RYERSON UNIVERSITY

Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	532
Course Title	Signal and Systems I
Semester/Year	F2021

Instructor	Soosan Beheshti

ASSIGNMENT No.	3

Assignment Title	Fourier Series Analysis Using Matlab

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Due Date	November 21, 2021

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A.1 and A.2:

ELE 532 LAB 3

A.1 $x_1(t) = \cos \frac{3\pi}{10} t + \frac{1}{2} \cos \frac{\pi}{10} t$

$$x_1(t) = \frac{1}{2} e^{j3\pi/10 t} + \frac{1}{2} e^{-j3\pi/10t} + \frac{1}{2}\left(\frac{1}{2} e^{j\pi/10t} + \frac{1}{2} e^{-j\pi/10t}\right)$$

$$x_1(t) = \frac{1}{2} e^{j3\pi/10t} + \frac{1}{2} e^{-j3\pi/10t} + \frac{1}{4} e^{j\pi/10t} + \frac{1}{4} e^{-j\pi/10t}$$

Fundamental Sequency:

$W_{01} = \frac{3\pi}{10}$ $\quad W_{02} = \frac{\pi}{10}$ $\quad T_0 = \frac{2\pi}{\left(\frac{\pi}{10}\right)} = 2\pi \cdot \frac{10}{\pi} = 20$

$\frac{3\pi}{10} \cdot \frac{10}{\pi} = 3$

$\frac{GCE}{LCM} = \frac{\pi}{10}$

$jn\frac{\pi}{10} t = j\frac{3\pi}{10} t$ $\qquad jn\frac{\pi}{10} t = j\frac{\pi}{10} t$

$n = 3$ $\quad n = -3$ $\quad n = 1$ $\quad n = -1$

$D_3 = \frac{1}{2}$ $\quad D_{-3} = \frac{1}{2}$ $\quad D_1 = \frac{1}{4}$ $\quad D_{-1} = \frac{1}{4}$

$$D_n = \frac{1}{20} \int_{-10}^{10} \left[\frac{1}{2} e^{j3\pi/10t} + \frac{1}{2} e^{-j3\pi/10t} + \frac{1}{4} e^{j\pi/10t} + \frac{1}{4} e^{-j\pi/10t}\right] e^{-j\pi/10 nt}\, dt$$

A.1 (continued)

$$D_n = \frac{1}{20}\left[\frac{e^{j(3-n)\pi} - e^{-j(3-n)\pi}}{2j(3-n)\pi/10} + \frac{e^{j(3+n)\pi} - e^{-j(3+n)\pi}}{2j(3+n)\pi/10} + \frac{e^{j(1+n)\pi} - e^{-j(1+n)\pi}}{4j(1+n)\pi/10} + \frac{e^{j(1-n)\pi} - e^{-j(1-n)\pi}}{4j(1-n)\pi/10}\right]$$

$$-D_n = \frac{1}{\pi}\left[\operatorname{Sinc}\left[(3-n)\pi\right] + \operatorname{Sinc}\left[(3+n)\pi\right] + \frac{1}{2}\operatorname{Sinc}\left[(1+n)\pi\right] + \frac{1}{2}\operatorname{sinc}\left[(1-n)\pi\right]\right]$$

A.2 $x_2(t)$: $W_0 = \frac{2\pi}{T_0}$ where $T_0 = 20$, $W_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$D_n = \frac{1}{20}\left[\int_{-5}^{5} (1)\, e^{-jn\pi/10\, t}\, dt\right] = \frac{1}{20}\left[\frac{1}{-jn\pi/10} e^{-jn\pi/10\, t}\right]_{-5}^{5}$$

$$D_n = \frac{1}{20}\left[\frac{-10}{jn\pi} e^{-jn\pi/2} + \frac{10}{jn\pi} e^{jn\pi/2}\right] = \frac{1}{n\pi}\sin\left(\frac{\pi n}{2}\right)$$

$$D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$x_3(t)$: $T_0 = 40$ $\quad W_0 = \frac{2\pi}{40} = \frac{\pi}{20}$ | $D_n = \frac{1}{40}\int_{-5}^{5}$

$$(1)\, e^{-jn\pi/20\, t}\, dt \rightarrow D_n = \frac{1}{40}\left[\frac{1}{-jn\pi/10} e^{-jn\pi/10\, t}\right]_{-5}^{5}$$

$$D_n = \frac{1}{40}\left[\frac{-20}{jn\pi} e^{-jn\pi/4} + \frac{20}{jn\pi} e^{jn\pi/4}\right]$$

$$\therefore D_n = \frac{1}{n\pi}\operatorname{Sin}\left(\frac{n\pi}{4}\right)$$

A.3:

```
%A.3:
function [D1,D2,D3]=Lab3(d,n)

D1 = (1/2)*(abs(n)==3)+(1/4)*(abs(n)==1);
D2 = (1/n*pi).*(sin(pi*n/2));
D3 = (1/n*pi).*(sin(pi*n/4));

if (d == 1)
D1 = (1/2)*(abs(n)==3)+(1/4)*(abs(n)==1);
end

if (d == 2)
D2 = (1/n*pi).*(sin(pi*n/2));
end

if (d == 3)
D3 = (1/n*pi).*(sin(pi*n/4));
end

end
```

A.4 (a):

