#Gate Syllabus [12 Marks]

- Fluid statics
- Newtonian & non-newtonian fluids.
- Shell balances including differential form of Bernoulli equation & energy balance.
- Macroscopic friction factors
- Dimensional analysis & similitude.
- Flow through pipelines system
- Flow meters.
- Pumps & compressors
- Elementary boundary layer theory
- Flow past immersed bodies including packed & fluidized beds
- Turbulent flow
- Fluctuating velocity
- Universal- velocity profile & ~~profile~~ pressure drop.

FLUID MECHANICS

Fluid mechanics is that branch of science that deals with behaviour of fluid at rest as well as in motion & the interaction of fluids with solids or other fluids at the boundaries.

Fluid Mechanics

Fluid static	Fluid kinematics	Fluid dynami
The study of fluid at rest is called fluid static	When pressure forces are not considered	If the pressure forces are considered.

Fluid

A fluid is a substance that does not permanentl resist distortion. An attempt to change the shape of mass of a fluid results in layers of fluid sliding over one another, until a new shape is attained.

During the change in shape, shear stress exist the magnitude of which depend upon the viscosit of the fluid & rate of sliding but when a final shape is attained all shear stresses have disappeared.

* A fluid in equilibrim is free from shear stresses.

Shear

Shear is the lateral displacement of materials relative to another layer by an external force.

Shear stress

It is defined as the ratio of this force to the area of the layer.

Property of fluid

1. Density or mass density

- It is defined as the ratio of mass of a fluid to its volume.
- Its S.I. unit is kg/m^3.
- The density of all fluids depends on the temp & pressure.
- If the density changes only slightly with moderate changes in temp. & pressure, the fluid is said to be incompressible.
- If the changes in density are significant, the fluid is said to be compressible.
- Liquids are generally incompressible in nature while gases are compressible in nature.

$$\boxed{\rho = \frac{\text{mass of fluid}}{\text{volume of fluid}}}$$

2. Specific weight or weight density

It is defined as the ratio of the weight of a fluid to its volume.

$$\boxed{w = \frac{\text{Weight of fluid}}{\text{vol}^m \text{ of fluid}} = \rho g}$$

3. Relative density or specific gravity

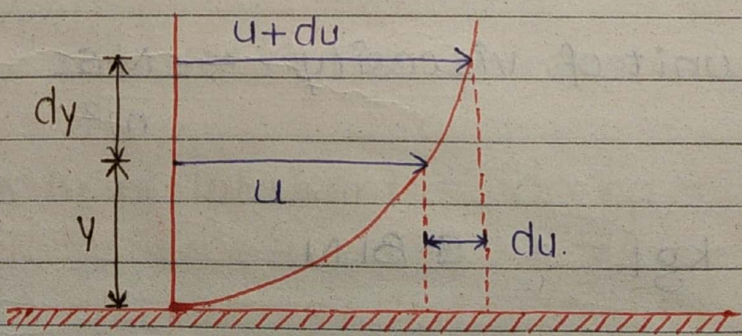
It is defined as the ratio of the density of a fluid to the density of a standard fluid.

- for liquids, the standard fluid is taken as water.
- for gases, the standard fluid is taken as air.

$$\boxed{S = \frac{\text{density of liquid}}{\text{density of water}}}$$

4. Viscosity

It is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.



When two layers of a fluid at a distance dy from each other at different velocities u & u+du respectively.

The top layer causes a shear stress on the

adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.
This shear stress is proportional to the rate of change of velocity w.r.t. small y.

$$\tau \propto \frac{du}{dy}$$

or,
$$\boxed{\tau = \mu \frac{du}{dy}}$$

This is known as Newton's law of viscosity.

- $\frac{du}{dy}$ = Rate of shear strain or velocity gradient

- M.K.S. unit of viscosity = $\frac{kgf \cdot sec}{m^2}$
- C.G.S. unit of viscosity = $\frac{dyne \cdot sec}{cm^2}$
- S.I. unit of viscosity = $\frac{N \cdot s}{m^2}$ = Pa. sec.
- 1 kgf = 9.81 N
- 1 N = 10^5 dyne
- $1 \frac{kgf\ sec}{m^2} = 9.81 \frac{N\ sec}{m^2} = \frac{9.81 \times 10^5}{10^4} \frac{dyne \cdot sec}{cm^2}$

- $1 \frac{kgf\ sec}{m^2} = 98.1$ dyne sec/cm^2 or poise

$\boxed{1 \text{ poise} = 1 \text{ dyne sec}/cm^2}$

$\boxed{1 \text{ poise} = 0.1 \text{ pa sec}}$

viscosity of water at 20°C is 0.01 poise or 1 centipoise or 1 cp.

<u>Ques(1):-</u> A lubricant 100 times more viscous than
<u>Gate (2003):-</u> water would have a viscosity (in Pa. sec).
(i) 0.01 (ii) 0.1 (iii) 1 (iv) 10.

Viscosity of water = 1 cp.

but, 1 poise = 0.1 pa. sec.

$\Rightarrow$ 1 cp = 10^{-3} pa. sec.

$\Rightarrow$ viscosity of lubricant = 100 × 1 cp.

$\Rightarrow$ 100 cp = $10^{-3} \times 100$ pa. sec

$\Rightarrow$ viscosity of lubricant = 100 cp = <u>0.1 pa.sec</u>.

<u>Ques(2):-</u> viscosity of water at 40°C lies in the range of
<u>Gate (2004):-</u> (i) $1 \times 10^{-3} - 2 \times 10^{-3}$ kg/m·s
(ii) $0.5 \times 10^{-3} - 1 \times 10^{-3}$ kg/m·s
(iii) $1 \times \cancel{10^{-3}} - 2$ kg/m·s
(iv) 0.5 – 1 kg/m.s.

