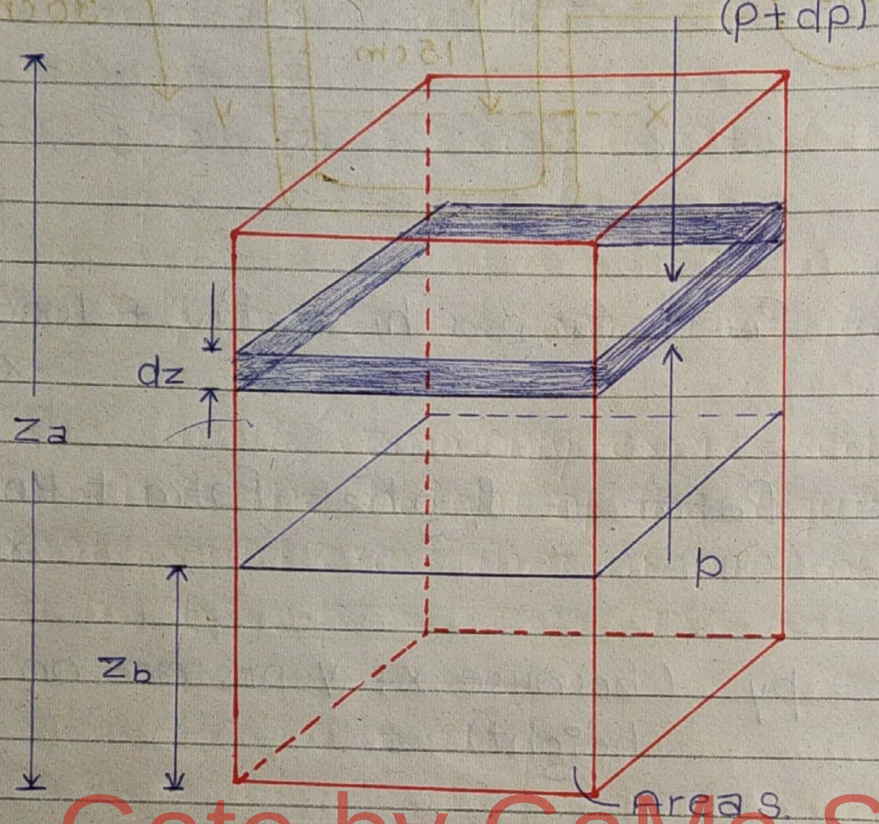
Fluid Static

Hydrostatic Equilibrim



In a stationary mass of a single static fluid the pressure is constant in any cross-section parallel to earth surface but it varies from height to height.

Consider a verticle element of fluid as shown in figure and assume that cross-sectional area is S.

At a height z above the base of the column, We assume an element of thickness dz.

The resultant of all forces on the small vol^m of fluid of height dz & cross-sectional area S must be zero.

3 verticle forces are acting on this element

→ The force from pressure p acting in an upward dir^n which is pS.

→ The force from pressure $p+dp$ acting in a downward dir^n which is $(p+dp)\cdot S$

→ Force of gravity acting downward which is $g\rho S dz$.

$$\Rightarrow pS - (p+dp)S - \rho g S dz = 0$$

$$\Rightarrow \boxed{dp + \rho g \, dz = 0.}$$

If density is constant,

$$\boxed{\frac{P}{\rho} + gz = 0.}$$

Barometric Equation

We know that,

$$\rho = \frac{PM}{RT}.$$

put the value of ρ in above eq^n.

$$\Rightarrow \frac{dp}{p} + g\frac{M}{RT} dz = 0$$

$$\Rightarrow \int_{pa}^{pb} \frac{dp}{p} = -\int_{za}^{zb} \frac{gM}{RT} dz.$$

$\Rightarrow$

$$\left[\ln p\right]_{p_a}^{p_b} = -\frac{gM}{RT}\left[z\right]_{z_a}^{z_b}$$

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$$\Rightarrow \boxed{\frac{p_b}{p_a} = \exp\left[-\frac{gM}{RT}(z_b - z_a)\right]}$$

This is known as Barometric equation.

