Assignment No 1

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SP-21-110
Linear Algebra

QNo1

a) As, According to the given scenerio Coeffecient matrix of the system has a pivot position in every row, i.e no row of the form

$\{0, 0, \cdots, 0 \quad a\}$, where $a \neq 0$

Then system will be consistent.

$$\begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

e.g;

$$Ad = \left[\begin{array}{cc|c} 1 & 5 & 2 \\ -1 & 2 & 4 \end{array}\right] \quad R_2 + R_1$$

$$Ad = \left[\begin{array}{cc|c} 1 & 5 & 2 \\ 0 & 7 & 6 \end{array}\right]$$

Since,

Rank (A) = Rank (Ad)

So, the solution exists & the system is consistent.

b) $Ax = b$

$$A = \begin{Bmatrix} 1 & c \\ a & 3 \end{Bmatrix}, \quad b = \begin{Bmatrix} 1 \\ h \end{Bmatrix}$$

Solve

Roll No 21110

3rd digit = 1 ; 4th digit = 1

$\boxed{a = 1}$, $\boxed{c = 1}$

$$Ad = \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 3 & h \end{array}\right] R_2 - R_1$$

$$Ad = \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & h-1 \end{array}\right]$$

So,

Rank (A) = Rank (Ad)

System is consistent & solution exists

And,

Rank (A) = Rank (Ad) = no. of unknowns.

So, The system is unique.

c) Using gauss's Jordan Method.

$$A_d = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ h \end{bmatrix}$$

$$A_d = \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & h-1 \end{array}\right] \quad R_2 - R_1$$

$1x_1 + x_2 = 1 \rightarrow$ eq 1

$2x_2 = h-1 \rightarrow$ eq 2

$\boxed{x_2 = \frac{h-1}{2}} \rightarrow$ eq 2

$x_1 = 1 - x_2 \rightarrow$ From eq 1

$X_1 = 1 - \left(\frac{h-1}{2}\right)$

$X_1 = 1 - \frac{(h-1)}{2}$

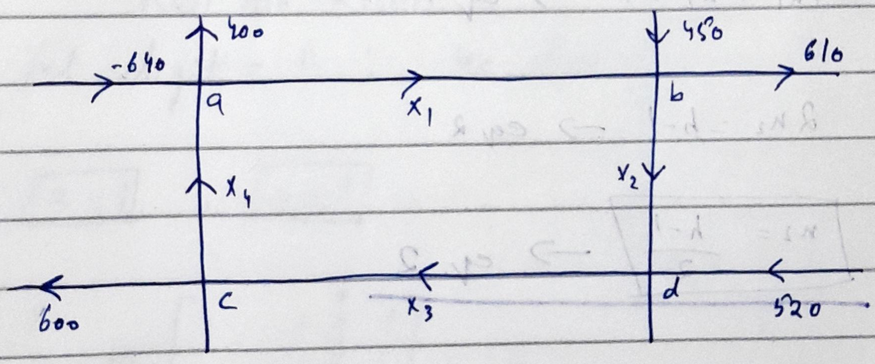
$X_1 = \frac{2-h+1}{2}$

$\boxed{X_1 = \frac{3-h}{2}} \rightarrow$ eq 1

d) Since all the columns are pivot, So the solution does not exist.

Since in $Ad = \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & h-1 \end{array}\right]$, there is no multiple of columns to another so the columns are linearly independent.

Q NO 2



at Junction (A)

$x_4 = -640 + 400 + x_1$

$x_4 - x_1 = -240$

$\boxed{x_1 - x_4 = 240}$

(b)

$450 + x_1 = x_2 + 610$

$$\boxed{X_1 - X_2 = 160}$$

(c)

$X_3 = X_4 + 600$

$$\boxed{X_3 - X_4 = 600}$$

(d)

$X_2 + 520 = X_3$

$$\boxed{X_2 - X_3 = -520}$$

Now, making a Matrix using these equations

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 240 \\ 1 & -1 & 0 & 0 & 160 \\ 0 & 0 & 1 & -1 & 600 \\ 0 & 1 & -1 & 0 & -520 \end{array}\right] \begin{array}{l} R_1 \leftrightarrow R_2 \\ \\ R_3 \leftrightarrow R_4 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 160 \\ 1 & 0 & 0 & -1 & 240 \\ 0 & 1 & -1 & 0 & -520 \\ 0 & 0 & 1 & -1 & 600 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - R_2' \\ R_4 - R_3' \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 160 \\ 0 & 1 & 0 & -1 & 80 \\ 0 & 0 & -1 & 1 & -600 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$Rank(A) = Rank(Ad) = 3$

So, solution exists & There exists a free variable.

ie, x_4

let,

$$\boxed{x_4 = t}$$

Now,

$$-x_3 + x_4 = -600$$

$$-x_3 = -600 - x_4$$

$$\boxed{x_3 = 600 + t}$$

$$x_2 - x_4 = 80$$

$$x_2 = 80 + x_4$$

$$\boxed{x_2 = 80 + t}$$

$$x_1 - x_2 = 160$$

$$x_1 = 160 + x_2$$

$$\boxed{x_1 = 240 + t}$$