Ryerson University

Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

ELE 202

Electric Circuit Analysis

LAB COVER PAGE for Part I submission.

Lab #:	6	Lab Title:	Transient Response of First Order R-C and R-L Circuits

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Section #:	10
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Document submission for Part I:

- A completed and signed "COVER PAGE – **Part I**" has to be included with your submission. The report will not be graded if the signed cover page is not included.
- Your completed handwritten pages of **Section 4.0** should be scanned (via a scanner or phone images), together with the required MultiSIM images. **Note:** *MultiSIM results must be generated using the Department's licensed version of MultiSIM, and the captured screenshots should show your name (at the center-top) and the timestamp (at the bottom-right corner of your screen).*
- Collate and create a *.pdf* or *.docx* file of the above, and upload it via D2L **any time prior to the start of your scheduled lab**. Upload instructions are provided on D2L.

Zero marks will be assigned for the entire lab if this Part I is not submitted prior to your scheduled lab.

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Department of Electrical, Computer, & Biomedical Engineering
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ELE 202
Laboratory #6

Transient Response of First Order R-C and R-L Circuits

1.0 INTRODUCTION

A circuit is classified as **First Order** when a first order differential equation is required to solve for all of the variables. A first order differential equation is an equation in which the highest derivative, or order, that it contains is the first derivative, i.e., $\mathrm{d}x/\mathrm{d}t$ type. Hence, a First Order circuit contains an equivalent **resistor** and only one type of "***storage***" element, either a **capacitor** or an **inductor**, to form **R-C** or **R-L** circuits.

Unlike resistors, which *dissipate* energy in the form of heat when connected to a power source, capacitors and inductors *store* energy which can be retrieved at a later time. When a capacitive circuit is disconnected from a power source, the capacitor will temporarily maintain voltage. Whereas when an inductive circuit is disconnected from the power source, the inductor temporarily maintains current. Another way of saying this is that the voltage across a capacitor or the current through an inductor cannot change abruptly, as the stored energy would require time to readjust to the new conditions. Hence, the response of a circuit immediately after an abrupt change is called the *transient response*.

The *transient* behavior of an R-C or R-L circuit is best understood and observed by applying a **step-input** (voltage or current) to the circuit. The **step response** results in a "*forced-response*" which is what the circuit does when the source is abruptly turned on but with initial conditions set to zero. The "*natural response"* or *"source-free response*" is what the circuit does, including initial conditions, when its power source is abruptly disconnected or suppressed, which causes the capacitor's, or the inductor's, stored energy to be released to the resistor.

The time-constant, usually denoted by the Greek letter, $\boldsymbol{\tau}$ (tau) is the key parameter that characterizes the response to a **step-input** of a First Order circuit or system. This time-constant, $\boldsymbol{\tau}$ gives the time required for the transient response: **(i)** to rise from zero to 63% (or $1 - 1/e$) of its final steady state value as shown in **Figure 1.0a**, or **(ii)** to fall to 37% (or $1/e$) of its initial value as shown in **Figure 1.0b**. Circuits that have a lower value of $\boldsymbol{\tau}$ can rapidly conform to new conditions, whereas those with higher value of $\boldsymbol{\tau}$ take a longer time to reach steady state conditions. Hence, calculating the time-constant is *essential* for a First-Order circuit as it allows for the determination of how long a transient response phase will last.

References: (i) Course Textbook: "*Fundamentals of Electric Circuits*" by C. K. Alexander and M. N. O. Sadiku

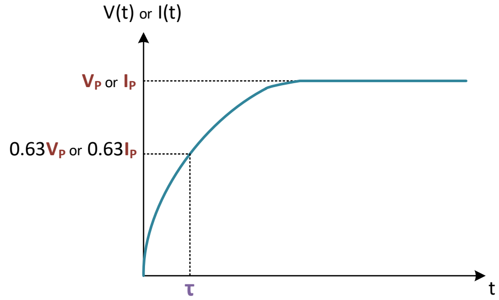


Figure 1.0a: A First-Order "*charging*" transient response

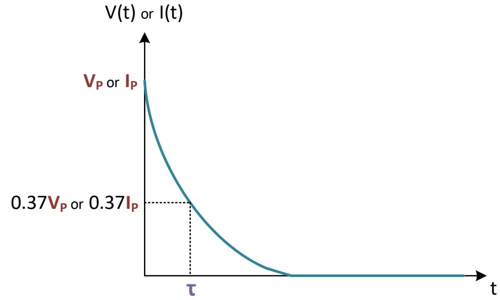


Figure 1.0b: A First-Order "*discharging*" transient response

2.0 OBJECTIVES

- To investigate the **step-input** transient characteristics of First Order **R-C** and **R-L** circuits.
- To understand the concept of the **time-constant**, **τ**.

