Faculty of Engineering and Architectural Science

Department of Computer and Electrical Engineering

Course Number	**ELE532**
Course Title	**Signal and Systems l**
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Instructor	**Dr.Beheshti**

Report Title	**Fourier Series Analysis using Matlab**

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A.1

$D_n = \frac{1}{T}\int_0^T x_1(t)\, e^{-jn\omega_0 t}\, dt$ $\qquad x_1(t) = \cos \overset{3\omega_0}{\frac{3\pi}{10}} t + \frac{1}{2}\cos\frac{\pi}{10} t$

$\omega_0 = \frac{\pi}{10} \qquad T = \frac{2\pi}{\frac{\pi}{10}} = 20$

general integral

$x_1(t) = \frac{1}{2}\left(e^{j3\omega_0 t} + e^{-j3\omega_0 t}\right) + \frac{1}{4}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$

$D_n = \frac{1}{T}\int_0^T \left[\frac{1}{2} e^{j3\omega_0 t} + \frac{1}{2} e^{-j3\omega_0 t} + \frac{1}{4} e^{j\omega_0 t} + \frac{1}{4} e^{-j\omega_0 t}\right] e^{-jn\omega_0 t}\, dt$

$D_n = \frac{1}{T}\left[\frac{1}{2}\int_0^T e^{j(3-n)\omega_0 t} + \frac{1}{2}\int_0^T e^{j(-3-n)\omega_0 t} + \frac{1}{4}\int_0^T e^{j(1-n)\omega_0 t} + \frac{1}{4}\int_0^T e^{j(1-n)\omega_0 t}\right] dt$

for each integral

$\int_0^T e^{jk\omega_0 t}\, dt = T$ if $k = 0$ or $\int_0^T e^{jk\omega_0 t} = \frac{e^{jk\omega_0 T} - 1}{jk\omega_0} = 0$ if $k = 0$

where k is some int

so n must be such that added w/ each n

$k - n = 0$

$D_n = \frac{1}{T}\left[\frac{1}{2}(T)\delta_{n,3} + \frac{1}{2}(T)\delta_{n,-3} + \frac{1}{4}(T)\delta_{n,1} + \frac{1}{4}(T)\delta_{n-1}\right]$

$D_{\pm 3} = \frac{1}{2} \qquad D_{\pm 1} = \frac{1}{4}$

A1: *Calculation for the value of Dn for function x1(t)*

A.2 general integral

$D_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt$

$D_n = \frac{1}{T}\int_{-5}^{5} e^{-jn\omega_0 t}\, dt = \frac{1}{T}\,\frac{e^{-jn\omega_0(5)} - e^{jn\omega_0(5)}}{-jn\omega_0} = \frac{2}{Tn\omega_0}\sin(n\omega_0 5),\ n \neq 0$

DC Term

$D_0 = \frac{1}{T}\int_{-5}^{5} 1\, dt = \frac{10}{T}$

Simplify

$D_n = \frac{2}{Tn(2\pi/T)}\sin(5n\omega_0) = \frac{1}{\pi n}\sin(5n\omega_0),\ n \neq 0$

for $x_2(t)$

$T = 20 \qquad \omega_0 = \frac{2\pi}{20} = \frac{\pi}{10} \qquad 5n\omega_0 = \frac{\pi n}{2}$

$D_n = \frac{10}{20} = \frac{1}{2},\quad n = 0 \qquad D_n = \frac{1}{\pi n}\sin\left(\frac{\pi n}{2}\right)\quad n \neq 0$

for $x_3(t)$

$T = 40 \qquad \omega_0 = \frac{2\pi}{40} = \frac{\pi}{20}$

$5n\omega_0 = \frac{\pi n}{4}$

$D_n = \frac{10}{40} = \frac{1}{4},\ n = 0 \qquad D_n = \frac{1}{\pi n}\sin\left(\frac{n\pi}{4}\right)\quad n \neq 0$

A2: *Calculations for the value of Dn for function x2(t), x3(t), subbed in n = 0 to the existing equation and found that n = ½ for x2(t) and n = ¼ for x3(t)*

```
% A.3
function Dn = compute_Dn(n_values)
% compute_Dn - Returns the Exponential Fourier Series coefficients Dn
% for the signal:
%   x1(t) = cos(3πt/10) + 0.5*cos(πt/10)
%
% Usage:
%   Dn = compute_Dn(n_values)
%
% Example:
%   Dn = compute_Dn(-5:5)
   % Initialize Dn as zeros
   Dn = zeros(size(n_values));
   % Loop through the user-specified n values
   for k = 1:length(n_values)
       n = n_values(k);
       if abs(n) == 1
           Dn(k) = 1/4;      % From 0.5*cos(πt/10)
       elseif abs(n) == 3
           Dn(k) = 1/2;      % From cos(3πt/10)
       else
           Dn(k) = 0;        % All other harmonics are zero
       end
   end
end
```

A3: *Code for A3 that computes dn and returns the fourier coefficients*

```
% A.4 - Fourier Coefficients and Reconstruction
% --- x1(t) ---
% a) Compute and plot |Dn| and Dn for n = -5:5
figure(1);
n = (-5:5);
D_func = Dn(1);          % this selects the Dn definition
D_n = D_func(n);% Evaluate coefficients for chosen n
D_n(n==0)=1/2;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
% b) Increase harmonic range
figure(4);
n = (-20:20);
D_func = Dn(1);
D_n = D_func(n);
D_n(n==0)=1/2;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
% c) Increase to n = -50:50
figure(7);
n = (-50:50);
D_func = Dn(1);
D_n = D_func(n);
D_n(n==0)=1/2;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
% d) increases n = -500:500
figure(10);
n = (-500:500);
D_func = Dn(1);
D_n = D_func(n);
D_n(n==0)=1/2;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
```

