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Subject:- CALCULUS

Instructor:- Usman Allam

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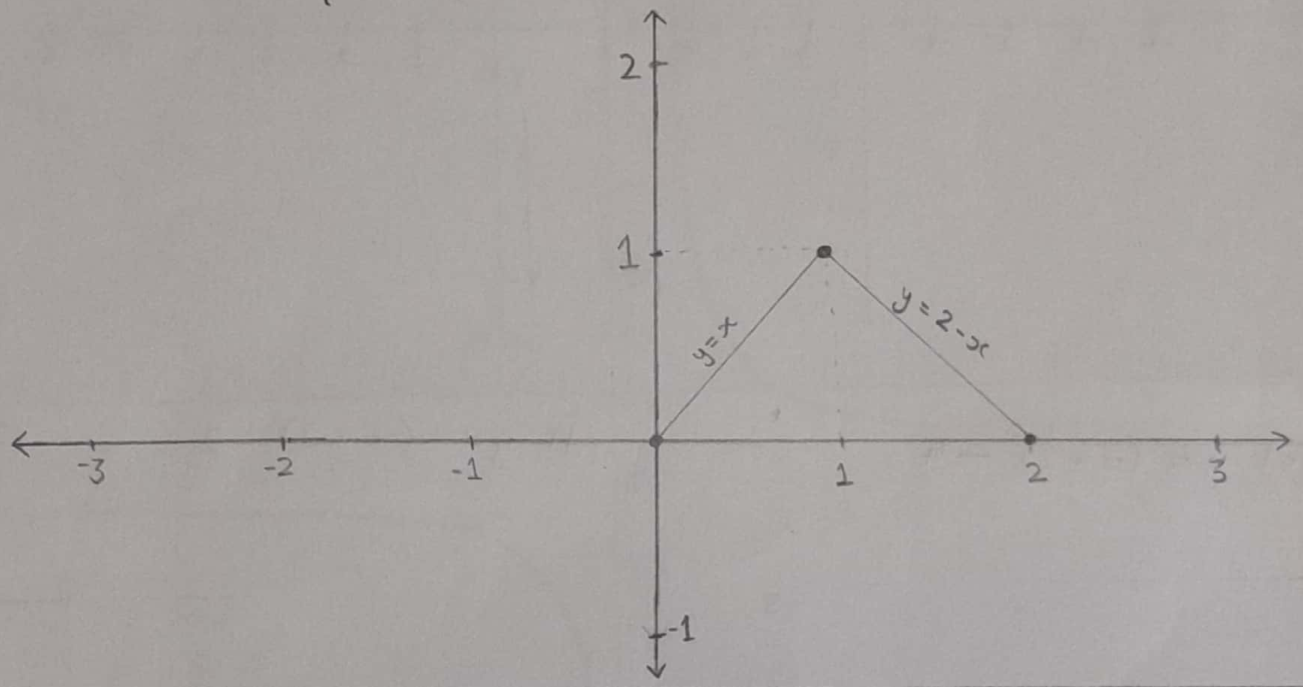
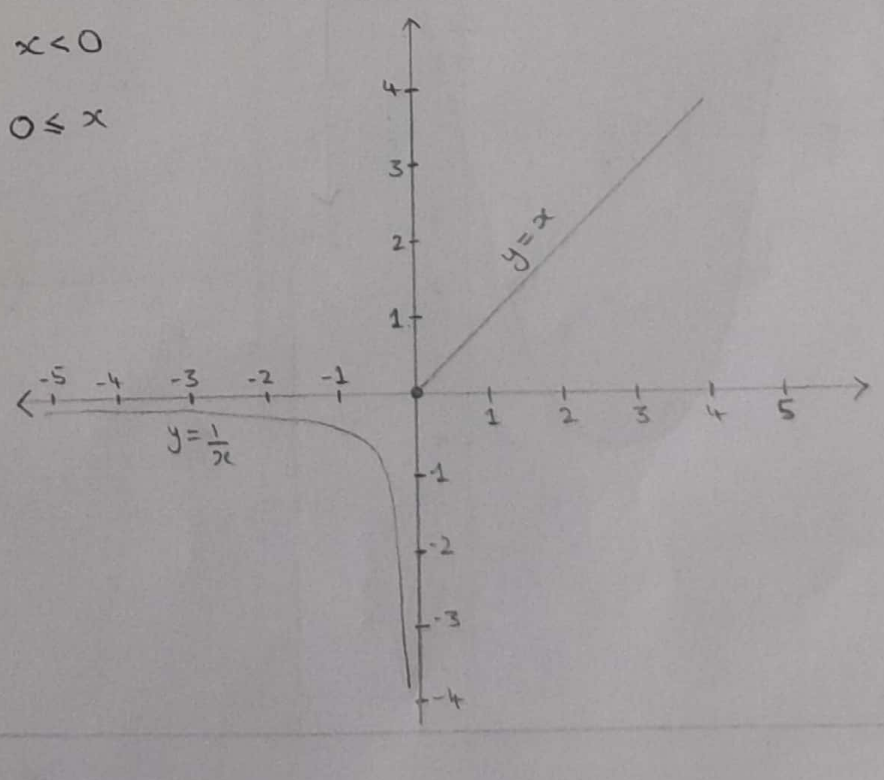
ASSIGNMENT 02

Q1

(i) $f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 < x \le 2 \end{cases}$