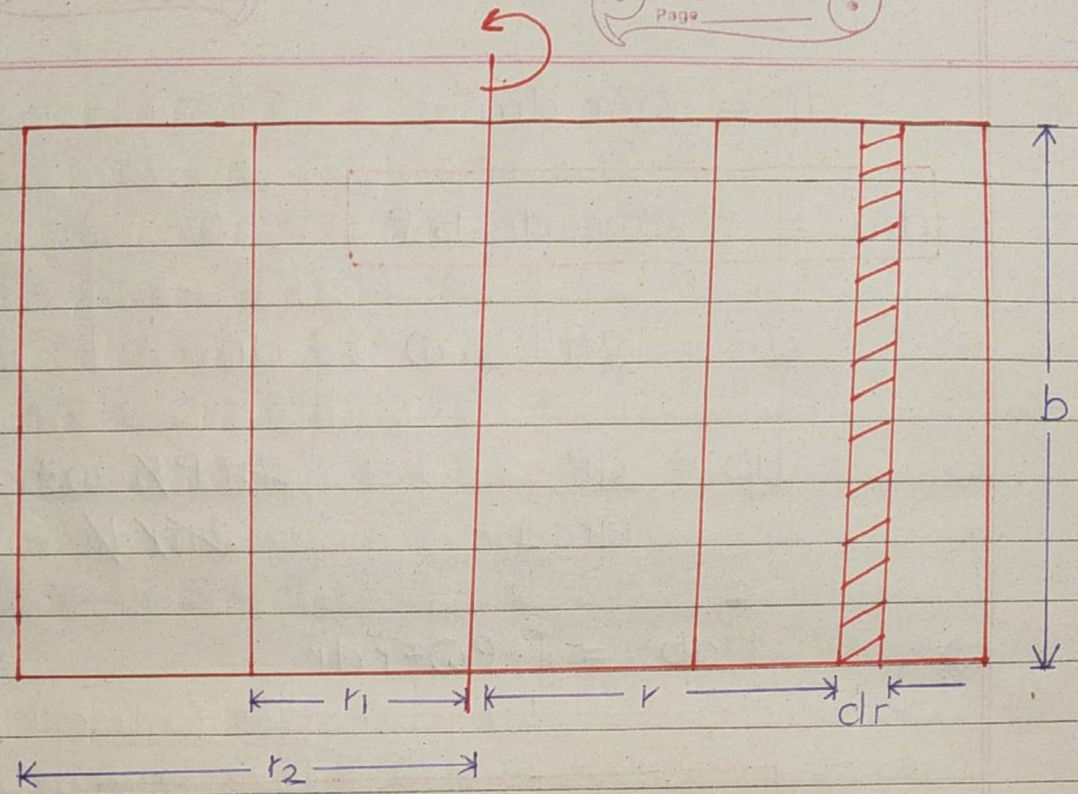
Pressure is commonly expressed as a head which is the height of the liquid column that would exert that amount of pressure at its base.

$$\boxed{z = \frac{P}{\rho g}}$$

Hydrostatic Equilibrim in a centrifugal field

In a rotating centrifuse a layer of liquid is thrown outward from the axis of rotation & is held against the wall of the bowl by centrifugal force.

The free surface of the liquid takes the shape of a paraboloid of revolution but in industrial centrifuses the rotational speed is so high & centrifugal force so much greater than the force of gravity that the liquid is virtually cylindrical.



r_1 = radial distance from the axis of rotation to the free liquid surface

r_2 =

The entire mass of liquid indicated in figure is rotating as a rigid body with no sliding of one layer of liquid over another.
Under these conditions the pressure distribution in liquid may be found from the principle of fluid static.

The pressure drop over any ring of rotating liquid is calculated in a following way :

Consider the ring of liquid as shown in figure & the vol^m element of thickness dr at a radius r.

$$dF = \omega^2 r\, dm$$

$$\boxed{dm = 2\pi r \cdot dr \cdot b\rho}$$

$$\Rightarrow \quad dF = 2\pi\rho b \cdot \omega^2 r^2\, dr$$

$$\Rightarrow \quad dp = \frac{dF}{\text{Area}} = \frac{2\pi\rho \cancel{b} \cdot \omega^2 r^{\cancel{2}} dr}{2\pi \cancel{r} \cancel{b}}$$

$$\Rightarrow \quad \int_1^2 dp = \int_1^2 \rho\omega^2 r\, dr$$

$$\Rightarrow \quad \boxed{p_2 - p_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2)} \quad [\because \rho = \text{constant}]$$

Ques(13):- The liq. surface in a cylindrical bucket of radius
2007:- R rotating about its axis acquire a parabolic profile given by the equation $\boxed{y = a + br^2}$ where y is the height of the liquid surface from the bottom of the bucket at a radial distance r from the bucket axis.
If the liq. has density ρ then the mass of the liquid in the bucket is.

(i) $\pi\rho R^2\left(\frac{a+bR^2}{2}\right)$ (ii) $\pi\rho R^2\left(a + \frac{bR^2}{2}\right)$

(iii) $\pi\rho R^2 a$ (iv) $\pi\rho R^2(a+bR^2)$

$$dM = 2\pi r\, dr \cdot y \cdot \rho$$

$$\Rightarrow \int_0^M dM = 2\pi\rho\int_0^R (a+br^2) r \cdot dr$$

$$M = 2\pi\rho \int_0^R (ar + br^3)\, dr$$

$$\Rightarrow M = 2\pi\rho \left[\frac{ar^2}{2} + \frac{br^4}{4} \right]_0^R$$

$$\Rightarrow M = 2\pi\rho \left[\frac{aR^2}{2} + \frac{bR^4}{4} \right]$$

$$\Rightarrow M = \cancel{2}\pi\rho \frac{R^2}{\cancel{2}} \left[a + \frac{bR^2}{2} \right]$$

$$\Rightarrow \boxed{M = \pi\rho R^2 \left[a + \frac{bR^2}{2} \right]}$$

Ques(14) :- A centrifugal filteration unit operating
2004 :- at a rotational speed of ω has inner surface of the liquid (density $= \rho_L$), located at a radial distance R from the axis of rotation. The thickness of the film is δ & no cake is formed. The initial pressure drop during filtration is

(i) $\frac{1}{2}\omega^2 R^2 \rho_L$ (ii) $\frac{1}{2}\omega^2 \delta^2 \rho_L$

(iii) $\frac{1}{2}\omega^2 \delta \rho_L (2R+\delta)$ (iv) $\frac{1}{2}\omega^2 R^2 \rho_L (R+2\delta)$

$$dp = \frac{dF}{2\pi r b} = \frac{\cancel{2\pi} r^{\cancel{2}} \omega^2 \rho\, dr\, \cancel{b}}{\cancel{2\pi r b}}$$

$$p_2 - p_1 = \rho\omega^2 \int_{r_1}^{r_2} r\, dr = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2)$$

$$\Rightarrow r_2 = R+\delta \;;\; r_1 = R$$

$$\Delta p = \frac{\rho\omega^2}{2}\left[(R+\delta)^2 - R^2\right]$$

$$\Delta p = \frac{\rho\omega^2}{2}\left[\cancel{R^2} + \delta^2 + 2R\delta - \cancel{R^2}\right]$$

$$\boxed{\Delta p = \frac{\rho\omega^2}{2}\cdot\delta\,(2R+\delta)}$$

continuous gravity decantor is used for the continuous separation of two immscible liquid of different densities.

