Fluid kinematics [2 Marks]

Kinematics of flow

Kinematic is defined as that branch of science which deals with motion of particles without considering the forces causing the motion.

The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics.

Methods of describing fluid motion

1. **Lagrangian method**

In this method, a single fluid particle is followed during its motion & its velocity, acceleration, density, etc are described.

2. **Eulerian method**

In this method, the velocity, acceleration, density, etc are described at a point in the fluid flow.

* Eulerian method is commonly used in fluid mechanics.

Types of fluid flow

(i) Steady & unsteady flow.
(ii) uniform & non-uniform flow
(iii) Laminar & turbulent flow
(iv) compressible & incompressible flow.

(v) 1, 2 & 3 – dimensional flow.

→ 3–Dimensional unsteady flow

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

→ 3– Dimensional steady flow

$$u = f_1(x, y, z)$$
$$v = f_2(x, y, z)$$
$$w = f_3(x, y, z)$$

(vi) Rotational & Irrotational flow

Rotational flow is that type of flow in which the fluid particles while flowing along stream lines also rotate about their ~~on~~ own axis. While in irrotational flow fluid particles do not rotate about their own axis.

Rate of flow or Discharge (Q)

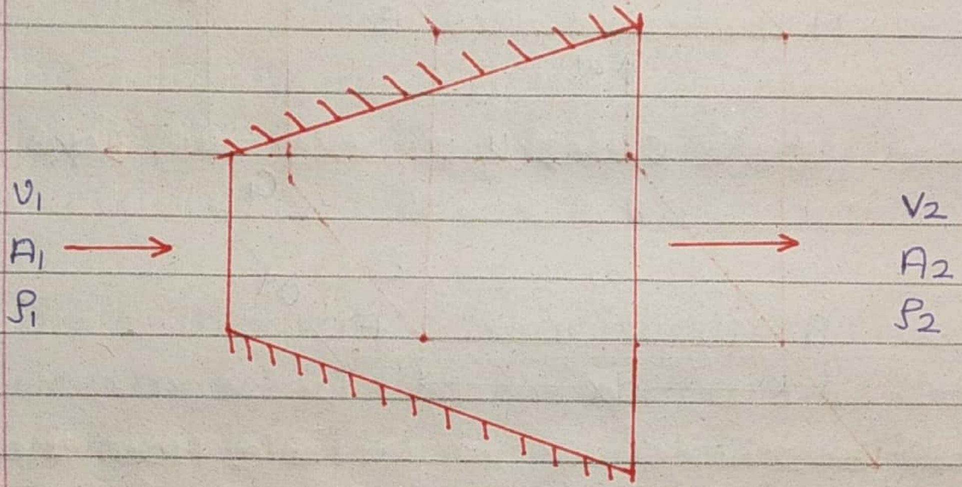
It is defined as the quantity of a fluid flowing per second through a section of pipe or a channel.

** $\boxed{\text{Discharge } (Q) = A \cdot V}$

where, A = cross-sectional area of pipe.
V = Average velocity of fluid across the section.

Continunity Equation

This eqn is based on the principle of conservation of mass. For a fluid flowing through the pipe at all the cross-section the quantity of fluid per second is constant.



Rate of flow at section 1 = $A_1 V_1 \rho_1$

Rate of flow at section 2 = $A_2 V_2 \rho_2$

from conservation of mass

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$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

This equation is applicable for compressible & incompressible fluid.

⇒ For incompressible ; i.e. ρ = constant.