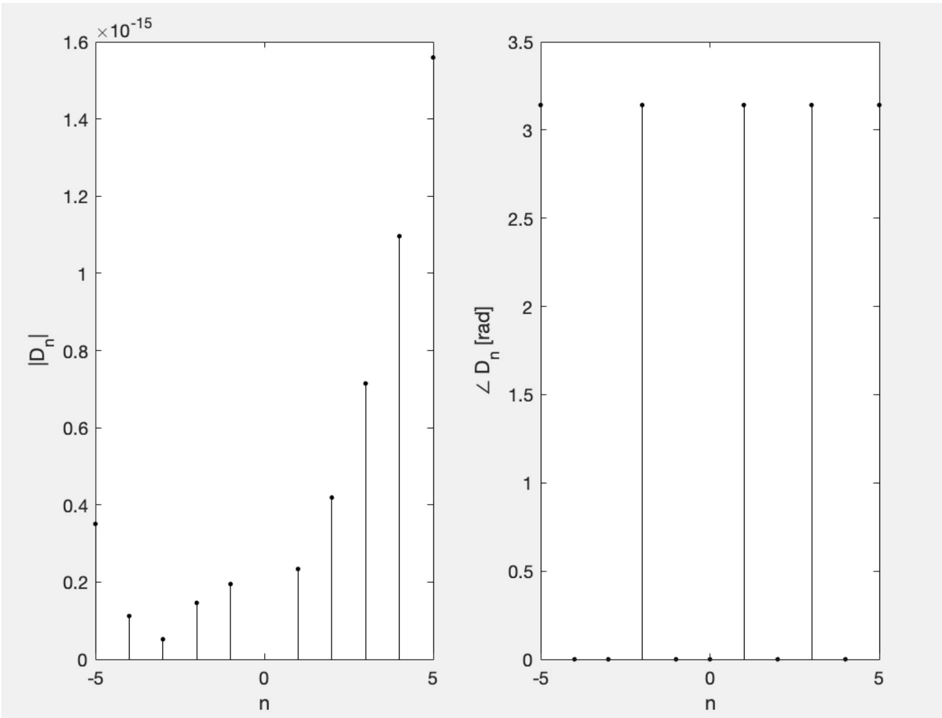
```
% A.4: (a)
%x1(t)
clf;
n = (-5:5);

D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi ))+(1./pi.*n).*sin((3+n).*pi)+(1./(2.*n.*pi).*sin((1+n).*pi))+(1./(2.*n.*pi).*sin((1-n).*pi));

subplot(1,2,1);
stem(n,abs(D_n),'.k');
xlabel('n');
ylabel('|D_n|');

subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n');
ylabel('\angle D_n [rad]');
```

x1(t):



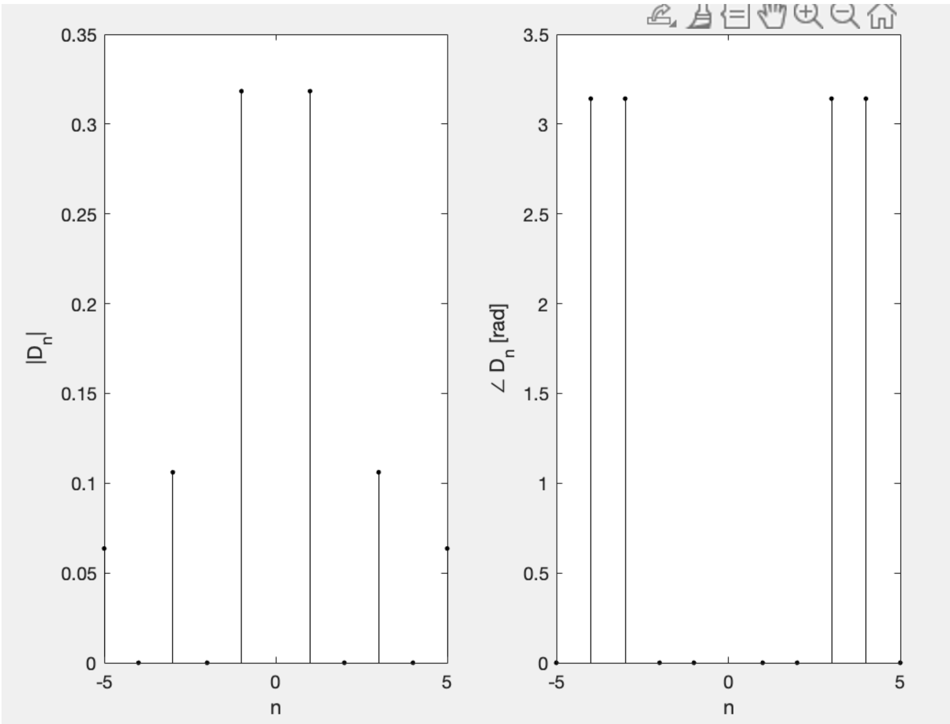

```
%x2(t)

clf;
n = (-5:5);

D_n = (1 ./(n.*pi).*sin((n.*pi)./2));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

x2(t):

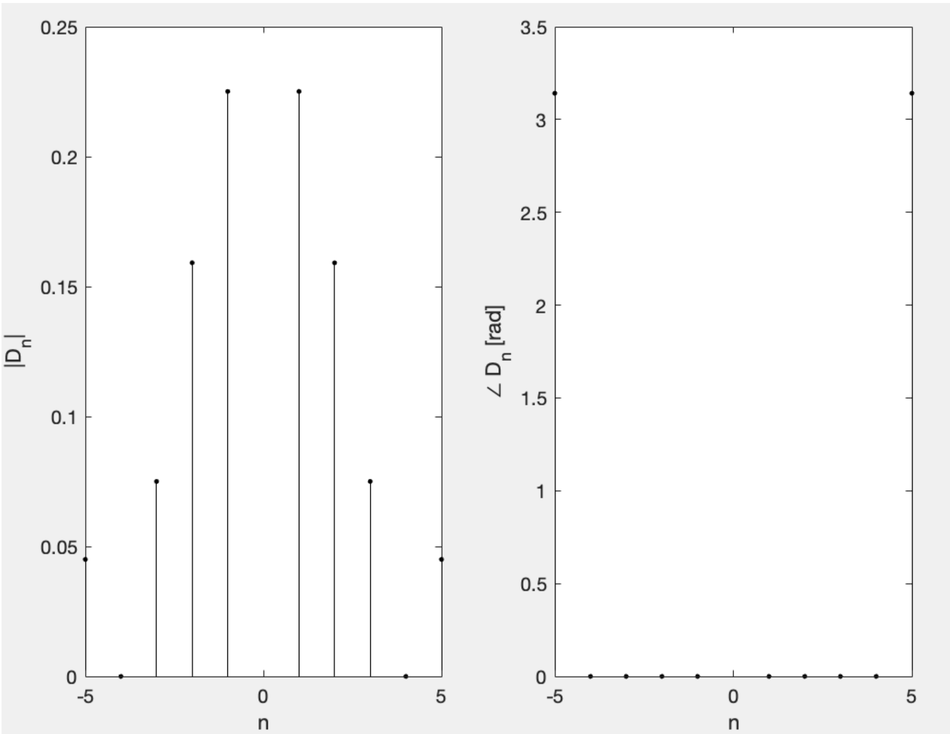