When the difference b/w the densities of the two liquid is very small then this is separated by centrifugal decantor.

Ideal fluid

Ideal fluid is one that is incompressible & has zero viscosity. The flow of such an ideal fluid is known as potential flow & is completly described by the principle of Newtonian mechanics & conservation of mass.

Ques(15):- The value of Reynolds number for ideal fluid will be

(i) 2100 (ii) 0 (iii) ∞ (iv) notdefined.

$\Rightarrow \boxed{Re = \frac{Dv\rho}{\mu}}$ $\because \mu = 0 \Rightarrow \boxed{Re = \infty}$

Reynolds number & transition from laminar to turbulent flow

We know that,

$$\boxed{Re = \frac{D\bar{v}\rho}{\mu} = \frac{D\bar{v}}{\nu}}$$

where, D = diameter

$\bar{v}$ = average velocity of the liquid.

ν = kinematic viscosity of liquid.

→ for Re NO < 2100 flow is laminar

→ for Re NO; $2100 <$ Re NO < 4000 transition

→ for Re NO > 4000 flow is Turbulent.

* Turbulence may be generated in other way by flow through a liquid.
In general, it can result either from contact of the flowing stream with solid boundaries or from contact b/w two layers of fluid moving at different velocities. The first kind of turbulence is called wall turbulence & second kind turbulence is called free-turbulence.

* Free turbulence is specially important in mixing.

* Turbulent flow consist of mass of eddies of various sizes co-existing in the flowing

Stream. Large eddies are continually formed. They breakdown into smallest eddies. At a given time & in a given volm, a wide spectrum of eddy size exist.

* Flow within a eddy is laminar.
* Since, even the smallest eddy contain 10^{12} molecules, all eddies are of macroscopic size & turbulent flow is not a molecular phenomena.

viscous heat dissipation.

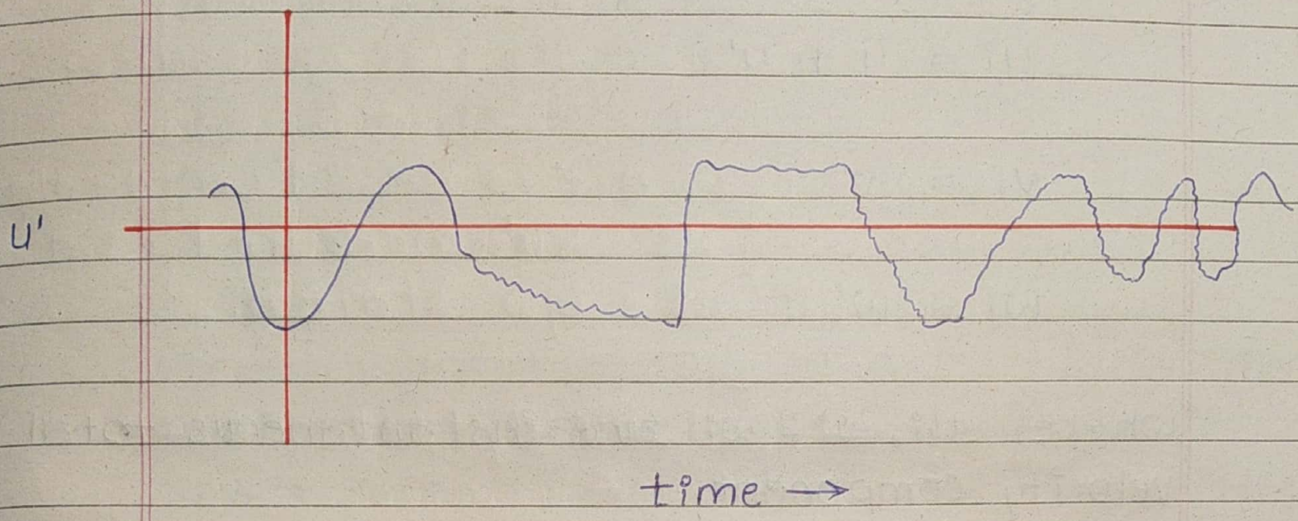
Any given eddy having a definate amount of mechanical energy. The energy of the largest eddy is supplied by the potential energy of the bulk flow of the fluid.

From an energy point of view, turbulence is a transfer process in which large eddies formed from bulk flow, passed their energy of rotation along a continuous series of smaller eddies.

This mechanical energy is not appreciably dissipated into heat during the breakup of large eddies into smaller ones but is passed almost to the smallest eddies.

It is finally converted into heat when the smallest eddies are destroyed by viscous action.
Energy conversion by this action is known as viscous heat dissipation.

Deviating velocity in turbulent flow



A typical picture of the variations in the instantaneous velocity at a given point in a turbulent flow field.

