Signal and Systems I
ELE532
Lecture 2

- ❖ Lecture starts at 12:10pm
- ❖ Before the lecture starts download, or have a soft, copy of the lecture PDF from the D2L
- ❖ Emailing regarding ELE532:
  1- Title of the email starts with "ELE532"
  2- CC lead TA in the email: Luella Marcos lgmarcos@torontomu.ca

Last Lecture:

Signal Classification

Important Signals

- $u(t)$: Picking up the causal part of the signal
- $\delta(t)$: Derivative of the $u(t)$, Zero everywhere except at zero
- $Sinc(t)$: Box in Frequency, Low-Pass Filter
- e^{st}: Defining zeros and poles in Fourier Transform

Exponential Signals: e^{st}

$s = \sigma + j\omega$

e^{st}

S plane

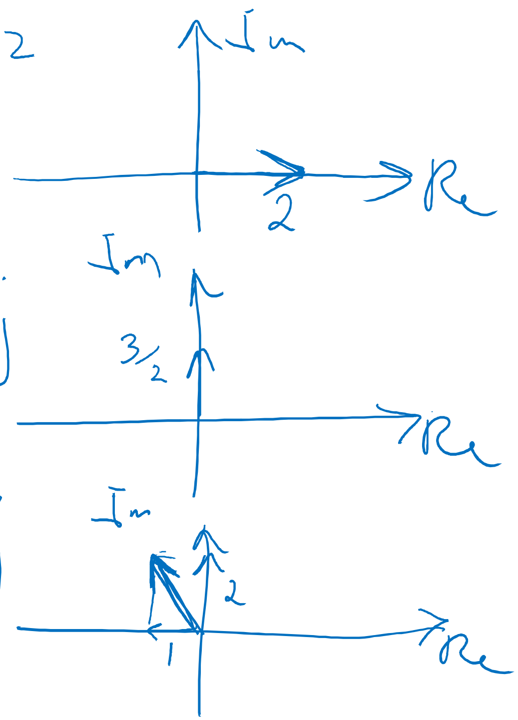
$e^{-\frac{1}{2}t}$

$s = -\frac{1}{2}$

$s = 2$

$s = \frac{3}{2}j$

$s = -1 + 2j$

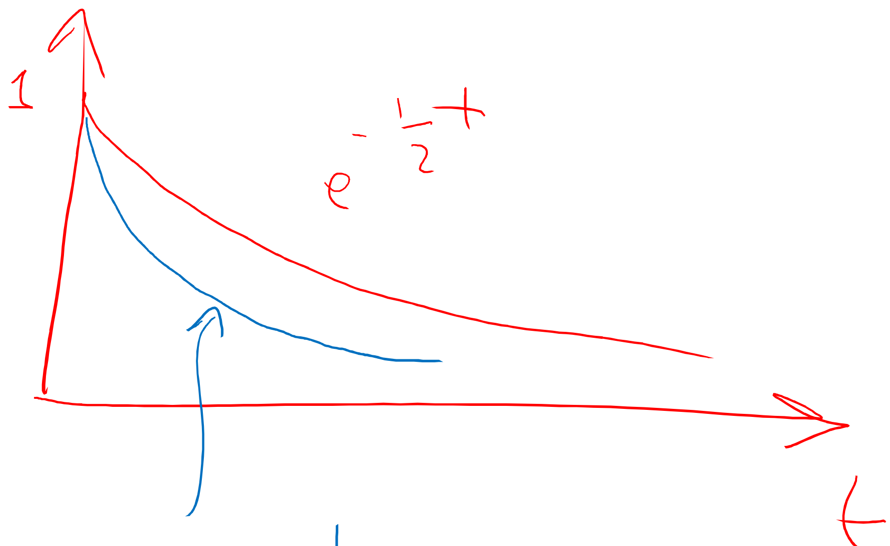


Exponential Signals:

$$e^{st}$$

$$s = \sigma + j\omega$$

$$e^{-\frac{1}{2}t}$$



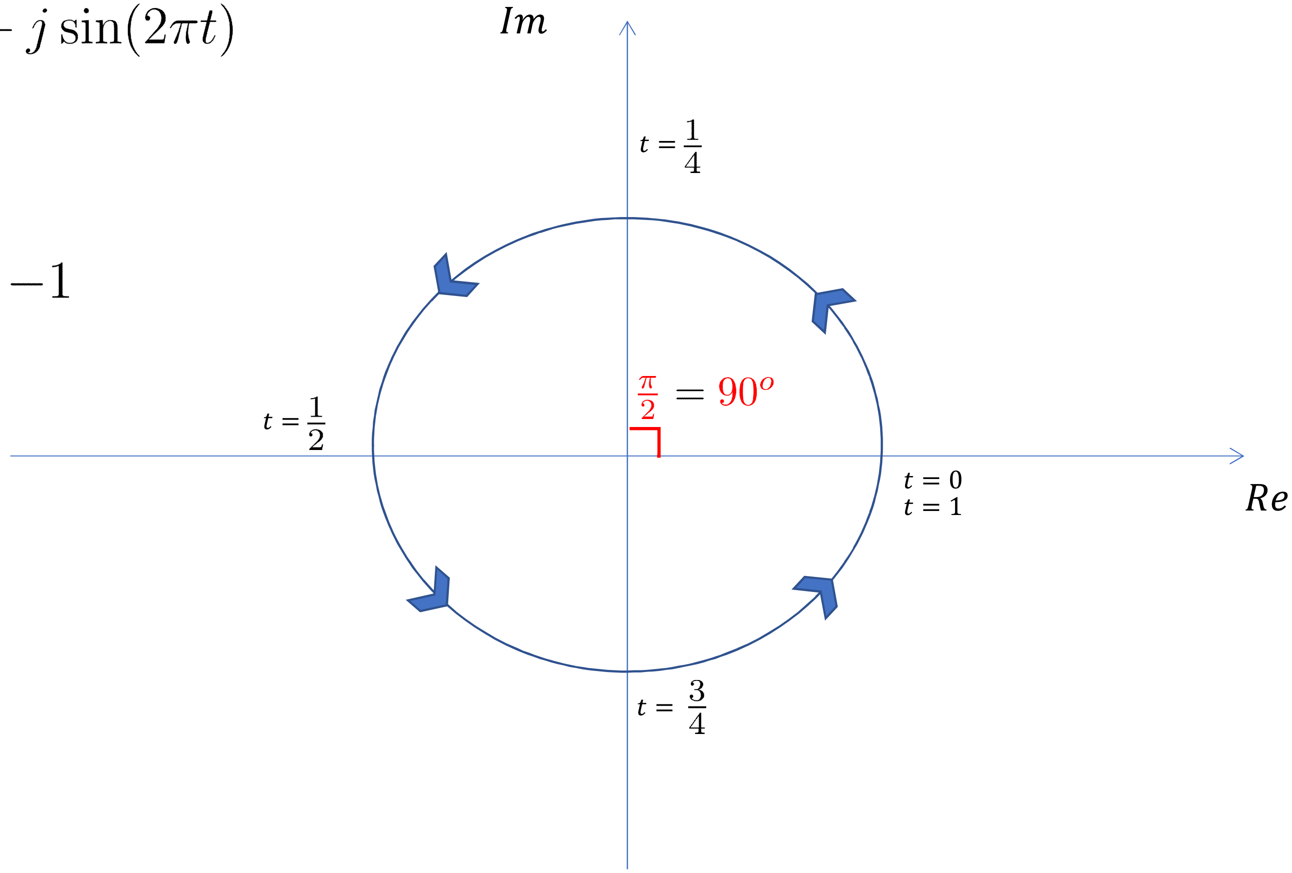
e^{-2t} (decays faster)

Exponential Signals:

Example: Consider the pure imaginary example of complex number: $e^{j2\pi t} = \cos(2\pi t) + j\sin(2\pi t)$

For :

- $t = 0 \rightarrow e^{j2\pi(0)} = 1$
- $t = \frac{1}{2} \rightarrow e^{j\frac{2\pi}{2}} = e^{j\pi} = -1$
- $t = 1 \rightarrow e^{j2\pi} = 1$



Exponential Signals:

General Case: $e^{\sigma t}.e^{j\omega t} = e^{(\sigma+j\omega)t}$

σ *indicate the decay or expansion,* ω *indicate the speed of rotation*

$$e^{-\frac{1}{2}t}e^{j2\pi t} = e^{-\frac{1}{2}t}(\cos(2\pi t) + j\sin(2\pi t))$$

For :

- $t = 0 \rightarrow e^{0}e^{j2\pi(0)} = 1$
- $t = \frac{1}{2} \rightarrow e^{-\frac{1}{2}\times\frac{1}{2}}e^{j\frac{2\pi}{2}} = e^{-\frac{1}{4}}e^{j\pi} = e^{-\frac{1}{4}} \times -1$
- $t = 1 \rightarrow e^{-\frac{1}{2}}e^{j\pi} = e^{-\frac{1}{2}}e^{j2\pi} = e^{-\frac{1}{2}}$

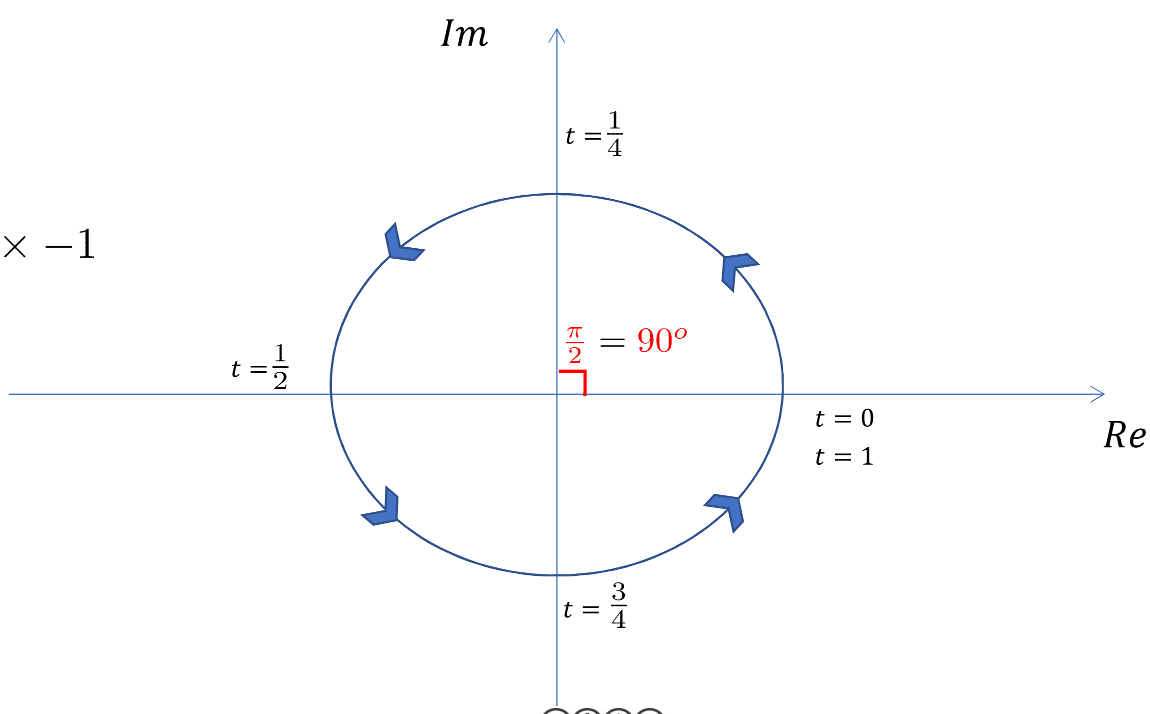
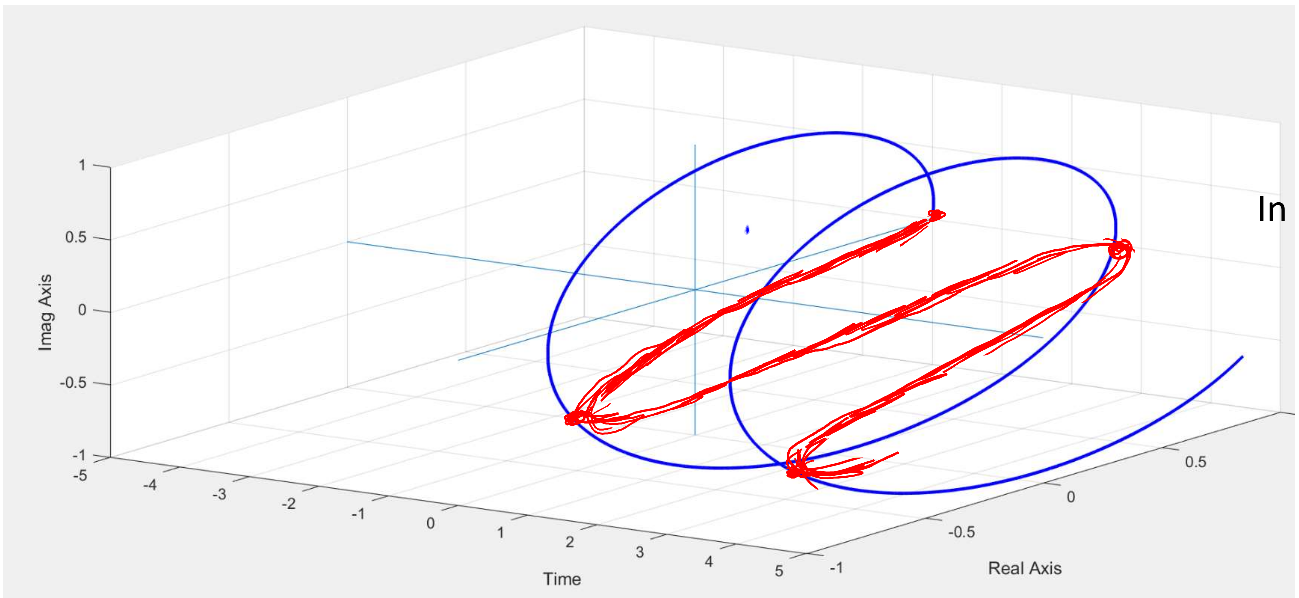


