Signals and Systems I

Lecture 4

Last Lecture

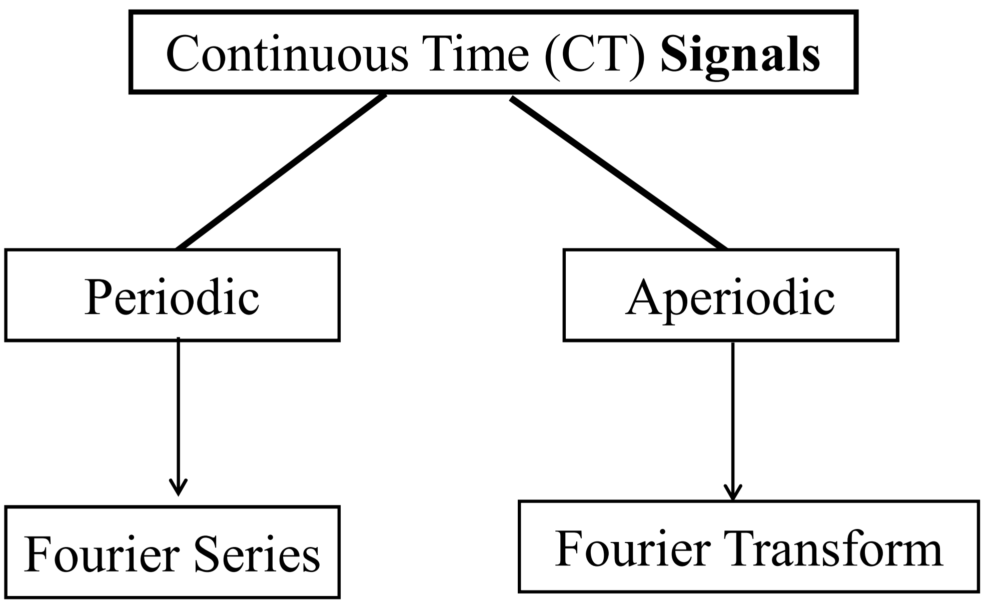
- Odd and even signals
- more on $u(t)$ & $\delta(t)$
- Building Signals from other Signals
- Closed form expression for signals

"Closed form expression" is a mathematical expression that can be evaluated in a finite number of operations.

Today

- System Classification:
    - Continuous Time/Discrete Time
    - Digital /Analog
    - Linear / Nonlinear
    - Time Invariant / Time Varying
    - Causal / Non-Causal
    - Memoryless/ With Memory
    - Invertible / Noninvertible

Course Subjects:



<u>In Time Domain</u>

1. Signal Classification
2. Signal Operation
3. Important Signals

Chap 1

1

<u>In Frequency Domain</u>

1. Fourier Series
2. Fourier Transform

Chap 6

3

Chap 7

4

Course Subjects:

$\boxed{\text{Continuous Time (CT) } \textbf{Systems}}$

<u>In Time Domain</u>

1. Properties
2. Linear Time-Invariant (LTI) Systems
3. Impulse Response
4. Convolution

Chap 1&2

2

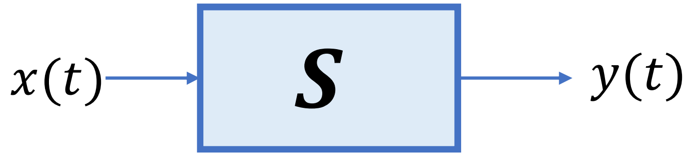
<u>In Laplace and Frequency Domain</u>

Laplace Transform & Frequency Analysis

Chap 4

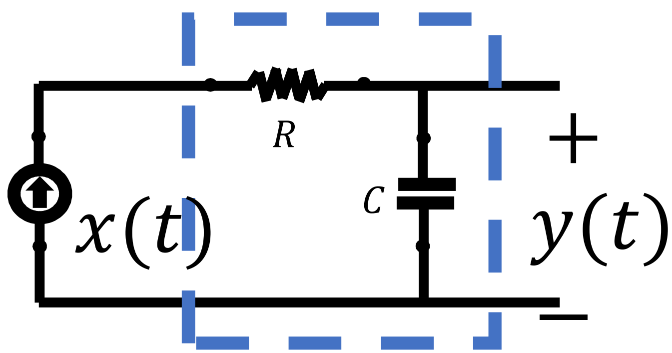
5

Systems: processes signals, operates on signals and generates signals



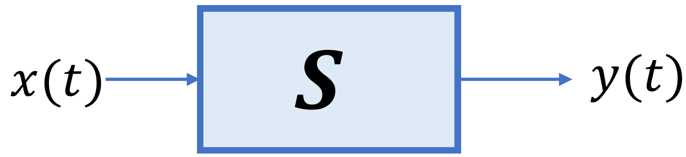
$$y(t) = \boldsymbol{S}(x(t))$$

$Example$: $y(t) = 3x(t-7)$ or RC circuit:



$y(t) = \frac{1}{C}\int_{t_0}^{t} x(t)dt + V_C(t_0)$ where $V_C(t_0)$ is the initial condition and t_0 is the start time

Systems: processes signals, operates on signals and generates signals



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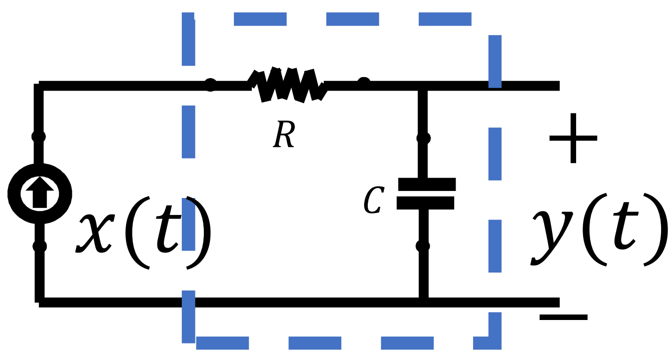


There are two types of systems around us, the existing ones in nature and the manmade ones. We study existing systems for the purpose of understanding and <u>analyzing</u> them and also to <u>design</u> new systems.

$y(t) = \frac{1}{C}\int_{t_0}^{t} x(t)dt + V_C(t_0)$ where $V_C(t_0)$ is the initial condition and t_0 is the start time

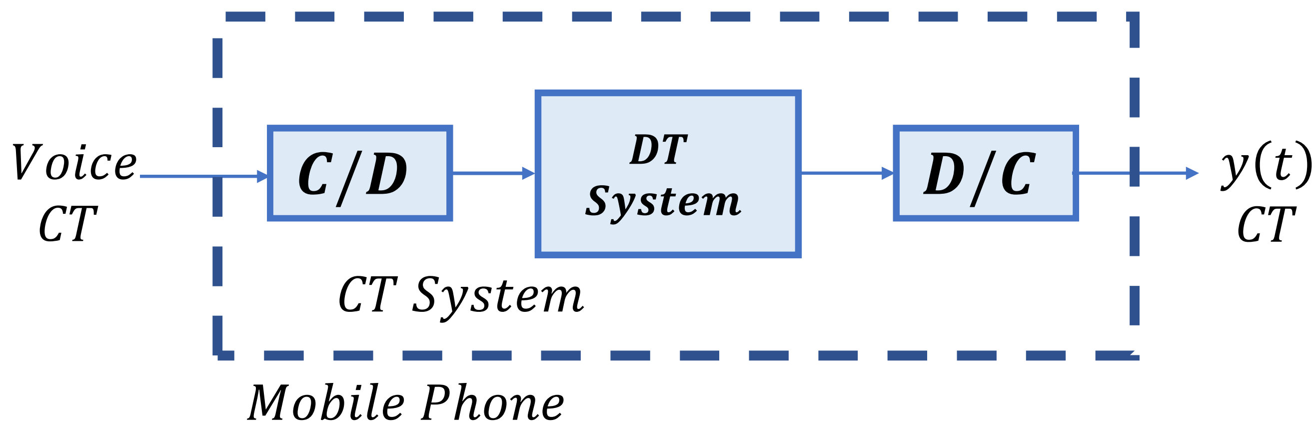
Systems Classification

Continuous Time (CT) or Discrete Time (DT)

- If input and output of a system are CT $\longrightarrow$ the system is CT
- If input and output of a system are DT $\longrightarrow$ the system is DT

Examples

CT system



Systems Classification

Analog or Digital Systems

- If input and output of a system are Analog $\longrightarrow$ the system is Analog

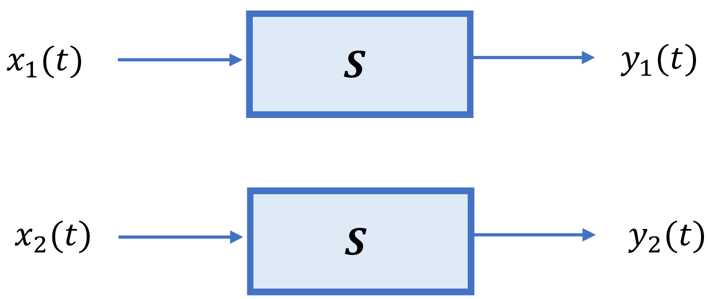
- If input and output of a system are Digital $\longrightarrow$ the system is Digital

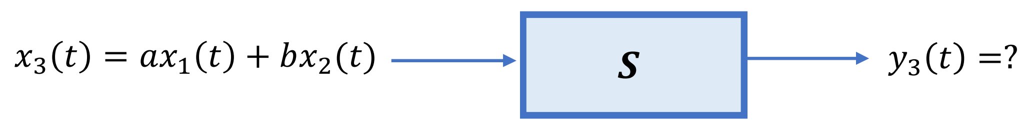
Analog example: Radio, Digital example: computers

Systems Classification

Linear or Non-Linear Systems

The system is called *Linear* if outputs of linear combinations of inputs is the same linear combination of linear combination of their outputs.





If the system is linear, then $y_3(t) = ay_1(t) + by_2(t)$

Systems Classification

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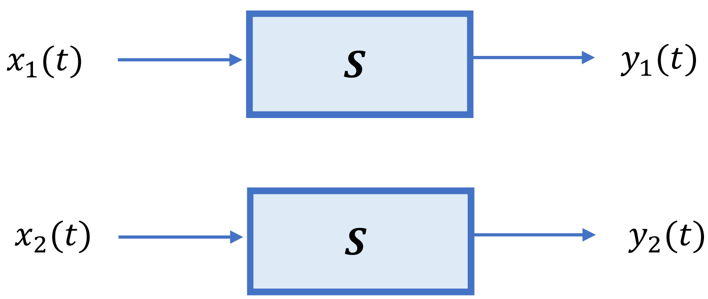
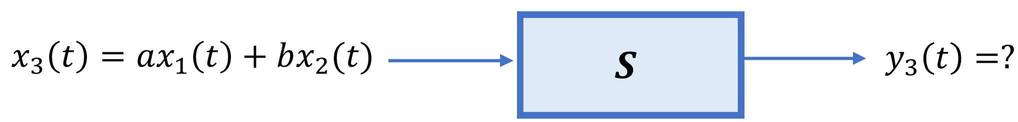




If the system is linear, then $y_3(t) = ay_1(t) + by_2(t)$

If a system is linear, then output of the system to $x(t) = 0$ is always $y(t) = 0$. why?

Can linear systems have initial conditions that generate nonzero output?

Linear/ Nonlinear Systems

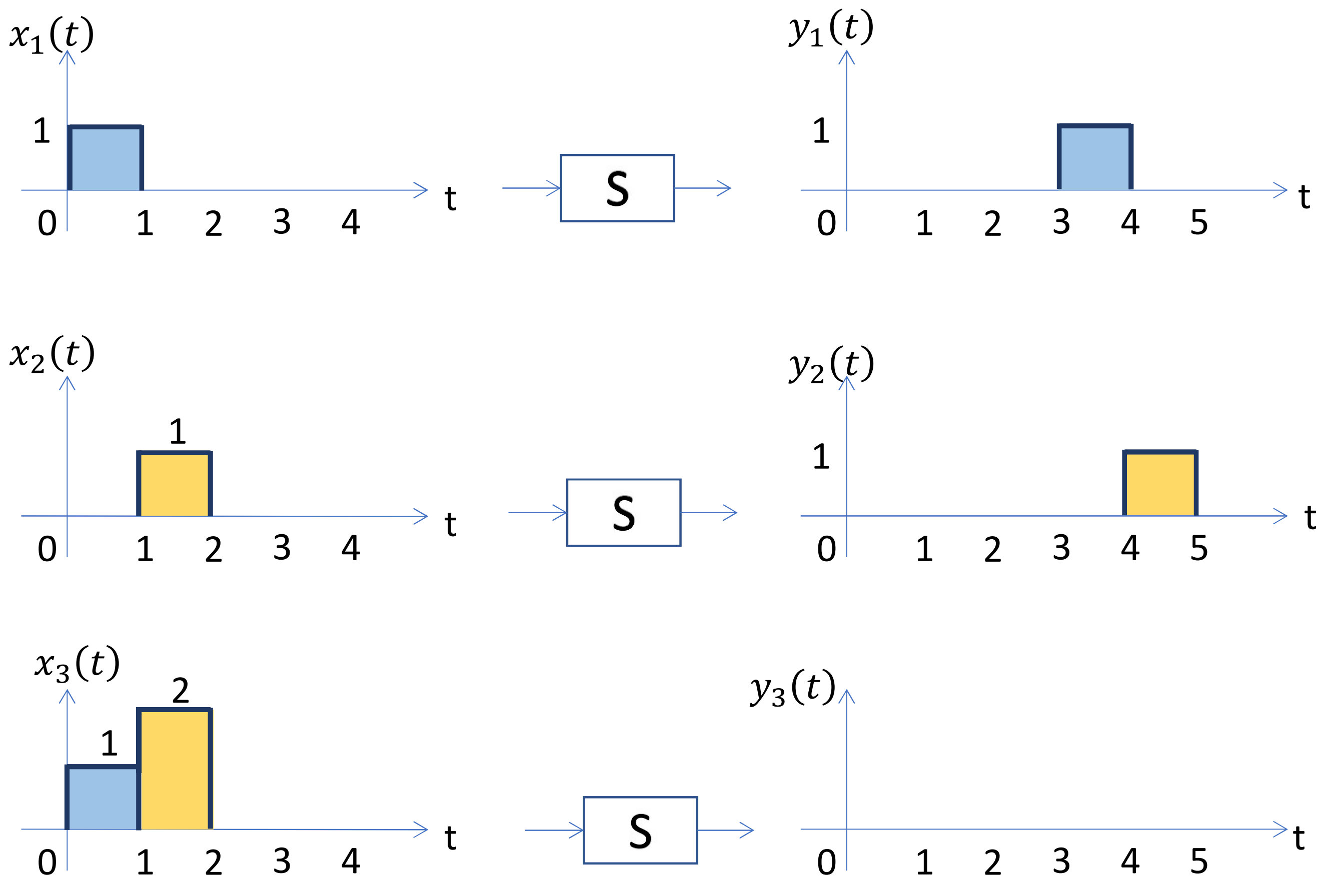
Example:

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t) = x(t-3)$$

$$x_3(t) = x_1(t) + 2x_2(t)$$

linear combination of $x_1(t)$ and $x_2(t)$.

what is $y_3(t)$?