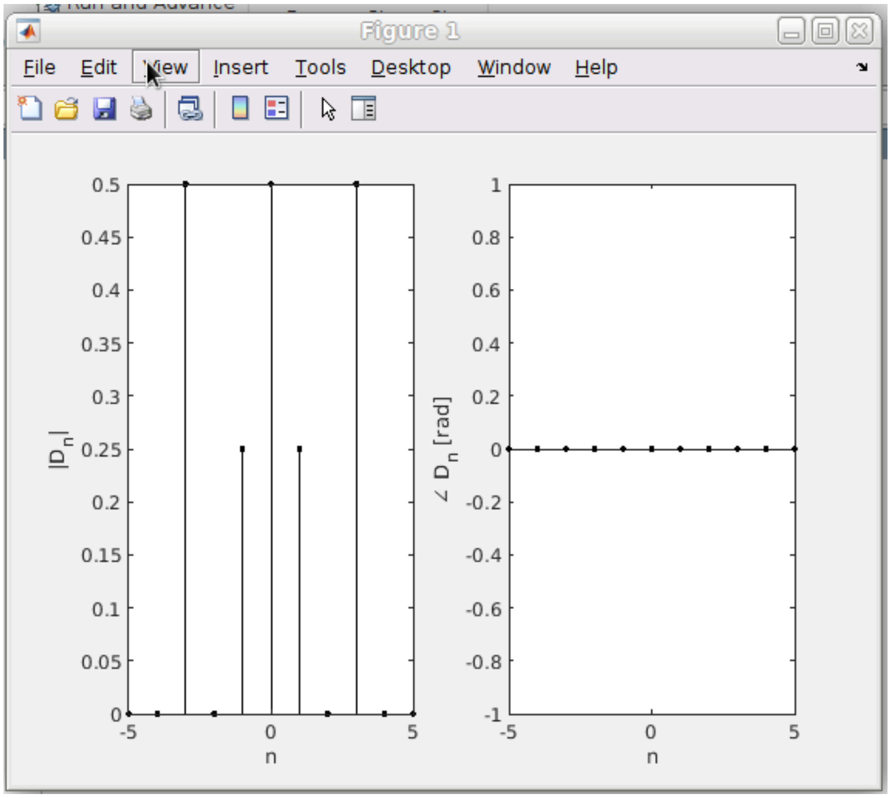


Figure 1
File Edit View Insert Tools Desktop Window Help
|D_n|
n
∠ D_n [rad]
n

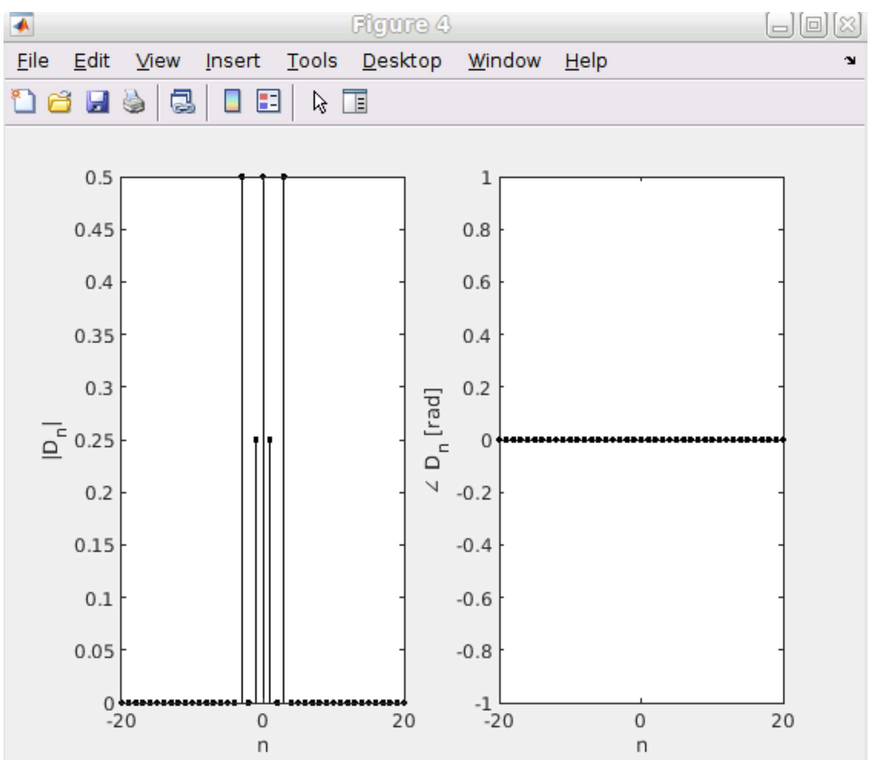


Figure 4
File Edit View Insert Tools Desktop Window Help
|D_n|
n
∠ D_n [rad]
n

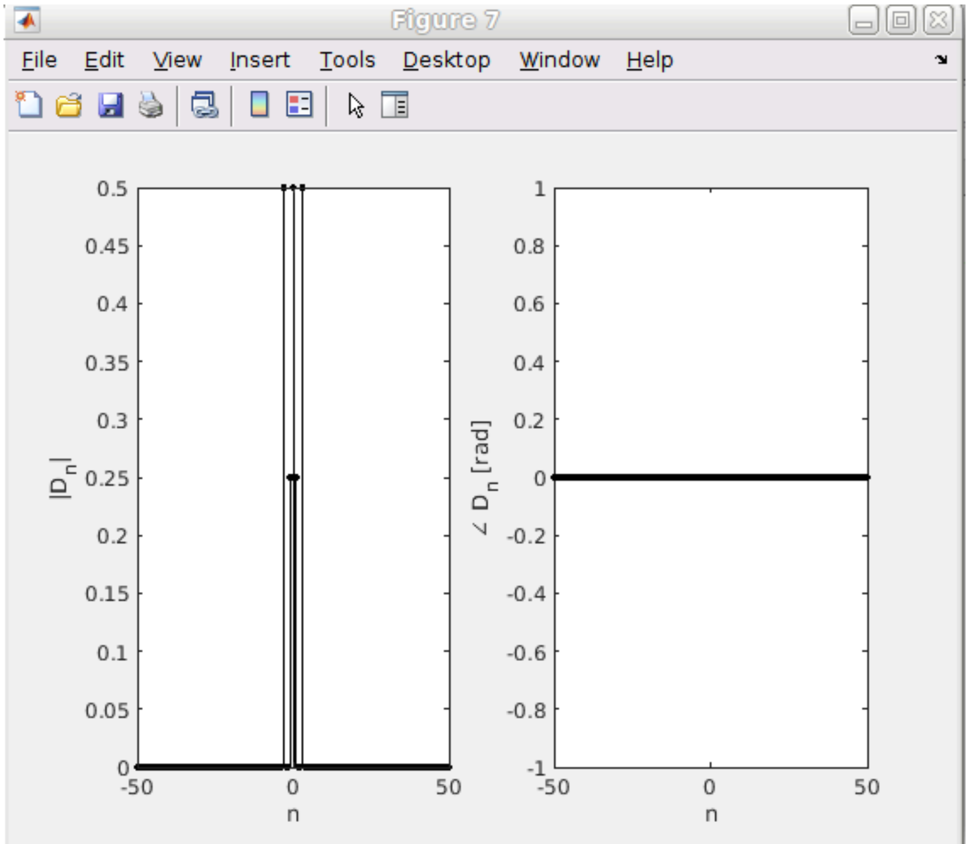


Figure 7
File Edit View Insert Tools Desktop Window Help
|D_n|
n
∠ D_n [rad]
n

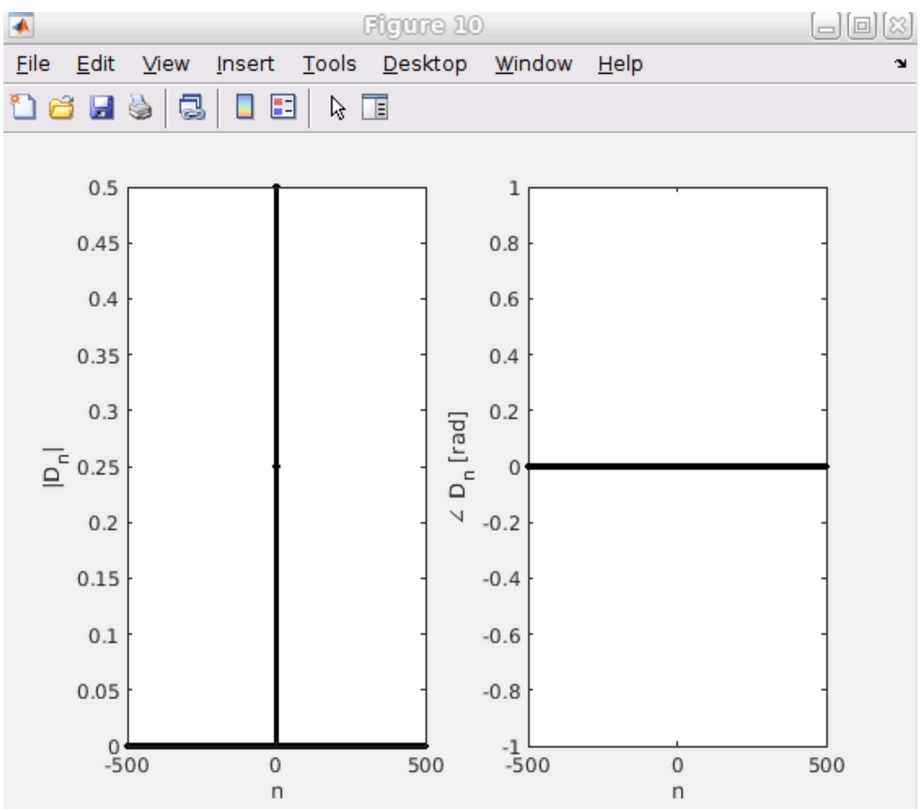
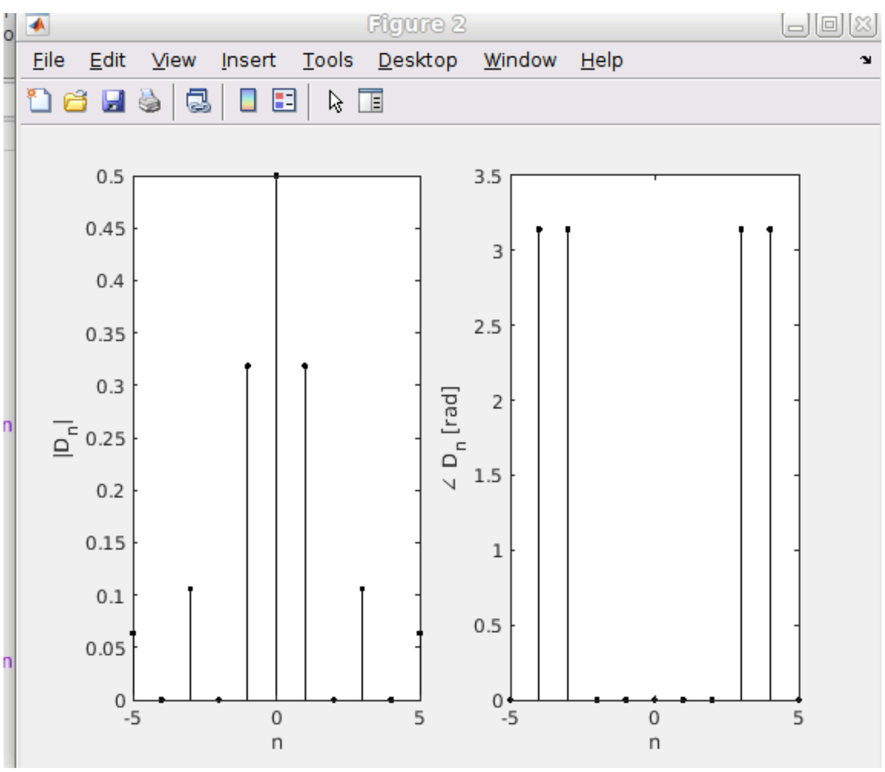


