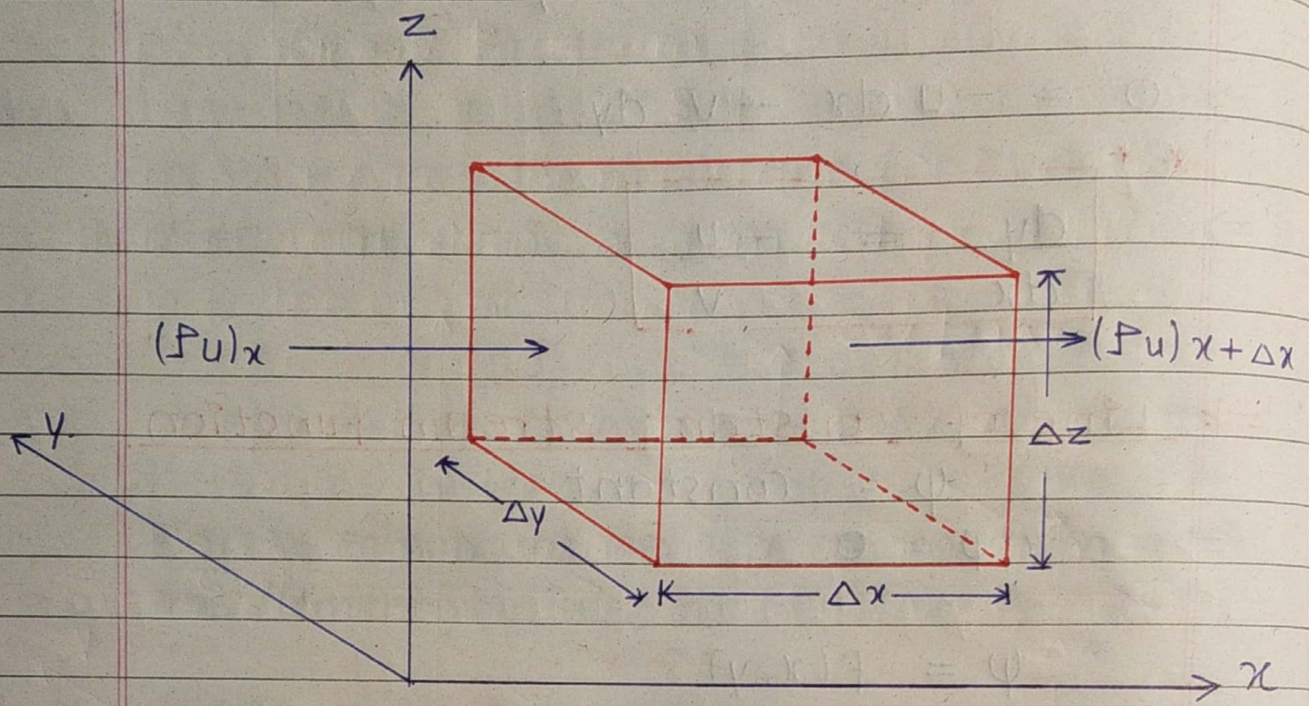
Continuity Equation



Region of vol^m $\Delta x \cdot \Delta y \cdot \Delta z$ fixed in space through which fluid is flowing.

(Rate of mass flow in) − (Rate of mass flow out) = (Rate of mass accumulation).

for fluid of density ρ, the mass flux in the x−dir^n at the face x = $(\rho u)_x$.

and at the face $x+\Delta x$ = $(\rho u)_{x+\Delta x}$.
where u is fluid velocity in x−dir^n.

→ flux is defined as rate of flow of any quantity per unit area.

Hence,
The rate of mass flow entering the element in x−dir^n = $(\rho u)_x \cdot \Delta y \cdot \Delta z$

The rate of mass flow leaving the element
$= (\rho u)_{x+\Delta x} \cdot \Delta y \cdot \Delta z$.

Rate of accumulation in the vol^m element

$$= \frac{\partial}{\partial t} (\rho\, \Delta x \cdot \Delta y \cdot \Delta z)$$

$$\Rightarrow [(\rho u)_x - (\rho u)_{x+\Delta x}] \cdot \Delta y \cdot \Delta z + [(\rho v)_y - (\rho v)_{y+\Delta y}] \Delta x \cdot \Delta z + [(\rho w)_z - (\rho w)_{z+\Delta z}] \Delta y \cdot \Delta z = \frac{\partial}{\partial t} (\rho\, \Delta x \cdot \Delta y \cdot \Delta z).$$

$$\Rightarrow -\frac{\partial}{\partial x}(\rho u) - \frac{\partial}{\partial y}(\rho v) - \frac{\partial}{\partial z}(\rho w) = \frac{\partial \rho}{\partial t}$$

**

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} = -\left[\frac{\partial (\rho u)}{\partial x} + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]}$$

**

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} = -\nabla(\rho \vec{V})}$$

Now

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = -\rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right]$$

**

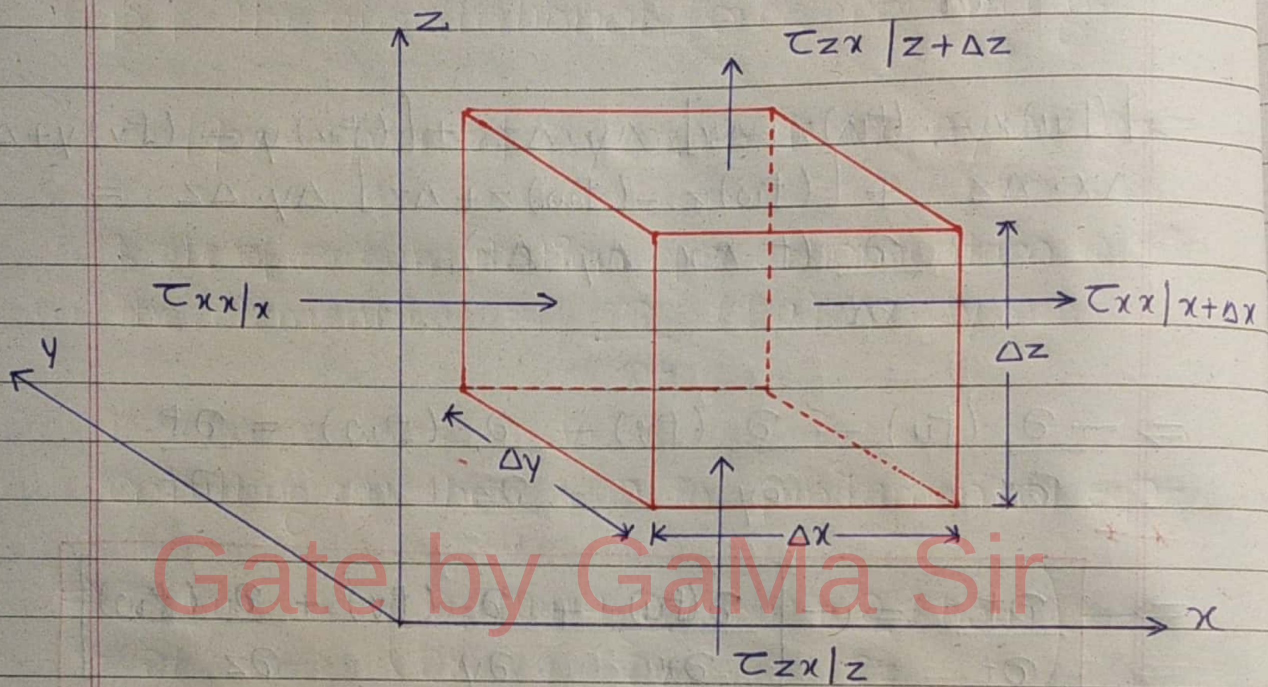
$$\Rightarrow \boxed{\frac{D\rho}{Dt} = -\rho(\nabla \cdot \vec{V}).}$$

where $\frac{D\rho}{Dt}$ is the Substaintial derivative.

Mass velocity

$$\boxed{G = \bar{v}\rho = \frac{\dot{m}}{S}}$$

Differential momentum balance (Eq^n of motion



Vol^m element $\Delta x \cdot \Delta y \cdot \Delta z$ with arrows indicating the dir^n in which the $x-$ component of momentum is transported through the surface.

$\Rightarrow$ [Rate of momentum entering] $-$ [Rate of momentum leaving] $+$ [Sum of forces acting on the system] $=$ [Rate of accumulation]

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Consider the flow rates of the x - component of momentum into & out of the vol^m element.

Momentum enters & leaves the vol^m element partly by convection from the flow of the bulk fluid & partly by viscous action as a result of velocity gradient.

Rate at which the x- component of momentum enters the face at x by convection is
$(\rho u \cdot u)_x \cdot \Delta y \cdot \Delta z.$

Rate at which it leaves at $x + \Delta x$ =
$(\rho u \cdot u)_{x+\Delta x} \cdot \Delta y \cdot \Delta z.$

Net convective flow into the vol^m element
$= [(\rho u \cdot u)_x - (\rho u \cdot u)_{x+\Delta x}] \cdot \Delta y \cdot \Delta z +$
$[(\rho v u)_y - (\rho v u)_{y+\Delta y}] \cdot \Delta x \, \Delta z +$
$[(\rho w u)_z - (\rho w u)_{z+\Delta z}] \, \Delta x \cdot \Delta y.$

Similarly the rate at which the x- component of momentum enters the face at x by molecular transport $= (\tau_{xx})_x \, \Delta y \cdot \Delta z.$

Rate at which it leaves $= (\tau_{xx})_{x+\Delta x} \cdot \Delta y \cdot \Delta z.$

Net flow of x- momentum into the vol^m element by viscous action
$= [\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x}] \, \Delta y \cdot \Delta z + [\tau_{yx}|_y - \tau_{yx}|_{y+\Delta y}]$
$\Delta x \, \Delta z + [\tau_{zx}|_z - \tau_{zx}|_{z+\Delta z}] \, \Delta x \cdot \Delta y.$

where τ_{yx} is the x – directed tangential stress or shear stress on the y-face resulting from the viscous force.