Linear/ Nonlinear Systems

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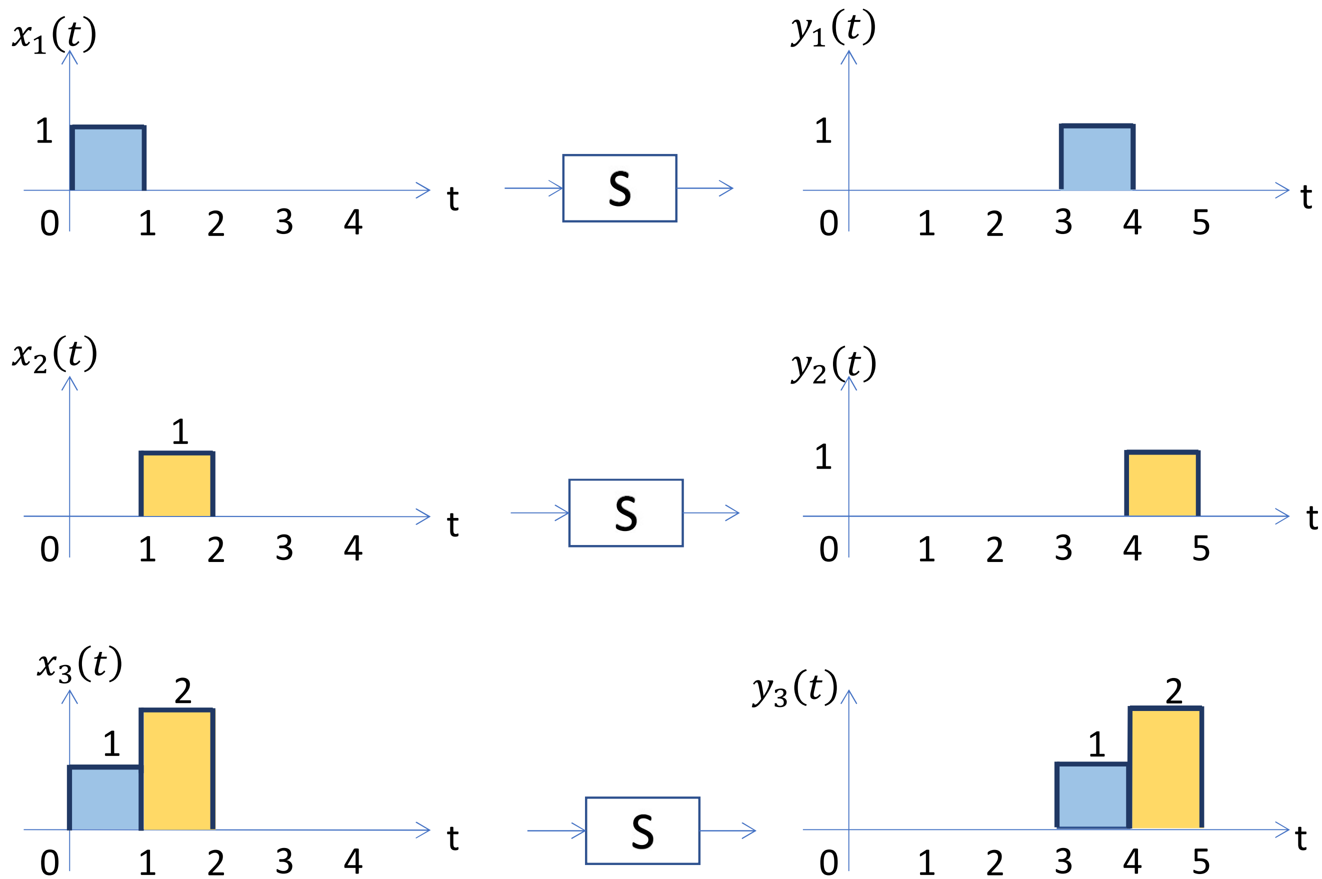
$y_3(t) = x_3(t-3)$



Linear/ Nonlinear Systems

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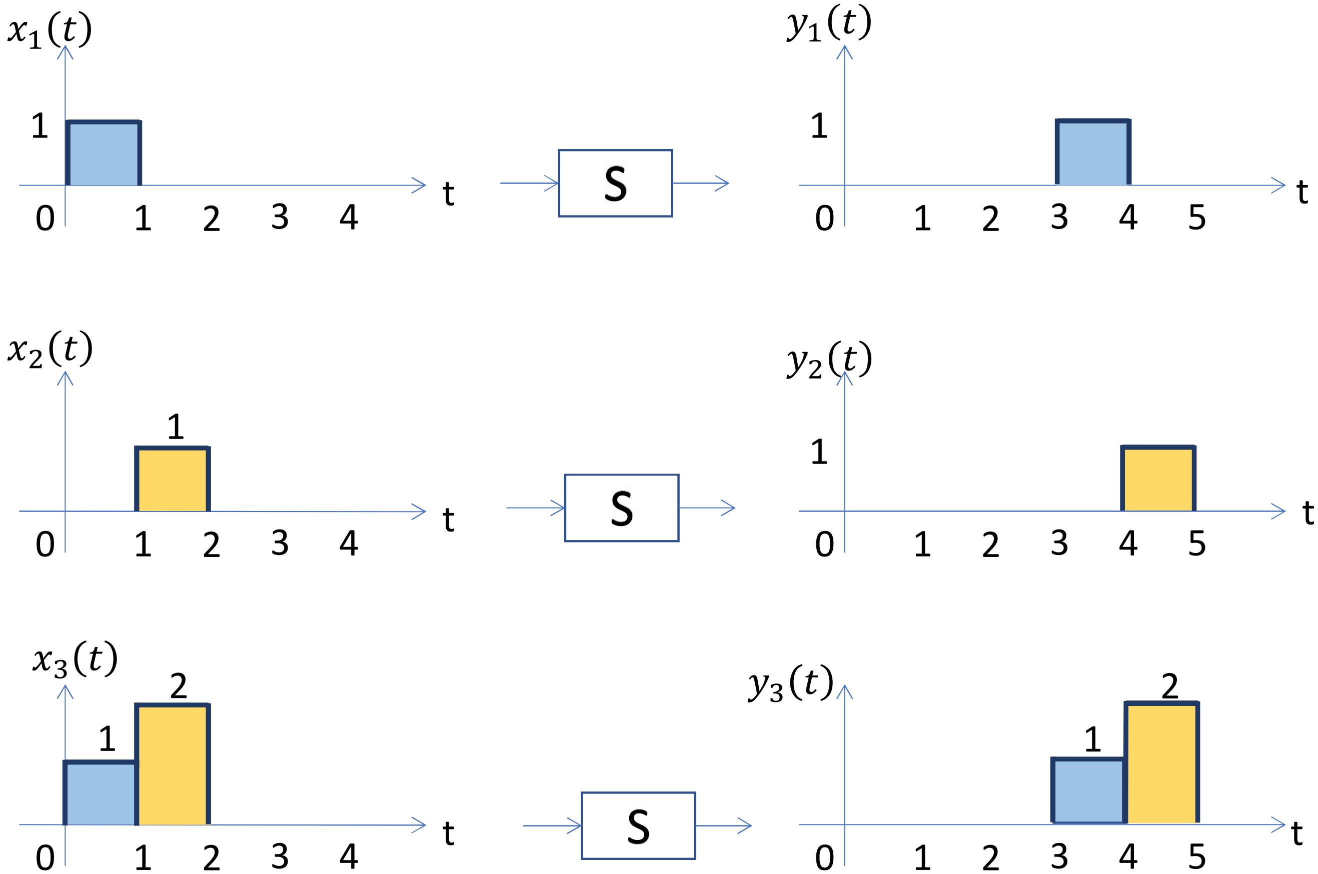
linear combination of $x_1(t)$ and $x_2(t)$.

what is $y_3(t)$?

$$y_3(t) = x_3(t-3)$$
$$= y_1(t) + 2y_2(t)$$

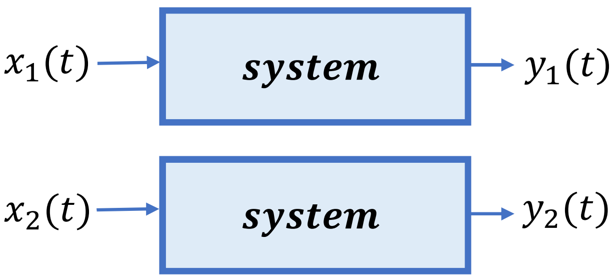


Can we conclude that the system is linear?

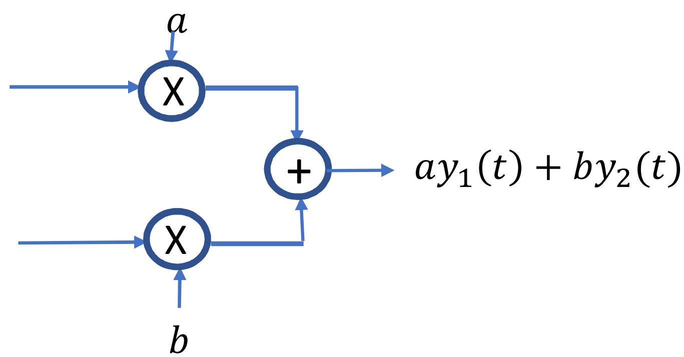
Linear/ Nonlinear Systems

Five steps for checking whether a system is linear or not

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$

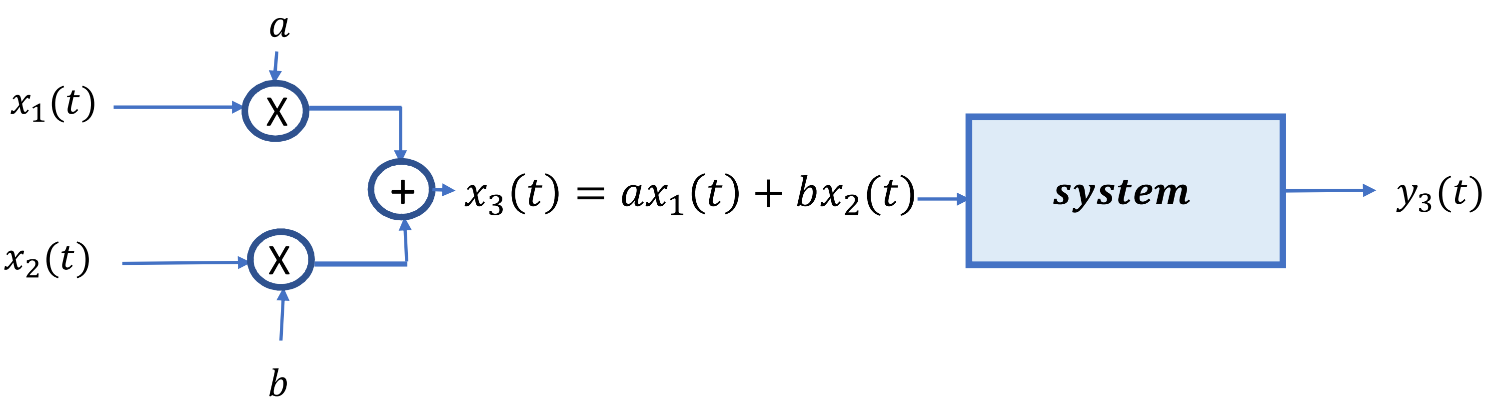


Step 5:
Check if result of step 2 is the same as result of step 4:

If the answer is yes, the system is **linear**

Step 3: Build $x_3(t)$

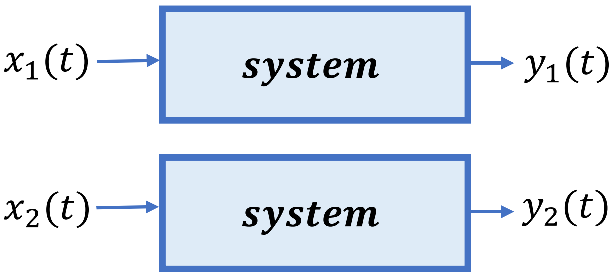
Step 4: Find the out put of the system to $x_3(t)$



Linear/ Nonlinear Systems

Check the steps for $y(t) = x(t-3)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



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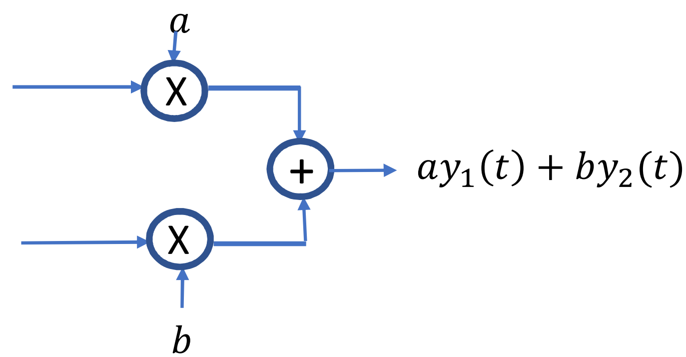

