Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Ryerson University

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Instructor:	S. Jassar

Assignment/Lab Number:	Lab 2 PreLab
Assignment/Lab Title:	Second Order Circuits

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Due Date:	Friday Oct 4, 2024 6pm

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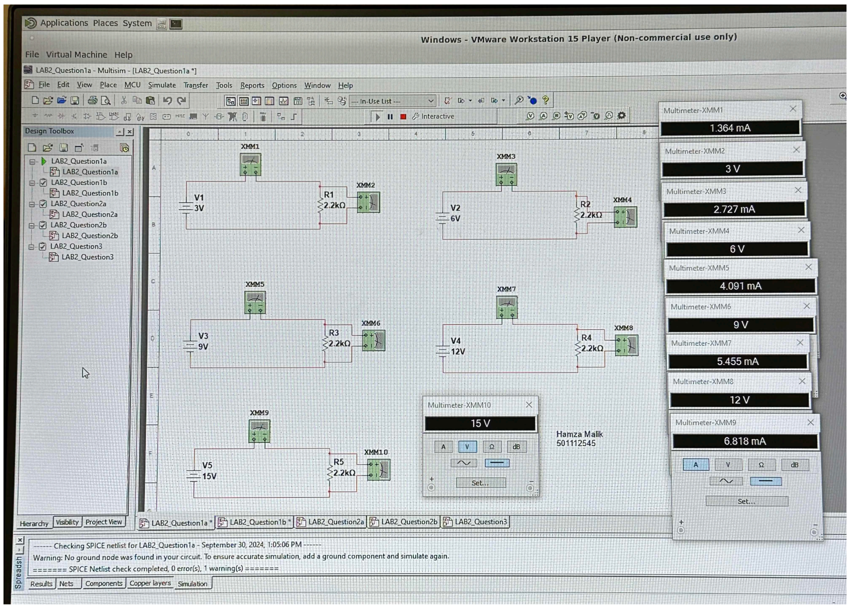
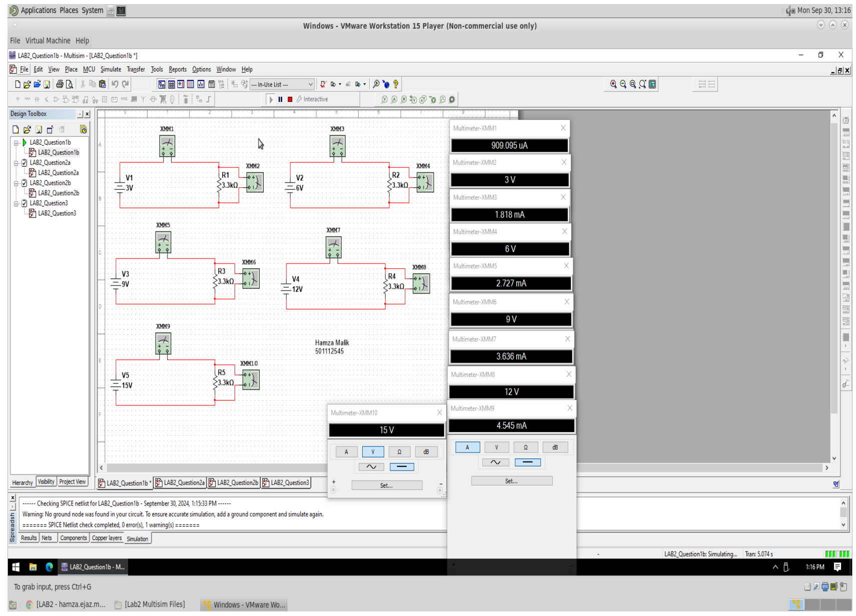
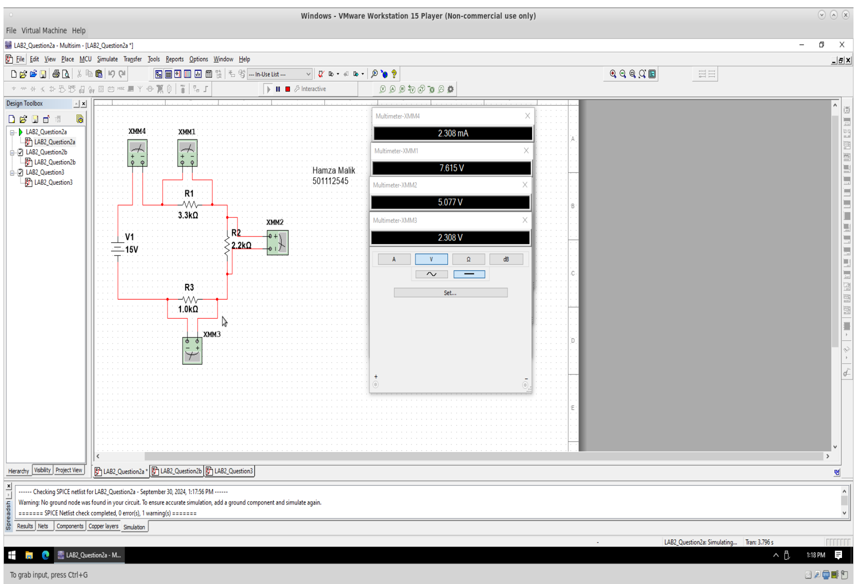
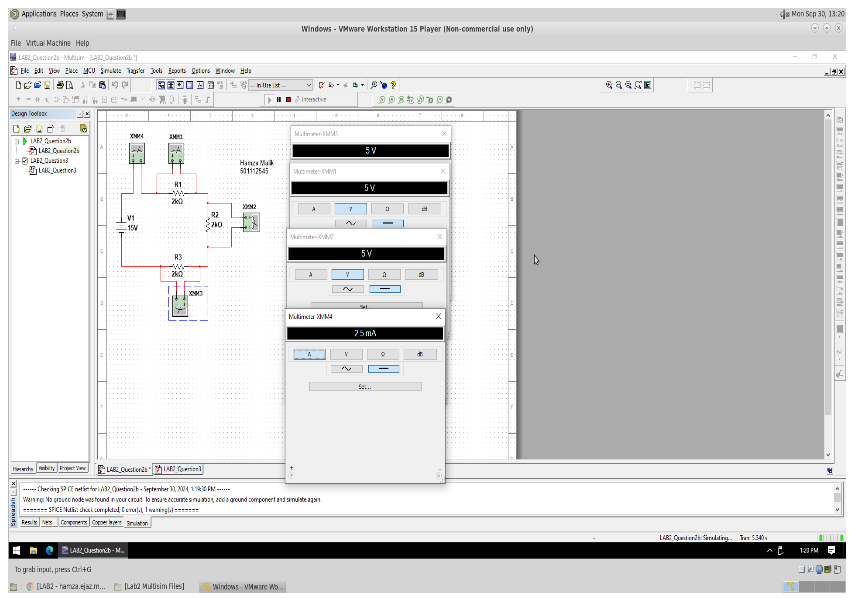
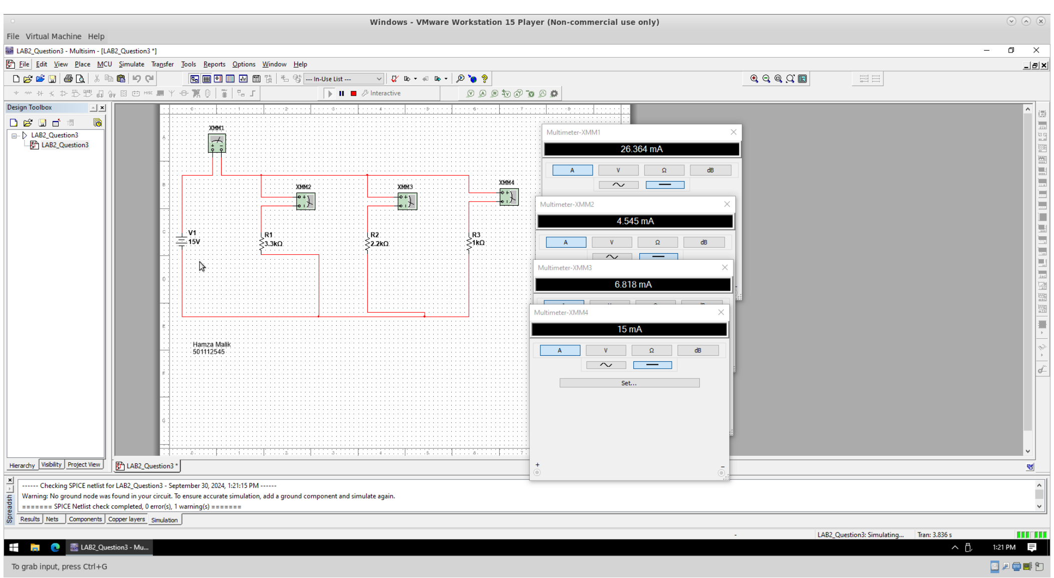
Multisim Files