Figure 10
File Edit View Insert Tools Desktop Window Help
|D_n|
n
∠ D_n [rad]
n

A4: *Code for x1(t) that computes dn and returns the fourier coefficients and its respective graphs (graph a, graph b, graph c, graph d)*

```
% --- x2(t) ---
% a) Compute coefficients for x2(t)
figure(2);
n = (-5:5);
D_func = Dn(2);
D_n = D_func(n);
D_n(n==0)=1/2;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
% b) n = -20:20
figure(5);
n = (-20:20);
D_func = Dn(2);
D_n = D_func(n);
D_n(n==0)=1/2; % we took the limit at n -> 0 and it is 1/2
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
% c) n = -50:50
figure(8);
n = (-50:50);
D_func = Dn(2);
D_n = D_func(n);
D_n(n==0)=1/2;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
% d) n = -500:500
figure(11);
n = (-500:500);
D_func = Dn(2);
D_n = D_func(n);
D_n(n==0)=1/2;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
```



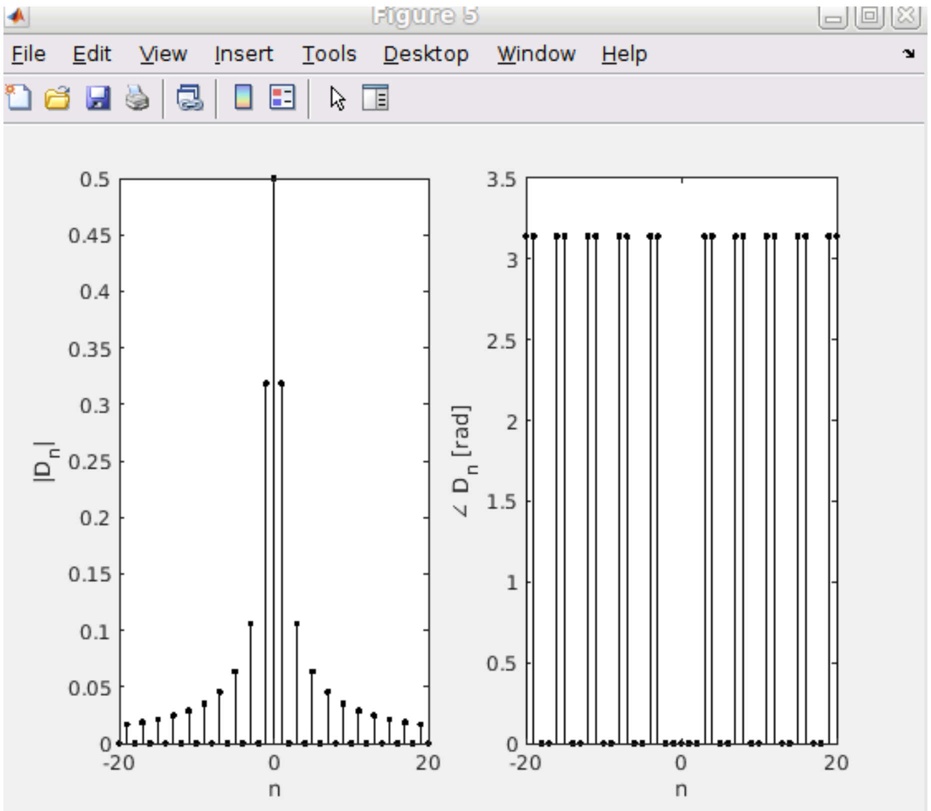


Figure 5
File Edit View Insert Tools Desktop Window Help
|D_n|
n
∠ D_n [rad]
n

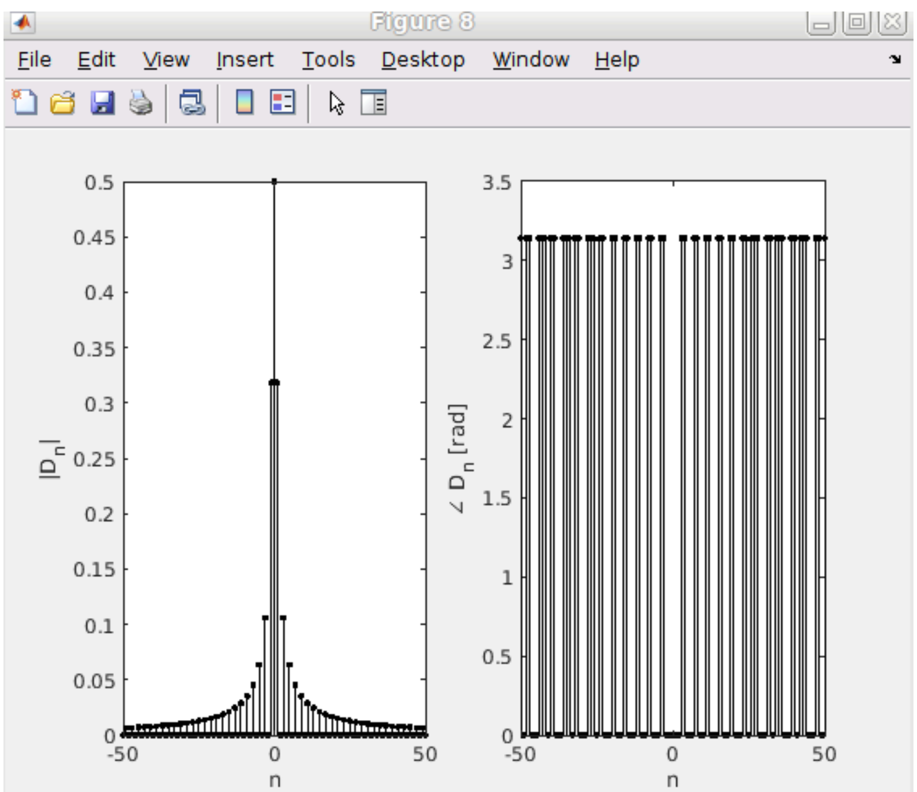
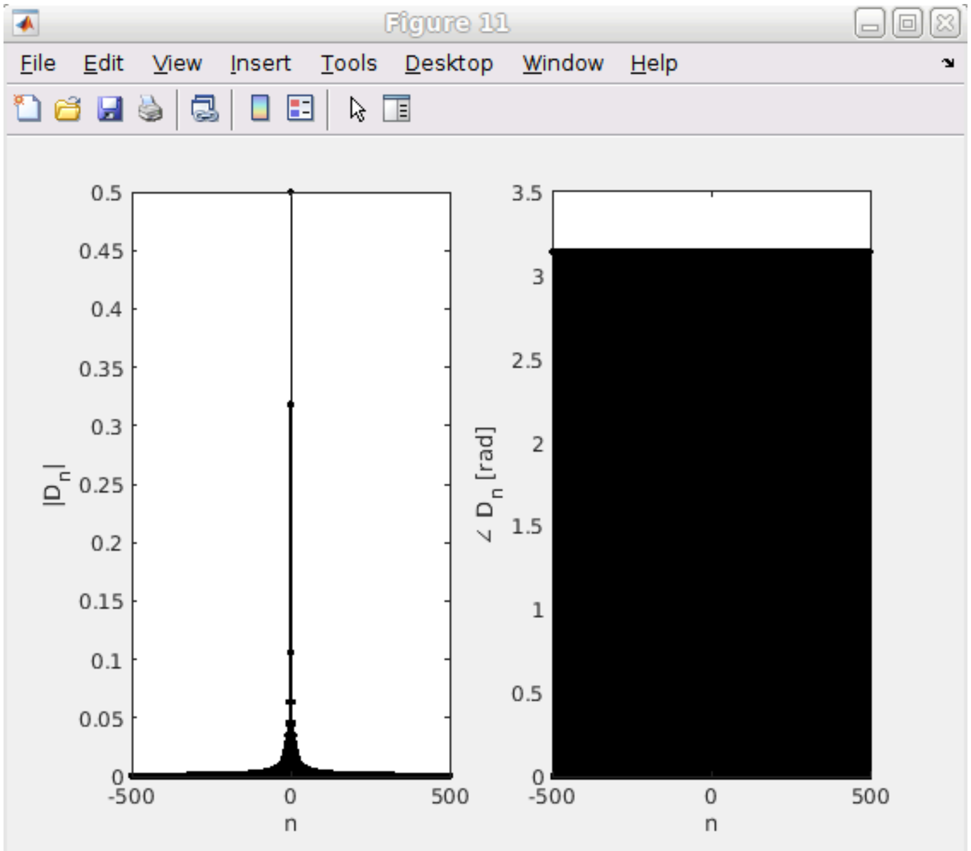


