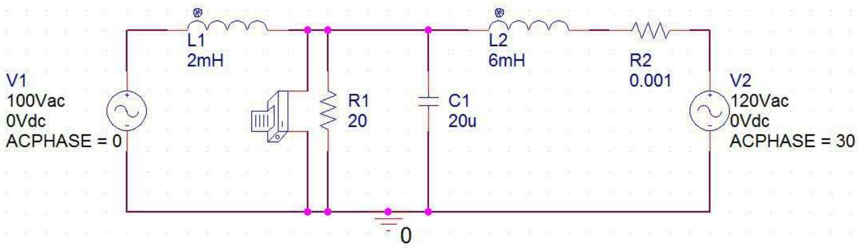
$$\omega L_2 = 30$$

$$L_2 = \frac{30}{5000} = 6 \text{ mH}$$

$$\frac{1}{\omega C} = 10$$

$$C = \frac{1}{(10)(5000)} = 20\ \mu\text{F}$$

The OrCAD simulation is shown below.



The corresponding output is shown below.

```
  FREQ        VM(N00180,0)VP(N00180,0)VR(N00180,0)VI(N00180,0)
   3.183E+03   9.196E+00  -1.638E+02  -8.830E+00  -2.568E+00
```

The results are $v_R(t) = 9.196\cos(20000t - 163.8°)$ V.

Exercise 8–29.

(a). Find the Thévenin equivalent circuit seen by the inductor in Figure 8–34.

Use the lookback technique to find the Thévenin impedance and then apply superposition to find the open-circuit voltage.

$$Z_T = 100 + (10 \parallel -j100) = 109.9 - j0.990 = 109.9\angle{-0.516°}\ \Omega$$

$$\mathbf{V}_{OC1} = (0.1\angle{-90°})(100) = 10\angle{-90°} = 0 - j10 \text{ V}$$

$$\mathbf{V}_{OC2} = \frac{-j100}{10 - j100}(-20\angle 45°) = -15.4 - j12.6 = 19.9\angle{-140.7°} \text{ V}$$

$$\mathbf{V}_{OC} = \mathbf{V}_{OC1} + \mathbf{V}_{OC2} = -15.4 - j22.6 = 27.35\angle{-124.27°} \text{ V}$$

(b). Use the Thévenin equivalent to calculate the current $\mathbf{I}_X$.

Compute the current

$$\mathbf{I}_X = \frac{\mathbf{V}_T}{Z_T + j75} = \frac{-15.4 - j22.6}{109.9 - j0.99 + j75} = -0.1917 - j0.07656 = 0.20643\angle{-158.23°} \text{ A}$$

Exercise 8–30. By inspection, determine the Thévenin equivalent circuit seen by the capacitor in Figure 8–30 for $\mathbf{V}_{\mathrm{S}} = 10\angle 0°$ V/

The impedance is a parallel combination of two equal branches, so the Thévenin impedance is half of the impedance of one branch.

$$Z_{\mathrm{T}} = \frac{50 + j100}{2} = 25 + j50\ \Omega$$

The open-circuit voltage can be found through voltage division, which occurs between two equal impedances, so it is half of the source voltage.

$$\mathbf{V}_{\mathrm{T}} = \frac{\mathbf{V}_{\mathrm{S}}}{2} = 5\angle 0°\ \mathrm{V}$$

Exercise 8–31. In the steady state the short-circuit current at an interface is observed to be

$$i_{\mathrm{SC}}(t) = 0.75\sin(\omega t)\ \mathrm{A}$$

When a 150-Ω resistor is connected across the interface, the interface current is observed to be

$$i(t) = 0.6\cos(\omega t - 53.1°)\ \mathrm{A}$$

Find the Norton equivalent phasor circuit at the interface.

The short-circuit current is the Norton current.

$$i_{\mathrm{SC}}(t) = 0.75\sin(\omega t) = 0.75\cos(\omega t - 90°)\ \mathrm{A}$$

$$\mathbf{I}_{\mathrm{N}} = \mathbf{I}_{\mathrm{SC}} = 0.75\angle{-90°} = 0 - j0.75\ \mathrm{A}$$

Find the interface current phasor, the interface voltage, the current through the Norton impedance, and, finally, the Norton impedance.

$$\mathbf{I} = 0.6\angle{-53.1°} = 0.36 - j0.48\ \mathrm{A}$$

$$\mathbf{V} = 150\mathbf{I} = (150)(0.36 - j0.48) = 54 - j72\ \mathrm{V}$$

$$\mathbf{I}_1 = \mathbf{I}_{\mathrm{N}} - \mathbf{I} = 0 - j0.75 - (0.36 - j0.48) = -0.36 - j0.27 = 0.45\angle{-143.1°}\ \mathrm{A}$$

$$Z_{\mathrm{N}} = \frac{\mathbf{V}}{\mathbf{I}_1} = \frac{54 - j72}{-0.36 - j0.27} = 0 + j200\ \Omega$$

Exercise 8–32. Repeat the problem in Example 8–23, except you cannot use capacitors.

Use an inductor in series with a resistor and take the output across the resistor. We have the following relationships:

$$\frac{\mathbf{V}_{\mathrm{O}}}{\mathbf{V}_{\mathrm{S}}} = \frac{R}{R + jX} = \frac{R}{\sqrt{R^2 + X^2}}\angle\left[-\tan^{-1}\left(\frac{X}{R}\right)\right]$$

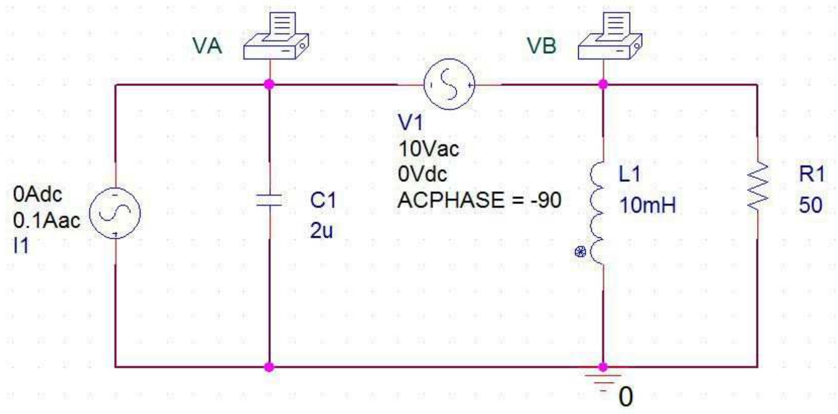
$$\frac{R}{\sqrt{R^2 + X^2}} = 0.707$$

$$-\tan^{-1}\left(\frac{X}{R}\right) = -45°$$

Pick $R = 1$ kΩ and then we solve for $X = 1$ kΩ. Solve for $L = X/\omega = 1000/1000 = 1$ H. Other solutions are possible.

Exercise 8–33. Use OrCAD to solve for the node voltages $\mathbf{V}_{\mathrm{A}}$ and $\mathbf{V}_{\mathrm{B}}$ in the circuit of Figure 8–49(a).

The radian frequency of the circuit is $\omega = 10$ krad/s. The corresponding capacitor is $C = 1/[(50)(10000)] = 2\ \mu$F and the corresponding inductor is $L = 100/10000 = 10$ mH. The OrCAD simulation is shown below.



The corresponding OrCAD output is shown below.

```
FREQ        VM(N00185)  VP(N00185)  VR(N00185)  VI(N00185)
  1.592E+03   1.265E+01   1.843E+01   1.200E+01   4.000E+00

FREQ        VM(N00238)  VP(N00238)  VR(N00238)  VI(N00238)
  1.592E+03   1.342E+01  -2.657E+01   1.200E+01  -6.000E+00
```

The results are $\mathbf{V}_{\mathrm{A}} = 12 + j4$ V and $\mathbf{V}_{\mathrm{B}} = 12 - j6$ V and agree with the solution in Example 8–24.

Exercise 8–34. Find the Thévenin equivalent circuit seen by the capacitor in the circuit of Figure 8–51

Replace the voltage source with a short circuit to find the lookback impedance. After redrawing the circuit, we can write the lookback impedance as follows

$$Z_{\mathrm{T}} = [(60 \parallel 50) + j200] \parallel 50 = \frac{(27.27 + j200)(50)}{27.27 + j200 + 50} = 45.80 + j10.88 = 47.07\angle 13.36^\circ\ \Omega$$

Apply voltage division twice to find the open-circuit voltage.

$$Z_{\mathrm{EQ}} = (50 + j200) \parallel 50 = 45 + j10\ \Omega$$

$$\mathbf{V}_{\mathrm{B}} = \frac{60}{60 + Z_{\mathrm{EQ}}}(75) = \frac{60}{60 + 45 + j10}(75) = 42.47 - j4.045\ \mathrm{V}$$

$$\mathbf{V}_{\mathrm{CB}} = \mathbf{V}_{\mathrm{C}} - \mathbf{V}_{\mathrm{B}} = 75 - (42.47 - j4.045) = 32.53 + j4.045\ \mathrm{V}$$

$$\mathbf{V}_{\mathrm{AB}} = \frac{j200}{50 + j200}(32.53 + j4.045) = 29.66 + j11.46\ \mathrm{V}$$

$$\mathbf{V}_{\mathrm{A}} = \mathbf{V}_{\mathrm{T}} = \mathbf{V}_{\mathrm{AB}} + \mathbf{V}_{\mathrm{B}} = 29.66 + j11.46 + 42.47 - j4.045 = 72.13 + j7.416 = 72.52\angle 5.87^\circ\ \mathrm{V}$$

Exercise 8–35. In the circuit of Figure 8–52, Z_1 is a 1-kΩ resistor and Z_2 is the parallel combination of a 10-kΩ resistor and a 1-μF capacitor. Determine the output voltage $v_{\mathrm{O}}(t)$ if the input is $v_{\mathrm{S}}(t) = 1\cos(100t)$ V.

Compute Z_2, the gain of the circuit, and the output voltage.

$$Z_2 = 10000 \parallel \frac{1}{j\omega C} = 10000 \parallel -j10000 = 5000 - j5000\ \Omega$$

$$K = -\frac{Z_2}{Z_1} = -\frac{5000 - j5000}{1000} = -5 + j5 = 7.07\angle 135^\circ$$

$$\mathbf{V}_{\mathrm{O}} = K\mathbf{V}_{\mathrm{S}} = (K)(1\angle 0^\circ) = K = 7.07\angle 135^\circ\ \mathrm{V}$$

$$v_{\mathrm{O}}(t) = 7.07\cos(100t + 135^\circ)\ \mathrm{V}$$

Exercise 8–36. Use the circuit of Figure 8–53 to design a high-pass filter with a pass-band gain of −100 and a cutoff frequency ω_C of 10000 rad/s. Use standard parts (See the inside rear cover).

Pick $R_1 = 1$ kΩ and use the cutoff frequency to solve for the capacitance. Then use the gain to solve for the other resistor.

$$\omega_C = \frac{1}{R_1 C}$$

$$C = \frac{1}{\omega_C R_1} = \frac{1}{(10000)(1000)} = 0.1\ \mu\text{F}$$

$$K = -\frac{R_2}{R_1} = -100$$

$$R_2 = 100 R_1 = 100\ \text{k}\Omega$$

Other solutions are possible.

Exercise 8–37. Design a bandpass filter with a lower frequency cutoff of 100 Hz and an upper frequency cutoff of 20 kHz.

Use the circuit in Figure 8-54. The low-pass filter controls the upper frequency cutoff and the high-pass filter controls the lower frequency cutoff. Select both resistors to be equal with $R_1 = R_2 = 1$ kΩ. Solve for the capacitor values.

$$\frac{1}{R_1 C_1} = (20000)(2\pi)$$

$$C_1 = \frac{1}{40000\pi R_1} = 7960\ \text{pF}$$

$$\frac{1}{R_2 C_2} = (100)(2\pi)$$

$$C_2 = \frac{1}{200\pi R_2} = 1.59\ \mu\text{F}$$

Exercise 8–38. Use the mesh-current or node-voltage method to find the branch currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in Figure 8–57.

Use node-voltage analysis, since only the upper right node is not defined by a voltage source. We have the following results:

$$\frac{\mathbf{V}_A - 120}{j10} + \frac{\mathbf{V}_A - 120\angle{-30^\circ}}{j20} + \frac{\mathbf{V}_A}{50} = 0$$

$$\left(\frac{1}{j10} + \frac{1}{j20} + \frac{1}{50}\right)\mathbf{V}_A = \frac{120}{j10} + \frac{120\angle{-30^\circ}}{j20}$$

$$\mathbf{V}_A = 110.02 - j34.67\ \text{V}$$

$$\mathbf{I}_1 = \frac{120 - 120\angle{-30^\circ}}{j5} = 12.4\angle{-15^\circ}\ \text{A}$$

$$\mathbf{I}_2 = \frac{120 - 110.02 + j34.67}{j10} = 3.61\angle{-16.1^\circ}\ \text{A}$$

$$\mathbf{I}_3 = \frac{120\angle{-30^\circ} - 110.02 + j34.67}{j20} = 1.303\angle 166.5^\circ\ \text{A}$$

Exercise 8–39. Use the mesh-current or node-voltage method to find the output voltage $\mathbf{V}_2$ and input impedance Z_{IN} in Figure 8–59.

Label the undesignated node as $\mathbf{V}_1$ and solve the node-voltage equations.

$$\frac{\mathbf{V}_1 - \mathbf{V}_{\text{S}}}{50} + \frac{\mathbf{V}_1}{-j50} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j100} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_2}{j100} + \frac{\mathbf{V}_2}{-j50} + \frac{\mathbf{V}_2}{50} = 0$$

$$\mathbf{V}_1 = 3.75 - j1.25 = 3.95\angle{-18.43^\circ} \text{ V}$$

$$\mathbf{V}_2 = -1.25 - j1.25 = 1.768\angle{-135^\circ} \text{ V}$$

$$\mathbf{I}_{\text{S}} = \frac{5 - 3.75 + j1.25}{50} = 25 + j25 \text{ mA}$$

$$Z_{\text{IN}} = \frac{\mathbf{V}_{\text{S}}}{\mathbf{I}_{\text{S}}} = \frac{5}{0.025 + j0.025} = 100 - j100\ \Omega$$

Exercise 8–40. Use MATLAB and either mesh-current or node-voltage analysis to find the current $\mathbf{I}_{\text{X}}$ in Figure 8–61.

The following MATLAB code provides the mesh-current solution:

```
syms IA IB IC
VS = 500;
Eqn1 = IA*60j + (IA-IC)*60 + (IA-IB)*60;
Eqn2 = IB*(-60j) + (IB-IA)*60 + (IB-IC)*60 + VS;
Eqn3 = -VS + (IC-IB)*60 + (IC-IA)*60 + IC*(-200j);
Soln = solve(Eqn1,Eqn2,Eqn3,IA,IB,IC);
IAnum = double(Soln.IA)
IBnum = double(Soln.IB)
ICnum = double(Soln.IC)
IX = IAnum
IXMag = abs(IX)
IXPhase = 180*angle(IX)/pi
```

The corresponding MATLAB output is shown below.

```
IAnum = -1.4262e+000 +213.9364e-003i
IBnum = -3.7694e+000 -  1.5179e+000i
ICnum = 702.9340e-003 +519.5599e-003i
IX = -1.4262e+000 +213.9364e-003i
IXMag = 1.4422e+000
IXPhase = 171.4692e+000
```

The result is $\mathbf{I}_{\text{X}} = -1.4262 + j0.2138 = 1.4422\angle 171.47^\circ$ A.

Exercise 8–41. Find the average power delivered to the 25-Ω load resistor in Figure 8–66.

Use node-voltage analysis to solve for the voltage across the 25-Ω resistor.

$$\frac{\mathbf{V}_{\text{A}} - 150}{j2} + \frac{\mathbf{V}_{\text{A}} - 125\angle{-90^\circ}}{j4} + \frac{\mathbf{V}_{\text{A}}}{25} = 0$$

$$\mathbf{V}_{\text{A}} = 97.5 - j46.87 \text{ V}$$

$$\mathbf{I}_{\text{A}} = \frac{\mathbf{V}_{\text{A}}}{25} = 3.9 - j1.875 = 4.327\angle{-25.67} \text{ A}$$

$$P = \frac{1}{2} R_{\text{L}} \left|\mathbf{I}_{\text{A}}\right|^2 = (0.5)(25)(4.327)^2 = 234 \text{ W}$$

Exercise 8–42. Calculate the maximum average power available at the interface in Figure 8–69.

Find the Thévenin equivalent circuit by finding the lookback impedance and the short-circuit current. Then compute the maximum average power available.

$$Z_{\mathrm{T}} = (50 + j100) \parallel -j50 = 25 - j75\ \Omega$$

$$\mathbf{I}_{\mathrm{SC}} = \frac{50}{50 + j100}(0.1) = 0.02 - j0.04\ \mathrm{A}$$

$$\mathbf{V}_{\mathrm{T}} = Z_{\mathrm{T}}\mathbf{I}_{\mathrm{SC}} = (25 - j75)(0.02 - j0.04) = -2.5 - j2.5 = 3.536\angle{-135^\circ}\ \mathrm{V}$$

$$P_{\mathrm{MAX}} = \frac{|\mathbf{V}_{\mathrm{T}}|^2}{8R_{\mathrm{T}}} = \frac{(3.536)^2}{(8)(25)} = 62.5\ \mathrm{mW}$$

8.2 Problem Solutions

Problem 8–1. Transform the following sinusoids into phasor form and draw a phasor diagram. Use the additive property of phasors to find $v_1(t) + v_2(t)$.

(a). $v_1(t) = 50\cos(\omega t - 30°)$ V

In phasor form, we have:

$$\mathbf{V}_1 = 50\angle{-30°} = 43.3 - j25 \text{ V}$$

(b). $v_2(t) = 200\cos(\omega t + 150°)$ V

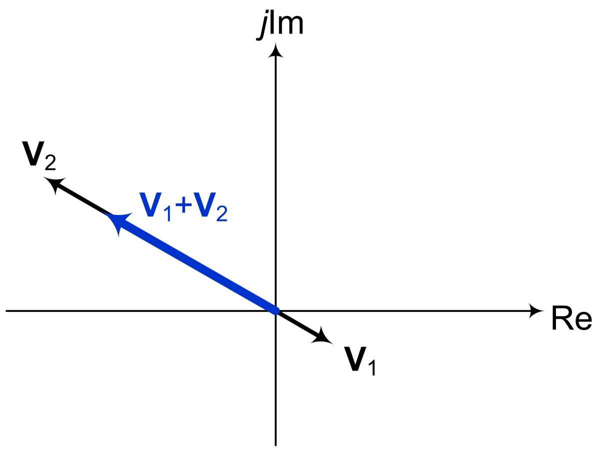
In phasor form, we have:

$$\mathbf{V}_2 = 200\angle 150° = -173.2 + j100 \text{ V}$$

Add the phasors

$$\mathbf{V}_1 + \mathbf{V}_2 = 43.3 - j25 - 173.2 + j100 = -129.9 + j75 = 150\angle 150° \text{ V}$$

The phasor diagram is sketched below.



Problem 8–2. Transform the following sinusoids into phasor form and draw a phasor diagram. Use the additive property of phasors to find $i_1(t) + i_2(t)$.

(a). $i_1(t) = -4\sin(\omega t)$ A

In phasor form, we have:

$$i_1(t) = -4\sin(\omega t) = -4\cos(\omega t - 90°) = 4\cos(\omega t + 90°) \text{ A}$$

$$\mathbf{I}_1 = 4\angle 90° = 0 + j4 \text{ A}$$

(b). $i_2(t) = 3\cos(\omega t - 26.6°)$ A

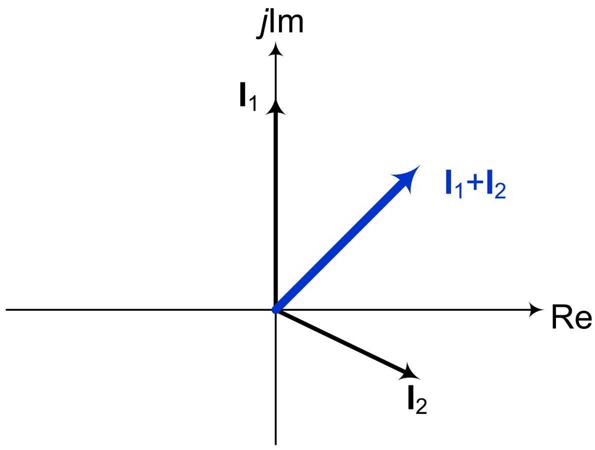
In phasor form, we have:

$$\mathbf{I}_2 = 3\angle{-26.6°} = 2.68 - j1.34 \text{ A}$$

Add the phasors

$$\mathbf{I}_1 + \mathbf{I}_2 = 2.68 + j2.66 = 3.775\angle 44.7° \text{ A}$$

The phasor diagram is sketched below.



Problem 8–3. The sum of the two voltage phasors shown in Figure P8–3 is $\mathbf{V}_3$. If the frequency is 100 Hz, write the sum in the time domain, $v_3(t)$.

We have the following relationships.

$$\mathbf{V}_1 = 20 + j20 \text{ V}$$

$$\mathbf{V}_2 = -10 + j10 \text{ V}$$

$$\mathbf{V}_3 = \mathbf{V}_1 + \mathbf{V}_2 = 20 + j20 - 10 + j10 = 10 + j30 = 31.62\angle 71.57^\circ \text{ V}$$

$$v_3(t) = 31.62\cos(100t + 71.57^\circ) \text{ V}$$

Problem 8–4. Convert the following phasors into sinusoidal waveforms.

(a). $\mathbf{V}_1 = 100e^{-j60^\circ}$ V, $\omega = 10^3$ rad/s

The corresponding waveform is

$$v_1(t) = 100\cos(1000t - 60^\circ) \text{ V}$$

(b). $\mathbf{V}_2 = 120e^{j45^\circ}$ V, $\omega = 10^3$ rad/s

The corresponding waveform is

$$v_2(t) = 120\cos(1000t + 45^\circ) \text{ V}$$

(c). $\mathbf{I}_1 = 240e^{-j26.6^\circ}$ mA, $\omega = 2\pi \times 10^4$ rad/s

The corresponding waveform is

$$i_1(t) = 240\cos(20000\pi t - 26.6^\circ) \text{ mA}$$

(d). $\mathbf{I}_2 = 150e^{-j45^\circ}$ mA, $\omega = 2\pi \times 10^4$ rad/s

The corresponding waveform is

$$i_2(t) = 150\cos(20000\pi t - 45^\circ) \text{ mA}$$

Problem 8–5. Use the phasors in Problem 8–4 and the additive property to find the sinusoidal waveforms $v_3(t) = v_1(t) - v_2(t)$ and $i_3(t) = 2i_1(t) + 3i_2(t)$.

We have the following results:

$$\mathbf{V}_1 = 100\angle{-60^\circ} = 50 - j86.6$$

$$\mathbf{V}_2 = 120\angle 45^\circ = 84.85 + j84.85$$

$$\mathbf{V}_3 = 50 - j86.6 - (84.85 + j84.85) = -34.85 - j171.46 = 174.96\angle{-101.5^\circ} \text{ V}$$

$$v_3(t) = 174.96\cos(1000t - 101.5^\circ) \text{ V}$$

$$\mathbf{I}_1 = 240\angle{-26.6^\circ} = 214.6 - j107.5$$

$$\mathbf{I}_2 = 150\angle{-45^\circ} = 106.1 - j106.1$$

$$\mathbf{I}_3 = 2(214.6 - j107.5) + 3(106.1 - j106.1) = 747.4 - j533.1 = 918\angle{-35.5^\circ} \text{ mA}$$

$$i_3(t) = 918\cos(20000\pi t - 35.5^\circ) \text{ mA}$$

Problem 8–6. The phasor representation of a sinusoid with $\omega = 500$ rad/s is $\mathbf{V} = 15 - j10$ V. Use the phasor derivative property to find the time derivative of the sinusoid.

Compute the derivative in the phasor domain and determine the waveform.

$$\mathbf{V} = 15 - j10 = 18.03\angle{-33.69^\circ} \text{ V}$$

$$j\omega\mathbf{V} = (j500)(15 - j10) = 5000 + j7500 = 9014\angle 56.31^\circ \text{ V/s}$$

$$\frac{dv(t)}{dt} = 9014\cos(500t + 56.31^\circ) \text{ V/s}$$

Problem 8–7. Convert the following phasors into sinusoids:

(a). $\mathbf{V}_1 = 4 + j5$ V, $\omega = 10$ Mrad/s

We have the following results:

$$\mathbf{V}_1 = 4 + j5 = 6.40\angle 51.34^\circ \text{ V}$$

$$v_1(t) = 6.40\cos(10^7 t + 51.34^\circ) \text{ V}$$

(b). $\mathbf{V}_2 = 6(8 - j8)$ V, $\omega = 200$ krad/s

We have the following results:

$$\mathbf{V}_2 = 48 - j48 = 67.88\angle{-45^\circ} \text{ V}$$

$$v_2(t) = 67.88\cos(200000t - 45^\circ) \text{ V}$$

(c). $\mathbf{I}_1 = 12 + j5 + \dfrac{5}{j}$ A, $\omega = 30$ rad/s

We have the following results:

$$\mathbf{I}_1 = 12 + j5 - j5 = 12 = 12\angle 0^\circ \text{ A}$$

$$i_1(t) = 12\cos(30t) \text{ A}$$

(d). $\mathbf{I}_2 = \dfrac{330 + j810}{220 - j560}$ A, $\omega = 60$ rad/s

We have the following results:

$$\mathbf{I}_2 = \frac{330 + j810}{220 - j560} = -1.0525 + j1.0028 = 1.4537\angle 136.39^\circ \text{ A}$$
$$i_2(t) = 1.4537\cos(60t + 136.39^\circ) \text{ A}$$

Problem 8–8. Use phasors to find the sinusoid $v_2(t)$, where

$$v_2(t) = \frac{1}{100}\frac{dv_1(t)}{dt} + 20v_1(t) \quad \text{and} \quad v_1(t) = 10\cos(100t + 90^\circ) \text{ V.}$$

We have the following results:

$$\mathbf{V}_1 = 10\angle 90^\circ = j10 \text{ V}$$
$$j\omega\mathbf{V}_1 = (j100)(j10) = -1000 = 1000\angle 180^\circ \text{ V/s}$$
$$\mathbf{V}_2 = \frac{1}{100}(-1000) + (20)(j10) = -10 + j200 = 200.25\angle 92.86^\circ \text{ V}$$
$$v_2(t) = 200.25\cos(100t + 92.86^\circ) \text{ V}$$

Problem 8–9. Given the sinusoids $v_1(t) = 500\cos(\omega t - 45^\circ)$ V and $v_2(t) = 750\sin(\omega t)$ V, use the additive property of phasors to find $v_3(t)$ such that $v_1 + v_2 + v_3 = 0$.

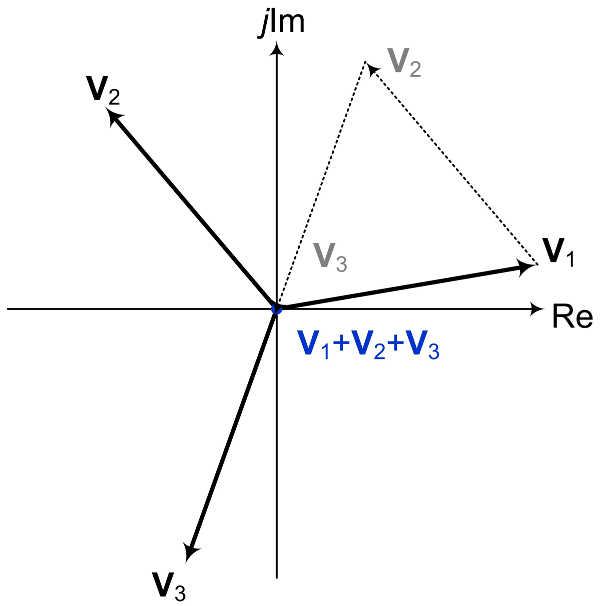
We have the following results:

$$\mathbf{V}_1 = 500\angle{-45^\circ} = 353.6 - j353.6$$
$$\mathbf{V}_2 = 750\angle{-90^\circ} = 0 - j750$$
$$\mathbf{V}_3 = -\mathbf{V}_1 - \mathbf{V}_2 = -353.6 + j353.6 + j750 = -353.6 + j1103.6 = 1158.8\angle 107.8^\circ \text{ V}$$
$$v_3(t) = 1158.8\cos(\omega t + 107.8) \text{ V}$$

Problem 8–10. Graphically add the following three phasors and determine their sum:

$$\mathbf{V}_1 = 9.85 + j1.74 \text{ V}$$
$$\mathbf{V}_2 = 10\angle 130^\circ \text{ V}$$
$$\mathbf{V}_3 = -3.42 - j9.40 \text{ V}$$

We can write the second phasor as $\mathbf{V}_2 = -6.43 + j7.66$. The results are shown in the following figure.



The sum of the three phasors is approximately zero.

Problem 8–11. Given a sinusoid $v_1(t)$ whose phasor is $\mathbf{V}_1 = 3 - j4$ V, use phasor methods to find a voltage $v_2(t)$ that leads $v_1(t)$ by 90° and has an amplitude of 10 V.

If $v_2(t)$ leads $v_1(t)$ by 90°, then its phase angle is 90° greater. We have the following results:

$$\mathbf{V}_1 = 3 - j4 = 5\angle{-53.1^\circ} \text{ V}$$

$$\mathbf{V}_2 = 10\angle(90 - 53.1)^\circ = 10\angle 36.87^\circ \text{ V}$$

$$v_2(t) = 10\cos(\omega t + 36.87^\circ) \text{ V}$$

Problem 8–12. A new parameter Z is defined as $\mathbf{V}/\mathbf{I}$. If $\mathbf{V} = 9.85 + j1.74$ V and $i_1(t) = -4\sin(\omega t)$ A, find Z.

We have the following results:

$$i_1(t) = -4\sin(\omega t) = -4\cos(\omega t - 90^\circ) = 4\cos(\omega t + 90^\circ)$$

$$\mathbf{I} = 4\angle 90^\circ = j4$$

$$\mathbf{V} = 9.85 + j1.74 = 10\angle 10^\circ$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{10\angle 10^\circ}{4\angle 90^\circ} = 2.5\angle{-80^\circ} = 0.435 - j2.4625\ \Omega$$

Problem 8–13. Complex power S is defined as $\mathbf{VI}^*$, where $\mathbf{I}^*$ is the complex conjugate of the current phasor. If $\mathbf{V} = 1200 + j1500$ V and $\mathbf{I} = 500 - j250$ mA, find S.

Compute S using the relationship provided.

$$S = \mathbf{VI}^* = (1200 + j1500)(0.5 - j0.25)^* = (1200 + j1500)(0.5 + j0.25) = 225 + j1050 = 1074\angle 77.9^\circ \text{ VA}$$

Problem 8–14. A design engineer needs to know what value of R, L, or C to use in circuits to achieve a certain impedance.

(a). At what radian frequency will a 0.033-μF capacitor's impedance equal $-j100\ \Omega$?

Calculate the frequency as follows:

$$Z_{\rm C} = \frac{1}{j\omega C}$$

$$\omega = \frac{1}{jZ_{\rm C}C}$$

$$\omega = \frac{1}{j(-j100)(0.033\,\mu)} = 303.03 \text{ krad/s}$$

(b). At what radian frequency will a 47-mH inductor's impedance equal $j100\ \Omega$?

Calculate the frequency as follows:

$$Z_{\rm L} = j\omega L$$

$$\omega = \frac{Z_{\rm L}}{jL}$$

$$\omega = \frac{j100}{j0.047} = 2.1277 \text{ krad/s}$$

(c). At what radian frequency will a 100-Ω resistor's impedance equal 100 Ω?

The resistor's impedance does not change with frequency, so it always equals 100 Ω.

Problem 8–15. Find the equivalent impedance Z in Figure P8–15 when $\omega = 2000$ rad/s. Express the result in both polar and rectangular forms.

Compute the component impedances and combine them.

$$Z_{\rm L} = j\omega L = j(2000)(0.02) = j40\ \Omega$$

$$Z_{\rm C} = \frac{1}{j\omega C} = \frac{1}{j(2000)(100\,\mu)} = -j5$$

$$Z = 10 + j40 + (20 \parallel -j5) = 10 + j40 + 1.177 - j4.71$$

$$Z = 11.177 + j35.29 = 37.02\angle 72.43^\circ\ \Omega$$

Problem 8–16. Find the equivalent impedance Z in Figure P8–16. If $\omega = 10$ krad/s, what two elements (R, L, and/or C) could be used to replace the phasor circuit?

Combine the individual impedances and then determine an equivalent circuit.

$$Z = 25 - j25 + [(20 + j50) \parallel -j100] = 25 - j25 + 68.97 + j72.41$$

$$Z = 93.97 + j47.41 = 105.25\angle 26.77^\circ\ \Omega$$

$$R = 93.97\ \Omega$$

$$X = 47.41\ \Omega = \omega L$$

$$L = \frac{X}{\omega} = \frac{47.41}{10000} = 4.74 \text{ mH}$$

We could use a 93.97-Ω resistor in series with a 4.74-mH inductor to replace the circuit when $\omega = 10$ krad/s.

Problem 8–17. Find the equivalent impedance Z in Figure P8–17 when $\omega = 50$ krad/s. What two elements (R, L, and/or C) could be used to replace the phasor circuit?

Compute the equivalent impedance and then determine an equivalent circuit.

$$Z_{\mathrm{C}} = \frac{1}{j\omega C} = \frac{1}{j(50000)(0.47\,\mu)} = -j42.55\ \Omega$$

$$Z_{\mathrm{L}} = j\omega L = j(50000)(0.002) = j100\ \Omega$$

$$Z = 33 + [-j42.55 \parallel (100 + j100)] = 46.61 - j50.37 = 68.63\angle{-47.2^\circ}\ \Omega$$

$$R = 46.61\ \Omega$$

$$X = -50.37\ \Omega = \frac{-1}{\omega C}$$

$$C = \frac{-1}{\omega X} = \frac{1}{(50000)(50.37)} = 0.397\ \mu\mathrm{F}$$

We could use a 46.61-Ω resistor in series with a 0.397-μF to replace the circuit when $\omega = 50$ krad/s.

Problem 8–18. Find the equivalent impedance Z in Figure P8–18. If $\omega = 100$ krad/s, what two elements (R, L, and/or C) could be used to replace the phasor circuit?

Compute the equivalent impedance and then determine an equivalent circuit.

$$Z = j600 + [-j300 \parallel (600 + j900)] = j600 + 75 - j375 = 75 + j225 = 237.2\angle 71.57^\circ\ \Omega$$

$$R = 75\ \Omega$$

$$X = 225\ \Omega = j\omega L$$

$$L = \frac{X}{\omega} = \frac{225}{100000} = 2.25\ \mathrm{mH}$$

We could use a 75-Ω resistor in series with a 2.25-mH inductor to replace the circuit when $\omega = 100$ krad/s.

Problem 8–19. The circuit in Figure P8–18 is operating in the sinusoidal steady-state with $\omega = 5$ krad/s.

(a). How would the element impedances change if the steady-state frequency were reduced to 500 rad/s?

The impedance for the resistor would not change, because it does not depend on frequency. The impedances for the inductors would decrease by a factor of 10 to $j60\ \Omega$ and $j90\ \Omega$. The impedance for the capacitor would increase by a factor of 10 to $-j3000\ \Omega$.

(b). What is the equivalent impedance Z at this new frequency?

Compute the new equivalent impedance.

$$Z = j60 + [-j3000 \parallel (600 + j90)] = j60 + 611.7 - j33.3 = 611.7 + j26.66 = 612.3\angle 2.5^\circ\ \Omega$$

(c). What two elements (R, L, and/or C) could be used to replace the phasor circuit?

We have the following relationships

$$R = 611.7\ \Omega$$

$$X = 26.66\ \Omega = \omega L$$

$$L = \frac{X}{\omega} = \frac{26.66}{500} = 53.33\ \mathrm{mH}$$

We could use a 611.7-Ω resistor in series with a 53.33-mH inductor to replace the circuit when $\omega =$ 500 rad/s.

Problem 8–20. The circuit in Figure P8–20 is operating in the sinusoidal steady state with $\omega = 100$ krad/s.

(a). Find the equivalent impedance Z.

Find the component impedances and then the equivalent impedance.

$$Z_{\rm C} = \frac{1}{j\omega C} = \frac{1}{j(100000)(5\,\mu)} = -j2\ \Omega$$

$$Z_{\rm L} = j\omega L = j(100000)(0.0025) = j250\ \Omega$$

$$Z = -j2 + [j250 \parallel (20 - j2)] = -j2 + 20.19 - j0.388$$

$$Z = 20.19 - j2.388 = 20.33\angle{-6.74^\circ}\ \Omega$$

(b). What circuit element can be added in series with the equivalent impedance to place the circuit in resonance?

In resonance, there is no reactance in the impedance, so we need to add a reactance of $j2.388\ \Omega$. Add an inductor with $L = X/\omega = 2.388/100000 = 23.88\ \mu$H.

Problem 8–21. The circuit of Figure P8–21 is operating at 60 Hz. Find the equivalent impedance Z.

The radian frequency is $\omega = 2\pi f = 120\pi = 377$ rad/s. The component impedances are

$$Z_{\rm L} = j\omega L = j(377)(0.0082) = j3.09\ \Omega$$

$$Z_{\rm C} = \frac{1}{j\omega C} = \frac{1}{j(377)(220\,\mu)} = -j12.06\ \Omega$$

Determine the impedance of the circuit by finding the ratio of the input voltage to the input current. Let $\mathbf{V}_1$ be the voltage across the resistor and inductor in series and $\mathbf{V}_{\rm X}$ be the voltage across the capacitor. The current through the capacitor is

$$\mathbf{I}_{\rm X} = \frac{\mathbf{V}_{\rm X}}{Z_{\rm C}} = \frac{\mathbf{V}_{\rm X}}{-j12.06}$$

The current through the resistor and inductor is

$$\mathbf{I}_{\rm S} = 3\mathbf{I}_{\rm X} + \mathbf{I}_{\rm X} = 4\mathbf{I}_{\rm X} = (4)\frac{\mathbf{V}_{\rm X}}{-j12.06} = \frac{\mathbf{V}_{\rm X}}{-j3.01}$$

The voltage across the resistor and inductor is

$$\mathbf{V}_1 = (15 + j3.09)(\mathbf{I}_{\rm S}) = (15 + j3.09)\frac{\mathbf{V}_{\rm X}}{-j3.01}$$

The source voltage is

$$\mathbf{V}_{\rm S} = \mathbf{V}_1 + \mathbf{V}_{\rm X} = (15 + j3.09)\frac{\mathbf{V}_{\rm X}}{-j3.01} + \mathbf{V}_{\rm X} = \frac{15 + j3.09 - j3.01}{-j3.01}\mathbf{V}_{\rm X}$$

Compute the input impedance

$$Z = \frac{\mathbf{V}_{\rm S}}{\mathbf{I}_{\rm S}} = \frac{\dfrac{15 + j0.08}{-j3.01}\mathbf{V}_{\rm X}}{\dfrac{\mathbf{V}_{\rm X}}{-j3.01}}$$

$$Z = 15 + j0.077 = 15\angle 0.294^\circ\ \Omega$$

Problem 8–22. The equivalent impedance in Figure P8–22 is known to be $Z = 60 + j180\ \Omega$. Find the impedance of the inductor.

Let the inductor impedance be $Z_{\rm L} = jX_{\rm L}$ and let the impedance of the series combination of the capacitor and inductor be $jX = jX_{\rm L} - j200 = j(X_{\rm L} - 200)$. We have the following relationships:

$$Z = 60 + j180 = 600 \parallel jX = \frac{(600)(jX)}{600 + jX}$$

$$(60 + j180)(600 + jX) = (600)(jX)$$

$$36000 + j108000 + (60 + j180)(jX) = (600)(jX)$$

$$36000 + j108000 = (540 - j180)(jX)$$

$$jX = \frac{36000 + j108000}{540 - j180} = j200$$

$$X = 200\ \Omega$$

$$Z_{\rm L} = jX + j200 = j400\ \Omega$$

Problem 8–23. A capacitor C is connected in parallel with a resistor R. Select values of R and C so that the equivalent impedance of the parallel combination is $300 - j400\ \Omega$ at $\omega = 2$ Mrad/s.

We have the following relationships:

$$Z = 300 - j400 = R \parallel \frac{1}{j\omega C} = \frac{\dfrac{R}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{R}{1 + j\omega RC}$$

$$(300 - j400)(1 + j\omega RC) = R$$

$$j300\omega RC + 300 + 400\omega RC - j400 = R$$

$$300 - j400 = R - 400\omega RC - j300\omega RC$$

$$300 = R - 400\omega RC$$

$$400 = 300\omega RC$$

$$\frac{1600}{3} = 400\omega RC$$

$$300 = R - \frac{1600}{3}$$

$$R = \frac{2500}{3}\ \Omega$$

$$C = \frac{1600}{(3)(400\omega R)} = 800\ \text{pF}$$

Problem 8–24. The circuit in Figure P8–24 is excited by a 1 krad/s sinusoidal source. As the circuit's designer, select a capacitor C such that the impedance Z looking into the circuit is all real.

We have the following relationships:

$$Z_{\rm L} = j\omega L = j(1000)(0.01) = j10\ \Omega$$

$$Z_{\rm C} = \frac{1}{j\omega C} = -jX$$

$$Z = (50 + j10) \parallel Z_{\rm C} = \frac{(50 + j10)(-jX)}{50 + j10 - jX}$$

$$\begin{aligned}
50Z + j(10 - X)Z &= 10X - j50X \\
50Z &= 10X \\
(10 - X)Z &= -50X \\
X &= 5Z \\
(10 - 5Z)Z &= -50(5Z) \\
10Z - 5Z^2 &= -250Z \\
2 - Z &= -50 \\
Z &= 52\ \Omega \\
X &= 260\ \Omega \\
\frac{1}{\omega C} &= 260 \\
C &= \frac{1}{260\omega} = 3.846\ \mu\text{F}
\end{aligned}$$

Problem 8–25. An 800-Ω resistor is connected in parallel with a 0.001-μF capacitor. The impedance of the parallel combination is $400 - j400\ \Omega$. Find the frequency.

We have the following relationships:

$$Z = R \parallel \frac{1}{j\omega C} = \frac{\dfrac{R}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{R}{1 + j\omega RC}$$

$$Z = 400 - j400 = 400\sqrt{2}\angle{-45^\circ}\ \Omega$$

Examine the expression for the impedance. To get a phase angle of -45°, we must have an angle of 45° in the denominator, which implies $1 = \omega RC$ or $\omega = 1/RC = 1/(800)(0.001\,\mu) = 1.25$ Mrad/s.

Problem 8–26. A voltage $v_{\rm S}(t) = 30\cos(2000t)$ V is applied to the circuit in Figure P8–26.

(a). Convert the circuit into the phasor domain.

The voltage source is $\mathbf{V}_{\rm S} = 30\angle 0^\circ$, the resistor remains 50 Ω, and the inductor impedance is $Z_{\rm L} = j\omega L = j(2000)(0.025) = j50\ \Omega$.

(b). Find the phasor current flowing through the circuit and the phasor voltages across the inductor and the resistor.

Add the impedances in series to get $Z_{\rm EQ} = 50 + j50\ \Omega$. The current and voltages are:

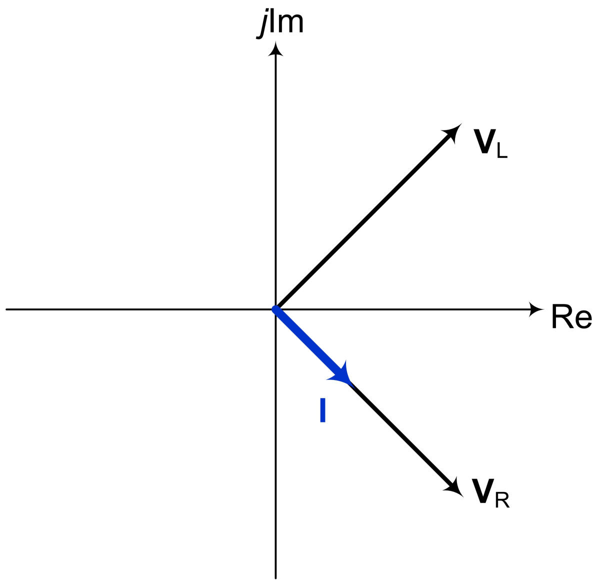
$$\mathbf{I} = \frac{\mathbf{V}_{\rm S}}{Z_{\rm EQ}} = \frac{30\angle 0^\circ}{50 + j50} = 300 - j300 \text{ mA} = 424.3\angle{-45^\circ} \text{ mA}$$

$$\mathbf{V}_{\rm R} = (50)(0.3 - j0.3) = 15 - j15 = 21.21\angle{-45^\circ} \text{ V}$$

$$\mathbf{V}_{\rm L} = (j50)(0.3 - j0.3) = 15 + j15 = 21.21\angle 45^\circ \text{ V}$$

(c). Plot all three phasors from (b) on a phasor diagram. Describe if the current leads or lags the inductor voltage.

The phasor diagram is shown below.



The current lags the inductor voltage by 90°.

Problem 8–27. The circuit in Figure P8–27 is operating in the sinusoidal steady state. Find the phasor current and the two element voltages. Is the phasor voltage across the capacitor leading or lagging the current?

We have the following results:

$$\mathbf{I} = \frac{5\angle 30^\circ}{33000 - j10000} = 99.15 + j105.8\ \mu\text{A} = 145.0\angle 46.9^\circ\ \mu\text{A}$$

$$\mathbf{V}_{\rm R} = Z_{\rm R}\mathbf{I} = (33000)(99.15 + j105.8\,\mu) = 3.27 + j3.49 \text{ V} = 4.785\angle 46.9^\circ \text{ V}$$

$$\mathbf{V}_{\rm C} = Z_{\rm C}\mathbf{I} = (-j10000)(99.15 + j105.8\,\mu) = 1.06 - j0.992 \text{ V} = 1.45\angle{-43.1^\circ} \text{ V}$$

The phasor voltage across the capacitor lags the current by 90°.

Problem 8–28. A voltage $v(t) = 100\cos(3000t)$ V is applied across a series connection of a 330-Ω resistor and 110-mH inductor. Find the steady-state current $i(t)$ through the series connection.

Find the impedances and then the current.

$$Z_{\rm L} = j\omega L = j(3000)(0.11) = j330\ \Omega$$

$$Z_{\rm R} = R = 330\ \Omega$$

$$Z_{\rm EQ} = R + j\omega L = 330 + j330\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}}{Z_{\rm EQ}} = \frac{100\angle 0^\circ}{330 + j330} = 151.5 - j151.5\ \text{mA} = 214.3\angle{-45^\circ}\ \text{mA}$$

$$i(t) = 214.3\cos(3000t - 45^\circ)\ \text{mA}$$

Problem 8–29. The circuit in Figure P8–29 is operating in the sinusoidal steady state with $v_{\rm S}(t) = V_{\rm A}\cos(\omega t)$. Derive a general expression for the phasor response $\mathbf{I}_{\rm L}$ and the voltage $\mathbf{V}_{\rm O}$.

Convert to the phasor domain and perform a source transformation to get a current source in parallel with three elements. Find the equivalent impedance, the output voltage, and the current through the inductor.

$$\mathbf{V}_{\rm S} = V_{\rm A}\angle 0^\circ$$

$$\mathbf{I}_{\rm S} = \frac{\mathbf{V}_{\rm S}}{R} = \frac{V_{\rm A}}{R}\angle 0^\circ$$

$$Z_{\rm EQ} = R \parallel j\omega L \parallel R = \frac{R}{2} \parallel j\omega L = \frac{\frac{R}{2} j\omega L}{\frac{R}{2} + j\omega L} = \frac{j\omega RL}{R + j2\omega L}$$

$$\mathbf{V}_{\rm O} = Z_{\rm EQ}\mathbf{I}_{\rm S} = \frac{j\omega RL}{R + j2\omega L}\left(\frac{V_{\rm A}}{R}\right) = \frac{j\omega L V_{\rm A}}{R + j2\omega L}$$

$$\mathbf{I}_{\rm L} = \frac{\mathbf{V}_{\rm O}}{j\omega L} = \frac{V_{\rm A}}{R + j2\omega L}$$

Problem 8–30. A current source delivering $i(t) = 120\cos(500t)$ mA is connected across a parallel combination of a 10-kΩ resistor and a 0.2-μF capacitor. Find the steady-state current $i_{\rm R}(t)$ through the resistor and the steady-state current $i_{\rm C}(t)$ through the capacitor. Draw a phasor diagram showing $\mathbf{I}$, $\mathbf{I}_{\rm C}$, and $\mathbf{I}_{\rm R}$.

Determine the impedances and then apply current division and KCL to find the currents.

$$Z_{\rm R} = R = 10000\ \Omega$$

$$Z_{\rm C} = \frac{1}{j\omega C} = -j10000\ \Omega$$

$$\mathbf{I} = 0.12\angle 0^\circ\ \text{A}$$

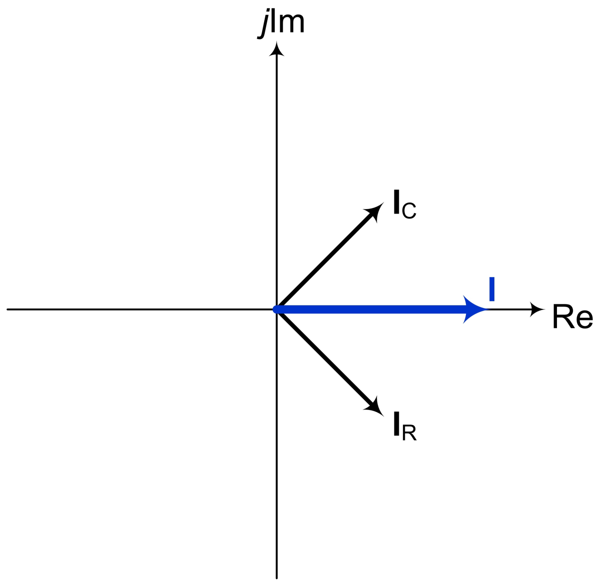
$$\mathbf{I}_{\rm R} = \frac{Z_{\rm C}}{Z_{\rm R} + Z_{\rm C}}\mathbf{I} = \frac{-j10000}{10000 - j10000}(0.12) = 60 - j60\ \text{mA} = 84.85\angle{-45^\circ}\ \text{mA}$$

$$\mathbf{I}_{\rm C} = \mathbf{I} - \mathbf{I}_{\rm R} = 120 - (60 - j60) = 60 + j60\ \text{mA} = 84.85\angle 45^\circ\ \text{mA}$$

$$i_{\rm R}(t) = 84.85\cos(500t - 45^\circ)\ \text{mA}$$

$$i_{\rm C}(t) = 84.85\cos(500t + 45^\circ)\ \text{mA}$$

The phasor diagram is shown below.



Problem 8–31. The circuit in Figure P8–31 is operating in the sinusoidal steady state with $i_S(t) = I_A \cos(\omega t)$. Derive general expressions for the steady-state responses $\mathbf{V}_R$ and $\mathbf{I}_C$.

Apply two-path current division in the phasor domain.

$$\mathbf{I}_S = I_A\angle 0^\circ$$

$$\mathbf{I}_C = \frac{Z_R}{Z_C + Z_R}\mathbf{I}_S = \frac{2R}{\dfrac{1}{j\omega C} + 2R} I_A = \frac{j\omega 2RC I_A}{1 + j\omega 2RC}$$

$$\mathbf{I}_R = \frac{Z_C}{Z_C + Z_R}\mathbf{I}_S = \frac{\dfrac{1}{j\omega C}}{\dfrac{1}{j\omega C} + 2R} I_A = \frac{I_A}{1 + j\omega 2RC}$$

$$\mathbf{V}_R = R\mathbf{I}_R = \frac{RI_A}{1 + j\omega 2RC}$$

Problem 8–32. A practical voltage source of $v_S(t) = 120\cos(2\pi 60t)$ V is in series with a 50-Ω resistor. Convert the source into the phasor domain and then do a source transformation into a current source in parallel with an impedance. Finally, convert the source back into the time domain.

The calculations are shown below.

$$\mathbf{V}_S = 120\angle 0^\circ \text{ V}$$

$$\mathbf{I}_S = \frac{\mathbf{V}_S}{R} = \frac{120\angle 0^\circ}{50} = 2.4\angle 0^\circ \text{ A}$$

$$Z_N = Z_T = R = 50\ \Omega$$

$$i_S(t) = 2.4\cos(2\pi 60t) \text{ A}$$

The parallel resistance is the same as the original series resistance.

Problem 8–33. A current source of $\mathbf{I}_N = 20\angle 33.8^\circ$ mA is in parallel with an impedance of $Z = 100 - j50\ \Omega$. Convert the practical current source into a voltage source in series with an impedance.

Apply a source transformation.

$$\mathbf{V}_\mathrm{T} = Z\mathbf{I}_\mathrm{N} = (100 - j50)(0.02\angle 33.8^\circ) = 2.218 + j0.2816 = 2.236\angle 7.23^\circ \text{ V}$$

$$Z_\mathrm{T} = Z = 100 - j50\ \Omega$$

Problem 8–34. A voltage $v(t) = 12\cos(3030t - 45^\circ)$ V is applied across a parallel connection of a 1.5-kΩ resistor and a 0.22-μF capacitor. Find the steady-state current $i_\mathrm{C}(t)$ through the capacitor and the steady-state current $i_\mathrm{R}(t)$ through the resistor. Draw a phasor diagram showing $\mathbf{V}$, $\mathbf{I}_\mathrm{C}$, and $\mathbf{I}_\mathrm{R}$.

Solve the problem in the phasor domain.

$$\mathbf{V} = 12\angle{-45^\circ} \text{ V}$$

$$Z_\mathrm{C} = \frac{1}{j\omega C} = -j1500\ \Omega$$

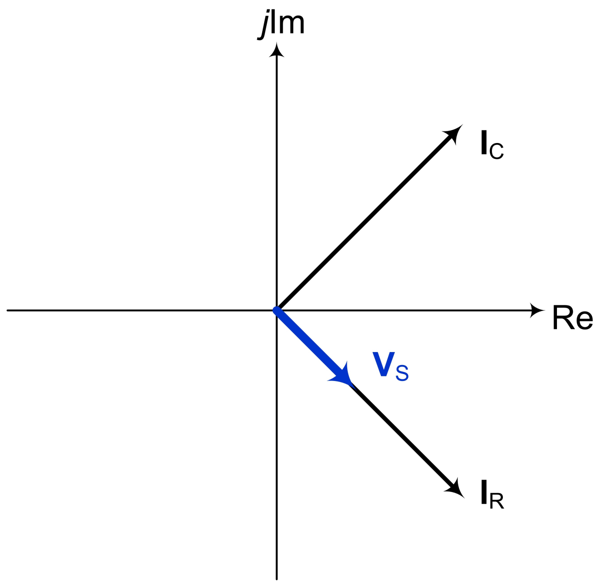
$$\mathbf{I}_\mathrm{C} = \frac{\mathbf{V}}{Z_\mathrm{C}} = \frac{12\angle{-45^\circ}}{-j1500} = 5.657 + j5.657 \text{ mA} = 8\angle 45^\circ \text{ mA}$$

$$\mathbf{I}_\mathrm{R} = \frac{\mathbf{V}}{Z_\mathrm{R}} = \frac{12\angle{-45^\circ}}{1500} = 5.657 - j5.657 \text{ mA} = 8\angle{-45^\circ} \text{ mA}$$

$$i_\mathrm{C}(t) = 8\cos(3030t + 45^\circ) \text{ mA}$$

$$i_\mathrm{R}(t) = 8\cos(3030t - 45^\circ) \text{ mA}$$

The phasor diagram is shown below.



Problem 8–35. The circuit in Figure P8–35 is operating in the sinusoidal steady state. Find the steady-state response $v_\mathrm{X}(t)$.

Find the component impedances and then the equivalent impedance to the right of the resistor in series with the source. Apply voltage division twice to find the output voltage.

$$Z_{\text{C}} = \frac{1}{j\omega C} = -j500\ \Omega$$

$$Z_{\text{L}} = j\omega L = j500\ \Omega$$

$$Z_{\text{EQ}} = -j500 \parallel (500 + j500) = 500 - j500\ \Omega$$

$$\mathbf{V}_1 = \frac{Z_{\text{EQ}}}{500 + Z_{\text{EQ}}}\mathbf{V}_{\text{S}} = \frac{500 - j500}{500 + 500 - j500}(20) = 12 - j4\ \text{V}$$

$$\mathbf{V}_{\text{X}} = \frac{500}{500 + j500}\mathbf{V}_1 = \frac{500}{500 + j500}(12 - j4) = 4 - j8\ \text{V} = 8.944\angle{-63.43^\circ}\ \text{V}$$

$$v_{\text{X}}(t) = 8.944\cos(2000t - 63.43^\circ)\ \text{V}$$

Problem 8–36. The circuit in Figure P8–36 is operating in the sinusoidal steady state. Find the steady-state response $v_{\text{X}}(t)$.

Apply two-path current division and then Ohm's law to find the output voltage.

$$\mathbf{I}_{\text{S}} = 2\ \text{A}$$

$$Z_{\text{C}} = \frac{1}{j\omega C} = \frac{1}{j(2500)(0.2\,\mu)} = -j2000\ \Omega$$

$$\mathbf{I}_{\text{X}} = \frac{1000}{1000 + 2000 - j2000}(2) = 0.4615 + j0.3077\ \text{A}$$

$$\mathbf{V}_{\text{X}} = 2000\mathbf{I}_{\text{X}} = 923.08 + j615.4 = 1109\angle 33.7^\circ\ \text{V}$$

$$v_{\text{X}}(t) = 1.109\cos(2500t + 33.7^\circ)\ \text{kV}$$

Problem 8–37. Use the unit-output method to find $\mathbf{V}_{\text{X}}$ and $\mathbf{I}_{\text{X}}$ in the circuit of Figure P8–37.

Let $\mathbf{I}_{\text{X}} = 1\angle 0^\circ$ A. Use the unit-output method to solve for $\mathbf{I}_{\text{X}}$ and then use KCL and Ohm's law to find $\mathbf{V}_{\text{X}}$.

$$\mathbf{V}_1 = 400\mathbf{I}_{\text{X}} = 400$$

$$\mathbf{V}_2 = -j200\mathbf{I}_{\text{X}} = -j200$$

$$\mathbf{V}_{\text{X}} = \mathbf{V}_1 + \mathbf{V}_2 = 400 - j200$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_{\text{X}}}{200} = 2 - j$$

$$\mathbf{I}_3 = \mathbf{I}_2 + \mathbf{I}_{\text{X}} = 3 - j$$

$$K = \frac{1}{3 - j} = 0.3 + j0.1$$

$$\mathbf{I}_{\text{S}} = 200\angle{-45^\circ}\ \text{mA}$$

$$\mathbf{I}_{\text{X}} = K\mathbf{I}_{\text{S}} = (0.3 + j0.1)(200\angle{-45^\circ}) = 56.6 - j28.3\ \text{mA} = 63.2\angle{-26.6^\circ}\ \text{mA}$$

$$\mathbf{I}_2 = \mathbf{I}_{\text{S}} - \mathbf{I}_{\text{X}} = 200\angle{-45^\circ} - (56.6 - j28.3) = 84.85 - j113.1\ \text{mA}$$

$$\mathbf{V}_{\text{X}} = 200\mathbf{I}_2 = 200(84.85 - j113.1\ \text{mA}) = 16.97 - j22.63 = 28.28\angle{-53.13^\circ}\ \text{V}$$

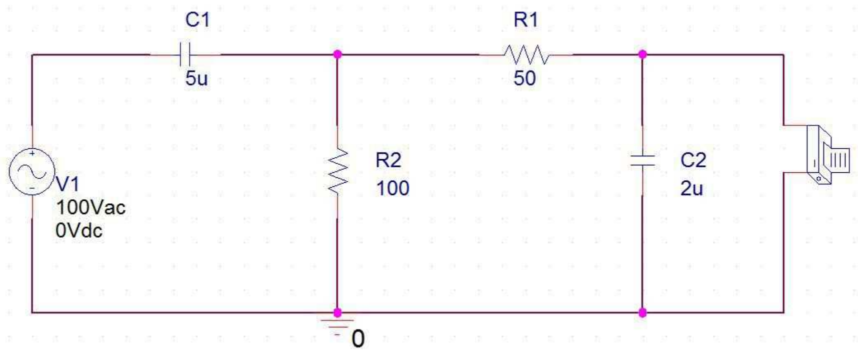
Problem 8–38. The circuit in Figure P8–38 is driven by a 10-krad/s source and is operating in the sinusoidal steady state. Use OrCAD to find the steady-state phasor response $\mathbf{V}_X$.

Compute the capacitor values.

$$C_1 = \frac{1}{j\omega Z_{C1}} = \frac{1}{j(10000)(-j20)} = 5\ \mu\text{F}$$

$$C_2 = \frac{1}{j\omega Z_{C2}} = \frac{1}{j(10000)(-j50)} = 2\ \mu\text{F}$$

The OrCAD simulation is shown below.



The corresponding output is shown below.

```
  FREQ        VM(N00168,0)VP(N00168,0)VR(N00168,0)VI(N00168,0)
   1.592E+03   5.590E+01  -2.657E+01   5.000E+01  -2.500E+01
```

The result is $\mathbf{V}_X = 55.9\angle{-26.57^\circ} = 50 - j25$ V.

Problem 8–39. The circuit in Figure P8–39 is operating in the sinusoidal steady state. Use superposition to find the phasor response $\mathbf{I}_X$.

Use current division to find the contribution from the current source. Find an equivalent impedance to find the contribution from the voltage source. The current from the voltage source will have a negative sign.

$$\mathbf{I}_{X1} = \frac{\frac{1}{50}}{\frac{1}{25} + \frac{1}{-j50} + \frac{1}{50}}(1\angle 45^\circ) = 0.2828 + j0.1414\ \text{A}$$

$$Z_{EQ} = (25 \parallel -j50) + 50 = 70 - j10\ \Omega$$

$$\mathbf{I}_{X2} = -\frac{100\angle{-45^\circ}}{Z_{EQ}} = -\frac{100\angle{-45^\circ}}{70 - j10} = -1.131 + j0.8485\ \text{A}$$

$$\mathbf{I}_X = \mathbf{I}_{X1} + \mathbf{I}_{X2} = -0.8485 + j0.9899 = 1.304\angle 130.6^\circ\ \text{A}$$

Problem 8–40. The circuit in Figure P8–40 is operating in the sinusoidal steady state. Use superposition to find the response $v_X(t)$. *Note*: The sources do not have the same frequency.

Since the sources have different frequencies, the impedances will vary and we cannot directly combine the results using superposition. With the top source active, we have the following results:

$$Z_{C1} = \frac{1}{j\omega C} = \frac{1}{j(1500)(2\,\mu)} = -j333.3\ \Omega$$

$$\mathbf{V}_{X1} = \frac{-j333.3}{500 - j333.3}(5) = 2.774\angle{-56.3°}\ \text{V}$$

$$v_{X1}(t) = 2.774\cos(1500t - 56.3°)\ \text{V}$$

With the bottom source active, we have the following results:

$$Z_{C2} = \frac{1}{j\omega C} = \frac{1}{j(500)(2\,\mu)} = -j1000\ \Omega$$

$$\mathbf{V}_{X1} = \frac{-j1000}{500 - j1000}(10) = 8.944\angle{-26.6°}\ \text{V}$$

$$v_{X1}(t) = 8.944\cos(500t - 26.6°)\ \text{V}$$

The complete response is

$$v_X(t) = 2.774\cos(1500t - 56.3°) + 8.944\cos(500t - 26.6°)\ \text{V}$$

Problem 8–41. An RC series circuit is excited by a sinusoidal source $v(t) = V_A\cos(\omega t + \phi)$ V. Determine the effects on the magnitudes of the current, voltages, and impedances caused by changes in the source parameters. Complete the following table.

We have the following relationships:

$$Z = R + \frac{1}{j\omega C} = \frac{j\omega RC + 1}{j\omega C}$$

$$\mathbf{I} = \frac{\mathbf{V}_S}{Z} = \frac{(j\omega C)V_A\angle\phi}{j\omega RC + 1}$$

$$\mathbf{V}_R = R\mathbf{I} = \frac{(j\omega RC)V_A\angle\phi}{j\omega RC + 1}$$

$$\mathbf{V}_C = Z_C\mathbf{I} = \frac{V_A\angle\phi}{j\omega RC + 1}$$

Source	$\lvert\mathbf{V}_R\rvert$	$\lvert\mathbf{V}_C\rvert$	$\lvert\mathbf{I}\rvert$	$\lvert Z_R\rvert$	$\lvert Z_C\rvert$
Increase/decrease V_A	Inc/dec $\lvert\mathbf{V}_R\rvert$	Inc/dec $\lvert\mathbf{V}_C\rvert$	Inc/dec $\lvert\mathbf{I}\rvert$	none	none
Increase/decrease ω	Inc/dec $\lvert\mathbf{V}_R\rvert$	Dec/inc $\lvert\mathbf{V}_C\rvert$	Inc/dec $\lvert\mathbf{I}\rvert$	none	Dec/inc $\lvert Z_C\rvert$
Increase/decrease ϕ	none	none	none	none	none

Problem 8–42. The circuit in Figure P8–42 is operating in the sinusoidal steady state. Use superposition to find the response $v_X(t)$.

Use current division and voltage division to determine the responses.

$$Z_C = \frac{1}{j(2000)(2\,\mu)} = -j250\ \Omega$$

$$\mathbf{I}_{X1} = \frac{50}{50+200-j250}(200\angle{-30^\circ}) = 27.3 + j7.32\ \text{mA}$$

$$\mathbf{V}_{X1} = 200\mathbf{I}_{X1} = 5.464 + j1.464\ \text{V}$$

$$\mathbf{V}_{X2} = -\frac{200}{200+50-j250}(10) = -4 - j4\ \text{V}$$

$$\mathbf{V}_X = \mathbf{V}_{X1} + \mathbf{V}_{X2} = 1.464 - j2.536 = 2.928\angle{-60^\circ}\ \text{V}$$

$$v_X(t) = 2.928\cos(2000t - 60^\circ)\ \text{V}$$

Problem 8–43. The circuit in Figure P8–43 is operating in the sinusoidal steady state. Use superposition to find the response $v_X(t)$. *Note*: The sources do not have the same frequency.

With the current source active, find the equivalent impedance.

$$Z_{C1} = \frac{1}{j(1000)(2\,\mu)} = -j500\ \Omega$$

$$Z_{EQ} = 500 \parallel -j500 \parallel 2000 = 243.9 - j195.1\ \Omega$$

$$\mathbf{V}_{X1} = Z_{EQ}\mathbf{I} = (243.9 - j195.1)(0.02\angle{-30^\circ}) = 2.273 - j5.819 = 6.247\angle{-68.66^\circ}\ \text{V}$$

$$v_{X1}(t) = 6.247\cos(1000t - 68.66^\circ)\ \text{V}$$

With the voltage source active, use voltage division.

$$Z_{C2} = \frac{1}{j(2000)(2\,\mu)} = -j250\ \Omega$$

$$Z_{EQ} = 500 \parallel -j250 = 100 - j200\ \Omega$$

$$\mathbf{V}_{X2} = -\frac{2000}{2000+100-j200}(100) = -94.382 - j8.989 = 94.81\angle{-174.56^\circ}\ \text{V}$$

$$v_{X2}(t) = 94.81\cos(2000t - 174.56^\circ)\ \text{V}$$

The final results is :

$$v_X(t) = v_{X1}(t) + v_{X2}(t) = 6.247\cos(1000t - 68.66^\circ) + 94.81\cos(2000t - 174.56^\circ)\ \text{V}$$

Problem 8–44. The circuit in Figure P8–44 is operating in the sinusoidal steady state.

(a). What impedance, if any, should be connected across $\mathbf{V}_X$ to cancel the reactance in the circuit?

Find the equivalent impedance of the circuit.

$$Z = (20 - j10) \parallel (20 + j40) = \frac{(20-j10)(20+j40)}{20 - j10 + 20 + j40} = 20 + j0 = 20\ \Omega$$

There is no reactance in the circuit, so no capacitors needs to be added.

(b). Is the bridge balanced, that is, $\mathbf{V}_X = 0$?

Apply voltage division to each branch and subtract the results to find $\mathbf{V}_{\text{X}}$.

$$\mathbf{V}_{\text{A}} = \frac{-j10}{20-j10}(100\angle{-30^\circ}) = -2.679 - j44.64 \text{ V}$$

$$\mathbf{V}_{\text{B}} = \frac{20}{20+j40}(100\angle{-30^\circ}) = -2.679 - j44.64 \text{ V}$$

$$\mathbf{V}_{\text{X}} = \mathbf{V}_{\text{A}} - \mathbf{V}_{\text{B}} = 0 \text{ V}$$

The bridge is balanced.

Problem 8–45. The circuit in Figure P8–45 is operating in the sinusoidal steady state. Use the unit output method to find the phasor responses $\mathbf{V}_{\text{X}}$ and $\mathbf{I}_{\text{X}}$.

Let the output voltage be $\mathbf{V}_{\text{X}} = 1$ V and compute the gain.

$$\mathbf{V}_{\text{X}} = 1 \text{ V}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{\text{X}}}{-j100000} = \frac{1}{-j100000} = j10\ \mu\text{A}$$

$$\mathbf{V}_1 = 100000\mathbf{I}_1 = j \text{ V}$$

$$\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{V}_{\text{X}} = 1 + j \text{ V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{-j20000} = -50 + j50\ \mu\text{A}$$

$$\mathbf{I}_3 = \mathbf{I}_1 + \mathbf{I}_2 = -50 + j60\ \mu\text{A}$$

$$\mathbf{V}_3 = 10000\mathbf{I}_3 = -0.5 + j0.6 \text{ V}$$

$$\mathbf{V}_{\text{S}} = \mathbf{V}_3 + \mathbf{V}_2 = 0.5 + j1.6 \text{ V}$$

$$K = \frac{1}{\mathbf{V}_{\text{S}}} = \frac{1}{0.5+j1.6} = 0.1779 - j0.5694$$

$$\mathbf{V}_{\text{X}} = K\mathbf{V}_{\text{S}} = (0.1779 - j0.5694)(100) = 17.79 - j56.94 = 59.66\angle{-72.65^\circ} \text{ V}$$

$$\mathbf{I}_1 = 569.4 + j177.9\ \mu\text{A}$$

$$\mathbf{V}_1 = 56.94 + j17.79 \text{ V}$$

$$\mathbf{V}_2 = 74.73 - j39.15 \text{ V}$$

$$\mathbf{I}_{\text{X}} = 1.957 + j3.737 = 4.218\angle 62.35^\circ \text{ mA}$$

Problem 8–46. Find the Thévenin equivalent of the source circuit to the left of the interface in Figure P8–46. Then use the equivalent circuit to find the steady-state voltage $v(t)$ and current $i(t)$ delivered to the load. Validate you answer using OrCAD.

Determine the component impedances and then find the open-circuit voltage and the lookback impedance.

$$Z_{\mathrm{L}} = j(10^6)(0.001) = j1000\ \Omega$$

$$Z_{\mathrm{C}} = \frac{1}{j(10^6)(250\,\mathrm{p})} = -j4000\ \Omega$$

$$\mathbf{V}_{\mathrm{S}} = 30\angle 0^\circ\ \mathrm{V}$$

$$\mathbf{V}_{\mathrm{OC}} = \mathbf{V}_{\mathrm{T}} = \frac{-j4000}{j1000 - j4000}(30) = 40\ \mathrm{V}$$

$$Z_{\mathrm{T}} = (j1000 \parallel -j4000) + 1000 = 1000 + j1333\ \Omega$$

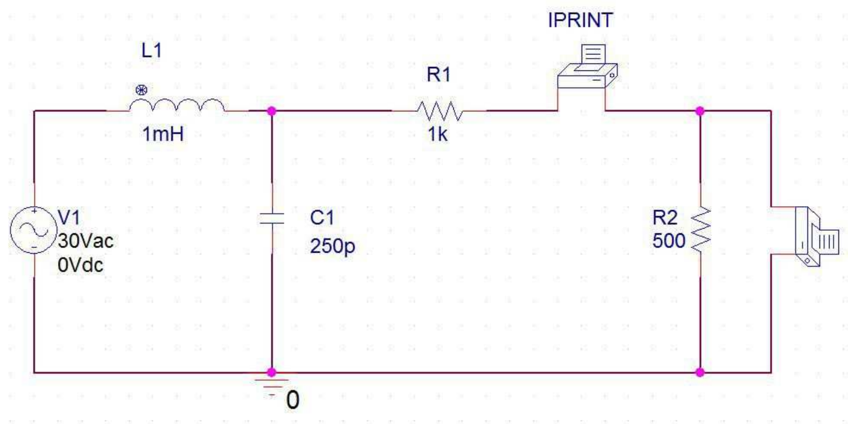
$$\mathbf{V} = \frac{500}{500 + Z_{\mathrm{T}}}(\mathbf{V}_{\mathrm{T}}) = \frac{500}{500 + 1000 + j1333}(40) = 7.448 - j6.621 = 9.965\angle -41.63^\circ\ \mathrm{V}$$

$$v(t) = 9.965\cos(10^6 t - 41.63^\circ)\ \mathrm{V}$$

$$\mathbf{I} = \frac{\mathbf{V}_{\mathrm{T}}}{500 + Z_{\mathrm{T}}} = \frac{40}{1500 + j1333} = 14.897 - j13.241 = 19.931\angle -41.63^\circ\ \mathrm{mA}$$

$$i(t) = 19.93\cos(10^6 t - 41.63^\circ)\ \mathrm{mA}$$

The following OrCAD simulation and results verify the answer.




```
  FREQ        VM(N00210,0)VP(N00210,0)VR(N00210,0)VI(N00210,0)
   1.592E+05   9.965E+00  -4.163E+01   7.448E+00  -6.621E+00
******************************************************************************
  FREQ        IM(V_IL)    IP(V_IL)    IR(V_IL)    II(V_IL)
   1.592E+05   1.993E-02  -4.163E+01   1.490E-02  -1.324E-02
```

Problem 8–47. Find the phasor Thévenin equivalent of the source circuit to the left of the interface in Figure P8–47. Then use the equivalent circuit to find the phasor voltage **V** and current **I** delivered to the load.

Determine the component impedances. Use node-voltage analysis to find the open-circuit voltage and then find the lookback impedance.

$$Z_C = \frac{1}{j(25000)(1\,\mu)} = -j40\ \Omega$$

$$Z_L = j(25000)(0.01) = j250\ \Omega$$

$$\mathbf{V}_S = 100\angle{-90^\circ}\ \text{V}$$

$$\mathbf{I}_S = 5\angle 0^\circ\ \text{A}$$

$$\frac{\mathbf{V}_T - 100\angle{-90^\circ}}{50} + \frac{\mathbf{V}_T}{-j40} - 5 = 0$$

$$\mathbf{V}_T\left(\frac{1}{50} + \frac{1}{-j40}\right) = 5 + 2\angle{-90^\circ} = 5 - j2$$

$$\mathbf{V}_T = 48.78 - j160.98 = 168.20\angle{-73.14^\circ}\ \text{V}$$

$$Z_T = (50 \parallel -j40) + 100 = 119.51 - j24.39\ \Omega$$

$$\mathbf{V} = \frac{100 + j250}{100 + j250 + 119.51 - j24.39}(168.20\angle{-73.14^\circ} = 91.08 - j111.38 = 143.88\angle{-50.73^\circ}\ \text{V}$$

$$\mathbf{I} = \frac{168.20\angle{-73.14^\circ}}{100 + j250 + 119.51 - j24.39} = -258.46 - j467.69 = 534.36\angle{-118.93^\circ}\ \text{mA}$$

Problem 8–48. The circuit in Figure P8–48 is operating in the sinusoidal steady state. When $Z_L = 0$, the phasor current at the interface is $\mathbf{I} = 4.8 - j3.6$ mA. When $Z_L = -j20$ kΩ, the phasor interface current is $\mathbf{I} = 10 + j0$ mA. Find the Thévenin equivalent of the source circuit.

With $Z_L = 0$, the phasor current is the short-circuit current, which is also the Norton current, so we have $\mathbf{I}_N = 4.8 - j3.6$ mA. With $Z_L = -j20$ kΩ and a current $\mathbf{I} = 10$ mA, the interface voltage is $\mathbf{V} = Z_L\mathbf{I} = (-j20000)(0.01) = -j200$ V. Calculate the Norton impedance.

$$Z_N = \frac{\mathbf{V}}{\mathbf{I}_N - \mathbf{I}} = \frac{-j200}{0.0048 - j0.0036 - 0.01} = 18 + j26\ \text{k}\Omega$$

Compute the Thévenin voltage.

$$\mathbf{V}_T = Z_N\mathbf{I}_N = (18000 + j26000)(0.0048 - j0.0036) = 180 + j60 = 189.74\angle 18.43^\circ\ \text{V}$$

Problem 8–49. Design a linear circuit that will deliver an output phasor $\mathbf{V}_O = 60\angle{-45^\circ}$ V when an input phasor $\mathbf{V}_S = 240\angle 0^\circ$ V is applied in Figure P8–49.

Use a voltage divider as the interface, with Z_1 in series with the source and Z_2 in parallel with the output.

$$\mathbf{V}_O = \frac{Z_2}{Z_1 + Z_2}\mathbf{V}_S$$

$$\frac{\mathbf{V}_O}{\mathbf{V}_S} = \frac{60\angle{-45^\circ}}{240\angle 0^\circ} = \frac{Z_2}{Z_1 + Z_2}$$

$$Z_2 = 60\angle{-45^\circ} = 42.43 - j42.43\ \Omega$$

$$Z_1 + Z_2 = 240$$

$$Z_1 = 240 - Z_2 = 240 - 60\angle{-45^\circ} = 197.57 + j42.43\ \Omega$$

Many other solutions are possible.

Problem 8–50. A load of $Z_L = 100 + j100\ \Omega$ is to be driven by a phasor source $\mathbf{V}_S = 120\angle 0^\circ$ V. The voltage across the load needs to be $\mathbf{V}_L = 100\angle 0^\circ$ V. Design an interface that will meet these conditions. Validate your answer using OrCAD.

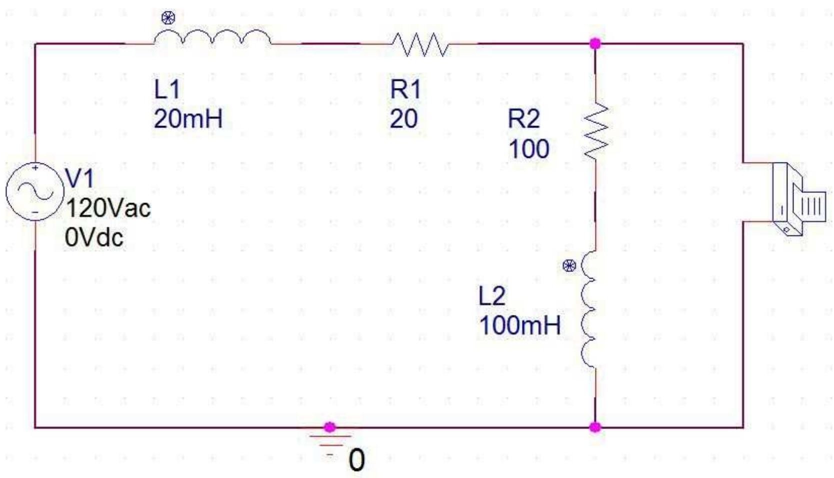
Find the load current. Design a series impedance that will deliver the desired current.

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{Z_L} = \frac{100}{100 + j100} = 0.5 - j0.5 \text{ A}$$

$$Z_{EQ} = \frac{\mathbf{V}_S}{\mathbf{I}_L} = \frac{120}{0.5 - j0.5} = 120 + j120\ \Omega$$

$$Z_S = Z_{EQ} - Z_L = 120 + j120 - (100 + j100) = 20 + j20\ \Omega$$

Use a frequency of $\omega = 1000$ rad/s to create the simulation. The following OrCAD simulation and results verify the answer.




```
FREQ        VM(N00210,0)VP(N00210,0)VR(N00210,0)VI(N00210,0)
 1.592E+02   1.000E+02   4.071E-15   1.000E+02   7.105E-15
```

Problem 8–51. Design an interface circuit so that an input voltage $v_S(t) = 100\cos(2 \times 10^4 t)$ V delivers a steady-state output current of $i_O(t) = 10\cos(2 \times 10^4 t - 30^\circ)$ mA to a 1-kΩ resistive load. Validate your answer using OrCAD.

Design a series interface to create the appropriate current for the load.

$$\mathbf{V}_S = 100\angle 0^\circ \text{ V}$$

$$\mathbf{I}_O = 10\angle{-30^\circ} \text{ mA}$$

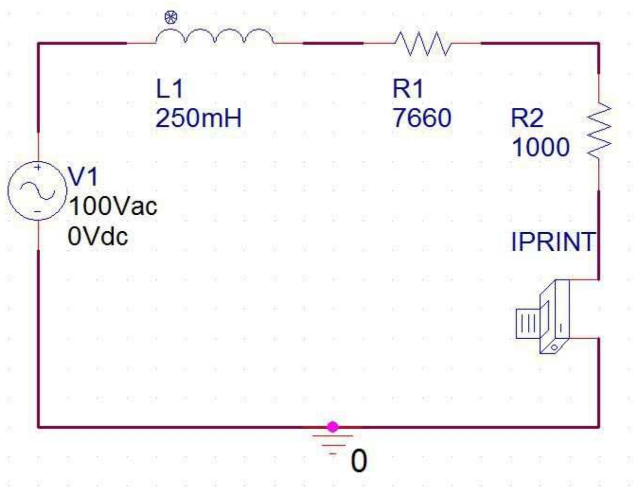
$$Z_{EQ} = \frac{\mathbf{V}_S}{\mathbf{I}_O} = \frac{100}{0.01\angle{-30^\circ}} = 10\angle 30^\circ = 8.66 + j5 \text{ k}\Omega$$

$$Z_S = Z_{EQ} - Z_L = 8.66 + j5 - 1 = 7.66 + j5 \text{ k}\Omega$$

$$R = 7.66 \text{ k}\Omega$$

$$L = \frac{5}{20000} = 250 \text{ mH}$$

The following OrCAD simulation and results verify the answer.




```
FREQ        IM(V_IL)    IP(V_IL)    IR(V_IL)    II(V_IL)
 3.183E+03   1.000E-02  -3.000E+01   8.660E-03  -5.000E-03
```

Problem 8–52. Design an interface circuit so that an input voltage $v_S(t) = 15\cos(10000t)$ V delivers a steady-state output voltage of $v_O(t) = 10\cos(10000t - 45°)$ V.

Use a voltage divider with Z_1 in series with the source and Z_2 in parallel with the output voltage.

$$\mathbf{V}_S = 15\angle 0° \text{ V}$$

$$\mathbf{V}_O = 10\angle -45° \text{ V}$$

$$\mathbf{V}_O = \frac{Z_2}{Z_1 + Z_2}\mathbf{V}_S$$

$$\frac{\mathbf{V}_O}{\mathbf{V}_S} = \frac{10\angle -45°}{15\angle 0°} = \frac{Z_2}{Z_1 + Z_2}$$

$$Z_2 = 10\angle -45° = 7.071 - j7.071 \text{ k}\Omega$$

$$Z_1 + Z_2 = 15000$$

$$Z_1 = 15 - Z_2 = 15 - 7.071 + j7.071 = 7.929 + j7.071 = 10.62\angle 41.73° \text{ k}\Omega$$

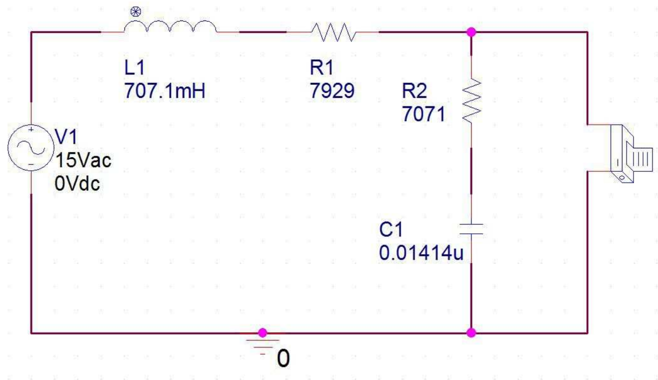
$$R_1 = 7.929 \text{ k}\Omega$$

$$L_1 = \frac{7071}{10000} = 707.1 \text{ mH}$$

$$R_2 = 7.071 \text{ k}\Omega$$

$$C_2 = \frac{1}{(10000)(7071)} = 0.01414\ \mu\text{F}$$

The following OrCAD simulation and results verify the answer.

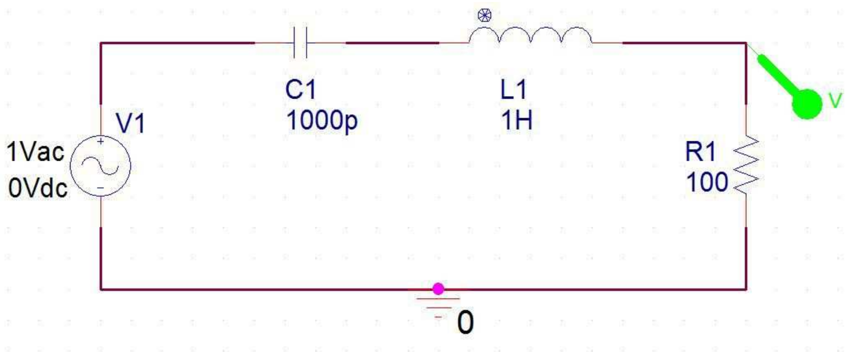



```
FREQ         VM(N00210,0)VP(N00210,0)VR(N00210,0)VI(N00210,0)
 1.592E+03   1.000E+01  -4.500E+01   7.072E+00  -7.072E+00
```

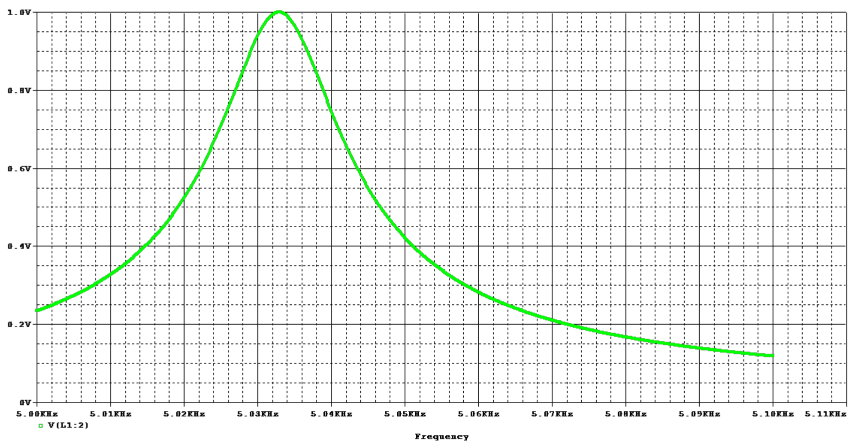
Problem 8–53. Refer to the *RLC* series circuit shown in Figure P8–53.

(a). What is the maximum output voltage and at what frequency does it occur? Consider using OrCAD and doing an ac sweep from 10 Hz to 1 MHz, and then narrow your sweep until you find the frequency at which the peak occurs and the output voltage at that frequency.

Use OrCAD to perform the analysis. The OrCAD circuit is shown below.



The corresponding plot is shown below.



The resonant peak occurs at $f = 5.0329$ kHz and the maximum output voltage has the same magnitude as the input voltage.

(b). Bandwidth is defined as $\mathrm{BW} = f_\mathrm{H} - f_\mathrm{L}$, where f_H is the higher frequency at which the magnitude of the output is exactly 0.707 of the maximum value, and f_L is the lower frequency at which the magnitude of the output is exactly 0.707 of the maximum value. What is the bandwidth of this circuit?

Using the cursor function in OrCAD, we can determine the following values:

$$f_\mathrm{L} = 5040.8 \text{ Hz}$$

$$f_\mathrm{H} = 5025.0 \text{ Hz}$$

$$\mathrm{BW} = f_\mathrm{H} - f_\mathrm{L} = 5040.8 - 5025.0 = 15.8 \text{ Hz}$$

Problem 8–54. The circuit in Figure P8–54 is operating in the sinusoidal steady state with $\omega = 5$ krad/s. Use node-voltage analysis to find the steady-state response $v_\mathrm{X}(t)$.

Find the component impedances and then apply node-voltage analysis.

$$Z_\mathrm{L} = j(5000)(0.5) = j2500\ \Omega$$

$$Z_\mathrm{C} = \frac{1}{j(5000)(0.1\,\mu)} = -j2000\ \Omega$$

$$\frac{\mathbf{V}_\mathrm{X} - 15}{j2500} + \frac{\mathbf{V}_\mathrm{X}}{10000} + \frac{\mathbf{V}_\mathrm{X} - 30\angle{-45^\circ}}{-j2000} = 0$$

$$\mathbf{V}_\mathrm{X}\left(\frac{1}{j2500} + \frac{1}{10000} + \frac{1}{-j2000}\right) = \frac{15}{j2500} + \frac{30\angle{-45^\circ}}{-j2000}$$

$$\mathbf{V}_\mathrm{X} = 76.07 - j30.0 = 81.77\angle{-21.52^\circ} \text{ V}$$

$$v_\mathrm{X}(t) = 81.77\cos(5000t - 21.52^\circ) \text{ V}$$

Problem 8–55. Use node-voltage analysis to find the steady-state phasor response $\mathbf{V}_\mathrm{O}$ in Figure P8–55.

Let $\mathbf{V}_\mathrm{A}$ be the center node. The node-voltage equations are as follows:

$$\frac{\mathbf{V}_\mathrm{A} - 120}{j20} + \frac{\mathbf{V}_\mathrm{A}}{j50} + \frac{\mathbf{V}_\mathrm{A} - \mathbf{V}_\mathrm{O}}{j100} = 0$$

$$\frac{\mathbf{V}_\mathrm{O} - \mathbf{V}_\mathrm{A}}{j100} + \frac{\mathbf{V}_\mathrm{O}}{200} = 0$$

Solve for the node voltages.

$$\mathbf{V}_\mathrm{A} = 83.08 - j4.62 \text{ V}$$

$$\mathbf{V}_\mathrm{O} = 64.62 - j36.92 = 74.42\angle{-29.74^\circ} \text{ V}$$

Problem 8–56. The circuit in Figure P8–56 is operating in the sinusoidal steady state.

(a). Find the node voltage phasors $\mathbf{V}_\mathrm{A}$ and $\mathbf{V}_\mathrm{B}$.

Use node-voltage analysis.

$$\frac{\mathbf{V}_\mathrm{A}}{-j200} + \frac{\mathbf{V}_\mathrm{A}}{100} - 2 + \frac{\mathbf{V}_\mathrm{A} - \mathbf{V}_\mathrm{B}}{400} = 0$$

$$\frac{\mathbf{V}_\mathrm{B}}{j200} + \frac{\mathbf{V}_\mathrm{A}}{-j50} + 2 + \frac{\mathbf{V}_\mathrm{B} - \mathbf{V}_\mathrm{A}}{400} = 0$$

Solve for the node voltages.

$$\mathbf{V}_{\mathrm{A}} = 141.18 - j35.29 = 145.52\angle{-14.04^\circ} \text{ V}$$

$$\mathbf{V}_{\mathrm{B}} = -23.53 + j105.88 = 108.47\angle 102.53^\circ \text{ V}$$

(b). If the circuit is operating with $\omega = 10$ krad/s, use OrCAD to verify your answer in part (a).

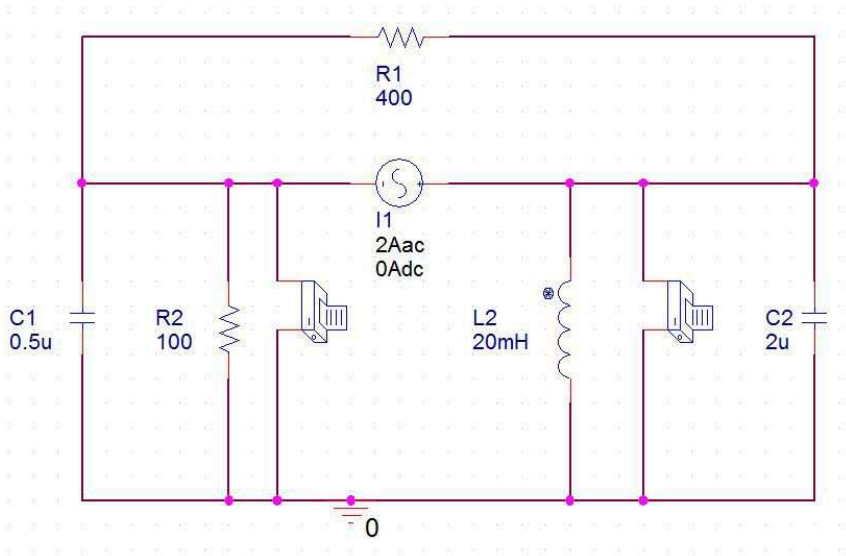
Solve for the component values.

$$C_1 = \frac{1}{(10000)(200)} = 0.5\ \mu\text{F}$$

$$C_2 = \frac{1}{(10000)(50)} = 2\ \mu\text{F}$$

$$L = \frac{200}{10000} = 20 \text{ mH}$$

The OrCAD simulation is shown below.



The simulation results confirm the node-voltage analysis.

```
  FREQ        VM(N00185,0)VP(N00185,0)VR(N00185,0)VI(N00185,0)
   1.592E+03   1.455E+02  -1.404E+01   1.412E+02  -3.529E+01
******************************************************************************
  FREQ        VM(N00210,0)VP(N00210,0)VR(N00210,0)VI(N00210,0)
   1.592E+03   1.085E+02   1.025E+02  -2.353E+01   1.059E+02
```

Problem 8–57. Use mesh-current analysis to find the phasor branch currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit shown in Figure P8–57.

Find the component values and write the mesh-current equations.

$$Z_{\mathrm{L}} = j(10000)(0.01) = j100\ \Omega$$

$$Z_{\mathrm{C}} = \frac{1}{j(10000)(0.04\,\mu)} = -j2500\ \Omega$$

$$-100\angle{-90^\circ} + 1000\mathbf{I}_\mathrm{A} + j100(\mathbf{I}_\mathrm{A} - \mathbf{I}_\mathrm{B}) = 0$$

$$j100(\mathbf{I}_\mathrm{B} - \mathbf{I}_\mathrm{A}) - j2500\mathbf{I}_\mathrm{B} - 100\angle 0^\circ = 0$$

Solve for the mesh currents and then compute the requested currents.

$$\mathbf{I}_\mathrm{A} = -14.43 - j98.50 \text{ mA}$$

$$\mathbf{I}_\mathrm{B} = 0.601 + j45.77 \text{ mA}$$

$$\mathbf{I}_1 = \mathbf{I}_\mathrm{A} = -14.43 - j98.50 = 99.55\angle{-98.33^\circ} \text{ mA}$$

$$\mathbf{I}_2 = \mathbf{I}_\mathrm{A} - \mathbf{I}_\mathrm{B} = -15.03 - j144.27 = 145.05\angle{-95.95^\circ} \text{ mA}$$

$$\mathbf{I}_3 = -\mathbf{I}_\mathrm{B} = -0.601 - j45.77 = 45.77\angle{-90.75^\circ} \text{ mA}$$

Problem 8–58. Use mesh-current analysis to find the phasor branch currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit shown in Figure P8–58.

Find the component values and write the mesh-current equations.

$$Z_\mathrm{L} = j(10000)(0.01) = j100\ \Omega$$

$$Z_\mathrm{C} = \frac{1}{j(10000)(0.04\,\mu)} = -j2500\ \Omega$$

$$-100\angle{-90^\circ} + 1000(\mathbf{I}_\mathrm{A} - \mathbf{I}_\mathrm{C}) + j100(\mathbf{I}_\mathrm{A} - \mathbf{I}_\mathrm{B}) = 0$$

$$j100(\mathbf{I}_\mathrm{B} - \mathbf{I}_\mathrm{A}) - j2500(\mathbf{I}_\mathrm{B} - \mathbf{I}_\mathrm{C}) - 100\angle 0^\circ = 0$$

$$2000\mathbf{I}_\mathrm{C} - j2500(\mathbf{I}_\mathrm{C} - \mathbf{I}_\mathrm{B}) + 1000(\mathbf{I}_\mathrm{C} - \mathbf{I}_\mathrm{A}) = 0$$

Solve for the mesh currents and then compute the requested currents.

$$\mathbf{I}_\mathrm{A} = 35.57 - j148.50 \text{ mA}$$

$$\mathbf{I}_\mathrm{B} = 50.60 - j4.23 \text{ mA}$$

$$\mathbf{I}_\mathrm{C} = 50 - j50 \text{ mA}$$

$$\mathbf{I}_1 = \mathbf{I}_\mathrm{A} - \mathbf{I}_\mathrm{C} = -14.43 - j98.50 = 99.55\angle{-98.33^\circ} \text{ mA}$$

$$\mathbf{I}_2 = \mathbf{I}_\mathrm{A} - \mathbf{I}_\mathrm{B} = -15.03 - j144.27 = 145.05\angle{-95.95^\circ} \text{ mA}$$

$$\mathbf{I}_3 = -\mathbf{I}_\mathrm{C} = -50 + j50 = 70.71\angle 135^\circ \text{ mA}$$

Problem 8–59. Use mesh-current analysis to find the phasor branch currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit shown in Figure P8–59.

Find the component values and write the mesh-current equations.

$$Z_\mathrm{L} = j(10000)(0.01) = j100\ \Omega$$

$$Z_\mathrm{C} = \frac{1}{j(10000)(0.04\,\mu)} = -j2500\ \Omega$$

$$-100\angle{-90^\circ} + 1000(\mathbf{I}_\mathrm{A} - \mathbf{I}_\mathrm{C}) + j100(\mathbf{I}_\mathrm{A} - \mathbf{I}_\mathrm{B}) = 0$$

$$j100(\mathbf{I}_\mathrm{B} - \mathbf{I}_\mathrm{A}) - j2500(\mathbf{I}_\mathrm{B} - \mathbf{I}_\mathrm{C}) + 2000\mathbf{I}_\mathrm{B} = 0$$

$$\mathbf{I}_\mathrm{C} = -2\mathbf{I}_2 = -2(\mathbf{I}_\mathrm{A} - \mathbf{I}_\mathrm{B}) = 2(\mathbf{I}_\mathrm{B} - \mathbf{I}_\mathrm{A})$$

Solve for the mesh currents and then compute the requested currents.

$$\mathbf{I}_{\rm A} = 47.04 - j15.44 \text{ mA}$$

$$\mathbf{I}_{\rm B} = 72.60 + j25.56 \text{ mA}$$

$$\mathbf{I}_{\rm C} = 51.14 + j82 \text{ mA}$$

$$\mathbf{I}_1 = \mathbf{I}_{\rm A} - \mathbf{I}_{\rm C} = -4.1 - j97.44 = 97.53\angle{-92.41^\circ} \text{ mA}$$

$$\mathbf{I}_2 = \mathbf{I}_{\rm A} - \mathbf{I}_{\rm B} = -25.57 - j41 = 48.32\angle{-121.95^\circ} \text{ mA}$$

$$\mathbf{I}_3 = \mathbf{I}_{\rm B} = 72.60 + j25.56 = 76.97\angle{19.39^\circ} \text{ mA}$$

Problem 8–60. Use mesh-current analysis to find the phasor currents $\mathbf{I}_{\rm A}$ and $\mathbf{I}_{\rm B}$ in Figure P8–60.

Write the mesh-current equations and solve.

$$(10 + j30)\mathbf{I}_{\rm A} + (20 + j10)(\mathbf{I}_{\rm A} - \mathbf{I}_{\rm B}) + 120 = 0$$

$$-120 + (20 + j10)(\mathbf{I}_{\rm B} - \mathbf{I}_{\rm A}) + (30 + j20)\mathbf{I}_{\rm B} = 0$$

$$(30 + j40)\mathbf{I}_{\rm A} - (20 + j10)\mathbf{I}_{\rm B} = -120$$

$$-(20 + j10)\mathbf{I}_{\rm A} + (50 + j30)\mathbf{I}_{\rm B} = 120$$

$$\mathbf{I}_{\rm A} = -0.96 + j1.44 = 1.731\angle{123.69^\circ} \text{ A}$$

$$\mathbf{I}_{\rm B} = 1.44 - j0.48 = 1.518\angle{-18.43^\circ} \text{ A}$$

Problem 8–61. The OP AMP circuit in Figure P8–61 is operating in the sinusoidal steady state.

(a). Show that

$$\frac{\mathbf{V}_{\rm O}}{\mathbf{V}_{\rm S}} = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{j\omega + \dfrac{1}{(R_1 + R_2)C}}{j\omega + \dfrac{1}{R_1 C}}\right)$$

The phasor voltage at the negative input terminal is $\mathbf{V}_{\rm S}$. Write a node-voltage equation at the negative input terminal.

$$\frac{\mathbf{V}_{\rm S}}{R_1 + \dfrac{1}{j\omega C}} + \frac{\mathbf{V}_{\rm S} - \mathbf{V}_{\rm O}}{R_2} = 0$$

$$R_2\mathbf{V}_{\rm S} + (\mathbf{V}_{\rm S} - \mathbf{V}_{\rm O})\left(R_1 + \frac{1}{j\omega C}\right) = 0$$

$$(j\omega R_1 C + j\omega R_2 C + 1)\mathbf{V}_{\rm S} = (j\omega R_1 C + 1)\mathbf{V}_{\rm O}$$

$$\frac{\mathbf{V}_\mathrm{O}}{\mathbf{V}_\mathrm{S}} = \frac{j\omega R_1C + j\omega R_2C + 1}{j\omega R_1C + 1}$$

$$\frac{\mathbf{V}_\mathrm{O}}{\mathbf{V}_\mathrm{S}} = \frac{j\omega(R_1 + R_2)C + 1}{j\omega R_1C + 1} = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{j\omega C + \dfrac{1}{R_1 + R_2}}{j\omega C + \dfrac{1}{R_1}}\right)$$

$$\frac{\mathbf{V}_\mathrm{O}}{\mathbf{V}_\mathrm{S}} = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{j\omega + \dfrac{1}{(R_1 + R_2)C}}{j\omega + \dfrac{1}{R_1C}}\right)$$

(b). Find the value of the magnitude of $\mathbf{V}_\mathrm{O}/\mathbf{V}_\mathrm{S}$ at $\omega = 0$ and as $\omega \to \infty$.

At $\omega = 0$, we have

$$\frac{\mathbf{V}_\mathrm{O}}{\mathbf{V}_\mathrm{S}} = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{\dfrac{1}{(R_1 + R_2)C}}{\dfrac{1}{R_1C}}\right) = \frac{\dfrac{1}{C}}{\dfrac{1}{C}} = 1$$

As $\omega \to \infty$, we have

$$\frac{\mathbf{V}_\mathrm{O}}{\mathbf{V}_\mathrm{S}} = \frac{R_1 + R_2}{R_1}$$

Problem 8–62. The circuit in Figure P8–62 is operating in the sinusoidal steady state.

(a). If $v_\mathrm{S}(t) = 2\cos(2128t)$ V, find the output $v_\mathrm{O}(t)$.

Find the component impedance and compute the gain of the OP AM amplifier.

$$Z_\mathrm{C} = \frac{1}{j(2128)(4700\,\mathrm{p})} = -j100000\ \Omega$$

$$\mathbf{V}_\mathrm{O} = -\frac{Z_2}{Z_1}\mathbf{V}_\mathrm{S} = -\frac{100000}{100000 - j100000}(2\angle 0^\circ) = 1.414\angle{-135^\circ}\ \mathrm{V}$$

$$v_\mathrm{O}(t) = 1.414\cos(2128t - 135^\circ)\ \mathrm{V}$$

(b). At what frequency is the magnitude of the output voltage equal to half of the magnitude of the input voltage in the circuit of Figure 8–62? Consider using OrCAD and doing an ac sweep from 1 Hz to 100 kHz, and then use OrCAD's cursor to find the desired frequency.

Compute the frequency directly.

$$\left|\frac{\mathbf{V}_\mathrm{O}}{\mathbf{V}_\mathrm{S}}\right| = \frac{1}{2} = \frac{|R|}{\left|R + \dfrac{1}{j\omega C}\right|} = \left|\frac{j\omega RC}{j\omega RC + 1}\right| = \frac{|\omega RC|}{\sqrt{1 + (\omega RC)^2}}$$

$$\frac{1}{4} = \frac{(\omega RC)^2}{1 + (\omega RC)^2}$$

$$4(\omega RC)^2 = 1 + (\omega RC)^2$$

$$(\omega RC)^2 = \frac{1}{3}$$

$$\omega = \frac{1}{\sqrt{3}RC} = 1.2284\ \mathrm{krad/s}$$

Problem 8–63. Use MATLAB to find the phasor current $\mathbf{I}_\mathrm{O}$ in Figure P8–63.
Use mesh-current analysis. The MATLAB code is shown below.

```
syms IA IB IC
Vs = 200;
Is = 2;
Eqn1 = -Vs+200*(IA-IB) - 50j*(IA-IC);
Eqn2 = (400-300j)*IB + 200j*(IB-IC) + 200*(IB-IA);
Eqn3 = IC + Is;
Soln = solve(Eqn1,Eqn2,Eqn3,IA,IB,IC);
IA = double(Soln.IA);
IC = double(Soln.IC);
IO = IA-IC
IOMag = abs(IO)
IOPhase = 180*angle(IO)/pi
```

The corresponding MATLAB output is shown below.

```
IO = 3.4462e+000 +430.7692e-003i
IOMag = 3.4730e+000
IOPhase = 7.1250e+000
```

The result is $\mathbf{I}_\mathrm{O} = 3.446 + j0.431 = 3.473\angle 7.125^\circ$ A.

Problem 8–64. The circuit in Figure P8–64 is operating with $\omega = 10$ krad/s.

(a). Find the phasor outputs $\mathbf{V}_\mathrm{O}$ and $\mathbf{I}_\mathrm{O}$ in Figure P8–64 when $\mu = 50$ and the phasor input is $\mathbf{I}_\mathrm{S} = 1 + j1$ mA.

Use node-voltage analysis.

$$\mathbf{I}_\mathrm{S} = 0.001 + j0.001 = 0.001414\angle 45^\circ \text{ A}$$

$$\mathbf{V}_\mathrm{X} = \mathbf{V}_\mathrm{A}$$

$$\mathbf{V}_\mathrm{B} = \mu\mathbf{V}_\mathrm{X}$$

$$-(0.001 + j0.001) + \frac{\mathbf{V}_\mathrm{A}}{2000} + \frac{\mathbf{V}_\mathrm{A} - \mu\mathbf{V}_\mathrm{A}}{j20000} = 0$$

$$\mathbf{V}_\mathrm{A} = 0.4718 - j0.3119 \text{ V}$$

$$\mathbf{V}_\mathrm{O} = \mu\mathbf{V}_\mathrm{X} = 50\mathbf{V}_\mathrm{A} = 23.59 - j15.59 = 28.28\angle{-33.47^\circ} \text{ V}$$

$$\mathbf{I}_\mathrm{O} = \frac{\mathbf{V}_\mathrm{O}}{-j50000} = 311.88 + j471.81 = 565.6\angle 56.53^\circ\ \mu\text{A}$$

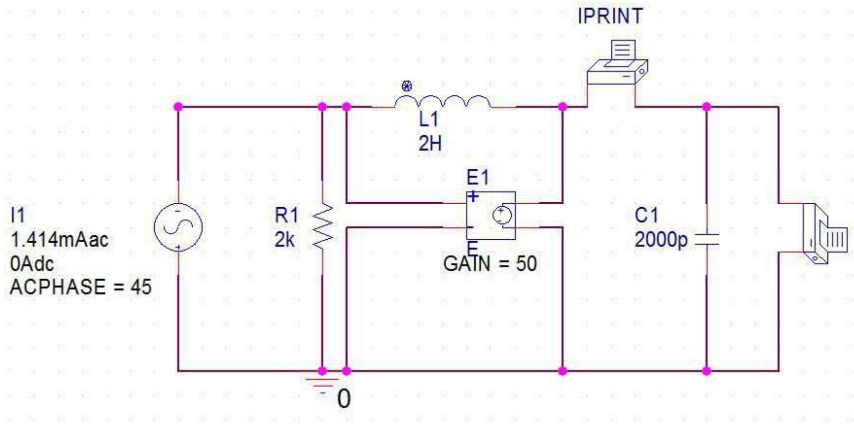
(b). Use OrCAD to verify your results above.

Solve for the component values.

$$C = \frac{1}{(50000)(10000)} = 2000 \text{ pF}$$

$$L = \frac{20000}{10000} = 2 \text{ H}$$

The OrCAD simulation is shown below.



The corresponding results are shown below.

```
  FREQ        VM(N00210,0)VP(N00210,0)VR(N00210,0)VI(N00210,0)
   1.592E+03   2.827E+01  -3.347E+01   2.359E+01  -1.559E+01
******************************************************************************
  FREQ        IM(V_IL)    IP(V_IL)    IR(V_IL)    II(V_IL)
   1.592E+03   5.655E-04   5.653E+01   3.118E-04   4.717E-04
```

The results agree with the calculations in part (a).

Problem 8–65. Find the phasor responses $\mathbf{I}_{\mathrm{IN}}$ and $\mathbf{V}_{\mathrm{O}}$ in Figure P8–65 when $\mathbf{V}_{\mathrm{S}} = 1 + j0$ V.

We have the following relationships:

$$\mathbf{I}_{\mathrm{IN}} = \frac{\mathbf{V}_{\mathrm{S}} + \dfrac{\mathbf{V}_{\mathrm{O}}}{20}}{10 + j10}$$

$$Z_{\mathrm{EQ}} = 50 \parallel -j50 = 25 - j25\ \Omega$$

$$\mathbf{V}_{\mathrm{O}} = -50\mathbf{I}_{\mathrm{IN}}Z_{\mathrm{EQ}} = -50\left(\frac{\mathbf{V}_{\mathrm{S}} + \dfrac{\mathbf{V}_{\mathrm{O}}}{20}}{10 + j10}\right)(25 - j25)$$

$$(200 + j200)\mathbf{V}_{\mathrm{O}} = (20\mathbf{V}_{\mathrm{S}} + \mathbf{V}_{\mathrm{O}})(-1250 + j1250)$$

$$(1450 - j1050)\mathbf{V}_{\mathrm{O}} = (-25000 + j25000)\mathbf{V}_{\mathrm{S}} = -25000 + j25000$$

$$\mathbf{V}_{\mathrm{O}} = -19.5 + j3.12 = 19.75\angle 170.9^\circ\ \mathrm{V}$$

$$\mathbf{I}_{\mathrm{IN}} = 9.048 + j6.552 = 11.17\angle 35.91\ \mathrm{mA}$$

Problem 8–66. For the circuit of Figure P8–66 find the Thévenin equivalent circuit seen at the output.

Use node-voltage analysis to find the open-circuit voltage.

$$\frac{\mathbf{V}_{\mathrm{X}}-1}{2000}+\frac{\mathbf{V}_{\mathrm{X}}-\mathbf{V}_{\mathrm{T}}}{j2000}=0$$

$$\frac{\mathbf{V}_{\mathrm{T}}-\mathbf{V}_{\mathrm{X}}}{j2000}+\frac{\mathbf{V}_{\mathrm{T}}-2\mathbf{V}_{\mathrm{X}}}{1000}=0$$

$$\left(\frac{1}{2000}+\frac{1}{j2000}\right)\mathbf{V}_{\mathrm{X}}-\frac{1}{j2000}\mathbf{V}_{\mathrm{T}}=\frac{1}{2000}$$

$$-\left(\frac{1}{j2000}+\frac{2}{1000}\right)\mathbf{V}_{\mathrm{X}}+\left(\frac{1}{j2000}+\frac{1}{1000}\right)\mathbf{V}_{\mathrm{T}}=0$$

$$\mathbf{V}_{\mathrm{X}}=0.6-j0.8\ \mathrm{V}$$

$$\mathbf{V}_{\mathrm{T}}=1.4-j1.2\ \mathrm{V}$$

Solve for the short-circuit current.

$$\mathbf{I}_1=\frac{1}{2000+j2000}=250-j250\ \mu\mathrm{A}$$

$$\mathbf{V}_{\mathrm{X}}=j2000\mathbf{I}_1=0.5+j0.5\ \mathrm{V}$$

$$\mathbf{I}_2=\frac{2\mathbf{V}_{\mathrm{X}}}{1000}=\frac{1+j1}{1000}=1+j1\ \mathrm{mA}$$

$$\mathbf{I}_{\mathrm{SC}}=\mathbf{I}_1+\mathbf{I}_2=1.25+j0.75\ \mathrm{mA}$$

$$Z_{\mathrm{T}}=\frac{\mathbf{V}_{\mathrm{T}}}{\mathbf{I}_{\mathrm{SC}}}=400-j1200\ \Omega$$

Problem 8–67. The OP AMP circuit in Figure P8–67 is operating in the sinusoidal steady state with $\omega = 1$ krad/s. Find the magnitude of the ratio of the output phasor $\mathbf{V}_{\mathrm{O}}$ to the input phasor $\mathbf{V}_{\mathrm{S}}$. Repeat for $\omega = 10$ krad/s, $\omega = 100$ krad/s, $\omega = 1$ Mrad/s, and $\omega = 10$ Mrad/s. Use MATLAB to plot the log of $|\mathbf{V}_{\mathrm{O}}/\mathbf{V}_{\mathrm{S}}|$ versus the log of the frequency ω. Comment on the plot.

Compute the gain of the OP AMP circuit.

$$K=\frac{\mathbf{V}_{\mathrm{O}}}{\mathbf{V}_{\mathrm{S}}}=-\frac{Z_2}{Z_1}$$

$$Z_1=R$$

$$Z_2=R\parallel\frac{1}{j\omega C}=\frac{\dfrac{R}{j\omega C}}{R+\dfrac{1}{j\omega C}}=\frac{R}{j\omega RC+1}$$

$$K=-\frac{Z_2}{Z_1}=-\frac{1}{j\omega RC+1}$$

$$|K|=\frac{1}{\sqrt{(\omega RC)^2+1}}$$

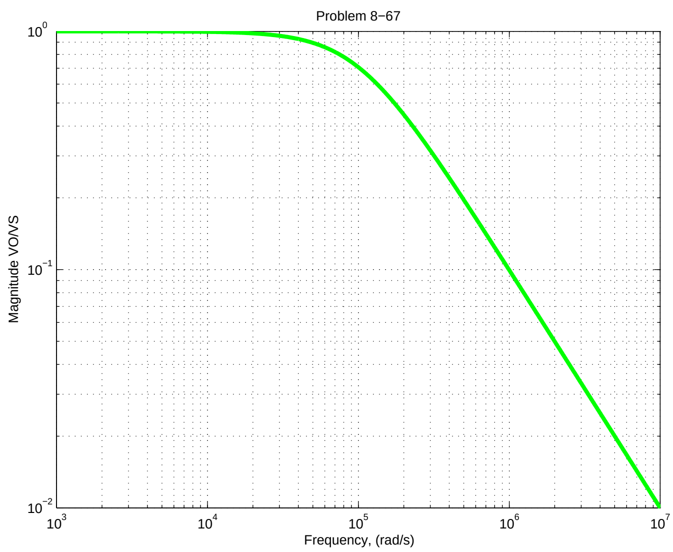
Use MATLAB to compute the gains at the requested frequencies and to plot the results. The appropriate MATLAB code is shown below.

```
R = 10e3;
Z1 = R;
C = 0.001e-6;
w = [1e3 10e3 100e3 1e6 10e6];
KMag = zeros(size(w));
for n = 1:length(w)
    ZC = 1/j/w(n)/C;
    Z2 = R*ZC/(R+ZC);
    KMag(n) = abs(-Z2/Z1);
end
Results = [w' KMag']

% Plot the response
w = logspace(3,7,1000);
ZC = 1/j./w/C;
Z2 = R*ZC./(R+ZC);
KMag = abs(-Z2/Z1);
figure
loglog(w,KMag,'g','LineWidth',3)
grid on
xlabel('Frequency, (rad/s)')
ylabel('Magnitude VO/VS')
title('Problem 8-67')
```

The corresponding output is shown below.

```
Results =
     1.0000e+003   999.9500e-003
    10.0000e+003   995.0372e-003
   100.0000e+003   707.1068e-003
     1.0000e+006    99.5037e-003
    10.0000e+006     9.9995e-003
```

The results are: for ω =[1, 10, 100, 1000, 10000] krad/s, $\mathbf{V}_\mathrm{O}/\mathbf{V}_\mathrm{S}$ =[0.999, 0.995, 0.707, 0.0995, 0.00999]. The plot is shown below.



The plot has a gain of nearly one for low frequencies. Above a frequency of ω = 100 krad/s, the gain decreases linearly on the log-log plot. This circuit is a low-pass filter.

Problem 8–68. Find the phasor input $\mathbf{V}_\mathrm{S}$ in Figure P8–68 when the phasor output is $\mathbf{V}_\mathrm{O} = 300 + j200$ V.

We have the following relationships:

$$Z_{\text{EQ}} = 100 \parallel -j200 = 80 - j40\ \Omega$$

$$\mathbf{I}_{\text{O}} = \frac{\mathbf{V}_{\text{O}}}{Z_{\text{EQ}}} = \frac{300 + j200}{80 - j40} = 2 + j3.5\ \text{A}$$

$$\mathbf{V}_1 = (10 + j20)\mathbf{I}_{\text{O}} = -50 + j75\ \text{V}$$

$$\mathbf{V}_{\text{S}} = \mathbf{V}_1 + \mathbf{V}_{\text{O}} = 250 + j275 = 371.65\angle 47.73^\circ\ \text{V}$$

Problem 8–69. The dependent source circuit in Figure P8–69 is operating in the sinusoidal steady state with $\omega = 1$ krad/s and $\mu = 10^3$. Find the phasor gain $K = \mathbf{V}_{\text{O}}/\mathbf{V}_{\text{S}}$ and the input impedance Z_{IN} seen by $\mathbf{V}_{\text{S}}$. Validate your answer using OrCAD.

Use node-voltage analysis to solve for the voltages and then determine the input impedance.

$$\frac{\mathbf{V}_{\text{A}} - \mathbf{V}_{\text{S}}}{10000} + \frac{\mathbf{V}_{\text{A}} - \mathbf{V}_{\text{X}}}{10000} + \frac{\mathbf{V}_{\text{A}} - \mu\mathbf{V}_{\text{X}}}{-j10000} = 0$$

$$\frac{\mathbf{V}_{\text{X}} - \mathbf{V}_{\text{A}}}{10000} + \frac{\mathbf{V}_{\text{X}}}{-j10000} = 0$$

$$\mathbf{V}_{\text{A}} = \frac{1}{997}(-1 + j)\mathbf{V}_{\text{S}}$$

$$\mathbf{V}_{\text{X}} = \frac{1}{997}(j)\mathbf{V}_{\text{S}}$$

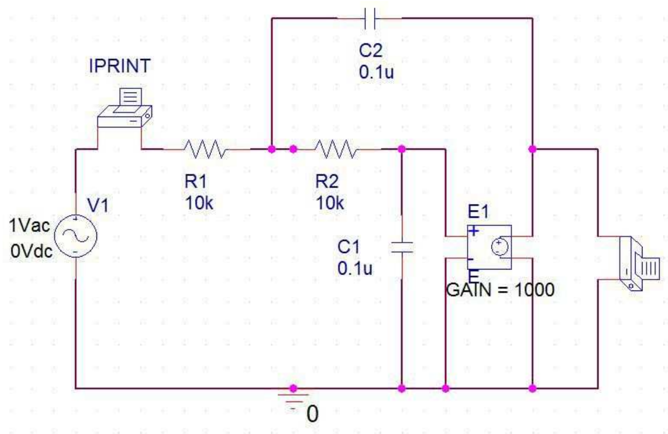
$$\mathbf{V}_{\text{O}} = \mu\mathbf{V}_{\text{X}} = j\frac{1000}{997}\mathbf{V}_{\text{S}}$$

$$K = j\frac{1000}{997} = j1.003$$

$$\mathbf{I}_{\text{IN}} = \frac{\mathbf{V}_{\text{S}} - \mathbf{V}_{\text{A}}}{10000} = (100.1 - j0.1003\ \mu)\mathbf{V}_{\text{S}}$$

$$Z_{\text{IN}} = \frac{\mathbf{V}_{\text{S}}}{\mathbf{I}_{\text{IN}}} = 9990 + j10.01\ \Omega$$

The OrCAD simulation is shown below.



The corresponding OrCAD results are shown below and are consistent with the results developed above.

```
  FREQ        VM(N00206,0)VP(N00206,0)VR(N00206,0)VI(N00206,0)
   1.592E+02   1.003E+00   9.000E+01  -7.194E-10   1.003E+00
****************************************************************************
  FREQ        IM(V_IL)    IP(V_IL)    IR(V_IL)    II(V_IL)
   1.592E+02   1.001E-04  -5.741E-02   1.001E-04  -1.003E-07
```

Problem 8–70. Find the phasor gain $K = \mathbf{V}_{\mathrm{O}}/\mathbf{V}_{\mathrm{S}}$ and input impedance Z_{IN} of the circuit in Figure P8–70.

Use node-voltage analysis.

$$\frac{\mathbf{V}_{\mathrm{A}} - \mathbf{V}_{\mathrm{S}}}{-j15000} + \frac{\mathbf{V}_{\mathrm{A}}}{33000} + \frac{\mathbf{V}_{\mathrm{A}} - \mathbf{V}_{\mathrm{O}}}{-j25000} = 0$$

$$\frac{\mathbf{V}_{\mathrm{O}} - \mathbf{V}_{\mathrm{A}}}{-j25000} + \frac{\mathbf{V}_{\mathrm{O}} - \mathbf{V}_{\mathrm{S}}}{10000} = 0$$

$$\mathbf{V}_{\mathrm{A}} = (0.8903 + j0.2517)\mathbf{V}_{\mathrm{S}}$$

$$\mathbf{V}_{\mathrm{O}} = (0.8981 - j0.0031)\mathbf{V}_{\mathrm{S}}$$

$$K = \frac{\mathbf{V}_{\mathrm{O}}}{\mathbf{V}_{\mathrm{S}}} = 0.8981 - j0.0031$$

$$\mathbf{I}_{\mathrm{IN}} = \frac{\mathbf{V}_{\mathrm{S}} - \mathbf{V}_{\mathrm{A}}}{-j15000} + \frac{\mathbf{V}_{\mathrm{S}} - \mathbf{V}_{\mathrm{O}}}{10000} = (26.98 + j7.63)\mathbf{V}_{\mathrm{S}}\ \mu\mathrm{A}$$

$$Z_{\mathrm{IN}} = \frac{\mathbf{V}_{\mathrm{S}}}{\mathbf{I}_{\mathrm{IN}}} = 34.32 - j9.71\ \mathrm{k}\Omega$$

Problem 8–71. Find the phasor gain $K = \mathbf{V}_{\mathrm{O}}/\mathbf{V}_{\mathrm{S}}$, input impedance Z_{IN} of the circuit, and the capacitor current $\mathbf{I}_{\mathrm{X}}$ in Figure P8–71.

Use node-voltage analysis.

$$\frac{\mathbf{V}_{\mathrm{A}} - \mathbf{V}_{\mathrm{S}}}{100} + \frac{\mathbf{V}_{\mathrm{A}}}{-j100} + \frac{\mathbf{V}_{\mathrm{A}} - \mathbf{V}_{\mathrm{O}}}{j100} = 0$$

$$\frac{\mathbf{V}_{\mathrm{O}} - \mathbf{V}_{\mathrm{A}}}{j100} + \frac{\mathbf{V}_{\mathrm{O}}}{100} = 0$$

$$\mathbf{V}_{\mathrm{A}} = (0.6 - j0.2)\mathbf{V}_{\mathrm{S}}$$

$$\mathbf{V}_{\mathrm{O}} = (0.2 - j0.4)\mathbf{V}_{\mathrm{S}}$$

$$K = 0.2 - j0.4$$

$$\mathbf{I}_{\mathrm{IN}} = \frac{\mathbf{V}_{\mathrm{S}} - \mathbf{V}_{\mathrm{A}}}{100} = (4 + j2)\mathbf{V}_{\mathrm{S}}\ \mathrm{mA}$$

$$Z_{\mathrm{IN}} = \frac{\mathbf{V}_{\mathrm{S}}}{\mathbf{I}_{\mathrm{IN}}} = 200 - j100\ \Omega$$

$$\mathbf{I}_{\mathrm{X}} = \frac{\mathbf{V}_{\mathrm{A}}}{-j100} = (2 + j6)\mathbf{V}_{\mathrm{S}}\ \mathrm{mA}$$

Problem 8–72. Given the circuit in Figure P8–72:

(a). Use node-voltage or mesh-current analysis to develop a set of matrix equations for the circuit.

Use node-voltage analysis and write the matrix equations by inspection.

$$\begin{bmatrix} \left(\frac{1}{15}+\frac{1}{60}+\frac{1}{j50}\right) & -\frac{1}{60} & -\frac{1}{j50} \\ -\frac{1}{60} & \left(\frac{1}{60}+\frac{1}{-j90}+\frac{1}{50}\right) & -\frac{1}{50} \\ -\frac{1}{j50} & -\frac{1}{50} & \left(\frac{1}{j50}+\frac{1}{50}+\frac{1}{100}\right) \end{bmatrix} \begin{bmatrix} \mathbf{V}_{\text{A}} \\ \mathbf{V}_{\text{B}} \\ \mathbf{V}_{\text{O}} \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{V}_{\text{S}}}{15} \\ 0 \\ 0 \end{bmatrix}$$

(b). Use MATLAB to solve the matrix equations and then find the phasor gain $K = \mathbf{V}_{\text{O}}/\mathbf{V}_{\text{S}}$ and input impedance Z_{IN} of the circuit.

The MATLAB code is shown below.

```
syms VS VA VB VO
% Node-voltage analysis by inspection
A = [1/15+1/60+1/50j, -1/60, -1/50j; ...
    -1/60, 1/60+1/(-90j)+1/50, -1/50; ...
    -1/50j, -1/50, 1/50j+1/50+1/100];
b = [VS/15; 0; 0];
x = A\b;
Va = vpa(x(1),6)
Vb = vpa(x(2),6)
Vo = vpa(x(3),6)
K = double(Vo/VS)
Iin = vpa((VS-Va)/15,6)
Zin = double(VS/Iin)
```

The corresponding results are shown below.

```
Va = VS*(0.83273 - 0.0396372*i)
Vb = VS*(0.603041 - 0.422101*i)
Vo = VS*(0.646134 - 0.405797*i)
K = 646.1342e-003 -405.7975e-003i
Iin = VS*(0.00264248*i + 0.0111514)
Zin = 84.9075e+000 - 20.1201e+000i
```

The results are $K = 0.6461 - j0.4058$ and $Z_{\text{IN}} = 84.91 = j20.12\ \Omega$.

(c). Without using the matrix equations, use the MATLAB command solve on the original node-voltage or mesh-current equations to solve the equations and then find the phasor gain and input impedance.

The MATLAB code is shown below.

```
% Using solve
Eqn1 = (VA-VS)/15 + (VA-VB)/60 + (VA-VO)/50j;
Eqn2 = (VB-VA)/60 + VB/(-90j) + (VB-VO)/50;
Eqn3 = (VO-VA)/50j + (VO-VB)/50 + VO/100;
Soln = solve(Eqn1,Eqn2,Eqn3,VA,VB,VO);
VA = vpa(Soln.VA,6)
VB = vpa(Soln.VB,6)
VO = vpa(Soln.VO,6)
K = double(VO/VS)
Iin = vpa((VS-VA)/15,6)
Zin = double(VS/Iin)
```

The corresponding results are shown below and agree with the previous results.

```
VA = VS*(0.83273 - 0.0396372*i)
VB = VS*(0.603041 - 0.422101*i)
VO = VS*(0.646134 - 0.405797*i)
K = 646.1342e-003 -405.7975e-003i
Iin = VS*(0.00264248*i + 0.0111514)
Zin = 84.9075e+000 - 20.1201e+000i
```

Problem 8–73. A load consisting of a 2.2-kΩ resistor in series with a 1-μF capacitor is connected across a voltage source $v_S(t) = 169.7\cos(377t)$ V. Find the phasor voltage, current, and average power delivered to the load.

The requested values are calculated below.

$$\mathbf{V}_\mathrm{S} = \mathbf{V}_\mathrm{L} = 169.7\angle 0^\circ = 169.7 + j0 \text{ V}$$

$$Z_\mathrm{C} = \frac{1}{j(377)(1\,\mu)} = -j2652.5\ \Omega$$

$$Z_\mathrm{EQ} = 2200 - j2652.5\ \Omega$$

$$\mathbf{I}_\mathrm{L} = \frac{\mathbf{V}_\mathrm{L}}{Z_\mathrm{EQ}} = \frac{169.7}{2200 - j2652.5} = 31.44 + j37.90 = 49.2\angle 50.3^\circ \text{ mA}$$

$$P = \frac{1}{2}R_\mathrm{L}\,|\mathbf{I}_\mathrm{L}|^2 = (0.5)(2200)(0.0492)^2 = 2.6674 \text{ W}$$

Problem 8–74. A load consisting of a 50-Ω resistor in parallel with a 0.47-μF capacitor is connected across a current source delivering $i_S(t) = 12\cos(3000t)$ mA. Find the average power delivered to the load.

The calculations are shown below.

$$\mathbf{I}_\mathrm{S} = \mathbf{I}_\mathrm{L} = 12\angle 0^\circ = 12 + j0 \text{ mA}$$

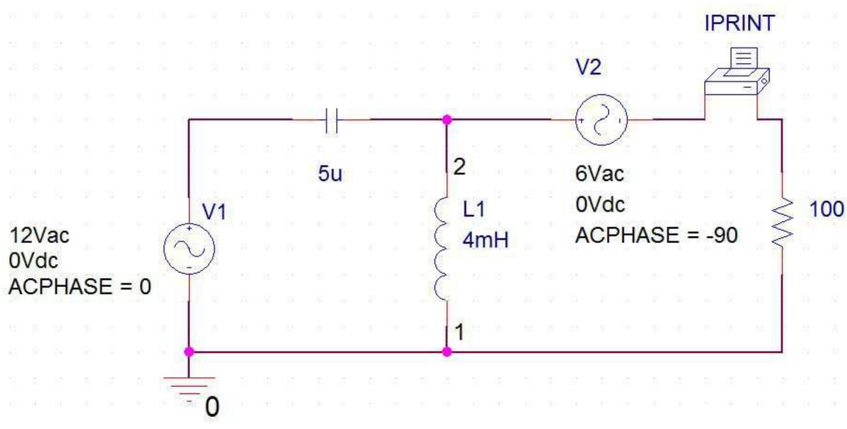
$$Z_\mathrm{C} = \frac{1}{j(3000)(0.47\,\mu)} = -j709.22\ \Omega$$

$$Z_\mathrm{EQ} = 50 \parallel -j709.22 = 49.753 - j3.508\ \Omega$$

$$P = \frac{1}{2}R_\mathrm{L}\,|\mathbf{I}_\mathrm{L}|^2 = (0.5)(49.753)(0.012)^2 = 3.5822 \text{ mW}$$

Problem 8–75. The circuit in Figure P8–75 is operating in the sinusoidal steady state at a frequency of 10 krad/s. Use OrCAD to find the average power delivered to the 100-Ω resistor.

The OrCAD simulation is shown below.



The corresponding output is shown below.

```
 FREQ        IM(V_PRINT1)IP(V_PRINT1)IR(V_PRINT1)II(V_PRINT1)
  1.592E+03   2.297E-01   3.584E+01   1.862E-01   1.345E-01
```

The magnitude of the load current is $|\mathbf{I}_\mathrm{L}| = 0.2297$ A. The average power delivered to the load is:

$$P = \frac{1}{2}R_\mathrm{L}\,|\mathbf{I}_\mathrm{L}|^2 = (0.5)(100)(0.2297)^2 = 2.638 \text{ W}$$

Problem 8–76. You have a task of designing a load that ensure maximum power is delivered to it. The load needs to be connected to a source circuit that is not readily observable, but that you can make measurements at its output terminals. You measure the open circuit voltage and read $120\angle 0^\circ$ V. You then connect a known load of $50 - j50\ \Omega$ and you measure $47.1\angle 11.3^\circ$ V across it.

(a). Design your load for maximum power transfer.

Find the Thévenin equivalent circuit. The Thévenin voltage is the open-circuit voltage, $\mathbf{V}_\mathrm{T} = 120\angle 0^\circ$ V. Compute the Thévenin impedance as follows:

$$\mathbf{I}_\mathrm{L} = \frac{\mathbf{V}_\mathrm{L}}{Z_\mathrm{L}} = \frac{47.1\angle 11.3^\circ}{50 - j50} = 0.3696 + j0.5542 \text{ A}$$

$$\mathbf{V}_1 = \mathbf{V}_\mathrm{T} - \mathbf{V}_\mathrm{L} = 120 - 47.1\angle 11.3^\circ = 73.81 - j9.23 \text{ V}$$

$$Z_\mathrm{T} = \frac{\mathbf{V}_1}{\mathbf{I}_\mathrm{L}} = \frac{73.81 - j9.23}{0.3696 + j0.5542} = 50 - j100\ \Omega$$

For maximum power transfer, we need $Z_\mathrm{L} = Z_\mathrm{T}^* = 50 + j100\ \Omega$.

(b). Find the maximum average power delivered to your load.

Compute the maximum average power transfer.

$$P_\mathrm{MAX} = \frac{|\mathbf{V}_\mathrm{T}|^2}{8R_\mathrm{T}} = \frac{(120)^2}{(8)(50)} = 36 \text{ W}$$

Problem 8–77.

(a). Find the average power delivered to the load in Figure P8–77.

Perform a source transformation and then use current division to find the load current.

$$Z_\mathrm{L} = j(10^6)(0.001) = j1000\ \Omega$$

$$Z_\mathrm{C} = \frac{1}{j(10^6)(500\,\mathrm{p})} = -j2000\ \Omega$$

$$\mathbf{I}_\mathrm{S} = \frac{\mathbf{V}_\mathrm{S}}{Z_\mathrm{L}} = \frac{30}{j1000} = -j0.03 \text{ A}$$

$$\mathbf{I}_\mathrm{L} = \frac{\frac{1}{1500}}{\frac{1}{j1000} + \frac{1}{-j2000} + \frac{1}{1500}}(-j0.03) = 14.4 - j19.2 = 24.0\angle{-53.1^\circ} \text{ mA}$$

$$P = \frac{1}{2}R_\mathrm{L}|\mathbf{I}_\mathrm{L}|^2 = (0.5)(500)(0.024)^2 = 144.0 \text{ mW}$$

(b). Find the maximum available average power at the interface shown in the figure.

Find the Thévenin equivalent source circuit.

$$\mathbf{V}_\mathrm{T} = \frac{-j2000}{j1000 - j2000}(30) = 60 \text{ V}$$

$$Z_\mathrm{T} = (j1000 \parallel -j2000) + 1000 = 1000 + j2000 \ \Omega$$

$$P_\mathrm{MAX} = \frac{|\mathbf{V}_\mathrm{T}|^2}{8R_\mathrm{T}} = \frac{(60)^2}{(8)(1000)} = 450 \text{ mW}$$

(c). Specify the load required to extract the maximum average power.

The impedance for the maximum load power is $Z_\mathrm{L} = Z_\mathrm{T}^* = 1000 - j2000\ \Omega$.

Problem 8–78.

(a). Find the maximum average power available at the interface in Figure P8–78.

Convert the circuit to the phasor domain and find the Thévenin equivalent source.

$$Z_\mathrm{L} = j(10000)(0.5) = j5000\ \Omega$$

$$Z_\mathrm{T} = 500 + j5000\ \Omega$$

$$\mathbf{V}_\mathrm{T} = 10\angle 0^\circ \text{ V}$$

$$P_\mathrm{MAX} = \frac{|\mathbf{V}_\mathrm{T}|^2}{8R_\mathrm{T}} = \frac{(10)^2}{(8)(500)} = 25 \text{ mW}$$

(b). Specify the values of R and C that will extract the maximum power from the source circuit.

For maximum power transfer, the load impedance must be $Z_\mathrm{L} = Z_\mathrm{T}^* = 500 - j5000\ \Omega$.

$$Z_\mathrm{L} = Z_\mathrm{C} \parallel R = \frac{\dfrac{R}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{R}{j\omega RC + 1} = \frac{R(1 - j\omega RC)}{1 + (\omega RC)^2}$$

$$500 - j5000 = \frac{R(1 - j\omega RC)}{1 + (\omega RC)^2}$$

$$500 = \frac{R}{1 + (\omega RC)^2}$$

$$5000 = \frac{\omega R^2 C}{1 + (\omega RC)^2}$$

$$\omega RC = 10$$

$$\frac{R}{1 + 10^2} = 500$$

$$R = 50.5 \text{ k}\Omega$$

$$C = \frac{10}{\omega R} = 0.0198\ \mu\text{F}$$

Problem 8–79. (A) AC Voltage Measurement

An ac voltmeter measurement indicates the amplitude of a sinusoid and not its phase angle. The magnitude and phase can be inferred by making several measurements and using KVL. For example, Figure P8–79 shows a relay coil of unknown resistance and inductance. The following ac voltmeter readings are taken with the circuit operating in the sinusoidal steady state at $f = 60$ Hz: $|\mathbf{V}_\text{S}| = 24$ V, $|\mathbf{V}_1| = 10$ V, and $|\mathbf{V}_2| = 18$ V. Find R and L.

Use voltage division to write expressions for the magnitudes of the measured voltages and solve the simultaneous equations.

$$\mathbf{V}_1 = \frac{200}{200 + R + j\omega L}\mathbf{V}_\text{S}$$

$$\mathbf{V}_2 = \frac{R + j\omega L}{200 + R + j\omega L}\mathbf{V}_\text{S}$$

$$|\mathbf{V}_1| = 10 = \frac{200|\mathbf{V}_\text{S}|}{\sqrt{(200+R)^2 + (\omega L)^2}}$$

$$|\mathbf{V}_2| = 18 = \frac{\sqrt{R^2 + (\omega L)^2}|\mathbf{V}_\text{S}|}{\sqrt{(200+R)^2 + (\omega L)^2}}$$

The following MATLAB code solves the equations.

```
f = 60;
w = 2*pi*f;
VSMag = 24;
V1Mag = 10;
V2Mag = 18;
syms R L
Eqn1 = V1Mag - 200*VSMag/sqrt((200+R)^2+(w*L)^2);
Eqn2 = V2Mag - sqrt(R^2 + (w*L)^2)*VSMag/sqrt((200+R)^2+(w*L)^2);
Soln = solve(Eqn1,Eqn2,R,L);
R = double(Soln.R)
L = double(Soln.L)
```

The resulting output is shown below.

```
R = 152.0000e+000
    152.0000e+000
L = 865.6364e-003
   -865.6364e-003
```

The solution is $R = 152\ \Omega$ and $L = 865.64$ mH.

Problem 8–80. (A,E) Home Power Distribution

The circuit of Figure P8–80 emulates a typical 60-Hz residential power system. There are three wires entering the house, two are called "hot" and the remaining one is called the return or "neutral." Each hot line is protected by a circuit breaker—but not the return. In the "house," appliances such as lights, toasters, and electronics are connected between one of the hot wires and the neutral. Large appliances like ovens and dryers are connected between the two hot wires. Appliances are designed to operate with either 120 V or 240 V within a few volts either way.

(a). Show that if R_1 and R_2 are equal, $\mathbf{I}_\text{N} = 0$ and $\mathbf{V}_1 = \mathbf{V}_2$.

Use node-voltage analysis, where $\mathbf{V}_\text{A}$ is the node at the top circuit breaker, $\mathbf{V}_\text{B}$ is in the center, and

$\mathbf{V}_\mathrm{C}$ is at the lower circuit breaker. Let $Z = 0.05 + j0.02\ \Omega$. The node-voltage equations are:

$$\frac{\mathbf{V}_\mathrm{A} - 120}{Z} + \frac{\mathbf{V}_\mathrm{A} - \mathbf{V}_\mathrm{B}}{R_1} + \frac{\mathbf{V}_\mathrm{A} - \mathbf{V}_\mathrm{C}}{R_3} = 0$$

$$\frac{\mathbf{V}_\mathrm{B} - \mathbf{V}_\mathrm{A}}{R_1} + \frac{\mathbf{V}_\mathrm{B}}{Z} + \frac{\mathbf{V}_\mathrm{B} - \mathbf{V}_\mathrm{C}}{R_2} = 0$$

$$\frac{\mathbf{V}_\mathrm{C} - \mathbf{V}_\mathrm{A}}{R_3} + \frac{\mathbf{V}_\mathrm{C} - \mathbf{V}_\mathrm{B}}{R_2} + \frac{\mathbf{V}_\mathrm{C} + 120}{Z} = 0$$

The following MATLAB code solves the node-voltage equations:

```
VS = 120;
Z1 = 0.05+0.02j;
syms R1 R2 R3 R VA VB VC
Eqn1 = (VA-VS)/Z1 + (VA-VB)/R1 + (VA-VC)/R3;
Eqn2 = (VB-VA)/R1 + VB/Z1 + (VB-VC)/R2;
Eqn3 = (VC-VA)/R3 + (VC-VB)/R2 + (VC-(-VS))/Z1;
Soln = solve(Eqn1,Eqn2,Eqn3,VA,VB,VC);
Va = Soln.VA;
Vb = Soln.VB;
Vc = Soln.VC;
V1 = simplify(subs((Va-Vb),{R1,R2},{R,R}))
V2 = simplify(subs((Vb-Vc),{R1,R2},{R,R}))
V1V2Check = simplify(V1-V2)
IN = simplify(subs((Vb/Z1),{R1,R2},{R,R}))
```

The corresponding results are:

```
V1 = (6000*R*R3)/(R3*(i + 5/2) + 50*R*R3 + R*(2*i + 5))
V2 = (6000*R*R3)/(R3*(i + 5/2) + 50*R*R3 + R*(2*i + 5))
V1V2Check = 0
IN = 0
```

If R_1 and R_2 are equal, the $\mathbf{I}_\mathrm{N} = 0$ and $\mathbf{V}_1 = \mathbf{V}_2$.

(b). Consider a typical power draw where a R_1 is, for example, a 130-Ω light bulb, R_2 is a 40-Ω toaster, and R_3 is a 10-Ω clothes dryer. Find the phasors $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{I}_\mathrm{N}$.

Use the results from part (a) and substitute in the values. The MATLAB code and results are shown below.

```
V1 = subs((Va-Vb),{R1,R2,R3},{130,40,10})
V2 = subs((Vb-Vc),{R1,R2,R3},{130,40,10})
IN = subs((Vb/Z1),{R1,R2,R3},{130,40,10})
```

```
V1 = 118.8677e+000 -447.4353e-003i
V2 = 118.5596e+000 -568.7960e-003i
IN = -2.0496e+000 + 10.7781e-003i
```

The results are:

$$\mathbf{V}_1 = 118.87 - j0.4474 = 118.87\angle{-0.216^\circ}\ \mathrm{V}$$

$$\mathbf{V}_2 = 118.56 - j0.5688 = 118.56\angle{-0.275^\circ}\ \mathrm{V}$$

$$\mathbf{I}_\mathrm{N} = -2.0496 + j0.01078 = 2.05\angle 179.7^\circ\ \mathrm{A}$$

(c). Based on your results in parts (a) and (b), is the neutral line even necessary? Open the neutral line, that is, force $\mathbf{I}_\mathrm{N} = 0$, and again find the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$. What is your answer? Would your home be better protected by adding a breaker to the return line?

The neutral is required since it carries a significant current. If we set $\mathbf{I}_\mathrm{N} = 0$ and solve the equations again, we have the following MATLAB code and results.

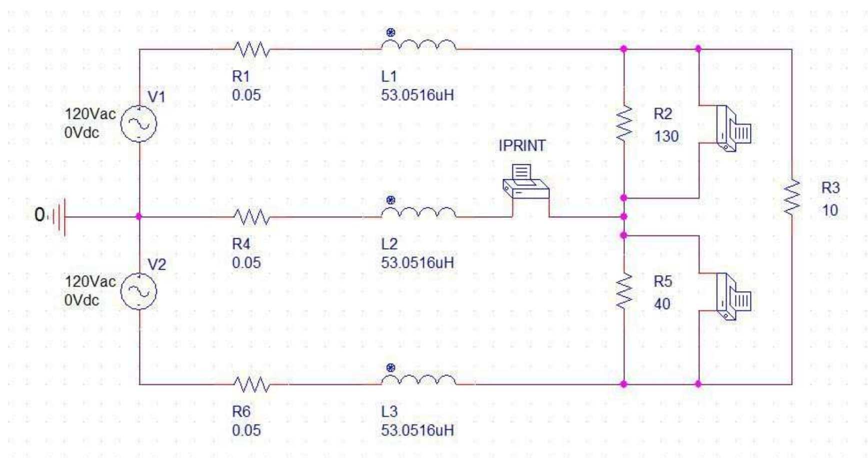
```
VS = 120;
Z1 = 0.05+0.02j;
syms R1 R2 R3 R VA VB VC
Eqn1 = (VA-VS)/Z1 + (VA-VB)/R1 + (VA-VC)/R3;
Eqn2 = (VB-VA)/R1 + (VB-VC)/R2;
Eqn3 = (VC-VA)/R3 + (VC-VB)/R2 + (VC-(-VS))/Z1;
Soln = solve(Eqn1,Eqn2,Eqn3,VA,VB,VC);
Va = Soln.VA;
Vb = Soln.VB;
Vc = Soln.VC;
V1 = subs((Va-Vb),{R1,R2,R3},{130,40,10})
V2 = subs((Vb-Vc),{R1,R2,R3},{130,40,10})
```

```
V1 = 181.6033e+000 -761.0850e-003i
V2 =  55.8779e+000 -234.1800e-003i
```

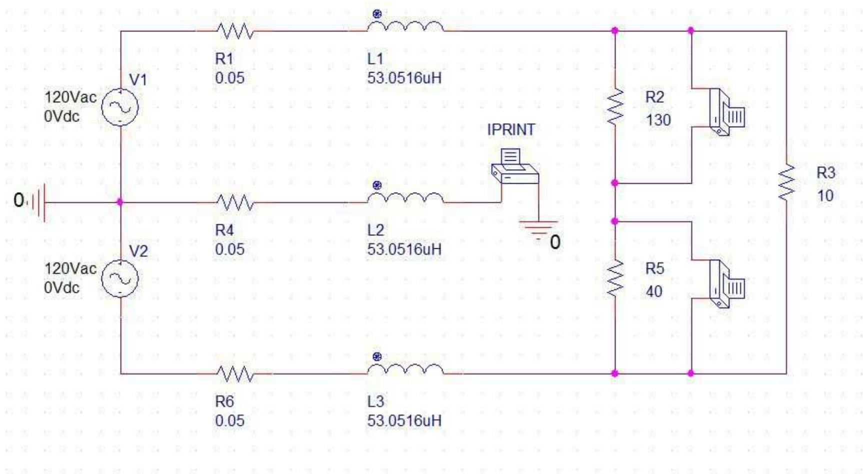
The results are significantly different and the appliances do not receive the expected voltages. Adding a circuit breaker to the return line would not significantly improve the protection of the home, since the current is alternating.

(d). Simulate the circuit in OrCAD and validate your results. The source frequency is 60 Hz.

The OrCAD simulations and results are shown below and they confirm the results shown above.




```
  FREQ        IM(V_I_N)   IP(V_I_N)   IR(V_I_N)   II(V_I_N)
   6.000E+01   2.050E+00   1.797E+02  -2.050E+00   1.078E-02
*****************************************************************************
  FREQ        VM(N00378,N00385)VP(N00378,N00385)VR(N00378,N00385)VI(N00378,N00385)
   6.000E+01   1.189E+02  -2.157E-01   1.189E+02  -4.474E-01
*****************************************************************************
  FREQ        VM(N00385,N00389)VP(N00385,N00389)VR(N00385,N00389)VI(N00385,N00389)
   6.000E+01   1.186E+02  -2.749E-01   1.186E+02  -5.688E-01
```




```
  FREQ        IM(V_I_N)   IP(V_I_N)   IR(V_I_N)   II(V_I_N)
   6.000E+01   1.000E-30   0.000E+00   0.000E+00   0.000E+00
******************************************************************************
  FREQ        VM(N00378,N00385)VP(N00378,N00385)VR(N00378,N00385)VI(N00378,N00385)
   6.000E+01   1.816E+02  -2.401E-01   1.816E+02  -7.611E-01
******************************************************************************
  FREQ        VM(N00385,N00389)VP(N00385,N00389)VR(N00385,N00389)VI(N00385,N00389)
   6.000E+01   5.588E+01  -2.401E-01   5.588E+01  -2.342E-01
```

Problem 8–81. (D) OP AMP Band Pass Filter

Use the analysis methods discussed in Example 8–28 to find the input-output relationship $\mathbf{V}_{\mathrm{O}}/\mathbf{V}_{\mathrm{S}}$ for the active band pass filter of Figure P8–81. Treat each stage separately and then multiply the input-output relationships from each stage to obtain the overall input-output relationship. Select R and C values so that the low cutoff frequency is 500 rad/s, the upper cutoff frequency is 50 krad/s, and the magnitude of the passband gain is 100.

We have the following results:

$$Z_1 = R_1$$

$$Z_2 = R_2 \parallel \frac{1}{j\omega C_2} = \frac{R_2}{j\omega R_2 C_2 + 1}$$

$$Z_3 = R_3 + \frac{1}{j\omega C_3} = \frac{j\omega R_3 C_3 + 1}{j\omega C_3}$$

$$Z_4 = R_4$$

$$K_1 = -\frac{Z_2}{Z_1} = -\frac{\dfrac{R_2}{R_1}\left(\dfrac{1}{R_2C_2}\right)}{j\omega + \dfrac{1}{R_2C_2}}$$

$$K_2 = -\frac{Z_4}{Z_3} = -\frac{\dfrac{R_4}{R_3}(j\omega)}{j\omega + \dfrac{1}{R_3C_3}}$$

$$K = K_1K_2 = \frac{Z_2Z_4}{Z_1Z_3}$$

The first stage is a low-pass filter and controls the upper cutoff frequency. The second stage is a high-pass filter and controls the lower cutoff frequency. Choose $R_1 = R_3 = 20$ kΩ, $R_2 = R_4 = 200$ kΩ, $C_2 = 100$ pF, and $C_3 = 0.1$ μF to meet the specifications.

Problem 8–82. (A) Power Transmission Efficiency

A power transmission circuit with a source voltage of $\mathbf{V}_\text{S} = 440 + j0$ V can be modeled as shown in Figure P8–82. Find the average power produced by the source, lost in the wires, and delivered to the load. What is the transmission efficiency?

The calculations are shown below:

$$Z_\text{EQ} = 0.4 + j5 + 50 + j150 + 0.4 + j5 = 50.8 + j160\ \Omega$$

$$\mathbf{I}_\text{S} = \frac{\mathbf{V}_\text{S}}{Z_\text{EQ}} = \frac{440}{50.8 + j160} = 0.7932 - j2.498 = 2.6211\angle{-72.39^\circ}\ \text{A}$$

$$P_\text{S} = \frac{1}{2}R|\mathbf{I}_\text{S}|^2 = (0.5)(50.8)(2.6211)^2 = 174.5\ \text{W}$$

$$P_\text{W} = (0.5)(0.8)(2.6211)^2 = 2.75\ \text{W}$$

$$P_\text{L} = (0.5)(50)(2.6211)^2 = 171.75\ \text{W}$$

$$\eta = \frac{P_\text{L}}{P_\text{S}} = \frac{171.75}{174.5} = 0.9843$$

Problem 8–83. (E) 60 Hz Filter

A 1-kΩ resistor models an important and sensitive laboratory instrument. The desired signal that the instrument measures, varies from 1 Hz to 500 Hz. However, interference from power lines in the laboratory causes the instrument to saturate. A vendor has designed a device that he claims will essentially eliminate a 60-Hz signal from the instrument with only the smallest attenuation to other frequencies. The interface circuit is shown in Figure P8–83. Connect the vendor's circuit to the model for the instrument and analyze its performance to determine if it will do the job.

Compute the gain of the circuit when the interface is connected to a 1-kΩ resistor.

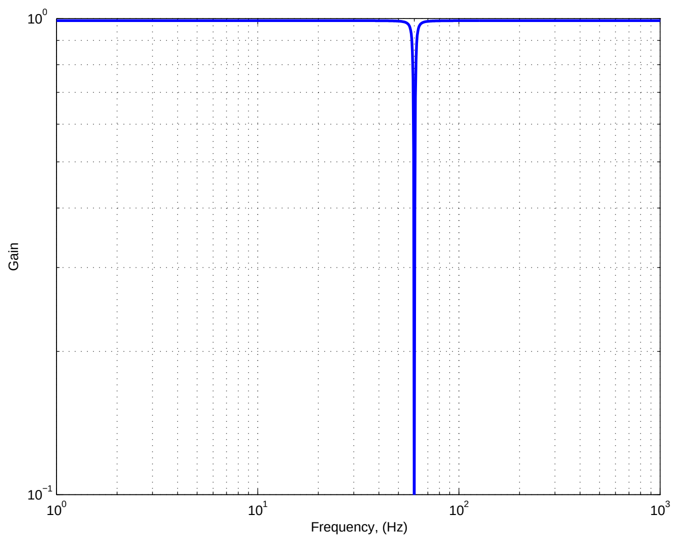
$$Z_\text{EQ} = 1000 \parallel \left(j\omega L + \frac{1}{j\omega C}\right)$$

$$K = \left|\frac{\mathbf{V}_\text{O}}{\mathbf{V}_\text{S}}\right| = \frac{Z_\text{EQ}}{10 + Z_\text{EQ}}$$

Use MATLAB to perform the calculations and plot the gain versus frequency.

```
syms w
C = 7.03e-6;
L = 1;
R = 10;
RL = 1e3;
ZC = 1/j/w/C;
ZL = j*w*L;
Zeq = RL*(ZC+ZL)/(RL+ZC+ZL)
K = simplify(Zeq/(R+Zeq))
Knum = vpa(K,5)
% Plot the gain versus frequency
f = logspace(0,3,1000);
ww = 2*pi*f;
Kww = subs(K,w,ww);
MagKww = abs(Kww);
loglog(f,MagKww,'b','LineWidth',2)
grid on
axis([1,1000,.1,1])
xlabel('Frequency, (Hz)')
ylabel('Gain')
```

The corresponding plot is shown below:



The gain response is nearly one except when $f = 60$ Hz, so the circuit performs the required function.

Problem 8–84. (D) AC Circuit Design

Select values of L and C in Figure P8–84 so that the input impedance seen by the voltage source is $50 + j0\ \Omega$ when the frequency is $\omega = 10^6$ rad/s. For these values of L and C, find the output Thévenin impedance seen by the 300-Ω load resistor.

Find the equivalent impedance and then solve for the component values and $Z_{\rm OUT}$.

$$Z_{\rm EQ} = 50 = j\omega L + \left(\frac{1}{j\omega C} \parallel R\right)$$

$$50 - j\omega L = \frac{R}{j\omega RC + 1}$$

$$R = j\omega 50RC + 50 + \omega^2 RLC - j\omega L$$

$$\omega^2 RLC = R - 50$$

$$\omega L = \omega 50RC$$

$$L = 50RC$$

$$LC = \frac{R-50}{R\omega^2}$$

$$C^2 = \frac{R-50}{50R^2\omega^2}$$

$$C = 7453.6 \text{ pF}$$

$$L = 111.80 \ \mu\text{H}$$

$$Z_{\text{OUT}} = (50 + j\omega L) \parallel \frac{1}{j\omega C} = (50 + j111.8) \parallel (-j134.2) = 300 \ \Omega$$

When the input impedance matches the source impedance, the output impedance matches the load impedance.

Problem 8–85. (E) AC Circuit Analysis

Ten years after graduating with a BSEE, you decide to go to graduate schools for a masters degree. In desperate need of income, you agree to sign on as a grader in the basic circuit analysis course. One of the problems asks the students to find $v(t)$ in Figure P8–85 when the circuit operates in the sinusoidal steady state. One of the students offers the following solution:

$$\begin{aligned} v(t) &= (R + j\omega L) \times i(t) \\ &= (20 + j20) \times 0.5\cos(200t) \\ &= 10\cos(200t) + j10\cos(200t) \\ &= 10\sqrt{2}\cos(200t + 45^\circ) \end{aligned}$$

Is the answer correct? If not what grade would you give the student? If correct, what comments would you give the student about the method of solution?

The student has the correct solution, but the approach is not valid. A correct approach is shown below.

$$\mathbf{I} = 0.5\angle 0^\circ \text{ A}$$

$$Z_{\text{L}} = j(200)(0.1) = j20 \ \Omega$$

$$Z_{\text{EQ}} = 20 + j20 \ \Omega$$

$$\mathbf{V} = Z_{\text{EQ}}\mathbf{I} = (20 + j20)(0.5) = 10 + j10 = 10\sqrt{2}\angle 45^\circ \text{ V}$$

$$v(t) = 10\sqrt{2}\cos(200t + 45^\circ) \text{ V}$$

In this case, the student's answer happens to be correct because the current flowing in the circuit has zero phase. If the current had non-zero phase, this approach would not always give the correct answer or the required analysis would be much more difficult. The fundamental error in the approach is that the impedances are in the phasor domain, which depends on frequency, and the sinusoid is in the time domain. The two domains are not compatible for analysis and calculations.

9 Laplace Transforms

9.1 Exercise Solutions

Exercise 9–1. Find the Laplace transform of $v(t) = -7u(t)$ V.

Apply the definition of a Laplace transform.

$$\begin{aligned}
V(s) &= \int_{0^-}^{\infty} -7u(t)e^{-st}\,dt \\
&= \int_{0^-}^{\infty} -7e^{-st}\,dt \\
&= \left.\frac{-7e^{-st}}{-s}\right|_{0^-}^{\infty} = \frac{7}{s}(0-1) = -\frac{7}{s} \text{ V-s}
\end{aligned}$$

Exercise 9–2. Find the Laplace transform of $v(t) = 8e^{-5t}u(t)$ V.

Apply the results of Example 9–2.

$$\begin{aligned}
V(s) &= \int_{0^-}^{\infty} 8e^{-5t}u(t)e^{-st}\,dt = 8\int_{0^-}^{\infty} e^{-5t}e^{-st}\,dt \\
&= (8)\frac{1}{s+5} = \frac{8}{s+5} \text{ V-s}
\end{aligned}$$

Exercise 9–3. Find the Laplace transform of $i(t) = 0.5\delta(t)$ A.

Apply the definition of a Laplace transform.

$$\begin{aligned}
I(s) &= \int_{0^-}^{\infty} 0.5\delta(t)e^{-st}\,dt \\
&= 0.5\int_{0^-}^{0^+} \delta(t)e^{-st}\,dt = 0.5\int_{0^-}^{0^+} \delta(t)\,dt \\
&= 0.5 \text{ A-s}
\end{aligned}$$

Exercise 9–4. Transform the response $v(t) = \left[10e^{-1000t} - 5\right]u(t)$ V of a particular RC circuit into the Laplace domain.

Apply the linearity property to transform each component.

$$V(s) = \frac{10}{s+1000} - \frac{5}{s} = \frac{10s - 5s - 5000}{s(s+1000)} = \frac{5(s-1000)}{s(s+1000)} \text{ V-s}$$

Exercise 9–5. Transform the sinusoid $i(t) = 100\left[\sin(200t)\right]u(t)$ mA into the Laplace domain.

Apply the results from Example 9–5.

$$I(s) = \frac{(100)(200)}{s^2 + 200^2} = \frac{20000}{s^2 + 40000} \text{ mA-s}$$

Exercise 9–6. Let $v_1(t) = V_A e^{-\alpha t}u(t)$ V. Show that the Laplace transform of $v_2(t) = \int_0^t V_A e^{-\alpha x}\,dx$ V is equal to $V_1(s)/s$.

The calculations are shown below.

$$\int_0^t V_A e^{-\alpha x}\,dx = \frac{V_A}{-\alpha}e^{-\alpha x}\Big|_0^t = \frac{V_A}{-\alpha}\left(e^{-\alpha t}-1\right) = \frac{V_A}{\alpha}\left(1-e^{-\alpha t}\right)$$

$$\mathscr{L}\left\{\frac{V_A}{\alpha}\left(1-e^{-\alpha t}\right)\right\} = \frac{V_A}{\alpha}\left(\frac{1}{s}-\frac{1}{s+\alpha}\right) = \frac{V_A}{\alpha}\left[\frac{s+\alpha-s}{s(s+\alpha)}\right] = \frac{V_A}{s(s+\alpha)}$$

$$V_1(s) = \frac{V_A}{s+\alpha}$$

$$\frac{V_1(s)}{s} = \frac{V_A}{s(s+\alpha)}$$

Exercise 9–7. If $i(t) = 6e^{-1000t}u(t)$ mA, find the Laplace transform of $v(t) = \frac{1}{10^{-6}}\int_0^t i(x)dx$ V.

Apply the integration property and linearity.

$$I(s) = \frac{6}{s+1000}\ \text{mA-s}$$

$$V(s) = 10^6\frac{I(s)}{s} = 10^6\frac{0.006}{s(s+1000)} = \frac{6000}{s(s+1000)}\ \text{V-s}$$

Exercise 9–8. Let $v_1(t) = V_A r(t)$ V. Show that the Laplace transform of $v_2(t) = dV_A r(t)/dt$ V is equal to $sV_1(s) - v_1(0^-)$.

The calculations are shown below.

$$\frac{dV_A r(t)}{dt} = \frac{dV_A tu(t)}{dt} = V_A t\delta(t) + V_A u(t) = V_A u(t)$$

$$\mathscr{L}\left\{V_A u(t)\right\} = \frac{V_A}{s}$$

$$sV_1(s) - v_1(0^-) = s\frac{V_A}{s^2} - 0 = \frac{V_A}{s}$$

Exercise 9–9. If $i(t) = 30e^{-1200t}u(t)$ mA, find the Laplace transform of $v(t) = 0.1\frac{di(t)}{dt}$ V.

We can solve this problem in two ways. First, find $I(s)$ and apply the differentiation property.

$$I(s) = \mathscr{L}\left\{i(t)\right\} = \mathscr{L}\left\{0.03e^{-1200t}u(t)\right\} = 0.03\,\mathscr{L}\left\{e^{-1200t}u(t)\right\} = \frac{0.03}{s+1200}$$

$$V(s) = \mathscr{L}\left\{0.1\frac{di(t)}{dt}\right\} = 0.1\,\mathscr{L}\left\{\frac{di(t)}{dt}\right\} = 0.1\left[sI(s) - i(0^-)\right] = 0.1\left[\frac{0.03s}{s+1200} - 0\right]$$

$$V(s) = \frac{0.003s}{s+1200}\ \text{V-s} = \frac{3s}{s+1200}\ \text{mV-s}$$

Second, confirm this answer by finding an expression for $v(t)$ in the time domain and taking the Laplace transform of the result.

$$v(t) = 0.1\frac{di(t)}{dt} = (0.1)\,\frac{d}{dt}\left[0.03e^{-1200t}u(t)\right] = \frac{d}{dt}\left[0.003e^{-1200t}u(t)\right]$$

$$v(t) = 0.003e^{-1200t}\delta(t) + (0.003)(-1200)e^{-1200t}u(t)$$

$$V(s) = \mathscr{L}\left\{v(t)\right\} = \mathscr{L}\left\{0.003e^{-1200t}\delta(t) - 3.6e^{-1200t}u(t)\right\}$$

$$V(s) = 0.003 + \frac{-3.6}{s+1200} = \frac{0.003s + 3.6 - 3.6}{s+1200}$$

$$V(s) = \frac{0.003s}{s+1200}\text{ V-s}$$

Exercise 9–10. Find the Laplace transforms of the following waveforms:

(a). $f(t) = \left[e^{-2t}\right]u(t) + 4tu(t) - u(t)$

$$F(s) = \frac{1}{s+2} + \frac{4}{s^2} - \frac{1}{s} = \frac{s^2 + 4s + 8 - s^2 - 2s}{s^2(s+2)} = \frac{2(s+4)}{s^2(s+2)}$$

(b). $f(t) = [2 + 2\sin(2t) - 2\cos(2t)]\,u(t)$

$$F(s) = \frac{2}{s} + \frac{4}{s^2+4} - \frac{2s}{s^2+4} = \frac{2s^2 + 8 + 4s - 2s^2}{s(s^2+4)} = \frac{4(s+2)}{s(s^2+4)}$$

Exercise 9–11. Find the Laplace transforms of the following waveforms:

(a). $f(t) = \left[e^{-4t}\right]u(t) + 5\int_0^t \sin(4x)\,dx$

$$F(s) = \frac{1}{s+4} + \frac{5}{s}\frac{4}{s^2+16} = \frac{s^3 + 16s + 20s + 80}{s(s+4)(s^2+16)} = \frac{s^3 + 36s + 80}{s(s+4)(s^2+16)}$$

(b). $f(t) = 5\left[e^{-40t}\right]u(t) + \dfrac{d[5te^{-40t}]u(t)}{dt}$

$$F(s) = \frac{5}{s+40} + s\frac{5}{(s+40)^2} = \frac{5s + 200 + 5s}{(s+40)^2} = \frac{10(s+20)}{(s+40)^2}$$

Exercise 9–12. Find the Laplace transforms of the following waveforms:

(a). $f_1(t) = A\left[\cos(\beta t - \phi)\right]u(t)$

$$\begin{aligned} f_1(t) &= A\cos\left[\beta\left(t - \frac{\phi}{\beta}\right)\right]u(t) \\ &= A\left[\cos(\beta t)\cos(\phi) + \sin(\beta t)\sin(\phi)\right]u(t) \\ F_1(s) &= A\cos(\phi)\frac{s}{s^2+\beta^2} + A\sin(\phi)\frac{\beta}{s^2+\beta^2} \\ &= A\cos(\phi)\left[\frac{s + \beta\tan(\phi)}{s^2+\beta^2}\right] \end{aligned}$$

(b). $f_2(t) = A\left[e^{-\alpha t}\cos(\beta t - \phi)\right]u(t)$

$$f_2(t) = e^{-\alpha t}f_1(t)$$

$$F_2(s) = F_1(s+\alpha) = A\cos(\phi)\left[\frac{s+\alpha+\beta\tan(\phi)}{(s+\alpha)^2+\beta^2}\right]$$

Exercise 9–13. Find the Laplace transforms of the following waveforms for $T > 0$:

(a). $f(t) = Au(t) - 2Au(t-T) + Au(t-2T)$

$$F(s) = \frac{A}{s} - \frac{2A}{s}e^{-Ts} + \frac{A}{s}e^{-2Ts} = \frac{A}{s}\left(1-e^{-Ts}\right)^2$$

(b). $f(t) = Ae^{-\alpha(t-T)}u(t-T)$

$$\mathscr{L}\left\{Ae^{-\alpha t}u(t)\right\} = \frac{A}{s+\alpha}$$

$$F(s) = \frac{A}{s+\alpha}e^{-Ts}$$

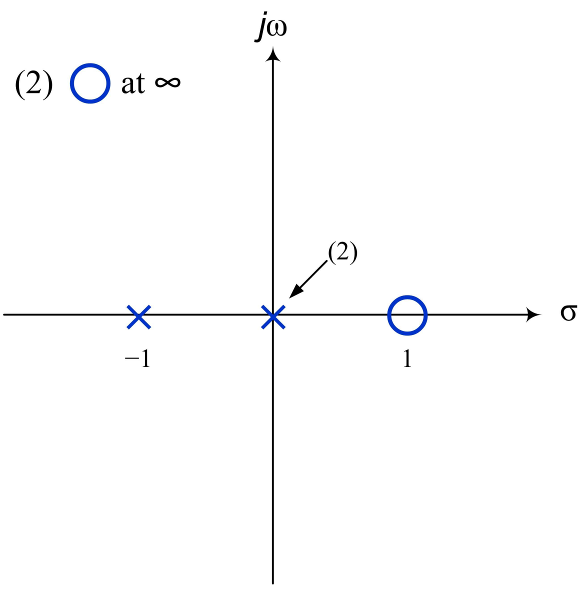
Exercise 9–14. Find the poles and zeros of the transform of the following waveform and plot the results on a pole-zero diagram.

$$f(t) = \left[-2e^{-t} - t + 2\right]u(t)$$

Compute the transform.

$$F(s) = \frac{-2}{s+1} - \frac{1}{s^2} + \frac{2}{s} = \frac{-2s^2 - s - 1 + 2s^2 + 2s}{s^2(s+1)} = \frac{s-1}{s^2(s+1)}$$

The zeros are at 1, ∞, and ∞. The poles are at -1, 0, and 0. The pole-zero diagram is shown below.



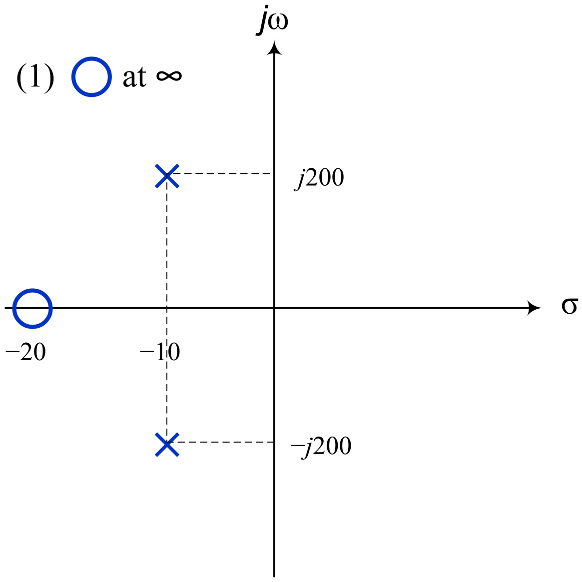
Exercise 9–15. Find the poles and zeros of the transform of the following waveform and plot the results on a pole-zero diagram.

$$f(t) = \left[e^{-10t}\cos(200t) + 0.05e^{-10t}\sin(200t)\right]u(t)$$

Compute the transform.

$$F(s) = \frac{s+10}{(s+10)^2+(200)^2} + \frac{(0.05)(200)}{(s+10)^2+(200)^2} = \frac{s+20}{(s+10)^2+(200)^2}$$

The zeros are at -20 and ∞. The poles are at $-10 \pm j200$. The pole-zero diagram is shown below.



Exercise 9–16. Find the waveforms corresponding to the following transforms:

(a). $F_1(s) = \dfrac{4}{(s+1)(s+3)}$

Apply partial fraction expansion.

$$F_1(s) = \frac{4}{(s+1)(s+3)} = \frac{k_1}{s+1} + \frac{k_2}{s+3}$$

$$k_1 = (s+1)F_1(s)\Big|_{s=-1} = \frac{4}{s+3}\Big|_{s=-1} = \frac{4}{2} = 2$$

$$k_2 = (s+3)F_1(s)\Big|_{s=-3} = \frac{4}{s+1}\Big|_{s=-3} = \frac{4}{-2} = -2$$

$$F_1(s) = \frac{2}{s+1} - \frac{2}{s+3}$$

$$f_1(t) = \left[2e^{-t} - 2e^{-3t}\right]u(t)$$

(b). $F_2(s) = e^{-5s}\left[\dfrac{2s}{(s+1)(s+3)}\right]$.

Perform partial fraction expansion on the term in square brackets and then account for the e^{-5s} term.

$$F_2(s) = e^{-5s}\left[\frac{2s}{(s+1)(s+3)}\right] = e^{-5s}\left[\frac{k_1}{s+1} + \frac{k_2}{s+3}\right]$$

$$k_1 = (s+1)\frac{2s}{(s+1)(s+3)}\bigg|_{s=-1} = \frac{2s}{s+3}\bigg|_{s=-1} = \frac{-2}{2} = -1$$

$$k_2 = (s+3)\frac{2s}{(s+1)(s+3)}\bigg|_{s=-3} = \frac{2s}{s+1}\bigg|_{s=-3} = \frac{-6}{-2} = 3$$

$$F_2(s) = e^{-5s}\left[\frac{-1}{s+1} + \frac{3}{s+3}\right]$$

$$f_2(t) = \left[-e^{-t} + 3e^{-3t}\right]u(t)\Big|_{t=t-5} = \left[-e^{-(t-5)} + 3e^{-3(t-5)}\right]u(t-5)$$

(c). $F_3(s) = \dfrac{4(s+2)}{(s+1)(s+3)}$.

Apply partial fraction expansion.

$$F_3(s) = \frac{4(s+2)}{(s+1)(s+3)} = \frac{k_1}{s+1} + \frac{k_2}{s+3}$$

$$k_1 = (s+1)F_3(s)\Big|_{s=-1} = \frac{4(s+2)}{s+3}\bigg|_{s=-1} = \frac{4}{2} = 2$$

$$k_2 = (s+3)F_3(s)\Big|_{s=-3} = \frac{4(s+2)}{s+1}\bigg|_{s=-3} = \frac{-4}{-2} = 2$$

$$F_3(s) = \frac{2}{s+1} + \frac{2}{s+3}$$

$$f_3(t) = \left[2e^{-t} + 2e^{-3t}\right]u(t)$$

Exercise 9–17. Find the waveforms corresponding to the following transforms:

(a). $F(s) = \dfrac{6(s+2)}{s(s+1)(s+4)}$

Apply partial fraction expansion.

$$F(s) = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+4}$$

$$k_1 = \frac{(6)(2)}{(1)(4)} = 3$$

$$k_2 = \frac{(6)(1)}{(-1)(3)} = -2$$

$$k_3 = \frac{(6)(-2)}{(-4)(-3)} = -1$$

$$f(t) = \left[3 - 2e^{-t} - e^{-4t}\right]u(t)$$

(b). $F(s) = \dfrac{4(s+1)}{s(s+1)(s+4)}$

Apply partial fraction expansion.

$$F(s) = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+4}$$

$$k_1 = \frac{(4)(1)}{(1)(4)} = 1$$

$$k_2 = \frac{(4)(0)}{(-1)(3)} = 0$$

$$k_3 = \frac{(4)(-3)}{(-4)(-3)} = -1$$

$$f(t) = \left[1 - e^{-4t}\right] u(t)$$

Exercise 9–18. Find the inverse transforms of the following rational functions:

(a). $F(s) = \dfrac{16}{(s+2)(s^2+4)}$

Apply partial fraction expansion.

$$F(s) = \frac{k_1}{s+2} + \frac{k_2}{s-j2} + \frac{k_2^*}{s+j2}$$

$$k_1 = \left.\frac{16}{s^2+4}\right|_{s=-2} = \frac{16}{8} = 2$$

$$k_2 = \left.\frac{16}{(s+2)(s+j2)}\right|_{s=j2} = \frac{16}{(2+j2)(j4)} = -1 - j = \sqrt{2}\angle{-3\pi/4} = \sqrt{2}e^{-j3\pi/4}$$

$$f(t) = \left[2e^{-2t} + 2\sqrt{2}\cos(2t - 3\pi/4)\right] u(t)$$

(b). $F(s) = \dfrac{2(s+2)}{s(s^2+4)}$

Apply partial fraction expansion.

$$F(s) = \frac{k_1}{s} + \frac{k_2}{s-j2} + \frac{k_2^*}{s+j2}$$

$$k_1 = \left.\frac{2(s+2)}{s^2+4}\right|_{s=0} = \frac{4}{4} = 1$$

$$k_2 = \left.\frac{2(s+2)}{s(s+j2)}\right|_{s=j2} = \frac{2(2+j2)}{j2(j4)} = -0.5 - j0.5 = \frac{1}{2}\sqrt{2}\angle{-3\pi/4} = \frac{1}{2}\sqrt{2}e^{-j3\pi/4}$$

$$f(t) = \left[1 + \sqrt{2}\cos(2t - 3\pi/4)\right] u(t)$$

Exercise 9–19. Find the inverse transforms of the following rational functions:

(a). $F(s) = \dfrac{8}{s(s^2+4s+8)}$

Apply partial fraction expansion

$$F(s) = \frac{k_1}{s} + \frac{k_2}{s+2-j2} + \frac{k_2^*}{s+2+j2}$$

$$k_1 = \left.\frac{8}{s^2+4s+8}\right|_{s=0} = \frac{8}{8} = 1$$

$$k_2 = \left.\frac{8}{s(s+2+j2)}\right|_{s=-2+j2} = \frac{8}{(-2+j2)(j4)} = \frac{1}{-1-j} = \frac{\sqrt{2}}{2}e^{j3\pi/4}$$

$$f(t) = \left[1+\sqrt{2}e^{-2t}\cos(2t+3\pi/4)\right]u(t)$$

(b). $F(s) = \dfrac{4s}{s^2+4s+8}$

Apply partial fraction expansion

$$F(s) = \frac{k}{s+2-j2} + \frac{k^*}{s+2+j2}$$

$$k = \left.\frac{4s}{s+2+j2}\right|_{s=-2+j2} = \frac{-8+j8}{j4} = 2+j2 = 2\sqrt{2}e^{j\pi/4}$$

$$f(t) = \left[4\sqrt{2}e^{-2t}\cos(2t+\pi/4)\right]u(t)$$

Exercise 9–20. Use the sum of residues to find the unknown residue in the following expansions:

(a). $\dfrac{21(s+5)}{(s+3)(s+10)} = \dfrac{6}{s+3} + \dfrac{k}{s+10}$

We have $n = m+1 = 2$, so the sum of residues is $K = 21$. If $6+k = 21$, the $k = 15$.

(b). $\dfrac{58s}{(s+2)(s^2+25)} = \dfrac{k}{s+2} + \dfrac{2+j5}{s+j5} + \dfrac{2-j5}{s-j5}$

We have $n = 3 > m+1 = 2$, so the sum of residues is zero. If $k+2+j5+2-j5 = 0$, then $k = -4$.

Exercise 9–21. Find the inverse transforms of the following functions:

(a). $F(s) = \dfrac{s^2+4s+5}{s^2+4s+3}$

Perform long division by rewriting the numerator of $F(s)$

$$F(s) = \frac{s^2+4s+5}{s^2+4s+3} = \frac{s^2+4s+3+2}{s^2+4s+3} = \frac{s^2+4s+3}{s^2+4s+3} + \frac{2}{s^2+4s+3}$$

$$= 1 + \frac{2}{s^2+4s+3} = 1 + \frac{k_1}{s+1} + \frac{k_2}{s+3}$$

$$k_1 = \frac{2}{2} = 1$$

$$k_2 = \frac{2}{-2} = -1$$

$$f(t) = \delta(t) + \left[e^{-t} - e^{-3t}\right]u(t)$$

(b). $F(s) = \dfrac{s^2 - 4}{s^2 + 4}$

Perform long division by rewriting the numerator of $F(s)$

$$F(s) = \frac{s^2 - 4}{s^2 + 4} = \frac{s^2 + 4 - 8}{s^2 + 4} = \frac{s^2 + 4}{s^2 + 4} + \frac{-8}{s^2 + 4} = 1 + \frac{-8}{s^2 + 4}$$

$$F(s) = 1 - 4\left(\frac{2}{s^2 + 2^2}\right)$$

$$f(t) = \delta(t) - 4\sin(2t)u(t)$$

Exercise 9–22. Find the inverse transforms of the following functions:

(a). $F(s) = \dfrac{2s^2 + 3s + 5}{s}$

Rewrite $F(s)$

$$F(s) = \frac{2s^2}{s} + \frac{3s}{s} + \frac{5}{s} = 2s + 3 + \frac{5}{s}$$

$$f(t) = 2\frac{d\delta(t)}{dt} + 3\delta(t) + 5u(t)$$

(b). $F(s) = \dfrac{s^3 + 2s^2 + s + 3}{s + 2}$

Perform long division.

$$\begin{array}{r|l} & s^2 \qquad + 1 \\ \hline s+2 & s^3 + 2s^2 + s + 3 \\ & -s^3 - 2s^2 \\ \hline & \qquad\qquad s + 3 \\ & \qquad\quad\; -s - 2 \\ \hline & \qquad\qquad\quad 1 \end{array}$$

Rewrite $F(s)$.

$$F(s) = s^2 + 1 + \frac{1}{s + 2}$$

$$f(t) = \frac{d^2\delta(t)}{dt^2} + \delta(t) + e^{-2t}u(t)$$

Exercise 9–23. Find the inverse transform of

$$F(s) = 400\frac{(s + 100)}{s(s + 200)^2}$$

Rewrite $F(s)$.

$$\begin{aligned}
F(s) &= \frac{400(s+100)}{s(s+200)^2} = \frac{1}{s+200}\left[\frac{400(s+100)}{s(s+200)}\right] \\
&= \frac{1}{s+200}\left[\frac{200}{s} + \frac{200}{s+200}\right] \\
&= \frac{200}{s(s+200)} + \frac{200}{(s+200)^2} \\
&= \frac{1}{s} + \frac{-1}{s+200} + \frac{200}{(s+200)^2} \\
f(t) &= \left[1 - e^{-200t} + 200te^{-200t}\right]u(t)
\end{aligned}$$

Exercise 9–24. Find the inverse transforms of the following functions:

(a). $F(s) = \dfrac{2}{(s+1)(s+2)^2}$

Rewrite $F(s)$.

$$\begin{aligned}
F(s) &= \frac{1}{s+2}\left[\frac{s}{(s+1)(s+2)}\right] = \frac{1}{s+2}\left[\frac{-1}{s+1} + \frac{2}{s+2}\right] \\
&= \frac{-1}{(s+1)(s+2)} + \frac{2}{(s+2)^2} = \frac{-1}{s+1} + \frac{1}{s+2} + \frac{2}{(s+2)^2} \\
f(t) &= \left[-e^{-t} + e^{-2t} + 2te^{-2t}\right]u(t)
\end{aligned}$$

(b). $F(s) = \dfrac{16}{s^2(s+4)}$

Rewrite $F(s)$.

$$\begin{aligned}
F(s) &= \frac{1}{s}\left[\frac{16}{s(s+4)}\right] = \frac{1}{s}\left[\frac{4}{s} + \frac{-4}{s+4}\right] \\
&= \frac{4}{s^2} + \frac{-4}{s(s+4)} = \frac{4}{s^2} + \frac{-1}{s} + \frac{1}{s+4} \\
f(t) &= \left[4t - 1 + e^{-4t}\right]u(t)
\end{aligned}$$

(c). $F(s) = \dfrac{800s(s+1)}{(s+2)(s+10)^2}$

Rewrite $F(s)$.

$$F(s) = \frac{1}{s+10}\left[\frac{800s(s+1)}{(s+2)(s+10)}\right]$$

$$\begin{array}{r}
800 \\
s^2+12s+20\overline{\smash{\big)}\;800s^2 \quad + 800s \qquad\qquad} \\
-\,800s^2 - 9600s - 16000 \\
\hline
-\,8800s - 16000
\end{array}$$

$$F(s) = \frac{1}{s+10}\left[800 + \frac{-8800s - 16000}{(s+2)(s+10)}\right] = \frac{1}{s+10}\left[800 + \frac{200}{s+2} + \frac{-9000}{s+10}\right]$$

$$= \frac{800}{s+10} + \frac{200}{(s+10)(s+2)} + \frac{-9000}{(s+10)^2}$$

$$= \frac{800}{s+10} + \frac{25}{s+2} + \frac{-25}{s+10} + \frac{-9000}{(s+10)^2}$$

$$f(t) = \left[25e^{-2t} + 775e^{-10t} - 9000te^{-10t}\right]u(t)$$

Exercise 9–25. Find the transform $F(s)$ from the pole-zero diagram of Figure 9–9. K is 3×10^4.

The seros are located at $s = -100 \pm j50$. The poles are located at $s = 0$, $s = -100$, and $s = -200$.

$$F(s) = \frac{K(s+100+j50)(s+100-j50)}{s(s+100)(s+200)^2} = \frac{30000(s^2+200s+12500)}{s(s+100)(s+200)^2}$$

Exercise 9–26. The inductor in Figure 9–11 is replaced by a capacitor C. The switch has been in position A for a long time. At $t = 0$ it is moved to position B. Find $V_C(s)$ and $v_C(t)$ for $t \geq 0$.

The initial condition is $v_C(0^-) = V_A$. After the switch moves, we have the following KVL relationship and results:

$$v_R(t) + v_C(t) = 0$$

$$Ri_C(t) + v_C(t) = 0$$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 0$$

$$\frac{dv_C(t)}{dt} + \frac{v_C(t)}{RC} = 0$$

$$\left[sV_C(s) - v_C(0^-)\right] + \frac{1}{RC}V_C(s) = 0$$

$$\left(s + \frac{1}{RC}\right)V_C(s) = V_A$$

$$V_C(s) = \frac{V_A}{s + \dfrac{1}{RC}} \text{ V-s}$$

$$v_C(t) = V_A e^{-t/RC}u(t) \text{ V}$$

Exercise 9–27. Find the transforms that satisfy the following equations and the given initial conditions.

(a). $\dfrac{dv(t)}{dt} + 6v(t) = 4u(t)$ V, $v(0^-) = -3$ V.

Write the equation in the Laplace domain.

$$sV(s) - (-3) + 6V(s) = \frac{4}{s}$$

$$(s+6)V(s) = \frac{-3s+4}{s}$$

$$V(s) = \frac{-3s+4}{s(s+6)} \text{ V-s}$$

(b). $4\dfrac{dv(t)}{dt} + 12v(t) = 16\cos(3t)$ V, $v(0^-) = 2$ V.

Write the equation in the Laplace domain.

$$\begin{aligned}
4\left[sV(s) - 2\right] + 12V(s) &= \frac{16s}{s^2+9} \\
(4s+12)V(s) &= \frac{16s}{s^2+9} + 8 \\
(s+3)V(s) &= \frac{4s}{s^2+9} + 2 \\
V(s) &= \frac{4s}{(s^2+9)(s+3)} + \frac{2}{s+3} \text{ V-s}
\end{aligned}$$

Exercise 9–28. Find the transforms that satisfy the following equations and the given initial conditions.

(a). $\int_0^t v(\tau)d\tau + 10v(t) = 10u(t)$ V.

Write the equation in the Laplace domain.

$$\begin{aligned}
\frac{1}{s}V(s) + 10V(s) &= \frac{10}{s} \\
(10s+1)V(s) &= 10 \\
V(s) &= \frac{10}{10s+1} = \frac{1}{s+\dfrac{1}{10}} \text{ V-s}
\end{aligned}$$

(b). $\dfrac{d^2v(t)}{dt^2} + 4\dfrac{dv(t)}{dt} + 3v(t) = 5e^{-2t}$ V, $v'(0^-) = 2$ V/s, $v(0^-) = -2$ V.

Write the equation in the Laplace domain.

$$\begin{aligned}
s^2V(s) - sv(0^-) - v'(0^-) + 4\left[sV(s) - v(0)\right] + 3V(s) &= \frac{5}{s+2} \\
(s^2+4s+3)V(s) &= \frac{5}{s+2} - 2s + 2 - 8 \\
(s+1)(s+3)V(s) &= \frac{5}{s+2} - 2s - 6 = \frac{5}{s+2} - 2(s+3) \\
V(s) &= \frac{5}{(s+1)(s+2)(s+3)} - \frac{2}{s+1} \text{ V-s}
\end{aligned}$$

Exercise 9–29. The RC circuit in Figure 9–13 has $R = 10$ kΩ, $C = 0.2$ μF, and $V_0 = -5$ V. The input is $v_S(t) = 10e^{-1000t}u(t)$ V. Find $v_C(t)$ for $t \geq 0$.

Follow the approach used in Example 9-18. The analysis and results are shown below.

$$\frac{1}{RC} = 500 \neq \alpha = 1000$$

$$\alpha RC = \frac{1000}{500} = 2$$

$$v_C(t) = \left[\frac{V_A}{1-\alpha RC}e^{-\alpha t} + \frac{V_A}{\alpha RC - 1}e^{-t/RC} + V_0 e^{-t/RC}\right]u(t)\ \text{V}$$

$$v_C(t) = \left[-10e^{-1000t} + 10e^{-500t} - 5e^{-500t}\right]u(t)$$

$$v_C(t) = \left[5e^{-500t} - 10e^{-1000t}\right]u(t)\ \text{V}$$

Exercise 9–30. The RL circuit of Figure 9–15 is in the zero state when the input $i_S(t) = [2\cos(1000t)]u(t)$ A is applied. Find $i_L(t)$ for $t \geq 0$.

We have the following results:

$$i_S(t) = i_R(t) + i_L(t)$$

$$i_S(t) = \frac{1}{R}v_L(t) + i_L(t)$$

$$i_S(t) = \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t)$$

$$\frac{2s}{s^2+1000^2} = \frac{L}{R}\left[sI_L(s) - i_L(0^-)\right] + I_L(s)$$

$$\left(s + \frac{R}{L}\right)I_L(s) = \frac{2\frac{R}{L}s}{s^2+1000^2}$$

$$I_L(s) = \frac{1000s}{(s+500)(s^2+1000^2)} = \frac{k_1}{s+500} + \frac{k_2}{s-j1000} + \frac{k_2^*}{s+j1000}$$

$$k_1 = \frac{(1000)(-500)}{(-500)^2+1000^2} = -0.4$$

$$k_2 = \frac{(1000)(j1000)}{(500+j1000)(j2000)} = \frac{1}{1+j2} = 0.2 - j0.4 = 0.447\angle{-63.4^\circ}$$

$$i_L(t) = \left[-0.4e^{-500t} + 0.894\cos(1000t - 63.4^\circ)\right]u(t)\ \text{A}$$

Exercise 9–31. Find the initial and final values of the waveforms corresponding to the following transforms:

(a). $F_1(s) = 100\dfrac{s+3}{s(s+5)(s+20)}$.

Apply the initial and final value properties.

$$f_1(0) = \lim_{s\to\infty} sF_1(s) = \lim_{s\to\infty}\left[100\frac{s+3}{(s+5)(s+20)}\right] = 0$$

$$f_1(\infty) = \lim_{s\to 0} sF_1(s) = \lim_{s\to 0}\left[100\frac{s+3}{(s+5)(s+20)}\right] = \frac{300}{100} = 3$$

(b). $F_2(s) = 80\dfrac{s(s+5)}{(s+4)(s+20)}$.

The function $F_2(s)$ is not a proper rational function, so we cannot determine the initial value.

$$f_2(\infty) = \lim_{s\to 0} sF_2(s) = \lim_{s\to 0}\left[80\frac{s^2(s+5)}{(s+4)(s+20)}\right] = 0$$

9.2 Problem Solutions

Problem 9–1. Find the Laplace transform of $f(t) = 500[1 - e^{-100t}]u(t)$. Locate the poles and zeros of $F(s)$.

Use the table of Laplace transform pairs.

$$F(s) = \frac{500}{s} - \frac{500}{s+100}$$
$$= \frac{500(s+100) - 500s}{s(s+100)} = \frac{50000}{s(s+100)}$$

The zeros are located at $s = [\infty,\ \infty]$ and the poles are located at $s = [0,\ -100]$.

Problem 9–2. Find the Laplace transform of $f(t) = 20\sin(377t)u(t)$. Locate the poles and zeros of $F(s)$.

Use the table of Laplace transform pairs.

$$F(s) = \frac{20(377)}{s^2+377^2} = \frac{7540}{s^2+377^2}$$

The zeros are located at $s = [\infty,\ \infty]$ and the poles are located at $s = \pm j377$.

Problem 9–3. Find the Laplace transform of $f(t) = -10\delta(t) + 10u(t)$. Locate the poles and zeros of $F(s)$.

Use the table of Laplace transform pairs.

$$F(s) = -10 + \frac{10}{s} = \frac{10 - 10s}{s} = \frac{10(1-s)}{s}$$

The zero is located at $s = 1$ and the poles is located at $s = 0$.

Problem 9–4. Find the Laplace transform of $f(t) = 20[e^{-200t} - 2e^{-100t}]u(t)$. Locate the poles and zeros of $F(s)$.

Use the table of Laplace transform pairs.

$$F(s) = \frac{20}{s+200} - \frac{40}{s+100} = \frac{20s + 2000 - 40s - 8000}{(s+100)(s+200)} = \frac{-20(s+300)}{(s+100)(s+200)}$$

The zeros are located at $s = [-300,\ \infty]$ and the poles are located at $s = [-100,\ -200]$.

Problem 9–5. Find the Laplace transform of $f(t) = A[(B + \alpha t)e^{-\alpha t}]u(t)$. Locate the poles and zeros of $F(s)$.

Use the table of Laplace transform pairs.

$$f(t) = \left[ABe^{-\alpha t} + A\alpha te^{-\alpha t}\right]u(t)$$
$$F(s) = \frac{AB}{s+\alpha} + \frac{A\alpha}{(s+\alpha)^2} = \frac{ABs + AB\alpha + A\alpha}{(s+\alpha)^2}$$

The zeros are located at $s = [-\alpha(B+1)/B,\ \infty]$ and the poles are located at $s = [-\alpha,\ -\alpha]$.

Problem 9–6. Find the Laplace transform of $f(t) = 0.005[10 - 10\cos(1000t)]u(t)$. Locate the poles and zeros of $F(s)$.

Use the table of Laplace transform pairs.

$$F(s) = \frac{0.05}{s} - \frac{0.05s}{s^2+1000^2} = \frac{50000}{s(s^2+1000^2)}$$

The zeros are located at $s = [\infty,\ \infty,\ \infty]$ and the poles are located at $s = [0,\ \pm j1000]$.

Problem 9–7. Find the Laplace transform of $f(t) = 5[4\cos(50t) - 5\sin(50t)]u(t)$. Locate the poles and zeros of $F(s)$.

Use the table of Laplace transform pairs.

$$F(s) = \frac{20s}{s^2+2500} - \frac{25(50)}{s^2+2500} = \frac{20s-1250}{s^2+2500}$$

The zeros are located at $s = [62.5,\ \infty]$ and the poles are located at $s = \pm j50$.

Problem 9–8. Find the Laplace transform of $f(t) = \delta(t) - 200e^{-20t}\cos(200t)u(t)$. Locate the poles and zeros of $F(s)$.

Use the table of Laplace transform pairs.

$$F(s) = 1 - \frac{200(s+20)}{(s+20)^2+200^2} = \frac{s^2-160s+36400}{s^2+40s+40400}$$

The zeros are located at $s = 80 \pm j173.2$ and the poles are located at $s = -20 \pm j200$.

Problem 9–9. Find the Laplace transform of $f(t) = 10[3 - 5t - 2e^{-15}]u(t)$. Locate the poles and zeros of $F(s)$.

Use the table of Laplace transform pairs.

$$F(s) = \frac{30}{s} - \frac{50}{s^2} - \frac{20}{s+15} = \frac{30s(s+15) - 50(s+15) - 20s^2}{s^2(s+15)}$$

$$= \frac{10(s^2+40s-75)}{s^2(s+15)}$$

The zeros are located at $s = [-41.79,\ 1.79,\ \infty]$ and the poles are located at $s = [-15,\ 0,\ 0]$.

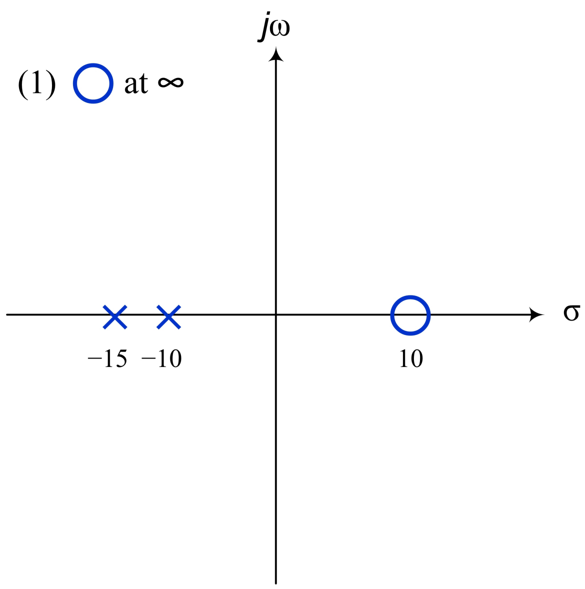
Problem 9–10. Find the Laplace transforms of the following waveforms and plot their pole-zero diagrams:

(a). $f_1(t) = [25e^{-15t} - 20e^{-10t}]u(t)$

Use the table of Laplace transform pairs.

$$F_1(s) = \frac{25}{s+15} - \frac{20}{s+10} = \frac{5(s-10)}{(s+10)(s+15)}$$

The pole-zero diagram is shown below

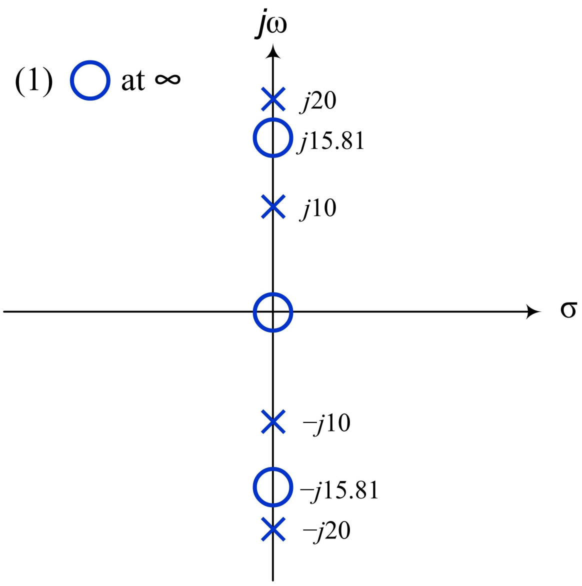


(b). $f_2(t) = 10[\cos(10t) + \cos(20t)]u(t)$

Use the table of Laplace transform pairs.

$$F_2(s) = \frac{10s}{s^2+100} + \frac{10s}{s^2+400} = \frac{20s(s^2+250)}{(s^2+100)(s^2+400)}$$

The pole-zero diagram is shown below



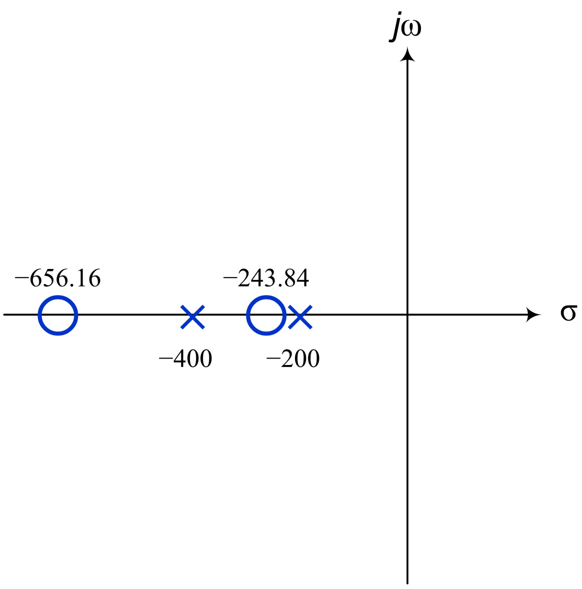
Problem 9–11. Find the Laplace transforms of the following waveforms and plot their pole-zero diagrams:

(a). $f_1(t) = 2\delta(t) + [200e^{-200t} + 400e^{-400t}]u(t)$

Use the table of Laplace transform pairs.

$$F_1(s) = 2 + \frac{200}{s+200} + \frac{400}{s+400} = \frac{2(s^2+900s+160000)}{(s+200)(s+400)}$$

The pole-zero diagram is shown below

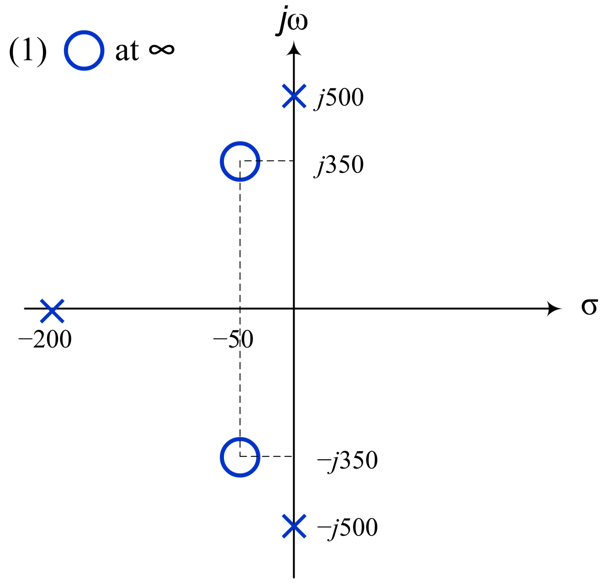


(b). $f_2(t) = [15e^{-200t} + 15\cos(500t)]u(t)$

Use the table of Laplace transform pairs.

$$F_2(s) = \frac{15}{s+200} + \frac{15s}{s^2+500^2} = \frac{30(s^2+100s+125000)}{(s+200)(s^2+500^2)}$$

The pole-zero diagram is shown below



Problem 9–12. Find the Laplace transforms of the following waveforms. Locate the poles and zeros of $F(s)$. Use MATLAB to verify your results.

