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QUIZ 03

Q Find the area bounded by $y = x^3$ and $y = x$ along the y-axis.

*First we make x the subject in both functions.

$y = x^3$

⇒ Taking Cuberoot $\sqrt[3]{y} = \cancel{\sqrt[3]{x^3}}$

$\sqrt[3]{y} = x$

$x = \sqrt[3]{y}$

$y = x$

$x = y$

Step 1:- Finding Intervals:-

⇒ Equating both functions

$$\sqrt[3]{y} = y$$

$$(\cancel{\sqrt[3]{y}})^{\cancel{3}} = (y)^3$$

$$y = y^3$$

$$y^3 - y = 0$$

$$y(y^2 - 1) = 0$$

$y = 0$. $y^2 - 1 = 0$

$y^2 = 1$

$y = \sqrt{1}$

$y = \pm 1$

Step 2:- Sign Test

-1 0 $+1$

⇒ Finding point between interval $[-1, 0]$

$$y = -0.5$$

$x = \sqrt[3]{y}$

$x = \sqrt[3]{-0.5}$

$\therefore$ Lower Curve $x = -0.79$

$x = y$

$x = -0.5$

$\therefore$ Upper Curve

Finding point between interval [0, 1]

$$y = +0.5$$

$x = \sqrt[3]{y}$	$x = y$
$x = \sqrt[3]{0.5}$	$x = 0.5$
$x = 0.79$	$\therefore$ Lower Curve
$\therefore$ Upper Curve	

Step 3:- Finding Area

$$A_T = \underset{[-1,0]}{A_1} + \underset{[0,1]}{A_2}$$

$$A_T = \left[\int_{-1}^{0}(y - \sqrt[3]{y})\,dy\right] + \left[\int_{0}^{1}(\sqrt[3]{y} - y)\,dy\right]$$

$$A_T = \left[\int_{-1}^{0} y\,dy - \int_{-1}^{0} \sqrt[3]{y}\,dy\right] + \left[\int_{0}^{1} \sqrt[3]{y}\,dy - \int_{0}^{1} y\,dy\right]$$

$$A_T = \left[\Big|_{-1}^{0} \frac{y^2}{2} - \Big|_{-1}^{0} \frac{y^{4/3}}{4/3}\right] + \left[\Big|_{0}^{1} \frac{y^{4/3}}{4/3} - \Big|_{0}^{1} \frac{y^2}{2}\right]$$

$$A_T = \left[\Big|_{-1}^{0} \frac{y^2}{2} - \Big|_{-1}^{0} \frac{3y^{4/3}}{4}\right] + \left[\Big|_{0}^{1} \frac{3y^{4/3}}{4} - \Big|_{0}^{1} \frac{y^2}{2}\right]$$

$$A_T = \left[\left[\frac{(0)^2}{2} - \frac{(-1)^2}{2}\right] - \left[\frac{3(0)^{4/3}}{4} - \frac{3(-1)^{4/3}}{4}\right]\right] + \left[\left[\frac{3(1)^{4/3}}{4} - 0\right] - \left[\frac{(1)^2}{2} - 0\right]\right]$$

$$A_T = \left[\left[0 - \frac{1}{2}\right] - \left[0 - \frac{3}{4}\right]\right] + \left[\left[\frac{3}{4}\right] - \left[\frac{1}{2}\right]\right]$$

$$A_T = 0.25 + 0.25$$

$$\boxed{A_T = 0.5}$$