Toronto Metropolitan University
Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

ELE 302
Laboratory #2

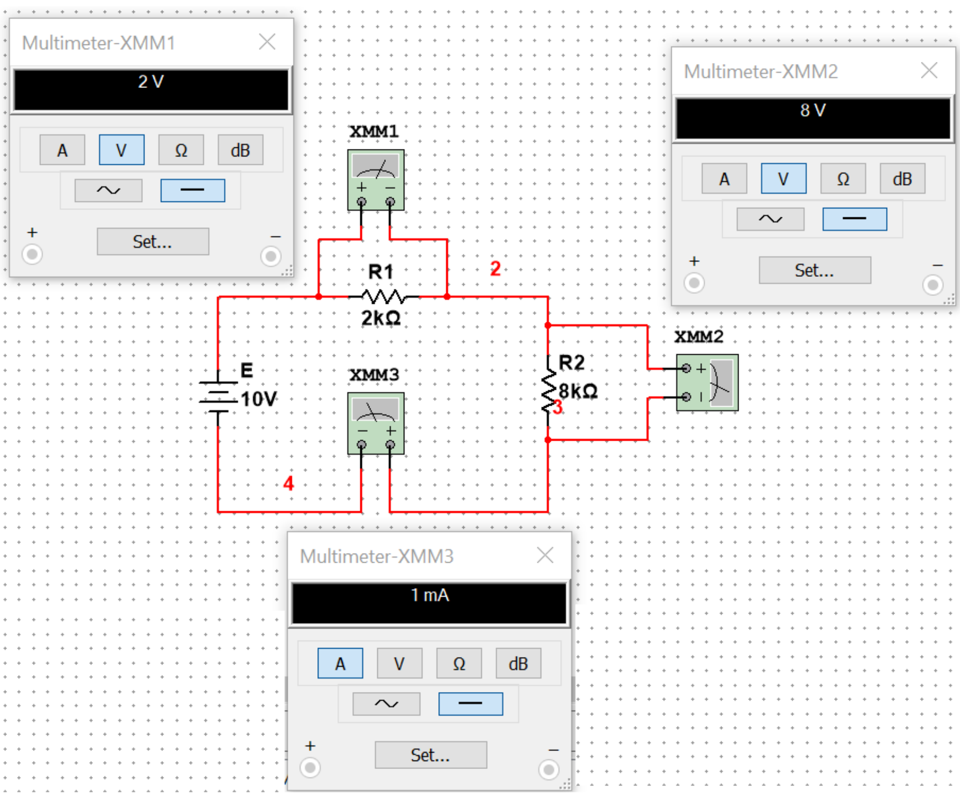
The Step Response of Second-Order Circuits

1.0 INTRODUCTION:

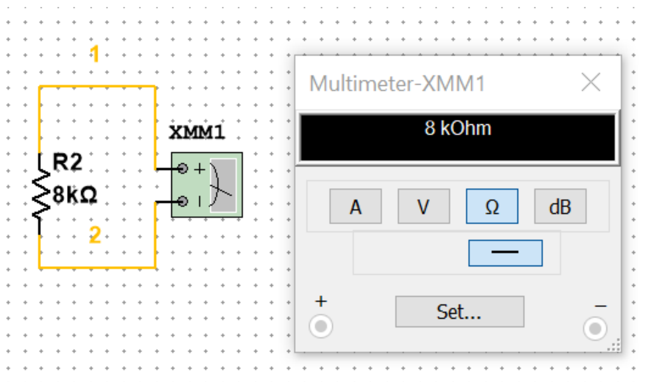
Circuits containing energy-storage elements (capacitors and/or inductors) are knows as dynamic circuits. When switching occurs in a dynamic circuit, the circuit response will go through a transition period prior to settling down to a steady-state value. In applications such as data-acquisition, instrumentation, and computer-control systems, the settling time is an important parameter, as circuits must be allowed to settle to steady state before readings are taken.

Dynamic circuits are often characterized by applying a step-function input. The resulting step-response provides important insights into the response of dynamic circuits in general. By investigating the step response, we discover that it consists of a dc-component called the forced response, and a rapidly vanishing time-varying component, called the natural-response. The form of the time function for the natural component depends on the order and composition of the circuit. The natural response of a second-order circuit is one-out-of-three possible functions known as over damped, critically damped, and under damped, with the under damped case being an exponentially-decaying sinusoid.

This experiment examines the step response of various second-order dynamic circuits. We begin by providing a brief review of the Multisim circuit simulation software and sinusoidal functions.



(i)



(ii)

Figure 1.0a: Examples of using MultiSIM

1.1 Multisim

MultiSIM is an electronic schematic capture and simulation program used to analyze circuit behavior. All AC/DC voltages, AC/DC currents, resistance, frequency, phase-shift, time-domain waveform, etc. can be determined using this software. An example circuit simulation measurement is shown below in **Figure 1.0a.** In this simulation, all components are visually laid out in a way that is the same as a circuit diagram. Each DMM configuration is connected the same way that a physical DMM would be connected on the breadboard. Results are obtained by running the simulation and then double clicking on each piece of equipment to read the desired output values. Refer to the MultiSIM software download procedures, related FAQs and video tutorials on the course website (D2L) to get acquainted with proper use of this simulation tool, and become proficient at it.

1.2 Sinusoidal Functions

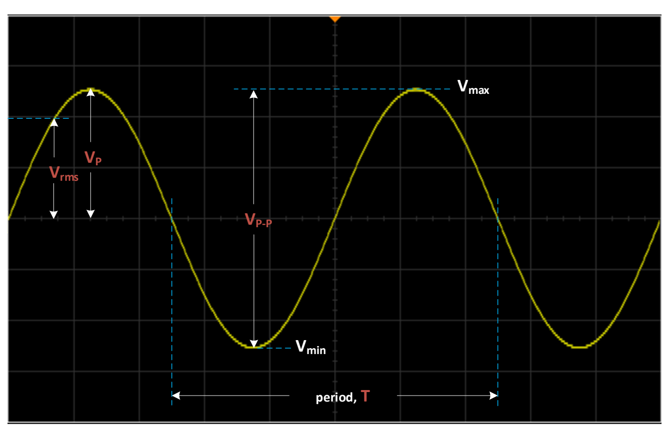


Figure 1.0b: Characterization of a Time-Varying Sinusoidal Signal

A ***sine-wave*** is shown in **Figure 1.0b**. The sinusoidal signal periodically varies with time. It can be characterized by a number of parameters, some of which are shown in **Figure 1.0b**:

- Peak-To-Peak Voltage: $\mathbf{V_{P-P}} = \mathbf{V_{max}} - \mathbf{V_{min}}$
- Amplitude of the sinusoidal signal is defined as its Peak Voltage: $\mathbf{V_P} = \frac{\mathbf{V_{P-P}}}{2}$
- Root-Mean-Squared Voltage: $\mathbf{V_{rms}} = \frac{\mathbf{V_P}}{\sqrt{2}} = 0.707\mathbf{V_P}$
- The period, **T** is defined as the time within which the signal repeats.

