The Hall Effect

Physics Topics

If necessary, review the following topics and relevant textbook sections from Serway / Jewett "Physics for Scientists and Engineers", 10th Ed.

- Electric Force (Serway 22.3)
- Potential Difference in a Uniform Field (Serway, 24.2)
- Electric Current (Serway, 26.1)
- Analysis Model: Particle in a Field (Magnetic) (Serway, 28.1)
- The Hall Effect (Serway, 28.6)

Introduction

Current is carried by charges. If a current carrying material is placed in a magnetic field, the charges within it will feel a magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ and will move in response to it. The magnetic force will push the charge carriers to one side of the sample (a lack of charges on the opposite side causes it to become oppositely charged). This separation of charges creates an electric field $\vec{E_H}$ and an associated electric potential ΔV_H which can be measured with a voltmeter. This effect was first observed by Edwin Hall in 1879.

The central concept relevant for the Hall Effect is a competition between electric and magnetic forces. Initially the charge carriers only feel a magnetic force, and so they get pushed to one side of the sample. However, as the concentration of charges on that side builds up, they will tend to electrically repel other charge carriers away. As long as the magnetic force on the charge carriers is larger than the electrical repulsion, the charge will continue to build up. Once the electric repulsion exactly balances the magnetic force, the system comes to equilibrium, the buildup of charge stops, and the Hall Voltage becomes constant. This occurs when the electric force has an equal magnitude as the magnetic force:

$$\cancel{|q|}|\vec{E_H}| \;=\; \cancel{|q|}|\vec{v}_d||\vec{B}| \tag{1}$$

Throughout this lab, we will assume the current carrying sample is a rectangular solid with current flowing along the length L direction, and the other two dimensions denoted as width W and thickness t. If the magnetic field $\vec{B}$ points along the thickness direction (t), and the current flows along the length direction (L), the Hall Electric field points along the "width" (W) of the sample. Thus the associated potential difference is $|\Delta V_H| = |\vec{E}_H|W$. Combining this with above, we have

$$|\Delta V_H| = W|\vec{v}_d||\vec{B}| \tag{2}$$

You may wonder how we know that electrons (negatively charged particles) are the current carriers in a metal. Conventional experiments which do not use the Hall Effect can only determine the direction of the current, but not the sign of the carriers. On the other hand, the sign of the Hall Voltage is sensitive to the sign of the charge carriers. The Hall Effect is also one of the main tools we have for studying the electrical properties of a semiconductor. In fact, the Hall Effect can tell us

- The sign of the charge carrier
- The density n of charge carriers
- The mobility μ of charge carriers. (Recall that mobility is defined as the ratio of the speed of the charge carriers to the applied electric field magnitude $|\vec{v}| = \mu|\vec{E}|$.)

Using (2) as a starting point, the Pre-Lab Questions will guide you through derivation of formulas which allow you to determine n, μ and the sign of the charge carriers.

Pre-Lab Questions

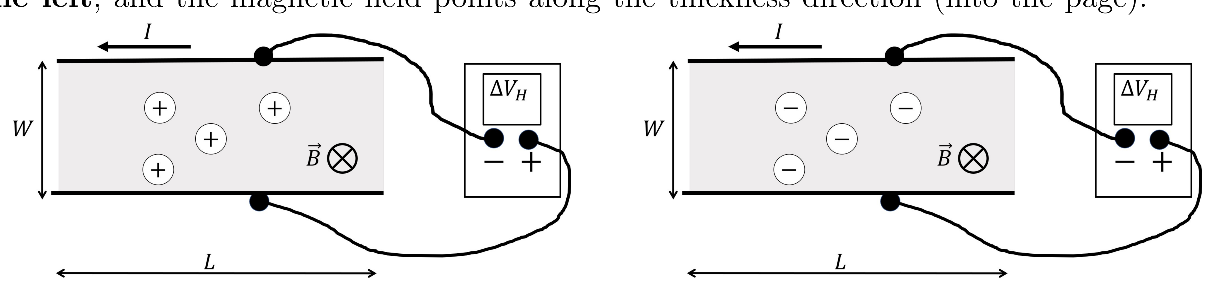
Please complete the following questions prior to coming to lab. They will help you prepare for both the lab and the pre-lab quiz (Found on D2L).

1.) Read through the entire lab writeup before beginning

2.) What is the **specific** goal of this lab? Exactly what question(s) are you trying to answer? Be as specific as possible. ("To learn about topic X..." is **not** specific!)

3.) What **specific** measurements or observations will you make in order to answer this question?

4.) The following set of questions will help you determine the sign of the charge carriers from the sign of the Hall Voltage. Shown below are two possible cases: one with the positive charge carriers, and one with negative charge carriers. A few charge carriers are shown in each figure. In both figures, the conventional current flows **to the left**, and the magnetic field points along the thickness direction (into the page).