In most cases the important forces acting on the system arises from the fluid pressure p & the gravitational force per unit g.

$$[p_x - p_{x+\Delta x}]\,\Delta y \Delta z + \rho\,\Delta x \cdot \Delta y \cdot \Delta z\, g_x$$

Rate of accumulation of x – momentum within the element is $= \frac{\partial}{\partial t}(\rho u)\,\Delta x \cdot \Delta y \cdot \Delta z$

on putting all these eqn in fundamental eqn:

$$\Rightarrow \frac{\partial(\rho u)}{\partial t} = -\left(\frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho v u) + \frac{\partial}{\partial z}(\rho w u)\right) - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

**

$$\Rightarrow \boxed{\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial}{\partial z}\tau_{zx}\right) + \rho g_x}$$

In vector form.

**

$$\boxed{\rho \frac{DV}{Dt} = -\nabla p - [\nabla \cdot \tau] + \rho g}$$

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This eqⁿ is known as eqⁿ of motion.

Navier stoke's equation

For a fluid of constant density & viscosity the eqⁿ of motion is known as Navier stoke's equation.

**

$$\boxed{\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\partial p}{\partial x} + \rho g_x}$$

In vector form,

**

$$\boxed{\rho\frac{DV}{Dt} = \mu\nabla^2 V + \rho g - \nabla p}$$

Euler's Equation

For a constant density & zero viscosity (as in potential flow) the eqⁿ of motion in rectangular coordinates known as Euler's Equation.

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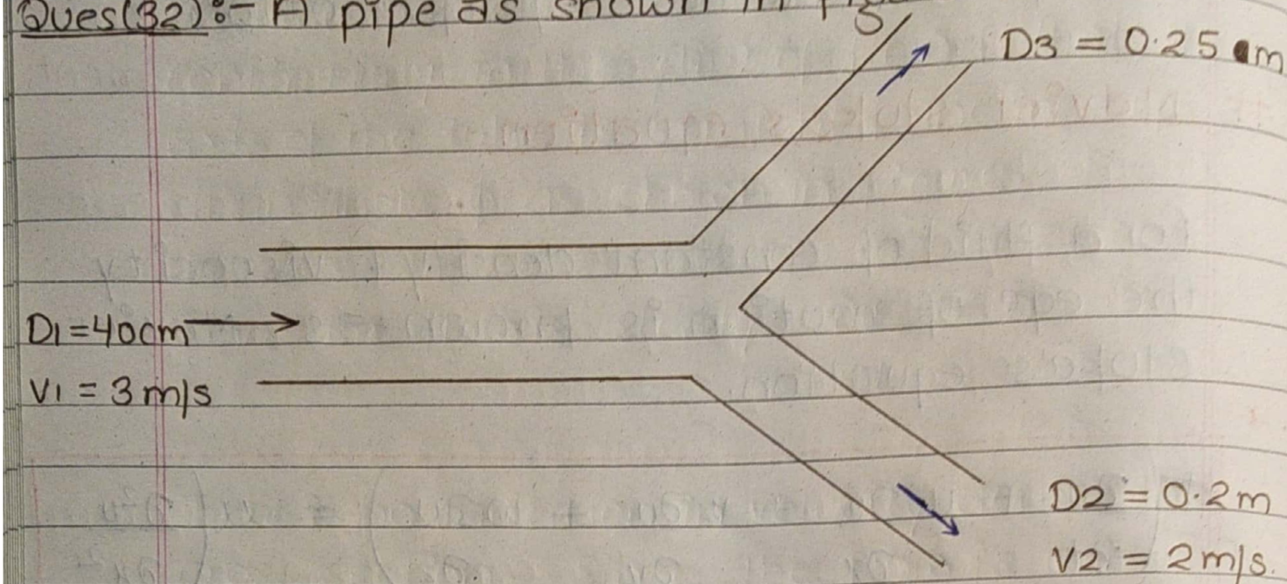
$$\boxed{\rho\frac{Dv}{Dt} = -\nabla p + \rho g}$$

Ques(32):- A pipe as shown in figure :



(i) Calculate discharge through 40 cm diameter pipe.

(ii) The average velocity in 25 cm diameter pipe.

Discharge, $Q_1 = A_1 V_1$

$$Q_1 = \frac{\pi}{4} D_1^2 V_1$$

$$Q_1 = \frac{3.14}{4} \times 0.4 \times 0.4 \times 3$$

$$\boxed{Q_1 = 0.377 \ m^3/sec}$$

Average velocity,

$$Q_1 = Q_2 + Q_3$$

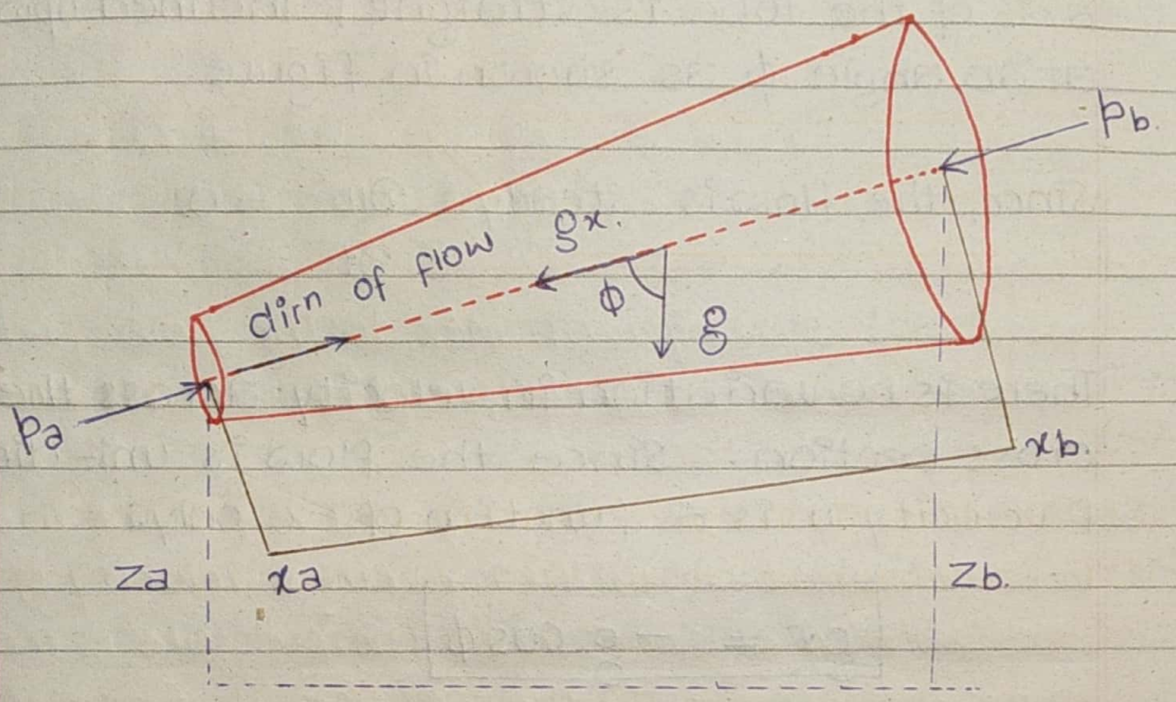
$$\Rightarrow A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$\Rightarrow \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} \left[D_2^2 V_2 + D_3^2 V_3 \right]$$

$$\Rightarrow 0.4 \times 0.4 \times 3 = 0.2 \times 0.2 \times 2 + 0.25 \times 0.25 \times V_3$$

$$\boxed{V_3 = 6.4 \ m/s}$$

Mechanical energy eqn (Bernoullie eqn without friction)



potential flow through inclined stream tube.

The x-component of Euler's equation is,

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x.$$

$$\Rightarrow \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\frac{\partial p}{\partial x} + \rho g_x.$$

This is the mechanical energy eqn for uni-dirn potential flow of fluids of constant density when the flow rate varies with time.

Consider a volm element of stream tube within a larger stream of fluid as shown in figure.

Assume that cross-section of tube increasing continuously in the dir^n of flow & that the axis of the tube is straight & inclined upward at an angle ϕ as shown in figure.

Since, the flow is steady $\frac{\partial u}{\partial t} = 0$.

There is no variation in velocity across the cross section. Since the flow is uni-dir^n & velocity u is a function of x only.

$$\boxed{g_x = -g\cos\phi}$$

$\Rightarrow z = z_a + x\cos\phi$

$dz = \cos\phi \, dx$.

$$\Rightarrow \boxed{\frac{dz}{dx} = \cos\phi}$$

$$\Rightarrow \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} + \rho g\cos\phi = 0.$$

$$\Rightarrow u\frac{\partial}{\partial x}(\rho u^2/2) + u\frac{\partial p}{\partial x} + \rho u g\cos\phi = 0.$$

**

$$\Rightarrow \boxed{\frac{\partial(u^2/2)}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} + g\frac{dz}{dx} = 0}$$

This is point form of bernoullie eq^n without friction.

on integration

** $\Rightarrow$ $$\boxed{\frac{U_a^2}{2} + \frac{p_a}{\rho g} + g z_a = \frac{U_b^2}{2} + \frac{p_b}{\rho} + g z_b}$$

on dividing by g.

$\Rightarrow$ $$\boxed{\frac{p}{\rho g} + \frac{u^2}{2g} + z = \text{constant}}$$

* The term $p/\rho g$ is the pressure head which represents the height of a fluid column that produces a static pressure p.

* $\frac{u^2}{2g}$ is the velocity head.

$\Rightarrow$ During steady, invicid flow of an incompressible fluid along a stream line total mechanical energy at any point is constant.

$\Rightarrow$ The total mechanical energy consist of flow energy, Kinetic energy & potential energy.

Assumptions made in the derivation of bernoullie eq^n :

→ The flow is steady

→ The flow is invicid.

→ The fluid is incompressible.

→ The flow is along a stream line.

* Bernoullie equation deals with flow of conservatio of mechnical energy.

⇒ The equation,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

holds true b/w any two points in the flow field provided that the flow field is irrotational.

Ques(33):- An oil of relative density 0.9 is flowing in a pipe of 10 cm diameter with an average velocity of 3 m/s. At a particular section 1 the pressure is measured to be 300 kN/m². If the section is 6 m above the datum. determine the total head of the oil.