(a). $f_1(t) = 5\delta(t) + (625te^{-25t})u(t)$.

Use the table of Laplace transform pairs.

$$F_1(s) = 5 + \frac{625}{(s+25)^2} = \frac{5(s^2+50s+750)}{(s+25)^2}$$

The zeros are located at $s = -25 \pm j11.18$ and the poles are located at $s = [-25,\ -25]$. The corresponding MATLAB code and output are shown below.

```
syms s t A B a
ft =5*dirac(t)+ (625*t*exp(-25*t))*heaviside(t)
Fs = simplify(laplace(ft))
Fs = factor(Fs)
z = roots([1 50 750])
```

```
Fs = 625/(s + 25)^2 + 5
Fs = (5*(s^2 + 50*s + 750))/(s + 25)^2
z =
   -25.0000e+000 + 11.1803e+000i
   -25.0000e+000 - 11.1803e+000i
```

(b). $f_2(t) = [10 + 5e^{-10t}(\cos(10t) + \sin(10t))]u(t)$.

Use the table of Laplace transform pairs.

$$F_2(s) = \frac{10}{s} + \frac{5(s+10)}{(s+10)^2+100} + \frac{50}{(s+10)^2+100} = \frac{15s^2+300s+2000}{s(s^2+20s+200)}$$

The zeros are located at $s = [-10 \pm j5.77,\ \infty]$ and the poles are located at $s = [0,\ -10 \pm j10]$. The corresponding MATLAB code and output are shown below.

```
syms s t A B a
ft = (10+5*exp(-10*t)*(cos(10*t)+sin(10*t)))*heaviside(t);
Fs = simplify(laplace(ft))
Fs = factor(Fs)
z = roots([15 300 2000])
```

```
Fs = 10/s + (5*s + 100)/(s^2 + 20*s + 200)
Fs = (5*(3*s^2 + 60*s + 400))/(s*(s^2 + 20*s + 200))
z =
   -10.0000e+000 +  5.7735e+000i
   -10.0000e+000 -  5.7735e+000i
```

Problem 9–13. Find the Laplace transforms of the following waveforms. Use MATLAB to verify your results.

(a). $f_1(t) = 5\delta(t-2)$.

Use the table of Laplace transform pairs.

$$\mathscr{L}\left\{5\delta(t)\right\} = 5$$

$$F_1(s) = 5e^{-2s}$$

The following MATLAB code and output verify the results.

```
syms s t A B a
ft =5*dirac(t-2);
Fs = simplify(laplace(ft))
Fs = factor(Fs)
```

```
Fs = 5/exp(2*s)
Fs = 5/exp(2*s)
```

(b). $f_2(t) = 10e^{-50(t-1)}u(t-1)$.

Use the table of Laplace transform pairs.

$$\mathscr{L}\left\{10e^{-50t}u(t)\right\} = \frac{10}{s+50}$$

$$F_2(s) = \frac{10e^{-s}}{s+50}$$

The following MATLAB code and output verify the results.

```
syms s t A B a
ft = (10*exp(-50*(t-1)))*heaviside(t-1);
Fs = simplify(laplace(ft))
Fs = factor(Fs)
```

```
Fs = 10/(exp(s)*(s + 50))
Fs = 10/(exp(s)*(s + 50))
```

(c). $f_3(t) = 20e^{-50(t-10)}u(t-10)$.

Use the table of Laplace transform pairs.

$$\mathscr{L}\left\{20e^{-50t}u(t)\right\} = \frac{20}{s+50}$$

$$F_3(s) = \frac{20e^{-10s}}{s+50}$$

The following MATLAB code and output verify the results.

```
syms s t A B a
ft = 20*exp(-50*(t-10))*heaviside(t-10);
Fs = simplify(laplace(ft))
FS = factor(Fs)
```

```
Fs = 20/(exp(10*s)*(s + 50))
FS = 20/(exp(10*s)*(s + 50))
```

Problem 9–14. Use MATLAB to find the Laplace transform of the following waveform

$$f(t) = [10 + 2e^{-10t}]u(t) + [5\cos(100(t-0.05))]u(t-0.05)$$

The MATLAB code and results are shown below.

```
syms s t A B a
ft = (10+2*exp(-10*t))*heaviside(t) + ...
    (5*cos(100*(t-0.05)))*heaviside(t-0.05);
Fs = simplify(laplace(ft))
```

```
Fs = 2/(s + 10) + 10/s + (5*s)/(exp(s/20)*(s^2 + 10000))
```

The result is:

$$F(s) = \frac{10}{s} + \frac{2}{s+10} + \frac{5se^{-s/20}}{s^2+100^2}$$

Problem 9–15. Find the Laplace transforms of the following waveforms.

(a). $f_1(t) = \dfrac{d}{dt}[50e^{-1000t}\cos(200000t)]u(t)$.

Use the table of Laplace transform pairs.

$$F(s) = \mathscr{L}\left\{50e^{-1000t}\cos(200000t)u(t)\right\} = \frac{50(s+1000)}{(s+1000)^2+(200000)^2}$$

$$F_1(s) = sF(s) - f(0^-) = \frac{50s(s+1000)}{(s+1000)^2+(200000)^2} - 50 = \frac{-50000(s+40001000)}{s^2+2000s+40001000000}$$

(b). $f_2(t) = \int_0^t 20e^{-10x}dx + 10u(t) + 20\dfrac{de^{-10t}}{dt}u(t)$.

Use the table of Laplace transform pairs.

$$F_2(s) = \frac{1}{s}\mathscr{L}\left\{20e^{-10t}\right\} + \frac{10}{s} + 20\left[s\mathscr{L}\left\{e^{-10t}\right\} - 1\right]$$

$$= \frac{20}{s(s+10)} + \frac{10}{s} + 20\left[\frac{s}{s+10} - 1\right] = \frac{-10(19s-12)}{s(s+10)}$$

Problem 9–16. Consider the waveform in Figure P9–16.

(a). Write an expression for the waveform $f(t)$ using step and ramp functions.

The waveform is a ramp function with a slope of A/T multiplied by step functions.

$$f(t) = \frac{At}{T}\left[u(t) - u(t-T)\right] = \frac{At}{T}u(t) - \frac{At}{T}u(t-T)$$
$$= \frac{At}{T}u(t) - \frac{A(t-T+T)}{T}u(t-T) = \frac{At}{T}u(t) - \frac{A(t-T)}{T}u(t-T) - Au(t-T)$$

(b). Use the time-domain translation property to find the Laplace transform of the waveform $f(t)$ found in part (a).

Use the table of Laplace transform pairs and the simplified version of $f(t)$ developed in part (a).

$$F(s) = \frac{A}{Ts^2} - \frac{Ae^{-Ts}}{Ts^2} - \frac{Ae^{-Ts}}{s} = \frac{A(1 - e^{-Ts} - sTe^{-Ts})}{Ts^2}$$

(c). Verify the Laplace transform found in part (b) by applying the definition of the Laplace transformation in Eq. (9-2) to the waveform $f(t)$ found in part (a).

Use the definition of the Laplace transform and integration by parts.

$$F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt = \int_0^T \frac{At}{T}e^{-st}dt$$
$$= \left.\frac{-Ate^{-st}}{Ts}\right|_0^T - \int_0^T \frac{-Ae^{-st}}{Ts}dt = -\frac{Ae^{-sT}}{s} + 0 - \left.\frac{A}{Ts^2}e^{-st}\right|_0^T$$
$$= -\frac{Ae^{-Ts}}{s} - \frac{Ae^{-Ts}}{Ts^2} + \frac{A}{Ts^2}$$

Problem 9–17. Consider the waveform in Figure P9–17.

(a). Write an expression for the waveform $f(t)$ in Figure P9–17 using step functions.

We have the following expression:

$$f(t) = Au(t) - 2Au(t-T) + Au(t-2T)$$

(b). Use the time-domain translation property to find the Laplace transform of the waveform $f(t)$ found in part (a).

The Laplace transform is:

$$F(s) = \frac{A}{s} - \frac{2A}{s}e^{-Ts} + \frac{A}{s}e^{-2Ts} = \frac{A(1 - 2e^{-Ts} + e^{-2Ts})}{s}$$

(c). Verify the Laplace transform found in part (b) by applying the definition of the Laplace transformation in Eq. (9-2) to the waveform $f(t)$ found in part (a).

Use the definition of the Laplace transform.

$$F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt = \int_0^T Ae^{-st}dt - \int_T^{2T} Ae^{-st}dt$$
$$= -\frac{A}{s}e^{-Ts} + \frac{A}{s} + \frac{A}{s}e^{-2Ts} - \frac{A}{s}e^{-Ts}$$
$$= \frac{A}{s} - \frac{2A}{s}e^{-Ts} + \frac{A}{s}e^{-2Ts}$$

Problem 9–18. For the following waveform: $f(t) = [500 + 100e^{-500t}tsin(1000t)]u(t)$

(a). Find the Laplace transform of the waveform. Locate the poles and zeros of $F(s)$.

Use the Laplace transform table and Euler's identity.

$$F(s) = \frac{500}{s} + \mathscr{L}\left\{100e^{-500t}t\sin(1000t)\right\}$$

$$\sin(1000t) = \frac{1}{j2}\left[e^{j1000t} - e^{-j1000t}\right]$$

$$\mathscr{L}\left\{100e^{-500t}t\sin(1000t)\right\} = \mathscr{L}\left\{100te^{-500t}\frac{1}{j2}\left[e^{j1000t} - e^{-j1000t}\right]\right\}$$
$$= \mathscr{L}\left\{-j50te^{-500t+j1000t} + j50te^{-500t-j1000t}\right\}$$
$$= \frac{-j50}{(s+500-j1000)^2} + \frac{j50}{(s+500+j1000)^2}$$
$$= \frac{-j50(s+500+j1000)^2 + j50(s+500-j1000)^2}{[(s+500-j1000)(s+500+j1000)]^2}$$
$$= \frac{-j50[(s+500)^2 - 1000^2 + 2(j1000)(s+500)]}{[(s+500)^2+1000^2]^2}$$
$$+ \frac{j50[(s+500)^2 - 1000^2 - 2(j1000)(s+500)]}{[(s+500)^2+1000^2]^2}$$
$$= \frac{200000(s+500)}{[(s+500)^2+1000^2]^2}$$

$$F(s) = \frac{500}{s} + \frac{200000(s+500)}{[(s+500)^2+1000^2]^2}$$

(b). Validate your result using MATLAB.

The following MATLAB code and results verify the calculations above.

```
syms s t
ft = (500+100*exp(-500*t)*t*sin(1000*t))*heaviside(t);
Fs = laplace(ft)
```

```
Fs = (100000*(2*s + 1000))/((s + 500)^2 + 1000000)^2 + 500/s
```

Problem 9–19. Consider the waveform in Figure P9–19.

(a). Write an expression for the waveform $f(t)$ in Figure P9–19 using a delayed exponential.

The waveform has an amplitude of A, a time constant of $T_{\rm C}/2$ and a delay of $T_{\rm C}/2$. Without the delay, the expression for the waveform is

$$g(t) = Ae^{-2t/T_{\rm C}}u(t)$$

With the delay included, the waveform is

$$f(t) = g(t - T_{\rm C}/2) = Ae^{-2(t-T_{\rm C}/2)/T_{\rm C}}u(t - T_{\rm C}/2)$$

(b). Use the time-domain translation property to find the Laplace transform of the waveform $f(t)$ found in part (a).

Use the Laplace transform table.

$$G(s) = \frac{A}{s + \dfrac{2}{T_{\rm C}}}$$

$$F(s) = e^{-T_{\rm C}s/2}G(s) = \frac{Ae^{-T_{\rm C}s/2}}{s + \dfrac{2}{T_{\rm C}}}$$

(c). Verify the Laplace transform found in part (b) by applying the definition of the Laplace transformation in Eq. (9-2) to the waveform $f(t)$ found in part (a).

We have the following results:

$$\mathscr{L}\{f(t)\} = \int_{0^-}^{\infty} Ae^{-2(t-T_{\rm C}/2)/T_{\rm C}}u(t - T_{\rm C}/2)e^{-st}dt$$

$$\begin{aligned}
F(s) &= \int_{T_{\rm C}/2}^{\infty} Ae^{-2(t-T_{\rm C}/2)/T_{\rm C}}e^{-st}dt \\
&= \int_{T_{\rm C}/2}^{\infty} Ae^{-(2/T_{\rm C}+s)t+1}dt = \int_{T_{\rm C}/2}^{\infty} Aee^{-(2/T_{\rm C}+s)t}dt \\
&= \frac{Ae}{-(2/T_{\rm C}+s)}e^{-(2/T_{\rm C}+s)t}\Big|_{T_{\rm C}/2}^{\infty} \\
&= \frac{Ae}{-(2/T_{\rm C}+s)}\left[0 - e^{-(2/T_{\rm C}+s)T_{\rm C}/2}\right] \\
&= \frac{A}{s + \dfrac{2}{T_{\rm C}}}e^{1}e^{-1-T_{\rm C}s/2} \\
&= \frac{A}{s + \dfrac{2}{T_{\rm C}}}e^{-T_{\rm C}s/2}
\end{aligned}$$

Problem 9–20. Find the inverse Laplace transforms of the following functions:

(a). $F_1(s) = \dfrac{50}{s(s+50)}$

Use partial fraction expansion.

$$F_1(s) = \frac{k_1}{s} + \frac{k_2}{s+50}$$

$$k_1 = \frac{50}{50} = 1$$

$$k_2 = \frac{50}{-50} = -1$$

$$F_1(s) = \frac{1}{s} - \frac{1}{s+50}$$

$$f_1(t) = \left[1 - e^{-50t}\right] u(t)$$

(b). $F_2(s) = \dfrac{s+1}{(s+2)(s+3)}$

Use partial fraction expansion.

$$F_2(s) = \frac{k_1}{s+2} + \frac{k_2}{s+3}$$

$$k_1 = \frac{-1}{1} = -1$$

$$k_2 = \frac{-2}{-1} = 2$$

$$F_2(s) = \frac{-1}{s+2} + \frac{2}{s+3}$$

$$f_2(t) = [-e^{-2t} + 2e^{-3t}]u(t)$$

Problem 9–21. Find the inverse Laplace transforms of the following functions:

(a). $F_1(s) = \dfrac{s+30}{s(s+40)}$

Use partial fraction expansion.

$$F_1(s) = \frac{k_1}{s} + \frac{k_2}{s+40}$$

$$k_1 = \frac{30}{40} = 0.75$$

$$k_2 = \frac{-10}{-40} = 0.25$$

$$F_1(s) = \frac{0.75}{s} + \frac{0.25}{s+40}$$

$$f_1(t) = [0.75 + 0.25e^{-40t}]u(t) = \frac{1}{4}[3 + e^{-40t}]u(t)$$

(b). $F_2(s) = \dfrac{(s+10)(s+20)}{s(s+50)(s+100)}$

Use partial fraction expansion.

$$F_2(s) = \frac{k_1}{s} + \frac{k_2}{s+50} + \frac{k_3}{s+100}$$

$$k_1 = \frac{(10)(20)}{(50)(100)} = 0.04$$

$$k_2 = \frac{(-40)(-30)}{(-50)(50)} = -0.48$$

$$k_3 = \frac{(-90)(-80)}{(-100)(-50)} = 1.44$$

$$F_2(s) = \frac{0.04}{s} - \frac{0.48}{s+50} + \frac{1.44}{s+100}$$

$$f_2(t) = [0.04 - 0.48e^{-50t} + 1.44e^{-100t}]u(t) = \frac{1}{25}\left[1 - 12e^{-50t} + 36e^{-100t}\right]u(t)$$

Problem 9–22. Find the inverse Laplace transforms of the following functions:

(a). $F_1(s) = \dfrac{5000(s+1000)}{(s+500)(s+5000)}$

Use partial fraction expansion.

$$F_1(s) = \frac{k_1}{s+500} + \frac{k_2}{s+5000}$$

$$k_1 = \frac{(5000)(500)}{4500} = \frac{5000}{9}$$

$$k_2 = \frac{(5000)(-4000)}{-4500} = \frac{40000}{9}$$

$$F_1(s) = \frac{1}{9}\left[\frac{5000}{s+500} + \frac{40000}{s+5000}\right]$$

$$f_1(t) = \frac{1}{9}[5000e^{-500t} + 40000e^{-5000t}]u(t)$$

(b). $F_2(s) = \dfrac{5s^2}{(s+100)(s+500)}$

Perform long division.

$$\begin{array}{r|l} & 5 \\ \hline s^2 + 600s + 50000 & 5s^2 \\ & -5s^2 - 3000s - 250000 \\ \hline & \quad -3000s - 250000 \end{array}$$

Use partial fraction expansion.

$$F_2(s) = 5 + \frac{-3000s - 250000}{(s+100)(s+500)}$$

$$F_2(s) = 5 + \frac{k_1}{s+100} + \frac{k_2}{s+500}$$

$$k_1 = \frac{300000 - 250000}{400} = 125$$

$$k_2 = \frac{1500000 - 250000}{-400} = -3125$$

$$F_2(s) = 5 + \frac{125}{s+100} - \frac{3125}{s+500}$$

$$f_2(t) = 5\delta(t) + [125e^{-100t} - 3125e^{-500t}]u(t)$$

Problem 9–23. Find the inverse Laplace transforms of the following functions:

(a). $F_1(s) = \dfrac{900}{(s+10)^2 + 30^2}$

Use the Laplace transform table.

$$F_1(s) = (30)\frac{30}{(s+10)^2 + 30^2}$$

$$f_1(t) = 30e^{-10t}\sin(30t)u(t)$$

(b). $F_2(s) = \dfrac{3(s+10)}{(s+10)^2 + 30^2}$

Use the Laplace transform table.

$$F_2(s) = (3)\frac{(s+10)}{(s+10)^2 + 30^2}$$

$$f_2(t) = 3e^{-10t}\cos(30t)u(t)$$

Problem 9–24. Find the inverse Laplace transforms of the following functions and sketch their waveforms for $\beta > 0$:

(a). $F_1(s) = \dfrac{\beta(s+\beta)}{s(s^2+\beta^2)}$

Use partial fraction expansion.

$$F_1(s) = \frac{k_1}{s} + \frac{k_2 s + k_3}{s^2 + \beta^2}$$

$$k_1 s^2 + k_1\beta^2 + k_2 s^2 + k_3 s = \beta s + \beta^2$$

$$k_1 + k_2 = 0$$

$$k_3 = \beta$$

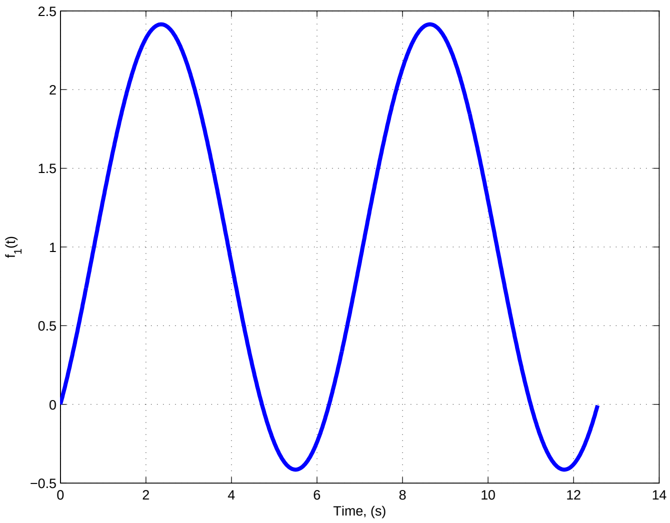
$$k_1 = 1$$

$$k_2 = -1$$

$$F_1(s) = \frac{1}{s} - \frac{s}{s^2 + \beta^2} + \frac{\beta}{s^2 + \beta^2}$$

$$f_1(t) = [1 - \cos(\beta t) + \sin(\beta t)]\, u(t)$$

A plot of the function with $\beta = 1$ is shown below.



(b). $F_2(s) = \dfrac{s(s+\beta)}{s^2 + \beta^2}$

Use partial fraction expansion.

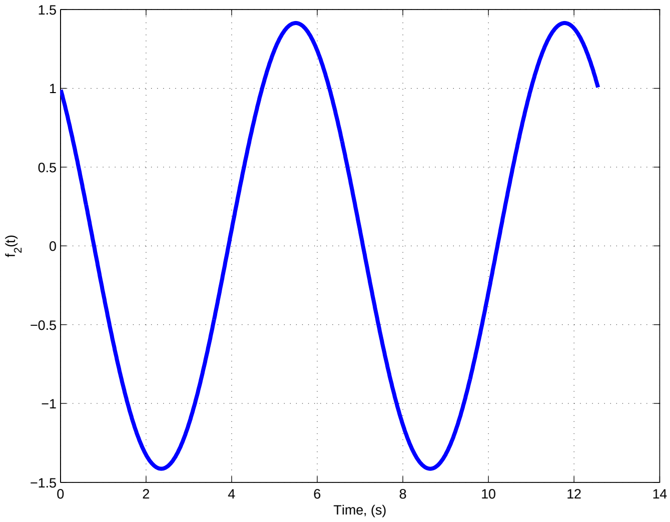
$$F_2(s) = \frac{s(s+\beta)}{s^2 + \beta^2} = \frac{s^2}{s^2 + \beta^2} + \frac{\beta s}{s^2 + \beta^2}$$

$$F_2(s) = \frac{s^2 + \beta^2 - \beta^2}{s^2 + \beta^2} + \frac{\beta s}{s^2 + \beta^2}$$

$$F_2(s) = 1 - \frac{\beta^2}{s^2 + \beta^2} + \frac{\beta s}{s^2 + \beta^2}$$

$$f_2(t) = \delta(t) + [-\beta \sin(\beta t) + \beta \cos(\beta t)]\, u(t)$$

A plot of the function with $\beta = 1$ is shown below.



Problem 9–25. Find the inverse Laplace transforms of the following functions:

(a). $F_1(s) = \dfrac{\alpha^2}{s^2(s+\alpha)}$

Use partial fraction expansion.

$$F_1(s) = \frac{1}{s}\left[\frac{k_1}{s} + \frac{k_2}{s+\alpha}\right]$$

$$k_1 = \frac{\alpha^2}{\alpha} = \alpha$$

$$k_2 = \frac{\alpha^2}{-\alpha} = -\alpha$$

$$F_1(s) = \frac{1}{s}\left[\frac{\alpha}{s} + \frac{-\alpha}{s+\alpha}\right]$$

$$F_1(s) = \frac{\alpha}{s^2} + \frac{-\alpha}{s(s+\alpha)} = \frac{\alpha}{s^2} + \frac{-1}{s} + \frac{1}{s+\alpha}$$

$$f_1(t) = [\alpha t - 1 + e^{-\alpha t}]u(t)$$

(b). $F_2(s) = \dfrac{\alpha^2}{s(s+\alpha)^2}$

Use partial fraction expansion.

$$F_2(s) = \frac{1}{s+\alpha}\left[\frac{k_1}{s} + \frac{k_2}{s+\alpha}\right]$$

$$k_1 = \frac{\alpha^2}{\alpha} = \alpha$$

$$k_2 = \frac{\alpha^2}{-\alpha} = -\alpha$$

$$F_2(s) = \frac{1}{s+\alpha}\left[\frac{\alpha}{s} + \frac{-\alpha}{s+\alpha}\right]$$

$$F_2(s) = \frac{-\alpha}{(s+\alpha)^2} + \frac{\alpha}{s(s+\alpha)} = \frac{-\alpha}{(s+\alpha)^2} + \frac{1}{s} + \frac{-1}{s+\alpha}$$

$$f_2(t) = [1 - \alpha t e^{-\alpha t} - e^{-\alpha t}]u(t)$$

Problem 9–26. Find the inverse Laplace transforms of the following functions:

(a). $F_1(s) = \dfrac{600}{(s+10)(s+20)(s+30)}$

Use partial fraction expansion.

$$F_1(s) = \frac{k_1}{s+10} + \frac{k_2}{s+20} + \frac{k_3}{s+30}$$

$$k_1 = \frac{600}{(10)(20)} = 3$$

$$k_2 = \frac{600}{(-10)(10)} = -6$$

$$k_3 = \frac{600}{(-20)(-10)} = 3$$

$$F_1(s) = \frac{3}{s+10} - \frac{6}{s+20} + \frac{3}{s+30}$$

$$f_1(t) = [3e^{-10t} - 6e^{-20t} + 3e^{-30t}]u(t)$$

(b). $F_2(s) = \dfrac{2(s+10)}{(s+15)(s+20)}$

Use partial fraction expansion.

$$F_2(s) = \frac{k_1}{s+15} + \frac{k_2}{s+20}$$

$$k_1 = \frac{(2)(-5)}{5} = -2$$

$$k_2 = \frac{(2)(-10)}{-5} = 4$$

$$F_2(s) = \frac{-2}{s+15} + \frac{4}{s+20}$$

$$f_2(t) = [-2e^{-15t} + 4e^{-20t}]u(t)$$

Problem 9–27. Find the inverse Laplace transforms of the following functions then validate your answers using MATLAB:

(a). $F_1(s) = \dfrac{16s}{(s+3)(s^2+11s+10)}$

Use partial fraction expansion.

$$F_1(s) = \frac{k_1}{s+3} + \frac{k_2}{s+1} + \frac{k_3}{s+10}$$

$$k_1 = \frac{-48}{(-2)(7)} = \frac{24}{7}$$

$$k_2 = \frac{-16}{(2)(9)} = -\frac{8}{9}$$

$$k_3 = \frac{-160}{(-7)(-9)} = -\frac{160}{63}$$

$$F_1(s) = \frac{24}{7}\frac{1}{s+3} - \frac{8}{9}\frac{1}{s+1} - \frac{160}{63}\frac{1}{s+10}$$

$$f_1(t) = \left[\frac{24}{7}e^{-3t} - \frac{8}{9}e^{-t} - \frac{160}{63}e^{-10t}\right]u(t)$$

The following MATLAB code and output verify the results.

```
syms s t
Fs = 16*s/(s+3)/(s^2+11*s+10);
ft = ilaplace(Fs)
```

```
ft = 24/(7*exp(3*t)) - 8/(9*exp(t)) - 160/(63*exp(10*t))
```

(b). $F_2(s) = \dfrac{5(s^2+9)}{s(s^2+25)}$

Use partial fraction expansion.

$$F_2(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 25}$$

$$As^2 + 25A + Bs^2 + Cs = 5s^2 + 45$$

$$A + B = 5$$

$$C = 0$$

$$25A = 45$$

$$A = \frac{9}{5}$$

$$B = \frac{16}{5}$$

$$F_2(s) = \frac{9}{5}\frac{1}{s} + \frac{16}{5}\frac{s}{s^2 + 25}$$

$$f_2(t) = \left[\frac{9}{5} + \frac{16}{5}\cos(5t)\right]u(t)$$

The following MATLAB code and output verify the results.

```
syms s t
Fs = 5*(s^2+9)/s/(s^2+25);
ft = ilaplace(Fs)
```

```
ft = (16*cos(5*t))/5 + 9/5
```

Problem 9–28. Find the inverse Laplace transforms of the following functions:

(a). $F_1(s) = \dfrac{(s + 10000)(s + 100000)}{s(s + 1000)(s + 50000)}$

Use partial fraction expansion.

$$F_1(s) = \frac{k_1}{s} + \frac{k_2}{s + 1000} + \frac{k_2}{s + 50000}$$

$$k_1 = \frac{(10000)(100000)}{(1000)(50000)} = 20$$

$$k_2 = \frac{(9000)(99000)}{(-1000)(49000)} = -\frac{891}{49}$$

$$k_3 = \frac{(-40000)(50000)}{(-50000)(-49000)} = -\frac{40}{49}$$

$$F_1(s) = \frac{20}{s} - \frac{891}{49}\frac{1}{s + 1000} - \frac{40}{49}\frac{1}{s + 50000}$$

$$f_1(t) = \left[20 - \frac{891}{49}e^{-1000t} - \frac{40}{49}e^{-50000t}\right]u(t)$$

(b). $F_2(s) = \dfrac{3(s^4 + 10s^2 + 4)}{s(s^2 + 1)(s^2 + 4)}$

Use partial fraction expansion.

$$F_2(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 1} + \frac{Ds + E}{s^2 + 4}$$

$$3(s^4 + 10s^2 + 4) = A(s^4 + 5s^2 + 4) + (Bs^2 + Cs)(s^2 + 4) + (Ds^2 + Es)(s^2 + 1)$$

$$A = 3$$

$$B = 5$$

$$C = 0$$

$$D = -5$$

$$E = 0$$

$$F_2(s) = \frac{3}{s} + \frac{5s}{s^2 + 1} + \frac{-5s}{s^2 + 4}$$

$$f_2(t) = [3 + 5\cos(t) - 5\cos(2t)]u(t)$$

Problem 9–29. Find the inverse Laplace transforms of the following functions:

(a). $F_1(s) = \dfrac{300(s + 50)}{s(s^2 + 40s + 300)}$

Use partial fraction expansion.

$$F_1(s) = \frac{k_1}{s} + \frac{k_2}{s + 10} + \frac{k_3}{s + 30}$$

$$k_1 = \frac{(300)(50)}{(10)(30)} = 50$$

$$k_2 = \frac{(300)(40)}{(-10)(20)} = -60$$

$$k_3 = \frac{(300)(20)}{(-30)(-20)} = 10$$

$$F_1(s) = \frac{50}{s} - \frac{60}{s + 10} + \frac{10}{s + 30}$$

$$f_1(t) = [50 - 60e^{-10t} + 10e^{-30t}]u(t)$$

(b). $F_2(s) = \dfrac{1000s}{(s + 5)(s^2 + 4s + 8)}$

Use partial fraction expansion.

$$F_2(s) = \frac{k_1}{s+5} + \frac{k_2}{s+2-j2} + \frac{k_2^*}{s+2+j2}$$

$$k_1 = \frac{(1000)(-5)}{25-20+8} = -\frac{5000}{13}$$

$$k_2 = \frac{(1000)(-2+j2)}{(3+j2)(j4)} = 192.31 + j38.46 = 196.12\angle 11.31^\circ$$

$$f_2(t) = \left[-\frac{5000}{13}e^{-5t} + 392.24e^{-2t}\cos(2t+11.31^\circ)\right]u(t)$$

$$f_2(t) = \frac{5000}{13}\left\{e^{-2t}\left[\cos(2t) - \frac{1}{5}\sin(2t)\right] - e^{-5t}\right\}u(t)$$

Problem 9–30. Find the inverse Laplace transforms of the following functions:

(a). $F_1(s) = \dfrac{16(s^2+256)}{s(s^2+8s+32)}$

Use partial fraction expansion.

$$F_1(s) = \frac{A}{s} + \frac{Bs+C}{s^2+8s+32}$$

$$16s^2 + 4096 = As^2 + 8As + 32A + Bs^2 + Cs$$

$$A + B = 16$$

$$8A + C = 0$$

$$32A = 4096$$

$$A = 128$$

$$B = -112$$

$$C = -1024$$

$$\begin{aligned} F_1(s) &= \frac{128}{s} + \frac{-112s - 1024}{s^2+8s+32} \\ &= \frac{128}{s} + \frac{-112(s+4) - 576}{(s+4)^2+4^2} \\ &= \frac{128}{s} - 112\frac{s+4}{(s+4)^2+4^2} - 144\frac{4}{(s+4)^2+4^2} \end{aligned}$$

$$f_1(t) = \left\{128 - e^{-4t}\left[112\cos(4t) + 144\sin(4t)\right]\right\}u(t)$$

(b). $F_2(s) = \dfrac{3(s^2+20s+400)}{s(s^2+50s+400)}$

Use partial fraction expansion.

$$F_2(s) = \frac{k_1}{s} + \frac{k_2}{s+10} + \frac{k_3}{s+40}$$

$$k_1 = \frac{(3)(400)}{(10)(40)} = 3$$

$$k_2 = \frac{(3)(100-200+400)}{(-10)(30)} = -3$$

$$k_3 = \frac{(3)(1600-800+400)}{(-40)(-30)} = 3$$

$$F_2(s) = \frac{3}{s} - \frac{3}{s+10} + \frac{3}{s+40}$$

$$f_2(t) = [3 - 3e^{-10t} + 3e^{-40t}]u(t)$$

Problem 9–31. Find the inverse Laplace transforms of the following functions then validate your answers using MATLAB:

(a). $F_1(s) = \dfrac{(s+100)^2}{(s+50)^2(s+200)}$

Use partial fraction expansion.

$$F_1(s) = \frac{1}{s+50}\left[\frac{(s+100)^2}{(s+50)(s+200)}\right] = \frac{1}{s+50}\left[1 + \frac{-50s}{(s+50)(s+200)}\right]$$

$$= \frac{1}{s+50}\left[1 + \frac{50/3}{s+50} + \frac{-200/3}{s+200}\right]$$

$$= \frac{50/3}{(s+50)^2} + \frac{1}{s+50} + \frac{-4/9}{s+50} + \frac{4/9}{s+200}$$

$$f_1(s) = \left[\frac{50}{3}te^{-50t} + \frac{5}{9}e^{-50t} + \frac{4}{9}e^{-200t}\right]u(t)$$

The following MATLAB code and output verify the results.

```
syms s t
Fs = (s+100)^2/(s+50)^2/(s+200);
ft = ilaplace(Fs)
```

```
ft =
5/(9*exp(50*t)) + 4/(9*exp(200*t)) + (50*t)/(3*exp(50*t))
```

(b). $F_2(s) = \dfrac{(s+50)^2}{(s+100)^2(s+200)}$

Use partial fraction expansion.

$$F_2(s) = \frac{1}{s+100}\left[\frac{s^2+100s+2500}{s^2+300s+20000}\right] = \frac{1}{s+100}\left[1+\frac{-200s-17500}{(s+100)(s+200)}\right]$$

$$= \frac{1}{s+100}\left[1+\frac{25}{s+100}+\frac{-225}{s+200}\right]$$

$$= \frac{25}{(s+100)^2}+\frac{1}{s+100}+\frac{-9/4}{s+100}+\frac{9/4}{s+200}$$

$$f_2(t) = \left[25te^{-100t}-\frac{5}{4}e^{-100t}+\frac{9}{4}e^{-200t}\right]u(t)$$

The following MATLAB code and output verify the results.

```
syms s t
Fs = (s+50)^2/(s+100)^2/(s+200);
ft = ilaplace(Fs)
```

```
ft =
9/(4*exp(200*t)) - 5/(4*exp(100*t)) + (25*t)/exp(100*t)
```

Problem 9–32. A certain transform has a simple pole at $s = -20$, a simple zero at $s = -\gamma$, and a scale factor of $K = 1$. Select values for γ so the inverse transform is

(a). $f(t) = \delta(t) - 5e^{-20t}$.

The transform has the form

$$F(s) = \frac{K(s+\gamma)}{s+20} = \frac{s+\gamma}{s+20}$$

Take the transform of $f(t)$

$$\mathscr{L}\{f(t)\} = \mathscr{L}\left\{\delta(t)-5e^{-20t}\right\} = 1-\frac{5}{s+20} = \frac{s+20-5}{s+20} = \frac{s+15}{s+20}$$

Therefore, $\gamma = 15$.

(b). $f(t) = \delta(t)$.

Take the transform of $f(t)$

$$\mathscr{L}\{f(t)\} = \mathscr{L}\{\delta(t)\} = 1 = \frac{s+20}{s+20}$$

Therefore, $\gamma = 20$.

(c). $f(t) = \delta(t) + 5e^{-20t}$.

Take the transform of $f(t)$

$$\mathscr{L}\{f(t)\} = \mathscr{L}\left\{\delta(t)+5e^{-20t}\right\} = 1+\frac{5}{s+20} = \frac{s+20+5}{s+20} = \frac{s+25}{s+20}$$

Therefore, $\gamma = 25$.

Problem 9–33. Find the inverse transforms of the following functions:

(a). $F_1(s) = \dfrac{s(s+10)(s+20)}{(s+5)(s+30)(s+50)}$.

Perform long division

$$\begin{array}{r|l} & 1 \\ \hline s^3+85s^2+1900s+7500 & s^3+30s^2 \quad +200s \\ & -s^3-85s^2-1900s-7500 \\ \hline & \quad -55s^2-1700s-7500 \end{array}$$

Use partial fraction expansion.

$$F_1(s) = 1 + \frac{-55s^2-1700s-7500}{(s+5)(s+30)(s+50)}$$

$$= 1 + \frac{-1/3}{s+5} + \frac{12}{s+30} + \frac{-200/3}{s+50}$$

$$f_1(t) = \delta(t) + \left[-\frac{1}{3}e^{-5t} + 12e^{-30t} - \frac{200}{3}e^{-50t}\right]u(t)$$

(b). $F_2(s) = \dfrac{(s+1000)(s+5000)}{(s+2000)}$.

Perform long division

$$\begin{array}{r|l} & \quad s \quad +4000 \\ \hline s+2000 & s^2+6000s+5000000 \\ & -s^2-2000s \\ \hline & \quad 4000s+5000000 \\ & \quad -4000s-8000000 \\ \hline & \quad -3000000 \end{array}$$

Use the Laplace transform table.

$$F_2(s) = s + 4000 - \frac{3000000}{s+2000}$$

$$f_2(t) = \frac{d\delta(t)}{dt} + 4000\delta(t) - 3000000e^{-2000t}u(t)$$

Problem 9–34. Find the inverse transforms of the following functions:

(a). $F_1(s) = \dfrac{s^2}{(s+5)}$.

Perform long division.

$$\begin{array}{r|l} & \quad s \quad -5 \\ \hline s+5 & s^2 \\ & -s^2-5s \\ \hline & \quad -5s \\ & \quad 5s+25 \\ \hline & \quad 25 \end{array}$$

Use the Laplace transform table.

$$F_1(s) = s - 5 + \frac{25}{s+5}$$

$$f_1(t) = \frac{d\delta(t)}{dt} - 5\delta(t) + 25e^{-5t}u(t)$$

(b). $F_2(s) = \dfrac{(s+1000)^2}{(s+2000)^2}$.

Perform long division.

$$\begin{array}{r|r} & 1 \\ \hline s^2+4000s+4000000 & s^2+2000s+1000000 \\ & -s^2-4000s-4000000 \\ \hline & -2000s-3000000 \end{array}$$

Use the Laplace transform table.

$$F_2(s) = 1 + \frac{-2000s - 3000000}{(s+2000)^2}$$

$$= 1 + \frac{A}{(s+2000)} + \frac{B}{(s+2000)^2}$$

$$As + 2000A + B = -2000s - 3000000$$

$$A = -2000$$

$$B = 1000000$$

$$F_2(s) = 1 - \frac{2000}{(s+2000)} + \frac{1000000}{(s+2000)^2}$$

$$f_2(t) = \delta(t) + \left[-2000e^{-2000t} + 1000000te^{-2000t}\right]u(t)$$

Problem 9–35. Find the inverse transforms of the following functions:

(a). $F_1(s) = \dfrac{e^{-5s}(s+20)}{(s+10)(s+30)}$.

Apply the time-domain translation after partial fraction expansion.

$$F_1(s) = e^{-5s}\left[\frac{1/2}{s+10} + \frac{1/2}{s+30}\right]$$

$$f_1(t) = \frac{1}{2}\left[e^{-10(t-5)} + e^{-30(t-5)}\right]u(t-5)$$

(b). $F_2(s) = \dfrac{se^{-5s} + 20}{(s+10)(s+30)}$.

Split the function to parts that have a time delay and parts that do not.

$$F_2(s) = e^{-5s}\left[\frac{s}{(s+10)(s+30)}\right] + \frac{20}{(s+10)(s+30)}$$

$$F_2(s) = e^{-5s}\left[\frac{-1/2}{s+10} + \frac{3/2}{s+30}\right] + \frac{1}{s+10} + \frac{-1}{s+30}$$

$$f_2(t) = \frac{1}{2}\left[-e^{-10(t-5)} + 3e^{-30(t-5)}\right]u(t-5) + \left[e^{-10t} - e^{-30t}\right]u(t)$$

(c). $F_3(s) = \dfrac{s+20e^{-5s}}{(s+10)(s+30)}$.

Split the function to parts that have a time delay and parts that do not.

$$F_3(s) = \left[\frac{s}{(s+10)(s+30)}\right] + e^{-5s}\left[\frac{20}{(s+10)(s+30)}\right]$$

$$F_3(s) = \left[\frac{-1/2}{s+10} + \frac{3/2}{s+30}\right] + e^{-5s}\left[\frac{1}{s+10} + \frac{-1}{s+30}\right]$$

$$f_3(t) = \frac{1}{2}\left[-e^{-10t} + 3e^{-30t}\right]u(t) + \left[e^{-10(t-5)} - e^{-30(t-5)}\right]u(t-5)$$

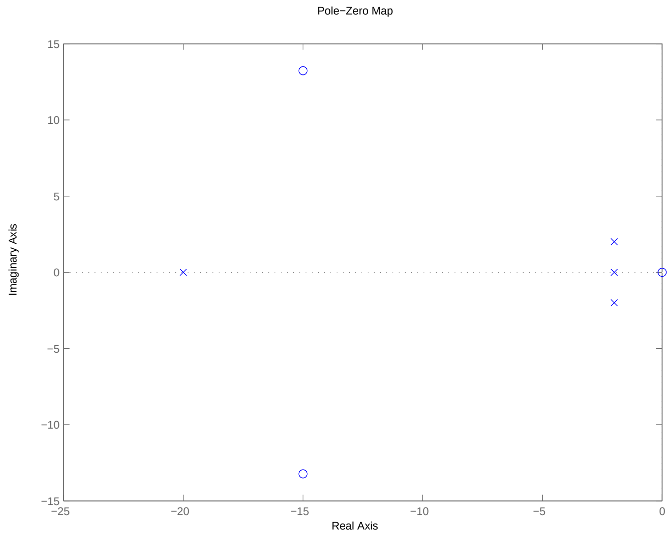
Problem 9–36. Use MATLAB to find the inverse transform and plot the poles and zeros of the following function:

$$F(s) = \frac{300s(s^2+30s+400)}{(s+20)(s^3+6s^2+16s+16)}$$

The MATLAB code and results are shown below.

```
syms s t
Fs = 300*s*(s^2+30*s+400)/(s+20)/(s^3+6*s^2+16*s+16);
ft = ilaplace(Fs)
z = roots([1 30 400 0])
p = roots(conv([1 20],[1 6 16 16]))

clear s
s = tf('s');
Fs = 300*s*(s^2+30*s+400)/(s+20)/(s^3+6*s^2+16*s+16);
pzplot(Fs)
```

```
ft = 25000/(123*exp(20*t)) - 8600/(3*exp(2*t))
     + (121500*(cos(2*t) + (373*sin(2*t))/405))/(41*exp(2*t))
z =
     0.0000e+000
   -15.0000e+000 + 13.2288e+000i
   -15.0000e+000 - 13.2288e+000i
p =
   -20.0000e+000
    -2.0000e+000 +  2.0000e+000i
    -2.0000e+000 -  2.0000e+000i
    -2.0000e+000
```



The corresponding waveform is

$$f(t) = \left\{ \frac{25000}{123} e^{-20t} - \frac{8600}{3} e^{-2t} + \frac{121500}{41} e^{-2t} \left[\cos(2t) + \frac{373}{405} \sin(2t) \right] \right\} u(t)$$

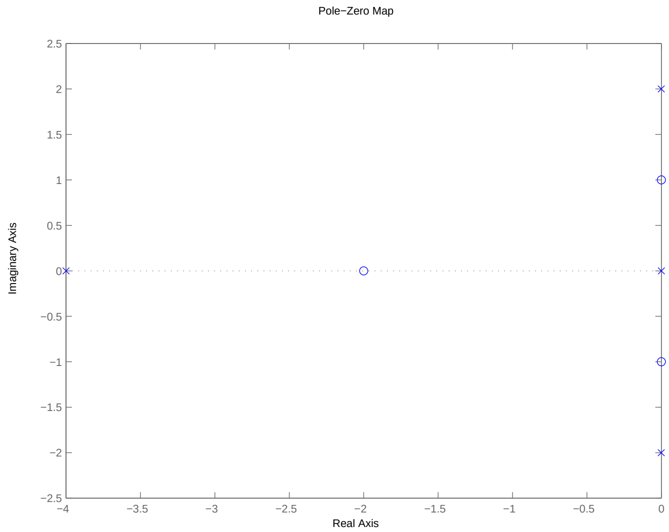
Problem 9–37. Use MATLAB to find the inverse transform and plot the poles and zeros of the following function:

$$F(s) = \frac{500(s^3 + 2s^2 + s + 2)}{s(s^3 + 4s^2 + 4s + 16)}$$

The MATLAB code and results are shown below.

```
syms s t
Fs = 500*(s^3+2*s^2+s+2)/s/(s^3+4*s^2+4*s+16);
ft = ilaplace(Fs)
z = roots([1 2 1 2])
p = roots([1 4 4 16 0])

clear s
s = tf('s');
Fs = 500*(s^3+2*s^2+s+2)/s/(s^3+4*s^2+4*s+16);
pzplot(Fs)
```

```
ft = 225*cos(2*t) + 425/(2*exp(4*t)) - 75*sin(2*t) + 125/2
z =
    -2.0000e+000
  -138.7779e-018 +  1.0000e+000i
  -138.7779e-018 -  1.0000e+000i
p =
     0.0000e+000
    -4.0000e+000
  -277.5558e-018 +  2.0000e+000i
  -277.5558e-018 -  2.0000e+000i
```



The corresponding waveform is

$$f(t) = \left[\frac{125}{2} + \frac{425}{2}e^{-4t} + 225\cos(2t) - 75\sin(2t)\right]u(t)$$

Problem 9–38. Find the transform $F(s)$ from the pole-zero diagram of Figure P9–38. K is 5.

With K given, we can write the transform by inspection.

$$F(s) = \frac{5(s+5)(s+10)}{s(s+5-j5)(s+5+j5)} = \frac{5(s+5)(s+10)}{s[(s+5)^2+25]}$$

Problem 9–39. Find the transform $F(s)$ from the pole-zero diagram of Figure P9–39. K is 5×10^6.

With K given, we can write the transform by inspection.

$$F(s) = \frac{5 \times 10^6(s)(s+100)^2}{[(s+50)^2+(250)^2][(s+100)^2+(500)^2]}$$

Problem 9–40. Find the transform $F(s)$ from the pole-zero diagram of Figure P9–40. K is 50.

With K given, we can write the transform by inspection.

$$F(s) = \frac{50(s)(s-50)(s+100)}{(s+50)(s^2+2500)}$$

Problem 9–41. Use the Laplace transformation to find the $v(t)$ that satisfies the following first-order differential equations:

(a). $250\dfrac{dv(t)}{dt} + 2500v(t) = 0$, $v(0^-) = 50$ V.

Convert the equation into the Laplace domain, solve for the transform, and then determine the inverse transform.

$$250\frac{dv(t)}{dt} + 2500v(t) = 0$$

$$250\left[sV(s) - 50\right] + 2500V(s) = 0$$

$$\left[sV(s) - 50\right] + 10V(s) = 0$$

$$(s+10)V(s) = 50$$

$$V(s) = \frac{50}{s+10}$$

$$v(t) = 50e^{-10t}u(t) \text{ V}$$

(b). $\dfrac{dv(t)}{dt} + 300v(t) = 600u(t)$, $v(0^-) = -150$ V.

Convert the equation into the Laplace domain, solve for the transform, and then determine the inverse transform.

$$\frac{dv(t)}{dt} + 300v(t) = 600u(t)$$

$$sV(s) - (-150) + 300V(s) = \frac{600}{s}$$

$$(s+300)V(s) = \frac{600}{s} - 150 = \frac{600 - 150s}{s}$$

$$V(s) = \frac{-150(s-4)}{s(s+300)} = \frac{2}{s} - \frac{152}{s+300}$$

$$v(t) = \left[2 - 152e^{-300t}\right]u(t) \text{ V}$$

Problem 9–42. Use the Laplace transformation to find the $i(t)$ that satisfies the following first-order differential equation:

$$\frac{di(t)}{dt} + 500i(t) = \left[0.100e^{-100t}\right]u(t), \quad i(0^-) = 0 \text{ A}$$

Convert the equation into the Laplace domain, solve for the transform, and then determine the inverse transform.

$$sI(s) - 0 + 500I(s) = \frac{0.1}{s+100}$$

$$(s+500)I(s) = \frac{0.1}{s+100}$$

$$I(s) = \frac{0.1}{(s+100)(s+500)} = \frac{1/4000}{s+100} - \frac{1/4000}{s+500}$$

$$i(t) = \frac{1}{4000}\left[e^{-100t} - e^{-500t}\right]u(t)$$

Problem 9–43. The switch in Figure P9–43 has been open for a long time and is closed at $t = 0$. The circuit parameters are $R = 1$ kΩ, $L = 100$ mH, and $V_{\text{A}} = 15$ V.

(a). Find the differential equation for the inductor current $i_{\rm L}(t)$ and initial condition $i_{\rm L}(0)$.

Since the switch has been open for a long time, the initial condition is zero, $i_{\rm L}(0) = 0$ A. Perform a source transformation and then write a KCL equation to develop the differential equation.

$$i_{\rm R}(t) + i_{\rm R}(t) + i_{\rm L}(t) = \frac{V_{\rm A}}{R}u(t)$$

$$\frac{v_{\rm R}(t)}{R} + \frac{v_{\rm R}(t)}{R} + i_{\rm L}(t) = \frac{V_{\rm A}}{R}u(t)$$

$$\frac{2}{R}v_{\rm R}(t) + i_{\rm L}(t) = \frac{V_{\rm A}}{R}u(t)$$

$$v_{\rm R}(t) = v_{\rm L}(t) = L\frac{di_{\rm L}(t)}{dt}$$

$$\frac{2L}{R}\frac{di_{\rm L}(t)}{dt} + i_{\rm L}(t) = \frac{V_{\rm A}}{R}u(t)$$

(b). Solve for $i_{\rm L}(t)$ using the Laplace transformation.

Convert the equation into the Laplace domain, solve for the transform, and then determine the inverse transform.

$$\frac{2L}{R}\left[sI_{\rm L}(s) - 0\right] + I_{\rm L}(s) = \frac{V_{\rm A}}{R}\frac{1}{s}$$

$$\left(s + \frac{R}{2L}\right)I_{\rm L}(s) = \frac{V_{\rm A}}{2L}\frac{1}{s}$$

$$(s + 5000)I_{\rm L}(s) = \frac{75}{s}$$

$$I_{\rm L}(s) = \frac{75}{s(s+5000)} = \frac{0.015}{s} - \frac{0.015}{s+5000}$$

$$i_{\rm L}(t) = 15\left(1 - e^{-5000t}\right)u(t) \text{ mA}$$

Problem 9–44. The switch in Figure P9–43 has been closed for a long time and is opened at $t = 0$. The circuit parameters are $R = 50\ \Omega$, $L = 200$ mH, and $V_{\rm A} = 50$ V.

(a). Find the differential equation for the inductor current $i_{\rm L}(t)$ and initial condition $i_{\rm L}(0)$.

Witht the switch closed, the inductor acts like a short circuit, so its initial current is $i_{\rm L}(0) = V_{\rm A}/R = 50/50 = 1$ A. Write the KCL equation after the swithc opens.

$$i_{\rm R}(t) + i_{\rm L}(t) = 0$$

$$\frac{1}{R}v_{\rm R}(t) + i_{\rm L}(t) = 0$$

$$\frac{L}{R}\frac{di_{\rm L}(t)}{dt} + i_{\rm L}(t) = 0$$

$$\frac{di_{\rm L}(t)}{dt} + \frac{R}{L}i_{\rm L}(t) = 0$$

$$\frac{di_{\rm L}(t)}{dt} + 250i_{\rm L}(t) = 0$$

(b). Solve for $i_{\rm L}(t)$ using the Laplace transformation.

Convert the equation into the Laplace domain, solve for the transform, and then determine the inverse transform.

$$sI_{\rm L}(s) - 1 + 250I_{\rm L}(s) = 0$$
$$(s+250)I_{\rm L}(s) = 1$$
$$I_{\rm L}(s) = \frac{1}{s+250}$$
$$i_{\rm L}(t) = e^{-250t}u(t) \text{ A}$$

Problem 9–45. The switch in Figure P9–45 has been open for a long time. At $t = 0$ the switch is closed.

(a). Find the differential equation for the capacitor voltage and initial condition $v_{\rm C}(0)$.

Since the switch has been open for a long time, the initial condition is zero, $v_{\rm C}(0^-) = 0$ V. Use node-voltage analysis to develop the differential equation. Let $R = 100$ kΩ and combine the two right resistors in series to get a single resistor $2R = 200$ kΩ.

$$\frac{v_{\rm C}(t) - v_{\rm S}(t)}{R} + i_{\rm C}(t) + \frac{v_{\rm C}(t)}{2R} = 0$$
$$C\frac{dv_{\rm C}(t)}{dt} + \frac{3}{2R}v_{\rm C}(t) = \frac{1}{R}v_{\rm S}(t)$$
$$\frac{dv_{\rm C}(t)}{dt} + \frac{3}{2RC}v_{\rm C}(t) = \frac{1}{RC}v_{\rm S}(t)$$
$$\frac{dv_{\rm C}(t)}{dt} + 15000v_{\rm C}(t) = 10000v_{\rm S}(t)$$

(b). Find $v_{\rm O}(t)$ using the Laplace transformation for $v_{\rm S}(t) = 25u(t)$ V.

Convert the equation into the Laplace domain, solve for the transform, and then determine the inverse transform. Apply voltage division to find $v_{\rm O}(t)$.

$$sV_{\rm C}(s) - 0 + 15000V_{\rm C}(s) = 10000V_{\rm S}(s)$$
$$(s+15000)V_{\rm C}(s) = 10000V_{\rm S}(s)$$
$$V_{\rm C}(s) = \frac{10000V_{\rm S}(s)}{s+15000}$$
$$V_{\rm C}(s) = \frac{250000}{s(s+15000)} = \frac{50/3}{s} - \frac{50/3}{s+15000}$$
$$v_{\rm C}(t) = \frac{50}{3}\left[1 - e^{-15000t}\right]u(t)$$
$$v_{\rm O}(t) = \frac{150}{150+50}v_{\rm C}(t) = 12.5\left[1 - e^{-15000t}\right]u(t) \text{ V}$$

Problem 9–46. Repeat Problem 9–45 for the input waveform $v_S(t) = 169[\cos(377t)]u(t)$ V.

The differential equation and initial condition do not change for a different source voltage. The general results are as follows.

$$V_C(s) = \frac{10000V_S(s)}{s + 15000}$$

$$V_O(s) = \frac{7500V_S(s)}{s + 15000}$$

Use these general results with the new voltage source.

$$V_O(s) = \left[\frac{7500}{s + 15000}\right]\left[\frac{169s}{s^2 + 377^2}\right]$$

$$V_O(s) = \frac{A}{s + 15000} + \frac{Bs + C}{s^2 + 377^2}$$

$$(7500)(169s) = As^2 + 377^2A + Bs^2 + 15000Bs + Cs + 15000C$$

$$A + B = 0$$

$$15000B + C = 1267500$$

$$142129A + 15000C = 0$$

$$A = -84.45$$

$$B = 84.45$$

$$C = 800.15$$

$$V_O(s) = \frac{-84.45}{s + 15000} + \frac{84.45s + 800.15}{s^2 + 377^2}$$

$$V_O(s) = \frac{-84.45}{s + 15000} + 84.45\frac{s}{s^2 + 377^2} + 2.12\frac{377}{s^2 + 377^2}$$

$$v_O(t) = \left[-84.45e^{-15000t} + 84.45\cos(377t) + 2.12\sin(377t)\right]u(t) \text{ V}$$

Problem 9–47. Repeat Problem 9–45 for the input waveform $v_S(t) = 24e^{-1000t}u(t)$ V.

The differential equation and initial condition do not change for a different source voltage. The general results are as follows.

$$V_C(s) = \frac{10000V_S(s)}{s + 15000}$$

$$V_O(s) = \frac{7500V_S(s)}{s + 15000}$$

Use these general results with the new voltage source.

$$V_O(s) = \left[\frac{7500}{s + 15000}\right]\left[\frac{24}{s + 1000}\right]$$

$$V_O(s) = \frac{-90/7}{s + 15000} + \frac{90/7}{s + 1000}$$

$$v_O(t) = \frac{90}{7}\left[e^{-1000t} - e^{-15000t}\right]u(t) \text{ V}$$

Problem 9–48. Use the Laplace transformation to find the $v(t)$ that satisfies the following second-order differential equation:

$$\frac{d^2v(t)}{dt^2} + 20\frac{dv(t)}{dt} + 1000v(t) = 0, \quad v(0^-) = 20 \text{ V and } \frac{dv(0^-)}{dt} = 0.$$

Convert the equation into the Laplace domain.

$$s^2V(s) - sv(0^-) - v'(0^-) + 20\left[sV(s) - v(0^-)\right] + 1000V(s) = 0$$

$$s^2V(s) - 20s + 20sV(s) - 400 + 1000V(s) = 0$$

$$(s^2 + 20s + 1000)V(s) = 20s + 400$$

$$V(s) = \frac{20s + 400}{s^2 + 20s + 1000}$$

$$V(s) = \frac{20(s+10) + 200}{(s+10)^2 + 30^2}$$

$$V(s) = 20\frac{s+10}{(s+10)^2 + 30^2} + \frac{20}{3}\frac{30}{(s+10)^2 + 30^2}$$

$$v(t) = 20e^{-10t}\left[\cos(30t) + \frac{1}{3}\sin(30t)\right]u(t) \text{ V}$$

Problem 9–49. Use the Laplace transformation to find the $v(t)$ that satisfies the following second-order differential equation:

$$\frac{d^2v(t)}{dt^2} + 50\frac{dv(t)}{dt} + 400v(t) = 0, \quad v(0^-) = 0 \text{ V and } \frac{dv(0^-)}{dt} = 1000 \text{ V/s}.$$

Convert the equation into the Laplace domain.

$$s^2V(s) - sv(0^-) - v'(0^-) + 50\left[sV(s) - v(0^-)\right] + 400V(s) = 0$$

$$s^2V(s) - 1000 + 50sV(s) + 400V(s) = 0$$

$$(s^2 + 50s + 400)V(s) = 1000$$

$$V(s) = \frac{1000}{s^2 + 50s + 400} = \frac{100/3}{s+10} - \frac{100/3}{s+40}$$

$$v(t) = \frac{100}{3}\left[e^{-10t} - e^{-40t}\right]u(t) \text{ V}$$

Problem 9–50. The switch in Figure P9–50 has been open for a long time and is closed at $t = 0$. The circuit parameters are $R = 500\ \Omega$, $L = 2.5$ H, $C = 2.5\ \mu$F, and $V_A = 1000$ V.

(a). Find the circuit differential equation in $i_L(t)$ and the initial conditions $i_L(0)$ and $v_C(0)$.

With the switch open, the capacitor acts as an open circuit and the inductor acts as a short circuit. The initial capacitor voltage is zero $v_C(0) = 0$ and the initial inductor current is $i_L(0) = V_A/2R = 1000/1000 = 1$ A. After the switch closes, the RLC parallel circuit is isolated from the source. Write a KCL equation to determine the differential equation.

$$v_{\rm L}(t) = L\frac{di_{\rm L}(t)}{dt}$$

$$i_{\rm R}(t) = \frac{v_{\rm L}(t)}{R} = \frac{L}{R}\frac{di_{\rm L}(t)}{dt}$$

$$i_{\rm C}(t) = C\frac{dv_{\rm C}(t)}{dt} = LC\frac{d^2 i_{\rm L}(t)}{dt^2}$$

$$i_{\rm C}(t) + i_{\rm R}(t) + i_{\rm L}(t) = 0$$

$$LC\frac{d^2 i_{\rm L}(t)}{dt^2} + \frac{L}{R}\frac{di_{\rm L}(t)}{dt} + i_{\rm L}(t) = 0$$

$$\frac{d^2 i_{\rm L}(t)}{dt^2} + \frac{1}{RC}\frac{di_{\rm L}(t)}{dt} + \frac{1}{LC}i_{\rm L}(t) = 0$$

$$\frac{d^2 i_{\rm L}(t)}{dt^2} + 800\frac{di_{\rm L}(t)}{dt} + 160000 i_{\rm L}(t) = 0$$

(b). Use Laplace transforms to solve for $i_{\rm L}(t)$ for $t \geq 0$.

Convert the equation into the Laplace domain.

$$s^2 I_{\rm L}(s) - s i_{\rm L}(0) - i'_{\rm L}(0) + 800\left[sI_{\rm L}(s) - i_{\rm L}(0)\right] + 160000 I_{\rm L}(s) = 0$$

$$(s^2 + 800s + 160000)I_{\rm L}(s) = s + 800$$

$$I_{\rm L}(s) = \frac{s+800}{(s+400)^2}$$

$$I_{\rm L}(s) = \frac{1}{s+400} + \frac{400}{(s+400)^2}$$

$$i_{\rm L}(t) = \left[e^{-400t} + 400te^{-400t}\right]u(t) \text{ A}$$

Problem 9–51. The switch in Figure P9–50 has been open for a long time and is closed at $t = 0$. The circuit parameters are $R = 500\ \Omega$, $L = 2.5$ H, $C = 2.5\ \mu$F, and $V_{\rm A} = 50$ V.

(a). Find the circuit differential equation in $v_{\rm C}(t)$ and the initial conditions $i_{\rm L}(0)$ and $v_{\rm C}(0)$.

With the switch open, the capacitor acts as an open circuit and the inductor acts as a short circuit. The initial capacitor voltage is zero $v_{\rm C}(0) = 0$ and the initial inductor current is $i_{\rm L}(0) = V_{\rm A}/2R = 50/1000 = 50$ mA. After the switch closes, the RLC parallel circuit is isolated from the source. Write a KCL equation to determine the differential equation.

$$i_{\rm C}(t) = C\frac{dv_{\rm C}(t)}{dt}$$

$$i_{\rm R}(t) = \frac{v_{\rm C}(t)}{R}$$

$$i_{\rm L}(t) = \frac{1}{L}\int_0^t v_{\rm C}(\tau)d\tau$$

$$i_{\mathrm{C}}(t) + i_{\mathrm{R}}(t) + i_{\mathrm{L}}(t) = 0$$

$$C\frac{dv_{\mathrm{C}}(t)}{dt} + \frac{v_{\mathrm{C}}(t)}{R} + \frac{1}{L}\int_0^t v_{\mathrm{C}}(\tau)d\tau = 0$$

$$C\frac{d^2v_{\mathrm{C}}(t)}{dt^2} + \frac{1}{R}\frac{dv_{\mathrm{C}}(t)}{dt} + \frac{1}{L}v_{\mathrm{C}}(t) = 0$$

$$\frac{d^2v_{\mathrm{C}}(t)}{dt^2} + \frac{1}{RC}\frac{dv_{\mathrm{C}}(t)}{dt} + \frac{1}{LC}v_{\mathrm{C}}(t) = 0$$

$$\frac{d^2v_{\mathrm{C}}(t)}{dt^2} + 800\frac{dv_{\mathrm{C}}(t)}{dt} + 160000v_{\mathrm{C}}(t) = 0$$

(b). Use Laplace transforms to solve for $v_{\mathrm{C}}(t)$ for $t \geq 0$.

First, determine the initial condition for the derivative of the capacitor voltage. The initial voltage is zero, so there is no current through the resistor at $t = 0$.

$$i_{\mathrm{C}}(0) = C\frac{dv_{\mathrm{C}}(0)}{dt} = -i_{\mathrm{L}}(0)$$

$$\frac{dv_{\mathrm{C}}(0)}{dt} = \frac{-i_{\mathrm{L}}(0)}{C} = \frac{-0.05}{2.5\,\mu} = -20000$$

Convert the differential equation into the Laplace domain.

$$s^2V_{\mathrm{C}}(s) - sv_{\mathrm{C}}(0) - v'_{\mathrm{C}}(0) + 800\,[sV_{\mathrm{C}}(s) - v_{\mathrm{C}}(0)] + 160000V_{\mathrm{C}}(s) = 0$$

$$(s^2 + 800s + 160000)V_{\mathrm{C}}(s) = -20000$$

$$V_{\mathrm{C}}(s) = \frac{-20000}{s^2 + 800s + 160000} = \frac{-20000}{(s+400)^2}$$

$$v_{\mathrm{C}}(t) = -20000te^{-400t}u(t)\text{ V}$$

Problem 9–52. The switch in Figure P9–52 has been closed for a long time and is opened at $t = 0$.

(a). Find the circuit differential equation in $v_{\mathrm{C}}(t)$ and the initial conditions $i_{\mathrm{L}}(0)$ and $v_{\mathrm{C}}(0)$.

With the switch closed, the series RLC part of the circuit is isolated from the source and both initial conditions are zero: $i_{\mathrm{L}}(0) = 0$ and $v_{\mathrm{C}}(0) = 0$. With the switch open, we have a series RLC circuit with a voltage source input. Let $R = R_1 + R_2$ to combine the resistors in series.

$$v_{\mathrm{R}}(t) + v_{\mathrm{L}}(t) + v_{\mathrm{C}}(t) = v_{\mathrm{S}}(t)$$

$$Ri_{\mathrm{C}}(t) + L\frac{di_{\mathrm{C}}(t)}{dt} + v_{\mathrm{C}}(t) = v_{\mathrm{S}}(t)$$

$$RC\frac{dv_{\mathrm{C}}(t)}{dt} + LC\frac{d^2v_{\mathrm{C}}(t)}{dt^2} + v_{\mathrm{C}}(t) = v_{\mathrm{S}}(t)$$

$$\frac{d^2v_{\mathrm{C}}(t)}{dt^2} + \frac{R}{L}\frac{dv_{\mathrm{C}}(t)}{dt} + \frac{1}{LC}v_{\mathrm{C}}(t) = \frac{1}{LC}v_{\mathrm{S}}(t)$$

(b). The circuit parameters are $L = 50$ H, $C = 0.25$ μF, $R_1 = 10$ kΩ, $R_2 = 20$ kΩ, and $v_S(t) = 10u(t)$ V. Use Laplace transforms and MATLAB to solve for $v_C(t)$ for $t \geq 0$.

Convert the differential equation into the Laplace domain.

$$s^2V_C(s) - sv_C(0) - v'_C(0) + \frac{R}{L}\left[sV_C(s) - v_C(0)\right] + \frac{1}{LC}V_C(s) = \frac{1}{LC}V_S(s)$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)V_C(s) = \frac{10}{LCs}$$

$$(s^2 + 600s + 80000)V_C(s) = \frac{800000}{s}$$

$$V_C(s) = \frac{800000}{s(s^2 + 600s + 80000)} = \frac{10}{s} - \frac{20}{s+200} + \frac{10}{s+400}$$

$$v_C(t) = \left[10 - 20e^{-200t} + 10e^{-400t}\right]u(t) \text{ V}$$

The MATLAB code and results are shown below.

```
syms s t
VCs = 800000/s/(s^2+600*s+80000);
vCt = ilaplace(VCs)
vCt2 = dsolve('12.5e-6*D2x+7.5e-3*Dx+x=10','x(0)=0','Dx(0)=0')
```

```
vCt =  10/exp(400*t) - 20/exp(200*t) + 10
vCt2 = 10/exp(400*t) - 20/exp(200*t) + 10
```

Problem 9–53. The switch in Figure P9–52 has been open for a long time and is closed at $t = 0$.

(a). Find the circuit differential equation in $i_L(t)$ and the initial conditions $i_L(0)$ and $v_C(0)$.

Assume the voltage source is a constant voltage to determine the initial conditions. With the switch open, no current flows, so $i_L(0) = 0$, and the capacitor voltage equals the source voltage $v_C(0) = v_S(0)$. With the switch closed, we have a series RLC circuit with no input. Apply KVL to develop the differential equation.

$$v_{R_2}(t) + v_L(t) + v_C(t) = 0$$

$$R_2i_L(t) + L\frac{di_L(t)}{dt} + \frac{1}{C}\int_0^t i_L(\tau)d\tau = 0$$

$$\frac{d^2i_L(t)}{dt} + \frac{R_2}{L}\frac{di_L(t)}{dt} + \frac{1}{LC}i_L(t) = 0$$

(b). The circuit parameters are $L = 50$ H, $C = 0.25$ μF, $R_1 = 10$ kΩ, $R_2 = 10$ kΩ, and $v_S(t) = 10u(t)$ V. Use Laplace transforms and MATLAB to solve for $i_L(t)$ for $t \geq 0$.

First, determine the initial condition for the derivative of the inductor current. The initial current is zero, so there is no voltage across the resistor at $t = 0$.

$$v_L(0) = L\frac{di_L(0)}{dt} = -v_C(0)$$

$$\frac{di_L(0)}{dt} = \frac{-v_C(0)}{L} = \frac{-10}{50} = -0.2$$

Convert the differential equation into the Laplace domain.

$$s^2 I_{\mathrm{L}}(s) - s i_{\mathrm{L}}(0) - i'_{\mathrm{L}}(0) + 200\left[s I_{\mathrm{L}}(s) - i_{\mathrm{L}}(0)\right] + 80000 I_{\mathrm{L}}(s) = 0$$

$$(s^2 + 200s + 80000) I_{\mathrm{L}}(s) = -0.2$$

$$I_{\mathrm{L}}(s) = \frac{-0.2}{s^2 + 200s + 80000} = \frac{-0.2}{(s+100)^2 + (100\sqrt{7})^2}$$

$$I_{\mathrm{L}}(s) = -\frac{\sqrt{7}}{3500}\left[\frac{100\sqrt{7}}{(s+100)^2 + (100\sqrt{7})^2}\right]$$

$$i_{\mathrm{L}}(t) = -\frac{\sqrt{7}}{3500}\left[e^{-100t}\sin(100\sqrt{7}t)\right]u(t)$$

The MATLAB code and results are shown below.

```
syms s t
ILs = -0.2/(s^2+200*s+80000);
iLt = ilaplace(ILs)
iLt2 = simplify(dsolve('D2x+200*Dx+80000*x=0','x(0)=0','Dx(0)=-0.2'))
```

```
iLt =  -(7^(1/2)*sin(100*7^(1/2)*t))/(3500*exp(100*t))
iLt2 = -(7^(1/2)*sin(100*7^(1/2)*t))/(3500*exp(100*t))
```

Problem 9–54. The *RLC* circuit in Figure P9–54 is in the zero state when at $t = 0$ an exponential source, $v_{\mathrm{S}}(t) = V_{\mathrm{A}} e^{-\alpha t}$ V, is suddenly connected to it.

(a). Find the circuit integrodifferential equation that describes the behavior of the current in the circuit.

Apply KVL to write the equation

$$v_{\mathrm{R}}(t) + v_{\mathrm{L}}(t) + v_{\mathrm{C}}(t) = v_{\mathrm{S}}(t)$$

$$R i_{\mathrm{L}}(t) + L\frac{di_{\mathrm{L}}(t)}{dt} + \frac{1}{C}\int_0^t i_{\mathrm{L}}(\tau)d\tau = v_{\mathrm{S}}(t)$$

(b). If $R = 100\ \Omega$, $L = 50$ mH, $C = 10\ \mu$F, $V_{\mathrm{A}} = 10$ V, and $\alpha = 500\ \mathrm{s}^{-1}$, use Laplace transforms and MATLAB to solve for $i(t)$ for $t \geq 0$.

Convert the equation into the Laplace domain. All of the initial conditions are zero, so do not include them in the conversion. Note that $i_{\mathrm{L}}(t) = i(t)$.

$$RI(s) + LsI(s) + \frac{1}{Cs}I(s) = V_{\mathrm{S}}(s)$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)I(s) = \left[\frac{s}{L}\right]\left[\frac{10}{s+500}\right]$$

$$\left(s^2 + 2000s + 2000000\right)I(s) = \frac{200s}{s+500}$$

$$I(s) = \frac{200s}{(s+500)(s^2+2000s+2000000)}$$

$$I(s) = \frac{A}{s+500} + \frac{Bs+C}{(s+1000)^2+1000^2}$$

$$200s = As^2 + 2000As + 2000000A + Bs^2 + 500Bs + Cs + 500C$$

$$A + B = 0$$

$$2000A + 500B + C = 200$$

$$2000000A + 500C = 0$$

$$A = -0.08$$

$$B = 0.08$$

$$C = 320$$

$$I(s) = \frac{-80}{s+500} + \frac{80s+320000}{(s+1000)^2+1000^2} \text{ mA}$$

$$I(s) = \frac{-80}{s+500} + 80\frac{s+1000}{(s+1000)^2+1000^2} + 240\frac{1000}{(s+1000)^2+1000^2} \text{ mA}$$

$$i(t) = \left\{-80e^{-500t} + e^{-1000t}\left[80\cos(1000t) + 240\sin(1000t)\right]\right\} u(t) \text{ mA}$$

The MATLAB code and results are shown below.

```
syms s t
R = 100;
L = 50e-3;
C = 10e-6;
VA = 10;
a = 500;
vSt = VA*exp(-a*t);
VSs = laplace(vSt)

ILs = s*VSs/L/(s^2+R*s/L+2e6)
iLt = ilaplace(ILs)
iLtnum = vpa(iLt,5)
```

```
VSs = 10/(s + 500)
ILs = (200*s)/((s + 500)*(s^2 + 2000*s + 2000000))
iLt = (2*(cos(1000*t) + 3*sin(1000*t)))/(25*exp(1000*t)) - 2/(25*exp(500*t))
iLtnum = (0.08*(cos(1000.0*t) + 3.0*sin(1000.0*t)))/exp(1000.0*t) - 0.08/exp(500.0*t)
```

Problem 9–55. Repeat problem 9–54 when an exponential source, $v_S(t) = 10(1 - e^{-500t})$ V, is suddenly connected to the circuit.

We have the following results.

$$I(s) = \frac{20s}{s^2 + 2000s + 2000000} V_S(s)$$

$$V_S(s) = \frac{10}{s} - \frac{10}{s+500} = \frac{5000}{s(s+500)}$$

$$I(s) = \left[\frac{20s}{s^2 + 2000s + 2000000}\right]\left[\frac{5000}{s(s+500)}\right]$$

$$I(s) = \frac{100000}{(s+500)(s^2 + 2000s + 2000000)}$$

$$I(s) = \frac{A}{s+500} + \frac{Bs + C}{(s+1000)^2 + 1000^2}$$

$$I(s) = \frac{80}{s+500} + \frac{-80s - 120000}{(s+1000)^2 + 1000^2} \text{ mA}$$

$$I(s) = \frac{80}{s+500} - 80\frac{s+1000}{(s+1000)^2 + 1000^2} - 40\frac{1000}{(s+1000)^2 + 1000^2} \text{ mA}$$

$$i(t) = \left\{80e^{-500t} - e^{-1000t}\left[80\cos(1000t) + 40\sin(1000t)\right]\right\} u(t) \text{ mA}$$

The MATLAB code and results are shown below.

```
syms s t
R = 100;
L = 50e-3;
C = 10e-6;
vSt = 10*(1-exp(-500*t));
VSs = laplace(vSt)

ILs = s*VSs/L/(s^2+R*s/L+2e6)
iLt = ilaplace(ILs)
iLtnum = vpa(iLt,5)
```

```
VSs = 10/s - 10/(s + 500)
ILs = -(20*s*(10/(s + 500) - 10/s))/(s^2 + 2000*s + 2000000)
iLt = 2/(25*exp(500*t)) - (2*(cos(1000*t) + sin(1000*t)/2))/(25*exp(1000*t))
iLtnum = 0.08/exp(500.0*t) - (0.08*(cos(1000.0*t) + 0.5*sin(1000.0*t)))/exp(1000.0*t)
```

Problem 9–56. Find $v_C(t)$ for $t \geq 0$ when the input to the RC circuit shown in Figure P9–56 is $v_S(t) = V_A r(t)$ V. Assume $v_C(0^-) = 0$ V.

Use KVL to develop the differential equation and then solve in the Laplace domain.

$$v_R(t) + v_C(t) = v_S(t)$$

$$Ri_C(t) + v_C(t) = V_A r(t)$$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = V_A r(t)$$

$$RCsV_C(s) + V_C(s) = \frac{V_A}{s^2}$$

$$\left(s + \frac{1}{RC}\right) V_C(s) = \frac{V_A/RC}{s^2}$$

$$V_C(s) = \frac{V_A/RC}{s^2(s+1/RC)}$$

$$V_C(s) = \frac{1}{s}\left[\frac{V_A}{s} - \frac{V_A}{s+1/RC}\right]$$

$$V_C(s) = \frac{V_A}{s^2} - \frac{V_A RC}{s} + \frac{V_A RC}{s+1/RC}$$

$$v_C(t) = \left[V_A t - V_A RC + V_A RC e^{-t/RC}\right] u(t)$$

The MATLAB code and results are shown below.

```
syms s t R C VA
VCs = VA/s^2/R/C/(s+1/R/C)
vCt = ilaplace(VCs)
```

```
VCs = VA/(C*R*s^2*(s + 1/(C*R)))
vCt = VA*t - C*R*VA + (C*R*VA)/exp(t/(C*R))
```

Problem 9–57. Use the initial and final value properties to find the initial and final values of the waveforms corresponding to the transforms below. If either property is not applicable, explain why.

(a). $F_1(s) = \dfrac{16(s+1)}{(s+2)(s+3)}$

Apply the initial and final value properties.

$$sF_1(s) = \frac{16s(s+1)}{(s+2)(s+3)}$$

$$f_1(0) = \lim_{s\to\infty} sF_1(s) = 16$$

$$f_1(\infty) = \lim_{s\to 0} sF_1(s) = 0$$

(b). $F_2(s) = \dfrac{s-100}{s(s+50)}$

Apply the initial and final value properties.

$$sF_2(s) = \frac{s-100}{s+50}$$

$$f_2(0) = \lim_{s\to\infty} sF_2(s) = 1$$

$$f_2(\infty) = \lim_{s\to 0} sF_2(s) = -2$$

Problem 9–58. Use the initial and final value properties to find the initial and final values of the waveforms corresponding to the transforms below. If either property is not applicable, explain why.

(a). $F_1(s) = \dfrac{8s^2}{(s+2)(s^2+12s+13)}$

Apply the initial and final value properties.

$$sF_1(s) = \frac{8s^3}{(s+2)(s^2+12s+13)}$$

$$f_1(0) = \lim_{s\to\infty} sF_1(s) = 8$$

$$f_1(\infty) = \lim_{s\to 0} sF_1(s) = 0$$

(b). $F_2(s) = \dfrac{s+1000}{s(s+50)(s+100)}$

Apply the initial and final value properties.

$$sF_2(s) = \frac{s+1000}{(s+50)(s+100)}$$

$$f_2(0) = \lim_{s\to\infty} sF_2(s) = 0$$

$$f_2(\infty) = \lim_{s\to 0} sF_2(s) = \frac{1}{5}$$

Problem 9–59. Use the initial and final value properties to find the initial and final values of the waveforms corresponding to the transforms below. If either property is not applicable, explain why.

(a). $F_1(s) = \dfrac{400s}{(s+10)^2+20^2}$

Apply the initial and final value properties.

$$sF_1(s) = \frac{400s^2}{(s+10)^2+20^2}$$

$$f_1(0) = \lim_{s\to\infty} sF_1(s) = 400$$

$$f_1(\infty) = \lim_{s\to 0} sF_1(s) = 0$$

(b). $F_2(s) = \dfrac{30(3s^4+10s^2+4)}{s(s^2+1)(s^2+4)}$

Apply the initial and final value properties.

$$sF_2(s) = \frac{30(3s^4+10s^2+4)}{(s^2+1)(s^2+4)}$$

$$f_2(0) = \lim_{s\to\infty} sF_2(s) = 90$$

The final value property does not apply in this case because $sF_2(s)$ has poles on the $j\omega$ axis, so it does not have a final value.

Problem 9–60. Use the initial and final value properties to find the initial and final values of the waveforms corresponding to the transforms below. If either property is not applicable, explain why.

(a). $F_1(s) = \dfrac{50s(s^2+5s+6)}{(s+2)(s+6)(s+12)}$

Apply the initial and final value properties.

$$sF_1(s) = \frac{50s^2(s^2+5s+6)}{(s+2)(s+6)(s+12)}$$

$$f_1(\infty) = \lim_{s\to 0} sF_1(s) = 0$$

The initial value property does not apply in this case because $F_1(s)$ is not a proper rational function.

(b). $F_2(s) = \dfrac{25(s^2+10s+40)}{s(s^2-625)}$

Apply the initial and final value properties.

$$sF_2(s) = \frac{25(s^2+10s+40)}{(s^2-625)}$$

$$f_2(0) = \lim_{s\to\infty} sF_2(s) = 25$$

The final value property does not apply in this case because $sF_2(s)$ has a pole in the right half plane at $s = 25$.

Problem 9–61. Use the initial and final value properties to find the initial and final values of the waveforms corresponding to the transforms below. If either property is not applicable, explain why.

(a). $F_1(s) = \dfrac{s(s+6)}{s^2+6s+9}$

Apply the initial and final value properties.

$$sF_1(s) = \frac{s^2(s+6)}{s^2+6s+9}$$

$$f_1(\infty) = \lim_{s\to 0} sF_1(s) = 0$$

The initial value property does not apply in this case because $F_1(s)$ is not a proper rational function.

(b). $F_2(s) = \dfrac{20(s^2+10s+100)}{s(s^2+20s+100)}$

Apply the initial and final value properties.

$$sF_2(s) = \frac{20(s^2+10s+100)}{(s^2+20s+100)}$$

$$f_2(0) = \lim_{s\to\infty} sF_2(s) = 20$$

$$f_2(\infty) = \lim_{s\to 0} sF_2(s) = 20$$

Problem 9–62. Use the initial and final value properties to find the initial and final values of the waveform corresponding to the following transform. If either property is not applicable, explain why.

$$F(s) = \frac{80(s^3+2s^2+s+2)}{s(s^3+4s^2+4s+16)}$$

Apply the initial and final value properties.

$$sF(s) = \frac{80(s^3 + 2s^2 + s + 2)}{(s^3 + 4s^2 + 4s + 16)}$$

$$f(0) = \lim_{s\to\infty} sF(s) = 80$$

The final value property does not apply in this case because $sF(s)$ has poles on the $j\omega$ axis, so it does not have a final value. The poles are located at $s = -4$ and $s = \pm j2$.

Problem 9–63. The MATLAB function `limit` can be used to take the limit of a symbolic expression. Use MATLAB and the initial and final value properties to find the initial and final values of the waveforms corresponding to the following transforms. If either property is not applicable, explain why. Use MATLAB again to compute the waveforms corresponding to each transform and then take limits in the time domain to verify the answers found using the initial and final value properties.

(a). $F_1(s) = \dfrac{(s + 200)^2}{(s + 20)^2(s + 100)}$

The MATLAB code and results are shown below.

```
syms s t
Fs = (s+200)^2/(s+20)^2/(s+100);
ft0 = limit(s*Fs,s,inf)
ftinf = limit(s*Fs,s,0)
ft = ilaplace(Fs)
ft02 = limit(ft,t,0)
ftinf2 = limit(ft,t,inf)
ValidAnswer = (ft0==ft02)&(ftinf==ftinf2)
```

```
ft0 = 1
ftinf = 0
ft = 25/(16*exp(100*t)) - 9/(16*exp(20*t)) + (405*t)/exp(20*t)
ft02 = 1
ftinf2 = 0
ValidAnswer = 1
```

The initial value is $f_1(0) = 1$ and the final value is $f_1(\infty) = 0$. Both approaches yield the same results.

(b). $F_2(s) = \dfrac{(s + 20)^2}{(s + 100)^2(s + 200)}$

The MATLAB code and results are shown below.

```
syms s t
Fs = (s+20)^2/(s+100)^2/(s+200);
ft0 = limit(s*Fs,s,inf)
ftinf = limit(s*Fs,s,0)
ft = ilaplace(Fs)
ft02 = limit(ft,t,0)
ftinf2 = limit(ft,t,inf)
ValidAnswer = (ft0==ft02)&(ftinf==ftinf2)
```

```
ft0 = 1
ftinf = 0
ft = 81/(25*exp(200*t)) - 56/(25*exp(100*t)) + (64*t)/exp(100*t)
ft02 = 1
ftinf2 = 0
ValidAnswer = 1
```

The initial value is $f_2(0) = 1$ and the final value is $f_2(\infty) = 0$. Both approaches yield the same results.

Problem 9–64. (A) The Dominant Pole Approximation

When a transform $F(s)$ has widely separated poles, then those closest to the $j\omega$-axis tend to dominate the response because they have less damping. An approximation to the waveform can be obtained by ignoring the contributions of all except the dominant poles. We can ignore the other poles simply by discarding their terms in the partial fraction expansion of $F(s)$. The purpose of this problem is to examine a dominant pole approximation of the transform

$$F(s) = 10^6 \frac{s + 4000}{(s + 10000)\left[(s + 25)^2 + 100^2\right]}$$

(a). Construct a partial-fraction expansion of $F(s)$ and find $f(t)$.

Apply partial fraction expansion.

$$F(s) = \frac{10^6(s + 4000)}{(s + 10000)\left[(s + 25)^2 + 100^2\right]}$$

$$F(s) = \frac{A}{s + 10000} + \frac{Bs + C}{(s + 25)^2 + 100^2}$$

$$10^6(s + 4000) = As^2 + 50As + 10625A + Bs^2 + 10000Bs + Cs + 10000C$$

$$A + B = 0$$

$$50A + 10000B + C = 10^6$$

$$10625A + 10000C = 4 \times 10^9$$

$$A = -60.295$$

$$B = 60.295$$

$$C = 400064$$

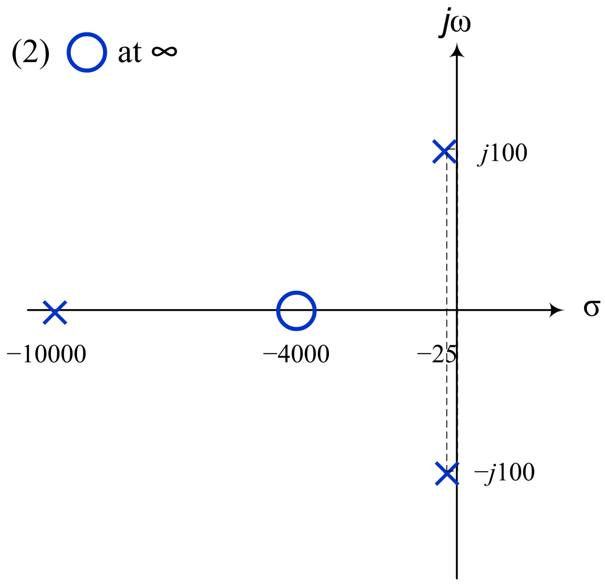
$$F(s) = \frac{-60.295}{s + 10000} + \frac{60.295s + 400064}{(s + 25)^2 + 100^2}$$

$$F(s) = \frac{-60.295}{s + 10000} + 60.295\frac{(s + 25)}{(s + 25)^2 + 100^2} + 3985.6\frac{100}{(s + 25)^2 + 100^2}$$

$$f(t) = \left\{-60.295e^{-10000t} + e^{-25t}\left[60.295\cos(100t) + 3985.6\sin(100t)\right]\right\}u(t)$$

(b). Construct a pole-zero diagram of $F(s)$ and identify the dominant poles.

The one zero is located at $s = -4000$ and two are located at infinity. The poles are located at $s = -10000$ and $s = -25 \pm j100$. The dominant poles are at $s = -25 \pm j100$. The pole-zero diagram approximation is shown below.



(c). Construct a dominant pole approximation $g(t)$ by discarding the other poles in the partial fraction expansion in part (a).

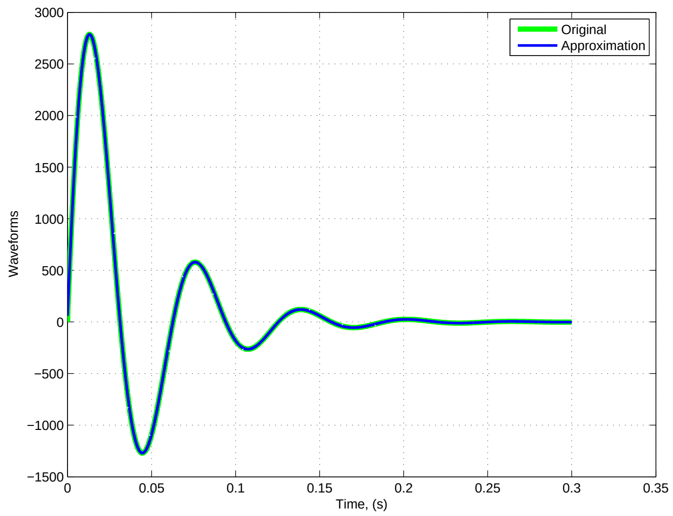
The new transform is $G(s)$.

$$G(s) = 60.295\frac{(s+25)}{(s+25)^2+100^2} + 3985.6\frac{100}{(s+25)^2+100^2}$$

$$g(t) = e^{-25t}\left[60.295\cos(100t) + 3985.6\sin(100t)\right]u(t)$$

(d). Plot $f(t)$ and $g(t)$ and comment on the accuracy of the approximation.

The plots are shown together in the following figure.



The original signal is plotted with a thick green line and the approximation is plotted with a thinner blue line. The waveforms are nearly identical, except for small differences during the initial portions of each signal. The differences are extremely small after 1 ms.

Problem 9–65. (A) First-Order Circuit Step Response

In Chapter 7 we found that the step response of a first-order circuit can be written as

$$f(t) = f(\infty) + [f(0) - f(\infty)]\, e^{-t/T_C}$$

where $f(0)$ is the initial value, $f(\infty)$ is the final value, and T_C is the time constant. Show that the corresponding transform has the form

$$F(s) = K\left[\frac{s+\gamma}{s(s+\alpha)}\right]$$

and relate the time-domain parameters $f(0)$, $f(\infty)$, and T_C to the s-domain parameters K, γ, and α.

Calculate the Laplace transform of $f(t)$.

$$\begin{aligned}
F(s) = \mathscr{L}\{f(t)\} &= \frac{f(\infty)}{s} + \frac{f(0)-f(\infty)}{s+1/T_C} \\
&= \frac{f(\infty)(s+1/T_C) + [f(0)-f(\infty)]s}{s(s+1/T_C)} \\
&= \frac{f(0)s + f(\infty)/T_C}{s(s+1/T_C)} \\
&= \frac{f(0)\left(s + \dfrac{f(\infty)}{f(0)T_C}\right)}{s\left(s+\dfrac{1}{T_C}\right)} \\
K &= f(0) \\
\gamma &= \frac{f(\infty)}{f(0)T_C} \\
\alpha &= \frac{1}{T_C}
\end{aligned}$$

Problem 9–66. (A) Inverse Transform for Complex Poles

In Section 9–4 we learned that complex poles occur in conjugate pairs and that for simple poles the partial fraction expansion of $F(s)$ will contain two terms of the form

$$F(s) = \cdots \frac{k}{s+\alpha-j\beta} + \frac{k^*}{s+\alpha+j\beta} + \cdots$$

Show that when the complex conjugate residues are written in rectangular form as

$$k = a + jb \quad \text{and} \quad k^* = a - jb$$

the corresponding term in the waveform $f(t)$ is

$$f(t) = \cdots + 2e^{-\alpha t}\left[a\cos(\beta t) - b\sin(\beta t)\right] + \cdots$$

Perform the substitution, simplify, and take the inverse Laplace transform.

$$F(s) = \frac{k}{s+\alpha-j\beta} + \frac{k^*}{s+\alpha+j\beta}$$

$$F(s) = \frac{a+jb}{s+\alpha-j\beta} + \frac{a-jb}{s+\alpha+j\beta}$$

$$F(s) = \frac{(a+jb)(s+\alpha+j\beta)+(a-jb)(s+\alpha-j\beta)}{(s+\alpha)^2+\beta^2}$$

$$F(s) = \frac{2a(s+\alpha)-2b\beta}{(s+\alpha)^2+\beta^2}$$

$$F(s) = 2a\frac{(s+\alpha)}{(s+\alpha)^2+\beta^2} - 2b\frac{\beta}{(s+\alpha)^2+\beta^2}$$

$$f(t) = 2e^{-\alpha t}\left[a\cos(\beta t) - b\sin(\beta t)\right]u(t)$$

The following MATLAB code and output confirm the results.

```
syms t s a b A B f F
F = (a+j*b)/(s+A-j*B) + (a-j*b)/(s+A+j*B);
f = simplify(ilaplace(F))
```

```
f = (2*(a*cos(B*t) - b*sin(B*t)))/exp(A*t)
```

Problem 9–67. (A) Solving State Variable Equations

With zero input, a series RLC circuit can be described by the following coupled first-order equations in the inductor current $i_L(t)$ and capacitor voltage $v_C(t)$.

$$\frac{dv_C(t)}{dt} = \frac{1}{C}i_L(t)$$

$$\frac{di_L(t)}{dt} = -\frac{1}{L}v_C(t) - \frac{R}{L}i_L(t)$$

(a). Transform these equations into the s domain and solve for the transforms $I_L(s)$ and $V_C(s)$ in terms of the initial conditions $i_L(0) = I_0$ and $v_C(0) = V_0$.

Take the Laplace transform of each equation and then solve for the state variable transforms.

$$sV_C(s) - v_C(0) = \frac{1}{C}I_L(s)$$

$$sI_L(s) - i_L(0) = -\frac{1}{L}V_C(s) - \frac{R}{L}I_L(s)$$

Write the equations so that we can use matrix techniques to solve.

$$sV_C(s) - \frac{1}{C}I_L(s) = V_0$$

$$\frac{1}{L}V_C(s) + \left(s+\frac{R}{L}\right)I_L(s) = I_0$$

Solve for the two transforms.

$$V_C(s) = \frac{CLV_0 s + I_0 L + RCV_0}{LCs^2 + RCs + 1}$$

$$I_L(s) = \frac{-C(V_0 - I_0 Ls)}{LCs^2 + RCs + 1}$$

(b). Find $i_L(t)$ and $v_C(t)$ for $R = 1$ kΩ, $L = 1$ H, $C = 1$ μF, $I_0 = 15$ mA and $V_0 = 10$ V.

Substitute and solve.

$$V_C(s) = \frac{10s + 25000}{s^2 + 1000s + 1000000}$$

$$V_C(s) = 10\frac{s+500}{(s+500)^2 + (500\sqrt{3})^2} + \frac{40\sqrt{3}}{3}\frac{500\sqrt{3}}{(s+500)^2 + (500\sqrt{3})^2}$$

$$v_C(t) = 10e^{-500t}\left[\cos(500\sqrt{3}t) + \frac{4\sqrt{3}}{3}\sin(500\sqrt{3}t)\right]u(t) \text{ V}$$

$$I_L(s) = \frac{15s - 10000}{s^2 + 1000s + 1000000} \text{ mA}$$

$$I_L(s) = 15\frac{s+500}{(s+500)^2 + (500\sqrt{3})^2} - \frac{35\sqrt{3}}{3}\frac{500\sqrt{3}}{(s+500)^2 + (500\sqrt{3})^2} \text{ mA}$$

$$i_L(t) = \frac{3}{200}e^{-500t}\left[\cos(500\sqrt{3}t) - \frac{7\sqrt{3}}{9}\sin(500\sqrt{3}t)\right]u(t) \text{ mA}$$

Problem 9–68. (A) Complex Differentiation Property

The complex differentiation property of the Laplace transformation states that

$$\text{If } \mathscr{L}\{f(t)\} = F(s) \quad \text{then} \quad \mathscr{L}\{tf(t)\} = -\frac{d}{ds}F(s)$$

Use this property to find the Laplace transforms of $f(t) = \{tg(t)\}\,u(t)$ when $g(t) = e^{-\alpha t}$. Repeat for $g(t) = \sin(\beta t)$ and $g(t) = \cos(\beta t)$.

First function.

$$g(t) = e^{-\alpha t}$$

$$G(s) = \frac{1}{s+\alpha}$$

$$f(t) = tg(t)u(t)$$

$$F(s) = -\frac{d}{ds}G(s) = -\frac{d}{ds}(s+\alpha)^{-1}$$

$$F(s) = (-1)(-1)(s+\alpha)^{-2} = \frac{1}{(s+\alpha)^2}$$

Second fucntion.

$$g(t) = \sin(\beta t)$$

$$G(s) = \frac{\beta}{s^2 + \beta^2}$$

$$f(t) = tg(t)u(t)$$

$$F(s) = -\frac{d}{ds}G(s) = -\frac{d}{ds}\beta(s^2 + \beta^2)^{-1}$$

$$F(s) = (-1)(-1)(\beta)(s^2 + \beta^2)^{-2}(2s) = \frac{2\beta s}{(s^2 + \beta^2)^2}$$

Third function.

$$g(t) = \cos(\beta t)$$

$$G(s) = \frac{s}{s^2 + \beta^2}$$

$$f(t) = tg(t)u(t)$$

$$F(s) = -\frac{d}{ds}G(s) = -\frac{d}{ds}\left[s(s^2 + \beta^2)^{-1}\right]$$

$$F(s) = (-1)\left[(-s)(s^2 + \beta^2)^{-2}(2s) + (s^2 + \beta^2)^{-1}\right]$$

$$F(s) = \frac{s^2 - \beta^2}{(s^2 + \beta^2)^2}$$

Problem 9–69. (A) Butterworth Poles

Steven Butterworth, a British engineer, 1885-1958, discovered a method of designing electric filters. He was quoted saying "An ideal electrical filter should not only completely reject the unwanted frequencies but should also have uniform sensitivity for the wanted frequencies." His algorithms are widely used in filter design, as we will see in Chapter 14. He based his design on locating the poles of his filters in a unique pattern around a circle of radius ω_C. The number of poles on the circle constitutes the order of the filter. The more poles, the better the filter. Odd-order filters include one real pole at $-\omega_C$ and pairs of complex-conjugate poles placed on a circle of radius ω_C at equal angular spacing. Even-order filters locate complex-conjugate poles placed on a circle of radius ω_C at equal angular spacing. Figure P9–69 shows the location of the poles in a third- and in a fourth-order Butterworth filter. Assuming an ω_C of 1 rad/s, what is the denominator of the Laplace transform, $F(s)$, associated with each Butterworth filter shown in the figure?

For the third-order filter, the poles are located at $s = -1$ and $s = -1/2 \pm j\sqrt{3}/2$.

$$d(s) = (s+1)\left(s + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\left(s + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$= (s+1)(s^2 + s + 1) = s^3 + 2s^2 + 2s + 1$$

For the fourth-order filter, the poles are located at $s = -0.9239 \pm j0.3827$ and $s = -0.3827 \pm j0.9239$.

$$d(s) = (s + 0.9239 - j0.3827)(s + 0.9239 + j0.3827)(s + 0.3827 - j0.9239)(s + 0.3827 + j0.9239)$$

$$= (s^2 + 1.8478s + 1)(s^2 + 0.7654s + 1)$$

$$= (s^4 + 2.6131s^3 + 3.4142s^2 + 2.6131s + 1)$$

10 s-Domain Circuit Analysis

10.1 Exercise Solutions

Exercise 10–1. Transform the circuit of Figure 10–7(a) into the s domain and solve for the voltage $v_C(t)$ if $v_S(t) = V_A e^{-\alpha t}u(t)$ V and $v_C(0) = V_0$.

In the s domain, replace the source with $V_A/(s+\alpha)$ and replace the capacitor with $1/Cs$ in series with a voltage source V_0/s. The resistor does not change. Solve for the current flowing in the circuit and then the voltage across the capacitor.

$$I(s) = \frac{[V_A/(s+\alpha)] - [V_0/s]}{R + 1/Cs}$$

$$\begin{aligned}
V_C(s) &= \frac{V_0}{s} + \frac{1}{Cs}I(s) = \frac{V_0}{s} + \frac{\left(\dfrac{V_A}{s+\alpha} - \dfrac{V_0}{s}\right)}{RCs+1} \\
&= \frac{V_0}{s} + \frac{V_A/RC}{(s+\alpha)(s+1/RC)} - \frac{V_0/RC}{s(s+1/RC)} \\
&= \frac{V_0}{s} + \frac{\dfrac{V_A/RC}{(1/RC-\alpha)}}{s+\alpha} - \frac{\dfrac{V_A/RC}{(1/RC-\alpha)}}{s+\dfrac{1}{RC}} - \frac{V_0}{s} + \frac{V_0}{s+\dfrac{1}{RC}}
\end{aligned}$$

$$v_C(t) = \left[\left(V_0 - \frac{\dfrac{V_A}{RC}}{\dfrac{1}{RC}-\alpha}\right)e^{-t/RC} + \left(\frac{\dfrac{V_A}{RC}}{\dfrac{1}{RC}-\alpha}\right)e^{-\alpha t}\right]u(t) \text{ V}$$

Exercise 10–2. The source of the t-domain circuit of Figure 10–8(a) is suddenly turned off. Use Laplace techniques to solve for the voltage across the resistor $v_R(t)$.

In the steady state, the inductor current is V_A/R, so this is the initial inductor current when the source is turned off. In the s domain, we have resistor R in series with Ls and voltage source $Li_L(0) = LV_A/R$. Apply voltage division to find the voltage across the resistor.

$$V_R(s) = \frac{R}{R+Ls}\left(\frac{LV_A}{R}\right) = \frac{V_A}{s+\dfrac{R}{L}}$$

$$v_R(t) = V_A e^{-Rt/L}u(t) \text{ V}$$

Exercise 10–3. Replace the capacitor in the circuit of Figure 10–9(a) with an inductor with an initial current of I_B A. Find $v_O(t)$ and identify the forced and natural responses.

Make the substitution and use mesh-current analysis to find the current $I(s)$.

$$-\frac{V_{\mathrm{S}}}{s} + RI(s) + LsI(s) - LI_{\mathrm{B}} = 0$$

$$(R+Ls)I(s) = \frac{V_{\mathrm{A}} + LsI_{\mathrm{B}}}{s}$$

$$I(s) = \frac{V_{\mathrm{A}} + LsI_{\mathrm{B}}}{s(Ls+R)} = \frac{V_{\mathrm{A}}/L + sI_{\mathrm{B}}}{s(s+R/L)}$$

$$V_{\mathrm{X}}(s) = LsI(s) - LI_{\mathrm{B}} = \frac{V_{\mathrm{A}} + LsI_{\mathrm{B}}}{(s+R/L)} - LI_{\mathrm{B}}$$

$$V_{\mathrm{X}}(s) = \frac{V_{\mathrm{A}}}{s+R/L} + \frac{LsI_{\mathrm{B}}}{s+R/L} - \frac{LI_{\mathrm{B}}(s+R/L)}{s+R/L}$$

$$V_{\mathrm{X}}(s) = \frac{V_{\mathrm{A}}}{s+R/L} - \frac{RI_{\mathrm{B}}}{s+R/L}$$

$$v_{\mathrm{O}}(t) = -\mu v_{\mathrm{X}}(t) = -\mu\left[V_{\mathrm{A}} - RI_{\mathrm{B}}\right]e^{-Rt/L}u(t)$$

Exercise 10–4. The circuit of Figure 10–13 is in the zero state.

(a). Find the equivalent impedance $Z_{\mathrm{IN}}(s)$ that the source sees.

Combine the two parallel components and then combine those results in series.

$$Z_1 = R_1 \parallel 1/C_1 s = \frac{R_1/C_1 s}{R_1 + 1/C_1 s} = \frac{1/C_1}{s + 1/R_1C_1}$$

$$Z_2 = R_2 \parallel 1/C_2 s = \frac{R_2/C_2 s}{R_2 + 1/C_2 s} = \frac{1/C_2}{s + 1/R_2C_2}$$

$$Z_{\mathrm{IN}}(s) = Z_1 + Z_2 = \frac{1/C_1}{s+1/R_1C_1} + \frac{1/C_2}{s+1/R_2C_2}$$

$$= \frac{\dfrac{1}{C_1}\left(s + \dfrac{1}{R_2C_2}\right) + \dfrac{1}{C_2}\left(s + \dfrac{1}{R_1C_1}\right)}{\left(s + \dfrac{1}{R_1C_1}\right)\left(s + \dfrac{1}{R_2C_2}\right)}$$

$$= \frac{\left(\dfrac{C_1+C_2}{C_1C_2}\right)s + \dfrac{R_1+R_2}{R_1R_2C_1C_2}}{\left(s + \dfrac{1}{R_1C_1}\right)\left(s + \dfrac{1}{R_2C_2}\right)}$$

(b). Find the output voltage $V_2(s)$.

Apply voltage division.

$$V_2(s) = \frac{Z_2}{Z_{\rm IN}(s)}V_1(s) = \left[\frac{1/C_2}{s+1/R_2C_2}\right]\left[\frac{\left(s+\dfrac{1}{R_1C_1}\right)\left(s+\dfrac{1}{R_2C_2}\right)}{\left(\dfrac{C_1+C_2}{C_1C_2}\right)s+\dfrac{R_1+R_2}{R_1R_2C_1C_2}}\right]V_1(s)$$

$$= \frac{\left[\dfrac{C_1}{C_1+C_2}s+\dfrac{1}{R_1(C_1+C_2)}\right]}{s+\left(\dfrac{1}{C_1+C_2}\right)\left(\dfrac{R_1+R_2}{R_1R_2}\right)}V_1(s) \text{ V-s}$$

Exercise 10–5. For the Figure 10–14, find the impedance $Z_2(s)$ seen looking into the $V_2(s)$ terminals with the input source removed (open circuited). What are its poles and zeros?

The output resistor is in parallel with the series combination of the other three elements.

$$Z_2(s) = 2000 \parallel \left(\frac{s}{2}+1000+\frac{10^6}{s}\right)$$

$$= \frac{2000\left(\dfrac{s}{2}+1000+\dfrac{10^6}{s}\right)}{2000+\dfrac{s}{2}+1000+\dfrac{10^6}{s}}$$

$$= \frac{2000\left(s^2+2000s+2000000\right)}{s^2+6000s+2000000}$$

The zeros are located at $s = -1000 \pm j1000$ and the poles are located at $s = -354$ and $s = -5646$.

Exercise 10–6. The circuit of Figure 10–16 is in the zero state.

(a). Find the output current transform $I_2(s)$.

The branch impedances are Ls and $R + 1/Cs$. Apply two-path current division in the s domain.

$$I_2(s) = \frac{Ls}{Ls+R+1/Cs}I_1(s) = \frac{s^2}{s^2+\dfrac{R}{L}s+\dfrac{1}{LC}}I_1(s) \text{ A-s}$$

(b). If $R = 1$ kΩ, select values of L and C such that $I_2(s)$ has two identical poles at -5000 rad/s.

The desired characteristic equation is

$$(s+5000)^2 = s^2+10000s+5000^2 = s^2+\frac{R}{L}s+\frac{1}{LC}$$

Solve for L and C.

$$R = 1000$$

$$\frac{R}{L} = 10000$$

$$L = \frac{1000}{10000} = 100 \text{ mH}$$

$$\frac{1}{LC} = 5000^2$$

$$C = \frac{1}{(5000^2)(0.1)} = 0.4\ \mu\text{F}$$

Exercise 10–7. The inductor current and capacitor voltage in Figure 10–17 are zero at $t = 0$.

(a). Find the equivalent impedance between terminals A and B.

Find the series impedance along each branch and combine the two branches in parallel.

$$Z_1 = R_1 + \frac{1}{Cs} = \frac{R_1Cs+1}{Cs}$$

$$Z_2 = R_2 + Ls$$

$$Z_{\mathrm{EQ}}(s) = Z_1 \parallel Z_2 = \frac{\left(\dfrac{R_1Cs+1}{Cs}\right)(R_2+Ls)}{\dfrac{R_1Cs+1}{Cs} + R_2 + Ls}$$

$$= \frac{(R_1Cs+1)(R_2+Ls)}{LCs^2 + (R_1+R_2)Cs + 1}$$

(b). Solve for the output voltage transform $V_2(s)$ in terms of the input voltage $V_1(s)$.

Apply voltage division across the second branch.

$$V_2(s) = \left[\frac{Ls}{R_2+Ls}\right] V_1(s)$$

Exercise 10–8. In Figure 10–19 find the network function relating the output $V_2(s)$ to the input $I_1(s)$.

Apply two-path current division to find the output current and then apply Ohm's law to determine the output voltage.

$$I_2(s) = \frac{R}{R+Ls+1/Cs} I_1(s)$$

$$V_2(s) = \frac{1}{Cs} I_1(s) = \frac{R}{LCs^2+RCs+1} I_1(s)$$

Exercise 10–9. The switch in the circuit of Figure 10–21(a) has been closed for a long time. At $t = 0$ the switch is suddenly opened.

(a). Find the transform $I_{\mathrm{R}}(s)$ for the current through the resistor.

With the switch closed, the current flows through the inductor and the initial voltage across the capacitor is zero. Once the switch opens, use two-path current division to find the current through the resistor.

$$I_{\mathrm{R}}(s) = \left(\frac{1/Cs}{R+1/Cs}\right)\frac{I_{\mathrm{A}}}{s}$$

$$= \frac{I_{\mathrm{A}}\left(\dfrac{1}{RC}\right)}{s\left(s+\dfrac{1}{RC}\right)}$$

$$= \frac{I_{\mathrm{A}}}{s} - \frac{I_{\mathrm{A}}}{s+\dfrac{1}{RC}}$$

$$i_{\mathrm{R}}(t) = I_{\mathrm{A}}\left(1-e^{-t/RC}\right)u(t) \text{ A}$$

(b). Select values of R, L, and C so that the current reaches at least 63% of its final value in 100 ms or less.

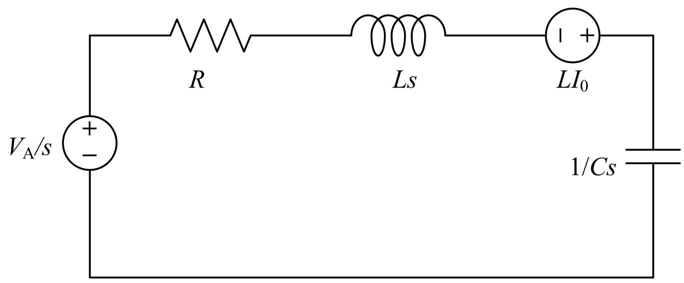
The time constant for the response is $T_C = RC$ and the specification implies $T_C \leq 100$ ms. Choose $C = 1\ \mu$F and solve for the resistance.

$$R \leq \frac{0.1}{1\,\mu} = 100\text{ k}\Omega$$

Choose $R = 56$ kΩ as one possible solution. The value for the inductor does not matter.

Exercise 10–10. The initial conditions for the circuit in Figure 10–23 are $v_C(0^-) = 0$ and $i_L(0^-) = I_0$. Transform the circuit into the s domain and find the zero-state and zero-input components of $V(s)$.

The s-domain circuit is shown below.



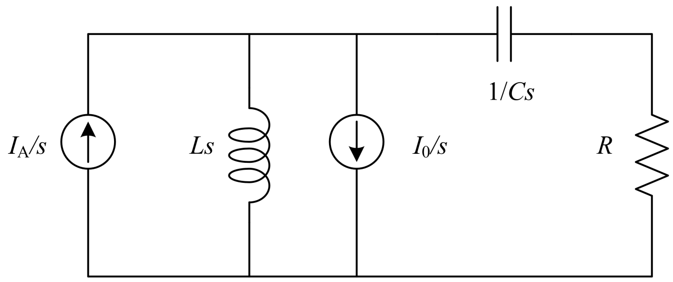
The zero-state response comes from the voltage source and the zero-input response comes from the initial current through the inductor. Apply voltage division to find the responses.

$$V_{zs}(s) = \left(\frac{1/Cs}{R + Ls + 1/Cs}\right)\frac{V_A}{s} = \frac{V_A}{s(LCs^2 + RCs + 1)}$$

$$V_{zi}(s) = \left(\frac{1/Cs}{R + Ls + 1/Cs}\right)LI_0 = \frac{LI_0}{LCs^2 + RCs + 1}$$

Exercise 10–11. The initial conditions for the circuit in Figure 10–24 are $v_C(0^-) = 0$ and $i_L(0^-) = I_0$. Transform the circuit into the s domain and find the zero-state and zero-input components of $I(s)$.

The s-domain circuit is shown below.



The zero-state response comes from the original current source and the zero-input response comes from the initial current through the inductor. Apply two-path current division to find the responses.

$$I_{zs}(s) = \left(\frac{Ls}{Ls + R + 1/Cs}\right)\frac{I_A}{s} = \frac{LCsI_A}{LCs^2 + RCs + 1}$$

$$I_{zi}(s) = \left(\frac{Ls}{Ls + R + 1/Cs}\right)\left(-\frac{I_0}{s}\right) = \frac{-LCsI_0}{LCs^2 + RCs + 1}$$

Exercise 10–12. For the circuit of Figure 10–25(a), let the input voltage be $V_S(s) = V_A/s$. Use superposition to find the zero-state and the zero-input components of the output. Then show that $v_O(t)$ is the same as that found in Example 10–3.

Apply voltage division with each independent source to find the output voltage components. Let $v_{\rm C}(0) = V_{\rm B}$.

$$V_{\rm Ozs}(s) = -\mu V_{\rm Xzs}(s) = \left(\frac{-\mu\dfrac{1}{Cs}}{R+\dfrac{1}{Cs}}\right)\frac{V_{\rm A}}{s} = \frac{-\dfrac{\mu V_{\rm A}}{RC}}{s\left(s+\dfrac{1}{RC}\right)} = -\frac{\mu V_{\rm A}}{s} + \frac{\mu V_{\rm A}}{s+\dfrac{1}{RC}}$$

$$V_{\rm Ozi}(s) = -\mu V_{\rm Xzi}(s) = \left(\frac{-\mu R}{R+\dfrac{1}{Cs}}\right)\frac{V_{\rm B}}{s} = -\frac{\mu V_{\rm B}}{s+\dfrac{1}{RC}}$$

$$V_{\rm O}(s) = V_{\rm Ozs}(s) + V_{\rm Ozi}(s) = -\frac{\mu V_{\rm A}}{s} + \frac{\mu V_{\rm A}}{s+\dfrac{1}{RC}} - \frac{\mu V_{\rm B}}{s+\dfrac{1}{RC}}$$

$$v_{\rm O}(t) = -\mu\left(\underbrace{V_{\rm A} - V_{\rm A}e^{-t/RC}}_{\text{zero state}} + \underbrace{V_{\rm B}e^{-t/RC}}_{\text{zero input}}\right)u(t)\ \text{V}$$

The result matches that of Example 10–3.

Exercise 10–13. The circuit of Figure 10–28 is in the zero state.

(a). Find the Thévenin equivalent circuit that the load sees.

Use voltage division to find the open-circuit voltage and then determine the lookback impedance.

$$V_{\rm T}(s) = \frac{1/C_1 s}{R+1/C_1 s}V_1(s) = \frac{\dfrac{1}{RC_1}}{s+\dfrac{1}{RC_1}}V_1(s)\ \text{V-s}$$

$$Z_{\rm T}(s) = \left(R \parallel \frac{1}{C_1 s}\right) + \frac{1}{C_2 s} = \frac{\dfrac{1}{C_1}}{s+\dfrac{1}{RC_1}} + \frac{1}{C_2 s}$$

$$= \frac{\dfrac{1}{C_1}s + \dfrac{1}{C_2}\left(s+\dfrac{1}{RC_1}\right)}{s\left(s+\dfrac{1}{RC_1}\right)} = \frac{\left(\dfrac{C_1+C_2}{C_1C_2}\right)\left(s+\dfrac{1}{R(C_1+C_2)}\right)}{s\left(s+\dfrac{1}{RC_1}\right)}\ \Omega$$

(b). Find the Norton equivalent of the same circuit.

Perform a source transformation on the Thévenin equivalent circuit.

$$Z_{\rm N}(s) = Z_{\rm T}(s)$$

$$I_{\rm N}(s) = \frac{V_{\rm T}(s)}{Z_{\rm T}(s)} = \left[\frac{\dfrac{1}{RC_1}}{s+\dfrac{1}{RC_1}}V_1(s)\right]\left[\frac{s\left(s+\dfrac{1}{RC_1}\right)}{\left(\dfrac{C_1+C_2}{C_1C_2}\right)\left(s+\dfrac{1}{R(C_1+C_2)}\right)}\right]$$

$$= \frac{\left(\dfrac{C_2}{C_1+C_2}\right)\left(\dfrac{V_1(s)}{R}\right)s}{s+\dfrac{1}{R(C_1+C_2)}}\ \text{A-s}$$

Exercise 10–14. Find the Norton equivalent of the s-domain circuits in Figure 10–30. Find the Thévenin equivalent of the s-domain circuits in Figure 10–30.

(a). Use two-path current division to find the short-circuit current and then find the lookback impedance. Use Ohm's law to find the open-circuit voltage.

$$I_{\mathrm{N}}(s) = \left(\frac{1/Cs}{R+1/Cs}\right)\left(\frac{I_{\mathrm{A}}}{s+\alpha}\right) = \frac{\left(\dfrac{1}{RC}\right)I_{\mathrm{A}}}{\left(s+\dfrac{1}{RC}\right)(s+\alpha)} = \frac{I_{\mathrm{A}}}{(RCs+1)(s+\alpha)}$$

$$Z_{\mathrm{N}}(s) = Z_{\mathrm{T}}(s) = R + \frac{1}{Cs} = \frac{RCs+1}{Cs}$$

$$V_{\mathrm{T}}(s) = \left(\frac{1}{Cs}\right)\left(\frac{I_{\mathrm{A}}}{s+\alpha}\right) = \frac{I_{\mathrm{A}}}{Cs(s+\alpha)}$$

(b). Use two-path current division to find the short-circuit current and then find the lookback impedance. Perform a source transformation to determine the Thévenin equivalent source voltage.

$$I_{\mathrm{N}}(s) = \left(\frac{R}{R+Ls}\right)\left(\frac{I_{\mathrm{A}}}{s+\alpha}\right) = \frac{RI_{\mathrm{A}}}{(Ls+R)(s+\alpha)}$$

$$Z_{\mathrm{N}}(s) = Z_{\mathrm{T}}(s) = \frac{\dfrac{R+Ls}{Cs}}{R+Ls+\dfrac{1}{Cs}} = \frac{Ls+R}{LCs^2+RCs+1}$$

$$V_{\mathrm{T}}(s) = Z_{\mathrm{N}}(s)I_{\mathrm{N}}(s) = \left[\frac{Ls+R}{LCs^2+RCs+1}\right]\left[\frac{RI_{\mathrm{A}}}{(Ls+R)(s+\alpha)}\right]$$

$$= \frac{RI_{\mathrm{A}}}{(s+\alpha)(LCs^2+RCs+1)}$$

Exercise 10–15. Using the nodes identified, write a set of node-voltage equations for the circuit of Figure 10–33.

Node A is defined by voltage source $V_{\mathrm{S}}(s)$. Sum the currents leaving nodes B and C.

$$\frac{V_{\mathrm{B}}(s)-V_{\mathrm{S}}(s)}{R_2} + \frac{V_{\mathrm{B}}(s)}{1/Cs} - Cv_{\mathrm{C}}(0) + \frac{i_{\mathrm{L}}(0)}{s} + \frac{V_{\mathrm{B}}(s)-V_{\mathrm{C}}(s)}{Ls} = 0$$

$$\frac{V_{\mathrm{C}}(s)-V_{\mathrm{S}}(s)}{R_1} - \frac{i_{\mathrm{L}}(0)}{s} + \frac{V_{\mathrm{C}}(s)-V_{\mathrm{B}}(s)}{Ls} + \frac{V_{\mathrm{C}}(s)}{R_3} = 0$$

Rewrite the equations in standard form.

$$V_{\mathrm{B}}(s)\left(\frac{1}{R_2}+Cs+\frac{1}{Ls}\right) - V_{\mathrm{C}}(s)\left(\frac{1}{Ls}\right) = \frac{V_{\mathrm{S}}(s)}{R_2} - \frac{i_{\mathrm{L}}(0)}{s} + Cv_{\mathrm{C}}(0)$$

$$-V_{\mathrm{B}}(s)\left(\frac{1}{Ls}\right) + V_{\mathrm{C}}(s)\left(\frac{1}{R_1}+\frac{1}{Ls}+\frac{1}{R_3}\right) = \frac{V_{\mathrm{S}}(s)}{R_1} + \frac{i_{\mathrm{L}}(0)}{s}$$

Exercise 10–16. For the circuit of Figure 10–32(a), find the zero-state current transforms through each passive element.

The inductor and capacitor share the same current. Use two-path current division to determine each current.

$$I_{\text{Rzs}}(s) = \frac{Ls + \dfrac{1}{Cs}}{R + Ls + \dfrac{1}{Cs}} I_{\text{S}}(s) = \frac{s^2 + \dfrac{1}{LC}}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}} I_{\text{S}}(s) \text{ A-s}$$

$$I_{\text{Lzs}}(s) = I_{\text{Czs}}(s) = \frac{R}{R + Ls + \dfrac{1}{Cs}} I_{\text{S}}(s) = \frac{\dfrac{R}{L}s}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}} I_{\text{S}}(s) \text{ A-s}$$

Exercise 10–17. Consider the circuit in Figure 10–36. Select values for the various components to produce a pair of complex poles defined by $\zeta = 0.5$ and $\omega_0 = 1$ krad/s. To produce your design you must assume unity gain ($\mu = 1$) for the dependent source, and that $R_1 = R_2 = R$.

Use the results from Example 10–15.

$$\begin{aligned}
\Delta(s) &= C_1C_2s^2 + \left(\frac{C_1}{R_2} + \frac{C_2}{R_1} + \frac{C_2}{R_2} - \mu\frac{C_1}{R_2}\right)s + \frac{1}{R_1R_2} \\
&= C_1C_2s^2 + \left(\frac{C_1}{R} + \frac{C_2}{R} + \frac{C_2}{R} - \frac{C_1}{R}\right)s + \frac{1}{R^2} \\
&= C_1C_2s^2 + \frac{2C_2}{R}s + \frac{1}{R^2} \\
\frac{\Delta(s)}{C_1C_2} &= s^2 + \frac{2}{RC_1}s + \frac{1}{R^2C_1C_2} \\
&= s^2 + 2\zeta\omega_0 s + \omega_0^2 \\
\omega_0^2 &= \frac{1}{R^2C_1C_2} \\
\omega_0 &= \frac{1}{R\sqrt{C_1C_2}} \\
2\zeta\omega_0 &= \frac{2}{RC_1} \\
\zeta &= \frac{1}{\omega_0 RC_1} = \frac{R\sqrt{C_1C_2}}{RC_1} = \sqrt{\frac{C_2}{C_1}} \\
\zeta^2 &= \frac{C_2}{C_1} \\
C_2 &= \zeta^2 C_1 = \frac{C_1}{4} \\
C_1 &= 4C_2 \\
\omega_0^2 &= \frac{1}{R^2 4C_2^2} \\
C_2 &= \frac{1}{2R\omega_0}
\end{aligned}$$

Pick $R = 10$ kΩ and solve for $C_2 = 0.05$ μF and $C_1 = 0.2$ μF. Other solutions are possible.

Exercise 10–18.

(a). For the s-domain circuit in Figure 10–38, solve for the zero-state output $V_{\rm O}(s)$ in terms of a general input $V_{\rm S}(s)$.

The positive input terminal of the OP AMP is grounded, so the voltage at the negative input terminal is zero. Write the node-voltage equation at the negative input terminal.

$$\frac{0-V_{\rm S}(s)}{R_1}+\frac{0-V_{\rm O}(s)}{R_2}+\frac{0-V_{\rm O}(s)}{1/Cs}=0$$

$$\left(\frac{1}{R_2}+Cs\right)V_{\rm O}(s)=-\frac{V_{\rm S}(s)}{R_1}$$

$$\left(\frac{R_2Cs+1}{R_2}\right)V_{\rm O}(s)=-\frac{V_{\rm S}(s)}{R_1}$$

$$V_{\rm O}(s)=\frac{-\dfrac{R_2}{R_1}}{R_2Cs+1}V_{\rm S}(s)=\frac{-\dfrac{1}{R_1C}}{s+\dfrac{1}{R_2C}}V_{\rm S}(s)\text{ V-s}$$

(b). Solve for the zero-state output when the input is a step function $v_{\rm S}(t)=V_{\rm A}u(t)$ V.

Find the output in the Laplace domain and perform an inverse transformation.

$$V_{\rm S}(s)=\frac{V_{\rm A}}{s}$$

$$V_{\rm O}(s)=\frac{-\dfrac{1}{R_1C}}{s+\dfrac{1}{R_2C}}\left(\frac{V_{\rm A}}{s}\right)$$

$$V_{\rm O}(s)=\frac{-\dfrac{V_{\rm A}}{R_1C}}{s\left(s+\dfrac{1}{R_2C}\right)}=\frac{-\dfrac{R_2}{R_1}V_{\rm A}}{s}+\frac{\dfrac{R_2}{R_1}V_{\rm A}}{s+\dfrac{1}{R_2C}}$$

$$v_{\rm O}(t)=-\frac{R_2}{R_1}V_{\rm A}\left(1-e^{-t/R_2C}\right)u(t)\text{ V}$$

Exercise 10–19. Formulate node-voltage equations for the circuit in Figure 10–39 and find the circuit determinant. Assume that the initial conditions are zero.

Write the node-voltage equations.

$$\frac{V_{\rm B}(s)-V_{\rm S}(s)}{R_1}+\frac{V_{\rm B}(s)}{Ls}+\frac{V_{\rm B}(s)-V_{\rm C}(s)}{R_2}=0$$

$$\frac{V_{\rm C}(s)-V_{\rm B}(s)}{R_2}+\frac{V_{\rm C}(s)-V_{\rm S}(s)}{1/Cs}=0$$

Convert the equations into standard form.

$$\left(\frac{1}{R_1}+\frac{1}{Ls}+\frac{1}{R_2}\right)V_{\rm B}(s)-\frac{1}{R_2}V_{\rm C}(s)=\frac{V_{\rm S}(s)}{R_1}$$

$$-\frac{1}{R_2}V_{\rm B}(s)+\left(\frac{1}{R_2}+Cs\right)V_{\rm C}(s)=CsV_{\rm S}(s)$$

Compute the determinant.

$$\begin{aligned}
\Delta(s) &= \left(\frac{1}{R_1}+\frac{1}{Ls}+\frac{1}{R_2}\right)\left(\frac{1}{R_2}+Cs\right)-\left(-\frac{1}{R_2}\right)\left(-\frac{1}{R_2}\right)\\
&= \frac{1}{R_1R_2}+\frac{Cs}{R_1}+\frac{1}{R_2^2}+\frac{Cs}{R_2}+\frac{1}{R_2Ls}+\frac{C}{L}-\frac{1}{R_2^2}\\
&= \frac{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)LCs^2+\left(\frac{L}{R_1R_2}+C\right)s+\frac{1}{R_2}}{Ls}
\end{aligned}$$

Exercise 10–20. Solve for the zero-state components of $I_{\rm A}(s)$ and $I_{\rm B}(s)$ in Figure 10–41(b).

Use the results developed in Example 10–17. Set both initial inductor currents equal to zero and apply Cramer's rules with only $V_{\rm S}(s)$ active.

$$\Delta(s) = L_1L_2s^2+(R_1L_2+R_1L_1+R_2L_1)s+R_1R_2$$

$$\begin{aligned}
I_{\rm Azs}(s) &= \frac{\begin{vmatrix} V_{\rm S}(s) & -R_1\\ 0 & L_2s+R_1+R_2\end{vmatrix}}{\Delta(s)}\\
&= \frac{L_2s+R_1+R_2}{\Delta(s)}V_{\rm S}(s)\ \text{V-s}\\
I_{\rm Bzs}(s) &= \frac{\begin{vmatrix} L_1s+R_1 & V_{\rm S}(s)\\ -R_1 & 0\end{vmatrix}}{\Delta(s)}\\
&= \frac{R_1}{\Delta(s)}V_{\rm S}(s)\ \text{V-s}
\end{aligned}$$

Exercise 10–21. For the circuit in Figure 10–42(b), let $I_{\rm S}(s) = 0.01/(s+500)$ A-s, $L = 250$ mH, $C = 1\ \mu$F, and $R = 1$ kΩ. Use partial fraction expansion or MATLAB to solve for the zero-state component of $i_{\rm A}(t)$.

Set the initial inductor current and capacitor voltage to zero.

$$\begin{aligned}
I_{\rm Azs}(s) &= \frac{RI_{\rm S}(s)}{R+Ls+\frac{1}{Cs}}\\
&= \left(\frac{0.01R}{s+500}\right)\left(\frac{\frac{1}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}\right)\\
&= \frac{40s}{(s+500)(s^2+4000s+4000000)} = \frac{40s}{(s+500)(s+2000)^2}\\
&= \frac{1}{s+2000}\left(\frac{-40/3}{s+500}+\frac{160/3}{s+2000}\right)\\
&= \frac{160/3}{(s+2000)^2}+\frac{4/450}{s+2000}-\frac{4/450}{s+500}\\
i_{\rm Azs}(t) &= \left[\left(\frac{160t}{3}+\frac{2}{225}\right)e^{-2000t}-\frac{2}{225}e^{-500t}\right]u(t)\ \text{A}
\end{aligned}$$

Exercise 10–22.

(a). Formulate mesh-current equations for the circuit in Figure 10–44. Assume that the initial conditions are zero.

Write the mesh-current equations by inspection.

$$(R_1 + Ls)I_{\mathrm{A}}(s) - R_1 I_{\mathrm{B}}(s) = V_{\mathrm{S}}(s)$$

$$-R_1 I_{\mathrm{A}}(s) + \left(R_1 + R_2 + \frac{1}{Cs}\right) I_{\mathrm{B}}(s) = 0$$

(b). Find the circuit determinant.

We have the following results:

$$\begin{aligned}\Delta(s) &= (R_1 + Ls)\left(R_1 + R_2 + \frac{1}{Cs}\right) - R_1^2 \\ &= R_1^2 + R_1 R_2 + \frac{R_1}{Cs} + R_1 Ls + R_2 Ls + \frac{L}{C} - R_1^2 \\ &= \frac{R_1 R_2 Cs + R_1 + R_1 LCs^2 + R_2 LCs^2 + Ls}{Cs} \\ &= \frac{(R_1 + R_2)LCs^2 + (R_1 R_2 C + L)s + R_1}{Cs}\end{aligned}$$

(c). Solve for the zero-state component of $I_{\mathrm{B}}(s)$.

Apply Cramer's rule.

$$\begin{aligned}I_{\mathrm{Bzs}}(s) &= \frac{\begin{vmatrix} Ls + R_1 & V_{\mathrm{S}}(s) \\ -R_1 & 0 \end{vmatrix}}{\Delta(s)} \\ &= \frac{R_1}{\Delta(s)} V_{\mathrm{S}}(s) \\ &= \frac{R_1 Cs V_{\mathrm{S}}(s)}{(R_1 + R_2)LCs^2 + (R_1 R_2 C + L)s + R_1} \text{ V-s}\end{aligned}$$

Exercise 10–23. Formulate mesh-current equations for the circuit in Figure 10–44 when a resistor R_3 is connected between nodes A and B. Assume that the initial conditions are zero.

Let current $I_{\mathrm{C}}(s)$ flow in the new mesh created by the addition of resistor R_3. Write the mesh-current equations by inspection.

$$(R_1 + Ls)I_{\mathrm{A}}(s) - R_1 I_{\mathrm{B}}(s) - Ls I_{\mathrm{C}}(s) = V_{\mathrm{S}}(s)$$

$$-R_1 I_{\mathrm{A}}(s) + \left(R_1 + R_2 + \frac{1}{Cs}\right) I_{\mathrm{B}}(s) - R_2 I_{\mathrm{C}}(s) = 0$$

$$-Ls I_{\mathrm{A}}(s) - R_2 I_{\mathrm{B}}(s) + (R_2 + R_3 + Ls) I_{\mathrm{C}}(s) = 0$$

Exercise 10–24. The circuit determinants of three circuits to be studied in subsequent chapters are given below. Determine the nature of the poles of each circuit and what conditions, if any, could cause the circuit to become unstable.

(a). $R^2C^2s^2 + 2RCs + 1$

The poles are located at

$$s_1 = s_2 = -\frac{1}{RC}$$

Poles are real, negative, and equal, so the circuit is always stable.

(b). $R^2C^2s^2 + RCs + 1$

The poles are located at

$$s = \frac{-1 \pm j\sqrt{3}}{2RC}$$

Poles are complex with negative real parts, so the circuit is always stable.

(c). $R^2C^2s^2 + (3 - \mu)RCs + 1$

The poles will vary depending on μ. In general, we have

$$s = \frac{(\mu - 3) \pm \sqrt{\mu^2 - 6\mu + 5}}{2RC}$$

If $\mu < 1$, the poles are real, distinct, and both negative, so the circuit is always stable. If $\mu = 1$, the poles are real, negative, and equal, so the circuit is always stable. If $1 < \mu < 3$, the poles are complex with negative real parts, so the circuit is always stable. If $\mu = 3$, the poles are pure imaginary and the circuit is marginally stable. If $\mu > 3$, the poles have positive real parts and the circuit is unstable.

10.2 Problem Solutions

Problem 10–1. For a series RC circuit find $Z_{\text{EQ}}(s)$ and then select R and C so that there is a pole at zero and a zero at -1 krad/s.

The equivalent impedance is the sum of the component impedances.

$$Z_{\text{EQ}}(s) = R + \frac{1}{Cs} = \frac{RCs+1}{Cs}$$

The pole is always located at zero. The zero is located at $s = -1/RC$. Choose $R = 1$ kΩ and solve for $C = 1$ μF to get $s = -1000$ rad/s.

Problem 10–2. For a parallel RC circuit find $Z_{\text{EQ}}(s)$ and then select R and C so that there is a pole at -10 krad/s.

The equivalent impedance is given below.

$$Z_{\text{EQ}}(s) = R \parallel \frac{1}{Cs} = \frac{R/Cs}{R+1/Cs} = \frac{R}{RCs+1} = \frac{\dfrac{1}{C}}{s+\dfrac{1}{RC}}$$

The pole is located at $s = -1/RC$. Choose $C = 1$ μF and solve for $R = 100$ Ω.

Problem 10–3. For the circuit of Figure P10–3:

(a). Find and express $Z_{\text{EQ}}(s)$ as a rational function and locate its poles and zeros.

Combine the resistor and capacitor in series and then combine the result in parallel with the other resistor.

$$Z_{\text{EQ}}(s) = 2R \parallel \left(2R + \frac{1}{Cs}\right) = \frac{2R\left(2R+\dfrac{1}{Cs}\right)}{2R+2R+\dfrac{1}{Cs}}$$

$$= \frac{4R^2 + \dfrac{2R}{Cs}}{4R+\dfrac{1}{Cs}} = \frac{Rs+\dfrac{1}{2C}}{s+\dfrac{1}{4RC}}$$

The zero is located at $s = -1/2RC$ and the pole is located at $s = -1/4RC$.

(b). Select values of R and C to locate a pole at -640 rad/s. Where is the resulting zero?

Pick $R = 1$ kΩ and solve for C

$$C = \frac{1}{(4)(640)(1000)} = 0.391\ \mu\text{F}$$

The zero is located at $s = -1280$ rad/s.

Problem 10–4. For the circuit of Figure P10–4:

(a). Find and express $Z_{\text{EQ}}(s)$ as a rational function and locate its poles and zeros.

Convert the circuit into the s domain. Combine the resistors in parallel, combine the capacitors in parallel, and then combine the two results together in series.

$$Z_{\text{EQ}}(s) = (R \parallel 2R) + \left(\frac{1}{Cs} \parallel \frac{2}{Cs}\right) = \frac{2R^2}{3R} + \frac{\dfrac{2}{C^2s^2}}{\dfrac{3}{Cs}}$$

$$= \frac{2R}{3} + \frac{2}{3Cs} = \frac{2RCs+2}{3Cs} = \frac{2}{3}\left(\frac{RCs+1}{Cs}\right)$$

The zero is located at $s = -1/RC$ and the pole is located at zero.

(b). Select values of R and C to locate a zero at -33 krad/s.

Pick $R = 1\ \text{k}\Omega$ and solve for C.

$$C = \frac{1}{(1000)(33000)} = 0.0303\ \mu\text{F}$$

Problem 10–5. For the circuit of Figure P10–5:

(a). Find and express $Z_{\text{EQ}}(s)$ as a rational function and locate its poles and zeros.

Combine the inductor and capacitor in series and then combine the result in parallel with the resistor.

$$Z_{\text{EQ}}(s) = \left(Ls + \frac{1}{Cs}\right) \parallel R = \left(\frac{LCs^2+1}{Cs}\right) \parallel R$$

$$= \frac{\dfrac{RLCs^2+R}{Cs}}{\dfrac{LCs^2+1}{Cs} + R} = \frac{RLCs^2+R}{LCs^2+RCs+1}$$

$$= \frac{R\left(s^2 + \dfrac{1}{LC}\right)}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}}$$

The zeros are located at $s = \pm j1/\sqrt{LC}$. The poles are located at

$$s = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC}$$

(b). If $R = 2\ \text{k}\Omega$ and $C = 0.1\ \mu\text{F}$, select a value of L to locate zeros at $\pm j5000$ rad/s.

We have the following results:

$$5000 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.1\,\mu)(L)}}$$

$$L = \frac{1}{(0.1\,\mu)(5000^2)} = 400\ \text{mH}$$

(c). Where are the poles located once you have selected the inductor in part (b)?

Solve for the poles.

$$s^2 + 5000s + 5000^2 = 0$$

$$s = -2500 \pm j4330\ \text{rad/s}$$

Problem 10–6. For the circuit of Figure P10–6:

(a). Find and express $Z_{\text{EQ}}(s)$ as a rational function and locate its poles and zeros.

Combine the inductor and resistor in parallel and then combine the result with the series resistor.

$$Z_{\text{EQ}}(s) = 2R + (Ls \parallel R) = 2R + \frac{RLs}{R+Ls}$$

$$= \frac{2RLs + 2R^2 + RLs}{R+Ls} = \frac{3RLs + 2R^2}{R+Ls} = \frac{3Rs + \dfrac{2R^2}{L}}{s + \dfrac{R}{L}}$$

The zero is located at $s = -2R/3L$ and the pole is located at $s = -R/L$.

(b). Select values of R and L to locate a pole at -150 rad/s. Where is the resulting zero?

Pick $L = 1$ H and solve for $R = 150\ \Omega$. The zero is located at $s = -2(150)/3 = -100$ rad/s.

Problem 10–7. For the circuit of Figure P10–7:

(a). Find and express $Z_{\text{EQ}}(s)$ as a rational function and locate its poles and zeros.

Combine the resistor and inductor in parallel and then combine the result in series with the other inductor.

$$Z_{\text{EQ}}(s) = 2Ls + (R \parallel 2Ls) = 2Ls + \frac{2RLs}{R + 2Ls}$$

$$= \frac{2RLs + 4L^2s^2 + 2RLs}{R + 2Ls} = \frac{4Ls(Ls + R)}{2Ls + R}$$

The zeros are located at zero and $s = -R/L$. The pole is located at $s = -R/2L$.

(b). Select values of R and L to locate a pole at -2 krad/s. Where are the resulting zeros?

Choose $R = 1$ kΩ and solve for $L = 250$ mH. The zeros are located at zero and $s = -4$ krad/s.

Problem 10–8. For the circuit of Figure P10–8:

(a). Find and express $Z_{\text{EQ}}(s)$ as a rational function and locate its poles and zeros.

Combine the inductor and capacitor in parallel and then combine the result in series with the resistor.

$$Z_{\text{EQ}}(s) = R + \left(Ls \parallel \frac{1}{Cs}\right) = R + \frac{(Ls)(1/Cs)}{Ls + 1/Cs}$$

$$= R + \frac{\frac{1}{C}s}{s^2 + \frac{1}{LC}} = \frac{Rs^2 + \frac{1}{C}s + \frac{R}{LC}}{s^2 + \frac{1}{LC}}$$

$$= \frac{R\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)}{s^2 + \frac{1}{LC}}$$

The poles are located at $s = \pm j1/\sqrt{LC}$ and the zeros are located at

$$s = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

(b). If $R = 10$ kΩ, select values of L and C to locate poles at $\pm j200$ krad/s. Where are the resulting zeros?

Pick $C = 0.01\ \mu$F. Solve for L.

$$(200000)^2 = \frac{1}{LC}$$

$$L = \frac{1}{(200000)^2(0.01\,\mu)} = 2.5 \text{ mH}$$

The zeros are located at $s = -5000 \pm j199937$ rad/s.

Problem 10–9. For the circuit of Figure P10–9:

(a). If $R = 1$ kΩ, $L = 2$ H, and $C = 0.5$ μF, locate the poles and zeros of $Z_{\text{EQ}}(s)$.

Combine the resistor and capacitor in parallel and then combine the result in series with the inductor.

$$Z_{\text{EQ}}(s) = Ls + \left(R \parallel \frac{1}{Cs}\right) = Ls + \frac{R/Cs}{R + 1/Cs}$$

$$= \frac{RLs + L/C + R/Cs}{R + 1/Cs} = \frac{Ls^2 + \dfrac{L}{RC}s + \dfrac{1}{C}}{s + \dfrac{1}{RC}}$$

$$= \frac{L\left(s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC}\right)}{s + \dfrac{1}{RC}}$$

$$= \frac{2(s^2 + 2000s + 10^6)}{s + 2000}$$

The zeros are locates at $s = -1000$ rad/s and $s = -1000$ rad/s. The pole is located at $s = -2000$ rad/s.

(b). If we were to increase the inductance to 5 H, how would the poles and zeros change?

The pole would remain at $s = -2000$ rad/s. The zeros would shift to $s = -1775$ rad/s and $s = -225.4$ rad/s.

Problem 10–10. Find $Z_{\text{EQ1}}(s)$ and $Z_{\text{EQ2}}(s)$ for the bridge-T circuit in Figure P10–10. Express each impedance as a rational function and locate its poles and zeros.

We have the following results for $Z_{\text{EQ1}}(s)$:

$$Z_{\text{EQ1}}(s) = \left[\frac{1}{Cs} \parallel \left(2R + \frac{2}{Cs}\right)\right] + R$$

$$= \left[\frac{1}{Cs} \parallel \frac{2RCs + 2}{Cs}\right] + R$$

$$= \frac{\dfrac{2RCs + 2}{C^2 s^2}}{\dfrac{1}{Cs} + \dfrac{2RCs + 2}{Cs}} + R$$

$$= \frac{2RCs + 2}{Cs(2RCs + 3)} + R$$

$$= \frac{2RCs + 2 + 2R^2C^2s^2 + 3RCs}{Cs(2RCs + 3)}$$

$$= \frac{2R^2C^2s^2 + 5RCs + 2}{Cs(2RCs + 3)}$$

$$= \frac{(2RCs + 1)(RCs + 2)}{Cs(2RCs + 3)}$$

The zeros are located at $s = -1/2RC$ and $s = -2/RC$. The poles are located at zero and $s = -3/2RC$.

For $Z_{\text{EQ2}}(s)$, we have the following results:

$$\begin{aligned}
Z_{\text{EQ2}}(s) &= \left[\frac{2}{Cs} \parallel \left(2R + \frac{1}{Cs}\right)\right] + R \\
&= \left[\frac{2}{Cs} \parallel \frac{2RCs+1}{Cs}\right] + R \\
&= \frac{\dfrac{4RCs+2}{C^2s^2}}{\dfrac{2}{Cs} + \dfrac{2RCs+1}{Cs}} + R \\
&= \frac{4RCs+2}{Cs(2RCs+3)} + R \\
&= \frac{4RCs + 2 + 2R^2C^2s^2 + 3RCs}{Cs(2RCs+3)} \\
&= \frac{2R^2C^2s^2 + 7RCs + 2}{Cs(2RCs+3)}
\end{aligned}$$

The poles are located at zero and $s = -3/2RC$. The zeros are located at

$$s = \frac{-7RC \pm \sqrt{49R^2C^2 - 16R^2C^2}}{4R^2C^2} = \frac{-7 \pm \sqrt{33}}{4RC}$$

Problem 10–11. For the circuit of Figure P10–11:

(a). Find and express $Z_{\text{EQ}}(s)$ as a rational function and locate its poles and zeros.

Working from right to left, combine the resistor and capacitor in series. Combine that result in parallel with the other resistor. Combine that result in series with the other capacitor.

$$\begin{aligned}
Z_{\text{EQ}}(s) &= \left[\left(2R + \frac{1}{Cs}\right) \parallel 2R\right] + \frac{1}{2Cs} \\
&= \frac{4R^2 + 2R/Cs}{4R + 1/Cs} + \frac{1}{2Cs} \\
&= \frac{4R^2Cs + 2R}{4RCs+1} + \frac{1}{2Cs} \\
&= \frac{4RCs + 1 + 8R^2C^2s^2 + 4RCs}{2Cs(4RCs+1)} \\
&= \frac{8R^2C^2s^2 + 8RCs + 1}{2Cs(4RCs+1)} \\
&= \frac{2RC\left(s^2 + \dfrac{1}{RC}s + \dfrac{1}{8R^2C^2}\right)}{2Cs\left(s + \dfrac{1}{4RC}\right)}
\end{aligned}$$

The poles are located at zero and $s = -1/4RC$. The zeros are located at

$$s = \frac{-\dfrac{1}{RC} \pm \sqrt{\left(\dfrac{1}{RC}\right)^2 - \dfrac{1}{2R^2C^2}}}{2} = \frac{1}{RC}\left(-\frac{1}{2} \pm \sqrt{\frac{1}{8}}\right)$$

(b). If its poles were located at -20 krad/s and zero, where would the poles move to if the value of R was reduced to 25% of its current value?

If R is reduced by a factor of four, the poles would increase in magnitude by a factor of four to be located at zero and $s = -80$ krad/s.

Problem 10–12. For the two-port circuit of Figure P10–12:

(a). Find $Z_{\mathrm{EQ1}}(s)$ and $Z_{\mathrm{EQ2}}(s)$, and express each impedance as a rational function and locate its poles and zeros.

We have the following results for $Z_{\mathrm{EQ1}}(s)$:

$$\begin{aligned} Z_{\mathrm{EQ1}}(s) &= [(2R+2Ls) \parallel R] + Ls \\ &= \frac{2R^2+2LRs}{3R+2Ls} + Ls = \frac{2L^2s^2+5RLs+2R^2}{2Ls+3R} \\ &= \frac{(2Ls+R)(Ls+2R)}{2Ls+3R} \end{aligned}$$

The zeros are located at $s = -R/2L$ and $s = -2R/L$. The pole is located at $s = -3R/2L$.

For $Z_{\mathrm{EQ2}}(s)$, the inductor with magnitude L does not contribute to the impedance, so we have the following results.

$$Z_{\mathrm{EQ2}}(s) = (2R+R) \parallel 2Ls = 3R \parallel 2Ls = \frac{6RLs}{2Ls+3R}$$

The zero is located at zero. The pole is located at $s = -3R/2L$.

(b). Select values of R and L to place a pole at -500 Hz.

A pole at -500 Hz is equivalent to a pole at -1000π rad/s. Pick $R = 1$ kΩ and solve for L

$$L = \frac{(3)(1000)}{(2)(1000\pi)} = 477 \text{ mH}$$

Problem 10–13. Find the equivalent impedance between terminals 1 and 2 in Figure P10–13. Select values of R and L so that $Z_{\mathrm{EQ}}(s)$ has a pole at $s = -3000$ rad/s. Locate the zeros of $Z_{\mathrm{EQ}}(s)$ for your choice of R and L.

We have the following results

$$\begin{aligned} Z_{\mathrm{EQ}}(s) &= 2Ls + (R \parallel Ls) + 2R = 2Ls + \frac{RLs}{R+Ls} + 2R \\ &= \frac{2RLs+2L^2s^2+2R^2+2RLs+RLs}{R+Ls} = \frac{2L^2s^2+5RLs+2R^2}{Ls+R} \end{aligned}$$

The pole is located at $s = -R/L$, so select $R = 3$ kΩ and $L = 1$ H to place the pole at $s = -3000$ rad/s. The zeros are located at the roots of the following equation

$$2L^2s^2+5RLs+2R^2 = 2s^2+15000s+18000000 = 2(s^2+7500s+9000000) = 2(s+6000)(s+1500)$$

The zeros are located at $s = -6000$ rad/s and $s = -1500$ rad/s.

Problem 10–14. For the circuit of Figure P10–14:

(a). Use voltage division to find $V_O(s)$.

Apply voltage division.

$$V_O(s) = \frac{1/Cs}{R + 1/Cs} V_S(s) = \frac{\dfrac{1}{RC}}{s + \dfrac{1}{RC}} V_S(s)$$

(b). Use the look-back method to find $Z_T(s)$.

We have the following results:

$$Z_T(s) = R + \left(R \parallel \frac{1}{Cs}\right) = R + \frac{R/Cs}{R + 1/Cs}$$

$$= \frac{R^2 + 2R/Cs}{R + 1/Cs} = \frac{Rs + \dfrac{2}{C}}{s + \dfrac{1}{RC}}$$

Problem 10–15. For the circuit of Figure P10–15:

(a). Use current division to find $I_2(s)$.

Apply two-path current division.

$$I_2(s) = \frac{R_1 + Ls}{R_1 + R_2 + Ls} I_1(s)$$

(b). Use the look-back method to find $Z_N(s)$.

We have the following results:

$$Z_N(s) = (R_1 + Ls) \parallel R_2 = \frac{(R_1 + Ls)(R_2)}{R_1 + R_2 + Ls}$$

Problem 10–16. The circuit of Figure P10–16 needs to produce an output transform of

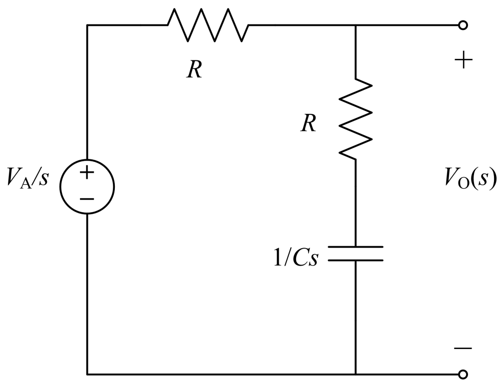
$$V_O(s) = \frac{V_A(RCs + 1)}{s(2RCs + 1)}$$

When the input is V_A/s. Design an appropriate circuit.

Rewrite the expression for the output voltage so that we can easily design a voltage divider circuit to meet the requirements.

$$V_O(s) = \frac{V_A(RCs + 1)}{s(2RCs + 1)} = \left(\frac{R + \dfrac{1}{Cs}}{2R + \dfrac{1}{Cs}}\right) \frac{V_A(s)}{s} = \left(\frac{R + \dfrac{1}{Cs}}{R + R + \dfrac{1}{Cs}}\right) \frac{V_A(s)}{s}$$

We can now design the following circuit.



Problem 10–17. Find the Thévenin equivalent for the circuit in Figure P10–17. Then select values for R and L so that the Thévenin voltage has a pole at -12 krad/s.

Combine the resistor and inductor in parallel and then apply voltage division to find the open-circuit voltage. Compute the lookback resistance by combining all three components in parallel.

$$Z_{\mathrm{EQ}} = R \parallel Ls = \frac{RLs}{R + Ls}$$

$$V_{\mathrm{T}}(s) = \frac{Z_{\mathrm{EQ}}}{R + Z_{\mathrm{EQ}}} V_{\mathrm{S}}(s) = \frac{\dfrac{RLs}{R+Ls}}{R + \dfrac{RLs}{R+Ls}} V_{\mathrm{S}}(s) = \frac{RLs}{R^2 + 2RLs} V_{\mathrm{S}}(s) = \frac{Ls}{R + 2Ls} V_{\mathrm{S}}(s)$$

$$Z_{\mathrm{T}}(s) = R \parallel R \parallel Ls = \frac{R}{2} \parallel Ls = \frac{\dfrac{RLs}{2}}{\dfrac{R}{2} + Ls} = \frac{RLs}{R + 2Ls}$$

The pole for the voltage is located at $s = -R/2L$, so pick $R = 3$ kΩ and solve for $L = 125$ mH, as one possible solution.

Problem 10–18. If the input to the RLC circuit of Figure P10–18 is $v_{\mathrm{S}}(t) = u(t)$:

(a). Find the output voltage transform across each element.

The inputs is $V_{\mathrm{S}}(s) = 1/s$. Apply voltage division to determine the output voltage transforms.

$$V_{\mathrm{R}}(s) = \left(\frac{R}{R + Ls + \dfrac{1}{Cs}} \right) \frac{1}{s} = \frac{R}{Ls^2 + Rs + \dfrac{1}{C}} = \frac{\dfrac{R}{L}}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}}$$

$$V_{\mathrm{L}}(s) = \left(\frac{Ls}{R + Ls + \dfrac{1}{Cs}} \right) \frac{1}{s} = \frac{Ls}{Ls^2 + Rs + \dfrac{1}{C}} = \frac{s}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}}$$

$$V_{\mathrm{C}}(s) = \left(\frac{\dfrac{1}{Cs}}{R + Ls + \dfrac{1}{Cs}} \right) \frac{1}{s} = \frac{\dfrac{1}{Cs}}{Ls^2 + Rs + \dfrac{1}{C}} = \frac{\dfrac{1}{LC}}{s\left(s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}\right)}$$

(b). Compare the three outputs with regards to their respective poles and zeros.

For the resistor voltage, there are two zeros at infinity and the poles are located at

$$s = \frac{-R \pm \sqrt{R^2 - \dfrac{4L}{C}}}{2L}$$

For the inductor, the zeros are at zero and infinity and the poles are the same as those for the resistor output. For the capacitor, three zeros are at infinity and it has two poles that match the other two outputs, plus an additional pole at zero.

(c). Use the initial and final value theorems to determine the value of the voltage across each element at $t = 0$ and $t = \infty$. What conclusions can one draw regarding the results?

We have the following results.

$$v_{\mathrm{R}}(0) = \lim_{s\to\infty} sV_{\mathrm{R}}(s) = 0$$

$$v_{\mathrm{R}}(\infty) = \lim_{s\to 0} sV_{\mathrm{R}}(s) = 0$$

$$v_{\mathrm{L}}(0) = \lim_{s\to\infty} sV_{\mathrm{L}}(s) = 1$$

$$v_{\mathrm{L}}(\infty) = \lim_{s\to 0} sV_{\mathrm{L}}(s) = 0$$

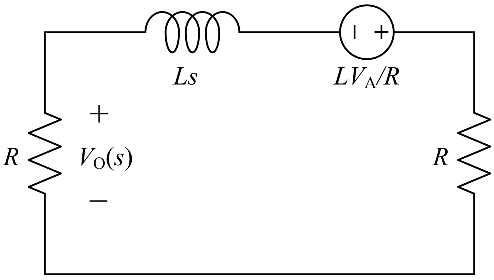
$$v_{\mathrm{C}}(0) = \lim_{s\to\infty} sV_{\mathrm{C}}(s) = 0$$

$$v_{\mathrm{C}}(\infty) = \lim_{s\to 0} sV_{\mathrm{C}}(s) = 1$$

In terms of conclusions, an RLC circuit with a step function input can be used to switch a voltage from zero to one, from one to zero, or to hold a value of zero, depending on how we connect the output terminals.

Problem 10–19. The switch in Figure P10–19 has been in position A for a long time and is moved to position B at $t = 0$. Transform the circuit into the s domain and solve for $I_{\mathrm{L}}(s)$, $i_{\mathrm{L}}(t)$, $V_{\mathrm{O}}(s)$, and $v_{\mathrm{O}}(t)$ in symbolic form.

The initial current through the inductor is $i_{\mathrm{L}}(0) = V_{\mathrm{A}}/R$. The transformed circuit is shown below.



Apply Ohm's law to find the current and the output voltage.

$$I_{\mathrm{L}}(s) = \frac{Li_{\mathrm{L}}(0)}{R+R+Ls} = \frac{\dfrac{LV_{\mathrm{A}}}{R}}{Ls+2R} = \frac{\dfrac{V_{\mathrm{A}}}{R}}{s+\dfrac{2R}{L}}$$

$$i_{\mathrm{L}}(t) = \frac{V_{\mathrm{A}}}{R}e^{-2Rt/L}u(t)$$

$$V_{\mathrm{O}}(s) = -RI_{\mathrm{L}}(s) = \frac{-V_{\mathrm{A}}}{s+\dfrac{2R}{L}}$$

$$v_{\mathrm{O}}(t) = -V_{\mathrm{A}}e^{-2Rt/L}u(t)$$

Problem 10–20. The switch in Figure P10–19 has been in position B for a long time and is moved to position A at $t = 0$. Transform the circuit into the s domain and solve for $I_{\mathrm{L}}(s)$ and $i_{\mathrm{L}}(t)$ in symbolic form.

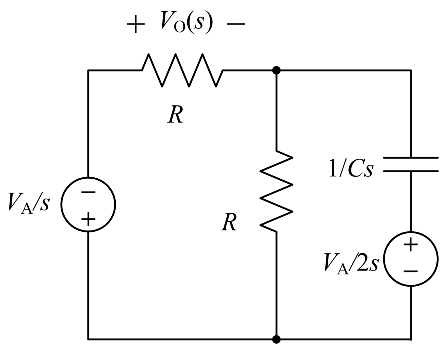
The initial inductor current is zero, so the transformed circuit after the switch moves is the source in series with the inductor and one resistor. The voltage source is $V_{\mathrm{S}}(s) = V_{\mathrm{A}}/s$. Apply Ohm's law to find the current.

$$I_{\rm L}(s) = \frac{\dfrac{V_{\rm A}}{s}}{Ls+R} = \frac{\dfrac{V_{\rm A}}{L}}{s\left(s+\dfrac{R}{L}\right)} = \frac{\dfrac{V_{\rm A}}{R}}{s} - \frac{\dfrac{V_{\rm A}}{R}}{s+\dfrac{R}{L}}$$

$$i_{\rm L}(t) = \frac{V_{\rm A}}{R}\left(1-e^{-Rt/L}\right)u(t)$$

Problem 10–21. The switch in Figure P10–21 has been in position A for a long time and is moved to position B at $t = 0$. Transform the circuit into the s domain and solve for $I_{\rm C}(s)$, $i_{\rm C}(t)$, $V_{\rm O}(s)$, and $v_{\rm O}(t)$ in symbolic form.

Using voltage division, the initial voltage across the capacitor is $v_{\rm C}(0) = V_{\rm A}/2$. The transformed circuit is shown below.



Apply node-voltage analysis to find the capacitor voltage.

$$\frac{V_{\rm C}(s)+V_{\rm A}/s}{R} + \frac{V_{\rm C}(s)}{R} + \frac{V_{\rm C}(s)-V_{\rm A}/2s}{1/Cs} = 0$$

$$V_{\rm C}(s)\left(\frac{1}{R}+\frac{1}{R}+Cs\right) = -\frac{V_{\rm A}}{Rs} + \frac{CV_{\rm A}}{2}$$

$$V_{\rm C}(s)\left(\frac{RCs+2}{R}\right) = \frac{(RCs-2)V_{\rm A}}{2Rs}$$

$$V_{\rm C}(s) = \frac{(RCs-2)V_{\rm A}}{2s(RCs+2)}$$

$$= \frac{(s-2/RC)\dfrac{V_{\rm A}}{2}}{s(s+2/RC)}$$

$$= \frac{-\dfrac{V_{\rm A}}{2}}{s} + \frac{V_{\rm A}}{s+2/RC}$$

Compute the capacitor current.

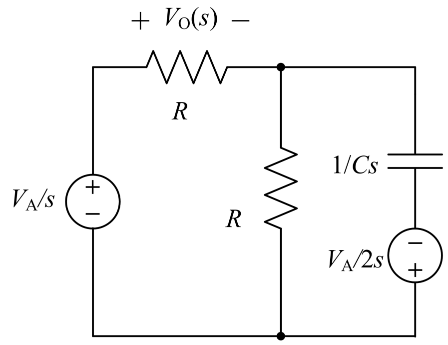
$$\begin{aligned}
I_C(s) &= \frac{V_C(s) - V_A/2s}{1/Cs} = CsV_C(s) - \frac{CV_A}{2} \\
&= \frac{C(RCs-2)V_A}{2(RCs+2)} - \frac{CV_A}{2} \\
&= \frac{\frac{CV_A}{2}(RCs-2) - \frac{CV_A}{2}(RCs+2)}{RCs+2} \\
&= \frac{-2CV_A}{RCs+2} = \frac{-2\frac{V_A}{R}}{s+\frac{2}{RC}} \\
i_C(t) &= -\frac{2V_A}{R}e^{-2t/RC}u(t)
\end{aligned}$$

Compute the output voltage.

$$\begin{aligned}
V_O(s) &= -\frac{V_A}{s} - V_C(s) = \frac{-\frac{V_A}{2}}{s} - \frac{V_A}{s+2/RC} \\
v_O(t) &= -\frac{V_A}{2}\left(1+2e^{-2t/RC}\right)u(t)
\end{aligned}$$

Problem 10–22. The switch in Figure P10–21 has been in position B for a long time and is moved to position A at $t = 0$. Transform the circuit into the s domain and solve for $V_C(s)$, $v_C(t)$, $V_O(s)$, and $v_O(t)$ in symbolic form.

Using voltage division, the initial voltage across the capacitor is $v_C(0) = -V_A/2$. The transformed circuit is shown below.



The analysis is simimlar to that in Problem 10-21, with two of the voltage source signs changed. Apply node-voltage analysis to find the capacitor voltage.

$$\frac{V_\text{C}(s)-V_\text{A}/s}{R}+\frac{V_\text{C}(s)}{R}+\frac{V_\text{C}(s)+V_\text{A}/2s}{1/Cs}=0$$

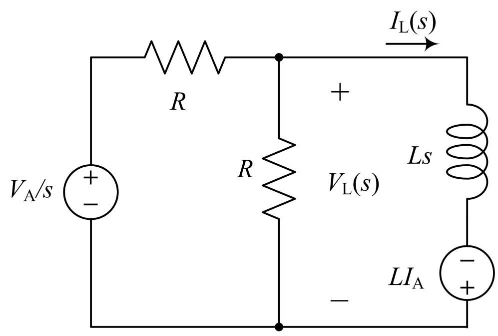
$$\begin{aligned}
V_\text{C}(s)\left(\frac{1}{R}+\frac{1}{R}+Cs\right) &= \frac{V_\text{A}}{Rs}-\frac{CV_\text{A}}{2}\\
V_\text{C}(s)\left(\frac{RCs+2}{R}\right) &= \frac{(2-RCs)V_\text{A}}{2Rs}\\
V_\text{C}(s) &= \frac{(2-RCs)V_\text{A}}{2s(RCs+2)}\\
&= \frac{-\frac{V_\text{A}}{2}(s-2/RC)}{s(s+2/RC)}\\
&= \frac{\frac{V_\text{A}}{2}}{s}-\frac{V_\text{A}}{s+2/RC}\\
v_\text{C}(t) &= \frac{V_\text{A}}{2}\left(1-2e^{-2t/RC}\right)u(t)
\end{aligned}$$

Compute the output voltage.

$$\begin{aligned}
V_\text{O}(s) &= \frac{V_\text{A}}{s}-V_\text{C}(s)=\frac{\frac{V_\text{A}}{2}}{s}+\frac{V_\text{A}}{s+2/RC}\\
v_\text{O}(t) &= \frac{V_\text{A}}{2}\left(1+2e^{-2t/RC}\right)u(t)
\end{aligned}$$

Problem 10–23. Transform the circuit in Figure P10–23 into the s domain and find: $I_\text{L}(s)$, $i_\text{L}(t)$, $V_\text{L}(s)$, and $v_\text{L}(t)$ when $v_1(t)=V_\text{A}u(t)$ and $i_\text{L}(0)=I_\text{A}$.

The transformed circuit is shown below.



Apply node-voltage analysis.

$$\frac{V_\mathrm{L}(s) - V_\mathrm{A}/s}{R} + \frac{V_\mathrm{L}(s)}{R} + \frac{V_\mathrm{L}(s) + LI_\mathrm{A}}{Ls} = 0$$

$$V_\mathrm{L}(s)\left(\frac{1}{R} + \frac{1}{R} + \frac{1}{Ls}\right) = \frac{V_\mathrm{A}/R - I_\mathrm{A}}{s}$$

$$V_\mathrm{L}(s)\left(\frac{2Ls+R}{RLs}\right) = \frac{V_\mathrm{A}/R - I_\mathrm{A}}{s}$$

$$V_\mathrm{L}(s) = \frac{(V_\mathrm{A}/R - I_\mathrm{A})RL}{2Ls+R} = \frac{\frac{1}{2}(V_\mathrm{A} - RI_\mathrm{A})}{s + \frac{R}{2L}}$$

$$v_\mathrm{L}(t) = \frac{1}{2}(V_\mathrm{A} - RI_\mathrm{A})e^{-Rt/2L}u(t)$$

Compute the inductor current.

$$I_\mathrm{L}(s) = \frac{V_\mathrm{L}(s) + LI_\mathrm{A}}{Ls} = \frac{\frac{1}{2}(V_\mathrm{A} - RI_\mathrm{A})}{Ls\left(s + \frac{R}{2L}\right)} + \frac{LI_\mathrm{A}}{Ls}$$

$$= \frac{V_\mathrm{A}/2 - RI_\mathrm{A}/2 + LI_\mathrm{A}s + RI_\mathrm{A}/2}{Ls\left(s + \frac{R}{2L}\right)}$$

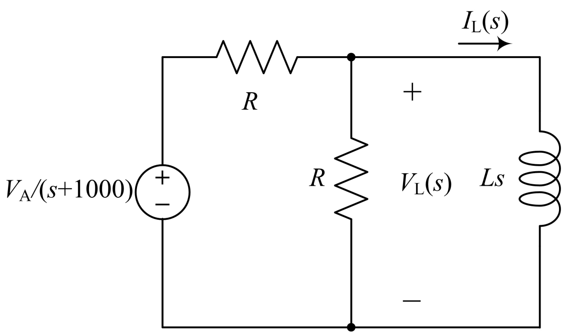
$$= \frac{V_\mathrm{A}/2L + I_\mathrm{A}s}{s\left(s + \frac{R}{2L}\right)}$$

$$= \frac{\frac{V_\mathrm{A}}{R}}{s} + \frac{I_\mathrm{A} - \frac{V_\mathrm{A}}{R}}{s + \frac{R}{2L}}$$

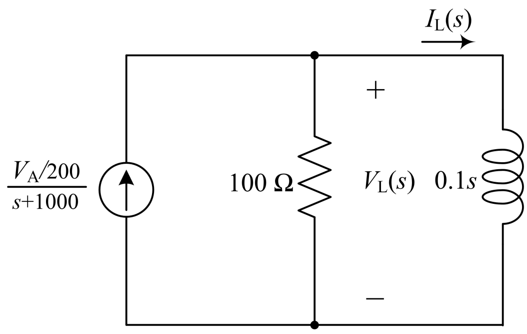
$$i_\mathrm{L}(t) = \left[\frac{V_\mathrm{A}}{R} + \left(I_\mathrm{A} - \frac{V_\mathrm{A}}{R}\right)e^{-Rt/2L}\right]u(t)$$

Problem 10–24. Transform the circuit in Figure P10–23 into the s domain and find $I_\mathrm{L}(s)$ and $i_\mathrm{L}(t)$ when $v_1(t) = V_\mathrm{A}e^{-1000t}u(t)$, $R = 200\ \Omega$, $L = 100$ mH, and $i_\mathrm{L}(0) = 0$ A.

The transformed voltage source is $V_1(s) = V_\mathrm{A}/(s + 1000)$. The transformed circuit is shown below.



Perform a source transformation on the circuit and combine the two resistors in parallel. Substitute in the component values to get the following simplified circuit.



Apply two-path current division.

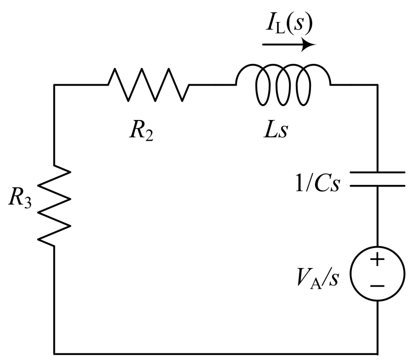
$$I_L(s) = \left(\frac{100}{0.1s + 100}\right)\left(\frac{V_A/200}{s + 1000}\right) = \frac{5V_A}{(s + 1000)^2}$$

$$i_L(t) = 5V_A te^{-1000t}u(t)$$

Problem 10–25. The switch in Figure P10–25 has been in position A for a long time and is moved to position B at $t = 0$.

(a). Transform the circuit into the s domain and solve for $I_L(s)$ in symbolic form.

The initial inductor current is zero and the initial capacitor voltage is $v_C(0) = V_A$. The transformed circuit is shown below.



Solve for the inductor current.

$$I_L(s) = \left(-\frac{V_A}{s}\right)\left(\frac{1}{Z_{EQ}}\right) = \frac{-V_A}{s(R_2 + R_3 + Ls + 1/Cs)}$$

$$= \frac{-\dfrac{V_A}{L}}{s^2 + \dfrac{R_2 + R_3}{L}s + \dfrac{1}{LC}}$$

(b). Repeat part (a) using MATLAB.

The MATLAB code is shown below.

```
syms s t
syms R1 R2 R3 L C VA positive
ZR = R2+R3;
ZL = L*s;
ZC = 1/C/s;
Zeq = ZR+ZL+ZC;
ILs = factor(-VA/s/Zeq)
```

The corresponding output is shown below and agrees with the results in part (a).

```
ILs = -(VA*C)/(C*R2*s + C*R3*s + C*L*s^2 + 1)
```

(c). Find $i_L(t)$ for $R_1 = R_2 = 500\ \Omega$, $R_3 = 1\ \text{k}\Omega$, $L = 500$ mH, $C = 0.2\ \mu$F, and $V_A = 15$ V.

Substitute in the component values and complete the square in the denominator to be able to take the inverse Laplace transform.

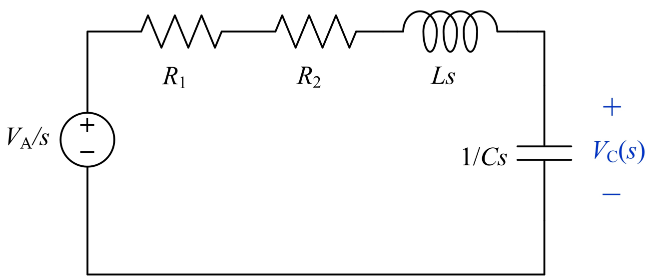
$$I_L(s) = \frac{-30}{s^2 + 3000s + 10^7} = \frac{(-0.010776)(2784)}{(s+1500)^2 + (2784)^2}$$

$$i_L(t) = -10.776e^{-1500t}\sin(2784t)u(t)\ \text{mA}$$

Problem 10–26. The switch in Figure P10–25 has been in position B for a long time and is moved to position A at $t = 0$.

(a). Transform the circuit into the s domain and solve for $V_C(s)$ in symbolic form.

The initial inductor current is zero and the initial capacitor voltage is zero. The transformed circuit is shown below.



Apply voltage division to solve for $V_C(s)$.

$$Z_{EQ}(s) = R_1 + R_2 + Ls + \frac{1}{Cs} = \frac{LCs^2 + (R_1+R_2)Cs + 1}{Cs}$$

$$V_C(s) = \left(\frac{\frac{1}{Cs}}{Z_{EQ}(s)}\right)\frac{V_A}{s} = \frac{V_A}{s(LCs^2 + (R_1+R_2)Cs + 1)}$$

$$= \frac{\frac{V_A}{LC}}{s\left(s^2 + \frac{R_1+R_2}{L}s + \frac{1}{LC}\right)}$$

(b). Find $v_{\rm C}(t)$ for $R_1 = 50\ \Omega$, $R_2 = 250\ \Omega$, $R_3 = 1\ \text{k}\Omega$, $L = 500$ mH, $C = 1\ \mu$F, and $V_{\rm A} = 15$ V. In addition, the inductor is not ideal, but has a parasitic resistance (in series) of 100 Ω.

The parasitic resistance adds to the other resistors in series. Substitute in the component values and perform partial fraction expansion.

$$V_{\rm C}(s) = \frac{3\times 10^7}{s(s^2+800s+2\times 10^6)} = \frac{A}{s} + \frac{Bs+C}{s^2+800s+2\times 10^6}$$

$$3\times 10^7 = As^2 + 800As + (2\times 10^6)A + Bs^2 + Cs$$

$$A = 15$$

$$B = -15$$

$$C = -12000$$

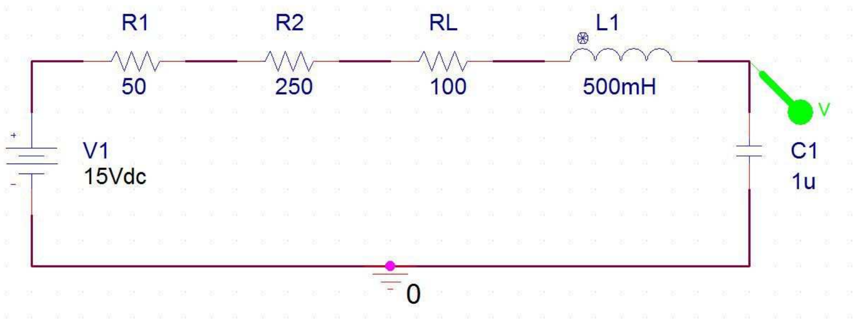
$$V_{\rm C}(s) = \frac{15}{s} - \frac{15s+12000}{s^2+800s+2\times 10^6} = \frac{15}{s} - \frac{15s+12000}{(s+400)^2+(1356)^2}$$

$$= \frac{15}{s} - \frac{15(s+400)}{(s+400)^2+(1356)^2} - \frac{(4.423)(1356)}{(s+400)^2+(1356)^2}$$

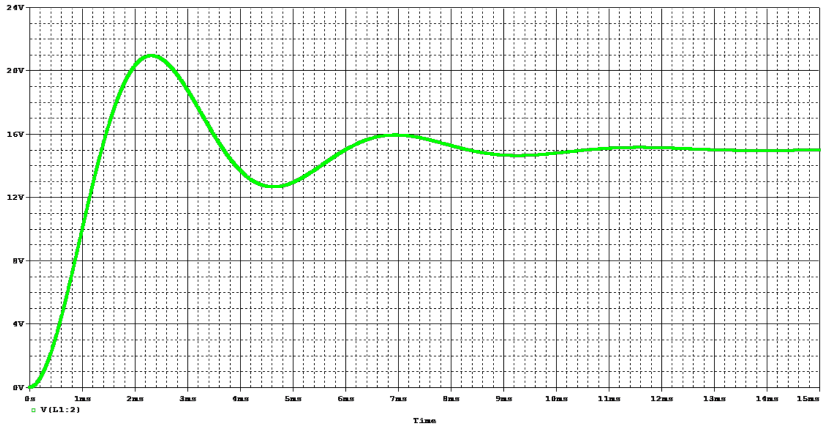
$$v_{\rm C}(t) = 15 - 15e^{-400t}\left[\cos(1356t) + 0.2949\sin(1356t)\right]u(t)\ \text{V}$$

(c). Repeat part (b) using OrCAD.

The OrCAD simulation is shown below. In the simulation, the initial current through the inductor is zero and the initial voltage across the capacitor is zero.



The simulation results are shown below and are consistent with the results in part (b).



Problem 10–27. The circuit in Figure P10–27 is in the zero state. The s-domain relationship between the input $I_1(s)$ and the output $I_R(s)$ is usually given as a ratio called a network function. Find $I_R(s)/I_1(s)$. Identify the poles and the zeros.

Apply two-path current division.

$$I_R(s) = \frac{Ls + \dfrac{1}{Cs}}{Ls + R + \dfrac{1}{Cs}} I_1(s)$$

$$\frac{I_R(s)}{I_1(s)} = \frac{LCs^2 + 1}{LCs^2 + RCs + 1} = \frac{s^2 + \dfrac{1}{LC}}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}}$$

The zeros are located at $s = \pm j/\sqrt{LC}$ and the poles are located at

$$s = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC}$$

Problem 10–28. The circuit in Figure P10–28 is in the zero state. Find the s-domain relationship between the input $I_1(s)$ and the output $V_O(s)$. Identify the poles and the zeros.

Combine Ohm's law with two-path current division to determine the output voltage.

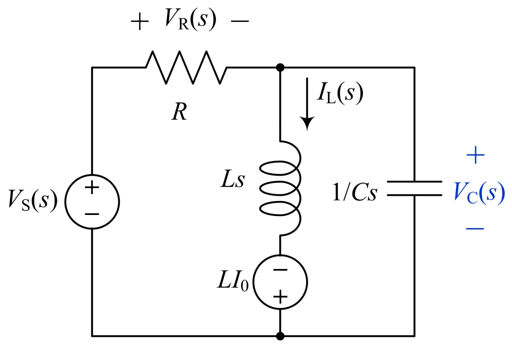
$$V_O(s) = \left(R + \frac{2}{Cs}\right) I_O(s) = \left(\frac{RCs + 2}{Cs}\right)\left(\frac{2R + \dfrac{1}{Cs}}{3R + \dfrac{3}{Cs}} I_1(s)\right)$$

$$\frac{V_O(s)}{I_1(s)} = \frac{(RCs + 2)(2RCs + 1)}{Cs(3RCs + 3)} = \frac{2R^2C^2s^2 + 5RCs + 2}{3Cs(RCs + 1)}$$

The zeros are located at $s = -2/RC$ and $s = -1/2RC$. The poles are located at zero and $s = -1/RC$.

Problem 10–29. The initial conditions for the circuit in Figure P10–29 are $v_C(0) = 0$ and $i_L(0) = I_0$. Transform the circuit into the s domain and use superposition and voltage division to find the zero-state and zero-input components of $V_C(s)$.

The transformed circuit is shown below.



To find the zero-state response, activate only $V_S(s)$. Combine the inductor and capacitor in parallel and apply voltage division.

$$V_{Czs}(s) = \frac{\dfrac{Ls/Cs}{Ls+1/Cs}}{R+\dfrac{Ls/Cs}{Ls+1/Cs}}V_S(s) = \frac{L/C}{RLs+R/Cs+L/C}V_S(s)$$

$$= \frac{\dfrac{1}{RC}s}{s^2+\dfrac{1}{RC}s+\dfrac{1}{LC}}V_S(s)$$

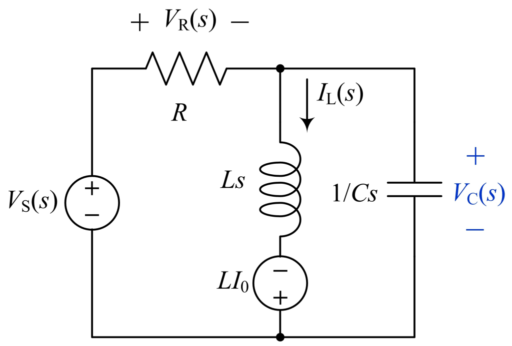
To find the zero-input response, activate the source associated with the inductor's initial current. Combine the resistor and capacitor in parallel and apply voltage division.

$$V_{Czi}(s) = \frac{-\dfrac{R/Cs}{R+1/Cs}}{Ls+\dfrac{R/Cs}{R+1/Cs}}LI_0 = \frac{-R/Cs}{RLs+L/C+R/Cs}LI_0$$

$$= \frac{-\dfrac{1}{C}I_0}{s^2+\dfrac{1}{RC}s+\dfrac{1}{LC}}$$

Problem 10–30. The initial conditions for the circuit in Figure P10–29 are $v_C(0) = 0$ and $i_L(0) = I_0$. Transform the circuit into the s domain and use superposition and voltage division to find the zero-state and zero-input components of $V_R(s)$.

The transformed circuit is shown below.



To find the zero-state response, activate only $V_S(s)$. Combine the inductor and capacitor in parallel and apply voltage division.

$$V_{\rm Rzs}(s) = \frac{R}{R + \dfrac{Ls/Cs}{Ls + 1/Cs}} V_{\rm S}(s) = \frac{RLs + R/Cs}{RLs + R/Cs + L/C} V_{\rm S}(s)$$

$$= \frac{s^2 + \dfrac{1}{LC}}{s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC}} V_{\rm S}(s)$$

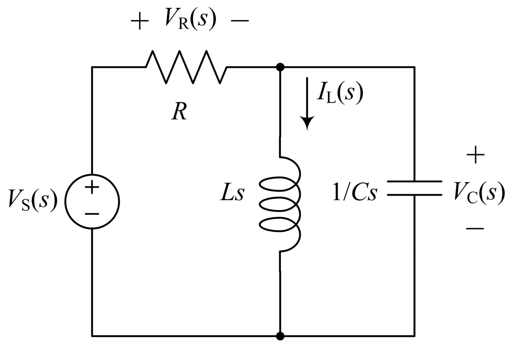
To find the zero-input response, activate the source associated with the inductor's initial current. Combine the resistor and capacitor in parallel and apply voltage division.

$$V_{\rm Rzi}(s) = \frac{\dfrac{R/Cs}{R + 1/Cs}}{Ls + \dfrac{R/Cs}{R + 1/Cs}} LI_0 = \frac{R/Cs}{RLs + L/C + R/Cs} LI_0$$

$$= \frac{\dfrac{1}{C} I_0}{s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC}}$$

Problem 10–31. The circuit in Figure P10–29 is in the zero state. Transform the circuit into the s domain and find the Thévenin equivalent circuit at the capacitor's terminals.

The transformed circuit is shown below.



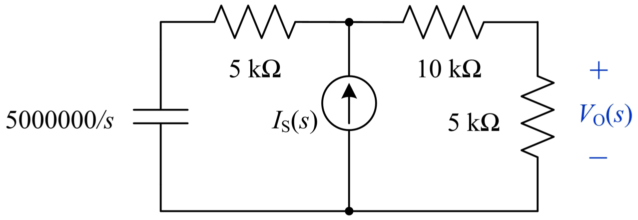
Apply voltage division to find the open-circuit voltage and then determine the lookback impedance.

$$V_{\rm T}(s) = \frac{Ls}{R + Ls} V_{\rm S}(s)$$

$$Z_{\rm T}(s) = R \parallel Ls = \frac{RLs}{R + Ls}$$

Problem 10–32. There is no energy stored in the capacitor in Figure P10–32 at $t = 0$. Transform the circuit into the s domain and use current division to find $v_{\rm O}(t)$ when the input is $i_{\rm S}(t) = 5e^{-500t}u(t)$ mA. Identify the forced and natural poles in $V_{\rm O}(s)$.

The transformed circuit is shown below.



Apply two-path current division and Ohm's law.

$$I_S(s) = \frac{0.005}{s+500}$$

$$I_O(s) = \frac{5000 + \frac{5000000}{s}}{20000 + \frac{5000000}{s}} I_S(s) = \frac{\frac{s}{4} + 250}{s+250} I_S(s)$$

$$= \frac{\frac{1}{4}(s+1000)(0.005)}{(s+250)(s+500)}$$

$$V_O(s) = 5000 I_O(s) = \frac{\frac{25}{4}(s+1000)}{(s+250)(s+500)} = \frac{18.75}{s+250} - \frac{12.5}{s+500}$$

$$v_O(t) = \left(\underbrace{18.75e^{-250t}}_{\text{Natural}} - \underbrace{12.5e^{-500t}}_{\text{Forced}}\right) u(t) \text{ V}$$

Problem 10–33. Repeat Problem 10–32 when $i_S(t) = 2.5\cos(2000t)u(t)$ mA.

Use the results from Problem 10–32 with the new source voltage.

$$I_S(s) = \frac{0.0025s}{s^2 + 2000^2}$$

$$I_O(s) = \frac{\frac{s}{4} + 250}{s+250} I_S(s) = \frac{\frac{1}{4}(s+1000)(0.0025s)}{(s+250)(s^2+2000^2)}$$

$$V_O(s) = 5000 I_O(s) = \frac{\frac{25}{8}(s+1000)(s)}{(s+250)(s^2+2000^2)}$$

$$= \frac{A}{s+250} + \frac{Bs+C}{s^2+2000^2} = \frac{-0.14423}{s+250} + \frac{3.2692s + 2307.7}{s^2+2000^2}$$

$$= \frac{-0.14423}{s+250} + \frac{3.2692s}{s^2+2000^2} + \frac{(1.1538)(2000)}{s^2+2000^2}$$

$$v_O(t) = \left(\underbrace{-0.14423e^{-250t}}_{\text{Natural}} + \underbrace{3.2692\cos(2000t) + 1.1538\sin(2000t)}_{\text{Forced}}\right) u(t) \text{ V}$$

Problem 10–34. The circuit in Figure P10–34 is in the zero state. Use a Thévenin equivalent to find the s-domain relationship between the input $I_S(s)$ and the interface current $I(s)$.

Perform a source transformation to create the Thévenin equivalent circuit. In the s domain, we have

$$V_{\mathrm{T}}(s) = \frac{I_{\mathrm{S}}(s)}{Cs}$$

$$Z_{\mathrm{T}}(s) = R + \frac{1}{Cs} = \frac{RCs+1}{Cs}$$

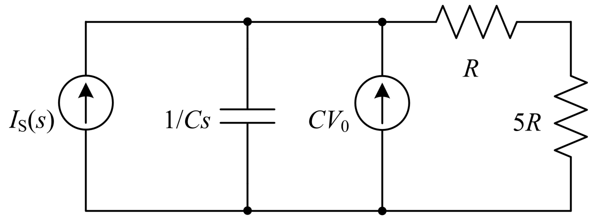
$$I(s) = \frac{V_{\mathrm{T}}(s)}{Z_{\mathrm{EQ}}(s)} = \frac{\frac{I_{\mathrm{S}}(s)}{Cs}}{R + \frac{1}{Cs} + 5R}$$

$$= \frac{I_{\mathrm{S}}(s)}{6RCs+1}$$

Problem 10–35. For the circuit of Figure 10–34:

(a). Find the Thévenin equivalent circuit that the $5R$ load resistor sees in when $v_{\mathrm{C}}(0) = V_0$ V.

The transformed circuit is shown below.



Add the current sources together to get a single source $I_{\mathrm{S}}(s) + CV_0$. Perform a source transformation with the capacitor to find the Thévenin equivalent.

$$V_{\mathrm{T}}(s) = \frac{I_{\mathrm{S}}(s) + CV_0}{Cs}$$

$$Z_{\mathrm{T}}(s) = R + \frac{1}{Cs} = \frac{RCs+1}{Cs}$$

(b). Then find the voltage delivered to the load $v_{\mathrm{O}}(t)$ if $v_{\mathrm{C}}(0) = 10$ V, $i_{\mathrm{S}}(t) = 10u(t)$ mA, $R = 1$ kΩ, and $C = 2\ \mu$F.

Use the Thévenin equivalent circuit and apply voltage division.

$$I_{\mathrm{S}}(s) = \frac{0.01}{s}$$

$$V_{\mathrm{O}}(s) = \frac{5R}{6R + \dfrac{1}{Cs}} V_{\mathrm{T}}(s) = \frac{5R[I_{\mathrm{S}}(s) + CV_0]}{6RCs + 1}$$

$$= \frac{5000\left(\dfrac{0.01}{s} + 20\,\mu\right)}{0.012s + 1} = \frac{50}{s(0.012s + 1)} + \frac{0.1}{0.012s + 1}$$

$$= \frac{4167}{s(s + 83.3)} + \frac{8.33}{s + 83.3}$$

$$= \frac{50}{s} - \frac{50}{s + 83.3} + \frac{8.33}{s + 83.3}$$

$$v_{\mathrm{O}}(t) = \left(50 - 50e^{-83.3t} + 8.33e^{-83.3t}\right) u(t) \text{ V}$$

(c). Identify the forced, natural, zero state, and zero input components of $v_{\mathrm{O}}(t)$.

The components are identified below.

$$v_{\mathrm{Ozs}}(t) = \left(50 - 50e^{-83.3t}\right) u(t) \text{ V}$$

$$v_{\mathrm{Ozi}}(t) = \left(8.33e^{-83.3t}\right) u(t) \text{ V}$$

$$v_{\mathrm{OForced}}(t) = 50u(t) \text{ V}$$

$$v_{\mathrm{ONatural}}(t) = \left(-50e^{-83.3t} + 8.33e^{-83.3t}\right) u(t) \text{ V}$$

Problem 10–36. The circuit in Figure P10–36 is in the zero state. Find the Thévenin equivalent to the left of the interface.

Apply voltage division to find the open-circuit voltage and then find the lookback impedance.

$$V_{\mathrm{T}}(s) = \left(\frac{20000}{40000 + \dfrac{10^7}{s}}\right)\left(\frac{100}{s}\right) = \frac{50}{s + 250}$$

$$Z_{\mathrm{T}}(s) = \left(20000 + \frac{10^7}{s}\right) \parallel 20000 = \frac{\left(20000 + \dfrac{10^7}{s}\right)(20000)}{40000 + \dfrac{10^7}{s}} = \frac{10000s + 5000000}{s + 250} = \frac{10000(s + 500)}{s + 250}$$

Problem 10–37. The Thévenin equivalent shown in Figure P10–37 needs to deliver

$$V_{\mathrm{O}}(s) = \frac{50000}{(s + 1500)(s + 10)} \text{ V}$$

to a 1-kΩ load. Design an interface to allow that to occur.

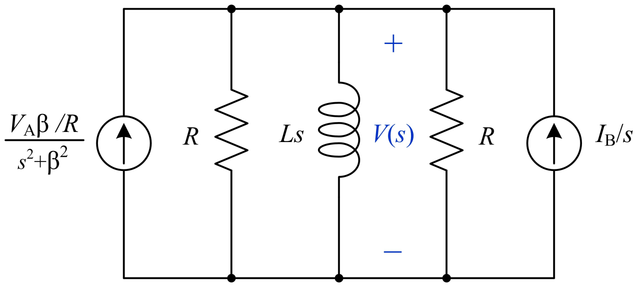
Let the interface be a series component. Apply voltage division.

$$V_{\rm O}(s) = \frac{50000}{(s+1500)(s+10)} = \left(\frac{1000}{Z_{\rm EQ}(s)}\right)\left(\frac{100}{s+10}\right)$$
$$= \frac{100000}{Z_{\rm EQ}(s)(s+10)}$$
$$Z_{\rm EQ}(s) = 2(s+1500) = 2s+3000 = s+2000+Z_{\rm I}(s)+1000$$
$$Z_{\rm I}(s) = s$$

The series interface component is a 1-H inductor.

Problem 10–38. There is no initial energy stored in the circuit in Figure P10–38. Transform the circuit into the s domain and use superposition to find $V(s)$. Identify the forced and natural poles in $V(s)$.

Transform the circuit into the s domain and perform a source transformation on the voltage source to get the circuit shown below.



Find the equivalent impedance to determine the voltage across the inductor.

$$Z_{\rm EQ}(s) = R \parallel R \parallel Ls = \frac{R}{2} \parallel Ls = \frac{\dfrac{RLs}{2}}{\dfrac{R}{2}+Ls} = \frac{RLs}{2Ls+R}$$

$$V_1(s) = Z_{\rm EQ}(s)I_1(s) = \left(\frac{RLs}{2Ls+R}\right)\left(\frac{\dfrac{V_{\rm A}}{R}\beta}{s^2+\beta^2}\right) = \frac{\beta Ls V_{\rm A}}{(s^2+\beta^2)(2Ls+R)}$$

$$V_2(s) = Z_{\rm EQ}(s)I_2(s) = \left(\frac{RLs}{2Ls+R}\right)\left(\frac{I_{\rm B}}{s}\right) = \frac{I_{\rm B}RL}{2Ls+R}$$

$$V(s) = V_1(s)+V_2(s) = \frac{\beta LsV_{\rm A} + I_{\rm B}RLs^2 + I_{\rm B}RL\beta^2}{(s^2+\beta^2)(2Ls+R)}$$

The forced poles are located at $s = \pm j\beta$ and the natural pole is located at $s = -R/2L$.

Problem 10–39. The equivalent impedance between a pair of terminals is

$$Z_{\rm EQ}(s) = 2000\left[\frac{s+3000}{s+2000}\right]\ \Omega$$

A voltage $v(t) = 10u(t)$ is applied across the terminals. Find the resulting current response $i(t)$.

We have the following results.

$$V(s) = \frac{10}{s}$$

$$I(s) = \frac{V(s)}{Z_{\rm EQ}(s)} = \left(\frac{10}{s}\right)\left(\frac{s+2000}{2000(s+3000)}\right) = \frac{\frac{1}{200}(s+2000)}{s(s+3000)}$$

$$= \frac{\frac{1}{300}}{s} + \frac{\frac{1}{600}}{s+3000}$$

$$i(t) = \frac{1}{600}\left(2 + e^{-3000t}\right)u(t)$$

Problem 10–40. There is no initial energy stored in the circuit in Figure P10–40. Use circuit reduction to find the output network function $V_2(s)/V_1(s)$. Then select values of R and C so that the poles of the network function are approximately -2618 and -382 rad/s.

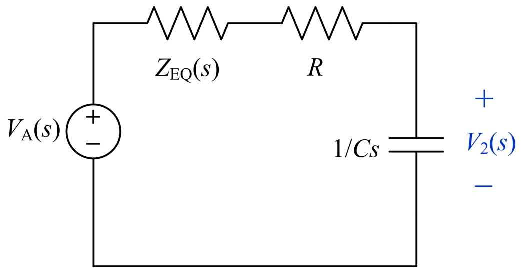
Perform a source transformation to get a current source $CsV_1(s)$ in parallel with the capacitor. Combine the capacitor in parallel with the resistor to get $Z_{\rm EQ}(s)$.

$$Z_{\rm EQ}(s) = \frac{R/Cs}{R+1/Cs} = \frac{R}{RCs+1}$$

Perform another source transformation to get a new voltage source $V_{\rm A}(s)$.

$$V_{\rm A}(s) = Z_{\rm EQ}(s)CsV_1(s) = \left[\frac{R}{RCs+1}\right][CsV_1(s)] = \frac{RCsV_1(s)}{RCs+1}$$

The resulting circuit is shown below.



Apply voltage division

$$V_2(s) = \left(\frac{\frac{1}{Cs}}{\frac{R}{RCs+1}+R+\frac{1}{Cs}}\right)\left(\frac{RCsV_1(s)}{RCs+1}\right)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{R}{R+R^2Cs+R+R+\frac{1}{Cs}} = \frac{\frac{1}{RC}s}{s^2+\frac{3}{RC}s+\frac{1}{R^2C^2}}$$

For the required poles, we have:

$$(s+2618)(s+382) = s^2+3000s+10^6$$

$$\frac{1}{RC} = 1000$$

Choose $R = 1$ kΩ and $C = 1$ μF to meet the specifications.

Problem 10–41. There is no initial energy stored in the circuit in Figure P10–41.

(a). Transform the circuit into the s domain and formulate mesh-current equations.

Write the mesh-current equations by inspection.

$$(Ls + R_1)I_\mathrm{A}(s) - R_1 I_\mathrm{B}(s) = V_1(s)$$

$$-R_1 I_\mathrm{A}(s) + \left(R_1 + R_2 + \frac{1}{Cs}\right) I_\mathrm{B}(s) = 0$$

(b). Show that the solution of these equations for $I_2(s)$ in symbolic form is

$$I_2(s) = I_\mathrm{B}(s) = \frac{R_1 C s V_1(s)}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

Solve the second mesh-current equation for $I_\mathrm{A}(s)$ and substitute into the first equation.

$$I_\mathrm{A}(s) = \frac{\left(R_1 + R_2 + \frac{1}{Cs}\right) I_\mathrm{B}(s)}{R_1}$$

$$V_1(s) = (Ls + R_1)\frac{\left(R_1 + R_2 + \frac{1}{Cs}\right) I_\mathrm{B}(s)}{R_1} - R_1 I_\mathrm{B}(s)$$

$$R_1 V_1(s) = \left[R_1 Ls + R_2 Ls + \frac{L}{C} + R_1^2 + R_1R_2 + \frac{R_1}{Cs} - R_1^2\right] I_\mathrm{B}(s)$$

$$R_1 C s V_1(s) = \left[(R_1 + R_2)LCs^2 + (L + R_1R_2C)s + R_1\right] I_\mathrm{B}(s)$$

$$I_2(s) = I_\mathrm{B}(s) = \frac{R_1 C s V_1(s)}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

(c). Identify the poles and zeros of $I_2(s)$.

Assume $V_1(s)$ does not contribute to the poles and zeros. The zero is located at zero. The poles are located at

$$s = \frac{-(L + R_1R_2C) \pm \sqrt{(L + R_1R_2C)^2 - 4(R_1 + R_2)LCR_1}}{2(R_1 + R_2)LC}$$

(d). Find $i_2(t)$ for $v_1(t) = 10u(t)$ V, $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $L = 2$ H, and $C = 0.5$ μF.

Compute the response.

$$I_2(s) = \left[\frac{R_1 Cs}{(R_1+R_2)LCs^2 + (R_1R_2C+L)s + R_1}\right]\left[\frac{10}{s}\right]$$

$$= \frac{5000}{3000s^2 + 3000000s + 10^9} = \frac{\frac{5}{3}}{s^2 + 1000s + \frac{10^6}{3}}$$

$$= \left(\frac{\sqrt{3}}{300}\right)\frac{\frac{500\sqrt{3}}{3}}{(s+500)^2 + \left(\frac{500\sqrt{3}}{3}\right)^2}$$

$$i_2(t) = \frac{\sqrt{3}}{300}e^{-500t}\sin\left(\frac{500\sqrt{3}}{3}t\right)u(t)\text{ A}$$

Problem 10–42. There is no initial energy stored in the circuit in Figure P10–41.

(a). Transform the circuit into the s domain and formulate node-voltage equations.

Write the node-voltage equations by inspection.

$$\left(\frac{1}{Ls} + \frac{1}{R_1} + \frac{1}{R_2}\right)V_{\text{A}}(s) - \frac{1}{R_2}V_{\text{B}}(s) = \frac{V_1(s)}{Ls}$$

$$-\frac{1}{R_2}V_{\text{A}}(s) + \left(\frac{1}{R_2} + Cs\right)V_{\text{B}}(s) = 0$$

(b). Show that the solution of these equations for $V_2(s)$ in symbolic form is

$$V_2(s) = \frac{R_1V_1(s)}{(R_1+R_2)LCs^2 + (R_1R_2C+L)s + R_1}$$

Solve the second node-voltage equation for $V_{\text{A}}(s)$ and substitute into the first equation.

$$\frac{1}{R_2}V_{\text{A}}(s) = \left(\frac{R_2Cs+1}{R_2}\right)V_{\text{B}}(s)$$

$$V_{\text{A}}(s) = (R_2Cs+1)V_{\text{B}}(s)$$

$$\frac{V_1(s)}{Ls} = \left(\frac{1}{Ls} + \frac{1}{R_1} + \frac{1}{R_2}\right)(R_2Cs+1)V_{\text{B}}(s) - \frac{1}{R_2}V_{\text{B}}(s)$$

$$R_1R_2V_1(s) = (R_1R_2^2Cs + R_1R_2 + R_2^2LCs^2 + R_2Ls + R_1R_2LCs^2 + R_1Ls - R_1Ls)V_{\text{B}}(s)$$

$$V_2(s) = V_{\text{B}}(s) = \frac{R_1V_1(s)}{(R_1+R_2)LCs^2 + (R_1R_2C+L)s + R_1}$$

(c). Identify the natural and forced poles of $V_2(s)$.