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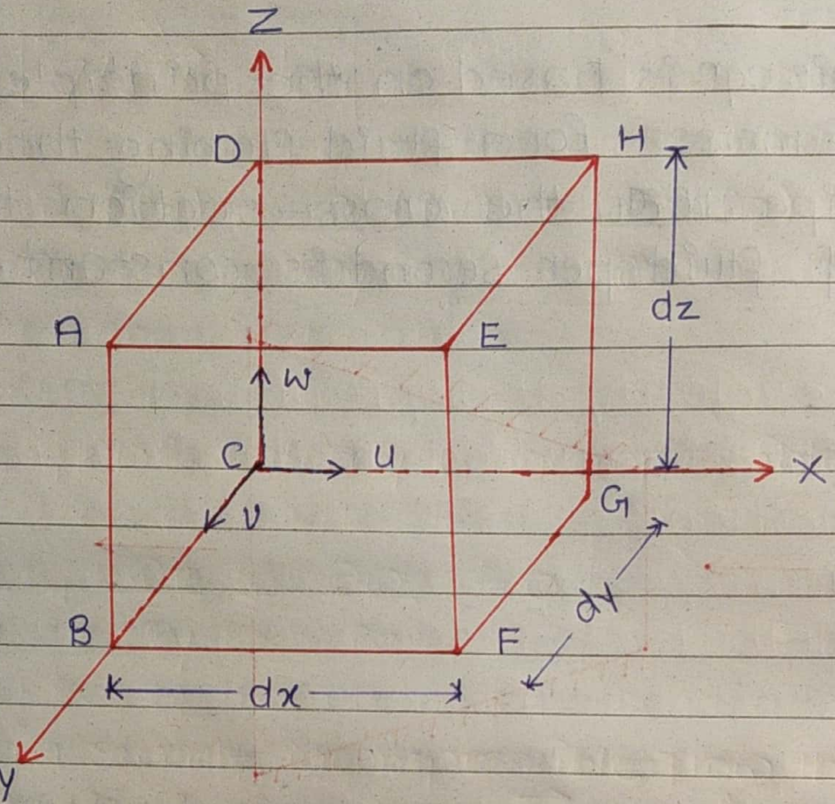
$$\boxed{A_1 V_1 = A_2 V_2}$$

continunity equation in 3 dimension



Consider a fluid element of lengths dx, dy, dz in x, y & z dir^n respectively & assume u, v & ω are the inlet velocity component in x, y & z dir^n respectively.

The mass of fluid entering the face ABCD per sec

$$= \rho \times (\text{velocity in } x\text{-dir}^n) \times \text{area of ABCD.}$$

$$= \rho u \, (dy \cdot dz)$$

The mass of fluid leaving the face EFGH per sec

$$= \rho u \, dy \, dz + \frac{\partial}{\partial x} (\rho u \, dy \, dz) \, dx.$$

Gain of mass in x – dirn = $\cancel{\rho u}\,\cancel{dy}\,dz - \cancel{\rho u\,dy dz}$

$$- \frac{\partial}{\partial x}(\rho u\, dy \cdot dz) \cdot dx$$

Net gain of masses $= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\, dy\, dz.$

Since, the mass is neither created nor be destroyed in the fluid element, the net inc. of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element.

Rate of increase of mass with time

$$= \frac{\partial}{\partial t}(\rho\, dx\, dy\, dz)$$

on equating,

** $$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0.}$$

This is known as continuity equation.

** $$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \nabla \rho \vec{V} = 0}$$

⇒ If the flow is steady

$$\boxed{\nabla(\rho \vec{V}) = 0}$$

∵ ** $\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$

⇒ If the flow is steady & incompressible

** $\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$

Ques(22):- In a steady, incompressible flow the velocity
2014:- distribution is given by, $\vec{V} = 3x\hat{i} - py\hat{j} + 5z\hat{k}$
where v is in (m/s) & x, y, z are in m.
In order to satisfy the mass conservation the value of constant p is ______.

$$\vec{V} = 3x\hat{i} - py\hat{j} + 5z\hat{k}$$

$$u = 3x \quad ; \quad v = -py \quad ; \quad w = 5z$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow 3 - p + 5 = 0$$

$$\Rightarrow \boxed{p = 8}$$

Ques(23):- The velocity field in a 2-D flow field is given by $\vec{V} = (x^2y + yz)\hat{i} - xy^2\hat{j}$. Verify that the above velocity field describes the motion of an incompressible flow.

for incompressible flow;