Given: $S = 0.9$; $\rho = 0.9 \times 1000 = 900$ ~~m~~ kg/m³

Total head at section 1 =

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1$$

Total head $= \frac{300 \times 10^3}{900 \times 9.8} + \frac{3 \times 3}{2 \times 9.8} + 6$

Total head $= \frac{10^3}{29.4} + \frac{9}{19.6} + 6$

$= 34.014 + 0.459 + 6$

$= \underline{40.47 \text{ m}}$

Ques(34):- Water is flowing through a pipe having diameters 30 cm & 20 cm at section 1 & 2 respectively. The velocity of water at section 1 is 4 m/s. Find the velocity at section 2 & also calculate rate of discharge.

Given : $D_1 = 0.3\ m$ $\quad D_2 = 0.2\ m$

$V_1 = 4\ m/s$ $\quad V_2 = ?$

from continunity eq^n :

$Q_1 = Q_2$

$\Rightarrow A_1 V_1 = A_2 V_2$

$\Rightarrow \frac{\pi}{4} D_1^2 \cdot 4 = \frac{\pi}{4} D_2^2 \times V_2$

$\Rightarrow 0.3 \times 0.3 \times 4 = 0.2 \times 0.2 \times V_2$

$\Rightarrow \boxed{V_2 = 9\ m/s}$

Discharge, $Q = A_1 V_1 = \frac{\pi}{4} \times D_1^2 \times V_1$

$Q = \frac{3.14 \times 0.3 \times 0.3 \times \cancel{4}}{\cancel{4}}$

$\boxed{Q = 0.283\ m^3/s}$

Ques(35):- An oil of density 850 kg/m³ is flowing through a pipe having diameter 30 cm & 15 cm at the bottom & upper end respectively. The intensity of pressure at bottom end is 200 kN/m² & upper end is 98 kN/m². If the rate of flow through the pipe is 50 litres/sec. Find the difference in datum head. Neglect friction.

Given : $\rho = 850\ kg/m^3$

$D_1 = 0.3\ m$ $\quad D_2 = 0.15\ m$

$p_1 = 200 \; KN/m^2$; $p_2 = 98 \; KN/m^2$

Discharge, $Q = 50$ litres/sec.

Since, $1 \; m^3 = 1000$ litres.

$$1 \text{ litre} = \frac{1}{1000} \; m^3$$

Therefore,

$$Q = \frac{50}{1000} \; m^3/sec$$

$$\boxed{Q = 0.05 \; m^3/sec}$$

Bernoullie eq^n without friction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$z_2 - z_1 = \frac{p_1 - p_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g}$$

$Q_1 = Q_2 = 0.05 \; m^3/sec$

$\Rightarrow A_1 V_1 = A_2 V_2$

$\Rightarrow \frac{\pi}{4} D_1^2 \times V_1 = \frac{\pi}{4} \times D_2^2 \times V_2 = 0.05$

$\Rightarrow 0.3 \times 0.3 \times V_1 = 0.05 \times \frac{4}{3.14}$

$\Rightarrow \boxed{V_1 = 0.708 \; m/s}$

$\Rightarrow V_2 = \frac{0.05 \times 4}{3.14 \times 0.15 \times 0.15}$

$\Rightarrow \boxed{V_2 = 2.831 \; m/s}$

$$z_2 - z_1 = \frac{(200-98)\times 10^3}{850 \times 9.8} + \frac{0.708^2 - 2.831^2}{2 \times 9.8}$$

$$z_2 - z_1 = 12.245 + \frac{(0.501 - 8.015)}{19.6}$$

$$\boxed{z_2 - z_1 = 11.862 \text{ m}}$$

Correction of Bernoullie eq^n for fluid friction

To extend the bernoullie eq^n to cover practicle situations, we need two modification.

→ The first one is usually of minor importance is a correction of the kinetic energy term for the variation of local velocity with position in the boundary layer.

→ The second is of major importance is the correction of the eq^n for the existence of fluid friction which appears whenever a boundary layer forms.

** ⇒

$$\boxed{\frac{p_a}{\rho} + g z_a + \alpha_a \frac{\bar{V}_a^2}{2} = \frac{p_b}{\rho} + g z_b + \alpha_b \frac{\bar{V}_b^2}{2} + h_f}$$

* The mechanical terms represent the cond^n of specific locations but h_f represent the loss of mechanical energy at all the points b/w station A & B.

* The sign of h_f is always (+ve) & in case of potential flow it is zero.

* Friction appears in boundary layer because the work done by shear forces in maintaining the velocity gradient in both laminar & turbulent flow is eventually converted to heat by viscous action. Friction generated in unseparated boundary layer is known as skin friction.

* When boundary layer separate & form wake an additional energy dissipation appears with the wake & friction of this type is known as form friction. It is the func of the position & the shape of the solid.

* The total friction h_f in the above eqn includes both type of friction loss.

Pump Work in Bernoullie equation

Assume that a pump is introduced b/w stations A & B and also assume that W_p be the work done by the pump per unit mass of the fluid.

Assume h_{fp} be the total friction in the pump per unit mass of fluid then net work to the fluid is

$$\Rightarrow \boxed{W_p - h_{fp} = \eta W_p}$$

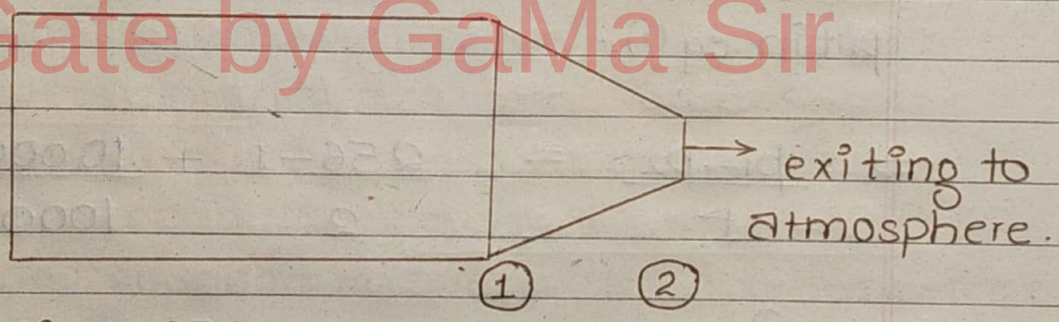
$$\boxed{\eta = \frac{W_p - h_{fp}}{W_p}}$$

The mechanical energy deliver to the fluid is ηW_p.

**

$$\Rightarrow \boxed{\frac{p_a}{\rho} + gZ_a + \alpha_a \frac{\bar{V}_a^2}{2} + \eta W_p = \frac{p_b}{\rho} + gZ_b + \alpha_b \frac{\bar{V}_b^2}{2} + h_f}$$

Ques(36):- Water ($\rho = 1000\ kg/m^3$) is flowing through a
2013:- nozzle as shown in figure. & exiting to atmosphere.



The relationship b/w diameters of the nozzle at location 1 & 2 is $D_1 = 4D_2$. The average velocity of stream at location 2 is 16 m/s. & frictional loss b/w location 1 & 2 is 10,000 pa. Assuming steady state & turbulent flow. The gauge pressure (in pa) at location 1 is

(i) 12250 (ii) 142500 (iii) 102500 (iv) 137500.

Given : $\rho = 1000\ kg/m^3$.

$D_1 = 4D_2$

$V_2 = 16\ m/s$

$\frac{h_f}{\rho} = \frac{10,000\ \text{pascal}}{1000\ kg/m^3}$ or $\frac{N/m^2}{kg/m^3}$

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Bernoullie eq^n with friction;

$$\frac{p_1}{\rho} + \cancel{g z_1} + \frac{V_1^2}{2} = \frac{p_2}{\rho} + \cancel{g z_2} + \frac{V_2^2}{2} + h_f$$

$$\frac{p_1 - p_2}{\rho} = \frac{V_2^2 - V_1^2}{2} + \frac{h_f}{\rho} \quad \text{—— (1)}$$

from continunity eq^n

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow D_1^2 V_1 = D_2^2 V_2$$

$$\Rightarrow \cancel{16}\ \cancel{D_2^2}\ V_1 = \cancel{D_2^2}\ \cancel{16}$$

$$\Rightarrow \boxed{V_1 = 1\ m/s}$$

put in eq (1).

$$\Rightarrow \frac{p_1 - p_2}{\rho} = \frac{256 - 1}{2} + \frac{10{,}000}{1000}$$

$$\Rightarrow \frac{P_{gauge}}{\rho} = \cancel{10127.5} \times \frac{10000}{1000} + \frac{255}{2}$$

$$\Rightarrow P_{gauge} = 10{,}000 + 127.5 \times 1000$$

$$\Rightarrow \boxed{P_{gauge} = 137500\ \text{pascal}}$$

Ques(37):- In a steady & incompressible flow of a fluid
2014 :- ($\rho = 1.25\ kg/m^3$). The difference b/w stagnation & static pressures at the same location in the flow is 30 mm of Hg. ($\rho_m = 13600\ kg/m^3$) Consider $g = 10\ m/s^2$. The fluid speed (in m/s) is ____.

Given : $\rho_f = 1.25 \ kg/m^3$

$\rho_m = 13600 \ kg/m^3$

$g = 10 \ m/s^2$

$$\Rightarrow \frac{p_1}{\rho_f g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho_f g}$$

$$\Rightarrow \frac{p_2 - p_1}{\rho_f \cancel{g}} = \frac{V_1^2}{2\cancel{g}}$$

$$\Rightarrow V_1^2 = \frac{2 \Delta P}{\rho_f} = \frac{2 \times 13600 \times \cancel{9.8}^{10} \times 30 \times 10^{-3}}{1.25}$$

$$\Rightarrow \boxed{V_1 = 80.792 \ m/s}$$

Power Requirement

The power supplied to pump drive from external source P_B & is defined as

** $$\boxed{P_B = \dot{m} W_p = \dot{m} \frac{\Delta H}{\eta}}$$

where, $\dot{m}$ = mass flow rate.

* The power deliver to the fluid is calculated from the mass flow rate & the head developed by the pump.