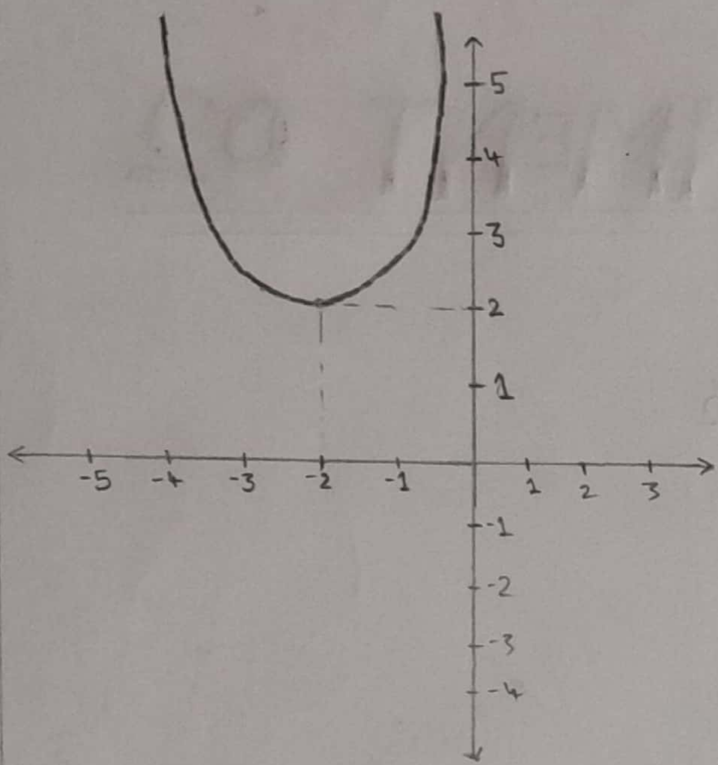


(ii) $G(x) = \begin{cases} 1/x & x < 0 \\ x & 0 \le x \end{cases}$

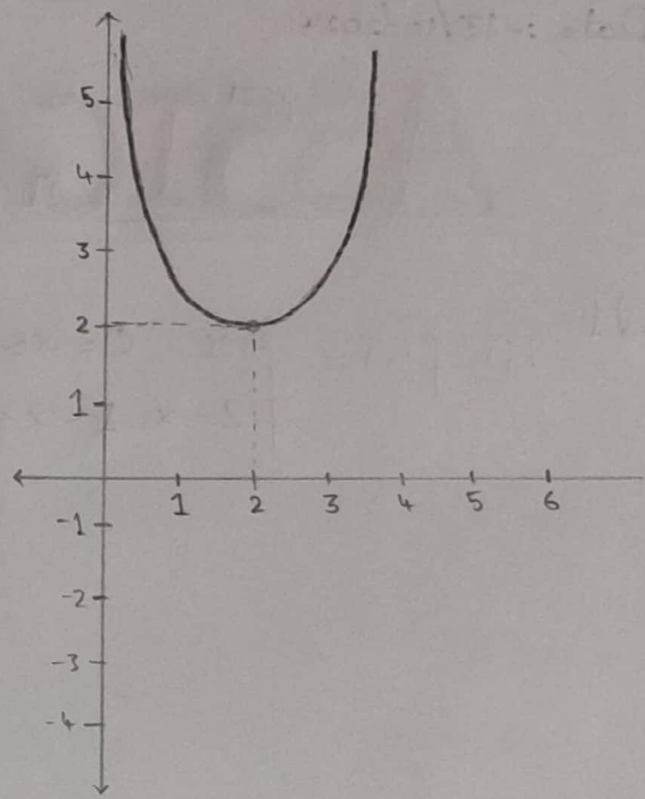


(iii)