$\Rightarrow$ 0.01 poise = $\frac{0.01}{10}$ pasec = 10^{-3} pasec or kg/ms.

5. Kinematic viscosity

It is defined as the ratio between dynamic viscosity and density of the fluid.

$$\boxed{\nu = \frac{\text{dynamic viscosity}}{\text{density of fluid}} = \frac{\mu}{\rho}}$$

- Dimension $= \frac{[ML^{-1}T^{-1}]}{[ML^{-3}]} = [L^2T^{-1}]$

- unit is m^2/sec.

- In M.K.S. or S.I. unit of kinematic viscosity is m^2/sec.

- While in C.G.S. unit is cm^2/sec.

In C.G.S. unit it is known as stoke.

$$\boxed{1 \text{ stoke} = \frac{cm^2}{sec} = 10^{-4}\ \frac{m^2}{sec}}$$

Fluids which follow newton's law of viscosity known as newtonian fluid & which do not follow are known as non-newtonian fluid.

Vapour pressure & Cavitation

Vapour pressure of a pure substance is defined as the pressure exerted by its vapour in phase eq^m with its liquid at a given temperature.

Consider a flowing liquid in a system if the pressure at any point in this flowing liquid

becomes equal or less than the vapour pressure, the vapourisation of liquid starts.
The bubbles of these vapour are carried by flowing liquid into the region of high pressure where they collapse giving rise to high impact pressure.
The pressure developed by the collapsing bubbles is so high that the material from adjoining boundaries get eroded & cavities are formed on them.
This process is known as cavitation.

* To avoid cavitation the pressure at any point in flow should be maintained greater than the vapour pressure.

No slip Condition
Consider the flow of a fluid in a stationary pipe or over a solid surface i.e. non-porous. All experimental observations indicate that a fluid in motion comes to a completely stop at the surface & assume a zero velocity relative to a surface.

* The fluid property responsible for no slip cond^n & the development of the boundary layer is viscosity.

Another consequence of the no-slip cond^n is the surface drag or skin friction drag which is the force, a fluid exertes on a surface in the flow dir^n.

* No slip condⁿ applies everywhere along the surface.

Variation of viscosity with temperature

The viscosity of liquids decreases with the increase of temperature while the viscosity of gases increases with increase of the temperature.

* for Gases :- In gases, the cohesive forces are very small & molecular momentum transfer dominates. With the increase in temperature, molecular momentum transfer increases & viscosity increases.

In gases, momentum is transfered by molecules moving relatively large distance to regions where the velocity is lower.

The viscosity depends on the average momentum of the molecules which is proportional to the molecular weight times the average velocity.

Since the velocity is proportional to $\left(\frac{T}{M}\right)^{1/2}$, the viscosity is proportional to

$$\boxed{\mu \propto (MT)^{1/2}}$$

* The viscosity also depends on mean free path with decrease as the size of molecule increase.

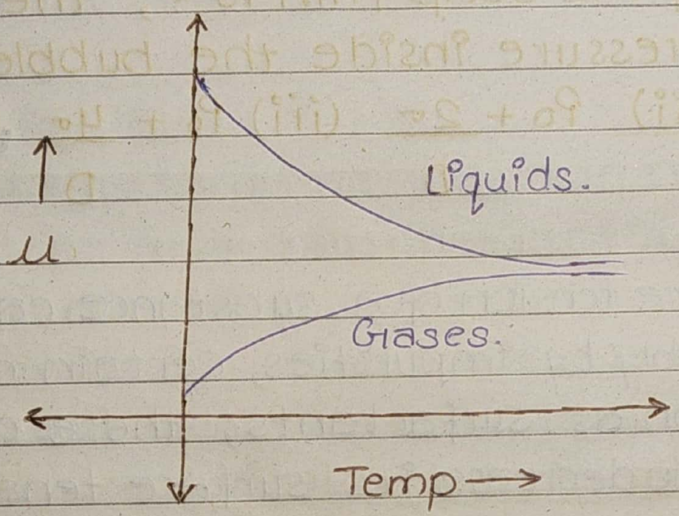
Gases viscosity at room temperature are generally b/w 0.005 cp to 0.02 cp at 20°C.

* The viscosity of air is 0.018 cp

$$\boxed{\mu \propto \sqrt{T}}$$

A simple theory of non-interacting molecule

$$\boxed{\mu = 0.00267 \frac{(MT)^{1/2}}{\sigma^2}}$$



* The viscosity of liquids are much greater than those of gases at the same temperature.

⇒ Surface Tension

* It is defined as the tensile force acting on the surface of liquid in contact with a gas or on the surface b/w two immscible liquids such that the contact surface behaves like a membrane under tension.
* The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area.