** $$\boxed{P_f = \dot{m} \Delta H}$$ or ** $$\boxed{P_B = \frac{P_f}{\eta}}$$

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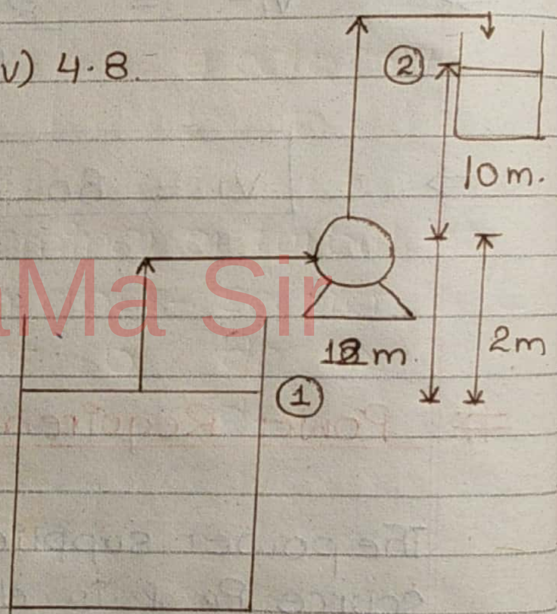
Ques(38):- Water ($\rho = 1000\ kg/m^3$) is pumped at a rate
2016 :- of $36\ m^3/hr$. from a tank 2m below the pump to an overhead pressurised vessel 10 m above. The pressure values at the point of suction from the bottom tank & discharge point to overhead vessel 120 kpa & 240 kpa respectively. All the pipes are of same diameter. Take $g = 10\ m/s^2$. Neglecting frictional losses.
What is the power (in kW) required to deliver the fluid is

(i) 1.2 (ii) 2.4 (iii) 3.6 (iv) 4.8



Given : $\rho = 1000\ kg/m^3$

Discharge, $Q = 36\ \frac{m^3}{hr}$

$$Q = \frac{36}{3600}\ \frac{m^3}{sec}$$

$$\boxed{Q = 0.01\ m^3/sec}$$

$p_1 = 120$ kpa
$p_2 = 240$ kpa

$$\frac{p_1}{\rho} + z_1 g + \cancel{\frac{v_1^2}{2}} + \eta W_p = \frac{p_2}{\rho} + z_2 g + \cancel{\frac{v_2^2}{2}}$$

$$\eta W_p = \frac{p_2 - p_1}{\rho} + g(z_2 - z_1)$$

$$\eta W_p = \frac{(240-120) \times \cancel{10^3}}{\cancel{10^3}} + 10 \times (12)$$

$$\boxed{\eta W_p = 240 \ J/kg}$$

$$P_f = \dot{m}\,\Delta H$$

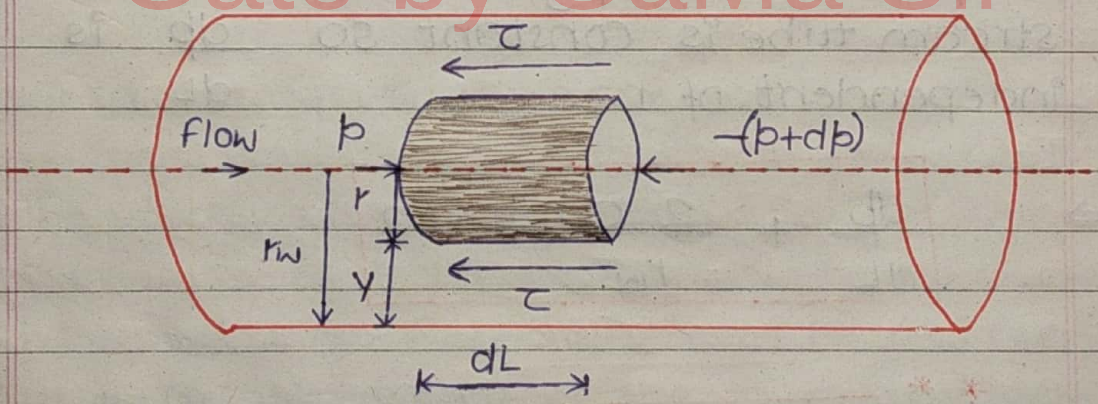
$$P_f = \rho Q \Delta H$$

$$P_f = 10^3 \times 0.01 \times 240 \quad \cancel{k}\,W$$

$$\boxed{P_f = 2.4 \ kW}$$

Incompressible flow in pipes & channels

Shear stress distribution



Fluid element in steady flow through pipe.

Consider the steady flow of fluid of constant density in fully developed flow through a horizontal pipe.
Consider a disc shape element of a fluid concentric with the axis of the tube of radius 'r' & length dL
Since, the fluid having a viscosity, a shear force

opposing flow will exist on the rim of the element.

Shearforce Fs acting on the rim of the element is the product of shear stress & cylindrical area = $(2\pi r\, dL)\cdot\tau$

Since, the channel is horizontal, $F_g = 0$.

$\Rightarrow p\pi r^2 - (p+dp)\pi r^2 - (2\pi r\, dL)\cdot\tau = 0$

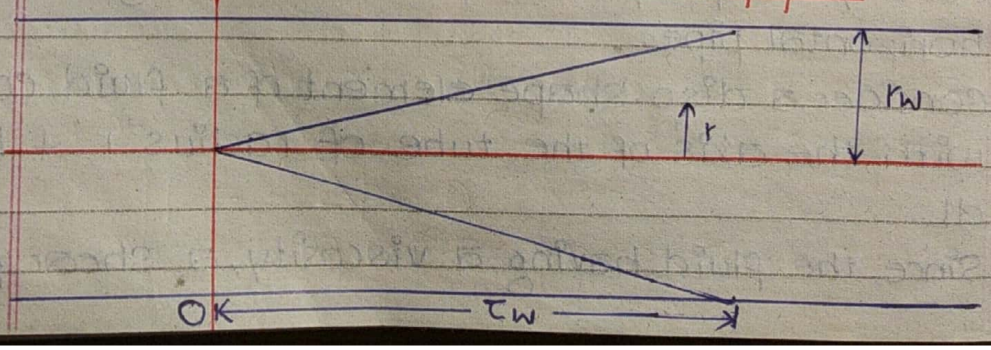
** $$\boxed{\frac{dp}{dL} + \frac{2\tau}{r} = 0}$$ @gate 2017

In steady flow either laminar or turbulent the pressure at any given cross-section of a stream tube is constant so $\frac{dp}{dL}$ is independent of r.

$$\Rightarrow \boxed{\frac{dp}{dL} + \frac{2\tau_w}{r_w} = 0}$$

** $$\Rightarrow \boxed{\frac{\tau}{r} = \frac{\tau_w}{r_w}}$$

Variation of shear stress in pipe



Ques(39):- 2010:- In Hagen Poiseullie flow through a cylindrical tube, the radial profile of shear stress is given by

(i) constant (ii) Cubic (iii) Linear (iv) Parabolic

Ques(40):- 2017:- oil is being delivered at a steady flow rate through a circular pipe of radius 1.25×10^{-2} m & length 10 m. The pressure drop across the pipe is 500 pascal. The shear stress at the pipe wall is given

$$-\frac{dp}{dL} + \frac{2\tau_w}{r_w} = 0.$$

$$\Rightarrow -\frac{500}{10} + \frac{2\tau_w}{1.25\times10^{-2}} = 0.$$

$$\Rightarrow 2\tau_w = 50\times1.25\times10^{-2}$$

$$\Rightarrow \boxed{\tau_w = 0.3125}$$

* Equation of Bernoullie can be written over a definate length L of the complete stream in previous section Δp was define as $(p_b - p_a)$ but usually p_a is greater than p_b & thus $(p_b - p_a)$ is usually negative.

The term Δp is commonly used for pressure drop i.e. $(p_a - p_b)$.

$$\frac{p_a}{\rho} = \frac{p_b}{\rho} + h_{fs}.$$

$$\Rightarrow \frac{p_a - p_b}{\rho} = h_{fs}$$

$$\Rightarrow \boxed{\frac{\Delta p_s}{\rho} = h_{fs}}$$

→ for a definate length L of pipe, $\frac{dp}{dL}$ becomes $\frac{\Delta p_s}{L}$.

**

$$\Rightarrow \boxed{h_{fs} = \frac{2}{\rho}\frac{\tau_w}{r_w}\cdot L = \frac{4}{\rho}\frac{\tau_w}{D}\cdot L}$$

where D = diameter of pipe.

Friction factor

It is defined as

$$\boxed{f = \frac{\tau_w}{\rho \bar{V}^2/2}}$$

Relation between skin friction parameter

$$h_{fs} = \frac{2}{\rho}\frac{\tau_w}{r_w}\cdot L = \frac{\Delta p_s}{\rho} = 4f\frac{L}{D}\frac{\bar{V}^2}{2}$$

from which we will get,

**

$$\boxed{f = \frac{\Delta p_s \cdot D}{2L\rho\bar{V}^2}}$$

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** $$\boxed{\frac{\Delta p_s}{L} = \frac{2 f \rho \bar{v}^2}{D}}$$

⇒ If boundary layer separation occurs h_f is greater than h_{fs}.

Ques(41):- A centrifugal pump delivers water at a rate
2015 :- of $0.22\ m^3/sec$ from a reservoir at ground level to another reservoir at a height H through a verticle pipe of 0.2 m diameter. Both the reservoirs are open to atmosphere. The power input to the pump is 90 kW & it operates with an efficiency of 0.75.
Given data : Faining friction factor for pipe flow is $f = 0.004$. Neglect other head losses $g = 9.8\ m/s^2$, $\rho_w = 1000\ kg/m^3$. The height H is (in m) to which water can be delivered.