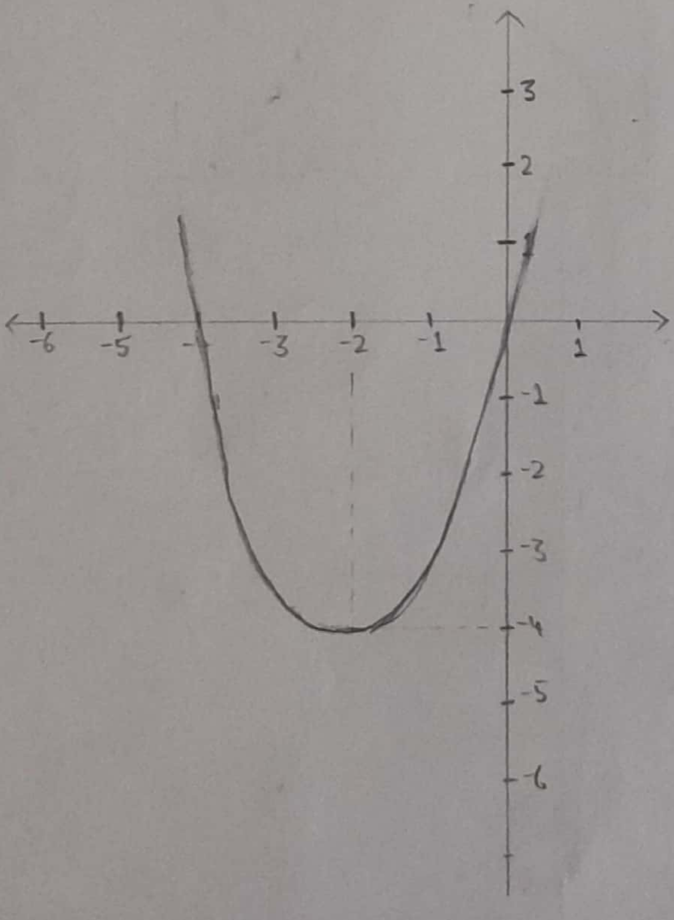
(a) $y = (x+2)^2 + 2$



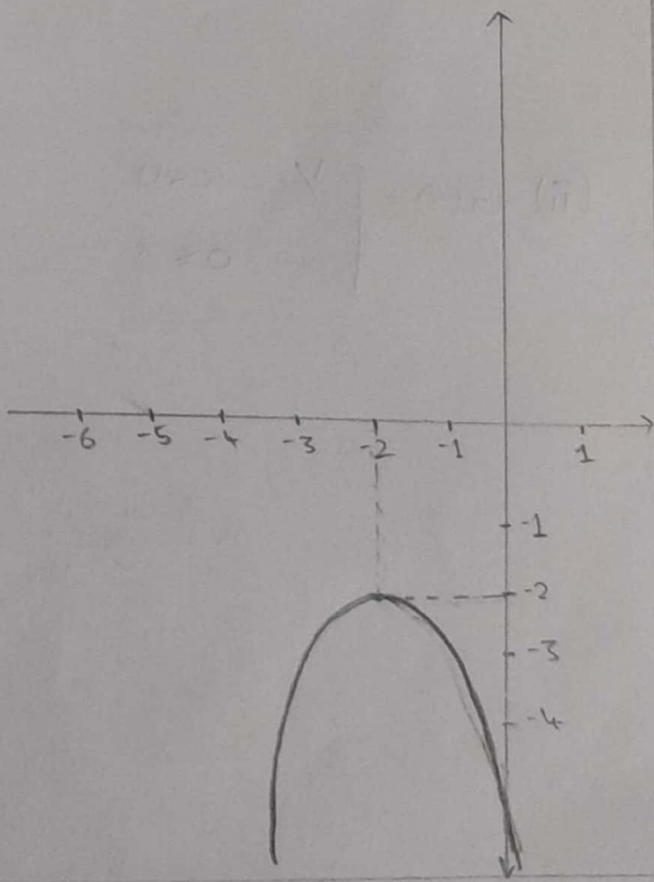
(b) $y = (x-2)^2 + 2$



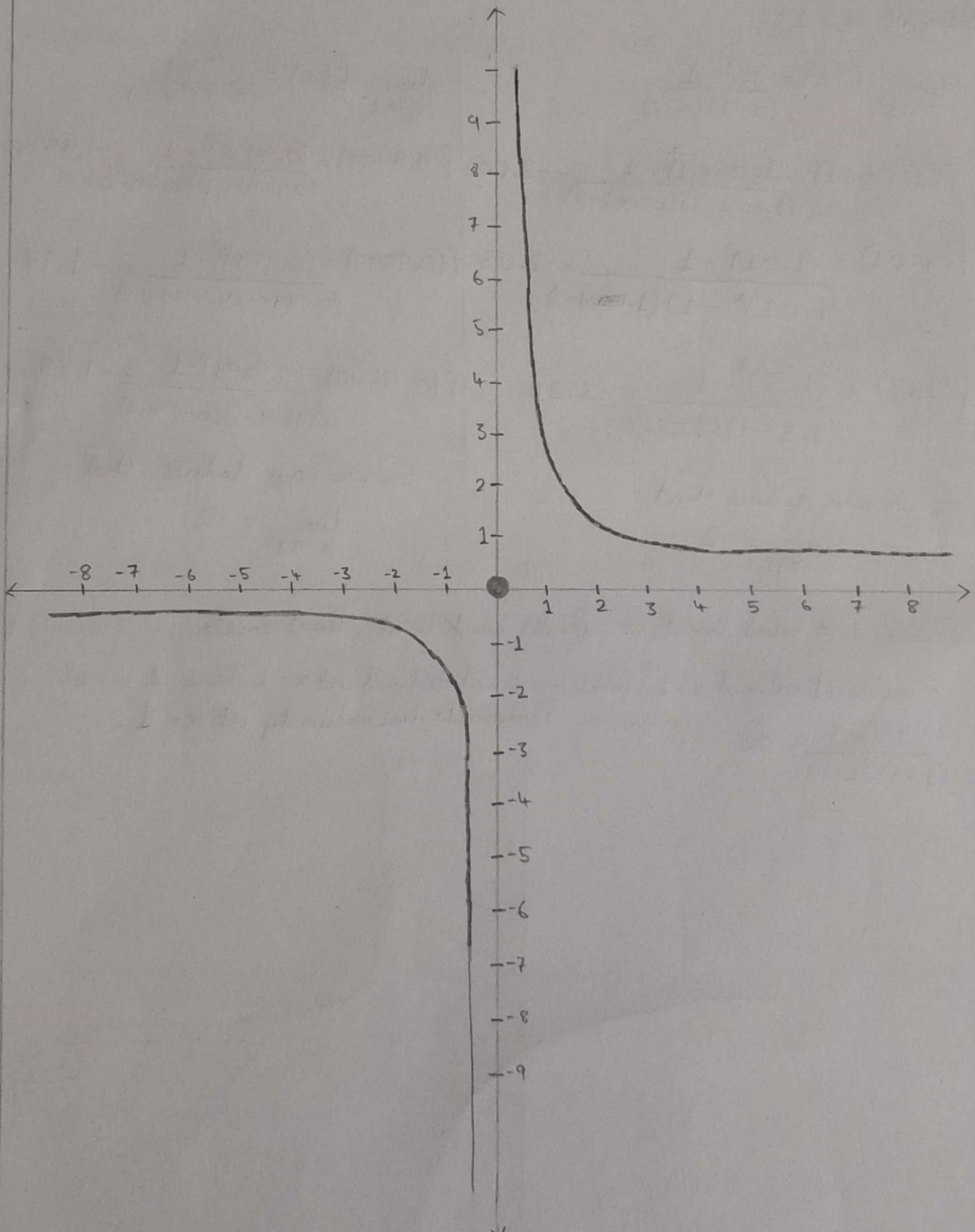
(c) $y = (x+2)^2 - 4$



(d) $y = -(x+2)^2 - 2$



(iv) $y = \begin{cases} \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$



Q2 How does the function behave near $x = 1$ and $x = 2$

$$f(x) = \frac{x^2 - 1}{(x-1)(x-2)}$$

Looking At $x = 1$;

$$\lim_{x \to 1^+} f(x) = \frac{x^2 - 1}{(x-1)(x-2)}$$

$$f(1.0001) = \frac{(1.0001)^2 - 1}{(1.0001-1)(1.0001-2)} = -2.000$$