Assume $V_1(s)$ contributes the forced poles, if any. The natural poles are located at

$$s = \frac{-(L+R_1R_2C) \pm \sqrt{(L+R_1R_2C)^2 - 4(R_1+R_2)LCR_1}}{2(R_1+R_2)LC}$$

(d). Find $v_2(t)$ for $v_1(t) = 25u(t)$ V, $R_1 = R_2 = 500\ \Omega$, $L = 0.5$ H, and $C = 2\ \mu$F.

Compute the response

$$\begin{aligned}
V_2(s) &= \left[\frac{R_1}{(R_1+R_2)LCs^2 + (R_1R_2C+L)s + R_1}\right]\left[\frac{25}{s}\right] \\
&= \frac{12500}{s(0.001s^2+s+500)} = \frac{12500000}{s(s^2+1000s+500000)} \\
&= \frac{A}{s} + \frac{Bs+C}{s^2+1000s+500000} = \frac{25}{s} + \frac{-25s-25000}{s^2+1000s+500000} \\
&= \frac{25}{s} - \frac{25(s+500)}{(s+500)^2+500^2} - \frac{(25)(500)}{(s+500)^2+500^2} \\
v_2(t) &= \left\{25 - 25e^{-500t}\left[\cos(500t)+\sin(500t)\right]\right\} u(t) \text{ V}
\end{aligned}$$

Problem 10–43. There is no initial energy stored in the circuit in Figure P10–43.

(a). Transform the circuit into the s domain and formulate node-voltage equations.

Write the node-voltage equations by inspection.

$$\left(C_1s + \frac{1}{R_1} + C_2s\right)V_{\rm A}(s) - C_2sV_{\rm B}(s) = C_1sV_1(s)$$

$$-C_2sV_{\rm A}(s) + \left(C_2s + \frac{1}{R_2}\right)V_{\rm B}(s) = 0$$

(b). Solve these equations for $V_2(s)$ in symbolic form.

Solve the second equation for $V_{\rm A}(s)$ and substitute into the first equation.

$$\begin{aligned}
C_2sV_{\rm A}(s) &= \left(C_2s + \frac{1}{R_2}\right)V_{\rm B}(s) = \left(\frac{R_2C_2s+1}{R_2}\right)V_{\rm B}(s) \\
V_{\rm A}(s) &= \left(\frac{R_2C_2s+1}{R_2C_2s}\right)V_{\rm B}(s) \\
C_1sV_1(s) &= \left(C_1s + \frac{1}{R_1} + C_2s\right)\left(\frac{R_2C_2s+1}{R_2C_2s}\right)V_{\rm B}(s) - C_2sV_{\rm B}(s) \\
R_1C_1sV_1(s) &= (R_1C_1s + 1 + R_1C_2s)\left(\frac{R_2C_2s+1}{R_2C_2s}\right)V_{\rm B}(s) - R_1C_2sV_{\rm B}(s) \\
R_1R_2C_1C_2s^2V_1(s) &= (R_1R_2C_1C_2s^2 + R_1C_1s + R_2C_2s + 1 + R_1C_2s)V_{\rm B}(s) \\
V_2(s) = V_{\rm B}(s) &= \frac{R_1R_2C_1C_2s^2V_1(s)}{R_1R_2C_1C_2s^2 + (R_1C_1+R_2C_2+R_1C_2)s + 1}
\end{aligned}$$

(c). Insert an OP AMP buffer at point A and solve for $V_2(s)$ in symbolic form. How did inserting the buffer change the denominator, and therefore, the location of the poles?

We can find the output by applying voltage division twice.

$$V_2(s) = \left(\frac{R_1}{R_1 + 1/C_1 s}\right)\left(\frac{R_2}{R_2 + 1/C_2 s}\right) V_1(s)$$

$$= \frac{R_1 R_2 C_1 C_2 s^2 V_1(s)}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)} = \frac{R_1 R_2 C_1 C_2 s^2 V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2)s + 1}$$

The buffer reduced the coefficient of the s term in the denominator. In general, this change reduces the damping coefficient.

(d). Select values of R_1, R_2, C_1, and C_2 to locate a pole at 10 krad/s and a second pole at 100 krad/s. You can choose either the circuit with or the one without the OP AMP for your design.

Use the OP AMP design, since the poles are visible in the development of the output voltage, with $s = -1/R_1C_1 = -10$ krad/s and $s = -1/R_2C_2 = -100$ krad/s. Choose $R_1 = R_2 = 1$ kΩ, $C_1 = 0.1$ μF, and $C_2 = 0.01$ μF to place the poles as specified.

Problem 10–44. There is no initial energy stored in the circuit in Figure P10–43. The Thévenin equivalent circuit to the left of point A when a unit step is applied is

$$V_{\mathrm{T}}(s) = \frac{1}{s+1000} \text{ V-s} \quad \text{and} \quad Z_{\mathrm{T}}(s) = \frac{10^6}{s+1000}\ \Omega$$

Select values for R_2 and C_2 such that the output transform is

$$V_2(s) = \frac{s}{s^2 + 3000s + 10^6} \text{ V-s}$$

Apply voltage division to solve for the output voltage.

$$V_2(s) = \left[\frac{R_2}{R_2 + 1/C_2 s + Z_{\mathrm{T}}(s)}\right] V_{\mathrm{T}}(s) = \left[\frac{R_2 C_2 s}{R_2 C_2 s + 1 + \dfrac{C_2 s 10^6}{s+1000}}\right]\left[\frac{1}{s+1000}\right]$$

$$= \frac{R_2 C_2 s}{R_2 C_2 s^2 + 1000 R_2 C_2 s + s + 1000 + 10^6 C_2 s}$$

$$= \frac{s}{s^2 + \left(1000 + \dfrac{1}{R_2 C_2} + \dfrac{10^6}{R_2}\right) s + \dfrac{1000}{R_2 C_2}}$$

$$= \frac{s}{s^2 + 3000s + 10^6}$$

$$1000 = \frac{1}{R_2 C_2}$$

$$3000 = 1000 + \frac{1}{R_2 C_2} + \frac{10^6}{R_2}$$

$$R_2 = 1\ \text{k}\Omega$$

$$C_2 = 1\ \mu\text{F}$$

Problem 10–45. There is no initial energy stored in the bridged-T circuit in Figure P10–45.

(a). Transform the circuit into the s domain and formulate mesh-current equations.

Write the mesh-current equations by inspection

$$\left(R_1 + \frac{1}{C_1 s}\right) I_{\text{A}}(s) - \frac{1}{C_1 s} I_{\text{B}}(s) = V_1(s)$$

$$-\frac{1}{C_1 s} I_{\text{A}}(s) + \left(R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right) I_{\text{B}}(s) = 0$$

(b). Use the mesh-current equations to find the s-domain relationship between the input $V_1(s)$ and the output $V_2(s)$.

Solve the equations for the mesh currents.

$$I_{\text{A}}(s) = \frac{(R_2 C_1 C_2 s + C_1 + C_2) s V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

$$I_{\text{B}}(s) = \frac{C_2 s V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

Compute the output voltage.

$$V_2(s) = R_1 I_{\text{A}}(s) + \frac{1}{C_2 s} I_{\text{B}}(s)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2)s + 1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

Problem 10–46. There is no initial energy stored in the bridged-T circuit in Figure P10–45.

(a). Transform the circuit into the s domain and formulate node-voltage equations.

Write the node-voltage equations by inspection.

$$\left(C_1 s + \frac{1}{R_1} + C_2 s\right) V_{\text{A}}(s) - C_2 s V_{\text{B}}(s) = C_1 s V_1(s)$$

$$-C_2 s V_{\text{A}}(s) + \left(\frac{1}{R_2} + C_2 s\right) V_{\text{B}}(s) = \frac{V_1(s)}{R_2}$$

(b). Use the node-voltage equations to find the s-domain relationship between the input $V_1(s)$ and the output $V_2(s)$.

Solve for the node voltages.

$$V_{\text{A}}(s) = \frac{R_1(C_1 + C_2 + R_2 C_1 C_2 s) s V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

$$V_{\text{B}}(s) = \frac{(R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2)s + 1) V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

The output voltage is $V_2(s) = V_{\text{B}}(s)$, so we have:

$$\frac{V_2(s)}{V_1(s)} = \frac{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2)s + 1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

Problem 10–47. Find the transform of the Thévenin equivalent circuit looking into the $v_2(t)$ terminals for the circuit of P10–45.

The open-circuit voltage is $V_2(s)$ and was found in Problems 10–45 and 10–46.

$$V_{\rm T}(s) = \frac{(R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2)s + 1)V_1(s)}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_2C_2 + R_1C_2)s + 1}$$

Compute the lookback impedance.

$$\begin{aligned}
Z_{\rm T}(s) &= R_2 \parallel \left[\frac{1}{C_2s} + \left(\frac{1}{C_1s} \parallel R_1\right)\right] \\
&= R_2 \parallel \left[\frac{1}{C_2s} + \frac{R_1}{R_1C_1s+1}\right] \\
&= R_2 \parallel \frac{R_1C_2s + R_1C_1s + 1}{C_2s(R_1C_1s+1)} \\
&= \frac{R_2\left[\dfrac{R_1C_2s + R_1C_1s + 1}{C_2s(R_1C_1s+1)}\right]}{R_2 + \left[\dfrac{R_1C_2s + R_1C_1s + 1}{C_2s(R_1C_1s+1)}\right]} \\
&= \frac{R_2[(R_1C_1 + R_1C_2)s + 1]}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2 + R_2C_2)s + 1}
\end{aligned}$$

Problem 10–48. There is no initial energy stored in the circuit in Figure P10–48.

(a). Find the zero-state mesh currents $i_{\rm A}(t)$ and $i_{\rm B}(t)$ when $v_1(t) = 50e^{-1000t}u(t)$ V.

The source voltage is

$$V_1(s) = \frac{50}{s+1000}$$

Write the mesh-current equations by inspection.

$$\left(1000 + \frac{10^5}{s} + \frac{10^5}{s}\right)I_{\rm A}(s) - \frac{10^5}{s}I_{\rm B}(s) = \frac{50}{s+1000}$$

$$-\frac{10^5}{s}I_{\rm A}(s) + \left(\frac{10^5}{s} + 250 + 750\right)I_{\rm B}(s) = 0$$

Solve for the mesh currents.

$$I_{\rm A}(s) = \frac{s(s+100)}{20(s+1000)(s^2+300s+10000)}$$

$$I_{\rm B}(s) = \frac{5s}{(s+1000)(s^2+300s+10000)}$$

Perform partial fraction expansion.

$$I_{\rm A}(s) = \frac{0.0634}{s+1000} - \frac{0.000549}{s+38.2} - \frac{0.0128}{s+261.8}$$

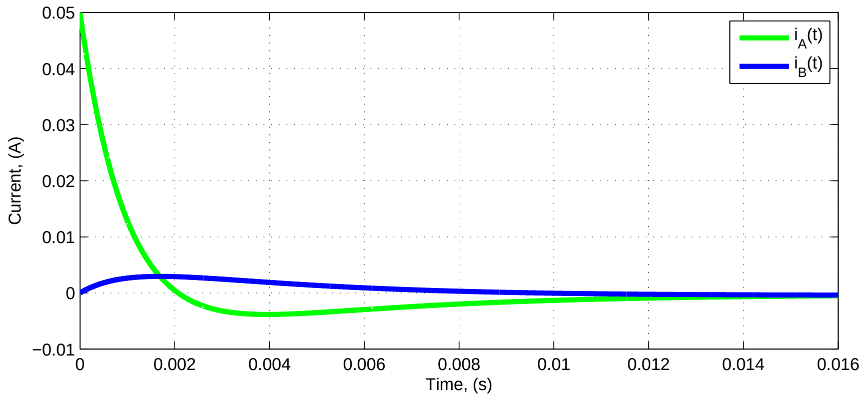
$$I_{\rm B}(s) = -\frac{0.00704}{s+1000} - \frac{0.000888}{s+38.2} + \frac{0.00793}{s+261.8}$$

Convert the currents into the time domain.

$$i_{\mathrm{A}}(t) = \left(63.4e^{-1000t} - 0.549e^{-38.2t} - 12.8e^{-261.8t}\right) u(t) \text{ mA}$$

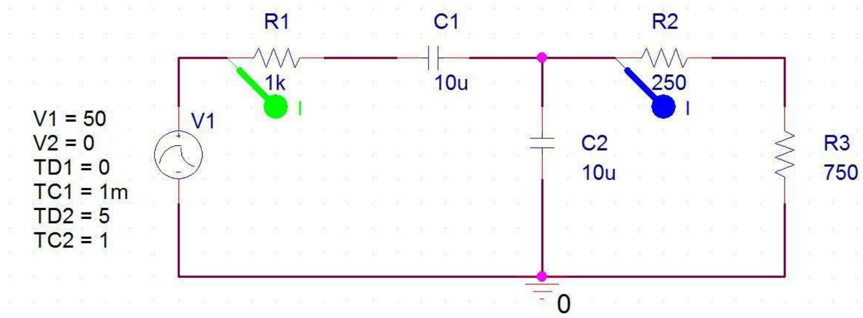
$$i_{\mathrm{B}}(t) = \left(-7.04e^{-1000t} - 0.888e^{-38.2t} + 7.93e^{-261.8t}\right) u(t) \text{ mA}$$

A MATLAB plot of the results is shown below.

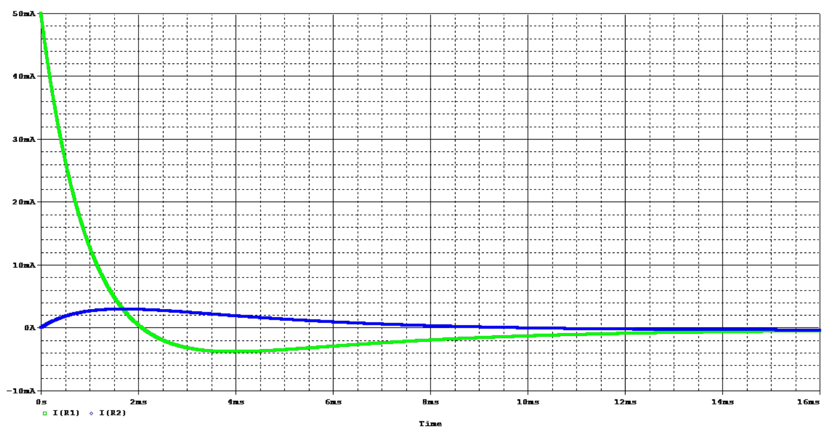


(b). Validate your answers using OrCAD.

The OrCAD simulation is shown below. In the simulation, both capacitors have initial voltages of zero.



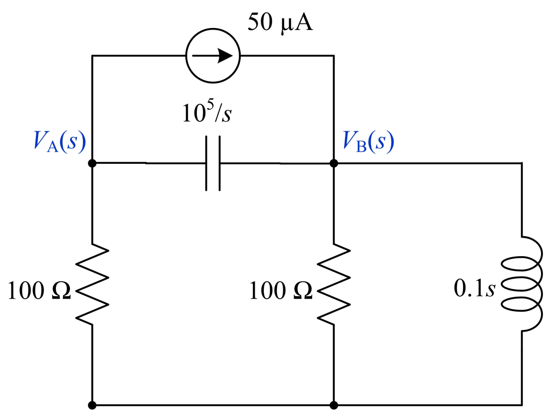
The corresponding results are shown below and agree with the results in part (a).



Problem 10–49. There is no external input in the circuit in Figure P10–49.

(a). Find the zero-input node voltages $v_A(t)$ and $v_B(t)$, and the voltage across the capacitor $v_C(t)$ when $v_C(0) = -5$ V and $i_L(0) = 0$ A.

Transform the circuit into the s domain.



Write the node-voltage equations by inspection and solve them.

$$\left(\frac{1}{100} + 10^{-5}s\right) V_A(s) - 10^{-5}sV_B(s) = -50\,\mu$$

$$-10^{-5}sV_A(s) + \left(10^{-5}s + \frac{1}{100} + \frac{10}{s}\right) V_B(s) = 50\,\mu$$

$$V_A(s) = \frac{-5(s+1000)}{2(s^2+1000s+500000)} = -\frac{5}{2}\frac{s+500}{(s+500)^2+500^2} - \frac{5}{2}\frac{500}{(s+500)^2+500^2}$$

$$V_B(s) = \frac{5s}{2(s^2+1000s+500000)} = \frac{5}{2}\frac{s+500}{(s+500)^2+500^2} - \frac{5}{2}\frac{500}{(s+500)^2+500^2}$$

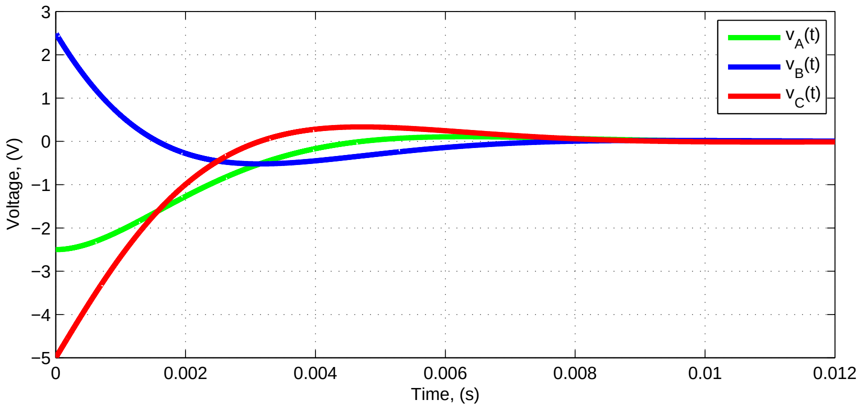
$$v_A(t) = -\frac{5}{2}e^{-500t}\left[\cos(500t) + sin(500t)\right]u(t) \text{ V}$$

$$v_B(t) = \frac{5}{2}e^{-500t}\left[\cos(500t) - sin(500t)\right]u(t) \text{ V}$$

$$v_C(t) = v_A(t) - v_B(t) = -5e^{-500t}\cos(500t)u(t) \text{ V}$$

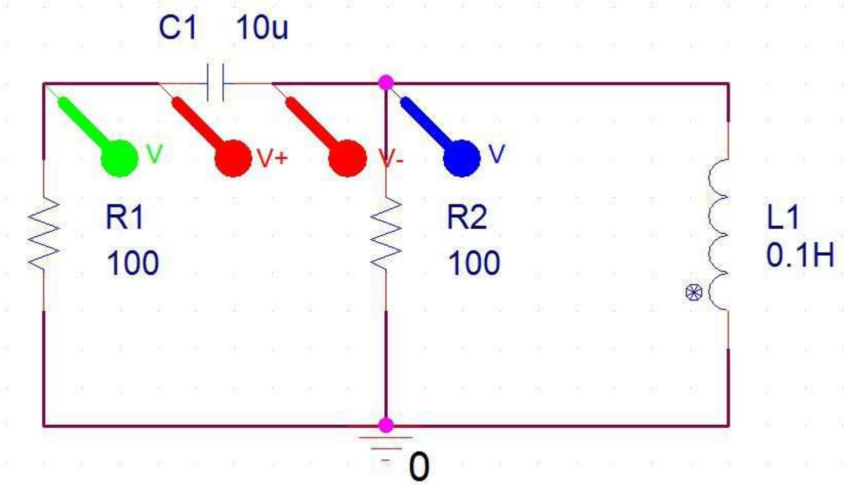
(b). Use MATLAB to plot your results in (a).

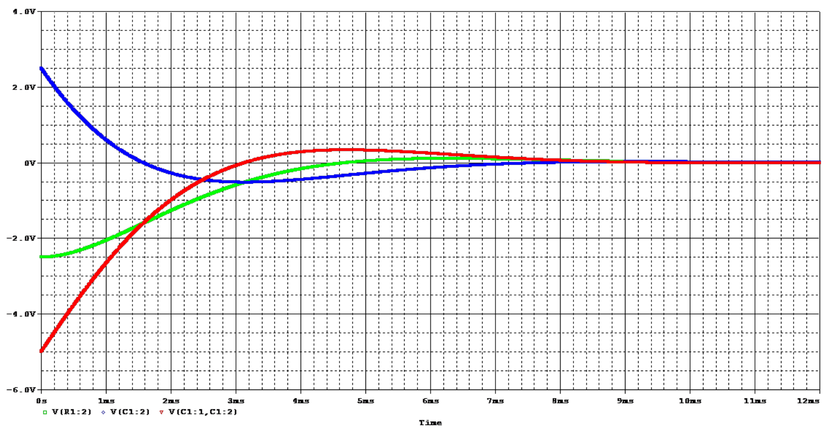
The plot is shown below.



(c). Use OrCAD to validate your results in (a).

The OrCAD simulation and results are shown below. In the simulation, the capacitor has an initial voltage of −5 V and the inductor has an initial value of zero.





(d). Compare the MATLAB and OrCAD plots. Are they the same?

The MATLAB and OrCAD results are the same, as they should be.

Problem 10–50. The two-OP AMP circuit in Figure P10–50 is a band-pass filter.

(a). Your task is to design such a filter so that the low-frequency cutoff is 1000 rad/s and the high-frequency cutoff is 100000 rad/s. (*Hint*: See Example 10–16 and Exercise 10–18.)

Applying the previous results, OP AMP 1 serves as a low-pass filter and will control the higher cutoff frequency. OP AMP 2 serves as a high-pass filter and will control the lower cutoff frequency. We have the following results.

$$Z_1 = R_1$$

$$Z_2 = \frac{\frac{1}{C_2}}{s + \frac{1}{R_2C_2}}$$

$$Z_3 = \frac{R_3C_3s + 1}{C_3s}$$

$$Z_4 = R_4$$

$$K_1 = -\frac{Z_2}{Z_1} = \frac{\left(-\frac{R_2}{R_1}\right)\left(\frac{1}{R_2C_2}\right)}{s + \frac{1}{R_2C_2}}$$

$$K_2 = -\frac{Z_4}{Z_3} = \frac{\left(-\frac{R_4}{R_3}\right)s}{s + \frac{1}{R_3C_3}}$$

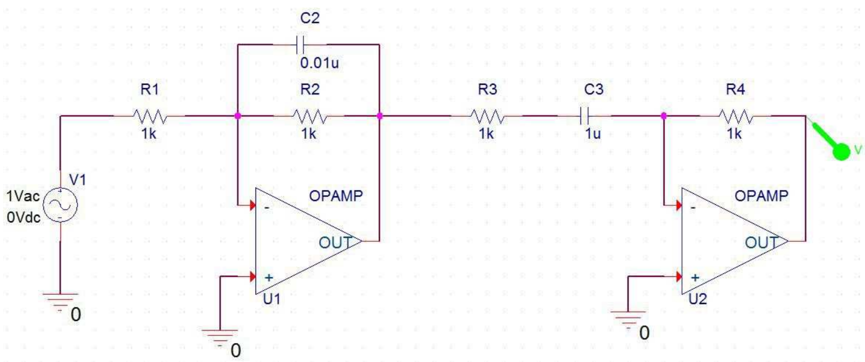
$$\frac{1}{R_2C_2} = 100000$$

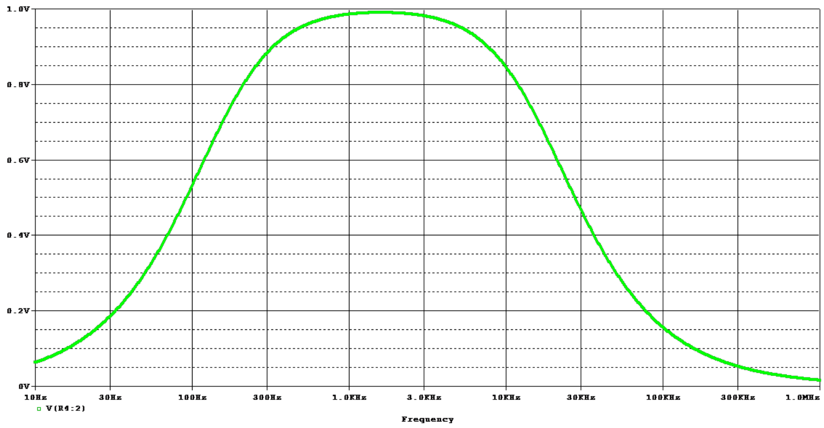
$$\frac{1}{R_3C_3} = 1000$$

Choose $R_1 = R_2 = R_3 = R_4 = 1\ \text{k}\Omega$, $C_2 = 0.01\ \mu\text{F}$, and $C_3 = 1\ \mu\text{F}$.

(b). Show that your design is correct using OrCAD.

The following OrCAD simulation and results verify the design.





Problem 10–51. The circuit in Figure P10–51 is in the zero state. Use node-voltage equations to find the circuit determinant. Select values of R, C, and μ so that the circuit has $\omega_0 = 5$ krad/s and $\zeta = 0.707$.

Write the node-voltage equations by inspection and solve for the circuit determinant.

$$\left(Cs + Cs + \frac{1}{R}\right) V_{\rm A}(s) - \left(Cs + \frac{\mu}{R}\right) V_{\rm X}(s) = CsV_1(s)$$

$$-CsV_{\rm A}(s) + \left(Cs + \frac{1}{R}\right) V_{\rm X}(s) = 0$$

$$\Delta(s) = \begin{vmatrix} 2Cs + \dfrac{1}{R} & -Cs - \dfrac{\mu}{R} \\ -Cs & Cs + \dfrac{1}{R} \end{vmatrix} = \begin{vmatrix} \dfrac{2RCs+1}{R} & \dfrac{-RCs-\mu}{R} \\ \dfrac{-RCs}{R} & \dfrac{RCs+1}{R} \end{vmatrix}$$

$$= \frac{1}{R^2}\left[R^2C^2s^2 + (3-\mu)RCs + 1\right]$$

$$= C^2\left[s^2 + \frac{3-\mu}{RC}s + \frac{1}{R^2C^2}\right]$$

$$2\zeta\omega_0 = \frac{3-\mu}{RC}$$

$$\omega_0^2 = \frac{1}{R^2C^2}$$

$$C = 0.1\ \mu\text{F}$$

$$R = 2\ \text{k}\Omega$$

$$\mu = 3 - 2\zeta = 3 - \sqrt{2} = 1.5858$$

Problem 10–52. The circuit in Figure P10–52 is in the zero state. Use mesh-current equations to find the circuit determinant. Select values of R, L, and C so that the circuit has $\omega_0 = 10$ krad/s and $\zeta = 1$.

Write the mesh-current equations by inspection and solve for the circuit determinant.

$$\left(R + Ls + \frac{1}{Cs}\right) I_{\rm A}(s) - \left(Ls + \frac{1}{Cs}\right) I_{\rm B}(s) = V_1(s)$$

$$-\left(Ls + \frac{1}{Cs}\right) I_{\rm A}(s) + \left(R + Ls + \frac{1}{Cs}\right) I_{\rm B}(s) = 0$$

$$\Delta(s) = \begin{vmatrix} R + Ls + \frac{1}{Cs} & -Ls - \frac{1}{Cs} \\ -Ls - \frac{1}{Cs} & R + Ls + \frac{1}{Cs} \end{vmatrix}$$

$$= \left[\frac{LCs^2 + RCs + 1}{Cs}\right]^2 - \left[\frac{LCs^2 + 1}{Cs}\right]^2$$

$$= \frac{2RLCs^2 + R^2Cs + 2R}{Cs} = \frac{2RL}{s}\left[s^2 + \frac{R}{2L}s + \frac{1}{LC}\right]$$

$$2\zeta\omega_0 = \frac{R}{2L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$C = 0.1\ \mu\text{F}$$

$$L = 100\ \text{mH}$$

$$R = 4\ \text{k}\Omega$$

Problem 10–53. The OP AMP circuit in Figure P10–53 is in the zero state. Use node-voltage equations to find the circuit determinant. Select values of R, C_1, and C_2 so that the circuit has $\omega_0 = 10$ krad/s and $\zeta = 1$.

Write the node-voltage equations by inspection and solve for the circuit determinant.

$$\left(\frac{1}{R} + \frac{1}{R} + C_1s\right)V_\text{A}(s) - \left(\frac{1}{R} + C_1s\right)V_2(s) = \frac{V_1(s)}{R}$$

$$-\frac{1}{R}V_\text{A}(s) + \left(\frac{1}{R} + C_2s\right)V_2(s) = 0$$

$$\Delta(s) = \begin{vmatrix} \frac{1}{R} + \frac{1}{R} + C_1s & -\frac{1}{R} - C_1s \\ -\frac{1}{R} & \frac{1}{R} + C_2s \end{vmatrix}$$

$$= \frac{R^2C_1C_2s^2 + 2C_2Rs + 1}{R^2} = C_1C_2\left[s^2 + \frac{2}{RC_1}s + \frac{1}{R^2C_1C_2}\right]$$

$$2\zeta\omega_0 = \frac{2}{RC_1}$$

$$\omega_0^2 = \frac{1}{R^2C_1C_2}$$

$$C_1 = 0.1\ \mu\text{F}$$

$$C_2 = 0.1\ \mu\text{F}$$

$$R = 1\ \text{k}\Omega$$

Problem 10–54. Compare the results of your designs of the circuits in Figures P10–52 and P10–53. Since both circuits purport to have the same response characteristics, what are the advantages and disadvantages of each?

In Problem 10–52, the circuit uses an inductor, which is usually larger and more difficult to construct than an equivalent design made with capacitors. The circuit converts an input voltage to an output current, which may not be desired in all cases. The circuit is passive, so it does not required additional power beyond the voltage source. Finally, if the circuit has to be connected to another circuit, it may experience loading and the response characteristic would change.

In Problem 10–53, the circuit uses only capacitors, which are generally a better design choice. The circuit converts an input voltage to an output voltage and the OP AMP will prevent loading at the output stage. The circuit is active, so it requires additional power to operate. In general, this approach is probably better than the one used in Problem 10–52.

Problem 10–55. Three node voltages are shown in Figure P10–55.

(a). Explain why only one of the node voltages is independent.

We have the following relationships:

$$\begin{aligned}
V_{\mathrm{A}}(s) &= V_{\mathrm{S}}(s) \\
V_{\mathrm{B}}(s) &= \mu V_{\mathrm{X}}(s) \\
V_{\mathrm{X}}(s) &= V_{\mathrm{A}}(s) - V_{\mathrm{B}}(s) = V_{\mathrm{A}}(s) - \mu V_{\mathrm{X}}(s) \\
(1+\mu)V_{\mathrm{X}}(s) &= V_{\mathrm{A}}(s) \\
V_{\mathrm{X}}(s) &= \frac{V_{\mathrm{A}}(s)}{1+\mu} = \frac{V_{\mathrm{S}}(s)}{1+\mu} \\
V_{\mathrm{B}}(s) &= \frac{\mu}{1+\mu}V_{\mathrm{S}}(s)
\end{aligned}$$

Node voltages $V_{\mathrm{A}}(s)$ and $V_{\mathrm{B}}(s)$ are controlled by the voltage source and the dependent voltage source. In addition, the definition of $V_{\mathrm{X}}(s)$ links these two node voltages together. Only $V_{\mathrm{C}}(s)$ is independent in that it depends on the values of the elements in the circuit

(b). Write a node voltage equation in the independent node voltage.

The node voltage equation is

$$\left(\frac{1}{R_2} + Cs\right)V_{\mathrm{C}}(s) - \frac{1}{R_2}V_{\mathrm{B}}(s) = 0$$

$$V_{\mathrm{C}}(s) = \frac{1}{R_2Cs+1}V_{\mathrm{B}}(s) = \frac{\frac{\mu}{1+\mu}}{R_2Cs+1}V_{\mathrm{S}}(s)$$

(c). If $V_{\mathrm{C}}(s)$ is the circuit's output, find the output-input ratio or network function, $V_{\mathrm{C}}(s)/V_{\mathrm{S}}(s)$.

Based on the results in part (b):

$$\frac{V_{\mathrm{C}}(s)}{V_{\mathrm{S}}(s)} = \frac{\frac{\mu}{1+\mu}}{R_2Cs+1}$$

Problem 10–56. Three mesh currents are shown in Figure P10–56.

(a). Explain why only two of these mesh currents are independent.

We have the following relationships:

$$
\begin{aligned}
I_{\mathrm{X}}(s) &= I_{\mathrm{A}}(s) - I_{\mathrm{B}}(s) \\
I_{\mathrm{B}}(s) &= -\beta I_{\mathrm{X}}(s) = -\beta[I_{\mathrm{A}}(s) - I_{\mathrm{B}}(s)] \\
(1-\beta)I_{\mathrm{B}}(s) &= -\beta I_{\mathrm{A}}(s) \\
I_{\mathrm{B}}(s) &= \frac{\beta I_{\mathrm{A}}(s)}{\beta - 1}
\end{aligned}
$$

Mesh currents $I_{\mathrm{A}}(s)$ and $I_{\mathrm{B}}(s)$ are linked through the definition of $I_{\mathrm{X}}(s)$ and the dependent current source. Only one of those two currents is independent. Mesh current $I_{\mathrm{C}}(s)$ is also independent.

(b). Write s-domain mesh-current equations in the two independent mesh currents.

Write the mesh-current equations.

$$
\begin{aligned}
-V_{\mathrm{S}}(s) + R_1[I_{\mathrm{A}}(s) - I_{\mathrm{C}}(s)] + Ls[I_{\mathrm{A}}(s) - I_{\mathrm{B}}(s)] &= 0 \\
\frac{1}{Cs}I_{\mathrm{C}}(s) + R_2[I_{\mathrm{C}}(s) - I_{\mathrm{B}}(s)] + R_1[I_{\mathrm{C}}(s) - I_{\mathrm{A}}(s)] &= 0
\end{aligned}
$$

Substitute in for $I_{\mathrm{B}}(s)$.

$$
\begin{aligned}
-V_{\mathrm{S}}(s) + R_1[I_{\mathrm{A}}(s) - I_{\mathrm{C}}(s)] + Ls\left[I_{\mathrm{A}}(s) - \frac{\beta I_{\mathrm{A}}(s)}{\beta - 1}\right] &= 0 \\
\frac{1}{Cs}I_{\mathrm{C}}(s) + R_2\left[I_{\mathrm{C}}(s) - \frac{\beta I_{\mathrm{A}}(s)}{\beta - 1}\right] + R_1[I_{\mathrm{C}}(s) - I_{\mathrm{A}}(s)] &= 0
\end{aligned}
$$

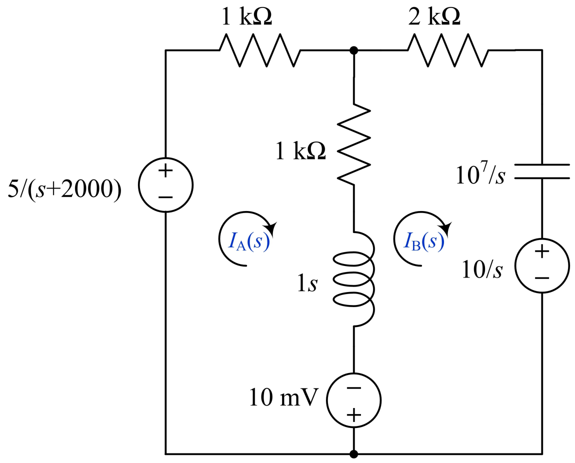
Simplify the equations.

$$
\begin{aligned}
\left[R_1 - \frac{Ls}{\beta - 1}\right] I_{\mathrm{A}}(s) - R_1 I_{\mathrm{C}}(s) &= V_{\mathrm{S}}(s) \\
-\left[R_1 + \frac{R_2\beta}{\beta - 1}\right] I_{\mathrm{A}}(s) + \left[\frac{1}{Cs} + R_2 + R_1\right] I_{\mathrm{C}}(s) &= 0
\end{aligned}
$$

Problem 10–57. The switch in Figure P10–57 has been in position A for a long time and is moved to position B at $t = 0$.

(a). Write an appropriate set of node-voltage or mesh-current equations in the s-domain.

The initial inductor current is $i_{\mathrm{L}}(0) = 10$ mA and the initial capacitor voltage is $v_{\mathrm{C}}(0) = 10$ V. The transformed circuit is shown below.



Write the mesh-current equations by inspection.

$$(1000 + 1000 + s)I_{\rm A}(s) - (1000 + s)I_{\rm B}(s) = 0.01 + \frac{5}{s + 2000}$$

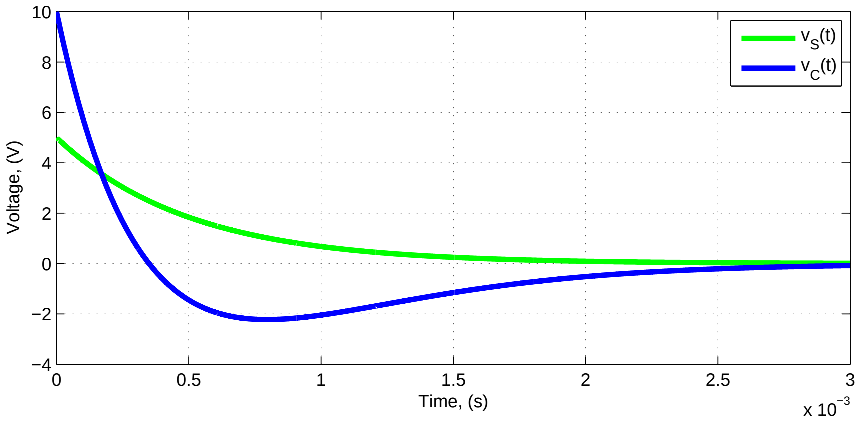
$$-(1000 + s)I_{\rm A}(s) + \left(1000 + 2000 + s + \frac{10^7}{s}\right) I_{\rm B}(s) = -0.01 - \frac{10}{s}$$

(b). Use MATLAB to solve for $V_{\rm C}(s)$ and $v_{\rm C}(t)$. Also using MATLAB, plot $v_{\rm C}(t)$ and the exponential source on the same axes.

The MATLAB code and results are shown below.

```
syms s t
vC0 = 10;
iL0 = 10e-3;
C = 0.1e-6;
A = [2000+s, -1000-s; -1000-s, 3000+s+1/C/s];
b = [iL0+5/(s+2000); -iL0-vC0/s];
x = A\b;
IAs = x(1);
IBs = x(2);
VCs = IBs/C/s+vC0/s
pretty(factor(VCs))
VCsnum = vpa(simplify(vpa(VCs,5)),5)
vCt = ilaplace(VCs)
vCtnum = vpa(simplify(vpa(vCt,5)),5)
```

```
VCs = 10/s - (50000*(3*s + 8000)*(s + 1000))/(s*(s + 2000)*(3*s^2 + 15000*s + 20000000))

                2
       10 (3 s  + 6000 s - 5000000)
  --------------------------------------
                   2
  (s + 2000) (3 s  + 15000 s + 20000000)

VCsnum = (35.0*s + 75000.0)/(s^2 + 5000.0*s + 6.6667*10^6) - 25.0/(s + 2000.0)
vCt = (35*(cos((500*3^(1/2)*5^(1/2)*t)/3) -
      (3^(1/2)*5^(1/2)*sin((500*3^(1/2)*5^(1/2)*t)/3))/7))/exp(2500*t) - 25/exp(2000*t)
vCtnum = (35.0*(cos(645.5*t) - 0.55328*sin(645.5*t)))/exp(2500.0*t) - 25.0/exp(2000.0*t)
```



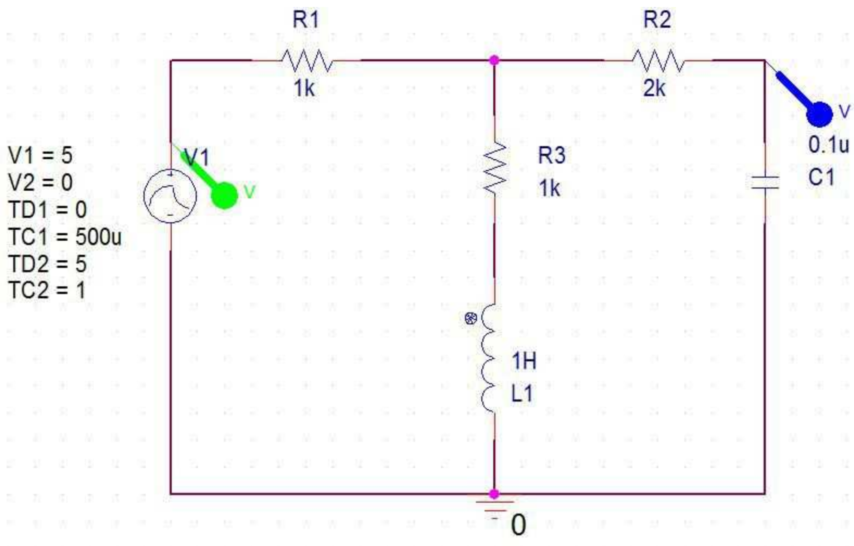
The results are:

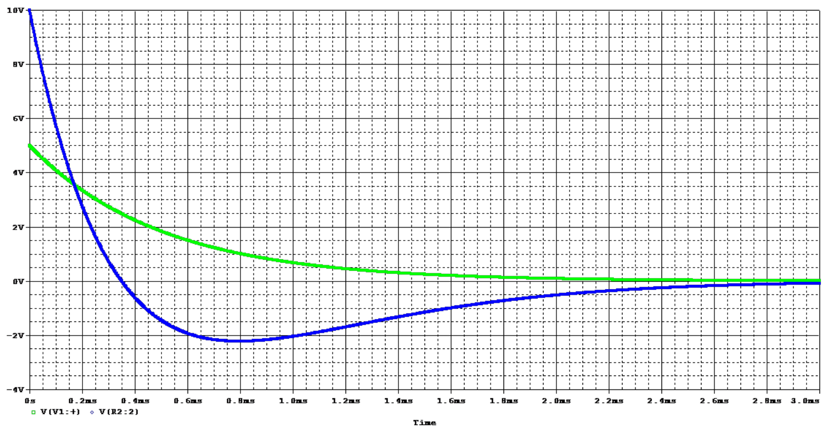
$$V_{\rm C}(s) = \frac{10(3s^2 + 6000s - 5000000)}{(s + 2000)(3s^2 + 15000s + 20000000)}$$

$$v_{\rm C}(t) = \left\{-25e^{-2000t} + 35e^{-2500t}\left[\cos(645.5t) - 0.55328\sin(645.5t)\right]\right\} u(t) \text{ V}$$

(c). Validate your results for $v_{\rm C}(t)$ using OrCAD. Include the exponential source (See Web Appendix C) in your Probe plot.

The OrCAD simulation and results are shown below. In the simulation, the initial current through the inductor is 10 mA and the initial voltage across the capacitor is 10 V.





Problem 10–58. There is no energy stored in the circuit in Figure P10–58 at $t = 0$. Transform the circuit into the s domain and solve for $V_O(s)$ and $v_O(t)$.

Use node-voltage analysis with $R = 1$ kΩ and $C = 0.2$ μF.

$$\left(\frac{1}{R} + Cs + Cs\right) V_A(s) - CsV_O(s) = \frac{V_S(s)}{R}$$

$$-CsV_A(s) + \left(Cs + Cs + \frac{1}{R}\right) V_O(s) = 0$$

Solve the equations for $V_O(s)$.

$$RCsV_A(s) = (2RCs + 1)V_O(s)$$

$$V_A(s) = \frac{2RCs + 1}{RCs} V_O(s)$$

$$(2RCs + 1)V_A(s) - RCsV_O(s) = \frac{100}{s}$$

$$(2RCs + 1)^2 V_O(s) - R^2C^2s^2V_O(s) = 100RC$$

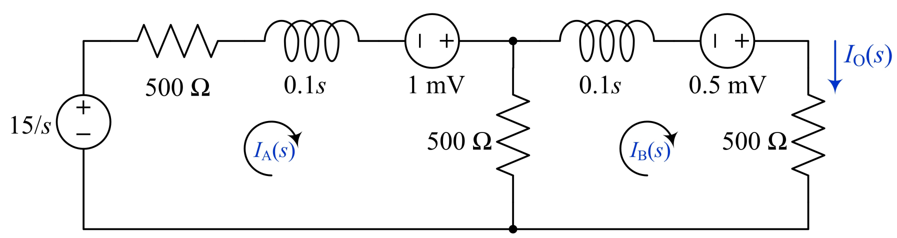
$$V_O(s) = \frac{100RC}{3R^2C^2s^2 + 4RCs + 1}$$

$$V_O(s) = \frac{166667}{(s + 5000)(s + 1667)} = \frac{50}{s + 1667} - \frac{50}{s + 5000}$$

$$v_O(t) = 50\left(e^{-1667t} - e^{-5000t}\right) u(t) \text{ V}$$

Problem 10–59. The switch in Figure P10–59 has been open for a long time and is closed at $t = 0$. Transform the circuit into the s domain and solve for $I_O(s)$ and $i_O(t)$.

The initial current through the left inductor is $i_{L1}(0) = 15/1500 = 10$ mA and the initial current through the right inductor is $i_{L2}(0) = 5$ mA. The transformed circuit is shown below.



The mesh-current equations are

$$(500 + 0.1s + 500)I_{\rm A}(s) - 500I_{\rm B}(s) = \frac{15}{s} + \frac{1}{1000}$$

$$-500I_{\rm A}(s) + (500 + 500 + 0.1s)I_{\rm B}(s) = \frac{1}{2000}$$

Solve for the output current.

$$I_{\rm O}(s) = I_{\rm B}(s) = \frac{s^2 + 20000s + 1.5 \times 10^8}{200s(s^2 + 20000s + 7.5 \times 10^7)}$$

$$i_{\rm O}(t) = \left(2.5e^{-15000t} - 7.5e^{-5000t} + 10\right) u(t) \text{ mA}$$

Problem 10–60. There is no initial energy stored in the circuit in Figure P10–60.

(a). Transform the circuit into the s domain and solve for $V_{\rm O}(s)$ in symbolic form.

Write the node-voltage equations by inspection.

$$\left(\frac{1}{R_1} + \frac{1}{Ls} + Cs\right) V_{\rm A}(s) - CsV_{\rm O}(s) = \frac{V_{\rm S}(s)}{R_1}$$

$$-CsV_{\rm A}(s) + \left(Cs + \frac{1}{R_2}\right) V_{\rm O}(s) = 0$$

Solve for the ouput voltage.

$$V_{\rm O}(s) = \frac{R_2LCs^2V_{\rm S}(s)}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

(b). If $R_1 = R_2 = 500\ \Omega$, select values of L and C to produce $\zeta = 0.707$ and $\omega_0 = 707$ rad/s. (*Hint*: Try values of L first, i.e., 100 mH, 200 mH, etc.)

We have the following results:

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = s^2 + \frac{R_1R_2C + L}{(R_1 + R_2)LC}s + \frac{R_1}{(R_1 + R_2)LC}$$

$$2\zeta\omega_0 = 1000 = \frac{250000C + L}{1000LC}$$

$$\omega_0^2 = 500000 = \frac{1}{2LC}$$

$$10^6LC = 250000C + L$$

$$10^6LC = 1$$

$$1 = \frac{250000}{10^6L} + L$$

$$L^2 - L + 0.25 = 0$$

$$(L - 0.5)^2 = 0$$

$$L = 500 \text{ mH}$$

$$C = 2\ \mu\text{F}$$

Problem 10–61. With the circuit in the zero state, the input to the integrator shown in Figure P10–61 is $v_1(t) = \cos(1000t)$ V. The desired output is $v_2(t) = -\sin(1000t)$ V. Use Laplace to select values of R and C to produce the desired output. If the capacitor had 10 V across it at $t = 0$, how would that affect the output?

We have the following relationships:

$$V_1(s) = \frac{s}{s^2 + 1000^2}$$

$$V_2(s) = \frac{-1000}{s^2 + 1000^2}$$

$$V_2(s) = \frac{-1/Cs}{R}V_1(s) = \left(\frac{-1}{RCs}\right)\left(\frac{s}{s^2 + 1000^2}\right) = \frac{-\frac{1}{RC}}{s^2 + 1000^2}$$

$$\frac{1}{RC} = 1000$$

$$R = 1\ \text{k}\Omega$$

$$C = 1\ \mu\text{F}$$

If the initial capacitor voltage was 10 V, we would have the following node-voltage equation and results:

$$-\frac{V_1(s)}{R} - CsV_2(s) - 10C = 0$$

$$V_2(s) = -\frac{V_1(s)}{RCs} - \frac{10}{s} = \frac{-\frac{1}{RC}}{s^2 + 1000^2} - \frac{10}{s}$$

$$v_2(t) = [-\sin(1000t) - 10]u(t)\ \text{V}$$

The initial capacitor voltage adds a constant value to the output.

Problem 10–62. Show that the circuit in Figure P10–62 has natural poles at $s = -4/RC$ and $s = -2/RC \pm j2/RC$ when $L = R^2C/4$.

Write the node-voltage equations for the circuit by inspection.

$$\left(\frac{1}{R + Ls} + Cs + \frac{1}{Ls}\right)V_\text{A}(s) - \frac{1}{Ls}V_\text{O}(s) = \frac{V_\text{S}(s)}{R + Ls}$$

$$-\frac{1}{Ls}V_\text{A}(s) + \left(\frac{1}{Ls} + \frac{1}{R}\right)V_\text{O}(s) = 0$$

Solve the equations for $V_\text{O}(s)$.

$$V_\text{O}(s) = \frac{RV_\text{S}(s)}{(R + Ls)(LCs^2 + RCs + 2)}$$

The poles are located at:

$$s = -\frac{R}{L}$$

$$s = \frac{-RC \pm \sqrt{R^2C^2 - 8LC}}{2LC}$$

Substitute in for L.

$$s = -\frac{R}{R^2C/4} = -\frac{4}{RC}$$

$$s = \frac{-RC \pm \sqrt{R^2C^2 - 2R^2C^2}}{R^2C^2/2} = \frac{-2 \pm j2}{RC}$$

Problem 10–63. Find the range of the gain μ for which the circuit's output $V_{\text{X}}(s)$ in Figure P10–63 is stable (i.e., all poles are in the left hand side of the s plane.)

Write the node-voltage equation for the circuit and solve for the poles.

$$\frac{1}{R}[V_{\text{X}}(s) - V_{\text{S}}(s)] + CsV_{\text{X}}(s) + \frac{1}{R}[V_{\text{X}}(s) - \mu V_{\text{X}}(s)] = 0$$

$$\left[\frac{2}{R} + Cs - \frac{\mu}{R}\right] V_{\text{X}}(s) = \frac{V_{\text{S}}(s)}{R}$$

$$[RCs + 2 - \mu]\, V_{\text{X}}(s) = V_{\text{S}}(s)$$

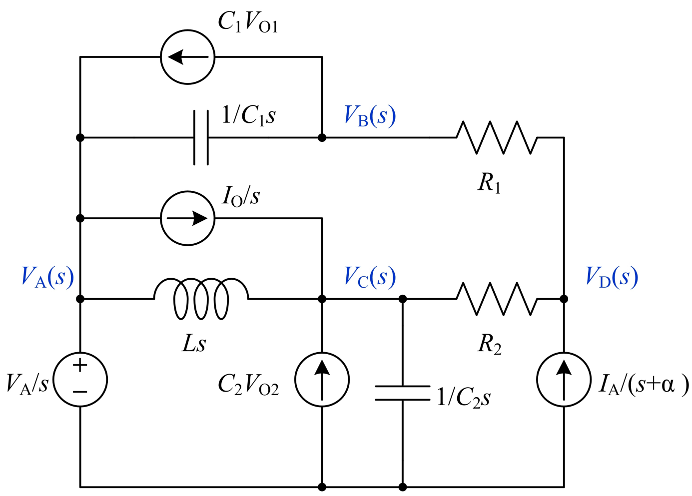
$$V_{\text{X}}(s) = \frac{V_{\text{S}}(s)}{RCs + 2 - \mu}$$

The pole is located at $s = (\mu - 2)/RC$. To maintain stability, the pole must be negative, so $\mu < 2$.

Problem 10–64. The circuit in Figure P10–64 is shown in the t domain with initial values for the energy storage devices.

(a). Transform the circuit into the s domain and write a set of node-voltage equations.

The transformed circuit is shown below.



Write the node-voltage equations by inspection.

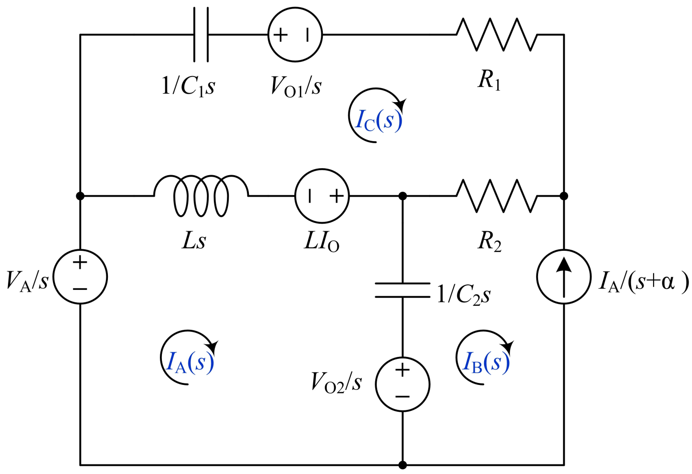
$$V_{\rm A}(s) = \frac{V_{\rm A}}{s}$$

$$\left(C_1 s + \frac{1}{R_1}\right) V_{\rm B}(s) - \frac{1}{R_1} V_{\rm D}(s) = -C_1 V_{\rm O1} + C_1 V_{\rm A}$$

$$\left(\frac{1}{Ls} + C_2 s + \frac{1}{R_2}\right) V_{\rm C}(s) - \frac{1}{R_2} V_{\rm D}(s) = \frac{I_{\rm O}}{s} + C_2 V_{\rm O2} + \frac{V_{\rm A}}{Ls^2}$$

$$-\frac{1}{R_1} V_{\rm B}(s) - \frac{1}{R_2} V_{\rm C}(s) + \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_{\rm D}(s) = \frac{I_{\rm A}}{s+\alpha}$$

(b). Transform the circuit into the s domain and write a set of mesh-current equations. The transformed circuit is shown below.



Write the mesh-current equations by inspection.

$$\left(Ls + \frac{1}{C_2 s}\right) I_{\rm A}(s) - \frac{1}{C_2 s} I_{\rm B}(s) - Ls I_{\rm C}(s) = \frac{V_{\rm A}}{s} + LI_{\rm O} - \frac{V_{\rm O2}}{s}$$

$$I_{\rm B}(s) = -\frac{I_{\rm A}}{s+\alpha}$$

$$-Ls I_{\rm A}(s) - R_2 I_{\rm B}(s) + \left(R_1 + R_2 + Ls + \frac{1}{C_1 s}\right) I_{\rm C}(s) = -\frac{V_{\rm O1}}{s} - LI_{\rm O}$$

(c). With the circuit in the zero state, use symbolic operations in MATLAB to solve for the node voltages. Set all of the initial conditions to zero in part (a). The MATLAB code and results are shown below.

```
syms s t R1 R2 L C1 C2 VA VB VC VD IA a
Eqn1 = (C1*s+1/R1)*VB - 1/R1*VD - C1*VA;
Eqn2 = (1/L/s+C2*s+1/R2)*VC- 1/R2*VD - VA/L/s^2;
Eqn3 = -1/R1*VB - 1/R2*VC + (1/R1 + 1/R2)*VD - IA/(s+a);
Soln = solve(Eqn1,Eqn2,Eqn3,VB,VC,VD);
VB = simplify(Soln.VB)
VC = simplify(Soln.VC)
VD = simplify(Soln.VD)
```

```
VB = ((C1*C2*L*R1*VA + C1*C2*L*R2*VA)*s^4 + (C1*L*VA + C2*IA*L*R2 + C1*C2*L*R1*VA*a +
      C1*C2*L*R2*VA*a)*s^3 + (IA*L + C1*R1*VA + C1*R2*VA + C1*L*VA*a)*s^2 +
     (VA + IA*R2 + C1*R1*VA*a + C1*R2*VA*a)*s + VA*a)/(s*(a + s)*(C1*R1*s + C1*R2*s +
      C1*L*s^2 + C2*L*s^2 + C1*C2*L*R1*s^3 + C1*C2*L*R2*s^3 + 1))

VC = ((C1*L*VA + C1*IA*L*R1)*s^3 + (IA*L + C1*R1*VA + C1*R2*VA + C1*L*VA*a)*s^2 +
     (VA + C1*R1*VA*a + C1*R2*VA*a)*s + VA*a)/(s*(a + s)*(C1*R1*s + C1*R2*s +
      C1*L*s^2 + C2*L*s^2 + C1*C2*L*R1*s^3 + C1*C2*L*R2*s^3 + 1))

VD = ((C1*C2*L*R2*VA + C1*C2*IA*L*R1*R2)*s^4 + (C1*L*VA + C1*IA*L*R1 + C2*IA*L*R2 +
      C1*C2*L*R2*VA*a)*s^3 + (IA*L + C1*R1*VA + C1*R2*VA + C1*IA*R1*R2 + C1*L*VA*a)*s^2 +
     (VA + IA*R2 + C1*R1*VA*a + C1*R2*VA*a)*s + VA*a)/(s*(a + s)*(C1*R1*s + C1*R2*s +
      C1*L*s^2 + C2*L*s^2 + C1*C2*L*R1*s^3 + C1*C2*L*R2*s^3 + 1))
```

Problem 10–65. (A) Thévenin's Theorem from Time-domain Data

A black box containing a linear circuit has an on-off switch and a pair of external terminals. When the switch is turned on, the open-circuit voltage between the external terminals is observed to be

$$v_{\rm OC}(t) = (10e^{-10t} - 10e^{-50t})u(t)\ \text{V}$$

The short circuit current was observed to be

$$i_{\rm SC}(t) = (100e^{-10t} + 200e^{-50t} + 50e^{-100t})u(t)\ \text{mA}$$

A 50-Ω load resistance is connected across the terminals and the switch turned on again. What is the voltage delivered to the load?

Convert the open-circuit voltage and the short-circuit current into the s domain and use their ratio to determine the Thévenin impendance. Then use the Thévenin equivalent circuit and voltage division to determine the output voltage.

$$V_{\rm OC}(s) = V_{\rm T}(s) = \frac{10}{s+10} - \frac{10}{s+50} = \frac{400}{(s+10)(s+50)}$$

$$I_{\rm SC}(s) = \frac{0.1}{s+10} + \frac{0.2}{s+50} + \frac{0.05}{s+100} = \frac{7s^2+800s+14500}{20(s+10)(s+50)(s+100)}$$

$$Z_{\rm T}(s) = \frac{V_{\rm OC}(s)}{I_{\rm SC}(s)} = \frac{8000(s+100)}{7s^2+800s+14500}$$

$$V_{\rm O}(s) = \frac{50}{50+Z_{\rm T}(s)}V_{\rm T}(s) = \frac{400(7s^2+800s+14500)}{(7s+610)(s+10)(s+50)^2}$$

$$v_{\rm O}(t) = \left(\frac{10}{3}e^{-10t} - \frac{750}{169}e^{-50t} + \frac{560}{507}e^{-610t/7} + \frac{4000t}{13}e^{-50t}\right)u(t)\ \text{V}$$

Problem 10–66. (D) Design a Load Impedance

In order to match the Thévenin impedance of a source, the load impedance in Figure P10–66 must be

$$Z_{\rm L}(s) = \frac{s+5}{s+10}$$

(a). What impedance $Z_2(s)$ is required?

We have the following relationships:

$$Z_{\mathrm{L}}(s) = \frac{s+5}{s+10} = 20 \parallel Z_2 = \frac{20Z_2}{20+Z_2}$$

$$20sZ_2 + 200Z_2 = 20s + sZ_2 + 100 + 5Z_2$$

$$19sZ_2 + 195Z_2 = 20(s+5)$$

$$Z_2 = \frac{20(s+5)}{19s+195}$$

(b). How would you realize $Z_2(s)$ using only resistors, inductors, and/or capacitors? (*Hint*: Write $Z_{\mathrm{L}}(s)$ as a sum of admittances, then solve for $Y_2(s)$.)

We have the following relationships:

$$Y_2 = \frac{1}{Z_2} = \frac{19s+195}{20(s+5)} = \frac{\frac{19}{20}(20s+100)+100}{20s+100}$$

$$= \frac{19}{20} + \frac{100}{20s+100} = \frac{19}{20} + \frac{1}{0.2s+1}$$

$$= \frac{1}{Z_3} + \frac{1}{Z_4}$$

$$Z_3 = \frac{20}{19}\ \Omega$$

$$Z_4 = 0.2s + 1 = Ls + R$$

$$L = 200\ \mathrm{mH}$$

$$R = 1\ \Omega$$

Create Z_2 as a parallel combination of Z_3 and Z_4. Z_3 is a 20/19-Ω resistor and Z_4 is a 1-Ω resistor in series with a 200-mH inductor.

Problem 10–67. (A,D) *RC* Circuit Analysis and Design

The *RC* circuits in Figure P10–67 represent the situation at the input to an oscilloscope. The parallel combination of R_1 and C_1 represent the probe used to connect the oscilloscope to a test point. The parallel combination of R_2 and C_2 represent the input impedance of the oscilloscope.

(a). Assuming zero initial conditions, transform the circuit into the s-domain and find the relationship between the test point voltage $V_{\mathrm{S}}(s)$ and the voltage $V_{\mathrm{O}}(s)$ at the oscilloscope's input.

Find the equivalent impedance of a resistor in parallel with a capacitor and then apply voltage division.

$$Z_{\mathrm{EQ}}(s) = R \parallel \frac{1}{Cs} = \frac{R/Cs}{R+1/Cs} = \frac{R}{RCs+1}$$

$$V_{\mathrm{O}}(s) = \frac{Z_2}{Z_1+Z_2}V_{\mathrm{S}}(s) = \frac{\dfrac{R_2}{R_2C_2s+1}}{\dfrac{R_1}{R_1C_1s+1} + \dfrac{R_2}{R_2C_2s+1}}V_{\mathrm{S}}(s)$$

$$= \frac{R_1R_2C_1s + R_2}{R_1R_2C_2s + R_1 + R_1R_2C_1s + R_2}V_{\mathrm{S}}(s)$$

(b). For $R_2 = 15$ MΩ and $C_2 = 3$ pF, determine the values of R_1 and C_1 that make the input voltage a scaled duplicate of the test point voltage.

There are many correct solutions. One approach is to satisfy the following expression, which makes $V_O(s) = V_S(s)/2$:

$$R_1R_2C_2s + R_1 = R_1R_2C_1s + R_2$$

Picking $R_1 = R_2$ and $C_1 = C_2$ will work.

Problem 10–68. (A) s-domain OP AMP Circuit Analysis

The OP AMP circuit in Figure P10–68 is in the zero-state. Transform the circuit into the s-domain and use the OP AMP circuit analysis techniques developed in Section 4–4 to find the relationship between the input $V_1(s)$ and the output $V_2(s)$.

Write the node-voltage equations for the circuit.

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + C_2s\right)V_A(s) - \left(\frac{1}{R_2} + C_2s\right)V_2(s) = \frac{V_1(s)}{R_1}$$

$$-\frac{1}{R_2}V_A(s) + \left(\frac{1}{R_2} + C_1s\right)V_2(s) = 0$$

Solve for the output voltage.

$$V_2(s) = \frac{V_1(s)}{R_1R_2C_1C_2s^2 + (R_1 + R_2)C_1s + 1}$$

Problem 10–69. (E) Pulse Conversion Circuit

The purpose of the test setup in Figure P10–69 is to deliver damped sine pulses to the test load. The excitation comes from a 1-Hz square wave generator. The pulse conversion circuit must deliver damped sine waveforms with $\zeta < 0.5$ and $\omega_0 > 10$ krad/s to 50-Ω and 600-Ω loads. The recommended values for the pulse conversion circuit are $L = 10$ mH and $C = 0.1$ μF. Verify that the test setup meets the specifications. (*Hint*: Compute the voltage across the load for an input signal equal to a unit step function, $u(t)$, and then again for a negative unit step function, $-u(t)$.) Note that the output of a square wave generator is the sum of a series of step functions.

Apply voltage division to determine the output for each of the two loads. For the 50-Ω load we have the following:

$$V_O(s) = \frac{50}{50 + 50 + Ls + 1/Cs}V_S(s) = \frac{50}{100 + 0.01s + 10^7/s}\left(\frac{1}{s}\right) = \frac{5000}{s^2 + 10000s + 10^9}$$

$$\omega_0 = \sqrt{10^9} = 31623 \text{ rad/s}$$

$$2\zeta\omega_0 = 10000$$

$$\zeta = 0.158$$

For the 600-Ω load we have the following:

$$V_O(s) = \frac{600}{600 + 50 + Ls + 1/Cs}V_S(s) = \frac{600}{650 + 0.01s + 10^7/s}\left(\frac{1}{s}\right) = \frac{60000}{s^2 + 65000s + 10^9}$$

$$\omega_0 = \sqrt{10^9} = 31623 \text{ rad/s}$$

$$2\zeta\omega_0 = 65000$$

$$\zeta = 1.028$$

Changing the input to be a negative step function will not change the damping coefficient or the frequency. The circuit meets the specification for the 50-Ω load but not for the 600-Ω.

Problem 10–70. (D) By-Pass Capacitor Design

In transistor amplifier design, a by-pass capacitor is connected across the emitter resistor $R_{\rm E}$ to effectively short out the emitter resistor at signal frequencies. This then, improves the gain of the transistor for the desired ac signals. The circuit in Figure P10–70(a) is a common-emitter amplifier. The shaded portion is a low-frequency model of the transistor in use. In this problem the task is to design a proper by-pass capacitor so that there is a pole at $s = -300$ rad/s. Reduce the circuit to its Thévenin equivalent as shown in Figure P10–70(b), and then select the proper capacitor. $R_{\rm S} = 12$ kΩ, $R_\pi = 2.5$ kΩ, $R_{\rm E} = 3.3$ kΩ, $R_{\rm L} = 1$ kΩ, and $\beta = 50$.

Remove the capacitor from the original circuit and write the mesh-current equations.

$$-V_{\rm S}(s) + (R_{\rm S} + R_\pi + R_{\rm E})I_1(s) - R_{\rm E}I_2(s) = 0$$

$$I_2(s) = -\beta I_{\rm B} = -\beta I_1$$

$$(R_{\rm S} + R_\pi + R_{\rm E})I_1(s) - R_{\rm E}[-\beta I_1(s)] = V_{\rm S}(s)$$

$$[R_{\rm S} + R_\pi + (1+\beta)R_{\rm E}]I_1(s) = V_{\rm S}(s)$$

$$I_1(s) = I_{\rm B}(s) = \frac{V_{\rm S}(s)}{R_{\rm S} + R_\pi + (1+\beta)R_{\rm E}}$$

$$V_{\rm OC}(s) = (1+\beta)R_{\rm E}I_{\rm B}(s) = \frac{(1+\beta)R_{\rm E}V_{\rm S}(s)}{R_{\rm S} + R_\pi + (1+\beta)R_{\rm E}}$$

Find the short-circuit current and compute the Thévenin impedance.

$$I_{\rm SC}(s) = (1+\beta)I_{\rm B}(s)$$

$$I_{\rm B}(s) = \frac{V_{\rm S}(s)}{R_{\rm S} + R_\pi}$$

$$I_{\rm SC}(s) = \frac{(1+\beta)V_{\rm S}(s)}{R_{\rm S} + R_\pi}$$

$$Z_{\rm T}(s) = \frac{V_{\rm OC}(s)}{I_{\rm SC}(s)} = \frac{R_{\rm E}(R_{\rm S} + R_\pi)}{R_{\rm S} + R_\pi + (1+\beta)R_{\rm E}} = 262\ \Omega$$

Find the pole associated with the capacitor voltage.

$$V_{\rm C}(s) = \frac{1/Cs}{Z_{\rm T}(s) + 1/Cs}V_{\rm T}(s) = \frac{1}{262Cs + 1}V_{\rm T}(s)$$

$$s = -\frac{1}{262C} = -300$$

$$C = 12.734\ \mu\text{F}$$

11 Network Functions

11.1 Exercise Solutions

Exercise 11–1. The network function for a circuit is

$$T(s) = \frac{10s}{s+100}$$

Find the zero-state response $v_2(t)$ when the input waveform is $v_1(t) = \cos(50t)$ V.

Find the response in the s domain

$$V_2(s) = T(s)V_1(s) = \left(\frac{10s}{s+100}\right)\left(\frac{s}{s^2+50^2}\right) = \frac{10s^2}{(s+100)(s^2+50^2)}$$

$$= \frac{A}{s+100} + \frac{Bs+C}{s^2+50^2} = \frac{8}{s+100} + \frac{2s-200}{s^2+50^2}$$

$$= \frac{8}{s+100} + (2)\frac{s}{s^2+50^2} - (4)\frac{50}{s^2+50^2}$$

$$v_2(t) = \left[8e^{-100t} + 2\cos(50t) - 4\sin(50t)\right] u(t) \text{ V}$$

Exercise 11–2. For the circuit of Figure 11–8, find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ and the driving-point impedance $Z(s)$.

Apply voltage division to find the transfer function. Sum the impedances in series to determine the driving-point impedance.

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{R}{R+Ls+1/Cs} = \frac{RCs}{LCs^2+RCs+1}$$

$$Z(s) = R + Ls + \frac{1}{Cs} = \frac{LCs^2+RCs+1}{Cs}$$

Exercise 11–3. For the circuit of Figure 11–10,

(a). Find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ and the driving-point impedance $Z(s)$.

Apply voltage division to find the transfer function. Sum the impedances in series to determine the driving-point impedance.

$$Z_1 = Ls + R_1$$

$$Z_2 = \frac{1}{Cs} + R_2 = \frac{R_2Cs+1}{Cs}$$

$$T_V(s) = \frac{Z_2}{Z_1+Z_2} = \frac{\dfrac{R_2Cs+1}{Cs}}{Ls+R_1+\dfrac{R_2Cs+1}{Cs}} = \frac{R_2Cs+1}{LCs^2+R_1Cs+R_2Cs+1}$$

$$Z(s) = Ls + R_1 + \frac{1}{Cs} + R_2 = \frac{LCs^2+(R_1+R_2)Cs+1}{Cs}$$

(b). Locate the poles and zeros of the transfer function when $R_1 = R_2 = 1$ kΩ, $L = 10$ mH, and $C = 0.1$ μF.

Compute the transfer function.

$$T_V(s) = \frac{10^{-4}s+1}{10^{-9}s^2+(2\times10^{-4})s+1} = \frac{10^5(s+10^4)}{s^2+200000s+10^9}$$

The zeros are located at infinity and $s = -10^4$ rad/s. The poles are located at $s = -194868$ rad/s and $s = -5132$ rad/s.

Exercise 11–4. Suppose that capacitor C_1 in the circuit of Figure 11–11 suddenly became shorted. What effect would it have on the circuit's voltage transfer function?

The transfer function would become

$$T_V(s) = \frac{-Z_2(s)}{R_1} = \frac{-\dfrac{R_2}{R_1}}{R_2C_2s + 1}$$

The transfer function no longer has a pole at $s = -1/R_1C_1$.

Exercise 11–5. Suppose that capacitor C_2 in the circuit of Figure 11–11 suddenly became open-circuited. What effect would it have on the circuit's voltage transfer function?

The transfer function would become

$$T_V(s) = \frac{-R_2}{Z_1(s)} = \frac{-R_2C_1s}{R_1C_1s + 1}$$

The transfer function no longer has a pole at $s = -1/R_2C_2$.

Exercise 11–6. For the circuit shown in Figure 11–12, insert a follower at point A and find the transfer function $T_V(s) = V_2(s)/V_1(s)$. Compare your result with that found in Example 11–5 for the same transfer function.

The follower effectively separates the two RC circuits. Find the transfer function on each side of the follower and multiply the results together.

$$T_1(s) = \frac{R}{R + 1/Cs} = \frac{RCs}{RCs + 1}$$

$$T_2(s) = \frac{1/Cs}{R + 1/Cs} = \frac{1}{RCs + 1}$$

$$T_V(s) = T_1(s)T_2(s) = \frac{RCs}{(RCs + 1)^2} = \frac{RCs}{(RCs)^2 + 2RCs + 1}$$

The difference appears in the middle term in the denominator, which is now a two instead of three. The change would affect the location of the poles and the nature of the circuit response. The follower eliminates the loading effects between the two RC components in the circuit.

Exercise 11–7.

(a). Find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure 11–14.

Let the node voltage between the resistor and capacitor be $V_{\text{A}}(s)$ and write a node-voltage equation.

$$\left(\frac{1}{R} + Cs\right) V_{\text{A}}(s) - (Cs)[-\mu V_{\text{X}}(s)] = \frac{V_1(s)}{R}$$

$$V_{\text{X}}(s) = V_{\text{A}}(s) - [-\mu V_{\text{X}}(s)] = V_{\text{A}}(s) + \mu V_{\text{X}}(s)$$

$$V_{\text{A}}(s) = (1-\mu)V_{\text{X}}(s)$$

$$(RCs+1)V_{\text{A}}(s) + \mu RCsV_{\text{X}}(s) = V_1(s)$$

$$(RCs+1)(1-\mu)V_{\text{X}}(s) + \mu RCsV_{\text{X}}(s) = V_1(s)$$

$$(RCs+1-\mu RCs-\mu+\mu RCs)V_{\text{X}}(s) = V_1(s)$$

$$(RCs+1-\mu)V_{\text{X}}(s) = V_1(s)$$

$$V_{\text{X}}(s) = \frac{V_1(s)}{RCS+1-\mu}$$

$$V_2(s) = -\mu V_{\text{X}}(s) = \frac{-\mu V_1(s)}{RCs+1-\mu}$$

$$T_{\text{V}}(s) = \frac{-\mu}{RCs+1-\mu}$$

(b). What are the conditions on μ that will ensure the circuit is stable?

The pole is located at $s = (\mu-1)/RC$. For stability, we need the pole to be negative, so the condition is $\mu < 1$.

Exercise 11–8. Figure 11–18 shows two cascade connections involving the same two stages but with their positions reversed. Does either of these connections involve loading? Find their voltage transfer functions and, if loading is present, determine the condition necessary to minimize the effect.

Both circuits exhibit loading effects. Circuit (a) has loading at the output stage and circuit (b) has loading at the input stage. Examine circuit (a) first. The OP AMP provides a gain

$$T_1(s) = \frac{R_{\text{A}}+R_{\text{B}}}{R_{\text{B}}}$$

The output stage has the following transfer function.

$$R_{\text{EQ}} = R \parallel R_{\text{L}} = \frac{RR_{\text{L}}}{R+R_{\text{L}}}$$

$$T_2(s) = \frac{R_{\text{EQ}}}{R_{\text{EQ}}+1/Cs} = \frac{\dfrac{RR_{\text{L}}}{R+R_{\text{L}}}}{\dfrac{RR_{\text{L}}}{R+R_{\text{L}}}+\dfrac{1}{Cs}} = \frac{RR_{\text{L}}Cs}{R+R_{\text{L}}+RR_{\text{L}}Cs}$$

Combine the stages to get the overall transfer function.

$$T_{\text{V}}(s) = T_1(s)T_2(s) = \left(\frac{R_{\text{A}}+R_{\text{B}}}{R_{\text{B}}}\right)\left(\frac{RR_{\text{L}}Cs}{R+R_{\text{L}}+RR_{\text{L}}Cs}\right) = \left(\frac{R_{\text{A}}+R_{\text{B}}}{R_{\text{B}}}\right)\left(\frac{s}{s+\dfrac{R+R_{\text{L}}}{RR_{\text{L}}C}}\right)$$

To minimize the loading effects, select $R \ll R_{\text{L}}$, so that the pole is located at approximately $s = -1/RC$.

Now examine circuit (b) using a similar approach.

$$T_1(s) = \frac{R}{R + R_S + 1/Cs} = \frac{RCs}{(R + R_S)Cs + 1}$$

$$T_2(s) = \frac{R_A + R_B}{R_B}$$

$$T_V(s) = T_1(s)T_2(s) = \left(\frac{RCs}{(R + R_S)Cs + 1}\right)\left(\frac{R_A + R_B}{R_B}\right) = \left(\frac{R_A + R_B}{R_B}\right)\left(\frac{\dfrac{R}{R + R_S}s}{s + \dfrac{1}{(R + R_S)C}}\right)$$

To minimize the loading effects, select $R \gg R_S$, so that the pole is located at approximately $s = -1/RC$.

Exercise 11–9. A certain circuit has the following voltage transfer function:

$$T_V(s) = \frac{10^5 s}{(s + 10^2)(s + 10^5)}$$

Find the circuits impulse response $h(t)$.

We have the following relationships:

$$h(t) = \mathscr{L}^{-1}\{H(s)\} = \mathscr{L}^{-1}\{T(s)\}$$

$$T_V(s) = \frac{10^5 s}{(s + 10^2)(s + 10^5)} = \frac{-100.1}{s + 100} + \frac{100100}{s + 100000}$$

$$h(t) = \left(-100.1e^{-100t} + 100100e^{-100000t}\right)u(t)$$

Exercise 11–10. The impulse response of a circuit is $h(t) = 100e^{-20t}u(t)$. Find the output when the input is a step function $x(t) = u(t)$.

Compute the transform of the input and the transfer function. Multiply the two transforms together and take the inverse transform to find the output waveform.

$$X(s) = \mathscr{L}\{u(t)\} = \frac{1}{s}$$

$$H(s) = T(s) = \mathscr{L}\left\{100e^{-20t}u(t)\right\} = \frac{100}{s + 20}$$

$$Y(s) = T(s)X(s) = \frac{100}{s(s + 20)} = \frac{5}{s} + \frac{-5}{s + 20}$$

$$y(t) = \left(5 - 5e^{-20t}\right)u(t)$$

Exercise 11–11. Find the impulse response of the circuit in Figure 11–21.

Find the transfer function and compute the inverse Laplace transform.

$$T_V(s) = \frac{1000 + 1/Cs}{1000 + 9000 + 1/Cs} = \frac{1000Cs + 1}{10000Cs + 1} = \frac{\dfrac{1}{10}\left(s + \dfrac{1}{1000C}\right)}{s + \dfrac{1}{10000C}}$$

$$= \frac{\dfrac{1}{10}(s + 1000)}{s + 100} = \frac{\dfrac{1}{10}(s + 100) + 90}{s + 100} = \frac{1}{10} + \frac{90}{s + 100}$$

$$h(t) = \frac{1}{10}\delta(t) + 90e^{-100t}u(t)$$

Exercise 11–12. Design a circuit that will produce the following step response output:

$$v_2(t) = \left[1 - e^{-1000t}\right] u(t) \text{ V}$$

Find the transfer function.

$$g(t) = \left[1 - e^{-1000t}\right] u(t)$$

$$G(s) = \frac{1}{s} - \frac{1}{s+1000} = \frac{s+1000-s}{s(s+1000)} = \frac{1000}{s(s+1000)}$$

$$H(s) = sG(s) = \frac{1000}{s+1000} = \frac{\dfrac{1000}{s}}{1+\dfrac{1000}{s}} = \frac{\dfrac{10^6}{s}}{1000+\dfrac{10^6}{s}}$$

The numerator of the transfer function has the form $1/Cs$, where $C = 1\ \mu$F, and the denominator has the form $R + 1/Cs$, where $R = 1$ kΩ and $C = 1\ \mu$F. Apply voltages division to design a series RC circuit with the output taken across the capacitor to produce the desired response. Other component values are possible as long as the pole is located at $s = -1/RC = -1000$ rad/s.

Exercise 11–13. A particular circuit has the following voltage transfer function:

$$T_{\text{V}}(s) = \frac{s+5}{s+10}$$

Find the circuit's step response $g(t)$, impulse response $h(t)$, and step response transform $G(s)$, and the impulse response transform $H(s)$.

We have the following relationships:

$$H(s) = T_{\text{V}}(s) = \frac{s+5}{s+10} = \frac{s+10-5}{s+10} = \frac{s+10}{s+10} - \frac{5}{s+10} = 1 - \frac{5}{s+10}$$

$$h(t) = \mathscr{L}^{-1}\left\{H(s)\right\} = \delta(t) - 5e^{-10t}u(t)$$

$$G(s) = \frac{H(s)}{s} = \frac{s+5}{s(s+10)} = \frac{\frac{1}{2}}{s} + \frac{\frac{1}{2}}{s+10}$$

$$g(t) = \mathscr{L}^{-1}\left\{G(s)\right\} = \frac{1}{2}\left(1 + e^{-10t}\right) u(t)$$

Exercise 11–14. The impulse response of a circuit is $h(t) = 8000(600t - 1)e^{-800t}u(t)$. Find and plot the step response. Find approximate values for the rise time, delay time, and overshoot.

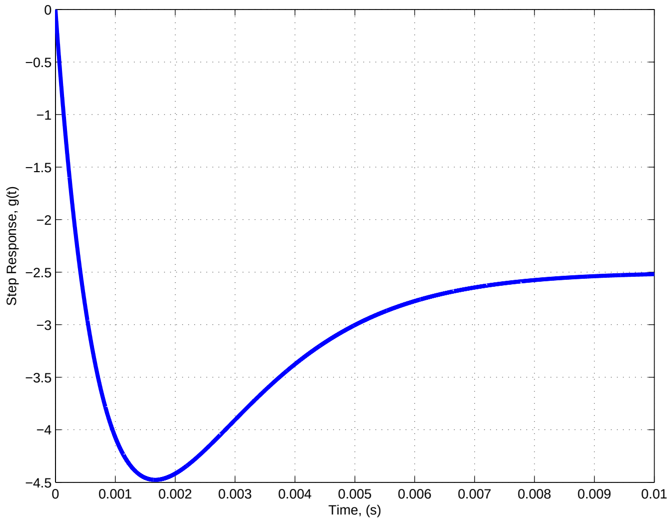
We have the following results.

$$H(s) = \mathscr{L}\left\{h(t)\right\} = \frac{4800000}{(s+800)^2} - \frac{8000}{s+800} = \frac{-8000s - 1600000}{(s+800)^2}$$

$$G(s) = \frac{H(s)}{s} = \frac{-8000s - 1600000}{s(s+800)^2} = -\frac{2.5}{s} + \frac{2.5}{s+800} - \frac{6000}{(s+800)^2}$$

$$g(t) = \mathscr{L}^{-1}\left\{G(s)\right\} = \left[-2.5 + (2.5 - 6000t)e^{-800t}\right] u(t)$$

The step response plot is shown below.



Based on the plot, we have the following approximate results:

$$g(\infty) = -2.5$$

$$g(0.000032) = 0.1g(\infty)$$

$$g(0.000180) = 0.5g(\infty)$$

$$g(0.000362) = 0.9g(\infty)$$

$$T_{\rm R} = 0.000362 - 0.000032 = 330\ \mu\text{s}$$

$$T_{\rm D} = 180\ \mu\text{s}$$

$$\text{Overshoot} = \frac{4.5 - 2.5}{2.5} = 80\%$$

Exercise 11–15. Find the steady-state output in Figure 11–28 for a general input $v_1(t) = V_{\rm A}\cos(\omega t + \phi)$ V.

Determine the magnitude and phase of the transfer function.

$$T_{\text{V}}(s) = -\frac{Z_2(s)}{Z_1(s)} = -\frac{\dfrac{R_2/Cs}{R_2 + 1/Cs}}{R_1} = \frac{-\dfrac{1}{R_1C}}{s + \dfrac{1}{R_2C}}$$

$$T_{\text{V}}(j\omega) = \frac{-\dfrac{1}{R_1C}}{j\omega + \dfrac{1}{R_2C}}$$

$$|T_{\text{V}}(j\omega)| = \frac{\dfrac{1}{R_1C}}{\sqrt{\left(\dfrac{1}{R_2C}\right)^2 + \omega^2}} = \frac{1}{R_1C\sqrt{\left(\dfrac{1}{R_2C}\right)^2 + \omega^2}}$$

$$\angle T(j\omega) = -180^\circ - \tan^{-1}(\omega R_2 C)$$

$$v_{2\text{SS}}(t) = \frac{V_{\text{A}}}{R_1C\sqrt{\left(\dfrac{1}{R_2C}\right)^2 + \omega^2}} \cos\left[\omega t + \phi - 180^\circ - \tan^{-1}(\omega R_2 C)\right] \text{ V}$$

The 180° is due to the inverting OP AMP. At dc, the capacitor is an open circuit and the OP AMP is operating as a simple inverter with gain of $-R_2/R_1$. As $\omega \to \infty$, the output approaches zero.

Exercise 11–16. Rather than purchasing the device in Example 11–14, design a circuit to achieve the transfer function given. (*Hint*: Use a series *RLC* circuit with the output voltage taken appropriately.)

The required transfer function is

$$T_{\text{V}}(s) = \frac{s^2 + 142129}{s^2 + 7.45s + 142129} = \frac{s + \dfrac{142129}{s}}{s + 7.45 + \dfrac{142129}{s}}$$

We can achieve this transfer function by using voltage division with the *RLC* circuit. Take the output across the inductor in series with the capacitor with

$$Ls + \frac{1}{Cs} = s + \frac{142129}{s}$$

Choose $L = 1$ H, $C = 7.04\ \mu$F, and $R = 7.45\ \Omega$.

Exercise 11–17. The transfer function of a linear circuit is $T(s) = 5(s + 100)/(s + 500)$. Find the steady-state output for

(a). $x(t) = 3\cos(100t)$

Find the magnitude and phase of $T(j\omega) = T(j100)$.

$$T(j100) = \frac{5(j100 + 100)}{j100 + 500} = 1.1539 + j0.7692 = 1.3868\angle 33.69^\circ$$

$$y_{\text{SS}}(t) = 4.16\cos(100t + 33.69^\circ)$$

(b). $x(t) = 2\sin(500t)$

Find the magnitude and phase of $T(j\omega) = T(j500)$.

$$x(t) = 2\sin(500t) = 2\cos(500t - 90^\circ)$$

$$T(j500) = \frac{5(j500 + 100)}{j500 + 500} = 3 + j2 = 3.606\angle 33.69^\circ$$

$$y_{\text{SS}}(t) = 7.212\cos(100t - 56.31^\circ)$$

Exercise 11–18. The impulse response of a linear circuit is $h(t) = \delta(t) - 100[e^{-100t}]u(t)$. Find the steady-state output for

(a). $x(t) = 25\cos(100t)$

Find the magnitude and phase of the transfer function at $s = j\omega$.

$$H(s) = 1 - \frac{100}{s + 100} = \frac{s + 100 - 100}{s + 100} = \frac{s}{s + 100}$$

$$T(j100) = \frac{j100}{100 + j100} = 0.5 + j0.5 = 0.707\angle 45^\circ$$

$$y_{\text{SS}}(t) = 17.68\cos(100t + 45^\circ)$$

(b). $x(t) = 50\sin(100t)$

The frequency has not changed, so use the results from part (a).

$$x(t) = 50\cos(100t - 90^\circ)$$

$$T(j100) = 0.707\angle 45^\circ$$

$$y_{\text{SS}}(t) = 35.36\cos(100t - 45^\circ)$$

Exercise 11–19. A linear circuit has an impulse response $h(t) = e^{-10t}u(t)$ and an input $x(t) = 5u(t)$. Find the zero-state response using the t-domain convolution integral.

We have the following results:

$$h(t) = e^{-10t}u(t)$$

$$x(t) = 5u(t)$$

$$y(t) = \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t e^{-10(t-\tau)}(5)u(\tau)d\tau = 5e^{-10t}\int_0^t e^{10\tau}d\tau$$

$$= \frac{5}{10}e^{-10t}\left(e^{10t} - 1\right) = \frac{1}{2}\left(1 - e^{-10t}\right) \quad t \geq 0$$

Exercise 11–20. A linear circuit has an impulse response $h(t) = Ae^{-at}u(t)$. Use the convolution integral to find the zero-state response for $x(t) = Be^{-bt}u(t)$. Assume $a \neq b$ and that both a and b are positive, real numbers.

We have the following results:

$$h(t) = Ae^{-at}u(t)$$

$$x(t) = Be^{-bt}u(t)$$

$$y(t) = \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t Ae^{-a(t-\tau)}u(t-\tau)Be^{-b\tau}u(\tau)d\tau = ABe^{-at}\int_0^t e^{-(b-a)\tau}d\tau$$

$$= \frac{AB}{a-b}e^{-at}e^{-(b-a)\tau}\Big|_0^t = \frac{AB}{a-b}e^{-at}\left(e^{-(b-a)t}-1\right) = \frac{AB}{a-b}\left(e^{-bt}-e^{-at}\right)$$

$$= \frac{AB}{b-a}\left(e^{-at}-e^{-bt}\right)u(t)$$

Exercise 11–21. Repeat Example 11–18 by mathematically, not geometrically, computing the convolution integral. Verify that the result is equivalent to the answer found for Example 11–18. You may use MATLAB.

We have the following results.

$$h(t) = 2e^{-t}u(t)$$

$$x(t) = 5\left[u(t)-u(t-2)\right]$$

$$y(t) = \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t 2e^{-(t-\tau)}u(t-\tau)5[u(\tau)-u(\tau-2)]d\tau$$

$$= 10e^{-t}\left[\int_0^t e^{\tau}u(t-\tau)u(\tau)d\tau - \int_0^t e^{\tau}u(t-\tau)u(\tau-2)d\tau\right]$$

$$= 10e^{-t}\left\{\left[\int_0^t e^{\tau}d\tau\right]u(t) - \left[\int_2^t e^{\tau}d\tau\right]u(t-2)\right\}$$

$$= 10e^{-t}[e^t-1]u(t) - 10e^{-t}[e^t-e^2]u(t-2)$$

$$= 10[1-e^{-t}]u(t) - 10[1-e^{-(t-2)}]u(t-2)$$

The corresponding MATLAB code and results are shown below.

```
syms s t tau
ht = 2*exp(-t);
xt = 5*(heaviside(t)-heaviside(t-2));
httau = subs(ht,t,t-tau);
xtau = subs(xt,t,tau);
yt = int(httau*xtau,tau,0,t)
```

```
yt = piecewise([t < 2, 10 - 10/exp(t)],
      [2 ≤ t, 10*heaviside(t - 2)*(exp(2 - t) - 1) - 10/exp(t) + 10])
```

Both results are consistent with Example 11–18.

Exercise 11–22. Use the convolution integral to find the zero-state response for $h(t) = 2u(t)$ and $x(t) = 5[u(t)-u(t-2)]$.

We have the following results:

$$h(t) = 2u(t)$$

$$x(t) = 5[u(t) - u(t-2)]$$

$$\begin{aligned}
y(t) &= \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t 2u(t-\tau)5[u(\tau) - u(\tau-2)]d\tau \\
&= 10\int_0^t u(t-\tau)u(\tau)d\tau - 10\int_0^t u(t-\tau)u(\tau-2)d\tau \\
&= 10\left[\int_0^t d\tau\right]u(t) - 10\left[\int_2^t d\tau\right]u(t-2) \\
&= 10tu(t) - 10(t-2)u(t-2) \\
&= 10r(t) - 10r(t-2)
\end{aligned}$$

Exercise 11–23. Design an *RC* circuit to realize the following transfer function:

$$T(s) = \frac{200}{s+1000}$$

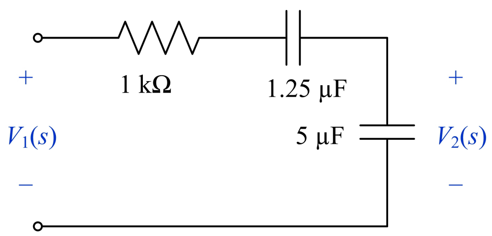
Rewrite the transfer function so that we can easily design the circuit using voltage division.

$$T(s) = \frac{200}{s+1000} = \frac{\frac{200}{s}}{1+\frac{1000}{s}} = \frac{\frac{200}{s}}{1+\frac{800}{s}+\frac{200}{s}}$$

The circuit is a series *RC* circuit with a resistor and two capacitors. The resistor has a value of 1 Ω and the capacitors have values of $1/800 = 1250\ \mu$F and $1/200 = 5000\ \mu$F. The output is taken across the 5000 μF capacitor. We can also get the same response by scaling the values appropriately. Consider the following equality:

$$T(s) = \frac{\frac{200}{s}}{1+\frac{800}{s}+\frac{200}{s}} = \frac{\frac{200000}{s}}{1000+\frac{800000}{s}+\frac{200000}{s}}$$

We can now design the circuit with a 1-kΩ resistor and 1.25-μF and 5-μF capacitors as shown below.



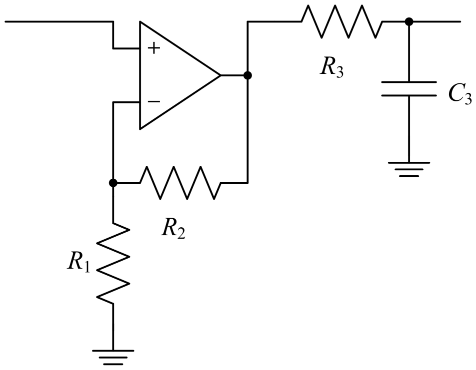
Exercise 11–24. Design an active *RC* circuit to realize the following transfer function

$$T(s) = \frac{2000}{s+1000}$$

Rewrite the transfer function so that we can easily design the circuit.

$$T(s) = \frac{2000}{s+1000} = \frac{\frac{2000}{s}}{1+\frac{1000}{s}} = 2\left[\frac{\frac{1000}{s}}{1+\frac{1000}{s}}\right]$$

Use the circuit shown in Figure 11–35. The OP AMP stage has a gain of 2 by making both resistors equal. Choose the components in the second stage voltage divider so that $R = 1\ \Omega$ and $C = 1000\ \mu\text{F}$. We can scale these values to be $R = 1\ \text{k}\Omega$ and $C = 1\ \mu\text{F}$. The resulting circuit is shown below with $R_1 = R_2 = R_3 = 1\ \text{k}\Omega$ and $C_3 = 1\ \mu\text{F}$.



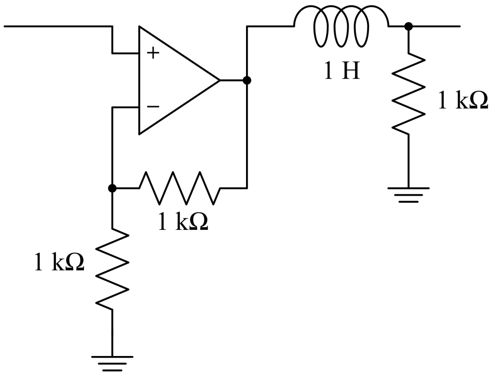
Exercise 11–25. Design an active RL circuit to realize the following transfer function

$$T(s) = \frac{2000}{s + 1000}$$

Rewrite the transfer function so that we can easily design the circuit.

$$T(s) = 2\left[\frac{1000}{s + 1000}\right]$$

The circuit is a noninverting amplifier with a gain of two followed by a resistor in series with an inductor. Take the output voltage across the resistor and set the values at $R = 1\ \text{k}\Omega$ and $L = 1$ H. The resulting circuit is shown below.



Exercise 11–26. Design a circuit to realize the following transfer function using only resistors, capacitors, and no more than one OP AMP.

$$T_{\text{V}}(s) = \frac{10^6}{(s + 10^3)^2}$$

Rewrite the transfer function as follows:

$$T_{\text{V}}(s) = \left(\frac{1000}{s + 1000}\right)\left(\frac{1000}{s + 1000}\right)$$

$$= \left(\frac{\frac{1000}{s}}{1 + \frac{1000}{s}}\right)\left(\frac{\frac{1000}{s}}{1 + \frac{1000}{s}}\right)$$

Design two identical series RC stages separated by an OP AMP buffer. Each RC stage has the output taken across the capacitor with $C = 1/1000$ F and $R = 1\ \Omega$. Figure 11–37 in the text shows the resulting circuit.

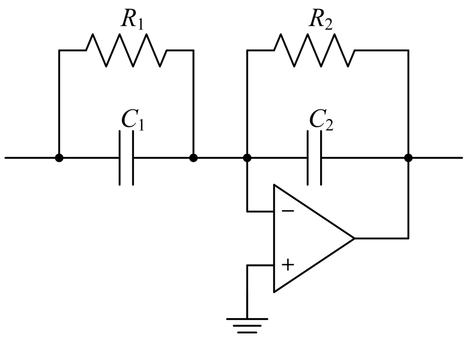
Exercise 11–27. Design an active RC prototype circuit to realize the following transfer function

$$T(s) = -100\frac{s+50}{s+100}$$

Compare the transfer function to transfer function used to design the circuit in Figure 11–38.

$$T(s) = -100\frac{s+50}{s+100} = -K\frac{s+\gamma}{s+\alpha}$$

Apply the results for Figure 11–38 to get the following prototype circuit with $R_1 = 1/5000\ \Omega$, $C_1 = 100$ F, $R_2 = 1/100\ \Omega$, and $C_2 = 1$ F.



Exercise 11–28. Design a circuit to realize the following transfer function using only resistors, capacitors, and no more than one OP AMP.

$$T_{\rm V}(s) = \frac{-10^6}{(s+10^3)^2}$$

Rewrite the transfer function as follows:

$$T_{\rm V}(s) = \left(\frac{-1000}{s+1000}\right)\left(\frac{1000}{s+1000}\right) = \left(\frac{-1}{\dfrac{1}{1}+\dfrac{s}{1000}}\right)\left(\frac{\dfrac{1000}{s}}{1+\dfrac{1000}{s}}\right)$$

Design the first stage as an inverting amplifier with $Z_1 = 1\ \Omega$ and Z_2 as a parallel combination of a 1-Ω resistor and a 1/1000 F capacitor. The second stage is a series RC circuit with the output taken across the capacitor and $R = 1\ \Omega$ and $C = 1/1000$ F. See Figure 11–41 in the text.

Exercise 11–29. Select a magnitude scale factor for each stage in Figure 11–36 so that both capacitances are 0.01 μF and all resistances are greater than 10 kΩ.

In the first stage, to scale the capacitor to 0.01 μF, we need to divide by $k_1 = 10^5$. In the second stage, to scale the smaller resistor to 10 kΩ, multiply by $k_2 = 10^4$. In the third stage, to scale the capacitor to 0.01 μF, divide by $k_3 = 25000$.

Exercise 11–30. Select a magnitude scale factor for the OP AMP circuit in Figure 11–39.

Scale the larger capacitor from 100 F to 1 μF by using a scale factor of $k_{\rm m} = 10^8$. We then have the following values: $R_1 = 20$ kΩ, $R_2 = 1$ MΩ, $C_1 = 1\ \mu$F, and $C_2 = 0.01\ \mu$F, all of which are reasonable for the circuit.

Exercise 11–31. Design a second-order circuit to realize the following transfer function:

$$T_{\rm V}(s) = \frac{10^6}{(s+10^3)^2}$$

Rewrite the transfer function as follows:

$$T_{\rm V}(s) = \frac{10^6}{s^2+2000s+10^6} = \frac{\dfrac{10^6}{s}}{s+2000+\dfrac{10^6}{s}}$$

Design a series RLC circuit with the output taken across the capacitor. Use values of $R = 2$ kΩ, $L = 1$ H, and $C = 1$ μF. Other designs are possible.

Exercise 11–32. The following transfer function was realized in different ways in Figures 11–37, 11–41 and 11–45:

$$T_V(s) = \frac{\pm 10^6}{(s + 10^3)^2}$$

Compare the various designs in a table similar to Table 11–1.

			Number of			
Example	Figure	Description	R	L	C	OP AMP
11–26	11–37	RC voltage-divider cascade	2	0	2	1
11–28	11–41	RC inverting and voltage-divider cascade	3	0	2	1
11–31	11–45	RLC voltage divider	1	1	1	0

Which would you recommend if

(a). There was no power available?

Use the RLC design, because it does not require an OP AMP.

(b). There was a desire not to invert the output and to avoid using inductors?

Use the RC voltage-divider cascade in Figure 11–37, since it does not invert the output and does not require an inductor.

(c). There was a concern about loading at the output?

None of the circuits prevents the possibility of loading at the output. One could add an OP AMP follower at the output of any of the three solutions to address loading concerns.

11.2 Problem Solutions

Problem 11–1. Find the driving point impedance seen by the voltage source in Figure P11–1 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

We have the following results:

$$Z(s) = 2R + 2R + \frac{1}{Cs} = \frac{4RCs+1}{Cs}$$

$$T_V(s) = \frac{2R + \dfrac{1}{Cs}}{2R + 2R + \dfrac{1}{Cs}} = \frac{2RCs+1}{4RCs+1}$$

Problem 11–2. Find the driving point impedance seen by the voltage source in Figure P11–2 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

We have the following results:

$$Z(s) = (Ls \parallel R) + 2R = \frac{RLs}{R+Ls} + 2R = \frac{RLs + 2R^2 + 2RLs}{R+Ls} = \frac{3RLs+2R^2}{R+Ls}$$

$$T_V(s) = \frac{2R}{\dfrac{RLs}{R+Ls} + 2R} = \frac{2RLs+2R^2}{3RLs+2R^2}$$

Problem 11–3. Find the driving point impedance seen by the voltage source in Figure P11–3 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

We have the following results:

$$Z(s) = Ls + \frac{1}{Cs} + R = \frac{LCs^2 + RCs + 1}{Cs}$$

$$T_V(s) = \frac{R}{Ls + R + \dfrac{1}{Cs}} = \frac{RCs}{LCs^2 + RCs + 1}$$

Problem 11–4. Find the driving point impedance seen by the voltage source in Figure P11–4 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

We have the following results:

$$Z(s) = Ls + \frac{1}{Cs} + R = \frac{LCs^2 + RCs + 1}{Cs}$$

$$T_V(s) = \frac{R + \dfrac{1}{Cs}}{Ls + R + \dfrac{1}{Cs}} = \frac{RCs+1}{LCs^2 + RCs + 1}$$

Problem 11–5. Find the driving point impedance seen by the voltage source in Figure P11–5 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

We have the following results:

$$Z(s) = R + \left(\frac{1}{Cs} \parallel Ls\right) = R + \frac{\dfrac{Ls}{Cs}}{Ls + \dfrac{1}{Cs}} = R + \frac{Ls}{LCs^2+1} = \frac{RLCs^2 + R + Ls}{LCs^2+1}$$

$$T_V(s) = \frac{\dfrac{Ls}{LCs^2+1}}{R + \dfrac{Ls}{LCs^2+1}} = \frac{Ls}{RLCs^2 + Ls + R}$$

Problem 11–6. Find the driving point impedance seen by the voltage source in Figure P11–6 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

We have the following results:

$$Z(s) = \frac{V_1(s)}{I_1(s)} = \frac{V_1(s)}{\dfrac{V_1(s) - 0}{R}} = R$$

$$T_V(s) = \frac{-Z_2}{Z_1}$$

$$Z_1 = R$$

$$Z_2 = 10R \parallel \frac{1}{Cs} = \frac{\dfrac{10R}{Cs}}{10R + \dfrac{1}{Cs}} = \frac{10R}{10RCs + 1}$$

$$T_V(s) = \frac{-10}{10RCs + 1}$$

Problem 11–7. Find the driving point impedance seen by the voltage source in Figure P11–7 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

We have the following results:

$$Z(s) = \frac{V_1(s)}{I_1(s)}$$

$$I_1(s) = 0$$

$$Z(s) = \infty$$

$$T_V(s) = \frac{Z_1 + Z_2}{Z_1}$$

$$Z_1 = \frac{1}{Cs} \parallel 2R = \frac{\dfrac{2R}{Cs}}{2R + \dfrac{1}{Cs}} = \frac{2R}{2RCs + 1}$$

$$Z_2 = 2R$$

$$T_V(s) = \frac{\dfrac{2R}{2RCs + 1} + 2R}{\dfrac{2R}{2RCs + 1}} = \frac{2R + 4R^2Cs + 2R}{2R} = 2RCs + 2$$

Problem 11–8. Find the driving point impedance seen by the voltage source in Figure P11–8 and the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$.

We have the following results:

$$Z_1 = R_1 \parallel \frac{1}{Cs} = \frac{\dfrac{R_1}{Cs}}{R_1 + \dfrac{1}{Cs}} = \frac{R_1}{R_1 Cs + 1}$$

$$Z_2 = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

$$Z(s) = Z_1 = \frac{R_1}{R_1 Cs + 1}$$

$$T_{\text{V}}(s) = \frac{-Z_2}{Z_1} = \frac{-\dfrac{R_2 R_3}{R_2 + R_3}}{\dfrac{R_1}{R_1 Cs + 1}} = \frac{-R_2 R_3 (R_1 Cs + 1)}{R_1 R_2 + R_1 R_3}$$

Problem 11–9. Find the voltage transfer function $T_{\text{V}}(s) = V_2(s)/V_1(s)$ in Figure P11–9.

Perform a source transformation to get a current source $V_1(s)/Ls$ in parallel with the inductor. Combine the inductor in parallel with the left resistor. Perform two-path current division and then apply Ohm's law to find the output voltage. We have the following results:

$$Z_{\text{EQ}}(s) = Ls \parallel R = \frac{RLs}{R + Ls}$$

$$I_2(s) = \left(\frac{Z_{\text{EQ}}(s)}{Z_{\text{EQ}}(s) + R + \dfrac{1}{Cs}} \right) \left(\frac{V_1(s)}{Ls} \right) = \left(\frac{\dfrac{RLs}{R + Ls}}{\dfrac{RLs}{R + Ls} + R + \dfrac{1}{Cs}} \right) \left(\frac{V_1(s)}{Ls} \right)$$

$$= \left(\frac{RLCs^2}{2RLCs^2 + R^2 Cs + Ls + R} \right) \left(\frac{V_1(s)}{Ls} \right) = \frac{RCsV_1(s)}{2RLCs^2 + R^2 Cs + Ls + R}$$

$$V_2(s) = \frac{1}{Cs} I_2(s) = \frac{RV_1(s)}{2RLCs^2 + R^2 Cs + Ls + R}$$

$$T_{\text{V}}(s) = \frac{V_2(s)}{V_1(s)} = \frac{R}{2RLCs^2 + R^2 Cs + Ls + R}$$

Problem 11–10. Find the driving point impedance seen by the voltage source in Figure P11–10 and the voltage transfer function $T_{\text{V}}(s) = V_2(s)/V_1(s)$. Insert a follower at A and repeat.

Find the driving point impedance of the original circuit.

$$Z_{\text{IN}} = R + \frac{(Ls)(R + Ls)}{Ls + R + Ls} = \frac{R^2 + 2RLs + RLs + L^2 s^2}{R + 2Ls} = \frac{L^2 s^2 + 3RLs + R^2}{R + 2Ls}$$

Without the follower inserted, perform a source transformation to get a current source $V_1(s)/R$ in parallel with the left resistor. Combine the left resistor in parallel with the left inductor. Perform two-path current division and then apply Ohm's law to find the output voltage. We have the following results:

$$Z_{\text{EQ}}(s) = Ls \parallel R = \frac{RLs}{R+Ls}$$

$$I_2(s) = \left(\frac{Z_{\text{EQ}}(s)}{Z_{\text{EQ}}(s)+R+Ls}\right)\left(\frac{V_1(s)}{R}\right) = \left(\frac{\dfrac{RLs}{R+Ls}}{\dfrac{RLs}{R+Ls}+R+Ls}\right)\left(\frac{V_1(s)}{R}\right)$$

$$= \left(\frac{RLs}{RLs+R^2+RLs+RLs+L^2s^2}\right)\left(\frac{V_1(s)}{R}\right) = \frac{LsV_1(s)}{L^2s^2+3RLs+R^2}$$

$$V_2(s) = LsI_2(s) = \frac{L^2s^2V_1(s)}{L^2s^2+3RLs+R^2}$$

$$T_{\text{V}}(s) = \frac{V_2(s)}{V_1(s)} = \frac{L^2s^2}{L^2s^2+3RLs+R^2}$$

Insert the follower. Find the driving point impedance.

$$Z_{\text{IN}} = R + Ls$$

Compute the transfer function on each side of the follower and multiply the results together. Note that the two sides are identical.

$$T_{\text{V}}(s) = \left(\frac{Ls}{R+Ls}\right)\left(\frac{Ls}{R+Ls}\right) = \frac{L^2s^2}{L^2s^2+2RLs+R^2}$$

Problem 11–11. Find the voltage transfer function $T_{\text{V}}(s) = V_2(s)/V_1(s)$ in Figure P11–11. Select values of R and C so that $T_{\text{V}}(s)$ has a pole at $s = -100$ krad/s.

The circuit is a noninverting amplifier.

$$Z_1 = R$$

$$Z_2 = R \parallel \frac{1}{Cs} = \frac{R}{RCs+1}$$

$$T_{\text{V}}(s) = \frac{Z_1+Z_2}{Z_1} = \frac{R+\dfrac{R}{RCs+1}}{R} = 1 + \frac{1}{RCs+1} = \frac{RCs+2}{RCs+1}$$

The pole is located at $s = -1/RC$, so pick $R = 10$ kΩ and solve for $C = 0.001$ μF.

Problem 11–12. Find the voltage transfer function $T_{\text{V}}(s) = V_2(s)/V_1(s)$ in Figure P11–12. Select values of R_1, R_2, and C so that $T_{\text{V}}(s)$ has a pole at $s = -250$ krad/s and $R_2/R_1 = 100$.

The circuit is an inverting amplifier.

$$Z_1 = R_1$$

$$Z_2 = R_2 \parallel \frac{1}{Cs} = \frac{R_2}{R_2Cs+1}$$

$$T_{\text{V}}(s) = \frac{-Z_2}{Z_1} = \frac{-\dfrac{R_2}{R_2Cs+1}}{R_1} = \frac{-\dfrac{R_2}{R_1}}{R_2Cs+1}$$

The pole is located at $s = -1/R_2C$, so pick $R_2 = 400$ kΩ and solve for $C = 10$ pF. Also solve for $R_1 = R_2/100 = 4$ kΩ.

Problem 11–13. Find the current transfer function $T_I(s) = I_2(s)/I_1(s)$ in Figure P11–13. Select values of R and L so that $T_I(s)$ has a pole at $s = -377$ rad/s.

Apply two-path current division.

$$I_2(s) = \frac{R}{R + 2R + Ls} I_1(s) = \frac{R}{Ls + 3R} I_1(s)$$

$$T_I(s) = \frac{I_2(s)}{I_1(s)} = \frac{R}{Ls + 3R}$$

The pole is located at $s = -3R/L$, so pick $R = 100\ \Omega$ and solve for $L = 796$ mH.

Problem 11–14. Find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ of the cascade connection in Figure P11–14. Locate the poles and zeros of the transfer function.

Each OP AMP stage has the same basic structure. Solve for the general transfer function for one stage and then multiply the results together using the appropriate component values.

$$Z_1 = R_1$$

$$Z_2 = R_2 \parallel \frac{1}{C_2 s} = \frac{R_2}{R_2 C_2 s + 1}$$

$$T_V(s) = \frac{-Z_2}{Z_1} = \frac{-\dfrac{R_2}{R_1}}{R_2 C_2 s + 1} = \frac{-\dfrac{1}{R_1 C_2}}{s + \dfrac{1}{R_2 C_2}}$$

$$T_{V1}(s) = \frac{-10000}{s + 10000}$$

$$T_{V2}(s) = \frac{-4545}{s + 454.5} = \frac{-50000}{11s + 5000}$$

$$T_V(s) = T_{V1}(s) T_{V2}(s) = \frac{5 \times 10^8}{(s + 10000)(11s + 5000)}$$

Both zeros are located at infinity. The poles are located at $s = -10000$ rad/s and $s = -5000/11$ rad/s.

Problem 11–15. Find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ of the cascade connection in Figure P11–15. Locate the poles and zeros of the transfer function.

The two stages do not load each other, so find the transfer function for each stage and multiply them together.

$$T_{V1}(s) = \frac{R_1}{R_1 + \dfrac{1}{C_1 s}} = \frac{R_1 C_1 s}{R_1 C_1 s + 1} = \frac{s}{s + \dfrac{1}{R_1 C_1}} = \frac{s}{s + 967.1}$$

$$T_{V2}(s) = \frac{R_2 + \dfrac{1}{C_2 s} + R_3}{R_2 + \dfrac{1}{C_2 s}} = \frac{\dfrac{R_2 + R_3}{R_2} s + \dfrac{1}{R_2 C_2}}{s + \dfrac{1}{R_2 C_2}} = \frac{11s + 10000}{s + 10000}$$

$$T_V(s) = T_{V1}(s) T_{V2}(s) = \frac{s(11s + 10000)}{(s + 10000)(s + 967.1)}$$

The zeros are located at $s = 0$ and $s = -10000/11$ rad/s. The poles are located at $s = -10000$ rad/s and $s = -967.1$ rad/s.

Problem 11–16. Find the input impedance $Z(s)$ in Figure P11–15.

We have the following results:

$$Z(s) = \frac{V_1(s)}{I_1(s)} = \frac{1}{C_1 s} + R_1 = \frac{R_1 C_1 s + 1}{C_1 s} = \frac{R_1 s + \dfrac{1}{C_1}}{s} = \frac{22000s + 21277000}{s}$$

Problem 11–17. Find the impulse response at $v_2(t)$ in Figure P11–17. Find the step response.

Find the transfer function and take the inverse Laplace transform to determine the impulse response. Then determine the step response.

$$H(s) = T(s) = \frac{R_2 + \dfrac{1}{Cs}}{R_1 + R_2 + \dfrac{1}{Cs}} = \frac{\dfrac{R_2}{R_1+R_2}s + \dfrac{1}{(R_1+R_2)C}}{s + \dfrac{1}{(R_1+R_2)C}} = \frac{\dfrac{47}{80}s + \dfrac{250}{3}}{s + \dfrac{250}{3}}$$

$$= \frac{47}{80} + \frac{\dfrac{275}{8}}{s + \dfrac{250}{3}}$$

$$h(t) = \frac{47}{80}\delta(t) + \frac{275}{8}e^{-250t/3}u(t)$$

$$g(t) = \int_0^t h(\tau)d\tau = \frac{47}{80}u(t) + \left(\frac{275}{8}\right)\left(\frac{-3}{250}\right)\left(e^{-250t/3} - 1\right)u(t)$$

$$= \left(1 - \frac{33}{80}e^{-250t/3}\right)u(t)$$

Problem 11–18. Find $v_2(t)$ in Figure P11–18 when $v_1(t) = \delta(t)$. Repeat for $v_1(t) = u(t)$.

The first response is the impulse response and the second response is the step response.

$$H(s) = T(s) = \frac{25}{25 + 100 + 0.1s} = \frac{250}{s + 1250}$$

$$h(t) = 250e^{-1250t}u(t)$$

$$G(s) = \frac{H(s)}{s} = \frac{250}{s(s+1250)} = \frac{\dfrac{1}{5}}{s} - \frac{\dfrac{1}{5}}{s + 1250}$$

$$g(t) = \frac{1}{5}\left(1 - e^{-1250t}\right)u(t)$$

Problem 11–19. Find $v_2(t)$ in Figure P11–19 when $v_1(t) = \delta(t)$. Repeat for $v_1(t) = u(t)$.

The first response is the impulse response and the second response is the step response.

$$H(s) = T(s) = \frac{R_2}{R_2 + \dfrac{R_1}{R_1Cs+1}} = \frac{R_1R_2Cs + R_2}{R_1R_2Cs + R_1 + R_2} = \frac{s + \dfrac{1}{R_1C}}{s + \dfrac{R_1+R_2}{R_1R_2C}}$$

$$= \frac{s+1000}{s+3000} = 1 - \frac{2000}{s+3000}$$

$$h(t) = \delta(t) - 2000e^{-3000t}u(t)$$

$$G(s) = \frac{H(s)}{s} = \frac{s+1000}{s(s+3000)} = \frac{\frac{1}{3}}{s} + \frac{\frac{2}{3}}{s+3000}$$

$$g(t) = \frac{1}{3}\left(1 + 2e^{-3000t}\right)u(t)$$

Problem 11–20. Find $h(t)$ and $g(t)$ for the circuit in Figure P11–20.

Combine the resistor and inductor in series to create Z_1 and combine the capacitor and resistor in parallel to create Z_2. Then apply voltage division.

$$Z_1 = R_1 + Ls = s + 1000$$

$$Z_2 = \frac{R_2}{R_2Cs+1} = \frac{\dfrac{1}{C}}{s + \dfrac{1}{R_2C}} = \frac{5000000}{s+5000}$$

$$H(s) = \frac{Z_2}{Z_1+Z_2} = \frac{\dfrac{5000000}{s+5000}}{s + 1000 + \dfrac{5000000}{s+5000}} = \frac{5000000}{s^2 + 6000s + 10^7}$$

$$= \frac{(5000)(1000)}{(s+3000)^2 + 1000^2}$$

$$h(t) = 5000e^{-3000t}\sin(1000t)u(t)$$

$$G(s) = \frac{H(s)}{s} = \frac{5000000}{s(s^2+6000s+10^7)} = \frac{\frac{1}{2}}{s} - \frac{\frac{1}{2}s + 3000}{s^2+6000s+10^7}$$

$$= \frac{\frac{1}{2}}{s} - \left(\frac{1}{2}\right)\frac{s+3000}{(s+3000)^2+1000^2} - \left(\frac{3}{2}\right)\frac{1000}{(s+3000)^2+1000^2}$$

$$g(t) = \frac{1}{2}\left\{1 - e^{-3000t}\left[\cos(1000t) + 3\sin(1000t)\right]\right\}u(t)$$

Problem 11–21. Find $h(t)$ and $g(t)$ for the circuit in Figure P11–21 if $R_F = 100$ kΩ.

We have the following results:

$$
\begin{aligned}
Z_1 &= R_1 \\
Z_2 &= \frac{\frac{R_F}{Cs}}{R_F + \frac{1}{Cs}} = \frac{\frac{1}{C}}{s + \frac{1}{R_F C}} \\
H(s) &= \frac{-Z_2}{Z_1} = \frac{-\frac{1}{R_1 C}}{s + \frac{1}{R_F C}} = \frac{-10000}{s+500} \\
h(t) &= -10000 e^{-500t} u(t) \\
G(s) &= \frac{H(s)}{s} = \frac{-10000}{s(s+500)} = -\frac{20}{s} + \frac{20}{s+500} \\
g(t) &= 20\left(e^{-500t} - 1\right) u(t)
\end{aligned}
$$

Problem 11–22. Select an appropriate R_F for the circuit of Figure P11–21 so that the step response of the circuit is $g(t) = (10e^{-1000t} - 10)u(t)$ V.

The pole is located at $s = -1000$ rad/s, so we have $1/R_F C = 1000$. Solve for $R_F = 1/1000C = 50$ kΩ. Use the results from Problem 11–21, to verify the step response.

$$
\begin{aligned}
H(s) &= \frac{-Z_2}{Z_1} = \frac{-\frac{1}{R_1 C}}{s + \frac{1}{R_F C}} = \frac{-10000}{s+1000} \\
G(s) &= \frac{H(s)}{s} = \frac{-10000}{s(s+1000)} = -\frac{10}{s} + \frac{10}{s+1000} \\
g(t) &= 10\left(e^{-1000t} - 1\right) u(t)
\end{aligned}
$$

Problem 11–23. Find $v_2(t)$ in Figure P11–23 when $v_1(t) = \delta(t)$. Repeat for $v_1(t) = u(t)$.

The first response is the impulse response, $h(t)$, and the second response is step response, $g(t)$. The circuit is a noninverting amplifier.

$$Z_1 = R_1$$

$$Z_2 = \frac{\frac{R_2}{Cs}}{R_2 + \frac{1}{Cs}} = \frac{\frac{1}{C}}{s + \frac{1}{R_2C}}$$

$$H(s) = \frac{Z_1 + Z_2}{Z_1} = \frac{R_1 + \frac{\frac{1}{C}}{s + \frac{1}{R_2C}}}{R_1} = 1 + \frac{\frac{1}{R_1C}}{s + \frac{1}{R_2C}} = 1 + \frac{10}{s+5}$$

$$h(t) = \delta(t) + 10e^{-5t}u(t)$$

$$G(s) = \frac{H(s)}{s} = \frac{s+15}{s(s+10)} = \frac{3}{s} - \frac{2}{s+5}$$

$$g(t) = \left(3 - 2e^{-5t}\right)u(t)$$

Problem 11–24. The impulse response of a linear circuit is $h(t) = (100e^{-200t} - 100e^{-1000t})u(t)$. Find the circuit's step response $g(t)$, impulse response transform $H(s)$, step response transform $G(s)$, and the circuit's transfer function $T(s)$.

We have the following results:

$$h(t) = (100e^{-200t} - 100e^{-1000t})u(t)$$

$$T(s) = H(s) = \frac{100}{s+200} - \frac{100}{s+1000} = \frac{80000}{(s+200)(s+1000)}$$

$$G(s) = \frac{H(s)}{s} = \frac{80000}{s(s+200)(s+1000)} = \frac{0.4}{s} - \frac{0.5}{s+200} + \frac{0.1}{s+1000}$$

$$g(t) = \left(\frac{2}{5} - \frac{1}{2}e^{-200t} + \frac{1}{10}e^{-1000t}\right)u(t)$$

Problem 11–25. The impulse response of a linear circuit is $h(t) = \delta(t) - 2000e^{-200t}u(t)$. Find the circuit's step response $g(t)$, impulse response transform $H(s)$, step response transform $G(s)$, and the circuit's transfer function $T(s)$.

We have the following results:

$$h(t) = \delta(t) - 2000e^{-200t}u(t)$$

$$T(s) = H(s) = 1 - \frac{2000}{s+200} = \frac{s-1800}{s+200}$$

$$G(s) = \frac{H(s)}{s} = \frac{s-1800}{s(s+200)} = \frac{-9}{s} + \frac{10}{s+200}$$

$$g(t) = \left(10e^{-200t} - 9\right)u(t)$$

Problem 11–26. The step response transform of a linear circuit is $G(s) = 1000/s(s+1000)$. Find the circuit's impulse response $h(t)$, step response $g(t)$, impulse response transform $H(s)$, and the circuit's transfer function $T(s)$.

We have the following results:

$$G(s) = \frac{1000}{s(s+1000)} = \frac{1}{s} - \frac{1}{s+1000}$$

$$g(t) = \left(1 - e^{-1000t}\right) u(t)$$

$$H(s) = T(s) = sG(s) = \frac{1000}{s+1000}$$

$$h(t) = 1000e^{-1000t}u(t)$$

Problem 11–27. The step response of a linear circuit is $g(t) = 15(e^{-20000t} - e^{-30000t})u(t)$. Find the circuit's impulse response $h(t)$, impulse response transform $H(s)$, step response transform $G(s)$, and the circuit's transfer function $T(s)$.

We have the following results:

$$g(t) = 15(e^{-20000t} - e^{-30000t})u(t)$$

$$G(s) = \frac{15}{s+20000} - \frac{15}{s+30000} = \frac{150000}{(s+20000)(s+30000)}$$

$$H(s) = T(s) = sG(s) = \frac{150000s}{(s+20000)(s+30000)} = \frac{-300000}{s+20000} + \frac{450000}{s+30000}$$

$$h(t) = 150000\left(3e^{-30000t} - 2e^{-20000t}\right)u(t)$$

Problem 11–28. Find $h(t) = dg(t)/dt$ when $g(t) = (3 - e^{-10t})u(t)$. Verify your answer by first transforming $g(t)$ into $G(s)$ and finding $H(s) = sG(s)$ and then taking the inverse transform of $H(s)$. Did you get the same answer?

We have the following results:

$$g(t) = \left(3 - e^{-10t}\right) u(t)$$

$$h(t) = \frac{dg(t)}{dt} = \left(3 - e^{-10t}\right)\frac{du(t)}{dt} + \left[\frac{d}{dt}\left(3 - e^{-10t}\right)\right]u(t) = \left(3 - e^{-10t}\right)\delta(t) + \left(10e^{-10t}\right)u(t)$$

$$= (3-1)\,\delta(t) + 10e^{-10t}u(t) = 2\delta(t) + 10e^{-10t}u(t)$$

$$G(s) = \frac{3}{s} - \frac{1}{s+10} = \frac{2s+30}{s(s+10)}$$

$$H(s) = sG(s) = \frac{2s+30}{s+10} = 2 + \frac{10}{s+10}$$

$$h(t) = 2\delta(t) + 10e^{-10t}u(t)$$

The results are the same, as expected.

Problem 11–29. The impulse response of a linear circuit is $h(t) = 1000[e^{-1000t}]u(t)$. Find the output waveform when the input is $x(t) = 5tu(t)$ V.

We have the following results:

$$h(t) = 1000e^{-1000t}u(t)$$

$$H(s) = \frac{1000}{s+1000}$$

$$x(t) = 5tu(t)$$

$$X(s) = \frac{5}{s^2}$$

$$Y(s) = H(s)X(s) = \frac{5000}{s^2(s+1000)} = \frac{5}{s^2} - \frac{0.005}{s} + \frac{0.005}{s+1000}$$

$$y(t) = \frac{1}{200}\left(1000t - 1 + e^{-1000t}\right)u(t)$$

Problem 11–30. The step response of a linear circuit is $g(t) = 0.25[1 - e^{-150t}]u(t)$. Find the output waveform when the input is $v_1(t) = [20e^{-200t}]u(t)$. Use MATLAB to find the Laplace transforms of $g(t)$ and $v_1(t)$. Then find $V_2(s)$. Finally, use the inverse Laplace function to find the waveform $v_2(t)$ and plot the results.

We have the following results:

$$g(t) = 0.25[1 - e^{-150t}]u(t)$$

$$G(s) = \frac{0.25}{s} - \frac{0.25}{s+150} = \frac{37.5}{s(s+150)}$$

$$H(s) = sG(s) = \frac{37.5}{s+150}$$

$$v_1(t) = 20e^{-200t}u(t)$$

$$V_1(s) = \frac{20}{s+200}$$

$$V_2(s) = H(s)V_1(s) = \frac{750}{(s+150)(s+200)} = \frac{15}{s+150} - \frac{15}{s+200}$$

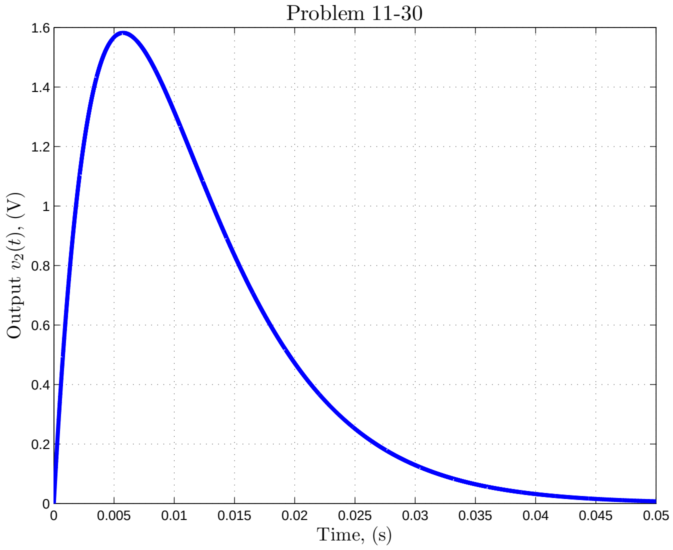
$$v_2(t) = 15\left(e^{-150t} - e^{-200t}\right)u(t)$$

The corresponding MATLAB code, results, and plot are shown below.

```
syms s t
gt = 0.25*(1-exp(-150*t));
Gs = laplace(gt)
Gs2 = simplify(Gs)
Hs = simplify(s*Gs)
v1t = 20*exp(-200*t);
V1s = laplace(v1t)
V2s = simplify(Hs*V1s)
v2t = ilaplace(V2s)
```

```
Gs = 1/(4*s) - 1/(4*(s + 150))
Gs2 = 75/(2*s*(s + 150))
Hs = 75/(2*(s + 150))
V1s = 20/(s + 200)
```

```
V2s = 750/((s + 150)*(s + 200))
v2t = 15/exp(150*t) - 15/exp(200*t)
```



Problem 11–31. The step response of a linear circuit is $g(t) = 10[e^{-50t}\cos(200t)]u(t)$. Find the circuit's impulse response $h(t)$, impulse response transform $H(s)$, step response transform $G(s)$, and the circuit's transfer function $T(s)$.

We have the following results:

$$g(t) = 10[e^{-50t}\cos(200t)]u(t)$$

$$G(s) = \frac{10(s+50)}{(s+50)^2+200^2}$$

$$H(s) = T(s) = sG(s) = \frac{10s(s+50)}{(s+50)^2+200^2}$$

$$= \frac{10(s^2+100s+42500)}{s^2+100s+42500} - \frac{500s+425000}{s^2+100s+42500}$$

$$= 10 - (500)\frac{(s+50)}{(s+50)^2+200^2} - (2000)\frac{200}{(s+50)^2+200^2}$$

$$h(t) = 10\delta(t) - 500e^{-50t}\left[\cos(200t) + 4\sin(200t)\right]u(t)$$

Problem 11–32. The impulse response transform of a linear circuit is $H(s) = (s+2000)/(s+1000)$. Find the output waveform when the input is $x(t) = 5e^{-1000t}u(t)$. Use MATLAB to find the Laplace transform of $x(t)$. Then find $Y(s)$. Finally, use the inverse Laplace function to find the waveform $y(t)$ and plot the results.

We have the following results:

$$H(s) = \frac{s+2000}{s+1000}$$

$$x(t) = 5e^{-1000t}u(t)$$

$$X(s) = \frac{5}{s+1000}$$

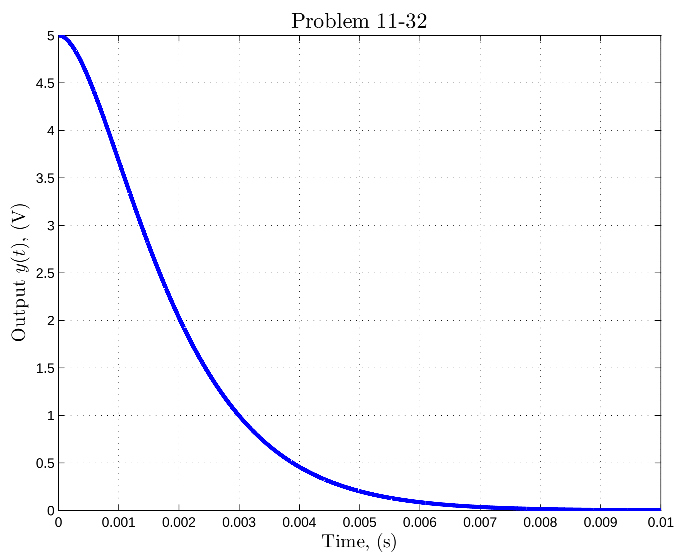
$$Y(s) = H(s)X(s) = \frac{5(s+2000)}{(s+1000)^2} = \frac{5000}{(s+1000)^2} + \frac{5}{s+1000}$$

$$y(t) = 5e^{-1000t}(1000t+1)u(t)$$

The corresponding MATLAB code, results, and plot are shown below.

```
syms s t
Hs = (s+2000)/(s+1000)
xt = 5*exp(-1000*t);
Xs = laplace(xt)
Ys = simplify(Hs*Xs)
yt = ilaplace(Ys)
```

```
Hs = (s + 2000)/(s + 1000)
Xs = 5/(s + 1000)
Ys = (5*(s + 2000))/(s + 1000)^2
yt = 5/exp(1000*t) + (5000*t)/exp(1000*t)
```



Problem 11–33. The impulse response of a linear circuit is $h(t) = 20u(t) + \delta(t)$. Find the output waveform $y(t)$ when the input is $x(t) = 2[e^{-20t}]u(t)$.

We have the following results:

$$h(t) = 20u(t) + \delta(t)$$

$$H(s) = \frac{20}{s} + 1 = \frac{s+20}{s}$$

$$x(t) = 2e^{-20t}u(t)$$

$$X(s) = \frac{2}{s+20}$$

$$Y(s) = H(s)X(s) = \left(\frac{s+20}{s}\right)\left(\frac{2}{s+20}\right) = \frac{2}{s}$$

$$y(t) = 2u(t)$$

Problem 11–34. The circuit in Figure P11–34 is in the steady state with $v_1(t) = 5\cos(1414.21t)$ V. Find $v_{2SS}(t)$. Repeat for $v_1(t) = 5\cos(1000t)$ V. And without doing any calculations, repeat for $v_1(t) = 5$ V.

Find the transfer function and then evaluate it at $s = j\omega$ for the specified frequencies. Use the magnitude and phase of the results to determine the sinusoidal, steady-state responses. We have the following results:

$$T(s) = \frac{Ls + \dfrac{1}{Cs}}{R + Ls + \dfrac{1}{Cs}} = \frac{s^2 + \dfrac{1}{LC}}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}} = \frac{s^2 + 2\times 10^6}{s^2 + 4000s + 2\times 10^6}$$

$$T(j\omega) = \frac{2\times 10^6 - \omega^2}{2\times 10^6 - \omega^2 + j4000\omega}$$

$$T(j1414.21) = 1.78\times 10^{-6}\angle{-90^\circ}$$

$$v_{2SS}(t) = 8.91\cos(1414.21t - 90^\circ)\ \mu\text{V}$$

$$T(j1000) = 0.2425\angle{-76^\circ}$$

$$v_{2SS}(t) = 1.2127\cos(1000t - 76^\circ)\ \text{V}$$

$$T(0) = 1$$

$$v_{2SS}(t) = 5\ \text{V}$$

Problem 11–35. The circuit in Figure P11–35 is in the steady state with $v_1(t) = 10\cos(500t)$ V. Find $v_{2SS}(t)$. Repeat for $v_1(t) = 10\cos(1000t)$ V, and for $v_1(t) = 10\cos(10000t)$ V. Where is the pole located?

Find the transfer function and then evaluate it at $s = j\omega$ for the specified frequencies. Use the magnitude and phase of the results to determine the sinusoidal, steady-state responses. We have the following results:

$$Z_1 = R_1$$

$$Z_2 = \frac{R_2/Cs}{R_2 + 1/Cs} = \frac{\frac{1}{C}}{s + \frac{1}{R_2C}}$$

$$T(s) = \frac{-Z_2}{Z_1} = \frac{-\frac{1}{R_1C}}{s + \frac{1}{R_2C}} = \frac{-2000}{s + 1000}$$

$$T(j\omega) = \frac{-2000}{1000 + j\omega}$$

$$T(j500) = 1.79\angle 153^\circ$$

$$v_{2\text{SS}}(t) = 17.89\cos(500t + 153^\circ)\ \text{V}$$

$$T(j1000) = 1.41\angle 135^\circ$$

$$v_{2\text{SS}}(t) = 14.14\cos(1000t + 135^\circ)\ \text{V}$$

$$T(j10000) = 0.2\angle 95.7^\circ$$

$$v_{2\text{SS}}(t) = 1.99\cos(10000t + 95.7^\circ)\ \text{V}$$

The pole is located at $s = -1000$ rad/s.

Problem 11–36. The circuit in Figure P11–36 is in the steady state with $v_1(t) = 25\cos(2000t)$ V. Find $v_{2\text{SS}}(t)$. Repeat for $v_1(t) = 25\cos(10000t)$ V. Where are the poles located?

Find the transfer function and then evaluate it at $s = j\omega$ for the specified frequencies. Use the magnitude and phase of the results to determine the sinusoidal, steady-state responses. We have the following results:

$$T(s) = \frac{Ls}{Ls + R + \frac{1}{Cs}} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{s^2}{s^2 + 2000s + 4 \times 10^6}$$

$$T(j\omega) = \frac{-\omega^2}{4 \times 10^6 - \omega^2 + j2000\omega}$$

$$T(j2000) = 1\angle 90^\circ$$

$$v_{2\text{SS}}(t) = 25\cos(2000t + 90^\circ)\ \text{V}$$

$$T(j10000) = 1.0198\angle 11.8^\circ$$

$$v_{2\text{SS}}(t) = 25.5\cos(10000t + 11.8^\circ)\ \text{V}$$

The poles are located at $s = -1000 \pm j1732$ rad/s.

Problem 11–37. The circuit in Figure P11–37 is in the steady state with $i_1(t) = 10\cos(50000t)$ mA, $R_1 = 100\ \Omega$, $R_2 = 400\ \Omega$, and $L = 100$ mH. Find $i_{2\text{SS}}(t)$. Repeat for $i_1(t) = 10\cos(5000t)$ mA. Where is the pole located?

Use two-path current division to determine the transfer function and note the direction of the output current. Evaluate the transfer function at $s = j\omega$. Use the magnitude and phase of the results to determine the sinusoidal, steady-state responses.

$$T(s) = \frac{-R_1}{R_1 + R_2 + Ls} = \frac{-\dfrac{R_1}{L}}{s + \dfrac{R_1 + R_2}{L}} = \frac{-1000}{s + 5000}$$

$$T(j\omega) = \frac{-1000}{5000 + j\omega}$$

$$T(j50000) = 0.0199\angle 95.7^\circ$$

$$i_{2\text{SS}}(t) = 199\cos(50000t + 95.7^\circ)\ \mu\text{A}$$

$$T(j5000) = 0.1414\angle 135^\circ$$

$$i_{2\text{SS}}(t) = 1.414\cos(5000t + 135^\circ)\ \text{mA}$$

The pole is located at $s = -5000$ rad/s.

Problem 11–38. The circuit in Figure P11–38 is in the steady state with $i_1(t) = 5\cos(1000t)$ mA, $R = 1$ kΩ, $L = 2$ H, and $C = 0.5$ μF.

(a). Find $i_{2\text{SS}}(t)$.

Apply current division to find the transfer function.

$$T(s) = \frac{\dfrac{1}{R}}{\dfrac{1}{R} + \dfrac{1}{Ls} + Cs} = \frac{\dfrac{1}{RC}s}{s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC}} = \frac{2000s}{s^2 + 2000s + 10^6} = \frac{2000s}{(s+1000)^2}$$

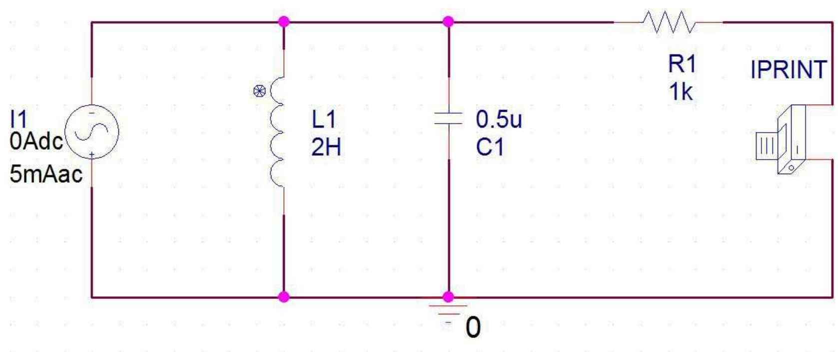
$$T(j\omega) = \frac{j2000\omega}{(1000 + j\omega)^2}$$

$$T(j1000) = 1\angle 0^\circ$$

$$i_{2\text{SS}}(t) = 5\cos(1000t)\ \text{mA}$$

(b). Verify your results using OrCAD. Use IAC for a source and note that the current comes out of the negative terminal (passive sign convention.). The IPRINT element on the right side of the figure is an ammeter and it comes from the Special Library. It has a polarity as indicated by the small "–" on the element. You should ensure that it will read the correct current direction. Before you can use the IPRINT element, it needs to be set up to display the current magnitude and phase. Double click on the element and in its property editor place a "y" in AC, MAG, and PHASE boxes. Click the Apply button and close the property editor. To obtain the simulation select AC Sweep/Noise. Set the start frequency and the stop frequency at 159.15 Hz (1000 rad/s). Place a 1 in Points/Decade. Run the simulation. Under View, select Output File. Scroll down until you see the IPRINT output.

The OrCAD simulation and the corresponding results are shown below.




```
FREQ        IM(V_PRINT1)IP(V_PRINT1)
 1.592E+02   5.000E-03  -2.049E-05
```

Problem 11–39. The circuit in Figure P11–39 is in the steady state with $i_1(t) = 10\cos(5000t)$ mA.

(a). Find the steady-state voltage $v_{2SS}(t)$. Repeat for $i_1(t) = 5\cos(2500t)$ mA.

Use two-path current division and Ohm's law to determine the output voltage.

$$I_2(s) = \frac{R}{R + R + 1/Cs} = \frac{\frac{1}{2}s}{s + \frac{1}{2RC}}$$

$$T(s) = RI_2(s) = \frac{\frac{R}{2}s}{s + \frac{1}{2RC}} = \frac{5000s}{s + 2500}$$

$$T(j\omega) = \frac{j5000\omega}{2500 + j\omega}$$

$$T(j5000) = 4472\angle 26.6^\circ$$

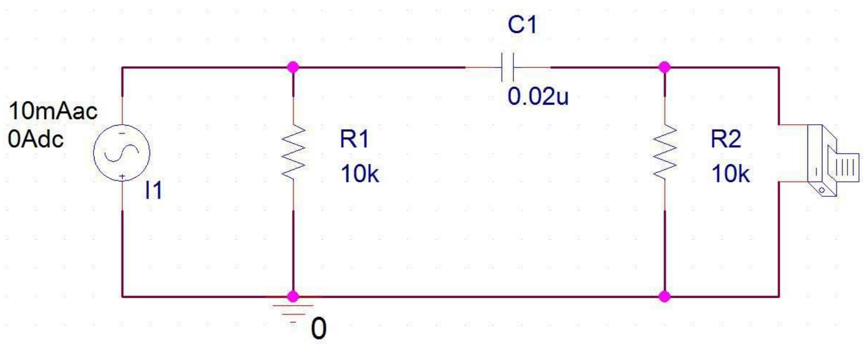
$$v_{2SS}(t) = 44.72\cos(5000t + 26.6^\circ) \text{ V}$$

$$T(j2500) = 3536\angle 45^\circ$$

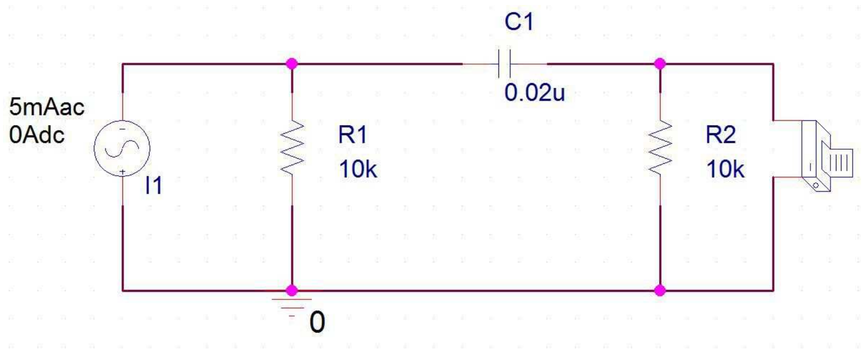
$$v_{2SS}(t) = 17.68\cos(2500t + 45^\circ) \text{ V}$$

(b). Verify your answer using OrCAD (see Problem 11–38 for help on OrCAD setup.)

The OrCAD simulations and the corresponding results are shown below.



```
FREQ        VM(N00148,0)VP(N00148,0)
 7.958E+02   4.472E+01   2.656E+01
```




```
FREQ        VM(N00148,0)VP(N00148,0)
 3.979E+02   1.768E+01   4.500E+01
```

Problem 11–40. The impulse response transform of a circuit is $H_{\mathrm{R}}(s) = \dfrac{V_2(s)}{I_1(s)} = \dfrac{5000s}{s+2500}$. Find $v_{2\mathrm{SS}}(t)$ if $i_1(t) = 10\cos(5000t)$ mA. Compare your answer to that found in Problem 11–39.

The transfer function is the same as the impulse response transform.

$$T(s) = H_{\mathrm{R}}(s) = \frac{V_2(s)}{I_1(s)} = \frac{5000s}{s+2500}$$

$$T(j\omega) = \frac{j5000\omega}{2500 + j\omega}$$

$$T(j5000) = 4472\angle 26.6^\circ$$

$$v_{2\mathrm{SS}}(t) = 44.72\cos(5000t + 26.6^\circ)\ \mathrm{V}$$

The answer is the same as the result in Problem 11–39 for the corresponding input.

Problem 11–41. The transfer function of a linear circuit is $T(s) = (s+100)/(s+10)$. Find the sinusoidal steady-state output for an input $x(t) = 5\cos(100t)$.

We have the following results:

$$T(s) = \frac{s+100}{s+10}$$

$$T(j\omega) = \frac{100 + j\omega}{10 + j\omega}$$

$$T(j100) = \frac{100 + j100}{10 + j100} = 1.4072\angle{-39.3^\circ}$$

$$x(t) = 5\cos(100t)$$

$$y_{\mathrm{SS}}(t) = 7.036\cos(100t - 39.3^\circ)$$

Problem 11–42. The step response of a linear circuit is $g(t) = [15e^{-500t}]u(t)$. Find the sinusoidal steady-state output for an input $x(t) = 5\cos(1000t)$.

We have the following results:

$$g(t) = 15e^{-500t}u(t)$$

$$G(s) = \frac{15}{s+500}$$

$$H(s) = T(s) = sG(s) = \frac{15s}{s+500}$$

$$T(j\omega) = \frac{j15\omega}{500+j\omega}$$

$$T(j1000) = \frac{j15000}{500+j1000} = 13.42\angle 26.6^\circ$$

$$x(t) = 5\cos(1000t)$$

$$y_{SS}(t) = 67.08\cos(1000t + 26.6^\circ)$$

Problem 11–43. The step response of a linear circuit is $g(t) = [2e^{+100t}]u(t)$. Find the sinusoidal steady-state output for an input $x(t) = 5\cos(500t)$.

We have the following results:

$$g(t) = 2e^{100t}u(t)$$

$$G(s) = \frac{2}{s-100}$$

$$H(s) = T(s) = sG(s) = \frac{2s}{s-100}$$

The pole is located at $s = 100$ rad/s, so the response is not stable. There is no steady-state response.

Problem 11–44. The impulse response of a linear circuit is $h(t) = [500e^{-5000t}]u(t) - \delta(t)$. Find the sinusoidal steady-state output for an input $x(t) = 10\cos(10000t)$.

We have the following results:

$$h(t) = 500e^{-5000t}u(t) - \delta(t)$$

$$H(s) = T(s) = \frac{500}{s+5000} - 1 = \frac{-s-4500}{s+5000}$$

$$T(j\omega) = \frac{-4500-j\omega}{5000+j\omega}$$

$$T(j10000) = \frac{-4500-j10000}{5000+j10000} = 0.981\angle{-177.7^\circ}$$

$$x(t) = 10\cos(10000t)$$

$$y_{SS}(t) = 9.81\cos(10000t - 177.7^\circ)$$

Problem 11–45. The impulse response of a linear circuit is $h(t) = 800[e^{-100t} - e^{-400t}]u(t)$. Use MATLAB to find the sinusoidal steady-state output for an input $x(t) = 8\cos(200t)$. Use MATLAB to plot $y(t)$.

We have the following results:

$$h(t) = 800\left(e^{-100t} - e^{-400t}\right)u(t)$$

$$H(s) = T(s) = \frac{800}{s+100} - \frac{800}{s+400} = \frac{240000}{(s+100)(s+400)}$$

$$T(j200) = \frac{240000}{(100+j200)(400+j200)} = 2.4\angle{-90^\circ}$$

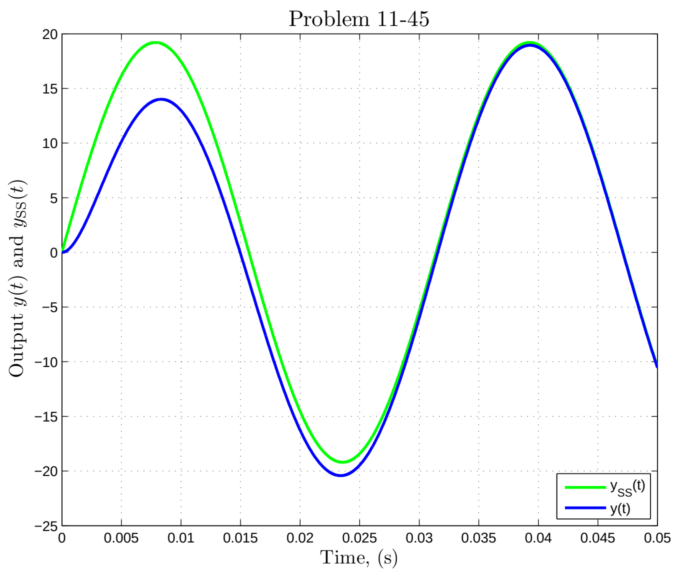
$$x(t) = 8\cos(200t)$$

$$y_{\mathrm{ss}}(t) = 19.2\cos(200t - 90^\circ) = 19.2\sin(200t)$$

The MATLAB code, results, and plot are shown below. The plot shows both the response $y(t)$ and the steady-state response $y_{SS}(t)$.

```
syms s t
xMag = [8];
xPhase = 0;
w = [200];
ht = 800*(exp(-100*t)-exp(-400*t));
Hs = laplace(ht);
Ts = Hs
Tjw = subs(Ts,s,j*w);
TjwMag = abs(Tjw);
TjwPhase = 180*angle(Tjw)/pi;
yssMag = xMag.*TjwMag;
yssPhase = xPhase+TjwPhase;
Results = [w' TjwMag' TjwPhase' yssMag' yssPhase']
```

```
Ts = 800/(s + 100) - 800/(s + 400)
Results = 200.0000e+000     2.4000e+000   -90.0000e+000    19.2000e+000   -90.0000e+000
```



Problem 11–46. The step response of a linear circuit is $g(t) = [-e^{-60t}\sin(80t)]u(t)$. Find the sinusoidal steady-state response for an input $x(t) = 20\cos(100t)$.

We have the following results:

$$g(t) = [-e^{-60t}\sin(80t)]u(t)$$

$$G(s) = \frac{-80}{(s+60)^2+80^2}$$

$$H(s) = T(s) = sG(s) = \frac{-80s}{(s+60)^2+80^2}$$

$$T(j100) = \frac{-j8000}{(60+j100)^2+80^2} = 0.6667\angle{-180^\circ}$$

$$x(t) = 20\cos(100t)$$

$$y_{\text{SS}}(t) = 13.33\cos(100t-180^\circ) = -\frac{40}{3}\cos(100t)$$

Problem 11–47. The step response of a linear circuit is $g(t) = [1-20te^{-10t}]u(t)$. Find the sinusoidal steady-state response for an input $x(t) = 50\cos(10t)$.

We have the following results:

$$g(t) = [1-20te^{-10t}]u(t)$$

$$G(s) = \frac{1}{s} - \frac{20}{(s+10)^2} = \frac{s^2+100}{s(s^2+20s+100)}$$

$$H(s) = T(s) = sG(s) = \frac{s^2+100}{s^2+20s+100}$$

$$x(t) = 50\cos(10t)$$

$$X(s) = \frac{50s}{s^2+100}$$

$$Y(s) = H(s)X(s) = \left(\frac{s^2+100}{s^2+20s+100}\right)\left(\frac{50s}{s^2+100}\right) = \frac{50s}{s^2+20s+100}$$

$$= \frac{50s}{(s+10)^2} = \frac{50}{s+10} - \frac{500}{(s+10)^2}$$

$$y(t) = (50-500t)\,e^{-10t}u(t)$$

$$y_{\text{SS}}(t) = 0$$

There is no steady-state response.

Problem 11–48. The impulse response of a linear circuit is $h(t) = u(t)$. Use the convolution integral to find the response due to an input $x(t) = u(t)$.

We have the following results:

$$
\begin{aligned}
h(t) &= u(t) \\
x(t) &= u(t) \\
y(t) &= \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t u(t-\tau)u(\tau)d\tau = \left[\int_0^t d\tau\right] u(t) \\
&= tu(t) = r(t)
\end{aligned}
$$

We can verify the result in the Laplace domain.

$$
\begin{aligned}
H(s) &= \frac{1}{s} \\
X(s) &= \frac{1}{s} \\
Y(s) &= H(s)X(s) = \frac{1}{s^2} \\
y(t) &= tu(t) = r(t)
\end{aligned}
$$

Problem 11–49. The impulse response of a linear circuit is $h(t) = [u(t) - u(t-3)]$. Use the convolution integral to find the response due to an input $x(t) = u(t-3)$.

We have the following results:

$$
\begin{aligned}
h(t) &= u(t) - u(t-3) \\
x(t) &= u(t-3) \\
y(t) &= \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t [u(t-\tau) - u(t-\tau-3)]u(\tau-3)d\tau \\
&= \int_0^t u(t-\tau)u(\tau-3)d\tau - \int_0^t u(t-\tau-3)u(\tau-3)d\tau \\
&= \left[\int_3^t d\tau\right] u(t-3) - \left[\int_3^{t-3} d\tau\right] u(t-6) \\
&= (t-3)u(t-3) - (t-6)u(t-6) = r(t-3) - r(t-6)
\end{aligned}
$$

Problem 11–50. The impulse response of a linear circuit is $h(t) = [u(t) - u(t-1)]$. Use the convolution integral to find the response due to an input $x(t) = u(t) - u(t-1)$.

We have the following results:

$$
\begin{aligned}
h(t) &= u(t) - u(t-1) \\
x(t) &= u(t) - u(t-1) \\
y(t) &= \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t [u(t-\tau) - u(t-\tau-1)][u(\tau) - u(\tau-1)]d\tau \\
&= \int_0^t u(t-\tau)u(\tau)d\tau - \int_0^t u(t-\tau)u(\tau-1)d\tau - \int_0^t u(t-\tau-1)u(\tau)d\tau + \int_0^t u(t-\tau-1)u(\tau-1)d\tau \\
&= \left[\int_0^t d\tau\right]u(t) - \left[\int_1^t d\tau\right]u(t-1) - \left[\int_0^{t-1} d\tau\right]u(t-1) + \left[\int_1^{t-1} d\tau\right]u(t-2) \\
&= tu(t) - (t-1)u(t-1) - (t-1)u(t-1) + (t-2)u(t-2) = r(t) - 2r(t-1) + r(t-2)
\end{aligned}
$$

Problem 11–51. The impulse response of a linear circuit is $h(t) = t[u(t) - u(t-1)]$. Use the convolution integral to find the response due to an input $x(t) = u(t-1)$.

We have the following results:

$$
\begin{aligned}
h(t) &= tu(t) - tu(t-1) \\
x(t) &= u(t-1) \\
y(t) &= \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t (t-\tau)[u(t-\tau) - u(t-\tau-1)]u(\tau-1)d\tau \\
&= t\int_0^t [u(t-\tau)u(\tau-1) - u(t-\tau-1)u(\tau-1)]d\tau - \int_0^t \tau[u(t-\tau)u(\tau-1) - u(t-\tau-1)u(\tau-1)]d\tau \\
&= t\left[\int_1^t d\tau\right]u(t-1) - t\left[\int_1^{t-1} d\tau\right]u(t-2) - \left[\int_1^t \tau d\tau\right]u(t-1) + \left[\int_1^{t-1} \tau d\tau\right]u(t-2) \\
&= t(t-1)u(t-1) - t(t-2)u(t-2) - \frac{1}{2}(t^2-1)u(t-1) + \frac{1}{2}[(t-1)^2 - 1]u(t-2) \\
&= \left(t^2 - t - \frac{t^2}{2} + \frac{1}{2}\right)u(t-1) + \left(-t^2 + 2t + \frac{t^2}{2} - t + \frac{1}{2} - \frac{1}{2}\right)u(t-2) \\
&= \frac{1}{2}\left[(t^2 - 2t + 1)u(t-1) - (t^2 - 2t)u(t-2)\right]
\end{aligned}
$$

Problem 11–52. The impulse response of a linear circuit is $h(t) = e^{-t}u(t)$. Use the convolution integral to find the response due to an input $x(t) = u(t)$.

We have the following results:

$$
\begin{aligned}
h(t) &= e^{-t}u(t) \\
x(t) &= u(t) \\
y(t) &= \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t e^{-(t-\tau)}u(t-\tau)u(\tau)d\tau \\
&= e^{-t}\left[\int_0^t e^{\tau}d\tau\right]u(t) \\
&= e^{-t}\left(e^t-1\right)u(t) = \left(1-e^{-t}\right)u(t)
\end{aligned}
$$

Problem 11–53. The impulse response of a linear circuit is $h(t) = 10[u(t) - u(t-1)]$. Use the convolution integral to find the response due to an input $x(t) = e^{-t}u(t)$.

We have the following results:

$$
\begin{aligned}
h(t) &= 10[u(t) - u(t-1)] \\
x(t) &= e^{-t}u(t) \\
y(t) &= \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t 10[u(t-\tau) - u(t-\tau-1)]e^{-\tau}u(\tau)d\tau \\
&= \left[\int_0^t 10e^{-\tau}d\tau\right]u(t) - \left[\int_0^{t-1} 10e^{-\tau}d\tau\right]u(t-1) \\
&= -10(e^{-t}-1)u(t) + 10(e^{-(t-1)}-1)u(t-1) \\
&= 10(1-e^{-t})u(t) + 10(e^{-(t-1)}-1)u(t-1)
\end{aligned}
$$

Problem 11–54. The impulse response of a linear circuit is $h(t) = e^{-t}u(t)$. Use the convolution integral to find the response due to an input $x(t) = tu(t)$.

We have the following results:

$$
\begin{aligned}
h(t) &= e^{-t}u(t) \\
x(t) &= tu(t) \\
y(t) &= \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t e^{-(t-\tau)}u(t-\tau)\tau u(\tau)d\tau \\
&= e^{-t}\left[\int_0^t \tau e^{\tau}d\tau\right]u(t) \\
&= e^{-t}\left[(t-1)e^t+1\right]u(t) \\
&= \left[(t-1)+e^{-t}\right]u(t)
\end{aligned}
$$

Problem 11–55. Show that $f(t) * \delta(t) = f(t)$. That is, show that convolving any waveform $f(t)$ with an impulse leaves the waveform unchanged.

We have the following results:

$$h(t) = f(t)$$

$$x(t) = \delta(t)$$

$$y(t) = \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t f(t-\tau)\delta(\tau)d\tau$$

$$= \int_0^t f(t)\delta(\tau)d\tau = f(t)\int_0^t \delta(\tau)d\tau$$

$$= f(t)(1) = f(t)$$

Problem 11–56. Show that if $h(t) = u(t)$, then output $y(t)$ for any input $x(t)$ is $y(t) = \int_0^t x(\tau)d\tau$. That is, a circuit whose impulse response is a step function operates as an integrator.

We have the following results:

$$h(t) = u(t)$$

$$x(t) = x(t)$$

$$y(t) = \int_0^t h(t-\tau)x(\tau)d\tau = \int_0^t u(t-\tau)x(\tau)d\tau$$

$$= \left[\int_0^t x(\tau)d\tau\right]u(t)$$

The output is the integral of the input multiplied by a step function.

Problem 11–57. Use the convolution integral to show that if the input to a linear circuit is $x(t) = u(t)$ then

$$y(t) = g(t) = \int_0^t h(\tau)d\tau$$

That is, show that the step response is the integral of the impulse response.

We have the following results:

$$h(t) = h(t)$$

$$x(t) = u(t)$$

$$y(t) = \int_0^t x(t-\tau)h(\tau)d\tau = \int_0^t u(t-\tau)h(\tau)d\tau$$

$$= \left[\int_0^t h(\tau)d\tau\right]u(t) = g(t)u(t)$$

Problem 11–58. If the input to a linear circuit is $x(t) = tu(t)$, then the output $y(t)$ is called the ramp response. Use the convolution integral to show that

$$\frac{dy(t)}{dt} = \int_0^t h(\tau)d\tau = g(t)$$

That is, show that the derivative of the ramp response is the step response.

We have the following results:

$$h(t) = h(t)$$

$$x(t) = tu(t)$$

$$y(t) = \int_0^t x(t-\tau)h(\tau)d\tau = \int_0^t (t-\tau)u(t-\tau)h(\tau)d\tau$$

$$= \left[t\int_0^t h(\tau)d\tau\right]u(t) - \left[\int_0^t \tau h(\tau)d\tau\right]u(t)$$

$$\frac{dy(t)}{dt} = \left[th(t) + \int_0^t h(\tau)d\tau - th(t)\right]u(t)$$

$$= \left[\int_0^t h(\tau)d\tau\right]u(t) = g(t)u(t)$$

Problem 11–59. The impulse response of a linear circuit is $h(t) = 2u(t)$. Use MATLAB to compute the convolution integral and find the response due to an input $x(t) = t[u(t) - u(t-1)]$.

The MATLAB code is shown below.

```
syms s t
syms tau positive
ht = 2*heaviside(t);
xt = t*(heaviside(t)-heaviside(t-1));
httau = subs(ht,t,t-tau);
xtau = subs(xt,t,tau);
yt = simplify(int(httau*xtau,tau,0,t))
```

The corresponding results are shown below.

```
yt = piecewise([t < 1, t^2], [1 ≤ t, t^2 - heaviside(t - 1)*(t^2 - 1)])
```

The result is:

$$y(t) = t^2u(t) - (t^2-1)u(t-1)$$

Problem 11–60. The impulse response of a linear circuit is $h(t) = 50e^{-5t}u(t)$ and $x(t) = tu(t)$. Use s-domain convolution to find the zero-state response $y(t)$.

We have the following results:

$$h(t) = 50e^{-5t}u(t)$$

$$x(t) = tu(t)$$

$$H(s) = \frac{50}{s+5}$$

$$X(s) = \frac{1}{s^2}$$

$$Y(s) = H(s)X(s) = \frac{50}{s^2(s+5)} = \frac{10}{s^2} - \frac{2}{s} + \frac{2}{s+5}$$

$$y(t) = \left[10t - 2 + 2e^{-5t}\right]u(t)$$

Problem 11–61. The impulse responses of two linear circuits are $h_1(t) = 2e^{-2t}u(t)$ and $h_2(t) = 5e^{-5t}u(t)$. What is the impulse response of a cascade connection of these two circuits?

We have the following results:

$$h_1(t) = 2e^{-2t}u(t)$$

$$h_2(t) = 5e^{-5t}u(t)$$

$$H_1(s) = \frac{2}{s+2}$$

$$H_2(s) = \frac{5}{s+5}$$

$$H(s) = H_1(s)H_2(s) = \frac{10}{(s+2)(s+5)} = \frac{\frac{10}{3}}{s+2} - \frac{\frac{10}{3}}{s+5}$$

$$h(t) = \frac{10}{3}\left(e^{-2t} - e^{-5t}\right)u(t)$$

Problem 11–62. Design a circuit to realize the transfer function below using only resistors, capacitors, and OP AMPs.

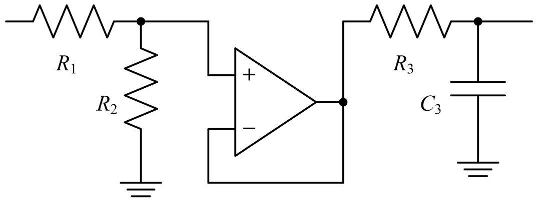
$$T_{\mathrm{V}}(s) = \frac{20000}{s+100000}$$

Scale the circuit so that all capacitors are exactly 1000 pF.

Rewrite the transfer function as follows:

$$T_{\mathrm{V}}(s) = \frac{20000}{s+100000} = \left[\frac{1}{5}\right]\left[\frac{100000}{s+100000}\right] = \left[\frac{1}{5}\right]\left[\frac{\frac{10^5}{s}}{1+\frac{10^5}{s}}\right]$$

We can create the circuit with a voltage divider, a buffer, and a series RC circuit with the output taken across the capacitor. In the RC circuit prototype, we have $R = 1\ \Omega$ and $C = 10\ \mu$F. Use a scale factor of 10^4 to get $R = 10$ kΩ and $C = 1000$ pF. The resulting circuit is shown below, where $R_1 = 4$ kΩ, $R_2 = 1$ kΩ, $R_3 = 10$ kΩ, and $C_3 = 1000$ pF.



Problem 11–63. Design a circuit to realize the transfer function below using only resistors, capacitors, and OP AMPs.

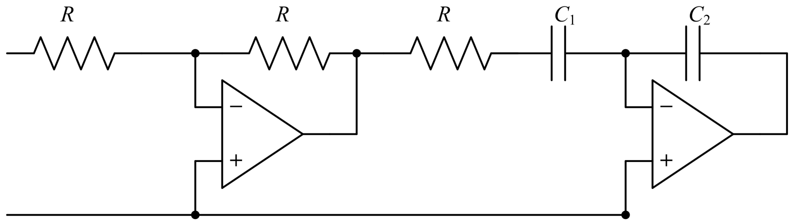
$$T_{\mathrm{V}}(s) = \frac{20000}{s+1000}$$

Scale the circuit so that all resistors are exactly 1 kΩ.

Rewrite the transfer function as follows:

$$T_{\mathrm{V}}(s) = \frac{20000}{s+1000} = [-1]\left[\frac{-\frac{20000}{s}}{1+\frac{1000}{s}}\right]$$

Use a cascade connection of two OP AMP amplifiers. The first stage has a gain of −1 and the second stage implements the remainder of the transfer function with two capacitors and a resistor. The design is shown below with $R = 1$ kΩ, $C_1 = 1\ \mu$F, and $C_2 = 0.05\ \mu$F.



Problem 11–64. Design a circuit to realize the transfer function below using only resistors, inductors, and OP AMPs.

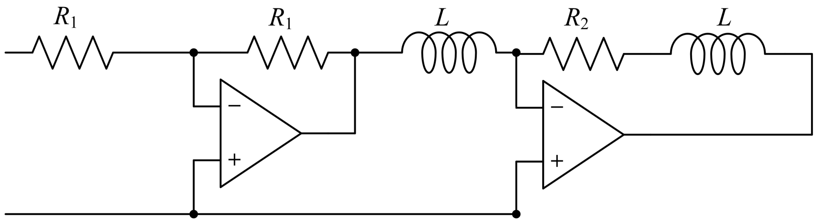
$$T_{\rm V}(s) = \frac{s+5000}{s}$$

Scale the circuit so that all inductors are exactly 100 mH.

Rewrite the transfer function as follows:

$$T_{\rm V}(s) = \frac{s+5000}{s} = [-1]\left[-\frac{0.1s+500}{0.1s}\right]$$

Use a cascade connection of two inverting amplifiers. The first stage has a gain of -1 and the second stage implements the remainder of the transfer function with two inductors and a resistor. The design is shown below with $R_1 = 1$ kΩ, $R_2 = 500$ Ω, and $L = 100$ mH.



Problem 11–65. Design a circuit to realize the transfer function below using only resistors, capacitors, and OP AMPs.

$$T_{\rm V}(s) = \frac{-50000s}{s+2500}$$

Scale the circuit so that all capacitors are exactly 0.1 μF.

Rewrite the transfer as follows:

$$T_{\rm V}(s) = \frac{-50000s}{s+2500} = [-500]\left[\frac{1}{1+\dfrac{2500}{s}}\right][100]$$

Use three stages connected in cascade. The first stage is an inverting amplifier with a gain of -500. The second stage is a series RC circuit with the output taken across the resistor. In the prototype RC circuit, we have $R = 1$ Ω and $C = 1/2500$ F. Scale by a factor of 4000 to get $R = 4$ kΩ and $C = 0.1$ μF. The third stage is a noninverting ammplifier with a gain of 100.

Problem 11–66. Design a circuit to realize the transfer function below using only resistors, capacitors, and OP AMPs.

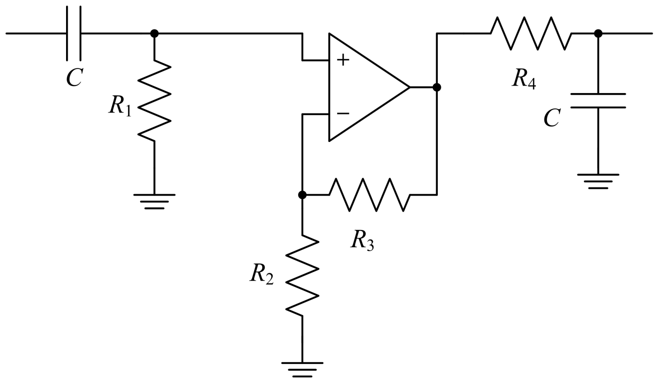
$$T_{\rm V}(s) = \frac{50000s}{(s+50)(s+1000)}$$

Scale the circuit so that all capacitors are exactly 0.1 μF.

Rewrite the transfer function as follows:

$$T_{\rm V}(s) = \frac{50000s}{(s+50)(s+1000)} = \left[\frac{s}{s+50}\right][50]\left[\frac{1000}{s+1000}\right] = \left[\frac{1}{1+\dfrac{50}{s}}\right][50]\left[\frac{\dfrac{1000}{s}}{1+\dfrac{1000}{s}}\right]$$

Use three stages connected in cascade. The first stage is a series RC circuit with the output taken across the resistor. The second stage is a noninverting amplifier with a gain of 50. The third stage is a series RC circuit with the output taken across the capacitor. The design is shown below with $R_1 = 200$ kΩ, $R_2 = 1$ kΩ, $R_3 = 49$ kΩ, $R_4 = 10$ kΩ, and $C = 0.1$ μF.



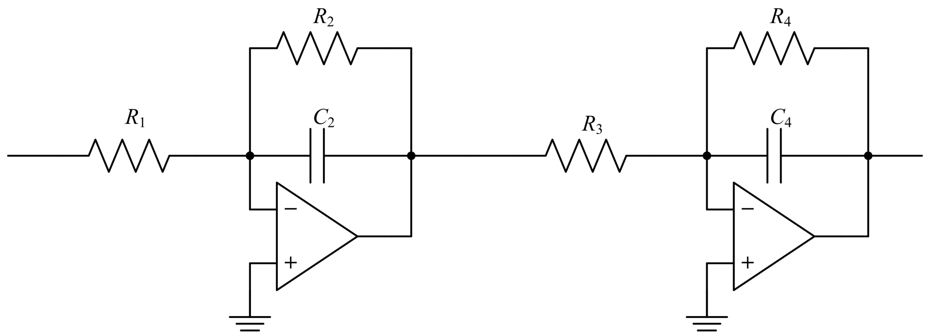
Problem 11–67. Design a circuit to realize the transfer function below using only resistors, capacitors, and OP AMPs. Scale the circuit so that all resistors are greater than 10 kΩ and all capacitors are less the 1 μF.

$$T_{\rm V}(s) = \pm \frac{5 \times 10^8}{(s+100)(s+10000)}$$

Rewrite the transfer function as follows:

$$T_{\rm V}(s) = \pm \frac{5 \times 10^8}{(s+100)(s+10000)} = \left[\frac{50000}{s+100}\right]\left[\frac{10000}{s+10000}\right] = \left[\frac{500}{\dfrac{s}{100}+1}\right]\left[\frac{1}{\dfrac{s}{10000}+1}\right]$$

Use two active low-pass RC OP AMP filters in cascade; the first filter has an input resistor $R_1 = 10$ kΩ, a feedback resistor $R_2 = 5$ MΩ, and a feedback capacitor $C_2 = 0.002$ μF; the second filter has an input resistor $R_3 = 10$ kΩ, a feedback resistor $R_4 = 10$ kΩ, and a feedback capacitor $C_4 = 0.01$ μF. The design is shown below.



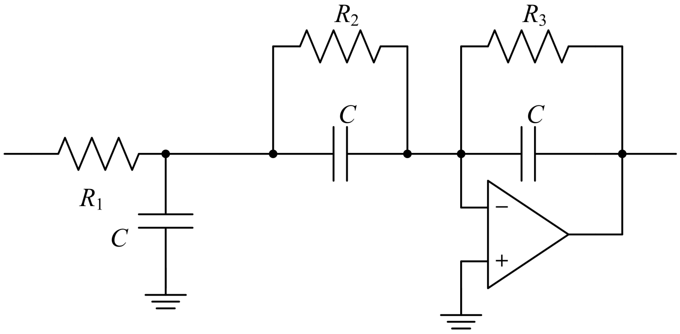
Problem 11–68. Design a circuit to realize the transfer function below using only resistors, capacitors and not more than one OP AMP. Scale the circuit so that all capacitors are exactly 0.01 μF.

$$T_{\rm V}(s) = \pm \frac{100(s+1000)}{(s+100)(s+10000)}$$

Rewrite the transfer function as follows:

$$T_{\rm V}(s) = \pm \frac{100(s+1000)}{(s+100)(s+10000)} = \left[\frac{100}{s+100}\right]\left[\frac{s+1000}{s+10000}\right] = \left[\frac{\dfrac{100}{s}}{1+\dfrac{100}{s}}\right]\left[\frac{s+1000}{s+10000}\right]$$

Use two stages in cascade. The first stage is a series RC circuit with the output taken across the capacitor. The second stage uses the approach presented in Figure 11–38(b) in the textbook. The design is shown below with $R_1 = 1$ MΩ, $R_2 = 100$ kΩ, $R_3 = 10$ kΩ, and $C = 0.01$ μF.



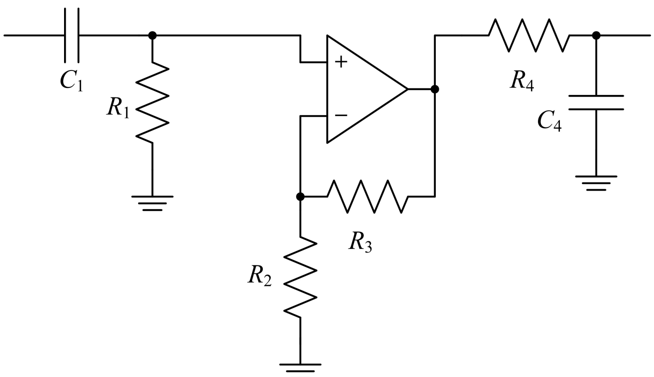
Problem 11–69. Design a circuit to realize the transfer function below using only resistors, capacitors and not more than one OP AMP. Scale the circuit so that the final design uses only 20-kΩ resistors.

$$T_{\rm V}(s) = \pm\frac{20000s}{(s+1000)(s+5000)}$$

Rewrite the transfer function as follows:

$$T_{\rm V}(s) = \frac{20000s}{(s+1000)(s+5000)} = \left[\frac{s}{s+1000}\right][4]\left[\frac{5000}{s+5000}\right] = \left[\frac{1}{1+\dfrac{1000}{s}}\right][4]\left[\frac{\dfrac{5000}{s}}{1+\dfrac{5000}{s}}\right]$$

Use three stages in cascade. The first stage is a series RC circuit with the output taken across the resistor. The second stage is a noninverting amplifier with a gain of four. The third stage is a series RC circuit with the output taken across the capacitor. The design is shown below with $R_1 = 20$ kΩ, $R_2 = 20$ kΩ, $R_3 = 60$ kΩ or three 20-kΩ resistors in series, $R_4 = 20$ kΩ, $C_1 = 0.05$ μF, and $C_4 = 0.01$ μF.



Problem 11–70. Design a passive circuit to realize the transfer function below using only resistors, capacitors, and inductors. Scale the circuit so that all inductors are 50 mH or less.

$$T_{\rm V}(s) = \frac{s^2}{(s+2000)^2}$$

Rewrite the transfer function as follows:

$$T_{\rm V}(s) = \frac{s^2}{(s+2000)^2} = \frac{s^2}{s^2+4000s+4000000} = \frac{s}{s+4000+\dfrac{4000000}{s}} = \frac{0.05s}{0.05s+200+\dfrac{200000}{s}}$$

Use a series RLC circuit with the output taken across the inductor and $R = 200$ Ω, $L = 50$ mH, and $C = 5$ μF.

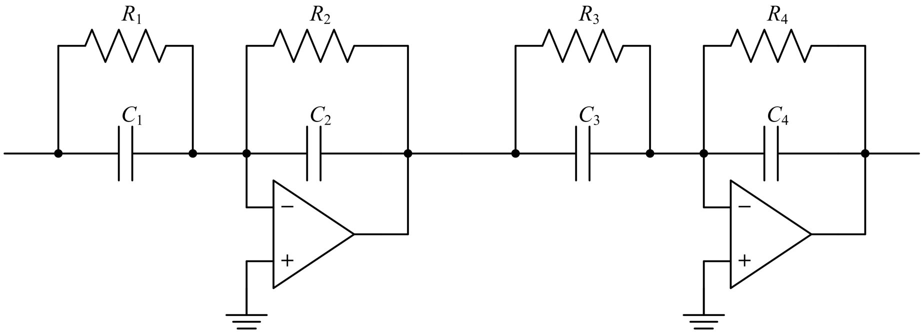
Problem 11–71. A circuit is needed to realize the transfer function listed below.

$$T_V(s) = \pm\frac{(s+125)(s+500)}{(s+250)(s+1000)}$$

In all cases, scale the circuit so that all parts use practical values.

(a). Design the circuit using two OP AMPs.

Use two stages following the design in Figure 11–38(b) in the textbook. The design is shown below with $R_1 = 8\ \text{k}\Omega$, $R_2 = 4\ \text{k}\Omega$, $R_3 = 2\ \text{k}\Omega$, $R_4 = 1\ \text{k}\Omega$, and $C_1 = C_2 = C_3 = C_4 = 1\ \mu\text{F}$.

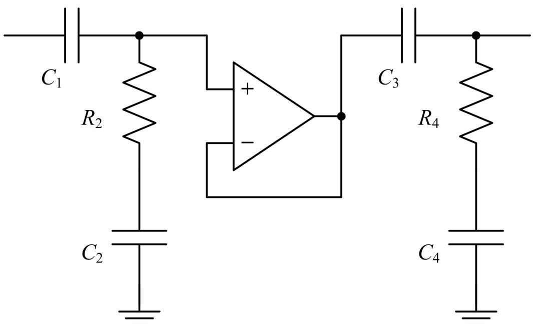


(b). Design the circuit using only one OP AMP.

Rewrite the transfer function as follows:

$$T_V(s) = \left[\frac{1+\dfrac{125}{s}}{1+\dfrac{125}{s}+\dfrac{125}{s}}\right]\left[\frac{1+\dfrac{500}{s}}{1+\dfrac{500}{s}+\dfrac{500}{s}}\right]$$

Use two RC circuits separated by a buffer. The design is shown below with $R_2 = 8\ \text{k}\Omega$, $R_4 = 2\ \text{k}\Omega$, and $C_1 = C_2 = C_3 = C_4 = 1\ \mu\text{F}$.

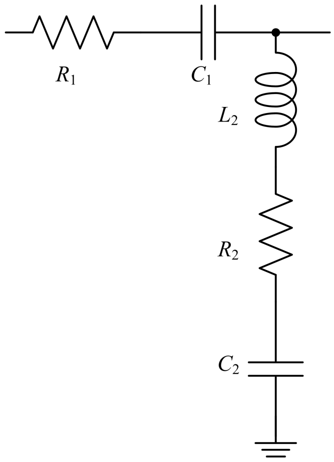


(c). Design the circuit using no OP AMPs.

Rewrite the transfer function as follows

$$T_V(s) = \frac{s^2+625s+62500}{s^2+1250s+250000} = \frac{10s+6250+\dfrac{625000}{s}}{10s+12500+\dfrac{2500000}{s}}$$

Use the following RLC circuit with $R_1 = R_2 = 6.25\ \text{k}\Omega$, $C_1 = 0.533\ \mu\text{F}$, $C_2 = 1.6\ \mu\text{F}$, and $L = 10$ H.



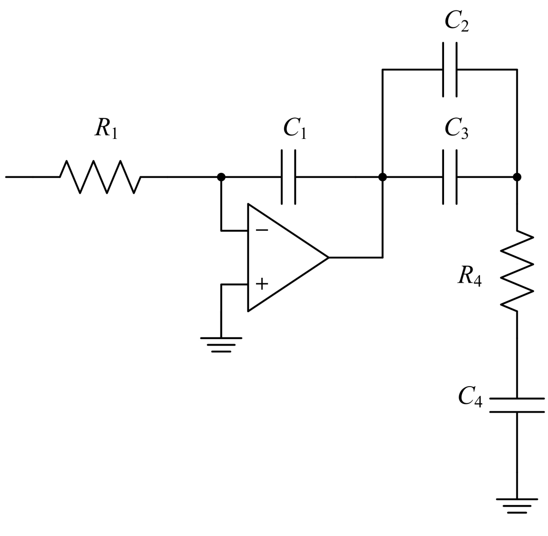
Problem 11–72. Design a circuit to realize the transfer function below using only resistors, capacitors, and OP AMPs. Use only values from the inside rear cover. Your design must be within ±10% of the desired response.

$$T_V(s) = -\frac{500(s+100)}{s(s+10000)}$$

Rewrite the transfer function as follows:

$$T_V(s) = -\frac{500(s+100)}{s(s+10000)} = \left[\frac{-500}{s}\right]\left[\frac{s+100}{s+1000}\right] = \left[\frac{-500}{s}\right]\left[\frac{1+\dfrac{100}{s}}{1+\dfrac{100}{s}+\dfrac{900}{s}}\right]$$

Use two stages in cascade. The first stage is an integrator with a gain of −500 by choosing $R_1 = 2$ kΩ and $C_1 = 1$ μF. The second stage is an RC circuit with the values $C_2 = 0.01$ μF, $C_3 = 0.1$ μF, $C_4 = 1$ μF, and $R_4 = 10$ kΩ. The design is shown below.



Problem 11–73. A circuit is needed to realize the impulse response transform listed below. Scale the circuit so that all parts use practical values.

$$H(s) = \pm\frac{200s+10^6}{s^2+200s+10^6}$$

Rewrite the transfer function as follows:

$$H(s) = \frac{200 + \dfrac{10^6}{s}}{s + 200 + \dfrac{10^6}{s}}$$

Use a series RLC circuit with the ouput taken across the series combination of the resistor and capacitor. Use values of $R = 200\ \Omega$, $L = 1$ H, and $C = 1\ \mu$F.

Problem 11–74. It is claimed that both circuits in Figure P11–74 realize the transfer function

$$T_V(s) = K\left(\frac{s+2000}{s+1000}\right)$$

(a). Verify that both circuits realize the specified $T_V(s)$.

For the first circuit, we have the following results:

$$T_V(s) = \frac{Z_1 + Z_2}{Z_1} = \frac{R + \dfrac{R/Cs}{R + 1/Cs}}{R}$$

$$= 1 + \frac{1/Cs}{R + 1/Cs} = 1 + \frac{\dfrac{1}{RC}}{s + \dfrac{1}{RC}}$$

$$= \frac{s + \dfrac{2}{RC}}{s + \dfrac{1}{RC}}$$

The first circuit realizes the desired transfer function with $K = 1$, since $1/RC = 1000$. For the second circuit, we have the following results:

$$T_V(s) = \frac{R + 1/Cs}{2R + 1/Cs} = \frac{\dfrac{1}{2}s + \dfrac{1}{2RC}}{s + \dfrac{1}{2RC}}$$

$$= \frac{1}{2}\,\frac{s + \dfrac{1}{RC}}{s + \dfrac{1}{2RC}}$$

The second circuit realizes the desired transfer function with $K = 1/2$, since $1/2RC = 1000$.

(b). Which circuit would you choose if the output must drive a 1-kΩ load?

To drive a 1-kΩ load, use the first circuit, because the output is the output of an OP AMP and there will be no loading issues. There will be loading issues with the second circuit.

(c). Which circuit would you choose if the input comes from a 50-Ω source?

If the input comes from a 50-Ω source, either circuit will be acceptable. Since the first circuit is a noninverting amplifier, the impedance of the source does not matter. The second circuit is a simpler design and its input impedance is high enough (10 kΩ) that the 50-Ω source will not matter.

(d). It is further claimed that connecting the two circuits in cascade produces an overall transfer function of $[T_V(s)]^2$ no matter which circuit is the first stage and which is the second stage. Do you agree or disagree? Explain.

The claim is true because the first circuit is a noninverting amplifier. Either way that the circuits are connected does not cause a loading issue between the two circuits, so the transfer function is the product of the individual transfer functions.

Problem 11–75. It is claimed that both circuits in Figure P11–75 realize the transfer function

$$T_{\text{V}}(s) = \frac{\pm 1000s}{(s+1000)(s+4000)}$$

(a). Verify that both circuits realize the specified $T_{\text{V}}(s)$.

For the first circuit, we have the following results:

$$T_{\text{V}}(s) = \frac{-\dfrac{R/C_2s}{R+1/C_2s}}{R+1/C_1s} = \frac{-\dfrac{R}{RC_2s+1}}{\dfrac{RC_1s+1}{C_1s}}$$

$$= \frac{-RC_1s}{(RC_1s+1)(RC_2s+1)} = \frac{-\dfrac{1}{RC_2}s}{\left(s+\dfrac{1}{RC_1}\right)\left(s+\dfrac{1}{RC_2}\right)}$$

The first circuit realizes the desired transfer function, since $1/RC_1 = 4000$ and $1/RC_2 = 1000$. For the second circuit, we have the following results:

$$T_{\text{V}}(s) = \frac{Z_2}{Z_1+Z_2} = \frac{\dfrac{R_2}{R_2C_2s+1}}{\dfrac{R_1C_1s+1}{C_1s}+\dfrac{R_2}{R_2C_2s+1}}$$

$$= \frac{R_2C_1s}{R_1R_2C_1C_2s^2+(R_1C_1+R_2C_2)s+1+R_2C_1s}$$

$$= \frac{\dfrac{1}{R_1C_2}s}{s^2+\dfrac{R_1C_1+R_2C_1+R_2C_2}{R_1R_2C_1C_2}s+\dfrac{1}{R_1R_2C_1C_2}}$$

$$= \frac{1000s}{s^2+5000s+4000000}$$

The second circuit realizes the desired transfer function.

(b). Which circuit would you choose if the output must drive a 1-kΩ load?

To drive a 1-kΩ load, use the first circuit, because the output is the output of an OP AMP and there will be no loading issues. There will be loading issues with the second circuit.

(c). Which circuit would you choose if the input comes from a 50-Ω source?

Either circuit will be acceptable if the source has an output resistance of 50 Ω. Both circuits have input impedances that are significantly higher than 50 Ω.

(d). It is further claimed that connecting the two circuits in cascade produces an overall transfer function of $[T_{\text{V}}(s)]^2$ no matter which circuit is the first stage and which is the second stage. Do you agree or disagree? Explain.

The claim is false. If the second circuit is connected in front of the first circuit, there will be loading between the circuits and the resulting transfer function will not be the product of the individual transfer functions.

Problem 11–76. Design a circuit that produces the following step response.

$$g(t) = 24[1 - e^{-50t} - 50te^{-50t}]u(t)$$

We have the following relationships to determine the transfer function:

$$g(t) = 24[1 - e^{-50t} - 50te^{-50t}]u(t)$$

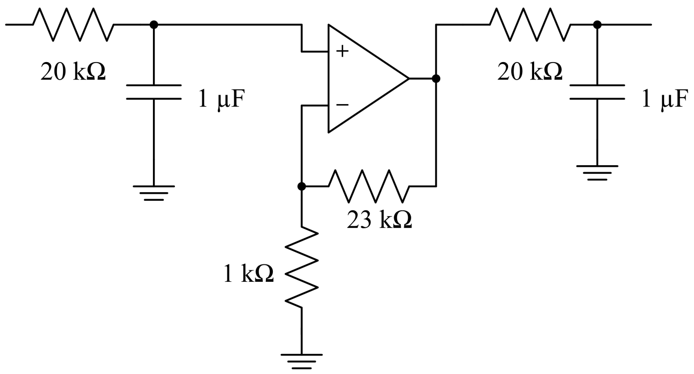
$$G(s) = 24\left[\frac{1}{s} - \frac{1}{s+50} - \frac{50}{(s+50)^2}\right] = 24\left[\frac{(s+50)^2 - (s^2+50s) - 50s}{s(s+50)^2}\right]$$

$$= 24\left[\frac{s^2+100s+2500-s^2-50s-50s}{s(s+50)^2}\right] = 24\left[\frac{2500}{s(s+50)^2}\right]$$

$$H(s) = sG(s) = 24\left[\frac{2500}{(s+50)^2}\right] = \left(\frac{50}{s+50}\right)(24)\left(\frac{50}{s+50}\right)$$

$$= \left(\frac{\frac{50}{s}}{1+\frac{50}{s}}\right)(24)\left(\frac{\frac{50}{s}}{1+\frac{50}{s}}\right) = \left(\frac{\frac{10^6}{s}}{20000+\frac{10^6}{s}}\right)(24)\left(\frac{\frac{10^6}{s}}{20000+\frac{10^6}{s}}\right)$$

The circuit is shown below.



Problem 11–77. A circuit is needed that will take an input of $v_1(t) = 25e^{-10t}u(t)$ mV and produce an output of $v_2(t) = 500e^{-200t}u(t)$ mV. Design such a circuit using practical parts values. Validate your design by using OrCAD.

We have the following results:

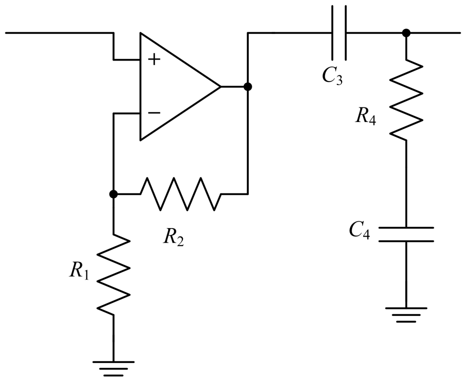
$$V_1(s) = \frac{25}{s+10}$$

$$V_2(s) = \frac{500}{s+200}$$

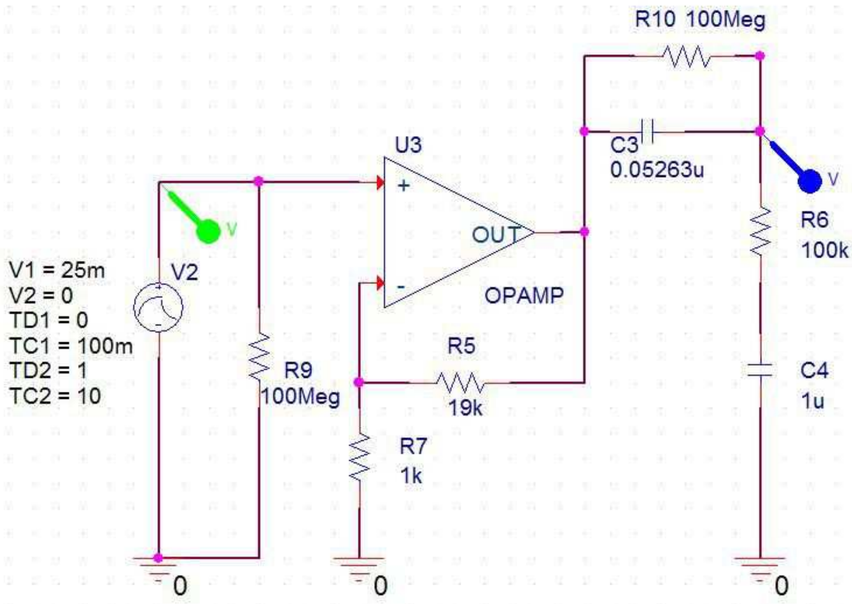
$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{20(s+10)}{s+200} = (20)\left(\frac{1+\frac{10}{s}}{1+\frac{10}{s}+\frac{190}{s}}\right)$$

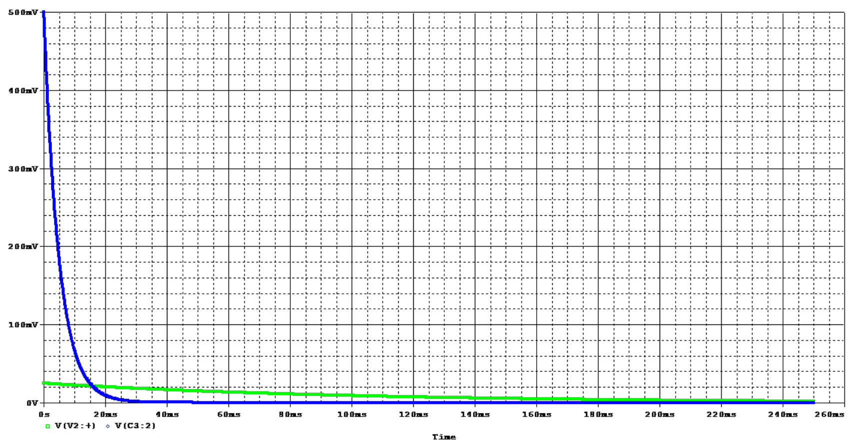
$$= (20)\left(\frac{10^5+\frac{10^6}{s}}{10^5+\frac{10^6}{s}+\frac{1.9\times10^7}{s}}\right)$$

The circuit is shown below with $R_1 = 1$ kΩ, $R_2 = 19$ kΩ, $R_4 = 100$ kΩ, $C_3 = 0.05263$ μF, $C_4 = 1$ μF.



The OrCAD simulation and the resulting output are shown below. In the simulation, the initial conditions on the capacitors are both zero. Note that the simulation requires two 100-MΩ resistors to avoid floating nodes. These resistors do not alter the results substantially.





Problem 11–78. A circuit is needed that will take an input of $v_1(t) = [1 - e^{-10000t}]u(t)$ V and produce a constant −5 V output. Design such a circuit using practical parts values. Validate your design using OrCAD.

We have the following results:

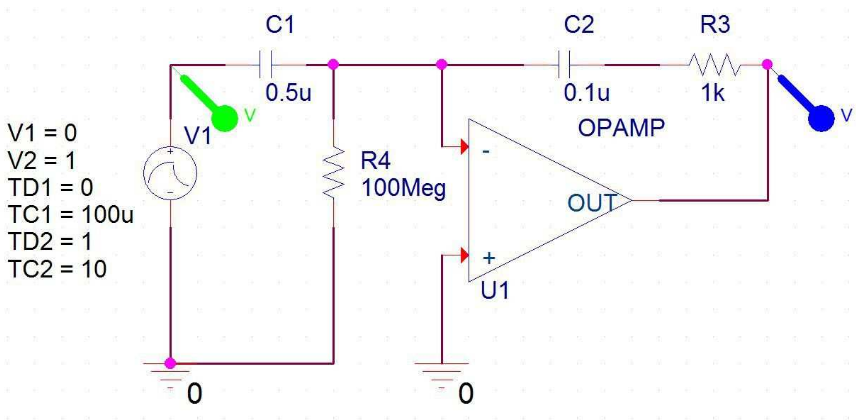
$$V_1(s) = \frac{1}{s} - \frac{1}{s+10000} = \frac{10000}{s(s+10000)}$$

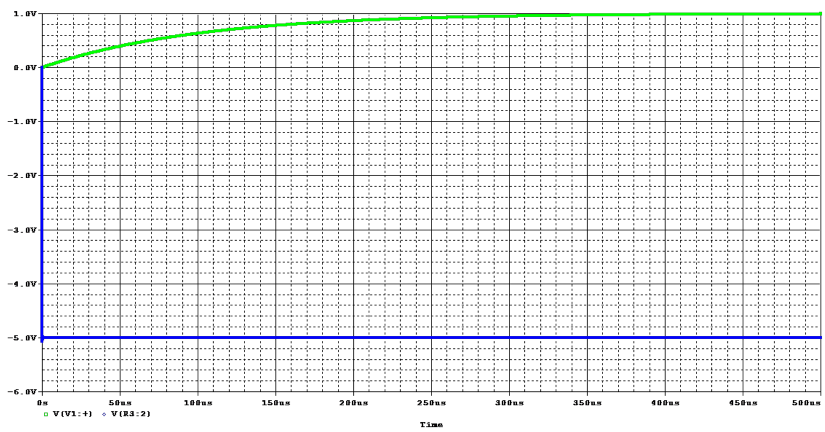
$$V_2(s) = -\frac{5}{s}$$

$$T(s) = \frac{V_2(s)}{V_1(s)} = \left[-\frac{5}{s}\right]\left[\frac{s(s+10000)}{10000}\right]$$

$$= -\frac{s+10000}{2000} = -\frac{1+\dfrac{10000}{s}}{\dfrac{2000}{s}} = -\frac{1000+\dfrac{10^7}{s}}{\dfrac{2000000}{s}}$$

The OrCAD simulation and the resulting output are shown below. In the simulation, the initial conditions on the capacitors are both zero. Note that the simulation requires one 100-MΩ resistor to avoid floating nodes. This resistor does not alter the results substantially.

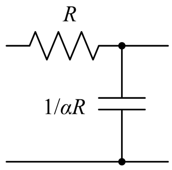
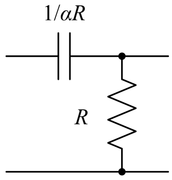
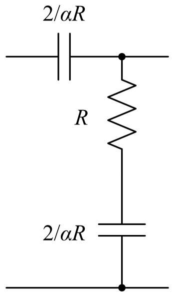




Problem 11–79. (A,D) First-Order Circuit Impulse and Step Responses

Each row in the table shown in Figure P11–79 refers to a first-order circuit with an impulse response $h(t)$ and a step response $g(t)$. Fill in the missing entries in the table.

The completed table is shown below and the required calculation follow below the table.

Circuit	$h(t)$	$g(t)$
	$\alpha e^{-\alpha t}u(t)$	$(1 - e^{-\alpha t})u(t)$
	$\delta(t) - (\alpha e^{-\alpha t})u(t)$	$e^{-\alpha t}u(t)$
	$\delta(t) - \dfrac{\alpha}{2}e^{-\alpha t}u(t)$	$\dfrac{1}{2}\left(1 + e^{-\alpha t}\right)u(t)$

For the first row in the table, we have the following:

$$H(s) = \frac{\alpha R/s}{R + \alpha R/s} = \frac{\alpha}{s+\alpha}$$

$$h(t) = \alpha e^{-\alpha t}u(t)$$

$$G(s) = \frac{H(s)}{s} = \frac{\alpha}{s(s+\alpha)} = \frac{1}{s} - \frac{1}{s+\alpha}$$

$$g(t) = (1 - e^{-\alpha t})u(t)$$

For the second row in the table, we have the following:

$$h(t) = \delta(t) - (\alpha e^{-\alpha t})u(t)$$

$$H(s) = 1 - \frac{\alpha}{s+\alpha} = \frac{s}{s+\alpha} = \frac{1}{1+\alpha/s} = \frac{R}{R+\alpha R/s}$$

$$G(s) = \frac{H(s)}{s} = \frac{1}{s+\alpha}$$

$$g(t) = e^{-\alpha t}u(t)$$

For the third row in the table, we have the following:

$$g(t) = \frac{1}{2}\left(1 + e^{-\alpha t}\right) u(t)$$

$$G(s) = \frac{0.5}{s} + \frac{0.5}{s+\alpha} = \frac{s+\alpha/2}{s(s+\alpha)}$$

$$H(s) = sG(s) = \frac{s+\alpha/2}{s+\alpha} = 1 - \frac{\alpha/2}{s+\alpha}$$

$$h(t) = \delta(t) - \frac{\alpha}{2}e^{-\alpha t}u(t)$$

$$H(s) = \frac{R+\alpha R/2s}{R+\alpha R/2s + \alpha R/2s}$$

Problem 11–80. (A) OP AMP Modules and Loading

Figure P11–80 shows an interconnection of three basic OP AMP modules.

(a). Does this interconnection involve loading?

The interconnection does not involve loading because the outputs of the first and second stages are the outputs of OP AMPs, so they do not load the third stage, and vice versa.

(b). Find the overall transfer function of the interconnection and locate its poles and zeros.