Given : $Q = 0.22\ m^3/sec$; diameter $= 0.2\ m$.
$P = 90 \times 10^3$ Watt ; $\eta = 0.75$; $f = 0.004$

Ques(42):- In a delivery line CCl_4 at constant flow rate of 4×10^{-5} m^3/sec. The first 1000 m long pipe is of 20 mm inside diameter & followed by another 1000 m long pipe of 50 mm inside diameter as shown in figure. Estimate the pressure drop over the entire length of the delivery line. Neglect minor losses due to sudden expansion. Given data : viscosity = 10^{-3} pasec., density = 1500 kg/m^3.

for laminar $f = 16/Re$ for turbulent $f = 0.079\,(Re)^{-0.25}$

$$Re_1 = \frac{\rho v_1 D_1}{\mu} = \frac{1500 \times 0.127 \times 20 \times 10^{-3}}{10^{-3}} = 3810$$

$$Re_2 = \frac{\rho v_2 D_2}{\mu} = \frac{1500 \times 0.02 \times 50 \times 10^{-3}}{10^{-3}} = 1500.$$

Given: $Q = 4 \times 10^{-5}$ m^3/sec

$D_1 = 20 \times 10^{-3}$ m ; $D_2 = 50 \times 10^{-3}$ m.

$\Rightarrow Q = A_1 v_1$

$\Rightarrow 4 \times 10^{-5} = \frac{3.14}{4} \times 20 \times 20 \times 10^{-6} \times v_1$

$\Rightarrow \underline{v_1 = 0.127 \text{ m/s}}$

$$v_2 = \frac{4 \times 4 \times 10^{-5}}{3.14 \times 50 \times 50 \times 10^{-6}}$$

$\underline{v_2 = 0.02 \text{ m/s}}$

$$f_1 = 0.079 \times (3810)^{-0.25}$$

$$f_1 = \frac{0.079}{(3810)^{0.25}} = \frac{0.079}{7.857} = \underline{0.01}$$

$$f_2 = \frac{16}{Re} = \frac{16}{1500} = \underline{0.011}$$

$$\Delta p_1 + \Delta p_2 = \frac{2 f_1 \rho v_1^2}{D_1} + \frac{2 f_2 \rho v_2^2}{D_2}$$

$$\frac{\Delta p_1 + \Delta p_2}{L_1 + L_2} = \frac{2 \times 0.01 \times 1500 \times (0.127)^2}{20 \times 10^{-3}} +$$

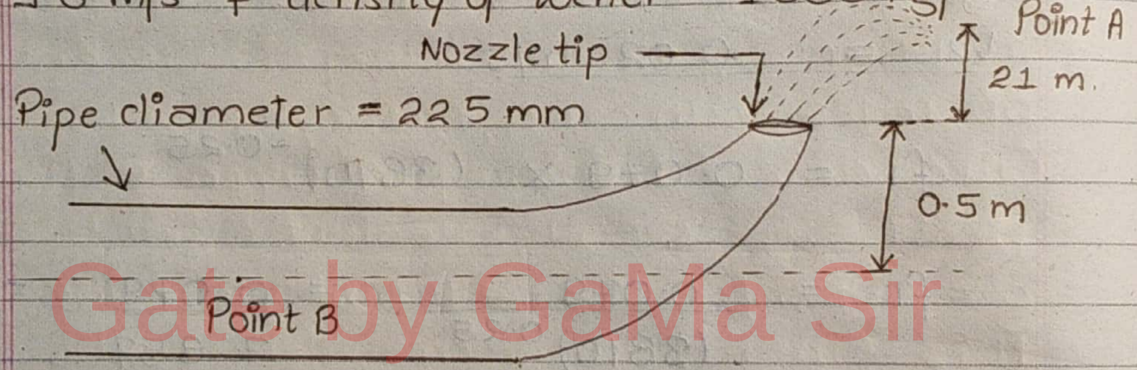
$$\frac{2 \times 0.011 \times 1500 \times (0.02)^2}{50 \times 10^{-3}}$$

$$= \frac{3000}{10^{-3}} \left[\frac{0.016 \times 0.01}{20} + \frac{0.0004 \times 0.011}{50} \right]$$

$$\Delta p_1 + \Delta p_2 = \frac{3000}{10^{-3}}\left[\frac{0.0088 + 0.000088}{1000}\right]$$

$$\boxed{\Delta p = 24.264 \text{ pascal}}$$

Ques(43):- 2009:- A free jet of water immerging from a nozzle (diameter = 75 mm) attached to a pipe (diameter = 225 mm) as shown in figure. The velocity of water at point A is 18 m/s. Neglect friction in the pipe & nozzle. Use g = 9.8 m/s² & density of water = 1000 kg/m³



(i) The velocity of water at the tip of the nozzle (m/s)

(i) 13.4 (ii) 18 (iii) 23.2 (iv) 27.1 ✓

(ii) The gauge pressure (kpa) at point B is

(i) 80 (ii) 100 (iii) 239.3 (iv) 367.6 ✓

Given: Nozzle tip diameter = 0.075 m.

Pipe diameter = 225 mm = 0.225 m.

V_A = 18 m/s.

g = 9.8 m/s²

ρ = 1000 kg/m³

Using Bernoullie,

$$\frac{\cancel{P_T}}{\cancel{\rho g}} + \frac{V_T^2}{2g} + Z_T = \frac{\cancel{P_A}}{\cancel{\rho g}} + \frac{V_A^2}{2g} + Z_A$$

$$\frac{V_T^2}{2g} = \frac{V_A^2}{2g} + (Z_A - Z_T)$$

$$\frac{V_T^2}{2g} = \frac{18 \times 18}{2 \times 9.8} + 21$$

$$V_T^2 = 37.531 \times 2 \times 9.8$$

(i) $\boxed{V_T = 27.13 \text{ m/s}}$

NOW,

$$Q_B = Q_T$$

$$\Rightarrow A_B V_B = A_T V_T$$

$$\Rightarrow 0.225 \times 0.225 \times V_B = 0.075 \times 0.075 \times 27.1$$

$$\Rightarrow V_B = 3.01$$

$$\frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B = \frac{P_T}{\rho g} + \frac{V_T^2}{2g} + Z_T$$

$$\Rightarrow \frac{P_B - P_T}{\rho g} = \frac{V_T^2 - V_B^2}{2g} + (Z_T - Z_B)$$

$$\Rightarrow \frac{P_B - P_T}{\rho g} = \frac{(27.1)^2 - (3.01)^2}{2 \times 9.8} + 0.5$$

$$\Rightarrow P_B - P_T = 367574.95 \text{ pascal}$$

$\Rightarrow$ (ii) $\boxed{P_B - P_T = \cancel{367} \; 367.6 \text{ kpa}}$

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Ques(44):- A pump draws oil (specific gravity = 0.8) from
2008:- a storage tank & discharges it into an overhead tank. The mechanical energy delivered by the pump to the fluid is 50 J/kg. The velocity at the suction & discharge points of the pump are 1 m/s & 7 m/s respectively. Neglecting frictional losses & assuming kinetic energy correction factor to be unity. The pressure developed by the pump in kN/m^2 is

(i) 19.2 (ii) 20.8 (iii) 40 (iv) 80.

Given : $S = 0.8$, $\rho = 800\ kg/m^3$

$$P_f = \dot{m}\,\Delta H$$

$$\Rightarrow \frac{P_f}{\dot{m}} = \Delta H$$

$$\Rightarrow \boxed{\Delta H = 50\ J/kg}$$

$V_1 = 1\ m/s$; $V_2 = 7\ m/s$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{g z_1} + \Delta H = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{g z_2}$$

$$\Rightarrow \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} - 50 = 0$$

$$\Rightarrow \frac{P_2 - P_1}{\rho} = 50 - \frac{(49-1)}{2}$$

$$\Rightarrow \frac{P_2 - P_1}{\rho} = 50 - 24$$

$$\Rightarrow P_2 - P_1 = 20800\ N/m^2 = \underline{20.8\ kN/m^2}$$

Ques(45):- A siphon tube having a diameter of 2 cm
2013 :- draws water from a large open reservoir & discharges it into atmosphere. Assume incompressible fluid & neglect frictional loss $g = 9.8\ m/s^2$. The velocity (in m/s) at the discharge point is
(i) 9.9 (ii) 11.7 (iii) 98 (iv) 136.9
The volumetric flow rate (in L/s) of water.

Given : Diameter = 2 cm ; $g = 9.8\ m/s^2$
using Bernoullie eq^n $z_1 - z_2 = 5\ m$.

$$\Rightarrow \cancel{\frac{P_1}{\rho}} + \frac{V_1^2}{2} + g z_1 = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + g z_2$$

$\Rightarrow$ Since, $V_1 = 0$.

$$\Rightarrow \quad V_2 = \sqrt{2g(z_1 - z_2)}$$

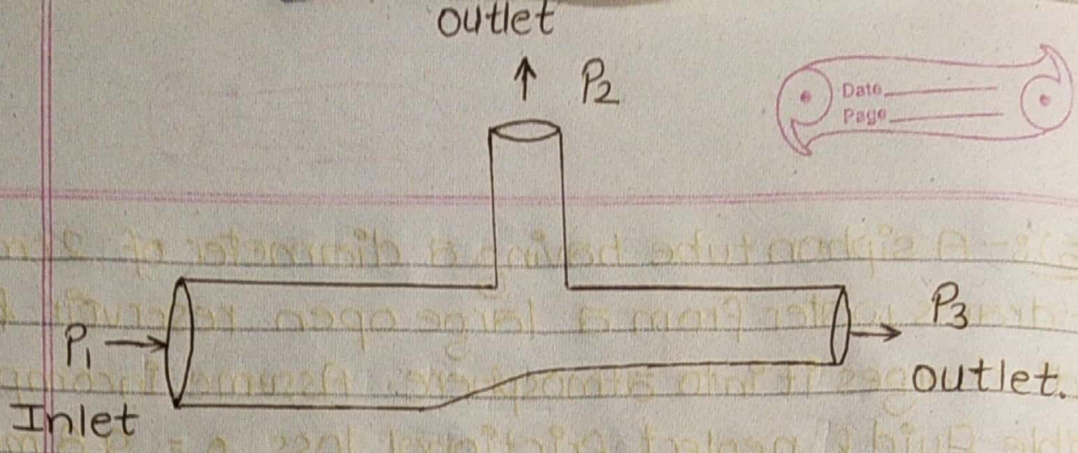
$$\Rightarrow \quad V_2 = \sqrt{2 \times 9.8 \times 5} = 9.8\ m/s.$$

$$Q = \text{Area} \times \text{velocity} = \frac{3.14 \times 0.02 \times 0.02 \times 9.8}{4}$$

$$Q = 30.772 \times 10^{-4} \times 10^3\ L/s$$

$$\boxed{Q = 3.1\ \text{Litre/sec}}$$

Ques(46):- A pipeline carries crude oil of density
2007 :- $800\ kg/m^3$. The volumetric flow rate at point 1 is $0.28\ m^3/sec$. The cross-sectional area of the branches are in a horizontal plane & the friction is negligible. If the pressure at point 1 & 3 are 270 kPa & 240 kPa. Calculate the pressure at point 2 (in kPa).