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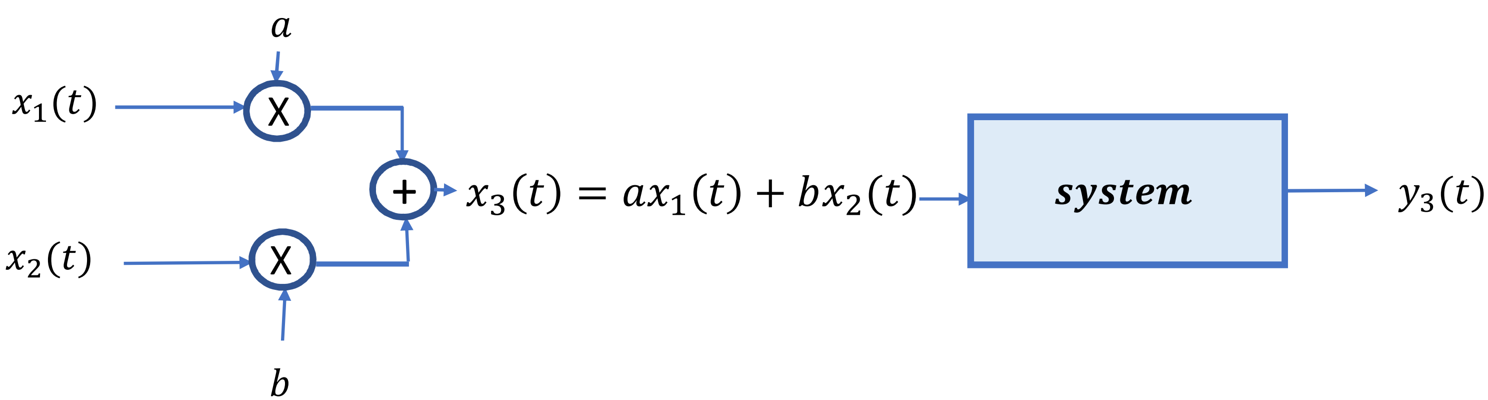

Linear/ Nonlinear Systems

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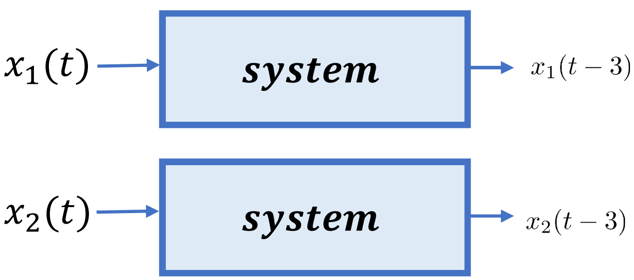

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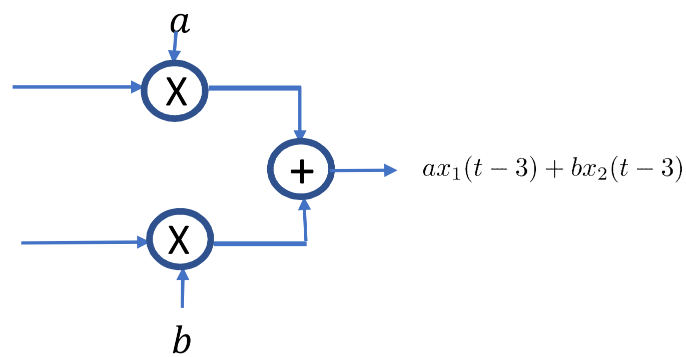

Step 5:
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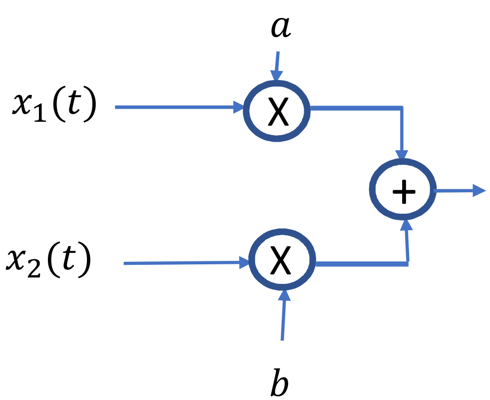
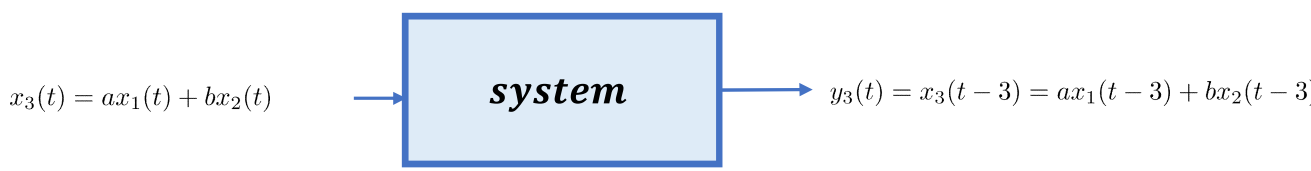
The two are the same so the system is **linear**

Step 3: Build $x_3(t)$



Step 4: Find the out put of the system to $x_3(t)$

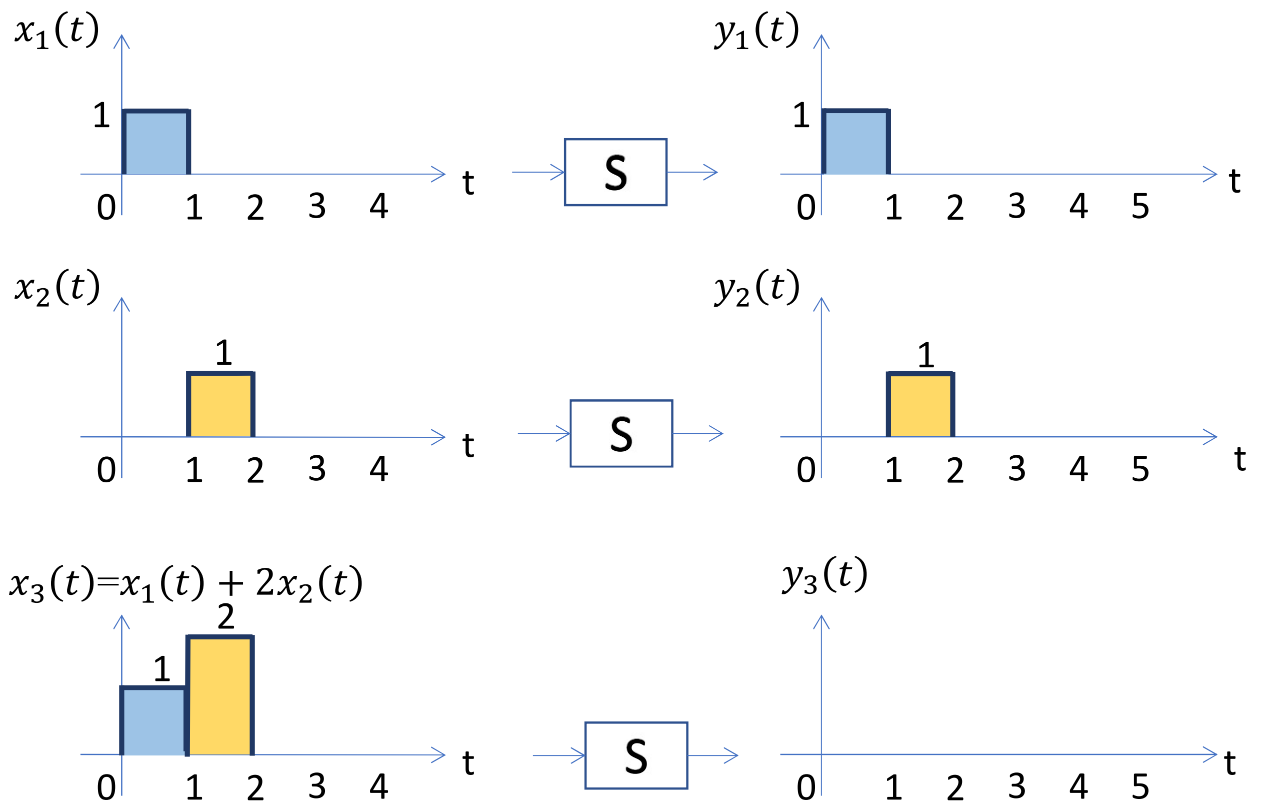


Linear/ Nonlinear Systems

Example 2: $y(t) = x^2(t)$

$x_3(t) = x_1(t) + 2x_2(t)$
linear combination of $x_1(t)$ and $x_2(t)$.

what is $y_3(t)$?



Linear/ Nonlinear Systems

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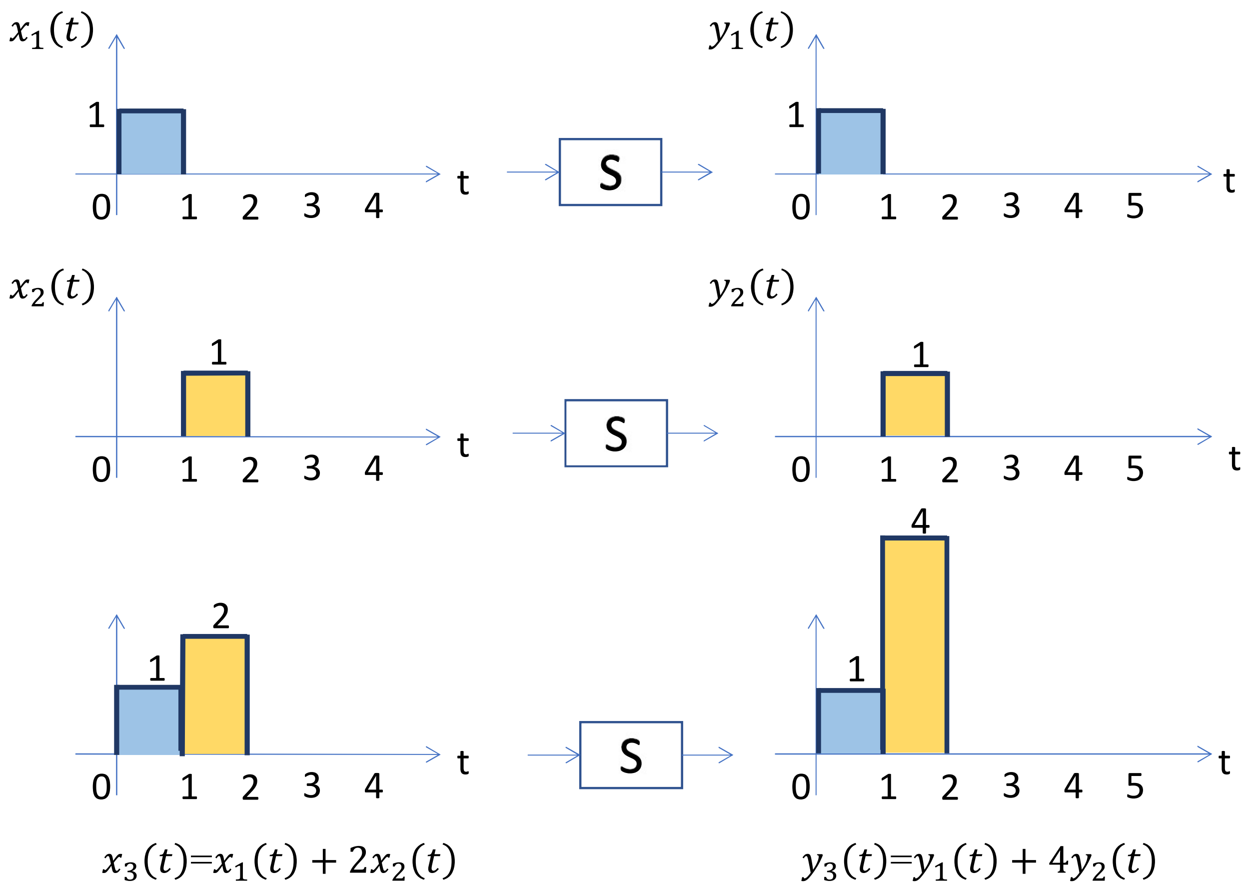
$y_3(t) = x_3^2(t)$

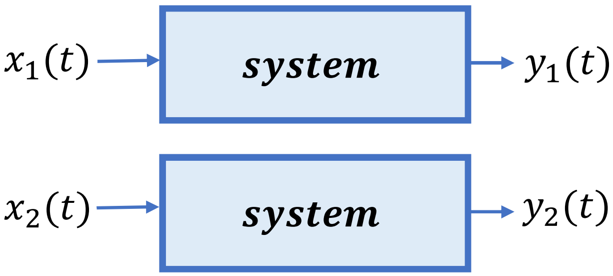
$= y_1^2 + 4y_2(t)$



Linear/ Nonlinear Systems

Check the steps for $y(t) = x^2(t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



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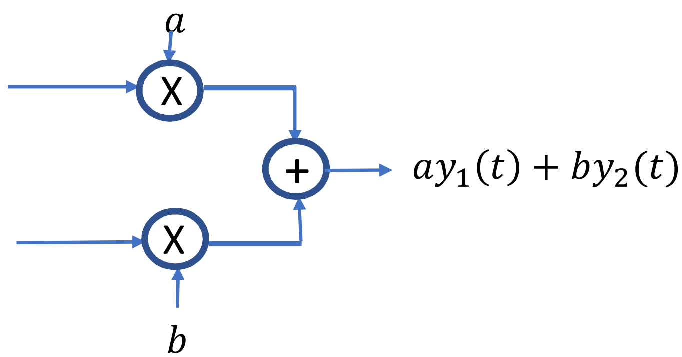

Step 5:
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If yes, the system is **Linear**