```
%x3(t)
clf;
n = (-5:5);

D_n = (1./(n.*pi).*sin((n.*pi)./4));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

x3(t):



A.4 (b):

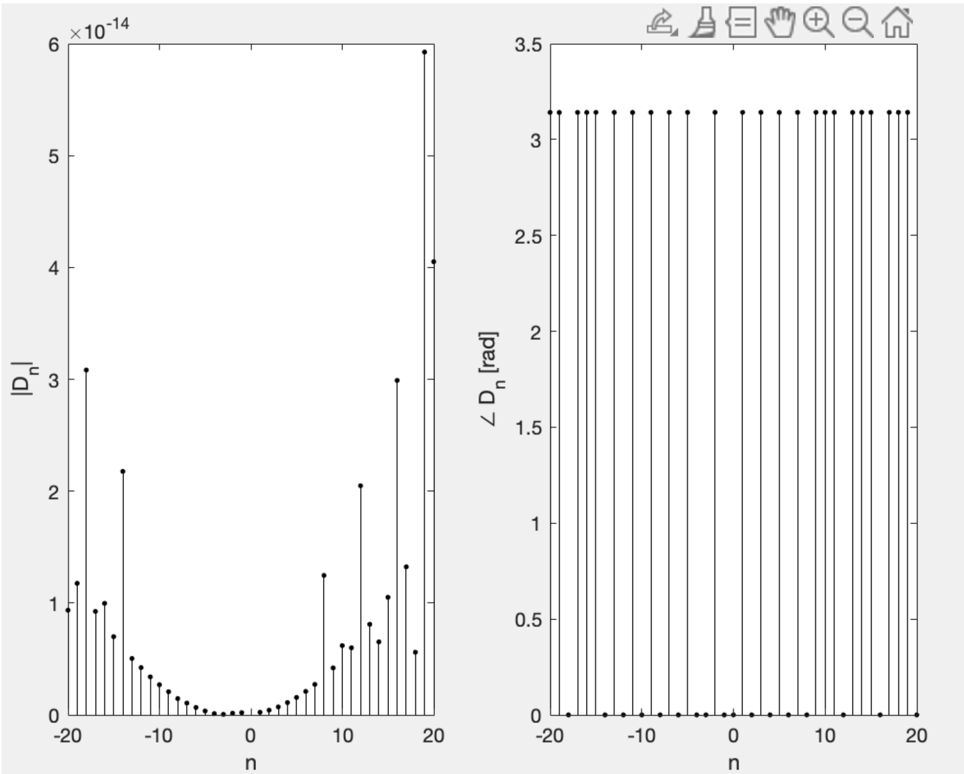
```
%x1(t)
clf;
n = (-20:20);

D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi ))+(1./pi.*n).*sin((3+n).*pi)+(1./(2.*n.*pi).*sin((1+n).*pi))+(1./(2.*n.*pi).*sin((1-n).*pi));

subplot(1,2,1);
stem(n,abs(D_n),'.k');
xlabel('n');
ylabel('|D_n|');

subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n');
ylabel('\angle D_n [rad]');
```

x1(t):

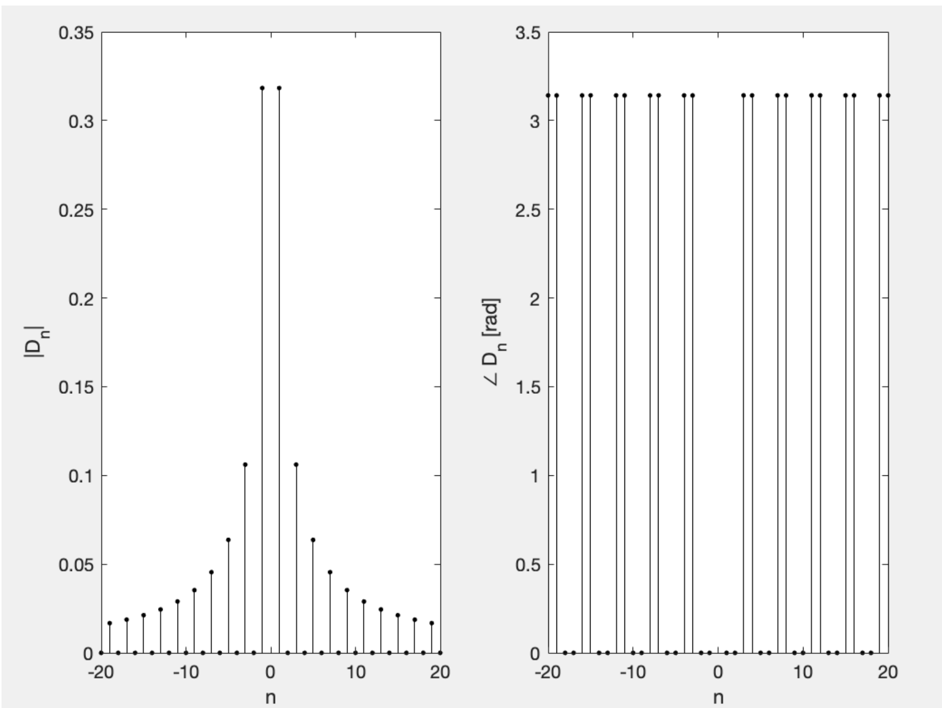



```
%x2(t)
clf;
n = (-20:20);

D_n = (1 ./(n.*pi).*sin((n.*pi)./2));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

x2(t):

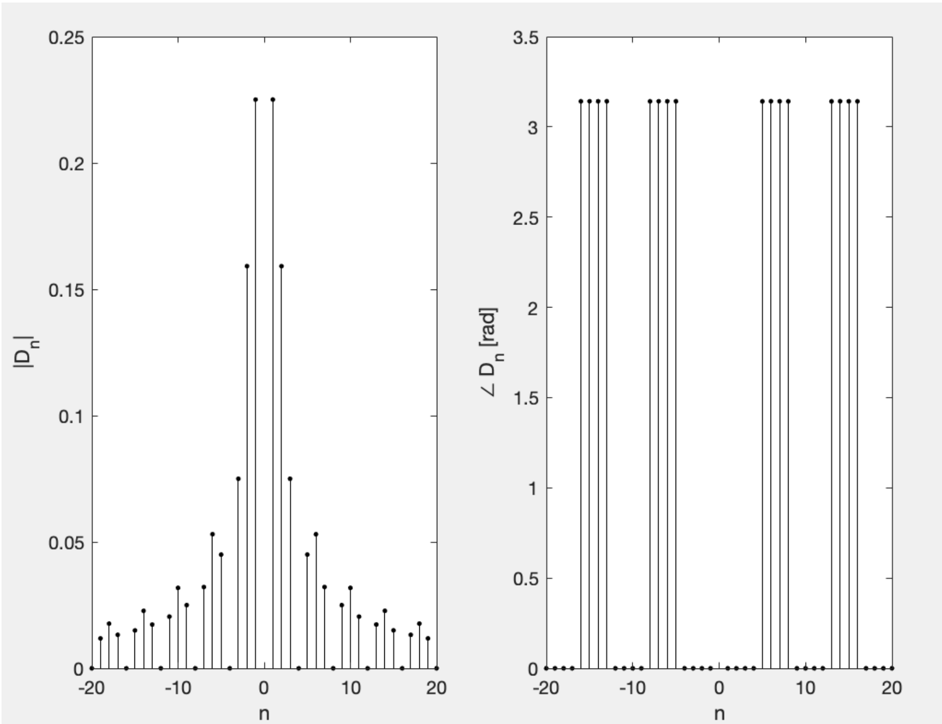