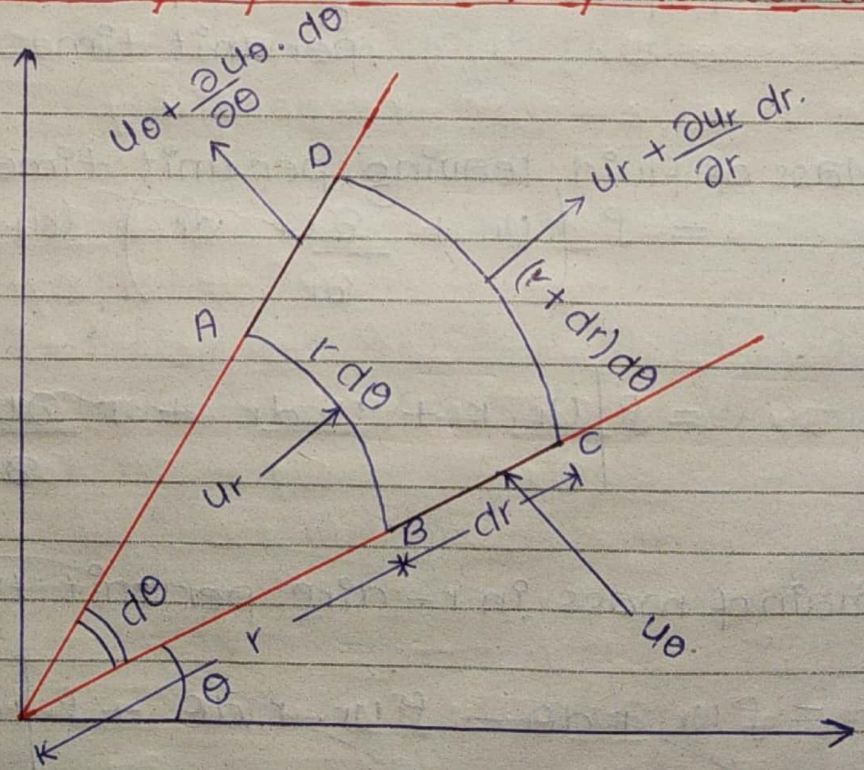
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

$$\Rightarrow u = x^2y + yz \quad ; \quad v = -xy^2$$

$$\Rightarrow \frac{\partial u}{\partial x} = 2xy \quad ; \quad \frac{\partial v}{\partial y} = -2xy$$

$$\Rightarrow \boxed{2xy - 2xy = 0}$$

continuity equation in cylindrical coordinate



Consider a 2 dimensional compressible flow field. The 2-D polar coordinates are r & θ. Consider a fluid element ABCD b/w the r & $r+dr$. The angle subtended by the element at the centre is $d\theta$.

The components of velocity, U_r in radial dirn & U_θ in the tangential dirn.

$$AB = r\,d\theta$$
$$CD = (r+dr)\,d\theta$$
$$BC = AD = dr$$

The thickness of the element perpendicular to the plane of the paper is assumed to be unity.

Consider the flow in radial dirn.

Mass of fluid entering the face AB per unit time $= \rho U_r (r\,d\theta \cdot 1)$

Mass of fluid leaving per unit time the face CD

$$= \rho \left(U_r + \frac{\partial U_r}{\partial r} dr \right) (r+dr)\,d\theta \cdot 1$$

$$= \rho \left[U_r \cdot r + U_r\,dr + r \frac{\partial U_r}{\partial r} dr \right] d\theta$$

Gain of mass in r-dirn per unit time

$$= \cancel{\rho U_r \cdot r\,d\theta} - \cancel{\rho U_r \cdot r\,d\theta} - \rho \left[U_r\,dr + r \frac{\partial U_r}{\partial r} dr \right] d\theta$$

$$= -\rho\left[\frac{u_r}{r} + \frac{\partial u_r}{\partial r}\right] r\,dr\,d\theta.$$

Consider the flow in θ - dirn.

Gain in mass in θ - dirn. per unit time

$$= \rho u_\theta dr\cdot 1 - \rho\left(u_\theta + \frac{\partial u_\theta}{\partial\theta}d\theta\right)dr\cdot 1$$

$$= -\rho\frac{\partial u_\theta}{\partial\theta}\frac{r}{r}dr\cdot d\theta.$$

Total gain in fluid mass per unit time

$$= -\rho\left[\frac{u_r}{r} + \frac{\partial u_r}{\partial r}\right] r\,dr\,d\theta - \rho\frac{\partial u_\theta}{\partial\theta}\,dr\,d\theta.\;\; r/r$$

Rate of increase of mass with time

$$= \frac{\partial}{\partial t}(\rho\,dr\cdot d\theta\cdot r\cdot 1)$$

**

$$\Rightarrow \boxed{\frac{\partial\rho}{\partial t} + \rho\left[\frac{u_r}{r} + \frac{\partial u_r}{\partial r}\right] + \rho\frac{\partial u_\theta}{\partial\theta}\frac{1}{r} = 0}$$

Velocity & Acceleration

Assume v is a resultant velocity at a point in a fluid flow. Assume u, v, w are its components in x, y & z-directions. The velocity components are function of space coordinate & time.