Step 3: Build $x_3(t)$

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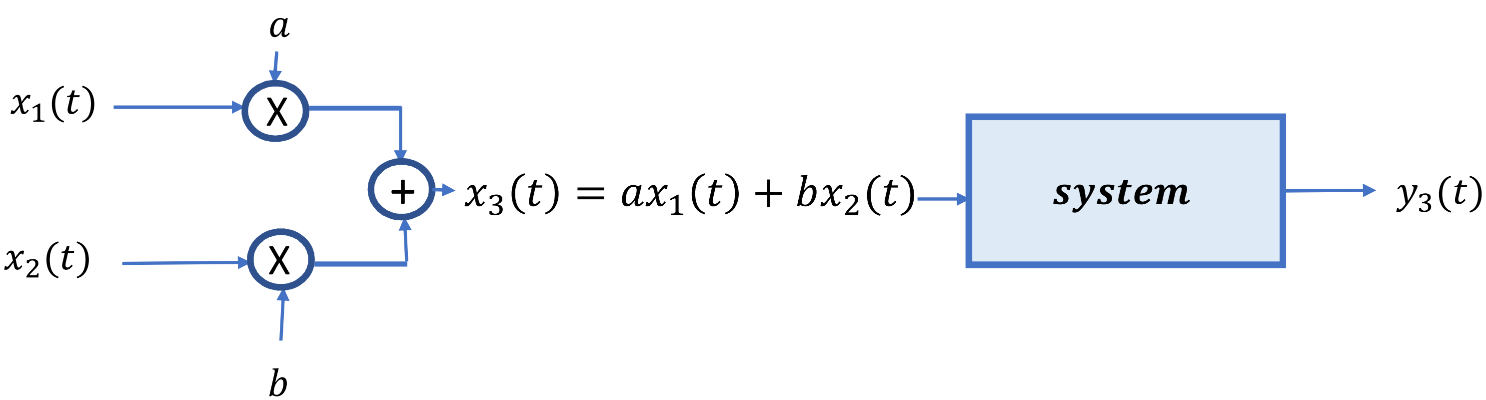

Linear/ Nonlinear Systems

Check the steps for $y(t) = x^2(t)$

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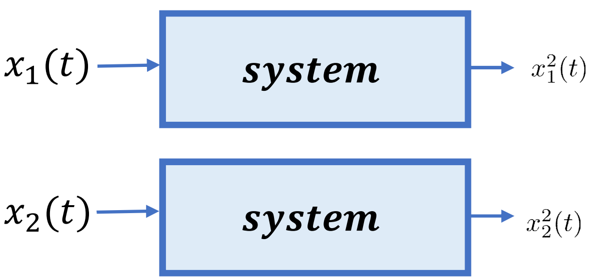

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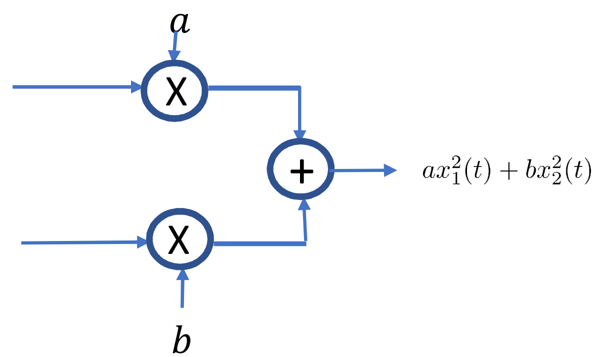

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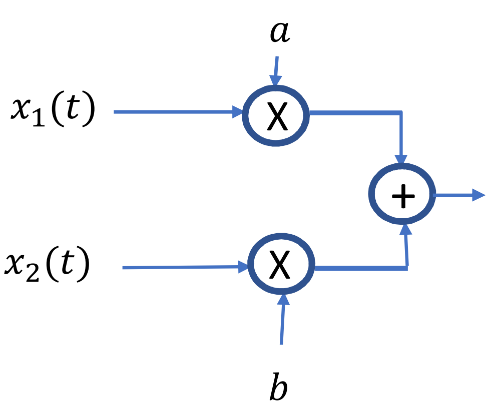
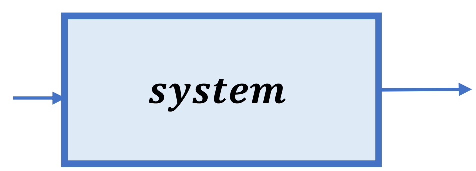
Step 3: Build $x_3(t)$



$$x_3(t) = ax_1(t) + bx_2(t)$$

Step 4: Find the out put of the system to $x_3(t)$

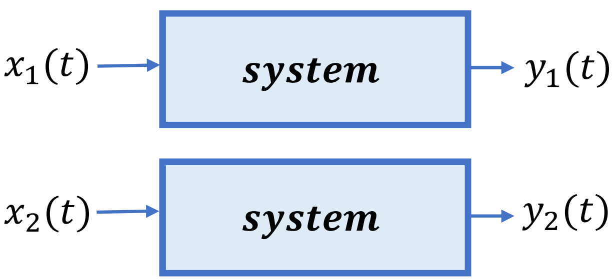


$$y_3(t) = x_3^2(t) = (ax_1(t) + bx_2(t))^2$$
$$= a^2x_1^2(t) + b^2x_2^2(t) + 2abx_1(t)x_2(t)$$

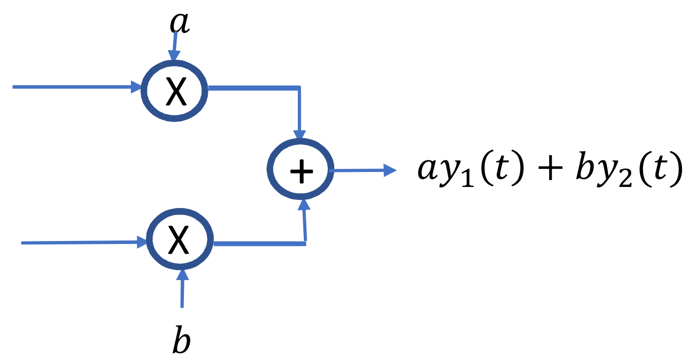
Linear/ Nonlinear Systems

Check the steps for $y(t) = x(t) - 3$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



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If yes, the system is **Linear**

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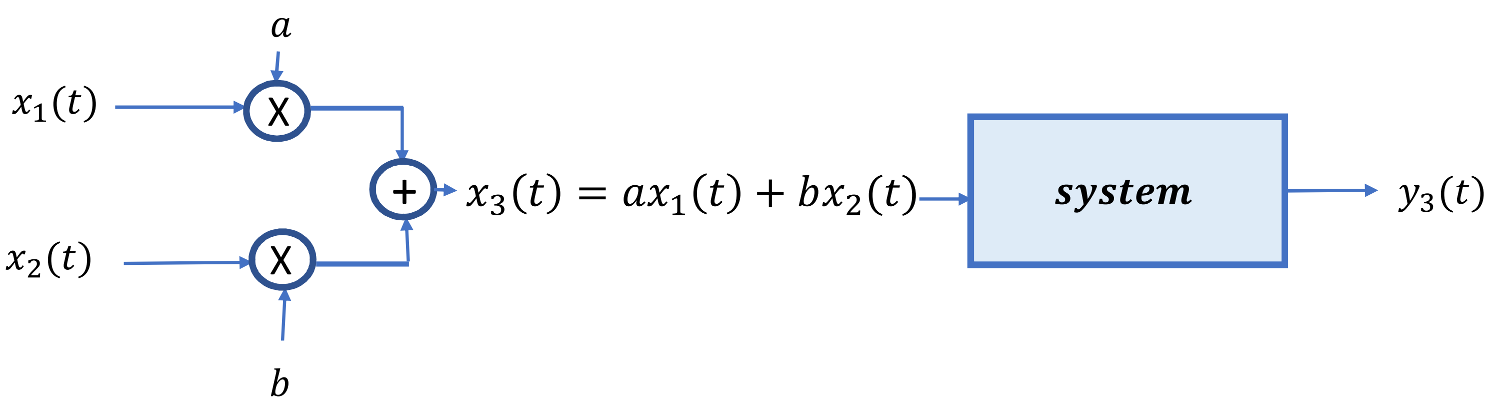

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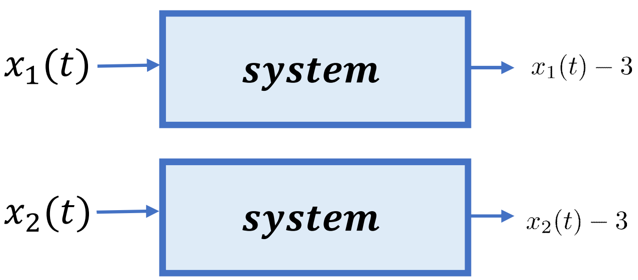

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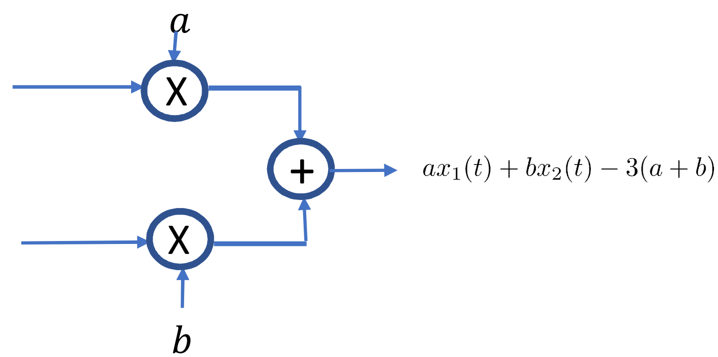

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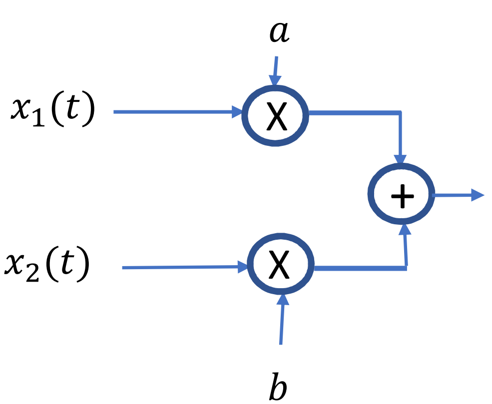

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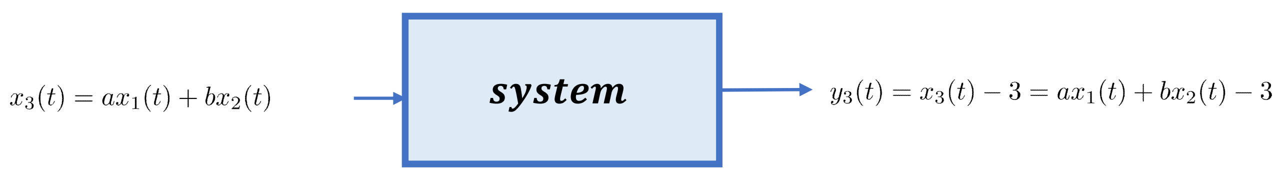
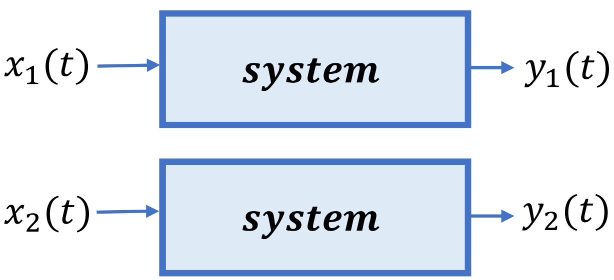


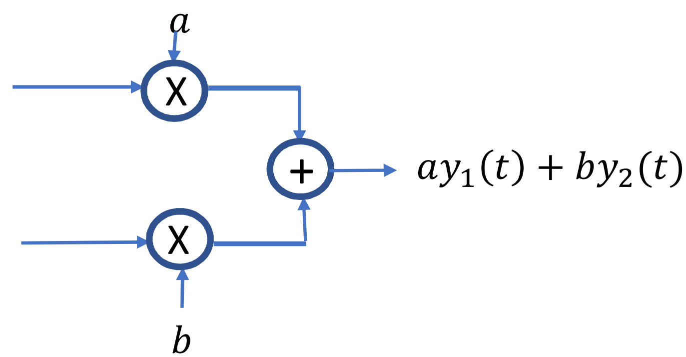
Linear/ Nonlinear Systems

Check the steps for $y(t) = x(2t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 5:
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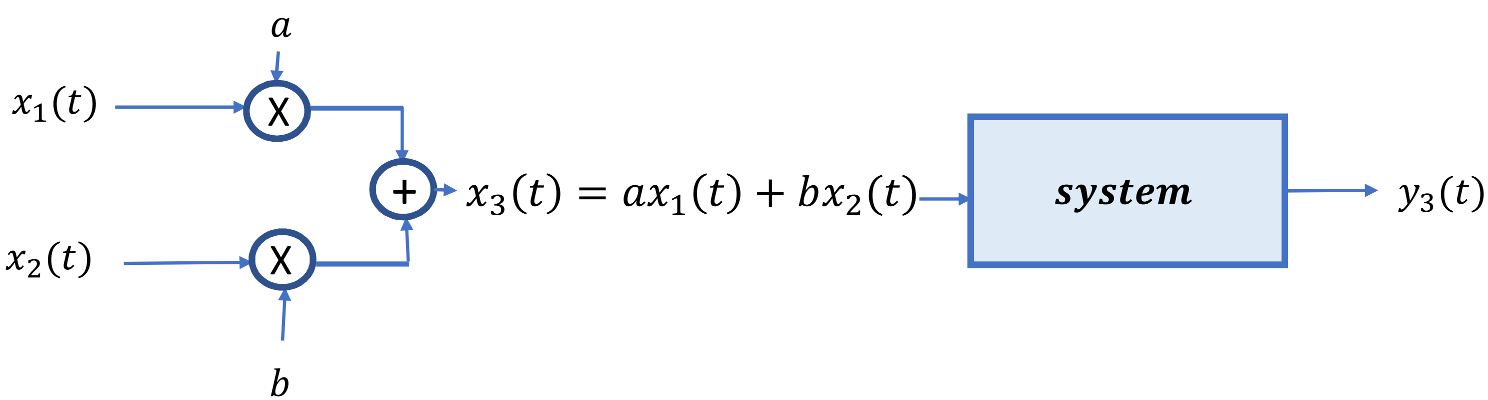
If yes, the system is **Linear**

Step 3: Build $x_3(t)$

Step 4: Find the out put of the system to $x_3(t)$



Linear/ Nonlinear Systems

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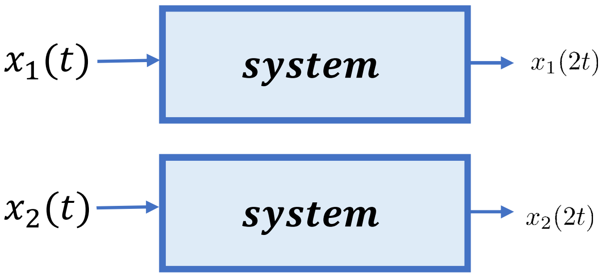