$f(1.01) = \dfrac{(1.01)^2 - 1}{((1.01)^2 - 1)\,}$(1.[redacted]01-2) $= -2.03$

$$f(1.3) = \frac{(1.3)^2 - 1}{(1.3-1)(1.3-2)} = -3.28$$

So we can deduce that :-

$$\lim_{x \to 1^+} = -2$$

$$\lim_{x \to 1^-} f(x) = \frac{x^2 - 1}{(x-1)(x-2)}$$

$$f(0.9998) = \frac{(0.9998)^2 - 1}{(0.9998 - 1)(0.9998-2)} = -1.9994$$

$$f(0.9899) = \frac{(0.9899)^2 - 1}{(0.9899-1)(0.9899-2)} = -1.97$$

$$f(0.9599) = \frac{(0.9599)^2 - 1}{(0.9599-1)(0.9599-2)} = -1.88$$

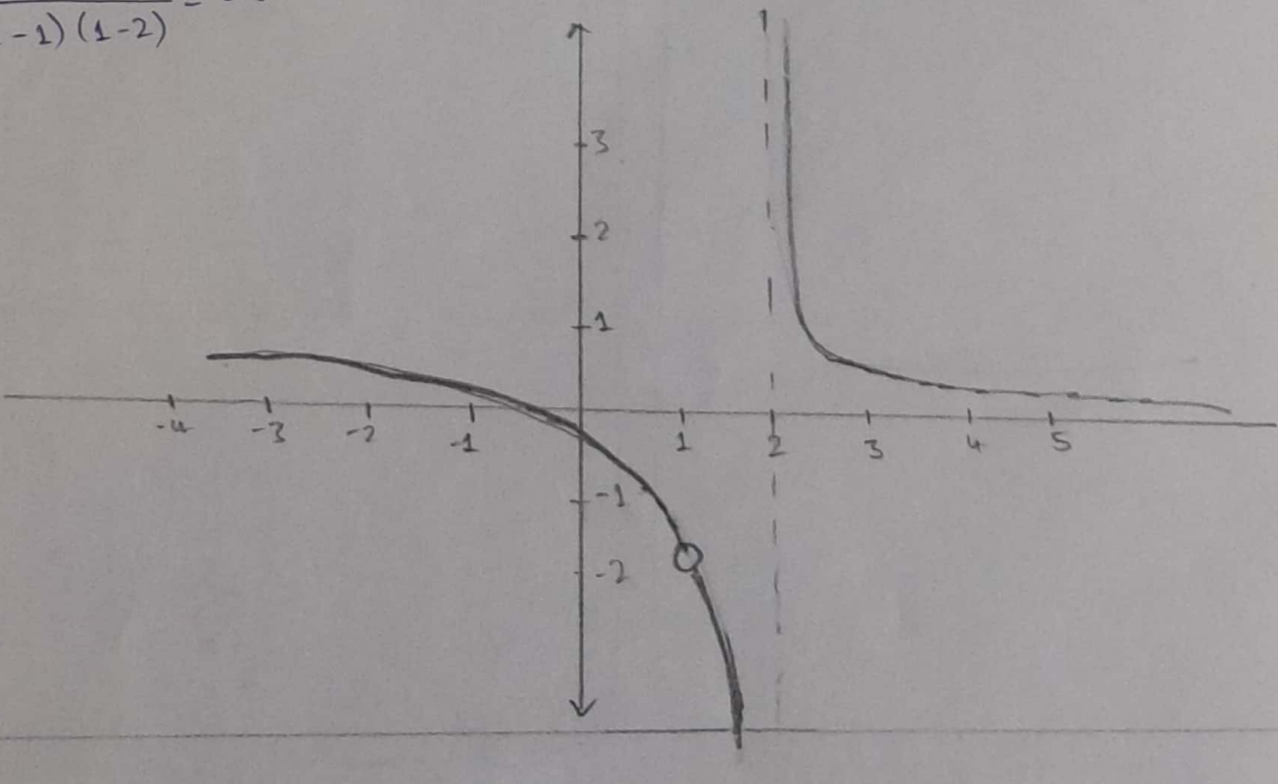
So we can deduce that :-

$$\lim_{x \to 1^-} = -2$$

Since Left hand limit = Right hand limit, limit exists.