```
%x3(t)
clf;
n = (-20:20);

D_n = (1./(n.*pi).*sin((n.*pi)./4));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

x3(t):



A.4 (c):

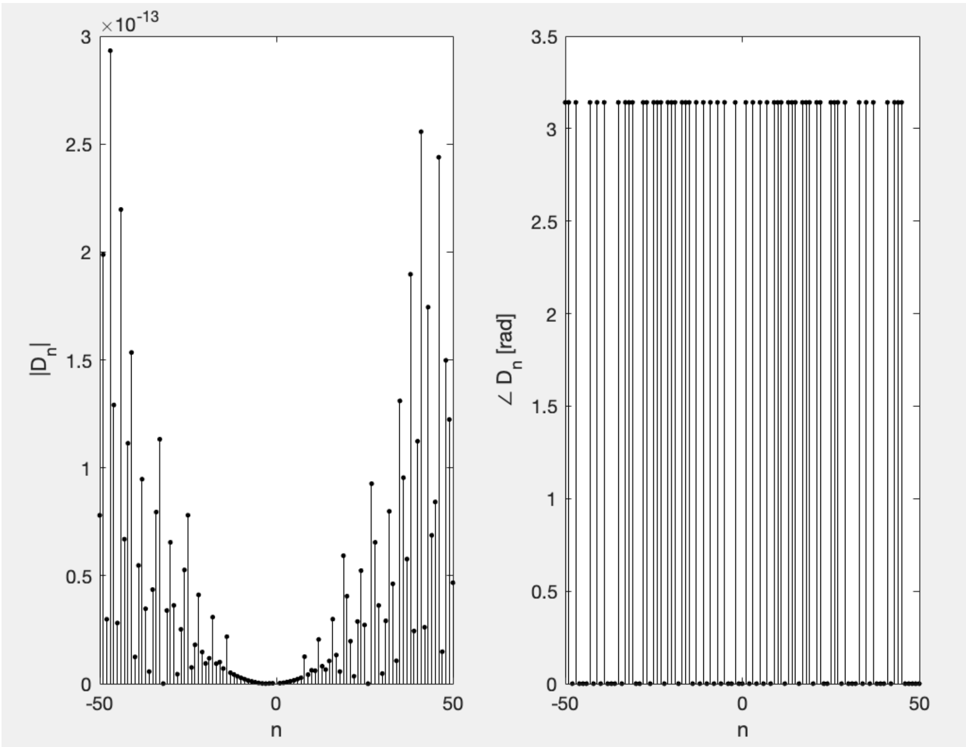
```
%x1(t)
clf;
n = (-50:50);

D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi ))+(1./pi.*n).*sin((3+n).*pi)+(1./(2.*n.*pi).*sin((1+n).*pi))+(1./(2.*n.*pi).*sin((1-n).*pi));

subplot(1,2,1);
stem(n,abs(D_n),'.k');
xlabel('n');
ylabel('|D_n|');

subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n');
ylabel('\angle D_n [rad]');
```

x1(t):