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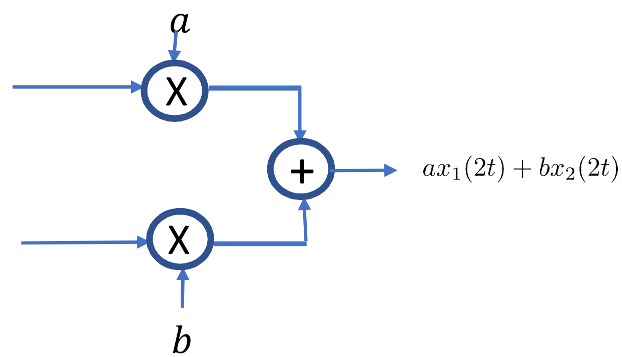

Step 5:
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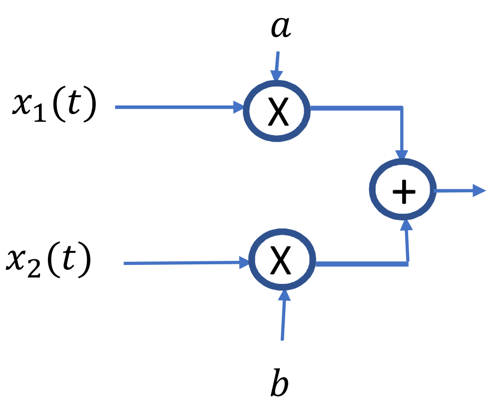
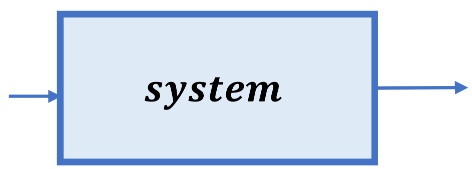
The two are the same so the system is **<u>Linear</u>**

Step 3: Build $x_3(t)$



$x_3(t) = ax_1(t) + bx_2(t)$

Step 4: Find the out put of the system to $x_3(t)$

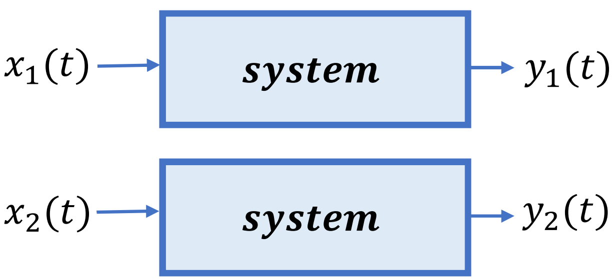


$y_3(t) = x_3(2t) = ax_1(2t) + bx_2(2t)$

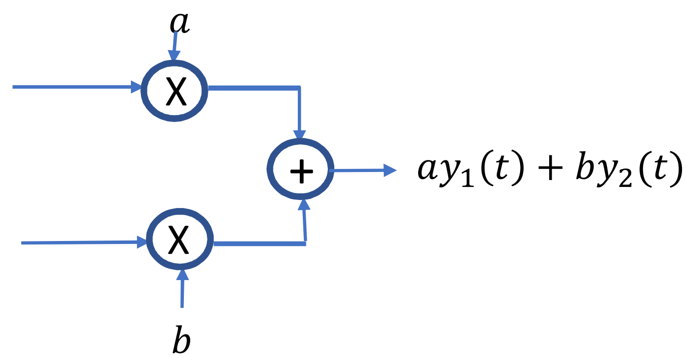
Linear/ Nonlinear Systems

Check the steps for $y(t) = t^2 x(t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



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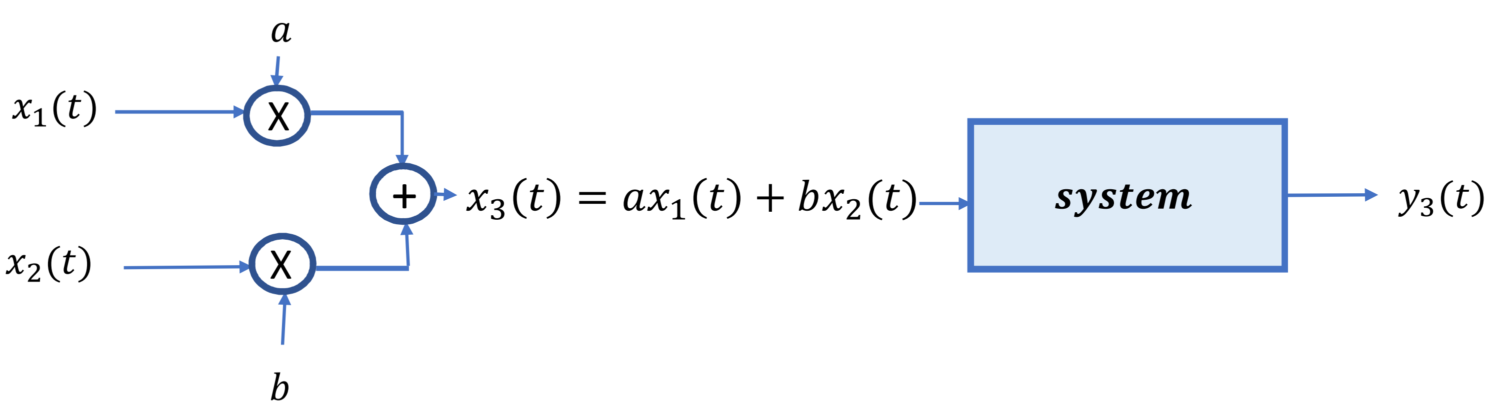
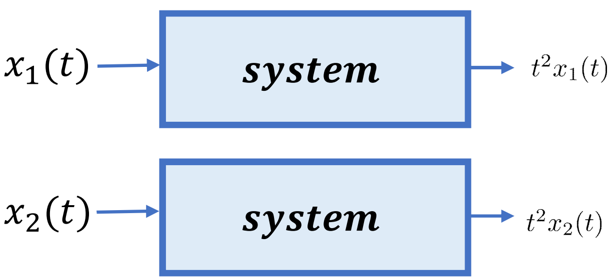


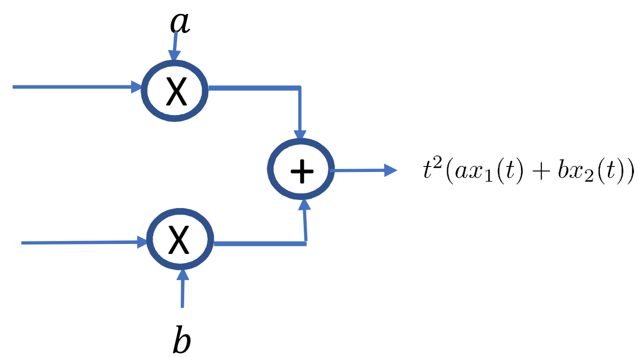
Linear/ Nonlinear Systems

Check the steps for $y(t) = t^2 x(t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



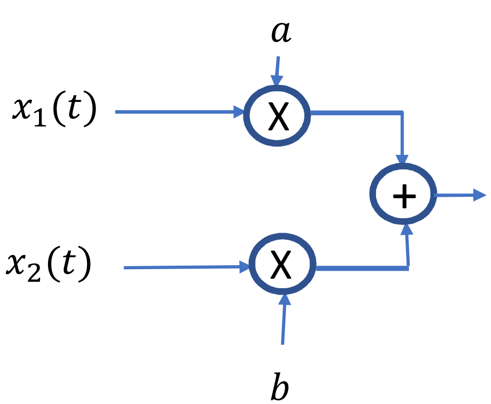
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



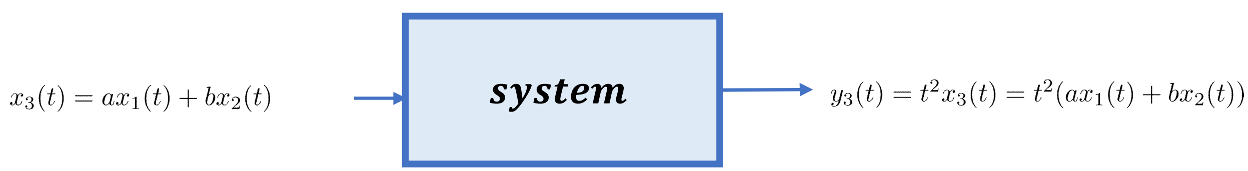
Step 5:
Check if result of step 2 is
the same as result of step 4:

The two are the same so the system is **Linear**

Step 3: Build $x_3(t)$



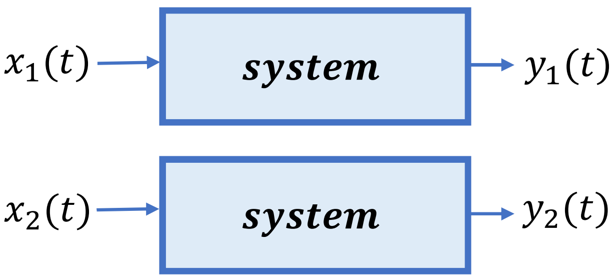
Step 4: Find the out put of the system to $x_3(t)$



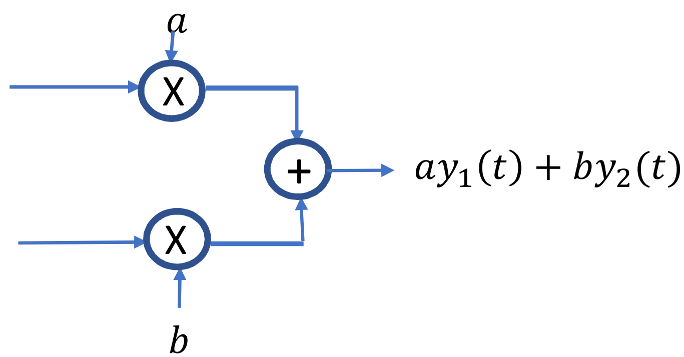
Linear/ Nonlinear Systems

Check the steps for a system with following equation $\frac{dy(t)}{dt} + t^2 y(t) = 2tx(t)$

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$

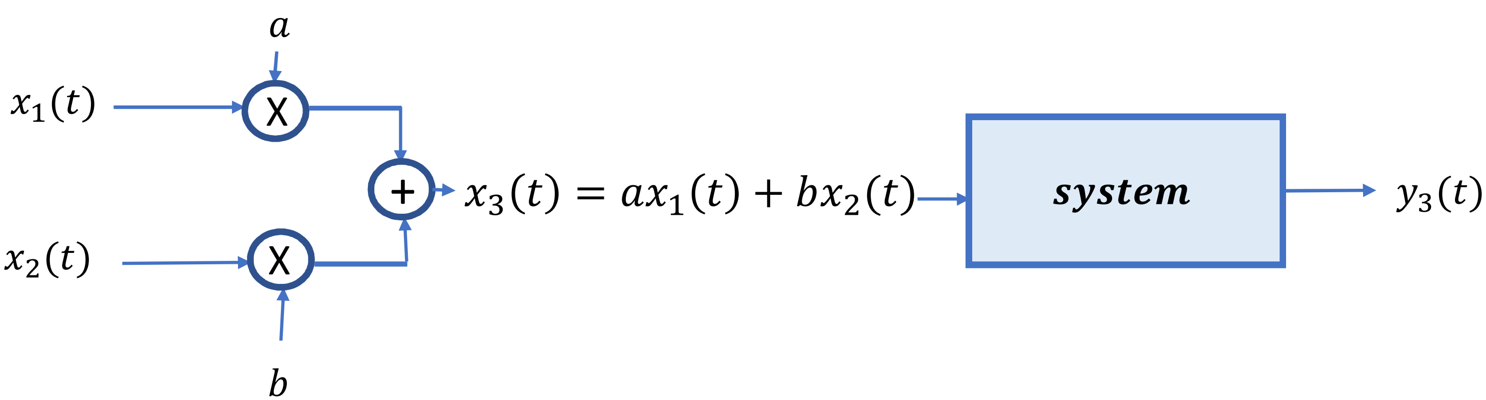


Step 5:
Check if result of step 2 is
the same as result of step 4:

If yes, the system is **Linear**

Step 3: Build $x_3(t)$

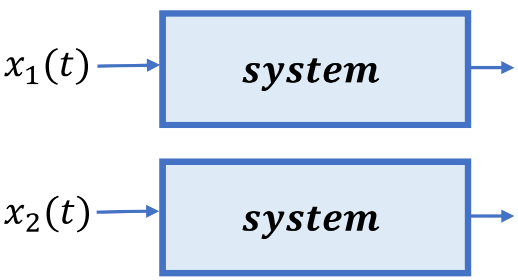
Step 4: Find the out put of the system to $x_3(t)$



Linear/ Nonlinear Systems

Check the steps for a system with following equation $\frac{dy(t)}{dt} + t^2 y(t) = 2tx(t)$

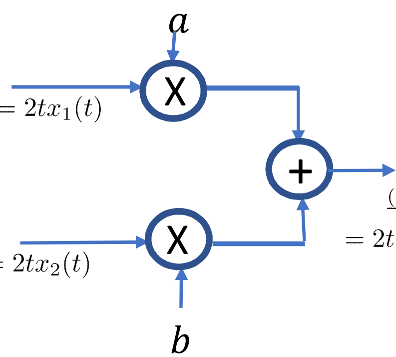
Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



$\frac{dy_1(t)}{dt} + t^2 y_1(t) = 2tx_1(t)$

$\frac{dy_2(t)}{dt} + t^2 y_2(t) = 2tx_2(t)$

Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



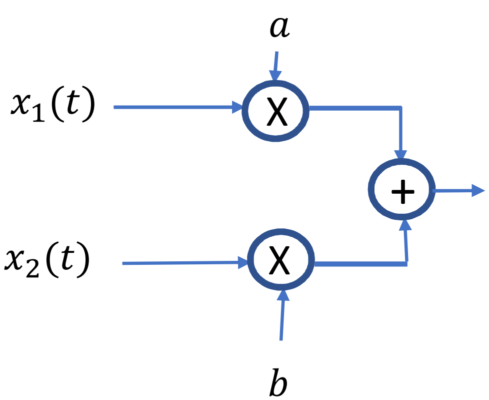
$\frac{(d(ay_1(t)+by_2(t))}{dt} + t^2(ay_1 + by_2(t))$
$= 2t(ax_1(t) + bx_2(t))$

Step 5:
Check if result of step 2 is the same as result of step 4:

Here we can replace $y_3(t)$ is step 4 with $ay_1(t) + by_2(t)$ in step 2 as both equations then become identical.