Functional value at $x = 1$, function is not defined at $x = 1$, hence it is not continuous. Removable discontinuity at $x = 1$.

$$\frac{(1)^2 - 1}{(1-1)(1-2)} = \infty$$



Looking at $x = 2$;

$\lim_{x \to 2^+} f(x) = \frac{x^2 - 1}{(x-1)(x-2)}$

$\lim_{x \to 2^-} = \frac{x^2 - 1}{(x-1)(x-2)}$

$f(2.0001) = \frac{(2.0001)^2 - 1}{(2.0001) - 1)(2.0001 - 2)} = 30001$

$f(1.998) = \frac{(1.998)^2 - 1}{(1.998) - 1)(1.998 - 2)} = -1499$

$f(2.01) = \frac{(2.01)^2 - 1}{(2.01 - 1)(2.01 - 2)} = 301$

$f(1.989) = \frac{(1.989)^2 - 1}{(1.989 - 1)(1.989 - 2)} = -271$

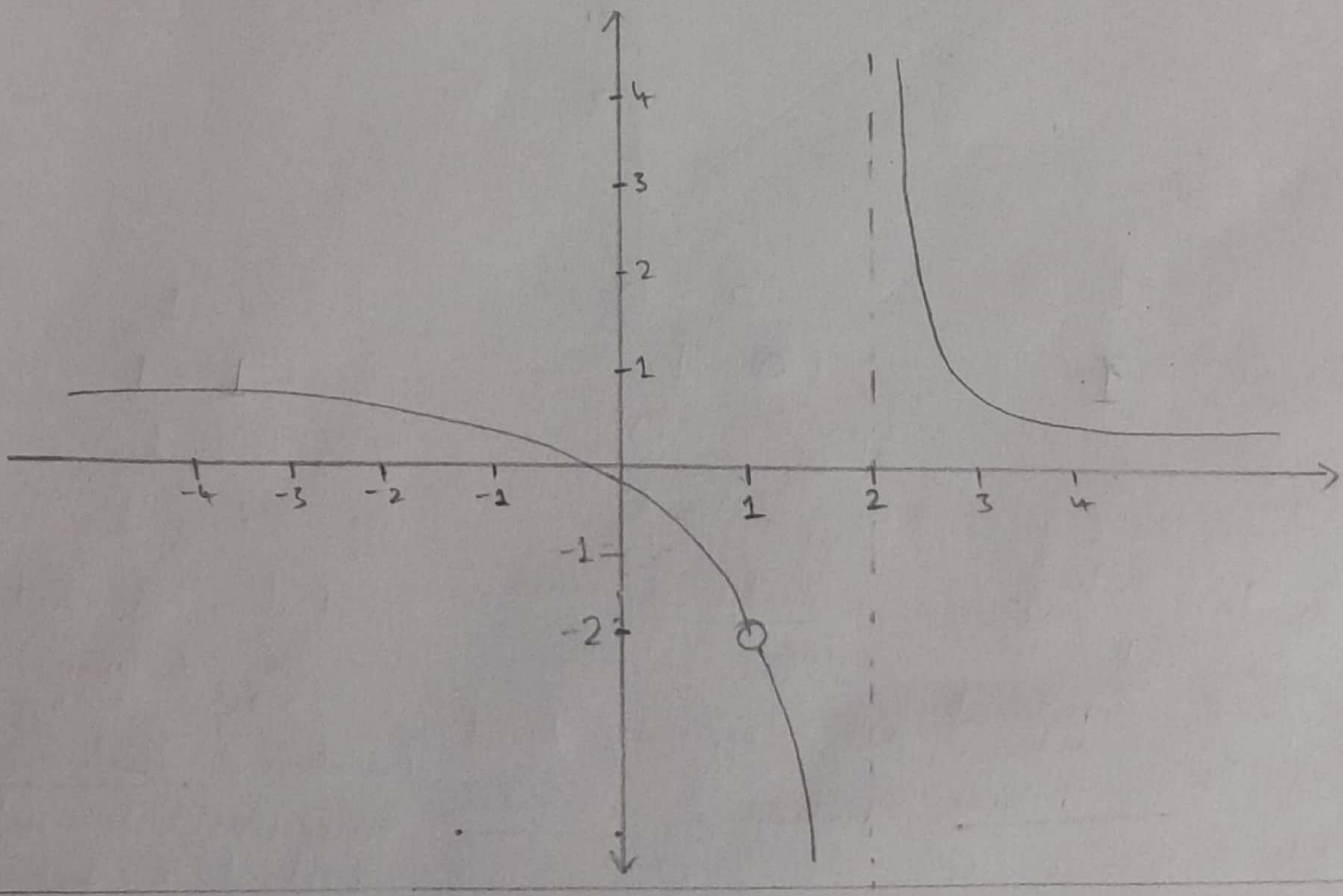
$f(2.3) = \frac{(2.3)^2 - 1}{(2.3) - 1)(2.3 - 2)} = 11$