Given : $\rho = 800\ kg/m^3$

$Q_1 = 0.28\ m^3/sec$

$Q_2 = ?$

$A_1 = 0.012\ m^2$

$A_2 = 0.008\ m^2$

$A_3 = 0.004\ m^2$

$P_1 = 270\ kpa$

$P_3 = 240\ kpa$

apply bernoullie at pt 1 & pt 3.

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + \cancel{z_2}$$

$$\Rightarrow \frac{270 \times 10^3}{800 \times \cancel{9.8}} + \frac{V_1^2}{2\cancel{g}} = \frac{240 \times 10^3}{800 \times \cancel{9.8}} + \frac{V_3^2}{2\cancel{g}}$$

$$\Rightarrow \frac{V_3^2 - V_1^2}{2} = \frac{10^3}{80\cancel{0}} \times 3\cancel{0}$$

$$\Rightarrow V_3^2 - V_1^2 = \frac{6}{80} \times 10^3$$

$$Q_1 = A_1 V_1 \Rightarrow 0.28 = 0.012 \times V_1$$

$$\Rightarrow V_1 = \frac{0.28}{0.012} = 23.33\ m/s$$

$$\Rightarrow V_3^2 = 544.444 + 75$$

$$\Rightarrow \boxed{V_3 = 24.89\ m/s}$$

$$Q_1 = Q_2 + Q_3$$

$$\Rightarrow 0.28 = 0.008 \times V_2 + 0.004 \times 24.89$$

$$\Rightarrow \boxed{V_2 = 22.56 \text{ m/s}}$$

apply bernoullie at pt 1 & pt 2.

$$\Rightarrow \frac{270 \times 10^3}{800} + \frac{544.44}{2} = \frac{P_2}{800} + \frac{508.95}{2}$$

$$\Rightarrow 337.5 + 272.22 = \frac{P_2}{800} + 254.47$$

$$\Rightarrow \boxed{P_2 = 284200 \text{ pa} = 284.2 \text{ kpa}}$$

Flow in non-circular channel

Hydraulic Radius

It is defined as ratio of cross-sectional area of the channel to the wetted perimeter of the channel in contact with fluid.

$$\boxed{r_H = \frac{S}{L_p}}$$

where, S = cross-sectional area of the channel.

L_p = perimeter of channel in contact with liquid.

$$\boxed{D_{eq} = 4 r_H}$$

⇒ for circular tube :

$$r_H = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

⇒ for annulus b/w two concentric pipes :

$$r_H = \frac{\pi D_o^2/4 - \pi D_i^2/4}{\pi D_i + \pi D_o} = \frac{D_o - D_i}{4}$$

where D_i & D_o are the inside & outside diameter of the annulus.

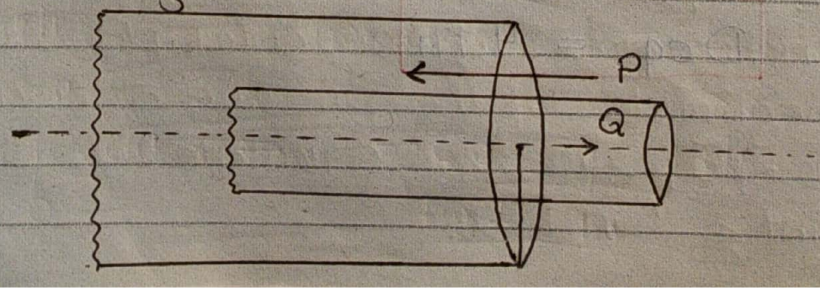
Ques(47) :- The equivalent diameter for flow through
2004 :- rectangle duct of width B & height H is given by :

(i) $\frac{BH}{2(B+H)}$ (ii) $\frac{HB}{(H+B)}$

(iii) $\frac{2HB}{(H+B)}$ (iv) $\frac{4HB}{(H+B)}$

$$D = 4 r_H = 4 \frac{B \times H}{2(H+B)} = \frac{2BH}{(H+B)}$$

Ques(48) :- Two liquids (P & Q) having same viscosity are
2011 :- flowing through a double heat exchanger as shown in figure.



Densities of P & Q are 1000 kg/m^3 & 800 kg/m^3. The average velocities of the liquid P & Q are 1 m/s & 2.5 m/s. The inner diameter of the pipes are 0.31 m & 0.1 m. Both pipes are 5 mm thick. The ratio $\frac{Re_P}{Re_Q}$ is ____.

(i) 2.5 (ii) 1.55 (iii) 1 (iv) 4

$$Re_P = \frac{\rho_P V_P D_P}{\mu}$$

$$Re_Q = \frac{\rho_Q V_Q D_Q}{\mu}$$

$$D_P = 0.31 - 0.1 - 2 \times 0.005$$

$$D_P = 0.2 \text{ m}$$

$$D_Q = 0.1 - 2 \times 0.005 = 0.09 \text{ m}$$

$$\Rightarrow \frac{Re_P}{Re_Q} = \frac{1000 \times 1 \times 0.2}{800 \times 2.5 \times 0.09}$$

$$\Rightarrow \boxed{\frac{Re_P}{Re_Q} = 1.11 = 1}$$

Laminar flow of Newtonian fluid

We will discuss four following quantities :

→ velocity distribution

→ Average velocity

→ α - kinetic energy correction factor

→ β - momentum correction factor.

velocity distribution

The relation between the local velocity & position in the stream is found in the following way.

In circular channels because of the symmetry about the axis of tube the local velocity u depends only on the radius.

Cosider a thin ring of radius r & width dr forming an element of cross-sectional area ds is given by $2\pi r dr$.

The velocity distribution is found by using the definition of viscosity

**

$$\boxed{\mu = \frac{-\tau}{du/dr}}$$

–ve sign in the above eqⁿ account for the fact that in the pipe u decreases as r increases

$$\frac{du}{dr} = -\frac{\tau_w}{r_w \cdot \mu} \cdot r$$

$$\Rightarrow \int_0^u du = \frac{-\tau_w}{r_w \mu} \int_{r_w}^{r} r\,dr.$$

$$\boxed{u = \frac{\tau_w}{2 r_w \mu}(r_w^2 - r^2)}$$

The maximum value of the local velocity is located at the centre of the pipe i.e. at $r=0$.

$$\boxed{U_{max} = \frac{\tau_w r_w}{2\mu}}$$

$$\boxed{\frac{u}{U_{max}} = 1 - \left(\frac{r}{r_w}\right)^2}$$

This eqn shows that in laminar flow the velocity distribution w.r.t. the radius is a parabola with the apex at the centre line of the pipe.

Average velocity

For flow of a real fluid, velocity is not uniform at any section varies from wall to wall.

The average velocity over a cross-section is given by

$$\boxed{\bar{u} = \frac{\int_A u \, dA}{Area}}$$

$$\Rightarrow \quad \bar{u} = \frac{\int_0^R 2\pi r \, dr \cdot u}{\pi R^2}$$

$$\Rightarrow \quad \bar{u} = \frac{2\pi U_{max}}{\pi R^2} \int_0^R r\left(1 - \frac{r^2}{R^2}\right) dr$$

$$\Rightarrow \bar{u} = \frac{2\,u_{max}}{R^2}\int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$\Rightarrow \bar{u} = \frac{2\,u_{max}}{R^2}\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$\Rightarrow \bar{u} = \frac{2\,u_{max}}{R^2}\left[\frac{R^2}{2} - \frac{R^2}{4}\right]$$

$$\Rightarrow \bar{u} = 2\,u_{max} \cdot \frac{1}{4}$$

**

$$\Rightarrow \boxed{\bar{u} = \frac{1}{2}\,u_{max}}$$

Ques(49):- The velocity distribution in a circular pipe is given by $u = u_{max}\left(1 - \frac{r}{R}\right)^n$ where u is the velocity at a distance r from the centre, u_{max} is the maximum velocity at the centre of pipe & R is the pipe radius. Find the ratio of average velocity of flow in the pipe to max. velocity.

$$\bar{u} = \frac{\int_0^R 2\pi r\,dr\, u}{\pi R^2}$$

$$\Rightarrow \bar{u} = \frac{2\cancel{\pi}\,u_{max}}{\cancel{\pi} R^2}\int_0^R r\left(1 - \frac{r}{R}\right)^n dr$$

Let $1 - \frac{r}{R} = z \Rightarrow r = (1-z)R$

$$\Rightarrow dz = -\frac{dr}{R}$$

$$\Rightarrow \boxed{dr = -dz \cdot R}$$

when $r = 0$; $z = 1$
$r = R$; $z = 0$

$$\Rightarrow \bar{u} = \frac{2\,U_{max}}{R^2} \int_1^0 -(1-z) R^2 \cdot z^n \, dz$$

$$\Rightarrow \bar{u} = \frac{2\,u_{max} \cancel{R^2}}{\cancel{R^2}} \int_0^1 (z^n - z^{n+1})\, dz$$

$$\Rightarrow \frac{\bar{u}}{U_{max}} = 2\left[\frac{z^{n+1}}{n+1} - \frac{z^{n+2}}{n+2}\right]_0^1$$

$$\Rightarrow \frac{\bar{u}}{U_{max}} = 2\left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$\Rightarrow \frac{\bar{u}}{U_{max}} = 2\left[\frac{\cancel{n}+2-\cancel{n}-1}{(n+1)(n+2)}\right]$$

**

$$\Rightarrow \boxed{\frac{\bar{u}}{U_{max}} = \frac{2}{(n+1)(n+2)}}$$

Ques(50):- 2015 :- Two different liquids are flowing through different pipes of the same diameter in the first pipe the flow is laminar with centre line velocity $V_{max\,1}$ whereas in 2nd pipe the flow is turbulent. For turbulent flow the average velocity 0.82 times the centre line velocity V_{max2}. For equal volumetric flow rates in both pipes the ratio V_{max1}/V_{max2}.