Illustration of e^{st} for different values of s

$$e^{st} = e^{(\sigma + j\omega)t}$$

$s = \sigma + j\omega$

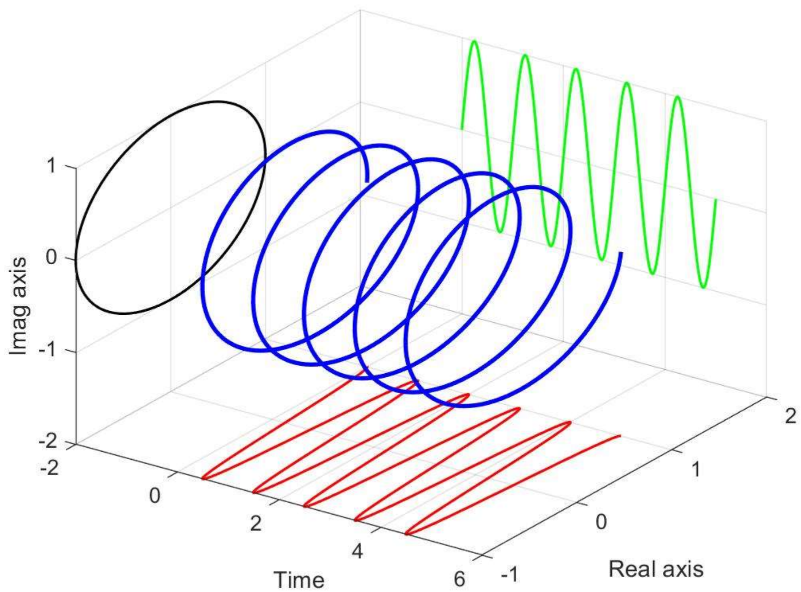
3. $\sigma = 0, s = j\omega, e^{st} = e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$

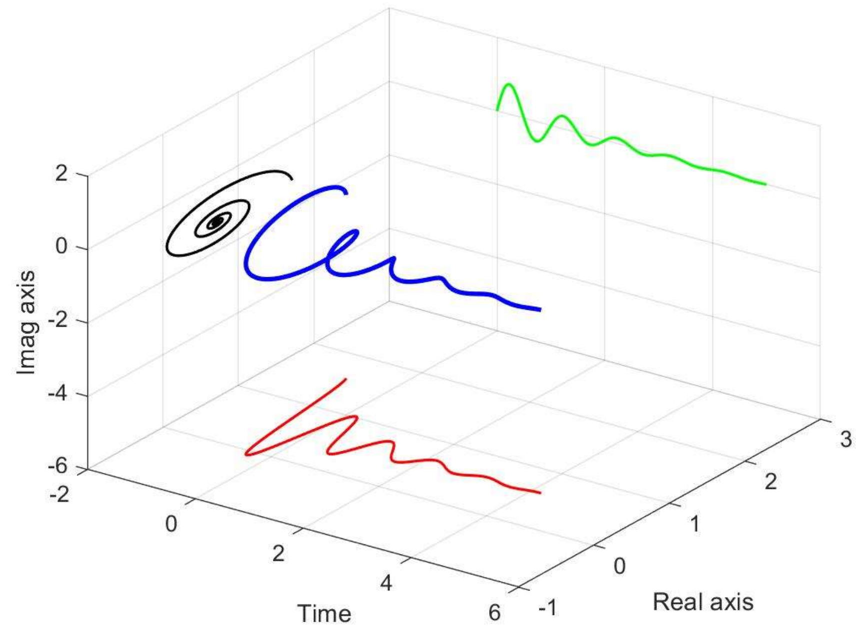


In red: real part

3-D View Plot

Exponential Signals





$$e^{j2\pi t} = \cos(2\pi t) + j\sin(2\pi t)$$

S=j2π

$$e^{-.85t}e^{j2\pi t} = e^{-.85t}\cos(2\pi t) + je^{-.85t}\sin(2\pi t)$$

S=-0.85-j2π

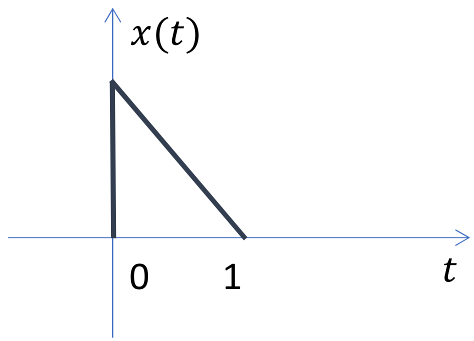
Today:

Useful Signal Operations

- Time Shift
- Amplitude Scaling
- Time Scaling
- Time Reversal
- Combined Operation

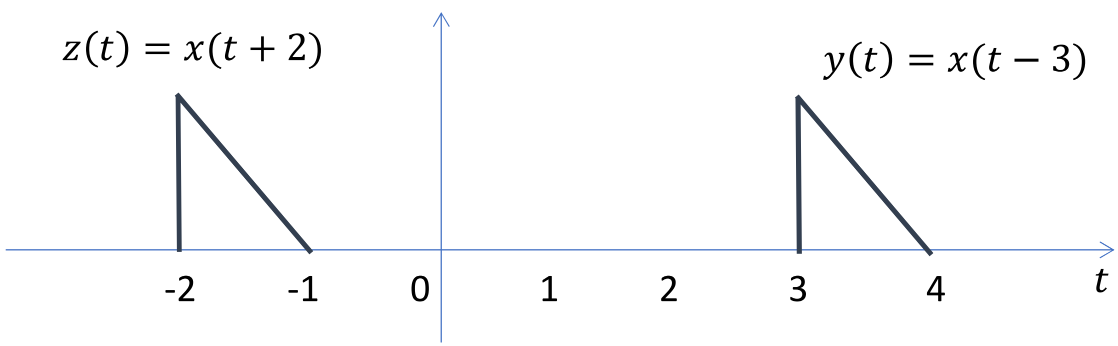
One more signal classitication: Odd and Even Signals

Time Shift:



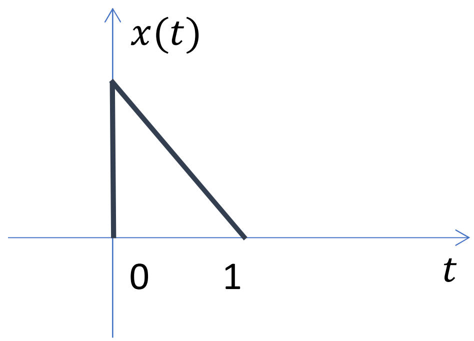
$x(t-T) \Rightarrow$ Shift to Right if $T > 0$ (Delayed, After, Forward)

$x(t-T) \Rightarrow$ Shift to Left if $T < 0$ (Advanced, Before, Backward)



Example: For $y_2(t) = x(t{+}10)$ find the values for $y_2(-10), y_2(0), y_2(5), y_2(9), y_2(10), y_2(11)$ and Plot $y_2(t)$

Time Shift:



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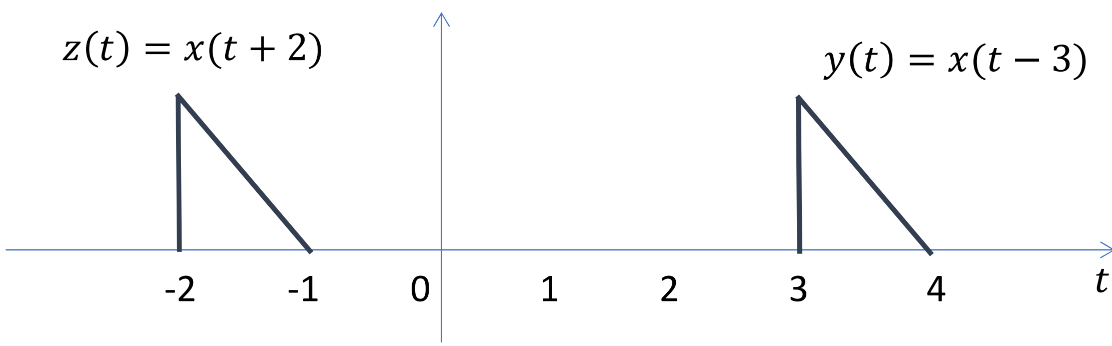
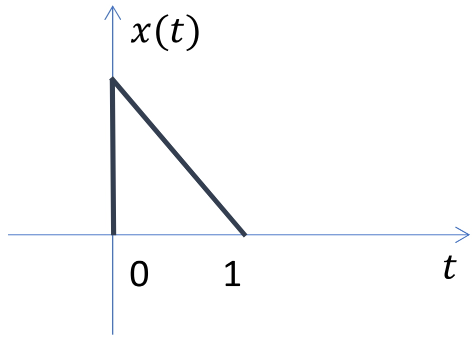


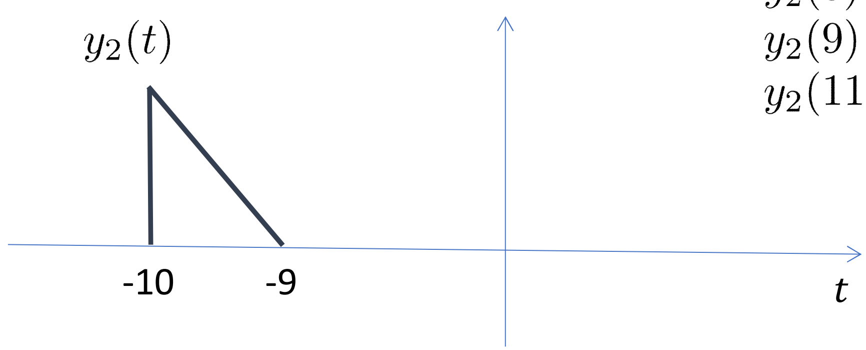
Note: get used to labeling any Transformed (operated) signal with a new name. For example here $y(t)$ and $z(t)$

Example: For $y_2(t) = x(t+10)$ find the values for $y_2(-10), y_2(0), y_2(5), y_2(9), y_2(10), y_2(11)$ and Plot $y_2(t)$

Time Shift:

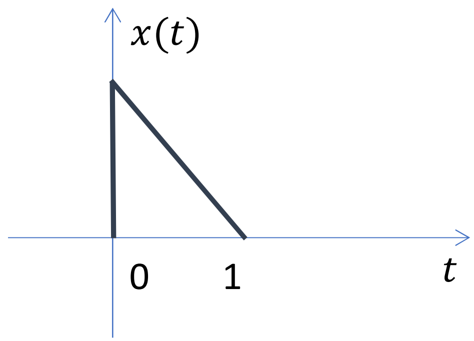


Example: For $y_2(t) = x(t{+}10)$ find the values for $y_2(0), y_2(5), y_2(9), y_2(10), y_2(11)$ and Plot $y_2(t)$



$$y_2(-10) = x(-10+10) = x(0)$$
$$y_2(0) = x(0+10) = x(10)$$
$$y_2(5) = x(5+10) = x(15)$$
$$y_2(9) = x(9+10) = x(19)$$
$$y_2(11) = x(11+10) = x(21)$$