- The frequency, **f** is equal to the number of repetitions per unit of time, and can be calculated from the period, **T**: $\mathbf{f}(\mathrm{Hz}) = \frac{1}{\mathbf{T}(\mathrm{sec})}$ or $\mathbf{T}(\mathrm{sec}) = \frac{1}{\mathbf{f}(\mathrm{Hz})}$.
  When frequency is given in radians/sec then the symbol, $\boldsymbol{\omega}$ is used, where $\boldsymbol{\omega} = 2\pi \mathbf{f}$
- Phase Shift, $\boldsymbol{\Theta}$ of one sinusoidal signal with respect to another (of the same frequency) occurs when there is time-offset, $\boldsymbol{\Delta}\mathbf{T}$ between them. Phase-Shift => $\boldsymbol{\theta} = \frac{\Delta \mathrm{T}}{\mathrm{T}} . 2\pi \text{ (radians)} = \frac{\Delta \mathrm{T}}{\mathrm{T}} . 360° \text{(degrees)}$
- Sinusoidal AC voltage as a function of time: $v(\mathrm{t}) = \mathbf{V_P} \cos(\boldsymbol{\omega}\mathrm{t} + \boldsymbol{\theta}) = \mathbf{V_P} \cos(2\pi \mathbf{f}\mathrm{t} + \boldsymbol{\theta})$

2.0 OBJECTIVES:

- To use Multisim circuit simulation software to plot the step response of second-order dynamic circuits.
- To use the oscilloscope to display the step response of second-order dynamic circuits.
- To measure the parameters that characterize the step response of second-order dynamic circuits.

3.0 REQUIRED LAB EQUIPMENT & PARTS:

- Function Generator (FG) and Oscilloscope.
- ELE202 Lab Kit and ELE302 Lab Kit: various components, breadboard, wires and jumpers.

4.0 PRE-LAB ASSIGNMENT (3 marks with 0.75 marks for each step):

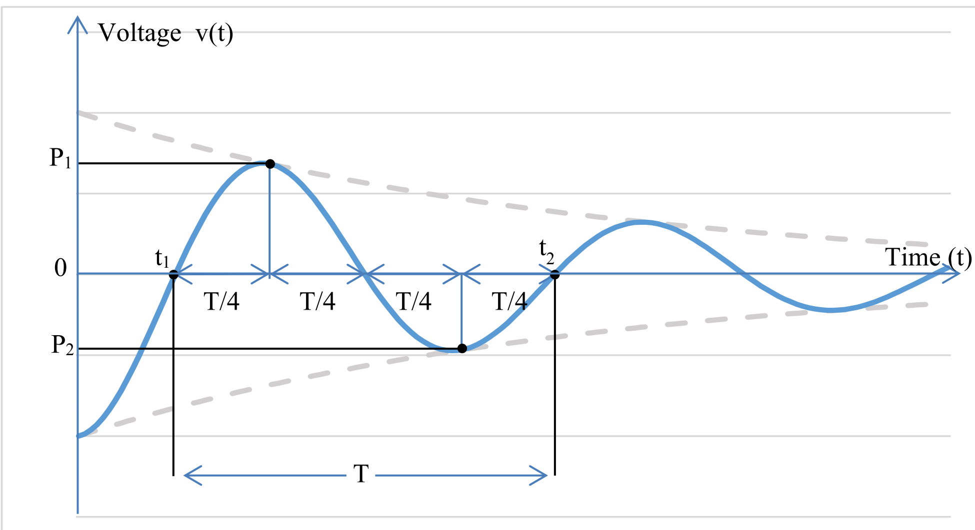


Figure 2.0: An Exponentially Decaying Sinusoidal Function

(a) Step 1: The circuit shown in **Figure 2.0** is an exponentially decaying sinusoidal function that can be characterized by the function: $\mathrm{v(t)} = Ae^{-\sigma t} \cos(\boldsymbol{\omega}\mathrm{t} + \boldsymbol{\theta})$. The figure defines three variables T, P_1, and P_2. t_1 is the time of the first zero-crossing of v(t) and t_2 is time for the third zero-crossing of v(t). T is the time difference between t_1 and t_2. P_1 is the value of v(t) one quarter of the way between the first zero-crossing and the third zero-crossing (i.e. $\mathrm{P_1} = \mathrm{v}(\mathrm{t_1} + \frac{\mathrm{T}}{4})$). Finally, P_2 is the value of v(t) three quarters of the way between the first zero-crossing and the third zero-crossing (i.e. $\mathrm{P_2} = \mathrm{v}(\mathrm{t_1} + 3\frac{\mathrm{T}}{4})$).

i) Find an expression for ω in terms of T.