Note: A repetitive square-wave voltage waveform source will be used to emulate a **step-input** voltage that generates a *forced-response* behavior from the low-to-high transition (**0**v-to-**V**$_P$) during the upper half-period, followed by a **source-free** condition to invoke a *natural-response* behavior from the high-to-low transition (**V**$_P$-to-**0**v) during the lower half-period.

3.0 REQUIRED LAB EQUIPMENT & PARTS

- Digital Multimeter (DMM), Function Generator (FG) and Oscilloscope
- ELE202 Lab Kit:- various components, breadboard, wires and jumpers.

4.0 PRE-LAB: ASSIGNMENT

(a) *R-C Circuit Transient Response*

(i) Referring to the **R-C** circuit shown in **Figure 2.0a**, assume the switch has been in position "x" long enough so that the capacitor is fully discharged. At time t = 0, the switch is abruptly moved to position "y" connecting the circuit to the voltage source, thereby creating a step-input voltage of V_P. It stays in this position long enough for the capacitor to be fully charged and beyond. Recall, since the voltage across the capacitor does not change instantaneously, then $V_C(t)$ becomes a more convenient variable to characterize the transient response in the "*charging*" phase than $I_C(t)$.

For the above stated conditions, sketch & label the **step-input** response of $V_C(t)$ and prove that this ***charging*** transient response can be expressed as:

$V_C(t) = V_P(1 - e^{-\frac{t}{\tau}})$ where $\tau = RC$

Pre-Lab workspace

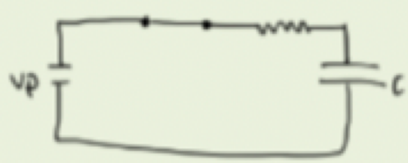
t<0

$= v_p(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$= v_p + (0 - v_p)e^{-t/\tau}$

$= v_p - v_p e^{-t/\tau}$

$= v_p(1 - e^{-t/\tau})$



$R_{eq} = R$
$\tau = RC$

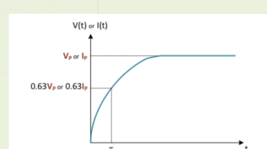


Figure 1.0a: A First-Order "*charging*" transient response

since the function resembles an expondtial function, this means that the capacitor gradually charges and not instantly.

(ii) For ***each*** set of values of **R** and **C** shown in **Table 2.0**, calculate the corresponding "*charging*" time-constant, $\boldsymbol{\tau}$ (*in µsec.*) and steady-state value of $\mathbf{V_C(t)}$. Record your results in the appropriate columns. **Note:** 1 *µsec.* = 10^{-6} *sec*.

Pre-Lab workspace

$\tau_1 = RC$
$= 2 \times 0.0047$
$\tau_1 = 9.4\ \mu sec$

$\tau_2 = RC$
$= 2 \times 0.015$
$\tau_2 = 30\ \mu sec$

$V_P = 10^V$
$V_C = (5\tau_1) = 10(1 - e^{-5\tau_1/\tau_1})$
$V_C(5\tau_1) = 9.95V$

$V_C(5\tau) = (1 - e^{-5\tau_2/\tau_2})$
$V_C(5\tau_1) = 9.93^V$

since at t=0, the voltage in the capacitor is at a steady state.

(iii) In order to observe the transient response on the MultiSIM or Lab oscilloscope, the **R-C** circuit will be driven with a periodic square-wave signal as illustrated in **Figure 2.1**.

Note: Use of this type of square-wave is equivalent to sequentially "opening" and "closing" the switch in the circuit of **Figure 2.0a**. The period of the square-wave waveform specified would be much longer than the time constant, $\boldsymbol{\tau}$ of the circuit to allow the transient response of the circuit to reach its steady state between successive waveform transitions. The upper half of the waveform represents a **step-input** to invoke "***charging***" phase (*forced-response*), while the lower half of the waveform will represent "***discharging***" phase (*natural response*).