⇒ The instantaneous local velocities at a given point can be measured by Laser Doppler Anemometer.

Local velocities can be analyzed by spiliting each component of the total instantaneous velocity into two parts — one a constant part that is time average or mean value of the component in the dir^n of the flow of the stream & the other one is deviating velocity, the instantaneous fluctuation around the mean.

Assume three components of the instantaneous velocity in dir^n x, y, z be u_i, v_i & w_i respectively.

Also assume that x is oriented in the

dir^n of flow of the stream.

$$u_i = u + u'$$

$$v_i = v'$$

$$w_i = w'$$

where, u_i, v_i & w_i are instantaneous total velocity component.

u = constant velocity of stream in x-dir^n.

u', v' & w' = deviating velocities in x, y, z dir^n.

In similar way,

$$\boxed{p_i = p + p'}$$

where, p_i = variable local pressure

p = constant pressure

p' = fluctuating part due to eddies.

* Because of the random nature of the fluctuations, the time average of the fluctuating components of velocity & pressure vanish when averaged over a time period t_0. (in order of few seconds)

$$\frac{1}{t_0}\int_0^{t_0} u'\,dt = \frac{1}{t_0}\int_0^{t_0} v'\,dt = \frac{1}{t_0}\int_0^{t_0} w'\,dt = \frac{1}{t_0}\int_0^{t_0} p'\,dt = 0.$$

for every positive value of a fluctuation there is an equal negative value & the algebric sum is zero.

The time average of the mean square of any of the velocity component is not equal to zero.

$$\Rightarrow \boxed{\frac{1}{t_0}\int_0^{t_0} (u')^2\,dt = (\overline{u'})^2}$$

In laminar flow, there are no eddies, the deviating velocities & pressure fluctuation donot exist.

Intensity & scale of Turbulence

Turbulence fields are characterised by two parameters. The first measure the intensity of field & refers to the speed of rotation of eddies, and the energy contained in an eddy of specific size.

Intensity is measured by the root mean

Square velocity of the component.

→ The second measures the size of eddies.

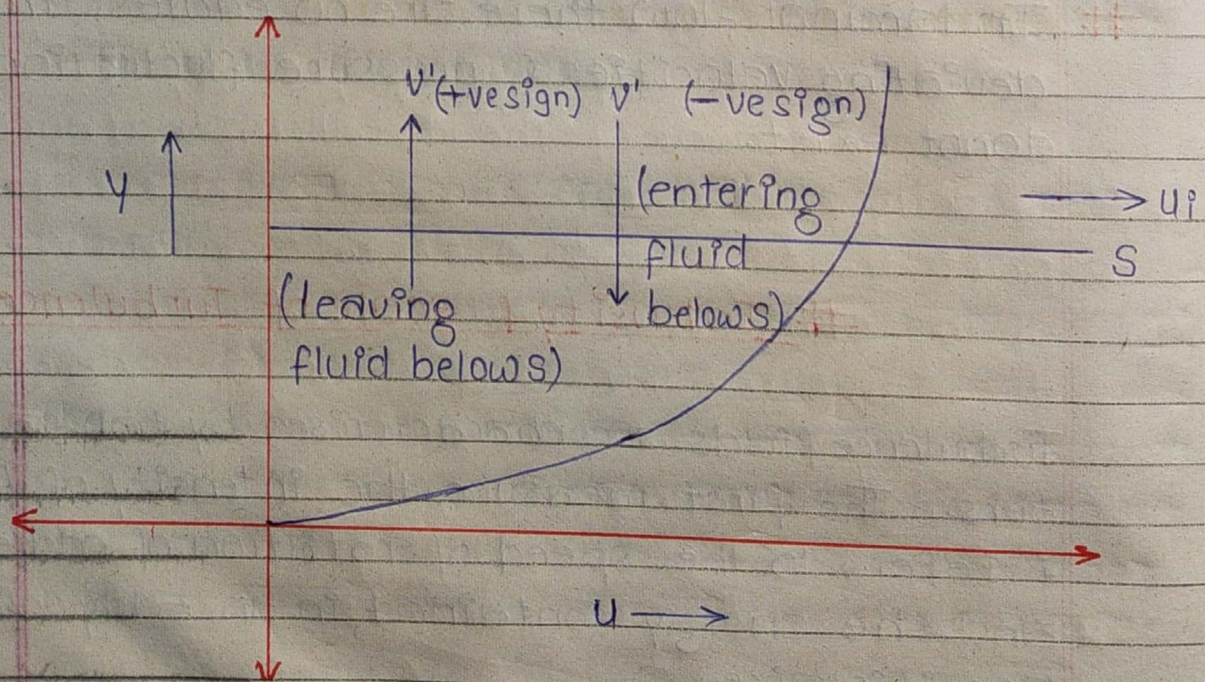
→ The scale of turbulence is based on co-relation coefficient such as $R_{u'}$ measured as a func of the distance b/w station.

$$\boxed{L_y = \int_0^{\infty} R_{u'}\, dy}$$

where,

$$\boxed{R_{u'} = \frac{\overline{u_1' u_2'}}{\sqrt{(\bar{u}_1')^2 \quad (\bar{u}_2')^2}}}$$

Reynold's stresses or turbulent shear stress



$$\boxed{\tau_t = \rho \overline{v'u'}}$$

$$\Rightarrow \quad \tau_t = E_v \frac{du}{dy}$$

quantity E_v is analogous to μ & this is known as eddy viscosity.