Figure 8
File Edit View Insert Tools Desktop Window Help
|D_n|
n
∠ D_n [rad]
n



A4: *Code for x2(t) that computes dn and returns the fourier coefficients and its respective graphs (graph a, graph b, graph c, graph d)*

```
% a) Compute coefficients for x3(t)
figure(3);
n = (-5:5);
D_func = Dn(3);
D_n = D_func(n);
D_n(n==0)=1/4;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
% b) n = -20:20
figure(6);
n = (-20:20);
D_func = Dn(3);
D_n = D_func(n);
D_n(n==0)=1/4;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
% c) n = -50:50
figure(9);
n = (-50:50);
D_func = Dn(3);
D_n = D_func(n);
D_n(n==0)=1/4;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
% d) n = -500:500
figure(12);
n = (-500:500);
D_func = Dn(3);
D_n = D_func(n);
D_n(n==0)=1/4;
subplot(1,2,1); stem(n,abs(D_n),'.k'); xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k'); xlabel('n'); ylabel('\angle D_n [rad]');
```

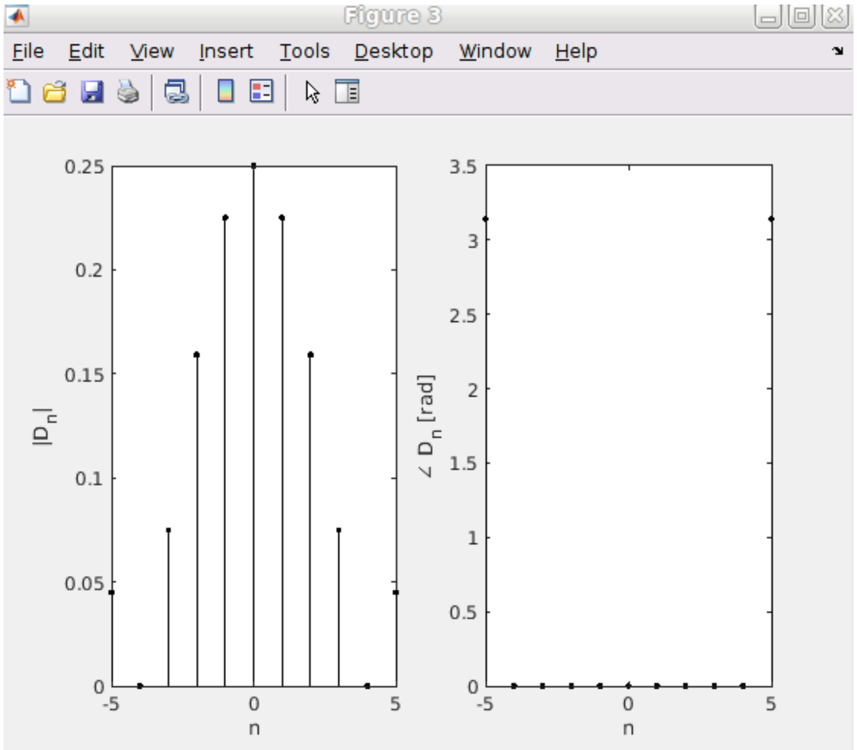


Figure 3
File Edit View Insert Tools Desktop Window Help
|D_n|
∠ D_n [rad]
n
n

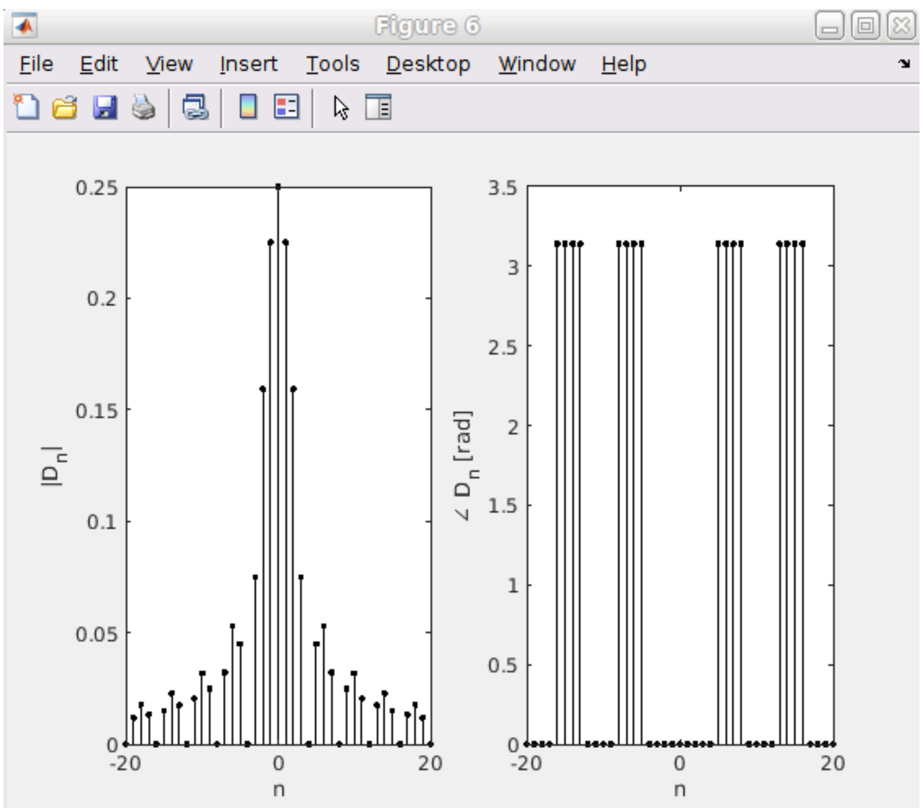
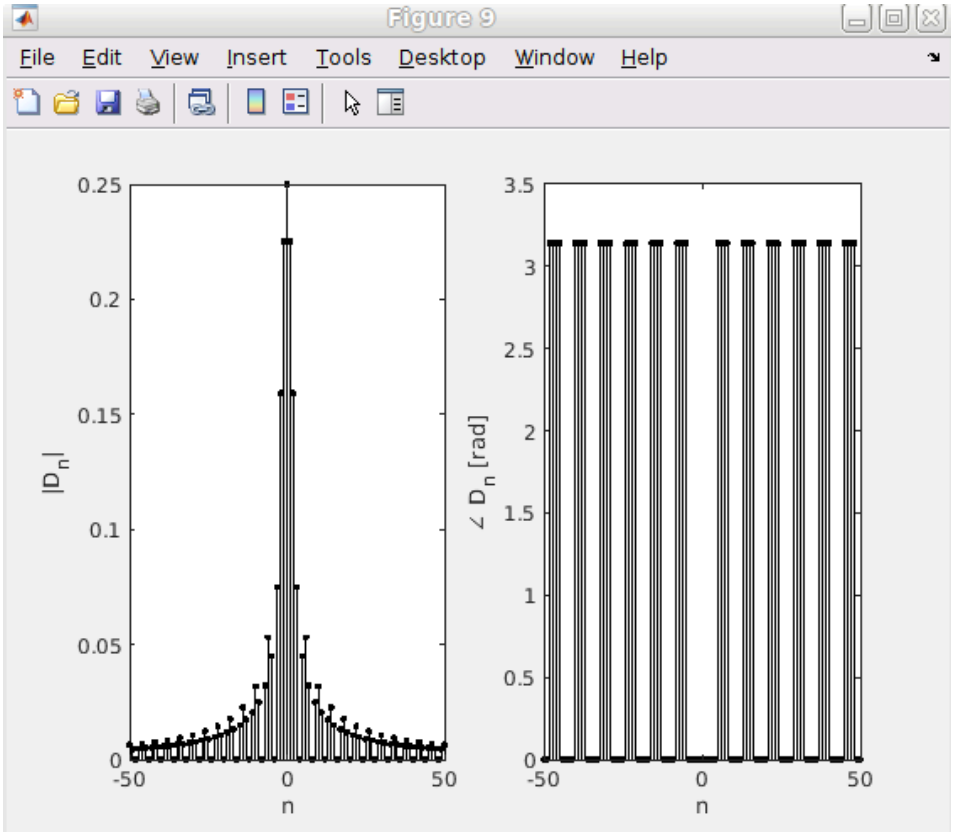
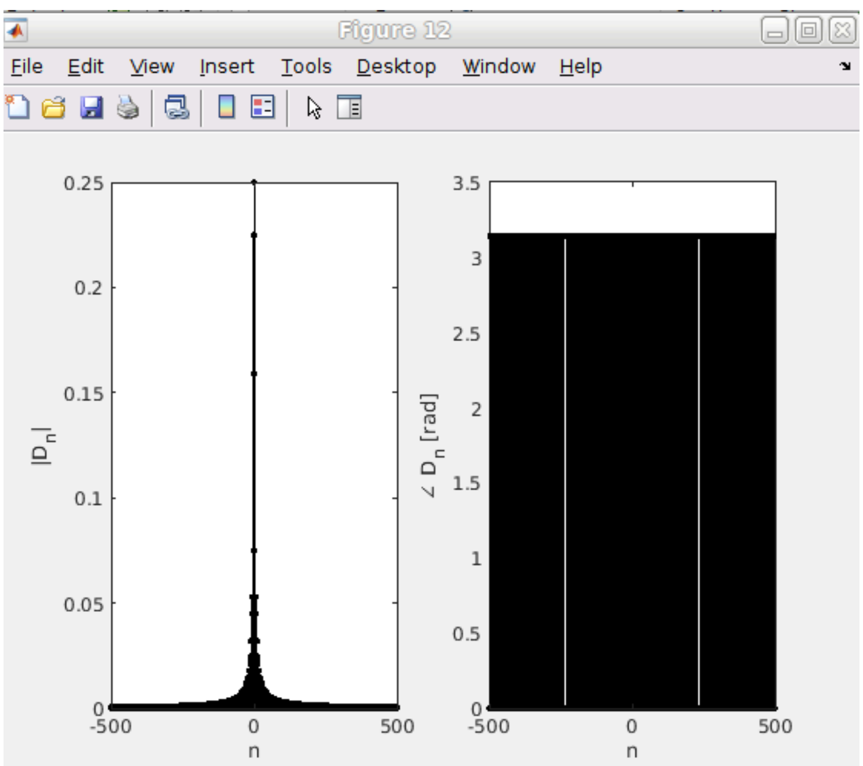


