PRESENTATION

TOPIC: APPLICATION'S OF LINEAR SYSTEM.

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· Linear Systems & Linear Algebra has many vital & important applications in real life. We will try to understand 2 main applications which are

1 → Economic Modeling
2 → Networking

1) Economic modeling

→ Every country has an economy in which they have many industries.
→ These industries generate output in the form of revenue.
→ This output is supplied to other industries as well as Industry it self.

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Example

Purchased by	Mining (P_m)	Lumber (P_L)	Energy (P_E)	Transport (P_T)
P_m	30	15	20	20
P_L	10	15	15	10
P_T	0	20	20	20
P_E	60	50	45	50

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Exchange Table

→ We have to find a set of equilibrium prices, we want income to be equal to expence in every industry.

So,

For P_m

$$30P_m + 15P_L + 20P_E + 20P_T = 100P_M$$

$$\rightarrow -70P_m + 15P_L + 20P_E + 20P_T = 0 \rightarrow eq\ (1)$$

P_L

$$10P_m + 15P_L + 15P_E + 10P_T = 100P_L$$

$$\rightarrow 10P_m - 85P_L + 15P_E + 10P_T = 0 \rightarrow eq\ (2)$$

P_T

$$20P_L + 20P_E + 10P_T = 100P_T$$

$$\rightarrow 20P_L + 20P_E - 90P_T = 0 \longrightarrow eq\ (3)$$

P_E

$$\rightarrow 60P_m + 50P_L - 55P_E + 50P_T = 0 \rightarrow eq\ (4)$$

Convert those equations into Matrix form

$$\left[\begin{array}{cccc|c} -70 & 15 & 20 & 20 & 0 \\ 10 & -85 & 15 & 10 & 0 \\ 0 & 20 & 20 & -80 & 0 \\ 60 & 50 & -55 & 50 & 0 \end{array}\right]$$

Dividing All by 10

$$\left[\begin{array}{cccc|c} -7 & 1.5 & 2 & 2 & 0 \\ 1 & -8.5 & 1.5 & 1 & 0 \\ 0 & 2 & 2 & -4 & 0 \\ 6 & 5 & -5.5 & 5 & 0 \end{array}\right]$$

by Rearranging

$$\left[\begin{array}{cccc|c} 1 & -8.5 & 1.5 & 1 & 0 \\ -7 & 1.5 & 2 & 2 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 6 & 5 & -5.5 & 5 & 0 \end{array}\right]$$

$$\begin{array}{cl} \left[\begin{array}{cccc|c} 1 & -8.5 & 1.5 & 1 & 0 \\ 0 & -58 & 12.5 & 9 & 0 \\ 0 & 1 & 1 & -4 & 0 \\ 0 & 56 & -14.5 & -1 & 0 \end{array}\right] & \begin{array}{l} \\ R_2+7(R_1) \\ \frac{1}{2}(R_3) \\ R_4-6(R_1) \end{array} \end{array}$$

by Rearranging

$$\left[\begin{array}{cccc|c} 1 & -8.5 & 1.5 & 1 & 0 \\ 0 & 1 & 1 & -4 & 0 \\ 0 & -58 & 12.5 & 9 & 0 \\ 0 & 56 & -14.5 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -8.5 & 1.5 & 1 & 0 \\ 0 & 1 & 1 & -4 & 0 \\ 0 & 0 & 70.5 & -223 & 0 \\ 0 & 0 & -70.5 & 223 & 0 \end{array}\right] \begin{array}{l} \\ \\ R_3+58(R_2) \\ R_4-56(R_2) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -8.5 & 1.5 & 1 & 0 \\ 0 & 1 & 1 & -4 & 0 \\ 0 & 0 & 70.5 & -223 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ R_4-R_3 \end{array}$$

$$\begin{array}{cccc} P_m & P_L & P_E & P_T \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & -8.5 & 1.5 & 1 & 0 \\ 0 & 1 & 1 & -4 & 0 \\ 0 & 0 & 1 & -3.16 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ \frac{1}{(70.5)}(R_3) \\ \\ \end{array}$$

P_t is free.

this means.

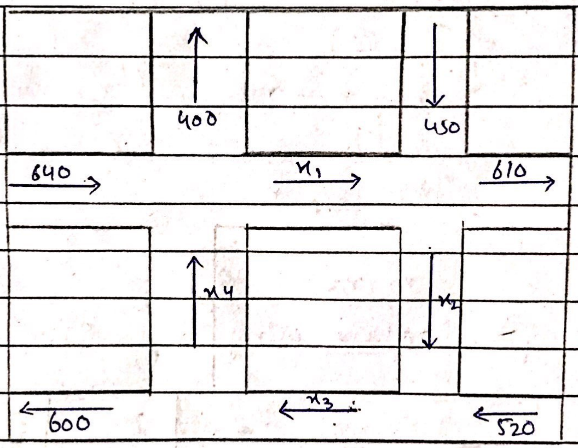
$P_E = 3.16 P_T$ $\rightarrow$ This means price of 1 unit of Energy is = 3.16 units of Transport.

$P_L = .84 P_T$

$P_m = 1.37 P_T$

Question NO 2

Find the general flow pattern of the network shown in the figure. Assumning that the flows are all non negatives, what is the smallest possible values for the unknown?