Pre-Lab workspace (show your analysis here)

$w = 2\pi f$ $\qquad f = \frac{1}{T}$

$w = 2\pi\left(\frac{1}{T}\right)$

$w = \frac{2\pi}{T}$

ii) Find an expression for σ in terms of P_1, P_2, and T.

Pre-Lab workspace (show your analysis here)

$P_1 = V(t_1 + \frac{T}{4})$

$P_2 = V(t_1 + 3\frac{T}{4})$

$P_1 = V(t + \frac{T}{4})$

$\frac{P1}{V} = t + \frac{T}{4}$

$\frac{P1}{V} - \frac{T}{4} = t_1$ — 1

$P_2 = V(t + 3\frac{T}{4})$

$\frac{P_2}{V} = \frac{t + 3(\frac{T}{4})}{3}$

$\frac{P_2}{3V} = t + \frac{T}{4}$

$\frac{P2}{3V} - \frac{T}{4} = t_2$

$V(t) = Ae^{-6t}\cos(\omega t + \theta)$

$v(t) = Ae^{-6t}\cos(\omega t + \theta)$

$v(t) = Ae^{-6t}\cos(\omega t + \theta)$

$Ae^{-6t}\cos(\omega t_1 + 6) = Ae^{-6t}\cos(\omega t_2 + \theta)$

$\frac{Ae^{-6t}\cos(\omega(\frac{P1}{V} - \frac{T}{4}) + \theta)}{A} = \frac{Ae^{-6t}\cos(\omega(\frac{P2}{3V} - \frac{T}{4}) + \theta)}{A}$

$\log e^{-6t}\cos/w(\frac{P1}{V} - \frac{T}{4}) + \theta) = \log e^{-6t}\cos(\omega(\frac{P2}{3V} - \frac{T}{4}) + \theta)$

$-6t\cos/\omega(\frac{P1}{V} - \frac{T}{4}) + \theta = -6t\cos(\omega(\frac{P2}{3V} - \frac{T}{4}) + \theta)$

$6t\cos/\omega(\frac{P1}{V} - \frac{T}{4}) + \theta) = 6t\cos(\omega(\frac{P2}{3V} - \frac{T}{4}) + \theta)$

$6t(\omega(\frac{P1}{V} - \frac{T}{4})) = 6t(\omega(\frac{P2}{3V} - \frac{T}{4})$

$6t\frac{P1}{V} - 6t\frac{T}{4} = 6t\frac{P2}{3V} - 6t\frac{T}{4}$

$6t\frac{P1}{V} = 6t\frac{P2}{3V}$

$6\frac{P1}{V} = 6 \cdot \frac{P2}{3V6} \longrightarrow 6\frac{P1}{V} = \frac{6 \cdot P2}{3V6} \longrightarrow 6 = \frac{P2 - P1}{6}$

iii) Find an expression of θ in terms of t_1 and T.

Pre-Lab workspace (show your analysis here)