* Surface tension on liquid droplet, $\boxed{\Delta p = \frac{4\sigma}{D}}$

* Surface tension on Hollow bubble (soap bubble)

$$\boxed{\Delta p = 8\sigma/D}$$

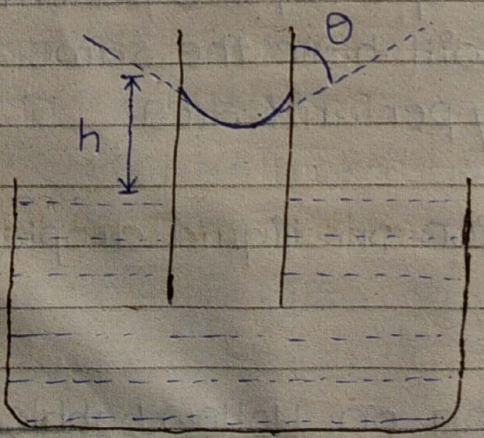
* Surface tension on a liquid jet $\boxed{\Delta p = \frac{2\sigma}{D}}$

Ques(3):- Consider a soap film bubble of diameter D
2007:- if the external pressure is P_0 & surface tension of the soap film is σ, the expression for the pressure inside the bubble is,
(i) P_0 (ii) $P_0 + \frac{2\sigma}{D}$ (iii) $P_0 + \frac{4\sigma}{D}$ ✓(iv) $P_0 + \frac{8\sigma}{D}$

The surface tension of a substance can be changed considerably by impurities, certain chemicals are known as surfactants, these can be added to liquid to decrease its surface tension.

⇒ Capillary
It is a phenomena of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

* Its value depends upon a specific weight of the liquid, diameter of the tube & the surface tension of the liquid.



$$\boxed{h = \frac{4\sigma \cos\theta}{\rho g d}}$$

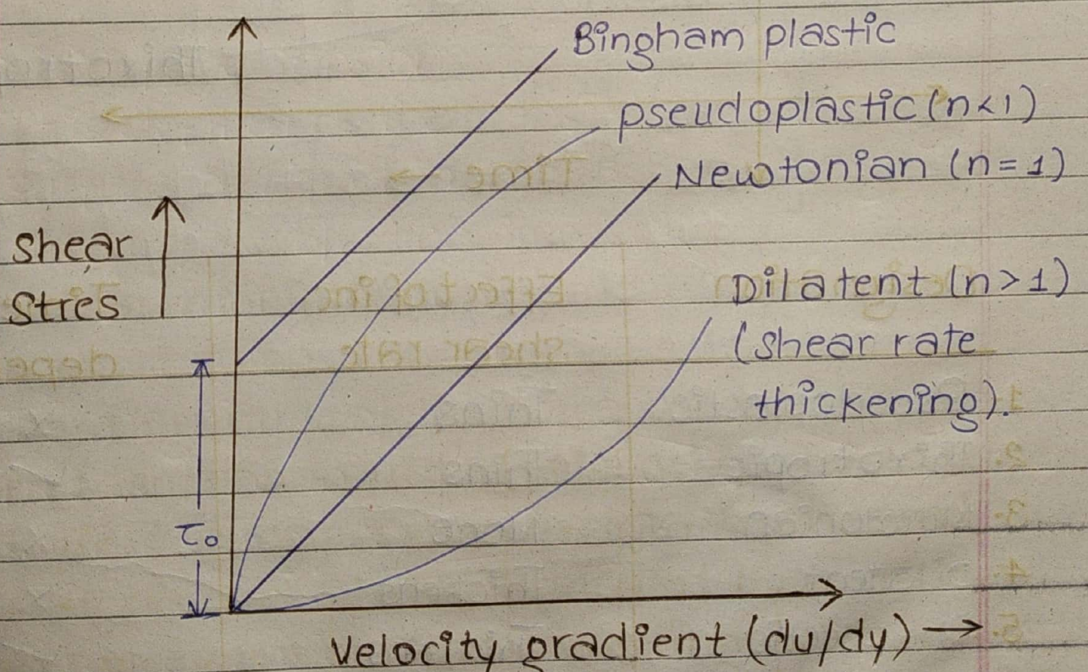
Meniscus :- The curved free surface of a liq. in a capillary tube is called meniscus.