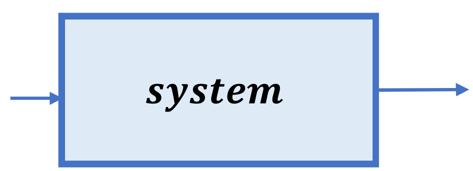
so the system is **Linear**

Step 3: Build $x_3(t)$



$x_3(t) = ax_1(t) + bx_2(t)$

Step 4: Find the out put of the system to $x_3(t)$



$\frac{dy_3(t)}{dt} + t^2 y_3(t) = 2tx_3(t)$

$\frac{dy_3(t)}{dt} + t^2 y_3(t) = 2t(ax_1(t) + bx_2(t))$

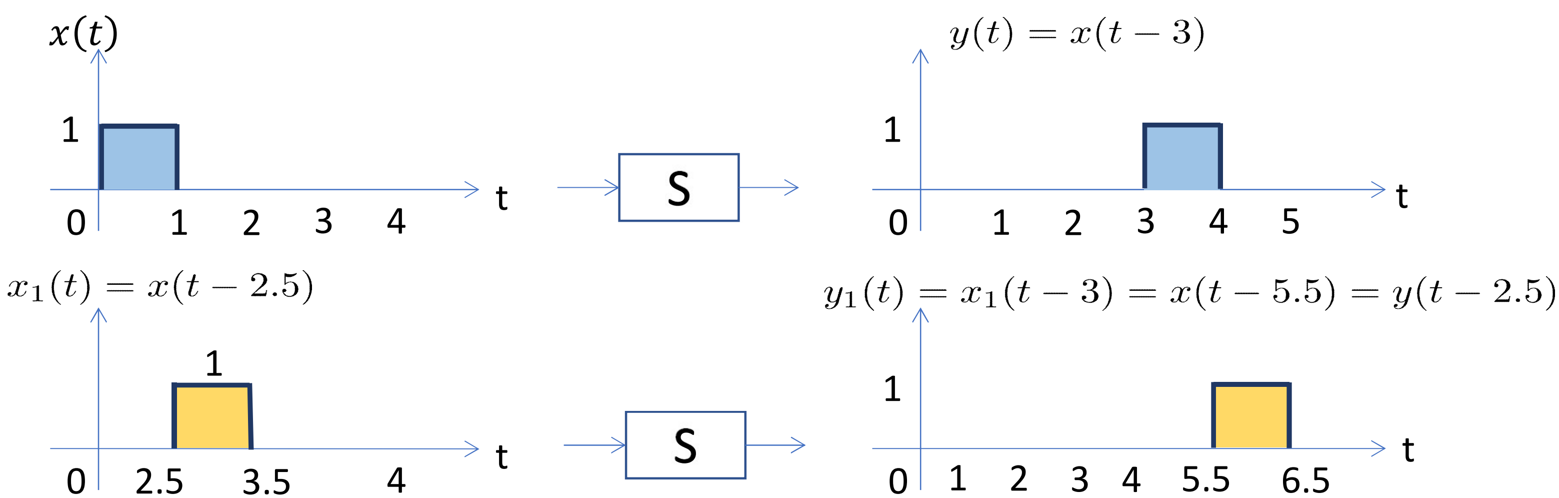
Time Invariant(TI) / Time Variant (TV) Systems

System S with output $y(t) = S((x(t))$ is TI if and only if

$$y(t-T) = S\big(x(t-T)\big)$$

"Time shift T in input results in time shift T in output."

Example:



Time Invariant(TI) / Time Variant (TV) Systems

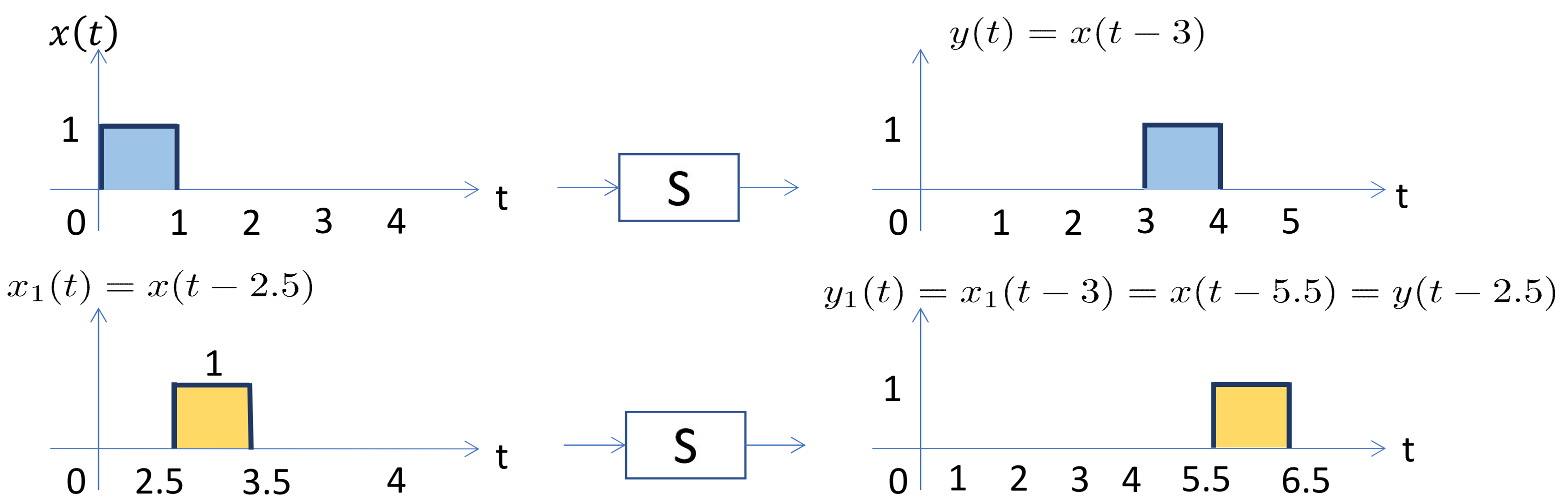
System S with output $y(t) = S((x(t))$ is TI if and only if

$$y(t-T) = S\big(x(t-T)\big)$$

"Time shift T in input results in time shift T in output."

Holds for $T = 2.5$
is the system TI?

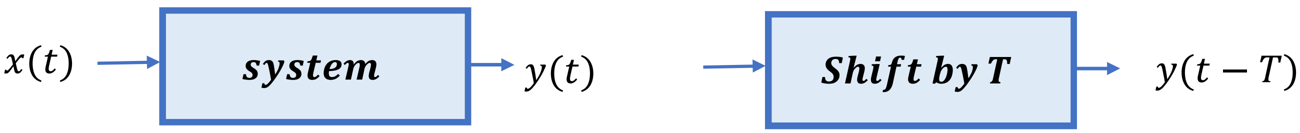
Example:



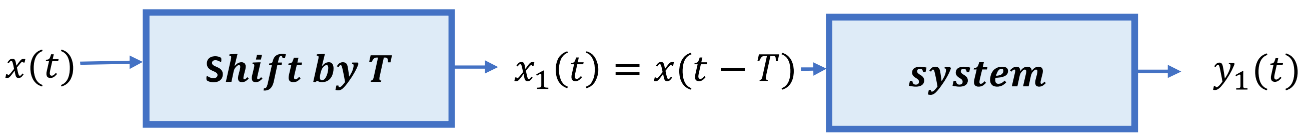
Time Invariant(TI) / Time Variant (TV) Systems

Three steps to check if a system is TI:

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



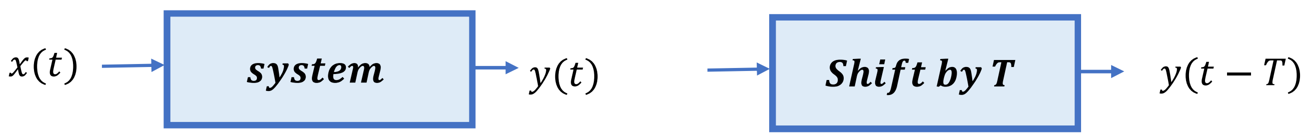
Step 3:
Check if result of step 1 is the same as result of step 2:

If the answer is yes,
the system is **Time Invariant**

Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(t-3)$

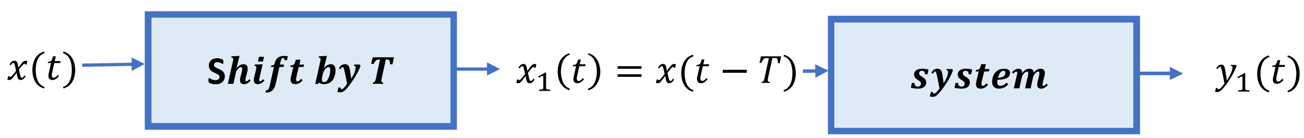
Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 3:
Check if result of step 1 is
the same as result of step 2:

If the answer is yes,
the system is **Time Invariant**

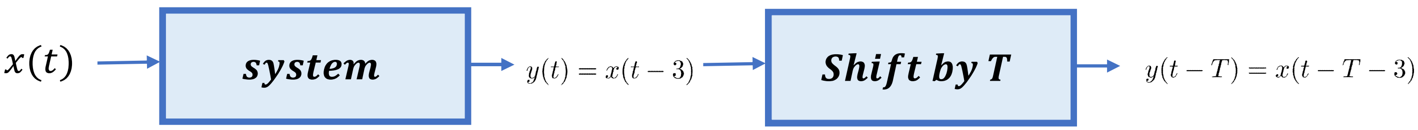
Step 2: Find the output of system to shifted version $x(t)$



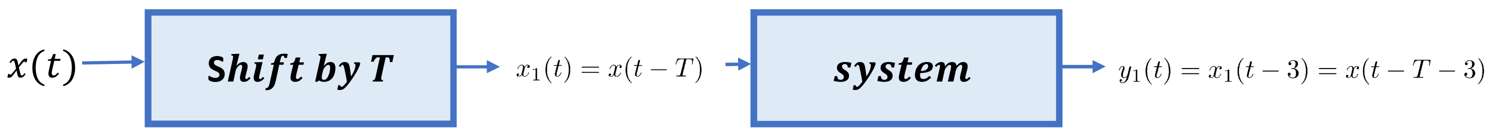
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(t-3)$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



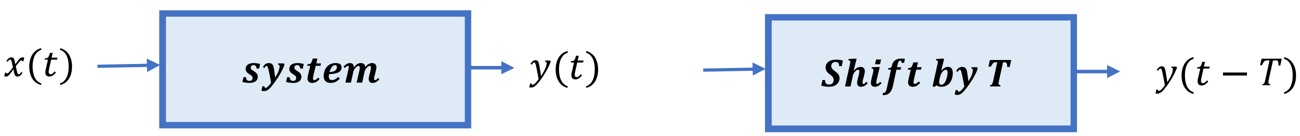
Step 3:
Check if result of step 1 is the same as result of step 2:

Resuts of step 1 and step 2 are the same the system is
Time Invariant

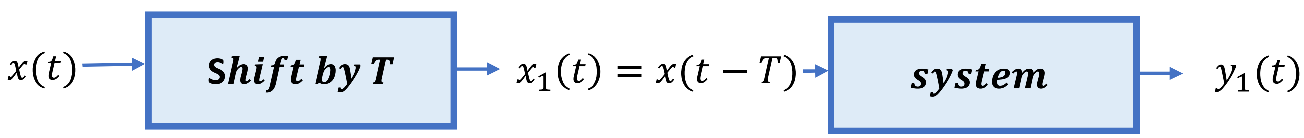
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x^2(t)$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



Step 3:
Check if result of step 1 is the same as result of step 2:

If the answer is yes, the system is **Time Invariant**

Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x^2(t)$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T

$$x(t) \rightarrow \boxed{\textit{system}} \rightarrow y(t) = x^2(t) \rightarrow \boxed{\textit{Shift by } T} \rightarrow y(t-T) = x^2(t-T)$$

Step 2: Find the output of system to shifted version $x(t)$

$$x(t) \rightarrow \boxed{\textit{Shift by } T} \rightarrow x_1(t) = x(t-T) \rightarrow \boxed{\textit{system}} \rightarrow y_1(t) = x_1^2(t) = x^2(t-T)$$

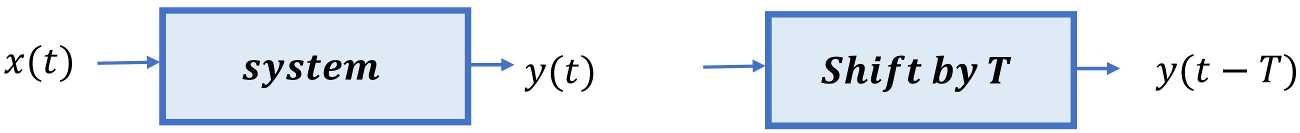
Step 3:
Check if result of step 1 is the same as result of step 2:

Resuts of step 1 and step 2 are the same the system is
Time Invariant

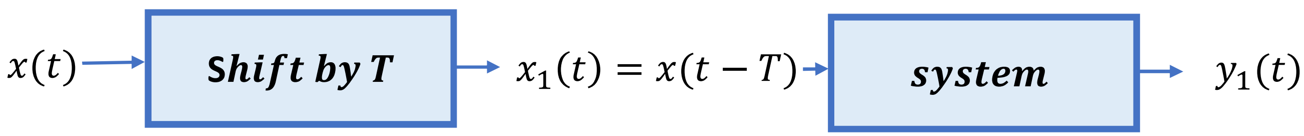
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(t) + 3$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$