$$\Rightarrow \quad \boxed{\varepsilon_M = \frac{E_v}{\rho} = \text{eddy diffusivity of momentum}}$$

Ques(16):- In an experimental data of turbulence flow that were recorded at 1 second interval at a particular point. Determine the value of reynold shear stress.

u (m/s)	22	34	12	42	17	29
v (m/s)	7	−8	−4	10	7	−6

Given : density = $1.2 \ kg/m^3$

$$\tau_t = \rho \overline{v'u'}$$

also, $\bar{u} = u + u'$

$$\Rightarrow \quad u' = u - \bar{u}$$

$$\bar{u} = \frac{22 + 34 + 12 + 42 + 17 + 29}{6}$$

$$\boxed{\bar{u} = 26 \ m/s}$$

$$\bar{v} = \frac{7 + (-8) + (-4) + 10 + 7 + (-6)}{6}$$

$$\boxed{\bar{v} = 1 \ m/s}$$

$u'(u-\bar{u})$	$v'(v-\bar{v})$	$u'v'$
-4	6	-24
8	-9	-72
-14	-5	70
16	9	144
-9	6	-54
3	-7	-21

$$\overline{v'u'} = \frac{-24-72+70+144-54-21}{6}$$

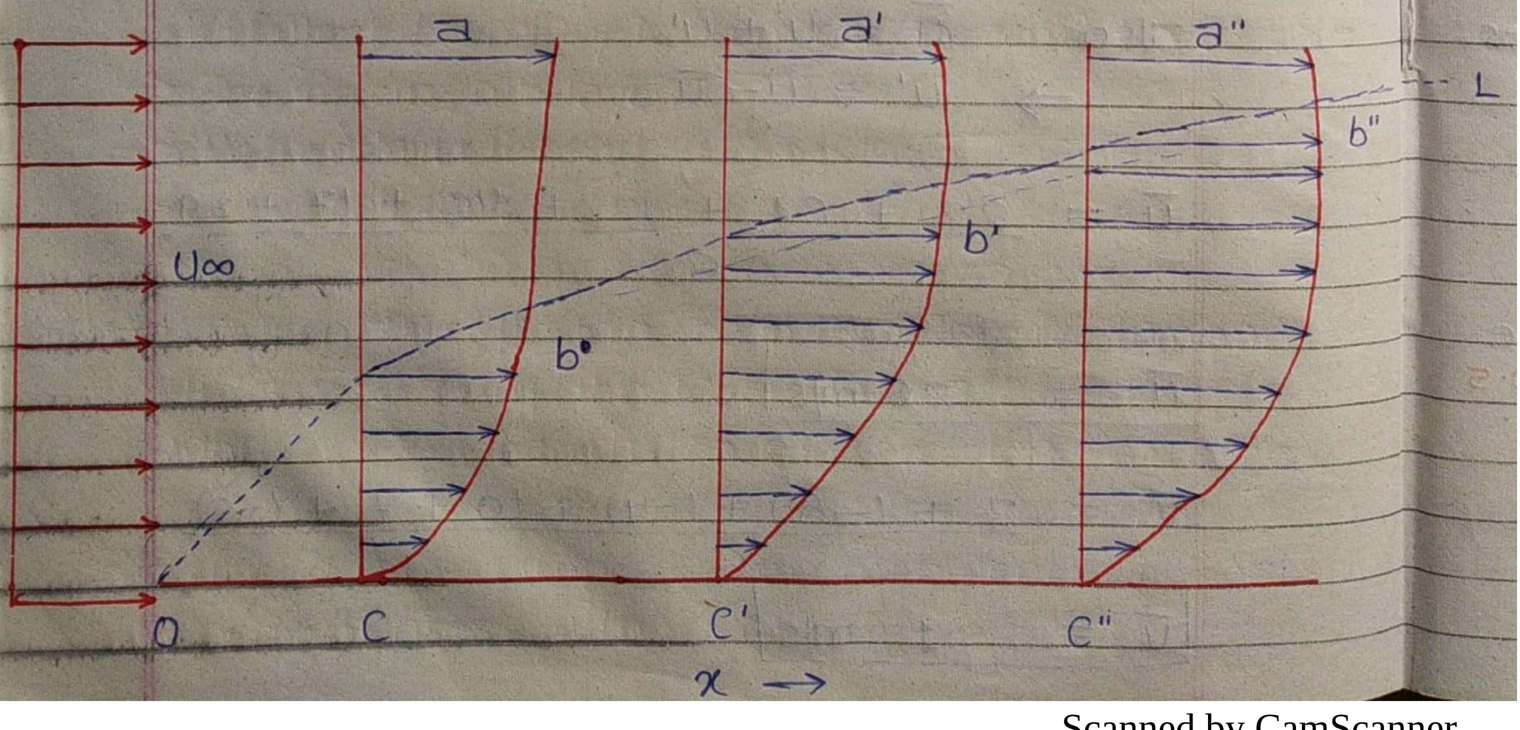
$$\boxed{\overline{v'u'} = 7.167}$$

$$\tau_t = \rho\,\overline{v'u'}$$

$$\tau_t = 1.2 \times 7.167$$

$$\boxed{\tau_t = 8.6 \ N/m^2}$$

Boundary layer



Date
Page

OL :- outer limit of boundary layer.

abc, a'b'c', a"b"c" :- curves of velocity vs distance from wall at c, c' & c".

x = distance from leading edge.

u_∞ = velocity of undisturbed stream.