The third stage acts as an inverting summer with gains of one along each path. Find the transfer functions for the first and second stages and add them together.

$$T_1(s) = \frac{-R}{R+1/C_1 s} = \frac{-s}{s+\dfrac{1}{RC_1}} = \frac{-s}{s+100000}$$

$$T_2(s) = \frac{-\dfrac{R/C_2 s}{R+1/C_2 s}}{R} = \frac{-\dfrac{1}{RC_2}}{s+\dfrac{1}{RC_2}} = \frac{-1000}{s+1000}$$

$$T(s) = -T_1(s) - T_2(s) = \frac{s}{s+100000} + \frac{1000}{s+1000} = \frac{s^2+2000s+10^8}{(s+1000)(s+100000)}$$

The zeros are located at $s = -1000 \pm j9950$ and the poles are located at $s = -1000$ and $s = -100000$.

(c). Find the steady-state output $v_2(t)$ when the input is $v_1(t) = \cos(500t)$ V. Repeat for $v_1(t) = \cos(10000t)$ V and again for $v_1(t) = \cos(200000t)$ V.

For each input frequency, find the magnitude and phase of the transfer function. We have the following results:

$$T(j\omega) = \frac{10^8 - \omega^2 + j2000\omega}{(1000 + j\omega)(100000 + j\omega)}$$

$$T(j500) = 0.892\angle{-26.3^\circ}$$

$$T(j10000) = 0.020\angle 0^\circ$$

$$T(j200000) = 0.892\angle 26.3^\circ$$

$$v_1(t) = 0.892\cos(500t - 26.3^\circ) \text{ V}$$

$$v_2(t) = 0.020\cos(10000t) \text{ V}$$

$$v_3(t) = 0.892\cos(200000t + 26.3^\circ) \text{ V}$$

(d). Can you think of a use for this circuit?

The filter passes low frequencies and high frequencies, but blocks a center frequency. It could be used to attenuate an interferrence signal.

Problem 11–81. (A) OP AMP Modules and Stability

Figure P11–81 shows an interconnection of three basic circuit modules. Does this interconnection involve loading? Find the overall transfer function of the interconnection and locate its poles and zeros. Is the circuit stable?

The circuit does not experience loading because the OP AMPs effectively isolate the RC circuit in the second stage. Let $V_{\text{A}}(s)$ be the output of the first stage and $V_{\text{B}}(s)$ be the output of the second stage. We have the following results:

$$V_{\text{A}}(s) = -\frac{V_2(s)}{RCs} - \frac{V_1(s)}{RCs} = -\frac{1}{RCs}[V_1(s) + V_2(s)]$$

$$V_{\text{B}}(s) = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} V_{\text{A}}(s) = \frac{V_{\text{A}}(s)}{RCs + 1} = -\frac{V_1(s) + V_2(s)}{RCs(RCs + 1)}$$

$$V_2(s) = \frac{R + R}{R} V_{\text{B}}(s) = 2V_{\text{B}}(s) = -\frac{2(V_1(s) + V_2(s))}{RCs(RCs + 1)}$$

$$V_2(s)[R^2C^2s^2 + RCs] = -2V_1(s) - 2V_2(s)$$

$$V_2(s)[R^2C^2s^2 + RCs + 2] = -2V_1(s)$$

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{-2}{R^2C^2s^2 + RCs + 2} = \frac{-\frac{2}{R^2C^2}}{s^2 + \frac{1}{RC}s + \frac{2}{R^2C^2}}$$

The two zeros are both located at infinity. The poles are located at:

$$s = \frac{-RC \pm \sqrt{R^2C^2 - 8R^2C^2}}{2R^2C^2} = \frac{-1 \pm \sqrt{1 - 8}}{2RC} = \frac{-1 \pm j\sqrt{7}}{2RC}$$

The poles have a negative real part, so the circuit is stable.

Problem 11–82. (A) Step Response and Fan-Out

The fan-out of a digital device is defined as the maximum number of inputs to similar devices that can be reliably driven by the device output. Figure P11–82 is a simplified diagram of a device's output driving n identical capacitive inputs. To operate reliably, a 5-V step function at the device output must drive the capacitive inputs to 3.7 V in 10 ns or less. Determine the device fan-out for R = 1 kΩ and C = 3 pF.

The capacitors are in parallel, so their capacitances add to get the equivalent capacitance. Compute the transfer function and the step response.

$$T(s) = H(s) = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

$$G(s) = \frac{H(s)}{s} = \frac{\frac{1}{RC}}{s\left(s + \frac{1}{RC}\right)} = \frac{1}{s} - \frac{1}{s + \frac{1}{RC}}$$

$$g(t) = (1 - e^{-t/RC})u(t)$$

To meet the specifications, we have the following requirement:

$$5(1 - e^{-10\,\mathrm{n}/RC}) = 3.7$$

$$e^{-10\,\mathrm{n}/RC} = 0.260$$

$$RC = \frac{10\,\mathrm{n}}{1.34707}$$

$$C = \frac{10\,\mathrm{n}}{(1.34707)(1000)} = 7.4235\ \mathrm{pF}$$

The fan-out level with 3-pF capacitors is two in this case. If the driver connects to more than two loads, the capacitance will be above 7.4235 pF, the RC time constant will increase, and the 5-V step function will not rise to 3.7 V in less than 10 ns.

Problem 11–83. (D) Designing to Specifications

A particular circuit needs to be designed that has the following transfer function requirements: Poles at $s = -100$ and $s = -10000$; zeros at $s = 0$ and $s = -1000$; and a gain of 10 as $s \to \infty$. Find the circuit's transfer function and use MATLAB to plot its step response. Then design a circuit that will meet that requirement. Finally, use OrCAD to validate that your circuit has the same step response as found using MATLAB.

Given the locations of the poles and zeros and the high-frequency gain, we can construct the transfer function as follows:

$$T(s) = \frac{10(s)(s+1000)}{(s+100)(s+10000)} = \left(\frac{s}{s+100}\right)(10)\left(\frac{s+1000}{s+10000}\right)$$

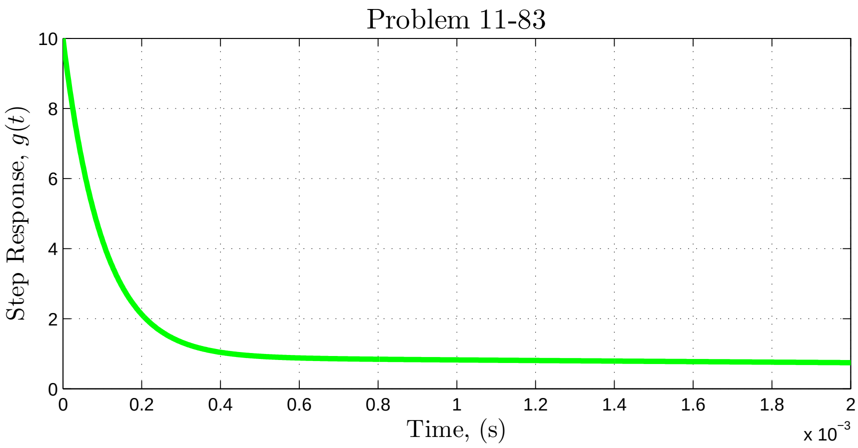
$$= \left(\frac{1}{1+\dfrac{100}{s}}\right)(10)\left(\frac{1+\dfrac{1000}{s}}{1+\dfrac{1000}{s}+\dfrac{9000}{s}}\right)$$

$$= \left(\frac{10000}{10000+\dfrac{1000000}{s}}\right)(10)\left(\frac{10000+\dfrac{10000000}{s}}{10000+\dfrac{10000000}{s}+\dfrac{90000000}{s}}\right)$$

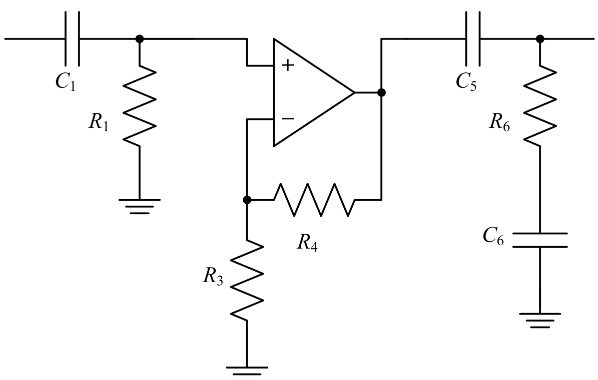
$$G(s) = \frac{T(s)}{s} = \frac{10(s+1000)}{(s+100)(s+10000)} = \frac{\dfrac{10}{11}}{s+100} + \frac{\dfrac{100}{11}}{s+10000}$$

$$g(t) = \frac{10}{11}\left(e^{-100t} + 10e^{-10000t}\right)u(t)$$

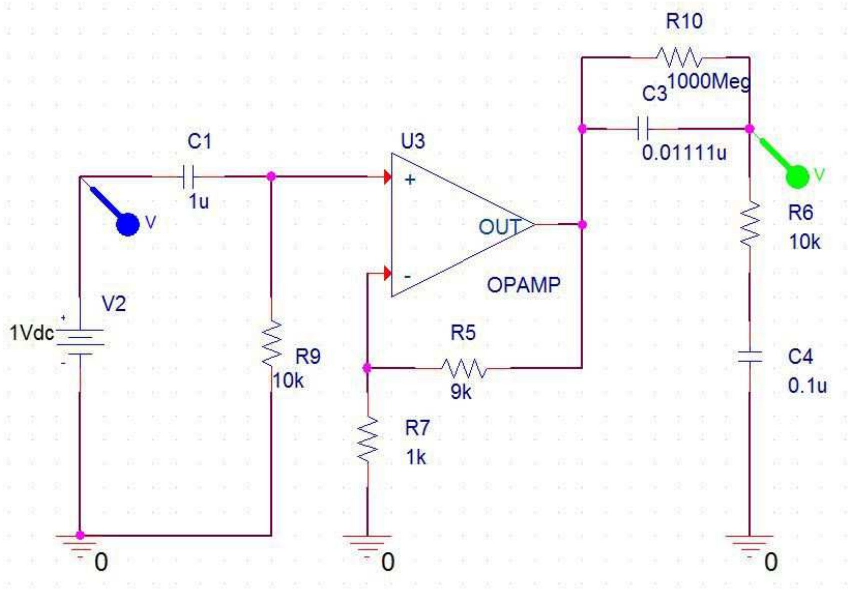
The MATLAB plot of the step response is shown below.

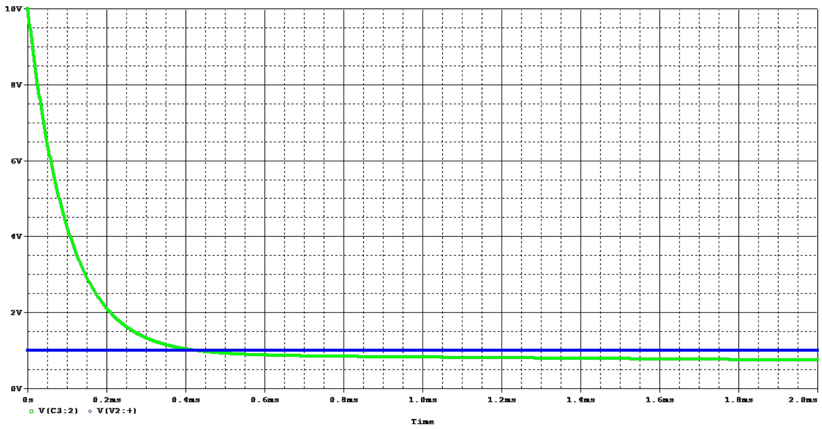


The circuit design is shown below with $R_1 = 10$ kΩ, $R_3 = 1$ kΩ, $R_4 = 9$ kΩ, $R_6 = 10$ kΩ, $C_1 = 1$ μF, $C_5 = 0.01111$ μF, $C_6 = 0.1$ μF:



The OrCAD simulation and results are shown below. In the simulation, the initial conditions on the capacitors are all zero. Note that the simulation requires an additional large resistor to prevent a floating node.





Problem 11–84. (E) Comparison of Sinusoidal Steady-State Analysis versus Phasor Analysis

A circuit designer often is faced with deciding which analysis technique to use when attempting to solve a circuit problem. In this problem we will look at the circuit in Figure P11–84 and choose which technique is the better one to use for different analysis scenarios. Explain why you selected the technique you did.

(a). You need to calculate the circuit's transfer function $T_V(s) = V_2(s)/V_1(s)$.

Use node-voltage analysis in the Laplace domain to solve for $V_2(s)/V_1(s)$. The result is

$$T_V(s) = \frac{10^9}{s^3 + 4000s^2 + 5000000s + 10^9}$$

(b). The input is given as $v_1(t) = 5\cos(1000t)$ V and you need to find $v_{2SS}(t)$.

Use the transfer function found in part (a), substitute in $s = j1000$, and find the magnitude and phase of the result. We can then find $v_{2SS}(t) = \cos(1000t - 126.9°)$ V.

(c). The input is given as $v_1(t) = 5\cos(1000t)$ V and you need to find $i_{XSS}(t)$.

Use mesh-current analysis in the phasor domain to find the mesh currents and then we have

$$i_{XSS}(t) = 1.4142\cos(1000t + 8.13°) \text{ mA}$$

(d). The input is given as $v_1(t) = V_A \cos(\omega t)$ V and you need to find $v_{2SS}(t)$.

Solve for $T_V(s)$ in the Laplace domain, substitute in $s = j\omega$, and compute the steady-state output:

$$v_{2SS}(t) = |T(j\omega)|\, V_A \cos\left[\omega t + \angle T(j\omega)\right]$$

(e). The input is given as $v_1(t) = 170\cos(377t)$ V and you need to find all of the voltages and currents in the circuit.

Use mesh-current analysis in the phasor domain, find the mesh-currents at the given frequency, and then compute the other currents and voltages.

(f). You need to find the poles and zeros of the circuit.

Solve for the transfer function in the Laplace domain, since we can directly solve for the poles and zeros.

(g). You need to find if the current leads or lags the voltage across the two resistors when the input is $5\cos(1000t)$ V.

Use the phasor domain, since we have a specific frequency and we can compute the magnitude and phase of each signal.

(h). You need to determine what type of filtering the circuit performs.

Solve for the transfer function in the Laplace domain, substitute $s = j\omega$ and determine the gain of the transfer function for different frequencies.

(i). You need to select a load for maximum power when the input is $v_1(t) = 170\cos(377t)$ V.

Use the phasor domain, find the lookback impedance, and design the load to be the complex conjugate of the lookback impedance at the specified frequency.

12 Frequency Response

12.1 Exercise Solutions

Exercise 12–1. A transfer function has a passband gain of 25. At a particular frequency in its stopband, the gain of the transfer function is only 0.0006. By how many decibels does the gain of the passband exceed that of the frequency in the stopband?

We have the following computation:

$$K = 20\log_{10}\left(\frac{25}{0.0006}\right) = (20)(4.61979) = 92.3958 \text{ dB}$$

Exercise 12–2. A particular filter is said to be 83 dB down at a desired stop frequency. How many times reduced is a signal at that frequency compared to a signal in the filter's passband?

We have the following calculations:

$$83 \text{ dB} = 20\log_{10}(T)$$

$$T = 10^{83/20} = 14125$$

The reduction factor is 14125 times.

Exercise 12–3. Select values of R and L for the circuit of Figure 12–4 so that the cutoff frequency occurs at 10 kHz.

The cutoff frequency is $\omega_\text{C} = R/L$. Select a value for R and solve for L. We have the following results:

$$f_\text{C} = 10000 \text{ Hz}$$

$$\omega_\text{C} = 2\pi f_\text{C} = 20000\pi \text{ rad/s}$$

$$R = 100 \text{ k}\Omega$$

$$\omega_\text{C} = \frac{R}{L}$$

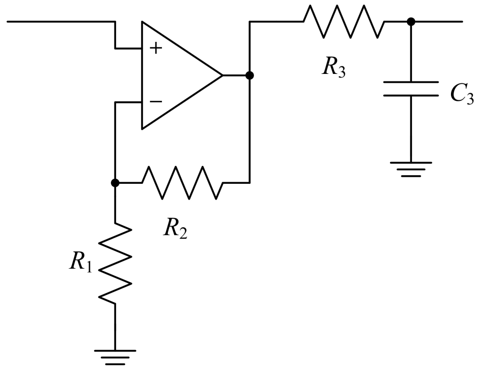
$$L = \frac{R}{\omega_\text{C}} = \frac{100000}{20000\pi} = 1.5915 \text{ H}$$

Exercise 12–4. Design an RC low-pass filter with a cutoff of 100 rad/s and a passband gain of +4.

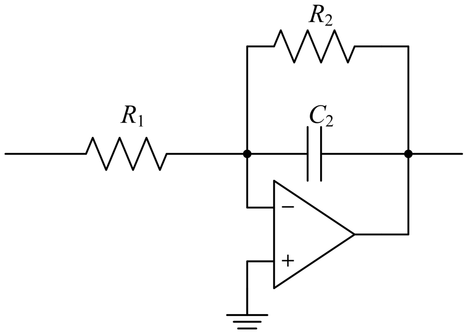
We have the following transfer function:

$$T(s) = (4)\left(\frac{100}{s+100}\right) = (4)\left(\frac{\frac{1}{RC}}{s+\frac{1}{RC}}\right) = (4)\left(\frac{\frac{1}{Cs}}{R+\frac{1}{Cs}}\right)$$

We need a noninverting amplifier with a gain of +4 and a series RC circuit with the output taken across the capacitor and $1/RC = 100$. One possible design is shown below with $R_1 = 1$ kΩ, $R_2 = 3$ kΩ, $R_3 = 1$ kΩ, and $C_3 = 10$ μF.



Exercise 12–5. Design a low-pass active filter that has a gain of -10 and a cutoff frequency of 10 krad/s. Use the following circuit.



To get a gain of -10, we need $R_2/R_1 = 10$. To get $\omega_C = 10000$ rad/s, we need $1/R_2C_2 = 10000$. Select $R_2 = 100$ kΩ, solve for $C_2 = 1000$ pF, and solve for $R_1 = 10$ kΩ.

Exercise 12–6. A particular uA741 OP AMP, a common all-purpose amplifier, has a guaranteed minimum gain-bandwidth product of 0.7 MHz. Using a straight-line approximation, what is the highest frequency that can be amplified by 40 dB without any loss of gain?

We have the following results:

$$G = Af_C = 0.7 \text{ MHz}$$

$$40 \text{ dB} \Rightarrow A = 100$$

$$f_C = \frac{700000}{100} = 7 \text{ kHz}$$

Exercise 12–7. The circuit shown in Figure 12–11 has $R = 2.2$ kΩ and $C = 0.33$ μF. What is the gain of the circuit at $\omega = 1$ krad/s in dB?

Compute the tranfer function and determine the gain at the requested frequency.

$$T(s) = \frac{R}{R + \dfrac{1}{Cs}} = \frac{s}{s + \dfrac{1}{RC}}$$

$$T(j\omega) = \frac{j\omega}{j\omega + \dfrac{1}{RC}}$$

$$|T(j\omega)| = \frac{\omega}{\sqrt{\left(\dfrac{1}{RC}\right)^2 + \omega^2}}$$

$$|T(j1000)| = 0.5874978$$

$$|T(j1000)|_{\text{dB}} = -4.62 \text{ dB}$$

Exercise 12–8. Design an RL high-pass filter with a cutoff of 10 krad/s and a passband gain of 1.

Use an RL voltage divider with the output taken across the inductor. The transfer function is

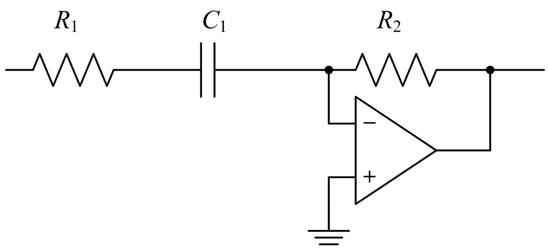
$$T(s) = \frac{Ls}{Ls + R} = \frac{s}{s + \dfrac{R}{L}}$$

The transfer function has the correct form for a high-pass filter and the cutoff frequency is $\omega_C = R/L$. Select $R = 1$ kΩ and solve for $L = 100$ mH to get $\omega_C = 10$ krad/s. Many other designs are possible.

Exercise 12–9. Design a high-pass filter that has the following transfer function:

$$T(s) = -\frac{200s}{s+5000}$$

Use the following circuit:



We need a gain of -200, which implies $R_2/R_1 = 200$. We need a cutoff frequency of 5000 rad/s, which implies $1/R_1C_1 = 5000$. Choose $R_1 = 1$ kΩ and solve for $R_2 = 200$ kΩ and $C_1 = 0.2$ μF.

Exercise 12–10. Suppose that the evaluation task of Example 12–8 included the requirement that the filter feed a 500-Ω recorder. Would the choice be different?

Yes. The 500-Ω recorder would load both of the passive filters. Only the OP AMP solution from vendor #2 would work.

Exercise 12–11. For each circuit in Figure 12–15, identify whether the gain response has low-pass or high-pass characteristics and find the passband gain and cutoff frequency.

(a). Determine the transfer function

$$T(s) = \frac{5000}{5000 + 10000 + \dfrac{1}{Cs}} = \frac{\frac{1}{3}s}{s + 66.7}$$

The circuit is a high-pass filter with a high-frequency gain of 1/3 and a cutoff frequency of 66.7 rad/s.

(b). Determine the transfer function.

$$Z = \frac{R/Cs}{R + 1/Cs} = \frac{R}{RCs + 1}$$

$$T(s) = \frac{Z}{Z + 5000} = \frac{\dfrac{R}{RCs+1}}{\dfrac{R}{RCs+1} + 5000}$$

$$= \frac{R}{5000RCs + 5000 + R} = \frac{\dfrac{1}{5000C}}{s + \dfrac{15000}{5000RC}} = \frac{200}{s + 300}$$

The circuit is a low-pass filter with a low-frequency gain of $200/300 = 2/3$ and a cutoff frequency of 300 rad/s.

(c). Determine the transfer function.

$$R_1 \parallel R_2 = 10000 \parallel 5000 = 3333\ \Omega$$

$$T(s) = \frac{3333}{3333 + 0.01s} = \frac{333300}{s + 333300}$$

The circuit is a low-pass filter with a low-frequency gain of $333300/333300 = 1$ and a cutoff frequency of 333 krad/s.

(d). Determine the transfer function.

$$Z = \frac{(5000)(s/100)}{5000 + s/100} = \frac{5000s}{s + 500000}$$

$$T(s) = \frac{Z}{Z + 10000} = \frac{\dfrac{5000s}{s + 500000}}{\dfrac{5000s}{s + 500000} + 10000}$$

$$= \frac{5000s}{15000s + 5 \times 10^9} = \frac{\frac{1}{3}s}{s + 333000}$$

The circuit is a high-pass filter with a high-frequency gain of 1/3 and a cutoff frequency of 333 krad/s.

Exercise 12–12. Consider the two circuits shown in Figure 12–16. Determine if each has a low-pass or high-pass characteristic. Then design each to have a cutoff frequency of 10 krad/s and a gain of 200. The input resistance must be greater than 1 kΩ.

(a). At low frequencies, the capacitor acts like an open circuit, so the overall circuit is an inverting amplifier with a gain of $-R_2/R_1$. At high frequencies, the capacitor acts like a short circuit and the overall gain is zero. The circuit is a low-pass filter with a cutoff frequency of $\omega_C = 1/R_2C$. To meet the design specifications, choose $R_1 = 2$ kΩ and then calculate $R_2 = 200R_1 = 400$ kΩ and $C = 1/(10000R_2) =$ 250 pF.

(b). At low frequencies, the inductor acts like a short circuit, so the overall circuit is an inverting amplifier with a gain of R_2/R_1. At high frequencies, the inductor acts like an open circuit and the overall gain is zero. The circuit is a low-pass filter with a cutoff frequency of $\omega_C = R_1/L$. To meet the design specifications, choose $R_1 = 2$ kΩ and then calculate $R_2 = 200R_1 = 400$ kΩ and $L = R_1/10000 =$ 200 mH.

Exercise 12–13. State whether the following transfer functions have low-pass or high-pass gain characteristics and find the passband gain and cutoff frequency.

(a). $T_1(s) = \dfrac{1}{10s^{-1} + 10^{-3}}$

Rewrite the transfer function.

$$T_1(s) = \frac{1000s}{s + 10000}$$

The circuit is a high-pass filter with a passband gain of 1000 and a cutoff frequency of 10 krad/s.

(b). $T_2(s) = \dfrac{100}{25s + 1000}$

Rewrite the transfer function.

$$T_2(s) = \frac{4}{s + 40}$$

The circuit is a low-pass filter with a passband gain of 0.1 and a cutoff frequency of 40 rad/s.

(c). $T_3(s) = \dfrac{20/s}{50 + 20/s}$

Rewrite the transfer function.

$$T_3(s) = \frac{0.4}{s + 0.4}$$

The circuit is a low-pass filter with a passband gain of 1 and a cutoff frequency of 0.4 rad/s.

Exercise 12–14. Select the element values in Figure 12–21 so that the passband gain is 6 dB and the cutoff frequencies are 1 krad/s and 50 krad/s.

A passband gain of 6 dB is equivalent to a gain of $K = 2$. To get a gain of 2 with the noninverting amplifier, choose $R_1 = R_2 = 1$ kΩ. The high-pass filter controls the lower cutoff frequency. Choose $R_C = 5$ kΩ and solve for $C = 1/(1000R_C) = 0.2$ μF. The low-pass filter controls the upper cutoff frequency. Choose $R_L = 20$ kΩ and solve for $L = R_L/50000 = 400$ mH.

Exercise 12–15. Two first-order circuits in a cascade connection have the following transfer functions.

$$T_1(s) = \frac{20}{\dfrac{s}{2000} + 1} \quad \text{and} \quad T_2(s) = \frac{s}{s+40}$$

What are the cutoff frequencies and the passband gain? Assume the chain rule applies.

Determine the overall transfer function.

$$T(s) = T_1(s)T_2(s) = \left(\frac{40000}{s+2000}\right)\left(\frac{s}{s+40}\right) = \left(\frac{2000}{s+2000}\right)(20)\left(\frac{s}{s+40}\right)$$

The cutoff frequencies are $\omega_{C1} = 40$ rad/s and $\omega_{C2} = 2000$ rad/s and the passband gain is 20.

Exercise 12–16. Following the analysis pattern in Example 12–10, design a circuit that realizes the following transfer function. Use no resistor smaller than 1 kΩ. What are the passband gain and the cutoff frequencies of the filter?

$$T(s) = \frac{200(s^2 + 200s + 10^6)}{(s+100)(s+10000)}$$

At high and low frequencies, the passband gains are 200. Rewrite the transfer function as follows:

$$\begin{aligned} T(s) &= \frac{200(s^2 + 200s + 10^6)}{(s+100)(s+10000)} = \frac{200s^2 + 20000s + 20000s + 2\times 10^8}{(s+100)(s+10000)} \\ &= \frac{200s(s+100) + 20000(s+10000)}{(s+100)(s+10000)} = \frac{200s}{s+10000} + \frac{200(100)}{s+100} \end{aligned}$$

The cutoff frequency for the low-pass filter is 100 rad/s and the cutoff frequency for the high-pass filter is 10 krad/s. Figure 12–24 in the textbook shows one possible design.

Exercise 12–17. A series RLC circuit has a center frequency of $\omega_0 = 500$ krad/s, a resistance of 40 Ω, and a bandwidth of 50 krad/s. Find Q, L, C, ω_{C1}, and ω_{C2}.

For a series RLC circuit, we have the following results:

$$\omega_0 = 500000 \text{ rad/s}$$

$$R = 40\ \Omega$$

$$B = 50000 \text{ rad/s}$$

$$Q = \frac{\omega_0}{B} = \frac{500000}{50000} = 10$$

$$L = \frac{R}{B} = \frac{40}{50000} = 800\ \mu\text{H}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(500000)^2(800\ \mu)} = 0.005\ \mu\text{F}$$

$$\omega_{C1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = 475625 \text{ rad/s}$$

$$\omega_{C2} = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = 525625 \text{ rad/s}$$

Exercise 12–18. Design a series RLC bandpass circuit that has a center frequency of $\omega_0 = 10$ krad/s, a maximum gain of 20 dB, and a bandwidth of 5 krad/s.

A gain of 20 dB is equivalent to $K = 10$. We have the following results, where we have selected a value for the resistor:

$$\omega_0 = 10 \text{ krad/s}$$

$$B = 5 \text{ krad/s}$$

$$Q = \frac{\omega_0}{B} = \frac{10000}{5000} = 2$$

$$R = 500\ \Omega$$

$$L = \frac{R}{B} = \frac{500}{5000} = 100 \text{ mH}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(10000)^2(0.1)} = 0.1\ \mu\text{F}$$

Use a series RLC circuit with the values calculated above and the output taken across the resistor. Connect the circuit to a noninverting amplifier with a gain of 10 by choosing $R_1 = 10$ kΩ and $R_2 = 90$ kΩ.

Exercise 12–19. Design a bandstop filter to eliminate a 13.5 kHz signal. The bandwidth of the notch should not exceed 10 kHz.

Convert the specification frequencies into radian frequencies and then select a value for the inductor.

$$\omega_0 = 2\pi f_0 = 2\pi(13500) = 84.823 \text{ krad/s}$$

$$B < 2\pi(10000) = 62.832 \text{ krad/s}$$

$$L = 100 \text{ mH}$$

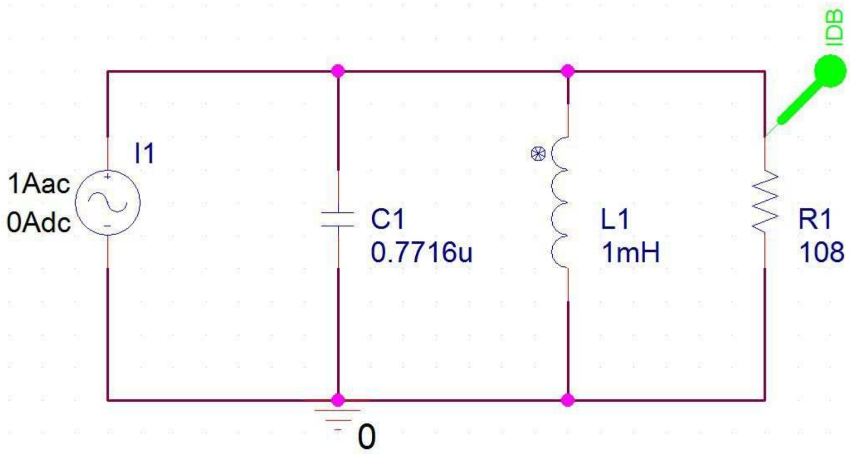
$$C = \frac{1}{\omega_0^2 L} = 1390 \text{ pF}$$

$$R = BL < 6.28 \text{ k}\Omega$$

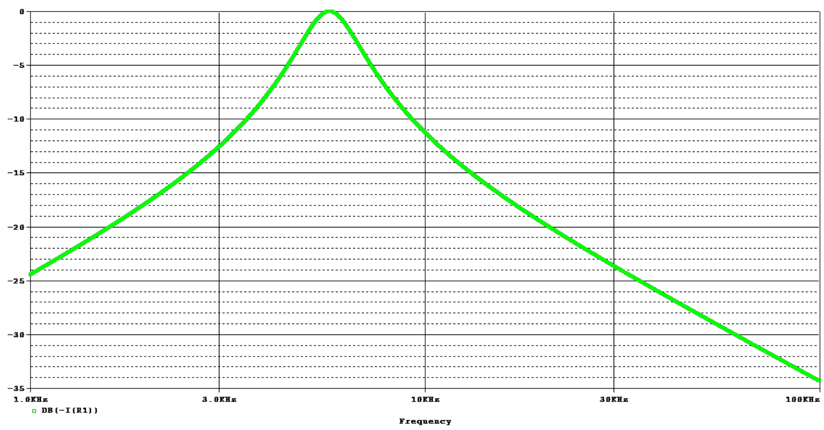
Use a series RLC circuit with the values calculated above and the output taken across the series combination of the inductor and capacitor.

Exercise 12–20. Verify with OrCAD that the circuit designed in Example 12–14 indeed meets the specifications.

The OrCAD simulation is shown below.



The resulting output is shown below.



Examining the plot, we find that $\omega_0 = 36$ krad/s, $\omega_{C1} = 30.49$ krad/s, $\omega_{C2} = 42.58$ krad/s, and $Q = \dfrac{36}{42.58 - 30.49} = 2.98$.

Exercise 12–21. A series RLC circuit has cutoff frequencies at 100 rad/s and 10 krad/s. Find the values of B, ω_0, and Q. Does the circuit have a wide-band or narrow-band response?

We have the following results for a series RLC circuit:

$$\omega_{C1} = 100 \text{ rad/s}$$

$$\omega_{C2} = 10000 \text{ rad/s}$$

$$\omega_0 = \sqrt{\omega_{C1}\omega_{C2}} = \sqrt{1000000} = 1000 \text{ rad/s}$$

$$B = \omega_{C2} - \omega_{C1} = 9900 \text{ rad/s}$$

$$Q = \frac{\omega_0}{B} = \frac{1000}{9900} = 0.101$$

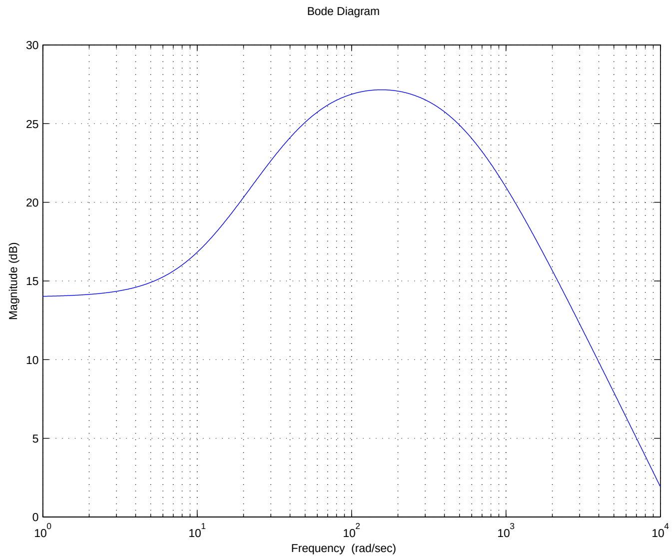
Since $Q < 1$, the circuit has a wide-band response.

Exercise 12–22. Use MATLAB to graph the Bode magnitude plot of the transfer function in Example 12–16.

The MATLAB code is shown below.

```
T = tf(12500*[1 10], [1 550 25000]);
w = logspace(0,4,1000);
bodemag(T,w);
grid on
```

The resulting output is shown below.



Exercise 12–23.

(a). Derive an expression for the straight-line approximation to the gain response of the following transfer function:

$$T(s) = \frac{500(s+50)}{(s+20)(s+500)}$$

Write the transfer function in standard form for a Bode plot.

$$T(j\omega) = \frac{(500)(50)}{(500)(20)} \frac{\left(\frac{j\omega}{50}+1\right)}{\left(\frac{j\omega}{20}+1\right)\left(\frac{j\omega}{500}+1\right)} = \frac{2.5\left(\frac{j\omega}{50}+1\right)}{\left(\frac{j\omega}{20}+1\right)\left(\frac{j\omega}{500}+1\right)}$$

The scale factor is $K_0 = 2.5$ and the corner frequencies are at $\omega_C = 20$ (a pole), 50 (a zero), and 500 (a pole) rad/s. There are no critical frequencies at zero, so the slope is zero up until $\omega = 20$ rad/s, when the slope becomes

$$|T(j\omega)|_{SL} = \frac{2.5(1)}{\left(\frac{\omega}{20}\right)(1)} = \frac{50}{\omega}$$

At $\omega = 50$ rad/s, the zero becomes active and the slope of the plot returns to zero and the magnitude is

$$|T(j\omega)|_{SL} = \frac{2.5\left(\frac{\omega}{50}\right)}{\left(\frac{\omega}{20}\right)(1)} = 1$$

At $\omega = 500$ rad/s, the second pole becomes active and the slope becomes

$$|T(j\omega)|_{SL} = \frac{2.5\left(\frac{\omega}{50}\right)}{\left(\frac{\omega}{20}\right)\left(\frac{\omega}{500}\right)} = \frac{500}{\omega}$$

We have the following results

$$|T(j\omega)|_{SL} = \begin{vmatrix} 2.5 & \text{if} & 0 < \omega \leq 20 \\ \frac{50}{\omega} & \text{if} & 20 < \omega \leq 50 \\ 1 & \text{if} & 50 < \omega \leq 500 \\ \frac{500}{\omega} & \text{if} & 500 < \omega \end{vmatrix}$$

(b). Find the straight-line gains at $\omega = 10$, 30, and 100 rad/s.

At $\omega = 10$ rad/s, the gain is 2.5 or 7.96 dB. At $\omega = 30$ rad/s, the gain is $50/\omega = 50/30 = 1.667$ or 4.44 dB. At $\omega = 100$ rad/s, the gain is 1 or 0 dB.

(c). Find the frequency at which the high-frequency gain asymptote falls below -20 dB.

A gain of -20 dB is 0.1, which occurs when $500/\omega = 0.1$ or $\omega = 5000$ rad/s.

(d). Compare your answers in parts (b) and (c) with MATLAB.

The following MATLAB code calculates more accurate results.

```
syms s w
Ts = 2.5*(s/50 + 1)/(s/20 + 1)/(s/500 + 1);
Tjw = subs(Ts,s,j*w);
ww = [10 30 100];
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
MagdBTjww = 20*log10(MagTjww)
MagT2 = 500*sqrt(50^2+w^2)/sqrt(20^2+w^2)/sqrt(500^2+w^2);
dBMagT2 = 20*log10(MagT2);
w20 = solve(dBMagT2+20);
w20_num = double(w20(1))
```

The results are:

```
MagdBTjww = 7.1583e+000     4.1597e+000   628.4333e-003
w20_num = 4.9752e+003
```

The straight-line approximations are reasonably close to the more accurate MATLAB results.

Exercise 12–24. Construct a Bode plot of the straight-line approximation to the phase response of the transfer function in Exercise 12–23. Use the plot to estimate the phase angles at $\omega = 1$, 15, 300, and 10^4 rad/s. Compare your results with those obtained using MATLAB.

From Exercise 12-23, we have the following transfer function:

$$T(j\omega) = \frac{2.5\left(\dfrac{j\omega}{50}+1\right)}{\left(\dfrac{j\omega}{20}+1\right)\left(\dfrac{j\omega}{500}+1\right)}$$

Use the transfer function to construct the following table which characterizes the slopes of the straight-line approximation for the phase plot.

Frequency (rad/s)	Pole or Zero	Change in Slope (Degrees)	Slope
0	None	None	0
2	Pole	−45	−45
5	Zero	+45	0
50	Pole	−45	−45
200	Pole	+45	0
500	Zero	−45	−45
5000	Pole	+45	0

Estimate the phase for the requested frequencies

$$\angle T(j1) = 0^\circ$$

$$\angle T(j15) = (-45)\left[\log_{10}(5) - \log_{10}(2)\right] = -17.91^\circ$$

$$\angle T(j300) = (-45)\left[\log_{10}(200) - \log_{10}(50) + \log_{10}(5) - \log_{10}(2)\right] = -45^\circ$$

$$\angle T(j10000) = -90^\circ$$

Compute the exact results using MATLAB

```
syms s w
Ts = 500*(s+50)/(s+20)/(s+500);
w = [1 15 300 10000];
Tjw = subs(Ts,s,j*w);
PhaseTjw = 180*angle(Tjw)/pi
```

The results are shown below:

```
PhaseTjw = -1.8312e+000   -21.8890e+000   -36.6120e+000   -87.3095e+000
```

The straight-line approximations are reasonable.

Exercise 12–25. Construct a straight-line graph of the gain function of the following transfer function. Then use MATLAB to plot the actual Bode magnitude plot.

$$T(s) = \frac{20s}{s^2 + 2s + 2500}$$

Rewrite the transfer function.

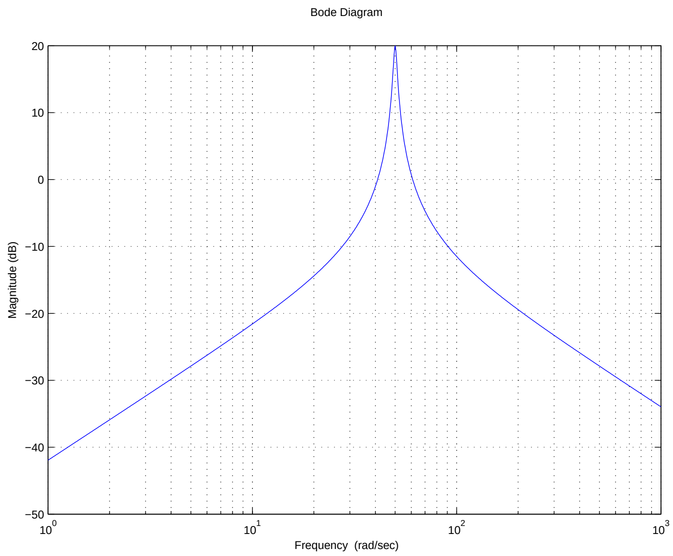
$$T(j\omega) = \frac{20(j\omega)}{2500\left[1 - \left(\dfrac{\omega}{50}\right)^2 + j\dfrac{\omega}{1250}\right]} = \frac{1}{125}\left[\frac{j\omega}{1 - \left(\dfrac{\omega}{50}\right)^2 + j\dfrac{\omega}{1250}}\right]$$

The center frequency is $\omega_0 = 50$ rad/s. We have the following results

$$|T(j\omega)|_{\rm SL} = \left| \begin{array}{ll} \dfrac{\omega}{125} & \text{if} \quad 0 < \omega \leq 50 \\ \dfrac{20}{\omega} & \text{if} \quad 50 < \omega \end{array} \right.$$

The magnitude of the straight-line approximation at $\omega = 5$ rad/s is $5/125 = -28$ dB. The straight-line plot has a slope of $+20$ dB/decade, so the magnitude at $\omega = 50$ rad/s is -8 dB. The corresponding MATLAB code and results are shown below.

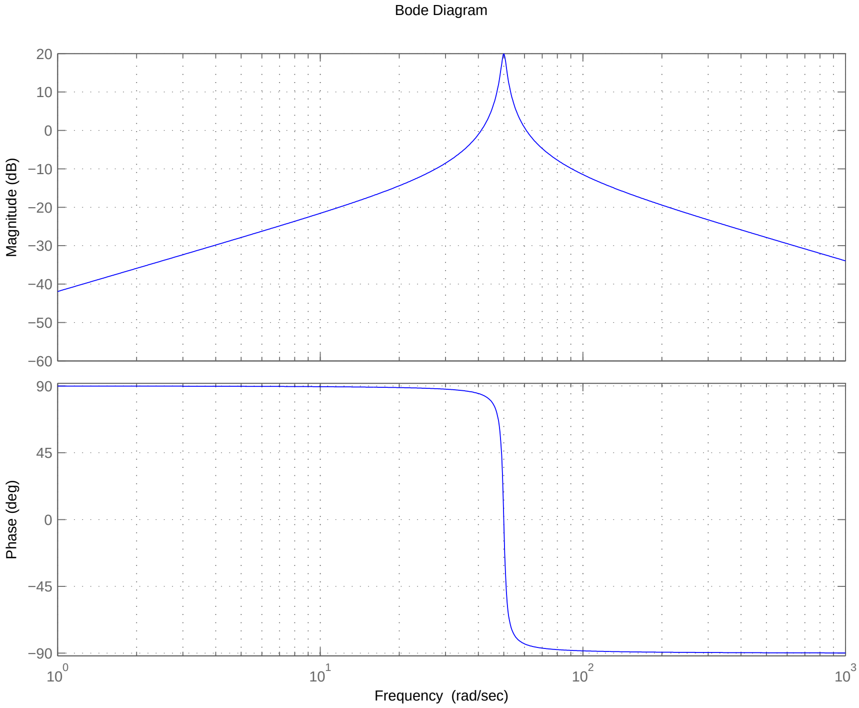
```
T = tf([20 0],[1 2 2500])
w = logspace(0,3,1000);
bodemag(T,w);
grid on
```



Exercise 12–26. Use MATLAB to plot the actual Bode phase plot for the transfer function in Exercise 12–25.

The MATLAB code and results are shown below.

```
T = tf([20 0],[1 2 2500])
w = logspace(0,3,1000);
bode(T,w);
grid on
```

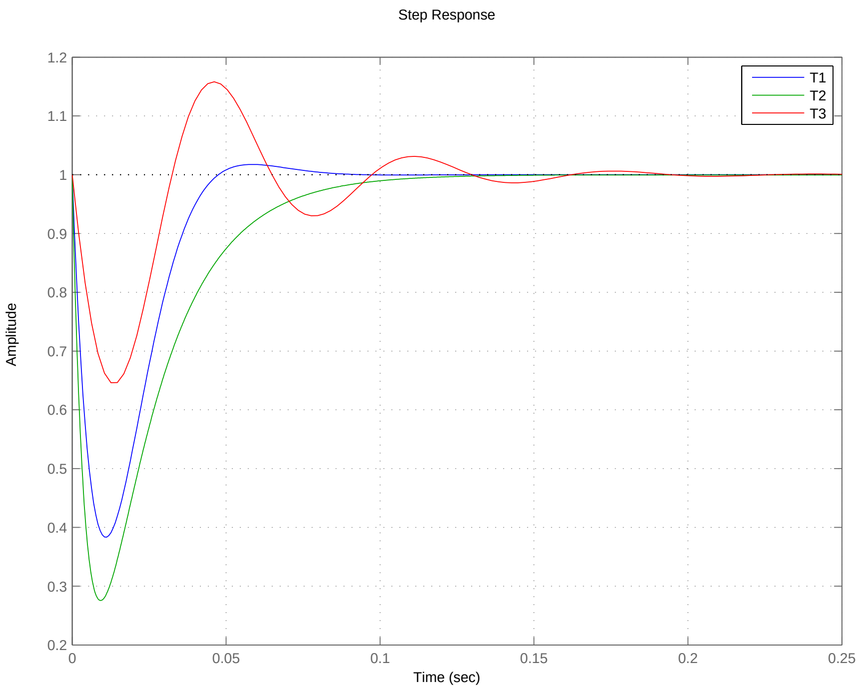


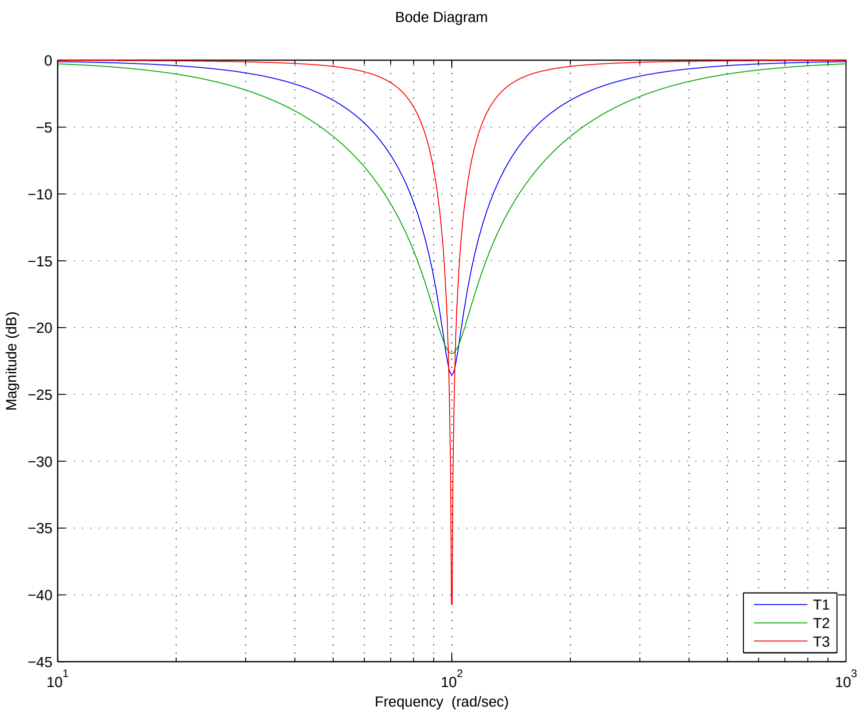
Exercise 12–27. There is a need for a passive notch filter at 100 rad/s. The narrower the notch, the better, but there should be minimal ringing of signals passing through. Shown below are the transforms of three filters submitted for consideration. Which would you recommend and why?

Use MATLAB to plot the step responses and Bode magnitude plots for each transfer function. The MATLAB code is shown below.

```
T1 = tf([1 10 10000],[1 150 10000]);
T2 = tf([1 20 10000],[1 250 10000]);
T3 = tf([1 0 10000],[1 50 10000]);
figure
step(T1,T2,T3);grid
legend('T1','T2','T3')

figure
w = logspace(1,3,1000);
bodemag(T1,T2,T3,w); grid
legend('T1','T2','T3','Location','SouthEast')
```

The corresponding plots are shown below





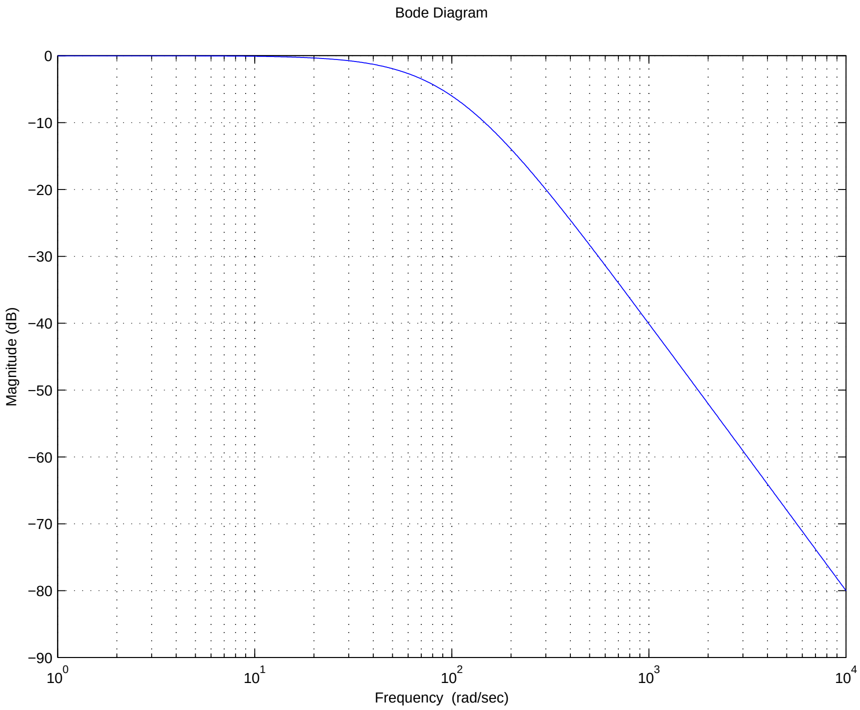
$T_1(s)$ offers the best compromise—a relatively narrow notch with only a slight overshoot. $T_2(s)$ would be the best choice if no ringing at all was desired, but the notch would be wider.

Exercise 12–28. The impulse response of a particular filter is $h(t) = 10^4 te^{-100t}u(t)$. What type of filter is this? What is its passband bandwidth?

Determine the transfer function.

$$H(s) = \frac{10000}{(s+100)^2} = \frac{10000}{s^2 + 200s + 10000}$$

The filter is a second-order low-pass filter. The Bode plot is shown below.



The cutoff frequency is about 64 rad/s, so that is also its bandwidth.

Exercise 12–29. A certain RLC series bandpass circuit has the following transfer function:

$$T(s) = \frac{200s}{s^2 + 200s + 640000}$$

Is it possible to alter a circuit element to keep the same center frequency while minimizing the ringing?

Yes. The transfer function for a series RLC circuit is shown below.

$$T(s) = \frac{R}{R + Ls + 1/Cs} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

The characteristic equation has the form:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\zeta\omega_0 s + \omega_0^2$$

Increasing the resistance will make the middle term in the denominator, $R/L = 2\zeta\omega_0 = 200$, larger, which will increase ζ but leave the center frequency at 800 rad/s. Multiplying the value of the resistance by 8 will increase the damping ratio to make it equal to 1 and all ringing will stop. Note that the bandwidth will increase with increasing ζ.

12.2 Problem Solutions

Problem 12–1. A transfer function has a passband gain of 500. At a particular frequency in its stopband, the gain of the transfer function is only 0.00025. By how many decibels does the gain of the passband exceed that of the frequency in the stopband?

We have the following calculations:

$$K = 20\log_{10}\left(\frac{500}{0.00025}\right) = (20)(6.301) = 126.02\text{ dB}$$

Problem 12–2. A particular filter is said to be 56 dB down at a desired stop frequency. How many times reduced is a signal at that frequency compared to a signal in the filter's pass-band?

We have the following calculations:

$$-56\text{ dB} = 20\log_{10}\left(\frac{x}{1}\right)$$

$$-2.8 = \log_{10}(x)$$

$$x = 10^{-2.8}$$

$$x = 0.00158 \approx \frac{1}{631}$$

Problem 12–3. A certain low-pass filter has the Bode diagram shown in Figure P12–3.

(a). How many dB down is the filter at 5000 rad/s?

The gain of the transfer function starts at 20 and drops to 0.02 at $\omega = 5000$ rad/s.

$$K = 20\log_{10}\left(\frac{0.02}{20}\right) = 20\log_{10}(0.001) = -60\text{ dB}$$

(b). Estimate where the cutoff frequency occurs, then determine how many dB down is the filter at one decade after the cutoff frequency?

The cutoff frequency is at approximately -3 dB or 0.7071 of the passband value. In this case, the cutoff frequency is at $(0.7071)(20) = 14.14$, which occurs at approximately $\omega = 50$ rad/s. One decade later, $\omega = 500$ rad/s and the gain is approximately 1.4.

$$K = 20\log_{10}\left(\frac{1.4}{20}\right) = (20)(-1.155) = -23.1\text{ dB}$$

Problem 12–4. Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12–4.

The transfer function is

$$T_V(s) = \frac{100}{100 + 100 + 0.1s} = \frac{1000}{s + 2000}$$

(a). Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

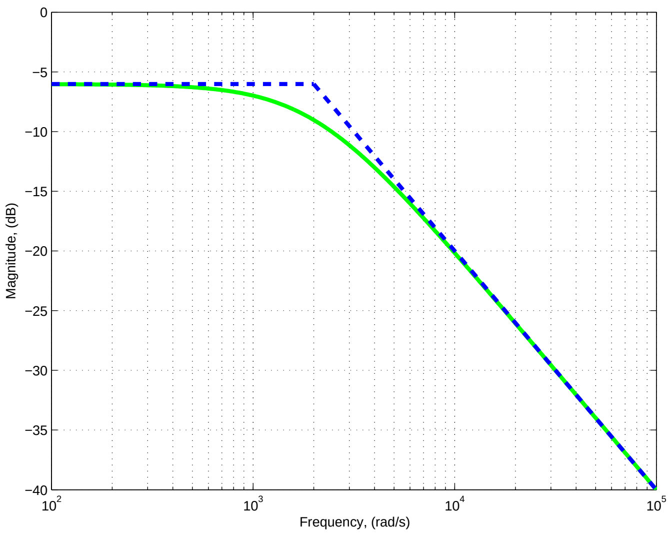
By inspection of the transfer function, the cutoff frequency is $\omega_C = 2000$ rad/s and it is a low-pass filter.

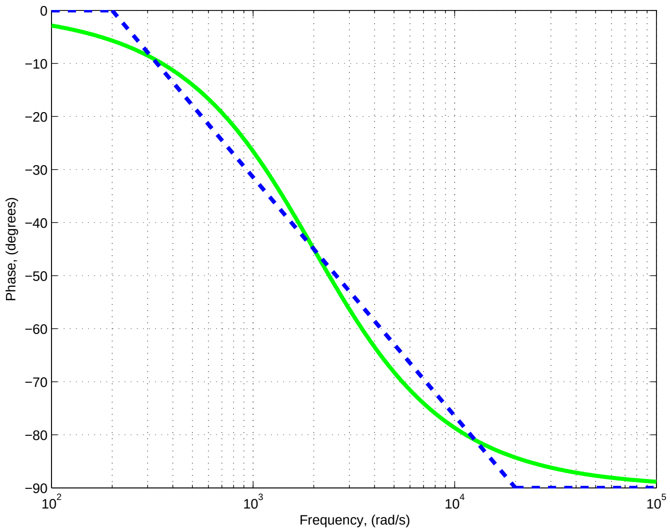
$$|T(j0)| = \frac{1000}{2000} = 0.5$$

$$|T(j\infty)| = 0$$

(b). Sketch the straight-line approximations of the gain and phase responses.

Both the straight-line approximations and the actual responses are shown below for the gain and phase.





(c). Calculate the gain at $\omega = 0.5\omega_C$, ω_C, and $2\omega_C$.

We have the following results:

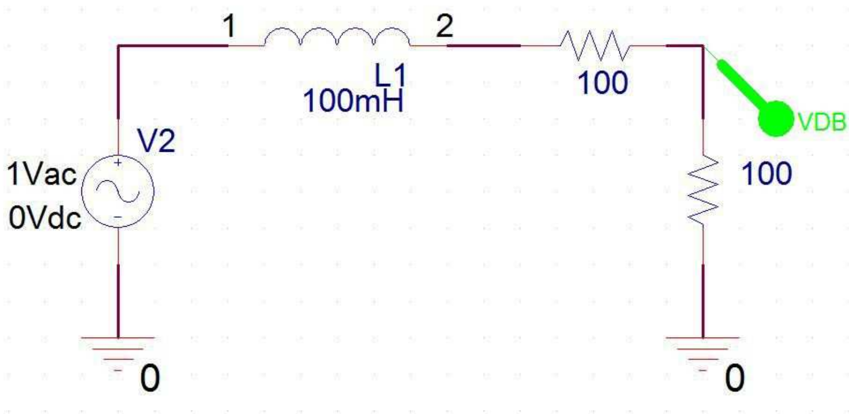
$$|T(j1000)| = \left|\frac{1000}{2000 + j1000}\right| = 0.447 = -6.99 \text{ dB}$$

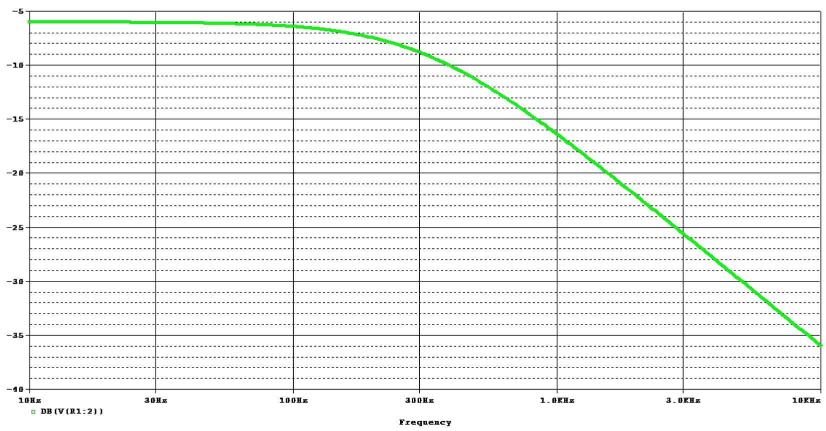
$$|T(j2000)| = \left|\frac{1000}{2000 + j2000}\right| = 0.354 = -9.03 \text{ dB}$$

$$|T(j4000)| = \left|\frac{1000}{2000 + j4000}\right| = 0.224 = -13.01 \text{ dB}$$

(d). Use OrCAD to plot the Bode magnitude gain response of the circuit. Validate your answers for part (c).

The OrCAD simulation and results are shown below and agree with answers in part (c).





(e). How many dB down from the passband is the filter at one octave past the cutoff?

At one octave past the cutoff, $\omega = 2\omega_C = 4000$ rad/s. We have the following results:

$$|T(j4000)| = \left|\frac{1000}{2000 + j4000}\right| = 0.224$$

$$K = 20\log_{10}\left(\frac{0.224}{0.5}\right) = -6.99 \text{ dB}$$

Problem 12–5. Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12–5.
The transfer function is

$$T_V(s) = \frac{\dfrac{RLs}{R+Ls}}{R + \dfrac{RLs}{R+Ls}} = \frac{RLs}{R^2 + 2RLs}$$

$$= \frac{\dfrac{1}{2}s}{s + \dfrac{R}{2L}} = \frac{\dfrac{1}{2}s}{s + 4700}$$

(a). Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

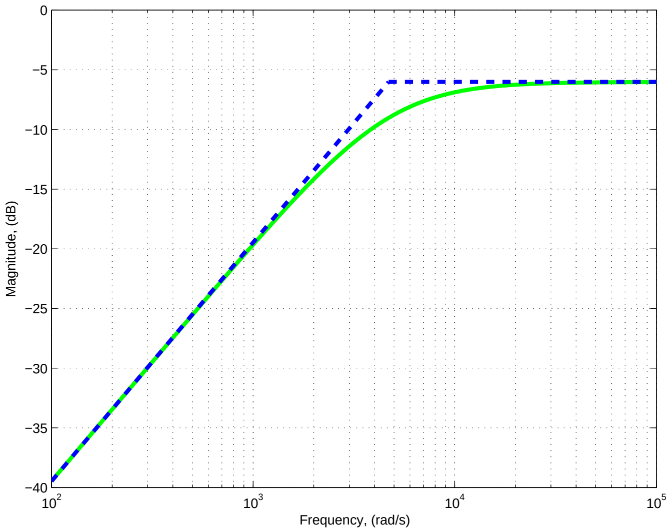
By inspection of the transfer function, the cutoff frequency is $\omega_C = 4700$ rad/s and it is a high-pass filter.

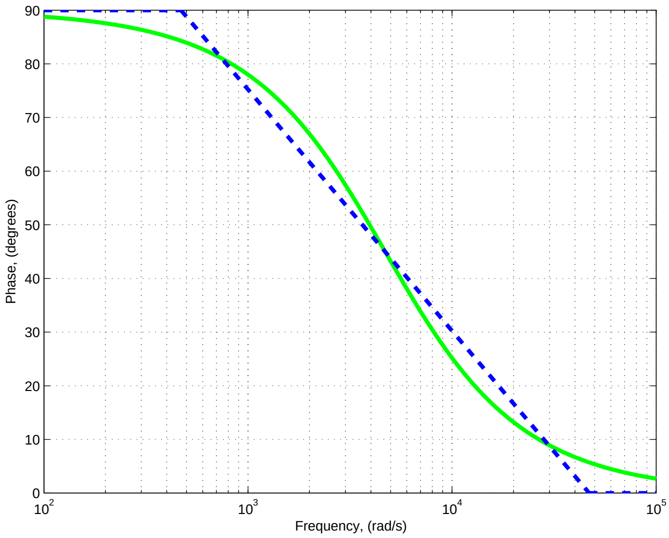
$$|T(j0)| = \frac{0}{4700} = 0$$

$$|T(j\infty)| = \frac{1}{2}$$

(b). Sketch the straight-line approximations of the gain and phase responses.

Both the straight-line approximations and the actual responses are shown below for the gain and phase.





(c). Calculate the gain at $\omega = 0.5\omega_{\mathrm{C}}$, ω_{C}, and $2\omega_{\mathrm{C}}$.

We have the following results:

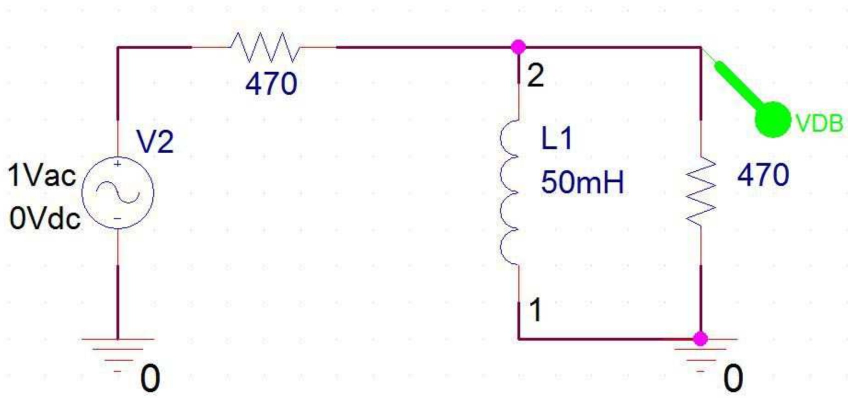
$$|T(j2350)| = \left|\frac{j1175}{4700 + j2350}\right| = 0.224 = -13.01 \text{ dB}$$

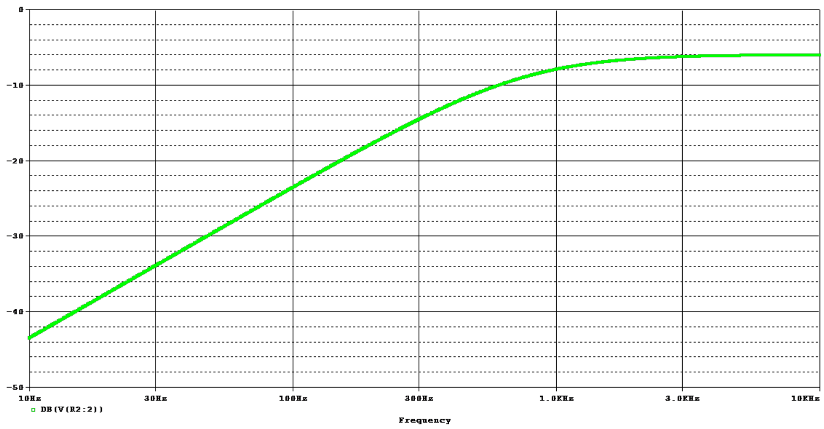
$$|T(j4700)| = \left|\frac{j2350}{4700 + j4700}\right| = 0.354 = -9.03 \text{ dB}$$

$$|T(j9400)| = \left|\frac{j4700}{4700 + j9400}\right| = 0.447 = -6.99 \text{ dB}$$

(d). Use OrCAD to plot the Bode magnitude gain response of the circuit. Validate your answers for part (c).

The OrCAD simulation and results are shown below and agree with answers in part (c).





(e). How many dB down from the passband is the filter at one octave before the cutoff?

At one octave before the cutoff, $\omega = \omega_C/2 = 2350$ rad/s. We have the following results:

$$|T(j2350)| = \left|\frac{j1175}{4700 + j2350}\right| = 0.224$$

$$K = 20\log_{10}\left(\frac{0.224}{0.5}\right) = -6.99 \text{ dB}$$

Problem 12–6. Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12–6.

The transfer function is

$$T_V(s) = \left(\frac{\frac{1}{Cs}}{R + \frac{1}{Cs}}\right)\left(\frac{R_3 + R_4}{R_3}\right) = \frac{4\left(\frac{1}{RC}\right)}{s + \frac{1}{RC}}$$

$$= \frac{4(333.3)}{s + 333.3}$$

(a). Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

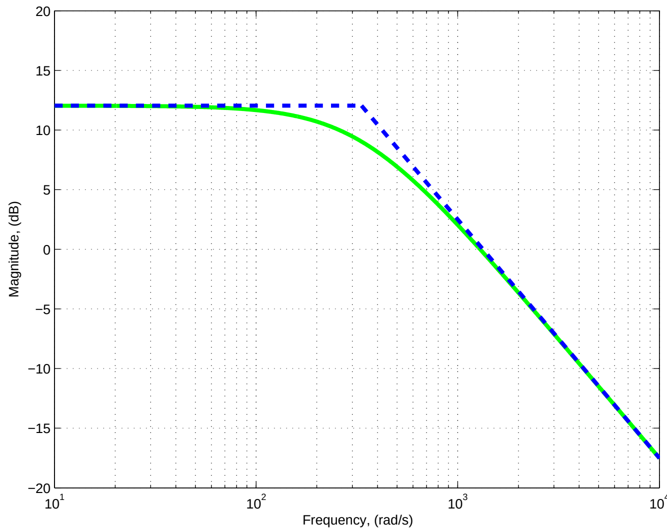
By inspection of the transfer function, the cutoff frequency is $\omega_C = 333.3$ rad/s and it is a low-pass filter.

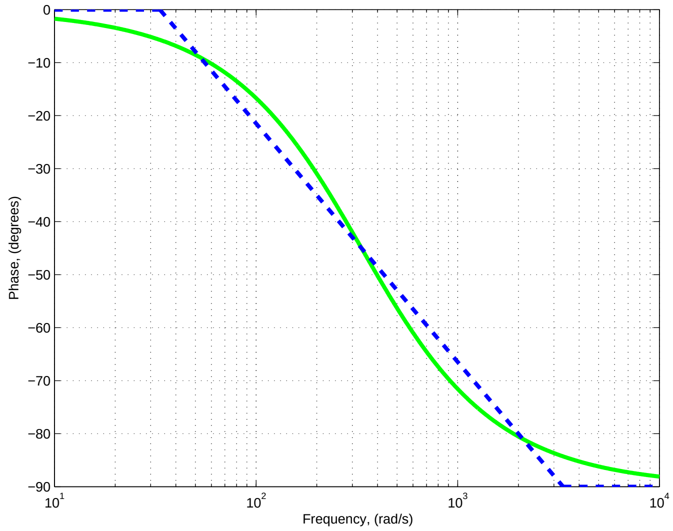
$$|T(j0)| = \frac{4(333.3)}{333.3} = 4$$

$$|T(j\infty)| = 0$$

(b). Sketch the straight-line approximations of the gain and phase responses.

Both the straight-line approximations and the actual responses are shown below for the gain and phase.





(c). Calculate the gain at $\omega = 0.1\omega_C$, ω_C, and $10\omega_C$.

We have the following results:

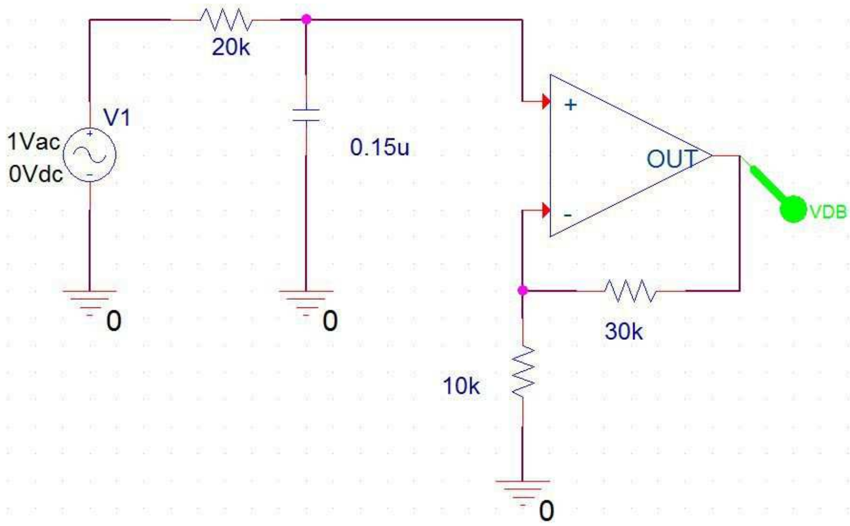
$$|T(j33.33)| = \left|\frac{4(333.3)}{333.3 + j33.33}\right| = 3.98 = 12.00 \text{ dB}$$

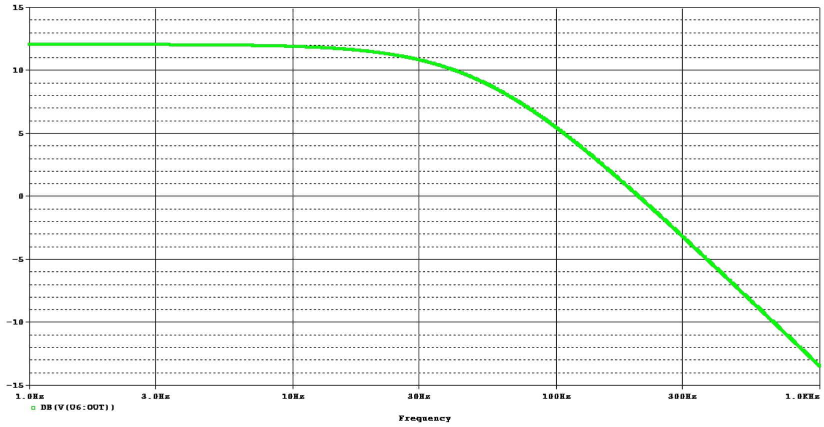
$$|T(j333.3)| = \left|\frac{4(333.3)}{333.3 + j333.3}\right| = 2.83 = 9.03 \text{ dB}$$

$$|T(j3333)| = \left|\frac{4(333.3)}{333.3 + j3333}\right| = 0.398 = -8.00 \text{ dB}$$

(d). Use OrCAD to plot the Bode magnitude gain response of the circuit. Validate your answers for part (c).

The OrCAD simulation and results are shown below and agree with answers in part (c).





(e). What element values would you change to increase the passband gain to 10?

Change the OP AMP's feedback resistor from 30 kΩ to 90 kΩ.

(f). How many dB down from the passband is the filter at one decade past the cutoff?

At one decade past the cutoff, $\omega = 10\omega_C = 3333$ rad/s. We have the following results:

$$|T(j3333)| = \left|\frac{4(333.3)}{333.3 + j3333}\right| = 0.398$$

$$K = 20\log_{10}\left(\frac{0.398}{4}\right) = -20 \text{ dB}$$

Problem 12–7. Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12–7.

The transfer function is

$$T_V(s) = \frac{-\dfrac{R_2}{R_1}\left(\dfrac{1}{R_2C_2}\right)}{s + \dfrac{1}{R_2C_2}} = \frac{-5(1000)}{s+1000}$$

(a). Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

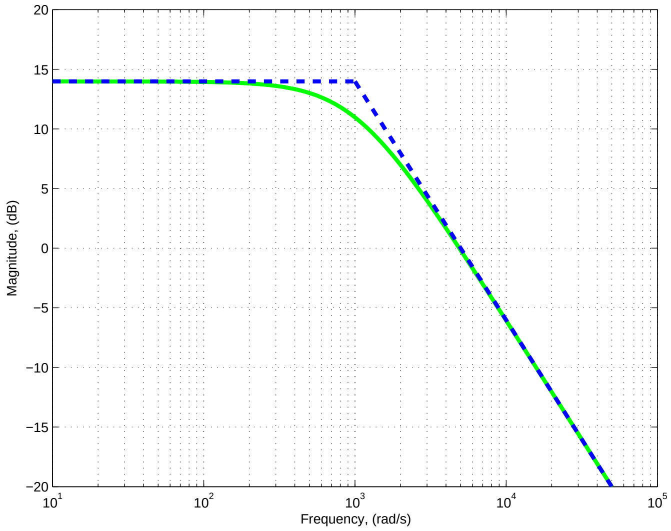
By inspection of the transfer function, the cutoff frequency is $\omega_C = 1000$ rad/s and it is a low-pass filter.

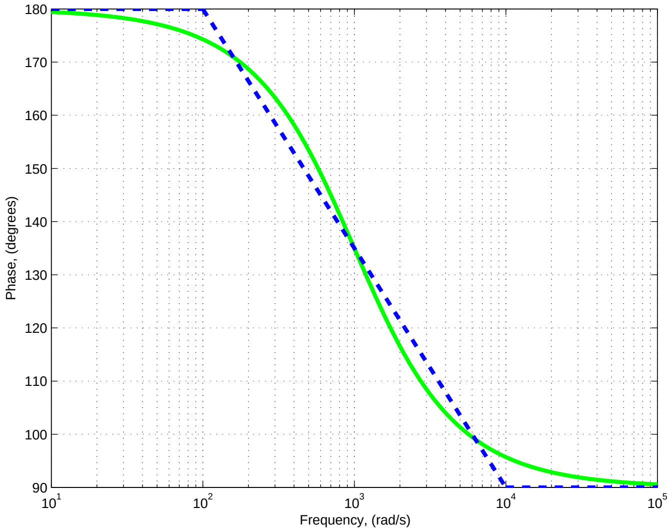
$$|T(j0)| = \frac{5(1000)}{1000} = 5$$

$$|T(j\infty)| = 0$$

(b). Sketch the straight-line approximations of the gain and phase responses.

Both the straight-line approximations and the actual responses are shown below for the gain and phase.





(c). Calculate the gain at $\omega = 0.1\omega_{\mathrm{C}}$, ω_{C}, and $10\omega_{\mathrm{C}}$.

We have the following results:

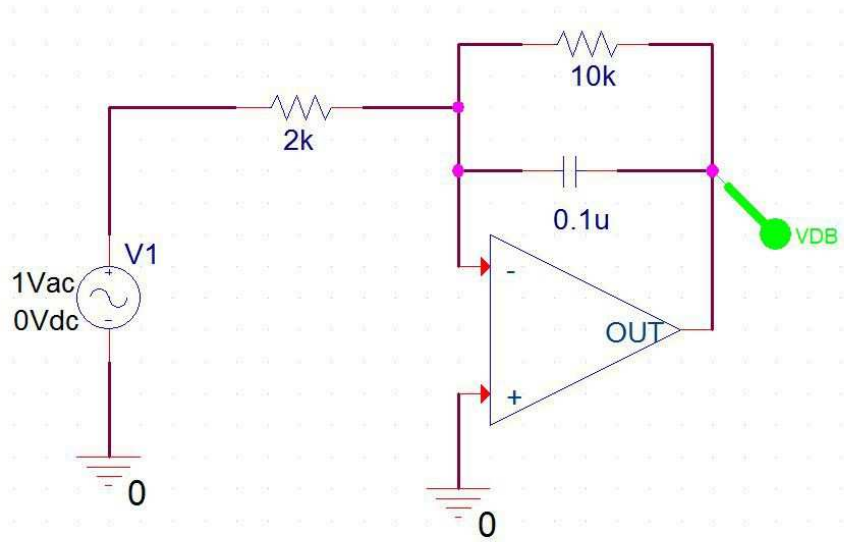
$$|T(j100)| = \left|\frac{5(1000)}{1000 + j100}\right| = 4.975 = 13.94 \text{ dB}$$

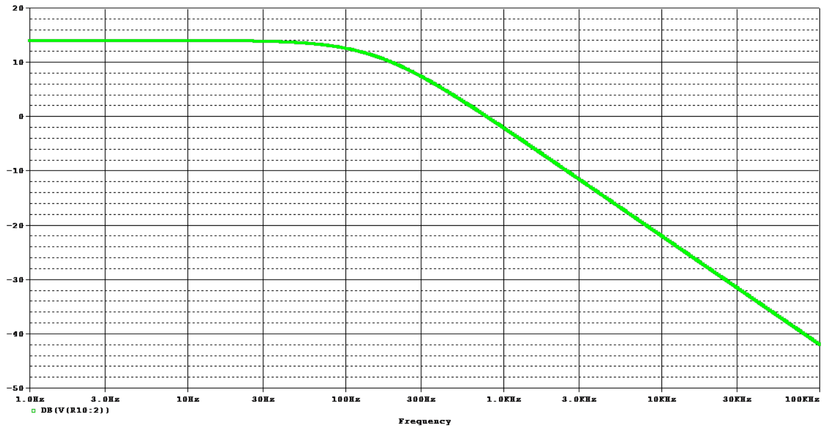
$$|T(j1000)| = \left|\frac{5(1000)}{1000 + j1000}\right| = 3.536 = 10.97 \text{ dB}$$

$$|T(j10000)| = \left|\frac{5(1000)}{1000 + j10000}\right| = 0.4975 = -6.06 \text{ dB}$$

(d). Use OrCAD to plot the Bode magnitude gain response of the circuit. Validate your answers for part (c).

The OrCAD simulation and results are shown below and agree with answers in part (c).





(e). What element values would you change to double the passband gain without changing the cutoff frequency?

Change the input resistor from 2 kΩ to 1 kΩ.

Problem 12–8. Your task is to connect the modules in Figure P12–8 so that the gain of the transfer function is 4 and the cutoff frequency of the filter is 500 rad/s when connected between the source and the load. Repeat if the gain is 3 and the cutoff frequency is 625 rad/s.

The gain stage has a gain of 8 when it operates independently. The transfer function for the filter stages is:

$$T(s) = \frac{R}{2R + \dfrac{1}{Cs}} = \frac{\dfrac{1}{2}s}{s + \dfrac{1}{2RC}} = \frac{\dfrac{1}{2}s}{s + 500}$$

The filter stage has a gain of 0.5 and a cutoff frequency of 500 rad/s. If we connect the stages as source—filter—gain—load, then source resistor does not change the filter stage significantly, the cutoff frequency remains at 500 rad/s, and the overall gain is $(0.5)(8) = 4$, which meets the specification.

To meet the second specification, swap the order of the filter and gain stages. The load resistor changes the properties of the filter stage so that the filter's transfer function is

$$T(s) = \frac{\dfrac{3}{8}s}{s + 625}$$

The overall gain is $(0.375)(8) = 3$ and the cutoff frequency is 625 rad/s, which meets the specification.

Problem 12–9. A young designer needed to design a low-pass filter with a cutoff of 1 krad/s and a gain of −5. The filter is to fit as an interface between the source and the load. The designer was perplexed when no matter how the stages are connected the results are not what were expected. Explain the problem and suggest a way to achieve the desired results.

The gain stage has a gain of −5 and the filter stage have the following transfer function:

$$T(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} = \frac{1000}{s + 1000}$$

The filter stage has a passband gain of one and a cutoff frequency of 1000 rad/s.

If the gain stage is connected before the filter stage, then the load resistor significantly changes the performance of the filter stage in terms of both its gain and its cutoff frequency. If the filter stage is connected before the gain stage, then the input resistor for the gain stage significantly influences the performance of the filter. In both cases, the specifications can be met by adding a buffer between the two stages that are interacting with each other in a negative manner. The design could also be changed to combine the gain and filtering into a single low-pass filter OP AMP circuit by placing a 0.02-μF capacitor in parallel with the 50-kΩ resistor.

Problem 12–10. Design a low-pass first-order filter with a cutoff frequency of 2000 rad/s and a passband gain of 1.

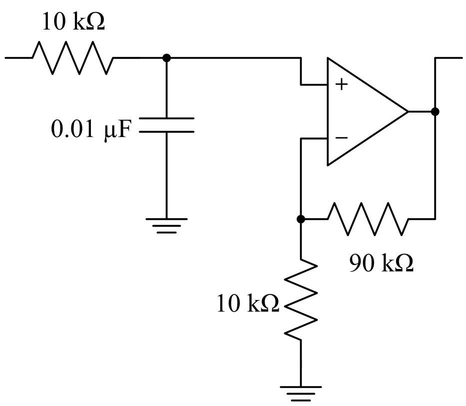
Write the transfer function for the design

$$T(s) = \frac{2000}{s + 2000} = \frac{\frac{2000}{s}}{1 + \frac{2000}{s}} = \frac{\frac{2000000}{s}}{1000 + \frac{2000000}{s}} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}}$$

The design is a series RC circuit with the output taken across the capacitor and $R = 1$ kΩ and $C = 0.5$ μF.

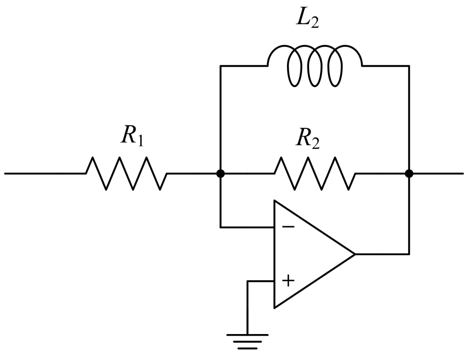
Problem 12–11. Design an RC low-pass first-order filter with a cutoff frequency of 10000 rad/s and a passband gain of +10.

To get a cutoff frequency of 10000 rad/s, select $R = 10$ kΩ and $C = 0.01$ μF. Connect the RC circuit to a noninverting amplifier with a gain of +10 as shown below.



Problem 12–12. Design an RL high-pass first-order filter with a cutoff frequency of 120 Hz and a passband gain of −15.

The following RL circuit is a basic high-pass filter design with gain



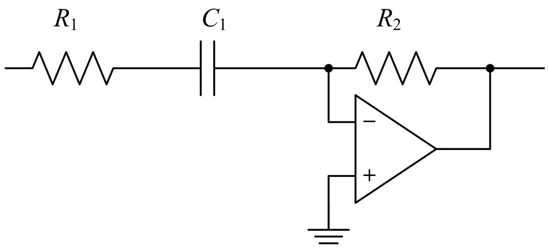
The transfer function is

$$T(s) = \frac{-\dfrac{R_2}{R_1}s}{s + \dfrac{R_2}{L_2}} = \frac{-15s}{s + 240\pi}$$

To get a gain of -15, choose $R_1 = 1$ kΩ and $R_2 = 15$ kΩ. Then solve for $L = 19.9$ H to get a cutoff frequency of 120 Hz, which is also $\omega_C = 240\pi = 754$ rad/s. Note that the inductor value is very large to achieve this design, which is one of the disadvantages of using inductor-based designs.

Problem 12–13. Design an RC high-pass first-order filter with a cutoff frequency of 2000 rad/s and a passband gain of -100.

Use the following RC high-pass filter design.



The tranfer function is

$$T(s) = \frac{-\dfrac{R_2}{R_1}s}{s + \dfrac{1}{R_1C_1}} = \frac{-100s}{s + 2000}$$

Choose $R_1 = 1$ kΩ and solve for $R_2 = 100$ kΩ and $C_1 = 0.5$ μF to meet the specifications.

Problem 12–14. Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12–14.

The tranfer function is

$$T_V(s) = \left(\frac{5+60}{5}\right)\left(\frac{R}{2R + \dfrac{1}{Cs}}\right) = \frac{6.5s}{s + \dfrac{1}{2RC}} = \frac{6.5s}{s + 333}$$

(a). Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

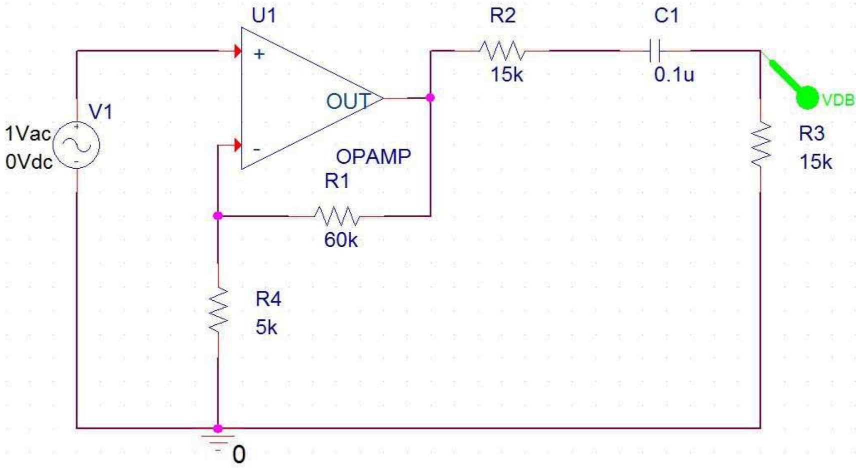
By inspection of the transfer function, the cutoff frequency is $\omega_C = 333$ rad/s and it is a high-pass filter.

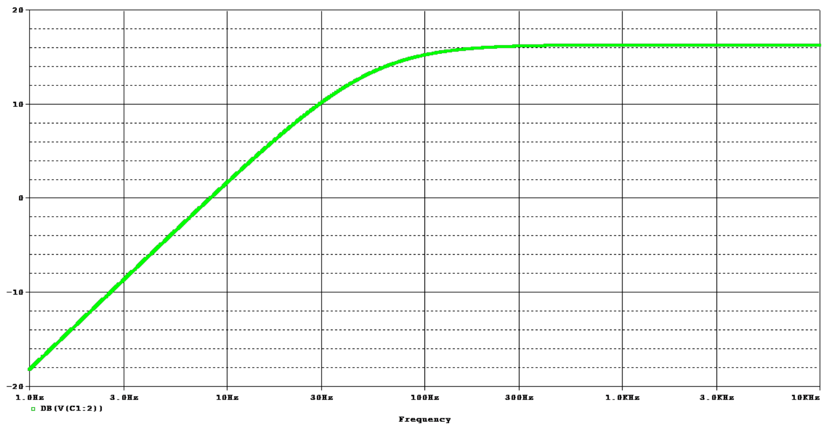
$$|T(j0)| = \frac{0}{333} = 0$$

$$|T(j\infty)| = 6.5$$

(b). Use OrCAD to plot the Bode magnitude gain response of the circuit.

The OrCAD simulation and gain response are shown below.





(c). What element value would you change to increase the cutoff frequency by one decade?

To increase the cutoff frequency by a factor of 10, decrease the capacitor by a factor of 10 to 0.01 μF.

Problem 12–15. Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12–15. What type of gain response does the circuit have? What is the passband gain? Select practical, standard values of R and C from the inside rear cover so that the cutoff frequency is 300 kHz $\pm 5\%$.

Determine the transfer function

$$T_V(s) = \left(\frac{R}{R + 1/Cs}\right)\left(\frac{R+R}{R}\right) = \frac{2s}{s + \dfrac{1}{RC}}$$

The circuit is a high-pass filter with a passband gain of 2 and a cutoff frequency $\omega_C = 1/RC$. We desired a cutoff frequency of $f_C = 300$ kHz or $\omega_C = 1.885$ Mrad/s, $\pm 5\%$. Choose $C = 33$ pF and R as a series combination of a 1-kΩ resistor and a 15-kΩ resistor to get a cutoff frequency of 1.894 Mrad/s.

Problem 12–16. A first-order high-pass circuit has a passband gain of 20 dB and a cutoff frequency of 1000 rad/s. Find the gain (in dB) at $\omega = 0$, 500, 1000, and 5000 rad/s.

A passband gain of 20 dB is a gain of 10. The corresponding transfer function is

$$T(s) = \frac{10s}{s + 1000}$$

We have the following results for the gains:

$$|T(j0)| = 0 = -\infty \text{ dB}$$

$$|T(j500)| = \left|\frac{j5000}{1000 + j500}\right| = 4.472 = 13.01 \text{ dB}$$

$$|T(j1000)| = \left|\frac{j10000}{1000 + j1000}\right| = 7.071 = 16.99 \text{ dB}$$

$$|T(j5000)| = \left|\frac{j50000}{1000 + j5000}\right| = 9.806 = 19.83 \text{ dB}$$

Problem 12–17. A first-order low-pass circuit has a passband gain of 0 dB and a cutoff frequency of 2 krad/s. Find the gain (in dB) at $\omega = 0$, 200 rad/s, 20 krad/s, and 200 krad/s.

A passband gain of 0 dB is a gain of 1. The corresponding transfer function is

$$T(s) = \frac{2000}{s + 2000}$$

We have the following results for the gains:

$$|T(j0)| = \left|\frac{2000}{2000}\right| = 1 = 0 \text{ dB}$$

$$|T(j200)| = \left|\frac{2000}{2000 + j200}\right| = 0.995 = -0.043 \text{ dB}$$

$$|T(j20000)| = \left|\frac{2000}{2000 + j20000}\right| = 0.0995 = -20.04 \text{ dB}$$

$$|T(j200000)| = \left|\frac{2000}{2000 + j200000}\right| = 0.010 = -40.0 \text{ dB}$$

Problem 12–18. The transfer function of a first-order circuit is

$$T(s) = \frac{10000}{s + 2000} = \frac{5(2000)}{s + 2000}$$

(a). Identify the type of gain response. Find the cutoff frequency and the passband gain.

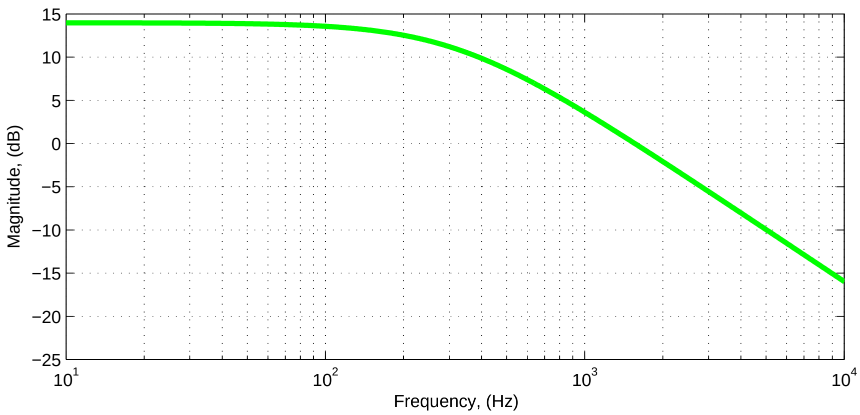
The gain response is a low-pass filter response with a cutoff frequency of $\omega_C = 2000$ rad/s and a passband gain of 5 or 14 dB.

(b). Use MATLAB to plot the magnitude of the Bode gain response.

The MATLAB code to plot the Bode gain response is shown below.

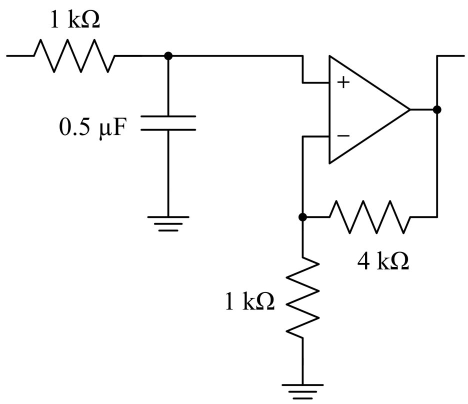
```
syms s
T = 10000/(s+2000);
w = logspace(1,5,1000);
Tjw = subs(T,s,j*w);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
semilogx(w/2/pi,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (Hz)')
ylabel('Magnitude, (dB)')
axis([10 1e4 -25 15])
```

The corresponding plot is shown below.



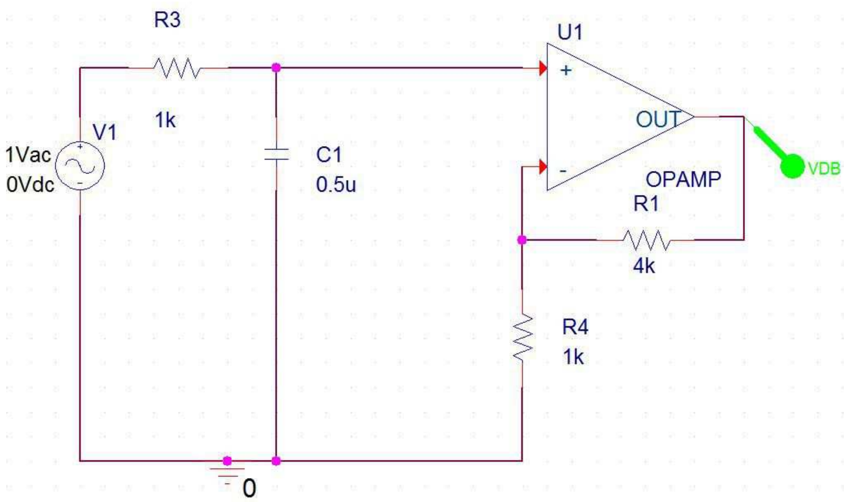
(c). Design a circuit to realize the transfer function.

The design is a first-order series RC circuit with the output taken across the capacitor connected to a noninverting amplifier with a gain of 5. The design is shown below.

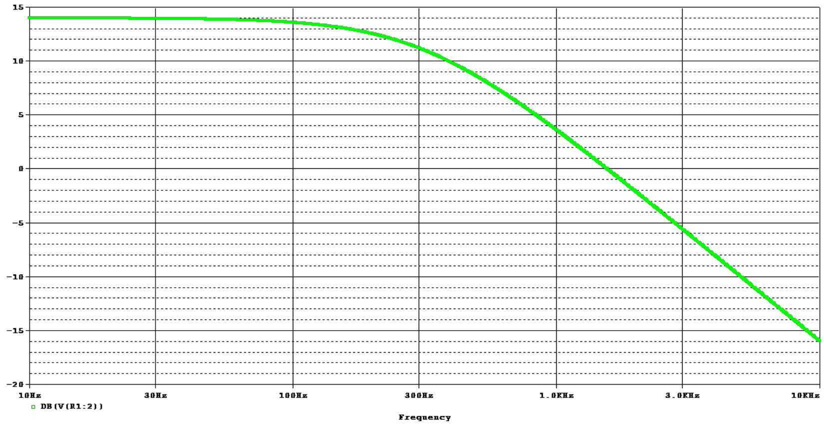


(d). Use OrCAD to validate your circuit design by comparing its frequency response to the MATLAB output.

The OrCAD simulation is shown below.



The corresponding output is shown below and agrees with the MATLAB plot.



Problem 12–19. The transfer function of a first-order circuit is

$$T(s) = \frac{5s}{s + 15000}$$

(a). Identify the type of gain response. Find the cutoff frequency and the passband gain.

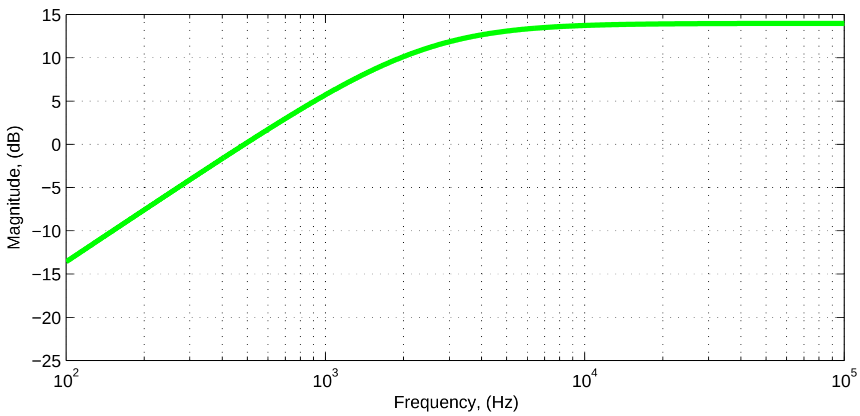
The gain response is a high-pass filter response with a cutoff frequency of $\omega_C = 15000$ rad/s and a passband gain of 5 or 14 dB.

(b). Use MATLAB to plot the magnitude of the Bode gain response.

The MATLAB code to plot the Bode gain response is shown below.

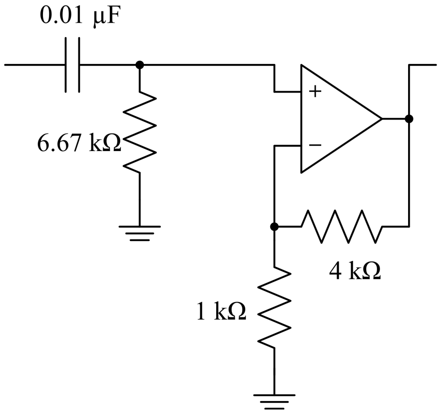
```
syms s
T = 5*s/(s+15000);
w = logspace(2,6,1000);
Tjw = subs(T,s,j*w);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
figure
semilogx(w/2/pi,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (Hz)')
ylabel('Magnitude, (dB)')
axis([100 1e5 -25 15])
```

The corresponding plot is shown below.



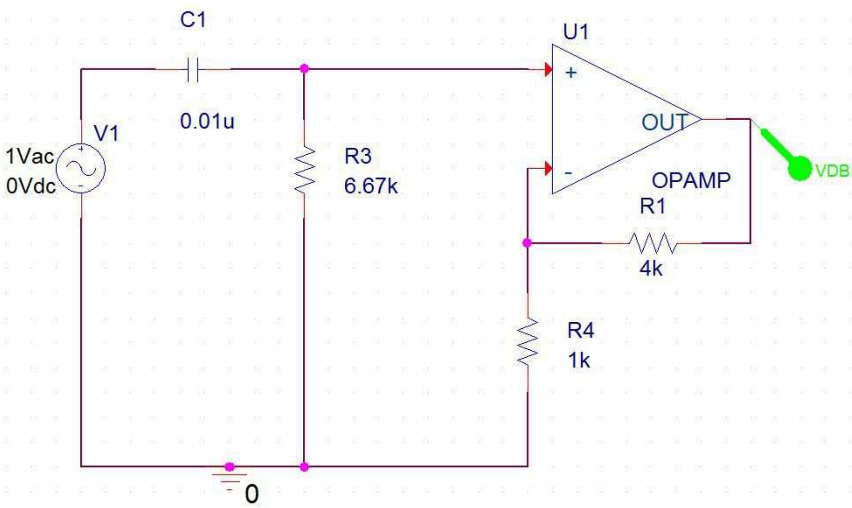
(c). Design a circuit to realize the transfer function.

The design is a first-order series *RC* circuit with the output taken across the resistor connected to a noninverting amplifier with a gain of 5. The design is shown below.

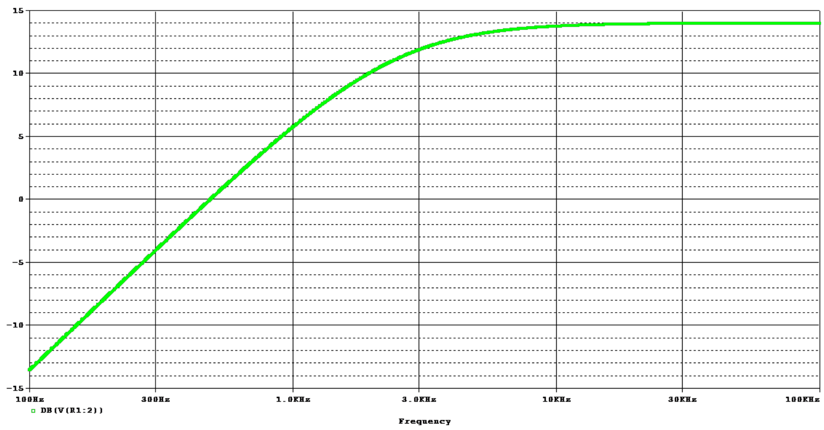


(d). Use OrCAD to validate your circuit design by comparing its frequency response to the MATLAB output.

The OrCAD simulation is shown below.



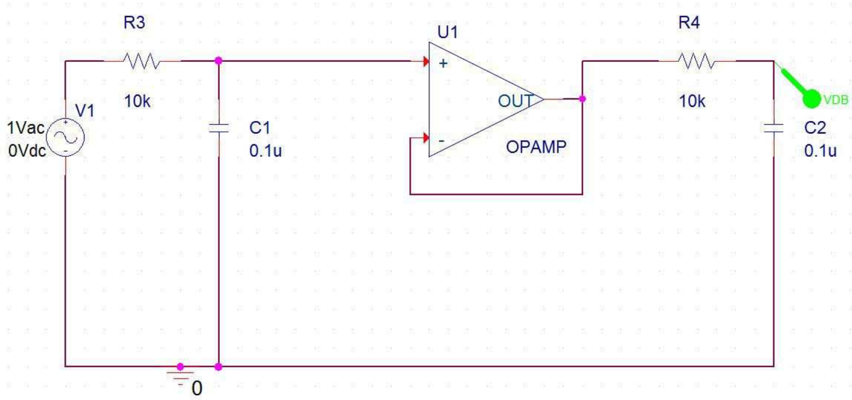
The corresponding output is shown below and agrees with the MATLAB plot.

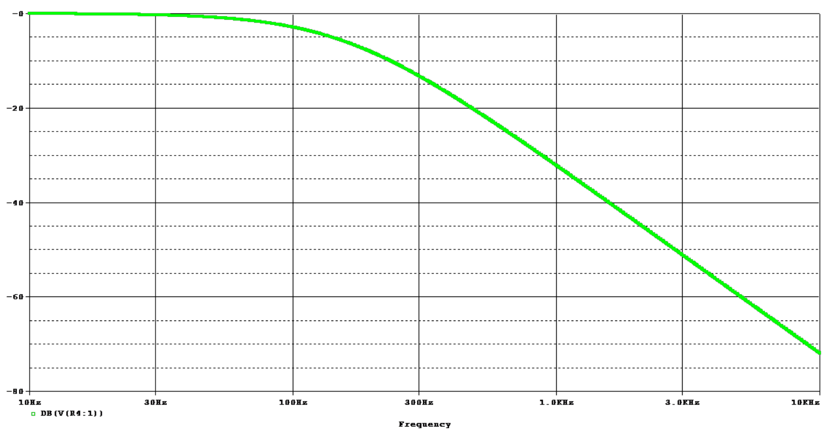


Problem 12–20. A student decided that she needed a low-pass filter that had a roll-off of −2 or −40 dB/decade, with a cutoff frequency of 1000 rad/s. She correctly designed two identical passive *RC* filters, each with a cutoff of 1000 rad/s, and connected them in cascade or one after the other. She cleverly separated them with a buffer to avoid loading. When she measured the cutoff frequency she was dismayed to discover that it was not 1000 rad/s although the roll-off was correct.

(a). Simulate the cascaded pair and determine where the actual cutoff frequency occurs.

The OrCAD simulation and results are shown below.

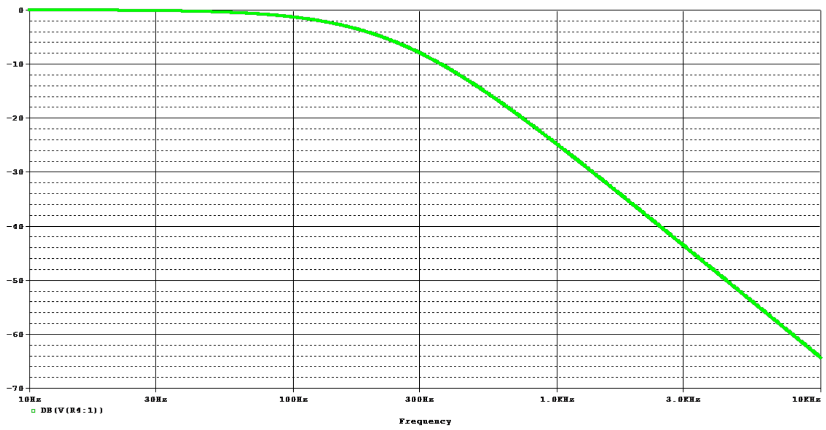




The measured cutoff frequency is $f_C = 102.7$ Hz or $\omega_C = 645.28$ rad/s.

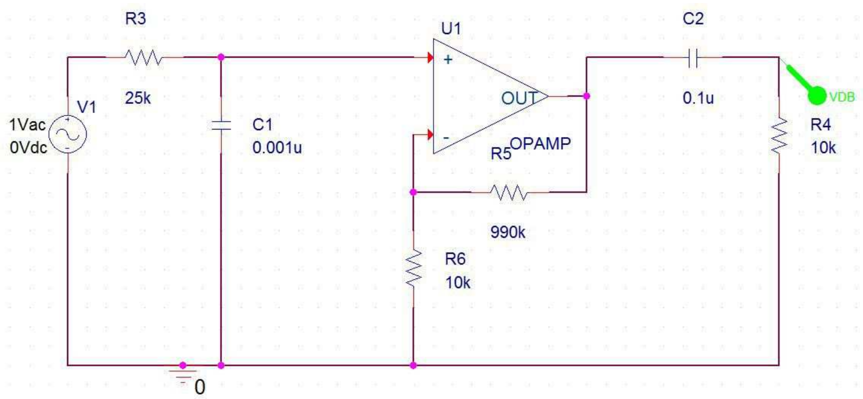
(b). Help the student by altering her design so that the two identical *RC* filters actually produce the desired outcome. Explain why the discrepancy occurred.

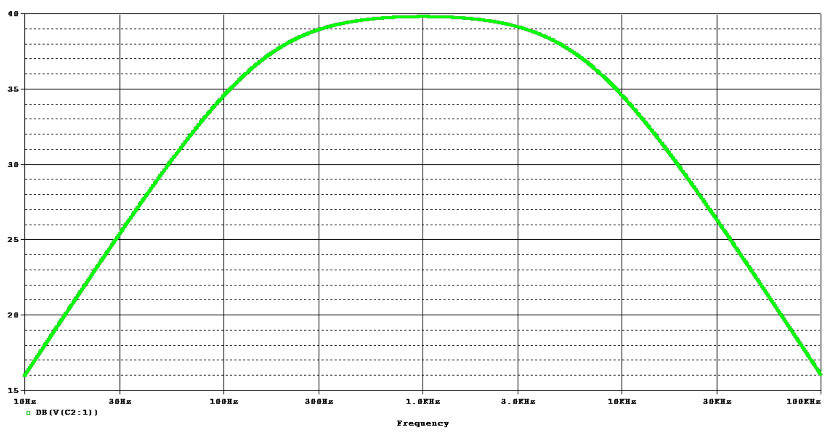
If we multiply two identical low-pass filters together, the resulting filter's gain response will roll off at a lower frequency. We need to shift the cutoff frequency of the first-order filters to a higher value so that the new cutoff frequency occurs at the desired point. In this case, shifting the first-order cutoff frequency to $\omega_C = 1554$ rad/s will yield a second-order cutoff frequency of 1000 rad/s. To achieve this result, replace the 10-kΩ resistors with 6.45-kΩ resistors. The resulting gain response is shown below.



Problem 12–21. The circuit in Figure P12–21 produces a bandpass response for a suitable choice of element values. Identify the elements that control the two cutoff frequencies. Select the element values so that the passband gain is 100 and the cutoff frequencies are 1000 rad/s and 40 krad/s. Use practical element values with $R \geq 10$ kΩ and $C \leq 1$ μF. Use OrCAD to validate your design.

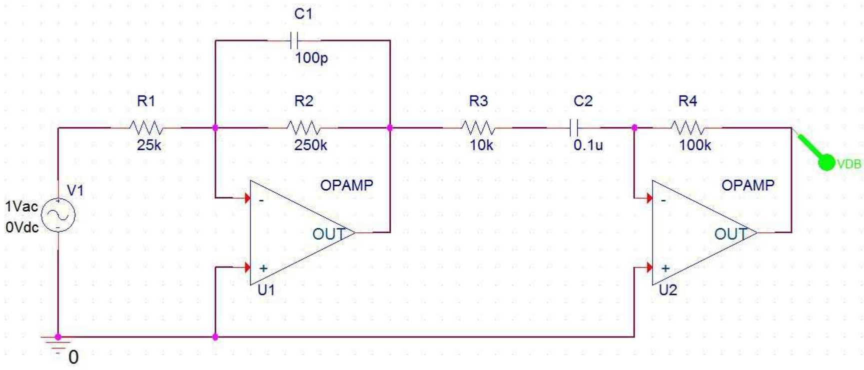
The first RC stage is a low-pass filter and it controls the upper cutoff frequency. We have $1/R_1C_1 = 40000$, so pick $C_1 = 0.001$ μF and solve for $R_1 = 25$ kΩ. The OP AMP stage controls the gain with $K = (R_3 + R_4)/R_4$, so pick $R_3 = 990$ kΩ and $R_4 = 10$ kΩ to get $K = 100$. The last RC stage is a high-pass filter and it controls the lower cutoff frequency. We have $1/R_2C_2 = 1000$, so pick $C_2 = 0.1$ μF and $R_2 = 10$ kΩ. The OrCAD simulation and the resulting gain response are shown below.

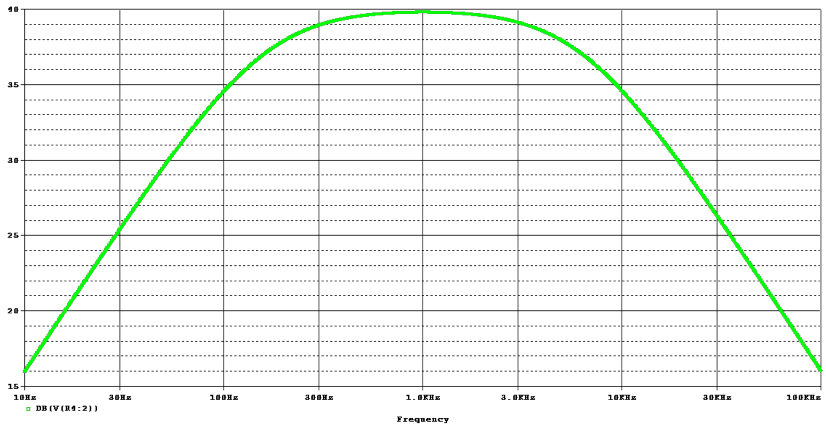




Problem 12–22. Repeat Problem 12–21 for the circuit in Figure P12–22. Use OrCAD to validate your design.

The first OP AMP stage is a low-pass filter with a gain of $-R_2/R_1$ and a cutoff frequency of $1/R_2C_1$. The low-pass filter controls the upper cutoff frequency. Pick $R_1 = 25$ kΩ and $R_2 = 250$ kΩ to give the first stage a gain of -10 and solve for $C_1 = 100$ pF to get a cutoff frequency of 40 krad/s. The second OP AMP stage is a high-pass filter with a gain of $-R_4/R_3$ and a cutoff frequency of $1/R_3C_2$. Pick $R_3 = 10$ kΩ and solve for $C_2 = 0.1$ μF to get a cutoff frequency of 1000 rad/s and then solve for $R_4 = 100$ kΩ so that the gain for the second stage is -10. The overall gain is $(-10)(-10) = 100$. The OrCAD simulation and the resulting gain response are shown below.





Problem 12–23. Suppose the circuits you designed in problems 12-21 and 12-22 had to feed a 500-Ω instrument. Which circuit would you select and why?

Choose the circuit in Problem 12–22, since its output is the output of an OP AMP and it will not cause any loading at the output. The circuit is Problem 12–21 will not meet the specifications when connected to a 500-Ω load.

Problem 12–24. The circuit in Figure P12–24 produces a bandstop response for a suitable choice of element values.

(a). Find the circuit's transfer function.

Find the transfer function for each input OP AMP stage and then add them together.

$$T_1(s) = \frac{s}{s + \dfrac{1}{R_1C_1}}$$

$$T_2(s) = \frac{\dfrac{1}{R_2C_2}}{s + \dfrac{1}{R_2C_2}}$$

$$T(s) = -\frac{R_\text{F}}{R}\left[T_1(s) + T_2(s)\right] = -\frac{R_\text{F}}{R}\left[\frac{s}{s + \dfrac{1}{R_1C_1}} + \frac{\dfrac{1}{R_2C_2}}{s + \dfrac{1}{R_2C_2}}\right]$$

$$= -\frac{R_\text{F}}{R}\left[\frac{s^2 + \left(\dfrac{2}{R_2C_2}\right)s + \dfrac{1}{R_1R_2C_1C_2}}{s^2 + \left(\dfrac{1}{R_1C_1} + \dfrac{1}{R_2C_2}\right)s + \dfrac{1}{R_1R_2C_1C_2}}\right]$$

(b). Identify the elements that control the two cutoff frequencies. Select the element values so that the cutoff frequencies are 200 krad/s and 4000 krad/s. Use practical element values with $R \geq 10$ k$\Omega$and $C \leq 1$ μF. Design your passband gain to be +20 dB.

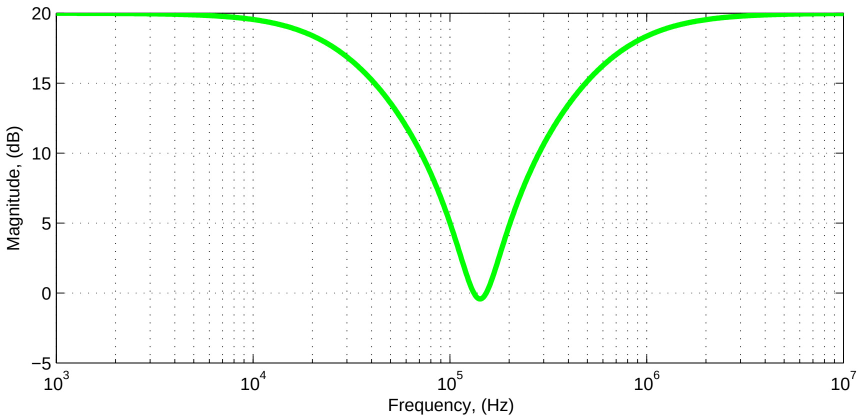
Elements R_1 and C_1 control the high-pass filter and the upper cutoff frequency. Elements R_2 and C_2 control the low-pass filter and the lower cutoff frequency. To meet the specifications, choose $R_1 = R_2 = 10$ kΩ, $C_1 = 25$ pF, and $C_2 = 500$ pF. The passband gain is +20 dB or 10, so choose $R = 10$ kΩ and $R_\text{F} = 100$ kΩ.

(c). Use MATLAB to plot the Bode magnitude plot for the values you selected.

The following MATLAB code will create the Bode magnitude response.

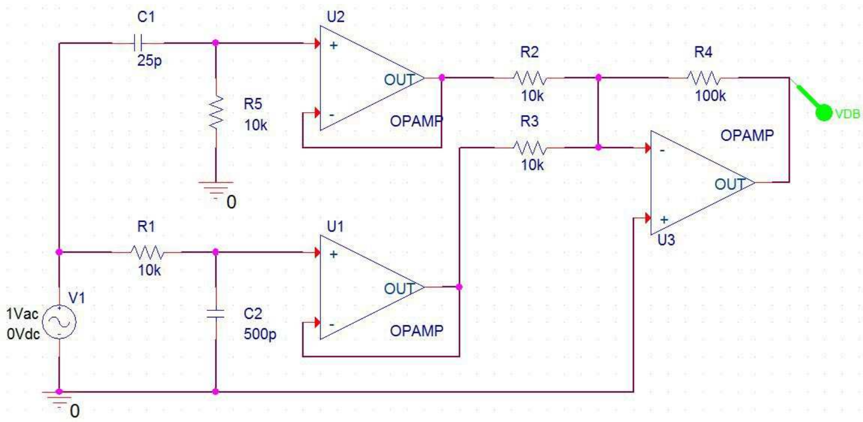
```
syms s
C1 = 25e-12;
C2 = 500e-12;
R1 = 10e3;
R2 = 10e3;
R = 10e3;
RF = 100e3;
T1s = R1/(R1+1/C1/s);
T2s = 1/C2/s/(R2+1/C2/s);
Ts = factor(-RF*(T1s+T2s)/R)

w = logspace(3,8,1000);
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
figure
semilogx(w/2/pi,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (Hz)')
ylabel('Magnitude, (dB)')
axis([1e3 1e7 -5 20])
```

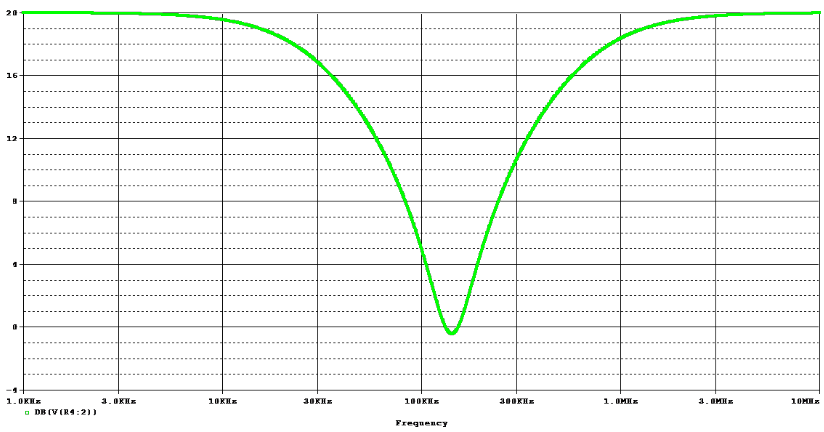
The corresponding plot is shown below.



(d). Simulate your circuit using OrCAD and compare the results to the MATLAB output.

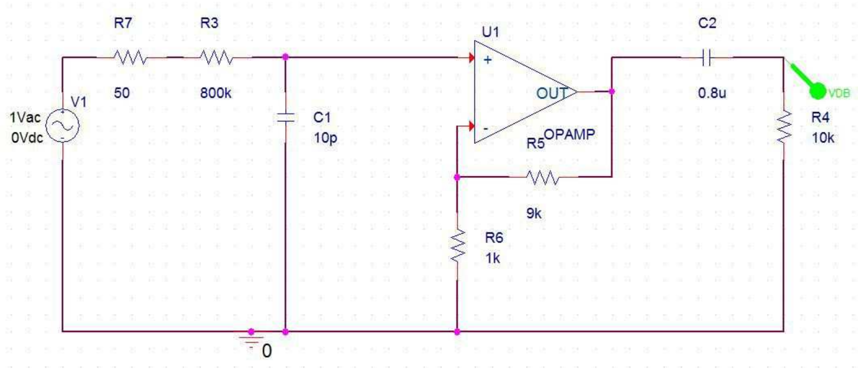
The OrCAD simulation and resulting plot are shown below. The plot agrees with the MATLAB results.



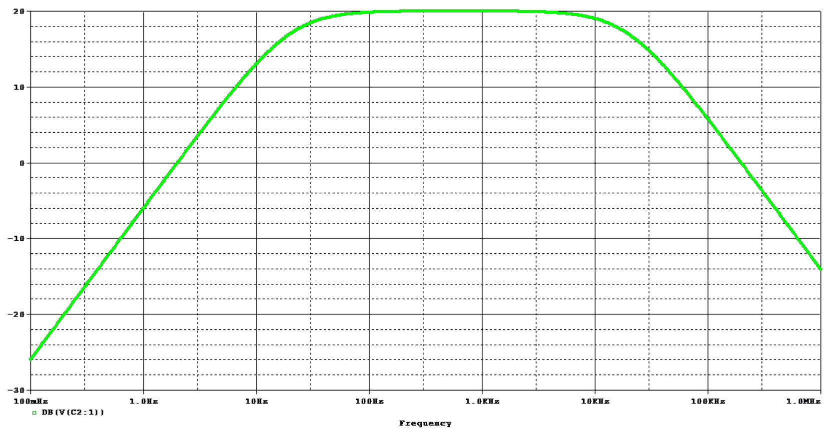


Problem 12–25. Design an audio amplifier that amplifies signals from 20 Hz to 20 kHz. Your approach should be to use a cascade connection of two first-order passive circuits separated by a non-inverting OP AMP. The source has a 50-Ω series resistor and the output of the filter feeds a 10-kΩ audio transducer. Design a bandpass circuit with the following specifications all ±5%: $f_{\text{CL}} = 20$ Hz, $f_{\text{CH}} = 20$ kHz, $B = 19980$ Hz. Passband gain +40 dB. Use OrCAD to validate your results.

Make the low-pass filter the first stage of the circuit. It will be responsible for the upper cutoff frequency of 20 kHz $= 40\pi$ krad/s and we will choose the resistance value such that the source resistance is not a significant factor. The non-inverting amplifier will provide the gain of 20 dB, which is equivalent to an absolute gain of 10. The high-pass filter will be the third stage and it will use the 10-kΩ load as part of its design to achieve the lower cutoff frequency of 20 Hz = 40π rad/s. The design is shown in the following OrCAD simulation.

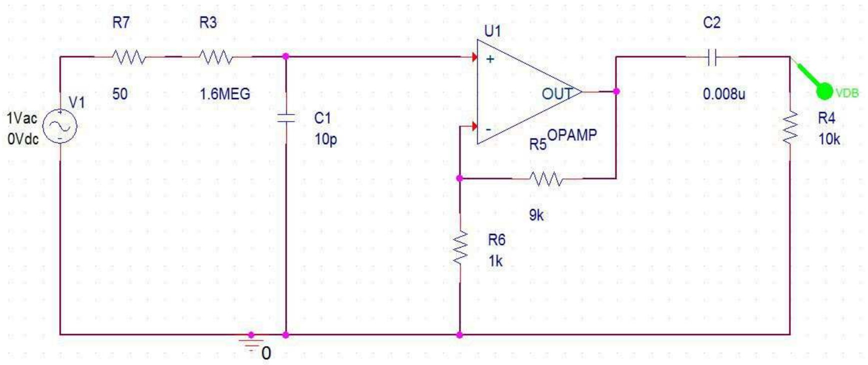


The corresponding gain response is shown below.

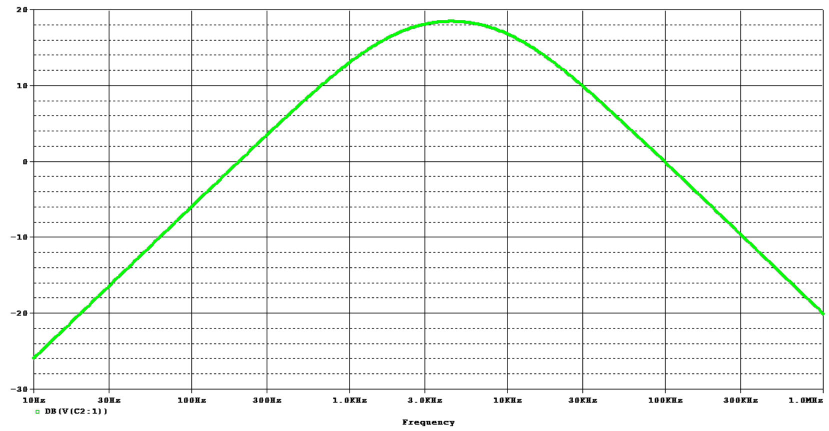


Problem 12–26. Design an audio amplifier that amplifies signals from 2 kHz to 10 kHz. Your approach should be to use a cascade connection of two first-order passive circuits separated by a non-inverting OP AMP. The source has a 50-Ω series resistor and the output of the filter feeds a 10-kΩ audio transducer. Design a bandpass circuit with the following specifications all ±5%: $f_{\mathrm{CL}} = 2$ kHz, $f_{\mathrm{CH}} = 10$ kHz, $B = 8$ kHz. Passband gain +40 dB. Use OrCAD to validate your results. Even though Problems 12–25 and 12–26 share similar constructs one cannot achieve the desired specifications in 12–26. What is the reason for the problem?

Make the low-pass filter the first stage of the circuit. It will be responsible for the upper cutoff frequency of 10 kHz = 20π krad/s and we will choose the resistance value such that the source resistance is not a significant factor. The non-inverting amplifier will provide the gain of 20 dB, which is equivalent to an absolute gain of 10. The high-pass filter will be the third stage and it will use the 10-kΩ load as part of its design to achieve the lower cutoff frequency of 2 kHz = 4π krad/s. The design is shown in the following OrCAD simulation.



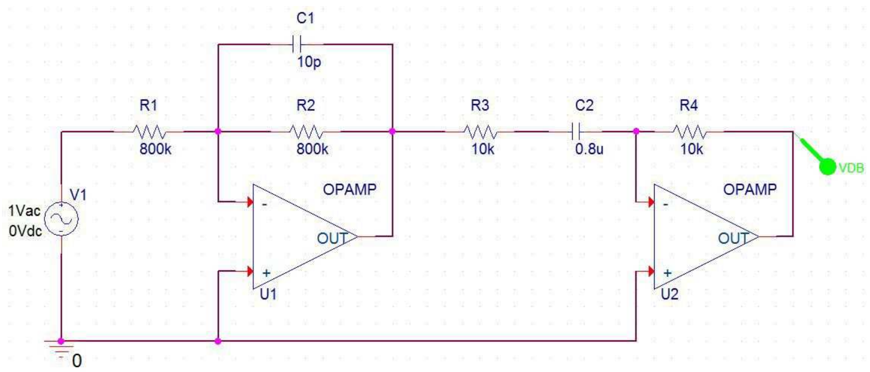
The corresponding gain response is shown below.



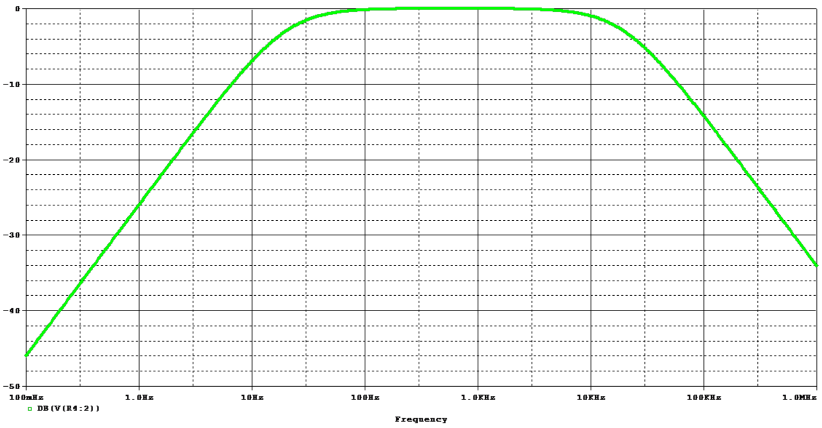
The simulation shows that the circuit does not meet all of the specifications. The passband gain is not within 5% of 20 dB. If we increase the gain in the non-inverting amplifier stage to achieve a passband gain of 20 dB, then the resulting magnitude gain response does not meet the bandwidth specification. With this magnitude response, the cutoff frequencies are not correct and the bandwidth is too large. We cannot achieve the specifications because the cutoff frequencies are too close to each other for a pair of first-order filters. In Problem 12-18, the bandwidth was large enough to allow the high-pass filter to approach its maximum gain before the low-pass filter began attenuating the signal. That is not the case with this circuit, so we cannot meet the specifications with the proposed circuit type.

Problem 12–27. Design an audio amplifier that amplifies signals from 20 Hz to 20 kHz. Your approach should be to use a cascade connection of two first-order active OP AMP circuits. The source has a 50-Ω series resistor and the output of the filter feeds a 10-kΩ audio transducer. Design a bandpass circuit with the following specifications all ±5%: $f_{\text{CL}} = 20$ Hz, $f_{\text{CH}} = 20$ kHz, $B = 19980$ Hz. Passband gain should be adjustable (use a potentiometer) from 0 dB to +40 dB. Use OrCAD to validate your results.

The circuit design is shown in the following OrCAD simulation.



The corresponding gain response is shown below.



To adjust the gain, vary the 10-kΩ resistor (R4 in the OrCAD simulation) from 10 kΩ to 1 MΩ.

Problem 12–28. A student needed to design a bandstop filter that was to block frequencies between 1000 rad/s and 10000 rad/s with unity gain in the passbands. His design is shown in Figure P12–28. As a teaching assistant, you are required to grade his design. What grade would you assign and what critique would you give him?

The general structure of the circuit is correct. The high-pass filter has a cutoff frequency of $1/RC = 1/[(0.1\ \mu)(10000)] = 1000$ rad/s and the low-pass filter has a cutoff frequency of $1/[0.01\,\mu)(10000)] = 10000$ rad/s. The cutoff frequencies have the correct values, but they are assigned to the wrong filters. As designed, the filter is a bandpass filer. To correct the design, for each OP AMP filter, swap the positions of the capacitor and the resistor. Since the error is relatively minor, a B would be an appropriate grade.

Problem 12–29. Determine the filter type for the circuit in Figure P12–29. Then find Q, B, $\omega_{\rm C1}$, $\omega_{\rm C2}$, and ω_0. Is the circuit a narrow-band or a wide-band filter?

The circuit is a series RLC bandpass filter with gain. The gain from the amplifier is +5. For the RLC circuit, we have the following results:

$$R = 410\ \Omega$$

$$L = 100\ \text{mH}$$

$$C = 25\ \mu\text{F}$$

$$K = 5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 632.5\ \text{rad/s}$$

$$B = \frac{R}{L} = 4100\ \text{rad/s}$$

$$Q = \frac{\omega_0}{B} = 0.1543$$

$$\omega_{\rm C1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = 95.34\ \text{rad/s}$$

$$\omega_{\rm C2} = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = 4195.3\ \text{rad/s}$$

The circuit is a wide-band filter because $Q < 1$.

Problem 12–30. Design an RLC bandstop filter with a center frequency of 33 krad/s and a bandwidth of 3.3 krad/s. The passband gain is 0 dB. Use practical values for R, L, and C.

Use a series RLC circuit with the output taken across the series combination of the inductor and capacitor. We have the following calculations, where we have picked a value for the inductor:

$$\omega_0 = 33 \text{ krad/s}$$

$$B = 3.3 \text{ krad/s}$$

$$K = 0 \text{ dB}$$

$$B = \frac{R}{L}$$

$$L = 100 \text{ mH}$$

$$R = 330\ \Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$C = 9180 \text{ pF}$$

A gain of 0 dB is an absolute gain of one, so an additional gain stage is not required.

Problem 12–31. Design an RLC bandpass filter with a center frequency of 1000 rad/s and a Q of 20. The passband gain is +20 dB. Use practical values for R, L, and C.

Use a series RLC circuit with the output taken across the resistor. Connect the filter to a noninverting amplifier with a gain of 10, which is +20 dB. We have the following results, where we have picked a value for the inductor:

$$\omega_0 = 1000 \text{ rad/s}$$

$$Q = 20$$

$$B = \frac{\omega_0}{Q} = 50 \text{ rad/s}$$

$$B = \frac{R}{L}$$

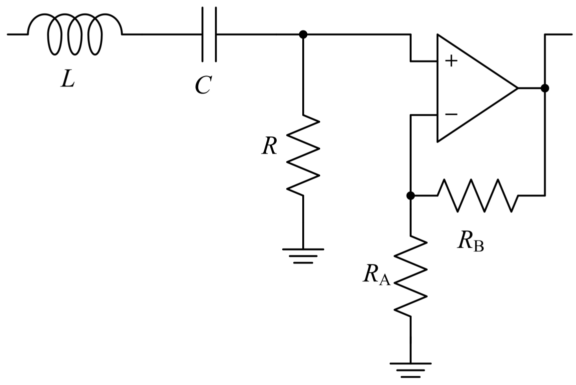
$$L = 1 \text{ H}$$

$$R = 50\ \Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$C = 1\ \mu\text{F}$$

The design is shown below, where $R_{\text{A}} = 1$ kΩ and $R_{\text{B}} = 9$ kΩ.



Problem 12–32. A series RLC bandpass circuit with $R = 30\ \Omega$ is designed to have a bandwidth of 5 Mrad/s and a center frequency of 50 Mrad/s. Find L, C, Q, and the two cutoff frequencies. Could you design this circuit using a cascade connection of two first-order filters separated by a follower? Why or why not?

We have the following results:

$$
\begin{aligned}
R &= 30\ \Omega \\
B &= 5 \text{ Mrad/s} \\
\omega_0 &= 50 \text{ Mrad/s} \\
B &= \frac{R}{L} \\
L &= \frac{R}{B} = 6\ \mu\text{H} \\
C &= \frac{1}{\omega_0^2 L} = 66.7 \text{ pF} \\
Q &= \frac{\omega_0}{B} = 10 \\
\omega_{C1} &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = 47.56 \text{ Mrad/s} \\
\omega_{C2} &= +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = 52.56 \text{ Mrad/s}
\end{aligned}
$$

The filter cannot be designed with a cascade connection of first-order filters because the quality factor is too high.

Problem 12–33. A parallel RLC bandpass circuit with $C = 0.01\ \mu$F and $Q = 10$ has a center frequency of 500 krad/s. Find R, L, and the two cutoff frequencies. Could you design this circuit using a cascade connection of two first-order filters separated by a follower? Why or why not?

We have the following results:

$$C = 0.01\ \mu\text{F}$$

$$Q = 10$$

$$\omega_0 = 500\ \text{krad/s}$$

$$B = \frac{\omega_0}{Q} = 50\ \text{krad/s} = \frac{1}{RC}$$

$$R = \frac{1}{BC} = 2\ \text{k}\Omega$$

$$L = \frac{1}{\omega_0^2 C} = 400\ \mu\text{H}$$

$$\omega_{C1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} = 475.6\ \text{krad/s}$$

$$\omega_{C2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} = 525.6\ \text{krad/s}$$

The filter cannot be designed with a cascade connection of first-order filters because the quality factor is too high.

Problem 12–34. A 20-mH inductor with an internal series resistance of 25 Ω is connected in series with a capacitor and a voltage source with a Thévenin resistance of 50 Ω.

(a). What value of C is needed to produce $\omega_0 = 5$ krad/s?

Use the analysis of a series RLC design, where $R = 25 + 50 = 75\ \Omega$.

$$C = \frac{1}{\omega_0^2 L} = 2\ \mu\text{F}$$

(b). Find the bandwidth and quality factor of the circuit.

We have the following results:

$$B = \frac{R}{L} = \frac{75}{0.02} = 3.75\ \text{krad/s}$$

$$Q = \frac{\omega_0}{B} = \frac{5000}{3750} = 1.333$$

Problem 12–35. In a series RLC circuit, which element would you adjust (and by how much) to

(a). Double the bandwidth without changing the center frequency?

We have the following relationships:

$$B = \frac{R}{L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

To double the bandwidth without changing the center frequency, double the value for R.

(b). Double the center frequency without changing the bandwidth?

Based on the relationships listed above, to double the center frequency without changing the bandwidth, reduce C by a factor of four.

(c). Repeat parts (a) and (b) for a parallel RLC circuit.

We have the following relationships:

$$B = \frac{1}{RC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

To double the bandwidth, reduced R by a factor of two. To double the center frequency, reduce L by a factor of four.

Problem 12–36. A parallel RLC circuit with $R = 1$ kΩ has a center frequency of 50 kHz and a bandwidth of 50 kHz. Find the values of L and C. Could you design this circuit using a cascade connection of two first-order filters separated by a follower? Why or why not?

We have the following relationships:

$$R = 1 \text{ k}\Omega$$

$$\omega_0 = 2\pi(50000) = 100\pi \text{ krad/s}$$

$$B = 2\pi(50000) = 100\pi \text{ krad/s}$$

$$C = \frac{1}{RB} = 3183 \text{ pF}$$

$$L = \frac{1}{\omega_0^2 C} = 3.183 \text{ mH}$$

$$\omega_{C1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} = 194 \text{ krad/s}$$

$$\omega_{C2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} = 508 \text{ krad/s}$$

The two cutoff frequencies are relatively close together. It would be difficult to directly design this filter using two first-order filters separated by a follower.

Problem 12–37. A series RLC bandpass filter is required to have resonance at $f_0 = 50$ kHz. The circuit is driven by a sinusoidal source with a Thévenin resistance of 60 Ω. The following standard capacitors are available in the stock room: 1 μF, 0.68 μF, 0.47 μF, 0.33 μF, 0.2 μF, and 0.12 μF. The inductor will be custom-designed to match the capacitor used. Select the available capacitor that minimizes the filter bandwidth.

For a series RLC circuit, we have the following relationships:

$$\omega_0 = 2\pi(50000) = 100\pi \text{ krad/s}$$

$$R = 60\ \Omega$$

$$B = \frac{R}{L}$$

$$L = \frac{1}{\omega_0^2 C}$$

$$B = RC\omega_0^2$$

To minimize the bandwidth, select the smallest capacitor, $C = 0.12\ \mu$F.

Problem 12–38. A series RLC bandstop circuit is to be used as a notch filter to eliminate a bothersome 120-Hz hum in an audio channel. The signal source has a Thévenin resistance of 300 Ω. Select values of L and C so the upper cutoff frequency of the stopband is below 180 Hz. Use OrCAD to verify your design.

For a series RLC bandstop filter, we have the following relationships, where we have selected an upper cutoff frequency of $f_{C2} = 150$ Hz to meet the specification:

$$\omega_0 = 240\pi \text{ rad/s}$$

$$R = 300\ \Omega$$

$$\omega_{C2} = 300\pi \text{ rad/s}$$

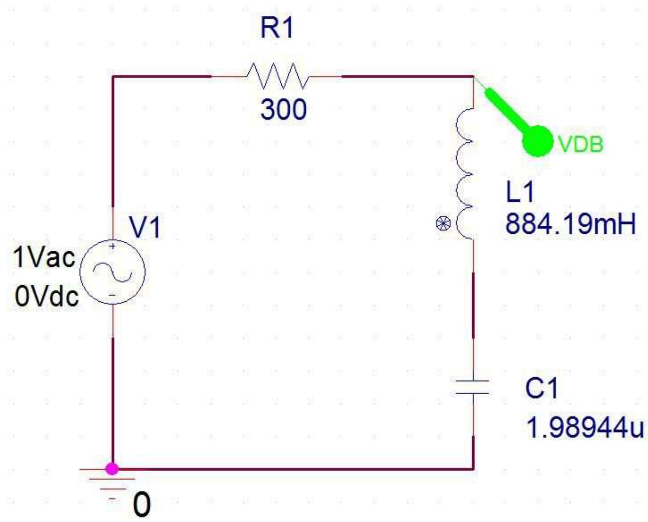
$$\omega_{C1} = \frac{\omega_0^2}{\omega_{C2}} = 192\pi \text{ rad/s}$$

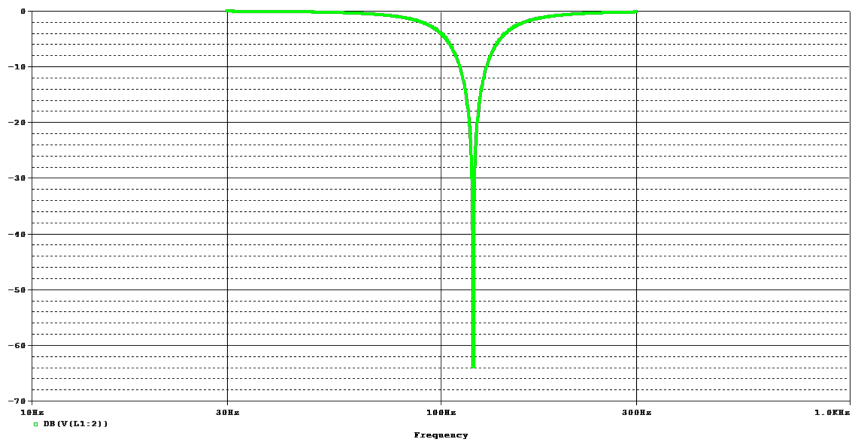
$$B = \omega_{C2} - \omega_{C1} = 108\pi \text{ rad/s}$$

$$L = \frac{R}{B} = 884 \text{ mH}$$

$$C = \frac{1}{\omega_0^2 L} = 1.99\ \mu\text{F}$$

The OrCAD simulation and corresponding results are shown below to verify the design.





Problem 12–39. Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ for the bandpass circuit in Figure P12–39. Use MATLAB to visualize the Bode characteristics if $R = 50\ \Omega$, $L = 500\ \mu$H, and $C = 0.002\ \mu$F. Design an active circuit to meet those characteristics. Verify your design using OrCAD.

Find the LC parallel impedance and then determine the transfer function.

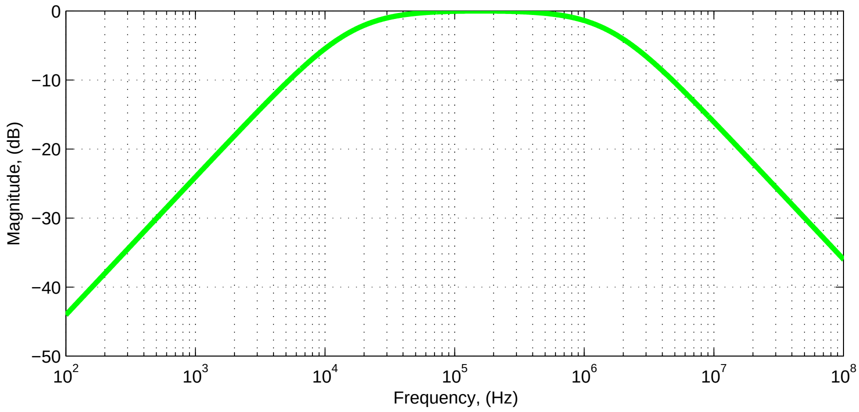
$$Z = \frac{Ls/Cs}{Ls + 1/Cs} = \frac{Ls}{LCs^2 + 1}$$

$$T(s) = \frac{\dfrac{Ls}{LCs^2+1}}{R + \dfrac{Ls}{LCs^2+1}} = \frac{Ls}{RLCs^2 + Ls + R} = \frac{\dfrac{1}{RC}s}{s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC}}$$

The MATLAB code to visualize the Bode characteristic is shown below:

```
syms s
R = 50;
L = 500e-6;
C = 0.002e-6;
Ts = factor(L*s/(R*L*C*s^2+L*s+R));
w0 = sqrt(1/L/C)
wc1 = -1/2/R/C + sqrt(1/4/R^2/C^2 + 1/L/C)
wc2 = 1/2/R/C + sqrt(1/4/R^2/C^2 + 1/L/C)
B = wc2-wc1
BHz = B/2/pi

w = logspace(2,9,2000);
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
figure
semilogx(w/2/pi,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (Hz)')
ylabel('Magnitude, (dB)')
axis([100 1e8 -50 0])
```

The corresponding plot is shown below.



Using the transfer function, we have the following results:

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = 1 \text{ Mrad/s}$$

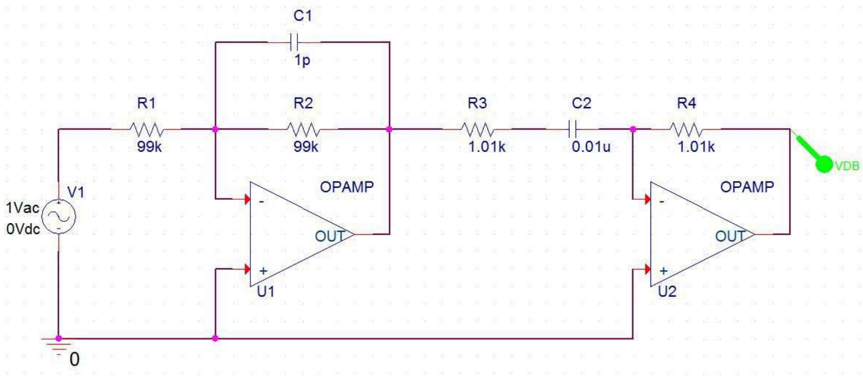
$$B = \frac{1}{RC} = 10 \text{ Mrad/s}$$

$$Q = \frac{\omega_0}{B} = 0.1$$

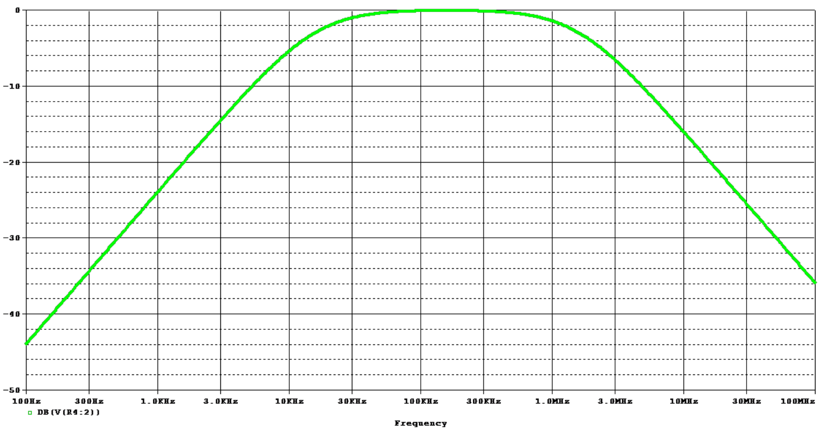
$$\omega_{C1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} = 99 \text{ krad/s}$$

$$\omega_{C2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} = 10.099 \text{ Mrad/s}$$

Design the circuit as an active low-pass filter with a cutoff frequency of 10.1 Mrad/s in cascade with an active high-pass filter with a cutoff frequency of 99 krad/s. The design is shown below in the OrCAD simulation.



The corresponding output is shown below and it matches the desired response.



Problem 12–40. Show that the transfer function $T_V(s) = V_2(s)/V_1(s)$ of the circuit in Figure P12–40 has a bandstop filter characteristic. Derive expressions relating the notch frequency and the cutoff frequencies to R, L, and C. Then select values of R, L, and C so that the bandwidth and the center frequency are both 10 krad/s. Validate you design using OrCAD.

Find the LC parallel impedance and then determine the transfer function.

$$Z = \frac{Ls/Cs}{Ls + 1/Cs} = \frac{Ls}{LCs^2 + 1}$$

$$T(s) = \frac{R}{R + \dfrac{Ls}{LCs^2 + 1}} = \frac{RLCs^2 + R}{RLCs^2 + Ls + R} = \frac{R(LCs^2 + 1)}{RLCs^2 + Ls + R}$$

The transfer function has the form of a bandstop filter. The notch occurs at the zero of the transfer function or $\omega_0 = 1/\sqrt{LC}$. The passband gain of the filter is one at both high and low frequencies. Determine the cutoff frequencies where the filter drops to $1/\sqrt{2}$.

$$T(j\omega) = \frac{R(1 - \omega^2 LC)}{R(1 - \omega^2 LC) + j\omega L}$$

$$|T(j\omega)| = \frac{1}{\sqrt{2}} = \frac{R(1 - \omega^2 LC)}{\sqrt{R^2(1 - \omega^2 LC)^2 + \omega^2 L^2}}$$

The cutoff frequency occurs when $R(1 - \omega^2 LC) = \omega L$, so we have the following results:

$$R(1 - \omega^2 LC) = \omega L$$

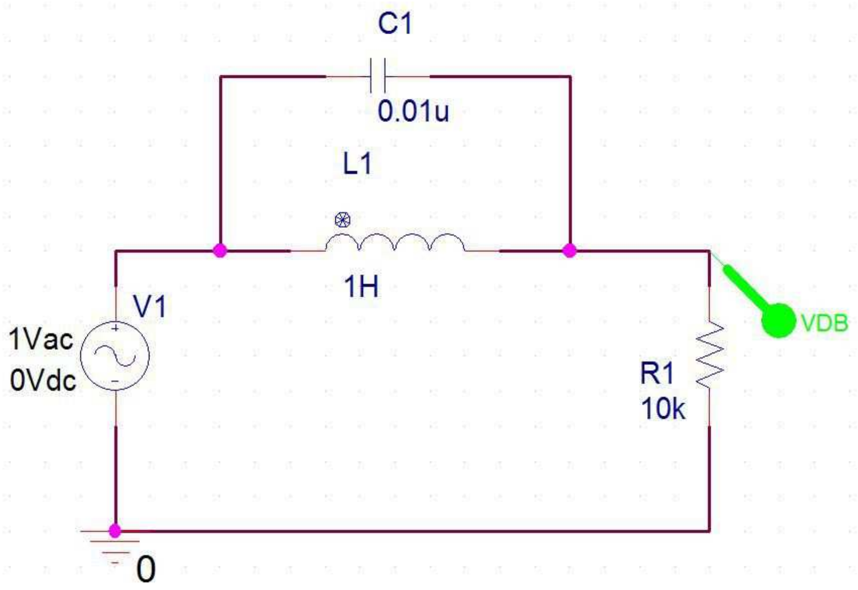
$$RLC\omega^2 + L\omega - R = 0$$

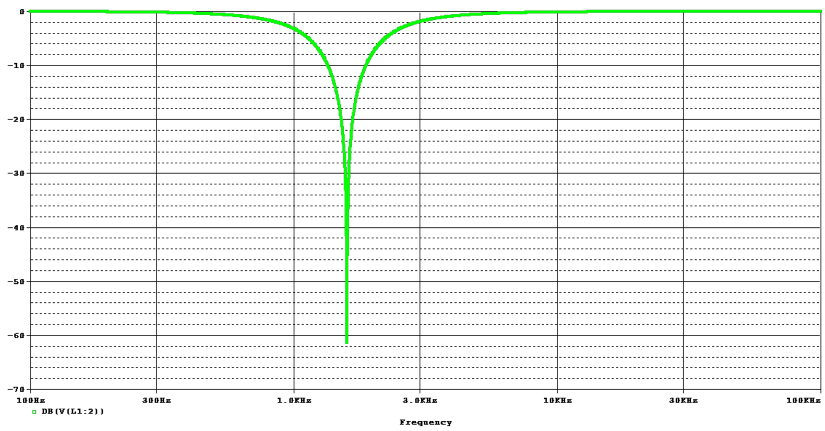
$$\omega_{C1} = \frac{-L + \sqrt{L^2 + 4R^2 LC}}{2RLC} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_{C2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \frac{1}{RC}$$

Pick $R = 10$ kΩ and solve for $C = 0.01$ μF and $L = 1$ H to meet the specifications. The OrCAD simulation and corresponding plot are shown below.





Problem 12–41. Figure P12–41 shows an RLC filter with an input current and an output voltage. The purpose of this problem is to determine the filter type using informal circuit analysis. Use the element impedances and basic analysis tools to find the magnitude of the output voltage $|V_2(j\omega)|$ at $\omega = 0$, $\omega = \infty$, and $\omega = \omega_0 = 1/\sqrt{LC}$. What is the filter type?

At $\omega = 0$, the capacitor acts like an open circuit and the inductor acts like a short circuit. The current flows through the resistor and the inductor and the output voltage is zero, so the gain is zero.

At $\omega = \infty$, the capacitor acts like a short circuit and the inductor acts like an open circuit. The current flows through the capacitor only and the output voltage is again zero, so the gain is zero.

At $\omega = \omega_0 = 1/\sqrt{LC}$, we can find the impedance of the capacitor and the inductor and then use current division to find the output current. Once we know the output current, we can find the output voltage.

$$Z_{\mathrm{C}} = \frac{1}{j\omega C} = -j\sqrt{\frac{L}{C}}$$

$$Z_{\mathrm{L}} = j\omega L = j\sqrt{\frac{L}{C}}$$

$$I_2 = \frac{-j\sqrt{\frac{L}{C}}}{-j\sqrt{\frac{L}{C}} + R + j\sqrt{\frac{L}{C}}}(I_1) = -j\frac{1}{R}\sqrt{\frac{L}{C}}I_1$$

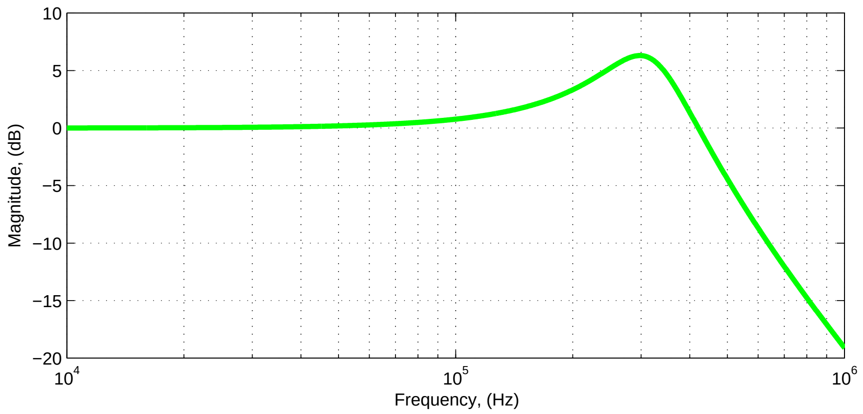
$$V_2 = Z_{\mathrm{L}}I_2 = -j\sqrt{\frac{L}{C}}j\frac{1}{R}\sqrt{\frac{L}{C}}I_1 = \frac{L}{RC}I_1$$

There is an output at the center frequency, but no output at low and high frequencies. The filter is a bandpass filter.

Problem 12–42. A student was designing a passive tuned filter using a series RLC circuit with $R = 10\ \Omega$, $L = 10\ \mu$H, and $C = 0.025\ \mu$F. When he connected the output of the filter across the capacitor he was surprised by an output near the cutoff point that was greater than the input signal. Since the filter was passive he was quite perplexed. Explain how this can be.

The output of a passive filter can have a gain above one near the cutoff frequency because of resonance with $\zeta < 0.5$. The following MATLAB code and plot shows that this circuit displays a resonant peak at the cutoff point.

```
syms s
R = 10;
L = 10e-6;
C = 0.025e-6;
Ts = factor(1/C/s/(R+L*s+1/C/s));
w0 = sqrt(1/L/C)
wc1 = -R/2/L + sqrt(R^2/4/L^2 + 1/L/C)
wc2 = R/2/L + sqrt(R^2/4/L^2 + 1/L/C)
w = [wc1 wc2];
Tjw = subs(Ts,s,j*w);
TjwMag = abs(Tjw)
TjwMagdB = 20*log10(TjwMag)

w = logspace(4,7,2000);
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
MagTjwdB = 20*log10(MagTjw);
figure
semilogx(w/2/pi,MagTjwdB,'g','LineWidth',3)
grid on
hold on
xlabel('Frequency, (Hz)')
ylabel('Magnitude, (dB)')
```



Problem 12–43. Find the transfer function $T_V(s) = V_2(s)/V_1(s)$ for the circuit in Figure P12–43.

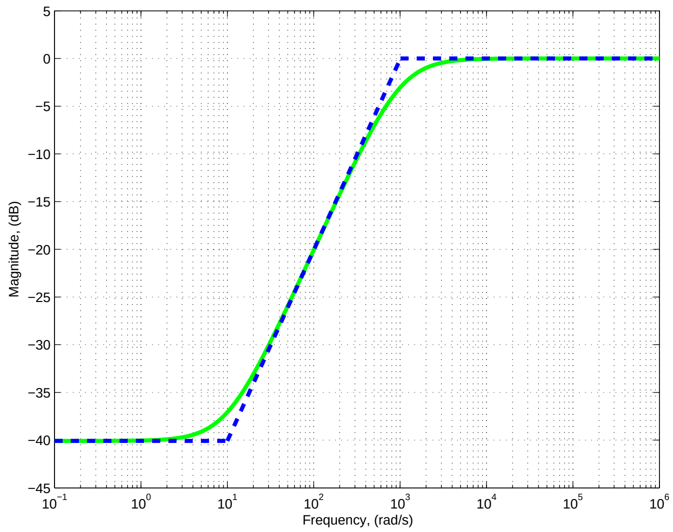
Determine the transfer function.

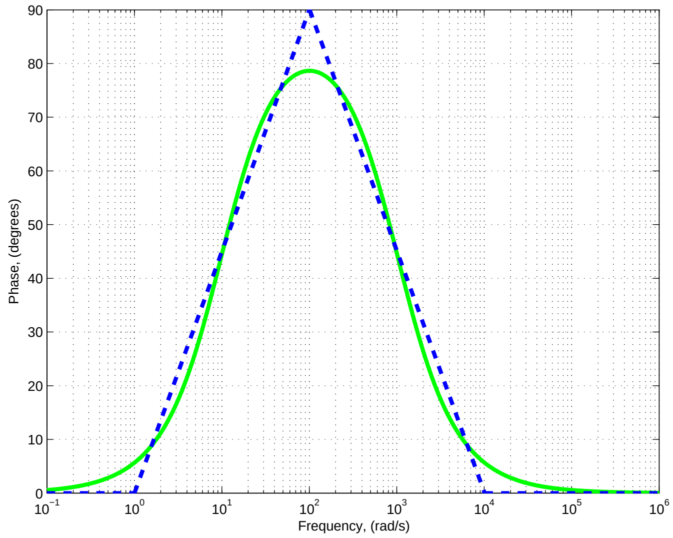
$$T_V(s) = \frac{R_2}{R_2 + \dfrac{R_1/C_1 s}{R_1 + 1/C_1 s}} = \frac{R_1 R_2 C_1 s + R_2}{R_1 R_2 C_1 s + R_1 + R_2}$$

$$= \frac{s + \dfrac{R_2}{R_1 R_2 C_1}}{s + \dfrac{R_1 + R_2}{R_1 R_2 C_1}} = \frac{s + 10}{s + 1010}$$

(a). Construct the straight-line Bode plot of the gain and phase of the transfer function. Use the straight-line plots to estimate the amplitude and phase of the steady-state output for $v_1(t) = 10\sin(100t)$ V.

The transfer function has a zero at $s = -10$ rad/s and a pole at $s = -1010$ rad/s. The low-frequency gain is $10/1010 = 0.0099$ or approximately -40 dB and the high-frequency gain is one or 0 dB. The following MATLAB plots show the actual and straight-line approximations for the gain and phase responses.





Based on the straight-line approximations at $\omega = 100$ rad/s, the gain is -20 dB or 0.1 and the phase shift is $+90°$. We have the following estimate for the steady-state output

$$v_1(t) = 10\sin(100t) = 10\cos(100t - 90°)$$

$$v_2(t) = (0.1)(10)\cos(100t - 90° + 90°) = \cos(100t)\ \text{V}$$

(b). Calculate the actual output amplitude and phase for this input and compare the two results.

Use the transfer function to determine the actual response at $\omega = 100$ rad/s.

$$T_{\text{V}}(j100) = \frac{10 + j100}{1010 + j100} = 0.099\angle 78.6°$$

$$v_2(t) = 0.99\cos(100t - 11.4°)\ \text{V}$$

The amplitude is within 1% of the approximation, but the phase is off by a significant amount.

(c). Use MATLAB to generate a Bode plot of your transfer function. Compare your answers from the straight-line plot and your hand calculations, with that produced by MATLAB.

The MATLAB plots are shown in part (a). The straight-line approximation is reasonably good, but has larger errors when the straight-line approximation changes slope.

Problem 12–44. Repeat Problem 12–43 using the circuit in Figure P12–44.
Determine the transfer function.

$$Z_1 = R_1$$

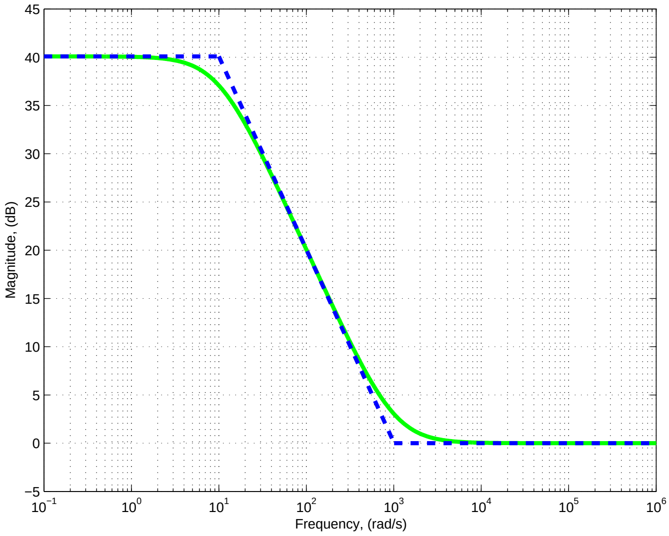
$$Z_2 = \frac{R_2/C_2 s}{R_2 + 1/C_2 s} = \frac{R_2}{R_2 C_2 s + 1}$$

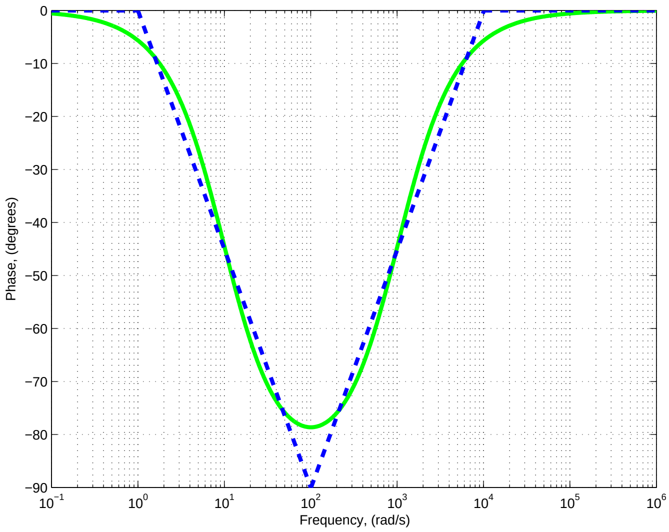
$$T_{\text{V}}(s) = \frac{Z_1 + Z_2}{Z_1} = \frac{R_1 + \dfrac{R_2}{R_2 C_2 s + 1}}{R_1}$$

$$= \frac{R_1 R_2 C_2 s + R_1 + R_2}{R_1 R_2 C_2 s + R_1} = \frac{s + 1010}{s + 10}$$

(a). Construct the straight-line Bode plot of the gain and phase of the transfer function. Use the straight-line plots to estimate the amplitude and phase of the steady-state output for $v_1(t) = 10\sin(100t)$ V.

The transfer function has a zero at $s = -1010$ rad/s and a pole at $s = -10$ rad/s. The low-frequency gain is $1010/10 = 101$ or approximately $+40$ dB and the high-frequency gain is one or 0 dB. The following MATLAB plots show the actual and straight-line approximations for the gain and phase responses.





Based on the straight-line approximations at $\omega = 100$ rad/s, the gain is $+20$ dB or 10 and the phase shift is $-90°$. We have the following estimate for the steady-state output

$$v_1(t) = 10\sin(100t) = 10\cos(100t - 90°)$$

$$v_2(t) = (10)(10)\cos(100t - 90° - 90°) = 100\cos(100t - 180°)\text{ V}$$

(b). Calculate the actual output amplitude and phase for this input and compare the two results.

Use the transfer function to determine the actual response at $\omega = 100$ rad/s.

$$T_{\mathrm{V}}(j100) = \frac{1010 + j100}{10 + j100} = 10.099\angle{-78.6^\circ}$$

$$v_2(t) = 101\cos(100t - 168.6^\circ)\ \mathrm{V}$$

The amplitude is within 1% of the approximation, but the phase is off by a significant amount.

(c). Use MATLAB to generate a Bode plot of your transfer function. Compare your answers from the straight-line plot and your hand calculations, with that produced by MATLAB.

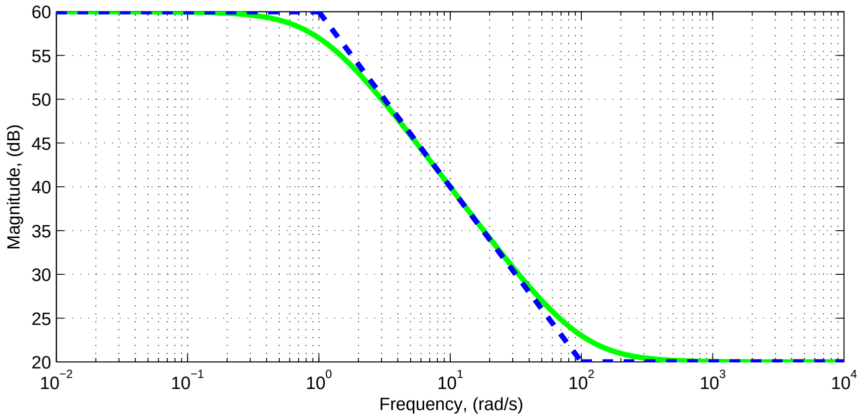
The MATLAB plots are shown in part (a). The straight-line approximation is reasonably good, but has larger errors when the straight-line approximation changes slope.

Problem 12–45. For the following transfer function

$$T(s) = \frac{10(s + 100)}{s + 1}$$

(a). Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency and passband gain.

The following MATLAB plot shows the actual and straight-line approximations for the gain response.



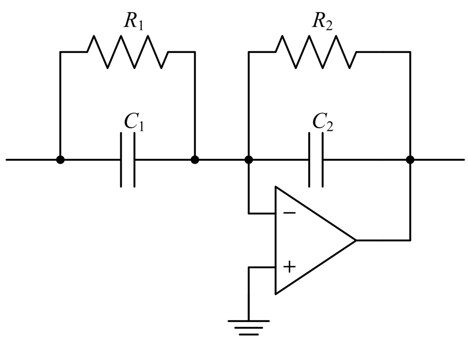
The function is a low-pass filter with a cutoff frequency of 1 rad/s and a passband gain of 60 dB or 1000.

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

Part (a) shows the actual Bode magnitude response.

(c). Design a circuit using practical values for the components that has the same transfer function.

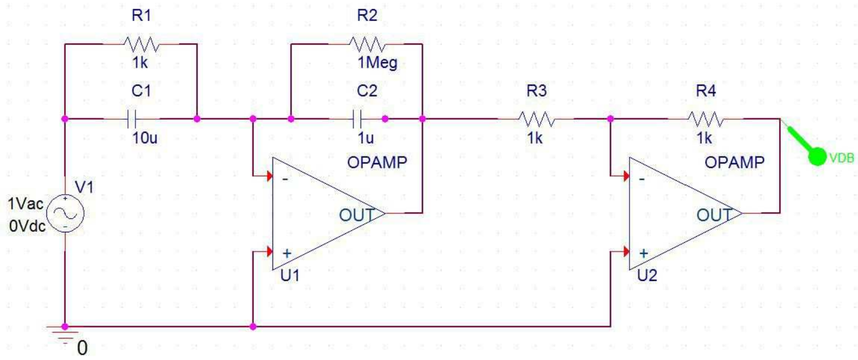
Use the following circuit to implement the transfer function with $R_1 = 1$ kΩ, $C_1 = 10\ \mu$F, $R_2 = 1$ MΩ, and $C_2 = 1\ \mu$F:

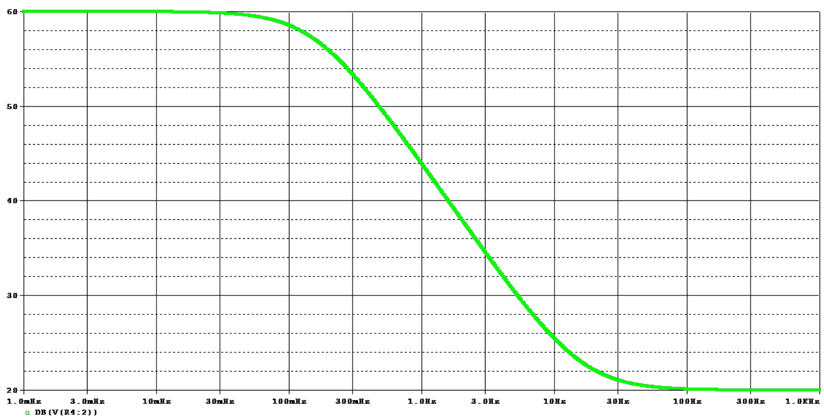


To match the sign of the transfer function, connect the output of the circuit to an inverting amplifier with a gain of -1.

(d). Use OrCAD to compare the frequency response of your designed circuit with the MATLAB Bode plot.

The OrCAD simulation and the corresponding output are shown below. The output agrees with the MATLAB results.



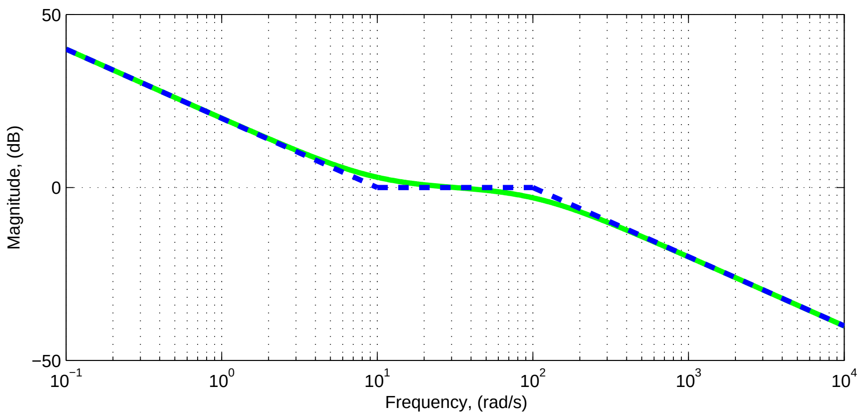


Problem 12–46. For the following transfer function

$$T(s) = \frac{100(s+10)}{s(s+100)}$$

(a). Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency and passband gain.

The following MATLAB plot shows the actual and straight-line approximations for the gain response.



The function is a low-pass filter with a cutoff frequency of 1 rad/s and a passband gain of 60 dB or 1000.

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

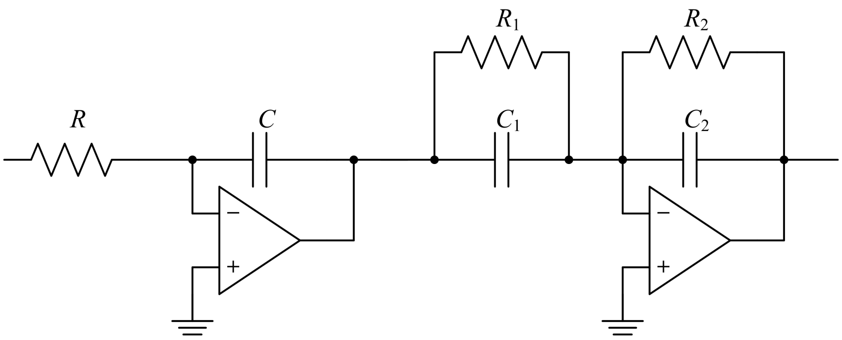
Part (a) shows the actual Bode magnitude response.

(c). Design a circuit using practical values for the components that has the same transfer function.

Write the transfer function as follows:

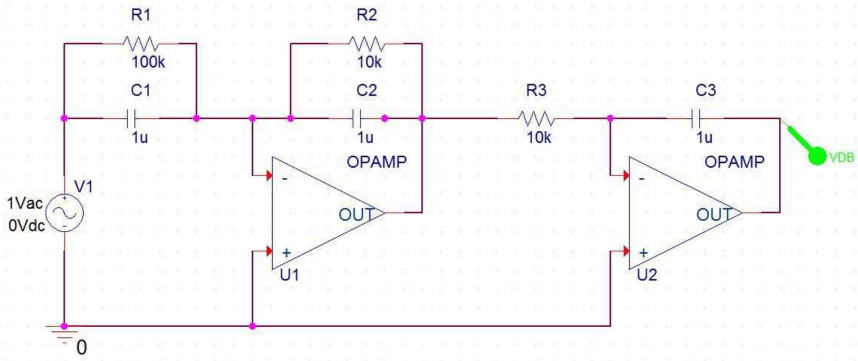
$$T(s) = \left(\frac{100}{s}\right)\left(\frac{s+10}{s+100}\right)$$

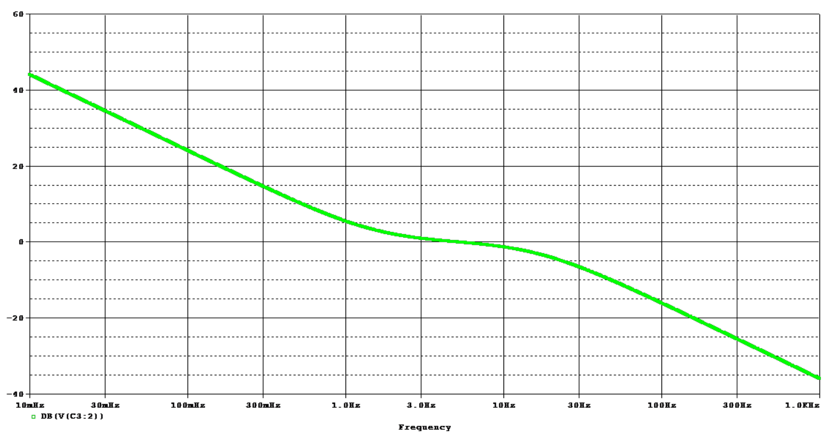
Use the following circuit to implement the transfer function with $R = 10$ kΩ, $C = 1$ μF, $R_1 = 100$ kΩ, $C_1 = 1$ μF, $R_2 = 10$ kΩ, and $C_2 = 1$ μF:



(d). Use OrCAD to compare the frequency response of your designed circuit with the MATLAB Bode plot.

The OrCAD simulation and the corresponding output are shown below. The output agrees with the MATLAB results.



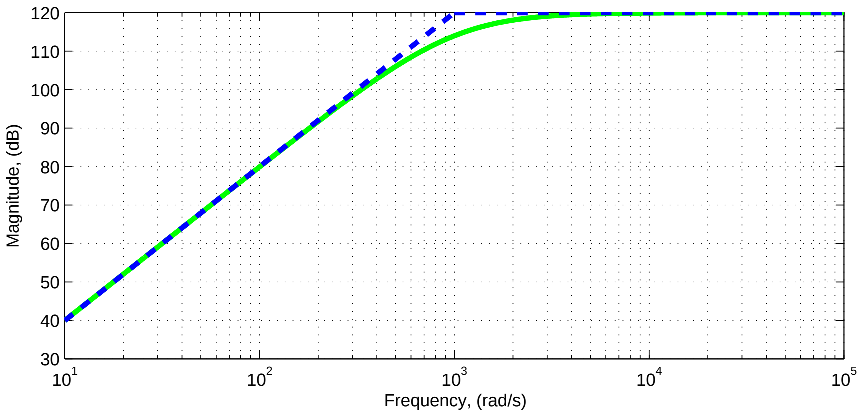


Problem 12–47. For the following transfer function

$$T(s) = \frac{10^6 s^2}{(s+1000)^2}$$

(a). Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency and passband gain.

The following MATLAB plot shows the actual and straight-line approximations for the gain response.



The function is a high-pass filter with a cutoff frequency of 1560 rad/s and a passband gain of 120 dB or 10^6. Note that having both poles at $\omega = 1000$ rad/s shifts the cutoff frequency to be higher than 1000 rad/s, even though 1000 rad/s is the corner frequency in the straight-line approximation.

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

Part (a) shows the actual Bode magnitude response.

(c). Design a circuit using practical values for the components that has the same transfer function.

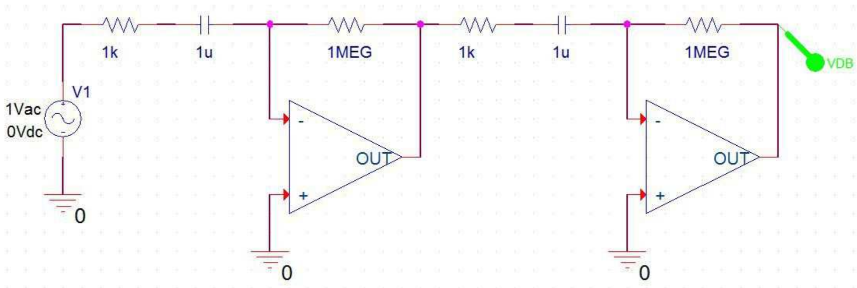
Write the transfer function as follows:

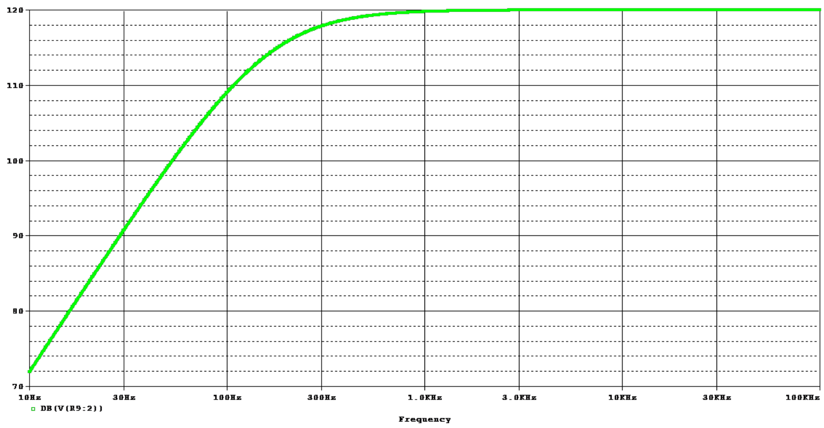
$$T(s) = \left(\frac{1000s}{s+1000}\right)^2 = \left(\frac{1000}{1+\dfrac{1000}{s}}\right)^2 = \left(\frac{10^6}{1000+\dfrac{10^6}{s}}\right)^2$$

Use two identical stages in cascade. Each stage is an active, first-order, high-pass RC filter with a 1-kΩ input resistor, a 1-μF input capacitor, and a 1-MΩ feedback resistor.

(d). Use OrCAD to compare the frequency response of your designed circuit with the MATLAB Bode plot.

The OrCAD simulation and the corresponding output are shown below. The output agrees with the MATLAB results.



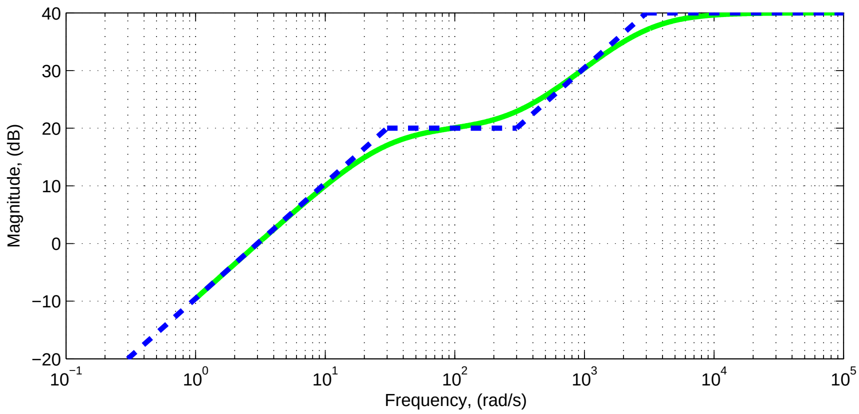


Problem 12–48. For the following transfer function

$$T(s) = \frac{100s(s+300)}{(s+30)(s+3000)}$$

(a). Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency and passband gain.

The following MATLAB plot shows the actual and straight-line approximations for the gain response.



The function is a high-pass filter with a cutoff frequency of 3 krad/s and a passband gain of 40 dB or 100.

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

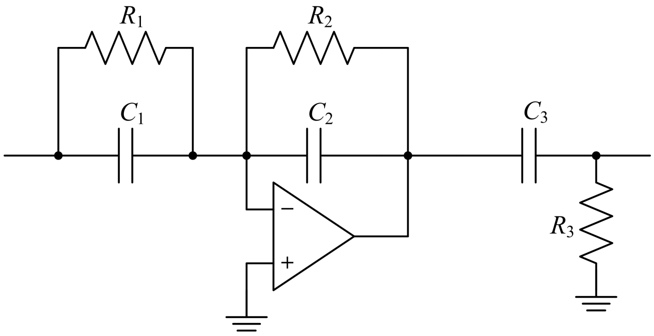
Part (a) shows the actual Bode magnitude response.

(c). Design a circuit using practical values for the components that has the same transfer function.

Write the transfer function as follows:

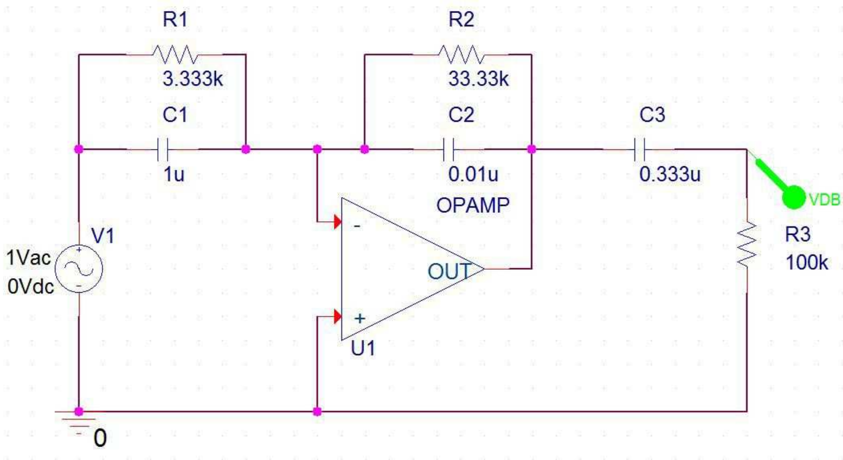
$$T(s) = \left(\frac{s}{s+30}\right)(100)\left(\frac{s+300}{s+3000}\right)$$

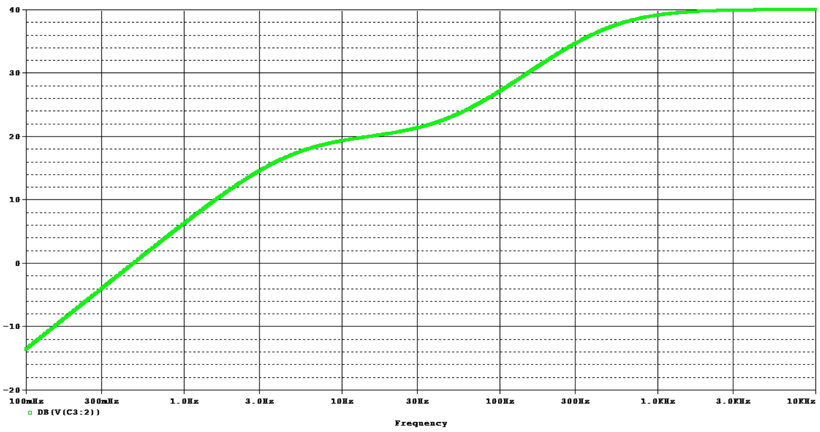
Use the following circuit to implement the transfer function with $R_1 = 3.333$ kΩ, $C_1 = 1$ μF, $R_2 = 33.33$ kΩ, $C_2 = 0.01$ μF, $R_3 = 100$ kΩ, and $C_3 = 0.333$ μF:



(d). Use OrCAD to compare the frequency response of your designed circuit with the MATLAB Bode plot.

The OrCAD simulation and the corresponding output are shown below. The output agrees with the MATLAB results.



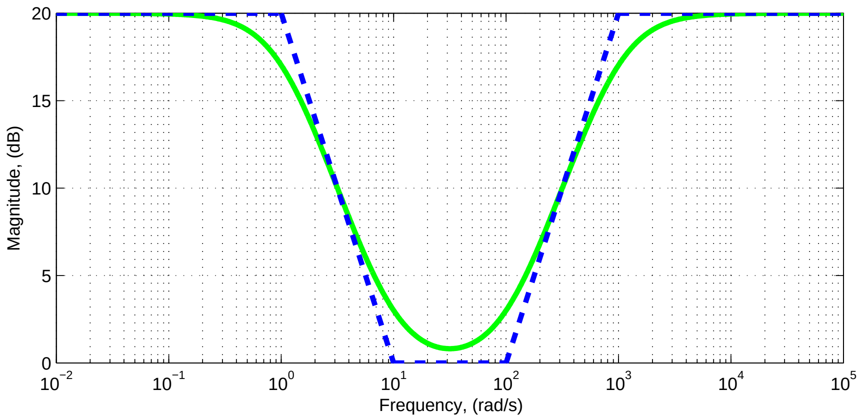


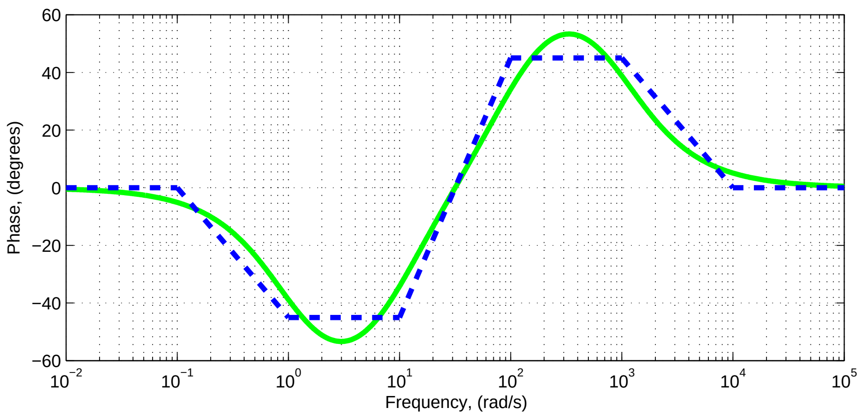
Problem 12–49. For the following transfer function $T_V(s) = V_2(s)/V_1(s)$

$$T_V(s) = \frac{10(s+10)(s+100)}{(s+1)(s+1000)}$$

(a). Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency(ies) and passband gain.

The following MATLAB plots show the actual and straight-line approximations for the gain and phase responses.





The function is a bandstop filter with cutoff frequencies at 1 rad/s and 1000 rad/s. The passband gain is 20 dB or 10.

(b). Use MATLAB to plot the Bode magnitude and phase of the transfer function.

Part (a) shows the actual Bode magnitude and phase responses.

(c). What is the output $v_2(t)$, if $v_1(t) = 10\cos(200t + 45^\circ)$ V?

We have the following results:

$$T_V(j200) = \frac{10(10 + j200)(100 + j200)}{(1 + j200)(1000 + j200)} = 2.195\angle 49.55^\circ$$

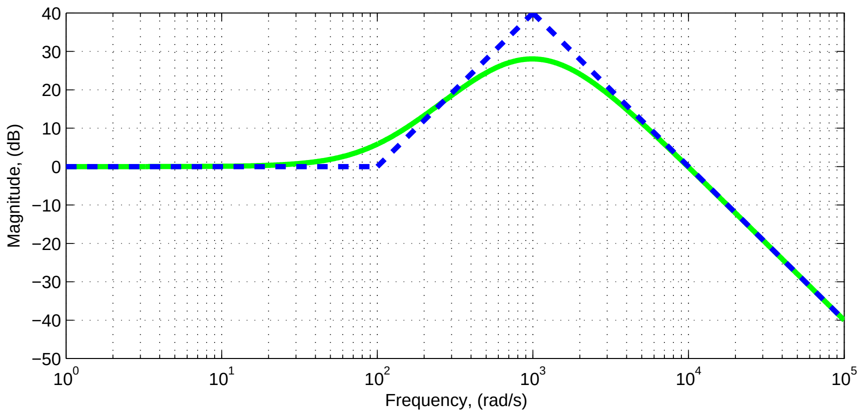
$$v_2(t) = 21.95\cos(200t + 94.55^\circ)\ \text{V}$$

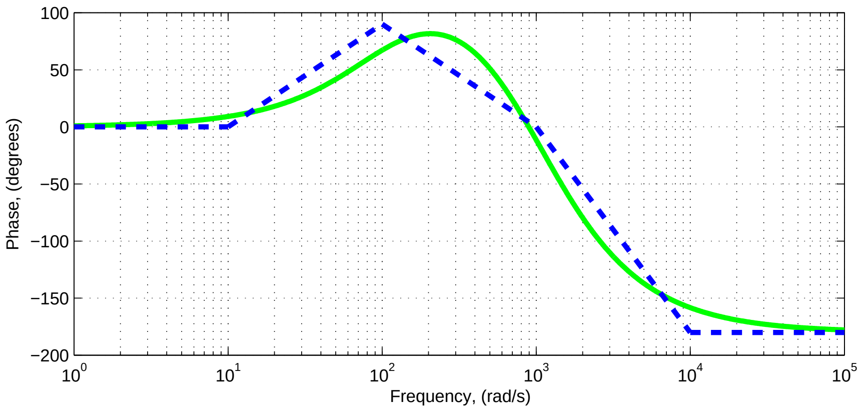
Problem 12–50. For the following transfer function $T_V(s) = V_2(s)/V_1(s)$

$$T_V(s) = \frac{10^8(s + 100)^2}{(s + 1000)^4}$$

(a). Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function? Estimate the cutoff frequency(ies) and passband gain.

The following MATLAB plots show the actual and straight-line approximations for the gain and phase responses.





The function is a low-pass filter with a cutoff frequency at 1000 rad/s. The passband gain is 0 dB or 1.

(b). Use MATLAB to plot the Bode magnitude and phase of the transfer function.

Part (a) shows the actual Bode magnitude and phase responses.

(c). What is the output $v_2(t)$, if $v_1(t) = 10\cos(1000t - 60^\circ)$ V?

We have the following results:

$$T_\mathrm{V}(j1000) = \frac{10^8(100 + j1000)^2}{(1000 + j1000)^4} = 25.25\angle{-11.42^\circ}$$

$$v_2(t) = 252.5\cos(1000t - 71.42^\circ)\ \mathrm{V}$$

Problem 12–51. For the following transfer function $T_\mathrm{V}(s) = V_2(s)/V_1(s)$

$$T_\mathrm{V}(s) = \frac{4s}{0.04s^2 + 0.2s + 1}$$

(a). Construct the straight-line Bode plot of the gain. Is this a low-pass, high-pass, bandpass, or bandstop function?

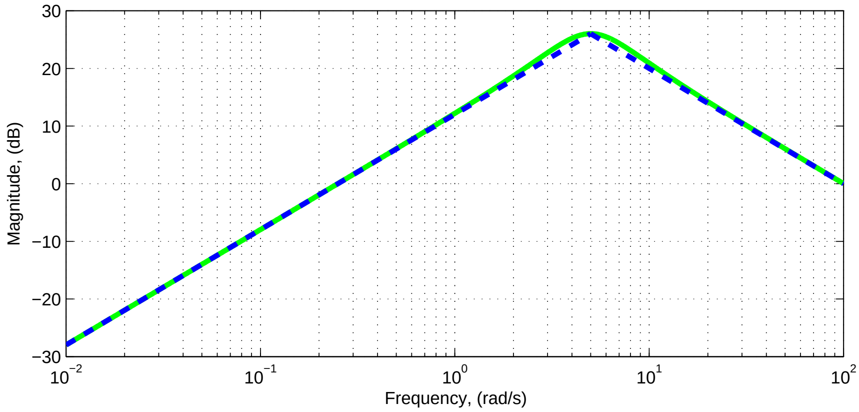
Write the transfer function as follows:

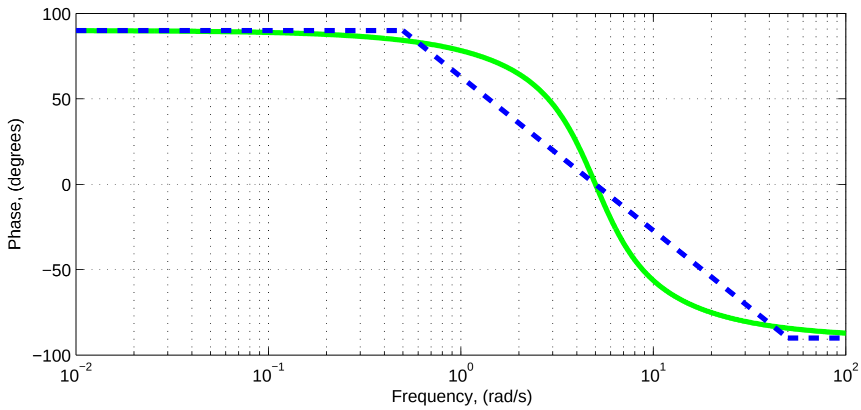
$$T_\mathrm{V}(s) = \frac{100s}{s^2 + 5s + 5^2}$$

The transfer function has a zero at 0 rad/s and complex poles at 5 rad/s. The straight-line approximation enters with a slope of +20 dB/decade up until $\omega = 5$ rad/s, where the complex poles contribute to the response. After $\omega = 5$ rad/s, the slope is -20 dB/decade. There are no other changes to the Bode magnitude plot. The damping ratio is 0.5, so we do not expect a significant resonant peak. To scale the magnitude of the plot, compute the actual magnitude at $\omega = 5$ rad/s.

$$|T_{\rm V}(j5)| = \frac{j500}{-25 + j25 + 25} = 20$$

The gain at $\omega = 5$ rad/s is 20 or 26 dB. The following MATLAB plots show the actual and straight-line approximations for the gain and phase responses.





The function is a bandpass filter with a center frequency of 5 rad/s.

(b). Use the straight-line plot to estimate the maximum gain and the frequency at which it occurs.

The maximum gain is approximately 26 dB or 20 and it occurs at $\omega = 5$ rad/s.

(c). Use MATLAB to plot the Bode magnitude and phase of the transfer function.

Part (a) shows the actual Bode magnitude and phase responses.

Problem 12–52. Consider the gain plot in Figure P12–52.

(a). Find the transfer function corresponding to the straight-line gain plot.

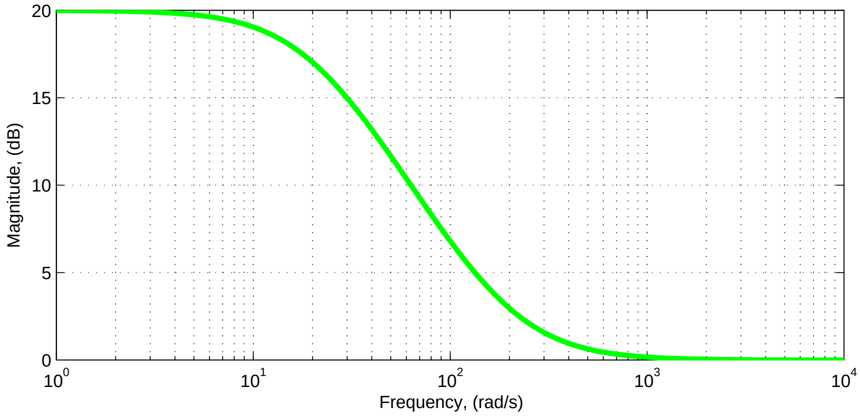
Since the gain plot is horizontal for low frequencies, there are no poles or zeros at 0 rad/s. Based on the changes in the plot, a single pole occurs at 20 rad/s and a single zero at 200 rad/s. The transfer function takes the following form:

$$T(s) = K\frac{s + 200}{s + 20}$$

The high-frequency gain is 0 dB, which is an absolute gain of one, so $K = 1$.

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

The MATLAB plot is shown below.

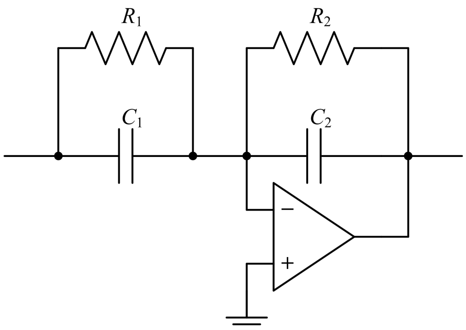


(c). Compare the straight-line gain and the actual gain at $\omega = 20$ and 200 rad/s.

At $\omega = 20$ and 200 rad/s, the straight-line gains are 20 and 0 dB, respectively. The actual gains at those frequencies are 17.03 and 2.97 dB, respectively. For both frequencies, the error is approximately 3 dB.

(d). Design a circuit to realize the Bode plot.

Use the following circuit to implement the transfer function with $R_1 = 5$ kΩ, $C_1 = 1$ μF, $R_2 = 50$ kΩ, and $C_2 = 1$ μF:



To match the sign of the transfer function, connect the output of the circuit to an inverting amplifier with a gain of −1.

Problem 12–53. Consider the gain plot in Figure P12–53.

(a). Find the transfer function corresponding to the straight-line gain plot.

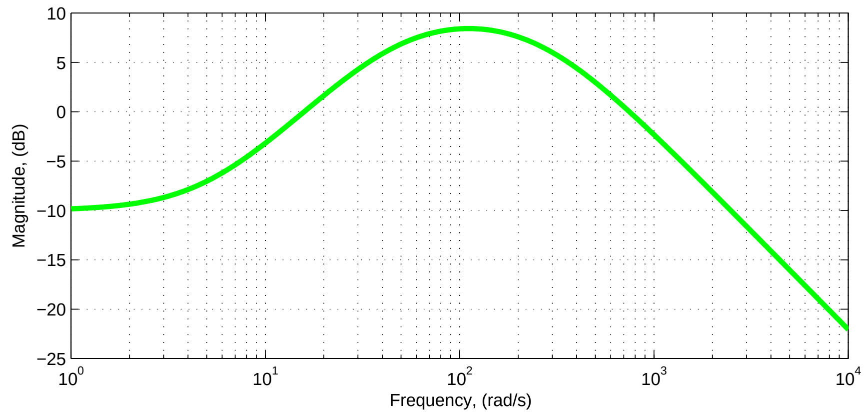
There are no zeros or poles at 0 rad/s. There is a zero located at 5 rad/s. The two poles are located at 50 rad/s and 250 rad/s. The transfer function has the following form:

$$T(s) = K\frac{s+5}{(s+50)(s+250)}$$

The gain at low frequencies is −10 dB, which is an absolute gain of 0.3162. If we substitute $s = j\omega$ for a small value of ω, the approximate gain of the transfer function is $5K/[(50)(250)] = K/2500 = 0.3162$. Solving for K, we have $K = 790.57$.

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

The MATLAB plot is shown below.



(c). Compare the straight-line gain and the actual gain at $\omega = 100$ and 500 rad/s.

At $\omega = 100$ and 500 rad/s, the straight-line gains are 10 and 3.98 dB, respectively. The actual gains at those frequencies are 8.40 and 2.97 dB, respectively. For both frequencies, the error is less than 2 dB.

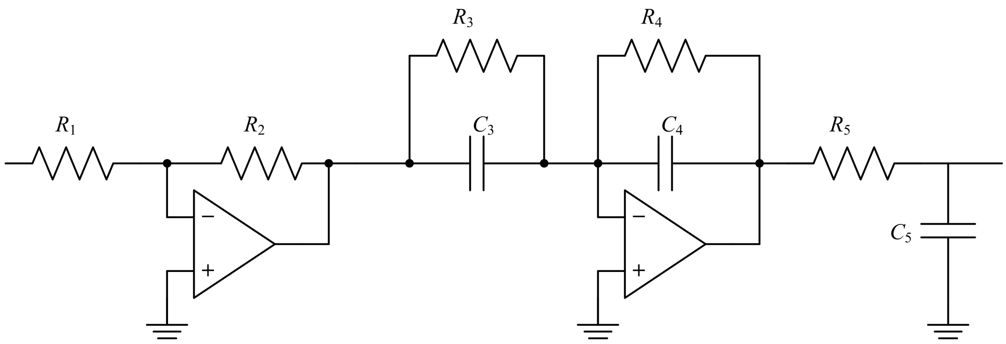
(d). Design a circuit that will realize the transfer function found in part (a).