Figure 6
File Edit View Insert Tools Desktop Window Help
|D_n|
∠ D_n [rad]
n
n

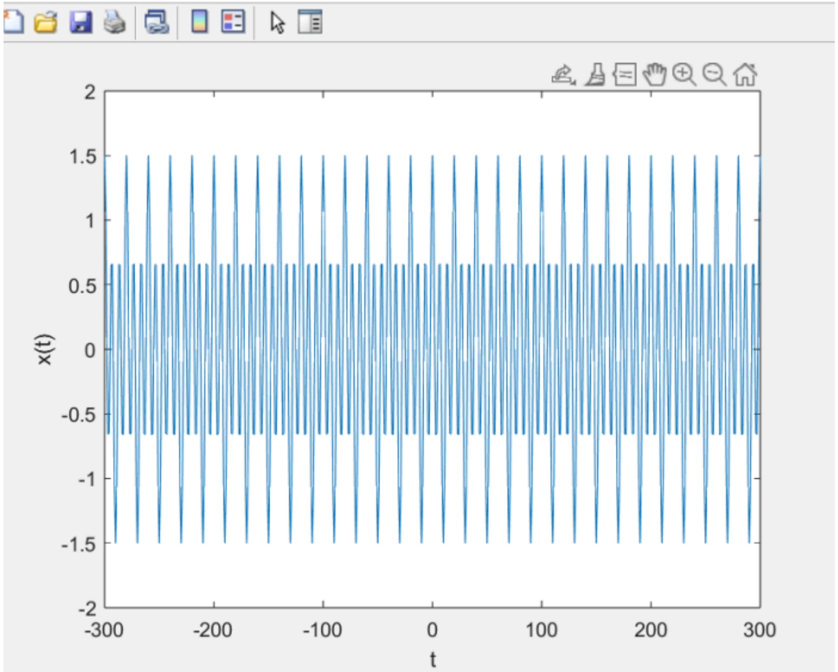


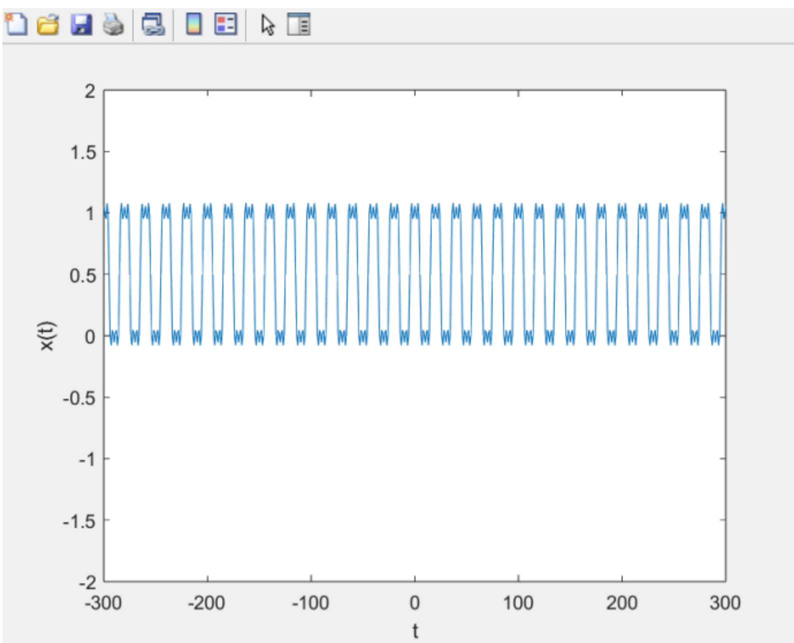


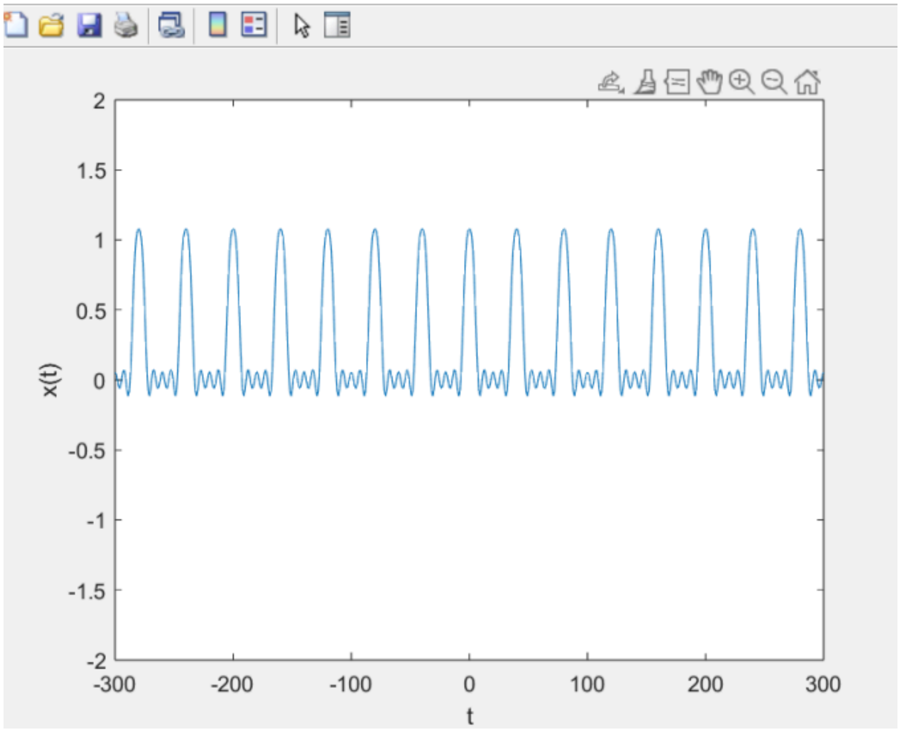
A4: *Code for x3(t) that computes dn and returns the fourier coefficients and its respective graphs (graph a, graph b, graph c, graph d)*