Z_x = Thickness of boundary layer at distance x.

u = local velocity.

⇒ **Boundary layer** :- A boundary layer is defined as that part of a moving fluid in which the fluid motion is influenced by the presence of a solid boundary.

The velocity of fluid at interface b/w the solid & fluid is zero. Each of these curves corresponds to a definate ~~curve~~ value of x.

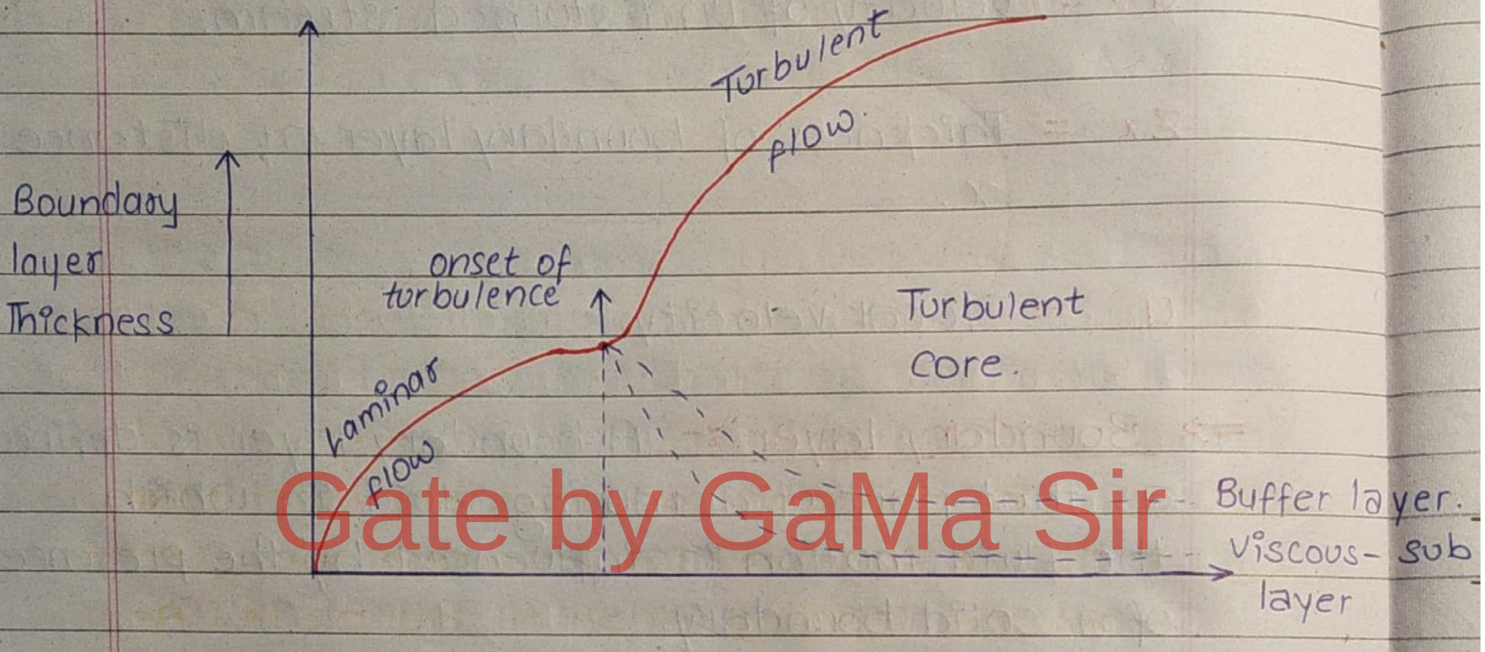
* The velocity increases with distance from the plate.

* The curve change slope rapidly near the plate.

* They also show that the local velocity approaches asynptotically the velocity of the bulk of the fluid stream.

* Line OL represents an imaginary surface that separate the fluid stream into two parts — one which fluid velocity is constant & the other in which velocity varies from 0 to 99 % of the free stream velocity.

Laminar & turbulent flow in boundary layer



Turbulent boundary is consider to consist of 3 zones:

→ The viscous sub-layer

→ Buffer layer

→ Turbulent zone

Imp * When the flow in boundary layer is laminar the thickness Z_x of the layer increases with $(x)^{0.5}$ where x is the distance from leading edge.

* For a short time, after turbulence appears Z_x increases with $(x)^{1.5}$

Date
Page

* After turbulence is fully developed z_x increases with $(x)^{0.8}$.

⇒ In laminar flow,

$$\boxed{\frac{\delta}{x} = \frac{4.67}{\sqrt{Re_x}}}$$

Where, δ = Boundary layer thickness.

$$\boxed{Re_x = \frac{\rho u_\infty \cdot x}{\mu}}$$

u_∞ = bulk stream velocity.

ver. Ques(17) :- The thickness of boundary layer (laminar)
sub 2017 :- over a flat plat varies along the distance from the leading edge of the plate. As the distance increases, the boundary layer thickness
(i) Decreases (ii) Intially inc. & then dec.
(iii) Initially dec & then inc. (iv) Increases

