RYERSON UNIVERSITY

Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	CPS843
Course Title	Introduction to Computer Vision
Semester/Year	F2023

Instructor	Guanghui Richard Wang

ASSIGNMENT No.	4

Assignment Title	Problem Set #4

Submission Date	November 19th, 2023
Due Date	November 20th, 2023

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**By signing above you attest that you have contributed to this written lab report and confirm that all work you have swung the lab contributed to this lab report is your own work.*

Part 1

Problem 1:

(1)/(2)/(3)

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HW 4

Problem 1

(1) At Plane of infinity, the canonical form is:

$$\pi_\infty = (0,0,0,1)^T$$

(2) If and only if H is an affinity, can we assume the plane at infinity is a fixed plane under projective transformation H.

3D affine transformations can be replaced as H for a given H, that is proved to be an affinity. They will have the same properties.

(3) A 3D plane can be transformed as $\pi' = H^{-T}\pi$ if $X' = HX$.

since 3 planes define point : $\begin{bmatrix} \pi_1^T \\ \pi_2^T \\ \pi_3^T \end{bmatrix} X = 0$, where $Ax = 0$, then in using the 3×4 matrix, the point is obtained as the 1D right null-space. $X' = HX$ represents the points under transformation in projective transformation. As a result, the plane transforms into $\pi' = H^{-T}\pi$.

Problem 2:

(1)

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Problem 2

(1) With a matrix, the finite projective camera is

$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$. The matrix $P = KR[I \mid -\tilde{C}]$ which is a finite projective camera with camera centre's coordinates $(\tilde{C}^T, 1)^T$. It has 11 d.o.f. There are the same camera matrices for finite projective cameras as for homogeneous 3x4 matrices.

The matrix can be decomposed using RQ decomposition which is the general form

$$P = K[R \mid t]$$

$$P = K[M \mid p_4]$$

$$P = M[I \mid M^{-1}p_4] = Kr[I \mid -\tilde{C}]$$

$$P = [M \mid p_4]$$

This will decompose to $M = KR$, where R is the orthogonal matrix, $\tilde{C}$ is the camera centre with world coordinate system coordinates, K is a camera with calibration matrix.

(2)

With the projection matrix $\mathrm{P} = [\mathrm{M} \mid \mathbf{p}_4]$, in order to recover the parameters from the projection matrix, RQ decomposition can be used where M = KR. Afterward, reverse the QR decomposition with RQ decomposition.

(3)

The image of a point at infinity is only affected by the sub-matrix M as if we assume the mapping point $D = (d^T, 0)^T$ which can be substituted into the formula x= PX for x, where the homogeneous image point is x, the camera projection matrix is P, and the world point is X.

This results in x = PD since the space point is mapped to an image point. Next P can also be substituted for $[\mathrm{M} \mid \mathbf{p}_4]$, since this represents the properties of the projective camera, where p4 is the coordinate origin image. Hence, x becomes

$$\mathbf{x} = \mathrm{P}\mathbf{D} = [\mathrm{M} \mid \mathbf{p}_4]\mathbf{D} = \mathrm{M}\mathbf{d}$$

This shows how an image of a point at infinity is only affected by the sub-matrix M.

Problem 3:

(1)

To calculate the optical center of a camera of a given projection matrix P, first use singular value decomposition to obtain PC = 0, then using direct computation, this gets:
x = det([p2, p3, p4])
y = -det([p1, p3, p4])
z = det([p1, p2, p4])
t = -det([p1, p2, p3])

(2)