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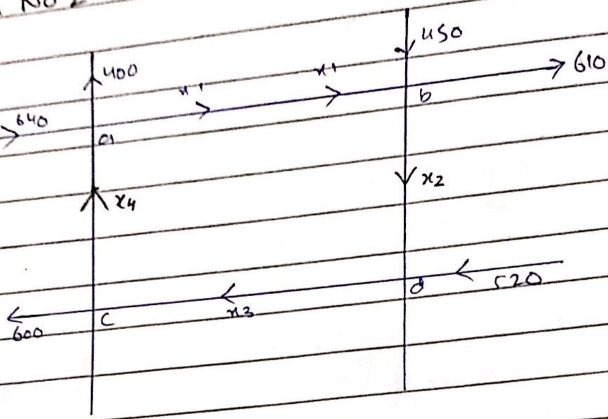
Question No 2



At Junction (A(a))

$$x_4 = -640 + 400 + x_1$$

$$x_4 - x_1 = -640 + 400 \;,\; x_4 - x_1 = -240$$

$$\boxed{x_1 - x_4 = 240}$$

(b)

$$450 + x_1 = x_2 + 610$$

$$\boxed{x_1 - x_2 = 160}$$

(c)

$$x_3 = x_4 + 600 \qquad \boxed{x_3 - x_4 = 600}$$

(d)

$$x_2 + 520 = x_3$$

$$\boxed{x_2 - x_3 = -520}$$

Making a Matrix using these equations.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 240 \\ 1 & -1 & 0 & 0 & 160 \\ 0 & 0 & 1 & -1 & 600 \\ 0 & 1 & -1 & 0 & -520 \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 160 \\ 1 & 0 & 0 & -1 & 240 \\ 0 & 1 & -1 & 0 & -520 \\ 0 & 0 & 1 & -1 & 600 \end{array}\right]$$

$$\begin{array}{c} \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 160 \\ 0 & 1 & 0 & -1 & 80 \\ 0 & 0 & -1 & 1 & -600 \\ 0 & 0 & 0 & \cancel{0} & 0 \end{array}\right] \begin{array}{l} \\ R_2 - R_1, \\ R_3 - R_2, \\ R_4 - R_3 \end{array} \\ \begin{array}{cccc} P & P & P & N.P \end{array} \end{array}$$

Rank A = Rank Ad = 3

So the Solution exists.
& There exists a free variable. i.e x_4
let

$\boxed{x_4 = t}$

Now.

$-x_3 + x_4 = -600 \longrightarrow$ from row 3
$-x_3 = -600 - x_4$, $\boxed{x_3 = 600 + t}$

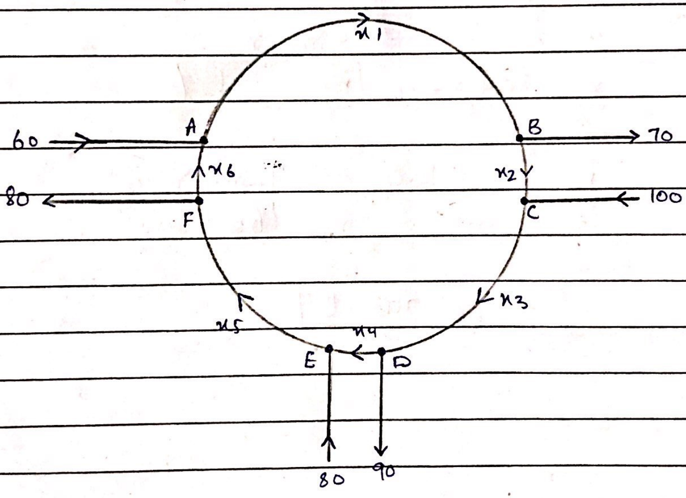
$x_2 - x_4 = 80 \longrightarrow$ from row 2
$x_2 = 80 + x_4$, $\boxed{x_3 = 80 + t}$

$x_1 - x_2 = 160 \longrightarrow$ from row 1
$x_1 = 160 + x_2$, $x_1 = 160 + 80 + t$

$\boxed{x_1 = 240 + t}$

Question No 3

Intersections in England are often constructed as one-way "roundabouts", such as the one shown in the figure. Assume that traffic must travel in the directions shown. Find the general solution of the network flow. Find the smallest possible value for x_6.



Junctions = ABCDE & F are called nodes or Junctions

Basic assumption.

Total flow in = Total flow out.

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because cars dont stop at a Junction so the total cars that come in goes out

For A

$x_6 + 60 = x_1$, $-x_1 + x_6 = -60$ — 1

For B

$x_1 = 70 + x_2$, $x_1 - x_2 = 70$. — 2

For C

$x_2 + 100 = x_3$, $x_2 - x_3 = -100$ — 3

For D

$x_3 = 90 + x_4$, $x_3 - x_4 = 90$ — 4

For E

$x_4 + 80 = x_5$, $x_4 - x_5 = -80$ — 5

For F

$x_5 = x_6 + 80$, $x_5 - x_6 = 80$ — 6

Combining eq 1,2,3,4,5,6 into Matrix form.

-1	0	0	0	0	1	-60
1	-1	0	0	0	0	70
0	1	-1	0	0	0	-100
0	0	1	-1	0	0	90
0	0	0	1	-1	0	-80
0	0	0	0	1	-1	80

$$
\begin{array}{c}
\begin{array}{cccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & & \end{array} \\
\left[\begin{array}{cccccc|c}
1 & 0 & 0 & 0 & 0 & -1 & +60 \\
0 & -1 & 0 & 0 & 0 & 1 & +10 \\
0 & 0 & -1 & 0 & 0 & 1 & -90 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & -80 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\begin{array}{l} -R_1, \\ R_2+R_1, \\ R_3+R_2, \\ R_4+R_3, \\ R_5+R_4, \\ R_6+R_5 \end{array}
\end{array}
$$

$\boxed{x_6 = t}$

$\boxed{x_5 = 80 + t}$

$\boxed{x_4 = t}$

$\boxed{x_3 = 90 + t}$

$\boxed{x_2 = t - 10}$

$\boxed{x_1 = 60 + t}$

Since x_6 can not be negative so x_6 should be minimum 10