$f(1.9799) = \frac{(1.9799)^2 - 1}{(1.9799 - 1)(1.989 - 2)} = -148.2$

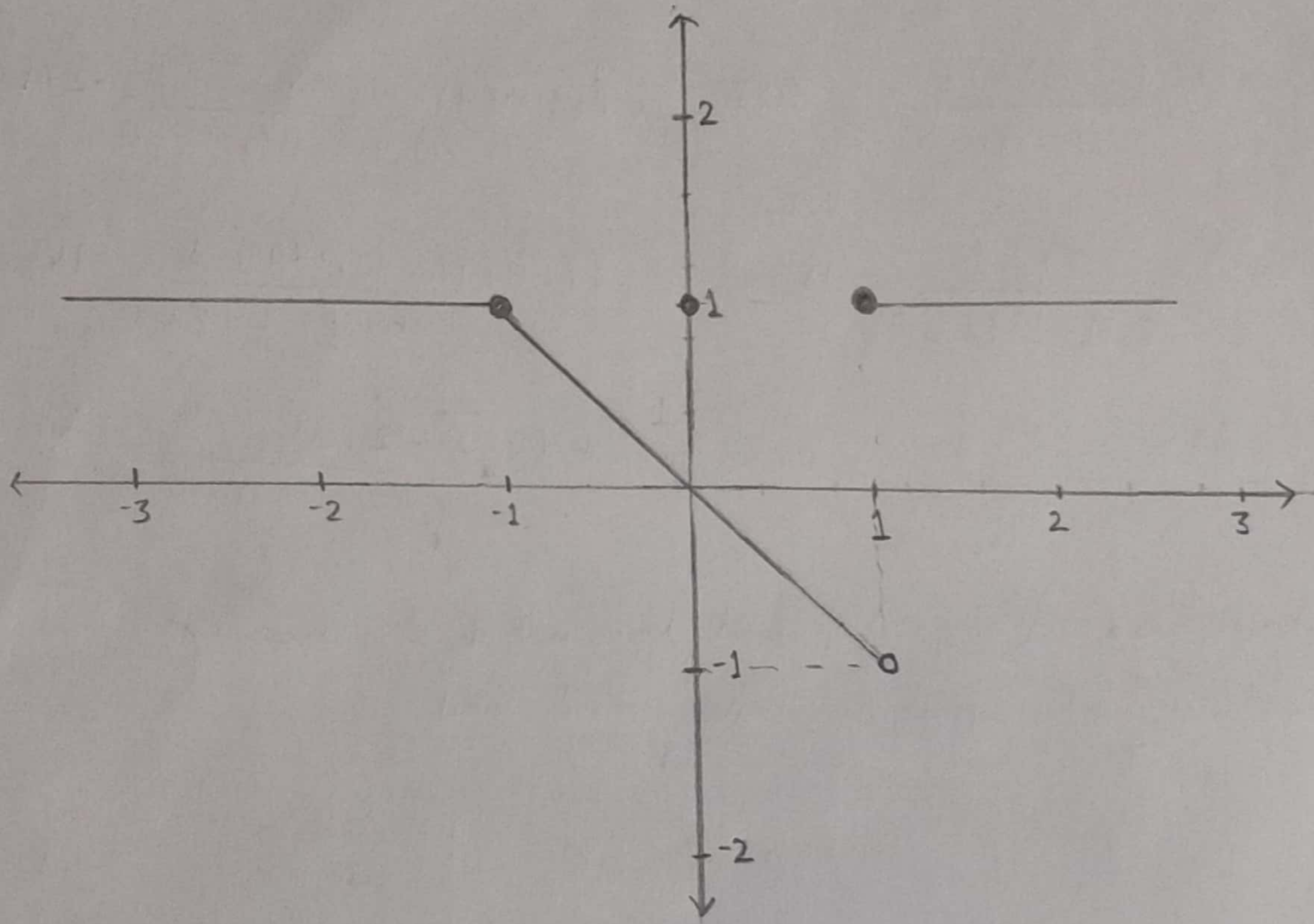
$f(2.0^+) = \frac{(2.0)^2 - 1}{(x-1)(x-2)} = +\infty$

$f(2.0^-) = \frac{(2.0)^2 - 1}{(2.0 - 1)(2.0 - 2)} = -\infty$

Asymptotes at $x = 2$, limit does not exist since left hand limit is not equal to right hand limit



Q3 $f(x) = \begin{cases} 1 & x \le -1 \\ -x & -1 < x < 0 \\ 1 & x = 0 \\ -x & 0 < x < 1 \\ 1 & x \ge 1 \end{cases}$



$x = -1$	$x = 0$	$x = 1$
$\lim_{x \to -1^+} f(x) = 1$ $\lim_{x \to -1^-} f(x) = 1$ $f(x) = 1$ Since $\lim_{x \to -1^+} = \lim_{x \to -1^-}$ and Limit = Functional value Function is <u>continuos</u> at $x = -1$	$f(0) = 1$ $\lim_{x \to 0^+} f(x) = 0$ $\lim_{x \to 0^-} f(x) = 0$ <u>Limit exists</u> since $\lim_{x \to 0^+} = \lim_{x \to 0^-}$ But Functional value $\neq$ Limit Function, is <u>discontinuos</u> at $x = 0$	$f(1) = 1$ $\lim_{x \to 1^+} f(x) = 1$ $\lim_{x \to 1^-} f(x) = -1$ Limit does not exist since $\lim_{x \to 1^+} \neq \lim_{x \to 1^-}$ Function is <u>discontinuos</u> since limit does not exist at $x = 1$