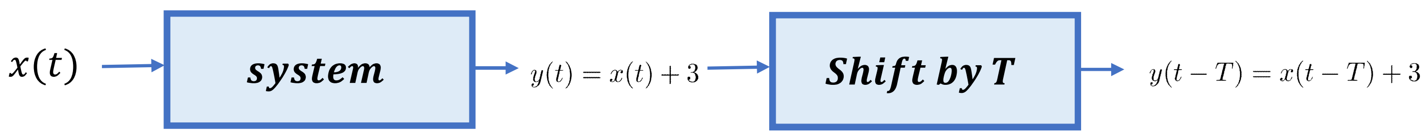
Step 3:
Check if result of step 1 is
the same as result of step 2:

If the answer is yes,
the system is **Time Invariant**

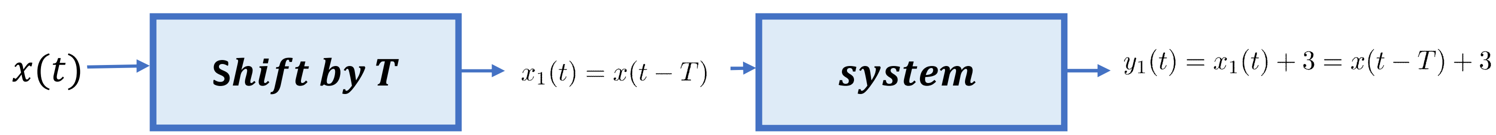
Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(t) + 3$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



Step 2: Find the output of system to shifted version $x(t)$

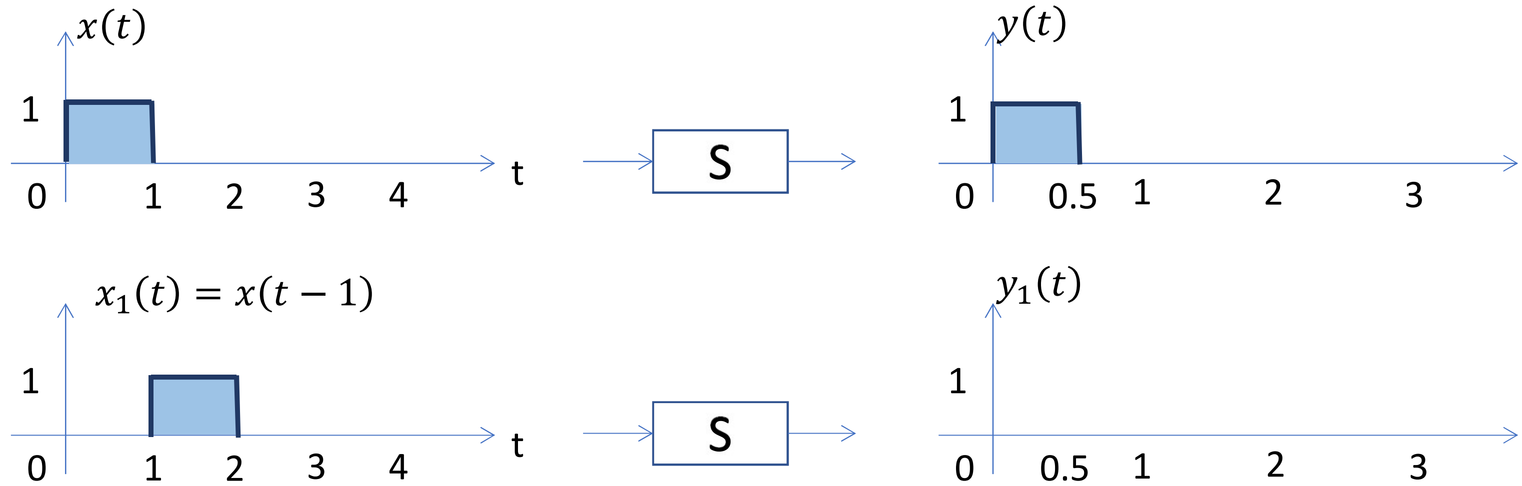


Step 3:
Check if result of step 1 is the same as result of step 2:

Resuts of step 1 and step 2 are the same the system is
Time Invariant

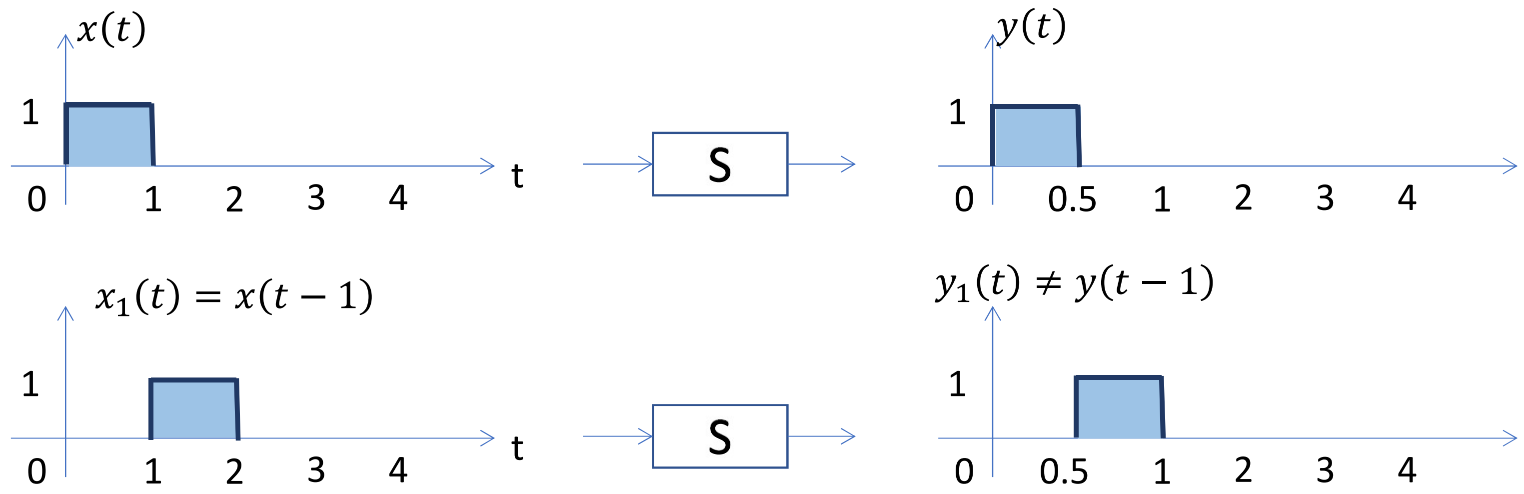
Time Invariant(TI) / Time Variant (TV) Systems

Is $y(t) = x(2t)$ Time Invariant?



Time Invariant(TI) / Time Variant (TV) Systems

Is $y(t) = x(2t)$ Time Invariant?



Time Invariant(TI) / Time Variant (TV) Systems

Check the steps for $y(t) = x(2t)$

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T

$$x(t) \rightarrow \boxed{\textit{system}} \rightarrow y(t) = x(2t) \rightarrow \boxed{\textit{Shift by } T} \rightarrow y(t-T) = x(2(t-T)) = x(2t-2T)$$

Step 2: Find the output of system to shifted version $x(t)$

$$x(t) \rightarrow \boxed{\textit{Shift by } T} \rightarrow x_1(t) = x(t-T) \rightarrow \boxed{\textit{system}} \rightarrow y_1(t) = x_1(2t) = x(2t-T)$$

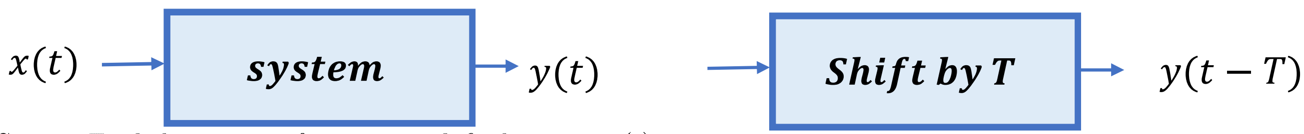
Step 3:
Check if result of step 1 is the same as result of step 2:

Resuts of step 1 and step 2 are not the same, the system is
Time Varying

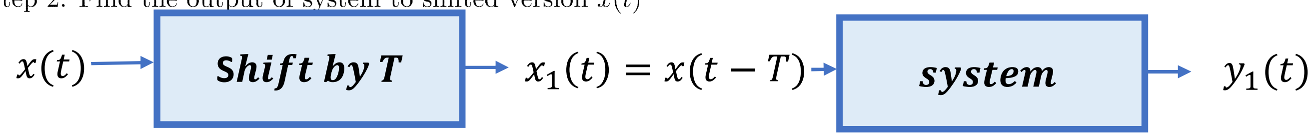
Time Invariant(TI) / Time Variant (TV) Systems

Is system $y(t) = x(-t)$ Linear? is it time invariant?

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T



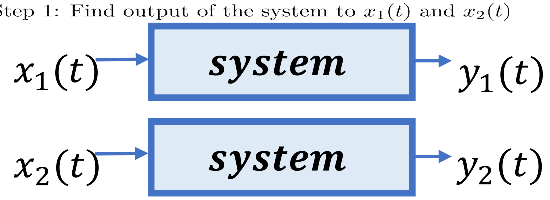
Step 2: Find the output of system to shifted version $x(t)$



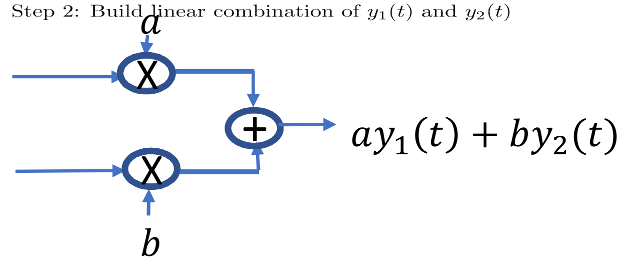
Step 3:
Check if result of step 1 is
the same as result of step 2:

If the answer is yes,
the system is **Time Invariant**

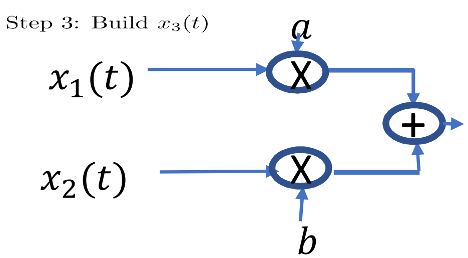
Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$



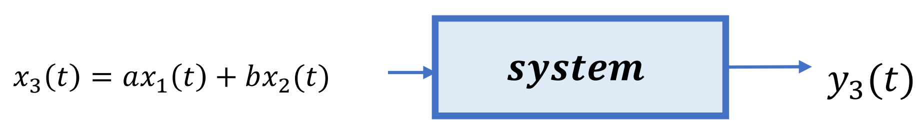
Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$



Step 3: Build $x_3(t)$



Step 4: Find the out put of the system to $x_3(t)$



Step 5:
Check if result of step 2 is
the same as result of step 4:

If yes, the system is **Linear**

Time Invariant(TI) / Time Variant (TV) Systems

Is system $y(t) = x(-t)$ Linear? is it time invariant?

$x(\alpha t - T)$: $\alpha = 1$ is LTI; $\alpha \neq 1$ is LTV

Step 1: Write $y(t)$, the output of the system to $x(t)$ and shift it by T

$$x(t) \rightarrow \boxed{\textbf{system}} \rightarrow y(t) = x(-t) \rightarrow \boxed{\textbf{Shift by } T} \rightarrow y(t-T) = x(-(t-T)) = x(-t+T))$$

Step 2: Find the output of system to shifted version $x(t)$

$$x(t) \rightarrow \boxed{\textbf{Shift by } T} \rightarrow x_1(t) = x(t-T) \rightarrow \boxed{\textbf{system}} \rightarrow y_1(t) = x_1(-t) = x(-t-T)$$

Step 3:
The result of step 1 is not the same as result of step 2:

The system is **Time Varying**

Step 1: Find output of the system to $x_1(t)$ and $x_2(t)$

$$x_1(t) \rightarrow \boxed{\textbf{system}} \rightarrow y_1(t) = x_1(-t)$$

$$x_2(t) \rightarrow \boxed{\textbf{system}} \rightarrow y_2(t) = x_2(-t)$$

Step 2: Build linear combination of $y_1(t)$ and $y_2(t)$

a, b

$$a x_1(-t) + b x_2(-t)$$

Step 3: Build $x_3(t)$

$x_1(t)$, $x_2(t)$, a, b

Step 4: Find the out put of the system to $x_3(t)$

$$x_3(t) = a x_1(t) + b x_2(t) \rightarrow \boxed{\textbf{system}} \rightarrow \begin{aligned} y(t) &= x_3(-t) \\ &= a x_1(-t) + b x_2(-t) \end{aligned}$$

Step 5:
The result of step 2 is the same as result of step 4:

The system is **Linear**

System Classification: Memoryless (instantaneous)/with memory(dynamic)

Memoryless system's output at time *t* only depends on input at time *t*. Otherwise the system is with memory!

Which of these systems are memoryless?