Write the transfer functions as follows:

$$T(s) = (790.57)\frac{s+5}{(s+50)(s+250)} = \left(\frac{250}{s+250}\right)(3.16)\left(\frac{s+5}{s+50}\right)$$

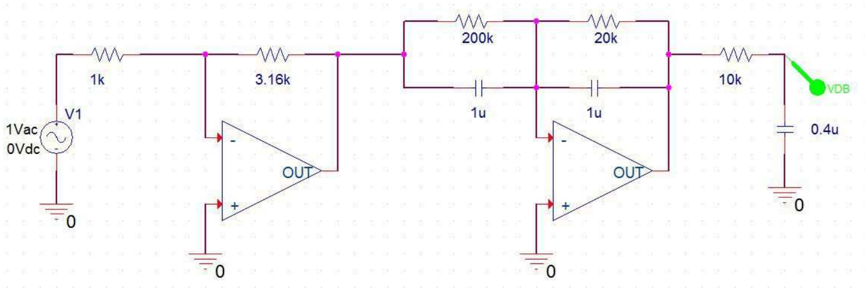
$$= \left(\frac{\dfrac{250}{s}}{1+\dfrac{250}{s}}\right)(3.16)\left(\frac{s+5}{s+50}\right)$$

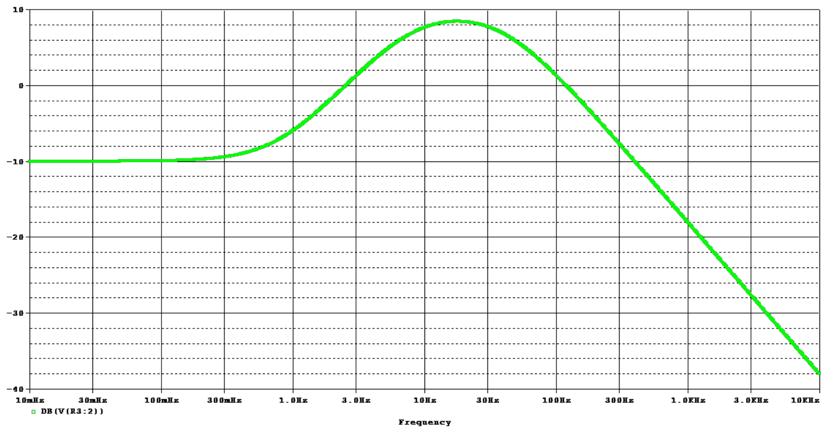
The circuit has three stages: a first-order low-pass filter, a gain stage, and a second-order filter. The following circuit will realize the transfer function with $R_1 = 1$ kΩ, $R_2 = 3.16$ kΩ, $R_3 = 200$ kΩ, $C_3 = 1$ μF, $R_4 = 20$ kΩ, $C_4 = 1$ μF, $R_5 = 10$ kΩ, $C_5 = 0.4$ μF



(e). Use OrCAD to verify your circuit design.

The OrCAD simulation and corresponding output are shown below. The output agrees with the MATLAB results.





Problem 12–54. Consider the gain plot in Figure P12–54.

(a). Find a transfer function corresponding to the straight-line gain plot. Note that the magnitude of the actual frequency response must be exactly 5 at the geometric mean of the two cutoff frequencies (2450 rad/s).

The transfer function has a zero at $\omega = 0$ and poles at $\omega = 100$ and 60000 rad/s. The form of the transfer function is

$$T(s) = \frac{Ks}{(s+100)(s+60000)} = \frac{Ks}{s^2 + 60100s + 6000000}$$

The geometric mean of the two cutoff frequencies is $\sqrt{(100)(60000)} = 2450$ rad/s. Find the gain at that frequency and solve for K.

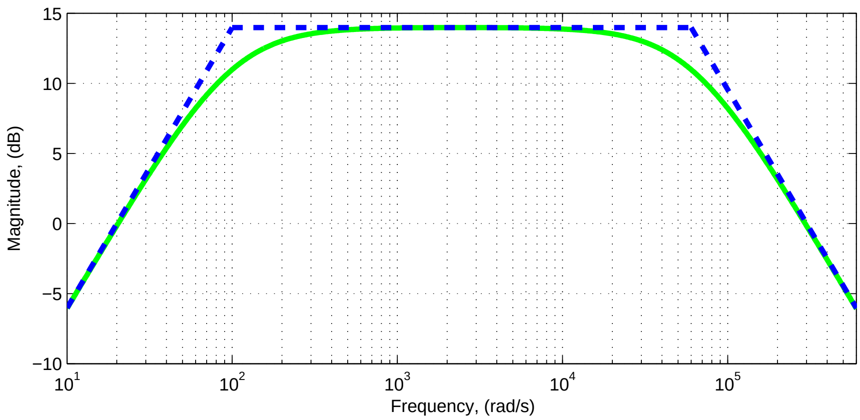
$$T(j2450) = \frac{j2450K}{(100 + j2450)(60000 + j2450)} = 0.00001664K\angle 0^\circ$$

$$K = \frac{5}{0.00001664} = 300500$$

$$T(s) = \frac{300500s}{(s+100)(s+60000)}$$

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

The MATLAB plot is shown below.



Problem 12–55. Consider the gain plot in Figure P12–55.

(a). Find the transfer function corresponding to the straight-line gain plot.

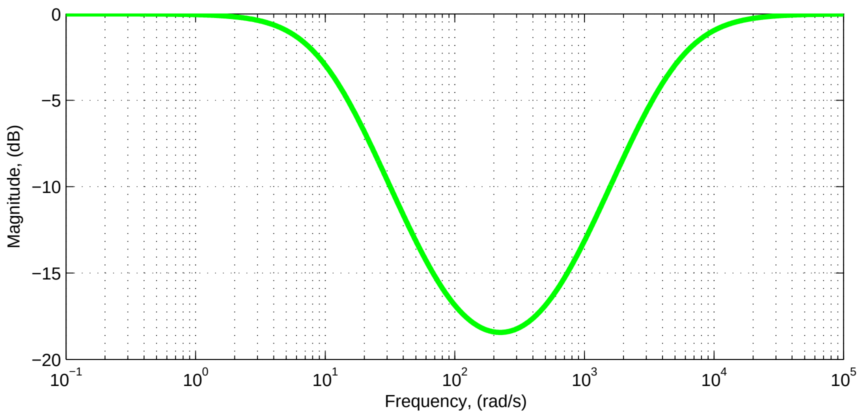
The transfer function has no poles or zeros at 0 rad/s. There is a pole at 10 rad/s and another pole at 5000 rad/s. The two zeros are located at 100 rad/s and 500 rad/s. The gain at 0 rad/s is 1. The transfer function has the following form.

$$T(s) = \frac{K(s+100)(s+500)}{(s+10)(s+5000)} = \frac{(s+100)(s+500)}{(s+10)(s+5000)}$$

If we evaluate the transfer function at 0 rad/s and set the gain equal to 1, the gain constant is $K = 1$, as shown above.

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

The MATLAB plot is shown below.

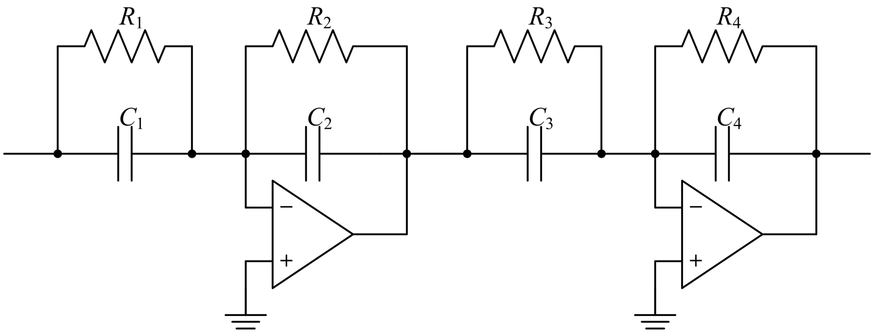


(c). Design a circuit that will realize the transfer function found in part (a).

Write the transfer function as follows:

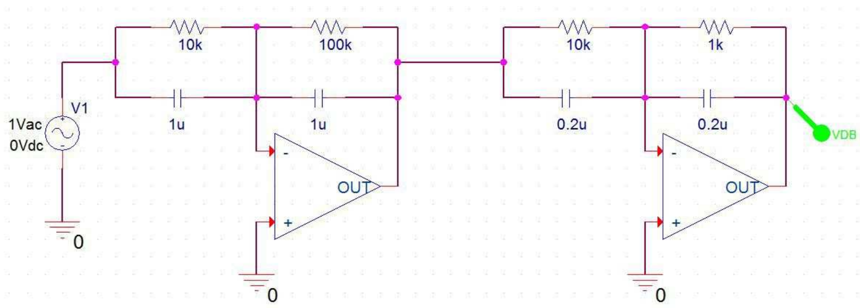
$$T(s) = \left(\frac{s+100}{s+10}\right)\left(\frac{s+500}{s+5000}\right)$$

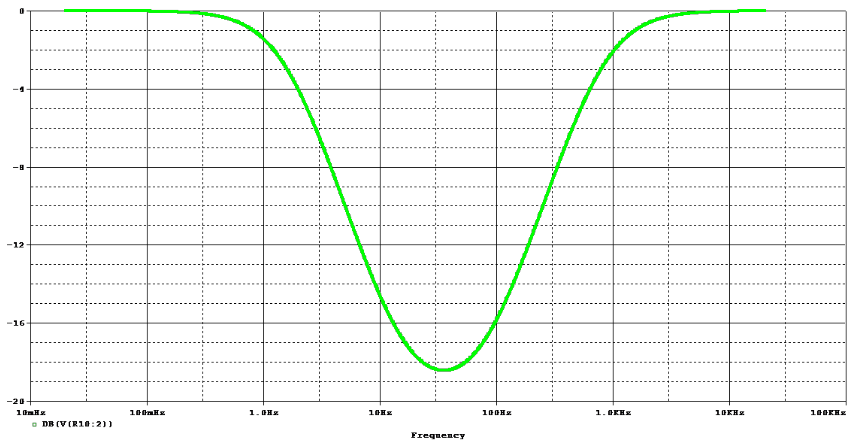
We can then design the two stages as shown below, where $R_1 = 10$ kΩ, $R_2 = 100$ kΩ, $R_3 = 10$ kΩ, $R_4 = 1$ kΩ, $C_1 = C_2 = 1\ \mu$F, $C_3 = C_4 = 0.2\ \mu$F



(d). Use OrCAD to verify your circuit design.

The following OrCAD simulation and results verify the design.





Problem 12–56. Consider the gain plot in Figure P12–56.

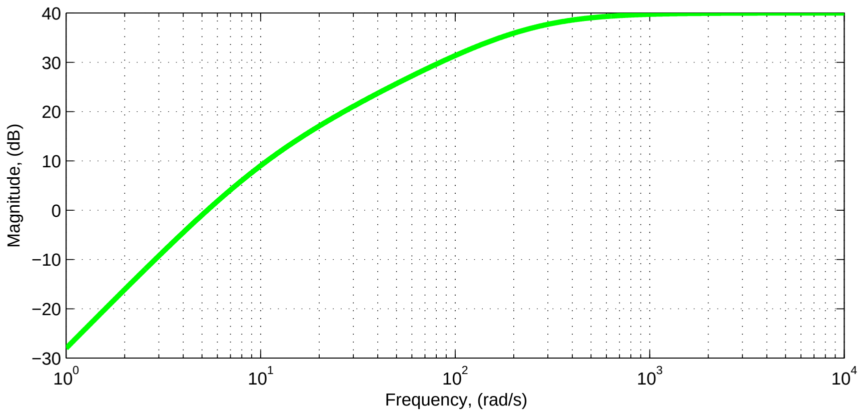
(a). Find the transfer function corresponding to the straight-line gain plot.

The transfer function has two zeros at 0 rad/s. There is a pole at 10 rad/s and another pole at 250 rad/s. The high-frequency gain is 40 dB or 100. The transfer function has the following form.

$$T(s) = \frac{100s^2}{(s+10)(s+250)}$$

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

The MATLAB plot is shown below.



(c). Design a circuit that will realize the transfer function found in part (a).

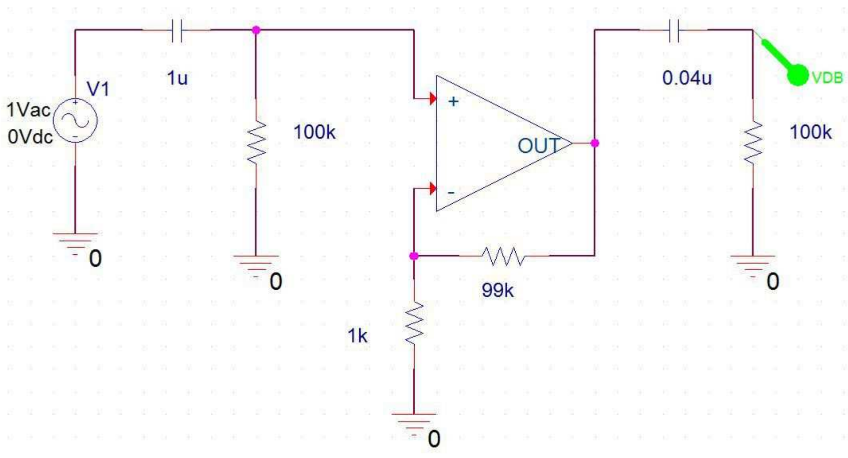
Write the transfer function as follows:

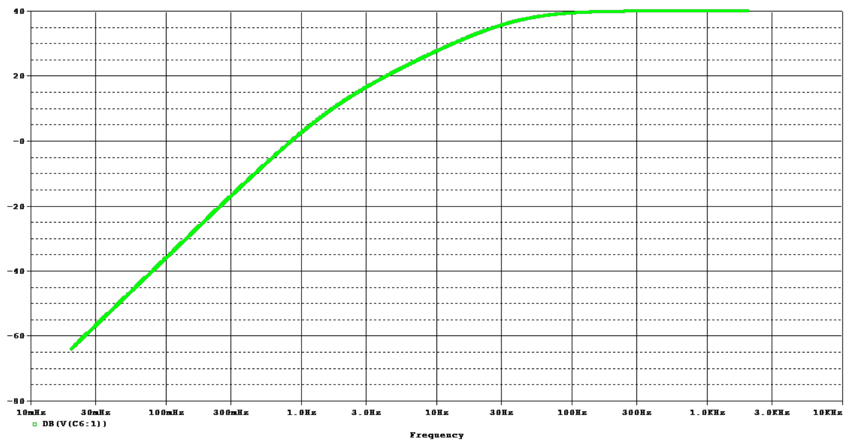
$$T(s) = \left(\frac{s}{s+10}\right)(100)\left(\frac{s}{s+250}\right) = \left(\frac{1}{1+\dfrac{10}{s}}\right)(100)\left(\frac{1}{1+\dfrac{250}{s}}\right)$$

We can design the circuit using three stages. The first stage is a series RC high-pass filter with $R = 100\ \text{k}\Omega$ and $C = 1\ \mu\text{F}$. The second stage is a noninverting amplifier with a gain of 100. The third stage is a series RC high-pass filter with $R = 100\ \text{k}\Omega$ and $C = 0.04\ \mu\text{F}$.

(d). Use OrCAD to verify your circuit design.

The following OrCAD simulation and results verify the design.





Problem 12–57. Consider the gain plot in Figure P12–57. The goal is to design a circuit that will result in the dashed curve shown on the plot.

(a). Find the transfer function corresponding to the straight-line gain plot.

The function has a zero at 0 rad/s and two poles at 100 rad/s. The gain is 0 dB or one at $\omega = 100$ rad/s. We have the following results for the transfer function:

$$T(s) = \frac{Ks}{(s+100)^2}$$

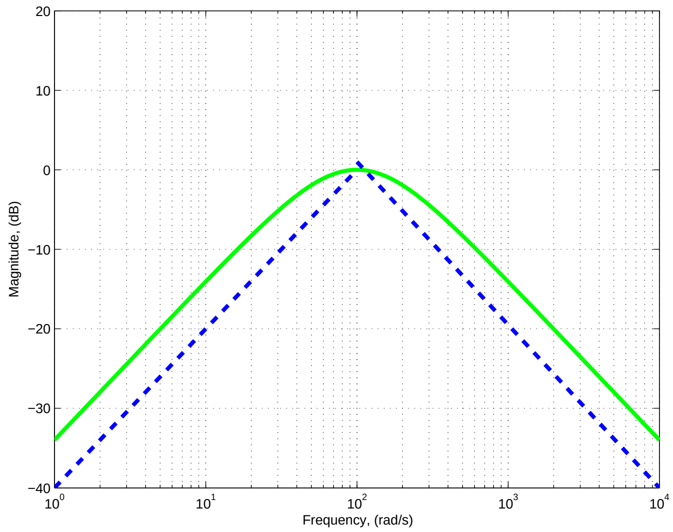
$$T(j100) = 1 = \frac{j100K}{(100+j100)^2} = \frac{j100K}{j20000} = \frac{K}{200}$$

$$K = 200$$

$$T(s) = \frac{200s}{(s+100)^2}$$

(b). Use MATLAB to plot the Bode magnitude of the transfer function.

The MATLAB plot is shown below.

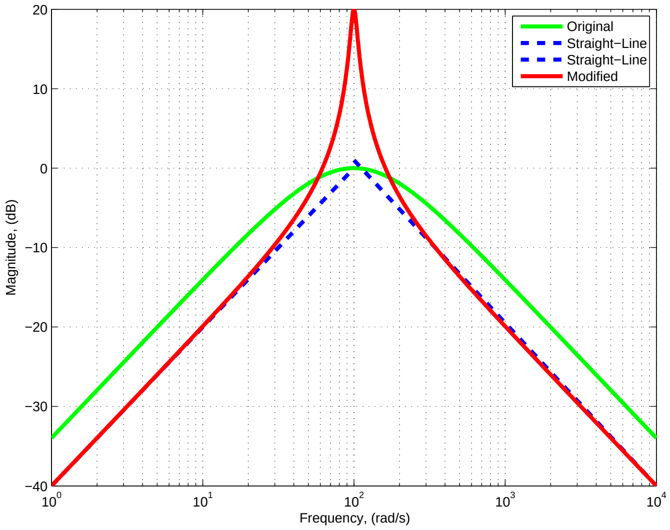


(c). Adjust the poles so that the transfer function results in the dashed line. (*Hint*: multiply the two poles into a quadratic expression. Then adjust the Q of the circuit to attain the desired result.)

Through trial and error with MATLAB, the following transfer function is a good fit:

$$T(s) = \frac{100s}{s^2 + 10s + 10000}$$

The MATLAB plot is shown below.



(d). Design a circuit that will realize the transfer function found in part (c).

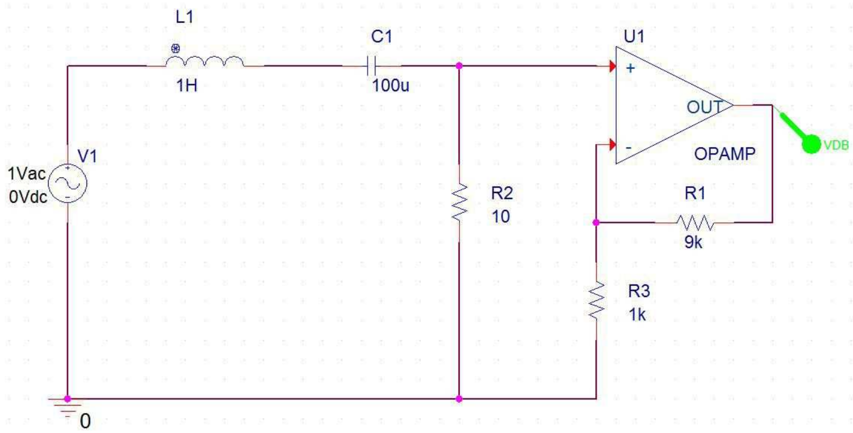
Write the transfer function as follows:

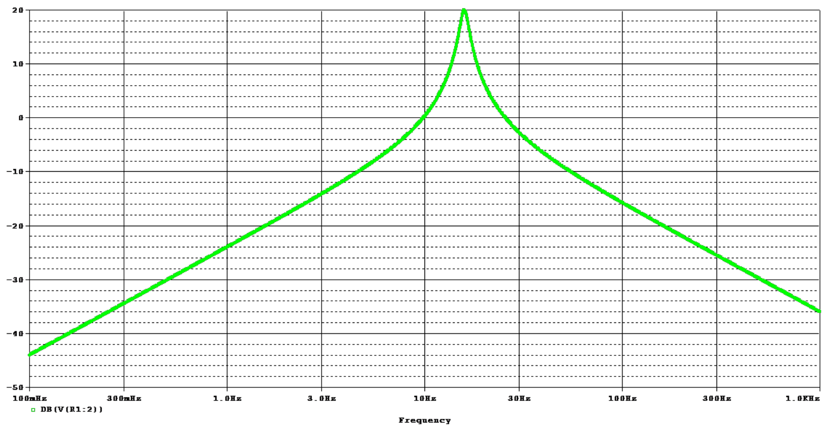
$$T(s) = \frac{10(10)}{s + 10 + \dfrac{10000}{s}}$$

Use a series RLC circuit with the output taken across the resistor and $R = 10\ \Omega$, $L = 1$ H, and $C = 100\ \mu$F. Connect the RLC circuit to a noninverting amplifier with a gain of 10.

(e). Use OrCAD to verify your circuit design.

The following OrCAD simulation and output verify the design.



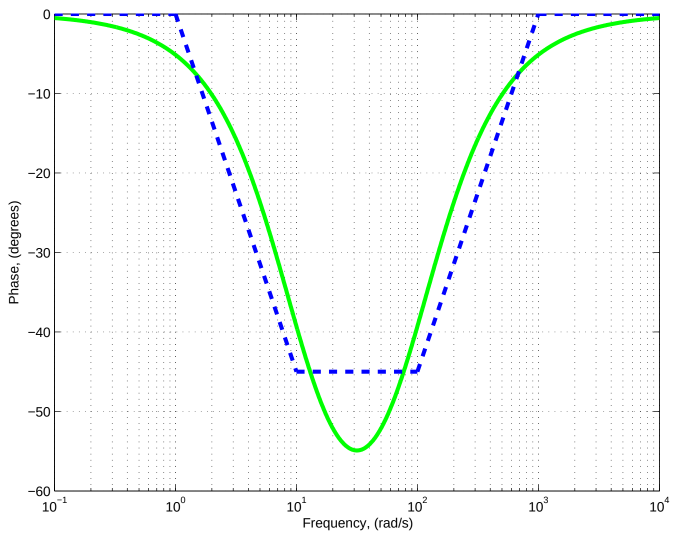


Problem 12–58. For the following transfer function $T_{\rm V}(s) = V_2(s)/V_1(s)$

$$T_{\rm V}(s) = \frac{10(s+100)}{s+10}$$

(a). Construct the straight-line Bode plot of the phase.

The following plot shows both the straight-line and actual Bode plots for the phase.



(b). Use the straight-line phase diagram to estimate the phase at $\omega = 1$, 10, 100, and 1000 rad/s.

For $\omega = 1$, 10, 100, and 1000 rad/s, the phase estimates are $0°$, $-45°$, $-45°$, and $0°$, respectively. The actual phase values at those frequencies are $-5.14°$, $-39.3°$, $-39.3°$, and $-5.14°$.

(c). Use MATLAB to plot the Bode phase and compare the phase plot to the straight-line estimate.

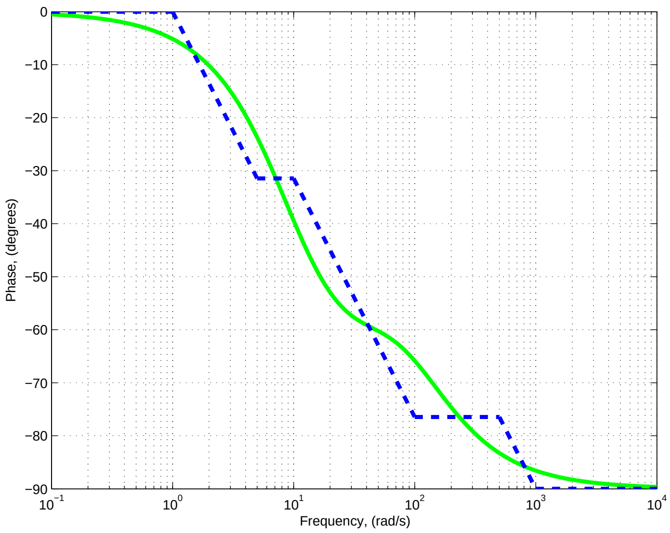
The plot is shown in part (a). The straight-line approximation captures the general trends in the phase plot and has significant differences at the center of the bandstop region and the corner frequencies.

Problem 12–59. For the following transfer function $T_{\rm V}(s) = V_2(s)/V_1(s)$

$$T_{\rm V}(s) = \frac{10(s+50)}{(s+10)(s+100)}$$

(a). Construct the straight-line Bode plot of the phase.

The following plot shows both the straight-line and actual Bode plots for the phase.



(b). Use the straight-line phase diagram to estimate the phase at $\omega = 1$, 10, 100, and 1000 rad/s.

For $\omega = 1$, 10, 100, and 1000 rad/s, the phase estimates are $0°$, $-31.5°$, $-76.5°$, and $-90°$, respectively. The actual phase values at those frequencies are $-5.14°$, $-39.4°$, $-65.9°$, and $-86.6°$.

(c). Use MATLAB to plot the Bode phase and compare the phase plot to the straight-line estimate.

The plot is shown in part (a). The straight-line approximation captures the general trends in the phase plot, but it is not a close match to the actual plot.

Problem 12–60. The step response of a linear circuit is

$$g(t) = \left[6 - 4e^{-100t} - 2e^{-1000t}\right] u(t)$$

(a). Is the circuit a low-pass, high-pass, bandpass, or bandstop filter?

Determine the transfer function.

$$G(s) = \frac{6}{s} - \frac{4}{s+100} - \frac{2}{s+1000} = \frac{6(s+100)(s+1000) - 4s(s+1000) - 2s(s+100)}{(s+100)(s+1000)}$$

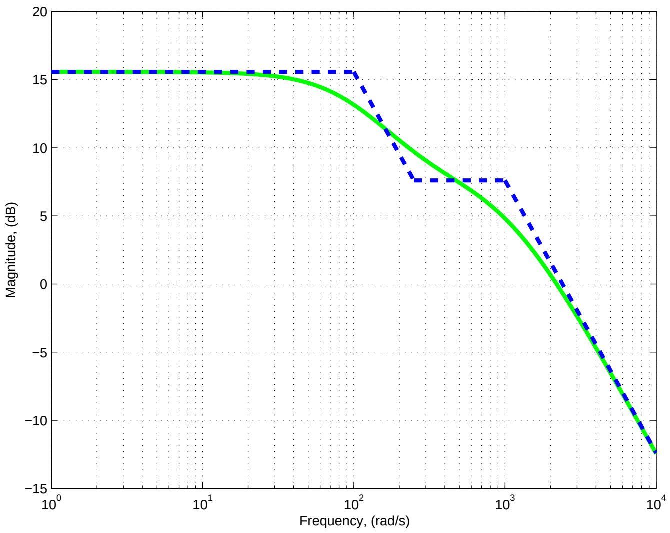
$$= \frac{2400(s+250)}{s(s+100)(s+1000)}$$

$$T(s) = sG(s) = \frac{2400(s+250)}{(s+100)(s+1000)}$$

The transfer function has gain at low frequencies, but not at high frequencies. It is a low-pass filter.

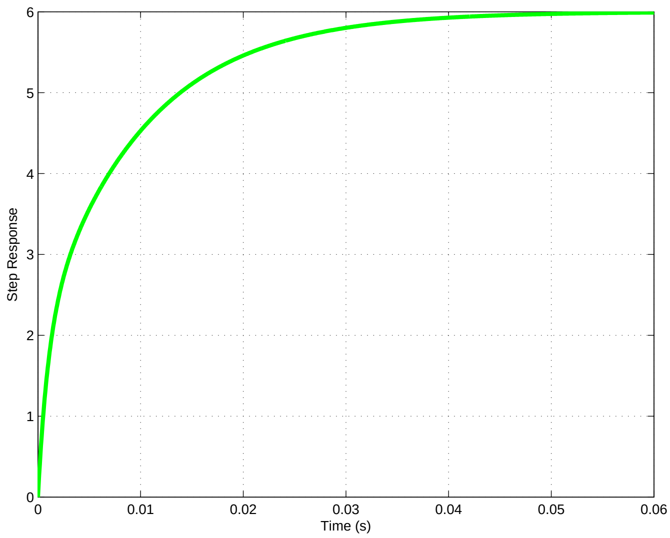
(b). Construct the straight-line Bode gain plot and estimate the cutoff frequency and passband gain.

The following plot shows both the straight-line and actual Bode plots for the gain. The cutoff frequency is approximately 100 rad/s and the passband gain is approximately 16 dB.



(c). Use MATLAB to plot the Bode magnitude and step responses.

The Bode magnitude response is shown in part (b). The step response is shown below.

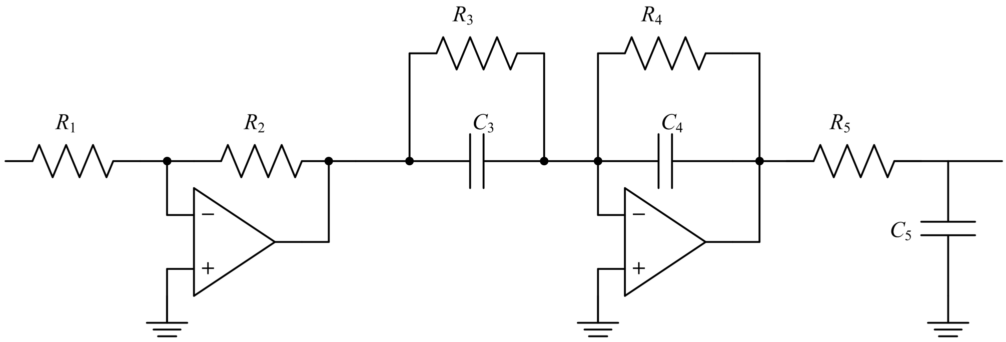


(d). Design a circuit to achieve the transfer function.

Write the transfer function as follows:

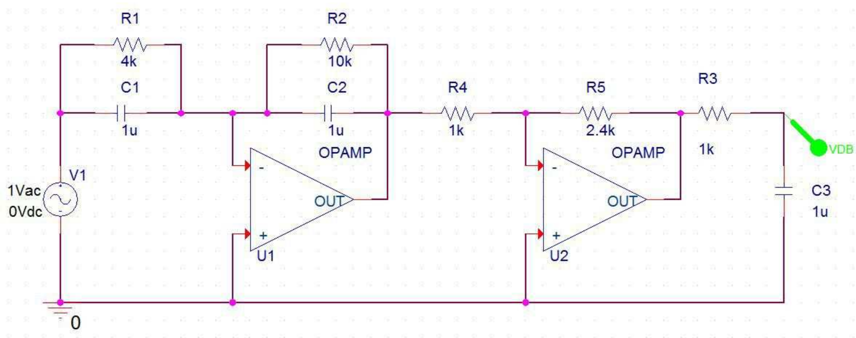
$$T(s) = -\left(\frac{s+250}{s+100}\right)(-2.4)\left(\frac{\dfrac{1000}{s}}{1+\dfrac{1000}{s}}\right)$$

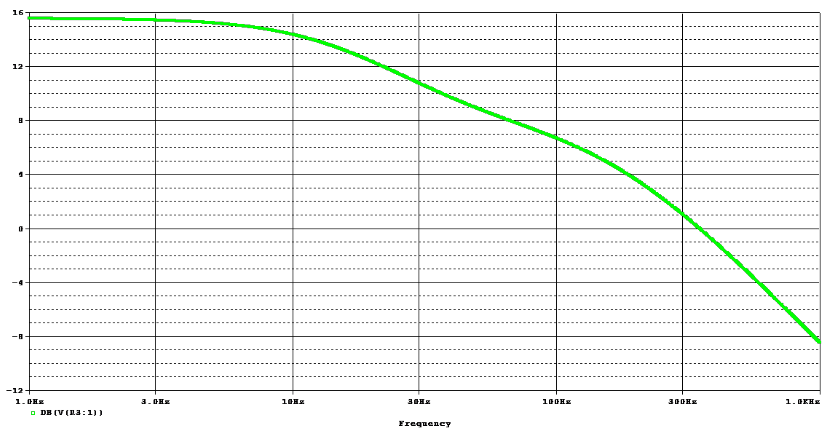
The following three-stage circuit will realize the transfer function with $R_1 = 1$ kΩ, $R_2 = 2.4$ kΩ, $R_3 = 4$ kΩ, $C_3 = C_4 = 1$ μF, $R_4 = 10$ kΩ, $R_5 = 1$ kΩ, and $C_5 = 1$ μF.

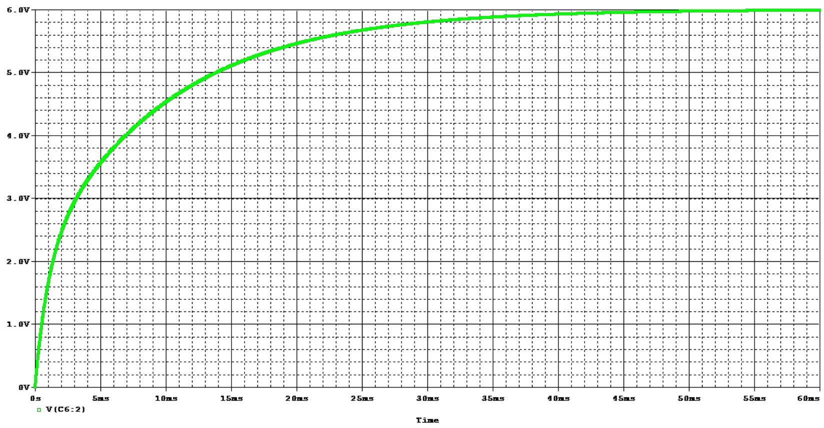


(e). Use OrCAD to verify the step and frequency responses.

The OrCAD simulation and results verify the responses.







Problem 12–61. The step response of a linear circuit is

$$g(t) = \left[2 - 2e^{-20t} + 2e^{-500t}\right] u(t)$$

(a). Is the circuit a low-pass, high-pass, bandpass, or bandstop filter?

Determine the transfer function.

$$G(s) = \frac{2}{s} - \frac{2}{s+20} + \frac{2}{s+500} = \frac{2(s+20)(s+500) - 2s(s+500) + 2s(s+20)}{s(s+20)(s+500)}$$

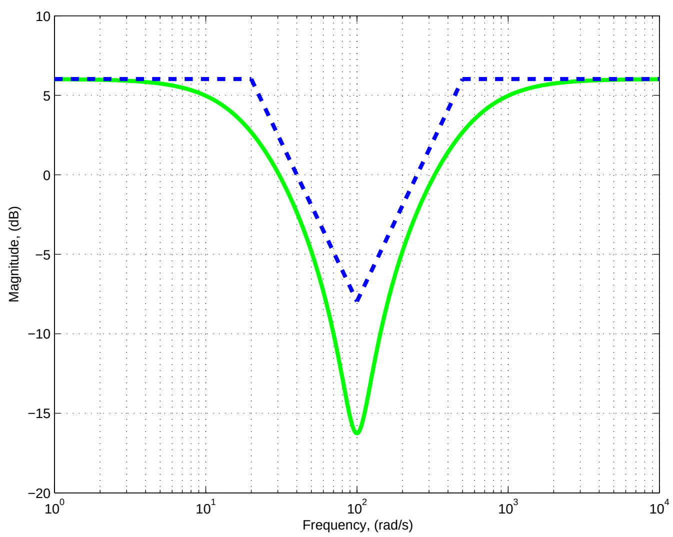
$$= \frac{2(s^2+40s+10000)}{s(s+20)(s+500)}$$

$$T(s) = sG(s) = \frac{2(s^2+40s+10000)}{(s+20)(s+500)}$$

The transfer function has a gain of two at both low and high frequencies and a notch at the center frequency of 100 rad/s. It is a bandstop filter.

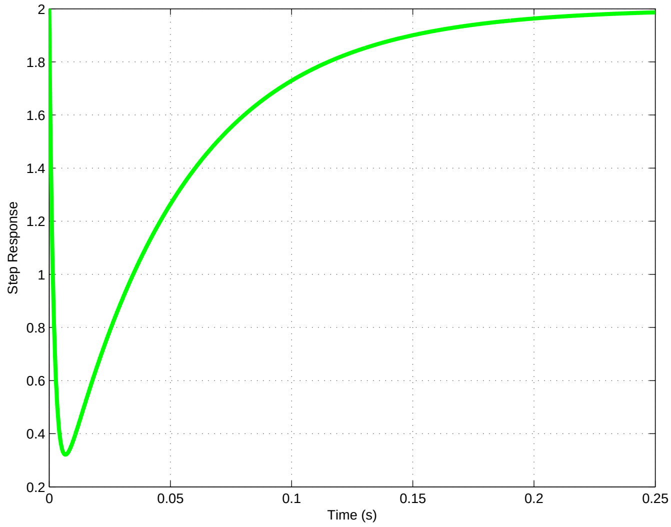
(b). Construct the straight-line Bode gain plot and estimate the cutoff frequency and passband gain.

The following plot shows both the straight-line and actual Bode plots for the gain. The cutoff frequencies are approximately 20 rad/s and 500 rad/s. The passband gain is approximately 6 dB.



(c). Use MATLAB to plot the Bode magnitude and step responses.

The Bode magnitude response is shown in part (b). The step response is shown below.

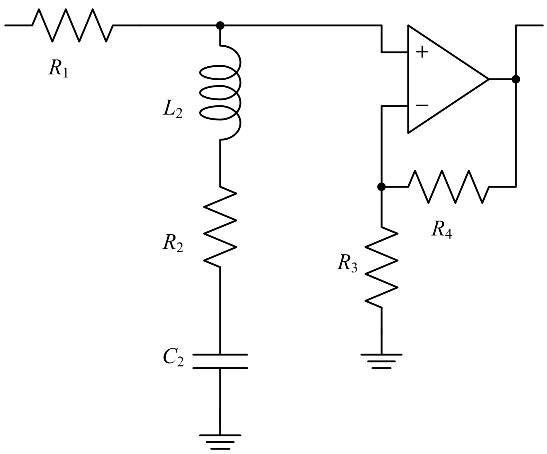


(d). Design a circuit to achieve the transfer function.

Write the transfer function as follows:

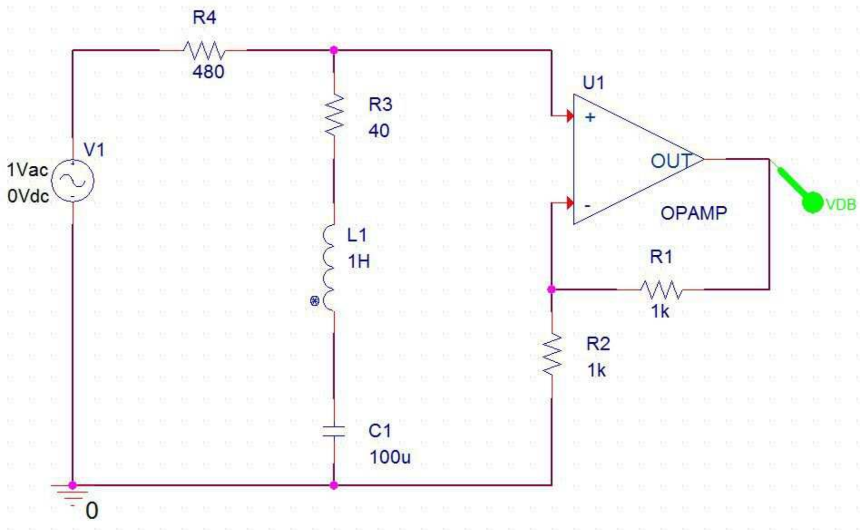
$$T(s) = (2)\left(\frac{s^2 + 40s + 10000}{s^2 + 520s + 10000}\right) = (2)\left(\frac{s + 40 + \dfrac{10000}{s}}{s + 520 + \dfrac{10000}{s}}\right)$$

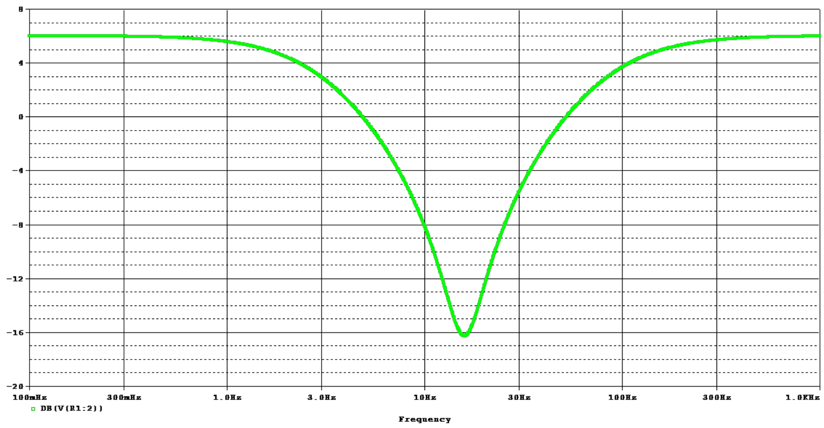
The following *RLC* circuit and amplifier will realize the transfer function with $R_1 = 480\ \Omega$, $R_2 = 40\ \Omega$, $L_2 = 1$ H, $C_2 = 100\ \mu$F, $R_3 = R_4 = 1$ kΩ.

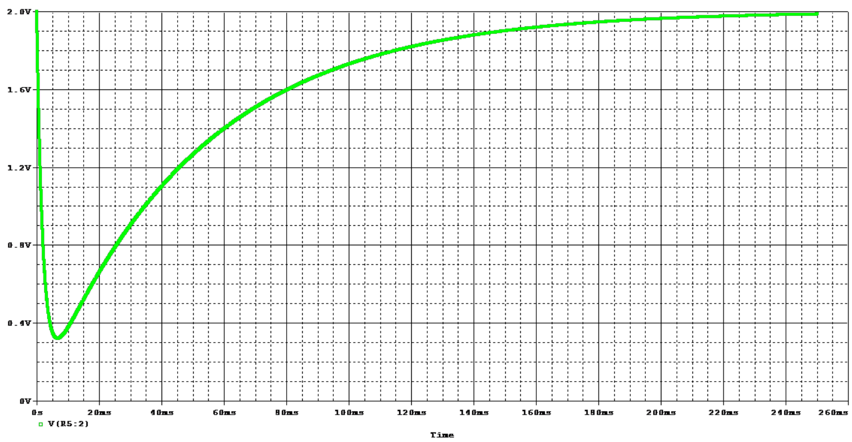


(e). Use OrCAD to verify the step and frequency responses.

The OrCAD simulation and results verify the responses.







Problem 12–62. The impulse response transform of a linear circuit is

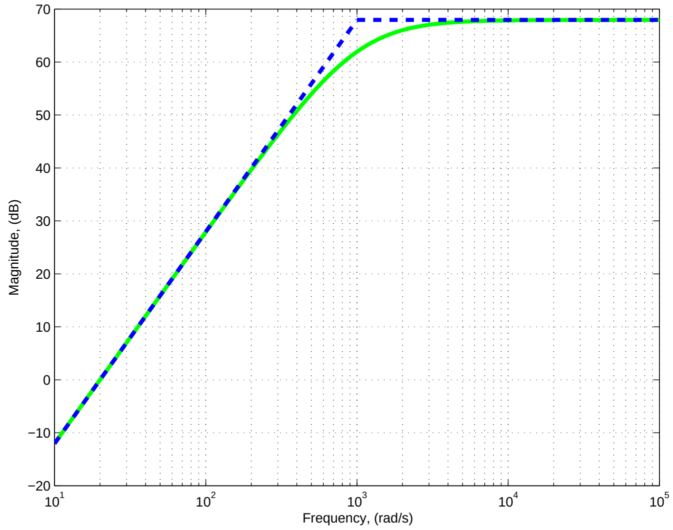
$$H(s) = \frac{2500s^2}{(s+1000)^2}$$

(a). Is the circuit a low-pass, high-pass, bandpass, or bandstop filter?

The transfer function has two zeros at 0 rad/s and two poles at 1000 rad/s. It is a high-pass filter.

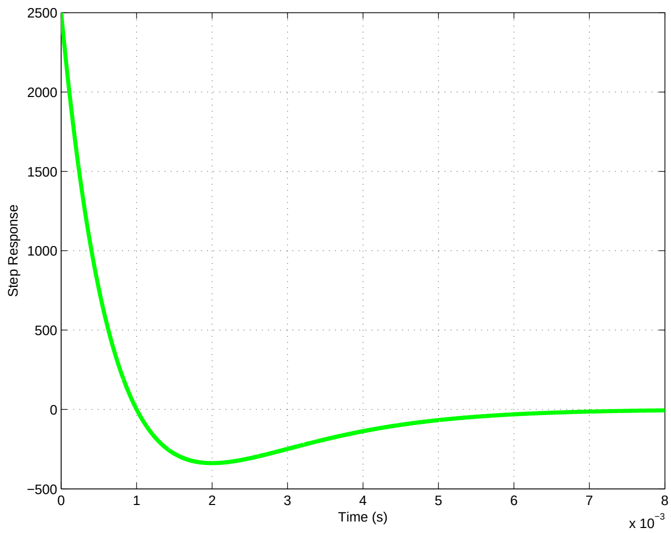
(b). Construct the straight-line Bode gain plot and estimate the cutoff frequency and passband gain.

The following plot shows both the straight-line and actual Bode plots for the gain. The cutoff frequency is approximately 1560 rad/s. The passband gain is approximately 68 dB.



(c). Use MATLAB to plot the Bode magnitude and step responses.

The Bode magnitude response is shown in part (b). The step response is shown below.

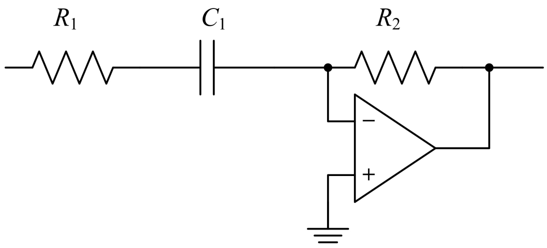


(d). Design a circuit to achieve the transfer function.

Write the transfer function as follows:

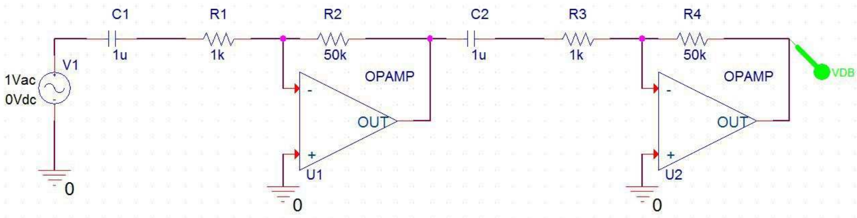
$$T(s) = \left(\frac{50s}{s+1000}\right)\left(\frac{50s}{s+1000}\right)$$

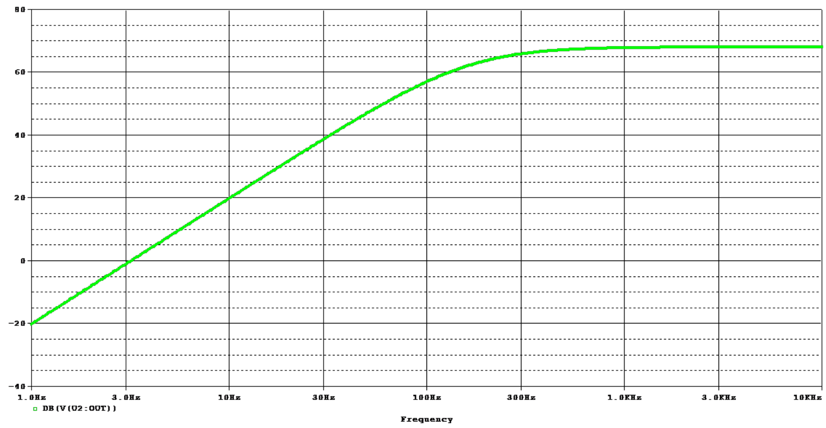
Use two identical first-order, high-pass RC OP AMP circuits, each with a gain of 50 and a cutoff frequency of 1000 rad/s. Each stage has the following design with $R_1 = 1$ kΩ, $C_1 = 1$ μF, and $R_2 = 50$ kΩ.

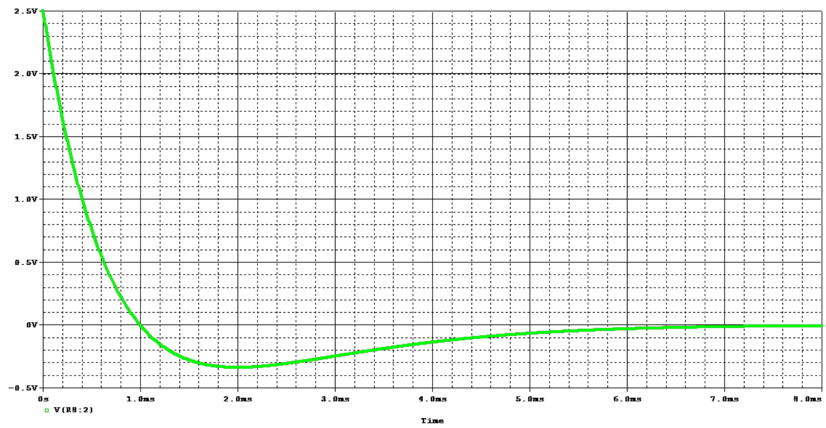


(e). Use OrCAD to verify the step and frequency responses.

The OrCAD simulation and results verify the responses. Note that the step response used an input of 1 mV to avoid saturating the OP AMPs. The results match the MATLAB results, but are scaled by a factor of 1000.







Problem 12–63. The straight-line gain response of a linear circuit is shown in Figure P12–52. What are the initial and final values of the circuit step response? What is the approximate duration of the transient response?

From Problem 12–52, the transfer function is

$$T(s) = \frac{s + 200}{s + 20}$$

Determine the step response.

$$\begin{aligned}
G(s) &= \frac{T(s)}{s} = \frac{s + 200}{s(s + 20)} = \frac{10}{s} - \frac{9}{s + 20} \\
g(t) &= \left(10 - 9e^{-20t}\right) u(t) \\
g(0) &= 1 \\
g(\infty) &= 10 \\
T_{\mathrm{C}} &= \frac{1}{20} = 50 \text{ ms} \\
5T_{\mathrm{C}} &= 250 \text{ ms}
\end{aligned}$$

The initial and final values of the step response are 1 and 10, respectively. The duration of the step response is approximately 250 ms.

Problem 12–64. The straight-line gain response of a linear circuit is shown in Figure P12–54. What are the initial and final values of the circuit step response? What is the approximate duration of the transient response?

From Problem 12–54, the transfer function is

$$T(s) = \frac{300500s}{(s + 100)(s + 60000)}$$

Determine the step response.

$$\begin{aligned}
G(s) &= \frac{T(s)}{s} = \frac{300500}{(s + 100)(s + 60000)} = \frac{\frac{3005}{599}}{s + 100} - \frac{\frac{3005}{599}}{s + 60000} \\
g(t) &= \frac{3005}{599}\left(e^{-100t} - e^{-60000t}\right) u(t) \\
g(0) &= 0 \\
g(\infty) &= 0 \\
T_{\mathrm{C}} &= \frac{1}{100} = 10 \text{ ms} \\
5T_{\mathrm{C}} &= 50 \text{ ms}
\end{aligned}$$

The initial and final values of the step response are both zero. The duration of the step response is approximately 50 ms.

Problem 12–65. There is a need for a passive notch filter at 10 krad/s. The narrower the notch the better, but there should be minimal ringing of the signals passing through. The transforms of three filters were submitted for consideration. Which would you recommend and why?

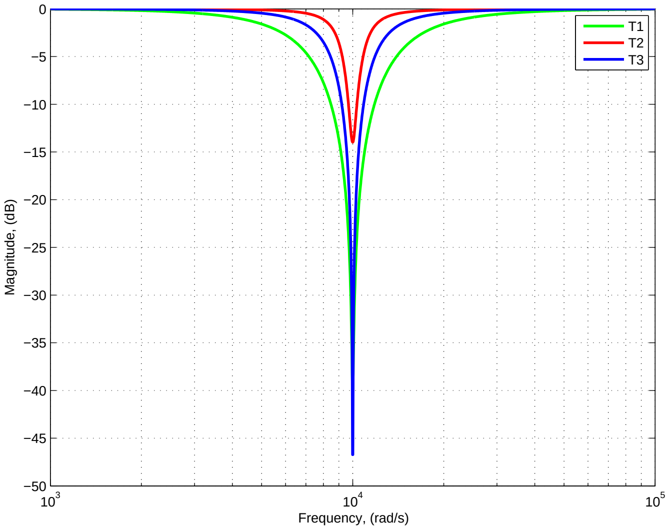
$$T_1(s) = \frac{s^2 + 100s + 10^8}{s^2 + 10^4 s + 10^8}$$

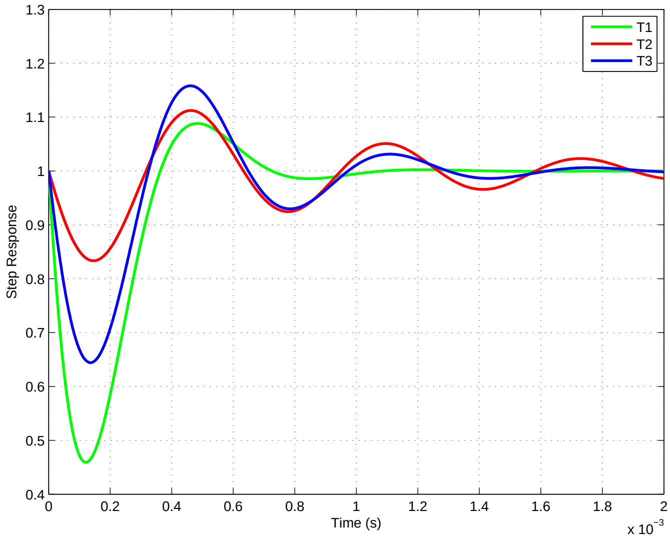
$$T_2(s) = \frac{s^2 + 500s + 10^8}{s^2 + 2500s + 10^8}$$

$$T_3(s) = \frac{s^2 + 10^8}{s^2 + 5000s + 10^8}$$

Use MATLAB to plot the magnitude response and step response for each transfer function. The MATLAB code and the corresponding results are shown below.

```
syms s t
% Form the transfer functions
T1 = (s^2+100*s+1e8)/(s^2+1e4*s+1e8);
T2 = (s^2+500*s+1e8)/(s^2+2500*s+1e8);
T3 = (s^2+1e8)/(s^2+5000*s+1e8);
% Find the step responses
g1 = ilaplace(T1/s);
g2 = ilaplace(T2/s);
g3 = ilaplace(T3/s);
% Compute the magnitude response and plot them
ww = logspace(3,5,2000);
MagT1jwdB = 20*log10(abs(subs(T1,s,j*ww)));
MagT2jwdB = 20*log10(abs(subs(T2,s,j*ww)));
MagT3jwdB = 20*log10(abs(subs(T3,s,j*ww)));
% Plot the actual gain response
semilogx(ww,MagT1jwdB,'g','LineWidth',2)
grid on
hold on
semilogx(ww,MagT2jwdB,'r','LineWidth',2)
semilogx(ww,MagT3jwdB,'b','LineWidth',2)
xlabel('Frequency, (rad/s)')
ylabel('Magnitude, (dB)')
legend('T1','T2','T3')

% Plot the step responses
figure
tt = 0:2e-6:2e-3;
g1tt = subs(g1,t,tt);
g2tt = subs(g2,t,tt);
g3tt = subs(g3,t,tt);
plot(tt,g1tt,'g','LineWidth',2)
grid on
hold on
plot(tt,g2tt,'r','LineWidth',2)
plot(tt,g3tt,'b','LineWidth',2)
xlabel('Time (s)')
ylabel('Step Response')
legend('T1','T2','T3')
```





From the two plots, transfer function $T_2(s)$ has the narrowest notch and the lowest amplitude ringing, although its ringing requires more time to attenuate than that from the other two transfer functions.

Problem 12–66. There is a need for a passive tuned filter at 10 krad/s. The higher the Q the better, but there should be no ringing of the signals passing through. The transform of a prototype filter is shown. Design the filter by selecting the middle term of the denominator to maximize the Q while assuring there is no ringing.

$$T_{\text{tuned}}(s) = \frac{10^8 s}{s^2 + xs + 10^8}$$

The transfer function has the following form:

$$T_{\text{tuned}}(s) = \frac{10^8 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

To ensure there is no ringing, we require $\zeta = 1$. We then have the following results:

$$\zeta = 1$$
$$\omega_0 = 10000$$
$$B = 2\zeta\omega_0 = 20000$$
$$Q = \frac{\omega_0}{B} = \frac{1}{2}$$

To maximize Q, we want to decrease B and, therefore, ζ. Picking a ζ less than one will allow ringing. The best design option is $x = 20000$.

Problem 12–67. (A,D) Step Response of an RLC bandpass circuit

The step response of a series RLC bandpass circuit is

$$g(t) = \left[\frac{4}{5}e^{-200t}\sin(500t)\right]u(t)$$

(a). Find the passband center frequency and the two cutoff frequencies.

Determine the transfer function and the response characteristics.

$$G(s) = \left(\frac{4}{5}\right)\left(\frac{500}{(s+200)^2+500^2}\right) = \frac{400}{s^2+400s+290000}$$
$$T(s) = sG(s) = \frac{400s}{s^2+400s+290000}$$
$$\omega_0 = \sqrt{290000} = 538.516 \text{ rad/s}$$
$$B = 400 \text{ rad/s}$$
$$\omega_0^2 = \omega_{C1}\omega_{C2}$$
$$B = \omega_{C2} - \omega_{C1}$$
$$\omega_{C1} = 374.4563 \text{ rad/s}$$
$$\omega_{C2} = 774.4563 \text{ rad/s}$$

(b). Design a circuit that would possess the above step response.

Write the transfer functions as follows:

$$T(s) = \frac{400}{s + 400 + \dfrac{290000}{s}}$$

Use a series RLC filter with the output taken across the resistor and $R = 400\ \Omega$, $L = 1$ H, and $C = 3.448\ \mu$F.

Problem 12–68. (A) A Tunable Tank Circuit

The RLC circuit in Figure P12–68 (often called a tank circuit) has $R = 4.7$ kΩ, $C = 680$ pF, and an adjustable (tunable) L ranging from 64 to 640 μH.

(a). Show that the circuit is a bandpass filter.

Determine the transfer function.

$$Z = \frac{Ls/Cs}{Ls + 1/Cs} = \frac{Ls}{LCs^2 + 1}$$

$$T(s) = \frac{Z}{R + Z} = \frac{\dfrac{Ls}{LCs^2+1}}{R + \dfrac{Ls}{LCs^2+1}} = \frac{Ls}{RLCs^2 + Ls + R}$$

$$= \frac{\dfrac{1}{RC}s}{s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC}}$$

The transfer function has the form of a bandpass filter response, where the gain is zero at both low and high frequencies, but there is a response at the center frequency.

(b). Find the frequency range (in Hz) over which the center frequency can be tuned.

We have the following results:

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_{0,\mathrm{min}} = 1.5158 \text{ Mrad/s}$$

$$\omega_{0,\mathrm{max}} = 4.7935 \text{ Mrad/s}$$

$$f_{0,\mathrm{min}} = 241.25 \text{ kHz}$$

$$f_{0,\mathrm{max}} = 762.91 \text{ kHz}$$

(c). Find the bandwidth (in Hz) at the end points of this range.

The bandwidth is constant with $B = 1/RC = 49.798$ kHz.

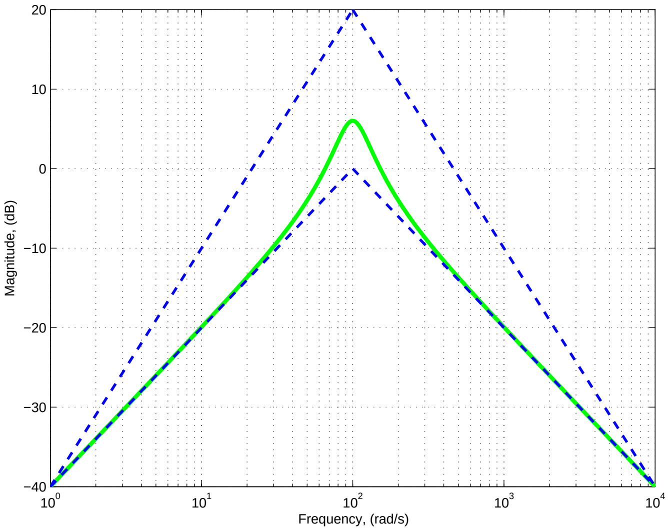
Problem 12–69. (D) Filter Design Specification

Construct a transfer function whose gain response lies entirely within the non-shaded region in Figure P12–69.

The transfer function has a zero at 0 rad/s and a second-order pole with a center frequency of 100 rad/s. The function will have the following form:

$$T(s) = \frac{Ks}{s^2 + Bs + 10000}$$

where K is the gain and B is the bandwidth. Use trial and error with MATLAB to find an appropriate response. The following plot uses $K = 100$ and $B = 50$ with acceptable results.



Problem 12–70. (E) Chip RC Networks

Integrated circuit (chip) RC networks are used at parallel data ports to suppress radio frequency noise. In a certain application, RF noise at 3.2 MHz is interfering with a 4-bit parallel data signal operating at 1.1 MHz. A chip RC network is to be used to reduce the RF noise on the parallel bus by at least 7 dB without reducing the data signals by more than 2 dB. A vendor offers a family of chip RC networks connected as shown in Figure P12–70. The available circuit parameters are shown in Table P12–70. Select the part number that best meets the noise suppression requirements.

The circuit for each bit is a low-pass filter with the following transfer function:

$$T(s) = \frac{\dfrac{1}{Cs}}{R + \dfrac{1}{Cs}} = \frac{\dfrac{1}{RC}}{s + \dfrac{1}{RC}}$$

The cutoff frequency is $1/RC$ and the passband gain is one. The magnitude of the transfer function is given by:

$$|T(j\omega)| = \frac{\dfrac{1}{RC}}{\sqrt{\left(\dfrac{1}{RC}\right)^2 + \omega^2}}$$

For each combination of resistors and capacitors, determine the gains at the signal and noise frequencies to see if they meet the specifications. The following MATLAB code performs the required calculations.

```
syms s t
w1 = 2*pi*1.1e6;
w2 = 2*pi*3.2e6;
R = [47 100 220 470];
C = [220 470 1000]*1e-12;
RC = R'*C;
RCinv = 1./RC;
RCinv2 = RCinv.^2;
Tjw1Mag = RCinv./sqrt(RCinv2+w1^2);
Tjw1MagdB = 20*log10(Tjw1Mag)
Tjw2Mag = RCinv./sqrt(RCinv2+w2^2);
Tjw2MagdB = 20*log10(Tjw2Mag)
Specs = (Tjw1MagdB≥-2)&(Tjw2MagdB<-7)
```

The resulting output is shown below.

```
Tjw1MagdB =
   -0.0221   -0.1001   -0.4357
   -0.0993   -0.4357   -1.6958
   -0.4607   -1.7919   -5.2009
   -1.7919   -5.2257  -10.6266
Tjw2MagdB =
   -0.1838   -0.7819   -2.7715
   -0.7761   -2.7715   -7.0265
   -2.8937   -7.2609  -13.1315
   -7.2609  -13.1653  -19.5569
Specs =
     0     0     0
     0     0     1
     0     1     0
     1     0     0
```

Three RC pairs meet the specifications, with the acceptable options being Part Numbers ZAEN26, ZAEN35, or ZAEN44.

Problem 12–71. (E) Design Evaluation

Your company issued a request for proposals listing the following design requirements and evaluation criteria.

Design Requirements: Design a low-pass filter with a passband gain of 9 ±10% and a cutoff frequency of 90 ±10% krad/s. A sensor drives the filter input with a 1-kΩ source resistance and an open-circuit voltage range of ±1.6 V.

Evaluation Criteria: Filter performance, parts count, use of standard parts, and cost. The two vendors have responded with the designs shown in Figure P12–71.

As a junior engineer, the project manager asks you evaluate the designs and recommend a vendor. Which vendor would you recommend and why?

Determine the transfer function and properties for each design option. We have the following results:

$$T_1(s) = \frac{(9.2)\dfrac{1}{RC}}{s+\dfrac{1}{RC}} = \frac{(9.2)(91827)}{s+91827}$$

$$K_1 = 9.2$$

$$\omega_{C1} = 91827 \text{ rad/s}$$

$$T_2(s) = \frac{(9.29)\dfrac{1}{(R+1000)C}}{s+\dfrac{1}{(R+1000)C}} = \frac{(9.29)(70472)}{s+70472}$$

$$K_2 = 9.29$$

$$\omega_{C2} = 70472 \text{ rad/s}$$

The second design does not meet the cutoff frequency requirement because of the loading from the source voltage output resistance. The first circuit meets the requirements for the gain and the cutoff frequency. In addition, in the first circuit, the maximum signal is $(9.2)(1.6) = 14.72$ which will not saturate the OP AMP. Choose the design from the First-Order Filter Company.

Problem 12–72. (E) Design Evaluation

In a research laboratory, you need a bandpass filter to meet the following requirements:

Design Requirements: Passband gain: 10 ±5%, $B = 10$ krad/s ±5%, $\omega_0 = 5$ krad/s ±2%, $\omega_{CL} = 2$ krad/s ±10%.

Evaluation Criteria: Filter performance, parts count, use of standard parts, and cost. The two vendors have responded with the designs shown in Figure P12–72.

As a research assistant, your supervisor asks you to evaluate the two designs and recommend a vendor. Which vendor would you recommend and why?

Determine the transfer function and properties for each design. We have the following results:

$$T_1(s) = (10)\frac{1000}{0.1s + 1000 + \dfrac{2500000}{s}} = \frac{100000s}{s^2 + 10000s + 25000000}$$

$$K_1 = 10$$

$$\omega_{01} = 5000 \text{ rad/s}$$

$$B_1 = 10000 \text{ rad/s}$$

$$\omega_{\text{CL1}} = 2071 \text{ rad/s}$$

$$T_2(s) = \left(\frac{s}{s+2062}\right)(11.7)\left(\frac{12136}{s+12136}\right)$$

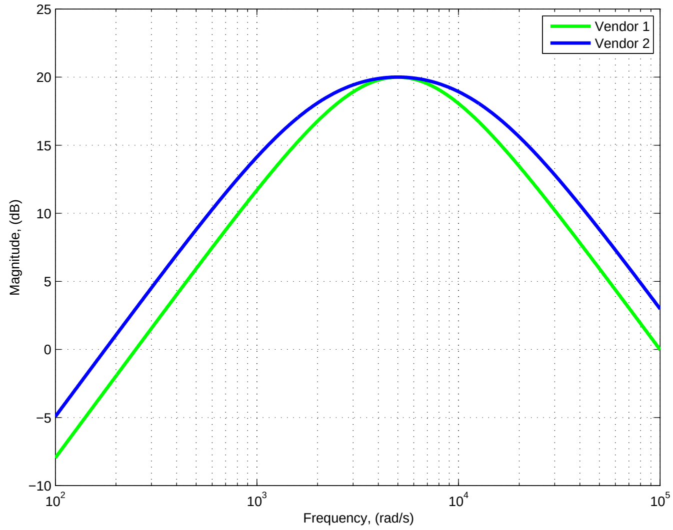
$$K_2 = 10.0009$$

$$\omega_{02} = 5002 \text{ rad/s}$$

$$B_2 = 14198 \text{ rad/s}$$

$$\omega_{\text{CL2}} = 1585 \text{ rad/s}$$

The following MATLAB plot show the gain responses for the two designs.



The first design meets all of the specifications, but the second design does not meet the requirements for the bandwidth or the lower cutoff frequency. Choose the design from Vendor #1.

Problem 12–73. (E) Design Evaluation

In a cable service distribution station, you need a bandstop filter to meet the following requirements:

Design Requirements: Passband gain: 10 $\pm 5\%$, $B = 3.3$ kHz $\pm 5\%$, $f_0 = 500$ Hz $\pm 2\%$, $f_{\text{CL}} = 75$ Hz $\pm 10\%$. Filter must interface with a 50-Ω source and a 500-Ω load.

Evaluation Criteria: Filter performance including depth of notch, parts count, power usage, ease of maintenance, use of standard parts, and cost.

The two vendors have responded with the designs shown in Figure P12–73. As an engineering cable guy, your supervisor asks you to evaluate the two designs and recommend a vendor. Which vendor would you recommend and why?

Vendor A uses a summer with a gain of five to combine a high-pass filter and a low-pass filter. Each filter has a gain of two. The high-pass filter has a cutoff frequency of 20 krad/s and the low-pass filter has a cutoff frequency of 500 rad/s. The transfer function for the circuit from Vendor A is shown below:

$$T_{\rm A}(s) = 5\left[\frac{2s}{s+20000} + \frac{2(500)}{s+500}\right] = 10\left[\frac{s^2+500s+500s+10^7}{(s+500)(s+20000)}\right]$$

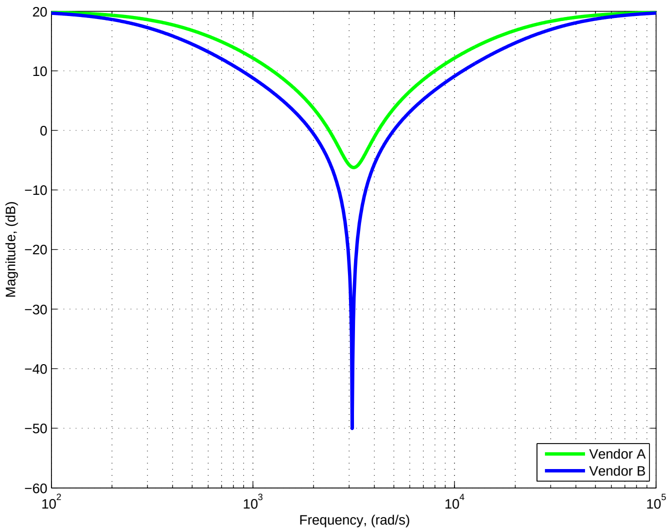
$$= 10\left[\frac{s^2+1000s+10^7}{s^2+20500s+10^7}\right]$$

From the transfer function, the low-frequency and high-frequency gains are both 10, the bandwidth is 20500 rad/s or 3.26 kHz, and the center frequency is 3162 rad/s or 503 Hz. With the OP AMP design, the circuit will work well for the impedances at the source and load.

Vendor B uses an *RLC* circuit with a noninverting amplifier with a gain of 10.1. The transfer function is shown below and accounts for the 50-Ω source resistance.

$$T_{\rm B}(s) = 10.1\left[\frac{0.0043s + \dfrac{41667}{s}}{0.0043s + (82+50) + \dfrac{41667}{s}}\right] = 10.1\left[\frac{s^2+9.6899\times10^6}{s^2+30698s+9.6899\times10^6}\right]$$

From the transfer function, the low-frequency and high-frequency gains are both 10.1, the bandwidth is 30698 rad/s or 4.89 kHz, and the center frequency is 3113 rad/s or 495 Hz. When connected to the source, the circuit does not meet the specification for the bandwidth. The MATLAB plot below confirms that the circuit from Vendor A meets all of the specifications, including the lower cutoff frequency, at approximately 80 Hz. Choose the circuit from Vendor A.



13 Fourier Analysis

13.1 Exercise Solutions

Exercise 13–1. Find the Fourier coefficients for the rectangular pulse wave in Figure 13–1.
Compute each coefficient.

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)dt = \frac{1}{T_0}\int_{-T/2}^{T/2} Adt = \frac{AT}{T_0}$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\cos(2\pi nt/T_0)dt = \frac{2}{T_0}\int_{-T/2}^{T/2} A\cos(2\pi nt/T_0)dt = \frac{2A}{n\pi}\sin\left(\frac{n\pi T}{T_0}\right)$$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\sin(2\pi nt/T_0)dt = \frac{2}{T_0}\int_{-T/2}^{T/2} A\sin(2\pi nt/T_0)dt = 0$$

Exercise 13–2. Given a rectangular pulse waveform as shown in Figure 13–1, let $A = 10$, $T_0 = 5$ ms, and $T = 2$ ms.

(a). Use MATLAB to calculate the Fourier coefficients for the first 10 harmonics.

The MATLAB code is shown below.

```
syms t
syms A T0 n T positive
ft = A;
a0 = int(ft,t,-T/2,T/2)/T0
an = 2*int(ft*cos(2*pi*n*t/T0),t,-T/2,T/2)/T0
bn = 2*int(ft*sin(2*pi*n*t/T0),t,-T/2,T/2)/T0
% Substitute numerical values
A = 10;
T0 = 5e-3;
T = 2e-3;
a0 = subs(a0)
an = subs(an)
bn = subs(bn)
nn = 1:10;
ann = subs(an,n,nn)'
```

The corresponding results are

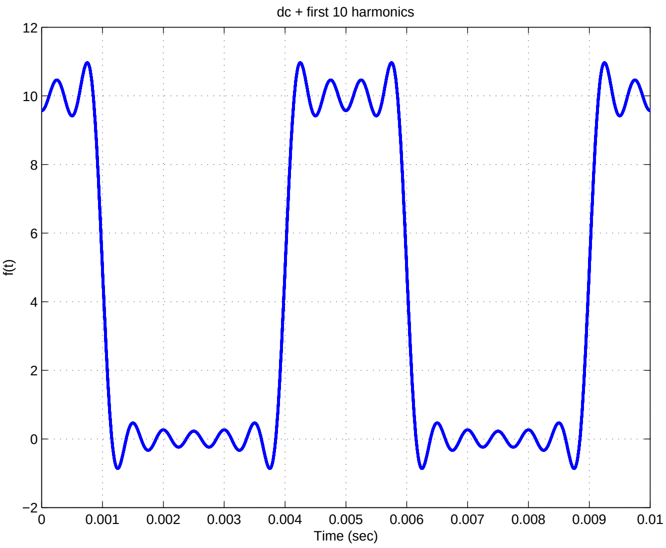
```
a0 = (A*T)/T0
an = (2*A*sin((pi*T*n)/T0))/(pi*n)
bn = 0
a0 = 4.0000e+000
an = (20*sin((2*pi*n)/5))/(pi*n)
bn = 0.0000e+000
ann =
     6.0546e+000
     1.8710e+000
    -1.2473e+000
    -1.5137e+000
  -311.8537e-018
     1.0091e+000
   534.5653e-003
  -467.7446e-003
  -672.7349e-003
  -311.8537e-018
```

(b). Use the 10 harmonics to plot a truncated series representation of the waveform.

The MATLAB code to plot the harmonics is shown below.

```
% Create vectors for the cosine and sine terms
cnt = cos(2*pi*nn*t/T0);
snt = sin(2*pi*nn*t/T0);
% Create the partial sums
ft10 = a0 + ann*cnt' + bn*snt';
% Substitute in a time vector
tt = 0:T0/500:2*T0;
ftt10 = subs(ft10,t,tt);
% Create the plots
figure
plot(tt,ftt10,'b','LineWidth',2)
xlabel('Time (sec)')
ylabel('f(t)')
title('dc + first 10 harmonics')
grid on
```

The MATLAB plot using the 10 harmonics is shown below.



Exercise 13–3. The triangular wave in Figure 13–4 has a peak amplitude of $A = 10$ and $T_0 = 2$ ms. Calculate the Fourier coefficients of the first nine harmonics.

Use the formula in the table and MATLAB to perform the calculations. Note that $a_0 = 0$ and $b_n = 0$ for all n.

```
A = 10;
n = 1:2:9;
an = 8*A./(n*pi).^2
```

The results are:

```
an = 8.1057e+000    900.6327e-003    324.2278e-003    165.4223e-003    100.0703e-003
```

The results show a_n for $n = 1$, 3, 5, 7, and 9, since $a_n = 0$ for even values of n.

Exercise 13–4. Derive expressions for the amplitude A_n and phase angle ϕ_n for the triangular wave in Figure 13–4 and write an expression for the first three nonzero terms in the Fourier series with $A = \pi^2/8$ and $T_0 = 2\pi/5000$ s.

For the triangular wave, $a_0 = 0$, $a_n = 0$ for even values of n, and $b_n = 0$ for all n. Therefore, $A_n = a_n$

and for odd values of n we have:

$$A = \frac{\pi^2}{8}$$

$$A_n = \frac{8A}{(n\pi)^2} = \frac{1}{n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) = \tan^{-1}(0) = 0^\circ$$

$$A_n \cos(2\pi n f_0 t) = \frac{1}{n^2}\cos(5000nt)$$

$$f(t) = \cos(5000t) + \frac{1}{9}\cos(15000t) + \frac{1}{25}\cos(25000t) + \cdots$$

Exercise 13–5.

(a). Identify the symmetries in the waveform $f(t)$ whose Fourier series is

$$f(t) = \frac{2\sqrt{3}A}{\pi}\left[\cos(\omega_0 t) - \frac{1}{5}\cos(5\omega_0 t) + \frac{1}{7}\cos(7\omega_0 t) - \frac{1}{11}\cos(11\omega_0 t) + \frac{1}{13}\cos(13\omega_0 t) + \cdots\right]$$

The waveform has no average value. There are only cosine terms, so it have even symmetry. Only the odd harmonics are present, so the waveform has half-wave symmetry.

(b). Write the corresponding terms of the function $g(t) = f(t - T_0/4)$.

Computer $g(t)$.

$$\begin{aligned}
g(t) &= f(t - T_0/4) \\
&= \frac{2\sqrt{3}A}{\pi}\Big[\cos(\omega_0 t - \omega_0 T_0/4) - \frac{1}{5}\cos(5\omega_0 t - 5\omega_0 T_0/4) + \frac{1}{7}\cos(7\omega_0 t - 7\omega_0 T_0/4) \\
&\quad - \frac{1}{11}\cos(11\omega_0 t - 11\omega_0 T_0/4) + \frac{1}{13}\cos(13\omega_0 t - 13\omega_0 T_0/4) + \cdots\Big] \\
&= \frac{2\sqrt{3}A}{\pi}\Big[\cos(\omega_0 t - \pi/2) - \frac{1}{5}\cos(5\omega_0 t - 5\pi/2) + \frac{1}{7}\cos(7\omega_0 t - 7\pi/2) \\
&\quad - \frac{1}{11}\cos(11\omega_0 t - 11\pi/2) + \frac{1}{13}\cos(13\omega_0 t - 13\pi/2) + \cdots\Big] \\
&= \frac{2\sqrt{3}A}{\pi}\Big[\sin(\omega_0 t) - \frac{1}{5}\sin(5\omega_0 t) - \frac{1}{7}\sin(7\omega_0 t) + \frac{1}{11}\sin(11\omega_0 t) + \frac{1}{13}\cos(13\omega_0 t) + \cdots\Big]
\end{aligned}$$

Exercise 13–6.

(a). Given the circuit in Figure 13–8, use Laplace-domain techniques to find the current $i(t)$ when the voltage source is a scaled ramp function $v_S(t) = (V_A/T_0)tu(t)$. (*Hint:* Recall the relationship between the impulse response, step response, and ramp response.)

In the Laplace domain, we have the following results:

$$H(s) = T(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)} = \frac{1}{R+Ls} = \frac{\frac{1}{L}}{s+\frac{R}{L}}$$

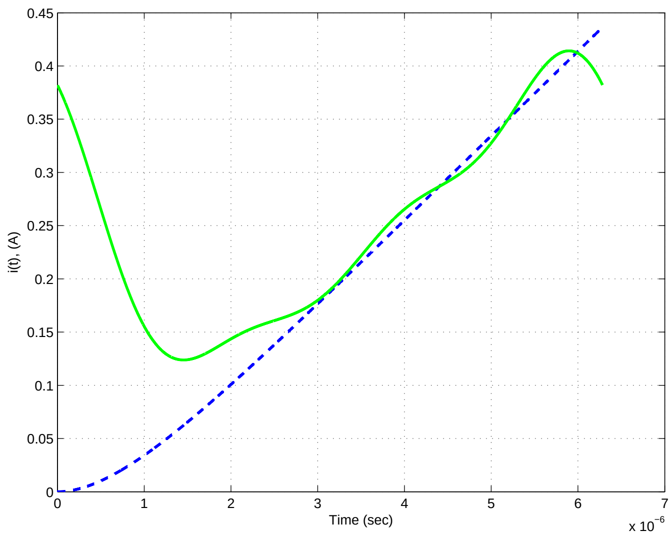
$$V(s) = \frac{V_A}{T_0}\frac{1}{s^2}$$

$$I(s) = H(s)V(s) = \frac{V_A}{T_0}\left(\frac{\frac{1}{L}}{s^2\left(s+\frac{R}{L}\right)}\right) = \frac{V_A}{R^2T_0}\left(\frac{R}{s^2} - \frac{L}{s} + \frac{L}{s+\frac{R}{L}}\right)$$

$$i(t) = \frac{V_A}{R^2T_0}\left[Rt - L\left(1-e^{-Rt/L}\right)\right]u(t)$$

(b). Using the same values as in Example 13–6, on the same axes plot the current $i(t)$ found above and the approximation for $i(t)$ found in Example 13–6 over one period.

The plot is shown below.



Exercise 13–7. Design a first-order low pass filter to allow only the fundamental of the square wave of Design Example 13–7 to pass with an attenuation of ≤ 3 dB. Verify your results by calculating the magnitude of the fundamental and of the third harmonic.

The fundamental frequency is $f_0 = 200$ kHz or $\omega_0 = 400\pi$ krad/s or 1.257 Mrad/s. Set the cutoff frequency to be the fundamental frequency. The transfer function of a series RC circuit, with the output taken across the capacitor is

$$T(s) = \frac{\frac{1}{RC}}{s+\frac{1}{RC}}$$

We then have the following results:

$$\omega_C = \frac{1}{RC}$$

$$R = 1 \text{ k}\Omega$$

$$C = 796 \text{ pF}$$

$$|T(j\omega_0)| = \frac{1}{\sqrt{2}} = -3 \text{ dB}$$

$$|T(j3\omega_0)| = 0.3162 = -10 \text{ dB}$$

The fundamental is attenuated by -3 dB and the third harmonic is attenuated by -10 dB.

Exercise 13–8. Derive an expression for the first three nonzero terms in the Fourier series of the steady-state output voltage in Example 13–8.

For the example, $b_n = 0$ for all n, so $A_n = |a_n|$ and $\phi_n = 0°$. The transfer function is

$$T(s) = \frac{200^2}{s^2 + 70s + 200^2}$$

The first three nonzero terms are for $n = 0$, 2, and 4. Determine the magnitude of the input, find the transfer function at the corresponding frequencies, and then find the magnitude and phase of the output. The results are:

$$A_0 = \frac{2V_A}{\pi} = 15.02$$

$$A_2 = \left|\frac{4V_A/\pi}{1-4}\right| = 10.02$$

$$A_4 = \left|\frac{4V_A/\pi}{1-16}\right| = 2.00$$

$$T(j0) = 1$$

$$T(j2\omega_0) = 0.0753\angle 5.7°$$

$$T(j4\omega_0) = 0.0179\angle 2.7°$$

$$v_O(t) = 15.02 + 0.754\cos(2\pi 120t + 5.7°) + 0.036\cos(2\pi 240t + 2.7°) \text{ V}$$

Exercise 13–9. The fullwave rectified sine wave shown in Figure 13–4 has an rms value of $A/\sqrt{2}$. What fraction of the average power that the waveform delivers to a resistor is carried by the first two nonzero terms in its Fourier series?

We have the following results with $R = 1$:

$$V_{\text{rms}} = \frac{A}{\sqrt{2}}$$

$$P_{\text{rms}} = V_{\text{rms}}^2 = \frac{A^2}{2}$$

$$A_0 = \frac{2A}{\pi}$$

$$A_2 = \frac{4A/\pi}{4-1}$$

$$P = \left(\frac{2A}{\pi}\right)^2 + \frac{1}{2}\left(\frac{4A/\pi}{4-1}\right)^2$$

$$\frac{P}{P_{\text{rms}}} = \frac{88}{9\pi^2} = 0.9907$$

Exercise 13–10. In the rectangular-pulse waveform shown in Figure 13–4, the width of the pulse is one-third the period, $T = T_0/3$. The waveform is to pass through a low-pass filter and then through a resistive load. The load must receive at least 97% of the average power in the original waveform. Determine the minimum value of N such that if the filter passes components V_0, V_1, V_2, …, V_N, the load will receive the required amount of power. (*Hint:* You may want to use MATLAB to perform the calculations.)

We have the following results with $R = 1$:

$$T = \frac{T_0}{3}$$

$$a_0 = \frac{AT}{T_0}$$

$$a_n = \frac{2A}{n\pi}\sin\left(\frac{n\pi T}{T_0}\right)$$

$$V_{\text{rms}}^2 = \frac{1}{T_0}\int_0^{T_0} |v(t)|^2\,dt = \frac{1}{T_0}\int_{-T_0/6}^{T_0/6} A^2 dt = \frac{A^2}{3}$$

$$P_{\text{rms}} = \frac{A^2}{3}$$

$$V_0 = \frac{A}{3}$$

$$V_n = \frac{2A}{n\pi}\sin\left(\frac{n\pi}{3}\right)$$

$$P = V_0^2 + \sum_{n=1}^{K}\frac{V_n^2}{2}$$

Find the value for K where P is at least 97% of P_{rms}. The following MATLAB code provides the solution.

```
syms A T0 positive
syms t
% Compute the RMS value
T = T0/3;
```

```
R = 1;
Vrms = sqrt(int(A^2,t,T0/6,-T0/6)/T0)
Prms = Vrms^2/R;
V0 = A*T/T0
n = 1:10;
Vn = 2*A./n/pi.*sin(n*pi*T/T0);
P = V0^2/R + (Vn*Vn')/2/R;
Ratio = simplify(P/Prms)
Ratio_num = double(Ratio)
```

The results are:

```
Vrms = (3^(1/2)*A)/3
V0 = A/3
Ratio = 197361/(31360*pi^2) + 1/3
Ratio_num = 970.9880e-003
```

Use $N = 10$ to pass the required amount of power.

Exercise 13–11. Use MATLAB to compute the Fourier transform of the rectangular pulse in Figure 13–13. Confirm the results of Example 13–11.

The following MATLAB code provides the solution:

```
% Create the symbolic variables
syms t A T w
% Compute the Fourier transform with two techniques
Fw1 = int(A*exp(-j*w*t),t,-T/2,T/2)
Fw2 = fourier(A*(heaviside(t+T/2)-heaviside(t-T/2)),t,w)
Fw2a = simplify(Fw2)
```

The results are:

```
Fw1 = (2*A*sin((T*w)/2))/w
Fw2 = -A*((1/exp((T*w*i)/2))*(pi*dirac(w) - i/w) - exp((T*w*i)/2)*(pi*dirac(w) - i/w))
Fw2a = (2*A*sin((T*w)/2))/w
```

Both options for calculating the Fourier transform yield answers that agree with the results in Example 13-11.

Exercise 13–12. Find the Fourier transform of the waveform in Figure 13-16.

We have the following results:

$$
\begin{aligned}
F(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \\
&= \int_{-T/2}^{0} (-A)e^{-j\omega t}dt + \int_{0}^{T/2} Ae^{-j\omega t}dt \\
&= \frac{A}{j\omega}\left[2 - \left(e^{j\omega T/2} + e^{-j\omega T/2}\right)\right] \\
&= \frac{2A}{j\omega}\left[1 - \cos\left(\frac{\omega T}{2}\right)\right]
\end{aligned}
$$

Exercise 13–13. Use the inversion integral to find the inverse transform of $F(\omega) = j\omega\pi[u(\omega+1) - u(\omega-1)]$.

We have the following results using integration by parts:

$$
\begin{aligned}
f(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \\
&= \frac{1}{2\pi}\int_{-1}^{1} j\omega\pi e^{j\omega t}d\omega \\
&= \frac{1}{2}\left[\frac{1}{t}\left(e^{jt}+e^{-jt}\right) - \frac{1}{t}\left(\frac{1}{jt}\right)\left(e^{jt}-e^{-jt}\right)\right] \\
&= \frac{\cos(t)}{t} - \frac{\sin(t)}{t^2}
\end{aligned}
$$

Exercise 13–14. Use Laplace transforms to find the Fourier transforms of the following causal waveforms. Assume that $\alpha > 0$.

(a). $f(t) = A[e^{-\alpha t}\sin(\beta t)]u(t)$

We have the following results:

$$
\begin{aligned}
F(s) &= \frac{A\beta}{(s+\alpha)^2+\beta^2} \\
F(\omega) &= F(s)\Big|_{\sigma=0} = \frac{A\beta}{(j\omega+\alpha)^2+\beta^2}
\end{aligned}
$$

(b). $f(t) = A[\alpha t e^{-\alpha t}]u(t)$.

We have the following results:

$$
\begin{aligned}
F(s) &= \frac{A\alpha}{(s+\alpha)^2} \\
F(\omega) &= F(s)\Big|_{\sigma=0} = \frac{A\alpha}{(j\omega+\alpha)^2}
\end{aligned}
$$

Exercise 13–15. Use the Laplace transform method to find the causal waveform corresponding to the following Fourier transform:

$$F(\omega) = \frac{5(j\omega-3)}{(j\omega+1)(j\omega+5)}$$

We have the following results:

$$
\begin{aligned}
F(s) &= \frac{5(s-3)}{(s+1)(s+5)} = \frac{-5}{s+1} + \frac{10}{s+5} \\
f(t) &= \left[-5e^{-t}+10e^{-5t}\right]u(t)
\end{aligned}
$$

Exercise 13–16. Explain why Laplace transforms cannot be used to find the Fourier transforms of the following waveforms. Assume $\alpha > 0$.

(a). $f(t) = A\alpha t u(t)$

$f(t)$ is not absolutely integrable or, equivalently, $F(s)$ has a double pole at $s = 0$.

(b). $f(t) = Ae^{-\alpha|t|}$

$f(t)$ is not causal or, equivalently, $F(s)$ has a pole in the right half plane.

(c). $f(t) = A\sin(\alpha t)$

$f(t)$ is not causal or, equivalently, $F(s)$ has poles on the $j\omega$ axis.

Exercise 13–17. Explain why Laplace transforms cannot be used to find the inverse transforms of the following functions. Assume $\alpha > 0$.

(a). $F(\omega) = \dfrac{2\alpha}{\alpha^2 + \omega^2}$

$F(s)$ has a right half plane pole at $s = +\alpha$.

(b). $F(\omega) = e^{-\alpha|\omega|}$

$F(s)$ is not a rational function.

Exercise 13–18. Use MATLAB to find the Fourier transform of $f(t) = Ae^{-|\alpha t|}\cos(\beta t)$ for $\alpha > 0$.

The MATLAB code is shown below.

```
syms A B t w real
syms a positive
Fw = simplify(int(A*exp(a*t)*cos(B*t)*exp(-j*w*t),t,-inf,0)...
    +int(A*exp(-a*t)*cos(B*t)*exp(-j*w*t),t,0,inf))
```

The results are:

```
Fw = (2*A*a*(Bˆ2 + aˆ2 + wˆ2))/(Bˆ4 + 2*Bˆ2*aˆ2 - 2*Bˆ2*wˆ2 + aˆ4 + 2*aˆ2*wˆ2 + wˆ4)
```

The answer is

$$F(\omega) = \frac{2A\alpha(\omega^2 + \beta^2 + \alpha^2)}{(\omega^2 + 2\beta\omega + \beta^2 + \alpha^2)(\omega^2 - 2\beta\omega + \beta^2 + \alpha^2)}$$

$$= \frac{2A\alpha(\omega^2 + \beta^2 + \alpha^2)}{\omega^4 + \alpha^4 + \beta^4 + 2\beta^2\alpha^2 - 2\beta^2\omega^2 + 2\alpha^2\omega^2}$$

Exercise 13–19. Use symbolic operations in MATLAB to confirm the time differentiation property of Fourier transforms.

The MATLAB code is:

```
syms t w real
Fw = fourier(diff(sym('f(t)')),t,w)
```

The results are:

```
Fw = w*transform::fourier(f(t), t, -w)*i
```

The results are consistent with the property. The MATLAB expression `transform::fourier` is part of the symbolic calculation engine and its frequency variable `w` is defined with the opposite sign of the frequency variable for the `fourier` command. See the MATLAB Help files for additional information.

Exercise 13–20. Use the reversal property to find the Fourier transform of $f(t) = e^{-\alpha|t|}$. Assume $\alpha > 0$.

We have the following results:

$$f(t) = e^{-\alpha|t|} = e^{-\alpha t}u(t) + e^{\alpha t}u(-t)$$

$$\mathscr{F}\left\{e^{-\alpha t}u(t)\right\} = \frac{1}{\alpha + j\omega}$$

$$\mathscr{F}\left\{e^{\alpha t}u(-t)\right\} = \frac{1}{\alpha - j\omega}$$

$$F(\omega) = \frac{1}{\alpha + j\omega} + \frac{1}{\alpha - j\omega} = \frac{2\alpha}{\omega^2 + \alpha^2}$$

Exercise 13–21. Use the duality property to find the Fourier transform of $g(t) = 1/t$.
We have the following results:

$$f(t) = \text{sgn}(t) = u(t) - u(-t)$$

$$F(\omega) = \frac{2}{j\omega}$$

$$\mathscr{F}\left\{F(t)\right\} = \mathscr{F}\left\{\frac{2}{jt}\right\} = 2\pi f(-\omega) = 2\pi\text{sgn}(-\omega)$$

$$\frac{2}{j}\mathscr{F}\left\{\frac{1}{t}\right\} = 2\pi\text{sgn}(-\omega)$$

$$\mathscr{F}\left\{\frac{1}{t}\right\} = j\pi\text{sgn}(-\omega)$$

Exercise 13–22. Use Tables 13–1 and 13–2 to find the Fourier transforms of

(a). $f_1(t) = \text{sgn}(t) + 2$

We have the following results:

$$F_1(\omega) = \mathscr{F}\left\{f_1(t)\right\} = \mathscr{F}\left\{\text{sgn}(t) + 2\right\} = \mathscr{F}\left\{\text{sgn}(t)\right\} + \mathscr{F}\left\{2\right\}$$

$$= \frac{2}{j\omega} + (2)[2\pi\delta(\omega)] = \frac{2}{j\omega} + 4\pi\delta(\omega)$$

(b). $f_2(t) = \cos(2t) + 2$

We have the following results:

$$F_2(\omega) = \mathscr{F}\left\{f_2(t)\right\} = \mathscr{F}\left\{\cos(2t) + 2\right\} = \mathscr{F}\left\{\cos(2t)\right\} + \mathscr{F}\left\{2\right\}$$

$$= \pi[\delta(\omega - 2) + \delta(\omega + 2)] + 4\pi\delta(\omega)$$

$$= \pi[\delta(\omega - 2) + 4\delta(\omega) + \delta(\omega + 2)]$$

Exercise 13–23. Use Tables 13–1 and 13–2 to find the inverse Fourier transforms of

(a). $F_1(\omega) = \dfrac{1}{j\omega} + 3\pi\delta(\omega)$

We have the following results:

$$f_1(t) = \mathscr{F}^{-1}\left\{F_1(\omega)\right\} = \frac{1}{2}\text{sgn}(t) + \frac{3}{2} = 1 + \frac{1}{2}[1 + \text{sgn}(t)] = 1 + u(t)$$

(b). $F_2(\omega) = \dfrac{2}{4+j\omega} - 2 + 4\pi\delta(\omega+2)$

We have the following results:

$$f_2(t) = \mathscr{F}^{-1}\left\{F_2(\omega)\right\} = 2e^{-4t}u(t) - 2\delta(t) + 2e^{-j2t}$$

Exercise 13–24. Use Fourier transforms to find $v_2(t)$ in the circuit in Figure 13–23 for $v_1(t) = Ae^{-\alpha t}u(t)$ V. Apply voltage division in the frequency domain.

$$\begin{aligned}
V_2(\omega) &= \left[\frac{\dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}}\right] V_1(\omega) \\
&= \left[\frac{\dfrac{1}{RC}}{j\omega + \dfrac{1}{RC}}\right]\left[\frac{A}{j\omega + \alpha}\right] \\
&= \frac{\dfrac{A}{\alpha RC - 1}}{j\omega + \dfrac{1}{RC}} - \frac{\dfrac{A}{\alpha RC - 1}}{j\omega + \alpha} \\
v_2(t) &= \frac{A}{\alpha RC - 1}\left(e^{-t/RC} - e^{-\alpha t}\right)u(t) \text{ V}
\end{aligned}$$

Exercise 13–25. Use Fourier transforms to find $v_2(t)$ in the circuit in Figure 13–23 for $v_1(t) = u(t) - 1$. We have the following results:

$$\begin{aligned}
v_1(t) &= u(t) - 1 \\
V_1(\omega) &= \frac{1}{j\omega} + \pi\delta(\omega) - 2\pi\delta(\omega) = \frac{1}{j\omega} - \pi\delta(\omega) \\
V_2(\omega) &= \left[\frac{\dfrac{1}{RC}}{j\omega + \dfrac{1}{RC}}\right] V_1(\omega) = \left[\frac{\dfrac{1}{RC}}{j\omega + \dfrac{1}{RC}}\right]\left[\frac{1}{j\omega} - \pi\delta(\omega)\right] \\
&= \frac{1}{j\omega} - \frac{1}{j\omega + \dfrac{1}{RC}} - \pi\delta(\omega) \\
v_2(t) &= \frac{1}{2}\text{sgn}(t) - e^{-t/RC}u(t) - \frac{1}{2} \\
&= -u(-t) - e^{-t/RC}u(t)
\end{aligned}$$

Exercise 13–26. The impulse response of a system is $h(t) = 2e^{-4t}u(t)$. Find the output $y(t)$ when $x(t) = -2 + 4u(t)$.

Solve in the Fourier domain.

$$h(t) = 2e^{-4t}u(t)$$

$$x(t) = -2 + 4u(t)$$

$$H(\omega) = \frac{2}{4 + j\omega}$$

$$X(\omega) = -4\pi\delta(\omega) + \frac{4}{j\omega} + 4\pi\delta(\omega) = \frac{4}{j\omega}$$

$$Y(\omega) = H(\omega)X(\omega) = \frac{8}{j\omega(4 + j\omega)} = \frac{2}{j\omega} - \frac{2}{j\omega + 4}$$

$$y(t) = \mathrm{sgn}(t) - 2e^{-4t}u(t)$$

Exercise 13–27.

(a). Find the transfer function of a system whose impulse response is $h(t) = \delta(t) - \alpha e^{-\alpha|t|}$.

Use the Fourier transform pairs.

$$H(\omega) = 1 - \frac{2\alpha^2}{\alpha^2 + \omega^2} = \frac{\alpha^2 - 2\alpha^2 + \omega^2}{\omega^2 + \alpha^2} = \frac{\omega^2 - \alpha^2}{\omega^2 + \alpha^2}$$

(b). Describe the frequency response of the system.

At both low and high frequencies, the magnitude of the gain approaches one. When the frequency approaches α, the magnitude of the gain goes to zero. The system has a bandstop characteristic with a notch at $\omega = \pm\alpha$.

Exercise 13–28. Derive an expression for the 1-Ω energy in the waveform $v(t) = A[u(t+T) - u(t-T)]$ V (a) in the time domain and (b) in the frequency domain.

(a). In the time domain, we have the following results:

$$W_{1\Omega} = \int_{-\infty}^{\infty} v^2(t)dt = \int_{-T}^{T} A^2 dt = 2A^2T$$

(b). In the frequency domain, we have the following results:

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

$$V(\omega) = A\left[e^{j\omega T}\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) - e^{-j\omega T}\left(\frac{1}{j\omega} + \pi\delta(\omega)\right)\right] = 2AT\frac{\sin(\omega T)}{\omega T}$$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|2AT\frac{\sin(\omega T)}{\omega T}\right|^2 d\omega = 2A^2T$$

Exercise 13–29. Find the percentage of the 1-Ω energy in $v(t) = A[u(t+T) - u(t-T)]$ V that is carried in the frequency band $|\omega| < \pi/T$, which represents the main lobe of the spectrum.

Integrate in the frequency domain for $|\omega| < \pi/T$. Since the frequency domain is symmetric, we can also double the integral over $0 < \omega < \pi/T$. Using the development in Exercise 13–28, we have the following results:

$$W_{1\Omega} = 2A^2T$$

$$W = \frac{1}{\pi}\int_0^{\pi/T} |V(\omega)|^2\, d\omega = \frac{1}{\pi}\int_0^{\pi/T} \left|2AT\frac{\sin(\omega T)}{\omega T}\right|^2 d\omega$$

Use MATLAB to compute the integral and the ratio:

```
syms t w real
syms A T positive
Vw = 2*A*sin(w*T)/w;
W = simplify(int(abs(Vw^2),w,-inf,inf)/2/pi)
W2 = simplify(int(abs(Vw^2),w,0,pi/T)/pi)
EnergyRatio = double(W2/W)
```

The results are:

```
W = 2*A^2*T
Warning: Explicit integral could not be found.
W2 = 2*A^2*T + (2*A^2*T*(Ei(1, -2*pi*i)*i - Ei(1, 2*pi*i)*i))/pi
EnergyRatio = 902.8233e-003
```

Therefore, the main lobe contains 90.28% of the power.

Exercise 13–30.

(a). Find the total 1-Ω energy carried by a double-sided exponential $f(t) = Ae^{-\alpha|t|}$.

We have the following results:

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)dt = 2\int_0^{\infty} A^2e^{-2\alpha t}dt = \frac{A^2}{\alpha}$$

(b). Find the fraction of the total energy carried by the frequency band $|\omega| < \alpha$.

We have the following results:

$$F(\omega) = \frac{2A\alpha}{\alpha^2+\omega^2}$$

$$W = \frac{1}{2\pi}\int_{-\alpha}^{\alpha} \left|\frac{2A\alpha}{\alpha^2+\omega^2}\right|^2 d\omega = \frac{A^2(\pi+2)}{2\alpha\pi}$$

$$\frac{W}{W_{1\Omega}} = \frac{\pi+2}{2\pi} = 0.8183$$

Exercise 13–31. What percentage of the total energy in $f(t) = 5e^{-10t}u(t)$ is carried by the frequency band $|\omega| > 5$ rad/s?

We have the following results:

$$f(t) = 5e^{-10t}u(t)$$

$$F(\omega) = \frac{5}{10 + j\omega}$$

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)dt = \int_0^{\infty} 25e^{-20t}dt = \frac{5}{4}$$

$$W = \frac{1}{\pi}\int_5^{\infty} \left|\frac{5}{10 + j\omega}\right|^2 d\omega = \frac{5\tan^{-1}(2)}{2\pi}$$

$$\frac{W}{W_{1\Omega}} = \frac{2\tan^{-1}(2)}{\pi} = 0.7048$$

13.2 Problem Solutions

Problem 13–1. Derive expressions for the Fourier coefficients of the periodic waveform in Figure P13–1.

Use the results in Figure 13–4 for the rectangular pulse and substitute the correct values. We have the following for all values of n:

$$a_0 = \frac{AT}{T_0} = \frac{(5)(250\,\mu)}{0.001} = \frac{5}{4}$$

$$a_n = \frac{2A}{n\pi}\sin\left(\frac{n\pi T}{T_0}\right) = \frac{10}{n\pi}\sin\left(\frac{n\pi}{4}\right)$$

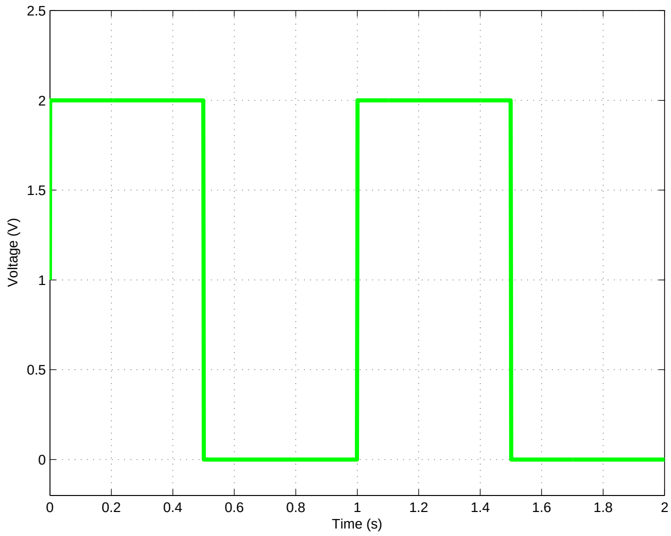
$$b_n = 0$$

Problem 13–2. The equation for the first cycle ($0 \leq t \leq T_0$) of a periodic pulse train is

$$v(t) = V_{\mathrm{A}}\left[2u(t) - 2u(t - T_0/2)\right]\ \mathrm{V}$$

(a). Sketch the first two cycles of the waveform.

The sketch is shown below.



(b). Derive expressions for the Fourier coefficients a_n and b_n.

We have the following results:

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v(t)dt = \frac{1}{T_0}\int_0^{T_0/2} 2V_{\mathrm{A}}dt = V_{\mathrm{A}}$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\cos(2\pi nt/T_0)dt = \frac{2}{T_0}\int_0^{T_0/2} 2V_{\mathrm{A}}\cos(2\pi nt/T_0)dt = \frac{2V_{\mathrm{A}}\sin(n\pi)}{n\pi} = 0$$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\sin(2\pi nt/T_0)dt = \frac{2}{T_0}\int_0^{T_0/2} 2V_{\mathrm{A}}\sin(2\pi nt/T_0)dt = \frac{4V_{\mathrm{A}}\sin^2(n\pi/2)}{n\pi}$$

Problem 13–3. Derive expressions for the Fourier coefficients of the periodic waveform in Figure P13–3.

We can write the first cycle of the waveform as

$$v(t) = V_{\mathrm{A}}\left[1 - \frac{2t}{T_0}\right]$$

Use MATLAB to perform the integration.

```
syms t VA real
syms T0 n positive
vt = VA*(1-2*t/T0);
a0 = int(vt,t,0,T0)/T0;
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0)/T0;
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0)/T0;
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
```

The results are:

```
a02 = 0
an2 = (VA*(2*sin(pi*n)^2 - pi*n*sin(2*pi*n)))/(pi^2*n^2)
bn2 = -(2*VA*cos(pi*n)*(sin(pi*n) - pi*n*cos(pi*n)))/(pi^2*n^2)
```

Equivalently, for all n we have:

$$a_0 = 0$$

$$a_n = 0$$

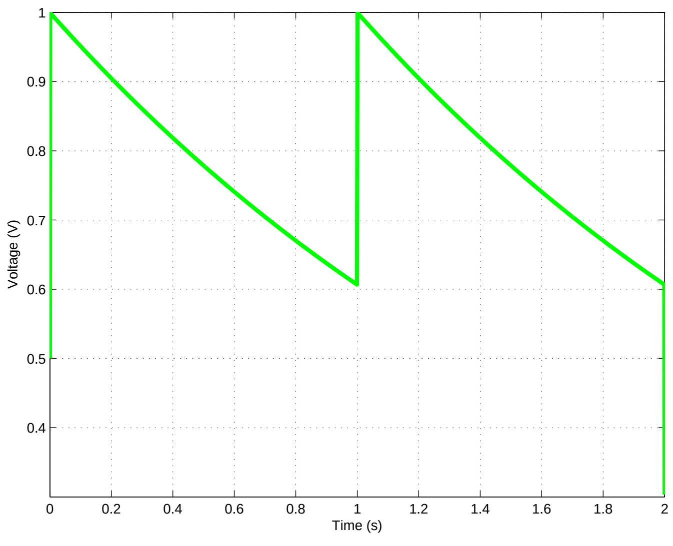
$$b_n = \frac{2V_\mathrm{A}}{n\pi}$$

Problem 13–4. The equation for the first cycle $(0 \le t \le T_0)$ of a periodic waveform is

$$v(t) = V_\mathrm{A} e^{-t/2T_0} \text{ V}$$

(a). Sketch the first two cycles of the waveform.

The sketch is shown below.



(b). Derive expressions for the Fourier coefficients a_n and b_n.

Use MATLAB to perform the integration.

```
syms t VA real
syms T0 n positive
vt = VA*exp(-t/T0/2);
a0 = int(vt,t,0,T0)/T0;
```

```
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0)/T0;
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0)/T0;
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
```

The results are:

```
a02 = (2 - 2/exp(1)^(1/2))*VA
an2 = (4*VA*(exp(1)^(1/2) - cos(2*n*pi) + 4*pi*n*sin(2*pi*n)))/...
      (exp(1)^(1/2)*(16*pi^2*n^2 + 1))
bn2 = -(4*VA*(sin(2*pi*n) - 4*pi*n*exp(1)^(1/2) + 4*pi*n*cos(2*pi*n)))/...
       (exp(1)^(1/2)*(16*pi^2*n^2 + 1))
```

Equivalently, for all n we have:

$$a_0 = V_{\rm A}\left[2 - \frac{2}{\sqrt{e}}\right]$$

$$a_n = \frac{4V_{\rm A}(\sqrt{e}-1)}{\sqrt{e}(16\pi^2 n^2 + 1)}$$

$$b_n = \frac{16\pi n V_{\rm A}(\sqrt{e}-1)}{\sqrt{e}(16\pi^2 n^2 + 1)}$$

Problem 13–5. Derive expressions for the Fourier coefficients of the periodic waveform in Figure P13–5.

On the interval $0 < t < T_0/2$, the waveforms is $v_{\rm S}(t) = 2V_{\rm A}t/T_0$. Use MATLAB to evaluate the integrals.

```
syms t VA real
syms T0 n positive
vt = 2*VA*t/T0;
a0 = int(vt,t,0,T0/2)/T0;
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0/2)/T0;
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0/2)/T0;
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
```

The results are

```
a02 = VA/4
an2 = (VA*(cos(pi*n) + pi*n*sin(pi*n) - 1))/(pi^2*n^2)
bn2 = (VA*(sin(pi*n) - pi*n*cos(pi*n)))/(pi^2*n^2)
```

Simplified, the coefficients for all n are:

$$a_0 = \frac{V_{\rm A}}{4}$$

$$a_n = \frac{V_{\rm A}[\cos(n\pi) - 1]}{n^2\pi^2}$$

$$b_n = \frac{-V_{\rm A}\cos(n\pi)}{n\pi}$$

Problem 13–6. Use the results in Figure 13–4 to calculate the Fourier coefficients of the square wave in Figure P13–6. Write an expression for the first four nonzero terms in the Fourier series.

For this square ware, $a_0 = 0$, $a_n = 0$ for all n, and $b_n = 0$ for even values of n. For odd values of n, we have:

$$b_n = \frac{4A}{n\pi} = \frac{40}{n\pi}$$

With $T_0 = 10$ ms, $f_0 = 100$ Hz. The first four nonzero terms in the Fourier series are:

$$v(t) = 12.73\sin(200\pi t) + 4.24\sin(600\pi t) + 2.55\sin(1000\pi t) + 1.82\sin(1400\pi t)$$

Problem 13–7. Use the results in Figure 13–4 to calculate the Fourier coefficients of the shifted triangular wave in Figure P13–7. Write an expression for the first four nonzero terms in the Fourier series.

For the unshifted version, the only nonzero coefficients are $a_n = 8A/(n\pi)^2$ for odd values of n. The shift is a $-\pi/2$ radian shift or $T_0/4$. Write the original expression and apply the shift.

$$\begin{aligned}
a_n &= \frac{8A}{(n\pi)^2} \quad \text{for} \quad \text{odd } n \\
v(t + T_0/4) &= \sum_{n \text{ odd}} a_n \cos\left(\frac{2\pi n t}{T_0}\right) \\
v(t) &= \sum_{n \text{ odd}} a_n \cos\left(\frac{2\pi n(t - T_0/4)}{T_0}\right) \\
&= \sum_{n \text{ odd}} a_n \cos\left(\frac{2\pi n t}{T_0} - \frac{\pi n}{2}\right) \\
&= \sum_{n \text{ odd}} a_n \left[\cos\left(\frac{2\pi n t}{T_0}\right)\cos\left(\frac{\pi n}{2}\right) + \sin\left(\frac{2\pi n t}{T_0}\right)\sin\left(\frac{\pi n}{2}\right)\right] \\
&= \sum_{n \text{ odd}} a_n \left[\sin\left(\frac{2\pi n t}{T_0}\right)\sin\left(\frac{\pi n}{2}\right)\right] \\
b_n &= a_n \sin\left(\frac{\pi n}{2}\right) = \frac{8A}{(n\pi)^2}\sin\left(\frac{\pi n}{2}\right) \quad \text{for} \quad \text{odd } n
\end{aligned}$$

Problem 13–8. Derive expressions for the Fourier coefficients of the periodic waveform in Figure P13–8.

The waveform is a modified rectangular pulse with no average value, $A = 4$, $T = T_0/2$, and multiplied by -1. Using the results in Figure 13–4, we have the following coefficients:

$$\begin{aligned}
a_0 &= 0 \\
a_n &= -\frac{8}{n\pi}\sin\left(\frac{n\pi}{2}\right) \\
b_n &= 0
\end{aligned}$$

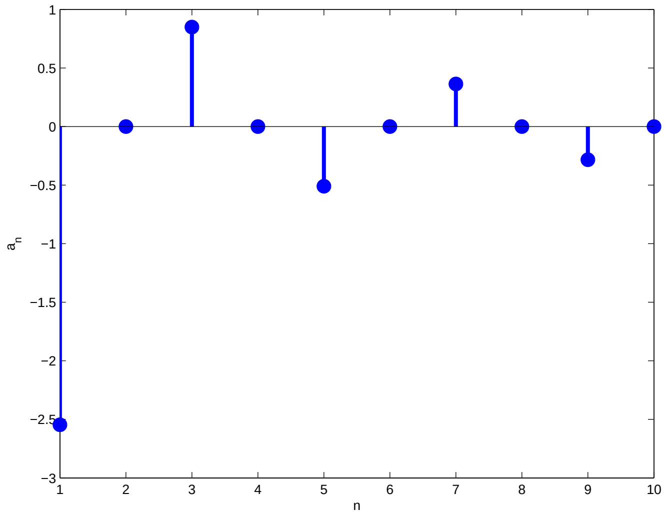
(a). Write an expression for the first four nonzero terms in the Fourier series.

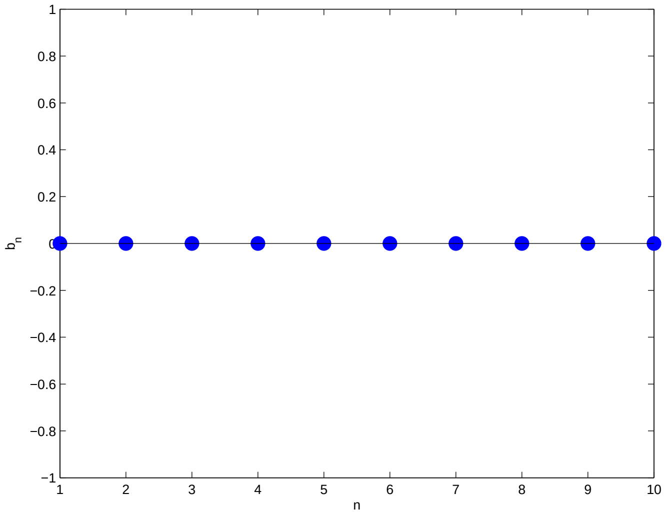
The expression is shown below.

$$v(t) = -2.55\cos(0.25\pi t) + 0.849\cos(0.75\pi t) - 0.509\cos(1.25\pi t) + 0.364\cos(1.75\pi t) \text{ V}$$

(b). Plot the spectrum of the Fourier coefficients a_n and b_n.

The spectral plots are shown below.





Problem 13–9. A particular periodic waveform with a period of 10 ms has the following Fourier coefficients

$$a_0 = 4, \quad a_n = \frac{8}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi}{4}\right), \quad \frac{-8}{n\pi}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{4}\right)$$

(a). Write an expression for the terms in the Fourier series a_0 and $n = 1$ through 8.

With $T_0 = 10$ ms, we have $f_0 = 100$ Hz. We have the following results:

$$v(t) \approx 4 + 1.8\cos(200\pi t) + 0.6\cos(600\pi t) - 0.36\cos(1000\pi t) - 0.257\cos(1400\pi t)$$

$$- 1.8\sin(200\pi t) + 0.6\sin(600\pi t) + 0.36\sin(1000\pi t) - 0.257\sin(1400\pi t)$$

(b). Convert your expression to amplitude and phase form and plot its spectrum.

The conversion is

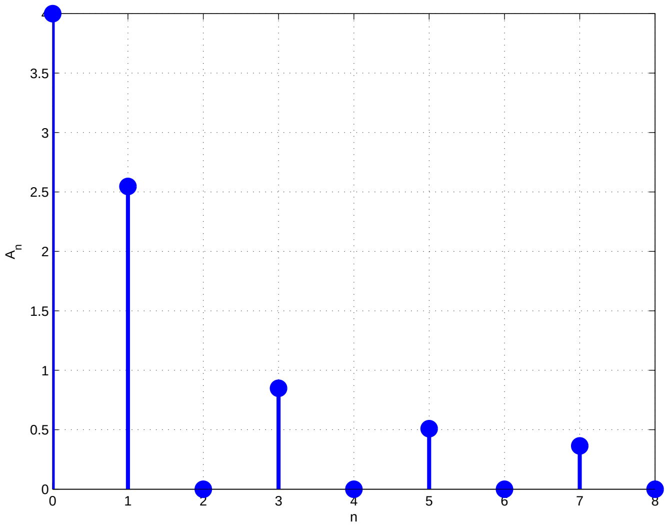
$$A_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

The results are

$$v(t) \approx 4 + 2.55\cos(200\pi t + 45^\circ) + 0.849\cos(600\pi t - 45^\circ)$$
$$+ 0.509\cos(1000\pi t - 135^\circ) + 0.364\cos(1400\pi t + 135^\circ)$$

The amplitude spectrum is shown below.



Problem 13–10. Use the results in Figure 13–4 to calculate the Fourier coefficients of the full-wave rectified sine wave in Figure P13–10. Use MATLAB to verify your results. Write an expression for the first four nonzero terms in the Fourier series.

Using Figure 13–4, $b_n = 0$ for all n, $a_n = 0$ for odd values of n, and for even values of n, we have:

$$a_0 = \frac{2A}{\pi} = \frac{340}{\pi}$$

$$a_n = \frac{4A/\pi}{1-n^2} = \frac{680}{\pi(1-n^2)}$$

The following MATLAB code and results verify the answer:

```
syms t VA real
syms T0 n positive
% The period of the original sinusoid is T0, but
% the period of the rectified signal is T0/2.
% Adjust the integrals accordingly
vt = VA*sin(2*pi*t/T0);
a0 = int(vt,t,0,T0/2)/(T0/2);
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0/2)/(T0/2);
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0/2)/(T0/2);
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
```

```
a02 = (2*VA)/pi
an2 = piecewise([n = 1, 0], [n <> 1, (2*VA + 2*VA*cos(pi*n))/(pi - pi*n^2)])
bn2 = piecewise([n = 1, VA], [n <> 1, -(2*VA*sin(pi*n))/(pi*(n^2 - 1))],...
     [T0 = 2*pi and 2 ≤ n and n in Z_, 0])
```

The first four nonzero terms in the Fourier series are:

$$v(t) \approx 108.23 - 72.15\cos(240\pi t) - 14.43\cos(480\pi t) - 6.18\cos(720\pi t) - 3.44\cos(960\pi t)$$

Problem 13–11. A half-wave rectified sine wave has an amplitude of 339 V and a fundamental frequency of 50 Hz. Use the results in Figure 13–4 to write an expression for the first four nonzero terms in the Fourier series. Use MATLAB to plot the amplitude spectrum of the signal.

From Figure 13–4, we have the following results:

$$a_0 = \frac{A}{\pi} = \frac{339}{\pi} = 107.9 \text{ V}$$

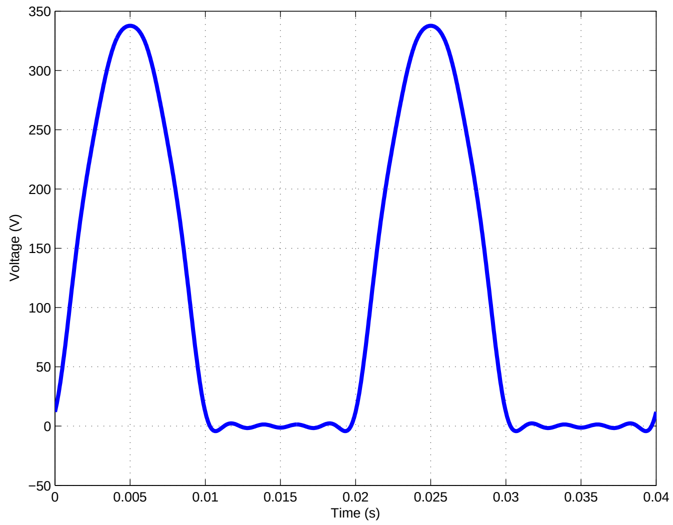
$$a_n = \frac{2A/\pi}{1-n^2} = \frac{678}{\pi(1-n^2)} \quad n \text{ even}$$

$$b_1 = \frac{A}{2} = \frac{339}{2} = 169.5 \text{ V}$$

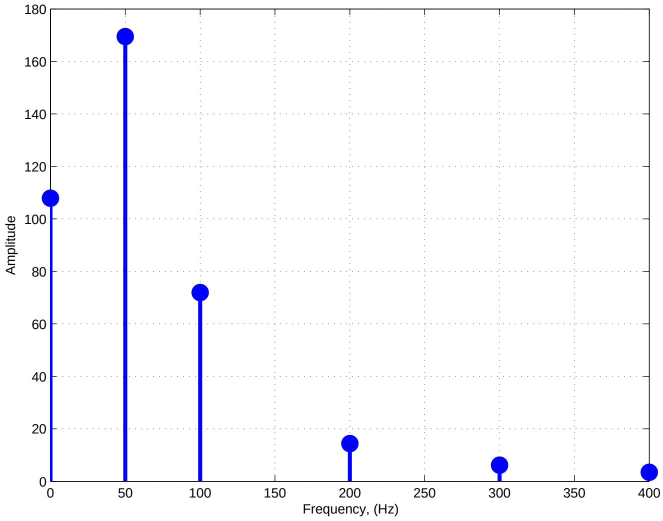
$$b_n = 0 \quad n \neq 1$$

$$v(t) \approx 107.9 + 169.5\sin(100\pi t) - 71.94\cos(200\pi t) - 14.39\cos(400\pi t) - 6.17\cos(600\pi t) \text{ V}$$

The approximate waveform is sketched below.



The amplitude spectrum is shown below.



Problem 13–12. The waveform $f(t)$ is a 2-kHz triangular wave with a *peak-to-peak* amplitude of 8 V. Use the results in Figure 13–4 to write an expression for the first four nonzero terms in the Fourier series of $g(t) = 4 + f(t)$ and plot its amplitude spectrum. Use MATLAB to plot two periods of $g(t)$ and an estimate for $g(t)$ using the first four nonzero terms in the Fourier series. Comment on the differences between the two waveforms.

From Figure 13–4, we have the following results:

$$A = 4 \text{ V}$$

$$a_0 = 4 \text{ V}$$

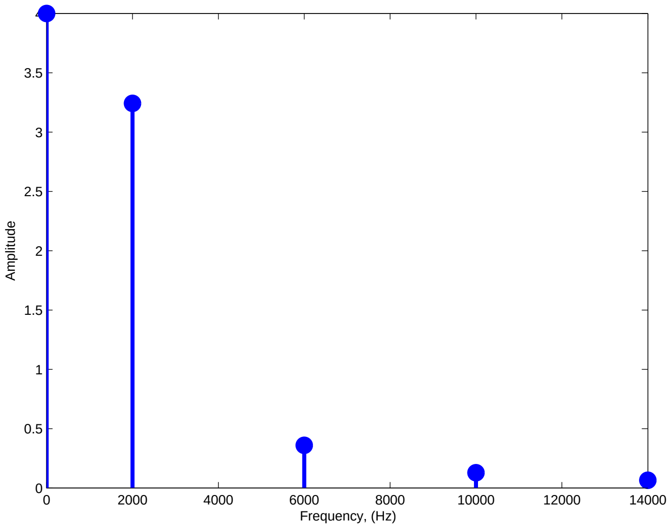
$$a_n = \frac{8A}{(n\pi)^2} = \frac{32}{(n\pi)^2} \quad n \text{ odd}$$

$$a_n = 0 \quad n \text{ even}$$

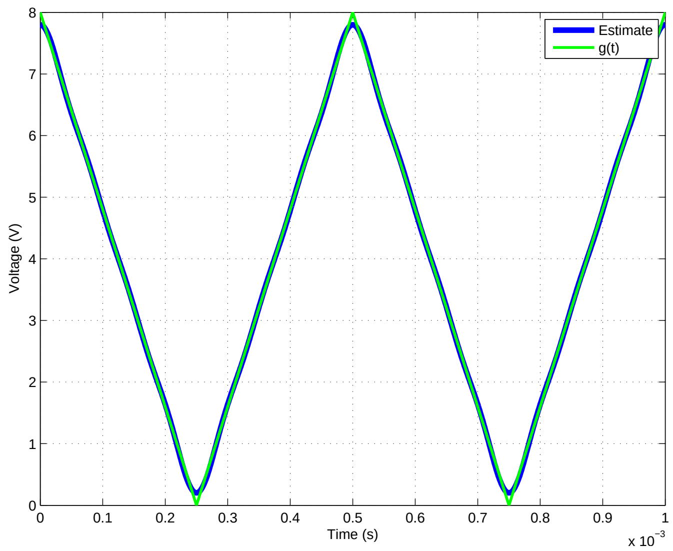
$$b_n = 0 \quad \text{all } n$$

$$g(t) \approx 4 + 3.24\cos(4000\pi t) + 0.36\cos(12000\pi t) + 0.13\cos(20000\pi t) + 0.066\cos(28000\pi t) \text{ V}$$

The amplitude spectrum is shown below.



The function $g(t)$ and an estimate for $g(t)$ are sketched below. The estimate with only four terms is a very good approximation, except at the times where $g(t)$ changes slope. At those times, the estimate is curved instead of a sharp transition.



Problem 13–13. A sawtooth wave has *peak-to-peak* amplitude of 24 V and a fundamental frequency of 2 kHz. Use the results in Figure 13–4 to write an expression for the first four nonzero terms in the Fourier series and plot the amplitude spectrum of the signal. Use MATLAB to plot two periods of the original signal and an estimate for the signal using the first four nonzero terms in the Fourier series. Comment on the differences between the two waveforms.

From Figure 13–4, we have the following results:

$$A = 24 \text{ V}$$

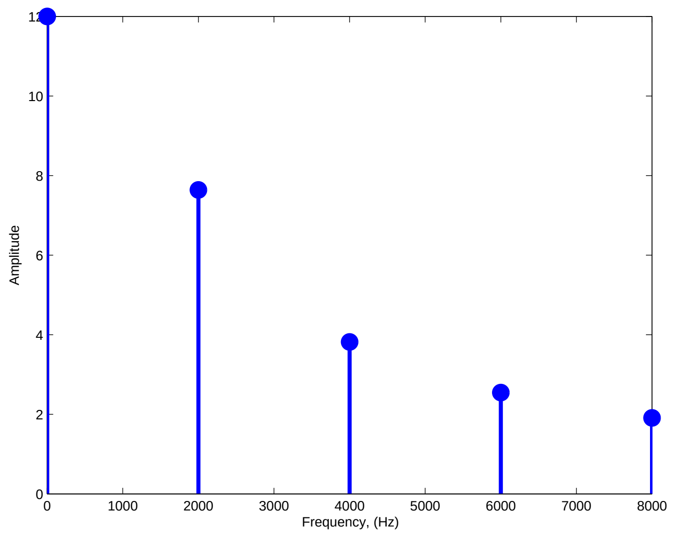
$$a_0 = \frac{A}{2} = 12 \text{ V}$$

$$a_n = 0 \quad \text{all } n$$

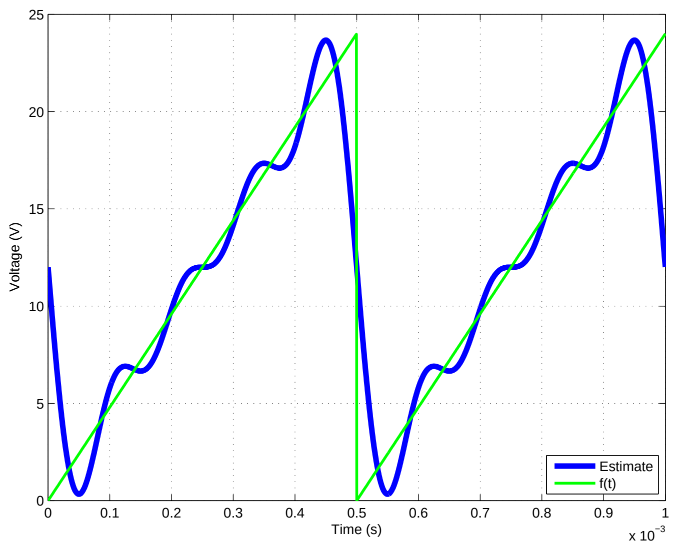
$$b_n = -\frac{A}{n\pi} \quad \text{all } n$$

$$f(t) = 12 - 7.6394\sin(4000\pi t) - 3.8197\sin(8000\pi t) - 2.5465\sin(12000\pi t) - 1.9099\sin(16000\pi t) \text{ V}$$

The amplitude spectrum is shown below.



The original signal and the estimate are sketched below. The estimate with only four terms is a fair approximation and captures the general trends in the original signal.

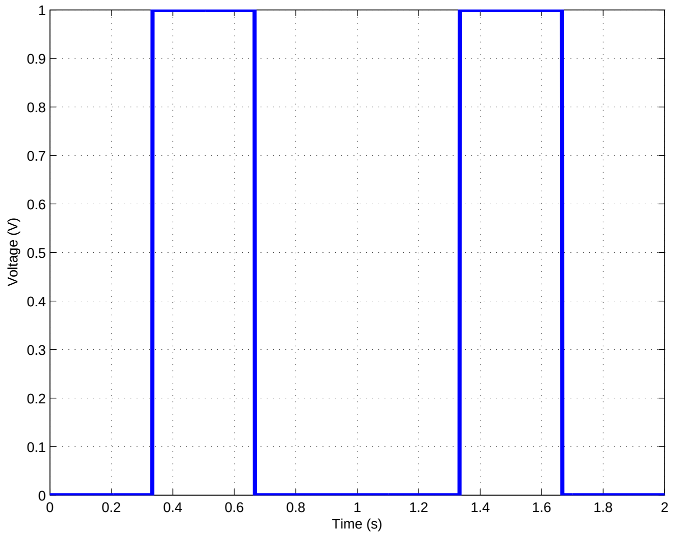


Problem 13–14. The equation for the first cycle ($0 \leq t \leq T_0$) of a periodic pulse train is

$$v(t) = V_{\mathrm{A}}\left[u(3t - T_0) - u(3t - 2T_0)\right] \text{ V}$$

(a). Sketch the first two cycles of the waveform and identify a related signal in Figure 13–4.

The waveform is sketched below with $V_{\mathrm{A}} = 1$ V. A related signal in Figure 13–4 is a rectangular pulse with $T = 2T_0/3$ that has also been multiplied by -1 and shifted by V_{A} to make the signal all positive.



(b). Use the Fourier series of the related signal to find the Fourier coefficients of $v(t)$.

Using the results in Figure 13–4 and the description of the related signal in part (a), we have the following results for all values of n:

$$a_0 = V_A - \frac{V_A T}{T_0} = V_A - \frac{2V_A}{3} = \frac{V_A}{3}\text{ V}$$

$$a_n = -\frac{2V_A}{n\pi}\sin\left(\frac{n\pi T}{T_0}\right) = -\frac{2V_A}{n\pi}\sin\left(\frac{2n\pi}{3}\right)$$

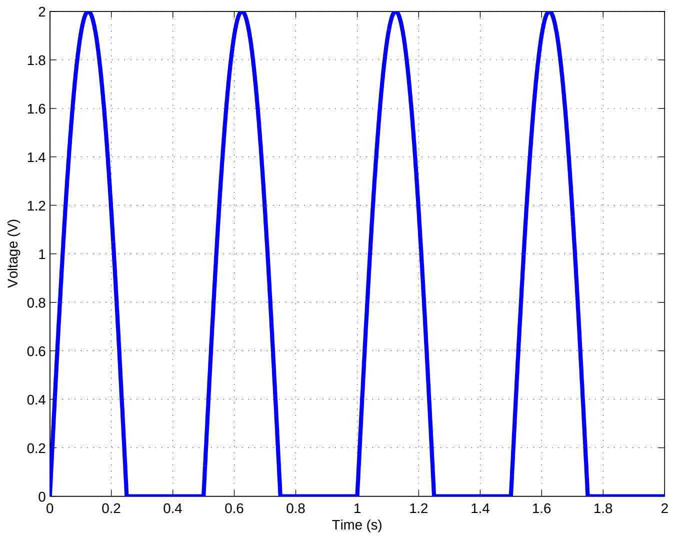
$$b_n = 0$$

Problem 13–15. The equation for a periodic waveform is

$$v(t) = V_A\left[\sin\left(\frac{4\pi t}{T_0}\right) + \left|\sin\left(\frac{4\pi t}{T_0}\right)\right|\right]$$

(a). Sketch the first two cycles of the waveform and identify a related signal in Figure 13–4.

The waveform is sketched below with $V_A = 1$ V and $T_0 = 1$ s. A related signal in Figure 13–4 is a half-wave rectified sine wave with an amplitude of $2V_A$ and a period of $T_0/2$



(b). Use the Fourier series of the related signal to find the Fourier coefficients of $v(t)$.

From Figure 13–4, we have the following results:

$$a_0 = \frac{2V_A}{\pi} \text{ V}$$

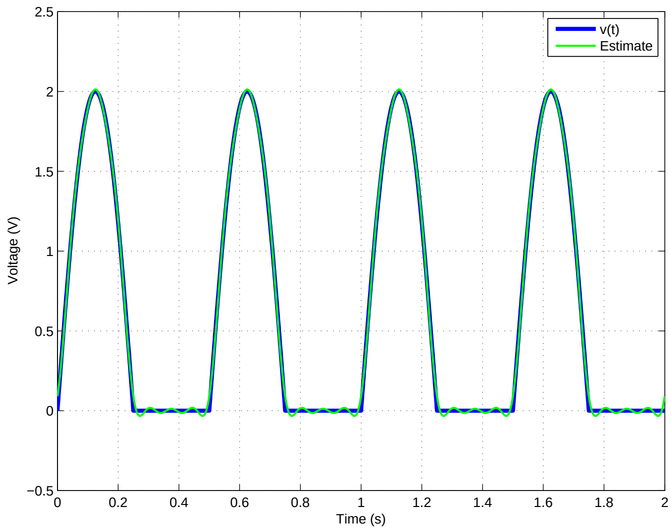
$$a_n = \frac{4V_A/\pi}{1-n^2} \quad n \text{ even}$$

$$b_1 = V_A \text{ V}$$

$$b_n = 0 \quad n \neq 1$$

(c). Use MATLAB to sketch an estimate for the signal using the Fourier coefficients for $n \leq 6$.

The original signal and its estimate are sketched below.



Problem 13–16. The first four terms in the Fourier series of a periodic waveform are

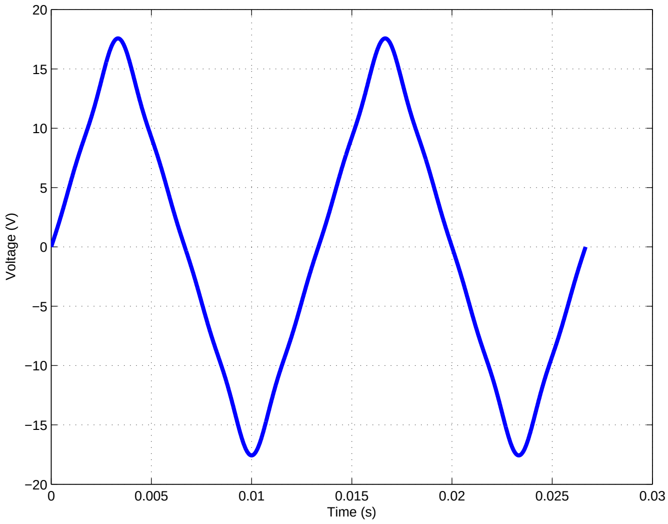
$$v(t) = 15\left[\sin(150\pi t) - \frac{1}{9}\sin(450\pi t) + \frac{1}{25}\sin(750\pi t) - \frac{1}{49}\sin(1050\pi t)\right] \text{ V}$$

(a). Find the period and fundamental frequency in rad/s and Hz. Identify the harmonics present in the first four terms.

The fundamental frequency is $f_0 = 75$ Hz or $\omega_0 = 150\pi$ rad/s. The period is $T_0 = 1/75 = 13.33$ ms. Harmonics are present for $n = 1$, 3, 5, and 7.

(b). Identify the symmetry features of the waveform.

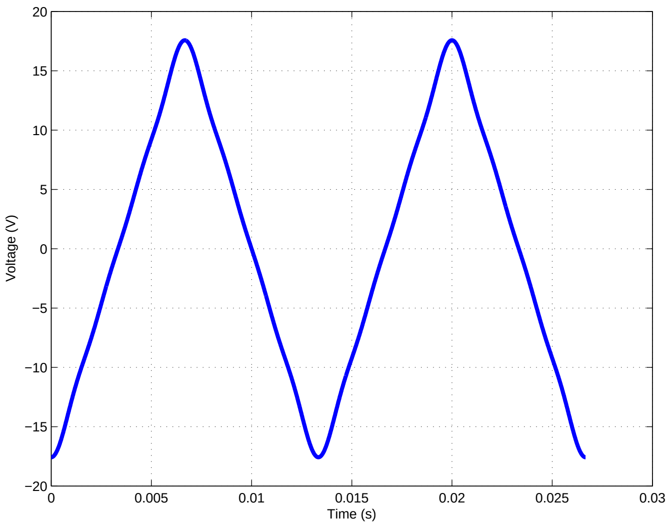
The waveform is sketched below.



The waveform only contains sine terms, so it has odd symmetry. Only the odd harmonics are present, so it has half-wave symmetry.

(c). Write the first four terms in the Fourier series for the waveform $v(t - T_0/4)$.

The new waveform is sketched below.



Shifting each sine term by a quarter period will result in replacing the sine terms with cosine terms.

We have the following results:

$$\begin{aligned}
\sin\left[150\pi n\left(t-\frac{T_0}{4}\right)\right] &= \sin\left(150\pi nt-\frac{\pi n}{2}\right)\\
&= \sin(150\pi nt)\cos\left(\frac{\pi n}{2}\right)-\cos(150\pi nt)\sin\left(\frac{\pi n}{2}\right)\\
&= -\cos(150\pi nt)\sin\left(\frac{\pi n}{2}\right)
\end{aligned}$$

$$\sin\left(\frac{\pi n}{2}\right) = 1 \quad n = 1,\ 5$$

$$\sin\left(\frac{\pi n}{2}\right) = -1 \quad n = 3,\ 7$$

The net result is that all of the cosine terms will have negative signs. This is also consistent with multiplying the triangular wave in Figure 13–4 by -1. The new expression is:

$$v(t-T_0/4) = -15\left[\cos(150\pi t)+\frac{1}{9}\cos(450\pi t)+\frac{1}{25}\cos(750\pi t)+\frac{1}{49}\cos(1050\pi t)\right]\ \text{V}$$

Problem 13–17. The first five terms in the Fourier series of a periodic waveform are

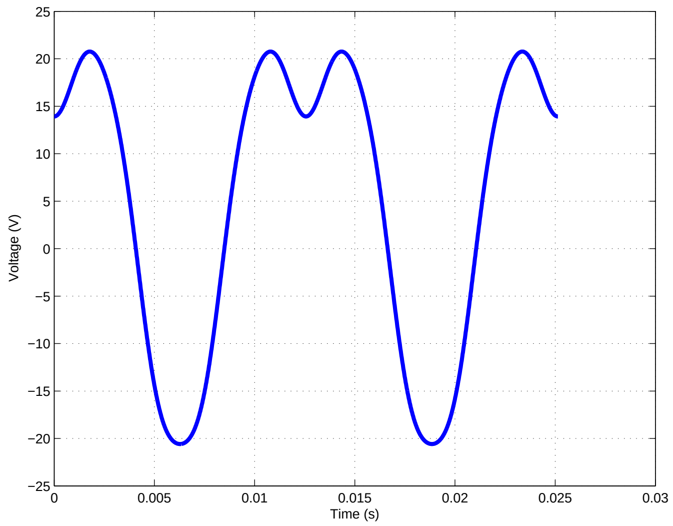
$$v(t) = 5+25\left[\frac{\pi}{4}\cos(500t)-\frac{1}{3}\cos(1000t)-\frac{1}{15}\cos(1500t)-\frac{1}{35}\cos(2500t)\right]\ \text{V}$$

(a). Find the period and fundamental frequency in rad/s and Hz. Identify the harmonics present in the first five terms.

The fundamental frequency is $f_0 = 250/\pi$ Hz or $\omega_0 = 500$ rad/s. The period is $T_0 = \pi/250 = 12.57$ ms. Harmonics are present for $n = 1,\ 2,\ 3$, and 5.

(b). Use MATLAB to plot two periods of $v(t)$.

The MATLAB plot is shown below.



(c). Identify the symmetry features of the waveform.

The waveform contains all cosine terms, so it has even symmetry.

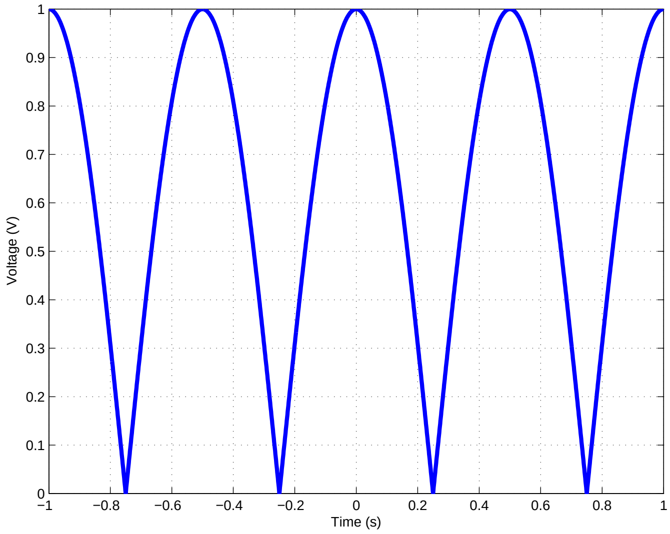
(d). Write the first five terms of the Fourier series for the waveform $v(-t)$.

Since the waveform has even symmetry, $v(t) = v(-t)$.

Problem 13–18. The equation for a full-wave rectified cosine is $v(t) = V_{\text{A}}\,|\cos(2\pi t/T_0)|$ V.

(a). Sketch $v(t)$ for $-T_0 \le t \le T_0$.

The sketch is shown below for $V_{\text{A}} = 1$ V and $T_0 = 1$ s.



(b). Compute the Fourier coefficients for $v(t)$.

The following MATLAB code computes the coefficients:

```
syms t VA real
syms T0 n positive
% The period of the original sinusoid is T0, but
% the period of the rectified signal is T0/2.
% Adjust the integrals accordingly
T02 = T0/2;
% Define the function on the interval -T02/2 to T02/2
vt = VA*cos(2*pi*t/T0);
a0 = int(vt,t,-T02/2,T02/2)/T02;
an = 2*int(vt*cos(2*pi*n*t/T02),t,-T02/2,T02/2)/T02;
bn = 2*int(vt*sin(2*pi*n*t/T02),t,-T02/2,T02/2)/T02;
a02 = simple(a0)
an2 = simple(an)
bn2 = simple(bn)
```

The results for all n are:

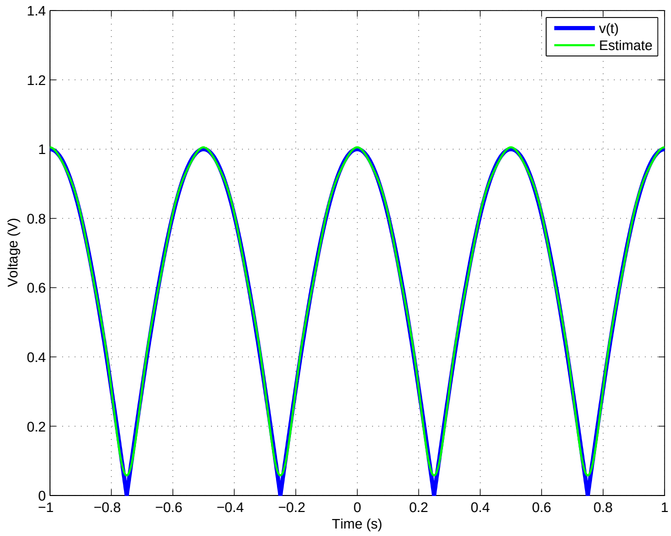
$$a_0 = \frac{2V_{\text{A}}}{\pi}$$

$$a_n = \frac{4V_{\text{A}}\cos(n\pi)}{\pi(1-4n^2)}$$

$$b_n = 0$$

(c). Use the Fourier coefficients to plot an estimate for $v(t)$.

The original waveform and its estimate are shown below.



Problem 13–19. Find the Fourier series for the waveform in Figure 13–19.

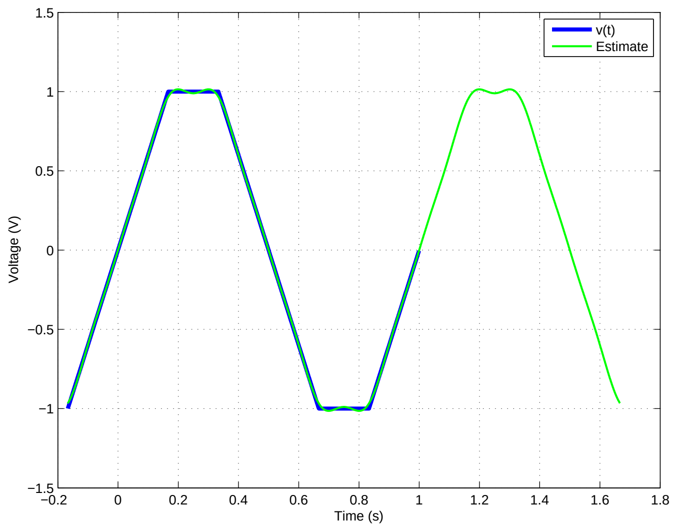
The waveform has an average value of zero, so $a_0 = 0$ V. The waveform has odd symmetry, so $a_n = 0$. The signal has half-wave symmetry, so $b_n = 0$ for even values of n. Use MATLAB to calculate the b_n coefficients as follows:

```
syms t VA real
syms T0 n positive
% Define the function on the interval -T0/6 to 5*T0/6
vt = 6*VA*t/T0...
    - 6*VA*(t-T0/6)/T0*heaviside(t-T0/6)...
    - 6*VA*(t-T0/3)/T0*heaviside(t-T0/3)...
    + 6*VA*(t-2*T0/3)/T0*heaviside(t-2*T0/3)...
    + 6*VA*(t-5*T0/6)/T0*heaviside(t-5*T0/6);
a0 = int(vt,t,0,T0)/T0;
an = 2*int(vt*cos(2*pi*n*t/T0),t,0,T0)/T0;
bn = 2*int(vt*sin(2*pi*n*t/T0),t,0,T0)/T0;

% Create bn terms with separate integrals
bn1 = 2*int(6*VA*t/T0*sin(2*pi*n*t/T0),t,-T0/6,T0/6)/T0;
bn2 = 2*int(VA*sin(2*pi*n*t/T0),t,T0/6,T0/3)/T0;
bn3 = 2*int((3*VA-6*VA*t/T0)*sin(2*pi*n*t/T0),t,T0/3,2*T0/3)/T0;
bn4 = 2*int(-VA*sin(2*pi*n*t/T0),t,2*T0/3,5*T0/6)/T0;
bnv2 = simple(bn1+bn2+bn3+bn4)
pretty(bnv2)
```

The results are:

$$b_n = \frac{V_{\rm A}\left[6\sin(\pi n/3) + 3\sin(2\pi n/3) - 3\sin(4\pi n/3) - \pi n\cos(\pi n/3) + \pi n\cos(5\pi n/3)\right]}{(n\pi)^2}$$

The original waveform and its estimate using the coefficients are shown below.



Problem 13–20. The periodic pulse train in Figure P13–20 is applied to the RL circuit shown in the figure.

(a). Use the results in text Figure 13–4 to find the Fourier coefficients of the input for $V_{\mathrm{A}} = 10$ V, $T_0 = \pi$ ms, and $T = T_0/4$.

We have the following results for all values of n:

$$V_{\mathrm{A}} = 10 \text{ V}$$

$$T_0 = \frac{\pi}{1000} \text{ s}$$

$$f_0 = \frac{1000}{\pi} \text{ Hz}$$

$$\omega_0 = 2000 \text{ rad/s}$$

$$T = \frac{T_0}{4}$$

$$a_0 = \frac{V_{\mathrm{A}}}{4} = \frac{5}{2}$$

$$a_n = \frac{20}{n\pi} \sin\left(\frac{n\pi}{4}\right)$$

$$b_n = 0$$

(b). Find the first four nonzero terms in the Fourier series of $v_{\mathrm{O}}(t)$ for $R = 400\ \Omega$, and $L = 100$ mH.

Apply voltage division to find the transfer function.

$$T(s) = \frac{\frac{R}{L}}{s + \frac{R}{L}} = \frac{4000}{s + 4000}$$

$$T(\omega) = \frac{4000}{j\omega + 4000}$$

For $\omega = 2000$, 4000, and 6000 rad/s, compute the magnitude and phase of the transfer function. Use those values to determine the magnitude scaling and phase shift for each component in the input signal. The following MATLAB code provides the solution.

```
syms s
syms w positive
VA = 10;
T0 = pi*1e-3;
T = T0/4;
% Compute the transfer function
R = 400;
L = 100e-3;
ZL = s*L;
Ts = simplify(R/(ZL+R));
% Find the input frequencies
n = [1 2 3];
w = 2*pi*n/T0
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
Tjw2 = Tjw.*conj(Tjw);
MagTjw = sqrt(Tjw2)
% The angle of the transfer function has the following form
AngleTjw = -180*atan2(w,R/L)/pi
% Find the output coefficients
a0 = VA*T/T0
an = 2*VA*sin(n*pi*T/T0)./n/pi
anOut = an.*MagTjw
% Since we only have a_n Fourier coefficients, all of the input
% signals are cosines and the output phase is the phase of the
% transfer function plus 180 degrees when the input coefficient
% has a negative sign
PhaseIn = 180*(an<0)
PhaseOut = PhaseIn+AngleTjw
```

We have the following results:

```
w = 2.0000e+003     4.0000e+003     6.0000e+003
MagTjw = 894.4272e-003   707.1068e-003   554.7002e-003
AngleTjw = -26.5651e+000   -45.0000e+000   -56.3099e+000
a0 = 2.5000e+000
an = 4.5016e+000     3.1831e+000     1.5005e+000
anOut = 4.0263e+000     2.2508e+000   832.3427e-003
PhaseIn = 0.0000e+000     0.0000e+000     0.0000e+000
PhaseOut = -26.5651e+000   -45.0000e+000   -56.3099e+000
```

In summary, we have:

$$v_S(t) = 2.5 + 4.5\cos(2000t) + 3.18\cos(4000t) + 1.5\cos(6000t)\text{ V}$$

$$v_O(t) = 2.5 + 4.03\cos(2000t - 26.6^\circ) + 2.25\cos(4000t - 45^\circ) + 0.832\cos(6000t - 56.3^\circ)\text{ V}$$

Problem 13–21. The periodic triangular wave in Figure P13–21 is applied to the RC circuit shown in the figure. The Fourier coefficients of the input are:

$$a_0 = 0 \quad a_n = 0 \quad b_n = \frac{8V_A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)$$

If $V_A = 20$ V and $T_0 = 2\pi$ ms, find the first four nonzero terms in the Fourier series of $v_O(t)$ for $R = 10$ kΩ, and $C = 0.05$ μF.

The transfer function for the circuit is

$$T(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} = \frac{2000}{s + 2000}$$

$$T(\omega) = \frac{2000}{j\omega + 2000}$$

For $\omega = 1000$, 3000, 5000, and 7000 rad/s, compute the magnitude and phase of the transfer function. Use those values to determine the magnitude scaling and phase shift for each component in the input signal. The following MATLAB code provides the solution.

```
VA = 20;
T0 = 2*pi/1000;
n = 1:8;
bn = 8*VA*sin(n*pi/2)./(n*pi).^2;
w = 2*pi*n/T0;
Results = [n' bn' w']

syms s
% Compute the transfer function
R = 10e3;
C = 0.05e-6;
ZC = 1/s/C;
Ts = simplify(ZC/(R+ZC));
% Find the input frequencies
n = 1:2:7;
w = 2*pi*n/T0
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Find the output coefficients
bn = 8*VA*sin(n*pi/2)./(n*pi).^2;
bnOut = abs(bn).*MagTjw;
% Since we only have bn Fourier coefficients, all of the input
% signals are sines.  The input phase is -90 degrees if the input
% coefficient is positive and the input phase is +90 degrees if
% the input coefficient is negative
PhaseIn = 180*(bn<0)-90;
PhaseOut = PhaseIn+PhaseTjw;
Results = [w' MagTjw' PhaseTjw' bn' bnOut' PhaseIn' PhaseOut']
Results = [w' bnOut' PhaseOut']
```

The results are:

$$v_O(t) = 14.5\cos(1000t - 116.6^\circ) + 0.999\cos(3000t + 33.7^\circ) + 0.241\cos(5000t - 158.2^\circ) + 0.091\cos(7000t + 15.95^\circ)$$

Problem 13–22. The periodic sawtooth wave in Figure P13–22 drives the OP AMP circuit shown in the figure.

(a). Use the results in text Figure 13–4 to find the Fourier coefficients of the input for $V_A = 5$ V and $T_0 = 4\pi$ ms.

Using the results in Figure 13–4, we have the following for all values of n:

$$a_0 = \frac{V_A}{2} = 2.5 \text{ V}$$

$$a_n = 0$$

$$b_n = -\frac{V_A}{n\pi} = -\frac{5}{n\pi}$$

We also have

$$T_0 = \frac{4\pi}{1000} \text{ s}$$

$$\omega_0 = 500 \text{ rad/s}$$

(b). Find the first four nonzero terms in the Fourier series of $v_O(t)$ for $R_1 = 20$ kΩ, $R_2 = 200$ kΩ, and $C = 0.1$ µF.

The transfer function for the circuit is

$$T(s) = \frac{-\frac{R_2}{R_1}s}{s + \frac{1}{R_1 C}} = \frac{-10s}{s+500}$$

$$T(\omega) = \frac{-j10\omega}{j\omega + 500}$$

For $\omega = 500$, 1000, 1500, and 2000 rad/s, compute the magnitude and phase of the transfer function. Use those values to determine the magnitude scaling and phase shift for each component in the input signal. The following MATLAB code provides the solution.

```
VA = 5;
T0 = 4*pi/1000;
n = 1:4;
a0 = VA/2
bn = -VA./n/pi;
w = 2*pi*n/T0;
Results = [n' bn' w']

syms s
% Compute the transfer function
R1 = 20e3;
R2 = 200e3;
C = 0.1e-6;
ZC = 1/s/C;
Ts = factor(-R2/(R1+ZC))
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Find the output coefficients
bn;
bnOut = abs(bn).*MagTjw;
% Since we only have bn Fourier coefficients, all of the input
% signals are sines.  The input phase is -90 degrees if the input
% coefficient is positive and the input phase is +90 degrees if
% the input coefficient is negative
PhaseIn = 180*(bn<0)-90;
PhaseOut = PhaseIn+PhaseTjw;
Results = [w' MagTjw' PhaseTjw' bn' bnOut' PhaseIn' PhaseOut']
```

The results are:

$$v_O(t) = 11.25\cos(500t-45^\circ)+7.12\cos(1000t-63.4^\circ)+5.03\cos(1500t-71.6^\circ)+3.86\cos(2000t-76.0^\circ) \text{ V}$$

Problem 13–23. The periodic sawtooth wave in Figure P13–22 above drives the OP AMP circuit shown in the figure.

(a). Use the results in text Figure 13–4 to find the Fourier coefficients of the input for $V_A = 8$ V and $T_0 = 20\pi$ ms.

We have the following coefficients for all values of n:

$$a_0 = 4$$

$$a_n = 0$$

$$b_n = \frac{-8}{n\pi}$$

We also have

$$T_0 = \frac{20\pi}{1000} \text{ s}$$

$$\omega_0 = 100 \text{ rad/s}$$

(b). Find the first four nonzero terms in the Fourier series of $i(t)$ for $R_1 = 100$ kΩ, $R_2 = 50$ kΩ, and $C = 0.4$ μF.

The transfer function is

$$T(s) = \frac{I(s)}{V_S(s)} = \frac{1}{R_1 + \dfrac{1}{Cs}} = \frac{\dfrac{1}{R_1}s}{s + \dfrac{1}{R_1C}} = \frac{10^{-5}s}{s+25}$$

$$T(\omega) = \frac{10^{-5}j\omega}{j\omega + 25}$$

For $\omega = 100$, 200, 300, and 400 rad/s, compute the magnitude and phase of the transfer function. Use those values to determine the magnitude scaling and phase shift for each component in the input signal. The following MATLAB code provides the solution.

```
VA = 8;
T0 = 20*pi/1000;
n = 1:4;
a0 = VA/2
bn = -VA./n/pi;
w = 2*pi*n/T0;
Results = [n' bn' w']

syms s
% Compute the transfer function
R1 = 100e3;
R2 = 50e3;
C = 0.4e-6;
ZC = 1/s/C;
Zin = R1+ZC;
Ts = factor(1/Zin)
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Find the output coefficients
bn;
bnOut = abs(bn).*MagTjw;
% Since we only have bn Fourier coefficients, all of the input
% signals are sines.  The input phase is -90 degrees if the input
```

```
% coefficient is positive and the input phase is +90 degrees if
% the input coefficient is negative
PhaseIn = 180*(bn<0)-90;
PhaseOut = PhaseIn+PhaseTjw;
disp('Input Current')
Results = [w' MagTjw' PhaseTjw' bn' bnOut' PhaseIn' PhaseOut']
Results = [w' bnOut' PhaseOut']
```

The results are:

$i(t) = 24.7\cos(100t+104.0^\circ)+12.63\cos(200t+97.1^\circ)+8.46\cos(300t+94.76^\circ)+6.35\cos(400t+93.58^\circ)\ \mu\text{A}$

(c). Find the first four nonzero terms in the Fourier series of $v_O(t)$ with the same element values as in part (b).

The transfer function for the circuit is

$$T(s) = \frac{-\dfrac{R_2}{R_1}s}{s+\dfrac{1}{R_1C}} = \frac{-0.5s}{s+25}$$

$$T(\omega) = \frac{-0.5j\omega}{j\omega+25}$$

The following MATLAB code provides the solution.

```
% Compute the transfer function
Ts = factor(-R2/(R1+ZC))
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Find the output coefficients
bn;
bnOut = abs(bn).*MagTjw;
% Since we only have bn Fourier coefficients, all of the input
% signals are sines.  The input phase is -90 degrees if the input
% coefficient is positive and the input phase is +90 degrees if
% the input coefficient is negative
PhaseIn = 180*(bn<0)-90;
PhaseOut = PhaseIn+PhaseTjw;
disp('Output Voltage')
Results = [w' MagTjw' PhaseTjw' bn' bnOut' PhaseIn' PhaseOut']
Results = [w' bnOut' PhaseOut']
```

The results are:

$$v_O(t) = 1.235\cos(100t-75.96^\circ)+0.632\cos(200t-82.88^\circ)+0.423\cos(300t-85.24^\circ)$$
$$+\,0.318\cos(400t-86.42^\circ)\ \text{V}$$

Problem 13–24.

(a). Design a low-pass OP AMP circuit to pass only the fundamental and the next nonzero harmonic of a 20π ms square wave. The gain of the OP AMP should be +10.

Use a series RC circuit with the ouput taken across the capacitor connected to a noninverting amplifier with a gain of +10. We have the following period and frequency:

$$T_0 = \frac{20\pi}{1000}\ \text{s}$$

$$\omega_0 = 100\ \text{rad/s}$$

A square ware includes only the odd harmonics, so the filter must pass $\omega = 100$ and 300 rad/s. Pick the filter cutoff frequency to be $\omega_C = 1/RC = 400$ rad/s. Choose $C = 1$ μF and $R = 2.5$ kΩ.

(b). Find the first four nonzero terms in the Fourier series of the output of your filter.

The transfer function for the filter is

$$T(s) = \frac{10(400)}{s+400} = \frac{4000}{s+400}$$

Let $V_A = 1$ and use MATLAB to perform the calculations.

```
VA = 1;
T0 = 20*pi/1000;
n = 1:2:7;
bn = 4*VA./n/pi;
w = 2*pi*n/T0;
Results = [n' bn' w']

syms s
% Compute the transfer function
R = 2.5e3;
C = 1e-6;
K = (1+9)/1;
ZC = 1/s/C;
Ts = factor(K*ZC/(R+ZC))
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Find the output coefficients
bnOut = abs(bn).*MagTjw;
% Since we only have bn Fourier coefficients, all of the input
% signals are sines.  The input phase is -90 degrees if the input
% coefficient is positive and the input phase is +90 degrees if
% the input coefficient is negative
PhaseIn = 180*(bn<0)-90;
PhaseOut = PhaseIn+PhaseTjw;
Results = [w' MagTjw' PhaseTjw' bn' bnOut' PhaseIn' PhaseOut']
Results = [w' bnOut' PhaseOut']
```

The results are:

$$v_O(t) = 12.35\cos(100t - 104^\circ) + 3.40\cos(300t - 126.9^\circ) + 1.59\cos(500t - 141.3^\circ)$$
$$+ 0.902\cos(700t - 150.3^\circ) \text{ V}$$

Problem 13–25.

(a). Design a passive low-pass *RC* filter to block the fundamental and all harmonics from a full-wave rectified sinusoidal waveform. Use the results of text Figure 13–4 to find the Fourier coefficients of the input for $V_A = 170$ V, $T_0 = 16.6$ ms.

The fundamental frequency of the input signal is $f_0 = 60$ Hz or $\omega_0 = 377$ rad/s. Design the filter cutoff frequency to be $\omega_C = 10$ rad/s by choosing $C = 1$ μF and $R = 100$ kΩ. The b_n coefficients are all zero, as are the a_n coefficients for odd values of n. We have the following coefficients for the input signal for even values of n:

$$a_0 = \frac{2V_A}{\pi} = 108.225 \text{ V}$$

$$a_n = \frac{4V_A/\pi}{1-n^2} = \frac{216.45}{1-n^2}$$

(b). Find the first four nonzero terms in the Fourier series of the output of your filter.

The transfer function for the filter is

$$T(s) = \frac{10}{s+10}$$

Use MATLAB to perform the calculations.

```
VA = 170;
T0 = 1/60;
n = 2:2:6;
a0 = 2*VA/pi
an = 4*VA/pi./(1-n.^2);
w = 2*pi*n/T0;
Results = [n' an' w']

syms s
% Compute the transfer function
R = 100e3;
C = 1e-6;
ZC = 1/s/C;
Ts = factor(ZC/(R+ZC))
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
PhaseTjw = 180*angle(Tjw)/pi;
% Find the output coefficients
anOut = abs(an).*MagTjw;
% Since we only have a_n Fourier coefficients, all of the input
% signals are cosines and the output phase is the phase of the
% transfer function plus 180 degrees when the input coefficient
% has a negative sign
PhaseIn = 180*(an<0);
PhaseOut = PhaseIn+PhaseTjw;
Results = [w' MagTjw' PhaseTjw' an' anOut' PhaseIn' PhaseOut']
Results = [w' anOut' PhaseOut']
```

The results are:

$$v_O(t) = 108.225 + 0.957\cos(240\pi t + 90.76^\circ) + 0.0957\cos(480\pi t + 90.38^\circ)$$
$$+ 0.0273\cos(720\pi t + 90.25^\circ)$$

Problem 13–26. The periodic triangular wave in Figure P13–26 is applied to the *RLC* circuit shown in the figure.

(a). Use the results in text Figure 13–4 to find the Fourier coefficients of the input for $V_A = 10$ V and $T_0 = 400\pi$ μs.

The Fourier coefficients are all zero except for the a_n terms for odd values of n. We have:

$$a_n = \frac{8V_A}{(n\pi)^2} = \frac{80}{(n\pi)^2}$$

$$T_0 = 400\pi\ \mu\text{s}$$

$$\omega_0 = 5000 \text{ rad/s}$$

(b). Find the amplitude of the first five nonzero terms in the Fourier series for $i(t)$ when $R = 1\ \Omega$, $L = 8$ mH, and $C = 0.2$ μF. What term in the Fourier series tends to dominate the response? Explain.

The transfer function is

$$T(s) = \frac{I(s)}{V_S(s)} = \frac{1}{R + Ls + \frac{1}{Cs}} = \frac{\frac{1}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$= \frac{125s}{s^2 + 125s + 625000000}$$

Evaluate the transfer function at $\omega = 5000n$ for $n = 1, 3, 5, 7,$ and 9 to determine the amplitude scaling for each component in the input signal. The following MATLAB code computes the results:

```
VA = 10;
T0 = pi*400e-6;
n = [1 3 5 7 9];
an = 8*VA./(n*pi).^2;
w = 2*pi*n/T0;
Results = [n' an' w']

syms s
% Compute the transfer function
R = 1;
L = 8e-3;
C = 0.2e-6;
ZL = s*L;
ZC = 1/s/C;
Zin = R+ZL+ZC;
Ts = simplify(1/Zin);
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw)
% Find the output coefficients
an
anOut = abs(an).*MagTjw
Results = [w' an' anOut']
```

The following table summarizes the results.

ω (krad/s)	Input a_n (V)	Output a_n (mV)
5	8.106	8.44
15	0.900	4.22
25	0.324	324.23
35	0.167	1.206
45	0.100	0.402

The output response is dominated by the term at $\omega = 25$ krad/s. The filter is a bandpass filter with a center frequency of 25 krad/s, so it makes sense that an input at this frequency would pass through while other components were significantly attenuated.

Problem 13–27. A sawtooth wave with $V_A = 12$ V and $T_0 = 10\pi$ ms drives a circuit with a transfer function $T(s) = s/(s + 400)$. Find the amplitude of the first five nonzero terms in the Fourier series of the steady-state output. Construct plots of the amplitude spectra for the input and output waveforms and comment on any differences.

For a sawtooth wave, we have the following coefficients for all values of n:

$$a_0 = \frac{V_A}{2} = 6 \text{ V}$$

$$a_n = 0$$

$$b_n = -\frac{A}{n\pi} = -\frac{12}{n\pi}$$

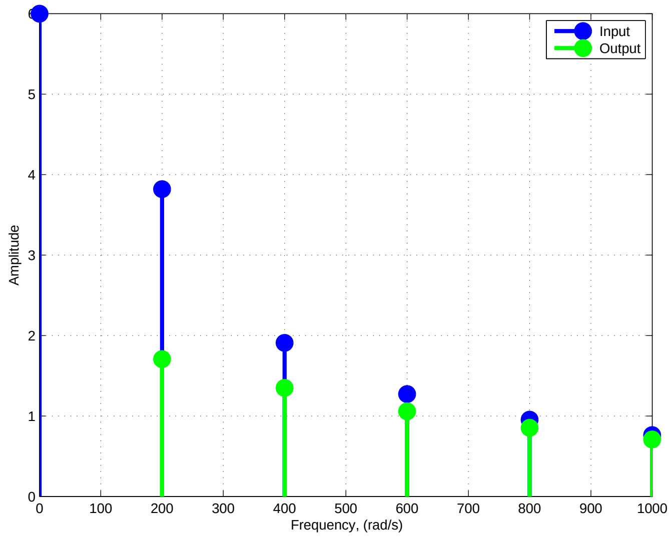
The following MATLAB code computes the magnitudes of the input and output coefficients.

```
VA = 12;
T0 = pi*10e-3;
n = 1:5;
bn = -VA./n/pi;
w = 2*pi*n/T0;
Results = [n' bn' w']

% Compute the transfer function
syms s
Ts = s/(s+400);
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
%PhaseTjw = 180*angle(Tjw)/pi
% Find the output coefficients
bnOut = abs(bn).*MagTjw;
disp('Results  =  Freq   |T(jw)|      b_n         b_n_Out')
Results = [w' MagTjw' bn' bnOut']
```

The results are

ω (rad/s)	$\lvert T(j\omega)\rvert$	Input $\lvert b_n\rvert$ (V)	Output $\lvert b_n\rvert$ (V)
0	0.000	6.000	0.000
200	0.447	3.820	1.708
400	0.707	1.910	1.351
600	0.832	1.273	1.059
800	0.894	0.955	0.854
1000	0.928	0.764	0.709

The plots of the input and output amplitude spectra are shown below. The filter is a high-pass filter, so the higher frequency signals have less attenuation than the lower frequency signals.



Problem 13–28. Repeat Problem 13–27 for $T(s) = 400/(s + 400)$.

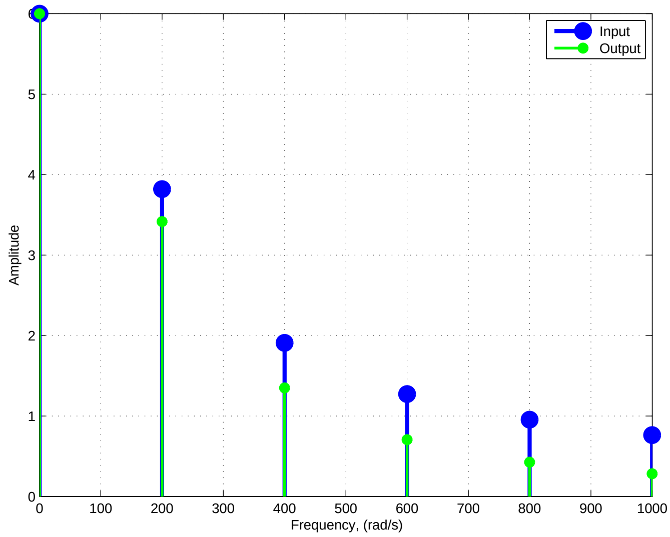
The transfer function now represents a low-pass filter. The following MATLAB code computes the magnitudes of the input and output coefficients.

```
VA = 12;
T0 = pi*10e-3;
n = 1:5;
bn = -VA./n/pi;
w = 2*pi*n/T0;
Results = [n' bn' w']

% Compute the transfer function
syms s
Ts = 400/(s+400);
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
%PhaseTjw = 180*angle(Tjw)/pi
% Find the output coefficients
bnOut = abs(bn).*MagTjw;
disp('Results  =  Freq    |T(jw)|       b_n          b_n_Out')
Results = [w' MagTjw' bn' bnOut']
```

The results are

ω (rad/s)	$\lvert T(j\omega)\rvert$	Input $\lvert b_n\rvert$ (V)	Output $\lvert b_n\rvert$ (V)
0	1.000	6.000	6.000
200	0.894	3.820	3.417
400	0.707	1.910	1.351
600	0.555	1.273	0.706
800	0.447	0.955	0.427
1000	0.371	0.764	0.284

The plots of the input and output amplitude spectra are shown below. Since the filter is a low-pass filter, lower frequencies have less attenuation than higher frequencies.



Problem 13–29. Design a tuned RLC filter to pass the third harmonic of a triangular wave.

(a). Use the results in Figure 13–4 to find the Fourier coefficients of the input for $V_{\mathrm{A}} = 8$ V and $T_0 = 20\pi$ ms. Design your filter with a Q of 10.

The Fourier coefficients are all zero except for the a_n terms for odd values of n. We have:

$$a_n = \frac{8V_{\mathrm{A}}}{(n\pi)^2} = \frac{64}{(n\pi)^2}$$

$$T_0 = 20\pi \text{ ms}$$

$$\omega_0 = 100 \text{ rad/s}$$

The third harmonic appears at $\omega = 300$ rad/s, which is the center frequency for the filter. We have the following design:

$$\omega_{\mathrm{C}} = 300 \text{ rad/s}$$

$$\omega_{\mathrm{C}}^2 = 300^2 = \frac{1}{LC}$$

$$Q = 10 = \frac{\omega_{\mathrm{C}}}{B}$$

$$B = 30 \text{ rad/s} = \frac{R}{L}$$

To meet the design specifications, choose $R = 30\ \Omega$, solve for $L = 1$ H, and $C = 11.11\ \mu$F. In the series RLC circuit, the output is taken across the resistor.

(b). Compare the magnitudes of the fundamental and of the fifth harmonic with that of the third harmonic at the input and output of the tuned filter.

The transfer function is

$$T(s) = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{30s}{s^2 + 30s + 300^2}$$

Use MATLAB to compute the magnitudes at the input and output for the three harmonics.

```
VA = 8;
T0 = pi*20e-3;
n = 1:2:5;
an = 8*VA./n.^2/pi^2;
w = 2*pi*n/T0;

% Compute the transfer function
syms s
w0 = 3*w(1);
R = 30;
L = 1;
C = 1/L/w0^2;
Ts = factor(R/(R+L*s+1/C/s))
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
% Find the output coefficients
anOut = abs(an).*MagTjw;
disp('Results  =  n     Freq    |T(jw)|       a_n         a_n_Out')
Results = [n' w' MagTjw' an' anOut']
```

The first and fifth harmonics are severely attenuated and the third harmonic is not changed between the input and output. Specifically, the input and output values for the first harmonic are 6.48 and 0.243, for the third harmonic are 0.721 and 0.721, and for the fifth harmonic are 0.259 and 0.024. The bandpass filter is correctly isolating the third harmonic.

Problem 13–30. Design a notch RLC filter to block the third harmonic of a triangular wave.

(a). Use the results in Figure 13–4 to find the Fourier coefficients of the input for $V_{\rm A} = 8$ V and $T_0 = 20\pi$ ms. Design your filter with a Q of 20.

Use a series RLC circuit with the output taken across the inductor and capacitor. The Fourier coefficients are all zero except for the a_n terms for odd values of n. We have:

$$a_n = \frac{8V_{\rm A}}{(n\pi)^2} = \frac{64}{(n\pi)^2}$$

$$T_0 = 20\pi \text{ ms}$$

$$\omega_0 = 100 \text{ rad/s}$$

The third harmonic appears at $\omega = 300$ rad/s, which is the center frequency for the filter. We have the following design:

$$\omega_{\rm C} = 300 \text{ rad/s}$$

$$\omega_{\rm C}^2 = 300^2 = \frac{1}{LC}$$

$$Q = 20 = \frac{\omega_{\rm C}}{B}$$

$$B = 15 \text{ rad/s} = \frac{R}{L}$$

To meet the design specifications, choose $R = 15\ \Omega$, solve for $L = 1$ H, and $C = 11.11\ \mu$F.

(b). Compare the magnitudes of the fundamental and of the fifth harmonic with that of the third harmonic at the input and output of the tuned filter.

The transfer function is

$$T(s) = \frac{s^2 + \dfrac{1}{LC}}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}} = \frac{s^2 + 300^2}{s^2 + 15s + 300^2}$$

Use MATLAB to compute the magnitudes at the input and output for the three harmonics.

```
VA = 8;
T0 = pi*20e-3;
n = 1:2:5;
an = 8*VA./n.^2/pi^2;
w = 2*pi*n/T0;

% Compute the transfer function
syms s
w0 = 3*w(1);
R = 15;
L = 1;
C = 1/L/w0^2;
Ts = factor((L*s+1/C/s)/(R+L*s+1/C/s))
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
% Find the output coefficients
anOut = abs(an).*MagTjw;
disp('Results  =  n     Freq   |T(jw)|      a_n         a_n_Out')
Results = [n' w' MagTjw' an' anOut']
```

At the output, the third harmonic is eliminated with a magnitude of zero. The input and output for the first harmonic are 6.485 and 6.483 and those for the fifth harmonic are 0.2594 and 0.2591, so both of these components are relatively unchanged.

Problem 13–31. The voltage across a 1000-pF capacitor is a triangular wave with $V_A = 150$ V and $f_0 = 1$ kHz. Construct plots of the amplitude spectra of the capacitor voltage and current. Discuss any differences in spectral content.

Across a capacitor, the transfer function from the voltage to the current is

$$T(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)} = Cs$$

$$T(\omega) = j\omega C$$

$$I(\omega) = T(\omega)V(\omega) = j\omega C V(\omega)$$

The transfer function indicates that as the frequency increases, the current will increase. The Fourier coefficients are all zero except for the a_n terms for odd values of n. We have:

$$a_n = \frac{8V_A}{(n\pi)^2} = \frac{1200}{(n\pi)^2}$$

$$f_0 = 1 \text{ kHz}$$

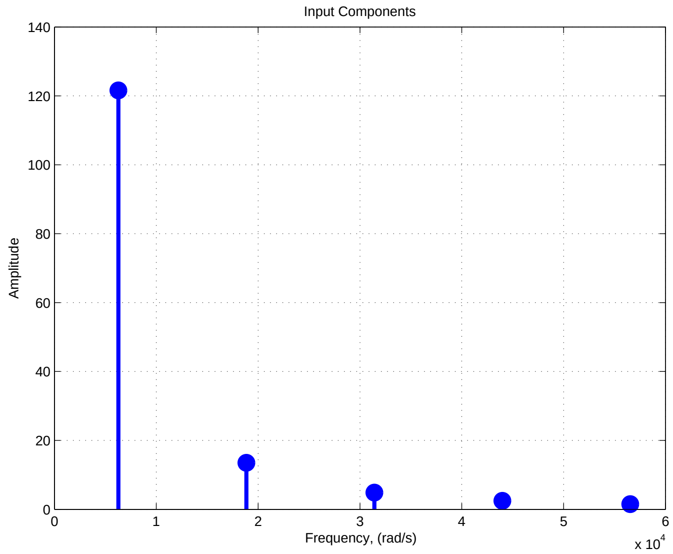
$$\omega_0 = 2\pi \text{ krad/s}$$

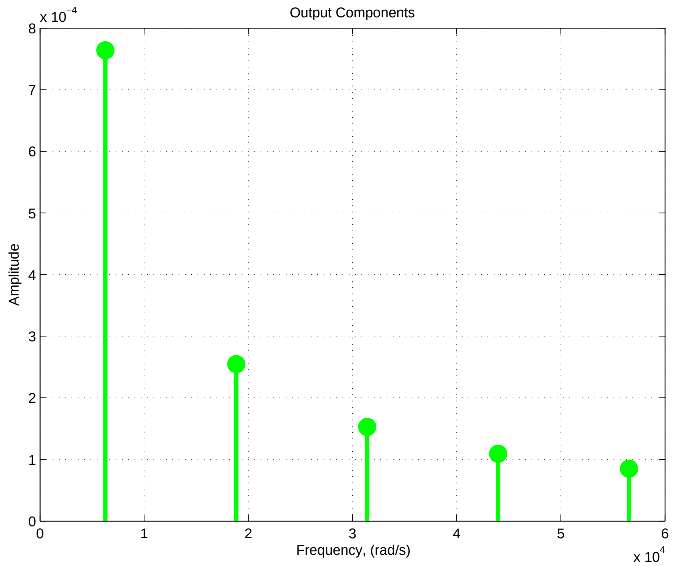
Use MATLAB to perform the calculations and construct the plots.

```
VA = 150;
f0 = 1e3;
C = 1000e-12;
T0 = 1/f0;
n = 1:2:9;
an = 8*VA./(n*pi).^2;
w = 2*pi*n/T0;

syms s
% Compute the transfer function
```

```
Ts = s*C;
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
% Find the output coefficients
anOut = abs(an).*MagTjw;
Results = [n' w' MagTjw' an' anOut']
```

The input and output spectra are shown below.





The harmonics of the voltage waveform (input) decay at a rate of $\dfrac{1}{n^2}$, but the harmonics in the current waveform (output) decay at a rate of $\dfrac{1}{n}$. The higher frequencies components of the current are more significant than those in the voltage.

Problem 13–32. An ideal time delay is a signal processor whose output is $v_{\rm O}(t) = v_{\rm IN}(t - T_{\rm D})$. Write an expression for $v_{\rm O}(t)$ for $T_{\rm D} = 0.5$ ms and

$$v_{\rm IN}(t) = 10 + 10\cos(2\pi 500t) + 2.5\cos(2\pi 1000t) + 0.625\cos(2\pi 4000t) \text{ V}$$

Discuss the spectral changes caused by the time delay.

We have the following results:

$$v_{\rm IN}(t) = 10 + 10\cos(2\pi 500 t) + 2.5\cos(2\pi 1000 t) + 0.625\cos(2\pi 4000 t)$$

$$\begin{aligned} v_{\rm O}(t) = v_{\rm IN}(t - T_{\rm D}) &= 10 + 10\cos[2\pi 500(t - T_{\rm D})] + 2.5\cos[2\pi 1000(t - T_{\rm D})] + 0.625\cos[2\pi 4000(t - T_{\rm D})] \\ &= 10 + 10\cos[2\pi 500 t - 2\pi 500 T_{\rm D}] + 2.5\cos[2\pi 1000 t - 2\pi 1000 T_{\rm D}] + 0.625\cos[2\pi 4000 t - 2\pi 4000 T_{\rm D}] \\ &= 10 + 10\cos[2\pi 500 t - \pi/2] + 2.5\cos[2\pi 1000 t - \pi] + 0.625\cos[2\pi 4000 t - 4\pi] \\ &= 10 + 10\sin[2\pi 500 t] - 2.5\cos[2\pi 1000 t] + 0.625\cos[2\pi 4000 t] \text{ V} \end{aligned}$$

The amplitude spectrum does not change. The phase spectrum changes, with the amount of phase shift being proportional to the frequency of the component. The linear phase shift allows for the constant time delay.

Problem 13–33. A sawtooth wave with $V_{\rm A} = 10$ V and $T_0 = 20\pi$ ms drives a circuit whose transfer function is

$$T(s) = \frac{100s}{(s+50)^2 + 400^2}$$

(a). Find the amplitude of the first four nonzero terms in the Fourier series of the steady-state output. What term in the Fourier series tends to dominate the response? Explain.

For this sawtooth wave, $a_0 = 5$ V, $a_n = 0$, and $b_n = -10/(n\pi)$ V for all values of n. The fundamental frequency is $\omega_0 = 100$ rad/s. Evaluate the transfer function at $\omega_0 n$ for $n = 1$ through 4. Use MATLAB to perform the calculations.

```
VA = 10;
T0 = pi*20e-3;
n = 1:6;
a0 = VA/2
bn = -VA./n/pi;
w = 2*pi*n/T0;

syms s
% Compute the transfer function
Ts = 100*s/((s+50)^2+400^2);
% Evaluate the transfer function at the input frequencies
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
% Find the output coefficients
bnOut = abs(bn).*MagTjw;
Results = [n' w' bn' bnOut']
```

The amplitudes of the first four output components are 0.208, 0.256, 0.406, and 0.794 V and correspond to frequencies $\omega = 100$, 200, 300, and 400 rad/s. The amplitudes decrease after the fourth component. The fourth component dominates the response because the transfer function is a bandpass filter with a center frequency at 400 rad/s. The dc component does not pass through the filter.

(b). Repeat Part (a) for a new transfer function $X(s) = T(s)/s$. What is the key difference between $T(s)$ and $X(s)$?

The new transfer function is a low-pass filter with a gain less than one. The following MATLAB code performs the calculations.

```
% Include the DC component
abn = [a0 bn];
w2 = [0 w];
```

```
% Compute the new transfer function
Rs = Ts/s;
% Evaluate the transfer function at the input frequencies
Tjw = subs(Rs,s,j*w2);
MagTjw = abs(Tjw);
% Find the output coefficients
abnOut = abs(abn).*MagTjw;
n2 = [0 n];
Results = [n2' w2' MagTjw' abn' abnOut']
```

The amplitudes of the first four output components are 3.08, 2.08, 1.28, 1.35 mV and correspond to frequencies $\omega = 0$, 100, 200, and 300 rad/s. The dc component dominates the response because the filter is now a low-pass filter. Note the slight increase in the output components between 200 and 300 rad/s. The increase occurs because the filter is resonant near the cutoff frequency of $\omega_C = 400$ rad/s, so gains greater than one are possible.

Problem 13–34. The current through a 500-Ω resistor is

$$i(t) = 50 + 36\cos(120\pi t - 30^\circ) - 12\cos(360\pi t + 45^\circ) \text{ mA}$$

Find the rms value of the current and the average power delivered to the resistor.

We have the following results:

$$I_{\text{rms}} = \sqrt{50^2 + \left(\frac{36}{\sqrt{2}}\right)^2 + \left(\frac{12}{\sqrt{2}}\right)^2} = 56.745 \text{ mA}$$

$$P = I_{\text{rms}}^2 R = (0.056745)^2(500) = 1.61 \text{ W}$$

Problem 13–35. The voltage across a 50-Ω resistor is

$$v(t) = 60 + 24\sin(200\pi t) - 8\sin(600\pi t) + 4.8\sin(1000\pi t) \text{ V}$$

(a). Find expressions for the current through the resistor and the power dissipated by the resistor.

We have the following results:

$$i(t) = \frac{v(t)}{R} = 1.2 + 0.48\sin(200\pi t) - 0.16\sin(600\pi t) + 0.096\sin(1000\pi t) \text{ A}$$

$$p(t) = i(t)v(t) = [1.2 + 0.48\sin(200\pi t) - 0.16\sin(600\pi t) + 0.096\sin(1000\pi t)]$$
$$\times [60 + 24\sin(200\pi t) - 8\sin(600\pi t) + 4.8\sin(1000\pi t)] \text{ W}$$

$$p(t) = \frac{v^2(t)}{R} = \frac{1}{50}\left[60 + 24\sin(200\pi t) - 8\sin(600\pi t) + 4.8\sin(1000\pi t)\right]^2 \text{ W}$$

(b). Find the average of the power expression by integrating over one period of the waveform and dividing by the period.

We have the following results:

$$P = \frac{1}{T_0}\int_0^{T_0} p(t)dt = 100\int_0^{0.01} \frac{1}{50}\left[60 + 24\sin(200\pi t) - 8\sin(600\pi t) + 4.8\sin(1000\pi t)\right]^2 dt$$

Use MATLAB to compute the integral:

```
syms t
R = 50;
```

```
vt = 60+24*sin(200*pi*t)-8*sin(600*pi*t)+4.8*sin(1000*pi*t)
it = vt/R
it_num = vpa(it,5)
pt = vt*it
pt_num = vpa(pt,5)
pt_num2 = vpa(expand(pt_num),5)
% pt = vt^2/R

% Average power
T0 = 1/100;
Pavg = double(int(pt,t,0,T0)/T0)
```

The result is $P = 78.63$ W.

(c). Find the rms value of the voltage by applying Eq (13–19) to the expression for $v(t)$.

We have the following:

$$V_{\text{rms}} = \sqrt{\frac{1}{T_0}\int_0^{T_0} [v(t)]^2\, dt}$$

Use MATLAB to perform the calculation:

```
% Vrms from integral
Vrms = double(sqrt(int(vt^2,t,0,T0)/T0))
```

The result is $V_{\text{rms}} = 62.70$ V.

(d). Find the rms value of the voltage by applying Eq (13–24) to the Fourier coefficients of $v(t)$ and compare the result to the answer in Part (c).

We have the following results:

$$V_{\text{rms}} = \sqrt{A_0^2 + \sum_{n=1}^{\infty} \frac{A_n^2}{2}}$$

$$= \sqrt{60^2 + \frac{24^2}{2} + \frac{8^2}{2} + \frac{4.8^2}{2}} = 62.70 \text{ V}$$

The results match those from Part (c), as expected.

(e). Compute the average power dissipated by the resistor using $P = V_{\text{rms}}^2/R$ and compare the result to the answer in Part (b).

We have the following:

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{62.70^2}{50} = 78.63 \text{ W}$$

The results match those from Part (b), as expected.

Problem 13–36. Find the rms value of a square wave. Find the fraction of the total average power carried by the first three nonzero ac components in the Fourier series.

We can determine the rms value of a square wave without completing the integral. The rms value is the square root of the average value of the square of the function. Once we square a square wave function, it has a constant value of A^2. The average value of a constant, is the constant. Take the square root to get the rms value of a square wave as A, its amplitude.

The nonzero components in the Fourier series are $b_n = 4A/n\pi$ for odd values of n. Find the rms value from the first three nonzero terms:

$$V_{\text{rms}} = \sqrt{\sum_{n=1,3,5}\left(\frac{1}{2}\right)\left(\frac{16A^2}{n^2\pi^2}\right)} = \sqrt{\frac{8A^2}{\pi^2}\left[1+\frac{1}{9}+\frac{1}{25}\right]}$$

$$= \sqrt{\frac{2072A^2}{225\pi^2}}$$

To find the fraction of total power in those components, square the voltage and divide by $V_{\text{rms}}^2 = A^2$ to get:

$$\frac{P_{\text{rms}}}{P} = \frac{2072}{225\pi^2} = 0.9331$$

Problem 13–37. Find the rms value of a triangular wave. Find the fraction of the total average power carried by the first three nonzero ac components in the Fourier series. Compare with the results found in Problem 13–36.

We can write the segments of a triangular wave as follows and then compute the rms value.

$$v_1(t) = A - \frac{4At}{T_0} \quad 0 \le t \le \frac{T_0}{2}$$

$$v_2(t) = -3A + \frac{4At}{T_0} \quad \frac{T_0}{2} \le t \le T_0$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T_0}\left[\int_0^{T_0/2} v_1^2(t)dt + \int_{T_0/2}^{T_0} f_2^2(t)dt\right]}$$

Use the following MATLAB code to perform the calculations:

```
syms A t T0
% Compute RMS values with integral
f1t = A-4*A*t/T0;
f2t = -3*A+4*A*t/T0;
Vrms = sqrt((int(f1t^2,t,0,T0/2)+int(f2t^2,t,T0/2,T0))/T0)
```

The rms values is $V_{\text{rms}} = A/\sqrt{3}$.

The nonzero components in the Fourier series are $b_n = 8A/(n\pi)^2$ for odd values of n. Find the rms value from the first three nonzero terms:

$$V_{\text{rms}} = \sqrt{\sum_{n=1,3,5}\left(\frac{1}{2}\right)\left(\frac{64A^2}{n^4\pi^4}\right)} = \sqrt{\frac{32A^2}{\pi^4}\left[1+\frac{1}{81}+\frac{1}{625}\right]}$$

$$= \sqrt{\frac{(32)(51331)A^2}{50625\pi^4}}$$

To find the fraction of total power in those components, square the voltage and divide by $V_{\text{rms}}^2 = A^2/3$ to get:

$$\frac{P_{\text{rms}}}{P} = \frac{(3)(32)(51331)}{50625\pi^4} = 0.9993$$

The first three terms of the Fourier series for the triangular wave carry a higher percentage of the power than the same results for the square wave.

Problem 13–38. Find the rms value of a parabolic wave. Find the fraction of the total average power carried by the first three nonzero ac components in the Fourier series. Compare with the results found in Problem 13–36.

Express the parabolic wave as two components:

$$f_1(t) = A - A\left[\frac{4t}{T_0} - 1\right]^2 \quad 0 \leq t \leq \frac{T_0}{2}$$

$$f_2(t) = -A + A\left[\frac{4t}{T_0} - 3\right]^2 \quad \frac{T_0}{2} \leq t \leq T_0$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T_0}\left[\int_0^{T_0/2} v_1^2(t)dt + \int_{T_0/2}^{T_0} f_2^2(t)dt\right]}$$

The coefficients in the Fourier series are only the b_n components for odd values of n:

$$b_n = \frac{32A}{(n\pi)^3}$$

Use MATLAB to perform all of the calculations to determine the rms value and the power ratio as follows:

```
syms A t T0
% Plot the functions to verify the form
f1t = A-A*(4*t/T0-1)^2;
f2t = -A+A*(4*t/T0-3)^2;
t1 = 0:1e-3:1;
t2 = 1:1e-3:2;
f1tn = subs(f1t,{A,T0,t},{2,2,t1});
f2tn = subs(f2t,{A,T0,t},{2,2,t2});
figure
plot(t1,f1tn,'g')
hold on
plot(t2,f2tn,'b')

Vrms = sqrt((int(f1t^2,t,0,T0/2)+int(f2t^2,t,T0/2,T0))/T0)
% Estimate RMS power
n = [1 3 5];
An = 32*A./(n*pi).^3;
% Assume R = 1
Pavg = Vrms^2;
% Estimate Pavg
P2 = sum(An.^2)/2;
Ratio = double(P2/Pavg)
```

The results are:

$$V_{\text{rms}} = \sqrt{\frac{8A^2}{15}}$$

$$\frac{P_{\text{rms}}}{P} = 0.9999887$$

The first three terms of the Fourier series for the parabolic wave carry an even higher percentage of the power than the same results for the square wave or the triangular wave.

Problem 13–39. Use MATLAB to find the rms value of a half-wave rectified sine wave. Find the fraction of the total average power carried by the dc component plus the first three nonzero ac components in the Fourier series.

The MATLAB code is shown below.

```
syms t
syms A T0 R positive
ft = A*sin(2*pi*t/T0);
Vrms = sqrt(int(ft^2,t,0,T0/2)/T0)
P = Vrms^2/R;

a0 = A/pi;
b1 = A/2;
n = [2 4];
an = 2*A/pi./(1-n.^2);
A0 = a0;
An = [b1 an];
Vrms2 = sqrt(A0^2+sum(An.^2)/2);
P2 = Vrms2^2/R;
P_ratio = double(P2/P)
```

The results are:

```
Vrms = A/2
P_ratio = 998.9505e-003
```

The rms value is $V_{\rm rms} = A/2$ and the percentage of power carried by the dc component and the first three terms in the Fourier series is 99.895%.

Problem 13–40. Find the rms value of the periodic waveform in Figure P13–40 and the average power the waveform delivers to a resistor. Find the dc component of the waveform and the average power carried by the dc component. What fraction of the total average power is carried by the dc component? What fraction is carried by the ac components?

The nonzero portion of the waveform can be expressed as follows:

$$v(t) = \frac{2V_{\rm A}t}{T_0}$$

Compute the rms value:

$$V_{\rm rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0/2}\left[\frac{2V_{\rm A}t}{T_0}\right]^2 dt}$$

The dc component is the average value, which is $V_{\rm A}/4$. Use the following MATLAB code to perform the remaining calculations:

```
syms t
syms VA T0 R positive
% Create the non-zero portion of one period of the signal
vt = 2*VA/T0*t;
% Find the rms value
Vrms = sqrt(int(vt^2,t,0,T0/2)/T0)
% Compute the average power
P = Vrms^2/R
% The dc component of the waveform is its average value
Vdc = int(vt,t,0,T0/2)/T0
% Average power of dc component
Pdc = Vdc^2/R
% Fraction of power carried by dc component
Pdc_ratio = double(Pdc/P)
% Fraction of power carried by ac components is the remainder
Pac_ratio = 1-Pdc_ratio
```

The results are:

```
Vrms = (6^(1/2)*VA)/6
P = VA^2/(6*R)
```

```
Vdc = VA/4
Pdc = VA^2/(16*R)
Pdc_ratio = 375.0000e-003
Pac_ratio = 625.0000e-003
```

In summary, we have the following results:

$$V_{\text{rms}} = \frac{V_{\text{A}}}{\sqrt{6}}$$

$$P = \frac{V_{\text{A}}^2}{6R}$$

$$P_{\text{dc}} = \frac{V_{\text{A}}^2}{16R}$$

The dc component carries 37.5% of the power and the ac components carry the remaining 62.5% of the power.

Problem 13–41. Repeat Problem 13–40 for the periodic waveform in Figure P13–41.

Express one period of the waveform as

$$v(t) = V_{\text{A}} - \frac{V_{\text{A}}t}{T_0}$$

Compute the rms value:

$$V_{\text{rms}} = \sqrt{\frac{1}{T_0}\int_0^{T_0}\left[V_{\text{A}} - \frac{V_{\text{A}}t}{T_0}\right]^2 dt}$$

The dc component is the average value, which is $V_{\text{A}}/2$. Use the following MATLAB code to perform the remaining calculations:

```
syms t
syms VA T0 R positive
% Create one period of the signal
vt = (VA-VA/T0*t);
% Find the rms value
Vrms = sqrt(int(vt^2,t,0,T0)/T0)
% Compute the average power
P = Vrms^2/R
% The dc component of the waveform is its average value
Vdc = int(vt,t,0,T0)/T0
% Average power of dc component
Pdc = Vdc^2/R
% Fraction of power carried by dc component
Pdc_ratio = double(Pdc/P)
% Fraction of power carried by ac components is the remainder
Pac_ratio = 1-Pdc_ratio
```

The results are:

```
Vrms = (3^(1/2)*VA)/3
P = VA^2/(3*R)
Vdc = VA/2
Pdc = VA^2/(4*R)
Pdc_ratio = 750.0000e-003
Pac_ratio = 250.0000e-003
```

In summary, we have the following results:

$$V_{\text{rms}} = \frac{V_{\text{A}}}{\sqrt{3}}$$

$$P = \frac{V_{\text{A}}^2}{3R}$$

$$P_{\text{dc}} = \frac{V_{\text{A}}^2}{4R}$$

The dc component carries 75% of the power and the ac components carry the remaining 25% of the power.

Problem 13–42. A first-order low-pass filter has a cutoff frequency of 600 rad/s and a passband gain of 20 dB. The input to the filter is $v(t) = 10\cos(300t) + 6\cos(1200t)$ V. Find the rms value of the steady-state output.

The transfer function for the filter is

$$T(s) = \frac{6000}{s + 600}$$

Determine the magnitudes of the output components by evaluating the transfer function at $\omega = 300$ and 1200 rad/s and then using the results to scale the two input components. Then determine the rms value of the output using the following expression on the output components:

$$V_{\text{rms}} = \sqrt{A_0^2 + \sum_{n=1}^{\infty} \frac{A_n^2}{2}}$$

The following MATLAB code performs the calculations:

```
syms s
Ts = 6000/(s+600);
w = [300 1200];
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
MagX = [10 6];
MagY = MagX.*MagTjw;
Yrms = sqrt(sum(MagY.^2)/2)
```

The transfer function magnitudes are 8.9443 and 4.4721. The output component magnitudes are 89.44 V and 26.83 V. The rms value of the steady-state output is 66.03 V.

Problem 13–43. Repeat Problem 13–42 for a first-order high-pass filter with the same cutoff frequency and passband gain.

The transfer function for the filter is

$$T(s) = \frac{10s}{s + 600}$$

Use the same approach as for Problem 13-42. The corresponding MATLAB code is shown below.

```
syms s
Ts = 10*s/(s+600);
w = [300 1200];
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw);
MagX = [10 6];
MagY = MagX.*MagTjw
Yrms = sqrt(sum(MagY.^2)/2)
```

The transfer function magnitudes are 4.4721 and 8.9443. The output component magnitudes are 44.72 V and 53.67 V. The rms value of the steady-state output is 49.40 V.