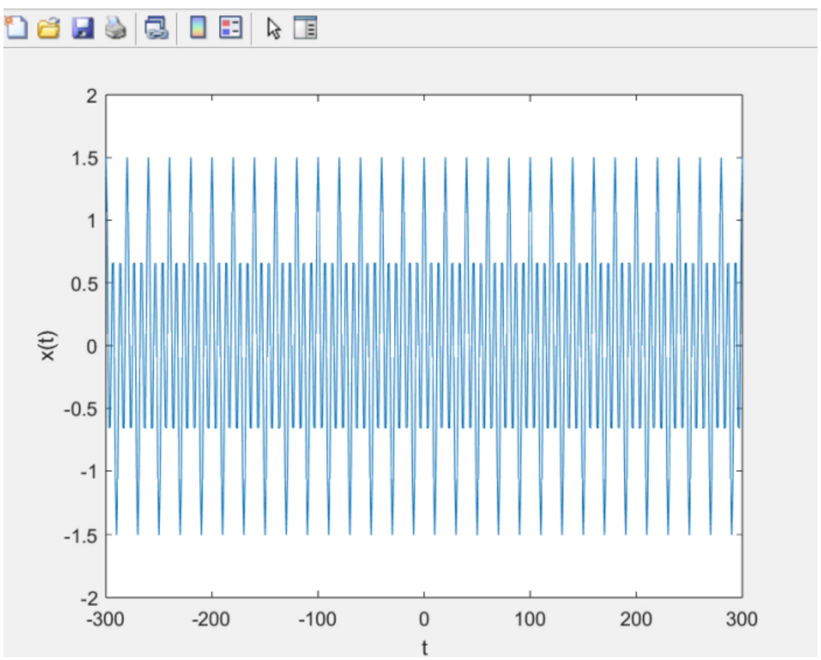
```
% A5(Dn): Reconstruct signal from Fourier coefficients
function [D] = A5(Dn)
   n = -200:200;
   D = Dn;
   t = -300:300;
   w = pi * 0.1;
   x = zeros(size(t));           % Initialize reconstructed signal
   % Compute partial Fourier sum
   for i = 1:length(n)
       x = x + D(i) * exp(1j * n(i) * w * t);
   end
   % Plot reconstructed signal
   figure;
   plot(t, real(x), 'k');
   xlabel('t (sec)');
   ylabel('x(t)');
   axis([-300 300 -1 2]);
   title('Reconstructed Signal from Fourier Coefficients');
   grid on;
end
```

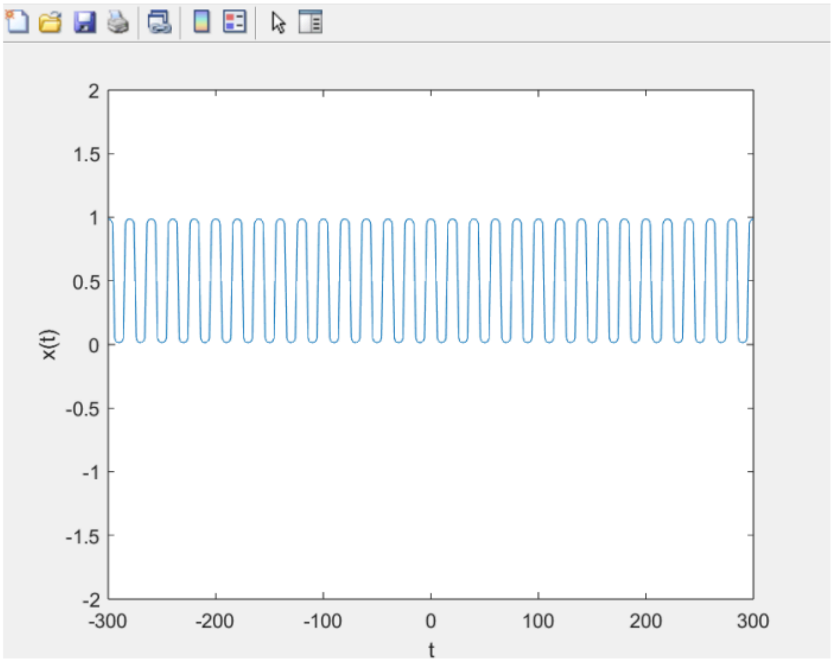
A5: *Code for A5 that computes dn and reconstructs original time-domain signal*

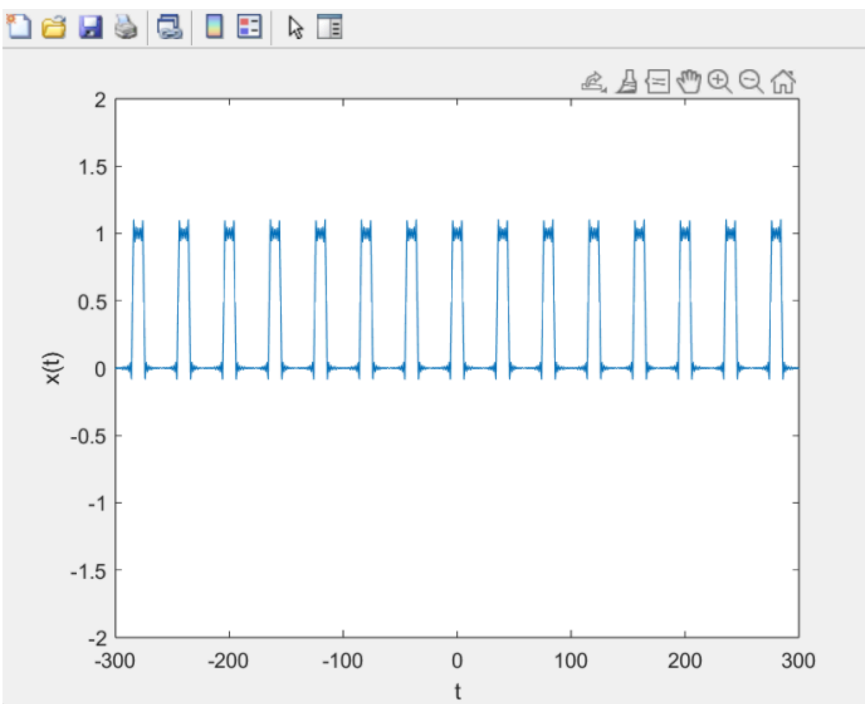

2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
x(t)
-300
-200
-100
0
100
200
300
t

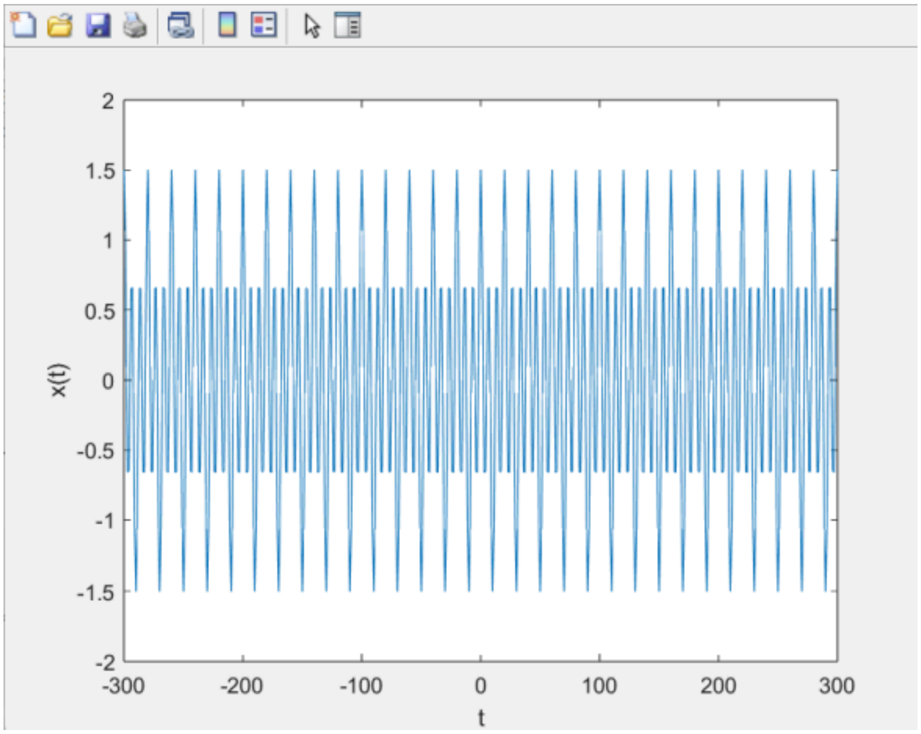

2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
x(t)
-300
-200
-100
0
100
200
300
t