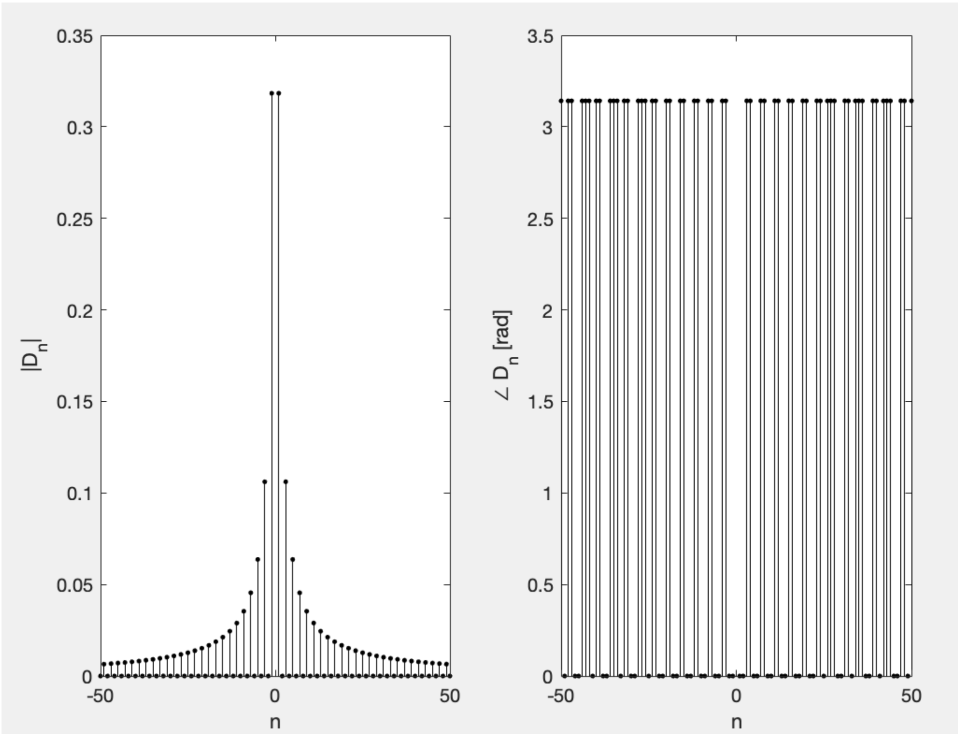



```
% %x2(t)
clf;
n = (-50:50);

D_n = (1 ./(n.*pi).*sin((n.*pi)./2));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

x2(t):

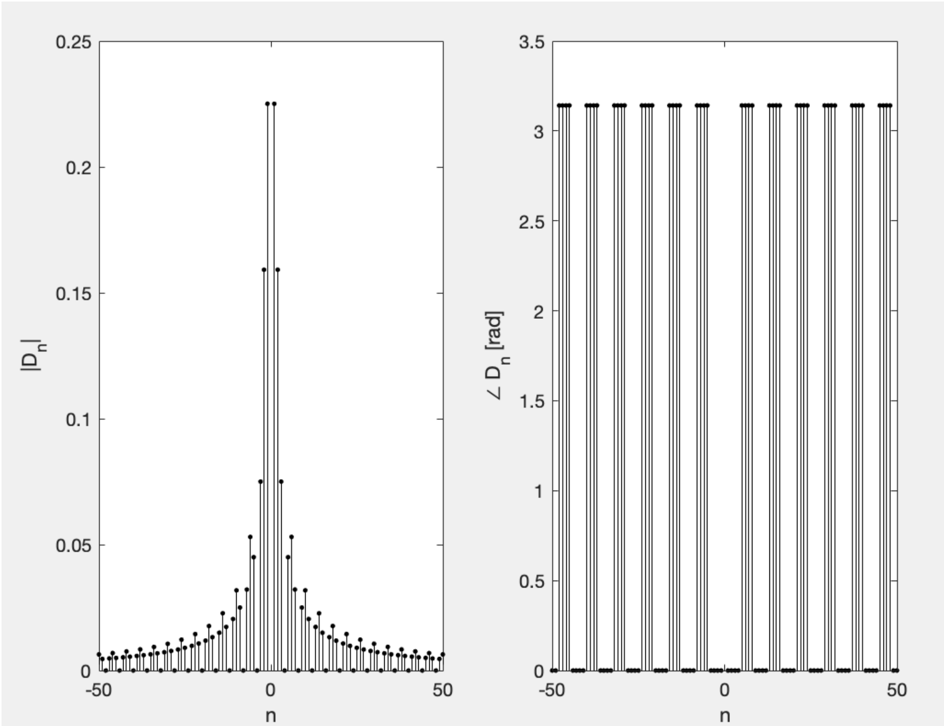



```
%x3(t)
clf;
n = (-50:50);

D_n = (1./(n.*pi).*sin((n.*pi)./4));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

x3(t):



A.4 (d):

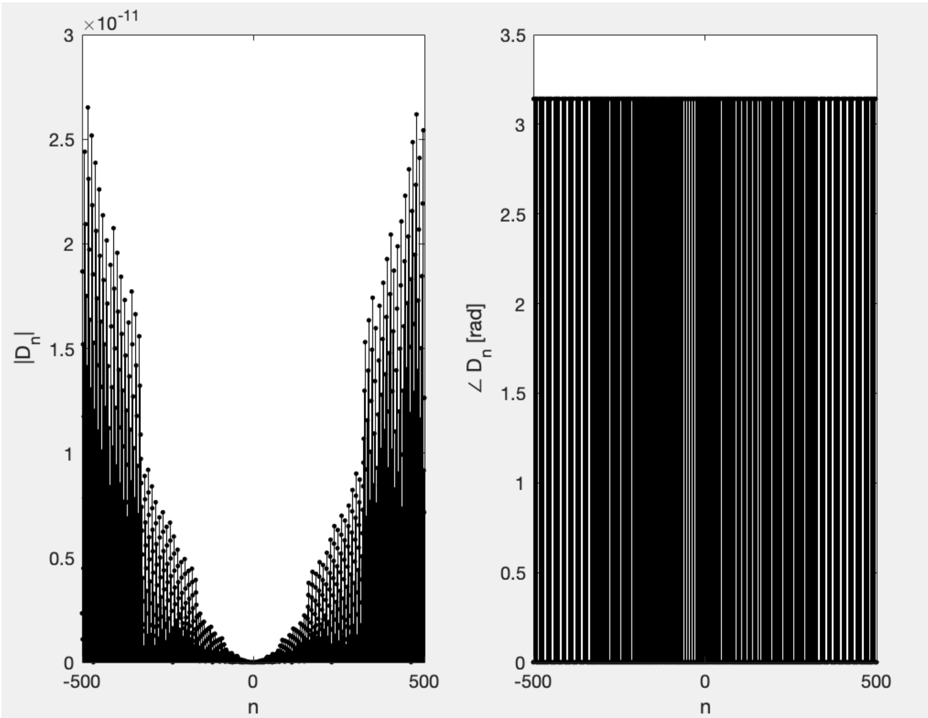
```
%x1(t)
clf;
n = (-500:500);

D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi ))+(1./pi.*n).*sin((3+n).*pi)+(1./(2.*n.*pi).*sin((1+n).*pi))+(1./(2.*n.*pi).*sin((1-n).*pi));

subplot(1,2,1);
stem(n,abs(D_n),'.k');
xlabel('n');
ylabel('|D_n|');

subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n');
ylabel('\angle D_n [rad]');
```

x1(t):