To construct and simulate the circuit of **Figure 2.0b** on MultiSIM, use the following procedures to set up the circuit for proper measurements: -

1. Set the function generator (**FG**) for a square-wave output with frequency of **5000 Hz** and $\mathbf{10V_{P\text{-}P}}$ voltage, and display the waveform directly on **CH-1** (or CH-A) of the Oscilloscope. Display at least two complete waveform cycles. On the **FG**, apply a **+5V** "DC-Offset" to set the square-wave amplitude from **0v** to **10V** (= $\mathbf{V_P}$), and confirm this on the Oscilloscope display.

2. For **each** set of values provided for **R** and **C** in **Table 2.0**, construct the circuit of **Figure 2.0b** on MultiSIM, connecting the **FG** as the input source signal and using **CH-1** (or CH-A) of the Oscilloscope to monitor this input signal. Simultaneously, display the capacitor voltage, $\mathbf{V_C(t)}$ on **CH-2** (or CH-B) of the Oscilloscope. Use proper voltage and time scales on **CH-1** and **CH-2** inputs to adequately display the waveforms over two complete cycles on the scope screen. Capture and record the scope screen signals.
Note: The Oscilloscope inputs (CH-1 & CH-2) should be set for **DC-coupled mode** to retain the critical DC information of the waveforms.

3. From the captured $\mathbf{V_C(t)}$ waveform (**CH-2**), determine the values of the time-constant, $\boldsymbol{\tau}$ during the "*charging*" and "*discharging*" phases. Refer to the MultiSIM FAQ in D2L on the use of ***vertical*** *and* ***horizontal cursors*** in combination to take reliable measurement of $\boldsymbol{\tau}$ from the first-order transient response waveform. Record these results in **Table 2.0** in the appropriate columns.
   - Copy and paste a screenshot showing one MultiSIM readings on the circuit. Include the MultiSIM circuit file (**.ms14**) in your Pre-Lab submission.
   - All screenshots should show your name printed on the center-top of the MultiSIM screen and the timestamp at the bottom-lower corner.

4. Compare and comment on your results in **Table 2.0**, and provide explanation for deviations in the time-constant, τ between the theoretical and MultSIM results. Were the measured time-constants in the "*charging*" and "*discharging*" phases the same? Why?

Pre-Lab workspace

→ Deviations could occur due to rounding errors/ and reading decimals.

→ Values are slightly different

→ The time constants in charging and discharging are very identical.

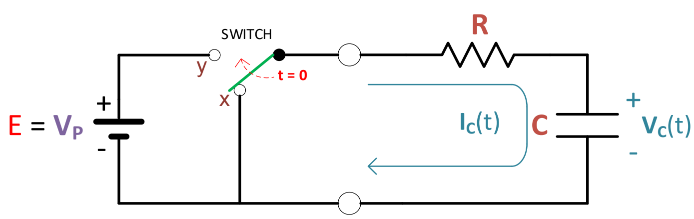


Figure 2.0a: R-C circuit with step voltage source

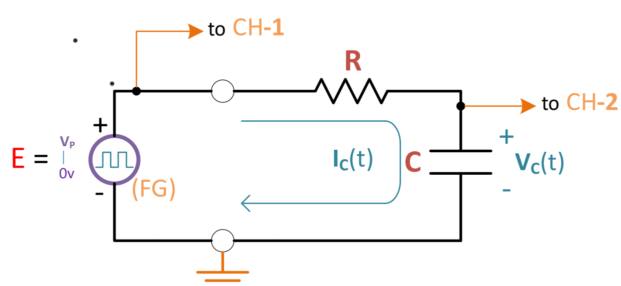


Figure 2.0b: R-C circuit with square-wave input source

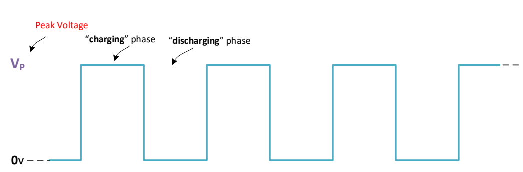


Figure 2.1: Square-wave waveform input voltage source

		Time-Constant, τ			Steady-state value of $V_C(t)$ during "*charging*" phase.	
R	**C**	Theoretical value (*μsec.*)	From MultiSIM result of the "*charging*" phase (*μsec.*)	From MultiSIM result of the "*discharging*" phase (*μsec.*)	Theoretical value (*volts*)	MultiSIM result (*volts*)
2 kΩ	**0.0047 μF**	9.4	9.295	9.667	10	10
2 kΩ	**0.015 μF**	30	30.671	33.475	10	9.685

Table 2.0: Theoretical and MultiSIM results of the Figure 2.0 circuit

(b) *R-L ~~Circuit~~ Transient Response*

(i) Referring to the **R-L** circuit shown in **Figure 3.0a**, assume the switch has been in position "**x**" long enough so that the inductor is fully discharged. At time t = 0, the switch is abruptly moved to position "**y**" connecting the circuit to the voltage source, thereby creating a step-input voltage of V_P. It stays in this position long enough for the inductor to be fully charged and beyond. Recall, since the current through the inductor does not change instantaneously, then $I_L(t)$ becomes a more convenient variable to characterize the transient response in the "*charging*" phase than $V_C(t)$.