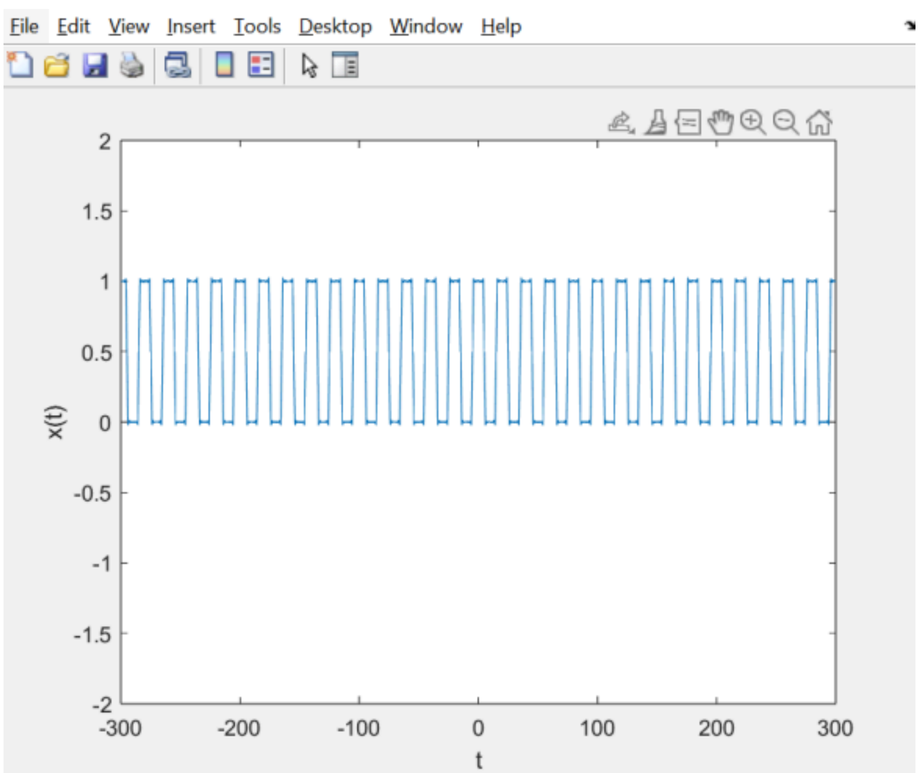

2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
x(t)
-300
-200
-100
0
100
200
300
t

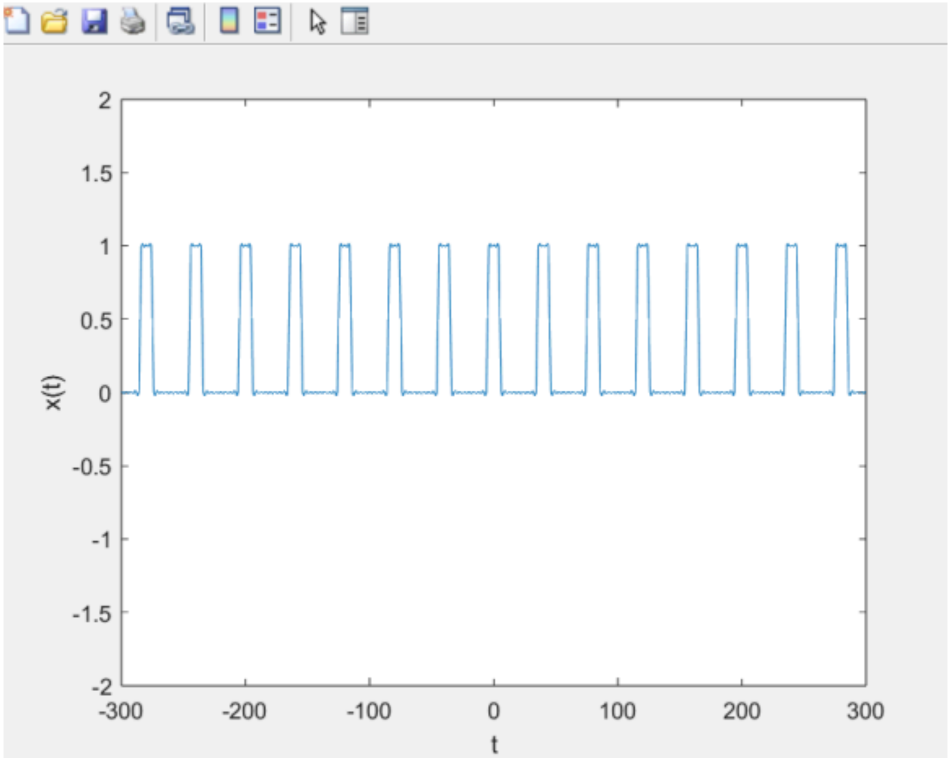

2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
x(t)
-300
-200
-100
0
100
200
300
t


x(t)
t

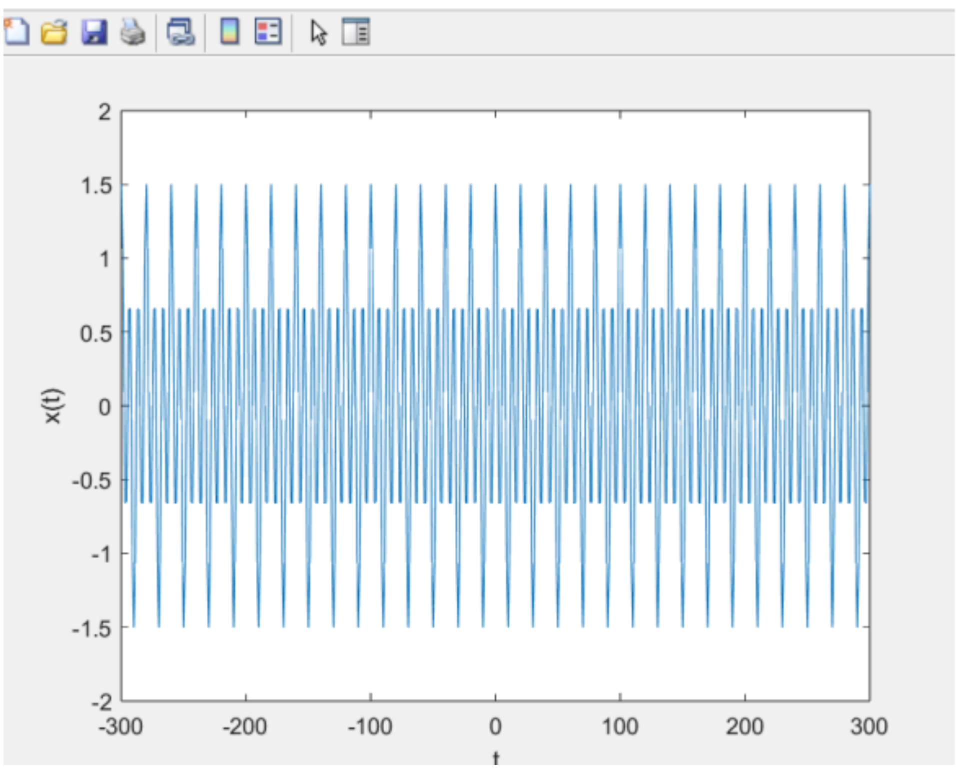

x(t)
t


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
x(t)
-300
-200
-100
0
100
200
300
t