(a) On each figure, draw the velocity vectors for the charges. Remember that conventional current is defined as the flow of *positive* charge.

(b) On each figure, draw the direction of the magnetic force on the charges.

(c) The magnetic force on the charge carriers will push them to either the upper or lower part of the sample. Indicate on each figure whether the upper surface of the sample becomes positively or negatively charged (and similarly for the lower surface).

(d) On each figure, draw a vector $\vec{E_H}$ in the direction of the Hall Electric field, pointing from the positively charged surface to the negatively charged surface.

(e) Given that the voltmeter is hooked up as shown in the figure, indicate whether it will read a positive or negative Hall Voltage. You may need to refer to these drawings when you complete your analysis.

5.) The following set of questions will help you derive an equation for the number density of charge carriers as a function of current and Hall Voltage.

(a) Using your knowledge of electric current, write an equation relating the current in a material to the density of free charge carriers n, their (drift) speed v_d, their charge, and the physical dimensions of the sample L, W and t.

(b) The formula you wrote contains v_d, the speed of the charges which is not directly measurable in lab. You'll need to eliminate it in favor of measurable quantities. Use equation (2) to do so.

(c) Re-write your equation to solve for n the number density of charge carriers in terms of directly measurable quantities in lab (you may assume the charge of the carriers is known to be $|q| = e$).

6.) Finally, the following set of questions will help you to derive an equation for the mobility μ of the charges in terms of directly measurable quantities.

(a) Begin with the definition of mobility $\mu = |\vec{v}_d|/|\vec{E_L}|$, where $\vec{E}_L$ is the electric field along the current direction (from the power supply) which is causing the current to flow. Remove the unmeasurable electric field in terms of the applied power supply voltage $|\Delta V_L|$. Be careful to distinguish this voltage from the Hall voltage!

(b) Finally remove the drift velocity in favor of measurable quantities using equation (2). Simplify your result as much as possible.

Apparatus

- Cables with banana plug terminations
- 0 - 200 mA current supply
- FW Bell Model 5070 Gauss/Tesla Meter and zero gauss chamber
- Dual multimeter (ideally 2/setup, though the experiment can be done with 1/setup)
- Unilab Germanium Hall Effect Wafer
- Strong permanent magnet

A rectangular wafer of germanium (a semiconductor) is provided on a circuit board. Handle the board carefully as the wafer is delicate. The dimensions of the germanium wafer are $L \times W \times t$ =10.0mm $\times$ 5.0mm $\times$ 1.0mm.

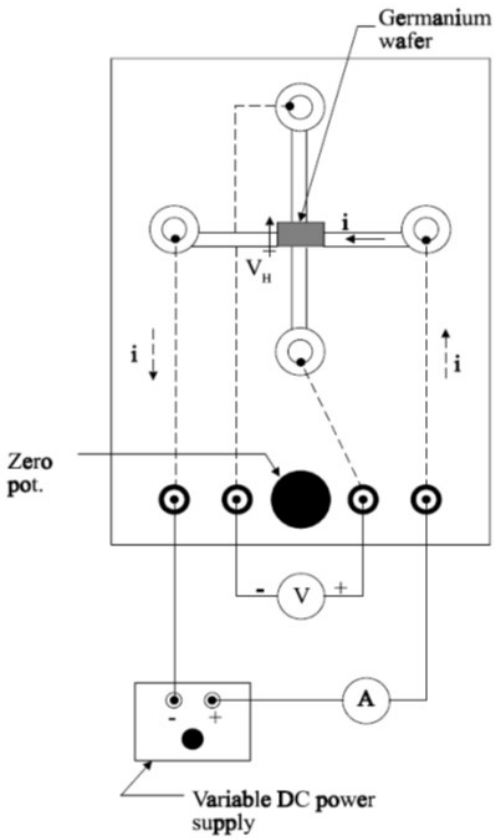


Figure 1: Circuit Board Connections for Hall Effect Lab

A DC power supply is used to drive a current along the length of the wafer. **CAUTION: do NOT exceed 50mA of current at any time during the experiment.** A multimeter can be used to measure the voltage drop ΔV_L across the length (L) of the wafer. The multimeter can also be used to measure the (Hall) Voltage drop along the width (W), ΔV_H.

A strong permanent magnet has poles shaped to provide a reasonably uniform magnetic field. Be careful not to allow any steel pieces to crash into the magnet (it has a very strong pull), as that will cause demagnetization.

Procedure

1.) The next two steps will allow you to measure the magnitude and direction of the magnetic field generated by the permanent magnet. Note that gauss meter probes are delicate; do not bend the probe. Place the tip of the probe into the zero-Gauss chamber (a small cylindrical hole in a small black piece of plastic). Push the "zero" button on the gauss meter to zero the probe.