Mathematically,
The velocity components are given by

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

$$\Rightarrow \quad \vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

**

$$\Rightarrow \quad \boxed{|\vec{V}| = \sqrt{u^2 + v^2 + w^2}}$$

Assume a_x, a_y & a_z are the total acceleration in x, y & z-dirn respectively.

$$\Rightarrow \quad a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}$$

**

$$\Rightarrow \quad \boxed{a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}}$$

Ques
2014 :

Similarly,

**

$$\boxed{a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}}$$

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** $$\boxed{a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}}$$

Accerelation vector ;

** $$\boxed{\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$$

** $$\boxed{|\vec{A}| = \sqrt{a_x^2 + a_y^2 + a_z^2}}$$

Local accerelation :- It is defined as rate of increase of velocity with respect to time at a given point in a flow field. in the above eq^n the terms — $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ are known as local accerelation terms.

Convective acceleration :- It is defined as rate of change of velocity due to change of position of fluid particles in a fluid flow. The terms other than $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ are known as convective acceleration terms.

$\frac{\partial u}{\partial t}$

Ques (24) :- 2014 :- An incompressible fluid is flowing through a cross-section of length L & has one-D (x-dir^n), steady state velocity distribution is given by $u = u_0\left[1 + \frac{2x}{L}\right]$ if $u_0 = 2\,m/s$ & $L = 3\,m$. The convective acceleration in m/s^2 of fluid particle at the length L is ____

Given, $u_0 = 2\ m/s$ $L = 3\ m$

$$a_x = u\frac{\partial u}{\partial x}$$

$$\Rightarrow \frac{\partial u}{\partial x} = u_0\left[\frac{2}{L}\right]$$

$$\Rightarrow a_x\Big|_{x=L} = u_0^2\left[1+\frac{2x}{L}\right]\left[\frac{2}{L}\right]$$

$$\Rightarrow a_x\Big|_{x=L} = u_0^2\left[1+2\right]\left[\frac{2}{L}\right]$$

at $L = 3\ m$ & $u_0 = 2\ m/s$

$$\Rightarrow a_x = 4(\cancel{3})\left(\frac{2}{\cancel{3}}\right)$$

$$\Rightarrow \boxed{a_x = 8\ m/s^2}$$

Ques(25) :- A steady flow field of an incompressible
2008 :- fluid is given by $\vec{V} = (Ax + By)\hat{i} - Ay\hat{j}$
where $A = 1\ sec^{-1}$ or $B = +1\ sec^{-1}$. x & y
are in (m). The magnitude of acceleration in
(m/s^2) of a fluid particle at (1, 2) is
(i) 1 (ii) $\sqrt{5}$ (iii) $\sqrt{2}$ (iv) $\sqrt{10}$

$u = x + y$; $v = -y$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$\Rightarrow a_x = (x+y)(1) - y(1)$$

$$\Rightarrow a_x\Big|_{(1,2)} = 1+2-2$$

$$\Rightarrow \boxed{a_x = 1 \ m/s^2}$$

$$a_y = (x+y)(0) + y(1)$$

$$a_y\Big|_{(1,2)} = 2 \ m/s^2$$

$$\Rightarrow \vec{A} = a_x\hat{i} + a_y\hat{j}$$

$$\Rightarrow \vec{A} = 1\hat{i} + 2\hat{j}$$

$$|\vec{A}| = \sqrt{1^2 + 2^2}$$

$$|\vec{A}| = \sqrt{5}$$

Ques(26):- The velocity field in a fluid flow is given by

$$\vec{V} = (6+2x+t^2)\hat{i} - (xy^2+10t)\hat{j} + 20\hat{k}$$

Calculate the accerelation of fluid particle at (1,0,2) at $t=1$.

$$u = 6+2x+t^2$$

$$v = -(xy^2+10t)$$

$$w = 20.$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_x = [6+2(1)+1^2](2) + (-10)(0) + 0 + 2(1)$$

$$\Rightarrow \boxed{a_x = 20 \ m/s^2}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_y = 9(0) + (-10)(0) + 20(0) + (-10)$$

$$\Rightarrow \boxed{a_y = -10 \ m/s^2}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$a_z = 9(0) + (-10)0 + 20(0) + 0$$

$$\boxed{a_z = 0}$$

$$\vec{A} = 20\hat{i} - 10\hat{j}$$

$$|\vec{A}| = \sqrt{20^2 + 10^2}$$

$$|\vec{A}| = \sqrt{400 + 100} = \sqrt{500}$$

$$\boxed{|\vec{A}| = 10\sqrt{5}}$$

Ques(27):- The velocity field in a fluid flow is given by $\vec{V} = x^2t\,\hat{i} + 2xyt\,\hat{j} + 2yzt\,\hat{k}$ where x, y & z are in (m) & t is in sec. Determine the velocity at point (2, −1, 1) at t = 1 sec. Also determine the magnitude of velocity & acceleration of the flow for the given location & time.