For the above stated conditions, <u>sketch & label</u> the **step-input** response of $I_L(t)$ and <u>**prove**</u> that this ***charging*** transient response can be expressed as:

$I_L(t) = I_P(1 - e^{-\frac{t}{\tau}})$, where $I_P = V_P/R$ and $\tau = L/R$

Pre-Lab workspace

t<0

$I_C(t) = i_C(\infty) + (i_L(0) - i_C(\infty))e^{-t/\tau}$
$= I_P - I_P e^{-t/\tau}$
$= I_P(1 - e^{-t/\tau})$

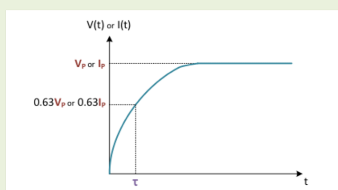
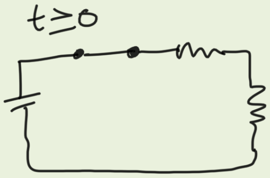


Figure 1.0a: A First-Order "*charging*" transient response

t≥0



$I_P(\infty) = \frac{V_P}{R}$

Since the equation for $I_L(t)$ represents an exponential function, This means that the inductor gradually charges and not instantly right away.

(ii) For each set of values of **R** and **L** shown in **Table 3.0**, calculate the corresponding "*charging*" time-constant, $\boldsymbol{\tau}$ (*in μsec.*) and steady-state value of **$I_L(t)$**. Record your results in the appropriate columns. **Note**: 1 *μsec.* = 10^{-6} *sec*.

Pre-Lab workspace

$I_1 = \frac{L}{R}$

$= \frac{47\times10^{-3}}{1\times10^{-6}(6.2\times10^{-3})}$

$= 7.58\ \mu sec$

$I_2 = \frac{L}{R}$

$= \frac{47\times10^{-3}}{1\times10^{-6}(2\times10^{-3})}$

$= 23.5\ \mu sec$

$I_L(5t_1) = \frac{10}{6.2}(1-e^{-5}) = 1.6$

$I_L(5T_1) = \frac{10}{2}(1-e^{-5}) = 4.96$

since at $t=\infty$, the value of $I_L(t)$ is at a steady state.

(iii) In order to observe the transient response on the MultiSIM or Lab oscilloscope, the **R-L** circuit will be driven with a periodic square-wave signal as illustrated in **Figure 2.1**.

Note 1: As noted earlier, use of this type of square-wave is equivalent to sequentially "opening" and "closing" the switch in the circuit of **Figure 3.0a**. The period of the square-wave waveform specified would be much longer than the time constant, $\boldsymbol{\tau}$ of the circuit to allow the transient response of the circuit to reach its steady state between successive waveform transitions. The upper half of the waveform represents a **step-input** to invoke "***charging***" phase (*forced-response*), while the lower half of the waveform will represent "***discharging***" phase (*natural response*).

Note 2: Because both vertical channels (**CH-1** & **CH-2**) of the oscilloscope need to have one input terminal grounded, all voltage measurements made by the oscilloscope use the ground as a reference and so no unreferenced "differential" voltage measurements (e.g. across **R** in the circuit) are possible using an oscilloscope. Hence, since a current cannot be measured directly with an oscilloscope, then to allow actual monitoring of the current **$I_L(t)$** using voltage measurement, a small "dummy" resistor, **R_d** is placed in series with the inductor, **L** (*in the return path to the ground connection*) as shown in **Figure 3.0b**. This arrangement generates a voltage proportional to **$I_L(t)$** that is monitored on the oscilloscope as **$V_d(t)$** on **CH-2**. It should be noted that as long as **R_d** value chosen is **much less** than **R**, the effect of the resistor **R_d** on the **R-L** circuit's transient responses is negligible, and **can be ignored**. **$I_L(t)$** current values can be determined from the **$V_d(t)$** transient response displayed on **CH-2** using the expression **$I_L(t) = V_d(t)/R_d$**.

