UNIVERSITÄT BASEL

Optimization of Student Time Slot Allocation using Hungarian Method and Mixed-Integer Linear Programming

Bachelor's Thesis

Natural Science Faculty of the University of Basel
Department of Mathematics and Computer Science
Artificial Intelligence
https://ai.dmi.unibas.ch/

Examiner: Prof. Dr. Malte Helmert
Supervisor: Salomé Eriksson and Claudia Grundke

Hamza Zarah
h.zarah@stud.unibas.ch
20-051-801

18.08.2024

Acknowledgments

First of all, I would like to express my gratitude to Prof. Dr. Malte Helmert for providing me the opportunity to write my Bachelor's thesis in the Artificial Intelligence Research Group and for offering me an interesting selection of thesis topics.
I am especially thankful to my supervisors, Salomé Eriksson and Claudia Grundke, for their unwavering support, patience, understanding, and guidance throughout this project.

Abstract

The assignment of students to exercise groups represents a classic assignment problem, which aims to optimize their preferences. While simple variants, where students only indicate a single preference, can be solved in polynomial time using algorithms such as weighted maximum matching, more complex variants become significantly more challenging. This thesis explores various versions of the assignment problem and applies combinatorial optimization techniques to find solutions. These include considering group and language preferences and evaluating algorithms like the Hungarian Method and advanced Mixed Integer Linear Programming (MILP) variant. The goal is to assess the efficiency and practicality of these approaches in different scenarios.

Table of Contents

1 Introduction

Assigning students to exercise groups is a typical example of an assignment problem that occurs in many academic and organizational contexts. Generally, it involves optimally assigning individuals to various groups or resources while considering their different preferences. These assignment problems are not only important in education but also in fields like logistics, human resource management, and resource planning.

In its simplest form, students can specify a preference for different time slots. Such simple variants can be solved in polynomial time, for example, using weighted maximum matching algorithms. These algorithms are known for providing efficient solutions for bipartite graphs, where the goal is to find an optimal pairing between two sets. However, when additional preferences and constraints are considered, such as language preferences or the desire for students to be in the same group, the problem becomes more complex.

The aim of this thesis is to formulate various variants of the assignment problem and solve them using different combinatorial optimization techniques. A particular focus is on the investigation and implementation of the Hungarian Method, also known as the Kuhn-Munkres algorithm, as well as advanced variants of Mixed Integer Linear Programming (MILP). The Hungarian Method is a well-known algorithm for solving the assignment problem in polynomial time. By implementing and applying this algorithm, it will be shown how efficient solutions for simple assignment problems can be found.

For more complex variants, where multiple preferences and constraints must be considered, MILP is a suitable method. MILP allows for the inclusion of both discrete and continuous variables in an optimization problem and is therefore particularly well-suited for problems where multiple preferences and constraints must be handled simultaneously. In this thesis, an extended variant of MILP, called the SmartAlloc method, will be implemented and studied. SmartAlloc is specialized in assigning students and extends conventional MILP approaches by considering specific preferences and constraints to improve the efficiency of the solution search.

An important aspect of this thesis is the evaluation of the different approaches in various scenarios. For this purpose, different test scenarios will be created using a generator to assess the performance and scalability of the algorithms. The results of these tests will provide insights into how well the different approaches work in practice and what advantages and

disadvantages they have in different application contexts. The methods and algorithms developed in this thesis are intended not only to contribute to scientific discussion but also to demonstrate practical implications and applications in various fields, with a focus on student assignment.

2
Background

In this section, all relevant definitions and concepts are presented to provide the theoretical framework for understanding the assignment problem and the optimization techniques used in this thesis. To illustrate these often abstract definitions, a simple example of a student assignment problem will be used throughout.

2.1 Combinatorial Optimization

Definition 1: Combinatorial Optimization Problems

Definition 2: Objective Functions and Constraints

Definition 3: Search vs. Optimization Problems

2.2 Linear Programming and Mixed-Integer Linear Programming

Definition 4: Linear Programming (LP)

Definition 5: Mixed-Integer Linear Programming (MILP)

Definition 6: Constraints in LP and MILP

2.3 Graph Theory and Matching Algorithms

Definition 7: Graph Theory Basics

Definition 8: Bipartite Graphs

Definition 9: Matching in Graphs

Definition 10: Maximum Matching

Definition 11: Hungarian Algorithm

2.4 Applications and Practical Considerations

Definition 12: Real-World Applications of Matching Algorithms

Definition 13: Computational Complexity

Definition 14: Practical Constraints and Heuristics

2.5 Example Cases

Example 1: Simple 4x4 Assignment Problem

Example 2: Real-World Case Study (School Slot Allocation)

3

Hungarian Method

4

Mixed-Integer Linear Programming (MILP)

5

Evaluation

6

Conclusion

Bibliography