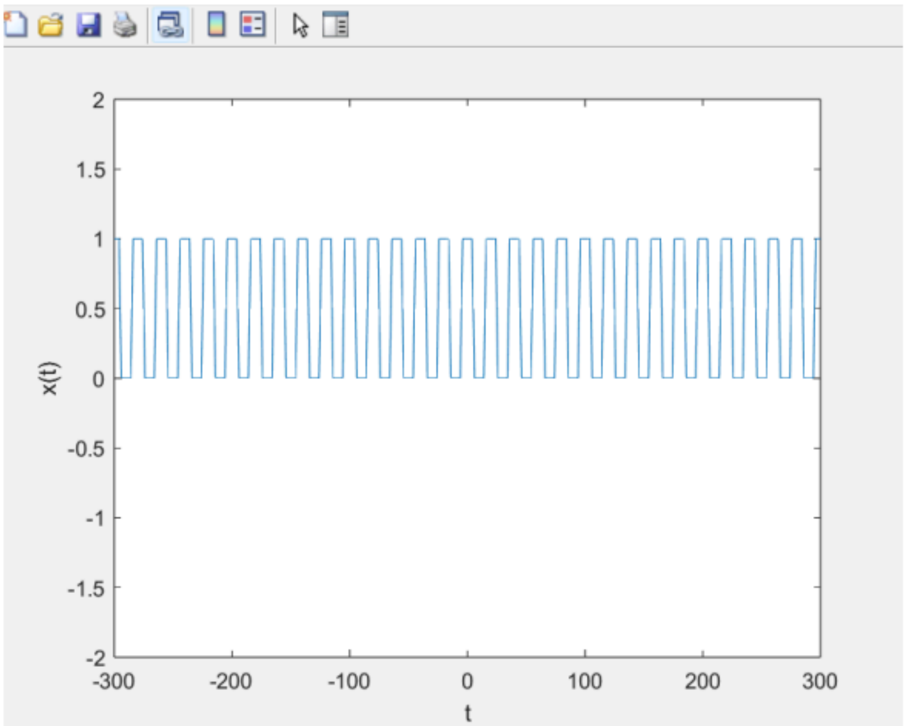

File Edit View Insert Tools Desktop Window Help
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
x(t)
-300
-200
-100
0
100
200
300
t

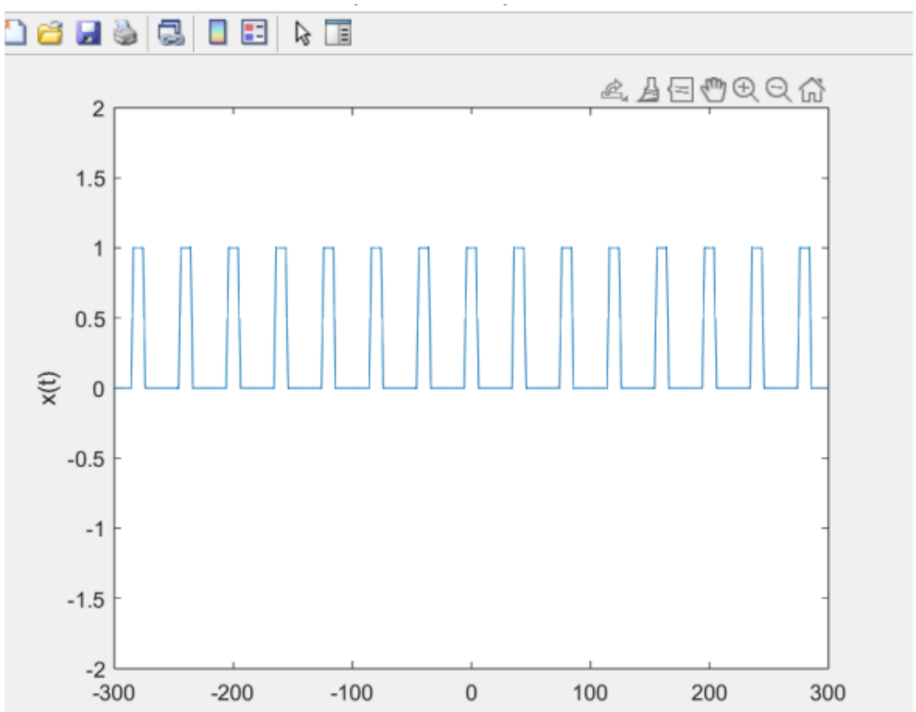


x(t)
t



x(t)
t





A6: *Graphs for the reconstructed time-domain signals x1(t), x2(t) and x3(t) with the fourier coefficients generated in Problem A4*

B1.

$$x_1(t) = \cos\left(\frac{3\pi}{10}t\right) + \frac{1}{2}\cos\left(\frac{\pi}{10}t\right)$$

$$W_{01} = \frac{3\pi}{10} \qquad W_{02} = \frac{\pi}{10}$$

$$W_0 = \frac{GCF}{LCM} = \boxed{\frac{\pi}{10}\ \text{rad/s}}$$

$x_2(t)$

$$T = 20$$

$$W_0 = \frac{2\pi}{T} = \boxed{\frac{\pi}{10}\ \text{rad/s}}$$

$x_3(t)$

$$T = 40s$$

$$W_0 = \frac{2\pi}{T} = \boxed{\frac{\pi}{20}\ \text{rad/s}}$$

B1: *Calculation for B2 values.*

B2:

The main differences between x1(t) and x2(t) are how the sin functions in x1(t) and sin functions in x2(t) are used. Additionally, x1(t) contains four distinct Fourier series coefficients, while x2(t) contains infinite fourier series as n grows to infinity. We also note that x2(t) has a DC value signal, a0, which is 0.5, while x1(t) does not have a DC value signal. Otherwise, the two signals share a fundamental frequency.

B3:

Since the period of x3(t) is greater, it will be inversely proportional to its fundamental frequency, which means that x3(t) has a smaller fundamental frequency compared to x2(t). Additionally, a shorter period will expand the change values of fourier coefficients. And vice versa; a greater period will compress the change in values of the fourier coefficients, as seen in signal x3(t).

B4:

B4. $\frac{1}{20}\int_{-10}^{10} x(t)\, e^{-jn(\frac{\pi}{10})t}\, dt$

$$= -\frac{1}{20}\int_{-10}^{-5} e^{-jn(\frac{\pi}{10})t} + \frac{1}{20}\int_{-5}^{5} e^{-jn(\frac{\pi}{10})t} - \frac{1}{20}\int_{5}^{10} e^{-jn(\frac{\pi}{10})t}$$

$$= -\frac{1}{20}\left[\frac{-10e^{-jn(\frac{\pi}{10})t}}{jn\pi}\right]_{-10}^{-5} + \frac{1}{20}\left[\frac{-10e^{-jn(\frac{\pi}{10})t}}{jn\pi}\right]_{-5}^{5} - \frac{1}{20}\left[\frac{-10e^{-jn(\frac{\pi}{10})t}}{jn\pi}\right]_{5}^{10}$$

$$= \frac{1}{2}\cancel{\frac{e^{jn(\frac{\pi}{2})t}}{jn\pi}} - \frac{1}{2}\cancel{\frac{e^{jn\pi}}{jn\pi}} - \frac{1}{2}\cancel{\frac{e^{-jn(\frac{\pi}{2})t}}{jn\pi}} + \frac{1}{2}\cancel{\frac{e^{jn(\frac{\pi}{2})t}}{jn\pi}}$$

$$+ \frac{1}{2}\cancel{\frac{e^{-jn\pi t}}{jn\pi}} - \frac{1}{2}\cancel{\frac{e^{jn(\frac{\pi}{2})t}}{jn\pi}}$$

$$= 0$$

$\boxed{\therefore D_n \text{ of } x_4(t) \text{ is } 0.}$

B5:

It is known that x1(t) has a finite number of fourier coefficients, while for x2(t), there are infinite numbers of fourier coefficients. Thus, increasing the number of fourier coefficients will increase the accuracy of the reconstructed signal for x2(t), but x1(t) will stay the same.

B6:

Referencing Problem B5, we only need four fourier coefficients to perfectly reconstruct x1(t). For x2(t) and x3(t), we will need an infinite number of fourier coefficients for a perfect reconstruction.

B7:

For signals that require a large amount of finite fourier coefficients, it is not recommended to be used to reconstruct a fourier series signal due to space and time limitations. Such as signals x2(t) and x3(t), they require an infinite number of fourier coefficients, so unless there are constraints set on the number of fourier coefficients required, it will use a lot of space and time. For signals such as x1(t), it is possible since it only requires four fourier coefficients to replicate a signal.