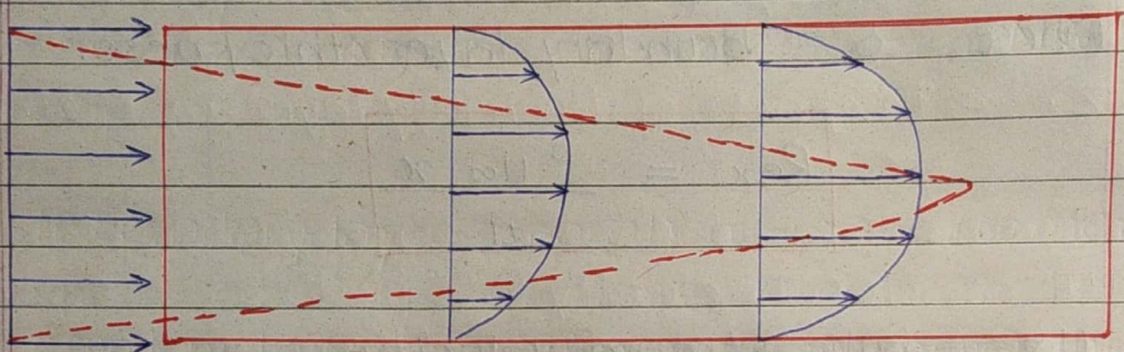
Ques(18) :- For uniform laminar flow over a flat plate the
2016 :- thickness of boundary layer δ at a distance x from the leading edge of the plate follows the relation.
(i) $\delta(x) \propto x^{-1}$ (ii) $\delta(x) \propto x$
(iii) $\delta(x) \propto x^{1/2}$ (iv) $\delta(x) \propto x^{-1/2}$

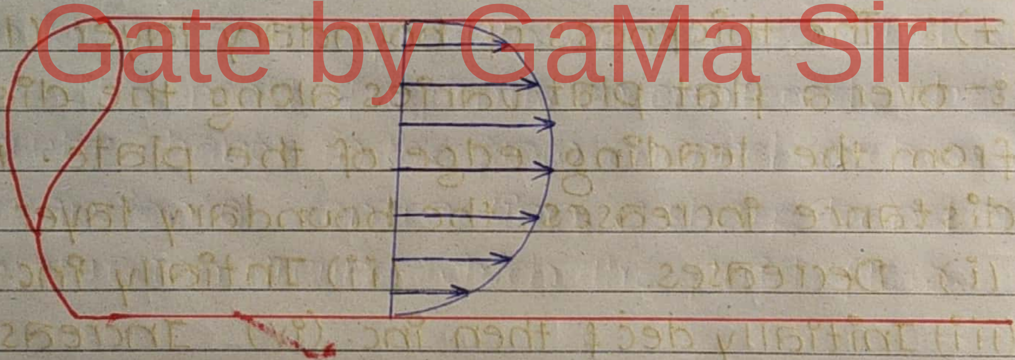
Ques(19) :- In the laminar boundary layer flow over a flat plate the ratio δ/x is directly proportional to
(i) Re (ii) $\sqrt{Re}$ (iii) $1/Re$ (iv) $1/\sqrt{Re}$

Ques(20):- for flow past a flat plate if x is the distance
2002:- along the plate in the dir^n of flow the boundary layer thickness is proportional to.

(i) $\sqrt{x}$ (ii) x (iii) $1/\sqrt{x}$ (iv) $1/x$

Development of boundary layer flow in pipe





Consider a straight thin walled tube with fluid entering it at a uniform velocity.

* A boundary layer begins to forms at the entrance of the tube & as the fluid moves through the channel the boundary layer thickness increases.

* During initial stage, the boundary layer occupies only a small cross-section of the tube.

* In the boundary layer the velocity increases from zero at the wall to the constant velocity existing in the core.

* As the stream moves farther down the tube

the boundary layer occupies an ~~interesting~~ increasing portion of the cross-section.

* Finally, at a point in the down stream from the entrance the boundary layer reaches the centre of the tube & boundary layer occupies the entire cross-section of the stream.
* At this point the velocity distribution in the tube reaches it final form & remain unchanged during the remaining length of the tube. Such flow with an unchanging velocity distribution is known as fully developed flow.

Transition length for laminar & turbulent flow

The length of the entrance region of the tube necessary for the boundary layer to reach centre of the tube & for fully developed flow to be established is known as transition length.

* The approximate length for straight pipe necessary for completion of the final velocity distribution is

⇒ for laminar flow; $\boxed{\dfrac{x_t}{D} = 0.05\, Re}$

where,

x_t = transition length

D = Diameter of pipe

Re = Reynold's number.

⇒ If the fluid entering the pipe is turbulent & velocity in the tube is above critical, the transition length is nearly independent of Reynold's number.

Ques(21):- Given a pipe of diameter D, the entrance length
2018:- necessary to achieve fully developed flow is proportional to (NRe is the reynold's number)

(i) $D\,N_{Re}$ (ii) $\frac{D}{N_{Re}}$ (iii) $D\,N_{Re}^2$ (iv) $\frac{D}{N_{Re}^2}$

Boundary layer separation & wake formation

* Boundary layer separation occurs whenever the change in velocity of the fluid in either magnitude or dir^n is too large for the fluid to adhere to solid surface.
* It is most frequently encountered, there is an abrubt change in the flow channel such as a sudden expansion or contraction, a sharp bend or an obstruction around which the fluid flow.