$$\vec{V} = x^2t\,\hat{i} + 2xyt\,\hat{j} + 2yzt\,\hat{k}$$

$u = x^2t = 4$

$v = 2xyt = -4$

$w = 2yzt = -2$

$$\boxed{\vec{V} = 4\hat{i} - 4\hat{j} - 2\hat{k}}$$

$$|\vec{V}| = \sqrt{4^2 + 4^2 + 2^2}$$

$$|\vec{V}| = \sqrt{16 + 16 + 4} = 6.$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_x = 4(2xt) - 4(0) - 2(0) + x^2$$

$$a_x = 4(4) + (2)^2 = \underline{20\ m/s^2}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}.$$

$$a_y = 4(2yt) - 4(2xt) - 2(0) + 2xy.$$

$a_y = 4(-2) - 4(4) - 4 = -8 - 16 - 4$

$a_y = \underline{-28 \ m/s^2}$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$a_z = 4(0) + (-4)(2zt) - 2(2yt) + 2yz$

$a_z = -4(2) - 2(-2) + (-2) = -8 + 4 - 2$

$a_z = \underline{-6 \ m/s^2}$

$$\vec{A} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$|\vec{A}| = \sqrt{20^2 + 28^2 + 6^2}$$

$$\boxed{|\vec{A}| = 34.9}$$

Stream lines :- stream line at any instant can be defined as an imaginary line in the flow field so that the tangent at any point represents the dir^n of instantaneous velocity at that point.

From the definition of stream line it can be written as

$$\boxed{\vec{V} \times \vec{ds} = 0}$$

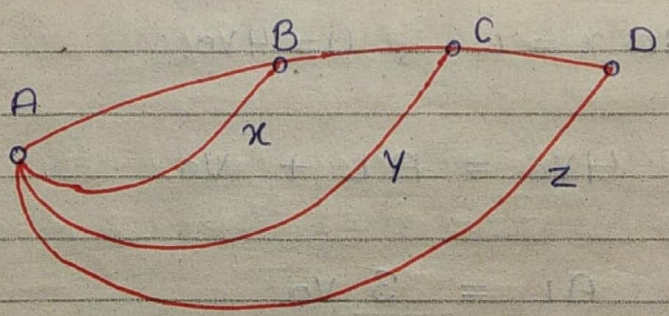
$\Rightarrow$ $\boxed{\dfrac{dx}{u} = \dfrac{dy}{v}}$ It represents the eq^n of stream line in the $x-y$ plane.

Pathline :- A pathline is the actual trajectory through space of selected fluid particles during a time of interval.

* Path lines & stream lines are identical in an steady flow.

* A pathline is a lagrangian concept because it is defined by the motion of fluid particles.

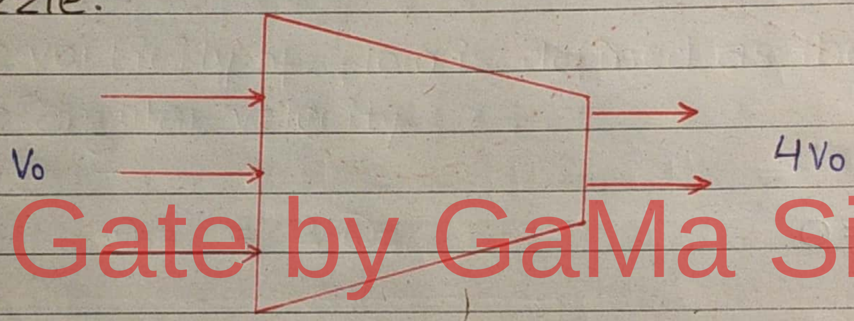
Streak line :- A streak line at any instant of time is the temprorary locations of all the particles that have passed through a fixed point in the flow field.



* Stream lines, pathlines & streak lines are identical in steady flow.

* Stream line is a instantaneous line while the streak line & path line are generated by the passage of time.