$\theta = \frac{\Delta T}{T}$

$\theta = \frac{T_2 - T_1}{T}$

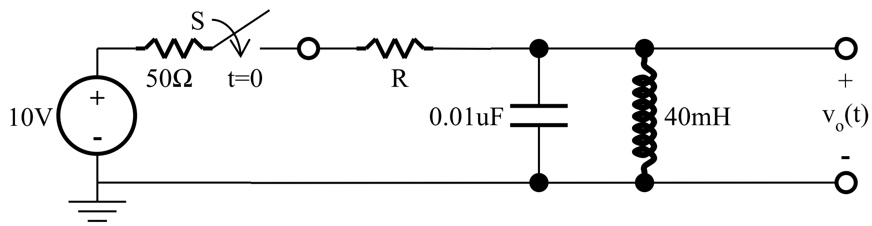
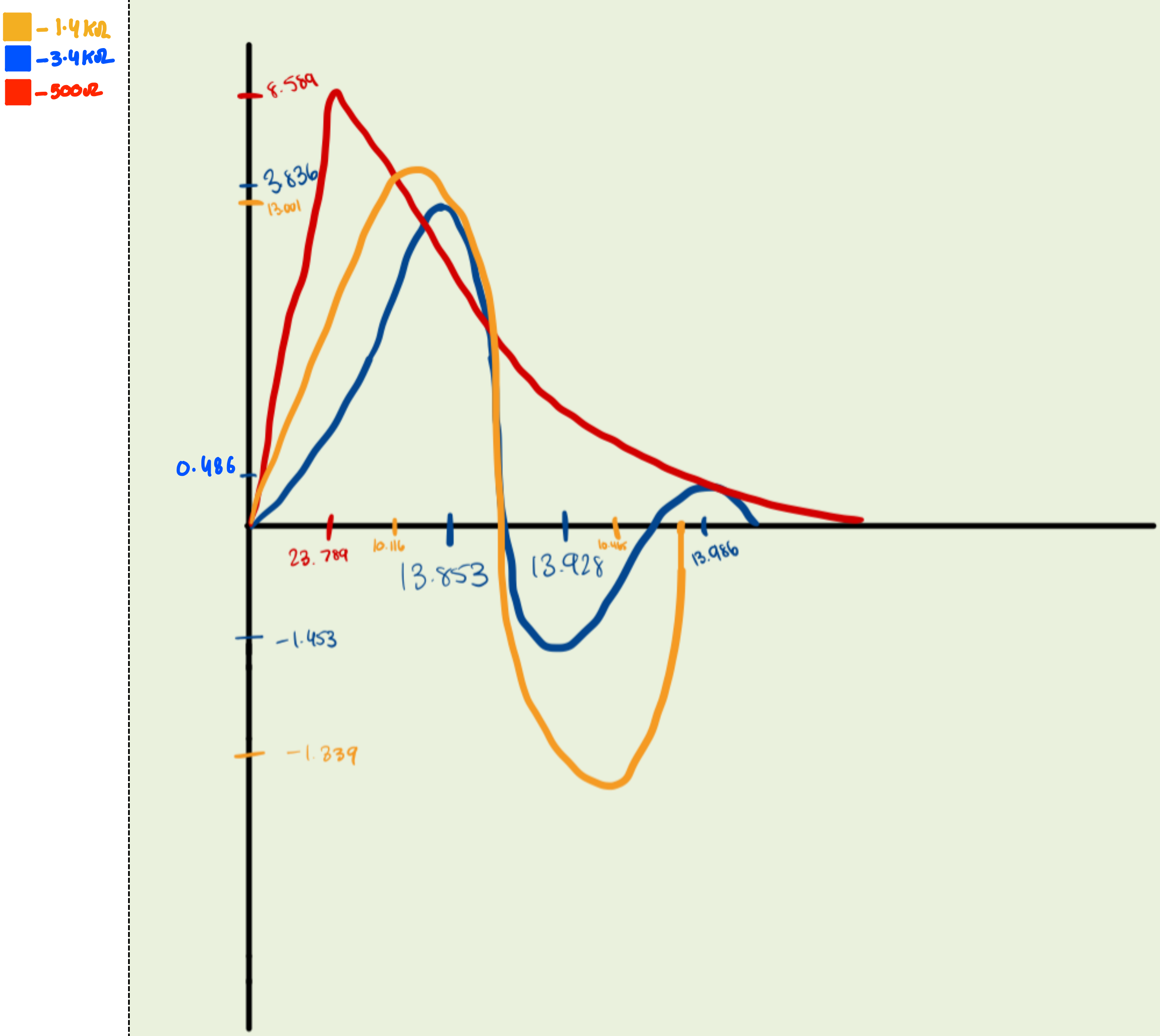


Figure 2.0: Second-Order Bandpass Circuit

(b) Step 2: Consider the dynamic circuit shown in **Figure 2.0**. The switch S has been open for a long time. At t=0, the switch is closed, where it remains for a long time.

i) Use Multisim to plot the step response [$v_o(t)$ for $t \geq 0^+$] when R=1.4kΩ, 3.4kΩ, and 500Ω.

Pre-Lab workspace (show your analysis here)



ii) Use the plots to calculate the parameters (σ and ω) that characterize the step response as: $\mathbf{v_o}(t) = A\boldsymbol{e}^{-\sigma t}\sin(\omega t)$ for R=3.4kΩ.

Pre-Lab workspace (show your analysis here)

Peak Voltage → $A = 3.836$

Angular frequency → $W = \frac{2\pi}{\Delta T} = \frac{2\pi}{T_2 - T_1} = \frac{2\pi}{13.986 - 13.853} = 47.242$

Damping constant → σ

$$\frac{d^2 v}{dt^2} + \frac{3400}{40\times10^{-3}}\frac{di}{dt} + \frac{1}{(40\times10^{-3})(1\times10^{-8})} = 0$$

$$\sigma^2 + 85000\sigma + 2.5\times10^9 = 0$$

$$\sigma = \frac{-85000 \pm \sqrt{(85000)^2 - 4(1)(2.5\times10^9)}}{2}$$

$$= -42500 \pm 13875.00i$$

$$\sigma = \frac{P_2 - P_1}{\Delta T} = \frac{3.836 - 0.486}{13.986 - 13.85} = 24.632$$