→ **Minimization**

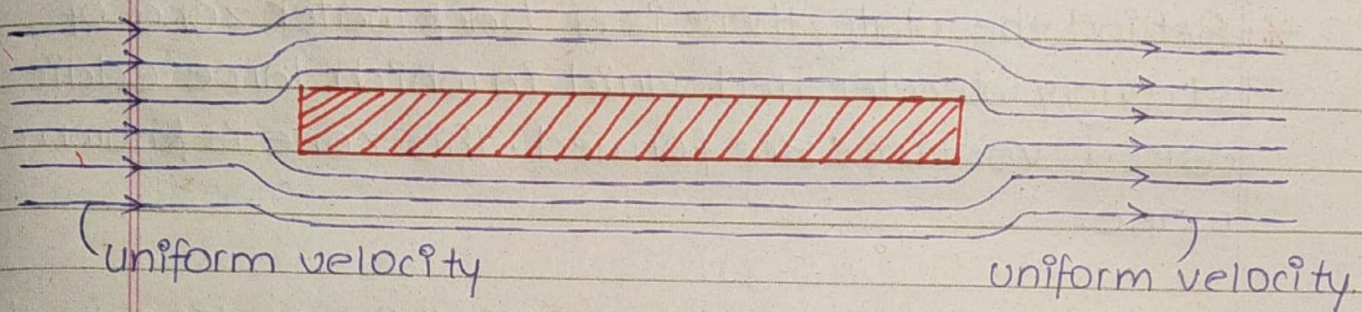
Separation is minimized by avoiding sharp changes in cross-sectional area of the flow channel & by stream lining any object over which the liquid must flow.

→ **Importance of boundary layer separation**

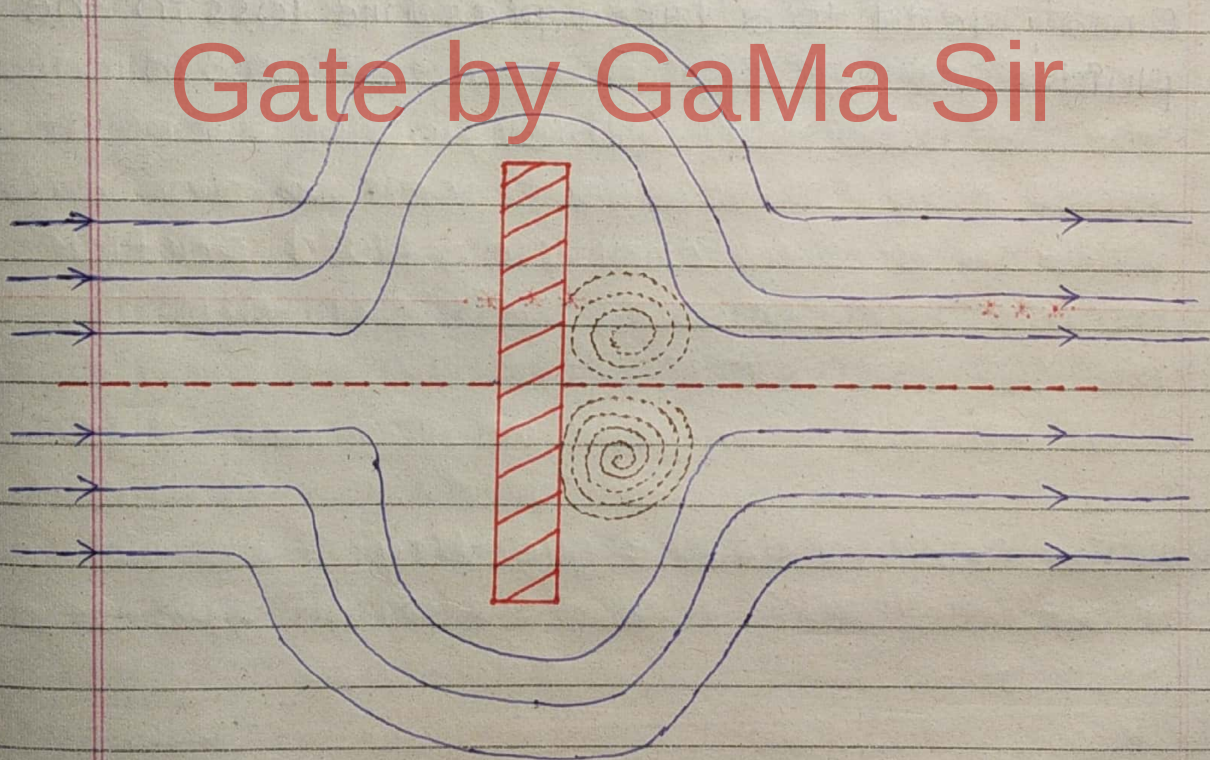
for some purpose, such as promotion of heat transfer on mixing in a fluid boundary layer separation may be diserable.

Date ____
Page ____

Case I :- Flow past flat plate (flow parallel with plate).



Case II :- Flow perpendicular to a flat plate.



* Suppose the plate is turn at right angle to the dirⁿ of flow, a boundary layer forms as before in the fluid flowing over the upstream face.

* When the fluid reaches the edge of the plate however its momentum prevents from making the sharp turn around the edge & it separate from the plate & proceeds outwards into the bulk of the fluid.

* Behind the plate there is a backwater zone of strongly decelerated fluid in which large eddies called vortices are formed. This zone is known as wake.

* The eddies in the wake are kept in motion by the shear stress b/w the wake & separated current. They consumed considerable mechanical energy & may lead to a large pressure loss in the fluid.

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