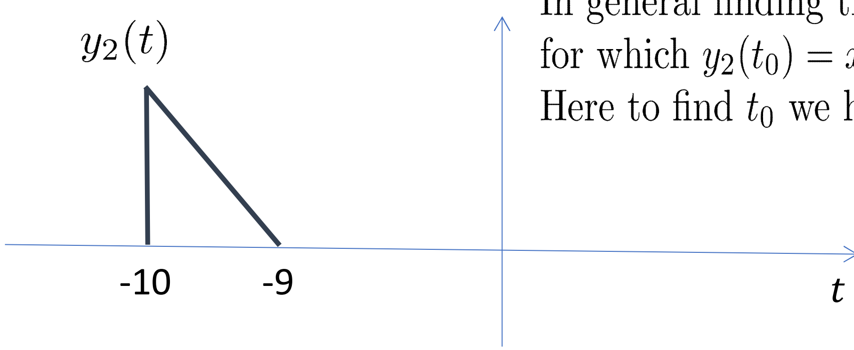
Time Shift:



Example: For $y_2(t) = x(t{+}10)$ find the values for $y_2(0), y_2(5), y_2(9), y_2(10), y_2(11)$ and Plot $y_2(t)$

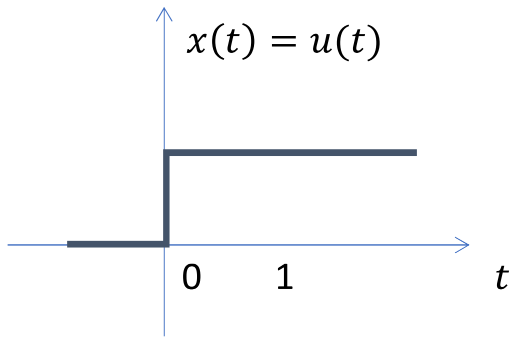
$$y_2(-10) = x(-10+10) = x(\mathbf{0})$$
$$y_2(\mathbf{0}) = x(0+10) = x(10)$$

In general finding the value of $y_2(0)$ and also value of t_0 for which $y_2(t_0) = x(0)$ are useful.
Here to find t_0 we have to have $t_0 + 10 = 0$ which means $t_0 = -10$



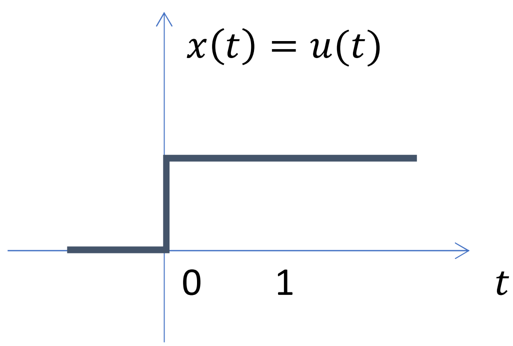
Time Shift:

Example: plot $y(t) = x(t-3)$



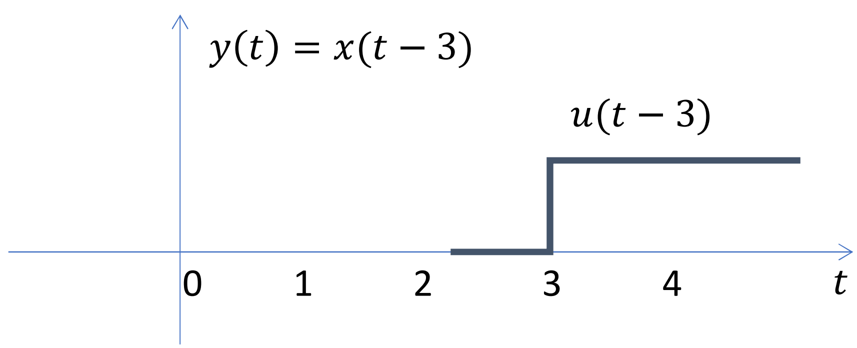
Time Shift:

Example: plot $y(t) = x(t-3)$



$y(3) = x(3-3) = x(\mathbf{0})$
$y(\mathbf{0}) = x(0-3) = x(-3)$

Finding the value of $y(0)$ and also value of t_0 for which $y(t_0) = x(0)$ are useful.
Here to find t_0 we have to have $t_0 - 3 = 0$ which means $t_0 = 3$



Time Shift:

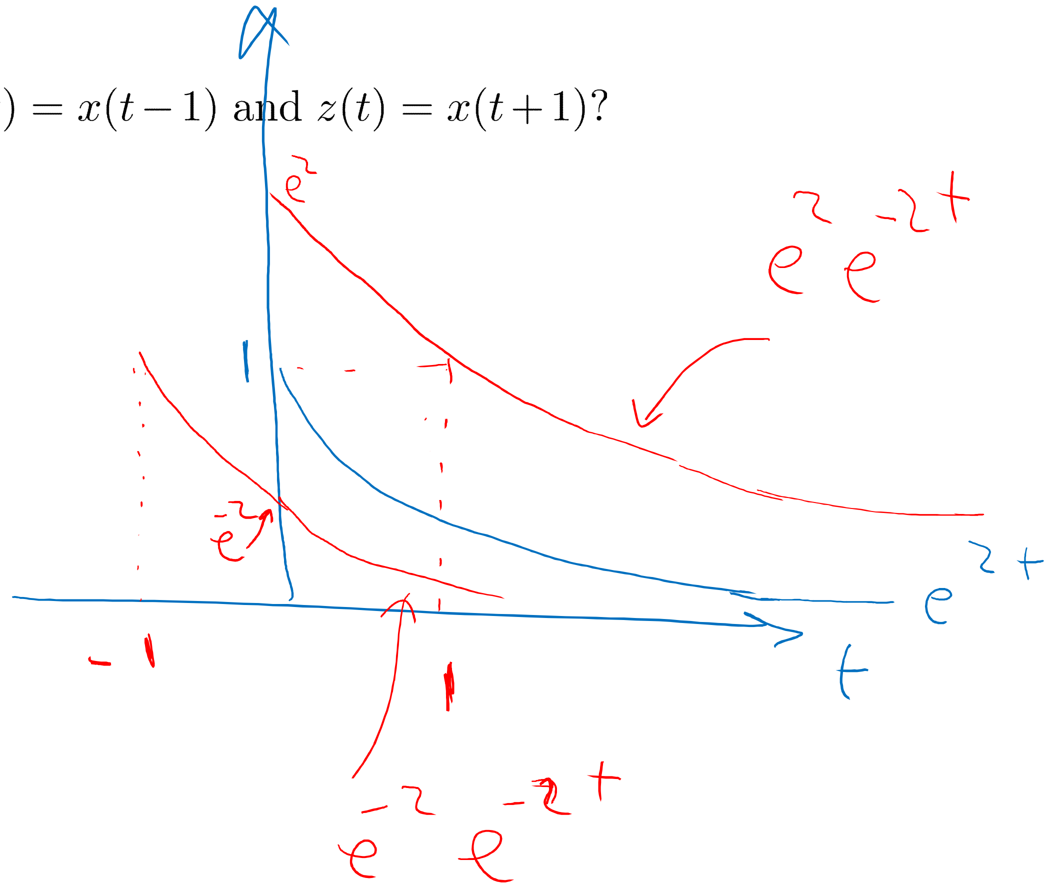
Example: If $x(t) = e^{-2t}$, then what are $y(t) = x(t-1)$ and $z(t) = x(t+1)$? Plot $y(t)$ and $z(t)$

Time Shift:

Example: If $x(t) = e^{-2t}$, then what are $y(t) = x(t-1)$ and $z(t) = x(t+1)$? Plot $y(t)$ and $z(t)$

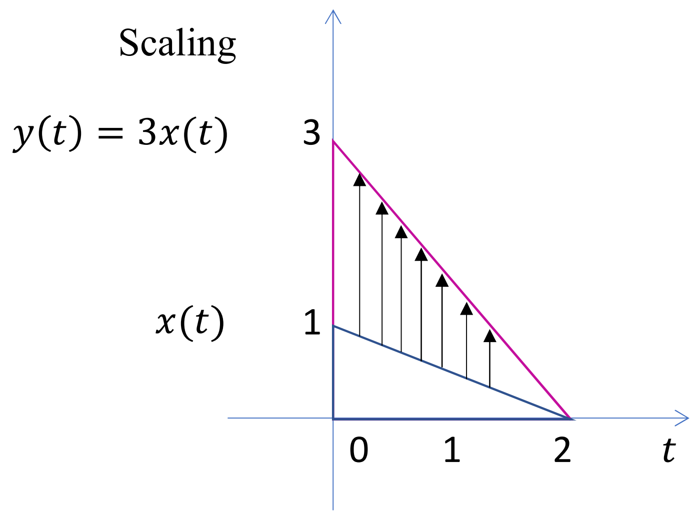
$$y(t) = x(t-1) = e^{-2((\mathbf{t-1}))} = e^{2}e^{-2t}$$

$$z(t) = x(t+1) = e^{-2((\mathbf{t+1}))} = e^{-2}e^{-2t}$$



Amplitude Scaling:

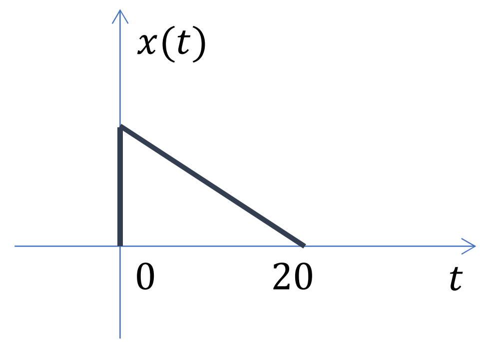
$$y(t) = Ax(t)$$



Time Reversal:

$y(t) = x(-t)$

Example: plot $y(t) = x(-t)$ first find $y(1)$, $y(2)$, $y(0)$, $y(-1)$, and $y(-2)$



Time Reversal:

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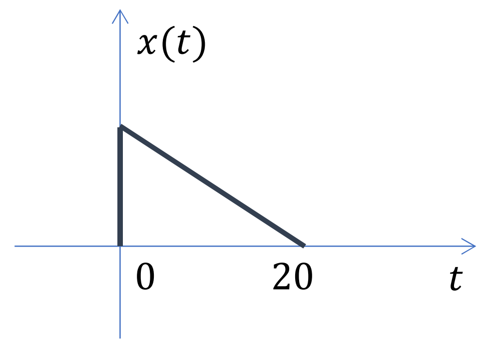
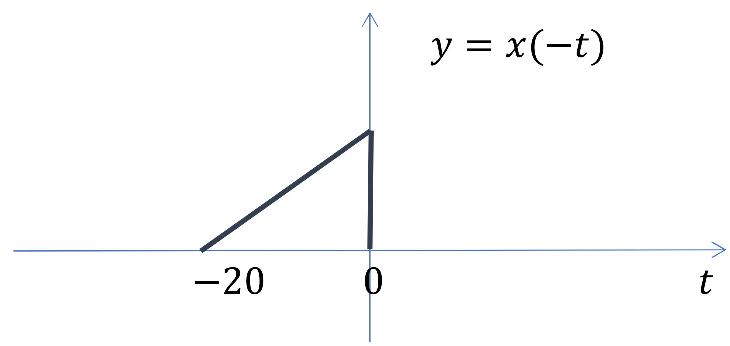
Example: plot $y(t) = x(-t)$ first find $y(1)$, $y(2)$, $y(0)$, $y(-1)$, and $y(-2)$



$$
\begin{aligned}
y(t) &= x(-t)\\
y(1) &= x(-1)\\
y(0) &= x(0)\\
y(-1) &= x(1)\\
y(-2) &= x(2)
\end{aligned}
$$

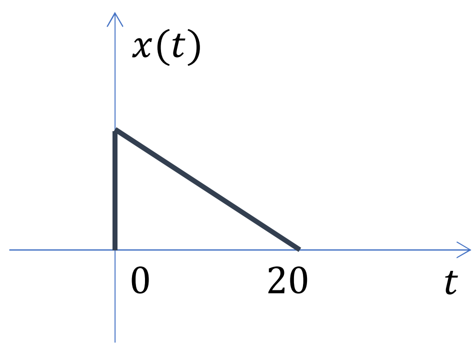


<u>Time Scaling:</u>

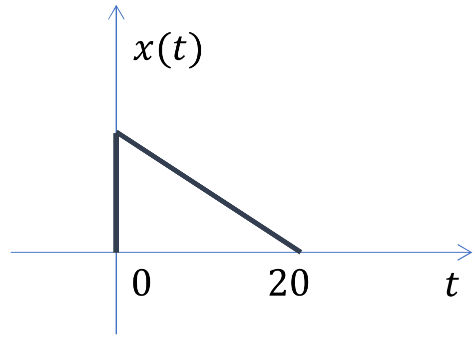
$$y(t) = x(\alpha t)$$

Note that Time Reversal is a special case of Time Scaling with $\alpha = -1$

Example: Find $y(t) = x(2t)$ and $z(t) = x(\frac{t}{2})$.