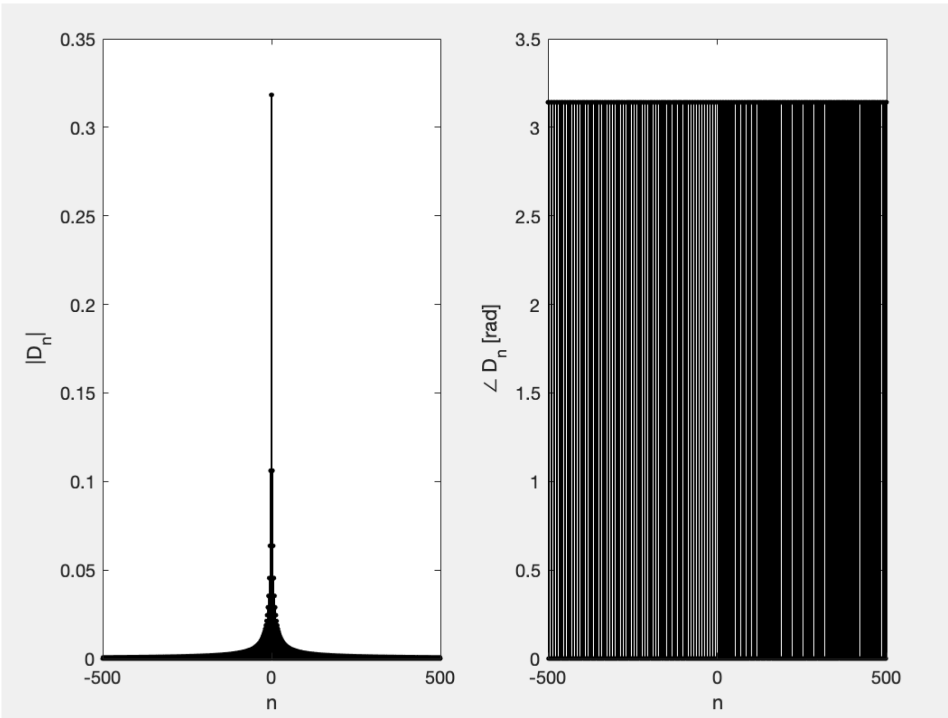



```
%x2(t)
clf;
n = (-500:500);

D_n = (1 ./(n.*pi).*sin((n.*pi)./2));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

x2(t):

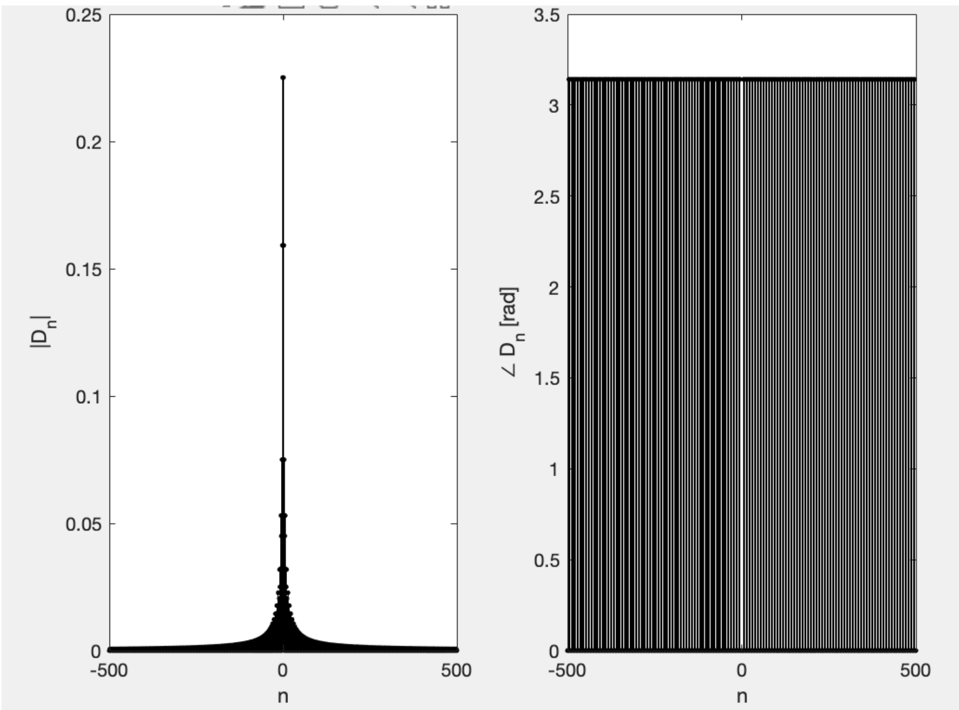



```
%x3(t)
clf;
n = (-500:500);

D_n = (1./(n.*pi).*sin((n.*pi)./4));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

x3(t):



A.5:

x1(t)

```
%A5 (x1):
function [D] = Lab3(Dn)
n=-500:500;
D=(1/2)*(abs(n)==3)+(1/4)*(abs(n)==1);
t=[-300:1:300];
w=pi*0.1;
x=zeros(size(t));
for i = 1:length(n)
 x=x+D(i)*exp(j*n(i)*w*t);
 't'
end

figure(5);
plot(t,x,'k')
xlabel('t(sec)');
ylabel('x(t)');
axis([-300 300 -1 2]);
title('Fourier Coefficients Reconstructed');
grid;
```

x2(t)

```
%A5 (x2):
function [D] = lab3(Dn)
n=-500:500;
D=0.5*sinc(n/2);
t=[-300:1:300];
w=pi*0.1;
x=zeros(size(t));
for i = 1:length(n)
 x=x+D(i)*exp(j*n(i)*w*t);
 't'
end

figure(5);
plot(t,x,'k')
xlabel('t(sec)');
ylabel('x(t)');
axis([-300 300 -1 2]);
title('Fourier Coefficients Reconstructed');
grid;
```

x3(t)

```
%A5 (x3):
function [D] = Lab3(Dn)
n=-500:500;
D=0.25.*(sinc(n/4));
t=[-300:1:300];
w=pi*0.1;
x=zeros(size(t));
for i = 1:length(n)
 x=x+D(i)*exp(j*n(i)*w*t);
 't'
end

figure(5);
plot(t,x,'k')
xlabel('t(sec)');
ylabel('x(t)');
axis([-300 300 -1 2]);
title('Fourier Coefficients Reconstructed');
grid;
```

A.6:

x1(t):