Problem 13–44. Estimate the rms value of the periodic voltage

$$v(t) = V_A \left[2 - \cos(\omega_0 t) + \frac{1}{3}\cos(3\omega_0 t) - \frac{1}{5}\cos(5\omega_0 t) + \frac{1}{7}\cos(7\omega_0 t) - \cdots \right] \text{ V}$$

We can estimate the rms value as follows:

$$V_{\text{rms}} = V_A \sqrt{2^2 + \frac{1}{2}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right)}$$

$$= 2.14869 V_A$$

Including more components will improve the estimate. The estimate above used 50000 components.

Problem 13–45. The input to the circuit in Figure P13–45 is the voltage

$$v_S(t) = 15\cos(2\pi 2000 t) + 5\cos(2\pi 6000 t) \text{ V}$$

Calculate the average power delivered to the 50-Ω load resistor.

Combine the resistor and capacitor in parallel and then apply voltage division to determine the transfer function.

$$T(s) = \frac{\dfrac{R/Cs}{R+1/Cs}}{R + \dfrac{R/Cs}{R+1/Cs}} = \frac{\dfrac{1}{RC}}{s + \dfrac{2}{RC}} = \frac{26667}{s + 53333}$$

Evaluate the magnitude of the transfer function at the input frequencies, determine the magnitudes of the output components, and then determine the rms voltage at the output. Finally, calculate the average power delivered to the load. The following MATLAB code performs the calculations:

```
syms s
R1 = 50;
R2 = 50;
C = 0.75e-6;
ZC = 1/s/C;
Z2 = 1/(1/ZC+1/R2);
Ts = Z2/(R1+Z2);
w = 2*pi*[2000 6000];
Tjw = subs(Ts,s,j*w);
MagTjw = abs(Tjw)
MagVs = [15 5];
MagVo = MagVs.*MagTjw;
Vorms = sqrt(sum(MagVo.^2)/2);
P = Vorms^2/R2
```

The transfer function magnitudes are 0.48667 and 0.40830. The output component magnitudes are 7.30 V and 2.04 V. The rms value of the output voltage is 5.36 V and the average power delivered to the load is $P = V_{\text{rms}}^2/R = 574.59$ mW.

Problem 13–46. Find an expression for the average power delivered to a resistor R by a triangular wave voltage with amplitude V_A and period T_0. How many components of the Fourier series are required to account for 98% of the average power carried by the waveform?

See the solution to Problem 13–37. The following MATLAB code performs the required calculations:

```
syms t
syms VA T0 R positive
% Create one period of the waveform
v1t = VA-4*VA*t/T0;
```

```
v2t = -3*VA+4*VA*t/T0;
% Find the rms voltage
vrms = simplify(sqrt((int(v1t^2,t,0,T0/2)+int(v2t^2,t,T0/2,T0))/T0))
% Find the average power delivered to a resistor
ptrms = vrms^2/R

% Triangular waveform
n = 1:2:9;
an = 8./(n*pi).^2;
% Find the square of the rms voltage for each n
Vrms2 = VA^2*cumsum(an.^2)/2;
P = Vrms2/R;
Pratio = double(P/ptrms);
Results = [n' Pratio']
```

The results are:

```
vrms = (3^(1/2)*VA)/3
ptrms = VA^2/(3*R)
Results =
     1.0000e+000   985.5343e-003
     3.0000e+000   997.7014e-003
     5.0000e+000   999.2782e-003
     7.0000e+000   999.6887e-003
     9.0000e+000   999.8389e-003
```

The rms voltage is $V_{\mathrm{A}}/\sqrt{3}$. The average power delivered to a resistor is $V_{\mathrm{A}}^2/3R$. Only one component is required to deliver over 98% of the average power carrier by the waveform.

Problem 13–47. Use the defining integral to find the Fourier transform of $f(t) = A[u(t) - u(t-3)]$.

We have the following results:

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \\ &= \int_{0}^{3} Ae^{-j\omega t}dt = \frac{A}{-j\omega}\left[e^{-j\omega 3} - 1\right] \\ &= \frac{A}{j\omega}\left[1 - e^{-j\omega 3}\right] \end{aligned}$$

Problem 13–48. Use the defining integral to find the Fourier transform of $f(t) = A[u(t+T_0) - u(t-T_0)]$.

We have the following results:

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \\ &= \int_{-T_0}^{T_0} Ae^{-j\omega t}dt = \frac{A}{-j\omega}\left[e^{-j\omega T_0} - e^{j\omega T_0}\right] \\ &= \frac{A}{j\omega}\left[e^{j\omega T_0} - e^{-j\omega T_0}\right] = \frac{2A\sin(\omega T_0)}{\omega} \end{aligned}$$

Problem 13–49. Use the defining integral to find the Fourier transform of $f(t) = At[u(t) - u(t-1)]$.

We have the following results:

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \\ &= \int_{0}^{1} Ate^{-j\omega t}dt = \frac{A}{\omega^2}\left(1 + j\omega t\right)e^{-j\omega t}\Big|_0^1 = \frac{A}{\omega^2}\left[(1 + j\omega)e^{-j\omega} - 1\right] \end{aligned}$$

Problem 13–50. Use MATLAB and the defining integral to find the Fourier transform of the following waveform:

$$f(t) = A\cos(\pi t/4)[u(t) - u(t-4)]$$

The MATLAB code is shown below.

```
syms t w A
ft = A*cos(pi*t/4)*(heaviside(t)-heaviside(t-4));
Fw = int(ft*exp(-j*w*t),t,-inf,inf)
Fw = simplify(Fw)
```

The results are:

```
Fw = (2*A*w*cos(2*w)*(cos(2*w) - sin(2*w)*i)*i)/(pi^2/16 - w^2)
Fw = (32*A*w*cos(2*w)*(cos(2*w) - sin(2*w)*i)*i)/(pi^2 - 16*w^2)
```

The answer is:

$$F(\omega) = \frac{32Aj\omega\cos(2\omega)[\cos(2\omega) - j\sin(2\omega)]}{\pi^2 - 16\omega^2}$$

Problem 13–51. Use the inversion integral to find the inverse transform of the following function:

$$F(\omega) = 10\pi[u(\omega+1) - u(\omega-1)]$$

We have the following results:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-1}^{1} 10\pi e^{j\omega t}d\omega$$

$$= \frac{5}{jt}[e^{jt} - e^{-jt}] = \frac{5}{jt}[2j\sin(t)] = \frac{10}{t}\sin(t)$$

Problem 13–52. Use MATLAB and the inversion integral to find the inverse transform of the following function:

$$F(\omega) = j\pi 8[2u(\omega+2) - 4u(\omega) + 2u(\omega-2)]$$

The MATLAB code is shown below.

```
syms t w
Fw = j*pi*8*(2*heaviside(w+2)-4*heaviside(w)+2*heaviside(w-2));
ft = 1/2/pi*int(Fw*exp(j*w*t),w,-inf,inf)
ft = vpa(simplify(ft),5)
```

The results are:

```
ft = (5734161139222659*pi*sin(t)^2)/(562949953421312*t)
ft = (32.0*sin(t)^2)/t
```

The answer is:

$$f(t) = \frac{32\sin^2(t)}{t}$$

Problem 13–53. Use MATLAB and the inversion integral to find the inverse transform of the following function:

$$F(\omega) = \sin(\pi\omega/4)[u(\omega+4) - u(\omega-4)]$$

The MATLAB code is shown below.

```
syms t w
Fw = sin(pi*w/4)*(heaviside(w+4)-heaviside(w-4));
ft = 1/2/pi*int(Fw*exp(j*w*t),w,-inf,inf)
ft = vpa(simplify(ft),5)
```

The results are:

```
ft = (5734161139222659*pi*sin(4*t)*i)/(4503599627370496*(pi^2 - 16*t^2))
ft = -(4.0*sin(4.0*t)*i)/(16.0*t^2 - 9.8696)
```

The answer is:

$$f(t) = \frac{4j\sin(4t)}{\pi^2 - 16t^2}$$

Problem 13–54. Find the inverse transforms of the following functions:

(a). $F_1(\omega) = \dfrac{5000}{(j\omega + 50)(j\omega + 100)}$

Rewrite the function using partial fraction expansion and perform the inverse transformation.

$$F_1(\omega) = \frac{100}{j\omega + 50} - \frac{100}{j\omega + 100}$$

$$f_1(t) = \left[100e^{-50t} - 100e^{-100t}\right] u(t)$$

(b). $F_2(\omega) = \dfrac{40j\omega}{(j\omega + 20)(j\omega + 40)}$

Rewrite the function using partial fraction expansion and perform the inverse transformation.

$$F_2(\omega) = \frac{-40}{j\omega + 20} + \frac{80}{j\omega + 40}$$

$$f_2(t) = \left[-40e^{-20t} + 80e^{-40t}\right] u(t)$$

Problem 13–55. Find the inverse transforms of the following functions:

(a). $F_1(\omega) = \dfrac{1600}{j\omega(j\omega + 20)(j\omega + 40)}$

Rewrite the function using partial fraction expansion and perform the inverse transformation.

$$F_1(\omega) = \frac{2}{j\omega} - \frac{4}{j\omega + 20} + \frac{2}{j\omega + 40}$$

$$f_1(t) = \mathrm{sgn}(t) + \left[-4e^{-20t} + 2e^{-40t}\right] u(t)$$

(b). $F_2(\omega) = \dfrac{-\omega^2}{j\omega(j\omega + 20)(j\omega + 40)}$

Rewrite the function using partial fraction expansion and perform the inverse transformation.

$$F_2(\omega) = \frac{0}{j\omega} - \frac{1}{j\omega + 20} + \frac{2}{j\omega + 40}$$

$$f_2(t) = \left[-e^{-20t} + 2e^{-40t}\right] u(t)$$

Problem 13–56. Find the Fourier transforms of the following waveforms:

(a). $f_1(t) = 2u(-t) - 2$

An equivalent expression for $f_1(t)$ is $-2u(t)$. Apply the Fourier transform pairs.

$$\mathscr{F}\{-2u(t)\} = -2\mathscr{F}\{u(t)\} = -2\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

(b). $f_2(t) = -2\text{sgn}(t) - 2u(-t)$

An equivalent expression for $f_2(t)$ is also $-2u(t)$, but use the Fourier transform pairs and properties in this case.

$$\mathscr{F}\{-2\text{sgn}(t) - 2u(-t)\} = -2\mathscr{F}\{\text{sgn}(t)\} - 2\mathscr{F}\{u(-t)\} = \frac{-4}{j\omega} - 2\left[\frac{-1}{j\omega} + \pi\delta(-\omega)\right]$$

$$= -2\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

(c). $f_3(t) = -\text{sgn}(t) - 1$

Again, an equivalent expression for $f_3(t)$ is also $-2u(t)$, but use the Fourier transform pairs and properties in this case.

$$\mathscr{F}\{-\text{sgn}(t) - 1\} = -\mathscr{F}\{\text{sgn}(t)\} - \mathscr{F}\{1\} = -\frac{2}{j\omega} - 2\pi\delta(\omega)$$

$$= -2\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

Problem 13–57. Find the Fourier transforms of the following waveforms:

(a). $f_1(t) = 2e^{-2t}u(t) - 2\text{sgn}(t)$

Apply the Fourier transform pairs and properties.

$$F_1(\omega) = \frac{2}{2+j\omega} - \frac{4}{j\omega} = \frac{-2j\omega - 8}{j\omega(j\omega+2)}$$

(b). $f_2(t) = 2e^{-2t}u(t) - 2u(t)$

Apply the Fourier transform pairs and properties.

$$F_2(\omega) = \frac{2}{2+j\omega} - 2\left[\frac{1}{j\omega} + \pi\delta(\omega)\right] = -2\pi\delta(\omega) - \frac{4}{j\omega(j\omega+2)}$$

Problem 13–58. Find the Fourier transforms of the following waveforms:

(a). $f_1(t) = 2\sin(t) - 2\cos(t)$

Apply the Fourier transform pairs and properties.

$$F_1(\omega) = -2j\pi\left[\delta(\omega-1) - \delta(\omega+1)\right] - 2\pi\left[\delta(\omega-1) + \delta(\omega+1)\right]$$

$$= -2\pi\left[(1+j)\delta(\omega-1) + (1-j)\delta(\omega+1)\right]$$

(b). $f_2(t) = (2/t)\sin(t) - 2\cos(t)$

Apply the Fourier transform pairs and properties.

$$F_2(\omega) = 2\pi\left[u(\omega+1) - u(\omega-1)\right] - 2\pi\left[\delta(\omega-1) + \delta(\omega+1)\right]$$

Problem 13–59. Find the Fourier transforms of the following waveforms:

(a). $f_1(t) = 5\cos[2\pi(t-3)]$

Apply the Fourier transform pairs and properties.

$$\mathscr{F}\{5\cos[2\pi t]\} = 5\pi[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$$

$$\mathscr{F}\{5\cos[2\pi(t-3)]\} = F_1(\omega) = 5\pi e^{-j\omega 3}[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$$

(b). $f_2(t) = 2e^{j4t}\text{sgn}(t)$

Apply the Fourier transform pairs and properties.

$$\mathscr{F}\{2\text{sgn}(t)\} = \frac{4}{j\omega}$$

$$\mathscr{F}\left\{2e^{j4t}\text{sgn}(t)\right\} = F_2(\omega) = \frac{4}{j(\omega-4)}$$

Problem 13–60. Find the inverse transforms of the following functions:

(a). $F_1(\omega) = 4\pi\delta(\omega) + 4\pi\delta(\omega-2) + 4\pi\delta(\omega-4)$

Apply the Fourier transform pairs and properties.

$$f_1(t) = 2 + 2e^{j2t} + 2e^{j4t}$$

(b). $F_2(\omega) = 4\pi\delta(\omega) - j2/\omega + 4\pi\delta(\omega-2)$

Apply the Fourier transform pairs and properties.

$$f_2(t) = 2 + \text{sgn}(t) + 2e^{j2t} = 1 + 2u(t) + 2e^{j2t}$$

(c). $F_3(\omega) = 2\pi\delta(\omega) - j2/\omega$

Apply the Fourier transform pairs and properties.

$$f_3(t) = 1 + \text{sgn}(t) = 2u(t)$$

Problem 13–61. Use the duality property to find the inverse transforms of the following functions:

(a). $F_1(\omega) = 3\cos(10\omega)$

Set $2\pi f(-\omega)$ equal to the transformed function and then solve for $f(\omega)$. Substitute t for ω to find $f(t)$. Take the transform of $f(t)$ to get $F(\omega)$. Substitute t for ω to find $F(t)$.

$$2\pi f(-\omega) = 3\cos(10\omega)$$

$$f(\omega) = \frac{3}{2\pi}\cos(-10\omega) = \frac{3}{2\pi}\cos(10\omega)$$

$$f(t) = \frac{3}{2\pi}\cos(10t)$$

$$F(\omega) = \frac{3}{2}[\delta(\omega-10) + \delta(\omega+10)]$$

$$f_1(t) = F(t) = \frac{3}{2}[\delta(t-10) + \delta(t+10)]$$

(b). $F_2(\omega) = 6u(\omega) - 3$

We have the following results:

$$2\pi f(-\omega) = 6u(\omega) - 3$$

$$f(\omega) = \frac{3}{\pi}u(-\omega) - \frac{3}{2\pi}$$

$$f(t) = \frac{3}{\pi}u(-t) - \frac{3}{2\pi}$$

$$F(\omega) = \frac{-3}{j\omega\pi} + 3\delta(\omega) - 3\delta(\omega) = \frac{-3}{j\omega\pi}$$

$$f_2(t) = F(t) = \frac{-3}{jt\pi} = \frac{j3}{t\pi}$$

(c). $F_3(\omega) = 6e^{-|3\omega|}$

We have the following results:

$$2\pi f(-\omega) = 6e^{-|3\omega|}$$

$$f(\omega) = \frac{3}{\pi}e^{-3|\omega|}$$

$$f(t) = \frac{3}{\pi}e^{-3|t|}$$

$$F(\omega) = \frac{18}{\pi(\omega^2 + 9)}$$

$$f_3(t) = F(t) = \frac{18}{\pi(t^2 + 9)}$$

Problem 13–62. Use the time-shifting property to find the inverse transforms of the following functions:

(a). $F_1(\omega) = [6\pi\delta(\omega) - j3/\omega]e^{-j3\omega}$

Find the transform without the time shift and then apply the time shift.

$$\mathscr{F}\left\{6\pi\delta(\omega) - j3/\omega\right\} = 3 + \frac{3}{2}\mathrm{sgn}(t)$$

$$f_1(t) = 3 + \frac{3}{2}\mathrm{sgn}(t-3) = \frac{3}{2} + 3u(t-3)$$

(b). $F_2(\omega) = 9e^{-j4\omega}/(j\omega + 5)$

We have the following results:

$$\mathscr{F}\left\{\frac{9}{5 + j\omega}\right\} = 9e^{-5t}u(t)$$

$$f_2(t) = 9e^{-5(t-4)}u(t-4)$$

(c). $F_3(\omega) = 2\cos(5\omega)/j\omega$

We have the following results:

$$\frac{2\cos(5\omega)}{j\omega} = \frac{e^{j\omega 5} - e^{-j\omega 5}}{j\omega}$$

$$\mathscr{F}\left\{\frac{1}{j\omega}\right\} = \frac{1}{2}\text{sgn}(t)$$

$$f_3(t) = \frac{1}{2}\left[\text{sgn}(t+5) - \text{sgn}(t-5)\right] = u(t+5) - u(t-5)$$

Problem 13–63. Given that the Fourier transform of $f(t)$ is

$$F(\omega) = \frac{1600}{(j\omega+20)(j\omega+40)}$$

use the integration property to find the waveform

$$g(t) = \int_{-\infty}^{t} f(x)dx$$

Apply the integration property:

$$\begin{aligned}
G(\omega) &= \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega) \\
&= \frac{1600}{j\omega(j\omega+20)(j\omega+40)} + 2\pi\delta(\omega) \\
&= \frac{2}{j\omega} - \frac{4}{j\omega+20} + \frac{2}{j\omega+40} + 2\pi\delta(\omega) \\
g(t) &= \text{sgn}(t) + 1 - 4e^{-20t}u(t) + 2e^{-40t}u(t) \\
&= \left[2 - 4e^{-20t} + 2e^{-40t}\right]u(t)
\end{aligned}$$

Problem 13–64. Use the reversal property to show that

$$\mathscr{F}\left\{Ae^{-\alpha|t|}\text{sgn}(t)\right\} = \frac{-2Aj\omega}{\omega^2+\alpha^2}$$

We have the following results:

$$f(t) = Ae^{-\alpha|t|}\text{sgn}(t) = Ae^{-\alpha t}u(t) - Ae^{\alpha t}u(-t)$$

$$F(\omega) = \frac{A}{\alpha+j\omega} - \frac{A}{\alpha-j\omega} = \frac{A(\alpha-j\omega) - A(\alpha+j\omega)}{\alpha^2+\omega^2} = \frac{-2Aj\omega}{\omega^2+\alpha^2}$$

Problem 13–65. Use the frequency shifting property to prove the modulation property

$$\mathscr{F}\left\{f(t)\sin(\beta t)\right\} = \frac{F(\omega-\beta)}{2j} - \frac{F(\omega+\beta)}{2j}$$

We have the following results:

$$\begin{aligned}
f(t)\sin(\beta t) &= \frac{1}{2j}f(t)[e^{j\beta t} - e^{-j\beta t}] \\
&= \frac{1}{2j}[f(t)e^{j\beta t} - f(t)e^{-j\beta t}]
\end{aligned}$$

$$\mathscr{F}\left\{f(t)\sin(\beta t)\right\} = \frac{1}{2j}\left[F(\omega-\beta) - F(\omega+\beta)\right]$$

Problem 13–66. Use the frequency shifting property to show that

$$\mathscr{F}\left\{\cos(\beta t)u(t)\right\} = \frac{j\omega}{\beta^2-\omega^2} + \frac{\pi}{2}[\delta(\omega-\beta)+\delta(\omega+\beta)]$$

We have the following results:

$$\cos(\beta t)u(t) = \frac{1}{2}\left[e^{j\beta t}+e^{-j\beta t}\right]u(t)$$

$$\begin{aligned}
\mathscr{F}\left\{\cos(\beta t)u(t)\right\} &= \frac{1}{2}\left[\frac{1}{j(\omega-\beta)}+\pi\delta(\omega-\beta)+\frac{1}{j(\omega+\beta)}+\pi\delta(\omega+\beta)\right] \\
&= \frac{1}{2}\left[\frac{j(\omega+\beta)+j(\omega-\beta)}{-(\omega^2-\beta^2)}+\pi\delta(\omega-\beta)+\pi\delta(\omega+\beta)\right] \\
&= \frac{1}{2}\left[\frac{2j\omega}{\beta^2-\omega^2}+\pi\delta(\omega-\beta)+\pi\delta(\omega+\beta)\right] \\
&= \frac{j\omega}{\beta^2-\omega^2}+\frac{\pi}{2}[\delta(\omega-\beta)+\delta(\omega+\beta)]
\end{aligned}$$

Problem 13–67. The input in Figure P13–67 is $v_1(t) = 5e^{-|t|}$ V. Use Fourier transforms to find $v_2(t)$.

The circuit is a low-pass filter with a gain of one and a cutoff frequency of $\omega_C = 1/RC = 1000$ rad/s. The transfer function is:

$$T(\omega) = \frac{1000}{j\omega+1000}$$

We have the following results for the output signal:

$$\begin{aligned}
V_1(\omega) &= \frac{10}{1+\omega^2} \\
V_2(\omega) &= T(\omega)V_1(\omega) = \frac{10000}{(1+\omega^2)(j\omega+1000)} = \frac{10000}{(1-j\omega)(1+j\omega)(j\omega+1000)} \\
&= \frac{4.995}{1-j\omega}+\frac{5.005}{1+j\omega}-\frac{0.01}{1000+j\omega} \\
v_2(t) &= 4.995e^{t}u(-t)+5.005e^{-t}u(t)-0.01e^{-1000t}u(t) \text{ V}
\end{aligned}$$

Problem 13–68. The input in Figure P13–67 is $v_1(t) = 10\text{sgn}(t)$ V. Use Fourier transforms to find $v_2(t)$.

The circuit is a low-pass filter with a gain of one and a cutoff frequency of $\omega_C = 1/RC = 1000$ rad/s. The transfer function is:

$$T(\omega) = \frac{1000}{j\omega+1000}$$

We have the following results for the output signal:

$$\begin{aligned}
V_1(\omega) &= \frac{20}{j\omega} \\
V_2(\omega) &= T(\omega)V_1(\omega) = \frac{20000}{j\omega(j\omega+1000)} = \frac{20}{j\omega}-\frac{20}{j\omega+1000} \\
v_2(t) &= 10\text{sgn}(t)-20e^{-1000t}u(t)
\end{aligned}$$

Problem 13–69. The input in Figure P13–69 is $v_1(t) = 2\text{sgn}(t)$ V. Use Fourier transforms to find $v_2(t)$.

Determine the transfer function and then compute the output waveform.

$$T(\omega) = \frac{R}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{1000j\omega}{(j\omega)^2 + 1000j\omega + 10^6}$$

$$V_1(\omega) = \frac{4}{j\omega}$$

$$V_2(\omega) = T(\omega)V_1(\omega) = \frac{4000}{(j\omega)^2 + 1000j\omega + 10^6}$$

$$V_2(s) = \frac{4000}{s^2 + 1000s + 10^6} = \frac{4000}{(s+500)^2 + 750000} = \left(\frac{8\sqrt{3}}{3}\right)\left(\frac{500\sqrt{3}}{(s+500)^2 + (500\sqrt{3})^2}\right)$$

$$v_2(t) = \frac{8\sqrt{3}}{3}e^{-500t}\sin(500\sqrt{3}t)u(t) \text{ V}$$

Problem 13–70. The input in Figure P13–70 is $v_1(t) = 10e^{4t}u(-t)$ V. Use Fourier transforms to find $v_2(t)$. Determine the transfer function and then compute the output waveform.

$$T(\omega) = \frac{\dfrac{R/j\omega C}{R + 1/j\omega C}}{R + \dfrac{R/j\omega C}{R + 1/j\omega C}} = \frac{\dfrac{1}{RC}}{j\omega + \dfrac{2}{RC}} = \frac{100}{j\omega + 200}$$

$$V_1(\omega) = \frac{10}{4 - j\omega}$$

$$V_2(\omega) = T(\omega)V_1(\omega) = \frac{1000}{(4 - j\omega)(j\omega + 200)} = \frac{\dfrac{250}{51}}{4 - j\omega} + \frac{\dfrac{250}{51}}{j\omega + 200}$$

$$v_2(t) = \frac{250}{51}\left[e^{4t}u(-t) + e^{-200t}u(t)\right] \text{ V}$$

Problem 13–71. The input in Figure P13–70 is $v_1(t) = 10\text{sgn}(t)$ V. Use Fourier transforms to find $v_2(t)$. Determine the transfer function and then compute the output waveform.

$$T(\omega) = \frac{\dfrac{R/j\omega C}{R + 1/j\omega C}}{R + \dfrac{R/j\omega C}{R + 1/j\omega C}} = \frac{\dfrac{1}{RC}}{j\omega + \dfrac{2}{RC}} = \frac{100}{j\omega + 200}$$

$$V_1(\omega) = \frac{20}{j\omega}$$

$$V_2(\omega) = T(\omega)V_1(\omega) = \frac{2000}{j\omega(j\omega + 200)} = \frac{10}{j\omega} - \frac{10}{j\omega + 200}$$

$$v_2(t) = 5\text{sgn}(t) - 10e^{-200t}u(t) \text{ V}$$

Problem 13–72. The input in Figure P13–72 is $v_1(t) = 5u(-t)$ V. Use Fourier transforms to find $v_2(t)$. Determine the transfer function and then compute the output waveform. The transfer function is a high-pass filter with a gain of -15 and a cutoff frequency of $\omega_C = 1/RC = 10000$ rad/s.

$$T(\omega) = \frac{-15j\omega}{j\omega + 10000}$$

$$V_1(\omega) = 5\left[\frac{1}{-j\omega} + \pi\delta(-\omega)\right]$$

$$V_2(\omega) = T(\omega)V_1(\omega) = \frac{75}{j\omega + 10000} - \frac{75\pi j\omega\delta(-\omega)}{j\omega + 10000} = \frac{75}{j\omega + 10000}$$

$$v_2(t) = 75e^{-10000t}u(t) \text{ V}$$

Problem 13–73. The input in Figure P13–72 is $v_1(t) = 10e^{-4|t|}$ V. Use Fourier transforms to find $v_2(t)$.

Determine the transfer function and then compute the output waveform. The transfer function is a high-pass filter with a gain of -15 and a cutoff frequency of $\omega_C = 1/RC = 10000$ rad/s.

$$T(\omega) = \frac{-15j\omega}{j\omega + 10000}$$

$$V_1(\omega) = \frac{80}{\omega^2 + 16} = \frac{10}{4 + j\omega} + \frac{10}{4 - j\omega}$$

$$V_2(\omega) = T(\omega)V_1(\omega) = \frac{-150j\omega}{(j\omega + 4)(j\omega + 10000)} + \frac{-150j\omega}{(-j\omega + 4)(j\omega + 10000)}$$

$$= \frac{\frac{600}{9996}}{j\omega + 4} - \frac{\frac{1500000}{9996}}{j\omega + 10000} - \frac{\frac{600}{10004}}{-j\omega + 4} + \frac{\frac{1500000}{10004}}{j\omega + 10000}$$

$$= \frac{\frac{50}{833}}{j\omega + 4} - \frac{\frac{600}{10004}}{-j\omega + 4} - \frac{\frac{250000}{2083333}}{j\omega + 10000}$$

$$v_2(t) = 0.06e^{-4t}u(t) - 0.06e^{4t}u(-t) - 0.12e^{-10000t}u(t) \text{ V}$$

Problem 13–74. The impulse response of a linear system is $h(t) = e^{-2t}u(t)$. Find the output for an input $x(t) = u(-t)$.

We have the following results:

$$h(t) = e^{-2t}u(t)$$

$$H(\omega) = \frac{1}{2 + j\omega}$$

$$x(t) = u(-t)$$

$$X(\omega) = \frac{1}{-j\omega} + \pi\delta(-\omega)$$

$$Y(\omega) = H(\omega)X(\omega) = \frac{-1}{j\omega(2 + j\omega)} + \frac{\pi\delta(-\omega)}{2 + j\omega}$$

$$= \frac{-\frac{1}{2}}{j\omega} + \frac{\frac{1}{2}}{2 + j\omega} + \frac{\pi\delta(\omega)}{2}$$

$$y(t) = -\frac{1}{4}\text{sgn}(t) + \frac{1}{2}e^{-2t}u(t) + \frac{1}{4} = \frac{1}{2}\left[u(-t) + e^{-2t}u(t)\right]$$

Problem 13–75. The impulse response of a linear system is $h(t) = e^{-2|t|}$. Find the output for an input $x(t) = u(-t)$.

We have the following results:

$$h(t) = e^{-2|t|}$$

$$H(\omega) = \frac{4}{4+\omega^2} = \frac{4}{(2+j\omega)(2-j\omega)}$$

$$x(t) = u(-t)$$

$$X(\omega) = \frac{1}{-j\omega} + \pi\delta(-\omega)$$

$$Y(\omega) = H(\omega)X(\omega) = \frac{-4}{j\omega(2+j\omega)(2-j\omega)} + \frac{4\pi\delta(-\omega)}{(2+j\omega)(2-j\omega)}$$

$$= \frac{-1}{j\omega} + \frac{\frac{1}{2}}{2+j\omega} - \frac{\frac{1}{2}}{2-j\omega} + \pi\delta(\omega)$$

$$y(t) = -\frac{1}{2}\text{sgn}(t) + \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{2t}u(-t) + \frac{1}{2}$$

$$= u(-t) + \frac{1}{2}\left[e^{-2t}u(t) - e^{2t}u(-t)\right]$$

Problem 13–76. The impulse response of a linear system is $h(t) = 2\delta(t) - 4e^{-t}u(t)$. Use MATLAB and Fourier transform techniques to find the output for an input $x(t) = \text{sgn}(-t)$.

The MATLAB code is shown below.

```
syms t w
ht = 2*dirac(t)-4*exp(-t)*heaviside(t);
Hw = fourier(ht,t,w)
xt = heaviside(-t)-heaviside(t);
Xw = fourier(xt,t,w)
Yw = Hw*Xw
yt = simplify(ifourier(Yw,w,t))
```

The corresponding output is shown below.

```
Hw = 2 - 4/(1 + w*i)
Xw = (2*i)/w
Yw = -(2*(4/(1 + w*i) - 2)*i)/w
yt = 4*heaviside(t) - (8*heaviside(t))/exp(t) - 2
```

The results are:

$$y(t) = \left[4 - 8e^{-t}\right]u(t) - 2$$

Problem 13–77. The impulse response of a linear system is $h(t) = A[\delta(t) - \alpha e^{-\alpha t}u(t)]$, with $\alpha > 0$. Let $A = 5$ and $\alpha = 2$ and use MATLAB to plot $|H(\omega)|$. On the same axes, plot $|H(\omega)|$ for $A = 5$ and $\alpha = 4$. Describe the system frequency response and the influence of the parameter α.

The MATLAB code is shown below.

```
syms t w
syms A a positive
ht = A*(dirac(t)-a*exp(-a*t)*heaviside(t));
Hw = fourier(ht,t,w);
```

```
% Solve for |H(w)| with the given values for A and alpha
H1w = subs(Hw,{A,a},{5,2})
H1wa = factor(H1w)
H2w = subs(Hw,{A,a},{5,4})
H2wa = factor(H2w)
```

The MATLAB output is shown below.

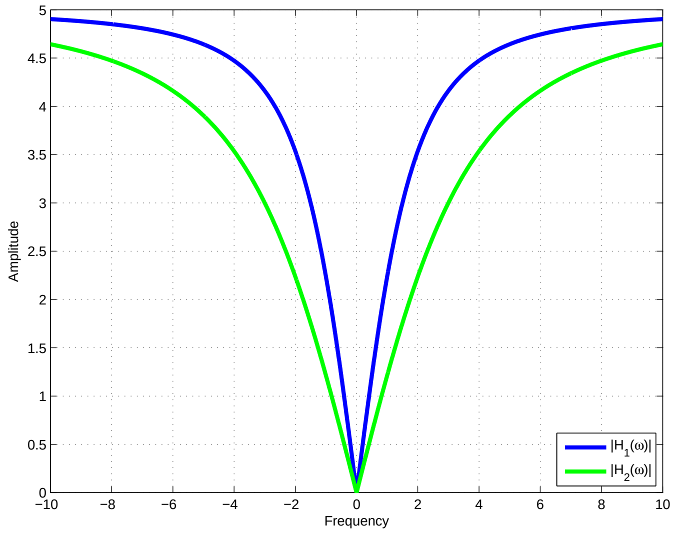
```
H1w = 5 - 10/(2 + w*i)
H1wa = (5*w)/(w - 2*i)
H2w = 5 - 20/(4 + w*i)
H2wa = (5*w)/(w - 4*i)
```

The results are:

$$H_1(\omega) = \frac{5j\omega}{j\omega + 2}$$

$$H_2(\omega) = \frac{5j\omega}{j\omega + 4}$$

The transfer functions are high-pass filters with the cutoff frequency controlled by the parameter α. The magnitude plots are shown below.



Problem 13–78. The impulse response of a linear system is $h(t) = A[\delta(t) - \sin(\beta t)/\pi t]$. Let $A = 5$ and $\beta = 2$ and use MATLAB to plot $|H(\omega)|$. On the same axes, plot $|H(\omega)|$ for $A = 5$ and $\beta = 4$. Describe the system frequency response and the influence of the parameter β. You may need to use the following equality in your solution: $\sin(\beta t)/\pi t = \dfrac{1}{2j}(e^{j\beta t} - e^{-j\beta t})$.

The MATLAB code is shown below.

```
syms t w
syms A B positive
ht = A*(dirac(t)-(exp(j*B*t)-exp(-j*B*t))/j/2/pi/t);
Hw = simplify(fourier(ht,t,w))
% Solve for |H(w)| with the given values for A and alpha
H1w = subs(Hw,{A,B},{5,2})
H2w = subs(Hw,{A,B},{5,4})
```

The MATLAB output is shown below.

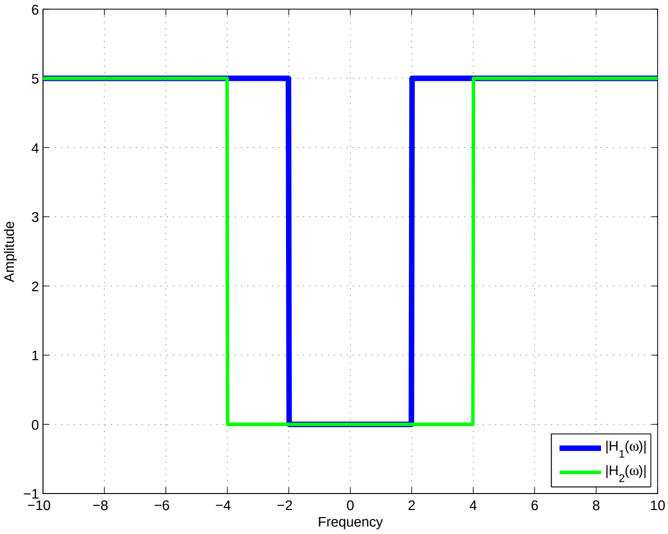
```
Hw = -A*(heaviside(B + w) + heaviside(B - w) - 2)
H1w = 10 - 5*heaviside(2 - w) - 5*heaviside(w + 2)
H2w = 10 - 5*heaviside(4 - w) - 5*heaviside(w + 4)
```

The results are:

$$H_1(\omega) = 10 - 5\left[u(2-\omega) + u(\omega+2)\right]$$

$$H_2(\omega) = 10 - 5\left[u(4-\omega) + u(\omega+4)\right]$$

The system frequency response is that of an ideal high-pass filter. The parameter β influences the cutoff frequency of the filter. As β increases, the cutoff frequency increases. The magnitude plots are shown below.



Problem 13–79. The impulse response of a linear system is $h(t) = -Ae^{-\alpha t}u(t) + Ae^{-\alpha(-t)}u(-t)$, with $\alpha > 0$. Let $A = 10$ and $\alpha = 2.5$ and use MATLAB to plot $|H(\omega)|$. On the same axes, plot $|H(\omega)|$ for $A = 10$ and $\alpha = 5$. Describe the system frequency response and the influence of the parameter α.

The MATLAB code is shown below.

```
syms t w real
syms A a positive
ht = -A*exp(-a*t)*heaviside(t)+A*exp(a*t)*heaviside(-t);
Hw = fourier(ht,t,w);
% Solve for |H(w)| with the given values for A and alpha
H1w = subs(Hw,{A,a},{10,2.5})
H1wa = simplify(H1w)
H2w = subs(Hw,{A,a},{10,5})
H2wa = simplify(H2w)
```

The MATLAB output is shown below.

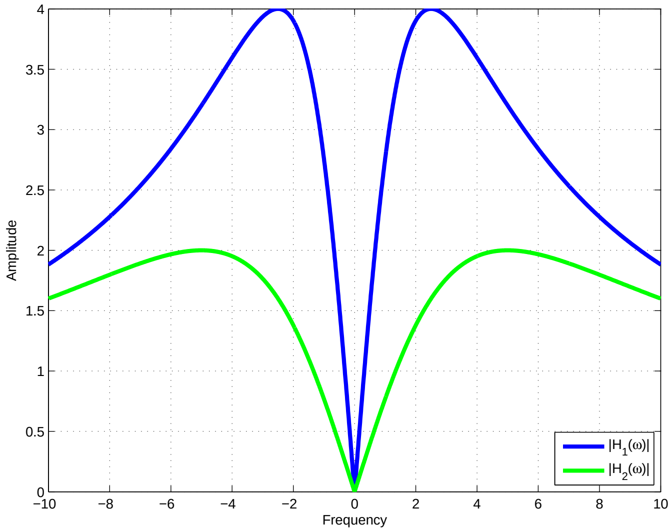
```
H1w = - 10/(- 5/2 + w*i) - 10/(5/2 + w*i)
H1wa = (80*w*i)/(4*w^2 + 25)
H2w = - 10/(- 5 + w*i) - 10/(5 + w*i)
H2wa = (20*w*i)/(w^2 + 25)
```

The results are:

$$H_1(\omega) = \frac{20j\omega}{\omega^2 + 6.25}$$

$$H_2(\omega) = \frac{20j\omega}{\omega^2 + 25}$$

The system frequency response is that of a bandpass filter. The parameter α influences the center frequency of the pass band. As α increases, the center frequency increases. In addition, as the parameter α increases, the magnitude of the pass band gain decreases. The magnitude plots are shown below.



Problem 13–80. The frequency response of a linear system is shown in Figure P13–80. Find the system impulse response $h(t)$.

Write an expression for the frequency response using step functions and then compute the inverse transform.

$$H(\omega) = 5u(\omega + 2\beta) + 5u(\omega + \beta) - 5u(\omega - \beta) - 5u(\omega - 2\beta)$$

$$= 5\left[u(\omega + 2\beta) - u(\omega - 2\beta)\right] + 5\left[u(\omega + \beta) - u(\omega - \beta)\right]$$

$$h(t) = 5\left[\frac{\sin(2\beta t)}{\pi t} + \frac{\sin(\beta t)}{\pi t}\right]$$

Problem 13–81. The frequency response of a linear system is shown in Figure P13–81. Find the system impulse response $h(t)$.

Write an expression for the frequency response using ramp functions and then compute the inverse transform.

$$H(\omega) = \frac{A}{\beta}r(\omega + \beta) - \frac{2A}{\beta}r(\omega) + \frac{A}{\beta}r(\omega - \beta)$$

Use MATLAB to calculate the integral.

```
syms t w A B
Hw = A*(w+B)/B*heaviside(w+B)-2*A*w/B*heaviside(w)+A*(w-B)/B*heaviside(w-B);
ht = simplify(ifourier(Hw,w,t))
```

The results are:

```
ht = -(A*(1/exp(B*t*i))*(exp(B*t*i) - 1)ˆ2)/(2*B*pi*tˆ2)
```

We have:

$$h(t) = \frac{-Ae^{-j\beta t}[e^{j\beta t} - 1]^2}{2\beta\pi t^2} = \frac{2A\sin^2\left(\dfrac{\beta t}{2}\right)}{\pi\beta t^2}$$

Problem 13–82. Find the 1-Ω energy carried by the signal $F(\omega) = 8/(\omega^2 + 16)$.

We have the following results:

$$F(\omega) = \frac{8}{\omega^2 + 16}$$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{8}{\omega^2 + 16}\right|^2 d\omega = \int_{-\infty}^{\infty} f^2(t)dt$$

$$f(t) = e^{-4|t|}$$

$$W_{1\Omega} = 2\int_{0}^{\infty} e^{-8t}dt = -\frac{1}{4}\left[e^{-8t}\right]\Big|_0^{\infty} = \frac{1}{4}\text{ J}$$

Problem 13–83. Compute the 1-Ω energy carried by the signal $f(t) = 5e^{2.5t}u(-t)$.

We have the following results:

$$W_{1\Omega} = \int_{-\infty}^{0} 25e^{5t}dt = 5e^{5t}\Big|_{-\infty}^{0} = 5\text{ J}$$

Problem 13–84. Find the 1-Ω energy carried by the signal

$$F(\omega) = \frac{j\omega A}{\omega^2 + \alpha^2}$$

Then find the percentage of the 1-Ω energy carried in the frequency band $|\omega| \leq \alpha$.

Use MATLAB to compute the following integrals and the ratio of the results.

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{j\omega A}{\omega^2 + \alpha^2}\right|^2 d\omega$$

$$W_{\alpha} = \frac{1}{2\pi}\int_{-\alpha}^{\alpha}\left|\frac{j\omega A}{\omega^2 + \alpha^2}\right|^2 d\omega$$

The MATLAB code is shown below.

```
syms w t
syms A a positive
Fw = j*w*A/(w^2+a^2);
% Compute the integral in the frequency domain
W = int((abs(Fw))^2,w,-inf,inf)/2/pi
% Check the results with the time-domain integral
ft = simplify(ifourier(Fw,w,t));
Wt = int(ft^2,t,-inf,inf);
% Find the energy in the requested frequency range
Wa = int((abs(Fw))^2,w,0,a)/pi
Wa_percent = Wa/W
Wa_percent_num = double(Wa_percent)
```

The results are:

```
W = A^2/(4*a)
Wa = (A^2*(pi - 2))/(8*pi*a)
Wa_percent = (pi - 2)/(2*pi)
Wa_percent_num = 181.6901e-003
```

The results are:

$$W_{1\Omega} = \frac{A^2}{4\alpha}$$

$$W_\alpha = \frac{A^2(\pi - 2)}{8\pi\alpha}$$

$$\frac{W_\alpha}{W_{1\Omega}} = \frac{\pi - 2}{2\pi} = 0.1817$$

Problem 13–85. The transfer function of an ideal high-pass filter is $H(\omega) = 1$ for $|\omega| \geq 2$ krad/s. The filter input signal is $x(t) = 10e^{-2000t}u(t)$. Find the 1-Ω energy carried by the input signal and the percentage of the input energy that appears in the output.

Compute the 1-Ω energy carried by the input signal.

$$W_{1\Omega} = \int_{-\infty}^{\infty} x^2(t)dt = \int_0^{\infty} 100e^{-4000t}dt = \frac{1}{40}\text{ J}$$

Convert to the frequency domain to find the energy in the output signal. The filter passes frequencies above 2 krad/s, so integrate the frequency response above that value.

$$X(\omega) = \frac{10}{2000 + j\omega}$$

$$W = \frac{1}{\pi}\int_{2000}^{\infty} \left|\frac{10}{2000 + j\omega}\right|^2 d\omega$$

Use MATLAB to perform the calculations and determine the energy ratio.

```
syms w t
syms A a positive
ft = 10*exp(-2000*t)*heaviside(t);
% Compute the integral in the time domain
Wt = int(ft^2,t,-inf,inf)
% Compute the integral in the frequency domain
Fw = fourier(ft,t,w);
W = int((abs(Fw))^2,w,-inf,inf)/2/pi
% Calculate the energy in the output signal by
% integrating across the pass band
Wout = int((abs(Fw))^2,w,2000,inf)/pi
% Find the percentage of energy in the output
Wout_percent = double(Wout/W)
```

The results are:

```
Wt = 1/40
W = 1/40
Wout = 1/80
Wout_percent = 500.0000e-003
```

The energy in the output signal is 1/80 J, so the output signal contains 50% of the energy in the input signal.

Problem 13–86. The impulse response of a filter is $h(t) = 100e^{-400t}u(t)$. Find the 1-Ω energy in the output signal when the input is $x(t) = 5e^{-100t}u(t)$.

We have the following results:

$$h(t) = 100e^{-400t}u(t)$$

$$H(\omega) = \frac{100}{400 + j\omega}$$

$$x(t) = 5e^{-100t}u(t)$$

$$X(\omega) = \frac{5}{100 + j\omega}$$

$$Y(\omega) = H(\omega)X(\omega) = \frac{500}{(400 + j\omega)(100 + j\omega)} = \frac{-\frac{5}{3}}{400 + j\omega} + \frac{\frac{5}{3}}{100 + j\omega}$$

$$y(t) = \frac{5}{3}\left[e^{-100t} - e^{-400t}\right]u(t)$$

$$W_{1\Omega} = \int_{-\infty}^{\infty} y^2(t)dt = \int_0^{\infty}\left[\frac{5}{3}\left(e^{-100t} - e^{-400t}\right)\right]^2 dt = \frac{1}{160}\text{ J}$$

The following MATLAB code verifies the results.

```
syms t w
ht = 100*exp(-400*t)*heaviside(t);
xt = 5*exp(-100*t)*heaviside(t);
% Compute the output signal
Hw = fourier(ht,t,w);
Xw = fourier(xt,t,w);
Yw = Hw*Xw
yt = simplify(ifourier(Yw,w,t))
yt2 = simplify(yt^2)
% Find the energy in the output signal
W = int((abs(Yw))^2,w,-inf,inf)/2/pi
W2 = int(yt^2,t,0,inf)
```

Problem 13–87. The impulse response of a filter is $h(t) = 100e^{-10t}u(t)$. Find the 1-Ω energy in the output signal when the input is $x(t) = \delta(t)$.

We have the following results:

$$h(t) = 100e^{-10t}u(t)$$

$$H(\omega) = \frac{100}{10 + j\omega}$$

$$x(t) = \delta(t)$$

$$X(\omega) = 1$$

$$Y(\omega) = H(\omega)X(\omega) = \frac{100}{10 + j\omega}$$

$$y(t) = 100e^{-10t}u(t)$$

$$W_{1\Omega} = \int_{-\infty}^{\infty} y^2(t)dt = \int_0^{\infty} 10000e^{-20t}dt = 500\text{ J}$$

Problem 13–88. The current in a 100-Ω resistor is $i(t) = -4u(t+2) + 8u(t) - 4u(t-2)$. Find the total energy delivered to the resistor.

We have the following results:

$$i(t) = -4u(t+2) + 8u(t) - 4u(t-2)$$

$$p(t) = i^2(t)$$

$$i^2(t) = 16[u(t+2) - u(t-2)]$$

$$p(t) = 1600[u(t+2) - u(t-2)]$$

$$W = \int_{-\infty}^{\infty} p(t) = (1600)(4) = 6400 \text{ J}$$

Problem 13–89. The transfer function of an ideal bandpass filter is $H(\omega) = 1$ for $500 \leq \omega \leq 1000$ rad/s. Use MATLAB to find the 1-Ω energy carried by the output signal when the input is $x(t) = 15e^{-1000t}u(t)$. What percentage of the input signal energy appears in the output?

The following MATLAB code performs the calculations.

```
syms w t
syms A a positive
ft = 15*exp(-1000*t)*heaviside(t);
% Find the energy of the input signal
% Compute the integral in the time domain
Wt = int(ft^2,t,-inf,inf)
% Compute the integral in the frequency domain
Fw = fourier(ft,t,w);
W = int((abs(Fw))^2,w,-inf,inf)/2/pi
% Calculate the energy in the output signal by
% integrating across the pass band
Wout = double(int((abs(Fw))^2,w,500,1000)/pi)
% Find the percentage of energy in the output
Wout_percent = double(Wout/W)
```

The MATLAB output is shown below.

```
Wt = 9/80
W = 9/80
Wout = 23.0437e-003
Wout_percent = 204.8328e-003
```

The 1-Ω energy in the input signal is $9/80 = 0.1125$ J. At the output, the 1-Ω energy is 0.02304 J, so 20.48% of the input energy appears in the output.

Problem 13–90. (A) Fourier Series from a Bode Plot

The transfer function of a linear circuit has the straight-line gain and phase Bode plots in Figure P13–90. The first four terms in the Fourier series of a periodic input $v_1(t)$ to the circuit are

$$v_1(t) = 42\cos(100t) + 14\cos(300t) + 8.4\cos(500t) + 6\cos(700t) \text{ V}$$

Estimate the amplitudes and phase angles of the first four terms in the Fourier series of the steady-state output $v_2(t)$.

The following table summarizes the results from the straight-line gain and phase Bode plots.

ω	$\lvert V_1(\omega)\rvert$ (V)	$\lvert T(j\omega)\rvert$ (dB)	$\lvert T(j\omega)\rvert$	$\angle T(j\omega)$ (°)	$\lvert Y(\omega)\rvert$ (V)
100	42	0	1	−27	42
300	14	−6	0.501	−27	7
500	8.4	−12	0.251	−27	2.1
700	6	−12	0.251	−27	1.5

The estimated output signal is:

$$v_2(t) = 42\cos(100t - 27^\circ) + 7\cos(300t - 27^\circ) + 2.1\cos(500t - 27^\circ) + 1.5\cos(700t - 27^\circ) \text{ V}$$

Based on the Bode plots, the actual transfer function is

$$T(\omega) = \frac{\frac{1}{4}(j\omega + 400)^2}{(j\omega + 200)^2}$$

The following MATLAB code calculates the actual response:

```
syms s w t
Ts = 1/4*(s+400)^2/(s+200)^2;
Tjw = subs(Ts,s,j*w);
ww = [100 300 500 700];
Magv1 = [42 14 8.4 6];
Tjww = subs(Tjw,w,ww);
MagTjww = abs(Tjww);
PhaseTjww = 180*angle(Tjww)/pi;
% Compute the output magnitudes
Magv2 = Magv1.*MagTjww;
% The input phases are all zero, so the output phases are the
% phases of the transfer function
Phasev2 = PhaseTjww;
Results = [ww' Magv2' Phasev2']
```

The results are:

```
Results =
   100.0000e+000    35.7000e+000   -25.0576e+000
   300.0000e+000     6.7308e+000   -38.8801e+000
   500.0000e+000     2.9690e+000   -33.7168e+000
   700.0000e+000     1.8396e+000   -27.5990e+000
```

The actual output signal is:

$$v_2(t) = 35.7\cos(100t - 25.1^\circ) + 6.73\cos(300t - 38.9^\circ) + 2.97\cos(500t - 33.7^\circ) + 1.84\cos(700t - 27.6^\circ) \text{ V}$$

The straight-line estimate is a fair approximation.

Problem 13–91. (A) Spectrum of a Periodic Impulse Train

A periodic impulse train can be written as

$$x(t) = T_0 \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

Find the Fourier coefficients of $x(t)$. Plot the amplitude spectrum and comment on the frequencies contained in the impulse train.

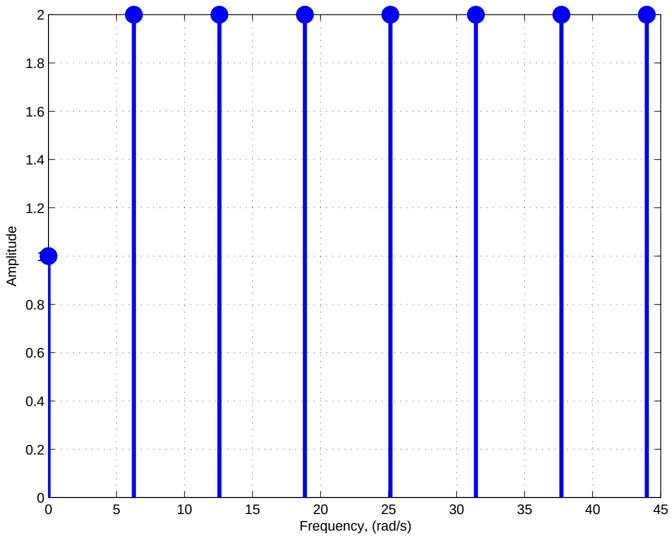
Use the definitions to compute the Fourier coefficients:

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} T_0\delta(t)dt = 1$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\cos(2\pi nt/T_0)dt = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} T_0\delta(t)\cos(2\pi nt/T_0)dt = 2$$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\sin(2\pi nt/T_0)dt = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} T_0\delta(t)\sin(2\pi nt/T_0)dt = 0$$

The coefficients are $a_0 = 1$, $a_n = 2$, and $b_n = 0$ for all values of n. The impulse train contains equal amounts of energy at every nonzero frequency. The amplitude spectrum is shown below.



Problem 13–92. (D) Power Supply Filter Design

The input to a power supply filter is a full-wave rectified sine wave with $f_0 = 60$ Hz. The filter is a first-order low pass with unity dc gain. Select the cutoff frequency of the filter so that all of the ac components in the filter output are all less than 1% of the dc component.

Using the results in Figure 13–4, a full-wave rectified sine wave has a dc component and ac components for even values of n. The gain of a first-order low-pass filter decreases as frequency increases. Therefore, if we design the filter such that the first ac component is less than 1% of the dc component, then all of the ac components will meet the specification. With unity dc gain, the magnitude of the dc component at the output will be $2A/\pi$. The first ac component appears at $n = 2$ or $\omega = 2\pi n/T_0 = 240\pi$, with an amplitude of $4A/(3\pi)$. We need the low-pass filter to have a gain at $\omega = 240\pi$ such that $|T(\omega)| \times 4A/(3\pi) < 2A/(100\pi)$. Therefore, $|T(\omega)| < 3/200$. For a first-order low-pass filter we have:

$$T(\omega) = \frac{\omega_C}{j\omega + \omega_C}$$

$$|T(\omega)| = \frac{\omega_C}{\sqrt{\omega^2 + \omega_C^2}} = \frac{3}{200}$$

$$\frac{\omega_C^2}{\omega^2 + \omega_C^2} = \frac{9}{40000}$$

$$40000\omega_C^2 = 9\omega^2 + 9\omega_C^2$$

$$\omega_C = \sqrt{\frac{9\omega^2}{39991}} = \sqrt{\frac{9(240\pi)^2}{39991}} = 11.311 \text{ rad/s}$$

Choose the cutoff frequency to be $\omega_C = 10$ rad/s.

Problem 13–93. (E) Spectrum Analyzer Calibration

A certain spectrum analyzer measures the average power delivered to a calibrated resistor by the individual harmonics of periodic waveforms. The calibration of the analyzer has been checked by applying a 1-MHz square wave and the following results reported

f (MHz)	1	3	5	7	9	11
P (dBm)	12.1	2.56	−1.88	−4.80	−6.98	−8.73

The reported power in dBm is $P = 10\log(P_n)$, where P_n is the average power delivered by the nth harmonic in mW. Is the spectrum analyzer correctly calibrated?

Using the results of Figure 13–4, the harmonics of a square wave are proportional to $1/n$, and appear only for odd values of n. Assume that the reported results at an input frequency of 1 MHz are correct and determine the expected power levels for the harmonics. If the voltage decreases by $1/n$ for the harmonics, then the power decreases by $1/(n^2)$. The following analysis predicts how the power levels should change:

$$P = 10\log(P_{\rm n}) = 10\log\left(\frac{P_1}{n^2}\right) = 10\log(P_1) - 10\log(n^2)$$

The power levels should decrease by $10\log(n^2)$ for each harmonic. The following MATLAB calculations predict the output power levels.

```
n = 1:2:11;
P1 = 12.1;
Pn2 = 10*log10(n.^2);
P_out = P1-Pn2'
```

The results are:

```
P_out =
    12.1000e+000
     2.5576e+000
    -1.8794e+000
    -4.8020e+000
    -6.9849e+000
    -8.7279e+000
```

The expected output power levels agree closely with the measured values. The spectrum analyzer is calibrated correctly.

Problem 13–94. (D) Virtual Keyboard Design

Electronic keyboards are designed using the following equation that assigns particular frequencies to each of the 88 keys in a standard piano keyboard,

$$f(n) = 440\left(\sqrt[12]{2}\right)^{n-49}\ \text{Hz}$$

where n is the key number. There is a need for an amplifier that can pass middle C, key 40, but block keys 39 and 41. Design such a filter using an RLC tuned circuit with a gain K of 100.

Find the frequency associated with middle C and the two adjacent keys.

$$f(39) = 440\left(\sqrt[12]{2}\right)^{39-49} = 440(2^{-10/12}) = 246.942\ \text{Hz}$$

$$f(40) = 440\left(\sqrt[12]{2}\right)^{40-49} = 440(2^{-9/12}) = 261.626\ \text{Hz}$$

$$f(41) = 440\left(\sqrt[12]{2}\right)^{41-49} = 440(2^{-8/12}) = 277.183\ \text{Hz}$$

The center frequency is $f_{\rm c} = 261.626$ Hz or $\omega_{\rm C} = 1643.84$ rad/s. The bandwidth must be less than 30 Hz or 188.5 rad/s. Choose a bandwidth of $B = 50$ rad/s. For the filter, use a series RLC circuit with the output taken across the resistor. Connect the RLC circuit to a noninverting amplifier with a gain of 100. The bandwidth is $B = R/L = 50$, so choose $R = 50\ \Omega$ and $L = 1$ H. The center frequency is $\omega_{\rm C}^2 = 1/LC$, so $C = 0.37\ \mu$F.

Problem 13–95. (A) Fourier Series and Fourier Transforms.

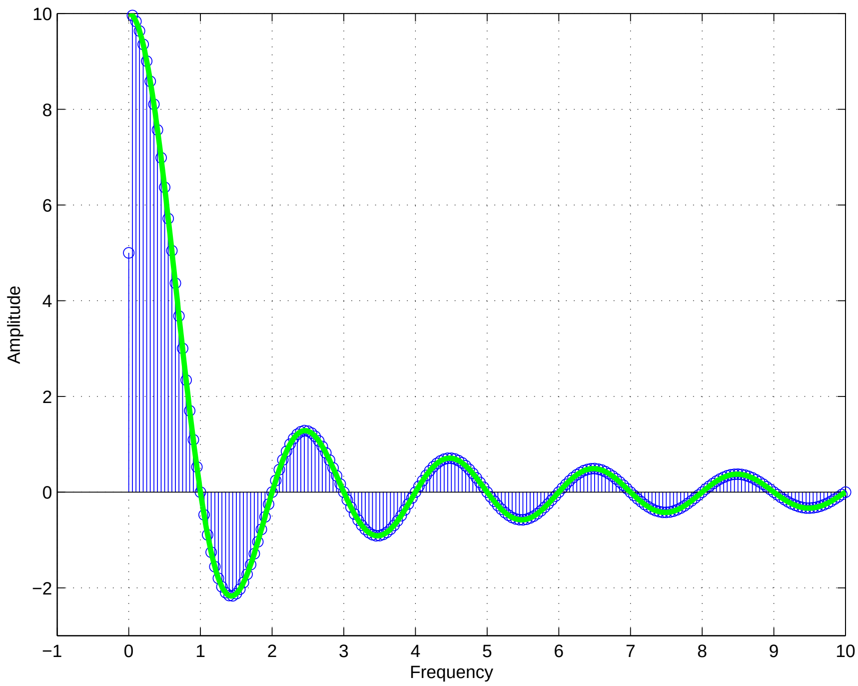
Given a rectangular pulse as shown in Figure 13–4, with amplitude A, width T, and period T_0, we can compute and plot the coefficients in the corresponding Fourier series. If we allow T_0 to increase to infinity, the waveform is a single pulse and the Fourier series approaches a scaled version of the Fourier transform.

To see this graphically, use MATLAB to create the following series of plots. Let $A = 5$, $T = 1$, and $T_0 = 2$. Compute the Fourier series coefficients for $n = 0, 1, 2, \ldots, 10T_0$. Create a stem plot of $(a_n \times T_0)$ on the vertical axis versus the (n/T_0) on the horizontal axis. Increment T_0 by one and repeat the stem plot. Create plots up until $T_0 = 20$ and comment on the behavior of the results. Now compute the Fourier transform of $f(t) = A[u(t + T/2) - u(t - T/2)]$. Evaluate $F(\omega)$ for $\omega = 0$ to 20π rad/s. On the same axes as your final stem plot, plot $2F(\omega)$ versus $\omega/2\pi$. Comment on the results.

The following MATLAB code performs the requested operations.

```
A = 5;
T = 1;
T0 = 2;

% Use a for loop to plot the amplitude spectra
for T0 = 2:20
    n = 1:10*T0;
    n0 = [0 n];
    a0 = A*T/T0;
    an = 2*A*sin(n*pi*T/T0)./n/pi;
    aAll = [a0 an];
    stem(n0/T0,aAll*T0)
    grid on
    if T0==2
        v = axis;
    end
    axis([-1 v(2)+1 v(3) 2*v(4)])
    pause(0.5)
end

% Calculate and plot the Fourier transform
syms t w

ft = A*(heaviside(t+T/2)-heaviside(t-T/2));
Fw = fourier(ft,t,w);
ww = 0:0.01:10;
Fww = subs(Fw,w,2*pi*ww);
hold on
plot(ww,2*real(Fww),'g','LineWidth',2)
xlabel('Frequency')
ylabel('Amplitude')
```

The final output plot is shown below.



As $T_0 \to \infty$, the Fourier series approaches the Fourier transform.

Problem 13–96. (A) Impulse Generator

Theoretically, an impulse has an amplitude spectrum that is constant at all frequencies. In practice, a constant spectrum across an infinite bandwidth cannot be achieved, nor is it really necessary. What is required is an amplitude spectrum that is "essentially" constant over the frequency range of interest. Under this concept, an impulse generator is a signal source that produces a pulse waveform whose amplitude spectrum does not vary more than a prescribed amount over a specified frequency range.

Consider a rectangular pulse:

$$f(t) = A[u(t + T/2) - u(t - T/2)]$$

Find $F(\omega)$ and sketch its amplitude spectrum $|F(\omega)|$. Select the pulse duration T such that the amplitude spectrum does not change by more than 10% over a frequency range from 1 MHz to 10 MHz.

Apply the duality property to determine $F(\omega)$.

$$g(t) = \frac{\sin(\beta t)}{\pi t}$$

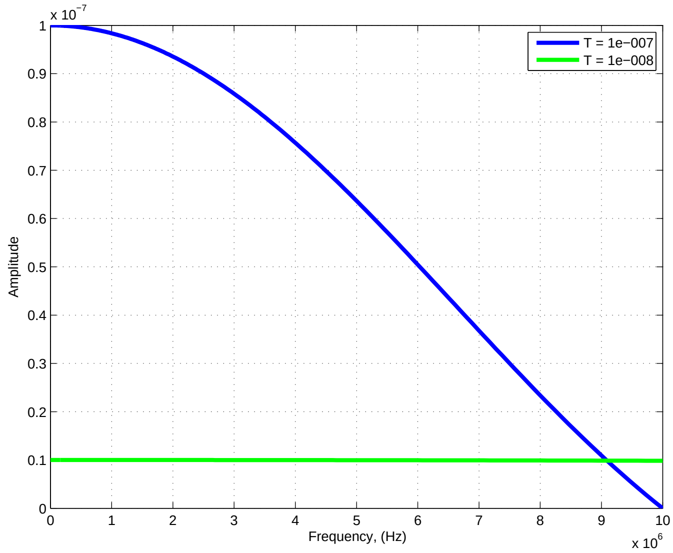
$$G(\omega) = u(\omega + \beta) - u(\omega - \beta)$$

$$G(t) = u(t + \beta) - u(t - \beta)$$

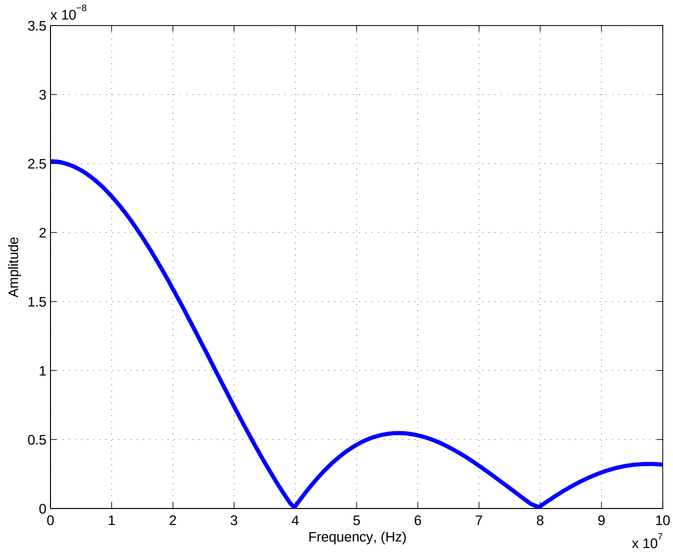
$$\mathscr{F}\{G(t)\} = 2\pi g(-\omega) = \frac{2\sin(-\beta\omega)}{-\omega} = \frac{2\sin(\beta\omega)}{\omega}$$

$$F(\omega) = \frac{2A\sin\left(\dfrac{T\omega}{2}\right)}{\omega}$$

Sketches of $|F(\omega)|$ are shown below for two values of T.



Based on the plots above, it appears that a pulse duration between 10 ns and 100 ns will meet the specification on maximum allowed change in the amplitude spectrum. An iterative search shows that a pulse duration less than 25.16 ns will meet the specifications. The corresponding plot is shown below. Note that the amplitude changes by less than 10% from 0 Hz to 10 MHz.



14 Active Filter Design

14.1 Exercise Solutions

Exercise 14–1. Develop a second-order low-pass transfer function with a corner frequency of 50 rad/s, a dc gain of 2, and a gain of 4 at the corner frequency. Validate your result by using MATLAB to plot the transfer function's absolute gain versus frequency.

The transfer function for a second-order low-pass filter has the following form:

$$T(s) = \frac{K}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$\omega_0 = 50 \text{ rad/s}$$

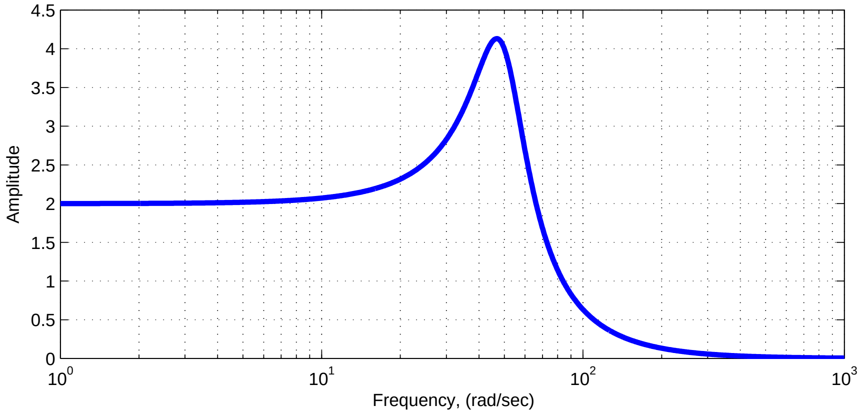
$$K = 2$$

$$|T(j\omega_0)| = 4 = \frac{|K|}{2\zeta} = \frac{2}{2\zeta} = \frac{1}{\zeta}$$

$$\zeta = \frac{1}{4}$$

$$T(s) = \frac{5000}{s^2 + 25s + 2500}$$

The plot of the transfer function is shown below and it meets the specifications.



Exercise 14–2. Design circuits using both the equal element design and unity gain design techniques to realize the transfer function in Exercise 14–1. Use OrCAD to simulate your designs and compare them to the MATLAB results shown in Figure 14–6.

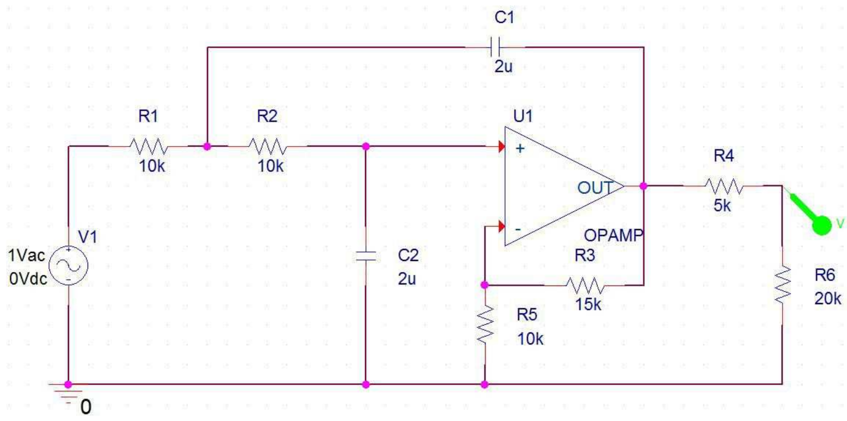
For the equal element approach, pick $R = 10$ kΩ and solve for the other values as follows:

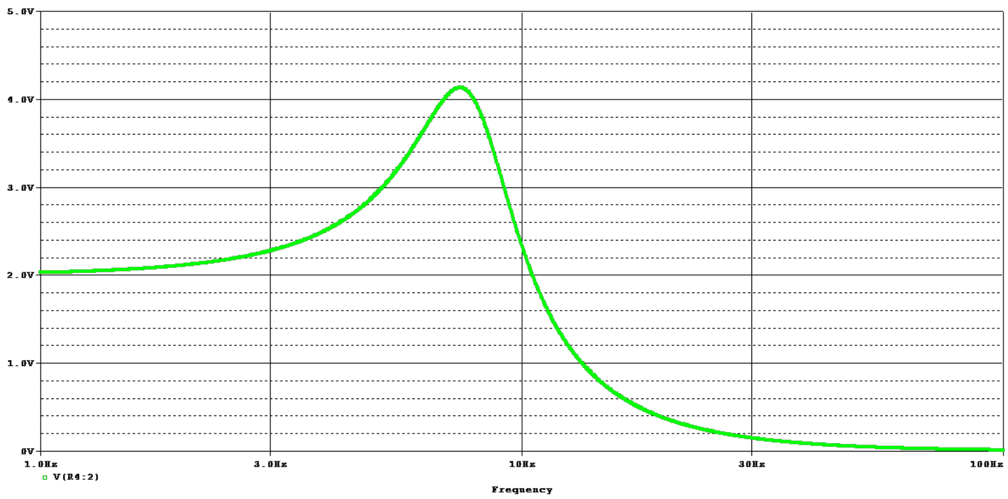
$$\omega_0 = \frac{1}{RC}$$

$$C = \frac{1}{\omega_0 R} = \frac{1}{(50)(10000)} = 2\ \mu\text{F}$$

$$\mu = 3 - 2\zeta = 3 - 2(0.25) = 2.5$$

Design the circuit using the resistor and capacitor specified and then use a voltage divider to reduce the gain from 2.5 to 2 to meet the original specifications in Exercise 14–1. The OrCAD circuit simulation is shown below and the output follows. The output agrees with the MATLAB plot.





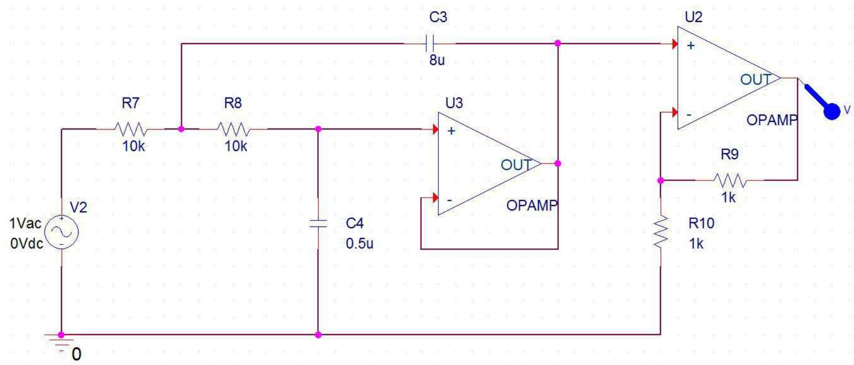
For the unity gain approach, pick $C_1 = 8\ \mu$F and solve for the other values as follows:

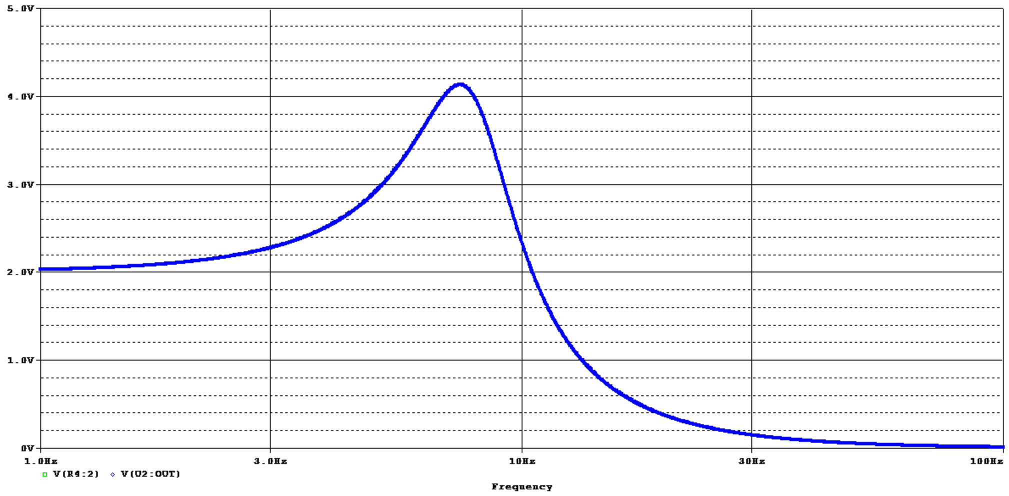
$$C_2 = \zeta^2 C_1 = \frac{8\,\mu\text{F}}{16} = 0.5\ \mu\text{F}$$

$$R = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 10\ \text{k}\Omega$$

$$\mu = 1$$

Design the circuit using the resistor and capacitors specified and then use a noninverting amplifier to increase the gain from 1 to 2 to meet the original specifications in Exercise 14–1. The OrCAD circuit simulation is shown below and the output follows. The output agrees with the MATLAB plot.





Exercise 14–3. Construct a second-order high-pass transfer function with a corner frequency of 20 rad/s, an infinite-frequency gain of 4, and a gain of 2 at the corner frequency.

The transfer function for a second-order high-pass filter has the following form:

$$T(s) = \frac{K(s/\omega_0)^2}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} = \frac{Ks^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$\omega_0 = 20 \text{ rad/s}$$

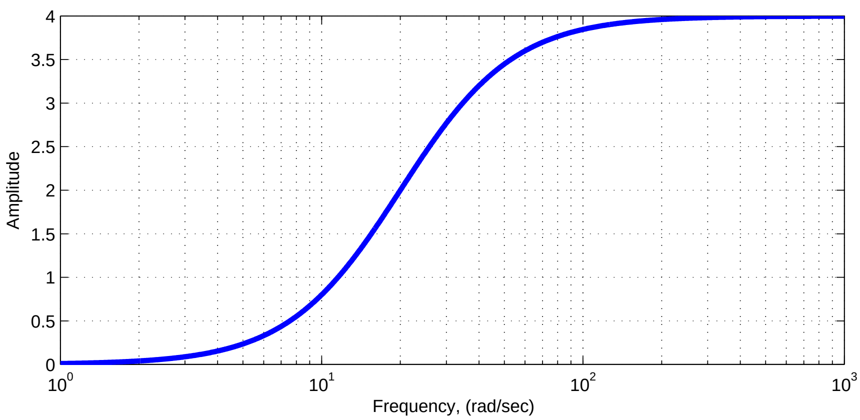
$$K = 4$$

$$|T(j\omega_0)| = 2 = \frac{|K|}{2\zeta} = \frac{4}{2\zeta} = \frac{2}{\zeta}$$

$$\zeta = 1$$

$$T(s) = \frac{4s^2}{s^2 + 40s + 400}$$

The plot of the transfer function is shown below and it meets the specifications.



Exercise 14–4. Rework the design in Example 14–3, starting with $C = 2000$ pF.

The design decisions and calculations are shown below.

$$C = 2000 \text{ pF}$$

$$\omega_0 = 20000\pi = 62832 \text{ rad/s}$$

$$B = 8000\pi = 25133 \text{ rad/s}$$

$$\zeta = \frac{B}{2\omega_0} = \frac{8000\pi}{40000\pi} = 0.2$$

$$R_1 = \zeta^2 R_2$$

$$\sqrt{R_1 R_2} = \frac{1}{\omega_0 C}$$

$$\sqrt{\zeta^2 R_2^2} = \frac{1}{\omega_0 C}$$

$$R_2 = \frac{1}{\zeta\omega_0 C} = 39.79 \text{ k}\Omega$$

$$R_1 = (0.2)^2(39790) = 1.592 \text{ k}\Omega$$

Use the active, second-order bandpass circuit with $C_1 = C_2 = C = 2000$ pF, $R_1 = 1.592$ kΩ, and $R_2 =$ 39.79 kΩ. The transfer function has the following form:

$$T(s) = \frac{-314000s}{s^2 + 25100s + 3.9456 \times 10^9}$$

which agrees with the results in Example 14–3.

Exercise 14–5. Construct a second-order bandpass transfer function with a corner frequency of 50 rad/s, a bandwidth of 10 rad/s and a center frequency gain of 4.

The transfer function for a second-order bandpass filter has the following form:

$$T(s) = \frac{K(s/\omega_0)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} = \frac{K\omega_0 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$\omega_0 = 50 \text{ rad/s}$$

$$B = 10 \text{ rad/s} = 2\zeta\omega_0$$

$$\zeta = \frac{B}{2\omega_0} = \frac{10}{100} = 0.1$$

$$|T(j\omega_0)| = 4 = \frac{|K|}{2\zeta}$$

$$K = (4)(0.2) = 0.8$$

$$T(s) = \frac{40s}{s^2 + 10s + 2500}$$

Exercise 14–6. Rework the circuit design in Example 14–4 starting with $C_1 = C_2 = C = 0.2\ \mu$F.

The design decisions and calculations are shown below.

$$C_1 = C_2 = C = 0.2\ \mu\text{F}$$
$$\omega_0 = 120\pi = 377\text{ rad/s}$$
$$B = 24\pi = 75.4\text{ rad/s}$$
$$R_1 = \zeta^2 R_2$$
$$\sqrt{R_1 R_2} = \frac{1}{\omega_0 C}$$
$$\sqrt{\zeta^2 R_2^2} = \frac{1}{\omega_0 C}$$
$$R_2 = \frac{1}{\zeta\omega_0 C} = 132.6\text{ k}\Omega$$
$$R_1 = 0.01 R_2 = 1.326\text{ k}\Omega$$
$$R_\text{A} = 2R_1 = 2.653\text{ k}\Omega$$
$$R_\text{B} = R_2 = 132.6\text{ k}\Omega$$

Exercise 14–7. Construct a second-order bandstop transfer function with a notch frequency of 50 rad/s, a notch bandwidth of 10 rad/s, and passband gains of 5.

The transfer function for a second-order bandstop filter has the following form:

$$T(s) = \frac{K[(s/\omega_0)^2 + 1]}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} = \frac{K[s^2 + \omega_0^2]}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$
$$\omega_0 = 50\text{ rad/s}$$
$$K = 5$$
$$B = 10\text{ rad/s} = 2\zeta\omega_0$$
$$\zeta = \frac{B}{2\omega_0} = \frac{10}{100} = 0.1$$
$$T(s) = \frac{5[s^2 + 2500]}{s^2 + 10s + 2500}$$

Exercise 14–8. Design a notch filter using the realization in Figure 14–19 to achieve a notch at 200 krad/s, a B of 20 krad/s, and a passband gain of 10.

The design decisions and calculations are shown below.

$$\omega_0 = 200 \text{ krad/s}$$

$$B = 20 \text{ krad/s}$$

$$K = 10$$

$$B = \frac{3}{(K_1+1)RC}$$

$$\frac{1}{RC} = \frac{B(K_1+1)}{3}$$

$$\omega_0 = \frac{1}{RC\sqrt{K_1+1}} = \frac{B(K_1+1)}{3\sqrt{K_1+1}} = \frac{B}{3}\sqrt{K_1+1}$$

$$\frac{3\omega_0}{B} = \sqrt{K_1+1}$$

$$K_1 = \frac{9\omega_0^2}{B^2} - 1 = \frac{(9)(200)^2}{20^2} - 1 = 899$$

$$K_2 = 10$$

$$RC = \frac{3}{(K_1+1)B} = \frac{3}{(900)(20000)} = 166.67 \times 10^{-9}$$

$$C = 1000 \text{ pF}$$

$$R = 167\ \Omega$$

$$K_1R = 149.8 \text{ k}\Omega$$

$$\alpha = \frac{K_1}{3} = 299.7$$

$$\frac{R_X}{\alpha} = 1 \text{ k}\Omega$$

$$R_X = 299.7 \text{ k}\Omega$$

$$\frac{K_2R_X}{\alpha} = 10 \text{ k}\Omega$$

Exercise 14–9. Construct a first-order cascade transfer function that meets the following requirements: $T_{MAX} = 0$ dB, $T_{MIN} = -30$ dB, $\omega_C = 200$ rad/s, and $\omega_{MIN} = 1$ krad/s.

Find the filter order that will meet the specifications. The gain decreases by 30 dB in the transition band with $\omega_{MIN}/\omega_C = 5$. In Figure 14–23, $n = 4$ appears to meet the specification. Calculate α:

$$\alpha = \frac{\omega_C}{\sqrt{2^{1/n}-1}} = \frac{200}{\sqrt{2^{1/4}-1}} = 459.79 \text{ rad/s}$$

The gain $T_{MAX} = 0$ dB is an absolute gain of 1, so $K = 1^{1/4} = 1$. The transfer function is:

$$T(s) = \left(\frac{K}{s/\alpha+1}\right)^n = \left(\frac{K\alpha}{s+\alpha}\right)^n = \left(\frac{460}{s+460}\right)^4$$

Exercise 14–10. The circuit design in Example 14–7 used the *equal element* method. Rework the problem using the *unity gain* technique. Use OrCAD to validate your design. Comment on the two approaches.

Based on the results in Example 14–7, we have the following specifications:

$$n = 4$$

$$K = 10$$

$$\omega_C = 1000 \text{ rad/s}$$

The transfer function has the following form:

$$T(s) = \left[\frac{1}{\left(\frac{s}{1000}\right)^2 + 0.7654\left(\frac{s}{1000}\right) + 1}\right]\left[\frac{1}{\left(\frac{s}{1000}\right)^2 + 1.848\left(\frac{s}{1000}\right) + 1}\right][10]$$

For the unity gain method, select all resistors to be 100 kΩ and apply the following relationships:

$$R\sqrt{C_1C_2} = \frac{1}{\omega_0}$$

$$\frac{C_2}{C_1} = \zeta^2$$

Design the first stage:

$$\zeta = \frac{0.7654}{2} = 0.3827$$

$$C_2 = \zeta^2 C_1$$

$$\zeta C_1 = \frac{1}{R\omega_0}$$

$$C_1 = \frac{1}{\zeta R\omega_0} = 0.0261\ \mu\text{F}$$

$$C_2 = 3830 \text{ pF}$$

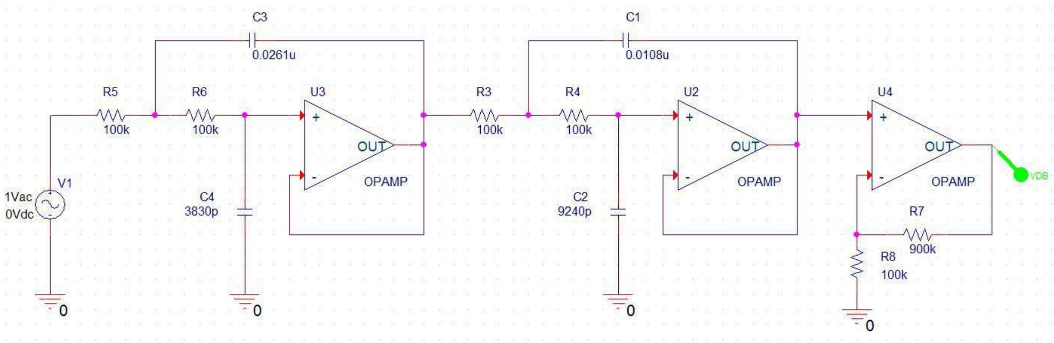
Design the second stage:

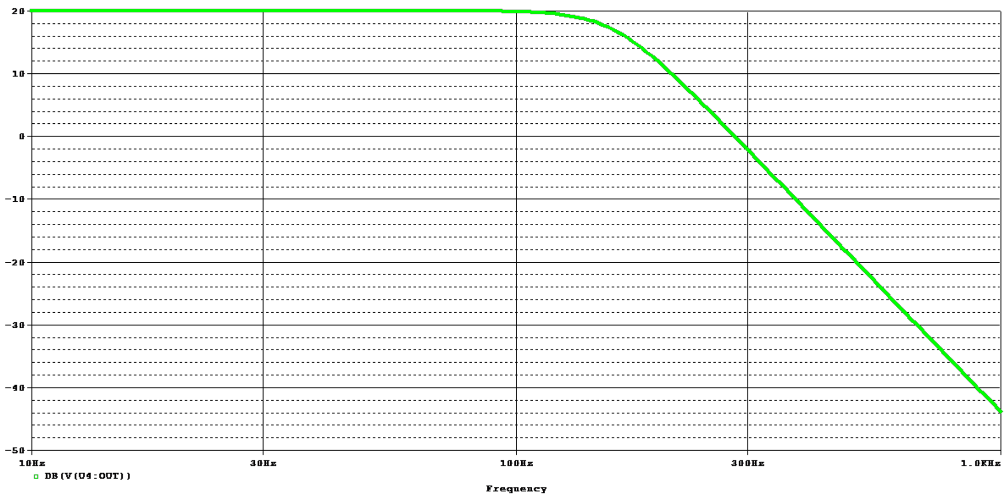
$$\zeta = \frac{1.848}{2} = 0.924$$

$$C_1 = \frac{1}{\zeta R\omega_0} = 0.0108\ \mu\text{F}$$

$$C_2 = 9240 \text{ pF}$$

The third stage is a noninverting amplifier with a gain of 10. The OrCAD design is shown below, followed by the simulation results.





The design meets all of the specifications. The unity gain technique requires fewer resistors than the equal element approach, but requires precise capacitors that may be difficult to find.

Exercise 14–11. Construct a Butterworth low-pass transfer function that meets the following requirements: $T_{\text{MAX}} = 0$ dB, $T_{\text{MIN}} = -40$ dB, $\omega_{\text{C}} = 250$ rad/s, and $\omega_{\text{MIN}} = 1.5$ krad/s.

Determine the filter order:

$$n \geq \frac{1}{2}\frac{\ln[(T_{\text{MAX}}/T_{\text{MIN}})^2 - 1]}{\ln[\omega_{\text{MIN}}/\omega_{\text{C}}]} = \frac{1}{2}\frac{\ln[10000-1]}{\ln[6]} = 2.57$$

$$n = 3$$

The following steps complete the transfer function using the table of Butterworth polynomials to find $q_n(s)$:

$$K = 0\text{ dB} = 1$$

$$q_3(s) = (s+1)(s^2+s+1)$$

$$T(s) = \frac{K}{q_3(s/250)} = \frac{1}{\left[\dfrac{s}{250}+1\right]\left[\left(\dfrac{s}{250}\right)^2 + \dfrac{s}{250}+1\right]}$$

$$= \frac{(250)^3}{[s+250][s^2+250s+(250)^2]}$$

Exercise 14–12. Rework the design in Example 14–8 using the unity gain method in Section 14–2 to design the required third-order low-pass circuit.

Based on the results in Example 14–8, we have the following specifications:

$$n = 3$$

$$K = 10$$

$$\omega_C = 10 \text{ rad/s}$$

The transfer function has the following form:

$$T(s) = \frac{K}{q_3(s/10)}$$

$$= \left[\frac{1}{\left(\frac{s}{2.98}\right)+1}\right][10]\left[\frac{1}{\left(\frac{s}{9.159}\right)^2 + 0.3254\left(\frac{s}{9.159}\right)+1}\right]$$

For the unity gain method, select all resistors to be 100 kΩ and apply the following relationships:

$$R\sqrt{C_1C_2} = \frac{1}{\omega_0}$$

$$\frac{C_2}{C_1} = \zeta^2$$

Design the first stage as an RC circuit followed by a noninverting amplifier with a gain of 10:

$$\omega_0 = \frac{1}{RC}$$

$$C = \frac{1}{(100000)(2.98)} = 3.36\ \mu\text{F}$$

Design the second stage:

$$\zeta = \frac{0.3254}{2} = 0.1627$$

$$C_2 = \zeta^2 C_1$$

$$\zeta C_1 = \frac{1}{R\omega_0}$$

$$C_1 = \frac{1}{\zeta R\omega_0} = 6.71\ \mu\text{F}$$

$$C_2 = 0.178\ \mu\text{F}$$

Figure 14–39 in the textbook shows the resulting circuits.

Exercise 14–13. Construct a Chebychev low-pass transfer function that meets the following requirements: $T_{\text{MAX}} = 0$ dB, $T_{\text{MIN}} = -30$ dB, $\omega_C = 250$ rad/s, and $\omega_{\text{MIN}} = 1.5$ krad/s.

Determine the filter order:

$$n \geq \frac{\cosh^{-1}\left[\sqrt{(T_{\text{MAX}}/T_{\text{MIN}})^2 - 1}\right]}{\cosh^{-1}[\omega_{\text{MIN}}/\omega_C]} = \frac{\cosh^{-1}\left[\sqrt{(31.623)^2 - 1}\right]}{\cosh^{-1}[6]} = 1.6733$$

$$n = 2$$

The following steps complete the transfer function using the table of Chebychev polynomials to find $q_n(s)$:

$$K = 0 \text{ dB} = 1$$

$$q_2(s) = [(s/0.8409)^2 + 0.7654(s/0.8409) + 1]$$

$$T(s) = \frac{K/\sqrt{2}}{q_2(s/250)} = \frac{1/\sqrt{2}}{\left(\frac{s}{210}\right)^2 + 0.7654\left(\frac{s}{210}\right) + 1}$$

$$= \frac{(210)^2/\sqrt{2}}{s^2 + 161s + (210)^2}$$

Exercise 14–14. Design a high-pass, first-order cascade filter with a cutoff frequency of 100 krad/s, a T_{MIN} of -65 dB, a ω_{MIN} of 10 krad/s, and a passband gain of 100.

First, find the filter order. We have the following relationships for a high-pass, first-order cascade design:

$$\alpha = \omega_{\text{C}}\sqrt{2^{1/n} - 1}$$

$$|T(j\omega_{\text{MIN}})| \geq \frac{|T_{\text{MAX}}|}{\left[\sqrt{1 + \left(\frac{\alpha}{\omega_{\text{MIN}}}\right)^2}\right]^n}$$

$$\frac{T_{\text{MAX}}}{T_{\text{MIN}}} \leq \left[1 + \left(\frac{\alpha}{\omega_{\text{MIN}}}\right)^2\right]^{n/2}$$

$$\frac{T_{\text{MAX}}}{T_{\text{MIN}}} \leq \left[1 + \left(\frac{\omega_{\text{C}}}{\omega_{\text{MIN}}}\right)^2 (2^{1/n} - 1)\right]^{n/2}$$

Find the smallest value of n that satisfies the last inequality. The following MATLAB code is helpful for determining filter orders for low-pass and high-pass filters with first-order, Butterworth, or Chebychev designs.

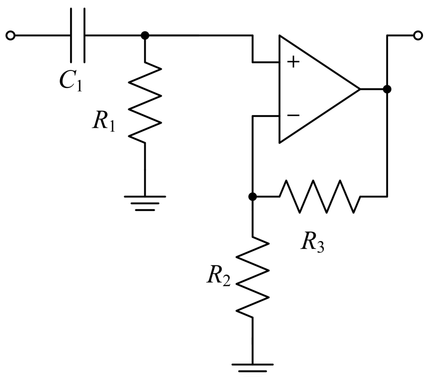
```
% Select the correct filter type via commenting
%FilterType = 'LowPass '
FilterType = 'HighPass'
% Set the filter specifications
TmaxdB = 40;
TmindB = -65;
wC = 100e3;
wmin = 10e3;
Tmax = 10^(TmaxdB/20);
Tmin = 10^(TmindB/20);
if FilterType == 'LowPass '
    % Compute ratios of filter specification values
    Tratio = Tmax/Tmin;
    wratio = wmin/wC;
    % Set the initial filter order to one
    n = 1;
    % While the filter order is too small to satisfy the specifications,
    % increment the filter order
    while (Tratio > (1 + wratio^2*(2^(1/n) - 1))^(n/2))&&(n<200)
        n = n + 1;
    end
```

```
    % Display the filter order
    FirstOrderCascadeOrder = n
    FirstOrderAlpha = wC/sqrt(2^(1/n) - 1)
    FirstOrderK = Tmax^(1/n)
    % Compute the Butterworth order
    ButterworthExact = log(Tratio^2-1)/log(wratio)/2
    ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
    % Compute the Chebychev order
    ChebychevExact = acosh(sqrt(Tratio^2-1))/acosh(wratio)
    ChebychevOrder = ceil(acosh(sqrt(Tratio^2-1))/acosh(wratio))
end
if FilterType == 'HighPass'
    % Compute ratios of filter specification values
    Tratio = Tmax/Tmin;
    wratio = wC/wmin;
    % Set the initial filter order to one
    n = 1;
    % While the filter order is too small to satisfy the specifications,
    % increment the filter order
    while (Tratio > (1 + wratio^2*(2^(1/n) - 1))^(n/2))&&(n<200)
        n = n + 1;
    end
    % Display the filter order
    FirstOrderCascadeOrder = n
    FirstOrderAlpha = wC*sqrt(2^(1/n) - 1)
    FirstOrderK = Tmax^(1/n)
    % Compute the Butterworth order
    ButterworthExact = log(Tratio^2-1)/log(wratio)/2
    ButterworthOrder = ceil(log(Tratio^2-1)/log(wratio)/2)
    % Compute the Chebychev order
    ChebychevExact = acosh(sqrt(Tratio^2-1))/acosh(wratio)
    ChebychevOrder = ceil(acosh(sqrt(Tratio^2-1))/acosh(wratio))
end
```

In this case, the minimum value is $n = 13$. We have

$$\alpha = \omega_C\sqrt{2^{1/13} - 1} = 23.4 \text{ krad/s}$$

The cutoff frequency for each stage is 23.4 krad/s and the gain for each stage is $100^{1/13} = 1.4251$. Each stage contains an RC high-pass filter with $R = 10$ kΩ and $C = 4273$ pF, connected to a noninverting amplifier with a gain of 1.4251. The design for a single stage is shown below, where $C_1 = 4273$ pF, $R_1 = R_2 = 10$ kΩ, and $R_3 = 4.251$ kΩ.



Exercise 14–15. Design a Butterworth high-pass filter using the equal element configuration that meets the following conditions: passband gain 100, $\omega_C = 5$ krad/s, $\omega_{MIN} = 500$ rad/s, $T_{MIN} = -40$ dB ± 1 dB. You must use 10-kΩ resistors as much as possible.

Determine the filter order:

$$n \geq \frac{1}{2}\frac{\ln[(T_{\text{MAX}}/T_{\text{MIN}})^2 - 1]}{\ln[\omega_{\text{C}}/\omega_{\text{MIN}}]} = \frac{1}{2}\frac{\ln[(100/0.01)^2 - 1]}{\ln[10]} = 4.00$$

$$n = 4$$

The following steps complete the transfer function using the table of Butterworth polynomials to find $q_n(s)$:

$$K = 40 \text{ dB} = 100$$

$$q_4(s) = (s^2 + 0.7654s + 1)(s^2 + 1.848s + 1)$$

$$T(s) = \frac{K}{q_4(5000/s)} = \frac{100}{\left[\left(\frac{5000}{s}\right)^2 + 0.7654\left(\frac{5000}{s}\right) + 1\right]\left[\left(\frac{5000}{s}\right)^2 + 1.848\left(\frac{5000}{s}\right) + 1\right]}$$

$$= \left[\frac{\mu_1}{\left(\frac{5000}{s}\right)^2 + 0.7654\left(\frac{5000}{s}\right) + 1}\right]\left[\frac{\mu_2}{\left(\frac{5000}{s}\right)^2 + 1.848\left(\frac{5000}{s}\right) + 1}\right]\left[\frac{100}{\mu_1\mu_2}\right]$$

The design requires three stages. The first two stages are second-order, high-pass filters with gain and the third stage is a noninverting amplifier to contribute the remaining gain. With the equal-element approach, choose $R_1 = R_2 = R = 10$ kΩ and $C_1 = C_2 = C$. Both filtering stages have the same resistors and capacitors to achieve a common cutoff frequency, but their gains will differ. We have the following results:

$$RC = \frac{1}{\omega_0}$$

$$C = \frac{1}{\omega_0 R} = 0.02\ \mu\text{F}$$

$$\mu = 3 - 2\zeta$$

$$\mu_1 = 3 - 0.7654 = 2.2346$$

$$\mu_2 = 3 - 1.848 = 1.152$$

$$\frac{100}{\mu_1\mu_2} = 38.85$$

Figure 14–52(a) in the textbook shows the design.

Exercise 14–16. Repeat Exercise 14–15, but design a Butterworth high-pass filter using the unity gain configuration. You must use 0.01 μF capacitors.

The transfer function remains the same, but the gains will be distributed differently, as follows:

$$T(s) = \left[\frac{1}{\left(\frac{5000}{s}\right)^2 + 0.7654\left(\frac{5000}{s}\right) + 1}\right]\left[\frac{1}{\left(\frac{5000}{s}\right)^2 + 1.848\left(\frac{5000}{s}\right) + 1}\right][100]$$

The design requires three stages. The first two stages are second-order, high-pass filters with unity gain and the third stage is a noninverting amplifier with a gain of 100. With the unity-gain approach, choose

$C_1 = C_2 = C = 0.01\mu\text{F}$. Design the first stage:

$$\begin{aligned}
\zeta &= \frac{0.7654}{2} = 0.3827 \\
R_1 &= \zeta^2 R_2 \\
C\sqrt{R_1 R_2} &= \frac{1}{\omega_0} \\
R_2 &= \frac{1}{\zeta\omega_0 C} = 52.26\ \text{k}\Omega \\
R_1 &= 7.654\ \text{k}\Omega
\end{aligned}$$

Design the second stage:

$$\begin{aligned}
\zeta &= \frac{1.848}{2} = 0.924 \\
R_2 &= \frac{1}{\zeta\omega_0 C} = 21.65\ \text{k}\Omega \\
R_1 &= 18.48\ \text{k}\Omega
\end{aligned}$$

Figure 14–52(b) in the textbook shows the design.

Exercise 14–17. Construct Butterworth and Chebychev high-pass transfer functions that meet the following requirements: $T_{\text{MAX}} = 10$ dB, $\omega_{\text{C}} = 50$ rad/s, $T_{\text{MIN}} = -40$ dB, and $\omega_{\text{MIN}} = 10$ rad/s.

Design the Butterworth transfer function first. Determine the filter order and then construct the transfer fucntion.

$$\begin{aligned}
n &\geq \frac{1}{2}\frac{\ln[(T_{\text{MAX}}/T_{\text{MIN}})^2 - 1]}{\ln[\omega_{\text{C}}/\omega_{\text{MIN}}]} = \frac{1}{2}\frac{\ln[(3.1623/0.01)^2 - 1]}{\ln[5]} = 3.577 \\
n &= 4 \\
K &= 10\ \text{dB} = \sqrt{10} \\
q_4(s) &= (s^2 + 0.7654s + 1)(s^2 + 1.848s + 1) \\
T(s) &= \frac{K}{q_4(50/s)} = \frac{\sqrt{10}}{\left[\left(\frac{50}{s}\right)^2 + 0.7654\left(\frac{50}{s}\right) + 1\right]\left[\left(\frac{50}{s}\right)^2 + 1.848\left(\frac{50}{s}\right) + 1\right]} \\
&= \frac{\sqrt{10}s^4}{[s^2 + 38.3s + 50^2]\,[s^2 + 92.4s + 50^2]}
\end{aligned}$$

Design the Chebychev transfer function. Determine the filter order and then construct the transfer function.

$$n \geq \frac{\cosh^{-1}\left[\sqrt{(T_{\rm MAX}/T_{\rm MIN})^2 - 1}\right]}{\cosh^{-1}[\omega_{\rm C}/\omega_{\rm MIN}]} = \frac{\cosh^{-1}\left[\sqrt{(3.1623/0.01)^2 - 1}\right]}{\cosh^{-1}[5]} = 2.8134$$

$$n = 3$$

$$K = 10 \text{ dB} = \sqrt{10}$$

$$q_3(s) = [(s/0.2980) + 1][(s/0.9159)^2 + 0.3254(s/0.9159) + 1]$$

$$T(s) = \frac{K}{q_3(50/s)} = \frac{\sqrt{10}}{\left[\left(\frac{50}{0.2980s}\right) + 1\right]\left[\left(\frac{50}{0.9159s}\right)^2 + 0.3254\left(\frac{50}{0.9159s}\right) + 1\right]}$$

$$= \frac{\sqrt{10}s^3}{(s + 168)(s^2 + 17.8s + 54.6^2)}$$

Exercise 14–18. Construct Butterworth low-pass and high-pass transfer functions whose cascade connection produces a bandpass function with cutoff frequencies at 20 rad/s and 500 rad/s, a passband gain of 0 dB, and a stopband gain less than −20 dB at 5 rad/s and 2000 rad/s.

The low-pass filter has the following specifications and results:

$$\omega_{\rm C} = 500 \text{ rad/s}$$

$$\omega_{\rm MIN} = 2000 \text{ rad/s}$$

$$T_{\rm MAX} = 1$$

$$T_{\rm MIN} = 0.1$$

$$n \geq \frac{1}{2}\frac{\ln[(T_{\rm MAX}/T_{\rm MIN})^2 - 1]}{\ln[\omega_{\rm MIN}/\omega_{\rm C}]} = \frac{1}{2}\frac{\ln[(1/0.1)^2 - 1]}{\ln[4]} = 1.657$$

$$n = 2$$

$$q_2(s) = s^2 + 1.414s + 1$$

$$T_{\rm LPF}(s) = \frac{1}{\left(\frac{s}{500}\right)^2 + 1.414\left(\frac{s}{500}\right) + 1} = \frac{500^2}{s^2 + 707s + 500^2}$$

The high-pass filter has the following specifications and results:

$$\omega_{\mathrm{C}} = 20 \text{ rad/s}$$

$$\omega_{\mathrm{MIN}} = 5 \text{ rad/s}$$

$$T_{\mathrm{MAX}} = 1$$

$$T_{\mathrm{MIN}} = 0.1$$

$$n \geq \frac{1}{2}\frac{\ln[(T_{\mathrm{MAX}}/T_{\mathrm{MIN}})^2 - 1]}{\ln[\omega_{\mathrm{C}}/\omega_{\mathrm{MIN}}]} = \frac{1}{2}\frac{\ln[(1/0.1)^2 - 1]}{\ln[4]} = 1.657$$

$$n = 2$$

$$q_2(s) = s^2 + 1.414s + 1$$

$$T_{\mathrm{HPF}}(s) = \frac{1}{\left(\dfrac{20}{s}\right)^2 + 1.414\left(\dfrac{20}{s}\right) + 1} = \frac{s^2}{s^2 + 283s + 400}$$

The complete transfer function is

$$T(s) = T_{\mathrm{LPF}}(s)T_{\mathrm{HPF}}(s) = \left[\frac{500^2}{s^2 + 707s + 500^2}\right]\left[\frac{s^2}{s^2 + 283s + 400}\right]$$

Exercise 14–19. Develop Butterworth low-pass and high-pass transfer functions whose parallel connection produces a bandstop filter with cutoff frequencies at 2 rad/s and 800 rad/s, passband gains of 20 dB, and stopband gains less than −30 dB at 20 rad/s and 80 rad/s.

The low-pass filter has the following specifications and results:

$$\omega_{\mathrm{C}} = 2 \text{ rad/s}$$

$$\omega_{\mathrm{MIN}} = 20 \text{ rad/s}$$

$$T_{\mathrm{MAX}} = 10$$

$$T_{\mathrm{MIN}} = 0.03162$$

$$n \geq \frac{1}{2}\frac{\ln[(T_{\mathrm{MAX}}/T_{\mathrm{MIN}})^2 - 1]}{\ln[\omega_{\mathrm{MIN}}/\omega_{\mathrm{C}}]} = \frac{1}{2}\frac{\ln[(10/0.03162)^2 - 1]}{\ln[10]} = 2.5$$

$$n = 3$$

$$q_3(s) = (s+1)(s^2 + s + 1)$$

$$T_{\mathrm{LPF}}(s) = \frac{10}{\left[\dfrac{s}{2} + 1\right]\left[\left(\dfrac{s}{2}\right)^2 + \left(\dfrac{s}{2}\right) + 1\right]} = \frac{80}{[s+2][s^2 + 2s + 4]}$$

The high-pass filter has the following specifications and results:

$$\omega_{\mathrm{C}} = 800 \text{ rad/s}$$

$$\omega_{\mathrm{MIN}} = 80 \text{ rad/s}$$

$$T_{\mathrm{MAX}} = 10$$

$$T_{\mathrm{MIN}} = 0.03162$$

$$n \geq \frac{1}{2}\frac{\ln[(T_{\mathrm{MAX}}/T_{\mathrm{MIN}})^2 - 1]}{\ln[\omega_{\mathrm{C}}/\omega_{\mathrm{MIN}}]} = \frac{1}{2}\frac{\ln[(10/0.03162)^2 - 1]}{\ln[10]} = 2.5$$

$$n = 3$$

$$q_3(s) = (s+1)(s^2+s+1)$$

$$T_{\mathrm{HPF}}(s) = \frac{10}{\left[\dfrac{800}{s}+1\right]\left[\left(\dfrac{800}{s}\right)^2+\left(\dfrac{800}{s}\right)+1\right]} = \frac{10s^3}{[s+800][s^2+800s+800^2]}$$

The complete transfer function is

$$T(s) = T_{\mathrm{LPF}}(s) + T_{\mathrm{HPF}}(s) = \left[\frac{80}{[s+2][s^2+2s+4]}\right] + \left[\frac{10s^3}{[s+800][s^2+800s+800^2]}\right]$$

14.2 Problem Solutions

Problem 14–1. Interchanging the positions of the resistors and capacitors converts the low-pass filter in Figure 14–3(a) into the high-pass filter in Figure 14–9(a). This CR-RC interchange involves replacing R_k by $1/C_k s$ and $C_k s$ by $1/R_k$. Show that this interchange converts the low-pass transfer function in Eq. (14–6) into the high-pass function in Eq. (14–11).

Start with Equation (14–6) and make the required substitutions. Simplify the results and verify that it matches Equation (14–11).

$$T_{\mathrm{LPF}}(s) = \frac{\mu}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2 + R_2C_2 - \mu R_1C_1)s + 1}$$

$$\begin{aligned} T_{\mathrm{HPF}}(s) &= \frac{\mu}{\frac{1}{C_1s}\frac{1}{C_2s}\frac{1}{R_1}\frac{1}{R_2} + \frac{1}{C_1s}\frac{1}{R_1} + \frac{1}{C_1s}\frac{1}{R_2} + \frac{1}{C_2s}\frac{1}{R_2} - \mu\frac{1}{C_1s}\frac{1}{R_1} + 1} \\ &= \frac{\mu R_1R_2C_1C_2s^2}{1 + (R_2C_2 + R_1C_2 + R_1C_1 - \mu R_2C_2)s + R_1R_2C_1C_2s^2} \end{aligned}$$

The result matches Equation (14–11). We can also use MATLAB to efficiently perform the substitution as follows.

```
syms u R1 R2 C1 C2 r1 r2 c1 c2 s
TLP = u/(R1*R2*C1*C2*s^2+(R1*C1+R1*C2+R2*C2-u*R1*C1)*s+1);
THP = simplify(subs(TLP,{R1,R2,C1,C2},{1/c1/s,1/c2/s,1/r1/s,1/r2/s}))
```

The results confirm the solution.

```
THP = (c1*c2*r1*r2*s^2*u)/(c1*r1*s + c2*r1*s + c2*r2*s - c2*r2*s*u + c1*c2*r1*r2*s^2 + 1)
```

Problem 14–2. Show that the circuit in Figure 14–14 has the bandpass transfer function in Eq. (14–16).

Use node-voltage analysis. Let the node between R_1 and C_2 be $V_{\mathrm{A}}(s)$.

$$\frac{V_{\mathrm{A}}(s) - V_1(s)}{R_1} + \frac{V_{\mathrm{A}}(s) - 0}{1/C_2s} + \frac{V_{\mathrm{A}}(s) - V_2(s)}{1/C_1s} = 0$$

$$\frac{-V_{\mathrm{A}}(s)}{1/C_2s} + \frac{-V_2(s)}{R_2} = 0$$

Solve the second equation for $V_{\mathrm{A}}(s)$ and substitute into the first equation.

$$V_{\mathrm{A}}(s) = \frac{-V_2(s)}{R_2C_2s}$$

$$0 = V_{\mathrm{A}}(s) - V_1(s) + R_1C_2sV_{\mathrm{A}}(s) + R_1C_1sV_{\mathrm{A}}(s) - R_1C_1sV_2(s)$$

$$V_{\mathrm{A}}(s)[1 + R_1C_2s + R_1C_1s] = V_1(s) + R_1C_1sV_2(s)$$

$$\frac{-V_2(s)}{R_2C_2s}[1 + R_1C_2s + R_1C_1s] = V_1(s) + R_1C_1sV_2(s)$$

$$-V_2(s)[1 + R_1C_2s + R_1C_1s] = R_2C_2sV_1(s) + R_1R_2C_1C_2s^2V_2(s)$$

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{-R_2C_2s}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2)s + 1}$$

Problem 14–3. Show that the circuit in Figure 14–17 has the bandstop transfer function in Eq. (14–20).

Use node-voltage analysis. Let the node between R_1 and C_2 be $V_A(s)$. Label the positive and negative terminals of the OP AMP as $V_P(s)$ and $V_N(s)$. The relevant equations are:

$$V_P(s) = V_N(s) = \frac{R_B}{R_A + R_B} V_1(s)$$

$$0 = \frac{V_A(s) - V_1(s)}{R_1} + \frac{V_A(s) - V_2(s)}{1/C_1 s} + \frac{V_A(s) - V_N(s)}{1/C_2 s}$$

$$0 = \frac{V_N(s) - V_A(s)}{1/C_2 s} + \frac{V_N(s) - V_2(s)}{R_2}$$

The following MATLAB code solves the equations for the transfer function.

```
syms s R1 R2 C1 C2 RA RB V1 V2 VN VA K
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
Eqn1 = (VA-V1)/R1 + (VA-V2)/ZC1 + (VA-VN)/ZC2;
Eqn2 = (VN-VA)/ZC2 + (VN-V2)/R2;
Eqn3 = (VN-V1)/RA + VN/RB;
Soln = solve(Eqn1,Eqn2,Eqn3,VA,VN,V2);
V2 = Soln.V2;
Ts = factor(V2/V1)
pretty(Ts)
```

The results are:

```
Ts = (RB + C1*R1*RB*s + C2*R1*RB*s - C2*R2*RA*s + C1*C2*R1*R2*RB*s^2)/...
     ((RA + RB)*(C1*R1*s + C2*R1*s + C1*C2*R1*R2*s^2 + 1))
                                                              2
  RB + C1 R1 RB s + C2 R1 RB s - C2 R2 RA s + C1 C2 R1 R2 RB s
  -------------------------------------------------------------
                                                    2
       (RA + RB) (C1 R1 s + C2 R1 s + C1 C2 R1 R2 s  + 1)
```

The transfer function is

$$T(s) = \left[\frac{R_B}{R_A + R_B}\right]\left[\frac{R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2 - R_2C_2R_A/R_B)s + 1}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2)s + 1}\right]$$

Problem 14–4. Show that the active filter in Figure P14–4 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{R_1R_2C_1C_2s^2 + R_2C_2s + 1}$$

Using $C_1 = C_2 = C$, develop a method of selecting values for C, R_1, and R_2. Then select values so that the filter has a cutoff frequency of 100 krad/s and a ζ of 0.6. Use MATLAB to plot the filter's Bode diagram. Determine the type of filter it is and its roll-off.

Use node-voltage analysis. Let $V_A(s)$ be the voltage at the positive input terminal for the left OP AMP and note that the input voltages for the right OP AMP are $V_2(s)$. We have the following equations and results:

$$0 = \frac{V_{\mathrm{A}}(s) - V_1(s)}{R_1} + \frac{V_{\mathrm{A}}(s) - V_2(s)}{1/C_1 s}$$

$$0 = \frac{V_2(s) - V_{\mathrm{A}}(s)}{R_2} + \frac{V_2(s)}{1/C_2 s}$$

$$0 = V_{\mathrm{A}}(s) - V_1(s) + R_1 C_1 s V_{\mathrm{A}}(s) - R_1 C_1 s V_2(s)$$

$$0 = V_2(s) - V_{\mathrm{A}}(s) + R_2 C_2 s V_2(s)$$

$$V_{\mathrm{A}}(s) = V_2(s)[1 + R_2 C_2 s]$$

$$0 = V_2(s)[1 + R_2 C_2 s][1 + R_1 C_1 s] - V_1(s) - R_1 C_1 s V_2(s)$$

$$V_1(s) = V_2(s)[1 + R_1 C_1 s + R_2 C_2 s - R_1 C_1 s + R_1 R_2 C_1 C_2 s^2]$$

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + R_2 C_2 s + 1}$$

For $C_1 = C_2 = C$, we have the following simplification:

$$T(s) = \frac{1}{R_1 R_2 C^2 s^2 + R_2 C s + 1} = \frac{\dfrac{1}{R_1 R_2 C^2}}{s^2 + \dfrac{1}{R_1 C} s + \dfrac{1}{R_1 R_2 C^2}}$$

To solve for the component values, select a value for C and then solve for R_1 and R_2 using the following two equations, in order:

$$\frac{1}{R_1 C} = 2\zeta\omega_0$$

$$\frac{1}{R_1 R_2 C^2} = \omega_0^2$$

For the given values, we have the following results:

$$\omega_0 = 100 \text{ krad/s}$$

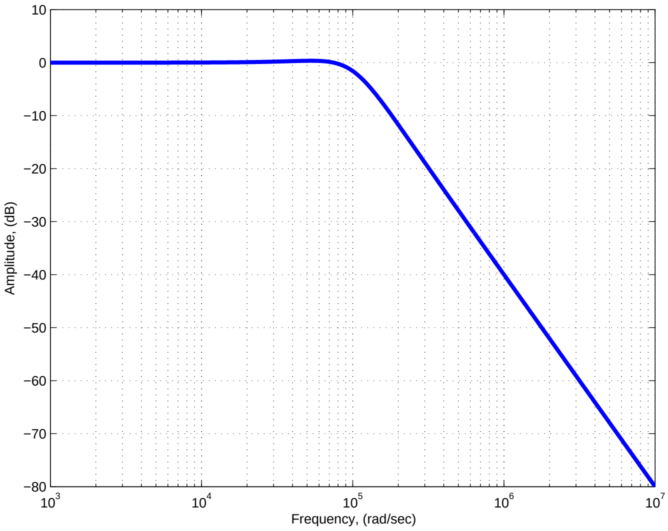
$$\zeta = 0.6$$

$$C = 0.001\ \mu\text{F}$$

$$R_1 = \frac{1}{2\zeta\omega_0 C} = 8.33 \text{ k}\Omega$$

$$R_2 = \frac{1}{R_1 C^2 \omega_0^2} = 12 \text{ k}\Omega$$

The Bode magnitude plot is shown below. The filter is a low-pass filter with a roll-off of 40 dB per decade.



Problem 14–5. The circuit in Figure 14–3(b) has a low-pass transfer function given in Eq. (14–6) and repeated below

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{\mu}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2 + R_2C_2 - \mu R_1C_1)s + 1}$$

In Section 14–2, we developed *equal element* and *unity gain* design methods for this circuit. This problem explores an *equal time constant* design method. Using $R_1C_1 = R_2C_2$ and $\mu = 2$, develop a method of selecting values for C_1, C_2, R_1, and R_2. Then select values so that the filter has a cutoff frequency of 2 krad/s and a ζ of 0.05. Use MATLAB to plot the filter's Bode diagram. Determine the location in rad/s and magnitude in dB of the peak in the frequency response.

Let $T = R_1C_1 = R_2C_2$ and $\mu = 2$. Substitute into the transfer function.

$$T(s) = \frac{2}{T^2s^2 + (T + R_1C_2 + T - 2T)s + 1} = \frac{\frac{2}{T^2}}{s^2 + \frac{R_1C_2}{T^2}s + \frac{1}{T^2}}$$

We now have the following equations:

$$\frac{1}{T^2} = \frac{1}{(R_1C_1)^2} = \omega_0^2$$

$$\omega_0 = \frac{1}{R_1C_1} = \frac{1}{R_2C_2}$$

$$R_1 = \frac{1}{\omega_0C_1}$$

$$\frac{R_1C_2}{T^2} = R_1C_2\omega_0^2 = 2\zeta\omega_0$$

$$R_1C_2\omega_0 = 2\zeta$$

$$C_2 = \frac{2\zeta}{\omega_0R_1}$$

$$R_2 = \frac{R_1C_1}{C_2}$$

The procedure to design the circuit is as follows: Select C_1 and solve for R_1 as shown above. Then solve for C_2 and R_2, in turn. For the specifications given above, choose $C_1 = 0.025\ \mu$F and solve for the other values. The results are:

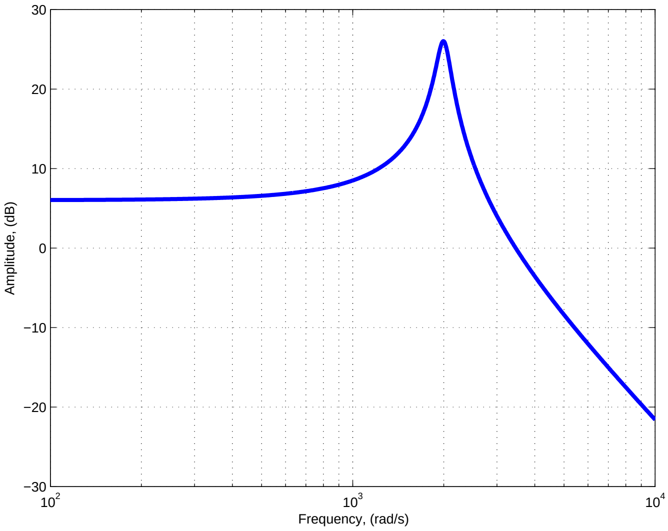
$$C_1 = 0.025\ \mu\text{F}$$

$$R_1 = \frac{1}{\omega_0 C_1} = 20\ \text{k}\Omega$$

$$C_2 = \frac{2\zeta}{\omega_0 R_1} = 0.0025\ \mu\text{F}$$

$$R_2 = \frac{R_1 C_1}{C_2} = 200\ \text{k}\Omega$$

The amplitude Bode plot is shown below. The peak occurs at $\omega = 1995$ rad/s and has a value of 26.0315 dB.



Problem 14–6. The active filter in Figure P14–6 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{-R_2/R_1}{R_2R_3C_1C_2s^2 + [R_3C_2 + R_2C_2(1 + R_3/R_1)]s + 1}$$

Using $R_1 = R_2 = R_3 = R$, develop a method of selecting values for C_1, C_2, and R. Then select values so that the filter has a cutoff frequency of 150 krad/s and a ζ of 0.01. Use MATLAB to plot the filter's Bode diagram. Build and simulate your circuit in OrCAD and compare the results with MATLAB's. Determine the location in rad/s and magnitude in dB of the peak in the frequency response.

Substitute $R_1 = R_2 = R_3 = R$ into the transfer function and simplify as follows:

$$T(s) = \frac{-1}{R^2C_1C_2s^2 + 3RC_2s + 1} = \frac{-\dfrac{1}{R^2C_1C_2}}{s^2 + \dfrac{3}{RC_1}s + \dfrac{1}{R^2C_1C_2}}$$

To solve for the component values, select a value for C_1 and then solve for R and C_2, in order, using the following equations:

$$\frac{3}{RC_1} = 2\zeta\omega_0$$

$$R = \frac{3}{2\zeta\omega_0 C_1}$$

$$\frac{1}{R^2 C_1 C_2} = \omega_0^2$$

$$C_2 = \frac{1}{\omega_0^2 R^2 C_1}$$

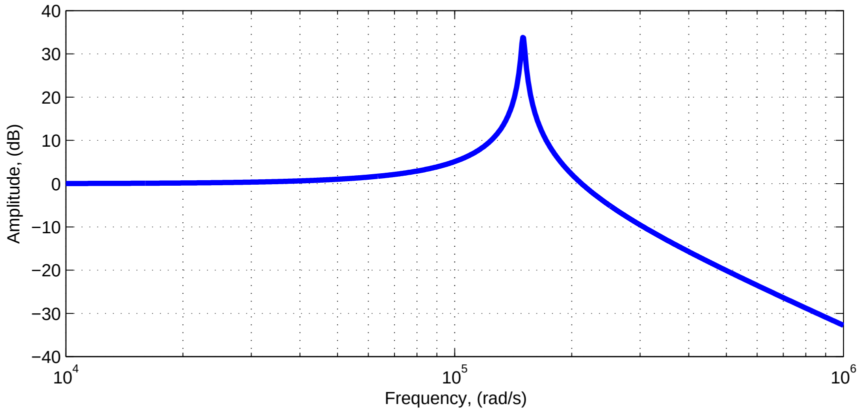
For the specifications given above, choose $C_1 = 0.1\ \mu$F and solve for the other values. The results are:

$$C_1 = 0.1\ \mu\text{F}$$

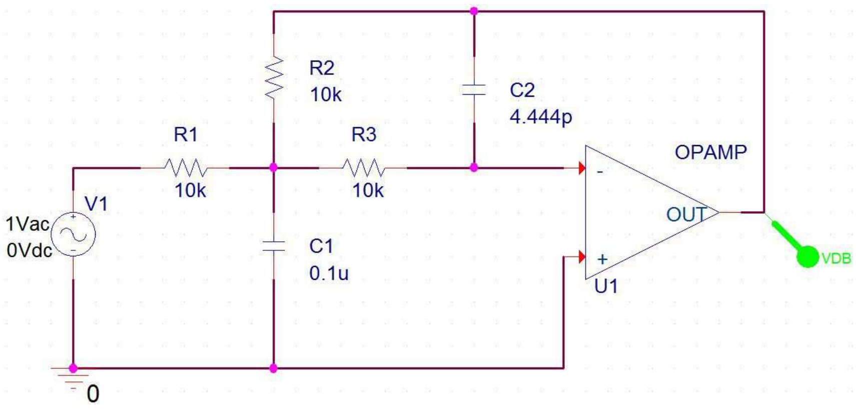
$$R = \frac{3}{2\zeta\omega_0 C_1} = 10\ \text{k}\Omega$$

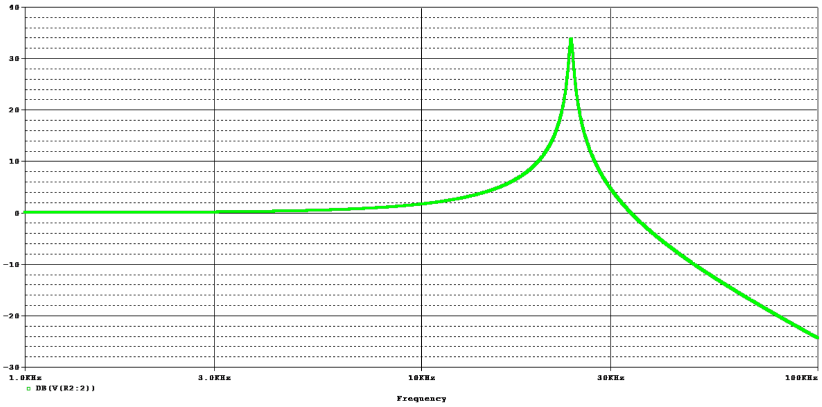
$$C_2 = \frac{1}{\omega_0^2 R^2 C_1} = 4.444\ \text{pF}$$

The amplitude Bode plot is shown below. The peak occurs at $\omega = 149.985$ krad/s and has a value of 33.9798 dB.



The OrCAD circuit and simulation results are shown below and agree with the MATLAB results.





Problem 14–7. Show that the active filter in Figure P14–7 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_1R_2C_1C_2s^2}{R_1R_2C_1C_2s^2 + R_1C_1s + 1}$$

Using $C_1 = C_2 = C$, develop a method of selecting values for C, R_1, and R_2. Then select values so that the filter has a cutoff frequency of 1 krad/s and a ζ of 0.1. Use MATLAB to plot the filter's Bode diagram. Determine the type of filter it is and its roll-off. How does the circuit and this transfer function compare with that of Problem 14–4?

Use node-voltage analysis. Let the positive input terminal of the left OP AMP have voltage $V_{\text{A}}(s)$.

$$0 = \frac{V_{\text{A}}(s) - V_1(s)}{1/C_1s} + \frac{V_{\text{A}}(s) - V_2(s)}{R_1}$$

$$0 = \frac{V_2(s) - V_{\text{A}}(s)}{1/C_2s} + \frac{V_2(s)}{R_2}$$

$$0 = R_1C_1sV_{\text{A}}(s) - R_1C_1sV_1(s) + V_{\text{A}}(s) - V_2(s)$$

$$0 = R_2C_2sV_2(s) - R_2C_2sV_{\text{A}}(s) + V_2(s)$$

$$V_{\text{A}}(s) = \frac{R_2C_2s + 1}{R_2C_2s}V_2(s)$$

$$R_1C_1sV_1(s) = V_2(s)\left[\frac{R_2C_2s + 1}{R_2C_2s}\right][R_1C_1s + 1] - V_2(s)$$

$$R_1R_2C_1C_2s^2V_1(s) = V_2(s)\left[R_1R_2C_1C_2s^2 + R_1C_1s + R_2C_2s - R_2C_2s + 1\right]$$

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_1R_2C_1C_2s^2}{R_1R_2C_1C_2s^2 + R_1C_1s + 1}$$

We have the following design relationships:

$$\omega_0^2 = \frac{1}{R_1R_2C_1C_2}$$

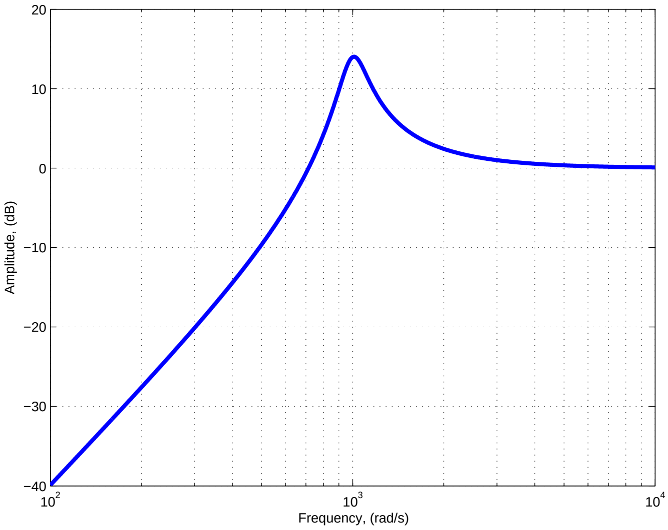
$$2\zeta\omega_0 = \frac{1}{R_2C_2}$$

Pick $C_1 = C_2 = C = 0.1\ \mu$F and solve for the resistor values.

$$R_2 = \frac{1}{2\zeta\omega_0 C} = 50\ \text{k}\Omega$$

$$R_1 = \frac{1}{R_2\omega_0^2 C^2} = 2\ \text{k}\Omega$$

The amplitude Bode plot is shown below. The filter is a high-pass filter with a roll-off of 40 dB per decade.



The circuit in Figure P14–7 is the same as the circuit in Figure P14–4, except that the resistors and capacitors are swapped. The swap causes the low-pass filter in Figure P14–4 to become a high-pass filter in Figure P14–7. The transfer functions are similar in their general structure, but the high-pass filter has s^2 in the numerator to achieve the correct gain response.

Problem 14–8. The circuit in Figure 14–9(b) has a high-pass transfer function given in Eq. (14–11) and repeated below

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{\mu R_1R_2C_1C_2s^2}{R_1R_2C_1C_2s^2 + (R_2C_2 + R_1C_1 + R_1C_2 - \mu R_2C_2)s + 1}$$

In Section 14–2, we developed *equal element* and *unity gain* design methods for this circuit. This problem explores an *equal time constant* design method. Using $R_1C_1 = R_2C_2$ and $\mu = 2$, develop a method of selecting values for C_1, C_2, R_1, and R_2. Then select values so that the filter has a cutoff frequency of 500 krad/s and a ζ of 0.25. Use MATLAB to plot the filter's Bode diagram.

Let $T = R_1C_1 = R_2C_2$ and $\mu = 2$. Substitute into the transfer function.

$$T(s) = \frac{2T^2s^2}{T^2s^2 + (T + T + R_1C_2 - 2T)s + 1} = \frac{2s^2}{s^2 + \dfrac{R_1C_2}{T^2}s + \dfrac{1}{T^2}}$$

We now have the following equations:

$$\frac{1}{T^2} = \frac{1}{(R_1C_1)^2} = \omega_0^2$$

$$\omega_0 = \frac{1}{R_1C_1} = \frac{1}{R_2C_2}$$

$$R_1 = \frac{1}{\omega_0 C_1}$$

$$\frac{R_1C_2}{T^2} = R_1C_2\omega_0^2 = 2\zeta\omega_0$$

$$R_1C_2\omega_0 = 2\zeta$$

$$C_2 = \frac{2\zeta}{\omega_0 R_1}$$

$$R_2 = \frac{R_1C_1}{C_2}$$

The procedure to design the circuit is as follows: Select C_1 and solve for R_1 as shown above. Then solve for C_2 and R_2, in turn. For the specifications given above, choose $C_1 = 500$ pF and solve for the other values. The results are:

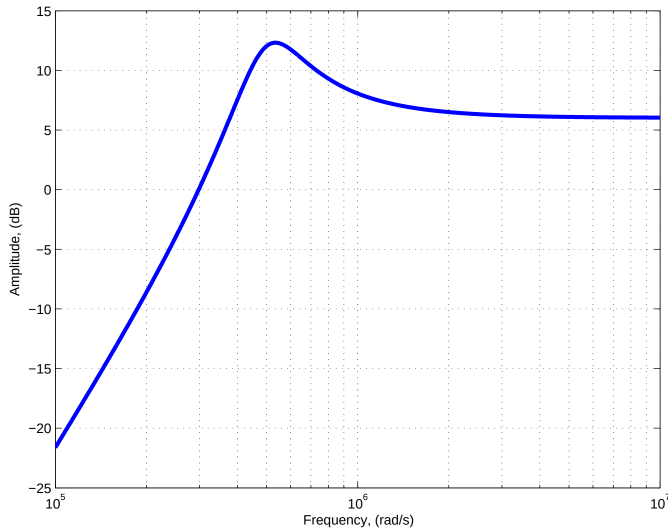
$$C_1 = 500 \text{ pF}$$

$$R_1 = \frac{1}{\omega_0 C_1} = 4 \text{ k}\Omega$$

$$C_2 = \frac{2\zeta}{\omega_0 R_1} = 250 \text{ pF}$$

$$R_2 = \frac{R_1C_1}{C_2} = 8 \text{ k}\Omega$$

The amplitude Bode plot is shown below.



Problem 14–9. Show that the active filter in Figure P14–9 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{-R_1C_1s}{R_1R_2C_1C_2s^2 + (R_1C_2 + R_2C_2)s + 1}$$

Using $R_1 = R_2 = R$, develop a method of selecting values for C_1, C_2, and R. Then select values so that the filter has a ω_0 of 50 krad/s and a ζ of 0.1. Use MATLAB to plot the filter's Bode diagram. What type of filter is this?

Use node-voltage analysis. Let the node between C_1 and R_2 be $V_\mathrm{A}(s)$.

$$0 = \frac{V_\mathrm{A}(s) - V_1(s)}{1/C_1s} + \frac{V_\mathrm{A}(s)}{R_2} + \frac{V_\mathrm{A}(s) - V_2(s)}{R_1}$$

$$0 = \frac{-V_\mathrm{A}(s)}{R_2} + \frac{-V_2(s)}{1/C_2s}$$

$$0 = R_1R_2C_1sV_\mathrm{A}(s) - R_1R_2C_1sV_1(s) + R_1V_\mathrm{A}(s) + R_2V_\mathrm{A}(s) - R_2V_2(s)$$

$$0 = -V_\mathrm{A}(s) - R_2C_2sV_2(s)$$

$$V_\mathrm{A}(s) = -R_2C_2sV_2(s)$$

$$R_1R_2C_1sV_1(s) = [-R_2C_2sV_2(s)][R_1R_2C_1s + R_1 + R_2] - R_2V_2(s)$$

$$R_1R_2C_1sV_1(s) = -V_2(s)[R_1R_2^2C_1C_2s^2 + R_1R_2C_2s + R_2^2C_2s + R_2]$$

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{-R_1C_1s}{R_1R_2C_1C_2s^2 + (R_1C_2 + R_2C_2)s + 1}$$

With $R_1 = R_2 = R$, we have the following design relationships:

$$\omega_0^2 = \frac{1}{R_1R_2C_1C_2} = \frac{1}{R^2C_1C_2}$$

$$2\zeta\omega_0 = \frac{R_1C_2 + R_2C_2}{R_1R_2C_1C_2} = \frac{2RC_2}{R^2C_1C_2} = \frac{2}{RC_1}$$

$$R = \frac{1}{\zeta\omega_0C_1}$$

$$C_2 = \frac{1}{\omega_0^2R^2C_1}$$

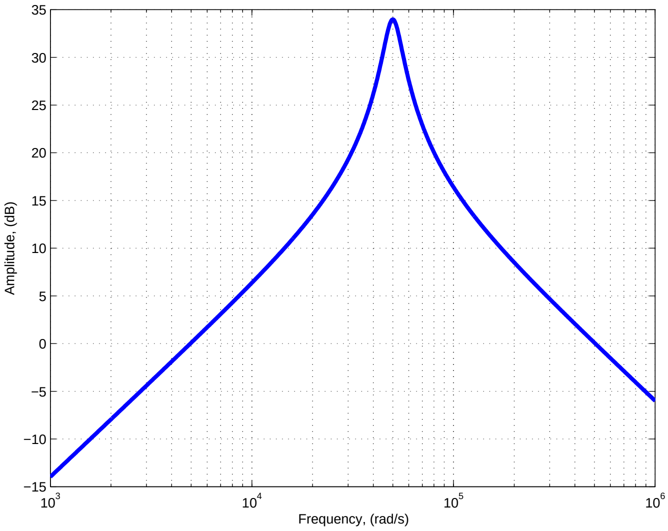
The design procedures is to select a value for C_1 and then solve for R and C_2, in turn, using the equations above. For the given specifications, we have the following design choices and results:

$$C_1 = 0.1\ \mu\mathrm{F}$$

$$R = \frac{1}{\zeta\omega_0C_1} = 2\ \mathrm{k}\Omega$$

$$C_2 = \frac{1}{\omega_0^2R^2C_1} = 1000\ \mathrm{pF}$$

The amplitude Bode plot is shown below. The filter is a bandpass filter.



Problem 14–10. The active filter in Figure P14–10 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{-R_3C_2s}{R_1R_3C_1C_2s^2 + (R_1C_1 + R_1C_2)s + 1 + R_1/R_2}$$

Using $C_1 = C_2 = C$ and $R_1 = R_2 = R$, develop a method of selecting values for C and R. Then select values so that the filter has a ω_0 of 10 krad/s and a ζ of 0.5. Use MATLAB to plot the filter's Bode diagram. What type of filter is this?

Substitute $C_1 = C_2 = C$ and $R_1 = R_2 = R$ into the transfer function and simplify.

$$T(s) = \frac{-R_3Cs}{RR_3C^2s^2 + 2RCs + 2} = \frac{-\dfrac{1}{RC}s}{s^2 + \dfrac{2}{R_3C}s + \dfrac{2}{RR_3C^2}}$$

We have the following design relationships:

$$2\zeta\omega_0 = \frac{2}{R_3C}$$

$$R_3 = \frac{1}{\zeta\omega_0C}$$

$$\omega_0^2 = \frac{2}{RR_3C^2}$$

$$R = \frac{2}{\omega_0^2R_3C^2}$$

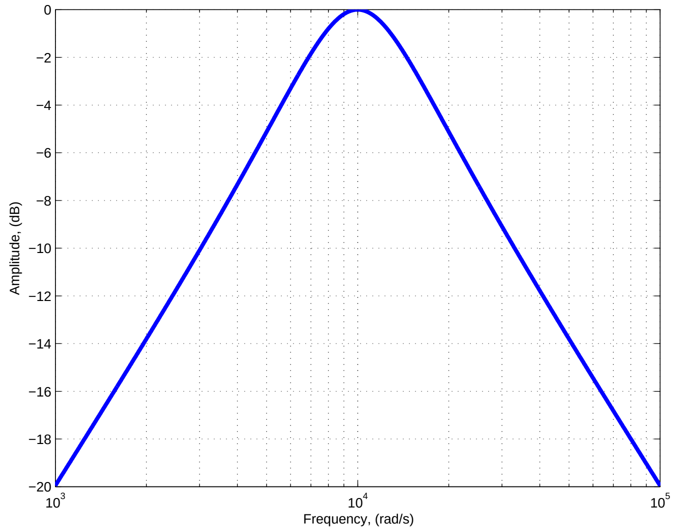
The design procedure is to select a value for C and then solve for R_3 and R, in turn, using the equations above. For the given specifications, we have the following design choices and results:

$$C = 0.01\ \mu\text{F}$$

$$R_3 = \frac{1}{\zeta\omega_0C} = 20\ \text{k}\Omega$$

$$R = \frac{2}{\omega_0^2R_3C^2} = 10\ \text{k}\Omega$$

The amplitude Bode plot is shown below. The filter is a bandpass filter.



Problem 14–11. The active filter in Figure P14–11 has a transfer function of the form

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{(RCs)^2 + 1}{(RCs)^2 + 2RCs + 1}$$

Select values of R and C so that the filter has a ω_0 of 377 krad/s. Use OrCAD to plot the filter's Bode magnitude diagram. What type of filter is this? With the gain equal to 1, is ζ selectable? What is ζ in this filter?

We have the following design relationships:

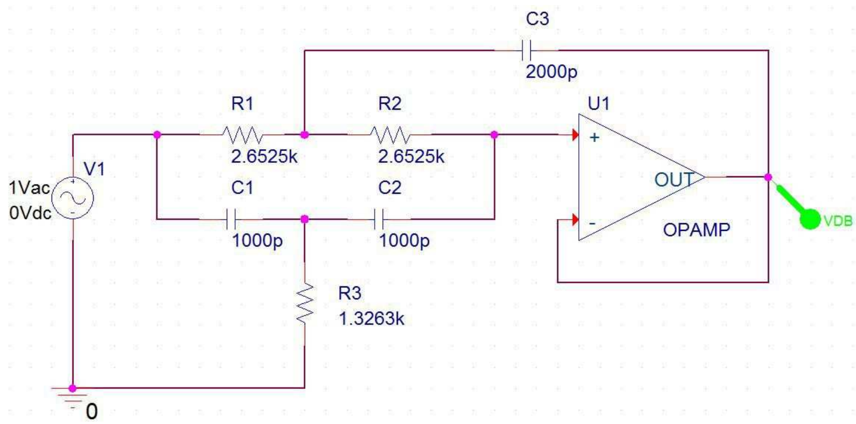
$$\omega_0^2 = \frac{1}{R^2C^2}$$

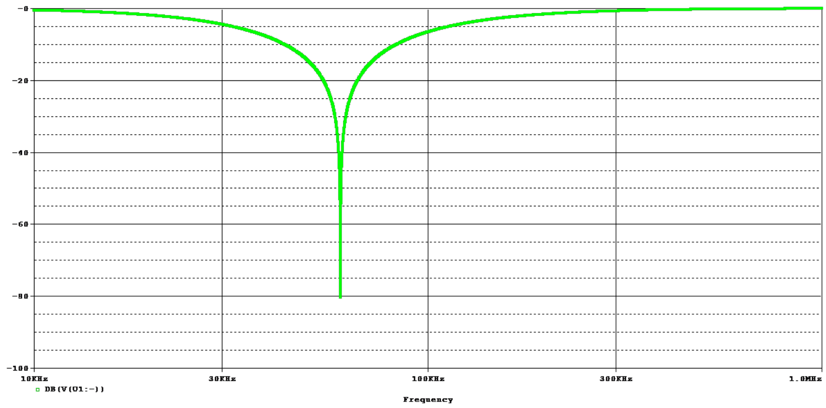
$$\omega_0 = \frac{1}{RC}$$

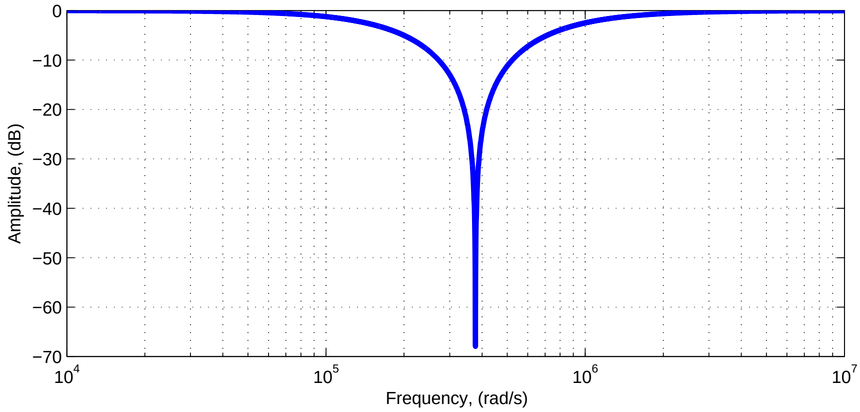
$$2\zeta\omega_0 = \frac{2}{RC}$$

$$\zeta = \frac{1}{\omega_0 RC} = 1$$

Pick $C = 1000$ pF and solve for $R = 1/(\omega_0 C) = 2.6525$ kΩ. The OrCAD circuit and simulation results are shown below. In addition, a MATLAB plot of the transfer function is shown. The filter is a bandstop filter with $\zeta = 1$. With a gain of one, the value for ζ is not selectable.







Problem 14–12. The task is to design a second-order low-pass filter using the three approaches shown in Problems 14–4, 14–5, and 14–6. The filter specs are a cutoff frequency of 100 krad/s and a ζ of 0.25. Using OrCAD, build your three designs and select the best one based on how well each meets the specs, the number of parts, and the gain.

Using the results from Problem 14–4, we have the following design choices and results:

$$\omega_0 = 100 \text{ krad/s}$$

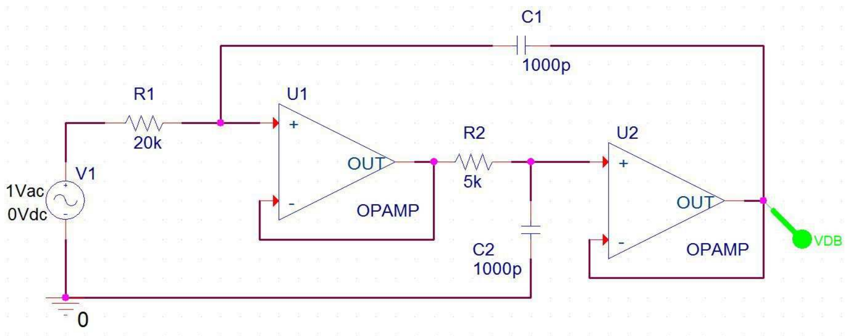
$$\zeta = 0.25$$

$$C = C_1 = C_2 = 1000 \text{ pF}$$

$$R_1 = \frac{1}{2\zeta\omega_0 C} = 20 \text{ k}\Omega$$

$$R_2 = \frac{1}{R_1 C^2 \omega_0^2} = 5 \text{ k}\Omega$$

The OrCAD circuit is shown below.



Using the results from Problem 14–5, we have the following design choices and results:

$$\mu = 2$$

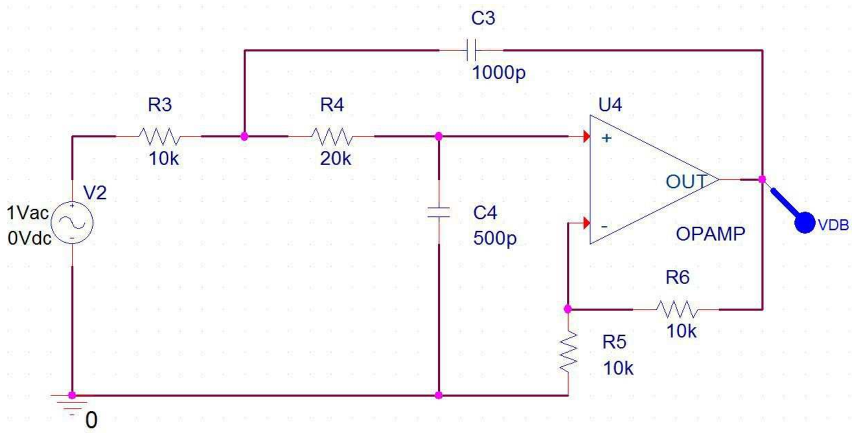
$$C_1 = 1000 \text{ pF}$$

$$R_1 = \frac{1}{\omega_0 C_1} = 10 \text{ k}\Omega$$

$$C_2 = \frac{2\zeta}{\omega_0 R_1} = 500 \text{ pF}$$

$$R_2 = \frac{R_1 C_1}{C_2} = 20 \text{ k}\Omega$$

The OrCAD circuit is shown below.



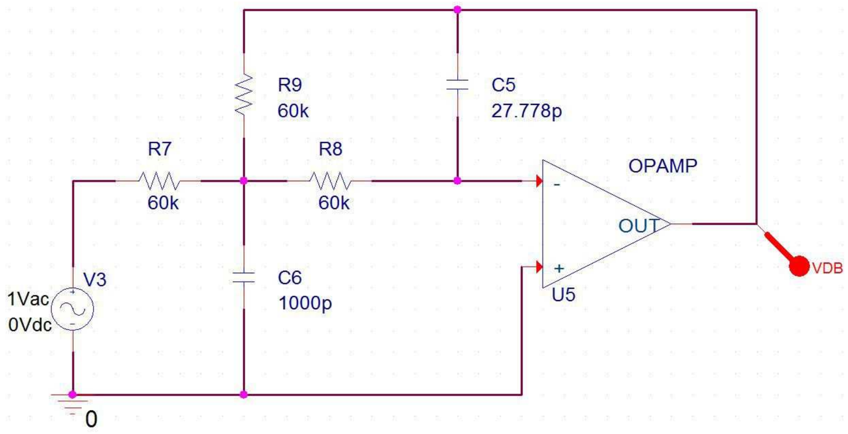
Using the results from Problem 14–6, we have the following design choices and results:

$$C_1 = 1000 \text{ pF}$$

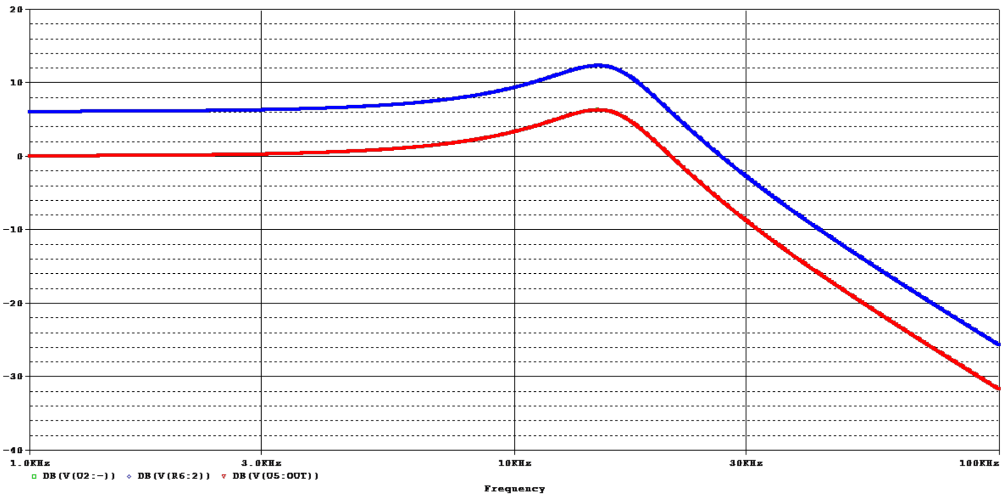
$$R = \frac{3}{2\zeta\omega_0 C_1} = 60 \text{ k}\Omega$$

$$C_2 = \frac{1}{\omega_0^2 R^2 C_1} = 27.78 \text{ pF}$$

The OrCAD circuit is shown below.



The simulation results from all three designs are shown below. The colors of the voltage markers in the circuits correspond to the colors of the curves in the plot. The results from Problems 14–4 and 14–6 are identical and the red curve overlays the green curve in the plot.



The following table summarizes the results.

Item	Problem 14–4	Problem 14–5	Problem 14–6
Meets Specs	Yes	Yes	Yes
Gain	1	2	1
OP AMPS	2	1	1
Resistors	2	4	3
Capacitors	2	2	2

If a larger gain is more important, then choose the circuit in Problem 14–5. If a lower part count is more important, then choose the circuit in Problem 14–6, since it requires only one OP AMP.

Problem 14–13. Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Type	ω_0 (rad/s)	ζ	$\lvert T(j\omega_0)\rvert$	Constraints
Low-pass	100000	$*$	50	dc gain of 100

We have the following results:

$$\omega_0 = 100000 \text{ rad/s}$$

$$K = 100$$

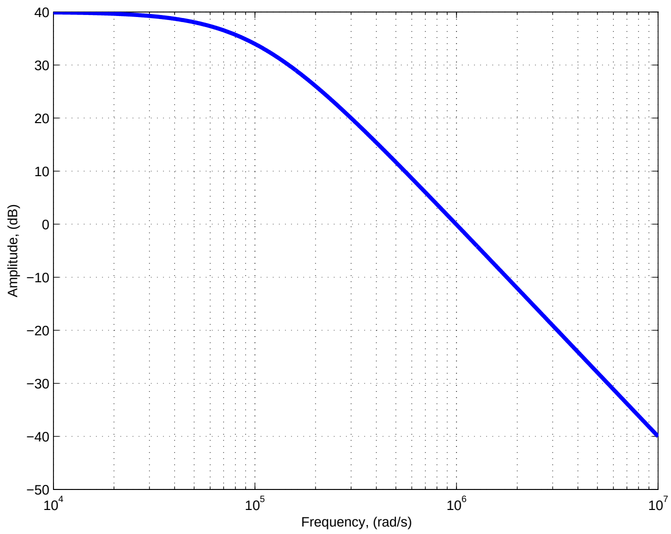
$$|T(j\omega_0)| = 50 = \frac{K}{2\zeta}$$

$$\zeta = \frac{100}{(2)(50)} = 1$$

$$T(s) = \frac{K}{\left(\dfrac{s}{\omega_0}\right)^2 + 2\zeta\left(\dfrac{s}{\omega_0}\right) + 1} = \frac{100}{\left(\dfrac{s}{10^5}\right)^2 + 2\left(\dfrac{s}{10^5}\right) + 1}$$

$$= \frac{10^{12}}{s^2 + 2 \times 10^5 s + 10^{10}}$$

The Bode diagram is shown below and validates that the transfer function meets the requirements.



Problem 14–14. Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Type	ω_0 (rad/s)	ζ	$\lvert T(j\omega_0)\rvert$	Constraints
High-pass	100	0.5	$*$	infinite frequency gain of 20

We have the following results:

$$\omega_0 = 100 \text{ rad/s}$$

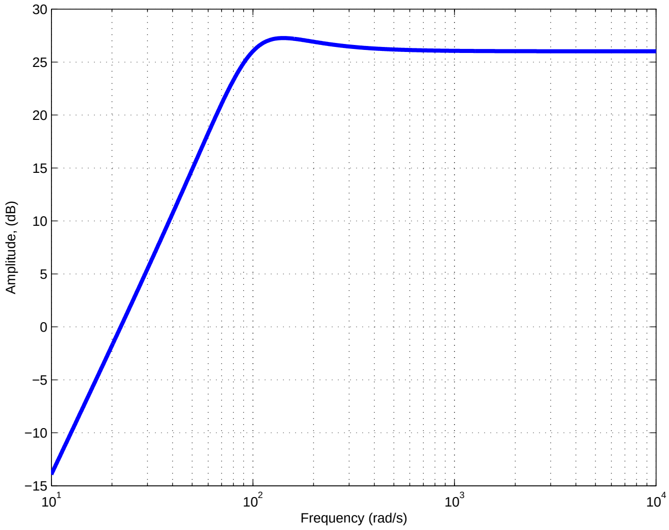
$$\zeta = 0.5$$

$$K = 20$$

$$T(s) = \frac{K\left(\dfrac{s}{\omega_0}\right)^2}{\left(\dfrac{s}{\omega_0}\right)^2 + 2\zeta\left(\dfrac{s}{\omega_0}\right) + 1} = \frac{Ks^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$= \frac{20s^2}{s^2 + 100s + 10000}$$

The Bode diagram is shown below and validates that the transfer function meets the requirements.



Problem 14–15. Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Type	ω_0 (rad/s)	ζ	$\lvert T(j\omega_0)\rvert$	Constraints
Bandpass	15000	$*$	5	$B = 250$ rad/s

We have the following results:

$$\omega_0 = 15000 \text{ rad/s}$$

$$B = 2\zeta\omega_0 = 250 \text{ rad/s}$$

$$\zeta = \frac{B}{2\omega_0} = 0.008333$$

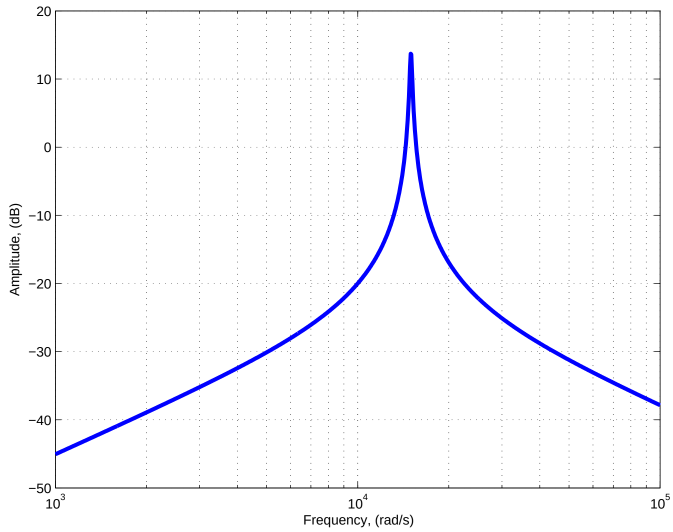
$$|T(j\omega_0)| = 5 = \frac{K}{2\zeta}$$

$$K = 10\zeta = 0.08333$$

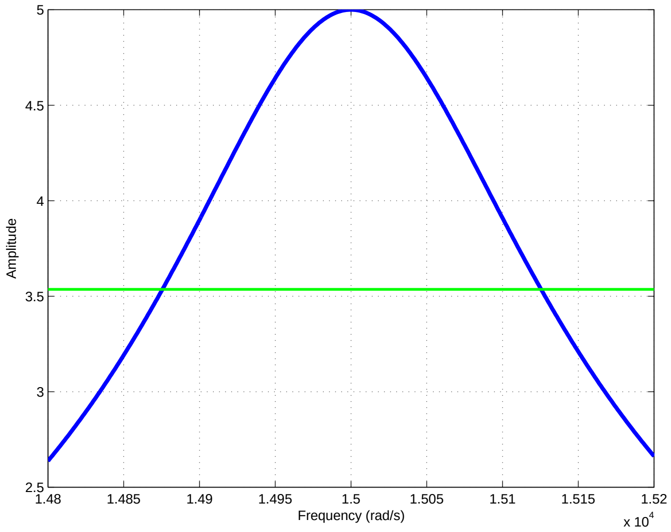
$$T(s) = \frac{K\left(\dfrac{s}{\omega_0}\right)}{\left(\dfrac{s}{\omega_0}\right)^2 + 2\zeta\left(\dfrac{s}{\omega_0}\right) + 1} = \frac{K\omega_0 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$= \frac{1250s}{s^2 + 250s + 225000000}$$

The Bode diagram is shown below and validates that the transfer function meets the requirements.



The plot below focuses on peak of the Bode plot and confirms the bandwidth specification.



5
4.5
4
3.5
3
2.5
Amplitude
1.48
1.485
1.49
1.495
1.5
1.505
1.51
1.515
1.52
Frequency (rad/s)
x 10^4

Problem 14–16. Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Type	ω_0 (rad/s)	ζ	$\lvert T(j\omega_0)\rvert$	Constraints
Bandpass	632	15.8	10	dc gain of zero

We have the following results:

$$\omega_0 = 632 \text{ rad/s}$$

$$\zeta = 15.8$$

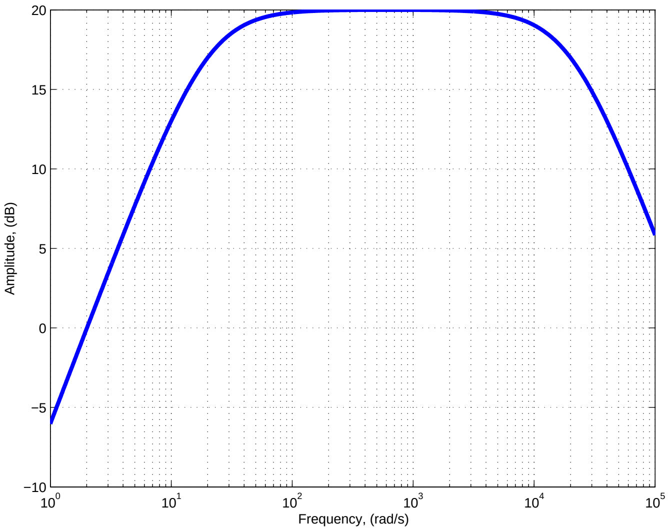
$$|T(j\omega_0)| = 10 = \frac{K}{2\zeta}$$

$$K = 20\zeta = 316$$

$$T(s) = \frac{K\left(\dfrac{s}{\omega_0}\right)}{\left(\dfrac{s}{\omega_0}\right)^2 + 2\zeta\left(\dfrac{s}{\omega_0}\right) + 1} = \frac{K\omega_0 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$= \frac{200000s}{s^2 + 20000s + 400000}$$

The Bode diagram is shown below and validates that the transfer function meets the requirements.



Problem 14–17. Construct a second-order transfer function that meets the following requirements. Use MATLAB to plot the transfer function's Bode diagram and validate the requirements.

Type	ω_0 (rad/s)	ζ	$\lvert T(j\omega_0)\rvert$	Constraints
Bandstop	377	0.01	0	passband gain of 10

We have the following results:

$$\omega_0 = 377 \text{ rad/s}$$

$$\zeta = 0.01$$

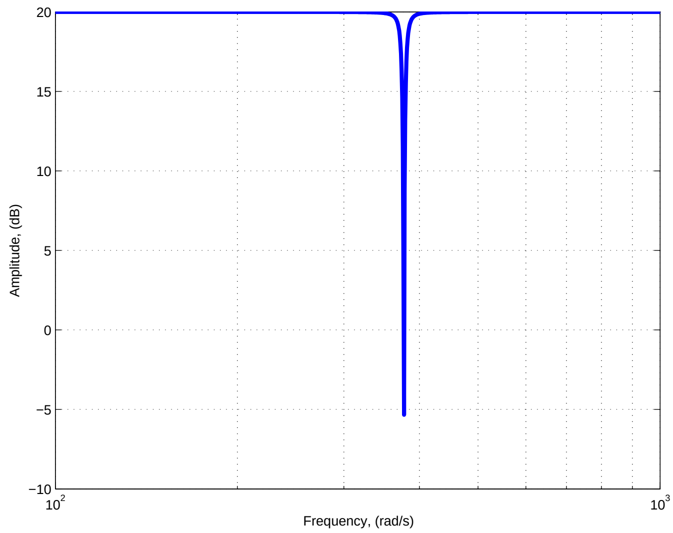
$$|T(j\omega_0)| = 0$$

$$K = 10$$

$$T(s) = \frac{K\left[\left(\dfrac{s}{\omega_0}\right)^2 + 1\right]}{\left(\dfrac{s}{\omega_0}\right)^2 + 2\zeta\left(\dfrac{s}{\omega_0}\right) + 1} = \frac{K[s^2+\omega_0^2]}{s^2+2\zeta\omega_0 s+\omega_0^2}$$

$$= \frac{10[s^2+377^2]}{s^2+7.54s+377^2}$$

The Bode diagram is shown below and validates that the transfer function meets the requirements.



Problem 14–18. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
Low-pass	20000	0.5	Use 10-kΩ resistors

Use an equal-element design approach with $R_1 = R_2 = R = 10$ kΩ. We have the following results.

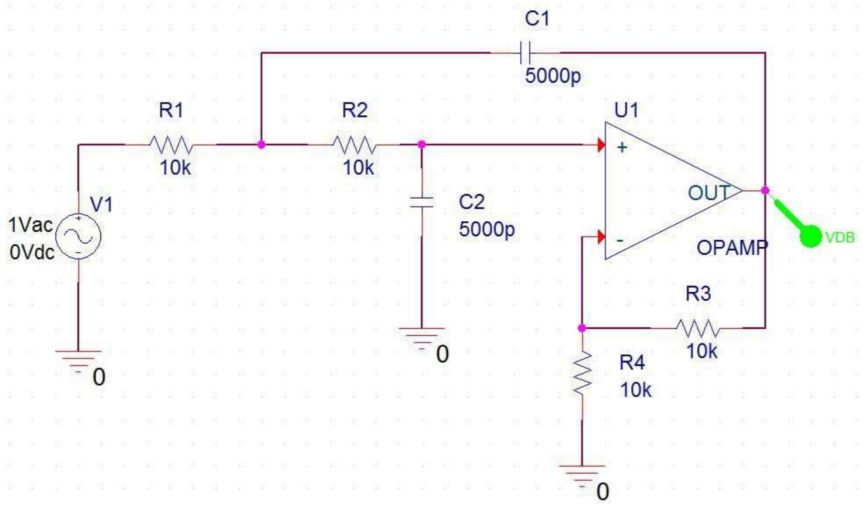
$$R = 10 \text{ k}\Omega$$

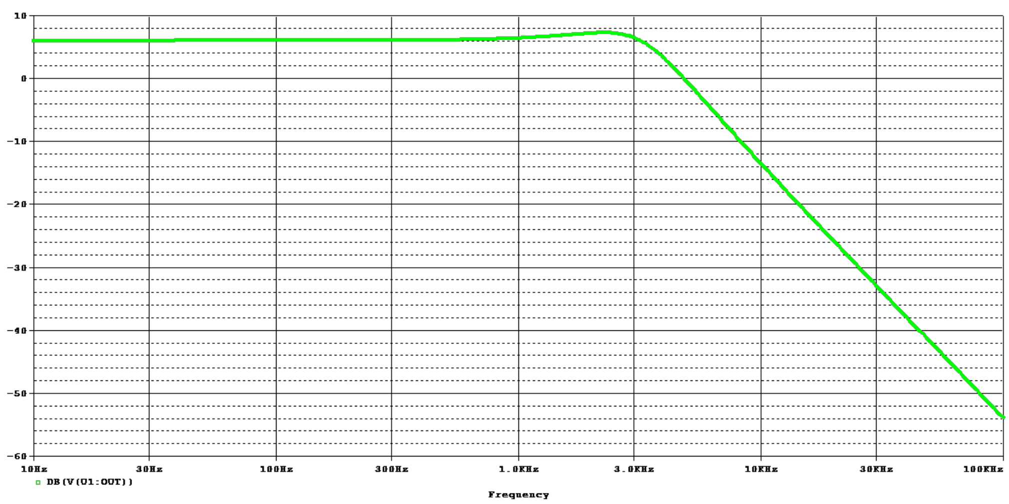
$$C = \frac{1}{\omega_0 R} = 5000 \text{ pF}$$

$$\mu = 3 - 2\zeta = 2$$

$$R_{\text{A}} = R_{\text{B}} = 10 \text{ k}\Omega$$

The OrCAD circuit and simulation are shown below.





Problem 14–19. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
Low-pass	2500	0.8	dc gain of 20 dB

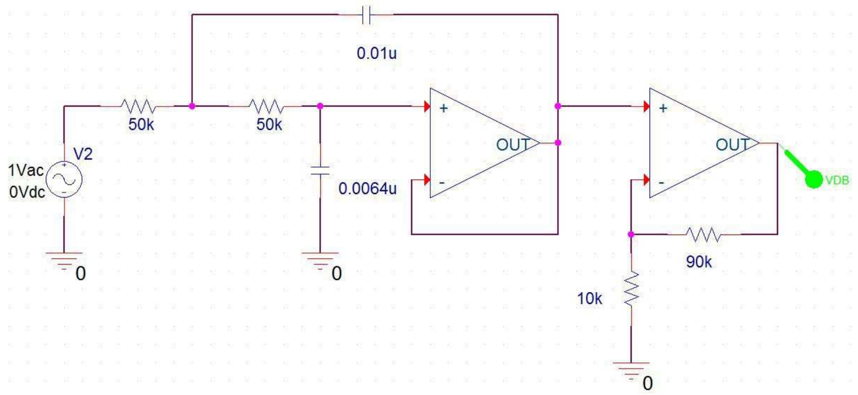
With a dc gain of 20 dB, which is a factor of 10, we will use the unity gain approach along with an amplifier with a gain of 10.

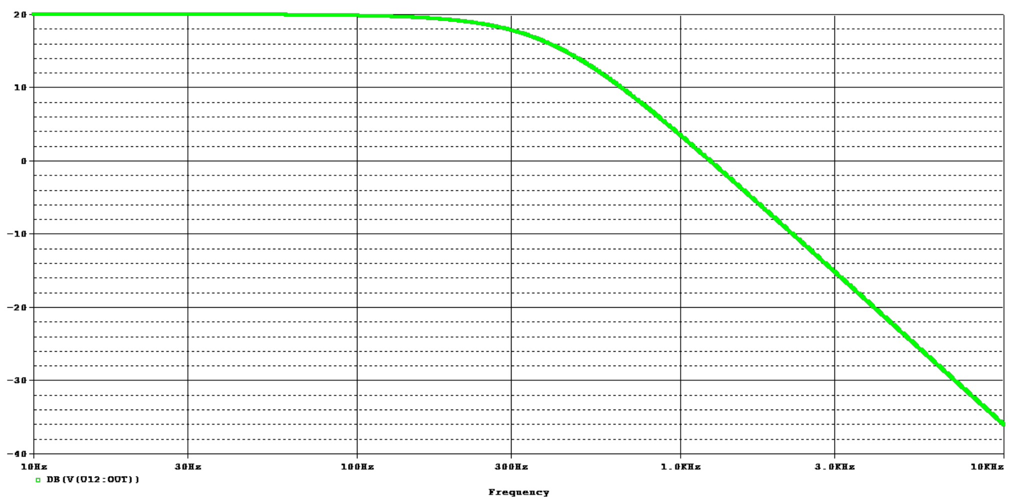
$$C_1 = 0.01\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 0.0064\ \mu\text{F}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 50\ \text{k}\Omega$$

The OrCAD circuit and simulation are shown below.





Problem 14–20. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
Low-pass	100000	0.25	Use 0.2-μF capacitors

With equal valued capacitors, we must use the equal element design approach.

$$C = 0.2\ \mu\text{F}$$

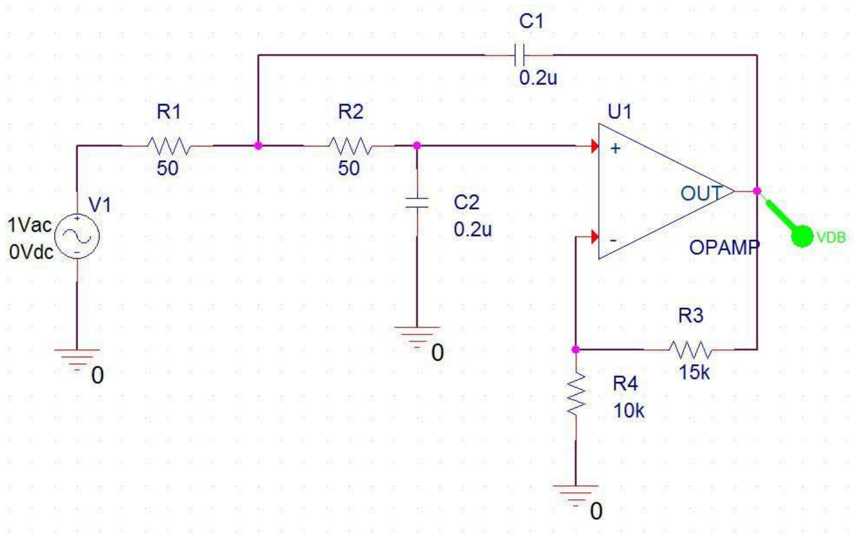
$$R = \frac{1}{\omega_0 C} = 50\ \Omega$$

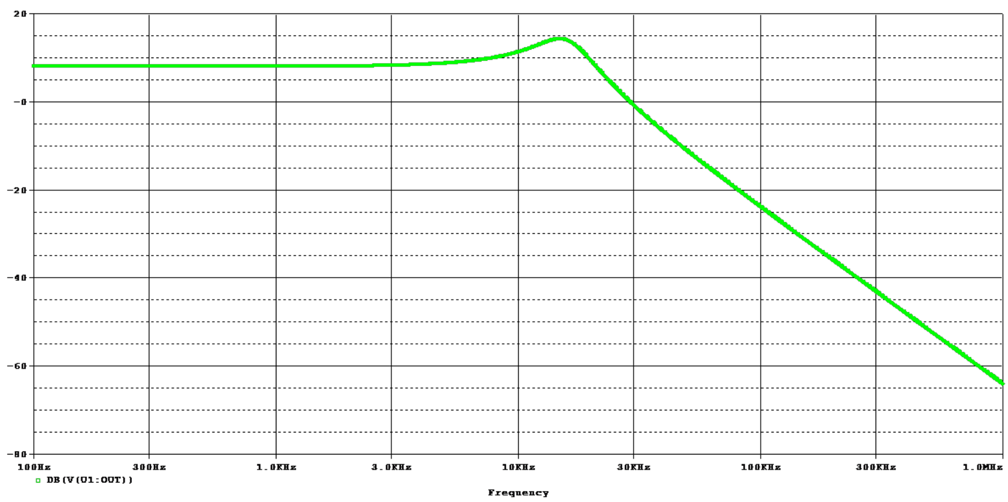
$$\mu = 3 - 2\zeta = 2.5$$

$$R_\text{B} = 10\ \text{k}\Omega$$

$$R_\text{A} = (\mu - 1)R_\text{B} = 15\ \text{k}\Omega$$

The OrCAD circuit and simulation are shown below.





Problem 14–21. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
High-pass	2500	0.75	Use 0.2-μF capacitors

With equal valued capacitors, we must use the equal element design approach.

$$C = 0.2\ \mu\text{F}$$

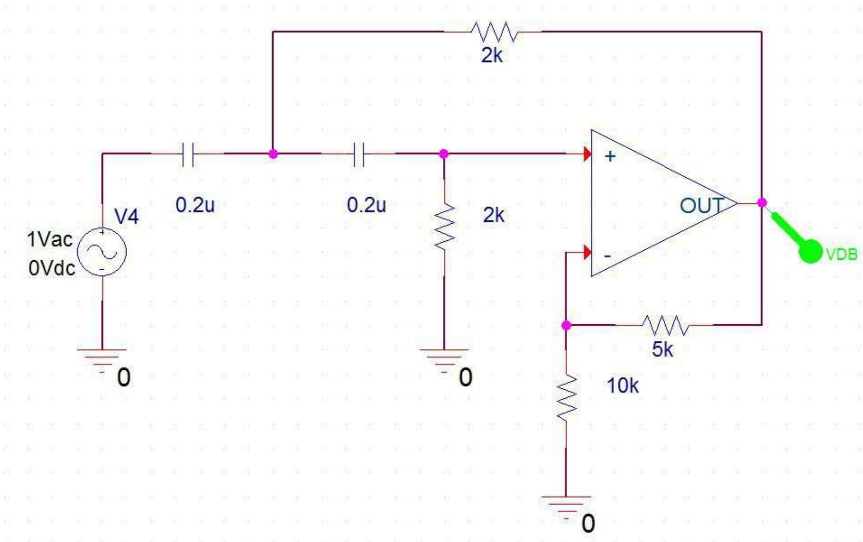
$$R = \frac{1}{\omega_0 C} = 2\ \text{k}\Omega$$

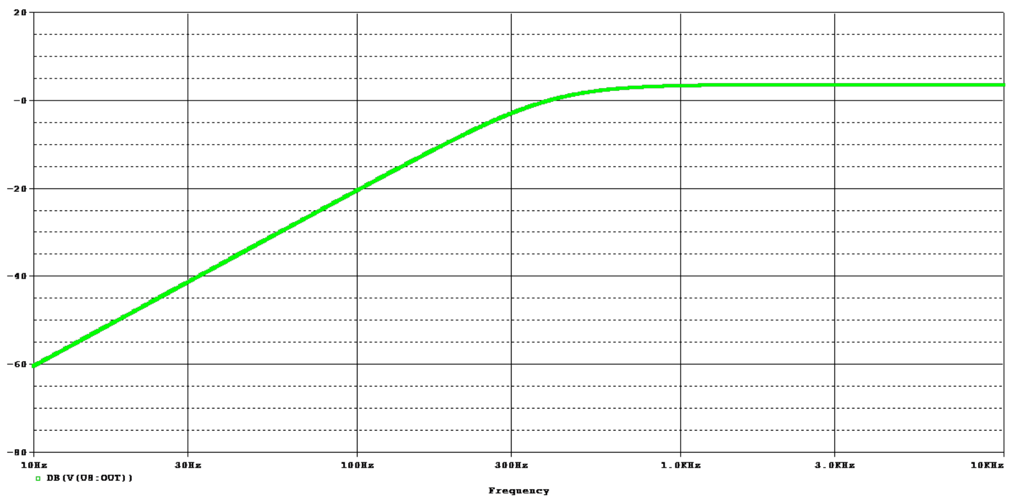
$$\mu = 3 - 2\zeta = 1.5$$

$$R_\text{B} = 10\ \text{k}\Omega$$

$$R_\text{A} = (\mu - 1)R_\text{B} = 5\ \text{k}\Omega$$

The OrCAD circuit and simulation are shown below.





Problem 14–22. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
High-pass	25000	0.25	High-frequency gain of 40 dB

Since we have a gain specification, we will use the unity gain design approach and include an amplifier to provide the gain of 40 dB, which is a factor of 100.

$$\omega_0 = 25000 \text{ rad/s}$$

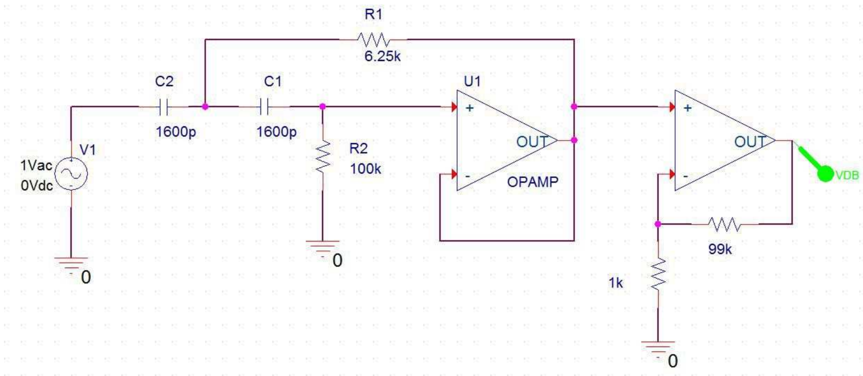
$$\zeta = 0.25$$

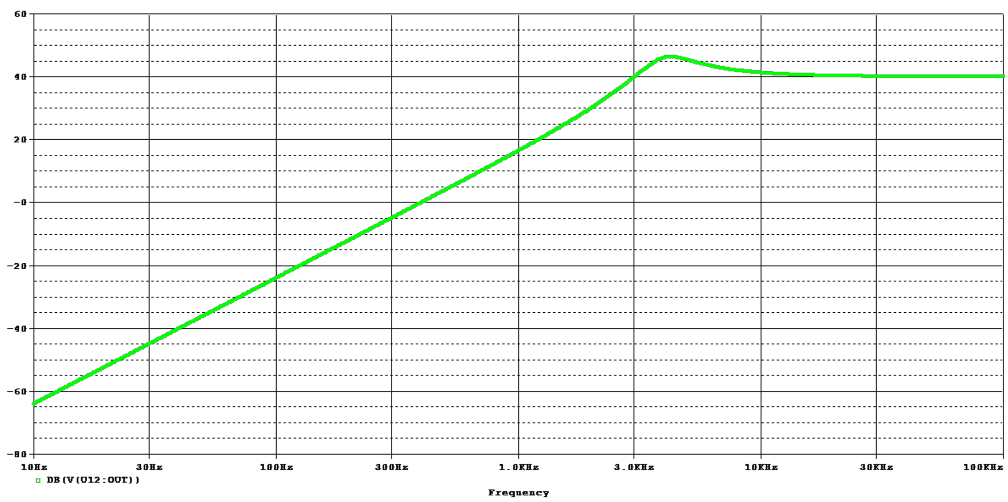
$$R_2 = 100 \text{ k}\Omega$$

$$R_1 = \zeta^2 R_2 = 6.25 \text{ k}\Omega$$

$$C = \frac{1}{\omega_0 \sqrt{R_1 R_2}} = 1600 \text{ pF}$$

The OrCAD circuit and simulation are shown below.





Problem 14–23. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
Bandpass	5000	$*$	Center frequency gain of 20 dB

The center-frequency gain is 10. If we use the equal-capacitor method, we need to choose ζ such that $1/(2\zeta^2) = 10$, which means $\zeta = 0.2236$. Now use the equal-capacitor method to design the circuit.

$$\omega_0 = 5000 \text{ rad/s}$$

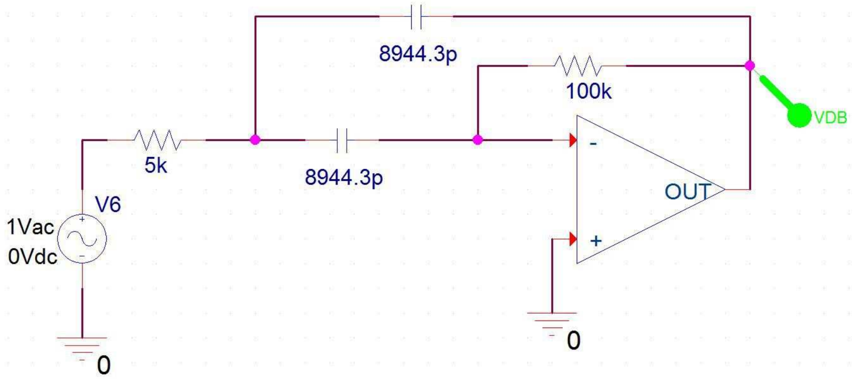
$$\zeta = 0.2236$$

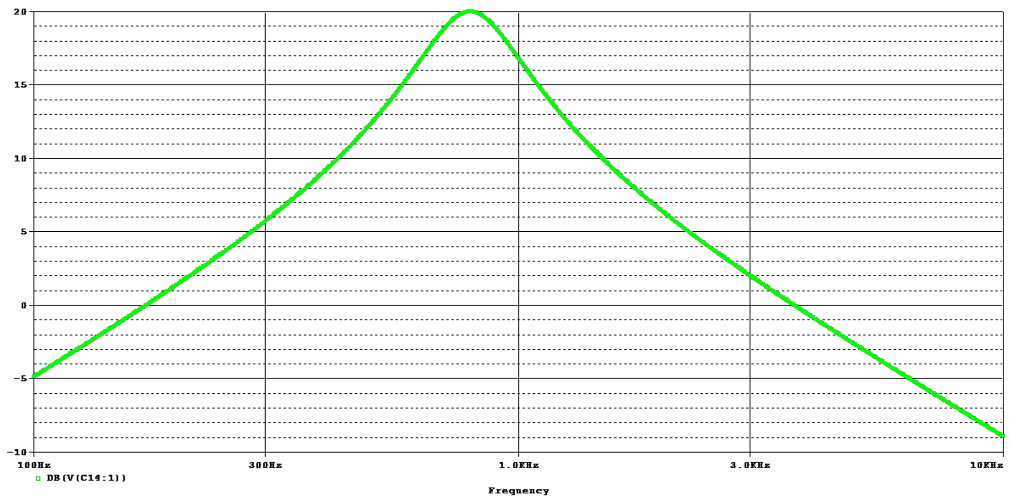
$$R_2 = 100 \text{ k}\Omega$$

$$R_1 = \zeta^2 R_2 = 5 \text{ k}\Omega$$

$$C = \frac{1}{\omega_0\sqrt{R_1 R_2}} = 8944.3 \text{ pF}$$

The OrCAD circuit and simulation are shown below.





Problem 14–24. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
Bandpass	3974	$*$	Bandwidth of 125.5 krad/s

With a bandwidth of 125.5 krad/s $= 2\zeta\omega_0$, we can calculate $\zeta = 15.7901$. Design the circuit using the equal-capacitor method.

$$\omega_0 = 3974 \text{ rad/s}$$

$$\zeta = 15.7901$$

$$R_2 = 1 \text{ k}\Omega$$

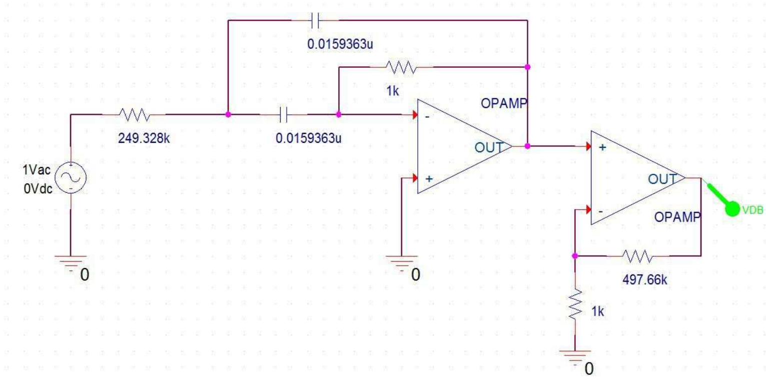
$$R_1 = \zeta^2 R_2 = 249.328 \text{ k}\Omega$$

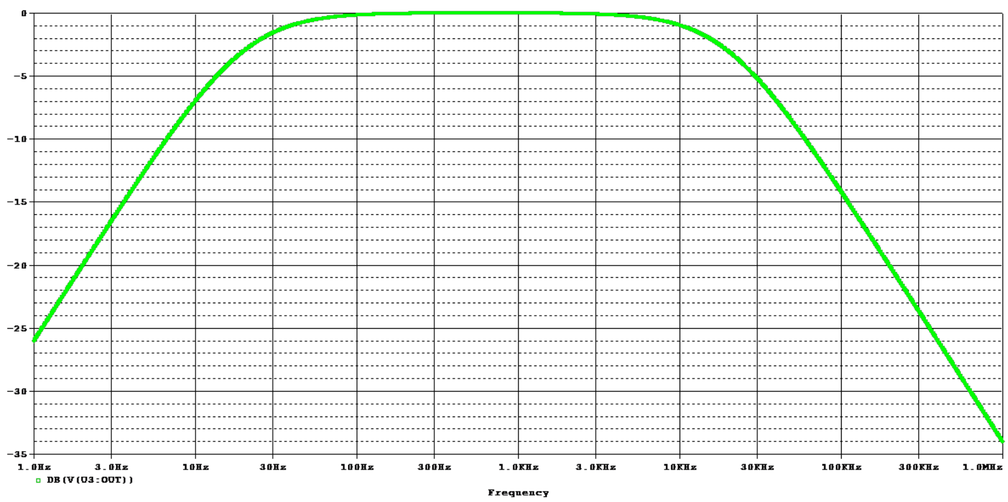
$$C = \frac{1}{\omega_0\sqrt{R_1R_2}} = 0.0159363\ \mu\text{F}$$

$$|T(j\omega_0)| = \frac{1}{2\zeta^2} = \frac{1}{498.66}$$

$$K = 498.66$$

The additional gain K is required to return the overall gain to one. The OrCAD circuit and simulation are shown below.





Problem 14–25. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
Tuned	3.45 M	0.005	Use 500-pF capacitors

Design the circuit using the equal-capacitor method.

$$\omega_0 = 3.45 \text{ Mrad/s}$$

$$\zeta = 0.005$$

$$R_1 = \zeta^2 R_2$$

$$C\sqrt{R_1 R_2} = \frac{1}{\omega_0}$$

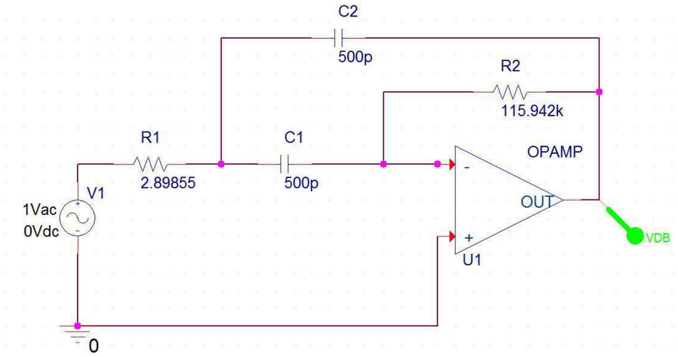
$$C\zeta R_2 = \frac{1}{\omega_0}$$

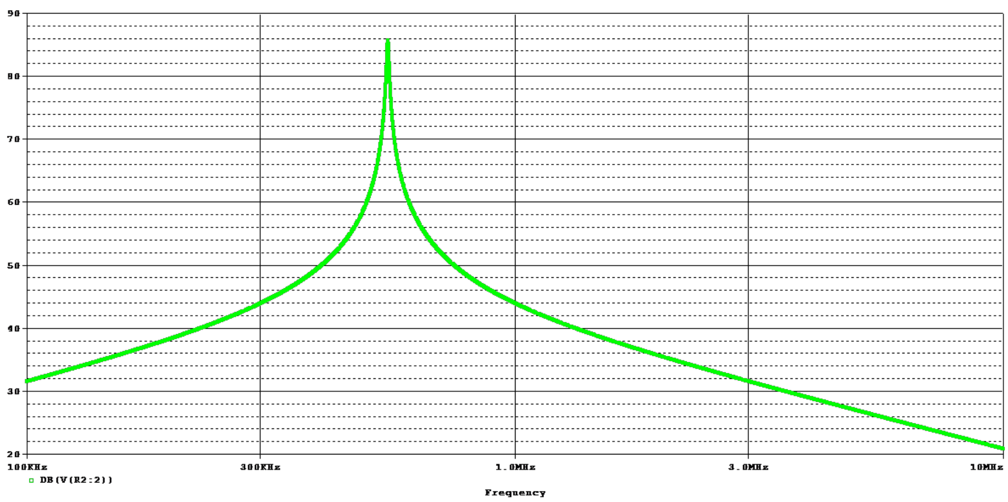
$$R_2 = \frac{1}{\zeta C \omega_0} = 115.942 \text{ k}\Omega$$

$$R_1 = \zeta^2 R_2 = 2.8986\ \Omega$$

$$|T(j\omega_0)| = \frac{R_2}{2R_1} = \frac{1}{2\zeta^2} = 20000 = 86.02 \text{ dB}$$

The OrCAD circuit and simulation are shown below.





Problem 14–26. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
Notch	377	0.01	Use 0.022-μF capacitors

Design the circuit using the equal-capacitor method.

$$\omega_0 = 377 \text{ rad/s}$$

$$\zeta = 0.01$$

$$R_1 = \zeta^2 R_2$$

$$C\sqrt{R_1 R_2} = \frac{1}{\omega_0}$$

$$C\zeta R_2 = \frac{1}{\omega_0}$$

$$R_2 = \frac{1}{\zeta C \omega_0} = 12.06 \text{ M}\Omega$$

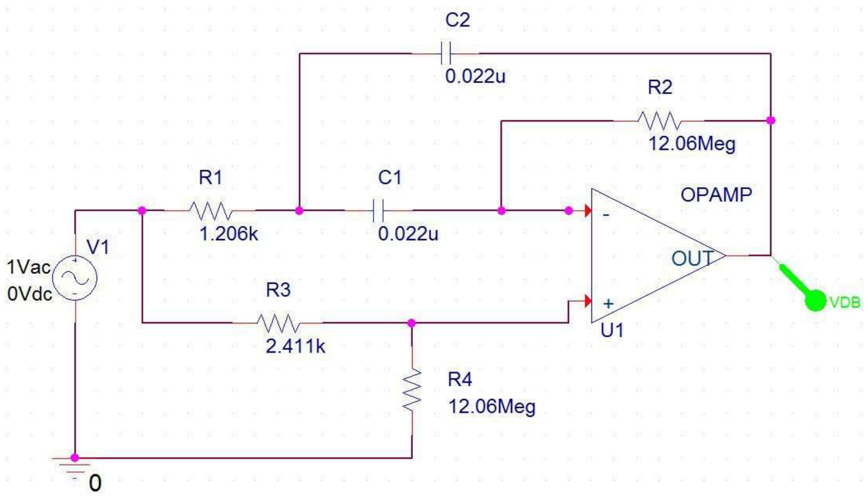
$$R_1 = \zeta^2 R_2 = 1.206 \text{ k}\Omega$$

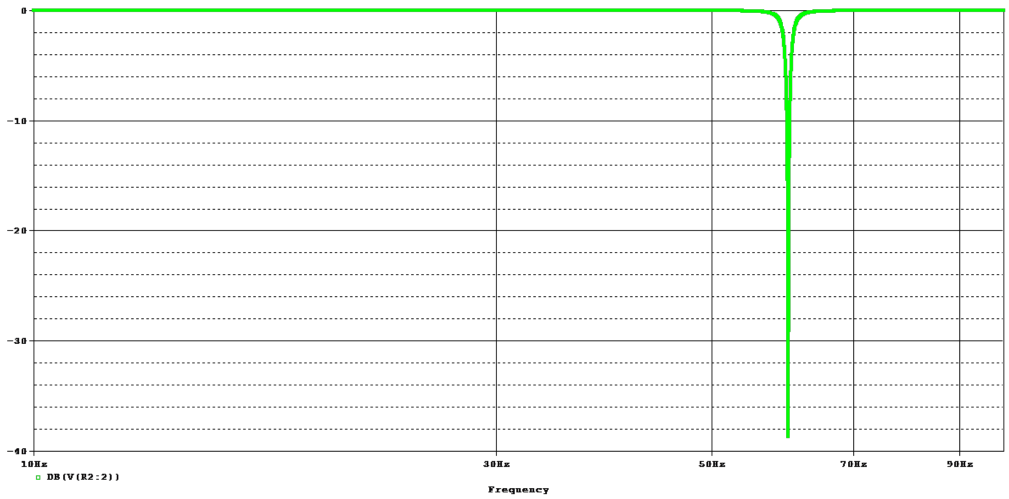
$$\frac{R_{\text{A}}}{R_{\text{B}}} = \frac{2R_1}{R_2}$$

$$R_{\text{A}} = 2R_1 = 2.411 \text{ k}\Omega$$

$$R_{\text{B}} = R_2 = 12.06 \text{ M}\Omega$$

The OrCAD circuit and simulation are shown below.




0
-10
-20
-30
-40
10Hz
30Hz
50Hz
70Hz
90Hz
DB(V(R2:2))
Frequency

Problem 14–27. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
Bandstop	5000	2	Use 0.2-μF capacitors

Design the circuit using the equal-capacitor method.

$$\omega_0 = 5000 \text{ rad/s}$$

$$\zeta = 2$$

$$R_1 = \zeta^2 R_2$$

$$C\sqrt{R_1 R_2} = \frac{1}{\omega_0}$$

$$C\zeta R_2 = \frac{1}{\omega_0}$$

$$R_2 = \frac{1}{\zeta C \omega_0} = 500\ \Omega$$

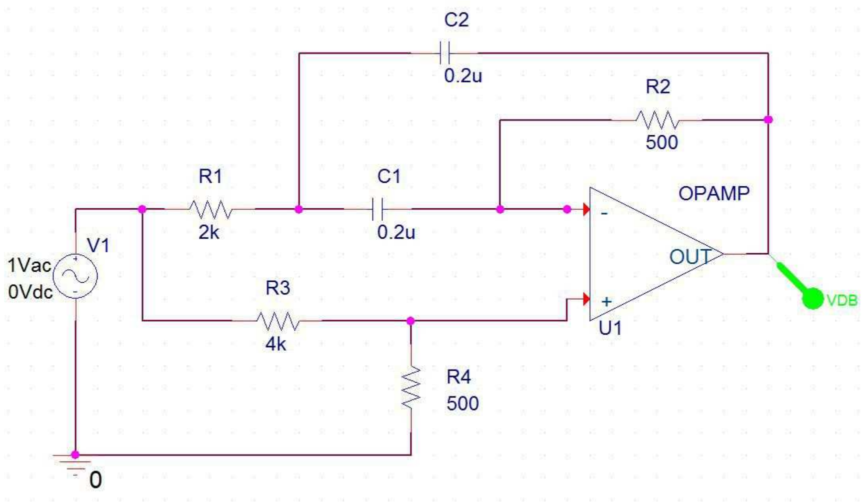
$$R_1 = \zeta^2 R_2 = 2\ \text{k}\Omega$$

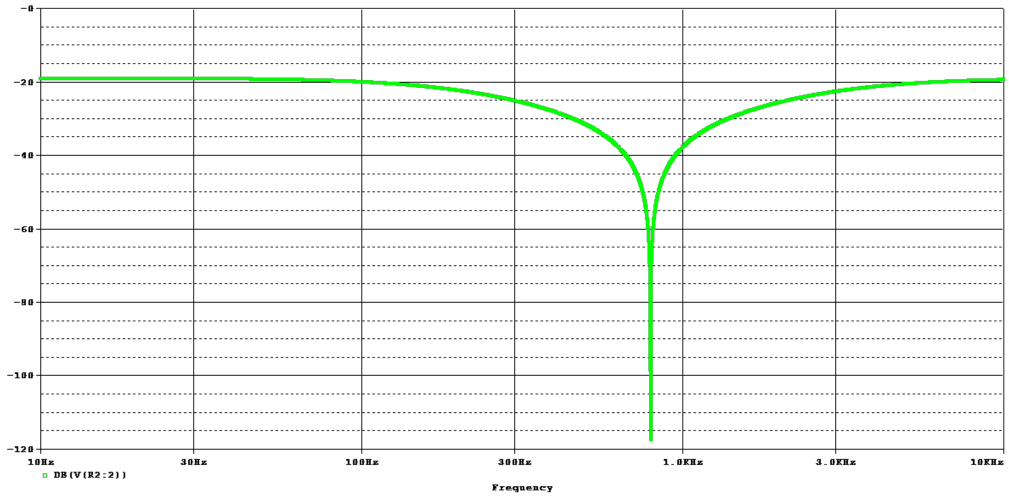
$$\frac{R_\text{A}}{R_\text{B}} = \frac{2R_1}{R_2}$$

$$R_\text{A} = 2R_1 = 4\ \text{k}\Omega$$

$$R_\text{B} = R_2 = 500\ \Omega$$

The OrCAD circuit and simulation are shown below.




-0
-20
-40
-60
-80
-100
-120
10Hz
30Hz
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
DB(V(R2:2))
Frequency

Problem 14–28. Design a second-order active filter that meets the following requirements. Simulate your design in OrCAD to validate the requirements.

Type	ω_0 (rad/s)	ζ	Constraints
Notch	754	0.05	Passband gain of 20 dB

Design the circuit using the equal-capacitor method.

$$\omega_0 = 754 \text{ rad/s}$$

$$\zeta = 0.05$$

$$C = 0.05\ \mu\text{F}$$

$$R_1 = \zeta^2 R_2$$

$$C\sqrt{R_1 R_2} = \frac{1}{\omega_0}$$

$$C\zeta R_2 = \frac{1}{\omega_0}$$

$$R_2 = \frac{1}{\zeta C \omega_0} = 530.5 \text{ k}\Omega$$

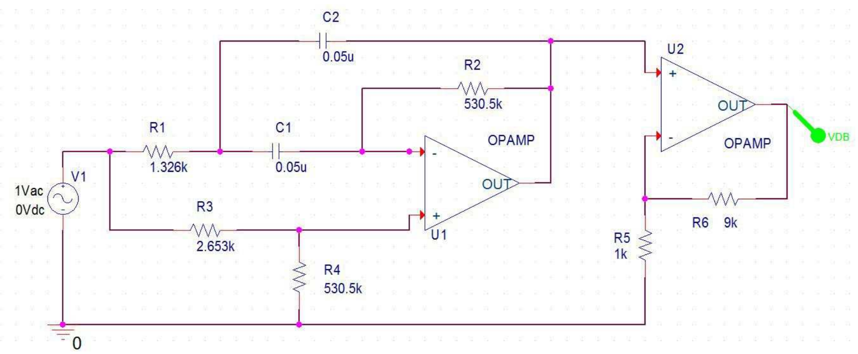
$$R_1 = \zeta^2 R_2 = 1.326 \text{ k}\Omega$$

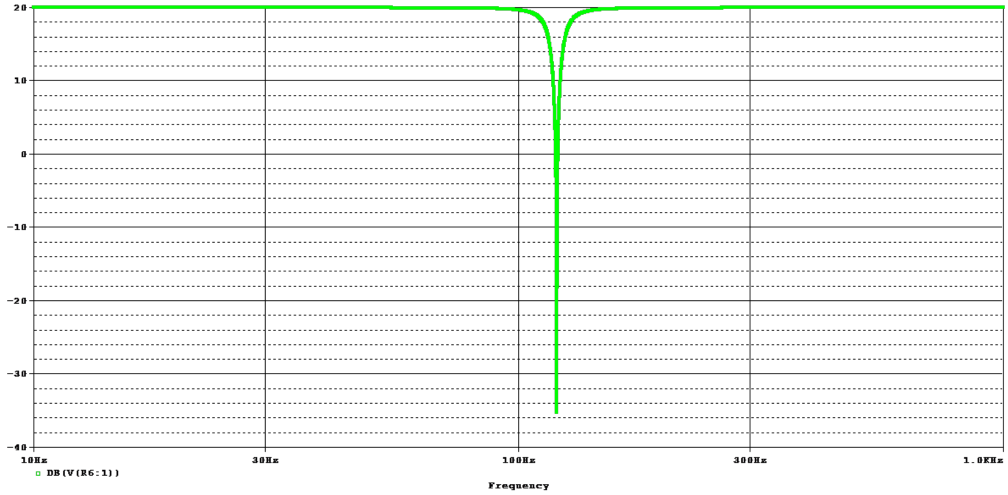
$$\frac{R_\text{A}}{R_\text{B}} = \frac{2R_1}{R_2}$$

$$R_\text{A} = 2R_1 = 2.653 \text{ k}\Omega$$

$$R_\text{B} = R_2 = 530.5 \text{ k}\Omega$$

Add a noninverting gain stage to reach the required gain of 10. The OrCAD circuit and simulation are shown below.