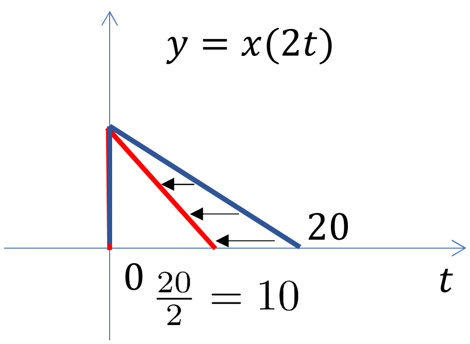


Time Scaling:

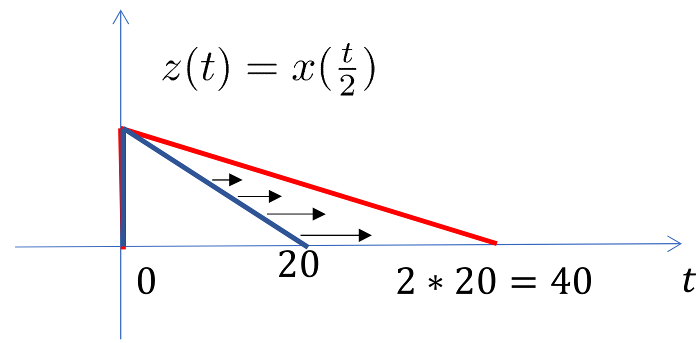


$$y(t) = x(\alpha t)\begin{cases}\text{Squeezing}, & \text{if} \quad \alpha > 1\\ \text{Expanding}, & \text{if} \quad 0 < \alpha < 1\end{cases}$$

$$\begin{aligned}
y(t) &= x(2t)\\
y(0) &= x(0)\\
y(1) &= x(2)\\
y(-1) &= x(-2)\\
y(2) &= x(4)\\
&\vdots\\
y(10) &= x(20)\\
y(11) &= x(22)
\end{aligned}$$



$$\begin{aligned}
z(t) &= x(\tfrac{t}{2})\\
z(0) &= x(0)\\
z(1) &= x(\tfrac{1}{2})\\
z(-1) &= x(\tfrac{-1}{2})\\
z(2) &= x(\tfrac{2}{2})\\
&\vdots\\
z(10) &= x(\tfrac{10}{2})\\
z(11) &= x(\tfrac{11}{2})
\end{aligned}$$



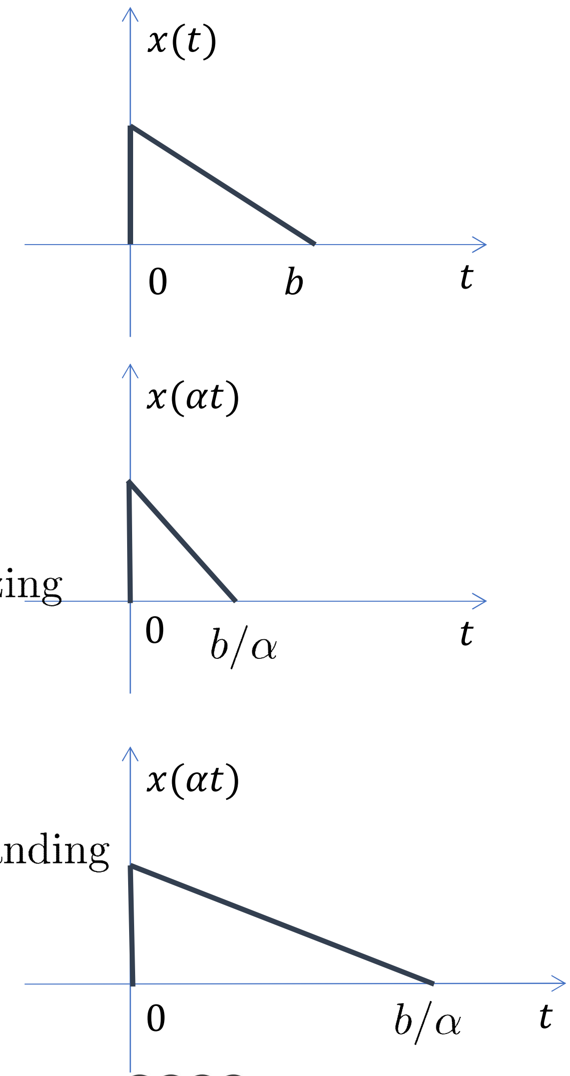
Generally, it is a good idea to check for couple of points when per-forming time scale operation.

Time Scaling:

$$y(t) = x(\alpha t) \begin{cases} \text{Squeezing,} & \text{if} \quad \mid \alpha \mid > 1 \\ \text{Expanding,} & \text{if} \quad 0 < \mid \alpha \mid < 1 \end{cases}$$

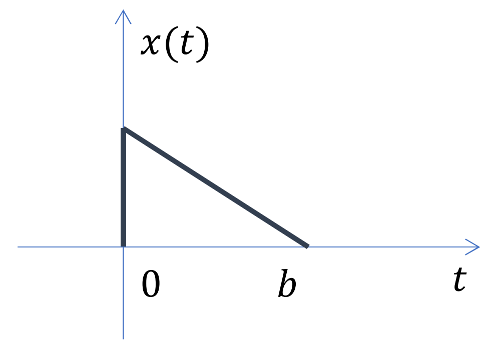
$\alpha > 1$ signal squeezing

$0 < \alpha < 1$ signal expanding



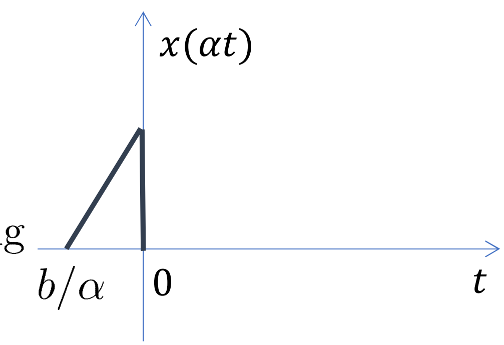
Time Scaling:

$$y(t) = x(\alpha t)\begin{cases}\text{Squeezing,} & \text{if} \quad |\ \alpha\ | > 1 \\ \text{Expanding,} & \text{if} \quad 0 < |\ \alpha\ | < 1\end{cases}$$



For $\alpha < 0$:

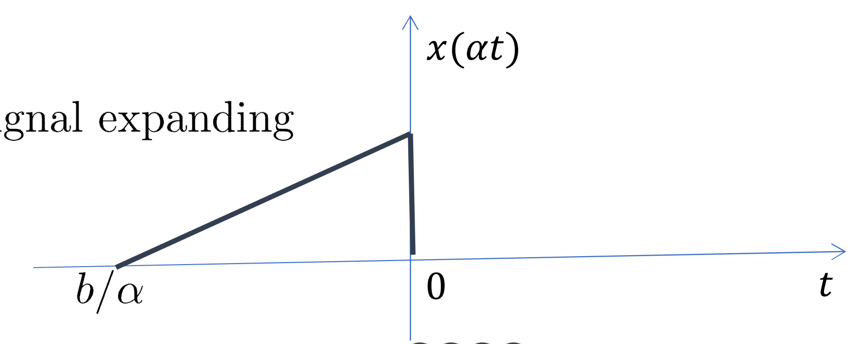
$\alpha < -1$ signal squeezing



In this case: $\alpha = -|\alpha|$

example: $-2 = -|-2|$

$-1 < \alpha < 0$ signal expanding

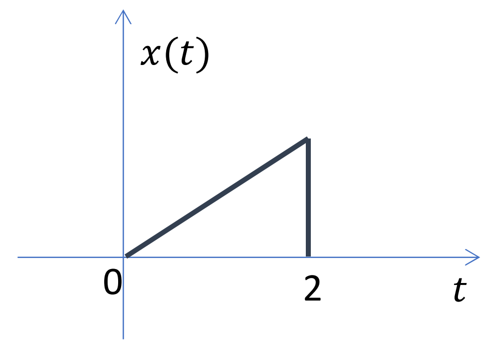


$y(t) = x(\alpha t) = x(-|\alpha| t)$

$x(-2t) = x(-(2t))$ additional flipping

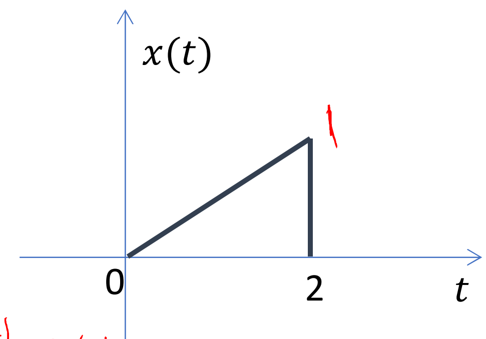
Time Scaling:

Plot $y(t) = x(3t)$ and $z(t) = x(-t/4)$



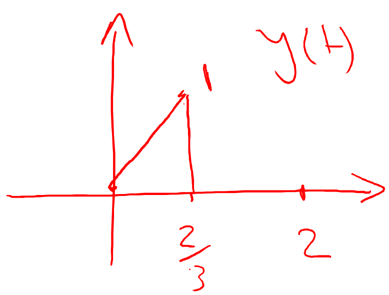
Time Scaling:

Plot $y(t) = x(3t)$ and $z(t) = x(-t/4)$

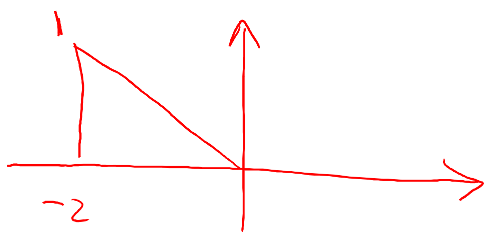


$2 = 3t$

$\frac{2}{3} = t$

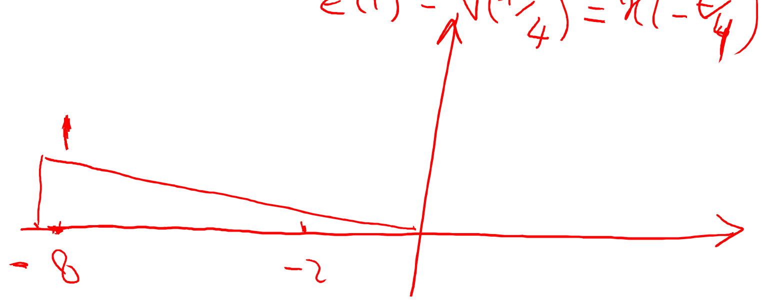


$v(t) = x(-t)$

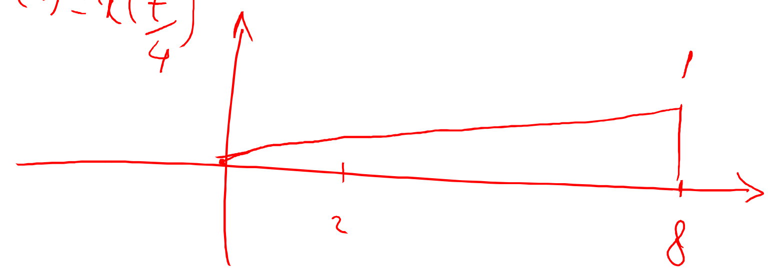


$\Downarrow$

$z(t) = v(t/4) = x(-t/4)$



$w(t) = x\left(\frac{t}{4}\right)$



$z(t) = w(-t) = x\left(-\frac{t}{4}\right)$

Combined Operations:

$$z(t) = Ax(\alpha t - T)$$

We first plot $y(t) = x(\alpha t - T)$ then plot $z(t) = Ay(t)$

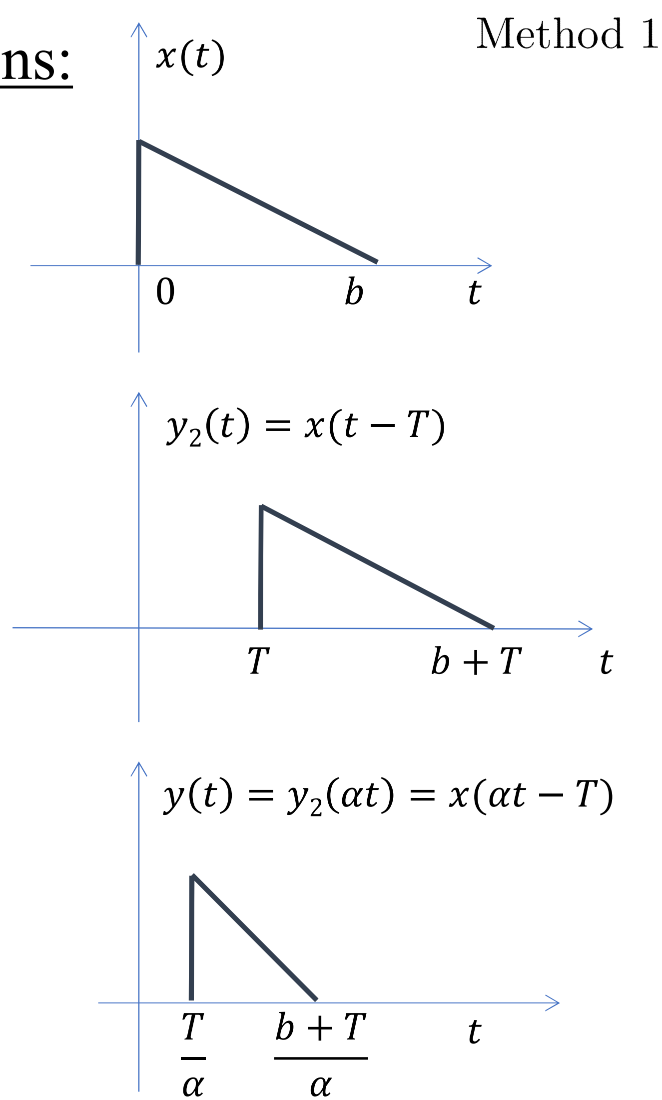
Two methods to plot $y(t)$

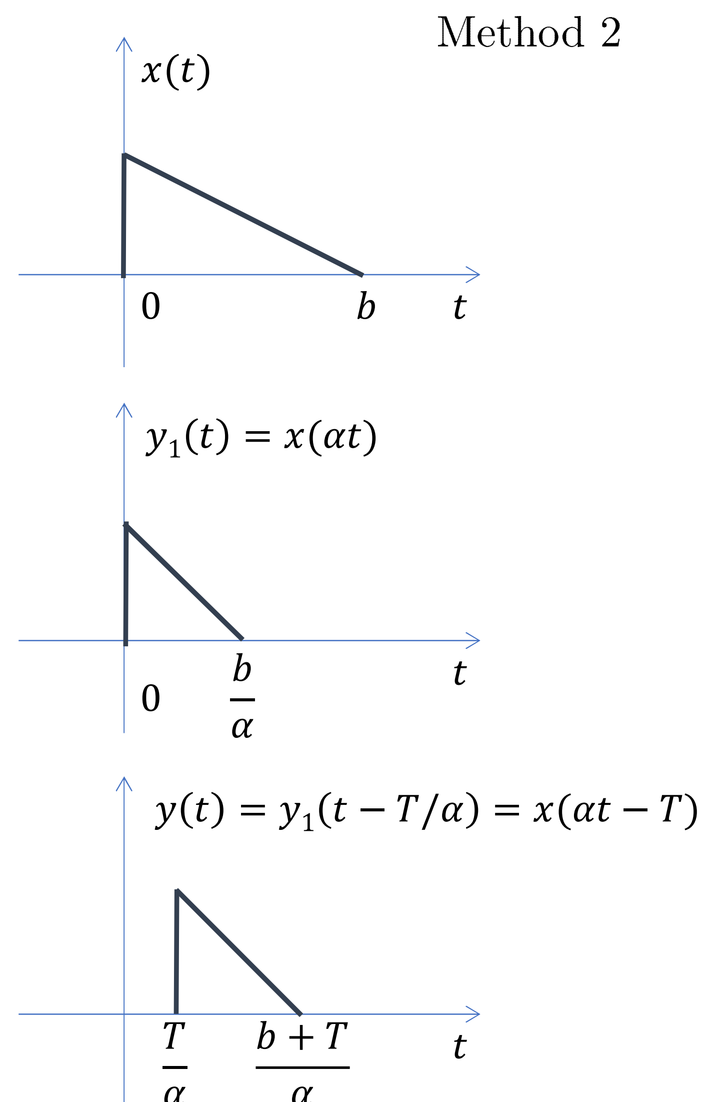
Method 1	Method 2
1- Shift by T	1- Time scale by α
$y_2(t) = x(t - T)$	$y_1(t) = x(\alpha t)$
2- Time scale by α	2- Shift by T/α
$y(t) = y_2(\alpha t) = x(\alpha t - T)$	$y(t) = y_1(t - T/\alpha) = x\left(\alpha(t - T/\alpha)\right) = x(\alpha t - T)$

Combined Operations:

$$y(t) = x(\alpha t - T)$$

Example $\alpha > 1$, $T > 0$

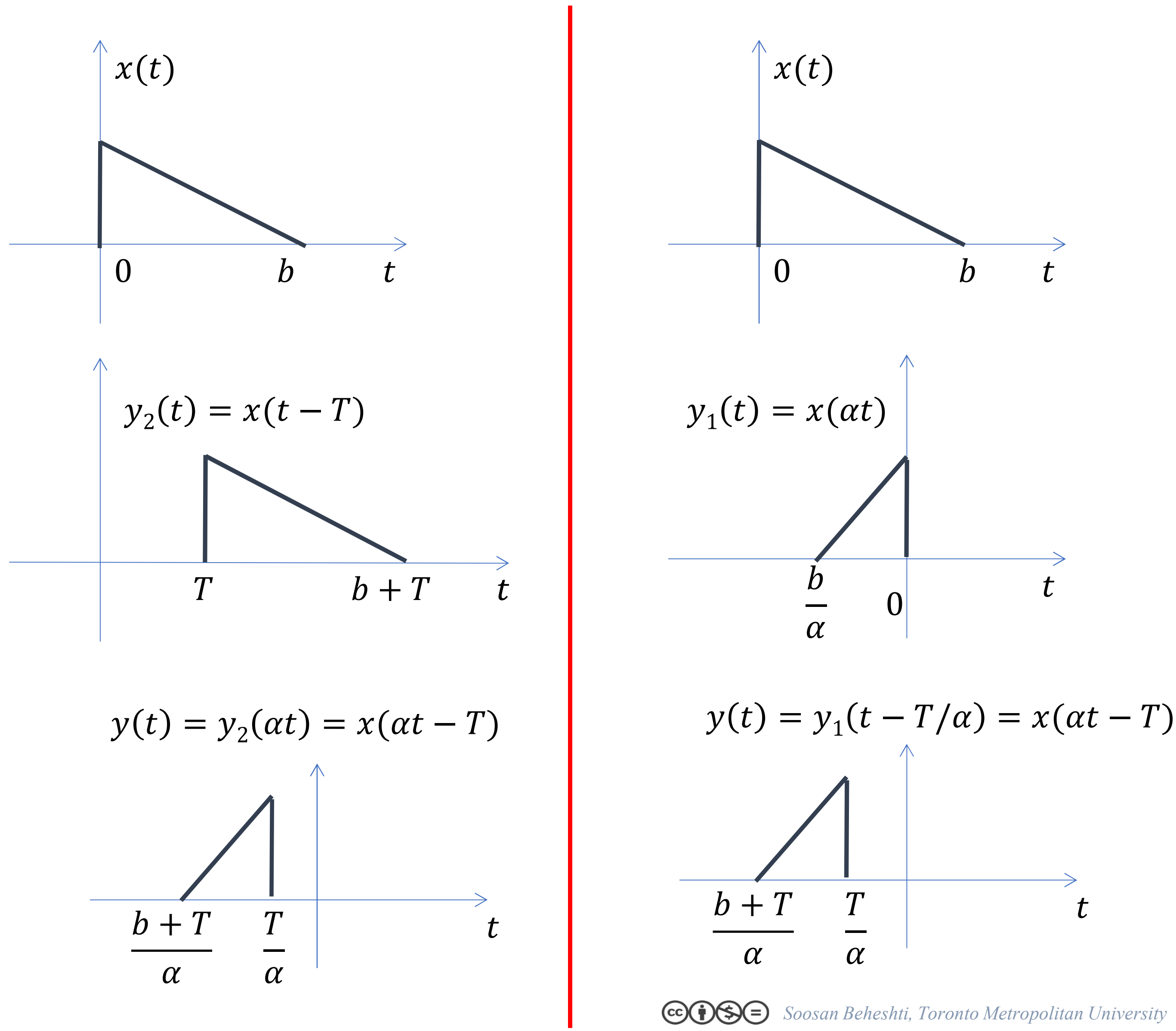




Combined Operations:

$y(t) = x(\alpha t - T)$

Example $\alpha < -1$**,** $T > 0$

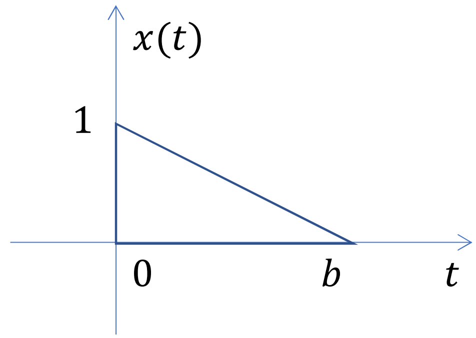


Combined Operations:

Easy steps for combined operations

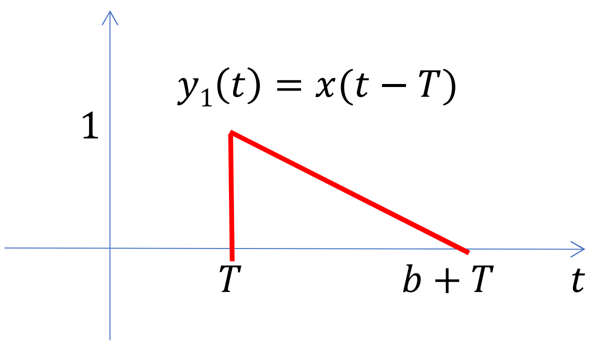
Given $x(t)$ plot $z(t) = Ax(\alpha t - T)$
1- Shift by T
2- Time scale by $\lvert\alpha\rvert$
3- If α is positive go to step 4. If α is negative, flip the signal (time reverse)
4- Scale the signal by A

Combined Operations:

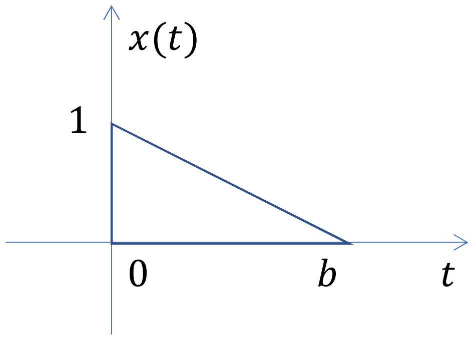


Find $y(t) = Ax(\alpha t - T)$

Step One

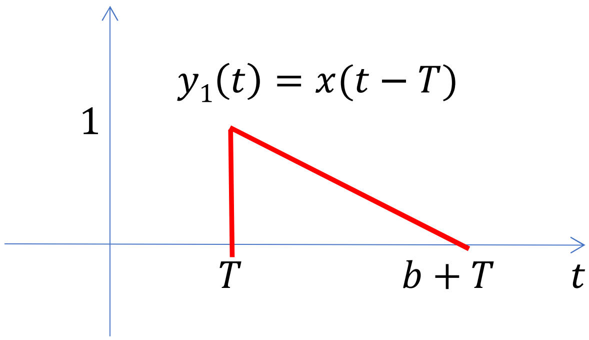


Combined Operations:

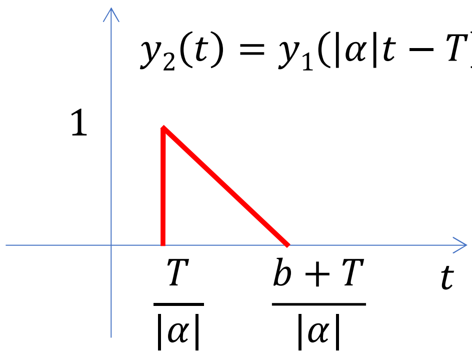


Find $y(t) = Ax(\alpha t - T)$

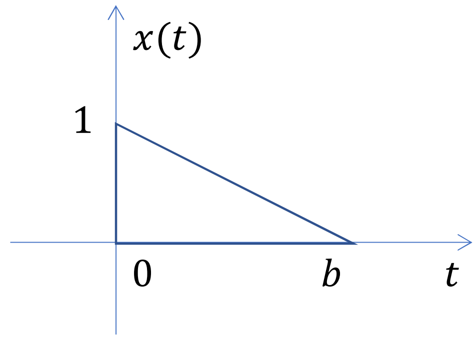
Step One



Step Two



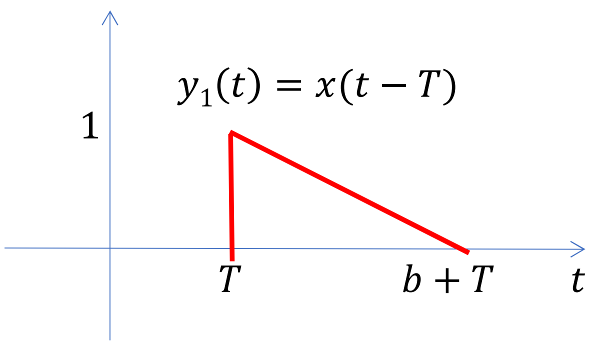
Combined Operations:



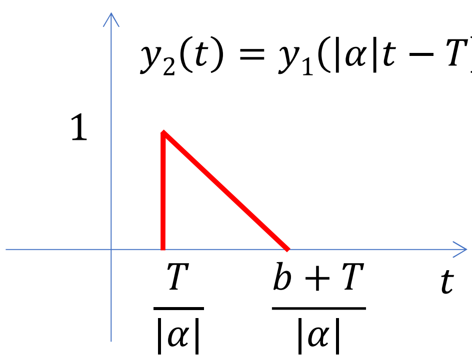
Find $y(t) = Ax(\alpha t - T)$

$\alpha < 0$

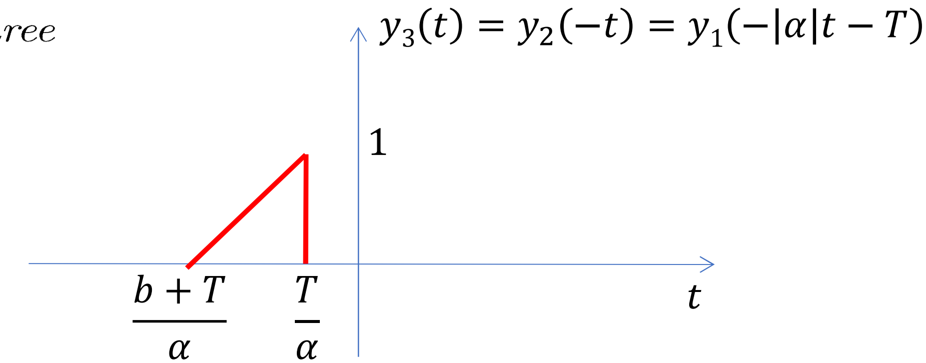
Step One



Step Two



Step Three



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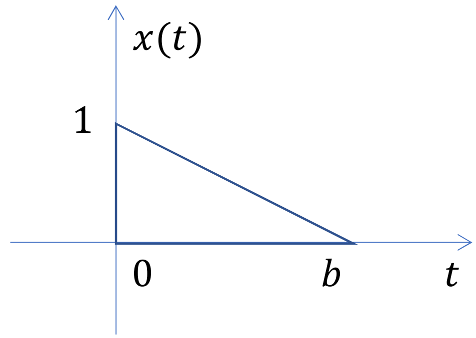

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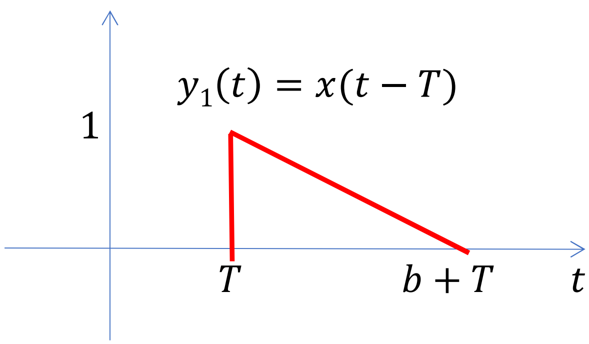
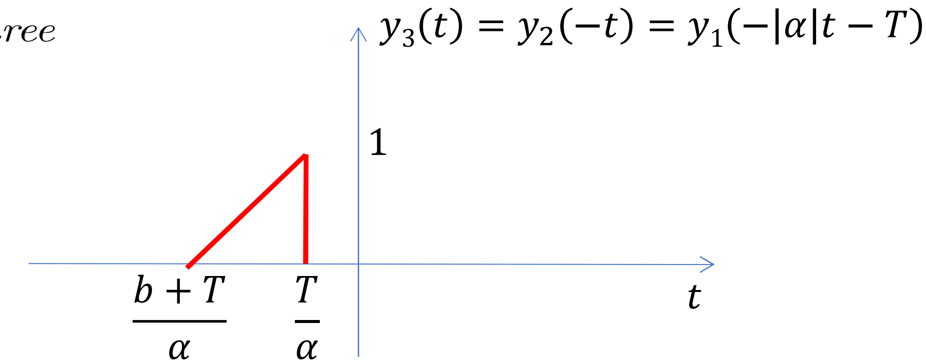
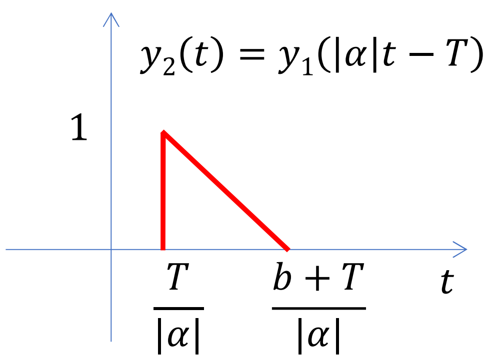


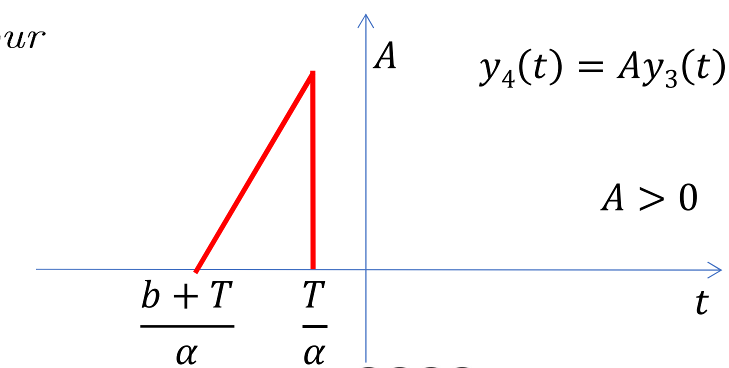
Step Three



Step Two

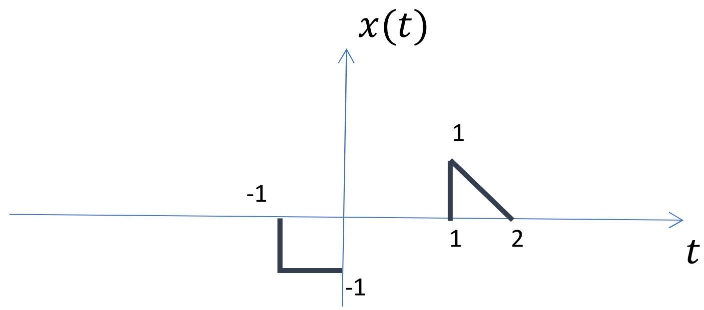


Step Four



Combined Operations:

Example: $Plot\ x(3t),\ x(t+2),\ -4x(3t+2), and\ x(\frac{-t}{2}-3)$



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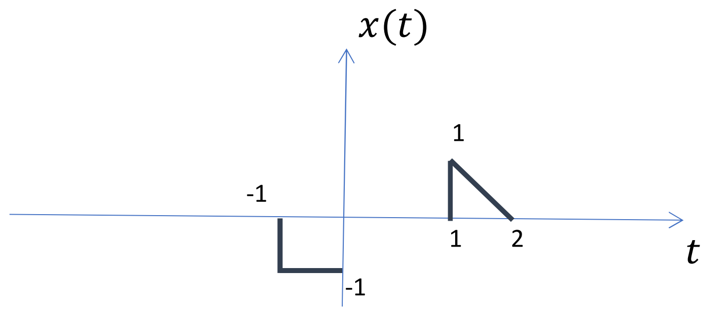
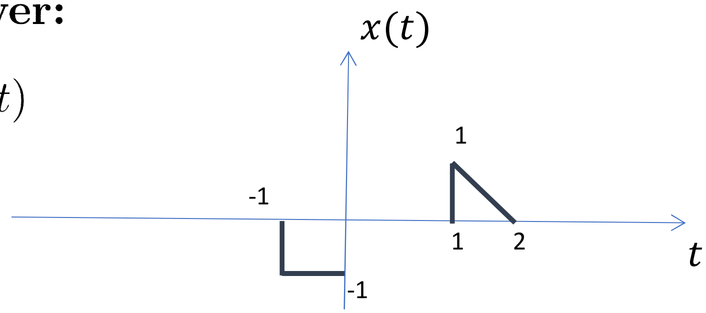
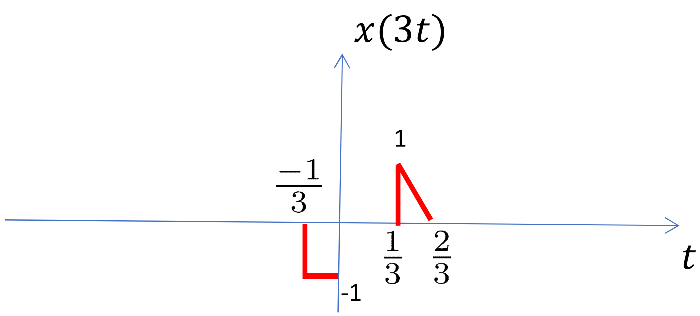


Answer:

- $x(3t)$





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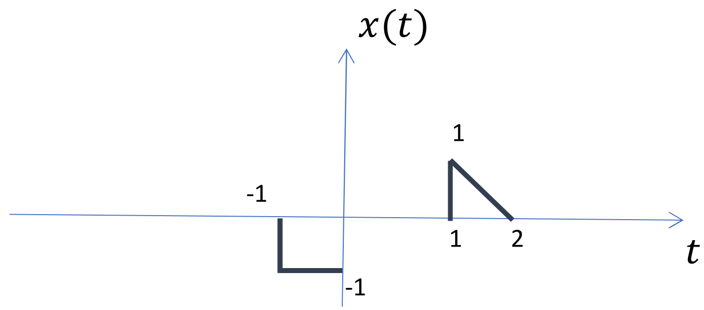

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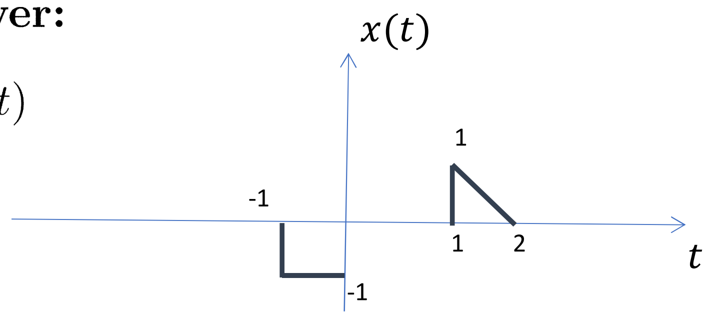
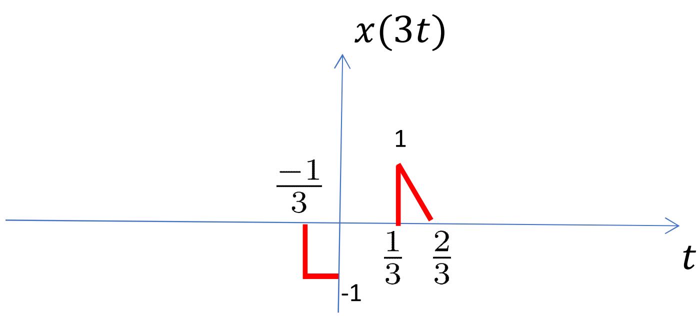
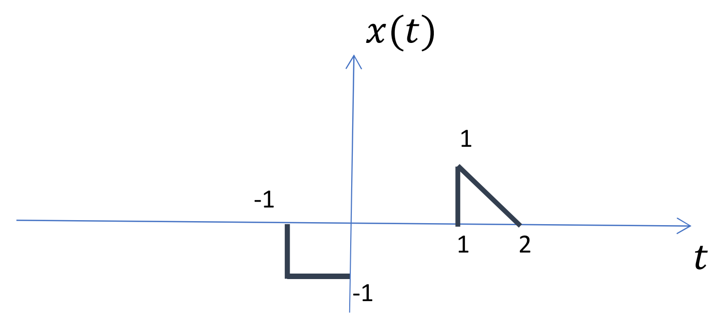
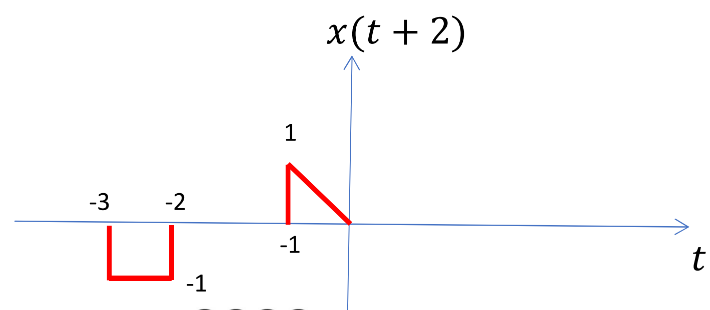