To construct and simulate the circuit of **Figure 3.0b** on MultiSIM, use the following procedures to set up the circuit for proper measurements: -

1. Set the function generator (**FG**) for a square-wave output with frequency of **5000 Hz** and **$10V_{P-P}$** voltage, and display the waveform directly on **CH-1** of the Oscilloscope. Display at least two complete waveform cycles. On the **FG**, apply a **+5V** "DC-Offset" to set the square-wave amplitude from **0v** to **10V** (= **V_P**), and confirm this on the Oscilloscope display.

2. For **each** set of values provided for **R** and **L** in **Table 3.0**, construct the circuit of **Figure 3.0b** on MultiSIM, connecting the **FG** as the input source signal and using **CH-1** of the Oscilloscope to monitor this input signal. Simultaneously, display the voltage, **$V_d(t)$** on **CH-2** of the Oscilloscope. Use proper voltage and time scales on **CH-1** and **CH-2** inputs to adequately display the waveforms over two complete cycles on the scope screen. Capture and record the scope screen signals.
Note: The Oscilloscope inputs (CH-1 & CH-2) should be set for **DC-coupled mode** to retain the critical DC information of the waveforms.

3. From the captured $V_d(t)$ waveform (**CH-2**) representing $I_L(t)$, determine the values of the time-constant, τ during the "*charging*" and "*discharging*" phases. Refer to the MultiSIM FAQ in D2L on the use of ***vertical*** *and* ***horizontal cursors*** in combination to take reliable measurement of τ from the first-order transient response waveform. Record these results in **Table 3.0** in the appropriate columns.

- Copy and paste a screenshot showing one MultiSIM readings on the circuit. Include the MultiSIM circuit file (**.ms14**) in your Pre-Lab submission.
- All screenshots should show your name printed on the center-top of the MultiSIM screen and the timestamp at the bottom-lower corner.

4. Compare and comment on your results in **Table 3.0**, and provide explanation for deviations in the time-constant, τ between the theoretical and MultSIM results. Were the measured time-constants in the "*charging*" and "*discharging*" phases the same? Why?

Pre-Lab workspace

- values are similar to another but only deviations are due to rounding errors/- leading decimals.
- charging and discharging are constant, similar.

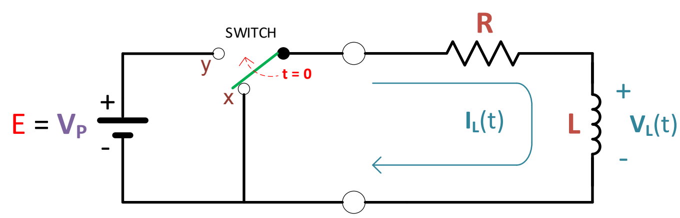


Figure 3.0a: R-L circuit with step voltage source

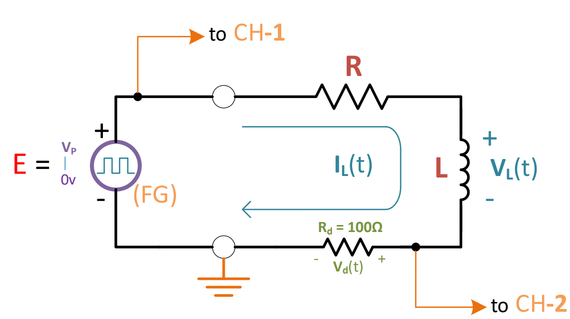


Figure 3.0b: R-L circuit with square-wave input source

		Time-Constant, τ			Steady-state value of $I_L(t)$ during "*charging*" phase.	
R	**L**	Theoretical value (*μsec.*)	From MultiSIM result of the "*charging*" phase (*μsec.*)	From MultiSIM result of the "*discharging*" phase (*μsec.*)	Theoretical value (*mA*)	MultiSIM result (*mA*) V.Ch2 div 100 Ω
6.2 kΩ	**47 mH**	7.58	7.435	7.436	10	9.993
2.0 kΩ	**47 mH**	23.5	23.222	22.305	10	9.998

Table 3.0: Theoretical and MultiSIM results of the Figure 3.0 circuit

*Additional Pre-Lab workspace (**if needed**)*

Time constant on Multisim
→ To find the time constant, I measured where the voltage was 63.2% of the total voltage. (10V)(63.2%) = 6.32V and measured the difference in time.

Time constant for Ic(t)
100% − 63.2% = 36.8%, (10)V(36.8%) = 3.68V

Multisim Screenshots