20
10
0
-10
-20
-30
-40
10Hz
30Hz
100Hz
300Hz
1.0KHz
DB(V(R6:1))
Frequency

Problem 14–29. Construct the lowest-order transfer function that meets the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Pole Type	ω_C (rad/s)	T_{MAX}	ω_{MIN} (rad/s)	T_{MIN}
First-order Cascade	5000	60 dB	50000	−20 dB

First, find the filter order. We have the following relationships for a low-pass, first-order cascade design:

$$\alpha = \frac{\omega_C}{\sqrt{2^{1/n} - 1}}$$

$$|T(j\omega_{MIN})| \geq \frac{|T_{MAX}|}{\left[\sqrt{1 + \left(\frac{\omega_{MIN}}{\alpha}\right)^2}\right]^n}$$

$$\frac{T_{MAX}}{T_{MIN}} \leq \left[1 + \left(\frac{\omega_{MIN}}{\alpha}\right)^2\right]^{n/2}$$

$$\frac{T_{MAX}}{T_{MIN}} \leq \left[1 + \left(\frac{\omega_{MIN}}{\omega_C}\right)^2 (2^{1/n} - 1)\right]^{n/2}$$

Find the smallest value of n that satisfies the last inequality. Using MATLAB to search for the value, we find $n = 8$. We then have:

$$\alpha = \frac{\omega_C}{\sqrt{2^{1/n} - 1}} = \frac{5000}{\sqrt{2^{1/8} - 1}} = 16.62 \text{ krad/s}$$

$$K = (T_{MAX})^{1/8} = (1000)^{1/8} = 2.3714$$

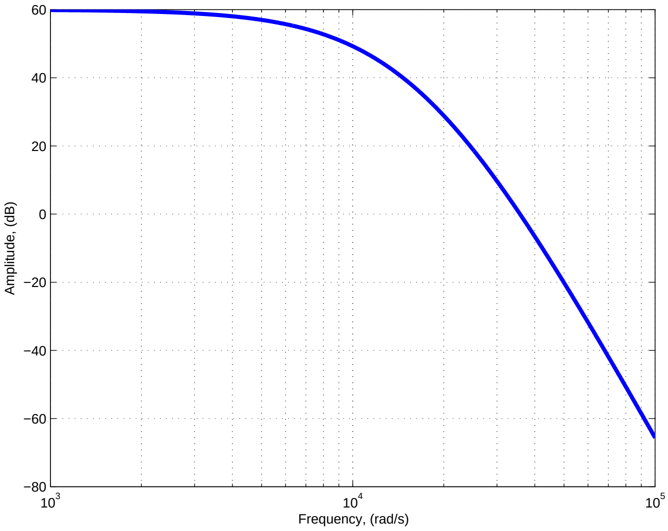
$$T(s) = \left[\frac{K\alpha}{s + \alpha}\right]^n = \left[\frac{(2.3714)(16620)}{s + 16620}\right]^8$$

The transfer function has the following gains, which meet the specifications:

$$|T(j\omega_C)| = 56.99 \text{ dB}$$

$$|T(j\omega_{MIN})| = -20.17 \text{ dB}$$

The following MATLAB plot validates the results.


60
40
20
0
−20
−40
−60
−80
Amplitude, (dB)
10^3
10^4
10^5
Frequency, (rad/s)

Problem 14–30. Construct the lowest-order transfer function that meets the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Pole Type	ω_C (rad/s)	T_{MAX}	ω_{MIN} (rad/s)	T_{MIN}
Butterworth	10000	0 dB	40000	−50 dB

Determine the filter order:

$$n \geq \frac{1}{2}\frac{\ln[(T_{MAX}/T_{MIN})^2 - 1]}{\ln[\omega_{MIN}/\omega_C]} = \frac{1}{2}\frac{\ln[100000 - 1]}{\ln[4]} = 4.15$$

$$n = 5$$

The following steps complete the transfer function using the table of Butterworth polynomials to find $q_n(s)$:

$$K = 0 \text{ dB} = 1$$

$$q_5(s) = (s+1)(s^2 + 0.6180s + 1)(s^2 + 1.618s + 1)$$

$$T(s) = \frac{K}{q_5(s/10000)}$$

$$= \frac{1}{\left[\left(\frac{s}{10000}\right) + 1\right]\left[\left(\frac{s}{10000}\right)^2 + 0.6180\left(\frac{s}{10000}\right) + 1\right]\left[\left(\frac{s}{10000}\right)^2 + 1.618\left(\frac{s}{10000}\right) + 1\right]}$$

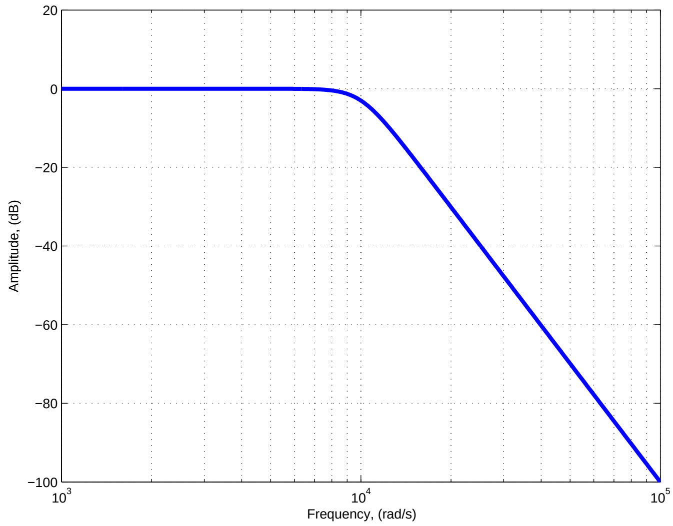
$$= \frac{10^{20}}{(s + 10^4)(s^2 + 6180s + 10^8)(s^2 + 16180s + 10^8)}$$

The transfer function has the following gains, which meet the specifications:

$$|T(j\omega_C)| = -3.01 \text{ dB}$$

$$|T(j\omega_{MIN})| = -60.2 \text{ dB}$$

The following MATLAB plot validates the results.



Problem 14–31. Construct the lowest-order transfer function that meets the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Pole Type	ω_C (rad/s)	T_{MAX}	ω_{MIN} (rad/s)	T_{MIN}
Chebychev	25000	20 dB	250000	−80 dB

Determine the filter order:

$$n \geq \frac{\cosh^{-1}\left[\sqrt{(T_{MAX}/T_{MIN})^2 - 1}\right]}{\cosh^{-1}[\omega_{MIN}/\omega_C]} = \frac{\cosh^{-1}\left[\sqrt{(10^5)^2 - 1}\right]}{\cosh^{-1}[10]} = 4.0779$$

$$n = 5$$

The following steps complete the transfer function using the table of Chebychev polynomials to find $q_n(s)$:

$$K = 20 \text{ dB} = 10$$

$$q_5(s) = [(s/0.1772) + 1][(s/0.9674)^2 + 0.1132(s/0.9674) + 1][(s/0.6139)^2 + 0.4670(s/0.6139) + 1]$$

$$T(s) = \frac{K}{q_5(s/25000)}$$

$$= \frac{10}{\left[\left(\frac{s}{4430}\right) + 1\right]\left[\left(\frac{s}{24185}\right)^2 + 0.1132\left(\frac{s}{24185}\right) + 1\right]\left[\left(\frac{s}{15348}\right)^2 + 0.4670\left(\frac{s}{15348}\right) + 1\right]}$$

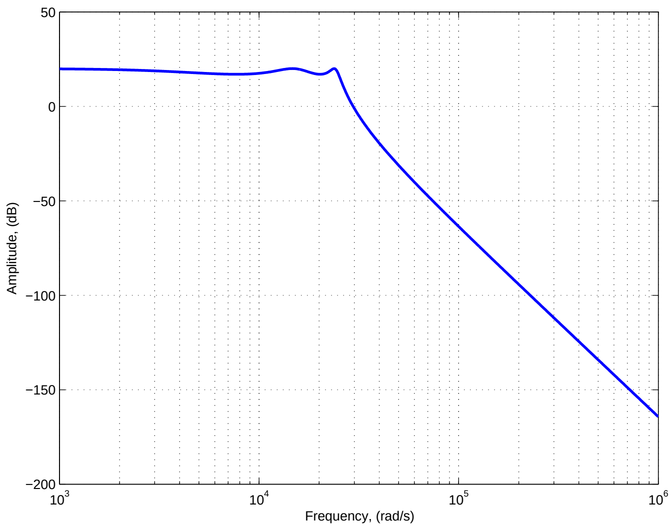
$$= \frac{(10)(4430)(24185)^2(15348)^2}{(s + 4430)(s^2 + 2738s + 24185^2)(s^2 + 7167s + 15348^2)}$$

The transfer function has the following gains, which meet the specifications:

$$|T(j\omega_C)| = 16.99 \text{ dB}$$

$$|T(j\omega_{MIN})| = -103.97 \text{ dB}$$

The following MATLAB plot validates the results.



Problem 14–32. Construct the lowest-order transfer function that meets the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Pole Type	ω_C (rad/s)	T_{MAX}	ω_{MIN} (rad/s)	T_{MIN}
Butterworth	300000	10 dB	600000	−10 dB

Determine the filter order:

$$n \geq \frac{1}{2}\frac{\ln[(T_{MAX}/T_{MIN})^2 - 1]}{\ln[\omega_{MIN}/\omega_C]} = \frac{1}{2}\frac{\ln[100-1]}{\ln[2]} = 3.31$$

$$n = 4$$

The following steps complete the transfer function using the table of Butterworth polynomials to find $q_n(s)$:

$$K = 10 \text{ dB} = \sqrt{10}$$

$$q_4(s) = (s^2 + 0.7654s + 1)(s^2 + 1.848s + 1)$$

$$T(s) = \frac{K}{q_4(s/300000)}$$

$$= \frac{\sqrt{10}}{\left[\left(\frac{s}{300000}\right)^2 + 0.7654\left(\frac{s}{300000}\right) + 1\right]\left[\left(\frac{s}{300000}\right)^2 + 1.848\left(\frac{s}{300000}\right) + 1\right]}$$

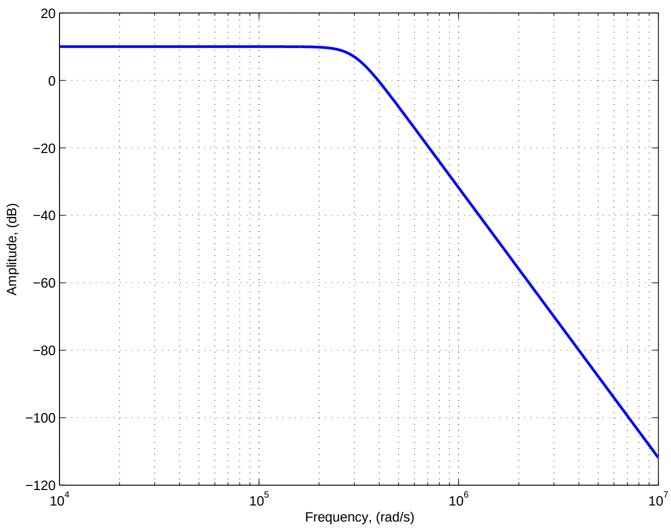
$$= \frac{\sqrt{10}(300000)^4}{(s^2 + 229620s + 300000^2)(s^2 + 554400s + 300000^2)}$$

The transfer function has the following gains, which meet the specifications:

$$|T(j\omega_C)| = 6.99 \text{ dB}$$

$$|T(j\omega_{MIN})| = -14.1 \text{ dB}$$

The following MATLAB plot validates the results.



Problem 14–33. Construct the lowest-order transfer function that meets the following low-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Pole Type	ω_C (rad/s)	T_{MAX}	ω_{MIN} (rad/s)	T_{MIN}
Chebychev	2000000	20 dB	4000000	−10 dB

Determine the filter order:

$$n \geq \frac{\cosh^{-1}\left[\sqrt{(T_{MAX}/T_{MIN})^2 - 1}\right]}{\cosh^{-1}[\omega_{MIN}/\omega_C]} = \frac{\cosh^{-1}\left[\sqrt{(31.63)^2 - 1}\right]}{\cosh^{-1}[2]} = 3.15$$

$$n = 4$$

The following steps complete the transfer function using the table of Chebychev polynomials to find $q_n(s)$:

$$K = 20 \text{ dB} = 10$$

$$q_4(s) = [(s/0.9502)^2 + 0.1789(s/0.9502) + 1][(s/0.4425)^2 + 0.9276(s/0.4425) + 1]$$

$$T(s) = \frac{K/\sqrt{2}}{q_4(s/2000000)}$$

$$= \frac{7.071}{\left[\left(\frac{s}{1900400}\right)^2 + 0.1789\left(\frac{s}{1900400}\right) + 1\right]\left[\left(\frac{s}{885000}\right)^2 + 0.9276\left(\frac{s}{885000}\right) + 1\right]}$$

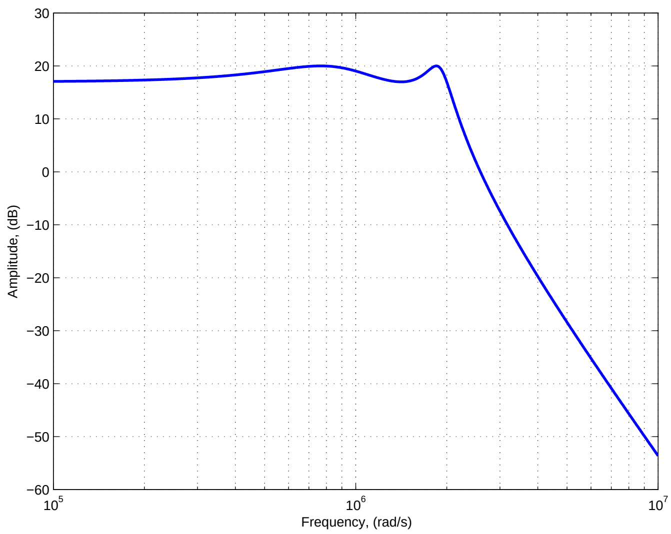
$$= \frac{(7.071)(1900400)^2(885000)^2}{(s^2 + 340000s + 1900400^2)(s^2 + 820900s + 885000^2)}$$

The transfer function has the following gains, which meet the specifications:

$$|T(j\omega_C)| = 16.99 \text{ dB}$$

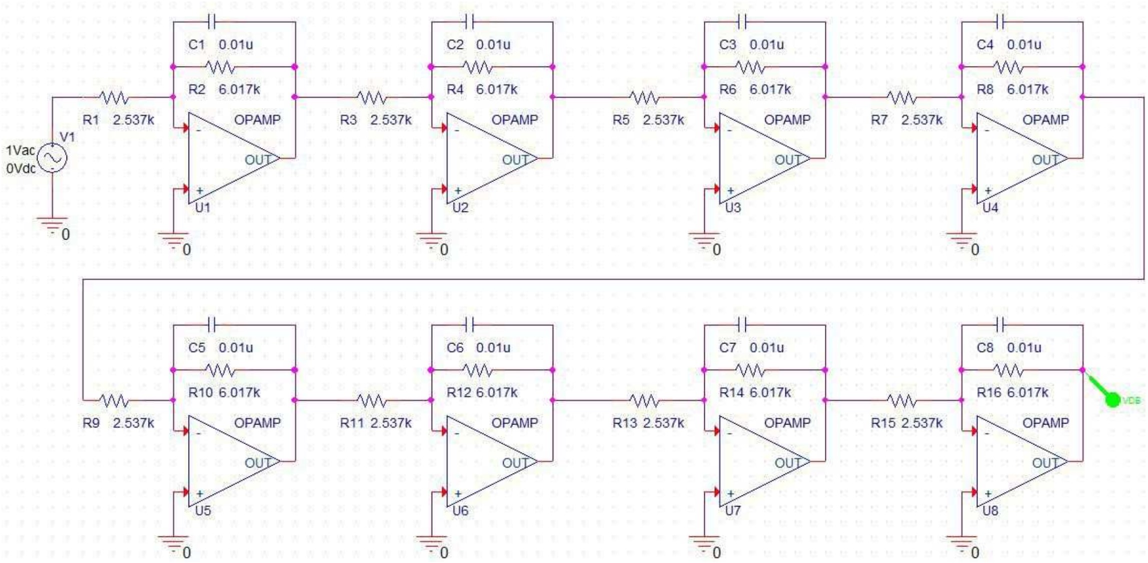
$$|T(j\omega_{MIN})| = -19.74 \text{ dB}$$

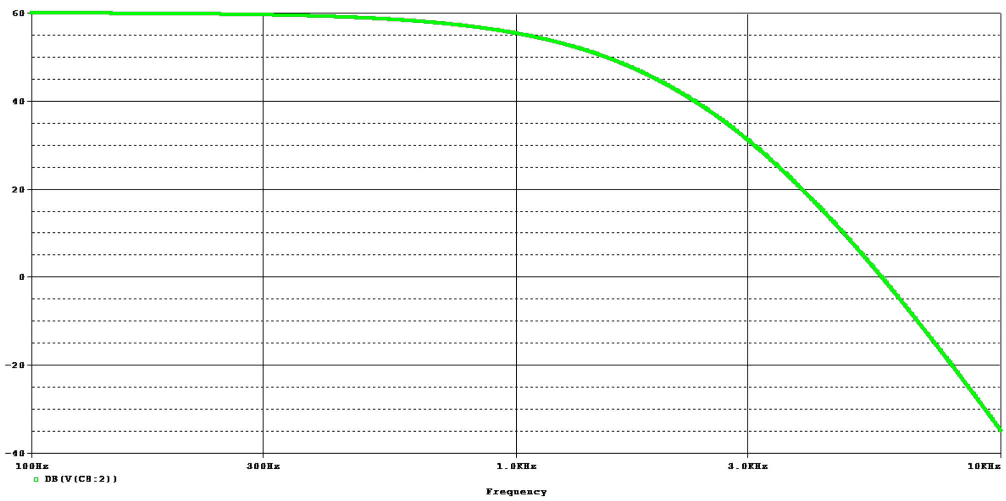
The following MATLAB plot validates the results.



Problem 14–34. Design an active low-pass filter to meet the specification in Problem 14–29. Use OrCAD to verify that your design meets the specifications.

Each stage has a cutoff frequency of $\omega_C = 16.62$ krad/s and a gain of $K = 2.3714$. Use a first-order, low-pass OP AMP design with gain. Pick $C_2 = 0.01\ \mu$F and solve for $R_2 = 1/(\omega_C C_2) = 6.017$ kΩ. Solve for $R_1 = R_2/K = 2.537$ kΩ. Use eight identical stages in cascade. The OrCAD simulation and results are shown below.





Problem 14–35. Design an active low-pass filter to meet the specification in Problem 14–30. Use OrCAD to verify that your design meets the specifications.

The first stage is a first-order, low-pass OP AMP filter with a gain of one. Pick $C = 0.01\ \mu$F and solve for $R_1 = R_2 = 1/(\omega_0 C) = 10\ \text{k}\Omega$. Use a unity-gain design approach for the other two second-order stages. For the second stage we have the following design choices and results:

$$\zeta = \frac{0.618}{2} = 0.309$$

$$C_1 = 0.01\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 954.8\ \text{pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 32.363\ \text{k}\Omega$$

For the third stage we have the following design choices and results:

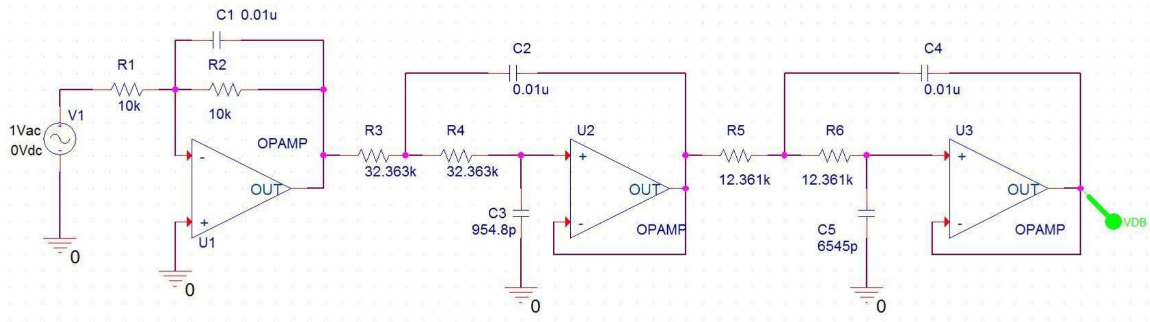
$$\zeta = \frac{1.618}{2} = 0.809$$

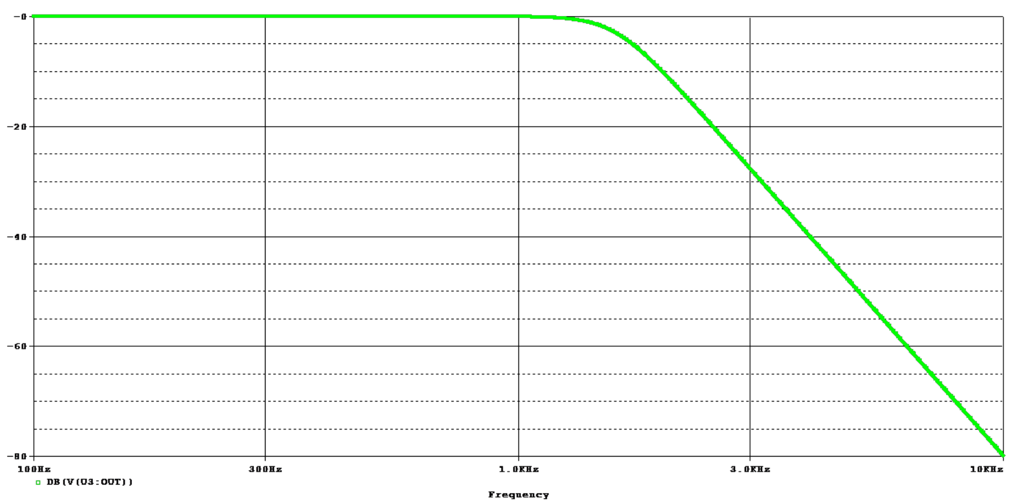
$$C_1 = 0.01\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 6545\ \text{pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 12.361\ \text{k}\Omega$$

The OrCAD simulation and results are shown below.





Problem 14–36. Design an active low-pass filter to meet the specification in Problem 14–31. Use OrCAD to verify that your design meets the specifications.

The first stage is a first-order, low-pass OP AMP filter with a cutoff frequency of 4430 rad/s and a gain of 10. Pick $C = 0.01\ \mu$F and solve for $R_2 = 1/(\omega_0 C) = 22.573$ kΩ. Solve for $R_1 = R_2/K = 2.2573$ kΩ. Use a unity-gain design approach for the other two second-order stages. For the second stage we have the following design choices and results:

$$\zeta = \frac{0.1132}{2} = 0.0566$$

$$\omega_0 = 24185 \text{ rad/s}$$

$$C_1 = 0.01\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 32.0356 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 73.0529 \text{ k}\Omega$$

For the third stage we have the following design choices and results:

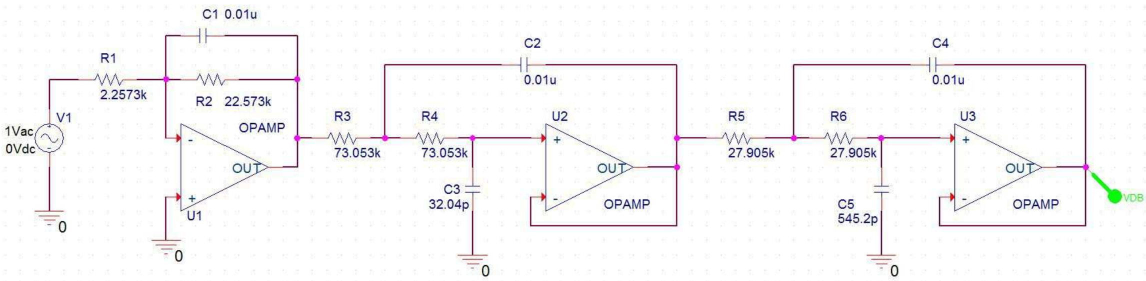
$$\zeta = \frac{0.4670}{2} = 0.2335$$

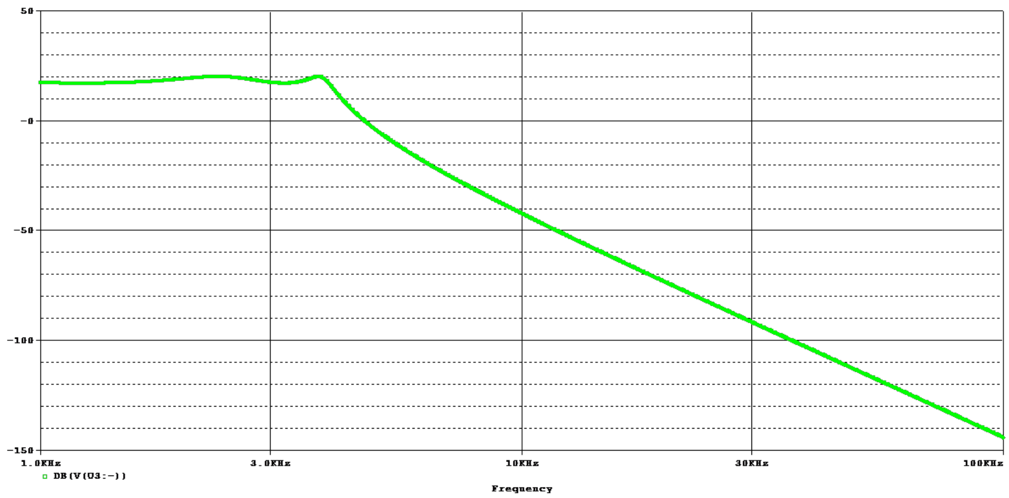
$$C_1 = 0.01\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 545.2225 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 27.9046 \text{ k}\Omega$$

The OrCAD simulation and results are shown below.





Problem 14–37. Design an active low-pass filter to meet the specification in Problem 14–32. Use OrCAD to verify that your design meets the specifications.

Use a noninverting amplifier with a gain of 3.1623 to provide all of the gain for the transfer function. Use a unity-gain design approach for the two second-order stages. For the first filter stage we have the following design choices and results:

$$\zeta = \frac{0.7654}{2} = 0.3827$$

$$\omega_0 = 300000 \text{ rad/s}$$

$$C_1 = 0.001\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 146.4593 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 8.71\ \text{k}\Omega$$

For the second filter stage we have the following design choices and results:

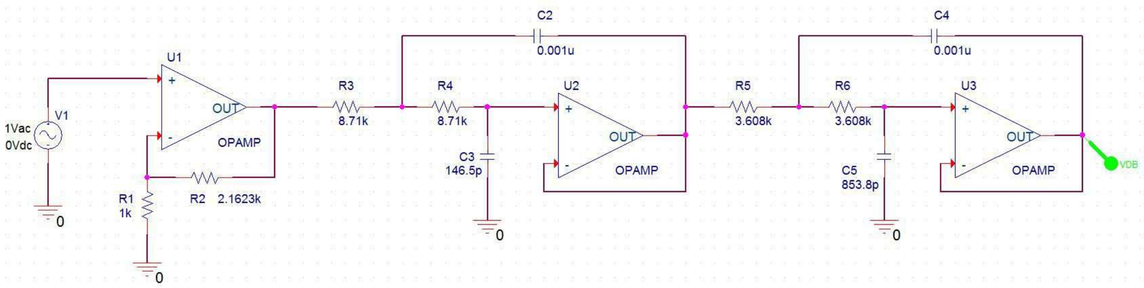
$$\zeta = \frac{1.848}{2} = 0.924$$

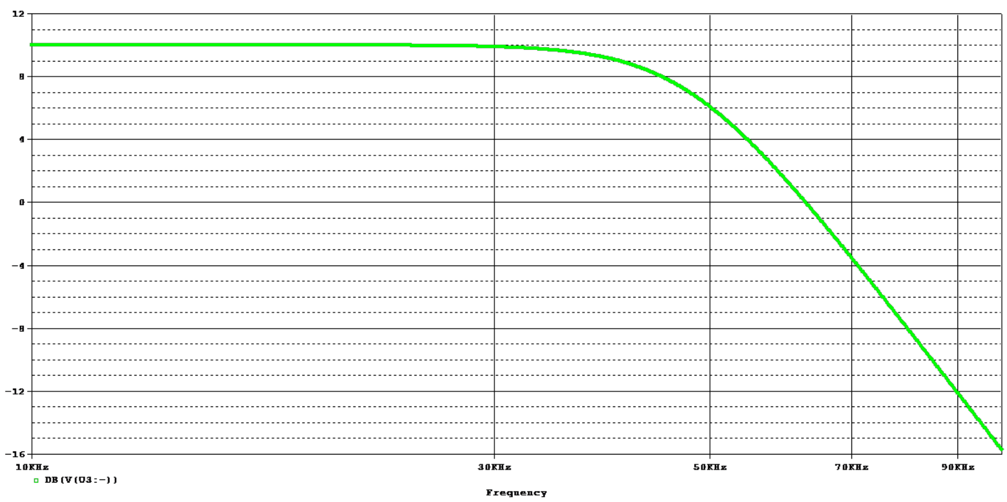
$$C_1 = 0.001\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 853.776 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 3.6075\ \text{k}\Omega$$

The OrCAD simulation and results are shown below.





Problem 14–38. Design an active low-pass filter to meet the specification in Problem 14–33. Use OrCAD to verify that your design meets the specifications.

Use a noninverting amplifier with a gain of 7.07 to provide all of the gain for the transfer function. Use a unity-gain design approach for the two second-order stages. For the first filter stage we have the following design choices and results:

$$\zeta = \frac{0.1789}{2} = 0.08945$$

$$\omega_0 = (0.9502)(2000000) = 1900400 \text{ rad/s}$$

$$C_1 = 0.001\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 8\text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 5.8827\text{ k}\Omega$$

For the second filter stage we have the following design choices and results:

$$\zeta = \frac{0.9276}{2} = 0.4638$$

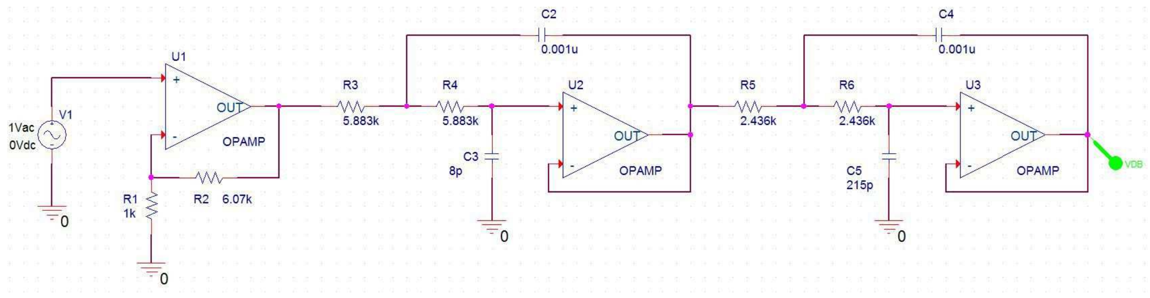
$$\omega_0 = (0.4425)(2000000) = 885000 \text{ rad/s}$$

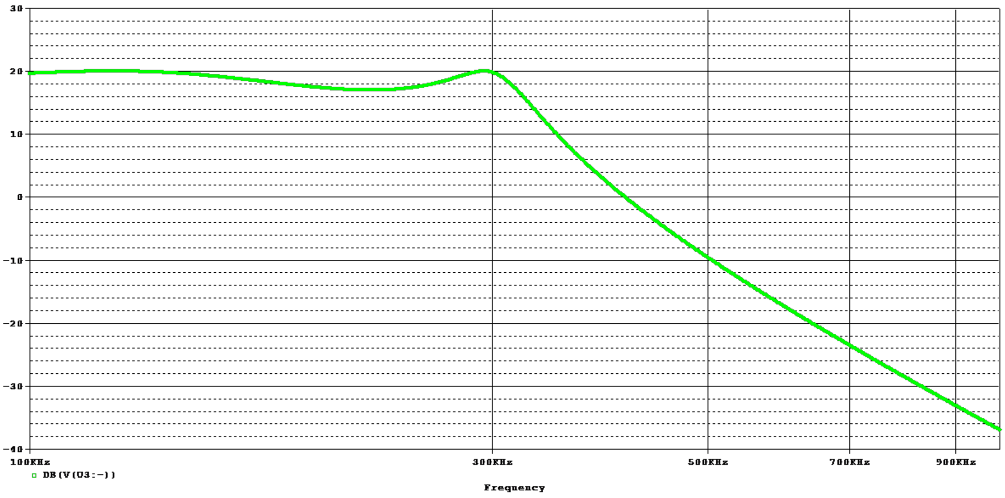
$$C_1 = 0.001\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 215\text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 2.4363\text{ k}\Omega$$

The OrCAD simulation and results are shown below.




30
20
10
0
−10
−20
−30
−40
100KHz
300KHz
500KHz
700KHz
900KHz
DB(V(U3:−))
Frequency

Problem 14–39. A low-pass filter is needed to suppress the harmonics in a periodic waveform with $f_0 =$ 1 kHz. The filter must have unity passband gain, less than -50 dB gain at the third harmonic, and less than -80 dB gain at the fifth harmonic. Since power is at a premium, choose a filter approach that minimizes the number of OP AMPs. Design a filter that meets these requirements. Verify your design using OrCAD.

The filter requires $\omega_0 = 2000\pi = 6283$ rad/s and $K = 1 = 0$ dB. At $3\omega_0$ the gain must be -50 dB and at $5\omega_0$ the gain must be -80 dB. Determine the filter order required for each design specification using both Butterworth and Chebychev designs. With a Butterworth design, both gain specifications require sixth-order filters. With a Chebychv design, the -50 dB critierion requires a fourth-order filter and the -80 dB critierion requires a fifth-order filter. To reduce the total number of OP AMPs, use a fifth-order Chebychev design, with the first-order portion of the filter using a passive RC design. The transfer function is:

$$q_5(s) = [(s/0.1772)+1][(s/0.9674)^2 + 0.1132(s/0.9674)+1][(s/0.6139)^2 + 0.4670(s/0.6139)+1]$$

$$\begin{aligned} T(s) &= \frac{K}{q_5(s/\omega_0)} \\ &= \frac{1}{\left[\left(\frac{s}{1113}\right)+1\right]\left[\left(\frac{s}{6078}\right)^2 + 0.1132\left(\frac{s}{6078}\right)+1\right]\left[\left(\frac{s}{3857}\right)^2 + 0.4670\left(\frac{s}{3857}\right)+1\right]} \\ &= \frac{(1)(1113)(6078)^2(3857)^2}{(s+1113)(s^2+688s+6078^2)(s^2+1801s+3857^2)} \end{aligned}$$

For the first-order RC stage, we have the following design choices and results:

$$\omega_0 = 1113 \text{ rad/s}$$

$$C = 0.1\ \mu\text{F}$$

$$R = \frac{1}{C\omega_0} = 8.982\ \text{k}\Omega$$

For the second filter stage we have the following design choices and results:

$$\zeta = \frac{0.1132}{2} = 0.0566$$

$$\omega_0 = (0.9674)(6283) = 6078 \text{ rad/s}$$

$$C_1 = 0.1\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 320 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 29.0667\ \text{k}\Omega$$

For the third filter stage we have the following design choices and results:

$$\zeta = \frac{0.4670}{2} = 0.2335$$

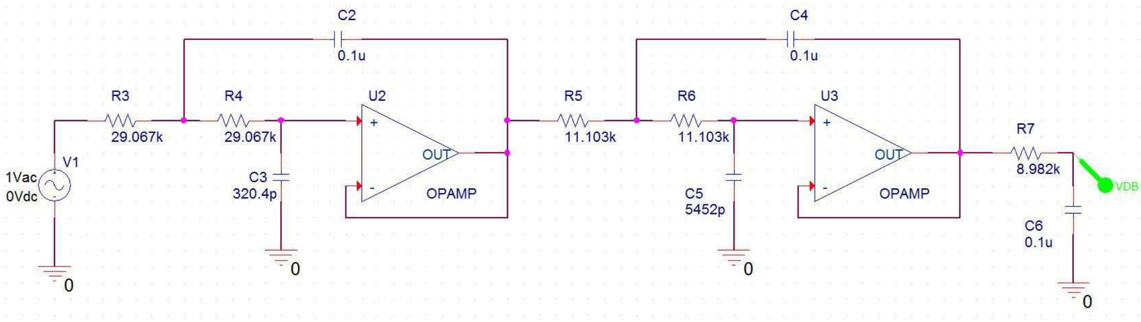
$$\omega_0 = (0.6139)(6283) = 3857 \text{ rad/s}$$

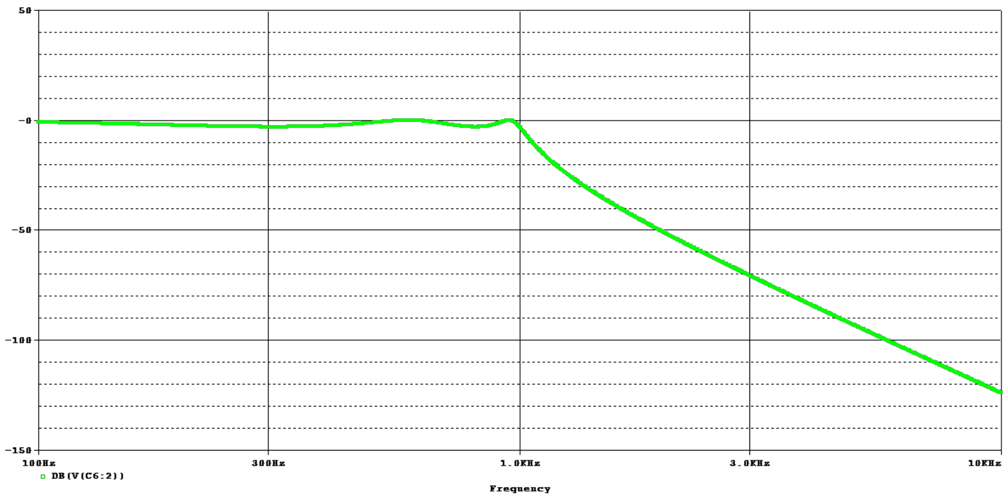
$$C_1 = 0.1\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 5452 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 11.1029\ \text{k}\Omega$$

The OrCAD simulation and results are shown below.





Problem 14–40. Design a low-pass filter with 6 dB passband gain, a cutoff frequency of 2 kHz, and a stopband gain of less than −14 dB at 6 kHz. The filter must not have an overshoot. Verify your design using OrCAD.

To guarantee that there is no overshoot, use a first-order cascade design. We have the following relationships for a low-pass, first-order cascade design:

$$\alpha = \frac{\omega_C}{\sqrt{2^{1/n}-1}}$$

$$|T(j\omega_{MIN})| \geq \frac{|T_{MAX}|}{\left[\sqrt{1+\left(\frac{\omega_{MIN}}{\alpha}\right)^2}\right]^n}$$

$$\frac{T_{MAX}}{T_{MIN}} \leq \left[1+\left(\frac{\omega_{MIN}}{\alpha}\right)^2\right]^{n/2}$$

$$\frac{T_{MAX}}{T_{MIN}} \leq \left[1+\left(\frac{\omega_{MIN}}{\omega_C}\right)^2(2^{1/n}-1)\right]^{n/2}$$

Find the smallest value of n that satisfies the last inequality. Using MATLAB to search for the value, we find $n = 7$. We then have:

$$\alpha = \frac{\omega_C}{\sqrt{2^{1/n}-1}} = \frac{4000\pi}{\sqrt{2^{1/7}-1}} = 38.95 \text{ krad/s}$$

$$K = (T_{MAX})^{1/7} = (1.9953)^{1/7} = 1.1037$$

$$T(s) = \left[\frac{K\alpha}{s+\alpha}\right]^n = \left[\frac{(1.1037)(38950)}{s+38950}\right]^7$$

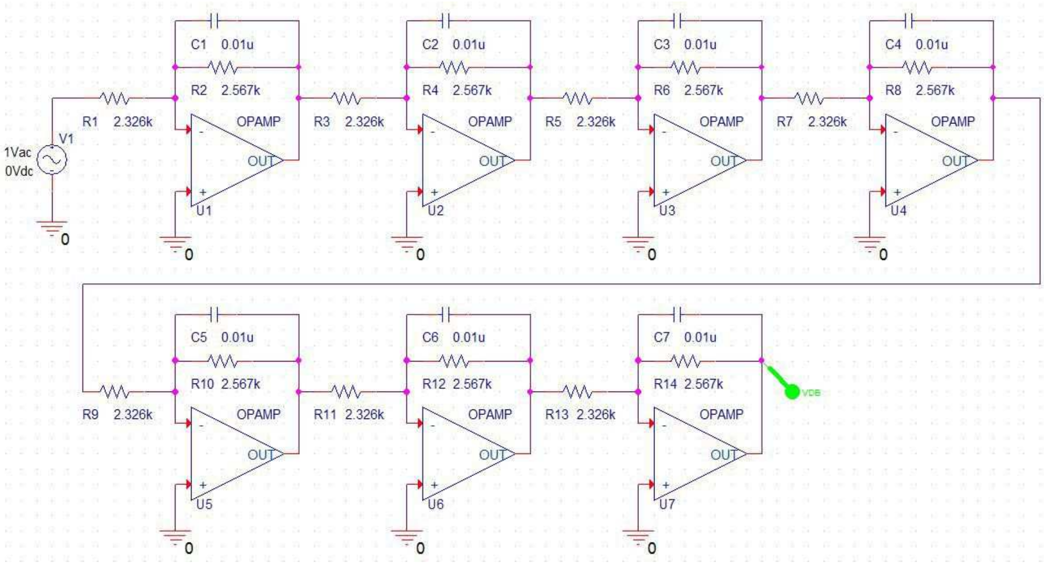
Use a first-order RC OP AMP filter with gain for each of the seven identical stages. Pick the capacitor value and solve for the resistor values.

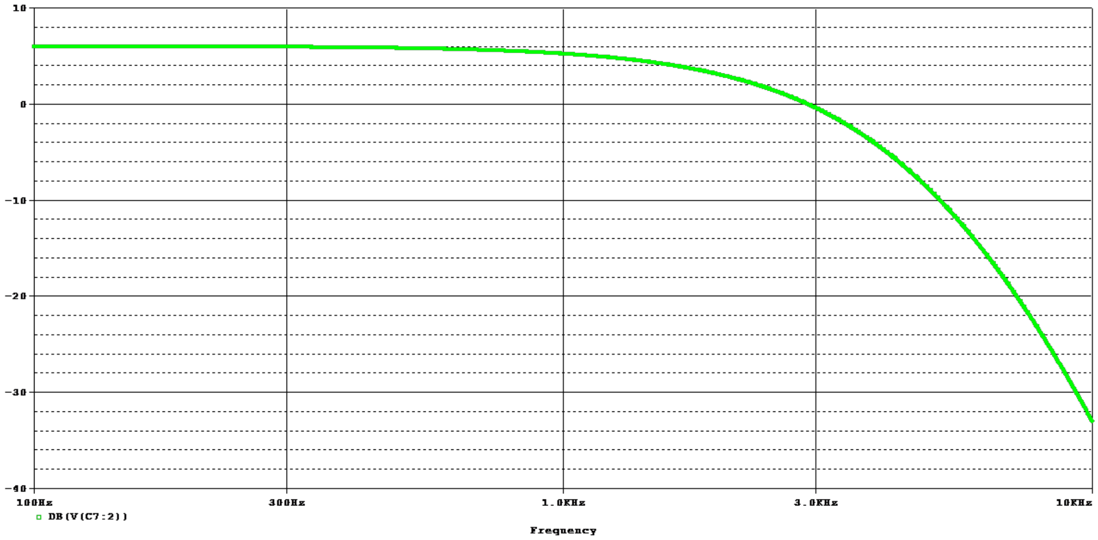
$$C_2 = 0.01\ \mu\text{F}$$

$$R_2 = \frac{1}{\alpha C_2} = 2.567\ \text{k}\Omega$$

$$R_1 = \frac{R_2}{K} = 2.326\ \text{k}\Omega$$

The following OrCAD simulation and output validate the results.


C1 0.01u
R2 2.567k
R1 2.326k
OPAMP
OUT
U1
V1
1Vac
0Vdc
0
C2 0.01u
R4 2.567k
R3 2.326k
U2
C3 0.01u
R6 2.567k
R5 2.326k
U3
C4 0.01u
R8 2.567k
R7 2.326k
U4
C5 0.01u
R10 2.567k
R9 2.326k
U5
C6 0.01u
R12 2.567k
R11 2.326k
U6
C7 0.01u
R14 2.567k
R13 2.326k
U7


10
0
-10
-20
-30
-40
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
DB(V(C7:2))
Frequency

Problem 14–41. Design a low-pass filter with 0 dB passband gain, a cutoff frequency of 1 kHz, and a stopband gain of less than −50 dB at 4 kHz. The filter must not have an overshoot greater than 13%. Verify your design using OrCAD.

A Butterworth design will maintain an overshoot of less than 13%. Solve for the filter order.

$$n \geq \frac{1}{2}\frac{\ln[(T_{\mathrm{MAX}}/T_{\mathrm{MIN}})^2 - 1]}{\ln[\omega_{\mathrm{MIN}}/\omega_{\mathrm{C}}]} = \frac{1}{2}\frac{\ln[100000 - 1]}{\ln[4]} = 4.15$$

$$n = 5$$

The following steps complete the transfer function using the table of Butterworth polynomials to find $q_n(s)$:

$$K = 0\text{ dB} = 1$$

$$q_5(s) = (s+1)(s^2 + 0.6180s + 1)(s^2 + 1.618s + 1)$$

$$T(s) = \frac{K}{q_5(s/6283)}$$

$$= \frac{1}{\left[\left(\frac{s}{6283}\right) + 1\right]\left[\left(\frac{s}{6283}\right)^2 + 0.6180\left(\frac{s}{6283}\right) + 1\right]\left[\left(\frac{s}{6283}\right)^2 + 1.618\left(\frac{s}{6283}\right) + 1\right]}$$

$$= \frac{6283^5}{(s+6283)(s^2 + 3883s + 6283^2)(s^2 + 10166s + 6283^2)}$$

The first stage is an active, first-order RC low-pass filter with a gain of one. The other two stages are second-order, active low-pass filters using the unity-gain design approach. For the first stage we have:

$$C_2 = 0.1\ \mu\text{F}$$

$$R_2 = \frac{1}{\omega_0 C} = 1.592\text{ k}\Omega$$

$$R_1 = R_2 = 1.592\text{ k}\Omega$$

For the second stage we have:

$$\zeta = \frac{0.618}{2} = 0.309$$

$$C_1 = 0.1\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 9548\text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 5.151\text{ k}\Omega$$

For the third stage we have:

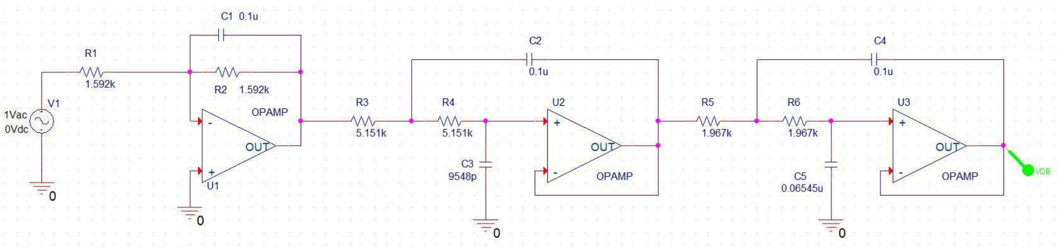
$$\zeta = \frac{1.618}{2} = 0.809$$

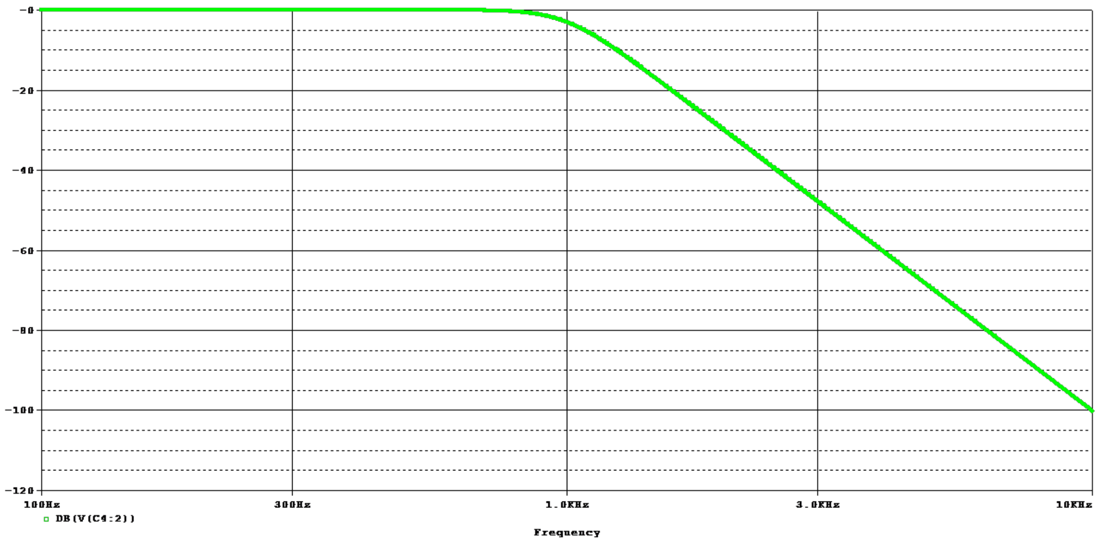
$$C_1 = 0.1\ \mu\text{F}$$

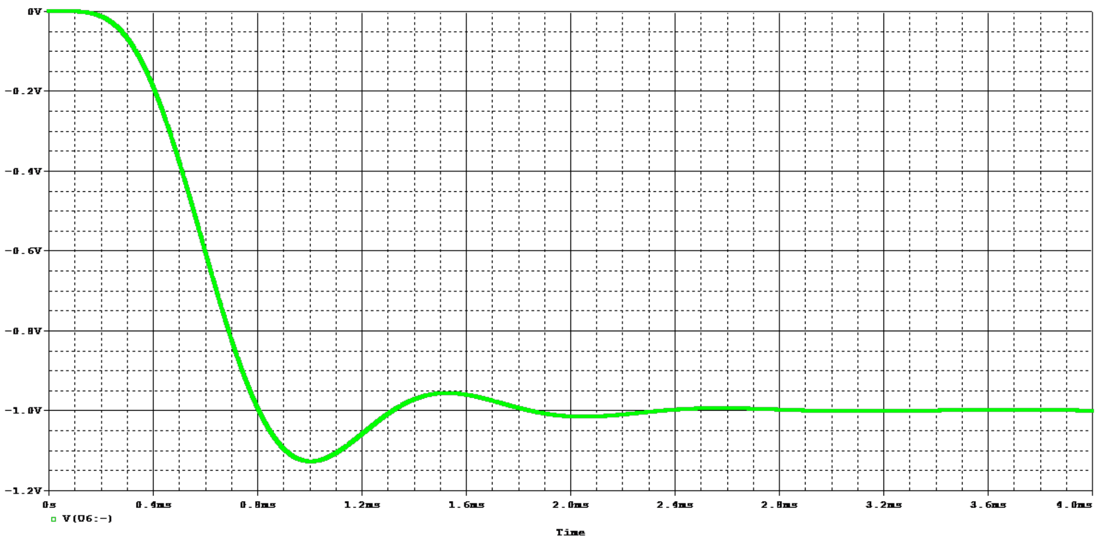
$$C_2 = \zeta^2 C_1 = 0.06545\ \mu\text{F}$$

$$R = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 1.967\text{ k}\Omega$$

The following OrCAD simulation and output plots validate the results.


C1 0.1u
R1
1.592k
R2 1.592k
1Vac
0Vdc
V1
OPAMP
OUT
U1
R3
5.151k
R4
5.151k
C3
9548p
C2
0.1u
U2
OUT
OPAMP
R5
1.967k
R6
1.967k
C5
0.06545u
C4
0.1u
U3
OUT
OPAMP
0


-0
-20
-40
-60
-80
-100
-120
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
DB(V(C4:2))
Frequency


0V
-0.2V
-0.4V
-0.6V
-0.8V
-1.0V
-1.2V
0s
0.4ms
0.8ms
1.2ms
1.6ms
2.0ms
2.4ms
2.8ms
3.2ms
3.6ms
4.0ms
V(U6:-)
Time

Problem 14–42. Design a low-pass filter with 10 dB passband gain, a cutoff frequency of 1 kHz, and a stopband gain of less than −20 dB at 2 kHz. Overshoot is not a problem, but a low filter order, least number of parts, and a maximum of two OP AMPs is desired. Verify your design using OrCAD.

To minimize the number of parts and use only two OP AMPs, explore a Chebychev design approach. Determine the filter order:

$$n \geq \frac{\cosh^{-1}\left[\sqrt{(T_{\text{MAX}}/T_{\text{MIN}})^2 - 1}\right]}{\cosh^{-1}[\omega_{\text{MIN}}/\omega_{\text{C}}]} = \frac{\cosh^{-1}\left[\sqrt{(31.63)^2 - 1}\right]}{\cosh^{-1}[2]} = 3.15$$

$$n = 4$$

The following steps complete the transfer function using the table of Chebychev polynomials to find $q_n(s)$:

$$K = 10 \text{ dB} = 3.162$$

$$q_4(s) = [(s/0.9502)^2 + 0.1789(s/0.9502) + 1][(s/0.4425)^2 + 0.9276(s/0.4425) + 1]$$

$$T(s) = \frac{K/\sqrt{2}}{q_4(s/6283)}$$

$$= \frac{2.236}{\left[\left(\frac{s}{5970}\right)^2 + 0.1789\left(\frac{s}{5970}\right) + 1\right]\left[\left(\frac{s}{2780}\right)^2 + 0.9276\left(\frac{s}{2780}\right) + 1\right]}$$

$$= \frac{(2.236)(5970)^2(2780)^2}{(s^2 + 1068s + 5970^2)(s^2 + 2579s + 2780^2)}$$

The two filter stages are second-order, active low-pass filters using the unity-gain design approach. For the first stage we have:

$$\zeta = \frac{0.1789}{2} = 0.08945$$

$$\omega_0 = 5970 \text{ rad/s}$$

$$C_1 = 0.1\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 800 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 18.725 \text{ k}\Omega$$

For the second stage we have:

$$\zeta = \frac{0.9276}{2} = 0.4638$$

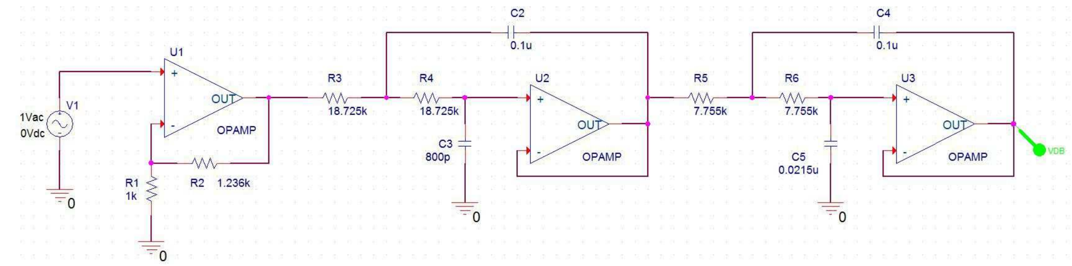
$$\omega_0 = 2780 \text{ rad/s}$$

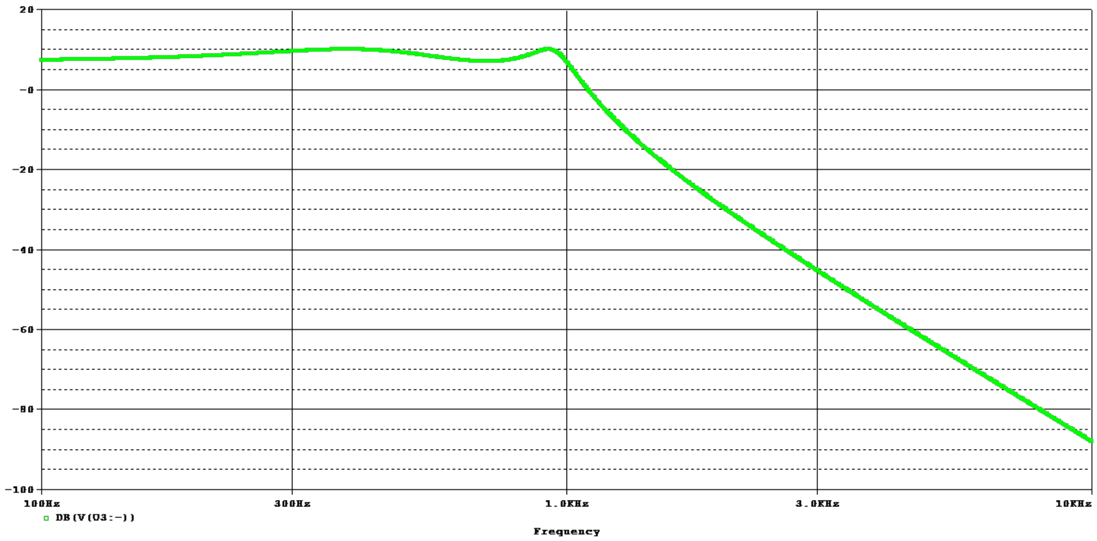
$$C_1 = 0.1\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 0.0215\ \mu\text{F}$$

$$R = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 7.755 \text{ k}\Omega$$

Include a noninverting amplifier as the third stage to add the required gain of 2.236. The following OrCAD simulation and output plot validate the results.


C2
0.1u
C4
0.1u
U1
R3
18.725k
R4
18.725k
U2
R5
7.755k
R6
7.755k
U3
1Vac
0Vdc
V1
OUT
OPAMP
C3
800p
C5
0.0215u
R1
1k
R2 1.236k
0
VDB


20
0
-20
-40
-60
-80
-100
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
DB(V(U3:-))
Frequency

Problem 14–43. A pesky signal at 100 kHz is interfering with a desired signal at 20 kHz. A careful analysis suggests that reducing the interfering signal by 72 dB will eliminate the problem, provided the desired signal is not reduced by more than 3 dB. Design an active RC filter that meets these requirements. Verify your design using OrCAD.

Design the filter with a passband gain of 20 dB, a cutoff frequency of $\omega_0 = 40\pi$ krad/s, $T_{\rm MIN} = -52$ dB, and $\omega_{\rm MIN} = 200\pi$ krad/s. The required filter order is either a sixth-order Butterworth design or a fourth-order Chebychev design. Since the desired signal can tolerate a 3-dB variation, use the Chebychev approach, since it will require fewer OP AMPs.

$$K = 20\text{ dB} = 10$$

$$q_4(s) = [(s/0.9502)^2 + 0.1789(s/0.9502) + 1][(s/0.4425)^2 + 0.9276(s/0.4425) + 1]$$

$$\begin{aligned}
T(s) &= \frac{K/\sqrt{2}}{q_4(s/125664)} \\
&= \frac{7.071}{\left[\left(\frac{s}{119406}\right)^2 + 0.1789\left(\frac{s}{119406}\right) + 1\right]\left[\left(\frac{s}{55606}\right)^2 + 0.9276\left(\frac{s}{55606}\right) + 1\right]} \\
&= \frac{(7.071)(119406)^2(55606)^2}{(s^2 + 21362s + 119406^2)(s^2 + 51580s + 55606^2)}
\end{aligned}$$

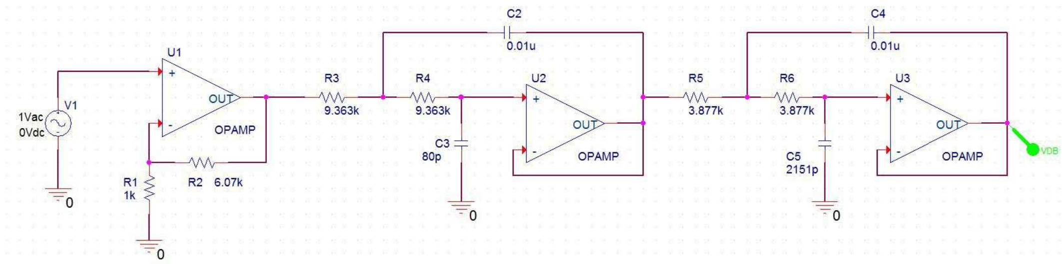
The two filter stages are second-order, active low-pass filters using the unity-gain design approach. For the first stage we have:

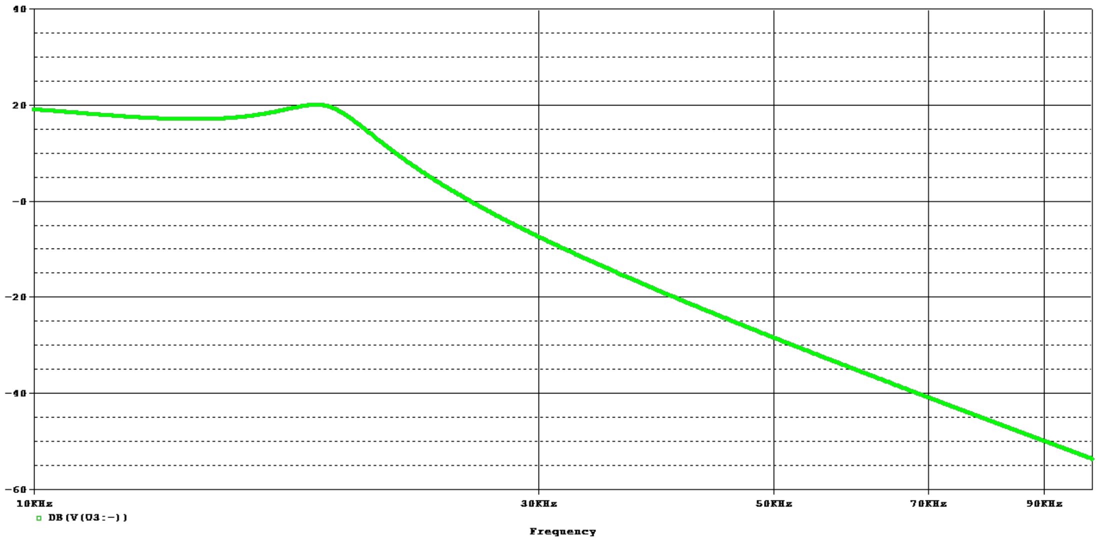
$$\begin{aligned}
\zeta &= \frac{0.1789}{2} = 0.08945 \\
\omega_0 &= 119406\text{ rad/s} \\
C_1 &= 0.01\ \mu\text{F} \\
C_2 &= \zeta^2 C_1 = 80\text{ pF} \\
R &= \frac{1}{\omega_0\sqrt{C_1C_2}} = 9.363\text{ k}\Omega
\end{aligned}$$

For the second stage we have:

$$\begin{aligned}
\zeta &= \frac{0.9276}{2} = 0.4638 \\
\omega_0 &= 55606\text{ rad/s} \\
C_1 &= 0.01\ \mu\text{F} \\
C_2 &= \zeta^2 C_1 = 0.00215\ \mu\text{F} \\
R &= \frac{1}{\omega_0\sqrt{C_1C_2}} = 3.877\text{ k}\Omega
\end{aligned}$$

Include a noninverting amplifier as the third stage to add the required gain of 7.071. The following OrCAD simulation and output plot validate the results.


C2
0.01u
C4
0.01u
U1
R3
9.363k
R4
9.363k
U2
R5
3.877k
R6
3.877k
U3
1Vac
0Vdc
V1
OUT
OPAMP
C3
80p
C5
2151p
R1
1k
R2 6.07k
0
VDB


40
20
0
-20
-40
-60
10KHz
30KHz
50KHz
70KHz
90KHz
DB(V(U3:-))
Frequency

Problem 14–44. A strong signal at 2.45 MHz is interfering with an AM signal at 980 kHz. Design a filter that will attenuate the undesired signal by at least 60 dB. Verify your design using OrCAD.

Design the filter with a passband gain of 0 dB, a cutoff frequency of $\omega_0 = 2\pi$ Mrad/s, $T_{\text{MIN}} = -60$ dB, and $\omega_{\text{MIN}} = 4.9\pi$ Mrad/s. The required filter order is either an eighth-order Butterworth design or a fifth-order Chebychev design. Use the Chebychev approach, since it will require fewer OP AMPs.

$$q_5(s) = [(s/0.1772) + 1][(s/0.9674)^2 + 0.1132(s/0.9674) + 1][(s/0.6139)^2 + 0.4670(s/0.6139) + 1]$$

$$\begin{aligned}
T(s) &= \frac{K}{q_5(s/\omega_0)} \\
&= \frac{1}{\left[\left(\frac{s}{1113000}\right)+1\right]\left[\left(\frac{s}{6078000}\right)^2 + 0.1132\left(\frac{s}{6078000}\right)+1\right]\left[\left(\frac{s}{3857000}\right)^2 + 0.4670\left(\frac{s}{3857000}\right)+1\right]} \\
&= \frac{(1)(1113000)(6078000)^2(3857000)^2}{(s+1113000)(s^2+688000s+6078000^2)(s^2+1801000s+3857000^2)}
\end{aligned}$$

For the first-order RC stage, we have the following design choices and results:

$$\omega_0 = 1113 \text{ krad/s}$$

$$C_2 = 100 \text{ pF}$$

$$R_2 = \frac{1}{C\omega_0} = 8.982 \text{ k}\Omega$$

$$R_1 = R_2 = 8.982 \text{ k}\Omega$$

For the second filter stage we have the following design choices and results:

$$\zeta = \frac{0.1132}{2} = 0.0566$$

$$\omega_0 = (0.9674)(6283) = 6078 \text{ krad/s}$$

$$C_1 = 0.001\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 3.204 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 2.907 \text{ k}\Omega$$

For the third filter stage we have the following design choices and results:

$$\zeta = \frac{0.4670}{2} = 0.2335$$

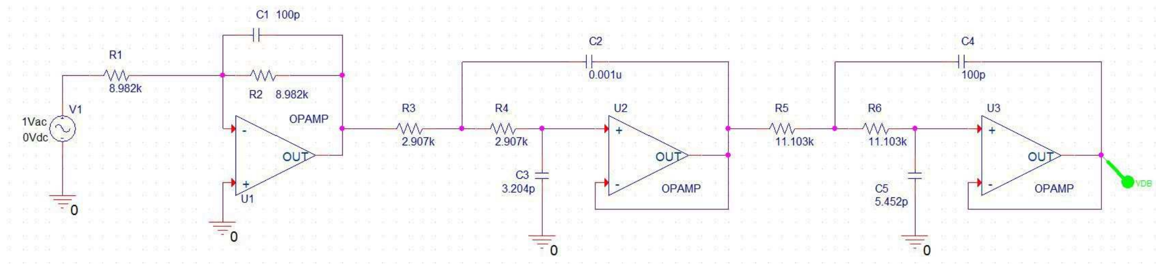
$$\omega_0 = (0.6139)(6283) = 3857 \text{ krad/s}$$

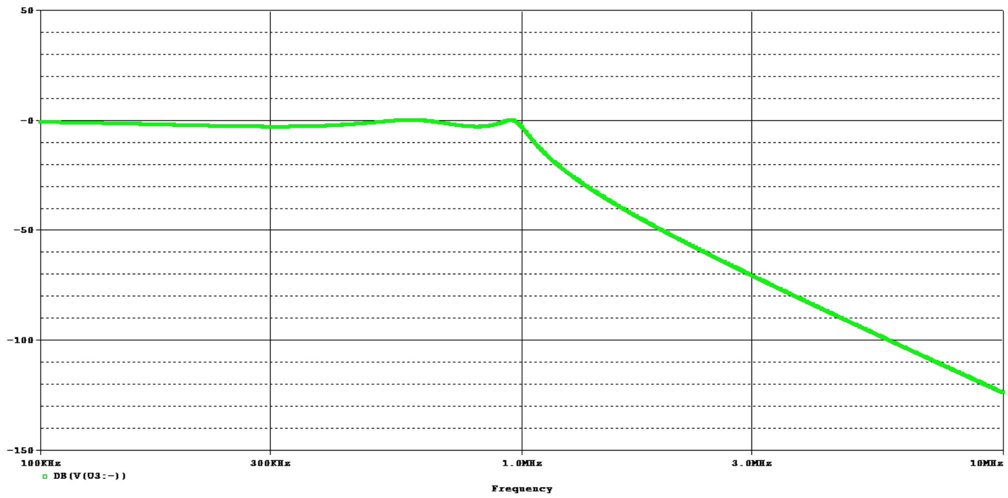
$$C_1 = 100 \text{ pF}$$

$$C_2 = \zeta^2 C_1 = 5.452 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 11.1029 \text{ k}\Omega$$

The OrCAD simulation and results are shown below.


C1 100p
R1
8.982k
R2 8.982k
1Vac
0Vdc
V1
OPAMP
OUT
U1
0
R3
2.907k
R4
2.907k
C2
0.001u
C3
3.204p
U2
OUT
OPAMP
R5
11.103k
R6
11.103k
C4
100p
C5
5.452p
U3
OUT
OPAMP


50
-0
-50
-100
-150
100KHz
300KHz
1.0MHz
3.0MHz
10MHz
DB(V(U3:-))
Frequency

Problem 14–45. A 10 kHz square wave must be bandwidth-limited by attenuating all harmonics after the third. Design a low-pass filter that attenuates the fifth harmonic and greater by at least 20 dB. The fundamental and third harmonic should not be reduced by more than 3 dB and the overshoot cannot exceed 13%.

The third harmonic occurs at $\omega_0 = 60\pi$ krad/s and the fifth harmonic occurs at 100π krad/s. Let the gain be 0 dB in the passband and no more than -20 dB by the fifth harmonic. Since the overshoot is limited to 13%, we can use a Butterworth design.

$$n \geq \frac{1}{2}\frac{\ln[(T_{\rm MAX}/T_{\rm MIN})^2 - 1]}{\ln[\omega_{\rm MIN}/\omega_{\rm C}]} = \frac{1}{2}\frac{\ln[100-1]}{\ln[10/6]} = 4.498$$

$$n = 5$$

The Butterworth polynomial is:

$$q_5(s) = (s+1)(s^2+0.6180s+1)(s^2+1.618s+1)$$

The first stage is an active, first-order RC low-pass filter with a gain of one. The other two stages are second-order, active low-pass filters using the unity-gain design approach. For the first stage we have:

$$\omega_0 = 188.496 \text{ krad/s}$$

$$C_2 = 0.001\ \mu\text{F}$$

$$R_2 = \frac{1}{\omega_0 C} = 5.305 \text{ k}\Omega$$

$$R_1 = R_2 = 5.305 \text{ k}\Omega$$

For the second stage we have:

$$\zeta = \frac{0.618}{2} = 0.309$$

$$C_1 = 0.001\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 95.48 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 17.169 \text{ k}\Omega$$

For the third stage we have:

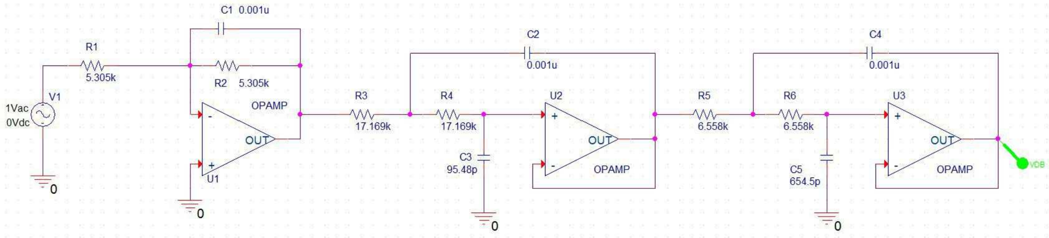
$$\zeta = \frac{1.618}{2} = 0.809$$

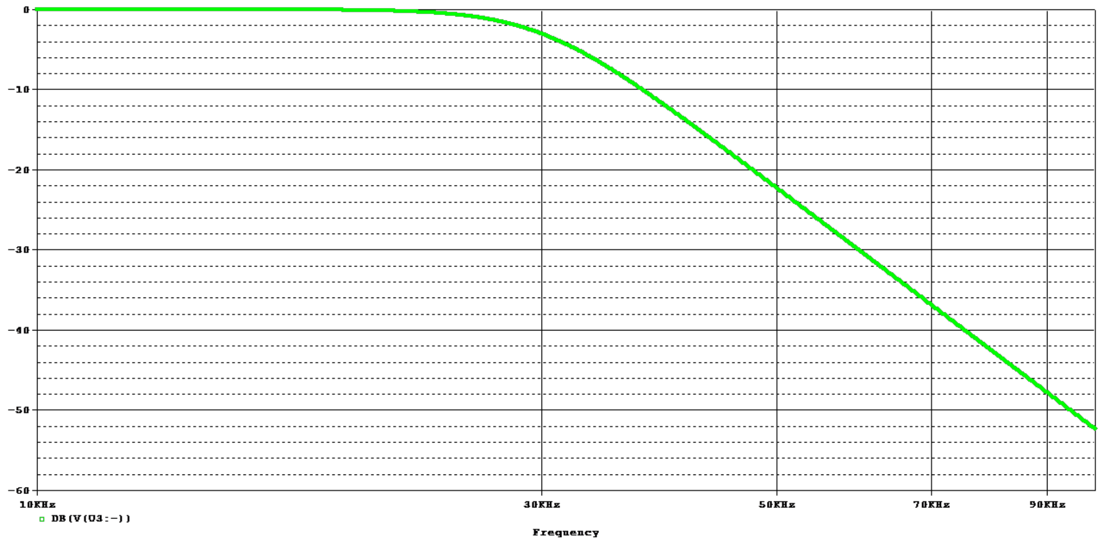
$$C_1 = 0.001\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 654.5 \text{ pF}$$

$$R = \frac{1}{\omega_0\sqrt{C_1C_2}} = 6.558 \text{ k}\Omega$$

The following OrCAD simulation and output plots validate the results.


C1 0.001u
R1
5.305k
R2 5.305k
1Vac
0Vdc
V1
OPAMP
OUT
U1
0
R3
17.169k
R4
17.169k
C2
0.001u
C3
95.48p
U2
OUT
OPAMP
R5
6.558k
R6
6.558k
C4
0.001u
C5
654.5p
U3
OUT
OPAMP


0
-10
-20
-30
-40
-50
-60
10KHz
30KHz
50KHz
70KHz
90KHz
DB(V(U3:-))
Frequency

Problem 14–46. Construct the lowest-order transfer function that meets the following high-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Pole Type	ω_C (rad/s)	T_{MAX}	ω_{MIN} (rad/s)	T_{MIN}
First-order cascade	10000	40 dB	1000	0 dB

Determine the filter order.

$$\alpha = \omega_C\sqrt{2^{1/n} - 1}$$

$$\frac{T_{MAX}}{T_{MIN}} \leq \left[1 + \left(\frac{\omega_C}{\omega_{MIN}}\right)^2 (2^{1/n} - 1)\right]^{n/2}$$

Find the smallest value of n that satisfies the last inequality. In this case, the minimum value is $n = 3$. We have

$$\alpha = \omega_C\sqrt{2^{1/3} - 1} = 5.098 \text{ krad/s}$$

$$K = (T_{MAX})^{1/3} = (100)^{1/3} = 4.6416$$

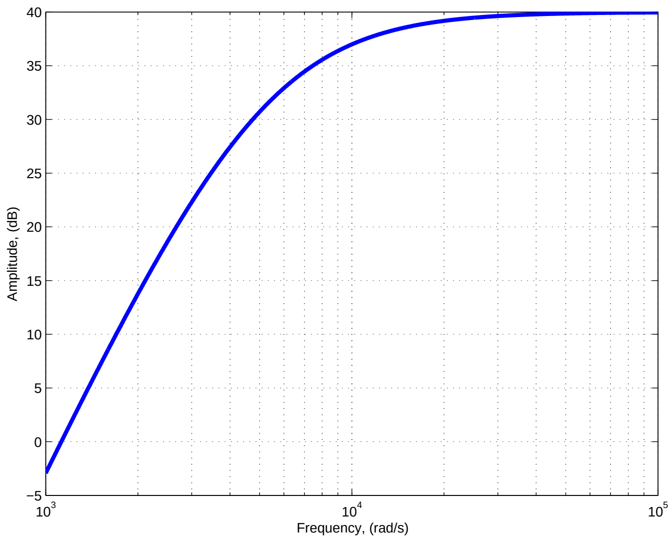
$$T(s) = \left[\frac{K}{\frac{\alpha}{s} + 1}\right]^n = \left[\frac{4.6416}{\frac{5098}{s} + 1}\right]^3 = \left[\frac{4.6416s}{s + 5098}\right]^3$$

The transfer function has the following gains, which meet the specifications:

$$|T(j\omega_C)| = 36.99 \text{ dB}$$

$$|T(j\omega_{MIN})| = -2.94 \text{ dB}$$

The following MATLAB plot validates the results.



Problem 14–47. Construct the lowest-order transfer function that meets the following high-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Pole Type	ω_C (rad/s)	T_{MAX}	ω_{MIN} (rad/s)	T_{MIN}
Butterworth	10000	20 dB	1000	−30 dB

Determine the filter order and transfer function.

$$n \geq \frac{1}{2}\frac{\ln[(T_{MAX}/T_{MIN})^2 - 1]}{\ln[\omega_C/\omega_{MIN}]} = \frac{1}{2}\frac{\ln[(10/0.03162)^2 - 1]}{\ln[10]} = 2.5$$

$$n = 3$$

$$q_3(s) = (s+1)(s^2+s+1)$$

$$T(s) = \frac{K}{q_3\left(\frac{\omega_C}{s}\right)}$$

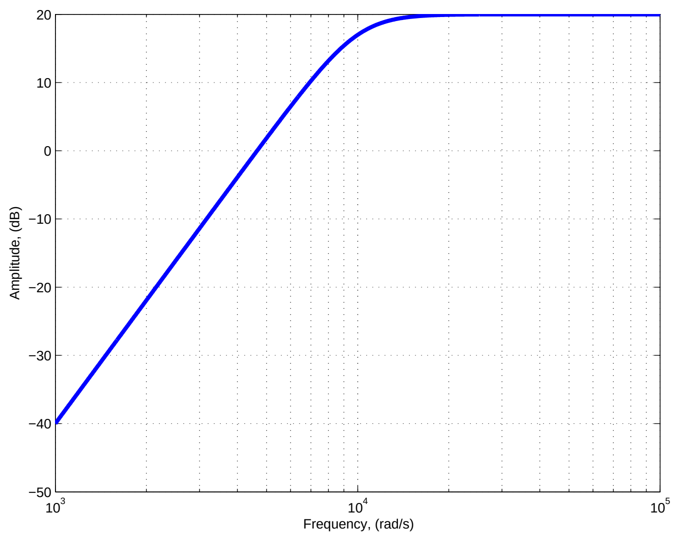
$$= \frac{10}{\left[\frac{10000}{s}+1\right]\left[\left(\frac{10000}{s}\right)^2 + \left(\frac{10000}{s}\right)+1\right]} = \frac{10s^3}{[s+10000][s^2+10000s+10^8]}$$

The transfer function has the following gains, which meet the specifications:

$$|T(j\omega_C)| = 16.99 \text{ dB}$$

$$|T(j\omega_{MIN})| = -40 \text{ dB}$$

The following MATLAB plot validates the results.



Problem 14–48. Construct the lowest-order transfer function that meets the following high-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Pole Type	ω_C (rad/s)	T_{MAX}	ω_{MIN} (rad/s)	T_{MIN}
Chebychev	10000	0 dB	5000	−40 dB

Determine the filter order and then construct the transfer function.

$$n \geq \frac{\cosh^{-1}\left[\sqrt{(T_{MAX}/T_{MIN})^2 - 1}\right]}{\cosh^{-1}[\omega_C/\omega_{MIN}]} = \frac{\cosh^{-1}\left[\sqrt{(1/0.01)^2 - 1}\right]}{\cosh^{-1}[2]} = 4.023$$

$$n = 5$$

$$K = 0\text{ dB} = 1$$

$$q_5(s) = [(s/0.1772) + 1][(s/0.9674)^2 + 0.1132(s/0.9674) + 1][(s/0.6139)^2 + 0.4670(s/0.6139) + 1]$$

$$T(s) = \frac{K}{q_5\left(\frac{\omega_C}{s}\right)}$$

$$= \frac{1}{\left[\left(\frac{56443}{s}\right) + 1\right]\left[\left(\frac{10337}{s}\right)^2 + 0.1132\left(\frac{10337}{s}\right) + 1\right]\left[\left(\frac{16289}{s}\right)^2 + 0.4670\left(\frac{16289}{s}\right) + 1\right]}$$

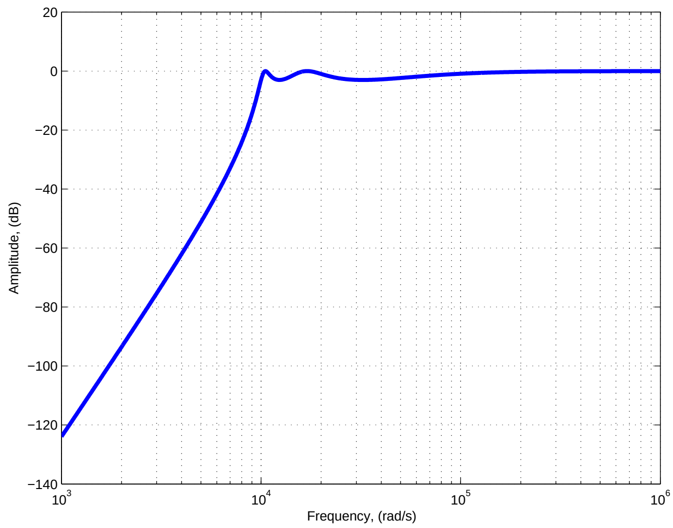
$$= \frac{s^5}{(s + 56443)(s^2 + 1170s + 10337^2)(s^2 + 7607s + 16289^2)}$$

The transfer function has the following gains, which meet the specifications:

$$|T(j\omega_C)| = -3.01\text{ dB}$$

$$|T(j\omega_{MIN})| = -51.17\text{ dB}$$

The following MATLAB plot validates the results.



Problem 14–49. Construct the lowest-order transfer function that meets the following high-pass filter specifications. Calculate the gain (in dB) of the transfer function at $\omega = \omega_C$ and ω_{MIN}. Use MATLAB to validate that your transfer function meets the design specifications.

Pole Type	ω_C (rad/s)	T_{MAX}	ω_{MIN} (rad/s)	T_{MIN}
You Decide	10000	10 dB	2000	−50 dB

Choose a Butterworth filter to simplify the design calculations. Determine the filter order and then construct the transfer function.

$$n \geq \frac{1}{2}\frac{\ln[(T_{MAX}/T_{MIN})^2 - 1]}{\ln[\omega_C/\omega_{MIN}]} = \frac{1}{2}\frac{\ln[1000000 - 1]}{\ln[5]} = 4.292$$

$$n = 5$$

The following steps complete the transfer function using the table of Butterworth polynomials to find $q_n(s)$:

$$K = 10\text{ dB} = 3.1623$$

$$q_5(s) = (s+1)(s^2 + 0.6180s + 1)(s^2 + 1.618s + 1)$$

$$T(s) = \frac{K}{q_5\left(\frac{\omega_C}{s}\right)}$$

$$= \frac{3.1623}{\left[\left(\frac{10000}{s}\right) + 1\right]\left[\left(\frac{10000}{s}\right)^2 + 0.6180\left(\frac{10000}{s}\right) + 1\right]\left[\left(\frac{10000}{s}\right)^2 + 1.618\left(\frac{10000}{s}\right) + 1\right]}$$

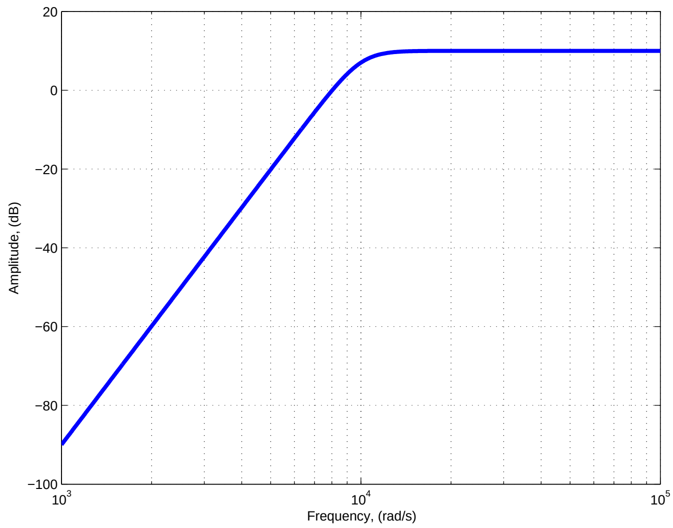
$$= \frac{3.1623s^5}{(s+10000)(s^2 + 6180s + 10^8)(s^2 + 16180s + 10^8)}$$

The transfer function has the following gains, which meet the specifications:

$$|T(j\omega_C)| = 6.99\text{ dB}$$

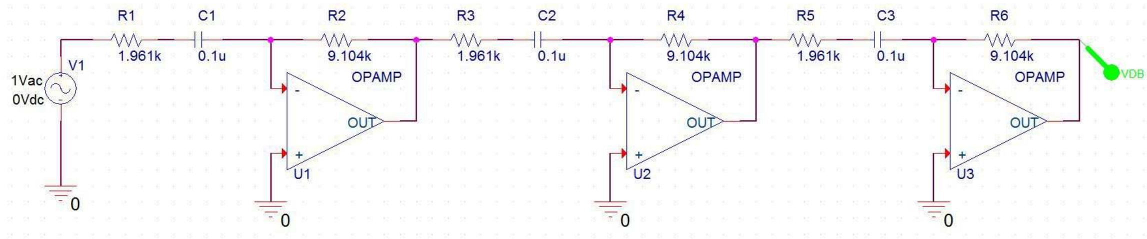
$$|T(j\omega_{MIN})| = -59.90\text{ dB}$$

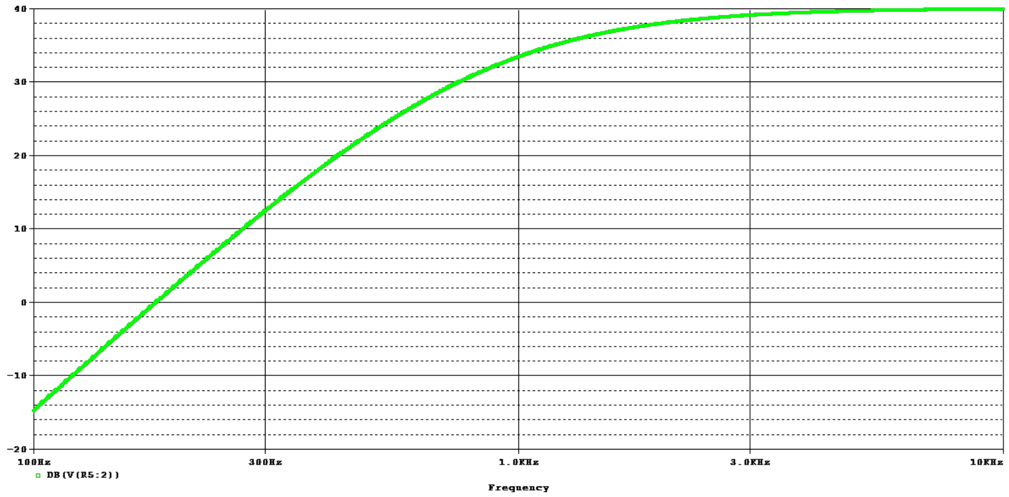
The following MATLAB plot validates the results.



Problem 14–50. Design an active high-pass filter to meet the specification in Problem 14–46. Use OrCAD to verify that your design meets the specifications.

Each stage has a cutoff frequency of $\omega_C = 5.098$ krad/s and a gain of $K = 4.6416$. Use a first-order, high-pass OP AMP design with gain. Pick $C_1 = 0.1\ \mu$F and solve for $R_1 = 1/(\omega_C C_1) = 1.961$ kΩ. Solve for $R_2 = KR_1 = 9.104$ kΩ. Use three identical stages in cascade. The OrCAD simulation and results are shown below.





Problem 14–51. Design an active high-pass filter to meet the specification in Problem 14–47. Use OrCAD to verify that your design meets the specifications.

Use two high-pass filter stages. The first is a first-order filter with a gain of 10. The second is a second-order filter with unity gain. For the first-order filter, we have:

$$C_1 = 0.1\ \mu\text{F}$$

$$R_1 = \frac{1}{\omega_0 C_1} = 1\ \text{k}\Omega$$

$$R_2 = 10R_1 = 10\ \text{k}\Omega$$

For the second-order filter, we have:

$$\omega_0 = 10000\ \text{rad/s}$$

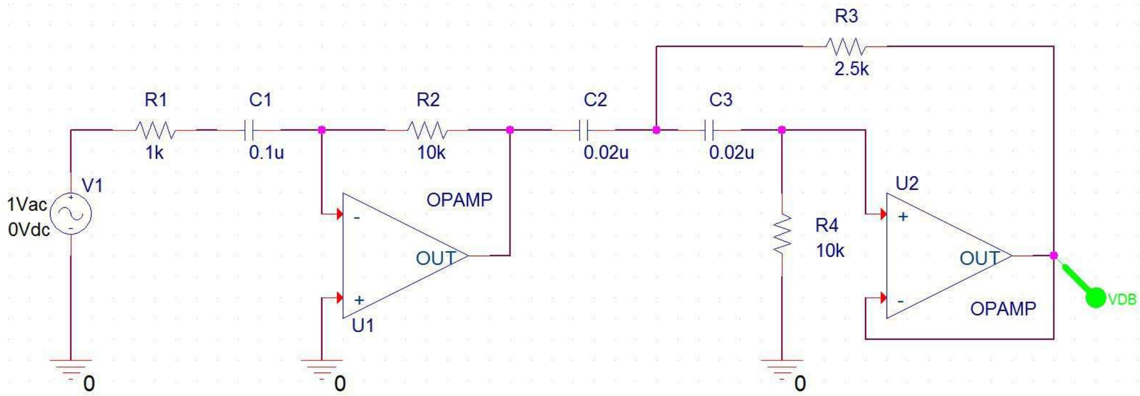
$$\zeta = 0.5$$

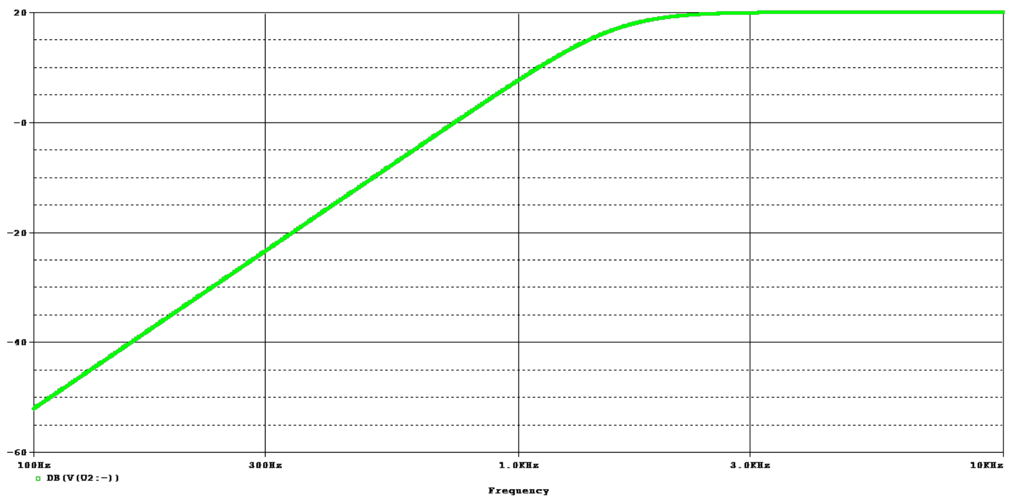
$$R_2 = 10\ \text{k}\Omega$$

$$R_1 = \zeta^2 R_2 = 2.5\ \text{k}\Omega$$

$$C = \frac{1}{\omega_0\sqrt{R_1 R_2}} = 0.02\ \mu\text{F}$$

The OrCAD simulation and results are shown below.





Problem 14–52. Design an active high-pass filter to meet the specification in Problem 14–48. Use OrCAD to verify that your design meets the specifications.

Use three high-pass filter stages, all with unity gain. The first stage is a first-order filter and the other two stages are second-order filters. For the first-order stage, we have the following design choices and results:

$$\omega_0 = 56443 \text{ rad/s}$$

$$C_1 = 0.01\ \mu\text{F}$$

$$R_1 = \frac{1}{\omega_0 C_1} = 1.772\ \text{k}\Omega$$

$$R_2 = R_1 = 1.772\ \text{k}\Omega$$

For the second filter stage we have the following design choices and results:

$$\zeta = \frac{0.1132}{2} = 0.0566$$

$$\omega_0 = \frac{10000}{0.9674} = 10.337 \text{ krad/s}$$

$$R_2 = 1\ \text{M}\Omega$$

$$R_1 = \zeta^2 R_2 = 3.204\ \text{k}\Omega$$

$$C = \frac{1}{\omega_0\sqrt{R_1 R_2}} = 1709 \text{ pF}$$

For the third filter stage we have the following design choices and results:

$$\zeta = \frac{0.4670}{2} = 0.2335$$

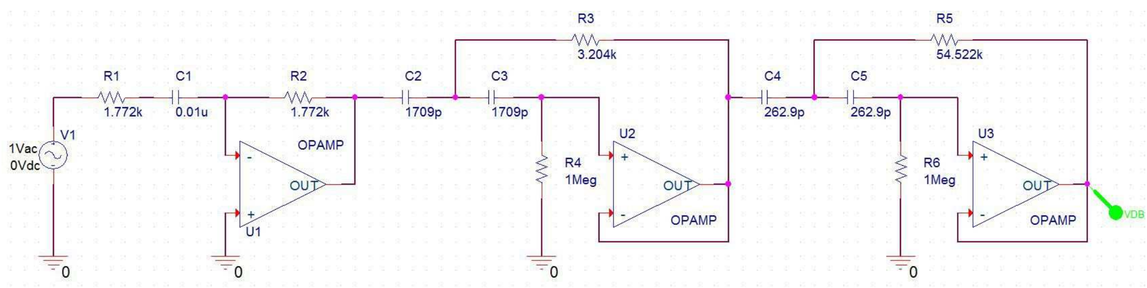
$$\omega_0 = \frac{10000}{0.6139} = 16.289 \text{ krad/s}$$

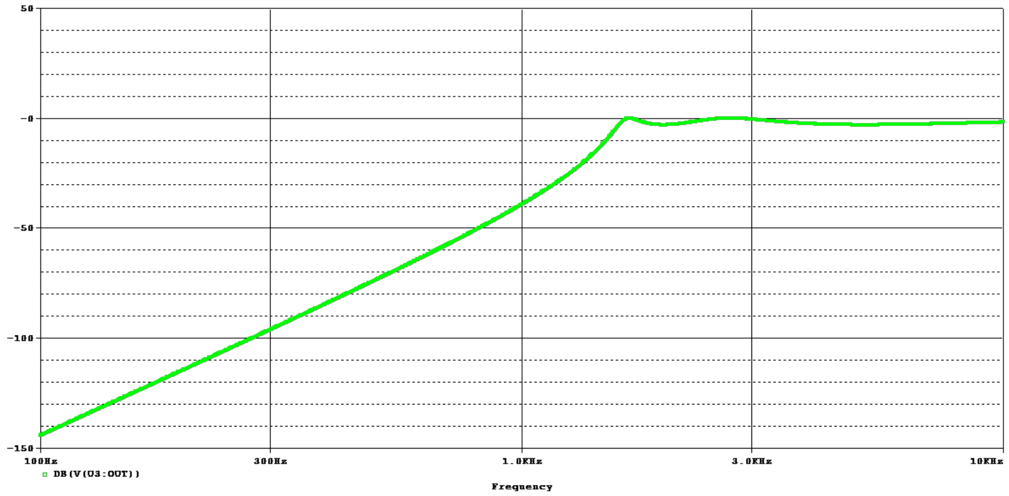
$$R_2 = 1\ \text{M}\Omega$$

$$R_1 = \zeta^2 R_2 = 54.522\ \text{k}\Omega$$

$$C = \frac{1}{\omega_0\sqrt{R_1 R_2}} = 262.9 \text{ pF}$$

The OrCAD simulation and results are shown below.




50
-0
-50
-100
-150
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
DB(V(U3:OUT))
Frequency

Problem 14–53. Design an active high-pass filter to meet the specification in Problem 14–49. Use OrCAD to verify that your design meets the specifications.

Use three high-pass filter stages. The first is a first-order filter with a gain of 3.1623. The other two filters are second-order filters with unity gain. For the first-order filter, we have:

$$C_1 = 0.1\ \mu\text{F}$$

$$R_1 = \frac{1}{\omega_0 C_1} = 1\ \text{k}\Omega$$

$$R_2 = 3.1623 R_1 = 3.1623\ \text{k}\Omega$$

For the second stage, we have:

$$\omega_0 = 10000\ \text{rad/s}$$

$$\zeta = \frac{0.6180}{2} = 0.309$$

$$R_2 = 10\ \text{k}\Omega$$

$$R_1 = \zeta^2 R_2 = 954.81\ \Omega$$

$$C = \frac{1}{\omega_0\sqrt{R_1 R_2}} = 0.03236\ \mu\text{F}$$

For the third stage, we have:

$$\omega_0 = 10000\ \text{rad/s}$$

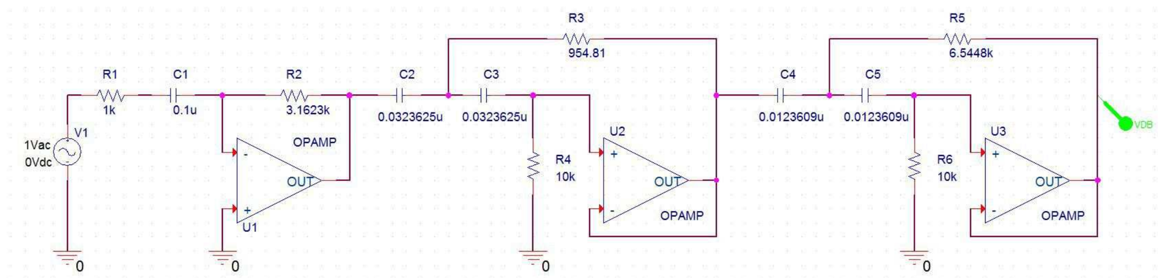
$$\zeta = \frac{1.6180}{2} = 0.809$$

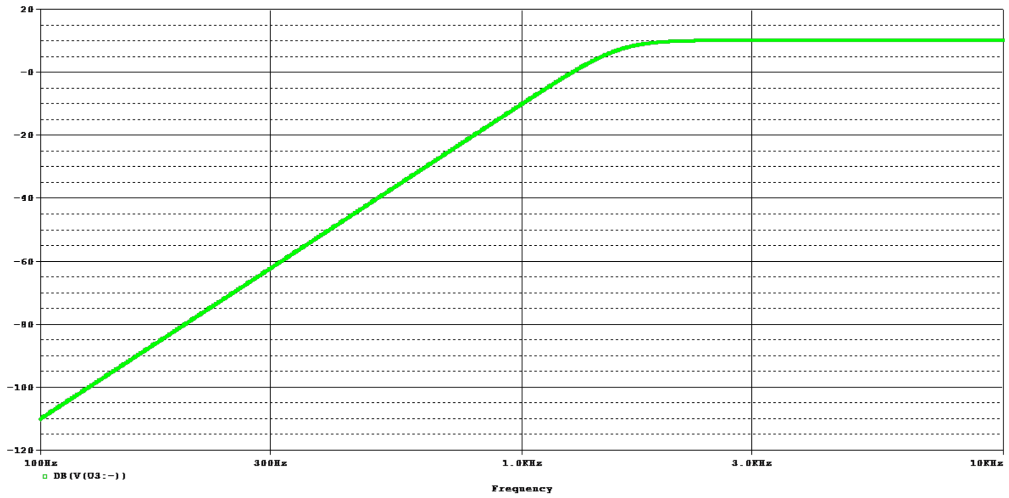
$$R_2 = 10\ \text{k}\Omega$$

$$R_1 = \zeta^2 R_2 = 6.545\ \text{k}\Omega$$

$$C = \frac{1}{\omega_0\sqrt{R_1 R_2}} = 0.01236\ \mu\text{F}$$

The OrCAD simulation and results are shown below.




20
-0
-20
-40
-60
-80
-100
-120
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
DB(V(U3:-))
Frequency

Problem 14–54. A certain instrumentation system for a new hybrid car needs a bandpass filter to limit its output bandwidth prior to digitization. The filter must meet the following specifications:

$$T_{\text{MAX}} = +20 \;\pm 1 \text{ dB}$$

$$T_{\text{MIN}} \leq -20 \text{ dB}$$

$$\omega_{\text{CH}} = 5.5 \text{ krad/s} = 875.4 \text{ Hz} \;\pm 10\%$$

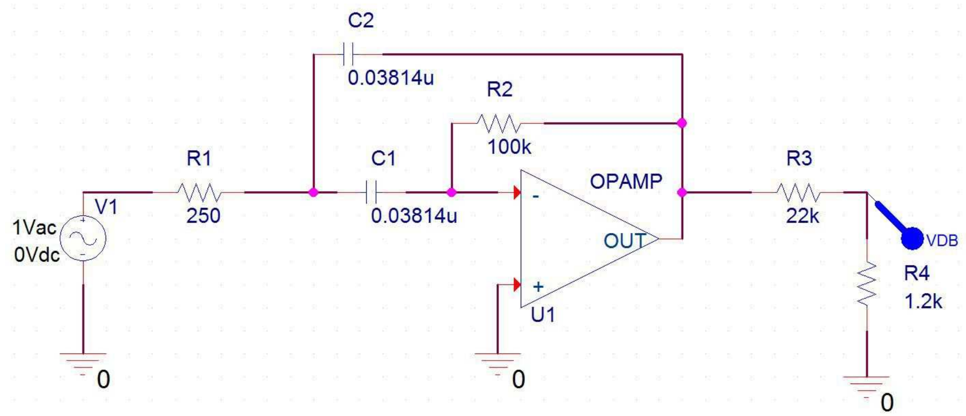
$$\omega_{\text{CHMIN}} = 55 \text{ krad/s} = 8.754 \text{ kHz}$$

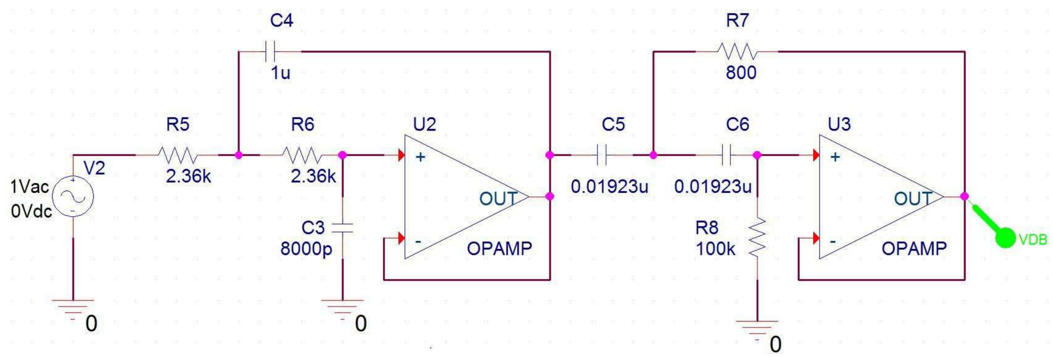
$$\omega_{\text{CL}} = 5 \text{ krad/s} = 795.8 \text{ Hz} \;\pm 10\%$$

$$\omega_{\text{CLMIN}} = 500 \text{ rad/s} = 79.6 \text{ Hz}$$

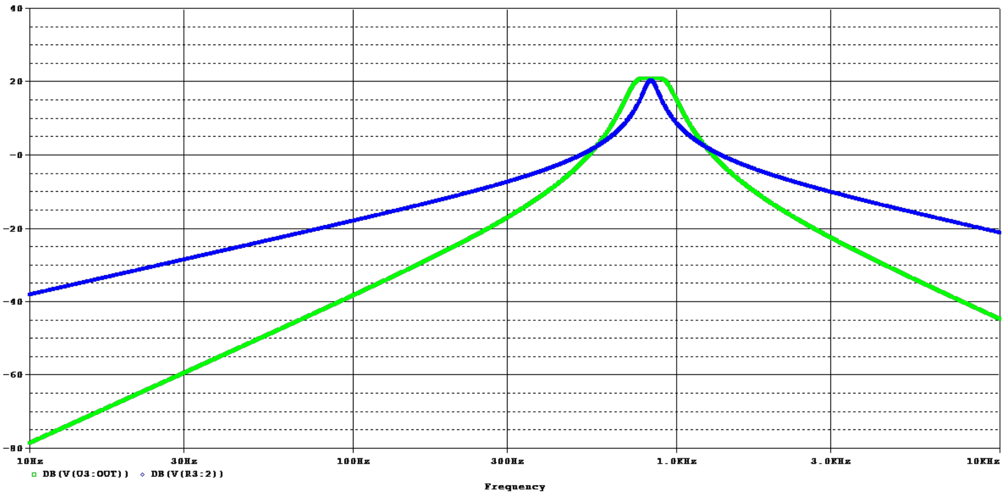
Two vendors have submitted proposed solutions in the form of OrCAD graphics, shown in Figure P14–54. As the requirements engineer on the project, you need to select the best one. You must check whether each circuit meets or fails every specification. You should also consider (1) parts count, (2) ease of maintainability and implementation (fewer parts, number of similar parts, adjustments of potentiometers, standard values), (3) frequency-domain response, and (4) cost. You should address each item for each design at least qualitatively. Attach whatever software output you think will support your decision.

Simulate both circuits in OrCAD to determine how well they meet the specifications.





The amplitude responses for both circuits are plotted on the same axes, with Vendor #1 in blue and Vendor #2 in green.



Use the OrCAD cursor to help determine the performance of each circuit. The results are summarized in the table below.

Item	Specification	Vendor #1	Vendor #2
$\lvert T_{\text{MAX}}\rvert$	20 dB	20.29 dB	20.90 dB
ω_{CL}	795.8 Hz	793.9 Hz	723.7 Hz
ω_{CH}	875.4 Hz	877.3 Hz	964.1 Hz
$\lvert T(\omega_{\text{CLMIN}})\rvert$	-20 dB	-20.04 dB	-42.4 dB
$\lvert T(\omega_{\text{CHMIN}})\rvert$	-20 dB	-20.04 dB	-42.4 dB
Specifications		Meets specs,	Meets specs
Passband		Peaked passband	Flat passband
Parts Count	Minimum	7	10
Similar Parts	Maximum	1 pair	3 pairs
Potentiometers	Minimum	0	2
Standard Values	Maximum	3	2
Cost	Minimum	$75	$60

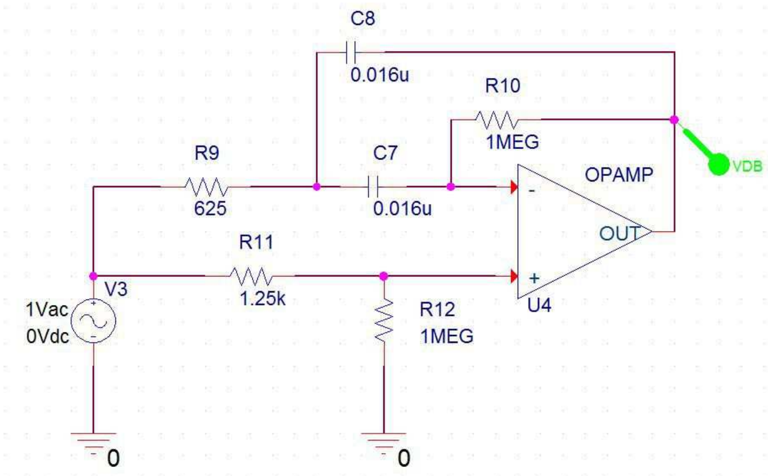
Strictly speaking, the response for Vendor #2 does not meet the specification for ω_{CH} by 1.16 Hz. The response is extremely close to meeting the specification and we would have to analyze actual circuits to determine their performance. From an overall perspective, the circuit from Vendor #1 clearly meets all of the specifications, requires fewer parts, has more similar parts, no potentiometers, and more standard parts. Even though the circuit from Vendor #1 is more expensive, it is probably the best option.

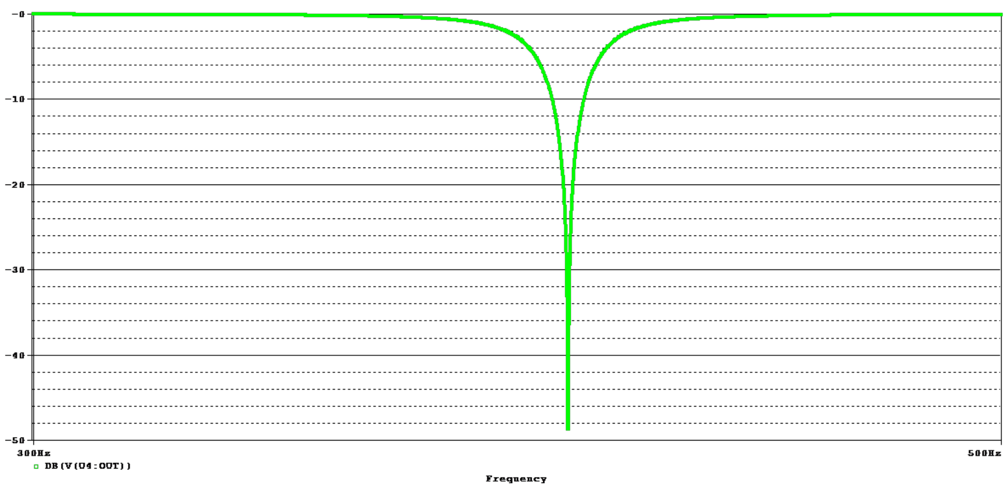
Problem 14–55. You are working at an aircraft manufacturing plant on an altitude sensor that eventually will be used to retrofit dozens of similar sensors on an upgrade to a current airframe. You are required to find a quality notch filter to eliminate an undesirable interference at 400 Hz. You find a vendor and ask for a suitable product specification and a bid. The specifications you require are:

1. Must block signals at 400 ±2% Hz
2. Pass-band gain should be 0 dB ±1 dB
3. Bandwidth should be 20 Hz ±5 % Hz

The vendor's proposal, shown as an OrCAD drawing in Figure P14–55, claims to meet all of the design requirements. Determine if it passes all of the stated requirements. Then explain if you would buy the filter and why or why not?

Simulate the following circuit in OrCAD and analyze its performance against the specifications. Note that the desired center frequency for the filter is $\omega_0 = 2\pi f_0 = 800\pi = 2513$ rad/s.





Using the cursor tools to explore the response, the filter has a notch located at 397.9 Hz with a gain of −48.7 dB. The passband gain is 0 dB. The cutoff frequencies occur at 388.0 Hz and 408.0 Hz, so the bandwidth is 20 Hz. From the simulation, the filter meets all of the specifications. It appears that the filter is an acceptable choice, based on the simulation. One concern with the filter is the 625-Ω resistor at the input. Depending on the nature of the sensor with which the filter will interface, there may be loading issues at the input. We would either have to isolate the filter with a buffer or examine the sensor in more detail to make sure it would not impact the performance of the filter.

Problem 14–56. An amplified portion of the radio spectrum is shown in Figure P14–56. You need to hear all of the signals from 1.0 MHz to 2.2 MHz, but there is an interfering signal at 1.8 MHz. Design a notch filter to reduce that signal by at least 50 dB and not reduce the desired signal at 1.7 MHz by more than 6 dB. Use OrCAD to validate your design.

Use a second-order bandstop filter with a center frequency of 1.8 MHz and a bandwidth of 0.3 MHz. Use the equal-capacitor design approach with $C = 2$ pF. We have the following design results.

$$\omega_0 = 3.6\pi \text{ Mrad/s}$$

$$B = 0.6\pi \text{ Mrad/s} = 2\zeta\omega_0$$

$$\zeta = \frac{B}{2\omega_0} = \frac{0.3}{3.6} = 0.08333$$

$$C = 2 \text{ pF}$$

$$R_1 = \zeta^2 R_2$$

$$C\sqrt{R_1 R_2} = C\zeta R_2 = \frac{1}{\omega_0}$$

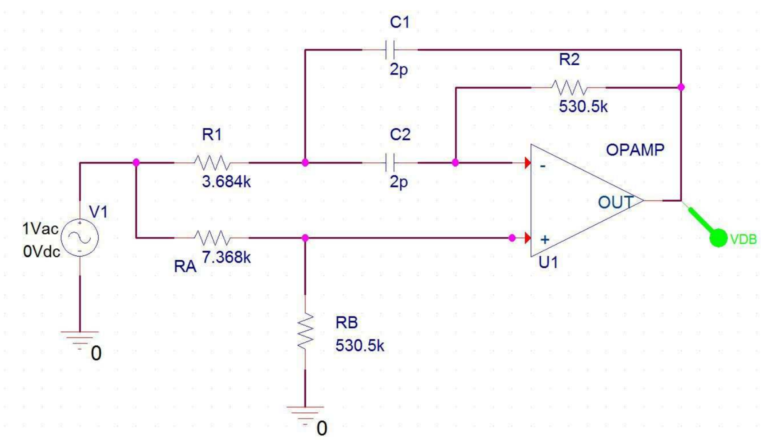
$$R_2 = \frac{1}{\zeta C\omega_0} = 530.5 \text{ k}\Omega$$

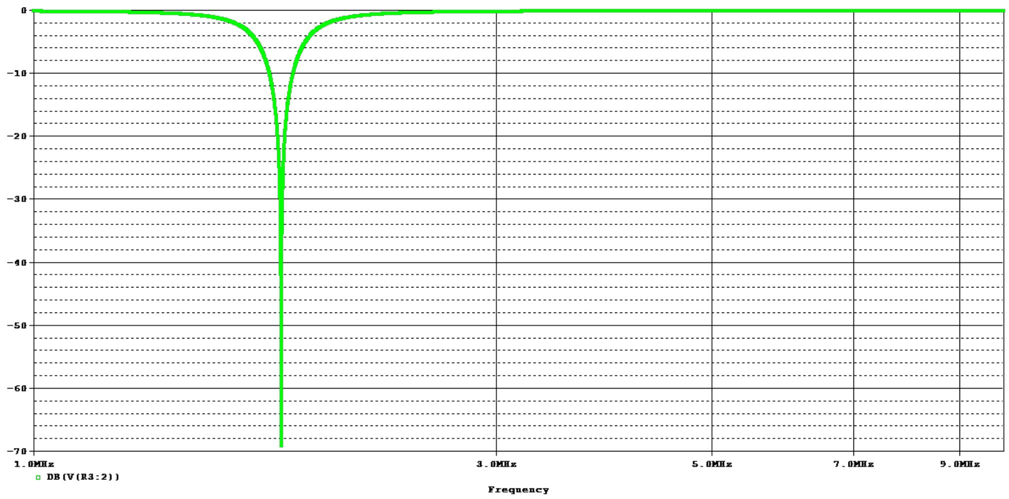
$$R_1 = 3.684 \text{ k}\Omega$$

$$R_\text{A} = 2R_1 = 7.368 \text{ k}\Omega$$

$$R_\text{B} = R_2 = 530.5 \text{ k}\Omega$$

The OrCAD simulation and amplitude response are shown below.





Using the cursor tools to explore the response, the filter has a center frequency of 1.8 MHz and cutoff frequencies of 1.652 MHz and 1.961 MHz, which yield a bandwidth of 0.3 MHz. The gain at the center frequency is −50.3 dB, the gain at 1.7 MHz is −5.05 dB, and the gain at 1.9 MHz is −5.42 MHz. The design meets all of the specifications.

Problem 14–57. A amplified portion of the radio spectrum is shown in Figure P14–56. You want to select the signal at 1.22 MHz but it is barely above the background noise. Design a tuned filter that has a Q of at least 50 and amplifies the signal by 20 dB. Use OrCAD to validate your design.

Use a second-order bandpass filter with a center frequency of 1.22 MHz. Choose a passband gain of 20 dB and $Q = 50$. Using the equal-capacitor method, we have the following design results:

$$\omega_0 = 2\pi(1.22\,\text{MHz}) = 2.44\pi \text{ Mrad/s}$$

$$Q = 50 = \frac{\omega_0}{B} = \frac{\omega_0}{2\zeta\omega_0} = \frac{1}{2\zeta}$$

$$\zeta = \frac{1}{2Q} = 0.01$$

$$C = 1 \text{ pF}$$

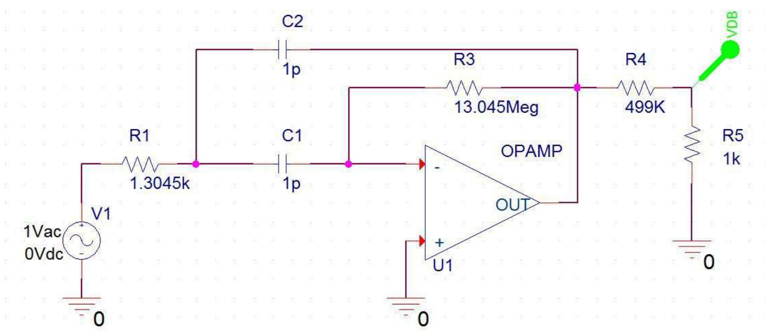
$$R_1 = \zeta^2 R_2$$

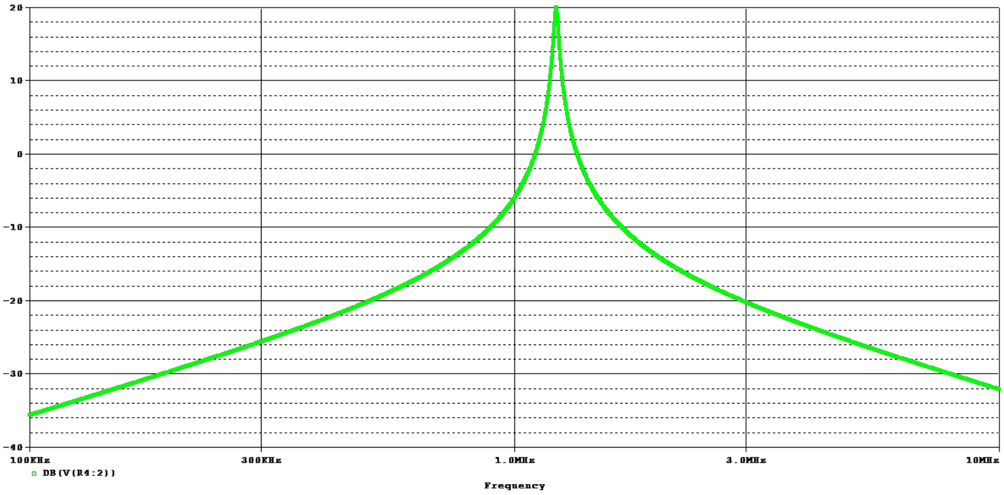
$$C\sqrt{R_1R_2} = C\zeta R_2 = \frac{1}{\omega_0}$$

$$R_2 = \frac{1}{\zeta C\omega_0} = 13.045 \text{ M}\Omega$$

$$R_1 = 1.3045 \text{ k}\Omega$$

The center frequency gain is $|T(j\omega_0)| = 1/(2\zeta^2) = 5000$, so we need a voltage divider with a gain of 1/500 to bring the overall gain down to 10 or 20 dB. The OrCAD simulation and amplitude response are shown below.





Using the cursor tools to explore the response, the filter has a center frequency of 1.2201 MHz and cutoff frequencies of 1.2077 MHz and 1.2325 MHz, which yield a bandwidth of 0.0248 MHz and $Q = 49.27$. The gain at the center frequency is 19.82 dB. The design value for Q is just below the required value of 50. We would have to build the actual circuit and make measurements to verify it meets specifications.

Problem 14–58. The portion of the radio spectrum shown in Figure P14–56 is the result you want after you design a suitable filter and amplifier. The passband gain desired is 100 dB and the "shoulders" of your filter should have a roll-off of −120 dB per decade. Design a wide-band filter and amplifier that is flat in the passband and has a bandwidth of 1 MHz with a lower cutoff frequency of 1 MHz that meets all of these criteria. Use OrCAD to validate your design.

A roll-off of −120 dB per decade requires sixth-order filters for both sides of the bandpass filter. The requirement for a flat passband indicates that a Butterworth filter design is appropriate. Cascade two, sixth-order Butterworth filters together, with one being a low-pass filter and the other a high-pass filter. Use a unity-gain design approach for the filters. Add gain stages at the end to achieve a passband gain of 100 dB. For both filters, the Butterworth polynomial is:

$$q_6(s) = (s^2 + 0.5176s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$$

For the low-pass filter, the cutoff frequency will be 2 MHz or 4π Mrad/s and the design equations are:

$$\mu = 1$$
$$R_1 = R_2 = R$$
$$R\sqrt{C_1C_2} = \frac{1}{\omega_0}$$
$$C_2 = \zeta^2 C_1$$
$$C_1 = \frac{1}{R\zeta\omega_0}$$

For the high-pass filter, the cutoff frequency will be 1 MHz or 2π Mrad/s and the design equations are:

$$\mu = 1$$
$$C_1 = C_2 = C$$
$$C\sqrt{R_1R_2} = \frac{1}{\omega_0}$$
$$R_1 = \zeta^2 R_2$$
$$R_2 = \frac{1}{C\zeta\omega_0}$$

The following MATLAB code performs the calculations for all filter stages.

```
wL = 2*pi*1e6;
wH = 2*pi*2e6;

syms s
z1 = 0.5176/2;
z2 = 1.414/2;
z3 = 1.932/2;
q6s = (s^2+0.5176*s+1)*(s^2+1.414*s+1)*(s^2+1.932*s+1);

disp('Low-pass filter design')
w0 = wH;
disp('First stage')
z = z1;
R = 10e3
C1 = 1/R/z/w0
C2 = z^2*C1
```

```
disp('Second stage')
z = z2;
R = 10e3
C1 = 1/R/z/w0
C2 = z^2*C1
disp('Third stage')
z = z3;
R = 10e3
C1 = 1/R/z/w0
C2 = z^2*C1

disp('High-pass filter design')
w0 = wL;
disp('First stage')
z = z1;
C = 10e-12
R2 = 1/z/C/w0
R1 = z^2*R2
disp('Second stage')
z = z2;
C = 10e-12
R2 = 1/z/C/w0
R1 = z^2*R2
disp('Third stage')
z = z3;
C = 10e-12
R2 = 1/z/C/w0
R1 = z^2*R2
```

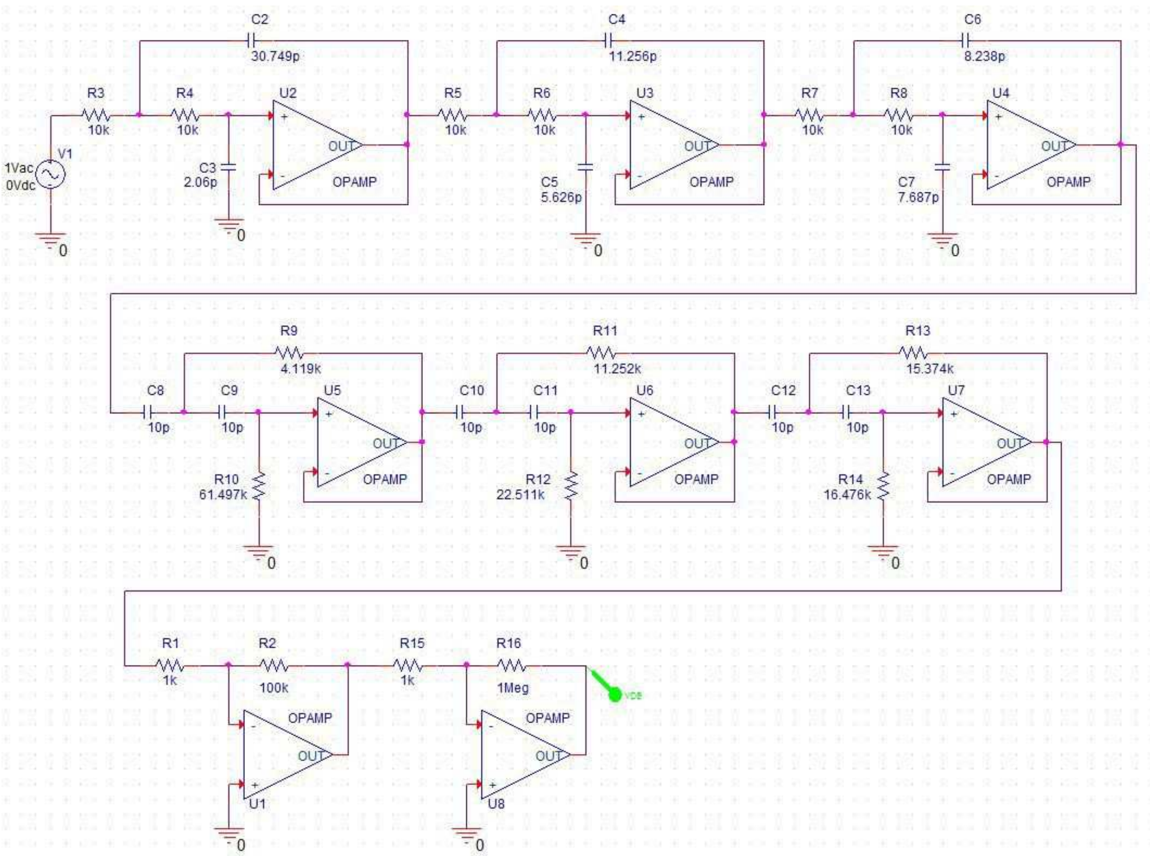
The results for the low-pass filter are

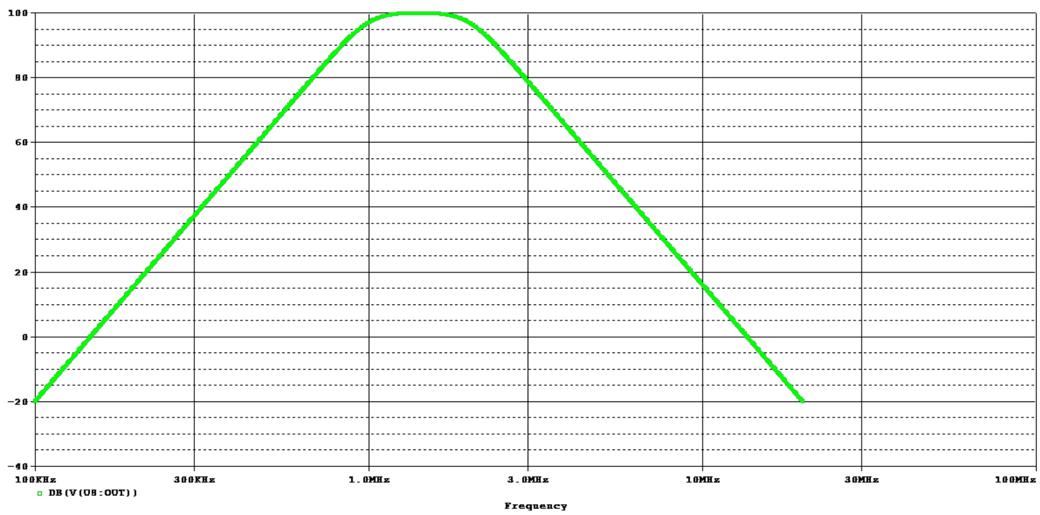
Component	Stage 1	Stage 2	Stage 3
R (kΩ)	10	10	10
C_1 (pF)	30.749	11.256	8.238
C_2 (pF)	2.060	5.626	7.687

The results for the high-pass filter are

Component	Stage 1	Stage 2	Stage 3
C (pF)	10	10	10
R_1 (kΩ)	4.119	11.252	15.374
R_2 (kΩ)	61.497	22.511	16.476

The OrCAD simulation and amplitude response are shown below.



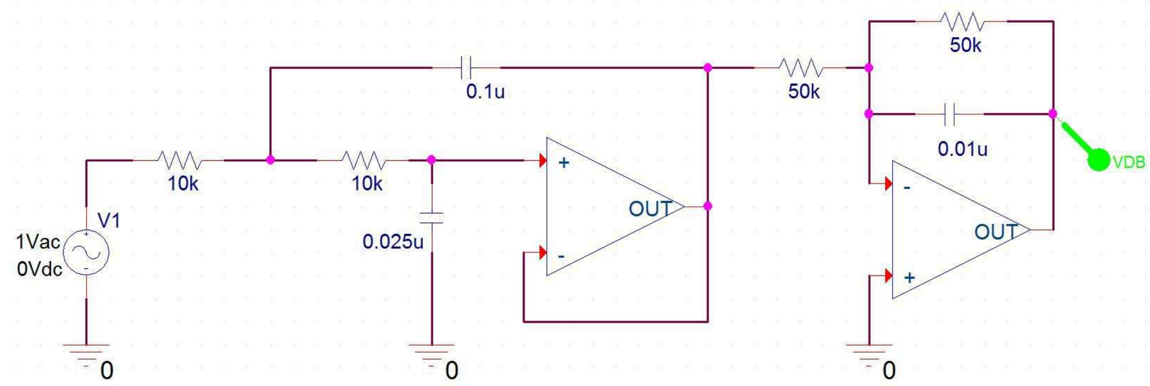


The circuit meets the design specifications.

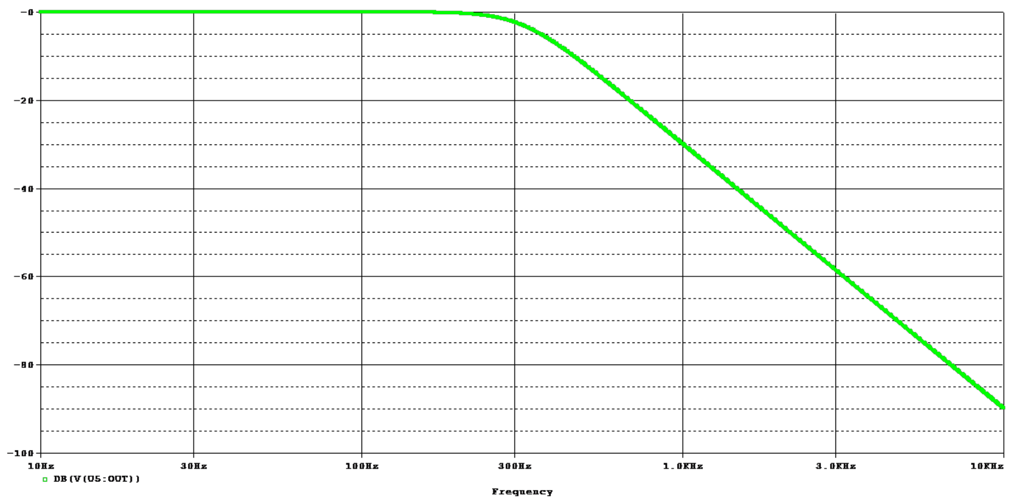
Problem 14–59. (E) Design Evaluation

A need exists for a third-order Butterworth low-pass filter with a cutoff frequency of 2 krad/s and a dc gain of 0 dB. The design department has proposed the circuit in Figure P14–59. As a junior engineer in the manufacturing department, you have been asked to verify the design and suggest modifications that would simplify production.

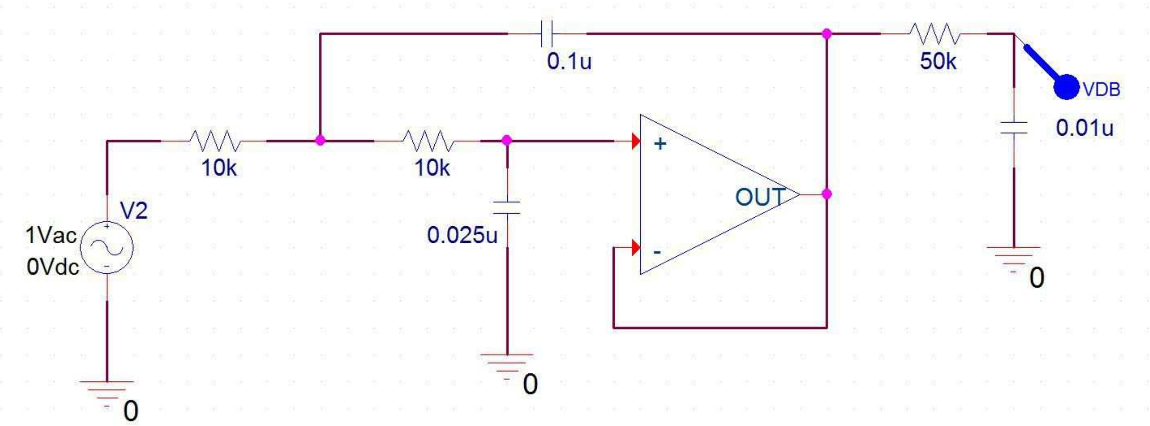
Verify the design by simulating the circuit in OrCAD.



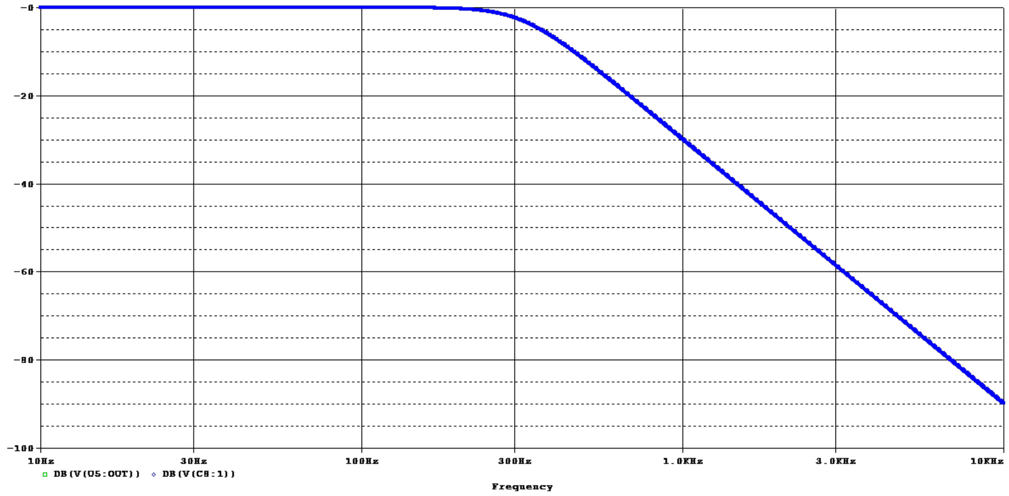
The amplitude response is shown below.



The circuit meets the specifications. Since the output stage is a unity-gain low-pass filter, it can be simplified by replacing it with an RC circuit with the correct cutoff frequency. The design is shown below.



The new amplitude response is shown below.



The amplitude response matches that of the original circuit, but the new circuit has fewer parts and a simpler design.

Problem 14–60. (D) Modifying an Existing Circuit

One of your company's products includes the passive *RLC* filter and OP AMP buffer circuit in Figure P14–60. The supplier of the inductor is no longer in business and a suitable replacement is not available, even on *eBay* or *Craig's List*. You have been asked to design a suitable inductorless replacement. To minimize production changes, your design must use the existing OP AMP as is and either the 1-kΩ resistor or the 0.1-μF capacitor or both, if possible.

Determine the transfer function for the circuit.

$$T(s) = \frac{\dfrac{1}{Cs}}{R + Ls + \dfrac{1}{Cs}} = \frac{\dfrac{1}{LC}}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}} = \frac{10^8}{s^2 + 10^4 s + 10^8}$$

The filter is a second-order, low-pass filter with the following characteristics:

$$\mu = 1$$

$$\omega_0 = 10000 \text{ rad/s}$$

$$2\zeta\omega_0 = 10000 \text{ rad/s}$$

$$\zeta = 0.5$$

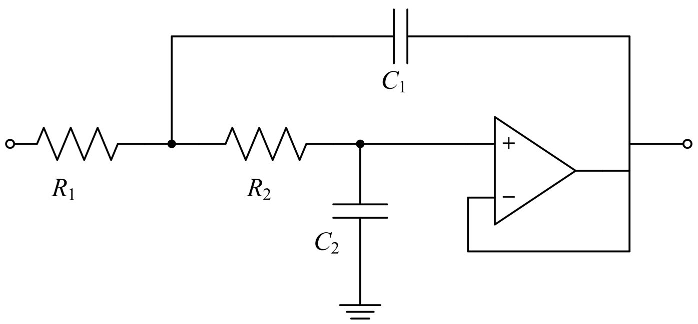
Design a replacement low-pass filter using just resistors and capacitors using the unity-gain approach.

$$C_1 = 0.1\ \mu\text{F}$$

$$C_2 = \zeta^2 C_1 = 0.025\ \mu\text{F}$$

$$R = R_1 = R_2 = \frac{1}{\omega_0\sqrt{C_1 C_2}} = 2\ \text{k}\Omega$$

The new circuit is shown below.



The alternate design uses the OP AMP, the capacitor, and four of the 1-kΩ resistors to make the two 2-kΩ resistors in the new filter.

Problem 14–61. (A) What is a High-Pass Filter

Ten years after earning a BSEE, you return for a master's degree and sign on as the laboratory instructor for the basic circuit analysis course. One experiment asks the students to build the active filter in Figure P14–61 and measure its gain response over the range from 150 Hz to 15 kHz. The lab instructions say the circuit is a high-pass filter with $\omega_0 = 10$ krad/s and an infinite frequency gain of 0 dB. Everything goes well until a student, intrigued by the concept of infinite frequency, inputs 1 MHz and measures a gain of only 0.7. The student then inputs 2 MHz and measures a gain of 0.45. The high-pass filter appears to be a bandpass filter! Motivated by an insatiable thirst for understanding, the student asks you for an explanation. You first check the student's circuit and find it to be correct. You next replace the OP AMP and get almost the same results. Desperate for an explanation (your credibility is on the line here), you read the course textbook (it is Thomas, Rosa, and Toussaint), and find the answer in Chapter 12 (Example 12–5). What do you tell the student?

Real OP AMPs have a finite gain-bandwidth product, typically on the order of 1 MHz. The gain-bandwidth limitation does not affect the high-pass filter gain for test frequencies in the range from 150 Hz to 15 kHz. At 1 MHz, the OP AMP begins to run out of bandwidth and its closed-loop gain decreases. The effect is even more noticeable at 2 MHz. In other words, the OP AMP eventually runs out of bandwidth, so active high-pass filters do not really work at infinite frequency. To design an active high-pass filter, you must choose an OP AMP with sufficient bandwidth to cover the frequency range of interest (150 Hz to 15 kHz in this lab), which must be a finite range.

Problem 14–62. (A) Bandpass to Bandstop Transformation

The three-terminal circuit in Figure P14–62(a) has a bandpass transfer function of the form

$$T(s) = \frac{V_{\rm O}(s)}{V_{\rm S}(s)} = \frac{2\zeta(s/\omega_0)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

Show that the circuit in Figure P14–62(b) has a bandstop transfer function of the form

$$T(s) = \frac{V_{\rm O}(s)}{V_{\rm S}(s)} = \frac{(s/\omega_0)^2 + 1)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

That is, show that interchanging the input and ground terminals changes a unity-gain bandpass circuit into a unity-gain bandstop circuit.

The original input to the circuit is $V_1(s) = V_{\rm S}(s) - 0$, and the original output is $V_2(s) = V_{\rm O}(s) - 0$. With the interchange of the input source and the ground, the new input signal is $V_1(s) = 0 - V_{\rm S}(s)$, and the new output is $V_2(s) = V_{\rm O}(s) - V_{\rm S}(s)$. Apply the original transfer function to the new input signal to generate a new output signal $V_2(s)$. Add $V_{\rm S}(s)$ to the new output to determine $V_{\rm O}(s)$. Divide the result by $V_{\rm S}(s)$ to determine the new transfer function. The steps are presented below.

$$T(s) = \frac{2\zeta(s/\omega_0)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

$$V_1(s) = -V_{\rm S}(s)$$

$$V_2(s) = T(s)V_1(s) = \frac{-2\zeta(s/\omega_0)V_{\rm S}(s)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

$$\begin{aligned} V_{\rm O}(s) &= V_2(s) + V_{\rm S}(s) \\ &= \frac{-2\zeta(s/\omega_0)V_{\rm S}(s)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} + \frac{[(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1]V_{\rm S}(s)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} \\ &= \frac{[(s/\omega_0)^2 + 1]V_{\rm S}(s)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} \end{aligned}$$

$$T_{\rm New}(s) = \frac{V_{\rm O}(s)}{V_{\rm S}(s)} = \frac{(s/\omega_0)^2 + 1}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

The following MATLAB code validates the solution.

```
syms s w0 z Vo Vs V1 V2
% Create the original transfer function
Ts1 = 2*z*s/w0/((s/w0)^2+2*z*(s/w0)+1);
% Create the new input signal
V1 = -Vs;
% Calculate the new output signal, V2(s)
V2 = Ts1*V1;
% Calculate the new output signal, Vo(s)
Vo = V2+Vs;
% Determine the new transfer function
Ts2 = simplify(Vo/Vs)
```

The corresponding output is shown below and is equivalent to the results shown above.

```
Ts2 = (s^2 + w0^2)/(s^2 + 2*z*s*w0 + w0^2)
```

Problem 14–63. (A) Third-Order Butterworth Circuit

Show that the circuit in Figure P14–63 produces a third-order Butterworth low-pass filter with a cutoff frequency of $\omega_C = 1/RC$ and a passband gain of $K = 4$.

Use node-voltage analysis to find the transfer function of the circuit. Label the input as $V_1(s)$ and the output as $V_2(s)$. Label the nodes to the right of the horizontal resistors as nodes A, B, and C, and note that node C has equations at both input terminals of the OP AMP. The node-voltage equations are:

$$\frac{V_A(s) - V_1(s)}{R} + \frac{V_A(s)}{1/2Cs} + \frac{V_A(s) - V_B(s)}{R} = 0$$

$$\frac{V_B(s) - V_A(s)}{R} + \frac{V_B(s) - V_2(s)}{2/Cs} + \frac{V_B(s) - V_C(s)}{R} = 0$$

$$\frac{V_C(s) - V_B(s)}{R} + \frac{V_C(s)}{1/Cs} = 0$$

$$\frac{V_C(s)}{R} + \frac{V_C(s) - V_2(s)}{3R} = 0$$

Solve the equations for $V_2(s)$ and then divide by $V_1(s)$ to determine the transfer function $T(s)$. The MATLAB code to find the answer is shown below.

```
syms s R C V1 V2 VA VB VC
ZC = 1/s/C;
Z2C = 1/s/(2*C);
ZC2 = 1/s/(C/2);
% Write the node-voltage equations
Eqn1 = (VA-V1)/R + VA/Z2C + (VA-VB)/R;
Eqn2 = (VB-VA)/R + (VB-V2)/ZC2 + (VB-VC)/R;
Eqn3 = (VC-VB)/R + VC/ZC;
Eqn4 = VC/R + (VC-V2)/(3*R);
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,Eqn4,VA,VB,VC,V2);
V2 = Soln.V2;
% Create the transfer function
Ts = simplify(V2/V1)
```

The results are:

```
Ts = 4/((C*R*s + 1)*(C^2*R^2*s^2 + C*R*s + 1))
```

The transfer function is

$$T(s) = \frac{4}{(RCs+1)(R^2C^2s^2+RCs+1)} = \frac{4\left(\dfrac{1}{RC}\right)^3}{\left[s+\dfrac{1}{RC}\right]\left[s^2+\dfrac{1}{RC}s+\left(\dfrac{1}{RC}\right)^2\right]}$$

The polynomial for a third-order Butterworth filter is shown below.

$$q_3(s) = (s+1)(s^2+s+1)$$

The transfer function matches the form of a third-order, low-pass Butterworth filter with a cutoff frequency of $1/RC$ and a passband gain of 4, as expected.

Problem 14–64. (E) Notch Filter Comparison

To eliminate an interfering signal at 10 krad/s on a new product design, your consulting firm needs to purchase a notch filter with the following specifications:

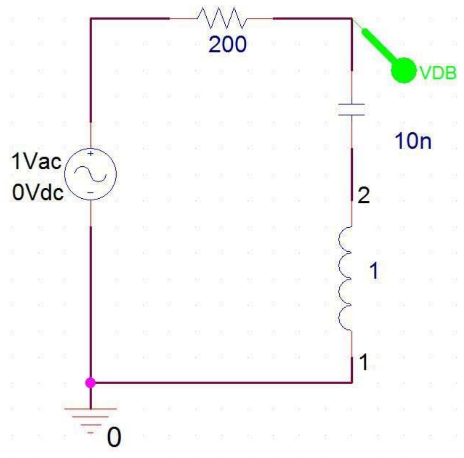
Center frequency = 10 krad/s ±0.5%

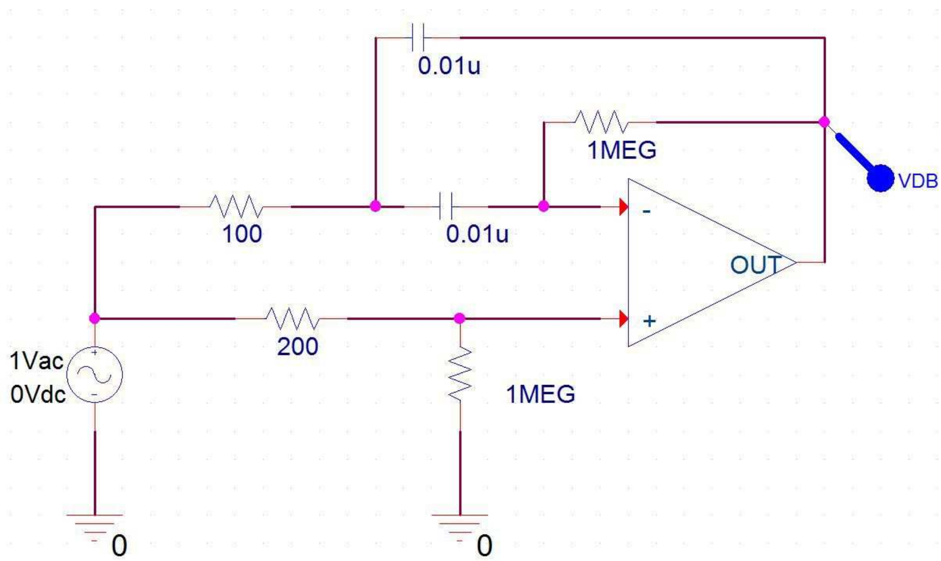
Bandwidth = 200 rad/s ±2%

Depth of notch (attenuation at ω_0) = 50 dB min.

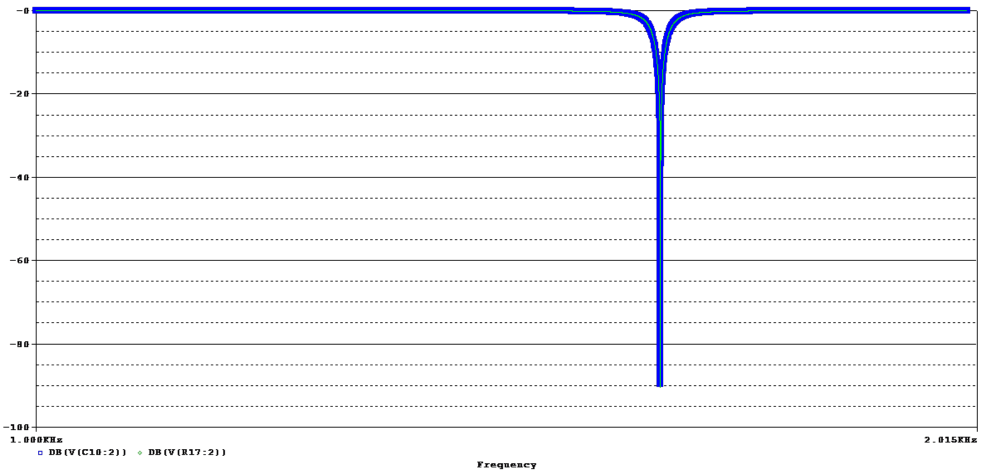
Two vendors have submitted proposed solutions as OrCAD graphics shown in Figure P14–64. As the owner, you want to select the best one. You must check that each circuit meets or fails every specification. You should also consider: (1) parts count, (2) ease of maintainability and implementation (fewer parts, number of similar parts, adjustments of potentiometers, standard values, and power usage), (3) frequency domain response, and (4) cost. You should address each item for each design at least qualitatively. Attach whatever software outputs you think will support your decision.

Simulate the circuits and analyze their amplitude responses.





The amplitude responses for both designs are shown below.



The two responses align with each other, so they share the same characteristics with respect to the specifications. The following table summarizes the results.

Item	Specification	Simplicity Plus	Filters-R-Us
ω_0	10 krad/s ±0.5%	10 krad/s	10 krad/s
B	200 rad/s ±2%	200.4 rad/s	200.4 rad/s
Notch Depth	50 dB	90.4 dB	90.4 dB
Response Summary		Meets Specs	Meets Specs
Parts Count	Minimum	3	7
Similar Parts	Maximum	0 pairs	2 pairs
Potentiometers	Minimum	0	0
Standard Values	Maximum	All	All
Power Usage (OP AMPs)	Minimum	0	1
Cost	Minimum	$15.00	$12.50

The circuits have the same frequency responses and both meet the specifications. If using an inductor is not a problem, then choose the design from Simplicity Plus, since it performs better than the other design with respect to most of the criteria. If using an inductor presents a problem, then choose the design from Filters-R-Us, since it meets the specifications.

Problem 14–65. (A) Biquad Filter

A biquad filter has the unique properties of having the ability to alter the filter's parameters, namely, gain K, quality factor Q, and resonant frequency ω_0. This is done in each case by adjusting one of the circuit's resistors. Analyze the biquad circuit in Figure P14–65 and determine which resistors influence each parameter. Explain how you can adjust three resistors in a specific order to set all three parameters.

Let $V_{\mathrm{A}}(s)$ be the output of the first OP AMP and $V_{\mathrm{B}}(s)$ be the output of the second OP AMP. The node-voltage equations at the negative input terminals for the three OP AMPs are:

$$\frac{-V_1(s)}{R_3} + \frac{-V_{\mathrm{A}}(s)}{R_1} + \frac{-V_{\mathrm{A}}(s)}{1/C_1 s} + \frac{-V_2(s)}{R_2} = 0$$

$$\frac{-V_{\mathrm{A}}(s)}{R_4} + \frac{-V_{\mathrm{B}}(s)}{1/C_2 s} = 0$$

$$\frac{-V_{\mathrm{B}}(s)}{R_5} + \frac{-V_2(s)}{R_5} = 0$$

Use the following MATLAB code to solve the equations for the transfer function.

```
syms s R1 R2 R3 R4 R5 C1 C2 V1 V2 VA VB
ZC1 = 1/s/C1;
ZC2 = 1/s/C2;
% Write the node-voltage equations
Eqn1 = -V1/R3 - VA/R1 - VA/ZC1 - V2/R2;
Eqn2 = -VA/R4 - VB/ZC2;
Eqn3 = -VB/R5 - V2/R5;
% Solve the equations
Soln = solve(Eqn1,Eqn2,Eqn3,VA,VB,V2);
V2 = Soln.V2;
% Create the transfer function
Ts = simplify(V2/V1)
```

The results are:

```
Ts = -(R1*R2)/(R3*(C1*C2*R1*R2*R4*s^2 + C2*R2*R4*s + R1))
```

The transfer function and its associated properties are:

$$T(s) = \frac{-R_1R_2}{R_3(R_1R_2R_4C_1C_2s^2 + R_2R_4C_2s + R_1)} = \frac{-\left(\dfrac{R_2}{R_3}\right)\dfrac{1}{R_2R_4C_1C_2}}{s^2 + \dfrac{1}{R_1C_1}s + \dfrac{1}{R_2R_4C_1C_2}}$$

$$K = -\frac{R_2}{R_3}$$

$$\omega_0^2 = \frac{1}{R_2R_4C_1C_2}$$

$$B = \frac{1}{R_1C_1}$$

$$Q = \frac{\omega_0}{B} = \sqrt{\frac{R_1^2C_1}{R_2R_4C_2}}$$

To adjust the parameter values, first set R_2 to specify the value of ω_0. Next, adjust R_1 to specify the value of Q. Finally, adjust R_3 to specify the value of K.

Problem 14–66. (A) Crystal Filters

Although not an active filter, crystal (Quartz) filters are very high-Q filters. Some can have Q's approaching 100000. High Q means high selectivity; hence, crystal filters are used extensively in communications where fine tuning is essential. In its simplest form, a quartz crystal is a rectangular cut crystal with metal plates attached to its flattest sides—imagine a parallel-plate capacitor with the crystal as the dielectric. An electronic equivalent of a quartz filter is shown in Figure P14–66. $C_{\rm O}$ is called the shunt capacitance and C_1, L_1, and R_1 the mechanical capacitance, inductance, and resistance of the crystal, respectively. A particular crystal has $C_{\rm O} = 1.0$ pF, $C_1 = 2.6$ fF, $L_1 = 6.0$ mH, and $R_1 = 35\ \Omega$. Assume $R_{\rm L} = 10\ \Omega$. Analyze this filter and determine its Q and ω_0. Use MATLAB or OrCAD to examine this circuit and help calculate the requested parameters.

Construct the transfer function for the circuit.

$$Z_{\rm O} = \frac{1}{C_{\rm O}s}$$

$$Z_1 = \frac{1}{C_1 s} + L_1 s + R_1 = \frac{L_1C_1s^2 + R_1C_1s + 1}{C_1 s}$$

$$Z_{\rm EQ} = \frac{Z_1Z_{\rm O}}{Z_1 + Z_{\rm O}} = \frac{\left(\dfrac{L_1C_1s^2 + R_1C_1s + 1}{C_1s}\right)\left(\dfrac{1}{C_{\rm O}s}\right)}{\dfrac{L_1C_1s^2 + R_1C_1s + 1}{C_1s} + \dfrac{1}{C_{\rm O}s}}$$

$$= \frac{L_1C_1s^2 + R_1C_1s + 1}{L_1C_1C_{\rm O}s^3 + R_1C_1C_{\rm O}s^2 + C_{\rm O}s + C_1s}$$

$$T(s) = \frac{R_{\rm L}}{R_{\rm L} + Z_{\rm EQ}} = \frac{R_{\rm L}L_1C_1C_{\rm O}s^3 + R_1R_{\rm L}C_1C_{\rm O}s^2 + R_{\rm L}(C_{\rm O} + C_1)s}{R_{\rm L}L_1C_1C_{\rm O}s^3 + (R_1R_{\rm L}C_1C_{\rm O} + L_1C_1)s^2 + [R_{\rm L}C_{\rm O}R_{\rm L}C_1 + R_1C_1]s + 1}$$

The following MATLAB code confirms the results:

```
syms R1 RL C1 CO L1 s
Z1 = 1/s/C1 + s*L1 + R1;
ZO = 1/s/CO;
Zeq = Z1*ZO/(Z1+ZO);
Ts = simplify(RL/(RL+Zeq))
```

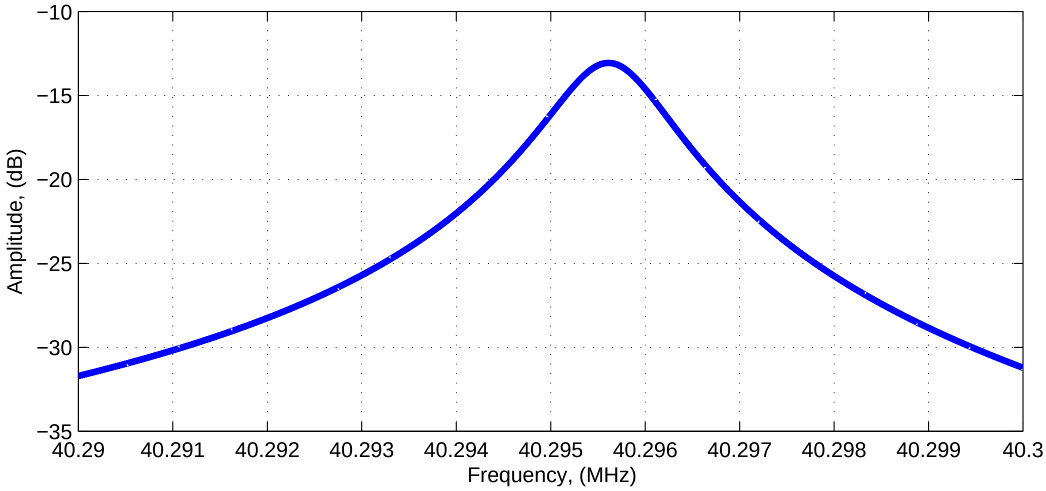
The output is:

```
Ts = (RL*s*(C1*CO*L1*s^2 + C1*CO*R1*s + C1 + CO))/...
     (C1*R1*s + C1*RL*s + CO*RL*s + C1*L1*s^2 + C1*CO*L1*RL*s^3 + C1*CO*R1*RL*s^2 + 1)
```

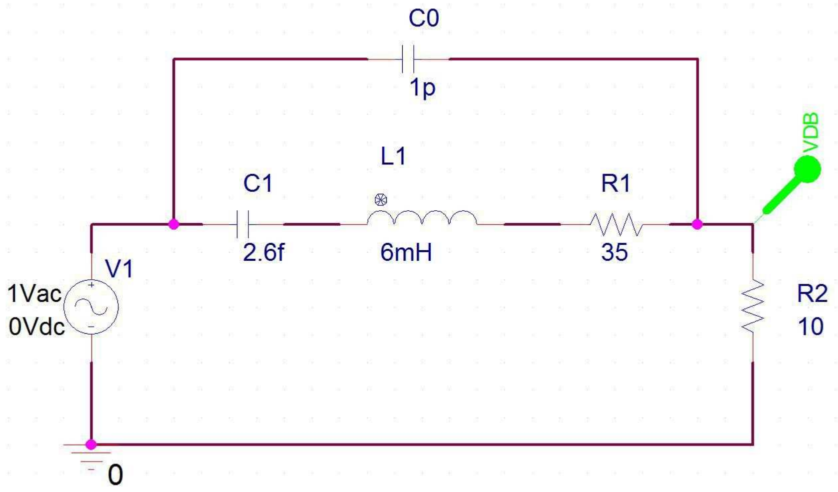
For the given values, the transfer function has the following value:

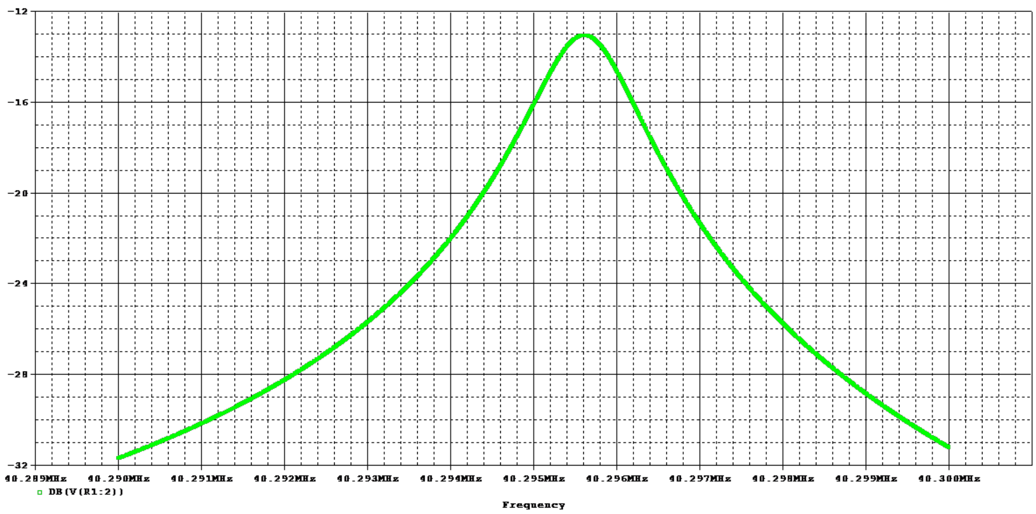
$$T(s) = \frac{s(1.98\times10^{16}s^2 + 1.15\times10^{20}s + 1.27\times10^{33})}{1.98\times10^{16}s^3 + 1.98\times10^{27}s^2 + 1.28\times10^{33}s + 1.27\times10^{44}}$$

The poles of the transfer function are $s = -10^{11}$ rad/s and $s = -3750 \pm j253.185\times10^6$ rad/s. Ignore the pole with the largest magnitude and determine the response from the dominate poles, which are the complex poles. With just the dominant poles, the denominator is approximately $s^2 + 7500s + 6.4102\times10^{16}$. The center frequency is 253.185 Mrad/s or 40.26 MHz. The bandwidth is 7500 rad/s or 1194 Hz. Calculate $Q = \omega_0/B = 33758$. The filter is very selective with an extremely high value for Q. A MATLAB plot of the amplitude response near the center frequency is shown below.



The OrCAD simulation and results are shown below.





15 Mutual Inductance and Transformers

15.1 Exercise Solutions

Exercise 15–1. In Figure 15–4, $i_1(t) = -20\cos(8000t)$ mA and $i_2(t) = 0$. Find $v_1(t)$ and $v_2(t)$.

From Figure 15–4, we have $L_1 = 10$ mH, $L_2 = 10$ mH, $M = 2$ mH, and additive coupling. The i-v equations are

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = 0.01\frac{di_1(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = 0.002\frac{di_1(t)}{dt}$$

Substitute in for $i_1(t)$ and solve for the two voltages.

$$v_1(t) = 0.01\frac{d}{dt}\left[-0.02\cos(8000t)\right] = (0.01)(-0.02)(8000)\left[-\sin(8000t)\right] = 1.6\sin(8000t)\,\text{V}$$

$$v_2(t) = 0.002\frac{d}{dt}\left[-0.02\cos(8000t)\right] = (0.002)(-0.02)(8000)\left[-\sin(8000t)\right] = 0.32\sin(8000t)\,\text{V}$$

Exercise 15–2. Find $v_1(t)$ and $v_2(t)$ for the circuit in Figure 15–6.

From Figure 15–6, we have $L_1 = 0.2$ mH, $L_2 = 0.5$ mH, $M = 0.3$ mH, and additive coupling. The two current sources define the inductor currents with $i_1(t) = 5\cos(10000t)$ A and $i_2(t) = -2\sin(5000t)$ A. The i-v equations are

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt}$$

Substitute in for inductance values and currents and solve for the two voltages.

$$v_1(t) = 0.0002\frac{d}{dt}[5\cos(10000t)] + 0.0003\frac{d}{dt}[-2\sin(5000t)] = -10\sin(10000t) - 3\cos(5000t)\,\text{V}$$

$$v_2(t) = 0.0003\frac{d}{dt}[5\cos(10000t)] + 0.0005\frac{d}{dt}[-2\sin(5000t)] = -15\sin(10000t) - 5\cos(5000t)\,\text{V}$$

Exercise 15–3. A pair of coupled inductors have self-inductances $L_1 = 4.5$ mH and $L_2 = 8$ mH. What is the maximum possible mutual inductance?

The energy constraints require $M^2 \leq L_1L_2$.

$$M^2 \leq L_1L_2 = (0.0045)(0.008) = 36\times10^{-6}$$

$$M \leq \sqrt{36\times10^{-6}} = 6\,\text{mH}$$

Exercise 15–4. For a transformer with perfect coupling, find the secondary voltage $v_2(t)$ when the primary voltage is $v_1(t) = 50\cos(3000t)$ V, $n = 50$, and the mutual inductance is subtractive.

For subtractive coupling, $v_2(t) = -nv_1(t) = (-50)(50)\cos(3000t) = -2500\cos(3000t)$ V.

Exercise 15–5. The turns ratio of the ideal transformer in Figure 15–10 is $n = 1/4$ and the source and load impedances are $Z_\text{S} = 50 + j0\ \Omega$ and $Z_\text{L} = 5 - j2\ \Omega$. Find the impedance seen by the voltage source.

Compute the transformer input impedance and add it to the source impedance in series.

$$Z = Z_\text{S} + Z_\text{IN} = Z_\text{S} + \frac{Z_\text{L}}{n^2} = 50 + j0 + \frac{5-j2}{\frac{1}{16}}$$

$$Z = 50 + 80 - j32 = 130 - j32\,\Omega$$

Exercise 15–6. The input on the primary side of an ideal transformer is 1250 Ω when the load connected to the secondary is 50 Ω. What is the transformer turns ratio?

Apply the relationship for the transformer input impedance.

$$Z_{\mathrm{IN}} = \frac{Z_{\mathrm{L}}}{n^2}$$

$$n = \sqrt{\frac{Z_{\mathrm{L}}}{Z_{\mathrm{IN}}}} = \sqrt{\frac{50}{1250}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$$

Exercise 15–7. Using the values of the mesh currents found in Example 15–8, find the average power delivered to the load Z_{L}.

From Example 15–8, the current through the load is $\mathbf{I}_{\mathrm{B}} = 20.959\angle{-44.18^\circ}$ A. The average power delivered to the load is given by the following expression:

$$P_L = \frac{1}{2}\left|\mathbf{I}_{\mathrm{B}}\right|^2 R_{\mathrm{L}} = \frac{1}{2}(20.959)^2(25) = 5.491\,\mathrm{kW}$$

Exercise 15–8. Using the results in Example 15–10, find the input impedance seen by the source in Figure 15–15.

The input impedance is the ratio of voltage $\mathbf{V}_{\mathrm{S}}$ to current $\mathbf{I}_{\mathrm{A}}$.

$$Z_{\mathrm{IN}} = \frac{\mathbf{V}_{\mathrm{S}}}{\mathbf{I}_{\mathrm{A}}} = \frac{500}{(n+1)\mathbf{I}_{\mathrm{B}}} = \frac{500}{(2.4)(10.3 - j11.9)} = 8.66 + j10.0\,\Omega$$

Exercise 15–9. Using the results found in Example 15–11, find the input impedance seen by the source in Figure 15–16.

The input impedance is the ratio of voltage $\mathbf{V}_{\mathrm{S}}$ to current $\mathbf{I}_1$.

$$Z_{\mathrm{IN}} = \frac{\mathbf{V}_{\mathrm{S}}}{\mathbf{I}_1} = \frac{100}{1.92 - j3.46} = 12.26 + j22.10\,\Omega$$

15.2 Problem Solutions

Problem 15–1. In Figure P15–1, $L_1 = 120$ mH, $L_2 = 400$ mH, $M = 180$ mH, and $v_2(t) = 60\sin(500t)$ V.

(a). Write the *i*-*v* relationships for the coupled inductors using the reference marks in the figure.

In the figure, the coupling is additive, so the appropriate equations are

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = 0.12\frac{di_1(t)}{dt} + 0.18\frac{di_2(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = 0.18\frac{di_1(t)}{dt} + 0.4\frac{di_2(t)}{dt}$$

(b). Find the source voltage $v_S(t)$ when the output terminals are open-circuited ($i_2 = 0$).

With the output terminals open-circuited, there is no current in the secondary, so $i_2 = 0$ and the $di_2(t)/dt$ terms drop out of the *i*-*v* equations. Solve the second equation in Part (a) for $di_1(t)/dt$ and substitute into the first equation.

$$v_1(t) = 0.12\frac{di_1(t)}{dt}$$

$$v_2(t) = 0.18\frac{di_1(t)}{dt}$$

$$\frac{di_1(t)}{dt} = \frac{v_2(t)}{0.18}$$

$$v_S(t) = v_1(t) = 0.12\frac{di_1(t)}{dt} = (0.12)\left(\frac{v_2(t)}{0.18}\right) = \left(\frac{2}{3}\right)60\sin(500t) = 40\sin(500t)\text{ V}$$

Problem 15–2. In Figure P15–1, $L_1 = 10$ mH, $L_2 = 5$ mH, $M = 7$ mH, and $v_S(t) = 100\sin(1000t)$ V.

(a). Write the *i*-*v* relationships for the coupled inductors using the reference marks in the figure.

In the figure, the coupling is additive, so the appropriate equations are

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = 0.01\frac{di_1(t)}{dt} + 0.007\frac{di_2(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = 0.007\frac{di_1(t)}{dt} + 0.005\frac{di_2(t)}{dt}$$

(b). Solve for $i_2(t)$ when the output terminals are short-circuited ($v_2 = 0$). Assume i_2 has no dc component.

With the output terminals short-circuited, the second equation in Part (a) can be used to solve for $di_1(t)/dt$ in terms of $di_2(t)/dt$.

$$0 = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = 0.007\frac{di_1(t)}{dt} + 0.005\frac{di_2(t)}{dt}$$

$$\frac{di_1(t)}{dt} = \frac{-0.005}{0.007}\frac{di_2(t)}{dt} = \frac{-5}{7}\frac{di_2(t)}{dt}$$

Substitute into the first equation in Part (a)

$$v_1(t) = v_S(t) = 100\sin(1000t) = (0.01)\left(\frac{-5}{7}\frac{di_2(t)}{dt}\right) + 0.007\frac{di_2(t)}{dt}$$

$$100\sin(1000t) = \left(\frac{-0.05}{7} + \frac{7}{1000}\right)\frac{di_2(t)}{dt} = \left(\frac{49-50}{7000}\right)\frac{di_2(t)}{dt} = \left(\frac{-1}{7000}\right)\frac{di_2(t)}{dt}$$

$$\frac{di_2(t)}{dt} = -700000\sin(1000t)$$

$$i_2(t) = \int -700000\sin(1000x)dx = 700\cos(1000t)\text{ A}$$

In the last integral, there is no constant of integration because $i_2(t)$ has no dc component.

Problem 15–3. In Figure P15–1, $L_1 = 10$ mH, $L_2 = 10$ mH, $M = 7$ mH, $i_1(t) = 2\sin(1000t)$ A, and $i_2(t) = -2\sin(1000t)$ A.

(a). Write the i-v relationships for the coupled inductors using the reference marks given.

In the figure, the coupling is additive, so the appropriate equations are

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = 0.01\frac{di_1(t)}{dt} + 0.007\frac{di_2(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = 0.007\frac{di_1(t)}{dt} + 0.01\frac{di_2(t)}{dt}$$

(b). Find the input voltage $v_1(t)$ and the output voltage $v_2(t)$.

Solve directly using the currents given to compute the voltages.

$$v_1(t) = (0.01)\frac{d}{dt}[2\sin(1000t)] + (0.007)\frac{d}{dt}[-2\sin(1000t)]$$

$$= 20\cos(1000t) - 14\cos(1000t) = 6\cos(1000t)\text{ V}$$

$$v_2(t) = (0.007)\frac{d}{dt}[2\sin(1000t)] + (0.01)\frac{d}{dt}[-2\sin(1000t)]$$

$$= 14\cos(1000t) - 20\cos(1000t) = -6\cos(1000t)\text{ V}$$

The corresponding MATLAB code is shown below.

```
syms t real
L1 = 10e-3;
L2 = 10e-3;
M = 7e-3;
i1 = 2*sin(1000*t);
i2 = -2*sin(1000*t);
di1 = diff(i1,t);
di2 = diff(i2,t);
v1 = L1*di1 + M*di2
v2 = M*di1 + L2*di2
```

The MATLAB results are shown below and agree with the calculations above.

```
v1 = 6*cos(1000*t)
v2 = -6*cos(1000*t)
```

Problem 15–4. In Figure P15–4, $L_1 = 30$ mH, $L_2 = 25$ mH, and $M = 25$ mH. The output current is observed to be $i_2(t) = 5\sin(100t)$ A when the output terminals are short-circuited ($v_2 = 0$). Find $v_1(t)$ and $i_S(t)$, assuming that $i_S(t)$ has no dc component.

In the figure, the coupling is subtractive, so the appropriate equations are

$$v_1(t) = L_1\frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} = 0.03\frac{di_1(t)}{dt} - 0.025\frac{di_2(t)}{dt}$$

$$v_2(t) = -M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = -0.025\frac{di_1(t)}{dt} + 0.025\frac{di_2(t)}{dt}$$

Apply the short-circuit condition and solve for the requested values.

$$v_2(t) = 0 = -0.025\frac{di_1(t)}{dt} + 0.025\frac{di_2(t)}{dt}$$

$$\frac{di_1(t)}{dt} = \frac{di_2(t)}{dt}$$

$$i_1(t) = i_S(t) = i_2(t) = 5\sin(100t) \text{ A}$$

$$v_1(t) = 0.03\frac{di_1(t)}{dt} - 0.025\frac{di_2(t)}{dt} = 0.005\frac{di_2(t)}{dt} = (0.005)\frac{d}{dt}[5\sin(100t)] = 2.5\cos(100t) \text{ V}$$

Problem 15–5. In Figure P15–4, $L_1 = 25$ mH, $L_2 = 30$ mH, and $M = 25$ mH. The output voltage is observed to be $v_2(t) = 50\cos(1000t)$ V when the output terminals are open-circuited ($i_2 = 0$). Find $v_1(t)$.

In the figure, the coupling is subtractive, so the appropriate equations are

$$v_1(t) = L_1\frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} = 0.025\frac{di_1(t)}{dt} - 0.025\frac{di_2(t)}{dt}$$

$$v_2(t) = -M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = -0.025\frac{di_1(t)}{dt} + 0.03\frac{di_2(t)}{dt}$$

Apply the open-circuit condition and solve for the requested values.

$$v_2(t) = -0.025\frac{di_1(t)}{dt}$$

$$\frac{di_1(t)}{dt} = -40v_2(t)$$

$$v_1(t) = 0.025\frac{di_1(t)}{dt} = (0.025)(-40)v_2(t) = -v_2(t) = -50\cos(1000t) \text{ V}$$

Problem 15–6. A pair of coupled inductors have $L_1 = 3.6$ H, $L_2 = 2.5$ H, and $k = 1.0$. When the output terminals are open-circuited ($i_2 = 0$) the output voltage is observed to be $v_2(t) = 30\sin(1000t)$ V. Find the input voltage $v_1(t)$ for additive coupling.

With additive coupling and $i_2 = 0$, the appropriate equations are:

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = L_1\frac{di_1(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = M\frac{di_1(t)}{dt}$$

$$\frac{di_1(t)}{dt} = \frac{v_2(t)}{M}$$

$$M = k\sqrt{L_1L_2} = (1)\sqrt{(3.6)(2.5)} = 3 \text{ H}$$

$$v_1(t) = \frac{L_1}{M}v_2(t) = \frac{3.6}{3}[30\sin(1000t)] = 36\sin(1000t) \text{ V}$$

Problem 15–7. In Figure P15–7, $L_1 = 6$ H, $L_2 = 3$ H, $M = 4$ H, and $i_1(t) = 5\sin(1000t)$ mA. Find the input voltage $v_X(t)$.

The coupling is additive and by tracing the current flow in the circuit, we have $i_1(t) = i_2(t)$. We have the following relationships and results:

$$
\begin{aligned}
v_X(t) &= v_1(t) + v_2(t) \\
v_1(t) &= L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = (L_1 + M)\frac{di_1(t)}{dt} \\
&= (10)\frac{d}{dt}[0.005\sin(1000t)] = 50\cos(1000t) \text{ V} \\
v_2(t) &= M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = (M + L_2)\frac{di_1(t)}{dt} \\
&= (7)\frac{d}{dt}[0.005\sin(1000t)] = 35\cos(1000t) \text{ V} \\
v_X(t) &= 85\cos(1000t) \text{ V}
\end{aligned}
$$

Problem 15–8. In Figure P15–8 show that $L_{EQ} = L_1(1 - k^2)$, where k is the coupling coefficient.

The circuit has additive coupling and $v_2(t) = 0$. We have the following relationships:

$$
\begin{aligned}
v_2(t) = 0 &= M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} \\
\frac{di_2(t)}{dt} &= -\frac{M}{L_2}\frac{di_1(t)}{dt} \\
v_1(t) &= L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = L_1\frac{di_1(t)}{dt} - \frac{M^2}{L_2}\frac{di_1(t)}{dt} = \left(L_1 - \frac{M^2}{L_2}\right)\frac{di_1(t)}{dt} \\
M &= k\sqrt{L_1L_2} \\
M^2 &= k^2L_1L_2 \\
v_1(t) &= \left(L_1 - \frac{k^2L_1L_2}{L_2}\right)\frac{di_1(t)}{dt} = L_1\left(1 - k^2\right)\frac{di_1(t)}{dt} \\
L_{EQ} &= L_1\left(1 - k^2\right)
\end{aligned}
$$

Problem 15–9. In Figure P15–9, show that the indicated open-circuit voltage is $v_{OC} = \left(k\sqrt{L_2/L_1}\right)v_1$, where k is the coupling coefficient.

The coupling is additive and with the open circuit, we have $i_2(t) = 0$. We have the following relationships:

$$
\begin{aligned}
v_1(t) &= L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = L_1\frac{di_1(t)}{dt} \\
v_2(t) &= M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = M\frac{di_1(t)}{dt} \\
v_{OC}(t) = v_2(t) &= \frac{M}{L_1}v_1(t) = \frac{k\sqrt{L_1L_2}}{L_1}v_1(t) = k\sqrt{\frac{L_2}{L_1}}v_1(t)
\end{aligned}
$$

Problem 15–10. In Figure P15-10, show that the indicated short-circuit current is $i_{SC} = \left(k\sqrt{L_1/L_2}\right)i_1$, where k is the coupling coefficient. Assume that i_1 has no dc component.

The coupling is additive and with the short circuit, we have $v_2(t) = 0$. We have the following relationships:

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$v_2(t) = 0 = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt}$$

$$\frac{di_2(t)}{dt} = -\frac{M}{L_2}\frac{di_1(t)}{dt}$$

$$i_2(t) = -\frac{M}{L_2}i_1(t)$$

$$i_{\mathrm{SC}}(t) = -i_2(t) = \frac{M}{L_2}i_1(t) = \frac{k\sqrt{L_1L_2}}{L_2}i_1(t) = k\sqrt{\frac{L_1}{L_2}}i_1(t)$$

Problem 15–11. The primary voltage of an ideal transformer is a 120-V, 60-Hz sinusoid. The secondary voltage is a 24-V, 60-Hz sinusoid. The secondary winding is connected to an 800-Ω resistive load.

(a). Find the transformer turns ratio.

We have the following results:

$$n = \frac{v_2(t)}{v_1(t)} = \frac{24}{120} = \frac{1}{5}$$

(b). Write expressions for the primary current and voltage.

With additive coupling, we have the following relationships:

$$i_2(t) = -\frac{v_2(t)}{R_{\mathrm{L}}} = -\frac{24\cos(120\pi t)}{800} = -30\cos(120\pi t)\text{ mA}$$

$$i_1(t) = -ni_2(t) = 6\cos(120\pi t)\text{ mA}$$

$$v_1(t) = 120\cos(120\pi t)\text{ V}$$

Problem 15–12. The number of turns in the primary and secondary of an ideal transformer are $N_1 = 50$ and $N_2 = 400$. The primary winding is connected to a 120-V, 60-Hz source with a source resistance of 50 Ω. The secondary winding is connected to a 1600-Ω load. Find the primary and secondary currents.

We have the following relationships:

$$n = \frac{N_2}{N_1} = \frac{400}{50} = 8$$

$$Z_{\mathrm{IN}} = \frac{Z_{\mathrm{L}}}{n^2} = \frac{1600}{8^2} = 25\ \Omega$$

$$i_1(t) = \frac{v_1(t)}{Z_{\mathrm{S}} + Z_{\mathrm{IN}}} = \frac{120\cos(120\pi t)}{50+25} = 1.6\cos(120\pi t)\text{ A}$$

$$i_2(t) = -\frac{i_1(t)}{n} = -\frac{1.6\cos(120\pi t)}{8} = -0.2\cos(120\pi t)\text{ A}$$

Problem 15–13. The turns ratio of the second ideal transformer in Figure P15–13 is $n = 5$. Find the equivalent resistance indicated in the figure.

Apply the relationship for the input impedance of a transformer twice as follows:

$$R_{\text{IN}} = \frac{R_{\text{L}}}{n^2}$$

$$R_{\text{EQ}} = \frac{1}{2^2}\left[10 + \left(\frac{1}{5^2}\right)(50)\right] = 3\ \Omega$$

Problem 15–14. The equivalent resistance in Figure P15–13 is $R_{\text{EQ}} = 15\ \Omega$. Find the turns ratio of the second ideal transformer.

We have the following results:

$$R_{\text{EQ}} = 15 = \frac{1}{2^2}\left[10 + \left(\frac{1}{n^2}\right)(50)\right]$$

$$60 = 10 + \left(\frac{1}{n^2}\right)(50)$$

$$50 = \left(\frac{1}{n^2}\right)(50)$$

$$1 = \frac{1}{n^2}$$

$$n = 1$$

Problem 15–15. Figure P15–15 shows an ideal transformer connected as an autotransformer. Find $i_{\text{L}}(t)$ and $i_{\text{S}}(t)$ when $v_{\text{S}}(t) = 120\sin(400t)$ and $R_{\text{L}} = 60\ \Omega$.

The circuit has additive coupling. We have the following relationships:

$$v_1(t) = v_{\text{S}}(t) = 120\sin(400t)\text{ V}$$

$$R_{\text{L}} = 60\ \Omega$$

$$v_2(t) = \frac{N_2}{N_1}v_1(t) = (1)[120\sin(400t)] = 120\sin(400t)\text{ V}$$

$$v_{\text{L}}(t) = v_1(t) + v_2(t) = 240\sin(400t)\text{ V}$$

$$i_{\text{L}}(t) = \frac{v_{\text{L}}(t)}{R_{\text{L}}} = 4\sin(400t)\text{ A}$$

$$i_2(t) = -i_{\text{L}}(t) = -4\sin(400t)\text{ A}$$

$$i_1(t) = -ni_2(t) = 4\sin(400t)\text{ A}$$

$$i_{\text{S}}(t) = i_2(t) - i_1(t) = -8\sin(400t)\text{ A}$$

Problem 15–16. The primary winding of an ideal transformer with $n = 1/5$ is connected to a voltage source with a source resistance of 2 kΩ. Find the load resistance connected across the secondary winding that will draw maximum power from the source.

For maximum power transfer, we need $R_{\text{IN}} = R_{\text{S}} = 2\text{ k}\Omega$. We have the following results:

$$R_{\text{IN}} = \frac{R_{\text{L}}}{n^2}$$

$$R_{\text{L}} = n^2 R_{\text{IN}} = \left(\frac{1}{5^2}\right)(2000) = 80\ \Omega$$

Problem 15–17. Select the turns ratio of an ideal transformer in the interface circuit shown in Figure P15–17 so that the input resistance seen by the voltage source is 150 Ω.

We have the following results:

$$R_{\text{IN}} = \frac{R_{\text{L}}}{n^2}$$

$$R_{\text{S}} = 150 = 50 + \left[200 \parallel \left(150 + \frac{1000}{n^2}\right)\right]$$

$$100 = \left[200 \parallel \left(150 + \frac{1000}{n^2}\right)\right] = [200 \parallel 200]$$

$$200 = 150 + \frac{1000}{n^2}$$

$$50 = \frac{1000}{n^2}$$

$$n^2 = \frac{1000}{50} = 20$$

$$n = \sqrt{20} = 4.4721$$

Problem 15–18. A voltage source with Thévenin parameters $v_{\text{T}} = 5\sin(1000t)$ V and $R_{\text{T}} = 25$ Ω drives the primary winding of an ideal transformer with $n = 2$. Write an expression for the instantaneous power delivered to a 300-Ω load connected across the secondary winding.

Find the input resistance and then the primary current. Then solve for the power delivered to the load.

$$R_{\text{IN}} = \frac{R_{\text{L}}}{n^2} = \frac{300}{2^2} = 75\ \Omega$$

$$i_1(t) = \frac{v_{\text{T}}}{R_{\text{T}} + R_{\text{IN}}} = \frac{5\sin(1000t)}{25+75} = 50\sin(1000t)\ \text{mA}$$

$$v_1(t) = \left(\frac{75}{25+75}\right)[5\sin(1000t)] = 3.75\sin(1000t)\ \text{V}$$

$$p_2(t) = p_1(t) = v_1(t)i_1(t) = [3.75\sin(1000t)]\,[0.05\sin(1000t)] = 187.5\sin^2(1000t)\ \text{mW}$$

Problem 15–19. Find $i_{\text{S}}(t)$ in Figure P15–19 when $v_{\text{S}}(t) = 15\sin(377t)$ V.

We have the following relationships and results:

$$v_{\text{S}}(t) = v_1(t) = 15\sin(377t)\ \text{V}$$

$$v_2(t) = nv_1(t) = 60\sin(377t)\ \text{V}$$

$$i_{\text{L}}(t) = \frac{v_2(t)}{12} = 5\sin(377t)\ \text{A}$$

$$\begin{aligned}
i_S(t) &= i_1(t) + \frac{v_1(t) - v_2(t)}{6} = i_1(t) + \frac{15\sin(377t) - 60\sin(377t)}{6} \\
&= i_1(t) - 7.5\sin(377t) \text{ A} \\
i_2(t) + i_L(t) &= -7.5\sin(377t) \text{ A} \\
i_2(t) &= -7.5\sin(377t) - 5\sin(377t) = -12.5\sin(377t) \text{ A} \\
i_1(t) &= -ni_2(t) = 50\sin(377t) \text{ A} \\
i_S(t) &= 42.5\sin(377t) \text{ A}
\end{aligned}$$

Problem 15–20. Show that the equivalent resistance in Figure P15–20 is

$$R_{EQ} = \left(\frac{N_1 + N_2}{N_2}\right)^2 R_L$$

We have the following relationships and results:

$$\begin{aligned}
R_{EQ} &= \frac{v_1(t) + v_2(t)}{i_1(t)} \\
N_1 v_2(t) &= N_2 v_1(t) \\
i_L(t) &= \frac{v_2(t)}{R_L} \\
i_2(t) &= -\frac{N_1}{N_2} i_1(t) \\
i_1(t) &= i_2(t) + i_L(t) = -\frac{N_1}{N_2} i_1(t) + \frac{v_2(t)}{R_L} \\
\left(1 + \frac{N_1}{N_2}\right) i_1(t) &= \frac{v_2(t)}{R_L} \\
\left(\frac{N_1 + N_2}{N_2}\right) i_1(t) &= \frac{v_2(t)}{R_L} \\
i_1(t) &= \left(\frac{N_2}{N_1 + N_2}\right) \frac{v_2(t)}{R_L} \\
R_{EQ} &= \frac{\frac{N_1}{N_2} v_2(t) + v_2(t)}{\left(\frac{N_2}{N_1 + N_2}\right) \frac{v_2(t)}{R_L}} = \left(\frac{N_1 + N_2}{N_2}\right)^2 R_L
\end{aligned}$$

Problem 15–21. In Figure P15–21, the impedances are $Z_1 = 25 - j45\ \Omega$, $Z_2 = 45 + j30\ \Omega$, and $Z_3 = 360 + j270\ \Omega$. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.

Apply the formula

$$Z_{IN} = \frac{Z_L}{n^2}$$

and account for the series impedances in the circuit to determine the overall input impedance. Then determine the currents in the circuit. We have the following results:

$$\begin{aligned}
Z_{\rm IN} &= Z_1 + \frac{1}{n_1^2}\left(Z_2 + \frac{Z_3}{n_2^2}\right) \\
&= 25 - j45 + \frac{1}{4}\left(45 + j30 + \frac{360 + j270}{9}\right) \\
&= 25 - j45 + \frac{1}{4}\left(45 + j30 + 40 + j30\right) \\
&= 25 - j45 + \frac{1}{4}(85 + j60) = 25 - j45 + 21.25 + j15 \\
&= 46.25 - j30\ \Omega
\end{aligned}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{\rm S}}{Z_{\rm IN}} = \frac{200}{46.25 - j30} = 3.044 + j1.974\ \text{A}$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1}{2} = 1.522 + j0.987\ \text{A}$$

$$\mathbf{I}_3 = \frac{\mathbf{I}_2}{3} = 0.507 + j0.329\ \text{A}$$

Problem 15–22. In Figure P15–21, the impedances are $Z_1 = 35 + j20\ \Omega$, $Z_2 = 70 + j20\ \Omega$, and $Z_3 = 270 - j90\ \Omega$. Find the average power delivered to Z_3.

Apply the formula

$$Z_{\rm IN} = \frac{Z_{\rm L}}{n^2}$$

and account for the series impedances in the circuit to determine the overall input impedance. Then determine the currents in the circuit. Finally, calculate the average power delivered to the load. We have the following results:

$$\begin{aligned}
Z_{\rm IN} &= Z_1 + \frac{1}{n_1^2}\left(Z_2 + \frac{Z_3}{n_2^2}\right) \\
&= 35 + j20 + \frac{1}{4}\left(70 + j20 + \frac{270 - j90}{9}\right) \\
&= 35 + j20 + \frac{1}{4}\left(70 + j20 + 30 - j10\right) \\
&= 35 + j20 + 25 + j2.5 = 60 + j22.5\ \Omega
\end{aligned}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{\rm S}}{Z_{\rm IN}} = \frac{200}{60 + j22.5} = 2.922 - j1.096\ \text{A}$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1}{2} = 1.461 - j0.548\ \text{A}$$

$$\mathbf{I}_3 = \frac{\mathbf{I}_2}{3} = 0.487 - j0.183 = 0.520\angle{-20.6^\circ}\ \text{A}$$

$$P_3 = \frac{1}{2}\left|\mathbf{I}_3\right|^2 R_3 = \frac{1}{2}(0.520)^2(270) = 36.53\ \text{W}$$

Problem 15–23. An ideal transformer has a turns ratio of $n = 10$. The secondary winding is connected to a load $Z_L = 600 + j100\ \Omega$. The primary is connected to a voltage source with a peak amplitude of 400 V and an internal impedance of $Z_S = j7\ \Omega$. Find the average power delivered to the load.

We have the following results:

$$Z_{IN} = \frac{Z_L}{n^2} = \frac{600 + j100}{10^2} = 6 + j\ \Omega$$

$$Z_1 = Z_S + Z_{IN} = j7 + 6 + j = 6 + j8\ \Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_S}{Z_1} = \frac{400}{6 + j8} = 24 - j32\ \text{A}$$

$$\mathbf{I}_L = -\mathbf{I}_2 = \frac{\mathbf{I}_1}{n} = 2.4 - j3.2 = 4\angle{-53.1^\circ}\ \text{A}$$

$$P_L = \frac{1}{2}\left|\mathbf{I}_L\right|^2 R_L = \frac{1}{2}(4)^2(600) = 4.8\ \text{kW}$$

Problem 15–24. The primary winding of an ideal transformer with $N_1 = 100$ and $N_2 = 250$ is connected to a 480-V source. A load impedance of $Z_L = 150 + j200\ \Omega$ is connected across the secondary windings. Find the amplitudes of the primary and secondary currents.

We have the following results:

$$n = \frac{N_2}{N_1} = 2.5$$

$$Z_{IN} = \frac{Z_L}{n^2} = \frac{150 + j200}{2.5^2} = 24 + j32\ \Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_S}{Z_{IN}} = \frac{480}{24 + j32} = 7.2 - j9.6 = 12\angle{-53.1^\circ}\ \text{A}$$

$$\mathbf{I}_2 = -\frac{\mathbf{I}_1}{n} = -2.88 + j3.84 = 4.8\angle 126.9^\circ\ \text{A}$$

The current amplitudes are $|\mathbf{I}_1| = 12$ A and $|\mathbf{I}_2| = 4.8$ A.

Problem 15–25. A transformer that can be treated as ideal has 480 turns in the primary winding and 240 turns in the secondary winding. The primary is connected to a 60-Hz source with peak amplitude of 440 V. The secondary winding delivers an average power of 5 kW to a resistive load. Find the amplitudes of the primary and secondary currents and the transformer input resistance.

We have the following relationships and results:

$$n = \frac{N_2}{N_1} = \frac{240}{480} = \frac{1}{2}$$

$$|\mathbf{V}_1| = |\mathbf{V}_\mathrm{S}| = 440 \text{ V}$$

$$|\mathbf{V}_2| = n|\mathbf{V}_1| = 220 \text{ V}$$

$$|\mathbf{V}_\mathrm{L}| = |\mathbf{V}_2| = 220 \text{ V}$$

$$P_\mathrm{L} = 5000 \text{ W} = \frac{1}{2}\frac{|\mathbf{V}_2|^2}{R_\mathrm{L}} = \frac{220^2}{2R_\mathrm{L}}$$

$$R_\mathrm{L} = \frac{220^2}{10000} = 4.84\ \Omega$$

$$|\mathbf{I}_2| = |\mathbf{I}_\mathrm{L}| = \frac{|\mathbf{V}_2|}{R_\mathrm{L}} = \frac{220}{4.84} = 45.455 \text{ A}$$

$$|\mathbf{I}_1| = n|\mathbf{I}_2| = 22.727 \text{ A}$$

$$R_\mathrm{IN} = \frac{R_\mathrm{L}}{n^2} = \frac{4.84}{0.25} = 19.36\ \Omega$$

Problem 15–26. The input voltage to the transformer in Figure P15–26 is a sinusoid $v_\mathrm{S}(t) = 60\cos(2500t)$ V. With the circuit operating in the sinusoidal steady state, use mesh-current analysis to find the phasor output voltage $\mathbf{V}_2$ and the input impedance Z_IN.

The circuit has additive coupling and the phasor-domain equations are:

$$\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2 = j75\mathbf{I}_1 + j150\mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2 = j150\mathbf{I}_1 + j300\mathbf{I}_2$$

The mesh-current equations are:

$$50\mathbf{I}_\mathrm{A} + \mathbf{V}_1 = \mathbf{V}_\mathrm{S}$$

$$-\mathbf{V}_2 + 600\mathbf{I}_\mathrm{B} = 0$$

$$\mathbf{I}_\mathrm{A} = \mathbf{I}_1$$

$$\mathbf{I}_\mathrm{B} = -\mathbf{I}_2$$

Substitute for the voltages and solve.

$$50\mathbf{I}_1 + j75\mathbf{I}_1 + j150\mathbf{I}_2 = 60$$

$$-j150\mathbf{I}_1 - j300\mathbf{I}_2 - 600\mathbf{I}_2 = 0$$

$$\mathbf{I}_1 = 480 - j360 \text{ mA}$$

$$\mathbf{I}_2 = -120 - j60 \text{ mA}$$

$$\mathbf{V}_2 = -\mathbf{I}_2 Z_\mathrm{L} = (0.12 + j0.06)(600) = 72 + j36 \text{ V}$$

$$Z_\mathrm{IN} = \frac{\mathbf{V}_\mathrm{S}}{\mathbf{I}_1} = \frac{60}{0.48 - j0.36} = 80 + j60\ \Omega$$

Problem 15–27. Repeat Problem 15–26 with $v_1(t) = 60\cos(500t)$ V.

The circuit has additive coupling and the phasor-domain equations are:

$$\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2 = j15\mathbf{I}_1 + j30\mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2 = j30\mathbf{I}_1 + j60\mathbf{I}_2$$

The mesh-current equations are:

$$50\mathbf{I}_{\mathrm{A}} + \mathbf{V}_1 = \mathbf{V}_{\mathrm{S}}$$

$$-\mathbf{V}_2 + 600\mathbf{I}_{\mathrm{B}} = 0$$

$$\mathbf{I}_{\mathrm{A}} = \mathbf{I}_1$$

$$\mathbf{I}_{\mathrm{B}} = -\mathbf{I}_2$$

Substitute for the voltages and solve.

$$50\mathbf{I}_1 + j15\mathbf{I}_1 + j30\mathbf{I}_2 = 60$$

$$-j30\mathbf{I}_1 - j60\mathbf{I}_2 - 600\mathbf{I}_2 = 0$$

$$\mathbf{I}_1 = 1076 - j310.3 \text{ mA}$$

$$\mathbf{I}_2 = -20.69 - j51.72 \text{ mA}$$

$$\mathbf{V}_2 = -\mathbf{I}_2 Z_{\mathrm{L}} = (0.02069 + j0.05172)(600) = 12.41 + j31.03 \text{ V}$$

$$Z_{\mathrm{IN}} = \frac{\mathbf{V}_{\mathrm{S}}}{\mathbf{I}_1} = \frac{60}{1.076 - j0.3103} = 51.49 + j14.85\ \Omega$$

Problem 15–28. A transformer has self-inductances $L_1 = 200$ mH, $L_2 = 200$ mH, and a coupling coefficient of $k = 0.99$. The transformer is operating in the sinusoidal steady state with $\omega = 500$ rad/s and a short-circuit connected across the secondary winding. Find the transformer input impedance. Assume additive coupling.

With the short-circuit at the secondary, $\mathbf{V}_2 = 0$. We have the following relationships and results:

$$M = k\sqrt{L_1L_2} = (0.99)\sqrt{(0.2)(0.2)} = 198 \text{ mH}$$

$$\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2$$

$$0 = \mathbf{V}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -\frac{j\omega M}{j\omega L_2}\mathbf{I}_1 = -\frac{M}{L_2}\mathbf{I}_1$$

$$\mathbf{V}_1 = \left(j\omega L_1 - j\omega\frac{M^2}{L_2}\right)\mathbf{I}_1$$

$$Z_{\mathrm{IN}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = j\omega L_1 - j\omega\frac{M^2}{L_2} = j\omega\left(L_1 - \frac{M^2}{L_2}\right) = j1.99\ \Omega$$

Problem 15–29. A transformer operating in the sinusoidal steady state with $\omega = 377$ rad/s has self inductances $L_1 = 200$ mH, $L_2 = 400$ mH, and $k = 0.95$. The load connected across the secondary winding is $Z_{\mathrm{L}} = 75 + j150\ \Omega$. Find the transformer input impedance. Assume additive coupling.

With additive coupling, the i-v equations are

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

The load determines the relationship between $\mathbf{V}_2$ and $\mathbf{I}_2$ as $\mathbf{V}_2 = -Z_{\mathrm{L}}\mathbf{I}_2$. Substitute this relationship into the second equation, solve for $\mathbf{I}_2$, and then substitute into the first equation to find the input impedance.

$$\mathbf{V}_2 = -Z_{\mathrm{L}}\mathbf{I}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

$$\mathbf{I}_2 = \frac{-j\omega M}{Z_{\mathrm{L}} + j\omega L_2}\mathbf{I}_1$$

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \left(\frac{-j\omega M}{Z_{\mathrm{L}} + j\omega L_2}\mathbf{I}_1\right) = \left[j\omega L_1 + j\omega M \left(\frac{-j\omega M}{Z_{\mathrm{L}} + j\omega L_2}\right)\right]\mathbf{I}_1$$

$$Z_{\mathrm{IN}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = j\omega L_1 + j\omega M \left(\frac{-j\omega M}{Z_{\mathrm{L}} + j\omega L_2}\right)$$

Use MATLAB to perform the calculations, noting that $M = k\sqrt{L_1 L_2}$.

```
w = 377;
L1 = 200e-3;
L2 = 400e-3;
k = 0.95;
M = k*sqrt(L1*L2)
ZL = 75+150j;
ZIN = j*w*L1+j*w*M*(-j*w*M/(ZL+j*w*L2))
```

The corresponding MATLAB output is shown below.

```
M = 268.7006e-003
ZIN = 8.0082e+000 + 43.2820e+000i
```

The input impedance is

$$Z_{\mathrm{IN}} = 8.0082 + j43.282\,\Omega$$

Problem 15–30. The circuit in Figure P15–30 is in the sinusoidal steady state with $\mathbf{V}_{\mathrm{S}} = 500$ V and $Z_{\mathrm{L}} = 16+j12\ \Omega$. Use mesh-current analysis to find the amplitude of the output voltage, the input impedance seen by the source, and the average power delivered to Z_{L}.