Q4 Find the horizontal and vertical asymptotes of the graph of

$$f(x) = \frac{8}{x^2-4}$$

$$f(x) = \frac{8}{(x)^2-(2)^2}$$

$$f(x) = \frac{8}{(x+2)(x-2)}$$

Vertical Asymtotes :-

$x + 2 = 0$

$\boxed{x = -2}$

$x - 2 = 0$

$\boxed{x = +2}$

Horizontal Asymtotes :-

We plug in $x = \pm\infty$

$$y = \frac{8}{(+\infty)^2 - 4} \quad \text{or} \quad y = \frac{8}{(-\infty)^2-4}$$

$$y = \frac{8}{+\infty} \qquad y = \frac{8}{-\infty}$$

$$\boxed{y = 0} \qquad \boxed{y = 0}$$

Final Answer :- Vertical Asymptote $x = +2, x = -2$

Horizotal Asymptote $y = 0$

Q5 Discuss Holes and Asymptotes of the following and also draw graph of each.

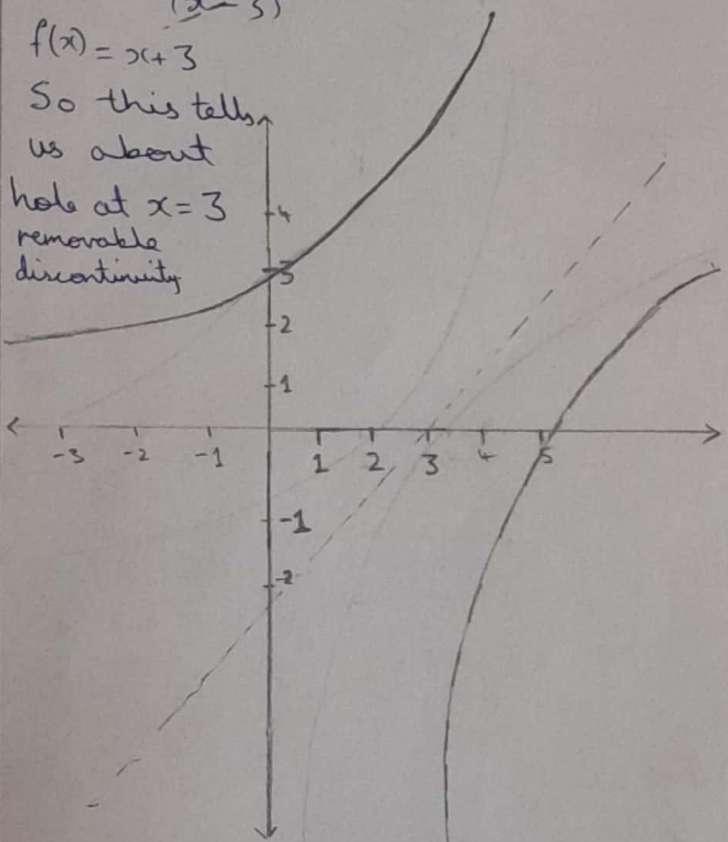
(a) $f(x) = \dfrac{x^2 - 6x - 9}{x - 3}$

$f(x) = \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$f(x) = x + 3$

So this tells us about hole at $x = 3$

removable discontinuity



Since the degree of the numerator is 1 greater than the denominator hence a slant asyntote is formed. To calculate this we use long division.

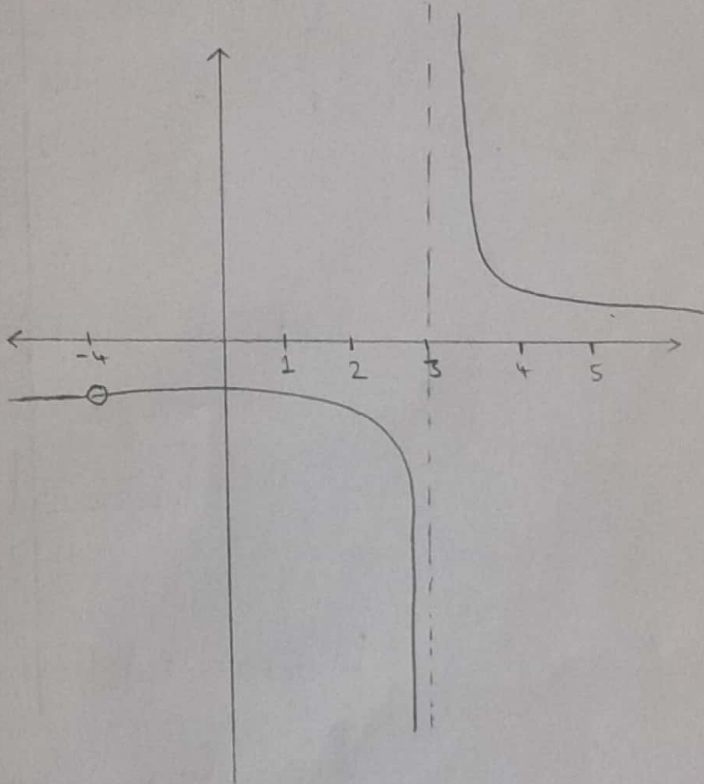
$$\begin{array}{r l} & x - 3 \\ x - 3 \,\big) & x^2 - 6x - 9 \\ & -(x^2 - 3x) \\ \hline & 0 - 3x - 9 \\ & -(-3x - 9) \\ \hline & 0 \quad 0 \end{array}$$