Yes, the first three columns of the projection matrix correspond to the vanishing points of the X, Y, and Z axes of the world system, as to determine the X, Y and/or Z values, the values are multiplied by the respective plane at infinity. This is showcased as

$$s_x \tilde{\boldsymbol{v}}_x = \boldsymbol{P}\tilde{\boldsymbol{x}}_w = [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4][1, 0, 0, 0]^{\mathrm{T}} = \boldsymbol{p}_1,$$
$$s_y \tilde{\boldsymbol{v}}_y = \boldsymbol{P}\tilde{\boldsymbol{y}}_w = [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4][0, 1, 0, 0]^{\mathrm{T}} = \boldsymbol{p}_2,$$
$$s_z \tilde{\boldsymbol{v}}_z = \boldsymbol{P}\tilde{\boldsymbol{z}}_w = [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4][0, 0, 1, 0]^{\mathrm{T}} = \boldsymbol{p}_3,$$
$$s_o \tilde{\boldsymbol{v}}_o = \boldsymbol{P}\tilde{\boldsymbol{o}}_w = [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4][0, 0, 0, 1]^{\mathrm{T}} = \boldsymbol{p}_4.$$

(3)

Yes the last row does correspond to the principal plane of the camera since in the projection matrix it represents a perpendicular vector to the axis, and geometrically, row vectors are world planes.

$$\mathsf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{1\mathsf{T}} \\ \mathbf{P}^{2\mathsf{T}} \\ \mathbf{P}^{3\mathsf{T}} \end{bmatrix}$$

Problem 4

(1)/(2)/(3)

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Problem 4

(1) Since $x = PX = [P_1\ P_2\ P_3\ P_4]\begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix} = [P_1\ P_2\ P_4]\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$

and $P = K[R|T]$, this shows coplaner 3D points and their images are related by a 2D homography given by $H = K[r_1, r_2, t]$

(2) The camera matrix that maps a set of points in space on a line through $\pi^T X = l^T P X$ is the back-projection of an image line.

(3) The zooming camera images are related by 2D homography given by $H = K'K^{-1}$, since you multiply by $x' = K'[I|0]X = K'K^{-1}x = (H)x$ when zooming in, hence they are related.

Problem 5

(1)(2)/(3)/(4)

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problem 5

(1) We can calculate the angle between two rays as the IAC depends only on internal parameters. This is given by $\cos\theta = \frac{x_1^T w x_2}{\sqrt{x_1^T w x_1}\sqrt{x_2^T w x_2}}$.

Both image points correspond to orthogonal directions $x_1^T w x_2 = 0$, resulting in dual image of absolute conic $w^* = w^{-1} = KK^T$. Performing the Cholesky decomposition from the DIAC gives the absolute conic.

(2) two image points correspond to orthogonal directions satisfy $x_1^T w x_2 = 0$ as w is an absolute conic and the dot product of the two vectors is zero when orthogonal.

(3) Using 3D homography, it can be used to acquire the constraints of an image of an absolute conic. Aswell as with $H = [h_1, h_2, h_3]$, which is the homography metric plane. $x = Hx$ $x' = Hx'$ which transforms to $I = Fx$ then to $F = H^{-T} F H^{-1}$

(4) Where $w = (KK^T)^{-1}$, the square pixels gather $w_{12} = w_{21} = 0$
$w_{11} = w_{22}$, $w = [(w_1\ 0\ w_2), (0\ w_1\ w_3), (w_2\ w_3\ w_4)]$

Part 2

AutoStich allows panoramic images to become stitched together. AutoStich can reduce redundancy as AutoStich provides image stitching by recognizing matching images automatically.

The user will capture images of a certain zone, and then feed the images to AutoStich which will then create the panoramic image by combining these images using the SIFT algorithm. The scale-invariant feature transform (SIFT) of Autostitch is important, as in digital images, this algorithm figures out and describes the key points.

With the given key points, it forms them using descriptors that hold quantitative information about the images. The information is used to further analyze information in images, such as the SIFT algorithm which is used commonly for object recognition.

Choogie im back
Image Stitching

6 Sample Original Images

Stitching Result (Panorama):

Based on the panorama image and effects of the SIFT algorithm, it is apparent that the stitching result appears quite smooth and blended together between the frames while keeping intricate details. The result is a full comprehensive image. This picture was able to capture the scene in a wider angle than a normal camera, providing more immersion.

References

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