- $x(t+2)$





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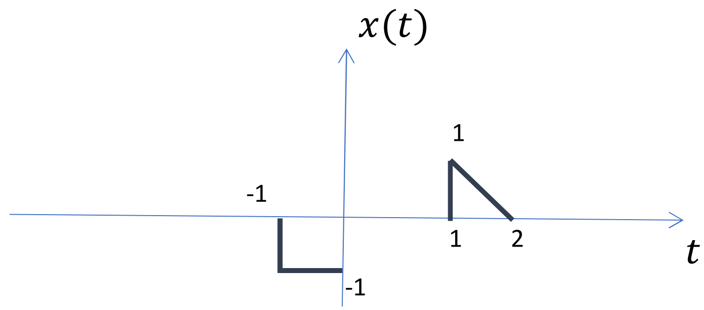

Answer:

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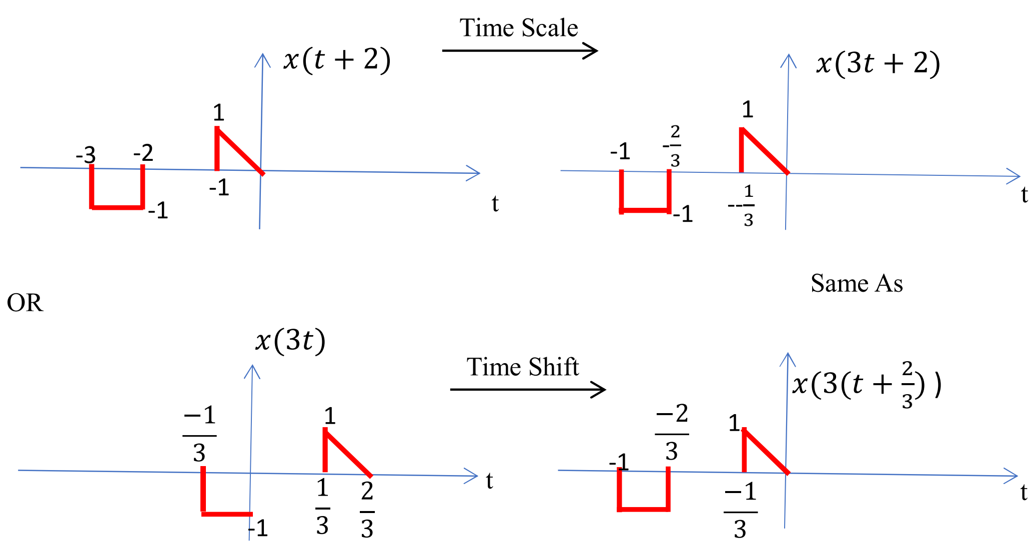

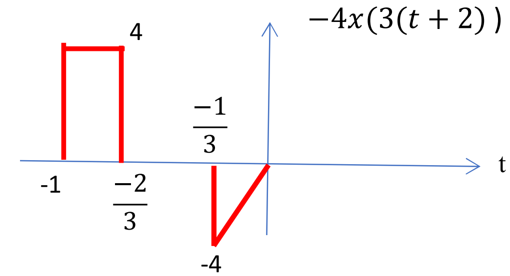

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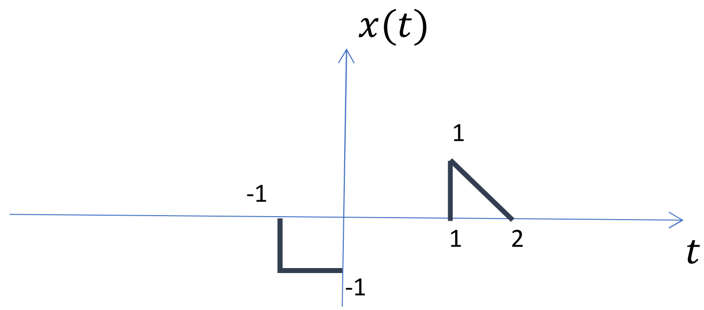

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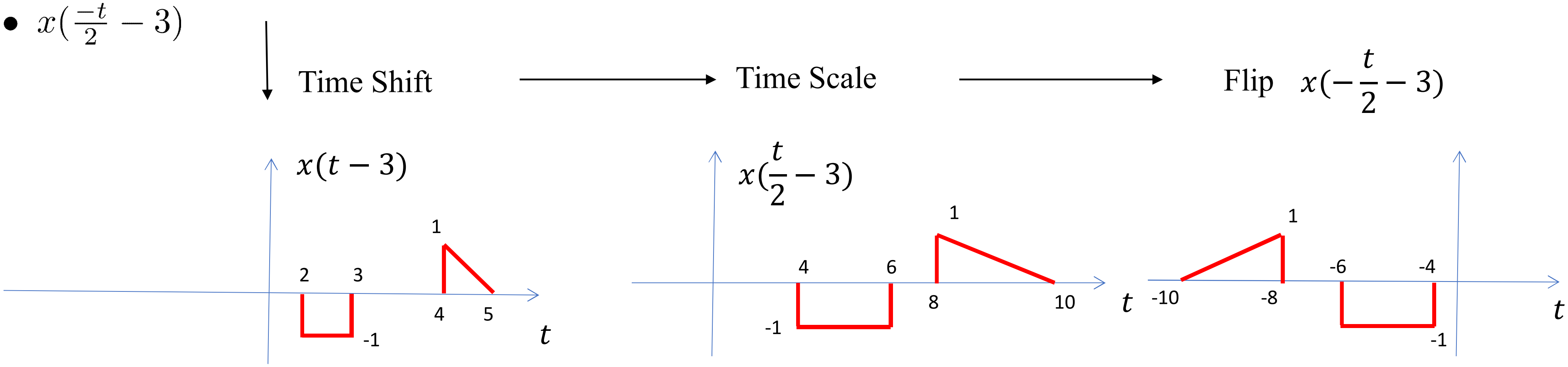
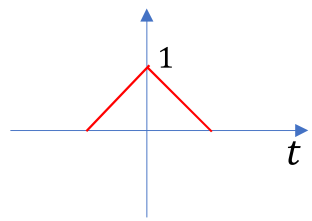
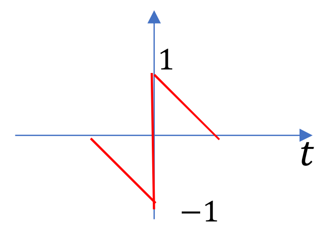
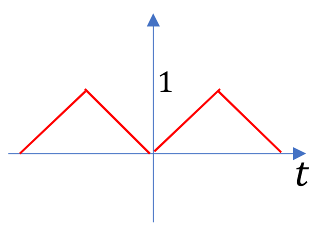


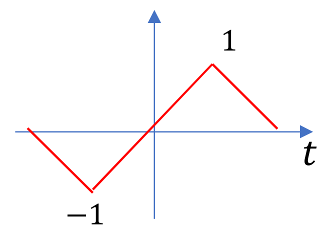
Odd and Even Functions (Signals):

Even functions	odd functions
$x_e(-t) = x_e(t)$ $\int_{-\infty}^{\infty} x_e(t)dt = 2\int_{0}^{\infty} x_e(t)dt$	$x_o(-t) = -x_o(t)$ $\int_{-\infty}^{\infty} x_o(t)dt = 0$
Examples	
$\cos(t)$	$\sin(t)$









Odd and Even Functions (Signals):

In general signals can be neither odd nor even. However, all signals can be represented as sum of their even & odd components!

For any signal $x(t)$ we have:

$$x(t) = x_e(t) + x_o(t) \tag{1}$$

How to find these components (?)

$$x_e(t) = \frac{x(t) + x(-t)}{2}, \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$

To prove the above claim we need to show the following facts:

1) $x_e(t) = x_e(-t)$ 2) $x_o(t) = -x_o(-t)$ 3) $x(t) = x_e(t) + x_o(t)$

showing 3):

$x_e(t) + x_o(t) = \frac{x(t)+x(-t)}{2} + \frac{x(t)-x(-t)}{2} = 2\frac{x(t)}{2} = x(t)$

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How about 1) and 2)?

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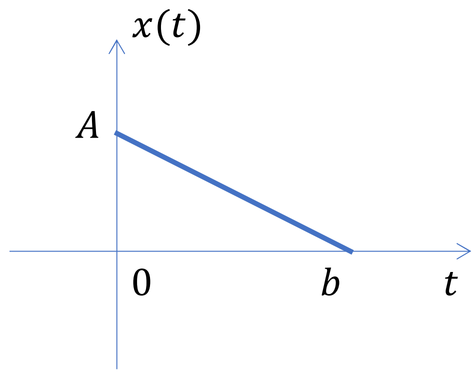
Odd and Even Signals:

Example: *Find odd & even parts of the following signal.*



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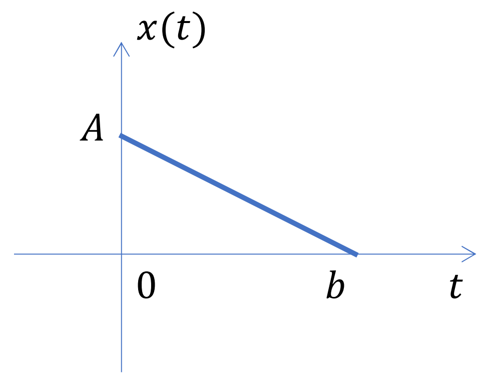