Given : $\bar{V}_1 = \frac{V_{max1}}{2}$.

$\bar{V}_2 = V_{max2} \times 0.82$.

$\because$ $Q_1 = Q_2$

$\Rightarrow$ $A_1\bar{V}_1 = A_2\bar{V}_2$

$\because$ $\boxed{A_1 = A_2}$

$\Rightarrow$ $\bar{V}_1 = \bar{V}_2$

$\Rightarrow$ $\frac{V_{max1}}{2} = V_{max2} \times 0.82$.

$\Rightarrow$ $\boxed{\frac{V_{max1}}{V_{max2}} = 1.64}$

α - kinetic energy correction factor

**

$$\boxed{\alpha = \frac{\int_S u^3 \, ds}{\bar{V}^3 S}}$$

for laminar ; $\alpha = 2$.

Momentum correction factor (β)

** $$\boxed{\beta = \frac{1}{S}\int_S \left(\frac{u}{V}\right)^2 ds}$$

$$\boxed{\beta = \frac{4}{3}}$$ for laminar.

For laminar flow

Fanning friction factor ;

** $$\boxed{f = \frac{16}{Re}}$$

Hagen- Poiseullie Eq^n

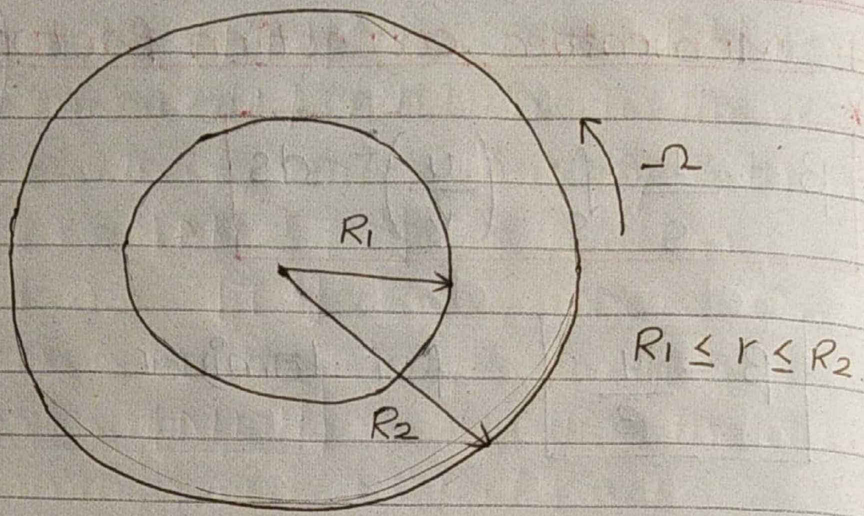
** $$\boxed{\frac{\Delta p_s}{L} = \frac{32\,\bar{V}\,\mu}{D^2}}$$

Ques (51) :- In case of pressure driven laminar flow
2014 :- of a newtonian fluid of viscosity (μ) through horizontal circular pipe, the velocity of the fluid is proportional to.

(i) μ (ii) $\mu^{0.5}$ (iii) μ^{-1} (iv) $\mu^{-0.5}$

Ques(52) :-

The outer cylinder is rotating with an angular velocity Ω while inner cylinder is stationary. Given that $(R_2 - R_1) << R_1$. The profile of the θ component of the velocity v_θ can be approximated by



(i) $R_2 \Omega$ (ii) $\frac{(r-R_2)}{(R_2-R_1)} \cdot r\Omega$

(iii) $\frac{(r+R_1)}{(R_1+R_2)} \cdot R_1\Omega$ (iv) $\frac{(r-R_1)}{(R_2-R_1)} \cdot R_2\Omega$

Ques(53):- Water is flowing under laminar condn in a pipe of length L. If the diameter of the pipe is doubled for a constant volumetric flow rate. The pressure drop across the pipe is

(i) decrease 2 times. (ii) decrease 16 times

(iii) Increase 2 times. (iv) Increase 16 times

using Hagen Poiseullie eqn :-

$$\frac{\Delta p_s}{L} = \frac{32 \bar{V} \mu}{D^2}$$

$$\Rightarrow Q = A_1 \bar{V}_1 = \frac{\pi}{4} D^2 \cdot \bar{V}_1$$

$$\Rightarrow \bar{V}_1 = \frac{4Q}{\pi D^2}$$

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$\Rightarrow$ Now, Diameter is doubled.

$$\Rightarrow \cancel{\bar{V}_2} = \frac{\cancel{4}Q}{\pi \cancel{4} D^2} = \frac{\cancel{Q}}{\pi \cancel{D}^2}.$$

Now.

$$\frac{\Delta p_{s1}}{L} = \frac{32 \mu \cdot 4Q}{\pi D^4} \quad \text{— (1).}$$

$$\frac{\Delta p_{s2}}{\cancel{L}} = \frac{\cancel{32} \mu Q}{\pi \cancel{D^2}}$$

$$\frac{\Delta p_{s1}}{L} = \frac{32 \mu \; 4Q}{\pi (2D)^2 \; (2D)^2} \quad \text{— (2).}$$

$$\Rightarrow \frac{\Delta p_{s1}}{\Delta p_{s2}} = 16.$$

$$\Rightarrow \boxed{\Delta p_{s2} = \frac{1}{16} \Delta p_{s1}}$$

Ques(54):- The relationship b/w the stress τ & the strain rate dU_x/dy for the rapid flow of a grannule material is given by $\tau = B\left(\frac{dU_x}{dy}\right)^2$ where B is a constant. If M, L & T are the mass, length & time dimension respectively. what is the dimension of the constant B ?

(i) $[ML^{-1}T^{-1}]$ (ii) $[ML^{-1}T^{-2}]$ (iii) $[MT^{-1}]$ (iv) $[ML^{-1}]$

$$\tau = \frac{F}{A} = \frac{[MLT^{-2}]}{L^2} = [ML^{-1}T^{-2}]$$

$$\left(\frac{dU_x}{dy}\right)^2 = \frac{[LT^{-1}]^2}{[L]^2} = [T^{-2}].$$

$$B = \frac{[ML^{-1}T^{-2}]}{[T^{-2}]}$$

$$\boxed{B = [ML^{-1}]}$$

Turbulent flow in pipes & channels

* The viscous sub-layer occupies only a very small fraction of the total cross-section.
  It has no sharp upper boundries & its thickness is difficult to define & in this region only viscous shear is important.

* A transition layer exist immediately adajacent to the viscous sub-layer in which both viscous shear & shear due to eddy diffusion exist.

* The bulk of the cross-section of the flowing stream is occupied by entirely turbulent flow known as turbulent core. In turbulent core viscous shear is negligible in comparison with that from eddy diffusion.

velocity distribution for turbulent flow

The curve for turbulent flow is clearly much flatter than that for laminar flow.
In turbulent as in laminar flow the velocity gradient is zero at the centre line. It is known

that the eddies in the turbulent core is large but of low intensity & those in the transition zone are small but intense.

Most of the K.E. content of the eddies lies in the buffer zone & the outer portion of the turbulent core.

**

$$\boxed{u^* = \bar{v}\sqrt{\frac{f}{2}} = \sqrt{\frac{\tau_w}{\rho}}}$$

$$\boxed{u^+ = \frac{u}{u^*}} \; ; \; \boxed{y^+ = \frac{y u^* \rho}{\mu} = \frac{y\sqrt{\tau_w \rho}}{\mu}}$$

where u^* = frictional velocity
u^+ = velocity quotient
y^+ = distance
y = distance from wall of tube.

* y^+ may be considered to be reynolds based on the frictional velocity & distance from the wall.

* Radius of the tube $\boxed{r_w = r + y}$

Equations relating u^+ to y^+ are called universal velocity distribution laws.

* In laminar sub-layer $u^+ = y^+$; $y^+ < 5$
* In buffer laminar $u^+ = 5 \ln y^+ - 3.05$
* In turbulent core $u^+ = 2.5 \ln y^+ + 5.5$; $y^+ > 30$

Ques(55):- Water flows through a smooth circular pipe
2016:- under turbulent condition. In the viscous sub-layer the velocity varies linearly with the distance from the wall. The faining friction factor is defined as $f = \frac{\tau_w}{\rho \bar{u}^2/2}$ where τ_w is the shear stress at the wall of the pipe, ρ is the density of the fluid & $\bar{u}$ is the average velocity in the pipe. Water ($\rho = 1000\ kg/m^3$, $\mu = 10^{-3}\ kg\ m^{-1}\ s^{-1}$) flows at an average velocity of 1 m/s through the pipe.
For this flow condition the friction factor $f = 0.005$. At a distance of 0.05 mm from the wall of the pipe (in the viscous sub-layer) the velocity is ______ (in m/s).

