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Q Find the area under $f(x) = x^2 + 1$ on $[0, 2]$ where x_k is left end point :-

Step 1 :-

$$\Delta x = \frac{b-a}{n}$$

$$= \frac{2-0}{n}$$

$$\boxed{\Delta x = \frac{2}{n}}$$

Step 2 :-

Left End Point

$$x_k = a + (k-1) \cdot \Delta x$$

$$x_k = 0 + (k-1)\left(\frac{2}{n}\right)$$

$$x_k = (k-1)\left(\frac{2}{n}\right)$$

Step 3 :-

$$f(x) = \left(2\left(\frac{(k-1)}{n}\right)\right)^2 + 1$$

$$\sum_{k=1}^{n} f(x_k)\,\Delta x$$

$$\sum_{k=1}^{n} \left(\frac{4k^2}{n^2} + 1\right)\left(\frac{2}{n}\right)$$

$$\frac{8}{n^3} \sum_{k=1}^{n} k^2 + \frac{2}{n} \sum_{k=1}^{n} 1$$

$$\frac{8}{n^3} \sum_{k=1}^{n} k^2 + \frac{2}{n} \sum_{k=1}^{n} 1$$

$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1 = n$$

$$= \frac{8}{n^3} \times \frac{n(n+1)(2n+1)}{6} + \frac{2}{n} \times n$$

$$= \frac{8\left(n(n+1)(2n+1)\right)}{6n^3} + 2$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} \frac{8\left(n(n+1)(2n+1)\right)}{6n^3} + 2$$

$$= \frac{8n^2}{6} + 2$$

$$= \frac{16}{6} + 2$$

$$= \frac{16+12}{6}$$

$$= \frac{28}{6}$$

$$\boxed{= \frac{14}{3}}$$