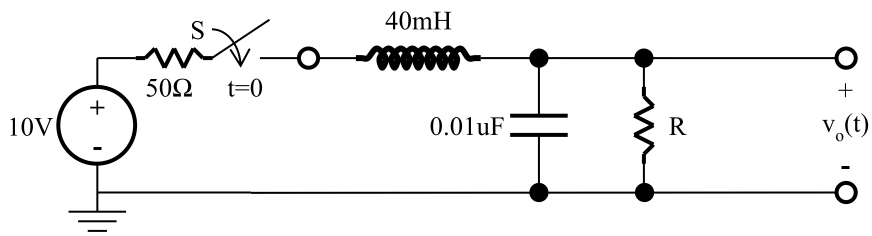
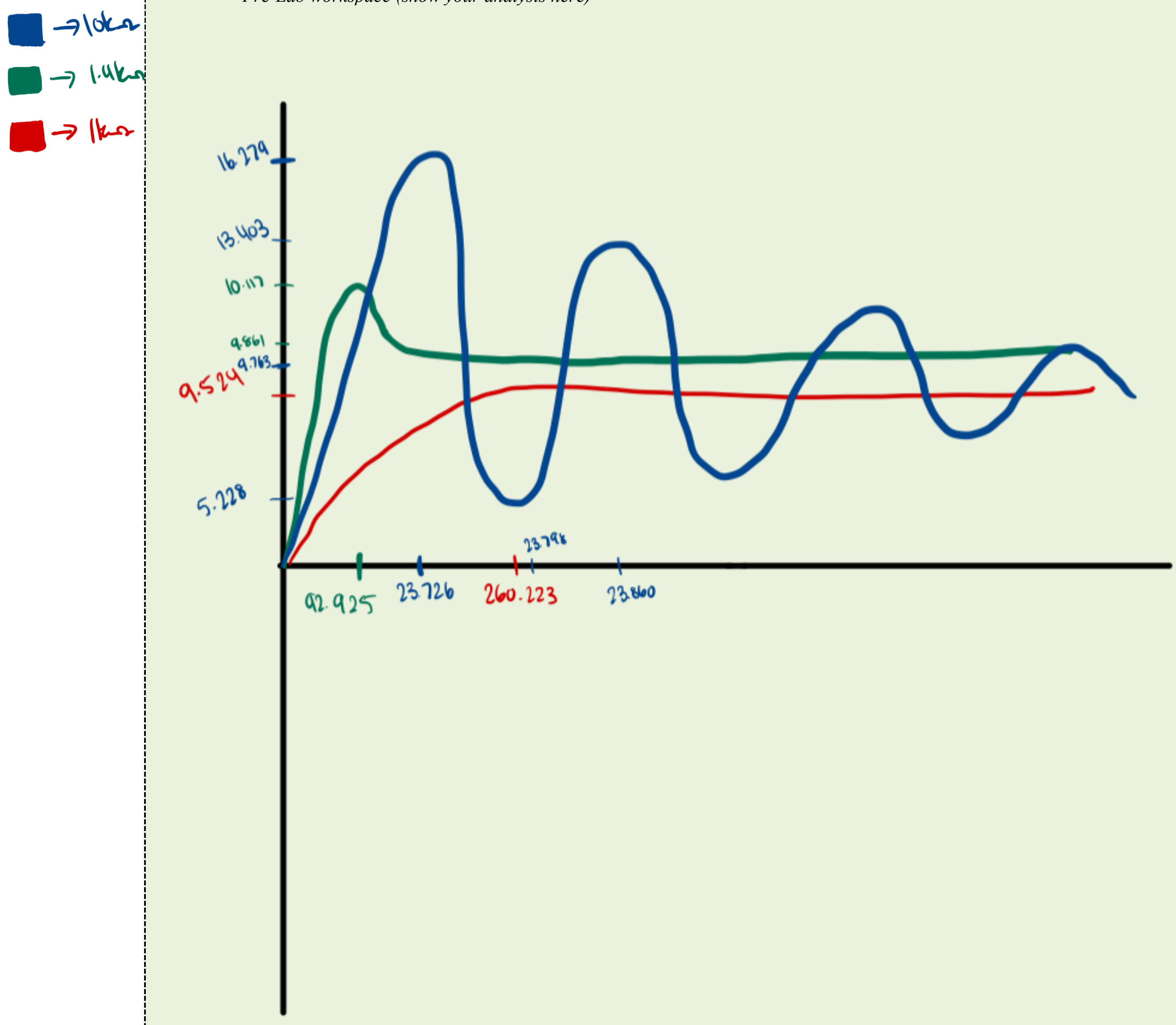


Figure 3.0: Second-Order Lowpass Circuit

(c) Step 3: Consider the dynamic circuit shown in **Figure 3.0.** The switch S has been open for a long time. At t=0, the switch is closed, where it remains for a long time.

i) Use Multisim to plot the step response [$v_o(t)$ for $t \geq 0^+$] when R=1kΩ, 1.4kΩ, and 10kΩ.

Pre-Lab workspace (show your analysis here)



ii) Use the plots to calculate the parameters (σ and ω) that characterize the step response as: $\mathbf{v_o}(t) = B + Ae^{-\sigma t}\cos(\omega t + \theta)$ for R=10kΩ.

Pre-Lab workspace (show your analysis here)

Peak to peak voltage → A = Vpeak − B

Voltage when stabilizes → B = 9.763

Angular Frequency → $\omega = \frac{2\pi}{\Delta T} = \frac{2\pi}{23.860 - 23.726} = 64.11$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = \frac{1}{LC}\frac{dv}{dt}$$

$$\frac{d^2 i}{dt^2} + \frac{10k\Omega}{40mH}\frac{di}{dt} + \frac{1}{(40mH)(0.01\mu F)} i = 0$$