Hence Slant Asymptote at $\boxed{y = x - 3}$

(b) $f(x) = \dfrac{2x + 8}{x^2 + x - 12}$

$\dfrac{2x - 8}{(x+4)(x-3)}$

$\dfrac{-2\cancel{(x+4)}}{\cancel{(x+4)}(x-3)}$

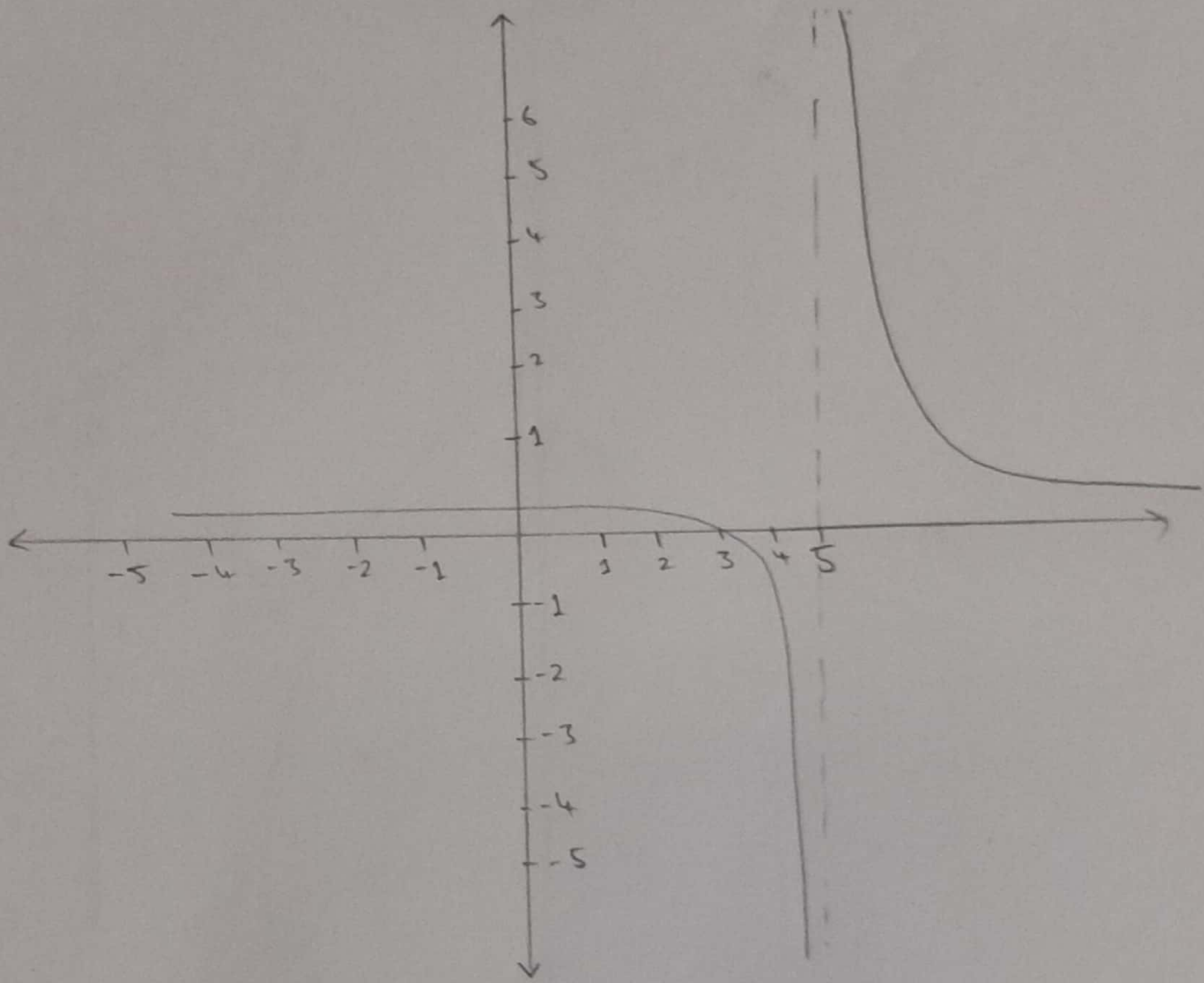


Hole at $x = -4$

Horizontal Asymptote at $y = 0$ since degree of the denominator is 1 greater than numerator

Vertical Asymptote at $x = 3$ since denominator contain $x - 3$

(c) $f(x) = \dfrac{x^2 - 3x - 10}{x^2 - 10x + 25}$



* Vertical Asymptote

$$f(x) = \frac{x^2 - 3x - 10}{x^2 - 10x + 25}$$

$$= \frac{(x+2)\cancel{(x-5)}}{(x-5)\cancel{(x-5)}}$$

$$= \frac{x+2}{x-5}$$

$x - 5 = 0$

* $\boxed{x = 5}$

* Since $x - 5$ cancels out there is a hole at $x = 5$

* Since powers are same horizontal asymptote at $y = 1$ because ratio of x^2 is 1.