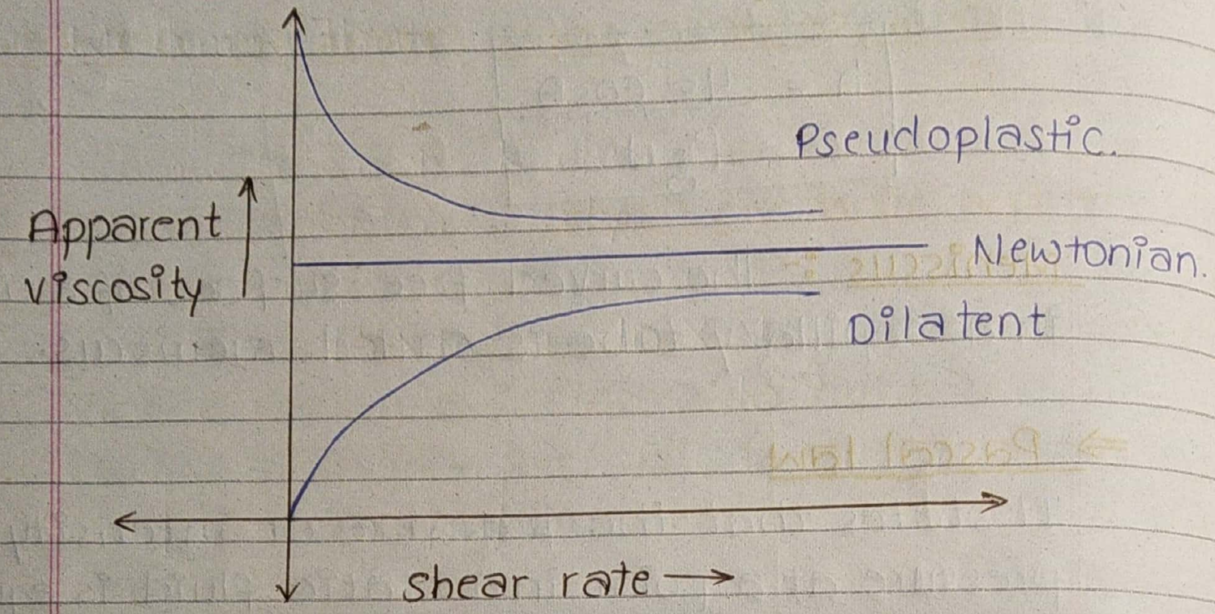
⇒ Pascal law

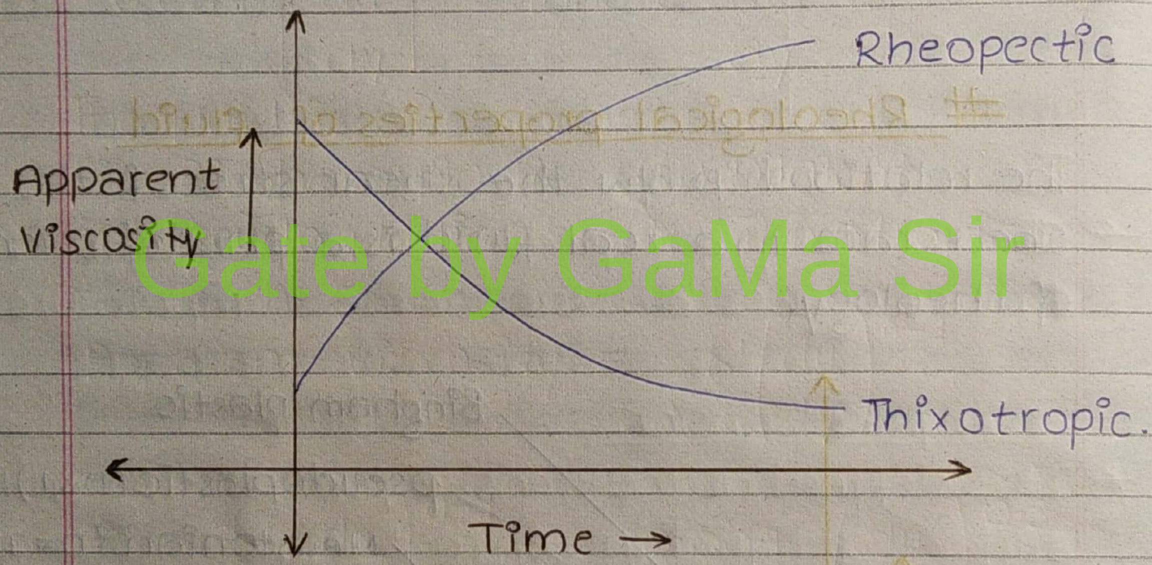
It states that the pressure or intensity of pressure at a point in a static fluid is equal in all direction.

Rheological properties of fluid

The relationship b/w the shear stress & the shear rate in a real fluid is a part of science of rheology.







Designation	Effect of inc. shear rate	Time dependent
1. Pseudoplastic	Thins	×
2. Thixotropic	Thins	✓
3. Newtonian	None	×
4. Dilatent	Thickens	×
5. Rheopectic	Thickens	✓

⇒ Rate of shear vs shear stress for non-newtonian fluids

→ for Bingham plastic

$$\boxed{\tau = \tau_0 + k\frac{du}{dy}}$$

where k is a constant.

* over some range of shear rates, dilatent & pseudoplastic fluid follows a power law, also known as ostwald-de-waele eqn

$$\boxed{\tau = k'\left(\frac{du}{dy}\right)^{n'}}$$

where, n' = flow behaviour index
k' = flow consistency index.

* $$\boxed{\tau = \left[k'\left(\frac{du}{dy}\right)^{n-1}\right]\left(\frac{du}{dy}\right)}$$

Apparent viscosity : $k'\left(\frac{du}{dy}\right)^{n-1}$

Ques(4):- A bingham fluid of viscosity $\mu = 10$ pa·sec
2007:- & yield stress $\tau_0 = 10$ kpa is sheared b/w flat parallel plates separated by a distance 10^{-3} m. The top plate is moving with a velocity of 1 m/s. The shear stress on the plate is
(i) 10 kpa (ii) 20 kpa (iii) 30 kpa (iv) 40 kpa

We know that,

$$\tau = \tau_0 + K\frac{du}{dy}$$

$$\tau = 10\times10^3 + 10\times\frac{1}{10^{-3}}$$

$$\boxed{\tau = 20\,kpa}$$

Ques(5):- The apparent viscosity of fluid is given by
2013:- $0.007\left|\frac{du}{dy}\right|^{0.3}$ where $\frac{du}{dy}$ is velocity gradient
The fluid is
(i) Bingham plastic (ii) Dilatent
(iii) Pseudo plastic (iv) Thixo tropic

$$\text{Apparent viscosity} = K'\left(\frac{du}{dy}\right)^{n'-1}$$

$$\Rightarrow K'\left(\frac{du}{dy}\right)^{n'-1} = 0.007\left(\frac{du}{dy}\right)^{0.3}$$

on compairing

$\Rightarrow n'-1 = 0.3$

$\Rightarrow \boxed{n' = 1.3}$ i.e. $n' > 1$

$\Rightarrow$ fluid is Dilatent....

Ques(6):- Which of the following statement are correct
2014 :- (P) For a rheopectic fluid the apparent viscosity increases with time under a constant applied shear stress.

(Q) For a pseudo plastic fluid, the apparent viscosity decreases with time under a constant applied shear stress.

(R) For a Bingham plastic the apparent viscosity increase exponentially with the deformation rate.

(S) For a dilatent fluid, the apparent viscosity increases with increasing deformation rate.