Given : viscous sub-layer,

$$\Rightarrow \quad u^+ = y^+ \ ; \ y^+ < 5$$

$$\Rightarrow \quad \frac{u}{u^*} = \frac{y u^* \rho}{\mu}$$

Now, $$u^* = \bar{V}\sqrt{\frac{f}{2}} = \sqrt{\frac{\tau_w}{\rho}}$$

$$\Rightarrow \quad u = \frac{y u^{*2} \rho}{\mu} = \frac{y \bar{V}^2 f \rho}{2\mu}$$

$$\Rightarrow \quad u = \frac{0.05 \times \cancel{10^{-3}} \times 1 \times 0.005 \times 1000}{2 \times \cancel{10^{-3}}}$$

$$\Rightarrow \quad \boxed{u = \frac{0.00025 \times 1000}{2} = 0.125\ m/s}$$

Average velocity

$$\boxed{\frac{\bar{V}}{U_{max}} = \frac{1}{1 + 3.75\sqrt{f/2}}}$$

* If nothing is given $\frac{\bar{V}}{U_{max}} = 0.82$

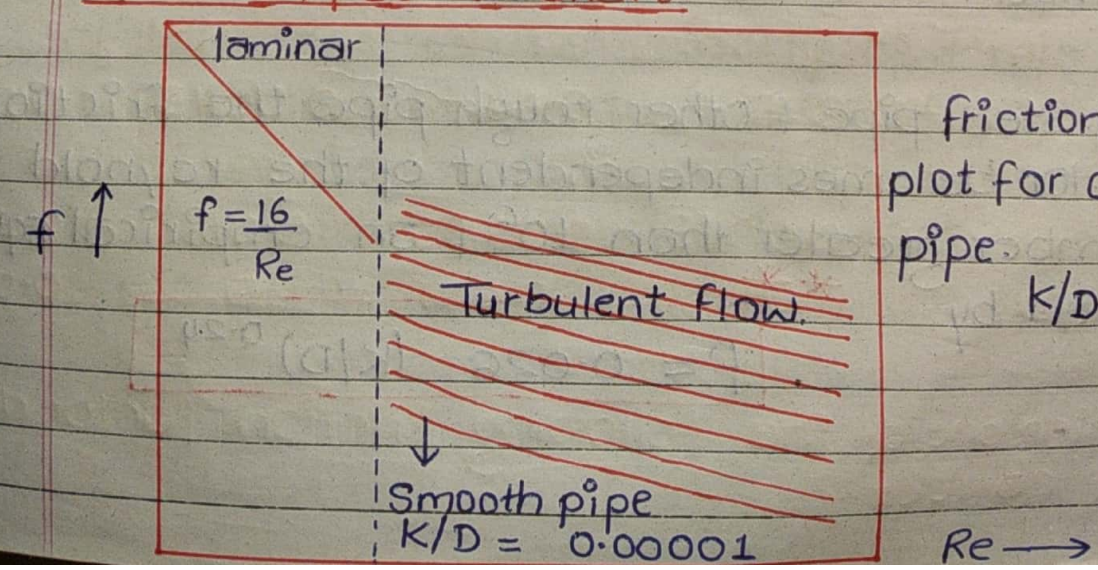
⇒ $\boxed{\alpha = 1.032}$

⇒ $\boxed{\beta = 1.011}$

We know that in turbulent flow a rough pipe leads to a larger friction factor for a given reynolds number than a smooth pipe.

⇒ If a rough pipe is smoothed, the friction factor is reduced. When further smoothing brings about no further reduction in the friction factor for a given reynolds number the tube is said to be hydraulically smooth.

Friction factor chart



friction factor plot for circular pipe.

$\frac{k}{D}$ = relative roughness.

k = roughness parameter

* for turbulent flow the lowest line represent the friction factor for smooth tubes & integral eqⁿ for this line is given by **

$$\boxed{f = 0.046\, Re^{-0.2}}$$

This applies over a range of reynolds number about 50,000 to 1×10^6.

* Another eqⁿ applicable over a range of reynolds from 3000 to 3×10^6 is given by **

$$\boxed{f = 0.0014 + \frac{0.125}{Re^{0.32}}}$$

* The other curved lines in the turbulent flow range represents the friction factor for various types of commercial pipe, each of which is characterised by a different value of reynold number.

* for steal pipe & other rough pipe the friction factor becomes independent of the reynold number greater than 10^6 & an empirical eqⁿ is given by **

$$\boxed{f = 0.026\, (k/D)^{0.24}}$$

* for different flow regimes in given system the variation of pressure drop with flow rate can be found as

→ for laminar flow ($Re < 2100$).

$$\boxed{\frac{\Delta p_s}{L} \propto \bar{V}}$$

→ For turbulent flow ($2500 < Re < 10^6$)

$$\boxed{\frac{\Delta p_s}{L} \propto \bar{V}^{1.8}}$$

→ for very turbulent flow ($Re > 10^6$)

$$\boxed{\frac{\Delta p_s}{L} \propto \bar{V}^2}$$

Ques (56) :- For a flow through a smooth pipe the
2016 :- faining friction factor is given by $f = m\,Re^{-0.2}$ in the turbulent flow regime where Re is reynolds number & m is a constant. Water flowing through a section of this pipe with a velocity of 1 m/s results in a frictional pressure drop of 10 kpa. What will be the pressure drop across this section (in kpa) when the velocity of 2 m/s.

(i) 11.5 (ii) 20 (iii) 34.8 ✓ (iv) 40.

$$\frac{\Delta p_{s1}}{\Delta p_{s2}} = \frac{\bar{V}_1^{1.8}}{\bar{V}_2^{1.8}}$$

$$\Rightarrow \frac{10\times10^3}{\Delta p_{s2}} = \frac{(1)^{1.8}}{(2)^{1.8}}$$

$$\Rightarrow \Delta p_{s2} = \frac{10^3\times 10\times(2)^{1.8}}{1}$$

$$\Rightarrow \boxed{\Delta p_{s2} = 34.8 \text{ kpa}}$$

Ques(57):- For a newtonian fluid flowing in a circular
2013:- pipe under steady state condn in fully developed laminar flow. The faining friction factor is given by

(i) $0.046\, Re^{-0.2}$ (ii) $0.0014 + \frac{0.125}{Re^{0.32}}$

(iii) $\frac{16}{Re}$ (iv) $\frac{24}{Re}$

Ques(58):- The following table provides four sets of faining
2017:- friction data for different values of reynolds number & roughness factor

	Re	10^2	10^3	10^5	10^5
Set-I	f	0.16	0.016	16×10^{-5}	16×10^{-5}
Set-II	f	0.016	0.16	0.0055	0.0045
Set-III	f	0.16	0.016	0.0045	0.00055
Set-IV	f	0.0045	0.0055	0.016	0.16
	K/D	0	0.001	0	0.001

Which one of the following friction factor data is correct?

(i) Set-I (ii) Set-II
(iii) Set-III (iv) Set-IV.

Friction from changes in velocity & dir^n

→ Friction loss from sudden expansion of cross-section :-

** $$\boxed{h_{fe} = K_e \frac{\bar{V}_a^2}{2}}$$

where, ** $$\boxed{K_e = \left(1 - \frac{S_a}{S_b}\right)^2}$$

K_e = expansion loss coefficient.
$\bar{V}_a$ = average velocity in larger section.

→ Friction loss from sudden contraction of cross-section :-

** $$\boxed{h_{fc} = K_c \frac{\bar{V}_b^2}{2}}$$

** $$\boxed{K_c = 0.4 \left(1 - \frac{S_b}{S_a}\right)}$$

where, K_c = contraction loss coefficient
$\bar{V}_b$ = average velocity in the smaller section or down stream section.

→ Effect of fittings & valves :-

**

$$\boxed{h_{ff} = k_f \frac{\bar{V}^2}{2}}$$

where,

k_f = loss factor for fitting

$\bar{V}$ = average velocity in pipe leading to fitting.

**

$$\Rightarrow \boxed{h_f = \left(4f\frac{L}{D} + k_c + k_e + k_f\right)\frac{\bar{V}^2}{2}}$$

Ques(59) :- water is pumped from a tank A to B using a 5 cm diameter pipe & a pump with the delivery head of 30 m. Both the tanks are uncovered & the water level in the tank 20 m above water level in the tank A. The total pressure loss coefficients for the piping is 50. Calculate the velocity of water through the pipe is ______.

Given : h_{pump} = 30. m

$z_2 - z_1 = 20 m$

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 + h_{pump} = \cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f$$

$$h_{pump} = (z_2 - z_1) + h_f.$$

$$h_f = 30 - 20 = \underline{10 \text{ m}}.$$

$$h_f = K_f \frac{\bar{V}^2}{2}$$

$$\frac{10}{50} \times 2 = \bar{V}^2$$

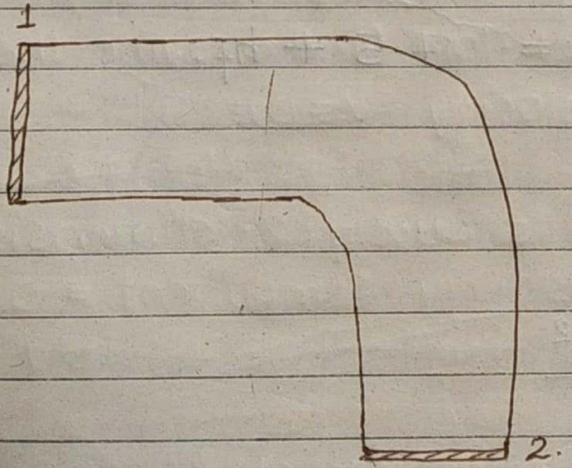
$$\Rightarrow \bar{V}^2 = \frac{2}{5} = 0.4$$

$$\Rightarrow \boxed{\bar{V} = 0.632 \text{ m/s}}$$

Ques(60) :- The inlet velocity of water ($\rho = 1000$ kg/m^3) in a right angled bend reduces $V_1 = 1$ m/s as shown in figure. The inlet diameter is 0.8m & the outlet diameter is 0.4 m. The flow is turbulent & the velocity profile at the inlet & outlet are flat. Gravitational forces are negligible.

(i) Find pressure drop $(P_1 - P_2)$ across the bend assuming negligible frictional losses.

(ii) If the actual pressure drop is $P_1 - P_2 = 82.5$ kpa. find out the friction factor loss coefficients based on velocity V_1.



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(i) $\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{g} z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{g z_2}$

$\Rightarrow \frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_1^2}{2}$

Since, $Q_1 = Q_2$.

$\Rightarrow A_1 V_1 = A_2 V_2$

$\Rightarrow D_1^2 V_1 = D_2^2 V_2$

$\Rightarrow V_2 = \frac{0.8 \times 0.8 \times 1}{0.4 \times 0.4}$

$\Rightarrow \boxed{V_2 = 4 \text{ m/s}}$

$\Rightarrow P_1 - P_2 = \frac{16-1}{2} \times 1000$

$\Rightarrow \boxed{P_1 - P_2 = 7.5 \text{ kpa}}$

(ii) $\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{g} z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{g} z_2 + h_f$

$\Rightarrow \frac{P_1 - P_2}{\rho} = 8 - 0.5 + h_f$

$\Rightarrow \frac{82.5 \times \cancel{10^3}}{\cancel{1000}} = 7.5 + h_f$.

$\Rightarrow h_f = 75.$

$\Rightarrow h_f = k_f \frac{V_1^2}{2}$

$\Rightarrow \boxed{k_f = 150}$