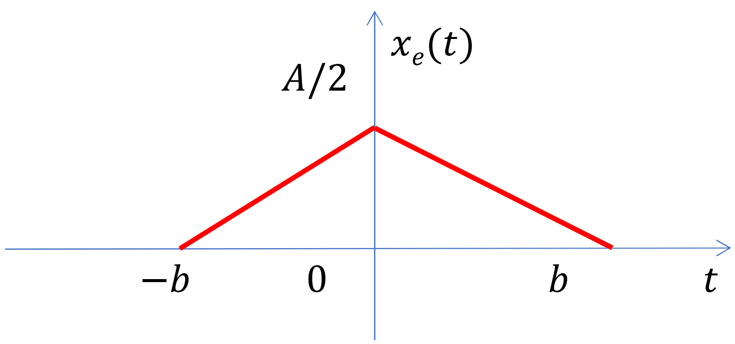

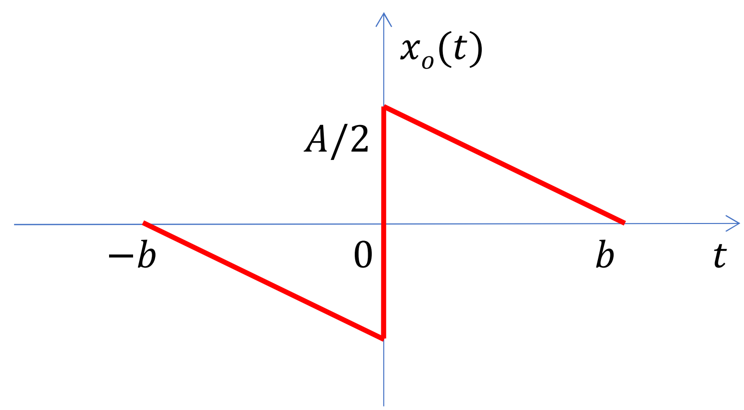

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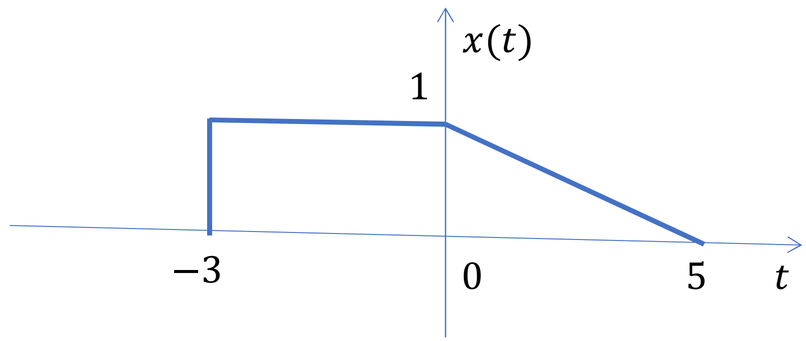
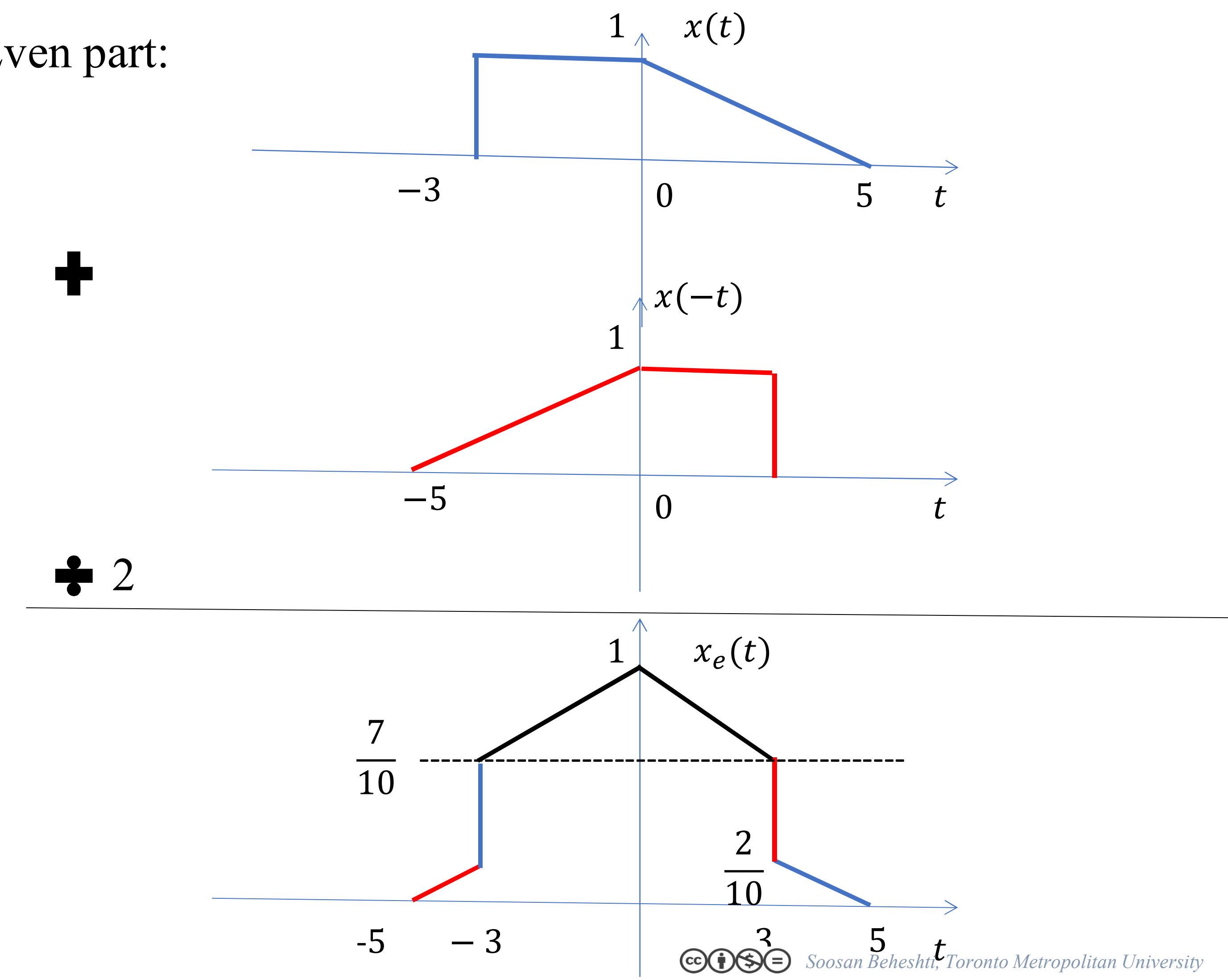


Odd and Even Signals:

Even part:



Odd and Even Signals:

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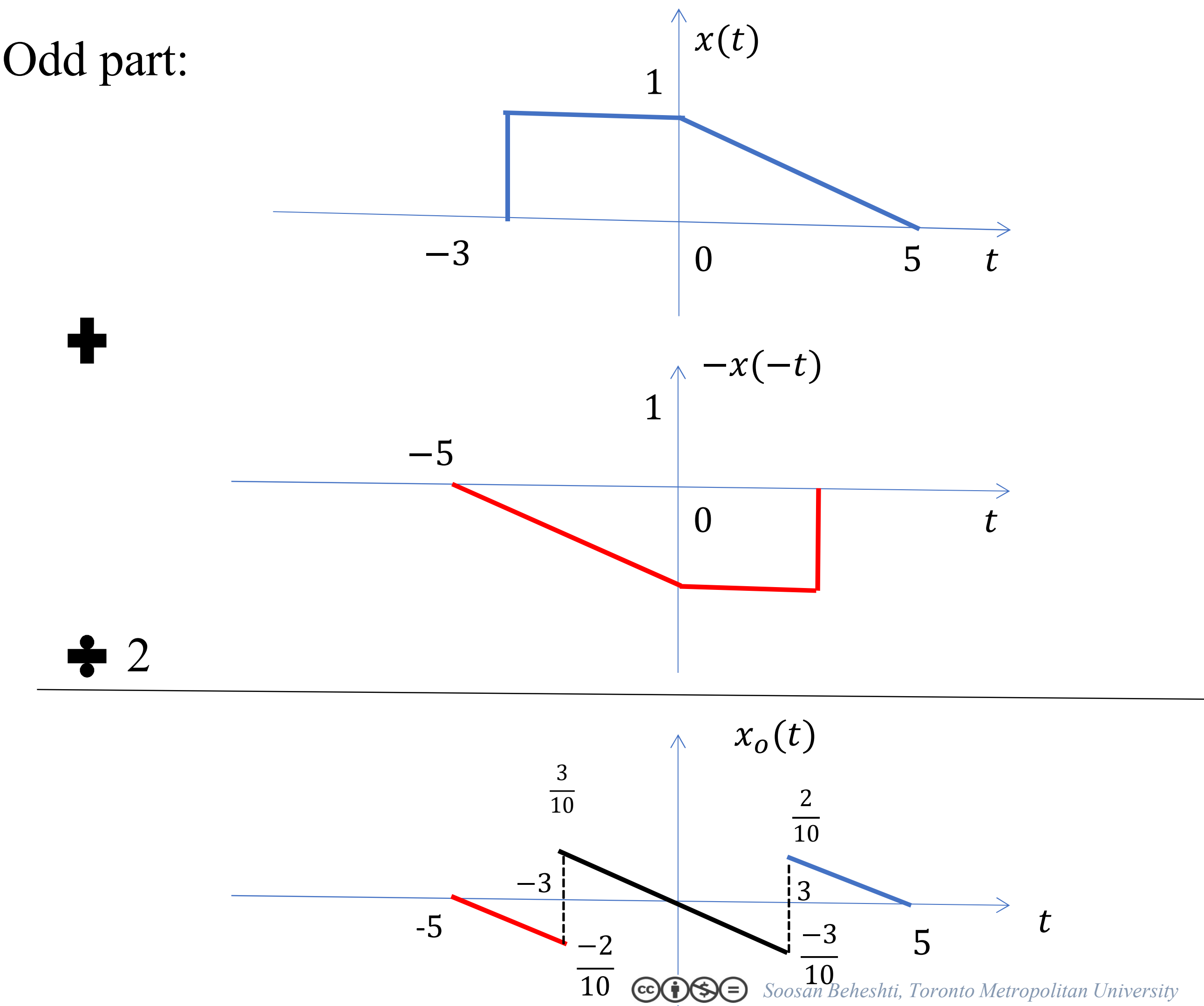

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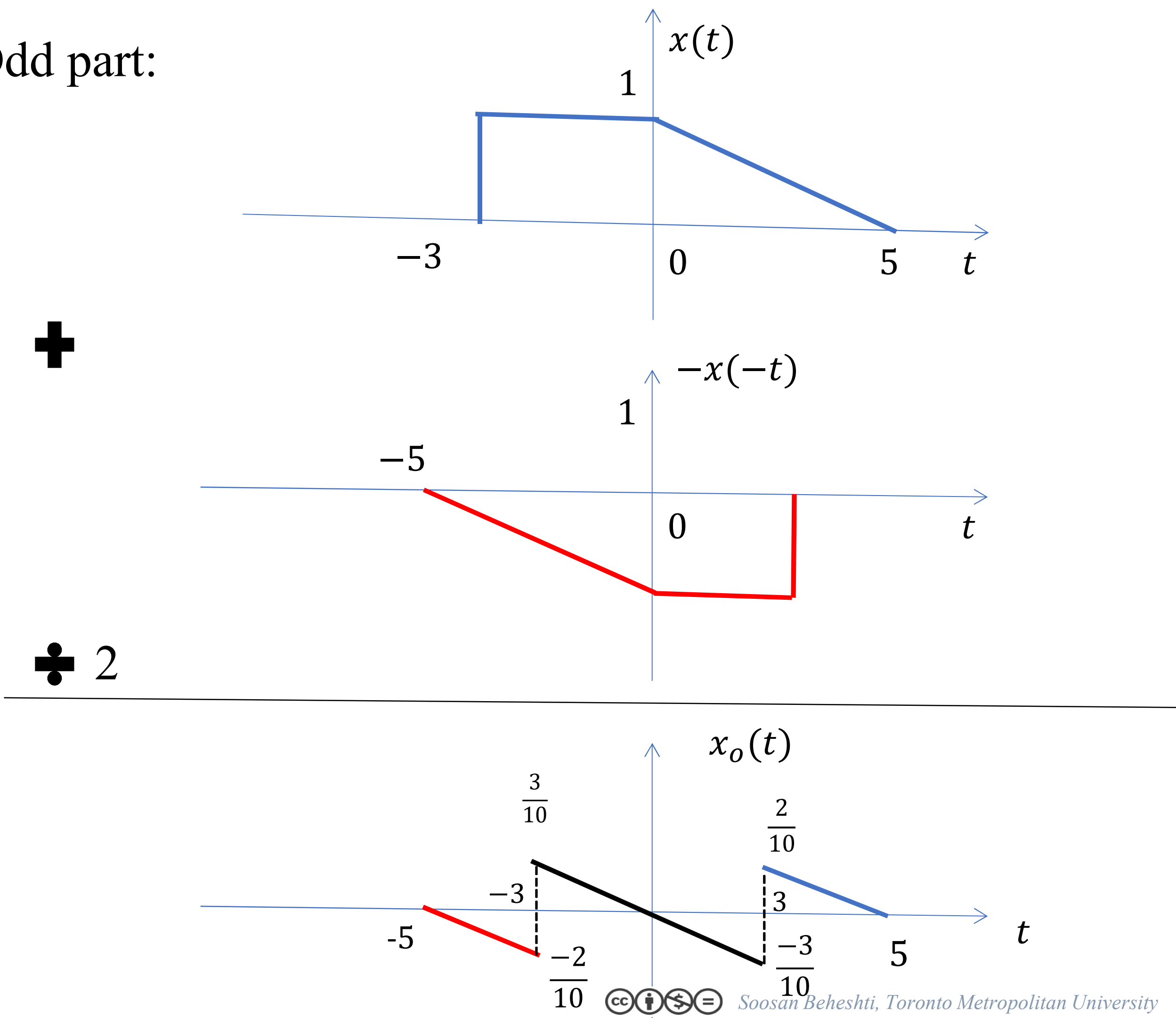

Try adding $x_e(t)$ and $x_o(t)$ to generate $x(t)$ itself!