Ques(28):- Fluid flows steadily through a conversing nozzle of length L. Flow can be approximated as 1-dimensional such that the axial velocity varies linearly from entrance to exit. The velocity at the entrance & exit are V_0 & $4V_0$ respectively. Find out an expression of the acceleration of the particle flowing through the nozzle.



Assume, $u = Ax + B$.

at $x = 0$; $u = V_0$.

$$\Rightarrow \boxed{B = V_0}$$

at $x = L$; $u = 4V_0$.

$$\Rightarrow 4V_0 = AL + V_0.$$

$$\Rightarrow AL = 3V_0.$$

$$\Rightarrow \boxed{A = 3\frac{V_0}{L}}$$

$$\Rightarrow \quad u = 3\frac{V_o}{L}x + V_o.$$

$$\Rightarrow \quad u = V_o\left[3\frac{x}{L} + 1\right]$$

Now,

$$a_x = u\frac{\partial u}{\partial x}.$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = V_o\left[\frac{3}{L}\right].$$

$$\Rightarrow \quad \boxed{a_x = \frac{3V_o^2}{L}\left[1 + \frac{3x}{L}\right]}$$

Rotation & vorticity

The angular velocity or rotation about the z-axis, ω_z is the arthimetic average of angular velocity of the line segment AB & CD, which were originally perpendicular to each other. The angular velocity at A can be written as

**

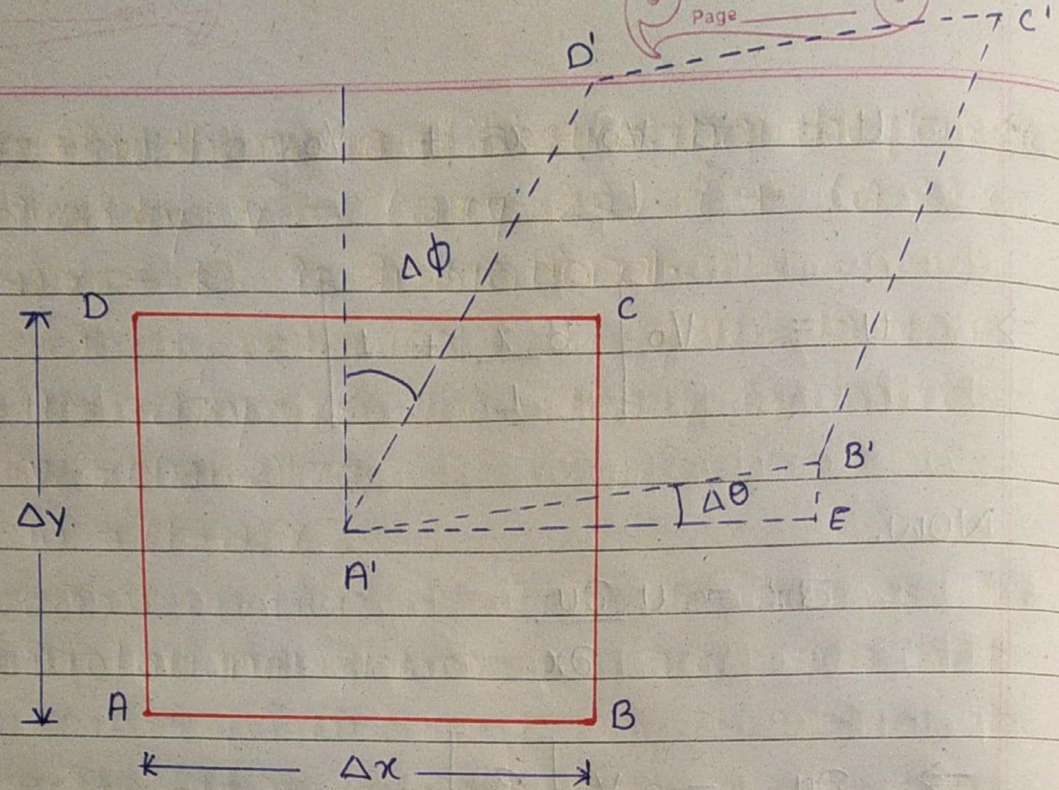
$$\boxed{\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}$$

**

$$\boxed{\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)}$$

**

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$



⇒ The vorticity of flow is defined as the twice the angular velocity.

$$\Omega_z = 2\omega_z$$

$$\boxed{\Omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}}$$

Rate of ~~linear~~ angular deformation

$$\boxed{\dot{\varepsilon}_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}}$$