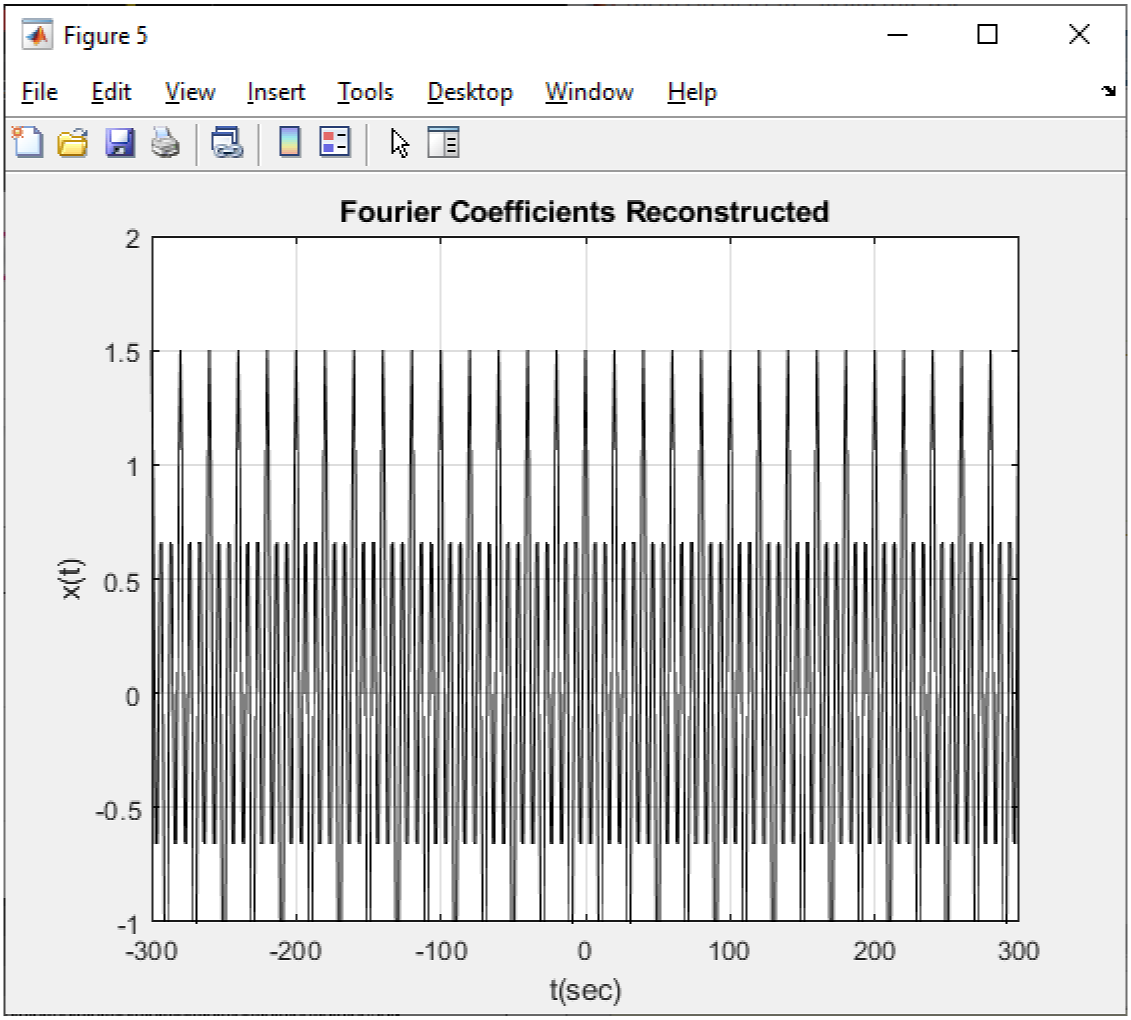
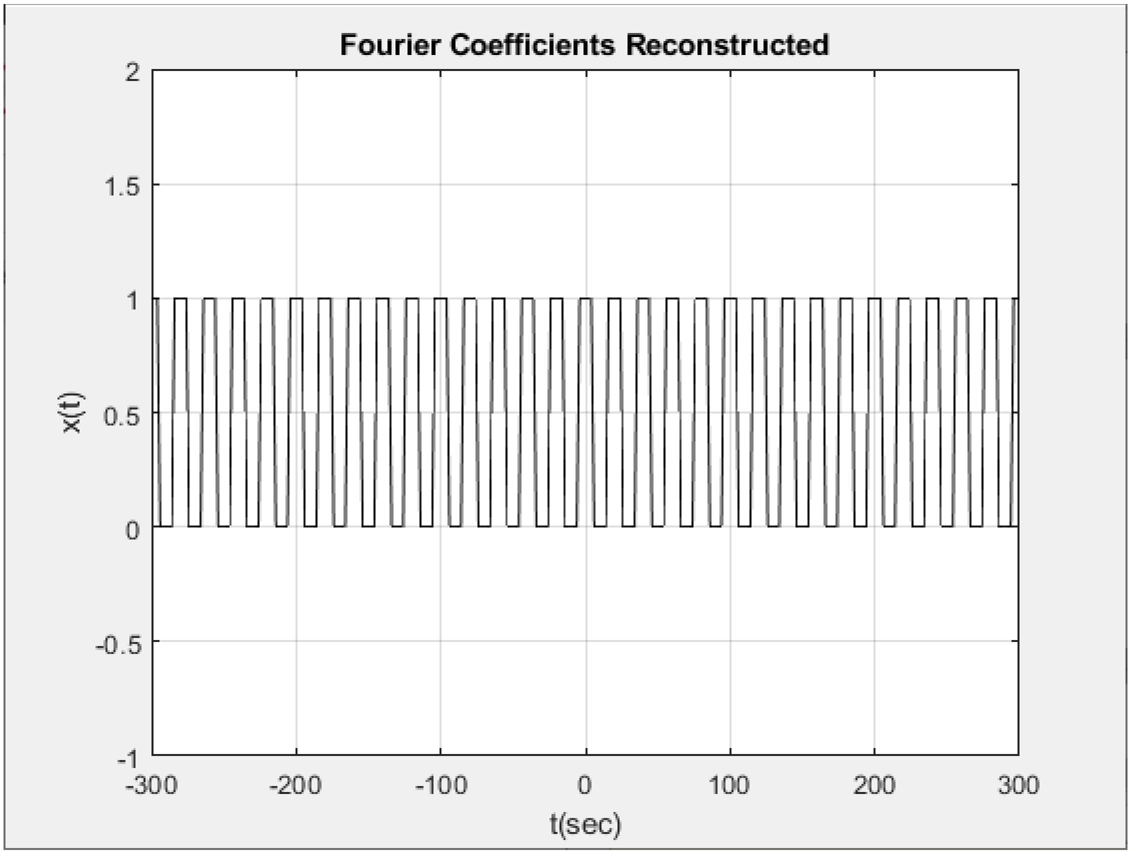
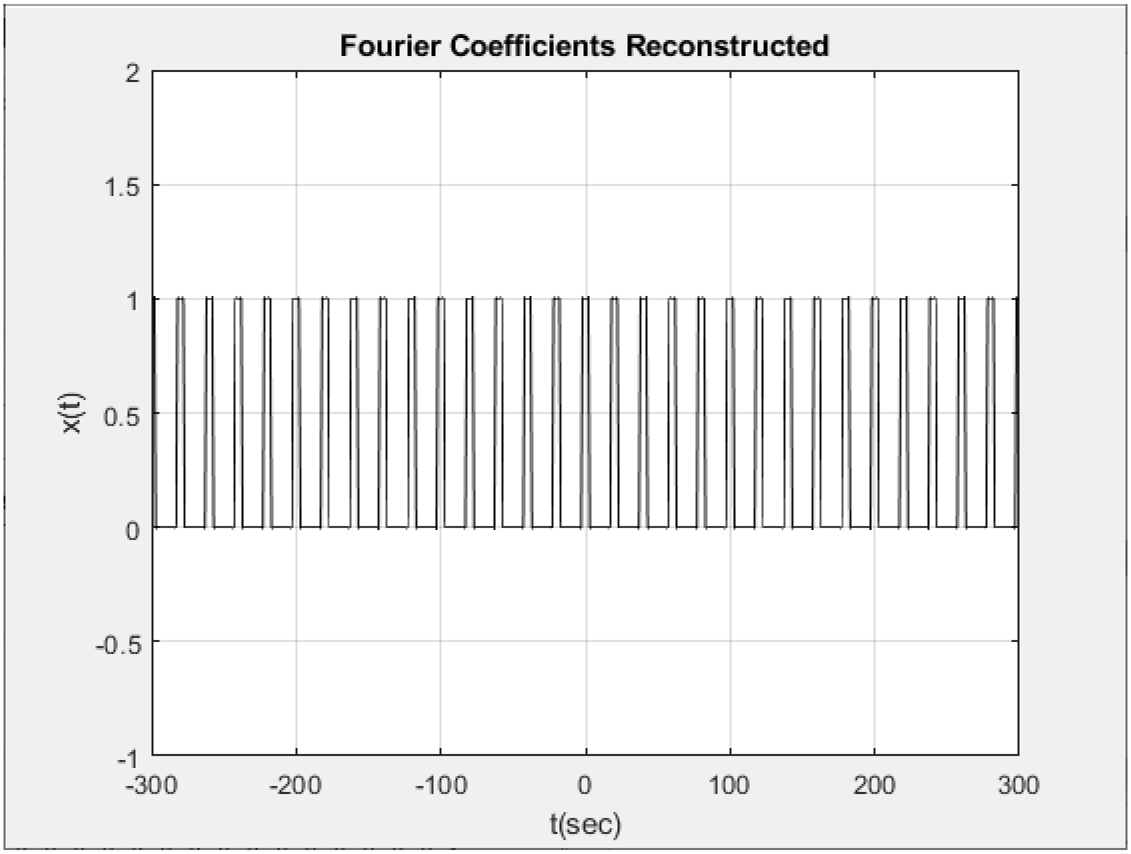