(i) P & Q only
(ii) Q & R only
(iii) R & S only
(iv) P & S only.

Ques(7):- Match the column :
2005 :-

I	II
P) Pseudo plastic	1) $\tau = \mu\gamma$
Q) Bingham plastic	2) $\tau = \tau_0 + k\gamma$
	3) $\tau = k\|\gamma)^n \quad n<1$
	4) $\tau = k\|\gamma\|^n \quad n>1$

where, τ = shear stress ; γ = strain rate.

Pressure & pressure measurement

Absolute pressure :- It is defined as the pressure which is measured with reference to vacuum absolute pressure.

Gauge Pressure :- Gauge pressure is defined as the pressure which is measured with the help of measuring instruments, in which the atmospheric pressure is taken as datum.

$$\boxed{P_{abs} = P_{atm} + P_{gauge}}$$

Vacuum Pressure :- Atmospheric pressure − Absolute pressure

Pressure measurement devices

- Manometer
  - Simple Manometer
    - Piezometer
    - U-tube manometer
    - Single Column manometer.
  - Differential Manometer
- Mechanical Gauges
  - Diaphragm pressure Gauge
  - Bourdon tube
  - Dead Weight
  - Bellow.

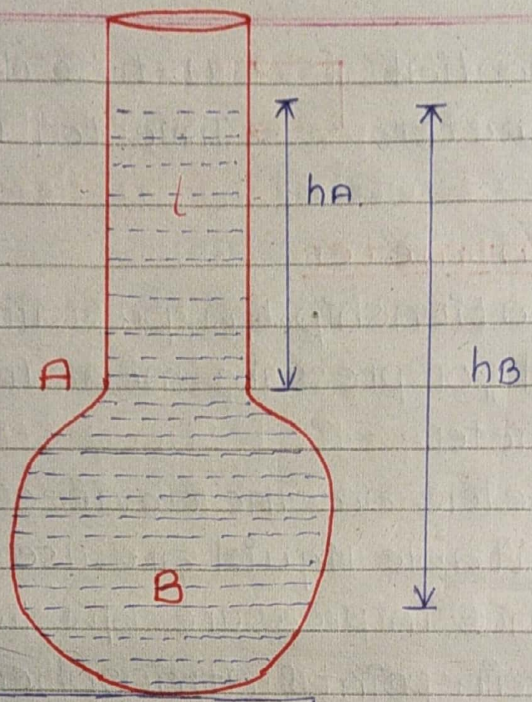
Differential Manometer → U-tube differential manometer
→ Inverted U-tube.

Manometer.

The relationship b/w pressure & the head is utilised for pressure measurement in the manometer.
Manometers are the devices in which column of a suitable liquid are used to measure the difference in pressure b/w two points or b/w a certain point & atmosphere.
Manometers are extensively used for measurement of pressure in various flow measuring devices.

Simple Manometer :- A simple manometer consists of a glass of plastic tube adding one end connected to the point where the pressure is to be measured while the other end open to the atmosphere.

Piezometer :- The most elementary manometer is called a piezometer. A piezometer is essentially a glass of plastic tube mounted vertically so that it is connected to the space within the container.
Liquid ~~rises~~ rises by the verticle distance from the meniscus to the point where the pressure is to be measured.



$$\Rightarrow \boxed{\begin{aligned} P_A &= \rho g h_A \\ P_B &= \rho g h_B \end{aligned}}$$

Limitations :-

→ It measures only positive gauge pressure.

→ The use of piezometer is also impracticle for measuring large pressure.

→ If the working fluid is the gas, the piezometer is not possible to use because gases don't have free surface.

U- tube manometer

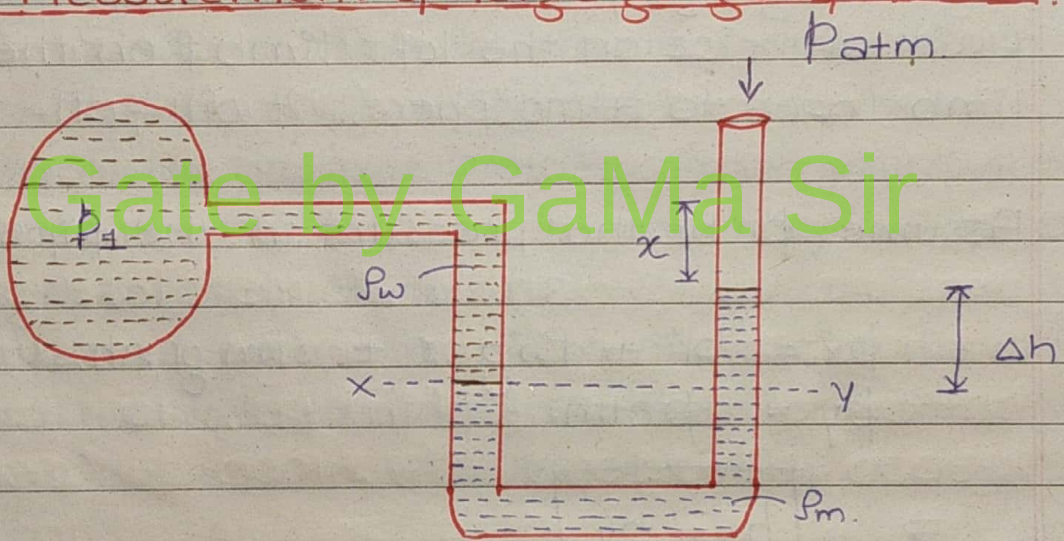
The most elementary manometer which is frequently used for the measurement of small negative or large gauge pressure is the U-tube manometer. The lower part of the U-tube contains a liquid which is immiscible with the working fluid. This fluid is known as manometric fluid.