$(a)\quad x(t) \rightarrow \boxed{S} \rightarrow t + x(t)$

$(b)\quad x(t) \rightarrow \boxed{S} \rightarrow x(t+2)$

$(c)\quad x(t) \rightarrow \boxed{S} \rightarrow x(t-1) + x(t)$

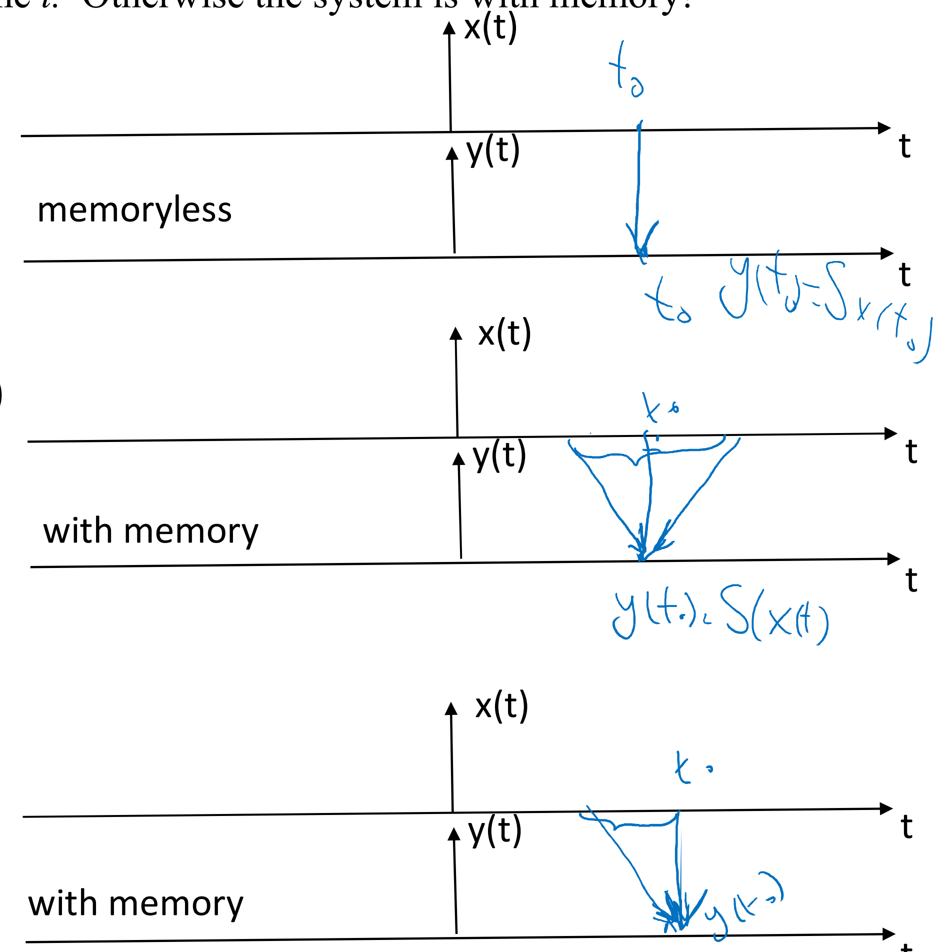
$(d)\quad x(t) \rightarrow \boxed{S} \rightarrow e^{x(t)}$

$(e)\quad x(t) \rightarrow \boxed{S} \rightarrow x(-t)$

$(f)\quad x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{\infty} x(t)dt$

$(g)\quad x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{t} x(t)dt$

$(h)\quad x(t) \rightarrow \boxed{S} \rightarrow x^2(t)$



System Classification: Memoryless (instantaneous)/with memory(dynamic)

Memoryless system's output at time *t* only depends on input at time *t*. Otherwise the system is with memory!

Which of these systems are memoryless?

✓ (a) $\quad x(t) \rightarrow \boxed{S} \rightarrow y(t) = t + x(t)$ $\quad y(2) = 2 + x(2)$

✗ (b) $\quad x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(t+2)$ $\quad y(2) = x(4)$

✗ (c) $\quad x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(t-1) + x(t)$ $\quad y(2) = x(1) + x(2)$

✓ (d) $\quad x(t) \rightarrow \boxed{S} \rightarrow y(t) = e^{x(t)}$ $\quad y(2) = e^{x(2)}$

✗ (e) $\quad x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(-t)$ $\quad y(2) = x(-2)$

✗ (f) $\quad x(t) \rightarrow \boxed{S} \rightarrow \mathrm{y(t)} = \int_{-\infty}^{\infty} x(t)dt$ $\quad y(2) = y(3) = y(4.2) = \int_{-\infty}^{\infty} x(t)dt$

✗ (g) $\quad x(t) \rightarrow \boxed{S} \rightarrow y(t) = \int_{-\infty}^{t} x(t)dt$ $\quad y(2) = \int_{-\infty}^{2} x(t)\,dt$

✓ (h) $\quad x(t) \rightarrow \boxed{S} \rightarrow y(t) = x^2(t)$ $\quad y(2) = x^2(2)$

System Classification: Causal/ Non-Causal Systems

Causal system's output at time t_0 only depends on input values at time t_0 and at times before t_0, i.e., only depends on $t \leq t_0$.

Which of these systems are causal?

(a) $x(t) \rightarrow \boxed{S} \rightarrow t + x(t)$

(b) $x(t) \rightarrow \boxed{S} \rightarrow x(t+2)$

(c) $x(t) \rightarrow \boxed{S} \rightarrow x(t-1) + x(t)$

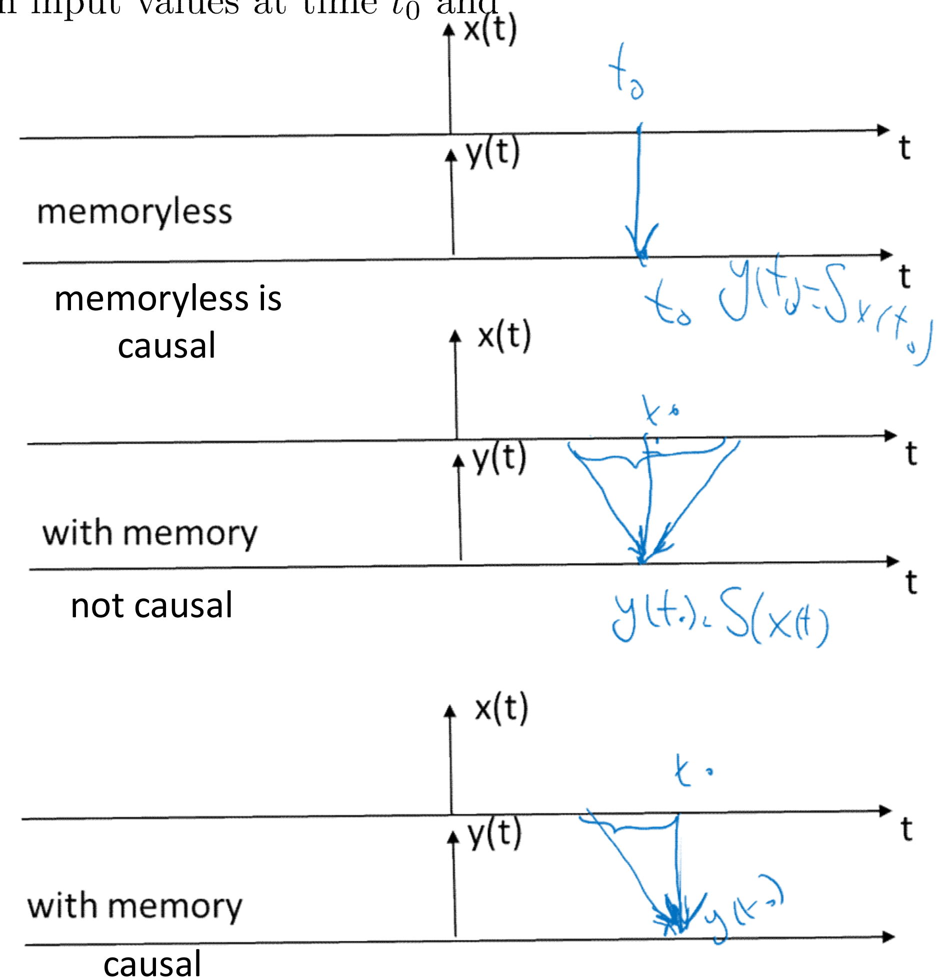
(d) $x(t) \rightarrow \boxed{S} \rightarrow e^{x(t)}$

(e) $x(t) \rightarrow \boxed{S} \rightarrow x(-t)$

(f) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{\infty} x(t)dt$

(g) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{t} x(t)dt$

(h) $x(t) \rightarrow \boxed{S} \rightarrow x^2(t)$



System Classification: Causal/ Non-Causal Systems

Causal system's output at time t_0 only depends on input values at time t_0 and at times before t_0, i.e., only depends on $t \leq t_0$.

Which of these systems are causal?

✓ (a) $x(t) \rightarrow \boxed{S} \rightarrow y(t) = t + x(t)$

✗ (b) $x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(t+2)$ $\quad y(2) = x(4)$

✓ (c) $x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(t-1) + x(t)$

✓ (d) $x(t) \rightarrow \boxed{S} \rightarrow y(t) = e^{x(t)}$

✗ (e) $x(t) \rightarrow \boxed{S} \rightarrow y(t) = x(-t)$ $\quad y(-2) = x(2)$

✗ (f) $x(t) \rightarrow \boxed{S} \rightarrow y(t) = \int_{-\infty}^{\infty} x(t)dt$

✓ (g) $x(t) \rightarrow \boxed{S} \rightarrow y(t) = \int_{-\infty}^{t} x(t)dt$

✓ (h) $x(t) \rightarrow \boxed{S} \rightarrow y(t) = x^2(t)$

System Classification: Invertible/ Non-invertible Systems

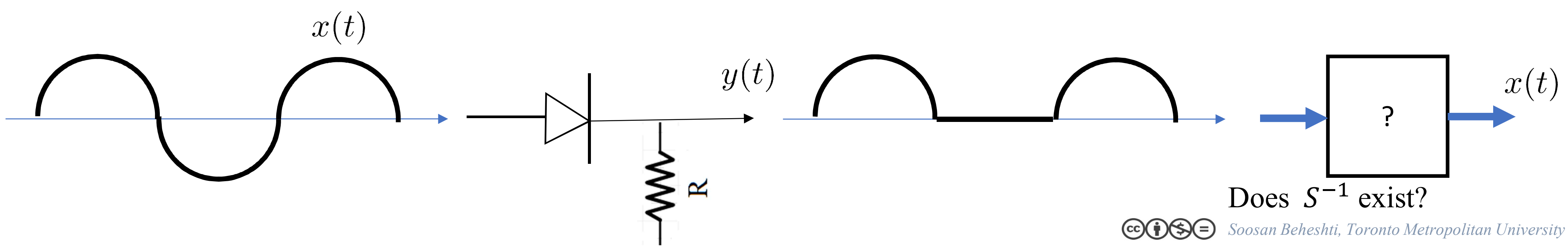
System S is invertible if there exists a system S^{-1} such that:

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t) \longrightarrow \boxed{S^{-1}} \longrightarrow x(t)$$

(Important desired property in design of many systems such as communications systems).

$$x(t) \longrightarrow \boxed{\textit{Mobile}} \longrightarrow \begin{matrix}\textit{Transmitted} \\ \textit{voice data}\end{matrix} \longrightarrow \boxed{\textit{Channel}} \longrightarrow \boxed{\textit{Receiver}} \longrightarrow \hat{x}(t)$$

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t) = x(t-2) \longrightarrow \boxed{S^{-1}} \longrightarrow y_1(t) = y(t+2) = x(t-2+2)$$



System Classification: Invertible/ Non-invertible Systems

Which of these systems are invertible?

✓ (a) $x(t) \rightarrow \boxed{S} \rightarrow y(t)=t+x(t)$ $x(t)=y(t)-t$

✓ (b) $x(t) \rightarrow \boxed{S} \rightarrow y(t)=x(t+2)$ $\longrightarrow$ $x(t)=y(t-2)$

✗ (c) $x(t) \rightarrow \boxed{S} \rightarrow y(t)=x(t-1)+x(t)$

✓ (d) $x(t) \rightarrow \boxed{S} \rightarrow y(t)=e^{x(t)}$ $\longrightarrow$ $x(t)=\ln y(t)$

✓ (e) $x(t) \rightarrow \boxed{S} \rightarrow y(t)=x(-t)$ $x(t)=y(-t)$

✗ (f) $x(t) \rightarrow \boxed{S} \rightarrow \mathrm{y(t)}=\int_{-\infty}^{\infty} x(t)dt$

✓ (g) $x(t) \rightarrow \boxed{S} \rightarrow y(t)=\int_{-\infty}^{t} x(t)dt$ $x(t)=y'(t)$

✗ (h) $x(t) \rightarrow \boxed{S} \rightarrow y(t)=x^2(t)$

System Classification: Stable/Unstable Systems

External stability or Bounded input/Bounded output (BIBO) stability

If $|x(t)| < \ C_1, \ \forall t \rightarrow \exists \ c > 0 : |y(t)| < C \ \forall t$

We will discuss Internal Stability later

Serious concepts in design of Control Systems (ELE639)

System Classification: Stable/Unstable Systems

Which of these systems are externally stable?

✗ (a) $x(t) \rightarrow \boxed{S} \rightarrow t + x(t)$

✓ (b) $x(t) \rightarrow \boxed{S} \rightarrow x(t+2)$

✓ (c) $x(t) \rightarrow \boxed{S} \rightarrow x(t-1) + x(t)$

✓ (d) $x(t) \rightarrow \boxed{S} \rightarrow e^{x(t)}$

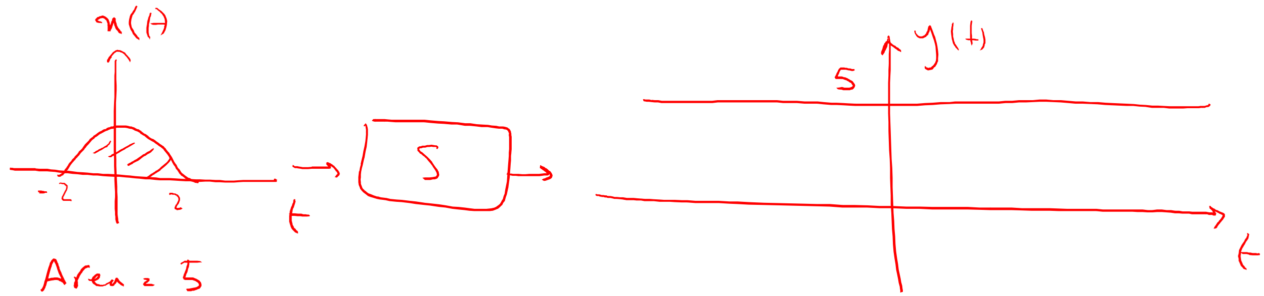
✓ (e) $x(t) \rightarrow \boxed{S} \rightarrow x(-t)$

✗ (f) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{\infty} x(t)dt$

✗ (g) $x(t) \rightarrow \boxed{S} \rightarrow \int_{-\infty}^{t} x(t)dt$

✓ (h) $x(t) \rightarrow \boxed{S} \rightarrow x^2(t)$

$x(t) \rightarrow \boxed{S} \rightarrow y(t) = \int_{-\infty}^{\infty} x(t)\,dt$



Area = 5

$x(t) \rightarrow \boxed{S_2}$ $\quad y(t) = \int_{-\infty}^{t} x(t)\,dt$