INTEGRATION BY SUBSTITUTION

Guidelines for u-Substitution :-

Step 1:- Look for some composition $f(g(x))$ within the integrand for which the substitution

$$u = g(x) \quad ; \quad du = g'(x)\,dx$$

produces an integral that is expressed entirely in terms of u and its differential du. This may or may not be possible

Step 2:- If you are successful in Step 1, then try to evaluate the resulting integral in terms of u. Again, this may or may not be possible

Step 3:- If you are successful in Step 2, then replace u by $g(x)$ to express your final answer in terms of x.

EXAMPLE 2 :-

(i) $\int \sin(x+9)\,dx$

(Diffrential of this is present) $u = x+9$ → Step 1

$du = 1 \cdot dx = dx$

$\int \sin u \, du$ → Step 2

$-\cos u + c \xrightarrow[\text{replacing } u]{} \boxed{-\cos(x+9)+c}$

Step3

(ii) $\int (x-8)^{23}\,dx$

$u = x-8$ → Step 1

$du = 1 \cdot dx = dx$

Step 3

$\int u^{23}\,du = \dfrac{u^{24}}{24} + C = \boxed{\dfrac{(x-8)^{24}}{24} + c}$

Step 2

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EXAMPLE 3 :-

$$\int \cos 5x \, dx$$

$u = 5x$
$du = 5\,dx$ → Step 1

$\frac{1}{5}\int \cos u \, du$ → Step 2

$\frac{1}{5}\sin u + c$

$\boxed{\frac{1}{5}\sin 5x + c}$ → Step 3

EXAMPLE 4 :-

$$\int \frac{dx}{\left(\frac{1}{3}x - 8\right)^5}$$

$u = \frac{1}{3}x - 8$
$du = \frac{1}{3}dx \rightarrow dx = 3\,du$ → Step 1

$\int \frac{3\,du}{u^5}$
$3\int u^{-5}\,du$ → Step 2

$\frac{-3}{4}u^{-4} + c$

$\boxed{\frac{-3}{4}\left(\frac{1}{3}x - 8\right) + c}$

EXAMPLE 5 :-

$$\int \frac{dx}{1+3x^2}$$

Step 1 :- $u = \sqrt{3}x \qquad du = \sqrt{3}\,dx$

$$\int \frac{dx}{1+3x^2}$$

Step2 ← $\frac{1}{\sqrt{3}} \int \frac{du}{1+u^2}$ * Replacing f(x) with u

$$= \frac{1}{\sqrt{3}} \tan^{-1} u + c$$

Step 3 ← $\boxed{\frac{1}{\sqrt{3}} \tan^{-1} \sqrt{3}x + c}$ * Replacing u with assinged value:

EXAMPLE 6:-

$$\int \frac{1}{x} + \sec^2 \pi x \, dx$$

Applying Addition ~~and~~ rule of integration $\int \frac{dx}{x} + \int \sec^2 \pi x \, dx$

$$= \ln|x| + \int \sec^2 \pi x \, dx$$

$u = \pi x$

$du = \pi dx \rightarrow dx = \frac{1}{\pi} du$

$$= \ln|x| + \frac{1}{\pi} \int \sec^2 u \, du$$

$$= \ln|x| + \frac{1}{\pi} \tan u + c$$

$$= \ln|x| + \frac{1}{\pi} \tan \pi x + c$$

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EXAMPLE 7 :-

$$\int \sin^2 x \cos x \, dx$$

Step1 ← $u = \sin x$

$\frac{du}{dx} = \cos x \rightarrow du = \cos dx$

Step2 ← $= \int u^2 \, du$

$= \frac{u^3}{3} + c$

Step3 --> $= \boxed{\frac{\sin^3 x}{3} + c}$

EXAMPLE 8 :-

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$du = \frac{1}{2\sqrt{x}} dx$

$\int 2e^u \, du$

$2 \cdot \int e^u \, du$

$2e^u + c$

$2e^{\sqrt{x}} + c$

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EXAMPLE 7 :-

$$\int \sin^2 x \cos x \, dx$$

Step1 ←

$u = \sin x$

$\frac{du}{dx} = \cos x \rightarrow du = \cos dx$

Step2 ← $= \int u^2 \, du$

$= \frac{u^3}{3} + C$

Step3 --> $= \boxed{\frac{\sin^3 x}{3} + c}$

EXAMPLE 8 :-

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$du = \frac{1}{2\sqrt{x}} \, dx$

$\int 2e^u \, du$

$2 \cdot \int e^u \, du$

$2e^u + C$

$2e^{\sqrt{x}} + c$

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EXAMPLE 9

$$\int t^4 \sqrt[3]{3 - 5t^5}\, dx$$

Step 1 ←

$$u = 3 - 5t^5$$

$$du = -25t^4 dt$$

$$-\frac{1}{25} du = t^4 dt$$

$$\frac{-du}{25} = t^4 dt$$

$$\int \sqrt[3]{u} \frac{-du}{25}$$

$$-\frac{1}{25} \int \sqrt[3]{u}\, du$$

$$-\frac{1}{25} \left(\frac{u^{4/3}}{4/3} \right) + C$$

Step 3 ⟶ $-\frac{3}{100} (3 - 5t^5)^{4/3} + C$

Replacing u with assigned variable

EXAMPLE 10

$$\int \frac{e^x}{\sqrt{1 - e^{2x}}}$$

Step1 ←

$$u = e^x$$

$$du = e^x dx$$

Step 2 ← $\int \frac{du}{\sqrt{1 - u^2}}$

$$\sin^{-1} u + C$$

Step3 ← $\sin^{-1}(e^x) + C$