* The choice of manometric fluid depends on the range of pressure to be measured.

Desirable property of manometric fluid

- It should have low vapour pressure.
- It should have a well-define meniscus at the interface.
- It should have low surface tension to avoid capillary rise.
- The fluid should be immiscible with the working fluid.
- Mercury has widely used as manometric fluid because of its low vapour pressure & high density.

Case(I): Measurement of large gauge pressure



$$p_x = p_1 + \rho_w g (x + \Delta h)$$

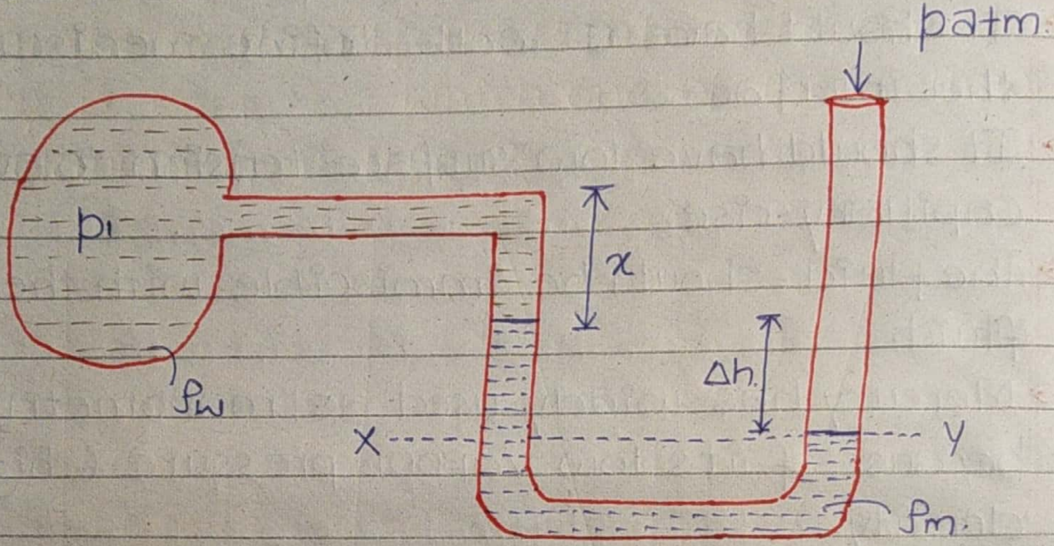
$$p_y = p_{atm} + \rho_m g (\Delta h)$$

from the fundamental law of fluid static.

$$\boxed{p_x = p_y}$$

** $$\boxed{p_1 - p_{atm} = (\rho_m - \rho_w) g \Delta h - \rho_w g x}$$

Case (II) :- Measurement of negative gauge pressure



When the manometer is used to measure the negative gauge pressure the level of manometric fluid will rise on the left limb & on the right limb (open to atmosphere), it will fall.

Assume p1 be the pressure in the pipe.

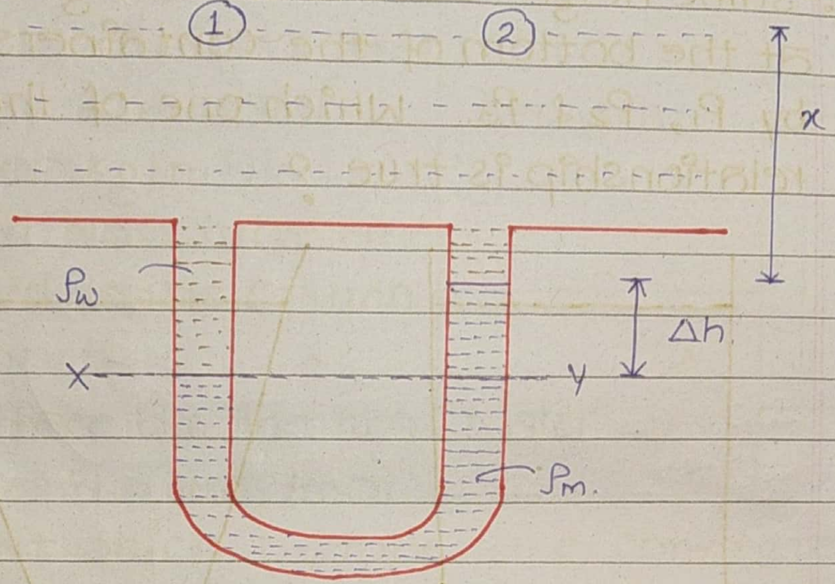
$$p_x = p_1 + \rho_w g\,x + \rho_m g\,\Delta h$$
$$p_y = p_{atm}$$

$$\Rightarrow \boxed{p_x = p_y}$$

$$\Rightarrow \boxed{p_{atm} - p_1 = \rho_w g x + \rho_m g \Delta h}$$

U-tube differential manometer

It is very handy in nature to measure the pressure difference between two points in the flow field directly.



A differential U-tube is connected b/w two points (1) & (2) in a pipe through which the fluid is flowing.

Since the fluid is flowing from point 1 to point 2, the pressure at point 1 will be higher than pressure at point 2.

Therefore, the level of meniscus in the left limb will fall down & in the right limb will go up.

Assume p_1 & p_2 be the pressure.

$$p_x = p_1 + \rho_w g (x + \Delta h)$$

$$p_y = p_2 + \rho_w g x + \rho_m g \Delta h$$

$$\boxed{p_1 - p_2 = (\rho_m - \rho_w) g \Delta h}$$

* Barometer is a special manometer used for measuring atmospheric air pressure.