Rate of linear deformation

$$\boxed{\dot{\varepsilon}_x = \frac{\partial u}{\partial x}}$$

Ques(29):- The velocity for a 2-D flow is given by:

$$\vec{V} = (x + 2y + 2)\hat{i} + (4 - y)\hat{j}$$

Check whether the flow is (i) compressible or incompressible. (ii) Rotational or irrotational.

$$u = x + 2y + 2 \qquad v = 4 - y$$

(i) for compressible & incompressible.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{(for incompressible)}$$

$$\Rightarrow \frac{\partial u}{\partial x} = 1 \quad ; \quad \frac{\partial v}{\partial y} = -1$$

$\Rightarrow 1 - 1 = 0 \quad \therefore$ flow is incompressible.

(ii) for rotational & irrotational.

$$\omega_z = \left[-\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] \times \frac{1}{2}$$

$$\frac{\partial u}{\partial y} = 2 \quad ; \quad \frac{\partial v}{\partial x} = 0.$$

$$\omega_z = \frac{1}{2}\left[0 - 2\right]$$

$$\omega_z = -1$$

$\therefore$ flow is rotational.

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Ques(30):- Find the value of 'a' such that the flow field is given by $\vec{V} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$. is irrotational.

$u = axy - z^3$
$v = (a-2)x^2$
$w = (1-a)xz^2$

for irrotational flow,

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0.$$

$$\Rightarrow \frac{\partial v}{\partial x} = 2(a-2)x$$

$$\Rightarrow \frac{\partial u}{\partial y} = ax$$

$$\Rightarrow 2(a-2)x - ax = 0.$$

$$\Rightarrow 2(a-2)\cancel{x} = a\cancel{x}$$

$$\Rightarrow \boxed{a = 2}$$

Ques(31):- If the velocity vector in a 2-D flow field is given by $\vec{V} = 2xy\,\hat{i} + (2y^2 - x^2)\hat{j}$. find vorticity.

$u = 2xy$
$v = 2y^2 - x^2$

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vorticity, $\Omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

$\frac{\partial u}{\partial y} = 2x$; $\frac{\partial v}{\partial x} = -2x$.

$\Omega_z = -2x - 2x$.

$\Rightarrow \boxed{\Omega_z = -4x}$

velocity potential function & stream func

velocity potential function :- It is defined as a scalar function of space & time such that its negative derivative w.r.t. any dirn gives the fluid velocity in that ~~any~~ dirn. It is denoted by ϕ & defined as

$$u = -\frac{\partial \phi}{\partial x} \; ; \; v = -\frac{\partial \phi}{\partial y} \; ; \; w = -\frac{\partial \phi}{\partial z}.$$

Continunity eqn for incompressible steady flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

$$\Rightarrow \boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0.}$$

$$\Rightarrow \boxed{\nabla^2 \phi = 0.}$$

* If velocity potential exist the flow should be irrotational.

* If velocity potential satisfy the la place eqn it represents the steady, incompressible irrotational flow.

Stream function :- It is defined as the scalar function of time & space such that its partial derivative w.r.t. any dirn gives the velocity component at right angle to that dirn. It is denoted by ψ & is defined as

** $$\boxed{\frac{\partial \psi}{\partial x} = v}$$ ** $$\boxed{\frac{\partial \psi}{\partial y} = -u}$$

* It is defined only for 2 dimensional flow. If stream funn exist it is a possible case of fluid flow which may be rotational or irrotational.

* If stream funn satisfy the laplace eqn it is a possible case of an irrotational flow.

* The equipotential lines are orthogonal to the stream lines at all points of the intersection.

For equipotential line

$$\phi = \text{constant}$$

$$\Rightarrow d\phi = 0.$$

$$\phi = f(x, y).$$

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy$$

$$\Rightarrow \quad 0 = -u\,dx - v\,dy.$$

**

$$\Rightarrow \quad \boxed{\frac{dy}{dx} = -\frac{u}{v}}$$

Line of constant stream function.

$$\psi = \text{constant}.$$

$$\Rightarrow \quad d\psi = 0.$$

$$\psi = f(x, y).$$

$$\Rightarrow \quad d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy.$$

$$\Rightarrow \quad 0 = v\,dx - u\,dy.$$

$$\Rightarrow \quad \boxed{\frac{dy}{dx} = \frac{v}{u}}$$

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