Figure 5
File Edit View Insert Tools Desktop Window Help
Fourier Coefficients Reconstructed
2
1.5
1
0.5
0
-0.5
-1
x(t)
-300
-200
-100
0
100
200
300
t(sec)

x2(t):


Fourier Coefficients Reconstructed
2
1.5
1
0.5
0
-0.5
-1
x(t)
-300
-200
-100
0
100
200
300
t(sec)

x3(t):



B.1:

x1(t) = cos $(3\pi/10)t + ½ \cos(\pi/10)t$,

$w_{o1} = 3\pi/10$
$w_{o2} = \pi/10$

GCF/LCM: $w_o = \pi/10 = 0.314$ rad/s

For x2(t)
$T_o = 20$ s

$T_o = 2\pi/w_o$

$w_o = \pi/10 = 0.314$ rad/s

For x3(t)
$T_o = 40$ s

$T_o = 2\pi/w_o$

$w_o = \pi/20 = 0.157$ rad/s

B.2:

The main difference between the fourier coefficients of $x_1(t)$ and $x_2(t)$ is that $x_1(t)$ has four distinct fourier series coefficients, while $x_2(t)$ has infinite fourier series coefficients for Dn. Additionally, one of them consists of sin ($x_2(t)$) function while the other consists of sinc ($x_1(t)$).

B.3:

The characteristics of the signals $x_2(t)$ and $x_3(t)$ having the same rectangular pulse shape, but different periods is reflected in their respective Fourier coefficients through the signal $x_3(t)$ having a smaller fundamental frequency value when compared to $x_2(t)$ for its Fourier coefficients.

B.4:

To derive D_o of $x_4(t)$ from D_o of $x_2(t)$ by letting $D_o = 0.5$ for signal $x_4(t)$ derived from $x_2(t)$.

B.5:

The reconstructed signals for $x_1(t)$ and $x_2(t)$ change as we increase the number of fourier coefficients, because $x_1(t)$ has a finite number of Dn values, nothing will change if the fourier coefficients are increased. Contrastingly, for $x_2(t)$ and $x_3(t)$, increasing values of Dn results in a higher accuracy.

B.6:

4 fourier coefficients are needed to perfectly reconstruct the periodic waveforms due to x1(t) having a finite range of Dn values. For x2(t) & x3(t), an infinite number of Dn would be required for perfect reconstruction.

B.7:

We know that a periodic signal has an infinite amount of Dn values so this is not viable. But, if it is finite such as $x_1(t)$, then the values of Dn can be stored. Although this also would not be efficient as you would end up wasting space for signals which have large amounts of finite Dn values.