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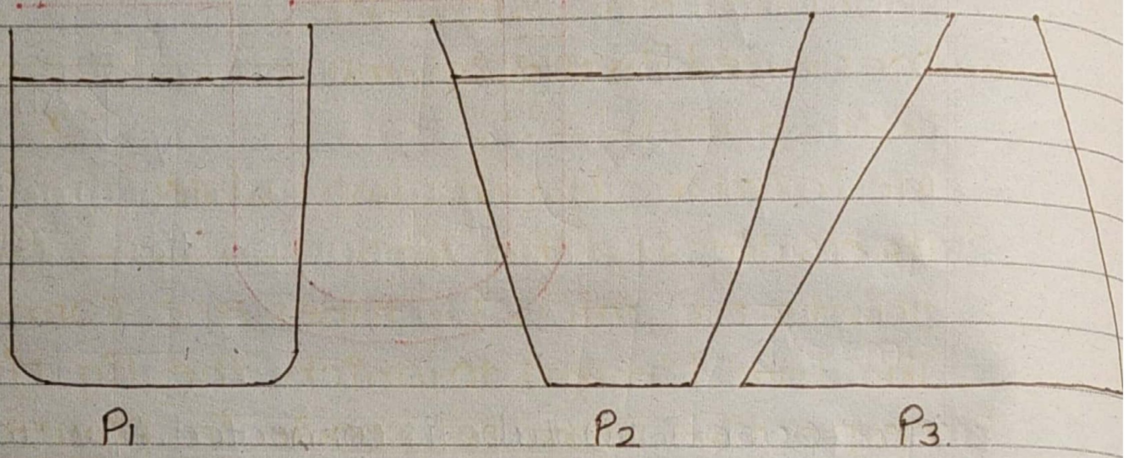
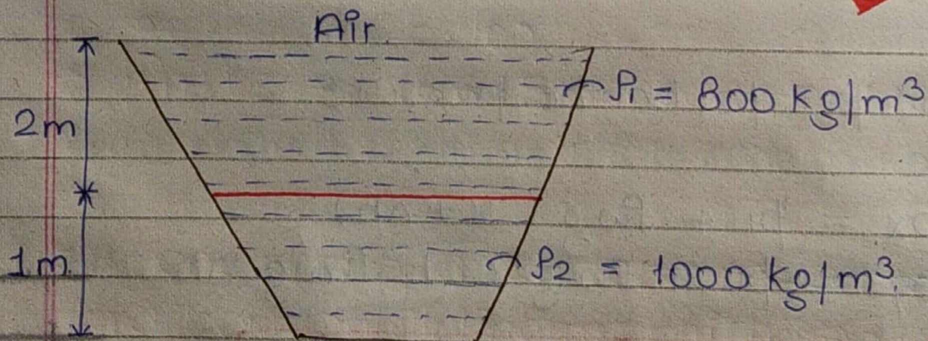
Ques(8):- 3 containers are filled with water upto the
2008:- same height as shown in figure. The pressure at the bottom of the containers are denoted by P_1, P_2 & P_3. Which one of the following relationship is true ?



(i) $P_3 > P_1 > P_2$ (ii) $P_2 > P_1 > P_3$
(iii) $P_1 > P_2 = P_3$ (iv) $P_1 = P_2 = P_3$.

Ques(9):- An open tank contains two immiscible liq of densities as shown in figure. Take $g = 10 m/s^2$ under static condⁿ the gauge pressure at the bottom of the tank (in pa) is
(i) 23000 (ii) 22000 (iii) 24000 (iv) 26000.

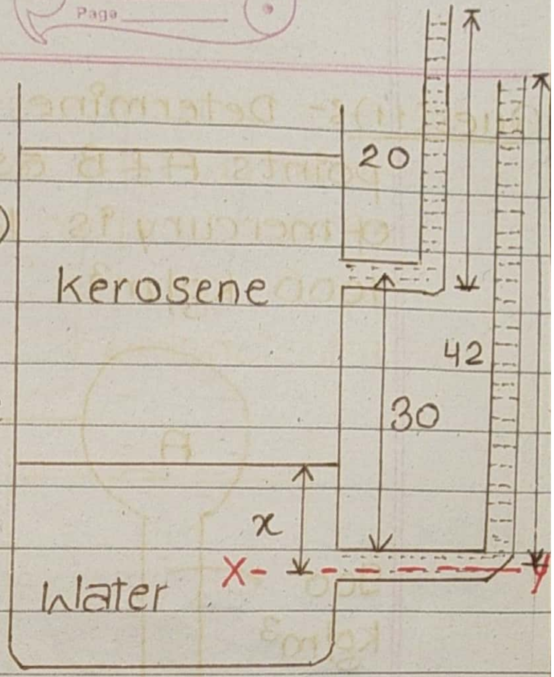


$P = \rho_1 g h_1 + \rho_2 g h_2$

$P = 800 \times 10 \times 2 + 1000 \times 10 \times 1$

$\boxed{P = 26000 \text{ pa}}$

Ques(10):- A verticle cylindrical vessel has a layer of kerosene ($\rho_k = 800\ kg/m^3$) over a layer of water ($\rho_w = 1000\ kg/m^3$).
L-shaped glass tube are connected to the column 30cm apart.
The interface b/w the two layers lies b/w the two points at which L-tubes are connected.
The level (in cm) to which the liq rise in respective tubes as shown in figure.
The distance x (in cm) of the interface from the point at which the lower L-tube is connected is ______.