2.) Insert the gaussmeter in between the poles of the permanent magnet. With the lettering on the blue part of the probe facing up, a $\vec{B}$-field in the **down** direction will give a positive reading on the meter. Record your measurement of the magnitude and direction of the magnetic field in between the poles of the magnet. Try moving the probe around in between the poles in order to get a sense of how much the field varies in between the poles. Use your observations to record an estimate of uncertainty in the magnetic field.

3.) Connect the power supply to the wafer board so that current will flow in the same direction as shown in Fig. 1. Connect the multimeter across the width of the wafer as shown in the figure and set it to measure DC volts.

4.) With the wafer far away from the magnet, turn on the power supply and adjust the current to 50mA [do **NOT** exceed 50mA at any time during this experiment!].

5.) Adjust the zero potentiometer ("zero pot" as shown in the figure) so that your multimeter reads 0V when the wafer is far away from the magnet.

6.) Insert the wafer between the poles of the magnet so that the field is perpendicular to the face of the wafer, and the wafer is centered. There is a scratch along the plexiglass which indicates the center of the poles; try to line up your wafer so that the current flows parallel to this scratch.

7.) Note the relative directions of the current, magnetic field, and how your have hooked up your multimeter. Draw a diagram in your notebook for future reference.

8.) Vary the current from 0mA to 50mA and record the Hall voltage at each measurement (make sure to take note of the sign). As always, record your data with an estimate of uncertainty.

9.) Remove the wafer from the magnetic field. Disconnect the multimeter from the wafer and reconnect it to measure the voltage drop across the length L of the wafer (ΔV_L).

10.) Replace the wafer in the magnetic field. Vary the current from 0mA to 50mA and record the values of ΔV_L at each step with associated uncertainty. **Be sure that you take data at the same current values as you did in step 8 so that at each value of the current you have both a measurement of ΔV_H and ΔV_L.**

11.) Remove the wafer from the magnetic field, disconnect all cables, and turn off all devices.

Analysis

1.) Using the sign of the hall voltage which you observed, and comparing your experimental setup with your work on the pre-lab questions, determine whether this germanium wafer is n-type or p-type.

2.) Plot your data of ΔV_H and I in such a way so that the result is a straight line. Add error bars to your plot. Fit the data with a line, and use your answers to the pre-lab questions to determine the carrier concentration n from the slope. Make sure to check the units!

3.) Plot your data of ΔV_H and ΔV_L in such a way so that the result is a straight line. Add error bars to your plot. Fit the data with a line, and use your answers to the pre-lab questions to determine the carrier mobility μ from the slope. Make sure to check the units!

4.) Compare your value of the mobility μ to the accepted value for germanium which has $\mu_n = 3900\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$ and $\mu_p = 1900\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$. Calculate a % error from the accepted value.

Wrap Up

The following questions are designed to make sure that you understand the physics implications of the experiment and also to extend your knowledge of the physical concepts covered.Your report should answer these questions in the noted section in a seamless manner.

1.) [Theory] Is it always the case that a positive hall voltage means the semiconductor is p type (and a negative hall voltage means the semiconductor is n type)? If so, explain why. If not, give an example of when this would not be the case.

2.) [Discussion] The theoretical value of the conductivity of intrinsic germanium is approximately 2.2 $(\Omega\cdot\mathrm{m})^{-1}$. Do you expect the conductivity of your wafer to be larger, smaller, or the same as this value? Explain why. Then, using your measured values, calculate the conductivity $\sigma = en\mu$ (where n is the concentration of charge carriers) to check your intuition. As always, watch out for units!

3.) [Discussion] If the magnitude of the magnetic field were increased (but all everything else in your lab setup stayed the same), would the *magnitude* of the Hall voltage increase, decrease, or stay the same? Explain.

4.) [Discussion] If the magnetic field were increased (but everything else in your setup stayed the same), would your result for the mobility increase, decrease, or stay the same? Explain.

Report

Labs will be completed in groups, you will enroll in a group with your lab partner at the beginning of each lab session. Each group will submit a single report through the assignment section on D2L.

- **Introduction**
  - What is the experiment's objective?
- **Theory**
  - You may be able to show a derivation of the physics you're investigating, or you may want to reference a source that provides a description/equation representing the physics you're investigating.
  - You may want to provide graphs that illustrate or predict how you expect the system under study to behave.
- **Procedure**
  - Explain the systematic steps required to take any measurements.
- **Results and Calculations**
  - Tabulate your measurements in an organized manner.
  - Based on your procedure, you should know what your tables
  - Provide examples of any calculations.
- **Discussion and Conclusions**
  - Discuss the main observations and outcomes of your experiment.
  - Summarize any significant conclusions.
- **References**
- **(Appendices)**