$$\sigma^2 + 250000\sigma + 2.5\times10^9 = 0$$

$$\sigma = \frac{-25000 \pm \sqrt{(250000)^2 - 4(1)(2\times10^9)}}{2}$$

$\sigma = -10435.61$ or -239564.4

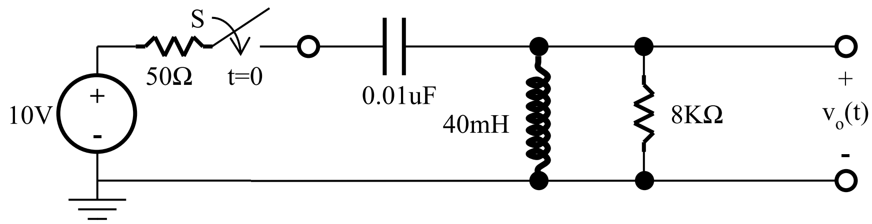
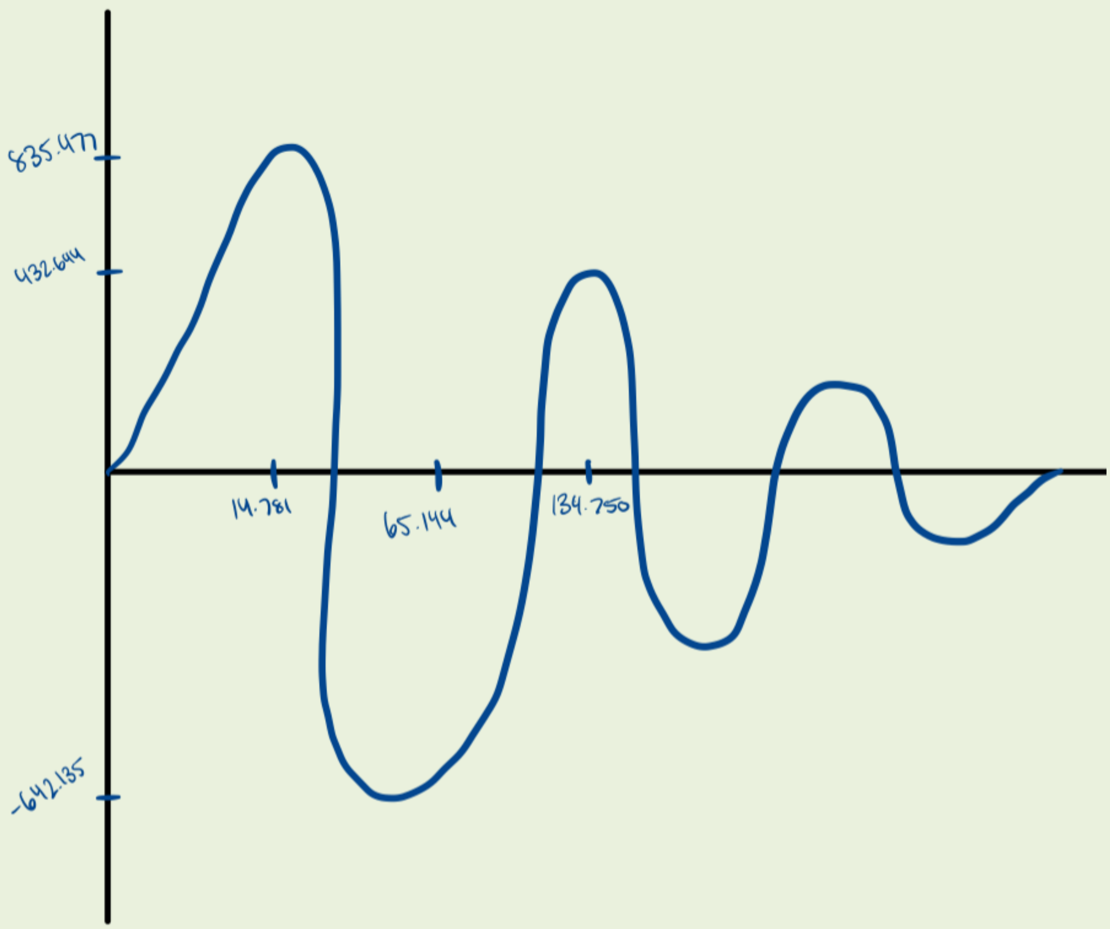


Figure 4.0: Second-Order Highpass Circuit

(d) Step 4: Consider the dynamic circuit shown in **Figure 4.0.** The switch S has been open for a long time. At t=0, the switch is closed, where it remains for a long time.

i) Use Multisim to plot the step response [$v_o(t)$ for $t \geq 0^+$].

Pre-Lab workspace (show your analysis here)



ii) Use the plot to calculate the parameters (σ and ω) that characterize the step response as: $\mathbf{v_o}(t) = Ae^{-\sigma t}\cos(\omega t + \theta)$.

Pre-Lab workspace (show your analysis here)

Peak Voltage → $A = 835.477$

Angular frequency → $w = \frac{2\pi}{\Delta T} = \frac{2\pi}{134.750 - 14.281} = 0.05215$

Phase Shift

↳ $\theta = \frac{\pi}{2}$ because sine function

Damping Constant → σ

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = \frac{1}{LC}\frac{dv}{dt}$$

$$\frac{d^2 i}{dt^2} + \frac{8k\Omega}{40mH}\frac{di}{dt} + \frac{1}{(40mH)(0.01\mu f)} = 0$$

$$\sigma^2 + 200000\sigma + 2.5\times10^9 = 0$$

$$\sigma = \frac{-200000 \pm \sqrt{(200000)^2 - 4(1)(2.5\times10^9)}}{2(1)}$$

$$= -13397.4 \text{ or } -186602.5$$