With additive coupling, the i-v equations are

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

The mesh-current equations are:

$$(50 + j10)\mathbf{I}_{\mathrm{A}} + \mathbf{V}_1 = \mathbf{V}_{\mathrm{S}}$$

$$-\mathbf{V}_2 + Z_{\mathrm{L}}\mathbf{I}_{\mathrm{B}} = 0$$

$$\mathbf{I}_{\mathrm{A}} = \mathbf{I}_1$$

$$\mathbf{I}_{\mathrm{B}} = -\mathbf{I}_2$$

Solve the equations for the primary and secondary voltages.

$$\mathbf{V}_1 = \mathbf{V}_\mathrm{S} - (50 + j10)\mathbf{I}_1$$

$$\mathbf{V}_2 = Z_\mathrm{L}\mathbf{I}_\mathrm{B} = -Z_\mathrm{L}\mathbf{I}_2$$

Substitute into the i-v equations.

$$\mathbf{V}_\mathrm{S} - (50 + j10)\mathbf{I}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2$$

$$-Z_\mathrm{L}\mathbf{I}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2$$

Solve the equations for $\mathbf{I}_1$ and $\mathbf{I}_2$ and then compute the other requested values.

$$\mathbf{I}_1 = 5.6967 - j2.9589 = 6.4193\angle{-27.45^\circ} \text{ A}$$

$$\mathbf{I}_2 = -6.8385 + j1.5759 = 7.0177\angle 167.02^\circ \text{ A}$$

$$\mathbf{V}_2 = 128.3268 + j56.8465 = 140.35\angle 23.89^\circ \text{ V}$$

$$Z_\mathrm{IN} = \frac{\mathbf{V}_\mathrm{S}}{\mathbf{I}_1} = \frac{500}{5.6967 - j2.9589} = 69.122 + j35.0924\ \Omega$$

$$P_\mathrm{L} = \frac{1}{2}|\mathbf{I}_2|^2 R_\mathrm{L} = \frac{1}{2}|7.0177|^2(16) = 393.99 \text{ W}$$

Problem 15–31. Repeat Problem 15–30 with $Z_\mathrm{L} = 16 - j12\ \Omega$.

The approach is identical to that used in Problem 15–30. The solution is given in the following MATLAB code.

```
VS = 500;
ZS = 50+10j;
X1 = 100;
X2 = 50;
XM = 70;
ZL = 16-12j;
% Solve for I1 and I2 using a matrix approach
A = [j*X1+ZS, j*XM; j*XM, j*X2+ZL];
b = [VS; 0];
x = A\b;
I1 = x(1)
I2 = x(2)
V2 = j*XM*I1+j*X2*I2
V2Mag = abs(V2)
ZIN = VS/I1
P2 = real(ZL)/2*abs(I2)^2
```

The results are

```
I1 =  5.2018e+000 - 25.4680e-003i
I2 = -8.1561e+000 -  3.3872e+000i
V2 = 171.1448e+000 - 43.6776e+000i
V2Mag = 176.6303e+000
ZIN = 96.1176e+000 +470.5882e-003i
P2 = 623.9654e+000
```

In summary, we have:

$$\mathbf{I}_1 = 5.2018 - j0.025468 \text{ A}$$

$$\mathbf{I}_2 = -8.1561 - j3.3872 \text{ A}$$

$$\mathbf{V}_2 = 171.1448 - j43.6776 = 176.6303\angle{-14.32^\circ} \text{ V}$$

$$Z_{\text{IN}} = 96.1176 + j0.4706\ \Omega$$

$$P_{\text{L}} = 623.97 \text{ W}$$

Problem 15–32. Find the phasor current $\mathbf{I}$ and impedance Z_{IN} in the circuit in Figure P15–32.
The source current is $\mathbf{V}_\text{S} = 200\angle 0^\circ$. Apply KVL to write the following equations:

$$\mathbf{V}_\text{S} = 100\mathbf{I}_1 + \mathbf{V}_1$$

$$\mathbf{V}_\text{S} = 200\mathbf{I}_2 + \mathbf{V}_2$$

The transformer has additive coupling, so we have

$$\mathbf{V}_1 = j4\mathbf{I}_1 + j8\mathbf{I}_2$$

$$\mathbf{V}_2 = j8\mathbf{I}_1 + j16\mathbf{I}_2$$

Substitute the second set of equations into the first set of equations.

$$\mathbf{V}_\text{S} = (100 + j4)\,\mathbf{I}_1 + j8\mathbf{I}_2$$

$$\mathbf{V}_\text{S} = j8\mathbf{I}_1 + (200 + j16)\,\mathbf{I}_2$$

Solve the simultaneous equations. The following MATLAB code uses a matrix approach.

```
VS = 200;
R1 = 100;
R2 = 200;
X1 = 4;
X2 = 16;
XM = 8;
% Solve for I1 and I2 using a matrix approach
A = [R1+j*X1, j*XM; j*XM, R2+j*X2];
b = [VS; VS];
x = A\b;
I1 = x(1)
I2 = x(2)
```

The corresponding MATLAB output is shown below.

```
I1 = 1.9811e+000 -157.7287e-003i
I2 = 981.0726e-003 -157.7287e-003i
```

The phasor current $\mathbf{I}$ is the sum of the transformer currents

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

and the input impedance is the ratio of the source voltage $\mathbf{V}_\text{S}$ to the current $\mathbf{I}$

$$Z_{\text{IN}} = \frac{\mathbf{V}_\text{S}}{\mathbf{I}}$$

Using MATLAB, we have

```
I = I1+I2
ZIN = VS/I
```

The corresponding MATLAB output is shown below.

```
I =  2.9621e+000 -315.4574e-003i
ZIN = 66.7615e+000 +  7.1098e+000i
```

The results are

$$\mathbf{I} = 2.9621 - j0.31546\,\text{A}$$

$$Z_{\text{IN}} = 66.7615 + j7.1098\,\Omega$$

Problem 15–33. The circuit in Figure P15–33 is in the sinusoidal steady state with $\mathbf{V}_\text{S} = 100\angle 0°$ V and $R_\text{L} = 45\ \Omega$. Use mesh-current analysis to find $\mathbf{V}_\text{O}$ and the average power delivered to R_L.

The transformer has additive coupling, so we can write

$$\mathbf{V}_1 = j40\mathbf{I}_1 + j60\mathbf{I}_2$$

$$\mathbf{V}_2 = j60\mathbf{I}_1 + j100\mathbf{I}_2$$

The mesh-current equations are

$$(j10 - j30)\mathbf{I}_\text{A} + \mathbf{V}_1 - (-j30)\mathbf{I}_\text{B} = \mathbf{V}_\text{S}$$

$$-(-j30)\mathbf{I}_\text{A} - \mathbf{V}_2 + (45 - j30)\mathbf{I}_\text{B} = 0$$

Apply KCL to get $\mathbf{I}_\text{A} = \mathbf{I}_1$ and $\mathbf{I}_\text{B} = -\mathbf{I}_2$. Substitute to get the following equations.

$$(j10 - j30 + j40)\mathbf{I}_1 + (-j30 + j60)\mathbf{I}_2 = \mathbf{V}_\text{S}$$

$$(j30 - j60)\mathbf{I}_1 + (-j100 - 45 + j30)\mathbf{I}_2 = 0$$

The output voltage and the average power delivered to the load are given by

$$\mathbf{V}_\text{O} = -\mathbf{I}_2 R_\text{L}$$

$$P_\text{L} = \frac{|\mathbf{I}_2|^2}{2} R_\text{L}$$

Use MATLAB and a matrix approach to compute the results.

```
VS = 100;
RL = 45;
% Solve for I1 and I2 using a matrix approach
A = [10j-30j+40j, -30j+60j; 30j-60j, -100j-RL+30j];
b = [VS; 0];
x = A\b;
I1 = x(1);
I2 = x(2);
VO = -I2*RL
PL = RL/2*abs(I2)^2
```

The corresponding MATLAB output is shown below.

```
VO = 114.6226e+000 - 63.6792e+000i
PL = 191.0377e+000
```

The results are

$$\mathbf{V}_{\mathrm{O}} = 114.623 - j63.679\,\mathrm{V}$$

$$P_{\mathrm{L}} = 191.038\,\mathrm{W}$$

Problem 15–34. In Figure P15–34, find $\mathbf{I}_{\mathrm{A}}$, $\mathbf{I}_{\mathrm{B}}$, and the input impedance seen by the voltage source.

We have the following relationships:

$$n\mathbf{V}_1 = \mathbf{V}_2$$

$$\mathbf{I}_1 = -n\mathbf{I}_2$$

$$\mathbf{I}_{\mathrm{A}} = \mathbf{I}_1$$

$$\mathbf{I}_{\mathrm{B}} = -\mathbf{I}_2$$

Write the mesh-current equations.

$$\mathbf{V}_1 + 5(\mathbf{I}_{\mathrm{A}} - \mathbf{I}_{\mathrm{B}}) = \mathbf{V}_{\mathrm{S}}$$

$$5(\mathbf{I}_{\mathrm{B}} - \mathbf{I}_{\mathrm{A}}) - \mathbf{V}_2 - j5\mathbf{I}_{\mathrm{B}} = 0$$

Rewrite the equations and solve.

$$\mathbf{V}_1 + 5(n\mathbf{I}_{\mathrm{B}} - \mathbf{I}_{\mathrm{B}}) = \mathbf{V}_{\mathrm{S}}$$

$$-n\mathbf{V}_1 + [5(1-n) - j5]\mathbf{I}_{\mathrm{B}} = 0$$

$$\mathbf{I}_{\mathrm{A}} = 37.3541 + j2.3346 \text{ A}$$

$$\mathbf{I}_{\mathrm{B}} = 7.4708 + j0.4669 \text{ A}$$

$$Z_{\mathrm{IN}} = \frac{\mathbf{V}_{\mathrm{S}}}{\mathbf{I}_{\mathrm{A}}} = \frac{120}{37.3541 + j2.3346} = 3.2 - j0.2\,\Omega$$

Problem 15–35. The ideal transformer in Figure P15-35 is connected as an auto transformer. Find $\mathbf{I}_{\mathrm{S}}$ and the average power delivered to R_{L} when $N_1 = N_2 = 500$ turns, $R_{\mathrm{L}} = 60\ \Omega$, and $\mathbf{V}_{\mathrm{S}} = 120\angle 0^\circ$ V.

We have the following relationships and results:

$$n = \frac{N_2}{N_1} = 1$$

$$\mathbf{V}_2 = \mathbf{V}_{\mathrm{S}}$$

$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{n} = \mathbf{V}_2 = \mathbf{V}_{\mathrm{S}}$$

$$\mathbf{V}_{\mathrm{L}} = \mathbf{V}_1 + \mathbf{V}_2 = 2\mathbf{V}_{\mathrm{S}}$$

$$\mathbf{I}_{\mathrm{L}} = \frac{\mathbf{V}_{\mathrm{L}}}{R_{\mathrm{L}}} = \frac{240}{60} = 4 \text{ A}$$

$$\mathbf{I}_1 = -\mathbf{I}_{\mathrm{L}}$$

$$\mathbf{I}_2 = -\frac{\mathbf{I}_1}{n} = -\mathbf{I}_1 = \mathbf{I}_{\mathrm{L}}$$

$$\mathbf{I}_{\mathrm{S}} = \mathbf{I}_2 - \mathbf{I}_1 = 2\mathbf{I}_{\mathrm{L}} = 8 \text{ A}$$

$$P_{\mathrm{L}} = \frac{1}{2}|\mathbf{I}_{\mathrm{L}}|^2 R_{\mathrm{L}} = (0.5)(4)^2(60) = 480 \text{ W}$$

Problem 15–36. The transformer in Figure P15–36 is operating in the ac steady-state with a voltage source connected at the input and the output shorted. Show that the input impedance is $Z_{\mathrm{IN}} = jX_1(1-k^2)$, where k is the coupling coefficient. (*Hint:* $k = X_{\mathrm{M}}/\sqrt{X_1 X_2}$.)

The circuit has additive coupling and $\mathbf{V}_2 = 0$. We have the following relationships and results:

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{X_{\mathrm{M}}}{\sqrt{X_1 X_2}}$$

$$\mathbf{V}_1 = jX_1\mathbf{I}_1 + jX_{\mathrm{M}}\mathbf{I}_2$$

$$0 = \mathbf{V}_2 = jX_{\mathrm{M}}\mathbf{I}_1 + jX_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -\frac{jX_{\mathrm{M}}}{jX_2}\mathbf{I}_1$$

$$\mathbf{V}_1 = jX_1\mathbf{I}_1 + jX_{\mathrm{M}}\left(-\frac{jX_{\mathrm{M}}}{jX_2}\right)\mathbf{I}_1$$

$$= \left(jX_1 - j\frac{X_{\mathrm{M}}^2}{X_2}\right)\mathbf{I}_1 = j\left(\frac{X_1X_2 - X_{\mathrm{M}}^2}{X_2}\right)\mathbf{I}_1$$

$$Z_{\mathrm{IN}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = j\left(\frac{X_1X_2 - X_{\mathrm{M}}^2}{X_2}\right) = j\left(\frac{X_1X_2 - k^2X_1X_2}{X_2}\right) = jX_1(1-k^2)$$

Problem 15–37. The transformer in Figure P15–36 is operating in the ac steady-state with a voltage source connected at the input and the output shorted. Show that the short-circuit current is

$$\mathbf{I}_{\mathrm{SC}} = \left(\frac{k^2}{1-k^2}\right)\frac{\mathbf{V}_{\mathrm{S}}}{jX_{\mathrm{M}}}$$

(*Hint:* Use the result derived in Problem 15-36 to find $\mathbf{I}_1$.)

The circuit has additive coupling and $\mathbf{V}_2 = 0$. We have the following relationships and results:

$$
\begin{aligned}
k &= \frac{X_\mathrm{M}}{\sqrt{X_1 X_2}} \\
Z_\mathrm{IN} &= jX_1(1-k^2) \\
\mathbf{V}_1 = \mathbf{V}_\mathrm{S} &= Z_\mathrm{IN}\mathbf{I}_1 = jX_1(1-k^2)\mathbf{I}_1 \\
\mathbf{I}_1 &= \frac{\mathbf{V}_\mathrm{S}}{jX_1(1-k^2)} \\
\mathbf{V}_1 &= jX_1\mathbf{I}_1 + jX_\mathrm{M}\mathbf{I}_2 \\
0 = \mathbf{V}_2 &= jX_\mathrm{M}\mathbf{I}_1 + jX_2\mathbf{I}_2 \\
\mathbf{I}_2 &= -\frac{jX_\mathrm{M}}{jX_2}\mathbf{I}_1 = \left(-\frac{X_\mathrm{M}}{X_2}\right)\left[\frac{\mathbf{V}_\mathrm{S}}{jX_1(1-k^2)}\right] \\
\mathbf{I}_\mathrm{SC} = -\mathbf{I}_2 &= \frac{X_\mathrm{M}\mathbf{V}_\mathrm{S}}{jX_1X_2(1-k^2)} = \frac{X_\mathrm{M}\mathbf{V}_\mathrm{S}}{j\left(\frac{X_\mathrm{M}^2}{k^2}\right)(1-k^2)} = \left(\frac{k^2}{1-k^2}\right)\frac{\mathbf{V}_\mathrm{S}}{jX_\mathrm{M}}
\end{aligned}
$$

Problem 15–38. The linear transformer in Figure P15–38 is in the sinusoidal steady-state with reactances of $X_1 = 32\ \Omega$, $X_2 = 48\ \Omega$, $X_\mathrm{M} = 36\ \Omega$, and a load impedance of $Z_\mathrm{L} = 150 - j75\ \Omega$. Find the input impedance seen by the input voltage source.

We have the following relationships and results:

$$
\begin{aligned}
Z_\mathrm{IN} &= \frac{\mathbf{V}_\mathrm{S}}{\mathbf{I}_1} = \frac{\mathbf{V}_1}{\mathbf{I}_1} \\
\mathbf{V}_1 &= jX_1\mathbf{I}_1 + jX_\mathrm{M}\mathbf{I}_2 = \mathbf{V}_\mathrm{S} \\
\mathbf{V}_2 &= jX_\mathrm{M}\mathbf{I}_1 + jX_2\mathbf{I}_2 = -Z_\mathrm{L}\mathbf{I}_2 \\
(-Z_\mathrm{L} - jX_2)\mathbf{I}_2 &= jX_\mathrm{M}\mathbf{I}_1 \\
\mathbf{I}_2 &= \frac{jX_\mathrm{M}}{-Z_\mathrm{L} - jX_2}\mathbf{I}_1 \\
\mathbf{V}_1 &= \left[jX_1 + jX_\mathrm{M}\left(\frac{jX_\mathrm{M}}{-Z_\mathrm{L} - jX_2}\right)\right]\mathbf{I}_1 \\
Z_\mathrm{IN} &= jX_1 + jX_\mathrm{M}\left(\frac{jX_\mathrm{M}}{-Z_\mathrm{L} - jX_2}\right) = 8.3688 + j33.5064\ \Omega
\end{aligned}
$$

Problem 15–39. The linear transformer in Figure P15–38 is in the sinusoidal steady-state with reactances of $X_1 = 15\ \Omega$, $X_2 = 60\ \Omega$, $X_\mathrm{M} = 27\ \Omega$. Find the transformer secondary response $\mathbf{V}_2$ and $\mathbf{I}_2$ when $Z_\mathrm{L} = 200 - j100\ \Omega$ and $\mathbf{V}_\mathrm{S} = 200\angle 0^\circ$ V.

We have the following relationships and results:

$$\mathbf{V}_1 = jX_1\mathbf{I}_1 + jX_\mathrm{M}\mathbf{I}_2 = \mathbf{V}_\mathrm{S}$$

$$\mathbf{V}_2 = jX_\mathrm{M}\mathbf{I}_1 + jX_2\mathbf{I}_2 = -Z_\mathrm{L}\mathbf{I}_2$$

$$(-Z_\mathrm{L} - jX_2)\mathbf{I}_2 = jX_\mathrm{M}\mathbf{I}_1$$

$$\mathbf{I}_2 = \frac{jX_\mathrm{M}}{-Z_\mathrm{L} - jX_2}\mathbf{I}_1$$

$$\mathbf{V}_1 = \left[jX_1 + jX_\mathrm{M}\left(\frac{jX_\mathrm{M}}{-Z_\mathrm{L} - jX_2}\right)\right]\mathbf{I}_1$$

$$Z_\mathrm{IN} = jX_1 + jX_\mathrm{M}\left(\frac{jX_\mathrm{M}}{-Z_\mathrm{L} - jX_2}\right) = 3.5048 + j15.7010\ \Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_\mathrm{S}}{Z_\mathrm{IN}} = 2.7085 - j12.1335\ \mathrm{A}$$

$$\mathbf{I}_2 = \frac{jX_\mathrm{M}}{-Z_\mathrm{L} - jX_2}\mathbf{I}_1 = -1.5047 - j0.6666\ \mathrm{A}$$

$$\mathbf{V}_2 = -Z_\mathrm{L}\mathbf{I}_2 = 367.6 - j17.1536\ \mathrm{V}$$

Problem 15–40. The self and mutual inductances of a transformer can be calculated from measurements of the steady-state ac voltages and currents with the secondary winding open circuited and short circuited. Suppose the measurements are $|\mathbf{V}_1| = 120$ V, $|\mathbf{I}_1| = 120$ mA, and $|\mathbf{V}_2| = 240$ V when the secondary is open. When the secondary is shorted, the measurements are $|\mathbf{I}_1| = 10$ A and $|\mathbf{I}_2| = 2.2$ A. All measurements were made at $f = 400$ Hz. Find L_1, L_2, and M.

For the open-circuit case, $\mathbf{I}_2 = 0$ and we have the following relationships:

$$\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2 = j\omega L_1\mathbf{I}_1$$

$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2 = j\omega M\mathbf{I}_1$$

$$120 = |j\omega L_1\mathbf{I}_1| = \omega L_1(0.12)$$

$$240 = |j\omega M\mathbf{I}_1| = \omega M(0.12)$$

$$L_1 = \frac{120}{(800\pi)(0.12)} = 0.3979\ \mathrm{H}$$

$$M = \frac{240}{(800\pi)(0.12)} = 0.7958\ \mathrm{H}$$

For the short-circuit case, $\mathbf{V}_2 = 0$ and we have the following relationships:

$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2 = 0$$

$$|j\omega M\mathbf{I}_1| = |-j\omega L_2\mathbf{I}_2|$$

$$\frac{M}{L_2} = \frac{|\mathbf{I}_2|}{|\mathbf{I}_1|} = \frac{2.2}{10}$$

$$L_2 = \frac{10M}{2.2} = 3.6172\ \mathrm{H}$$

Problem 15–41. (A) Transformer Sawtooth Response

The linear transformer in Figure P15–41 has inductances of $L_1 = 25$ mH, $L_2 = 100$ mH, and $M = 50$ mH. The input voltage $v_S(t)$ has a sawtooth waveform with a peak amplitude of 5 V and a period of 50 μs. Derive an expression for the first three terms of the Fourier series of the open-circuit ($i_2 = 0$) output voltage $v_2(t)$.

The first four terms of the Fourier series for a sawtooth waveform are

$$v_S(t) = \frac{A}{2} - \frac{A}{\pi}\sin(2\pi f_0 t) - \frac{A}{2\pi}\sin(4\pi f_0 t) - \frac{A}{3\pi}\sin(6\pi f_0 t)\text{ V}$$

where $A = 5$ and $f_0 = 1/T_0 = 1/50\ \mu s = 20$ kHz. With $i_2 = 0$ and additive coupling, we have the following equations for the transformer

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = L_1\frac{di_1(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} = M\frac{di_1(t)}{dt}$$

Solve for $v_2(t)$ in terms of $v_1(t)$.

$$v_2(t) = \frac{M}{L_1}v_1(t) = \frac{50}{25}v_1(t) = 2v_1(t)$$

The dc component of $v_S(t)$ does not pass through the transformer, so we have the following output using the three ac components in $v_S(t)$:

$$v_2(t) = -\frac{10}{\pi}\left[\sin(40000\pi t) + \frac{1}{2}\sin(80000\pi t) + \frac{1}{3}\sin(120000\pi t)\right]\text{ V}$$

Problem 15–42. (A) Transformer Thévenin Equivalent

In the time domain, the i-v relationships for a linear transformer are

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt}$$

Assuming zero initial conditions, transform these equations into the s-domain and show that the s-domain parameters of the Thévenin equivalent at the output are

$$Z_T(s) = (1 - k^2)L_2 s$$

$$V_T(s) = k\sqrt{\frac{L_2}{L_1}}V_1(s)$$

where k is the coupling coefficient.

Take the Laplace transform of the i-v relationships.

$$V_1(s) = L_1 sI_1(s) + MsI_2(s)$$

$$V_2(s) = MsI_1(s) + L_2 sI_2(s)$$

Let the secondary have an open circuit so that $I_2(s) = 0$. We then have:

$$V_T(s) = V_{OC}(s) = V_2(s)$$

$$V_1(s) = L_1 sI_1(s)$$

$$V_2(s) = MsI_1(s) = \frac{Ms}{L_1 s}V_1(s) = \frac{k\sqrt{L_1L_2}}{L_1}V_1(s) = k\sqrt{\frac{L_2}{L_1}}V_1(s)$$

Let the secondary have a short circuit so that $V_2(s) = 0$. We then have:

$$0 = MsI_1(s) + L_2sI_2(s)$$

$$I_{\mathrm{SC}}(s) = -I_2(s) = \frac{Ms}{L_2s}I_1(s) = \frac{M}{L_2}I_1(s)$$

$$V_1(s) = L_1sI_1(s) - \frac{M^2s}{L_2}I_1(s) = \left(L_1 - \frac{M^2}{L_2}\right)sI_1(s) = \left(L_1 - \frac{k^2L_1L_2}{L_2}\right)sI_1(s)$$

$$I_1(s) = \frac{V_1(s)}{L_1(1-k^2)s}$$

$$I_2(s) = -\frac{M}{L_2}I_1(s) = \frac{MV_1(s)}{L_1L_2(k^2-1)s}$$

$$Z_{\mathrm{T}} = \frac{V_{\mathrm{T}}(s)}{I_{\mathrm{SC}}(s)} = \frac{\dfrac{MV_1(s)}{L_1}}{\dfrac{MV_1(s)}{L_1L_2(1-k^2)s}} = (1-k^2)L_2s$$

Problem 15–43. (A) Perfectly Coupled Transformer

Figure P15–43 is an equivalent circuit of a perfectly coupled transformer. This model is the basis for the transformer equivalent circuits used in the analysis of power systems. The inductance L_{m} is called the magnetizing inductance. The current through this inductance represents the current needed to establish the magnetic field in the transformer at no load ($i_2 = 0$). Show that the i-v equations for this circuit are:

$$v_1(t) = L_{\mathrm{m}}\frac{di_1(t)}{dt} + nL_{\mathrm{m}}\frac{di_2(t)}{dt}$$

$$v_2(t) = nL_{\mathrm{m}}\frac{di_1(t)}{dt} + n^2L_{\mathrm{m}}\frac{di_2(t)}{dt}$$

Use these equations to show that $k = 1$ and $v_2/v_1 = n$.

Apply KCL to find the current through L_{m} and then develop the requested relationships.

$$i_{\mathrm{L_m}}(t) = i_1(t) + ni_2(t)$$

$$v_{\mathrm{L_m}}(t) = v_1(t) = L_{\mathrm{m}}\frac{di_{\mathrm{L_m}}(t)}{dt} = L_{\mathrm{m}}\frac{d}{dt}[(i_1(t) + ni_2(t)]$$

$$= L_{\mathrm{m}}\frac{di_1(t)}{dt} + nL_{\mathrm{m}}\frac{di_2(t)}{dt}$$

$$v_2(t) = nv_1(t) = nL_{\mathrm{m}}\frac{di_1(t)}{dt} + n^2L_{\mathrm{m}}\frac{di_2(t)}{dt}$$

$$k = \frac{M}{\sqrt{L_1L_2}} = \frac{nL_{\mathrm{m}}}{\sqrt{L_{\mathrm{m}}n^2L_{\mathrm{m}}}} = \frac{nL_{\mathrm{m}}}{nL_{\mathrm{m}}} = 1$$

$$\frac{v_2(t)}{v_1(t)} = n$$

16 AC Power Systems

16.1 Exercise Solutions

Exercise 16–1. Using the reference marks in Figure 16–1, calculate the average and reactive power for the following voltages and currents.

(a). $v(t) = 168\cos(377t + 45°)$ V, $i(t) = 0.88\cos(377t)$ A

We have the following results:

$$P = \frac{V_\mathrm{A} I_\mathrm{A}}{2}\cos(\theta) = \frac{(168)(0.88)}{2}\cos(45°) = 52.2693 \text{ W}$$

$$Q = \frac{V_\mathrm{A} I_\mathrm{A}}{2}\sin(\theta) = \frac{(168)(0.88)}{2}\sin(45°) = 52.2693 \text{ VAR}$$

(b). $v(t) = 285\cos(2500t + 68°)$ V, $i(t) = 0.66\cos(2500t)$ A

We have the following results:

$$P = \frac{V_\mathrm{A} I_\mathrm{A}}{2}\cos(\theta) = \frac{(285)(0.66)}{2}\cos(68°) = 35.2318 \text{ W}$$

$$Q = \frac{V_\mathrm{A} I_\mathrm{A}}{2}\sin(\theta) = \frac{(285)(0.66)}{2}\sin(68°) = 87.2016 \text{ VAR}$$

Exercise 16–2. Determine the average power, reactive power, and apparent power for the following voltage and current phasors. State whether the power factor is lagging or leading.

(a). $\mathbf{V} = 208\angle{-90°}$ V(rms), $\mathbf{I} = 1.75\angle{-75°}$ A(rms)

We have the following results:

$$S = \mathbf{V}\mathbf{I}^* = P + jQ$$

$$S = [208\angle{-90°}]\,[1.75\angle{+75°}] = 351.597 - j94.21 = 364\angle{-15°} \text{ VA}$$

$$P = 351.597 \text{ W}$$

$$Q = -94.21 \text{ VAR}$$

$$|S| = 364 \text{ VA}$$

The reactive power is negative, so the power factor is leading.

(b). $\mathbf{V} = 277\angle{+90°}$ V (rms), $\mathbf{I} = 11.3\angle 0°$ A (rms)

We have the following results:

$$S = \mathbf{V}\mathbf{I}^* = P + jQ$$

$$S = [277\angle{+90°}]\,[11.3\angle 0°] = 0 + j3130.1 = 3130.1\angle 90° \text{ VA}$$

$$P = 0 \text{ W}$$

$$Q = 3130.1 \text{ VAR}$$

$$|S| = 3130.1 \text{ VA}$$

The reactive power is positive, so the power factor is lagging.

Exercise 16–3. Find the impedance of a two-terminal load under the following conditions.

(a). $\mathbf{V} = 120\angle 30°$ V (rms) and $\mathbf{I} = 20\angle 75°$ A (rms)

We have the following results:

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle 30°}{20\angle 75°} = 6\angle{-45°} = 4.2426 - j4.2426\ \Omega$$

(b). $|S| = 3.3$ kVA, $Q = -1.8$ kVAR, and $I_{\text{rms}} = 7.5$ A

We have the following results:

$$|S| = V_{\text{rms}} I_{\text{rms}}$$

$$V_{\text{rms}} = \frac{|S|}{I_{\text{rms}}} = \frac{3300}{7.5} = 440\ \text{V(rms)}$$

$$X = \frac{Q}{I_{\text{rms}}^2} = \frac{-1800}{7.5^2} = -32\ \Omega$$

$$P = \sqrt{|S|^2 - Q^2} = 2765.9\ \text{W}$$

$$R = \frac{P}{I_{\text{rms}}^2} = \frac{2765.9}{7.5^2} = 49.17\ \Omega$$

$$Z = R + jX = 49.17 - j32.00\ \Omega$$

Exercise 16–4. A load consisting of a 50-Ω resistor in parallel with an inductor whose reactance is 75 Ω is connected across a 500-V (rms) source. Find the complex power delivered to the load and the load power factor. State whether the power factor is leading or lagging.

Compute the load impedance:

$$Z_{\text{L}} = \frac{(R)(j\omega L)}{R + j\omega L} = \frac{(50)(j75)}{50 + j75} = 34.6154 + j23.0769 = 41.6025\angle 33.69°\ \Omega$$

Compute the magnitude of the current through the load:

$$|\mathbf{I}| = \frac{|\mathbf{V}|}{|Z_{\text{L}}|} = \frac{500}{41.6025} = 12.0185\ \text{A(rms)}$$

Find the complex power delivered to the load and the load power factor:

$$S_{\text{L}} = |\mathbf{I}|^2 Z_{\text{L}} = P_{\text{L}} + jQ_{\text{L}} = (12.0185)^2(34.6154 + j23.0769) = 5000 + j3333\ \text{VA}$$

$$\text{pf} = \frac{P_{\text{L}}}{|S_{\text{L}}|} = \frac{5000}{|5000 + j3333|} = 0.83205$$

The power factor is lagging since Q_{L} is positive.

Exercise 16–5. In Figure 16–6 the load Z_{L} is an 80-Ω resistor and the source voltage is 220 V (rms). Find the complex power produced by the *source*.

Find the impedance seen by the source and then compute the magnitude of the source current and the complex power.

$$Z_{\text{T}} = 50 + (j40 \parallel 80) = 50 + (16 + j32) = 66 + j32\ \Omega$$

$$|\mathbf{I}_{\text{S}}| = \frac{|\mathbf{V}_{\text{S}}|}{|Z_{\text{T}}|} = \frac{220}{|66 + j32|} = 2.9994\ \text{A(rms)}$$

$$S_{\text{S}} = |\mathbf{I}_{\text{S}}|^2 Z_{\text{T}} = (2.9994)^2(66 + j32) = 593.7546 + j287.881\ \text{VA}$$

Exercise 16–6. A single-phase source supplies a load through a two-wire line with an impedance of $Z_W = 2 + j10\ \Omega$ per wire. The rms load voltage is 2.4 kV and the load receives an apparent power of 25 kVA at a lagging power factor of 0.85. Find the required source power and rms voltage.

Find the magnitude of the load current.

$$|\mathbf{I}_L| = \frac{|S_L|}{|\mathbf{V}_L|} = \frac{25000}{2400} = 10.4167\text{ A(rms)}$$

Find the complex load power.

$$S_L = P_L + jQ_L = |S_L|\left(\text{pf} + j\sqrt{1 - \text{pf}^2}\right) = 21250 + j13169.6\text{ VA}$$

Find the total complex power absorbed by the line.

$$S_W = (2)\left(|\mathbf{I}_L|^2 Z_W\right) = (2)(10.4167)^2(2 + j10) = 434.03 + j2170.1\ \Omega$$

Compute the complex power at the source.

$$S_S = S_W + S_L = 21684 + j15340\text{ VA}$$

$$|S_S| = 26561.3\text{ VA}$$

Find the rms value of the source voltage.

$$|\mathbf{V}_S| = \frac{|S_S|}{|\mathbf{I}_L|} = \frac{26561.3}{10.4167} = 2550\text{ V(rms)}$$

Exercise 16–7. For the load conditions in Example 16–11, find the capacitance needed to raise the power factor to unity.

Compute the complex power delivered to the load

$$S_L = P_L + jQ_L = |S_L|\left(\text{pf} + j\sqrt{1 - \text{pf}^2}\right) = 1600 + j1200\text{ VA}$$

To raise the power factor to unity, we need to cancel the reactive power completely, so

$$Q_C = -1200\text{ VAR}$$

Compute the capacitance.

$$C = \frac{-Q_C}{2\pi f|\mathbf{V}_L|^2} = \frac{-(-1200)}{120\pi(880)^2} = 4.1104\ \mu\text{F}$$

Exercise 16–8. In a balanced three-phase circuit the rms line voltage is $V_L = 7.2$ kV (rms). Find all of the phase and line voltages for a positive phase sequence using $\angle\mathbf{V}_{AN} = 0°$ as the phase reference.

With $V_L = 7.2$ kV (rms), $V_P = V_L/\sqrt{3} = 4157$ V (rms). The phase voltages are:

$$\mathbf{V}_{AN} = 4157\angle 0°\text{ V(rms)}$$

$$\mathbf{V}_{BN} = 4157\angle{-120°}\text{ V(rms)}$$

$$\mathbf{V}_{CN} = 4157\angle{-240°}\text{ V(rms)}$$

The line voltages are:

$$\mathbf{V}_{AB} = 7200\angle{+30°}\text{ V(rms)}$$

$$\mathbf{V}_{BC} = 7200\angle{-90°}\text{ V(rms)}$$

$$\mathbf{V}_{CA} = 7200\angle{-210°}\text{ V(rms)}$$

Exercise 16–9. In a balanced three-phase circuit $\mathbf{V}_{\mathrm{BC}} = 480\angle{-120^\circ}$ V (rms). Find the phase voltages for a positive phase sequence.

With $V_{\mathrm{L}} = 480$ V (rms), $V_{\mathrm{P}} = V_{\mathrm{L}}/\sqrt{3} = 277$ V (rms). The phase voltages are:

$$\mathbf{V}_{\mathrm{AN}} = 277\angle{-30^\circ}\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{BN}} = 277\angle{-150^\circ}\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{CN}} = 277\angle{-270^\circ}\ \mathrm{V(rms)}$$

Exercise 16–10. Two balanced Δ-connected loads are connected in parallel. Their phase impedances are $Z_{\Delta 1} = 50 + j24\ \Omega$ and $Z_{\Delta 2} = 60 + j25\ \Omega$. Find the equivalent Y-connected load for the two parallel loads.

We have the following results:

$$Z_{\Delta\mathrm{EQ}} = Z_{\Delta 1} \parallel Z_{\Delta 2} = (50 + j24) \parallel (60 + j25) = 27.295 + j12.296\ \Omega$$

$$Z_{\mathrm{YEQ}} = \frac{Z_{\Delta\mathrm{EQ}}}{3} = 9.098 + j4.099\ \Omega$$

Exercise 16–11. A balanced Y-Y circuit operates with $V_{\mathrm{L}} = 4160$ V (rms) and phase impedances of $Z_{\mathrm{Y}} = 100 + j40\ \Omega$ per phase. Using $\angle\mathbf{V}_{\mathrm{AB}} = 0^\circ$ as the phase reference, find $\mathbf{I}_{\mathrm{A}}$ and $\mathbf{V}_{\mathrm{AN}}$ for a positive phase sequence.

We have the following calculations and results:

$$V_{\mathrm{P}} = \frac{V_{\mathrm{L}}}{\sqrt{3}} = \frac{4160}{\sqrt{3}} = 2402\ \mathrm{V(rms)}$$

$$\angle\mathbf{V}_{\mathrm{AN}} = \angle\mathbf{V}_{\mathrm{AB}} - 30^\circ = -30^\circ$$

$$\mathbf{V}_{\mathrm{AN}} = 2402\angle{-30^\circ} = 2080 - j1201\ \mathrm{V(rms)}$$

$$\mathbf{I}_{\mathrm{A}} = \frac{\mathbf{V}_{\mathrm{AN}}}{Z_{\mathrm{Y}}} = \frac{2402\angle{-30^\circ}}{100 + j40} = 22.30\angle{-51.8^\circ}\ \mathrm{A(rms)}$$

Exercise 16–12. In a balanced Y-Y circuit the load and line impedances are $Z_{\mathrm{Y}} = 16 + j12\ \Omega$ per phase and $Z_{\mathrm{W}} = 0.25 + j1.5\ \Omega$ per phase. The line current is $I_{\mathrm{L}} = 14.2$ A (rms). Find the line voltage phasors at the source using $\angle\mathbf{I}_{\mathrm{A}} = 0^\circ$ as the phase reference.

We have the following calculations and results:

$$\mathbf{V}_{\mathrm{AN}} = \mathbf{I}_{\mathrm{A}}(Z_{\mathrm{Y}} + Z_{\mathrm{W}}) = 14.2(16 + j12 + 0.25 + j1.5) = 230.75 + j191.7 = 300\angle 39.7^\circ\ \mathrm{V(rms)}$$

$$V_{\mathrm{P}} = 300\ \mathrm{V(rms)}$$

$$V_{\mathrm{L}} = \sqrt{3}V_{\mathrm{P}} = 519.6\ \mathrm{V(rms)}$$

$$\angle\mathbf{V}_{\mathrm{AB}} = \angle\mathbf{V}_{\mathrm{AN}} + 30^\circ = 69.7^\circ$$

$$\mathbf{V}_{\mathrm{AB}} = V_{\mathrm{P}}\angle\mathbf{V}_{\mathrm{AB}} = 519.6\angle 69.7^\circ\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{BC}} = 519.6\angle{-50.3^\circ}\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{CA}} = 519.6\angle{-170.3^\circ}\ \mathrm{V(rms)}$$

Exercise 16–13. The line voltage at a Δ-connected load with $Z_{\Delta} = 520 + j400\ \Omega$ per phase is $V_{\mathrm{L}} = 1300$ V (rms). Find $\mathbf{I}_{\mathrm{A}}$ and $\mathbf{I}_{\mathrm{AB}}$ using $\angle\mathbf{V}_{\mathrm{AB}} = 0^\circ$ as the phase reference.

We have the following calculations and results:

$$\mathbf{V}_{\mathrm{AB}} = V_{\mathrm{L}}\angle 0^\circ = 1300\angle 0^\circ \text{ V(rms)}$$

$$\mathbf{I}_{\mathrm{AB}} = \frac{\mathbf{V}_{\mathrm{AB}}}{Z_\Delta} = \frac{1300\angle 0^\circ}{520 + j400} = 1.57 - j1.21 = 1.98\angle{-37.6^\circ} \text{ A(rms)}$$

$$I_{\mathrm{P}} = 1.98 \text{ A(rms)}$$

$$\mathbf{I}_{\mathrm{A}} = \sqrt{3} I_{\mathrm{P}}\angle(\angle \mathbf{I}_{\mathrm{AB}} - 30^\circ) = 3.43\angle{-67.6^\circ} \text{ A(rms)}$$

Exercise 16–14. The phase B line current in a Δ-connected load with $Z_\Delta = 14 + j9\ \Omega$ per phase is $\mathbf{I}_{\mathrm{B}} = 26\angle{-165^\circ}$ A (rms). Find $\mathbf{I}_{\mathrm{AB}}$ and $\mathbf{V}_{\mathrm{AB}}$ for a positive phase sequence.

We have the following calculations and results:

$$I_{\mathrm{P}} = \frac{I_{\mathrm{L}}}{\sqrt{3}} = \frac{26}{\sqrt{3}} = 15.01 \text{ A(rms)}$$

$$\mathbf{I}_{\mathrm{AB}} = I_{\mathrm{P}}\angle(\angle \mathbf{I}_{\mathrm{A}} + 30^\circ) = I_{\mathrm{P}}\angle(\angle \mathbf{I}_{\mathrm{B}} + 120^\circ + 30^\circ) = 15.01\angle{-15^\circ} \text{ A(rms)}$$

$$\mathbf{V}_{\mathrm{AB}} = \mathbf{I}_{\mathrm{AB}} Z_\Delta = (15.01\angle{-15^\circ})(14 + j9) = 249.8\angle 17.7^\circ \text{ V(rms)}$$

Exercise 16–15. In Figure 16–23, the load is Δ-connected with a phase impedance of $Z_\Delta = 26 + j8\ \Omega$ per phase and the line voltage at the load is $V_{\mathrm{L}} = 1.0$ kV (rms). Find the line current I_{L} and the complex power delivered to the load.

We have the following calculations and results:

$$V_{\mathrm{P}} = \frac{V_{\mathrm{L}}}{\sqrt{3}} = \frac{1000}{\sqrt{3}} = 577.35 \text{ V(rms)}$$

$$Z_{\mathrm{Y}} = \frac{Z_\Delta}{3} = 8.6667 + j2.6667\ \Omega$$

$$I_{\mathrm{L}} = \frac{V_{\mathrm{P}}}{|Z_{\mathrm{Y}}|} = \frac{577.35}{|8.6667 + j2.6667|} = 63.6715 \text{ A(rms)}$$

$$S_{\mathrm{L}} = 3 I_{\mathrm{L}}^2 Z_{\mathrm{Y}} = (3)(63.6715)^2(8.6667 + j2.6667) = 105.4054 + j32.4324 \text{ kVA}$$

Exercise 16–16. In a balanced three-phase circuit, the line voltage at the load is 4160 V (rms) and the apparent power delivered to the load is 60 kVA. Find the line current.

We have the following results:

$$I_{\mathrm{L}} = \frac{|S_{\mathrm{L}}|}{\sqrt{3} V_{\mathrm{L}}} = \frac{4160}{(3)(60000)} = 8.327 \text{ A(rms)}$$

Exercise 16–17. In Figure 16–23, the line current is $I_{\mathrm{L}} = 10$ A (rms), the line impedance is $Z_{\mathrm{W}} = 0.6 + j3.7\ \Omega$ per phase, and the phase impedance of the load is $Z_{\mathrm{Y}} = 15 + j28\ \Omega$ per phase. Find the complex power produced by the source.

We have the following calculations and results:

$$S_{\mathrm{L}} = 3 I_{\mathrm{L}}^2 Z_{\mathrm{Y}} = (3)(10)^2(15 + j28) = 4500 + j8400 \text{ VA}$$

$$S_{\mathrm{W}} = 3 I_{\mathrm{L}}^2 Z_{\mathrm{W}} = (3)(10)^2(0.6 + j3.7) = 180 + j1110 \text{ VA}$$

$$S_{\mathrm{S}} = S_{\mathrm{L}} + S_{\mathrm{W}} = 4680 + j9510 \text{ VA}$$

Exercise 16–18. In Figure 16–23, the line current is $I_{\mathrm{L}} = 5$ A (rms), the line impedance is $Z_{\mathrm{W}} = 2 + j6\ \Omega$ per phase, and the load absorbs $S_{\mathrm{L}} = 3 + j2$ kVA. Find the complex power produced by the source.

We have the following calculations and results:

$$S_{\mathrm{W}} = 3I_{\mathrm{L}}^2 Z_{\mathrm{W}} = (3)(5)^2(2 + j6) = 150 + j450 \text{ VA}$$

$$S_{\mathrm{S}} = S_{\mathrm{W}} + S_{\mathrm{L}} = 3.15 + j2.45 \text{ kVA}$$

Exercise 16–19. In Figure 16–26, the load at bus 2 draws a complex power of $S_2 = 125 + j60$ kVA. The line current is $I_{\mathrm{L}} = 40$ A(rms), and the line impendance is $Z_{\mathrm{W}} = 2 + j10\ \Omega$ per phase. Find the line voltages at bus 1 and bus 2.

We have the following calculations and results:

$$V_{\mathrm{L2}} = \frac{|S_2|}{\sqrt{3}I_{\mathrm{L}}} = \frac{|125000 + j60000|}{(\sqrt{3})(40)} = 2001 \text{ V(rms)}$$

$$S_{\mathrm{W}} = 3I_{\mathrm{L}}^2 Z_{\mathrm{W}} = (3)(40)^2(2 + j10) = 9600 + j48000 \text{ VA}$$

$$S_1 = S_{\mathrm{W}} + S_2 = 134600 + j108000 \text{ VA}$$

$$V_{\mathrm{L1}} = \frac{|S_1|}{\sqrt{3}I_{\mathrm{L}}} = \frac{|134600 + j108000|}{(\sqrt{3})(40)} = 2491 \text{ V(rms)}$$

16.2 Problem Solutions

Problem 16–1. The following sets of $v(t)$ and $i(t)$ apply to the load circuit in Figure P16–1. Find the average power, reactive power, and instantaneous power delivered to the load.

(a). $v(t) = 1500\cos(\omega t - 45^\circ)$ V, $i(t) = 2\cos(\omega t + 50^\circ)$ A

We have the following calculations and results:

$$\theta = \phi_{\mathrm{V}} - \phi_{\mathrm{I}} = -45^\circ - 50^\circ = -95^\circ$$

$$P = \frac{V_{\mathrm{A}} I_{\mathrm{A}}}{2}\cos(\theta) = \frac{(1500)(2)}{2}\cos(-95^\circ) = -130.73 \text{ W}$$

$$Q = \frac{V_{\mathrm{A}} I_{\mathrm{A}}}{2}\sin(\theta) = \frac{(1500)(2)}{2}\sin(-95^\circ) = -1494.3 \text{ VAR}$$

$$p(t) = P[1 + \cos(2\omega t)] - Q\sin(2\omega t) = -130.73\,[1 + \cos(2\omega t)] + 1494.3\sin(2\omega t) \text{ W}$$

(b). $v(t) = 90\cos(\omega t + 60^\circ)$ V, $i(t) = 10.5\cos(\omega t - 20^\circ)$ A

We have the following calculations and results:

$$\theta = \phi_{\mathrm{V}} - \phi_{\mathrm{I}} = 60^\circ + 20^\circ = 80^\circ$$

$$P = \frac{V_{\mathrm{A}} I_{\mathrm{A}}}{2}\cos(\theta) = \frac{(90)(10.5)}{2}\cos(80^\circ) = 82.05 \text{ W}$$

$$Q = \frac{V_{\mathrm{A}} I_{\mathrm{A}}}{2}\sin(\theta) = \frac{(90)(10.5)}{2}\sin(80^\circ) = 465.32 \text{ VAR}$$

$$p(t) = P[1 + \cos(2\omega t)] - Q\sin(2\omega t) = 82.05\,[1 + \cos(2\omega t)] - 465.32\sin(2\omega t) \text{ W}$$

Problem 16–2. The following sets of $v(t)$ and $i(t)$ apply to the load circuit in Figure P16–1. Find the average power, reactive power, and instantaneous power delivered to the load.

(a). $v(t) = 135\cos(\omega t)$ V, $i(t) = 2\cos(\omega t + 30^\circ)$ A

We have the following calculations and results:

$$\theta = \phi_{\mathrm{V}} - \phi_{\mathrm{I}} = 0^\circ - 30^\circ = -30^\circ$$

$$P = \frac{V_{\mathrm{A}} I_{\mathrm{A}}}{2}\cos(\theta) = \frac{(135)(2)}{2}\cos(-30^\circ) = 116.91 \text{ W}$$

$$Q = \frac{V_{\mathrm{A}} I_{\mathrm{A}}}{2}\sin(\theta) = \frac{(135)(2)}{2}\sin(-30^\circ) = -67.5 \text{ VAR}$$

$$p(t) = P[1 + \cos(2\omega t)] - Q\sin(2\omega t) = 116.91\,[1 + \cos(2\omega t)] + 67.5\sin(2\omega t) \text{ W}$$

(b). $v(t) = 370\sin(\omega t)$ V, $i(t) = 10\cos(\omega t + 20^\circ)$ A

We have the following calculations and results:

$$\theta = \phi_{\text{V}} - \phi_{\text{I}} = -90^\circ - 20^\circ = -110^\circ$$

$$P = \frac{V_{\text{A}}I_{\text{A}}}{2}\cos(\theta) = \frac{(370)(10)}{2}\cos(-110^\circ) = -632.74 \text{ W}$$

$$Q = \frac{V_{\text{A}}I_{\text{A}}}{2}\sin(\theta) = \frac{(370)(10)}{2}\sin(-110^\circ) = -1738.4 \text{ VAR}$$

$$p(t) = P[1+\cos(2\omega t)] - Q\sin(2\omega t) = -632.74\,[1+\cos(2\omega t)] + 1738.4\sin(2\omega t) \text{ W}$$

Problem 16–3. The following voltage and current phasors apply to the circuit in Figure P16–3. Calculate the average power and reactive power delivered to the impedance Z. Find the power factor and state whether the power factor is lagging or leading.

(a). $\mathbf{V} = 250\angle 0^\circ$ V (rms), $\mathbf{I} = 0.25\angle{-15^\circ}$ A (rms)

We have the following calculations and results:

$$\theta = \phi_{\text{V}} - \phi_{\text{I}} = 0^\circ + 15^\circ = 15^\circ$$

$$S = \mathbf{VI}^* = P + jQ$$

$$= (250\angle 0^\circ)(0.25\angle 15^\circ) = 62.5\angle 15^\circ = 60.37 + j16.18 \text{ VA}$$

$$P = 60.37 \text{ W}$$

$$Q = 16.18 \text{ VAR}$$

$$\text{pf} = \cos(\theta) = 0.9659$$

The reactive power is positive, so the power factor is lagging.

(b). $\mathbf{V} = 120\angle 135^\circ$ V (rms), $\mathbf{I} = 12.5\angle 165^\circ$ A (rms)

We have the following calculations and results:

$$\theta = \phi_{\text{V}} - \phi_{\text{I}} = 135^\circ - 165^\circ = -30^\circ$$

$$S = \mathbf{VI}^* = P + jQ$$

$$= (120\angle 135^\circ)(12.5\angle{-165^\circ}) = 1500\angle{-30^\circ} = 1299 - j750 \text{ VA}$$

$$P = 1299 \text{ W}$$

$$Q = -750 \text{ VAR}$$

$$\text{pf} = \cos(\theta) = 0.8660$$

The reactive power is negative, so the power factor is leading.

Problem 16–4. The following sets of $\mathbf{V}$ and $\mathbf{I}$ apply to the circuit in Figure P16–3. Calculate the complex power and the power factor. State whether the power factor is lagging or leading.

(a). $\mathbf{V} = 120\angle 30^\circ$ V (rms), $\mathbf{I} = 3.3\angle{-15^\circ}$ A (rms)

We have the following calculations and results:

$$\theta = \phi_V - \phi_I = 30^\circ + 15^\circ = 45^\circ$$

$$S = \mathbf{VI}^* = P + jQ$$

$$= (120\angle 30^\circ)(3.3\angle 15^\circ) = 396\angle 45^\circ = 280.01 + j280.01 \text{ VA}$$

$$\text{pf} = \cos(\theta) = 0.7071$$

The reactive power is positive, so the power factor is lagging.

(b). $\mathbf{V} = 480\angle 45^\circ$ V (rms), $\mathbf{I} = 8.5\angle 90^\circ$ A (rms)

We have the following calculations and results:

$$\theta = \phi_V - \phi_I = 45^\circ - 90^\circ = -45^\circ$$

$$S = \mathbf{VI}^* = P + jQ$$

$$= (480\angle 45^\circ)(8.5\angle{-90^\circ}) = 4080\angle{-45^\circ} = 2885 - j2885 \text{ VA}$$

$$\text{pf} = \cos(\theta) = 0.7071$$

The reactive power is negative, so the power factor is leading.

Problem 16–5. The conditions in this problem apply to the circuit in Figure P16–3. Calculate the complex power for each condition given below.

(a). $\mathbf{V} = 15\angle 45^\circ$ kV (rms), $Z = 500\angle{-15^\circ}$ Ω

We have the following results:

$$\mathbf{I} = \frac{\mathbf{V}}{Z} = \frac{15000\angle 45^\circ}{500\angle{-15^\circ}} = 30\angle 60^\circ = 15 + j25.98 \text{ A(rms)}$$

$$S = \mathbf{VI}^* = (15000\angle 45^\circ)(30\angle{-60^\circ}) = 450000\angle{-15^\circ} = 434667 - j116469 \text{ VA}$$

(b). $Z = 40 - j30$ Ω, $\mathbf{I} = 10\angle 25^\circ$ A (rms)

We have the following results:

$$\mathbf{V} = Z\mathbf{I} = (40 - j30)(10\angle 25^\circ) = 500\angle{-11.87^\circ} = 489.31 - j102.85 \text{ V(rms)}$$

$$S = \mathbf{VI}^* = (500\angle{-11.87^\circ})(10\angle{-25^\circ}) = 5000\angle{-36.87^\circ} = 4000 - j3000 \text{ VA}$$

Problem 16–6. Find the power factor under the following conditions. State whether the power factor is lagging or leading.

(a). $S = 1000 + j250$ kVA

We have the following results:

$$\text{pf} = \frac{P}{|S|} = \frac{1000}{|1000 + j250|} = \frac{1000}{\sqrt{1000^2 + 250^2}} = 0.970$$

The reactive power is positive, so the power factor is lagging (b). $|S| = 15$ kVA, $P = 12$ kW, $Q < 0$

We have the following results:

$$\text{pf} = \frac{P}{|S|} = \frac{12000}{15000} = 0.8$$

The reactive power is negative, so the power factor is leading.

Problem 16–7. A load draws an apparent power of 30 kVA at a power factor of 0.8 lagging from a 2400-V (rms) source. Find P, Q, and the load impedance.

We have the following results:

$$P = \text{pf}|S| = (0.8)(30000) = 24000 \text{ W}$$

$$Q = \sqrt{|S|^2 - P^2} = \sqrt{(30000)^2 - (24000)^2} = 18000 \text{ VAR}$$

$$S = P + jQ = 24000 + j18000 \text{ VA}$$

$$|\mathbf{I}| = \frac{|S|}{|\mathbf{V}|} = \frac{30000}{2400} = 12.5 \text{ A(rms)}$$

$$Z = \frac{S}{|\mathbf{I}|^2} = \frac{24000 + j18000}{(12.5)^2} = 153.6 + j115.2\ \Omega$$

Problem 16–8. A load draws 8 kW at a power factor of 0.8 leading from a 880-V (rms) source. Find Q and the load impedance.

We have the following results:

$$|S| = \frac{P}{\text{pf}} = \frac{8000}{0.8} = 10000 \text{ VA}$$

$$Q = -\sqrt{|S|^2 - P^2} = -\sqrt{(10000)^2 - (8000)^2} = -6000 \text{ VAR}$$

$$S = 8000 - j6000 \text{ VA}$$

$$|\mathbf{I}| = \frac{|S|}{|\mathbf{V}|} = \frac{10000}{880} = 11.36 \text{ A(rms)}$$

$$Z = \frac{S}{|\mathbf{I}|^2} = \frac{8000 - j6000}{(11.36)^2} = 61.95 - j46.46\ \Omega$$

Problem 16–9. A load draws 15 A (rms), 5 kW, and 2.5 kVAR (lagging) from a 60-Hz source. Find the load power factor and impedance.

We have the following results:

$$S = P + jQ = 5000 + j2500 \text{ VA}$$

$$|S| = \sqrt{(5000)^2 + (2500)^2} = 5590.2 \text{ VA}$$

$$\text{pf} = \frac{P}{|S|} = \frac{5000}{5590.2} = 0.8944$$

$$Z = \frac{S}{|\mathbf{I}|^2} = \frac{5000 + j2500}{(15)^2} = 22.22 + j11.11\ \Omega$$

Problem 16–10. Find the impedance of a load that is rated at 440 V (rms), 5 A (rms), and 2.2 kW.

We have the following results:

$$R = \frac{P}{|\mathbf{I}|^2} = \frac{2200}{5^2} = 88\ \Omega$$

$$|Z| = \frac{|\mathbf{V}|}{|\mathbf{I}|} = \frac{440}{5} = 88\ \Omega$$

$$X = \sqrt{|Z|^2 - R^2} = \sqrt{88^2 - 88^2} = 0\ \Omega$$

$$Z = R + jX = 88 + j0\ \Omega$$

Problem 16–11. A load made up of a 100-Ω resistor in series with a 150-mH inductor is connected across a 240-V (rms), 60-Hz voltage source. Find the complex power delivered to the load and the load power factor. State whether the power factor is lagging or leading.

We have the following calculations and results:

$$\omega = 2\pi f = 376.99\ \text{rad/s}$$

$$Z_{\text{L}} = R + j\omega L = 100 + j56.55\ \Omega$$

$$|\mathbf{I}| = \frac{|\mathbf{V}|}{|Z_{\text{L}}|} = \frac{240}{|100 + j56.55|} = 2.0891\ \text{A(rms)}$$

$$S_{\text{L}} = Z_{\text{L}}|\mathbf{I}|^2 = (100 + j56.55)(2.0891)^2 = 436.438 + j246.8\ \text{VA}$$

$$\text{pf} = \frac{P_{\text{L}}}{|S_{\text{L}}|} = \frac{436.438}{|436.438 + j246.8|} = 0.87046$$

The reactive power is positive, so the power factor is lagging.

Problem 16–12. A load made up of a 50-Ω resistor in parallel with a 10-μF capacitor is connected across a 400-Hz source that delivers 110 V (rms). Find the complex power delivered to the load and the load power factor. State whether the power factor is lagging or leading.

We have the following calculations and results:

$$\omega = 2\pi f = 2513.3\ \text{rad/s}$$

$$Z_{\text{C}} = \frac{1}{j\omega C} = -j39.79\ \Omega$$

$$Z_{\text{L}} = R \parallel Z_{\text{C}} = 50 \parallel -j39.79 = 19.386 - j24.362\ \Omega$$

$$|\mathbf{I}| = \frac{|\mathbf{V}|}{|Z_{\text{L}}|} = \frac{110}{|19.386 - j24.362|} = 3.5331\ \text{A(rms)}$$

$$S_{\text{L}} = Z_{\text{L}}|\mathbf{I}|^2 = (19.386 - j24.362)(3.5331)^2 = 242 - j304.1\ \text{VA}$$

$$\text{pf} = \frac{P_{\text{L}}}{|S_{\text{L}}|} = \frac{242}{|242 - j304.1|} = 0.6227$$

The reactive power is negative, so the power factor is leading.

Problem 16–13. A load made up of a resistor R in series with an inductor L draws a complex power of $1200 + j800$ VA when connected across a 440-V (rms), 60-Hz voltage source. Find the value of R and L.

We have the following calculations and results:

$$\omega = 2\pi f = 376.99 \text{ rad/s}$$

$$|S_\text{L}| = |1200 + j800| = 1442.2 \text{ VA}$$

$$|\mathbf{I}| = \frac{|S_\text{L}|}{|\mathbf{V}|} = \frac{1442.2}{440} = 3.2778 \text{ A(rms)}$$

$$Z_\text{L} = \frac{S_\text{L}}{|\mathbf{I}|^2} = \frac{1200 + j8000}{(3.2778)^2} = 111.69 + j74.46\ \Omega$$

$$R_\text{L} = 111.69\ \Omega$$

$$X_\text{L} = 74.46\ \Omega$$

$$L = \frac{X_\text{L}}{\omega} = \frac{74.46}{376.99} = 197.52 \text{ mH}$$

Problem 16–14. In Figure P16–14, the load Z_L is a 60-Ω resistor in series with a capacitor whose reactance is $-30\ \Omega$. The source voltage is 440 V (rms). Find the complex power produced by the source and the complex power delivered to the load.

First, determine the input impedance seen by the source.

$$Z_\text{IN} = 20 + j25 + [-j20 \parallel (60 - j30)] = 20 + j25 + [3.9344 - j16.7213] = 23.9344 + j8.2787\ \Omega$$

Find the magnitude of the source current.

$$|\mathbf{I}_\text{S}| = \frac{|\mathbf{V}_\text{S}|}{|Z_\text{IN}|} = \frac{440}{|23.9344 + j8.2787|} = 17.3736 \text{ A(rms)}$$

Find the complex power produced by the source.

$$S_\text{S} = Z_\text{IN}|\mathbf{I}_\text{S}|^2 = (23.9344 + j8.2787)(17.3736)^2 = 7224.4 + j2498.9 \text{ VA}$$

Find the magnitude of the voltage across the load.

$$|\mathbf{V}_\text{L}| = (|\mathbf{I}_\text{S}|)(|3.9344 - j16.7213|) = 298.4432 \text{ V(rms)}$$

Find the magnitude of the current through the load.

$$|\mathbf{I}_\text{L}| = \frac{|\mathbf{V}_\text{L}|}{|60 - j30|} = 4.4489 \text{ A(rms)}$$

Calculate the complex power delivered to the load.

$$S_\text{L} = Z_\text{L}|\mathbf{I}_\text{L}|^2 = (60 - j30)(4.4489)^2 = 1187.6 - j593.8 \text{ VA}$$

Problem 16–15. Repeat Problem 16–14 when Z_L is a 50-Ω resistor in parallel with an impedance of $40 - j60\ \Omega$.

First, determine the input impedance seen by the source.

$$Z_\text{L} = 50 \parallel (40 - j60) = 30.7692 - j12.8205\ \Omega$$

$$Z_\text{IN} = 20 + j25 + [-j20 \parallel Z_\text{L}] = 20 + j25 + [6.0811 - j13.5135] = 26.0811 + j11.4865\ \Omega$$

Find the magnitude of the source current.

$$|\mathbf{I}_\text{S}| = \frac{|\mathbf{V}_\text{S}|}{|Z_\text{IN}|} = \frac{440}{|26.0811 + j11.4865|} = 15.4394 \text{ A(rms)}$$

Find the complex power produced by the source.

$$S_{\rm S} = Z_{\rm IN}|\mathbf{I}_{\rm S}|^2 = (26.0811 + j11.4865)(15.4394)^2 = 6217.1 + j2738.1 \text{ VA}$$

Find the magnitude of the voltage across the load.

$$|\mathbf{V}_{\rm L}| = (|\mathbf{I}_{\rm S}|)(|6.0811 - j13.5135|) = 228.7927 \text{ V(rms)}$$

Find the magnitude of the current through the load.

$$|\mathbf{I}_{\rm L}| = \frac{|\mathbf{V}_{\rm L}|}{|Z_{\rm L}|} = \frac{|\mathbf{V}_{\rm L}|}{|30.7692 - j12.8205|} = 6.8638 \text{ A(rms)}$$

Calculate the complex power delivered to the load.

$$S_{\rm L} = Z_{\rm L}|\mathbf{I}_{\rm L}|^2 = (30.7692 - j12.8205)(6.8638)^2 = 1449.6 - j603.99 \text{ VA}$$

Problem 16–16. In Figure P16–16, the load $Z_{\rm L}$ is a 100-Ω resistor and the source voltage is 240 V (rms). Find the complex power produced by the source.

Determine the input impedance seen by the source.

$$Z_{\rm IN} = j100 + [-j25 \parallel (100 + j100)] = 4 + j72 \ \Omega$$

Find the magnitude of the source current.

$$|\mathbf{I}_{\rm S}| = \frac{|\mathbf{V}_{\rm S}|}{|Z_{\rm IN}|} = \frac{240}{|4 + j72|} = 3.3282 \text{ A(rms)}$$

Find the complex power produced by the source.

$$S_{\rm S} = Z_{\rm IN}|\mathbf{I}_{\rm S}|^2 = (4 + j72)(3.3282)^2 = 44.3077 + j797.5385 \text{ VA}$$

Problem 16–17. Repeat Problem 16–16 when the load $Z_{\rm L}$ is a 50-Ω resistor in series with a capacitor whose reactance is −100 Ω.

Determine the input impedance seen by the source.

$$Z_{\rm IN} = j100 + [-j25 \parallel (50 - j100 + j100)] = 10 + j80 \ \Omega$$

Find the magnitude of the source current.

$$|\mathbf{I}_{\rm S}| = \frac{|\mathbf{V}_{\rm S}|}{|Z_{\rm IN}|} = \frac{240}{|10 + j80|} = 2.9768 \text{ A(rms)}$$

Find the complex power produced by the source.

$$S_{\rm S} = Z_{\rm IN}|\mathbf{I}_{\rm S}|^2 = (10 + j80)(2.9768)^2 = 88.6154 + j708.9231 \text{ VA}$$

Problem 16–18. In Figure P16–18, the three load impedances are: $Z_1 = 20 + j15\ \Omega$, $Z_2 = 25 + j10\ \Omega$, and $Z_3 = 75 + j50\ \Omega$. Find the total complex power produced by the two sources and the overall circuit power factor.

Find the complex power delivered to each load.

$$S_1 = \frac{|\mathbf{V}_1|^2}{Z_1^*} = \frac{110^2}{20 - j15} = 387.2 + j290.4 \text{ VA}$$

$$S_2 = \frac{|\mathbf{V}_2|^2}{Z_2^*} = \frac{110^2}{25 - j10} = 417.2 + j166.9 \text{ VA}$$

$$S_3 = \frac{|\mathbf{V}_1 + \mathbf{V}_2|^2}{Z_3^*} = \frac{220^2}{75 - j50} = 446.8 + j297.8 \text{ VA}$$

Sum the individual complex powers to determine the total complex power produced by the two sources

$$S_{\text{Total}} = S_1 + S_2 + S_3 = 1251.2 + j755.1 \text{ VA}$$

Compute the power factor.

$$\text{pf} = \frac{P_{\text{Total}}}{|S_{\text{Total}}|} = \frac{1251.2}{|1251.1 + j755.1|} = 0.8562$$

The reactive power is positive, so the power factor is lagging.

Problem 16–19. In Figure P16–18, the complex powers delivered to each load are: $S_1 = 400 + j270$ VA, $S_2 = 550 + j150$ VA, and $S_3 = 1000 + j0$ VA. Find the line currents $\mathbf{I}_\text{A}$, $\mathbf{I}_\text{N}$, and $\mathbf{I}_\text{B}$.

Compute the current through each load and then apply KCL to find the requested currents.

$$S = \mathbf{VI}^*$$

$$\mathbf{I} = \left(\frac{S}{\mathbf{V}}\right)^*$$

$$\mathbf{I}_1 = \left(\frac{S_1}{\mathbf{V}_1}\right)^* = \left(\frac{400 + j270}{110}\right)^* = 3.6364 - j2.4545 \text{ A(rms)}$$

$$\mathbf{I}_2 = \left(\frac{S_2}{\mathbf{V}_2}\right)^* = \left(\frac{550 + j150}{110}\right)^* = 5 - j1.3636 \text{ A(rms)}$$

$$\mathbf{I}_3 = \left(\frac{S_3}{\mathbf{V}_1 + \mathbf{V}_2}\right)^* = \left(\frac{1000 + j0}{220}\right)^* = 4.5455 + j0 \text{ A(rms)}$$

$$\mathbf{I}_\text{A} = \mathbf{I}_1 + \mathbf{I}_3 = 8.1818 - j2.4545 \text{ A(rms)}$$

$$\mathbf{I}_\text{N} = \mathbf{I}_2 - \mathbf{I}_1 = 1.3636 + j1.0909 \text{ A(rms)}$$

$$\mathbf{I}_\text{B} = -\mathbf{I}_2 - \mathbf{I}_3 = -9.5455 + j1.3636 \text{ A(rms)}$$

Problem 16–20. Two loads are connected in parallel across an 880-V (rms) line. The first load draws an average power of 20 kW at a lagging power factor of 0.8. The second load draws 16 kW at a lagging power factor of 0.9. Find the overall power factor of the circuit and the rms current drawn from the line.

Compute the complex power for each load.

$$S_1 = P_1 + jP_1\sqrt{\left(\frac{1}{\text{pf}_1}\right)^2 - 1} = 20 + j15 \text{ kVA}$$

$$S_2 = P_2 + jP_2\sqrt{\left(\frac{1}{\text{pf}_2}\right)^2 - 1} = 16 + j7.7492 \text{ kVA}$$

Sum the load powers to determine the total power provided by the source.

$$S_\text{S} = S_1 + S_2 = 36 + j22.7492 \text{ kVA}$$

Compute the overall power factor.

$$\text{pf} = \frac{P_\text{S}}{|S_\text{S}|} = \frac{36}{|36 + j22.7492|} = 0.84536$$

Determine the source current.

$$\mathbf{I}_\text{S} = \left(\frac{S_\text{S}}{\mathbf{V}_\text{S}}\right)^* = \left(\frac{36000 + j22749.2}{880}\right)^* = 40.9091 - j25.8513 \text{ A(rms)}$$

$$|\mathbf{I}_\text{S}| = \frac{|S_\text{S}|}{|\mathbf{V}_\text{S}|} = \frac{|36000 + j22749.2|}{880} = 48.3926 \text{ A(rms)}$$

Problem 16–21. The apparent power delivered to the load Z_{L} in Figure P16–21 is 46 kVA at a lagging power factor of 0.84. The load voltage is 2.4 kV (rms) and the line has an impedance of $Z_{\text{W}} = 1 + j8\ \Omega$ per wire. Find the required apparent source power, the source power factor, and the magnitude of the source voltage.

Determine the complex power delivered to the load

$$S_{\text{L}} = |S_{\text{L}}|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = 46000\left(0.84 + j\sqrt{1-0.84^2}\right) = 38640 + j24959\text{ VA}$$

Find the magnitude of the load current.

$$|\mathbf{I}_{\text{L}}| = \frac{|S_{\text{L}}|}{|\mathbf{V}_{\text{L}}|} = \frac{46000}{2400} = 19.1667\text{ A(rms)}$$

Find the complex power dissipated by the wires.

$$S_{\text{W}} = 2Z_{\text{W}}|\mathbf{I}_{\text{L}}|^2 = (2)(1+j8)(19.1667)^2 = 734.7222 + j5877.8\text{ VA}$$

Compute the requested values.

$$S_{\text{S}} = S_{\text{W}} + S_{\text{L}} = 39374.7 + j30836.8\text{ VA}$$

$$|S_{\text{S}}| = 50012.7\text{ VA}$$

$$\text{pf} = \frac{P_{\text{S}}}{|S_{\text{S}}|} = \frac{39374.7}{50012.7} = 0.7873$$

$$|\mathbf{V}_{\text{S}}| = \frac{|S_{\text{S}}|}{|\mathbf{I}_{\text{L}}|} = \frac{50012.7}{19.1667} = 2609.4\text{ V(rms)}$$

Problem 16–22. Repeat Problem 16–21 with the load power factor increased to 0.95.

Determine the complex power delivered to the load

$$S_{\text{L}} = |S_{\text{L}}|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = 46000\left(0.95 + j\sqrt{1-0.95^2}\right) = 43700 + j14363.5\text{ VA}$$

Find the magnitude of the load current.

$$|\mathbf{I}_{\text{L}}| = \frac{|S_{\text{L}}|}{|\mathbf{V}_{\text{L}}|} = \frac{46000}{2400} = 19.1667\text{ A(rms)}$$

Find the complex power dissipated by the wires.

$$S_{\text{W}} = 2Z_{\text{W}}|\mathbf{I}_{\text{L}}|^2 = (2)(1+j8)(19.1667)^2 = 734.7222 + j5877.8\text{ VA}$$

Compute the requested values.

$$S_{\text{S}} = S_{\text{W}} + S_{\text{L}} = 44434.7 + j20241.3\text{ VA}$$

$$|S_{\text{S}}| = 48827.8\text{ VA}$$

$$\text{pf} = \frac{P_{\text{S}}}{|S_{\text{S}}|} = \frac{44434.7}{48827.8} = 0.91003$$

$$|\mathbf{V}_{\text{S}}| = \frac{|S_{\text{S}}|}{|\mathbf{I}_{\text{L}}|} = \frac{48827.8}{19.1667} = 2547.5\text{ V(rms)}$$

Problem 16–23. The average power delivered to the load $Z_{\rm L}$ in Figure P16–21 is 250 kW at a lagging power factor of 0.85. The load voltage is 7.2 kV (rms) and the line has an impedance of $Z_{\rm W} = 2 + j12\ \Omega$ per wire. Find the apparent power supplied by the source and the magnitude of the source voltage.

Compute the complex power delivered to the load.

$$S_{\rm L} = P_{\rm L} + jP_{\rm L}\sqrt{\left(\frac{1}{{\rm pf}_L}\right)^2 - 1} = 250 + j154.9361 \text{ kVA}$$

$$|S_{\rm L}| = 294.1176 \text{ kVA}$$

Find the magnitude of the load current.

$$|\mathbf{I}_{\rm L}| = \frac{|S_{\rm L}|}{|\mathbf{V}_{\rm L}|} = \frac{294117.6}{7200} = 40.8497 \text{ A(rms)}$$

Find the complex power dissipated by the wires.

$$S_{\rm W} = 2Z_{\rm W}|\mathbf{I}_{\rm L}|^2 = (2)(2 + j12)(40.8497)^2 = 6674.8 + j40048.7 \text{ VA}$$

Compute the requested values.

$$S_{\rm S} = S_{\rm W} + S_{\rm L} = 256674.8 + j194984.8 \text{ VA}$$

$$|S_{\rm S}| = 322336.8 \text{ VA}$$

$${\rm pf} = \frac{P_{\rm S}}{|S_{\rm S}|} = \frac{256674.8}{322336.8} = 0.79629$$

$$|\mathbf{V}_{\rm S}| = \frac{|S_{\rm S}|}{|\mathbf{I}_{\rm L}|} = \frac{322336.8}{40.8497} = 7890.8 \text{ V(rms)}$$

Problem 16–24. The complex power delivered to the load $Z_{\rm L}$ in Figure P16–21 is $20 + j15$ kVA. The source produces an average power of 21 kW and the line has an impedance of $Z_{\rm W} = 2.1 + j12\ \Omega$ per wire. Find the magnitude of the source and load voltages.

Determine the average power in the load and the wires.

$$P_{\rm L} = {\rm Re}(S_{\rm L}) = 20000 \text{ W}$$

$$P_{\rm W} = P_{\rm S} - P_{\rm L} = 21000 - 20000 = 1000 \text{ W}$$

Find the current through the wires.

$$|\mathbf{I}_{\rm L}| = \sqrt{\frac{P_{\rm W}}{2{\rm Re}(Z_{\rm W})}} = \sqrt{\frac{1000}{(2)(2.1)}} = 15.4303 \text{ A(rms)}$$

Find the magnitude of the voltage at the load.

$$|\mathbf{V}_{\rm L}| = \frac{|S_{\rm L}|}{|\mathbf{I}_{\rm L}|} = \frac{|20000 + j15000|}{15.4303} = 1620.2 \text{ V(rms)}$$

Find the total complex power delivered by the source.

$$S_{\rm W} = 2Z_{\rm W}|\mathbf{I}_{\rm L}|^2 = (2)(2.1 + j12)(15.4303)^2 = 1000 + j5714.3 \text{ VA}$$

$$S_{\rm S} = S_{\rm W} + S_{\rm L} = 21000 + j20714.3 \text{ VA}$$

Compute the magnitude of the source voltage.

$$|\mathbf{V}_{\rm S}| = \frac{|S_{\rm S}|}{|\mathbf{I}_{\rm L}|} = \frac{|21000 + j20714.3|}{15.4303} = 1911.6 \text{ V(rms)}$$

Problem 16–25. In Figure P16–25, the voltage across the two loads is $|\mathbf{V}_{\rm L}| = 4.8$ kV (rms). The load Z_1 draws an average power of 12 kW at a lagging power factor of 0.85. The load Z_2 draws an apparent power of 15 kVA at a lagging power factor of 0.8. The line has an impedance of $Z_{\rm W} = 9 + j50\ \Omega$ per wire. Find the apparent power produced by the source and the rms value of the source voltage.

Compute the complex power delivered to load Z_1.

$$S_{\rm L1} = P_{\rm L1} + jP_{\rm L1}\sqrt{\left(\frac{1}{\rm pf_1}\right)^2 - 1} = 12000 + j7436.9 \text{ VA}$$

$$|S_{\rm L1}| = 14117.6 \text{ VA}$$

Compute the complex power delivered to load Z_2.

$$P_{\rm L2} = {\rm pf}_2|S_{\rm L2}| = (0.8)(15000) = 12000 \text{ W}$$

$$S_{\rm L2} = P_{\rm L2} + jP_{\rm L2}\sqrt{\left(\frac{1}{\rm pf_2}\right)^2 - 1} = 12000 + j9000 \text{ VA}$$

$$|S_{\rm L2}| = 15000 \text{ VA}$$

Find the total complex power delivered to the loads and the magnitude of the current to the loads.

$$S_{\rm L} = S_{\rm L1} + S_{\rm L2} = 24000 + j16436.9 \text{ VA}$$

$$|\mathbf{I}_{\rm L}| = \frac{|S_{\rm L}|}{|\mathbf{V}_{\rm L}|} = \frac{|24000 + j16436.9|}{4800} = 6.0602 \text{ A(rms)}$$

Determine the complex power dissipated by the wires.

$$S_{\rm W} = 2Z_{\rm W}|\mathbf{I}_{\rm L}|^2 = (2)(9 + j50)(6.0602)^2 = 661.0724 + j3672.6 \text{ VA}$$

Find the complex power at the source and the magnitude of the source voltage.

$$S_{\rm S} = S_{\rm W} + S_{\rm L} = 24661.1 + j20109.6 \text{ VA}$$

$$|S_{\rm S}| = 31820.8 \text{ VA}$$

$$|\mathbf{V}_{\rm S}| = \frac{|S_{\rm S}|}{|\mathbf{I}_{\rm L}|} = \frac{31820.8}{6.0602} = 5250.8 \text{ V(rms)}$$

Problem 16–26. The two loads in Figure P16–25 draw apparent powers of $|S_1| = 16$ kVA at a lagging power factor of 0.8 and $|S_2| = 25$ kVA at unity power factor. The voltage across the loads is 3.8 kV and the line has an impedance of $Z_{\rm W} = 5 + j26\ \Omega$ per wire. Find the apparent power produced by the source and the rms value of the source voltage.

Compute the complex power for each load.

$$P_{\rm L1} = {\rm pf}_1|S_1| = (0.8)(16000) = 12800 \text{ W}$$

$$S_1 = P_{\rm L1} + jP_{\rm L1}\sqrt{\left(\frac{1}{\rm pf_1}\right)^2 - 1} = 12800 + j9600 \text{ VA}$$

$$P_{\rm L2} = {\rm pf}_2|S_2| = (1)(25000) = 25000 \text{ W}$$

$$S_2 = P_{\rm L2} + jP_{\rm L2}\sqrt{\left(\frac{1}{\rm pf_2}\right)^2 - 1} = 25000 + j0 \text{ VA}$$

Determine the total complex power dissipated by the loads.

$$S_{\rm L} = S_1 + S_2 = 37800 + j9600 \text{ VA}$$

Find the magnitude of the current through the loads.

$$|\mathbf{I}_{\rm L}| = \frac{|S_{\rm L}|}{|\mathbf{V}_{\rm L}|} = \frac{|37800 + j9600|}{3800} = 10.2632 \text{ A(rms)}$$

Determine the complex power dissipated by the wires.

$$S_{\rm W} = 2Z_{\rm W}|\mathbf{I}_{\rm L}|^2 = (2)(5 + j26)(10.2632)^2 = 1053.3 + j5477.3 \text{ VA}$$

Find the complex power at the source and the magnitude of the source voltage.

$$S_{\rm S} = S_{\rm W} + S_{\rm L} = 38853.3 + j15077.3 \text{ VA}$$

$$|S_{\rm S}| = 41676.2 \text{ VA}$$

$$|\mathbf{V}_{\rm S}| = \frac{|S_{\rm S}|}{|\mathbf{I}_{\rm L}|} = \frac{41676.2}{10.2632} = 4060.8 \text{ V(rms)}$$

Problem 16–27. A 60-Hz voltage source feeds a two-wire line with $Z_{\rm W} = 0.6 + j3.4\ \Omega$ per wire. The load at the receiving end of the line draws an apparent power of 5 kVA at a leading power factor 0.8. The voltage across the load is 500 V (rms). Find the apparent power produced by the source and the rms value of the source voltage.

Compute the complex power for the load.

$$P_{\rm L} = \text{pf}|S_{\rm L}| = (0.8)(5000) = 4000 \text{ W}$$

$$S_{\rm L} = P_{\rm L} - jP_{\rm L}\sqrt{\left(\frac{1}{\text{pf}}\right)^2 - 1} = 4000 - j3000 \text{ VA}$$

Find the magnitude of the current for the loads.

$$|\mathbf{I}_{\rm L}| = \frac{|S_{\rm L}|}{|\mathbf{V}_{\rm L}|} = \frac{5000}{500} = 10 \text{ A(rms)}$$

Determine the complex power dissipated by the wires.

$$S_{\rm W} = 2Z_{\rm W}|\mathbf{I}_{\rm L}|^2 = (2)(0.6 + j3.4)(10)^2 = 120 + j680 \text{ VA}$$

Find the complex power at the source and the magnitude of the source voltage.

$$S_{\rm S} = S_{\rm W} + S_{\rm L} = 4120 - j2320 \text{ VA}$$

$$|S_{\rm S}| = 4728.3 \text{ VA}$$

$$|\mathbf{V}_{\rm S}| = \frac{|S_{\rm S}|}{|\mathbf{I}_{\rm L}|} = \frac{4728.3}{10} = 472.83 \text{ V(rms)}$$

Problem 16–28. In Figure P16–28, the load voltage is $|\mathbf{V}_{\rm L}| = 4160$ V (rms) at 60 Hz and the load $Z_{\rm L}$ draws an average power of 10 kW at a lagging power factor of 0.725. Find the overall power factor of the combination if the parallel capacitance is 1 μF. Find the value of the capacitance needed to raise the overall power factor to unity.

Compute the complex power for the load.

$$S_{\rm L} = P_{\rm L} + jP_{\rm L}\sqrt{\left(\frac{1}{\text{pf}}\right)^2 - 1} = 10000 + j9500 \text{ VA}$$

Determine the reactive power of the capacitor.

$$Q_{\mathrm{C}} = -\omega C|\mathbf{V}_{\mathrm{L}}|^2 = (-120\pi)(1\,\mu)(4160)^2 = -6524.1\ \Omega$$

Find the total complex power delivered by the rest of the circuit.

$$S = S_{\mathrm{L}} + jQ_{\mathrm{C}} = 10000 + j2975.9\ \mathrm{VA}$$

Determine the overall power factor.

$$\mathrm{pf} = \frac{\mathrm{Re}(S)}{|S|} = \frac{10000}{|10000 + j2975.9|} = 0.9585$$

To get an overall power factor of one, we require $Q_{\mathrm{C}} = -9500$ to cancel the reactive component of the load's complex power.

$$Q_{\mathrm{C}} = -9500 = -\omega C|\mathbf{V}_{\mathrm{L}}|^2$$

$$C = \frac{9500}{\omega|\mathbf{V}_{\mathrm{L}}|^2} = \frac{9500}{(120\pi)(4160)^2} = 1.4561\ \mu\mathrm{F}$$

Problem 16–29. In Figure P16–28, the load voltage is $|\mathbf{V}_{\mathrm{L}}| = 2400$ V (rms) at 60 Hz. The load Z_{L} draws an apparent power of 25 kVA at a lagging power factor of 0.7. Find the value of the capacitance required to raise the overall power factor of the parallel combination to 0.95. Repeat for an overall power factor of unity.

Compute the complex power for the load.

$$S_{\mathrm{L}} = |S_{\mathrm{L}}|\left(\mathrm{pf} + j\sqrt{1 - \mathrm{pf}^2}\right) = (25000)\left(0.7 + j\sqrt{1 - 0.7^2}\right) = 17500 + j17853.6\ \mathrm{VA}$$

$$P_{\mathrm{L}} = \mathrm{Re}(S_{\mathrm{L}}) = 17500\ \mathrm{W}$$

Compute the complex power of the parallel combination with the new power factor.

$$S_{\mathrm{P}} = \frac{P_{\mathrm{L}}}{\mathrm{pf}_2}\left(\mathrm{pf}_2 + j\sqrt{1 - \mathrm{pf}_2^2}\right) = \frac{17500}{0.95}\left(0.95 + j\sqrt{1 - 0.95^2}\right) = 17500 + j5752\ \mathrm{VA}$$

The capacitor causes the change in the reactive power.

$$Q_{\mathrm{C}} = \mathrm{Im}(S_{\mathrm{P}}) - \mathrm{Im}(S_{\mathrm{L}}) = 5752 - 17853.6 = -12101.6\ \mathrm{VAR}$$

$$C = \frac{-Q_{\mathrm{C}}}{\omega|\mathbf{V}_{\mathrm{L}}|^2} = \frac{12101.6}{(120\pi)(2400)^2} = 5.573\ \mu\mathrm{F}$$

If the desired power factor is one, then we have the following results:

$$S_{\mathrm{P}} = \frac{P_{\mathrm{L}}}{\mathrm{pf}_2}\left(\mathrm{pf}_2 + j\sqrt{1 - \mathrm{pf}_2^2}\right) = \frac{17500}{1}\left(1 + j\sqrt{1 - 1^2}\right) = 17500\ \mathrm{VA}$$

$$Q_{\mathrm{C}} = \mathrm{Im}(S_{\mathrm{P}}) - \mathrm{Im}(S_{\mathrm{L}}) = 0 - 17853.6 = -17853.6\ \mathrm{VAR}$$

$$C = \frac{-Q_{\mathrm{C}}}{\omega|\mathbf{V}_{\mathrm{L}}|^2} = \frac{17853.6}{(120\pi)(2400)^2} = 8.2219\ \mu\mathrm{F}$$

Problem 16–30. A load draws 5 A (rms) and 4 kW at a power factor 0.8 (lagging) from a 60-Hz source. Find the capacitance needed in parallel with the load to raise the overall power factor to unity.

Compute the complex power for the load.

$$|S_L| = \frac{P_L}{\text{pf}} = \frac{4000}{0.8} = 5000 \text{ VA}$$

$$S_L = |S_L|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = (5000)\left(0.8 + j\sqrt{1-0.8^2}\right) = 4000 + j3000 \text{ VA}$$

Find the magnitude of the load voltage.

$$|\mathbf{V}_L| = \frac{|S_L|}{|\mathbf{I}_L|} = \frac{5000}{5} = 1000 \text{ V(rms)}$$

Compute the complex power of the parallel combination with a power factor of one.

$$S_P = \frac{P_L}{\text{pf}_2}\left(\text{pf}_2 + j\sqrt{1-\text{pf}_2^2}\right) = \frac{4000}{1}\left(1 + j\sqrt{1-1^2}\right) = 4000 \text{ VA}$$

The capacitor causes the change in the reactive power.

$$Q_C = \text{Im}(S_P) - \text{Im}(S_L) = 0 - 3000 = -3000 \text{ VAR}$$

$$C = \frac{-Q_C}{\omega|\mathbf{V}_L|^2} = \frac{3000}{(120\pi)(1000)^2} = 7.9577\ \mu\text{F}$$

Problem 16–31. In a balanced three-phase circuit, the phase voltage magnitude is $V_P = 277$ V (rms). For a positive phase sequence:

(a). Find all of the line and phase voltage phasors using $\mathbf{V}_{AN}$ as the phase reference.

The line voltage magnitude is $V_L = \sqrt{3}V_P = \sqrt{3}(277) = 480$ V (rms). We have the following phase and line voltages, using $\mathbf{V}_{AN}$ as the phase reference:

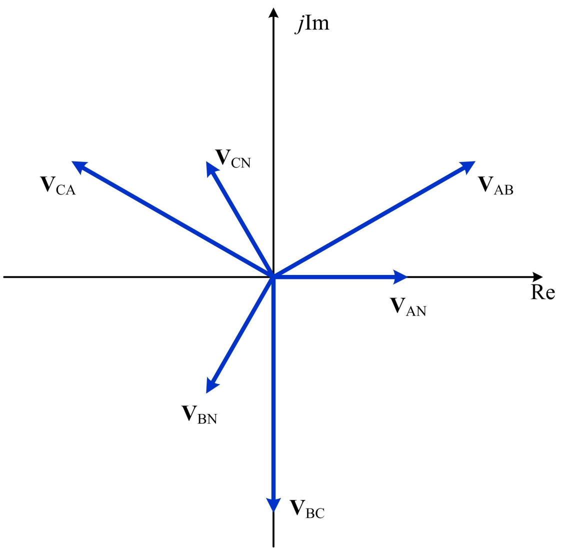
$$\mathbf{V}_{AN} = 277\angle 0^\circ \text{ V(rms)} \qquad \mathbf{V}_{AB} = 480\angle 30^\circ \text{ V(rms)}$$

$$\mathbf{V}_{BN} = 277\angle{-120^\circ} \text{ V(rms)} \qquad \mathbf{V}_{BC} = 480\angle{-90^\circ} \text{ V(rms)}$$

$$\mathbf{V}_{CN} = 277\angle{-240^\circ} \text{ V(rms)} \qquad \mathbf{V}_{CA} = 480\angle{-210^\circ} \text{ V(rms)}$$

(b). Sketch a phasor diagram of the line and phase voltages.

The sketch is shown below.



Problem 16–32. In a balanced three-phase circuit, the line voltage magnitude is $V_{\rm L} = 2.4$ kV (rms). For a positive phase sequence:

(a). Find all of the line and phase voltage phasors using $\mathbf{V}_{\rm AB}$ as the phase reference.

The phase voltage magnitude is $V_{\rm P} = V_{\rm L}/\sqrt{3} = 2400/\sqrt{3} = 1385.6$ V (rms). We have the following phase and line voltages, using $\mathbf{V}_{\rm AB}$ as the phase reference:

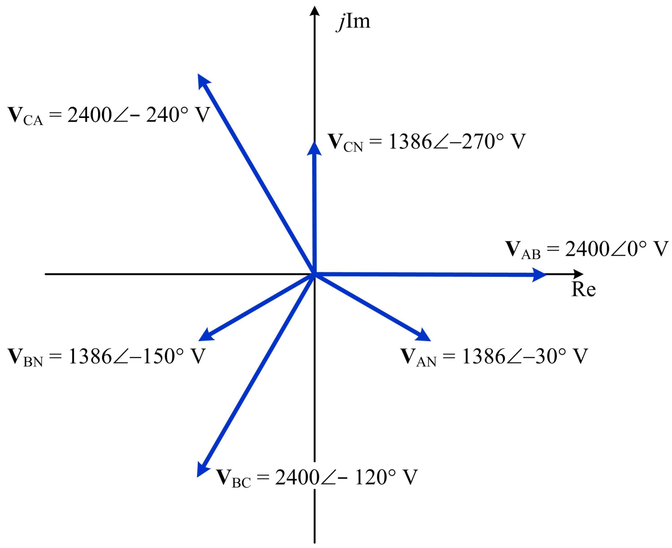
$$\mathbf{V}_{\rm AN} = 1385.6\angle{-30^\circ}\text{ V(rms)} \qquad \mathbf{V}_{\rm AB} = 2400\angle 0^\circ\text{ V(rms)}$$

$$\mathbf{V}_{\rm BN} = 1385.6\angle{-150^\circ}\text{ V(rms)} \qquad \mathbf{V}_{\rm BC} = 2400\angle{-120^\circ}\text{ V(rms)}$$

$$\mathbf{V}_{\rm CN} = 1385.6\angle{-270^\circ}\text{ V(rms)} \qquad \mathbf{V}_{\rm CA} = 2400\angle{-240^\circ}\text{ V(rms)}$$

(b). Sketch a phasor diagram of the line and phase voltages.

The sketch is shown below.



Problem 16–33. In a balanced three-phase circuit $\mathbf{V}_{\rm AB} = 208 + j0$ V (rms). Find all of the line and phase voltage phasors in polar form for a positive phase sequence.

We have $\mathbf{V}_{\rm AB} = 208\angle 0^\circ$ V (rms), which implies $V_{\rm L} = 208$ V (rms) and $V_{\rm P} = V_{\rm L}/\sqrt{3} = 208/\sqrt{3} = 120$ V (rms). We have the following phase and line voltages:

$$\mathbf{V}_{\rm AN} = 120\angle{-30^\circ}\text{ V(rms)} \qquad \mathbf{V}_{\rm AB} = 208\angle 0^\circ\text{ V(rms)}$$

$$\mathbf{V}_{\rm BN} = 120\angle{-150^\circ}\text{ V(rms)} \qquad \mathbf{V}_{\rm BC} = 208\angle{-120^\circ}\text{ V(rms)}$$

$$\mathbf{V}_{\rm CN} = 120\angle{-270^\circ}\text{ V(rms)} \qquad \mathbf{V}_{\rm CA} = 208\angle{-240^\circ}\text{ V(rms)}$$

Problem 16–34. In a balanced three-phase circuit $\mathbf{V}_{\rm BN} = 200 + j150$ V (rms). Find all of the line and phase voltage phasors in polar form for a positive phase sequence.

We have $\mathbf{V}_{\rm BN} = 200 + j150 = 250\angle 36.9^\circ$ V (rms). The phase voltage is $V_{\rm P} = 250$ V (rms) and the line voltage is $V_{\rm L} = \sqrt{3}V_{\rm P} = 433$ V (rms). We have the following phase and line voltages:

$$\mathbf{V}_{\rm AN} = 250\angle 156.9^\circ\text{ V(rms)} \qquad \mathbf{V}_{\rm AB} = 433\angle 186.9^\circ\text{ V(rms)}$$

$$\mathbf{V}_{\rm BN} = 250\angle 36.9^\circ\text{ V(rms)} \qquad \mathbf{V}_{\rm BC} = 433\angle 66.9^\circ\text{ V(rms)}$$

$$\mathbf{V}_{\rm CN} = 250\angle{-83.1^\circ}\text{ V(rms)} \qquad \mathbf{V}_{\rm CA} = 433\angle{-53.1^\circ}\text{ V(rms)}$$

Problem 16–35. A balanced Δ-connected three-phase source has $\mathbf{V}_{AB} = 208\angle 30°$ V (rms) and a positive phase sequence. Find the three source voltages of the equivalent Y-connected source.

The line voltage is $V_L = 208$ V (rms), so the phase voltage is $V_P = V_L/\sqrt{3} = 120$ V (rms). The phase voltages of the Y-connected source are:

$$\mathbf{V}_{AN} = 120\angle 0° \text{ V(rms)}$$

$$\mathbf{V}_{BN} = 120\angle -120° \text{ V(rms)}$$

$$\mathbf{V}_{CN} = 120\angle -240° \text{ V(rms)}$$

Problem 16–36. In a balanced three-phase circuit, $\mathbf{V}_{BN} = V_P\angle 90°$. Show that $\mathbf{V}_{BC} = \sqrt{3}V_P\angle 120°$ for a positive phase sequence.

For a positive phase sequence, $\mathbf{V}_{BC}$ leads $\mathbf{V}_{BN}$ by 30°, so $\mathbf{V}_{BC} = V_L\angle(90° + 30°) = V_L\angle 120°$. The line voltage and phase voltage are related as follows: $V_L = \sqrt{3}V_P$. Therefore, $\mathbf{V}_{BC} = \sqrt{3}V_P\angle 120°$. As an alternate approach, consider the following calculations:

$$\mathbf{V}_{BN} = V_P\angle 90° = jV_P$$

$$\mathbf{V}_{CN} = \mathbf{V}_{BN}e^{-j2\pi/3} = \frac{V_P}{2}(\sqrt{3} - j)$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = jV_P - \frac{V_P}{2}(\sqrt{3} - j) = \frac{V_P}{2}\left(-\sqrt{3} + j3\right)$$

$$= \sqrt{3}V_P\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_P\angle 120°$$

Problem 16–37. A balanced Y-connected load with $Z_Y = 10 - j10\ \Omega$ per phase is connected in parallel with a balanced Δ-connected load with $Z_\Delta = 60 + j30\ \Omega$ per phase. Find the phase impedance of an equivalent Δ-connected load.

Convert the Y-connected load into its equivalent Δ-connected load and then combine the two Δ-connected loads in parallel.

$$Z_{Y1} = 10 - j10\ \Omega$$

$$Z_{\Delta 1} = 3Z_{Y1} = 30 - j30\ \Omega$$

$$Z_{\Delta 2} = 60 + j30\ \Omega$$

$$Z_{\Delta EQ} = Z_{\Delta 1} \parallel Z_{\Delta 2} = (30 - j30) \parallel (60 + j30) = 30 - j10\ \Omega$$

Problem 16–38. A balanced Y-connected load with $Z_Y = 30 + j30\ \Omega$ per phase is connected in parallel with a balanced Δ-connected load with $Z_\Delta = 60 - j900\ \Omega$ per phase. Find the phase impedance of an equivalent Y-connected load.

Convert the Y-connected load into its equivalent Δ-connected load, combine the two Δ-connected loads in parallel, and then determine the equivalent Y-connected load.

$$Z_{Y1} = 30 + j30\ \Omega$$

$$Z_{\Delta 1} = 3Z_{Y1} = 90 + j90\ \Omega$$

$$Z_{\Delta 2} = 60 - j900\ \Omega$$

$$Z_{\Delta EQ} = Z_{\Delta 1} \parallel Z_{\Delta 2} = (90 + j90) \parallel (60 - j900) = 109.34 + j86.42\ \Omega$$

$$Z_{YEQ} = \frac{Z_{\Delta EQ}}{3} = 36.45 + j28.81\ \Omega$$

Problem 16–39. A balanced Y-connected load with $Z_{\mathrm{Y1}} = 12 + j6\ \Omega$ per phase is connected in parallel with a second balanced Y-connected load with $Z_{\mathrm{Y2}} = 24 + j6\ \Omega$ per phase. Find the phase impedance of the equivalent Y-connected load.

Convert both Y-connected loads into their equivalent Δ-connected loads, combine the two Δ-connected loads in parallel, and then determine the equivalent Y-connected load.

$$Z_{\mathrm{Y1}} = 12 + j6\ \Omega$$

$$Z_{\Delta 1} = 3Z_{\mathrm{Y1}} = 36 + j18\ \Omega$$

$$Z_{\mathrm{Y2}} = 24 + j6\ \Omega$$

$$Z_{\Delta 2} = 3Z_{\mathrm{Y2}} = 72 + j18\ \Omega$$

$$Z_{\Delta\mathrm{EQ}} = Z_{\Delta 1} \parallel Z_{\Delta 2} = (36 + j18) \parallel (72 + j18) = 24.3 + j9.9\ \Omega$$

$$Z_{\mathrm{YEQ}} = \frac{Z_{\Delta\mathrm{EQ}}}{3} = 8.1 + j3.3\ \Omega$$

Problem 16–40. In a balanced Δ-Δ circuit, the Δ-connected source produces $\mathbf{V}_{\mathrm{AB}} = 2400\angle 45^\circ$ V (rms) and a positive phase sequence. The phase impedance of the load is $Z_\Delta = 200\angle 45^\circ\ \Omega$ per phase. Find the three source voltages and the phase impedance in the equivalent Y-Y circuit.

The given line voltage is $V_{\mathrm{L}} = 2400$ V (rms), so the corresponding phase voltage is $V_{\mathrm{P}} = V_{\mathrm{L}}/\sqrt{3} = 2400/\sqrt{3} = 1386$ V (rms). The phase voltages of the Y-connected source are:

$$\mathbf{V}_{\mathrm{AN}} = 1386\angle 15^\circ\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{BN}} = 1386\angle -105^\circ\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{CN}} = 1386\angle -225^\circ\ \mathrm{V(rms)}$$

Compute the equivalent Y-connected load.

$$Z_{\mathrm{Y}} = \frac{Z_\Delta}{3} = \frac{200\angle 45^\circ}{3} = 66.7\angle 45^\circ = 47.1405(1 + j)\ \Omega$$

Problem 16–41. In a balanced Δ-Y circuit, the line voltage and phase impedance are $V_{\mathrm{L}} = 560$ V (rms) and $Z_{\mathrm{Y}} = 25 + j10\ \Omega$ per phase. Using $\angle\mathbf{V}_{\mathrm{AB}} = 0^\circ$ as the phase reference, find the line current and phase voltage phasors in polar form for a positive phase sequence.

Compute the phase voltage.

$$V_{\mathrm{P}} = \frac{V_{\mathrm{L}}}{\sqrt{3}} = \frac{560}{\sqrt{3}} = 323.3\ \mathrm{V(rms)}$$

The phase voltages are:

$$\mathbf{V}_{\mathrm{AN}} = 323.3\angle -30^\circ\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{BN}} = 323.3\angle -150^\circ\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{CN}} = 323.3\angle -270^\circ\ \mathrm{V(rms)}$$

Calculate the line currents.

$$\mathbf{I}_{\mathrm{A}} = \frac{\mathbf{V}_{\mathrm{AN}}}{Z_{\mathrm{Y}}} = \frac{323.3\angle -30^\circ}{25 + j10} = 12\angle -51.8^\circ\ \mathrm{A(rms)}$$

$$\mathbf{I}_{\mathrm{B}} = \frac{\mathbf{V}_{\mathrm{BN}}}{Z_{\mathrm{Y}}} = \frac{323.3\angle -150^\circ}{25 + j10} = 12\angle -171.8^\circ\ \mathrm{A(rms)}$$

$$\mathbf{I}_{\mathrm{C}} = \frac{\mathbf{V}_{\mathrm{CN}}}{Z_{\mathrm{Y}}} = \frac{323.3\angle -270^\circ}{25 + j10} = 12\angle -291.8^\circ\ \mathrm{A(rms)}$$

Problem 16–42. In a balanced Y-Y circuit, the line voltage is $V_L = 480$ V (rms). The phase impedance is $Z_Y = 40 + j25\ \Omega$ per phase. Using $\angle \mathbf{I}_A = 0°$ as the phase reference, find $\mathbf{I}_A$ and $\mathbf{V}_{AB}$ in polar form for a positive phase sequence.

We have the following calculations and results:

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{480}{\sqrt{3}} = 277.1281 \text{ V(rms)}$$

$$I_L = \frac{V_P}{|Z_Y|} = \frac{277.1281}{|40 + j25|} = 5.8751 \text{ A(rms)}$$

$$\mathbf{I}_A = I_L\angle 0° = 5.8751\angle 0° \text{ A(rms)}$$

$$\mathbf{V}_{AN} = Z_Y\mathbf{I}_A = (40 + j25)(5.8751\angle 0°) = 277.1281\angle 32° \text{ V(rms)}$$

$$\mathbf{V}_{AB} = \sqrt{3}\mathbf{V}_{AN}\left(e^{j\pi/6}\right) = 480\angle 62° \text{ V(rms)}$$

Problem 16–43. In a balanced Y-Δ circuit, the line voltage and phase impedance are $V_L = 440$ V (rms) and $Z_\Delta = 16 + j12\ \Omega$ per phase. Using $\angle \mathbf{V}_{AN} = 0°$ as the phase reference, find the line current and phase current phasors in polar form for a positive phase sequence.

We have the following calculations and results:

$$Z_Y = \frac{Z_\Delta}{3} = \frac{16 + j12}{3} = 5.3333 + j4\ \Omega$$

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.0341 \text{ V(rms)}$$

$$\mathbf{V}_{AN} = V_P\angle 0° = 254.0341\angle 0° \text{ V(rms)}$$

$$\mathbf{I}_A = \frac{\mathbf{V}_{AN}}{Z_Y} = \frac{254.0341\angle 0°}{5.3333 + j4} = 38.11\angle -36.87° \text{ A(rms)}$$

$$I_P = \frac{I_L}{\sqrt{3}} = \frac{38.11}{\sqrt{3}} = 22.00 \text{ A(rms)}$$

The line and phase currents are:

$$\mathbf{I}_A = 38.11\angle -36.87° \text{ A(rms)} \qquad \mathbf{I}_{AB} = 22\angle -6.87° \text{ A(rms)}$$

$$\mathbf{I}_B = 38.11\angle -156.87° \text{ A(rms)} \qquad \mathbf{I}_{BC} = 22\angle -126.87° \text{ A(rms)}$$

$$\mathbf{I}_C = 38.11\angle -276.87° \text{ A(rms)} \qquad \mathbf{I}_{CA} = 22\angle -246.87° \text{ A(rms)}$$

Problem 16–44. In a balanced Y-Δ circuit, the line impedances connecting the source and load are $Z_W = 1.5 + j6.5\ \Omega$ per phase. The phase impedances of the Δ load are $Z_\Delta = 15 + j8\ \Omega$ per phase and the line voltage at the source is $V_L = 250$ V (rms). Using $\angle \mathbf{V}_{AB} = 0°$ as the phase reference, find the line current $\mathbf{I}_A$ and phase current $\mathbf{I}_{AB}$ for a positive phase sequence.

The line voltage $\mathbf{V}_{AB}$ provides the phase reference.

$$\mathbf{V}_{AB} = V_L\angle 0° = 250\angle 0° \text{ V(rms)}$$

Calculate the phase voltage $\mathbf{V}_{AN}$.

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{250}{\sqrt{3}} = 144.3376 \text{ V(rms)}$$

$$\mathbf{V}_{AN} = V_P\angle -30° = 144.3376\angle -30° \text{ V(rms)}$$

Convert the load to an equivalent Y-connected impedance.

$$Z_{\mathrm{Y}} = \frac{Z_{\Delta}}{3} = \frac{15 + j8}{3} = 5 + j2.6667\ \Omega$$

Account for the line impedance.

$$Z_{\mathrm{EQ}} = Z_{\mathrm{W}} + Z_{\mathrm{Y}} = 6.5 + j9.1667\ \Omega$$

Find the line currrent.

$$\mathbf{I}_{\mathrm{A}} = \frac{\mathbf{V}_{\mathrm{AN}}}{Z_{\mathrm{EQ}}} = \frac{144.3376\angle{-30^\circ}}{6.5 + j9.1667} = 12.8445\angle{-84.66^\circ}\ \mathrm{A(rms)}$$

Find the phase current.

$$\mathbf{I}_{\mathrm{AB}} = \frac{\mathbf{I}_{\mathrm{A}}}{\sqrt{3}}\left(e^{j\pi/6}\right) = 7.4158\angle{-54.66^\circ}\ \mathrm{A(rms)}$$

Problem 16–45. In a balanced Y-Δ circuit, the line voltage and phase impedance are $V_{\mathrm{L}} = 4.16$ kV (rms) and $Z_{\Delta} = 250\angle 30^\circ\ \Omega$ per phase. Using $\angle\mathbf{V}_{\mathrm{AB}} = 0^\circ$ as the phase reference, find the phase current $\mathbf{I}_{\mathrm{AB}}$ and line current $\mathbf{I}_{\mathrm{A}}$ for a positive phase sequence.

The line voltage $\mathbf{V}_{\mathrm{AB}}$ provides the phase reference.

$$\mathbf{V}_{\mathrm{AB}} = V_{\mathrm{L}}\angle 0^\circ = 4160\angle 0^\circ\ \mathrm{V(rms)}$$

Calculate the phase voltage $\mathbf{V}_{\mathrm{AN}}$.

$$V_{\mathrm{P}} = \frac{V_{\mathrm{L}}}{\sqrt{3}} = \frac{4160}{\sqrt{3}} = 2402\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{AN}} = V_{\mathrm{P}}\angle{-30^\circ} = 2402\angle{-30^\circ}\ \mathrm{V(rms)}$$

Convert the load to an equivalent Y-connected impedance.

$$Z_{\mathrm{Y}} = \frac{Z_{\Delta}}{3} = \frac{250\angle 30^\circ}{3} = 83.33\angle 30^\circ\ \Omega$$

Find the line currrent.

$$\mathbf{I}_{\mathrm{A}} = \frac{\mathbf{V}_{\mathrm{AN}}}{Z_{\mathrm{Y}}} = \frac{2402\angle{-30^\circ}}{83.33\angle 30^\circ} = 28.82\angle{-60^\circ}\ \mathrm{A(rms)}$$

Find the phase current.

$$\mathbf{I}_{\mathrm{AB}} = \frac{\mathbf{I}_{\mathrm{A}}}{\sqrt{3}}\left(e^{j\pi/6}\right) = 16.64\angle{-30^\circ}\ \mathrm{A(rms)}$$

Problem 16–46. A balanced three-phase source with $\mathbf{V}_{\mathrm{AB}} = 440\angle 30^\circ$ V (rms) supplies a balanced Δ-connected load with a phase impedance of $Z_{\Delta} = 20\angle{-60^\circ}\ \Omega$ per phase. Find the phase current $\mathbf{I}_{\mathrm{AB}}$ and line current $\mathbf{I}_{\mathrm{A}}$ for a positive phase sequence.

Find the phase voltage $\mathbf{V}_{\mathrm{AN}}$.

$$\mathbf{V}_{\mathrm{AN}} = \frac{\mathbf{V}_{\mathrm{AB}}}{\sqrt{3}}\left(e^{-j\pi/6}\right) = \frac{440\angle 30^\circ}{\sqrt{3}}\left(e^{-j\pi/6}\right) = 254.03\angle 0^\circ\ \mathrm{V(rms)}$$

Convert the load to an equivalent Y-connected impedance.

$$Z_{\mathrm{Y}} = \frac{Z_{\Delta}}{3} = \frac{20\angle{-60^\circ}}{3} = 6.6667\angle{-60^\circ}\ \Omega$$

Find the line currrent.

$$\mathbf{I}_{\mathrm{A}} = \frac{\mathbf{V}_{\mathrm{AN}}}{Z_{\mathrm{Y}}} = \frac{254.03\angle 0^\circ}{6.6667\angle -60^\circ} = 38.11\angle 60^\circ \text{ A(rms)}$$

Find the phase current.

$$\mathbf{I}_{\mathrm{AB}} = \frac{\mathbf{I}_{\mathrm{A}}}{\sqrt{3}} \left(e^{j\pi/6}\right) = 22\angle 90^\circ \text{ A(rms)}$$

Problem 16–47. In a balanced Y-connected load, the line current and phase impedance are $I_{\mathrm{L}} = 6$ A (rms) and $Z_{\mathrm{Y}} = 20 + j15\ \Omega$ per phase. Using $\angle \mathbf{V}_{\mathrm{AB}} = 0^\circ$ as the phase reference, find the line current $\mathbf{I}_{\mathrm{A}}$ and phase voltage $\mathbf{V}_{\mathrm{AN}}$ for a positive phase sequence.

Compute the phase voltage.

$$V_{\mathrm{P}} = |Z_{\mathrm{Y}}| I_{\mathrm{L}} = |20 + j15|(6) = 150 \text{ V(rms)}$$

Find the line voltage.

$$V_{\mathrm{L}} = \sqrt{3} V_{\mathrm{P}} = (\sqrt{3})(150) = 259.8 \text{ V(rms)}$$

$$\mathbf{V}_{\mathrm{AB}} = V_{\mathrm{L}}\angle 0^\circ = 259.8\angle 0^\circ \text{ V(rms)}$$

Calculate the phase voltage.

$$\mathbf{V}_{\mathrm{AN}} = \frac{\mathbf{V}_{\mathrm{AB}}}{\sqrt{3}} \left(e^{-j\pi/6}\right) = 150\angle -30^\circ \text{ V(rms)}$$

Find the line current.

$$\mathbf{I}_{\mathrm{A}} = \frac{\mathbf{V}_{\mathrm{AN}}}{Z_{\mathrm{Y}}} = \frac{150\angle -30^\circ}{20 + j15} = 6\angle -66.87^\circ \text{ A(rms)}$$

Problem 16–48. In a balanced Δ-connected load, the phase current and phase impedance are $I_{\mathrm{P}} = 12$ A (rms) and $Z_{\Delta} = 200\angle 60^\circ\ \Omega$ per phase. Using $\angle \mathbf{V}_{\mathrm{AN}} = 0^\circ$ as the phase reference, find the line current $\mathbf{I}_{\mathrm{A}}$ and line voltage $\mathbf{V}_{\mathrm{AB}}$ for a positive phase sequence.

Convert the load to an equivalent Y-connected impedance.

$$Z_{\mathrm{Y}} = \frac{Z_{\Delta}}{3} = \frac{200\angle 60^\circ}{3} = 66.6667\angle 60^\circ\ \Omega$$

Find the line current and the phase voltage.

$$I_{\mathrm{L}} = \sqrt{3} I_{\mathrm{P}} = 20.78 \text{ A(rms)}$$

$$V_{\mathrm{P}} = |Z_{\mathrm{Y}}| I_{\mathrm{L}} = 1385.6 \text{ V(rms)}$$

$$\mathbf{V}_{\mathrm{AN}} = V_{\mathrm{P}}\angle 0^\circ = 1385.6\angle 0^\circ \text{ V(rms)}$$

Calculate the line current and line voltage.

$$\mathbf{I}_{\mathrm{A}} = \frac{\mathbf{V}_{\mathrm{AN}}}{Z_{\mathrm{Y}}} = \frac{1385.6\angle 0^\circ}{66.6667\angle 60^\circ} = 20.78\angle -60^\circ \text{ A(rms)}$$

$$\mathbf{V}_{\mathrm{AB}} = \sqrt{3}\mathbf{V}_{\mathrm{AN}} \left(e^{j\pi/6}\right) = \sqrt{3}\,(1385.6\angle 0^\circ) \left(e^{j\pi/6}\right) = 2400\angle 30^\circ \text{ V(rms)}$$

Problem 16–49. An average power of 6 kW is delivered to a balanced three-phase load with a phase impedance of $Z_{\mathrm{Y}} = 40 + j30\ \Omega$ per phase. Find V_{L} and the complex power delivered to the load.

Find the magnitude of the line current.

$$P_{\mathrm{L}} = 3I_{\mathrm{L}}^2 R_{\mathrm{L}}$$

$$I_{\mathrm{L}} = \sqrt{\frac{P_{\mathrm{L}}}{3R_{\mathrm{L}}}} = \sqrt{\frac{6000}{(3)(40)}} = 7.071\ \mathrm{A(rms)}$$

Calculate the complex power delivered to the load.

$$S_{\mathrm{L}} = 3Z_{\mathrm{Y}}I_{\mathrm{L}}^2 = (3)(40+j30)(7.071)^2 = 6000 + j4500\ \mathrm{VA}$$

Find the line voltage.

$$V_{\mathrm{L}} = \frac{|S_{\mathrm{L}}|}{\sqrt{3}I_{\mathrm{L}}} = \frac{|6000+j4500|}{\sqrt{3}(7.071)} = 612.37\ \mathrm{V(rms)}$$

Problem 16–50. An apparent power of 12 kVA is delivered to a balanced three-phase load with a phase impedance of $Z_{\mathrm{Y}} = 120 + j90\ \Omega$ per phase. Find I_{L}, V_{L}, and the complex power delivered to the load.

Find the magnitude of the line current.

$$|S_{\mathrm{L}}| = 3|Z_{\mathrm{Y}}|I_{\mathrm{L}}^2$$

$$I_{\mathrm{L}} = \sqrt{\frac{|S_{\mathrm{L}}|}{3|Z_{\mathrm{Y}}|}} = \sqrt{\frac{12000}{(3)(|120+j90|)}} = 5.164\ \mathrm{A(rms)}$$

Find the magnitude of the line voltage.

$$|S_{\mathrm{L}}| = \sqrt{3}V_{\mathrm{L}}I_{\mathrm{L}}$$

$$V_{\mathrm{L}} = \frac{|S_{\mathrm{L}}|}{\sqrt{3}I_{\mathrm{L}}} = \frac{12000}{\sqrt{3}(5.164)} = 1.342\ \mathrm{kV(rms)}$$

Calculate the complex power delivered to the load.

$$S_{\mathrm{L}} = 3Z_{\mathrm{Y}}I_{\mathrm{L}}^2 = (3)(120+j90)(5.164)^2 = 9600 + j7200\ \mathrm{VA}$$

Problem 16–51. A balanced three-phase load has a phase impedance of $Z_{\mathrm{Y}} = 60 - j20\ \Omega$ per phase. The line voltage at the load is $V_{\mathrm{L}} = 440$ V (rms). Find I_{L} and the complex power delivered to the load.

Find the magnitude of the line current.

$$I_{\mathrm{L}} = \frac{V_{\mathrm{L}}}{\sqrt{3}|Z_{\mathrm{Y}}|} = \frac{440}{\sqrt{3}(|60-j20|)} = 4.0166\ \mathrm{A(rms)}$$

Compute the complex power delivered to the load.

$$S_{\mathrm{L}} = 3Z_{\mathrm{Y}}I_{\mathrm{L}}^2 = (3)(60-j20)(4.0166)^2 = 2904 - j968\ \mathrm{VA}$$

Problem 16–52. A balanced three-phase load has a phase impedance of $Z_{\Delta} = 200 + j100\ \Omega$ per phase. The line voltage at the load is $V_{\mathrm{L}} = 2.4$ kV (rms). Find I_{L} and the complex power delivered to the load.

Find the magnitude of the line current.

$$I_{\mathrm{P}} = \frac{V_{\mathrm{L}}}{|Z_{\Delta}|} = \frac{2400}{|200+j100|} = 10.7331\ \mathrm{A(rms)}$$

$$I_{\mathrm{L}} = \sqrt{3}I_{\mathrm{P}} = \sqrt{3}(10.7331) = 18.5903\ \mathrm{A(rms)}$$

Compute the complex power delivered to the load.

$$S_{\mathrm{L}} = Z_{\Delta}I_{\mathrm{L}}^2 = (200+j100)(18.5903)^2 = 69120 + j34560\ \mathrm{VA}$$

Problem 16–53. A balanced three-phase load has a phase impedance of $Z_Y = 200 + j100\ \Omega$ per phase. The line current at the load is $I_L = 16$ A (rms). Find V_L and the complex power delivered to the load.

We have the following calculations and results:

$$V_P = |Z_Y|I_L = |200 + j100|(16) = 3577.7 \text{ V(rms)}$$

$$V_L = \sqrt{3}V_P = 6196.8 \text{ V(rms)}$$

$$S_L = 3Z_Y I_L^2 = (3)(200 + j100)(16)^2 = 153600 + j76800 \text{ VA}$$

Problem 16–54. The balanced three-phase source in Figure P16–54 produces an average power of 50 kW. The line impedance is $Z_W = 1 + j6\ \Omega$ per phase and the balanced load has a phase impedance of $Z_Y = 200 + j100\ \Omega$ per phase. Find the line voltage V_L at the load and the complex power delivered to the load.

Sum the line and load impedances to get an equivalent impedance seen by the source.

$$Z_{EQ} = Z_W + Z_Y = 201 + j106\ \Omega$$

Calculate the line current.

$$P_S = 50000 = 3I_L^2[\text{Re}(Z_{EQ})]$$

$$I_L = \sqrt{\frac{50000}{(3)[\text{Re}(Z_{EQ})]}} = \sqrt{\frac{50000}{(3)(201)}} = 9.106 \text{ A(rms)}$$

Find the complex power delivered to the load.

$$S_L = 3Z_Y I_L^2 = (3)(200 + j100)(9.106)^2 = 49751 + j24876 \text{ VA}$$

Find the line voltage at the load.

$$V_P = |Z_Y|I_L = |200 + j100|(9.106) = 2036 \text{ V(rms)}$$

$$V_L = \sqrt{3}V_P = 3527 \text{ V(rms)}$$

Problem 16–55. The balanced three-phase source in Figure P16–54 produces an average power of 15 kW at a power factor of 0.85 lagging. The line impedance is $Z_W = 3 + j15\ \Omega$ per phase and the balanced load has a phase impedance of $Z_\Delta = 250 + j55\ \Omega$ per phase. Find the line voltage V_L at the load and the complex power delivered to the load.

Find the equivalent Y-connect load.

$$Z_Y = \frac{Z_\Delta}{3} = 83.33 + j18.33\ \Omega$$

Find the equivalent impedance and resistance.

$$Z_{EQ} = Z_W + Z_Y = 86.33 + j33.33\ \Omega$$

$$R_{EQ} = \text{Re}(Z_{EQ}) = 86.33\ \Omega$$

Calculate the line current.

$$I_L = \sqrt{\frac{P_S}{3R_{EQ}}} = \sqrt{\frac{15000}{(3)(86.33)}} = 7.6102 \text{ A(rms)}$$

Find the complex power delivered to the load.

$$S_L = 3Z_Y I_L^2 = (3)(83.33 + j18.33)(7.6102)^2 = 14480 + j3185 \text{ VA}$$

Find the line voltage at the load.

$$V_L = \frac{|S_L|}{\sqrt{3}I_L} = \frac{|14480 + j3185|}{\sqrt{3}(7.6102)} = 1124.7 \text{ V(rms)}$$

Problem 16–56. The balanced three-phase source in Figure P16–54 produces an apparent power of 25 kVA at a power factor of 0.9 lagging. The line impedance is $Z_{\rm W} = 2 + j10\ \Omega$ per phase and the line current is 10 A (rms). Find the line voltage $V_{\rm L}$ at the load and complex power delivered to the load.

Determine the complex power provided by the source.

$$S_{\rm S} = |S_{\rm S}|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = (24000)\left(0.9 + j\sqrt{1-0.9^2}\right) = 22500 + j10897\ \text{VA}$$

Compute the complex power dissipated by the wires.

$$S_{\rm W} = 3Z_{\rm W}I_{\rm L}^2 = (3)(2+j10)(10)^2 = 600 + j3000\ \Omega$$

Find the complex power delivered to the load.

$$S_{\rm L} = S_{\rm S} - S_{\rm W} = 21900 + j7897\ \text{VA}$$

Find the line voltage.

$$V_{\rm L} = \frac{|S_{\rm L}|}{\sqrt{3}I_{\rm L}} = \frac{|21900 + j7897|}{\sqrt{3}(10)} = 1344.1\ \text{V(rms)}$$

Problem 16–57. The balanced three-phase source in Figure P16–54 produces an average power of 50 kW at a power factor of 0.75 lagging. The line impedance is $Z_{\rm W} = 2 + j12\ \Omega$ per phase and the line voltage at the source is 4160 V (rms). Find the line voltage $V_{\rm L}$ at the load and complex power delivered to the load.

Find the apparent power at the source and the line current.

$$|S_{\rm S}| = \frac{P_{\rm S}}{\text{pf}} = \frac{50000}{0.75} = 66667\ \text{VA}$$

$$I_{\rm L} = \frac{|S_{\rm S}|}{\sqrt{3}V_{\rm S}} = \frac{66667}{\sqrt{3}(4160)} = 9.2524\ \text{A(rms)}$$

Determine the complex power provided by the source.

$$S_{\rm S} = |S_{\rm S}|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = (66667)\left(0.75 + j\sqrt{1-0.75^2}\right) = 50000 + j44096\ \text{VA}$$

Compute the complex power dissipated by the wires.

$$S_{\rm W} = 3Z_{\rm W}I_{\rm L}^2 = (3)(2+j12)(9.2524)^2 = 514 + j3082\ \Omega$$

Find the complex power delivered to the load.

$$S_{\rm L} = S_{\rm S} - S_{\rm W} = 49486 + j41014\ \text{VA}$$

Find the line voltage.

$$V_{\rm L} = \frac{|S_{\rm L}|}{\sqrt{3}I_{\rm L}} = \frac{|49486 + j41014|}{\sqrt{3}(9.2524)} = 4011\ \text{V(rms)}$$

Problem 16–58. Two balanced three-phase loads are connected in parallel. The first load absorbs 25 kW at a lagging power factor of 0.9. The second load absorbs an apparent power of 30 kVA at a leading power factor of 0.1. The line voltage at the parallel loads is $V_{\rm L} = 880$ V (rms). Find the line current into the combined load.

Find the complex power at each load.

$$|S_{\rm L1}| = \frac{P_{\rm L1}}{\text{pf}_1} = \frac{25000}{0.9} = 27778\ \text{VA}$$

$$S_{\rm L1} = |S_{\rm L1}|\left(\text{pf}_1 + j\sqrt{1-\text{pf}_1^2}\right) = (27778)\left(0.9 + j\sqrt{1-0.9^2}\right) = 25000 + j12108\ \text{VA}$$

$$S_{\rm L2} = |S_{\rm L2}|\left(\text{pf}_2 - j\sqrt{1-\text{pf}_2^2}\right) = (30000)\left(0.1 - j\sqrt{1-0.1^2}\right) = 3000 - j29850\ \text{VA}$$

Find the total complex power delivered to the loads.

$$S_{\mathrm{L}} = S_{\mathrm{L1}} + S_{\mathrm{L2}} = 28000 - j17742 \text{ VA}$$

Calculate the line current into the combined load.

$$I_{\mathrm{L}} = \frac{|S_{\mathrm{L}}|}{\sqrt{3}V_{\mathrm{L}}} = \frac{|28000 - j17742|}{\sqrt{3}(880)} = 21.75 \text{ A(rms)}$$

Problem 16–59. The average power delivered to a balanced Y-connected load is 20 kW at a lagging power factor of 0.8. The line voltage at the load is $V_{\mathrm{L}} = 480$ V (rms). Find the phase impedance Z_{Y} of the load.

Find the complex power at the load.

$$|S_{\mathrm{L}}| = \frac{P_{\mathrm{L}}}{\mathrm{pf}} = \frac{20000}{0.8} = 25000 \text{ VA}$$

$$S_{\mathrm{L}} = |S_{\mathrm{S}}|\left(\mathrm{pf} + j\sqrt{1-\mathrm{pf}^2}\right) = (25000)\left(0.8 + j\sqrt{1-0.8^2}\right) = 20000 + j15000 \text{ VA}$$

Calculate the line current.

$$I_{\mathrm{L}} = \frac{|S_{\mathrm{L}}|}{\sqrt{3}V_{\mathrm{L}}} = \frac{25000}{\sqrt{3}(480)} = 30.07 \text{ V(rms)}$$

Find the phase impedance.

$$Z_{\mathrm{Y}} = \frac{S_{\mathrm{L}}}{3I_{\mathrm{L}}^2} = \frac{20000 + j15000}{(3)(30.07)^2} = 7.3728 + j5.5296\ \Omega$$

Problem 16–60. The apparent power delivered to a balanced Δ-connected load is 30 kVA at a lagging power factor of 0.72. The line voltage at the load is $V_{\mathrm{L}} = 2.4$ kV (rms). Find the phase impedance Z_{Δ} of the load.

Find the line current.

$$I_{\mathrm{L}} = \frac{|S_{\mathrm{L}}|}{\sqrt{3}V_{\mathrm{L}}} = \frac{30000}{\sqrt{3}(2400)} = 7.2169 \text{ A(rms)}$$

Calculate the complex power delivered to the load.

$$S_{\mathrm{L}} = |S_{\mathrm{L}}|\left(\mathrm{pf} + j\sqrt{1-\mathrm{pf}^2}\right) = (30000)\left(0.72 + j\sqrt{1-0.72^2}\right) = 21600 + j20819 \text{ VA}$$

Find the load impedances.

$$Z_{\mathrm{Y}} = \frac{S_{\mathrm{L}}}{3I_{\mathrm{L}}^2} = \frac{21600 + j20819}{(3)(7.2169)^2} = 138.24 + j133.24\ \Omega$$

$$Z_{\Delta} = 3Z_{\mathrm{Y}} = 414.72 + j399.73\ \Omega$$

Problem 16–61. In Figure P16–61, the source and load busses are interconnected by a transmission line with $Z_{\mathrm{W}} = 70 + j400\ \Omega$ per phase. The load at bus 2 draws an apparent power of $|S_2| = 3$ MVA at a lagging power factor of 0.85 and the line voltage at bus 2 is $V_{\mathrm{L2}} = 230$ kV (rms). Find the apparent power produced by the source at bus 1, the source power factor, and the line voltage at bus 1.

Find the complex power at bus 2.

$$S_2 = |S_2|\left(\mathrm{pf} + j\sqrt{1-\mathrm{pf}^2}\right) = (3)\left(0.85 + j\sqrt{1-0.85^2}\right) = 2.55 + j1.5803 \text{ MVA}$$

Find the line current.

$$I_{\mathrm{L}} = \frac{|S_2|}{\sqrt{3}V_{\mathrm{L2}}} = \frac{3}{\sqrt{3}(0.230)} = 7.5307 \text{ A(rms)}$$

Find the complex power dissipated by the transmission line.

$$S_{\mathrm{W}} = 3Z_{\mathrm{W}}I_{\mathrm{L}}^2 = (3)(70 + j400)(7.5307)^2 = 11909 + j68053 \text{ VA}$$

Find the complex power at bus 1, the apparent power at bus 1, the source power factor, and the line voltage at bus 1.

$$S_1 = S_{\mathrm{W}} + S_2 = 2.5619 + j1.6484 \text{ MVA}$$

$$|S_1| = 3.0464 \text{ MVA}$$

$$\text{pf} = \frac{P_1}{|S_1|} = \frac{2.5619}{3.0464} = 0.841$$

$$V_{\mathrm{L1}} = \frac{|S_1|}{\sqrt{3}I_{\mathrm{L}}} = \frac{3.0464 \times 10^6}{\sqrt{3}(7.5307)} = 233558 \text{ V(rms)}$$

Problem 16–62. In Figure P16–61, the source and load busses are interconnected by a transmission line with $Z_{\mathrm{W}} = 2 + j10\ \Omega$ per phase. The load at bus 2 draws an average power of $P_2 = 50$ kW at a lagging power factor of 0.8 and the line voltage at bus 2 is $V_{\mathrm{L2}} = 4.16$ kV (rms). Find the apparent power produced by the source at bus 1, the source power factor, and the line voltage at bus 1.

Find the complex power at bus 2.

$$|S_2| = \frac{P_2}{\text{pf}} = \frac{50000}{0.8} = 62500 \text{ VA}$$

$$S_2 = |S_2|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = (62500)\left(0.8 + j\sqrt{1-0.8^2}\right) = 50000 + j37500 \text{ VA}$$

Find the line current.

$$I_{\mathrm{L}} = \frac{|S_2|}{\sqrt{3}V_{\mathrm{L2}}} = \frac{62500}{\sqrt{3}(4160)} = 8.6741 \text{ A(rms)}$$

Find the complex power dissipated by the transmission line.

$$S_{\mathrm{W}} = 3Z_{\mathrm{W}}I_{\mathrm{L}}^2 = (3)(2 + j10)(8.6741)^2 = 451 + j2257 \text{ VA}$$

Find the complex power at bus 1, the apparent power at bus 1, the source power factor, and the line voltage at bus 1.

$$S_1 = S_{\mathrm{W}} + S_2 = 50451 + j39757 \text{ VA}$$

$$|S_1| = 64234 \text{ VA}$$

$$\text{pf} = \frac{P_1}{|S_1|} = \frac{50451}{64234} = 0.785$$

$$V_{\mathrm{L1}} = \frac{|S_1|}{\sqrt{3}I_{\mathrm{L}}} = \frac{64234}{\sqrt{3}(8.6741)} = 4275 \text{ V(rms)}$$

Problem 16–63. In Figure P16–61, the source and load busses are interconnected by a transmission line with $Z_{\mathrm{W}} = 10 + j75\ \Omega$ per phase. The load at bus 2 draws an average power of $P_2 = 600$ kW at a lagging power factor of 0.8 and the line current 2 is $I_{L1} = 14$ A (rms). Find the source power factor and the line voltages at bus 1 and bus 2.

Find the complex power at bus 2.

$$|S_2| = \frac{P_2}{\text{pf}} = \frac{600000}{0.8} = 750000 \text{ VA}$$

$$S_2 = |S_2|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = (750000)\left(0.8 + j\sqrt{1-0.8^2}\right) = 600000 + j450000 \text{ VA}$$

Find the line voltage at bus 2.

$$V_{L2} = \frac{|S_2|}{\sqrt{3}I_L} = \frac{750000}{\sqrt{3}(14)} = 30930 \text{ V(rms)}$$

Find the complex power dissipated by the transmission line.

$$S_W = 3Z_W I_L^2 = (3)(10 + j75)(14)^2 = 5880 + j44100 \text{ VA}$$

Find the complex power at bus 1, the apparent power at bus 1, the source power factor, and the line voltage at bus 1.

$$S_1 = S_W + S_2 = 605880 + j494100 \text{ VA}$$

$$|S_1| = 781809 \text{ VA}$$

$$\text{pf} = \frac{P_1}{|S_1|} = \frac{605880}{781809} = 0.775$$

$$V_{L1} = \frac{|S_1|}{\sqrt{3}I_L} = \frac{781809}{\sqrt{3}(14)} = 32241 \text{ V(rms)}$$

Problem 16–64. In Figure P16–64, the three buses are interconnected by transmission lines with wire impedances of $Z_{W1} = 100 + j600\ \Omega$ per phase and $Z_{W2} = 120 + j800\ \Omega$ per phase. The source at bus 2 produces an apparent power of $|S_2| = 300$ kVA at a lagging power factor of 0.85. The load at bus 3 draws an apparent power of $|S_3| = 600$ kVA at a lagging power factor of 0.8. The line voltage at bus 3 is $V_{L3} = 161$ kV (rms). Find the apparent power produced by the source at bus 1, the source power factor, and the line voltages at bus 1 and bus 2.

Find the complex power at bus 3.

$$S_3 = |S_3|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = (600000)\left(0.8 + j\sqrt{1-0.8^2}\right) = 480000 + j360000 \text{ VA}$$

Find the current in line 2.

$$I_{L2} = \frac{|S_3|}{\sqrt{3}V_{L3}} = \frac{600000}{\sqrt{3}(161000)} = 2.1516 \text{ A(rms)}$$

Find the complex power in line 2.

$$S_{W2} = 3Z_{W2}I_{L2}^2 = (3)(120 + j800)(2.1516)^2 = 1667 + j11111 \text{ VA}$$

The line voltage at bus 2 supplies power to S_{W2} and S_3.

$$V_{L2} = \frac{|S_{W2} + S_3|}{\sqrt{3}I_{L2}} = 163160 \text{ V(rms)}$$

Find the complex power provided by the source at bus 2 and the complex power flowing into bus 2.

$$S_2 = |S_2|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = (300000)\left(0.85 + j\sqrt{1-0.85^2}\right) = 255000 + j158035 \text{ VA}$$

$$S_{2\text{IN}} = S_{\text{W}2} + S_3 - S_2 = 226667 + j213076 \text{ VA}$$

Find the current in line 1.

$$I_{\text{L}1} = \frac{|S_{2\text{IN}}|}{\sqrt{3}V_{\text{L}2}} = \frac{|226667 + j213076|}{\sqrt{3}(163160)} = 1.101 \text{ A(rms)}$$

Find the complex power in line 1.

$$S_{\text{W}1} = 3Z_{\text{W}1}I_{\text{L}1}^2 = (3)(100 + j600)(1.101)^2 = 364 + j2181 \text{ VA}$$

Find the complex power and apparent power at bus 1.

$$S_1 = S_{\text{W}1} + S_{2\text{IN}} = 227030 + j215257 \text{ VA}$$

$$|S_1| = 312855 \text{ VA}$$

Find the line voltage and power factor at bus 1.

$$V_{\text{L}1} = \frac{|S_1|}{\sqrt{3}I_{\text{L}1}} = \frac{312855}{\sqrt{3}(1.101)} = 164084 \text{ V(rms)}$$

$$\text{pf} = \frac{P_1}{|S_1|} = \frac{227030}{312855} = 0.726$$

Problem 16–65. In Figure P16–64, the three buses are interconnected by transmission lines with wire impedances of $Z_{\text{W}1} = 120 + j800\ \Omega$ per phase and $Z_{\text{W}2} = 200 + j1200\ \Omega$ per phase. The source at bus 2 produces an apparent power of $|S_2| = 400$ kVA at a leading power factor of 0.9. The load at bus 3 draws an apparent power of $|S_3| = 650$ kVA at a lagging power factor of 0.95. The line current in line 2 is $I_{\text{L}2} = 3.25$ A (rms). Find the apparent power produced by the source at bus 1, the source power factor, and the line voltages at bus 1, bus 2, and bus 3.

Find the complex power at bus 3.

$$S_3 = |S_3|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = (650000)\left(0.95 + j\sqrt{1-0.95^2}\right) = 617500 + j202962 \text{ VA}$$

Find the line voltage at bus 3.

$$V_{\text{L}3} = \frac{|S_3|}{\sqrt{3}I_{\text{L}2}} = \frac{650000}{\sqrt{3}(3.25)} = 115470 \text{ V(rms)}$$

Find the complex power in line 2.

$$S_{\text{W}2} = 3Z_{\text{W}2}I_{\text{L}2}^2 = (3)(200 + j1200)(3.25)^2 = 6338 + j38025 \text{ VA}$$

The line voltage at bus 2 supplies power to $S_{\text{W}2}$ and S_3.

$$V_{\text{L}2} = \frac{|S_{\text{W}2} + S_3|}{\sqrt{3}I_{\text{L}2}} = 118804 \text{ V(rms)}$$

Find the complex power provided by the source at bus 2 and the complex power flowing into bus 2.

$$S_2 = |S_2|\left(\text{pf} - j\sqrt{1-\text{pf}^2}\right) = (400000)\left(0.9 - j\sqrt{1-0.9^2}\right) = 360000 - j174356 \text{ VA}$$

$$S_{2\text{IN}} = S_{\text{W}2} + S_3 - S_2 = 263838 + j415343 \text{ VA}$$

Find the current in line 1.

$$I_{\mathrm{L1}} = \frac{|S_{\mathrm{2IN}}|}{\sqrt{3}V_{\mathrm{L2}}} = \frac{|263838 + j415343|}{\sqrt{3}(118804)} = 2.3912 \text{ A(rms)}$$

Find the complex power in line 1.

$$S_{\mathrm{W1}} = 3Z_{\mathrm{W1}}I_{\mathrm{L1}}^2 = (3)(120 + j800)(2.3912)^2 = 2058.51 + j13723.37 \text{ VA}$$

Find the complex power and apparent power at bus 1.

$$S_1 = S_{\mathrm{W1}} + S_{\mathrm{2IN}} = 265896 + j429067 \text{ VA}$$

$$|S_1| = 504776 \text{ VA}$$

Find the line voltage and power factor at bus 1.

$$V_{\mathrm{L1}} = \frac{|S_1|}{\sqrt{3}I_{\mathrm{L1}}} = \frac{504776}{\sqrt{3}(2.3912)} = 121875 \text{ V(rms)}$$

$$\mathrm{pf} = \frac{P_1}{|S_1|} = \frac{265896}{504776} = 0.527$$

Problem 16–66. In Figure P16–66, the three buses are interconnected by transmission lines with wire impedances of $Z_{\mathrm{W1}} = 100 + j850\ \Omega$ per phase and $Z_{\mathrm{W2}} = 50 + j250\ \Omega$ per phase. The load at bus 1 draws an apparent power of $|S_1| = 400$ kVA at a lagging power factor of 0.85. The line voltage at bus 1 is $V_{\mathrm{L1}} = 138$ kV (rms). The load at bus 3 draws an apparent power of $|S_3| = 475$ kVA at a lagging power factor of 0.95. The line current in line 2 is $I_{\mathrm{L2}} = 2.0$ A (rms). Find the apparent power produced by the source at bus 2, the source power factor, and the line voltages at bus 2 and bus 3.

Find the complex power at bus 1.

$$S_1 = |S_1|\left(\mathrm{pf} + j\sqrt{1 - \mathrm{pf}^2}\right) = (400000)\left(0.85 + j\sqrt{1 - 0.85^2}\right) = 340000 + j210713 \text{ VA}$$

Find the current in line 1.

$$I_{\mathrm{L1}} = \frac{|S_1|}{\sqrt{3}V_{\mathrm{L1}}} = \frac{400000}{\sqrt{3}(138000)} = 1.6735 \text{ A(rms)}$$

Find the complex power in line 1.

$$S_{\mathrm{W1}} = 3Z_{\mathrm{W1}}I_{\mathrm{L1}}^2 = (3)(100 + j850)(1.6735)^2 = 840 + j7141 \text{ VA}$$

Find the complex power at bus 3.

$$S_3 = |S_3|\left(\mathrm{pf} + j\sqrt{1 - \mathrm{pf}^2}\right) = (475000)\left(0.95 + j\sqrt{1 - 0.95^2}\right) = 451250 + j148319 \text{ VA}$$

Find the line voltage at bus 3.

$$V_{\mathrm{L3}} = \frac{|S_3|}{\sqrt{3}I_{\mathrm{L2}}} = \frac{475000}{\sqrt{3}(2)} = 137121 \text{ V(rms)}$$

Find the complex power in line 2.

$$S_{\mathrm{W2}} = 3Z_{\mathrm{W2}}I_{\mathrm{L2}}^2 = (3)(50 + j250)(2)^2 = 600 + j3000 \text{ VA}$$

Find the complex power produced by the source, the apparent power, the power factor, and the line voltage at bus 2.

$$S_2 = S_1 + S_{W1} + S_3 + S_{W2} = 792690 + j369173 \text{ VA}$$

$$|S_2| = 874441 \text{ VA}$$

$$\text{pf} = \frac{P_2}{|S_2|} = \frac{792690}{874441} = 0.907$$

$$V_{L2} = \frac{|S_1 + S_{W1}|}{\sqrt{3}I_{L1}} = 139558 \text{ V(rms)}$$

Problem 16–67. In Figure P16–67, the source at bus 1 supplies two load buses through transmission lines with impedances of $Z_{W1} = 6 + j33\ \Omega$ per phase and $Z_{W2} = 3 + j15\ \Omega$ per phase. The load at bus 2 draws an apparent power of 4 MVA at a lagging power factor of 0.95. The load at bus 3 draws an apparent power of 3 MVA at a lagging power factor of 0.9. The line current in line 2 is $I_{L2} = 12.5$ A (rms). Find the apparent power produced by the source at bus 1, the source power factor, and the line voltages at bus 1, bus 2 and bus 3.

Find the complex power at bus 3.

$$S_3 = |S_3|\left(\text{pf} + j\sqrt{1 - \text{pf}^2}\right) = (3)\left(0.9 + j\sqrt{1 - 0.9^2}\right) = 2.7 + j1.308 \text{ MVA}$$

Find the line voltage at bus 3.

$$V_{L3} = \frac{|S_3|}{\sqrt{3}I_{L2}} = \frac{3 \times 10^6}{\sqrt{3}(12.5)} = 138564 \text{ V(rms)}$$

Find the complex power in line 2.

$$S_{W2} = 3Z_{W2}I_{L2}^2 = (3)(3 + j15)(12.5)^2 = 1406 + j7031 \text{ VA}$$

The line voltage at bus 2 supplies power to S_{W2} and S_3.

$$V_{L2} = \frac{|S_{W2} + S_3|}{\sqrt{3}I_{L2}} = 138764 \text{ V(rms)}$$

Find the complex power provided by the source at bus 2 and the complex power flowing into bus 2.

$$S_2 = |S_2|\left(\text{pf} - j\sqrt{1 - \text{pf}^2}\right) = (4)\left(0.95 - j\sqrt{1 - 0.95^2}\right) = 3.8 + j1.249 \text{ MVA}$$

$$S_{2IN} = S_{W2} + S_3 + S_2 = 6.501 + j2.564 \text{ MVA}$$

Find the current in line 1.

$$I_{L1} = \frac{|S_{2IN}|}{\sqrt{3}V_{L2}} = \frac{|6.501 + j2.564|}{\sqrt{3}(0.138764)} = 29.077 \text{ A(rms)}$$

Find the complex power in line 1.

$$S_{W1} = 3Z_{W1}I_{L1}^2 = (3)(6 + j33)(29.077)^2 = 15219 + j83703 \text{ VA}$$

Find the complex power, apparent power, and power factor at bus 1.

$$S_1 = S_{W1} + S_{2IN} = 6.517 + j2.647 \text{ MVA}$$

$$|S_1| = 7.034 \text{ MVA}$$

$$\text{pf} = \frac{P_1}{|S_1|} = \frac{6.517}{7.034} = 0.926$$

Find the line voltage and power factor at bus 1.

$$V_{\mathrm{L1}} = \frac{|S_1|}{\sqrt{3}I_{\mathrm{L1}}} = \frac{7.034 \times 10^6}{\sqrt{3}(29.077)} = 139663\ \mathrm{V(rms)}$$

Problem 16–68. (A) Three-Phase Voltages in the Time Domain

A balanced 60-Hz three-phase system has a line voltage of $V_{\mathrm{L}} = 208$ V (rms). Using $\angle\mathbf{V}_{\mathrm{AB}} = 0°$ as the phase reference, derive time-domain expressions for the phase voltages $v_{\mathrm{AN}}(t)$, $v_{\mathrm{BN}}(t)$, and $v_{\mathrm{CN}}(t)$ for a positive phase sequence.

Compute the phase voltages.

$$V_{\mathrm{P}} = \frac{V_{\mathrm{L}}}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{AN}} = 120\angle{-30°}\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{BN}} = 120\angle{-150°}\ \mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{CN}} = 120\angle{-270°}\ \mathrm{V(rms)}$$

Convert the rms voltage to an amplitude.

$$V_{\mathrm{A}} = \sqrt{2}V_{\mathrm{P}} = 170\ \mathrm{V}$$

The radian frequency is $\omega = 2\pi f = 120\pi = 377$ rad/s. Write the time-domain expressions for the phase voltages.

$$v_{\mathrm{AN}}(t) = 170\cos(377t - 30°)\ \mathrm{V}$$

$$v_{\mathrm{BN}}(t) = 170\cos(377t - 150°)\ \mathrm{V}$$

$$v_{\mathrm{CN}}(t) = 170\cos(377t - 270°)\ \mathrm{V}$$

Problem 16–69. (A) Single-Phase Motor Power Factor

When a 0.5-hp (1 hp = 746 W) single-phase induction motor delivers its rated mechanical output, it is 65% efficient and draws a current of 4.8 A (rms) from a 200 V (rms) line. Find the power factor of the motor.

When the motor delivers its rated output, it is providing $746/2 = 373$ W. Since it is 65% efficient, the average input power is $P = 373/0.65 = 573.8$ W. The apparent power at the rated output is

$$|S| = |\mathbf{V}||\mathbf{I}| = (200)(4.8) = 960\ \mathrm{VA}$$

Compute the power factor.

$$\mathrm{pf} = \frac{P}{|S|} = \frac{573.8}{960} = 0.598$$

Problem 16–70. (A) Three-Phase Transformer

Figure P16–70 shows three identical ideal transformers connected to form a three-phase transformer. The windings on the primary (source) side are Δ-connected and the windings on the secondary (load) side are Y-connected. On the secondary side of the transformer $V_{\mathrm{L2}} = 830$ V (rms) and $I_{\mathrm{L2}} = 40$ A (rms). The turns ratio of each transformer is $n = 0.2$. Find the line voltage and line current on the primary side.

The line voltage on the secondary is $V_{\mathrm{L2}} = 830$ V (rms). The secondary is Y-connected, so the voltages across the secondary windings are phase voltages.

$$V_{\mathrm{P2}} = \frac{V_{\mathrm{L2}}}{\sqrt{3}} = 479.2\ \mathrm{V(rms)}$$

The primary is Δ-connected, so the voltages across the primary windings are the line voltages V_{L1}. Apply the transformer turns ratio to compute the primary voltages.

$$V_{\text{L1}} = \frac{V_{\text{P2}}}{n} = \frac{479.2}{0.2} = 2396 \text{ V(rms)}$$

The currents through the secondary windings are the line currents $I_{\text{L2}} = 40$ A (rms). The primary is Δ-connected, so the currents through the primary windings are the phase currents $I_{\text{P1}} = nI_{\text{L2}} = (0.2)(40) = 8$ A (rms). The line current on the primary side is $I_{\text{L1}} = \sqrt{3}I_{\text{P1}} = 13.86$ A (rms).

Problem 16–71. (A) Phase Converter Efficiency

Three-phase motors are often used in equipment because they are more efficient and reliable than single-phase motors. Such equipment may be installed in locations where only single-phase power is available and the cost of installing three-phase service is prohibitive. The rotary phase converter in Figure P16–71 is one way of providing three-phase power from a single-phase source. Simply stated, the converter is a rotating transformer that shifts the phase of a portion of the single-phase input to produce a balanced set of three-phase voltages. In a certain application a converter supplies three-phase power to a 30-hp motor (1 hp = 746 W) that is 85% efficiency at full load. At full load the single-phase inputs are 220 V (rms) and 130 A (rms) at a power factor of 0.95 lagging. Find the efficiency of the converter.

For a 30-hp motor, the power at the output $P_{\text{OUT}} = (30)(746) = 22.38$ kW. Since the motor is 85% efficient, the three-phase power provided to the motor is $P_3 = P_{\text{OUT}}/0.85 = 22.38/0.85 = 26.329$ kW. Now examine the power associated with the converter. Compute the single-phase apparent and complex power provided to the converter.

$$|S_1| = |V_1||I_1| = (220)(130) = 28.6 \text{ kVA}$$

$$S_1 = |S_1|\left(\text{pf} + j\sqrt{1-\text{pf}^2}\right) = (28.6)\left(0.95 + j\sqrt{1-0.95^2}\right) = 27.170 + j8.9303 \text{ kVA}$$

Efficiency of the converter is the ratio of the average power provided to the three-phase motor to the average single-phase power provided to the converter.

$$\eta = \frac{P_3}{\text{Re}(S_1)} = \frac{26.329}{27.17} = 0.969$$

The converter is 96.9% efficient.

17 Two-Port Networks

17.1 Exercise Solutions

Exercise 17–1. Find the impedance parameters of the circuit in Figure 17–3.

Let port 2 have an open circuit so that $I_2 = 0$. Solve for z_{11} and z_{21}.

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 75 + 50 = 125\ \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{75I_1}{I_1} = 75\ \Omega$$

Let port 1 have an open circuit so that $I_1 = 0$. Solve for z_{12} and z_{22}.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{75I_2}{I_2} = 75\ \Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 100 + 75 = 175\ \Omega$$

Exercise 17–2. The impedance parameters of a two-port network are $z_{11} = 25\ \Omega$, $z_{12} = 50\ \Omega$, $z_{21} = 75\ \Omega$, and $z_{22} = 75\ \Omega$. Find the port currents I_1 and I_2 when a 15-V voltage source is connected at port 1 and port 2 is short circuited.

The i-v relationships are:

$$V_1 = z_{11}I_1 + z_{12}I_2 = 25I_1 + 50I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2 = 75I_1 + 75I_2$$

With port 2 short circuited, we have $V_2 = 0$:

$$0 = 75I_1 + 75I_2$$

$$I_1 = -I_2$$

Apply the voltage at port 1 and solve for the currents.

$$15 = 25I_1 + 50I_2 = 25I_1 - 50I_1 = -25I_1$$

$$I_1 = -0.6\ \text{A}$$

$$I_2 = -I_1 = 0.6\ \text{A}$$

Exercise 17–3. Find the admittance parameters of the circuit in Figure 17–5.

Let port 2 be short circuited so that $V_2 = 0$. Solve for y_{11} and y_{21}.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{V_1/j50}{V_1} = -j20\ \text{mS}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-V_1/j50}{V_1} = j20\ \text{mS}$$

Let port 1 be short circuited so that $V_1 = 0$. Solve for y_{12} and y_{22}.

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-V_2/j50}{V_2} = j20\ \text{mS}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{j50} + \frac{1}{200} = 5 - j20\ \text{mS}$$

Exercise 17–4. The admittance parameters of a two-port network are $y_{11} = 20$ mS, $y_{12} = 0$, $y_{21} = 100$ mS, and $y_{22} = 40$ mS. Find the output voltage V_2 when a 5-V voltage source is connected at port 1 and port 2 is connected to a 100-Ω load resistor.

The i-v relationships are:

$$I_1 = y_{11}V_1 + y_{12}V_2 = 0.02V_1 + (0)V_2 = 0.02V_1$$

$$I_2 = y_{21}V_1 + y_{22}V_2 = 0.1V_1 + 0.04V_2$$

With a 100-Ω load at port two, we have $V_2 = -100I_2$. Substitute and solve.

$$I_2 = (0.1)(5) + (0.04)(-100I_2) = 0.5 - 4I_2$$

$$I_2 = 100 \text{ mA}$$

$$V_2 = -100I_2 = -10 \text{ V}$$

Exercise 17–5. Find the h-parameters of the circuit in Figure 17–8.

Let port 2 be short circuited so that $V_2 = 0$. Write an expression for V_1 in terms of current I_1.

$$V_1 = 100000I_1 + (1000)(I_1 + 50I_1) = 151000I_1$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 151 \text{ k}\Omega$$

Solve for h_{21}.

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{50I_1}{I_1} = 50$$

Let port 1 have an open circuit so that $I_1 = 0$. We have $I_2 = 50I_1 = 0$, so no current flows in the circuit, regardless of the value of V_2. Solve for h_{12} and h_{22}.

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{0}{V_2} = 0$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{0}{V_2} = 0$$

Exercise 17–6. The h-parameters of a two-port network are $h_{11} = j200\ \Omega$, $h_{12} = 1$, $h_{21} = -1$, and $h_{22} = 2$ mS. Find the input impedance when the output port is open and the output admittance when the input port is short circuited.

The i-v relationships are:

$$V_1 = h_{11}I_1 + h_{12}V_2 = j200I_1 + V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2 = -I_1 + 0.002V_2$$

With the output port open, we have $I_2 = 0$. We have the following results:

$$0 = -I_1 + 0.002V_2$$

$$V_2 = 500I_1$$

$$V_1 = j200I_1 + 500I_1 = (500 + j200)I_1$$

$$Z_{\text{IN}} = \frac{V_1}{I_1} = 500 + j200\ \Omega$$

With the input port short circuited, we have $V_1 = 0$. We have the following results:

$$0 = j200I_1 + V_2$$

$$I_1 = j0.005V_2$$

$$I_2 = -j0.005V_2 + 0.002V_2$$

$$Y_{\text{OUT}} = \frac{I_2}{V_2} = 2 - j5 \text{ mS}$$

Exercise 17–7. The t-parameters of a two-port network are $A = 2$, $B = 200\ \Omega$, $C = 10$ mS, and $D = 1.5$. Find the Thévenin equivalent circuit at the output port when a 10-V voltage source is connected at the input port.

The i-v relationships are:

$$V_1 = AV_2 - BI_2 = 2V_2 - 200I_2$$

$$I_1 = CV_2 - DI_2 = 0.01V_2 - 1.5I_2$$

With a 10-V source at the input, we have:

$$10 = AV_2 - BI_2 = 2V_2 - 200I_2$$

$$V_2 = 5 + 100I_2$$

At the output port, apply an open circuit so that $I_2 = 0$. The open-circuit voltage is then:

$$V_2 = V_{\text{OC}} = 5 + (100)(0) = 5 \text{ V}$$

At the output port, apply a short circuit so that $V_2 = 0$. The short-circuit current is then:

$$0 = 5 + 100I_2$$

$$I_2 = -50 \text{ mA}$$

$$I_{\text{SC}} = -I_2 = 50 \text{ mA}$$

Calculate Thévenin equivalent resistance.

$$R_{\text{T}} = \frac{V_{\text{OC}}}{I_{\text{SC}}} = \frac{5}{0.05} = 100\ \Omega$$

The Thévenin equivalent at the output is $V_{\text{T}} = V_{\text{OC}} = 5$ V and $R_{\text{T}} = 100\ \Omega$.

Exercise 17–8. Find the t-parameters of the OP AMP circuit in Figure 17–11.

The i-v relationships are:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

The OP AMP circuit is an inverting amplifier, so we have the following gain relationships when the output is an open circuit and $I_2 = 0$:

$$V_2 = -\frac{Z_2}{Z_1}V_1$$

$$V_1 = -\frac{Z_1}{Z_2}V_2$$

Compare the gain relationship with the corresponding i-v relationship to determine:

$$A = -\frac{Z_1}{Z_2}$$

$$B = 0$$

Examine the input impedance to determine the other two parameters.

$$Z_1 = \frac{V_1}{I_1}$$

$$I_1 = \frac{V_1}{Z_1} = \frac{-Z_1V_2}{Z_1Z_2} = \frac{-V_2}{Z_2} = -I_2$$

Compare the gain relationship with the corresponding i-v relationship to determine:

$$C = -\frac{1}{Z_2}$$

$$D = 0$$

Exercise 17–9. The h-parameters of a two-port are $h_{11} = 1$ kΩ, $h_{12} = 0.02$, $h_{21} = -50$, and $h_{22} = 10^{-4}$ S. Find the t-parameters of the two-port.

Use the results in Table 17–2.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{-\Delta_{\mathrm{h}}}{h_{21}} & \dfrac{-h_{11}}{h_{21}} \\ \dfrac{-h_{22}}{h_{21}} & \dfrac{1}{h_{21}} \end{bmatrix}$$

where $\Delta_{\mathrm{h}} = h_{11}h_{22} - h_{12}h_{21}$. Use the following MATLAB code to perform the calculations:

```
h11 = 1e3;
h12 = 0.02;
h21 = -50;
h22 = 1e-4;

Dh = h11*h22-h12*h21
t11 = -Dh/h21
t12 = -h11/h21
t21 = -h22/h21
t22 = -1/h21
```

The results are:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.022 & 20\,\Omega \\ 2\,\mu\mathrm{S} & 0.02 \end{bmatrix}$$

Exercise 17–10. The t-matrices of networks N_{a} and N_{b} in Figure 17–15 are

$$[\mathbf{t}_{\mathrm{a}}] = \begin{bmatrix} 3 & 200 \\ 0.01 & 2 \end{bmatrix} \qquad \text{and} \qquad [\mathbf{t}_{\mathrm{b}}] = \begin{bmatrix} 2 & 0 \\ 0.04 & 1 \end{bmatrix}$$

Find the t-parameters of the cascade connection.

Multiply the two matrices to determine the parameters for the cascade connection.

$$[\mathbf{t}] = [\mathbf{t}_{\mathrm{a}}]\,[\mathbf{t}_{\mathrm{b}}] = \begin{bmatrix} 3 & 200 \\ 0.01 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0.04 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 200\,\Omega \\ 0.1\,\mathrm{S} & 2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Exercise 17–11. What is the current gain of the series connection in Figure 17–16 when $R = 0$ and $R = 1\ \text{k}\Omega$?

From Example 17–11, we have

$$[\mathbf{z}_\text{a}] = \begin{bmatrix} 500 & 0 \\ 5000 & 10 \end{bmatrix} \qquad \text{and} \qquad [\mathbf{z}_\text{b}] = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$

For a series connection, add the impedance matrices.

$$[\mathbf{z}_\text{a} + \mathbf{z}_\text{b}] = \begin{bmatrix} 500 + R & R \\ 5000 + R & 10 + R \end{bmatrix}$$

From Table 17–3, we have

$$T_\text{I} = -\frac{z_{21}}{z_{22}} = -\frac{5000 + R}{10 + R}$$

For $R = 0$, we have

$$T_\text{I} = -\frac{5000 + 0}{10 + 0} = -500$$

For $R = 1\ \text{k}\Omega$, we have

$$T_\text{I} = -\frac{5000 + 1000}{10 + 1000} = -5.94$$

17.2 Problem Solutions

Problem 17–1. Find the z-parameters of the two-port network in Figure P17–1.

Let port 2 have an open circuit so that $I_2 = 0$. Solve for z_{11} and z_{21}.

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 100 + [600 \parallel (400+200)] = 400\ \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{(200)(I_1/2)}{I_1} = 100\ \Omega$$

Let port 1 have an open circuit so that $I_1 = 0$. Solve for z_{12} and z_{22}.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{(600)(2I_2/12)}{I_2} = 100\ \Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 200 \parallel (400+600) = 166.7\ \Omega$$

Problem 17–2. Find the y-parameters of the two-port network in Figure P17–1.

Let port 2 be short circuited so that $V_2 = 0$. Solve for y_{11} and y_{21}.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{100 + (400 \parallel 600)} = 2.94118\ \text{mS}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-600I_1/1000}{V_1} = \frac{(-600)(y_{11}V_1)/1000}{V_1} = -1.7647\ \text{mS}$$

Let port 1 be short circuited so that $V_1 = 0$. Solve for y_{12} and y_{22}.

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = y_{21} = -1.7647\ \text{mS}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{[(100 \parallel 600) + 400] \parallel 200} = 7.0588\ \text{mS}$$

Above, we have applied the fact that a circuit with linear resistors is reciprocal, so $y_{12} = y_{21}$.

Problem 17–3. Find the z-parameters of the two-port network in Figure P17–3.

Let port 2 have an open circuit so that $I_2 = 0$. Solve for z_{11} and z_{21}.

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = j50 \parallel (100 - j100) = 20 + j60\ \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{(100)[j50I_1/(100-j50)]}{I_1} = -20 + j40\ \Omega$$

Let port 1 have an open circuit so that $I_1 = 0$. Solve for z_{12} and z_{22}.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{(j50)[100I_2/(100-j50)]}{I_2} = -20 + j40\ \Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 100 \parallel -j50 = 20 - j40\ \Omega$$

Problem 17–4. Find the y-parameters of the two-port network in Figure P17–3.

Let port 2 be short circuited so that $V_2 = 0$. Solve for y_{11} and y_{21}.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{j50 \parallel -j100} = -j10\ \text{mS}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{(-1)(j50)I_1/(j50-j100)}{V_1} = \frac{(-j50)(y_{11}V_1)/(-j50)}{V_1} = y_{11} = -j10\ \text{mS}$$

Let port 1 be short circuited so that $V_1 = 0$. Solve for y_{12} and y_{22}.

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = y_{21} = -j10 \text{ mS}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{-j100 \parallel 100} = 10 + j10 \text{ mS}$$

Above, we have applied the fact that a circuit with linear resistors is reciprocal, so $y_{12} = y_{21}$.

Problem 17–5. Find the z-parameters of the two-port network in Figure P17–5.

Let port 2 have an open circuit so that $I_2 = 0$. Solve for z_{11} and z_{21}. Write KVL around the left loop with V_1.

$$V_1 = R_1 I_1 + R_2(I_1 + \beta I_1) = [R_1 + (\beta + 1)R_2]I_1$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = R_1 + (\beta + 1)R_2$$

Apply Ohm's law at the output.

$$V_2 = -\beta I_1 R_3$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -\beta R_3$$

Let port 1 have an open circuit so that $I_1 = 0$. Solve for z_{12} and z_{22}. With $I_1 = 0$, no current flows through the dependent source. Therefore, I_2 flows through R_3.

$$V_2 = R_3 I_2$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = R_3$$

With $I_1 = 0$ and the dependent source is turned off, $V_1 = 0$ and we have:

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{0}{I_2} = 0$$

Problem 17–6. Find the y-parameters of the two-port network in Figure P17–5.

Let port 2 be short circuited so that $V_2 = 0$. Solve for y_{11} and y_{21}. Write KVL around the left loop with V_1.

$$V_1 = R_1 I_1 + R_2(I_1 + \beta I_1) = [R_1 + (\beta + 1)R_2]I_1$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{R_1 + (\beta + 1)R_2}$$

$$I_2 = \beta I_1$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{\beta I_1}{[R_1 + (\beta + 1)R_2]I_1} = \frac{\beta}{R_1 + (\beta + 1)R_2}$$

Let port 1 be short circuited so that $V_1 = 0$. Solve for y_{12} and y_{22}. Write KVL around the left loop with V_1.

$$V_1 = 0 = R_1 I_1 + R_2(I_1 + \beta I_1) = [R_1 + (\beta+1)R_2]I_1$$

$$I_1 = 0$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = 0$$

$$V_2 = R_3 I_2$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{R_3}$$

Problem 17–7. The z-parameters of a two-port circuit are $z_{11} = 1\text{ k}\Omega$, $z_{12} = z_{21} = 500\ \Omega$, and $z_{22} = 1.5\text{ k}\Omega$. Find the port currents I_1 and I_2 when a 12-V voltage source is connected across the input port and a load resistor $R_\text{L} = 250\ \Omega$ is connected across the output port.

The connections to the two-port circuit impose the following constraints:

$$V_1 = 12\text{ V}$$

$$R_\text{L} = 250\ \Omega$$

$$V_2 = -R_\text{L} I_2 = -250 I_2$$

We have the following i-v relationships:

$$V_1 = 12 = z_{11}I_1 + z_{12}I_2 = 1000I_1 + 500I_2$$

$$V_2 = -250I_2 = z_{21}I_1 + z_{22}I_2 = 500I_1 + 1500I_2$$

Solve the second equation for I_1 and substitute into the first equation.

$$500I_1 = -1750I_2$$

$$I_1 = -3.5I_2$$

$$12 = -3500I_2 + 500I_2$$

$$I_2 = \frac{12}{-3000} = -4\text{ mA}$$

$$I_1 = -3.5I_2 = 14\text{ mA}$$

Problem 17–8. The z-parameters of a two-port circuit are $z_{11} = 80\text{ k}\Omega$, $z_{12} = 3\text{ M}\Omega$, $z_{21} = -400\text{ k}\Omega$, and $z_{22} = 5\text{ k}\Omega$. Find the open-circuit ($I_2 = 0$) voltage gain $T_\text{V} = V_2/V_1$.

We have the following i-v relationships:

$$V_1 = z_{11}I_1 + z_{12}I_2 = 80000I_1 + 3000000I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2 = -400000I_1 + 5000I_2$$

Set $I_2 = 0$ to get the following equations and results:

$$V_1 = 80000I_1$$

$$V_2 = -400000I_1$$

$$T_\text{V} = \frac{V_2}{V_1} = \frac{-400000I_1}{80000I_1} = -5$$

Problem 17–9. The y-parameters of a two-port circuit are $y_{11} = 4$ mS, $y_{12} = y_{21} = -2$ mS, and $y_{22} = 2$ mS. A 15-V voltage source is connected at the input port and a load resistor $R_L = 2500\ \Omega$ is connected across the output port. Find the port variable responses V_2, I_1, and I_2.

The i-v relationships and circuit constraints are:

$$I_1 = y_{11}V_1 + y_{12}V_2 = 0.004V_1 - 0.002V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2 = -0.002V_1 + 0.002V_2$$

$$V_1 = 15\text{ V}$$

$$V_2 = -R_L I_2 = -2500I_2$$

Solve for the requested values.

$$I_2 = -0.002(15) + 0.002(-2500I_2)$$

$$6I_2 = -0.03$$

$$I_2 = -5\text{ mA}$$

$$V_2 = -2500I_2 = 12.5\text{ V}$$

$$I_1 = 0.004(15) - 0.002(12.5) = 35\text{ mA}$$

Problem 17–10. The y-parameters of a two-port circuit are $y_{11} = 15 + j20$ mS, $y_{12} = y_{21} = -j20$ mS, and $y_{22} = 40 + j20$ mS. Find the short-circuit ($V_2 = 0$) current gain $T_I = I_2/I_1$.

The i-v relationships with $V_2 = 0$ are:

$$I_1 = y_{11}V_1 + y_{12}V_2 = (0.015 + j0.02)V_1$$

$$I_2 = y_{21}V_1 + y_{22}V_2 = -j0.02V_1$$

Compute the current gain.

$$T_I = \frac{I_2}{I_1} = \frac{-j0.02V_1}{(0.015 + j0.02)V_1} = -0.64 - j0.48$$

Problem 17–11. In Figure P17–11, a load impedance Z_L is connected across the output port. Show that the input impedance $Z_{IN} = V_1/I_1$ is

$$Z_{IN} = z_{11} - \frac{z_{12}z_{21}}{Z_L + z_{22}}$$

The load impedance introduces the following constraint:

$$V_2 = -Z_L I_2$$

Apply the constraint to the i-v relationships.

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2 = -Z_L I_2$$

$$-(Z_L + z_{22})I_2 = z_{21}I_1$$

$$I_2 = -\frac{z_{21}}{Z_L + z_{22}}I_1$$

$$V_1 = z_{11}I_1 + z_{12}\left(-\frac{z_{21}}{Z_L + z_{22}}\right)I_1$$

$$Z_{IN} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{Z_L + z_{22}}$$

Problem 17–12. In Figure P17–11, a load impedance $Z_{\rm L}$ is connected across the output port. Show that the voltage gain $T_{\rm V} = V_2/V_1$ is

$$T_{\rm V} = \frac{-y_{21}}{Y_{\rm L} + y_{22}}$$

The load impedance introduces the following constraint:

$$V_2 = -Z_{\rm L} I_2$$

$$I_2 = -Y_{\rm L} V_2$$

Apply the constraint to the i-v relationships.

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2 = -Y_{\rm L}V_2$$

$$y_{21}V_1 = -(y_{22} + Y_{\rm L})V_2$$

$$T_{\rm V} = \frac{V_2}{V_1} = \frac{-y_{21}}{Y_{\rm L} + y_{22}}$$

Problem 17–13. Find the h-parameters of the two-port networks in Figure P17–13.

(a). The i-v relationships are:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Let port 2 be short circuited so that $V_2 = 0$. We then have $V_1 = V_2 = 0$ and

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{0}{I_1} = 0$$

Now solve for h_{21}. With port 2 short circuited, $I_2 = -I_1$.

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-I_1}{I_1} = -1$$

Let port 1 have an open circuit so that $I_1 = 0$. We have $V_1 = V_2$ and $I_2 = YV_2$. Solve for h_{12} and h_{22}.

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{V_2}{V_2} = 1$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{YV_2}{V_2} = Y$$

(b). The i-v relationships are:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Let port 2 be short circuited so that $V_2 = 0$. We then have $V_1 = ZI_1$ and $I_2 = -I_1$.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{ZI_1}{I_1} = Z$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-I_1}{I_1} = -1$$

Let port 1 have an open circuit so that $I_1 = 0$. We have $I_2 = -I_1 = 0$ and $V_2 = V_1$. Solve for h_{12} and h_{22}.

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{V_2}{V_2} = 1$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{0}{V_2} = 0$$

Problem 17–14. Find the t-parameters of the two-port networks in Figure P17–13.

(a). The i-v relationships are:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Let port 2 have an open circuit so that $I_2 = 0$. We then have $V_2 = V_1$ and $I_1 = YV_1$.

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{-I_2=0} = \frac{V_1}{V_1} = 1$$

$$A = 1$$

$$\frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{-I_2=0} = \frac{V_1}{YV_1} = \frac{1}{Y}$$

$$C = Y$$

Let port 2 have a short circuit so that $V_2 = 0$. We then have $V_1 = V_2 = 0$ and $I_2 = -I_1$. The i-v relationships allow us to solve for B and D:

$$V_1 = AV_2 - BI_2$$

$$0 = A(0) - BI_2$$

$$B = 0$$

$$\frac{1}{D} = \left.\frac{-I_2}{I_1}\right|_{V_2=0} = \frac{I_1}{I_1} = 1$$

(b). The i-v relationships are:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Let port 2 have an open circuit so that $I_2 = 0$. We then have $I_1 = -I_2 = 0$ and $V_1 = V_2$.

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{-I_2=0} = \frac{V_1}{V_1} = 1$$

$$A = 1$$

$$I_1 = CV_2 - DI_2$$

$$0 = CV_2 - D(0)$$

$$C = 0$$

Let port 2 have a short circuit so that $V_2 = 0$. We then have $V_1 = ZI_1$ and $I_2 = -I_1$. The i-v relationships allow us to solve for B and D:

$$\frac{1}{B} = \left.\frac{-I_2}{V_1}\right|_{V_2=0} = \frac{I_1}{ZI_1} = \frac{1}{Z}$$

$$B = Z$$

$$\frac{1}{D} = \left.\frac{-I_2}{I_1}\right|_{V_2=0} = \frac{I_1}{I_1} = 1$$

$$D = 1$$

Problem 17–15. Find the h-parameters of the two-port network in Figure P17–15.
The i-v relationships are:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Let port 2 be short circuited so that $V_2 = 0$. We then have $I_2 = \beta I_1$ and

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{(R_1 + R_2)I_1}{I_1} = R_1 + R_2$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{\beta I_1}{I_1} = \beta$$

Let port 1 have an open circuit so that $I_1 = 0$. We have $V_1 = 0$ and $V_2 = R_3I_2$. Solve for h_{12} and h_{22}.

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{0}{V_2} = 0$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{\frac{1}{R_3}V_2}{V_2} = \frac{1}{R_3}$$

Problem 17–16. Find the t-parameters of the two-port network in Figure P17–15.
The i-v relationships are:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Let port 2 have an open circuit so that $I_2 = 0$. We then have $V_1 = (R_1 + R_2)I_1$ and $V_2 = -\beta R_3I_1$.

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{-I_2=0} = \frac{-\beta R_3I_1}{(R_1 + R_2)I_1} = \frac{-\beta R_3}{R_1 + R_2}$$

$$A = -\frac{R_1 + R_2}{\beta R_3}$$

$$\frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{-I_2=0} = \frac{-\beta R_3I_1}{I_1} = -\beta R_3$$

$$C = -\frac{1}{\beta R_3}$$

Let port 2 have a short circuit so that $V_2 = 0$. We then have $V_1 = (R_1 + R_2)I_1$ and $I_2 = \beta I_1$.

$$\frac{1}{B} = \frac{-I_2}{V_1}\bigg|_{V_2=0} = \frac{-\beta I_1}{(R_1+R_2)I_1} = -\frac{\beta}{R_1+R_2}$$

$$B = -\frac{R_1+R_2}{\beta}$$

$$\frac{1}{D} = \frac{-I_2}{I_1}\bigg|_{V_2=0} = \frac{-\beta I_1}{I_1} = -\beta$$

$$D = -\frac{1}{\beta}$$

Problem 17–17. Find the h-parameters of the two-port network in Figure P17–17.

The i-v relationships are:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Let port 2 be short circuited so that $V_2 = 0$. The OP AMP acts as a buffer, so voltage V_1 appears at the output of the OP AMP. We then have $V_1 = R_1 I_1$ and $I_2 = (V_2 - V_1)/R_2$.

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \frac{R_1 I_1}{I_1} = R_1$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = \frac{\dfrac{0-V_1}{R_2}}{I_1} = \frac{-\dfrac{R_1}{R_2}I_1}{I_1} = -\frac{R_1}{R_2}$$

Let port 1 have an open circuit so that $I_1 = 0$. We have $V_1 = 0$ and $I_2 = (V_2 - V_1)/R_2 = V_2/R_2$. Solve for h_{12} and h_{22}.

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{0}{V_2} = 0$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{\dfrac{V_2}{R_2}}{V_2} = \frac{1}{R_2}$$

Problem 17–18. Find the t-parameters of the two-port network in Figure P17–17.

The i-v relationships are:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Let port 2 have an open circuit so that $I_2 = 0$. The OP AMP acts as a buffer, so voltage V_1 appears at the output of the OP AMP. We then have $V_1 = R_1 I_1$ and $V_2 = V_1$, since $I_2 = 0$.

$$\frac{1}{A} = \frac{V_2}{V_1}\bigg|_{-I_2=0} = \frac{V_1}{V_1} = 1$$

$$A = 1$$

$$\frac{1}{C} = \frac{V_2}{I_1}\bigg|_{-I_2=0} = \frac{R_1 I_1}{I_1} = R_1$$

$$C = \frac{1}{R_1}$$

Let port 2 have a short circuit so that $V_2 = 0$. We then have $V_1 = R_1 I_1$ and $I_2 = (0 - V_1)/R_2$.

$$\frac{1}{B} = \left.\frac{-I_2}{V_1}\right|_{V_2=0} = \frac{\frac{V_1}{R_2}}{V_1} = \frac{1}{R_2}$$

$$B = R_2$$

$$\frac{1}{D} = \left.\frac{-I_2}{I_1}\right|_{V_2=0} = \frac{\frac{V_1}{R_2}}{I_1} = \frac{\frac{R_1}{R_2}I_1}{I_1} = \frac{R_1}{R_2}$$

$$D = \frac{R_2}{R_1}$$

Problem 17–19. The h-parameters of a two-port network are $h_{11} = 500\ \Omega$, $h_{12} = 1$, $h_{21} = -1$, and $h_{22} = 2$ mS. Find the Thévenin equivalent circuit at the output port when a 12-V voltage source is connected at the input port.

The i-v relationships are:

$$V_1 = 12 = h_{11}I_1 + h_{12}V_2 = 500I_1 + V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2 = -I_1 + 0.002V_2$$

Find the open-circuit voltage and short-circuit current at the output. For the open-circuit voltage, we have $I_2 = 0$, which yields:

$$0 = -I_1 + 0.002V_2$$

$$I_1 = 0.002V_2$$

$$12 = 500I_1 + V_2 = (500)(0.002V_2) + V_2 = V_2 + V_2 = 2V_2$$

$$V_2 = V_{\mathrm{OC}} = V_{\mathrm{T}} = 6\ \mathrm{V}$$

For the short-circuit current, we have $V_2 = 0$, which yields:

$$12 = 500I_1 + V_2 = 500I_1$$

$$I_1 = 24\ \mathrm{mA}$$

$$I_2 = -I_1 + 0.002V_2 = -I_1 = -24\ \mathrm{mA}$$

$$I_{\mathrm{SC}} = -I_2 = 24\ \mathrm{mA}$$

$$R_{\mathrm{T}} = \frac{V_{\mathrm{OC}}}{I_{\mathrm{SC}}} = \frac{6}{0.024} = 250\ \Omega$$

The Thévenin equivalent source is $V_{\mathrm{T}} = 6$ V and $R_{\mathrm{T}} = 250\ \Omega$.

Problem 17–20. The h-parameters of a two-port network are $h_{11} = 6\ \mathrm{k}\Omega$, $h_{12} = 0$, $h_{21} = 50$, and $h_{22} = 0.2$ mS. Find the voltage gain V_2/V_1 when a 20-kΩ load resistor is connected across the output port.

The i-v relationships are:

$$V_1 = h_{11}I_1 + h_{12}V_2 = 6000I_1$$

$$I_2 = h_{21}I_1 + h_{22}V_2 = 50I_1 + 0.0002V_2$$

The load resistor introduces the following constraint:

$$V_2 = -R_L I_2 = -20000 I_2$$

Substitute into the second i-v relationship to solve for the voltage gain.

$$I_2 = 50 I_1 + 0.0002 V_2$$

$$-\frac{V_2}{20000} = (50)\left(\frac{V_1}{6000}\right) + 0.0002 V_2$$

$$-V_2 = 166.7 V_1 + 4 V_2$$

$$-5 V_2 = 166.7 V_1$$

$$\frac{V_2}{V_1} = -33.3333$$

Problem 17–21. The t-parameters of a two-port network are $A = 2$, $B = 400\ \Omega$, $C = 2.5$ mS, and $D = 1$.

(a). Find the output resistance V_2/I_2 when the input port is short-circuited.

The i-v relationships are:

$$V_1 = AV_2 - BI_2 = 2V_2 - 400 I_2$$

$$I_1 = CV_2 - DI_2 = 0.0025 V_2 - I_2$$

With the input port short-circuited, $V_1 = 0$ and we have:

$$0 = 2V_2 - 400 I_2$$

$$\frac{V_2}{I_2} = \frac{400}{2} = 200\ \Omega$$

(b). Find the output resistance V_2/I_2 when the input port is open-circuited.

With the input port open-circuited, $I_1 = 0$ and we have:

$$0 = 0.0025 V_2 - I_2$$

$$\frac{V_2}{I_2} = \frac{1}{0.0025} = 400\ \Omega$$

Problem 17–22. The t-parameters of a two-port network are $A = 0$, $B = -j100\ \Omega$, $C = -j20$ mS, and $D = 1 - j0.25$. Find the voltage gain V_2/V_1 when a 50-Ω load resistor is connected across the output port.

The i-v relationships are:

$$V_1 = AV_2 - BI_2 = j100 I_2$$

$$I_1 = CV_2 - DI_2 = -j0.02 V_2 - (1 - j0.25) I_2$$

The load resistor introduces the following constraint:

$$V_2 = -R_L I_2 = -50 I_2$$

Substitute into the first i-v relationship and solve for the voltage gain.

$$V_1 = j100I_2$$

$$V_1 = (j100)\left(-\frac{V_2}{50}\right)$$

$$\frac{V_2}{V_1} = \frac{-50}{j100} = j0.5$$

Problem 17–23. When a voltage V_{X} is applied across the input port, the short-circuit current at the output port is $I_{2\text{SC}}$. When the same voltage is applied across the output port, the short-circuit current at the input port is $I_{1\text{SC}}$. Show that reciprocity ($I_{1\text{SC}} = I_{2\text{SC}}$) requires that $h_{12} = -h_{21}$.

The i-v relationships are:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Apply V_{X} to the input port and short-circuit the output port.

$$V_{\text{X}} = h_{11}I_1 + h_{12}(0) = h_{11}I_1$$

$$I_{2\text{SC}} = h_{21}I_1 + h_{22}(0) = h_{21}I_1$$

$$I_1 = \frac{I_{2\text{SC}}}{h_{21}}$$

$$V_{\text{X}} = \frac{h_{11}}{h_{21}}I_{2\text{SC}}$$

$$I_{2\text{SC}} = \frac{h_{21}}{h_{11}}V_{\text{X}}$$

Apply V_{X} to the output port and short-circuit the input port.

$$0 = h_{11}I_{1\text{SC}} + h_{12}V_{\text{X}}$$

$$I_{1\text{SC}} = -\frac{h_{12}}{h_{11}}V_{\text{X}}$$

Comparing the equations for $I_{1\text{SC}}$ and $I_{2\text{SC}}$, for reciprocity to hold, we must have $h_{12} = -h_{21}$.

Problem 17–24. When a load impedance Z_{L} is connected across the output port, show that the input impedance is

$$Z_{\text{IN}} = \frac{AZ_{\text{L}} + B}{CZ_{\text{L}} + D}$$

The i-v relationships are:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

The load impedance introduces the following constraint:

$$V_2 = -Z_{\text{L}}I_2$$

Substitute and solve for the input impedance.

$$I_1 = -CZ_\mathrm{L}I_2 - DI_2 = -(CZ_\mathrm{L} + D)I_2$$

$$I_2 = \frac{-I_1}{CZ_\mathrm{L} + D}$$

$$V_2 = -Z_\mathrm{L}I_2 = \frac{Z_\mathrm{L}I_1}{CZ_\mathrm{L} + D}$$

$$V_1 = AV_2 - BI_2 = \frac{AZ_\mathrm{L}I_1}{CZ_\mathrm{L} + D} + \frac{BI_1}{CZ_\mathrm{L} + D} = \left(\frac{AZ_\mathrm{L} + B}{CZ_\mathrm{L} + D}\right) I_1$$

$$Z_\mathrm{IN} = \frac{V_1}{I_1} = \frac{AZ_\mathrm{L} + B}{CZ_\mathrm{L} + D}$$

Problem 17–25. Starting with the h-parameter i-v relationships in Eq. (17–9), show that:

$$A = -\frac{\Delta_\mathrm{h}}{h_{21}}$$

$$B = -\frac{h_{11}}{h_{21}}$$

$$C = -\frac{h_{22}}{h_{21}}$$

$$D = -\frac{1}{h_{21}}$$

$$\Delta_\mathrm{h} = h_{11}h_{22} - h_{12}h_{21}$$

The i-v relationships are:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Solve the second equation for I_1:

$$I_1 = -\frac{h_{22}}{h_{21}}V_2 + \frac{1}{h_{21}}I_2 = CV_2 - DI_2$$

Substitute the results for I_1 into the first i-v relationship.

$$V_1 = -\frac{h_{11}h_{22}}{h_{21}}V_2 + \frac{h_{11}}{h_{21}}I_2 + h_{12}V_2$$

$$V_1 = \left(\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right) V_2 + \frac{h_{11}}{h_{21}}I_2 = AV_2 - BI_2$$

Problem 17–26. Starting with the t-parameter i-v relationships in Eq. (17–13), show that:

$$y_{11} = \frac{D}{B}$$

$$y_{12} = -\frac{\Delta_{\rm t}}{B}$$

$$y_{21} = -\frac{1}{B}$$

$$y_{22} = \frac{A}{B}$$

$$\Delta_{\rm t} = AD - BC$$

The i-v relationships are:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Solve the first equation for I_2.

$$I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2 = y_{21}V_1 + y_{22}V_2$$

Substitute the results for I_2 into the second i-v relationship.

$$I_1 = CV_2 + \frac{D}{B}V_1 - \frac{AD}{B}V_2$$

$$I_1 = \frac{D}{B}V_1 - \frac{AD - BC}{B}V_2 = y_{11}V_1 + y_{12}V_2$$

Problem 17–27. Starting with the h-parameter i-v relationships in Eq. (17–9), show that for $I_2 = 0$, the voltage gain is

$$T_{\rm V} = \frac{V_2}{V_1} = \frac{-h_{21}}{\Delta_{\rm h}}$$

The i-v relationships are:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2 = 0$$

Solve the second equation for I_1 and substitute into the first equation.

$$I_1 = -\frac{h_{22}}{h_{21}}V_2$$

$$V_1 = -\frac{h_{11}h_{22}}{h_{21}}V_2 + h_{12}V_2 = -\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{21}}V_2$$

$$T_{\rm V} = \frac{V_2}{V_1} = -\frac{h_{21}}{h_{11}h_{22} - h_{12}h_{21}} = -\frac{h_{21}}{\Delta_{\rm h}}$$

Problem 17–28. Starting with the t-parameter i-v relationships in Eq. (17–13), show that for $V_2 = 0$, the current gain is

$$T_{\rm I} = \frac{I_2}{I_1} = -\frac{1}{D}$$

The i-v relationships are:

$$V_1 = AV_2 - BI_2 = -BI_2$$

$$I_1 = CV_2 - DI_2 = -DI_2$$

Solve the second equation for the current gain.

$$T_\mathrm{I} = \frac{I_2}{I_1} = -\frac{1}{D}$$

Problem 17–29. The t-parameters of the two-port networks N_a and N_b in Figure P17–29 are

$$[\mathbf{t}_\mathrm{a}] = \begin{bmatrix} 60 & 10 \\ 5 & 40 \end{bmatrix} \qquad \text{and} \qquad [\mathbf{t}_\mathrm{b}] = \begin{bmatrix} 10 & 10 \\ 2.5 & 5 \end{bmatrix}$$

Find the t-parameters of the cascade connection.

Multiply the two matrices to determine the parameters for the cascade connection.

$$[\mathbf{t}] = [\mathbf{t}_\mathrm{a}]\,[\mathbf{t}_\mathrm{b}] = \begin{bmatrix} 60 & 10 \\ 5 & 40 \end{bmatrix}\begin{bmatrix} 10 & 10 \\ 2.5 & 5 \end{bmatrix} = \begin{bmatrix} 625 & 650\,\Omega \\ 150\,\mathrm{S} & 250 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Problem 17–30. The t-parameters of the two-port networks N_a and N_b in Figure P17–29 are

$$[\mathbf{t}_\mathrm{a}] = \begin{bmatrix} 4 & 25 \\ 0.01 & 4 \end{bmatrix} \qquad \text{and} \qquad [\mathbf{t}_\mathrm{b}] = \begin{bmatrix} 10 & 500 \\ 0.2 & 10 \end{bmatrix}$$

Find the impedance looking into the input port of N_a when a 50-Ω resistive load is connected across the output port of N_b. Note: The result in Problem 17–24 will prove useful.

First, multiply the two matrices to determine the t-parameters for the cascade connection.

$$[\mathbf{t}] = [\mathbf{t}_\mathrm{a}]\,[\mathbf{t}_\mathrm{b}] = \begin{bmatrix} 4 & 25 \\ 0.01 & 4 \end{bmatrix}\begin{bmatrix} 10 & 500 \\ 0.2 & 10 \end{bmatrix} = \begin{bmatrix} 45 & 2250\,\Omega \\ 0.9\,\mathrm{S} & 45 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Now apply the results from Problem 17–24:

$$Z_\mathrm{IN} = \frac{AZ_\mathrm{L} + B}{CZ_\mathrm{L} + D} = \frac{(45)(50) + 2250}{(0.9)(50) + 45} = 50\ \Omega$$

Problem 17–31. The cascade connection in Figure P17–29 is a two-stage amplifier with identical two-port stages, each having the h-parameters $h_{11} = 1$ kΩ, $h_{12} = 0$, $h_{21} = -10$, and $h_{22} = 1$ mS. Find the t-parameters of the cascade connection and then calculate the current gain of the two-stage amplifier.

Use Table 17–2 to convert the h-parameters into t-parameters and then determine the t-parameters for the cascade connection by multiplying the two transmission matrices. Then apply the results in Table 17–3 to determine the current gain from the t-parameters. We have the following calculations and results:

$$\Delta_{\rm h} = h_{11}h_{22} - h_{12}h_{21} = 1 - 0 = 1$$

$$A = -\frac{\Delta_{\rm h}}{h_{21}} = \frac{-1}{-10} = 0.1$$

$$B = -\frac{h_{11}}{h_{21}} = \frac{-1000}{-10} = 100\ \Omega$$

$$C = -\frac{h_{22}}{h_{21}} = \frac{-0.001}{-10} = 0.0001\ \text{S}$$

$$D = -\frac{1}{h_{21}} = \frac{-1}{-10} = 0.1$$

$$[\mathbf{t}] = [\mathbf{t}_{\rm a}]\,[\mathbf{t}_{\rm b}] = \begin{bmatrix} 0.1 & 100 \\ 0.0001 & 0.1 \end{bmatrix}\begin{bmatrix} 0.1 & 100 \\ 0.0001 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.02 & 20\,\Omega \\ 0.00002\,\text{S} & 0.02 \end{bmatrix}$$

$$T_{\rm I} = \frac{-1}{D} = \frac{-1}{0.02} = -50$$

Problem 17–32. Find the voltage gain of the two-stage amplifier defined in Problem 17–31.

Using the results from Problem 17–31 and Table 17–3, we have the following:

$$[\mathbf{t}] = \begin{bmatrix} 0.02 & 20\,\Omega \\ 0.00002\,\text{S} & 0.02 \end{bmatrix}$$

$$T_{\rm V} = \frac{1}{A} = \frac{1}{0.02} = 50$$

Problem 17–33. The two-port parameters of the series connection in Figure P17-33 are

$$[\mathbf{z}_{\rm a}] = \begin{bmatrix} 60 & 10 \\ 10 & 10 \end{bmatrix}\ \Omega \qquad \text{and} \qquad [\mathbf{y}_{\rm b}] = \begin{bmatrix} 60 & -40 \\ -20 & 80 \end{bmatrix}\ \text{mS}$$

Find the z-parameters of the series connection. Is the network reciprocal?

To find the z-parameters of a series connection, convert the admittance matrix into an impedance matrix and then add the two impedance matrices together.

$$[\mathbf{z}_{\rm b}] = [\mathbf{y}_{\rm b}]^{-1} = \begin{bmatrix} 60 & -40 \\ -20 & 80 \end{bmatrix}^{-1} = \begin{bmatrix} 20 & 10 \\ 5 & 15 \end{bmatrix}\ \Omega$$

$$[\mathbf{z}] = [\mathbf{z}_{\rm a}] + [\mathbf{z}_{\rm b}] = \begin{bmatrix} 60 & 10 \\ 10 & 10 \end{bmatrix} + \begin{bmatrix} 20 & 10 \\ 5 & 15 \end{bmatrix} = \begin{bmatrix} 80 & 20 \\ 15 & 25 \end{bmatrix}\ \Omega$$

The network is not reciprocal since $z_{12} \neq z_{21}$.

Problem 17–34. In Figure P17–33, the network $N_{\rm a}$ is an active device with h-parameters $h_{11} = 20$ kΩ, $h_{12} = 0$, $h_{21} = -5000$, and $h_{22} = 50$ mS. The network $N_{\rm b}$ is a resistive feedback circuit with z-parameters $z_{11} = 2R$ and $z_{12} = z_{21} = z_{22} = R$. Find a value of R such that the voltage gain of the series connection is $T_{\rm V} = 2$.

To combine the networks in series, convert the hybrid parameters into impedance parameters using the results in Table 17–2 and then add the impedance parameters for the two networks together.

$$[\mathbf{z}_\mathrm{a}] = \begin{bmatrix} \dfrac{\Delta_\mathrm{h}}{h_{22}} & \dfrac{h_{12}}{h_{22}} \\ \\ \dfrac{-h_{21}}{h_{22}} & \dfrac{1}{h_{22}} \end{bmatrix} = \begin{bmatrix} 20000 & 0 \\ 100000 & 20 \end{bmatrix} \Omega$$

$$[\mathbf{z}] = [\mathbf{z}_\mathrm{a}] + [\mathbf{z}_\mathrm{b}] = \begin{bmatrix} 20000 & 0 \\ 100000 & 20 \end{bmatrix} + \begin{bmatrix} 2R & R \\ R & R \end{bmatrix} = \begin{bmatrix} 20000+2R & R \\ 100000+R & 20+R \end{bmatrix} \Omega$$

Using Table 17–3, the voltage gain with z-parameters is $T_\mathrm{V} = z_{21}/z_{11}$.

$$T_\mathrm{V} = 2 = \frac{z_{21}}{z_{11}} = \frac{100000+R}{20000+2R}$$

$$40000 + 4R = 100000 + R$$

$$3R = 60000$$

$$R = 20\ \mathrm{k}\Omega$$

Problem 17–35. The two-port parameters of the parallel connection in Figure P17–35 are

$$[\mathbf{z}_\mathrm{a}] = \begin{bmatrix} 30 & 10 \\ 10 & 20 \end{bmatrix} \Omega \qquad \text{and} \qquad [\mathbf{y}_\mathrm{b}] = \begin{bmatrix} 60 & -40 \\ -40 & 80 \end{bmatrix} \mathrm{mS}$$

Find the y-parameters of the parallel connection. Is the network reciprocal?

Find the equivalent admittance matrix for the impedance parameters and then add the two admittance matrices together to find the admittance matrix for the parallel connection.

$$[\mathbf{y}_\mathrm{a}] = [\mathbf{z}_\mathrm{a}]^{-1} = \begin{bmatrix} 30 & 10 \\ 10 & 20 \end{bmatrix}^{-1} = \begin{bmatrix} 40 & -20 \\ -20 & 60 \end{bmatrix} \mathrm{mS}$$

$$[\mathbf{y}] = [\mathbf{y}_\mathrm{a}] + [\mathbf{y}_\mathrm{b}] = \begin{bmatrix} 40 & -20 \\ -20 & 60 \end{bmatrix} + \begin{bmatrix} 60 & -40 \\ -40 & 80 \end{bmatrix} = \begin{bmatrix} 100 & -60 \\ -60 & 140 \end{bmatrix} \mathrm{mS}$$

The network is reciprocal since $y_{12} = y_{21}$.

Problem 17–36. The h-parameters of a two-port amplifier are $h_{11} = 10\ \mathrm{k}\Omega$, $h_{12} = 0$, $h_{21} = -40$, and $h_{22} = 1$ mS. Find the forward current gain of a parallel connection of two such amplifiers. Assume the connection does not change the parameters of either amplifier.

For a parallel connection, use the results in Table 17–2 to find the admittance parameters and then sum two identical networks together. Then use the results in Table 17–3 to find the forward current gain.

$$[\mathbf{y}_\mathrm{a}] = \begin{bmatrix} \dfrac{1}{h_{11}} & \dfrac{-h_{12}}{h_{11}} \\ \\ \dfrac{h_{21}}{h_{11}} & \dfrac{\Delta_\mathrm{h}}{h_{11}} \end{bmatrix} = \begin{bmatrix} 0.1 & 0 \\ -4 & 1 \end{bmatrix} \mathrm{mS}$$

$$[\mathbf{y}_\mathrm{b}] = [\mathbf{y}_\mathrm{a}]$$

$$[\mathbf{y}] = [\mathbf{y}_\mathrm{a}] + [\mathbf{y}_\mathrm{b}] = \begin{bmatrix} 0.2 & 0 \\ -8 & 2 \end{bmatrix} \mathrm{mS}$$

$$T_\mathrm{I} = \frac{y_{21}}{y_{11}} = \frac{-8}{0.2} = -40$$

Problem 17–37. A two-port network is said to be **unilateral** if excitation applied at the input port produces a response at the output port, but the same excitation applied at the output port produces no response at the input port. Show that a two-port is unilateral if $AD - BC = 0$.

The i-v relationships are:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

From Table 17–2, we have the following relationships:

$$z_{12} = \frac{\Delta_t}{C} = z_{21}\Delta_t$$

$$y_{12} = \frac{-\Delta_t}{B} = y_{21}\Delta_t$$

$$h_{12} = \frac{\Delta_t}{D} = -h_{21}\Delta_t$$

If an excitation at the input produces a nonzero output, then z_{21}, y_{21}, and h_{21} are not zero. When $\Delta_t = AD - BC = 0$, the reverse transfer functions z_{12}, y_{12}, and h_{12} are all zero. Therefore, an excitation applied at the output port produces zero response at the input port.

We can also rewrite the i-v relationships in matrix form.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

If $AD - BC = 0$, then the inverse matrix $[\mathbf{t}]^{-1}$ does not exist, so there is no way to relate input responses to output excitations.

Problem 17–38. A load impedance Z_L is connected at the output of a two-port network. Show that the input impedance $Z_{IN} = V_1/I_1 = Z_L$ when $A = D$ and $C = B/(Z_L)^2$. (*Hint:* The result shown in Problem 17–24 is the place to start.)

The result from Problem 17–24 is:

$$Z_{IN} = \frac{AZ_L + B}{CZ_L + D}$$

Substitute $A = D$ and $C = B/(Z_L)^2$.

$$Z_{IN} = \frac{AZ_L + B}{\dfrac{BZ_L}{Z_L^2} + A} = \frac{AZ_L + B}{\dfrac{B}{Z_L} + A} = \frac{Z_L(AZ_L + B)}{AZ_L + B} = Z_L$$

Problem 17–39. The y-parameters of a two-port network operating in the sinusoidal steady state are $y_{11} = 20 - j15$ mS, $y_{12} = y_{21} = -j12$ mS, and $y_{22} = 10 - j20$ mS. The input port is driven by an ac voltage source $\mathbf{V}_S = 120\angle 25°$ V (rms). Find the output port load impedance Z_L that will draw the maximum average power from the network.

To extract the maximum average power, the load impedance must be the conjugate of the Thévenin impedance seen at the output port. The Thévenin impedance is the impedance seen at the output port with the voltage source at the input turned off. Turning the source off creates a short circuit at the input port. The admittance seen at the output port with the input port shorted is, by definition, y_{22}. Therefore, the Thévenin impedance is $z_{22} = 1/y_{22}$.

$$Z_T = \frac{1}{y_{22}} = \frac{1}{0.01 - j0.02} = 20 + j40\ \Omega$$

$$Z_L = Z_T^* = 20 - j40\ \Omega$$

Problem 17–40. In the phasor domain, the **I-V** relationships of a linear transformer with positive coupling are

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

Find the phasor-domain transmission parameters of the linear transformer.

Apply the results in Table 17–2 to find the transmission parameters.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{z_{11}}{z_{21}} & \dfrac{\Delta_{\mathrm{z}}}{z_{21}} \\ \dfrac{1}{z_{21}} & \dfrac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} \dfrac{L_1}{M} & \dfrac{-j\omega(M^2 - L_1 L_2)}{M} \\ \dfrac{1}{j\omega M} & \dfrac{L_2}{M} \end{bmatrix}$$

We used the following MATLAB code to perform the calculations:

```
syms w L1 L2 M
z11 = j*w*L1;
z12 = j*w*M;
z21 = j*w*M;
z22 = j*w*L2;
Dz = z11*z22-z12*z21;
t = simplify([z11/z21, Dz/z21; 1/z21, z22/z21])
```

The results are:

```
t =
[     L1/M, -(w*(M^2 - L1*L2)*i)/M]
[ -i/(M*w),                   L2/M]
```