Pressure at point x

$$p_x = P_{atm} + \rho_k g(20+30-x) + \rho_w g x.$$

$$p_y = P_{atm} + \rho_w g \cdot 42$$

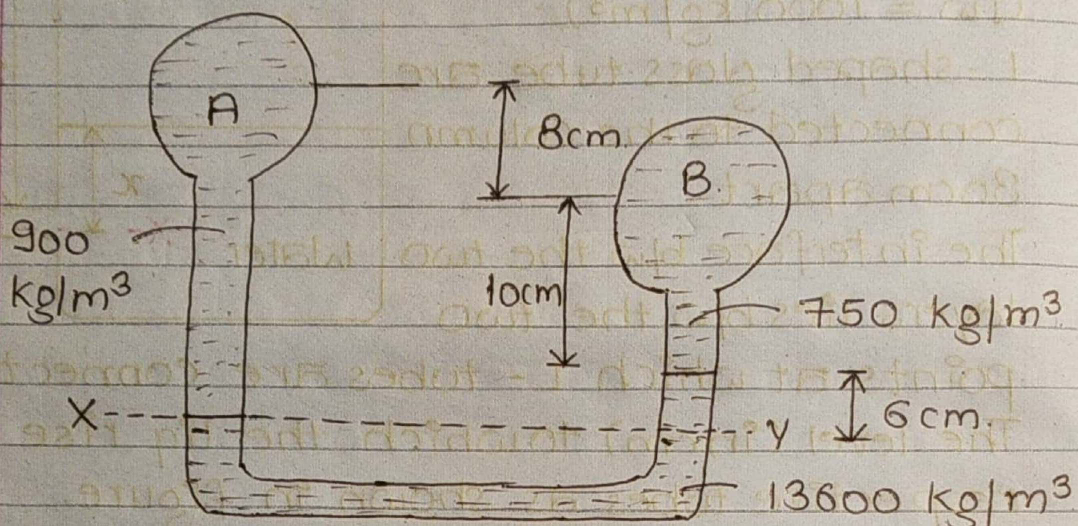
$\Rightarrow \quad p_x = p_y$

$$\Rightarrow \cancel{P_{atm}} + 800 \times 9.8 \times (50-x) + 1000 \times 9.8 \times x = \cancel{P_{atm}} + 1000 \times 9.8 \times 42$$

$$\Rightarrow 392000 - 7840x + 9800x = 411600$$

$$\Rightarrow \boxed{x = 10\ cm}$$

Ques(11):- Determine the pressure difference b/w points A & B as shown in figure. Assume density of mercury is 13600 kg/m³ & density of water 1000 kg/m³.



$p_x = p_A + 900 \times 9.8 \times 24$

Gate by GaMa Sir

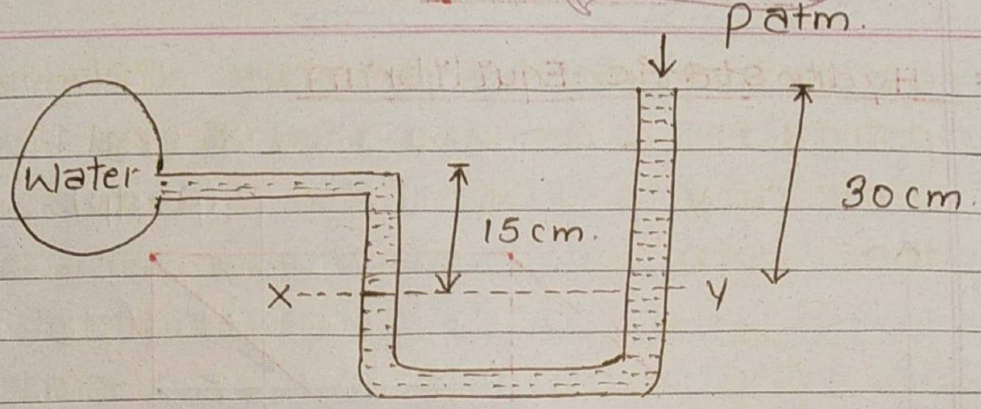
$p_y = p_B + 750 \times 9.8 \times 10 + 13600 \times 9.8 \times 6$

$p_x = p_y$

$\Rightarrow p_A + 900 \times 9.8 \times \frac{24}{100} = p_B + 750 \times 9.8 \times \frac{10}{100} + 13600 \times \frac{6}{100} \times 9.8$

$\Rightarrow \boxed{p_A - p_B = 6615 \text{ pascal or } N/m^2}$

Ques(12):- one end of a U-tube manometer is connected to a horizontal line in which water is flowing & its other end is opened to atmosphere as shown in figure. If the density of the manometric fluid is 4000 kg/m³. find the gauge pressure & absolute pressure in the pipe. Take atmosphere pressure as 101 kN/m².



$$p_x = P_w + \rho_w g \times h_1 = P_w + 1000 \times 9.8 \times 0.15$$

$$p_y = P_{atm} + \rho_f g h_2 = P_{atm} + 4000 \times 9.8 \times 0.3$$

$p_x = p_y$ (because x & y pts are on same height).

$\Rightarrow P_w + 1000 \times 9.8 \times 0.15 = P_{atm} + 4000 \times 9.8 \times 0.3$

$\Rightarrow P_w - P_{atm} = P_{gauge}$

8 $\Rightarrow P_{gauge} = P_w - 101 \times 10^3 = 111.29 - 101$
$= \underline{10.3\ kN/m^2}$

$\Rightarrow P_w = 101 \times 10^3 + 11760 - 1470$

$\Rightarrow P_{abs} = P_w = \underline{111.29}\ kN/m^2$ or kPa

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